Calculation of the deviation coefficients for marine magnetic compass
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ABSTRACT
Deviation coefficients of magnetic compass are a basis to determine the compass deviation as well as to establish the table for compass deviation which is considered as technical certificate for magnetic compass equipped on board the ship. Normally, magnetic compass correction expert needs to observe the deviation values in main course of compass and then insert them into deviation equation system for obtaining the deviation coefficients. In the existing method, the special courses for requirement are on eight main directions such as (N, S, E, W, NE, SE, SW, and NW). These special courses and respective deviations are replaced into the deviation equation to obtain eight equations. Then, by expanding the equations, the deviation coefficients are found out. However, this method has some drawbacks: The observed courses are special ones; the solution of equation is impossible if the number of observation is less than eight. In this paper, the author suggests a new method to calculate the deviation coefficients of compass quickly by observing the compass’s deviation in any course. This method is easy to carry out and without requirement of deviation observation on special courses.

Introduction
Nowadays, with the development of technology science, the role of finding course exactly has changed into the electrical equipment such as gyroscopic and satellite compasses. But, the disadvantage of these apparatuses is out of work as electrical power is cut. Therefore, magnetic compass, which works only based on the magnetic field of earth, has been used on board due to an operational ability without an electrical power. According to requirements of SOLAS convention (Safety of Life at Sea), the magnetic compass is an obligated apparatus which is required to install on board of sea-going ship.

The compass needle is oriented to north direction by a magnetic force produced by the magnetic field of earth. However, when using this equipment, it finds that the magnetic of earth also acts on the metal materials on ship and magnetize them to cause another magnetic field which is called as the magnetic field of ship. This magnetic field also affects to the needle of magnetic compass. Therefore, the orientation of magnetic compass is influenced simultaneously by both earth and on-board iron magnetic fields. The total of these forces makes the magnetic compass to have low accuracy and significant error in course.

To ensure the compass getting enough conditions before operation, the deviation, and error of magnetic compass must be compensated by adjusting available instruments such as: magnet bars, Flinder bar, and soft-iron spheres. When ship is newly launched or as the large repairs after operation period, the magnetic compass needs to be adjusted so that the compass’ deviation is under the permission range. As regulation, the remaining deviation after correction of magnetic compass should be limited within (±3 deg). After compensating the deviation, the remained deviations are observed and recorded to establish the deviation table for the magnetic compass of ship. This table then is printed out and pasted on bridge of ship as certificate of compass standard as in Figure 1. To build up this certificate, the deviation coefficients must be first determined by the calculation method, the deviation curve is then plotted for the deviations in arbitrary course.

Until now, there is only a unique method to calculate the deviation coefficients based on the deviations recorded in typical courses (Denne 1979). However, this method has some drawbacks as: The observed courses are special ones; the solution by this method is impossible if the number of observation is under and over eight. In this paper, the author suggests a new method to calculate the deviation coefficients of compass quickly by observing the compass’s deviation in arbitrary courses. In order to realize that work, the deviations of compass need to be observed and the recorded values are inserted into the deviation equation for calculating the deviation coefficients such as A, B, C, D, and E. Then the least square technique is used to determine these coefficients. In comparison with the existing method, this proposed method does not require the constraint conditions for
recorded courses. In addition, this method can be applied as the equation number of system is over and under eight.

The existing method for finding the deviation coefficients of magnetic compass

The existing method to calculate the deviation coefficients requires the special courses as eight main directions such as (North, South, East, West, North East, South East, South West, and North West). These special courses and respective deviations are replaced into the deviation equation system to obtain eight equations. Then, by expanding the equations, the deviation coefficients are found out.

The basic formula (Denne 1979) for finding the deviation coefficients of magnetic compass is presented as follows:

\[ \delta = A + B \sin \theta + C \cos \theta + D \sin 2\theta + E \cos 2\theta \]  (1)

where: (A, B, C, D, and E) are, respectively, the deviation coefficients of compass, the (Hd) symbol denotes the compass course observed.

