Inflation on a Pair of D3-brane and $\bar{\text{D}}$3-brane in Klebanov-Strassler Background

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We explain how to obtain Klebanov-Strassler solution in the low-energy limit of type IIB superstring theory and describe slow-roll inflation on the system of parallelly-separated D3-brane and $\bar{\text{D}}$3-brane in the Klebanov-Strassler background.

Keywords: flux compactification, D-brane inflation

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1. Introduction

Systematic derivation of a viable cosmological model including a natural inflationary era in the context of superstring theory has been a challenging problem. Particularly, in string cosmology, construction of a bridge between string-inspired brane-world scenario based on a warped geometry and obtaining a supergravity solution including this warped geometry has been an attractive subject. If such supergravity solution supports nonvanishing fluxes and is consistent with moduli stabilization, it is more intriguing.

In this note, we explain slow-roll inflation in the system of D3-brane and anti-D3-brane in the background of Klebanov-Strassler (KS) $^1$. In section $^2$, we briefly introduce the massless bosonic fields in the low energy limit of type IIB superstring theory and then summarize the KS solution involving warped geometry, deformed conifold, constant axion-dilaton, and various NS-NS and R-R form fields with non-vanishing fluxes. In section $^3$, the effective field theoretic description of the system of D3-brane and $\bar{\text{D}}$3-brane is given, of which action is the sum of Dirac-Born-Infeld (DBI) type term and Wess-Zumino (WZ) type R-R coupling. In the KS background, homogeneous time evolution of the separated D3$\bar{\text{D}}$3 shows slow-roll inflation for a wide range of parameter space. We conclude this paper with a few discussions.
2. Klebanov-Strassler Solution

In this section we briefly summarize the background geometry and fluxes on which the system of separated D3-brane and D3-brane lives. The specific form of (1+9)-dimensional spacetime of consideration is from the KS solution with various fluxes and warp factor, which is obtained by solving the supergravity equations given in the low energy limit of type IIB superstring theory.

2.1. IIB superstring theory and low energy limit

In (1 + 9) dimensions, five superstring theories are known:

| type IIB | type IIA | heterotic $E_8 \times E_8$ | heterotic SO(32) | type I |

We are interested in the type IIB superstring theory involving only closed oriented strings, of which characteristic mass scale is given by the string tension $1/\sqrt{\alpha'}$ (the corresponding string length scale is $l_s = \sqrt{2\pi\alpha'}$) and of which mutual interaction is proportional to the square of string coupling $g_s^2$.

When the string tension approaches infinity ($\alpha' \to \infty$), all the massive modes of higher nodes decouple and we obtain type IIB supergravity in ten dimensions as low energy effective theory which involves only massless fields (zero modes) and quadratic derivatives. The bosonic sector of the closed strings is composed of six fields completing $N = 2$ supergraviton multiplet as summarized in Table 1.

In the subsequent subsection we deal with classical equations of motion of type IIB supergravity and obtain KS solution.
2.2. Deformed conifold and Klebanov-Strassler background

Superstring theories are given in (1+9) dimensions but the present Universe we live is (1+3) dimensions. To be consistent with the observed Universe, six spatial dimensions in a superstring theory should be unobservable and a usual method is to assume that six spatial dimensions are compactified. Ten-dimensional coordinates we use are given in Table 2.

In the IIB superstring theory of consideration, the deformed conifold is utilized for the construction of a tip of compact six dimensions, of which the metric is

$$ds_\text{def}^2 = \frac{1}{2} \epsilon^2 K \left[ \frac{1}{3K^3} (d\rho^2 + g_i^2) + \sinh^2 \left( \frac{\rho}{2} \right) (g_1^2 + g_2^2) + \cosh^2 \left( \frac{\rho}{2} \right) (g_3^2 + g_4^2) \right],$$

where the function $K$ is a function of radial coordinate $\rho$ and decreasing,

$$K(\rho) = \frac{(\sinh 2\rho - 2\rho)^{\frac{1}{2}}}{2^{\frac{1}{2}} \sinh \rho} \approx \begin{cases} \left( \frac{4}{3} \right)^{\frac{1}{2}} \left( 1 - \frac{\rho^2}{10} \right) + \cdots & \text{as } \rho \to 0 \\ 2^{\frac{1}{2}} e^{-\frac{\rho}{4}} + \cdots & \text{as } \rho \to \infty \end{cases},$$

