Reduced-order dynamic model for droop-controlled inverter/converter-based low-voltage hybrid AC/DC microgrids – Part 2: DC sub-microgrid and power exchange

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Abstract: This study focuses on reduced-order dynamical modelling of droop controlled converter-based DC sub-microgrid (MG) in a hybrid AC/DC MG. In hybrid MGs, electrical power is exchanged between the AC and DC sub-MGs by a bidirectional AC/DC converter. The authors aim to develop a comprehensive reduced-order dynamical model for the DC side in this part, incorporating standard classes of electrical loads including constant current, constant power, and constant resistance loads. Furthermore, dynamical behaviour of the power exchange between the AC and DC sub-MGs is modelled, considering that the bidirectional power converter controller aims to equalise the load ratios of AC and DC sub-MGs in order to facilitate overall decentralised control over the hybrid MG. Analytical derivations of steady-state values of main variables are given and the overall dynamical and algebraic equations are determined. In order to validate the developed model, a hybrid MG is implemented in PSCAD. Then, the proposed model for the case study is implemented in Matlab/Simulink and the results are compared with the PSCAD outputs. The comparative results show the validity of the developed reduced-order comprehensive model. The reduced-order models are preferred in designing observers such as model-based fault detection and diagnosis observers.

Nomenclature

- $K_{conv,i}$: small-signal gain of the $i$th converter
- $K_{excon}$: small-signal gain of the main bidirectional converter
- $\tau_{excon}$: time constant of the main bidirectional converter
- $v_{dc,i}$: voltage setpoint for the $i$th converter
- $v_{dc,ref}$: voltage reference for DC link
- $R_D$: droop coefficient for the $i$th converter
- $\delta v_{dc}$: microgrid (MG) voltage deviation compensation which is calculated at secondary control
- $v_{dc}$: DC-link voltage
- $P_{ex,dc}$: power of constant power loads
- $R_{ex,dc}$: resistance of constant resistant loads
- $i_{dc}$: current of constant current loads
- $K_{p,i}$: proportional gain of the $i$th converter controller
- $K_{i,i}$: integral gain of the $i$th converter controller
- $K_{p,excon}$: proportional gain of the main converter controller
- $K_{I,excon}$: integral gain of the main converter controller
- $i_{o,dc}$: output current of the $i$th converter
- $v_{j}$: output voltage of the $j$th inverter in AC sub-MG
- $f_{j}$: frequency of the $j$th inverter in AC sub-MG
- $\delta_{j}$: phase angle with reference to AC-link voltage for the $j$th inverter in AC sub-MG
- $P_{ex}$: exchanging power between the AC and DC sub-MGs
- $R_{ex}$: output resistance of bidirectional AC–DC converter
- $v_{ex}$: DC voltage of bidirectional AC–DC converter
- $\pm$ - small-signal sign for variables, inputs and outputs
- $\delta_{max}$: maximum permissible sign for variables, inputs and outputs
- $v_{dc,max}$: maximum permissible value of $v_{dc}$
- $v_{dc,min}$: minimum permissible value of $v_{dc}$
- $i_{o,dc,max}$: maximum permissible value of $i_{o,dc}$
- $p = (d/dr)$: derivative operator

