D=10 CHIRAL TENSIONLESS SUPER p-BRANES

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We consider a model for tensionless (null) super p-branes with N chiral supersymmetries in ten dimensional flat space-time. After establishing the symmetries of the action, we give the general solution of the classical equations of motion in a particular gauge. In the case of a null superstring (p=1) we find the general solution in an arbitrary gauge. Then, using a harmonic superspace approach, the initial algebra of first and second class constraints is converted into an algebra of Lorentz-covariant, BFV-irreducible, first class constraints only. The corresponding BRST charge is as for a first rank dynamical system.

1 Introduction

The tensionless (null) p-branes correspond to usual p-branes with their tension $T_p$ taken to be zero. This relationship between null p-branes and the tensionful ones may be regarded as a generalization of the massless-massive particles correspondence. On the other hand, the limit $T_p \to 0$ corresponds to the energetic scale $E >> M_{Planck}$. In other words, the null p-brane is the high energy limit of the tensionful one. There exist also an interpretation of the null and free p-branes as theories, corresponding to different vacuum states of a p-brane, interacting with a scalar field background [1]. So, one can consider the possibility of tension generation for null p-branes (see [2] and references therein). Another viewpoint on the connection between null and tensionful p-branes is that the null one may be interpreted as a "free" theory opposed to the tensionful "interacting" theory [3]. All this explains the interest in considering null p-branes and their supersymmetric extensions.

Models for tensionless p-branes with manifest supersymmetry are proposed in [4]. In [4] a twistor-like action is suggested, for null super-p-branes with N-extended global supersymmetry in four dimensional space-time. In the recent work [5], the quantum constraint algebras of the usual and conformal tensionless spinning p-branes are considered.

In a previous paper [6], we announced for a null super p-brane model, and here we are going to formulate it, and to consider its classical properties. After establishing

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the symmetries of the action, we give the general solution of the classical equations of motion in a particular gauge. In the case of a null superstring, \((p=1)\), we find the general solution in an arbitrary gauge. Then, in the framework of a harmonic superspace approach, the initial algebra of first and second class constraints is converted into an algebra of Lorentz-covariant, BFV-irreducible, first class constraints only. The corresponding BRST charge is as for a first rank dynamical system.

2 Lagrangian formulation

We define our model for \(D = 10\) \(N\)-extended chiral tensionless super \(p\)-branes by the action:

\[
S = \int d^{p+1}\xi L, \quad L = V^J V^K \Pi^\mu_J \Pi^K_{\mu} \eta_{\mu\nu}, \tag{1}
\]

\[
\Pi^\mu_J = \partial_J x^\mu + i \sum_{A=1}^{N} (\theta^A \sigma^\mu \partial_J \theta^A), \quad \partial_J = \partial/\partial \xi^J,
\]

\[
\xi^J = (\xi^0, \xi^j) = (\tau, \sigma^j), \quad \text{diag}(\eta_{\mu\nu}) = (-,+,\ldots,+),
\]

\(J, K = 0, 1, \ldots, p\), \(j, k = 1, \ldots, p\), \(\mu, \nu = 0, 1, \ldots, 9\).

Here \((x^\nu, \theta^{A\alpha})\) are the superspace coordinates, \(\theta^{A\alpha}\) are \(N\) left Majorana-Weyl spacetime spinors \((\alpha = 1, \ldots, 16\), \(N\) being the number of the supersymmetries\) and \(\sigma^\mu\) are the 10-dimensional Pauli matrices (our spinor conventions are given in the Appendix). Actions of this type are first given in [7] for the case of tensionless \(\alpha\) time spinors \((\text{of motion in a particular gauge})\). In the case of a null superstring, \((\text{the symmetries of the action})\), we give the general solution of the classical equations only. The corresponding BRST charge is as for a first rank dynamical system.

As a consequence \(\delta^A_\eta = \eta^{A\alpha}\), \(\delta_\eta x^\mu = i \sum_A (\theta^A \sigma^\mu \delta_\eta \theta^A)\), \(\delta_\eta V^J = 0\).

To prove the invariance of the action under infinitesimal diffeomorphisms, we first write down the corresponding transformation law for the \((r,s)\)-type tensor density of weight \(a\)

\[
\delta_\varepsilon T^{J_1 \ldots J_r}_{K_1 \ldots K_s}[a] = \varepsilon^L \partial_L T^{J_1 \ldots J_r}_{K_1 \ldots K_s}[a] + \varepsilon^K \partial_K T^{J_1 \ldots J_r}_{K_1 \ldots K_s}[a] \partial^{K\mu} \varepsilon^\mu + \cdots + \partial T^{J_1 \ldots J_r}_{K_1 \ldots K_s}[a] \partial_{\mu} \varepsilon^\mu + \cdots + \partial T^{J_1 \ldots J_r}_{K_1 \ldots K_s}[a] \partial_{\mu} \varepsilon^\mu,
\tag{2}
\]

where \(L_\varepsilon\) is the Lie derivative along the vector field \(\varepsilon\). Using (2), one verifies that if \(x^\mu(\xi), \theta^{A\alpha}(\xi)\) are world-volume scalars \((a = 0)\) and \(V^J(\xi)\) is a world-volume \((1,0)\)-type tensor density of weight \(a = 1/2\), then \(\Pi^\mu_J\) is a \((0,1)\)-type tensor, \(\Pi^\mu_J \Pi^K_{\mu}\) is a \((0,2)\)-type tensor and \(L\) is a scalar density of weight \(a = 1\). Therefore,

\[
\delta_\varepsilon S = \int d^{p+1}\xi \partial_J (\varepsilon^J L)
\]
and this variation vanishes under suitable boundary conditions.

