TOP WIDTH EFFECTS IN SOFT GLUON RADIATION

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ABSTRACT

Soft gluons radiated in top quark production and decay can interfere in a way that is sensitive to the top width. We show how the width affects the gluon distribution in $e^+e^- \rightarrow t\bar{t}$ and discuss prospects for measuring $\Gamma$ from gluons radiated near $t\bar{t}$ threshold.

1. Soft Gluon Radiation in $e^+e^- \rightarrow t\bar{t}$

Because the top quark is so heavy that it can decay to a real $W$ and a $b$, it has a very large width: for large $m_t$, $\Gamma(t \rightarrow Wb) \approx (175$ MeV)$ (m_t/m_W)^3$. Widths in the GeV range can give rise to interesting effects involving the interplay between the strong and weak interactions. For example, if top is heavy enough, it can decay before forming bound states, and there is not much resonant structure at the $t\bar{t}$ threshold, making it difficult to measure $\Gamma$. In this talk we consider the effect of the top width on soft gluon radiation in $e^+e^- \rightarrow t\bar{t}$, at arbitrary collision energies$^1$ and near $t\bar{t}$ threshold.$^2$ For complete discussions see Refs. [1] and [2].

Consider a gluon emitted in a $t\bar{t}$ event at an $e^+e^-$ collider. Because of the top decays, the gluon can be radiated by the $t, \bar{t}, b,$ or $\bar{b}$. In the limit of soft gluons, the matrix element $\mathcal{M}$ factorizes and can be written as a product of the zeroth-order matrix element (with no gluon) and a term associated with the gluon emission. Schematically, we have $\mathcal{M} \sim \mathcal{M}^{(0)} J \cdot \epsilon$, where $J^\mu$ and $\epsilon^\mu$ are the gluon current and polarization, respectively. We can then define a gluon emission probability density, which is just the differential cross section for radiating a gluon normalized to the zeroth-order cross section. It is given by

$$dN \equiv \frac{1}{\sigma_0} d\sigma_g = \frac{d\omega}{\omega} \frac{d\Omega}{4\pi} \frac{C_F \alpha_s}{\pi} R,$$

where $\sigma_0$ is the zeroth-order cross section, $C_F = 4/3$, and $\alpha_s$ is the strong coupling constant.

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$^2$Presented by L.H. Orr at the 1992 Meeting of the Division of Particles and Fields, Fermilab, November 10-14, 1992.
Figure 1: Soft gluon distribution in $e^+e^- \rightarrow t\bar{t}$ for c.m. energy 1 TeV, $m_t = 140$ GeV, $\omega = 5$ GeV and $\phi = 0^\circ$. $\theta$ is the $t$-$g$ angle; the $t$ and $\bar{b}$ are at $0^\circ$ and the $\bar{t}$ and $b$ are at $180^\circ$.

where $\omega$ and $\Omega$ denote the gluon energy and solid angle. $R$ is obtained by integrating the absolute square of the current over the virtualities of the $t$ and $\bar{t}$.

The important point is that the current can be decomposed in a gauge-invariant way into terms corresponding to order $\alpha_s$ corrections to $t\bar{t}$ production, to $t$ decay, and to $\bar{t}$ decay. Therefore in $R$ we can unambiguously identify each contribution: (production)$^2$, (t or $\bar{t}$ decay)$^2$, production–decay interference and decay–decay interference. The production and decay squared terms are independent of the width, but both interference terms have $\Gamma$ dependence, including an overall factor of $\Gamma^2$.

2. Width Effects at High Energies

At high collision energies, the top width dependence arises from production–decay interference; the decay–decay interference is negligible. The production–decay interference is largest for large $b$–$t$ angular separations, and is destructive, so that the effect of the width is to suppress the gluon radiation.

This is illustrated in Fig. 1, where we show the gluon emission probability as a function of the angle $\theta$ of the gluon with respect to the top quark. We vary the top width and take $m_t = 140$ GeV, $\omega = 5$ GeV, and center-of-mass energy 1 TeV. The $t$ and $\bar{t}$ are produced back-to-back, and we have chosen a configuration in which the $t(\bar{t})$ decays to a backward $b(\bar{b})$. We see that for the SM case ($\Gamma = 0.7$ GeV) the peaks are suppressed compared to the case with no interference ($\Gamma = 0$) and as the width increases the peaks disappear altogether.

Now, energetic top quarks do not often decay to backward $b$’s. If we take a slightly more likely $b$–$t$ angle such as $90^\circ$, we obtain similar sensitivity. However, in the most probable configuration — $t$ and $b$ collinear — there is almost no interference and therefore no sensitivity to the top width. Thus at high energies, the production–decay interference can be substantial, but the most sensitive configurations are the least likely to occur.

