Two-Step mmWave Positioning Scheme with RIS-Part I: Angle Estimation and Analysis

Tuo Wu, Cunhua Pan, Yijin Pan, Sheng Hong, Hong Ren, Maged Elkashlan, Feng Shu and Jiangzhou Wang, Fellow, IEEE

Abstract

In this series of work, we propose a comprehensive two-step three-dimensional (3D) positioning scheme in a millimeter wave (mmWave) system, where the reconfigurable intelligent surface (RIS) is leveraged to enhance the positioning performance of mobile users (MUs). Specifically, the first step is the estimation error modeling and analysis based on the two-dimensional discrete Fourier transform (2D-DFT) angle estimation technique, while the second step is the corresponding positioning algorithm design and bias analysis. The first step is introduced in this paper, and the second step is investigated in Part II of this series work. Based on 2D-DFT angle estimation, the angle estimation error is modeled and analyzed by deriving its probability density functions (PDF). More specifically, we first derive the PDF by using the geometric relationship between the angles of arrival (AOAs) and their triangle functions. Then, we simplify the intricate expression of the PDF of the AOA estimation error by employing the first-order linear approximation of triangle functions. Finally, we derive a complex expression for the variance based on the derived PDF. Distinctively, for the azimuth estimation error, the variance is separately integrated according to the different non-zero intervals of the PDF, which will be used in the second part of this series work for the analysis of the position estimation error. Extensive simulation results are also presented to verify the accuracy of the derived results.

Index Terms

Reconfigurable intelligent surface (RIS), intelligent reflecting surface, positioning, radio localization.
I. INTRODUCTION

As an important requirement for the sixth generation (6G) wireless networks, high-accuracy positioning service is of great value in a wide range of applications [1], such as automated driving vehicles [2], smart factory [3] and virtual reality [4]. For example, it is predicted that the 2020s will be the first decade for automated driving vehicles with the positioning accuracy at decimeter level [5]. However, the positioning accuracy of the prevalent global positioning system (GPS) is about 5 meters even in ideal conditions [6], which cannot meet the stringent requirement on positioning accuracy for these thriving applications [7]. Therefore, network-based positioning systems are emerging as a promising alternative to GPS in 6G networks [8].

In current wireless positioning networks, millimeter wave (mmWave) technique [9] can provide extremely highly-accurate estimation of radio positioning parameters, such as the channel gain, time delays and angle of arrival (AOA). However, due to the high frequency, it is sensitive to blockage in complex propagation environments. For example, in smart factories, due to densely-deployed equipment such as metal machinery, random movement of objects (robots and trucks), wireless signals in the mmWave band are vulnerable to blockages, which can degrade the positioning reliability [3]. Therefore, the emerging technology of reconfigurable intelligent surface (RIS) [10], has been proposed to be integrated into the existing radio positioning systems with various benefits [11]. First, since the direct communication link between the base station (BS) and the mobile user (MU) may be blocked by obstacles [12], RIS can provide a secondary communication link [13]–[15], which introduces an extra positioning reference and additional positioning parameters for positioning networks. Furthermore, the large number of reflecting elements in RIS panel can ensure the high estimation accuracy for the positioning parameters, especially the AOA parameters, which thus can improve the final positioning accuracy [16]. Moreover, the deployment of RIS is usually more convenient and cheaper for positioning than the BSs [17]. Finally, different from other fixed scatterers in the positioning environment, a higher beamforming gain can be achieved by tuning the phase shifts of the reflecting elements in RIS. Therefore, it is of much interest to explore RIS-aided positioning.

Typically, positioning schemes work through the following two steps. First, radio positioning parameters can be obtained through some estimation methods [11], [18], [19]. Second, positioning algorithms can be designed accordingly. Specifically, expressions of the complex non-linear geometric relationships between the radio positioning parameters and the position coordinates
are first derived [20]. Then, the estimation error of the positioning parameters and the non-
linear geometric relationships are jointly utilized to derive the non-linear equations [21]. Finally,
the position of the MU is determined by solving the equations using iterative or non-iterative
algorithms.

From the above-mentioned positioning steps, the estimation error of the radio positioning
parameters can directly determine the positioning accuracy. In general, different methods for
estimating different channel parameters lead to different types of parameter estimation errors
[22]. Besides, the radio positioning parameter estimation is a new challenge due to the passive
property of RIS [23]. Therefore, it is essential to investigate different RIS-aided positioning
systems based on the different types of radio positioning parameter estimation errors.

To date, positioning systems have attracted extensive research attention, which mainly focused
on performance analysis [24]–[27] and algorithm design [27]–[30]. To provide an analytical
performance validation, [24] studied the theoretical performance bounds (i.e., Cramér-Rao lower
bounds (CRLB)) for positioning, and evaluated the impact of the number of reflecting elements
and the phase shifts on the positioning estimation accuracy. The authors in [25] derived the CRLB
in terms of the location parameters of the target node, and obtained the theoretic relationship
between the optimal power and the phase parameters of the RIS. The authors in [26] studied a
multiple-RIS-aided mmWave positioning system and derived the CRLB. Alouini et al. derived the
CRLB for assessing the performance of synchronous and asynchronous signaling schemes and
proposed an optimal closed-form reflecting phase shifts for joint communication and localization
[27]. For the algorithms design, the authors in [27] considered an RIS-assisted 3D positioning
system, where the AOAs and AODs were estimated by the maximum likelihood estimation
(MLE) algorithm. The authors in [28] proposed a received signal strength (RSS) based positioning
scheme enabled by an RIS and designed an iterative algorithm to derive the optimal phase shifts.
In [29], a reduced-complexity maximum-likelihood based estimation procedure was devised to
jointly recover the user position and the synchronization offset. The authors in [30] developed
an RIS-aided positioning framework to locate a MU in the scenarios where the LoS path may
or may not be available.

However, the above-mentioned contributions mainly focus on performance analysis, such
as CRLB analysis, and do not provide positioning algorithms. Besides, these contributions
assume that the error of the positioning parameters follows the Gaussian distribution, which
is inconsistent with practical scenarios. As for contributions that focus on algorithm design,
the positioning parameters were assumed to be known with estimation error following the Gaussian distribution. However, in practice, the distribution of these parameters depends on the practical estimation methods (e.g., 2D-DFT algorithm [23]), which may not follow the Gaussian distribution. Therefore, existing positioning algorithms and performance analyses may not be applicable when considering practical positioning parameters estimation method.

In this paper, we aim to design a comprehensive framework of a two-step three-dimensional (3D)-positioning algorithm for RIS-aided mmWave systems. In the first step, based on the 2D-DFT angle estimation, the angle estimation error is modeled and analyzed in terms of the probability density functions (PDF). We also derive the angle estimation error variance using the variance calculation formula. In the second step, we aim to obtain the closed-form expression of the MU’s location based on the DFT-based angle estimation and the angle error analysis obtained in the first step. The second step is detailed in Part II of this series work.

In this paper, we consider the first step. Our main contributions are summarized as follows.

1) The estimated AOAs are first derived in closed-form based on the 2D-DFT method. Based on the expression of estimation, we find that the angle estimation error at the RIS depends on the search grid, the panel size of the RIS, and the number of elements of the RIS. Moreover, due to the property of 2D-DFT, the error of the angle estimation follows the uniform distribution.

2) According to the uniform distribution of the estimation error of azimuth and elevation, the angle estimation error is characterized in terms of the PDF. To be specific, we first derive the PDF by using the geometric relationship between the AOAs and their triangle functions. Then, we simplify the complex geometric expression of the PDF by employing the first-order linear approximation of the triangle function. For the azimuth angle estimation, we provide an algorithm to derive and approximate the PDF expression of its estimation error.

3) Based on the PDF of estimation error, we theoretically derive the variance of the angle estimation error. Since the PDF of the azimuth estimation error has three different non-zero intervals, we separately calculate the integral in the different intervals according to the variance calculation formula.

4) Simulation results verify the accuracy of the derived results. We observe that the variance decreases with the number of elements, which means that increasing the number of RIS elements improves the estimation accuracy.

The remainder of the paper is organized as follows. The system model for the two-step
positioning aided by multiple RISs is described in Section II. The procedures of estimating
angles are given in Section III. Section IV derives the PDF in more details. Section V calculates
the variance. Simulation results are given in Section VI. Section VII concludes the work of this
paper.

Fig. 1: RIS-aided positioning system model.

**II. SYSTEM MODEL**

Consider a RIS-aided positioning system, where a MU sends pilot signals to the BS to locate
the MU with the assistance of an RIS. The BS is equipped with a uniform linear array (ULA)
with $N_b$ antennas, and the MU is equipped with a single antenna. Moreover, we assume that
there are $M$ RISs, and each RIS is a uniform planar array (UPA).

The BS is placed parallel to the x-axis with the center located at $p = [x_p, y_p, z_p]^T$. The $i$th RIS
is placed parallel to the y-o-z plane with their centers located at $s_i = [x_i, y_i, z_i]^T, i = 1, 2, \cdots, M$.
The UPA-based RIS has $N_{y,z} = N_y \times N_z$ reflecting elements, where $N_y$ and $N_z$ denote the
numbers of reflecting elements along the y-axis and z-axis, respectively. The real location of the MU is \( \mathbf{q} = [x_q, y_q, z_q]^T \) and is assumed to be placed parallel to the x-o-y plane. The estimated location of the MU is denoted as \( \hat{\mathbf{q}} = [\hat{x}_q, \hat{y}_q, \hat{z}_q]^T \). Generally, once the RISs and BS have been deployed, the coordinates \( s_i \) and \( p \) are known and invariant. In order to locate the MU, we need to obtain the estimated \( \hat{\mathbf{q}} \).

Take the reflecting link from the MU through the first RIS to the BS as an example. For simplicity, the RIS index is ignored in the following derivations. When the MU transmits the pilot signals via the RIS to the BS, the azimuth and elevation AOA from the MU to the RIS could be denoted as \( \Theta_{RA} \) and \( \Phi_{RA} \) as shown in Fig. 2-(a), respectively. As a result, the array steering vector is modeled as

\[
\mathbf{a}_{RA}(u_{RA}, v_{RA}) = \text{vec}\{\mathbf{A}_{RA}(u_{RA}, v_{RA})\} = \text{vec}\{\mathbf{a}_{Ny}(u_{RA})\mathbf{a}_{Nz}^T(v_{RA})\},
\]

where \( \mathbf{a}_{Ny}(u_{RA}) \) and \( \mathbf{a}_{Nz}(v_{RA}) \) are given by

\[
\mathbf{a}_{Ny}(u_{RA}) = [1, e^{j u_{RA}}, \ldots, e^{j (N_y - 1) u_{RA}}]^T, \tag{2}
\]
\[
\mathbf{a}_{Nz}(v_{RA}) = [1, e^{j v_{RA}}, \ldots, e^{j (N_z - 1) v_{RA}}]^T. \tag{3}
\]
with
\[
\begin{align*}
u_{RA} &= \frac{2\pi d_1 \cos \Theta_{RA} \cos \Phi_{RA}}{\lambda}, \\
u_{RA} &= \frac{2\pi d_1 \sin \Phi_{RA}}{\lambda}.
\end{align*}
\] (4) (5)

In (4), \(\lambda\) denotes the carrier wavelength and \(d_1\) denotes the distance between the adjacent elements of the RIS. Then, the channel between the MU and the RIS, denoted by \(h\), is modeled as [31]
\[
h = \alpha_{MR} \mathbf{a}_{RA}(u_{RA}, v_{RA}),
\] (6)

where \(\alpha_{MR}\) denotes the channel gain of \(h\).

The RISs are assumed to be placed in the vicinity of the BS [32]. Then, we can use the ray-tracing line of sight (LoS) channel model according to [33], [34]. Fig. 2-(b) illustrates the transmission from the RIS to the BS. The inter-element spacing is assumed to be \(d_1\) while the inter-antenna spacing is assumed to be \(d_2\). For the sake of analysis, in Fig. 2-(b), the first antenna of the BS is assumed to be placed at the original point. \(R\) represents the distance from the first antenna of the BS to the first reflecting element of the RIS, while \(\Phi_{BR}\) and \(\Theta_{BR}\) are the angles of the local spherical coordinate system at the RIS and the BS.

