INVENTORY REPLENISHMENT POLICIES FOR TWO SUCCESSIVE GENERATIONS PRICE-SENSITIVE TECHNOLOGY PRODUCTS

Gaurav Nagpal*
Department of Management Studies,
BITS Pilani, Pilani Campus,
Rajasthan, India

Udayan Chanda and Nitant Upasani
Department of Management Studies and Department of Mathematics
BITS Pilani, Pilani Campus,
Rajasthan, India

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Abstract. The high technology products come in generations, where the demand for newer technology generations is strongly influenced by the installed base of earlier generations (such as computers, cameras, notebooks, etc). However, the effect of technology substitution on inventory replenishment policies has received little attention in the supply chain literature. In the hi-technology market, consumers’ purchasing capability, the utility of a product along with the entry of the advanced generation product influence the market expansion/contraction of the products. In this study, the impact of parallel diffusion of two successive generations’ products on inventory policies of the monopolist has been analysed. The demand models have been characterised by considering the life-cycle dynamics for a P-type inventory system. The purpose of this paper is to develop a model for joint pricing and replenishment of technology generation products. The model has been solved by using a genetic algorithm technique. The impact of yearly price drop and the price sensitivity of demand on the profit margins vis-à-vis on replenishment policies has also been studied. The paper also brings forward the dynamics of the launch of newer generations and the pricing strategies on optimal inventory replenishment policies. Numerical illustrations have also been covered in the paper.

1. Introduction. In this world of ever-increasing digitalization, technological upgradation is no more an option, but has become an indispensable reality. Therefore, technological gadgets play an important role in our daily life. Furthermore, the impact of these kinds of products on the economy of a nation can not be ignored. Hence, it becomes all the more important to achieve the operational efficiencies in the supply chain for such technology products. The traditional economic ordered quantity (EOQ) models are based on the assumption that the demand rate of a

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* Corresponding author: Gaurav Nagpal.
product remains constant all through the planning horizon. However, for technology products, the product life cycle is very short with fast-changing consumer preferences and evolving product features. The demand for such kind of products not only varies with time but can also characterize by adoption-substitution between two competing technology generations of products. The classical multi-generation adoption model by Norton and Bass [25] suggested that the demand curve of new technology generation follows the S-shaped growth pattern. Nunez-Lopez [24] coupled the launch and the diffusion of competing technology products with the supply of financial resources using a biological analogy and taking example from television sets industry and renewable energy technologies industry.

The multi-generation products are a subset of substitutable products that are advanced versions of the same platform-based product and intended for the same function and scope of work. A new technology product often cannibalizes the market of the older generation product due to its better competence level. Also, the rapid technological advancement reduces the time-gap between the entries of two technology generations in the market significantly. As a result, several products of the technological generation can co-exist in the consumer marketplace at least for a short period, and the price of the older generations product declines over time. In a hi-technology market, especially for multiple generations of products, sellers use the price as a strategic tool to influence the sales of any particular generation of the product. Because of the cross-elasticity of demand, the decline in the price of one generation can lead to a decline in the sales of another generation product, which in turn, influences the inventory costs.

Due to shorter product life cycles and faster obsolescence of technology, the players in the technology domains experience faster price erosion. If they make the right pricing decisions, it can be their key to growth and profitability. The importance of pricing in the case of technology products is higher because of the need to cover up the research and innovation development expenses. A lot of caution is supposed to be exercised in the pricing of the product since that affects the demand for the product. Keeping this into consideration, this study has used an extended form of Norton and Bass Model [25] using the selling price of the product as one of the decision variables to develop the demand model for a two-successive generation technology product.

In general, the inventory replenishment policies in an organization are neither very long term in nature, not too short term. Generally, the norms related to replenishment cycle length are fixed for a certain period of intermediate length and then reviewed again at the end of the period to fix them at another appropriate level for another fixed period, referred to as the planning horizon here. The conventional multi-period inventory model is based upon the assumption that the demand does not vary with time. Since the demand for technology products is not constant as explained above, it makes sense to change the conventional p-model for the technology generations. The objective of this work is to develop a model that maximizes the total profit in different planning horizons for the successive generations of technology products under the demand that is governed by innovation diffusion and is also price-dependent.

This paper considers the supply chain of the technology generation products where the manufacturer replenishes the supplies to the stockist. The paper looks
at optimisation of the profit of the stockist. The profit of the stockist would be the selling revenues net of the basic purchase costs, and inventory related costs.

In the next section, the existing literature has been discussed, and a tabular summary of research papers on multi-item inventory modeling for price-dependent demand has been mentioned. Section 3 illustrates the assumptions, notations and a brief explanation of the price model and the demand model being used in this research. Section 4 lays down the equations for the costs, contribution margin and the profit function; and also puts forward some of the important theorems or generalizations that can be made and provides a justification for them. Section 5 performs the numerical illustration as a proof of concept for the model developed and shows the behavior of optimal costs and optimal replenishment cycle length over the product life cycle. Section 6 discusses the implications of this work for inventory and pricing policies. Section 7 draws insights for the managers and practitioners. Finally, Section 8 pens down the broad conclusions of this research, along with the future extensions and directions of the research.

2. Literature review. It is a fact that the price of a product has a predominant influence over its demand. Some of the pioneering works in this regard have been by Simon [30], Parker [28], and Tam and Hui [31]. There also exist a good number of research studies that have been done on inventory optimization for price-dependent demand. While some research studies have taken demand as a linear function of price, others have taken it as a polynomial function or as an exponential function of price. Nascimento and Vanhonacker [22] concluded that in the absence of protection, the skimming price strategies prove to be the most optimal ones. Kar, et al. [12] formulated the inventory model for multiple items with stochastic price-dependent demand where the objective function and constraints lack exactness. Jain et al. [10] explored the inventory optimization for the multiple items under price breaks and multiple set-ups for procurement and recovery. Tsao and Sheen [35] studied the inventory model for multiple items under a discount on freight costs. Chakraborty et al. [2] developed an inventory optimization framework for deteriorating multiple items with discounted pricing and fuzzy demand. Yang et al. [37] developed a model to determine the optimal special order quantity and retail price to maximize profit and established an algorithm to find the optimal solution. Tayal et al. [34] incorporated the time to expiry and also allowable shortages to formulate the inventory model for multiple items. Talebian et al. [32] illustrated how pricing can be used as a tool for demand learning during the assortment planning of perishable products. Mousavi et al. [18] formulated the multi-period inventory model for multiple items with price discounting. Paul et al. [29] studied how the multiple imperfect items under price discounts can be replenished jointly. Ghoreishiz et al. [7] developed the EOQ Model under inflation induced and price dependent demand while allowing the customer returns and gradual deterioration. Giri et al. [8] worked on pricing decisions for substitutable and complementary products. Fang [5] also worked on joint pricing and replenishment for substitutable products with uncertain demand rates. Wei and Zhao [36] considered manufacturing cost and customer demand as fuzzy variables to explore the effect of pricing strategies in the case of duopolistic manufacturers.
Table 1: Tabular review of existing literature on multi-item inventory modeling under price-dependent demand

