Excited heavy baryon masses to order $\Lambda_{QCD}/m_Q$ from QCD sum rules

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Masses of the p-wave excited heavy baryons have been calculated to the $\Lambda_{QCD}/m_Q$ order using QCD sum rule method within the framework of heavy quark effective theory. Numerical results for kinetic energy $\lambda_1$ and chromo-magnetic interaction $\lambda_2$ are presented. The splitting between spin $1/2$ and $3/2$ doublet derived from our calculation is given, for which the agreement with the current experiment is desirable.

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I. INTRODUCTION

In the past decade continuous progress has been made in the investigation of excited heavy baryons. The lowest lying orbitally excited charmed states $\Lambda_c(2593)$ and $\Lambda_c(2625)$ have been observed by several collaborations,[1] the excited states of $\Sigma_c$ have also been reported recently,[2] and more data are expected in the near future. From the theoretical prospect of view those data need to be studied comprehensively. Furthermore, with the collection of more experimental data for excited heavy baryon states it is useful to make some theoretical predictions on their spectroscopies.

Heavy baryons containing a single heavy quark can be described exactly by heavy quark effective theory (HQET) in the heavy quark limit. This fact should be contributed to the spin-flavor symmetry of system comprised of infinitely heavy quarks. HQET has been applied successfully to learn about the properties of heavy mesons and baryons, including the spectroscopy and weak decays. The mass formula for a spin symmetry doublet of heavy baryons up to order $1/m_Q$ corrections can be written as

$$M = m_Q + \bar{\Lambda} - \frac{1}{2m_Q}(\lambda_1 + d_M \lambda_2),$$

(1)

where parameter $\bar{\Lambda}$ is the effective mass of the light degrees of freedom in the $m_Q \to \infty$ limit, $\lambda_1$ and $\lambda_2$ are related to the heavy quark kinetic energy and the chromomagnetic energy of HQET, respectively

$$\lambda_1 = \langle B(v) | \bar{h}_v (iD^\perp)^2 h_v | B(v) \rangle,$$

$$d_M \lambda_2 = \langle B(v) | \bar{h}_v \sigma_{\mu\nu} \frac{g_s}{2} G^{\mu\nu} h_v | B(v) \rangle.$$

(2)

The constant $d_M$ characterizes the spin-orbit interaction of the heavy quark and the gluon field, it is zero for singlet and $1$, $-\frac{1}{2}$ for spin $1/2$, spin $3/2$ doublet, respectively. Thus the splitting of the spin $1/2$ and $3/2$ doublets is

$$M_{B_1^0}^2 - M_{B_0^*}^2 = \frac{3}{2} \lambda_2,$$

(3)

where $B_Q$, $B_0^*$ denote spin $1/2$ and $3/2$ doublet, respectively.

The heavy baryon mass parameters $\lambda_1$ and $\lambda_2$ play a most significant role in our understanding of the spectroscopy and inclusive decay rates. They must be estimated in some nonperturbative approaches due to the asymptotic freedom property of QCD. A viable approach is the QCD sum rules formulated in the framework of HQET. This method allows us to relate hadronic observable to QCD parameters.
via the operator product expansion (OPE) of the correlator. In the case of ground state heavy baryon
predictions on the mass spectroscopy have been made to leading and next-to-leading order in αS [10, 11]
and to order 1/mQ [12, 13] using QCD sum rule method. As to the excited baryon mass spectroscopy,
only results to leading order in 1/mQ expansion have been obtained from QCD sum rule [14, 15]. In [13]
we have calculated the baryonic parameters λ1 and λ2 for the ground state ΛQ and ΣQ baryons using
QCD sum rules in the HQET. Employing the baryonic currents from [14] we now derive these parameters
for excited Λ- and Σ-type baryons following the same procedure.

The remainder of this paper is organized as follows. In Sec. II A we introduce the interpolating currents
for excited state heavy baryons and briefly present the two-point sum rules. The direct Laplace sum rules
analysis for the matrix elements is presented in Sec. II B. The Sec. II is devoted to numerical results and
our conclusions. Some comments are also available in Sec. III.

