On brane-world black holes and short scale physics

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Abstract

There is evidence that trans-Planckian physics does not affect the Hawking radiation in four dimensions and, consequently, deviations from the linear dispersion relation (for massless particles) at very high energies cannot be revealed using four-dimensional black holes. We study this issue in the context of models with extra spatial dimensions and show that small black holes that could be produced in accelerators might also provide a chance of testing the high energy regime where non-linear dispersion relations are generally expected.

1 Introduction

In recent years some progress has been achieved toward a better understanding of the role played by short distance physics in the semiclassical treatment of gravitational systems. Trans-Planckian modes show up in quantum field theory on the curved background of a black hole due to the Hawking radiation [1]. The spectrum of the radiated quanta measured far away from the horizon \( r_h = 2M \) of a Schwarzschild black hole is expected to be Planckian with a typical energy \( \omega \) of the order of the Hawking temperature,

\[
\omega \sim k_B T_H = \left( 8 \pi M \right)^{-1},
\]

where \( M = (\ell_p/m_p) \tilde{M} \) is the Arnowitt Deser Misner (ADM) mass parameter of the Schwarzschild black hole (\( k_B \) is Boltzmann constant and we use units with \( c = \hbar = 1 \), \( m_p = \ell_p^{-1} \) is the Planck mass). However, such (usually very soft) quanta were unboundedly energetic if they originated at a radius \( r_s \) very close to the horizon. In fact, neglecting the backreaction, the gravitational blue-shift is given by

\[
z = \sqrt{\frac{r_s}{r_s-r_h}} - 1,
\]

and, in the geometrical optics approximation, the energy of Hawking quanta at the point of emission

\[
\omega_s = (1+z) \omega,
\]

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diverges for \( r_s \rightarrow r_h \).

Numerical studies of wave-packets moving toward the black hole have shown that any modification of the radiation spectrum at energies around the Planck mass \( m_p \) or above do not change the spectrum of the asymptotic flux \[2, 3\]. This could mean that either quanta originated at trans-Planckian frequencies do not contribute or they are not excited at all. Investigations based on the statistical mechanics of the Hawking radiation (in the microcanonical ensemble) \[4\] and on the uncertainty principle applied to the location of the source of the radiation \[5\] have further supported this conclusion. All such investigations were carried out in four space-time dimensions, where the fact that \( \text{\scalebox{0.7}{\textbf{M}}} \) must be larger than \( m_p \) in order to have a black hole plays a key role.

Models with extra spatial dimensions have raised some interest as possible scenarios in String Theory. Their novel feature is that Standard Model fields are degrees of freedom of open strings confined on a (negligibly thin) four-dimensional submanifold (the so called \textit{brane-world}), while gravity is mediated by closed strings which propagate in the whole space-time (the \textit{bulk}). Such models basically fall into two classes: 

\begin{enumerate}
\item compact (flat) extra dimensions (the ADD scenario of Refs. \[6\]) and
\item (possibly infinite) warped extra dimensions (the RS scenario of Refs. \[7\]).
\end{enumerate}

The understanding of black holes in such models is still rather incomplete to date (a partial list of References is given by \[8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23\]). It is however clear that, since one of the main consequences of the existence of extra dimensions is that the fundamental scale of gravity \( m_g = (\ell_g^{-1}) \) can be much smaller than \( m_p \) (perhaps as small as a few TeV’s), black holes might exist with proper mass \( m_g < \text{\scalebox{0.7}{\textbf{M}}} \ll m_p \) that could be produced in accelerators or by cosmic rays (see, e.g., Refs. \[24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\]). The Hawking temperature for such black holes could be of the order of \( m_g \), the value at which non-standard dispersion relations \[2, 3, 35\] might already be needed to effectively describe the physics by means of quantum field theory (for a description of trans-Planckian physics within the framework of String Theory, see, e.g. Refs. \[36\]). Deviations from the behaviour corresponding to a purely linear dispersion relation in the Hawking spectrum at energies of the order of a few TeV, which are within the range of planned experiments, might therefore signal the existence of extra spatial dimensions.