| Research Study               | Multiple items inventory | Price dependent demand | Demand Substitution | Joint replenishment | Innovation Diffusion demand | Multi-generation |
|------------------------------|--------------------------|------------------------|---------------------|---------------------|-----------------------------|------------------|
| Chakraborty et al., 2013 [2] | ✓                        | ✓                      | ✓                   | X                   | X                           | X                |
| Duran and Luis, 2013 [4]     | ✓                        | ✓                      | ✓                   | X                   | X                           | X                |
| Otori et al., 2016 [26]      | ✓                        | ✓                      | ✓                   | ✓                   | X                           | X                |
| Feng, 2016 [5]               | ✓                        | ✓                      | ✓                   | ✓                   | X                           | X                |
| Girli et al., 2016 [8]       | ✓                        | ✓                      | ✓                   | X                   | X                           | X                |
| Jain et al., 2012 [10]       | ✓                        | ✓                      | ✓                   | X                   | X                           | X                |
| Kar et al., 2001 [12]        | ✓                        | ✓                      | ✓                   | X                   | X                           | X                |
| Kuo and Huang, 2012 [14]     | ✓                        | ✓                      | ✓                   | X                   | ✓                           | ✓                |
| Lee and Lee, 2018 [15]       | ✓                        | ✓                      | ✓                   | X                   | X                           | X                |
| Liu et al., 2015 [16]        | ✓                        | ✓                      | ✓                   | ✓                   | X                           | X                |
| Malanoodi, 2016 [17]         | ✓                        | ✓                      | ✓                   | ✓                   | X                           | X                |
| Mounavi et al., 2014 [18]    | ✓                        | ✓                      | X                   | X                   | X                           | X                |
| Neda et al., 2016 [23]       | ✓                        | ✓                      | ✓                   | ✓                   | X                           | X                |
| Panda and Maiti, 2009 [27]   | ✓                        | ✓                      | X                   | X                   | X                           | X                |
| Paul et al., 2014 [29]       | ✓                        | ✓                      | ✓                   | ✓                   | X                           | X                |
| Talebian et al., 2013 [32]   | ✓                        | ✓                      | ✓                   | X                   | X                           | X                |
| Tayal et al., 2014 [34]      | ✓                        | ✓                      | X                   | X                   | X                           | X                |
| Tsao and Sheen, 2012 [35]    | ✓                        | ✓                      | X                   | X                   | X                           | X                |

Table 1: Tabular review of existing literature on multi-item inventory modeling under price-dependent demand

Feng et al. [6] suggested that the joint dynamic pricing in case of multiple substitutable products results in better pay-off as compared to static pricing. Jana and Das [11] studied the inventory model for multiple items in a two-warehouse system under nested discount on the costs. Taleizadeh et al. [33] proposed the inventory model for multiple serviceable items under contrasting inventory replenishment norms. Giri et al. [9] showed that pure bundling of complementary products offers higher benefits than when the products are sold at individual prices, but was silent on the pricing dynamics of substitutable products.

The work that has considered the influence of price on the inventories of multi-generational products is very rare. Kuo and Huang [14] formulated the dynamic pricing models for limited inventories of multi-generational products. Chanda and Aggarwal [3] discussed optimal replenishment policies for two-generation technology products. Nagpal and Chanda [20] developed the inventory optimization model for technology generations under trade credits but did not consider the impact of the selling price. Nagpal and Chanda [21] also pointed out the lack of inventory models on the innovation diffusion governed demand in the review of the literature on inventory modeling for multi-generational products. Table 1 summarizes the work of the research studies that have been done on the inventory optimization for the price-dependent demand.

Most of the studies cited above have considered conventional products only. Also, as shown in Table 1, there does not exist much work on inventory modeling for multiple generation products and in fact, no EOQ model for the multi-generation model is available that uses price as one of the decision variables. Nagarajan and Rajagopalan [19] suggested that the pattern of adoption-substitution can play an important role in inventory control for technology generations product. Kreng and Wang (2013) argued that most of the classical multi-generation models are of little
use for policy decisions; as they do not consider explicitly the effect of marketing
variables on the demand dynamics. The above discussion gave us enough impetus to
model the p-type replenishment cycles for substitutable technology products by con-
sidering a price dependent multi-generation adoption-substitution demand model.
The proposed framework has been characterized to give marketing-operational in-
sights to optimize supply chain efficiency.

3. Modeling framework. The purpose of this model is to help the supply chain
managers and practitioners of technology products in making sound business de-
cisions related to pricing and replenishment. We have considered here the part of
the supply chain that spans from manufacturer to stockist. The stockist plays a
very important role in making the product enter the consumer markets. It is very
important for the stockist to maximize his/her profit through operational excellence
so that there can be a sustainable business for the stockist, and the supplies can
continue to the end consumer. Going forward, a detailed discussion on the develop-
ment of the two-generation demand model is presented, which will be further used
for inventory modeling and cost modeling in the upcoming sections.

The assumptions and notations used in this research while developing the model
have been outlined in Sections 3.1 and Section 3.2 respectively. Section 3.3 discusses
in detail, the demand model for the two generations of technology products, which
will be further utilized to model the inventory and costs in Section 4.

3.1. Assumptions.

• The demand of the technology product is governed by the product lifecycle
dynamics, demand interplay with the substitutes as well as the selling price.
• The supply gets replenished instantaneously
• Shortages are not allowed
• The selling price of the product is dependent upon the stage of the life cycle.
• Logistics for both the generations of the product may be pooled or un-pooled,
depending upon the nature of ordering costs
• The fixed ordering cost can be broken into two components—one being product-
specific, and the other being product-non-specific.
• The number of replenishment cycles, and thus, the length of each replenish-
ment cycle is fixed for each planning horizon at the beginning of that period.