1 Introduction

The wide penetration of renewable energy sources has changed the structure of conventional power systems for good [1]. Specifically, advancement of power electronic devices has facilitated a great opportunity for Distributed Energy Resources facing the local loads and the grid through power converters to create a fully controlled microgrid (MG). While MGs reduce the inherent problems with traditional power systems including transmission and distribution losses, hybrid MGs can also improve energy saving by reducing consecutive conversions of AC to DC and vice versa, as they benefit from separate AC and DC sub-MGs. In the DC sub-MG, DC power resources, storage and DC loads are connected to the main DC bus directly or via DC/DC converters. The DC side is connected to the AC side through a bi-directional power converter [2, 3]. Given the rising trend in the development of hybrid MGs, deriving a comprehensive model for design and analysis is vital. Since MGs are regarded as some kinds of cyber-physical systems, the dynamic model consists of both the cyber part and physical part. In which cyber part mostly contributes to control strategies. Hence a literature review on both parts is needed. Hybrid MGs can be categorised into AC-coupled, DC-coupled or AC/DC-coupled networks [4]. Each of these structures can have grid-connected or stand-alone operations. The grid-connected mode can be on dispatched mode or undispached mode. A detailed study on distributed control for autonomous operation of hybrid MGs is reported in [5]. This study proposes coordination of the AC and DC MGs’ droop controls, or in other words, globalising of the droop control for the whole MG. The authors in [6] have reviewed hierarchical control strategies of DC MGs. Another hierarchical control scheme based on droop-controlled AC and DC MGs has been proposed in [7]. In [8], a model for a DC MG is developed which is based on the line impedances and the node capacitor. The control strategy is based on droop control, and the correction coefficient of secondary control is calculated by a load sharing algorithm. However, in [9] the authors try to derive the relation of voltages and currents in the DC MG based on the admittance matrix of the MG, while the loads are assumed to be constant current loads. The control strategy given in [10] represents an AC–DC interfacing converter control approach, which is used to control
MG in connected mode. There are also some reports on models for DC MGs, which include the algebraic equations in steady state, or only consider the droop controller in [11, 12]. The model which is derived in [13] is based on the admittance matrix for the MG as a network. Modelling of voltage source inverters interacting constant power loads on DC link is given in [14], in which a transfer function from source voltage, the reference voltage and load current to load voltage is determined. The authors in [15] have derived a transfer function from reference voltage to output voltage for each converter in order to analyse system stability. Moreover, the authors in [16] presented linear dynamical equations for parallel buck converters in a DC MG comprising inner voltage and current loops using proportional integral (PI) controllers, and primary or outer controller equation. The authors have derived a global DC MG dynamic model in transfer function form in [17]. Since the model has used much Laplace transfer function for controllers, converters and lines, it is not an optimised reduced-order model. In addition to it, the state variables of the model cannot be physical variables. A set of parallel sources and their corresponding transmission lines are modelled by an ideal voltage source in series with an equivalent resistance and inductance in [18]. This approximate model is used for stability analysis. In [19], a distributed control strategy is introduced for DC MGs, named DC-bus signalling, in order to control the MG in a distributed approach. The authors in [20] have represented small-signal dynamic equations in the time domain for each component in a low-voltage DC MG including sources, loads, and distribution lines are derived. Moreover, the interconnection structure of the system is expressed by Kirchhoff’s voltage law (KVL) and Kirchhoff’s current law (KCL) of the network. An up-down operation model of a hybrid MG is proposed in [21] which consists of the system- and device-level models, in order to balance the generation and load, and also to control the voltage variations in AC and DC sub-grids. In [22], the average converter models, the storage device internal structure and the other system component dynamics are augmented to implement a dynamical model of a DC MG with resistive loads. The problem of optimal voltage and power regulation is formulated in [23] for DGs in DC MGs, in which the power flow equations of the DC MG are given by

\[ P_i - P_e = V_i^2 \sum_j \frac{y_{ij}}{V_j} - P_i \]

where \( y_{ij} \) is the line admittance of node \( i \) and node \( j \).

In this paper, the resources in the DC side are connected to the DC link via power converters. As the loads can have constant current, constant power or constant resistance characteristics, all these load types are regarded in this paper. Hence, the DC-link voltage is related to the loads as given by the following equation:

\[ v_{dc} = R_{dc} \left( \sum_i i_{dc,i} - i_{dc} - P_{dc} + P_{ex} \right) \]

where \( i_{dc,i} \) represents the injected current to the DC bus by the \( i \)th DC/DC converter. The constant power load \( P_{dc} \) can also represent an energy storage system like a battery, while it takes negative value in discharging mode. In order to attain a linear dynamical model, we use the following approximations for any \( \theta \) and \( \phi \):

\[ (1 - \theta)^{-1} \cong 1 + \theta, \quad 0 < \theta < 1 \]

Therefore, (1) can be approximated as follows:

\[ v_{dc} = R_{dc} \sum_i i_{dc,i} + \frac{v_{dc}}{R_{dc}} - \frac{R_{dc} P_{dc}}{v_{dc}} + \frac{R_{dc} P_{ex}}{v_{dc}} \]

The variations of exchanged power between DC and AC sides, and injected currents by the inverters are assigned as state variables in dynamical model derivation, therefore, we have

\[ v_{dc} = R_{dc} \sum_i i_{dc,i} + \frac{v_{dc}}{R_{dc}} - \frac{R_{dc} P_{dc}}{v_{dc}} + \frac{R_{dc} P_{ex}}{v_{dc}} \]

The variations of exchanged power between DC and AC sides, and injected currents by the inverters are assigned as state variables in dynamical model derivation, therefore, we have
As shown in Fig. 3, the inner loop regulates the output voltage of the converter based on a PI controller as given by

\[
\delta v_{dc,i} = v_{dc,i} - v_{dc,i}^{ref}
\]