In these notes we intend to analyze such a possibility by studying in a rather general framework the problem of Hawking radiation in \( 4 + d \) dimensions. In Section 2 we shall review why trans-Planckian modes are not generated in four dimensions by appealing to the non-adiabaticity of the time evolution of wave modes on a Schwarzschild background. By repeating essentially the same argument in higher dimensional space-times, we shall then show that for \( d > 0 \) black holes with a mass close to \( m_g \) can emit radiation with energy of that order of magnitude. In Section 3 we explicitly consider the smooth dispersion relation introduced in Ref. \[35\] and show that its main effect is to reduce the Hawking flux near \( m_g \). According to recent estimates \[27\] (see, however, the remarks in Ref. \[28\]), such small black holes are expected to decay instantaneously and the suppression of high energy modes should therefore appear in the spectrum of the decay products.

## 2 Scalar waves in \textit{brane-world} black holes

A (minimally coupled) scalar wave in \( 4 + d \) dimensions satisfies the equation \((\mu, \nu = 0, \ldots, 3 + d)\)

\[
\partial_{\mu} \left( \sqrt{-g} g^{\mu \nu} \partial_{\nu} \right) \Phi = 0 ,
\]

\[2.1\]

\(^1\)The relation between the proper mass and the ADM mass of a very small black hole is a subtle issue \[11, 12, 14, 21\]. We shall presently trust Newtonian reasoning and assume they are equal \[8, 10, 18\].
where, for the case of interest, the metric tensor $g_{\mu\nu}$ (whose determinant is $g$) represents a black hole in the brane-world.

### 2.1 ADD scenario

In the ADD scenario, the brane tension is neglected. Hence, at least in principle, one should just supply the $4 + d$-dimensional vacuum Einstein equations with suitable boundary conditions at the edges of the extra dimensions [11] [12]. However, this task proves already difficult enough and one considers approximate (asymptotic) forms. One usually writes the metric as

$$ds^2 = g_{tt} dt^2 + g_{rr} dr^2 + r^2 d\Omega^2_{(2+d)},$$

(2.2)

where $g_{tt}$ and $g_{rr}$ are the non-trivial metric elements and $d\Omega_{(2+d)}$ the line element of a unit $2+d$-sphere. The $3+d$-dimensional radial coordinate $r$ can be decomposed as $r^2 = r_b^2 + y^2$, where $r_b$ is a three-dimensional radial coordinate on the brane and $y$ is bounded from above by the characteristic size $L$ of the extra dimensions. Let us also denote by $r = r_h$ the horizon radius (this quantity will in general depend on off-brane angles, since the shape of the horizon cannot be exactly a sphere [11]).

Outside the horizon of a large black hole, $L \ll r_h < r$, we can neglect the extra dimensions. One then simply takes the standard Schwarzschild line element on the brane [8],

$$ds^2 \simeq -\Delta_0(r_h) dt^2 + \frac{dr_b^2}{\Delta_0(r_b)} + r_b^2 (d\theta^2 + \sin^2 \theta d\phi^2) + \sum_{i=1}^{d} dy_i^2,$$

(2.3)

where

$$\Delta_0(x) = 1 - \left(\frac{r_h}{x}\right)^{1+d}.$$  

(2.4)

This is in fact a black string.

For small black holes and $r_h < r \ll L$ [8], one can analogously consider a spherically symmetric black hole in $4 + d$ dimensions [37]. However, in order to take into account the fact that $d$ spatial dimensions have size $L$, we shall instead use the following approximate form [11]

$$ds^2 \simeq -\Delta_0(r) dt^2 + \frac{dr_b^2}{\Delta_0(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + r^2 \sum_{i=1}^{d} d\phi_i^2,$$

(2.5)

where the $\phi_i$’s are off-brane angles such that $dy_i \simeq r d\phi_i$.

### 2.2 RS scenario

In the RS scenario there is one extra dimension but the brane, located at $y = 0$, has a tension $\sigma$ which warps the bulk,

$$ds^2 = e^{-\sigma |y|} g_{ij} dx^i dx^j + dy^2,$$

(2.6)

where $g_{ij} = g_{ij}(x^i, y)$ reduces to flat four-dimensional Minkowski in the absence of sources other than the brane itself [7] and $\sigma^{-1}$ (roughly) plays the same role as $L$ in the ADD case.

