Nonperturbative calculation of the anomalous magnetic moment in the Yukawa model within truncated Fock space

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Within the covariant formulation of light-front dynamics, we calculate the state vector of a physical fermion in the Yukawa model. The state vector is decomposed in Fock sectors and we consider the first three ones: the single constituent fermion, the constituent fermion coupled to one scalar boson, and the constituent fermion coupled to two scalar bosons. This last three-body sector generates nontrivial and nonperturbative contributions to the state vector, which are calculated numerically. Field-theoretical divergences are regularized using Pauli-Villars fermion and boson fields. Physical observables can be unambiguously deduced using a systematic renormalization scheme we have developed previously. As a first application, we consider the anomalous magnetic moment of the physical fermion.

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I. INTRODUCTION

The understanding of hadronic systems in terms of their elementary degrees of freedom have been, and still is, one of the most challenging problems in particle and nuclear physics over the last ten years. The phenomenological properties of hadrons are now rather well understood in terms of models, like the constituent quark model or the bag model. The understanding of their properties from the original Lagrangian of QCD is however still under active debate.

In nuclear physics, the properties of nuclear structure in terms of the exchanges of pions are also well known. They are described by using a phenomenological nucleon-nucleon potential expressed in terms of the exchanges of one pion, two correlated pions and so on. However, their complete description from an effective chiral Lagrangian is still missing.

A common difficulty in both domains is the description of relativistic bound systems. This description should be nonperturbative from the start in order to be able to find, for instance, the physical mass of the bound state from the pole of the scattering amplitude. The problem is especially acute when the interaction coupling constant is large.

One of the most relevant approaches aimed at studying relativistic systems of interacting particles is light-front dynamics (LFD) proposed initially by Dirac. LFD is a form of Hamiltonian dynamics which deals with the state vector defined not at a fixed time moment, but on the light front plane \( t + z = 0 \), in its traditional form. The state vector is then usually decomposed in a series of Fock sectors, each containing a fixed number of particles.

The use of LFD to investigate relativistic bound states has been advocated for a long time. However, while the dynamics of few-body systems, based on a phenomenologically constructed interaction, has developed rapidly, application of LFD to field theory beyond a perturbative framework is not yet under complete theoretical control. This is due to the fact that any practical calculation relies on taking into account only a restricted number of Fock sectors in the state vector decomposition or, in other words, on the Fock space truncation. This approximation strongly complicates the renormalization procedure, in contrast to that in standard perturbation theory. Indeed, the full cancellation of field-theoretical divergences which appear in a given Fock sector requires taking into account contributions from other sectors. If even a part of the latter is beyond our approximation, some divergences may leave uncanceled. Mathematically, it reflects itself in possible dependence of approximately calculated observables on the regularization parameters (e.g., cutoffs). This prevents to make any physical predictions if we cannot control the renormalization procedure in one way or another.

In a previous study [2] (see also references therein) we have developed an appropriate renormalization procedure – the so-called Fock sector dependent renormalization (FSDR) scheme – in order to keep the cancellation of field-theoretical divergences under perma-
nant control. Our approach is based on the covariant formulation of LFD (CLFD), where the state vector is defined on an arbitrary light-front plane characterized by a light-like four-vector $\omega$ and given by the equation $\omega \cdot x = 0$. The covariant formulation is necessary in order to control any violation of rotational invariance, including that which is caused by the Fock space truncation. In particular, this is important in order to formulate, in an unambiguous way, the renormalization conditions one should impose on the bare coupling constant (BCC) to relate it to the physical one.

In Ref. [2] we calculated the fermion state vector and the electromagnetic form factors within the Yukawa model and QED in the lowest nontrivial approximation, when the state vector includes only two Fock sectors given by one constituent fermion and one constituent fermion coupled to one boson. For this two-body Fock space truncation, the electromagnetic form factors are identical to those obtained in the second order of perturbation theory, giving rise to a Schwinger-type correction to the fermion magnetic moment. Note that this result is not surprising, in spite of the fact we have not done any expansion in powers of the coupling constant, since no other contributions to the fermion electromagnetic vertex, apart from the perturbative ones (resummed to all orders) is generated in the two-body truncation.

We shall present in this work the calculation of the fermion anomalous magnetic moment (AMM) within the same Yukawa model, but for the three-body Fock space truncation, when the state vector includes an additional Fock sector containing one constituent fermion coupled to two scalar bosons. The presence of three-body states gives rise to nontrivial nonperturbative contributions to the AMM, which can not be fully incorporated in perturbation theory. Besides that, the Yukawa model is a quite nontrivial one from the point of view of the renormalization procedure, since it exhibits simultaneously mass, vertex, and wave function renormalization.

The plan of the article is the following. We recall in Sec. II the main features of our nonperturbative approach and the renormalization procedure. The eigenvalue equations are derived in the three-body truncation in Sec. III. We calculate the fermion electromagnetic form factors in Sec. IV and present our numerical results for the AMM in Sec. V. Our conclusions are drawn in Sec. VI. The appendices collect all necessary details to calculate the AMM.
where \( I_n \) is the contribution of the \( n \)-body Fock sector to the full norm. Explicit formula for \( I_n \) can be found in Ref. [2].

In the following, we shall use CLFD [3] as a general framework. The covariance of our approach is due to the invariance of the light front plane equation. This implies that \( \omega \) is not the same in any reference frame, but varies according to Lorentz transformations, like the coordinate \( x \). It is not the case in the standard formulation of LFD where \( \omega \) is fixed to \( \omega = (1, 0, 0, -1) \) in any reference frame.

The light-front momentum operator \( \hat{P}_\rho \) can be constructed from the energy-momentum tensor. It is decomposed according to

\[
\hat{P}_\rho = \hat{P}_\rho^{(0)} + \hat{P}_\rho^{\text{int}},
\]

where the two terms on the r.-h. s. are, respectively, the free (i.e. independent of the coupling constant and counterterms) and interaction parts of the four-momentum operator. The operator \( \hat{P}_\rho^{\text{int}} \) is related to the interaction Hamiltonian \( H^{\text{int}}(x) \) on the light front by

\[
\hat{P}_\rho^{\text{int}} = \omega_\rho \int H^{\text{int}}(x) \delta(\omega x) d^4x.
\]

From the general transformation properties of the light-front plane \( \omega x = 0 \), one can derive the following conservation law [3] for each Fock component:

\[
k_1 + k_2 + \cdots + k_n = p + \omega \tau_n,
\]

where the quantity \( \tau_n \) is a measure of how far the \( n \)-body system is off the energy shell\(^1\). It is completely determined by the conservation law [9] and the on-mass shell condition for each individual particle momentum:

\[
2\omega \cdot p \tau_n = (s_n - M^2),
\]

where \( s_n = (k_1 + \cdots + k_n)^2 \).

It is convenient to introduce, instead of the wave functions \( \phi_n \), the vertex functions \( \Gamma_n \) (which we will also refer to as Fock components), defined by

\[
\bar{u}(k_1) \Gamma_n u(p) = (s_n - M^2) \phi_n \equiv 2\omega \cdot p \tau_n \phi_n.
\]

The vertex function \( \Gamma_n \) will be represented graphically by the diagram of Fig. 1. With the definition

\[
\mathcal{G}(p) = \sum_{n=1}^{\infty} \int dD_n \bar{u}(k_1) \Gamma_n(k_1, \ldots, k_n, p) u(p) |n\rangle,
\]

the eigenvalue equation [1] writes [2]

\[
\mathcal{G}(p) = \frac{1}{2\pi} \int \left[-\hat{H}^{\text{int}}(\omega \tau)\right] \frac{dx}{\tau} \mathcal{G}(p),
\]

where \( \hat{H}^{\text{int}} \) is the interaction Hamiltonian in momentum space:

\[
\hat{H}^{\text{int}}(p) = \int H^{\text{int}}(x) e^{-ipx} d^4x.
\]

FIG. 1: Vertex function of order \( n \) for the Fock space truncation of order \( N \).

With the form [13], the eigenvalue equation can thus be represented graphically, using the same rules as those derived in Ref. [3] for the calculation of matrix elements of the \( S \)-matrix. This graph techniques was developed by Kadyshevsky [4] and transformed to the case of CLFD in Ref. [5].

The substitution of the decomposition [12] into the eigenvalue equation [13] results in an (infinite) system of equations for the Fock components. In order to solve this system in practice, we should make it finite, i.e., truncate the decomposition [12], or equivalently [2], by retaining only those Fock sectors where the number of particles does not exceed some maximal value \( N \). The finite system can be solved numerically and nonperturbatively, that is, for any value of the coupling constant. This approach was developed in a series of papers [2] [6]. In Ref. [2] it was applied to the case of the two-body truncation, i.e. for \( N = 2 \).

B. Renormalization conditions

In order to be able to make definite predictions for physical observables, one should also define a proper
B Renormalization conditions

The renormalization scheme which allows to express, like in perturbation theory, observables through the physical coupling constant and masses and exclude the bare ones. The basis of the state vector decomposition, i.e. the states \( |n\rangle \) in Eq. (4) is constructed from free physical fermion and boson fields, with their physical masses \( m \) and \( \mu \), respectively. The interaction Hamiltonian contains the corresponding mass counterterms (MC’s) \( \delta m \) and \( \delta \mu^2 \) responsible for the fermion and boson mass renormalization. Since we will not consider here the fluctuations of the boson in terms of fermion-antifermion pairs, we have to set \( \delta \mu^2 = 0 \). The MC \( \delta m \) is determined from the eigenvalue equation (13) by demanding that the bound state mass \( M \) is equal, for the ground state, to the physical mass \( m \) of the constituent fermion. For this reason, we will distinguish \( M \) and \( m \) only when it is necessary. Otherwise, we will set \( M = m \).

Besides MC’s, the interaction Hamiltonian includes also the BCC \( g_0 \). The latter is determined, as in perturbation theory, by relating the on-energy-shell two-body vertex function \( \Gamma_2 \) to the physical coupling constant \( g \). As follows from Eq. (9), taking \( \Gamma_2 \) on the energy shell is equivalent to setting \( \tau_2 = 0 \). Once \( M \) is identified with \( m \), the latter condition reduces to \( s_2 = (k_1 + k_2)^2 = m^2 \) [see Eq. (10)]. Below, for brevity, we will denote the on-energy-shell two-body vertex function as \( \Gamma_2(s_2 = m^2) \) to indicate that its arguments are connected by the corresponding kinematical constraint.

Being a solution of the system of eigenvalue equations (13), \( \Gamma_2 \) depends on the BCC \( g_0 \). Hence, relating \( \Gamma_2 \) to \( g \) is equivalent to relating \( g_0 \) to \( g \), which just means coupling constant renormalization. This can be most easily done starting from the three point function with all undressed on-mass-shell external lines, called \( \tilde{\Gamma}_2 \). It is connected with the physical coupling constant by the following standard relation (see e.g. Ref. [9]):

\[
\sqrt{Z_f} \tilde{\Gamma}_2(s_2 = m^2)\sqrt{Z_f} \sqrt{Z_b} = g, \tag{15}
\]

where the \( Z \)-factors are the so-called field strength renormalization constants for the fermion (\( f \)) (both in the initial and final state) and boson (\( b \)) lines, respectively. This condition can also be recovered by demanding that the residue of the fermion-boson elastic scattering amplitude at \( s_2 = m^2 \) equals \( g^2 \). One can thus deduce the expression for \( Z_f \) in terms of the full fermion self-energy \( \Sigma(\not{p}) \):

\[
Z_f = \left[ 1 - \frac{\partial \Sigma(\not{p})}{\partial \not{p}} \right]^{-1}_{\not{p} = m} \tag{16}
\]

and similarly for \( Z_b \) as a function of the full boson self-energy.