For notation brevity, let \(n_{y,z} = (n_y, n_z)\) denote the reflecting element of the RIS at column \(n_y \in \{0, 1, \cdots, N_y - 1\}\) and row \(n_z \in \{0, 1, \cdots, N_z - 1\}\). Thus, we can calculate the distance between BS antenna \(n_b \in \{0, 1, \cdots, N_b - 1\}\) and reflecting element \(n_{y,z}\) at the RIS as
\[
r_{n_b,n_{y,z}} = \sqrt{(R \sin \Phi_{BR} \cos \Theta_{BR} + n_b d_2)^2 + (R \cos \Phi_{BR} + n_y d_1)^2 + (R \sin \Phi_{BR} \sin \Theta_{BR} - n_z d_1)^2}.
\]

The array vector from reflecting element \(n_{y,z}\) to the \(N_b\) antennas at the BS is written as
\[
h_{n_{y,z}} = [e^{j \frac{2\pi}{\lambda} r_{0,n_{y,z}}}, \cdots, e^{j \frac{2\pi}{\lambda} r_{N_b-1,n_{y,z}}}].
\] (7)

Thus, the channel matrix [33] from the RIS to the BS is given by [33]
\[
H = \sqrt{\rho}[h_0, h_1, \cdots, h_{N_{y,z}-1}],
\] (8)

where \(\rho\) is the common power attenuation.

Let \(e_t \in \mathbb{C}^{N_{y,z} \times 1}\) denote the phase shift vector of the RIS at time slot \(t\), which satisfies \(|e_t|_{n_{y,z}} = 1\) for \(1 \leq n_{y,z} \leq N_{y,z}\). Accordingly, the received signal from the MU via the RIS to
the BS at time slot $t$ could be expressed as

$$y(t) = HD_{\text{Diag}}(e_t)h\sqrt{p}s(t) + n(t),$$

(9)

where $s(t)$ denotes the transmitted pilot signal of the MU and $n(t) \in \mathbb{C}^{N_b \times 1}$ is additive white Gaussian noise (AWGN) following the distribution of $\mathcal{CN}(0, \delta^2 I)$. Moreover, $p$ is the transmit power of the MU.

Since the positions of the BS and the RISs are fixed and known at the BS, the channel matrix $H$ can be directly calculated based on (8). In this paper, we consider the massive mmWave multiple input multiple output (MIMO) system, where the number of antennas at the BS is larger than the number of reflecting elements at the RIS. As a result, we can obtain the pseudo-inverse matrix of the channel matrix $H$, which is given by $H_p = (H^H H)^{-1} H^H$. By multiplying both sides of (9) by $H_p$, one obtains

$$\tilde{y}(t) = \text{Diag}(e_t)h\sqrt{p}s(t) + \tilde{n}(t),$$

(10)

where $\tilde{y}(t) = H_p y(t)$, and $\tilde{n}(t) = H_p n(t)$ that follows the distribution of $\mathcal{CN}(0, \delta^2 (H^HH)^{-1})$.

Assuming that the phase shifts of the reflecting elements are available at the BS, by multiplying both sides of (10) by the inverse matrix of $\text{Diag}(e_t)$, we have

$$\hat{y}(t) = h\sqrt{p}s(t) + \hat{n}(t),$$

(11)

where $\hat{y}(t) = (\text{Diag}(e_t))^{-1}\tilde{y}(t)$ and $\hat{n}(t) = (\text{Diag}(e_t))^{-1}\tilde{n}(t)$. Here, $\hat{n}(t)$ follows the distribution of $\mathcal{CN}(0, \delta^2 ((\text{Diag}(e_t))H^H H \text{Diag}(e_t))^{-1})$.

Finally, it is assumed that channel gain $\alpha_{MR}$ can be perfectly estimated by using the scheme in [23]. Then, we can multiply both sides of (11) by $\frac{1}{\alpha_{MR}}$, leading to

$$\hat{y}(t) = a_{RA}(u_{RA}, v_{RA})\sqrt{p}s(t) + \hat{n}(t) = \text{vec}\{a_{RA}(u_{RA}, v_{RA})\} \sqrt{p}s(t) + \hat{n}(t),$$

(12)

where $\hat{y}(t) = \frac{1}{\alpha_{MR}}\hat{v}(t)$ and $\hat{n}(t) = \frac{1}{\alpha_{MR}}\hat{n}(t)$. Additionally, $\hat{n}(t)$ follows the distribution of $\mathcal{CN}(0, \frac{\delta^2}{\alpha_{MR}^2} ((\text{Diag}(e_t))H^H H \text{Diag}(e_t))^{-1})$.

Therefore, the received signal expression in (12) could be utilized to extract the angle information $(\theta_{RA}, \phi_{RA})$ at the RIS via the 2D-DFT algorithm [23], which will be described in the next section.
III. ANGLE ESTIMATION ALGORITHM

For the sake of illustration, we consider the estimation errors only for the noise-free scenario similar to [35] and [36], the performance of which is roughly the same as the general scenarios with sufficiently high received signal to noise ratio (SNR).

A. Initial Angle Estimation

Firstly, we define two normalized DFT matrices $F_Ny$ and $F_Nz$, elements of which are written as

$$[F_Ny]_{pp'} = e^{-j \frac{2\pi}{N_y} pp'} \quad (p, p' = 0, 1, \ldots, N_y - 1)$$

and

$$[F_Nz]_{qq'} = e^{-j \frac{2\pi}{N_z} qq'} \quad (q, q' = 0, 1, \ldots, N_z - 1),$$

respectively. Meanwhile, we define the normalized 2D-DFT of the matrix $a_{RA}$ in (12) as

$$a_{RA}^{DFT} = F_Ny a_{RA} F_Nz,$$

whose $(p, q)$th element is calculated as

$$[a_{RA}^{DFT}]_{pq} = \sum_{n_y=0}^{N_y-1} \sum_{n_z=0}^{N_z-1} [a_{RA}]_{pq} e^{-j2\pi \left( \frac{p n_y}{N_y} + \frac{q n_z}{N_z} \right)}$$

$$= e^{j \frac{N_y-1}{2} \left( u_{RA} - \frac{2\pi}{N_y} \right)} e^{j \frac{N_z-1}{2} \left( v_{RA} - \frac{2\pi}{N_z} \right)}$$

$$\times \frac{\sin(\pi p - \frac{N_y u_{RA}}{2})}{\sin((\pi p - \frac{N_y u_{RA}}{2})/N_y)} \cdot \frac{\sin(\pi q - \frac{N_z v_{RA}}{2})}{\sin((\pi q - \frac{N_z v_{RA}}{2})/N_z)}. \quad (13)$$

When the number of reflecting elements becomes infinite, i.e., $N_y \to \infty$, $N_z \to \infty$, there always exist some integers $p_n = \frac{N_y u_{RA}}{2\pi}$, $q_n = \frac{N_z v_{RA}}{2\pi}$ such that $[a_{RA}^{DFT}]_{pnqn} = 1$, while the other elements of $a_{RA}^{DFT}$ are all zero. Therefore, all power is concentrated on the $(p_n, q_n)$th element and $a_{RA}^{DFT}$ is a sparse matrix. However, the RIS size could not be infinitely large, thus $\frac{N_y u_{RA}}{2\pi}$ and $\frac{N_z v_{RA}}{2\pi}$ may not be integers in general, which leads to the channel power leakage from the $(p_n, q_n)$th element to its adjacent element. However, $a_{RA}^{DFT}$ can still be approximated as a sparse matrix with the most power concentrated around the $(p_n, q_n)$th element. Therefore, the peak power position of $a_{RA}^{DFT}$ is still useful for estimating the AOAs at the RIS. Then, the initial estimation is derived as follows:

$$\cos \hat{\Theta}_{RA}^{ini} \cos \hat{\Phi}_{RA}^{ini} = \frac{\lambda p_n}{N_y d_1},$$

$$\sin \hat{\Phi}_{RA}^{ini} = \frac{\lambda q_n}{N_z d_1}, \quad (14)$$

where $\hat{\Theta}_{RA}^{ini}$ and $\hat{\Phi}_{RA}^{ini}$ denote the initial estimated angles at the RIS.
B. Fine Angle Estimation

The resolution of the estimated $\sin \hat{\Phi}_{RA}$ and $\cos \hat{\Theta}_{RA} \cos \hat{\Phi}_{RA}$ is limited by the half of the DFT interval. In order to improve the estimation accuracy, angle rotation is provided to solve the mismatch issue in this subsection [23].

Let us define the angle rotation matrix of $a_{RA}$ as $a_{RA}^{ro}$, expressed as

$$a_{RA}^{ro} = U_{N_y}(\tilde{\omega}_1)a_{RA}U_{N_z}(\tilde{\omega}_2),$$

where the diagonal matrices $U_{N_y}(\tilde{\omega}_1)$ and $U_{N_z}(\tilde{\omega}_2)$ are given by

$$U_{N_y}(\tilde{\omega}_1) = \text{Diag}\{1, e^{j\tilde{\omega}_1}, ..., e^{j(N_y-1)\tilde{\omega}_1}\},$$

$$U_{N_z}(\tilde{\omega}_2) = \text{Diag}\{1, e^{j\tilde{\omega}_2}, ..., e^{j(N_z-1)\tilde{\omega}_2}\},$$

with $\tilde{\omega}_1 \in [-\pi/N_y, \pi/N_y]$ and $\tilde{\omega}_2 \in [-\pi/N_z, \pi/N_z]$ being the angle rotation parameters. By using the angle rotation operation, the 2D-DFT of the rotated matrix $a_{RA}^{ro}_{DFT}$ is calculated as

$$[a_{RA}^{ro}_{DFT}]_{pq} = \sum_{n_y=0}^{N_y-1} \sum_{n_z=0}^{N_z-1} [a_{RA}]_{pq} e^{-j2\pi(p \frac{N_y}{N_y} + q \frac{N_z}{N_z})} e^{j2\pi(p \frac{\tilde{\omega}_1}{2\pi} + q \frac{\tilde{\omega}_2}{2\pi})}$$

$$= e^{j\frac{N_y-1}{2}(u_{RA}+\tilde{\omega}_1-2\pi\frac{p}{N_y})} e^{j\frac{N_z-1}{2}(u_{RA}+\tilde{\omega}_2-2\pi\frac{q}{N_z})}$$

$$\times \frac{\sin(\pi p - N_y u_{RA} - N_y \tilde{\omega}_1)}{\sin((\pi p - N_y u_{RA} - N_y \tilde{\omega}_1)/N_y)} \frac{\sin(\pi q - N_z v_{RA} - N_z \tilde{\omega}_2)}{\sin((\pi q - N_z v_{RA} - N_z \tilde{\omega}_2)/N_z)},$$

where $\tilde{\omega}_1$ and $\tilde{\omega}_2$ could be optimized with the one-dimensional search. Then, we could obtain the estimated results as

$$\cos \hat{\Theta}_{RA} \cos \hat{\Phi}_{RA} = \frac{\lambda p_n}{N_y d_1} - \frac{\lambda \tilde{\omega}_1}{2\pi d_1},$$

$$\sin \hat{\Phi}_{RA} = \frac{\lambda q_n}{N_z d_1} - \frac{\lambda \tilde{\omega}_2}{2\pi d_1},$$

where $\hat{\Theta}_{RA}$ and $\hat{\Phi}_{RA}$ denote the final estimated azimuth and elevation AOAs after angle rotation, respectively. Furthermore, $\hat{\Theta}_{RA}$ and $\hat{\Phi}_{RA}$ could be expressed as

$$\hat{\Phi}_{RA} = \arcsin\left(\frac{\lambda q_n}{N_z d_1} - \frac{\lambda \tilde{\omega}_2}{2\pi d_1}\right),$$

$$\hat{\Theta}_{RA} = \arccos\left(\frac{\lambda p_n}{N_y d_1} - \frac{\lambda \tilde{\omega}_1}{2\pi d_1}\right) \sqrt{1 - \left(\frac{\lambda q_n}{N_z d_1} - \frac{\lambda \tilde{\omega}_2}{2\pi d_1}\right)^2}\right).$$
IV. PDF OF ANGLE ESTIMATION ERROR

In this section, we aim to derive the PDF of the angle estimation error, which will be used for deriving the angle estimation errors’ variance that is useful for the position error analysis in Part II of this series of work.

