Analysis of vibro-acoustic response of un-baffled laminated composite conical shell panel with varying thickness

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Abstract. This article presents the vibro-acoustic modeling and analysis of un-baffled laminated composite conical shell panels with varying thickness subjected to harmonic point load. The variation of thickness is considered to be unidirectional. A simulation model for the vibrating panel is developed using the commercial finite element package (ABAQUS). The natural frequencies are computed and compared with available published values for validation purpose. Then, the modal values are exported to LMS Virtual.Lab environment where an indirect boundary element approach has been adopted to extract the coupled vibro-acoustic responses. The sound power radiated by the vibrating structure computed using the present scheme is compared with the results available in the published literature. A comprehensive study has been performed to highlight the effect of thickness variation, excitation location, lamination scheme and support conditions on the acoustic radiation responses of the laminated conical shell panel. The variation of thickness is observed to greatly influence the mode shapes of the panel thereby influencing the sound radiation characteristics. The stiffness of the panels is affected by the lamination scheme, which in addition to stiffness variation due to varying thickness alters the radiation characteristics of the panels significantly.

1. Introduction

Present day engineering structures are designed by taken into consideration their sound radiation and transmission characteristics that are tuned so as to have minimal acoustic disturbance in the surroundings. Further, this also keeps a check on acoustic induced loading that could have significant impact on the dynamic characteristics of vibrating structures. Laminated composite structures with uniform [1] and varying thickness [2] have been the subject of worldwide scrutiny so far as their vibration and sound radiation characteristics are considered. Vibro-acoustic behaviour of the vibrating isotropic plate structures with arbitrary boundary conditions has been studied by Nowak and Zielinski [3] numerically. Various studies have been performed to study the free vibration responses of rectangular plates with varying thickness [4,5]. Isotropic plates of varying thickness have also been studied for their acoustic behaviour using coupled finite and boundary elements [6]. Further, a paraboloidal shell with variable thickness has been analysed by Leissa and Kang [7] for its free vibration characteristics. The concept of varying thickness has also been applied to cylindrical shell to the study its effect on the free vibration behaviour of cylindrical shells [8,9]. Also, studies on the free vibration characteristics of circular and annular membranes with varying thickness have also been

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performed [10]. Several studies have been reported on the free vibration analysis of isotropic, laminated composite and FGM conical shell panels of constant and varying thickness [11,12].

From this brief review of literature, it is clear that no study investigating the vibro-acoustic behaviour of laminated composite conical shell panels with varying thickness has been reported till date. So, the present work aims at analysing the vibration and acoustic response of un-baffled laminated composite conical shell panels with unidirectional variation in thickness and subjected to harmonic point excitation using a coupled FEM-BEM implemented in ABAQUS and LMS Virtual.Lab software package. The free vibration responses (natural frequency) of the panels are obtained and validated with the published literature. Further, the acoustic radiation from the panels is also compared to the published results to establish the validity of the proposed model. Finally, the influence of excitation location, lamination scheme and support condition on the vibro-acoustic behaviour of the panels is investigated and discussed in detail.

2. Theoretical Background

Fig. 1 depicts the basic geometry and lamination scheme of the orthotropic layered conical shell panels considered for the present analysis. The free vibration responses of the conical shell panels have been computed using a simulation model developed in ABAQUS environment. An eight nodded serendipity element with six degrees freedom per each node has been chosen from the ABAQUS element library for the discretization of the present model. The mid-plane kinematic for the panel structure is defined based on the first order shear deformation theory (FSDT) and conceded as:

\[
[u \ v \ w]^T = [u_0 + z\theta_x \ v_0 + z\theta_y \ w_0 + z\theta_z]^T
\]  

(1)

where, \(u\), \(v\) and \(w\) are the displacements of any point on \(k^{th}\) layer at time \(t\) along the \(x\), \(y\) and \(z\) coordinate axes, respectively; \(u_0\), \(v_0\) and \(w_0\) are the corresponding displacements of a point on the mid-plane; \(\theta_x\) and \(\theta_y\) are the rotations of normal to the mid-surface (\(z = 0\)) about the \(y\) and \(x\)-axes, respectively; \(\theta_z\) is the higher order term in the Taylor series expansion which accounts for the linear variation of displacement function along thickness direction. The thickness of the panels is varied unidirectionally in \(y\)-direction. The thickness variation is assumed to be: (a) Linear, and (b) Parabolic. Fig. 2 shows the cross section of the panels for the aforementioned thickness variation schemes. The edges of the panels have been labeled as 1, 2, 3 and 4, as shown in Fig. 2. The thickness is varied such that the volume of the panel remains equal to the volume of the conical panel with uniform thickness and the details of the procedure can be seen in Akiyama and Kuroda [5].

