We propose a bicosmology model which is the classical analog of noncommutative quantum mechanics. From this point of view the sources of the modified FRW equations are dark energy ones governed by a Chapligyn’s equation state. The parameters of noncommutativity $\theta$ and $B$ are interpreted in terms of the Planck area and a like-magnetic field, presumably the magnetic seed of magnetogenesis.

I. INTRODUCTION

In the last forty years a lot of observational evidence has been accumulated showing a remarkable agreement with the standard cosmological model. These observations also show that the universe is expanding at an accelerated rate and then, the model requires to incorporate dark energy in order to explain such acceleration $\ddot{a}$.

The mere existence of dark energy requires ideas beyond the standard cosmological model, in a similar way that the standard model of particles needs to be modified in order to incorporate the dark matter $\ddot{a}$.

However, the incorporation of dark energy might shed some light on other long-standing problems in cosmology. One of such issues is the origin of the magnetic fields in galaxies and the mechanism to originate a magnetic seed in the universe. These magnetic seeds requires, in principle the break of a symmetry, which is possibly hidden, appearing as an effective degree of freedom. This would be similar to the magnetic field of a magnet, which has a purely quantum origin.

From the cosmological point of view, the scale factor in the Friedmann-Robertson-Walker (FRW) solution describes our universe as a bubble that evolves according to the FRW equations.

The assumed large scale homogeneity of the Universe leading to the FRW solution also implies that, seeing from each point of spacetime, it evolves as a patch causally disconnected from the rest of the Universe. Therefore, the assumptions of the existence of other universes causally disconnected can not be theoretically ruled out.

The possibility that these patches evolve and interchange information between them would require that the cosmological principle be just a small distance approximation, otherwise the formation of these structures would not be entangled.

In this sense, if the cosmological principle is not an exact symmetry, the transference of information between patches should be a process that would leave some observable traces. This poses the question about the information transfer mechanism.

One of the goals of this letter is to offer an approach that combines the idea of bimetric gravity $\ddot{a}$ and a relationship with noncommutative quantum mechanics (for a general discussion on bimetric gravity as cosmology see e.g. $\ddot{a}$).

The idea sketched above allows to incorporate considerations about gauge invariance, causality and evolution of states, thus providing a different approach to those modern cosmological problems.

The approach has interesting implications because allows to give a physical interpretation to the infrared and ultraviolet scales.

The letter is organized as follows: In section II we will shortly review some basic aspects of the FRW metric in order to fix notations and to establish some basic features which will be necessary to our model.

Sections III and IV are devoted to present and develop the model. The last section V contains the final remarks.

II. BASICS ISSUES

Let us start discussing the notation: Firstly we consider the metric

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (1)$$

where coordinates are $\{t, r, \theta, \varphi\}$, $k = \{-1, 0, 1\}$ is the spatial curvature and $N(t)$ is the lapse shift.

The equations of motion for metric (1) reduce to the FLRW equations which, in the gauge $N = 1$, turn out to be

$$2 \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 = \Lambda + \cdots, \quad (2)$$

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{3} \Lambda + \cdots, \quad (3)$$

where $\cdots$ denote contributions from matter fields. From here on we restrict to the case $k = 0$, which is also consistent with present observations $\ddot{a}$ and, for the sake of clarity, we will also omit the matter contributions.
The previous equations can also be obtained from the Lagrangian
\[ L = \frac{1}{2N} \dot{a}^2 + \frac{N}{3} a^3. \]  
Indeed, while equation (2) results from variations with respect to the variable \( a \), the second one (3) results from variations respect to \( N \).

Consider now the following change of variables
\[ x = \frac{2}{3\sqrt{G}} a^{3/2}. \]  
In terms of this new variable, which has dimensions of (energy)^{-1/2}, the Lagrangian in (1) changes into
\[ G^{3/2} L = \frac{1}{2N} x^2 - \frac{N}{2} \omega^2 x^2, \]
where the frequency is defined as
\[ \omega^2 = -\frac{3}{4} \Lambda. \]

From here on we will omit the global factor \( G \) in the Lagrangian.

