Nonlocalized cluster dynamics and container picture

Bo Zhou
Department of Physics, Nanjing University, Nanjing 210093, China
Meme Media Laboratory, Hokkaido University, Sapporo 060-0810, Japan
E-mail: bo@nucl.sci.hokudai.ac.jp

Abstract. A container picture is proposed for understanding cluster dynamics where the clusters make nonlocalized motion occupying the lowest orbit of the cluster mean-field potential characterized by the size parameter “B” in the THSR (Tohsaki-Horiuchi-Schuck-Röpke) wave function. The nonlocalized cluster aspects of the inversion-doublet bands in $^{20}$Ne which have been considered as a typical manifestation of localized clustering are discussed. In the $\alpha+^{16}$O system, localization is entirely of kinematical origin, that is, due to the inter-cluster Pauli repulsion. It is concluded that this feature is general for nuclear cluster states.

1. Introduction
The formation of clusters is one of the most important features in light nuclei together with the formation of the nucleon mean field [1–4]. The $\alpha$-condensate-like state is a very novel cluster state in light nuclei which has attracted increasing interest in recent years (For example, see the reviews Ref. [5] and Ref. [6]). This state can be considered as a gas-like state of clusters in which the center-of-mass motion of each $\alpha$ cluster in nuclei occupies the same $0s$ orbit. The proposed THSR (Tohsaki-Horiuchi-Schuck-Röpke) wave function [7] has been proved to be very suitable for the realistic description of the dilute gas-like state of clusters. Actually in the case of the Hoyle state (the second $0^+$ state) of $^{12}$C, it was found that the full microscopic solutions [8] of the $3\alpha$ RGM (resonating group method) [9] and that [10] of the GCM (generator coordinate method) with the Brink wave function [9] are almost 100% equivalent to single $3\alpha$ THSR wave functions [11]. Also in the case of the ground-state band of $^8$Be, the full microscopic solutions of the $2\alpha$ RGM or its equivalent GCM with the Brink wave function were found to be practically 100% equivalent to single $2\alpha$ THSR wave functions [12].

Although the THSR wave function was devised for describing the gas-like state of clusters, it was already found a decade ago in Ref. [11] that the wave functions of the ground state of $^{12}$C with normal density obtained by $3\alpha$ RGM and $3\alpha$ Brink-GCM calculations have a large value of about 0.93 for the squared overlap with single $3\alpha$ THSR wave functions. Recently the present authors found [13] that the $^{16}$O + $\alpha$ Brink-GCM wave functions of the states of the ground-state rotational band of $^{20}$Ne with normal density are almost 100% equivalent to single $^{16}$O + $\alpha$ THSR wave functions. These results show that the THSR wave function has an ability which was not expected at first, namely it can be used to study not only the gas-like cluster states with low density but also (cluster) states with normal density.

In $^{20}$Ne, the ground-state rotational band with $K^\pi = 0^+_1$ is known to constitute an inversion doublet together with the negative-parity rotational band with $K^\pi = 0^-_1$ built upon the $1^-$ state at the excitation energy $E_x = 5.79$ MeV. The existence of the inversion doublet bands has been
regarded as being a clear manifestation of the existence of the parity-violating intrinsic state due to the $^{16}$O + $\alpha$ localized clustering. In general, in non-identical two-cluster systems, the existence of inversion-doublet bands has been regarded as a clear indication of the existence of the localized cluster structure together with the observation of the large cluster decay widths. This argument implies that we have to regard the states of the ground-state rotational band of $^{20}$Ne as having a $^{16}$O + $\alpha$ localized clustering. However, it was found that, also in this negative-parity rotational band, the $^{16}$O + $\alpha$ Brink-GCM wave functions are almost 100% equivalent to single $^{16}$O + $\alpha$ THSR wave functions. Thus both rotational bands constituting the inversion doublet are found to have nonlocalized cluster structure of $^{16}$O and $\alpha$.

Thus, the typical cluster systems, 2$\alpha$, 3$\alpha$, and $\alpha+^{16}$O, are all well described by a single THSR wave function. These facts urged us to introduce the container picture of cluster dynamics underlying the THSR wave function. Different from the traditional cluster models, in the container picture, the clusters make nonlocalized motion occupying the lowest orbit of the cluster mean-field potential characterized by the size parameter (See Ref. [14] for details).