A. PDF of $\hat{\Phi}_{RA}$

From the above section, we can find that $\tilde{\omega}_2$ could be obtained by the one-dimensional search in the interval of $[-\frac{\pi}{N_z}, \frac{\pi}{N_z}]$. We assume that there are $S_2$ grids points in the interval $[-\frac{\pi}{N_z}, \frac{\pi}{N_z}]$ and $s_2 \in \{1, \cdots, S_2\}$ is the optimal point. Therefore, the optimal solution for the one-dimensional search is $\tilde{\omega}_2 = \frac{2\pi s_2}{N_z S_2}$, and the estimation of $\sin \Phi_{RA}$ is thus written as

$$\sin \hat{\Phi}_{RA} = \frac{\lambda q_n}{N_z d_1} - \frac{\lambda \tilde{\omega}_2}{2\pi d_1} = \frac{\lambda q_n}{N_z d_1} - \frac{\lambda s_2}{N_z d_1 S_2}. \quad (20)$$

For notation simplicity, let us define $\hat{Y} = \sin \hat{\Phi}_{RA}$ and $Y = \sin \Phi_{RA}$. Then, the estimation of $\Phi_{RA}$ could be expressed as

$$\hat{\Phi}_{RA} = \arcsin \hat{Y} = \arcsin \left( \frac{\lambda q_n}{N_z d_1} - \frac{\lambda s_2}{N_z d_1 S_2} \right). \quad (21)$$

According to (20) and the property of the one-dimensional search method, the value of $Y$ follows the uniform distribution within the region of $[\hat{Y} - a, \hat{Y} + a]$, where $a = \frac{\lambda}{2N_z d_1 S_2}$, which is given by

$$f_Y(y) = \begin{cases} 
\frac{1}{2a}, & \hat{Y} - a \leq y \leq \hat{Y} + a \\
0, & \text{others}. 
\end{cases} \quad (22)$$

By denoting the estimation error of $Y$ as $\tilde{Y}$, we have $\hat{Y} = Y + \tilde{Y}$. Then, the distribution of $\tilde{Y}$ is given by

$$f_{\tilde{Y}}(\tilde{y}) = \begin{cases} 
\frac{1}{2a}, & -a \leq \tilde{y} \leq a \\
0, & \text{others}. 
\end{cases} \quad (23)$$

Since $\Phi_{RA} = \arcsin Y$, the cumulative density function (CDF) of $\Phi_{RA}$ is derived as

$$F_{\Phi_{RA}}(\phi_{RA}) = \Pr(\Phi_{RA} \leq \phi_{RA}) = \Pr(\arcsin Y \leq \phi_{RA}) = \Pr(Y \leq \sin \phi_{RA}). \quad (24)$$
By assuming that $\Phi_{RA} \in (-\frac{\pi}{2}, \frac{\pi}{2})$, the PDF of $\Phi_{RA}$ could be derived as

$$f_{\Phi_{RA}}(\phi_{RA}) = \frac{\partial F_{\Phi_{RA}}(\phi_{RA})}{\partial \phi_{RA}} = \cos \phi_{RA} f_Y(\sin \phi_{RA})$$

$$= \begin{cases} 
\frac{\cos \phi_{RA}}{2a}, & \arcsin(\hat{Y} - a) \leq \phi_{RA} \leq \arcsin(\hat{Y} + a) \\
0, & \text{others.}
\end{cases} \quad (25)$$

As we have $\hat{\Phi}_{RA} = \Phi_{RA} + \tilde{\Phi}_{RA}$, the CDF of the estimation error $\tilde{\Phi}_{RA}$ is calculated as

$$F_{\tilde{\Phi}_{RA}}(\tilde{\phi}_{RA}) = \Pr(\tilde{\Phi}_{RA} \leq \tilde{\phi}_{RA}) = \Pr(\hat{\Phi}_{RA} - \Phi_{RA} \leq \tilde{\phi}_{RA})$$

$$= \Pr(\Phi_{RA} \geq \hat{\Phi}_{RA} - \tilde{\phi}_{RA}) = 1 - \Pr(\Phi_{RA} \leq \hat{\Phi}_{RA} - \tilde{\phi}_{RA}). \quad (26)$$

Define $a_1 = \arcsin(\hat{Y} + a)$ and $a_2 = \arcsin(\hat{Y} - a)$. Based on (26), the PDF of $\tilde{\Phi}_{RA}$ is written as

$$f_{\tilde{\Phi}_{RA}}(\tilde{\phi}_{RA}) = \frac{\partial F_{\tilde{\Phi}_{RA}}(\tilde{\phi}_{RA})}{\partial \tilde{\phi}_{RA}} = f_{\Phi_{RA}}(\hat{\Phi}_{RA} - \tilde{\phi}_{RA})$$

$$= \begin{cases} 
\frac{\cos(\Phi_{RA} - \tilde{\phi}_{RA})}{2a}, & \hat{\Phi}_{RA} - a_1 \leq \tilde{\phi}_{RA} \leq \hat{\Phi}_{RA} - a_2 \\
0, & \text{others.}
\end{cases} \quad (27)$$

Moreover, as the estimation techniques are relatively mature, it is assumed that the estimation error is very small. Therefore, we have $\sin(\tilde{\phi}_{RA}) \approx \tilde{\phi}_{RA}$ and $\cos(\tilde{\phi}_{RA}) \approx 1$. Consequently, we have the following approximation:

$$\cos(\hat{\Phi}_{RA} - \tilde{\phi}_{RA}) = \cos \hat{\Phi}_{RA} \cos \tilde{\phi}_{RA} + \sin \hat{\Phi}_{RA} \sin \tilde{\phi}_{RA} \approx \cos \hat{\Phi}_{RA} + \tilde{\phi}_{RA} \sin \hat{\Phi}_{RA}. \quad (28)$$

Then, the PDF of $\tilde{\Phi}_{RA}$ is approximated as

$$f_{\tilde{\Phi}_{RA}}(\tilde{\phi}_{RA}) \approx \begin{cases} 
\frac{\cos \hat{\Phi}_{RA} + \tilde{\phi}_{RA} \sin \hat{\Phi}_{RA}}{2a}, & \hat{\Phi}_{RA} - a_1 \leq \tilde{\phi}_{RA} \leq \hat{\Phi}_{RA} - a_2 \\
0, & \text{others.}
\end{cases} \quad (29)$$

**B. PDF of $\tilde{\Theta}_{RA}$**

In this subsection, we derive the PDF of the estimation error $\tilde{\Theta}_{RA}$. As the derivations are complicated, we summarize the main procedure in Algorithm [1].

1) **PDF of $\cos \Phi_{RA}$**: First of all, we need to derive the PDF of $\cos \Phi_{RA}$, so that the PDF of $\cos \Theta_{RA}$ could be calculated.
Algorithm 1 Algorithm of deriving the PDF of $\tilde{\Phi}_{RA}$

1: Derive the PDF of $\cos \Phi_{RA}$ using the PDF of $\tilde{Y}$ in (23);
2: Derive the PDF of $\cos \Theta_{RA}$ for two cases by using the PDF of $\cos \Phi_{RA}$ in (36) and the PDF of $\cos \Phi_{RA} \cos \Theta_{RA}$ in (38);
3: Derive the PDF of $\tilde{\Theta}_{RA}$ based on the two cases of the PDF of $\cos \Theta_{RA}$ in (47) and (48).

Let $X = \cos \Phi_{RA}$, which can be expressed as a function of $\tilde{Y}$ as follows:

$$X = \cos \Phi_{RA} = \sqrt{1 - \sin^2 \Phi_{RA}} = \sqrt{1 - Y^2} = \sqrt{1 - (\hat{Y} - \tilde{Y})^2}. \quad (30)$$

By defining the estimation of $X$ as $\hat{X} = \cos \hat{\Phi}_{RA} = X + \tilde{X}$, the estimation error $\tilde{X}$ could be calculated as:

$$\tilde{X} = \hat{X} - \sqrt{1 - (\hat{Y} - \tilde{Y})^2}. \quad (31)$$

However, the expression (31) is complicated and thus challenging to derive a compact form of the PDF of $\tilde{X}$. Fortunately, since the value of $\tilde{Y}$ is relatively small, we can approximate $\tilde{X}$ in (31) by using the Taylor expansion, which is given by

$$\tilde{X} = \sqrt{1 - \tilde{Y}^2} - \sqrt{1 - (\hat{Y} - \tilde{Y})^2} \approx \sqrt{1 - \hat{Y}^2} - (1 - \hat{Y}^2) \frac{\hat{Y}}{1 - \hat{Y}^2} = -\frac{\hat{Y}}{\hat{X}} \tilde{Y}. \quad (32)$$

Utilizing (32), the CDF of $\tilde{X}$ could be calculated by

$$F_{\tilde{X}}(\tilde{x}) = \Pr(\tilde{X} \leq \tilde{x}) \approx \Pr\left(-\frac{\hat{Y}}{\hat{X}} \tilde{Y} \leq \tilde{x}\right) = \Pr\left(\tilde{Y} \geq -\frac{\hat{X}}{\hat{Y}} \tilde{x}\right) = 1 - \Pr\left(\tilde{Y} \leq -\frac{\hat{X}}{\hat{Y}} \tilde{x}\right). \quad (33)$$

Then, by using (33), the PDF of $\tilde{X}$ could be calculated as

$$f_{\tilde{X}}(\tilde{x}) = -\left(-\frac{\hat{X}}{\hat{Y}}\right) f_{\tilde{Y}}\left(-\frac{\hat{X}}{\hat{Y}} \tilde{x}\right) = \begin{cases} \frac{\tilde{x}}{2aY}, & -\frac{\hat{Y}}{\hat{X}} a \leq \tilde{x} \leq \frac{\hat{Y}}{\hat{X}} a \\ 0, & \text{others}. \end{cases} \quad (34)$$

Furthermore, by using $\hat{X} = X + \tilde{X}$, the CDF of $X$ can be calculated as

$$F_X(x) = \Pr(X \leq x) = \Pr(\hat{X} - \tilde{X} \leq x) = \Pr(\tilde{X} \geq \hat{X} - x) = 1 - \Pr(\tilde{X} \leq \hat{X} - x). \quad (35)$$
By using (34) and (35), the PDF of $X$ could be written as

$$f_X(x) = \frac{\partial F_X(x)}{\partial x} = f_X(\hat{X} - x) = \begin{cases} \frac{\hat{x}}{2a\sqrt{\hat{X}}}, & \frac{\hat{x}}{a} \leq x \leq \hat{X} - \frac{\hat{y}}{\hat{X}}a \\ 0, & \text{others.} \end{cases} \quad (36)$$

2) PDF of $\cos \Theta_{RA}$: By using the PDF of $\cos \Phi_{RA}$ and $\cos \Phi_{RA} \cos \Theta_{RA}$, we can derive the PDF of $\cos \Theta_{RA}$ as follows.

Firstly, from the above section, we know that $\tilde{\zeta}_1$ could be obtained by using the one-dimensional search in the interval of $[-\frac{\pi}{N_y}, \frac{\pi}{N_y}]$. Similarly, we assume that there are $S_1$ grids points in the interval $[-\frac{\pi}{N_y}, \frac{\pi}{N_y}]$ and $s_1 \in \{1, \cdots, S_1\}$ is the optimal point. Therefore, the optimal solution for the one-dimensional search is $\tilde{\zeta}_1 = \frac{2\pi s_1}{N_yS_1}$, and the estimated $\cos \Phi_{RA} \cos \Theta_{RA}$ is given by

$$\hat{Z} = \cos \Theta_{RA} \cos \Phi_{RA} = \frac{\lambda p_n}{N_yd_1} - \frac{\lambda \tilde{\zeta}_1}{2\pi d_1} = \frac{\lambda p_n}{N_yd_1} - \frac{\lambda s_1}{N_yd_1S_1}. \quad (37)$$

By using (37) and the nature of the one-dimensional search method, the real value of $Z = \cos \Theta_{RA} \cos \Phi_{RA}$ follows uniform distribution within the region of $[\hat{Z} - b, \hat{Z} + b]$, where $b = \frac{\lambda}{2Nyd_1S_1}$. The PDF of $Z$ is thus given by

$$f_Z(z) = \begin{cases} \frac{1}{2b}, & \hat{Z} - b \leq z \leq \hat{Z} + b \\ 0, & \text{others.} \end{cases} \quad (38)$$

By denoting the estimation error of $Z$ as $\tilde{Z}$, we have $\hat{Z} = Z + \tilde{Z}$. Then, the distribution of $\tilde{Z}$ is given by

$$f_{\tilde{Z}}(\tilde{z}) = \begin{cases} \frac{1}{2b}, & -b \leq \tilde{z} \leq b \\ 0, & \text{others.} \end{cases} \quad (39)$$

Next, for simplicity, let us denote $U = \cos \Theta_{RA}$. Then, by utilizing the definition of $Z$ below (37) and $X$ above (30), we have $Z = UX$. By combining the PDF of $X$ in (36) and the PDF of $Z$ in (38), we could derive the PDF of $U$ as follows

$$f_U(u) = \int_{\hat{X} - \frac{\hat{y}}{\hat{X}}a}^{\hat{X} + \frac{\hat{y}}{\hat{X}}a} xf_X(x)f_Z(ux)dx. \quad (40)$$