From the above equation and (4), the deviation of the output voltage from the steady-state value for each resource converter is determined to be

\[
\delta v_{dc,i} = \left( K_{P,i} + \frac{K_{I,i}}{s} \right) i_{dc,i} + \delta v_{dc,j}
\]

Let us define the DC control vector as follows:

\[
U_{dc} = \begin{bmatrix} \delta v_{dc} \\ \delta i_{dc} \end{bmatrix}
\]

From the definition of \(X_{dc}, W_{dc}\), and \(U_{dc}\), and using (7), the following dynamical equations are obtained:

\[
\dot{X}_{dc,i} = \begin{bmatrix} M_{ex,di} & M_{dc,di} \end{bmatrix} X_{dc} + \begin{bmatrix} A_{ex,di} & A_{dc,di} \end{bmatrix} U_{dc}
\]

where

\[
M_{ex,di} = \begin{bmatrix} m_{ex,i} \end{bmatrix}_{i=1}^{n}, \quad M_{dc,di} = \begin{bmatrix} m_{dc,i} \end{bmatrix}_{j=1}^{n}
\]

in which \(m_{ex,i} = F_{di}(1 + K_{conv,K_{P,i}})\)

\[
A_{ex,di} = \begin{bmatrix} a_{ex,i} \end{bmatrix}_{i=1}^{n}, \quad A_{dc,di} = \begin{bmatrix} a_{dc,i} \end{bmatrix}_{j=1}^{n}
\]

\[
\]

in which

\[
a_{dc,i,j} = \begin{cases} K_{conv,1} (R_{ij} + c_i) & \text{for } i = j \\ K_{conv,1} c_i & \text{for } i \neq j \end{cases}
\]

\[
B_{dc,i} = \begin{bmatrix} b_{dc,i} \end{bmatrix}_{j=1}^{n}, \quad E_{dc,i} = \begin{bmatrix} e_{dc,i} \end{bmatrix}_{j=1}^{n}
\]

\[
C_{dc,ni} = \begin{bmatrix} C_1 & C_2 & \cdots & C_n \end{bmatrix}_{i=1}^{n},
\]

\[
D_{dc,ni} = \begin{bmatrix} D_1 & \cdots & D_n \end{bmatrix}_{i=1}^{n}
\]

To have a lucid vision of dynamic model, MGs are defined as some kinds of cyber-physical systems. The equivalent circuit of the physical layer of MG is depicted in Fig. 3, while the cyber layer
which contains converters' control loop and also exchanging power reference control loop are defined in (7) and (25) (Fig. 4).

2.3 Transformation of the input and disturbance vectors

The columns of $B_{dc}$ and $E_{dc}$ matrices are linearly dependent. By defining new input and disturbance state variables as follows, the number of columns in the disturbance matrix can be reduced. Hence the dynamic equations in (8) can be rewritten as follows:

\[
\begin{bmatrix}
[M_Z \quad M_{dc \text{eq}}] \cdot X_{dc} = [A_{dc \text{eq}} \quad A_{dc \text{eq}}] \cdot X_{dc} \\
\dot{v}_{dc} = [C_{dc \text{eq}} \quad C_{dc \text{eq}}] \cdot X_{dc} + D_{dc \text{eq}} \cdot \dot{v}_{dc} + F_{dc \text{eq}} \cdot \dot{P}_{dc}
\end{bmatrix}
\]

(9)

Regarding $\dot{v}_{dc \text{ref}} = \dot{v}_{dc \text{ref}} + \dot{v}_{dc}$ and $\dot{P}_{dc} = \dot{F}_{dc \text{eq}} + \dot{F}_{dc \text{eq}} + F_{dc \text{eq}}$, yield

\[
\dot{B}_{dc \text{eq}} = [\dot{b}_{dc \text{eq}}] \quad \text{for } i = 1, \ldots, n, j = 1
\]

in which $\dot{b}_{dc \text{eq}} = -K_{\text{conv}} \cdot K_{P, \text{dc}} \cdot \dot{P} - K_{\text{conv}} \cdot K_{I, \text{eq}}$

\[
\dot{E}_{dc \text{eq}} = [\dot{e}_{dc \text{eq}}] \quad \text{for } i = 1, \ldots, n, j = 1
\]

in which $\dot{e}_{dc \text{eq}} = (K_{P, \text{conv}} + 1) \cdot \dot{P} + K_{I, \text{conv}}$

\[
D_{dc \text{eq}} = [0], \quad F_{dc \text{eq}} = [1]
\]

3 Globalising of the local AC and DC droop control schemes

To achieve the goal of globalised droop control, two control loops should be used. The outer control loop calculates a proper setpoint for exchanging power while the inner control loop regulates the converter's voltage such that the exchanging power reaches the calculated setpoint.