2. Hybrid-Brink-THSR wave function

Recently, to extend and further clarify the concept of nonlocalized clustering even in non-gas-like cluster states with more compact density, we proposed a new type of microscopic cluster wave function [15], which we call hybrid-Brink-THSR wave function,

$$
\Phi_{\text{cluster}}(\beta, S) = \int d^3R_1 \ldots d^3R_n \exp\left\{- \sum_{i=1}^{n} \sum_{k=x,y,z} \frac{R_{ik}^2}{\beta_{ik}^2}\right\} \Phi_{\text{cluster}}^{B}(R_1 + S_1, \ldots, R_n + S_n) \tag{1}
$$

$$
\propto A \prod_{i=1}^{n} \exp\left\{-A_i \sum_{k=x,y,z} \frac{\left(X_{ik} - S_{ik}\right)^2}{2B_{ik}^2}\right\} \phi(C_i) \right\}, \tag{2}
$$

$$
\Phi_{\text{cluster}}^{B}(S_1, \ldots, S_n) = A \prod_{i=1}^{n} \exp\left\{-A_i \frac{\left(X_i - S_i\right)^2}{2b^2}\right\} \phi(C_i). \tag{3}
$$

Here $\beta_i \equiv (\beta_{ix}, \beta_{iy}, \beta_{iz})$, and $X_i$ and $\phi(C_i)$ are the center-of-mass coordinate and the internal wave function of the cluster $C_i$, respectively. Different clusters $C_i$ can have different mass numbers $A_i$ and variational parameters $\beta_i$. The oscillator parameter of the cluster $C_i$ is called $b$, which also can be adopted so as to have different values for different clusters. $\Phi_{\text{cluster}}^{B}$ is the corresponding general Brink model wave function [16].

In Eq. (1), another generator coordinate $S_i$ is introduced to the original THSR wave function. It can be seen from Eq. (2) that this hybrid wave function combines the important characters of the Brink model as in Eq. (3) and the THSR wave function in a very simple way. When $S_i = 0$, Eq. (1) corresponds to the THSR wave function and $\beta_i$ or $B_i$ becomes the size parameter. When $\beta_{ik} = 0$, i.e., $B_{ik} = b (k = x, y, z)$, this equation is nothing more than the Brink wave function Eq. (3) and $S_i$ is the position parameter of the cluster $C_i$.

As we know, the THSR model provides a nonlocalized clustering picture for the cluster structure rather than the localized clustering represented by the Brink model [17]. Since these two different kinds of pictures for clustering are both included in the hybrid-Brink-THSR wave function as the aforementioned two limits, this hybrid wave function provides a very nice way for verifying which picture is more adequate for understanding the relative motions of the cluster structures in nuclei.

Now, based on the above hybrid-Brink-THSR wave function, the following cluster wave function of $^{20}$Ne can be obtained, as it was considered in Ref. [15],

$$
\Phi_{Ne}(\beta, S) = \int d^3R \exp\left\{-\left(\frac{4R_{x}^2}{5\beta_{x}^2} + \frac{4R_{y}^2}{5\beta_{y}^2} + \frac{4R_{z}^2}{5\beta_{z}^2}\right)\right\} \Phi_{Ne}^{B}(\frac{4}{5}(R + S), \ldots, \frac{1}{5}(R + S)). \tag{4}
$$
\[ \propto \exp(-\frac{10X_G^2}{b^2})\tilde{\Phi}_{Ne}(\beta, S), \]

\[ \tilde{\Phi}_{Ne}(\beta, S) = A[\exp(-\sum_{k=x,y,z} \frac{8(r_k - S_k)^2}{5B_k^2})\phi(\alpha)\phi^{(16O)}], \tag{5} \]

where \(B_k^2 = b^2 + 2\beta_k^2\), \((k = x, y, z)\), \(r = X_1 - X_2\), \(X_G = (4X_1 + 16X_2)/20\), and \(\tilde{\Phi}_{Ne}(\beta, S)\) is the intrinsic wave function where the spurious center-of-mass motion is eliminated from \(\Phi_{Ne}(\beta, S)\) in Eq. (4). \(X_1\) and \(X_2\) represent the center-of-mass coordinates of the \(\alpha\) cluster and the \(^{16}\text{O}\) cluster, respectively. All calculations are performed with restriction to axially symmetric deformation, that is, \(\beta_x = \beta_y \neq \beta_z\) and \(S \equiv (0,0,S_z)\). The spin and parity eigenfunctions can be obtained by the angular-momentum projection technique [13].

