Comments on the proposal of Dark Energy Stars by Chapline

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Abstract

We point out that the contentions of Chapline (astro-ph/0503200) that (i) General Relativity (GR) is in conflict with Quantum Mechanics (QM) and (ii) GR fails even on macroscopic length scales are incorrect and based on (i) misunderstanding of GR and (ii) completely misconceived “thought experiments”. Even without introduction of any QM, GR actually does not allow formation of any Event Horizon and is competent to figure out the actual nature of the supposed Black Hole candidates. The resultant picture is not in conflict with QM at all.

1 Introduction

In a recent preprint (astro-ph/0503200) Chapline[1] has proposed that the supposed Black Holes(BHs) are actually compact stars filled with dark energy and having a physical surface unlike BHs. He has termed such compact objects as “Dark Energy Stars”. Actually it was already shown several years ago that the supposed BH candidates cannot be true BHs and must be Ultra Compact Objects (UCOs) with physical surface[2,3,4,5]. Here Chapline has insisted for a specific QM model of such compact objects on the plea that GR is in conflict with QM and hence must be invalid. We systematically show below that this contention of Chapline is completely unfounded and his specific line of thinking behind “Dark Energy Stars” is based on elementary but serious errors. This is not to tell that there cannot be any Quantum Gravity (QG) inspired model of UCOs.

2 Synchronous Time

Chapline insists that ordinary QM requires an universal time which is present in Minkowski spacetime. While it is obvious that the non local large scale correlations implied by QM could be easily explained in terms of an univeral time, this, however, does not mean that in the absence of an universal time, such correlations would cease to exist. On the other hand, it would just be more difficult to formulate and work out those correlations.

In fact even if there would be no gravity and no GR, and electrons or anthing else would have accelerated motion with respect to each other, the clocks attached to them would
not be universally synchronized. Will it mean that QM is in conflict with the occurrence of accelerated motion? Will it mean that there cannot be any accelerated motion?

A logical extension of Chapline’s concern would be that, in a strict sense, in the presence of gravity, one needs to have an appropriate theory of Quantum Gravity (QG). But coming back to Chapline’s immediate concern, effect of varying gravity could actually be not much significant for long range QM correlations. Further, in the context of Friedmann model, the cosmic time may actually serve as a universal time (if one would ignore lack of clock synchronization due to the far more serious proper mutual accelerations).

To show that GR may not always admit universal clock synchronization, Chapline, rather unnecessarily mentions of Godel’s universe. Chapline argues that since Godel’s universe allows closed time like curves GR must be erroneous. Godel’s universe is filled with pure “dust” having pressure $p \equiv 0[6]$. If “dust” is to be treated as a fluid, as is the requirement for Einstein (hydrodynamic) equations, thermodynamics would tell that one can strictly have $p = 0$ only if the energy density of the fluid $\rho = 0$. In other words, Godel’s universe is only a mathematical model which corresponds to no mass energy at all. When there is no mass energy, there is no rotation, no physical universe either! Thus it is not GR which allowed closed time like curves; on the other hand, it is the incorrect assumption of a dust model ($p \equiv 0$) for a physical cosmic fluid which created this incorrect impression.

Chapline correctly points out that whenever rotation is involved, universal clock synchronization would not be allowed in GR. But this will be true for not only GR but probably for all relativistic theories of gravity. In fact what Chapline appears to be unaware of is that even if one would ignore GR or any other relativistic theory of gravity and consider only Special Theory of Relativity, the spacetime cross term in the metric cannot be done away with and there would be no universal time[7]. So, then, will Chapline demand that even special theory of relativity be abandoned?

And Chapline’s comment that GR is not valid in cosmology context (too) is simply unsubstantiated.

3 Spacetime Superfluidity?

Here Chapline starts with the assumption that ordinary matter be viewed as excited states of vacuum. Why? Quantum vacuum fluctuations may give rise to various virtual pairs and those pairs may be considered excited states of vacuum. But why normal matter? What about baryon and lepton number conservation in QM? In fact, here, Chapline seems to overlook conservation principles of QM altogether.

