Deformation of T-shaped beams under creep conditions with different properties in tension and compression

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Abstract. An algorithm is implemented to simulate a pure bending of a beam of a symmetric section under creep conditions. The beam is subjected to a bending moment with a constant absolute value. The algorithm is applied to the experimental data on the pure bending of T-beams made from a Ti-Al-Sn-V alloy at a temperature \( T = 700 \, ^\circ C \). The algorithm takes into account the accumulation of damage in the material, the effect of temperature exposure and the difference properties of the material under tension and compression. The model is based on kinetic creep theory by Rabotnov. The Nelder–Mead procedure is used to find the bending moment by the curvature of a beam.

1. Introduction
Direct and inverse shaping problems in the creep mode when modeling the forming of parts are considered. The stress-strain state during the predetermined time under the action of given external force and kinematic effects is determined by solving the direct forming problem [1-2]. Inelastic deformation is determined by solving the inverse problem. It occurs as a result of external power and kinematic effects. The objective is to determine the forces and the shape of stamp, which will provide the desired curvature of the product after removing the load. The complexity of the tasks is to take into account the different creep properties under tension and compression, as well as the anisotropy of the creep properties of the workpiece material.

Approaches to solving creep problems are described in [3]. A satisfactory agreement between the calculations and the experimental data is obtained by using the creep model to describe various tensile and compressive properties [4]. A numerical solution of problems in three dimensions related to forming a double curvature structure element for metal plates under creep processes under the action of constant concentrated forces applied at its corners is presented in [4]. Inaccurate modelling may lead to differences between the calculated deflections of the structure and the deflections observed in the experiment.

The modeling of deformation of T-beam from a Ti-Al-Sn-V alloy at \( T = 700 \, ^\circ C \) under creep conditions under the action of a moment constant in absolute value is considered in this paper. The search for the bending moment \( M \) by the target curvature through the solution of an inverse problem was implemented. The iteration steps of Nelder-Mead algorithm to determine the value of the bending moment for the achievement of the desired curvature during the forming of structural elements are considered.

2. Experimental data on the deformation of the material under creep
The solution of the direct and inverse problems is analysed through T-beam from a high-strength Ti-Al-Sn-V alloy at a temperature close to superplasticity (\( T = 700 \, ^\circ C \)) with various constant loads. Ti-Al-
Sn-V alloy treated a huge advantage over weight reduction in the aerospace advancement, shipbuilding, automotive industry and other areas requiring a good ratio of strength, weight and high corrosion resistance to the material.

The experiments were carried out on samples coupon representative of the plate with a thickness of 60 mm. Creep diagrams are given in [5].

The process of deforming a non-reinforcing material under creep and the damage of Y.N. Rabotnova in general form can be represented as [1, 2, 6-9]:

\[ W = \frac{dA}{dt} = f(\sigma) \frac{\varphi(\omega)}{\psi(\omega)}, \quad 0 \leq \omega \leq 1, \]

where \( A = \int_0^t W dt \) is specific energy scattering; functions \( f(\sigma) \) and \( \varphi(\omega) \) depending on \( T \);
\( \psi(\omega) = (1 - \omega)^m \) is softening function. The value \( W \) used for value of dissipation power under creep. The damage parameter \( \omega \) of the uniaxial deformation is the ratio of current work scattering to its value at the time of destruction \( A_0 \).

Established of creep process is expected to described by a power law
\( f(\sigma) = B_\omega \sigma^n \), \( \varphi(\omega) = B_\omega \omega^k \), where \( B_\omega, B_\omega, n_i \) and \( k_i \) are material characteristics under tension \((i = 1)\) and compression \((i = 2)\). Material characteristics are shown in table 1. They are derived from a series of creep curves for tension and compression at \( \sigma = \text{const} \).

