A Self-Stabilizing Phase Decoder for Quantum Key Distribution

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Abstract: Self-stabilization quantum key distribution (QKD) systems are often based on the Faraday magneto-optic effect such as “plug and play” QKD systems and Faraday–Michelson QKD systems. In this article, we propose a new anti-quantum-channel disturbance decoder for QKD without magneto-optic devices, which can be a benefit for the photonic integration and applications in magnetic environments. The decoder is based on a quarter-wave plate reflector–Michelson (Q–M) interferometer, with which the QKD system can be free of polarization disturbance caused by quantum channel and optical devices in the system. The theoretical analysis indicates that the Q–M interferometer is immune to polarization-induced signal fading, where the operator of the Q–M interferometer corresponding to Pauli Matrix $\sigma_2$ makes it satisfy the anti-disturbance condition naturally. A Q–M interferometer based time-bin phase encoding QKD setup is demonstrated, and the experimental results show that the QKD setup works stably with a low quantum bit error rate about 1.3% for 10 h over 60.6 km standard telecommunication optical fiber.

Keywords: quantum key distribution; anti-quantum-channel disturbance; quarter-wave plate reflector

1. Introduction

Quantum key distribution (QKD) [1] allows two authenticated distant participants Alice and Bob to securely share a long random string often called cryptographic keys even in the presence of an eavesdropper Eve. The keys can be used to carry out perfectly secure communication via one-time-pad and perfectly secure authentication via Wigman–Carter authentication scheme [2]. The first and the best-known QKD protocol is BB84 proposed by C.H. Bennett and G. Brassard in 1984 [3]. Since then, QKD technologies have made significant progresses. The unconditional security of QKD has been proved through a series of outstanding works [4–6]. Practical security of QKD has also been fully studied, such as the decoy state method [7–9] for beating the photon-number-splitting attack, and the measurement-device-independent QKD for removing detector side channel attacks [10]. Up to now, the transmission distance reaches 421 km in optical fiber [11], and 1200 km in free space from the Micius satellite to the Xinglong ground station [12]. Several QKD network testbeds have been built and demonstrated in metro areas [13–16]. Scientists hope to build a global quantum network through quantum satellites connecting terrestrial quantum networks over commercial optical fiber in the future.

To build terrestrial QKD networks over commercial optical fiber, the stability of QKD systems or exactly anti-quantum-channel disturbance is especially crucial and has received extensive attention from both scientific researchers and engineers. Polarization encoding QKD systems rely on complicated feedback compensation because polarization states of photons are randomly disturbed in optical fiber quantum channels due to environmental vibration and/or temperature variation, and it is not suitable for strong environmental disturbance. Owing to the fact that phase information encoded in quantum
states can be maintained in environmental disturbance, phase encoding or time-bin phase encoding QKD systems are more competitive in such situations as overhead and tube optical cables along roads or bridges. However, decoding the phase information with unbalanced-arm Mach–Zehnder or Michelson interferometers also suffers from polarization disturbance in transmission fiber that results in the fringe visibility of the interferometers varying fast [17]. To solve the problem, Muller et al. proposed the “plug and play” bidirectional QKD system which can automatically compensate the polarization disturbance besides the phase drifting in the channel [18], and has been applied in QKD products by ID Quantique, Inc., a well-known quantum company (located in Geneva, Switzerland). Moreover, Mo et al. proposed the Faraday–Michelson (F–M) unidirectional QKD system [19], which has been applied in the phase encoding QKD products by Anhui Asky Quantum Technology Co., Ltd. (Anhui, China), and designed in the latest time-bin phase QKD scheme in a patent proposed by the University of Science and Technology of China [20]. According to Ref. [17], for phase encoding QKD systems, the disturbances of quantum channel are collected in the system if there is polarization-induced fading at the receiver’s interferometer. Thus, it is crucial to construct an unbalanced-arm interferometer that can self-compensate quantum channel disturbance. Up to now, the effective and widely used solutions are mainly based on the two schemes mentioned above or their variants. Both of the schemes are based on the Faraday magneto-optic devices, which is not conducive to photonic integration and applications in magnetic environments.