3.1 Outer control loop

In the outer control loop, it is tried to regulate the exchanging power reference $P_{\text{ex}}^e$, such that the normalised DC-link voltage gets equal to the normalised AC-link voltage.

If the droop controlled variables get normalised, an attempt by a PI controller in exchanging power between AC and DC MGs can elevate the local droop control schemes to global droop control. While in [5, 25, 26] $f$ is used for droop control of active power, here $v_{ac}$ is used instead; since in the low-voltage AC sub-MG, the dominating impedance of electrical lines is resistive instead of inductive, as explained in Section 1. Thus

\[
P_{\text{ex}}^e = \left( K_{P, \text{ex}} + \frac{K_{I, \text{ex}}}{p} \right) \cdot (v_{\text{dc} \text{eq}} - \left( v_{\text{ac} \text{eq}} \right))
\]

(10)

In which $P_{\text{ex}}^e$ is the exchanging power reference from the DC side to AC side and normalised values can be obtained by the following equation:

\[
(\gamma) = \frac{\gamma - \gamma_{\text{min}}}{0.5(\gamma_{\text{max}} - \gamma_{\text{min}})}, \quad \gamma \in \{v_{\text{dc}}, v_{\text{ac}}\}
\]

(11)

The small-signal form of (10) is given by

\[
p_{\text{ex}}^e = 2(pK_{P, \text{ex}} + K_{I, \text{ex}}) \cdot \left( \frac{v_{\text{dc}}}{v_{\text{max}} - v_{\text{min}}} - \frac{v_{\text{ac}}}{v_{\text{max}} - v_{\text{min}}} \right)
\]

(12)

To prove that (12) yields in an equal load ratio for both the DC and AC sides, the following justification can be provided. The steady-state deviation of the AC and DC bus voltages from their maximum values can be estimated by droop equations. This deviation for the AC side is $v_{\text{ac} \text{eq}} = -R_{\text{ac}} \cdot P_{\text{ex}}$ and for the DC side is $v_{\text{dc} \text{eq}} = -R_{\text{dc}} \cdot P_{\text{ex}}$. Therefore, from (5), it can be determined that the normalised deviation voltages are determined to be

\[
\frac{-R_{\text{ac}} P_{\text{ex}}}{v_{\text{max}} - v_{\text{min}}} = \frac{-R_{\text{dc}} P_{\text{ex}}}{v_{\text{max}} - v_{\text{min}}}
\]

Thus, the controller in (10) aims to equalise the load ratios of AC and DC sub-MGs in order to facilitate overall decentralised control over the hybrid MG.

3.2 Inner control loop

In order to facilitate cooperation of the AC and DC MGs in power sharing, a bi-directional AC–DC converter is used. A prominent choice for this converter is the constant power factor one cycle control (CPF-OCC), which makes the AC voltage and current in phase and also regulates both AC and DC voltages. This converter can change the direction of power flow automatically without changing the switching mode [27].

Regarding the dynamic of such AC–DC converters for regulating output voltage $v_{\text{ex}}$ is given as (13), we should try to find a relation between output voltage and exchanging power

\[
(v_{\text{ex}} - v_{\text{ac}}) \cdot (K_{P, \text{ex}} + \frac{K_{I, \text{ex}}}{p}) + \frac{K_{\text{ex}}}{1 + T_{\text{ex}}p} + \dot{v}_{\text{ex}} = v_{\text{ex}}
\]

(13)

Knowing that the exchanging power can be adjusted by regulating the voltage of the converter at the DC side, the relation between the exchanging power and the voltage of converter at the DC side is given by

\[
P_{\text{ex}} = v_{\text{ex}} \cdot \left( \frac{v_{\text{dc}} - v_{\text{ex}}}{R_{\text{dc}}} \right)
\]

and therefore

\[
v_{\text{ex}} = \frac{v_{\text{dc}} + \sqrt{v_{\text{dc}}^2 - 4R_{\text{dc}}P_{\text{ex}}}}{2}
\]

(14)

in which $R_{\text{dc}}$ is the equivalent resistance between the converter and the DC link. Also, the corresponding voltage reference value for the DC side of AC–DC converter is given by

\[
v_{\text{dc}} = \frac{v_{\text{ac}} + \sqrt{v_{\text{ac}}^2 - 4R_{\text{ac}}P_{\text{ex}}}}{2}
\]

