Common Neighbourhood and Common Neighbourhood Domination in Fuzzy Graphs

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ABSTRACT

In this paper the concepts of common neighbourhood and common neighbourhood domination in fuzzy graph $G$ was introduced and investigated and denoted by $N_{cn}$ and $\gamma_{cn}$. We obtained many results related to $\gamma_{cn}(G)$ and $N_{cn}$. Finally we give the relationship of $\gamma_{cn}(G)$ with some other parameters in fuzzy graphs.

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1 INTRODUCTION

In the last 60 years, Graph theory has seen an explosive growth due to interaction with areas like computer science, electrical and communication engineering, Operations Research etc.

In (2011) A. Alwardi and N. D. Soner [1] introduced and studied common neighborhood dominating set \( CN - \text{domination} \), after two year A. Alwardi and N. D. Soner [2] introduced and investigated the concept of common neighborhood edge dominating set \( CN - \text{edge domination} \), all the graph considered here are finite and undirected with no loops and multiple edges. In (2017) P. Dunder, A. Aytac and E. Kilic [3] introduced and investigated the concept of common neighborhood \( CN - \text{neighbourhood} \) [3] after one year (Asma et. al.) introduced definition of common neighbourhood graph [4].

In (1973), Kaufmann [5] introduced definition of fuzzy graphs. Rosenfeld [6] introduced another elaborated definition including fuzzy vertex and fuzzy edges and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness, etc.

Perhaps the fastest growing area within graph and fuzzy graph is the study of domination, the reason being its many and varied applications in such fields as social sciences, communication networks, algorithm designs, computational complexity etc. There are several types of domination depending upon the nature of domination which motivated us to introduce the concepts common neighborhood \( CN - \text{neighbourhood} \) and the concept of common neighborhood dominating set also common neighborhood domination number \( CN - \text{domination number} \) \( \gamma_{cn} \) in fuzzy graph. The concept of domination in fuzzy graphs was investigated by Somasundaram and Somasundaram [7] and A. Somasundaram [8]. In this paper we introduce the concept of concepts common neighborhood \( CN - \text{neighbourhood} \) and the concept of common neighborhood dominating set and \( CN - \text{domination number} \) in fuzzy graphs using effective edges, we obtain some interesting results for this Parameter in fuzzy graphs.

2 PRELIMINARIES

In this section we review some basic definitions related to common neighbourhood and common neighbourhood domination of graphs, also basic definitions related to fuzzy graphs and domination in fuzzy graphs.

Let \( G \) be simple graph with vertex set \( V(G) = \{v_1, v_2, ..., v_n\} \).

For \( i \neq j \), the common neighborhood of the vertices \( v_i \) and \( v_j \), denoted by \( \Gamma(v_i, v_j) \), is the set of vertices, different from \( v_i \) and \( v_j \), which are adjacent to both \( v_i \) and \( v_j \). Let \( G = (V, E) \).

For any vertex \( u \in V \) the CN-neighborhood of \( u \) denoted by \( N_{cn}(u) \) is defined as \( N_{cn}(u) = \{v \in N(u) : |\Gamma(u, v)| \geq 1\} \). The cardinality of \( N_{cn}(u) \) is called the common neighbourhood degree \( CN - \text{degree} \) of \( u \) and denoted by \( deg_{cn}(u) \) in \( G \), and \( N_{cn}[u] = N_{cn}(u) \cup \{u\} \). The maximum and minimum common neighbourhood degree of a vertex in \( G \) are denoted respectively by \( \Delta_{cn}(G) \) and \( \delta_{cn}(G) \). That is \( \Delta_{cn}(G) = \max u \in V[N_{cn}(u)] \) and \( \delta_{cn}(G) = \min u \in V[N_{cn}(u)] \).

If \( u \) and \( v \) are any two adjacent vertices in \( V \) such that \( |\Gamma(u, v)| \geq 1 \), then we say \( u \) is common neighbourhood adjacent \( CN - \text{adjacent} to v \) or \( u \) is CN-dominate \( v \).

