The Effect of Number of Pole Pairs on Torque Ripple of Magnetic Gear

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Abstract—Field modulation magnetic gear is a transmission device with broad development prospects. It has the advantages of no friction, no pollution, low maintenance, and easy installation. Magnetic gear models with different gear ratios are established. The input and output torque waveforms of different models are compared. The influences of the number of pole pairs of the inner rotor \( P_1 \) and the number of pole pairs of the outer rotor \( P_2 \) on torque ripple are analyzed. According to the principle of magnetic field modulation, the torque ripple of magnetic gear is greatly affected by \( P_1 \) and \( P_2 \). Research results show that the torque ripple can be effectively reduced by selecting the magnetic gear with \( P_1 = 4 \), \( P_1/P_2 = 1/(n + 0.25) \) or \( 1/(n + 0.75) \) (\( n \) is a natural number).

1. INTRODUCTION

Magnetic gears have attracted the attention of many researchers due to its advantages of friction-free, oil-free, low maintenance, and easy installation [1–4]. The concept of magnetic gear was first put forward in 1913, but the traditional magnetic gear has a low torque density, which cannot meet the needs of industrial production. Until 2001, British scholars Atallah and Howe put forward a magnet-modulated magnetic gear [5], which makes full use of all permanent magnets on the inner and outer rotors, improves the utilization of permanent magnets, and makes up for the shortcomings of traditional magnetic gear in torque density at once. Magnetic gear with magnetic field regulation not only has high torque density, long service life, low maintenance cost, but also has broad application prospects in artificial heart [6], wind power [7], motor [8], electric vehicle [9], and other aspects.

Scholars have done a lot of research on magnetic gear. Pakdelian et al. studied a trans-rotating magnetic gear by two-dimensional and three-dimensional finite element method, analyzed piecewise spiral effect, permanent magnet diamagnetic, permanent magnet size, proportion, and flux density distribution [10]. Some studies investigated the vibration and noise characteristics of the field modulation magnetic gear by the electromagnetic forces in the field modulation magnetic gear [11]. Some scholars studied a Halbach array concentric magnetic gear prototype with a gear ratio of 4/17, calculated the magnetic field distribution of concentric magnetic gear by two-dimensional accurate analytical method, analyzed the relationship between the maximum static torque of magnetic gear and such parameters as the width and height of modulation magnet ring and the thickness of outer rotor yoke [12]. Kim et al. proposed a method to reduce torque pulsation by changing the shape of the modulation magnet ring, which is mainly based on the torque pulsation generated by different magnetic resistance of the modulation magnet ring when the inner and outer rotors are rotating [13]. Authors in [14] compared five kinds of magnetic gears with different permanent magnet arrangements. The first type is traditional table stick type; the second type is the spoke type; the third type is the simplified Halbach type; the fourth type is the commutator pole type; the fifth type is the commutator pole type Halbach type. The results show that the latter four types have higher torque density than the traditional...
type. Authors in [15] established three concentric permanent magnet gear finite element models with the same size and different gear ratios by transient finite element method and analyzed the starting characteristics under different gear ratios.

Authors in [16] proposed a novel topology of flux-focusing magnetic gear, which makes use of both axial and transverse magnetic fluxes, and performances are validated by a 3-D finite-element method. By reducing the saturation in soft magnetic materials and the flux leakage, the new topology has a significantly high torque density within a wide length-diameter ratio. In [17], the loss of inner and outer rotors of magnetic gear was calculated by using the finite element method, and a new rotor structure considering subharmonic was proposed to reduce the loss of the rotor. Authors in [18] adopted a stationary permanent-magnet (PM) ring between the inner and outer rotors, aiming at removing PMs and only retaining the salient-pole-iron on the outer rotor to improve its mechanical reliability. In [19], using the genetic algorithm optimization toolbox of Matlab, an exact analytical method was used to calculate the magnetic field distribution and electromagnetic torque. However, there are few papers on the output torque pulsation of magnetic gear, especially on the effect of the number of pole pairs of the inner and outer rotors on the torque pulsation of magnetic gear.

