Research on Trajectory Tracking Control Method of Omnidirectional Moving Platform Based on Backstepping

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Abstract. According to the relevant theoretical methods of omnidirectional mobile mechanism, the kinematic model of omnidirectional mobile platform under theoretical conditions is established, and the dynamic model of omnidirectional mobile mechanism in actual environment is analyzed according to the specific situation of platform body structure and actual influencing factors. Based on the kinematic model, a backstepping based trajectory tracking control method for omnidirectional mobile platform is designed, and the trajectory tracking error equation is solved. In order to further improve the stability of the system, a fuzzy control system based on backstepping method is designed based on the dynamic model, and the control torque of the platform is selected. Finally, the effectiveness of this method is verified by numerical simulation.

1. Introduction
Mobile robot is an important branch of robot field, and its motion modes include wheeled, legged, tracked and serpentine [1,2]. Among them, wheel type is the earliest and most widely used mobile mode of robot. Its structure is relatively simple, and it can achieve smooth, high-speed and accurate motion effect in a plane environment. Omnidirectional motion is a kind of motion mode of wheeled robot, which can make the robot move in any direction or rotate in place under the condition of unchanged posture [3]. Omnidirectional mobile robot has three degrees of freedom, which can move along all directions in a straight line and rotate around a fixed axis flexibly. It is especially suitable for the logistics industry where the working space is narrow and the environment changes a lot. It has a broader development prospect than the traditional differential drive mode of AGV. The wheels that can realize omnidirectional movement include ball wheel [4], continuous switching wheel [5], mecanum wheel [6], orthogonal wheel [7], etc.

Mecanum wheel was designed by mecanum company of Sweden in 1973. It is a typical omnidirectional mobile wheel [8-9]. Based on its unique and flexible structure, mecanum wheel has been adopted by many scholars at home and abroad. The research focuses include the design, motion control, kinematics, dynamics and so on [10]. After years of research and improvement, the structure of mecanum wheel has gradually diversified. According to the hub support mode, it is divided into two end support and middle support [11]. Wang Yizhi and others derived the kinematic model based on mecanum’s four-wheel drive system and summarized the motion conditions of the four-wheel omni-
directional mobile platform [12]. According to Hertz theory, Zhu Hao and others studied the roller deformation and put forward the improvement scheme of the roller [13]. The Institute of Chinese Academy of Sciences combines robust trajectory tracker with magnetic navigation AGV for intelligent logistics [14]. The related research institutions have more and more in-depth research on the mecanum wheel, which further improves the operation stability of the mecanum wheel under the working environment, obtains greater bearing capacity, and then improves the motion performance of the omnidirectional mobile platform.

Trajectory tracking control is the key part of robot control, which directly affects the overall performance of the robot. The backstepping control method designs different controllers through virtual feedback, which is a common method in robot trajectory tracking control [15]. In the backstepping, the control system is divided into several subsystems, and then the Lyapunov equation and local virtual control variables are designed for each subsystem, and then the whole system is reversely deduced. Finally, the controller design is completed jointly. Japanese scholar Tago and others designed the trajectory tracking control system of four wheeled omnidirectional mobile robot based on PID control algorithm with position information as the feedback of the system [16]. When the parameters of PID control are uncertain, it is difficult to ensure the system stability and achieve certain control performance index. The nonlinear robot controller is designed by using backstepping method, which can solve the unmatched uncertainty in the system.

2. Kinematic Model and Dynamic Model of Omnidirectional Mobile Platform

It is the most intuitive and effective method to analyze the motion performance of omnidirectional mobile platform by establishing the kinematic model and dynamic model of omnidirectional mobile platform, and it is also the preparatory work for the research of trajectory tracking control.

2.1. Kinematic Model of Omnidirectional Mobile Platform

For the kinematic analysis of omnidirectional mobile platform, first of all, a single mecanum wheel should be analyzed in detail, and the parametric relationship between roller and driving wheel, driving wheel and vehicle body should be deduced, so as to obtain the kinematic model of omnidirectional mobile platform. The configuration of omnidirectional mobile platform is shown in figure 1.