and $g_i$’s ($i = 1, ..., 5$) stand for the fundamental one-forms (vielbeins) of the five angular coordinates. In the metric $\epsilon$ is the deformation parameter which smooths the singular $S^3$ of $(g_3, g_4, g_5)$ at the tip of conifold as schematically shown in Fig. 1.
Our \((1+3)\)-dimensional spacetime is described by \(X^a\)'s \((a, b = 0, 1, 2, 3)\) and the metric \(G_{ab}\) is assumed to be flat for the KS solution, \(G_{ab} = \eta_{ab}\). In synthesis, the ansatz of the \((1+9)\)-dimensional metric is
\[
ds^2 = H^{-\frac{1}{2}}G_{ab}dX^adX^b + H^{\frac{1}{2}}ds_{\text{def}}^2,
\]
where \(H\) is a warp factor,
\[
H(\rho) = 2^{\frac{3}{2}}e^{-\frac{2}{3}(g_s\mathcal{M}\alpha')^2I(\rho)}.
\]
In the warp factor \(H\), the constant \(\mathcal{M}\) is R-R three-form flux and \(I(\rho)\) is decreasing exponentially for large \(\rho\)
\[
I(\rho) = \int_\rho^\infty dx \frac{x \coth x - \frac{1}{2\sinh^2 x}}{(\sinh 2x - 2x)^{\frac{1}{2}}} \approx \begin{cases} I(0) - 2 \left(\frac{1}{6}\right)^{\frac{1}{2}} \rho^2 + \cdots & \text{as } \rho \to 0, \quad I(0) \approx 0.71805 \\ 3 \cdot 2^{-\frac{1}{3}} \rho e^{-\frac{2}{3}\rho^2} + \cdots & \text{as } \rho \to \infty \end{cases}
\]
The KS solution involves various NS-NS and R-R form fluxes and their field configurations are given in Table 3. Since the deformed conifold is not compact along the \(\rho\)-coordinate, the NS-NS 3-form flux from the NS-NS 2-form field \(B_2\) is not explicitly given. In the phenomenological viewpoint of superstring theory, this \(\rho\)-direction should also be compactified, which means that the large \(\rho\) region is chopped and is replaced by a compact geometry. An appropriate known candidate is compact Calabi-Yau (CY) orientifold of which a schematic shape is shown in Fig. 2. The R-R 3-form flux lives along compact A cycle (red line) and the NS-NS 3-form does along compactified B cycle (green line) in compact CY orientifold. This surgery is not important to describe cosmological evolution of the early Universe which will be discussed in the subsequent section.

3. A Pair of D-brane and \(\bar{D}\)-brane in KS background
In addition to perturbative degrees of which bosonic fields are summarized in Table 1 type IIB superstring theory also involves various branes as nonperturba-
field or field strength | solution
---|---
dilaton axion | \( \Phi = \Phi_0 \)
R-R three-form field strength | \( C = 0 \)

\[
F_3 = \frac{M\alpha'}{2} \left\{ g_5 \wedge g_3 \wedge g_4 + d[F(g_1 \wedge g_3 + g_2 \wedge g_4)] \right\}
\]
with \( F(\rho) = \frac{\sinh \rho - \rho}{2 \sinh \rho} \)

NS-NS two-form field | \( B_2 = g_s M\alpha' \left( f g_1 \wedge g_2 + k g_3 \wedge g_4 \right) \)
with \( f(\rho) = \frac{\rho \coth \rho - 1}{2 \sinh \rho} (\cosh \rho - 1) \)
with \( k(\rho) = \frac{\rho \coth \rho - 1}{2 \sinh \rho} (\cosh \rho + 1) \)

self-dual R-R five-form field strength | \( \tilde{F}_5 = F_5 + *F_5, \quad F_5 = B_2 \wedge F_3 \)
with \( l(\rho) = f(1 - F) + kF \)

Table 3. NS-NS and R-R form fields and corresponding fluxes.

