A Novel Color Edge Detection Algorithm Based on Quaternion Hardy Filter

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Abstract

A study on color edge detection using the improved Di Zenzo’s gradient operator associated with quaternion Hardy filter is presented. The Di Zenzo gradient operator recently achieved great attentions in edge detection, but it is sensitive to image with additive noise. The quaternion Hardy filter demonstrates its advantage in denoising image. A robust color image edge detection algorithm called Quaternion Hardy filter with the improved Di Zenzo’s gradient operator, namely QHFIDZ, is presented in this paper. It preprocesses the color image to quaternion Hardy filter which reduces the noise effect and then apply the improved Di Zenzo’s gradient algorithm for edge map. The proposed technique is a robust algorithm for noisy image detection comparison to other image detection techniques.

Keywords: Quaternion Hardy filter, Quaternion Fourier transform, Color edge detection, Improved Di Zenzo’s gradient operator.

1 Introduction

Color edge detection is designed to identify discontinuities in the digital image where the brightness or color changes drastically or more formally. Color edge detection in image segmentation, pattern recognition, computer vision and many other aspects is crucial. It is significant in various types of image processing such as the high-level feature extraction, feature description, image understanding and target recognition, etc.

Canny [1], Sobel [2], Prewitt [3], differential phase congruence (DPC) [4] and modified differential phase congruence (MDPC) [5] detectors have drawn wide attention and achieved great success in grayscale edge detection. Given a color image, converting it to grayscale and then applied these grayscale-based image detectors which have been well developed. The grayscaling is a three-dimensional to one-dimensional process, which will inevitably lose some information about the image.

As shown in Fig. 1, the left half of the rectangle has RGB values of 2, 170 and 2, and the right half has RGB values of 250, 2 and 240, respectively. Fig. 1 (a) is the original image, Fig. 1 (d)-(f) are the grayscale-based edge detectors obtained by the Canny, Sobel and Prewitt algorithms, respectively. With the exception of (c), they all failed to extract the outline of different regions because only the

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brightness information was kept in the grayscale process. This makes the grayscale-based edge detection algorithms are more passive. Secondly, when dealing with color images, most of them use the gray images estimator to calculate each color component separately and then combine them to get the operator without considering the correlation information between each color channel. What we need to pay attention is that compared with grayscale images and binary images, each pixel of a color image includes not only brightness information but also color information. The role of these color information in edge detection can not be ignored.

As early as 1986, Di [6] proposed a multi-channel gradient operator and has been widely used. In 2012, Jin [7] solved the uncertainty of the Di’s gradient direction, namely improved Di Zenzo (IDZ) gradient operator. In this work, we propose a novel quaternion-based IDZ gradient algorithm combining with the quaternion Hardy filter, which can enhance the border effect in a holistic manner by extracting the main features of the color image. It naturally extends the IDZ gradient algorithm in the quaternion setting.

A growing body of research [8, 9, 10, 11, 12, 13] has shown that quaternions are well adapted to color images by encoding color channels into three imaginary parts. The quaternion analytic signal [14] is the non-tangential boundary values of quaternion Hardy filter. It is shown that [14] the quaternion analytic signal is a robust gray image detector with additive noises. Despite its initial success, it is original designed for gray image. To the author’s knowledge, the study of quaternion Hardy filter to color-based edge detector is not carried out. In this work, we apply the Quaternion Hardy filter with the IDZ gradient operator to detect the edge of the color image.

We develop a novel quaternion-based IDZ gradient operator with visual and quantitative analysis for color edge detection. The contributions of this paper are summarized as follows.

1. We propose a novel IDZ gradient algorithm based on quaternion Hardy filter, namely QHFIDZ algorithm, with application to color edge detection. Different from the classic IDZ gradient algorithm, QHFIDZ treats a color image in a holistic manner.

2. We set up a series of experiments to verify the denoising performance of the proposed algorithm in various environments. Visual and quantitative analysis are both considered. In term of peak SNR (PSNR) and similarity index measure (SSIM), compared with the classic five detec-
tion algorithms, namely Canny, Sobel, Prewitt, differential phase congruence (DPC) and modified differential phase congruence (MDPC). The proposed QHFIDZ algorithm consistently shows the superiority in color edge detection.

The remainder of this paper is structured as follows. In order to make it self-contained, Section 2 gives a brief introduction to some general definitions and basic properties of quaternions, quaternion Fourier transform, quaternion analytic signal and the improved Di Zeno gradient operator. Section 3 presents the main result of the paper, it defined the novel algorithm for color-based edge detection of real-world image. Finally, visual and quantitative analysis of the proposed algorithm are drawn in Section 4.

2 Preliminaries

The present section collects some basic facts about quaternions, quaternion Fourier transform [16], quaternion analytic signal, quaternion Hardy space and the improved Di Zenzo’s gradient operator which will be used throughout the text.

2.1 Quaternions

As a natural extension of the complex space $\mathbb{C}$, the quaternion space $\mathbb{H}$ was first proposed by Hamilton in 1843 [17]. A complex number consists of two components: one real part and one imaginary part. The quaternion $q \in \mathbb{H}$ has four components, i.e., one real part and three imaginary parts:

$$q = q_0 + q_1 i + q_2 j + q_3 k,$$

where $q_n \in \mathbb{R}, n = 0, 1, 2, 3$, and the basis elements $\{i, j, k\}$ obey the Hamilton’s multiplication rules:

$$i^2 = j^2 = k^2 = ijk = -1;$$
$$ij = k, jk = i, ki = j;$$
$$ji = -k, kj = -i, ik = -j.$$

(2)

Given a quaternion $q = q_0 + q_1 i + q_2 j + q_3 k$, its quaternion conjugate is $\bar{q} := q_0 - q_1 i - q_2 j - q_3 k$. We write $\text{Sc}(q) := \frac{1}{2}(q + \bar{q}) = q_0$ and $\text{Vec}(q) := \frac{1}{2}(q - \bar{q}) = q_1 i + q_2 j + q_3 k$, which are the scalar and vector parts of $q$, respectively. This leads to a modulus of $q \in \mathbb{H}$ defined by

$$|q| := \sqrt{q \bar{q}} = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2},$$

where $q_n \in \mathbb{R}, n = 0, 1, 2, 3$. By (1), an $\mathbb{H}$-valued function $f: \mathbb{R}^2 \rightarrow \mathbb{H}$ can be expressed as

$$f(x_1, x_2) = f_0(x_1, x_2) + f_1(x_1, x_2)i + f_2(x_1, x_2)j + f_3(x_1, x_2)k,$$

where $f_n : \mathbb{R}^2 \rightarrow \mathbb{R}(n = 0, 1, 2, 3)$.

