Tetrad gravity, electroweak geometry and conformal symmetry

Daniel Canarutto

Dipartimento di Matematica Applicata “G. Sansone”,
Via S. Marta 3, 50139 Firenze, Italia
email: daniel.canarutto@unifi.it
http://www.dma.unifi.it/~canarutto

v2 – 2 November 2010

Abstract

A partly original description of gauge fields and electroweak geometry is proposed. A discussion of the breaking of conformal symmetry and the nature of the dilaton in the proposed setting indicates that such questions cannot be definitely answered in the context of electroweak geometry.

2010 MSC: 53C07, 53Z05, 81Q99, 81R25, 81R40.
Keywords: Tetrad gravity, 2-spinors, electroweak geometry, conformal symmetry, dilaton

Contents

1 Mathematical preliminaries 2
  1.1 Tangent-valued forms, brackets and connections . . . . . . . . . . . . . . . . . . 2
  1.2 Hermitian spaces . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4
  1.3 Unit spaces and scaling . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4

2 Electrodynamics 6
  2.1 Two-spinors and Lorentzian geometry . . . . . . . . . . . . . . . . . . . . . . . 6
  2.2 Two-spinors and Einstein-Cartan-Maxwell-Dirac fields . . . . . . . . . . . . . 8
  2.3 Tetrad and QED interactions . . . . . . . . . . . . . . . . . . . . . . . . . . . . 8

3 Gauge fields 9
  3.1 Gauge fields in classical and pre-quantum theories . . . . . . . . . . . . . . . . 9
  3.2 Gauge fields and charges . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 11

4 Electroweak field theory 13
  4.1 The fermion bundle . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 13
  4.2 The fields . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 14
  4.3 Symmetry breaking . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 15
  4.4 Electroweak geometry and the dilaton . . . . . . . . . . . . . . . . . . . . . . . 17
Introduction

Several papers discussing electroweak geometry, and proposing modifications, have appeared even recently [Fa08, FKD09, LM94, ILM10, MT10, Mo10, RySh09, Ta05]. Actually the geometric background of the EW theory, in its current formulation, lacks the tidiness of the Dirac theory of spinors. One can see various unsatisfactory features: the meaning of the mixing angle, the origin of the potential term related to symmetry breaking, the need for various ad hoc choices. Furthermore, from a physical point of view, at least one crucial aspect of the electroweak theory (the existence of the Higgs particle) is still waiting for experimental confirmation, and some parameters still have to be precisely measured.

In this paper I attempt to give a partly original presentation of electroweak geometry, in the hope that some features can be seen more clearly by trying and somewhat changing the roles of various objects.

In §1 I’ll briefly review some basic mathematical notions which, although by now widely discussed in the literature, cannot be seen as standard knowledge; in particular, §1.3 contains a “not too short” discussion of the unit spaces approach to scaling. In §2 I’ll sketch the main ideas of a certain two-spinor formulation of electrodynamics in curved spacetime [C00b, C07] which I regard as specially neat from a geometrical point of view.

Those ideas suggest a somewhat non-standard general approach, discussed in §3, to gauge fields and their relation to the classical geometry underlying a field theory. Finally, in §4, I apply those ideas to a partly original formulation of electroweak geometry. The different role of spin with respect to other internal degrees of freedom is discussed, and the Higgs field is seen to arise naturally in this context.

In §4.4 I discuss the question of the breaking of conformal invariance and the nature of the dilaton in connection to electroweak geometry and to the Higgs field in particular, briefly commenting about some ideas which have appeared in the literature; here I argue that the question cannot actually be given a definite, convincing answer in the context of electroweak geometry: some substantial extension is needed.

By the way, it should be noted that the geometric language used here is somewhat less group-oriented than standard approaches (though eventually the two languages are essentially equivalent).

1 Mathematical preliminaries

1.1 Tangent-valued forms, brackets and connections

We’ll deal essentially with smooth finite-dimensional manifolds and bundles, and smooth maps. By T and V we denote the tangent and vertical functors.

The Frölicher-Nijenhuis algebra of tangent-valued forms provides us with a general framework for connections and related topics; though based on firmly established results in the literature [FN56, FN60, Mo91, MK, Mi01], this framework may be less familiar to the reader than the more usual language of principal bundles; hence, a brief account of the basics could be appropriate.

A tangent valued r-form on a manifold $M$, $r \in \{0\} \cup \mathbb{N}$, is a (local) smooth section $M \to \wedge^r T^*M \otimes_M TM$. The sheaf of all tangent valued forms has a natural structure of a graded Lie algebra determined by the Frölicher-Nijenhuis bracket. If $\phi$ is a t.v. $r$-form and $\psi$ is a t.v. $s$-form then their FN bracket $[\phi, \psi]$ is the t.v. $r+s$-form which has the coordinate
1.1 Tangent-valued forms, brackets and connections

expression\(^1\)

\[
[\phi, \psi]_{a_1...a_{r+3}}^b = (\phi^c_{a_1...a_r} \partial_c \psi^b_{a_{r+1}...a_{r+3}} - (-1)^{rs} \psi^c_{a_1...a_r} \partial_c \phi^b_{a_{r+1}...a_{r+3}} - r \phi^b_{a_1...a_{r-1}c} \partial_a \psi^c_{a_{r+1}...a_{r+3}} + (-1)^{rs} s \psi^b_{a_1...a_{r-1}c} \partial_a \phi^c_{a_{r+1}...a_{r+3}}) .
\]

The Frölicher-Nijenhuis algebra on a fibered manifold \( E \rightarrow M \) has a special interest. In general this is very complicated, however one deals mostly with the subalgebra of projectable forms. A tangent valued \( r \)-form \( \phi \) on \( E \) is said to be basic if it is a section

\[
\phi : E \rightarrow \wedge^r T^*M \otimes \mathcal{E} \subset \wedge^r T^*E \otimes \mathcal{E} ,
\]

and projectable over \( \phi : M \rightarrow \wedge^r T^*M \otimes_M TM \) if it is basic and the diagram

\[
\begin{array}{ccc}
E & \xrightarrow{\phi} & \wedge^r T^*M \otimes \mathcal{E} \\
\downarrow & & \downarrow \\
M & \xrightarrow{\phi} & \wedge^r T^*M \otimes_M TM
\end{array}
\]

commutes. The FN bracket of two projectable tangent valued forms turns out to be projectable. In particular, vertical-valued basic forms \( E \) commutes. The FN bracket of two projectable tangent valued forms turns out to be projectable. In particular, a linear connection is characterized in linear

\[
\nabla \gamma = d\gamma + \gamma^a \partial_a \gamma
\]

where

\[
\gamma = \alpha \otimes u, \beta \otimes v = \alpha \wedge (u, \beta) \otimes v - (v, \alpha) \wedge \beta \otimes u,
\]

and \( [\alpha \otimes u, \beta \otimes v] = \alpha \wedge \beta \otimes [u, v] + \alpha \wedge (u, \beta) \otimes v - (v, \alpha) \wedge \beta \otimes u \)

where \( \alpha : M \rightarrow \wedge T^*M, \beta : M \rightarrow \wedge T^*M, u, v : M \rightarrow TM \) and \([u, v]\) is the Lie bracket of \( u \) and \( v \).

\(^1\)The FN bracket can be characterized, in coordinate-free language, by
1.2 Hermitian spaces

If $V$ is a complex vector space of finite-dimension $n$, then we denote its dual and antidual spaces respectively as $V^\star$ and $V^\bar{x}$, mutually anti-isomorphic via the correspondence\(^2\) $\lambda \mapsto \bar{\lambda}$. Moreover we indicate as $\bar{V} := V^\bar{x}$ the conjugate space of $V$. Up to natural isomorphisms one gets only four distinct spaces $V \leftrightarrow \bar{V}$, $V^\star \leftrightarrow V^\bar{x} \cong \bar{V}^\star$ (the arrows indicate the conjugation anti-isomorphisms). Accordingly we use four different index types,\(^3\) with ‘dotted’ indices referring to ‘conjugated’ spaces $\bar{V}$ and $\bar{V}^\star$.

The space $V \otimes \bar{V}$ has a natural real linear (complex anti-linear) involution $w \mapsto w^\dagger$, which on decomposable tensors reads $(u \otimes \bar{v})^\dagger := v \otimes \bar{u}$. Hence one has the natural decomposition of $V \otimes \bar{V}$ into the direct sum of the real eigenspaces of the involution with eigenvalues $\pm 1$, respectively called the Hermitian and anti-Hermitian subspaces, namely

$$V \otimes \bar{V} = H(V \otimes \bar{V}) \oplus i H(V \otimes \bar{V}).$$

In other terms, the Hermitian subspace $H(V \otimes \bar{V})$ is constituted by all $w \in V \otimes \bar{V}$ such that $w^\dagger = w$, while an arbitrary $w$ is uniquely decomposed into the sum of an Hermitian and an anti-Hermitian tensor as

$$w = \frac{1}{2}(w + w^\dagger) + \frac{1}{2}(w - w^\dagger).$$

Then $w = w^{A\bar{B}} b_A \otimes \bar{b}_B$ is Hermitian (anti-Hermitian) iff the matrix $(w^{A\bar{B}})$ of its components is such, namely $\bar{w}^{\bar{B}A} = \mp w^{A\bar{B}}$. One has the natural isomorphism $[H(V \otimes \bar{V})]^{\star} \cong H(V^\star \otimes \bar{V}^\star)$, where $\star$ denotes the real dual.

A Hermitian 2-form is defined to be a Hermitian tensor $h \in H(V^\star \otimes V^\star)$. The associated quadratic form $v \mapsto h(v, v)$ is real-valued. The notions of signature and non-degeneracy of Hermitian 2-forms are introduced similarly to the case of real bilinear forms. If $h$ is non-degenerate then it yields the isometries $h^\#: \bar{V} \to V^\star : \bar{v} \mapsto h(\bar{v}, \_)$ and $\bar{h}^\#: V \to \bar{V}^\star : v \mapsto h(\_, v)$. The inverse isometries are denoted as $h^\#$ and $\bar{h}^\#$.