**Table 1. Creep characteristics of Ti-Al-Sn-V**

| Mode  | \( B_\omega \), MPa \( \cdot h^{-1} \) | \( B_\omega \), MPa \( \cdot h^{-1} \) | \( n \) | \( k \) | \( m \) |
|-------|---------------------------------|---------------------------------|-------|-------|-------|
| tension | \( 1.48 \cdot 10^{-13} \) | \( 3.22 \cdot 10^{-11} \) | 3.59  | 2.92  | 1.3   |
| compression | \( 1.37 \cdot 10^{-11} \) | \( 0 \) | 4.5  | 0     | 0     |

3. The calculation of the stress-strain state of the T-beam. Direct problem solution
Mathematical modeling of pure bending of T-beam with different creep properties in tension and compression is based on the equations of the energy version of the theory of creep (1). The creep process in material taking into account its damage [4, 10, 11] is described by a system of equations using the power law of creep

\[
\frac{dA}{dt} = \begin{cases} 
\frac{B_{\omega \sigma} \sigma^{n-1} \sigma}{(1 - \omega)^m}, & \sigma \geq 0; \\
\frac{B_{\omega \sigma} \sigma^{n-1} \sigma}{(1 - \omega)^m}, & \sigma \geq 0; \\
\frac{B_{\omega \sigma} \sigma^{n-1} \sigma}{(1 - \omega)^m}, & \sigma \geq 0;
\end{cases}
\]

\[
\frac{d\omega}{dt} = \begin{cases} 
\frac{B_{\omega \sigma} \sigma^{n-1} \sigma}{(1 - \omega)^m}, & \sigma \geq 0; \\
\frac{B_{\omega \sigma} \sigma^{n-1} \sigma}{(1 - \omega)^m}, & \sigma \geq 0; \\
\frac{B_{\omega \sigma} \sigma^{n-1} \sigma}{(1 - \omega)^m}, & \sigma \geq 0;
\end{cases}
\]

\[
\frac{d\varepsilon^c}{dt} = \varepsilon^c_{\text{cr}} \]

where \( B_{\omega} \) are creep characteristics under tension \((i = 1)\) and compression \((i = 2)\), \( \frac{d\varepsilon^c}{dt} = \varepsilon^c_{\text{cr}} \) is creep strain rate under tension \((\sigma \geq 0)\) or compression \((\sigma \leq 0)\) for some fiber beams at time \( t \), \( \omega \) is damage parameter what \( \omega(0) = 0, \omega(1) = 1 \).

Considering that Bernoulli hypothesis is true. Continuous displacement of the neutral axis occurs as a result of the material’s different resistance to tension and compression under creep. Thus, we are

\[
\varepsilon(z, t) = \frac{\sigma(z, t)}{E} + \varepsilon^c(z, t) = \chi(t)(z - \delta(t)),
\]

(3)
where \( \varepsilon(z,t) = \varepsilon^e + \varepsilon^c \) is the total strain of the beam; \( \varepsilon^e = \frac{\sigma(z,t)}{E} \) is the elastic strain; \( \varepsilon^c \) is the creep strain; \( \sigma(z,t) \) is the actual stress at point with a coordinate \( z \) at the moment \( t \); \( E \) is the modulus of elasticity of the material; \( \chi(t) \) is the curvature at a particular point on the axis of a beam at the time \( t \); \( \delta(t) \) is the displacement of the neutral axis at the time \( t \), \( z \) is the coordinate in the thickness direction of the beam.

The displacement of the neutral axis \( \delta(t) \) is based on the equilibrium equation

\[
\int_S \sigma dS = 0,
\]

where \( S = b_h + b_h \) is the cross sectional area of the T-beam.

Numerical integration procedure is to replace the integral with a series of sums in accordance with the formula Simpson (2). The stress along the height of the beam \( \sigma(z,t) \) is found by the above-mentioned values. The system was solved using the Runge – Kutta – Merson method with an automatic time step [13]. The algorithm for solving the direct problem was developed and written in C++.

The following criterion was used to select the integration step: if the decision error at each step \( \left| \text{err} \right| \geq \text{eps} \) (\( \text{eps} \) is the permissible specified error), then the step size is halved. If the value is \( \left| \text{err} \right| \leq \text{eps} / 32 \) then the step size is doubled [13].