2.2 Quaternion Fourier transform

Suppose that $f$ is an absolutely integrable complex function defined on $\mathbb{R}$, then the Fourier transform [18] of $f$ is given by

$$\hat{f}(w) := \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} f(x)e^{-iwx} dx,$$
where \( w \) denote the angular frequency. Moreover, if \( \hat{f} \) is an absolutely integrable complex function defined on \( \mathbb{R} \), then \( f \) can be reconstructed by the Fourier transform of \( f \) and is expressed by

\[
f(x) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \hat{f}(w)e^{iwx}dw. \tag{6}\]

The quaternion Fourier transform, which can be regarded as an extension of Fourier transform in quaternion domain, plays a vital role in grayscale image processing. The first definition of the quaternion Fourier transform was given in 1992 [19] and the first application to color images was discussed in 1996 [20]. It was recently applied to find the envelope of the image [21]. The application of quaternion Fourier transform on color images was discussed in [13, 22]. The Plancherel and inversion theorems of quaternion Fourier transform in the square integrable signals class was established in [23]. Due to the non-commutativity of the quaternions, there are various types of quaternion Fourier transforms. In the following, we focus our attention on the two-sided quaternion Fourier transform (QFT).

Suppose that \( f \) is an absolutely integrable \( \mathbb{H} \)-valued function defined on \( \mathbb{R}^2 \), then the continuous quaternion Fourier transform of \( f \) is defined by

\[
(Ff)(w_1, w_2) := \frac{1}{2\pi} \int_{\mathbb{R}^2} e^{-iw_1x_1}f(x_1, x_2)e^{-iw_2x_2}dx_1dx_2, \tag{7}\]

where \( w_1 \) and \( x_1 \) denote the 2D angular frequency and 2D space \((l = 1, 2)\), respectively.

Furthermore, if \( f \) is an absolutely integrable \( \mathbb{H} \)-valued function defined on \( \mathbb{R}^2 \), then the continuous inverse quaternion Fourier transform of \( f \) is defined by

\[
(F^{-1}f)(x_1, x_2) := \frac{1}{2\pi} \int_{\mathbb{R}^2} e^{iw_1x_1}f(w_1, w_2)e^{iw_2x_2}dw_1dw_2, \tag{8}\]

where \( w_1 \) and \( x_1 \) denote the 2D angular frequency and 2D space \((l = 1, 2)\), respectively.

### 2.3 Quaternion analytic function

In the following, we review the concept of analytic signal. Given a real signal \( f \), combined with its own Hilbert transform, then the analytic signal of \( f \) is given by

\[
f_a(x) := f(x) + i\mathcal{H}[f](x), \quad x \in \mathbb{R}, \tag{9}\]

where \( \mathcal{H}[f] \) denotes the Hilbert transform of \( f \) and is defined by

\[
\mathcal{H}[f](x) := \lim_{\varepsilon \to 0^+} \int_{x-\varepsilon}^{x+\varepsilon} \frac{f(s)}{s-x}ds. \tag{10}\]

The Fourier transform of analytic \( f_a \) defined in (9) is given by

\[
\hat{f}_a(w) = (1 + \text{sgn}(w))\hat{f}(w),
\]

where \( w \in \mathbb{R} \).

As a natural extension of the analytic signal from 1D to 4D space in the quaternion setting is called quaternion analytic signal. It was proposed by Hu [14] using partial and total Hilbert transform associated with QFT.

Given a 2D quaternion valued signal \( f \), combined with its own quaternion partial and total Hilbert transform, then we get a quaternion holomorphic signal \( f_q \) [24] as follows
\[ f_q(x_1, x_2) := f(x_1, x_2) + i\mathcal{H}_{x_1}[f](x_1) + \mathcal{H}_{x_2}[f](x_2) + i\mathcal{H}_{x_1,x_2}[f](x_1, x_2), \]  

where
\[
\begin{align*}
\mathcal{H}_{x_1}[f](x_1) &:= \frac{1}{\pi} \lim_{\tau \to 0} \int \frac{f(t_1, x_2)}{x_1 - t_1} dt_1; \\
\mathcal{H}_{x_2}[f](x_2) &:= \frac{1}{\pi} \lim_{\tau \to 0} \int \frac{f(x_1, t_1)}{x_2 - t_1} dt_1
\end{align*}
\]

are the quaternion partial Hilbert transform of \( f \) along the \( x_1 \)-axis, \( x_2 \)-axis, respectively. While
\[
\mathcal{H}_{x_1,x_2}[f](x_1, x_2) := \frac{1}{\pi} \lim_{\tau_1, \tau_2 \to 0} \int \frac{f(t_1, t_2)}{(x_1 - t_1)(x_2 - t_2)} dt_1 dt_2
\]

is the quaternion total Hilbert transform along the \( x_1 \) and \( x_2 \) axes.

### 2.4 Quaternion Hardy space

Let \( \mathbb{C} := \{ z \mid z = x + si, x, s \in \mathbb{R} \} \) be the complex plane and a subset of \( \mathbb{C} \) is defined by \( \mathbb{C}^+ := \{ z \mid z = x + si, x, s \in \mathbb{R}, s > 0 \} \), namely upper half complex plane. The Hardy space \( \mathcal{H}^2(\mathbb{C}^+) \) on the upper half complex plane consists of functions \( c \) satisfying the following conditions
\[
\begin{align*}
\frac{\partial}{\partial \overline{z}} c(z) &= 0; \\
(\sup_{s > 0} \int_{\mathbb{R}} |c(x + si)|^2 dx) &< \infty.
\end{align*}
\]

The generalization [14] to the higher dimension is given as follows. Let \( \mathbb{C}_{ij} := \{ (z_1, z_2) \mid z_1 = x_1 + s_1i, z_2 = x_2 + s_2j, x_1, s_i \in \mathbb{R}, l = 1, 2 \} \) and a subset of \( \mathbb{C}_{ij} \) is defined by \( \mathbb{C}_{ij}^+ := \{ (z_1, z_2) \mid z_1 = x_1 + s_1i, z_2 = x_2 + s_2j, x_1, s_l \in \mathbb{R}, s_l > 0, l = 1, 2 \} \). The quaternion Hardy space \( Q^2(\mathbb{C}_{ij}^+) \) consists of all functions satisfying the following conditions
\[
\begin{align*}
\frac{\partial}{\partial \overline{z_1}} h(z_1, z_2) &= 0; \\
h(z_1, z_2) \frac{\partial}{\partial z_2} &= 0; \\
(\sup_{s_1 > 0} \int_{\mathbb{R}^2} |h(x_1 + s_1i, x_2 + s_2j)|^2 dx_1 dx_2) &< \infty.
\end{align*}
\]