The equations of motion obtained through variations of the variables \( x \) and \( N \) are
\[ \ddot{x} + \omega^2 x = 0, \quad \dot{x}^2 + \omega^2 x^2 = 0. \]
Note that the constraint (3) is obtained also from (1) as a first integral. Indeed, multiplying this last equation by \( \dot{x} \) we get the equivalent equation
\[ \frac{d}{dt} [\dot{x}^2 + \omega^2 x^2] = 0. \]
Therefore, the constant \( \dot{x}^2 + \omega^2 x^2 \) must be chosen equal to zero (in which case the energy is zero) and non trivial solutions are obtained for \( \omega \) an imaginary number, as it is upon the identification with the cosmological constant.

### III. MODIFIED FRW EQUATIONS

In this section we will modify the FRW equations by assuming more than one scale factor. This could happen, for example, in bubbles models (these bubbles have been extensively studied in recent literature, see e.g. \textsuperscript{13}).

The assumptions of homogeneity and isotropy in the present model implies that these scale factors \( a_i(t), i = 1, 2, \cdots \) satisfy
\[ [a_i(t), a_j(t)] = 0, \]
where \( [ , ] \) denotes a Poisson bracket.

Note that this “microcausality” principle implies the possibility of choosing only one lapse function and, therefore, the existence of one cosmological time.

The key observation is to note that in the harmonic oscillator representation \textsuperscript{14}, the model with more than one scale factor admits a straightforward generalization.

Indeed, instead of the usual Poisson’s bracket of momenta \( p_i(t) \) satisfying \([p_i(t), p_j(t)] = 0\), we can choose \([p_i(t), p_j(t)] = F_{ij}\) where \( F_{ij}\) is a constant tensor which can be identified with some internal degree of freedom which acts as an effective infrared cut-off. From now on, we will consider just two patches and write \( F_{ij} = \epsilon_{ijk} B_k \) for a constant \( B \).

In the harmonic oscillator representation of (6), we can choose the Lagrangian
\[ L_0 = \frac{1}{2N} (\dot{x}_1^2 + \dot{x}_2^2) - \frac{N}{2} \left( \omega_1^2 x_1^2 + \omega_2^2 x_2^2 \right), \]
which is the generalization of (9) which incorporates two scale factors with frequencies
\[ \omega_1^2 = -\frac{3}{4} \Lambda_1, \quad \omega_2^2 = -\frac{3}{4} \Lambda_2. \]

We also add an interaction term
\[ \tilde{L} = \frac{B}{2} (x_1 \dot{x}_2 - x_2 \dot{x}_1) \]
to get the total Lagrangian
\[ L = L_0 + \tilde{L}. \]

This Lagrangian formally describes a charged nonrelativistic particle moving in a constant magnetic field \( B \) perpendicular to the \( (x_1, x_2) \) plane, pointing in the direction of an \( x_3 \)-axis. Note that one could define the ratio \( B = \frac{\mu}{\sqrt{G}} \) with dimensions (energy)^{-1} where \( \sqrt{G} \) is the Plack mass.

The equations of motion (in the gauge \( N = 1 \)) are
\[ \ddot{x}_1 + \omega_1^2 x_1 - B \dot{x}_2 = 0, \quad \ddot{x}_2 + \omega_2^2 x_2 + B \dot{x}_1 = 0, \]
while the constraint, a consequence of time reparametrization invariance, reads as
\[ \frac{1}{2} (\dot{x}_1^2 + \dot{x}_2^2) + \frac{1}{2} (\omega_1^2 x_1^2 + \omega_2^2 x_2^2) = 0. \]

As in the previous section, the condition \textsuperscript{15} gives nontrivial solutions for imaginary frequencies, which is just the case at hand.

In order to make contact with the cosmological description and following the analogy with (14), if we define
\[ x_1 = \frac{2}{3\sqrt{G}} a^{3/2}, \quad x_2 = \frac{2}{3\sqrt{G}} b^{3/2}, \]
equations (14) become
\[ \frac{2}{a} \ddot{a} + \left( \frac{\dot{a}}{a} \right)^2 = -\frac{4}{3} \omega_1^2 + B \sqrt{a} \dot{b} \frac{b}{a}, \]
\[ \frac{2}{b} \ddot{b} + \left( \frac{\dot{b}}{b} \right)^2 = -\frac{4}{3} \omega_2^2 - B \sqrt{a} \dot{a} \frac{a}{b}. \]
and the constraint \((15)\) now reads as
\[
\left(\frac{\dot{a}}{a}\right)^2 = -\left(\frac{2}{3} \omega_1\right)^2 - \frac{1}{a^2} \left(\frac{3}{4} \omega B^2 \tilde{b} + \tilde{b}^2 b\right). \tag{18}
\]

The expressions in \((17)\) can be considered as the FRW equations for two patches of the Universe that interact through a *constant external like-magnetic field* while \((15)\) is the analog of \(G_{00} = -8\pi G T_{00}\) in the one-metric conventional gravity.