1.3 Unit spaces and scaling

An algebraically precise treatment of physical scales was introduced around 1995 after an idea of M. Modugno, and has been systematically used, since then, in papers of various authors [CJM95, JM02, JM06, JMV10, MST05, SV00, Vi99, Vi00]. The basic notion is that of a positive space (or scale space, or unit space), namely a 1-dimensional ‘semi-vector space’ without the zero element. Though physical scales (or ‘dimensions’) are usually dealt with in an ‘informal’ way, without a precise mathematical setting, the notion of a scale space arises naturally from simple arguments. The distance of two points in Euclidean space, for example, can be expressed as a real number only if a length unit has been fixed, and the set $L$ of lengths is naturally endowed with a free and transitive left action $\mathbb{R}^+ \times L \to L$; this determines an algebraic structure of semi-vector space over $\mathbb{R}^+$ (note that $L$ has no distinguished element).

A rigorous study of this matter [JMV10] turns out to be more delicate than one may expect at first sight, but the basic notions needed for “everyday use” can be easily sketched.\(^2\) $V^\star$ is the space of all antilinear maps $V \to \mathbb{C}$. In general, if $f$ is any function then $f^\#: x \mapsto \overline{f(x)}$.

\(^3\) Let $(b_A)$, $1 \leq A \leq n$, be a basis of $V$ and $(b^\dagger_A)$ its dual basis of $V^\star$. Then we have the induced bases $(\bar{b}_A)$ of $\bar{V}$ and $(\bar{b}^\dagger_A)$ of $\bar{V}^\star$, with $\bar{b}_A := \overline{b_A}$ and the like. For $v \in V$, for example, we have $v = \bar{v}^\dagger b_A$ and $\bar{v} = \bar{v}^\dagger b_A$, with $\bar{v}^\dagger = \overline{v^\dagger}$. The conjugation morphism can be extended to tensors of any rank and type, exchanging dotted and non-dotted index types. Observe that dotted indices cannot be contracted with non-dotted indices. In particular if $K \in \text{Aut}(V) \subset V \otimes V^\star$ then $K \in \text{Aut}(\bar{V}) \subset \bar{V} \otimes \bar{V}^\star$ is the induced conjugated transformation (under a basis transformation, dotted indices transform with the conjugate matrix).
A semi-vector space is defined to be a set $A$ equipped with an addition map $A \times A \to A$ and a multiplication map $\mathbb{R}^+ \times A \to A$, fulfilling the usual axioms of vector spaces except those properties which involve opposites and the zero element. Then, in particular, any vector space is a semi-vector space, and the set of linear combinations over $\mathbb{R}^+$ of $n$ fixed independent vectors in a vector space is a semi-vector space. Semi-linear maps between semi-vector spaces are defined in an obvious way; in particular we obtain the semi-dual space $A^*$ (or simply the ‘dual space’) of any semi-vector space.

A semi-vector space $U$ is called a positive space if the multiplication $\mathbb{R}^+ \times U \to U$ is a transitive left action of the multiplicative group $\mathbb{R}^+$ on $U$ (then a positive space cannot have a zero element). If $b \in U$ then any other element $u \in U$ can be written as $u^0 b$ with $u^0 \in \mathbb{R}^+$. Quite naturally we can write $u^0 \equiv u \cdot b$, that is $u = (u/b) b$, $(u/b) \in \mathbb{R}^+$. So we might also say that a positive space is a ‘1-dimensional’ semi-vector space.

Several concepts and results of standard linear and multi-linear algebra related to vector spaces can be easily repeated for semi-vector spaces and positive semi-vector spaces (including linear and multi-linear maps, bases, dimension, tensor products and duality). The main caution to be taken is to avoid formulations which involve the zero element. In particular, one can define the tensor product (over $\mathbb{R}^+$) of semi-vector spaces; the tensor product of a semi-vector space and a real or complex vector space becomes naturally also a vector space.

A 1-dimensional semi-vector space will be called a unit space. In particular, let $U$ be a positive unit space; then the trace yields the natural identification $U \otimes U^* \cong \mathbb{R}^+$. Moreover for any $m \in \mathbb{N}$ there is, up to isomorphism, a unique $m$-root of $U$; this is a positive unit space, denoted as $U^{1/m}$, with the property $\otimes^m(U^{1/m}) \cong U$. Introducing the notation

\[
U^{p/m} := \otimes^p(U^{1/m}) \cong (\otimes^p U)^{1/m}, \quad p \in \mathbb{N},
\]

\[
U^{-1} := U^*,
\]

one obtains the definition of $U^{r}$ for all rational exponents $r$.

Accordingly, one may adopt a ‘number-like’ notation for elements of unit spaces. Namely one writes $u^{-1} := u^* \in U^{-1}$ (the dual element) and $uv := u \otimes v$ for $u \in U$, $v \in V$. This allows a rigorous treatment of measure units in physics, maintaining a notation close to the traditional one.

In many physical theories it is convenient to assume the spaces $T$ of time scales, $L$ of length scales and $M$ of mass scales as the basic spaces of scales. An arbitrary scale space is defined to be a positive space of the type $S = T^{d_1} \otimes L^{d_2} \otimes M^{d_3}$, with $d_i \in \mathbb{Q}$. An element $s \in S$ is also called a scale, or a unit of measurement.

A ‘scaled’ version of a vector bundle $E \to B$ is a fibered tensor product $S \otimes E \to B$. Fibered linear algebraic or differential operations on $E$ determine analogue scaled operations. In particular, a linear connection $\Gamma$ of $E \to B$ determines a linear connection of $S \otimes E \to B$.

Two sections $\sigma : B \to E$ and $\sigma' : B \to S \otimes E$ of differently scaled vector bundles can be compared if we avail of a scale factor $s : B \to S$, called a coupling scale, or possibly a coupling constant. The commonest coupling constants are the speed of light $c \in T^{-1} \otimes L$; Planck’s constant $h \in T^{-1} \otimes L^2 \otimes M$; Newton’s gravitational constant $\epsilon \in T^{-2} \otimes L^3 \otimes M^{-1}$; the positron charge $e \in T^{-1} \otimes L^{3/2} \otimes M^{1/2}$; a particle’s mass $m \in M$. By viewing $c$ as an isomorphism $T \to L$ and then $h$ as an isomorphism $M \to L^*$ one can express all physical scales as powers of $L$ (this is the familiar ‘natural units’ setting, usually introduced by the condition $c = h = 1$).

Certain scale spaces arise quite naturally. If $X$ is an $n$-dimensional real vector space then the choice of an orientation amounts to the choice of a positive subspace of $\wedge^n X$. In the context
of complex geometry we have another interesting construction: if $Z$ is a 1-dimensional complex vector space then the Hermitian subspace $H(Z \otimes \overline{Z}) \subset Z \otimes \overline{Z}$ (§1.2) is a 1-dimensional real vector space which has the distinguished positive subspace

$$H(Z \otimes \overline{Z})^+ := \{ z \otimes \overline{z} : z \in Z \setminus \{0\} \}.$$ 

Thus, any $n$-dimensional complex vector space $V$ yields the positive space $H(\wedge^n V \otimes \wedge^n \overline{V})^+.$

The metric, either Euclidean or Lorentzian, is appropriately described as a scaled tensor field. Focusing our attention on the spacetime $(M, g)$ of General Relativity, the metric is a section

$$g : M \rightarrow \mathbb{L}^2 \otimes T^*M \otimes T^*M,$$

so that the scalar product of vectors is valued into $\mathbb{R} \otimes \mathbb{L}^2 \equiv \mathbb{R} \otimes \mathbb{L} \otimes \mathbb{L}.$ Correspondingly, the volume form induced by $g$ is a scaled 4-form $\eta : M \rightarrow \mathbb{L}^4 \otimes \wedge^4(T^*M).$ Note that $g$ and $\eta$ can also be seen as unscaled objects on the fibers of $H \equiv \mathbb{L}^{-1} \otimes TM \rightarrow M.$

The notion of a positive space can be extended to that of a positive bundle over $M.$ This will be the natural context in which one may address the questions of running constants, conformal symmetry and the like (§4.4). By the way, we observe that the assignment of an orientation of $M$ amounts to the choice of a positive bundle

$$\nabla M \equiv (\wedge^4 TM)^+ \subset \wedge^4 TM,$$

so that $\eta : M \rightarrow \mathbb{L}^4 \otimes \nabla^{-1}M.$ A section $M \rightarrow \mathbb{R} \otimes \nabla^{-1/2}M$ is called a half-density.

## 2 Electrodynamics

### 2.1 Two-spinors and Lorentzian geometry

The geometry of Dirac fields can be conveniently expressed in the language of 2-spinors. In previous papers [C98, C00b, C07] I treated such matters according a partly original approach which uses minimal geometric assumptions; this approach has a number of differences with the classical Penrose formalism [PR84, PR88], and includes an integrated treatment of Einstein-Cartan-Maxwell-Dirac fields (see also [HCMN95]). The starting point is the realization that a classical Penrose formalism [PR84, PR88], and includes an integrated treatment of Einstein-Cartan-Maxwell-Dirac fields (see also [HCMN95]). The starting point is the realization that a classical Penrose formalism [PR84, PR88], and includes an integrated treatment of Einstein-Cartan-Maxwell-Dirac fields (see also [HCMN95]). The starting point is the realization that a classical Penrose formalism [PR84, PR88], and includes an integrated treatment of Einstein-Cartan-Maxwell-Dirac fields (see also [HCMN95]). The starting point is the realization that a classical Penrose formalism [PR84, PR88], and includes an integrated treatment of Einstein-Cartan-Maxwell-Dirac fields (see also [HCMN95]). The starting point is the realization that a classical Penrose formalism [PR84, PR88], and includes an integrated treatment of Einstein-Cartan-Maxwell-Dirac fields (see also [HCMN95]). The starting point is the realization that a classical Penrose formalism [PR84, PR88], and includes an integrated treatment of Einstein-Cartan-Maxwell-Dirac fields (see also [HCMN95]). The starting point is the realization that a classical Penrose formalism [PR84, PR88], and includes an integrated treatment of Einstein-Cartan-Maxwell-Dirac fields (see also [HCMN95]).