![Figure 1. (a) Geometry [11], (b) lay-up scheme of laminated composite conical shell panel.](image)

The modal analysis is performed using this simulation model and the modal values of the structure corresponding to each natural frequency of vibration are extracted to subsequently use as the input for the acoustic analysis.

The vibrating structure under the in-vacuo modes can be obtained by solving the eigenvalue equations: 

\[
[(K) - \omega^2(M)]\{\Phi\} = 0
\]

(2)
where, \([K]\) and \([M]\) are the stiffness and mass matrices, respectively, \(\omega\) is the natural frequency of vibration, and \(\{\Phi\}\) is the corresponding mode shape vector. The mode shapes are imported to LMS Virtual.Lab environment where an indirect boundary element method (BEM) approach is utilized to account for the sound radiation in the medium due to the vibrating structure.

\[
K + i\omega\Theta - \omega^2 M \begin{pmatrix} C \\ C^T \end{pmatrix} \begin{pmatrix} U \\ F \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\]

The governing equation for the acoustic problems is the well-known Helmholtz wave equation expressed as: \[\nabla^2 p + k^2 p = 0\] (3) where, \(k = \omega/c\) is the wave number, \(p\) is the acoustic pressure, \(\omega\) is the angular frequency, \(c\) is the speed of sound in the surrounding acoustic medium. The Eq. (3) is solved by imposing Neumann boundary condition on the entire surface of the vibrating laminated composite conical panel. The coupled vibro-acoustic response of the structure is obtained by solving the coupled equation of the system [13]:

\[ \begin{pmatrix} \frac{1}{\rho \omega^2} D(\omega) \end{pmatrix} \begin{pmatrix} U \\ F \end{pmatrix} = \begin{pmatrix} 0 \end{pmatrix} \]

The thickness of the panel is varied through the sections to achieve a linear/parabolic thickness variation. Figure 2 shows the natural frequency computed using the present model along with the reference values. It is clear that the present values are in excellent agreement with the results given by Jeyaraj [6] for both the values of taper ratio. Then, a clamped isotropic plate with parabolically varying thickness is analyzed by Jeyaraj [6] is considered for the validation of sound power level. Table 2 shows the natural frequency computed using the present model along with the reference values. It is evident that the present results conform closely with the reference for both of the excitation cases.

In this section, the effects of excitation location, lamination scheme and support condition on the vibro-acoustic responses of laminated composite conical shell panels are investigated using the present scheme and discussed in detail. The support conditions utilized in the present analysis are described in Table 1. The material properties used in the analysis are as provided by [14]. The following geometrical parameters and a constant structural damping ratio of 0.01 is considered throughout the analysis unless stated otherwise: \(L=1m, R_1=0.25m, L/h=100, \alpha = 15^\circ\) and \(\theta_0 = 60^\circ\). For validation purpose, a clamped isotropic plate with parabolically varying thickness as in Jeyaraj [6] is considered. The variable thickness laminated composite shell panels are modeled in ABAQUS commercial package. The thickness of the panel is varied through the sections to achieve a linear/parabolic variation. Three taper ratios (\(T_j\)) are considered (0.2, 0.5 and 0.7). In addition, the excitation location is...
varied to lie on thick and thin portions of the panels. Three different load cases are considered namely, (a) Load case-I (b) Load case-II and (c) Load case-III with excitation force acting at the central node, respectively, as shown in Fig. 3.

**Table 1. Support conditions**

| Condition | Support Conditions |
|-----------|--------------------|
| SSSS:     | $v_0=w_0=\theta_1=\theta_2=\theta_3=\theta_4=0$ at edges 1, 3; $u_0=w_0=\theta_5=\theta_6=0$ at edges 2, 4 |
| CCCC:     | $u_0=v_0=w_0=\theta_1=\theta_2=\theta_3=\theta_4=0$ at edges 1, 2, 3 and 4 |
| CFFF:     | $v_0=w_0=\theta_1=\theta_2=\theta_3=\theta_4=0$ at edge 1 |
| SCSC:     | $v_0=w_0=\theta_1=\theta_2=\theta_3=\theta_4=0$ at edges 1, 3; $u_0=v_0=w_0=\theta_1=\theta_2=\theta_3=\theta_4=0$ at edges 1, 2 |