3.2. Notations. O fixed cost of ordering per order for any shipment exclusive of
the product-specific costs (j = 1,2)
O_j fixed product-specific ordering cost per order for j^{th} generation product (j =
1,2)
C_j inventory holding expense (as a % of basic purchase cost) for j^{th} generation
product (j = 1,2)
M_j the size of market potential for j^{th} generation product (j = 1,2)
\lambda_1(t) and \lambda'_1(t) demand rate at time t of 1st generation product for t \leq \tau
and t \geq \tau respectively
\lambda_2(t) demand rate at time t of 2nd generation product
p_j coefficient of innovation for j^{th} generation product (j = 1,2)
q_j coefficient of imitation for j^{th} generation product (j = 1,2)
\tau be the time instant at which the second-generation product is introduced in the
market
\phi(t) is the conditional probability of a prospective adopter (who has not yet adopted
the product till time $t$) adopting the product in the time interval $(t, t + \delta t)$

$F_j(t)$ is the cumulative fraction of adopters that have adopted the $j^{th}$ generation product until time $t$.

$f_j(t)$ is the fraction of adopters who adopt the $j^{th}$ generation product at time $t$.

$x_j(t)$ is the influence of price changes of the $j^{th}$ generation product at time $t$.

$X_j(t)$ is the cumulative influence of price changes of the $j^{th}$ generation product till time $t$.

$\Upsilon$ is a factor proportional to the decline in selling price per unit time due to technology adoption.

$P_{tj}$ is the selling price of $j^{th}$ generation product at the time instant $t$.

$P_{oj}$ is the selling price of $j^{th}$ generation product at the time of launch.

$\rho$ is the length of the planning time horizon for which the inter-replenishment time interval is fixed.

$n$ is the number of inventory replenishment cycles in the $m^{th}$ planning horizon in a single generation scenario.

$HC$ is the inventory holding cost in any planning horizon.

$OC$ is the inventory ordering cost in any planning horizon.

$m$ and $m'$ denote the sequence of the planning horizon before and after the launch of second generation respectively.

$TR_m$ is the total revenue in the $m^{th}$ planning horizon in the single generation scenario.

$TCM_m$ is the total contribution margin, i.e. Revenue, net off material cost for the goods sold in the $m^{th}$ planning horizon in the single generation scenario.

$TC(m, n)$ is the total cost of ordering and holding in $m^{th}$ planning horizon with $n$ replenishment cycles in the single generation scenario.

$TP(m, n)$ is the total profit incurred from the goods sold in the $m^{th}$ planning horizon with $n$ replenishment cycles in the single generation scenario.

$TR'_m$ is the total revenue in the $m^{th}$ planning horizon from the first generation product.

$TR'_{m2}$ is the total revenue in the $m^{th}$ planning horizon from the second generation product.

$n_1$ and $n_2$ are the number of replenishment cycles for the first generation and second generation product respectively in the $m''$ planning horizon post the launching of the next generation product.

$y_m' = 0$, if first first generation product is discontinued by the $m''$ planning horizon; or 1 if it is still in business.

$TCM_{(m', 0)}$ is the total contribution margin, i.e. revenue, net off material cost for the goods sold in the $m''$ planning horizon in the single generation scenario.

$TC'_{(m', n_1, n_2)}$ is the total cost of ordering and holding in $m''$ planning horizon with $n_1$ and $n_2$ replenishment cycles of the first and the second generation product respectively.

$TP_{(m', n_1, n_2)}$ is the total profit incurred from the goods sold in the $m''$ planning horizon with $n_1$ and $n_2$ replenishment cycles of the first and the second generation product respectively.

$\beta$ is the influence of the change in price on the adoption rate, and is analogous to price elasticity of demand.

The key decision variables in this model are the replenishment frequency in each planning horizon, and the selling price of the product at any time, i.e. the rate of
Figure 1. The behavior of the price of the technology products with the time elapsed after the launch for different values of annual % price drop “Υ”

3.3. Price model. The demand is dependent upon price as well as the innovation diffusion dependent. Therefore, the basic demand model needs to be appended to incorporate the influence of the selling price. Also, there is a need to model the selling price. In the case of innovative products, the firms follow the price skimming strategy in the introduction stages since the huge research and development costs get amortized over the smaller volumes. Also, the target segment in the initial stages is the innovators and early adopters, who are ready to pay a premium for innovations. The lack of economies of scale in production and distribution coupled with the lack of competition enables the product to be sold at a higher price. However, gradually, the price of the product starts falling with time as the firm adopts the penetration pricing strategy, and has a learning curve on cutting down the costs. Since the rate of price drop due to technology maturation falls with time, it can best be expressed as a logarithmic function of time instant \( t \).

\[
P_{t1} = \begin{cases} 
P_{01} & \text{for } t \leq 1 \\
P_{01}(1 - \Upsilon \ln(t - \delta t)) & \text{for } t > 1 
\end{cases}
\]

where, \( \delta t \) is very small.

Thus, the price of the technology product remains constant for a small period post the launch after which it starts reducing due to multiple reasons such as the learning curve, increasing economies of scale, increasing competition, recovery of initial costs, etc.

The behavior of the price with time is thus shown in Figure 1.

3.4. Demand model. In this subsection, we discuss the demand model for two consecutive generations of a technology product. The demand rate is based on the following underlying assumptions
a) The product gets adopted by the innovation diffusion process. The diffusion process is governed by both the effects: the innovation effect (influence of mass media or advertising) as well as the imitation effect (influence of feedback sharing or word of mouth publicity).

b) The demand is also influenced by the selling price of the product.

3.4.1. Adoption model for a single generation. Bass et al. [1] proposed the following hazard-rate based adoption model (also called as Generalized Bass Model or GBM) by considering a variable \( x(t) \) that influences the conditional probability of adoption. The hazard rate as per the GBM is given by:

\[
\frac{f(t)}{1 - F(t)} = [p + qF(t)]x(t)
\]  

(1)

Solution of the GBM [1] for a single-generation can be stated as:

\[
F_1(t) = (1 - e^{-\frac{(X_1(t) - X_1(0))}{b_1}})
\]

(2)

\[
f_1(t) = x(t) \left[ \frac{(\frac{\beta}{p_1}) \cdot e^{-(X_1(t) - X_1(0))} + b_1}{(a_1 \cdot e^{-(X_1(t) - X_1(0))} + b_1+1)} \right]
\]

(3)

where \( a_1 = \frac{q}{p_1} \) and \( b_1 = (p_1 + q_1) \); According to the GBM, the influence of price \( x(t) \) is given by

\[
x_1(t) = 1 + \beta[P_1(t) - 1]
\]

(4)

