Fast-convolution (FC)-based filtering has been recently proposed as an efficient tool for spectrum control of single-carrier and multi-carrier waveforms [7]–[18]. In general, the objective of the filtering is to improve the spectral utilization of the channel by improving the localization of the waveform in frequency direction, that is, maximizing the transmission bandwidth for a given channel bandwidth. FC-based filter-bank solutions have superior flexibility when compared with the conventional polyphase-type filter banks [19]. FC processing approximates a linear (aperiodic) convolution through effective fast Fourier transform (FFT)-based circular convolutions using partly overlapping processing blocks (so-called FC blocks). With FC processing, it is very straightforward to adjust the bandwidths and the center frequencies of the subbands with possibly different numerologies individually [12] or even at the symbol level.

In original continuous FC-based filtered-OFDM processing model derived in [10], [12], continuous stream of CP-OFDM symbols are divided into overlapping FC-processing blocks of the same length and the overlap between FC blocks is fixed (typically 50%). Since the CP length in 5G NR is non-zero (and both the OFDM symbol length and the FC-processing block length typically take power-of-two values), FC blocks are not time synchronized to CP-OFDM symbols. The drawback of this approach is that, when the filter configuration changes, i.e., bandwidth or center frequency of the subband (or bandwidth part (BWP) in the 5G-NR terminology) is modified, or for some other reason filtering parameters need to be adjusted between two OFDM symbols, then this change typically occurs within a FC-processing block degrading the performance of the filtering during this block.

In discontinuous symbol-synchronized FC processing as detailed in [13], [16], each symbol is divided into fixed number of processing blocks (e.g. two). These FC-processing blocks are then filtered using FC-based circular convolutions and the filtered FC-processing blocks are concatenated by using overlap-and-add (OLA) processing to form a stream of filtered CP-OFDM symbols. In this case, the change in filtering configuration does not induce any additional intrinsic interference, since the OFDM symbol boundaries are also boundaries of the payload part of the FC blocks. However, for this approach, the FC processing is aligned only with one numerology at the time which may cause problems in supporting mixed numerology. Also, the needed OLA scheme may introduce additional constrains in time-critical applications due to the overlapping needed at the output side.

In the continuous symbol-synchronized processing model...
proposed in this article, the continuous stream of symbols is divided into overlapping blocks such that the overlap is dynamically adjusted based on the CP lengths thus guaranteeing the synchronous processing of all CP-OFDM symbols for all numerologies with normal CP. Therefore, the proposed approach avoids the drawbacks of the original continuous and discontinuous symbol-synchronized FC-based filtered-OFDM models. The only drawback is that the smallest possible forward transform length is somewhat higher when compared to earlier approaches.

The main contributions of this manuscript can be itemized as follows:

- Proposed processing is optimized to 5G NR and LTE physical-layer numerologies, where all supported subcarrier spacings align in time with 0.5 ms time resolution.
- FC blocks are also aligned between LTE and all numerologies with 5G NR, allowing smooth carrier combining in multi-technology or multi-radio transmitter (TX) and corresponding carrier separating receiver (RX) processing.
- Only one FC block within the 0.5 ms time window has different overhead, all other FC blocks have common overhead.
- Processing can be done using either overlap-and-save (OLS) or OLA, or even mix of these, providing additional degree of flexibility for implementation.
- By using FC-processing bin spacing of 60 kHz, we can support dynamic changes in the filter parameterization with time resolution corresponding to the 60 kHz subcarrier spacing OFDM symbols.

The presented solution supports all different use cases envisioned for the flexible, BWP-based 5G-NR radio interface, allowing filter re-configuration time resolution equal to one OFDM symbol while maintaining high quality separation of different frequency blocks. The solutions presented here are especially important for below-7 GHz communications due to scarce spectral resources, but there is no limitation in applying the solutions also for higher carrier frequencies if seen necessary.

The remainder of this paper is organized as follows. Section II shortly reviews the 5G NR numerology and relevant terminology for reference. Then, the proposed continuous symbol-synchronized FC-based filtered-OFDM processing models for TX and RX are described in Section III. This section also describes how to define the frequency-domain windows for reducing the out-of-band emissions (OOBES) and inter-numerology interference (INI). Section IV introduces the key metrics and requirements used for evaluating the performance of the TX processing. In Section V, the performance of the proposed processing is demonstrated in various mixed-numerology scenarios. Finally, the conclusions are drawn in Section VI.
TABLE II
5G-NR MIXED NUMERLOGY IN FREQUENCY RANGE 1 (FR1).

| Subcarrier spacing, \( f_{SCS,m,n} \) | 15 kHz | 30 kHz | 60 kHz | 2\( \times \)15 kHz |
|-------------------------------------|--------|--------|--------|-----------------|
| OFDM symbol duration, \( T_{OFDM,m,n} \) | 66.7 μs | 33.3 μs | 16.7 μs | 2\( \times \)66.7 μs |
| Cyclic prefix duration, \( T_{CP,m,n} \) | 4.69 μs | 2.34 μs | 1.17 μs | 2\( \times \)4.69 μs |
| Number of OFDM symbols per slot, \( N_{sym}^{m,n} \) | 14 | 14 | 14 or 12 | 14 or 12 |
| Number of slots per subframe, \( N_{slot}^{subframe} \) | 1 | 2 | 4 | 2\( \times \) |
| Slot duration, \( T_{slot} \) | 1 ms | 0.5 ms | 0.25 ms | 2\( \times \)1 ms |

5G NR supports subcarrier spacings (SCSs) of \( 2^\alpha \times 15 \) kHz where \( \alpha = 0, 1, \ldots, 4 \) while only 15 kHz is supported by LTE. Similar to LTE technology, a radio frame of 10 ms is divided into 10 subframes, each having 1 ms duration while each subframe has \( 2^\alpha \) slots. Each slot consists of either 14 or 12 OFDM symbols for the normal CP or extended CP, respectively [1], [20]. The slot duration varies based on the SCS as \( T_{slot} = 2^\alpha \times 1 \) ms, i.e., it is 1 ms for 15 kHz SCS, 0.5 ms for 30 kHz SCS and so on. The numerology for 15 kHz, 30 kHz, and 60 kHz SCS is summarized in Table II.

Fig. 1 exemplifies the time alignment of different numerologies within a half subframe. In general, 5G NR does not specify the minimum number of consecutive symbols of certain SCS and, therefore, in extreme time-multiplexing cases, it is possible that the SCS changes even at the symbol level as illustrated in Fig. 1(d).