As the nuclear interaction, we adopt the same effective nuclear force, Volkov No.1 with the Majorana parameter \(M = 0.59\), and the same oscillator parameter \(b = 1.46\) fm, as were used in our previous papers [13, 15].

3. Nonlocalized cluster dynamics

![Figure 1. Energy curves of \(J^n = 0^+, 2^+, 1^-,\) and \(3^-\) states with different widths of Gaussian relative wave functions in the hybrid model.](image)

After variational calculations for the parameters \(S_z\) and \(\beta\) in the projected hybrid-Brink-THSR wave function, it is surprising to find that the inter-cluster distance parameter \(S_z\) becomes zero for the inversion doublet bands in \(^{20}\text{Ne}\). This means this hybrid-Brink-THSR wave function tends to a pure THSR wave function in describing the cluster states of the inversion doublet bands in \(^{20}\text{Ne}\). Thus, in spite of the fact that a pure Brink wave function gives a distinct energy minimum point with non-zero \(S_z\), the localized clustering picture cannot be supported.

Fig. 1 shows the energy curves of the lower excited states of the inversion doublet bands with different widths of the Gaussian relative wave functions in the hybrid model. If \(\beta\) is fixed at 0, the hybrid-Brink-THSR wave function becomes the Brink wave function. In this case, \(S_z\) is the inter-cluster distance parameter and is usually regarded as a parameter describing the dynamics of the cluster system. For instance, the minimum energy of the ground state of \(^{20}\text{Ne}\) appears at
$S_z = 3.0$ fm. For the $J^\pi = 1^-$ state, the optimum position appears at $S_z = 3.9$ fm. The non-zero values of $S_z$ seem to indicate that the $\alpha+^{16}\text{O}$ structure of $^{20}\text{Ne}$ favours localized clustering. This is just the traditional concept of localized clustering. Now, we believe that this argument is misleading [15]. The non-zero minimum point $S_z$ simply occurs since the width of the Gaussian wave function of the relative motion in the Brink model is fixed to a narrow wave packet, characterized by the parameter $b$. If we take non-zero values for $\beta$, namely, $\beta_x = \beta_y = \beta_z = 1.8$ fm, 1.5 fm, 2.4 fm, and 1.9 fm for $J^\pi = 0^+, 2^+, 1^-$, and $3^-$ states, respectively, according to their minimum positions in the contour maps, then we find that the minimum points appear at $S_z = 0$ in Fig. 1. This indicates that the separation distance parameter $S_z$ does not play any physical role in describing the $\alpha+^{16}\text{O}$ cluster structure, even for the negative-parity states. Instead of that, the new parametrization by $\beta$, which characterizes nonlocalized clustering, is more appropriate for describing the cluster structure in $^{20}\text{Ne}$.

It should be noted that although $S_z$ does not give any contribution to the energy gain, it still plays an important role in providing negative-parity states. We can prove that the negative-parity states can be constructed even in the limiting situation, $S_z \to 0$.

**Table 1.** $E_{\text{min}}(\beta_x = \beta_y, \beta_z)$ are the minimum energies at the corresponding values of $\beta_x = \beta_y$ and $\beta_z$ in the hybrid model. The squared overlaps between the single normalized projected THSR-type wave functions $\hat{\Phi}_{\text{THSR}}$ corresponding to the minimum energies and the normalized Brink GCM wave functions are also listed. Units of energies are MeV.

| State | $E_{\text{min}}(\beta_x = \beta_y, \beta_z)$ | $|\langle \hat{\Phi}_{\text{THSR}}|\Phi_{\text{GCM}} \rangle|^2$ |
|-------|------------------------------------------|-------------------------------|
| $0^+$ | -159.85(0.9, 2.5)                        | 0.9929                        |
| $2^+$ | -158.53(0.0, 2.2)                        | 0.9879                        |
| $4^+$ | -155.50(0.0, 1.8)                        | 0.9775                        |
| $1^-$ | -155.38(3.7, 1.4)                        | 0.9998                        |
| $3^-$ | -153.07(3.7, 0.0)                        | 0.9987                        |