According to (38), it is observed that $f_Z(ux)$ is non-zero when $\hat{Z} - b \leq ux \leq \hat{Z} + b$, which determines the PDF of $U$. Therefore, we need to discuss the different conditions according to
the non-zero intervals of \( f_Z(ax) \) and \( f_X(x) \) in the following. For notation brevity, we denote 
\[ \alpha_1 = \hat{X} - \frac{\hat{Y}}{X} a, \quad \alpha_2 = \hat{X} + \frac{\hat{Y}}{X} a, \quad \beta_1 = \hat{Z} - b \quad \text{and} \quad \beta_2 = \hat{Z} + b. \]

Condition 1: If \( \alpha_1 < \frac{\beta_1}{u} \leq \alpha_2 < \frac{\beta_2}{u} \), the integral interval of \( x \) in (40) can be recast as \( \left[ \frac{\beta_1}{u}, \alpha_2 \right] \), thereby yielding the PDF of \( U \) as 
\[ f_U(u) = \int_{\frac{\beta_1}{u}}^{\alpha_2} \frac{1}{2b} \cdot \frac{\hat{X}}{2aY} \cdot x dx = \frac{\hat{X}}{4abY} \int_{\frac{\beta_1}{u}}^{\alpha_2} x dx = \frac{\hat{X}}{8abY} \left( \alpha_2^2 - \left( \frac{\beta_1}{u} \right)^2 \right). \] (41)

Furthermore, the interval of \( u \) is given by 
\[ \left[ \frac{\beta_1}{\alpha_2}, \frac{\beta_1}{\alpha_1} \right] \cap \left( -\infty, \frac{\beta_2}{\alpha_2} \right). \] (42)

Based on (42), it is necessary to compare \( \frac{\beta_1}{\alpha_1} \) and \( \frac{\beta_1}{\alpha_2} \), so that the interval of \( u \) can be further determined. If \( \frac{\beta_1}{\alpha_2} > \frac{\beta_1}{\alpha_1} \) holds, the interval can be rewritten as \( \left[ \frac{\beta_1}{\alpha_2}, \frac{\beta_1}{\alpha_1} \right] \). Otherwise, the interval is \( \left[ \frac{\beta_1}{\alpha_1}, \frac{\beta_1}{\alpha_2} \right] \).

Condition 2: If \( \frac{\beta_1}{u} \leq \alpha_1 < \frac{\beta_2}{u} \leq \alpha_2 \), the integral interval of \( x \) in (40) can be recast as \( \left[ \alpha_1, \frac{\beta_2}{u} \right] \), thus the PDF of \( U \) is given by 
\[ f_U(u) = \int_{\alpha_1}^{\frac{\beta_2}{u}} \frac{1}{2b} \cdot \frac{\hat{X}}{2aY} \cdot x dx = \frac{\hat{X}}{4abY} \int_{\alpha_1}^{\frac{\beta_2}{u}} x dx = \frac{\hat{X}}{8abY} \left( \left( \frac{\beta_2}{u} \right)^2 - \alpha_1^2 \right). \] (43)

The interval of \( u \) is given by 
\[ \left[ \frac{\beta_2}{\alpha_2}, \frac{\beta_2}{\alpha_1} \right] \cap \left[ \frac{\beta_1}{\alpha_1}, +\infty \right]. \] (44)

As a result, if \( \frac{\beta_2}{\alpha_2} > \frac{\beta_1}{\alpha_1} \) holds, the interval can be further recast as \( \left[ \frac{\beta_2}{\alpha_2}, \frac{\beta_2}{\alpha_1} \right] \). Otherwise, the interval can be derived as \( \left[ \frac{\beta_1}{\alpha_1}, \frac{\beta_2}{\alpha_2} \right] \).

Condition 3: If \( \frac{\beta_1}{u} \leq \alpha_1 < \alpha_2 < \frac{\beta_2}{u} \), the integral interval of \( x \) in (40) can be derived as \( \left[ \alpha_1, \alpha_2 \right] \). Thus, the PDF of \( U \) is 
\[ f_U(u) = \int_{\alpha_1}^{\alpha_2} \frac{1}{2b} \cdot \frac{\hat{X}}{2aY} \cdot x dx = \frac{\hat{X}}{4abY} \int_{\alpha_1}^{\alpha_2} x dx = \frac{\hat{X}}{8abY} \left( \alpha_2^2 - \alpha_1^2 \right) \] 
\[ = \frac{\hat{X}}{8abY} \left( \left( \frac{\hat{Y}a}{X} \right)^2 - \left( \frac{\hat{Y}a}{X} \right)^2 \right) = \frac{\hat{X}}{2b}. \] (45)

The interval of \( u \) is accordingly given by \( \left[ \frac{\beta_1}{\alpha_1}, \frac{\beta_2}{\alpha_2} \right] \).

Condition 4: If \( \alpha_1 < \frac{\beta_1}{u} < \frac{\beta_2}{u} \leq \alpha_2 \), the integral interval of \( x \) in (40) can be derived as
Accordingly, the PDF of \( U \) is

\[
f_U(u) = \int_{\frac{\beta_2}{\alpha_1}}^{\frac{\beta_1}{\alpha_1}} \frac{\hat{X}}{4abY} \cdot x \text{d}x = \frac{\hat{X}}{4abY} \int_{\frac{\beta_2}{\alpha_1}}^{\frac{\beta_1}{\alpha_1}} x \text{d}x = \frac{\hat{X}}{8abY} \left( \left( \frac{\beta_2}{u} \right)^2 - \left( \frac{\beta_1}{u} \right)^2 \right) = \frac{\hat{X} \hat{Z}}{2aY u^2}. \tag{46}\]

Additionally, after some mathematical manipulations, we can derive the interval of \( u \) as \( \left[ \frac{\beta_2}{\alpha_2}, \frac{\beta_1}{\alpha_1} \right] \).

Based on the above discussions, by comparing \( \frac{\beta_2}{\alpha_2} \) with \( \frac{\beta_1}{\alpha_1} \), the PDF of \( U \) can be simplified as the following two cases:

Case 1: If \( \frac{\beta_2}{\alpha_2} > \frac{\beta_1}{\alpha_1} \) holds, the intervals of \( u \) in Condition 1 and Condition 2 are given by \( \left[ \frac{\beta_1}{\alpha_1}, \frac{\beta_2}{\alpha_2} \right] \) and \( \left[ \frac{\beta_2}{\alpha_2}, \frac{\beta_1}{\alpha_1} \right] \), respectively. Moreover, Condition 4 is valid, whereas Condition 3 is invalid. Therefore, the PDF of \( U \) is written as

\[
f_U(u) = \begin{cases} \frac{\hat{X}}{8abY} \left( \alpha_2^2 - \left( \frac{\beta_1}{u} \right)^2 \right), & \frac{\beta_1}{\alpha_1} \leq u < \frac{\beta_2}{\alpha_2} \\ \frac{\hat{X}}{2b}, & \frac{\beta_2}{\alpha_2} \leq u < \frac{\beta_1}{\alpha_1} \\ \frac{\hat{X}}{8abY} \left( \left( \frac{\beta_2}{u} \right)^2 - \alpha_1^2 \right), & \frac{\beta_2}{\alpha_2} \leq u \leq \frac{\beta_1}{\alpha_1} \\ 0, & \text{others}. \end{cases} \tag{47}\]

Case 2: If \( \frac{\beta_2}{\alpha_2} < \frac{\beta_1}{\alpha_1} \) holds, the intervals of \( u \) in Condition 1 and Condition 2 are given by \( \left[ \frac{\beta_1}{\alpha_1}, \frac{\beta_2}{\alpha_2} \right] \) and \( \left[ \frac{\beta_2}{\alpha_2}, \frac{\beta_1}{\alpha_1} \right] \), respectively. Moreover, Condition 4 is valid, while Condition 3 is invalid. Accordingly, the PDF of \( U \) can be derived as

\[
f_U(u) = \begin{cases} \frac{\hat{X}}{8abY} \left( \alpha_2^2 - \left( \frac{\beta_1}{u} \right)^2 \right), & \frac{\beta_1}{\alpha_1} \leq u < \frac{\beta_2}{\alpha_2} \\ \frac{\hat{X} \hat{Z}}{2aY u^2}, & \frac{\beta_2}{\alpha_2} \leq u < \frac{\beta_1}{\alpha_1} \\ \frac{\hat{X}}{8abY} \left( \left( \frac{\beta_2}{u} \right)^2 - \alpha_1^2 \right), & \frac{\beta_2}{\alpha_2} \leq u \leq \frac{\beta_1}{\alpha_1} \\ 0, & \text{others}. \end{cases} \tag{48}\]

3) PDF of \( \Theta_{RA} \): By using the PDF of \( \cos \Theta_{RA} \), the PDF of \( \Theta_{RA} \) can be derived as follows.

First of all, the CDF of \( \Theta_{RA} \) can be derived as follows

\[
F_{\Theta_{RA}}(\theta_{RA}) = \Pr(\Theta_{RA} \leq \theta_{RA}) = \Pr(\arccos U \leq \theta_{RA}) = 1 - \Pr(U \leq \cos \theta_{RA}). \tag{49}\]

Then, the PDF of \( \Theta_{RA} \) is obtained as follows

\[
f_{\Theta_{RA}}(\theta_{RA}) = \frac{\partial F_{\Theta_{RA}}(\theta_{RA})}{\partial \theta_{RA}} = \sin \theta_{RA} f_U(\cos \theta_{RA}). \tag{50}\]

Furthermore, we consider the indoor positioning system in this paper, where the RISs are supposed to be mounted on the wall. Hence we have \( \Theta_{RA} \in (0, \pi) \). Thus, \( \Theta_{RA} \) decreases...
monotonically with $U$. According to the PDF of $U$ in the aforementioned two cases, we can derive the PDF of $\Theta_{RA}$ in the following.

Case 1: When $\frac{\beta_2}{\alpha_2} > \frac{\beta_1}{\alpha_1}$, according to (47), we can derive the PDF of $\Theta_{RA}$, which is given by

$$f_{\Theta_{RA}}(\theta_{RA}) = \begin{cases} \frac{X \sin \theta_{RA}}{8ab Y} \left( \frac{\beta_2}{\cos \theta_{RA}} \right)^2 - \alpha_1^2, & \text{arccos} \frac{\beta_2}{\alpha_1} \leq \theta_{RA} \leq \text{arccos} \frac{\beta_2}{\alpha_2} \\ \frac{X \sin \theta_{RA}}{2b}, & \text{arccos} \frac{\beta_2}{\alpha_2} < \theta_{RA} \leq \text{arccos} \frac{\beta_1}{\alpha_2} \\ \frac{X \sin \theta_{RA}}{8ab Y} \left( \alpha_2^2 - \left( \frac{\beta_1}{\cos \theta_{RA}} \right)^2 \right), & \text{arccos} \frac{\beta_1}{\alpha_2} < \theta_{RA} \leq \text{arccos} \frac{\beta_1}{\alpha_1} \\ 0, & \text{others}. \end{cases}$$ (51)

Case 2: When $\frac{\beta_2}{\alpha_2} < \frac{\beta_1}{\alpha_1}$, we can derive the PDF of $\Theta_{RA}$ based on (48), which is given by

$$f_{\Theta_{RA}}(\theta_{RA}) = \begin{cases} \frac{X \sin \theta_{RA}}{8ab Y} \left( \frac{\beta_2}{\cos \theta_{RA}} \right)^2 - \alpha_1^2, & \text{arccos} \frac{\beta_2}{\alpha_1} \leq \theta_{RA} \leq \text{arccos} \frac{\beta_2}{\alpha_2} \\ \frac{X \sin \theta_{RA}}{2aY(\cos \theta_{RA})^2}, & \text{arccos} \frac{\beta_1}{\alpha_1} \leq \theta_{RA} < \text{arccos} \frac{\beta_2}{\alpha_2} \\ \frac{X \sin \theta_{RA}}{8ab Y} \left( \alpha_2^2 - \left( \frac{\beta_1}{\cos \theta_{RA}} \right)^2 \right), & \text{arccos} \frac{\beta_2}{\alpha_2} < \theta_{RA} \leq \text{arccos} \frac{\beta_1}{\alpha_2} \\ 0, & \text{others}. \end{cases}$$ (52)

4) PDF of $\Theta_{RA}$: By using the CDF and PDF of $\Theta_{RA}$ in (49), (51) and (52), we can calculate the CDF and PDF of $\Theta_{RA}$ as follows.