For large black holes (i.e. $M \sigma \gg 1$), the horizon is flattened on the brane in a pancake shape [18] [21] and the brane metric departs but slightly from the four-dimensional Schwarzschild form. Small black holes are instead believed to be closer in shape to the corresponding ADD case, so that in their vicinity the metric can be approximated as in Eq. (2.5) with $d = 1$ [18] [22] (note that the warp factor $e^{-\sigma |y|}$ is essentially constant for $y \ll \sigma^{-1}$).
2.3 Origin of Hawking radiation

We are now ready to study the solutions of Eq. (2.1) in some generality. Let us then assume that the scalar field $\Phi$ can be factorized according to

$$\Phi = e^{i \omega t} R(r) S(\theta) e^{i m \phi} e^{i \sum n_i \phi_i},$$

with the $n_i$'s integers, so that $\Phi$ is periodic in the off-brane angles. Of course, such a field is not confined on the brane and cannot therefore truly represent matter fields but can be used to analyze gravitational waves. In order to adjust for matter fields, one should replace the last exponential above with a product of Dirac $\delta(\phi_i)$ or some smooth realization of it (for more details, see [1]). That would just affect the coefficient $A$ which appears below, and which we shall usually set to zero.

The radial equation obtained from Eq. (2.1) for the metric (2.5) then becomes

$$\left[-\Delta \frac{d}{dr}\left(r^2 + \Delta \frac{d}{dr}\right) + \omega^2 - \frac{\Delta(d)}{r^2} A\right] R = 0 \quad (2.8)$$

and the angular equation is

$$\left[\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta}\right) - \frac{m^2}{\sin^2 \theta}\right] S = \lambda S, \quad (2.9)$$

where $A = \lambda + \sum n_i^2$ (unless confinement is enforced, see [1]), the separation constant $\lambda = l(l + 1)$ and $S = Y_l^m$ is a standard three-dimensional spherical harmonic.

The radial equation can be further simplified by defining the “tortoise” coordinate

$$dr_* \equiv \Delta^{-1}(d) dr, \quad (2.10)$$

and introducing a rescaled radial function,

$$W \equiv r^{1+d/2} R, \quad (2.11)$$

which then satisfies a Schrödinger-like equation

$$\left[-\frac{d^2}{dr_*^2} + V_d\right] W = \omega^2 W, \quad (2.12)$$

where the potential $V_d$ is given by

$$V_d = \left[(1 + d) \left(1 + \frac{d}{2}\right) + A\right] \frac{\Delta(d)}{r^2} - \left(1 + \frac{d}{2}\right)^2 \frac{\Delta^2(d)}{r^2}. \quad (2.13)$$

This potential generates a barrier located outside the horizon which suppresses the gray-body factor for all modes of the scalar field including those with zero angular momentum. The effect for $d = 0$ is mild, however for $d > 0$ the barrier increases, significantly reducing the black hole decay rate [12] (see also [1]).

There is now a relatively easy way to understand the reason why trans-Planckian modes do not play any role in four dimensions. Consider a spherically symmetric wave-packet $\Psi$ built up with in-falling modes which shrinks toward the center of a Schwarzschild black hole from a very large
radius. Since for $r \gg r_h = 2M$ the solutions to the radial equation (2.8) can be approximated by spherical waves, $R \sim k e^{i k r}$, the packet can be written as

$$\Psi(r, t) = \int_0^\infty dk k^{2+d} f(k) e^{i \omega t + i k r}, \quad (2.14)$$

where $\omega = k$ and $f(k)$ is, e.g., a Gaussian in momentum space. It is actually impossible to follow the evolution of such a packet analytically in time, however one can use numerical methods to show that out-going modes start to be generated as the packet approaches the horizon [2]. One can look at this effect in two equivalent ways, i.e. as a violation of the WKB approximation in a time-independent context or as a breakdown of the adiabatic approximation in a time-dependent framework.

For the latter viewpoint, consider that each mode in the sum (2.14) moves in time according to the equation of motion (2.1) and hence experiences the potential $V_d$. According to the adiabatic approximation, such a potential does not appreciably affect a mode $k$ if it does not change significantly during the typical time $k^{-1}$ of oscillation of the mode. During this time the mode itself will move a distance of order $k^{-1}$ toward the horizon, so that the adiabatic approximation is satisfied if the difference $|V_d(r - k^{-1}) - V_d(r)|$ is negligible.