II. DERIVATION OF THE SUM RULES

A. Heavy baryonic currents and two-point sum rules

In this work we adopt those currents constructed from Bethe-Salpeter equation in Ref. [14] as

\[ j_{\Sigma Qk} = \epsilon_{abc} (q_1^T a \tau C \gamma_5 D_{\mu i} \Gamma_i D_{\nu j}^2) \Gamma' h_{\nu}^{\mu} , \]

\[ j_{\Lambda Qk} = \epsilon_{abc} (q_1^T a \tau C \gamma_\mu D_{\nu i} \Gamma_i D_{\mu j}^2) \Gamma' h_{\nu}^{\mu} , \]

\[ j_{\Lambda Kk} = \epsilon_{abc} (q_1^T a \tau C \gamma_\mu d_{\nu i} \Gamma_i D_{\mu j}^2) \Gamma' h_{\nu}^{\mu} , \]

\[ j_{\Sigma Qk} = \epsilon_{abc} (q_1^T a \tau C \gamma_5 D_{\mu i} \Gamma_i D_{\nu j}^2) \Gamma' h_{\nu}^{\mu} , \]

\[ j_{\Sigma Kk} = \epsilon_{abc} (q_1^T a \tau C \gamma_5 D_{\mu i} \Gamma_i D_{\nu j}^2) \Gamma' h_{\nu}^{\mu} , \]

\[ \Gamma' = \gamma_\mu \gamma_5 \]

for ΣQk1, ΣQk1, ΛQk1, ΣKk1 doublets’ spin 1/2 baryon, and

\[ \Gamma' = \Gamma_{\mu_1 \rho_1} = \frac{1}{3} (g_{\mu_1 \rho_1} + \gamma_{\mu_1} \gamma_{\rho_1} ) , \]

for ΣQk1, ΛQk1, ΣKk1, ΣKk1 doublets’ spin 3/2 baryon, in which \( g_{\mu_1 \rho_1} \) and \( \gamma_{\mu_1} \) are perpendicular to
heavy quark velocity \( v \), defined as \( g_{\mu \nu} = g_{\mu \nu} - v_{\mu} v_{\nu} \), \( \gamma_{\mu} = \gamma_{\mu} - \beta v_{\mu} \). For the singlets ΛQk0 and ΣKk0
the \( \Gamma' \) is simply unit matrix \( I \). Notations used here to describe excited state heavy baryons are the same
as those used in Ref. [10, 11]: k denotes \( l_k = 1 \) and \( l_K = 0 \) whereas K denotes \( l_k = 0 \) and \( l_K = 1 \),
in which \( l_k \) is the orbital angular momentum describes the relative motion of the two light quarks and
the orbital angular momentum \( l_K \) describes orbital motion of the center of mass of the two light quarks
relative to the heavy quark. \( Q \) denotes heavy quark and the number in subscript is the total angular
momentum of the light diquark system. As that for ground state baryons, the flavor configuration of Λ
type baryon is antisymmetric and \( \Sigma \) type is symmetric. Depending on the number of derivatives or the
form of \( \Gamma' \), interpolating currents can have forms different from those listed above and may also be used
in applications [10, 15].

In the following analysis we would use those currents to interpolate excited heavy baryon states. At
the leading order of the \( 1/mQ \) expansion they do not mix with each other even with the same quantum
number, but to the next-to-leading order the mixing of interpolating currents will appear. In our subsequent
calculations we would only use those currents and did not consider the effect resulting from the mixing of interpolating currents for references 10, 16, 17 have shown that the stability criterion for QCD sum rule applications excludes the existence of interpolating currents mixing or though there does exist the mixing the numerical result will not change drastically compared with the case without mixing.

The baryonic coupling constant in HQET are also needed in our calculation, they are defined in form as follows

\[ \langle 0 \mid j \mid B(v) \rangle = Fu, \]

where \( | B(v) \rangle \) denotes excited baryon state and \( u \) can be the ordinary spinor \( u \) or the Rarita-Schwinger spinor \( u_\alpha \) in the HQET corresponding to spin 1/2 or spin 3/2 doublet, respectively. Irrespective of an irrelevant constant factor in the leading order, the coupling constants for spin 1/2 and spin 3/2 doublet are equivalent for their identical spin-parity of the light degrees of freedom.