Let \( f \) be a 2D \( \mathbb{H} \)-valued function with zero scalar part which is given by \( f := \text{Vec}(f) = f_1i + f_2j + f_3k \). The Cauchy integral of \( f \) on \( \mathbb{R}^2 \),
\[
h(z_1, z_2) = \frac{1}{2\pi i} \left( \int_{\mathbb{R}^2} \frac{f(r_1, r_2)}{(r_1 - z_1)(r_2 - z_2)} dr_1 dr_2 \right)
\]

is some quaternion Hardy filter [15] \( h \in Q^2(\mathbb{C}_{ij}^+) \) which satisfies the followings, \( z_1 = x_1 + s_1i, z_2 = x_2 + s_2j \).

i) The non-tangential boundary value of quaternion Hardy filter \( h \) is the quaternion analytic signal of \( f \), that is
\[
f_q(x_1, x_2) = \lim_{s_1 \to 0 \atop s_2 \to 0} h(z_1, z_2).
\]
ii) Suppose that \( h(z_1, z_2) = \xi + \eta_1 i + \eta_2 j + \eta_3 k \), then
\[
\begin{align*}
\xi(z_1, z_2) &= f \ast O_{p_1, p_2}(x_1, x_2) \\
\eta_1(z_1, z_2) &= f \ast O_{q_1, p_2}(x_1, x_2) \\
\eta_2(z_1, z_2) &= f \ast O_{p_1, q_2}(x_1, x_2) \\
\eta_3(z_1, z_2) &= f \ast O_{q_1, q_2}(x_1, x_2)
\end{align*}
\]
(15)

where \( O_{fg}(x_1, x_2) := f(x_1)g(x_2) \), \( p_{s_1}(x_1) := \frac{s_1}{s_1^2 + x_1^2} \) and \( q_{s_1}(x_1) := \frac{x_1}{s_1^2 + x_1^2} \) are the Poisson
and conjugate Poisson kernels, respectively. Here \( \ast \) denotes the 2D convolution operator of \( \mathbb{H} \)-valued function \( f \) and real valued function \( g \), i.e.,
\[
f \ast g(r_1, r_2) := \int_{\mathbb{R}^2} f(m, n)g(r_1 - m, r_2 - n)dmdn.
\]
(16)

2.5 The improved Di Zenzo’s gradient operator

In the following, we recall the improved Di Zenzo’s gradient operator, namely IDZ gradient operator,
which will be combined with the quaternion Hardy filter to establish the novel edge detection algorithm
in next section.

Let \( f \) be an \( M \times N \) color image that maps a point \((x_1, x_2)\) to a vector \((f_1(x_1, x_2), f_2(x_1, x_2), f_3(x_1, x_2))\).
Then the square of the variation of \( f \) at the position \((x_1, x_2)\) with the distance \( \gamma \) in the direction \( \theta \) is
given by
\[
df^2 := ||f(x_1 + \gamma \cos \theta, x_2 + \gamma \sin \theta) - f(x_1, x_2)||^2_2 \\
\approx \sum_{i=1}^{3} \left( \frac{\partial f_i}{\partial x_1} \gamma \cos \theta + \frac{\partial f_i}{\partial x_2} \gamma \sin \theta \right)^2 \\
= \gamma^2 f(\theta),
\]

where
\[
f(\theta) := 2 \sum_{i=1}^{3} \frac{\partial f_i}{\partial x_1} \frac{\partial f_i}{\partial x_2} \cos \theta \sin \theta \\
+ \sum_{i=1}^{3} \left( \frac{\partial f_i}{\partial x_1} \right)^2 \cos^2 \theta + \sum_{i=1}^{3} \left( \frac{\partial f_i}{\partial x_2} \right)^2 \sin^2 \theta.
\]

Let
\[
\begin{align*}
A &:= \sum_{i=1}^{3} \left( \frac{\partial f_i}{\partial x_1} \right)^2 \\
B &:= \sum_{i=1}^{3} \left( \frac{\partial f_i}{\partial x_2} \right)^2 \\
C &:= \sum_{i=1}^{3} \frac{\partial f_i}{\partial x_1} \frac{\partial f_i}{\partial x_2}
\end{align*}
\]
(17)
Then the improved Di Zenzo's gradient operator, the gradient magnitude $f_{\text{max}}$ is given by:

$$f_{\text{max}}(\theta_{\text{max}}) := \max_{0 \leq \theta \leq 2\pi} f(\theta)$$

$$= \frac{1}{2} \left( A + C + \sqrt{(A - C)^2 + (2B)^2} \right).$$

The gradient direction is defined as the value $\theta_{\text{max}}$ that maximizes $f(\theta)$ over $0 \leq \theta \leq 2\pi$

$$\theta_{\text{max}} := \text{sgn}(B) \arcsin \left( \frac{f_{\text{max}} - A}{2f_{\text{max}} - A - C} \right) + K\pi,$$

where $\text{sgn}(B) = \begin{cases} -1, & B \geq 0; \\ 1, & B < 0. \end{cases}$ When $(A - C)^2 + B^2 = 0$, $\theta_{\text{max}}$ is undefined.

## 3 Main Results

To propose the novel color edge detection algorithms, we first review the discrete quaternion Fourier transform pairs. Then we construct the quaternion Hardy filter. Finally, we apply the IDZ gradient operator to obtain the edge map.

Figure 2: The flow chart of the DQFT algorithm.
3.1 Discrete quaternion Fourier transform pairs

3.1.1 Discrete quaternion Fourier transform

To proceed, we first review the discrete quaternion Fourier transform (DQFT) algorithm [8] in the following.

