Extended Multi-directional Crack Model Applied to the RC Precast Joint Interface with Shear Dowel

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Abstract

The extended multi-directional non-orthogonal crack model was applied to the joint interface area between RC members connected by shear dowel and the experimental verification was conducted in use of the low cycle fatigue experiments. The splitting tension field induced by the dowel action around the reinforcing bar is successfully reproduced with the coupled joint crack model and the beam finite elements, and the splitting tension cracking under cyclic fatigue loading and the joint degradation were fairly captured computationally. The effect of spiral steel confinement on the shear dowel performance can be quantitatively evaluated by the generic full 3D nonlinear analysis.

1. Introduction

For behavioral simulation of entire reinforced concrete (RC), joint interface finite elements are placed in between members represented by 3D solids, and its interfacial kinetics has been proposed as shown in Fig. 1. For computing the drift of RC columns in practice, the pullout of main reinforcing bars from the footing and the sliding along it are represented by the joint interface elements together with the deflection of the main body of the RC column. We also place joint finite elements at the junction places of precast RC (Takahashi et al. 2021) as well as concrete pavements connected by dowel bars.

For formulating the interface elements where joint opening and shear sliding are related to the transferred normal and shear stresses, the pullout and the dowel action of reinforcing bars have been integrated with the shear transfer of cracked concrete interface (Maekawa et al. 2003). This scheme works under both monotonic and cyclic actions. It must be noted herein that some assumptions exist as to the scope of this approach. The pullout and the dowel action of deformed bars are idealized by the constitutive modeling under the sound concrete without any bond cracking (Vintzeleou and Tassios 1986; Randel 2007; Maekawa and Qureshi 1996, 1997), and the shear transfer modeling of cracked concrete is formulated by assuming that contact point of a pair of crack planes is supported by the sound non-damaged concrete (Li et al. 1987, 1989). These assumptions are acceptable when concrete cover is thick in general.

However, if the concrete cover is small and/or the high cycle loads are applied to the joint planes, surrounding solid concrete deteriorates as shown in Fig. 1. These cracks appearing close to the RC junction planes are named bond splitting cracks, which are made by the splitting concrete tension caused by the deformed reinforcing bars subjected to the combined axial tension and bending moment (Jimenez et al. 1982; Gambarova et al. 1989; Gambarova and Rosati 1997). Also, experimental investigations showed that the width and the depth of concrete cover highly affect the dowel action. When the concrete cover is insufficient, the bar diameter is large and the concrete strength is low, concrete cover splitting is the main governing failure mode when the dowel action acts as a main mechanism to convey shear (Soroushian et al. 1987; Moradi et al. 2015). This phenomenon is highly intensified in the presence of pullout tension (Soltani and Maekawa 2008) as well.

The localized flexure of rebars and associated bearing shear as mentioned above has been converted to the joint opening and shear slide of the conventional joint interface elements. This mechanistic equivalent way was built based upon the assumption of no-splitting crack and the fictitious spring to support the shear force which is equilibrated with the gradient of flexural moment along dowel bars. Thus, the splitting crack, which is caused by the pullout and the dowel action of reinforcing bars supported by the local bearing stress, cannot be taken into account by mere simple summation of RC solid elements connected by joint interfaces as shown in Fig. 1. In fact, the deformation of RC solid elements is just associated with the nodal displacement in finite discretization but not linked with the local deformation of reinforcing bars. Similarly, the bar’s pull-out model in the joint interface modeling (Shima et al. 1987; Soltani and Maekawa 2008) is not linked with the damage of adjacent concrete. Within this scheme, the kinetics of confinement around the bars nearby the joint cannot be considered systematically. Then, it is neces-
sary to take into account the high nonlinearity of coupled reinforcing bars and concrete solid over the 3D domain including the joint and junction planes.