Following this, Chapline proposes that all atomic motions must be irrotational. But we know that it is possible to have eddies and vorticities in air, water and all ordinary fluids where $\nabla \times \vec{v} \neq 0$. Most likely Chapline is inspired here by a footnote of Landau & Lifshitz[8] which points out the simple fact that if the fluid is moving in such a manner that it is possible to have a global synchronization of clocks (i.e., if the spacetime cross term can be eliminated, which is possible when there is no rotation), the fluid motion will be irrotational: $\nabla \times \vec{v} = 0$. Actually, this fluid motion does not at all refer to any mysterious vacuum motion. But Chapline has attributed this natural and obvious result to the motion of the vacuum itself, when, actually we are concerned here the motion of a fluid in the spacetime. Clearly, Chapline’s formulation of the model has hardly
any physical basis. Even if one, momentarily, admits that all motions in the universe is irrotational, Chapline’s conclusion that, hence “an appropriate model for the vacuum of spacetime is a superfluid” is too far fetched at the best. In any case, this result is obtained by confusing the motion of the fluid with the motion of the vacuum!

And even if we accept this fluid motion (equated with vacuum motion) is irrotational why should vacuum be considered as a superfluid? If such an equivalence would exist, then any superfluid may be considered as vacuum as well! Further, in the first place, why not any irrotational fluid (which need not be a superfluid at all) be considered as a superfluid? Thus there is no physical basis for such a connection between vacuum and superfluidity.

4 Chapline’s Thought Experiment

Chapline purports to do a thought experiment to show “why it is wrong to assume that classical GR is always correct on macroscopic length scales” He imagines a vertical column of Bose superfluid; then he conceives that “As a result of the increasing pressure in the fluid as a function of depth it could happen that at a certain depth the speed of sound vanishes”! How? With increasing depth, density can only increase and elementary physics tells that sound speed will increase rather than decrease. It becomes clear subsequently, why, despite this, Chapline wished the sound speed to decrease and that too to zero value: because he thinks that speed of light becomes zero at the EH of a BH described by the metric:

\[ ds^2 = -\left(1 - \frac{\alpha_0}{R}\right) dt^2 + \left(1 - \frac{\alpha_0}{R}\right)^{-1} dR^2 + R^2(d\theta^2 + \sin^2 \phi d\phi^2); \quad R \geq R_0 = 0 \]  

Here, \( \theta \) and \( \phi \) are the polar angles and the integration constant \( \alpha_0 \) is interpreted as twice the gravitational mass of the “Massenpunkt”: \( \alpha_0 = 2M_0 \) \( (G = c = 1) \). The metric coefficients are

\[ g_{\bar{T}T} = -\left(1 - \frac{\alpha_0}{R}\right); \quad g_{RR} = \left(1 - \frac{\alpha_0}{R}\right)^{-1} \]  

It follows that as a test particle moves in the spacetime described by this metric its coordinate speed, \( dR/dT = 0 \) at the EH, not only for photons but for anything. On the other hand, the physical 3-speed:

\[ v = \frac{\sqrt{g_{RR}} \, dR}{\sqrt{-g_{\bar{T}T}}} \, dT = 1 \]  

for photons everywhere including the EH, by definition. Incidentally, the 3-speed of any material particle too acquires the speed of light at the EH.

In a complete misunderstanding of GR Chapline thought that the physical speed \( v = 0 \) for photons at the EH and also wished \( c_s = 0 \) at the same place to further build up his idea that vacuum behaves like a superfluid with strange critical properies. In reality, neither \( v \) nor \( c_s \) is zero at the EH. Also, even if they were zero, there would not be any physical basis for connecting the two different phenomenon.
Now from all angles, we have found that the inevitability of “Dark Energy Stars”, as discussed by Chapline, is based on elementary but severe misunderstanding and misconception. Thus there is no basis for thinking that GR is not valid even on macroscopic scales, vacuum behaves like a superfluid and there could be dramatic QM effects at large $R$.