Based on the principle of magnetic field modulation, this paper describes the working principle of magnetic gear and introduces the relationship among the number of pole pairs of magnetic gear, the number of magnetic cores, and harmonic order. A number of magnetic gear models with the same electromagnetic size, different \( P_1 \) and \( P_2 \) are established and simulated. The inner rotor torque ripple \( T_{\text{ripple},1} \), outer rotor torque ripple \( T_{\text{ripple},2} \), inner rotor average torque \( T_{\text{avg},1} \), outer rotor average torque \( T_{\text{avg},2} \), and the ratio of torque ripple to the average torque were calculated. Combined with the calculated results and the input and output torque waveforms of the model, the influence of the number of pole pairs of the inner and outer rotors on the torque pulsation is given.

2. MECHANISM OF TORQUE RIPPLE

Field modulation magnetic gear is composed of inner and outer rotors, inner and outer rotor magnetic poles, modulation magnet ring, and air gap, as shown in Fig. 1.

![Diagram of magnetic gear](image)

**Figure 1.** Field modulated magnetic gear.

The relationship between the number of cores of modulation magnet ring and \( P_1, P_2 \) can be written as [20] \( n_s = P_1 + P_2 \) (1)

where \( n_s \) is the number of cores of the modulation magnet ring, \( P_1 \) the number of pole pairs of inner rotor permanent magnet poles, and \( P_2 \) the number of pole pairs of outer rotor permanent magnet poles.
The magnetic poles of the inner and outer rotors of the field modulated magnetic gear are modulated through the modulation magnet ring to generate first harmonic and harmonics in the air gap. The first harmonic transmit torque and the harmonics produce the torque ripple. The formula for calculating the air gap harmonic number of field modulation magnetic gear is shown as [20]

\[ P_{ikm} = kn_s + mp_i \]  \hspace{1cm} (2)

where \( i = 1, 2 \), \( k = 0, 1, 2, ..., \infty \). \( m = 1, 3, 5, ..., \infty \). When \( i = 1 \), \( P_1 \) is the pole number of pole pairs of the permanent magnet of the inner rotor, and \( P_{1km} \) is the number of the inner air gap magnetic density harmonic components. When \( i = 2 \), \( P_2 \) is the number of pole pairs of the outer rotor permanent magnet, and \( P_{2km} \) is the number of the outer air gap magnetic density harmonic components.

Through the inner and outer air gap harmonic components, the inner and outer rotor torque ripples can be calculated [20]

\[ T_{\text{ripple}_i} = \sum_{n=1}^{\infty} S_{n_i} \sin(nU_i(\omega_it - \theta_{i0})) \]  \hspace{1cm} (3)

where \( n \) is any natural number; when \( i = 1 \), \( T_{\text{ripple}_1} \) is the torque ripple of the inner rotor; \( S_{n_1} \) is the Fourier coefficient; \( U_1 \) is the minimum common multiple of the number of pole pairs of the inner rotor \( P_1 \) and the number of cores of modulation magnet ring \( n_s \); \( \omega_1 \) is the speed of inner rotor; \( \theta_{10} \) is the angle of the initial stage of the inner rotor. When \( i = 2 \), \( T_{\text{ripple}_2} \) is the torque ripple of the outer rotor; \( S_{n_2} \) is the Fourier coefficient; \( U_2 \) is the minimum common multiple of the number of pole pairs of the outer rotor \( P_2 \) and the number of cores of modulation magnet ring \( n_s \); \( \omega_2 \) is the speed of the outer rotor; \( \theta_{20} \) is the angle of the initial stage of the outer rotor.