4. Approximate calculations of the pure bending

Slow deformation processes are promising from the point of view of resource preservation in the technology of forming body parts, in which creep deformations form an essential part, thereby reducing the formation loads, the accuracy of manufacturing increases.

**Method 1.** The method of characteristic parameters for assessing the behaviour of elements of a complex profile can be used. The behaviour of the structure is estimated by the stress value at the skeleton point [14-16].

Consider this method on a two-stage T-beam with one axis of symmetry in the section, loaded with a constant moment along the length \( M \).

The equilibrium equations are the following

\[
b_1 \int_0^h \sigma dz + b_2 \int_h^H \sigma dz = 0, \quad b_1 \int_0^h \sigma dz + b_2 \int_h^H \sigma dz = M, \tag{4}
\]

where \( H = h_1 + h_2 \) is the total height of the beam.

In the earliest possible time instance \( \varepsilon^c(t) = 0 \), then we get from (3)

\[
\sigma = E\chi(z - \delta_0), \tag{5}
\]

where \( \delta_0 = \frac{1}{S} \left( b_1 h_1 + \frac{h_2}{h} b_1 h_2 \right) \) is the initial position of neutral plane.

The constitutive relations of creep in the uniaxial case without taking into account the damage of the material will be taken in the form of the Norton power law: \( \frac{d\varepsilon^c}{dt} = B\sigma |\sigma|^{n-1} \), where \( B, n \) are material constants.

The steady-state stresses are determined using the time-differentiated equation (3) and the equations of creep

\[
\sigma = \left( \frac{1}{B} \frac{d\chi}{dt} \right)^\frac{1}{n} (z - \delta)^\frac{1}{n}. \tag{6}
\]
The stress at the characteristic point \( z \) remains constant during the deformation process [14]. Then, equating (5) and (6) with \( z = \hat{z} \), we get:

\[
\hat{\sigma} = E\chi \left( \hat{z} - \delta \right) = \left( \frac{1}{B} \frac{d\chi}{dt} \right)^\frac{1}{n} \left( \hat{z} - \delta \right)^\frac{1}{n} = \left( \frac{M}{J} \right) \left( \hat{z} - \delta_0 \right).
\]

The steady-state position of the neutral plane from the first equilibrium equation (4) is determined by:

\[
b_1 \int_0^h (z - \delta)^\frac{1}{n} \, dz + b_2 \int_0^h (z - \delta)^\frac{1}{n} \, dz = 0.
\]

The coordinates of the characteristic point \( \hat{z} \) are determined from the equation

\[
\left( \hat{z} - \delta \right)^\frac{1}{n} = b_1 \int_0^h (z - \delta)^\frac{1}{n} \, dz + b_2 \int_0^h (z - \delta)^\frac{1}{n} \, dz \left( \left( b_1 + b_2 \right) \hat{z}^2 \, dz - \left( b_1 h_1 + b_2 h_2 \right) \delta_0 \hat{z} \right) \left( \hat{z} - \delta_0 \right)^\frac{1}{n}.
\]

The coordinate of the characteristic point \( \hat{z} \) can be determined by solving the nonlinear equation (7). It depends on the creep index \( n \) and the geometric dimensions of the beam. Stress can be calculated knowing the value of the bending moment. The intensity of the construction process as a whole can be estimated from the intensity of the process in this fiber.

**Method 2.** Another approximate approach we consider. In [17, 18], it was shown that in the case of creep at a fixed temperature, the duration of the process to failure \( t^* \) is inversely proportional to the power of scattering \( W \):

\[
W t^* = \text{const}.
\]

We consider that the total work \( A(t) \) for unit volume is equal to the average value over the body volume in the general case of loading: \( A(t) = \frac{1}{V} \left( P \cdot \Delta l(t) + M \varphi(t) \right) \), where \( V \) is the working volume of the sample, \( P \) is the axial load; \( \Delta l(t) \) is sample elongation, \( \varphi(t) \) is sample twisting, \( M \) is bending moment.