Let us now check the kappa-invariance of the action. We define the \( \kappa \)-variations of \( \theta^A(\xi), x^\nu(\xi) \) and \( V^J(\xi) \) as follows:

\[
\delta_\kappa \theta^A = i(\Gamma^A)^\alpha = iV^J(\Pi_J^A)^\alpha, \quad \delta_\kappa x^\nu = -i \sum_A (\theta^A \sigma^\nu \delta_\kappa \theta^A), \quad \delta_\kappa V^K = 2V^K V^L \sum_A (\partial_L \theta^A )^A.
\]

Therefore, \( \kappa^A(\xi) \) are left Majorana-Weyl space-time spinors and world-volume scalar densities of weight \( a = -1/2 \).

From (3) we obtain:

\[
\delta_\kappa (\Pi_J^\nu \Pi_K^\nu) = -2i \sum_A [\partial_J \theta^A \Pi_K \theta^A \Pi_J] \delta_\kappa \theta^A
\]

and

\[
\delta_\kappa L = 2V^J \Pi_J \Pi_K^\nu \delta_\kappa V^K - 2V^K V^L \sum_A (\partial_L \theta^A )^A = 0.
\]

The algebra of kappa-transformations closes only on the equations of motion, which can be written in the form:

\[
\partial_J (V^J V^K \Pi_K^\nu) = 0, \quad V^J V^K (\partial_J \theta^A \Pi_K^\nu) = 0, \quad V^J \Pi_J^\nu \Pi_K^\nu = 0.
\]

As usual, an additional local bosonic world-volume symmetry is needed for its closure. In our case, the Lagrangian, and therefore the action, are invariant under the following transformations of the fields:

\[
\delta_\lambda \theta^A(\xi) = \lambda V^J \partial_J \theta^A, \quad \delta_\lambda x^\nu(\xi) = -i \sum_A (\theta^A \sigma^\nu \delta_\lambda \theta^A), \quad \delta_\lambda V^J(\xi) = 0.
\]

Now, checking the commutator of two kappa-transformations, we find:

\[
[\delta_\kappa_1, \delta_\kappa_2] \theta^A(\xi) = \delta_\kappa_1 \theta^A(\xi) + \text{terms } \propto \text{eqs. of motion},
\]

\[
[\delta_\kappa_1, \delta_\kappa_2] x^\nu(\xi) = (\delta_\kappa_1 \delta_\lambda + \delta_\kappa + \delta_\lambda) x^\nu(\xi) + \text{terms } \propto \text{eqs. of motion},
\]

\[
[\delta_\kappa_1, \delta_\kappa_2] V^J(\xi) = \delta_\kappa_2 V^J(\xi) + \text{terms } \propto \text{eqs. of motion}.
\]

Here \( \kappa^A(\xi), \lambda(\xi) \) and \( \varepsilon(\xi) \) are given by the expressions:

\[
\kappa^A = -2V^K \sum_B [\kappa_1^B \kappa_2^A \mu_2^B \mu_1^A - (\partial_K \theta^B \kappa_2^B) \kappa_1^A],
\]

\[
\lambda = 4iV^K \sum_A (\kappa_1^A \Pi_K \kappa_2^A), \quad \varepsilon^J = -V^J \lambda.
\]

We note that \( \Gamma_{\alpha\beta} = (V^J \Pi_J){\alpha\beta} \) in (3) has the following property on the equations of motion

\[
\Gamma^2 = 0.
\]
This means, that the local kappa-invariance of the action indeed eliminates half of the components of θA as is needed.

For transition to Hamiltonian picture, it is convenient to rewrite the Lagrangian density (3) in the form (∂τ = ∂/∂τ, ∂j = ∂/∂σj):

\[ L = \frac{1}{4μ^0} \left[ (\partial_τ - μ^j \partial_j)x + i ∑_A θ^A σ(\partial_τ - μ^j \partial_j)θ^A \right]^2, \]

where

\[ V^J = (V^0, V^j) = \left( -\frac{1}{2√μ^0}, \frac{μ^j}{2√μ^0} \right) \]

The equations of motion for the Lagrange multipliers μ0 and μj which follow from (3) give the constraints (pν and p_{θα}^A are the momenta conjugated to xν and θ^Aα):

\[ T_0 = p^2, \quad T_j = p_ν \partial_j x^ν + ∑_A p_{θα}^A \partial_j θ^Aα. \]

The remaining constraints follow from the definition of the momenta p_{θα}^A:

\[ D^A_α = -ip_{θα}^A - (fθ^A)_α. \]

3 Hamiltonian formulation

The Hamiltonian which corresponds to the Lagrangian density (5) is a linear combination of the constraint (3) and (5):

\[ H_0 = ∫ d^p σ[μ^0 T_0 + μ^j T_j + ∑_A μ^{Aα} D^A_α] \]

It is a generalization of the Hamiltonians for the bosonic null p-brane and for the N-extended Green-Schwarz superparticle.

The equations of motion which follow from the Hamiltonian (8) are:

\[ (\partial_τ - μ^j \partial_j)x^ν = 2μ^0 p^ν - ∑_A (μ^A σ^ν θ^A), \quad (\partial_τ - μ^j \partial_j)p_ν = (\partial_j μ^j)p_ν, \]

\[ (\partial_τ - μ^j \partial_j)θ^A_α = iμ^{Aα}, \quad (\partial_τ - μ^j \partial_j)p^A_{θα} = (\partial_j μ^j)p^A_{θα} + (μ^A f_α). \]

In (9), one can consider μ^0, μ^j and μ^{Aα} as depending only on σ = (σ^1, ..., σ^p), but not on τ as a consequence from their equations of motion.

