Jackiw-Teitelboim Gravity from the Karch-Randall Braneworld

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In this letter, we show that Jackiw-Teitelboim (JT) gravity can be naturally realized in the Karch-Randall braneworld. Notably the role of the dilaton in JT gravity is played by the radion in a suitably orbifolded version of the setup. In the classical entanglement entropy calculation, there is an apparent degeneracy of Ryu-Takayanagi surfaces. We demonstrate how quantum fluctuations of the radion/dilaton resolves this would-be classical puzzle regarding entanglement wedge reconstruction.

INTRODUCTION

Recent utilization of the doubly holographic Karch-Randall braneworld \cite{1, 2} has yielded substantial insights into quantum gravity. These include analytical models that have been used to produce Page curves describing black hole radiation in higher-dimensional (i.e in \(d > 2\) spacetime dimensions) setups \cite{3, 4}, demonstrating a connection between the existence of entanglement islands and massive gravity theories \cite{4, 9, 10}, providing simple holographic models to study quantum field theory in black hole backgrounds \cite{11, 12}, and motivating a novel holographic setup—wedge holography \cite{9, 10, 13, 14}.

Wedge holography is a simple canonical deformation of the original Karch-Randall braneworld which manifests a codimension-two holographic duality \cite{9, 10, 13, 14}. This setup starts with two AdS\(_d\) Karch-Randall branes in an ambient AdS\(_{d+1}\) with the two branes intersecting each other at the conformal boundary of the ambient spacetime. These branes are treated as “end-of-the-world” (EOW) branes, meaning that the part of the ambient spacetime behind them is excised. Thus, the leftover bulk spacetime is a wedge (see Fig. 1a). In analogy with the original Karch-Randall braneworld, the wedge system is doubly holographic in that there are three equivalent descriptions related to each other by the AdS/CFT correspondence \cite{15–17} (see Fig. 1):

- **The bulk description**: Einstein-Hilbert gravity in the wedge.
- **The intermediate description**: Two CFT\(_d\)'s coupled with gravity living on separate AdS\(_d\)'s and glued to each other along their asymptotic boundaries by imposing transparent boundary conditions.
- **The boundary description**: A CFT\(_{d-1}\) living on the common asymptotic boundary of the two branes (which is the corner of the wedge in Fig. 1a).

In this letter, we study wedge holography when the bulk is AdS\(_3\) with two AdS\(_2\) branes. As opposed to the original wedge holography setup in higher dimensions where the brane fluctuations can be neglected, we will consider fluctuating branes with orbifold projection so that there is only one fluctuating scalar degree of freedom. This scalar is the radion in the language of the Randall-Sundrum I setup \cite{18}. We will derive an action for the radion which is coupled with the 2d localized braneworld gravity. This can be thought of as the low-energy effective theory of the intermediate description. We will see that this theory is precisely the Jackiw-Teitelboim (JT) gravity \cite{19, 20}, in which the dilaton field is the radion.

The importance of our study is that it ensures the consistency of wedge holography in AdS\(_3\). This is because when we ignore the brane fluctuations and study the entanglement entropy in AdS\(_3\) wedge holography using the Ryu-Takayanagi (RT) formula, we find an infinite number of degenerate RT surfaces. This renders the entanglement wedge, which is an essential element in holography \cite{21}, as undefined. Taking into account the brane fluctuations, this degeneracy is lifted, as we show explicitly using our effective JT description.

We note that JT gravity \cite{22–25} has also played an important role in the recent development of low-dimensional (\(d = 2\)) quantum gravity \cite{5, 6, 26}. This includes provid-
In this section, we briefly review the Karch-Randall braneworld and wedge holography, focusing on the geometric pictures for later holographic study. The original theory of Karch and Randall is described by the following action

\[ S = -\frac{1}{16\pi G_{d+1}} \int d^{d+1}x \sqrt{-g_{\text{bulk}}} (R[g_{\text{bulk}}] - 2\Lambda) \]

\[ - \frac{1}{8\pi G_{d+1}} \int d^d x \sqrt{-h} (K - T), \]