- **The** Hermitian subspace of $\wedge^2 S \otimes \wedge^2 \overline{S}$ is a real 1-dimensional vector space with a distinguished orientation; its positively oriented semispace $\mathbb{L}^2$ (whose elements are of the type $w \otimes \overline{w}, w \in \wedge^2 S$) has the square root semispace $\mathbb{L},$ which will can be identified with the space of length units.

- **The** $2$-spinor space is defined to be $U := \mathbb{L}^{-1/2} \otimes S.$ The space $\wedge^2 U$ is then naturally endowed with a Hermitian metric, defined as the identity element in

$$H((\wedge^2 U^*) \otimes (\wedge^2 U^*)) \cong \mathbb{L}^2 \otimes H((\wedge^2 S^*) \otimes \wedge^2 S^*),$$

so that normalized ‘symplectic forms’ $\varepsilon \in \wedge^2 U^*$ constitute a $U(1)$-space (any two of them are related by a phase factor). Each $\varepsilon$ yields the isomorphism $\varepsilon^\#: U \rightarrow U^* : u \mapsto u^\#: = \varepsilon(u, \cdot).$

- The identity element in $H((\wedge^2 U^*) \otimes (\wedge^2 U^*))$ can be written as $\varepsilon \otimes \overline{\varepsilon}$ where $\varepsilon \in \wedge^2 U^*$ is any normalized element. This natural object can also be seen as a bilinear form $g$ on $U \otimes \overline{U},$ via the rule $g(p \otimes \overline{q}, r \otimes \overline{s}) = \varepsilon(p, r) \overline{\varepsilon}(q, s)$ extended by linearity. Its restriction to the Hermitian subspace $H \equiv H(U \otimes \overline{U})$ turns out to be a Lorentz metric. Null elements in $H$ are of the form $\pm u \otimes \overline{u}$ with $u \in U$ (thus there is a distinguished time-orientation in $H$).
• Let $W \equiv U \oplus \overline{U}^*$. The linear map $\gamma : U \otimes \overline{U} \to \text{End}(W) : y \mapsto \gamma(y)$ acting as

$$\tilde{\gamma}(p \otimes \bar{q})(u, \chi) = \sqrt{2}(\chi, \bar{q} \cdot p, \langle p^\gamma, \bar{u} \rangle)$$

is well-defined independently of the choice of the normalized $\varepsilon \in \wedge^2 U^*$ yielding the isomorphism $\varepsilon^\flat$. Its restriction to $H$ turns out to be a Clifford map. Thus one is led to regard $W \equiv U \oplus \overline{U}^*$ as the space of Dirac spinors, decomposed into its Weyl subspaces. The anti-isomorphism $W \to W^* : (u, \chi) \mapsto (\bar{\chi}, \bar{u})$ is called the Dirac adjunction ($\psi \mapsto \psi^\dagger$ in traditional notation), and is associated with a natural Hermitian structure $k \in W^* \otimes W^*$ which turns out to have the signature $(+, +, -)$.

The above constructions and results can be read in coordinates as follows. Consider an arbitrary basis $(\xi_\alpha)$ of $S$, $\alpha = 1,2$. This yields induced bases of the various associated spaces. In particular we consider the bases $l \in \mathbb{L}$ (a length unit), $(\zeta_\alpha) \equiv (I^{-1/2} \xi_\alpha) \subset U$, $\varepsilon \in \wedge^2 U^*$. We have $\varepsilon = \varepsilon_{AB} z^A \land z^B$, where $(z^A) \subset U^*$ is the dual basis of $(\zeta_\alpha)$ and $(\varepsilon_{AB})$ denotes the antisymmetric Ricci matrix.\footnote{In contrast to the usual 2-spinor formalism, no symplectic form is fixed. The 2-form $\varepsilon$ is unique up to a phase factor which depends on the chosen 2-spinor basis, and determines isomorphisms}

$$g(w, w) = \varepsilon_{AB} \bar{w}^A \varepsilon_{AB} = 2 \det w. $$

As for the basis of $H \equiv H(U \otimes \overline{U})$ associated with $(\zeta_\alpha)$ one usually considers the Pauli basis $(\tau_\lambda)$, given by $\tau_\lambda = \frac{1}{\sqrt{2}} \sigma^A_{\lambda} \zeta_A \otimes \zeta^*_A$, where $(\sigma^A_{\lambda})$, $\lambda = 0, 1, 2, 3$, denotes the $\lambda$-th Pauli matrix. This is readily seen to be orthonormal, namely $g_{\lambda\mu} \equiv g(\tau_\lambda, \tau_\mu) = 2 \delta^0_{\lambda}\delta^0_\mu - \delta_{\lambda\mu}$.

The associated Weyl basis of $W$ is defined to be the basis $(\zeta_\alpha)$, $\alpha = 1, 2, 3, 4$, given by

$$(\zeta_1, \zeta_2, \zeta_3, \zeta_4) := (\zeta_1, \zeta_2, -\bar{z}^1, -\bar{z}^2) .$$

Above, $\zeta_1$ is a simplified notation for $(\zeta_1, 0)$, and the like. Another important basis is the Dirac basis $(\zeta'_\alpha)$, $\alpha = 1, 2, 3, 4$, where

$$(\zeta'_1 \equiv \frac{1}{\sqrt{2}}(\zeta_1 - \zeta_3), \ z'_2 \equiv \frac{1}{\sqrt{2}}(\zeta_2 - \zeta_4), \ z'_3 \equiv \frac{1}{\sqrt{2}}(\zeta_1 + \zeta_3), \ z'_4 \equiv \frac{1}{\sqrt{2}}(\zeta_2 + \zeta_4).$$

Setting

$$\gamma_\lambda := \gamma(\tau_\lambda) \in \text{End}(W)$$

one recovers the usual Weyl and Dirac representations as the matrices $(\gamma_\lambda)$, $\lambda = 0, 1, 2, 3$, in the Weyl and Dirac bases respectively.

In standard expositions of electrodynamics one usually considers some other structures which, however, should not be seen as part of the basic assumptions but rather depend on further choices. In particular the assignment of a positive Hermitian metric on $U$ and consequently on $W$ (the related adjunction map is usually denoted as $\psi \mapsto \psi^\dagger$) is readily seen to be equivalent to the assignment of an observer, that is of a timelike future-oriented element in $H^* \equiv H(U^* \otimes U^*)$. Also parity is associated with the choice of an observer. Charge conjugation, on the other hand, is the anti-involution $(u, \chi) \mapsto (\varepsilon^\# \bar{\chi}, \varepsilon^\# \bar{u})$ associated with the choice of a normalized 2-form $\varepsilon$, and as such is unique up to a phase factor. Time reversal requires both an observer and a normalized symplectic form.

\footnote{In contrast to the usual 2-spinor formalism, no symplectic form is fixed. The 2-form $\varepsilon$ is unique up to a phase factor which depends on the chosen 2-spinor basis, and determines isomorphisms}

$$\begin{align*}
\varepsilon^b : U & \to U^* : u \mapsto u^b, \quad \langle u^b, v \rangle := \varepsilon(u, v) \quad \Rightarrow \quad (u^b)^A_B = \varepsilon_{AB} v^A, \\
\varepsilon^\# : U^* & \to U : \lambda \mapsto \lambda^\#, \quad \langle \mu, \lambda^\# \rangle := \varepsilon^{-1}(\mu, \lambda) \quad \Rightarrow \quad (\lambda^\#)^A_B = \varepsilon^{AB} \lambda_A .
\end{align*}$$

\footnote{In contrast to the usual 2-spinor formalism, no symplectic form is fixed. The 2-form $\varepsilon$ is unique up to a phase factor which depends on the chosen 2-spinor basis, and determines isomorphisms}
2.2 Two-spinors and Einstein-Cartan-Maxwell-Dirac fields

Let $S \mapsto M$ be a complex vector bundle with 2-dimensional fibers, and consider the vector bundles obtained by performing the above constructions fiberwise. A linear connection $\Gamma$ on $S$ determines linear connections on the associated bundles, and, in particular, connections $G$ of $\mathcal{L}, Y$ of $\wedge^2 U$ and $\hat{\Gamma}$ of $H$. Conversely $\Gamma$ can be expressed in terms of these as

$$\Gamma_{a\bar{b}} = (G_a + i Y_a)\delta^a_{\bar{b}} + \frac{i}{2} \hat{\Gamma}_{a\bar{b}A} = (G_a + i Y_a)\delta^a_{\bar{b}} + \hat{\Gamma}_{a\bar{b}A},$$

with $G_a, Y_a : M \to \mathbb{R}, \hat{\Gamma}_{a\bar{b}A} : M \to \mathbb{C}, \hat{\Gamma}_{a\bar{b}A} = 0$.

If $M$ is 4-dimensional then a tetrad (or soldering form) is defined to be a linear morphism $\Theta : TM \to \mathbb{L} \otimes H$. An invertible tetrad determines, by pull-back, a scaled Lorentz metric $\Theta^*g$ on $M$ and a metric connection of $TM \to M$, as well as a scaled Dirac morphism $\gamma \circ \Theta : TM \to \mathbb{L} \otimes \text{End} W$. The Dirac operator $\psi \mapsto \nabla \psi$ is defined by natural contractions in $\Theta \otimes (\gamma \nabla \psi)$, where $\Theta : \mathbb{L} \otimes H \to TM$ is the inverse morphism of $\Theta$.