The two-body Fock component \( \Gamma_2 \) being a solution of the eigenvalue equation (13) does not coincide with \( \tilde{\Gamma}_2 \). By definition, \( \Gamma_2 \) has no radiative corrections to any of its three legs, while \( \tilde{\Gamma}_2 \), on the contrary, includes such corrections. The relation between these two vertex functions taken off the energy shell is rather complicated in LFD. Fortunately, we need to know it on the energy shell only, where it simplifies strongly, because the on-shell radiative corrections mentioned above reduce to \( \epsilon \)-number factors. Indeed, \( \Gamma_2 \) is a particular case of the general vertex function shown in Fig. 1 corresponding to \( n = 2 \), i.e. two of its legs are represented by single external lines (one for the constituent fermion and one for the constituent boson), while the third leg, for the physical fermion, is shown by a double line. Radiative corrections to each of the two external single lines are given by insertions of self-energy parts with their subsequent summation. The latter, of course, can be done directly within LFD by using the graph techniques rules, but we will choose a simpler way.

Each on-energy-shell amplitude calculated in LFD must coincide with that found in the standard four-dimensional Feynman approach and taken on the mass shell. Hence, \( \Gamma_2(s_2 = m^2) \) coincides with its on-mass-shell Feynman counterpart.

The summation of radiative corrections to external lines in the Feynman approach is technically easier than in LFD, since it can be done for each of the two lines independently. We allow the external particle momenta being off the mass shell (in order to avoid intermediate singularities), then sum up the radiative corrections and finally perform a limiting transition to the mass shell. Thus, summing a chain series of self-energy blocks - together with the mass-counterterm insertion - on the constituent fermionic line with the four-momentum \( k_1 \) brings the factor

\[ \tau_2 \]

A three-leg vertex which enters, as an internal sub-block, in physically observed amplitudes is always off-shell. Taking it on shell, we imply its analytical continuation into a nonphysical kinematical region.
B Renormalization conditions

\[
\lim_{\theta \to m} \left[ 1 + \frac{\Sigma_r(\theta_1)}{\theta_1 - m} + \frac{\Sigma_g(\theta_1)}{\theta_1 - m} \times \frac{\Sigma_r(\theta_1)}{\theta_1 - m} + \ldots \right] = \lim_{\theta \to m} \left( \frac{\theta_1 - m}{\theta_1 - m - \Sigma_r(\theta_1)} \right) \\
= \left[ 1 - \frac{\partial \Sigma_r(\theta_1)}{\partial \theta_1} \right]_{\theta = m}^{-1},
\]

as shown in Appendix A

C. Renormalization scheme

The above conditions imposed on the BCC and MC are necessary in order to express physical observables, like the electromagnetic form factors, through the measurable coupling constant and masses. As a consequence, one should expect full cancellation of divergences.

Such a program could be realized in perturbation theory or nonperturbatively if the Fock space is not truncated. The latter case is hardly achieved in practice. Usually, Fock space is truncated to a finite order of admitted Fock sectors, and the cancellation of divergences is not anymore guaranteed. For instance, looking at Fig. 2 for the calculation of the fermion propagator in the second order of perturbation theory, one immediately realizes that the cancellation of divergences between the self-energy contribution (of order two in the Fock decomposition) and the fermion MC (of order one) involves two different Fock sectors. This means that, as a necessary condition for the cancellation of divergences, any MC and, more generally, any BCC should be associated with the number of particles present (or “in flight”) in a given Fock sector. In other words, all MC’s and BCC’s must depend on the Fock sector under consideration. The original MC, \( \delta m \), and the fermion-boson BCC, \( g_0 \), should thus be extended each to a whole series:

\[
\begin{align*}
g_0 & \to g_0 \delta m, \\
\delta m & \to \delta m_1
\end{align*}
\]
with \( l = 1, 2, \ldots, N \). The quantities \( g_{0l} \) and \( \delta m_l \) are calculated by solving the systems of equations for the vertex functions in the \( N = 1, N = 2, N = 3, \ldots \) approximations successively. This FSDR scheme has been proposed initially in Ref. [11] and developed as a full renormalization scheme in Ref. [2]. An alternative approach, also in the Pauli-Villars (PV) regulated Yukawa model with the two-boson truncation, but with a sector-independent renormalization scheme, was developed in Ref. [12].

Note that the series (22) does not imply that we have an infinite number of counterterms or bare parameters. We still have the original ones \( g_0 \) and \( \delta m \) in the Hamiltonian we start with, but they have different values according to the level of approximation used in the calculation. In the limit of an infinite \( N \), and if the Fock sector expansion converges, \( g_{0N} \) and \( \delta m_N \) should turn to the true BCC and the MC, respectively. This is completely analogous to the case of perturbation theory where, at each order \( n \), one determines \( g_0^{(n)} \) and \( \delta m^{(n)} \).

Apart from the mass and vertex radiative corrections, the third type of divergences arises from the field renormalization, i.e., from the constants \( Z_f \) and \( Z_0 \). The values of these constants should also depend on the maximal number \( N \) of particles kept in a given truncation. Consider for instance the vertex function \( \Gamma_2 \) represented by Fig. 1 for \( n = 2 \). The dressing of the physical fermion leg (the factor \( \sqrt{\Gamma_f} \)) should be calculated for the truncation to the \( N \)-th order. The situation changes however for the constituent (single) fermion line. The state in which the constituent fermion is considered already contains one constituent boson. Hence, even if the boson line is not dressed, the dressing of the constituent fermion leg involves radiative corrections of order \( (N - 1) \). In other words, the dressing factor \( Z_f \) for the constituent fermion leg must be calculated for the lower, \( (N - 1) \)-body truncation. Otherwise, we would go beyond our approximation, since the effective number of particles in which the physical fermion can fluctuate would exceed \( N \).

Taking this into account, the relations (15) and (18) for a finite order truncation \( N \) (and in the absence of boson dressing) obtain the following form:

\[
\sqrt{Z_f^{(N)} \Gamma_2(s_2 = m^2) Z_f^{(N-1)}} = g, \quad (23a)
\]

\[
\Gamma_2(s_2 = m^2) = \sqrt{I_1^{(N)} \Gamma_2(s_2 = m^2) Z_f^{(N-1)}}. \quad (23b)
\]

The superscripts \( (N) \) and \( (N - 1) \) here and below just indicate the order of the Fock space truncation in which the corresponding quantities are calculated.

It follows from Eqs. (23a) and (21) that the renormalization condition (20) simply writes

\[
\Gamma_2(s_2 = m^2) = g \sqrt{I_1^{(N-1)}}, \quad (24)
\]

in the absence of boson dressing.

For the simplest case of the two-body truncation, \( N = 2 \), one thus gets

\[
\Gamma_2^{(2)}(s_2 = m^2) = g, \quad (25)
\]

since \( I_1^{(1)} = 1 \). We recover here the condition given in Ref. [2]. This condition is however valid only for \( N = 2 \).

### III. YUKAWA MODEL IN THREE-BODY TRUNCATED FOCK SPACE

#### A. Eigenvalue equations

We consider in this study the Yukawa model: a spin-1/2 fermion interacting with massive spinless bosons. The regularization is provided by the PV method. In addition to physical particles, we introduce therefore one PV fermion and one PV boson with (large) masses \( m_1 \) and \( \mu_1 \), respectively. This amounts to extend the physical Fock space to embrace negatively normalized PV particles [2]. The interaction Hamiltonian in Eq. (13) is given by

\[
H^{int} = -g_0 \psi^\dagger \psi \varphi' - \delta m \tilde{\psi}^\dagger \tilde{\psi}',
\]

with

\[
\psi' = \psi + \psi_{PV}, \quad \varphi' = \varphi + \varphi_{PV}, \quad (26)
\]

where \( \psi \) and \( \varphi \) are the free physical fermion and boson field operators, while \( \psi_{PV} \) and \( \varphi_{PV} \) are their PV partners, with negative norm. The bosons are supposed to be neutral.

In the three-body truncation, the system of coupled equations for the vertex functions, derived from the eigenvalue equation (13), is shown graphically in Fig. 3. On the r.-h. s. of the last equation, the sum of the diagrams with permutated boson legs appears, reflecting the symmetrization of the amplitude due to the identity of bosons. Expressing \( \Gamma_3 \) through \( \Gamma_2 \) by means of this equation, and substituting the result into the second equation, we can exclude the highest
A Eigenvalue equations

\[ \Gamma_1 = \Gamma_1 \delta m_3 + \Gamma_2 \delta m_2 + \Gamma_3 \delta m_3 \]

\[ \Gamma_2 = \Gamma_1 g_{03} + \Gamma_2 \delta m_2 + \Gamma_3 g_{02} \]

\[ \Gamma_3 = \Gamma_2 g_{02} + \Gamma_2 g_{02} \]

FIG. 3: System of equations for the vertex functions in the case of three-body Fock space truncation.

\[ \bar{u}(p_1) \Gamma_1 u(p) = \bar{u}(p_1) (V_1 + V_2) u(p), \] 
\[ \bar{u}(k_1) \Gamma_2 u(p) = \bar{u}(k_1) (V_3 + V_{45} + V_6) u(p), \]

where the indices \( i \) and \( j \) refer to whether the line of a constituent fermion \( (i) \) or a constituent boson \( (j) \) corresponds to a physical particle \( (i,j = 0) \) or to a PV one \( (i,j = 1) \). The term \( V_{45} \) means the sum of \( V_4 \) and \( V_5 \). The explicit expressions for \( V_{1-6} \) are given in Appendix B.

Analytically, these equations read

\[ \bar{u}(p_1) \Gamma_1 u(p) = \bar{u}(p_1) (V_1 + V_2) u(p), \] 
\[ \bar{u}(k_1) \Gamma_2 u(p) = \bar{u}(k_1) (V_3 + V_{45} + V_6) u(p), \]

where the indices \( i \) and \( j \) refer to whether the line of a constituent fermion \( (i) \) or a constituent boson \( (j) \) corresponds to a physical particle \( (i,j = 0) \) or to a PV one \( (i,j = 1) \). The term \( V_{45} \) means the sum of \( V_4 \) and \( V_5 \). The explicit expressions for \( V_{1-6} \) are given in Appendix B.

Note that due to the covariance of our approach, we can identify the contribution \( \sim \omega \) to the self-energy which explicitly depends on the light front plane orientation. If not regularized, the coefficient \( C(k^2) \) is quadratically divergent and needs a priori both PV fermion and boson regularization. After this regularization however, \( C(k^2) \equiv 0 \) for any values of the PV fermion and boson masses. This makes the two-body
self-energy identical to the result obtained in perturbation theory in the Feynman approach \[17\]. The contributions \(A\) and \(B\) do respect chiral symmetry in the sense that they are equal to zero when the constituent mass \(m\), as well as the physical mass \(M\) (which, in our case, coincides with \(m\)), goes to zero, without the need of an extra PV boson. This is at variance with the standard formulation of LFD where it is claimed that an additional PV boson is needed, if the PV fermion mass is kept finite \[13\].