On the other hand, the exact solution of the $\alpha+^{16}\text{O}$ cluster system can be obtained by superposing the single Brink wave functions, that is the Brink-GCM wave function.

$$\sum_j \langle \Phi_{\text{Brink}}^J(R_i)|\hat{H} - E|\Phi_{\text{Brink}}^J(R_j)\rangle f(R_j) = 0. \quad (6)$$

Here, $\Phi_{\text{Brink}}^J(R_i)$ can be obtained directly from the projected Brink wave function $\Phi_{\text{Brink}}^J(\frac{4}{5}R, -\frac{1}{5}R)$ with $R = (0, 0, R_z)$. Thus, by solving the Hill-Wheeler equation Eq. (6), we can obtain the following Brink-GCM wave function,

$$\Phi_{\text{GCM}}^J = \sum_i f(R_i)\Phi_{\text{Brink}}^J(R_i). \quad (7)$$

Thus, we can compare the single THSR-type wave function with the exact Brink-GCM wave function [13] for the description of the $\alpha+^{16}\text{O}$ cluster system by calculating their squared overlap $|\langle \Phi_{\text{THSR}}^J|\Phi_{\text{Brink}}^J \rangle|^2$. In Table 1, we find that the obtained single THSR-type wave functions have 99.29\%, 98.79\%, 97.75\%, 99.98\%, and 99.87\% squared overlaps for $J^\pi = 0^+, 2^+, 4^+, 1^-$, and $3^-$ states of $^{20}\text{Ne}$, respectively, with the corresponding Brink-GCM solutions. These high squared overlaps mean that the single THSR-type wave functions are almost 100\% equivalent to the corresponding RGM/GCM wave functions, thus, these obtained single angular-momentum projected THSR-type wave functions can accurately describe the states of the inversion doublet bands in $^{20}\text{Ne}$ [13,15]. Moreover, the concept of nonlocalized clustering proposed by the THSR-type wave function obtained from the hybrid-Brink-THSR wave function is essential to correctly understand the $\alpha+^{16}\text{O}$ cluster structure in $^{20}\text{Ne}$. 

4
Figure 2. Density distribution of the $^{16}\text{O} + \alpha$ hybrid-Brink-THSR wave function with $S_z = 0.6$ fm and $(\beta_x, \beta_y, \beta_z) = (0.9$ fm, 0.9 fm, 2.5 fm).

But how to understand the rotational band based on the concept of nonlocalized clustering? Since the THSR wave function is a state of good parity, a pure THSR wave function is not suitable for expressing a parity-breaking density distribution of the $^{16}\text{O}-\alpha$ clustering. However, as we will see below, if we use a hybrid-Brink-THSR wave function with small $S_z$ parameter (for small values of $S_z$ the energies in Fig. 1 are practically degenerate), the density distribution of this hybrid-Brink-THSR wave function which is quite close to a prolate THSR wave function shows clearly the effective spatial localization of $^{16}\text{O}$ and $\alpha$ clusters, which can lead to the rotational bands in $^{20}\text{Ne}$. In Fig. 2 we show the density distribution of the hybrid-Brink-THSR wave function with $S_z = 0.6$ fm and $(\beta_x, \beta_y, \beta_z) = (0.9$ fm, 0.9 fm, 2.5 fm). We observe in this figure that, in spite of the small value of $S_z = 0.6$ fm, the inter-cluster distance between $^{16}\text{O}$ and $\alpha$ is about 3.6 fm. Namely the large inter-cluster distance of about 3.6 fm between $^{16}\text{O}$ and $\alpha$ is not due to the parameter $S_z$ but due to the effective spatial localization of $^{16}\text{O}$ and $\alpha$ clusters in the prolate THSR wave function with $(\beta_x, \beta_y, \beta_z) = (0.9$ fm, 0.9 fm, 2.5 fm) which is just the wave function reduced from the hybrid-Brink-THSR wave function by letting go $S_z$ to zero. Thus, the non-zero inter-cluster separation is a natural result of the Pauli repulsion in the THSR wave function while it is artificial to express this separation by the localized parameter in Brink wave function.