Where \( \beta \) is the sensitivity of \( x_j(t) \) for the \( j^{th} \) generation product in the GBM with the change in the price of the product \( P_j(t) \). Thus, the cumulative influence of price changes at time \( t \) can be derived by integrating equation [4] w.r.t. \( t \) and written as:

\[
X_1(t) = t + \beta[ln(1 + t)(ln(1 + t) - 1)]
\]

(5)

Demand rate for the first generation product before the launch of the next-generation is given as:

\[
\lambda_1(t) = M_1 f_1(t)
\]

(6)

Once the second-generation product is introduced at time \( \tau \), the demand for the first generation product gets impacted negatively because of the cannibalization by the advanced generation product. Thus, as proposed by Nagpal and Chanda [20], as the demand functions can be defined as:

\[
\lambda_1(t) = M_1 f_1(t) \quad \text{for } t \leq \tau
\]

(7)

\[
\lambda_1'(t) = M_1 f_1(t) - M_1 f_1(t)F_2(t) \quad \text{for } t > \tau
\]

(8)

where \( M_1 f_1(t)F_2(t - \tau) \) is the interaction effect between the potential adopters of the first-generation product, and the cumulative adopters of the second generation product. Similarly, the demand for the second-generation product can be given by:

\[
\lambda_2(t) = M_2 f_2(t) + M_1 f_1(t)F_2(t)
\]

(9)
3.4.2. Adoption model after the launching of the second generation product. Similarly, in case of the second-generation product that gets launched after time \( \tau \), the fraction of adopters at any time \( t \) (without considering any interaction effect with the potential buyers of the first generation) can be written as:

\[
f_2(t) = \frac{x_2(t) [b_2]}{a_2} \cdot e^{-((X_2(t) - X_2(\tau)) + b_2)}
\]

\[
F_2(t) = \frac{(1 - e^{-(X_2(t) - X_2(\tau)) + b_2})}{a_2 \cdot e^{-(X_2(t) - X_2(\tau)) + b_2) + 1}}
\]

And the cumulative adopters till time \( t \) can be written as:

\[
P_2(t) = P_{02}(1 - \Upsilon ln(t - \tau + 1))
\]

We define,

\[
x_2(t) = 1 + \beta \left[ \frac{P_2(t)}{P_{02}} - 1 \right]
\]

Thus,

\[
X_2 = t + \beta[-\Upsilon(1 + t - \tau)(ln(1 + t - \tau) - 1) - t]
\]

4. Cost modelling.

4.1. Cost modeling for the single generation product scenario. If there is no deterioration of the product, the consumption of inventory takes place on account of the demand usage only. Therefore, we can say that

\[
\lambda_j(t) = -\frac{d(I_j(t))}{dt}; \quad (j = 1, 2)
\]

If time \( t \) lies in the \( i \)th replenishment cycle of the \( m \)th planning horizon, in which a total of \( n \) replenishments are done. Then inventory at time \( t \) (as given in Figure 2) is the demand of the product from time \( t \) till the end of the corresponding replenishment cycle. It is visible from Figure 2 that the time point corresponding to the end of the \( i \)th replenishment cycle in such case is given by \( [(m - 1) + \frac{i}{n}] \rho \), we can say that inventory at time \( t \) is:

\[
I_1(t) = \int_{\rho}^{[(m - 1) + \frac{i}{n}] \rho} \lambda_1(t) dt
\]

The Total Cost in the \( m \)th planning horizon with \( n \) replenishment cycles can be stated as the sum of ordering costs and the inventory carrying costs

\[
TC_{m,n} = n(O + O_1) + I_1 C_1 \left[ \frac{I_1(t)}{[(m - 1) \rho]} \right]
\]

\[
TC_{m,n} = n(O + O_1) + I_1 C_1 \sum_{i=1}^{\frac{n}{m - 1}} \int_{\frac{i}{m - 1} \rho}^{\frac{i + 1}{m - 1} \rho} I_1(t) dt
\]

From equations [16], [17] and [18], we get:

\[
TC_{m,n} = n(O + O_1) + I_1 C_1 \sum_{i=1}^{\frac{n}{m - 1}} \int_{\frac{i}{m - 1} \rho}^{\frac{i + 1}{m - 1} \rho} \int_{\frac{i}{m - 1} \rho}^{\frac{i + 1}{m - 1} \rho} \lambda_1(t) dt dt
\]
4.2. Cost modelling after the second generation product. After the second generation product introduced, let the new policies be worked out in terms of several replenishment cycles for each subsequent planning horizon. For simplicity, we can assume that the next generation product is launched at a time when the first generation product has just got replenished by the vendor. The inventories of the products can now be stated as:

\[ I_1(t) = \int_{t}^{\tau + \left\{\frac{(m' - 1) + \frac{1}{m}}{m} \right\} \rho} \lambda_1'(t)dt \quad \text{for} \ t \geq \tau \]  

\[ I_2(t) = \int_{t}^{\tau + \left\{\frac{(m' - 1) + \frac{1}{m}}{m} \right\} \rho} \lambda_2(t)dt \]  

The \( m'^{th} \) planning horizon post the introduction of the second generation product starts at \( t = \tau + (m' - 1)\rho \) and ends at \( t = \tau + m'\rho \). The contribution margin during this period is the area under the contribution margin graph drawn on the time axis, between these two instants of time.
As illustrated in Figure 3, the period corresponding to the $i^{th}$ cycle of the first-generation product in the $m^{th}$ period post the introduction of the second generation product starts at $t = \tau + \left\{ (m' - 1) + \left( \frac{i - 1}{n_1} \right) \right\} \rho$ and ends at $t = \tau + \left\{ (m' - 1) + \left( \frac{i}{n_1} \right) \right\} \rho$.

Similarly, the period corresponding to the $i^{th}$ cycle of the second generation product in the $m^{th}$ period after the launch of the second generation starts at $t = \tau + \left\{ (m' - 1) + \left( \frac{i - 1}{n_2} \right) \right\} \rho$ and ends at $t = \tau + \left\{ (m' - 1) + \left( \frac{i}{n_2} \right) \right\} \rho$.