**Table 2. Validation of natural frequencies of a clamped square isotropic plate with parabolically varying thickness**

| Taper ratio | Present | Jeyaraj [6] | Present | Jeyaraj [6] |
|-------------|---------|-------------|---------|-------------|
| 0.25        | 221.43  | 222         | 202.58  | 201         |
| 0.75        | 448.29  | 450         | 396.53  | 404         |
|             | 449.14  | 456         | 411.71  | 412         |
|             | 659.36  | 668         | 614.07  | 622         |
|             | 795.75  | 810         | 666.62  | 700         |

**Figure 4. Validation of sound power level: (a) Eccentric excitation, (b) Central excitation.**

**Figure 5. Radiated sound power for linear thickness variation: Load case-I, (b) Load case-III.**

The effect of excitation location on the vibro-acoustic response of a clamped laminated composite symmetric angle-ply ($45^\circ/-45^\circ$), conical shell panel with varying thickness is investigated. Fig. 5 (a) and (b) show the variation of sound power level from linearly varying thickness panel for Load case-I and III, respectively. The panels considered in the analysis are thin panels ($L/h=100$). As expected, the
panels exhibit significantly distinct behavior for different load cases. The $T_y=0.7$ case radiates the least power for Load case-I and III. In general, the resonant peaks cascade to lower frequencies with decreasing thickness ratio for all of the load cases considered. Fig. 6 shows the radiation efficiency of the panels for Load case-II. It is evident that the panel with $T_y=0.2$ is the most efficient radiator over the entire frequency range under consideration. From here onward parabolic thickness variation is considered. Fig. 7, 8(a) and 8(b) show the variation of radiation efficiency with thickness ratio for Load case-I, II and III, respectively. The behavior of the panels is substantially different when subjected to Load case-II and III as compared to that in Load case-I. This is attributed to the fact that different modes are excited when the load is applied at a node other than the central node. The behavior of panels with $T_y=0.5$ and 0.7 is quite similar when subjected to Load case-II with $T_y=0.2$ being the least radiating case, as shown in Fig. 8(b). On the other hand, for Load case-III the $T_y=0.2$ and 0.5 exhibit similar radiation behavior with $T_y=0.7$ being the least radiating case.

![Figure 6. Radiation efficiency: Load case-II.](image)

![Figure 7. Radiation efficiency: Load case-I.](image)

Four different lamination schemes namely, anti-symmetric cross-ply (0º/90º)$_2$, symmetric cross-ply (0º/90º)$_s$, anti-symmetric angle-ply (45º/-45º), and symmetric angle-ply (45º/-45º), are considered. The varying thickness conical shell panels with parabolic thickness variation ($T_y=0.5$) are subjected to Load case-I and CCCC support condition. The behavior of symmetric and anti-symmetric angle plies has striking similarity throughout the frequency range under consideration and the same can be verified from the plot of the radiated sound power level shown in Fig. 9. The peaks in the radiated sound power cascade to lower frequencies for symmetric and anti-symmetric angle plies and the symmetric cross-ply radiates the least sound power throughout the frequency range under consideration. Further, The symmetric angle-ply (45º/-45º)$_s$ conical shell panels with parabolic thickness variation ($T_y=0.5$) and subjected to Load case-I are considered under CCCC, SSSS, SCSC and CFFF support conditions. The thickness variation is taken as parabolic ($T_y=0.5$). The peaks in the radiated sound power curve shift to lower frequencies with decreasing number of edge constraints as shown in Fig. 10. Also, the magnitude of the power increases with decreasing number of edge constraints.

### 4. Conclusion

The acoustic radiation characteristic of vibrating laminated composite conical shell panels with varying thickness has been analysed in the framework of the first order shear deformation theory in conjunction with coupled FEM-BEM technique by using commercially available software ABAQUS and LMS Virtual.Lab. The variation of thickness is considered to be unidirectional and the panels are subjected to harmonic point load at the locations lying on the thick and thin portions. Convergence behaviour of the present model is established and the natural frequencies and the radiated sound power computed using the present scheme is validated with the results published in the available literature.
Subsequently, the influence of taper ratio, excitation location, lamination scheme and support condition on the vibro-acoustic behaviour of the conical panels with varying thickness is investigated. It is observed that the anti-symmetric cross-ply laminated conical panels with taper ratio 0.2 are the most efficient radiator. The radiated sound power decreases with the increasing number of constraints at the support.

![Diagram](image)

**Figure 8.** Radiation efficiency of the panel: (a) Load case-II, (b) Load case-III

![Diagram](image)

**Figure 9.** Variation of power with lay-up.

**Figure 10.** Influence of support conditions.

5. References
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