Radiatively induced leptogenesis in a minimal seesaw model

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We study the possibility that the baryon asymmetry of the universe is generated in a minimal seesaw scenario where two right-handed Majorana neutrinos with degenerate masses are added to the standard model particle content. In the usual framework of thermal leptogenesis, a nonzero $CP$ asymmetry can be obtained through the mass splitting induced by the running of the heavy Majorana neutrino masses from their degeneracy scale down to the seesaw scale. Although, in the light of the present neutrino oscillation data, the produced baryon asymmetry turns out to be smaller than the experimental value, the present mechanism could be viable in simple extensions of the standard model.

I. INTRODUCTION

The outstanding advances witnessed by experimental cosmology have brought us to a new era where an unprecedented precision has been achieved in measuring several cosmological parameters. Of particular importance is the value of the baryon-to-photon ratio, which is determined to be \[ \eta_B = 6.1^{+0.3}_{-0.2} \times 10^{-10}, \] from the latest measurements of the Wilkinson Microwave Anisotropy Probe (WMAP) satellite. Several studies have been aimed at the explanation of this small but non-vanishing quantity. Among them, leptogenesis [2] has become the most attractive mechanism to generate the baryon asymmetry, not only because of its simplicity, but also due to its possible connection with low energy neutrino physics [3–6]. Indeed, it is conceivable that the physical effects responsible for the existence of suppressed neutrino masses may simultaneously play a crucial role in the thermal history of our universe.

From the theoretical point of view, the addition of heavy neutrino singlets to the standard model particle content is an elegant and economical way to account for small neutrino masses via the seesaw mechanism. In this framework, the simplest possibility which can lead to a successful baryogenesis relies on the out-of-equilibrium $L$-violating decays of heavy Majorana neutrinos (thermal leptogenesis). The excess of lepton number produced in these decays is then partially converted into a baryon asymmetry by the $(B + L)$-violating sphaleron interactions, which are in thermal equilibrium for temperatures $10^2 \text{ GeV} \lesssim T \lesssim 10^{12} \text{ GeV}$.

The heavy Majorana neutrino masses are rather unconstrained in the sense that they can range from a few TeV up to $10^{16} \text{ GeV}$, depending on the model considered. Moreover, their spectrum may be either hierarchical, quasi-degenerate or even exactly degenerate. If the heavy Majorana neutrinos have a hierarchical mass spectrum, a lower bound of $10^8 - 10^9 \text{ GeV}$ can be inferred on the leptogenesis scale [7]. On the other hand, the resonant enhancement of the leptonic $CP$ asymmetries through the mixing of two nearly degenerate heavy Majorana neutrinos [8, 9] can lower their mass scale up to TeV energies [9, 10].

Due to the proliferation of free parameters in the high energy neutrino sector, the link between leptogenesis and low energy phenomenology can be only established in a model dependent way. In the framework of the SM extended with three heavy Majorana neutrinos one has eighteen parameters in the high-energy neutrino sector which have to be confronted with the nine low-energy parameters of the light neutrino mass matrix. A minimal version of the seesaw mechanism, based on the existence of just two heavy neutrino states, reduces the number of parameters from eighteen to eleven. But even in this case, additional assumptions (such as the introduction of texture zeros in the Dirac Yukawa matrix) are required to completely determine the high energy neutrino sector from the low energy neutrino observables [11]. The effective light neutrino mass matrix is in this case defined by seven independent quantities since one of the neutrinos is predicted to be massless.