In this article, we propose a new decoder based on a quarter-wave plate reflector-Michelson (Q–M) interferometer, which is free of polarization disturbance caused by quantum channel and optical devices in QKD systems. The theoretical analysis is given to reveal how the Q–M interferometer works to eliminate the polarization-induced signal fading automatically. We also build a time-bin phase encoding QKD system based on the Q–M interferometer. The system exhibits high degree of stability with a low quantum bit error rate (QBER) about 1.3% in 10 h over 60.6 km standard telecommunication optical fiber in the outfield. Compared with the schemes using Faraday magneto-optic devices, our scheme is easier to realize photonic integration as shown in our subsequent work [21] and can be applied in magnetic environments owing to the absence of magnetic-optic devices.

2. The Q–M Interferometer Scheme and Theoretical Analysis

According to Ref. [17], for the phase encoding QKD systems with an unbalanced-arm Mach–Zehnder or Michelson interferometer as the decoder, the polarization disturbance in optical fiber quantum channel affects the stability of the systems, and the anti-disturbance condition is $L^* \cdot S = I$ or $L = S$, where $L$ and $S$ represent the operators of the whole long and short arms of the unbalanced-arm interferometer, respectively, and they are unitary. We propose the Q–M interferometer in this paper which satisfies the anti-disturbance condition. The Q–M interferometer is composed of a polarization maintaining coupler (PMC) and two unbalanced arms (the upper and lower arms), as shown in Figure 1. Both the upper and lower arms are comprised of the polarization maintaining coupler (PM) optical fiber, a quarter-wave plate (QWP), and a reflector. The QWP and the reflector can be fabricated into an integral optical component, i.e., a quarter-wave plate reflector (QWPR). We denote that the slow and fast axes of the QWP are along $x$- and $y$-directions, respectively, and the slow and fast axes of the PM optical fiber are along $X$- and $Y$-directions, respectively. The angle between the slow axes of the PM optical fiber and the QWP is 45 degrees. In addition, there is a phase shifter (PS, as shown in Figure 1) or a phase modulator in one arm, for example in the upper arm, for compensating phase drifting or implementing phase encoding. Through the analysis below, we find that either arm of the Q–M interferometer corresponds to Pauli Matrix $\sigma_2$ while that of the Faraday Michelson interferometer corresponds to Pauli Matrix $\sigma_3$. The two Pauli matrices will be defined below. It is easy to confirm that the Pauli Matrix $\sigma_2$ naturally satisfies the anti-disturbance condition and makes QKD systems immune to polarization-induced signal fading [17]. In the article, the mathematical notations are the same as Ref. [22].
Physically, a QWPR can turn an $X$-direction linear polarization light to $Y$-direction, and vice versa, when the angle between the polarization direction of the linear light and the slow axis of the QWP equals 45 degrees. As shown in Figure 2, a forward $X(Y)$ polarization incident light along the slow (fast) axis of PM optical fiber can be transformed into a backward $Y(X)$ polarization output light along the fast (slow) axis of PM optical fiber after reflected by the QWPR. Due to the same phase accumulation during the round-trip transmission, only the exchange of the $X$- and $Y$-polarization states happens between the input and output light, namely, the operator of the long or short arm can be written as

\[
\sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}
\]

Therefore, the output polarization state is independent of PM optical fiber in the interferometer arms and can be expressed as the product of the incident polarization state and Pauli Matrix $\sigma_2$.