The parameters \(g_02\) and \(\delta m_2\) are taken from the \(N=2\) calculation \[2\]. They are given by

\[
\begin{align*}
  g_0^2 & = \frac{g^2}{1 - g^2 J_2}, \\
  \delta m_2 & = g^2 [A(m^2) + B(m^2)],
\end{align*}
\]

(29a)

(29b)

where

\[
J_2 = -\frac{B(m^2)}{m} - z_0
\]

(30)

with \(z_0 = 2m [A'(m^2) + B'(m^2)]\). The norms of the one- and two-body Fock sectors, entering the normalization condition \[6\], are

\[
\begin{align*}
  I_1^{(2)} & = 1 - g^2 J_2, \\
  I_2^{(2)} & = g^2 J_2.
\end{align*}
\]

(31a)

(31b)

In the two-body approximation mentioned above and discussed in detail in Ref. \[2\], the two-body vertex function is automatically independent of \(\omega\) since \(\mathcal{C}(k^2) \equiv 0\). Moreover, it is a constant (i.e. it does not depend on the momenta of the constituent). Due to all these, the renormalization condition \[25\] directly leads to the relation \[29a\] between the bare and physical coupling constants. In principle, nothing prevents \(\Gamma_2\) to be \(\omega\)-dependent, since it is an off-shell object, but this dependence must completely disappear on the energy shell, i.e. for \(s_2 = m^2\). It would be indeed so if no Fock space truncation occurs. The latter, in approximations higher than the two-body one (i.e. for \(N = 3, 4, \ldots\)), may cause some \(\omega\)-dependence of \(\Gamma_2\) even on the energy shell, which immediately makes the general renormalization condition \[19\] ambiguous. If so, one has to insert into the light-front interaction Hamiltonian new counterterms which explicitly depend on \(\omega\) and cancels the \(\omega\)-dependence of \(\Gamma_2(s_2 = m^2)\). Its explicit form will be given in the next subsection. Note that the explicit covariance of CLFD allows to separate the terms which depend on the light front plane orientation (i.e. on \(\omega\)) from other contributions and establish the structure of these counterterms. This is not possible in ordinary LFD.

### B. Calculation of the two-body Fock component

The method of solution is similar to that used in the calculation \[2\] for \(N = 2\). We first decompose the vertex functions in invariant amplitudes. The vertex functions on the l.h.s. of Eqs. \[27\], being matrices in the spin indices, can be decomposed in a full set of spin matrices. This decomposition is very simple in CLFD and takes the form

\[
\begin{align*}
  \bar{u}(p_{1i}) \Gamma_1^{ij} u(p) & = (m^2 - M^2) \psi_i^1 \bar{u}(p_{1i}) u(p), \\
  \bar{u}(k_{1i}) \Gamma_2^{ij} u(p) & = \bar{u}(k_{1i}) \left[ b_{1}^{ij} + b_{2}^{ij} \frac{m^2}{\omega^2} \right] u(p),
\end{align*}
\]

(32)

where \(\psi_i^1\) is a constant, and \(b_{i}^{ij}\) are invariant functions of particle momenta, with \(m_0 \equiv m\) and \(\mu_0 \equiv \mu\). We denote temporarily, in the first of the above equations, the physical fermion mass by \(M\) in order to avoid singularities, since the equations contain the combination \(\Gamma_1^i/(m_i^2 - M^2)\) which becomes indeterminate for \(i = 0\) at \(M = m\). Using \(M \neq m\) allows to take a smooth limit \(\lim_{M \to m}[\Gamma_1^i/(m_i^2 - M^2)] = \psi_i^1\); after that one may set \(M = m\).

Each of the functions \(b_{i}^{ij}\) depends on two invariant kinematical variables. As usual, we define a pair of variables, consisting of the longitudinal and transverse (with respect to the three-vector \(\omega\) ) momenta:

\[
x = \frac{\omega \cdot k_2}{\omega \cdot p}, \quad R_\perp = k_\perp - x p_\perp.
\]

(33)

where \(k_2\) is the boson four-momentum. Then \(b_{i}^{ij}\) are functions of \(x\) and \(R_\perp^2\).

The renormalization condition \[24\], for \(N = 3\), implies two conditions

\[
\begin{align*}
  b_{10}^{ij}(s_2 = m^2) & = g \sqrt{I_1^{(2)}}, \\
  b_{20}^{ij}(s_2 = m^2) & = 0
\end{align*}
\]

(34a)

(34b)

for the spin components of \(\Gamma_2\) at \(s_2 = m^2\), where the two-body invariant energy squared \(s_2\) is expressed through \(R_\perp\) and \(x\) as follows:

\[
s_2 = \frac{R_\perp^2 + \mu^2}{x} + \frac{R_\perp^2 + m^2}{1-x}.
\]

(35)
B Calculation of the two-body Fock component

One should emphasize that the renormalization conditions are imposed on the two-body vertex function $\Gamma^2_{AB}$ corresponding to both physical constituents. The condition (33a) defines unambiguously $g_{03}$. The condition (34b) is not verified automatically if the Fock space is truncated for $N \geq 3$, unlike the case $N = 2$. We should thus enforce it by introducing an appropriate counterterm, as explained above (see also Ref. [2]). It corresponds to the following additional structure in the interaction Hamiltonian:

$$\delta \mathcal{H}^{int}_\omega = -Z_\omega \bar{\psi} \frac{m}{\omega \partial} \psi \varphi', \quad (36)$$

where $Z_\omega$ is a constant adjusted to make Eq. (34b) true. The operator $\varphi/\omega \partial$, in momentum space, leads to the appearance of a new three-leg vertex $\bar{\psi}/(\omega \cdot \partial)$ on each fermion-boson vertex with total incoming momentum $k$. In principle, a similar new $\omega$-dependent counterterm should be also added to the Hamiltonian in order to cancel the $\omega$-dependence of $\delta m_3$, in full analogy with the cancellation of $b_{00}^3(s_2 = m^2)$ [7]. However, as we will see below, $\delta m_3$ is needed only for a calculation in the four-body Fock space truncation. For this reason, working within the three-body truncation only, we may not bother about additional counterterms excepting that given by Eq. (36).

To solve the system of equations (27), we substitute the decompositions (32) into the expressions for $V_{1-6}$ given in Appendix B, then multiply Eqs. (27a) and (27b) by $u(p_{1i})$ and $u(k_{1i})$, respectively, to the left and each of them by $\bar{u}(p)$ to the right, and sum over spin projections. We thus get

$$\begin{align*}
(m_i^2 - m^2) (\bar{\psi}_{1i} + m_i) \psi_1^i (\bar{\psi} + m) &= (\bar{\psi}_{1i} + m_i) (V_1 + V_2) (\bar{\psi} + m), \quad (37a) \\
(\bar{\psi}_{1j} + m_j) \left( b_{1j}^i + b_{2j}^i \frac{m \bar{\psi}_j}{\omega \cdot p} \right) (\bar{\psi} + m) &= (\bar{\psi}_{1j} + m_j) (V_3 + V_4 + V_5) (\bar{\psi} + m). \quad (37b)
\end{align*}$$

The system of matrix equations (37a) and (37b) can be transformed into a homogeneous system of ten linear integral equations for ten unknown functions (two $\psi_1^i$, four $b_{1j}^i$, and four $b_{2j}^i$). These equations are obtained by taking the trace of Eqs. (37a) and (37b) (six equations), and by taking the trace of Eq. (37b) after the multiplication of its both sides by $\bar{\psi}$ (four equations).

In order to achieve the limit $m_1 \to \infty$, it is convenient to replace the functions $\psi_1^i$ and $b_{1j}^i$ by the new functions $\alpha_i$, $h_i^0$, and $H_i$ according to

$$\begin{align*}
\psi_1^i &= \frac{m_i}{m(m + m_i)} \alpha_i, \quad b_{1j}^i &= \frac{m_i h_j^0}{m}, \\
b_{2j}^i &= \frac{m_i H_i^0 - (1 - x + \frac{m_i}{m}) h_j^0}{2(1 - x)}.
\end{align*} \quad (38)$$

A careful analysis shows that in this limit the PV mass $m_1$ disappears from the equations written in terms of $\alpha_i$, $h_i^0$, and $H_i$. These functions have therefore a finite limit. Below we will imply that the limit $m_1 \to \infty$ is taken and $\alpha_i$, $h_i^0$, and $H_i$ denote the limiting values.

For further simplification of the equations, it is convenient to introduce new functions $\tilde{h}_{0,1}^0$ and $\tilde{H}_{0,1}^0$ by means of the relation

$$\begin{align*}
\begin{pmatrix} h_{0,1}^0 \\ H_{0,1}^0 \end{pmatrix} &= \alpha_0 \kappa \begin{pmatrix} \tilde{h}_{0,1}^0 \\ \tilde{H}_{0,1}^0 \end{pmatrix}, \quad (39)
\end{align*}$$

with

$$\kappa = \frac{1 - g^2 J_2}{1 + g^2 J_2}. \quad (40)$$

Using the substitution (39) and denoting

$$Z'_\omega = \frac{2Z_\omega - \alpha_1}{\alpha_0}, \quad (41)$$

the initial system of ten equations splits into two subsystems. The first one contains two equations involving the ratio $\alpha_1/\alpha_0$. It is not interesting for our study in the three-body approximation, since $\alpha_1$, as will be seen below, drops out from the observables we calculate here, while $\alpha_0$ is uniquely determined by the normalization condition for the state vector. As already mentioned, we also do not need to calculate $\delta m_3$ itself. It is used as an input in the calculation at the next, $N = 4$, truncation.
The second subsystem of eight equations involves the eight functions $\tilde{h}_1^j$ and $\tilde{H}_1^j$ only, since the ratio $\alpha_1/\alpha_0$ is absorbed into the definition of $Z'_2$ in Eq. (41). We thus get

$$\tilde{h}_0^j = 1 + g^2 \left( K_1^j \tilde{h}_0^j + K_2^j \tilde{h}_1^j \right) + g^2 i_0^j,$$

$$\tilde{h}_1^j = g^2 \left( -K_1^j \tilde{h}_0^j + K_2^j \tilde{h}_0^j + K_1^j \tilde{h}_1^j \right) + g^2 i_1^j,$$

$$\tilde{H}_0^j = Z'_2 (1 - x) + 2 - x$$

$$+ g^2 \left( K_1^j \tilde{H}_0^j + K_2^j \tilde{H}_1^j \right) + g^2 I_0^j,$$

$$\tilde{H}_1^j = 1 + g^2 \left( -K_1^j \tilde{H}_0^j + K_2^j \tilde{H}_1^j \right) + g^2 I_1^j,$$

where

$$g' = \frac{g^2}{(1 + g^2 z_0)},$$

and

$$K_1^j = \frac{1}{m} \left\{ B_c(s_1) - 2[A_c(s_1) + B_c(s_1)]m^2 \right\},$$

$$K_2^j = \frac{A_c(s_1) + B_c(s_1)}{m},$$

$$K_3^j = \frac{[A_c(s_1) + B_c(s_1)]m}{m^2 - s_1},$$

$$K_4^j = \frac{B_c(s_1)}{m}.$$

The substracted self-energy contributions $A_c(s_1)$ and $B_c(s_1)$, are given by

$$A_c(s_1) = A(s_1) - A(m^2),$$

$$B_c(s_1) = B(s_1) - B(m^2)$$

with

$$s_1 = \frac{R_1^2}{x} + (1 - x)m^2 - \frac{1 - x}{x} \mu_j^2.$$

The functions $A$ and $B$ are given in Appendix B, while the integral terms $i_{0,1}^j$ and $I_{0,1}^j$ are given in Appendix D.