In the above geometric environment we can formulate a non-singular field theory even if $\Theta$ is not required to be invertible everywhere [C98]. If the invertibility requirement is satisfied then the theory turns out to be essentially equivalent to the standard theory of Einstein-Cartan-Maxwell-Dirac fields, with some redefinition of the fundamental fields: these are now the 2-spinor connection $\Gamma$, the tetrad $\Theta$, the electromagnetic field $F : M \to \mathbb{L}^{-2} \otimes \wedge^2 H^*$ and the Dirac field $\psi : M \to \mathbb{L}^{-3/2} \otimes W$. The gravitational field is represented by the couple $(\Theta, \hat{\Gamma})$. The connection $G$ induced on $\mathbb{L} \to M$ is assumed to have vanishing curvature, $dG = 0$, so that we can find local charts such that $G_a = 0$; this amounts to ‘gauging away’ the conformal ‘dilaton’ symmetry. Coupling constants then arise as covariantly constants sections of $\mathbb{L}^r, r \in \mathbb{Q}$.

A natural unscaled Lagrangian density depending on the said fields can be now introduced. This uses the following observation: if $\xi : M \to \wedge^r T^* M \otimes \wedge^r H, r = 1, 2, 3$, then

$$\Theta^{(d-r)} \wedge \xi : M \to \mathbb{L}^{d-r} \otimes \wedge^4 T^* M \otimes \wedge^4 H, \quad \Theta^{(p)} \equiv \Theta \wedge \cdots \wedge \Theta \ (p \text{ factors});$$

by contraction with the volume form of the fibers of $H$ (determined by the Lorentz structure) we then get a scaled density $\tilde{\Theta}(\xi) : M \to \mathbb{L}^{d-r} \otimes \wedge^4 T^* M$. So we obtain a Lagrangian density $\mathcal{L} = \mathcal{L}_g + \mathcal{L}_{\text{em}} + \mathcal{L}_D$, with

$$\mathcal{L}_g = \frac{1}{8} \tilde{\Theta}(R[\hat{\Gamma}]), \quad \mathcal{L}_{\text{em}} = \left( \frac{1}{4} F^2 - \frac{1}{2} \tilde{\Theta}(dY \otimes F) \right) \eta,$$

$$\mathcal{L}_D = \frac{i}{\sqrt{2}} \tilde{\Theta}(\langle \bar{\psi}, \nabla \psi \rangle - \langle \nabla \bar{\psi}, \psi \rangle) - m \langle \bar{\psi}, \psi \rangle \eta.$$

The details of the above expressions and of the calculations of the Euler-Lagrange operator [C98, C00b] are not essential here. Eventually one gets the following field equations: the Einstein equation and the equation for torsion; the equation $F = 2\,dY$ (thus $Y$ is identified with the electromagnetic potential up to a charge factor) and the other Maxwell equation; the Dirac equation, also involving the torsion of the induced spacetime connection.

2.3 Tetrad and QED interactions

Generally speaking, perturbative QFT requires certain basic ingredients: time (namely the choice of some kind of observer), free-particle states and the quantum interaction; the latter, on turn, is formed out of a specifically quantum ingredient (a certain distribution on the bundle of particle momenta) and of the classical interaction deduced from the Lagrangian of the classical
field theory (essentially, a distinguished feature of the underlying finite-dimensional geometric structure). We are not going into many details here; rather we’ll make a few observations, about the classical part of the interaction, which will be relevant in the subsequent discussion.

Let \((M, g)\) be a Lorentzian spacetime and \(P_m \subset T^* M\) the subbundle over \(M\) whose fibers are the future hyperboloids (‘mass-shells’) corresponding to mass \(m \in \{0\} \cup \mathbb{L}^{-1}\). If \(p \in (P_m)_{x}, x \in M\), then

\[
W_x = W^+_p \oplus W^-_p, \quad W^\pm := \text{Ker}(\gamma[p^\#] \pm m),
\]

where \(p^\# \equiv g^\#(p) \in \mathbb{L}^{-2} \otimes T^* M\) is the contravariant form of \(p\). Thus one has 2-fibered bundles \(W^\pm_\bullet \rightarrow P_m \rightarrow M\), where

\[
W^\pm_\bullet := \bigsqcup_{p \in P_m} W^\pm_p \subset P_m \times M.
\]

We call \(W^+_m\) and \(\overline{W}^-_m\) the electron bundle and the positron bundle, respectively.\(^5\)

In standard presentations the classical interaction among electron, positron and electromagnetic field is read from the Dirac Lagrangian as the term

\[
- e \langle \bar{\psi}, \gamma[A] \psi \rangle \equiv -k(\bar{\psi}, \gamma[A] \psi),
\]

where \(e\) is the positron’s charge (see also §2.2). When an electromagnetic gauge is chosen (see also §3) then \(A\) can be viewed as a spacetime 1-form and, through the tetrad and the metric, as a section \(M \rightarrow \mathbb{L}^{-1} \otimes H\). Then we see that the classical interaction is related to a natural contraction \(\ell_{\text{int}} : \overline{W} \times_M H \times_M W \rightarrow \mathbb{C}\), that is a tensor field

\[
\ell_{\text{int}} : M \rightarrow \overline{W}^* \otimes H^* \otimes W^*, \quad \langle \ell_{\text{int}}, \bar{\phi} \otimes A \otimes \psi \rangle = -k(\bar{\phi}, \gamma[A] \psi).
\]

By using the Hermitian structure \(k\) of \(W\) (§2.1) and the metric \(g\) of \(H\) one can “raise indices” in \(\ell_{\text{int}}\), thus obtaining eight “clones” of this tensor, with different index types. In each type, a contravariant index corresponds to the creation of a particle, and a covariant index corresponds to annihilation. This mechanism (together with its quantum counterpart acting on particle momenta) essentially determines the allowed particle interactions.

**Remark.** A peculiar aspect of the resulting quantum theory is that the interactions can be written solely in terms of “internal” spinor variables: up to rearrangements of the index positions we may only have contractions of the types \(u^B A_{BG} \bar{u}^G\) and \(\bar{\chi}_B A^{BG} \chi_G\). Furthermore, also the momentum variables in the propagators can be expressed in terms of spinor variables. In other words, any Feynman diagram can be seen as “living” just in the space of the “internal” particle states, with the soldering form \(\Theta\) (the “tetrad”) eventually relating it to spacetime geometry.

### 3 Gauge fields

#### 3.1 Gauge fields in classical and pre-quantum theories

In a classical theory, “matter fields” and “gauge fields” are respectively described as a section and a connection of some bundle \(E \rightarrow M\) (where \(M\) is the spacetime manifold). If one assumes that the true physical meaning of the gauge fields is encoded in the curvature tensor, then one sees the connection itself as partly undetermined, in the sense that different connections can yield the same physical field. This is the essence of “gauge freedom”.

\(^5\) Electron and positron quantum states can be then respectively introduced as certain \(W^+_m\)-valued and \(\overline{W}^-_m\)-valued distributions on \(P_m\). Details can be seen in a previous paper [C05] where I presented some basic ideas regarding quantum interactions, and QED in particular.
In the corresponding theory of quantum particles the situation is different. In order to treat quantum interactions one has to “choose a gauge”, so that the gauge field becomes a section of a vector bundle and gets more degrees of freedom; for real (asymptotic) gauge particles these must be reduced by a suitable formalism.

In this paper I won’t discuss constraints, BRST symmetry or similar developments; rather I’ll take the provisional attitude of considering the classical description of gauge fields as connections, and the pre-quantum description of gauge fields in terms of unconstrained sections of certain vector bundles, as complementary descriptions.

In standard presentations, the choice of a gauge essentially amounts to the choice of a local “flat” connection \( \gamma_0 \) (vanishing curvature tensor). Then an arbitrary connection \( \gamma \) is characterized by the difference \( \alpha \equiv \gamma - \gamma_0 \), a true tensor field. If \( E \rightarrow M \) is a vector bundle, and we deal with linear connections, then \( \alpha : M \rightarrow T^*M \otimes E \otimes E^* \), and the curvature of \( \gamma \) can be expressed as

\[
R[\gamma] = -[\gamma, \gamma] = -2 [\gamma_0, \alpha] - [\alpha, \alpha].
\]

Conversely, if \( \alpha \) is seen as the “true” physical field then the choice of a gauge \( \gamma_0 \) determines the connection \( \gamma = \gamma_0 + \alpha \).

**Remark.** If one has a local flat linear connection \( \gamma_0 \), then the \( \gamma_0 \)-constant local sections of \( E \rightarrow M \) determine a trivialization of \( E \) over any sufficiently small open subset of \( M \). Thus one also has \( \gamma_0 \)-constant local frames. Conversely, the assignment of a local frame determines a flat \( \gamma_0 \) by the condition that its coefficients vanish in that frame. A gauge transformation is a section \( S : M \rightarrow \text{End} E \). This transforms the family of \( \gamma_0 \)-constant sections to a new family of sections, which determines a new flat connection\(^6\) \( \gamma_0' = \gamma_0 + (\nabla[\gamma_0]S) \uparrow S \).

Now the observations made in the remark concluding §2.3 suggest a more general, somewhat non-standard approach to the interactions between fermions and gauge particles. Suppose we describe fermions as sections of a vector bundle

\[
W \equiv W_r \oplus W_l \equiv (F \otimes U) \oplus (F \otimes U^*)
\]

(all tensor product and direct sums are fibered over \( M \)), where the vector bundle \( F \rightarrow M \) is endowed with a fibered Hermitian structure. Then, taking into account \( U \otimes U^* = \mathbb{C} \otimes H \) and \( U^* \otimes U^* = \mathbb{C} \otimes H^* \), we get natural linear inclusions

\[
\begin{align*}
W_r \otimes W_r & \equiv F \otimes F \otimes H \hookrightarrow W \otimes W, \\
W_l \otimes W_l & \equiv F^* \otimes F \otimes H^* \hookrightarrow W \otimes W .
\end{align*}
\]

Using the tetrad, the Lorentz metric of \( H \) and the Hermitian structure of \( F \), sections of the above bundles can be seen as sections

\[
M \rightarrow T^*M \otimes F^* \otimes F \cong T^*M \otimes \text{End}(F) .
\]

Let me elaborate a little further, in order to elucidate the essential idea contained in the above considerations. If the classical fields corresponding to certain particles are sections of a vector bundle \( E \rightarrow M \), then we also consider “gauge particles” interacting with them. The classical fields corresponding to these could be, in principle, sections of any vector bundles whose geometric structures yield suitable scalar-valued contractions; given a Hermitian

---

\(^6\)If \( \nabla[\gamma_0]S = 0 \) then \( \gamma_0' = \gamma_0 \) (the two families of covariantly constant sections coincide). In that case one uses to say that \( S \) is a “global” gauge transformation.
structure on the fibers, $\mathcal{E} \otimes \mathcal{E} \cong \text{End}(\mathcal{E})$ seems to be the simplest such item. In a sense, we might see all this as roughly analogue to chemistry, where the various kinds of atoms can bind together according to their valences.

