Exploration of student’s quantitative reasoning in solving mathematical problem: case study of field-dependent cognitive style

M Muzaini1*, D Juniati2, and T Y E Siswono2

1Fakultas Keguruan dan Ilmu Pendidikan (FKIP), Universitas Cukoroaminoto Palopo, Jl. Latammacelling, No. 19, Palopo91911, Indonesia
2Fakultas Matematika dan Ilmu Pengetahuan Alam (FMIPA), Universitas Negeri Surabaya, Jl. Ketintang Wiyata No. 48, Surabaya 60231, Indonesia

* muzaini_pmath@uncp.ac.id

Abstract. Quantitative reasoning plays a key role in the problem-solving process. This skill ensures that understanding a problem will be more effective, so the problem-solving process will be more productive. This study aimed at identifying the student’s quantitative reasoning in solving mathematical problems. The type of this research was explorative research with qualitative approach. The subject of this research was eighth-grade student who was male and had Field-dependent (FD) cognitive style. The main data of this research was obtained by interview technique. Time triangulation was used to generate credible data. Data were categorized from interview and task result, reduced, analyzed and interpreted to make conclusions. The results of this study indicate that the quantitative reasoning of subjects in the problem-solving process focuses on quantity. During the interview process, the subject demonstrated the ability to identify and interpret quantities, determine relationships among the quantities, represent all the quantities and their interrelationships.

1. Introduction
One of the math skills that need to be mastered is how the students' ability solves the problem. Problem solving is very important in teaching and learning mathematics [1]. The first problem-solving framework was developed by [2]. Many research results on problem solving emphasize the students' mental actions relating to the problem-solving steps developed by [2]. They are (1) understanding the problem; (2) designing a plan; (3) implementing the plan; and (4) looking back. It should be understood now that problem solving is not just a mathematical goal in school but also how to interpret mathematics learning [3]. Therefore, problem-solving strategies and ways of reasoning in the problem-solving process should be focused, not only when the students answer correctly then he is given praise [3]. We know that in order to develop students' reasoning, the role of problem solving is very important. It is intended to give them plenty of experiences to develop their own reasoning through the problem-solving process [4]. Therefore problem solving is an important thing that can be used to develop students' mathematical reasoning such as quantitative reasoning [5]. A student provides a reasonable reason quantitatively in the problem-solving process, it allows him to understand his problem better and can utilize better the daily experience he has [6]. It can be concluded that problem solving and quantitative reasoning play a key role in improving students' reasoning at school [1].
There are some definitions related to quantitative reasoning given by experts, Moore [7], quantitative reasoning includes the mental actions of a person such as understanding the context of the problem, identifying quantities, showing a relationship between quantities, and analyzing quantity changes. Quantitative reasoning is a way of describing a student's mental action in understanding mathematical situations, constructing quantities, and then connecting, manipulating and using that quantity to find the linkage of problems [8]. Quantitative reasoning involves quantity analysis, the relationship between quantity and problem, making a new quantity and making conclusions with quantity [9]. A person involved in quantitative reasoning will be able to identify the quantities that exist automatically, determine the relationship between quantities, and from the quantities available, he is able to build a new quantity. Quantitative reasoning is when students work with quantities and the relationship of the quantity [10]. Quantitative reasoning in this study was the mental activity doing by a person in understanding the mathematical situation, identify quantity, show the relationship between the quantities, and make conclusions based on quantity. The quality of something formed from a measurement process is called quantity [11]. In the process of constructing a quantity, one must be aware of the object (like a car), the attribute of a measured object (such as how far a car runs), and then specify that the attribute has a unit of measure (such as km) [12].

This research explore the quantitative reasoning of the students in solving the problem. In this research, the quantitative reasoning in solving problems observed through some characteristics: (1) conceiving a problem situation; (2) identifying and interpreting quantities in the problem situation; (3) making a plan in terms of quantitative relationships; (4) executing calculations with respect to the quantities in the plan; and (5) checking calculations, as calculations are planned in terms of quantitative relationships.

Cognitive style is the tendency of students in processing informations and it greatly affects the performance in solving mathematics problems [13]. Psychologically there is a difference on how people process and organize their activities. Each student has a cognitive style of FD and FI [14]. The difference in cognitive style has influenced students' ability in reasoning and problem solving in which this cognitive style contributes to the problem-solving reasoning done by the students. This study report that students’ quantitative reasoning in solving mathematical problems must also consider the student's cognitive style.

