Spin-dependent structure functions $\hat{g}_1$ and $\hat{g}_2$ for inclusive spin-half baryon production in electron-positron annihilation

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Two spin-dependent structure functions $\hat{g}_1$ and $\hat{g}_2$ for the inclusive spin-half baryon production in electron-positron annihilation are studied in the context of QCD factorization as well as in the naive quark parton model. As a result, it is found that the sum of $\hat{g}_1$ and $\hat{g}_2$ is related to $\hat{h}_1$ and $\hat{h}_T$, two quark fragmentation functions defined by Jaffe and Ji. In connection with the measurement of quark fragmentation functions, the possible phenomenological consequences are discussed.

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1. INTRODUCTION

In the development of our understanding of the hadron production mechanism, the inclusive one-particle production by electron-positron annihilation plays the same role as the deeply inelastic scattering does in our learning of the nucleon structures. Experimentally, a number of electron-positron colliders have been operated and will be constructed at various center of mass energies. On the theoretical side, a series of parton fragmentation functions have been identified by Jaffe and Ji [12] which characterize the information about the inclusive hadron production. Therefore, it is worthwhile to explore the possibility to measure them, especially those spin-dependent ones, by the inclusive one-particle production in electron-positron annihilation. To study spin physics related to the inclusive spin-half baryon production, it is usually needed to monitor the polarization of the produced particles. If we are restricted with the inclusive hyperon production, it is quite feasible to measure the polarization of the produced particles. In fact, a series of experiments have already been done at Fermilab measuring the polarization of the inclusive hyperons in fixed-target experiments [3]. We also note that some preliminary results [4] on the measurement of the inclusive spin-half baryon production by electron-positron annihilation, and the quark fragmentation functions defined by Jaffe and Ji.

Without loss of any generalities, we consider the inclusive Λ hyperon production for among the hyperons the Λ particle is of the largest production cross section and its polarization is the easiest to measure. The advantages of measuring the polarization of the inclusive Λ hyperon are also realized by several groups of authors in different cases. Among others, Burkardt and Jaffe [5] investigated the possibility to measure the s → Λ fragmentation functions at LEP, and Chen et al. [6] discussed the more complicated semi-inclusive processes e+e− → ΛXAX. For simplicity, we work in the energy region not too high so that only the photon channel needs to be considered. Then, all the information about the hyperon production is entailed by the photon fragmentation tensor, which is defined as

\[ \hat{W}_{\mu\nu}(q, p, s) = \frac{1}{4\pi} \sum_X \int d^4 \xi \exp(\text{i} q \cdot \xi) \langle 0 | j_\mu(0) | \Lambda(p, s) \rangle \langle \Lambda(p, s) , X | j_\nu(\xi) | 0 \rangle, \]

where \( \sum_X \) represents the summation over all the possible final states that contain the inclusive Λ hyperon. Throughout the work, we normalize the spin vector in such a way that \( s \cdot s = -1 \) for a pure state. The electromagnetic current is defined as \( j_\mu = \sum_f \bar{\psi}_f \gamma^\mu \psi_f \), with \( f \) being the quark flavor index and \( \gamma^\mu \) being the electric charge of the quark in unit of the electron charge. In our presentation, we will suppress the flavor index whenever possible. Subjected to the gauge invariance, hermiticity and parity conservation, \( \hat{W}(q, p, s) \) assumes the following general Lorentz decomposition [8]:

\[ \hat{W}_{\mu\nu}(q, p, s) = \frac{1}{2} \left[ (-g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) \hat{F}_1(z_B, Q^2) + (p_\mu - \frac{q \cdot q}{q^2} q_\mu)(p_\nu - \frac{q \cdot q}{q^2} q_\nu) \hat{F}_2(z_B, Q^2) \right] \\
+ i M z_B \epsilon_{\mu\nu\rho\sigma} q^\rho \hat{g}_1(z_B, Q^2) + i M \epsilon_{\mu\nu\rho\sigma} q^\rho \hat{g}_2(z_B, Q^2) \\
+ M \left[ (p_\mu - \frac{p \cdot q}{q^2} q_\mu) \epsilon_{\nu\rho\sigma\tau} p^\rho q^\sigma s^\tau + (p_\nu - \frac{p \cdot q}{q^2} q_\nu) \epsilon_{\mu\tau\rho\sigma} p^\rho q^\sigma s^\tau \right] \hat{F}(z_B, Q^2) \]