Now if this generic $\mathcal{E}$ is of the type of the above $\mathcal{W}$ then, because of the structure of the spinor bundle (§2) and of the Hermitian structure of $\mathcal{F}$, one actually recovers all the standard gauge fields and, possibly, also further fields.\(^7\) In the case of electroweak theory, in fact, we’ll (provisionally) leave out some of the occurring sectors.

Note how the Hermitian structure of $\mathcal{W}$ partly derives from that of $\mathcal{F}$, and partly from the natural Hermitian structure of $\mathcal{U} \oplus \mathcal{U}^*$. On the other hand, like in the particular case of electrodynamics, no Hermitian structure of $\mathcal{U}$ itself has to be assumed (this would imply the choice of an observer, see §2.1). This observation stresses an important difference between spin and other internal symmetries, besides the fact that only spin is directly connected to spacetime geometry via the tetrad.

Next, continuing along this line of thought, note that we have two further natural sub-bundles

$$
\begin{align*}
\mathcal{W}_r \otimes \mathcal{W}_l &\equiv \overline{\mathcal{F}} \otimes \mathcal{F} \otimes \overline{\mathcal{U}} \otimes \mathcal{U}^* \hookrightarrow \mathcal{W} \otimes \mathcal{W}, \\
\mathcal{W}_l \otimes \mathcal{W}_r &\equiv \mathcal{F} \otimes \mathcal{F} \otimes \mathcal{U}^* \otimes \mathcal{U} \hookrightarrow \mathcal{W} \otimes \mathcal{W}.
\end{align*}
$$

We see no reason why sections of the above bundles should be excluded from the set of fields of our theory. Actually it will turn out that the Higgs field of electroweak theory can be described in this way.

**Remark.** For simplicity, in the above preliminary discussion, we considered the same bundle $\mathcal{F}$ as a tensor factor in $\mathcal{W}_r$ and $\mathcal{W}_l$, but our formulation can be easily extended to the case of a fermion bundle of the type $\mathcal{W} \equiv (\mathcal{F}_r \otimes \mathcal{U}) \oplus (\mathcal{F}_l \otimes \overline{\mathcal{U}}^*)$, where $\mathcal{F}_r \rightarrow M$ and $\mathcal{F}_l \rightarrow M$ are distinct vector bundles.

A further important issue, in the comparison between a classical field theory and its pre-quantum counterpart, is that of the *scaling* of the fields. In general the fields of a classical theory are scaled (as we saw in the case of electrodynamics, §2.2). Using natural units, this means that they are sections of certain vector bundles tensorialized by powers of the unit space $\mathbb{L}$, needed in order to obtain, eventually, an unscaled Lagrangian density. However scaling actually disappears in the contributions of internal vertices and lines, hence considering unscaled fields in the pre-quantum formulation seems reasonable. We may obtain the scaled version of a field by tensor product with appropriate rational powers of the spacetime volume form $\eta$.

### 3.2 Gauge fields and charges

In usual presentations, gauge fields such as the electromagnetic potential bear *charge* factors,\(^8\) which depend on the type of the particle they interact with. Strictly speaking this only makes sense *after* a gauge has been chosen, because multiplying a connection by an arbitrary real number is not a geometrically well-defined operation. The fact that the same gauge field can interact differently with different particles is the basic reason why one cannot simply absorb the charge into the gauge field.

---

\(^7\)This approach could be extended further by taking higher tensor powers of $\mathcal{W}$ and of the bundles associated with it.

\(^8\)In natural units, a charge is a pure number.
Now there is, actually, a natural geometric way of describing different charge values without choosing a gauge, at least as long as they are all integer multiples of a basic unit. In fact, let $X$ be a linear connection of a line bundle $Q \to M$, and let $Y_a$ be its coefficient in a local frame $b$ (namely $\nabla_b b = -Y_a b$); then the coefficient of the induced connection on $Q^p \equiv Q \otimes \cdots \otimes Q$ ($p$ factors) is $pY_a$ in the induced frame $b^p$. More generally, we can naturally accommodate for different interaction strengths by tensorializing any vector bundle with powers of $Q$.

A further question concerns the numeric value of the basic charge, namely why it can’t simply absorbed into the field’s strength: the answer is that this numeric value plays an essential role in extracting the physical meaning of a theory of quantum particles. The precise mathematical place of the basic charge deserves a thorough discussion (see also Weinberg [We96], Vol.II, §15.1-15.2). Gauche fields are represented as 1-forms valued into the subbundle $L \subset \text{End} F$ of all $h$-anti-Hermitian endomorphisms (the fibers are Lie sub-algebras). The symmetric bilinear form$^9$

$$K(X, Y) := -2 \text{Tr}(X \circ Y), \quad X, Y \in \text{End} F,$$

restricts to a positive scalar product of $L$; moreover the $h$-contraction of $X, Y \in L$, defined as

$$\langle \bar{X}, Y \rangle \equiv \bar{X}^\alpha \beta Y^\alpha \beta h_{\alpha \beta} = -X^\beta \alpha Y^\alpha \beta,$$

can also be written as

$$\langle \bar{X}, Y \rangle = \frac{1}{2} K(X, Y).$$

Let now $(t_i)$ be a K-orthonormal frame of $L$, $q \in \mathbb{R} \setminus \{0\}$, and write $X : M \to T'M \otimes L$ as

$$X = q X_a^i dx^a \otimes t_i \quad X_a^i : M \to \mathbb{R}.$$ 

Then $X_a^i$ is assumed to be the physical ‘field strength’. If $X$ represents a connection of $F$ in some chosen gauge, then the curvature tensor of that connection has the expression

$$R[X] \equiv R_{abcd} dx^a \wedge dx^b \otimes t_i = \left( -q \partial_{[a} X_{b]}^i + q^2 X_b^a X^j \partial_{[a} c_{b]}^j \right) dx^a \wedge dx^b \otimes t_i =$$

$$= \left( -q \partial_{[a} X_{b]}^i + q^2 X_b^a X^j c_{h_{ij}} \right) dx^a \wedge dx^b \otimes t_i,$$

where $c_{h_{ij}} = c_{h_{ji}} \equiv K(t_i, [l_j, t_k])$ are the ‘structure constants’ of $L$ (the index $i$ is raised via $K$ and we are working with an orthonormal frame). Next one introduces a Lagrangian density $\mathcal{L}[X] \equiv \ell[X] \eta$, where $\eta$ is the spacetime volume form and

$$\ell[X] := -\frac{1}{2 q^2} g^{ac} g^{bd} K_{ij} R_{ab}^i R_{cd}^j, \quad K_{ij} = \delta_{ij}.$$ 

In this way eventually one gets ‘kinetic terms’ of the kind $g^{ac} g^{bd} \delta_{ij} \partial_a X^i \partial_c X^j$ which are not affected by $q$, 3-factors interaction terms (of the kind $g^{ac} g^{bd} c_{h_{ij}} \partial_a X^i X^h X^j$) multiplied by $q$, and 4-factors terms multiplied by $q^2$.

$^9$We recall that the Killing 2-form of a Lie algebra $\mathfrak{L}$ is defined by $K(A, B) := \text{Tr}([A \circ B])$, where for each $A \in \mathfrak{L}$ we define $A \circ B \in \text{End} \mathfrak{L}$ by $A \circ B(X) := A X$. In particular, if $\mathfrak{L}$ is the Lie algebra of all endomorphisms of an $n$-dimensional vector space, then $K(A, B) = 2 n \text{Tr}(A \circ B) - (\text{Tr} A) \cdot (\text{Tr} B)$. Note that the restriction of $K$ to a Lie subalgebra $\mathfrak{L}' \subset \mathfrak{L}$ is not, in general, the Killing metric of $\mathfrak{L}'$. In the case of a simple Lie algebra there is, up to a factor, a unique invariant symmetric 2-form; for a simple Lie algebra of endomorphisms this is $(A, B) \mapsto \text{Tr}(A \circ B)$. 


4 Electroweak field theory

4.1 The fermion bundle

The starting step for building up the geometric background of the electroweak theory consists in the introduction of a complex vector bundle $W \to M$ whose sections are the theory’s fermion fields. This is obtained, most naturally, by a suitable modification of the fermion bundle $W \equiv U \oplus \overline{U}^\ast$ of electrodynamics (§2.2). If one chooses to mimic the usual presentations closely, then one is led to set

$$W \equiv W_R \oplus W_L \equiv U \oplus (I \otimes \overline{U}^\ast),$$

where $I \to M$ is a new complex vector bundle with 2-dimensional fibers which must be endowed with a Hermitian structure (all tensor product and direct sums are fibered over $M$).

However on finds at once a slight complication: the needed charge values for gauge fields, when seen from the point of view expressed in §3.2, imply a precise relation between the fibers of the bundles of “complex volumes” of $U$ and $I$; namely one must have

$$\iota^2 U^\ast \cong \iota^2 I \otimes \iota^2 I.$$

Equivalently, this can expressed\(^\text{10}\) as a similar relation

$$\iota^2 W_R \cong \iota^4 W_L$$

between the “complex volume” bundles of the right-handed and left-handed fermion sectors. I find that the most natural construction that meets the above said requirement is obtained as follows: first, by assuming that the two-spinor connection $\Gamma$, differently from the standard setting of electrodynamics (§2.2), determines a curvature-free connection of $\iota^2 U$; second, by taking the fermion bundle of the electroweak theory to be

$$W \equiv W_R \oplus W_L \equiv (\iota^2 I \otimes U) \oplus (I \otimes \overline{U}^\ast),$$

with no further assumption.

