A REFINEMENT OF THE SIMPLE CONNECTIVITY AT INFINITY OF GROUPS

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Abstract. We give another proof for a result of Brick ([2]) stating that the simple connectivity at infinity is a geometric property of finitely presented groups. This allows us to define the rate of vanishing of \( \pi_\infty^1 \) for those groups which are simply connected at infinity. Further we show that this rate is linear for cocompact lattices in nilpotent and semi-simple Lie groups, and in particular for fundamental groups of geometric 3-manifolds.

Keywords: Simple connectivity at infinity, quasi-isometry, colored Rips complex, Lie groups, geometric 3-manifolds.

MSC Subject: 20 F 32, 57 M 50.

1. Introduction

The first aim of this note is to prove the quasi-isometry invariance of the simple connectivity at infinity for groups, in contrast with the case of spaces. We recall that:

Definition 1. The metric spaces \((X,d_X)\) and \((Y,d_Y)\) are quasi-isometric if there are constants \(\lambda, C\) and maps \(f : X \to Y, g : Y \to X\) (called \((\lambda,C)\)-quasi-isometries) such that the following:
\[
d_Y(f(x_1), f(x_2)) \leq \lambda d_X(x_1, x_2) + C,
\]
\[
d_X(g(y_1), g(y_2)) \leq \lambda d_Y(y_1, y_2) + C
\]
\[
d_X(fg(x), x) \leq C,
\]
\[
d_Y(gf(y), y) \leq C,
\]
hold true for all \(x, x_1, x_2 \in X, y, y_1, y_2 \in Y\).

Definition 2. A connected, locally compact, topological space \(X\) with \(\pi_1 X = 0\) is simply connected at infinity (abbreviated s.c.i. and one writes also \(\pi_\infty^1 X = 0\)) if for each compact \(k \subseteq X\) there exists a larger compact \(k \subseteq K \subseteq X\) such that any closed loop in \(X - K\) is null homotopic in \(X - k\).

Remark 1. The simple connectivity at infinity is not a quasi-isometry invariant of spaces ([15]). In fact \((S^1 \times \mathbb{R}) \cup_{s^1 \times \mathbb{Z}} D^2\) and \((S^1 \times \mathbb{R}) \cup_{s^1 \times \{0\}} D^2\) are simply connected, quasi-isometric spaces although the first is simply connected at infinity while the second is not.

This notion extends to a group-theoretical framework as follows (see [3], p.216):
Definition 3. A group $G$ is simply connected at infinity if for some (equivalently any) finite complex $X$ such that $\pi_1 X = G$ one has $\pi_1^{\infty} \widetilde{X} = 0$, where $\widetilde{X}$ denotes the universal covering of $X$.

The independence on the particular complex $X$ is proved in [17] and [15]. Roughly speaking the simple connectivity at infinity depends only on the 2-skeleton and any finite 2-complex corresponds to a presentation of $G$. Since Tietze transformations act transitively on the set of group presentations it suffices to check the invariance under such moves. We refer to [17] for details.

All groups considered in the sequel will be finitely generated and a system of generators determines a word metric on the group. Although this depends on the chosen generating set the different word metrics are quasi-isometric. Therefore properties which are invariant under quasi-isometries are independent on the particular word metric and will be called geometric properties. It is well-known that being finitely presented, word hyperbolic or virtually free are geometric properties, while being virtually solvable or virtually torsion-free are not geometric (see [5]).

Our main result is:

Theorem 1 ([2]). The simple connectivity at infinity of groups is a geometric property.

This was originally proved by Brick in [2]. We provide a simpler and more conceptual proof, by analyzing the colored Rips complex.

Remark 2. It seems that the fundamental group at infinity, whenever it is well-defined (see [9] for an extensive discussion on this topic), should also be a quasi-isometry invariant of the group.

Definition 4. Let $X$ be a simply connected non-compact metric space with $\pi_1^{\infty} X = 0$. The rate of vanishing of $\pi_1^{\infty}$, denoted $V_X(r)$, is the infimal $N(r)$ with the property that any loop which sits outside the ball $B(N(r))$ of radius $N(r)$ bounds a 2-disk outside $B(r)$.

Remark 3. It is easy to see that $V_X$ can be an arbitrary large function.