In order to determine the effective mass of the excited baryons, we analyze the two-point correlator defined as

\[ i \int d^4x e^{ik \cdot x} \langle 0 \mid T\{j(x)\bar{j}(0)\} \mid 0 \rangle = \frac{1 + \frac{\beta}{2}}{2} Tr[\tau^+ \tau] \Pi(\omega), \]

where \( k \) is the residual momentum and \( \omega = v \cdot k \). For large negative value of \( \omega \) \( \Pi(\omega) \) can be expressed in terms of perturbative and nonperturbative contributions. The nonperturbative effects can be accounted for by including quark and gluon condensates ordered by increasing dimension, which are the series of power corrections in the “small” \( 1/\omega \) variable. The Borel transformation in the variable \( \omega \) can help to improve the convergence of these nonperturbative series.

With those interpolating currents listed in Eq. 4 it is straightforward to obtain the two-point sum rules:

\[
\begin{align*}
F_{\Lambda_{QK}}^2 e^{-\Lambda_{QK}/T} & = \frac{18 T^8}{4 \pi^4} \delta(\omega_c/T) + \frac{T^4}{4 \pi^2} \left( \frac{\alpha_s}{\pi} G^2 \right) + \frac{m_0^2}{16} \langle \bar{q}q \rangle^2 e^{-\frac{m_0^2}{\pi T^2}}, \\
F_{\Sigma_{QK}}^2 e^{-\Lambda_{QK}/T} & = \frac{90 T^8}{4 \pi^4} \delta(\omega_c/T) - \frac{3T^4}{2 \pi^2} \left( \frac{\alpha_s}{\pi} G^2 \right) + \frac{m_0^2}{16} \langle \bar{q}q \rangle^2 e^{-\frac{m_0^2}{\pi T^2}}, \\
F_{\Lambda_{QK}}^2 e^{-\Lambda_{QK}/T} & = \frac{216 T^8}{4 \pi^4} \delta(\omega_c/T) - \frac{T^4}{4 \pi^2} \left( \frac{\alpha_s}{\pi} G^2 \right) + \frac{m_0^2}{8} \langle \bar{q}q \rangle^2 e^{-\frac{m_0^2}{\pi T^2}}, \\
F_{\Sigma_{QK}}^2 e^{-\Lambda_{QK}/T} & = \frac{216 T^8}{4 \pi^4} \delta(\omega_c/T) - \frac{T^4}{4 \pi^2} \left( \frac{\alpha_s}{\pi} G^2 \right) + \frac{m_0^2}{8} \langle \bar{q}q \rangle^2 e^{-\frac{m_0^2}{\pi T^2}}, \\
F_{\Sigma_{QK}}^2 e^{-\Lambda_{QK}/T} & = \frac{288 T^8}{4 \pi^4} \delta(\omega_c/T) - \frac{3T^4}{2 \pi^2} \left( \frac{\alpha_s}{\pi} G^2 \right) + \frac{m_0^2}{4} \langle \bar{q}q \rangle^2 e^{-\frac{m_0^2}{\pi T^2}}, \\
F_{\Sigma_{QK}}^2 e^{-\Lambda_{QK}/T} & = \frac{504 T^8}{4 \pi^4} \delta(\omega_c/T) - \frac{T^4}{4 \pi^2} \left( \frac{\alpha_s}{\pi} G^2 \right) + \frac{m_0^2}{4} \langle \bar{q}q \rangle^2 e^{-\frac{m_0^2}{\pi T^2}}.
\end{align*}
\]

In calculations we adopted the gaussian ansatz for the nonlocal quark condensate to get the dimension 6 condensate contribution. Dimension \( D > 6 \) condensates are not included. The functions \( \delta_n(\omega_c/T) \) arise from the continuum subtraction and are defined in 13.