In the gauge when μ^0, μ^j and μ^{Aα} are constants, the general solution of (9) is

\[ x^ν(τ, σ) = x^ν(τ, σ) + τ[2μ^0 p^ν(σ) - ∑_A (μ^A σ^ν θ^A(σ, σ))], \]

\[ = x^ν(σ) + τ[2μ^0 p^ν(σ) - ∑_A (μ^A σ^ν θ^A(σ))] \]

\[ p_ν(τ, σ) = p_ν(σ), \quad θ^{Aα}(τ, σ) = θ^{Aα}(σ) + iτ μ^{Aα}, \]

\[ p^A_{θα}(τ, σ) = p^A_{θα}(σ) + τ(μ^A σ^ν) p_ν(σ), \]
where \( x^\nu(\zeta) \), \( p_\nu(\zeta) \), \( \theta^{A\alpha}(\zeta) \) and \( p^A_{\alpha\sigma}(\zeta) \) are arbitrary functions of their arguments

\[ \zeta^j = \mu^j \tau + \sigma^j. \]

In the case of null strings \((p = 1)\), one can write explicitly the general solution of the equations of motion in an arbitrary gauge: \( \mu^0 = \mu^0(\sigma), \mu^1 \equiv \mu = \mu(\sigma), \mu^{A\alpha} = \mu^{A\alpha}(\sigma) \). This solution is given by

\[
x^\nu(\tau, \sigma) = g^\nu(w) - 2 \int_0^\sigma \frac{\mu^0(s)}{\mu^2(s)} ds f^\nu(w) + \sum_A \int_0^\sigma \frac{\mu^{A\alpha}(s)}{\mu(s)} ds [\sigma^\nu \zeta^A(w)]_\alpha - \text{i} \int_0^\sigma ds \frac{\mu^{A\alpha}(s)}{\mu(s)} ds,
\]

\[
p_\nu(\tau, \sigma) = \mu^{-1}(\sigma) f_\nu(w), \quad \theta^{A\alpha}(\tau, \sigma) = \zeta^{A\alpha}(w) - \text{i} \int_0^\sigma \frac{\mu^{A\alpha}(s)}{\mu(s)} ds,
\]

\[
p^A_{\alpha\sigma}(\tau, \sigma) = \mu^{-1}(\sigma) \left[ h^A_\alpha(w) - \int_0^\sigma \frac{\mu^A(s)\sigma^\nu}{\mu(s)} ds f^\nu(w) \right] - \text{i} \int_0^\sigma ds \frac{\mu^{A\alpha}(s)}{\mu(s)} ds.
\]

Here \( g^\nu(w), f_\nu(w), \zeta^{A\alpha}(w) \) and \( h^A_\alpha(w) \) are arbitrary functions of the variable

\[ w = \tau + \int_0^\sigma \frac{ds}{\mu(s)}. \]

The solution \((10)\) at \( p = 1 \) differs from \((11)\) by the choice of the particular solutions of the inhomogeneous equations. As for \( z \) and \( w \), one can write for example \((\mu^0, \mu, \mu^{A\alpha} \text{ are now constants})\)

\[ p_\nu(\tau, \sigma) = \mu^{-1} f_\nu(\tau + \sigma) = \mu^{-1} f_\nu[\mu^{-1}(\mu \tau + \sigma)] = p_\nu(\zeta) \]

and analogously for the other arbitrary functions in the general solution of the equations of motion.

Let us now consider the properties of the constraints \((3), (7)\). They satisfy the following (equal \( \tau \)) Poisson bracket algebra

\[
\{T_0(\sigma_1), T_0(\sigma_2)\} = 0, \quad \{T_0(\sigma_1), D^A_\alpha(\sigma_2)\} = 0,
\]

\[
\{T_0(\sigma_1), T_j(\sigma_2)\} = [T_0(\sigma_1) + T_0(\sigma_2)] \partial_{\delta^0} \delta^\nu (\sigma_1 - \sigma_2),
\]

\[
\{T_j(\sigma_1), T_k(\sigma_2)\} = [\delta^j_k T_0(\sigma_1) + \delta^j_k T_0(\sigma_2)] \partial_{\delta^\nu} \delta^\nu (\sigma_1 - \sigma_2),
\]

\[
\{T_j(\sigma_1), D^A_\alpha(\sigma_2)\} = D^A_\alpha(\sigma_1) \partial_j \delta^\nu (\sigma_1 - \sigma_2),
\]

\[
\{D^A_\alpha(\sigma_1), D^B_\beta(\sigma_2)\} = 2i \delta^{AB} \delta^\nu \delta_{\alpha\beta} (\sigma_1 - \sigma_2).
\]

From the condition that the constraints must be maintained in time, i.e. \((8)\)

\[
\{T_0, H_0\} \approx 0, \quad \{T_j, H_0\} \approx 0, \quad \{D^A_\alpha, H_0\} \approx 0, \quad (12)
\]
it follows that in the Hamiltonian $H_0$ one has to include the constraints

$$T^A_\alpha = \hat{p}_{\alpha \beta} D^{A\beta}$$

instead of $D^A_\alpha$. This is because the Hamiltonian has to be first class quantity, but $D^A_\alpha$ are a mixture of first and second class constraints. $T^A_\alpha$ has the following non-zero Poisson brackets

$$\{T_j(\sigma_1), T^A_\alpha(\sigma_2)\} = [T^A_\alpha(\sigma_1) + T^A_\alpha(\sigma_2)] \partial_j \delta^p(\sigma_1 - \sigma_2),$$
$$\{T^A_\alpha(\sigma_1), T^B_\beta(\sigma_2)\} = 2i \delta^{AB} \hat{p}_{\alpha \beta} T^0 \delta^p(\sigma_1 - \sigma_2).$$