The above is also the condition for the validity of the WKB approximation. If one further considers the blue-shift (1.3) of the wave modes as they approach toward the horizon, one finds the following bound

$$\Sigma_d \equiv \left| \frac{V_d(r - \hat{k}^{-1}) - V_d(r + \hat{k}^{-1})}{V_d(r)} \right| \ll 1, \quad (2.15)$$

where $\hat{k}^{-1} = \sqrt{\Delta_d} k^{-1}$ is the blue-shifted wavelength. The ratio $\Sigma_d$ can be easily computed, although its explicit expression appears rather cumbersome and we omit it.

### 2.3.1 Four-dimensional case

The quantity $\Sigma_0$ (for $s$-waves with $A = 0$) as a function of $r$ for the two wavelengths $k^{-1} = \lambda = r_h$ and $r_h/10$ is plotted in Fig. 1. For $\lambda = r_h$ the relative change in the potential becomes of order 1 already at $r \simeq 5 r_h$. This suggests that backscattered (out-going) modes really come from a region of radius appreciably larger than the horizon, that is $2 r_h \lesssim r_s \lesssim 10 r_h$. The corresponding blue-shift [defined in Eq. (1.3)] lies therefore in the range $0.05 \lesssim z \lesssim 0.41$. Its average is in good quantitative agreement with the result of Ref. [5], where an effective blue-shift $z \simeq 0.23$ was obtained by a different averaging prescription over the source position. It is also clear from the plot that the modes with wavelength much shorter than $r_h$ practically fall in a constant potential (the relative change $\Sigma_0 < 0.15$ for $\lambda = r_h/10$ and it is smaller for even shorter $\lambda$) and are not appreciably backscattered.

Considering again the packet (2.14) one is thus led to conclude that when it approaches the horizon, it is progressively distorted, with in-falling components of wavelength $\lambda \sim r_h$ being (partially) backscattered, while higher frequency components fall across the horizon undisturbed. The same effect occurs to the vacuum surrounding a black hole: virtual particles with wavelength of order $r_h$ are backscattered by the potential barrier, while their peers tunnel through inside the horizon; higher frequency modes behave instead as in flat space.
Figure 1: The ratio $\Sigma_{d=0}$ in four dimensions for $\lambda = r_h$ (solid line) and $0.1 r_h$ (dashed line).

### 2.3.2 Extra-dimensional cases

With increasing $d = 1, \ldots, 6$, the situation for $\lambda = r_h$ and $r_h/10$ is that of Fig. 2, which is qualitatively the same as in four dimensions. Let us also recall that $r_h$ in the ADD scenario depends on $d$, roughly according to the expression \[ r_h \simeq \left( 2C_d L^d M \right)^{1/1+d}, \] where $C_d$ is a numerical coefficient which accounts for the geometry of the extra dimensions in relating $m_g$ to $m_p$ \cite{8, 11, 12}, and \[ L \simeq \left( \frac{1 \text{TeV}}{m_g} \right) 10^{31/31} \text{m}. \] (2.17)

is the size of the extra dimensions (the case $d = 1$ is ruled out since it would imply corrections to the Newton law at solar system scale and below \cite{6}). On assuming $m_g \sim 1 \text{TeV}$, $C_d \sim 1$ and $M \sim 10^{-48} \text{m}$ ($\bar{M} \simeq 10 m_g$), one has the typical horizon radius (see Table 1)

\[ r_h \sim 10^{-19} \text{m} \sim \ell_g. \] (2.18)

Thus, $\bar{M} \sim 10 m_g$ is the minimum allowed mass for a black hole in this context. Since $\Sigma_d(\lambda = r_h) \sim 1$ for $r \sim 5 r_h$, and irrespectively of the value of $d > 0$, one concludes that quanta with the typical Hawking energy

\[ \omega \sim k_B T_H = m_p \frac{\ell_p}{L} \left( \frac{\ell_p}{M} \right)^{1/1+d} \sim 1 \text{TeV}, \] (2.19)

are again originated quite far away from the horizon and their blue-shift is negligibly small (see Table 1).

