Confinement due to the spin-orbit interaction in quantum point contacts

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The properties of a quantum point contact with spin-orbit coupling are analysed within scattering theory. It is demonstrated that an applied magnetic field results in the creation of quasi-localized states which become bound at high magnetic fields. These states correspond to solutions of the one-dimensional Dirac equation, carry spin current, and are analogous to edge and surface states in two- and three-dimensional topological insulators. This effect is predicted to occur in hole point contacts with Rashba interaction, and the extreme sensitivity of the properties of the localized states to the applied magnetic field makes the effect easily distinguishable and provides a straightforward way to measure the spin-orbit coupling in the channel.

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One dimensional (1D) spin-orbit coupled systems have recently garnered huge interest in the context of quantum information and spintronics, playing a key role in the search for emergent Majorana fermions and the generation of spin polarized current. The measurement of the strength of the spin-orbit interaction (SOI) in such systems, however, remains a crucial and open challenge which has been reflected in previous studies considering the modified conductance properties of such systems. In these studies the role of an external magnetic field in providing experimental control over the shape of the 1D dispersion was also recognized. In this work I show that an applied magnetic field leads to confinement of particles in a quantum point contact (QPC) at sufficiently strong spin-orbit coupling. This effect is a direct consequence of dynamics near the anticrossing of spin-split bands described by the one-dimensional Dirac equation. The spin-orbit interaction generates quasi-bound states which carry spin current and are thus analogous to surface and edge states in three- and two-dimensional topological insulators. The lifetime of these states is extremely sensitive to the applied magnetic field and may vary over nine orders of magnitude in the typical experimental range, allowing particles to be truly localized and transforming the QPC into a chiral quantum dot.

In this work I consider a QPC defined by lateral gates in a two-dimensional (2D) semiconductor heterostructure in the presence of an external magnetic field and Rashba spin-orbit interaction. In this system electrons flow through a one-dimensional channel smoothly connected to two-dimensional leads. Quantization of motion in the transverse direction forms a series of 1D modes, with the conductance of the system at zero temperature being equal to the sum of transmission probabilities at the Fermi energy in units of $\frac{e^2}{h}$ (see Refs. [17,18]). The Hamiltonian of the system is

$$H = \frac{p_x^2}{2m} - \alpha p_x \sigma_y - \beta \sigma_x + U(x). \quad (1)$$

where $m$ is the band mass, $\alpha$ is the Rashba coefficient, $U(x)$ is a smooth scattering potential with maximum $U_0$ at the center of the QPC ($x = 0$) and the Zeeman interaction is due to a parallel magnetic field, $\beta = \frac{1}{2} g \mu_B B_x$, with $g$ being the Landé $g$-factor. The 2D system lies in the $x-y$ plane, with the channel parallel to the $x$-axis. We note that $\alpha$ is constant, so that the spin-orbit interaction is spatially homogeneous in our case, and equal in the channel and the reservoirs. The conductance of the QPC is therefore determined solely by scattering of the electron from the smooth potential barrier $U(x)$, although the result may be generalized straightforwardly to systems with inhomogeneous spin-orbit coupling.

The dispersion of spin-split bands is given by

$$\epsilon_k^\pm = \frac{\hbar^2 k^2}{2m} \pm \sqrt{\alpha^2 \hbar^2 k^2 + \beta^2}. \quad (2)$$

For magnetic fields below a critical value $\beta < \beta_c = m \alpha^2$, the lower band has a “mexican hat” shape, with a local maximum at $k = 0$; the dispersion in this case is shown in Fig. 1a. The shape of the upper and lower branches near $k = 0$ is governed by an anticrossing of the spin-split bands, and in this region the terms in the Hamiltonian which are linear in $p_x$ are qualitatively most important. The wavefunction can therefore be modelled by a Dirac equation

$$(-\alpha p_x \sigma_y - \beta \sigma_x + U(x))\psi(x) = E\psi(x). \quad (3)$$

with $\alpha$ and $\beta$ playing the role of the effective speed of light and rest energy respectively. The existence of bound states in a repulsive scattering potential $U(x)$ may be seen as a consequence of the positronic physics present in the Dirac equation \[\Box\]. In order to illustrate this fact consider first the “non-relativistic” regime, in which $\alpha \hbar k \ll \beta$. The non-relativistic limit of the Dirac equation is a Schrodinger equation with negative effective mass,