In this form, our constraints are first class and the Dirac consistency conditions (12) (with $D^A_\alpha$ replaced by $T^A_\alpha$) are satisfied identically. However, one now encounters a new problem. The constraints $T^0$, $T_j$ and $T^A_\alpha$ are not BFV-irreducible, i.e. they are functionally dependent:

$$(\hat{p} T^A)^\alpha - D^{A\alpha} T^0 = 0.$$ 

It is known, that in this case after BRST-BFV quantization an infinite number of ghosts for ghosts appear, if one wants to preserve the manifest Lorentz invariance. The way out consists in the introduction of auxiliary variables, so that the mixture of first and second class constraints $D^{A\alpha}$ can be appropriately covariantly decomposed into first class constraints and second class ones. To this end, here we will use the auxiliary harmonic variables introduced in [10] and [11]. These are $u^a_\mu$ and $v^\pm_\alpha$, where superscripts $a = 1, \ldots, 8$ and $\pm$ transform under the ‘internal’ groups $SO(8)$ and $SO(1,1)$ respectively. The just introduced variables are constrained by the following orthogonality conditions

$$u^a_\mu u^b_\nu = C^{ab}, \quad u^\pm_\mu u^{a\mu} = 0, \quad u^\pm_\mu u^{-\mu} = -1,$$

where

$$u^\pm_\mu = v^\pm_\alpha \sigma^\alpha_\mu v^\pm_\beta,$$

$C^{ab}$ is the invariant metric tensor in the relevant representation space of $SO(8)$ and $(u^\pm)^2 = 0$ as a consequence of the Fierz identity for the 10-dimensional $\sigma$-matrices. We note that $u^a_\mu$ and $v^\pm_\alpha$ do not depend on $\sigma$.

Now we have to ensure that our dynamical system does not depend on arbitrary rotations of the auxiliary variables ($u^a_\mu$, $u^\pm_\mu$). It can be done by introduction of first class constraints, which generate these transformations

$$I^{ab} = -(u^a_\nu p^{b\nu} - u^b_\nu p^{a\nu} + \frac{1}{2} v^+ \sigma^{ab} p^{\nu}_v + \frac{1}{2} v^- \sigma^{ab} p^{-\nu}_v), \quad \sigma^{ab} = u^a_\mu u^b_\nu \sigma^{\mu\nu},$$
$$I^{-+} = -\frac{1}{2} (v^+ p^{+\alpha}_v - v^- p^{-\alpha}_v),$$
$$I^{\pm a} = -(u^\pm_\mu p^{a\mu}_v + \frac{1}{2} u^\pm \sigma^\pm \sigma^a p^{\pm}_v), \quad \sigma^\pm = u^\pm_\nu \sigma^{\nu}, \quad \sigma^a = u^a_\nu \sigma^{\nu}.$$
In the above equalities, $p^\alpha_\nu$ and $p^\pm_\alpha$ are the momenta canonically conjugated to $u^a_\nu$ and $u^\pm_\alpha$.

The newly introduced constraints (13) obey the following Poisson bracket algebra
\[
\{I^{ab}, I^{cd}\} = C^{bc}I^{ad} - C^{ac}I^{bd} + C^{ad}I^{bc} - C^{bd}I^{ac},
\]
\[
\{I^{-+}, I^{\pm a}\} = \pm I^{\pm a},
\]
\[
\{I^{ab}, I^{\pm c}\} = C^{bc}I^{\pm a} - C^{ac}I^{\pm b},
\]
\[
\{I^{+a}, I^{-b}\} = C^{ab}I^{-+} + I^{ab}.
\]

This algebra is isomorphic to the $SO(1,9)$ algebra: $I^{ab}$ generate $SO(8)$ rotations, $I^{-+}$ is the generator of the subgroup $SO(1,1)$ and $I^{\pm a}$ generate the transformations from the coset $SO(1,9)/SO(1,1) \times SO(8)$.

Now we are ready to separate $D^{\alpha a}$ into first and second class constraints in a Lorentz-covariant form. This separation is given by the equalities (14):
\[
D^{\alpha a} = \frac{1}{p^+}[(\sigma^a v^+)^\alpha D^\alpha_a + (\not\sigma^+ v^-)^\alpha G^\alpha_a], \quad p^+ = p^\nu u^\nu_\nu,
\]
\[
D^{\alpha a} = (v^+ \sigma^a \not\sigma)_{\beta} D^{\beta \alpha}, \quad G^{\alpha a} = \frac{1}{2}(v^- \sigma^a \sigma^+)_{\beta} D^{\beta \alpha}.
\]

Here $D^{\alpha a}$ are first class constraints and $G^{\alpha a}$ are second class ones:
\[
\{D^{\alpha a}(\tilde{\sigma}_1), D^{\beta b}(\tilde{\sigma}_2)\} = -2i\delta^{\alpha \beta} C^{ab} p^+ T_0 \delta^{\nu}(\tilde{\sigma}_1 - \tilde{\sigma}_2),
\]
\[
\{G^{\alpha a}(\tilde{\sigma}_1), G^{\beta b}(\tilde{\sigma}_2)\} = i\delta^{\alpha \beta} C^{ab} p^+ \delta^{\nu}(\tilde{\sigma}_1 - \tilde{\sigma}_2).
\]