In this paper we assume an exact degeneracy of the heavy Majorana neutrinos at a scale which is higher than their decoupling scale. We show that, within the minimal seesaw scenario, the heavy neutrino mass splitting induced by renormalization group effects can lead to appropriate values of the $CP$ asymmetries relevant for the computation of the cosmological baryon asymmetry in the thermal leptogenesis framework. This provides us with a simple explanation for the extremely small mass splitting which is typically required in this context. The compatibility of this framework with the presently available low-energy neutrino data is also addressed. We show that the solar and atmospheric neutrino data puts strong constraints on this scenario and already excludes its minimal version. Since the radiatively induced lepton asymmetry turns out to be proportional to the charged-lepton $\tau$ Yukawa coupling, an enhancement of this coupling could be sufficient to obtain the required baryon asymmetry. This can be achieved, for instance, in simple extensions of the standard model with more than one Higgs doublet. In this case, the present framework could easily accommodate the low-energy neutrino data and lead to a successful leptogenesis.
II. SEESAW RECONSTRUCTION

In the framework of the SM extended with two heavy right-handed neutrino singlets, and before the decoupling of the heavy Majorana neutrino states, the leptonic sector is characterized by the Lagrangian

\[ -\mathcal{L} = \ell_L Y_\ell \ell_R \phi^0 + \nu_L Y_\nu \nu_R \phi^0 + \frac{1}{2} \nu_R^T C M_R \nu_R + h.c., \]

where \( \ell_L(R) \) and \( \nu_L(R) \) are the left-handed (right-handed) charged-lepton and neutrino fields, respectively; \( \phi^0 \) is the neutral component of the SM Higgs doublet. The Dirac neutrino and charged-lepton Yukawa coupling matrices are denoted by \( Y_\ell \) and \( Y_\nu \), while \( M_R \) stands for the \( 2 \times 2 \) symmetric mass matrix of the heavy right-handed neutrinos. In the basis where \( Y_\ell \) and \( M_R \) are diagonal, the neutrino sector is characterized, in general, by eleven parameters, namely the six moduli and three phases in the \( 3 \times 2 \) Dirac neutrino Yukawa coupling \( Y_\nu \) plus the two heavy neutrino masses, \( M_1 \) and \( M_2 \). This number may be further reduced by making assumptions about the structure of \( Y_\nu \) \[11\text{–}13\].

Let us now assume that at a scale \( \Lambda_D \) the heavy neutrinos are degenerate in mass, i.e., \( M_1 = M_2 = M \), with \( M < \Lambda_D \). We remark that in the limit of exact heavy Majorana neutrino mass degeneracy, \( CP \) is not necessarily conserved. Indeed, the non-vanishing of the weak-basis invariant \[17, 18\]

\[ \mathcal{J}_1 = M^{-6} \text{Tr} \left[ Y_\nu Y_\nu^T Y_\ell Y_\ell^T Y_\nu^\dagger Y_\nu^\dagger Y_\ell Y_\ell^T \right]^3, \]

which is not proportional to \( M_2^2 - M_1^2 \), would signal a violation of \( CP \).

It is convenient to define the \( 3 \times 3 \) seesaw operator at \( \Lambda_D \),

\[ \kappa = Y_\nu \, M_R^{-1} \, Y_\nu^T = \frac{1}{M} Y_\nu \, Y_\nu^T. \]  (4)

An important feature of \( \kappa \) is that, at any renormalization scale \( \mu \), one of its eigenvalues is zero leading to a massless neutrino at low energies. Therefore, this minimal seesaw scenario predicts a hierarchical neutrino mass spectrum and, as a consequence, a detection of neutrino masses with \( m_i > \sqrt{2} \Delta m^2_{\odot} \) would automatically rule it out, independently of the seesaw parameters.

For the cases we will be interested in, the decoupling scale of the heavy neutrino singlets is approximately \( M \). Moreover, the RG effects in \( Y_\nu \) and \( M_R \) can be neglected for the purposes of the seesaw reconstruction, although they will be crucial for leptogenesis. Thus, the seesaw operator at the scale \( M \) is well described by Eq. (4), apart from an overall factor of order one.

In order to reconstruct the high energy neutrino sector in terms of the low energy parameters, we shall consider the following structure for the Dirac neutrino Yukawa matrix\(^1\):

\[ Y_\nu = y_0 \begin{pmatrix} y_1 & 0 \\ y_3 & y_4 \\ y_5 & y_6 \end{pmatrix}, \]  (5)

where \( y_{1\ldots6} \) are complex and \( y_0 \) is chosen, without loss of generality, to be real.

