CYCLE-TO-CYCLE FLUCTUATIONS OF BURNED FUEL MASS IN SPARK IGNITION COMBUSTION ENGINES

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Abstract

We examine a simple, fuel-air, model of combustion in spark ignition (si) engine with indirect injection. In our two fluid model, variations of fuel mass burned in cycle sequences appear due to stochastic fluctuations of a fuel feed amount. We have shown that a small amplitude of these fluctuations affects considerably the stability of a combustion process strongly depending on the quality of air-fuel mixture. The largest influence was found in the limit of a lean combustion. The possible effect of nonlinearities in the combustion process were also discussed.

Keywords: stochastic noise, combustion, engine control

1 Introduction

Cyclic combustion variability, found in 19th century by Clerk (1886) in all spark ignition (si) engines, has attracted great interest of researchers during last years (Heywood 1988, Hu 1996, Daw \textit{et al.} 1996, 1998, 2000, Wendeker \textit{et al.} 2003, 2004). Its elimination would give 10\% increase in the power output of the engine (Heywood 1988). The key source of their existence may be associated with either stochastic disturbances (Roberts \textit{et al.} 1997, Wendeker \textit{et al.} 1999) or nonlinear dynamics (Daw \textit{et al} \ldots)
1996, 1998) of the combustion process. Daw et al. (1996, 1998) and more recently Wendeker et al. (2003, 2004) have done the nonlinear analysis of the experimental data of such a process. Changing an advance spark angle they observed the considerable increase of the noise level (Wendeker et al. 2003) claiming that it is due to chaotic dynamics of the process. On the other hand the main sources of cyclic variability were classified by Heywood (1988) as the aerodynamics in the cylinder during combustion, the amount of fuel, air and recycled exhaust gases supplied to the cylinder and a mixture composition near the spark plug. In this paper we will model the variation of fuel ignition amount as the most common source of instability in indirect injection.

The present paper is organized as follows. After the introduction in the present section we define the model by a set of difference equations in the next section (Sec. 2). This model, in deterministic and stochastic forms, will be applied in Sec. 3, where we analyze the oscillations of burned mass. Finally we derive conclusions and remarks in Sec. 4.

2 Two fluid model of fuel-air mixture combustion

Starting from fuel-air mixture we define the time evolution of the corresponding amounts. Namely, we will follow the time histories of the masses of fuel $m_f$, and air $m_a$.

Firstly, we assume the initial value of $m_a(i), m_f(i)$ automatically their ratio $r(i)$:

$$r(i) = \frac{m_a(i)}{m_f(i)}$$

for $i = 0$.

Secondly, depending on parameter $r$ with reference to a stoichiometric constant $s$ we have two possible cases: fuel and air deficit, respectively. For a deterministic model, the first case lead to

$$r(i) > 1/s$$

we calculate next masses using following difference equations:

$$m_f(i+1) = (1 - \alpha) \left( m_f(i) - \frac{1}{s} m_a(i) \right) + \delta m_f$$

$$m_a(i+1) = \delta m_f,$$
Table 1: Constants and variables of the model.

| Description                                           | Symbol | Value |
|-------------------------------------------------------|--------|-------|
| stoichiometric coefficient                           | $s$    | 14.63 |
| exhaust ratio                                        | $\alpha$ | 0.92  |
| air mass in a cylinder                                | $m_a$  |       |
| fuel mass in a cylinder                               | $m_f$  |       |
| fresh air amount                                      | $\delta m_a$ |       |
| fresh fuel amount                                     | $\delta m_f$ |       |
| air/fuel ratio                                        | $r = m_a/m_f$ |       |
| burned fuel mass                                      | $\Delta m_f$ |       |
| combusted air mass                                    | $\Delta m_a$ |       |
| air/fuel equivalence ratio                            | $\lambda$ |       |
| random number generator                               | $N(0,1,i)$ |       |
| mean value of fresh fuel amount                       | $\delta m_{f_o}$ |       |
| standard deviation of fresh fuel amount               | $\sigma_{f}$ |       |
| mean value of the equivalence ratio                   | $\lambda$ |       |
| standard deviation of the equivalence ratio           | $\sigma_{\lambda}$ |       |
Figure 1: The combustion curve $\Delta m(\lambda)$ for the constant fresh air feed $\delta m_a = 200$ mg.

where $\alpha$ is the exhaust ratio of the engine, $\delta m_f$ and $\delta m_a$ denotes fresh fuel and air amounts added in each combustion cycle $i$. In the opposite (to Eq. 2.2) case

$$r(i) < 1/s \quad (4)$$

we use the different formula

$$m_f(i + 1) = \delta m_f$$
$$m_a(i + 1) = (1 - \alpha)(m_a(i) - sm_f(i)) + \delta m_a \quad (5)$$

Note that variables $m_a$ and $m_f$ are the minimal set of our interest. From the above equations one can easily calculate other interesting quantities as the combusted masses of fuel $\Delta m_f(i)$ and air $\Delta m_a(i)$ and air-fuel equivalence ratio before each combustion event $i$:

$$\lambda \approx s\frac{m_a(i) + (1 - \alpha)\Delta m_a(i - 1)}{m_f(i) + (1 - \alpha)\Delta m_f(i - 1)} \quad (6)$$

Basing on experimental results we use the additional necessary condition (Kowalewicz 1984) of combustion process

$$0.6 < \lambda < 1.3. \quad (7)$$
For better clarity our notations of system parameters: constants and variables are summarized in Tab. 1.

