Infrared Acceleration Radiation

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Abstract
We present an exactly soluble electron trajectory that permits an analysis of the soft (deep infrared) radiation emitted, the existence of which has been experimentally observed during beta decay via lowest order inner bremsstrahlung. Our treatment also predicts the time evolution and temperature of the emission, and possibly the spectrum, by analogy with the closely related phenomenon of the dynamic Casimir effect.

Keywords Moving mirrors · Beta decay · Black hole evaporation · Acceleration radiation · Dressed electrons

1 Introduction

The transmutation of a nucleus via beta decay involves the abrupt creation of an electron or positron, followed by its expulsion from the nucleus. Although this process can be fully understood only in the context of quantum field theory, there is a long history of classical treatments [1]. Viewed classically, beta decay involves the sudden appearance of a charged particle, which has been modeled by assigning a step function trajectory to a classical charge [2]. The resulting acceleration might be expected to produce electromagnetic radiation, see Fig. 1, and indeed, such radiation has been observed [3]. The process of photon production accompanying beta decay is sometimes referred to as ‘inner bremsstrahlung (IB)’.

The use of a step function is unrealistic, but convenient mathematically [4]. Fortunately, there is a smoother acceleration function that nevertheless permits
an exact treatment of the radiation emission, and we give that treatment here. By extending the period of acceleration being modeled, we find a classical similarity to the well-known quantum Davies–Fulling–Unruh effect [5–7]: in the frame of the charged particle, there is a thermal bath of photons with a temperature proportional to acceleration. Closely related is the emission of quanta by an accelerating mirror (moving mirror radiation) [8–11] and the correspondence to black hole radiation [12]. The interconnection of charged particle acceleration and the above mentioned quantum field theory effects has been the subject of much investigation. In this paper we will not attempt to review these linkages at a fundamental level, but instead we use the known results phenomenologically to extend the discussion of inner bremsstrahlung.

2 Step Function Example

If the electron is initially at rest and imagined to be instantaneously accelerated to a final constant speed, \( s = |\vec{\beta}_f| \) where \( 0 < s < 1 \), then,

\[
\nu(t) = \begin{cases} 
  s, & t > 0, \\
  0, & t < 0.
\end{cases}
\]

Working with unit charge, the angular differential distribution of radiated energy is found to be [4]:

\[
\frac{d^2E}{d\omega d\Omega} = \frac{1}{16\pi^3} \left( \frac{s \sin \theta}{1 - s \cos \theta} \right)^2,
\]

where \( \theta \) is the angle between the final velocity \( \vec{\beta}_f \) and the observation point of the radiation. The total energy radiated is obtained by integration of Eq. (2) over the solid angle \( d\Omega = \sin \theta \ d\varphi \ d\phi \) and over the relevant frequencies. The frequency range, IR/UV-limit \( \mathcal{E} \equiv \omega_{\text{max}} - \omega_{\text{min}}, \) is set by the initial and final energies of the emitted electron, which are determined by the physics of the beta decay process [13] and cannot be derived from this classical phenomenological model. Accordingly, consider a maximum cutoff in this context that stems from the experimentally measured value for free neutron decay of \( \mathcal{E} = m_n - m_p - m_e = 782.3 \) keV. The total energy is rendered finite in this interval (see e.g. [14]).
Equation (3) is lowest order IB energy \[2\], and has been observed to great accuracy \[3\]. The foregoing treatment is sometimes referred to as the instantaneous collision formalism \[13, 15\]. Not only is it physically desirable to avoid the infinite acceleration of Eq. (1), but the mathematical use of the discrete step velocity limits the final results to quantities independent of time. The radiated energy Eq. (3), is characterized by ‘IR universality’ \[16\], which also arises in the dynamical Casimir effect and the theory of dressed electrons. This energy is also found via a classical limit from a corresponding time-dependent trajectory \[1\].

3 Smooth Acceleration

Under the above motivations, we consider the rectilinear trajectory,

\[
\frac{dr}{dr} = \frac{1}{\kappa r} + \frac{1}{s}. \tag{4}
\]

The asymptotic speeds are \(v = (0, s)\) as \(r \to (0, \infty)\) (the electron moves to the right by convention\(^1\) \[4\]), matching Eq. (1). The proper acceleration, \(\alpha = d\gamma/dr\), has time-dependence, \(\alpha(t) = \kappa \beta r^3(1 - \beta/s)^2\), and possesses asymptotic inertia (i.e.