Step 1. Given

\[ f = f_1 \mathbf{i} + f_2 \mathbf{j} + f_3 \mathbf{k}. \]

Step 2. Decompose \( f \) as

\[ f = f_a + f_b \mathbf{j}, \]

where

\[ f_a = f_1 \mathbf{i}, \]
\[ f_b = f_2 + f_3 \mathbf{i}. \]

Step 3. Compute the 2-D fast Fourier transform of \( f_a \) and \( f_b \), namely \( F[f_a] \) and \( F[f_b] \), respectively. Here

\[
F[f](w_1, w_2) := \sum_{x_1=0}^{N-1} \sum_{x_2=0}^{N-1} e^{-i2\pi \left( \frac{w_1 x_1}{M} + \frac{w_2 x_2}{N} \right)} f(x_1, x_2)
\]

is the 2-D fast Fourier transform of \( f \).
Step 4. Then construct the DQFT of \( f \) by
\[
\mathcal{F}_D[f] := F[f_a] + F[f_b]j.
\] (20)

Therefore we obtain the DQFT of \( f, \mathcal{F}_D[f] \), with nonzero scalar part in general.

### 3.1.2 Inverse discrete quaternion Fourier transform

We next perform the inverse discrete quaternion Fourier transform (IDQFT) algorithm as follows.

**Step 1.** Given an \( \mathbb{H} \)-valued function
\[
g = g_0 + g_1i + g_2j + g_3k.
\]

**Step 2.** Decompose \( g \) as follows
\[
g := g_a + g_bj.
\] (21)

where
\[
g_a := g_0 + g_1i,
g_b := g_2 + g_3j.
\] (22)

**Step 3.** Compute 2-D inverse fast Fourier transform of \( g_a \) and \( g_b \), then we respectively obtain
\[
F^{-1}[f_a] \quad \text{and} \quad F^{-1}[f_b],
\]

where
\[
F^{-1}[f](x_1, x_2) := \sum_{w_1=0}^{M-1} \sum_{w_2=0}^{N-1} e^{i2\pi(w_1x_1 + w_2x_2)/M} f(w_1, w_2).
\] (23)

**Step 4.** Construct the IDQFT as follows
\[
\mathcal{F}_D^{-1}[g] := F^{-1}[f_a] + F^{-1}[f_b]j.
\] (24)

Therefore we obtain the IDQFT of \( f \).

### 3.2 Color edge detection algorithm

To proceed, we first give the convolution theorem of DQFT.

**Theorem 3.1** (Convolution theorem of DQFT) Suppose that \( g = g_0 + g_1i + g_2j + g_3k \) is a 2D \( \mathbb{H} \)-valued function and \( f \) is a 2D \( \mathbb{R} \)-valued function, then
\[
\mathcal{F}_D[f \ast g] = \mathcal{F}_D[f] \mathcal{F}_D[g],
\] (25)

where \( \ast \) is the 2D convolution operator defined in (?).

**Proof.**

\[
LHS = \mathcal{F}_D[f \ast (g_0 + g_1i + g_2j + g_3k)]
= \mathcal{F}_D[f \ast g_0 + (f \ast g_1)i + (f \ast g_2)j + (f \ast g_3)k]
= \mathcal{F}_D[f \ast g_0] + \mathcal{F}_D[f \ast g_1]i + \mathcal{F}_D[f \ast g_2]j
+ \mathcal{F}_D[f \ast g_3]k,
\]
by the convolution theorem of 2-D fast Fourier transform of real-valued functions

\[ \mathcal{F}_D[f * g] = F[f * g] = F[f]F[g], \quad l = 0, 1, 2, 3. \]

Therefore, the left hand side of (25) becomes

\[
\begin{align*}
\mathcal{F}_D[f * g] &= F[f](F[g_0] + F[g_1]i + F[g_2]j + F[g_3]k) \\
&= \mathcal{F}_D[f] \mathcal{F}_D[g] \\
&= RHS.
\end{align*}
\]

\[\Box\]

In the following, we give the color edge detection algorithm, namely **QHFIDZ algorithm**.

![Figure 4: The flow chart of the QHFIDZ algorithm.](image-url)
Step 1. Given an input color image \( f \), obtain the quaternion valued signal with zero scalar part
\[
f(x_1, x_2) := f_1(x_1, x_2)i + f_2(x_1, x_2)j + f_3(x_1, x_2)k,
\]
where \( f_1, f_2 \) and \( f_3 \) represent the red, green and blue components of color image \( f \), respectively.

Step 2. Compute the DQFT of the \( f \), i.e., \( \mathcal{F}_D[f] \).

Step 3. For fixed \( s_1 > 0, s_2 > 0 \), apply the one-dimensional Fourier transform to Poisson kernel \( p_{s_1} \) and conjugate Poisson kernel \( q_{s_1} \), then
\[
\begin{align*}
  P_{s_1}(w_1) &:= \hat{p}_{s_1}(w_1) = e^{-|w_1|s_1}; \\
  Q_{s_1}(w_1) &:= \hat{q}_{s_1}(w_1) = (-\frac{w_1}{|w_1|})e^{-|w_1|s_1}.
\end{align*}
\]
Here \( P_{s_1} \) and \( Q_{s_1} \) are the Fourier transform of Poisson and conjugate Poisson kernels.

Step 4. Compute the following equations:
\[
\begin{align*}
  \alpha(w_1, w_2; s_1, s_2) &:= \mathcal{F}_D[f] P_{s_1}(w_1) P_{s_2}(w_2); \\
  \beta_1(w_1, w_2; s_1, s_2) &:= \mathcal{F}_D[f] Q_{s_1}(w_1) P_{s_2}(w_2); \\
  \beta_2(w_1, w_2; s_1, s_2) &:= \mathcal{F}_D[f] P_{s_1}(w_1) Q_{s_2}(w_2); \\
  \beta_3(w_1, w_2; s_1, s_2) &:= \mathcal{F}_D[f] Q_{s_1}(w_1) Q_{s_2}(w_2).
\end{align*}
\]

Step 5. Compute the IDQFT of equations in (26), then obtain
\[
\begin{align*}
  \xi(x_1, x_2; s_1, s_2) &:= \mathcal{F}^{-1}_D[\alpha](x_1, x_2; s_1, s_2); \\
  \eta_1(x_1, x_2; s_1, s_2) &:= \mathcal{F}^{-1}_D[\beta_1](x_1, x_2; s_1, s_2); \\
  \eta_2(x_1, x_2; s_1, s_2) &:= \mathcal{F}^{-1}_D[\beta_2](x_1, x_2; s_1, s_2); \\
  \eta_3(x_1, x_2; s_1, s_2) &:= \mathcal{F}^{-1}_D[\beta_3](x_1, x_2; s_1, s_2).
\end{align*}
\]

Step 6. Form the quaternion Hardy filter:
\[
h := \xi + \eta_1 i + \eta_2 j + \eta_3 k,
\]
where \( \xi, \eta_1, \eta_2 \) and \( \eta_3 \) are \( \mathbb{H} \)-valued function defined in (27).