Let \( G = (V, E) \) be a graph and \( u \in V \) such that \( |\Gamma(u, v)| = 0 \) for all \( v \in N(u) \). Then \( u \) is in every common neighbourhood dominating set, such points are called common neighbourhood isolated vertices. Let \( I_{cn} \) denote the set of all common neighbourhood isolated vertices of \( G \). Hence \( I_{cn} \subseteq I_{min} \subseteq D \), where \( I_{min} \) is the set of isolated vertices and \( D \) is the minimum \( CN - \text{dominating set} \) of \( G \). A subset \( S \) of \( V \) is called a common neighbourhood independent set \( CN - \text{independent set} \), if for every \( u \in S ; v \notin N_{cn}(u) \) for all \( v \in S - \{u\} \). It is clear that every independent set is \( CN - \text{independent set} \). The \( CN - \text{independent set} \) \( S \) is called maximal if any vertex set properly containing \( S \) is not \( CN - \text{independent set} \).

The maximum cardinality of \( CN - \text{independent set} \) is called common neighbourhood independence number \( CN - \text{independence number} \) and denoted by \( \beta_{cn} \), and the lower \( CN - \text{independence number} \) \( i_{cn} \) is the minimum
cardinality of the $CN - \text{maximal independent set.}$

Let $G = (V, E)$ be a subset $S$ of $V$ is called Common
eighbourhood vertex covering $CN - \text{vertex covering}$
of $G$ if for any $CN - \text{edge} e = uv$
either $u \in S$ or $v \in S$. The minimum cordiality
of $CN - \text{vertex covering}$ of $G$ is called the
$CN - \text{covering number of} G$ and denoted by
$\alpha_{cn}(G)$. Let $G = (V, E)$ be a graph a subset $D$
of $V$ is called common neighbourhood dominating set
$CN - \text{dominating set}$ if for every vertex
$v \in V - D$, there exists a vertex $u \in D$ such that
$uv \in E(G)$ and $|\Gamma(u, v)| \geq 1$, where $|\Gamma(u, v)|$
is the number of common neighbourhood between
the vertices $u$ and $v$.

The common neighbourhood domination number $\gamma_{cn} CN - \text{domination number}$ is the
minimum cardinality of a common neighbourhood
dominating set of $G$.

A fuzzy graph $G = (V, \mu, \rho)$ is a non-empty set $V$
together with a pair of functions $\mu : V \rightarrow [0, 1]$
and $\rho : V \times V \rightarrow [0, 1]$ such that for all $x, y \in V$,
$\rho(x, y) \leq \mu(x) \wedge \mu(y)$. We call $\mu$ and $\rho$ the
fuzzy vertex set and the fuzzy edge set of $G$,
respectively. Let $G = (\mu, \rho)$ be fuzzy graph with
the underlying set $V$, the order of $G$ is defined as
$\sum_{v \in V} \mu(v)$, and is denoted by $p$. The size of $G$ is defined as $\sum_{(v, v' \in E) \in \rho} \mu(v, v')$ and is
denoted by $q$. The maximum degree of $G$ is
$\Delta(G) = \forall \{d(v) : v \in V\}$, and the minimum
degree of $G$ is $\delta(G) = \wedge \{d(v) : v \in V\}$. Let
$G = (\mu, \rho)$ be fuzzy graph and let $v \in V(G)$.
The edge between any vertices $u$ and $V$ in $G$ is
called effective edge if $(\rho(u, v) = \mu(u) \wedge \mu(v))$.
The vertex $v$ is adjacent to a vertex $u$, if they
reach between the effective edge. The effective
degree of vertex $v \in V(G)$ is defined as
d$e(v) = \sum_{u \neq v} \rho(u, v)$ and is denoted by $d(v)$.

