The doubling of stellar black hole nuclei

Mher V. Kazandjian1⋆ and J. R. Touma2⋆

1Leiden Observatory, Leiden University, PO Box 9513, NL-2300 RA Leiden, the Netherlands
2Department of Physics, American University of Beirut, Beirut, Lebanon

Accepted 2013 January 10. Received 2013 January 9; in original form 2012 July 3

ABSTRACT

It is strongly believed that Andromeda’s double nucleus signals a disc of stars revolving around its central supermassive black hole on eccentric Keplerian orbits with nearly aligned apsides. A self-consistent stellar dynamical origin for such apparently long-lived alignment has so far been lacking, with indications that cluster self-gravity is capable of sustaining such lopsided configurations if and when stimulated by external perturbations. Here, we present results of N-body simulations which show unstable counter-rotating stellar clusters around supermassive black holes saturating into uniformly precessing lopsided nuclei. The double nucleus in our featured experiment decomposes naturally into a thick eccentric disc of apo-apse aligned stars which is embedded in a lighter triaxial cluster. The eccentric disc reproduces key features of Keplerian disc models of Andromeda’s double nucleus; the triaxial cluster has a distinctive kinematic signature which is evident in Hubble Space Telescope observations of Andromeda’s double nucleus, and has been difficult to reproduce with Keplerian discs alone. Our simulations demonstrate how the combination of an eccentric disc and a triaxial cluster arises naturally when a star cluster accreted over a preexisting and counter-rotating disc of stars drives disc and cluster into a mutually destabilizing dance. Such accretion events are inherent to standard galaxy formation scenarios. They are here shown to double stellar black hole nuclei as they feed them.

Key words: instabilities – galaxies: kinematics and dynamics – galaxies: nuclei.

1 INTRODUCTION

Over the years, the nucleus of the Andromeda galaxy (M31) went from being asymmetric to doubling, then tripling. The asymmetry was first noted in the balloon-born Stratoscope observatory (Light, Danielson & Schwarzschild 1974). It was photometrically resolved into a double nucleus by Hubble Space Telescope (HST) observations (Lauer et al. 1993); asymmetry hence turned into lopsidedness, with a luminous feature (referred to as P1) shining a few parsecs away from a dimmer one (referred to as P2), which is closer to the centre of the host bulge. The resolved double nucleus had already been suspected to host a supermassive black hole (SMBH), located somewhere close to P2, with a neighbourhood that was known to shine brighter than the rest of the nucleus in the ultraviolet (Dressler & Richstone 1988; Kormendy 1988). Detailed HST spectroscopy added a third component (known as P3), a disc of young massive stars, whose size and rotation speed rule out viable alternatives to a central SMBH of $\sim 10^8 M_\odot$ (Bender et al. 2005).

In the currently favoured model of M31’s lopsided double nucleus (Tremaine 1995; Peiris & Tremaine 2003), the brighter peak P1 (Lauer et al. 1993) is thought to coincide with the common apocentric region of an eccentric disc of stars revolving on nearly apse-aligned Keplerian ellipses, around M31’s central SMBH. Similar eccentric discs are thought to underlie lopsided nuclei detected around SMBHs in the centres of nearby galaxies (Lauer et al. 1996, 2005; Gultekin et al. 2011). That such kinematic configurations can be sustained in self-gravitating discs around SMBHs was clarified in a series of dynamical studies. Indeed, the investigation of modes of hot nearly Keplerian discs indicated that slow ($m=1$, lopsided) modes can be stably excited (Tremaine 2001). This conclusion was corroborated by results of N-body simulations of discs around SMBHs which showed that, when started in asymmetric conditions, discs can relax into long-lived uniformly precessing lopsided configurations (Bacon et al. 2001; Jacobs & Sellwood 2001). Subsequently, razor-thin, self-gravitating, lopsided equilibria were constructed, yielding encouraging agreement with photometric and kinematic observations of M31’s nucleus (Salow & Statler 2001; Sambhus & Sridhar 2002). These and related studies left open questions about how such modes are excited, how they ultimately saturate into global lopsided configurations, and whether constructed equilibria were stable or not. The early suggestion (Tremaine 1995) that an initially circular disc could become eccentric under the influence of dynamical friction with the host bulge has not been thoroughly explored. A recently proposed scenario...
(Hopkins & Quataert 2010a,b) has eccentric discs forming in the notoriously complex—and poorly understood (Silk 2011)—environment which couples star formation and its feedback to gas accretion in the early stages of SMBH growth.