Extract the vector part of \( h \), we obtain
\[
\text{Vec}(h) := h_1 i + h_2 j + h_3 k,
\]
where \( h_l, l = 1, 2, 3 \) are real-valued functions.

Step 7. Perform the IDZ gradient operator to \( \text{Vec}(h) \). Applying equation (17), we obtain
\[
\begin{align*}
  A &= \left( \frac{\partial h_1}{\partial x_1} \right)^2 + \left( \frac{\partial h_2}{\partial x_1} \right)^2 + \left( \frac{\partial h_3}{\partial x_1} \right)^2; \\
  B &= \left( \frac{\partial h_1}{\partial x_2} \right)^2 + \left( \frac{\partial h_2}{\partial x_2} \right)^2 + \left( \frac{\partial h_3}{\partial x_2} \right)^2; \\
  C &= \frac{\partial h_1}{\partial x_1} \frac{\partial h_1}{\partial x_2} + \frac{\partial h_2}{\partial x_1} \frac{\partial h_2}{\partial x_2} + \frac{\partial h_3}{\partial x_1} \frac{\partial h_3}{\partial x_2},
\end{align*}
\]
then substitute them into equation (18), we obtain
\[
\text{Vec}(h)_{\text{max}} = \frac{1}{2} \left( A + C + \sqrt{(A - C)^2 + (2B)^2} \right).
\]

Step 8. Finally, we obtain the final processed result, edge map, by applying the nonmaxmum suppress.
4 Experiments

In this section, we aim to evaluate the performance of the proposed QHFIDZ algorithm for color edge detection.

Figure 5: Original images. The six randomly chosen noiseless images are Lena, Men, House (from left to the right on the 1st row), Image A, Image B and Image C (from left to the right on the 2nd row) respectively.

Figure 6: Noisy images. From the first row to the last row, they were obtained by adding I- Gaussian noise, II- Poisson noise, III- Salt and Pepper noise and IV- Speckle noise to the original images (the 1st row of Fig. 5).
Figure 7: Noisy images. From the first row to the last row, they were obtained by adding I- Gaussian noise, II- Poisson noise, III- Salt and Pepper noise and IV- Speckle noise to the original images (the 2nd row of Fig. 5).

Figure 8: The noiseless House image (left). The edge maps obtained by IDZ gradient algorithm (middle) and QHFIDZ algorithm (right).

Figure 9: The first row is the noisy House image with additive Gaussian, Poisson, salt and pepper, and speckle noises noise from left to right. The second and third rows are the edge maps which are captured by IDZ gradient algorithm and QHFIDZ algorithms, respectively.
Figure 10: The edge maps of the noiseless test images of Lena, Men and House by Canny, Sobel, Prewitt, DPC, MDPC and QHFIDZ algorithm, from the 1st to the last row, respectively.
Figure 11: The edge maps of the noiseless test images of A, B and C by Canny, Sobel, Prewitt, DPC, MDPC and QHFIDZ algorithm, from the 1st to the last row, respectively.
Figure 12: The edge maps of the noisy images of Lena (the 1st column of Fig. 6) by Canny, Sobel, Prewitt, DPC, MDPC and QHFIDZ algorithms, from the 1st to the last row, respectively.
Figure 13: The edge maps of the noisy images of Men (the 2nd column of Fig. 6) by Canny, Sobel, Prewitt, DPC, MDPC and QHFIDZ algorithms, from the 1st to the last row, respectively.
Figure 14: The edge maps of the noisy images of House (the 3rd column of Fig. 6) by Canny, Sobel, Prewitt, DPC, MDPC and QHFIDZ algorithms, from the 1st to the last row, respectively.
Figure 15: The edge maps of the noisy Images A (the 1st column of Fig. 7) by Canny, Sobel, Prewitt, DPC, MDPC and QHPIDZ algorithms, from the 1st to the last row, respectively.
Figure 16: The edge maps of the noisy Images B (the 2nd column of Fig. 7) by Canny, Sobel, Prewitt, DPC, MDPC and QHFIDZ algorithms, from the 1st to the last row, respectively.
Figure 17: The edge maps of the noisy Images C (the 3rd column of Fig. 7) by Canny, Sobel, Prewitt, DPC, MDPC and QHFIDZ algorithms, from the 1st to the last row, respectively.

We conducted this study to compare the visual and quantitative analysis for detection of edge. MATLAB is applied to give the performance of various comparisons. Six randomly test images (Fig. 5) with different environmental settings are considered. They are from smart phone image (Image B in the 2nd row of Fig. 5), printed images (Images A and C in the 2nd row of Fig. 5 [25]), and frequently used images, namely Lena, Men and House (the 1st row of Fig. 5), respectively.

Digital image distorted due to different type of noises such as I- Gaussian noise, II- Poisson noise, III- Salt and Pepper noise and IV- Speckle noise et al. We applied these four fundamental noises (I to IV)
into the six noiseless images (Fig. 6 and 7). The ideal noiseless (Fig. 5) and noisy images (Fig. 6 and 7) are both taken into account. Tables 1 - 6 summaries the parameter settings in terms of the signal-to-noise (SNR) ratio, scale values \([26, 27]\) \(s_1\), \(s_2\), and \(s\) respectively for QHFIDZ and (DPC, MDPC) algorithms, respectively.

### 4.1 Visual comparisons

For visual analysis, the color-based method is used, namely IDZ gradient algorithm. Five grayscale-based methods are also applied, namely Canny, Sobel, Prewitt, Differential Phase Congruence (DPC) and Modified Differential Phase Congruence (MDPC) methods.

#### 4.1.1 Color-based algorithm

In this section, we first compare the proposed QHFIDZ algorithm with the IDZ gradient algorithm. Fig. 8 presents the edge map of the noiseless House image, while Fig. 9 presents the edge map of the House image corrupted with four different noises. It can be seen from the second row of Fig. 9 that the IDZ gradient algorithm can hardly identify the house under these fundamental noises interference. They illustrate that the IDZ gradient algorithm is sensitive to the noises. The third row of Fig. 9 shows the detection result of the proposed QHFIDZ algorithm. They preserve more clearly details than the second row. They demonstrate that the QHFIDZ algorithm gives robust performance than the IDZ gradient algorithm.