The contribution margin in the $m^{th}$ planning horizon is given by

$$TCM_{m'} = y'_m \int_{\tau + (m' - 1)\rho}^{\tau + m'\rho} \left[ P_1(t) - C_1 \right] \lambda'_1(t) \, dt + \int_{\tau + (m' - 1)\rho}^{\tau + m'\rho} \left[ P_2(t) - C_2 \right] \lambda_2(t) \, dt$$

$$TC_{m', n_1, n_2} = n_1(O + O_1) + n_2(O + O_2) - z.O$$

$$+ y'_m I_1 C_1 \left[ \sum_{i=1}^{i=n} \int_{\tau + (m' - 1)\rho}^{\tau + (m' - 1)\rho + \left( \frac{i - 1}{n_1} \right) \rho} \left[ \tau + (m' - 1) + \left( \frac{i - 1}{n_1} \right) \rho \right] \lambda'_1(t) \, dt \right]$$

$$+ I_2 C_2 \left[ \sum_{i=1}^{i=n} \int_{\tau + (m' - 1)\rho}^{\tau + (m' - 1)\rho + \left( \frac{i - 1}{n_2} \right) \rho} \left[ \tau + (m' - 1) + \left( \frac{i - 1}{n_2} \right) \rho \right] \lambda_2(t) \, dt \right]$$

where $z = n_1$ in case of pooled logistics, i.e. if $n_1 = n_2$, else zero.

$$TP_{m', n_1, n_2} = TCM_{m'} - TC_{m', n_1, n_2}$$

The objective function is to minimize $TP_{m', n_1, n_2}$.
Subject to the constraints \( n \) is a positive integer.

In the above formulation, there are four decision variables \( n_1, n_2, z \) and \( y_m' \). Our objective is to maximize the profit function \( TP_{m',n_1,n_2} \). Also, after considering the % price drop per unit time \( \gamma \) as the decision variable, and the price sensitivity of demand \( \beta \) as another decision variable, the optimization problem becomes maximization of profit, which can be written as

\[
\text{Max. } TP_{m',n_1,n_2} = \text{Max. } [TCM_{m'} - TC_{m',n_1,n_2}]
\]

where \( n_1 \) and \( n_2 \) are positive integers.

The above model can be generalized through the following theorems.

**Theorem 4.1**. The higher the market potential and the innovation and imitation coefficients of the second generation, the earlier the phase-out of the first generation.

**Proof.** As shown in Figure 4, the higher market potential and the faster spread of the next generation product bring the sales of the first generation to an unsustainable level (when the contribution margin is not sufficient enough to cover the holding costs and product-specific ordering costs).

Considering \( e^{-(x)} \approx 1 - x \), we have

\[
F_2(t) = (X_2(t) - X_2(\tau))b_2/(a_2(1 - (X_2(t) - X_2(\tau))b_2 + 1)
\]

\[
\frac{\delta(\lambda_1(t))}{\delta(p_2)} = \frac{\delta(\lambda_1(t))}{\delta(F_2(t))} \cdot \frac{\delta(F_2(t))}{\delta(p_2)} = -M_1f_1(t) \left[ \frac{\delta(F_2(t))}{\delta(a_2)} \cdot \frac{\delta(a_2)}{\delta(p_2)} + \frac{\delta(F_2(t))}{\delta(b_2)} \cdot \frac{\delta(b_2)}{\delta(p_2)} \right]
\]

\[
= -M_1f_1(t) \left[ \frac{(X_2(t) - X_2(\tau))b_2}{(a_2(1 - (X_2(t) - X_2(\tau))b_2 + 1)^2} + \frac{(X_2(t) - X_2(\tau))}{a_2(1 - (X_2(t) - X_2(\tau))b_2 + 1}
\]

\[
= -M_1f_1(t) \left[ \frac{(X_2(t) - X_2(\tau))b_2}{(a_2(1 - (X_2(t) - X_2(\tau))b_2 + 1)^2} + \frac{(X_2(t) - X_2(\tau))}{a_2(1 - (X_2(t) - X_2(\tau))b_2 + 1}
\]

Since \( (X_2(t) - X_2(\tau))b_2 \) \( < 1 \), and since \( (X_2(t) - X_2(\tau))a_2(1 + b_2) < 1 + a_2 \) and we can infer that \( \frac{\delta(\lambda_1(t))}{\delta(q_2)} < 1 \).

\[
\frac{\delta(\lambda_1(t))}{\delta(q_2)} = \frac{\delta(\lambda_1(t))}{\delta(F_2(t))} \cdot \frac{\delta(F_2(t))}{\delta(q_2)} = -M_1f_1(t) \left[ \frac{\delta(F_2(t))}{\delta(a_2)} \cdot \frac{\delta(a_2)}{\delta(q_2)} + \frac{\delta(F_2(t))}{\delta(b_2)} \cdot \frac{\delta(b_2)}{\delta(q_2)} \right]
\]

\[
= -M_1f_1(t) \left[ \frac{F_2(t)(1)}{a_2^2 p_2^2} + \frac{F_2(t)}{b_2^2} + \frac{(X_2(t) - X_2(\tau))^2 a_2 b_2}{a_2(1 - (X_2(t) - X_2(\tau))b_2 + 1)^2} \right]
\]

Since \( (q_2)^2 \gg (q_2 + p_2) \), we can infer that \( \frac{\delta(\lambda_1(t))}{\delta(q_2)} < 0 \).

While the first part of the above expression is always \( \neq 0 \), the second part is \( \neq 0 \), and therefore, the demand rate for the first generation product falls with the higher coefficients of innovation and imitation of the second generation.

This theorem can help understand that the earlier generation product will have to be phased out faster if the newer generation product is diffusing faster.
Figure 4. Influence of diffusion rate of second generation product on phase-out timing of 1st generation

\textbf{Theorem 4.2.} If newer generation product is sold at a premium, the optimal exit time of the earlier generation product gets stretched.

\textit{Proof.} The higher the price of the newer generation product, the lesser the cannibalization of older generation products due to the price elasticity of the demand. Therefore, it may make a business sense to continue with the older generation product for a longer length of time.

\[
\frac{\delta [X_1(t)]}{\delta [P_2(t)]} = -M_1 f_1(t) \frac{\delta [F_2(t)]}{\delta [P_2(t)]} > 0
\]

\[
\frac{\delta [F_2(t)]}{\delta [P_2(t)]} = \frac{\delta [F_2(t)]}{\delta [X_2(t)]} \cdot \frac{\delta [X_2(t)]}{\delta [P_2(t)]}
\]

Considering $e^{-x} \approx 1 - x$, we have

\[
F_2(t) = \frac{(X_2(t) - X_2(\tau))b_2}{a_2(1 - (X_2(t) - X_2(\tau))b_2) + 1}
\]

\[
\frac{\delta [F_2(t)]}{\delta [X_2(t)]} = \frac{b_2}{a_2(1 - (X_2(t) - X_2(\tau))b_2) + 1} - \frac{(X_2(t) - X_2(\tau))a_2b_2^2}{a_2(1 - (X_2(t) - X_2(\tau))b_2) + 1)^2}
\]

\[
= \frac{b_2}{a_2(1 - (X_2(t) - X_2(\tau))b_2) + 1} \left[ 1 - \frac{(X_2(t) - X_2(\tau))a_2b_2}{a_2(1 - (X_2(t) - X_2(\tau))b_2) + 1} \right]
\]

\[
\frac{\delta [X_2(t)]}{\delta [P_2(t)]} = \beta_2 - Y(1 + t - \tau)(ln(1 + t - \tau) - 1) - t
\]