It is customary to introduce the following equivalence relation on functions: $f \sim g$ if there exists constants $c_1, C_j$ (with $c_1, c_2 > 0$) such that

$$c_1 f(c_2 R) + c_3 \leq g(R) \leq C_1 f(C_2 R) + C_3.$$ 

It is an easy consequence of the proof of theorem 1 that the equivalence class of $V_X(r)$ is a quasi-isometry invariant. In particular $V_G = V_{\widetilde{X}_G}$ is a quasi-isometry invariant of the group $G$, where $\widetilde{X}_G$ is the universal covering space of a compact simplicial complex $X_G$, with $\pi_1(X_G) = G$ and $\pi_1^{\infty}(G) = 0$.

Remark 4. For most groups $G$ coming from geometry $V_G$ is trivial, i.e. linear. Obviously if $M$ has an Euclidean structure then $V_{\pi_1(M)}$ is linear. Since metric balls in the hyperbolic space are diffeomorphic to standard balls in $\mathbb{R}^n$ one derives that $V_{\pi_1(M)}$ is linear for any compact hyperbolic manifold $M$.

Remark 5. Notice that there exists (see [4]) word hyperbolic groups $G$ (necessary of dimension $n \geq 4$ by [4]) which are not simply connected at infinity and hence $V_G$ is not defined. Moreover if $G$ is a word hyperbolic torsion-free group with $\pi_1^{\infty}(G) = 0$ then it seems that $V_G$ is linear.
Theorem 2. \( V_G \) is linear for uniform lattices in:
1. semi-simple Lie groups.
2. nilpotent groups.
3. solvable stabilizers of horospheres in product of symmetric spaces of rank at least two, or generic horospheres in products of rank one symmetric spaces.

Interesting examples of groups for which \( \pi_1^\infty(G) = 0 \) are the (infinite) fundamental groups of geometric 3-manifolds (and conjecturally of all 3-manifolds). We can show that:

Corollary 1. The fundamental groups of geometric 3-manifolds have linear \( V_G \).

Remark 6. The existence of groups \( G \) acting freely and cocompactly on \( \mathbb{R}^n \), which have super-linear \( V_G \) seems most likely. The examples described in [11], section 4), which have large acyclicity radius, strongly support this claim. The first point is that the rate of vanishing of \( \pi_1^\infty \) is rather related to higher (i.e. dimension \( n-2 \)) connectivity radii, which are less understood. The second difficulty is that these groups are not s.c.i. The simplest way to overcome it is to consider group extensions. For instance \( \pi_1^\infty(G \times \mathbb{Z}^2) = 0 \), for any finitely presented group \( G \); alternatively \( \pi_1^\infty(V^n \times \mathbb{R}) = 0 \) for any contractible manifold \( V^n \) \((n \geq 2)\). However this idea does not work because \( V_{G \times \mathbb{Z}^2} \) is always linear.

Remark 7. One needs some extra arguments in order to extend the proof to all solvable Lie groups. However, it seems very likely that cocompact lattices in all connected Lie groups have linear rate of vanishing of \( \pi_1^\infty \).

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2. Proof of Theorem 1

For positive \( d \), set \( P_d(G) \) for the simplicial complex defined as follows:
- its vertices are the elements of \( G \),
- the elements \( x_1, \ldots, x_n \) of \( G \) span an \( n \)-simplex, if \( d(x_i, x_j) \leq d \) for all \( i, j \) (where \( d(.,.) \) is the word metric).

Lemma 1. If \( 2d > r \), then \( \pi_1(P_d(G)) = 0 \).

Proof. Let \( l = [1, \gamma_1, \gamma_2, \ldots, \gamma_n, 1] \) be a (simplicial) loop in \( P_d(G) \) based at the identity. Two successive vertices of \( l \) are at distance at most \( d \). One can interpolate between two consecutive \( \gamma_j \)'s a sequence of elements of \( G \) of length at most \( d \), consecutive ones being adjacent when viewed as elements of the Cayley graph (hence at distance one). The product of elements corresponding to these edges of length one of \( l \) is trivial in \( G \). Therefore it is a product of conjugates of relators: \( \prod g_i^{-1} R_i g_i \). The diameter of each \( R_i \) is at most \( r/2 \), and the assumption \( 2d > r \), implies that each loop \( g_i^{-1} R_i g_i \) is contractible in \( P_d(G) \). This ends the proof. \( \square \)
The natural group action of $G$ on itself by left translations gives rise to an action on $P_d(G)$. In particular, if $G$ has no torsion then it acts freely on $P_d(G)$ and $P_d(G)/G = X$ is a compact simplicial complex with $\pi_1(P_d(G)/G) = G$.