where the first term is the standard Einstein-Hilbert action describing the gravitational in the ambient AdS\(_{d+1}\) with a cosmological constant \(\Lambda = -\frac{d(d-1)}{2}\) and the second term describes the embedding of the brane in the ambient AdS\(_{d+1}\), \(h_{ab}\) is the induced metric on the brane, \(K\) is the trace of the extrinsic curvature of the brane and \(T\) is the tension of the brane and satisfies \(T \leq (d-1)\) as a Karch-Randall brane.\(^3\) The bulk metric fluctuations satisfy the Neumann boundary condition near the brane:

\[ \nabla_n \delta g_{\text{bulk}} \mu \nu |_{\text{near brane}} = 0, \]

where \(n\) denotes the normal direction of the brane. With this boundary condition taken into account, setting the variation of the action Equ. (1) to zero we obtain two equations of motion—the usual Einstein’s field equation which determines the bulk geometry and the brane-localized equation of motion describing the embedding of the brane in the bulk. The latter is given by [29]

\[ K_{ab} = (K - T) h_{ab}. \]

This system of equations can be solved by

\[ ds^2_{\text{bulk}} = dr^2 + \cosh^2(r) ds^2_{\text{AdS}_d}, \]

where \(r \in (-\infty, \infty)\), with the brane given by \(r = r_1 = \text{const.}\) and \(T = (d-1)|\tanh r_1|\) (we only consider positive tension branes).\(^4\) The bulk metric is just empty AdS\(_{d+1}\)

\(^2\) We have set the AdS curvature length to one.

\(^3\) The critical value \(d = d - 1\) corresponds to the Randall-Sundrum brane [18, 28].

\(^4\) If the brane is given by \(r = r_1 < 0\) understood as an EOW brane with positive tension the domain of the bulk is \(r \in (r_1, \infty)\) and if the brane is given by \(r = r_2 > 0\) understood as an EOW brane with positive tension the bulk domain is \(r \in (-\infty, r_2)\) [30].
is a dynamical variable called the radion, which has important phenomenological implications [33]. Hence in the case of wedge holography, we should not neglect the possibility of brane fluctuations.

In this section, we consider small brane fluctuations (relative to the bulk AdS curvature scale) and derive the effective action for the coupling between these small fluctuations and the brane-localized graviton mode by doing dimensional reduction. While brane fluctuations can be neglected in higher dimensions, we will see that this small contribution plays an important role when the bulk is AdS$_3$. The detailed derivation and a more careful analysis will be presented in [27].

To be precise, we will consider two branes with tensions $T_1$ and $T_2$ sitting at $r = r_1 + \delta \phi_1(x)$ and $r = r_2 + \delta \phi_2(x)$ and $\phi$ describes the lowest 2d graviton mode (the higher modes would depend on the coordinate $r$ as $g_{ab}(r,x)$).

To obtain the low-energy effective action, we have to plug the above ansatz of the metric and brane locations into Eqn. (5) and keep terms only up to order $\mathcal{O}(\delta \phi^2)$. This yields the following action:

$$S_{\text{eff}} = S_0 - \frac{1}{16\pi G_3} \int d^2 x \sqrt{-g} \phi(x) \left[ R[g] + 2 \right] + S_{\text{dilaton}},$$

where $\phi(x) = \delta \phi_2(x) - \delta \phi_1(x)$ is the radion mode, $S_0$ is a purely topological term

$$S_0 = -\frac{T_2 - r_1}{16\pi G_3} \int d^2 x \sqrt{-g} R[g],$$

and $S_{\text{dilaton}}$ is the kinetic term for the brane fluctuations

$$S_{\text{dilaton}} = -\frac{1}{8\pi G_3} \int d^2 x \sqrt{-g} \left[ \frac{T_2}{2} \nabla_{\mu} \delta \phi_2 \nabla^{\mu} \delta \phi_2 + T_2 (\delta \phi_2)^2 + \frac{T_1}{2} \nabla_{\mu} \delta \phi_1 \nabla^{\mu} \delta \phi_1 + T_1 (\delta \phi_1)^2 \right].$$