In two-cluster systems, cluster states generally have effective localization of clusters because of the inter-cluster Pauli repulsion. However, in three or more cluster systems, the spatial arrangement of clusters are not necessarily geometrical, namely clusters can be nonlocalized, although the inter-cluster separations are non-zero simultaneously because of the inter-cluster Pauli repulsion. However, as is discussed in Ref. [18], if a cluster state is forced to have strongly-prolate deformation, the state can have effective localization of clusters like in the case of $3\alpha$ linear-chain structure. When the inter-cluster separations are large, the spatial arrangement of clusters can be non-rigid and gas-like.

4. Container picture
Clusters in the THSR wave function in low density systems make mutually independent nonlocalized motion occupying the lowest orbit of the harmonic-oscillator-like mean-field potential of clusters characterized by the size parameter $B$ whose magnitude is similar to the radius of the system. In systems of $3\alpha$’s [7,11,19] and $4\alpha$’s [7,20] we know that the excitation mode of the system is well described by the Hill-Wheeler equation of the size parameter $B$ treated
as the generator coordinate. Therefore we see that the excitation of the system is described firstly by the dynamics of the size parameter $B$ which is adopted as the generator coordinate and secondly by the excitation of the single-particle motion of clusters in the cluster mean-field potential. We will call our new understanding of nuclear cluster dynamics the container picture of nuclear clustering, by which we aim to stress that the central quantity of cluster dynamics is the size parameter $B$ of the self-consistent mean-field potential of clusters which we call the container. The name “container picture” may sound more appropriate for three (or more)-cluster systems because, for example, in the 3α system it describes the ground state (small $B$-parameter) and 3α-gas states (large $B$-parameter) on the same footing. In this container picture the existence of cluster-gas states is natural and the formation mechanism of cluster-gas states is just the spatial expansion of the container ($B$-parameter going from small to large). When we compare the container picture of cluster dynamics with the traditional description of cluster dynamics which uses explicitly wave functions with inter-cluster separation coordinates, the new understanding corresponds to a collective-motion picture characterized by the size parameter $B$. When $B$ has obtained a large value the clusters become more or less independent. They have, however, to respect the excluded volume which is due to the Pauli principle, what leads to scattering processes among the clusters. It is in this way that, e.g., the α condensate is depleted by about 30% in the Hoyle state of $^{12}$C [21].

Now we explain how the idea of the parity-violating deformation of localized $^{16}$O + α clustering for the inversion-doublet bands of $^{20}$Ne can be justified in this container picture of cluster dynamics which assumes nonlocalized clusters. The parity-violating deformation is a property of the intrinsic state which is the instantaneous (or adiabatic) quantum state of the rotation of the nucleus. Since the instantaneous configuration of two clusters is of prolate shape, the prolate THSR wave function is the intrinsic state of the system and the oblate THSR wave function is not the intrinsic state but rather a mathematical object which expresses the rotation-average of the intrinsic state. The spherical THSR wave function expresses the time average of the fully three-dimensional rotational motion, namely the angular-momentum projected state of the intrinsic state (the prolate THSR wave function). We, however, also need to notice the fact that two clusters can not come close to each other because, as just mentioned, of the inter-cluster Pauli repulsion, which implies that two clusters in the intrinsic state (the prolate THSR wave function) are effectively localized in space. Thus, the prolate THSR wave function has the parity-violating deformation of localized $^{16}$O + α clustering. We can say that dynamics prefers nonlocalized clustering but kinematics makes the system look like localized clustering. Of course, this localization is most pronounced in the necessarily strongly prolate two cluster systems. In systems with low density α clusters in number more than two have more space to move independently and are, therefore, less localized in spherical containers.

In a word, the container picture of cluster dynamics has three important ingredients. The first is to regard the motion of clusters as being mutually independent and described by the nonlocalized lowest orbit of the self-consistent mean-field potential of clusters. The second is the collective excitation of the system which is described by the Hill-Wheeler equation with respect to the size parameter(s) $B$ of the mean-field potential. The third is the inter-cluster Pauli repulsion which, in the case of two-cluster systems, is the origin of the molecular structure of clusters and which, in cases of more α clusters, like in the Hoyle state of $^{12}$C, leads to $\alpha - \alpha$ scattering processes which somewhat depopulate the α condensate.

