Relay Beamforming Design with SIC Detection for MIMO Multi-Relay Networks with Imperfect CSI

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Abstract—In this paper, we consider a dual-hop Multiple Input Multiple Output (MIMO) wireless multi-relay network, in which a source-destination pair both equipped with multiple antennas communicates through multiple half-duplex amplify-and-forward (AF) relay terminals which are also with multiple antennas. Since perfect channel state information (CSI) is difficult to obtain in practical multi-relay network, we consider imperfect CSI for all channels. We focus on maximizing the signal-to-interference-plus-noise ratio (SINR) at the destination. We propose a novel robust linear beamforming at the relays, based on the QR decomposition filter at the destination node which performs successive interference cancellation (SIC). Using Law of Large Number, we obtain the asymptotic rate in the presence of imperfect CSI, upon which, the proposed relay beamforming is optimized. Simulation results show that the asymptotic rate matches with the ergodic rate well. Analysis and simulation results demonstrate that the proposed beamforming outperforms the conventional beamforming schemes for any power of CSI errors and SNR regions.

Index Terms—MIMO relay, successive interference cancellation (SIC) detection, relay beamforming, channel state information (CSI), rate.

I. INTRODUCTION

Relay communications can extend the coverage of wireless networks and improve spatial diversity of cooperative systems [11]. Meanwhile, MIMO technique is well verified to provide significant improvement in the spectral efficiency and link reliability because of the multiplexing and diversity gains [2], [3]. Combining the relaying and MIMO techniques can make use of both advantages to increase the data rate in the cellular edge and extend the network coverage [4].

MIMO relay networks and MIMO broadcasting relay networks have been extensively investigated in [5]–[11]. In addition MIMO multi-relay networks have been studied in [12]–[18]. In [12], the authors show that the corresponding network capacity scales as $C = (M/2)\log(K) + O(1)$, where $M$ is the number of antennas at the source and $K \to \infty$ is the number of relays. The authors also propose a simple protocol to achieve the upper bound as $K \to \infty$ when perfect channel state informations (CSIs) of both backward channels (BC) and forward channels (FC) are available at the relay nodes. When CSIs are not available at the relays, a simple AF beamforming protocol is proposed at the relays, but the distributed array gain is not obtained. In [14], [15], the authors design three relay beamforming schemes based on matrix triangularization which have superiority over the conventional zero-forcing (ZF) and amplify-and-forward (AF) beamformers. The proposed beamforming scheme can both fulfill intranode gain and distributed array gain. In [16], the authors design a beamforming scheme that achieves the upper bound of capacity with a small gap when $K$ is significantly large. But it has bad performance for small $K$ and the source needs CSI which increases overhead. A unified algorithm is proposed in [17] for the optimal linear transceivers at the source and relays for both one-way and two-way networks. In [18], two efficient relay-beamformers for the dual-hop MIMO multi-relay networks have been presented, which are based on matched filter (MF) and regularized zero-forcing (RZF), and utilize QR decomposition (QRD) of the effective system channel matrix at the destination node [19]. The beamformers at the relay nodes can exploit the distributed array gain by diagonalizing both the backward and forward channels. The QRD can exploit the intranode array gain by successive interference cancellation (SIC) detection. These two beamforming schemes have lower complexity because they only need one QR decomposition at destination.

On the other hand, all the works for multi-relay MIMO system only consider perfect CSI to design beamformers at the relays or successive interference cancelation matrices at the destination. For the multi-relay networks, imperfect CSI is a practical consideration [12]. Especially, knowledge for the CSI of FC at relays will result in large delay and significant training overhead, because the CSI of FCs at relays are obtained through feedback links to multiple relays [20]. The imperfect CSI of BC at relays is also practical because of channel estimation error.

For the works on imperfect CSI, the ergodic capacity and BER performance of MIMO with imperfect CSI is considered in [21]–[23]. In [21], the authors investigated lower and upper bounds of mutual information under CSI error. In [22], the authors studied BER performance of MIMO system under combined beamforming and maximal ratio combining (MRC) with imperfect CSI. In [23], bit error probability (BEP) is analyzed based on Taylor approximation. Some optimization problem has been investigated with imperfect CSI in [24]–[26]. In [24], the authors maximize a lower bound of capacity by optimally configuring the number of antennas with imperfect CSI. In [26], the authors studied the trade-off between accuracy of channel estimation and data transmission, and show that the optimal number of training symbols is equal to the number of transmit antennas. In [27], the authors investigate the effects of channel estimation error on the receiver of

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This work is supported by the National 973 Project #2012CB316106 and #2009CB824904, by NSF China #60972031 and #61161130529.
MIMO AF two-way relaying networks.

In this paper, we propose a new robust beamforming schemes for dual-hop MIMO multi-relay networks under the condition of imperfect CSI. SIC is also implemented at the destination by QR decomposition. The proposed beamformer at relay is based on the minimum mean square error (MMSE) receiver and the RZF precoder. We focus on optimizing the regularizing factors in them. We first optimize the factor $\alpha_\text{MMSE}$ in MMSE, and then optimize the factor $\alpha_\text{RZF}$ in RZF for a given $\alpha_\text{MMSE}$. In the derivation, using Law of Large Number, we obtain the asymptotic rate capacity for the MMSE-RZF beamformer, based on which, the performance of the beamformer for imperfect CSI can be easily analyzed. Simulation results show that the asymptotic rate capacity matches with the ergodic capacity well. The asymptotic rate also validates the scaling law in [12], when the imperfect CSI presents. Analysis and simulations demonstrate that the rate of MMSE-RZF outperforms other schemes whether CSI is perfect or not. The ceiling effect of the rate capacity and the situation that CSI error increases with the number of relays are also discussed in this paper.

The remainder of this paper is organized as follows. In Section II, the system model of a dual-hop MIMO multi-relay network is introduced. In Section III, we explain the MMSE-RZF based beamforming scheme and QR decomposition. In Section IV, we optimize the MMSE-RZF and obtain the asymptotic rate of the system. Section V devotes to simulation results followed by conclusion in Section VI.

In this paper, boldface lowercase letter and boldface uppercase letter represent vectors and matrices, respectively. Notations $(\cdot)_i$ and $(\cdot)_{i,j}$ denote the $i$-th row and $(i,j)$-th entry of the matrix $\mathbf{A}$. Notations $\text{tr}(\cdot)$, $(\cdot)^\dagger$, $(\cdot)^*$ and $(\cdot)^H$ denote trace, pseudo-inverse, conjugate and conjugate transpose operation of a matrix respectively. Term $I_N$ is an $N \times N$ identity matrix. The diag $\{a_m\}_{m=1}^M$ denotes a diagonal matrix with diagonal entries of $a_1, \ldots, a_M$. $\|\cdot\|$ stands for the Euclidean norm of a vector $\mathbf{a}$, and $\rightarrow \text{P}$ represents convergence with probability one. Finally, we denote the expectation operation by $E\{\cdot\}$.