**Remark.** We recover essentially the previous setting as

$$W = U' \oplus (I' \otimes \overline{U}'^\ast),$$

with $U' \equiv \iota^2 I \otimes U$ and $I' \equiv \iota^2 T \otimes I$. Further suitable settings are actually possible, the chosen one being the most natural in my opinion.

We are going to formulate a field theory in which the fields are sections valued in $W$ and in $W \otimes W$, according to the ideas sketched in §3.1. In principle there is no obstruction to including also the gravitational field, represented by the tetrad $\Theta$ and the spinor connection\(^\text{11}\) of $U$: essentially, one only has to include the usual gravitational term (§2.2) in the Lagrangian density. But for the moment we leave out this kind of extension, and choose to work in a fixed (curved) gravitational background. Then $\Gamma$ and $\Theta$ are seen as a-priori fixed structures. A further fixed structure is the Hermitian metric of $I$, denoted as $h : M \to T^* \otimes I^*$ and assumed to have positive signature.

\(^{10}\) If $A$ and $B$ are vector spaces, respectively of dimension $m$ and $n$, then it is not difficult to see that there is a natural isomorphism $\wedge^{m+n} (A \oplus B) \cong \wedge^m A \otimes \wedge^n B$.

\(^{11}\) Since the induced connection of $\iota^2 U$ is now assumed to be curvature-free, the assignment of $\Gamma$ is equivalent to that of the connection $\tilde{\Gamma}$ of $H$. 
As in §2.1 we’ll denote by \((ζ_A), A = 1, 2\), a two-spinor frame, namely a local frame of \(U \rightarrow M\). Since the induced connection of \(∧^2U\) is now assumed to be curvature-free, we can choose the frame in such a way that \(∇ζ = ∇(ζ^1 ∧ ζ^2) = 0\), so that \(Γ_{aA} = 0\) (namely \(Γ\) is purely “gravitational”). Note how the existence of locally covariantly constant sections \(M \rightarrow ∧^2U^*\) does not imply that there exist a distinguished such 2-form (this is only unique up to a constant phase factor).

Moreover we’ll denote by \((ξ_α), α = 1, 2\), an isospin frame, namely an \(h\)-orthonormal local frame of \(I \rightarrow M\). We also set \(ω = ξ^1 ∧ ξ^2 : M \rightarrow ∧^2I^*\) (whenever no confusion arises, we distinguish dual frames simply by upper indices).

Let \(X\) be a linear connection of \(I \rightarrow M\). Then its coefficients in the frame \((ξ_α)\) can be written as

\[
X^α_a = X^λ_a σ^α_{aλ}, \quad X^λ_a : M \rightarrow \mathbb{C}, \quad a = 1, 2, 3, 4, \quad λ = 0, 1, 2, 3.
\]

One finds that the coefficients \(X^λ_a\) are imaginary if and only if \(X\) fulfills the condition \(∇[X]h = 0\) (we then say that \(X\) is Hermitian, though the matrices \((X^α_a)\) are actually anti-Hermitian).

Namely after the choice of a gauge one views the connection as a 1-form of \(M\) valued into the Lie algebra of \(U(2)\).

### 4.2 The fields

The fermion field is a section

\[
ψ ≡ ψ_R + ψ_L : M \rightarrow W_R \oplus W_L ≡ W.
\]

Its coordinate expression is written as

\[
ψ = ψ^λ ω^{-1} ⊗ ζ_A + ψ_α^A ξ_α ⊗ ξ^A,
\]

with \(ω^{-1} = ξ^1 ∧ ξ^2 : M \rightarrow ∧^2I\).

The boson fields will be introduced according to the ideas sketched in §3.1, with some restrictions aimed at reproducing essentially the standard electroweak theory (and no further fields). First we expand \(W ⊗ W\) and reorder some tensor products, obtaining

\[
W ⊗ W \cong (W_R ⊗ W_R) ⊕ (W_L ⊗ W_L) ⊕ (W_R ⊗ W_L) ⊕ (W_L ⊗ W_R) =
\]

\[
\cong (∧^2\overline{T} ⊗ ∧^2I ⊗ H) ⊕ (\overline{T} ⊗ I ⊗ H^*) ⊕
\]

\[
⊕ (∧^2\overline{T} ⊗ I ⊗ \text{End} \overline{U}) ⊕ (∧^2I ⊗ \overline{T} ⊗ \text{End} U).
\]

We first focus our attention to the two last bundles, which are mutually conjugate. We’ll consider sections having the special form

\[
φ ⊗ 1_U : M \rightarrow ∧^2\overline{T} ⊗ I ⊗ \text{End} U \quad \text{where} \quad φ : M \rightarrow ∧^2\overline{T} ⊗ I
\]

(here \(1_U : M \rightarrow U ⊗ U^* \cong \text{End} \overline{U}\) denotes identity of \(U \rightarrow M\)). Then also

\[
\overline{φ} ⊗ 1_U : M \rightarrow ∧^2I ⊗ \overline{T} ⊗ \text{End} U.
\]

A section \(φ\) as above will be called a Higgs field. We could consider more general fields in this sector, by dropping the restriction of proportionality to the identity, but for the moment we won’t broaden our investigation too much.
4.3 Symmetry breaking

The gauge fields are represented by a section

\[ W : M \to T \otimes I \otimes H^* \cong W_\ell \otimes W_\ell, \]

By contraction with the Hermitian metric \( h \) of \( I \) this also yields a section

\[ \hat{W} : M \to \wedge^2 T \otimes \wedge^2 I \otimes H^*, \]

which, using the natural Lorentz metric of \( H \), can also be seen as a section

\[ M \to \overline{W}_\ell \otimes W_\ell \cong \wedge^2 T \otimes \wedge^2 I \otimes H. \]

More precisely, we indicate as \( \tilde{h} : M \to \wedge^2 T^* \otimes \wedge^2 I^* \) the Hermitian metric induced on \( \wedge^2 I \) (then \( \tilde{h} \) is a positive object), and set

\[ \hat{W} := \tilde{h}^\# \otimes (h)W : M \to \wedge^2 T \otimes \wedge^2 I \otimes H^*. \]

In order to express the above field in component notation we consider the frame \((\iota_\lambda)\) of \( T \otimes I \) defined by

\[ \iota_\lambda := \sigma^\alpha_\lambda \xi_\alpha \otimes \xi_\alpha, \quad \lambda = 0, 1, 2, 3 \]

(then \( \langle \iota_\lambda, \iota_\mu \rangle = 2 \eta_{\lambda\mu} \)), and write

\[ W = W^\mu_\lambda \iota_\mu \otimes \tau^\lambda \Rightarrow \hat{W} = 2 W^0_\lambda \tilde{h}^\# \otimes \tau^\lambda. \]

If the field \( W \) is known, then the choice of an isospin gauge determines an isospin connection with components \( X^\prime_{a\beta} = i q W^\mu_\lambda \sigma^\alpha_{\mu\beta} \); the (unique) component of the induced connection \( \hat{X} \) of \( \wedge^2 I \) is \( \hat{X}_a = 2 i q W^0_\lambda \).

### 4.3 Symmetry breaking

At this point we introduce the symmetry breaking associated with the Higgs field according to the standard view (more or less): one assumes that there is one special section \( \phi_0 : M \to \wedge^2 T \otimes I \), seen as the “vacuum expectation value” of \( \phi \), supposedly arising as a minimum of the “Higgs potential”

\[ V[\phi] := \lambda (2 \mu^2 \langle \tilde{\phi}, \phi \rangle - \langle \tilde{\phi}, \phi \rangle^2), \]

where \( \lambda, \mu \in \mathbb{R}^+ \). We can choose the \( h \)-orthonormal isospin frame \((\xi_\alpha)\) in such a way that \( \phi_0 = \mu \xi_2 \); thus \( \xi_2 \) is determined by \( \phi_0 \), while \( \xi_1 \) is determined up to a phase factor. Next we observe that there is a unique \( h \)-preserving endomorphism \( S_\phi : M \to \text{End} I \) such that \( \phi = S_\phi(\|\phi\| \xi_2) \), with \( \|\phi\| = \langle \tilde{\phi}, \phi \rangle^{1/2} \); its matrix \((S^\alpha_{\beta})\) in any \( h \)-orthonormal frame is valued into \( \text{SU}(2) \). Moreover we write \( f \equiv \|\phi\| - \mu \) and represent \( \phi \) as the couple of fields

\[ (f, (S_\phi)) : M \to \mathbb{R} \times \text{SU}(2). \]

In the classical description, \( S_\phi \) is usually “absorbed” into the isospin connection through the following argument. Let \((\xi'_\alpha) \equiv (S_\phi(\xi_\alpha))\) be the “rotated” frame; then \( \phi = \|\phi\| \xi'_2 = (\mu + f) \xi'_2 \). We also express the left-handed fermion field in the new frame, namely we write \( \psi_L = \psi'^{\alpha}_X \xi'_\alpha \otimes \zeta'^{\dot{\alpha}} \) with \( \psi'^{\alpha}_X \equiv \bar{S}^\alpha_{\beta} \psi^\beta_X \). At the same time we consider the new connection \( X' \) whose components \( X'^{\prime}_{a\beta} \) in the frame \((\xi'_\alpha)\) are the same\(^1\) as the components of \( X \) in the

\(^1\)Namely \( X' \) is characterized by \( \nabla_a[X'](S_\sigma \xi'_2) = S(\nabla_a[X] \sigma) \) for all sections \( \sigma : M \to I \). The components of \( X' \) in the frame \((\xi_\alpha)\) are \((S X_\alpha \bar{S} + (\partial_a S) \bar{S})^{\alpha}_{\beta}\).
frame \((\xi_\alpha)\). The two connections have the same curvature tensor, but the gauge freedom of \(X'\) is reduced; now, in fact, we only obtain a classically equivalent theory by an \(U(1)\) gauge transformation affecting \(\xi_1',\) while \(\xi_2'\) is fixed. Eventually, one drops all the primes and says that the 3 real degrees of freedom, eliminated from \(\phi\) through the gauge transformation \(S_\phi\), have been “eaten up” by the connection.