5G NR supports also non-slot based scheduling (so-called mini slots), where the transmission length can be configured between 1 and 13 symbols [21, Section 8.1]. This mini-slot concept is especially essential for low-latency ultra-reliable low-latency communications (URLLC) and for dynamic time-domain multiplexing. Such transmissions can pre-empt the ongoing slot-based transmission and, therefore, also the time-domain flexibility of the filtering solutions becomes crucial in the 5G-NR context.

5G NR allows the use of a mixed numerology, i.e., using different SCSs in different subbands (or BWPs) within a single channel. However, the use of different SCSs within an OFDM multiplex harms the orthogonality of subcarriers, introducing INI. To cope with INI in mixed-numerology scenarios with basic CP-OFDM waveform, relatively wide guard bands (GBs) should be applied between adjacent BWPs, which would reduce the spectral efficiency. Alternatively, the third generation partnership project (3GPP) allows to use spectrum enhancement techniques for CP-OFDM, but this should be done in a transparent way and without performance loss with respect to plain CP-OFDM. The transparency means that a TX or a RX does not need to know whether spectrum enhancement is used at the other end. The spectrum enhancement techniques should also be compatible with each other, allowing different techniques to be used in the TX and RX. Transparent enhanced CP-OFDM techniques have been considered in [22]–[24].

5G NR is designed to operate in two operating bands where frequency range 1 (FR1) corresponds to 410–7125 MHz while frequency range 2 (FR2) corresponds to the 24.25–52.6 GHz [25, Table 5.1-1]. Basically, FR1 supports SCSs of \( f_{SCS} = 2^\alpha \times 15 \) kHz with \( \alpha = \{0, 1, 2, 3, 4\} \) for FR2. SCS of \( f_{SCS} = 2^\alpha \times 15 \) kHz with \( \alpha = \{0, 1, 3, 4\} \) are used for synchronization blocks (primary synchronization signal (PSS), secondary synchronization signal (SSS), and physical broadcast channel (PBCH)) and \( \mu = \{0, 1, 2, 3\} \) for data channels (physical downlink shared channel (PDSCH), physical uplink shared channel (PUSCH), etc.).

For the approaches proposed in this manuscript, the SCS for each CP-OFDM symbol on each subband can be independently adjusted. Therefore, we denote the SCS for the \( m \)th symbol on the \( n \)th subband by \( f_{SCS,m,n} \) for \( n \in \{0, 1, \ldots, B_m - 1\} \) and \( m \in \{0, 1, \ldots, M - 1\} \), where \( B_m \) and \( M \) are the number of CP-OFDM symbols on subband \( m \) and number of subbands.
respectively. Here, $f_{SCS,m,n}$ can be selected as $2^{\mu} \times 15$ kHz for $\mu = 0, 1, \ldots, 4$ and the SCS scaling factor of $n$th symbol on subband $m$ is denoted by $\mu_{m,n}$.

Let $f_s$ be the OFDM waveform sample rate as tabulated in Table III for 5G-NR channel bandwidths in FR1. Without loss of generality, we assume that the number of samples to be processed is multiple of

$$\begin{align*}
N_{samp,HSF} &= 0.5 \times 10^{-3} f_s, 
(1)
\end{align*}$$

i.e., number of samples per half subframe corresponding to seven CP-OFDM symbols with baseline SCS. The baseline SCS is $f_{BL} = 15$ kHz and $f_{BL} = 60$ kHz in FR1 and FR2, respectively. The OFDM transform length can now be determined as a ratio of sample rate and SCS as

$$\begin{align*}
N_{OFDM,m,n} &= \frac{f_s}{f_{SCS,m,n}} = \frac{f_s}{2\mu_{m,n} \times 15 \text{ kHz}}. 
(2)
\end{align*}$$

The maximum available OFDM inverse fast Fourier transform (IFFT) length in 5G NR is restricted to be smaller than or equal to $N_{IFFT,max} = 4096$, therefore, the maximum supported channel bandwidth, e.g., for 15 kHz SCS is 50 MHz while 240 kHz SCS supports 800 MHz.

The normal CP length in samples is determined as

$$\begin{align*}
N_{CP,m,n} &= \begin{cases} 
\frac{9}{128} N_{OFDM,m,n} + \alpha, & \text{for } \xi(n) = 0 \\
\frac{9}{128} N_{OFDM,m,n}, & \text{otherwise},
\end{cases} 
(3a)
\end{align*}$$

where

$$\begin{align*}
\alpha &= \text{mod} \left( N_{samp,HSF}, 9 + 128 \right) 
(3b)
\end{align*}$$

and

$$\begin{align*}
\xi(n) &= \text{mod} \left( N_{OFDM,m,0} - \sum_{k=0}^{n} N_{OFDM,m,k}, \frac{7 f_s}{f_{BL}} \right) 
(3c)
\end{align*}$$

for $n = 0, 1, \ldots, B_m - 1$ is equal to zero for the first symbol of each half subframe. In 5G-NR numerology (as well as in LTE), longer CP for the first symbol is needed to balance the excess samples for each half subframe such that

$$\begin{align*}
\sum_{n=0}^{B_m-1} \left( N_{OFDM,m,n} + N_{CP,m,n} \right) = N_{samp,HSF} 
(4)
\end{align*}$$

for given number of CP-OFDM symbols $B_m$. For example, in 10 MHz channel ($f_s = 15.36$ MHz) with 15 kHz SCS and seven ($B_7 = 7$) OFDM symbols per half subframe, $\alpha = 8$ and the CP length is $N_{CP,m,n} = 72 + \alpha = 80$ for mod($n, 7$) = 0 and $N_{CP,m,n} = 72$ otherwise, such that $7(1024 + 72) + 8 = N_{samp,HSF}$.

In LTE and 5G NR, the frequency-domain resources are allocated in physical resource blocks (PRBs) corresponding to 12 subcarriers or resource elements (REs). The transmission bandwidth configuration defining the maximum number of active PRBs for given channel bandwidth and given SCS are tabulated in [25, Tables 5.3.2-1 and 5.3.2-2] for FR1 and FR2, respectively.

The following processing model supports mixed SCSs and allocation bandwidths. Therefore, we denote by $L_{act,m,n}$, the number of active subcarriers of the $n$th symbol on subband $m$ and $S_{m,n} \subset \{0, 1, \ldots, B_m - 1\}$ for $\nu = 0, 1, \ldots, Y_m - 1$ is the set of symbol indices having the same symbol length and the same number of active subcarriers while $Y_m$ is the number of symbol sets with different numerology on subband $m$. Here, the number of active subcarriers can be selected as $L_{act,m,n} \leq 12 \times N_{PRB,max}$, where $N_{PRB,max}$ is the corresponding transmission bandwidth configuration.