2. Method

2.1. Subject

The research subject was a male student of class VIII with Field-dependent (FD) cognitive style who had high math skills and good communication skill. Subject selection was done by giving mathematic test to all students and classifying each student based on their scores. The scores were > 60 (high ability) and < 60 (low ability). Furthermore, the GEFT (Group Embedded Figure Test) test was used to determine students' cognitive style (Field-dependent and Field-independent). A male student with cognitive style FD and high mathematic ability was selected.

2.2. Instruments

This qualitative research used main instrument and aid instruments. The aid instruments were used to obtain the characteristic of subject that was appropriate and to collect data. The main instrument was the researcher himself. The aid instruments were mathematics competency exam, GEFT (Group Embedded Figures Test), unstructured interview guide and non-routine task problems. Mathematics competency test was built from the problems of National Exam (UN) in 2014, 2015, and 2016 and had been converted into an essay test to determine student's mathematical achievement. GEFT (Group Embedded Figures Test) was used to determine student’s cognitive style. An unstructured interview guide consisted of questions to explore the student’s quantitative reasoning in solving problems. Non-routine task problems were as math problems.
2.3. Procedure
Through non-routine problems and interview guide, the subject was interviewed to investigate aspects of quantitative reasoning in solving problems. Each interview spent 45 minutes. It was recorded in audio and video ways. Each interview was recorded with a video focusing on gesture, interaction among the researcher, student, and work result while audio was used to record the subject’s sound when solving problems. All interviews were conducted in the school hall during break time. Non-routine problems were contextual problems that were familiar to the subject. Time triangulation was used to generate credible data. Data were categorized from interview and task results, reduced, analyzed and interpreted to make conclusions.

3. Results and Discussion
The research focused on how the subject used quantitative reasoning in the problem-solving process. The results of interview and task data about the characteristic of quantitative reasoning included: (1) Conceiving a problem situation; (2) Identifying and interpreting quantities in the problem situation; (3) Making a plan in terms of quantitative relationships; (4) Executing calculations with respect to the quantities in the plan; and (5) Checking calculations, as calculations were planned in terms of quantitative relationships.

3.1. Conceiving a problem situation
To understand the context of the problem, first of all, the subject tried to understand the context of the problem encountered by reading repeatedly. The research subject explained the problem by imagining the problem situation and explained the quantity that was in the problem by expressing verbally. Next, the subject focused on the quantity of the given problem by observing all the quantities that existed and then the subject connected those quantities. The information presented in table form could be read by the subject. It was proven since the subject mentioned the information from the table and pointed to the part where the information was obtained. The subject of the study could also identify concepts related to the given problem. The concept was ratio. The above explanation showed that the subject was male Field-dependent (FD) who tried to build his understanding of the problem by reading the problem in more than one occasion. To ensure understanding related to mathematics problems, the subject explained the problem again by expressing verbally through his own words. To understand this problem, the subject focused on observing the quantities that existed and then he connected between the quantities in the problem. Furthermore, from the results of connecting between the quantities, the subject realized that the concept of comparison would be used to solve the problems encountered.

3.2. Identifying and interpreting quantities in the problem situation
Subject identified and performed mental activities by finding a relationship between quantities, changes between quantities and knowing the relationship between quantities. The subject did mental activity by finding the relationship of quantities involving ratios. The subject linked between known quantity (12 packs of cake) and the quantity being asked (18 packs of cake), then based on these two quantities the subject drew the conclusion that the ratio of known quantity to the quantity being asked was 3: 2. Furthermore, the subject used the ratio of quantity (3: 2) procedurally to obtain a new quantity, namely the ratio to the quantity of materials (bananas, eggs, granulated sugar and thick coconut milk), so that each material quantity changed based on the ratio obtained. The subject also explained quantitatively by saying that in order to make 18 packs of barongko cake of the same quality, the quantity of the ingredients used should be proportional to the quantity of the existing cake by looking at the ratio between the quantity of the existing cake and the quantity of the cake to be made. From the interview transcript we can also see that the subject could analyze the quantity change associated with the context of the problems encountered. This is illustrated from the result of the interview between the subject (PK1) and the researcher:
Interview Content (Conversation)

PK1: If you want to make 18 packs; it is known that 12 packs of cake need the following ingredients: to make 12 packs of cake it needs 10 banana kepok, of both compared that 18 packs of cake compared to 12 cakes is 3: 2. Thus it needs 18/12 multiplied by 10 or 3/2 multiplied by 10 is 15 banana kapok, to make 18 packs of barongko cake. For eggs, to make 12 cakes, it is required 6 eggs, so to make 18 packs, 18/12 is multiplied by 6 eggs is 9 eggs. I simplify 18/12 into 3/2. The third part is sugar. To make 12 packs of cake it is needed 300 grams sugar. Thus, to make 18 packs of cake, it is required 3/2 multiplied by 300 grams is 450 grams of sugar, to make 18 packs. Fourth, thick coconut milk, it is needed 250 ml of thick coconut milk to make 12 packs of barongko cake. Thus, it is needed 3/2 times 250 ml that is 375 ml thick coconut milk to make 18 packs of barongko cake.