$$(-\beta + \frac{p_x^2}{2m^*} + U(x))\psi(x) = E\psi(x) \quad (4)$$

where $m^* = -m(\frac{\alpha^2}{m^2} - 1)^{-1} < 0$. Such an equation is equivalent to one in which the mass is positive, but the
The probability is given by \( W \) in the relativistic regime, was first investigated by Schwinger in an electric field as particles with opposite charge and played by spin states in the lower branch, which behaves for forbidden region. In our case, the role of the positron is due to the Schwinger mechanism results in the transfer of a sign of the electric charge is reversed; thus an initially repulsive potential barrier \( U(x) \) becomes attractive for an electron moving in the lower branch of the Dirac dispersion; this leads to the formation of bound states in the channel.

While (1) is derived in an approximation which treats the upper and lower branches of the Dirac dispersion independently, we must also account for tunnelling between branches. This effect, which becomes important in the relativistic regime, was first investigated by Schwinger and has recently been discussed in the context of graphene. In the presence of quasibound states leads to resonances in the QPC conductance, which we shall now calculate based on general considerations in the absence of interactions. The conductance of the QPC is given by the sum of transmission probabilities for states injected in the upper (\( \epsilon_+^U \)) and lower (\( \epsilon_+^L \)) spin branches at the Fermi energy \( E_F \) of the 2D leads. (Since in the experimental situation the Zeeman splitting is typically much smaller than the Fermi energy, \( \beta \ll E_F \), the Fermi level intersects both branches; the situation when the Fermi level lies asymptotically in the Zeeman gap \( -\beta < E_F < \beta \) may be treated in a similar fashion.) Let us consider the transmission coefficient in both cases. First, consider a state injected into the lower branch. The semiclassical wavefunction is given by \( \psi(x) = \frac{e^{i\int_{x}^{b}kd\xi}}{\sqrt{A(x)}} \chi_k^+ \) where \( \chi_k^+ \) is an adiabatically varying spinor and remains nonsingular unless the semiclassical momentum touches the bottom of the lower band. This implies that for barrier heights satisfying \( E - U_0 > E_{\text{min}} = -\frac{\hbar^2}{2m_a^2} - \frac{\pi \gamma}{\alpha \lambda} \), the particle is fully transmitted, while for the case \( E - U_0 < E_{\text{min}} \) the particle is reflected. Thus the contribution to the conductance from the lower spin branch has the form of a single step at \( E = E_F \) when \( U_0 = E_F + |E_{\text{min}}| \).

The case of a particle injected in the upper spin branch is considerably more interesting. When injected from the left lead, the semiclassical wavefunction on the right of the barrier has the form (as a transmitted wave) \( \psi(x) = \frac{e^{i\int_{x}^{a}kd\xi}}{\sqrt{A(x)}} \chi_k^- \). The semiclassical wavefunction becomes singular when the particle approaches the bottom of the upper branch of the dispersion, \( E - U(x) = \epsilon_+^L = \beta \), signalling reflection from a turning point \( x = b \). Since reflection occurs at energies \( E - U_0 < \beta \), the contribution to the conductance from particles injected from the upper spin branch will possess a single step at \( U_0 = E_F - \beta \). Due to the Schwinger mechanism, a particle approaching the turning point from the left can tunnel through the classically forbidden region and reappear in the negative energy branch near the top of the barrier. In this region the semiclassical wavefunction takes the form \( \psi(x) = A \frac{e^{i\int_{x}^{a}kd\xi}}{\sqrt{A(x)}} \chi_k^- + B \frac{e^{-i\int_{x}^{a}kd\xi}}{\sqrt{A(x)}} \chi_k^+ \) and becomes singular at turning points \( x = \pm a \), when the particle undergoes reflection from the top of the lower branch. Thus the structure of the scattering wavefunction is defined by two pairs of turning points \( x = \pm a, x = \pm b \), corresponding to points in space at which the semiclassical momentum goes to zero either on the upper or lower spin branch (see Fig. 1b). A standing wave is formed in the region \( -a < x < a \).