The limit of infinite PV mass $\mu_1$ is not easy to perform analytically, as it was done for $m_1$. Setting $\mu_1 \to \infty$ directly in Eqs. (42) makes some integration kernels singular (they decrease too slowly at $R_\perp \to \infty$). The dependence of physical observables, like the AMM, on $\mu_1$ will therefore be studied numerically.

Note that although $g_{02}^2$ in Eq. (29a) can become infinite (for $J_2 = 1/g^2$) and changes sign from positive to negative at sufficiently large values of the PV boson mass $\mu_1$, the eigenvalue equations (42) do not show any singularity when $g_{02}^2$ goes to infinity. Indeed, $g_{02}^2$ does not appear in the equations (42). These equations depend only on $g^2$, given by Eq. (43), with $z_0$ strictly positive. Therefore, $g^2$ is strictly positive and finite. We shall come back in Sec. V to the interpretation of the limit of large $\mu_1$, when both $g_{02}^2$ and the norm of the one-body sector $I_1^{(2)}$ are negative, while the norm of the two-body sector $I_2^{(2)}$ is larger than 1, from Eqs. (31).

The constant $Z'_2$ entering the system of equations (42) is determined from the renormalization condition (34b), while $g_{03}$ needed to calculate the renormalized vertex functions in Eq. (39) is determined from the renormalization condition (34a).

The components $b_{01,2}^0(s_2 = m^2)$ entering these renormalization conditions are expressed through the solution of the system of equations (42) by means of Eqs. (36) and (39). The kinematical point $s_2 = m^2$ belongs to a nonphysical region, but there is no need to make an analytical continuation to this region of the solution $\tilde{h}_1^j$ and $\tilde{H}_1^j$ found numerically. Indeed, the integral terms in Eqs. (42) involve integrations within the physical domain only. One can simply set $j = 0$, $R_\perp = R_\perp^*$, $x = x^*$, where $R_\perp^*$ and $x^*$ are determined by the condition $s_2 = m^2$, and calculate the integral terms by substituting there the previously found solution $\tilde{h}_1^j(R_\perp^*, x)$ and $\tilde{H}_1^j(R_\perp^*, x)$ for physical values $R_\perp$ and $x$. After that Eqs. (42) reduces to a system of four ordinary linear inhomogeneous equations for $\tilde{h}_1^0(R_\perp^*, x^*)$ and $\tilde{H}_1^0(R_\perp^*, x^*)$. Finally, relating the calculated quantities $\tilde{h}_1^0(R_\perp^*, x^*)$ and $\tilde{H}_1^0(R_\perp^*, x^*)$ to $b_{01,2}^0(s_2 = m^2)$ we get $g_{03}$ from Eq. (34a) and $Z'_2$ from Eq. (34b).

The condition $s_2 = m^2$ however does not determine $R_\perp^*$ and $x^*$ simultaneously. It is convenient to fix $x^*$ somehow and then find $R_\perp^*$ from Eq. (39):

$$R_\perp^{*2} = - \left[ x^* m^2 + (1 - x^*) \mu_2^2 \right].$$

Since the two-body vertex function (32) on the energy shell must turn into a constant, the functions $b_{1,2}^0(s_2 = m^2)$...
C. Representation of the three-body component

We can find the three-body component by calculating the amplitude corresponding to the r.-h. s. of the equation shown by the last line in Fig. 3.

The general form of the relativistic vertex function of a system composed from one constituent fermion and two spinless bosons with total spin 1/2 reads

\[ \bar{u}_{\sigma_1}(k_1)\Gamma_{\alpha\beta}(1, 2, 3)u^\beta_\sigma(p), \]

where \( \Gamma_{\alpha\beta}(1, 2, 3) \) is a 4 \times 4-matrix in the indices \( \alpha, \beta \). The arguments of \( \Gamma_3 \), denoted symbolically by numbers, mean three pairs of the standard variables

\[ R_{l\perp} = k_{l\perp} - x_l p_{\perp}, \quad x_l = \frac{\omega k_l}{\omega p}, \]

with \( l = 1 \) corresponding to the fermion and \( l = 2, 3 \) to bosons. Here \( \bar{u}_{\sigma_1}(k_1) \) is the bispinor of the constituent fermion, \( u^\beta_\sigma(p) \) is the bispinor of the physical fermion (of the composite system), \( \sigma_1, \sigma \) are their spin projections in the corresponding rest frame. Since \( \sigma_1 = \pm 1/2 \) and \( \sigma = \pm 1/2 \), we have in general \( 2 \times 2 = 4 \) matrix elements. Usually, parity conservation reduces this number by a factor of two. However, this is not the case in relativistic calculations, for a \( n \)-body wave function with \( n \geq 3 \) \([13]\). This wave function is determined by four independent matrix elements or, equivalently, by four scalar functions \( g_{1-4} \) like

\[ \bar{u}(k_1)\Gamma_3(1, 2, 3)u(p) = \bar{u}(k_1)\left(g_{1} S_1 + g_{2} S_2 + g_{3} S_3 + g_{4} S_4\right)u(p). \]

For simplicity, we omitted the bispinor indices and the indices marking the particle type (either physical or PV one). It is convenient to construct the four basis spin structures as follows:

\[
S_1 = 2x_1 - (m_i + x_1 m) \frac{\varphi}{\omega p}, \\
S_2 = m \frac{\varphi}{\omega p}, \\
S_3 = iC_{ps} \left[2x_1 - (m_i - x_1 m) \frac{\varphi}{\omega p}\right] \gamma_5, \\
S_4 = i m C_{ps} \frac{\varphi}{\omega p} \gamma_5.
\]

with \( x_1 = \frac{\omega k_1}{\omega p} \), \( m_i \) being the internal fermion mass (either physical or PV one, depending on which type of fermion the momentum \( k_1 \) corresponds to), while \( C_{ps} \) is the following pseudoscalar:

\[
C_{ps} = \frac{1}{m^2\omega p} e^{\mu\nu\gamma\delta} k_{2\mu} k_{3\nu} p_\gamma p_\delta. \tag{49}
\]

The function \( C_{ps} \) can only be constructed with four independent four-vectors. This is the case in LFD for \( n \geq 3 \). In the non-relativistic limit, one would need \( n \geq 4 \). We can then construct two additional spin structures \( S_3 \) and \( S_4 \) of the same parity as \( S_1 \) and \( S_2 \) by combining \( C_{ps} \) with parity negative matrices constructed from \( S_1 \), \( S_2 \), and \( \gamma_5 \) matrices. Instead of \( k_{2\mu} k_{3\nu} \) one could have taken any pair of momenta \( (k_{1\mu} k_{3\nu} \) or \( k_{1\mu} k_{2\nu} \). We take the boson momenta for symmetry. With this definition

\[
C_{ps}^2 = \frac{1}{m^4} \left[R_{2\perp}^2 R_{3\perp}^2 - (R_{2\perp} R_{3\perp})^2\right]. \tag{50}
\]

The three-body vertex function \( \Gamma_3(1, 2, 3) \) is completely determined by the four scalar functions \( g_{1-4}(1, 2, 3) \) in Eq. (47). They depend on \( R_{1-3\perp} \) in the form of their scalar products among themselves and on \( x_{1-3} \). Since

\[
R_{1\perp} + R_{2\perp} + R_{3\perp} = 0, \quad x_1 + x_2 + x_3 = 1, \tag{51}
\]

we can exclude, for instance, \( R_{1\perp} \) and \( x_1 \). The functions \( g_{1,2} \) are symmetric relative to the permutation \( 2 \leftrightarrow 3 \), whereas \( g_{3,4} \) are antisymmetric:

\[
g_{1,2}(1, 2, 3) = g_{1,2}(1, 3, 2), \\
g_{3,4}(1, 2, 3) = -g_{3,4}(1, 3, 2),
\]

so that the product \( C_{ps} g_{3,4}(1, 2, 3) \) which appears in \( S_3 \) \( g_3 \) and \( S_4 \) \( g_4 \) is symmetric.

Each component \( g_n \) is represented as a sum or a difference of two terms:

\[
g_{1,2}(1, 2, 3) = \tilde{g}_{1,2}(1, 2, 3) + \bar{g}_{1,2}(1, 3, 2), \\
g_{3,4}(1, 2, 3) = \tilde{g}_{3,4}(1, 2, 3) - \bar{g}_{3,4}(1, 3, 2), \tag{52}
\]
where the permutation 2 ↔ 3 means
\[ R_{2\perp} \leftrightarrow R_{3\perp}, \quad x_2 \leftrightarrow x_3, \quad \mu_{j_2} \leftrightarrow \mu_{j_3}. \]
In their turn, \( \bar{g}_n(1, 2, 3) \), according to the last line in Fig. 3, are linearly expressed through the functions \( \bar{h}_{0,1}^{j_3} \) and \( \bar{H}_{0,1}^{j_3} \) which form a solution of the equations [42].

\[
\bar{g}_n(1, 2, 3) = \alpha_0 \kappa g_{02} \left[ a_{n0}(1, 2, 3) \bar{h}_{0,1}^{j_3}(2) + a_{n1}(1, 2, 3) \bar{H}_{0,1}^{j_3}(2) + A_{n1}(1, 2, 3) \bar{H}_{1}^{j_3}(2) \right]. \tag{53}
\]

The coefficients \( a \) and \( A \) in this formula are given in Appendix E.

We can finally calculate the three-body normalization integral. It is given by

\[ I_3 = \frac{1}{2} \sum_{j_2, j_3=0}^1 (-1)^{j_2+j_3} \int \frac{n_3^{j_2j_3}}{(s_3 - m^2)^2} dD_3 \tag{54} \]

with

\[ n_3^{j_2j_3} = \frac{1}{2} \text{Tr} \left[ \Gamma_3(\gamma_1 + m) \Gamma_3(\not{p} + m) \right] \tag{55} \]

\[ = 4x_1[R_{1\perp}g_1^2 + m^2g_2^2 + C_{ps}^2(R_{1\perp}^2g_3^2 + m^2g_4^2)], \]

and, as usual, \( \bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0 \). The factor \( \frac{1}{2} \) in Eq. (55) results from averaging over initial state spin projections, while the factor \( \frac{1}{2} \) in Eq. (54) is the combinatorial factor \( \frac{1}{n-1} \) originating from the identity of the two bosons. The contribution of PV fermion is omitted since it disappears in the limit \( m_1 \to \infty \). The phase space volume element has the form (see Eq. (3.23) from Ref. [3]):

\[ dD_3 = 2(2\pi)^3\delta(2)(R_{1\perp} + R_{2\perp} + R_{3\perp}) \]

\[ \times \delta(x_1 + x_2 + x_3 - 1) \prod_{l=1}^3 d^2R_{l\perp} dx_l. \tag{56} \]

\section{Electromagnetic Form Factors}

\subsection{Electromagnetic vertex in CLFD}

The electromagnetic vertex contains contributions of one-, two-, and three-body Fock sectors, as shown in Fig. 6. They are expressed, in our FSDF scheme, in terms of the external electromagnetic BCC \( \bar{e}_0 \), as explained in Ref. [2]. These coupling constants are all identical to the physical fermion charge, i.e., \( \bar{e}_0 = e \) for all \( l \)'s. Note that this important property of QED is not preserved in general if FSDF is not used.

