Dark Matter Jets of Rotating Black Holes

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We present a novel approach which produces Dark Matter jets along the rotation axis of Kerr black holes utilizing the Penrose process. The properties of these jets are investigated, as well as their potential to create Dark Matter overdensities in the solar system and in the vicinity of the black hole. We discover a highly collimated and long-range Dark Matter jet with a density that is most sensitive to the mass and distance to the black hole.

I. INTRODUCTION

Astrophysical jets produced by supermassive black holes are usually explained by the magnetic Blandford-Znajek process [1]. Due to its electromagnetic nature, only charged particles can be ejected as a jet in this way. In this work, we quantitatively analyze a gravitational mechanism for jet production. Through the Penrose process [2], particles which have fallen into a supermassive black hole’s ergosphere from the accretion disk can extract energy from the black hole and be ejected along the rotation axis [3]. It should be noted that while this is subdominant with respect to the Blandford-Znajek mechanism, it can affect Dark Matter (DM) particles as well, due to its gravitational nature.

The main goal of this article is to confirm the presence of a DM beam produced by the mechanism described above quantitatively. It will be shown that the DM beam is highly collimated. Furthermore, the dependence of the DM density on the mass of the black hole and on the distance to it will be analyzed as well.

The existence of such a DM beam renders this phenomenon potentially interesting for future DM detection. Indeed, a number of recent tests have yielded promising results for indirect DM detection: records from the PAMELA satellite [4] suggest a surprisingly high fraction of positrons in cosmic ray measurements over 10 GeV. Similar results were obtained by the AMS collaboration [5], the DAMPE collaboration [6] reported a peak in the flux of electron and positron cosmic rays at around 4 TeV, the H.E.S.S. telescopes [7] detected a point-like source of very high-energy γ rays from Sagittarius A*, and the FERMI/LAT data hinted at an excess in γ rays at energies of 130 GeV [8]. Of course, it is unknown whether the detected signals are truly the result of DM particle annihilation, but it is an interesting possibility.

The usual WIMP cross section is too small to explain the observed fluxes of DM-annihilation products, so a boost factor $B$, which usually lies in the range between $10^2$ and $10^4$, must be artificially added to DM models in order to explain the observed measurements. [9–13] . Several potential possibilities for the physical origin of this overdensity have been discussed in the literature such as density inhomogeneities at tiny scales [14] and Sommerfeld enhanced annihilation cross sections [15]. When an incoming beam of DM particles annihilates with some target DM particles in the proximity of the Earth, the boost factor $B$ is defined as

$$B = \frac{\rho_B \rho_T}{\rho_0 \rho_0}$$

where $\rho_B$ and $\rho_T$ are the densities of the beam and the target, respectively, and $\rho_0 \approx 0.4$ GeV/cm$^3$ is the macroscopic DM energy density in the solar system, as defined in [16] and [17].

With this motivation in mind, we study whether such DM overdensities in our vicinity could be caused by rotating black holes via jet production. In this situation, the target DM particles are those in our immediate vicinity, therefore $\rho_T/\rho_0 = 1$. We will show that even though this mechanism produces a DM beam, the corresponding DM densities are not sufficient to obtain a significant boost factor.

This article is organized as follows: we describe geodesics in a spinning black hole geometry in Sec. [II] and gather the relevant ones in Sec. [III] to determine the overdensities in the black hole jet. We introduce approximations in Sec. [IV] which allow us to numerically evaluate the DM density in the beam and show the findings in Sec. [V]. We conclude in Sec. [VI]. Throughout this article, we use units where $G = c = 1$.

II. GEODESICS IN A KERR GEOMETRY

The system under consideration is a galaxy with a rotating Kerr black hole in the center. We concentrate on particles which follow geodesics leading them from the accretion disk into the ergosphere. There, they can scatter with other particles or simply decay. As depicted in Fig. [I], some of the products of these interactions can then follow a geodesic leading out of the ergosphere and moving parallel to the rotation axis due to the Penrose process as shown in Ref. [3]. Collecting all geodesics with similar behaviour results in the black hole agglomerating particles from the accretion disk and releasing them along its rotation axis and thus forming a jet.
We consider the usual Kerr metric of a black hole with mass $M$ and angular momentum $Ma$. The motion of a particle for a non-rotating neutral black hole ($a = 0$) is completely determined by the particle’s mass, energy and angular momentum. However, for the case of a Kerr black hole ($a \neq 0$) a fourth constant is needed, the Carter constant, which for a particle of unit mass reads

$$Q = (u_\theta)^2 + a^2 \cos^2(\theta)(1 - E^2) + \frac{\cos^2(\theta)}{\sin^2(\theta)}L^2$$

(2)

where $u_\theta = dx_\theta/d\tau$ is the $\theta$ component of the proper four-velocity, $E = -u_t$ is the particle’s energy and $L = u_\phi$ is its angular momentum. With the help of the constants of motion together with the initial position and velocity of the DM particle in the accretion disk we can determine whether it has a chance of ending up in the DM jet together with its final position and velocity for a given time. In the next section we take all these geodesics into account to obtain the DM density in the jet.