According to Eq. (3), the minimum common multiple \( U_i \) of the number of pole pairs of the inner and outer rotors and the number of cores of modulation magnet ring affects the torque ripple of the magnetic gear. According to the Maxwell stress tensor method, torque pulsation is proportional to the integral of the product of the radial harmonic component and tangential harmonic component. The greater the minimum common multiple of the number of pole pairs of the inner and outer rotors and the number of cores of modulation magnet ring is, the smaller the torque ripple is.

3. MAGNETIC FIELD ANALYSIS

In this paper, Ansys Maxwell was used to establishing multiple magnetic gear models. Other electromagnetic dimensions of fixed field modulated magnetic gear was only changed by \( P_1 \) and \( P_2 \) to conduct finite element simulation. The size of the magnetic gear model is shown in Table 1.

| Parameter                              | Value |
|----------------------------------------|-------|
| Inside radius of inner rotor           | 20    |
| Outside radius of inner rotor          | 40    |
| Outside radius of inner permanent magnet | 45   |
| Outside radius of modulation magnet ring | 54  |
| Outside radius of outer permanent magnet | 60  |
| Outside radius of outer rotor          | 70    |
| Length of air gap                      | 1     |
| Axial length                           | 80    |

Because the ratio of pole pairs of inner and outer rotors is relatively small, the field modulated magnetic gear is not suitable for industrial production. In this paper, the ratio between the ratio of pole pair of inner and outer rotors is only selected to be between 1/3 and 1/10 to simulate and analyze the torque ripple of field modulated magnetic gear.
When $P_1 = 2$, field modulated magnetic gears with gear ratios of 2/10 and 2/11 are selected for analysis. Fig. 2 shows the distribution of the inner air gap magnetic density of the two magnetic gears. Figure 3 is the harmonic contrast diagram after Fourier decomposition of inner air gap density. Higher harmonics have less influence on torque ripple, so only the first three harmonics are selected for calculation and comparison. In the field modulated magnetic gear with a gear ratio of 2/10, the number of the air gap magnetic density harmonic components that affect the torque ripple of the inner rotor is $P_{113} = 18$, $P_{115} = 22$, and $P_{117} = 26$. In the field modulated magnetic gear with a gear ratio of 2/11, the number of air gap magnetic density harmonic components that affect the inner rotor torque ripple is $P_{113} = 19$, $P_{115} = 23$, and $P_{117} = 27$. It can be seen from Fig. 3 that the product of radial harmonic component and the tangential harmonic component of the field modulated magnetic gear
with a gear ratio of 2/10 is significantly larger than that of the magnetic gear with a gear ratio of 2/11. According to the Maxwell stress tensor method, the greater the product of radial and tangential harmonic components is, the greater the torque ripple is, and the smaller the torque ripple of the inner rotor with a gear ratio of 2/11 is. Meanwhile, magnetic gear \( U_1 = 12 \) with a gear ratio of 2/10; magnetic gear \( U_1 = 26 \) with a gear ratio of 2/11. It indicates that when \( U_1 \) is smaller, the torque ripple gradually decreases, which further indicates that the torque ripple of magnetic gear with a gear ratio of 2/11 is smaller.

Figure 4 shows the distribution of the outer air gap magnetic density of the two magnetic gears. Fig. 5 is the harmonic contrast diagram after Fourier decomposition of outer air gap density.

**Figure 4.** Flux density distribution of the outer air gap. (a) Radial magnetic density; (b) Tangential magnetic density.

**Figure 5.** Harmonic order of the outer air gap. (a) Radial magnetic density; (b) Tangential magnetic density.
Similarly, in the field modulated magnetic gear with a gear ratio of 2/10, the number of air gap magnetic density harmonic components affecting the torque ripple of the outer rotor is $P_{213} = 42$, $P_{223} = 54$, and $P_{215} = 62$. In a magnetic gear with a gear ratio of 2/11, the number of air gap magnetic density harmonic components affecting the torque ripple of the outer rotor is $P_{213} = 46$, $P_{223} = 59$, and $P_{215} = 68$. The harmonic component that affects the torque ripple of the outer rotor in Fig. 5 is calculated. The product of radial harmonic component and the tangential harmonic component of the field modulated magnetic gear with a gear ratio of 2/10 is greater than that of the magnetic gear with a gear ratio of 2/11. The conclusion is the same as that obtained in harmonic analysis of the inner rotor. Meanwhile, $U_2 = 60$ for magnetic gear with a gear ratio of 2/10 $U_2 = 143$ for magnetic gear with a gear ratio of 2/11. It further indicates that the outer rotor torque ripple of magnetic gear with a gear ratio of 2/11 is smaller.