where \( z_B \equiv 2(p \cdot q)/q^2, Q \equiv \sqrt{q^2}, M \) is the mass of the Λ hyperon, and \( \hat{F}_1, \hat{F}_2, \hat{g}_1, \hat{g}_2, \) and \( \hat{F} \) are the scaling function structures. It should be noted that due to the final-state interactions, the time reversal invariance cannot exert any constraints on the Lorentz decomposition of \( \hat{W}_{\mu\nu}(q, p, s) \).

Obviously, there exists one-to-one correspondence between \( \hat{F}_1, \hat{F}_2, \hat{g}_1, \) and \( \hat{g}_2, \) on the one hand, and four structure functions \( F_1, F_2, g_1 \) and \( g_2 \) in the deeply inelastic scattering, on the other hand. \( F_1 \) and \( F_2 \) have been vastly discussed in the literature [8], so we will simply ignore them in our discussion. As for the structure function \( F \), which arises from the final-state interaction, we will not address it, to avoid dispersing the reader’s sight. Our attention will be focused on \( \hat{g}_1 \) and \( \hat{g}_2 \) so that our results can be compared with their counterparts in the deeply inelastic scattering case [8].

We establish our coordinate system by letting the \( \hat{z} \) axis be along the outgoing direction of the inclusive hyperon and the \( \hat{x}-\hat{z} \) plane in the production plane. We adopt the light-cone coordinates and parameterize the Λ momentum as

\[ p^\mu = P^\mu + \frac{1}{2} M^2 n^\mu, \]

where

\[ P^\mu = \frac{1}{\sqrt{2}}(\sqrt{M^2 + |p|^2} + |p|)(1^+, 0^-, 0_8), \]

\[ n^\mu = \frac{\sqrt{2}}{M^2}(\sqrt{M^2 + |p|^2} - |p|)(0^+, 1^-, 0_8). \]

(5)

with \( |p| \) being the magnitude of the Λ momentum. We will work in the frame in which \( |p| \) has a large value. Obviously, \( P \) and \( n \) are light-like and they satisfy \( P \cdot n = 1 \).
II. NAIVE PARTON MODEL RESULTS

We feel it desirable to present a naive parton model approach to $\hat{g}_1$ and $\hat{g}_2$, since the corresponding results can supply us with a benchmark for comparison. In the naive parton model, the fragmented hadrons will be completely collinear to their parent quark parton, with no transverse momentum effects. Correspondingly, we need to assign a mass $m_f = M/z$ to the quark of flavor $f$, where $z$ is the momentum fraction of the quark carried by the inclusive $\Lambda$ particle. $\hat{W}^{\mu\nu}(q, p, s)$ can be obtained from the convolution of the underlying tensor $\hat{w}^{\mu\nu}_f(q, z, s_f)$ for photon fragmentation into a quark-antiquark pair with the $\Lambda$ multiplicity distribution $N_{f\rightarrow\Lambda}(z, s_f, s)$ in the jet initiated by the quark of flavor $f$ and spin four-vector $s_f$:

$$\hat{W}^{\mu\nu}(q, p, s) = \sum_f \int \frac{dz}{z} \delta(z - z_B) \hat{w}^{\mu\nu}_f(q, z, s_f) N_{f\rightarrow\Lambda}(z, s_f, s).$$

By analogy with the well-known leptonic tensor, we know

$$\hat{w}^{\mu\nu}_f(q, z, s_f) = e_f^2 \left[ -g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2} \right] q^2 - \frac{4}{z^2} (p^\mu - \frac{p^\nu q^\nu}{q^2}) (p^\nu - \frac{p^\mu q^\mu}{q^2}) + 2 i m_f e^{\mu\nu\tau\rho} q_{s_f}^\tau q_{s_f}^\rho.$$

Therefore, we have the following results for $\hat{g}_1$ and $\hat{g}_2$:

$$\hat{g}_1(z_B) = \frac{1}{z_B} \sum_f e_f^2 \Delta D_f(z_B, s),$$

$$\hat{g}_2(z_B) = 0,$$

where

$$\Delta D_f(z_B, s) = N_{f\rightarrow\Lambda}(z, s, s) - N_{f\rightarrow\Lambda}(z, -s, s)$$

is the $\Lambda$ number density difference with spin parallel and antiparallel to the quark spin in the $f$-flavored quark jet. In the naive parton model, if one boosts into the quark rest frame, all its fragmenting products will be at rest, too. Hence, $\Delta D_f(z_B, s)$ does not depend on the spin quantization direction.