Fig. 2 illustrates a single subchannel ($M = 1$) time-multiplexed mixed-numerology scenario with three ($Y_0 = 3$) SCSs. In this example, $S_{0,0} = \{2, 7, 10\}$ is the set of symbol indices with 15 kHz SCS, $S_{0,1} = \{0, 1, 8, 9\}$ are the indices for symbols with 30 kHz SCS, and $S_{0,2} = \{3, 4, 5, 6, 11, 12, 13, 14\}$ for 60 kHz SCS. The number of active subcarriers in 10 MHz channel for symbols with 15 kHz, 30 kHz, and 60 kHz SCSs are 624, 288, and 132, respectively [20].

III. FAST-CONVOLUTION-BASED FILTERED-OFDM

The basic principle of the proposed FC-based waveform TX processing for 5G NR is illustrated in Fig. 3. In original FC-based filtered-OFDM (FC-OFDM), filtering is applied at subband level, utilizing normal CP-OFDM waveform with one or multiple contiguous PRBs with same SCS [10], [12],

| Frequency (MHz) | Allocation with 15 kHz SCS | Allocation with 60 kHz SCS |
|----------------|---------------------------|---------------------------|
| 7.68           | 5488                      | 6584                      |
| 23.04          | 2304                      | 2200                      |
| 30.72          | 3072                      | 3072                      |
| 46.08          | 4608                      | 4608                      |
Fig. 3. Proposed transmitter processing using the FC synthesis filter bank (SFB) with \( M \) subbands. In FC-based filtered-OFDM, filtering is applied at subband level, which means one or multiple contiguous PRBs with possible mixed SCS, while utilizing normal CP-OFDM waveform for the PRBs. FC-processing consist of forward transforms of length \( L_m \) for \( m = 0, 1, \ldots, M \) and inverse transform of length \( N \). The center frequency of each symbol may be adjusted independently by simply mapping the corresponding frequency-domain bins.

[13], [16], [26]. In the proposed model, each subband can have mixed numerology, that is, SCS and/or number of active PRBs may change from one OFDM symbol to another. These OFDM symbols are generated with IFFTs of length \( L_{OFDM,m,n} \) for \( m = 0, 1, \ldots, M - 1 \) and \( n = 0, 1, \ldots, B_m - 1 \). CP of length \( L_{CP,m,n} \) is inserted to each symbol and the resulting CP-OFDM symbols are filtered using FC-based synthesis filter bank (SFB) consisting of forward transforms (FFTs) of length \( L_m \) for \( m = 0, 1, \ldots, M - 1 \), frequency-domain windowing, and inverse transform (IFFT) of length \( N \). The center frequency of each subband can be adjusted simply by mapping the resulting output bins of the forward transforms (FFTs) to the desired input bins of the inverse transform (IFFT).

The FC bin spacing, i.e., the resolution of the FC processing, is determined as a ratio of output sample rate and FC inverse transform length as

\[
f_{BS} = f_s / N.
\]

The FC bin spacing can be selected independent to OFDM SCS. FC processing provides the sampling-rate conversion factor determined by the ratio of the inverse transform and forward transform length as expressed by

\[
I_m = N / L_m.
\]

Therefore, the OFDM symbol and CP lengths on the high-rate side (at the SFB output) are \( N_{OFDM,m,n} = I_m L_{OFDM,m,n} \) and \( N_{CP,m,n} = I_m L_{CP,m,n} \), respectively. By following the proposed segmentation of the subband waveforms into the overlapping FC blocks and then carrying out the overlapped circular convolutions with the aid of FC-based SFB (FFT/IFFT pair with windowing) in conjunction of OLA or OLS schemes, even the center frequency of each symbol may adjusted independently.

### A. FC Filtered-OFDM TX Processing

Let us denote the \( m \)th frequency-domain multi-carrier symbol on \( m \)th subband with \( L_{act,m,n} \) active subcarriers by

\[
x_{m,n} = [x_{m,n}(0), x_{m,n}(1), \ldots, x_{m,n}(L_{act,m,n} - 1)]^T.
\]

Here, \( x_{m,n}(\ell)'s \) are the quadrature phase-shift keying (QPSK) or \( M \)-ary quadrature amplitude modulation (\( M \)-QAM) symbols to be transmitted on subcarrier \( \ell \). Further,

\[
x_m = [x_{m,0}^T, x_{m,1}^T, \ldots, x_{m,B_m-1}^T]^T
\]

is the column vector of length \( L_{act, tot,m} \)

\[
L_{act, tot,m} = \sum_{n=0}^{B_m-1} (L_{OFDM,m,n} + L_{CP,m,n})
\]

obtained by vertically stacking all symbols \( x_{m,n} \) for \( n = 0, 1, \ldots, B_m - 1 \). The corresponding low-rate CP-OFDM waveform length is

\[
L_{samp,m} = \sum_{n=0}^{B_m-1} (L_{OFDM,m,n} + L_{CP,m,n})
\]

The CP-OFDM TX processing of the \( m \)th subband can now be expressed as

\[
y_m = [y_{m,0}^T, y_{m,1}^T, \ldots, y_{m,B_m-1}^T]^T = T_m x_m,
\]

where the block diagonal OFDM modulation matrix of size \( L_{samp,m} \times L_{act, tot,m} \) is expressed as

\[
T_m = \text{diag}(T_{m,0}, T_{m,1}, \ldots, T_{m,B_m-1})
\]

with

\[
T_{m,n} = \hat{K}_{m,n} \hat{W}_{m,n}^H.
\]

Here, \( \hat{W}_{m,n} \) is the pruned unitary discrete Fourier transform (DFT) matrix of size \( L_{act,m,n} \times L_{OFDM,m,n} \) as given by

\[
\hat{W}_{m,n}_{p,q} = \frac{1}{\sqrt{L_{OFDM,m,n}}} \exp \left(-j \pi q \frac{2p}{L_{act,m,n}} \right)
\]

for \( p = 0, 1, \ldots, L_{act,m,n} - 1 \) and \( q = 0, 1, \ldots, L_{OFDM,m,n} - 1 \). In (10c), \( \hat{W}_{m,n} \in \mathbb{C}^{L_{OFDM,m,n} \times L_{OFDM,m,n}} \) is the CP insertion matrix as given by

\[
\hat{K}_{m,n} = \begin{bmatrix} 0_{L_{CP,m,n} \times (L_{OFDM,m,n} - L_{CP,m,n})} & I_{L_{CP,m,n}} \end{bmatrix} I_{L_{OFDM,m,n}}
\]

which copies \( L_{CP,m,n} \) last samples of the \( n \)th OFDM symbol in the beginning of the symbol. The block diagonal structure of the resulting OFDM modulation matrix \( T_m \) is illustrated in Fig. 4.