Firstly, based on $\Theta_{RA} = \Theta_{RA} + \tilde{\Theta}_{RA}$, the CDF of $\Theta_{RA}$ can be derived as follows

$$F_{\Theta_{RA}}(\theta_{RA}) = \Pr(\Theta_{RA} \leq \tilde{\theta}_{RA}) = \Pr(\Theta_{RA} - \Theta_{RA} \leq \tilde{\Theta}_{RA}) = \Pr(\Theta_{RA} \geq \tilde{\Theta}_{RA} - \tilde{\theta}_{RA}) = 1 - \Pr(\Theta_{RA} \leq \tilde{\Theta}_{RA} - \tilde{\theta}_{RA}).$$ (53)
Case 1: When $\frac{\beta_2}{\alpha_2} > \frac{\beta_1}{\alpha_1}$, by using (51), the PDF of $\tilde{\Theta}_{RA}$ is calculated as

$$f_{\tilde{\Theta}_{RA}}(\tilde{\Theta}_{RA}) = \frac{\partial F_{\tilde{\Theta}_{RA}}(\tilde{\Theta}_{RA})}{\partial \tilde{\Theta}_{RA}} = f_{\Theta_{RA}}(\tilde{\Theta}_{RA} - \tilde{\Theta}_{RA})$$

$$= \begin{cases} \frac{\dot{X} \sin(\Theta_{RA} - \tilde{\Theta}_{RA})}{8abY} \left[ \alpha_1^2 - \left( \frac{\beta_1}{\cos(\Theta_{RA} - \tilde{\Theta}_{RA})} \right)^2 \right], & \tilde{\Theta}_{RA} - \arccos \frac{\beta_1}{\alpha_1} \leq \tilde{\Theta}_{RA} < \tilde{\Theta}_{RA} - \arccos \frac{\beta_2}{\alpha_2} \\ \frac{\dot{X} \sin(\Theta_{RA} - \tilde{\Theta}_{RA})}{2b \sqrt{\alpha_2}}, & \tilde{\Theta}_{RA} - \arccos \frac{\beta_1}{\alpha_1} \leq \tilde{\Theta}_{RA} < \tilde{\Theta}_{RA} - \arccos \frac{\beta_2}{\alpha_2} \\ \frac{\dot{X} \sin(\Theta_{RA} - \tilde{\Theta}_{RA})}{8abY} \left[ \frac{\beta_2}{\cos(\Theta_{RA} - \tilde{\Theta}_{RA})} \right]^2 - \alpha_1^2, & \tilde{\Theta}_{RA} - \arccos \frac{\beta_1}{\alpha_1} \leq \tilde{\Theta}_{RA} \leq \tilde{\Theta}_{RA} - \arccos \frac{\beta_2}{\alpha_2} \\ 0, & \text{others.} \end{cases}$$

(54)

Case 2: When $\frac{\beta_2}{\alpha_2} < \frac{\beta_1}{\alpha_1}$, we can derive the PDF of $\tilde{\Theta}_{RA}$ by using (52), which is given by

$$f_{\tilde{\Theta}_{RA}}(\tilde{\Theta}_{RA}) = \frac{\partial F_{\tilde{\Theta}_{RA}}(\tilde{\Theta}_{RA})}{\partial \tilde{\Theta}_{RA}} = f_{\Theta_{RA}}(\tilde{\Theta}_{RA} - \tilde{\Theta}_{RA})$$

$$= \begin{cases} \frac{\dot{X} \sin(\Theta_{RA} - \tilde{\Theta}_{RA})}{8abY} \left[ \alpha_2^2 - \left( \frac{\beta_1}{\cos(\Theta_{RA} - \tilde{\Theta}_{RA})} \right)^2 \right], & \tilde{\Theta}_{RA} - \arccos \frac{\beta_1}{\alpha_1} \leq \tilde{\Theta}_{RA} < \tilde{\Theta}_{RA} - \arccos \frac{\beta_2}{\alpha_2} \\ \frac{\dot{X} \sin(\Theta_{RA} - \tilde{\Theta}_{RA})}{2aY \sqrt{\cos(\Theta_{RA} - \tilde{\Theta}_{RA})}} \left( \frac{\beta_2}{\cos(\Theta_{RA} - \tilde{\Theta}_{RA})} \right)^2, & \tilde{\Theta}_{RA} - \arccos \frac{\beta_1}{\alpha_1} \leq \tilde{\Theta}_{RA} < \tilde{\Theta}_{RA} - \arccos \frac{\beta_2}{\alpha_2} \\ \frac{\dot{X} \sin(\Theta_{RA} - \tilde{\Theta}_{RA})}{8abY} \left[ \frac{\beta_1}{\cos(\Theta_{RA} - \tilde{\Theta}_{RA})} \right]^2 - \alpha_1^2, & \tilde{\Theta}_{RA} - \arccos \frac{\beta_1}{\alpha_1} \leq \tilde{\Theta}_{RA} \leq \tilde{\Theta}_{RA} - \arccos \frac{\beta_2}{\alpha_2} \\ 0, & \text{others.} \end{cases}$$

(55)

To facilitate the error analysis, we now aim to derive the approximation of $f_{\tilde{\Theta}_{RA}}(\tilde{\Theta}_{RA})$ in this paper. As we have assumed that the estimation error is very small, we have $\sin \tilde{\Theta}_{RA} \approx \tilde{\Theta}_{RA}$ and $\cos \tilde{\Theta}_{RA} \approx 1$, leading to

$$\sin(\tilde{\Theta}_{RA} - \tilde{\Theta}_{RA}) = \sin \tilde{\Theta}_{RA} \cos \tilde{\Theta}_{RA} - \sin \tilde{\Theta}_{RA} \cos \tilde{\Theta}_{RA} \approx \sin \tilde{\Theta}_{RA} - \tilde{\Theta}_{RA} \cos \tilde{\Theta}_{RA},$$

$$\cos(\tilde{\Theta}_{RA} - \tilde{\Theta}_{RA}) = \cos \tilde{\Theta}_{RA} \cos \tilde{\Theta}_{RA} + \sin \tilde{\Theta}_{RA} \sin \tilde{\Theta}_{RA} \approx \cos \tilde{\Theta}_{RA} + \tilde{\Theta}_{RA} \sin \tilde{\Theta}_{RA}.$$  

(56)

Using the approximations in (56), we can derive the approximation of $f_{\tilde{\Theta}_{RA}}(\tilde{\Theta}_{RA})$ according to the above two cases. Furthermore, by denoting $\sin \tilde{\Theta}_{RA} = \hat{V}$, $\cos \tilde{\Theta}_{RA} = \hat{U}$, $B_1 = \tilde{\Theta}_{RA} - \arccos \frac{\beta_1}{\alpha_2}$, $B_2 = \tilde{\Theta}_{RA} - \arccos \frac{\beta_1}{\alpha_1}$, $B_3 = \tilde{\Theta}_{RA} - \arccos \frac{\beta_2}{\alpha_2}$, $B_4 = \tilde{\Theta}_{RA} - \arccos \frac{\beta_2}{\alpha_1}$, the expression could be further simplified in the following.

Case 1: When $\frac{\beta_2}{\alpha_2} > \frac{\beta_1}{\alpha_1}$, based on (54), we can derive the approximation of $f_{\tilde{\Theta}_{RA}}(\tilde{\Theta}_{RA})$, which
is written as

\[
\varphi_{\tilde{\theta}_{RA}}(\tilde{\theta}_{RA}) \approx \begin{cases} 
\frac{\hat{X}(\sin \tilde{\theta}_{RA} - \tilde{\theta}_{RA} \cos \tilde{\Theta}_{RA})}{8abY} \left[ \alpha_2^2 - \left( \frac{\beta_1}{\cos \tilde{\Theta}_{RA} + \tilde{\theta}_{RA} \sin \tilde{\Theta}_{RA}} \right)^2 \right], & B_1 \leq \tilde{\theta}_{RA} < B_2 \\
\frac{\hat{X}(\sin \tilde{\theta}_{RA} - \tilde{\theta}_{RA} \cos \tilde{\Theta}_{RA})}{2b} \left[ \left( \frac{\beta_2}{\cos \tilde{\Theta}_{RA} + \Delta \tilde{\theta}_{RA} \sin \tilde{\Theta}_{RA}} \right)^2 - \alpha_1^2 \right], & B_2 \leq \tilde{\theta}_{RA} < B_3 \\
0, & B_3 \leq \tilde{\theta}_{RA} \leq B_4 
\end{cases}
\]

Case 2: When \( \frac{\beta_2}{\alpha_2} < \frac{\beta_1}{\alpha_1} \), as we have (55), we could derive the approximation of \( \varphi_{\tilde{\theta}_{RA}}(\tilde{\theta}_{RA}) \), which is written as

\[
\varphi_{\tilde{\theta}_{RA}}(\tilde{\theta}_{RA}) \approx \begin{cases} 
\frac{\hat{X}(\sin \tilde{\theta}_{RA} - \tilde{\theta}_{RA} \cos \tilde{\Theta}_{RA})}{8abY} \left[ \alpha_2^2 - \left( \frac{\beta_1}{\cos \tilde{\Theta}_{RA} + \tilde{\theta}_{RA} \sin \tilde{\Theta}_{RA}} \right)^2 \right], & B_1 \leq \tilde{\theta}_{RA} < B_3 \\
\frac{\hat{X}(\sin \tilde{\theta}_{RA} - \tilde{\theta}_{RA} \cos \tilde{\Theta}_{RA})}{2aY(\cos \tilde{\Theta}_{RA} + \tilde{\theta}_{RA} \sin \tilde{\Theta}_{RA})^2} \left[ \left( \frac{\beta_2}{\cos \tilde{\Theta}_{RA} + \Delta \tilde{\theta}_{RA} \sin \tilde{\Theta}_{RA}} \right)^2 - \alpha_1^2 \right], & B_3 \leq \tilde{\theta}_{RA} < B_2 \\
0, & B_2 \leq \tilde{\theta}_{RA} \leq B_4 
\end{cases}
\]

V. VARIANCE OF ANGLE ESTIMATION ERROR

In this section, we aim to calculate the variance of \( \tilde{\Phi}_{RA} \) and \( \tilde{\Theta}_{RA} \) by using the PDF in the above section, which will be used for the positioning estimation error analysis in Part II of this series of work.
A. Variance of $\tilde{\Phi}_{RA}$

In this subsection, we provide the variance expression of $\tilde{\Phi}_{RA}$. Based on the PDF of $\tilde{\Phi}_{RA}$ in (29), the variance of $\tilde{\Phi}_{RA}$ can be calculated as

$$D(\tilde{\phi}_{RA}) = E(\tilde{\phi}_{RA}^2) - (E(\tilde{\phi}_{RA}))^2$$

$$= \int_{\hat{\phi}_{RA} - a_1}^{\hat{\phi}_{RA} - a_2} \tilde{\phi}_{RA}^2 f_{\tilde{\phi}_{RA}}(\tilde{\phi}_{RA}) d\tilde{\phi}_{RA} - \left( \int_{\hat{\phi}_{RA} - a_1}^{\hat{\phi}_{RA} - a_2} \tilde{\phi}_{RA} f_{\tilde{\phi}_{RA}}(\tilde{\phi}_{RA}) d\tilde{\phi}_{RA} \right)^2$$

$$= \frac{1}{2a} \left( \frac{\tilde{\phi}_{RA}^3}{3} \cos \hat{\Phi}_{RA} + \frac{\tilde{\phi}_{RA}^4}{4} \sin \hat{\Phi}_{RA} \right) \bigg|_{\hat{\phi}_{RA} - a_1}^{\hat{\phi}_{RA} - a_2}$$

$$- \frac{1}{4a^2} \left[ \left( \frac{\tilde{\phi}_{RA}^3}{2} \cos \hat{\Phi}_{RA} + \frac{\tilde{\phi}_{RA}^4}{3} \sin \hat{\Phi}_{RA} \right) \bigg|_{\hat{\phi}_{RA} - a_1}^{\hat{\phi}_{RA} - a_2} \right]^2,$$  \hspace{1cm} (59)

where $f(x)|_{x_2}^{x_1} = f(x_1) - f(x_2)$ and $E(\tilde{\phi}_{RA})$ denotes the expectation of $\tilde{\phi}_{RA}$.

B. Variance of $\tilde{\Theta}_{RA}$

In this subsection, we aim to derive the variance of $\tilde{\Theta}_{RA}$. Different from the PDF of $\tilde{\Phi}_{RA}$, the PDF of $\tilde{\Theta}_{RA}$ is more complicated. Therefore, we need to analyze the variance according to the aforementioned two cases as follows.