**Proposition 1.** The vanishing of $\pi_1^\infty$ is a geometric property of torsion-free groups.

**Proof.** One has to show that if the group $H$ is quasi-isometric to $G$ then $\pi_1^\infty P_d(G) = 0$ implies that $\pi_1^\infty P_a(H) = 0$ for large enough $a$.

Let $f : H \to G$ and $g : G \to H$ be $(k, C)$-quasi-isometries between $G$ and $H$.

Fix $x_0 \in P_a(H)$ and $f(x_0) \in P_d(G)$ as base points.

**Lemma 2.** If $\pi_1^\infty P_d(G) = 0$ then $\pi_1^\infty P_D(G) = 0$ for $D \geq d$.

**Proof.** An edge in $P_D(G)$ corresponds to a path (of length uniformly bounded by $\frac{D}{a} + 1$) in $P_d(G)$. Thus a loop $l$ in $P_D(G)$ at distance at least $R$ from a given point corresponds to a loop $l'$ in $P_d(G)$ at distance at least $R - \frac{D}{a} - 1$ from the same point. By assumption $l'$ will bound a 2-disk $D^2$ far away in $P_d(G)$.

Now the union of an edge $[x_1, x_n]$ in $P_D(G)$ and its corresponding path $[x_1, x_2, \ldots, x_n]$ in $P_d(G) \subset P_D(G)$ form the boundary of a 2-disk in $P_D(G)$, which is triangulated by using the triangles $[x_1, x_i, x_n]$. Consider one such triangulated 2-disk for each edge of $l$ and glue to the previously obtained $D^2$ to get a 2-disk in $P_D(G)$ bounding $l$ and far away.

By hypothesis for each $r$ there exists $N(r) > 0$ such that every loop $l$ in $P_d(G)$ satisfying $d(l, f(x_0)) > N(r)$ bounds a disk outside $B(f(x_0), r)$. This means that there exists a simplicial map $\varphi : D^2 \to P_d(G) - B(f(x_0), r)$ such that $\varphi(\partial D^2)$ is the given loop $l$, when $D^2$ is suitably triangulated. A loop $l = [x_1, x_2, \ldots, x_n, x_1]$ in $P_d(G)$, based at $x_1$, is the one-dimensional simplicial sub-complex with vertices $x_j$ and edges $[x_i, x_{i+1}]$, $i = 1, n$ (with the convention $n + 1 = 1$).

Set $M(R) = kN(kR + kC + 3C) + 3C$. We claim that:

**Lemma 3.** Any loop $l$ in $P_a(H)$ sitting outside the ball $B(x_0, M(R))$ bounds a 2-disk not intersecting $B(x_0, R)$.

**Proof.** Set $l = [x_1, \ldots, x_n]$. Using the previous lemma one can assume that $d$ is large enough such that $\frac{D - C}{k} > 1$. As in lemma 1 one can add extra vertices between the consecutive ones such that $d(x_i, x_{i+1}) \leq \varepsilon$ holds, where $\varepsilon + C = d$.

The image $f(l) = [f(x_1), \ldots, f(x_n), f(x_1)]$ of the loop $l$ has the property that $d(f(x_i), f(x_{i+1})) \leq k\varepsilon + C$. Using $d(x, gf(x)) \leq C$ one obtains that $d(gf(x), gf(y)) \geq d(x, y) - 2C$, which implies $d(x, y) \leq kd(f(x), f(y)) + 3C$ and thus:

$$d(f(x), f(y)) \geq \frac{d(x, y) - 3C}{k}, \text{ for all } x, y \in P_a(H).$$

From this inequality one derives that:

$$d(f(x_i), f(x_0)) \geq (M(R) - 3C)/k = N(kR + kC + 3C)$$

and thus the loop $f(l)$ sits outside the ball $B(f(x_0), N(kR + kC + 3C))$, and hence by assumption $f(l)$ bounds a disk which does not intersect $B(f(x_0), kR + kC + 3C)$.

