Determining Bidding Strategies in Sequential Auctions: Quasi-linear Utility and Budget Constraints

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ABSTRACT
In this paper, we develop a new method for finding an optimal bidding strategy in sequential auctions, using a dynamic programming technique. The existing method assumes the utility of a user is represented in an additive form. Thus, the remaining endowment of money must be explicitly represented in each state. On the other hand, our method assumes the utility of a user can be represented in a quasi-linear form, and representing the payment as a state-transition cost. Accordingly, we can obtain more than an m-fold speed-up in the computation time, where m is the initial endowment of money. Furthermore, we have developed a method for obtaining a semi-optimal bidding strategy under budget constraints.

1. INTRODUCTION
[1] proposed an approach for finding an optimal bidding strategy in a sequential auction, using a dynamic programming technique. This approach, however, assumes that the utility of the user is represented in an additive form, and accordingly, the remaining endowment of money must be explicitly represented for each state. Therefore, the larger the initial endowment of money becomes, the more time-consuming the calculation of the optimal bidding strategy gets. To put it concretely, suppose that there are n items and the initial endowment of money is m, then the number of states considered is O(m × 2^n).

In this paper, we develop a new problem formalization that can reduce the number of states by 1/m. In this formalization, we assume that the utility of the user can be represented in a quasi-linear form. By representing the payment of the user as a state-transition cost, we can avoid explicitly representing the remaining endowment of money in each state. Furthermore, there exists one practically important case where the quasi-linear representation fails to formalize, i.e., the case with budget constraints. To resolve this problem, we have developed a method for obtaining a semi-optimal bidding strategy under budget constraints.

2. BASIC MODELS
We assume that there are n items denoted by r_1, r_2, ..., r_n. The individual auction A_i for each item r_i is executed sequentially in the increasing order of i, and all bidders know the order in advance. To simplify the problem, we assume that A_i is a first-price sealed-bid auction [2].

In this paper, we focus on a specific agent and consider a method for finding the optimal bidding strategy for the agent. For a set of items R_s, which is a subset of all items R = {r_1, r_2, ..., r_n}, we represent the valuation of the subset R_s for the agent as v(R_s). In addition, we assume that this agent has a distribution function F_t(h) to predict the highest bids of other agents in A_i.

We also assume that the distributions of the highest bids of multiple items are mutually independent. For simplicity, we assume that the agent can win in cases of ties. Therefore, when the agent bids z for an item r_i, the probability that the agent wins the item r_i is given by F_t(z).

3. DYNAMIC PROGRAMMING IN A QUASI-LINEAR FORM
The utility of an agent is called quasi-linear if it can be represented in the following form.

v(R_s) = z R_s

Here, we represent the sum of payments for the subset R_s as Z R_s. We define the agent’s utility as the difference between the sum of the valuation for the allocated items and the payment. The assumption that an agent’s utility can be represented in a quasi-linear form has been widely used in many microeconomics studies [2].

To find the optimal bidding strategy, the auction process is divided into n + 1 stages, i.e., n stages at which bidding decisions must be made, and a terminal stage at the end of all the auctions. We use time index 0 ≤ t ≤ n to refer to stages. We can avoid explicitly representing the amount of remaining endowment of money in each state by representing the payment as a state-transition cost. Let < R_s >^t denote a state where an agent obtains R_s at stage t.
\[ \pi(< R_0 >) = z \] means that according to bidding strategy \( \pi \), the agent should bid \( z \) for item \( r_{t+1} \) if the agent obtains \( R_0 \) so far. Furthermore, let \( V^*(< R_0 >) \) indicate the expected utility obtained by executing strategy \( \pi \) from state \( < R_0 > \). The expected utility of strategy \( \pi \) of the initial state is represented as \( V^*(< \emptyset >) \).

We set \( V(< R_0 >) = v(R_0) \) for state \( < R_0 > \) in the terminal stage \( n \).

\[
Q(< R_0 >^t, z) = F_{t+1}(z) \cdot (V(< R_0 \cup \{r_{t+1}\} >^{t+1}) - z) + (1 - F_{t+1}(z)) \cdot V(< R_0 >^{t+1})
\]

\[
V(< R_0 >^t) = \max_z Q(< R_0 >^t, z)
\]

\[
\pi(< R_0 >^t) = \arg \max_z Q(< R_0 >^t, z)
\]

where \( V(< R_0 >^t) \) denotes the expected utility of state \( < R_0 >^t \). We can calculate the optimal bidding strategy using value iteration. In performing the value iteration, we can set the upper-bound of bidding price \( z \) to \( V(< R_0 \cup \{r_{t+1}\} >^{t+1}) - V(< R_0 >^{t+1}) \). Clearly, bidding more than this value gives a smaller expected utility than a bid of bidding 0.