Two vertices $v_1$ and $v_2$ are said to be neighbors in
a fuzzy graph $G$. Then $N(v) = \{u \in V : \rho(u, v) =
\mu(u) \wedge \mu(v)\}$ is called the open neighborhood
set of $v$ and $N[v] = N(v) \cup \{v\}$ is called the
closed neighborhood set of $v$. A fuzzy graph
$G = (\mu, \rho)$ is said to be strong fuzzy graph
if $\rho(u, v) = \mu(u) \wedge \mu(v)$ for all $(u, v) \in \rho^*$. A
complete fuzzy graph is a fuzzy graph $G = (\mu, \rho)$
such that $\rho(u, v) = \mu(u) \wedge \mu(v)$ for all $u$ and $v$.
A fuzzy graph $G = (\mu, \rho)$ is said to be bipartite
if the vertex set $V$ can be partitioned into two
nonempty sets $V_1$ and $V_2$ such that $\rho(u, v) = 0$
if $u, v \in V_1$ or $u, v \in V_2$. Further, if $\rho(u, v) =
\mu(u) \wedge \mu(v)$ for all $u \in V_1$ and $v \in V_2$ then
$G$ is called complete bipartite fuzzy graph and is
denoted by $K_{\mu_1, \mu_2}$ where $\mu_1$ and $\mu_2$ are,
respectively, the restrictions of $\mu$ to $V_1$ and $V_2$.

Let $G = (V, \mu, \rho)$ be a fuzzy graph. Then we call
fuzzy vertices $(u, \mu(u))$ and $(v, \mu(v))$ adjacent if
and only if $\rho(u, v) = \mu(u) \wedge \mu(v) > 0$.

In a fuzzy graph $G = (\mu, \rho)$ a fuzzy vertex and a
fuzzy edge are said to be incident if a fuzzy vertex
is the end vertex of a fuzzy edge and if they are
incident, then they are said to cover each other.
For any threshold $t$, $0 \leq t \leq 1$, $\mu_t = \{x \in V :
\mu(x) \geq t\}$ and $\rho_t = \{(x, y) \in V \times V :
\rho(x, y) \geq t\}$. Since
$\rho(x, y) \leq \mu(x) \wedge \mu(y), \forall x, y \in V$ we have
$\rho_t \subseteq \mu_t \times \mu_t$, so that $(\mu_t, \rho_t)$ is a graph with
the vertex set $\mu_t$ and edge set $\rho_t$ for all $t \in [0, 1]$. Let
$G = (\mu, \rho)$ be a fuzzy graph, if $0 \leq \alpha \leq \beta \leq 1$,
then $(\mu, \rho)$ is a subgraph of $(\mu_\alpha, \rho_\alpha)$. A path
$P$ in a fuzzy graph $G = (\mu, \rho)$ is a sequence of
distinct vertices $v_0, v_1, v_2, ..., v_n$ (except possibly
$v_0$ and $v_n$) such that $\mu(v_1) > \alpha, \rho(v_{i-1}, v_i) >
0, 0 \leq i \leq n$. Here $n \geq 1$ is called the length of
the path $P$. The consecutive pairs $(v_{i-1}, v_i)$ are
called the edges of the path.

Let $G = (\mu, \rho)$ be a fuzzy graph on $V$. Let
$u, v \in V$. We say that $u$ dominates $v$ in $G$ if
$\rho(u, v) = \mu(u) \wedge \mu(v)$. A subset $D$ of $V$ is called
a dominating set in $G$ if for every $v \in V - D$,
there exists $u \in D$ such that $u$ dominates $v$. The
minimum fuzzy cardinality of dominating sets in
$G$ is called the domination number and is
denoted by $\gamma(G)$. A dominating set $D$ of a fuzzy
graph $G$ is said to be a minimal dominating set
if no proper subset of $S$ is dominating set of
$G$. The maximum fuzzy cardinality of a minimal
dominating set is called the upper domination
number of $G$ and is denoted by $\Gamma(G)$.

3 THE COMMON NEIGHBOURHOOD IN FUZZY GRAPH

Definition 3.1. Let $G = (\mu, \rho)$ be fuzzy graph
with vertex set $V(G) = \{v_1, v_2, ..., v_n\}$. For $i \neq j$,
the common neighborhood of the vertices $v_i$ and
$v_j$, denoted by $\Gamma(v_i, v_j)$, is the set of vertices,
different from $v_i$ and $v_j$, which are adjacent to
both $v_i$ and $v_j$.
Definition 3.2. Let $G = (\mu, \rho)$ be a fuzzy graph for any vertex $u \in V$ the CN-degree of $u$ denoted by $N_{cn}(u)$ is defined as $N_{cn}(u) = \{v \in N(u) : |\Gamma(u,v)| > 0\}$.

