The Higgs mechanism from an extra dimension

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Abstract

The standard $SU(2) \times U(1)$ fields are considered in 4D plus one extra compact dimension. As a result two basic effects are obtained. First, four Goldstone-like scalars are produced, three of them are used to create longitudinal modes of the $W, Z$ fields, while the fourth becomes the Higgs-like scalar. Second, $W$ and $Z$ get their masses from the extra compact dimension with the standard pattern of symmetry violation. The resulting theory has the same fields as in the standard model, but without the Higgs vacuum average. The properties of the new Higgs scalar and its interaction with fermions are briefly discussed.

1 Introduction

The possible role of extra dimensions in the physics of our world was suggested almost a century ago [1] and is now a topic of a wide interest [2, 3, 4], see also [5, 6] for recent reviews. Possible extensions of the Standard Model in general and of the Higgs phenomenon in particular in 5D are now intensively discussed, often in connection with gravitation [6] (for a recent study see [7]).

It is a purpose of the present paper to investigate the explicit result of the inclusion of an extra compact dimension for the Standard Model Lagrangian without Higgs field. In particular, we shall see, that new degrees of freedom
due to an extra dimension, $A_{5}$, $B_{5}$, play the same role, as the Goldstone-like modes in the Higgs mechanism, producing longitudinal modes of $W$, $Z$. Moreover, one extra mode can get massive and play the role of the standard Higgs boson. All this can happen without introduction of the scalar Higgs field, and its vacuum average, and $W$ and $Z$ get their masses due to the compact extra dimension as in the Kaluza-Klein (KK) modes.

In this way one obtains seemingly the same results as in the Standard Model, but without the vacuum average of the scalar field. However, some features differ, e.g. the interaction of the new scalar field with fermions, which is proportional to fermion masses in the Standard Model, is now universal.

The plan of the paper is as follows. In next section the case of the $U(1)$ fields will be considered, while the section 3 is devoted to the standard $SU(2) \times U(1)$ case. In section 4 the inclusion of the scalar mode, its properties and interaction with fermions are discussed, The section 5 contains conclusions and prospectives.

## 2 The $U(1)$ Higgs mechanism from 5D

We consider below the vector fields $A_{\mu}$ which obtain masses due to an extra (fifth) dimension in the case of the local symmetry $U(1)$, in full analogy with the corresponding $U(1)$ Higgs mechanism [8]. We start with the $D = 5$ Lagrangian, containing the gauge fixing term $\mu, \nu = 0, 1, 2, 3, x_{5} \equiv y$,

$$L_{5} = -\frac{1}{4} F_{\mu\nu}^{2} - \frac{1}{4} F_{5\mu}^{2} - \frac{1}{2\xi} (\partial_{\mu} A_{\mu})^{2},$$  \hspace{1cm} (1)

where

$$F_{\mu5} = -F_{5\mu} = \partial_{\mu} A_{5} - \partial_{5} A_{\mu}, \quad F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}. \hspace{1cm} (2)$$

In what follows we shall use the 5D $U(1)$ extension of the Lagrangian [1], where we introduce

$$A_{\nu}(x, y) = \exp(\imath m_{5} y) A_{\nu}(x), \quad A_{5} = \imath \varphi_{5}(x) \exp(\imath m_{5} y) \hspace{1cm} (3)$$

and

$$\tilde{L}_{5} = -\frac{1}{4} |F_{\mu\nu}|^{2} - \frac{1}{2} |F_{5\mu}|^{2} - \frac{1}{2\xi} |\partial_{\mu} A_{\mu}|^{2}. \hspace{1cm} (4)$$

One can see that [1] acquires the form

$$\tilde{L}_{5} = \frac{1}{2} (\partial_{\mu} \varphi_{5}(x) - m_{5} A_{\mu}(x))^{2} - \frac{1}{4} F_{\mu\nu}^{2}(x) - \frac{1}{2\xi} (\partial_{\mu} A_{\mu})^{2}. \hspace{1cm} (5)$$