II. SYSTEM MODEL

The considered MIMO multi-relay network consists of a single source and destination node both equipped with $M$ antennas, and $K$ $N$-antenna relay nodes distributed between the source-destination pair as illustrated in Fig. 1. When the source node implements spatial multiplexing, the requirement $N \geq M$ must be satisfied if each relay node is supposed to support all the $M$ independent data streams. We assume $M = N$ in this paper, while the proposed beamforming scheme and the results can be easily expanded to the case $N > M$. We consider half-duplex non-regenerative relaying throughout this paper, where it takes two non-overlapping time slots for the data to be transmitted from the source to the destination node via the backward channels and forward channels. Due to deep large-scale fading effects produced by the long distance, we assume that there is no direct link between the source and destination. In a practical system, each relay needs to transmit training sequences or pilots to acquire the CSI of all channels. Imperfect channel estimation and limited feedback are also practical considerations. So in this paper, imperfect CSIs of BC and FC are assumed to be available at relay nodes. Assume that $\hat{\mathbf{H}}_k \in \mathbb{C}^{M \times M}$ and $\hat{\mathbf{G}}_k \in \mathbb{C}^{M \times M}$ stands for the available imperfect CSIs of BC and FC at the $k$-th relay. We model the CSIs of BC and FC of the $k$-th relay as

$$\mathbf{H}_k = \hat{\mathbf{H}}_k + \epsilon_1 \Omega_{1,k},$$
$$\mathbf{G}_k = \hat{\mathbf{G}}_k + \epsilon_2 \Omega_{2,k},$$

where $\mathbf{H}_k \in \mathbb{C}^{M \times M}$ and $\mathbf{G}_k \in \mathbb{C}^{M \times M}$ ($k = 1, \ldots, K$) stand for the backward and forward MIMO channel matrix of the $k$-th relay node respectively. $\Omega_{1,k}$ and $\Omega_{2,k}$ are matrices respectively independent of $\hat{\mathbf{H}}_k$ and $\hat{\mathbf{G}}_k$, whose entries are i.i.d zero-mean complex Gaussian, with unity variance [21]. Therefore the power of CSI errors of BC and FC can be $e_1^2$ and $e_2^2$. Since $e_1^2$ is the power of channel estimation error, it can be made very small. $e_2^2$ is the power of the channel error majorly coming from channel quantization, which is bounded by $2^{-B/M}$ if $B$ bits is used to do quantization. In this paper, we therefore assume $e_1^2 < 1$ and $e_2^2 < 1$, which are reasonable assumptions in a practical system. In this paper, all the relay nodes are supposed to be located in a cluster. Then all the channels $\mathbf{H}_1, \ldots, \mathbf{H}_K$ and $\mathbf{G}_1, \ldots, \mathbf{G}_K$ can be supposed to be independently and identically distributed (i.i.d) and experience the same Rayleigh flat fading. Assume that the entries of $\mathbf{H}_k$ and $\mathbf{G}_k$ are zero-mean complex Gaussian random variables with variance one. In the first time slot, the source node broadcasts the signal to all the relay nodes through BCs. Let $M \times 1$ vector $\mathbf{s}$ be the transmit signal vector satisfying the power constraint $E\{\mathbf{s}\mathbf{s}^H\} = \frac{P}{M} \mathbf{I}_M$, where $P$ is defined as the transmit power at the source node. Then the corresponding received signal at the $k$-th relay can be written as

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{s} + \mathbf{n}_k,$$

where the term $\mathbf{n}_k$ is the spatio-temporally white zero-mean complex additive Gaussian noise vector, independent across $k$, with the covariance matrix $E\{\mathbf{n}_k\mathbf{n}_k^H\} = \sigma_k^2 \mathbf{I}_M$. Therefore, noise variance $\sigma_k^2$ represents the noise power at each relay node.

In the second time slot, firstly each relay node performs linear processing by multiplying $\mathbf{r}_k$ with an $N \times N$ beamforming
matrix $\mathbf{F}_k$. This $\mathbf{F}_k$ is based on its imperfect CSIs $\hat{\mathbf{H}}_k$ and $\bar{\mathbf{G}}_k$. Consequently, the signal vector sent from the $k$-th relay node is

$$
t_k = \mathbf{F}_k \mathbf{r}_k.
$$

From more practical consideration, we assume that each relay node has its own power constraint satisfying $\mathbb{E}\{t_k^H t_k\} \leq Q$, which is independent of power $P$. Hence a power constraint condition of $t_k$ can be derived as

$$
\rho(t_k) = \text{tr} \left\{ \mathbf{F}_k \left( \frac{P}{M} \mathbf{H}_k \mathbf{H}_k^H + \sigma_1^2 \mathbf{I}_N \right) \mathbf{F}_k^H \right\} \leq Q.
$$

After linear relay beamforming processing, all the relay nodes forward their data simultaneously to the destination. Thus the signal vector received by the destination can be expressed as

$$
y = \sum_{k=1}^{K} \mathbf{G}_k t_k + \mathbf{n}_d
$$

(6)

$$
= \sum_{k=1}^{K} \mathbf{G}_k \mathbf{F}_k \mathbf{H}_k \mathbf{s} + \sum_{k=1}^{K} \mathbf{G}_k \mathbf{F}_k \mathbf{n}_k + \mathbf{n}_d,
$$

where $\mathbf{n}_d \in \mathbb{C}^M$, satisfying $\mathbb{E}\{\mathbf{n}_d\mathbf{n}_d^H\} = \sigma_2^2 \mathbf{I}_M$, denotes the zero-mean white circularly symmetric complex additive Gaussian noise vector at the destination node with the noise power $\sigma_2^2$.

III. RELAY BEAMFORMING DESIGN

In this section, the QR detector at the destination node for SIC detection is introduced and a relay beamforming scheme based on MMSE receiver and RZF precoder is proposed.

A. QR Decomposition and SIC Detection

QR-decomposition (QRD) detector is utilized as the destination receiver $\mathbf{W}$ in this paper, which is proved to be asymptotically equivalent to that of the maximum-likelihood detector (MLD) [19]. Let $\sum_{k=1}^{K} \mathbf{G}_k \mathbf{F}_k \mathbf{H}_k = \mathbf{H}_{SD}$ be the effective channel between the source and destination node, which can be estimated at the destination node by using the AF relay channel estimation methods [29]–[31]. Then (6) can be rewritten as

$$
y = \mathbf{H}_{SD} \mathbf{s} + \tilde{\mathbf{n}},
$$

(7)

where

$$
\tilde{\mathbf{n}} \approx \sum_{k=1}^{K} e_1 \mathbf{G}_k \mathbf{F}_k \mathbf{\Omega}_{1,k} \mathbf{s} + \sum_{k=1}^{K} e_2 < 2 \mathbf{G}_k \mathbf{F}_k \mathbf{H}_k \mathbf{s} + \sum_{k=1}^{K} \mathbf{G}_k \mathbf{F}_k \mathbf{n}_k + \mathbf{n}_d
$$

(8)

is the effective noise vector cumulated from the CSI errors, the noise $\mathbf{n}_k$ at the $k$-th relay node, and the noise vector $\mathbf{n}_d$ at the destination. In the derivation, we omit the term including $e_1 e_2$. Even if we retain the term $e_1 e_2$ in (8), it will result in some terms involving $e_1^2 e_2$, $e_1 e_2^2$ and $e_1^2 e_2^2$ when calculating the covariance of the effective noise $\tilde{\mathbf{n}}$. The first two terms are always zero after taking expectation, while the only terms left are those involving $e_1^2 e_2^2$. Since $e_1^2 \ll 1$ and $e_2^2 < 1$, we have $e_1^2 e_2^2 \ll 1$. Therefore, it is reasonable to omit the term including $e_1 e_2$ in (8).