Definition 3.3. The fuzzy cardinality of $N_{cn}(u)$ is called the common neighbourhood degree $CN$-degree of $u$ and denoted by $d_{cn}(u)$ in $G$, and $N_{cn}[u] = N_{cn}(u) \cup \{u\}$ is called the closed common neighbourhood degree $CN$-degree of $u$. The maximum and minimum common neighbourhood degree of a fuzzy graph $G$ are denoted respectively by $\Delta_{cn}(G)$ and $\delta_{cn}(G)$. That is $\Delta_{cn}(G) = \max \{d_{cn}(u); u \in |N_{cn}(u)|\}$ and $\delta_{cn}(G) = \min \{d_{cn}(u); u \in |N_{cn}(u)|\}$.

Example 3.4. Consider the fuzzy graph $G$ given in the Fig. 1.

![Fig 1.](image)

Then $N_{cn}(a) = \emptyset$, $N_{cn}(b) = \{c,d\}$, $N_{cn}(c) = \{b,d\}$, $N_{cn}(d) = \{b,c\}$, $\delta_{cn}(a) = 0$, $\delta_{cn}(b) = 1.3$, $\delta_{cn}(c) = 0.9$ and $\delta_{cn}(d) = 0.8$ the vertex $a$ is $CN$-isolated $\Delta_{cn}(G) = 1.3$, and $\delta_{cn}(G) = 0.8$

4 COMMON NEIGHBOURHOOD DOMINATION IN FUZZY GRAPHS

Definition 4.5. Let $G = (\mu, \rho)$ be a fuzzy graph and let $u$ and $v$ are any two adjacent vertices in $G$ such that $\rho(u,v) = \mu(u) \land \mu(v)$ and $|\Gamma(u,v)| > 0$, then we say $u$ is common neighbourhood adjacent to $v$ or $u$ is $CN$-dominate $v$.

Definition 4.6. Let $G = (\mu, \rho)$ be a fuzzy graph a subset $D$ of $V$ is called common neighbourhood dominating set $CN$-dominating if for every vertex $v \in V - D$ there exists a vertex $u \in D$, such that $\rho(u,v) = \mu(u) \land \mu(v)$ and $|\Gamma(u,v)| > 0$, where $\Gamma(u,v)$ is the number of common neighbourhood between the vertices $u$ and $v$, the common neighbourhood domination number $CN$-domination number is the minimum fuzzy cardinality taken over all minimal common neighbourhood dominating sets of $G$ and is denoted by $\gamma_{cn}(G)$ or $\gamma_{cn}$.

Definition 4.7. Let $G = (\mu, \rho)$ be a fuzzy graph a common neighbourhood dominating set $D$ is said to be minimal common neighbourhood dominating set if $D - \{u\}$ is not common neighbourhood dominating set of $G$ for all $v \in D$. A minimal common neighbourhood dominating set $D$ is called minimum common neighbourhood dominating set of $G$ if $|D| = \gamma_{cn}(G)$ and is denoted by $\gamma_{cn} - set$. 

4
Example 4.8. Consider the fuzzy graph $G$ given in the Fig. 1.

We have, $D_{c1} = \{a, b\}$, $D_{c2} = \{a, c\}$ and $D_{c3} = \{a, d\}$ are minimal CN-dominating sets. Then the minimum common neighbourhood number $\gamma_{cn} = \min\{|D_{c1}|, |D_{c2}|, |D_{c3}|\} = \min\{0.5, 0.9, 1\} = 0.5$.

Theorem 4.9. A common neighbourhood dominating set $D_{cn}$ of a fuzzy graph $G$, is minimal common neighbourhood dominating set if and only if one of the following condition holds:

(i). $N_{cn}(u) \cap D_{cn} = \emptyset$

(ii). There is a vertex $v \in V - D_{cn}$, such that $N_{cn}(v) \cap D_{cn} = \{u\}$.

Proof. Let $G$ be a fuzzy graph and let $D_{cn}$ be a minimal common neighbourhood dominating set. Then $D_{cn} - \{v\}$ is not common neighbourhood dominating set. Then there exists a vertex $v$ in $V - D_{cn} - \{v\}$ such that $u$ is not CN-dominated by any vertex of $D_{cn} - \{v\}; u \in V$ if $u = v$, then $N(u) \cap D_{cn} = \phi$ if $u \neq v$, then $N(v) \cap D_{cn} = \{u\}$.