3. Width Effects Near $t\bar{t}$ Threshold

At lower energies, near the $t\bar{t}$ threshold, the total cross section is higher and the $t$’s are produced nearly at rest, so that the relative orientations of the $t$ and $b$
momenta are irrelevant. We might expect, then, that top width effects could be more pronounced. On the other hand, if the $t'$'s are nearly at rest and only the $b$'s can radiate, it is not obvious that the top width enters at all. Naively, one would expect the $b$'s to radiate as if they were produced directly and the $t$'s never existed.

That the top width does influence the radiation from the $b$'s can be understood by considering the following extreme cases. As $\Gamma \to \infty$, the top lifetime becomes very short, the $b$ and $\bar{b}$ appear almost instantaneously, and they radiate coherently, as though produced directly. In particular, gluons from the $b$ and $\bar{b}$ interfere. In the other extreme, for $\Gamma \to 0$, top has a long lifetime and the $b$ and $\bar{b}$ appear at very different times and therefore radiate independently, with no interference. Clearly, the top width controls the interference between gluons emitted by the $b$ and $\bar{b}$.

The situation for finite width is between the two extremes. Let $v$ be the $b$ (or $\bar{b}$) velocity, $\theta_{1(2)}$ be the angle between the $b$ ($\bar{b}$) and the gluon, and $\theta_{12}$ the angle between the $b$ and $\bar{b}$. Then

$$ R = \frac{v^2 \sin^2 \theta_1}{(1 - v \cos \theta_1)^2} + \frac{v^2 \sin^2 \theta_2}{(1 - v \cos \theta_2)^2} + 2\chi \frac{v^2 (\cos \theta_1 \cos \theta_2 - \cos \theta_{12})}{(1 - v \cos \theta_1)(1 - v \cos \theta_2)}, \tag{2} $$

where $\chi \equiv \frac{\Gamma^2}{4 \Gamma \omega}$. The interference is the term proportional to $\chi$. Note that $0 \leq \chi \leq 1$ and $\chi = 0$ for $\Gamma = 0$ (independent emission) and $\chi = 1$ for $\Gamma = \infty$ (coherent emission). Thus a finite top width suppresses the interference compared to the naive expectation of full coherent emission. And from the form of $\chi$ we see that the radiation pattern exhibits maximum sensitivity to $\Gamma$ when $\Gamma$ is comparable to the gluon energy $\omega$.

The width effects are discussed in detail in Ref. 2; here we give two examples. First consider gluons emitted perpendicular to the $bb$ plane. Then $\theta_1 = \theta_2 = \pi/2$; $R$ is simply proportional to $1 - \chi \cos \theta_{12}$ and $\chi$ regulates the $\theta_{12}$ dependence — see Fig. 2. Now for a 5 GeV gluon, a 140 GeV top quark with $\Gamma = 0.7$ GeV has $\chi \approx 0.02$, which means the distribution is much closer to the independent emission case ($\chi = 0$) than the coherent case we would naively expect – the interference is almost completely absent. If we could detect 1 GeV gluons, we would have $\chi \approx 0.3$ and the distribution would be very sensitive to the width. (Conversely, we would get the same sensitivity for more energetic gluons if the width were larger: $\chi \approx 0.3$.)
As a second example we show in Fig. 3 the gluon distribution integrated over the gluon solid angle (and in a slight nod to reality, integrated over gluon energies from 5 to 10 GeV). For independent emission the radiation probability does not depend on the angle between the $b$ and $\bar{b}$, but in the coherent case the interference is destructive for small $\theta_{12}$ and constructive for large $\theta_{12}$. Again, we see that the 140 GeV case is much closer to the independent than the expected coherent case, and that as the width reaches the few GeV range we become increasingly sensitive.

### 4. Discussion

Is looking at soft gluon radiation a useful method for measuring the top width, an alternative to studying the threshold structure\cite{3} of the lowest order cross section? Each method has its disadvantages: The threshold structure is subject to large uncertainties due to beam energy spread; the soft gluon radiation is not, but it is a higher order process with a lower event rate. The two methods should be considered complementary, because the threshold cross section loses sensitivity with increasing width, but as we have seen, the gluon radiation pattern becomes more sensitive at larger $\Gamma$ for accessible gluon energies. The bottom line is that for most of the expected top mass range, the threshold structure is probably better, but if $m_t$ and $\Gamma$ are large, examining soft gluons may be more useful.

In summary, we have seen that the top quark’s large width gives rise to new effects from the interplay between the strong and weak interactions, and that the top width affects the distributions of soft gluons radiated in top events. At high collision energies, production–decay interference can suppress gluon radiation. Near the $tt$ threshold, the effect of the width is to suppress the interference between gluons radiated by the $b$ and $\bar{b}$, in contrast to the expectation of coherent radiation from the $b\bar{b}$ pair. Finally, if the width and gluon energy are comparable, the radiation pattern is quite sensitive to the value of $\Gamma$.

### References

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