In order to determine the transmission coefficient it is necessary to calculate the wavefunction in the tunnelling
regions, \(a < |x| < b\) (see Fig. 1b). Approximating the barrier by a linear potential with slope \(U'(x) = \lambda(-\lambda)\) for the region on the left (right) side of the barrier, the Dirac equation may be solved analytically in terms of parabolic cylinder functions. The resulting transmission probability is given by

\[
T = \left| \frac{1}{e^{2\pi\gamma} + (e^{2\pi\gamma} - 1)e^{i\int kdx - i\delta\varphi}} \right|^2 .
\]  

where \(\delta\varphi = 2\text{Arg}\Gamma(i\gamma) - 4\gamma \ln 2\) is a phase shift which disappears in the non-relativistic limit \(\alpha \hbar k \ll \beta\). Note that in deriving the transmission probability we assume that the energy lies in the range \(E_{\text{min}} < E - U_0 < -\beta\). For energies above this range, the turning points do not exist and the contribution to the conductance is trivially \(\frac{2}{\pi}\), with no resonant structure. For energies below this range, the particle encounters reflection from the bottom of the lower band, which significantly affects the structure of the quasibound state and ultimately destroys the resonance when the energy is too low. Thus we expect to observe resonances on the plateau \(E_F - \beta < U_0 < E_F + |E_{\text{min}}|\).

The transmission probability is of Breit-Wigner form, with resonances appearing when orbits exist in the lower branch satisfying the quantization condition \(\int kdx = 2\pi(n + \frac{1}{2}) + \delta\varphi\). Approximating the QPC potential as a parabolic barrier, \(U(x) \approx U_0 - \frac{m\omega^2 x^2}{2}\), we find in the case \(\alpha \hbar k \ll \beta\),

\[
\int kdx = \frac{2\pi(E - U_0 + \beta)}{\omega \sqrt{\frac{k}{\hbar} - 1}}
\]

and \(\delta\varphi \to 0\). This yields a spectrum

\[
E_n = U_0 - \beta - \omega^*(n + \frac{1}{2})
\]

where \(\omega^* = \sqrt{\frac{m}{\hbar^2}}\). This spectrum may also be obtained by solution of the non-relativistic Schrodinger equation, upon reversing the sign of the energy in a parabolic barrier becomes a Schrodinger equation for a harmonic oscillator with mass \(-m^*\) and oscillator frequency \(\omega^*\). The \(n = 0\) mode possesses the highest energy, with higher modes forming an inverted tower of oscillator states extending downward in energy. The condition \(E - U_0 > E_{\text{min}}\) implies that the spectrum terminates for a finite value of \(n\). This condition may be expressed in terms of parameters \(\zeta = \frac{n}{\sqrt{m^*\hbar}}\), \(\eta = \frac{m^*\hbar}{\omega^*}\) as

\[
\zeta < \frac{1}{2n + 1}(1 - \eta^2)\eta^2 .
\]  

The number of quasibound states predicted by (6) in different regions of the space of parameters \((\eta, \zeta)\) is shown in Fig. 2. In order to observe resonances we require the variation of the potential inside the channel to be smooth compared to the energy scale of the spin-orbit interaction, \(\zeta \ll 1\). The optimal regime, in which a large number of electrons is trapped in the constriction, occurs when \(\eta \approx \frac{1}{\sqrt{2}}\). Note that the number of quasibound states is much less sensitive to the size of the magnetic field than to the shape of the confining potential.

While (6) remain valid in the non-relativistic regime \(\alpha \hbar k \ll \beta\), implying that \(n\) is not too large, (6) nevertheless offers a simple and straightforward estimate for the number of resonances in the QPC. I now address the possibility of observing quasibound states in the typical experimental situation. Near pinchoff, the barrier height is tuned to the Fermi energy, \(U_0 \approx E_F\). We may parametrize the barrier in terms of the QPC length, \(U_0 = \frac{1}{2}m\omega^2(\frac{l}{2})^2\), to obtain

\[
\zeta = \frac{\hbar}{m\omega^2} \sqrt{\frac{8E_F}{m\omega^2}} = \frac{2}{k_F l} \frac{\hbar k_F}{m\omega^2} .
\]  