5 Chapline’s Dark Energy Star

Apart from the erroneous formulation of the concept behind “Dark Energy Stars”, there is a general inconsistency in the whole scheme: While Chapline starts with the premises that “EHs cannot exist in real world”, he eventually gives the sketch of a model of a star whose boundary is nothing but the EH! Chapline simultaneously rejects and accepts the EH. The latter necessity arises because he wants the boundary of the proposed “Dark Energy Star” to be the QM critical surface where time dilation factor approaches zero. Chapline mentions that time dilation factor ($\sqrt{-g_{TT}}$) inside the EH of a BH is “negative” whereas it is actually imaginary (for the metric [1]).

Probably Chapline is unaware of the fact that the entire region beneath the EH is a trapped region. And hence his dark fluid may not stay put and keep filling in the interior region. In a trapped region, for all realistic equation of state, all mass energy must undergo collapse inexorably to the central singularity[9]. Thus within no time, his Dark Energy Star may reduce to a BH, if there would be an EH at a finite $R$!

6 GR or Event Horizon to be blamed?

There is however one apparently valid basis for some of the conjectures made by Chapline. He notes that, although, it is asserted that the EH is a mere coordinate singularity and, physics-wise, nothing unusual happens there, actually, lot of unusual physical effects take place at the EH. Chapline mentions that QM Green’s function develops a “cusp-like behaviour” at the EH, where one can make $K_{EH}$ arbitrarily small if $M$ were really arbitrary parameter for a BH.

From this Chapline concludes that “it is wrong to assume that classical GR is always correct on macroscopic length scales”. This also leads him to “question whether quantum corrections to classical GR can be important under circumstances where at least locally spacetime appears to be quite ordinary”.

First let us see that unusual physical results appear at the EH not only from the point of view of QM. For instance, as noted by Chapline, the covariantly defined quantity surface gravitational redshift (which for Earth, is $z_E \approx 0$)

$$z_{EH} = \infty$$

for a BH. $z$ can be measured by any distant observer and is very much a physical quantity. Then why does it diverge at EH? And this is definitely not due to any QM effect.

From the acceleration 4-vector of a test particle, one can construct an Acceleration Scalar:
\[ a = \frac{M}{R^2 \sqrt{1 - 2M/R}} \] (5)

On the EH, this scalar diverges, \( a_{EH} = \infty \)\[^4,10\]! Neither is this due to any QM effect. To see more of it consider a dilute non-interacting gas of dimension \( R_{\text{max}} \) around an object of circumference radius \( R_0 \). Let \( m \) be the rest mass and \( E \) be the constant energy of the molecules of the gas.

The phase volume of this gas, an INVARIANT, is given by\[^11\]:

\[ \Gamma(E) = \frac{16\pi^2}{3} \int_{R_0}^{R_{\text{max}}} \frac{R^2 dR}{\sqrt{1 - 2M/R}} \left( \frac{E^2}{1 - 2M/R} - m^2 \right)^{3/2} \] (6)

The derivative of \( \Gamma(E) \) w.r.t. \( E \) gives the entropy, \( g(E) \) of this gas, and which is another INVARIANT for the system.

As long as \( R_0 > 2M \), i.e., if the object is not a BH, then \( \Gamma(E) \) and \( g(E) \) behave nicely. But if this object would be assumed to be a BH with finite \( M \), i.e., if \( R_0 = 2M > 0 \), then, both these scalars would diverge\[^11\].

So, if it would be assumed that there is a BH of arbitrary finite mass \( M \), then any number of unacceptable physical effects would happen even at a pure classical level. Thus occurrence of such unphysical behaviours cannot be attributed to any QM phase transitions.

On the other hand, it is obvious that, it is the assumption of occurrence of EH at arbitrary finite value of \( R \), which is responsible for such occurrences. At this stage one would argue that by virtue of the completely vacuum Schwarzschild solution (1) GR gives rise to arbitrary finite value of \( R_0 = 2M \). Therefore, ultimately, it is GR which is responsible for these unphysical behaviours. And this would be cited as a proof for “failure of classical GR on macroscopic length scales”. The basic problem lies at a more fundamental classical level. Let us analyze the scenario more closely.