III. QCD FACTORIZATION APPROACH

In the QCD field theory approach, the photon fragmentation tensor is composed of the contributions of an infinite series of the so-called cut Feynman diagrams. The cut diagram is formed by piecing together the Feynman diagram of one amplitude with the complex conjugate of another one with the same initial and final states. Owing to lack of the methods to treat nonperturbative interactions, one usually adopt the strategy to factorize the hadron-involved process into short- and long-distance parts. The former can be calculated perturbatively whereas the latter can be measured experimentally. In such a factorization approach, QCD long-distance interactions are conventionally represented by blobs in the cut diagram. In general, these cut diagrams begin to contribute at the different powers of the hard scattering scale. If a physical quantity goes like $Q^{\tau-2}$ or begins to make contributions at $O(Q^{\tau})$, it is referred to as being twist-$\tau$. According to Qiu and Sterman, the twist at which a cut diagram begins to contribute is controlled by the number of physical parton lines that attach to the long-distance interaction blobs. Now it is well known that some potentially power-suppressed twist-three spin asymmetries are relatively large, so in this paper we will work up to the twist-three level, i.e., to the first nonleading power. In addition, we work in the light-cone gauge specified by $n \cdot A = 0$. Correspondingly, the cut diagrams shown in figs. 1, and 2 consist of our subjects.

A. Longitudinally-polarized $\Lambda$ hyperon

We begin with the case in which the inclusive $\Lambda$ is of plus helicity. In this case, the leading spin dependence of $\hat{W}^{\mu\nu}(q, p, s)$ comes about at twist two, while its next-to-leading spin dependence at twist four. To protrude the main physics, we work at leading twist and accordingly only the lowest-order diagram shown in fig. 1 gets into work. As a result,

$$\hat{W}^{\mu\nu}(q, p, s_{||}) = \frac{1}{4\pi N} \int \frac{d^4k}{(2\pi)^4} \text{Tr}_D \text{Tr}_C \left[ H^{\mu\nu}_{(1)}(q, k) T(k, p, s_{||}) \right],$$

where $\text{Tr}_D$ and $\text{Tr}_C$ stands for making traces in the Dirac and color spaces, and

$$T_{\alpha\beta}(k, p, s_{||}) = \sum_X \int d^4\xi \exp(-ik \cdot \xi) \langle \psi_s(0)|\Lambda(p, s_{||}), X \rangle \langle \Lambda(p, s_{||}), X |\bar{\psi}_s(\xi)|0 \rangle.$$
Throughout all, we reserve the subscripts $\alpha$ and $\beta$ for the Dirac indices. The color summation is implicit in our parton fragmentation matrices, so there is a color factor $1/N$ ($N = 3$) on the right-hand side of Eq. (11). Since we work in the physical gauge, the gauge linkages of the form $P \exp[-ig \int_0^\infty dy^\mu A^\mu(y')]$ are identical to unity and hence suppressed in our parton fragmentation matrices.

Since high-energy reactions are light-cone dominant, the most efficient way to extract the leading contributions from each diagram is to make an expansion about the components of the parton momenta that are collinear to the corresponding hadron momentum. After that, one must further decouple the discrete indices between the long-distance matrices and the associated short-distance parts. For completing these tasks, one can employ the collinear technique developed by Ellis, Furmanski and Petonzio [13].