It is convenient to pass from second class constraints $G^{\alpha a}$ to first class constraints $\hat{G}^{\alpha a}$, without changing the actual degrees of freedom (12), (13):
\[
G^{\alpha a} \rightarrow \hat{G}^{\alpha a} = G^{\alpha a} + (p^+)^{1/2} \Psi^{\alpha a} \quad \Rightarrow \quad \{\hat{G}^{\alpha a}(\tilde{\sigma}_1), D^{\beta b}(\tilde{\sigma}_2)\} = 0,
\]
where $\Psi^{\alpha a}(\tilde{\sigma})$ are fermionic ghosts which abelianize our second class constraints as a consequence of the Poisson bracket relation
\[
\{\Psi^{\alpha a}(\tilde{\sigma}_1), \Psi^{\beta b}(\tilde{\sigma}_2)\} = -i\delta^{\alpha \beta} C^{ab} \delta^{\nu}(\tilde{\sigma}_1 - \tilde{\sigma}_2).
\]

It turns out, that the constraint algebra is much more simple, if we work not with $D^{\alpha a}$ and $\hat{G}^{\alpha a}$ but with $\hat{T}^{\alpha a}$ given by
\[
\hat{T}^{\alpha a} = (p^+)^{-1/2}[(\sigma^a v^+)^\alpha D^\alpha_a + (\not\sigma^+ v^-)^\alpha \hat{G}^\alpha_a]
\]
\[
= (p^+)^{1/2} D^{\alpha a} + (\not\sigma^+ v^-)^\alpha \Psi^\alpha_a.
\]

After the introduction of the auxiliary fermionic variables $\Psi^{\alpha a}$, we have to modify some of the constraints, to preserve their first class property. Namely $T_j$, $I^{ab}$ and $I^{-a}$ change as follows
\[
\hat{T}_j = T_j + \frac{i}{2} C^{ab} \sum_{\alpha} \Psi^A_a \partial_j \Psi^A_b,
\]
\[
\hat{I}^{ab} = I^{ab} + J^{ab}, \quad J^{ab} = \int d^p \sigma j^{ab}(\sigma), \quad j^{ab} = \frac{i}{4} (v^- \sigma^\alpha \sigma^b \sigma^+ \sigma_d v^-) \sum_{\beta} \Psi^{\alpha \beta} \Psi^{\beta a},
\]
\[
\hat{I}^{-a} = I^{-a} + J^{-a}, \quad J^{-a} = \int d^p \sigma j^{-a}(\sigma), \quad j^{-a} = -(p^+)^{-1} j^{ab} p_b.
\]
As a consequence, we can write down the Hamiltonian for the considered model in the form:

\[ H = \int d^p \sigma [\lambda^0 T_0(\sigma) + \lambda^j \hat{T}_j(\sigma) + \sum_A \lambda^{A\alpha} \hat{T}_\alpha^A(\sigma)] + \lambda_{ab} \hat{I}^{ab} + \lambda_{-+} I^{-+} + \lambda_{++} I^{++} + \lambda_{-a} \hat{I}^{-a}. \]

The constraints entering \( H \) are all first class, irreducible and Lorentz-covariant. Their algebra reads (only the non-zero Poisson brackets are written):

\[
\begin{align*}
\{ T_0(\sigma_1), \hat{T}_j(\sigma_2) \} &= (T_0(\sigma_1) + T_0(\sigma_2)) \partial_j \delta^p(\sigma_1 - \sigma_2), \\
\{ \hat{T}_j(\sigma_1), \hat{T}_k(\sigma_2) \} &= (\delta_j^i \hat{T}_i(\sigma_1) + \delta_k^i \hat{T}_i(\sigma_2)) \partial_i \delta^p(\sigma_1 - \sigma_2), \\
\{ \hat{T}_j(\sigma_1), \hat{T}_\alpha^A(\sigma_2) \} &= (\hat{T}_\alpha^A(\sigma_1) + \frac{1}{2} \hat{T}_\alpha^A(\sigma_2)) \partial_j \delta^p(\sigma_1 - \sigma_2), \\
\{ \hat{T}_\alpha^A(\sigma_1), \hat{T}_\beta^B(\sigma_2) \} &= i \delta^{AB} \sigma_\alpha^+ T_0 \delta^p(\sigma_1 - \sigma_2), \\
\{ I^{-+}, \hat{T}_\alpha^A \} &= \frac{1}{2} \hat{T}_\alpha^A, \\
\{ I^{-a}, \hat{T}_\alpha^A \} &= (2p^+)^{-1} \left[ p^a \hat{T}_\alpha^A + (\sigma^+ \sigma^{ab} v)_a \Psi^b T_0 \right], \\
\{ \hat{I}^{ab}, \hat{I}^{cd} \} &= C^{bc} \hat{I}^{ad} - C^{ac} \hat{I}^{bd} + C^{bd} \hat{I}^{ac} - C^{ad} \hat{I}^{bc}, \\
\{ I^{-+}, I^{+a} \} &= I^{+a}, \\
\{ I^{-+}, \hat{I}^{-a} \} &= -\hat{I}^{-a}, \\
\{ \hat{I}^{ab}, \hat{I}^{+c} \} &= C^{bc} \hat{I}^{a+} - C^{ac} \hat{I}^{b+}, \\
\{ I^{+a}, \hat{I}^{-b} \} &= \hat{C}^{ab} I^{++} + \hat{I}^{ab}, \\
\{ \hat{I}^{-a}, \hat{I}^{-b} \} &= -\int d^p \sigma (p^+)^{-2} j^{ab} T_0.
\end{align*}
\]