Without taking into account the loss of mechanical energy, we assume that the power of the internal \( W = \sigma \eta \) and external generalized forces \( W = Q \frac{dq}{dt} \) coincide, where \( \sigma \) and \( \eta \) are generalized stresses and generalized creep strain rates, \( Q, q \) correspond to the generalized forces and generalized displacements respectively.

We assume the material is incompressible, that is \( V = S_0 l_0 = S l \), where \( S_0 \) is the initial sample area, \( l_0 \) and \( l \) correspond to the initial and current sample length respectively. The creep behavior of this material in tension can be approximated by \( W = BQ^\alpha \).

The T-beam bending under the action of a constant bending moment \( dA = \frac{1}{V} (M d\varphi) = \frac{M}{S} \frac{d\varphi}{l_0} = Q_0 dq \), where \( Q_0 = M/S \) is generalized force of beam in bending, \( dq = d\varphi/l_0 = d\chi \) is generalized displacement.

The strain rate \( \eta \) representable in the form of a linear functions of the coordinates of to the beam height \( \eta = \frac{d\chi}{dt} \hat{z} \). By equal \( W = \sigma \eta = B \sigma^n \) we end up with the following equations for \( \sigma \):

\[
\sigma = \left( \frac{\eta}{B} \right)^{\frac{1}{n-1}}.
\]
An expression to find the moment \( M = \int_s |\sigma| y \, dS \) at the steady-state creep stage \( \left( \frac{d\chi}{dt} = \text{const} \right) \) according to (8) is: \( M = \left( \frac{d\chi}{dt} B \right) \int_s |z|^{n-1} \, dS \). Then the curvature rate equals

\[
\frac{d\chi}{dt} = M^{n-1} B / J^{n-1} .
\] (9)

where \( J = \int_s |z|^{n-1} \, dS \) is the second moment of area of the T-beam.

Knowing the parameters of the structural element, creep exponent and other parameters the intensity of the creep process of a structural element can be used for the estimation of the average dissipation.

Figure 1 presents the curvature versus time under the action of a constant bending moment \( M \) for T-beams. The curves were obtained through the estimation methods: 1) on a skeleton point \( (\dot{\chi}_{XT} (t) \) is the green lines); 2) on average by volume dissipation \( (\dot{\chi}_W (t) \) is the blue lines), and also numerically taking into account the combined creep properties under tension and compression and damage of material \( (\dot{\chi}_{mod} (t) \) is the red lines). The dots at the above chart denote experimental data under pure bending of the T-beam with tension (figure 1a) and compression (figure 1b).

![Figure 1](image_url)

**Figure 1.** The dependence of the beam curvature at steady creep on the time in the web in tension (a), and compression (b).

Table 2 shows the numerical and experimental values \( \dot{\chi}_{exp} (t) \) of the curvatures on the time \( t \). A comparison of the absolute deviations is shown, where

\[
\Delta_1 = (\dot{\chi}_{XT} (t) - \dot{\chi}_{exp} (t)) / \dot{\chi}_{exp} (t), \quad \Delta_2 = (\dot{\chi}_W (t) - \dot{\chi}_{exp} (t)) / \dot{\chi}_{exp} (t), \quad \Delta_3 = (\dot{\chi}_{mod} (t) - \dot{\chi}_{exp} (t)) / \dot{\chi}_{exp} (t) .
\]

Table 2 indicates that the curvature of the T-beam resulted from estimates for the skeleton point and the average dissipation, differs on average by 31.75% and 11.75% from the experimental data respectively.

The absolute deviation between the results of the numerical analysis and the experiment bias of 1.19%. This indicates that the good description of the creep process under modeling of the bending of the T-beam, taking into account the different relations in tension and compression and damage of the material. However, the impact of damage in this calculation is small due to the low level of accumulated creep strains, but the parameter itself in numerical calculations was laid down.