Fig. 2. From noncompact deformed conifold to compact Calabi-Yau orientifold

Tableau degrees. They are \( p \)-dimensional Dirichlet branes (Dp-branes), 5-dimensional Neveu-Schwarz brane (NS5-brane), and fundamental string (F1) summarized in the following:

| (D(-1)) | D1 | D3 | D5 | D7 | NS5 | F1 |

Though each D-brane is stable and supersymmetric, a pair of D-brane and anti-D-brane (D-brane) does not possess the supersymmetry and becomes unstable. Between the Dp-brane and Dp-brane, open string degrees live, of which low energy modes are a complex tachyon field \( T = \tau e^{ix} (\bar{T} = \tau e^{-ix}) \) depicting instability, two gauge fields \( A_a^a \) living on each brane, and two sets of transverse coordinates \( X_{(n)}^i \) representing the positions of Dp-brane and Dp-brane with distance \( \ell^i = X_{(1)}^i - X_{(2)}^i \). Dynamics of the system of Dp-brane and Dp-brane is described by effective action
which consists of DBI type term $\mathbb{D}$ and WZ type R-R coupling $\mathbb{W}$, respectively,

$$
S_{\text{DD}} = -T_p \int d^{p+1} \xi \left[ V_1(\tau, \ell) e^{-\Phi(X_1)} \sqrt{-\det A(1)} + V_2(\tau, \ell) e^{-\Phi(X_2)} \sqrt{-\det A(2)} \right],
$$

(7)

$$
S_{\text{WZ}} = T_p \int V(\tau) C \wedge \text{Str} e^{B_{2 \times 2} + 2\pi \alpha' \tilde{\Phi}}.
$$

(8)

The DD potential in (7)–(8) is based on the tachyon potential of an unstable D-brane $V(\tau, \ell)$ as $V_{(n)}(\tau, \ell) = V(\tau) \sqrt{-\det Q_{(n)}}$, and $A_{(n)}$ in the square roots are two $(1 + p) \times (1 + p)$ matrices as

$$
A_{(n)ab} = P_{(n)ab} \left[ E_{\mu\nu}(X_{(n)}) - \frac{\tau^2}{2\pi\alpha'} \det Q_{(n)} E_{\mu i}(X_{(n)}) \, e^i E_{\nu j}(X_{(n)}) \right] + 2\pi\alpha' F_{(n)ab}
$$

$$
+ \frac{1}{\det Q_{(n)}} \left\{ \frac{2\pi\alpha'}{2} (D_a TD_b T + D_b TD_a T)
$$

$$
+ \frac{i}{2} \left[ E_{ai}(X_{(n)}) + \partial_a X_{(n)j} E_{ji}(X_{(n)}) \right] i^i (TD_b T - TD_a T)
$$

$$
+ \frac{i}{2} (TD_a T - TD_a T) i^i \left[ E_{ib}(X_{(n)}) - E_{ij}(X_{(n)}) \partial_b X_{(n)j} \right] \right\}.
$$

(9)

In the previous expression, $P_{(n)ab} \{ \cdots \}$ means pull-back of the closed string fields on the $n$-th brane, $E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}$ raises and lowers the indices in the action, and

$$
\det Q_{(n)} = 1 + \frac{\tau^2}{2\pi\alpha'} \epsilon^i \epsilon^j G_{ij}(X_{(n)}).
$$

(10)

The field strength tensor of a U(1) gauge field on the $n$-th brane is $F_{(n)ab} = \partial^a A_{(n)b} - \partial^b A_{(n)a}$ and the covariant derivative of complex tachyon field is $D\Phi = \partial^a T - i(A_{(1)}^a - A_{(2)}^a) T$. In (9), Str denotes supertrace and

$$
\tilde{\Phi} = \left( \begin{array}{c}
F_{(1)} - i\Phi_1 \\
\frac{1}{2} \left[ DT + i T (i\Phi_1 - i\Phi_2) \right] F_{(2)} - i\Phi_2
\end{array} \right),
$$

(11)

where $i\Phi_n$ denotes the interior product by $\Phi_n$ regarded as a vector in the transverse space.