Conversely, Suppose that $D_{cn}$ is CN-dominating set and for each vertex $u$ in $D_{cn}$, one of the two condition holds. Now, we want to prove that $D_{cn}$ is minimal. Suppose $D_{cn}$ is not minimal. Then there exists a vertex $v \in D_{cn}$ such that $D_{cn} - \{v\}$ is CN-dominating set. Thus, $u$ is CN-adjacent to at least one vertex in $D_{cn} - \{v\}$. Hence condition (i) does not hold, also if $D_{cn} - \{v\}$ is CN-dominating set, then every vertex in $V - D_{cn}$ is CN-adjacent to at least one vertex in $D_{cn} - \{v\}$. That means condition (ii) does not hold. So we get contradiction. Hence $D_{cn}$ is minimal common neighbourhood dominating set.

\[ \Box \]

Theorem 4.10. Let $G$ be a fuzzy graph with common neighbourhood isolated vertices if $D_{cn}$ is minimal common neighbourhood dominating set. Then $V - D_{cn}$ is CN-dominating set.

Proof. Let $D_{cn}$ be a minimal CN-dominating set of $G$. Suppose that $V - D_{cn}$ is not CN-dominating set. Then there exists a vertex $u$ in $D_{cn}$ such that $u$ is not CN-dominated by any vertex in $V - D_{cn}$. Then $u$ is CN-dominated by at least one vertex $v$ in $D_{cn} - \{u\}$. Thus $D_{cn} - \{u\}$ is common neighbourhood dominating set of $G$ which contradicts the common neighbourhood dominating set of $D_{cn}$. Then every vertex in $D_{cn}$ is CN-adjacent with at least one vertex in $V - \{D_{cn}\}$. Hence $V - \{D_{cn}\}$ is CN-dominating set.

\[ \Box \]

Theorem 4.11. For any fuzzy graph $G$,

$\gamma(G) \leq \gamma_{cn}(G)$

Proof. Since every CN-dominating set of a fuzzy graph $G$ is dominating set of $G$. Then $\gamma(G) \leq \gamma_{cn}(G)$.

\[ \Box \]

In the following we give $\gamma_{cn}$ for some standard fuzzy graphs.

Proposition 4.12. For any fuzzy graph $G$,

1. If $G = P_n$ is a path. Then $\gamma_{cn}(P_p) = p$.

2. If $G = K_n$ is an complete cycle fuzzy graph. Then $\gamma_{cn}(C_p) = p$.

3. If $G = K_{2,2}$ be a complete cycle fuzzy graph. Then $\gamma_{cn}(K_{2,2}) = \min\{\mu(v) : v \in V(K_p)\}$.

Theorem 4.13. For a complete bipartite fuzzy graph $K_{p1,p2}$ with $|V_1| = p_1$ and $|V_2| = p_2$,

$\gamma_{cn}(K_{p1,p2}) = p_1 + p_2 = p$.

Proof. Let $G$ be complete bipartite fuzzy graph; Then $\rho(v1v2) = 0$ and $\Gamma(v1v2) = 0$ for all $(v1v2) \in V_1 \times V_2$ and $\rho(u,v) = \mu(u) \wedge \mu(v), \forall u \in V_1 \times V_2$. Thus every vertex in $V_1$ has not common neighbourhood in $V_2$; also similarly every vertex in $V_2$. Hence $\gamma_{cn} = p_1 + p_2 = p$.

\[ \Box \]

Theorem 4.14. For any fuzzy graph.

$\gamma_{cn}(G) \leq p - \Delta_{cn}(G)$

Proof. Let $G = (\mu, \rho)$ be any fuzzy graph and let $v \in V(G)$, such that $d_{cn}(v) = \Delta_{cn}(G)$. Then there exists at least one vertex $u \in V - N_{cn}(v)$ such that $\rho(u, v) = \mu(u) \wedge \mu(v)$ and $|\Gamma(w, v)| > 0$. Thus $V - N_{cn}(v)$ is CN-dominating of $G$. Hence

\[ \gamma_{cn}(G) \leq |V - N_{cn}(v)| \]

\[ \gamma_{cn}(G) \leq p - \Delta_{cn}(G) \]

\[ \Box \]
Corollary 4.15. For any fuzzy graph.

$$\gamma_{cn}(G) \leq p - \delta_{cn}(G)$$

Proof. Since $$\delta_{cn} \leq \Delta_{cn}$$ and by the above theorem then $$\gamma_{cn}(G) \leq p - \delta_{cn}(G)$$.