The block segmentation of the proposed continuous symbol-synchronized FC-processing scheme is exemplified in Fig. 5. Time-domain input sample stream is processed in overlapped FC-processing blocks of length \( L_m \) as in earlier schemes. However, now the overlap between the processing blocks is adjusted such that the length of the non-overlapping part for the FC blocks containing the CP part of the first symbol in a half subframe is longer than others.

Let us denote by \( R_{HSF} \) the number of half-subframes to be processed. For the proposed scheme, the non-overlapping part length for FC blocks for \( r = 0, 1, \ldots, R_{HSF} R_m - 1 \) is given by

\[
L_{S,m,r} = \begin{cases} 2^d_a (9 + 128) + \alpha, & \text{for } \text{mod}(r, R_m) = 0 \\ 2^d_a (9 + 128), & \text{otherwise} \end{cases}
\]
The overall waveform to be transmitted is obtained by summing all the M subband waveforms as
\[ z = \sum_{m=0}^{M-1} z_m. \]  

The leading and tailing overlapping part lengths for FC blocks are given as
\[ L_{L,m,n} = \left[ \left( L_m - L_{S,m,n} \right) / 2 \right] \]  

and
\[ L_{T,m,n} = L_m - L_{L,m,n}, \]  

respectively. In this case, exactly two FC-processing blocks are needed to process one, two, or four OFDM symbols with 15 kHz, 30 kHz, or 60 kHz SCS, respectively, as shown in Fig. 5. For \( L_m = 512 \), the corresponding number of FC-processing blocks is four, that is, the filtering can be re-configured with the shortest (60 kHz SCS) OFDM symbol time resolution.

In the FC SFB case, the block processing of \( m \)th CP-OFDM subband signal \( y_m \) for the generation of high-rate subband waveform \( z_m \) can now be represented as
\[ z_m = F_m \hat{y}_m, \]  

where \( F_m \) is the block diagonal synthesis processing matrix of the form
\[ F_m = \text{bdiag}(F_{m,0}, F_{m,1}, \ldots, F_{m,R_m-1})_{q_m,p_m}. \]  

This CP-OFDM waveform of (10a) with \( L_{L,m,0} \) and \( L_{T,m,R_m}-1 \) samples zero padding before and after the CP-OFDM symbols, respectively. Here, \( \text{bdiag}()_{q,p} \) is an operator for constructing block-diagonal matrix with overlapping blocks of its arguments. The column and row indices of the first element of the \( r \)th block are \( q_r \) and \( p_r \), respectively. In order to align the FC-processing blocks with CP-OFDM symbols, the row and column indices for the first elements of the \( r \)th block are given as
\[ p_{m,r} = \begin{cases} \left( rL_{S,m,n} + \frac{\alpha}{2} \right) \frac{r}{R_m} & \text{for } \text{mod}(r, R_m) = 0 \\ \left( rL_{S,m,n} + \frac{\alpha}{2} \right) \frac{r}{R_m} & \text{otherwise} \end{cases} \]  

and
\[ q_{m,r} = L_m p_{m,r}. \]  

respectively. The block diagonal structure of \( F_m \) with overlapping blocks is depicted in Fig. 6.

The overall waveform to be transmitted is obtained by summing all the \( M \) subband waveforms as
\[ z = \sum_{m=0}^{M-1} z_m. \]  

FC SFB can be represented using block processing by decomposing the \( F_{m,r} \)'s as follows:
\[ F_{m,r} = S_r W_r^T M_{m,r} D_{m,r} P_L W_L A_{m,r}. \]  

where \( A_{m,r} = \text{diag}(a_{m,r}) \) and \( S_r = \text{diag}(s_r) \) are the time-domain analysis and synthesis windowing matrices with the analysis.
and synthesis window weights \( a_{m,r} \in \mathbb{R}^{L_x \times 1} \) and \( s_r \in \mathbb{R}^{N_x \times 1} \), respectively. \( W_{m} \in \mathbb{C}^{N_x \times L_x} \) and \( W_{m}^H \in \mathbb{C}^{N_x \times N} \) are the unitary DFT and inverse discrete Fourier transform (IDFT) matrices, respectively. \( P_{m} \in \mathbb{C}^{N_x \times L_m} \) is the FFT-shift matrix and \( M_{m,r} \in \mathbb{C}^{N \times L_m} \) maps \( L_m \) consecutive frequency-domain bins of the input signal to \( L_m \) consecutive frequency-domain bins of the output signal as well as the rotates the phases for maintaining the phase continuity. Finally, the frequency-domain window is determined by diagonal \( D_{m,r} \). For OLA scheme, the analysis and synthesis time-domain windows are given as

\[
a_{m,r} = \begin{bmatrix} 0_{L_m \times 1} \\ L_{m,0} \end{bmatrix} \quad \text{and} \quad s_r = \begin{bmatrix} 1_{N \times 1} \end{bmatrix}, \tag{20a}
\]

respectively, whereas for OLS scheme these windows are given by

\[
a_{m,r} = \begin{bmatrix} 1_{L_m \times 1} \end{bmatrix} \quad \text{and} \quad s_r = \begin{bmatrix} 0_{N \times 1} \\ 1_{N \times 1} \\ 0_{N \times 1} \end{bmatrix}. \tag{20b}
\]

respectively.

The CP-OFDM waveforms can be generated, e.g., with lowest power-of-two transform length larger than \( L_{act,m,n} \), i.e.,

\[
L_{OFDM,m,n} = 2^\lceil \log_2(L_{act,m,n}) \rceil \tag{21}
\]

and the FC processing interpolates the CP-OFDM waveform at the desired output rate. However, for continuous FC-processing alternatives (as opposed to [16]), the sampling-rate conversion factor has to be selected such that the CP length on the low-rate side is still an integer, i.e., the shortest OFDM transform length is \( L_{OFDM_{\min}} = 128 \) corresponding to CP length of \( L_{CP_{\min}} = 9 \) samples. Furthermore, the non-overlapping part length \( L_{S,m,n} \) has also to be integer, restricting the \( L_m \) to be larger than equal to 256.

### B. FC Filtered-OFDM RX Processing

FC-F-OFDM waveform can be received transparently, e.g., with (i) basic CP-OFDM receiver, (ii) by first filtering the received waveform, either using the FC-based analysis filter bank (AFB) or conventional time-domain filter, or (iii) by using windowed overlap-and-add (WOLA) processing in connection with OFDM processing.

The FC-based AFB processing can be described as

\[
\tilde{y}_m = G_m \tilde{z}, \tag{22}
\]

where the analysis processing matrix is \( G_m = F_m^H \) and \( \tilde{z} \) is the received FC-F-OFDM waveform. Analogous to SFB case, the decimation factor \( D_m \) provided by the analysis processing is the ratio of long forward transform and short inverse transform sizes.