GR says that the best measure for the strength of gravitation is the value of the Kretschmann Scalar, \( K \). For a static spherical object of circumference radius \( R_0 \), we have

\[ K = \frac{48M^2}{R^6}; \quad R \geq R_0 \] (7)

Let on the surface of Earth, \( K = K_E \). In case the object is a Schwarzschild BH, the interior region too would be vacuum and the foregoing relation would be valid at any \( R \). In particular, at the EH, \( R = 2M \), we have

\[ K_{EH} = \frac{3}{M^4} \] (8)

Chapline correctly points out that for a sufficiently massive BH, \( K_{EH} \) could be very small. Let

\[ K_{EH} = bK_E \] (9)

where \( b \) could be arbitrarily small, i.e., gravity on the EH or in the interior region of a BH could be infinitely weaker than on the surface of Earth and yet one can have

- (i) covariantly defined and physically measurable \( z = \infty \),
- (ii) Physically measurable scalar \( a = \infty \),
• (iii) physically measurable scalar \( g(E) = \infty \) and as mentioned by Chapline
• (iv) QM Green function blowing up too.

Let us now recall the popular notion of a BH/EH:
“...where gravity is so strong(!) that even light cannot escape.”.

One should wonder then how can a gravity which could be infinitely weaker than the gravity on Earth’s surface be “so strong and do all such tricks (i-iv). And if such unusual things can indeed happen in arbitrarily weak gravity, then

“Why our Earth with an infinitely stronger gravity would not be able to trap photons permanently and why should we not have \( z = a = g = \infty \) on Earth’s, Moon’s or Sun’s surface too?”

But we know that no such spectacular things happen in the weak gravitational field of Earth: GR really does not do such odd things or fail at large \( R \).

Hence the big question here ought to be “Does GR Really Allow Existence of Finite Radius EHs or Finite Mass BHs?”

The very fact that the scalar \( a \) blows up at the EH immediately signals the fact that the EH is not region of mere “coordinate singularity”, on the other hand, it must be the true singularity! But the true singularity is at \( R = 0 \), Thus one must have \( R_g = 0 \), or, \( M = 0 \). And when this is so, it can be found that

\[
K_{EH} \equiv \infty
\] (10)
as well as

\[
a = \lim_{R \to 2M \to 0} \frac{1}{4M \sqrt{1 - 2M/R}} = \infty
\] (11)

And this is the reason why all those unusual physical things were found to be happening at the EH! Gravity is not arbitrarily weak there, but, on the other hand, gravity is indeed infinitely strong there! Therefore, in a consistent manner, at the EH,

- The Lorentz Factor of an Incident Particle, \( \gamma = \infty \)
- The Local Energy of an Incident Particle, \( E = \infty \)
- The Spectral Lines suffer a Redshift of \( z = \infty \)
- Acceleration Scalar \( a = \infty \)

And most importantly, in complete consistency,
- Kretschmann Scalar \( K = \infty \)
- Only when, Mass of BHs \( M = M_0 = 0 \)

Thus GR did not fail when it allowed those unusual physical events at the EH.

Incidentally, when the BH mass \( M = 0 \), the the invariant phase space volume of the surrounding gas becomes non-singular:

\[
\Gamma(E) = \frac{16\pi^2}{3}(E^2 - m^2)^{3/2} \int_0^{R_{max}} R^2 \, dR = \frac{16\pi^2}{9}(E^2 - m^2)^{3/2} R_{max}^3 
\] (12)

Accordingly, the invariant entropy of the gas too would become finite only if \( M = 0 \) (note that, if \( R > 2M \), then neither \( \Gamma \) nor \( g \) vanish for finite \( M \)). Also bear in mind that \( \Gamma \) and \( g \) are not the scalars associated with the EH and hence they must not blow up.

On the other hand, they are properties of the surrounding gas and must be finite (for a finite \( R_{max} \)) provided the underlying physics problem is formulated consistently, i.e., when, \( R_g = 0 \).