In the RS scenario, one expects $L \sim \sigma^{-1} < 10^{-3} \text{m}$ and for a 10 TeV black hole Eq. (2.16) with $d = 1$ yields the same order of magnitude as in Eq. (2.18) for the ADD case.
Figure 2: The ratio $\Sigma_d$ for $\lambda = r_h$ (solid lines) and $0.1 r_h$ (dashed lines). $d = 1, \ldots, 6$ (lower up).

| $d$ | 1  | 2  | 3  | 4  | 5  | 6  |
|-----|----|----|----|----|----|----|
| $r_h$ | $1 \times 10^{-18}$ | $5 \times 10^{-19}$ | $3 \times 10^{-19}$ | $3 \times 10^{-19}$ | $2 \times 10^{-19}$ | $2 \times 10^{-19}$ |
| $z$   | $2 \times 10^{-2}$    | $4 \times 10^{-3}$    | $8 \times 10^{-4}$    | $2 \times 10^{-4}$    | $3 \times 10^{-5}$    | $6 \times 10^{-6}$    |
| $\mathcal{L}_1$ | 1.0 | 0.99 | 0.98 | 0.98 | 0.96 | 0.96 |
| $\mathcal{L}_{1/2}$ | 0.99 | 0.98 | 0.93 | 0.93 | 0.86 | 0.86 |

Table 1: Horizon radius $r_h$ (in meters) and blue-shift $z$ for modes with $k^{-1} \sim \ell_g$ of a 10 TeV black hole with $d$ extra dimensions. The luminosity per particle specie $\mathcal{L}_\alpha$ is computed using the Epstein dispersion relation (3.2) and expressed in units of the corresponding four-dimensional canonical luminosity for $\alpha = 1$ and $1/2$. The case with $d = 1$ in the ADD scenario is ruled out by measurements [6].
Figure 3: The spectrum of Eq. (3.2) for $\alpha = 1$ (solid line) and $\alpha = 1/2$ (dashed line) compared with the linear dispersion relation (dotted line). The variables $\omega$ and $k_{\text{out}}$ are in TeV.

3 Non-linear dispersion relations

Since we have found that black holes can be used to produce quanta with energy close to the fundamental scale $m_g \sim 1\text{ TeV}$, we shall now study how modified dispersion relations for such quanta affect the evaporation. We shall basically follow what was done in Ref. [4] and replace $m_p$ with the much smaller $m_g$.

In Ref. [4] we showed that deviations from linearity in the spectrum yield a modified occupation number density for the Hawking quanta with energy close to the fundamental scale. The relation between the occupation number and the dispersion relation is given by [4, 38]

$$n(\omega) = \frac{1}{e^{4\pi r_h k_{\text{out}}(\omega)} - 1}.$$  \hspace{1cm} (3.1)

Note that the four-dimensional canonical number density [4] is formally recovered for the linear dispersion relation $k_{\text{out}} = \omega$.

As an example of dispersion relation we shall again take the Epstein function [35] with a maximum at $\omega \simeq \alpha m_g = \alpha \times 1\text{ TeV}$,

$$\omega = k_{\text{out}} \text{sech} \left( \frac{2k_{\text{out}}}{3\alpha} \right),$$  \hspace{1cm} (3.2)

which is plotted in Fig. 3 for $\alpha = 1$ and $1/2$ (the parameter $\alpha$ accounts for our ignorance of the details at high energy). Note that, at this point, all the dependence on $d$ is contained in $r_h$ as a function of the proper mass $M$ as given in Eq. (2.16).

From the number density the luminosity of the black hole per particle specie follows straightforwardly,

$$L = A \int_0^\infty \Gamma(\omega) n(\omega) \omega^3 d\omega \equiv A \int_0^\infty \Gamma(\omega) E(\omega) d\omega,$$  \hspace{1cm} (3.3)

where $\Gamma \sim 1$ is the gray-body factor for the given particle specie and $A = 16\pi r_h^2$ the four-dimensional horizon area. This expression neglects gravitational waves emitted into the bulk, since there is evidence that they contribute negligibly to the overall luminosity, because there are far less
Figure 4: The energy $E$ (divided by the horizon area) radiated in the range of frequencies between $\omega$ and $\omega + d\omega$ by a black hole in $4 + d$ dimensions according to the dispersion relation (3.2) with $\alpha = 1$ (solid line) and $\alpha = 1/2$ (dashed line) and to the linear dispersion relation (dotted line). Vertical units are arbitrary.

