Optimal design of the gear transmission system based on the modal response surface method

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Abstract. In order to conduct the lightweight optimization design of the gear transmission system, this paper uses the experimental design response surface method to construct the modal response explicit function of the gear transmission system based on the modal analysis results. Furthermore, the optimized design model of the gear transmission system (including gears, gear shafts, and bearings) is established on the base of the results of the modal response surface, and the optimized design model is solved and optimized through the Fmincon toolbox of MATLAB. By comparing the optimized results, it is shown that the optimization design method proposed in this paper can achieve the optimal design of the gear transmission system, which is of great significance to the optimization design of mechanical engineering.

1. Introduction
In the modern industry, the application of the gear transmission system is very common, it is often used to transfer torque and speed between the working machine and the prime mover. The gear transmission system is only composed of gears, shafts, bearings, etc., but it has the characteristics of high transmission efficiency and large transmission power, so it has an irreplaceable role in mechanical transmission. How to design a gear transmission system with better quality and better transmission performance is one of the important problems in the field of mechanical engineering in my country[1].

The accuracy and efficiency of the traditional optimization design method to design the reducer are very low, because the design process is more complicated, especially the optimization of mechanical products with implicit response, which makes the design more difficult and easy to make mistakes. At present, most research institutions use Pro/E for the secondary development of CAD systems[2]. Using Pro/E software as the platform, a system combining assembly design analysis and product conceptual design was developed through C language programming. The system was composed of five modules: design feature library, analysis module, interaction module, search module and assembly module. Due to the long-term research of 3D design by researchers[3] and their application in related industries, the application of independent development technology is very mature. Chen Chen of Nanjing University of Aeronautics and Astronautics and Astronautics and others participated in a more comprehensive design study of shaft parts (3D models), part drawings and part processing systems. Some scholars[4] have studied the three-dimensional standard parts library of diesel locomotives. This research is based on Pro/E for parametric modeling and secondary development to establish a three-dimensional standard parts
library of diesel locomotives, which makes the design and development of new products more convenient and fast.

However, there are currently no practical methods that can solve the mechanical optimization design with the modal implicit response problems. To solve this problem, this article will construct an explicit polynomial based on modal analysis and response surface method, providing an effective way for the optimal design model of the gear transmission system.

2. Modal response surface

2.1. Modal theory

Assuming that the structure has no external force, the governing equation of the free vibration of the linear structure is

\[ [M] \ddot{u} + [K]u = \{0\} \]

The free vibration control equation of the damped linear structure is

\[ [M] \ddot{u} + [C] \dot{u} + [K]u = \{0\} \]

Let its solution is

\[ \{x\} = \{\nu\} e^{\lambda t} \]

Substituting into equation (2), the necessary and sufficient condition for a non-zero solution is

\[ D(\lambda) = \lambda^2 [m] + \lambda [c] + [k] = 0 \]

2.2. Response surface method

According to the above modal theory, modal analysis is implicit analysis, and it is difficult to directly optimize the design of gear transmission systems with modal constraints. In order to realize the optimal design of the gear transmission system, this paper selects the polynomial response surface method for the optimal design. Suppose the random parameter vector \( X=(X_1, X_2, ..., X_{NR}) \) is a quadratic function, as shown in (5). Sampling to obtain \( N_s \) sample points, testing the sample points to obtain a set of sample points \( (y_1, y_2, ..., y_{Ns}) \), through regression analysis to obtain the least squares estimate in the function, to obtain the response surface function.

\[ \hat{Y} = C_0 + \sum_{i=1}^{N_s} C_i X_i + \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} C_{ij} X_i X_j \]

where, \( C_0, C_i, C_{ij} (i=1, ..., N_R; J=i, ..., N_R) \) are undetermined coefficients, in total, \( n+l+n(n+l)/2 \).

The sampling method used is the Box-Behnken sampling method. Each random variable takes three horizontal points, and the center point is combined with the sample point according to certain rules. Perform numerical calculation on \( N_s \) sample points of random parameters to obtain \( N_s \) output points \( (y_1, y_2, ..., y_{Ns}) \), and perform regression analysis on these data using the least square method

\[ s = \sum_{i=1}^{N_s} \varepsilon^2 \]

where \( N_s \) is the number of sample points, \( i \) is the number of random input variables, and \( \varepsilon \) is the error term. To make the error the minimum, there is

2.3. Modal response surface r

The modal analysis of the gear transmission system (parameter 3.1 of the calculation example). Figure 1 shows the modal analysis results of the gear, gear shaft and gear shaft gear of the gear transmission system.