The value of \(l\) is limited by the ballistic mean free path, which is approximately one micron in 2D GaAs. Taking \(l = 1\mu\)m, at typical experimental density \(n = 10^{11}\text{cm}^{-2}\) in the 2D regions, we have \(k_F l \approx 10^2\); in the strongly spin-orbit coupled materials InSb and InAs, the conduction band effective mass \((m \approx 10^{-2}m_e)\) is prohibitively small despite the strong Rashba effect, since \(\frac{h k_F}{m\omega^2} \approx 10^2\) at typical electric fields \(\alpha \propto E_z \approx 10^6\text{Vcm}^{-1}\); the resulting value of \(\zeta\) implies that no quasibound states exist in the channel regardless of the size of the magnetic field.

Since this effect is parametrically unobservable in electron systems, we next consider hole systems, which possess a much larger value of \(m\) and a stronger spin-orbit interaction. In the hole case the Rashba interaction in the Hamiltonian originates from a 2D Hamiltonian which is cubic rather than linear in momentum:

\[
H_{R^2}^{(2D)} = \frac{iC_3}{2}(\hat{p}_+^3 \sigma^- - \hat{p}_-^3 \sigma^+) .
\]
The sensitivity of the resonant width to the applied magnetic field is a direct consequence of Schwinger’s formula for pair creation \(^4\), in which the magnetic field plays the role of the usual Dirac mass; at high magnetic fields, decay of the quasibound state becomes exponentially suppressed. The measurement of the resonant width therefore provides a direct observation of Schwinger’s mechanism in a condensed matter system. Since previous studies have shown that both the \(g\)-factor\(^22\) and potential profile\(^23\) may be reliably extracted from conductance measurements, the exponential dependence of the lifetime would provide a highly accurate means of determining the spin-orbit constant \(\alpha\) which does not rely on coherent spin precession along a ballistic path as in previously suggested experiments\(^12\)\(^\text{–}^15\).

For low values of \(\eta\) (i.e. \(\beta \ll m\alpha^2\)), chiral quasibound states appear in the channel, with a wavefunction proportional to \(\psi(x) \propto e^{i\int kdx} \psi_+ + e^{-i\int kdx} \psi_-\) where \(\psi_{\pm}\) are spinors with polarization along the \(\pm y\) axis. Since the forward and backward components of the standing wave carry opposite spin, the average magnetization is zero. However the state carries a nonvanishing spin current \(J_{xy}(x) = \langle v_x, s_y \rangle\) which is concentrated at the top of the barrier, \(J_{xy} = 1/2 \langle v_x \psi | \psi \rangle\). This implies that the quasi-localized state may be used as a new kind of qubit which relies on spin current, rather than spin, for its operation. It is worth noting that, while the magnetic field does not lead to a total spin polarization, a relatively small magnetic field is necessary to create gap in the dispersion which is crucial for the binding of the chiral wavefunction to the centre of the QPC. For the effect described here the magnetic field plays a role similar to the band gap in topological insulators, and the “chiral quantum dot” which appears at the centre of the QPC may be considered the lower dimensional analogue of the

\[ H_R^{(1D)} = -3Cp^2 \hat{p}_x \sigma_y + C \hat{p}_z^3 \sigma_y \]  

(12)

The first term is the Rashba interaction, \(\alpha \hat{p}_x \sigma_y\) with \(\alpha = 3Cp^2\) while the second term provides a qualitatively unimportant modification of the dispersion. The size of the coefficient \(C\) may be determined from the densities of spin-split subbands observed in magnetic oscillations; we obtain values \(0.6 \hbar k_F \leq \alpha \leq 0.8 \hbar k_F\) from such studies in GaAs-AlGaAs heterojunctions\(^25\)\(^\text{–}^27\). Taking typical values for the hole density \(n = 10^{11} \text{cm}^{-2}\) and \(m = 0.4m_e\) (see\(^26\)), we obtain \(\eta = 1/6, \zeta = 1/4\) at \(gB_x = 3T\) for the low value \(m\alpha = 0.6\hbar k_F\). In this regime\(^10\) predicts \(N = 3\).