At low energies the effective neutrino mass matrix is given by

\[ \mathcal{M} = m_3 \, U \, \text{diag}(0, \rho e^{i\alpha}, 1) \, U^T, \]  (6)

where \( m_3 \) is the mass of the heaviest neutrino, \( \alpha \) is a Majorana phase and \( \rho \equiv m_2/m_3 \). The matrix \( U \) is the leptonic mixing matrix which can be conveniently parametrized in the form

\[ U = \begin{pmatrix} \rho & c_{\delta} & s_{\delta} \\ -s_{\delta} & c_{\delta} & 0 \\ c_{\delta} & -s_{\delta} \rho & \rho \end{pmatrix}, \]

where \( c_{\delta} = \cos \theta_\delta \), \( s_{\delta} = \sin \theta_\delta \) and \( \delta \) is the \( CP \)-violating Dirac phase.

We will restrict our analysis to the case of neutrinos with normal hierarchy \( 0 = m_1 < m_2 \ll m_3 \). In this case

\[ m_2 = \sqrt{\Delta m^2_{\odot}}, \quad m_3 \simeq \sqrt{\Delta m^2_{\odot}}, \quad \rho \simeq \frac{\Delta m^2_{\odot}}{\Delta m^2_{\odot}}. \]

The case of inverted hierarchy can be analyzed in a similar way. In all our numerical estimates we shall use the best-fit values \[15, 16\] \( \Delta m^2_{\odot} = 8.3 \times 10^{-5} \text{ eV}^2 \) and \( \Delta m^2_{\odot} = 2.2 \times 10^{-3} \text{ eV}^2 \), so that \( \rho \simeq 0.19 \). The atmospheric and solar mixing angles are taken as \( \theta_\alpha = \pi/4 \) and \( \tan^2 \theta_\alpha = 0.37 \), respectively. The mixing angle \( \theta_\nu \) is presently constrained to be \( |\sin \theta_\nu| \lesssim 0.2 \) at 95\% C.L..

The effective neutrino mass operator at the decoupling scale \( M \) can be found by running \( \mathcal{M} \) from the electroweak scale \( m_Z \) up to \( M \). However, in the case when the spectrum of the light neutrinos is hierarchical, these effects turn out to be irrelevant. In particular, the mixing angles and the ratio \( \rho \) are stable under the RG evolution. Therefore, one has

\[ m_3 \, U \, \text{diag}(0, \rho e^{i\alpha}, 1) \, U^T \simeq \frac{v^2}{M} Y_\nu \, Y_\nu^T, \]  (9)

with \( v \simeq 174 \text{ GeV} \). This relation leads to the following simple approximation for the reconstructed Dirac neutr-
trino matrix

\[ H \equiv Y_v^\dagger Y_v \simeq \frac{y_0^2}{\sqrt{1 + 2x^2 \cos \alpha + x^4}} \times \left( x \left[ e^{i\alpha/2} - \rho e^{-i\alpha/2} \right] \right), \tag{10} \]

where

\[ y_0^2 = \frac{M \sqrt{\Delta m^2_{\text{atm}}}}{v^2}, \quad x = \frac{\tan \theta_c}{\sqrt{\rho s_a}} \simeq \frac{s_r}{\sqrt{\rho s_a}}. \tag{11} \]

As expected, the assumption of one texture zero allows us to reconstruct the Dirac neutrino Yukawa matrix (5) in terms of low energy observables up to an overall factor which depends on the heavy Majorana neutrino mass \( M \). To determine the latter, additional assumptions would be required.