Let $y_1, \ldots, y_t$ be the vertices of the simplicial complex $\varphi(D^2)$ bounded by the loop $f(l)$. The vertices $f(x_1), \ldots, f(x_n)$ are contained among the $y_i$’s. One can suppose that any triangle $[y_i, y_j, y_m]$ of $\varphi(D^2)$ has edge length at most $d$. Therefore we have:

$$d(g(y_j), x_i) \leq d(g(y_j), gf(x_i)) + C \leq kd(y_j, f(x_i)) + 2C \leq k^2 \varepsilon + (k + 2)C.$$
This proves that \( x_i, x_j, g(y_m) \) span a simplex of \( P_a(H) \) (for all \( i, j, m \)) whenever we choose \( a \) larger than \( k^2 \varepsilon + (k + 2)C \). Moreover:
\[
d(x_0, g(y_i)) \geq d(gf(x_0), g(y_i)) - C \geq \frac{d(f(x_0), y_i) - 3C}{k} - C \geq R.
\]

Further there is a simplicial map \( \psi: \varphi(D^2) \to P_a(H) \) which sends \( f(x_j) \) into \( x_j \) and all other vertices \( y_k \) into the corresponding \( g(y_k) \). It is immediate now that \( \psi \varphi(D^2) \) is a simplicial sub-complex bounded by \( l \), which has the required properties. □

This proves proposition 1. □

When \( G \) has torsion, one can construct a highly connected polyhedron with a free and cocompact \( G \)-action as follows (see [1]):

**Definition 5.** The colored Rips complex \( P(d, m, G) \) (for natural \( m \)) is the sub-complex of the \( m \)-fold join \( G \ast G \ast \ldots \ast G \) consisting of those simplexes whose vertices are at distance at most \( d \) in \( G \).

**Lemma 4.** For \( m \geq 3 \) and \( d \) large enough, \( G \) acts freely on the 2-skeleton of \( P(d, m, G) \) and \( \pi_1(P(d, m, G)) = 0 \).

**Proof.** Clearly \( G \) acts freely on the vertices of \( P_2(G) \), hence any non-trivial \( g \in G \) fixing a simplex has to permute its vertices. Adding \( m \geq 3 \) colors prevents therefore the action from having fixed simplexes of dimension less than 3. Now using the proof of lemma 1 one obtains also the simple connectivity. □

The theorem 1 follows now from the proof of proposition 1, suitably adapted to the 2-skeleton of \( P(d, m, G) \).

**Remark 9.** The same technique shows that the higher connectivity at infinity is also a quasi-isometry invariant of groups.

We have then:

**Corollary 2.** The equivalence class of \( V_X(r) \) is a quasi-isometry invariant of \( X \).

**Proof.** The result is implied by theorem 1 and lemma 3. □

### 3. Uniform lattices in Lie groups

**Proposition 2.** Uniform lattices in (non-compact) semi-simple Lie groups have linear rate of vanishing of \( \pi_1^{\infty} \).

**Proof.** We will denote below by \( d_X \) the distance function and by \( B_X \) the respective metric balls for the space \( X \).

Let \( K \) be the maximal compact subgroup of the simple Lie group \( G \) and \( G/K \) the associated symmetric space. It is well-known that the Killing metric on \( G/K \) is non-positively curved, and hence the metric balls are diffeomorphic to standard balls, by the Hadamard theorem.

If \( G \) is not \( SL(2, \mathbb{R}) \) then \( K \) is different from \( S^1 \) and therefore it has finite fundamental group. In particular the universal covering \( \tilde{K} \) is compact. The Iwasawa decomposition \( G = KAN \) yields a canonical diffeomorphism \( G \to \tilde{K} \times G/K \).

Furthermore we have a an induced canonical quasi-isometry \( \tilde{G} \to \tilde{K} \times G/K \). Large
implies that there are two constants \(a>0\) and \(b>0\) such that:

\[
\frac{1}{a} d_{H^2 \times R}(x,y) - b \leq d_{\widetilde{SL(2,R)}}(x,y) \leq ad_{H^2 \times R}(x,y) + b,
\]
for all \(x, y \in H^2 \times R\), holds true. In particular we have the following inclusions between the respective metric balls:

\[
B_{H^2 \times R}\left(\frac{r}{c}\right) \subset B_{\widetilde{SL(2,R)}}(r) \subset B_{H^2 \times R}(cr) \subset B_{\widetilde{SL(2,R)}}(c^2 r),
\]
for \(r\) large enough and \(c > 0\). The claim follows. \(\square\)

**Remark 10.** The same argument shows that the acyclicity radius for semisimple Lie groups is linear (see [11], section 4).