![Fig. 6: One-, two-, and three-body contributions to the electromagnetic vertex.](image)

The decomposition of the spin-1/2 electromagnetic vertex in CLFD is given by [8, 15]

\[
\bar{u}(p')G^\rho u(p) = e\bar{u}(p') \left[ F_1 \gamma^\rho + \frac{if_2}{2m} \sigma^{\mu\nu} q_{\nu} + B_1 \left( \frac{\not{\omega} - 2\gamma^\rho}{\omega - p} \right) \right] + B_2 \left( m \not{\omega} + B_3 \frac{m^2 \not{\omega}}{(\omega - p)^2} \right) u(p) \tag{57} \]

with \( P = p + p' \), and \( q = p' - p \). \( F_1 \) and \( F_2 \) are the physical form factors, while \( B_{1,2,3} \) are spurious (nonphysical) contributions which appear if rotational invariance is broken, e.g., by Fock space truncation. The decomposition (57) enables to separate unambiguously the physical form factors from the nonphysical ones. Under the condition \( \omega q = 0 \), all \( F_{1,2}, B_{1-3} \) depend on \( Q^2 \equiv -q^2 \) only.

We shall represent \( q = (q_0, \mathbf{q}, q_\parallel) \), where \( q_\parallel \) and \( \mathbf{q} \) are, respectively, the longitudinal and transverse components of the momentum transfer with respect to the three-vector \( \omega \). Since \( \omega q = \omega_0(q_0 - q_\parallel) = 0 \), we have \( Q^2 = \Delta^2 \).

After construction of the matrix

\[ O^\rho = \frac{1}{4m^2} (\not{p} + m)G^\rho(\not{q} + m), \tag{58} \]
and calculation of the traces
\[ c_4 = \text{Tr}[O\rho_\omega \rho]m/\omega p, \quad c_5 = \text{Tr}[O\rho_\omega \rho \rho]m^2/(\omega p)^2, \]
the electromagnetic form factors write [7]
\[ F_1 = \frac{1}{2} c_5, \quad F_2 = \frac{2m^2}{Q^2} (c_5 - c_4). \]
The value \( F_1(Q^2 = 0) \) equals one, since it coincides with the norm of the state vector. The value \( F_2(Q^2 = 0) \) is just the AMM.

**B. One-body contribution**

The one-body contribution to the form factor \( F_1 \) is given by the first diagram in Fig. 6. It does not depend on \( Q^2 \) and coincides with the norm of the one-body sector:
\[ F_{1,1b} = a_0^2 \equiv I_1. \]
There is no one-body contribution to the form factor \( F_2 \):
\[ F_{2,1b} = 0. \]

**C. Two-body contribution**

The two-body contribution to the electromagnetic vertex, as given by the second diagram in Fig. 6 writes
\[
\bar{u}(p')G_{a_0}^u(p) = \frac{1}{(2\pi)^3} \sum_{i',j=0}^{1} (-1)^{i'j} \int d^2 R_\perp \int_0^1 dx \frac{\bar{u}(p')\Gamma_2^{ij}(k_{i'1} + m_1')\gamma^2(k_i + m_i)\Gamma_2^{ij} u(p)}{(s^{i'j} - m_i^2)(s^{ij} - m_i^2)},
\]
where \( \Gamma_2^{ij} \) is given by Eq. (42) with \( b_{1i2}^{ij} = b_{12}^{ij}(R_\perp, x), \quad k_{1i}(k_{1i'}) \) is the momentum of the constituent fermion incoming to (outgoing from) the elementary electromagnetic vertex, \( s^{ij} = (k_i + k_{i'} + m_2)^2 \), \( s^{i'j} = (k_{i'} + k_{i2})^2 \), and \( k_{i2} \) is the constituent boson momentum. \( \Gamma_2^{ij} \) has the same decomposition as \( \Gamma_2^{ij} \), with the replacement \( b_{1'}^{ij} = b_{12}^{ij}(R_\perp, x) \to b_{1'}^{ij}(R_\perp + x\Delta) \)

Using the relations (43) and (49), we can calculate the two-body contribution to the electromagnetic form factors in terms of the solutions \( \tilde{h}_i^1, \tilde{H}_i^j \) of the system of eigenvalue equations [12]. After taking the limit \( m_1 \to \infty \) the result is as follows:
\[
F_{1,2b} = \frac{\alpha_0^2\kappa^2}{16\pi^3} \sum_{j=0}^{1} (-1)^j \int d^2 R_\perp \int_0^1 dx \frac{x[R_{1\perp} + x^2 m + (1 - x)\mu_j^2]}{[R_{1\perp}^2 + x^2 m + (1 - x)\mu_j^2]^2}.
\]

Functions with primes depend on \( R_\perp^2 \) and \( x \). The value \( F_{1,2b}(Q^2 = 0) \) coincides with the two-body contribution to the normalization integral,
\[
F_{2,2b} = \frac{\alpha_0^2\kappa^2 m^2}{4\pi^3\Delta^2} \sum_{j=0}^{1} (-1)^j \int d^2 R_\perp \int_0^1 dx \frac{x(R_{1\perp} + \Delta)\tilde{h}_i^j \tilde{H}_i^j}{[R_{1\perp}^2 + x^2 m + (1 - x)\mu_j^2]^2}.
\]

To calculate the two-body contribution to the AMM, which is given by the value \( F_{2,2b}(Q^2 = 0) \), one should go over to the limit \( \Delta \to 0 \) in Eq. (64). The corresponding analytic formula includes derivatives over \( R_\perp \) from the Fock components. For numerical calculations it is however more convenient to find \( F_{2,2b} \) at small but finite \( Q^2 \) and then, decreasing the latter, to reach desired accuracy. The result of this numerical limiting procedure is very stable.

**D. Three-body contribution**

The three-body contribution to the electromagnetic vertex reads
\[
\bar{u}(p')G_{a_0}^u(p) = \frac{1}{2} \sum_{j_3,s_3} (-1)^{s_3} \int dD_3 \frac{\bar{u}(p')\Gamma_3^{i_3}(k_{i} + m)\gamma^2(k_i + m)\Gamma_3^{i_3} u(p)}{4x^2 (s_3 - m_2)^2(s_3 - m_2 dD_3},
\]
where $dD_3$ is defined by Eq. (56) and

$$s_3 = \frac{R_{1,1}^2 + m^2}{x_1} + \frac{R_{2,1}^2 + \mu_2^2}{x_2} + \frac{R_{3,1}^2 + \mu_3^2}{x_3}. \quad (66)$$

$s'_3$ differs from $s_3$ by the following shift of the arguments:

$$\begin{align*}
R_{1,1} &\rightarrow R'_{1,1} = R_{1,1} + (1 - x_1)\Delta, \\
R_{2,1} &\rightarrow R'_{2,1} = R_{2,1} - x_2\Delta, \\
R_{3,1} &\rightarrow R'_{3,1} = R_{3,1} - x_3\Delta. \quad (67)
\end{align*}$$

$\Gamma'_3$ is obtained from $\Gamma_3$ by the same shift of arguments. From the decomposition (47), we can calculate $G^\rho$, and construct the matrix $O^\rho$ by means of Eq. (68).

The form factors are thus given by Eqs. (60) and read

$$\begin{align*}
F_{1,3b} &= \int \left( C_{11}^{(1)} g_{11}' + C_{22}^{(1)} g_{22}' + C_{33}^{(1)} g_{33}' \right. \\
&\left. + C_{44}^{(1)} g_{44}' + 2C_{31}^{(1)} g_{31}' \right) \frac{dD_3}{d_1}, \quad (68a)
\end{align*}$$

$$\begin{align*}
F_{2,3b} &= 2 \int \left( C_{12}^{(2)} g_{12}' + C_{41}^{(2)} g_{41}' + C_{32}^{(2)} g_{32}' \right. \\
&\left. + C_{34}^{(2)} g_{34}' \right) \frac{dD_3}{d_2}, \quad (68b)
\end{align*}$$

where

$$d_1 = m^4 x_2 x_3 (m^2 - s_3)(m^2 - s'_3), \quad d_2 = \frac{2\Delta^2}{m^2 d_1}.$$ 

$g_n'$ differs from $g_n$ by the shift of the arguments (67). The coefficients $C_{ij}^{(1,2)}$ in Eqs. (68a) and (68b) are given in Appendix B.

The value $F_{1,3b}(Q^2 = 0)$ coincides with the norm, $I_3$, of the three-body sector given by Eq. (54). The quantity $a_0$ which has been unknown up to now, is determined from the normalization condition for the state vector:

$$a_0^2 + I_2 + I_3 = 1. \quad (69)$$

Since both $I_2$ and $I_3$ are proportional to $a_0^2$, then, denoting $I_{2,3} = a_0^2 \kappa^2 I_{2,3}$, where $\kappa$ is defined by Eq. (40), we immediately get

$$a_0^2 = \frac{1}{1 + \kappa^2 (I_2 + I_3)}. \quad (70)$$

V. NUMERICAL RESULTS

The solution of Eqs. (42) is found by a matrix inversion after discretization of the integrals, using Gaussian method. All integrals are finite at finite PV boson mass $\mu_1$. As already mentioned, the limit of infinite PV fermion mass $m_1$ has been done analytically, while the Fock components $\hat{h}^l$, $\hat{R}^l$ and, hence, $b_{1,2}^{ij}$ in Eq. (32) do depend on the PV boson mass $\mu_1$. The numerical calculations have been performed on an ordinary modern laptop.

The AMM is calculated for a typical set of physical parameters $m = 0.938$ GeV, $\mu = 0.138$ GeV, and two values of the coupling constant $\alpha = \frac{g^2}{4\pi} = 0.2$ and 0.5. This mimics, to some extent, a physical nucleon coupled to scalar "pions". The typical pion-nucleon coupling constant is given by $g = \frac{g_A}{2\pi f_\pi}$ where $(k)$ is a typical momentum scale, and $g_A$ and $F_\pi$ are the axial coupling constant and the pion decay constant, respectively. For $(k) = 0.2$ GeV we just get $\alpha \simeq 0.2$.

![FIG. 7: The anomalous magnetic moment in the Yukawa model as a function of the PV mass $\mu_1$, for two different values of the coupling constant. $\alpha = 0.2$ (upper plot) and 0.5 (lower plot). The dashed and dotted lines are, respectively, the two- and three-body contributions, while the solid line is the total result. The AMM value calculated in the $N = 2$ approximation is shown by the thin line on the right axis.](image-url)
We plot in Fig. 7 the AMM as a function of $\log \left( \frac{\mu_1^2}{\mu^2} \right)$, for the two different values of $\alpha$ pointed out above. We show also on each of these plots the value of the AMM calculated for the $N = 2$ truncation, which coincides with the AMM obtained in the second order of perturbation theory. The results for $\alpha = 0.2$ show rather good convergence as $\mu_1 \to \infty$. The contribution of the three-body Fock sector to the AMM is sizeable but small, indicating that the Fock decomposition (2) converges rapidly. This may show that once higher Fock components are small, we can achieve practically converging calculation of the AMM. Note that this value of $\alpha$ is not particularly small: it is about 30 times the electromagnetic coupling, and is about the size of the typical pion-nucleon coupling in a nucleus.