In this work, we opt for a minimalist and dynamically self-consistent route to three-dimensional, lopsided nuclei, out of counter-rotating (CR hereafter) collisionless stellar distributions dominated by SMBHs. Such CR distributions are prone to violent \( n = 1 \) instabilities in the presence of moderate counter-rotation (Touma 2002; Jog & Combes 2009), hence the suggestion that the lopsided structure in Andromeda’s nucleus may have been triggered by the accretion of a retrograde globular cluster in a preexisting disc of stars (Sambhus & Sridhar 2002). With the intention of exploring the outcomes of such accretion events, we performed a series of \( N \)-body simulations of CR stellar distributions evolving in the sphere of influence of an SMBH. Below, we report on results which show unstable CR distributions evolving into stable, uniformly precessing lopsided nuclei. We follow the instability in our featured simulation from multiple complementary perspectives. We then probe the ensuing lopsided nucleus, and show that, in addition to displaying all the key observational signatures which Keplerian discs are meant to model, it recovers asymmetries in the tail of Andromeda’s line-of-sight velocity dispersion (Bender et al. 2005) which a thick lopsided disc alone is unable to reproduce (Peiris & Tremaine 2003). This said, the lopsided nucleus in our featured proof of concept experiment differs with observations on various counts (e.g. slower rotation in the outer part of the nucleus, dimmer P2 peak). We discuss these shortcomings arguing that rather than signalling anything fundamental about the process itself, they point to limitations of our experimental setup which we expect more realistic simulations (with more powerful tools) to overcome. We conclude by reviewing a suite of simulations which point to the rich variety of CR unstable configurations, and then assess the likelihood of such configurations in the context of galaxy formation scenarios.

2 COUNTER-ROTATING INSTABILITY: FROM LINEAR GROWTH TO SATURATION

Our \( N \)-body simulations were performed with \textsc{gadget-2} (Springel 2005), an octree code, which is popular with the cosmological structure formation community. We pushed this tool to the limit of extreme mass ratios, one for which it was not specifically designed, one in which it did the desired job, albeit with stringent force accuracy, and time stepping criteria. With M31 in mind, all experiments have an SMBH with a mass of \( M_\bullet = 10^7 M_\odot \) and a main disc component (prograde in our convention) with a tenth of SMBH mass (Peiris & Tremaine 2003; Bender et al. 2005). An exhaustive exploration, with sufficient realism, of a scenario in which a CR cluster (10\(^{-2}\)) decays under dynamical friction to the eccentricity–inclination dynamics of the particles up to \( 1 \) Myr (Fig. 1b). Viewed from the side, the gradual in-plane dispersal of the RP occurs along with a dramatic excitation of out-of-plane motion; by \( 1 \) Myr the RP unfolds into a triaxial structure (Fig. 1c). Scattering plots of particle eccentricities and inclinations (Fig. 1d) reveal how, by 0.72 Myr, the now highly eccentric RP is well on its way to complete triaxial disruption. Tucked in this emerging cluster, the PP, which has absorbed much of the RP’s (negative) angular momentum, heats up slightly in inclination as it consolidates its eccentric lump, and then adjusts slowly to whatever little angular momentum is left in the RP.

A look at the mean eccentricity and inclination of both populations (Figs 2a and b) confirms these observations, as it reveals three distinct phases: phase I of near-coplanar growth of the mean

![Figure 1](https://academic.oup.com/mnras/article-abstract/430/4/2732/1098219/2733)

**Figure 1.** The eccentricity–inclination dynamics of the particles up to 1 Myr is shown. In row (a), we follow a top view of the prograde particles, from initial axisymmetric annular configuration, through a phase of growth of lopsidedness, with growing mean eccentricity (0.41 around 1 Myr), and alignment of apsides. Row (b) shows a top view of retrograde particles over the same period; they experience greater increase in mean eccentricity (around 0.73 by 0.72 Myr), and show signs of apse alignment by 0.5 Myr, before dispersing (by 1 Myr) into a discy blob. A side view of retrograde particles in row (c) shows how the in-plane dispersed seen in row (b) reflects in projection a burst of out-of-plane dynamics which puffs the initially retrograde disc into a triaxial cluster of stars. The eccentricity–inclination dynamics of the full cluster is captured in row (d), with prograde in black and retrograde in blue; the mean eccentricity of both populations grows as the instability develops; by 1 Myr, the mean inclination of the prograde particles has hardly changed, while the highly eccentric retrograde population is widely dispersed in inclination.
M. V. Kazandjian and J. R. Touma