### B. Sum rules for \( \lambda_1 \) and \( \lambda_2 \)

For the evaluation of the matrix elements \( \lambda_1 \) and \( \lambda_2 \) we consider the three-point correlators as follows

\[
\begin{align*}
i^2 \int d^4x \int d^4y e^{ik \cdot x - i k' \cdot y} \langle 0 \mid T\{j(x) \bar{h}_v (iD^+) h_v(0) \bar{j}(y)\} \mid 0 \rangle & = \frac{1 + \frac{\beta}{2}}{2} Tr[\tau^+ \tau] T_K(\omega, \omega') , \\
i^2 \int d^4x \int d^4y e^{ik \cdot x - i k' \cdot y} \langle 0 \mid T\{j(x) \bar{h}_v \sigma_{\mu \nu} \frac{g_s}{2} G^{\mu \nu} h_v(0) \bar{j}(y)\} \mid 0 \rangle & = d_M \frac{1 + \frac{\beta}{2}}{2} Tr[\tau^+ \tau] T_S(\omega, \omega') ,
\end{align*}
\]
where the coefficients $T_K(\omega, \omega')$ and $T_S(\omega, \omega')$ are analytic functions in the “off-shell energies” $\omega = v \cdot k$ and $\omega' = v \cdot k'$ with discontinuities for positive values of these variables. Saturating the three-point functions with complete set of baryon states, one can isolate the part of interest, the contribution of the lowest-lying baryon states associated with the heavy-light currents, as one having poles in both the variables $\omega$ and $\omega'$ at the value $\omega = \omega' = \bar{\Lambda}$.

Confining us to take into account these leading contributions of perturbation and the operators with dimension $D \leq 6$ in OPE, the relevant diagrams in our theoretical calculations for the kinetic energy are shown in Fig. 1. The relevant diagrams for the chromo-magnetic interaction do not differ from those for the ground state baryons [13], so we do not show them here. Using dispersion relations $T_K(\omega, \omega')$ and $T_S(\omega, \omega')$ can be casted into the form of integrals of the double spectral densities. Following Refs. [18, 19, 20], introduce new variables $\omega_+ = \frac{1}{2}(\omega + \omega')$ and $\omega_- = \omega - \omega'$, perform the integral over $\omega_-$, assume quark-hadron duality in $\omega_+$, and employ Borel transformation $B_T^2$, $B_T^{2'}$ to suppress the continuum contributions and subtractions, we then obtained the desired sum rules. Considered the symmetry of the correlator it is natural to set the parameters $\tau, \tau'$ to be the same and equal to $2T$, where $T$ is the Borel parameter of the two-point functions. We ended up with the set of sum rules