When $\alpha$ increases, we see that the contribution of the three-body sector considerably increases. For $\alpha = 0.5$ the three-body contribution to the AMM starts to dominate at large values of $\mu_1$. The dependence of the AMM on the PV boson mass $\mu_1$ becomes more appreciable, although it keeps rather small.

In order to have a more physical insight into the relative importance of different Fock sectors in the decomposition (2) for the state vector, we plot in Fig. 8 the contributions of the one-, two-, and three-body Fock sectors to the norm of the state vector for the two values of the coupling constant, considered in this work. We see again that at $\alpha = 0.2$ the three-body contribution to the norm is small, while it is not negligible, and increases with $\mu_1$, when $\alpha = 0.5$.

At very large values of $\mu_1$, and for large $\alpha$, $I_1$ becomes negative. As already mentioned, we still get a well defined solution of Eqs. (42), and there is no discontinuity whatsoever in the value of the AMM. As shown in Fig. 7, the convergence of the AMM as a function of the PV boson mass is expected in any case to settle much before we enter into this regime. According to renormalization theory, the mass of the PV boson should be much larger than any intrinsic momentum scale present in the calculation of physical observables. With this limitation, physical observables should be independent of any variation of the PV boson mass, within an accuracy which can be increased at will. This is what we found in our numerical calculation for small enough values of $\alpha$.

In order to understand the possible origin of the residual dependence of the AMM on $\mu_1$, we plot in Figs. 9 and 10 the two-body spin components $b_{10}^{00}$ and $b_{20}^{00}$ calculated at $s_2 = m^2$, as a function of $x$, for $\alpha = 0.2$ (dashed line) and $\alpha = 0.5$ (solid line), for a typical value of $\mu_1 = 100$ GeV.

In Figs. 9 and 10 the two-body spin components $b_{10}^{00}$ and $b_{20}^{00}$ calculated at $s_2 = m^2$, as a function of $x$. As we already mentioned in Sec. III, $b_{10}^{00}(s_2 = m^2)$ and $b_{20}^{00}(s_2 = m^2)$ should be independent of $x$ in an exact calculation. Moreover, $b_{10}^{00}$ should be zero. It is here fixed to zero at a given value of $x = x^*$, $x^* \equiv \frac{\mu}{m+\mu}$ by the adjustment of the constant $Z'$ in the system of equations (42). We clearly see in these figures that $b_{10}^{00}$ is not a constant, although its dependence on $x$ is always weak, while $b_{20}^{00}$ is not identically zero, although...
FIG. 10: The same as in Fig. 9 but for the spin component $b_{200}^{00}$.

its value is relatively smaller than that of $b_{100}^{00}$ for $\alpha = 0.2$, and starts to be not negligible for $\alpha = 0.5$.

FIG. 11: The component $h_{00}^0$ defined by Eq. (39), as a function of $R_{\perp}$ at $x = \frac{\mu}{m+\mu}$, for a typical value of $\mu_1 = 100$ GeV and for $\alpha = 0.2$ (dashed line) and $\alpha = 0.5$ (solid line).

We plot in Figs. 11 and 12 the two physical components $h_{00}^0$ and $H_{00}^0$ as a function of $R_{\perp}$, at $x = \frac{\mu}{m+\mu}$. As expected from the system of eigenvalue equations (42), the functions $h_{00}^0$ and $H_{00}^0$ tend to constants at large $R_{\perp}$. Hence, the functions $h_{00}^0$ and $H_{00}^0$ related to them by Eq. (39) tend to constants too. Note that the two-body wave function $\phi_2$ related to $\Gamma_2$ by Eq. (11) goes to zero at large momenta due to the rapidly decreasing kinematical factor $(s_2 - m^2)^{-1}$.

For completeness, we plot in Figs. 13 and 14 the two physical components $h_{00}^0$ and $H_{00}^0$ as a function of $x$, at $R_{\perp} = 0$.

FIG. 12: The same as in Fig. 11 but for the component $H_{00}^0$.

FIG. 13: The component $h_{00}^0$ defined by Eq. (19), as a function of $x$ at $R_{\perp} = 0$, for a typical value of $\mu_1 = 100$ GeV and for $\alpha = 0.2$ (dashed line) and $\alpha = 0.5$ (solid line).

FIG. 14: The same as in Fig. 13 but for the component $H_{00}^0$.

VI. CONCLUSION

We calculated the anomalous magnetic moment of a fermion in the Yukawa model, in the first non-trivial approximation, incorporating a constituent fermion
coupled to zero, one, and two scalar bosons, i.e. within
the three-body Fock state truncation. We applied a
general formalism based on the covariant formulation
of light-front dynamics and an appropriate Fock sector
dependent renormalization scheme which enables us
to control uncancelled divergences when Fock space is
truncated. We paid particular attention to the renor-
malization conditions necessary to relate the bare cou-
pling constant to the physical one. To do this, we need
to identify all spurious contributions originating from
the violation of rotational invariance, coming from the
Fock space truncation. This is possible in our covari-
ant formulation.

The anomalous magnetic moment shows a very nice
convergence as a function of the regularization scale
(the Pauli-Villars boson mass $\mu_1$), for the coupling
constant value $\alpha = 0.2$ which mimics a nucleon cou-
ped to a scalar "$\pi$on". For this value of $\alpha$, the two-
body component gives a dominant contribution to the
anomalous magnetic moment. As $\alpha$ increases, we see
the onset of higher Fock components.

This shows up in the large contribution of the
three-body component, and in the dependence of the
anomalous magnetic moment as a function of the reg-
ularization scale. We believe that this latter depen-
dence should be largely, if not completely, removed by
incorporating the relevant fermion-antifermion contri-
butions to the three-body Fock components. We are
presently investigating these contributions [10].

As we have seen in our study, the calculation of
nonperturbative properties of bound state systems de-
mand to control all approximations in a quantitative
way, in order to be able to make physical predictions
order by order in the Fock space truncation. We think
that the combination of the covariant formulation of
light-front dynamics with an appropriate Fock sector
dependent renormalization scheme is a quite promis-
ing method to investigate these properties in a very
elegant way, with a minimum of Fock components and
computational time.

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was performed.

Appendix A: Relation between the Fock
component normalization and the field strength
renormalization factors

We shall prove here the relation [21] in the general
case, i.e. without Fock space truncation. We omit
for a moment antifermion contributions generated by
the process $b \to ff$. They will be incorporated be-
low. The fermion self-energy is given by a sum of ir-
reducible graphs with all possible intermediate states:

$$\Sigma(\not{p}) = \sum_{n=1}^{\infty} \Sigma_n(\not{p}). \quad (A1)$$

Here $\Sigma_n(\not{p})$ is the contribution from a graph with
$n$ intermediate states. For example, the first graph
in Fig. 5 for the self-energy contains one interme-
diate state, the second one contains three intermediate
states while the third one contains 11 intermediate
states.

For the calculations we will use the 3D light-front
graph techniques exposed in Ref. [3]. It can be repre-
sented in different equivalent forms. We use the form
in which an amplitude is represented as a product of ener-
ggetic denominators (one denominator for each in-
termate state) multiplied by appropriate spin ma-
trices. The amplitude (with a conventional minus sign
for the self-energy) corresponding to a graph with $n$
intermediate states has the following form:

$$\Sigma_n(\not{p}) = -g_0^{n+1} \int \prod_{i=1}^{n} d^3 k_i \prod_{i=1}^{n} \prod_{k=1}^{s_i} \not{k}_i + m_i \not{s}_i - p^2. \quad (A2)$$

Since we do not truncate Fock space, we use the same
BCC in all vertices. $s_i$ is the square of the invariant
energy of $i$-th intermediate state. The product $\prod_{i=1}^{n}$
runs through all $n$ intermediate states. The integra-
tion in Eq. \(A2\) is performed over all independent
variables.

The decomposition of $\Sigma(\not{p})$ is similar to that given
by Eq. \(28\):

$$\Sigma(\not{p}) = \mathcal{A}(p^2) + \mathcal{B}(p^2) \frac{\not{p}}{m} + \mathcal{C}(p^2) \frac{m \not{\omega}}{\omega p} + \mathcal{C}_1(p^2) \sigma, \quad (A3)$$

where

$$\sigma = \frac{1}{4\omega p} (\not{p} \not{\omega} - \not{\omega} \not{p}).$$

Eq. \(A3\) is the most general form of the fermion self-
energy in CLFD. The term with the function $\mathcal{C}_1(p^2)$
does not appear for the two-body ($N = 2$) Fock space.
truncation, but it may appear for $N \geq 3$. We give
here the coefficient $C$ which will be used below:
\begin{align}
C &= \frac{1}{4M} \text{Tr} \left[ \left( \not{p} - \gamma^\mu \not{p} \right) \Sigma(\not{p}) \right]. \tag{A4}
\end{align}

To find $\frac{\partial \Sigma(\not{p})}{\partial \not{p}} \bigg|_{\not{p} = M}$, we first replace in Eq. (A3) $p^2$
by $M^2$, $\not{p}$ by $M$ (that is, replace $\gamma^\nu$ by $p^\nu/M$) and then calculate
the derivative over $M$. It is convenient
to make this replacement by using the formula
\begin{align}
\Sigma(M) &= \frac{1}{4M} \text{Tr} \left[ (\not{p} + M) \Sigma(\not{p}) \right]_{\not{p} = M^2}.
\end{align}

We get
\begin{align}
\frac{\partial \Sigma(\not{p})}{\partial \not{p}} \bigg|_{\not{p} = M} &= \frac{\partial}{\partial M} \left\{ \frac{1}{4M} \text{Tr} [ (\not{p} + M) \Sigma(\not{p}) ]_{\not{p} = M^2} \right\}.
\end{align}

We substitute here $\Sigma(\not{p})$ from Eqs. (A1) and (A2).

The contribution of derivative from the $j$-th factor
in the last equation is introduced for averaging over initial spin
projections. We therefore get

\begin{align}
\frac{\partial \Sigma_{nj}(\not{p})}{\partial \not{p}} \bigg|_{\not{p} = M} &= -g_0^{-1} 2M \int dD \left\{ \frac{1}{4M} (\not{p} + M) \left[ \prod_{i_1 = 1}^{j-1} \left( \frac{k_{i_1} + m_{i_1}}{(s_{i_1} - M^2)} \right) \right] \left( \frac{k_j + m_j}{(s_j - M^2)^2} \right)^n \left[ \prod_{j_2 = j+1}^n \left( \frac{k_{j_2} + m_{j_2}}{(s_{j_2} - M^2)} \right) \right] \right\}.
\end{align}

The factor
\begin{align}
\Gamma_j = g_0 \int dD' \prod_{i_1 = 1}^{j-1} \left( \frac{k_{i_1} + m_{i_1}}{(s_{i_1} - M^2)} \right)
\end{align}
is a contribution of the graph with $j - 1$ intermediate
states into the vertex function, and similarly for
the second product. In contrast to Eq. (A6), where the
integration $dD$ runs over the phase volumes of all the
intermediate states, the integration $dD'$ in Eq. (A7)
runs over the phase volumes of the intermediate states $i_1 = 1, \ldots, j - 1$ only.