(15)

3.3 Deriving steady-state value of exchanging power

In order to derive the steady-state value of $P_{\text{ex}}$, the following parameters are defined and determined. Here, the ratio of the total load to the capacity of hybrid MG generation is defined as follows:

\[
r_j = \frac{\sum P_{\text{load, ac}} + \sum P_{\text{load, dc}}}{P_{\text{cap}}} = \frac{\sum P_{\text{load, ac}}}{P_{\text{cap}}} + \frac{\sum P_{\text{load, dc}}}{P_{\text{cap}}}
\]

(16)

The individual ratios of AC and DC side loads to the capacity of AC and DC sub-MGs generation can be defined as follows:

\[
r_{j, \text{dc}}(\text{ac}) = \frac{\sum P_{\text{load, dc}}}{P_{\text{cap}}} = \frac{\sum P_{\text{load, dc}}}{P_{\text{cap}}} + \frac{\sum P_{\text{load, dc}}}{P_{\text{cap}}}
\]

(17)

The exchanging power of the bi-directional AC/DC converter should be calculated such that the normalised total, AC and DC sides' loads are equal, i.e. $r_j = r_{j, \text{dc}}(\text{ac})$, where

\[
r_{j, \text{dc}}(\text{ac}) = \frac{\sum P_{\text{load, dc}} + \sum P_{\text{ex}}}{P_{\text{cap}}}
\]

(18)
Therefore

\[
\dot{P}_{ex} = \frac{\sum_{i=1}^{n} P_{dc,i}^{\text{load}}}{\sum_{i=1}^{n} P_{ac,i}^{\text{load}}} (r_i - r_{dc}) \sum_{i=1}^{n} P_{dc,i}^{\text{load}} \tag{20}
\]

3.4 Derivation of power exchange dynamics

Substituting \( v_{ex} \) and \( \dot{v}_{ex} \) from (14) and (15) into (13) and taking a partial derivative, results in

\[
\left( \dot{\tilde{P}}_{ex} - \tilde{P}_{ex} \right) \left( \frac{K_{P,\text{excon}} + K_{L,\text{excon}}}{p} \right) \frac{K_{\text{excon}}}{1 + \tau_{\text{excon}} p} = - \frac{\ddot{v}_{ex}}{R_{Ex}} \dot{v}_{dc} + \tilde{P}_{ex} \tag{21}
\]

To have a reduced-order model, regarding that \( \dot{v}_{dc} \) is not a rich signal, the second-order derivative of it can be ignored. Substituting \( \dot{v}_{dc} \) in (21) from (9) yields to

\[
\left( \frac{\ddot{v}_{ex}}{R_{Ex}} C_{\text{ex,}14} - \frac{K_{P,\text{excon}} K_{\text{excon}} - 1}{p} \right) \dot{p}_{ex} - \tau_{\text{excon}} \dot{p}_{ex} \dot{\tilde{P}}_{ex} + \frac{\dot{v}_{ex}}{R_{Ex}} C_{\text{ex,}16} \left( \delta_{1.1,dc} \right) + \frac{\dot{v}_{ex}}{R_{Ex}} C_{\text{ex,}16} \left( \delta_{1.2,dc} \right) + \cdots + \frac{\dot{v}_{ex}}{R_{Ex}} C_{\text{ex,}16} \left( \delta_{n,dc} \right) + K_{P,\text{excon}} K_{\text{excon}} \dot{p}_{ex} + K_{L,\text{excon}} K_{\text{excon}} \dot{\tilde{P}}_{ex} \tag{22}
\]

Defining the following state, input, output and disturbance vectors (see equation below)

\[
U = \begin{bmatrix} \dot{v}_{\text{ref, ac}} & \dot{f}_{\text{ref}} & \dot{v}_{\text{ref, dc}} \end{bmatrix}^T
\]

\[
Y = \begin{bmatrix} \dot{v}_{\text{dc}} & \dot{\dot{v}}_{\text{dc}} \end{bmatrix}^T
\]

\[
W = \begin{bmatrix} \dot{P}_{\text{load}} & \dot{Q}_{\text{load}} & \dot{I}_{\text{load}} & \dot{Z}_{\text{load}} & \dot{\tilde{P}}_{\text{dc}} \end{bmatrix}^T
\]

where \( \dot{P}_{\text{load}}, \dot{Q}_{\text{load}}, \dot{I}_{\text{load}}, \) and \( \dot{Z}_{\text{load}} \) represent cumulative active and reactive power of AC loads and constant current and constant impedance loads in AC side, respectively. The new set of dynamical equations is derived as follows:

\[
\begin{bmatrix} 0 & 0 & M_{EX,74} & M_{EX,75} & M_{DC,76} & M_{EX,77} \end{bmatrix} \dot{X} = \begin{bmatrix} 0 & 0 & A_{EX,74} & 0 & 0 & A_{EX,77} \end{bmatrix} X + \begin{bmatrix} 0 & B_{DC,74} & U & 0 & 0 & E_{DC,76} \end{bmatrix} W \tag{23}
\]

in which

\[
M_{EX,74} = \frac{v_{ex}}{R_{ex}} C_{\text{ex,}34} - K_{P,\text{excon}} K_{\text{excon}} - 1
\]

\[
M_{EX,75} = - \tau_{\text{excon}}
\]

\[
M_{DC,76} = v_{ex} C_{\text{dc,16}}
\]

\[
M_{EX,77} = K_{P,\text{excon}} K_{\text{excon}}
\]

\[
A_{EX,74} = K_{L,\text{excon}} K_{\text{excon}}
\]

\[
A_{EX,77} = - K_{L,\text{excon}} K_{\text{excon}}
\]

\[
B_{DC,74} = - v_{ex} p D_{DC,74}
\]

\[
E_{DC,76} = - v_{ex} p F_{DC,76}
\]

3.5 Dynamical behaviour of the reference of exchanging power

The exchanging power dynamics is given by (12).

Here, we aim to realise the following state equations for the dynamics

\[
\begin{bmatrix} M_{AC,41} & M_{AC,42} & M_{AC,43} & M_{EX,44} \end{bmatrix} X = \begin{bmatrix} 0 & 0 & 0 & M_{DC,46} \end{bmatrix} X + \begin{bmatrix} B_{AC,4} & B_{DC,4} & U & E_{AC,4} & E_{DC,4} \end{bmatrix} W \tag{24}
\]

Having \( \dot{v}_{dc} \) and \( \dot{\dot{v}}_{dc} \) algebraic equations, (12) can be transformed into the following form (see page 25) at the top of the next page:
\[
\frac{2K_{P,ex}}{v_{dc}^{\max}} C_{ac,i}(m + 1:3m) + i_{ex}^2 + i_{ex}^1 + i_{ex}^0
\]

\[
\frac{2K_{P,ex}}{v_{dc}^{\max}} C_{ac,i}(3m + 1) - \frac{2K_{P,ex}}{v_{dc}^{\max}} C_{ac,i}(m + 1:3m) \hat{P}_{ex}
\]

\[
\frac{2K_{I,ex}}{v_{dc}^{\max}} C_{dc,i}(m + 1:3m) + \frac{2K_{I,ex}}{v_{dc}^{\max}} C_{dc,i}(3m + 1) - \frac{2K_{I,ex}}{v_{dc}^{\max}} C_{dc,i}(m + 1:3m) \hat{P}_{dc}
\]

Determination of all the seven groups of state equations, the total system can be integrated in one general state-space form as follows:

\[
\begin{bmatrix}
M_{ac,11} & M_{ac,12} & M_{ac,13} & M_{ac,14} & 0 & 0 & 0 \\
M_{ac,21} & M_{ac,22} & M_{ac,23} & M_{ac,24} & 0 & 0 & 0 \\
M_{ac,31} & M_{ac,32} & M_{ac,33} & M_{ac,34} & 0 & 0 & 0 \\
0 & M_{ac,41} & M_{ac,42} & M_{ac,43} & M_{ac,44} & M_{ac,45} & 1 \\
0 & 0 & 0 & M_{ac,64} & M_{ac,65} & 0 & 0 \\
0 & 0 & 0 & M_{ac,74} & M_{ac,75} & M_{ac,76} & M_{ac,77} \\
\end{bmatrix}
\begin{bmatrix}
\dot{v}_1 \\
\dot{v}_2 \\
\dot{v}_3 \\
\dot{v}_4 \\
\dot{v}_5 \\
\dot{v}_6 \\
\hat{p}_{ex} \\
\hat{p}_{dc} \\
\end{bmatrix}
\]

\[
A_0 = A_1 + p A_2
\]

\[
B_0 = B_1 + p B_2
\]

\[
B_0 = B_{ac,1} + p B_{dc,4}
\]

\[
E_{ac,1} = E_{ac,2} + p E_{ac,3}
\]

\[
E_{dc,6} = E_{dc,7}
\]

\[
C_0 = C_1 + p C_2
\]