\[
\begin{align*}
\lambda_2^{\Sigma K_1} F^2 e^{-\Lambda/T} &= \frac{213}{\pi^4} \delta_0(\omega_c/T) - \frac{27}{\pi^2} \langle \alpha_s G^2 \rangle + \frac{16 m_2^2 T^2 \alpha_s}{3\pi} \langle q\bar{q} \rangle^2 e^{-\frac{m_2^2}{T^2}} , \\
\lambda_2^{\Lambda K_1} F^2 e^{-\Lambda/T} &= \frac{213}{\pi^4} \delta_0(\omega_c/T) - \frac{37}{\pi^2} \langle \alpha_s G^2 \rangle + \frac{4 m_2^2 T^2 \alpha_s}{3\pi} \langle q\bar{q} \rangle^2 e^{-\frac{m_2^2}{T^2}} , \\
\lambda_2^{\Sigma K_1} F^2 e^{-\Lambda/T} &= \frac{28}{\pi^4} \delta_0(\omega_c/T) - \frac{7}{2\pi^2} \langle \alpha_s G^2 \rangle + \frac{m_2^2 T^2 \alpha_s}{3\pi} \langle q\bar{q} \rangle^2 e^{-\frac{m_2^2}{T^2}} , \\
\lambda_2^{\Lambda K_1} F^2 e^{-\Lambda/T} &= \frac{28}{\pi^4} \delta_0(\omega_c/T) - \frac{37}{\pi^2} \langle \alpha_s G^2 \rangle + \frac{4 m_2^2 T^2 \alpha_s}{3\pi} \langle q\bar{q} \rangle^2 e^{-\frac{m_2^2}{T^2}} , \\
-\lambda_1^{\Lambda K_1} F^2 e^{-\Lambda/T} &= \frac{28}{\pi^4} \delta_0(\omega_c/T) - \frac{7}{2\pi^2} \langle \alpha_s G^2 \rangle + \frac{5 m_2^4}{16} \langle q\bar{q} \rangle^2 e^{-\frac{m_2^2}{T^2}} , \\
-\lambda_1^{\Sigma K_0} F^2 e^{-\Lambda/T} &= \frac{28}{\pi^4} \delta_0(\omega_c/T) - \frac{113}{4\pi^2} \langle \alpha_s G^2 \rangle + \frac{5 m_2^4}{16} \langle q\bar{q} \rangle^2 e^{-\frac{m_2^2}{T^2}} , \\
-\lambda_1^{\Lambda K_1} F^2 e^{-\Lambda/T} &= \frac{28}{\pi^4} \delta_0(\omega_c/T) - \frac{37}{\pi^2} \langle \alpha_s G^2 \rangle + \frac{5 m_2^4}{8} \langle q\bar{q} \rangle^2 e^{-\frac{m_2^2}{T^2}} , \\
-\lambda_1^{\Sigma K_1} F^2 e^{-\Lambda/T} &= \frac{28}{\pi^4} \delta_0(\omega_c/T) - \frac{21}{8\pi} \langle \alpha_s G^2 \rangle + \frac{3 m_2^4}{64} \langle q\bar{q} \rangle^2 e^{-\frac{m_2^2}{T^2}} , \\
-\lambda_1^{\Lambda K_0} F^2 e^{-\Lambda/T} &= \frac{28}{\pi^4} \delta_0(\omega_c/T) - \frac{17}{6\pi} \langle \alpha_s G^2 \rangle + \frac{3 m_2^4}{64} \langle q\bar{q} \rangle^2 e^{-\frac{m_2^2}{T^2}} , \\
-\lambda_1^{\Lambda K_1} F^2 e^{-\Lambda/T} &= \frac{28}{\pi^4} \delta_0(\omega_c/T) - \frac{5}{32} \langle \alpha_s G^2 \rangle + \frac{3 m_2^4}{32} \langle q\bar{q} \rangle^2 e^{-\frac{m_2^2}{T^2}} .
\end{align*}
\]

The unitary normalization of flavor matrix $Tr[\tau \tau^+] = 1$ has been applied to get those sum rules, as what has been done for the two-point sum rules.

### III. NUMERICAL RESULTS AND CONCLUSIONS

In the following analysis, the standard value for the condensates are adopted [3]. From two-point sum rules the effective mass can be obtained via a derivative of the Borel parameter as $\Lambda = T^2 d\ln E/dT$, where $E$ denotes the right hand side of the obtained two-point sum rule. Then comply to the standard procedure of sum rule analysis, we changed the continuum threshold $\omega_c$ and Borel parameter $T$ to find the optimal stability window and the numerical value of the effective mass will be determined within this window. For those sum rules obtained above, we found the typical value for the continuum threshold is $\omega_c \sim 1.6$ GeV and typical interval for the Borel parameter is $\Delta T \sim 0.4$ GeV, which is a narrower one than the window for the ground state baryon. The only exception of this assertion is $\Lambda_{QK_0}$, for which we found a much
lower continuum threshold \( \omega_c \sim 1.2 \) GeV and a narrower interval \( \Delta T \sim 0.3 \) GeV. Also it is worth noting that for \( \Lambda_{QK0} \), the main contribution does not come from the perturbative part. The condensate ones play an important role in the determination of stability window. Indeed, if one keeps only gluon condensate contribution, there will be no stability window at all, just like the case in Ref.\[14\]. But if assuming the perturbative dominance and omitting condensate contributions, we only ended up with a better stability and the numerical value is almost exactly the same. For the other sum rules for the effective mass the case is different. The dominant contribution to those sum rules comes from the perturbative part, and the dimension 4 and dimension 6 operators in the OPE only amount to 20% and 10% within the stability windows, respectively. The numerical results are presented in Fig. 3. For the aim of clarity we give our numerical average for the effective masses in Table \[II\]