The vacuum value of the Higgs field selects a subbundle \(\mathbb{I} \subset I \rightarrow M\) whose fibers are the positive spaces generated by that value. Thus symmetry breaking determines a decomposition of \(I\) into \(h\)-orthogonal subbundles

\[
I = I_1 \oplus I_2 = I_1 \oplus (\mathbb{C} \otimes \mathbb{I}) ,
\]

with \(\xi_1 : M \rightarrow I_1\) and \(\xi_2 : M \rightarrow \mathbb{I}\).

Let \(\hat{h} \equiv h \otimes h\) denote the Hermitian metric of \(T \otimes I\) determined by \(h\), and observe that the inverse of \(h\) itself is a section \(h^\# : M \rightarrow T \otimes I\). Hence \(T \otimes I\) can be decomposed as the direct sum

\[
T \otimes I = (T \otimes I)_h \oplus (T \otimes I)^\perp_h ,
\]

of the sub-bundles constituted of all elements proportional to \(h^\#\) and of all elements orthogonal to \(h^\#\), respectively. Symmetry breaking determines a further decomposition of \(T \otimes I\) into four mutually \(h\)-orthogonal subbundles with complex 1-dimensional fibers:

\[
T \otimes I = (T_1 \otimes I_1) \oplus (T_2 \otimes I_2) \oplus (T_1 \otimes I_2) \oplus (T_2 \otimes I_1) .
\]

In order to describe the gauge fields of e.w. theory we are also interested in real subbundles of \(T \otimes I\) with 1-dimensional fibers. We have, of course, the Hermitian subbundle \(E_0 \subset (T \otimes I)_h\), that is the real bundle generated by \(h^\#\), and the Hermitian subbundles \(E_1 \subset T_1 \otimes I_1\) and \(E_2 \subset T_2 \otimes I_2\). Then

\[
\bar{\xi}_1 \otimes \xi_1 = \frac{1}{2} (\iota_1 + \iota_3) , \quad \bar{\xi}_2 \otimes \xi_2 = \frac{1}{2} (\iota_1 - \iota_3) ,
\]

are distinguished frames of \(E_1\) and \(E_2\), respectively.

We’ll need a further real subbundle \(E' \subset E_1 \oplus E_2\). This is not completely determined by the geometric structures assumed up to now, but needs a new ‘ingredient’: the Weinberg angle \(\theta_w \in (0, \pi/2)\). Now \(E'\) is defined as the subbundle of \(E_1 \oplus E_2\) generated by

\[
\iota' := -\frac{1}{2} \bar{\xi}_1 \otimes \xi_1 + \frac{1}{2} \cos(2 \theta_w) \bar{\xi}_2 \otimes \xi_2 \equiv -\frac{1}{2} [\sin^2(\theta_w) \iota_0 + \cos^2(\theta_w) \iota_3] .
\]

The subbundle of \(T \otimes I\) composed of all elements orthogonal to \(E_1\) and \(E_2\) is

\[
E^+ \oplus E^- \equiv (\bar{T}_1 \otimes I_2) \oplus (\bar{T}_2 \otimes I_1) .
\]

These subbundles \(E^\pm\) have complex 1-dimensional fibers, and we do not try to select a real subbundle. We observe that the map \(w \rightarrow w^\dagger\) (§1.2) determines an anti-isomorphism \(E^+ \leftrightarrow E^-\).

Eventually, the gauge fields of electroweak theory are the real fields

\[
A : M \rightarrow H^* \otimes E_2 , \quad Z : M \rightarrow H^* \otimes E' ,
\]

and the complex fields

\[
W^+ : M \rightarrow H^* \otimes E^+ , \quad W^- : M \rightarrow H^* \otimes E^- .
\]

---

\[13\] Here also expressed in terms of the above introduced (§4.2) frame \((i_\lambda) \equiv (\sigma^\alpha_\lambda \bar{\xi}_\alpha \otimes \xi_\alpha)\).
However $W^-$ is assumed to be the Hermitian adjoint of $W^+$ so that we actually have just one independent complex field. The coordinate expressions of the gauge fields are

$$A = A_\lambda \tau^\lambda \otimes \bar{\xi}_2 \otimes \xi_2,$$

$$Z = Z_\lambda \tau^\lambda \otimes \iota \equiv \frac{1}{2} Z_\lambda \tau^\lambda \otimes (-\bar{\xi}_1 \otimes \xi_1 + \cos(2 \theta_w) \bar{\xi}_2 \otimes \xi_2),$$

$$W^+ = W^+_\lambda \tau^\lambda \otimes \bar{\xi}_1 \otimes \xi_2 \equiv \frac{1}{2} W^+_\lambda \tau^\lambda \otimes (\iota_1 + i \iota_2),$$

$$W^- = W^-_\lambda \tau^\lambda \otimes \bar{\xi}_2 \otimes \xi_1 \equiv \frac{1}{2} W^-_\lambda \tau^\lambda \otimes (\iota_1 - i \iota_2),$$

with

$$W^-_\lambda = \bar{W}^+_\lambda : \mathcal{M} \to \mathbb{C}.$$ 

Thus the set of gauge fields is composed essentially by two real fields and one complex field.

**Remark.** One could introduce the gauge-field target bundle $E \equiv E_2 \oplus E' \oplus E^+$ as fully unrelated to the spin and isospin bundles, and describe the interactions via further geometric assumptions (see also [De02]).

### 4.4 Electroweak geometry and the dilaton

As we saw in §2.2, the “breaking of local conformal invariance” occurring in real physics actually amounts to the choice of a curvature-free connection $G$ of the bundle $L \rightarrow M$ of length units. We also saw how a natural candidate for $L$ arises in the context of electrodynamics. In the extended context of electroweak geometry one could associate $L$ with different constructions, but that is not an essential point since any space can be tensorialized by arbitrary powers of $L$. The point, instead, is that the various terms in the Lagrangian density must be “padded” with powers of $L$, so that all have the same “conformal weight”. This is unavoidable because some of the fields, notably the metric and the induced volume form, are inherently scaled.

In the Einstein-Cartan-Maxwell-Dirac field theory, the curvature-free connection $G$ of $L \rightarrow M$ must be just assumed, while in the context of electroweak geometry one is intrigued to look for possible mechanisms determining the breaking of local conformal invariance in relation to the Higgs field and the breaking of isospin symmetry (also encouraged by the evasive nature of the Higgs). Recent proposals suggest seeing the $h$-norm of the Higgs field as a conformal factor for the spacetime metric [Fa08, FKD09, RySh09], or the Hermitian metric of the isospin bundle itself as an independent field rather than a fixed structure [Ta05]. Here I wish to frame the question in the context of the above said point of view of determining a curvature-free connection $G$. Then I’ll argue, somewhat differently from other proposals, that any convincing solution of the said question would require a substantial extension of the electroweak theory.

Let’s start from the Lagrangian density of the electroweak theory in a “classical” field context; as usual we write it as the sum

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_\phi + \mathcal{L}_X + \mathcal{L}_{\text{int}},$$

---

14For example, one could see $L$ as the positive Hermitian subspace of $\wedge^2 \mathbb{T} \otimes \wedge^2 \mathbb{I}$.

15One can set up a more general “running constants” formalism by assuming bundles $\mathcal{M}, L, T \rightarrow \mathcal{M}$ and letting sections $c : \mathcal{M} \rightarrow L \otimes T^{-1}, h : \mathcal{M} \rightarrow M \otimes L^2 \otimes T^{-1}$ not to be fixed.
with

\[ L_\phi = \frac{1}{\sqrt{2}} \tilde{\Theta} \left( \nabla \psi_R \otimes \tilde{\psi}_R - \psi_R \otimes \nabla \tilde{\psi}_R + \bar{g}^2 \left( \langle \nabla \psi_L \otimes \tilde{\psi}_L \rangle - \langle \psi_L \otimes \nabla \tilde{\psi}_L \rangle \right) \right), \]

\[ L_\phi = \left( (g^\# \otimes \nabla \bar{\phi} \otimes \nabla \phi) + \lambda (2 m^2 \langle \bar{\phi} \otimes \phi \rangle - \langle \bar{\phi} \otimes \phi \rangle^2) \right) \eta, \quad m \in \mathbb{L}^{-1}, \lambda \in \mathbb{R}^+, \]

\[ L_X = - \langle (\tilde{\psi}_L \otimes \phi \otimes \psi_R) + (\tilde{\psi}_R \otimes \bar{\phi} \otimes \psi_L) \rangle \eta. \]

Here, \( R[X] \) denotes the curvature tensor of the connection \( X \) of \( I \rightarrow M \), the angular bracket denotes \( h \)-contraction and \( \tilde{\Theta} \) is the tetrad-related operation introduced in \( \S 2.2 \). In coordinates, using an \( h \)-orthonormal frame \( (\xi_\alpha) \) of \( I \), we get \( L = (\ell_\psi + \ell_\phi + \ell_X + \ell_{\text{int}}) d^4x \), where

\[ \ell_\psi = -\frac{1}{\sqrt{2}} \tilde{\Theta} \left( \frac{\tilde{\psi}_L}{\bar{\psi}_L} \right) \left( \langle \psi_L \otimes \nabla \bar{\psi}_L \rangle - \langle \psi_L \otimes \nabla \psi_L \rangle \right), \]

\[ \ell_\phi = \left( g^{ab} \nabla_\alpha \tilde{\psi}_L \nabla_\beta \phi^\alpha + 2 \lambda m^2 (\tilde{\phi}_L \phi^\alpha - \lambda (\bar{\phi}_L \phi^\alpha)^2) \right) \det \Theta, \]

\[ \ell_X = -g^{ac} g^{bd} \tilde{\psi}_L \tilde{\psi}_L R_{\alpha \beta} R_{\beta \alpha}, \quad \ell_{\text{int}} = -\left( \tilde{\psi}_L \phi^\alpha \psi^\beta + \bar{\psi}^\beta \tilde{\phi}_L \phi^\alpha \right) \det \Theta \]

(shorthands \( \tilde{\psi}_L \equiv h_{\alpha \beta} \tilde{\psi}_L^\alpha \), \( \tilde{\phi}_L \equiv h_{\alpha \beta} \tilde{\phi}_L^\alpha \) and the like were used).