Finally, the filtered and possibly decimated subband signals \( \tilde{y}_m \) for \( m = 0, 1, \ldots, M - 1 \) are demodulated by using the conventional CP-OFDM RX processing as expressed by

\[
\tilde{x}_m = Q_m \tilde{y}_m, \tag{23a}
\]

where \( Q_m = \text{diag}(Q_{m,0}, Q_{m,1}, \ldots, Q_{m,B_m-1}) \) (23b) with

\[
Q_{m,n} = \tilde{W}_{m,n} R_{CP,m,n}^{(\tau_{m,n})}. \tag{23c}
\]

Here, \( \tilde{W}_{m,n} \) is the pruned unitary DFT matrix as given by (11) and \( R_{m,n}^{(\tau_{m,n})} \in \mathbb{Z}^{L_{OFDM,m,n} \times (L_{OFDM,m,n} + 1)} \) is the following CP removal matrix

\[
R_{m,n}^{(\tau_{m,n})} = \begin{bmatrix} 0_{L_{CP,m,n} \times \tau_{m,n}} & \cdots & 0_{L_{CP,m,n} \times 1} \\ I_{L_{OFDM,m,n}}^{(\tau_{m,n})} & 0_{L_{OFDM,m,n}} \\ 0_{L_{OFDM,m,n}} & 0_{L_{OFDM,m,n}} \end{bmatrix}^T, \tag{23d}
\]

where \( C_{p}^{(\theta)} \) is the circular shift matrix of size \( p \) used to shift the elements of a column vector downward by \( q \) elements. Here, parameter \( \tau_{m,n} \) is used to control the sampling instant within the CP-OFDM symbol as described in Section IV.

The complexity of this scheme, in terms of multiplications, is the same as for symbol-synchronized discontinuous FC processing described in [16], i.e., \( 2 - 5 \) times the complexity of plain CP-OFDM. The channel estimation and equalization functionalities can be realized as for the conventional CP-OFDM waveform.

### C. Frequency-Domain Window

The frequency-domain characteristics of the FC processing are determined by the frequency-domain window. Basically, the frequency-domain window can be adjusted at the granularity of FC bin spacing, as given by (5). For the proposed and the earlier approaches, the frequency-domain window consist of ones on the passband, zeros on the stopband, and two symmetric transition bands with \( N_{TB,m,n} \) non-trivial optimized prototype transition-band values. The same optimized transition band weights can be used for realizing all the transmission band weights.
bandwidths by properly adjusting the number of one-valued weights between the transition bands.

According to [25, Table 5.3.3-1], the minimum guard bands for base station channel bandwidths are determined as

$$f_{GB} = \frac{1}{2} \left[ f_{Ch,BW} - f_{SCS}(L_{act,max} + 1) \right],$$  

(24)

where \( L_{act,max} = 12 \times N_{PRB,max} \) with \( N_{PRB,max} \) being the transmission bandwidth configuration. For example, in 10MHz channel with 15kHz SCS, the maximum number of active PRBs is \( N_{PRB,max} = 52 \) (\( L_{act,max} = 624 \)) and the resulting guard band is \( f_{GB} = 312.5 \)kHz. Assuming FC bin spacing of \( f_{BS} = 15 \)kHz, i.e., \( N = N_{OFDM} = 1024 \), the guard band corresponds to the \( f_{GB} / f_{BS} = 20.83 \) frequency-domain bins. Now, the frequency-domain window can be determined such that 624 frequency-domain window values corresponding to active subcarriers are equal to one, \( [20,83] = 20 \) window values on both sides of the active subcarriers can be optimized for achieving the desired spectral characteristics while the remaining frequency-domain window values are equal to zero. Same frequency-domain window can be used for filtering the higher SCSs as well, since for given channel bandwidth the guard band increases as the SCS increases.

Let us denote the desired center frequency of each OFDM symbol by \( f_{m,n}^{(center)} \). The lower and higher passband (PB) edge frequencies of each symbol can then be expressed as

$$f_{PB,m,n}^{(low)} = f_{m,n}^{(center)} - f_{s} \frac{L_{act,m,n}}{2} \frac{1}{L_{OFDM,m,n}}$$  

(25a)

and

$$f_{PB,m,n}^{(high)} = f_{m,n}^{(center)} + f_{s} \frac{L_{act,m,n}}{2} - 1 \frac{1}{L_{OFDM,m,n}},$$  

(25b)

respectively. Let us assume for simplicity that the center frequency of each subband is fixed and the subbands are sorted based on their center frequencies such that subband \( m \) for \( m = 0 \) has the lowest center frequency, subband \( m \) for \( m = 1 \) has the next lowest center frequency, and so on. Now, the stopband (SB) edge frequencies for the subbands can be expressed in the following three cases:

1) One subband \( (M = 1) \): In this case, the stopband edges are determined based on the channel edges, that is, the lower and upper stopband edge frequencies are determined as

$$f_{SB,0,n}^{(low)} = -f_{Ch,BW}/2$$  

(26a)

and

$$f_{SB,0,n}^{(high)} = f_{Ch,BW}/2,$$  

(26b)

respectively.

2) Two subbands \( (M = 2) \): In this case, the lower (higher) stopband edge of the first (second) subband is determined by channel bandwidth while the higher (lower) stopband edge of the first (second) subband is determined by lower (higher) passband edge of the second (first) subband, that is, the lower and higher stopband edge frequencies are determined as

$$f_{SB,m,n}^{(low)} = \begin{cases} -f_{Ch,BW}/2 & \text{for } m = 0 \\ f_{PB,0,n}^{(high)} & \text{for } m = 1 \end{cases}$$  

(27a)

and

$$f_{SB,m,n}^{(high)} = \begin{cases} f_{PB,1,n}^{(low)} & \text{for } m = 0 \\ f_{Ch,BW}/2 & \text{for } m = 1. \end{cases}$$  

(27b)

3) Three or more subbands \( (M \geq 3) \): Now, the lower and higher stopband edges are determined by

$$f_{SB,m,n}^{(low)} = \begin{cases} -f_{Ch,BW}/2 & \text{for } m = 0 \\ f_{PB,m-1,n}^{(high)} & \text{for } m = 1, 2, \ldots, M - 2 \\ f_{Ch,BW}/2 & \text{for } m = M - 1. \end{cases}$$  

(28a)

and

$$f_{SB,m,n}^{(high)} = \begin{cases} f_{PB,1,n}^{(low)} & \text{for } m = 0 \\ f_{PB,m+1,n}^{(low)} & \text{for } m = 1, 2, \ldots, M - 2 \\ f_{Ch,BW}/2 & \text{for } m = M - 1. \end{cases}$$  

(28b)

that is, the upper (lower) stopband edge of the \( m \)th subband is determined by the \((m + 1)\)th ((\(m - 1)\)th) subband except for the edgemost subbands, where the channel edge specifies the upper (lower) stopband edge of the last (first) subband.