Now we introduce a D3-brane and an $\bar{D}$3-brane in the KS background. If we consider the total action $S_{\text{DD}} + S_{\text{WZ}}$, the potential terms of the D3-brane inversely proportional to the warp factor, $H^{-1}(\rho_{(1)})$, do not appear due to cancellation between the contribution from the DBI type action $\mathbb{D}$ (or the NS-NS coupling) and that from the WZ type action $\mathbb{W}$ (or the R-R coupling), while the contributions are added up for the $\bar{D}$3-brane, proportional to $2 H^{-1}(\rho_{(2)})$. It means that the D3-brane experiences no net force from the background, but the $\bar{D}$3-brane does the attractive force from the background. Resultantly, as a natural initial configuration, the D3-brane is located at a position of the warped throat, while the $\bar{D}$3-brane is located at the tip of the deformed conifold as shown in Fig. 4. Here we introduce the distance
Fig. 3. D3 at $\rho = \rho_{(1)0} \gtrsim l_s$ (blue color) and $\bar{D}3$ at $\rho = \rho_{(2)0} \approx 0$ (red color) in a warped throat of the deformed conifold.

$\ell$ between the D3-brane and the anti-D3-brane as

$$\ell^i = \begin{cases} 
\rho_{(1)} - \rho_{(2)} & \text{for } i = \rho \\
0 & \text{otherwise}
\end{cases} .$$

(12)

In order to study cosmological implication of the D $\bar{D}$ system, of which main topic is realization of inflationary era, we choose the static gauge $\xi^a = X^a$, assume absence of nontrivial gauge fields, $A^a_{(n)} = 0$, and consider homogeneous open string fields

$$\tau = \tau(t), \quad \chi = \chi(t), \quad \ell = \ell(t).$$

(13)

In the world-volume of D $\bar{D}$ system we assume the flat Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (k = 0),$$

(14)

instead of the (1+3)-dimensional Minkowski spacetime in (3). In the 6-dimensional space of deformed conifold, we also read and substitute the string coupling $g_s = e^{\Phi_0}$, the trivial axion field $C = 0$, nonvanishing NS-NS 2-form field $B_2$, R-R 2-form field $C_2$, and R-R 4-form field $C_4$ from the self-dual R-R 5-form field $\tilde{F}_5$, summarized in Table 3.

Dynamics of the closed string degrees in the weak coupling limit is of order $1/g_s^2$ but that of open string degrees is of order $T_p = g_s^{-1}(2\pi)^{-p}(\alpha')^{-\frac{p+1}{2}}$. Therefore, the interaction between the D$p$-brane and $\bar{D}p$-brane from the closed string degrees which is a $1/g_s$ correction should be taken into account. When the transverse distance is large enough ($\ell > \ell_c$), the 1-loop correction provides $O(1/\ell^4)$ order corrections from gravitation and R-R coupling of which magnitudes and signatures are exactly the same. As the distance between D and $\bar{D}$ reaches a critical distance, the 1-loop amplitude diverges. A natural assumption for the coincident D$\bar{D}$ ($\ell = 0$) is to introduce a finite binding energy per unit area $E_b$. Interpolation of both limits suggests the following correction to the tachyon potential in the last square bracket

$$V_{(1)}(\tau, \ell) = V_{(2)}(\tau, \ell) = V(\tau, \ell) = \frac{1}{\cosh(\sqrt{\pi} \tau)} \sqrt{1 + \frac{\tau^2 \ell^2}{2\pi \alpha'}} \left[ 1 - \frac{E_b/2T_3}{1 + (\ell/\ell_c)^{1.5}} \right],$$

(15)
reflecting the gravitational and R-R attractions between the D-brane and \( \bar{D} \)-brane. In (15),
\[
\ell_c = \left( \frac{2\kappa^2_{10} T_3^2}{E_b} \right)^{1/4},
\]
where \( \kappa^2_{10} \) is ten-dimensional gravitational constant.

In Fig. 3, the buoyant D3-brane starts to move to the sunken \( \bar{D} \)-brane by these attractive forces, however they are weak enough due to the nontrivial warp factor. We assume that \( \rho_{(2)} \) is always sufficiently small. Furthermore, to perform the numerical analysis, we set the position of anti D3-brane \( \rho_{(2)} \) to be fixed at the tip of warped throat in the KS background, which naturally gives \( \rho_{(2)} = 0 \). Note that we consider only the motion along the radial coordinate \( \rho \) and omit the dynamics of the angular variables \( (\theta_1, \theta_2, \phi_1, \phi_2, \psi) \) in the deformed conifold for simplicity, which may not lose generality. In addition, we do not consider dynamics of the tachyon phase field \( \chi(t) = 0 \).