Finally, in order to cancel the interference from other antennas, QR decomposition of the effective channel is implemented as

$$
\mathbf{H}_{SD} = \mathbf{Q}_{SD} \mathbf{R}_{SD},
$$

(9)

where $\mathbf{Q}_{SD}$ is an $M \times M$ unitary matrix and $\mathbf{R}_{SD}$ is an $M \times M$ right upper triangular matrix. Therefore the QRD detector at destination node is chosen as: $\mathbf{W} = \mathbf{Q}_{SD}^H$, and the signal vector after QRD detection becomes

$$
\tilde{\mathbf{y}} = \mathbf{Q}_{SD}^H \mathbf{y} = \mathbf{R}_{SD} \mathbf{s} + \mathbf{Q}_{SD}^H \tilde{\mathbf{n}}.
$$

(10)

A power control factor $\rho_k$ is set with $\mathbf{F}_k$ in (5) to guarantee that the $k$-th relay transmit power is equal to $Q$. The transmit signal from each relay node after linear beamforming and power control becomes

$$
t_k = \rho_k \mathbf{F}_k \mathbf{r}_k,
$$

(11)

where the power control factor $\rho_k$ can be derived from (5) as

$$
\rho_k = \left( \frac{Q}{\mathbb{E}\left[ \text{tr}\left\{ \mathbf{F}_k \left( \frac{P}{M} \mathbf{H}_k \mathbf{H}_k^H + \sigma_1^2 \mathbf{I}_N \right) \mathbf{F}_k^H \right\} \right] } \right)^{\frac{1}{2}}.
$$

(12)

B. Beamforming at Relay Nodes

The MF beamformer is used in [18] according to maximum ratio transmission (MRT) and maximum ratio combining (MRC) which are advantageous to the beamformers based on matrix decomposition in [15]. If MF beamformer is used, then

$$
\mathbf{F}_k^{MF-MF} = \tilde{\mathbf{G}}_k^H \hat{\mathbf{H}}_k^H.
$$

(13)

Another choice is to diagonalize the effective channel between the source and destination, for example,

$$
\mathbf{F}_k^{ZF-ZF} = \tilde{\mathbf{G}}_k^H \hat{\mathbf{H}}_k^H = \hat{\mathbf{G}}_k^H \left( \hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H \right)^{-1} \left( \hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k \right)^{-1} \hat{\mathbf{H}}_k^H.
$$

(14)

MF-MF outperforms ZF-ZF in low SNR, while ZF-ZF outperforms MF-MF in high SNR [18]. But these two schemes are not optimized.

In this paper, we propose a robust MMSE-RZF beamformer at the relay nodes. When MMSE-RZF is chosen, beamforming at the $k$-th relay is

$$
\mathbf{F}_k^{MMSE-RZF} = \hat{\mathbf{G}}_k^H \left( \hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \alpha_k^{\text{RZF}} \mathbf{I}_M \right)^{-1} \left( \hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_M \right)^{-1} \hat{\mathbf{H}}_k^H.
$$

(15)

Note that MF-MF and ZF-ZF are two extreme cases for $\alpha_k^{\text{MMSE}} = \alpha_k^{\text{RZF}} = \infty$ and $\alpha_k^{\text{MMSE}} = \alpha_k^{\text{RZF}} = 0$ respectively. Generally, if the $\alpha$ (either regularizing factor in MMSE or RZF) is too large, the effective channel matrix will far deviate from a diagonal matrix, which results in power consumption and interference across different data. If $\alpha$ is too small, the MMSE receiver and RZF precoder will perform like a ZF receiver or precoder which have the power penalty problem due to its inverse Wishart distribution term in its transmit
We aim to obtain the optimal $\alpha_k^{\text{MMSE}}$ and $\alpha_k^{\text{RZF}}$ to maximize the rate in this paper. However, to directly get the global optimal closed-form solution is difficult. In the following, we derive an optimized solution by two steps. We first derive an optimized $\alpha_k^{\text{MMSE}}$ by maximizing the SINR at the relay nodes, and then we derive an optimized $\alpha_k^{\text{RZF}}$ dependent on the given optimized $\alpha_k^{\text{MMSE}}$ by maximizing the rate at the destination.

IV. ROBUST MMSE-RZF BEAMFORMER

In this section, we derive the optimized $\alpha_k^{\text{MMSE}}$ and $\alpha_k^{\text{RZF}}$ in the MMSE-RZF beamformer by two steps. We first derive the optimized $\alpha_k^{\text{MMSE}}$ by maximizing the SINR at relay nodes, and then derive the optimized $\alpha_k^{\text{RZF}}$ for a given $\alpha_k^{\text{MMSE}}$ based on the asymptotic rate. Although the derived solution is not global optimum, it is observed quite efficient in terms of rate in the simulations.

A. Optimization of $\alpha_k^{\text{MMSE}}$

We optimize $\alpha_k^{\text{MMSE}}$ by maximizing the SINR at relay nodes. For the $k$-th relay, the signal vector after processed by an MMSE receiver is

$$v_k = (\hat{H}_k^H \hat{H}_k + \alpha_k^{\text{MMSE}} I_M)^{-1} \hat{H}_k^H r_k$$

$$= (\hat{H}_k^H \hat{H}_k + \alpha_k^{\text{MMSE}} I_M)^{-1} \hat{H}_k^H s + e_1 (\hat{H}_k^H \hat{H}_k + \alpha_k^{\text{MMSE}} I_M)^{-1} \hat{H}_k^H \Omega_{1,k}s + (\hat{H}_k^H \hat{H}_k + \alpha_k^{\text{MMSE}} I_M)^{-1} \hat{H}_k^H n_k. \tag{16}$$

The first term in (16) is the signal vector, which contains inter-stream interference, because matrix $(\hat{H}_k^H \hat{H}_k + \alpha_k^{\text{MMSE}} I_M)^{-1} \hat{H}_k^H \hat{H}_k$ is not diagonal if $\alpha_k^{\text{MMSE}} \neq 0$. So we need to calculate the power of desired signal and the interference. We use the diagonal decompositions in the following analysis, i.e.,

$$\hat{H}_k \hat{H}_k^H = P_k \text{diag} \{\theta_{k,1}, \ldots, \theta_{k,M}\} P_k^H \Theta_k P_k^H,$$

$$\hat{G}_k \hat{G}_k^H = Q_k \text{diag} \{\lambda_{k,1}, \ldots, \lambda_{k,M}\} Q_k^H \Lambda_k Q_k^H,$$

where $P_k$ and $Q_k$ are unitary matrices. To divides the interference from the desired signal, we introduce the following two lemmas.