The way to prove the claim for nilpotent and solvable groups consists in the large scale comparison with some other metrics, whose balls are known to be diffeomorphic to standard balls. While locally the Riemannian geometry of a nilpotent Lie group is Euclidean, globally it is similar to the Carnot-Caratheodory non-isotropic geometry.

**Proposition 3.** If \(G\) is a torsion-free nilpotent group then \(V_G\) is linear.

**Proof.** It is known (see [13]) that \(G\) is a cocompact lattice in a real simply connected nilpotent Lie group \(G_R\), called the Malcev completion of \(G\). We have also a nice characterization of the metric balls in real, nilpotent Lie groups given by Karidi (see [12]), as follows. Since \(G_R\) is diffeomorphic to \(R^n\) it makes sense to talk about parallelepipeds with respect to the usual Euclidean structure on \(R^n\). Next, there exists some constant \(a > 0\) (depending on the group \(G_R\) and on the left invariant Riemannian structure chosen, but not on the radius \(r\)) such that the radius \(r\)-balls \(B_{G_R}(r)\) are sandwiched between two parallelepipeds, which are homothetic at ratio \(a\), so:

\[
P_r \subset B_{G_R}(r) \subset aP_r \subset B_{G_R}(ar), \text{ for any } r \geq 1.
\]
This implies that we can take \( V_G(r) \sim ar \), and hence \( V_G \) is linear.

**Remark 11.** Metric balls in solvable Lie groups are quasi-isometric with those of discrete solvgroups, and so they are highly concave (see [7]): there exist pairs of points at distance \( c \), sitting on the sphere of radius \( r \), which cannot be connected by a path within the ball of radius \( r \), unless its length is at least \( r^{0.9} \). Further it can be shown that there are arbitrarily large metric balls which are not simply connected. Nevertheless we will prove that metric balls contain large slices of hyperbolic balls.

**Proposition 4.** Cocompact lattices in solvable stabilizers of horospheres in product of symmetric spaces of rank at least two, or generic horospheres in products of rank one symmetric spaces have linear \( V_G \).

**Proof.** We give the proof for our favorite solvable group, namely the 3-dimensional group \( \text{Sol} \). It is well-known that \( \text{Sol} \) is isometric to a generic horosphere \( \mathcal{H} \) in the product \( H^2 \times H^2 \) of two hyperbolic planes. Generic means here that the horosphere is associated to a geodesic ray which is neither vertical nor horizontal. The argument in ([11] 3.D.), or its generalization from [6], shows that such horospheres \( \mathcal{H} \) are undistorted in the ambient space i.e. there exists \( a \geq 1 \), such that

\[
\frac{1}{a} d_{H^2 \times H^2}(x, y) \leq d_{\mathcal{H}}(x, y) \leq a d_{H^2 \times H^2}(x, y), \text{ for all } x, y \in \mathcal{H},
\]

holds true. Here \( d_{H^2 \times H^2} \) and \( d_{\mathcal{H}} \) denote the distance functions in \( H^2 \times H^2 \) and \( \mathcal{H} \), respectively. In particular we have the following inclusions between the respective metric balls:

\[
\mathcal{H} \cap B_{H^2 \times H^2}(r) \subset B_{\text{Sol}}(r) \subset \mathcal{H} \cap B_{H^2 \times H^2}(ar) \subset B_{\text{Sol}}(a^2 r).
\]

Since the horoballs in \( H^2 \times H^2 \) are convex it follows that the intersections \( \mathcal{H} \cap B_{H^2 \times H^2}(r) \) are diffeomorphic to standard balls. This proves that \( V_{\text{Sol}} \) is linear.

The linearity result extends without modifications to lattices in solvable stabilizers of generic horospheres in symmetric spaces of rank at least 2 (see [6]).

**Remark 12.** It is known that finitely presented solvable groups are either simply connected at infinity or are of a very special form, as described in [14]. On the other hand it is a classical result that any simply connected solvable Lie group is diffeomorphic to the Euclidean space. It would be interesting to know whether a simply connected solvable Lie group can be isometrically embedded as a horosphere in a symmetric space.