\(^1\) In the closely related moving mirror model, (see \[17\]), the usual convention is to move to the left (see Fig. 3). The difference is a sign change in the angular distribution. The energy remains invariant.
\( \alpha \to 0 \) as \( t \to \pm \infty \). See Figs. 2 and 3 for illustration. Here \( \beta = dr/\,dt \) is the speed, and \( \gamma = 1/\sqrt{1 - \beta^2} \) is the Lorentz factor.

Here \( \kappa \) is the dimensionful acceleration parameter of the model and sets the scale: \( \kappa \leftrightarrow 12\varepsilon/\pi \). We will use Eq. (4) to obtain Eqs. (2) and (3). With large \( \kappa \), the speed of Eq. (4) approaches the step function example, Eq. (1). However, no such approximation on Eq. (4) is needed to obtain the exact previously studied results of Eqs. (2) or (3). As we shall see, the smooth trajectory, without approximation, leads to new time-dependent results, Eqs. (5) and (6).

Notably, \( \bar{\kappa}(u) = \partial_u \ln \nu'(u) \), a useful measure of the ‘local’ \([18]\) or ‘peel’ \([19, 20]\) acceleration; is constant at high speeds and late times. That is, expressed in null coordinates, where \( v = t + r \) is the advanced time trajectory, and \( u = t - r \) is the retarded time, one sees that after taking two separate limits \( s \to 1 \) and \( u \to \infty \), then \( \bar{\kappa}(u) \to \kappa \). It is suggestive that the two-dimensional quantum field theory of moving mirrors also predicts a thermal emission spectrum for mirror trajectories with constant peel (see e.g. \([11, 21]\)), hinting at a deep underlying duality. Supporting evidence that this is indeed the case comes from, e.g. \([22–28]\). In fact, this intuition is confirmed by a direct calculation of the spectrum, which contains a Planck factor and resembles thermal radiation in \( 1 + 1 \) dimensions \([29]\).

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4 Time-Distribution and Power

The time-dependent power distribution is computed using Eq. (4) with straightforward vector algebra (see the procedure in [30]),

\[
\frac{dP}{d\Omega} = \frac{k^2 \beta^2 (\beta - s)^4 \sin^2 \theta}{16\pi^2 s^4 (1 - \beta \cos \theta)^5},
\]  

(5)

where \(s\) again, is the final constant speed, and \(\beta = \beta(t)\) is the time-dependent velocity. Integration of \(dP/d\Omega\), Eq. (5), over time, gives \(dE/d\Omega\), the time-independent angular differential distribution of energy. Accounting for unit bandwidth via \(\kappa \sim \omega\) by dividing \(dE/d\Omega\) by \(d\omega\), gives an identical result, \(\frac{d^2E}{d\omega d\Omega}\), as obtained from the step-function trajectory, Eq. (2).

Moreover, using the Lorentz-invariant proper acceleration in \(P = a^2/6\pi\), or equivalently integration of Eq. (5) over \(d\Omega\), we obtain the total power radiated,

\[
P = \frac{k^2 \gamma^6 \beta^2}{6\pi} \left(1 - \frac{\beta}{s}\right)^4.
\]  

(6)

The total radiated energy for the entire trajectory is readily obtained by integrating Eq. (6) over time, which again yields an identical result to the step-function case, given by Eq. (3) and associating \(\kappa \leftrightarrow 12E/\pi\). The fact that the more realistic smooth trajectory recapitulates the earlier results justifies the use of our choice of Eq. (4). However, our model has the advantage that we can examine the behavior of the accelerated charge over time.

5 Equilibrium Emission

Interestingly, a period of constant emission is present in the power measured by a far away observer. Best represented as the change of energy with respect to retarded time \(u = t - r\), and written as \(\dot{P} = \frac{dE}{du}\), such that

\[
E = \int_{-\infty}^{\infty} \dot{P}(u) \, du,
\]  