Since \( A_0 \), \( B_0 \), \( C_0 \), \( D_0 \), \( E_{ac,0} \) and \( E_{dc,0} \) in the above system, include \( p \)-operator. To exclude \( p \)-operator from these matrices, they are decomposed into the without and with \( p \)-operator parts. Hence the above system can be transformed into the following equation:

\[
4 \text{ Augmenting models for AC and DC MGs}
\]
\[ \begin{align*}
M \dot{X} &= A_1 X_1 + A_1 X_2 + B_1 U + B_1 \dot{U} + E_{d_1} W + E_{d_2} W \\
Y &= C_1 X_1 + C_1 X_2 + D_1 U + D_1 \dot{U} + F_{d_1} W + F_{d_2} W 
\end{align*} \]  

(27)

Regarding \( M = M_b - A_2 \) and substituting \( X \) in the output equation from the state equation, it can be concluded that

\[ \begin{align*}
Y &= (C_1 + C_1 M_1^{-1} A) X_1 + (C_1 + C_1 M_1^{-1} A) M_1^{-1} A_i X_2 \\
& + ((C_1 + C_1 M_1^{-1} A) M_1^{-1} B_1 + D_1 + C_1 M_1^{-1} B_2) U \\
& + (D_2 + C_1 M_1^{-1} B_2) \dot{U} \\
& + (C_1 + C_1 M_1^{-1} A) M_1^{-1} E_{d_1} + F_{d_1} + C_1 M_1^{-1} E_{d_2}) W \\
& + (F_{d_2} + C_1 M_1^{-1} E_{d_2}) W 
\end{align*} \]

(28)

4.1 Generating state-space form

To generate a state-space formed model, the state vector \( X \) is decomposed by \( X = X_1 + X_2 \). Consequently, using this state decomposition, (27) can be rewritten in (29), while the number of states is doubled

\[ \begin{align*}
M \dot{X}_1 &= A_1 X_1 + B_1 U + E_{d_1} W \\
M \dot{X}_2 &= A_1 X_2 + B_1 U + E_{d_2} W 
\end{align*} \]  

(29)

and

\[ \begin{align*}
Y &= (C_1 + C_1 M_1^{-1} A) X_1 + (C_1 + C_1 M_1^{-1} A) M_1^{-1} A_i X_2 \\
& + ((C_1 + C_1 M_1^{-1} A) M_1^{-1} B_1 + D_1 + C_1 M_1^{-1} B_2) U \\
& + (D_2 + C_1 M_1^{-1} B_2) \dot{U} \\
& + (C_1 + C_1 M_1^{-1} A) M_1^{-1} E_{d_1} + F_{d_1} + C_1 M_1^{-1} E_{d_2}) W \\
& + (F_{d_2} + C_1 M_1^{-1} E_{d_2}) W 
\end{align*} \]

(30)

In which \( A_i, M_1, B_1, B_2, E_{d_1}, E_{d_2}, C_1, C_2, D_1, D_2, F_{d_1} \) and \( F_{d_2} \) does not contain any \( p \)-operator. If the direct effect of \( U \) and \( W \) on outputs are ignored, in case that \( U \) and \( W \) are not rich signals, we get to the following standard form:

\[ \begin{align*}
M \dot{x} &= Ax + Bu + E_{d_1} W \\
Y &= Cx + Du + F_{d_1} W 
\end{align*} \]  

(31)

In which

\[ \begin{align*}
x &= \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \\
M &= \begin{bmatrix} M_1 & 0 \\ 0 & M_1 \end{bmatrix} \\
A &= \begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix} \\
B &= \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \\
E &= \begin{bmatrix} E_{d_1} \\ E_{d_2} \end{bmatrix} \\
C &= \begin{bmatrix} (C_1 + C_1 M_1^{-1} A) & (C_1 + C_1 M_1^{-1} A) M_1^{-1} A_i \end{bmatrix} \\
D &= \begin{bmatrix} (C_1 + C_1 M_1^{-1} A) M_1^{-1} B_1 + D_1 + C_1 M_1^{-1} B_2 \\ (C_1 + C_1 M_1^{-1} A) M_1^{-1} B_1 + D_1 + C_1 M_1^{-1} B_2 \end{bmatrix} \\
F_{d_1} &= \begin{bmatrix} (C_1 + C_1 M_1^{-1} A) M_1^{-1} E_{d_1} + F_{d_1} + C_1 M_1^{-1} E_{d_2} \\ (C_1 + C_1 M_1^{-1} A) M_1^{-1} E_{d_1} + F_{d_1} + C_1 M_1^{-1} E_{d_2} \end{bmatrix} \\
F_{d_2} &= \begin{bmatrix} (C_1 + C_1 M_1^{-1} A) M_1^{-1} E_{d_1} + F_{d_1} + C_1 M_1^{-1} E_{d_2} \\ (C_1 + C_1 M_1^{-1} A) M_1^{-1} E_{d_1} + F_{d_1} + C_1 M_1^{-1} E_{d_2} \end{bmatrix} 
\end{align*} \]