The projected density of the saturated configuration at the end of peaking around 0.72 Myr. Cycles of increasingly smaller amplitudes and inclination $\langle m \rangle$ mode, as the full cluster transits contributions. Triaxial dispersal of the RP $\langle h \rangle$ mode, along with non-negligible $m = 2$ and $m = 3$ contributions. Triaxial dispersal of the RP is responsible for the slight growth of the $m = 0$ amplitude, and the related slight decrease in the $m = 1$ mode amplitude after 0.72 Myr. In panel (d), we follow the precession rate of the $m = 1$ mode in time: initial large-amplitude oscillations reflect back and forth librations of prograde and retrograde $m = 1$ excitations about the uniform precession state; these oscillations die out with the dispersal of the retrograde bunch, leaving an $m = 1$ mode which precesses at a near-constant rate of 19 km s$^{-1}$ pc$^{-1}$.

eccentricity of both populations which lasts till about 0.5 Myr; phase II of continued mean eccentricity growth, but now coupled to growth in the mean inclination of the RP, both of which reach a maximum around 0.72 Myr; phase III, with cycles of increasingly small amplitudes leading to a nearly steady regime. As emphasized in the caption of Fig. 2, transitions in mean eccentricity and inclination are neatly imprinted on the modal structure of the projected density and the pattern speed of the dominant $m = 1$ mode, as the full cluster transitions from a thin axisymmetric initial state to a saturated, lopsided and slowly precessing mode (Figs 2c, d and 3).

Saturation of the $m = 1$ mode is evidently correlated with the dispersal of the initially retrograde population into a triaxial cluster of high-eccentricity orbits. The dispersal appears to be associated with collective dynamics along eccentricity–inclination cycles. These cycles are excited when a population of stars (the retrograde population) develops sufficient eccentricity (through a CR instability) in the presence of a dominant eccentric perturber (the more massive eccentric and precessing prograde disc) (Touma, Tremaine & Kazandjian 2009). One can presumably describe the inception of these cycles in a generalized Kozai–Lidov framework (Naoz et al. 2011), but then one is left with the harder task of accounting for the dispersal of the population on these cycles. We gained valuable insight into dispersal then saturation by modelling a closely related process in a 2D analogue of the 3D cluster (Kazandjian & Touma, in preparation). The model in question permits a blow-by-blow account of the mutual sculpting of planar prograde and retrograde

Figure 2. The linearly unstable CR configuration saturates into a lopsided uniformly precessing disc. Panels (a) and (b) show the mean eccentricity ($e$) and inclination ($f$) of PP (black) and RP (blue). ($e$) grows, at near-constant ($f$), till about 0.5 Myr, at which point ($f$) takes off, with both ($e$) and ($f$) peaking around 0.72 Myr. Cycles of increasingly smaller amplitudes follow this initial growth phase, leading to a saturated eccentric prograde disc which is shrouded by a triaxial halo. In panel (c), the power of the four dominant modes in projected density is displayed; an initially axisymmetric configuration ($m = 0$) lends way to a dominant $m = 1$ mode, along with non-negligible $m = 2$ and $m = 3$ contributions. Triaxial dispersal of the RP is responsible for the slight growth of the $m = 0$ amplitude, and the related slight decrease in the $m = 1$ mode amplitude after 0.72 Myr. In panel (d), we follow the precession rate of the $m = 1$ mode in time: initial large-amplitude oscillations reflect back and forth librations of prograde and retrograde $m = 1$ excitations about the uniform precession state; these oscillations die out with the dispersal of the retrograde bunch, leaving an $m = 1$ mode which precesses at a near-constant rate of 19 km s$^{-1}$ pc$^{-1}$.

populations in terms of capture in, then escape from a drifting, trapping region in phase space (Sridhar & Touma 1997). It can naturally explain the near-coplanar growth and apse alignment in phase I of 3D dynamics. Furthermore, it clarifies the collisionless damping process at work in the decaying cycles of phase III. How the eccentricity growth of phase I paves the way for inclination growth in phase II cannot be captured in a planar analogue, and shall await a more sophisticated treatment of the 3D deployment of unstable CR clusters.

3 CONFRONTATION WITH M31: PRELIMINARY RESULTS

The saturated distribution of stars, which is followed in projection, and over a full precessional cycle in Fig. 3, generalizes Keplerian disc models of M31’s nucleus (Peiris & Tremaine 2003) in two obvious ways. On one hand, the eccentric disc in our nucleus is shrouded by a lighter triaxial cluster, whereas currently preferred models tend to work with the eccentric disc alone: as discussed below, the additional triaxial cluster contributes a crucial improvement to model kinematics. On the other hand, successful eccentric disc models (Peiris & Tremaine 2003) are essentially kinematic in nature (their origin and dynamical evolution remain uncertain), whereas ours is a dynamically stable nucleus to which an unstable configuration saturates.