Now one must check that all the terms in \( L \) bear the same conformal weight. The most natural way to achieve this is by assuming that all terms are conformally invariant, namely non-scaled (this is the standard assumption anyway). Taking into account the scaling (or “conformal weight”) of the involved objects we find that the Fermion field and the Higgs field must be respectively \( \mathbb{L}^{-3/2} \)-scaled and \( \mathbb{L}^{-1} \)-scaled, namely

\[ \psi \equiv \psi_L + \psi_R : M \rightarrow \mathbb{L}^{-3/2} \otimes W \equiv \mathbb{L}^{-3/2} \otimes (W_R \oplus W_L), \]

\[ \phi : M \rightarrow \mathbb{L}^{-1} \otimes \mathbb{T} \otimes I. \]

Hence also \( m \in \mathbb{L}^{-1} \), and

\[ \| \phi \|^2 \equiv \langle \bar{\phi} \otimes \phi \rangle : M \rightarrow \mathbb{L}^{-2}. \]

All the above setting makes sense even if conformal symmetry is not broken, with the only addendum that any scaled coupling factors can’t be seen as constants; in particular \( m : M \rightarrow \mathbb{L}^{-1} \). Any given section \( \rho : M \rightarrow \mathbb{L}^r, r \in \mathbb{Z} \setminus \{0\} \), determines a curvature-free connection of \( \mathbb{L} \rightarrow M \) by the condition \( \nabla \rho = 0 \). It’s then clear that a “vacuum value” of \( \phi \), namely a special section \( \phi_0 : M \rightarrow \mathbb{L}^{-1} \otimes \mathbb{T} \otimes I \), also determines such a connection (fulfilling \( \nabla \| \phi_0 \|^2 = 0 \)). This observation, however, does not settle our question, since assigning the Higgs potential \( \lambda (2 m^2 \langle \bar{\phi} \otimes \phi \rangle - \langle \bar{\phi} \otimes \phi \rangle^2) \) also means fixing the section \( m \); namely, the connection \( G \) is already determined by the condition \( \nabla |G|m = 0 \). Now rather than shifting between essentially equivalent choices, which is not so interesting, we’d like to find an independent mechanism.

Also note that, more generally, if \( \phi \) is indeed scaled then we can’t write a polynomial expression in \( \| \phi \| \) and/or \( \| \phi \|^{-1} \), having any positive root, without fixing some scaled factor. On the other hand, if we let \( m \) (say) be an independent field, variation of the Lagrangian with respect to it yields a relation between \( m \) and \( \phi \) but no condition on \( G \).

So we see that the standard setting of the electroweak theory is not sufficient to give the question of conformal symmetry breaking a final answer. Instead we should extend the theory
4.4 Electroweak geometry and the dilaton

in some way. The most obvious extension would be allowing $G$ as a dynamical field and adding to the Lagrangian the non-scaled term

$$\langle g^\# \otimes g^\#, dG \otimes dG \rangle \eta = g^{ac} g^{bd} \partial_a G_b \partial_c G_d \det \Theta d^4x .$$

Now $G_a$ also appears in the covariant derivatives of the various scaled fields; these act as sources for $G$, and we cannot expect $dG = 0$. Moreover, since the spacetime metric is scaled, it seems that gravitation must be fully included in the dynamical problem; with regard to this aspect, let me say that I’m not convinced by proposals which involve expressing the metric as a fixed background metric multiplied by some further conformal factor (at least, that would be against Ockam’s principle—entities shouldn’t be multiplied without necessity). For similar reasons, if one is considering the generalization of viewing the Hermitian metric $h$ of $I$ as an independent field (e.g. see Talmadge’s proposal [Ta05]), I’d rather not take the new field as a variation of some fixed background.
References

[C98] Canarutto, D.: ‘Possibly degenerate tetrad gravity and Maxwell-Dirac fields’, J. Math. Phys. 39, N.9 (1998), 4814–4823.

[C00b] Canarutto, D.: ‘Two-spinors, field theories and geometric optics in curved spacetime’, Acta Appl. Math. 62 N.2 (2000), 187–224.

[C05] Canarutto, D.: ‘Quantum bundles and quantum interactions’, Int. J. Geom. Met. Mod. Phys., 2 N.5, (2005), 895–917. arXiv:math-ph/0506058v2.

[C07] Canarutto, D.: ‘“Minimal geometric data” approach to Dirac algebra, spinor groups and field theories’, Int. J. Geom. Met. Mod. Phys., 4 N.6, (2007), 1005–1040. arXiv:math-ph/0703003.

[CJM95] Canarutto, D., Jadczyk A. and Modugno, M.: ‘Quantum mechanics of a spin particle in a curved spacetime with absolute time’, Rep. Math. Phys. 36 (1995), 95–140.

[De02] Derdzinski, A.: ‘Geometry of the Standard Model’, notes of a talk at the Conference on Geometry in Bedlewo, Poland, September 2002.

[Fa08] Faddeev, L.D.: ‘An alternative interpretation of the Weinberg-Salam model’. arXiv:hep-th/0811.3311v2.

[FKD09] Foot, R., Kobakhidze, A. and McDonald, K.L.: ‘Dilaton as the Higgs boson’. arXiv:hep-th/0812.1604v2.

[FN56] Frölicher, A. and Nijenhuis, A.: ‘Theory of vector valued differential forms, I’, Indag. Math. 18 (1956), 338–360.

[FN60] Frölicher, A. and Nijenhuis, A.: ‘Invariance of vector form operations under mappings’, Communicationes Mathematicae Helveticae 34 (1960), 227–248.

[HCMN95] Hehl, F.W., McCrea, J.D., Mielke, E.W. and Ne’eman, Y.: ‘Metric-affine gauge theory of gravity: field equations, Noether identities, world spinors, and breaking of dilaton invariance’, Phys. Rep. 258 (1995), 1–171.

[ILM10] Ilderton, A., Lavelle, M. and McMullan, D.: ‘Symmetry breaking, conformal geometry and gauge invariance’. Phys. Lett. B 347 (1995), 89. arXiv:hep-th/9412145v1.

[JM02] Janyška, J. and Modugno, M.: ‘Covariant Schrödinger operator’, J. Phys. A 35 (2002), 8407–8434.

[JM06] Janyška, J. and Modugno, M.: ‘Hermitian vector fields and special phase functions’, Int. J. Geom. Met. Mod. Phys., 3 N.4 (2006), 1–36. arXiv:math-ph/0507070v1.

[JMV10] Janyška, J., Modugno, M. and Vitolo, R.: ‘An algebraic approach to physical scales’, Acta Appl. Math. 110 N.3 (2010), 1249–1276. arXiv:0710.1313v1.

[LM94] Lavelle, M. and McMullan, D.: ‘Observables and Gauge Fixing in Spontaneously Broken Gauge Theories’. Phys. Lett. B 347 (1995), 89. arXiv:hep-th/9412145v1.
REFERENCES

[MT10] Masson, T. and Wallet, J.C.: ‘A Remark on the Spontaneous Symmetry Breaking Mechanism in the Standard Model’. arXiv:hep-th/1001.1176v1.

[Mi01] Michor, P.W.: ‘Frölicher-Nijenhuis bracket’, in Hazewinkel, Michiel, *Encyclopedia of Mathematics*, Springer (2001).

[Mo91] Modugno, M.: ‘Torsion and Ricci tensor for non-linear connections’, Diff. Geom. and Appl. 2 (1991), 177–192.

[MK] Modugno, M. and Kolář, I.: ‘The Frölicher-Nijenhuis bracket on some functional spaces’, Ann. Pol. Math. LXVIII.2 (1998), 97–106.

[MST05] Modugno, M., Saller, D. and Tolksdorf, J.: ‘Classification of infinitesimal symmetries in covariant classical mechanics’, J. Math. Phys. 47 062903 (2006).

[Mo10] Moffat, J.W.: ‘Ultraviolet Complete Electroweak Model Without a Higgs Particle’, arXiv:hep-ph/1006.1859v2.

[PR84] Penrose, R. and Rindler, W.: *Spinors and space-time. I: Two-spinor calculus and relativistic fields*, Cambridge Univ. Press, Cambridge (1984).

[PR88] Penrose, R. and Rindler, W.: *Spinors and space-time. II: Spinor and twistor methods in space-time geometry*, Cambridge Univ. Press, Cambridge (1988).

[RySh09] Ryskin, M.G. and Shuvaev, A.G.: ‘Higgs Boson as a Dilaton’. arXiv:hep-th/0909.3374v1.

[SV00] Saller, D. and Vitolo, R.: ‘Symmetries in covariant classical mechanics’, J. Math. Phys. 41, 10 (2000), 6824–6842, arXiv:math-ph/0003027.

[Ta05] Talmadge, A.: ‘Symmetry breaking via internal geometry’, Int. J. Math. and Math. Sci. 2005:13 (2005), 2023–2030.

[Vi99] Vitolo, R.: ‘Quantum structures in Galilei general relativity’, Annales de l’Institut H. Poincaré, 70 (1999), 239–258.

[Vi00] Vitolo, R.: ‘Quantum structures in Einstein general relativity’, Lett. Math. Phys. 51 (2000), 119–133.

[We96] Weinberg, S.: *The Quantum Theory of Fields*, Vol. I-II, Cambridge University press (1996).