By the traditional method, the compass’s deviations are observed on eight different directions. Then, the observed values are inserted into the Equation (1) to calculate the results (A, B, C, D, E).

Assumption that \( \delta_N, \delta_{NE}, \delta_E, \delta_{SE}, \delta_S, \delta_{SW}, \delta_W, \) and \( \delta_{NW} \) are the observed values of compass’s deviation on (N, S, E, W, NE, SE, SW, and NW) courses. Rewriting Equation (1), we have following system:

\[ A = \delta_N + \delta_{NE} + \delta_E + \delta_{SE} + \delta_S + \delta_{SW} + \delta_W + \delta_{NW} \]

\[ B = \frac{\delta_N - \delta_S}{4} + \frac{\delta_{NE} - \delta_{SE}}{4} + \frac{\delta_E + \delta_{SW}}{4} \sin 45^0 \]

\[ C = \frac{\delta_N + \delta_S}{4} + \frac{\delta_{NE} + \delta_{SE}}{4} + \frac{\delta_E - \delta_{SW}}{4} \sin 45^0 \]

\[ D = \frac{\delta_N + \delta_S}{4} - \frac{\delta_{NE} + \delta_{SE}}{4} \sin 45^0 \]

\[ E = \frac{\delta_N - \delta_S}{4} + \frac{\delta_{NE} - \delta_{SE}}{4} \sin 45^0 \]

By using a same way to find the coefficients (A, B, C, D, E). In the equations which contain the solving unknown, two sides of each equation are multiplied with trigonometrical function including the sign of that function which is beside the solving unknown. Then, the expanding equations are summed together to obtain the results, detail as shown as follows:

\[ A = \delta_N + \delta_{NE} + \delta_E + \delta_{SE} + \delta_S + \delta_{SW} + \delta_W + \delta_{NW} \]

\[ B = \frac{\delta_N - \delta_S}{4} + \frac{\delta_{NE} - \delta_{SE}}{4} + \frac{\delta_E + \delta_{SW}}{4} \sin 45^0 \]

\[ C = \frac{\delta_N + \delta_S}{4} + \frac{\delta_{NE} + \delta_{SE}}{4} + \frac{\delta_E - \delta_{SW}}{4} \sin 45^0 \]

\[ D = \frac{\delta_N + \delta_S}{4} - \frac{\delta_{NE} + \delta_{SE}}{4} \sin 45^0 \]

\[ E = \frac{\delta_N - \delta_S}{4} + \frac{\delta_{NE} - \delta_{SE}}{4} \sin 45^0 \]
From the results presented in Equation (4), it can see that by the existing method, the deviation coefficients of compass is only determined if the observations are carried out on special courses as (N, S, E, W, NE, SE, SW, and NW). This is difficult to determine the deviation coefficient on arbitrary courses. To overcome that limitation, the new method is proposed based on the least square algorithm (Least square algorithm).

**The proposed method for calculating the deviation coefficients of magnetic compass**

Observing the deviation values on arbitrary courses and obtaining values $\delta_i$. Assumption that $S$ is a sum of mean square error between the observed values and desired ones which include the finding coefficients:

$$S = \sum_{i=1}^{n} (\delta_i - \delta)^2 = \sum_{i=1}^{n} (\delta_i - A \cdot B \sin Hd - C \cos Hd - D \sin 2Hd - E \cos 2Hd)^2 \quad (5)$$

The objective of problem is to find out the coefficients such that mean square error between the observed values and desired ones is minimum. For this purpose, derivating the $S$ function with respect to every deviation coefficients, and taking those derivate values to zero as follows:

$$\frac{\partial S}{\partial A} = 0; \frac{\partial S}{\partial B} = 0; \frac{\partial S}{\partial C} = 0; \frac{\partial S}{\partial D} = 0; \frac{\partial S}{\partial E} = 0 \quad (6)$$