**Remark 13.** Notice that horospheres in hyperbolic spaces (and hence non-generic horospheres in products of hyperbolic spaces) have exponential distortion, namely \( d_{H}(x, y) \sim \log d_{H^{-1}}(x, y) \), for \( x, y \in \mathcal{H}^{n-1} \). This highly contrast with the higher rank and/or generic case.

This ends the proof of theorem 2.

**Remark 14.** One might notice a few similarities between \( V_G \) and the isodiametric function considered by Gersten (see [11]).

**Corollary 3.** The rate of vanishing of \( \pi_1^{\infty} \) is linear for the fundamental groups of geometric 3-manifolds.
Proof. There are eight geometries in the Thurston classification (see [16]): the sphere $S^3$, $S^2 \times \mathbb{R}$, the Euclidean $E^3$, the hyperbolic 3-space $H^3$, $H^2 \times \mathbb{R}$, $SL(2, \mathbb{R})$, $Nil$ and $Sol$. Manifolds covered by $S^3$ have finite fundamental groups. Further the compact manifolds without boundary covered by $S^2 \times \mathbb{R}$ are the two $S^2$ bundles over $S^1$, $RP^2 \times S^1$ or the connected sum $RP^3 \# RP^3$, and the claim can be checked easily. As we already observed, this is the also the case for the Euclidean and hyperbolic geometries. The same holds for the product $H^2 \times \mathbb{R}$, in which case metric balls are diffeomorphic to standard balls. The remaining cases are covered by theorem 2. □

References

[1] M.Bestvina and G.Mess, The boundary of negatively curved groups, J.A.M.S. 4(1991), 469-481.
[2] S.Brick, Quasi-isometries and ends of groups, J.Pure Appl. Algebra 86(1993), 23-33.
[3] S.Brick and M.Mihalik, The QSF property for groups and spaces, Math. Zeitschrift 220(1995), 207-217.
[4] M.Davis, Groups generated by reflections and aspherical manifolds not covered by Euclidean space, Ann. of Math. 117(1983), 293–324.
[5] A.Dyubina-Erschler, Instability of the virtual solvability and the property of being virtually torsion-free for quasi-isometric groups, I.M.R.N. 2000, no. 21, 1097–1101.
[6] C.Druţu, Nondistorsion des horosphères dans des immeubles euclidiens et dans des espaces symétriques, G.A.F.A. 7(1997), 712–754.
[7] L.Funar, Discrete cocompact solegroups and Poénaru’s condition, Archiv Math.(Basel) 72(1999), 81-85.
[8] E.Ghys and P.de la Harpe (Ed.), Sur les groupes hyperboliques d’après M. Gromov, Progress in Math., vol. 3, Birkhäuser, 1990.
[9] R.Geoghegan and M.Mihalik, The fundamental group at infinity, Topology 35(1996), 655-669.
[10] M.Gromov, Hyperbolic groups, Essays in Group Theory (S. Gersten Ed.), MSRI publications, no. 8, Springer-Verlag (1987).
[11] M.Gromov, Asymptotic invariants of infinite groups, Geometric group theory, Vol. 2 (Sussex, 1991), 1–295, London Math. Soc. Lecture Note Ser., 182, (G.A.Niblo and M.A.Roller, Ed.), Cambridge Univ. Press, Cambridge, 1993.
[12] R.Karidi, Geometry of balls in nilpotent Lie groups, Duke Math. J. 74(1994), 301–317.
[13] A.Malcev, On a class of homogeneous spaces, Izvestiya Akad. Nauk. SSSR. Ser. Mat. 13(1949), 9–32, Amer. Math. Soc. Translation 39(1951).
[14] M.Mihalik, Solvable groups that are simply connected at $\infty$, Math. Zeitschrift 195 (1987), 79–87.
[15] D.E.Otera, On the simple connectivity of groups, preprint 139(2001), Univ. Palermo, Bull.U.M.I. (to appear).
[16] P.Scott, The geometries of 3-manifolds, Bull.London Math. Soc., 15(1983), 401-487.
[17] C.Tanasi, Groups simply connected at infinity, (Italian), Rend. Ist. Mat. Univ. Trieste 31 (1999), 61-78.

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