Let us parameterize the quark momentum as

$$k^\mu = \frac{1}{z} P^\mu + k_T^\mu + \frac{k_T^2 - |k_T|^2}{2k \cdot n} n^\mu.$$  \hspace{1cm} (13)

Then, the collinear expansion for the lowest-order diagram reads

$$H_{\mu\nu}(q, k) = H_{\mu\nu}(q, P/z) + \frac{\partial H_{\mu\nu}(q, k)}{\partial k^\tau}|_{k=P/z} (k - P/z)^\tau + \cdots.$$  \hspace{1cm} (14)

Since working at twist two, here we need only to consider the contributions associated with the leading term. As a result, we obtain

$$\hat{W}_{\mu\nu}(q, p, s_{||}) = \frac{1}{4\pi N} \int \frac{dz}{z} Tr_D Tr_C \left[ H_{\mu\nu}(q, P/z) T(z, p, s_{||}) \right],$$  \hspace{1cm} (15)

with

$$T_{\alpha\beta}(z, p, s_{||}) = z \sum_x \int \frac{d\lambda}{2\pi} \exp(-i\lambda/z) \langle 0 | \psi_{\alpha}(0) | \Lambda(p, s_{||}) \rangle \langle \Lambda(p, s_{||}) , X | \tilde{\psi}_{\beta}(\lambda n) | 0 \rangle.$$  \hspace{1cm} (16)

Since the color index has been decoupled in Eq. (14), now the only task for us is to decouple the Dirac indices between the expanded hard-interaction part $H_{(1)}^{\mu\nu}(q, P/z)$ and the simplified quark fragmentation matrix $T(z, p, s_{||})$. Expanding the latter in the Dirac space, we have

$$T(z, p, s_{||}) = M(s_{||} \cdot n) \gamma_5 P \tilde{g}_1(z) + M(s_{||} \cdot n) (P \not\! n - \not\! P) \gamma_5 \tilde{h}_L(z) + M^2(s_{||} \cdot n) \not\! n \gamma_5 \tilde{g}_3(z) + \cdots,$$  \hspace{1cm} (17)

where we have suppressed the spin-independent terms as well as those arising from the hadronic final-state interaction because we do not address $F_1$ and $F_2$ as well as $\tilde{F}_1(z), \tilde{h}_L(z)$ and $\tilde{g}_3(z)$ are quark fragmentation functions (matrix elements) at twist two, three, and four respectively, and their definitions can be found in Ref. [15]. From the chiral-odd structure of $\tilde{h}_L(z)$, one can immediately see that only by a quark mass insertion in the hard part can it make a nonzero contribution, so its contribution is actually at twist four.

Keeping the first term in Eq. (17), we will have

$$\hat{W}_{\mu\nu}(q, p, s_{||}) = \frac{i}{z_0 P \cdot q} \gamma_\nu \gamma_\tau q_s s_{||} \sum_f c_f^2 \tilde{g}_1(z_0),$$  \hspace{1cm} (18)

where $z_0 \equiv 2(P \cdot q)/q^2$. Notice that we have recovered the quark flavor index here. To confront with the general decomposition of $\hat{W}_{\mu\nu}(q, p, s_{||})$, we can simply substitute $p$ for $P$ and correspondingly $z_0$ for $z_B$. Such replacements amount to including some effects at two higher twist so they are allowed. As a consequence, we have the following twist-two formulas:

$$\tilde{g}_1(x_B) = \frac{1}{z_B} \sum_f c_f^2 \tilde{g}_1(z_B),$$  \hspace{1cm} (19)

$$g_2(x_B) = 0.$$  \hspace{1cm} (20)

In fact, our factorization at leading twist reproduced the naive parton model results about $\tilde{g}_1$ and $g_2$. The easiest way to recognize this point is to quantize the quark filed in the light-cone quantization formalism [13] and carry out the integration over the unobserved hadron system [15]. Physically, if one notices that the quark transverse momentum with respect to the $\Lambda$ momentum has been integrated out, it is not difficult to understand the equivalence of this twist-two factorization result to the naive parton model approach.
B. Transversely-polarized Λ hyperon

Now we turn to the case in which the spin of the inclusive Λ hyperon is aligned along the normal of the production plane, namely, the \( \hat{y} \) axis. In this case, \( s \cdot q = 0 \), so the \( \hat{g}_1 \) and \( \hat{g}_2 \) terms in Eq. (1) become degenerate. Therefore, we hope to express the sum of \( \hat{g}_1 \) and \( \hat{g}_2 \) in terms of some spin-dependent quark fragmentation functions. Related to the transverse spin of the inclusive Λ particle, the twist-two quark fragmentation function \( \hat{h}_1 \) is chiral-odd. As a result, its contribution comes about via a quark mass insertion in the hard part and correspondingly is at twist three. Hence, we need to work at least at twist three and the lowest-order diagram as well as those two in fig. 2 construct our subject.