6
Since much of the impetus for Chapline’s model comes from the apparent idea that GR predicts existence of finite mass BHs and finite radius EHs, it would be worthwhile to clinically show that the value of the integration constant occurring in the completely vacuum Schwarzschild Soln. (i.e., a BH) is \( \alpha_0 \equiv 0 \).

7 Invariance of 4-volume

The original identification of \( \alpha_0 = 2M \) was done by matching the vacuum solution in Metric (1) with the corresponding Newtonian solution at large \( R \).

Newtonian gravitation allows for the existence of a spherical “point mass”, i.e., \( R_0 = 0 \), and since in Newtonian gravitation, mass is essentially of baryonic or leptonic origin (bare mass) with no negative “dressing” due to gravity or any self-energy, all masses including that of a “point” is necessarily finite, \( M_0 > 0 \). And, in GR too, it is has so far been assumed that \( M_0 \) would continue to be finite even when the spacetime is completely empty, i.e., \( R_0 = 0 \) (mass point). It is this expectation which gave rise to the concept of Black Holes in the GR era[12].

The determinant of the Schwarzschild metric (1) can easily be found to be

\[
g_1 = -R^4 \sin^2 \theta \tag{13}
\]

When the Schwarzschild solution represents pure vacuum, both in the interior as well as in the exterior, then, by integrating, the vacuum photon propagation equation \((dR/dT)\), from \( R = R_0 = 0 \) to \( R = R \) (a procedure which would not be valid if the Schwarzschild solution would represent only the vacuum spacetime outside a spherical object filled with mass energy, i.e., if \( R_0 > 2M = \text{finite} \)) one obtains various special coordinates to study the BH spacetime. One such coordinate system is the so-called Eddington-Finkelstein coordinates:

\[
t_* = t \mp \alpha_0 \log \left( \frac{R}{\alpha_0} - 1 \right) \tag{14}
\]

\[
R_* = R; \quad \theta_* = \theta; \quad \phi_* = \phi \tag{15}
\]

In terms of these coordinates, the BH metric becomes

\[
ds^2 = -\left( 1 - \frac{\alpha_0}{R} \right) dt_*^2 \mp \frac{2\alpha_0}{R} dt_* dR + \left( 1 + \frac{\alpha_0}{R} \right) dR^2 + R^2 (d\theta^2 + \sin^2 \phi d\phi^2) \tag{16}
\]

The corresponding metric coefficients are

\[
g_{t_*t_*} = -(1 - \alpha_0/R), \quad g_{RR} = (1 + \alpha_0/R), \quad g_{t_*R} = g_{Rt_*} = \pm \alpha_0/R \tag{17}
\]

In this case the determinant is same as \( g_1 \):

\[
g_* = -g_{\theta\phi}g_{\phi\theta}(g_{t,R}^2 - g_{t,t}g_{R,R}) = -R^4 \sin^2 \theta = g_1 \tag{18}
\]

Now let us apply the principle of invariance of 4-volume[7,9] for the coordinate systems \((t, R, \theta, \phi)\) and \((t_*, R, \theta, \phi)\) at arbitrary \( R \) and not necessarily at \( R \leq 2M \):

\[
\int \sqrt{-g_1} \ dt_* dR d\theta d\phi = \int \sqrt{-g_1} \ dt dR d\theta d\phi \tag{19}
\]
Since \( g_1 = g_2 = -R^4 \sin^2 \theta \), after doing the \( \theta \) and \( \phi \) integrations we promptly obtain,

\[
\int R^2 \, dR \, dt_* = \int R^2 \, dR \, dt
\]  \hspace{1cm} (20)

From Eq.(14), we also have

\[
dt_* = dt \mp \frac{\alpha_0}{R - \alpha_0} dR
\]  \hspace{1cm} (21)

By substitution of this foregoing simple relation in Eq.(20) we find

\[
\alpha_0 \int \frac{R^2 \, dR}{R - \alpha_0} = 0
\]  \hspace{1cm} (22)

And this shows that

\[
\alpha_0 \equiv 0
\]  \hspace{1cm} (23)

(Remember again that, if there would be mass energy upto \( R = R_0 > 2M \), i.e., for the case which involves mass-energy, this result would not be valid).