Lemma 1: Assume that $\Lambda \in \mathbb{C}^{M \times M}$ is a random matrix. If there is a diagonal decomposition $\Lambda = QAQ^H$, where $A = \text{diag} \{\lambda_1, \ldots, \lambda_M\} \in \mathbb{R}^{M \times M}$ and the matrix $Q$ is unitary, we have

$$E\{(A)_{m,m}\} = \frac{1}{M(M+1)} \left( \sum_{\ell=1}^M \lambda_{\ell} \right)^2, \tag{17}$$

for any $m$, where the conditional expectation is taken with respect to the distribution $Q$ conditioned on $\Lambda$.

The proof of Lemma 1 can be directly obtained from [33] which considers perfect CSI. Although matrix $A$ in this paper is a multiplication of an imperfect channel matrix and its conjugate transpose, whose entries have covariance $1 - e_1^2$ or $1 - e_2^2$, the distribution of $Q$ is not changed. So is the expectation in Lemma 1. Note that the conditional expectation is taken with respect to $Q$ conditioned on $\Lambda$ is valid because $Q$ and $\Lambda$ are independent [35].

Lemma 2: Assume that $A \in \mathbb{C}^{M \times M}$ is a random matrix. If there is a diagonal decomposition $A = QAQ^H$, with $A = \text{diag} \{\lambda_1, \ldots, \lambda_M\} \in \mathbb{R}^{M \times M}$ and unitary matrix $Q$, we have

$$E\{|(A)_{m,j}|^2\} = \frac{1}{(M-1)(M+1)} \sum_{\ell=1}^{M} \lambda_{\ell} - \frac{1}{(M-1)M(M+1)} \left( \sum_{\ell=1}^{M} \lambda_{\ell} \right)^2 \stackrel{\nu}(\nu(\lambda)), \tag{18}$$

for any $m \neq j$, where the conditional expectation is taken with respect to the distribution $Q$ conditioned on $\Lambda$.

Proof: Because $A$ is a conjugate symmetric matrix, the conditional expectation with respect to the distribution $Q$ is

$$E\left\{ \sum_{j=1, j \neq m}^{M} |(A)_{m,j}|^2 \right\} = E\{(A)_{m,m}^2\},$$

$$= E\left\{ (QA^2Q^H)_{m,m} \right\} = \frac{1}{M} \sum_{\ell=1}^{M} \lambda_{\ell}^2. \tag{19}$$

Since $E\{|(A)_{k,j}|^2\}$ are all equal for $j \neq k$, we have

$$E\{|(A)_{k,j}|^2\} = \frac{1}{(M-1)} \left( \frac{1}{M} \sum_{\ell=1}^{M} \lambda_{\ell}^2 - E\{(A)_{m,m}^2\} \right) = \frac{1}{(M-1)M(M+1)} \left( \sum_{\ell=1}^{M} \lambda_{\ell} \right)^2. \tag{20}$$

Now we return to derive the signal-to-interference noise ratio (SINR) for each stream at each relay. The first term in the right hand side of (16) can be rewritten as

$$\left( \hat{H}_k^H \hat{H}_k + \alpha_k^{\text{MMSE}} I_M \right)^{-1} \hat{H}_k^H \hat{H}_k s = P_k \Theta_k \hat{H}_k^H \hat{H}_k s \tag{21}$$

Therefore, from Lemma 1, the power of the desired signal of the $m$-th stream can be calculated by conditional expectation as

$$E\left\{ \left| \frac{P_k \Theta_k \hat{H}_k^H \hat{H}_k s}{m,m} \right|^2 \right\} = \frac{P}{m} \mu \left( \frac{\theta_k + \alpha_k^{\text{MMSE}}}{\theta_k} \right). \tag{22}$$
where $\theta_k$ denotes the set of all the diagonal entries in $\Theta_k$. From Lemma 2, the interference from other streams by conditional expectation are

$$E \left\{ \sum_{j=1, j \neq m}^{M} \left( P_k \frac{\Theta_k}{\theta_k + \alpha_k^{\text{MMSE}}} I_N \right) s_j \right\} = \frac{P(M-1)}{M} \theta_k \left( \frac{\theta_k}{\theta_k + \alpha_k^{\text{MMSE}}} \right)^2. \quad (23)$$

The effective noise of the $m$-th stream is

$$n_{\text{eff},k} = c_1 \left( \hat{H}_k^H \hat{H}_k + \alpha_k^{\text{MMSE}} I_M \right)^{-1} \hat{H}_k^H \Omega_{1,k}s + \left( \hat{H}_k^H \hat{H}_k + \alpha_k^{\text{MMSE}} I_M \right)^{-1} \hat{H}_k^H n_k, \quad (24)$$

whose covariance matrix by conditional expectation can be calculated as

$$E \left\{ n_{\text{eff},k} n_{\text{eff},k}^H \right\} = \left( \sigma_1^2 P + \sigma_1^2 \right) \sum_{\ell=1}^{M} \theta_{k,\ell} \left( \frac{\theta_{k,\ell}}{\theta_{k,\ell} + \alpha_k^{\text{MMSE}}} \right)^2 I_M, \quad (25)$$

where we used the fact $E \{ \Omega \Omega^H \} = \text{tr} (\Omega)$ for an $N \times N$ matrix $\Omega$ and a unitary random matrix $\Omega$. In (22), (23), and (25), the conditional expectations are taken with respect to their respective unitary matrices. Combining (22), (24), and (25), the SINR of the $m$-th stream at the $k$-th relay is

$$\text{SINR}^R_{k,m} = \frac{P k^2 + \sigma_1^2}{\sum_{\ell=1}^{M} \left( \frac{\theta_{k,\ell}}{\theta_{k,\ell} + \alpha_k^{\text{MMSE}}} \right)^2 I_M}. \quad (26)$$

The derived SINR in (26) is neither the instantaneous SINR, nor the average SINR. It is the average SINR over the channels corresponding to the fixed Eigenmode $\Theta$. To maximize the SINR expression, we introduce the following lemma which is a conclusion of the Appendix B in [33].

**Lemma 3:** For an SNR in terms of $\alpha$,

$$\text{SNR}(\alpha) = A \left( \sum_{\ell=1}^{M} \frac{\lambda_\ell}{\lambda_\ell + \alpha} \right)^2 + B \sum_{\ell=1}^{M} \frac{\lambda_\ell^2}{\lambda_\ell + \alpha}, \quad (27)$$

is maximized by $\alpha = C/D$.

The optimum value of $\alpha$ can be obtained by differentiating (27) and setting it to zero, which results in

$$\sum_{\ell < k} \frac{\lambda_\ell \lambda_k (\lambda_k - \lambda_\ell)^2 (C/D - \alpha)}{(\lambda_\ell + \alpha)^3 (\lambda_k + \alpha)^3} = 0. \quad (28)$$

Since the eigenvalues are not all equal, the SINR is maximized only when $\alpha = C/D$.