According to the simulation results of magnetic gear with gear ratios of 2/10 and 2/11, when $P_1 = 2$, the magnetic gear with a gear ratio of fraction has a smaller torque ripple. Similarly, when $P_1 = 3$ and $P_1 = 4$, field modulated magnetic gears with gear ratios of 3/22, 3/15, 4/13, and 4/20 are selected for analysis, and the conclusions are the same as that when $P_1 = 2$.

Through the analysis of air gap density and harmonic components in magnetic gear, it is shown that the torque ripple can be smaller when the gear ratio of field modulated magnetic gear is a fractional ratio.

4. ANALYSIS OF TORQUE PULSATION

Draw multiple magnetic gear models with different $P_1$ and $P_2$. $P_1$ is an integer between 2 and 4, and $P_2$ is an integer between 8 and 20. Calculate the average torque of inner and outer rotors, the torque ripple and the proportion of torque ripple, respectively. Fig. 6 shows a three-dimensional diagram of average rotor torque in magnetic gears.

As can be seen from Fig. 6, the average torque of the inner rotor increases with the increase of $P_1$. As the number of pole pairs of outer rotor $P_2$ decreases, the average torque of the inner rotor increases continuously.

![Figure 6. The average torque of the inner rotor.](image1)

![Figure 7. The proportion of torque ripple of the inner rotor.](image2)

Figure 7 shows the proportion of inner rotor torque ripple. When $P_1 = 2$, the minimum common multiple of the number of pole pairs of the inner rotor and the number of modulation magnet ring cores is significantly smaller. It can be seen from Fig. 7 that the proportion of torque ripple of the inner rotor between 9.57% and 203.07%, and the torque ripple of the inner rotor is generally larger, including the fraction ratio of a magnetic gear with a gear ratio of $1/(n + 0.5)$.

When $P_1 = 3$, the minimum common multiple of the number of pole pairs of the inner rotor and the number of cores of modulation magnet ring is increased. The proportion of torque ripple of the
inner rotor is between 3.89% and 165.97%, where the gear ratio is $1/(n + 0.333...)$, and $1/(n + 0.666...)$ is a smaller fraction of the magnetic gear torque ripple, between 3.89% and 13.90%.

Figure 8 shows a three-dimensional diagram of the average torque of the outer rotor. As shown in Fig. 8, with the increase of $P_1$, the average torque of the outer rotor first increases and then decreases. With the increase of $P_2$, the average torque of the outer rotor increases.

Figure 9 shows that when $P_1 = 2$, the proportion of torque ripple of the outer rotor is significantly larger, ranging from 0.08% to 21.59%. When $P_1 = 3$ and the gear ratio is a fraction ratio, the torque ripple of the outer rotor occupies a relatively small proportion, ranging from 0.11% to 1.86%. When a gear ratio is an integral value, the torque ripple of the outer rotor accounts for a large proportion, ranging from 0.26% to 15.27%. When $P_1 = 4$ and the gear ratio is $1/(n + 0.25)$ and $1/(n + 0.75)$, the proportion of torque ripple of the outer rotor is the minimum, ranging from 0.08% to 0.52%. The gear ratio of $1/(n + 0.5)$ is the next, ranging from 0.14% to 4.59%. The gear ratio is the largest at an integer value, ranging from 0.48% to 11.37%. It is the same as the proportion of the torque ripple of the inner rotor.