Researcher: Give reasons why you use the comparison (3: 2) to find the materials needed.

PK1: Because what is asked is a cake of the same quality and type, so I use the comparison the amount of available cakes and the amount of cakes to make in order to compare the same ingredients used.

Researcher: If the shop owner wants to make a cake with 10 eggs, how many cakes can be made?

PK1: The shop owner wants to make a cake with 10 eggs. Specify how many cake packs can be made (the subject read the problem again). As it is known that 12 packs equals 6 eggs. That is the first comparison: 12 packs equal 6 eggs, so that 10 items equal to x (x=the packs of cake) 10 items per 6 items equals $\frac{x}{2}$ packs; 5/3 times 12 packs equals x. x equals 20 packs.

3.3. Making a plan in terms of quantitative relationships
At this phase, the subject designed a plan related to quantity, connected between the quantity and the quantity structure existed in the problem. The subject mentioned there was a relationship between the known quantity value and the matter being asked, the relationship between quantity and the material and concluded the interpretation result of the values of the known quantity by estimating the probability of an answer to be obtained or not based on the problem situation in question. The subject determined a meaningful relationship between quantities by saying if the number of cakes increased or decreased, it would affect many ingredients. It can be concluded that the Subject in designing a plan began by building up his knowledge about what relationship that happened between quantities. From his knowledge, every solution of the given problem would be solved using the concept of ratio.

3.4. Executing calculations with respect to the quantities in the plan
Furthermore, in carrying out the calculation, first the subject manipulated and defined the quantity that existed in the problem and used the concept of relationship between quantities to solve the problem. When the subject was asked to find out how much thick coconut milk was needed to make 25 packs of barongko cake, the subject reconnected the quantity known namely 12 packs of barongko along with the ingredients used and then he related them to the quantity being asked. With the ratio concept, the subject performed procedural operations to solve the problem. The ratio concept that the subject meant was equality ratio. Next, in the process of problem solving, the subject did not longer focus on the quantity, but the subject did the calculation using the formula to solve the problems encountered. The obtained formula was inseparable from a series of activities done by the subject, such as finding the relationship between quantities, seeing a fixed relationship between quantities and knowing that there were changes that happened in quantity.

3.5. Checking calculations, as calculations were planned in terms of quantitative relationships
In the process of rechecking completion, the subject rechecked by counting, seeing the similarity of procedures used beginning from understanding the quantities that were known and quantity that was asked, seeing the same relationship between quantities. After that he used the concept of ratio in solving the problem until the use of the formula found explicitly by the subject. It gave beliefs to the subject that what he accomplished was true.
The research results above showed that the male subject with FD cognitive style showed problem-solving behaviour related to quantitative reasoning: First of all, attempting to understand the context of the problem, focusing on the quantities in the context of the problem, identifying the quantities in the context of the problem, Meaningfully expressing the quantities in the context of the problem, Determining relationships among the quantities or analysing the changes in the quantities in the context of the problem, Devising a plan in terms of quantitative relationships or the changes in the quantities, Executing a calculation focusing on the quantities and relationships among the quantities in a devised plan, obtaining the solution of the problem, Justifying the validity of the solution in terms of quantitative meaning, Exhibiting confidence in the solutions/he obtained. This is in line with the opinion Tangul & Kabael [1], Ramful [9] that the quantitative reasoning of subjects in the problem-solving process focuses on quantity. It illustrates that the use of quantitative reasoning skill that the subject had significantly influenced the behaviour of problem-solving and the subject's ability to provide a meaningful solution. During the planning phase in the problem-solving process, the subject spent a lot of time in reasoning problems encountered. In the process of planning again to get a solution, the subject identified the relationship between quantities. By reasoning the relationship between quantities without performing the numerical operation, the subjects were able to engage in an alleged-imagining-evaluation cycle that mentally had a major role in providing a solution. This is in line with the research results done by [7, 10, 12, and 13]. In planning phase, the subject found the formula and explained it in relation to the situation. This allowed the subject to use the formula to represent the relationships that happened. When the subject ran a plan for obtaining a solution, the subject was able to explain the calculation that he obtained. It was because the calculation that the subject did was based on the quantitative relationship. Here the subject built a quantitative meaning first before obtaining numerical results. It can be concluded that this subject uses quantitative reasoning skill in solving problems that focuses on quantity. Thus, understanding a problem through a quantitative reasoning approach will be more effective and solving problems will be more productive. This is in accordance with the results of research conducted by many researchers Ellis [10], Moore [7] that quantitative reasoning plays a key role in the problem-solving process and this skill also ensures the problem-solving process is done efficiently. Good performance in solving mathematical problem requires beliefs [16, 17], motivation [18] as well as quantitative reasoning.