To the order at which we work,

\[
\begin{align*}
W^{\mu\nu}(q, p, s_{\perp}) &= \frac{1}{4\pi N} \int \frac{d^4 k}{(2\pi)^4} \text{Tr}D \text{Tr}_C \left[ H^{\mu\nu}_{(1)}(q, k) T(k, p, s_{\perp}) \right] \\
&\quad + \frac{1}{4\pi N} \int \frac{d^4 k}{(2\pi)^4} \left( \frac{d^4 k_1}{(2\pi)^4} \text{Tr}D \text{Tr}_C \left[ H^{\mu\nu}_{(2a)}(q, k, k_1) X_{\alpha}(k, k_1, p, s_{\perp}) \right] \right) \\
&\quad + \frac{1}{4\pi N} \int \frac{d^4 k}{(2\pi)^4} \left( \frac{d^4 k_1}{(2\pi)^4} \text{Tr}D \text{Tr}_C \left[ H^{\mu\nu}_{(2b)}(q, k, k_1) Y_{\alpha}(k, k_1, p, s_{\perp}) \right] \right), \tag{21}
\end{align*}
\]

where

\[
T_{\alpha\beta}(k, p, s_{\perp}) = \sum_X \int d^4 \xi \exp(-ik \cdot \xi) \langle 0|\psi_{\alpha}(0)|\Lambda(p, s_{\perp}), X \rangle \langle \Lambda(p, s_{\perp}), X |\bar{\psi}_{\beta}(\xi)|0, \tag{22}
\]

\[
X_{\alpha\beta}^{\mu\nu}(k_1, k, p, s_{\perp}) = \sum_X \int d^4 \xi d^4 \xi_1 \exp(-i(k - k_1) \cdot \xi_1 - ik \cdot \xi) \times \langle 0|(-g)A^\sigma(\xi_1)\psi_{\alpha}(0)|\Lambda(p, s_{\perp}), X \rangle \langle \Lambda(p, s_{\perp}), X |\bar{\psi}_{\beta}(\xi n)|0, \tag{23}
\]

\[
Y_{\alpha\beta}^{\mu\nu}(k_1, k, p, s_{\perp}) = \sum_X \int d^4 \xi d^4 \xi_1 \exp(-i(k_1 - k) \cdot \xi - i\xi_1 \cdot \xi_1) \times \langle 0|\psi_{\alpha}(0)|\Lambda(p, s_{\perp}), X \rangle \langle \Lambda(p, s_{\perp}), X |\bar{\psi}_{\beta}(\xi_1 n)(-g)A^\sigma(\xi)|0. \tag{24}
\]

Here we note that in writing down Eq. (21), we have absorbed the color matrix along with a minus strong coupling \(-gT_{ij}^a\) into \(X_{\alpha\beta}^{\mu\nu}(k_1, k, p, s_{\perp})\) and \(Y_{\alpha\beta}^{\mu\nu}(k_1, k, p, s_{\perp})\). Obviously, the gauge invariance of Eq. (21) is not manifest.

Since we work up to twist three, we need to expand the hard part for the lowest-order diagram to the nonleading term. As for those shown in fig. 2, it is enough to take their leading contributions. Making use of the Ward identity, the twist-three contributions associated with the first derivative term in the expansion of the lowest-order hard part can be combined with the leading contributions of the two diagrams shown in fig. 2. The net effect of such a combination is to replace the gluon field tensor by the covariant derivative operator in the two-variable fragmentation matrices coupled with the hard parts of the diagrams in fig. 2.