Case 1: When $\frac{\beta_2}{\alpha_2} > \frac{\beta_1}{\alpha_1}$, based on the PDF of $\tilde{\theta}_{RA}$ in (57), the variance of $\tilde{\Theta}_{RA}$ is derived as

$$D(\tilde{\theta}_{RA}) = \int_{D_1} \tilde{\theta}_{RA}^2 f_{\tilde{\theta}_{RA}}(\tilde{\theta}_{RA}) d\tilde{\theta}_{RA} - \left( \int_{D_1} \tilde{\theta}_{RA} f_{\tilde{\theta}_{RA}}(\tilde{\theta}_{RA}) d\tilde{\theta}_{RA} \right)^2.$$  \hspace{1cm} (60)

For $D_1$ in (60), we divide it into three different non-zero intervals that can be expressed as

$$D_1 = D_{11} + D_{12} + D_{13},$$  \hspace{1cm} (61)

where $D_{11}$, $D_{12}$ and $D_{13}$ are the integral expressions in the intervals of $[B_1, B_2)$, $[B_2, B_3)$ and $[B_3, B_4)$, respectively. The expressions of $D_{11}$, $D_{12}$ and $D_{13}$ are given in Appendix A.

For $D_2$ in (60), it is the expectation of $\tilde{\theta}_{RA}$, which is denoted by $E(\tilde{\theta}_{RA})$. It can be divided into three different non-zero intervals that are given by

$$D_2 = D_{21} + D_{22} + D_{23},$$  \hspace{1cm} (62)
where $D_{21}$, $D_{22}$ and $D_{23}$ are the integral expressions in the intervals of $[B_1, B_2)$, $[B_2, B_3)$ and $[B_3, B_4)$, respectively. The expressions of $D_{21}$, $D_{22}$ and $D_{23}$ are given in Appendix B.

Case 2: When $\frac{\beta_2}{\alpha_2} < \frac{\beta_1}{\alpha_1}$, according to the PDF of $\tilde{\Theta}_{RA}$ in (58), the variance of $\tilde{\Theta}_{RA}$ can be calculated as

$$D'(\tilde{\theta}_{RA}) = \int_{D_1'}^{D_2'} \tilde{\theta}_{RA}^2 f_{\tilde{\Theta}_{RA}}(\tilde{\theta}_{RA}) d\tilde{\theta}_{RA} - \left( \int_{D_1'}^{D_2'} \tilde{\theta}_{RA} f_{\tilde{\Theta}_{RA}}(\tilde{\theta}_{RA}) d\tilde{\theta}_{RA} \right)^2.$$  (63)

For $D'_1$, we divide it into three different non-zero intervals that are given by

$$D'_1 = D'_{11} + D'_{12} + D'_{13},$$  (64)

where $D'_{11}$, $D'_{12}$ and $D'_{13}$ are the integral expressions in the intervals of $[B_1, B_3)$, $[B_3, B_2)$ and $[B_2, B_4)$. The expressions of $D'_{11}$, $D'_{12}$ and $D'_{13}$ are given in Appendix C.

For $D'_2$ in (63), it is the expectation of $\tilde{\theta}_{RA}$, which is denoted by $E(\tilde{\theta}_{RA})$. It is also divided into three different non-zero intervals that are given by

$$D'_2 = D'_{21} + D'_{22} + D'_{23},$$  (65)

where $D'_{21}$, $D'_{22}$ and $D'_{23}$ are the integral expressions in the intervals of $[B_1, B_3)$, $[B_3, B_2)$ and $[B_2, B_4)$. The expressions of $D'_{21}$, $D'_{22}$ and $D'_{23}$ are given in Appendix D.

VI. SIMULATION RESULTS

This section presents simulation results to validate the accuracy of our derivations and approximations. In our simulation, we consider a mmWave multiple input single output (MISO) channel from the MU to the RIS. Moreover, the MU, the BS and the RISs are assumed to be placed in a 3D area. The location of the BS is $p = [0, 0, 40]^T$, while the locations of three RISs are $s_1 = [0, 5, 39]^T$, $s_2 = [1, 2, 38]^T$ and $s_3 = [0, 0, 34]^T$. The phase shift matrix of the RIS is set to a unit matrix. Additionally, we only consider the first RIS $s_1$ in the simulation of Part I. Furthermore, the noise power spectrum density is $N_0 = -174$ dBm/Hz, bandwidth is $B_w = 20$ MHz. The pass loss exponent is 2.5. The carrier frequency is 4.9 GHz and the number of subcarriers is 128. It is assumed that the inter-element spacing of UPA at the RIS is $d_2 = \lambda/2$. The following results are obtained by averaging over 10,000 random estimation error realizations. Unless otherwise stated, we assume $S = S_1 \times S_2 = 64 \times 64$ for rotation angle search grids, and the SNR is assumed to be 10 dB.
Fig. 3-(a) and Fig. 3-(b) illustrate the PDF of estimation errors. It is assumed that the RIS size is $N_y = N_z = 16$. It can be observed from Fig. 3-(a) and Fig. 3-(b) that our derived results match well with the simulation results, which verify the accuracy of our derived results.

Fig. 4-(a) and Fig. 4-(b) display the variance $\sigma^2_{\Theta_{RA}}$ of estimation error $\tilde{\Theta}_{RA}$ and the variance $\sigma^2_{\Phi_{RA}}$ of estimation error $\tilde{\Phi}_{RA}$ as the functions of RIS $N_y(N_z)$, respectively. Fig. 4-(a) and Fig. 4-(b) show that the theoretical results coincide with the simulation results, which validates the
correctness of the derived results. Moreover, it is observed that the variances of $\tilde{\Theta}_{RA}$ and $\tilde{\Phi}_{RA}$ decrease with the RIS size, which means that increasing the number of elements could improve the estimation accuracy.

VII. Conclusion

In this series of work, we proposed a comprehensive two-step 3D positioning scheme aided by an RIS. Here, we investigated the first step including estimation error modeling and analysis based on the 2D-DFT. Using the geometric relationship between the AOAs and their triangle functions, we derived the PDF of the angle estimation error. We then simplified the intricate geometric expression of the error PDF by employing the first-order linear approximation of triangle functions. We also derived the variance expression of the error by using the error PDF and theoretically derived the variance from the error PDF. Extended simulation results demonstrated the accuracy of the theory analysis.

Appendix A

Firstly, we derive the expression of $D_{11}$ as

$$D_{11} = \int_{B_1}^{B_2} \frac{\hat{X}(\hat{V} - \hat{\theta}_{RA}\hat{U})}{8abY} \left[ \alpha_2^2 - \left( \frac{\beta_1}{U + \theta_{RA}\hat{V}} \right)^2 \right] \hat{\theta}_{RA}^2 d\hat{\theta}_{RA}$$

$$= \frac{\hat{X}\alpha_2^2}{8abY} \int_{B_1}^{B_2} (\hat{V}\hat{\theta}_{RA}^2 - \hat{U}\hat{\theta}_{RA}^3) d\hat{\theta}_{RA} - \frac{\hat{X}\beta_1^2}{8abY} \int_{B_1}^{B_2} (\hat{V}\hat{\theta}_{RA}^2 - \hat{U}\hat{\theta}_{RA}^3) d\hat{\theta}_{RA}$$

$$= \frac{\hat{X}\alpha_2^2}{8abY} \int_{B_1}^{B_2} (\hat{V}\hat{\theta}_{RA}^2 - \hat{U}\hat{\theta}_{RA}^3) d\hat{\theta}_{RA}$$

$$- \left[ \frac{\hat{X}\beta_1^2}{8abYU} \left( \int_{B_1}^{B_2} \frac{\hat{V}\hat{\theta}_{RA}^2}{(1 + \frac{\theta_{RA}}{U})^2} d\hat{\theta}_{RA} - \int_{0}^{B_2} \frac{\hat{\theta}_{RA}^3}{(1 + \frac{\theta_{RA}}{U})^2} d\hat{\theta}_{RA} \right) \right]$$

$$= \frac{\hat{X}\alpha_2^2}{8abY} \left( \frac{\hat{U}\hat{\theta}_{RA}^4}{4} - \frac{\hat{V}\hat{\theta}_{RA}^3}{3} \right) \bigg|_{B_1}^{B_2}$$

$$- \left[ \left( - \frac{\hat{X}\beta_1^2}{8abYU} \cdot \frac{B_1^4}{4} \cdot 2F_1 \left( 2, 4; 5; -\frac{\hat{V}}{U}B_2 \right) + \frac{\hat{X}\beta_1^2\hat{V}}{8abYU^2} \cdot \frac{B_2^3}{3} \cdot 2F_1 \left( 2, 3; 4; -\frac{\hat{V}}{U}B_2 \right) \right) \right.$$

$$- \left. \left( - \frac{\hat{X}\beta_1^2}{8abYU} \cdot \frac{B_1^4}{4} \cdot 2F_1 \left( 2, 4; 5; -\frac{\hat{V}}{U}B_1 \right) + \frac{\hat{X}\beta_1^2\hat{V}}{8abYU^2} \cdot \frac{B_1^3}{3} \cdot 2F_1 \left( 2, 3; 4; -\frac{\hat{V}}{U}B_1 \right) \right) \right],$$

(66)
where \( \int_0^u \frac{x^{\mu-1}dx}{(1+\beta')^\nu} = 2 \) \( F_1(\nu, \mu; 1 + \mu; -\beta u) \) is a generalized hypergeometric series [37].

Then, \( D_{12} \) in (61) is derived as

\[
D_{12} = \int_{B_2}^{B_3} \frac{\hat{X}(\hat{V} - \hat{\theta}_{RA}\hat{U})}{2b} \theta_{RA}^2 d\theta_{RA} \\
= \int_{B_2}^{B_3} \frac{\hat{X}(\hat{V}\hat{\theta}_{RA}^2 - \hat{U}\hat{\theta}_{RA}^3)}{2b} d\theta_{RA} \\
= \left. \frac{\hat{X}}{2b} \left( \hat{V}\hat{\theta}_{RA}^2 - \hat{U}\hat{\theta}_{RA}^3 \right) \right|_{B_2}^{B_3}. \tag{67}
\]

Finally, \( D_{13} \) in (61) is given by

\[
D_{13} = \int_{B_2}^{B_3} \frac{\hat{X}(\hat{V} - \hat{\theta}_{RA}\hat{U})}{8abY} \left[ \left( \frac{\beta_2}{\hat{U} + \hat{\theta}_{RA}\hat{V}} \right)^2 - \alpha_1^2 \right] \theta_{RA} d\theta_{RA} \\
= \frac{\hat{X} \beta_2^2}{8abY} \int_{B_2}^{B_3} \left. \frac{\hat{V}\hat{\theta}_{RA}^2}{(\hat{U} + \hat{\theta}_{RA}\hat{V})^2} d\theta_{RA} \right|_{B_2}^{B_3} \\
= \left[ \frac{\hat{X} \beta_2^2}{8abY U^2} \left( \int_{B_2}^{B_3} \frac{\hat{V}\hat{\theta}_{RA}^2}{(1 + \hat{\theta}_{RA}\hat{U})^2} d\theta_{RA} \right) \\
- \frac{\hat{X} \beta_2^2}{8abY U} \left( \int_{B_2}^{B_3} \frac{\hat{\theta}_{RA}^2}{(1 + \hat{\theta}_{RA}\hat{U})^2} d\theta_{RA} \right) \right] \\
- \frac{\hat{X} \alpha_1^2}{8abY} \int_{B_2}^{B_3} \left. \frac{\hat{V}\hat{\theta}_{RA}^2}{(1 + \hat{\theta}_{RA}\hat{U})^2} d\theta_{RA} \right|_{B_2}^{B_3} \\
= \left[ \left( \frac{\hat{X} \beta_2^2}{8abY U} \right) \left( \frac{B_4}{4} \right) \cdot 2F_1 \left( 2, 4; 5; -\frac{\hat{V}}{U} B_4 \right) + \frac{\hat{X} \beta_2^2 \hat{V}}{8abY U^2} \left( \frac{B_4^3}{3} \right) \cdot 2F_1 \left( 2, 3; 4; -\frac{\hat{V}}{U} B_4 \right) \right] \\
- \left( \frac{\hat{X} \beta_2^2}{8abY U} \right) \left( \frac{B_4}{4} \right) \cdot 2F_1 \left( 2, 4; 5; -\frac{\hat{V}}{U} B_3 \right) + \frac{\hat{X} \beta_2^2 \hat{V}}{8abY U^2} \left( \frac{B_4^3}{3} \right) \cdot 2F_1 \left( 2, 3; 4; -\frac{\hat{V}}{U} B_3 \right) \\
- \frac{\hat{X} \alpha_1^2}{8abY} \left( \frac{\hat{V}\hat{\theta}_{RA}^2}{3} \right) \right|_{B_2}^{B_3}. \tag{68}
\]
Appendix B