Substituting $\mu(\lambda)$ and $\nu(\lambda)$ into (26) and using Lemma 3, we obtain

$$\alpha_k^{\text{MMSE, opt}} = \frac{\rho_1^2 (P + \sigma_1^2)}{\text{det}(\hat{H}_k^H \hat{H}_k + \alpha_k^{\text{MMSE}} I_M)} = \frac{(M + 1) \left( \sigma_1^2 + \sigma_2^2 \right)}{P}. \quad (29)$$

We see that the derived $\alpha_k^{\text{MMSE, opt}}$ is a closed-form value independent of the instantaneous channel. It is a function of the power of CSI error ($\sigma_1^2$) and the SNR ($P/\sigma_1^2$) of the BC. $\alpha_k^{\text{MMSE, opt}}$ increases with $C_1^2$, which means that a large regularization is needed to balance the desired signal and the additional noise inherited from the CSI error.

**B. Optimization of $\alpha_k^{\text{RZF}}$**

To optimize $\alpha_k^{\text{RZF}}$, we need to derive the rate of the system. In the rest of the analysis, we write $F_k^{\text{MMSE-RZF}}$ in (15) as $F_k$ for simplicity. By adding the power control factor at the relays, we have

$$H_{\text{SD}} = \sum_{k=1}^{K} \rho_k \hat{G}_k F_k \hat{H}_k. \quad (30)$$

The effective noise vector in (8) is

$$\hat{n} = \sum_{k=1}^{K} \sigma_1^2 \rho_k \hat{G}_k F_k \Omega_{1,k}s + \sum_{k=1}^{K} \rho_k \hat{G}_k F_k n_k + n_d, \quad (31)$$

which, after the QR decomposition of the effective channel, has a covariance matrix as

$$E \{ \hat{n} \hat{n}^H \} = \text{diag} \left\{ \left( \sigma_1^2 P + \sigma_1^2 \right) \right\} + \sum_{k=1}^{K} \left( \frac{\rho_k^2 \text{tr} \left( F_k \hat{H}_k \hat{H}_k^H F_k^H \right) \right) \sigma_1^2 + \sigma_2^2 \left( \hat{H}_k^H \hat{H}_k + \alpha_k^{\text{MMSE}} I_M \right). \quad (32)$$

Finally, we obtain the SINR of the $m$-th data stream at the destination after QR decomposition as

$$\text{SNR}^D_m = \frac{P \text{tr} (R_{\text{SD}})_{m,m}^2}{\sum_{j=1}^{m} \text{tr} (R_{\text{SD}})_{m,j}^2 + (N_{\text{cov}})_{m,m}}. \quad (33)$$

The ergodic rate is derived by summing up all the data rates on each antenna link, i.e.,

$$C = \text{E} \left\{ \log_2 \left( 1 + \text{SNR}^D_m \right) \right\} \sum_{k=1}^{K} \left( \frac{1}{2} \log_2 \left( 1 + \text{SNR}^D_m \right) \right). \quad (34)$$

where the $\frac{1}{2}$ penalty is due to the two time-slot transmission. From (33), we see that it is difficult to obtain the optimal
\[
E\left\{\rho_k^{-2}\right\} = \frac{1}{Q} E\left\{ \frac{P}{M^2} \text{tr}\left( F_k (\hat{\mathbf{H}}_k \hat{\mathbf{H}}_k^H + c_1^2 \Omega_{1,k} \Omega_{1,k}^H) F_k^H \right) + \sigma_1^2 \text{tr}\left( F_k F_k^H \right) \right\}
\]
\[
= \frac{P}{QM} E\left\{ \text{tr}\left( Q_k \frac{\Lambda_k}{(\Lambda_k + \zeta \mathbf{RZF})^2} Q_k^H P_k \frac{\Theta_k}{(\Theta_k + \zeta \mathbf{MMSE})^2} P_k^H \right) \right\}
\]
\[
+ \frac{P_c^2 + \sigma_1^2}{Q} E\left\{ \text{tr}\left( Q_k \frac{\Lambda_k}{(\Lambda_k + \zeta \mathbf{RZF})^2} Q_k^H P_k \frac{\Theta_k}{(\Theta_k + \zeta \mathbf{MMSE})^2} P_k^H \right) \right\}
\]
\[
= \frac{P}{Q} E\left\{ \theta^2 \right\} E\left\{ \frac{\lambda}{(\zeta + \alpha \mathbf{MMSE})^2} \right\} + \frac{1}{Q} E\left\{ \theta \right\} E\left\{ \frac{\lambda}{(\zeta + \alpha \mathbf{MMSE})^2} \right\}
\]
\[
\triangleq \rho^{-2}, \text{ (35)}
\]

\[
(H_{SD})_{i,i} \overset{w.p.}{\to} K \left( E\left\{ \left( \mathbf{G}_k \mathbf{F}_k \hat{\mathbf{H}}_k \right)_{i,i} \right\} \right) = \frac{K}{M N} \sum_{m=1}^{M} \frac{\theta_{k,m}}{\delta_{k,m} + \alpha \mathbf{MMSE}} \sum_{m=1}^{M} \frac{\lambda_{k,m}}{\delta_{k,m} + \alpha \mathbf{RZF}} = \frac{K}{M N} E\left\{ \frac{\theta}{\delta + \alpha \mathbf{MMSE}} \right\} E\left\{ \frac{\lambda}{\delta + \alpha \mathbf{RZF}} \right\}, \text{ (36)}
\]

solution directly. We derive asymptotic rate for large $K$ and then get the optimized $\alpha_k^{\mathbf{RZF}}$. Since all terms in (30) and (31) include $\rho_k$ except for $n_k$, we first consider the expectation of $\rho_k^{-2}$. From (12), substituting the perfect CSIs with (1) and (2), and taking the conditional expectation with respect to $P_k$ and $Q_k$ and conditioned on $\lambda$ and $\theta$, we have (35). Here we denote $\alpha$, $\lambda$, and $\theta$ without subscript $k$ and $m$ for simplicity, because all the channels for different relays are $i.i.d.$, and $\lambda_m$ and $\theta_m$ for every $m$ are identically distributed. (35) implies that the expectation of $\rho_k^{-2}$ results in a uniform fixed $\rho^{-2}$ for all relays. Therefore we approximate $\rho_k^{-2}$ by $\rho^{-2}$ in the following analysis. The performance with such approximation varies little compared with using the dynamic power control factors [30]. Since all terms in the numerator and the denominator of (35) except for $n_k$ will generate $\rho_k^{-2}$, the following analysis, we can omit $\rho_k$ in calculation and multiply $\rho^{-2}$ to $\sigma^2$ after calculating the power of $n_k$ in (35).