The objective of this paper is to overcome this limitation of conventional joint interface elements by using the 3D multi-directional non-orthogonal crack modeling (Maekawa and Fukuura 2014) as an extended manner like the masonry structural analysis (Yamanoi et al. 2021), and by introducing 3D in-plane Timoshenko’s beam elements of axial, flexural and shear stiffness for dowel bars. Regarding the bond of concrete and reinforcing bars, the microscopic approach including the bond cracks (Goto 1971) was discussed in an exact full 3D extent (Salem et al. 2004; Lura et al. 2002; Lundgren and Gylltoft 2000), and the splitting tension field has been verified to be developed.

In this study, the authors focus on the shear dowel action and related 3D splitting tension field for computational verification, in which the strong coupling among the joint elements, the beam elements for dowel bars and the connected 3D solid elements can be examined with the complex section forces of penetrating steel reinforcing bars. Splitting crack of concrete along reinforcing bars has been investigated and structurally discussed mainly in terms of the bond along the bars, but it is still under investigation in terms of the transverse shear dowel and associated 3D field to cause splitting cracks. This will be in use for the fatigue performance of concrete pavement on the foundation in future.

2. Extended multi-directional crack modeling

The multi-directional fixed crack model, which may handle non-orthogonally intersecting crack planes up to 6 inside the finite domain of solid, was developed for RC structural analysis (Maekawa and Fukuura 2014). Yamanoi et al. (2021) extended this framework for analyzing the masonry structures consisting of bricks and mortar joints as shown in Fig. 2. This extended version may handle different material properties for different crack planes. For example, we may define the mortar’s mechanical properties (strength, size of aggregates, mix proportion etc.) when the crack plane is made normal and parallel to the mortar joints in between bricks. If not, the cracks are thought to develop inside of bricks. Then, the material properties for bricks are addressed to those crack planes.

The original multi-directional crack modeling (Maekawa et al. 2003, 2014) can handle the non-orthogonal crack to crack interaction inside the finite-sized solids. Then, we do not need to define the directions of each intersecting crack in advance unlike discrete crack based methods which can strictly identify the location of cracking (Galvez et al. 2002; Ooi and Yang 2011). This extended version to be applied to masonry structures (Yamanoi et al. 2021) define the three directions of possible cracking along the location of mortar joints in advance and the rest of three crack
planes are reserved for cracking in the bricks under loads. Then, we may say that this extended version of space-averaging is the hybrid of discrete methods like the rigid body spring method, RBSM (Nagai et al. 2005), and the applied element method (Tagel-Din and Meguro 2000) with the original smeared multi-directional crack model.

The authors propose to place the extended finite elements on the joint areas as shown in Fig. 3, where the left and the right sides of blocks are connected with the center block by penetrating reinforcing bar elements. In this case, just one crack plane of the extended elements is defined as the precast joint with its mechanical properties which can be different from those of the solid. In considering the splitting tension of concrete and the associated splitting bond crack along the reinforcement, Timoshenko’s beam finite elements having section forces as generalized stresses (Tsuchiya et al. 2001, 2006) are placed together with the 3D solid concrete elements as shown in Fig. 3. Since the beam element reduces its degree-of-freedom of geometry, the occupied volume of the beams is not explicitly reflected on the

(a) Original 3D multi-directional crack modeling in consideration of crack-to-crack interaction.

(b) Explanation of extended modeling reduced to 2D extent for simplicity (Yamanoi et al. 2021).

Fig. 2 Original multi-directional crack model and its extended version.
3D extent although its volume is numerically considered. In order to avoid the duplication of the volume to store the strain energy as well as the consumed fracture one, we extract the overlapped volume of the beam element from the connecting solid ones as shown in Fig. 3.

As an example, the nodal displacement and the profile of the maximum principal strain of the specimen under shear are shown in Fig. 4. A single reinforcing bar is located at the gravity center of the cross-section of the beam subjected to the mixed bending moment and the shear force along the joint planes. We define the non-frictional flat plane of the joint with zero frictional
coefficient defined inside the extended model elements. The sliding of the joint can be seen on the surface of the beam. When the beam is split into two parts, we can see the nodal displacement close to the intersecting point between the joint area and the reinforcing bar element. Reasonably, there is no relative displacement of nodes. In other words, the conventional approach with the 2D joint interface elements cannot identify the local shear location and the joint gap opening.