One of the magnetic gear models $P_1 = 2$, $P_1 = 3$, and $P_1 = 4$ is selected to analyze the torque. A magnetic gear with a gear ratio of 2/17 is selected from the field modulated magnetic gear model with $P_1 = 2$. $T_{\text{ripple,1}} = 0.89$ Nm, $T_{\text{avg1}} = 9.31$ Nm, and the torque ripple of the inner rotor accounted for 9.57%. $T_{\text{ripple,2}} = 0.21$ Nm, $T_{\text{avg2}} = 79.17$ Nm, and the proportion of the torque ripple of the outer rotor is 0.27%. Because $P_1 = 2$ and $P_2$ is between 8 and 20, the magnetic gear $U_1$ with $P_1 = 2$ is generally small. However, as shown in Fig. 7, the proportion of rotor torque ripple in the field modulated magnetic gear $P_1 = 2$ is generally large, which is not conducive to the operation of the magnetic gear. Therefore, the magnetic gear $P_1 = 2$ is not selected.

A model with a gear ratio of 3/22 is selected from the field modulated magnetic gear model with $P_1 = 3$. $T_{\text{ripple,1}} = 0.43$ Nm, $T_{\text{avg1}} = 11.15$ Nm, and the proportion of torque ripple of the inner rotor is 3.89%. $T_{\text{ripple,2}} = 0.09$ Nm, $T_{\text{avg2}} = 81.85$ Nm, and the proportion of torque ripple of the outer rotor is 0.11%. The magnetic gear $U_1 = 75$ with a gear ratio of 3/22 has a larger minimum common multiple and a smaller torque ripple than the magnetic gear $P_1 = 2$.

Combined with Fig. 7 and Fig. 9, it can be seen that when $P_1 = 3$, the proportion of the torque ripple of the inner rotor decreases somewhat compared with that when $P_1 = 2$, but there is still room for further reduction.

A model with a gear ratio of 4/13 is selected from the field modulated magnetic gear model with $P_1 = 4$. $T_{\text{ripple,1}} = 0.221$ Nm, $T_{\text{avg1}} = 26.27$ Nm, and the proportion of torque ripple of the inner rotor is 0.84%. $T_{\text{ripple,2}} = 0.07$ Nm, $T_{\text{avg2}} = 85.33$ Nm, and the proportion of torque ripple of the outer rotor is 0.08%. $U_1 = 84$ of the magnetic gear with a gear ratio of 4/17 is significantly larger than that of the magnetic gear with a gear ratio of 3/22. Meanwhile, $U_2 = 357$ and the torque ripple of the outer rotor is significantly smaller than that of the magnetic gear with a gear ratio of $P_1 = 2$ and $P_1 = 3$. 

In the magnetic gear model with a gear ratio of 4/17, the proportion of torque ripple of the inner rotor decreases significantly, and the proportion of torque ripple of the outer rotor also decreases further. Based on the above figure and analysis, field modulated magnetic gear with $P_1 = 4$, the gear ratio of $1/(n + 0.25)$ and $1/(n + 0.75)$ is more conducive to stable operation, which is consistent with the conclusion of magnetic field analysis in the previous article.

5. CONCLUSION

Through the establishment of magnetic gear models with multiple pole pair of inner and outer rotors, the simulation analysis of multiple magnetic gear models with different $P_1$ is conducted, and the torque ripple, average torque, and the proportion of torque ripple to average torque are calculated. The following conclusions are drawn:

1. The proportion of torque ripple of field modulation magnetic gear with $P_1 = 2$ is generally larger than the others, which is not conducive to stable operation.
2. When the gear ratio is an integral value or $1/(n + 0.5)$, the proportion of torque ripple is significantly larger.
3. When the number of pole pairs of the inner and outer rotors is a fraction ratio other than $1/(n + 0.5)$, the proportion of torque ripple is generally small. When the gear ratio is $1/(n + 0.25)$ and $1/(n + 0.75)$, the proportion of torque ripple is the minimum.

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