For the case of large $K$, using Law of Large Number, we have the approximations [30]. Note that

\[
E\left\{ \begin{bmatrix} \mathbf{G}_k \mathbf{F}_k \hat{\mathbf{H}}_k \end{bmatrix}_{i,j} \right\} = E\left\{ \begin{bmatrix} Q_k \frac{\Lambda_k}{(\Lambda_k + \zeta \mathbf{RZF})^2} Q_k^H P_k \frac{\Theta_k}{(\Theta_k + \zeta \mathbf{MMSE})^2} P_k^H \end{bmatrix}_{i,j} \right\}
\]
\[
= \sum_{\ell,m,n} E\left\{ \begin{bmatrix} Q_k \frac{\Lambda_k}{(\Lambda_k + \zeta \mathbf{RZF})^2} Q_k^H P_k \frac{\Theta_k}{(\Theta_k + \zeta \mathbf{MMSE})^2} P_k^H \end{bmatrix}_{\ell,m} \right\} \begin{bmatrix} (P_k)_{m,n} \left( \frac{\Theta_k}{(\Theta_k + \zeta \mathbf{MMSE})^2} \right) n (P_k^H)_{n,j} \end{bmatrix} \right\}
\]
\[
= 0
\]

for $i \neq j$, because $(Q_k)_{i,i} (Q_k^H)_{i,m} = 0$ for $i \neq m$ and

\[ (P_k)_{m,n} (P_k^H)_{n,j} = 0 \text{ for } m \neq j. \] Then we have

\[
(H_{SD})_{i,i} \overset{w.p.}{\to} \frac{\sum_{k=1}^{K} \begin{bmatrix} \mathbf{G}_k \mathbf{F}_k \hat{\mathbf{H}}_k \end{bmatrix}_{i,i}}{K} = \frac{K}{M N} E\left\{ \begin{bmatrix} \mathbf{G}_k \mathbf{F}_k \hat{\mathbf{H}}_k \end{bmatrix}_{i,i} \right\} = 0 \text{ (38)}
\]

for large $K$. Therefore, from (36) and (38), we have

\[
(H_{SD})_{i,i} = O(K) \text{ (39)}
\]

\[
(H_{SD})_{i,j} = o(K), \text{ (40)}
\]

which results in that $H_{SD}$ is asymptotically diagonal for large $K$. So we have $Q_{SD} \overset{w.p.}{\to} \mathbf{I}_M$ and $R_{SD} \overset{w.p.}{\to} \mathbf{H}_{SD}$ for large $K$.

To obtain the power of interference, we calculate the non-diagonal entries of the effective channel matrix which is included in (52) in the appendix. Let us define the following expectations.

\[
E_1^{\theta} \triangleq E\left\{ \begin{bmatrix} \theta \end{bmatrix} \right\},
\]

\[
E_2^{\theta} \triangleq E\left\{ \begin{bmatrix} \theta \end{bmatrix} \right\},
\]

\[
E_3^{\theta} \triangleq E\left\{ \begin{bmatrix} \theta^2 \end{bmatrix} \right\},
\]

\[
E_4^{\theta} \triangleq E\left\{ \begin{bmatrix} \theta \end{bmatrix} \right\},
\]

\[
E_5^{\lambda} \triangleq E\left\{ \begin{bmatrix} \lambda \end{bmatrix} \right\},
\]

\[
E_6^{\lambda} \triangleq E\left\{ \begin{bmatrix} \lambda \end{bmatrix} \right\},
\]

and

\[
e_1^{\theta} \triangleq E\left\{ \begin{bmatrix} \theta \end{bmatrix} \right\},
\]

\[
e_2^{\lambda} \triangleq E\left\{ \begin{bmatrix} \lambda \end{bmatrix} \right\},
\]

\[
e_3^{\lambda} \triangleq E\left\{ \begin{bmatrix} \lambda \end{bmatrix} \right\},
\]

\[
e_4^{\lambda} \triangleq E\left\{ \begin{bmatrix} \lambda \end{bmatrix} \right\},
\]
\[ \mathcal{E}_3^\lambda = \mathbb{E} \left\{ \frac{\lambda^2}{(\lambda + \alpha^{RZF})^2} \right\}, \]
\[ \mathcal{E}_4^\lambda = \mathbb{E} \left\{ \frac{\lambda \lambda'}{(\lambda + \alpha^{RZF})(\lambda' + \alpha^{RZF})} \right\}. \]

Substituting (42), (45), (46) and (52) into (43) and (44), we obtain the asymptotic rate of the system as
\[ C \xrightarrow{w.p.} \frac{M}{2} \log_2 \left( 1 + \frac{\frac{P}{M} (K e_0^\lambda)^2}{\mathcal{I}(\mathcal{E}_3^\lambda, \mathcal{E}_4^\lambda) + N(\mathcal{E}_3^\lambda, \mathcal{E}_4^\lambda)} \right), \quad (41) \]
where
\[ \mathcal{I}(\mathcal{E}_3^\lambda, \mathcal{E}_4^\lambda) = \frac{PK(M-1)(M+2) e_0^\lambda e_3^\lambda}{M(M+1)^2} - \frac{PK(M-1)M e_0^\lambda e_4^\lambda}{M(M+1)^2} \]
\[ - \frac{PK(M-1)M^2 e_0^\lambda e_3^\lambda}{M(M+1)^2} \]
\[ - \frac{PK(M-1)M^2 e_0^\lambda e_4^\lambda}{M(M+1)^2} \quad (42) \]
\[ N(\mathcal{E}_3^\lambda, \mathcal{E}_4^\lambda) = (e_1^2 P + \sigma_1^2) K e_0^\lambda e_3^\lambda + PK e_2^0 e_0^\lambda e_2^\lambda + e_2^2 e_0^2 K M e_0^\lambda e_2^\lambda + \sigma_2^2 e_0^{-2} \quad (43) \]
is the power of interference, and
\[ \mathcal{I}(\mathcal{E}_3^\lambda, \mathcal{E}_4^\lambda) = \frac{PK(M-1)(M+2) e_3^\lambda e_3^\lambda}{M(M+1)^2} - \frac{PK(M-1)M e_3^\lambda e_4^\lambda}{M(M+1)^2} \]
\[ - \frac{PK(M-1)M^2 e_3^\lambda e_3^\lambda}{M(M+1)^2} \]
\[ - \frac{PK(M-1)M^2 e_3^\lambda e_4^\lambda}{M(M+1)^2} \quad (42) \]
is the asymptotic power of noise for large K, which can be calculated by taking expectations over \( P_k \) and \( Q_k \) to (42).

Generally, the expectations in the asymptotic rate are difficult to obtain. Fortunately, if we write the expectations by the arithmetic mean with random samples \( \lambda(\ell) \) for \( \ell = 0, \ldots, \infty \) as
\[ E_1^\lambda = \lim_{L \to \infty} \frac{1}{L} \sum_{\ell=1}^{L} \frac{\lambda(\ell)}{\lambda(\ell) + \alpha^{RZF}} \quad (44) \]
\[ E_2^\lambda = \lim_{L \to \infty} \frac{1}{L} \sum_{\ell=1}^{L} \left( \frac{\lambda(\ell)}{\lambda(\ell) + \alpha^{RZF}} \right)^2 \quad (45) \]
\[ E_3^\lambda = \lim_{L \to \infty} \frac{1}{L} \sum_{\ell=1}^{L} \left( \frac{\lambda(\ell)}{\lambda(\ell) + \alpha^{RZF}} \right)^2 \quad (46) \]
\[ E_4^\lambda = \lim_{L \to \infty} \frac{1}{L(L-1)} \left( \left( \sum_{\ell=1}^{L} \frac{\lambda(\ell)}{\lambda(\ell) + \alpha^{RZF}} \right)^2 \right. \]
\[ - \left. \left( \sum_{\ell=1}^{L} \frac{\lambda(\ell)}{\lambda(\ell) + \alpha^{RZF}} \right)^2 \right) \quad (47) \]
the asymptotic rate can be maximized by using Lemma 3.