Expanding and rearranging the detail of the equations, the matrix for deviation coefficients obtain as follows:

$$\begin{bmatrix} n & \sum \sin Hd & \sum \cos Hd & \sum \sin 2Hd & \sum \cos 2Hd \\ \sum \sin Hd & \sum \sin^2 Hd & \sum \cos Hd \sin Hd & \sum \sin 2Hd \sin Hd & \sum \cos 2Hd \sin Hd \\ \sum \cos Hd & \sum \cos Hd \sin Hd & \sum \cos^2 Hd & \sum \sin 2Hd \cos Hd & \sum \cos 2Hd \cos Hd \\ \sum \sin 2Hd & \sum \sin 2Hd \sin Hd & \sum \sin 2Hd \cos Hd & \sum \sin^2 2Hd & \sum \cos 2Hd \sin 2Hd \\ \sum \cos 2Hd & \sum \cos 2Hd \sin Hd & \sum \cos 2Hd \cos Hd & \sum \cos^2 2Hd & \sum \sin 2Hd \cos 2Hd \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \\ E \end{bmatrix} = \begin{bmatrix} \sum \delta_i \\ \sum \delta_i \sin Hd \\ \sum \delta_i \cos Hd \\ \sum \delta_i \sin 2Hd \\ \sum \delta_i \cos 2Hd \end{bmatrix} \quad (7)$$

Placing the elements as:

$$[X] = \begin{bmatrix} \sum \sin Hd & \sum \sin^2 Hd & \sum \cos Hd \sin Hd & \sum \sin 2Hd \sin Hd & \sum \cos 2Hd \sin Hd \\ \sum \cos Hd & \sum \cos Hd \sin Hd & \sum \cos^2 Hd & \sum \sin 2Hd \cos Hd & \sum \cos 2Hd \cos Hd \\ \sum \sin 2Hd & \sum \sin 2Hd \sin Hd & \sum \sin 2Hd \cos Hd & \sum \sin^2 2Hd & \sum \cos 2Hd \sin 2Hd \\ \sum \cos 2Hd & \sum \cos 2Hd \sin Hd & \sum \cos 2Hd \cos Hd & \sum \cos^2 2Hd & \sum \sin 2Hd \cos 2Hd \end{bmatrix}$$

$[Y] = \begin{bmatrix} \sum \delta_i \\ \sum \delta_i \sin Hd \\ \sum \delta_i \cos Hd \\ \sum \delta_i \sin 2Hd \\ \sum \delta_i \cos 2Hd \end{bmatrix}$

where: $n$ is the number of observation of compass’s deviations

The Equation (7) is rewritten as following form:

$$[X]_n [Y] = [Z]$$

Solving the Equation (8), the deviation coefficients obtain as below result:

$$[Y] = [X]^{-1} [Z] \quad (9)$$

where:

$[X]^{-1}$ is the transpose matrix of the matrix $[X]$

$[Y]$ is the vector of the deviation coefficients $A, B, C, D, E$ (finding unknowns)

To carry out this problem by using the proposed method, we only need to calculate 25 components in X and Y matrix. This is easy when program by computer. Additionally, in science and engineering, the least square technique is most popular to solve the equation system when the number of equation is under and over variable number with minimum mean square error. Therefore, the proposed method ensures the confident for this problem.

On the basic of the proposed method, it can see that the deviation coefficients can be calculated by using the observed deviation on arbitrary courses. This method helps the seafarers easily and quickly to determine the deviation coefficients.
However, the operational experiments have not yet carried out in fact. In the next time, the author will perform to determine the deviation coefficients by the proposed algorithm. The evaluations about the accuracy and shortcomings will be shown.

**Conclusions**

In this paper, the new method is proposed to calculated the deviation coefficients of magnetic compass using least square algorithm. The deviation coefficients can be calculated by using the observed deviation on arbitrary courses without requirement special ones. Particularly, this method can be useful when the yaw rate of ship is large, the seafarer, and the compass engineer cannot see the compass deviations on special courses.

By the way, this method can be used easily and automatically in personal computer to find the deviation coefficients.

**Disclosure statement**

No potential conflict of interest was reported by the author.

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