#### 4.1.2 Grayscale-based algorithms

The performance of the proposed QHFIDZ algorithm is compared with five grayscale-based algorithms, called Canny, Sobel, Prewitt, differential phase congruence (DPC) and modified differential phase congruence (MDPC) methods. The ideal noiseless (Fig. 5) and noisy images (Fig. 6 and 7) are both taken into account.

- **Noiseless case:** Figure 5 shows six noiseless test images whose edge maps of various detectors are shown in Fig. 10 and 11, respectively. Fig. 10 demonstrates the edge maps of the noiseless test
images of Lena, Men and House by Canny, Sobel, Prewitt, DPC, MDPC and QHFIDZ algorithms, from the first to the last row, respectively. While Fig. 11 demonstrates the edge maps of the noiseless test Images A, B and C by Canny, Sobel, Prewitt, DPC, MDPC and QHFIDZ algorithms, from the first to the last row, respectively.

- **Noisy case:** Performing four additional noises (I-IV) to the first row and the second row of Fig. 5 yields Fig. 6 and Fig. 7, respectively. The edge maps obtained by performing the Canny, Sobel, Prewitt, DPC and MDPC methods on Fig. 6 are shown in Fig. 12, 13 and 14, respectively. While the edge maps by these various detectors on Fig. 7 are given in Fig. 14, 15 and 16, respectively. From the third columns of edge maps in Fig. 12, 13 and 14, we can clearly see that the QHFIDZ algorithm is able to extract edge maps that are resistance to noise. The QHFIDZ algorithm is clearly superior to the other detectors on the images with Salt and Pepper noise. From these experimental results, we can clearly see that the QHFIDZ algorithm can easily extract boundary details from the noisy images, while the other five methods are comparably difficult to achieve. This shows that the QHFIDZ algorithm is robust than the other detectors.

### 4.2 Quantitative analysis

To show the accuracy of the proposed edge detector, the PSNR and SSIM [28] values of various type of edge detectors on noisy images (I- Gaussian noise, II- Poisson noise, III- Salt and Pepper noise and IV- Speckle noise) are calculated (Table 7 - 10).

| Noise | SNR | QHFIDZ | MDPC, DPC |
|-------|-----|--------|-----------|
| I     | 15.5126 | \(s_1 = 8.0, s_2 = 8.0\) | \(s = 3.5\) |
| II    | 22.0561 | \(s_1 = 6.0, s_2 = 6.0\) | \(s = 2.5\) |
| III   | 13.7424 | \(s_1 = 8.0, s_2 = 8.0\) | \(s = 4.5\) |
| IV    | 13.5357 | \(s_1 = 8.0, s_2 = 8.0\) | \(s = 4.5\) |

| Noise  | SNR | QHFIDZ | MDPC, DPC |
|--------|-----|--------|-----------|
| I      | 19.3066 | \(s_1 = 5.5, s_2 = 5.0\) | \(s = 2.0\) |
| II     | 26.7146 | \(s_1 = 5.5, s_2 = 5.0\) | \(s = 2.0\) |
| III    | 17.5371 | \(s_1 = 5.5, s_2 = 5.0\) | \(s = 2.0\) |
| IV     | 18.4826 | \(s_1 = 5.5, s_2 = 5.0\) | \(s = 2.0\) |

| Noise | SNR | QHFIDZ | MDPC, DPC |
|-------|-----|--------|-----------|
| I     | 25.0439 | \(s_1 = 2.0, s_2 = 2.0\) | \(s = 0.5\) |
| II    | 34.4851 | \(s_1 = 2.0, s_2 = 2.0\) | \(s = 0.5\) |
| III   | 21.3117 | \(s_1 = 2.0, s_2 = 2.0\) | \(s = 0.5\) |
| IV    | 29.5454 | \(s_1 = 2.0, s_2 = 2.0\) | \(s = 0.5\) |
Table 6: The image C in Fig. 7.

| Noise | SNR | QHFIDZ | MDPC, DPC |
|-------|-----|--------|-----------|
| I     | 21.6328 | $s_1 = 7.0$, $s_2 = 7.0$ | $s = 7.0$ |
| II    | 27.7044 | $s_1 = 6.0$, $s_2 = 6.0$ | $s = 5.5$ |
| III   | 19.8884 | $s_1 = 8.0$, $s_2 = 8.0$ | $s = 8.0$ |
| IV    | 18.6187 | $s_1 = 8.0$, $s_2 = 8.0$ | $s = 8.0$ |

Table 7: The PSNR comparison values for Fig. 12, 13 and 14. Type of noises: I- Gaussian noise, II- Poisson noise, III- Salt and Pepper noise and IV- Speckle noise.

|                | QHFIDZ | Canny | Sobel | Prewitt | MDPC | DPC | IDZ |
|----------------|--------|-------|-------|---------|------|-----|-----|
| **LENA**       |        |       |       |         |      |     |     |
| I              | 64.8622| 56.6985| 64.7819| 64.6047| 57.6375| 57.6707| 53.9715|
| II             | 65.8570| 57.5531| 68.2679| 68.1332| 59.9609| 60.0363| 56.0557|
| III            | 63.0016| 55.8291| 62.4449| 62.6426| 57.7647| 57.6952| 54.2721|
| IV             | 64.6239| 56.4122| 64.4007| 64.3951| 58.8433| 58.7682| 53.8232|
| **MEN**        |        |       |       |         |      |     |     |
| I              | 62.9965| 59.1716| 62.7258| 62.7488| 58.7690| 58.8037| 53.7155|
| II             | 65.2541| 63.4944| 66.3657| 66.2739| 60.9465| 61.0934| 59.2063|
| III            | 62.0697| 57.3177| 58.9356| 59.2481| 58.2379| 58.2515| 54.4117|
| IV             | 63.0900| 60.6136| 62.9773| 62.9611| 60.3605| 60.4286| 56.3922|
| **HOUSE**      |        |       |       |         |      |     |     |
| I              | 61.4998| 56.3984| 64.9506| 65.1998| 57.7217| 57.7132| 54.0252|
| II             | 63.4681| 63.4558| 67.6564| 67.9324| 60.0384| 60.1424| 56.0046|
| III            | 61.0881| 56.7973| 62.9551| 63.1633| 58.0498| 57.9419| 54.3644|
| IV             | 61.0322| 56.0536| 63.8801| 64.0296| 58.2135| 58.0122| 53.7992|

1. The **PSNR** is the most common and widely used method of objective evaluation of two images. It is based on the error between the corresponding pixel, which is based on the error-sensitive image quality evaluation. The PSNR between the original image (the edge map of noiseless image) $X$ and the reference image (the edge map of noisy image) $Y$ is defined by

$$
\text{PSNR}(X, Y) := 10 \log_{10} \left( \frac{(2^n - 1)^2}{\text{MSE}(X, Y)} \right),
$$

where $n$ is the number of bits per pixel and $\text{MSE}(X, Y)$ denotes the mean square error of those two images, that is $\text{MSE}(X, Y) := \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} (X_{ij} - Y_{ij})^2$. The PSNR value obtained by this method is the degree of similarity or fidelity of the edge maps. The unit of PSNR is dB, the bigger the value is, the stronger the robustness is.