We would like to emphasize that, in this picture, dark energy emerges as a consequence of the incorporation of a sort of interaction between neighboring patches in space-time.

From the point of view of the energy and momentum content in sector 1, eqs. \((17)\) and \((18)\) turn out to be
\[
T_{11} = T_{22} = T_{33} = -\sqrt{a b} \frac{\dot{b}}{a^2},
\]
and
\[
T_{00} = -\left(\frac{b}{a}\right)^3 \left[\left(\frac{\dot{b}}{b}\right)^2 + \left(\frac{2}{3} \omega_2\right)^2 \right] - \left(\frac{2}{3} \omega_1\right)^2.
\]

From these results one has the following state equation,
\[
\rho_b + \frac{6\pi}{B^2 \Omega^2} = \frac{A_2}{8\pi G} \left(\frac{b}{a}\right)^3,
\]
which describes a Chapligyn gas. This gas has been discussed extensively in cosmology in connection with the dark energy problem \([14, 15]\).

It is also interesting to discuss this problem from the point of view of a Hamiltonian system with modified Poisson Brackets. Indeed, the mechanical system described by the Lagrangian in \((12)\) can also be described by the Hamiltonian
\[
H = \frac{N}{2} \left(p_1^2 + p_2^2 + \omega_1^2 x_1^2 + \omega_2^2 x_2^2\right), \tag{20}
\]
with the Poisson’s bracket algebra
\[
[x_i, x_j] = 0, \quad [x_i, p_j] = \delta_{ij}, \quad [p_1, p_2] = \epsilon_{ij} B. \tag{21}
\]
Note that \(x\) has dimensions of \((\text{energy})^{-1/2}\) while \(p\) has its inverse dimensions. Then \(B\) has dimensions of \((\text{energy})^{1/2}\) and therefore, the magnetic field is \(B/\sqrt{G}\).

### IV. NONCOMMUTATIVE CLASSICAL COSMOLOGY

We observe that it is still possible to consider a more general deformation of the Poisson’s algebra, that is, the introduction of a noncommutative parameter in the bracket coordinates.

With this deformation (as in \((20)\)) we have two energy scales, namely the Planck energy \(E_P = G^{-\frac{1}{2}}\) and the “magnetic energy” \(B^2\) (or, equivalently, the Planck and magnetic lengths), since \(G\) (the Newton constant) and \(B\) (a magnetic-like seed) define two natural scales for the modified Poisson bracket structure.

In order to explore this system we restrict ourselves to the case with \(\omega_1 = \omega_2 = \omega\) and rewrite \((20)\) as
\[
H = N \mathcal{H}, \quad \tag{22}
\]
where the constraint reads now as
\[
\mathcal{H} = \frac{\omega}{2} \left(p_1^2 + p_2^2 + x_1^2 + x_2^2\right), \quad \tag{23}
\]
with original phase space variables \(\{x_i, p_j\}\) rescaled according to \(x_i \rightarrow \tilde{x}_i = \sqrt{\omega} x_i = \tilde{x}\) and \(p_j \rightarrow \tilde{p}_j = p_j/\sqrt{\omega}\).

The modified Poisson brackets structure, in view of our previous discussion \([20, 22]\), is
\[
[x_i, \tilde{x}_j] = \delta_{ij} \tilde{G}, \quad \tag{24}
[x_i, \tilde{p}_j] = \delta_{ij} \tilde{B}, \quad \tag{25}
[y_i, \tilde{p}_j] = \epsilon_{ij} \tilde{B}, \quad \tag{26}
\]
where \(\tilde{G}\) and \(\tilde{B}\) are the rescaled Newton’s constant and magnetic-like seed, respectively, according to \(\tilde{G} = G\omega\) and \(\tilde{B} = B = \sqrt{\mathcal{B}_{\text{seed}}/\omega}\) with \(\mathcal{B}_{\text{seed}} = B/\sqrt{G}\).