While $\frac{\delta [F_2(t)]}{\delta [X_2(t)]} > 0$, $\frac{\delta [X_2(t)]}{\delta [P_2(t)]} < 0$, and therefore, $\frac{\delta [F_2(t)]}{\delta [P_2(t)]} < 0$

Since $\frac{\delta [F_2(t)]}{\delta [P_2(t)]} < 0$, $\frac{\delta [X_2(t)]}{\delta [P_2(t)]} > 0$ as it becomes the product of two negative quantities.
Thus, we can say that the higher selling price of the advanced generation product enhances the sales volumes of the first generation product.

This theorem can help the managers use price skimming from the newer generation product and treat the older generation product as a cash cow. There is no point in penetration pricing of the second generation product since that will lead to loss of opportunity sales for the earlier generation product and overall loss of consumer surplus.

**Theorem 4.3.** When the second generation product caters to a niche segment with very little market potential, the $n^*_m$ will increase with its launch unless it is sold at a substantial premium.

**Proof.** If the second-generation product is a niche product, the ordering cost per unit is high due to lower volumes. In that case, the revenue from the product may not be sufficient enough to cover all the costs.

Total Profit per unit quantity is given by $[P - C - \frac{n(A + A_1)}{\lambda(t) dt}]$

In a bid to make it profitable, the inventory holding costs need to be reduced by decreasing the frequency of inventory replenishment in the planning horizon, or the selling price of the product needs to be increased.

$$\lim_{\lambda(t) \to 0^+} \frac{n(A + A_1)}{\lambda(t) dt} = M$$

where $M$ is a very large number.

Therefore for the profit to be retained at normal levels, the overall value of $P - C$ should be large enough, and therefore, the product needs to be sold at a premium.

This theorem helps the managers understand that they need to strike a fine balance between the profitability and cash velocity in the supply chain.

**Theorem 4.4.** Penetration pricing is a better strategy than price skimming when price elasticity of demand is very high. On the other hand, if the price elasticity of the products is high, it makes better sense to follow the price skimming for the second generation.

**Proof.** There are two conflicting effects of the technology price softening, the reduction in the price per unit, and the increase in the sales quantity due to negative price elasticity of demand. For the large values of the coefficient $\beta$, the positive impact of the price softening on the volumes outweighs the negative effect of the same on the drop in price per product. As a result of this, the overall revenues tend to increase. Also, an increase in volumes will tend to bring in replenishment efficiencies through more frequent replenishments. Figure 5 exhibits the variation in revenue with the price elasticity of demand.

If the demand is highly elastic, a fall in price leads to a much larger % of the increase in the volumes as compared to the % drop in the selling price per unit. Thus, a fall in price leads to an increase in revenues, making the penetration pricing a more suitable proposition. On the other hand, for the larger price elasticity of demand, a fall in price results in a much smaller % of the increase in the volumes as compared to the % drop in the selling price per unit; and therefore, the price skimming would be a better strategy. This is also substantiated with the results of
Figure 5. Influence of price elasticity of demand on the diffusion pattern and revenues

Numerical illustration depicted in Table 4 and Table 5.

\[
\frac{\delta [\lambda_i'(t)]}{\delta [\bar{Y}]} = \frac{\delta [\lambda_i'(t)]}{\delta [x_1(t)]} \cdot \frac{\delta [x_1(t)]}{\delta [\bar{Y}]}
\]

While \( \frac{\delta [\lambda_i'(t)]}{\delta [x_1(t)]} \) is always positive, the \( \frac{\delta [x_1(t)]}{\delta [\bar{Y}]} \) is also positive, implying that \( \frac{\delta [\lambda_i'(t)]}{\delta [\bar{Y}]} \) is positive.

Also, we can see that \( \frac{\delta [P_i(t)]}{\delta [\bar{Y}]} \) is always negative since \( \frac{\delta [P_i(t)]}{\delta [\bar{Y}]} = -P_{01} \ln(t + 1) \).

\[
\frac{\delta [TR_1(t)]}{\delta [\bar{Y}]} = \lambda_1'(t) \cdot \frac{\delta [P_1(t)]}{\delta [\bar{Y}]} + P_1(t) \cdot \frac{\delta [\lambda_1'(t)]}{\delta [\bar{Y}]}
\]

Where \( \frac{\delta [\lambda_1'(t)]}{\delta [\bar{Y}]} \) is proportional to \( \beta \).

In the above expression, the first pair is negative and the second pair is positive. For larger values of \( \beta \), the second part of the above expression is greater than the first part, making the overall \( \frac{\delta [TR_1(t)]}{\delta [\bar{Y}]} \) positive.

\[
\frac{\delta [TR_1(t)]}{\delta [\beta]} = P_1(t) \cdot \frac{\delta [\lambda_1'(t)]}{\delta [\beta]} > 0
\]

Thus, with the increase in \( \beta \), the total revenue increases.

This theorem helps the managers appreciate that penetration pricing would be more optimal for the more price elastic products, and the skim pricing would be more optimal for the inelastic products.

5. Numerical illustrations. These are the data points that we are going to consider in the numerical illustration. \( p = 0.5, p_1 = 0.5, 1 = 2.5, p_2 = 0.6, q_2 = 3.5, M_1 = 100000, M_2 = 120000, O = 1500000INR, O_1 = O_2 = 250000INR, C_1 = 1500 INR,
The optimization problem formulated above is a Mixed Integer Non-Linear Programming Problem (MINLP). This paper proposes the use of genetic algorithms (GAs) in determining the optimal number of replenishments. The use of GA proves to be appropriate in solving the model because of its highly non-linear demand pattern and several discontinuities in the objective function. The fitness function used has been formulated in a way that covers the revenues net off the basic purchase costs, inventory carrying costs and the ordering costs. The results of GA have been validated with the results obtained from a greedy search throughout the solution domain.

This paragraph illustrates how the genetic algorithm has been executed. At first, a population of chromosomes was randomly generated and ranked based on their fitness values. The fittest chromosomes were selected to create a mating pool. Genetic operations, i.e. the crossover and mutation, were performed on these chromosomes in the mating pool and a set of child chromosomes are generated, completing one generation. The following steps explain how the GA unfolded in steps to give the optimum solution to the proposed problem.