Basing on the relations (Eqs. 2.1-2.7) we plotted the combustion curve for the assumed constant fresh air feed $\delta m_a = 200$ mg.

Finally, in the case of stochastic injection, instead of constant $\delta m_f(i)$ (Eqs. 2.3 and 2.5) (for each cycle $i$) we introduce its mean value $\delta m_{fo} = \text{const.}$, while $\delta m_f$ in the following way:

$$\delta m_f(i) = \delta m_{fo} + \sigma_{mf} N(0, 1, i), \tag{8}$$

where $N(0, 1, i)$ represents random number generator giving a sequence $i$ of numbers with a unit-standard deviation of normal (Gaussian) distribution and the nodal mean. The scaling factor $\sigma_{mf}$ corresponds to the mean standard deviation of the fuel injection amount. The cyclic variation of $\delta m_f(i)$ can be associated with such phenomena as fuel vaporization and fuel-injector variations.
3 Oscillations of burned fuel mass

Here we describe the results of simulations. Using Eqs. 2.1-2.8 we have performed recursive calculations for deterministic and stochastic conditions and obtained time histories of various system parameters: \( m_f, m_a, \Delta m_f, \Delta m_a \) and \( \lambda \). The results for \( \lambda \) are shown in Fig. 2. The upper panel (Fig. 2a), corresponding to deterministic combustion for three different values of fuel injection parameter \( \delta m_f \), shows \( \lambda \) as straight lines versus cycle \( i \), while the lower (Fig. 2b) one reflects the variations of \( \lambda \) in stochastic conditions. The order of curves appearing in the Fig. 2b is the same as in Fig. 2a stating from the smallest value of considered fuel injection amounts from the top. In stochastic simulations we used input of random fuel injection with standard deviation equal to 10% of its mean value \( \sigma_{mf} = 0.1 \delta m_{fo} \). The obtained results clearly indicate that the fluctuations of \( \lambda \) are growing with larger \( \lambda \). This can be also found by analytical evaluation of Eq. 2.6. It is not difficult to check that

\[
\sigma_\lambda \sim \lambda^2 \sigma_{mf}.
\]

The results for burned fuel mass \( \Delta m_f \) are presented in Fig. 3. Starting from deterministic conditions \( \delta m_f = \delta m_{fo} = \text{const.} \) we obtain the constant fraction of the burned fuel mass \( \Delta m_f \) represented by the three straight lines in Fig. 3a lying very close to each other. In Fig 3 b-c we show the same, \( \Delta m_f \), for the considered case of assumed fuel injection \( \delta m_{fo} = 13.50 \text{ mg} \) - Fig. 3b, \( \delta m_{fo} = 14.63 \text{ mg} \) - Fig. 3c, \( \delta m_{fo} = 21.00 \text{ mg} \) - Fig. 3d) and stochastic conditions. Due to different magnitudes parameter \( \lambda \) fluctuations, and dependence of combustion curve Fig. 1 it is not surprise that the fluctuations of \( \Delta m_f \) have different character in all these cases. For lean combustion, which is a stable process in deterministic case, the fuel injection fluctuations introduce considerable instabilities to the combustion process leading to the suppression of combustion because in some cycles (Fig. 3b) where \( \lambda \) is larger that 1.3. Then Equation 2.7 is not satisfied. In the next case (Fig. 3c) the effect of stochasticity is much smaller. Here we have optimal air-fuel mixture. First of all one should note that fluctuations of \( \lambda \) are smaller than in previous case (Fig. 2b). Moreover \( \lambda \) oscillate around the region \( \lambda \approx 1 \) in combustion curve (Fig. 1) which does not have big changes comparing to previous case. Finally, Fig. 3d shows the sequence of \( \Delta m_f \) for
Figure 3: The dependence of burned fuel mass on sequential cycles $i$ for deterministic (a) and stochastic (b-d) processes. $\delta m_a = 200$ mg while $\delta m_f$ takes different values: 13.50 mg, 14.63 mg, 21.00 mg denoted in particular figures a-c.
the large $\delta m_{f_0}$ (rich fuel-air mixture). The fluctuations of $\lambda$ are the smallest of all three ones but $\lambda \approx 0.7$ causes suppressions of combustions in some cycles similarly to the case shown in Fig. 3b.

4 Conclusions

In this paper we examined the origin of combusted mass fluctuations. In case of stochastic conditions we have shown that depending on the quality of fuel-air mixture the final effect is different. The worse situation is for lean combustion. The consequences of it can be observed for idle speed regime of engine work. Unstable engine work, interrupted by the cycles without combustion lead to a large increase of fuel use.

Although the presented two component model is very simple it can reflect the underlying nature of engine working conditions. In spite of fact that the model is characterized by the nonlinear transform (Eqs. 2.3, 2.5 and 2.7) similar to logistic one, we have not found any chaotic region. Possibly that such solutions can be found for non realistic model parameters like $\lambda$ and $\alpha$. The other strong limitation was concerned with the sharp edges of combustion curve ($\Delta m_f$ versus $\lambda$ Fig.1) modelled by a Heaviside step $\Theta(x)$ function Eq. 2.7. We used such an approximation as a simplest one but modeling with the exponential growth $\exp(-1/x)$ is more realistic and possible. Similar assumptions of the exponential dependence led to chaotic behaviour in papers (Daw et al. 1996, 1998, Wendeker 2003). From a physical point of view mixture gasoline-air is not uniform before ignition and that can cause nonuniform combustion smearing the edges of the combustion curve Fig. 1. Calculations considering this effect are in progress and the results will be reported in a separate future publication.

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