Definition 4.16. Let G = (µ, ρ) be a fuzzy graph a subset D of V is called common neighbourhood independent set $$CN$$-independent if for every pair of vertices v, u ∈ D and u /∈ N_{cn}(v) and v /∈ N_{cn}(u). The maximum fuzzy cardinality taken over all $$CN$$-independent sets in a fuzzy graph G is called the $$CN$$-independent number of G and is denoted by $$\beta_{cn}(G)$$ or $$\beta_{cn}$$. 

Definition 4.17. Let G = (µ, ρ) be a fuzzy graph a vertex subset S of V is called common neighbourhood vertex covering set $$CN$$-vertex covering set of G, $$CN$$-edge e = uv such that $$\rho(u, v) = \mu(u) \land \mu(v)$$ either u ∈ S or v ∈ S. The minimum fuzzy cardinality taken over all $$CN$$-vertex covering sets in a fuzzy graph G is called the $$CN$$-vertex covering number of G and is denoted by $$\alpha_{cn}(G)$$ or $$\alpha_{cn}$$. 

Remark 4.18. If G a fuzzy graph has no $$CN$$-edge, Then $$\alpha_{cn}(G) = 0$$ 

Example 4.19. For the fuzzy graph G given in Fig. 2.

In Fig. (2), vertex subsets $$\{v_1, v_4, v_5\}$$, $$\{v_2, v_4, v_5\}$$, $$\{v_3, v_4, v_5\}$$ are $$CN$$-dominating sets. Then the minimum fuzzy cardinality of minimal $$CN$$-dominating sets is 0.8. Hence $$\gamma_{cn} = 0.8$$.

The $$CN$$-vertex covering set is $$\{v_1, v_3\}$$. Then $$\alpha_{cn} = 0.3$$.

The maximal $$CN$$-independent set is $$\{v_2, v_4, v_5\}$$, So $$\beta_{cn} = 1.1$$

Theorem 4.20. Let G be a fuzzy graph of order p, Then

$$\alpha_{cn}(G) + \beta_{cn}(G) = p$$
Proof. Let $S$ be $CN$-independent set in $G$ and $e = uv$ such that $\rho(u,v) = \mu(u) \land \mu(v)$ be any $CN$-edge. Then either $u$ or $v$ are in $V - S$. That is $V - S$ is common neighbourhood vertex cover of $G$.

Therefore, $|V - S| \geq \alpha_{cn}(G)$. Hence

$$p \geq \alpha_{cn} + \beta_{cn} \text{..............}(1)$$

Similarly, Let $S$ be $CN$-vertex covering set in $G$ and $e = uv$, such that $\rho(u,v) = \mu(u) \land \mu(v)$ be any be $CN$-edge. So one of the vertices $u$ or $v$ most belongs to $S$. Then $V - S$ in $G$ is common neighborhood independent.

Therefore, $|V - S| \leq \beta_{cn}$. Hence

$$p \leq \alpha_{cn} + \beta_{cn} \text{..............}(2)$$

From 1 and 2 we get

$$p = \alpha_{cn} + \beta_{cn}$$

\[\square\]

**Theorem 4.21.** For any fuzzy graph $G$,

$$\gamma_{cn} \leq \beta_{cn}$$

**Proof.** Let $G$ be a fuzzy graph, with $S$ is $CN$-independent set of $V$ such that $|S| = \beta_{cn}(G)$. Then every vertex $v \in V - S$ is $CN$-adjacent to at least one vertex of $S$.

Thus $S$ is $CN$-dominating set. Hence

$$\gamma_{cn} \leq \beta_{cn}$$

\[\square\]

**Remark 4.22.** Every $CN$-neighbourhood set in Fuzzy graph is $CN$-neighbourhood set in crisp graph

**Theorem 4.23.** Every $CN$-dominating set in fuzzy graph is $CN$-dominating set in crisp graph, but the converse is not true.