To perform the numerical analysis for the slow-roll inflation in the D\( \bar{D} \) system, we introduce the dimensionless quantities
\[
\frac{t}{\ell_s}, \quad a, \quad \tau, \quad \frac{\rho(1)}{\ell_s}, \quad \tilde{T}_3 = \frac{T_3 \ell_s^2}{M_P^2}, \quad e_b = \frac{E_b}{\tilde{T}_3}, \quad \frac{\ell_c}{\ell_s}, \quad \mathcal{M}, \quad \epsilon = \exp \left( -\frac{\pi K}{g_s \mathcal{M}} \right),
\]
where \( M_P \) is Planck mass and the deformation parameter \( \epsilon \) is given by the R-R 3-form flux \( \mathcal{M} \) and NS-NS 3-form flux \( K \). An appropriate set of initial conditions is
\[
\tau(0) = \tau_0, \quad \ell(0) = \ell_0, \quad \dot{\tau}(0) = \dot{\tau}_0, \quad \dot{\ell}(0) = \dot{\ell}_0,
\]
since \( a(0) \) can always be fixed to be unity for \( k = 0 \) from the Einstein equations. Actual numerical analysis will be performed under a natural condition of no initial time derivatives, \( \tau_0 = \dot{\tau}_0 = \ell_0 = \dot{\ell}_0 = 0 \). In synthesis, the expansion rate represented by the value of e-folding is studied in the space of four dimensionless parameters, \( (\tilde{T}_3, e_b, \tau_0, \ell_0) \).

As shown in Figure 4, the shaded area by thick-grey color stands for the region of sufficient slow-roll inflation over 60 e-folding in the KS background. For comparison, the area bounded by the dashed line represents the region of 60 e-folding condition in the flat background. As the tension \( \tilde{T}_3 \) increases, the initial distance \( \ell_0 \) decreases and the initial tachyon amplitude \( \tau_0 \) increases.

4. Discussion

The KS solution has constant dilaton and axion configuration, and it is proven to be a dynamically-favorable configuration and other moduli including the volume moduli can also be stabilized by adding D7-branes and D3-branes. Though we obtained a slow-roll inflation model based on D\( \bar{D} \) system in the IIB superstring theory, it is suffered by huge supergravity corrections, and thus its present form does not generate a natural inflationary era.

When the fluxes were also generated on the D3-brane and are left in the present Universe, it may also jeopardize the cosmological model. As far as the dilaton moduli is fixed, they can sufficiently be diluted through the cosmological expansion. Since
Fig. 4. The tachyon potential with $\xi_b = 0.5$, $T_3 = 1$, $\ell_c = 1$, and $\rho(1)_{\text{min}} = 3.18$. this result is obtained without R-R form fluxes, an intriguing research direction may be inclusion of fluxes both in D-brane and compactified directions.

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References

1. I. R. Klebanov and M. J. Strassler, JHEP 0008, 052 (2000) [arXiv:hep-th/0007191].
2. A. Sen, Int. J. Mod. Phys. A 20, 5513 (2005) [arXiv:hep-th/041103].
3. A. Sen, Phys. Rev. D 68, 066008 (2003) [arXiv:hep-th/0303057];
   M. R. Garousi, JHEP 0501, 029 (2005) [arXiv:hep-th/0411222].
4. H. B. Kim, T. Kim, Y. Kim, H. Nakajima, J. S. Song, in preparation.
5. T. Banks and L. Susskind, [arXiv:hep-th/9511194].
6. S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D 66, 106006 (2002)
   [arXiv:hep-th/0105097].
7. S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D 68, 046005 (2003)
   [arXiv:hep-th/0301240].
8. S. Kachru, R. Kallosh, A. Linde, J. M. Maldacena, L. McAllister and S. P. Trivedi,
   JCAP 0310, 013 (2003) [arXiv:hep-th/0308055].
9. D. Baumann, A. Dymarsky, I. R. Klebanov, L. McAllister and P. J. Steinhardt, Phys.
   Rev. Lett. 99, 141601 (2007) [arXiv:hep-th/0705.3837].
10. E. J. Chun, H. B. Kim and Y. Kim, JHEP 0503, 036 (2005) [arXiv:hep-ph/0502051].
    I. Cho, E. J. Chun, H. B. Kim and Y. Kim, Phys. Rev. D 74, 126001 (2006)
    [arXiv:hep-th/0603174].