Having in mind the above algebra, one can construct the corresponding BRST charge \( \Omega \) (\( *=\)complex conjugation)

\[ \Omega = \Omega^{\text{min}} + \pi_M \bar{F}^M, \quad \{ \Omega, \Omega \} = 0, \quad \Omega^* = \Omega, \quad (15) \]

where \( M = 0, j, A\alpha, ab, -+, +a, -a \). \( \Omega^{\text{min}} \) in (13) can be written as

\[
\begin{align*}
\Omega^{\text{min}} &= \Omega^{\text{brane}} + \Omega^{\text{aux}}, \\
\Omega^{\text{brane}} &= \int d^p \sigma \{ T_0 \eta^0 + \hat{T}_j \eta^j + \sum_A \hat{T}_\alpha^A \eta^{A\alpha} + \mathcal{P}_0 (\partial_j \eta^0) \eta^0 + (\partial_j \eta^0) \eta^j \} + \\
&+ \mathcal{P}_k (\partial_j \eta^k) \eta^j + \sum_A \mathcal{P}_A [\eta^j \partial_j \eta^{A\alpha} - \frac{1}{2} \eta^{A\alpha} \partial_j \eta^j] - \frac{i}{2} \mathcal{P}_0 \sum_A \eta^{A\alpha} \sigma_\alpha^+ \eta^{A\beta}, \\
\Omega^{\text{aux}} &= \hat{I}^{ab} \eta_{ab} + I^{-+} \eta_{-+} + I^{+a} \eta_{+a} + \hat{I}^{-a} \eta_{-a} + \mathcal{P}_a \eta^{a} - \mathcal{P}_a \eta_{-a} + \mathcal{P}_a \eta_{+a} + (\mathcal{P}_a \eta^{a}) \eta_{-a} + (\mathcal{P}_a \eta^{a}) \eta_{+a} + \\
&+ \frac{1}{2} \int d^p \sigma \left\{ \sum_A \mathcal{P}_A \eta^{A\alpha} \eta_{-+} + (p^+)^{-1} \sum_A [p^a \mathcal{P}_A - (\sigma^+ \sigma^{ab} v^a) \Psi^b \mathcal{P}_0] \eta^{A\alpha} \eta_{-a} - \right. \\
&\left. - (p^+)^{-2} j^{ab} \mathcal{P}_0 \eta_{-b} \eta_{-a} \right\}.
\end{align*}
\]

These expressions for \( \Omega^{\text{brane}} \) and \( \Omega^{\text{aux}} \) show that we have found a set of constraints which ensure the first rank property of the model.
\( \Omega^{\text{min}} \) can be represented also in the form
\[
\Omega^{\text{min}} = \int d^p \sigma [(T_0 + \frac{1}{2} T_0^{gh}) \eta^0 + (\hat{T}_j + \frac{1}{2} T_j^{gh}) \eta^j + \sum_A (\hat{T}^A_\alpha + \frac{1}{2} T^A_{gh} \eta^A) ]
\]
\[+(\hat{t}_{ab} + \frac{1}{2} I_{gh}^{ab}) \eta_{ab} + (I^{--} + \frac{1}{2} I_{gh}^{-+}) \eta_{-+} + (I^{+a} + \frac{1}{2} I_{gh}^{+a}) \eta_{+a} + (\hat{i}^{-a} + \frac{1}{2} I_{gh}^{-a}) \eta_{-a}
\]
\[+ \int d^p \sigma \partial_j (\frac{1}{2} \mathcal{P}_k \eta^k \eta^j + \frac{1}{4} \sum_A \mathcal{P}^A \eta^{A\alpha} \eta^{\alpha} ) .
\]

Here a super(sub)script \( gh \) is used for the ghost part of the total gauge generators

\( G^\text{tot} = \{ \Omega, \mathcal{P} \} = \{ \Omega^{\text{min}}, \mathcal{P} \} = G + G^{gh} . \)

We recall that the Poisson bracket algebras of \( G^\text{tot} \) and \( G \) coincide for first rank systems. The manifest expressions for \( G^{gh} \) are:

\[
T_0^{gh} = 2 \mathcal{P}_0 \partial_j \eta^j + (\partial_j \mathcal{P}) \eta^j,
\]
\[
T_j^{gh} = 2 \mathcal{P}_0 \partial_j \eta^0 + (\partial_j \mathcal{P}_0) \eta^0 + \mathcal{P}_j \partial_k \eta^k + \mathcal{P}_k \partial_j \eta^k + (\partial_k \mathcal{P}_j) \eta^k
\]
\[+ \frac{3}{2} \sum_A \mathcal{P}^A_\alpha \partial_j \eta^{A\alpha} + \frac{1}{2} \sum_A (\partial_j \mathcal{P}^A_\alpha) \eta^{A\alpha},
\]
\[
T_{\alpha}^{Agh} = -\frac{3}{2} \mathcal{P}^A_\alpha \partial_j \eta^j - (\partial_j \mathcal{P}^A_\alpha) \eta^j - i \mathcal{P}_0 \sigma^{++}_\alpha \eta^{A\beta} +
\]
\[+ \frac{1}{2} \mathcal{P}^A_\alpha \eta_{-+} + (2p^+)^{-1} \left[ \sigma^{++}_\alpha \mathcal{P}^A_\alpha - (\sigma^{+a} \sigma^{ab} v^-)_{\alpha} \mathcal{P}_b \right] \eta_{-a},
\]
\[
I_{gh}^{ab} = 2 (\mathcal{P}^{ac} \eta_{c}^{-} - \mathcal{P}^{bc} \eta_{c}^{+} ) + (\mathcal{P}^{+a} \eta_{+}^{-} - \mathcal{P}^{+b} \eta_{+}^{+} ) + (\mathcal{P}^{-a} \eta_{-}^{-} - \mathcal{P}^{-b} \eta_{-}^{+} ),
\]
\[
I_{gh}^{+} = \mathcal{P}^{+a} \eta_{+a} - \mathcal{P}^{-a} \eta_{-a} + \frac{1}{2} \int d^p \sigma \sum_A \mathcal{P}^A \eta^{A\alpha},
\]
\[
I_{gh}^{-} = 2 \mathcal{P}^{-b} \eta_{-b}^{a} - \mathcal{P}^{+a} \eta_{-a}^{-} + \mathcal{P}^{-+} \eta_{-}^{-} + \mathcal{P}^{ab} \eta_{-b},
\]
\[
I_{gh}^{-a} = 2 \mathcal{P}^{-b} \eta_{-b}^{a} + \mathcal{P}^{+a} \eta_{-a}^{a} - \mathcal{P}^{-+} \eta_{-}^{a} + \mathcal{P}^{ab} \eta_{+b} +
\]
\[+ \int d^p \sigma \left((2p^+)^{-1} \sum_A [p^{a} \mathcal{P}^A_{\alpha} - (\sigma^{+a} \sigma^{ab} v^-)_{\alpha} \mathcal{P}^A_b] \eta^{A\alpha} - (p^+)^{-2} j^{ab} \mathcal{P}_0 \eta_{-b} \right) .
\]