(7)

we write \(\dot{P} = P \frac{du}{du} = P/(1 - \beta)\). Formulating \(\dot{P}(u)\) in terms of retarded time, gives a lengthy result, but we plot the measure \(\dot{P}(u)\) at high final asymptotic speeds \(s \sim 1\) and reveal a constant power plateau indicative of thermal emission in line with \(P \sim T^2\) [31]. We plot the power as a function of time in Fig. 4. We note that the same trajectory, Eq. (4) applied to a moving mirror in a two-dimensional quantum field theory description explicitly yields a Planck spectrum in the high-speed limit, see Eq. (18).
Having computed the power, \( P = \alpha^2 / 6\pi \), we now turn to the magnitude of self-force or the well-known Lorentz–Abraham-Dirac expression, \( F = \alpha^2 / 6\pi \), where \( \tau \) is the proper time. For our trajectory, it is analytically tractable\(^3\), and a concise expression is given in terms of speed \( \beta \),

\[
F = \frac{k^2\gamma^6\beta^2}{6\pi} \left( 1 - \frac{\beta}{s} \right)^3 \left( 2\beta + \frac{1}{\beta} - \frac{3}{s} \right). \tag{8}
\]

The self-force is zero at the time the maximum power is given off. Integrating over distance gives the work done,

\[
W = \int_0^\infty F(r) \, dr = - \int_{-\infty}^\infty P(t) \, dt = -E. \tag{9}
\]

That is, taking Eq. (8) over \( d\beta \) using, \( d\beta / dr = k(1 - \beta / s)^2 \), where \( \beta \) ranges from \( (0, s) \), one obtains the energy associated with the self-force. The resulting work is \( W = -E \), the equal and opposite of Eq. (3). This demonstrates consistency between the radiation reaction and conservation of energy.

Furthermore, the power associated with the self-force \( \tilde{F}(u) = F \frac{dr}{du} = F\beta / (1 - \beta) \), or ‘Feynman power’ [33], if you will, expressed as a function of retarded time \( u \), has

\(^3\) Since the acceleration is non-zero for all \( t > -\infty \), cf. Fig. 2, the old puzzle of acausal pre-acceleration associated with radiation reaction in classical electrodynamics remains. Zhang [32] points out the possibility for an exact model like Eq. (8) with a classical-quantum correspondence.
an extended period of horizontal leveling present (see Fig. 4 for a similar illustration) when the electron asymptotically approaches light speed. Constant radiation reaction is in harmony with blackbody emission, previously in line with uniform peel $\tilde{k}(u)$, and the stable plateau of Larmor power $\tilde{P}(u)$.

7 Universality, Spectra and Temperature

The preceding results derived in the context of IB are the same\(^4\) for the scattering of Faddeev–Kulish electrons in QED where a cloud of soft photons exist in the dressed state [34]. Moreover, the same results hold true for the perfectly reflecting moving mirror [17] of the dynamical Casimir effect. In turn, the accelerated boundary correspondence between mirrors and black holes [10, 11], demonstrates trajectory Eq. (4) leads to an exact analog of black hole evaporation, possibly refined to include a remnant, e.g. [35], say as opposed to complete evaporation; e.g. [36, 37]. It therefore appears that the deep infrared is characterized by universal features that are manifested across several disparate investigations: IB, clouds, moving mirrors, and remnants.

In light of these universal features, it is interesting to compute the spectrum of a moving mirror in 1 + 1 dimensions with the same trajectory as the accelerating charged particle, e.g. [23]. In the presence of a mirror the mode functions that correspond to the in-vacuum state,

$$\phi_{\text{in}}^{\omega} = \frac{1}{\sqrt{4\pi\omega'}} [e^{-i\omega'v} - e^{-i\omega'p(u)}],$$

\hspace{1cm} (10)

and the mode functions that correspond to the out-vacuum state,

$$\phi_{\text{out}}^{\omega} = \frac{1}{\sqrt{4\pi\omega}} [e^{-i\omega f(v)} - e^{-i\omega u}],$$

\hspace{1cm} (11)

form the two sets of incoming and outgoing modes needed for the Bogolubov coefficients. It proves useful to write the trajectory of Eq. (4), in both spacetime coordinates,

$$r(t) = \frac{s}{\kappa} W(e^{\kappa t}), \quad t(r) = \frac{1}{\kappa} \ln \left( \frac{\kappa r}{s} \right) + \frac{r}{s},$$