Given that the eccentric disc component in our saturated lopsided nucleus is 10 times more massive than the triaxial cluster, the photometry and kinematics of this nucleus will naturally display all the significant qualitative features which eccentric disc models seek to explain (Fig. 4, left-hand column): a double peak in surface brightness, an asymmetry in the rotation velocity (with an off-centre zero velocity point) and an off-centred peak in the LOSDV $\sigma$ (Peiris & Tremaine 2003; Bender et al. 2005). Pushing our luck, we seek a preliminary quantitative confrontation of simulation results with bulge-subtracted HST observations of M31’s nucleus (Bender et al. 2005). In so doing, we work with a rescaled and dynamically similar version of our fiducial experiment (roughly speaking, we rescale from fiducial simulation length scale, $\sim 1$ pc, to M31 nucleus scale, $\sim 3$ pc). The resulting model nucleus is then observed under HST conditions (Peiris & Tremaine 2003; Bender et al. 2005) (details
Photometry and kinematics of saturated configurations are compared to $\sigma$-HST data (Kormendy & Bender 1999; HST observations by Bender et al. 2005), and which has proven difficult to recover with eccentric discs alone (Peiris & Tremaine 2003, hereafter PT03) are also displayed as dot–dashed lines. Quantitative agreement, between a dynamically similar copy of the featured eccentric disc and cluster parameters as well as the sky angles of the saturated cluster stars, will hopefully resolve this. For the adopted mass-to-light ratio, the brightness profile in their best-fitting model falls off too sharply to fit the bulge-subtracted photometry in the outer parts of the disc, and this precisely in the region where our model’s triaxial cluster makes its noteworthy impact on kinematics. The current masses and initial radial structure of both the PP and RP, for the adopted mass-to-light ratio, the brightness profile in the model nucleus shares the difficulties of PT03’s beyond 4 pc. We look forward to a more realistic analogy of the present simulation which, with careful stellar population and kinematic priming of both the preexisting disc and the accreted cluster, will hopefully resolve this.


document is not included the resulting rotation and velocity dispersion curves here, preferring to discuss such discs in the context of more realistic simulations with CR clusters decaying into the nucleus.

---

Figure 4. Photometry and kinematics of saturated configurations are probed, and favourable agreement with the kinematics of Andromeda’s nucleus is demonstrated. In the left-hand column, we show the results of photometric and kinematic observations of our fiducial simulation: a double peak is evident in the surface brightness distribution (top), with a brighter (P1-like) peak on the left of the SMBH at the origin, and a fainter (P2-like) peak on its right; the zero-point in the rotation velocity curve (centre) is shifted towards brighter peak and slower rotation; the peak velocity dispersion is displaced towards fainter peak and faster rotation (bottom); rotation velocity and velocity dispersion are shown without (dashed) and with (solid) contributions from the dispersed CR disc. These contributions make for a slower rotation velocity on the anti-P1 side of the nucleus, and introduce a significant flattening of and asymmetry in the wings of the velocity dispersion profile. The best-fitting bulge-subtracted (non-aligned) models of Peiris & Tremaine (2003) (labelled PT03) are also displayed as dot–dashed lines. Quantitative agreement, between a dynamically similar copy of the featured simulation (observed under HST conditions) (solid line) and M31’s bulge-subtracted kinematics (blue and red dots) (Bender et al. 2005), is evident in the right column’s centre and bottom panels. The doubling of stellar black hole nuclei

1 These expectations seem to be confirmed in preliminary experiments with an extended prograde disc destabilized by a ring-like CR distribution. We do not include the resulting rotation and velocity dispersion curves here, preferring to discuss such discs in the context of more realistic simulations with CR clusters decaying into the nucleus.