2. The **SSIM** is a method of comparing two images from three aspects of brightness, contrast and structure. In general, the SSIM between two images $X$ and $Y$ is defined by

$$
\text{SSIM}(X, Y) := [L(X, Y)]^\alpha[C(X, Y)]^\beta[S(X, Y)]^\gamma,
$$

where $\alpha, \beta, \gamma > 0$. They are used to adjust the importance of the three modules.

- **Luminance contrast function** uses the average gray level $\mu_x$ and $\mu_y$ as an estimate of the luminance measurement, which is defined by

$$
L(X, Y) := \frac{2\mu_x \mu_y + c_1}{\mu_x^2 + \mu_y^2 + c_1},
$$
where \( \mu_X = \frac{1}{N} \sum_{i=1}^{N} X_i \) is the average grayscale of image X. The parameter \( c_1 \) is to avoid system instability when \( \mu_X^2 + \mu_Y^2 \) approaches zero. In particular, we choose small constants \( c_1 = (k_1 l)^2, k_1 \ll 1 \), and l represents the grayscale of one image.

- **Contrast comparison function** uses the standard deviation \( \sigma_X \) and \( \sigma_Y \) as the contrast measure, which is
  \[
  C(X, Y) := \frac{2\sigma_X \sigma_Y + c_2}{\sigma_X^2 + \sigma_Y^2 + c_2},
  \]
  where \( \sigma_X = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (X_i - \mu_X)^2}, c_2 = (k_2 l)^2, k_2 \ll 1 \).

- **Structure contrast function** is defined by
  \[
  S(X, Y) := \frac{\sigma_{XY}}{\sigma_X \sigma_Y + c_3}
  \]
  where \( \sigma_{XY} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (X_i - \mu_X)(Y_i - \mu_Y)} \).

  In this paper, assume \( \alpha = \beta = \gamma = 1, c_3 = \frac{1}{2} c_2 \), by straightforward computation, we have
  \[
  \text{SSIM}(X, Y) := \frac{(2\mu_X \mu_Y + c_1)(2\sigma_{XY} + c_2)}{(\mu_X^2 + \mu_Y^2 + c_1)(\sigma_X^2 + \sigma_Y^2 + c_2)}.
  \]

  The SSIM\((X, Y)\) value is between 0 and 1. The larger the value, the greater the similarity between the two images X and Y.

Table 8: The PSNR comparison values for Fig. 15, 16 and 17. Type of noises: I- Gaussian noise, II- Poisson noise, III- Salt and Pepper noise and IV- Speckle noise.

|        | QHFIDZ | Canny | Sobel | Prewitt | MDPC | DPC  | IDZ  |
|--------|--------|-------|-------|---------|------|------|------|
| Image A | I      | 69.7830 | 54.6441 | 71.3263 | 72.8126 | 64.3338 | 61.0803 | 53.9459 |
|        | II     | 74.1626 | 57.7629 | 78.5871 | 77.5450 | 68.2203 | 69.1860 | 57.4738 |
|        | III    | 68.8984 | 54.8312 | 67.8883 | 68.4996 | 59.1397 | 57.6659 | 54.3844 |
|        | IV     | 70.2784 | 55.7384 | 71.3680 | 72.0796 | 62.1912 | 59.4687 | 53.8051 |
| Image B | I      | 67.7956 | 57.9033 | 67.5161 | 62.0191 | 61.6269 | 61.7502 | 53.6154 |
|        | II     | 70.8856 | 66.8262 | 70.8302 | 63.8282 | 60.8068 | 61.8739 | 54.6231 |
|        | III    | 66.8058 | 55.8679 | 66.7100 | 60.4750 | 61.0374 | 61.2672 | 54.1302 |
|        | IV     | 68.7155 | 61.7685 | 68.6092 | 61.7823 | 61.7823 | 61.8209 | 53.9745 |
| Image C | I      | 62.9955 | 53.3384 | 61.5886 | 61.6987 | 62.9903 | 62.8951 | 54.0625 |
|        | II     | 64.3923 | 54.0415 | 62.2696 | 62.2132 | 66.1757 | 66.3745 | 55.6372 |
|        | III    | 61.8965 | 53.7885 | 58.5834 | 58.4737 | 63.8826 | 63.8012 | 54.3543 |
|        | IV     | 63.0900 | 53.0448 | 61.7655 | 61.8349 | 61.8975 | 62.0452 | 53.7313 |

According to the definitions of PSNR and SSIM, the edge maps in Fig. 10 and 11 are marked as Image X, while the edge maps in Fig. 12 - 17 are marked as Image Y in both formulas (28) and (29). Tables 7 - 10 show in details the PSNR and SSIM values for Fig. 12 - 17. Each value in the table represents the similarity of the edge map of the noisy image and the edge map of the original noiseless image. That is, the larger the value, the stronger the denoising ability. From the results in Tables 7 - 8, we have the following conclusions.