4. Conclusion
The results of this study indicate that the quantitative reasoning of male FD subject in the problem-solving process focuses on quantity. Problem-solving behaviors associated with quantitative reasoning are attempting to understand the context of the problem, focusing on the quantities in the context of the problem, identifying the quantities in the context of the problem, meaningfully expressing the quantities in the context of the problem, determining relationships among the quantities or analyzing the changes in the quantities in the context of the problem, devising a plan in terms of quantitative relationships or the changes in the quantities, executing a calculation focusing on the quantities and relationships among the quantities in a devised plan, obtaining the solution of the problem, justifying the validity of the solution in terms of quantitative meaning, exhibiting confidence in the solutions/he obtained. The results of this study are important as input materials for teachers and the development of mathematics education; in this case, developing and utilizing the quantitative reasoning that students have in the learning process. In addition, the other researcher should do a research about the quantitative reasoning for middle school students on other aspects.

Acknowledgments
The authors would like to thank to the Director of Research and Community Service, Directorate General for Strengthening Research and Development, Ministry of Research, Technology and Higher Education that has provided research costs. Indeed, the Chairman of the institute of Research and Chairman of the Mathematics Education Cokroaminoto University Palopo who have given permission for research.
References

[1] Tangul and Kabael, A 2018 Prospective Middle-School Mathematics Teachers’ Quantitative Reasoning and Their Support for Students’ Quantitative Reasoning Int. J. Res. Educ. Sci. 178–97

[2] Polya G 1957 How to Solve It: A new aspect of mathematical methods (2nd Ed.). Garden City, NJ: Doubleday 1–242

[3] Harel G 2007 the DNR System as a Conceptual Framework for Curriculum Development and Instruction, pp. 263-280.

[4] Cai J & Knuth E 2012 Early algebraization: a global dialogue from multiple perspectives Res. Math. Educ. 14 301–5

[5] Cai J and Nie B 2007 Problem solving in Chinese mathematics education: research and practice Zdm 39 459–73

[6] Jhon P. Smith III P W T 2007 Quantitative Reasoning and the Development of Algebraic Reasoning John Narrat. Inq. 15 95–132

[7] Moore K C 2010 The role of quantitative reasoning in pre-calculus students learning central concepts of trigonometry (Arizona State University)

[8] Eric Weber, Amy Ellis T K and Z O 2014 ix Principles for Quantitative Reasoning and Modelling Natl. Council. Teach. Math. 108 24–30

[9] Ramful A 2015 Quantitative reasoning in problem solving Res. Guide. Pract. 20 15–20

[10] Ellis A B 2007 The Influence of Reasoning with Emergent Quantities on Students ’ Generalizations Linked references are available on JSTOR for this article : The Influence of Reasoning with Emergent Quantities on Students ’ Generalizations Cong. Intruction 25 439–78

[11] Thompson P W 1990 Theoretical Model of Quantity-Based Reasoning in Arithmetic and Algebra.

[12] Thompson P W 2011 Quantitative Reasoning and Mathematical Modelling 1,2 Patrick W. Thompson School of Mathematical and Statistical Sciences Arizona State University 1 33–57

[13] Syukriani A, Juniati D and Siswono T Y E 2017 Strategic competence of senior secondary school students in solving mathematics problem based on cognitive style AIP Conference Proceedings Volume 050009 pp. 1–6

[14] Kuo F, Hwang G, Chen S and Chen S Y 2012 A Cognitive Apprenticeship Approach to Facilitating Web-based Collaborative Problem Solving Educ. Technol. Soc. 15 319–31

[15] Kevin C. Moore 2011 Relationships between Quantitative Reasoning and Students’ Problem Solving Behaviours Proceedings of the 14th Annual Conference on Research in Undergraduate Mathematics Education volume 3 p 129

[16] Muhtarom, Juniati D and Siswono T Y E 2017 Global Journal of Engineering Education 19(2) 130–136.

[17] Muhtarom, Juniati D and Siswono T Y E 2017 AIP Conference Proceedings 1868 1–6.

[18] Nizaruddin, Muhtarom and Sugiyanti 2017 World Transactions on Engineering and Technology Education 15(2) 102-107