After 25 simulations, 25 response values are obtained. According to the modal analysis results and Eq. (5) and Eq.(6), the response surface function of the gear transmission system is obtained

\[ y = 0.785(2732.75x_1^2 x_2 + 0.92x_2 x_4^2 - x_2 x_3^2 + 12x_1 x_2 x_4 + 98x_3^2 + 39x_4) \]
3. Optimized design of gear transmission system

3.1. Parameters of examples
The parameters of the gear transmission system of the first-stage reducer are: input speed is 1500r/min; input power is 280kw; transmission ratio is 3.5; allowable stress is 975.4MPa; small gear bending condition is 261.7MPa; large gear bending condition is 213.3MPa, other parameters are shown in Table 1.

| Modulus | \( m = 1.5 \) |
|---------|----------------|
| Addendum coefficient | \( h_a^* = 1 \) |
| Head gap coefficient | \( c^* = 0.25 \) |
| Tooth width | \( B = 20\text{mm} \) |
| Number of teeth of driving gear | \( z_1 = 17 \) |
| Number of teeth of driven gear | \( z_2 = 60 \) |

3.2. Optimized design

3.2.1. Design variables. \( x = [x_1, x_2, x_3, x_4]^T = [m, B, d_1, d_2]^T \), where \( m \) is the modulus, \( B \) is the tooth width of the gear, \( d_1 \) is the diameter of the pinion shaft, and \( d_2 \) is the diameter of the large gear shaft.

3.2.2. Objective function. With the goal of reducing the weight of the gear transmission system, the objective function is

\[
\min f(x) = 0.785(2732.75x_1^2x_2 + 0.92x_2x_4^2 - x_2x_3^2 + 12x_1x_2x_4 + 98x_3^2 + 39x_4^2)
\] (8)

3.2.3. Constraints.
(1) The parameters of the gear transmission system of the first-stage reducer are: the input speed is 1500r/min; the input power is 280kw; the transmission ratio is 3.5; the allowable stress is 975.4MPa; the bending resistance condition of the small gear is 261.7MPa; the large gear resistance The bending condition is 213.3MPa.
(2) The range of gear tooth width coefficient is \( 16 \leq B \leq 24 \).
(3) According to the technological requirements of the diameter of the large gear, \( 17x1 - 30 \leq 0 \). The diameter range of the main and driven shafts is: \( 15 \leq d_1 \leq 25 \).

3.3. Results analysis
The SQP optimization method of Fmincon function in MATLAB is used to optimize the design of the gear transmission system in this example. Figures 2 and 3 show the model diagrams of the gear
transmission system before and after optimization. Table 2 shows the structural parameters of the gear transmission system before and after optimization and Constraint parameters. The results show that by optimizing the gear transmission system, its mass is reduced by 479.66g, the weight reduction ratio is 3.45%, the fundamental frequency is increased, and the bending and basic stress are slightly increased, which still meets the strength requirements.

![Fig.2 Model before optimization](image1) ![Fig.3 Model after optimization](image2)

|                  | Before optimized results | After optimized results |
|------------------|--------------------------|-------------------------|
| Mass/g           | 13885                    | 13405                   |
| Contact stress/MPa | 865.7                    | 791.3                   |
| Bending stress/MPa | 212.4                    | 210.8                   |
| Gear mode/Hz     | 76383                    | 94907                   |
| Gear shaft mode/Hz | 9473                     | 9365                    |
| Modulus m/mm     | 1.5                      | 1                       |
| Tooth width B/mm | 20                       | 16                      |
| gear 1 shaft diameter d1/mm | 20 | 22 |
| gear 2 shaft diameter d2/mm | 17 | 23 |

4. Conclusion

Based on the modal analysis and the response surface method, this paper builds an optimized model of the gear transmission system, and completes the lightweight design of the gear transmission system with implicit constraints. In this paper, the proposed optimization design method is versatile and suitable for the optimization design of other mechanical products. It has an important research significance on the mechanical design.

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