---

Appendix A). The outcome of this procedure is then overlaid with M31’s bulge-subtracted kinematics (Fig. 4, right-hand column). We closely match the observed shift in the zero-point of the rotation curve [$x = -0.485$ pc compared to $x = -0.448$ pc in observations (Bender et al. 2005)], the difference between the peaks in the rotation velocity ($\Delta v = 585$ km s$^{-1}$ compared to $\Delta v = 532 \pm 41$ km s$^{-1}$), and the overall magnitude and direction of the offset of the peak in $\sigma$ [$x = +0.187$ pc, $\sigma = +355.7$ km s$^{-1}$ compared to $x = 0.298$ pc, $\sigma = +373 \pm 48$ km s$^{-1}$). The triaxial cluster of our model endows its kinematics with the flattening of and marked asymmetry between the wings of the velocity dispersion profile, which is clearly observed in HST data (Kormendy & Bender 1999; Bender et al. 2005), and which has proven difficult to recover with eccentric discs alone (Peiris & Tremaine 2003, hereafter PT03) (compare PT03 and the dashed ‘no CR’ curve to the solid curve in the bottom-right panel of Fig. 4). Recall that this cluster consists of eccentric orbits which result from the triaxial dispersal of the initially retrograde cluster (rows b and d of Fig. 1). When superposed over the more massive lopsided disc, stars in this cluster will leave their kinematic signature mostly in the outer parts of the nucleus, which is where they linger the most on their eccentric orbits, and thus contribute to the flattening of the wings of the $\sigma$ profile. That signature is particularly pronounced where the lopsided prograde disc contributes least, i.e. on the anti-P1 side of the nucleus, hence the resulting asymmetry between the flattened wings. This said, the contribution of the triaxial cluster to the anti-P1 tail of the rotation velocity curve appears to worsen the agreement with HST data and PT03 results (middle-right panel of Fig. 4). At first sight, one may worry that a triaxial cluster will take away from rotation the benefits it contributes to dispersion. In fact, it is the ring-like structure of the massive prograde disc in our simulation which is mainly to blame for this feature. We expect that a more realistic model for that disc would significantly improve (and even fit) the tail of the rotation curve, while leaving the velocity dispersion profile largely unaffected. Indeed, an extended prograde disc will host a sizeable population of stars with semi-major axes beyond the current mean of 2.7 pc, each having an azimuthal velocity which, on average, remains larger than apsidal speeds of the eccentric triaxial cluster stars. Such a disc will contribute to a higher rotation velocity in the outer anti-P1 side of the nucleus, while still dominated by the velocity dispersion of the triaxial cluster in the same region. Incidentally, a more realistic prograde disc will also host an inner population of stars on near-circular low-angular-momentum orbits which are expected to remain largely unaffected by the CR instability; these stars will on average temper the periapse speed of eccentric orbits on the inner anti-P1 side of the nucleus, and hence help pull the rotation velocity down to values which are hopefully closer to observations. These additional stars will surely leave a kinematic signature on the P1 side of the nucleus which can be controlled in a global fit to kinematic observations involving disc and cluster parameters as well as the sky angles of the saturated nucleus.

A surface brightness profile of the model nucleus along a slit extending from brighter to fainter peak has maxima around 13.3 and 14.2 mag arcsec$^{-2}$, respectively. The brightness at the P1 peak compares favourably with the 13.2 mag arcsec$^{-2}$ observed by HST; on the other hand, and as expected in our ring-like prograde disc, the P2 peak in the model nucleus is significantly dimmer than the observed 13.57 mag arcsec$^{-2}$. Moving to the tail, PT03 noted that the surface brightness profile in their best-fitting model falls off too sharply to fit the bulge-subtracted photometry in the outer parts of the disc, and this precisely in the region where our model’s triaxial cluster makes its noteworthy impact on kinematics. With the current masses and initial radial structure of both the PP and RP, and for the adopted mass-to-light ratio, the brightness profile in the model nucleus shares the difficulties of PT03’s beyond 4 pc. We look forward to a more realistic analogue of the present simulation which, with careful stellar population and kinematic priming of both the preexisting disc and the accreted cluster, will hopefully resolve this.
photometric discrepancy as it maintains and improves the desirable features noted in the present proof of concept experiment.

Last but not least, we note that for the adopted similarity transformation\(^2\) the pattern speed of the lopsided mode scales down from the 19 km s\(^{-1}\) pc\(^{-1}\) of the fiducial nucleus (Fig. 2d) to the slower 4.3 km s\(^{-1}\) pc\(^{-1}\) of the model nucleus. At this relatively slow pattern speed, gravitational perturbations by the model nucleus are expected to drive and confine gas in a disc close to the SMBH (Chang et al. 2007). When fuelled by stellar mass-loss from the nucleus, such a disc can regenerate starbursts at a rate which is consistent with the observed compact cluster of early-type stars (P3) around M31’s SMBH (Bender et al. 2005; Chang et al. 2007). Thus, M31’s P3 could very well be a direct consequence of P1 and P2 in our dynamical model.