Finally we obtain
\[ \alpha^{RZF, opt} = \frac{(PK e_2^0 + \sigma_2^2 P) e_0^\lambda}{(e_1^2 P + \sigma_1^2) K e_2^0 + \frac{PK(M-1)(M+2)}{M(M+1)^2} e_3^\lambda + \frac{PK(M-1)M^2}{M(M+1)^2} e_4^\lambda} \]
\[ - \frac{(PK e_2^0 + \sigma_2^2 P) e_0^\lambda}{(e_1^2 P + \sigma_1^2) K e_2^0 + \frac{PK(M-1)(M+2)}{M(M+1)^2} e_3^\lambda + \frac{PK(M-1)M^2}{M(M+1)^2} e_4^\lambda} \quad (48) \]
\[ C. \text{ Remarks} \]

From (48) we can observe that \( \alpha^{RZF, opt} \) is independent of the instantaneous CSIs. This is very practical because once we have \( \alpha^{MMSE, opt} \) in terms of SNRs of BC and FC which we call as PNR \( (\frac{P}{\sigma_1^2}) \) and QNR \( (\frac{Q}{\sigma_2^2}) \), \( e_1^2 \) and \( e_2^2 \), \( \alpha^{RZF, opt} \) can be easily calculated although the solution is not in closed-form. The derived \( \alpha^{MMSE, opt} \) in (29) and \( \alpha^{RZF, opt} \) in (48) are respectively monotonically increasing functions of \( e_1 \) and \( e_2 \). So the robust MMSE-RZF balances the desired signal and the additional noise inherited from CSI error through larger regularizing factors. On the other hand, the proposed scheme is of low complexity because it needs only one QRD at the destination while in the work of [15] \( K \) or \( 2K \) QRD is needed at the relay nodes.

From the asymptotic rate in (41), we see that they satisfies the scaling law in [12], i.e., \( C \sim (M/2) \log_2(K) + O(1) \) for large \( K \). Therefore, the proposed scheme achieves the intranode array gain \( M \) and the distributed array gain \( K \). Intranode array gain is the gain obtained from the introduction of multiple antennas in each node of the dual-hop networks. Distributed array gain results from the implementation of multiple relay nodes and needs no cooperation among them. Note that although the MMSE-RZF with QR SIC is optimized for large \( K \), it also has efficient performance for small \( K \) which is validated by the simulations. Also from (41), it is observed that when QNR grows to infinity for a fixed PNR, the rate will reach a limit. When PNR and QNR both grow to infinity, the capacity will grow linearly with PNR and QNR (dB) for perfect CSIs, or reach a limit for imperfect CSIs. The limit of the rate performance is always referred as the “ceiling effect” [28] and will be confirmed by simulations.

Consider the case where CSI error varies with the number of relays \( (K) \). Generally, the CSI errors of BC are caused by the estimation error. The CSI errors of FC are caused by the estimation error, the quantization error and feedback delay. As in [26], [28], and [21], we assume that
\[ e_1^2 = \sigma_e^2 = \frac{1}{1 + \frac{M}{\rho^2} \tau}, \quad (49) \]
\[ e_2^2 = \sigma_q^2 + \sigma_d^2 = \frac{1}{1 + \frac{M}{\rho^2} \tau} + 2^{-B/M} \]
\[ + \left( 1 - J_0 \left( \frac{K+1}{2} \cdot 2\pi f_D \tau \right) \right) \quad (50) \]
where \( \sigma_q^2 \) denotes the estimation error during the training phase for TDD mode, \( \sigma_d^2 \) denotes the quantization error due to limited bits of feedback, and \( \sigma_d^2 \) denotes the error weight caused by feedback delay in each relay. The term \( K+1 \) scales the average delay for each relay when there is \( K \) relay nodes. \( \rho, \tau \) is the SNR of pilot signals in the training phase, \( T_r \) is the duration of training phase, \( B \) is the number of feedback bits for each relay, \( J_0 \) is a Bessel function of the first kind of order 0, \( f_D \) is the maximum Doppler shift, and \( \tau \) is the feedback delay. Note that in (49) and (50), the calculations are approximations for analysis and numerical simulations. Substituting such \( e_1^2 \) and \( e_2^2 \) into the asymptotic capacities and divides the numerator and the denominator by \( K+2 \), we see that the denominator will first decrease when \( e_2^2 \) is small and then increase when \( e_2^2 \) becomes large, which implies that the denominator will reach a minimum value at some \( K \). Therefore, there exists an optimal number of relays to maximize the asymptotic capacities, which will be confirmed by simulations.

V. Simulation Results

In this section, numerical results are carried out to validate what we draw from the analysis in the previous sections for
the proposed beamforming design. We compare the robust MMSE-RZF with MF and MF-RZF in [13] and QR in [14]. The $\alpha_{\text{RZF}}$ of MF-RZF in [13] is fixed to 1. ZF mentioned in Section III is also plotted for reference. In all these figures, we set $M = N = 4$, and focus on the performance of rates for various number of relays, SNR of BC and FC, and power of CSI errors. The ergodic rates are plotted by simulations through 1000 different channel realizations.

A. Capacity Versus Number of Relays

In Fig. 2, we compare the ergodic and asymptotic rate capacities of the MMSE-RZF beamforming schemes for various regularizing factors. We set PNR=QNR=10dB and $e_1^2 = e_2^2 = 0.01$. The curves are the asymptotic rates, and the dots are the ergodic rates obtained from simulation. The ergodic rate converges to the derived asymptotic rate for various $\alpha_{\text{MMSE}}$ and $\alpha_{\text{RZF}}$. In Fig. 3, we compare the rate performance of the five beamforming schemes. The advantage of the proposed robust MMSE-RZF can be observed. Note that MF, MF-RZF and ZF are all special cases of MMSE-RZF, which are not optimized with the system condition. The bad performance of QR can be explained as that its effective channel matrix is not diagonal but upper triangular, which results in power consumption. The poor performance of ZF comes from the inverse Wishart distribution term in its power control factor at the relays, especially when $M = N$. We find that the ergodic capacities still satisfy the scaling law in [12], i.e., $C = (M/2) \log(K) + O(1)$ for large $K$ in the presence of CSI errors. This is also consistent with the asymptotic capacities derived for MMSE-RZF with QR SIC detection.