The frequency-domain window for \( r \)th FC block on subband \( m \) is determined as

$$f_{m,r} = \begin{bmatrix} 0 & \cdots & 0 \\ h_{m,n}^{TB} & \cdots & h_{m,n}^{TB} \\ 1 & \cdots & 1 \end{bmatrix},$$  

(29)

where \( n \in \{0, 1, \ldots, B_{OFDM,m} - 1\} \) is now the index of the symbol processed by the \( r \)th FC block. Here, \( h_{m,n} \in \mathbb{R}^{N_{TB,n} \times 1} \) is the transition-band weight vector and \( J_{N_{TB,n}} \) is the reverse identity matrix of size \( N_{TB,n} \times \) essentially reversing the order of transition-band weight vector. The lower and higher stopband indices of each subband in (29) are determined as

$$k_{m,n}^{(low)} = \max \left( \left\{ \left. \left( f_{SB,m,n}^{(low)} - f_{m,n}^{(center)} \right) N/f_s \right| \right. f_{m,n}^{(low)} \right) + L_m/2, 0 \right)$$  

(30a)

and

$$k_{m,n}^{(high)} = \min \left( \left\{ \left. \left( f_{SB,m,n}^{(high)} - f_{m,n}^{(center)} \right) N/f_s \right| \right. f_{m,n}^{(high)} \right) + L_m/2, L_m - 1 \right),$$  

(30b)

respectively.

For channelization purposes, the frequency-domain window values are real. FC processing can also be used for shifting the output of each subband by fractional delay if needed. In this case, an additional phase term as given by

$$[d_{m,r}]_q = \exp \left\{ -2j\pi(q - L_m/2)\Phi_{FD,m}/L_m \right\}$$  

(31)

for \( q = 0, 1, \ldots, L_m - 1 \) is included in the coefficients. Here, \( \Phi_{FD,m} \in [0, 1] \) is the desired fractional delay value on subband \( m \). The characteristic responses for the FC-based fractional delay filter are illustrated in Fig. 7.
IV. WAVEFORM REQUIREMENTS

The performance of the FC-F-OFDM waveforms are evaluated with respect to requirements defined for the 5G-NR waveform in 3GPP specification for base stations [20]. The quality of the transmitted waveform is specified by the error vector magnitude (EVM) requirements, defining the maximum allowable deviation of the transmitted symbols with respect to ideal ones. The OOB E requirements, on the other hand, give the requirements for the tolerable spectral emissions of TX waveform. In addition to these two key metrics, there are other measures, e.g., EVM equalizer flatness requirements, in-band emissions (IBEs), occupied bandwidth (OBW), among others, however, these measures are beyond the scope of this paper.

A. Error Vector Magnitude

The quality of the TX processing in 5G NR is measured by evaluating the mean-squared error (MSE) between the transmitted and ideal symbols. For the proposed approach, \( Y_m \) symbol sets with difference numerology are allowed on subband \( m \), and therefore, the MSE is evaluated separately for each numerology as

\[
\epsilon_{\text{MSE},m,o} = \frac{1}{|S_{m,o}|} \sum_{n \in S_{m,o}} |\tilde{x}_{m,n} - x_{m,n}|^2
\]

for \( o = 0, 1, \ldots, Y_m - 1 \) as illustrated in Fig. 8. The corresponding error vector magnitude (EVM) in percents is expressed as

\[
\epsilon_{\text{EVM},m,o} = 100 \sqrt{\epsilon_{\text{MSE},m,o}}
\]

In this contribution, the EVM is expressed in decibels as

\[
\epsilon_{\text{EVM},m,o} = 10 \log_{10}(\epsilon_{\text{MSE},m,o}).
\]

Here, MSE and EVM are measured after executing zero-forcing equalizer (ZFE), as defined in [20, Annex B].

The average MSE is defined as the arithmetic mean of the MSE values on active subcarriers, as given by

\[
\bar{\epsilon}_{\text{MSE},m,o} = \frac{1}{L_{\text{act},o}} \sum_{n \in S_{\text{act}}} \epsilon_{\text{MSE},m,o}
\]

where \( L_{\text{act},o} \) is the number of active subcarriers for the symbols in index set \( S_{\text{act},o} \), that is, \( L_{\text{act},m,n} \) for \( n \in S_{m,o} \). The corresponding average EVM is denoted by \( \bar{\epsilon}_{\text{EVM},m,o} \).

In general, EVM can be measured at \( N_{\text{CP},m,n} \) timing instants by modifying the CP removal matrix as expressed by (23d). In this case, the timing adjustment has to be compensated by circularly shifting the OFDM symbols before taking the FFT. According to [20], the timing instant in the middle of the CP is selected as a reference point and the EVM performance before and after the reference point is measured in order to characterize the EVM performance degradation with respect to timing errors.

Fig. 9 illustrates the EVM evaluation for LTE and 5G-NR waveforms. In the case of 5G-NR waveform, the EVM is measured \( N_{\text{EVM},NR}/2 \) samples before and after the reference point, where \( N_{\text{EVM},NR} \) is the EVM window length, and the corresponding EVM values are denoted as \( \epsilon_{\text{EVM},NR}^{(\text{low})} \) and...
\( \epsilon_{\text{EVM,5G-NR}} \), respectively. The requirements for the EVM can be interpreted in the context of the EVM requirements of 5G NR, stated as \{17.5 \%, 12.5 \%, 8.0 \%, 3.5 \%\} or \{-15 \text{ dB}, -18 \text{ dB}, -22 \text{ dB}, -29 \text{ dB}\} for QPSK, 16-QAM, 64-QAM, 256-QAM, respectively [20, Table 6.5.2.2-1].

For LTE, \( \epsilon_{\text{EVM,LTE}} \) and \( \epsilon_{\text{EVM,LTE}} \) are evaluated in a same manner, however, while the 5G-NR EVM window lengths are 40–60 \% of the CP length [20, Tables B.5.2.1–B.5.2.3 for FR1], the corresponding LTE EVM windows are considerably longer, that is, 55.6–94.4 \% [25, Table E.5.1-1] implying relaxed time-synchronization requirements although more stringent requirements for waveform purity.

