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Relative Positioning Evaluation of a Tetrahedral Flight Formation’s Satellites

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Abstract. This paper presents a study about the tetrahedral layout of four satellites in a way that every half-orbital period this set groups together while flying in formation. The formation is calculated analyzing the problem from a geometrical perspective and disposed by precisely adjusting the orbital parameters of each satellite. The dynamic modelling considers the orbital motion equations. The results are analyzed, compared and discussed. A detection algorithm is used as flag to signal the regular tetrahedron’s exact moments of occurrence. To do so, the volume calculated during the simulation is compared to the real volume, based on the initial conditions of the exact moment of formation and respecting a tolerance. This tolerance value is established arbitrarily depending on the mission and the formation’s geometrical parameters. The simulations will run on a computational environment.

1. Introduction

Satellite formation flight is a set of spacecrafts acting cooperatively in a mission, for example NASA’s mission MMS [1]. However, to have cluster of satellites in formation flight is a precise and complex task, due to the number of factors that affect the orbital movement, such as disturbances, attitude, the satellites’ individual position etc.

This paper proposes the geometrical position of a four-satellite formation flight and the initial conditions needed to attain this arrangement. The satellites are positioned in a tetrahedral geometry formation in a way that it re-forms periodically without the need for orbital maneuvers or control action acting upon the movement prioritizing such formation. This arrangement occurs around the Earth and the dynamics of movement is relative to a geocentric frame of reference. The orbital elements of the four satellites are calculated and provided to the simulation’s computational environment. This computational environment is known as Spacecraft Trajectory Simulator (STRS) [2] [3] [4] [5] [6], it will be responsible for the orbital motion dynamics simulation. The body’s relative position during the formation of a regular polyhedron is analyzed by a detector proposed in order to indicate every regular tetrahedral geometry occurrence, within the tolerances admitted in an arbitrary manner and which vary according to the mission’s needs. This work is illustrated in Figure 1.
2. Mathematical Modelling

To form the tetrahedron’s base the satellites $S_1$, $S_2$, e $S_3$ are considered to have circular orbits around the Earth. For the tetrahedron to be regular, the orbit of the satellite $S_4$ has to be elliptical [7]. The orbital motion is described in a geocentric frame of reference, whereas the satellites’ relative movement is analyzed through reference frames centered on the satellites. Figure 2 presents a coordinate system with inertially defined axes positioned on the center of the Earth and a relative coordinate system positioned on the center of mass of all four satellites. The satellites $S_1$ and $S_2$ are in the same orbit and $S_3$ has inclination ($i_3$) from the orbital plane of $S_1$ and $S_2$. The forth satellite’s inclination from the orbital plane is $i_4 = i_3/3$ and because it’s an elliptical orbit, the orbit of satellite $S_4$ has low eccentricity, but the same orbital period as the other satellites, that is, $T_1 = T_2 = T_3 = T_4$.

To determinate the body’s mean anomalies, $S_1$ e $S_2$ were defined as being on the same circular orbit and the angle between them is the mean anomaly’s variation, according to the equation.

$$\Delta M = \left(\frac{\mu}{r^3}\right)^{\frac{1}{2}}(t_2 - t_1)$$  \hspace{1cm} (1)

Where $t_1$ and $t_2$ are the times relatives to $S_1$ and $S_2$ and $\mu$ the standard gravitational parameter. The satellite $S_3$ locates in the midpoint of the distance between $S_1$ and $S_2$ and its mean anomaly is calculated as following:
The orbit of \( S_4 \) is elliptical and it was defined that the tetrahedron would form at the apogee, that is, \( M_4 = 180^\circ \).

The tetrahedron volume varies according to the satellites’ motion. However, this volume is known as the regular tetrahedron forms. During the entire simulation, the calculated volume is compared to the nominal volume \( V_n \) of a regular tetrahedron. When both volumes are equal, the flag signals 1 showing that the tetrahedron has formed within an admissible tolerance. The algorithm responsible for this flag is shown on Figure 3.

![Diagram](image)

**Figure 3.** Regular tetrahedron formation detection algorithm based on volume.

### 3. Results
For the satellites in tetrahedral formation flight orbital motion simulation, the perturbative effects were not considered a priori, because the relevant fact is to evaluate the satellites’ geometrical arrangement during flight. Simulation data is presented in Table 1.
The apparent motion of the satellites is observable from the reference frame on the center of $S_1$. It is noted from that reference frame the movement of the other satellites, while $S_1$ remains stationary. It is observable that $S_2$ finds itself stationary relative to $S_1$, while $S_3$ and $S_4$ are not. That happens because $S_2$ is on the same orbit as $S_1$, while $S_3$ and $S_4$ are not and, besides, both possess orbital inclination relative to $S_1$ orbital plane. The period of the four bodies’ orbits is the same, demonstrating for a fact that there is occurrence of a tetrahedron (Figure 4).

Table 1. First simulation input parameters.

| Simulation Data – Simulation I | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|-------------------------------|------|------|------|------|
| $h$ (m) | 800000 | 800000 | 800000 | 800000 |
| $r$ (m) | 5000 | 5000 | 5000 | 5000 |
| $a$ (m) | 7178100 | 7178100 | 7178100 | 7178100 |
| $e$ | 0 | 0 | 0 | 0.0005687383463373053 |
| $i$ ($^\circ$) | 0 | 0 | 0.034562999271458 | 0.011520999757153 |
| $\Omega$ ($^\circ$) | 0 | 0 | 0 | 0 |
| $\omega$ ($^\circ$) | 0 | 0 | 0 | 270 |
| $M$ ($^\circ$) | 0 | 0.0399099558281 | 0.01995497791405 | 90.01995497791405 |

Simulation time = 6055 s (one nominal orbital period)

Tetrahedron’s volume = 14.73 km$^3$

Maximum error allowed relative to the nominal edge (5000 m) = 50 m

Maximum error allowed relative to the differences between positions = 100 m

Maximum error allowed relative to the nominal volume = 0.05 km$^3$

The detection flag (Figure 5) shows that the regular tetrahedron happens twice during orbital motion.
The regular tetrahedron formation’s periodicity during the orbital motion can be seen in Figure 6.

The movement of satellite $S_3$ is of deviation and elevation from $S_1$ orbital plane and at 1500 s, the segment $S_1S_3 = 5000$ m and the triangle formed by $S_1S_2S_3$ is formed. A few moments after this conjunction of satellites, their relative position degrades and the bodies reach that configuration again at 4500 s (Figure 7).
The composition of the ascension movement and the transversal velocity of $S_3$ satellite’s orbit, when observed from the reference frame of $S_1$ generates a Lissajous figure (Figure 8), due to the apparent motion.

![Figure 8. $S_3$ position’s variation relative to $S_1$, from a local perspective.]

4. Conclusion
This paper enables an analysis of the relative behavior between satellites in a geometrical tetrahedral flight formation’s satellites. The performed simulation shows that the precise positioning of the satellites is relevant for the tetrahedron’s formation and its periodical maintenance. The simulation demonstrates that the satellites group together at each half period of orbit, first on the apogee then on the perigee. Even though it is most of the time in a tetrahedral configuration during orbital motion, the regular tetrahedron only occurs twice. It is worth noting that the results obtained consider ideal situations, thus allowing the four bodies’ orbital parameters calculation.

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