Assuming that the roller does not slip and the discussion on the roller speed can be neglected, then the relationship between the rotational speed of the wheel train and the center of the car body is as follows:
Equation (1) reflects the relationship between the speed of the gear train and the motion state of the platform, so that the speed and direction of the whole platform can be controlled by controlling the rotation speed of four wheels.

The chassis structure parameters of omnidirectional mobile platform are set as follows:

\[
\begin{align*}
\ell_1 &= \ell_2 = \ell_3 = \ell_4 = l, \alpha_1 = \frac{\pi}{4}, \alpha_2 = -\frac{\pi}{4}, \alpha_3 = \frac{\pi}{4}, \alpha_4 = -\frac{\pi}{4} \\
\theta_1 &= \theta_2 = \theta_3 = \theta_4 = 0, \beta_1 = \frac{2}{3} \pi, \beta_2 = \frac{4}{3} \pi, \beta_3 = \frac{5}{3} \pi
\end{align*}
\]

The kinematic model of omnidirectional mobile platform can be obtained by introducing the parameters into equation (1). Through the analysis of the motion performance, the speed constraint relationship between each wheel is given.

\[
\begin{align*}
\omega_1 &= \frac{1}{4} (3\omega_1 - \omega_2 + \omega_3 + \omega_4), \omega_2 = \frac{1}{4} (-\omega_1 + 3\omega_2 + \omega_3 + \omega_4) \\
\omega_3 &= \frac{1}{4} (\omega_1 + \omega_2 + 3\omega_3 - \omega_4), \omega_4 = \frac{1}{4} (\omega_1 + \omega_2 - \omega_3 + 3\omega_4)
\end{align*}
\]

Equation (2) shows that there is a constraint relationship between the speeds of each wheel under the normal motion without sliding, and the constraint condition should be considered when controlling the speed of the gear train, so as to avoid the wheel slipping caused by the inconsistent speed of the gear train.

2.2. Dynamic Model of Omnidirectional Mobile Platform

The force of driving wheel and driven roller of the platform is analyzed by surface contact mechanics. The dynamic model of omnidirectional mobile platform is given by Lagrange equation after the friction force is taken as the actual environmental factor. It is expressed as:

\[
\begin{pmatrix}
F_x \\
F_y \\
F_\omega
\end{pmatrix} = \begin{pmatrix}
m + \frac{4F_w}{r^2} \\
0 \\
0
\end{pmatrix} \begin{pmatrix}
\ell^2 (\sqrt{3}+1)^2 F_w + I_z \\
0 \\
0
\end{pmatrix} \begin{pmatrix}
\frac{v_x}{r^2} \\
\frac{v_y}{r^2} \\
\frac{\omega}{2r^2}
\end{pmatrix} + \begin{pmatrix}
2F_\omega \\
0 \\
0
\end{pmatrix} \begin{pmatrix}
\frac{v_x}{r^2} \\
\frac{v_y}{r^2} \\
\frac{\omega}{2r^2}
\end{pmatrix}
\]

Equation (3) reflects the dynamic model of omnidirectional mobile platform considering friction factor.

3. Trajectory Tracking Control of Omnidirectional Mobile Platform Based on Backstepping
The above work completes the motion analysis and control of the platform. For the trajectory deviation in the actual motion process, this paper uses the tracking control method to study, and designs the trajectory tracking control law of omnidirectional mobile platform based on backstepping. By establishing the position and attitude tracking error equation of the platform, the control output equations of linear velocity and angular velocity of omnidirectional mobile platform are derived. The trajectory control law of the platform designed by backstepping is shown in figure 2.