However, the Ellis-Furnanzs-Petronzio scheme is not a satisfactory procedure to extract nonleading twist contributions. The basic reason is that the leading term in the collinear expansion contains as well the nonleading contributions. To isolate such nonleading effects implicit in the leading term, the efficient way is to pull down a certain number of “special” propagators into the hard part. In our twist-three case, we need to pull down one special propagator along with the connected quark-gluon vertex into the hard part, either on the left-hand side or on the right-hand side of the final-state cut. In other words, twist-three contributions hidden in the leading term of the collinear expansion of the lowest-order diagram can be taken into account by including the two diagrams in fig. 3. See what follows.

The notation of the special propagator can be introduced as follows. Consider the momentum carried by the quark propagator in fig. 2 that links the electromagnetic vertex to the quark-gluon one, one can parameterize it as

\[
k^\mu = \hat{k}^\mu + \frac{k^2}{2k \cdot n} n^\mu, \tag{25}
\]

where

\[
\hat{k}^\mu = \frac{1}{z} P^\mu + k^\mu - \frac{k^2}{2k \cdot n} n^\mu \tag{26}
\]

is the on-shell part of \(k^\mu\). Correspondingly, the quark propagator is decomposed into two parts:

\[
\frac{i\hat{k}}{k^2 + i\varepsilon} = \frac{i\hat{k}}{k^2 + i\varepsilon} + \frac{i\hat{n}}{2k \cdot n}. \tag{27}
\]
Technically, $in \cdot \gamma/(2k \cdot n)$ is termed the special propagator by Qiu and labelled by adding a bar on the normal propagator in the graphic representation. To unravel the physical content of the special propagator, let us consider the Fourier transformation

$$
\int \frac{dk^-}{2\pi} \exp[-ik^- (\xi_1 - \xi_2)] \frac{-ik}{k^2 + i\varepsilon} = \delta(\xi_1 - \xi_2) \frac{i\gamma^+}{2k^+} + \theta(\xi_1 - \xi_2) \frac{\hat{k}}{2k^+} \exp[-i \frac{k^2}{2k^+} (\xi_1 + \xi_2)],
$$

(28)

from which it can be seen that the special propagator describes a short-distance, or “contact” interaction in the light-cone. Hence, as one goes beyond the leading twist for the lowest-order diagram, a special propagator along with the connected quark-gluon vertex should be pulled down into the hard part. Considering that the diagrams shown in fig. 3 can be obtained from the lowest-order diagram by pulling out a quark propagator and its connected quark-gluon vertex, it can also be stated that the non-contact part of its linking propagator should be discarded, because they have been included in $T_{\alpha\beta}(k, p, s_{\perp})$. It should be stressed that Qiu [16] has demonstrated that such a cure can naturally reserve the gauge invariance for the hard part.

From the above discussions, we know that the formula for calculating the hadronic tensor reads

$$
\tilde{W}^{\mu\nu}(q, p, s_{\perp}) = \frac{1}{4\pi N} \int \frac{dz}{z} Tr_{D} Tr_{C} \left[ H_{(1)}^{\mu\nu}(q, P/z) T(z, p, s_{\perp}) \right]
+ \frac{1}{4\pi N} \int \frac{dz}{z} Tr_{D} Tr_{C} \left[ \left( H_{(2a)}^{\mu\nu}(q, P/z, P/z_{1}) + H_{(3a)}^{\mu\nu}(q, P/z) \right) X_{\alpha}(z, z_{1}, p, s_{\perp}) \right]
+ \frac{1}{4\pi N} \int \frac{dz}{z} Tr_{D} Tr_{C} \left[ \left( H_{(2b)}^{\mu\nu}(q, P/z, P/z_{1}) + H_{(3b)}^{\mu\nu}(q, P/z) \right) Y_{\alpha}(z, z_{1}, p, s_{\perp}) \right],
$$

(29)

where

$$
T_{\alpha\beta}(z, p, s_{\perp}) = \sum X_{\alpha\beta}(z_{1}, z, p, s_{\perp}) = \frac{1}{2\pi} \sum_{X} \frac{d\lambda}{(2\pi)^2} \exp(-i\lambda(1/z - 1/z_{1}) - i\lambda/z)
\times (0|D^{\alpha}(\lambda n)\psi_{\alpha}(0)|\Lambda(p, s_{\perp}, X)\Lambda(p, s_{\perp}, X)|\tilde{\psi}_{\beta}(\lambda n)^{\alpha}\rangle 0),
$$