Starting with chromosomes: To begin with, a population of chromosomes was randomly generated, where each chromosome was an arbitrary solution to the defined optimisation problem. The chromosomes were designed as a 1 X 4 vector, where four corresponds to each decision variable. A population pool of fifty chromosomes was initialized with random values.

Defining the Fitness function: The fitness function used has been formulated in a way that covers the revenues net off the basic purchase costs, inventory carrying costs and the ordering costs. Thus, the total profit was set as the fitness function. It was evaluated corresponding to each chromosome - more the profit, the fitter the chromosome. After this, the chromosomes were ranked in the order of fitness value.

Crossover: The crossover fraction or rate was kept as 0.6. Moreover, the single-point crossover function was used.

Mutation: The mutation rate was 2% and a uniform mutation function was used which randomly selected 2% of genes to mutate arbitrarily.

Setting the Stopping Criteria: It was observed that the child chromosomes obtained after performing the genetic operations on parent chromosomes had a better fitness value. This resulted in the eventual convergence of GA to an approximate optimal solution after a few generations. After 20 generations, it was observed that the value of fitness function ceased improving. The genetic algorithm got terminated and the fittest chromosome was said to be the optimal solution. With the presented formulation and given parameter settings, the numerical illustration considered here, converged to the optimal solution in as few as twenty generations, proving GA to be extremely efficient.

First, let us consider the scenario in which the second-generation product has been launched at time instant = 0.5. And we find the optimal replenishments as well as the continuity of the business of the first-generation product in each planning horizon. We also observe the behavior of revenue and profitability metrics with time in Table 3. The summary of the results is shown in Table 2. The cells with the lowest replenishment costs have been shaded green in Table 2. We take the non-specific ordering cost per order as a large number so that the model will tend to yield the same replenishment frequency for the products of both the generations.
Table 2: Optimal number of replenishments ($n_1$ and $n_2$, both of them, say equal $n$,) in pooled logistics determined by minimizing the sum of holding cost and carrying costs for each planning horizon (All the costs are in Mn INR)

We have shown the behavior of the ordering costs and holding costs for different replenishment frequencies in the different planning horizons in Table 2. For the initial planning horizons, the overall demand for both the products is increasing, leading to increased volumes and more number of optimal replenishment cycles. In the later planning periods, the volumes start declining and therefore, the optimal number of replenishment cycles within these planning horizons falls.

When the first generation product starts giving a negative contribution to the overall profit under the reduced scale, then it can be optimal to discontinue this product. It is also evident that the profit margins reduce over the years. This is in line with the established construct that with the technology becoming more common and more adopted by the masses, the profit margins start declining. Let $(OL_{m',n_1,n_2})$ be the opportunity loss of discontinuing the first generation product which is

$$(OL_{m',n_1,n_2}) = TP_{m',n_1,n_2} \text{ for } y_m = 1 - TP_{m',n_1,n_2} \text{ for } y_m = 0$$

Therefore, when $(OL_{m',n_1,n_2}) \geq 0$, $y_m = 1$; else, $y_m = 0$

Table 3: Total Revenue, Profits and Opportunity Loss for two generations scenario (The Revenue and Profit figure are in Mn INR)

Now, let’s understand Table 3, justify the findings and draw insights from it. Here, $TR_{1m'}$ and $TR_{2m'}$ denote the revenues from the first and the second generation product respectively in the $m^{th}$ planning horizon.

As shown in Table 3, the total Revenue from the first generation keeps declining after the launch of the second generation, while that of the second generation initially increases, reaches a peak, and then, starts declining. The contribution margin % which is simply the revenue net off the basic product cost keeps falling as the technology becomes more popular. After the launch of the second generation, the optimal number of replenishment cycles increases till the second generation product reaches the maturity stage, because of large volumes, and therefore, the over-play
of the inventory holding costs. After the second generation product reaches the maturity stage, the optimal number of replenishment cycles again starts falling due to lower volumes and therefore, the downplay of the inventory holding costs. Also, it can be observed in Table 3 that \((OL_{m',n1,n2})\) is declining with each successive planning horizon. This indicates that the first generation product will need to be phased out in one of the upcoming planning horizons.

6. Inventory replenishment and pricing dynamics. It can be observed from the numerical illustration section that the total Revenue from the first generation keeps declining after the launch of the second generation, while that of the second generation initially increases, reaches a peak, and then, starts declining. The contribution margin \% which is simply the revenue net of the basic product cost keeps falling as the technology becomes more popular. After the launch of the second generation, the optimal number of replenishment cycles increases till the second generation product reaches the maturity stage, because of large volumes, and therefore, the over-play of the inventory holding costs. After the second generation product reaches the maturity stage, the optimal number of replenishment cycles again starts falling due to lower volumes and therefore, the downplay of the inventory holding costs. Also, when the first generation product starts giving a negative contribution to the overall profit under the reduced scale, then it can be optimal to discontinue this product. It is also evident that the profit margins reduce over the years. This is in line with the established construct that with the technology becoming more common and more adopted by the masses, the profit margins start declining.

Now, we consider the price drop \% per unit time \(\Upsilon\) as well as the price sensitivity \(\beta\) as variables. So, with the change in these variables, our revenues and hence profits also get impacted. We used genetic algorithms to study the behavior of total profit since there are multiple variables in the optimization problem and we need to find the global optimum.

| \(\Upsilon\) | \(\beta = 10\) | \(\beta = 20\) | \(\beta = 100\) |
|-------------|-------------|-------------|-------------|
| 0.05        | 118.5       | 34.6        | 98.5        |
| 0.10        | 33.2        | 50.3        | 155.3       |
| 0.15        | 20.9        | 74.0        | 218.2       |
| 0.20        | 49.2        | 86.2        | 285.7       |
| 0.25        | 63.1        | 97.2        | 356.0       |

Table 4: Total Profit values \(TP_{m',n1,n2}\) for different values of yearly price drop \% with the change in \(\beta\) (The Profit figures are in Mn INR)

From Table 4, it is evident that the total profit initially falls with the increase in \% drop in the price until a point at which the increase in volumes due to price drop nullifies the negative effect of the price drop on the revenues; and the profits start increasing. The Table 4 also shows that for smaller values of \(\beta\), the total profit either gets reduced or witnesses a mild increase with the increase in the \(\Upsilon\) (the \% drop in selling price); while for larger values of \(\beta\), the increase in the profits is much
higher for the same % drop in price. This is in line with the expectation since the larger price sensitivity leads to a higher increase in sales volumes for a given decline in the selling price.