B. Unwanted emissions

In the base station case, out-of-band emissions (OOBEs) are unwanted emissions immediately outside the channel bandwidth. The OOBE requirements for the base station transmitter are specified both in terms of operating band unwanted emissions (OBUE) and adjacent channel leakage power ratio (ACLR). OBUE define all unwanted emissions in each supported downlink operating band as well as the frequency ranges \( \Delta f_{\text{OBUE}} \) above and \( \Delta f_{\text{OBUE}} \) below each band. In 5G NR, \( \Delta f_{\text{OBUE}} = 10 \text{ MHz} \) and \( \Delta f_{\text{OBUE}} = 40 \text{ MHz} \) in FR1 and FR2, respectively [20, Table 6.6.1-1]. ACLR is the ratio of the filtered mean power centred on the assigned channel frequency to the filtered mean power centred on an adjacent channel frequency [20].

Fig. 10 illustrates typical OBUE requirements adopting the limits defined in [20, Table 6.6.4.2.1-2]. Thin (red) response shows the non-averaged power spectral density (PSD) estimate and thick (blue) solid response shows the averaged PSD estimate with measurement bandwidth (MBW) of \( \Delta f_{\text{MBW}} \). Now, the requirements are stated such that for frequency offset of \( f_{\text{offset}} = \Delta f_{\text{MBW}}/2 \) from the channel edge, the maximum allowed power for the averaged PSD estimate is \(-7 \text{ dBm}. \) This requirement increases linearly to \(-14 \text{ dBm} \) for frequency offset of \( f_{\text{offset}} = 5 \text{ MHz} + \Delta f_{\text{MBW}}/2 \) and maintains a constant value until \( f_{\text{offset}} = 10 \text{ MHz} + \Delta f_{\text{MBW}}/2 \). The maximum carrier output power of the (wide-area) base station is vendor specific, e.g., \( P_{\text{max}} = 33 \text{ dBm} \) in 10 MHz channel, translating into 40 dB attenuation requirement at the channel edge with respect to in-band level for the waveform generation. In addition to these requirements, some additional margin is needed to cope with performance degradation due to implementation non-idealities, i.e., finite-precision arithmetic, power amplifier (PA) non-linearity, etc. Similarly, on the RX side, certain level of frequency selectivity is needed in order to limit the INI as well as to provide sufficient rejection from RF blockers and other interferences.

The PSD estimate (or the sample spectrum) of the transmitted waveform can be evaluated by taking the DFT of the time-domain waveform and then squaring the absolute value of the resulting frequency-domain response as given by [27]

\[
\tilde{\mathbf{s}}_z = \frac{1}{N_{\text{PSD}}} \left| \mathbf{W}_{N_{\text{PSD}}} \begin{bmatrix} z_n \end{bmatrix} \right|^2 .
\]  

(36)

Here, the signal is first zero padded to desired (e.g., power-of-two) length \( N_{\text{PSD}} \) and \( \mathbf{W}_{N_{\text{PSD}}} \) is the DFT matrix of size \( N_{\text{PSD}}. \)

Adopting the specifications in [20], [25], the MBW of \( \Delta_{\text{MBW}} = 100 \text{ kHz} \) is commonly used for 5G-NR bands below 1 GHz whereas for 5G-NR bands above 1 GHz, \( \Delta_{\text{MBW}} = 1 \text{ MHz} \) is also used for large frequency offsets from measurement filter centre frequency.

V. Numerical Examples

The performance and the flexibility of the proposed processing is demonstrated in terms of four examples. In all examples, FC-based filtering is also used on the RX side prior to OFDM demodulation if not stated otherwise.

A. Channelization of bandwidth parts (BWPs)

In this example, we demonstrate the division of a channel into non-contiguous BWPs with mixed-numerology. We consider 50 MHz channel with four \( (M = 4) \) BWPs such that transmission bandwidth is divided into 5 MHz, 10 MHz, 20 MHz, and 15 MHz BWPs with 30 kHz, 15 kHz, 60 kHz, and 30 kHz SCSs, respectively. According to [25, Table 5.3.2-1], the number of active subcarriers are \( L_{\text{act},0,n} = 132 \) for \( n = 0, 1, \ldots, 13 \) (11 PRBs), \( L_{\text{act},1,n} = 624 \) for \( n = 0, 1, \ldots, 6 \) (52 PRBs), \( L_{\text{act},2,n} = 288 \) for \( n = 0, 1, \ldots, 27 \) (24 PRBs), and \( L_{\text{act},3,n} = 456 \) for \( n = 0, 1, \ldots, 13 \) (38 PRBs), respectively.

The OFDM transform lengths needed to obtain sampling rate of \( f_s = 61.44 \text{ MHz} \) are \( 2N_{\text{OFDM,0,n}} = 4N_{\text{OFDM,1,n}} = 3N_{\text{OFDM,2,n}} = 5N_{\text{OFDM,3,n}} = 11N_{\text{OFDM,4,n}} \).
N_{\text{OFDM,2,n}} = 2N_{\text{OFDM,3,n}} = 1024. However, the complexity can be reduced by using interpolating FC processing with shortest power-of-two OFDM transform lengths for given number of active subcarriers, as given by (21). In this case, the OFDM transform lengths on the low-rate side can be reduced to L_{\text{OFDM,0,n}} = 2L_{\text{OFDM,1,n}} = L_{\text{OFDM,2,n}} = L_{\text{OFDM,3,n}} = 512, by using interpolating FC processing with 2I_0 = 2I_1 = I_2 = 2I_3 = 2, i.e., the FC-processing inverse and forward transform lengths are selected as 4N = L_0 = L_1 = L_2 = L_3 = 1024.

In this example, we have used guard band of 720 kHz (4 PRBs with 15 kHz SCS) between the BWPs such that the center frequencies of the BWPs are \( f_{1,\text{center}} = -21240 \) kHz, \( f_{2,\text{center}} = -2880 \) kHz, \( f_{3,\text{center}} = -13860 \) kHz, and \( f_{4,\text{center}} = 16380 \) kHz, respectively. The sub-modulations used on 5 MHz, 10 MHz, 20 MHz, and 15 MHz BWPs are 64-QAM, 16-QAM, QPSK, and 16-QAM, respectively.

The PSD of the resulting FC-based filtered-OFDM waveform is shown in Fig. 11 and the EVMs for the BWPs are shown in Fig. 12. The emission level at channel edge is \( A_s = -85.7 \) dB and the simulated EVMs values for the BWPs are \( \bar{e}_{\text{EVM,0,0}} = -51.8 \) dB, \( \bar{e}_{\text{EVM,1,0}} = -57.8 \) dB, \( \bar{e}_{\text{EVM,2,0}} = -47.0 \) dB, and \( \bar{e}_{\text{EVM,3,0}} = -52.5 \) dB. The EVM values at EVM low and high timing instances are shown in Fig. 12. As seen, the simulated EVM values are at least 25 dB better than the requirements stated in [20, Table 6.5.2.2-1].