In order to get the numerical results for two \( 1/m_Q \) order parameters, we divide our three-point sum rules by two-point functions to obtain \( \lambda_1 \) and \( \lambda_2 \) as functions of the continuum threshold \( \omega_c \) and the Borel parameter \( T \). This procedure can eliminate the systematic uncertainties and cancel the parameter \( \bar{\Lambda} \). From the experiences of QCD sum rule applications in the field of heavy quark physics it is well known that three-point sum rule receives heavier contamination from the continuum modes than two-point one, and the stability is not as good as that for two-point sum rule\[10, 13, 17, 21\]. Our results for three-point sum rule receives heavier contamination from the continuum modes than two-point one, and the experiences of QCD sum rule applications in the field of heavy quark physics it is well known that those excited heavy baryons are shown in Fig. 3 and in Fig. 4 results for \( T \) parameter rules by two-point functions to obtain \( \lambda \) windows for those sum rules we would like to state some facts. For the \( \omega \) results are listed in Table I, too.

The splitting between spin 1/2 and spin 3/2 doublet can be obtained by multiplying \( \lambda_2 \) by a factor

| \( \Lambda_{QK0} \) | \( \Sigma_{QK1} \) | \( \Lambda_{QK1} \) | \( \Lambda_{QK1} \) | \( \Sigma_{QK1} \) | \( \Sigma_{QK0} \) |
|----------------|----------------|----------------|----------------|----------------|----------------|
| \( \bar{\Lambda} \) | 1.04 ± 0.17 | 1.12 ± 0.22 | 1.36 ± 0.18 | 1.30 ± 0.13 | 1.27 ± 0.11 | 1.21 ± 0.09 |
| \( -\lambda_1 \) | 1.20 ± 0.26 | 1.02 ± 0.25 | 0.84 ± 0.15 | 1.16 ± 0.11 | 1.48 ± 0.18 | 1.42 ± 0.25 |
| \( \lambda_2 \) | 0.13 ± 0.03 | 0.11 ± 0.03 | 0.09 ± 0.01 | 0.13 ± 0.01 | 0.13 ± 0.01 | 0.13 ± 0.01 |

TABLE I: Effective mass in GeV, kinetic energy and chromo-magnetic interaction energy in GeV\(^2\) for excited heavy baryons. Errors quoted are due to the variation of the Borel parameter \( T \) and continuum threshold \( \omega_c \).

With those values and heavy quark masses given in \[13\], \( m_c = 1.41 \) GeV and \( m_b = 4.77 \) GeV, masses of excited heavy baryons to order \( 1/m_Q \) can be obtained immediately. We give those masses in Table \[II\].

Our results are comparable to the prediction on the excited heavy baryon masses obtained by using quark potential model\[22\].

| \( \Lambda_{QK0} \) | \( \Lambda_{QK1} \) | \( \Lambda_{QK1} \) | \( \Sigma_{QK1} \) | \( \Lambda_{QK1} \) | \( \Lambda_{QK1} \) | \( \Sigma_{QK1} \) | \( \Sigma_{QK1} \) | \( \Sigma_{QK0} \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( Q=c \) | 2.863 | 3.019 | 3.076 | 2.831 | 2.900 | 3.083 | 3.129 | 3.148 | 3.219 | 3.113 |
| \( Q=b \) | 5.934 | 6.205 | 6.222 | 5.977 | 5.998 | 6.185 | 6.199 | 6.182 | 6.203 | 6.127 |