In order to verify the stability of the system, the Lyapunov function is used to derive the following results:

$$\begin{align*}
V &= \left( x_r + c_1 x_e \right)^2 + \left( y_r + c_2 y_e \right)^2 \right)^{1/2} \\
\alpha &= \arctan \frac{x_r + c_1 x_e}{y_r + c_2 y_e}
\end{align*}$$

(4)

where $c_1$ and $c_2$ are normal numbers, trajectory tracking error and expected trajectory are defined as:

$$\begin{align*}
x &= v \cos \alpha = x_r + c_1 x_e \\
y &= v \sin \alpha = y_r + c_2 y_e \\
e &= \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} x(t) - x_r(t) \\ y(t) - y_r(t) \\ \theta(t) - \theta_r(t) \end{bmatrix} \\
x_r &= v \cos \theta_r \\
y_r &= v \sin \theta_r
\end{align*}$$

(5)

where $x_r$ and $y_r$ represent the ideal velocity along the x and y axes respectively, and $\theta_r$ is the ideal pose angle.

Based on the feedback attitude tracking error and velocity tracking error, the trajectory tracking control law is designed by using the backstepping algorithm. Based on the analysis of the control variables, the fuzzy system is used to design the control parameters to further improve the stability of the system.

$$
V = V_2 + \frac{1}{2 \gamma} (\dot{\theta^*})^T \dot{\theta^*} = -c_e e^T e - c_3 v_e^T v_e + v_e^T (F - \ddot{\xi} \dot{\theta^*}) + v_e^T (\ddot{\xi} \dot{\theta^*} - \ddot{\xi} \theta) - \frac{1}{\gamma} (\dot{\theta^*})^T \dot{\theta^*}
$$

$$
\leq -c_e e^T e - c_3 v_e^T v_e + \frac{1}{2} \left\| v_e \right\|^2 + \frac{1}{2} \left\| F - \ddot{\xi} \dot{\theta^*} \right\|^2 + (\dot{\theta^*})^T \left( v_e^T \ddot{\xi} - \frac{1}{\gamma} \theta \right) + \frac{1}{2} \left\| v_e \right\|^2
$$

(6)

$$
\leq k \left( \theta^* \cdot \frac{1}{2} (\dot{\theta^*}) \right) + \frac{1}{2} \left\| F - \ddot{\xi} \dot{\theta^*} \right\|^2
$$
Therefore, the system is asymptotically stable.

4. Simulation Experiment and Result Analysis
On the basis of the above theoretical analysis, the trajectory control simulation is carried out, and the effectiveness of this method is further verified by experiments.

4.1. Simulation of Trajectory Tracking Control Based on Kinematics Backstepping

4.1.1. Track A Straight Line. The setting parameters are as follows: the initial attitude of the platform is \((1 \ 2 \ \pi/2)\), the initial linear velocity is \(v = 0.5m/s\), initial angular velocity is \(\omega = 0.3 rad/s\), tracking trajectory is \(y = \sqrt{3}(x-1)\), the simulation results of linear trajectory tracking based on backstepping are shown in figure 3.

![Figure 3](image)

**Figure 3.** Linear trajectory tracking results, control variables and deviation curves based on backstepping.

It can be seen from figure 3(a) that when the car body runs for 1.25m and its attitude is adjusted, it runs according to the expected trajectory; from figure 3(b), the control speed and angular velocity gradually stabilize after 1.5 seconds; from figure 3(c), the deviation of each parameter is stable and approaches to 0 after 1.5 seconds.

4.1.2. Tracking Arc Trajectory. The setting parameters are as follows: initial attitude of platform is \((1 \ 0 \ \pi)\), the initial linear velocity is \(v = 0.1m/s\), initial angular velocity is \(\omega = 0.3 rad/s\), tracking trajectory is \(x^2 + y^2 = 1\), the simulation results of arc trajectory tracking are shown in figure 4.

![Figure 4](image)

**Figure 4.** Arc trajectory tracking results, control variables and deviation curves based on backstepping.

It can be seen from figure 4(a) that the car body can run along the arc track after a short period of adjustment from the initial position; from figure 4(b) that the control speed and angular velocity are gradually stable after 10 seconds; from figure 4(c), the deviation of each parameter is stable after 1 seconds, and approaches to 0 at 3 seconds.