As stated previously, the extended multi-directional crack model has its own volume unlike the 2D joint interface without any volume. In this paper, we use the extended element having 5 mm thickness of the maximum size of fine aggregates where the zero-frictional plane of Teflon sheet is embedded. The joint-extended element represents the kinetics of finite volume of contacting left and right sides of concrete blocks nearby the interface plane (see Fig. 4). In this experimental series, coarse aggregates hardly touch the joint plane but most of particle’s contact is made by sand fine aggregates. Then, the localized nonlinear kinetics just close to the interface is assumed to be covered by the joint extended elements and the volume to include the coarse aggregates is to be represented by the adjacent standard multi-directional crack modeling as shown in Fig. 4.

Since the shape of the element is slender, we did not use conventional iso-parametric interpolation but applied the enhanced strain formulation in order to avoid shear and volumetric locking of numerical discretization (Simo and Almero 1992). In fact, provided that the enhanced strain would not be utilized but the isoparametric one would be applied, the computation exhibits several times higher stiffness caused by shear locking of ill-condition with 5 mm slender elements. In the case of isoparametric interpolation, displacement continuity along the peripheral lines of boundaries shared by connected elements is satisfied (C1-continuity), but the strain continuity (gradient of displacement field) along the shared boundary lines is inevitably broken. Then, the material nonlinearity becomes discontinuous. The enhanced strain formulation relaxes this limit by applying the higher nonlinear strain interpolation from the nodal displacements. Then, the element size of 5 mm brings the dowel displacement which exhibits 2 to 5% difference from the case of 1 to 10 mm. The broken symmetry of the deformation can be seen in Fig. 4 where the joint deformation in the right and left blocks is different. This was also observed in the experiments under the large deformational states. The broken symmetry is generally driven by the strain localization, which leads to the minimized fracture energy consumption for solid mechanics and thermodynamics (Gebreyouhannes and Maekawa 2016). The numerical simulation presented in this paper may capture this mechanism as well.

3. Experiments for verification and validation

In order to evaluate the capability of the proposed model for simulating the joint interface area and verification, a specific experimental study was conducted. Figure 5 shows the outline of the specimens with 5 cases to be used for checking the performance of the numerical approach. As stated in the previous section, the focal point is the splitting crack which has much to
do with the nonlinear action of the reinforcing bar. In fact, this type of damage can never be reproduced with conventional 2D interface modeling. The Teflon sheet was placed in between concrete blocks to eliminate shear transfer along the joint, so that we may produce greater dowel forces which create the splitting tension field in the neighboring concrete blocks. The greater dowels may cause high localized compression just close to the interface. Then, we may validate the model in line with the compression of concrete with and without 3D confinement.

The cyclic vertical forces were applied to the specimens in accordance with the precast and concrete pavement joints where the splitting crack and the rupture of concrete under high cyclic actions may take place. A little bending moment is intentionally applied to the joint planes by allocating the support and bearing plates shown in Fig. 5. After the loading, the reinforcing bars were removed from the specimen and for each one, the local deformation profile of the bar was compared with the computed one since it is the representative of the bending and shear over the reinforcement.

The joint displacement was measured by the vertical relative displacement of the center and sides blocks. The displacement transducers were attached at the anchor points whose location is shown in Fig. 5 for the center and sides blocks. The relative displacement of the anchor points was compared with the measurements of the contact displacement gauges attached close to the joint plane, and both joint displacements were confirmed to match closely within 5%.