### III. RADIATIVELY INDUCED \( CP \) ASYMMETRIES

A non-zero leptonic asymmetry can be generated if and only if the \( CP \)-odd invariant

\[ J_2 = \text{Im} \text{Tr} \left[ HM_R^\dagger M_R M_R^\dagger H^T M_R \right] = M_1 M_2 (M_2^2 - M_1^2) \text{Im} [H_{12}^2] \tag{12} \]

does not vanish \([9]\). This requires not only the heavy neutrino mass degeneracy to be lifted, but also the non-vanishing of \( \text{Im} [H_{12}^2] \) at the leptogenesis scale \( M \). Although the first condition is easily guaranteed by the running alone of \( M_R \) from \( \Lambda_D \) to \( M \), to achieve the second condition we must also include the quantum corrections due to the Dirac neutrino Yukawa matrix \( Y_\nu \). Indeed, the running of the right-handed neutrino mass matrix is governed by the renormalization group equation (RGE) \([19, 20]\)

\[ \frac{dM_R}{dt} = H^T M_R + M_R H, \quad t = \frac{\ln (\mu/\Lambda_D)}{16 \pi^2}. \tag{13} \]

From this equation, and neglecting the running of \( Y_\nu \), one has in the leading-log approximation \( M_R(t) \propto 1 + (H^T + H) t \). It is then possible to show that the trace in Eq. (12) is a real quantity, which leads to \( J_2 = 0 \). Consequently, the \( CP \)-violating effects, to which the \( CP \) asymmetries are sensitive, vanish at the decoupling scale.\(^2\) Thus, the corrections coming from the running of \( Y_\nu \) from \( \Lambda_D \) to \( M \) must be taken into account.

In the basis where \( M_R \) is diagonal, and assuming that the charged-lepton Yukawa matrix \( Y_\ell \) is diagonal, the evolution of the right-handed neutrino mass matrix and the Dirac neutrino Yukawa matrix is given at one-loop\(^3\) by [19, 20]

\[ \frac{dM_i}{dt} = 2M_i H_{ii}, \quad \frac{dY_\nu}{dt} = \left[ -\frac{3}{4} g_\nu^2 - \frac{9}{2} g_\nu^2 - \frac{3}{2} \left( Y_\nu Y_\nu^\dagger - Y_\nu Y_\nu^\dagger \right) \right] Y_\nu + Y_\nu R, \tag{14} \]

where \( T = 3 \text{Tr}(Y_\nu Y_\nu^\dagger) + 3 \text{Tr}(Y_\nu Y_\nu^\dagger) + \text{Tr}(Y_\nu Y_\nu^\dagger) + \text{Tr}(Y_\nu Y_\nu^\dagger); Y_{u,d} \) are the up-quark and down-quark Yukawa matrices and \( g_{Y,2} \) are the gauge couplings. The matrix \( R \) is antihermitian with

\[ R_{11} = R_{22} = 0, \quad R_{21} = -R_{12}^*, \quad R_{12} = \frac{2 + \delta_N}{\delta_N} \text{Re} (H_{12}) + i \frac{\delta_N}{2 + \delta_N} \text{Im} (H_{12}). \tag{15} \]

The parameter

\[ \delta_N \equiv \frac{M_2}{M_1} - 1. \tag{16} \]

quantifies the degree of degeneracy between \( M_1 \) and \( M_2 \).

An important feature of the system (14)-(15) is the fact that, if the heavy Majorana neutrinos are exactly degenerate in mass (\( \delta_N = 0 \)) at a given scale \( \Lambda_D \), then the second RGE in (14) becomes singular, unless one imposes \( \text{Re} (H_{12}) = 0 \). The latter condition can always be satisfied by performing an additional rotation in the Dirac Yukawa matrix,

\[ Y_\nu' = Y_\nu O, \quad O = \left( \begin{array}{cc} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{array} \right), \tag{17} \]

with the rotation angle \( \theta \) determined by the relation

\[ \tan 2\theta = \frac{2 \text{Re}(H_{12})}{H_{22} - H_{11}}. \tag{18} \]

The rotated matrix \( H' = O^\dagger H O \) becomes

\[ H' = \left( \begin{array}{cc} H_{11} - \Delta & i \text{Im} (H_{12}) \\ -i \text{Im} (H_{12}) & H_{22} + \Delta \end{array} \right), \tag{19} \]

where \( \Delta = \tan \theta \text{Re} (H_{12}) \).