Up to now, we introduced canonically conjugated ghosts \((\eta^M, \mathcal{P}_M), (\bar{\eta}_M, \bar{\mathcal{P}}^M)\) and momenta \(\pi_M\) for the Lagrange multipliers \(\Lambda^M\) in the Hamiltonian. They have Poisson brackets and Grassmann parity as follows \((\epsilon_M\) is the Grassmann parity of the corresponding constraint):

\[
\{ \eta^M, \mathcal{P}_N \} = \delta^M_N, \quad \epsilon(\eta^M) = \epsilon(\mathcal{P}_M) = \epsilon_M + 1,
\]
\[
\{ \bar{\eta}_M, \mathcal{P}^N \} = -(1)^{\epsilon_M \epsilon_N} \delta^M_N, \quad \epsilon(\bar{\eta}_M) = \epsilon(\mathcal{P}^M) = \epsilon_M + 1,
\]
\[
\{ \lambda^M, \pi_N \} = \delta^M_N, \quad \epsilon(\lambda^M) = \epsilon(\pi_M) = \epsilon_M .
\]

The BRST-invariant Hamiltonian is
\[
H_\chi = H^{\text{min}} + \{ \chi, \Omega \} = \{ \chi, \Omega \}, \quad (16)
\]

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because from $H_{\text{canonical}} = 0$ it follows $H^{\text{min}} = 0$. In this formula $\tilde{\chi}$ stands for the gauge fixing fermion ($\tilde{\chi}^* = -\chi$). We use the following representation for the latter

$$\tilde{\chi} = \chi^{\text{min}} + \eta_M(\chi^M + \frac{1}{2} \rho(M) \pi^M), \quad \chi^{\text{min}} = \chi^M \mathcal{P}_M,$$

where $\rho(M)$ are scalar parameters and we have separated the $\pi^M$-dependence from $\chi^M$. If we adopt that $\chi^M$ does not depend on the ghosts ($\eta^M, \mathcal{P}_M$) and ($\eta_M, \bar{\mathcal{P}}_M$), the Hamiltonian $H_\chi$ from (13) takes the form

$$H_\chi = H^{\text{min}} + \mathcal{P}_M \bar{\mathcal{P}}^M - \pi_M(\chi^M + \frac{1}{2} \rho(M) \pi^M) + \eta_M \left[ \{\chi^M, G_N\} \eta^N + \frac{1}{2}(\chi)^N \mathcal{P}_Q \{\chi^M, U^Q_{NP}\} \eta^P \eta^N \right],$$

where

$$H^{\text{min}}_\chi = \{\chi^{\text{min}}, \Omega^{\text{min}}\},$$

and generally $\{\chi^M, U^Q_{NP}\} \neq 0$ as far as the structure coefficients of the constraint algebra $U^Q_{NP}$ depend on the phase-space variables.

One can use the representation (17) for $H_\chi$ to obtain the corresponding BRST invariant Lagrangian

$$L_\chi = L + L_{\text{GH}} + L_{\text{GF}}.$$

Here $L_{\text{GH}}$ stands for the ghost part and $L_{\text{GF}}$ - for the gauge fixing part of the Lagrangian. If one does not intend to pass to the Lagrangian formalism, one may restrict oneself to the minimal sector ($\Omega^{\text{min}}_\chi, \chi^{\text{min}}, H^{\text{min}}_\chi$). In particular, this means that Lagrange multipliers are not considered as dynamical variables anymore. With this particular gauge choice, $H^{\text{min}}_\chi$ is a linear combination of the total constraints

$$H^{\text{min}}_\chi = H^{\text{min}}_{\text{brane}} + H^{\text{min}}_{\text{aux}} =$$

$$= \int d^p \sigma \left[ \Lambda^0 T^0_0 (\sigma) + \Lambda^j T^j_0 (\sigma) + \sum_A \Lambda^{A0} T^A_0 (\sigma) \right] +$$

$$+ \Lambda_{ab} I_{tot}^+ - \Lambda_{-a} I_{tot}^- + \Lambda_{+a} I_{tot}^+ + \Lambda_{-a} I_{tot}^-,$$

and we can treat here the Lagrange multipliers $\Lambda^0, ..., \Lambda_{-a}$ as constants. Of course, this does not fix the gauge completely.