Since all the four-momenta are on the corresponding
mass shells $k_j^2 = m_j^2$, we have
\begin{align}
(k_j + m_j) &= \sum_{\sigma = \pm 1/2} u_\sigma(k_j) \bar{u}_\sigma(k_j), \\
\frac{1}{2} \text{Tr} [(\not{p} + M)O] &= \frac{1}{2} \sum_{\lambda, \pm 1/2} \bar{u}_\lambda(p) O u_\lambda(p),
\end{align}
for an arbitrary matrix $O$. The factor $\frac{1}{2}$ in the last
equation is introduced for averaging over initial spin
projections.

We therefore get

Here the integration $dD_j$ runs over the phase volume
of the $j$-th intermediate state not included in the
integral for $\Gamma_j$. The vertex function $\Gamma_j$ may correspond
to any fixed number of particles in the state $j$ allowed
by a given graph. We took into account that the fac-
tor $1/(s_j - M^2)$ turns each $\Gamma$ into $\phi$, according to

Eq. (11). Taking the sum over all the graphs and over
all the intermediate states $j$, we recover in Eq. (A8)
the contribution to the normalization integral $I_{N \geq 2}$
from all the $N$-body states with $N \geq 2$ (each inter-
mediate state in irreducibles graphs for $\Sigma$ contains at
least two particles). Since the rules of the graph tech-
niques used to calculate $\Sigma$ imply that the one-body states are normalized to 1, this means that $I_{N \geq 2}$ corresponds to a state vector normalized by the condition $I_1 = 1$. If $I_1 \neq 1$, then Eq. (A6) determines the ratio $-I_{N \geq 2}/I_1$. That is

$$\left. \frac{\partial \Sigma^{\text{den}}(\vec{p})}{\partial \vec{p}} \right|_{\vec{p}=M} = -\frac{I_{N \geq 2}}{I_1}. \quad (A9)$$

The contribution of the derivative of other factors in Eq. (A5), except for $\prod_{i=1}^{n_{\text{f}}} \frac{1}{s_i - M^2}$, reads

$$\left. \frac{\partial \Sigma^{\text{num}}(\vec{p})}{\partial \vec{p}} \right|_{\vec{p}=M} = -g_0^{n+1} \int dD \frac{1}{2M} \frac{1}{\Pi_{i=1}^{n} (s_i - M^2)} \left. \frac{\left[ \frac{1}{4M} \text{Tr} \{ (\vec{p} + M) \prod_{i=1}^{n} (k_i + m_i) \} \right]_{\hat{p}^2 = M^2}}{\prod_{i=1}^{n} (s_i - M^2)} \right. \quad (A10)$$

where the variables $R_{i \perp}$ and $x_i$ are constructed according to Eq. (33). Calculating then the derivative over $M$, we find

$$\left. \frac{\partial \Sigma^{\text{num}}(\vec{p})}{\partial \vec{p}} \right|_{\vec{p}=M} = -g_0^{n+1} \int dD_2 \frac{1}{2\pi M^2} \left. \frac{1}{\Pi_{i=1}^{n} (s_i - M^2)} \right. \left. \left( \frac{1}{2x_i M^2} \right) \right. \left. \frac{1}{\Pi_{i=1}^{n} (s_i - M^2)} \right. \left. \frac{1}{\Pi_{i=1}^{n} (s_i - M^2)} \right. \quad (A11)$$

Comparing Eq. (A11) with Eq. (A12), we find the relation

$$\left. \frac{\partial \Sigma^{\text{num}}(\vec{p})}{\partial \vec{p}} \right|_{\vec{p}=M} = -\frac{m}{M^2} \mathcal{C}. \quad (A13)$$

It turns out that Eq. (A13) is valid in the most general case. In the latter case, but still without antifermions, we get in the numerator in Eqs. (A11) and (A12) a product of the matrices $\prod_{i=1}^{n} (k_i + m_i)$, instead of the single term $(k_i + m_i)$. This product can be decomposed in the full set of the $4 \times 4$ matrices as follows:
\[ \prod_{i=1}^{n}(k_i + m_i) = G_0 + \sum_i G_i^{1} k_i + \sum_{i_1,i_2} G_{i_1}^{2} \sigma(k_{i_1}, k_{i_2}) + G_\gamma \gamma_5 + \sum_i G_i^{4} \gamma_5 k_i, \quad (A14) \]

where \( \sigma(k_{i_1}, k_{i_2}) = \frac{1}{2}(k_{i_1}k_{i_2} - k_{i_2}k_{i_1}) \). The coefficients \( G_{1-3} \) depend on the scalar products of the four-momenta \( k_1, \ldots, k_n \):

\[ k_i \cdot k_j = \frac{1}{2x_i x_j} [x_i^2 m_j^2 + (x_j R_{i\perp} - x_i R_{j\perp})^2 + x_j^2 m_i^2]. \]

It is important that these scalar products and, hence, the functions \( G_{1-3} \) do not depend on \( M \). Therefore \( G_{1-3} \) can be extracted from the operator \( \frac{\partial}{\partial M} \). We replace \( (k_i + m_i) \) in Eqs. (A11) and (A12) by the product \( \prod_{i=1}^{n}(k_i + m_i) \) represented in the form (A14) (and take the product of the denominators). The matrices \( \gamma_5 \) and \( \gamma_5 k_i \) give zero contributions to both Eqs. (A11) and (A12), whereas with the matrices 1, \( k_i \), and \( \sigma(k_{i_1}, k_{i_2}) \) we reproduce the relation (A13).

The incorporation of antifermions (say, the \( \bar{f} \bar{f} \bar{f} \) intermediate state, in addition to bosons) does not change the form of the denominator (though the energies \( s_i \) incorporate now the antifermion momenta).

That results in Eq. (A9). The corresponding numerator contains now the spin matrices of all the fermions + antifermions [we get one factor \( 1/(s_i - p^2) \) and a product of three matrices \( (\pm k_i + m_i) \) for the \( \bar{f} \bar{f} \bar{f} \) state; the signs plus and minus stand for fermions and antifermions, respectively]. We still can decompose the full numerator according to Eq. (A14) and again reproduce the formula (A13).

In this way, taking the sum of Eqs. (A9) and (A13), we finally find

\[ \frac{\partial \Sigma(\bar{p})}{\partial \bar{p}} \bigg|_{\bar{p} = M} = -\frac{I_{N \geq 2}}{I_1} - \frac{m}{M^2} \mathcal{C}. \quad (A15) \]

If rotational invariance is preserved (it can be violated for instance by omitting some time-ordered graphs or by using rotationally non-invariant cutoffs), \( \mathcal{C} \) is zero. It is indeed zero, for example, in the two-body approximation with the PV regularization [see Eq. (B6) in Appendix B]. If \( \mathcal{C} = 0 \), substituting Eq. (A15) into Eq. (16) and taking into account that \( I_1 + I_{N \geq 2} = 1 \), we prove the relation (21).

**Appendix B: Self-energy coefficients**

We give here explicit formulas for the coefficients \( A(k^2), B(k^2), \) and \( C(k^2) \) entering Eq. (28) for the two-body self-energy in the Yukawa model. If \( \Sigma(k) \) is known, these coefficients can be found as follows:

\[ g_{02}^A A(k^2) = \frac{1}{4} \text{Tr} \left[ \Sigma(k) \right], \quad (B1) \]
\[ g_{02}^B B(k^2) = \frac{m}{4\omega_k} \text{Tr} \left[ \Sigma(k) \omega \right], \quad (B2) \]
\[ g_{02}^C C(k^2) = \frac{1}{4m} \text{Tr} \left[ \Sigma(k) \left( k - \frac{\omega}{\omega_k} k^2 \right) \right]. \quad (B3) \]

In the Yukawa model, the self-energy regularized by one PV boson and one PV fermion reads, in CLFD,

\[ \Sigma(k) = -\frac{g_{02}^A A(k^2)}{(2\pi)^3} \int d^2 R_\perp \int_0^1 \frac{dx}{2x(1-x)} \times \sum_{i,j=0}^1 (-1)^{i+j} \frac{(k_1 + m_i)}{(s_{ij} - k^2)}, \quad (B4) \]

where \( k_1 \) is the internal fermion four-momentum. The light-front variables are, as usual, \( R_{1\perp} = k_{2\perp} - x k_{1\perp} \), \( x = \omega \cdot k_2 / \omega \cdot k \) (\( k_2 \) is the boson four-momentum), and

\[ s_{ij} = \frac{R^2_{i\perp} + m_j^2}{x} + \frac{R^2_{j\perp} + m_i^2}{1-x}. \quad (B5) \]

The coefficients \( A \) and \( B \) converge without PV fermion (i.e., they have finite limit when \( m_1 \to \infty \)). Substituting Eq. (B4) into Eqs. (B1) and (B2), integrating over \( d^2 R_\perp \) and omitting the PV fermion, we get
Notice that in the limit $\mu_1 \to \infty$ and for fixed $k^2$ the values of $A_r(k^2) = A(k^2) - A(m^2)$ and $B_r(k^2) = B(k^2) - B(m^2)$ are finite.

A similar calculation of $C(k^2)$ requires, for convergence, not only one PV boson, but also one PV fermion. We thus find

$$C(k^2) = -\frac{1}{32m^2} \int_0^1 \frac{dx}{1-x} \int_0^\infty dR_{\perp} \sum_{i,j=0}^1 (-1)^{i+j} \frac{R_{\perp}^2 - (1-x)^2k^2 + m_i^2}{R_{\perp}^2 - (1-x)k^2 + (1-x)\mu_1^2} \equiv 0. \tag{B6}$$

Appendix C: Right-hand sides of the eigenvalue equations \([27]\)

The system of equations \([27]\) determines the one- and two-body Fock components $\Gamma_{1}^i$, $\Gamma_{2}^{ij}$. The r.-h. s.'s of these equations are denoted by $\tilde{u}(p_1,v_1 + V_2) u(p)$ and $\tilde{u}(k_{1i},V_3 + V_{45} + V_6) u(p)$, respectively. The explicit form of $V_{1-6}$ is the following:

\begin{align*}
V_1 &= \delta m_3 \sum_{i'j'} (-1)^{i'} \frac{(p_{i'}^2 + m_{i'})}{m_{i'}^2 - M^2} \Gamma_{1}^{i'}, \quad \tag{C1a} \\
V_2 &= g_{03} \sum_{i'j'} (-1)^{i'+j'} \int \frac{d^2 R_{\perp}'}{2(2\pi)^3} \int_0^1 \frac{dx'}{2x'} \frac{(k_{i'}^{i'} + m_{i'})}{2(\omega \cdot p)\tau_{i'j'}} \Gamma_{2}^{i'j'}, \quad \tag{C1b} \\
V_3 &= g_{03} \sum_{i'} (-1)^{i'} \frac{(p_{i'}^2 + m_{i'})}{m_{i'}^2 - M^2} \Gamma_{1}^{i'}, \quad \tag{C1c} \\
V_{45} &= [-\Sigma(p_i - k_{2j}) + \delta m_2] \sum_{i''} (-1)^{i''} \frac{(k_{i''}^{i''} + m_{i''})}{2(\omega \cdot p)(1-x)\tau_{i''j}} \Gamma_{2}^{i''j}, \quad \tag{C1d} \\
V_6 &= g_{02} \sum_{i'j'i''} (-1)^{i'+j'+i''} \int \frac{d^2 R_{\perp}'}{2(2\pi)^3} \int_0^1 \frac{dx'}{2x'} \frac{(k_{i''}^{i''} + m_{i''})(k_{i'}^{i'} + m_{i'})}{4(\omega \cdot p)^2(1-x')(1-x-x')\tau_{i'j'}\tau_{i''j''}} \Gamma_{2}^{i'j'} \Gamma_{2}^{i''j''} \tag{C1d}
\end{align*}

with $g_{03} = g_{03} + Z_{\omega \cdot p}$ and obvious notations for the momenta of the particles in the intermediate states. The term $V_{45}$ stands for the sum of the contributions
of the graphs $V_4$ and $V_5$ in Fig. 5. The two-body vertex functions $\Gamma_2$ inside the integrands depend on $\mathbf{R}_\perp$ and $\mathbf{x}$, while those which are not integrated depend on $\mathbf{R}_\perp$ and $x$. After calculating the traces taken from the equations (37a) and (37b), we obtain scalar products which are expressed through the variables $\mathbf{R}_\perp$, $x$ and $\mathbf{R}_\perp^\prime$, $\mathbf{x}^\prime$. Examples are given in appendix C of Ref. 2.