5. Summary

It is found that the single THSR wave functions for the inversion doublet bands of $^{20}$Ne are nearly 100% equivalent to the corresponding exact RGM solution. We believe that the concept of nonlocalized clustering proposed by a THSR-type wave function is essential in correctly understanding the cluster structures in nuclei. We further proposed the container picture for
understanding cluster dynamics where the clusters make nonlocalized motion occupying the lowest orbit of the cluster mean-field potential characterized by the size parameter.

Acknowledgments
The author is very grateful to his collaborators, Dr. Y. Funaki, Profs. H. Horiuchi, Z. Ren, G. Röpke, P. Schuck, A. Tohsaki, C. Xu, and T. Yamada for lots of helpful comments and discussions in this work. This work is supported by the National Natural Science Foundation of China (Nos. 11035001, 10975072, 10735010, 11375086, 11175085, 11235001, and 11120101005), by the 973 Program of China (No. 2010CB327803 and No. 2013CB834400), by CAS Knowledge Innovation Project No. KJCX2-SW-N02, by Research Fund of Doctoral Point (RFDP) (No. 20100091110028), and by the Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD).

References
[1] Wildermuth K and Tang Y C 1977 *A Unified Theory of the Nucleus* (Braunschweig: Vieweg)
[2] von Oertzen W, Freer M and Kanada-En’yo Y 2006 *Phys. Rep.* **432** 43
[3] Freer M 2007 *Rep. Prog. Phys.* **70** 2149
[4] Ren Y and Ren Z 2012 *Phys. Rev.* **C 85** 044608
[5] Horiuchi H, Ikeda K and Kato K 2012 *Prog. Theor. Phys. Supple.* **192** 1
[6] Yamada T, Funaki Y, Horiuchi H, Röpke G, Schuck P and Tohsaki A 2012 *Lecture Notes in Physics* **848** 229
[7] Tohsaki A, Horiuchi H, Schuck P and Röpke G 2001 *Phys. Rev. Lett.* **87** 192501
[8] Fukushima Y and Kamimura M 1978 *Supple. of J. Phys. Soc. Japan* **44** 225 ; Kamimura M 1981 *Nucl. Phys. A* **351** 456
[9] Horiuchi H, Ikeda K, Kamimura M, Saito S, Tamagaki R and Tohsaki A 1977 *Prog. Theor. Phys. Supple. No.* **62**
[10] Uegaki E, Okabe S, Abe Y and Tanaka H 1977 *Prog. Theor. Phys.* **57** 1262 ; Uegaki E, Abe Y, Okabe S and Tanaka H 1978 *Prog. Theor. Phys.* **59** 1031
[11] Funaki Y, Tohsaki A, Horiuchi H, Schuck P and Röpke G 2003 *Phys. Rev. C* **67** 051306(R)
[12] Funaki Y, Horiuchi H, Tohsaki A, Schuck P and Röpke G 2002 *Prog. Theor. Phys.* **108** 297
[13] Zhou B, Ren Z, Xu C, Funaki Y, Yamada T, Tohsaki A, Horiuchi H, Schuck P and Röpke G 2012 *Phys. Rev. C* **86** 014301
[14] Zhou B, Funaki Y, Horiuchi H, Ren Z, Röpke G, Schuck P, Tohsaki A, Xu C and Yamada T 2014 *Phys. Rev. C* **89** 034319
[15] Zhou B, Funaki Y, Horiuchi H, Ren Z, Röpke G, Schuck P, Tohsaki A, Xu C and Yamada T 2013 *Phys. Rev. Lett.* **110** 262501
[16] Brink D M 1966 *Proc. Intern. School of Physics ‘Enrico Fermi’* course 36 ed Block C (Academic Press)
[17] Funaki Y, Horiuchi H, von Oertzen W, Röpke G, Schuck P, Tohsaki A and Yamada T 2009 *Phys. Rev. C* **80** 064326
[18] Suhara T, Funaki Y, Zhou B, Horiuchi H and Tohsaki A 2014 *Phys. Rev. Lett.* **112** 062501
[19] Funaki Y, Tohsaki A, Horiuchi H, Schuck P and Röpke G 2005 *Eur. Phys. J. A* **24** 321
[20] Funaki Y, Yamada T, Tohsaki A, Horiuchi H, Röpke G and Schuck P 2010 *Phys. Rev. C* **82** 024312
[21] Yamada T and Schuck P 2005 *Eur. Phys. J. A* **26** 185