TABLE II: Masses in GeV for excited heavy baryons.
$3/2$, cf. Eq. 4. It is $0.13 \pm 0.02 \text{GeV}^2$, $0.16 \pm 0.05 \text{GeV}^2$, $0.20 \pm 0.02 \text{GeV}^2$ and $0.19 \pm 0.04 \text{GeV}^2$ for $\Lambda_{QK1}$, $\Lambda_{Qk1}$, $\Sigma_{QK1}$ and $\Sigma_{Qk1}$ doublets, respectively. The approximately equal value for $\Lambda_{QK1}$ and $\Lambda_{Qk1}$, $\Sigma_{QK1}$ and $\Sigma_{Qk1}$ may be interpreted as the signal which implies that current mixing effect cannot be large. If taking the middle value as theoretical predictions for the physical state, then the splitting for excited baryon states are

$$\Lambda_{Q1}^* - \Lambda_{Q1} = 0.15 \pm 0.03 \text{GeV}^2,$$
$$\Sigma_{Q1}^* - \Sigma_{Q1} = 0.20 \pm 0.03 \text{GeV}^2. \quad (12)$$

Based on the current experimental data, the splitting of excited $\Lambda_c$ doublet is $0.17 \text{GeV}^2$, which is in agreement with our theoretical result. When it is scaled up to the bottom quark mass scale there will be a factor $\sim 0.8$ approximately due to the renormalization group improvement.

To conclude, we have calculated the $1/m_Q$ order corrections to the excited heavy baryon masses from QCD sum rules within the framework of the HQET. From thus obtained spectrum for the $c$ quark case, we found that $\Lambda_{QK0}$ and $\Sigma_{QK1}$ baryons lie $\sim 600 \text{MeV}$ above the ground state baryon $\Lambda_c$, while $\Lambda_{QK1}$, $\Lambda_{QK1}$, $\Sigma_{QK1}$ and $\Sigma_{Qk1}$ baryon this value is $\sim 900 \text{MeV}$, typically with an error $\sim 300 \text{MeV}$. When it comes to the $b$ quark case, the result is that $\Lambda_{QK0}$ and $\Sigma_{QK1}$ baryons lie $\sim 300 \text{MeV}$ above the ground state baryon $\Lambda_b$, while $\Lambda_{QK1}$, $\Lambda_{QK1}$, $\Sigma_{QK1}$ and $\Sigma_{Qk1}$ lie $\sim 550 \text{MeV}$ above $\Lambda_b$, for which the typical error is $\sim 200 \text{MeV}$. For the $c$ quark case, $1/m_Q$ order corrections are $\sim 400 \text{MeV}$, which is not a small one due to the large value of the kinetic energy. When it comes to the $b$ quark case those corrections will be suppressed by the still larger $b$ quark mass. Our theoretical predictions for the doublet splitting are in agreement with the current experimental data.

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[1] H. Albrecht et al., ARGUS collaboration, Phys. Lett. B 317, 227 (1993); Phys. Lett. B 402, 207 (1997); P.L. Frabetti et al., E687 collaboration, Phys. Rev. Lett. 72, 961 (1994); Phys. Lett. B 365, 461 (1996); K.W. Edwards et al., CLEO collaboration, Phys. Rev. Lett. 74, 3331 (1995).
[2] J.P. Alexander et al., CLEO Collaboration, Phys. Rev. Lett. 83, 3390 (1999).
[3] B. Grinstein, Nucl. Phys. B 339, 253 (1990); E. Eichten and B. Hill, Phys. Lett. B 234, 511 (1990); A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. B343, 1 (1990); F. Hussain, J. G. Körner, K. Schilcher, G. Thompson, and Y. L. Wu, Phys. Lett. B 249, 295 (1990); J. G. Körner and G. Thompson, *ibid.* 264, 185 (1991).
[4] M. Neubert, Phys. Rep. 245, 259 (1994).
[5] J.G. Körner, M. Krämer, and D. Pirjol, Prog. Part. Nucl. Phys. 33, 787 (1994).
[6] A.F. Falk and M. Neubert, Phys. Rev. D 47, 2965 and 2982 (1993).
[7] I.I. Bigi, N.G. Uraltsev and A.I. Vainshtein, Phys. Lett. B 293, 430 (1992); I.I. Bigi, M. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. Lett. 71, 496 (1993); A.V. Manohar and M.B. Wise, Phys. Rev. D 49, 1310 (1994); M. Luke and M.J. Savage, Phys. Lett. B 321, 88 (1994); M. Neubert and C. T. Sachrajda, Nucl. Phys. B483, 339 (1997).
[8] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147, 385 (1979); B147, 448 (1979); V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Fortschr. Phys. 32, 11 (1984).
[9] A recent review on the QCD sum rule method, see P. Colangelo and A. Khodjamirian, *At the Frontier of Particle Physics/Handbook of QCD*, edited by M. Shifman (World Scientific, Singapore, 2001), pp. 1495-1576, hep-ph/0010175.