bulk graviton modes than Standard Model particles [10, 11] (see, however, Refs. [39] for different pictures, and [13] for an estimate of the bulk emission). One then gets

$$L_{(d)}(\bar{M}) \simeq 4 \pi r_h^2(\bar{M}) \int_0^\infty \text{sech}^4 \left( \frac{2k}{3\alpha} \right) \left| 1 - \frac{2k}{3\alpha} \tanh \left( \frac{2k}{3\alpha} \right) \right| \frac{k^3 dk}{e^4 \pi r_h(\bar{M}) k - 1}. \quad (3.4)$$

Note that, since $r_h > 1 \text{TeV}^{-1}$, the contribution from $k > k_m$, ($k_m \simeq 1.8 \alpha \text{TeV}$ is the value of $k_{\text{out}}$ which maximizes $\omega$), is negligible because of the exponential suppression at large $k$ and could be omitted. The luminosity can be integrated numerically and, for $\bar{M} = 10 \text{TeV}$, one finds the values given in Table 1 where we display the ratio

$$L_{\alpha} = \frac{L_{(d)}}{L_H}, \quad (3.5)$$

between the computed luminosity and the canonical luminosity $L_H$ of a four-dimensional black hole of the same mass [1] and $\alpha = 1$ and $1/2$. For larger values of $\bar{M}$ (hence of $r_h$) the ratio between the two quantities quickly approaches one (the same result was previously obtained in four dimensions for $\bar{M} \geq m_p$ [4]).

This result implies that, in order to test non-linear dispersion relations, one should better look for exclusive (rather than inclusive) effects, such as the probability of emitting particles with energy in a given range (close to $m_g$) or the energy $E(\omega)$ emitted in the range of frequencies between $\omega$ and $\omega + d\omega$. The latter quantity is easily evaluated for the canonical number density, we shall call it $E_H$, and can be used for comparison. It can also be numerically determined for the dispersion relation (3.2) and the result is plotted in Figs. 4 and 5 for $\alpha = 1$ and $1/2$. If one trusts the picture according to which such small black holes decay instantaneously [27], then one expects an energy spectrum for the decay products of the form in Fig. 4.
Figure 5: The ratio between the energy radiated according to the dispersion relation with $\alpha = 1$ (solid line) and $\alpha = 1/2$ (dashed line) and to the linear dispersion relation.

4 Conclusions

We have studied the high energy behaviour of Hawking radiation in space-times with extra spatial dimensions in order to highlight the possibility of using small black holes as probes of non-linear dispersion relations. Such deviations from linearity are in general expected within the framework of String Theory and other theories of Quantum Gravity, for which there is no certain experimental evidence at present.

The result of Section 2, that black holes in the brane-world could emit particles with energy near the fundamental scale of gravity $m_g$, is not surprising. In fact, it simply means that the effect scales with $m_g$ naively: as Planckian frequencies would be emitted in four dimensions (where $m_g = m_p$) by a Planck size black hole, energies of the order of $m_g$ are generated by black holes with a mass close to $m_g$. That is where non-linear dispersion relations should become visible [4, 5, 36].

In Section 3 we have explicitly analysed one of such non-linear dispersion relations [35] and shown that it induces a (relatively mild) suppression of high energy modes which might be found in the decay products of small black holes produced in accelerators. Unfortunately, the exponential factor in the occupation number density (3.1) dominates over the modified dispersion relation for all values of $d = 0, \ldots, 6$ and makes it hard to detect any effect in the high energy regime ($\omega \sim m_g$).

Let us conclude by remarking that the results we have obtained concern objects which can be treated as classical black holes, i.e. whose horizon radius is larger than the Compton wavelength. This condition is doomed to break down during the evaporation process. However, when the mass of the object has become sufficiently small, it is still unlikely that trans-Planckian modes get excited since there is not enough energy stored in the remnant. At that point the canonical description obviously fails and one should rely on the microcanonical description (see, e.g. [40] and References therein). In four space-time dimensions, the latter approach was shown in Ref. [4] to change but slightly the overall picture obtained for non-linear dispersion relations within the canonical description and one does not expect a different situation with extra spatial dimensions.
Table 2: The contribution to the separation constant $A_N$ from the off-brane angle dependence for $N = 2$ and 10, and the bulk luminosity per particle specie (in units of the four-dimensional luminosity) for a black hole with $M = 10 \text{TeV}$ and $\alpha = 1$. We recall that the case with $d = 1$ in the ADD scenario is ruled out by measurements $[6]$.