In analogy to the Randall-Sundrum scenario, to isolate the dynamics of the radion we have to orbifold our setup. We can project out either $\delta \phi_1(x)$ or $\delta \phi_2(x)$ by a canonical orbifold prescription: start with an array of an infinite number of paired wedges, mod out the translation symmetry on the array to a single pair of wedges with $\mathbb{Z}_2$ symmetry, and then mod out this $\mathbb{Z}_2$ symmetry (see [27] for details) to get a single wedge with only one fluctuating brane. The possibility of projecting out either $\delta \phi_1(x)$ or $\delta \phi_2(x)$ are gauge-equivalent. To see this gauge equivalence, without loss of generality we can project out $\delta \phi_1(x)$ and redefine $g_{ab}(x)$ such that the final action is independent of $r_1$ and $r_2$. Specifically, by setting

$$\delta \phi_1(x) = 0, \quad g_{ab}(x) \to e^{\lambda_2 \delta \phi_2(x)} g_{ab}(x),$$

the total action becomes

$$S_{\text{eff}} = S_0 - \frac{1}{16\pi G_3} \int d^2 x \sqrt{-g} \phi(x) \left[ R[g] + 2 \right],$$

where now the dilaton field is $\phi(x) = \delta \phi_2(x)$. This is precisely Jackiw-Teitelboim gravity [19, 20] for AdS$_2$.

However, there is no nontrivial profile for the dilaton on the AdS$_2$ background without a cutoff [34]. This cutoff breaks the asymptotic conformal symmetry and allows the theory to have finite energy excitations [24]. Hence we fix a small cutoff $\epsilon$ and fix the boundary metric for the nearly-AdS$_2$ and the dilaton field to be

$$g_{ab}(x)|_{\text{bdy}} \, dx^a dx^b = -\frac{du^2}{\epsilon^2}, \quad \phi(x)|_{\text{bdy}} = \phi_b = \frac{\phi_r}{\epsilon}.$$  

A subtlety of our derivation is that we have to take $\phi_b \sim \epsilon$ (or $\phi_r \sim \epsilon^2$) to ensure the brane fluctuation is always small, i.e. $\delta \phi_b(x) \ll 1$.

Translating to the AdS$_3$ language Eqn. (6), we have to take a cutoff of the AdS$_3$ wedge and fix the metric there to be (see Fig. 3 for an indication)

$$ds_{\text{bdy}}^2 = dr^2 - \cosh^2(r) \frac{du^2}{\epsilon^2}.$$  

Now that we have identified this cutoff and boundary conditions for the metric in the AdS$_3$ bulk, we have to add the corresponding boundary term to the total action Eqn. (5). The boundary term is the usual Gibbons-Hawking term

$$S_{\text{bdy}} = S_{\text{GH}} = -\frac{1}{8\pi G_3} \int d^2 x_{\text{bdy}} \sqrt{-h_{\text{bdy}}} K^{(3)},$$  

FIG. 3. The shapes of the two branes are fluctuating (though more rigorously we have also to impose the orbifold projection). The AdS$_2$ cutoff is induced by the AdS$_3$ cutoff (the red curve). The shaded regions are excised. Now the regulated asymptotic boundary of the wedge are two intervals.
A surface is an RT surface and they all have equal area leads to the area functional

gument entropy from the wedge. However, this would be should use the HRT formula \[41\] to compute the entan-
tions to lift the degeneracy of the RT surfaces. Because
brane fluctuations is in general time-dependent, we
should use the HRT formula \[41\] to compute the entan-
glement entropy from the wedge. However, this would be

\[
S_{\text{bdy}} = \frac{2(r_2 - r_1)}{16\pi G_3} \int_{\partial \mathcal{M}} dx \sqrt{-h}K = \frac{2\phi_b}{16\pi G_3} \int_{\partial \mathcal{M}} dx \sqrt{-h}K ,
\]

where \(K\) is the trace of the extrinsic curvature of the
cutoff boundary of the AdS\(2\). With this term added to
Equ. (11), we get the full-fledged JT gravity action as
that in \[35\]. For more details we refer the readers to \[27\].