According to Eqs. (14), the running of the mass splitting parameter \( \delta_N \) is determined by the equation

\[ \frac{d\delta_N}{dt} = 2(1 + \delta_N)(H_{22} - H_{11}^{'}). \tag{20} \]

Given that Eqs. (10), (18) and (19) imply \( H_{22} - H_{11}^{'2} \simeq y_0^2 (1 - \rho) \), with \( y_0 \) defined in Eq. (11), the radiatively

\(^2\) This was first pointed out in Ref. [21].

\(^3\) In general, the two-loop contribution to the running of \( M_R \) enters in the calculation at the same level as the one-loop running of \( Y_\nu \). We have numerically verified that this contribution has a negligible effect on our results.
induced mass splitting at the decoupling scale \( M \) will be approximatively given by

\[
\delta_N \simeq \frac{y_0^2}{8\pi^2} (1 - \rho) \ln (\Lambda_D/M) .
\]

(21)

Moreover, even if at the degeneracy scale \( \Lambda_D \) one has \( \text{Re}(H'_{12}) = 0 \), a nonvanishing real part will be generated by quantum corrections. The latter are easily estimated from Eqs. (14), and one finds

\[
\text{Re}(H'_{12}) \simeq \frac{3y_0^2}{32\pi^2} \sqrt{\rho} y_\tau^2 \text{Re}(e^{i\alpha/2}U_{32}U_{33}^*) \ln (\Lambda_D/M) ,
\]

(22)

where \( y_\tau \) is the \( \tau \) Yukawa coupling and \( U \) is the neutrino mixing matrix defined in Eq. (7).

The crucial ingredient for the computation of the baryon asymmetry in a leptogenesis scenario are the \( CP \) asymmetries generated by the interference of the one-loop and tree-level heavy Majorana decay diagrams. In the case when the heavy Majorana neutrinos are nearly degenerate in mass, these asymmetries are approximately given by [10]

\[
\epsilon_j = \frac{\text{Im} [H'_{21}]}{16\pi \delta_N H'_{jj}} \left( 1 + \frac{\Gamma_i^2}{4M^2\delta_N^2} \right)^{-1} , \quad j = 1, 2,
\]

(23)

where

\[
\Gamma_i = \frac{H'_{11} M_i}{8\pi}
\]

(24)

are the tree-level decay widths\(^4\). We notice that the expressions (23) for the \( CP \) asymmetries \( \epsilon_i \) exhibit the expected enhancement due to the mixing of two nearly degenerate heavy Majorana neutrinos [8, 9].

In terms of the low-energy neutrino parameters, we obtain

\[
\epsilon_1 \simeq \frac{-3y_\tau^2}{64\pi} \sqrt{\rho} \frac{(1 + \rho) x}{(1 - \rho)(\rho + x^2 - \Delta)} \sin(\alpha/2) [\epsilon_s \cos(\delta - \alpha/2) + \sqrt{\rho} s^2 \cos(\alpha/2)] ,
\]

\[
\epsilon_2 \simeq \frac{\rho + x^2 - \Delta}{\rho x^2 + \Delta} 
\]

\[
\Delta = \frac{1}{2} (1 - \rho) \left[ -1 + x^2 + \sqrt{1 + 2x^2 \cos \alpha + x^4} \right] .
\]

(25)

\( \Delta = 0 \) for \( \alpha = \pi \) and \( \delta = \pi/2 \), the \( CP \) asymmetry \( \epsilon_1 \) reaches its maximum value for \( x = \sqrt{\rho} \), as

\(^4\) One may wonder whether finite temperature effects could modify the perturbative result. Since the thermally induced mass splitting is approximately given by \( M_2(T) - M_1(T) \approx \frac{4}{3} \text{Re}(H'_{21} T^2) / M \) [10], at \( T \gg M \) this contribution is very suppressed with respect to the zero-temperature mass splitting \( \delta_N \).
from Eqs. (10), (25) and (27) it follows that $|\varepsilon_2/K_2| \ll |\varepsilon_1/K_1|$, one expects

$$\eta_B \simeq -10^{-2} d_1 \varepsilon_1.$$  

(28)