[10] A.G. Grozin and O.I Yakovlev, Phys. Lett. B 285, 254 (1992); 291, 441 (1992).

[11] S. Groote, J.G. Körner, and O.I. Yakovlev, Phys. Rev. D 55, 3016 (1997).

[12] Y.-B. Dai, C.-S. Huang, C. Liu, and C.-D Liu, Phys. Lett. B 371, 99 (1996).

[13] D.-W. Wang, M.-Q. Huang, and C.-Z. Li, Phys. Rev. D 65, 094036 (2002).

[14] C.-S. Huang, A.-L. Zhang, and S.-L. Zhu, Phys. Lett. B 492, 288 (2000).

[15] S.-L. Zhu, Phys. Rev. D 61, 114019 (2000); J.P. Lee, C. Liu and H.S. Song, Phys. Lett. B 476, 303 (2000); M.-Q. Huang, J.P. Lee, C. Liu and H.S. Song, *ibid.* 502, 133 (2001).

[16] M.A. Ivanov, J.G. Körner, V.E. Lyubovitskij, M.A. Pisarev, and A.G. Rusetsky, Phys. Rev. D 61, 114010 (2000).

[17] D.-W. Wang and M.-Q. Huang, Phys. Rev. D 67, 074025 (2003).

[18] M. Neubert, Phys. Rev. D 45, 2451 (1992); 46, 3914 (1992).

[19] Y.-B. Dai, C.-S. Huang, M.-Q. Huang, and C. Liu, Phys. Lett. B387, 379 (1996).

[20] M. Neubert, Phys. Rev. D 46, 1076 (1992).

[21] P. Ball and V.M. Braun, Phys. Rev. D 49, 2472 (1994); M. Neubert, Phys. Lett. B389, 727 (1996).

[22] L.A. Copley, N. Isgur, and G. Karl, Phys. Rev. D 20, 768 (1979); S. Capstick and N. Isgur, *ibid.* 34, 2809 (1986).

[23] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D 66, 010001 (2002).
Figure Captions

Fig. 1. Non-vanishing diagrams for the kinetic energy. The kinetic energy operator is denoted by a white square, the interpolating baryon currents by black circles. Heavy-quark propagators are drawn as double lines.

Fig. 2. Sum rules of the effective mass $\bar{\Lambda}$ for: (a) $\Lambda_{Qk0}$, (b) $\Lambda_{Qk1}$, (c) $\Sigma_{Qk1}$, (d) $\Lambda_{QK1}$, (e) $\Sigma_{QK1}$ and (f) $\Sigma_{QK0}$ baryons. The different choices of the continuum threshold $\omega_c$ corresponding to different curves are designated in individual figures respectively. Curves are plotted against the Borel parameter $T$.

Fig. 3. Sum rules of the kinetic energy for: (a) $\Lambda_{Qk0}$, (b) $\Lambda_{Qk1}$, (c) $\Sigma_{Qk1}$, (d) $\Lambda_{QK1}$, (e) $\Sigma_{QK1}$ and (f) $\Sigma_{QK0}$ baryons. Others are the same as those in Fig. 2.

Fig. 4. Sum rules of the chromo-magnetic interaction for: (a) $\Lambda_{Qk1}$, (b) $\Sigma_{Qk1}$, (c) $\Lambda_{QK1}$ and (d) $\Sigma_{QK1}$ baryons. Others are the same as those in Fig. 2.
Fig. 1.

Fig. 2.
Fig. 3.

Fig. 4.