| $d$ | 1   | 2   | 3   | 4   | 5   | 6   |
|-----|-----|-----|-----|-----|-----|-----|
| $A_2$ | 1.59 | 2.53 | 4.03 | 6.41 | 10.2 | 16.3 |
| $A_{10}$ | 123 | $1.5 \times 10^4$ | $1.8 \times 10^6$ | $2.3 \times 10^8$ | $2.8 \times 10^{10}$ | $3.4 \times 10^{12}$ |
| $L_1$ | $4.8 \times 10^{-1}$ | $2.5 \times 10^{-1}$ | $1.3 \times 10^{-1}$ | $7.7 \times 10^{-2}$ | $4.3 \times 10^{-2}$ | $2.7 \times 10^{-2}$ |

A Confined fields

As mentioned in Section 2, a field confined on the brane cannot have a dependence on the extra spatial coordinates of the form in Eq. (2.7) and the separation constant $A$ must be correspondingly adjusted. We shall here bring some evidence in favour of the fact that the extra coordinate dependence does not appreciably affect the results given in Sections 2 and 3 and that setting $A = 0$ is not a serious limitation.

We focus on the dependence of the field $\Phi$ on the off-brane angles $\phi_i$’s for which we assume a smoothed out $\delta(\phi_i)$ in the form of the Fourier sum

$$\Phi \propto \prod_{i=1}^{d} \frac{1}{\sqrt{2\pi}} \sum_{n_i=-N}^{N} e^{i n_i \phi_i},$$

(A.1)

where $N \gtrsim 2$ to ensure that $\Phi$ is peaked on the brane ($\phi_i \simeq 0$ for $i = 1, \ldots, d$). Then

$$\partial^2_{\phi_i} \Phi \propto -\prod_{i=1}^{d} \frac{1}{\sqrt{2\pi}} \sum_{n_i=-N}^{N} n_i^2 e^{i n_i \phi_i}.$$ (A.2)

Such a form clearly introduces a coupling between the different $n_i$-modes which can be taken care of by integrating over the angles $\phi_i$’s. This introduces an effective separation constant

$$A_N \simeq -\int_{-\pi}^{+\pi} \prod_{i=1}^{d} d\phi_i \Phi \partial^2_{\phi_i} \Phi = \left[ \frac{N (1 + N) (1 + 2N)}{6\pi} \right]^d.$$ (A.3)

The numerical values are displayed in Table 2 for $N = 2$ and 10. It appears that the more modes are included (i.e. the larger is $N$) and the larger is $d$, the more the corresponding $A_N$ dominates over the contribution given in Eq. (2.13). The overall effect is to increase enormously the potential barrier which surrounds the horizon (thus suppressing the Hawking emission), since the maximum of $V_d$ is roughly of the same order of magnitude as $A_N$ for $A_N \gg 1$.

However, the ratios $\Sigma_d$ are just mildly affected. For instance, the peak in the curve for $d = 6$ as shown in Fig. 2 moves down from 2.9 to 2.2 for $A_{10}$ and $\Sigma_6$ is still roughly 1 for $r \simeq 5 r_h$. To conclude, the qualitative analysis of Sections 2 and 3 is not changed by the presence of $A \sim A_N \neq 0$, however large it is.

\footnote{One cannot take this result too literally because of the great simplifications we have assumed for the background metric. It is however suggestive to note that the more confined the fields are, the weaker appears the Hawking radiation.}
B  Bulk emission

The luminosity in Eqs. (3.3) and (3.4) describes the emission of four-dimensional particles (along the brane). In order to take into account gravitational waves emitted into the bulk, one must adjust both the horizon area

\[ A_{(d)} \simeq \frac{2\pi^{\frac{d+4}{2}} r_{h}^{2+d}}{\Gamma\left(\frac{3+d}{2}\right)}, \tag{B.1} \]

and the phase space measure which becomes \( d^{2+d}k \). One then obtains the values given in Table 2 for \( \alpha = 1 \), which, being smaller than the corresponding entries in Table 1, are negligible. This is in agreement with the results of Refs. [10, 11], according to which bulk gravitons do not contribute appreciably.

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