Table 1 indicates the strength of concrete used. The water to cement ratio of concrete was 43% by weight and the unit water content was 200 kgf/m³. The strength of concrete was different because of the different ages in the experiment after casting. The yield strength of D16 was 400 MPa and that of D13 is 385 MPa.

| Table 1 Strength of materials used. |
|-------------------------------------|
|                                       |
| Compressive strength [MPa]           |
| Center | Left | Right |
| BEAM 1 | 25.8 | 28.7  |
| BEAM 2 | 56.6 | 36.6  |
| BEAM 3 | 56.6 | 44.8  |
| BEAM 4 | 56.6 | 48.0  |
| BEAM 5 | 56.6 | 41.8  |

W/C=0.43, Normal Portland cement, W=200kgf/m³. The compressive strength of concrete was different according to the age at the test was different. Maximum sizes of gravel and sand: 15mm and 5mm

4. Model verification and validation

4.1 Splitting crack and effect of confinement

Figure 6 shows the experimentally obtained displacement in progress under lower cyclic loading. When splitting cracking reached the surface of members, a bit jump of displacement was evolved in progress. According to the systematically arranged experiments by Maekawa and Qureshi (1996, 1997), the bar’s dowel capacity, which is almost the same as the vertical capacity of the test, is mainly governed by the flexural moment of the dowel bar and the bearing stiffness of concrete close to the joint plane. These analysis results based on the constitutive models as shown in Fig. 2 (Maekawa et al. 2003, 2008) are not far from the reality except for the beginning stage of the cyclic loading as shown in Fig. 6. As in the case of RC beams under high cyclic loading (Gebreyohannes et al. 2008), the increasing rate of deflection varies exponentially with amplitude, the cyclic evolution of joint displacement similarly does under the low cycle repetition of shear force.

The rigidity of the specimen is rather smaller than the computation which reflects the initial elasticity of the whole system although the higher loading may bring about the similarity. It may attribute to the initial bedding of embedded Teflon sheet which is similar to the case of bedding of the soft rock experiment (Goto et al. 1991). In fact, the Teflon sheet is idealized as the perfect flat and non-deformed plane with perfect non-friction on it. The confined concrete specimen gives rise to a higher capacity and longer fatigue life, and the analysis can fairly capture this tendency quantitatively as shown in Fig. 6.

Figures 7 and 8 show the profiles of the computed averaged normal strains in the thickness direction of the specimens. The dark red indicated in the figure corresponds to the magnitude of the maximum strain. The splitting crack along the reinforcing bars was clearly detected with the strain profile and fairly reproduced by the extended multi-directional crack models and the beam elements for dowel bars. Furthermore, the spiral rings may successfully confine the progress of the splitting crack which is equilibrated with the bearing shear transferred from the dowel bars. It may attribute to the constitutive modeling of concrete which can consider the effect of 3D confinement (Irawan and Maekawa 1993) and the combined flexure-shear forces developing in the dowel bar as well. The overall behaviors of the specimens including the local joint kinetics and the dowel shear may be reproduced with the extended modeling under 3D stress extension although the accuracy shall be improved quantitatively in the future study.

Let us examine and verify the behaviors in more detail of locality. Figure 9 shows the profile of the normal compressive stress indicated in blue. The computed specimen is split into two pieces so that we may see the location of the bar and the joint area. The deformed
shape of the bar is indicated by the weight lines. The high stress concentration in compression can be seen just close to the bar at the interface when the sliding slip comes up to 2 mm. However, the vertical compressive stress is decayed when the deformation proceeds to 8 mm as shown in Fig. 9. This is the mark of compression softening owing to the greater local deformation of bearing actions.

Fig. 6 Low cycle loading to confined (BEAM-1) and unconfined (BEAM-4) concrete specimens.

Zoom-up of confined concrete close to the joint

Fig. 7 Splitting bond cracking appearing on the surface of the unconfined specimen (BEAM-1).
Figure 10 shows the close-up of the concrete nearby the joint and the dowel bars. The concrete surrounded by the spirals looks sound and no graveling was found. However, in the case of non-confinement, bearing concrete is almost ruptured and the deteriorated cementing of aggregates was seen with the graveling. The computed compressive stress profile is also shown in Fig. 10.