We also note that $K_1$ is independent of $M$ and approximately given by

$$K_1 \simeq \frac{44 (\rho + x^2 - \Delta)}{\sqrt{1 + 2x^2 \cos \alpha + x^4}}.$$  

(29)

In this washout regime, a simple estimate of the dilution factor $d_1$ can be obtained from the fit $d_1 \simeq 0.6 [\ln(K_1/2)]^{-0.6}/K_1$ [22]. For $\alpha \simeq \pi$, $\delta \simeq \pi/2$, the maximal value of the baryon asymmetry is then attained for $x \simeq 3\rho/(1 + \rho)/3 \simeq 0.23$ and $s_\tau \simeq 0.05$. From Eq. (25) we find $\varepsilon_1 \simeq -8 \times 10^{-7}$. Moreover, since Eq. (29) implies $K_1 \simeq 11$, then $d_1 \simeq 4 \times 10^{-2}$, using the above fit. Inserting these values into Eq. (28) one finally gets

$$\eta_B^{\text{max}} \simeq 3 \times 10^{-10},$$  

(30)

which is by a factor two smaller than the observed baryon asymmetry [cf. Eq. (1)]. It is worth noticing that this result is weakly dependent (through the renormalization effects on $y^2$) on the heavy Majorana neutrino mass scale $M$, as can be seen from the approximate expressions given in Eqs. (25). In Fig. 2 we present the numerical estimate for the baryon-to-photon ratio $\eta_B$ as a function of $s_\tau$.

We remark that a more accurate computation of the dilution factors requires the solution of the full set of Boltzmann equations. This question has been readdressed in recent works, where the effects on leptogenesis due to the $\Delta L = 1$ processes involving gauge bosons [10, 23] and thermal corrections at high temperature [23] have been discussed. However, since the present neutrino data fixes the decay parameters $K_i$ to be $K_i \gg 1$, the above effects turn out to be negligible and do not alter significantly the final value of the baryon asymmetry.

V. CONCLUSIONS

We have considered a minimal seesaw scenario with two heavy Majorana neutrinos which are degenerate in mass at a scale higher than their decoupling scale. We have shown that the heavy neutrino mass splitting, induced by radiative corrections, leads to non-zero CP asymmetries, and subsequently, to thermal leptogenesis even for heavy Majorana neutrino masses as low as a few TeV. However, we have seen that in the minimal extension of the SM with the addition of only two heavy Majorana neutrinos, it seems difficult to reconcile the present solar and atmospheric neutrino data with the observed cosmological baryon asymmetry. On the other hand, the analytical expressions (25) already suggest that these constraints can be easily relaxed, for e.g., in extensions of the SM with more than one Higgs doublet, since in this case the charged-lepton Yukawa couplings can be substantially enhanced.

We also remark that the results obtained here depend on the specific structure of the Dirac neutrino Yukawa matrix. In particular, it would be interesting to disentangle the texture considered here from other seesaw textures which can lead to the same low-energy observables. A more general analysis regarding this issue will be presented elsewhere [14]. It would also be desirable to find a symmetry which naturally leads to degenerate heavy Majorana neutrinos at high scales and, simultaneously, to the required Dirac neutrino Yukawa couplings.

Although low-energy (TeV-scale) leptogenesis is viable in the present framework, it requires tiny Dirac neutrino Yukawa couplings of the order of $10^{-6} - 10^{-7}$, which makes it difficult to be testable at future colliders. On the other hand, the fact that the final baryon asymmetry weakly depends on the heavy Majorana neutrino scale allows one to lower this scale below the typical values ($\sim 10^8 - 9$ GeV) required in the standard thermal leptogenesis scenarios. Thus, the potential problems related with the overproduction of relic abundances, which could jeopardize the successful nucleosynthesis predictions, are avoided.