The equations of motion for the variables \(\tilde{x}_i\) turn out to be
\[
\ddot{x}_i + \Omega^2 \tilde{x}_i - B \epsilon_{ij} \tilde{x}_j = 0. \tag{27}
\]

where
\[
\Omega^2 = \omega^2 (1 - \tilde{G} \tilde{B}) = \omega^2 (1 - GB_{\text{seed}}), \quad \tag{28}
\]
\[
\mathcal{B} = \omega (\tilde{B} + \tilde{G}) = \sqrt{\mathcal{B}_{\text{seed}}} (\omega^2 + \bar{\omega}^2). \quad \tag{29}
\]

Comparing equations \((27)\) with \((14)\) (with \(\omega_1 = \omega_2 = \omega\) for the last case) we see that an effective magnetic background field, given by
\[
\mathcal{B}/\sqrt{G} = \mathcal{B}_{\text{seed}} + \bar{\omega}^2, \quad \tag{30}
\]

is generated.

We identify \(\mathcal{B}/\sqrt{G}\) as the effective magnetic seed in the universe, \(\mathcal{B}_{\text{eff}}\), while \(\Omega^2\) can be seen as an effective cosmological constant.

Note that, as in noncommutative quantum mechanics \([20, 22]\), there are two phases with \(G\mathcal{B} > 1\) and \(G\mathcal{B} < 1\) respectively, separated by a critical point at \(G\mathcal{B} = 1\).

If we demand \(GB_{\text{seed}} \ll 1\), then
\[
\mathcal{B}_{\text{seed}} \approx \mathcal{B}_{\text{eff}} + \frac{3}{4} \Lambda.
\]

This formula provides a simple and direct link between the magnetogenesis \([16, 17]\) and the cosmological constant problem.

Note that the modified Poisson brackets in \((24), 25\), and \((26)\) can be mapped to a canonical form under a noncanonical change of variables \(\{\tilde{x}_i, \tilde{p}_j\} \rightarrow \{X_i, P_j\}\) where

\[
X_i = \xi_i, \quad P_j = \frac{\mathcal{B}}{\sqrt{G}} \xi_j + \lambda_j (\bar{\omega}^2 - \omega^2) X_j,
\]

with \(\lambda_j = \frac{\mathcal{B}}{\sqrt{G}} \epsilon_{ij} \xi_j\) and \(\xi_j = \sqrt{\frac{\mathcal{B}}{\sqrt{G}}} x_j\), such that the Poisson brackets between \(X_i\) and \(P_j\) take the form
\[
\{X_i, P_j\} = \mathcal{B} \delta_{ij}. \tag{31}
\]
the new Hamiltonian is still diagonal \[ 17, 18 \]. Thus, \( \mathcal{H} \) can be written as

\[
\frac{H}{N} = \frac{1}{2} \left( P_1^2 + \Omega_+^2 X_1^2 \right) + \frac{1}{2} \left( P_2^2 + \Omega_-^2 X_2^2 \right),
\]

where variables \( \{X_1, P_1\} \) and \( \{X_2, P_2\} \) are canonically conjugated, while the variables of the sector labeled as ‘1’ have zero Poisson bracket with those of sector ‘2’. The frequencies in \[ 31 \] are \[ 20, 22 \]

\[
\Omega_\pm = \pm \omega \left[ 1 + \frac{1}{4} (B - \bar{G})^2 \mp \frac{1}{2} (\bar{B} + \bar{G}) \right],
\]

\approx \pm \left[ \omega^2 + \left( \frac{\bar{G} B_{\text{eff}}}{2} \right)^2 \mp \frac{\bar{G} B_{\text{eff}}}{2} \right],
\]

where, in the last line, we used the fact that \( B_{\text{seed}} - \omega^2 = B_{\text{eff}} - 2\omega^2 \approx B_{\text{eff}} \) when \( \omega^2 \approx -3\Lambda/4 = \Omega^2 \). With same approximations, we finally obtain \( \Omega^2_\pm \approx \omega^2 \).