(30)

$$
X_{\alpha\beta}(z_{1}, z, p, s_{\perp}) = \sum X_{\alpha\beta}(z_{1}, z, p, s_{\perp}) = \frac{1}{2\pi} \sum_{X} \frac{d\lambda}{(2\pi)^2} \exp(-i\lambda(1/z - 1/z_{1}) - i\lambda/z)
\times (0|D^{\alpha}(\lambda n)\psi_{\alpha}(0)|\Lambda(p, s_{\perp}, X)\Lambda(p, s_{\perp}, X)|\tilde{\psi}_{\beta}(\lambda n)^{\alpha}\rangle 0),
$$

(31)

$$
Y_{\alpha\beta}(z_{1}, z, p, s_{\perp}) = \sum X_{\alpha\beta}(z_{1}, z, p, s_{\perp}) = \frac{1}{2\pi} \sum_{X} \frac{d\lambda}{(2\pi)^2} \exp(-i\lambda(1/z - 1/z_{1}) - i\lambda/z)
\times (0|\psi_{\alpha}(0)|\Lambda(p, s_{\perp}, X)\Lambda(p, s_{\perp}, X)|\tilde{\psi}_{\beta}(\lambda n)^{\alpha}\rangle 0).
$$

(32)

In our work, the covariant derivative operator is defined as $D^{\alpha} = i\partial^{\alpha} - gA^{\alpha}$ and $D^{\alpha}_{\perp} = i\partial^{\alpha}_{\perp} - gA^{\alpha}_{\perp}$.

To arrive at factorized expressions, we decompose the above three fragmentation matrices in the Dirac and Lorentz spaces. Again, we suppress the spin-independent terms as well as those arising from the final-state interactions for the brevity of formulas. As a result,

$$
T_{\alpha\beta}(z, p, s_{\perp}) = [\hat{h}_{1}(z)\gamma_{5}\gamma_{\perp}P + M\hat{g}_{T}(z)\gamma_{5}\gamma_{\perp} + \cdots]_{\alpha\beta},
$$

(33)

$$
X_{\alpha\beta}(z_{1}, z, p, s_{\perp}) = \frac{iM}{2z} \hat{G}_{1}(z_{1}, z)\epsilon_{\sigma\rho\eta\gamma}^{\star}P^{\sigma}n^{\eta}P_{\alpha\beta} + \frac{M}{2z} \hat{G}_{2}(z_{1}, z)\eta_{\alpha\beta}^{\star}(\gamma_{5}P)_{\alpha\beta} + \cdots,
$$

(34)

$$
Y_{\alpha\beta}(z_{1}, z, p, s_{\perp}) = -\frac{iM}{2z} \hat{G}_{1}(z_{1}, z)\epsilon_{\sigma\rho\eta\gamma}^{\star}P^{\sigma}n^{\eta}P_{\alpha\beta} + \frac{M}{2z} \hat{G}_{2}(z_{1}, z)\eta_{\alpha\beta}^{\star}(\gamma_{5}P)_{\alpha\beta} + \cdots,
$$

(35)

where $\hat{h}_{1}(z)$, $\hat{g}_{T}(z)$, $\hat{G}_{1}(z_{1}, z)$, and $\hat{G}_{2}(z_{1}, z)$ are Jaffe and Ji’s parton fragmentation matrix elements, whose definitions can be easily projected out from the above decompositions.

Using QCD equation of motion, one can prove [1] that

$$
\int d(\frac{1}{z_{1}})[\hat{G}_{1}(z_{1}, z) + \hat{G}_{2}(z_{1}, z)] = -\frac{1}{z}\hat{g}_{T}(z) + \frac{m}{M}\hat{h}_{1}(z),
$$

(36)

where $m$ is the quark mass.