This form should be compared with the \( U(1) \) Higgs Lagrangian

\[
L_{\text{Higgs}} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2\xi} (\partial_\mu A_\mu)^2 + |\partial_\mu - ieA_\mu|\phi|^2 - \frac{\lambda^2}{2} (\phi^* \phi - \eta^2)^2, \tag{6}
\]

where the field \( \phi \) can be written in the standard way

\[
\phi = \frac{1}{\sqrt{2}} (\eta + \rho + i\varphi_H(x)) \tag{7}
\]

and the Goldstone field \( \varphi_H(x) \) enters in the combination

\[
L_{\text{Higgs}} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2\xi} (\partial_\mu A_\mu)^2 + \frac{1}{2} (\partial_\mu \varphi_H - e\eta A_\mu)^2 + \Delta L_{\text{Higgs}}, \tag{8}
\]

\[
\Delta L_{\text{Higgs}} = \frac{1}{2} (\partial_\mu \rho)^2 - \frac{\lambda^2 \eta^2 \rho^2}{2} + eA_\mu \varphi_H \partial_\mu \rho + \frac{1}{2} e^2 A_\mu^2 (\varphi_H^2 + \rho^2) +
\]

\[
e^2 \eta \rho A_\mu^2 - e\rho A_\mu \partial_\mu \varphi_H - \frac{\lambda^2}{2} \left( \eta \rho (\rho^2 + \varphi_H^2) + \frac{(\rho^2 + \varphi_H^2)^2}{4} \right). \tag{9}
\]

One can see, that the first three terms in (8) coincide with \( \tilde{L}_5 \), when we identify the Goldstone fields \( \varphi_5 = \varphi_H \), and put \( m_5 = e\eta \), while \( \Delta L_{\text{Higgs}} \) contains extra terms, describing interaction of the Higgs meson field \( \rho \) and the Goldstone field \( \varphi_H \), and \( A_\mu \), which are not important in our further discussion. One can now calculate the propagator of the field \( A_\mu \), using \( \tilde{L}_5 \), and as in the case of the Higgs Lagrangian (6) in the zeroth approximation \( e = 0 \), one obtains in the Landau gauge, \( \xi = 0 \),

\[
G_{\mu\nu}^{(0)} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \quad G_{\varphi}^{(0)} = \frac{1}{k^2}. \tag{10}
\]

In the \( e^2 \) order, writing

\[
G_{\mu\nu} = G_{\mu\nu}^{(0)} + G_{\mu\nu}^{(0)} \prod_{\rho\sigma} G_{\sigma\nu}^{(0)} + \ldots \tag{11}
\]

with \( \Pi_{\mu\nu} = e^2 \eta^2 \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \), one has

\[
G_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2 - e^2 \eta^2}, \quad e^2 \eta^2 = m_5^2. \tag{12}
\]

In this way one obtains the Higgs mechanism in the \( U(1) \) case via the fifth dimension dependence, namely, \( A_5(x, y) = i\varphi(x) \exp(i m_5 y) \), \( A_\mu(x, y) = A_\mu(x) \exp(i m_5 y) \), but without the Higgs meson terms, i.e. the Higgs mechanism without the Higgs vacuum average.
3 The $SU(2) \times U(1)$ Higgs mechanism from an extra dimension

We now turn to the case of the $SU(2)_L \times U(1)$ symmetry, with the fields $\hat{A}_\mu = A_\mu t_i$, $t_i = \frac{1}{2}\sigma_i$, and $B_\mu$, so that

$$F^A_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - ig[\hat{A}_\mu, \hat{A}_\nu],$$

(13)

and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. It is convenient to unify both fields $\hat{A}_\mu$ and $\frac{1}{2}B_\mu$ in one $(2 \times 2)$ matrix $\hat{C}_\mu \equiv \hat{A}_\mu + \frac{1}{2}B_\mu$, where $\hat{1}$ is the unit $(2 \times 2)$ matrix, and write

$$-\frac{1}{2}tr(\hat{F}^A_{\mu\nu}\hat{F}^A_{\mu\nu}) - \frac{1}{2}tr\hat{B}_{\mu\nu}\hat{B}_{\mu\nu} = -\frac{1}{2}tr\hat{C}_{\mu\nu}\hat{C}_{\mu\nu}. $$

(14)

For the mass generation process we shall specifically need the terms

$$-\frac{1}{2}tr\hat{C}_{\mu5}\hat{C}^{\mu5} - \frac{1}{2}tr\hat{C}_{5\mu}\hat{C}^{5\mu} \equiv \mathcal{L}_m.$$  