The higher compressive stress supports the bars firmly unlike the case of unconfined specimen. This computational difference attributes to the concrete constitutive model which may express the confinement effect. This may be consistent with the macroscopic behaviors of these two cases.
4.2 Local strain and curvature of reinforcing bar

Figure 11 shows the deformational mode of the reinforcing bar of Beam-1 in comparison with the bar taken from the concrete specimen. It can be seen that the most curved portions are located inside the left, right and the center blocks and the bar section at the interface looks non-curved owing to the zero flexure. It implies the maximum dowel shear of the bar section since it is the inflection point of the moment distribution. The most curved section of steel is about 2.5 cm from the joint interface. It is about 1.5 times the bar’s diameter, and it matches the experimental results by Maekawa and Fig. 11 The deformed shape of reinforcing bars.
Qureshi (1996, 1997). The dowel shear force carried by the bar at the interface is equal to the first differentiation of the bending moment according to the beam in-plane theory.

**Figure 12** shows the profile of the extreme local strain of reinforcing bars. The local strain of the steel bar increases in accordance with the shear sliding of the joint planes. The location where the maximum strain develops is about 1.5 to 2.5 times the bar diameter from the joint plane, and the distance at the peak point of strain is gradually increasing which may attribute to the splitting tension failure of concrete. In general, the computed strain is greater than the measurement about 1.5 to 2 times although the profile matches the observation of the deformed bars. The strain sharply changes in the analysis but the strain gauge’s reading is the average of the entire length of it. This shall be the future discussion for more detailed validation.

The local extreme strain of the bar for spiral confinement at the 2D location from the joint gets less than the one of the non-confined case (Beam-4) even though the higher capacity is produced by the confinement. It appears contradictory, but it can be reasonably explained as shown in [Fig. 13](#). The spiral steel confinement introduces the greater dowel actions which results in the gradient of moment distribution and associated bending curvature as shown in [Fig. 13](#). When the axial force is zero as in this experiment, the curvature is proportional to the extreme fiber strain. In fact, the peak strain is greater than the one of the unconfined case, and the location of the peak strain section becomes closer to the joint plane. Then, the strain at the 2D location does not correspond to the peak one. This local nonlinearity related to the dowel actions can be treated by the full 3D nonlinear analysis but not conducted solely by the conventional joint interface modeling.

### 4.3 Local stiffness of surrounding concrete

As discussed in Section 4.1, the initial rigidity of the load-displacement relation is less than the computation which may exhibit the elasticity despite the validity of the higher nonlinearity. Bedding error caused by the distorted shape of the Teflon sheet is thought to be one of the reasons and actually, the broken sheet can be seen in the photos of [Fig. 13](#). The sheet rupture is not considered in the analysis. Another possible cause is the local stiffness of concrete which receives intensive bearing stresses. In fact, the bleeding water is kept beneath the bars and a few aggregates remain. This can be a substantial impact on the initial rigidity.

As for an extreme case, the reinforcing bar was provided with the soft rubber to dramatically change the rigidity of the solid media surrounding the reinforcing bars at the interface (see [Fig. 14](#)). The load versus displacement relation of the dowel shear was greatly changed with much reduced rigidity around the high bearing stress zones. The profile of the curvature along the bar became milder than the standard case of Beam-1.

For considering the weak bearing resistance owing to the soft rubber in the analysis, the authors used the sleeve element to consider the interaction of the beam...
The sleeve element consists of both translational and rotational nodes of the beam and the ones of 3D solid elements. This element has been used for expressing the interaction between piles represented by beam elements and soil foundation, and the discretization and interpolation of these degree-of-freedoms were conducted by Maki et al. (2006). Here, the general strains are the relative displacement between nodes of the beam and those of the solid elements. In the case of analysis of this paper, translational

![Diagram showing relationship between curvature, moment, strain, and shear force.](image)

A: large gradient of moment at the interface and the greater peak point closer to the joint

![Diagram showing bending moment profile and influence of confinement on the peak strain location.](image)

**Fig. 13** The profile of bending moment along the axis of the bar and influence of the confinement (BEAM-4) on the location of the peak strain section.