- Table 7 and 8 show the PSNR comparison values for Fig. 12, 13 and 14. The larger PSNR values demonstrate better denoising performance among various detectors. For the convenience of observation, we mark them as "bold letter" under each type of noises.
Table 9: The SSIM comparison values for Fig. 12, 13 and 14. Type of noises: I- Gaussian noise, II- Poisson noise, III- Salt and Pepper noise and IV- Speckle noise.

|         | QHFIDZ | Canny | Sobel | Prewitt | MDPC | DPC | IDZ   |
|---------|--------|-------|-------|--------|------|-----|-------|
| LENA    |        |       |       |        |      |     |       |
| I       | 0.8058 | 0.6568| 0.8041| 0.8011 | 0.2986| 0.2876| 0.1079 |
| II      | 0.9275 | 0.8593| 0.9165| 0.9124 | 0.6200| 0.6198| 0.2400 |
| III     | 0.7155 | 0.5299| 0.5406| 0.5611 | 0.1710| 0.1711| 0.0840 |
| IV      | 0.7872 | 0.5658| 0.7604| 0.7613 | 0.4959| 0.4953| 0.0980 |
| MEN     |        |       |       |        |      |     |       |
| I       | 0.6878 | 0.6850| 0.6673| 0.6722 | 0.4343| 0.4342| 0.0769 |
| II      | 0.8843 | 0.8739| 0.8575| 0.8572 | 0.6522| 0.6523| 0.6177 |
| III     | 0.4669 | 0.3973| 0.1668| 0.1726 | 0.1376| 0.1375| 0.0626 |
| IV      | 0.7463 | 0.7670| 0.7200| 0.7281 | 0.5677| 0.5633| 0.3976 |
| HOUSE   |        |       |       |        |      |     |       |
| I       | 0.8518 | 0.4250| 0.8475| 0.8522 | 0.2292| 0.2289| 0.0879 |
| II      | 0.9192 | 0.8503| 0.9183| 0.9134 | 0.5106| 0.5101| 0.1869 |
| III     | 0.6723 | 0.3759| 0.5767| 0.5921 | 0.1423| 0.1419| 0.0631 |
| IV      | 0.6707 | 0.3387| 0.7502| 0.7756 | 0.2672| 0.2691| 0.0713 |

Table 10: The SSIM comparison values for Fig. 15, 16 and 17. Type of noises: I- Gaussian noise, II- Poisson noise, III- Salt and Pepper noise and IV- Speckle noise.

|         | QHFIDZ | Canny | Sobel | Prewitt | MDPC | DPC | IDZ   |
|---------|--------|-------|-------|--------|------|-----|-------|
| Image A |        |       |       |        |      |     |       |
| I       | 0.9196 | 0.1449| 0.9562| 0.9309 | 0.7128| 0.3935| 0.0575 |
| II      | 0.9652 | 0.4470| 0.9270| 0.9456 | 0.8435| 0.9191| 0.1990 |
| III     | 0.8833 | 0.1227| 0.7497| 0.8045 | 0.1679| 0.0637| 0.0389 |
| IV      | 0.9139 | 0.3621| 0.8970| 0.9121 | 0.6074| 0.4338| 0.0477 |
| Image B |        |       |       |        |      |     |       |
| I       | 0.9248 | 0.4350| 0.8217| 0.8314 | 0.3204| 0.3271| 0.0243 |
| II      | 0.9664 | 0.9034| 0.9202| 0.9311 | 0.5635| 0.5865| 0.6465 |
| III     | 0.9098 | 0.1942| 0.7720| 0.7880 | 0.3153| 0.3206| 0.0228 |
| IV      | 0.9483 | 0.7197| 0.8914| 0.8901 | 0.3584| 0.3780| 0.0414 |
| Image C |        |       |       |        |      |     |       |
| I       | 0.7560 | 0.0827| 0.3983| 0.4001 | 0.0964| 0.1116| 0.0944 |
| II      | 0.8434 | 0.1872| 0.6195| 0.6175 | 0.3330| 0.3285| 0.1845 |
| III     | 0.7192 | 0.0675| 0.2326| 0.2430 | 0.1745| 0.1729| 0.0863 |
| IV      | 0.6850 | 0.0386| 0.3714| 0.3497 | 0.0725| 0.0803| 0.0712 |

• Among various algorithms listed in Fig. 10, we see that from the values of the top two rows in Table 7, QHFIDZ method performs the best in all the noisy images, except Poisson noise. While we studied the various detectors under Poisson noise, we found that the top three algorithm are Sobel, prewitt, and QHFIDZ. For the last row in Table 7, it corresponds to house image in Fig. 6, although the PSNR values are not the best, but still in the top three. Therefore, for these three classic graphs, our method performed very well overall, especially for the effects of Lena and Men in Fig. 6.

• From the results shown in Fig. 11, we found that not all methods are suitable for testing these images in Fig. 7. Under these conditions, if we want to analyze their denoising effect, we need to analyze the significance of PSNR values in combination with visual effects. Let’s first rank first two methods in terms of visual effects, follow the order from excellent to poor, respectively, the QHFIDZ method and the Canny method. Now we only need to analyze the numbers of the two
methods in Tables 8 and choose the largest number among them. It is not difficult to find that the algorithm of QHFIDZ has achieved excellent results compared with Canny. This is why, despite the fact that we find that the QHFIDZ’s PSNR values, though not the largest of all methods, still marks it as bold. In particular, the PSNR values for the QHFIDZ method in Table 8 are optimal regardless of the conditions. On the whole, using the QHFIDZ method to do color edge detection on this type of graph, the effect is obvious and excellent.

Tables 9 and 10 show the SSIM values between the edge map of original noiseless image and edge map of noisy images under various kinds of noises. When the SSIM value is close to 1, it shows robust denoising performance among various algorithms. From the SSIM values in these tables, we have the followings.

- From the SSIM values in Table 9, our proposed QHFIDZ algorithm give better performance than the other methods under four different noises. In particular, the noise reduction effect of salt and pepper is obviously robust than the other five methods. In addition, despite the speckle noise corresponding to the house image in Fig. 6, the SSIM value of the QHFIDZ algorithm is not the largest, but it is still the top three.

- As can be seen from Table 10 the method of QHFIDZ is obviously did better work than the other methods. Although the SSIM values of method QHFIDZ and Prewitt are closely, combined with Fig. 11 we found that Prewitt’s edge detection results are not perform well within or without noise. Therefore, the value of SSIM does not have reference meaning. In contrast, the SSIM values of our method are close to 1. In other words, its noise immunity is the best.

5 Conclusions and Discussions

In this paper, we have proposed a novel quaternion representation-based methods referred as QHFIDZ for color-based edge detection. The proposed QHFIDZ algorithm naturally extends the IDZ gradient operator from noiseless to noisy setting in quaternion space. The proposed algorithm in this paper shows a powerful effectiveness and stability when processing color image edge detection. The noisy images considered in this article only involve one single kind of noise disturbance. A forthcoming paper will be devoted to explain what this could be considered when deal with mixed noises [29, 30, 31] situation. Another significant area being developed which involves edge detection and recognizing textures is finding pathological objects in medical image processing.

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