B. Capacity Versus Power of CSI Error

Fig. 4 compare the ergodic rate capacities versus the power of CSI error. We set $K = 3$ and PNR=10dB, QNR=10dB. In this case, MMSE-RZF also outperforms others as the power of CSI error increases. The superiority of the MMSE-RZF compared with MF and MF-RZF decreases as $e_1$ and $e_2$ increase, and ZF outperforms MF for small $e_1(e_2)$ while underperforms MF for big $e_1(e_2)$. This is because that when $e_1$ and $e_2$ are small, the optimal $\alpha_{\text{MMSE}}$ and $\alpha_{\text{RZF}}$ should be small, e.g., $\alpha_{\text{MMSE,opt}} = 0.5$ for PNR=10dB and $e_1 = 0$. While in the MF and MF-RZF, $\alpha_{\text{MMSE}} = \infty$ and $\alpha_{\text{RZF}} = 1$, which are far away from the optimal values. When $e_1$ and $e_2$ grows, $\alpha_{\text{MMSE,opt}}$ and $\alpha_{\text{RZF,opt}}$ increases, which get close to the MF and MF-RZF.

C. Capacity Versus PNR and QNR

In Fig. 5, we increase PNR and QNR simultaneously for $K = 5$. When CSIs are perfect, the rates of all the five beamformings grow linearly with the PNR (=QNR) in dB. When CSI error occurs, we see the capacity limits. This is the “ceiling effect” discussed in Section V. This can also be seen in (41). If CSIs are perfect, SNR of each stream grows linearly with PNR (=QNR), so the rate grows linearly with PNR (=QNR) in dB. When CSI is imperfect, the numerator and denominator will simultaneously grows, resulting in a limit of SNR and the rate. The rate of ZF beamformer converges to the proposed robust MMSE-RZF beamformer at high SNR (PNR, QNR) for perfect CSI. This can be explained by the derived $\alpha_{\text{MMSE,opt}}$ in (29) and $\alpha_{\text{RZF,opt}}$ in (48), which both converge to zero at high SNR (large $P/\sigma_i^2$ and $Q/\sigma_j^2$) and perfect CSI ($e_1 = e_2 = 0$). Since ZF is known to be the
optimal beamforming for high SNR, the proposed MMSE-RZF is asymptotically optimal at high SNR with the QR SIC at the destination.

**D. Capacity Versus Relay Number for Dynamic CSI Error**

It is interesting to see Fig. 6 which shows the rates versus the relay number $K$ when CSI errors varies with $K$. This is a very practical scenario when we assume that the error caused by channel estimation is $\sigma^2_e = 0.05$, $B = 24$ for feedback, $f_D = 10Hz$ for $f = 2.4GHz$ and $v = 4.5Km/h$ for pedestrian speed ($f_D = vf/C$, $C$ denotes the speed of light), and $\tau = 5ms$ which is available in practical transmission. The feedback is based on the Lloyd VQ algorithm as in [43]. It is observed that the rates achieve maximum at some optimal relay number in the presence of CSI error. For the assumption in this figure, the optimal $K$ is 4. For multi-relay networks, the processing and feedback delay is always large. So we can save power and improve performance by only choosing the optimal number of relays to forward the signal. The selected relays can be constant or based on the instantaneous CSI.

**E. Capacity Versus Number of Feedback Bits**

In Fig. 7, using the same model as in Fig. 6, we focus on the effect of limited feedback. As can be observed, the rates increases fast with the number of feedback bits at the beginning, but slowly when the bits are enough to restore the CSIs. In Fig. 8, we further compare the performance under different number of feedback bits with perfect CSI case. It is observed that as the number of feedback bits increases, the rate approaches that of the perfect CSI case, and the rate is very close to that of perfect CSI case when the feedback bits are 12.

**VI. CONCLUSION**

In this paper, based on linear beamformer at the relay and QR SIC at the destination, we propose a robust MMSE-RZF beamformer optimized in terms of rate in a dual-hop MIMO multi-relay network with Amplify-and-Forward (AF) relaying protocol in the presence of imperfect CSI. Since it is difficult to obtain the global optimal MMSE-RZF beamformer, we solve the optimization in two steps. The MMSE receiver is optimized by maximizing the SINR at relay nodes. The RZF precoder is optimized by maximizing the asymptotic
demonstrate that the proposed robust MMSE-RZF outperforms rate matches with the ergodic rate. Analysis and simulations show that the asymptotic rate derived upon a given MMSE receiver using Law of Large Number. Simulation results show that the asymptotic rate, given MMSE receiver using Law of Large Number, for a given MMSE receiver using Law of Large Number, is equal to

\[
\frac{\lambda^2}{\lambda + \alpha_{RZF}} \cdot \frac{\lambda^2}{\lambda + \alpha_{MMSE}} \cdot \frac{\theta^2}{\theta + \alpha_{MMSE}} \cdot \frac{\theta^2}{\theta + \alpha_{MMSE}}
\]

(51)

\[
\frac{K(M + 2)}{(M + 1)^2} E \left\{ \frac{\lambda^2}{\lambda + \alpha_{RZF}} \cdot \frac{\lambda^2}{\lambda + \alpha_{RZF}} \cdot \frac{\theta^2}{\theta + \alpha_{MMSE}} \cdot \frac{\theta^2}{\theta + \alpha_{MMSE}} \right\}
\]

(52)

rate derived upon a given MMSE receiver using Law of Large Number. Simulation results show that the asymptotic rate matches with the ergodic rate. Analysis and simulations demonstrate that the proposed robust MMSE-RZF outperforms other coexistent beamforming schemes.

**APPENDIX**

We calculate square of the norm of the non-diagonal as (51). Since the fact in [33] that

\[
E \left\{ \left( Q_r \right)_{i,k} \right\} \left( Q_r \right)_{i,k} \right\} = \frac{2}{M(M + 1)} \text{ if } i = \ell, \quad (53)
\]

\[
E \left\{ \left( Q_r \right)_{i,k} \right\} \left( Q_r \right)_{i,k} \right\} = \frac{1}{M(M + 1)} \text{ if } i \neq \ell, \quad (54)
\]

then when \( i \neq m, \ell \neq r \) we have

\[
E \left\{ \left( Q_k \right)_{i,\ell} \left( Q_k \right)_{m,\ell} \left( Q_k \right)_{i,r} \left( Q_k \right)_{m,r} \right\}
\]

\[
= \frac{1}{M(M - 1)} E \left\{ \sum_{\ell, r = 1, \ell \neq r}^M \left( Q_k \right)_{i,\ell} \left( Q_k \right)_{m,\ell} \left( Q_k \right)_{i,r} \left( Q_k \right)_{m,r} \right\}
\]

\[
= \frac{1}{M(M - 1)} E \left\{ \sum_{\ell = 1}^M \left( Q_k \right)_{i,\ell} \left( Q_k \right)_{m,\ell} \right\} \left( \sum_{r = 1}^M \left( Q_k \right)_{i,r} \left( Q_k \right)_{m,r} \right)
\]

\[
= \frac{1}{M(M - 1)} \cdot \frac{1}{M(M + 1)}
\]

(55)

Substituting (53), (54) and (55) into (51), and through some manipulation, we have (52), where \( \lambda \) and \( \lambda' \), \( \theta \) and \( \theta' \) are different singular values within one decomposition. Obviously, (52) equals zero when \( \alpha_{MMSE} \) and \( \alpha_{RZF} \) are zero.