| Planning Horizon | \( \beta=10 \) | \( \beta=20 \) | \( \beta=30 \) | \( \beta=100 \) |
|------------------|----------------|----------------|----------------|----------------|
| 1st              | 20.87          | 74.10          | 92.26          | 218.15         |
| 2nd              | 4.40           | 25.89          | 20.26          | 233.27         |
| 3rd              | 4.04           | 20.35          | 18.60          | 404.46         |
| 4th              | 12.19          | 19.30          | 20.59          | 59.30          |
| 5th              | 32.00          | 20.92          | 23.82          | 97.45          |

Table 5: Total Profit values \( TP_{m',n_1,n_2} \) for different planning horizons with different values of price sensitivity \( \beta \) (The Profit figures are in Mn INR)

Table 5 shows how the % drop in price over time is accompanied by an increase in demand, and therefore, an increase in profit. The higher the value of \( \beta \), the more is the spike in the demand and therefore the spike in the total profit.

| Planning Horizon | \( Y=1.5, \beta=1.5 \) with order synergies (\( O \) is large) | \( Y=1.5, \beta=1.5 \) without order synergies (\( O=0 \)) | \( Y=1.5, \beta=10 \) with order synergies (\( O \) is large) | \( Y=1.5, \beta=10 \) without order synergies (\( O=0 \)) |
|------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| 1st              | \( n_1 \) 2, \( n_2 \) 1 | \( TP_{m',n_1,n_2} \) 164.72 | \( n_1 \) 3, \( n_2 \) 6 | \( TP_{m',n_1,n_2} \) 168.72 |
| 2nd              | \( n_1 \) 2, \( n_2 \) 1 | \( TP_{m',n_1,n_2} \) 202.72 | \( n_1 \) 1, \( n_2 \) 7 | \( TP_{m',n_1,n_2} \) 207.29 |
| 3rd              | \( n_1 \) 1, \( n_2 \) 1 | \( TP_{m',n_1,n_2} \) 208.85 | \( n_1 \) 1, \( n_2 \) 3 | \( TP_{m',n_1,n_2} \) 210.97 |
| 4th              | \( n_1 \) 1, \( n_2 \) 1 | \( TP_{m',n_1,n_2} \) 182.00 | \( n_1 \) 1, \( n_2 \) 1 | \( TP_{m',n_1,n_2} \) 183.55 |
| 5th              | \( n_1 \) 1, \( n_2 \) 1 | \( TP_{m',n_1,n_2} \) 156.20 | \( n_1 \) 1, \( n_2 \) 1 | \( TP_{m',n_1,n_2} \) 157.74 |

Table 6: Comparison of the replenishment dynamics: Joint vs Dis-joint for both the generations of products (The Profit figures are in Mn INR)

As shown in Table 6, when the non-specific ordering cost \( “O” \) is made nil, the optimal number of replenishments for both the generations may not be equal due to missing economies of pooling. On the contrary, when the non-specific ordering cost is a significant proportion of the overall ordering costs, the model will always suggest the pooled logistics solution where \( n_1 = n_2 \) to contain the ordering costs through pooled replenishments.

7. Managerial implications. This research has demonstrated how to find the optimal inventory policies over multiple periods for a multi-generation technology product. The managers can understand the complexity of making inventory optimization decisions in case of innovative products that get diffused in a non-linear
fashion and also undergo price softening over some time, resulting in faster-changing demand. The study has also illustrated the effect of pricing phenomenon on the inventory replenishment norms for the multi-generational technology products. The study shows that the changes in the price of one generation of technology products have a bearing on the demand rate of the other substitutable generations of the product.

The foremost important learning that this work has to offer to the practicing managers is that the higher replenishment efficiencies can be achieved with the launch of newer and more advanced generations products, on the account of higher demand rate leading to faster movement of inventories. This research study also illustrates that if the demand for the product is highly price-sensitive then the penetration pricing can be preferred to the price skimming in the initial product life cycle stages. This is because the positive effect of the increased demand rate far outweighs the negative effect of the price drop on the revenues. However, if its demand is relatively insensitive to the selling price, it makes sense to use price skimming until a possible later point in the product life cycle. This is because the positive effect of the increased price on the revenues is much higher than the negative effect of the fall in the demand rate.

On similar lines, the managers will not find it hard to infer that the perfect quality is not always of the best quality. Since quality has a trade-off with price, it may make business sense to limit the quality to an optimal extent in case of price elastic products. The study also sheds light upon certain factors that can expedite the phase-out of the first generation product. Such factors can be mentioned as the higher market potential of the second generation product, the higher innovation and imitation coefficients of the second generation product, the earlier launch of the second generation product, and the higher product-specific ordering costs of the first generation product.

The practitioners also need to acknowledge that when the non-specific ordering costs (the ones that yield synergies in joint replenishments) are a significant portion of the overall ordering costs, it makes sense to go for the consolidated replenishment of the multiple generations products. Another important observation is the behavior of the optimal inventory policy with the price softening. With the price softening of the technology, the profits initially decline before starting to rise as the price softening exceeds a threshold limit. However, if the demand is highly price-sensitive, the profits increase through-out with the price softening.

8. Conclusions and future directions. This study is the first one to formulate a multi-period inventory model for multi-generational technology products. Also, the study worked upon the influence of the price on the inventory policies for such products. The study also shed light upon the phase-out timing of the earlier generations. It also brought about some useful insights for the managers and the practitioners as listed in the earlier section. There can be further extensions to this work.

One possible direction can be incorporating the repeat purchase factor in the model, since a few of the consumers like to purchase the new product even when they have purchased the earlier generation product earlier. Another possible area can be working on the un-pooled logistics where the replenishment frequencies for the different generations of the product can be varied. The third possible extension of this research can be considering the price elasticity factors as fuzzy variables,
since the price elasticity can vary with time depending upon the consumer sentiments, the growth rate of the economy, and the stage of the product life cycle. The fourth possible research direction can be considering the inventory carrying cost per unit item per unit time as a variable cost which increases for higher replenishment quantities due to the need to rent a space from an agent, which is generally costlier than the own space. Last but not the least, taking the discounted values of the future cash flows to consider the inflation can be another possible area for extension of this work.

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E-mail address: gaurav.nagpal@pilani.bits-pilani.ac.in
E-mail address: udayanchanda@pilani.bits-pilani.ac.in
E-mail address: nitantupasani@gmail.com