\hspace{1cm} (12)

and null coordinates, $v = t + r$ and $u = t - r$, where the advanced time trajectory is,

$$p(u) = u + \frac{2s}{\kappa} \frac{W[(1 - s)e^{\kappa u}]}{1 - s},$$

\hspace{1cm} (13)

and the retarded time trajectory is,

\hspace{1cm} (14)

\[^4\] See [34, Eqs. 11, 14] for the photon count and the relevant expressions that match Eq. (3).
A tractable form of the beta integral is \[38\],

\[
\beta_{\omega\omega'} = \int_0^\infty dr \frac{e^{i\omega_p r - i\omega_{\prime} t(r)}}{4\pi \sqrt{\omega\omega'}} \left[ \omega_p - \omega_{\prime} t(r) \right],
\]

where \(\omega_p = \omega + \omega'\) and \(\omega_{\prime} = \omega - \omega'\). Combining the results for each side of the mirror [17] by adding the squares of the beta Bogolubov coefficients,\(^5\) the overall count per mode per mode is

\[
|\beta_{\omega\omega'}|^2 = \frac{2s^2 \omega\omega'}{\pi \kappa (\omega + \omega')} \frac{a^{-2} + b^{-2}}{e^{2\pi(\omega+\omega')/\kappa} - 1}.
\]

Here \(a = \omega(1+s) + \omega'(1-s)\), and \(b = \omega(1-s) + \omega'(1+s)\). A numerical integration of

\[
E = \int_0^\infty \int_0^\infty \omega |\beta_{\omega\omega'}|^2 d\omega d\omega',
\]

results in the total energy radiated, Eq. (3); \(\hbar = 1\). This integration is an example and explicit confirmation of the duality between accelerating mirrors and charges; e.g. [24–27]. Given this close association, see also [39–41], we note that the IB spectrum in beta decay is likely to be of a similar or related form as Eq. (16). We have plotted the spectrum of the moving mirror radiation in Fig. 5. If experiment confirms our

\(^5\) This accounts for the generalization from 1+1 dimensions to 3+1 dimensions of spacetime [28].

\[\text{Fig. 5} \quad \text{The} |\beta_{\omega\omega'}|^2 \text{ spectrum of Eq. (16). Here} \omega' = \kappa = 1 \text{ and} s = 1/2. \text{ The vertical axis has been scaled by} 10^5 \text{ for visual clarity. The qualitative black-body shape is indicative of the explicit Planck factor in Eq. (16).} \]
prediction then soft IB from beta decay is an analogue of the dynamical Casimir effect. We point out that accelerations experienced by an electron during the process of radiative neutron beta decay as measured by the RDK collaboration [42, 43] give precision tests of the shape of the photon energy spectrum over a broad range\(^6\) of photon energies.

The Planck factor in Eq. (16), is made more explicit by considering high final speeds, \(s \sim 1\) and the mode approximation \(\omega' \ll \omega\) which gives to leading order, the familiar form (see e.g. [44])

\[
|\beta_{\omega\omega'}|^2 = \frac{1}{2\pi \kappa \omega'} e^{2\pi \kappa / \omega'} - 1,
\]

(18)

demonstrating the particle spectrum,

\[
N(\omega) = \int |\beta_{\omega\omega'}|^2 d\omega',
\]

(19)
is Planck-distributed with a temperature,

\[
T = \frac{\kappa}{2\pi},
\]

(20)
in the ultra-relativistic regime and to \(\omega' \ll \omega\) leading order [12]. This is \(\mathcal{E}/T = \pi^2/6\); recalling \(\mathcal{E} \equiv \omega_{\text{max}} - \omega_{\text{min}}\) sets the scale of the system by the UV/IR range of the particular system of interest [13]. Thermal radiation, Eq. (20), as measured by the beta Bogolubov Planck-distribution, Eq. (18), associated with the spin-statistics of a boson heat bath consistently characterizes the extended period of constant power (Fig. 4) and force; tying together the analogy between quantum and classical measures of powers [28, 45] and self-forces [22, 46] connecting mirrors and electrons.

8 Conclusion

We have investigated the deep infrared radiation emitted by a rapidly accelerating classical point charge using a smooth trajectory that permits analytic results for relevant time-dependent quantities. We have derived the novel time-dependent power and angular distribution formulas. The soft self-force was computed, universality was highlighted across several distinct systems, and analog Bogolubov coefficient spectra were obtained, demonstrating consistency with the observed energy. The temperature of the light is found via a Planck distribution. The key result, from which the others flow, is an analytic continuous equation of motion for infrared acceleration radiation.

\(^6\) The range can be observed by two detector types: one composed of bismuth germanium oxide (BGO) scintillators and an avalanche photo diode (APD). A low energy spectrum, 0.3 keV to 20 keV, can be measured by the APD and a higher energy spectrum, 10 keV to 1000 keV, can be measured by the BGO.
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