5 Case study

In this section, a hybrid MG is modelled using PSCAD and also the proposed mathematical model is developed in Matlab/Simulink. The results are compared to validate the proposed model. The DC sub-MG and the main converter are chosen as follows:

- Power exchanging converter: Bi-directional CPF-OCO AC–DC converter which connects AC MG to DC MG.
- DC resource 1: DC–DC full-bridge converter which is connected to a DC voltage resource.
- DC resource 2: DC–DC full-bridge converter which is connected to a DC voltage resource.
- DC consumer 1: Constant resistive load.
- DC consumer 2: Constant current load.
- DC consumer 3: Constant power load.

There are also some other AC resources and loads on AC bus, which parameters were given in Section 1. Fig. 5 illustrates this configuration. The parameters of the system for simulation are brought in Tables 1–3.

To determine the small-signal gain of each converter, the derivative of the output voltage with respect to the duty cycle at a nominal operating voltage should be taken. Table 4 shows the different small-signal gains for different DC/DC converter types, in which \( V_{in} \) is the converter’s input, \( D \) represents the duty cycle and \( n \) is the transformer ratio.

The results of step response for a 5 V setpoint command change are depicted in the left side of Figs. 6–8. Also, disturbance response for a 1000 W constant power load increase and 16.7% resistance load decrease at the same time have been shown on the
right side of Figs. 5–7. As it can be seen the generated DRs’ power is changed automatically by changing in load. In other words, the model is also tested under the scenario in which the outputs of the DRs are varying. The signals of the derived dynamical model are shown in red-dashed and the signals of PSCAD simulation are shown in blue colour. These comparative results show that the proposed dynamical model approximates an acceptable reduced-order dynamic for the real system and can be utilised for MG analysis and design purposes. However, harmonic distortion makes the power equivalents inaccurate. Hence active and reactive powers have some deviations from their calculated value. Although the derived model is a reduced order dynamic, it can be very useful in model based fault detection and diagnosis observers. The application of this dynamical model will be approved in future studies.

Comparing this step response of proposed model with the step response of previous models such as models of [18, 20], it can be seen that although they may have shown more accurate step response, our proposed model has considered the much more complex condition for simulation. The previous models have not considered the power deal with AC sub-MG, voltage setpoint command from PMCU, converter control loops and three types of load. Also, the input of previous models is current/power changes of sources/loads which is very simpler to model.

6 Concluding remarks and future work

A comprehensive reduced-order dynamical model for the DC side in a hybrid AC/DC MG is derived which incorporates droop and inverter control loops, and power exchange between AC and DC sides. The inputs of the model are the setpoints which are sent from secondary control loop, while the load changes are considered as disturbances. This configuration for modelling can be used in cyber-attack detection, in which the data sent from PMCU to DGs may be deviated by an attacker. Moreover, the steady-state values of the main variables are analytically derived and finally, the state-space form of the MG is given. The dynamical model considers all load types, droop and inverter control loops, dynamics of exchanging power between both sides and its reference, derivation of steady-state values, and the overall dynamical and algebraic equations. Although the reduced-order models make some limitations on accuracy and robustness of the model, the comparative tests between PSCAD and derived reduced-order model justify its feasibility for utilisation in analysis, design, modifications, predictions, and operational management of such systems. The reduced-order models are more desirable in designing observers than the high-order models. In future studies, the model will be used for designing a model-based fault detection observer in order to detect a setpoint cyber attack.

Fig. 6 State variables’ variations for the dynamical model and PSCAD simulation. (Left) +5v command change in reference voltage of inverters; (Right) 16.7% decrease in $R_{load}$ magnitude and 1000 W load increase

Fig. 7 Outputs’ variations for the dynamical model and PSCAD simulation. (Left) +5v command change in reference voltage of inverters; (Right) 16.7% decrease in $R_{load}$ magnitude and 1000 W load increase
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Fig. 8 Active and reactive power of inverters for the dynamical model and PSCAD simulation.(Left) +5v command change in reference voltage of inverters; (Right) 16.7% decrease in \( R_{\text{load}} \) magnitude and 1000 W load increase

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