Inserting Eqs. (33), (35) into (28) and completing the algebra, we obtain
\[ \bar{W}_{\mu\nu}(q,p,s) = \frac{i}{z_B(P,q)} \sum_f e_f^2 \left[ z_0 m_f \bar{h}_f^z(z_0) + \bar{M} \bar{g}_f^z(z_0) \right] \varepsilon_{\mu\nu\rho\sigma} q^\rho s^\sigma. \] (37)

Here we note this result is complete up to twist three. Again, we make substitutions \( p \to P \) and \( z_0 \to z_B \). By confronting with Eq. (37), we attain for \( \hat{g}_1 \) and \( \hat{g}_2 \) the following relation:

\[ \hat{g}_1(x_B) + \hat{g}_2(x_B) = \frac{1}{z_B} \sum_f e_f^2 \left[ \frac{m_f}{\bar{M}} z_B \bar{h}_f^z(x_B) + \hat{g}_f^z(x_B) \right]. \] (38)

IV. CONCLUSION

We conclude this paper by discussing the phenomenological implications of our factorization results about \( \hat{g}_1 \) and \( \hat{g}_2 \). Here we assume that the \( \Lambda \) hyperon is predominantly produced via the strange quark fragmentation. Then, both in Eq. (19) and in (35) the flavor summation can be dropped. Furthermore, the terms associated with the quark mass in (35) can be ignored at a first-order approximation because \( m_s/M \approx 0.1 \). Although not discussed here, we can believe that by controlling judiciously the polarization of the initial-state electron beam and analyzing the final-state \( \Lambda \) polarization, it is possible to measure \( \hat{g}_1 \) and \( \hat{g}_2 \). Therefore, the measurement of these two structure functions will allow for the determination of two \( s \to \Lambda \) fragmentation functions \( \bar{g}_1(z) \) and \( \bar{g}_T(z) \).

It should be noted that Eq. (38) might be not so helpful. The reason is that \( \hat{g}_1(z) \) can be independently measured at the LEP, which Burkardt and Jaffe have discussed in Ref. [8]. At the Z\(^0\) resonance, the parity violation can naturally generate longitudinally-polarized strange quarks so one does not need to polarize the initial-state beam. However, Eq. (38) is by no means trivial. So far, the measurement of chiral-odd nonperturbative matrix elements, to which \( \bar{g}_1 \) and \( \bar{g}_2 \) belong, have been a challenge to the particle physics community. So far, the general strategy to treat them has been tackling them in pairs [1], [2], [7]. Our results, Eq. (38), indicates that once we have known \( \bar{h}_1(z) \) from other sources or it can be neglected at the first approximation, the measurement of \( \hat{g}_1 \) and \( \hat{g}_2 \) present us a relatively clean situation, in which information about twist-three fragmentation function \( \bar{g}_T(z) \) can be obtained. The penalty in such a scheme is to polarize one of the initial-state beams. As is well known, it is now quite feasible to obtain highly polarized electron beams.

In summary, we worked out a QCD field theory study of \( \hat{g}_1 \) and \( \hat{g}_2 \), two spin-dependent structure functions for the inclusive spin-half baryon production in electron-positron annihilation. Making use of the collinear expansion procedure by Ellis, Furmanski and Petronzio, and the special propagator concept by Qiu, we derived a formula relating the sum of \( \bar{h}_1 \) and \( \bar{g}_T \) to the transverse-spin-dependent quark fragmentation functions \( \bar{h}_1(z) \) and \( \bar{g}_T(z) \). On the basis of this finding, we pointed out that it is possible to extract data about \( \bar{g}_T(z) \) in the inclusive \( \Lambda \) hyperon production by electron-positron annihilation. Considering the common believe that chiral-odd hadron matrix elements and/or parton fragmentation matrix elements should be measured in pairs, our results can be taken as an interesting counter example.

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Figure Captions

1. The lowest-order cut diagram for the inclusive $\Lambda$ hyperon production by a time-like photon.

2. The cut diagram for the inclusive hyperon production by a time-like photon with one gluon correlation at the stage of parton fragmentation.

3. The cut diagram for the inclusive $\Lambda$ hyperon production by a time-like photon with one gluon radiation in the quark fragmentation. One “special” propagator is pulled down into the hard partonic interaction part, either (a) on the left-hand side or (b) on the right-hand side of the final-state cut.
Fig. 2(b)