4 DISCUSSION

Andromeda’s kinematics shows a marked asymmetry between the tails of its velocity dispersion which is clearly unaccounted for in Keplerian disc models, and appears to be cleanly resolved by combining an eccentric disc with a triaxial cluster of highly eccentric orbits. To be sure, thorough modelling is called for before we can rigorously quantify the improvements of a disc–cluster combination over the thick eccentric disc model of PT03. Still, we find it remarkable that this combination achieves, with minimal probing, the level of qualitative and quantitative agreement described above, and to see it emerge as a natural end product of our process can only strengthen the case for CR stimuli of double nuclei (Lauer et al. 2005). That case is made stronger when one learns that a rich variety of CR configurations are equally prone to developing lopsidedness (details in Appendix B). Experiments with CR point masses suggest that a cluster, initially on a circular trajectory, can drive a coplanar disc unstable provided its orbital radius is less than a critical radius (which is roughly equal to twice the mean radius of the disc); clusters on highly inclined circular trajectories can still drive a CR disc into a lopsided state; on the other hand, clusters with too large an initial eccentricity will end up with little negative angular momentum to drive a coplanar disc lopsided. The CR perturbation envisaged in this work is delivered by a CR cluster which spirals deep into the nucleus by dynamical friction with bulge stars (Tremaine et al. 1975) before it disrupts in the combined tidal field of bulge and SMBH (Quillen & Hubbard 2003). The orbital radius at which the cluster disrupts depends critically on the cluster’s core density. For a cluster migrating on a near-circular trajectory into M31’s nucleus, we estimate (Quillen & Hubbard 2003) that a core density \(\geq 5 \times 10^5 M_\odot\) pc\(^{-3}\) is sufficient for the cluster to cross past the critical radius for instability. Mass segregation and/or intermediate-mass black hole formation (Kim & Morris 2003; Fujii et al. 2010) can amplify core density significantly, thus further delaying disruption till the cluster migrates into the overlapping configurations explored here. Such configurations may also arise when a cluster penetrates the nucleus on an eccentric orbit, and disrupts upon a close encounter with the SMBH. CR clusters will likely approach a preexisting nuclear disc on eccentric and inclined trajectories; they may thus find themselves on the eccentricity–inclusion cycles observed above, and then disperse as they drive the disc in a saturated lopsided configuration. Given on one hand the strong likelihood of CR excitations in galactic centres (Jog & Combes 2009), and on the other hand the robustness and efficiency of the proposed mechanism, double nuclei are likely to prove ubiquitous in stellar clusters dominated by SMBHs (Lauer et al. 2005). More generally (and irrespective of whether or not a given lopsided nucleus results from a CR instability), the proposed mechanism can be deployed to customize triaxial equilibrium configurations with which to model observed kinematics of stellar black hole nuclei (Alexander 2005) (with the Milky Way’s included) and improve estimates of the mass and feeding rate\(^3\) of the black hole within (Margerian et al. 1998; Kormendy 2004). Whether exploring realistic CR scenarios or tailoring triaxial equilibrium models, extensive numerical simulations of CR stellar systems with state-of-the-art solvers (Touma et al. 2009; Fujii et al. 2010) will surely contribute invaluable insights into the dynamics and structure of stellar black hole nuclei.

ACKNOWLEDGEMENTS

We thank referee Eric Emsellem for his careful reading of our manuscript and for insightful remarks which significantly improved the presentation of results pertaining to M31. MVK expresses his gratitude to the faculty, staff and students of the Department of Physics at the American University of Beirut (AUB), where the bulk of this work was conducted. Computations were performed on the Ibsina cluster at the Center of Advanced Mathematical Sciences (AUB), and on the Aurora cluster at the Institute of Advanced Studies (Princeton). Fruitful discussions with S. Sridhar, Scott Tremaine and Jarle Brinchmann are gratefully acknowledged. The support of NSF grants AST-0206038 and AST-0507401 was invaluable during all phases of this work.

REFERENCES

Alexander T., 2005, Phys. Rep., 419, 65
Bacon R., Emsellem E., Combes F., Copin Y., Monnet G., Martin P., 2001, A&A, 371, 409
Bender R. et al., 2005, ApJ, 631, 280
Chang P., Murray-Clay R., Chiang E., Quataert E., 2007, ApJ, 668, 236
Dressler A., Richstone D. O., 1988, ApJ, 324, 701
Emsellem E., Combes F., 1997, A&A, 323, 674
Fujii M., Iwasawa M., Funato Y., Makino J., 2010, ApJ, 716, L80
Gerhard O., 2001, ApJ, 546, L39
Gultekin K., Richstone D. O., Gebhardt K., Faber S. M., Lauer T. R., Bender R., Kormendy J., Pinkney J., 2011, ApJ, 741, 38
Hopkins P. F., Quataert E., 2010a, MNRAS, 407, 1529
Hopkins P. F., Quataert E., 2010b, MNRAS, 405, L41
Jacobs V., Sellwood J. A., 2001, ApJ, 555, L25
Jog C. J., Combes F., 2009, Phys. Rep., 471, 75
Kim S. S., Morris M., 2003, ApJ, 597, 312
Kormendy J., 1988, ApJ, 325, 128
Kormendy J., 2004, in Ho L. C., ed., Carnegie Observatories Astrophysics Series Vol. 1, Coevolution of Black Holes and Galaxies. Cambridge Univ. Press, Cambridge, p. 1
Kormendy J., Bender R., 1999, ApJ, 522, 772
Lauer T. R. et al., 1996, ApJ, 471, L79

\(^2\) In this transformation, time is rescaled by \(s^2\), whenever length is rescaled by \(s\). The adopted \(s \approx 2.7\) accounts for the slower precession rate of the model nucleus.