The values of $\tau_j$'s, which appear in the above formulas, are related to the invariant energies in the corresponding intermediate states. For example, $\tau_i^{\nu,j'}$ in Eq. (C1d) for $V_6$ has the form
\[
\tau_i^{\nu,j'} = \frac{s_i^{\nu,j'} - m_i^2}{2\omega_p},
\]
where
\[
s_i^{\nu,j'} = (k_1\nu + k_2j + k_2j')^2.
\]
k_1\nu, k_2j, and $k_2j'$ are the four-momenta in the intermediate states while $s_i$ for any intermediate state, is expressed through the light-front variables as follows:
\[
s = \left(\sum_i k_i\right)^2 = \sum_i \frac{R_i^2}{x_i} + m_i^2,
\]
where $\mathbf{R}_\perp$ and $x_i$ are constructed according to Eq. (33). They satisfy the conservation laws similar to Eq. (51).

Appendix D: The integral terms in the equations (42)

The numerators and denominators of the kernels in the integrals in Eqs. (42) are linear functions of the scalar products $\mathbf{R}_\perp \cdot \mathbf{R}_\perp^\prime = R_\perp R_\perp^\prime \cos \phi'$, where $\mathbf{R}_\perp^\prime$ is the integration variable. We can therefore analytically integrate over $d\phi'$, using the formulas
\[
J_0 = \int_0^{2\pi} \frac{d\phi'}{2\pi D(A + B \cos \phi')} = \frac{\text{sign}(A)}{D \sqrt{A^2 - B^2}},
\]
\[
J_1 = \int_0^{2\pi} \frac{\cos \phi' d\phi'}{2\pi D(A + B \cos \phi')} = \frac{1}{DB} - \frac{A}{B} J_1.
\]
One should substitute here
\[
A = R_\perp^2 (1 - x) x + x' [x(x + x') m^2 + R_\perp^2 (1 - x')]
- (x + x' - 1) (x' \mu_j^2 + x \mu_j^2),
\]
\[
B = 2R_\perp R_\perp^\prime x',
\]
\[
D = -8\pi^2 (1 - x').
\]

With
\[
\eta_1 = m^2 x^2 + (1 - x') \mu_j^2 + R_\perp^2,
\]
the integral terms obtain the form
\[
i_n^j = \int_0^\infty R_\perp dR_\perp^\prime \int_0^{1-x} dx' \sum_{i,j'=0}^1 (-1)^{j'} \left( c_n \hat{h}_i^j + C_n \hat{H}_i^j \right),
\]
\[
I_n^j = \int_0^\infty R_\perp dR_\perp^\prime \int_0^{1-x} dx' \sum_{i,j'=0}^1 (-1)^{j'} \left( c_n' \hat{h}_i^j + C_n' \hat{H}_i^j \right),
\]
for $n = 0, 1$. These integrals converge due to the PV regularization (the sum over $j'$). The sixteen coefficients $c$, $C$, $c'$, and $C'$ depend on $j'$. They are given below.

\[
c_00 = \frac{R_\perp'}{R_\perp \eta_1} \left\{ R_\perp' R_\perp x x' J_0 + J_1 \left[ -R_\perp^2 (x - 1) x + x' [x(-x(x - 3) + 3x - 4) m^2 + R_\perp^2 (1 - x')] + (x - 1)(x' - 1)(x' \mu_j^2 + x \mu_j^2) \right] \right\},
\]
\[
c_{01} = -\frac{R_\perp'}{R_\perp} x (2x + x' - 2) J_1,
\]
\[
c_{00} = m^2 \frac{R_\perp}{R_\perp \eta_1} \left[ x x' \left\{ J_0 R_\perp (3x' - 2) + R_\perp' (2 - 3x) J_1 \right\} \right],
\]
\[
c_{01} = \frac{x}{R_\perp} \left[ R_\perp' (x - 1) J_1 - R_\perp x' J_0 \right],
\]
\[
c_{10} = \frac{R_\perp' m^2}{R_\perp \eta_1} x x' (x + 2x' - 2) J_1,
\]
\[
c_{11} = -\frac{R_\perp'}{R_\perp} x (x + x' - 1) J_1,
\]
\[
c_{10} = -\frac{m^2}{R_\perp \eta_1} x x' \left\{ R_\perp (1 - x') J_0 + R_\perp' x J_1 \right\},
\]
\[
c_{11} = 0.
\]
\[ c_{00} = \frac{R'_{\perp}xx'}{\eta_1}[R'_{\perp}(3x - 2)J_0 + R_{\perp}(2 - 3x')J_1], \]
\[ c'_{01} = -\frac{R'_{\perp}x}{m_0^2}[R_{\perp}'(x - 1)J_0 - R_{\perp}xx'J_1], \]
\[ C'_{00} = -\frac{1}{\eta_1}\left\{ R_{\perp}^2(x - 1)xJ_0 - R_{\perp}'R_{\perp}xx'J_1, \right. \]
\[ \left. + \left[ x(x(x' - 3) - 3x' + 4)m^2 + R_{\perp}^2(x - 1)\right]x'J_0 \right. \]
\[ \left. - (x - 1)(x' - 1)(x'm_0^2 + x_1^2)J_0 \right\}, \]
\[ C'_{01} = -x(2x + x' - 2)J_0, \]
\[ c'_{10} = \frac{R'_{\perp}xx'}{\eta_1}[R'_{\perp}xJ_0 - R_{\perp}(x' - 1)J_1], \]
\[ c'_{11} = 0, \]
\[ C'_{10} = \frac{m^2}{\eta_1}xx'(x + 2x' - 2)J_0, \]
\[ C'_{11} = -x(x + x' - 1)J_0. \]

Appendix E: Coefficients in the equations (53)

The three-body Fock component (one fermion + two bosons) is decomposed in four spin structures by Eq. (17) with the coefficients \( g_{1-4} \) being scalar functions. These coefficients are linear combinations [53] of the functions \( h \) and \( \bar{H} \) which are the solution of the equations (42). The coefficients of these linear combinations are given below.

With the notation
\[ \eta_2 = m^2 x_2^2 + (1 - x_2)\eta_2^2 + R_{\perp 2}^2, \]
we have
\[ a_{10} = -\frac{m(R_{1\perp \cdot R_{2\perp}})x_2(1 + x_1 - x_2)}{2R_{1\perp}^2x_1(1 - x_2)\eta_2}, \]
\[ a_{11} = \frac{(R_{1\perp \cdot R_{2\perp}})}{2mR_{1\perp}^2(1 - x_2)}, \]
\[ A_{10} = \frac{m x_2[R_{1\perp}^2(1 - x_2) + (R_{1\perp \cdot R_{2\perp}})x_1]}{2R_{1\perp}^2x_1(1 - x_2)\eta_2}, \]
\[ a_{20} = \frac{x_2(1 - x_2)(R_{1\perp \cdot R_{2\perp}}) + R_{2\perp}^2x_1)}{2mx_1(1 - x_2)\eta_2}, \]
\[ A_{20} = \frac{mx_2(1 + x_1 - x_2)}{2x_1(1 - x_2)\eta_2}, \]
\[ A_{21} = -\frac{1}{2m(1 - x_2)}, \]
\[ a_{30} = -\frac{m x_2(1 + x_1 - x_2)}{2R_{1\perp}^2x_1(1 - x_2)\eta_2}, \]
\[ a_{31} = -\frac{2R_{1\perp}^2(1 - x_2)}{m}, \]
\[ A_{30} = \frac{m x_2}{2R_{1\perp}^2(1 - x_2)\eta_2}, \]
\[ A_{40} = \frac{mx_2}{2x_1\eta_2}, \]
\[ A_{11} = a_{21} = A_{31} = a_{41} = A_{40} = A_{41} = 0. \]

Appendix F: Coefficients entering Eq. (68)

The three-body contributions to the form factors \( F_1 \) and \( F_2 \) are integrals from bi-linear combinations of the four spin components \( g_{1-4} \). The coefficients determining the three-body contribution to the form factor \( F_1 \), Eq. (68a) have the form
\[ C^{(1)}_{11} = m^4[R_{1\perp}^2 + (R_{1\perp \cdot \Delta})(1 - x_1)], \]
\[ C^{(1)}_{22} = m^6, \]
\[ C^{(1)}_{44} = -m^2[(R_{1\perp \cdot R_{2\perp}})^2 + [(1 - x_1)(R_{2\perp \cdot \Delta}) - x_2(R_{1\perp \cdot \Delta})](R_{1\perp \cdot R_{2\perp}}) \]
\[ + (R_{1\perp \cdot \Delta})R_{2\perp}^2(x_1 - 1) + R_{1\perp}^2[(R_{2\perp \cdot \Delta})x_2 - R_{2\perp}^2], \]
\[ C^{(1)}_{33} = \frac{1}{m^6}C^{(1)}_{11}C^{(1)}_{44}, \]
\[ C^{(1)}_{31} = m^2[(R_{1\perp \cdot \Delta})(R_{1\perp \cdot R_{2\perp}}) - R_{1\perp}^2(R_{2\perp \cdot \Delta})](x_1 - 1). \]

The coefficients determining the three-body contribution to the form factor \( F_2 \), Eq. (68b), have the form
\[ C^{(2)}_{12} = -4m^4(R_{1\perp \cdot \Delta}), \]
\[ C^{(2)}_{41} = -4m^2[(R_{1\perp \cdot \Delta})(R_{1\perp \cdot R_{2\perp}}) - R_{1\perp}^2(R_{2\perp \cdot \Delta})], \]
\[ C^{(2)}_{32} = -C^{(2)}_{41}, \]
\[ C^{(2)}_{34} = \frac{4}{m^2}(R_{1\perp \cdot \Delta})C^{(1)}_{44}. \]

We remind that \( Q^2 = \Delta^2 \).
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