III. THE DM DENSITY OF THE EJECTED PARTICLES

To investigate the jet formation we obtain the overall density of the outgoing particles by taking into account the contributions due to all the possible outgoing velocities,

$$\rho_{\text{out}} (r_{\text{out}}, \theta) = \int \frac{d^3v_{\text{out}}}{r_{\text{out}}^2 \sin(\theta) v_r,\text{out}} \frac{d^3N_{\text{out}} (r_{\text{out}}, \theta, v_{\text{out}})}{dv_r,\text{out} dv_\phi,\text{out} dv_\theta,\text{out}}$$

(3)

where $N_{\text{out}} (r_{\text{out}}, \theta, v_{\text{out}})$ is the total number of particles in the beam at $(r_{\text{out}}, \theta)$ with velocity $v_{\text{out}}$. The number of outgoing particles is computed by taking the number of particles that fall into the ergosphere and filtering out all of those that follow trajectories that do not end up in the beam,

$$N_{\text{out}} (x, v_{\text{out}}) = \eta P_{\text{scat}} \int D^3v_{\text{in}} \rho_{\text{in}} V_{\text{in}} (x, v_{\text{in}}, v_{\text{out}}).$$

(4)

Here $V_{\text{in}}$ is the volume occupied by the infalling particles with initial velocities $v_{\text{in}} = (v_r, v_\phi, v_\theta)$ which end up in the beam and reach $x$ with final velocity $v_{\text{out}}$. $\eta$ is the efficiency of the Penrose process. $P_{\text{scat}}$ gives the probability of an infalling particle scattering in the ergosphere and not before. The density of the infalling particles is denoted by $\rho_{\text{in}}$. The integration runs over $D^3v = d^3vf(v)$, where $f(v)$ is the distribution function for the velocities of the infalling particles. In the next Section, we will introduce approximations to simplify Eqs. (3) and (4), and afterwards compute $\rho_{\text{out}}$ numerically.

IV. APPROXIMATIONS AND ASSUMPTIONS

Since the system under consideration has a high dimensional parameter space and partially includes unknown initial conditions, approximations and assumptions are needed to numerically compute Eqs. (3) and (4) for any specific galaxy. This section provides a discussion of all such approximations and assumptions used in this article.

Volume of infalling particles:

The volume occupied by the incoming particles is for convenience approximated to be a ring of radius $r_{\text{in}}$, height $\Delta z = z_{\text{max}} - z_{\text{min}}$, and width $v_r \Delta t$, i.e. $V_{\text{in}} (x, v_{\text{in}}, v_{\text{out}}) = 2\pi r_{\text{in}} v_r \Delta t \Delta z (x, v_{\text{in}}, v_{\text{out}})$. A more sophisticated procedure would be to start from the detection of the particles with given velocities in some time interval and trace them back to the ergosphere and to the accretion disk. In this way one would collect all relevant geodesics such as the one in Fig. 1 and could ask where the detected particles originated in the accretion disk, which DM densities were present at that time and thus how many particles are actually taking this geodesic. By approximating this with an infalling ring, there are different counteracting effects, e.g. a larger radius accounts for a bigger volume but also a smaller density, there is a region in the accretion disk in which the most particles relevant for jet formation originate. Therefore, a suitable choice of $r_{\text{in}}$ has to lie within this region in order to gather the most relevant geodesics and thus to be a reasonable approximation to the above procedure. To obtain an upper bound, we choose $r_{\text{in}}$ so that the maximum feasible $\rho_{\text{out}}$ is achieved. For example, for the Andromeda galaxy the correct choice is $r_{\text{in}} = 0.1$ pc.

Carter constant:

We assume that the value of the Carter constant does not change due to the scattering in the ergosphere, as was done in Ref. [3]. Furthermore, we assume that outgoing
particles with the correct Carter constant $Q_{out}$ do indeed end up in the beam and do not follow some other geodesic with the same value of $Q_{out}$. This leads to an upper bound for the maximal obtainable overdensity. Keeping all other parameters fixed and solving $Q_{in} = Q_{out}$ for $z(x,v_{in},v_{out})$ results in the position the particles initially must have in order to end up in the beam at $x$. Here, $Q_{in}$ is the Carter constant for the ingoing particles. If it has no solution, there is no geodesic connecting the point in the beam with the point in the accretion disk. This makes sure that all and only the particles that can end up in the beam are considered for each set of parameters $\{x,v_{in},v_{out}\}$.