\(^3\) We note in passing that, as it drives a substantial fraction of stars to near-radial orbits, the CR instability populates the loss cone of the SMBH (Margerian & Tremaine 1999). Repeated CR accretion events will thus enhance the feeding rate of the central SMBH as they sculpt stellar distributions in its sphere of influence.
APPENDIX A: METHODS

We performed our simulations using the cosmological numerical tool GADGET-2 (Springel 2005). Forces were evaluated using the octree; cosmological, smoothed particle hydrodynamics and grid capabilities were switched off. Although this narrowed down the parameter space significantly, one had to be careful while fine tuning parameters which control tree walk and time stepping. The depth of the tree to be explored is controlled by the GADGET-2 parameter ErrTolForceAcc; daughter cells are opened until a certain criterion, set by ErrTolForceAcc, is satisfied, thus terminating the tree walk (we refer the reader to the user guide of GADGET-2 for more details about this criterion). Once the accelerations of all the particles are at our disposal, the positions of these particles are advanced with a time-step of \( \Delta t = \sqrt{2\hbar/|\alpha|} \), where \( |\alpha| \), \( \eta \) and \( \epsilon \) are the magnitude of the computed acceleration, ErrTolIntAccuracy (which controls the step size) and the gravitational softening, respectively. Setting ErrTolForceAcc to 0.001 and ErrTolIntAccuracy to 0.001 assured accurate enough simulations for our purposes.

Our featured simulations consist of a massive prograde disc, on which a light retrograde disc is overlaid, with both residing in the sphere of influence of a central SMBH. Such a configuration is a likely outcome of the disruption of clusters that can migrate deep in the galactic centres (Gerhard 2001; Kim & Morris 2003; Fujii et al. 2010). Initial positions and velocities of disc particles are picked from radial and vertical distributions which are standard to disc dynamics simulations (Emsellem & Combes 1997). Since GADGET-2 has difficulties in dealing with high mass density contrast, we further chose our disc to be annular in order to avoid close encounters with the SMBH. Thus, we introduced an inner and outer radial cutoff when sampling the discs. The SMBH is modelled as a Plummer-softerned particle with a softening length of \( b = 0.01 \) pc. To minimize sources of instability which are not related to counter-rotation, disc components were separately virialized before coupling them to each other or to perturbers. In the course of virializing, the more massive discs (prograde in our simulations) underwent short-lived gravitational instabilities, before relaxing to a hotter near-axisymmetric equilibrium state.

The central SMBH with a mass of \( 10^6 M_\odot \) is 10 times more massive than the preexisting disc of stars, itself 10 times more massive than the disturbing CR disc. The mean eccentricities of the virialized prograde and retrograde disc were \( \langle e_p \rangle = 0.22 \) and \( \langle e_r \rangle = 0.25 \), respectively, with corresponding standard deviations \( \sigma_p = 0.12 \) and \( \sigma_r = 0.14 \). In both discs, the mean of the semi-major axis distribution was \( \langle a \rangle \sim 1.06 \) pc with a standard deviation of \( \sim 0.3 \) pc whereas the spread in inclination was around \( 8^\circ \).

Simulation particles revolve around the central SMBH on perturbed Keplerian ellipses; associated osculating orbital elements [namely semi-major axis (\( a \)), eccentricity (\( e \)), inclination (\( I \)), argument of periapse (\( g \)) and longitude of the node (\( h \))] are simply recovered from particle position and velocity. In addition to particle orbit elements, our dynamical analysis required the time evolving Fourier modes of the surface (projected) density of the CR cluster, and associated potentials. The surface density \( \Sigma(r, \theta, \varpi) \) is Fourier expanded in \( \theta \):

\[
\Sigma(r, \theta, \varpi) = \sum_{m=\infty}^{m=0} a_m(r, \theta) \cos(m\theta + \phi_m(r, \varpi))
\]