## 4 Comments and conclusions

To ensure that the harmonics and their conjugate momenta are pure gauge degrees of freedom, we have to consider as physical observables only such functions on the phase space which do not carry any $SO(1, 1) \times SO(8)$ indices. More precisely, these functions are defined by the following expansion

$$F(y, u, v; p_y, p_u, p_v) = \sum [u^{\alpha_1}_{\nu_1} \cdots u^{\alpha_k}_{\nu_k} p^{\nu_{k+1}}_{\nu_{k+1}} \cdots p^{\nu_{k+l}}_{\nu_{k+l}}]_{SO(8)\text{singlet}}$$

$$= v_{\alpha_1}^+ \cdots v_{\alpha_m}^+ v_{\alpha_{m+1}}^- \cdots v_{\alpha_{m+n}}^- p_{\beta_1}^{\beta_1} \cdots p_{\beta_r}^{\beta_r} p_{\beta_{r+1}}^{-\beta_{r+1}} \cdots p_{\beta_{m-n+r}}^{-\beta_{m-n+r}} F_{\bar{\beta}_1 \cdots \bar{\beta}_{m-n+r}}^{\alpha_1 \cdots \alpha_{m+n} \nu_1 \cdots \nu_{k+l}}(y, p_y),$$

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where \((y,p_y)\) are the non-harmonic phase space conjugated pairs.

In this letter we consider a model for tensionless super \(p\)-branes with \(N\) global chiral supersymmetries in 10-dimensional Minkowski space-time. We show that the action is reparametrization and kappa-invariant. After establishing the symmetries of the action, we give the general solution of the classical equations of motion in a particular gauge. In the case of null superstrings \((p=1)\) we find the general solution in an arbitrary gauge. Starting with a Hamiltonian which is a linear combination of first and mixed (first and second) class constraints, we succeed to obtain a new one, which is a linear combination of first class, BFV-irreducible and Lorentz-covariant constraints only. This is done with the help of the introduced auxiliary harmonic variables. Then we give manifest expressions for the classical BRST charge, the corresponding total constraints and BRST-invariant Hamiltonian. It turns out, that in the given formulation our model is a first rank dynamical system.

**Appendix**

We briefly describe here our 10-dimensional conventions. Dirac \(\gamma\)-matrices obey

\[
\Gamma_\mu \Gamma_\nu + \Gamma_\nu \Gamma_\mu = 2\eta_{\mu\nu}
\]

and are taken in the representation

\[
\Gamma^\mu = \begin{pmatrix}
0 & (\sigma^\mu)^{\dot{\beta}}_{\dot{\alpha}} \\
(\bar{\sigma}^\mu)^{\alpha}_{\dot{\alpha}} & 0
\end{pmatrix}
\]

\(
\Gamma^{11}
\)

and charge conjugation matrix \(C_{10}\) are given by

\[
\Gamma^{11} = \Gamma^0 \Gamma^1 ... \Gamma^9 = \begin{pmatrix}
\delta^{\beta}_{\alpha} & 0 \\
0 & -\delta^{\dot{\beta}}_{\dot{\alpha}}
\end{pmatrix}
\]

\[
C_{10} = \begin{pmatrix}
0 & C^{\alpha\dot{\beta}} \\
(-C)^{\dot{\alpha}}_{\beta} & 0
\end{pmatrix}
\]

and the indices of right and left Majorana-Weyl fermions are raised as

\[
\psi^{\dot{\alpha}} = C^{\alpha\dot{\beta}} \psi_{\dot{\beta}} \quad , \quad \phi^{\dot{\alpha}} = (-C)^{\dot{\alpha}}_{\beta} \phi_{\beta}.
\]

We use \(D=10\) \(\sigma\)-matrices with undotted indices

\[
(\sigma^\mu)^{\alpha\beta} = C^{\alpha\dot{\alpha}} (\bar{\sigma}^\mu)^{\dot{\beta}}_{\dot{\alpha}} \quad , \quad (\sigma^\mu)^{\dot{\alpha}}_{\beta} = (-C)^{-1}_{\beta\dot{\beta}} (\bar{\sigma}^\mu)^{\dot{\beta}}_{\dot{\alpha}}
\]

and the notation

\[
\sigma^{\mu_1...\mu_n} \equiv \sigma^{[\mu_1...\mu_n]}
\]

for their antisymmetrized products.
From the corresponding properties of $D = 10$ $\gamma$-matrices, it follows:

\[
(\sigma^\mu)_{\alpha\gamma}(\sigma^\nu)_{\beta\gamma} + (\sigma^\nu)_{\alpha\gamma}(\sigma^\mu)_{\gamma\beta} = -2\delta^\beta_\alpha \eta^\mu\nu, \\
(\sigma_{\mu_1\ldots\mu_{2s+1}})_{\alpha\beta} = (-1)^s (\sigma_{\mu_1\ldots\mu_{2s+1}})_{\beta\alpha}, \\
\sigma^\mu \sigma^{\nu_1\ldots\nu_n} = \sigma^{\mu\nu_1\ldots\nu_n} + \sum_{k=1}^{n} (-1)^k \eta^{\mu\nu_k} \sigma^{\nu_1\ldots\nu_{k-1}\nu_{k+1}\ldots\nu_n}.
\]

The Fierz identity for the $\sigma$-matrices reads:

\[
(\sigma_\mu)^{\alpha\beta}(\sigma^\mu)^{\gamma\delta} + (\sigma_\mu)^{\beta\gamma}(\sigma^\mu)^{\alpha\delta} + (\sigma_\mu)^{\gamma\alpha}(\sigma^\mu)^{\beta\delta} = 0.
\]

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