(15)

For $\hat{C}_{\mu5}$ one can write

$$\hat{C}_{\mu5} = (\partial_\mu A_{5i} - \partial_5 A_{\mu i} + ge_{kl}A_{\mu k}A_{5l})t_i + \frac{1}{2}(\partial_\mu B_5 - \partial_5 B_\mu).$$

(16)

As before in (3), but now for the $SU(2)$ group, we define for $i = 1, 2$

$$A_{5i} = i\varphi_i(x)I(y); A_{53} = i\varphi_3(x)I_B(y),$$

(17)

$$\partial_5 A_{\mu i}(x,y) = im_A A_{\mu i}(x)I(y) \ (i = 1, 2),$$

$$I(y) = \exp(im_A y) = \exp(i\pi n\xi), \quad \xi = \frac{y}{y_+(m_A)}.$$  

(18)

Similarly to (17) one can write

$$B_5 = i\varphi_5(x)I_B(y), \quad I_B(y) = \exp(im_B y) = \exp(i\pi n\xi), \quad \xi = \frac{y}{y_+(m_B)}.$$  

(19)

The mixing of $A_{\mu3}$ and $B_\mu$ is defined in the standard way, introducing fields $Z_\mu$ and electromagnetic $\Gamma_\mu$,

$$A_{\mu3} = \cos \theta \ Z_\mu + \sin \theta \ \Gamma_\mu, \quad B_\mu = \cos \theta \ \Gamma_\mu - \sin \theta \ Z_\mu.$$  

(20)
We now assume that only $Z_\mu$ has the $y$ dependence,

$$\partial_5 A_\mu^3 = i \cos \theta \ m_B Z_\mu I_B; \quad \partial_5 B_\mu = -i \sin \theta \ m_B Z_\mu I_B.$$  

(21)

As a result, $\mathcal{L}_m$, Eq. (15) can be written as

$$\mathcal{L}_m = \frac{1}{2} \sum_{i=1,2} (\partial_\mu \varphi_i - m_A A_\mu^i)^2 + \frac{1}{2} (\partial_\mu \varphi_5 + \sin \theta m_B Z_\mu)^2 +$$

$$+ \frac{1}{2} (\partial_\mu \varphi_3 - \cos \theta m_B Z_\mu)^2 + \Delta \mathcal{L}_m \equiv \mathcal{L}_m^{(0)} + \Delta \mathcal{L}_m$$

(22)

where $\Delta \mathcal{L}_m$ is

$$\Delta \mathcal{L}_m = \frac{1}{2} \sum_{i=1,2,3} g^2 A_{\mu i} \varphi_k \epsilon_{lki} A_{\mu n} \varphi_m \epsilon_{nmi} +$$

$$+ g \sum_{i=1,2} (\partial_\mu \varphi_i - m_A A_\mu^i) A_{\mu l} \varphi_k \epsilon_{lki} \frac{I(y) + I^+(y)}{2} +$$

$$+ g (\partial_\mu \varphi_3 - \cos \theta m_B A_\mu^3) A_{\mu l} \varphi_k \epsilon_{lki} \frac{I_B(y)(I^*(y))^2 + I_B^+(y)I^2(y)}{2}.$$  

(23)

Note, that introducing $I_B(y) = I(y) = \exp(i n \pi \xi)$ and integrating over $d\xi$ in the cyclic interval $-1 \leq \xi \leq 1$, one has only the first term in (23) left nonvanishing.

Now we redefine the Goldstone modes $\varphi_3, \varphi_5$, namely,

$$\varphi_3 = \cos \theta \varphi_Z, \quad \varphi_5 = -\sin \theta \varphi_Z$$

(24)

and $\mathcal{L}_m^{(0)}$ acquires the final form

$$\mathcal{L}_m^{(0)} = \frac{1}{2} \sum_{i=1,2} (\partial_\mu \varphi_i - m_A A_\mu^i)^2 + \frac{1}{2} (\partial_\mu \varphi_Z - m_B Z_\mu)^2.$$  

(25)