For reference, the corresponding emission level for the WOLA-based TX is \( A_s = -60.7 \) dB while the simulated EVMs values for the BWPs when WOLA is used on TX and RX sides are \( \bar{e}_{\text{EVM,0,0}} = -31.9 \) dB, \( \bar{e}_{\text{EVM,1,0}} = -34.0 \) dB, \( \bar{e}_{\text{EVM,2,0}} = -30.8 \) dB, and \( \bar{e}_{\text{EVM,3,0}} = -33.0 \) dB. In this case, WOLA extension lengths are \( L_{\text{ext}} = L_{\text{CP,m,n}}/4 \).

### B. Wide-Band Carrier with Guard-Band IoT

In this example, we demonstrate the co-existence of 5G-NR wide-band carrier and fourth generation (4G)-based narrow-band (NB) internet-of-things (IoT) carriers in a same channel. Here, we consider 20 MHz channel with NB-IoT on the guard-band of the wide-band carrier. The wide-band carrier has \( L_{\text{act,0,n}} = 312 \) active SCs with 30 kHz SCSSs for 30 kHz SCSSs for \( n = 0, 1, \ldots, 13 \) while the NB-IoT carriers have \( L_{\text{act,1,n}} = L_{\text{act,2,n}} = L_{\text{act,3,n}} = L_{\text{act,4,n}} = 12 \) SCs with 15 kHz SCSSs for \( n = 0, 1, \ldots, 6 \). In this case, each pair of NB-IoT carriers is filtered as a single subband such that the total number of subbands (and frequency-domain windows) is three. The guard band between the narrow-band carriers with 15 kHz SCS and wide-band carrier with 30 kHz SCS is 180 kHz. In this case, 64-QAM is used on the wide-band carrier and QPSK on NB-IoT carriers. The time-frequency allocation of this example is illustrated in Fig. 13. The OFDM transform lengths are now \( 4L_{\text{OFDM,0,0}} = L_{\text{OFDM,1,n}} = L_{\text{OFDM,2,n}} = 256 \), that is, the IoT subcarriers are interpolated by eight while the FC-processing transform lengths are \( N = 8L_0 = L_1 = L_2 = 256 \).

The PSD of the resulting FC-based filtered-OFDM waveform is shown in Fig. 14 and the EVMs for the subbands are shown in Fig. 15. The power level at channel edge is \( A_s = -78.1 \) dB and the simulated EVM values for the carriers are \( \bar{e}_{\text{EVM,0,0}} = -48.9 \) dB, \( \bar{e}_{\text{EVM,1,0}} = -39.7 \) dB, and \( \bar{e}_{\text{EVM,2,0}} = -44.7 \) dB. Here, the EVM values of each pair of NB-IoT carriers are combined for simplicity. Again, the simulated EVM values are at least 25 dB above the requirements. The corresponding EVM low and high values are shown in Fig. 15.

### C. SSB-like Mixed-Numerology Scenario

In this example, we consider a synchronization signal block (SSB)-like scenario where a wide-band carrier with one SCS is punctured by the symbols with another SCS. In this case, four OFDM symbols with 15 kHz SCS (\( L_{\text{act,0,n}} = 624 \) active SCs for \( n = 0, 1, 2, 3 \)) and six symbols with 30 kHz SCS (\( L_{\text{act,0,n}} = 288 \) active SCs for \( n = 4, 5, \ldots, 9 \)) are transmitted in 10 MHz channel. Second and third symbol with 15 kHz SCS is punctured by the eight OFDM symbols with 60 kHz SCS (120 active SCs) such that the 528 innermost subcarriers...
of the symbols with 15 kHz SCS are deactivated. Third and fourth OFDM symbol with 30 kHz SCS is punctured by one OFDM symbol with 15 kHz SCS (432 active SCs) such that 240 SCs of the symbols with 30 kHz SCSS are deactivated. The resulting guard-band between the allocations with different SCSS is about 360 kHz. In this example, QPSK is used for all allocations. The time-frequency allocation of this mixed-numerology scenario is detailed in Fig. 16.

The simulated power level at the channel edge is 76.9 dB. The EVMs for the symbols for each numerology are shown in Fig. 17. As seen, the symbols with 60 kHz SCSS have the worst EVM since, for these subcarriers, the guard-band relative to SCS is the smallest. The peaks seen in the EVM responses of Fig. 17(a) are due to the time-domain transients resulting from filtering the symbols with 60 kHz SCS. Similarly, the peaks in Fig. 17(b) are the transients of allocation with 432 SCs, i.e., the improved frequency-domain localization increases the dispersion in time domain, however, even with this inter-symbol interference (ISI), the EVM levels are still within the requirements.

D. Adjustable BWPs

In this example, we demonstrate the flexibility of the proposed scheme in the case where FC-based filtering is reconfigured for each symbol. In this case, we have two variable subbands: First subband has 15 kHz SCS and $L_{act,0,n} = 192$ active SCs for $n = 0, 1, \ldots, 6$ while second has 30 kHz SCS and $L_{act,1,n} = 72$ active SCs for $n = 0, 1, \ldots, 13$. The center frequency of the first subband is adjusted as $f_{cen}(center) = 64(n - 3) \times 15$ kHz for $n = 0, 1, \ldots, 6$ and the center frequency of the second subband is $f_{cen}(center) = 218 \times 15$ kHz for $n = 0, 1, \ldots, 6$ and $f_{cen}(center) = -218 \times 15$ kHz for $n = 7, 8, \ldots, 13$. This configuration is depicted in Fig. 18.

The PSD of resulting FC-based filtered-OFDM waveform is shown in Fig. 19 while the corresponding EVMs are shown
in Fig. 20. As seen from these figures, the performance of the proposed scheme meets the requirements even at this most challenging scenario.

The presented solution allows unseen flexibility in supporting changing allocations in mixed-numerology scenarios with OFDM symbol resolution. This allows to support all envisioned use cases for 5G NR and provides a flexible starting point for sixth generation (6G) development.

VI. CONCLUSIONS

In this article, continuous symbol-synchronized fast-convolution-processing scheme was proposed, with particular emphasis on the physical-layer processing in fifth-generation new radio and beyond mobile radio networks. The proposed scheme was shown to offer various benefits over the basic continuous and discontinuous processing models, especially in providing excellent performance in reducing the unwanted emissions and inter-numerology interference in 5G-NR mixed-numerology scenarios while keeping in-band interference level well below the requirements stated in 3GPP specifications. Both dynamic and static filtering configurations are supported for all numerologies simultaneously providing greatly improved flexibility over the FC-processing schemes proposed earlier. The benefits are particularly important in specific application scenarios, like transmission of single or multiple narrow subbands, or in mini-slot type transmission, which is a core element in the ultra-reliable low-latency transmission service of 5G-NR networks.
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