V. FINAL REMARKS

We should also observe that there are differences between the quantum Hall effect approach and the magnetogenesis as discussed in \[ 14, 17, 19 \]. Indeed, as argued in \[ 19 \], the causal connection between two spacetime regions take place in the presence of an external constant magnetic-like field (at first sight presumably from the formation of some galactic halo). This external magnetic-like field would be very small, but not necessarily a magnetic seed. The magnetic seed would be created much earlier, as can be seen from the magnetic displacement \[ 20 \].

However, this posses some intriguing questions: Is \( B \) a real magnetic field?, What is the mechanism responsible for the creation of this \( B \)? At first glance, it is interesting to think that the origin of this field could be similar to that of the magnetic field produced by a magnet and, therefore, of purely quantum origin.

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[1] A. Joyce, B. Jain, J. Khoury and M. Trodden, Phys. Rept. 568, 1 (2015)
[2] For a nice recent review see, I. Debono and G. F. Smoot, Universe 2, no. 4, 23 (2016).
[3] J. Frieman, M. Turner and D. Huterer, Ann. Rev. Astron. Astrophys. 46 (2008) 385.
[4] D. F. Mota and D. J. Shaw, Phys. Rev. D 75, 063501 (2007); F. K. Hansen, A. J. Banday and K. M. Gorski, Mon. Not. Roy. Astron. Soc. 354, 641 (2004); R. Bousso, R. Harnik, G. D. Kribs and G. Perez, Phys. Rev. D 76, 043513 (2007).
[5] For example, P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 594, A20 (2016); P. A. R. Ade et al. [BICEP2 and Planck Collaborations], Phys. Rev. Lett. 114, 101301 (2015).
[6] Y. Akrami, S. F. Hassan, F. Könnig, A. Schmidt-May and A. R. Solomon, Phys. Lett. B 748, 37 (2015).
[7] Y. Akrami, T. S. Koivisto, D. F. Mota and M. Sandstad, JCAP 1310, 046 (2013).
[8] G. Cusin, R. Durrer, P. Guarato and M. Motta, JCAP 1505, no. 05, 030 (2015).
[9] S. Deser, K. Izumi, Y. C. Ong and A. Waldron, Mod. Phys. Lett. A 30, 1540006 (2015).
[10] C. de Rham, L. Heisenberg and R. H. Ribeiro, Class. Quant. Grav. 32, 035022 (2015).
[11] K. Bamba, A. N. Makarenko, A. N. Myagky, S. Nojiri and S. D. Odintsov, JCAP 1401, 008 (2014) and references therein.
[12] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update.
[13] M. Kleban, Class. Quant. Grav. 28, 204008 (2011); R. Gobbetti and M. Kleban, JCAP 1205, 025 (2012); J. L. Lehners, Class. Quant. Grav. 28, 204004 (2011); A. Aguirre, M. C. Johnson and A. Shomer, Phys. Rev. D 76, 063509 (2007); S. Chang, M. Kleban and T. S. Levi, JCAP 0804, 034 (2008).
[14] V. Gorini, A. Kamenshchik and U. Moschella, Phys. Rev. D 67, 063509 (2003); V. Gorini, A. Kamenshchik, U. Moschella and V. Pasquier, gr-qc/0403062 V. Gorini, A. Y. Kamenshchik, U. Moschella, O. F. Piattella and A. A. Starobinsky, JCAP 0802, 016 (2008).
[15] A. Melchiorri, L. Bersini-Houghton, C. J. Odman and M. Trodden, Phys. Rev. D 68, 043509 (2003).
[16] A. Kandus, K. E. Kunze and C. G. Tsagas, Phys. Rept. 505, 1 (2011).
[17] B. Ratra, Astrophys. J. 391 (1992) L1.
[18] H. Falomir, J. Gamboa, F. Mendez and P. Gondolo, Phys. Rev. D 96, 083534 (2017).
[19] H. Falomir, J. Gamboa, F. Mendez and P. Gondolo, preprint 2018.
[20] V. P. Nair and A. P. Polychronakos, Phys. Lett. B 505, 267 (2001).
[21] S. Bellucci, A. Nersessian and C. Sochichiu, Phys. Lett. B 522, 345 (2001).
[22] J. M. Carmona, J. L. Cortes, J. Gamboa and F. Mendez, JHEP 0303, 068 (2003).