Following the same procedure, as in (10)-(12), one arrives at the transverse form of the $W, Z$ propagator,

$$G^{(W)}_{\mu \nu} = \frac{g_{\mu \nu} - \frac{k_\mu k_\nu}{k^2}}{k^2 - m_W^2}, \quad G^{(Z)}_{\mu \nu} = \frac{g_{\mu \nu} - \frac{k_\mu k_\nu}{k^2}}{k^2 - m_Z^2}.$$  

(26)

where

$$m_W^2 = m_A^2 = m_Z^2 \cos^2 \theta, \quad m_Z^2 = m_B^2.$$  

(27)

In this way all Goldstone modes $\varphi_1, \varphi_2, \varphi_z$ are eaten by the now massive fields $A_{\mu i}, i = 1, 2$ and $Z_\mu$. The fields $\varphi_i (i = 1, 2)$ and $\varphi_Z$ can be removed by the gauge transformations of the vector fields and no Higgs-like mesons are left in the Lagrangian.
4 New scalar mode from extra dimension

However, we have lost one mode, since we have replaced \( \varphi_3, \varphi_5 \) by one mode \( \varphi_Z \) in (24). In a more general case one can write

\[
\varphi_3 = \varphi_Z \cos \theta + \varphi_H \sin \theta, \quad \varphi_5 = - \sin \theta \varphi_Z + \varphi_H \cos \theta. \tag{28}
\]

Substituting (28) into (22), one obtains, instead of (25), the following result

\[
\mathcal{L}'_{m(0)} = \frac{1}{2} \sum_{i=1,2} (\partial_\mu \varphi_i - m_A A_{\mu i})^2 + \frac{1}{2} (\partial_\mu \varphi_Z - m_B Z_\mu)^2 + \frac{1}{2} (\partial_\mu \varphi_H)^2. \tag{29}
\]

Now one can see that again the fields \( \varphi_i, i = 1, 2 \) and \( \varphi_Z \) can be absorbed by the fields \( A_{\mu i} \) and \( Z_\mu \), generating longitudinal components, but \( \varphi_H \) appears as a valid physical mode. Neglecting now the absorbed modes \( \varphi_1, \varphi_2, \varphi_Z \) in \( \Delta \mathcal{L}_m \), one obtains in (23)

\[
\Delta \mathcal{L}'_m = \frac{1}{2} g^2 \sin^2 \theta \varphi_H^2 ((A_{\mu 1})^2 + (A_{\mu 2})^2). \tag{30}
\]

It is clear that the field \( \varphi_H \), having no intrinsic mass term, in contrast to the Higgs field, acquires the mass through radiative corrections, e.g. from the quadratically divergent \( W \) loops in (30), and the higher order diagrams, as well as from the interaction of \( \varphi_H \) with fermions.

For the following we need some modifications of the original expressions for the field strength and long derivatives, to include the factors coming from the fifth coordinate. Namely, as in (18), (19) we define the 5D extensions of the fields \( A_{\mu i}(x) \) and \( B_\mu(x) \).

\[
A_{\mu i}(x) \rightarrow \hat{A}_{\mu i}(x, y) = A_{\mu i}(x) I_i(y), \tag{31}
\]

\[
B_\mu(x) \rightarrow \hat{B}_\mu(x, y) = B_\mu(x) I_B(y), \tag{32}
\]

while \( A_{5i}(x, y) \) and \( B_5(x, y) \) are given in (16), (19)

Now the usual long derivative can be rewritten in terms of fields \( A_{\mu}(x, y) \), \( B_{\mu}(x, y) \) as

\[
\hat{D}_\mu = \partial_\mu - ig_i A_{\mu i}(x, y)t_i - ig' B_{\mu}(x, y) Y \frac{1}{2}, \tag{33}
\]

where

\[
\hat{g}_i = g I^i(y) (i = 1, 2), \quad \hat{g}' = g' I_B^Y(y), \quad \hat{g}_3 = I_B(y)(I^*)^2; \tag{34}
\]

and
\[ \hat{F}^A_{\mu\nu}(x, y) = \partial_\mu \hat{A}_\nu(x, y) - \partial_\nu \hat{A}_\mu(x, y) - ig[\hat{A}_\mu(x, y), \hat{A}_\nu(x, y)]. \]  

(35)

Note, that expressing \( I(y) = I_B(y) = I(\xi) = \exp(in\pi\xi) \), all extra factors in \( g, g' \) reduce to one factor \( I^*(\xi) \).