Firstly, $D_{21}$ in (62) is derived as

\[
D_{21} = \int_{B_1}^{B_2} \frac{\hat{X}(\hat{V} - \hat{\theta}_{RA}\hat{U})}{8abY} \left[ \alpha_2^2 - \left( \frac{\beta_1}{\hat{U} + \hat{\theta}_{RA}\hat{V}} \right)^2 \right] \hat{\theta}_{RA} d\hat{\theta}_{RA}
\]

\[
= \frac{\hat{X}\alpha_2^2}{8abY} \int_{B_1}^{B_2} (-\hat{U}\hat{\theta}_{RA}^2 + \hat{V}\hat{\theta}_{RA}) d\hat{\theta}_{RA} - \frac{\hat{X}\beta_1^2}{8abY} \int_{B_1}^{B_2} \frac{-\hat{U}\hat{\theta}_{RA}^2 + \hat{V}\hat{\theta}_{RA}}{(\hat{U} + \hat{V}\hat{\theta}_{RA})^2} d\hat{\theta}_{RA}
\]

\[
= \frac{\hat{X}\alpha_2^2}{8abY} \int_{B_1}^{B_2} (-\hat{U}\hat{\theta}_{RA}^2 + \hat{V}\hat{\theta}_{RA}) d\hat{\theta}_{RA}
\]

\[
- \left[ \frac{\hat{X}\beta_1^2}{8abY\hat{U}} \left( \int_{0}^{B_2} \frac{\hat{V}\hat{\theta}_{RA}}{(1 + \hat{V}\hat{\theta}_{RA})^2} d\hat{\theta}_{RA} - \int_{0}^{B_1} \frac{\hat{\theta}_{RA}^2}{(1 + \hat{V}\hat{\theta}_{RA})^2} d\hat{\theta}_{RA} \right) \right]
\]

\[
= \frac{\hat{X}\alpha_2^2}{8abY} \left( -\frac{\hat{U}\hat{\theta}_{RA}^3}{3} + \frac{\hat{V}\hat{\theta}_{RA}^2}{2} \right) \bigg|_{B_1}^{B_2}
\]

\[
- \left[ \left( -\frac{\hat{X}\beta_1^2}{8abY\hat{U}} \cdot \frac{B_3^2}{3} \cdot {}_2F_1(2, 3; 4; -\frac{\hat{V}}{\hat{U}}B_2) + \frac{\hat{X}\beta_1^2\hat{V}}{8abY\hat{U}^2} \cdot \frac{B_3^2}{2} \cdot {}_2F_1(2, 2; 3; -\frac{\hat{V}}{\hat{U}}B_2) \right) \right]
\]

\[
- \left( -\frac{\hat{X}\beta_1^2}{8abY\hat{U}} \cdot \frac{B_3^2}{3} \cdot {}_2F_1(2, 3; 4; -\frac{\hat{V}}{\hat{U}}B_1) + \frac{\hat{X}\beta_1^2\hat{V}}{8abY\hat{U}^2} \cdot \frac{B_3^2}{2} \cdot {}_2F_1(2, 2; 3; -\frac{\hat{V}}{\hat{U}}B_1) \right) \bigg].
\]

Then, $D_{22}$ in (62) is written as

\[
D_{22} = \int_{B_2}^{B_3} \frac{\hat{X}(\hat{V} - \hat{\theta}_{RA}\hat{U})}{2b} \hat{\theta}_{RA} d\hat{\theta}_{RA}
\]

\[
= \frac{\hat{X}}{2b} \int_{B_2}^{B_3} (\hat{V}\hat{\theta}_{RA} - \hat{U}\hat{\theta}_{RA}^2) d\hat{\theta}_{RA}
\]

\[
= \frac{\hat{X}}{2b} \left( -\frac{\hat{U}\hat{\theta}_{RA}^3}{3} + \frac{\hat{V}\hat{\theta}_{RA}^2}{2} \right) \bigg|_{B_2}^{B_3}.
\]
Finally, $D_{23}$ in (62) could be formulated as

$$D_{23} = \int_{B_3}^{B_4} \frac{\dot{X}(\dot{V} - \dot{\theta}_{RA}\dot{U})}{8abY}\left[\left(\frac{\beta_2}{U + \theta_{RA}\dot{V}}\right)^2 - \alpha_1^2\right]\dot{\theta}_{RA}d\dot{\theta}_{RA}$$

$$= \frac{\dot{X}^2}{8abY} \int_{B_3}^{B_4} \frac{\dot{U}\dot{\theta}_{RA}^2 + \dot{V}\theta_{RA}^2}{(U + \theta_{RA}\dot{V})^2} d\theta_{RA} - \frac{\dot{X}\alpha_1^2}{8abY} \int_{B_3}^{B_4} (-\dot{U}\theta_{RA}^2 + \dot{V}\theta_{RA}) d\dot{\theta}_{RA}$$

$$= \left[-\frac{\dot{X}^2}{8abY} \int_{B_3}^{B_4} \frac{B_3^4}{3} \cdot 2F_1(2, 3; 4; -\dot{V}\dot{U}) + \int_{B_3}^{B_4} \frac{\dot{X}^2}{8abY} \int_{B_3}^{B_4} \frac{B_3^4}{3} \cdot 2F_1(2, 3; 4; -\dot{V}\dot{U})\right]$$

$$= \left[-\frac{\dot{X}^2}{8abY} \int_{B_3}^{B_4} \frac{B_3^4}{3} \cdot 2F_1(2, 3; 4; -\dot{V}\dot{U})\right]$$

(71)

**APPENDIX C**

As for $D'_{11}$ in (64), it could be expressed as

$$D'_{11} = \int_{B_1}^{B_3} \frac{\dot{X}(\dot{V} - \dot{\theta}_{RA}\dot{U})}{8abY}\left[\alpha_2^2 - \left(\frac{\beta_1}{\dot{U} + \theta_{RA}\dot{V}}\right)^2\right]\dot{\theta}_{RA}d\dot{\theta}_{RA}$$

$$= \frac{\dot{X}^2}{8abY} \int_{B_1}^{B_3} (-\dot{U}\dot{\theta}_{RA}^2 + \dot{V}\theta_{RA}^2) d\dot{\theta}_{RA} - \frac{\dot{X}\alpha_1^2}{8abY} \int_{B_1}^{B_3} (-\dot{U}\theta_{RA}^2 + \dot{V}\theta_{RA}) d\dot{\theta}_{RA}$$

$$= \frac{\dot{X}^2}{8abY} \left(-\frac{\dot{U}\theta_{RA}^4}{4} + \frac{\dot{V}\theta_{RA}^3}{3}\right)\bigg|_{B_1}^{B_3}$$

$$= \left[-\frac{\dot{X}^2}{8abY} \int_{B_1}^{B_3} \frac{B_3^4}{4} \cdot 2F_1(2, 4; 5; -\dot{V}\dot{U}) + \int_{B_1}^{B_3} \frac{\dot{X}^2}{8abY} \int_{B_1}^{B_3} \frac{B_3^4}{3} \cdot 2F_1(2, 3; 4; -\dot{V}\dot{U})\right]$$

(72)

Then, $D'_{12}$ in (64) is given by

$$D'_{12} = \int_{B_2}^{B_3} \frac{\dot{X}\dot{Z}(\dot{V} - \dot{\theta}_{RA}\dot{U})}{2aY(U + \theta_{RA}\dot{V})^2}\dot{\theta}_{RA}d\dot{\theta}_{RA}$$

$$= \left[-\frac{\dot{X}\dot{Z}}{2aY\dot{U}} \cdot \frac{B_2^3}{4} \cdot 2F_1(2, 4; 5; -\dot{V}\dot{U})\right]$$

(73)
Finally, $D'_{13}$ in (64) is derived as

\[
D'_{13} = \int_{B_2}^{B_3} \frac{\dot{X}(\dot{V} - \ddot{\theta}_{RA}\dot{U})}{8abY} \left[ \left( \frac{\beta_2}{U + \theta_{RA}\dot{V}} \right)^2 - \alpha_1^2 \right] \ddot{\theta}_{RA} d\ddot{\theta}_{RA}
\]
\[
= \frac{X\beta_2^2}{8abY} \int_{B_2}^{B_3} \frac{-\dot{U}\ddot{\theta}_{RA} + \dot{V}\ddot{\theta}_{RA}}{(U + \theta_{RA}\dot{V})^2} d\ddot{\theta}_{RA} - \frac{X\alpha_1^2}{8abY} \int_{B_2}^{B_3} (-\dot{U}\ddot{\theta}_{RA} + \dot{V}\ddot{\theta}_{RA}) d\ddot{\theta}_{RA}
\]
\[
= \left[ \left( \frac{-\ddot{X}\beta_2^2}{8abYU} \cdot \frac{B_4^3}{4} \cdot 2F_1 \left( 2, 4; 5; -\frac{\dot{V}}{U} \cdot B_4 \right) + \frac{\ddot{X}\beta_1^2\dot{V}}{8abYU^2} \cdot \frac{B_4^3}{3} \cdot 2F_1 \left( 2, 3; 4; -\frac{\dot{V}}{U} \cdot B_4 \right) \right)
\right.
\]
\[
\left. \right] - \left( \left( \frac{-\ddot{X}\beta_1^2}{8abYU} \cdot \frac{B_4^3}{3} \cdot 2F_1 \left( 2, 4; 5; -\frac{\dot{V}}{U} \cdot B_4 \right) + \frac{\ddot{X}\beta_1^2\dot{V}}{8abYU^2} \cdot \frac{B_4^3}{3} \cdot 2F_1 \left( 2, 3; 4; -\frac{\dot{V}}{U} \cdot B_4 \right) \right) \right]
\]
\[
= \frac{X\alpha_1^2}{8abY} \left( \frac{-\dot{U}\ddot{\theta}_{RA}}{4} + \frac{\dot{V}\ddot{\theta}_{RA}}{3} \right) \bigg|_{B_2}^{B_3}.
\] (74)

**APPENDIX D**

Firstly, $D'_{21}$ in (65) is formulated as

\[
D'_{21} = \int_{B_1}^{B_2} \frac{\dot{X}(\dot{V} - \ddot{\theta}_{RA}\dot{U})}{8abY} \left[ \alpha_2^2 - \left( \frac{\beta_1}{U + \theta_{RA}\dot{V}} \right)^2 \right] \ddot{\theta}_{RA} d\ddot{\theta}_{RA}
\]
\[
= \frac{X\alpha_2^2}{8abY} \left( \frac{-\dot{U}\ddot{\theta}_{RA}}{3} + \frac{\dot{V}\ddot{\theta}_{RA}}{2} \right) \bigg|_{B_1}^{B_2} - \left[ \left( \frac{-\ddot{X}\beta_1^2}{8abYU} \cdot \frac{B_3^3}{4} \cdot 2F_1 \left( 2, 3; 4; -\frac{\dot{V}}{U} \cdot B_3 \right) + \frac{\ddot{X}\beta_1^2\dot{V}}{8abYU^2} \cdot \frac{B_3^3}{2} \cdot 2F_1 \left( 2, 2; 3; -\frac{\dot{V}}{U} \cdot B_3 \right) \right)
\right.
\]
\[
\left. \right] - \left( \left( \frac{-\ddot{X}\beta_1^2}{8abYU} \cdot \frac{B_3^3}{3} \cdot 2F_1 \left( 2, 3; 4; -\frac{\dot{V}}{U} \cdot B_3 \right) + \frac{\ddot{X}\beta_1^2\dot{V}}{8abYU^2} \cdot \frac{B_3^3}{2} \cdot 2F_1 \left( 2, 2; 3; -\frac{\dot{V}}{U} \cdot B_3 \right) \right) \right].
\] (75)