**Proof.** Let $G = (\mu, \rho)$ be a fuzzy graph, with $D$ is $CN$-dominating set and let $x \in D_{cn}$. Then there exists $y \in N_{cn}$ and $y \in N_{cn}(x) = \{y \in N(x); |\Gamma(x,y)| > 0\}$.

Therefore, $y \in CN$-neighbourhood set in $G$. By the above remark $y \in CN$-neighbourhood set in crisp $G^*$ so $y \in N_{cn} = \{y \in N(x); |\Gamma(x,y)| \geq 1 \text{ and } x \text{ dominates } y \text{ in } G^* \}$ and $x \in D_{cn}$ in $G$ in $G^*$. Thus $D_{cn}$ is a CN-dominating set in $G^*$.

In the following example, we show that the converse of the above theorem is not true.

**Example 4.24.** For the fuzzy graph $G$ given in Fig. 4.
The vertex subset $D_{cn} = \{v_1, v_3, v_4\}$ is $CN$-dominating of $G^*$, but it is not a $CN$-dominating set of $G$ and

$D_{cn} = \{v_1, v_2, v_3, v_4, v_5\}$ is $CN$-dominating set of $G$.

\[\Box\]

**Theorem 4.25.** Let $G$ be a fuzzy graph, with $CN$-dominating of $G$, then $\gamma_{cn}(G) \leq \gamma_{cn}(G^*)$. Furthermore, equality holds, if $|v| = 1, \forall v \in V(G)$.

**Proof.** Since $\gamma(G) \leq \gamma_{cn}(G)$ and $\gamma(G^*) \leq \gamma_{cn}(G^*)$ also $\gamma(G) \leq \gamma(G^*)$. Then

$\gamma(G) \leq \gamma(G^*) \leq \gamma_{cn}(G^*)$

Hence

$\gamma_{cn}(G) \leq \gamma_{cn}(G^*)$

\[\Box\]

**Theorem 4.26.** For any fuzzy graph,

$\gamma_{cn}(G) + \gamma_{cn}(\bar{G}) \leq 2p$

**Proof.** Since $\gamma_{cn}(G) \leq p$ and $\gamma_{cn}(\bar{G}) \leq p$. Then

$\gamma_{cn}(G) + \gamma_{cn}(\bar{G}) \leq 2p$

\[\Box\]

5 CONCLUSION

In this paper, the concepts of common neighbourhood and common neighbourhood domination was introduced and investigated in fuzzy graphs $G$. We obtained many results related to common neighbourhood domination number $\gamma_{cn}(G)$ and common neighbourhood $N_{cn}$ in fuzzy graph $G$ were discussed with the suitable example. In the last we gave the relationship of common neighbourhood domination number with some other parameters in fuzzy graphs and some suitable examples have given.

COMPETING INTERESTS

Authors have declared that no competing interests exist.
REFERENCES

[1] Alwardi A, Soner ND, KaramM Ebadi. On the common neighbourhood domination number. J. Comp. Math. Sci. 2011;2(3):547-556.

[2] Alwardi A, Soner ND. CN-edge domination in graphs. Vladikavkaz. Mat. Zh. 2013;15(2):1117.

[3] Dundar P, Aytac A, Kilic E. Common-neighbourhood of a graph. Bol. Soc. Paran. Mat. 2017;35(1):2332.

[4] Hameh A, Ali Irnmanesh, (et al). On common neighborhood graphs. Iranian J. Math. Chem. 2018;9(1):37-46.

[5] Kaufman A. Introduction a la theorie des sous-ensembles flous, Paris. Masson etcie Editeurs; 1973.

[6] Rosenfeld A. Fuzzy graphs. In. Zadeh LA, Fu KS, Shimura M, (Eds). Fuzzy sets and their applications. Academic Press. 1975;77-95.

[7] Somasundaram A, Somasundaram S. Domination in fuzzy graphs-I. Pattern Recognition Letters. 1998;19:787-791.

[8] Somasundaram A. Domination in fuzzy graphs-II. Journal of Fuzzy Mathematics. 2000;20:281-289.