Thus, in a most direct manner, we obtain the fundamental result that the mass of the Schwarzschild BHs:

\[
M_0 = M(R_0 = 0) = \alpha_0 / 2 \equiv 0
\]  \hspace{1cm} (24)

8 Implications

Since all BH candidates or any other objects in the universe have \( M > 0 \), they cannot be BHs. Thus most of the concerns of Chapline actually get solved by properly interpreting GR and there is no need to belittle or suspect GR.

With \( \alpha_0 \equiv 0 \), the EH merges with the central singularity at \( R = 0 \) and hence the metric (1) has only one singularity, the singularity at \( R = \alpha = \alpha_0 = 0 \). This fundamental result is not at all in conflict with QG because many Supersymmetric String Theories obtain copious solutions for Extremal BHs where the horizon merges with the central singularity[13] as found here classically!

We may recall that there is only one exact analytical solution of spherical gravitational collapse where an uniform dust of mass \( M \), initially \( (t = 0) \) at rest with a radius \( R_i \), collapses to a SBH in a proper time[14]

\[
\tau_c = \frac{\pi}{2} \left( \frac{R_i^3}{2GM} \right)^{1/2}
\]  \hspace{1cm} (25)

Since the dust ball is at rest at \( t = 0 \), we can use the equation for hyrdo-static balance[15] at \( t = 0 \):

\[
\frac{dp}{dR} = -\frac{p + \rho_i}{R(R - 2M)} (4\pi R^3 p + M)
\]  \hspace{1cm} (26)

For a dust \( p \equiv 0 \), and therefore, we have \( dp/dR \equiv 0 \), and then the foregoing Eq. yields \( \rho_i = 0 \). From thermodynamical point of view too, whether at rest or not, a \( p = 0 \) equation of state is physically obtained only if \( \rho = 0 \). Therefore, trivially, the mass of the dust ball is zero for a finite \( R_i \). Hence the mass of the resultant SBH is indeed \( M = 0 \). But this was always ignored, it was pretended that eventhough, \( p = 0 \), we must have \( \rho = finite \).
and $M$ too would be finite in tune with the idea that the integration constant $\alpha_0 > 0$ (when it was actually zero).

And when $M = 0$, from Eq.(25), it follows that, $\tau_c = \infty$. Thus though the $M = 0$ SBHs are, mathematically, allowed by GR, they cannot be realized in an universe with finite proper age.

Since there is no EH in a finite proper time, there is no trapping or loss of quantum information either and hence another worry of Chapline also gets solved in a purely classical manner.

It was shown earlier that even for the most general case of spherical collapse (i.e, not necessarily for uniform dust), no trapped surface is ever formed[3,4,5,16]. Thus a collapsing fluid must always radiate and if it would be assumed to undergo continued collapse, we would have $M \to 0$ asymptotically. This is the reason that the integration constant $\alpha_0 = 2M_0$ turned out be identically zero. Hence the observed BH candidates with masses often much higher than the upper mass limit of cold baryonic bodies in hydrostatic equilibrium, cannot be SCBs. Detailed analysis of recent observational data indeed suggests that the BH candidates have strong intrinsic magnetic moments rather than any EH[17,18,19]. And it has been suggested that the BH candidates are Magnetized Eternally Collapsing Objects (MECOs). These are extremely hot objects in quasistatic equilibrium due to extremely strong radiation and magnetic pressure[17,18,19]. However, if Quantum Gravity would be invoked, the supposed Black Holes could be cold and static configurations with hard surfaces[20] where this boundary hard surface must not be any EH.

Chapline firmly thinks that GR should not be taken seriously “on any macroscopic length scale” and by seeing this not to be the case, he writes

“Indeed I am sure it will be a puzzle to future historians of science as to why it took so long to realize this”!

Future historians of science might also wonder when so much literature showing that GR actually does not allow formation of EH (at a finite $R$) was already available (as shown in the bibliography of this preprint), how Chapline was unaware about the same.

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