![Diagram showing fictitious spring displacement and rubber stiffness decay.](image)

**Fig. 14** Lapped reinforcing bar and the gently varying profile of the curvature along the bar for the case of BEAM-5.
nodal degree of freedom is associated with the circumferential fictitious spring whose rigidity reflects the rubber stiffness and its thickness in the transverse direction as shown in Fig. 14. The longitudinal directional relative displacement corresponds to the bond-slip of the bar in concrete. Then, we can consider both the zero frictional slip and the perfect bond. In the analysis, the no bond was assumed.

If the rubber would behave as a perfect elasticity, the transverse rigidity of the sleeve element becomes the elastic stiffness multiplied by the thickness of the rubber zone. First, we assumed 100 MPa/cm and the linear dowel response was reproduced as shown in Fig. 14. However, as the rubber used in the experiment is a nonlinear material whose averaged stiffness becomes smaller when the high strain develops. Then, the dowel shear response deviates from the elastic response at the beginning of loading and approaches to the dowel shear of the reduced stiffness of the rubber. This is to be linked with the rubber nonlinear properties in the future. Here, it must be noted that the relative displacement may not exceed the initial thickness of rubber zone, because overlapping of bar and concrete never occurs. Then, we assumed the transverse stiffness recovery close to the rubber thickness as shown in Fig. 14 and in fact, the dowel stiffness recovery was experienced under large dowel displacement.

The reduced bearing stiffness leads to a wider range of the large curvature zone similar to the experiment as shown in Fig. 14. The related weak zone of concrete close to the free surface of concrete interface has been also observed in the past research referring to the bond deterioration zone. The invalid initial stiffness, which was observed at the beginning of dowel loading, shall be investigated in using mortar and pre-packed concrete which may greatly change the stiffness rather than the strength in the future.

4.4 Interaction of reinforcing bars and sensitivity analysis

Figure 15 shows the experimental and analytical load-displacement relation under cyclic loading. The splitting cracks, which did not extend much, can be seen on the top surface of the specimen in the analysis as well as the experiment did. Furthermore, smaller cracking was also found on the side face of the specimen, and the analysis may predict this crack location as well. It may attribute to the affected splitting tension field developing over the section of concrete owing to the double arrangement of the bars. The capacity of the double bar arrangement is almost double of the one of single bar specimen of Beam-1. As discussed in the previous section, the initial rigidity of the experiment is almost two times less than the analysis. In order to investigate this difference, we conducted the sensitivity analysis where the initial stiffness of concrete was set to be one half as a trial. Although we have a bit different life of low cycle fatigue, the unloading rigidity of cyclic actions is consistently changed when the initial stiffness of concrete for computational material property is reduced by half. In other words, the confinement is the opposite to the reduced stiffness caused by the bleeding, settlement of aggregates, bedding and rupture of Teflon sheets used in the
5. Conclusions

The extended multi-directional non-orthogonal crack model was applied to the joint interface area including the discontinuous joint plane, and the experimental verification and validation were conducted in use of the low cycle fatigue experiments. The findings can be concluded as follows:

1. The splitting tension field induced by the dowel action around the reinforcing bar is reproduced with the coupled extended multi-directional crack model and the beam finite elements.

2. The splitting tension cracking under cyclic fatigue loading and degradation of dowel actions can be captured computationally.

3. The fatigue modeling for concrete compression and tension was experimentally validated under the highly bearing stress fields close to the intersecting points of the joints and the rebars.

4. The effect of spiral confinement on the dowel action can be considered in the 3D nonlinear analysis as an agent to resist against the ring tension stresses around the dowel bars. The quantitative validation is required under various types of bar arrangement and frictional states of joint contact.

5. The experimental rigidity of the dowel shear of joint is smaller than the analytical one at the beginning of loading and the residual dowel shear proceeds rapidly. In highly inelastic conditions of large dowel shear, analytical and experimental dowel shear stiffness becomes closer. This was thought to be caused by the weak bearing stiffness of concrete or bleeding just beneath dowel bars. Future study is required.

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