As a result, the total Lagrangian of vector, scalar and fermion fields can be written as follows:

\[ L = L_A + L_B + L_m + L_f, \]

(36)

\[ L_A + L_B = -\frac{1}{2} \text{tr} \hat{F}^A_{\mu\nu} \hat{F}^{A+}_{\mu\nu} - \frac{1}{2} \text{tr} \hat{B}_{\mu\nu} \hat{B}^{+\mu\nu}, \]

(37)

\[ L_m = \frac{1}{2} \sum_{i=1,2} (\partial_\mu \phi_i - m_A \hat{A}_{\mu i}(x, y))(\partial_\mu \phi^+_i - m_A \hat{A}^+_{\mu i}(x, y)) + \]

\[ + \frac{1}{2}(\partial_\mu \varphi_Z(x, y) - m_B Z_\mu(x, y))(\partial_\mu \varphi^*_Z(x, y) - m_B Z^*_\mu(x, y)) + \frac{1}{2}(\partial_\mu \phi_H - \partial_\mu \phi^*_H) + \Delta L'_m, \]

(38)

and \( \Delta L'_m \) is given in (30). With the notation

\[ L_A + L_B = L_{\text{neutr}} + L_{\text{ch}} + L_{\text{interf}}, \]

(39)

and using (20), the contribution of neutral fields \( B_\mu, A_{\mu 3} \) in (37) can be written as

\[ L_{\text{neutr}} = -\left\{ \frac{1}{2}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + \frac{1}{2}(\partial_\mu A_{\nu 3} - \partial_\nu A_{\mu 3})^2 \right\} = \]

\[ = -\left\{ \frac{1}{2}(\partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu)^2 + \frac{1}{2}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2 \right\}. \]

(40)

For the charged fields and their interference with neutrals one has

\[ L_{\text{ch}} = -\left\{ \frac{1}{2} \sum_{i=1,2} (\partial_\mu A_{\nu i} - \partial_\nu A_{\mu i})^2 + (g A_{\mu k} A_{\nu l} e_{3kl})^2 \right\} \]

(41)

and finally \( L_{\text{interf}} \) is

\[ L_{\text{interf}} = -\left\{ [\cos \theta(\partial_\mu Z_\nu - \partial_\nu Z_\mu) + 2 \sin \theta(\partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu)]g e_{kl3} A_{\mu k} A_{\nu l} + \right\}. \]

7
\[\sum_{i=1,2} (\partial_{\mu} A_{\nu i} - \partial_{\nu} A_{\mu i}) e_{ki} g A_{\mu k} A_{li} + g^2 (\cos \theta Z_{\mu} + \sin \theta \Gamma_{\mu}) (\cos \theta Z_{\nu} + \sin \theta \Gamma_{\nu}) \times \]
\[\times (\delta_{\mu \nu} (A_{\lambda 1} A_{\lambda 1} + A_{\lambda 2} A_{\lambda 2}) - (A_{\mu 1} A_{\nu 1} + A_{\mu 2} A_{\nu 2})) \}.
\]

One can see that (40), (41), (42) coincide with the vector field part of the standard Lagrangian.

In the fermion part of the Lagrangian, \(L_f\) in addition to the standard expression, one can write for the quarks

\[L_Q = \bar{Q} L i \gamma_{\mu} \left( \partial_{\mu} - ig A_{\mu i} t_i - i g' \frac{1}{6} B_{\mu} \right) Q_L,\]  
(43)

the term, containing the fifth components,

\[\Delta L_Q = i \bar{q}_{L,R} \left( \partial_5 - ig A_{53} t_3 - i g' \frac{1}{6} B_5 \right) q_{R,L},\]  
(44)

where

\[A_{53} = i (\varphi_Z \cos \theta + \varphi_H \sin \theta) I_B(y)\]  
(45)

\[B_5 = i (\varphi_H \cos \theta - \varphi_Z \sin \theta) I_B(y).\]  
(46)

Inserting in (43) the terms (45), (46) for \(\mu = 5\) and neglecting \(\varphi_Z\), one obtains the following interaction of the field \(\varphi_H\) with \(f \bar{f}\)

\[\Delta L_{Hf \bar{f}} = \bar{f}_{L,R} \varphi_H \left( g \sin \theta t_3 + g' \cos \theta \frac{\hat{Y}_f}{2} \right) f_{R,L},\]  
(47)

violating the \(SU(2)\) invariance in (47), which, however, is already violated by different up and down quark masses. In (47) \(g = \bar{g} \cos \theta, \ g' = \bar{g} \sin \theta; \ \bar{g} = 0.74\).