4.1.3. Tracking Spiral Trajectory. The setting parameters are as follows: initial attitude of platform is \((1.2 \ 0.8 \ \pi/4)\), the initial linear velocity is \(v = 0.4m/s\), initial angular velocity is \(\omega = 0.3 rad/s\), tracking trajectory is \(x = 0.1 \times \frac{t+150}{t+50} \times \cos\left(\frac{\pi}{2} + 0.1 \times \frac{3\ t + 50}{t + 50}\right), \ y = 0.1 \times \frac{t+150}{t+50} \times \sin\left(\frac{\pi}{2} + 0.1 \times \frac{3\ t + 50}{t + 50}\right)\), the simulation results of arc trajectory tracking based on backstepping are shown in figure 5.
The initial error has little effect on the trajectory tracking of the platform, and the platform can travel along the predetermined trajectory well. As can be seen from figure 5(a) that at the beginning, the control inputs $F_x$ and $F_y$ tend to 0, and $F_w$ begins to approach 0 about 2 seconds. As can be seen from figure 6(b), the initial error has little effect on the trajectory tracking of the platform, and the platform can travel along the predetermined trajectory well.

**4.2. Simulation of Trajectory Tracking Control of Fuzzy System Based on Dynamic Backstepping**

**4.2.1. Track A Straight Line.** The setting parameters are as follows: the initial attitude of the platform is $(1 1 1)$, $c_1 = c_2 = 500$, $\gamma = 2$, $k = 1.5$, tracking trajectory is $x = y = t$, $\theta = t$, the simulation results of trajectory tracking are shown in figure 6.

**Figure 6.** Linear trajectory control input and trajectory tracking curve.

It can be seen from figure 6(a) that due to the adjustment of fuzzy membership function, before 5 seconds, the amplitude of control input is relatively large, and the control value is unstable. However, in about 8 seconds, the control inputs $F_x$ and $F_y$ tend to 0, and $F_w$ begins to approach 0 about 2 seconds. As can be seen from figure 6(b), the initial error has little effect on the trajectory tracking of the platform, and the platform can travel along the predetermined trajectory well.

**4.2.2. Tracking Sinusoidal Trajectory.** The setting parameters are as follows: the initial attitude of the platform is $(1 1 1/2)$, $c_1 = c_2 = 500$, $\gamma = 2$, $k = 1.5$, tracking trajectory is $x = y = \sin t$, $\theta = \cos t$, the simulation results of sinusoidal trajectory tracking based on backstepping are shown in figure 7.
Figure 7. Sinusoidal trajectory control input and trajectory tracking curve.

As can be seen from figure 7(a), due to the adjustment of fuzzy membership function, before 10 seconds, the amplitude of control inputs $F_x$ and $F_y$ is relatively large, and the control value is unstable. However, at about 12 seconds, the control inputs $F_x$ and $F_y$ tend to 0, and $F_w$ begin to approach 0 about 3 seconds. It can be seen from figure 7(b) that before 5 seconds, the initial error has a certain impact on the trajectory tracking of the platform, and the platform is unstable around the predetermined trajectory. After 5 seconds, the platform basically completes the attitude adjustment and can travel along the sinusoidal trajectory.

5. Conclusion

The introduction of wheeled omnidirectional mobile robot into the automatic logistics system can fully adapt to the characteristics of narrow space of logistics workshop, many uncertain factors of working environment and frequent vehicle steering, so as to realize efficient, economical and flexible "unmanned" production. The backstepping fuzzy control law based on dynamic model design needs to be further studied by parameter adaptive law or other effective methods. This research work is all around trajectory tracking control, and path planning is the premise of trajectory tracking control, which can be carried out from the related content of path planning, combined with trajectory tracking control to achieve a complete control system.

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