Note added: While this work was being revised, Ref. [24] appeared, where it is shown that in the supersymmetric minimal seesaw model the present mechanism is indeed viable and thermal leptogenesis is successful.

Acknowledgments

We are grateful to G. C. Branco for valuable discussions and to T. Hambye and M. Raidal for useful comments. The work of R.G.F., F.R.J. and B.M.N. was sup-

FIG. 2: Numerical result for the baryon-to-photon ratio $\eta_B$ as a function of $s_\tau = \sin \theta_\tau$, obtained by integrating the full set of RGEs from the heavy neutrino degeneracy scale $\Lambda_B$ to the decoupling scale $M$. 

\[ \delta = \pi/2, \quad \alpha = \pi, \quad \Lambda_B = 10^{16} \text{GeV}, \quad M = 1 \text{ TeV} \]
ported by Fundação para a Ciência e a Tecnologia (FCT, Portugal) under the grants SFRH/BPD/1549/2000, SFRH/BPD/14473/2003 and SFRH/BD/995/2000, respectively.

[1] D. N. Spergel et al., Astrophys. J. Suppl. 148, 175 (2003).
[2] M. Fukugita and T. Yanagida, Phys. Lett. B 174, 45 (1986).
[3] W. Buchmüller, P. Di Bari and M. Plümacher, Nucl. Phys. B 665, 445 (2003).
[4] G. C. Branco, T. Morozumi, B. M. Nobre and M. N. Rebelo, Nucl. Phys. B 617, 475 (2001); G. C. Branco, R. González Felipe, F. R. Joaquim and M. N. Rebelo, Nucl. Phys. B 640, 202 (2002).
[5] G. C. Branco, R. González Felipe, F. R. Joaquim, I. Masina, M. N. Rebelo and C. A. Savoy, Phys. Rev. D 67, 073025 (2003).
[6] E. K. Akhmedov, M. Frigerio and A. Y. Smirnov, JHEP 0309, 021 (2003).
[7] S. Davidson and A. Ibarra, Phys. Lett. B 535, 25 (2002); W. Buchmüller, P. Di Bari and M. Plumacher, Nucl. Phys. B 643, 367 (2002).
[8] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B 384, 169 (1996).
[9] A. Pilaftsis, Phys. Rev. D 56, 5431 (1997).
[10] A. Pilaftsis and T. E. J. Underwood, Nucl. Phys. B 692, 303 (2004).
[11] P. H. Frampton, S. L. Glashow and T. Yanagida, Phys. Lett. B 548, 119 (2002).
[12] M. Raidal and A. Strumia, Phys. Lett. B 553, 72 (2003).
[13] V. Barger, D. A. Dicus, H. J. He and T. j. Li, Phys. Lett. B 583, 173 (2004).
[14] G. C. Branco, R. González Felipe, F. R. Joaquim and B. M. Nobre, in preparation.
[15] J. N. Bahcall, M. C. Gonzalez-Garcia and C. Penagaray, hep-ph/0406294; A. Bandyopadhyay, S. Choubey, S. Goswami, S. T. Petcov and D. P. Roy, hep-ph/0406328.
[16] M. C. Gonzalez-Garcia and M. Maltoni, hep-ph/0406056.
[17] F. del Aguila, J. A. Aguilar-Saavedra and M. Zralek, Comput. Phys. Commun. 100, 231 (1997).
[18] G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, Phys. Rev. Lett. 82, 683 (1999).
[19] J. A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, Nucl. Phys. B 556, 3 (1999).
[20] P. H. Chankowski and S. Pokorski, Int. J. Mod. Phys. A 17, 575 (2002).
[21] T. Hambye, J. March-Russell and S. M. West, JHEP 0407, 070 (2004).
[22] E. W. Kolb and M. S. Turner, The Early Universe, (Addison-Wesley, Reading, MA, 1990).
[23] G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B 685, 89 (2004).
[24] K. Turzynski, Phys. Lett. B 589, 135 (2004).