For comparison with the semiclassical solution previously described, the scattering problem was also solved by explicit numerical integration of the Schrodinger equation, with the corresponding QPC conductance shown in Fig. 3. The potential barrier was modelled as a Gaussian, \(U(x) = U_0 e^{-x^2/w^2}\) with \(w = 0.5 \mu m\), and the conductance is shown values of \(gB_x = 1T, 2T, 3T\). The variation of the magnetic field leads to a relatively small shift in the position of the resonances, but dramatically alters the resonant width. The lifetime of the rightmost resonance on the solid trace in Fig. 3 (indicated by the arrow) was calculated to be \(\tau = 7.4 \times 10^{-10}\) s. Upon decreasing the magnetic field, the resonances become significantly broader. At \(5T\) \((N = 2)\), the lifetime is \(\sim 10^{-8}\) s, and at \(10T\), the lifetime is \(\sim 10^{-1}\) s for the single resonance \((N = 1)\). Since the lifetime is much longer than the operation time of a typical experiment, the QPC can effectively function as a quantum dot at high magnetic fields.

\[ G \]

\[ U_0 \]  

\[ 0 \]

\[ 1.5 \]

\[ 2 \]

\[ 0.5 \]

\[ 0 \]

\[ 0.5 \]

\[ 1 \]

\[ 1.5 \]

\[ 2 \]

\[ \beta \]

\[ \alpha \]

\[ m\alpha^2 \]

\[ \eta \]

\[ \zeta \]

\[ \hbar k_F \]

\[ n \]

\[ m_e \]

\[ U_0 \]

\[ e^{-x^2/w^2} \]

\[ 50 \mu m, \]

\[ E_F = 6.0 \text{meV}, \]

\[ \alpha = 0.6 \hbar k_F, \]

\[ 3T \]

\[ 10T \]

\[ \sim \]

\[ \text{broader} \]

\[ 5T \]

\[ \text{dashed, blue} \]

\[ \text{dotted, green} \]

\[ \text{indicated in Fig. 3} \]

\[ \text{only nonvanishing component of the} \]

\[ \text{field for such studies in GaAs-AlGaAs heterojunctions}\]

\[ \text{low value of } m\alpha = 0.6 \hbar k_F \]

\[ \text{regime}\]

\[ \text{predicts } N = 3\]

\[ \text{without lateral confinement; we may make the replacement } \langle p_y \rangle = 0, \langle p_y^2 \rangle \approx (\hbar k_F)^2, \]

\[ H_R^{(1D)} = -3Cp^2 \hat{p}_x \sigma_y + C \hat{p}_z^3 \sigma_y \]  

(12)

\[ \text{The sensitivity of the resonant width to the applied} \]

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The spin current is shown in Fig.4 for the situation corresponding the parameters corresponding to the rightmost peak in the solid trace shown in Fig.3. In this case the lowest energy resonance (ground state) has $n = 2$: the distribution of spin current roughly mirrors the probability density. Although the state is not fully chiral in this case, it nevertheless possesses large spin current, $J_{xy} \approx -\alpha \psi^\dagger \psi$. While I have so far considered a low value of the spin-orbit interaction, taking the upper end of the experimental range $\alpha \approx \frac{\hbar k_F}{m}$, numerics show that trapping of chiral states for long times is possible. In order to detect the spin current it is necessary to first convert it into spin-polarized current, for example, by instantaneously switching off the potential $U(x)$, which causes the two opposite spin components to separate spatially, transferring states with opposite spin into opposite leads.

These results illustrate the important role of Dirac physics in ordinary semiconductor QPCs, and its profound influence on motion inside the channel. The effect discussed bears similarities with those previously discussed in emergent Dirac fermion systems and gives rise to wide possibilities for further study of spin-orbit coupled QPCs, including their interactions, which have been the subject of intense study for several decades.

While the small band-mass in n-type systems suggests that the effect will not be observed in electron QPCs, the effect is predicted to occur in hole QPCs due to the stronger Rashba interaction and higher mass. I also note that upon varying the orientation of the magnetic field, one changes the nature of the anticrossing, causing the effect to appear or disappear. This suggests that this mechanism may be responsible for the recently reported Kondo effect associated with an orientation-dependent resonance on the lowest conductance plateau in a hole QPC.

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