In this problem formalization, the number of states at stage \( t \) is given by \( 2^t \), and therefore, the total number of states becomes \( 2^{n+1} - 1 \). Accordingly, compared with the case of an agent's utility represented in an additive form, the number of states in a quad-linear form is reduced by \( 1/m \).

We show evaluation results to clarify the efficiency of our formalization. We assume that the valuation of the set of items \( \{r_1, r_2, \ldots, r_m\} \) and the set of items \( \{r_{t+1}, r_{t+2}, \ldots, r_n\} \) is 100 \( \times n/2 \). We also assume that having any additional items to these sets does not increase the utility. If any item in each set cannot be obtained, the utility becomes 0. Furthermore, we assume that the highest bids of other agents for each item are randomly distributed in the range of \([0, 100]\).

Figure 1 shows the computation time for the quad-linear form and for the additive form, where \( m \) is set to 500, 1000, and 1500, by varying the number of items \( n \). In the problem formalization using the quad-linear form, we don’t consider bids larger than 100 \( \times n/2 \). We ran our experiments on a workstation (296 MHz Sun UltraSparc II) with a program written in Lisp. We can see that in the quad-linear form, we can reduce the number of states by \( 1/m \), and we can obtain more than an \( m \)-fold speed-up in the computation time. This is because not only the number of states, but also the number of bids considered in each state is reduced.

## 4. INCORPORATING BUDGET CONSTRAINTS

In this section, we develop a method for obtaining a semi-optimal bidding strategy \( \pi’ \) that satisfies budget constraints, by modifying the strategy \( \pi \) that is obtained using the method described in Section 3. More specifically, based on the bids specified by \( \pi \), we calculate the upper-bound of a bid in each state, so that budget constraints can be satisfied, and find the optimal bid under this upper-bound using a dynamic programming procedure.

This method applies the dynamic programming procedure twice: once for obtaining strategy \( \pi \), and once for obtaining \( \pi’ \) by modifying \( \pi \). Therefore, we can expect our method to still attain about an \( m/2 \)-fold speed-up in total computation time.

### Figure 1: Comparison on Computation Time

![Figure 1: Comparison on Computation Time](image)

For a state \( < R_0 >^t \), \( Z_{vpt} \) denotes the sum of the payments based on \( \pi \) from the initial state \( < \emptyset >^0 \) until this state (excluding the payment in this state), \( z_{opt} \) denotes the optimal amount of the bid in this state specified by \( \pi \), and \( Z_{opt} \) denotes the maximal value of the sum of the payments for each of the possible paths branching from the state \( < R_0 \cup \{r_{t+1}\} >^{t+1} \). Note that the payments used for calculating \( Z_{opt} \) have already been adjusted to consider budget constraints. In addition, we denote the upper-bound of the total budget as \( Z_{bud} \).

For each state \( < R_0 >^t \), we set the upper-bound of a bid \( z_{max} \) as follows.

\[
z_{max} = z_{opt} \times (Z_{bud} - Z_{pt})/(Z_{vpt} + z_{opt})
\]

The meaning of this formula is as follows. For all states after \( t + 1 \), the amounts of the bids have already been adjusted. Therefore, to satisfy the budget constraints for all cases, the sum of the payments from the initial state until the state \( < R_0 \cup \{r_{t+1}\} >^{t+1} \) must be smaller than or equal to \( Z_{bud} - Z_{pt} \). The problem is how to distribute this amount among states from \( < \emptyset >^0 \) to \( < R_0 >^t \). In this method, we simply prorate this amount based on the bids specified in \( \pi \).

For all bids \( z \leq z_{max} \), we calculate \( Q(< R_0 >^t, z) \), choose the best bid, and update \( V(< R_0 >^t) \).

## 5. CONCLUSIONS

In this paper, we have proposed a method for determining an optimal strategy, using a dynamic programming technique in sequential auctions.

One future direction of this study is to develop a method for learning the optimal bidding strategy from experience, without assuming that the agent knows the distributions of the highest bids of other agents in advance.

## 6. REFERENCES

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