**Distribution function:**

The velocity distributions of the infalling particles are assumed to be Gaussian, i.e.

$$f(v_{in}) = \frac{\exp\{-\frac{1}{2}(v_{in} - v_0)^T \Sigma^{-1}(v_{in} - v_0)\}}{\sqrt{(2\pi)^3|\det(\Sigma)|}}. \quad (5)$$

We assumed the covariance matrix to be diagonal and isotropic such that it reads $\Sigma = \text{diag}(\sigma^2, \sigma^2, \sigma^2)$ with standard deviation $\sigma$. The orbiting of the black hole dominates the mean velocity of the DM particles in the halo, and consequently $v_0 = (0, v_{\phi,0}, 0)$. We take explicit values for $v_{\phi,0}$ from data on the rotation curve of the Milky Way in [15] and assume that other galaxies have similar velocity distributions. In order to estimate the value of $\sigma$ we refer to Fig. 2 of Ref. [19] to obtain the mass accretion rate $dM/dt$ of supermassive black holes. By setting this equal to the infall rate of particles around the black hole we obtain a value for $\sigma$. Explicitly, the equation to be solved is

$$\frac{dM}{dt} = 4\pi r_{in}^2 \rho_{in} \int_0^{\infty} dr_v e^{-\frac{r_v^2}{2\sigma^2}} v_r. \quad (6)$$

We solve Eq. (6) for $\sigma$ numerically with the appropriate parameters depending on the specific case considered. The velocity distribution must then be multiplied with the density of DM at $r_{in}$ in order to obtain the total number of particles with velocity $v_{in}$ per unit of volume. As an example, for the Andromeda galaxy with $r_{in} = 0.1$ pc and assuming a cored DM profile [20], the DM density can be approximated by $\rho_{in}(0.1 \text{ pc}) = 30 \rho_0$.

**Penrose efficiency:**

The efficiency of the Penrose process depends on the angular momentum of the black hole and can reach for maximally rotating black holes a value of $\eta = 0.29$ [21]. Since the angular momentum of the black holes under consideration is not known we assume a Penrose efficiency $\eta = 0.01$. Since the DM density depends linearly on $\eta$, one can easily adjust it for other angular momenta.

**Mean free path:**

In order to describe the path of incoming particles solely with one geodesic, the probability for avoiding any scattering in the accretion disk has to be implemented. Taking the mean free path of the particles such that in average one scattering event occurs within the ergosphere and assuming no significant change in DM density along the geodesic we take $P_{scat} = \lambda_{mfp}/r_{in} \approx 2M/r_{in}$.

**V. RESULTS**

With the approximations in place, we determine the DM density in a black hole DM beam numerically, i.e. $\rho_{out}(r_{out}, \theta)$ in Eq. (5). To this end we integrate over the initial velocities of the DM particles in Eq. (1) as a Riemann sum. In order to reduce the computation time as much as possible the integration limits were chosen such that the neglected portion of the parameter space contributes at most 0.1% to the final result. Furthermore, when deciding on the step size for the integration, a compromise had to be taken. However, while the final result can indeed change by as much as two orders of magnitude when decreasing the size of the integration steps, the qualitative analysis remains unchanged.

In the code provided in Ref. [22] we implemented the steps just described. The integration limits and step size are numerically adjusted such that we use the sweet spot between accuracy and computation time required for each specific black hole in question. Additionally, for the outgoing radial velocity there is a lower bound since particles arriving to us today had to be sent by the black hole at the earliest during the black hole formation such that $v_{r, out} > r_{out}/t_{age}$, where $t_{age}$ is the age of the black hole. DM particles that are slower than this bound might reach us in the future, but cannot have reached us yet and thus do not contribute to the boost factor.

In Fig. 2 we show the density profile of the DM beam created by the Andromeda black hole. There are immediate lessons to be learned from this result. First, the beam is highly collimated with an opening angle of $2\theta_B \approx 10^{-5}$ which for visibility is stretched. Second, the beam is far ranging such that even for a distance of 1 Mpc the beam can still be distinguished. Third, the DM density of the beam increases by many orders of magnitudes if evaluated closer to the black hole and its rotation axis but never reaches a significant overdensity. With these observations we can conclude that the Andromeda black hole is in principle capable to produce a sharp, far ranging but faint DM beam. As a sanity check we analyzed the qualitative behavior of the DM beam for different values of the black hole’s angular momentum $a$. For a Schwarzschild black hole, i.e. $a = 0$, without adjusting $\eta$, the DM density is 8 magnitudes smaller compared to the case with $a = M/2$ [22]. This background contribution is due to geodesics reaching the target location without using any Kerr-black-hole-related effects such as the Penrose process. This contribution is negligible compared to the DM density in the beam and thus effects uniquely attributed to the Kerr metric are causing the DM beam. Furthermore, the larger $a$ is chosen, the more collimated is the DM beam as expected [22].

It should be noted that the obtained densities are much smaller than the local DM density. As an example, we
FIG. 2. DM density profile in the beam represented in units of $\rho_0$ for various distances to the black hole $r_{\text{out}}$ and the rotation axis. In addition, the opening angle of the beam $2\theta_B$ is schematically indicated. Due to the inapplicability of our approximations, for the white pixels there is no data.

We hope that our study inspires future calculations. Moreover, a closer analysis of the produced overdensities near the rotating black hole at the center of the Milky Way might allow to find a source of DM annihilation which is relevant for local DM detection.
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