### 5. Solution of the inverse problem

The bending moment \( M \) should be found on the target curvature \( \chi \) in inverse problems. The moment search is carried out by solving a sequence of direct problems of pure bending of T-beams.
Comparisons are made between the obtained curvatures and the required experimental value at each step. The refinement of the curvature values using the Nelder–Mead method \[19\] with the addition of time constraints on the formation of structural elements is carried out.

### Table 2. The results of the calculations

| web in | $M$, Nm | $t$, h | $\chi_{\text{exp}}(t)$, $10^{-3}$,mm$^{-1}$ | $\chi_{\text{ST}}(t)$, $10^{-3}$,mm$^{-1}$ | $\chi_{w}(t)$, $10^{-3}$,mm$^{-1}$ | $\chi_{\text{mod}}(t)$, $10^{-3}$,mm$^{-1}$ | $\Delta_1$, % | $\Delta_2$, % | $\Delta_3$, % |
|--------|---------|--------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|--------|--------|--------|
| tension | 75.6 4.0 | 4.08   | 5.7                                      | 4.61                                      | 4.14                                      | 39                                        | 13     | 1.4    |
|         | 137.8 0.39 | 3.47   | 4.43                                     | 3.82                                      | 3.51                                      | 28                                        | 10     | 0.98   |
| compression | 75.6 3.5 | 3.07   | 4.19                                     | 3.47                                      | 3.13                                      | 36                                        | 13     | 1.4    |
|         | 137.8 0.5 | 5.27   | 6.51                                     | 5.81                                      | 5.49                                      | 24                                        | 11     | 0.98   |

The steps of the algorithm iteration to determine the value of the bending moment to achieve the desired curvature during the formation of structural elements will be considered. Data origination at the first stage are prepared. The initial simplex of the Nelder–Mead method with the calculated values of the integral quadratic criterion at these points is given. The second stage involves sort in the second stage. Three pairs of bending moment $M$ and curvature $\chi$ from the initial simplex are selected: the first with the maximum value, the second with the next value and the third with the smallest. The center of gravity is further defined. We do the reflection operation, i.e. the point $x_{Mh}$ with the maximum value of the deflection through the center of gravity is projected. Next, the operation of stretching or compression is performed. Compression if not carried out, then add a point $x_{Mr}$ instead of a point $x_{Mh}$. The iteration process is completed after we had satisfied the convergence conditions \[19\] and have obtained the required curvature of the part.

The algorithm is written and implemented in the C++ programming language, as a separate subroutine.

Figure 2 shows three lines of curvature of the growth process on the time, which we have gained while solving when solving the inverse problem of 1 - 243, 2 - 284 and 3 - 263 iterations of method. Dots denote experimental data.

The discrepancy between the exact experimental dependence and the numerical solution is about 2.3% as a result of the calculations was obtained.

It is possible to proceed to the modeling of the formation of finned structures on the basis of such an approach and to solve both direct and inverse problems.

### 6. Conclusion

The simulation of the pure bending of a beam of a T-section for the Ti-Al-Sn-V alloy was demonstrated in this study. The numerical model takes into account the difference in tensile and compressive properties of the material, as well as the presence of accumulated creep damage in the material. It is shown that taking into account the different properties of the material gives the smallest deviation from the experimental data, and also allows one to fairly accurately describe all three stages of the creep process of the beam.

Evaluation methods can be used at the steady-state creep stage, the error was 11.75%. This was shown when comparing approaches to modeling the stress-strain state. The calculation on the basis of the specific dissipation allows one to satisfactorily describe the processes of high-temperature creep up to destruction. We note that in the evaluation methods, the calculation of the constants of only on tension or compression is laid.
The damage of the material and the difference in the properties of creep in tension and compression must be taken into account for the most accurate assessment of the intensity of the creep process of T-beams, which is confirmed by the results. The absolute deviation was 1.19%.

It should be noted that the analysis for T-beams was carried out, but the results of this work for more complex, multi-stage T-beams, parts of ribbed panels may be applicable.

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