Then, $D'_{22}$ in (65) is written as

\[
D'_{22} = \int_{B_2}^{B_3} \frac{\dot{X}(\dot{V} - \ddot{\theta}_{RA}\dot{U})}{2aY(U + \theta_{RA}\dot{V})^2} \ddot{\theta}_{RA} d\ddot{\theta}_{RA}
\]
\[
= \left( \left( \frac{-\ddot{X}\beta_2}{2aYU} \cdot \frac{B_2^3}{3} \cdot 2F_1 \left( 2, 3; 4; -\frac{\dot{V}}{U} \cdot B_2 \right) + \frac{\ddot{X}\beta_2\dot{V}}{2aYU^2} \cdot \frac{B_2^3}{2} \cdot 2F_1 \left( 2, 2; 3; -\frac{\dot{V}}{U} \cdot B_2 \right) \right)
\right.
\]
\[
\left. \right] - \left( \left( \frac{-\ddot{X}\beta_2}{2aYU} \cdot \frac{B_2^3}{3} \cdot 2F_1 \left( 2, 3; 4; -\frac{\dot{V}}{U} \cdot B_2 \right) + \frac{\ddot{X}\beta_2\dot{V}}{2aYU^2} \cdot \frac{B_2^3}{2} \cdot 2F_1 \left( 2, 2; 3; -\frac{\dot{V}}{U} \cdot B_2 \right) \right) \right].
\] (76)
Finally, $D'_{23}$ in (65) is obtained as

$$D'_{23} = \int_{B_2}^{B_4} \frac{\hat{X}(\hat{V} - \hat{\theta}_{RA}\hat{U})}{8ab\hat{Y}} \left[ \left( \frac{\beta_2}{\hat{U} + \hat{\theta}_{RA}\hat{V}} \right)^2 - \alpha_1^2 \right] \hat{\theta}_{RA} d\hat{\theta}_{RA}$$

$$= \frac{\hat{X}\beta_2^2}{8ab\hat{Y}} \int_{B_2}^{B_4} \frac{-\hat{U}\hat{\theta}_{RA}^2 + \hat{V}\hat{\theta}_{RA}}{(\hat{U} + \hat{\theta}_{RA}\hat{V})^2} d\hat{\theta}_{RA} - \frac{\hat{X}\alpha_1^2}{8ab\hat{Y}} \int_{B_2}^{B_4} (-\hat{U}\hat{\theta}_{RA}^2 + \hat{V}\hat{\theta}_{RA}) d\hat{\theta}_{RA}$$

$$= \left[ \left( -\frac{\hat{X}\beta_2^2}{8ab\hat{Y}\hat{U}} \cdot \frac{B_3^4}{3} \cdot 2F_1(2, 3; 4; -\frac{\hat{V}}{\hat{U}} B_4) + \frac{\hat{X}\beta_2^2\hat{V}}{8ab\hat{Y}\hat{U}^2} \cdot \frac{B_4^2}{2} \cdot 2F_1(2, 2; 3; -\frac{\hat{V}}{\hat{U}} B_4) \right) \right]$$

$$- \left[ \left( -\frac{\hat{X}\beta_2^2}{8ab\hat{Y}\hat{U}} \cdot \frac{B_3^4}{3} \cdot 2F_1(2, 3; 4; -\frac{\hat{V}}{\hat{U}} B_4) + \frac{\hat{X}\beta_2^2\hat{V}}{8ab\hat{Y}\hat{U}^2} \cdot \frac{B_4^2}{2} \cdot 2F_1(2, 2; 3; -\frac{\hat{V}}{\hat{U}} B_4) \right) \right]$$

$$- \frac{\hat{X}\alpha_1^2}{8ab\hat{Y}} \left( -\frac{\hat{U}\hat{\theta}_{RA}^3}{3} + \frac{\hat{V}\hat{\theta}_{RA}^2}{2} \right) \bigg|_{B_2}^{B_4}.$$  

(77)

**REFERENCES**

[1] Y. Zhou, L. Liu, L. Wang, N. Hui, X. Cui, J. Wu, Y. Peng, Y. Qi, and C. Xing, “Service-aware 6G: An intelligent and open network based on the convergence of communication, computing and caching,” *Digital Communications and Networks*, 2020.

[2] Z. Zhang, Y. Xiao, Z. Ma, M. Xiao, Z. Ding, X. Lei, G. K. Karagiannidis, and P. Fan, “6G wireless networks: Vision, requirements, architecture, and key technologies,” *IEEE Vehicular Technology Magazine*, vol. 14, no. 3, pp. 28–41, 2019.

[3] H. Ren, K. Wang, and C. Pan, “Intelligent reflecting surface-aided URLLC in a factory automation scenario,” *IEEE Transactions on Communications*, vol. 70, no. 1, pp. 707–723, 2022.

[4] F. Hu, Y. Deng, W. Saad, M. Bennis, and A. H. Aghvami, “Cellular-connected wireless virtual reality: Requirements, challenges, and solutions,” *IEEE Communications Magazine*, vol. 58, no. 5, pp. 105–111, 2020.

[5] T. G. Reid, S. E. Houts, R. Cammarata, G. Mills, S. Agarwal, A. Vora, and G. Pandey, “Localization requirements for autonomous vehicles,” *SAE International Journal of Connected and Automated Vehicles*, vol. 2, no. 3, sep 2019.

[6] H. Sarieddeen, N. Saeed, T. Y. Al-Naffouri, and M.-S. Alouini, “Next generation terahertz communications: A rendezvous of sensing, imaging, and localization,” *IEEE Communications Magazine*, vol. 58, no. 5, pp. 69–75, 2020.

[7] R. Mendrzik, F. Meyer, G. Bauch, and M. Z. Win, “Enabling situational awareness in millimeter wave massive MIMO systems,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 13, no. 5, pp. 1196–1211, 2019.

[8] H. Liu, H. Darabi, P. Banerjee, and J. Liu, “Survey of wireless indoor positioning techniques and systems,” *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, vol. 37, no. 6, pp. 1067–1080, 2007.

[9] C. Pan, H. Ren, K. Wang, J. F. Kolb, M. Elkashlan, M. Chen, M. Di Renzo, Y. Hao, J. Wang, A. L. Swindlehurst, X. You, and L. Hanzo, “Reconfigurable intelligent surfaces for 6G systems: Principles, applications, and research directions,” *IEEE Communications Magazine*, vol. 59, no. 6, pp. 14–20, 2021.

[10] C. Pan, H. Ren, K. Wang, W. Xu, M. Elkashlan, A. Nallanathan, and L. Hanzo, “Multicell MIMO communications relying on intelligent reflecting surfaces,” *IEEE Transactions on Wireless Communications*, vol. 19, no. 8, pp. 5218–5233, 2020.
[11] D. Fan, F. Gao, Y. Liu, Y. Deng, G. Wang, Z. Zhong, and A. Nallanathan, “Angle domain channel estimation in hybrid millimeter wave massive MIMO systems,” *IEEE Transactions on Wireless Communications*, vol. 17, no. 12, pp. 8165–8179, 2018.

[12] C. Pan, H. Ren, K. Wang, M. Elkashlan, A. Nallanathan, J. Wang, and L. Hanzo, “Intelligent reflecting surface aided MIMO broadcasting for simultaneous wireless information and power transfer,” *IEEE Journal on Selected Areas in Communications*, vol. 38, no. 8, pp. 1719–1734, 2020.

[13] G. Zhou, C. Pan, H. Ren, K. Wang, and A. Nallanathan, “A framework of robust transmission design for IRS-aided MISO communications with imperfect cascaded channels,” *IEEE Transactions on Signal Processing*, vol. 68, pp. 5092–5106, 2020.

[14] K. Zhi, C. Pan, H. Ren, and K. Wang, “Uplink achievable rate of intelligent reflecting surface-aided millimeter-wave communications with low-resolution adc and phase noise,” *IEEE Wireless Communications Letters*, vol. 10, no. 3, pp. 654–658, 2021.

[15] Q. Wu and R. Zhang, “Towards smart and reconﬁgurable environment: Intelligent reﬂecting surface aided wireless network,” *IEEE Communications Magazine*, vol. 58, no. 1, pp. 106–112, 2020.

[16] K. Liu, Z. Zhang, L. Dai, S. Xu, and F. Yang, “Active reconﬁgurable intelligent surface: Fully-connected or sub-connected?” *IEEE Communications Letters*, vol. 26, no. 1, pp. 167–171, 2022.

[17] Z. Zhang and L. Dai, “A joint precoding framework for wideband reconﬁgurable intelligent surface-aided cell-free network,” *IEEE Transactions on Signal Processing*, vol. 69, pp. 4085–4101, 2021.

[18] D. Dardari, P. Closas, and P. M. Djurić, “Indoor tracking: Theory, methods, and technologies,” *IEEE Transactions on Vehicular Technology*, vol. 64, no. 4, pp. 1263–1278, 2015.

[19] Q. Li, B. Chen, and M. Yang, “Improved two-step constrained total least-squares TDOA localization algorithm based on the alternating direction method of multipliers,” *IEEE Sensors Journal*, vol. 20, no. 22, pp. 13666–13673, 2020.

[20] S. Rao, A. Mezghani, and A. L. Swindlehurst, “Channel estimation in one-bit massive MIMO systems: Angular versus unstructured models,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 13, no. 5, pp. 1017–1031, 2019.

[21] R. Ramlall, J. Chen, and A. L. Swindlehurst, “Non-line-of-sight mobile station positioning algorithm using TOA, AOA, and doppler-shift,” in *2014 Ubiquitous Positioning Indoor Navigation and Location Based Service (UPINLBS)*, 2014, pp. 180–184.

[22] C. Hu, L. Dai, S. Han, and X. Wang, “Two-timescale channel estimation for reconﬁgurable intelligent surface aided wireless communications,” *IEEE Transactions on Communications*, vol. 69, no. 11, pp. 7736–7747, 2021.

[23] G. Zhou, C. Pan, H. Ren, P. Popovski, and A. L. Swindlehurst, “Channel estimation for RIS-aided multiuser millimeter-wave systems,” 2021.

[24] J. He, H. Wyneersch, L. Kong, O. Silvén, and M. Juntti, “Large intelligent surface for positioning in millimeter wave MIMO systems,” in *2020 IEEE 91st Vehicular Technology Conference*, 2020, pp. 1–5.

[25] Z. Feng, B. Wang, M. Luan, and F. Hu, “Power optimization for target localization with reconﬁgurable intelligent surfaces,” *Signal Processing*, vol. 189, no. 5, p. 108252, 2021.

[26] Y. Liu, S. Hong, C. Pan, Y. Wang, Y. Pan, and M. Chen, “Optimization of RIS conﬁgurations for multiple-RIS-aided mmWave positioning systems based on crlb analysis,” 2021.

[27] A. Elzanaty, A. Guerra, F. Guidi, and M.-S. Alouini, “Reconﬁgurable intelligent surfaces for localization: Position and orientation error bounds,” *IEEE Transactions on Signal Processing*, vol. 69, pp. 5386–5402, 2021.

[28] H. Zhang, H. Zhang, B. Di, K. Bian, Z. Han, and L. Song, “Towards ubiquitous positioning by leveraging reconﬁgurable intelligent surface,” *IEEE Communications Letters*, vol. 25, no. 1, pp. 284–288, 2021.
[29] A. Fascista, M. F. Keskin, A. Coluccia, H. Wymeersch, and G. Seco-Granados, “RIS-aided joint localization and synchronization with a single-antenna receiver: Beamforming design and low-complexity estimation,” 2022.
[30] W. Zhang and W. P. Tay, “Using reconfigurable intelligent surfaces for user positioning in mmWave MIMO systems,” 2021.
[31] M. R. Akdeniz, Y. Liu, M. K. Samimi, S. Sun, S. Rangan, T. S. Rappaport, and E. Erkip, “Millimeter wave channel modeling and cellular capacity evaluation,” IEEE Journal on Selected Areas in Communications, vol. 32, no. 6, pp. 1164–1179, 2014.
[32] A. Papazafeiropoulos, C. Pan, P. Kourtessis, S. Chatzinotas, and J. M. Senior, “Intelligent reflecting surface-assisted MU-MISO systems with imperfect hardware: Channel estimation, beamforming design,” 2021.
[33] F. Bohagen, P. Orten, and G. E. Oien, “Design of optimal high-rank line-of-sight MIMO channels,” IEEE Transactions on Wireless Communications, vol. 6, no. 4, pp. 1420–1425, 2007.
[34] F. Bohagen, P. Orten, and G. Oien, “Construction and capacity analysis of high-rank line-of-sight MIMO channels,” in IEEE Wireless Communications and Networking Conference, 2005, vol. 1, 2005, pp. 432–437 Vol. 1.
[35] D. Fan, F. Gao, Y. Liu, Y. Deng, and A. Nallanathan, “Angle domain channel estimation in hybrid mmWave massive MIMO systems,” IEEE Transactions on Wireless Communications, vol. PP, no. 99, 2018.
[36] X. Y. Yang and B. X. Chen, “A high-resolution method for 2D DOA estimation,” Journal of Electronics and Information Technology, vol. 32, no. 4, pp. 953–958, 2010.
[37] I. S. Gradshteyn and I. M. Ryzhik, “Table of integrals, series, and products,” Mathematics of Computation, vol. 20, no. 96, p. 1157, 2007.