Note the difference between (47) and the standard Higgs – fermion interaction \(\Delta L_{\text{stand}} = \frac{m_f}{\eta} \bar{f} f H\), where \(m_f\) is the fermion mass and the Higgs condensate \(\eta = 246\) GeV. Thus for the \((\bar{t} t)\) quarks the coupling coefficient is 0.2, while in \(\Delta L_{\text{stand}}\) it is 0.71. At the same time for lighter quarks our result holds the same, while the standard coefficient is negligible for all \(m_f\) except \(m_t\).

One can see that on the r.h.s. of (44) the first term creates the intrinsic mass of the quarks,

\[i \partial_5 q(x, y) = m_q q(x, y),\]  
(48)
and the relation (48) automatically violates the $SU(2)$ symmetry, if $i\hat{\partial}_5$ is not $SU(2)$ doublet. As to the quark mass operator $i\hat{\partial}_5$, it can be written in the $SU(2)$ form as $\begin{pmatrix} \cos \phi & i\partial_5 \\ \sin \phi & i\partial_5 \end{pmatrix}$, with $\phi$ different, being applied to $\begin{pmatrix} \text{up} \\ \text{down} \end{pmatrix}$ or $\begin{pmatrix} \text{down} \\ \text{up} \end{pmatrix}^+$ states, as it is done in the Higgs condensate formalism, however, not lifting the mass difference problem.

Now turning to the fermion term $L_f$, one should take into account that fermions have their own factors $I_f(y)$, which can differ from the factors $I(\xi)$ entering the vector fields $W_\mu, Z_\mu$. As a result the corresponding term $L^{(0)}_f(x, y)$ simply coincides with the standard 4D expressions, since the factors $I_f(y)$ cancel in the diagonal product of fields, and $I(\xi)$ cancels in the products $\hat{g}\hat{A}, \hat{g}'\hat{B}$:

$$L^{(0)}_f = i\bar{f}_k \left( \partial_\mu - i\hat{g}\hat{A}_{\mu}(x, y)t_i - i\hat{g}'\hat{B}_{\mu}(x, y)\frac{Y_f}{2} \right) \gamma_\mu f_k(x, y), \ k = L, R. \ (49)$$

However, in the charged fields case, $W_\mu^{(\pm)}$, the fermions and antifermions may belong to different generations. This results in different factors $I_k(\xi)$ and $I_k(\xi)$ of the fermions $f_k$ and $\bar{f}_k$, and one can see that with our present definition of the norm in the $\xi$ space these matrix elements vanish after integration over $d\xi$. Thus in this approximation the mixing between generations does not appear and the CKM matrix is diagonal.

5 Conclusion and prospectives

We have shown that the extension of the standard higgsless Lagrangian, including new components of the vector fields $F_{\mu\nu}, F_{5\mu}$, as well as the fifth coordinate $x_5 = y$ dependence of all fields, $F_{\mu\nu}(x) \rightarrow F_{\mu\nu}(x, y) q(x) \rightarrow q(x, y)$, automatically produces Nambu-Goldstone modes, which create longitudinal components of the vector fields, while present in $F_{\mu5}$ derivatives in $y$ produce the masses of $W, Z$ in the needed proportion to reconstruct symmetries of the Lagrangian.

Surprisingly there appears the fourth scalar mode, which acquires the mass radiatively via interaction with vectors and fermions. This scalar can contest (or complement) the standard Higgs boson. However, it is different
from the latter in the interaction with fermions, which is not proportional to
the fermion masses.

The results of the paper open many questions and require additional steps
to clarify the details of the 5D construction. In particular, the aspects of
the fermion and (possibly) vector generations and mixings were not touched
upon.

Many aspects of dynamics in the extra dimension were not discussed
above, and only its circular compactification was exploited for simplicity,
which assumes possible KK excitations. This topic is crucial for the quarks
and will be treated in a separate paper [9].

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