(A1)

with \( [a_m(r, \theta), \phi_m(r, \varpi)] \) the \( m \)th mode amplitude and phase, respectively. This is practically performed with Fast Fourier Transforms (FFTs) over a grid in \( \theta \) at various judiciously chosen radii. The relative power \( K_m \) in a given mode \( m \) is then computed as

\[
K_m(t) \equiv \frac{\sum_{m=0}^{m=\infty} \int_0^\infty \int_0^\infty a_m^2(r, \varpi) 2\pi r dr}{\sum_{m=0}^{m=\infty} \int_0^\infty \int_0^\infty a_m^2(r, \varpi) 2\pi r dr}.
\]

(A2)

Cluster kinematics were determined from a snapshot of our benchmark 3D simulation at \( T = 4 \) Myr (well into the saturated regime). The scalelength of that simulation is of the order of 1 pc, whereas that of M31’s nucleus is more like 3 pc. To compare with M31 observations, we rescaled positions (and with them the softening length) by a factor \( s \), and time by a factor \( s^2 \); velocities are naturally rescaled by \( s^{-1/2} \) to leave equations of motion in the new variables invariant. The adopted scaling (\( s \approx 2.7 \)) was first constrained within the interval \( 2.0 < s < 3.1 \), by matching peaks in the rotation curve, and then fine tuned to fit the kinematics, while adjusting the sky angles as well.

While searching for the optimal sky angles (Peiris & Tremaine 2003) of our rescaled snapshot, we took into account artefacts of the HST STIS (Space Telescope Imaging Spectrograph) instrument by convolving all the particles with a double Gaussian (Bender et al. 2005). The slit width was set to 0.1 arcsec with a position angle (PA) of 39\(^\circ\). Both the fiducial and rescaled snapshots were observed with the same sky angles \( \theta_p = -59.1^\circ , \theta_p = 65.3^\circ \). We note that larger \( \theta_p \) permits an almost perfect match with the observed line of sight velocity dispersion (remarkably so for \( \theta_p \) close to M31’s inclination of 77\(^\circ\)), with a deteriorating rotation curve, whereas smaller \( \theta_p \) improved the fit to the rotation curve dramatically while disturbing agreement in the line of sight velocity dispersion.

APPENDIX B: THE CR INSTABILITY EXPLORED

Here, we summarize the results of a battery of experiments conducted with CR configurations that are likely to occur in galactic centres.

First, we explore dynamics in highly inclined configurations, by considering an extreme situation: a lower resolution (50 \( \times \) 10\(^3\) particles) analogue of the 3D benchmark simulation was evolved with equal mass annuli (\( M_p = 0.05 \) \( M_\star \), \( M_p \) are the masses of the retrograde and prograde discs, respectively) where...
the PP and RP discs have a relative inclination of 90°. This experiment was motivated by our concern about non-coplanar CR rings in galactic centres, and whether such configuration may result in \( m = 1 \) instabilities. The configuration that was violently unstable resulted in both components merging, and then relaxing into a thick, precessing, and lopsided disc.

Secondly, we explored the effect of an in-spiralling cluster on our annular disc (Fujii et al. 2010). Evolving an actual realization of a Plummer sphere with \textsc{gadget}-2 around the SMBH with enough accuracy is prohibitively expensive; thus, the cluster was modelled as a point mass with a large softening radius of 0.1 pc. The point mass was put on a retrograde Keplerian orbit around the SMBH. The massive disc of our 3D simulation was used as the prograde component \( (M_p = 0.1 M_\star, \langle a \rangle = 0.92 \text{ pc}) \), but with \( 25 \times 10^3 \) particles instead of \( 250 \times 10^3 \). The default values of the point mass (cluster) orbit are \( M_c = 0.01 M_\star \) (\( M_c \) is the mass of the cluster), semi-major axis \( a_c = 0.9 \text{ pc} \), eccentricity \( e_c = 0 \) and inclination \( I_c = 180^\circ \). The simulations for this scenario consisted of four suites. In each suite, one parameter was varied while keeping the remaining parameters to their default value.

(i) \textit{Suite 1}. \( M_c \) was varied from 0.001 to 0.1 \( M_\star \). The prograde disc developed an \( m = 1 \) mode for \( M_c > 0.01 M_\star \).

(ii) \textit{Suite 2}. The inclination of cluster was varied from 90° to 180°. In all cases the cluster and the disc ended up in the same plane, and with an \( m = 1 \) growing instability mode when the relative inclination was larger than 90°.

(iii) \textit{Suite 3}. The eccentricity was varied from 0 to 0.9. The low-eccentricity \( m = 1 \) growing instability shuts off for \( e_c > 0.6 \).

(iv) \textit{Suite 4}. The semi-major axis was varied from 0.8 to 10 pc. For \( a_c > 2\langle a \rangle \) the annulus remained axisymmetric, with an \( m = 1 \) mode growing for \( a_c < 2\langle a \rangle \).