We emphasize that the scaling \(\phi_b \sim \epsilon\) is special in our
setup. For example, it does not appear in dimensional
reduction for the near-horizon region of four-dimensional
near-extremal black holes \[36, 37\]. This indicates that
our theory has a different UV completion than previously
studied. In our case, the UV completion is a conformal
quantum mechanics system \[27\].

A PUZZLE AND ITS RESOLUTION

A canonical quantity to study in AdS\(_3\) wedge holo-
graphy is the entanglement entropy between the two asym-
ptotic defects which support the dual quantum mechanics
(see Fig. 2b). This can be calculated using the Ryu-
Takayanagi (RT) formula \[38, 39\] by studying the RT
surfaces that connect the two branes \[9, 13\]. Choos-
ing global AdS\(_2\) for those AdS\(_2\) slices, the bulk metric
Equ. (4) can be written as

\[
ds^2 = dr^2 + \cosh^2(r)\left[-\cosh^2(\eta)dr^2 + d\eta^2\right],
\]

if we ignore the brane fluctuations \(\delta \phi_b(x)\). The RT sur-
face can be parametrized as \(\eta = \eta(r), \tau = \text{const.}\) which
leads to the area functional

\[
A = \int_{r_1}^{r_2} dr \sqrt{1 + \cosh^2(r)\eta'(r)^2}.
\]

It is easy to show that, with the boundary term carefully
taken into account, the variation of the area functional
vanishes if and only if \(\eta'(r) = 0\). Therefore, any constant
\(\eta\) surface is an RT surface and they all have equal area
\(A = r_2 - r_1\) (see Fig. 4 for an illustration). According to
this result, there would be no definite RT surface for the
entanglement entropy between the two defects and as a
 corollary there would be no definite entanglement wedge
for either of the defects. This would be problematic be-
cause it would preclude a holographic interpretation from
the point of view of bulk (or more precisely, entanglement
wedge \[40\]) reconstruction.

Fortunately, this puzzling result occurs only when the
branes are treated as rigid. We expect brane fluctua-
tions to lift the degeneracy of the RT surfaces. Because
the brane fluctuation is in general time-dependent, we
should use the HRT formula \[41\] to compute the entan-
glement entropy from the wedge. However, this would be

\[
S_{\text{EE}} = \max_{\tau} \min_{\eta} \left(\frac{\phi_0 + \phi(x)}{4G_3}\right) = \frac{\phi_0 + \phi_\tau}{4G_3},
\]

where the minimization has a solution \(\eta = 0, \tau = 0\). As
a result, from the bulk AdS\(_3\) perspective Equ. (16), the
RT surface is just the \(\eta = 0\) surface which sits precisely
in the middle of the bulk (i.e. the middle green surface
in Fig. 4). Thus, the entanglement wedge of either defect
is the half bulk region next to it, which is consistent with
the symmetry between the two defects.

CONCLUSIONS

In this letter, we studied wedge holography when the
bulk is three-dimensional. We found that it is necessary
to consider brane fluctuations, identified as the radion
mode in the braneworld language \[18, 28\], for the ap-
parent consistency of the holographic setup. We found that
the low-energy dynamics of the brane fluctuations with
orbifold projection is precisely Jackiw-Teitelboim gravity.
ACKNOWLEDGEMENTS

We are grateful to Andreas Blommaert, Kristian Jensen, Hong Liu and Douglas Stanford for useful discussions. HG is very grateful to his parents and Recommenders. The work of HG is supported by the grant (272268) from the Moore Foundation “Fundamental Physics from Astronomy and Cosmology.” The work of AK and MR was supported, in part, by a grant from the Simons Foundation (Grant 651440, AK). The work of CP was supported in part by the National Science Foundation under Grant No. PHY-1620806 and PHY-1915071, the Chau Energy grant RTI4001. The work of LR is supported by the government of India through the Department of Atomic Energy grant RTH4001. The work of LR is supported by NSF grants PHY-1620806 and PHY-1915071, the Chau Foundation HS Chau postdoc award, the Kavli Foundation grant “Kavli Dream Team,” and the Moore Foundation Award 8342. SS is supported by NSF Grants No. PHY-1914679 and PHY-2112725.

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