\[
U(\gamma, s_1, s_2, s_3) = \sigma_0 \cos \gamma + i(s_1 \sigma_1 + s_2 \sigma_2 + s_3 \sigma_3) \sin \gamma
\]

\[
= \begin{bmatrix} \cos \gamma + i s_1 \sin \gamma & is_2 \sin \gamma - s_3 \sin \gamma \\ is_2 \sin \gamma + s_3 \sin \gamma & \cos \gamma - is_1 \sin \gamma \end{bmatrix}
\]

\[
\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}
\]

\[ (1) \]

where the angle $\gamma/2$ corresponds to the birefringence strength of the transmission medium, and Stokes parameters, $s_1, s_2, s_3$ originate from the $X$–$Y$ and $+45^\circ$ and $-45^\circ$ components of rectangular birefringence, and from the circular birefringence, respectively. Then, for the Q–M interferometer, the operator...
of PM optical fiber in the long arm (short arm) can be expressed as \( l(s) = U(\delta/2,1,0,0) \), where \( \delta \) is the birefringence strength of the PM fiber, and the operator of the QWPR can be expressed as \( QR_{1/4} = U(\pi/4,0,1,0)^4 \cdot U(\pi/4,0,1,0) = U(\pi/2,0,1,0) \). Then, the long arm operator \( L \) is:

\[
L = I \cdot QR_{1/4} \cdot I
= U(\frac{\pi}{2},1,0,0)^4 \cdot U(\frac{\pi}{2},0,1,0) \cdot U(\frac{\pi}{2},1,0,0)
= U(\frac{\pi}{2},0,1,0) = QR_{1/4} = i\sigma_2
\]  

(2)

where the arrows ← and → indicate the backward and forward propagation, respectively, and the superscript \( t \) designates the transposed matrix. For reciprocal optical element, the backward propagation notation \( I \) equals the transposed matrix of the notation \( I \) in the forward coordinate system. The conclusion also applies to the short arm operator \( S \).

Now, consider double Q–M interferometers in two distant participants Alice and Bob, respectively, connected with each other by commercial optical fiber quantum channel as seen in Figure 3. We only set up a phase shifter in the Q–M interferometer on Bob’s side because the phase shifter is used for compensating the relative phase drifting of the two Q–M interferometers. When Alice sends a quantum pulse to Bob, there are two paths for the pulse to interfere on Bob’s side [17]:

Path 1: \( L_a \rightarrow \text{channel} \rightarrow S_b \)

Path 2: \( S_a \rightarrow \text{channel} \rightarrow L_b \)

where the subscripts \( a \) and \( b \) represent the operators of the components on Alice’s and Bob’s side, respectively. \( L_i \) and \( S_i \) (\( i = a \) or \( b \)) are the long and short arm operators of the Q–M interferometer on Alice’s or Bob’s side, respectively. According to Equation (2) and considering the phase resulting from transmission along the long and short arms, the transformation matrices of the two paths can be described respectively by

\[
Path_1: \quad (s_b \cdot QR_{1/4} \cdot s_b^t) \cdot e^{i\phi} \cdot C \cdot (l_a \cdot QR_{1/4} \cdot l_a^t)
= e^{i\theta_b} \cdot QR_{1/4} \cdot e^{i\phi} \cdot C \cdot e^{i\lambda_b} \cdot QR_{1/4}
\]

(3)

\[
Path_2: \quad (l_b \cdot QR_{1/4} \cdot l_b^t) \cdot e^{i\phi} \cdot C \cdot (s_a \cdot QR_{1/4} \cdot s_a^t)
= e^{i\lambda_a} \cdot QR_{1/4} \cdot e^{i\phi} \cdot C \cdot e^{i\theta_a} \cdot QR_{1/4} \cdot e^{i\psi}
\]

where \( C \) is the operator of quantum channel and can represent arbitrary birefringence resulting from quantum channel, \( l_i \) and \( s_i \) (\( i = a, b \)) represent the operators of the long and short PM optical fiber in the arms of the Q–M interferometer, \( \alpha_i \) and \( \beta_i \) are the phase caused by the interferometer’s long and short arms, respectively, \( \phi \) is the phase of transmission fiber, and \( \varphi \) is the phase shift from the phase shifter in Bob’s interferometer. Supposing that the input Jones vector is \( E_{in} \) on Alice’s side, the output of Bob’s interferometer can be written as

\[
E_{out} = \frac{1}{4} \left[ e^{i(\alpha_a + \beta_b + \phi)} + e^{i(\alpha_a + \beta_b + \psi + \phi)} \cdot QR_{1/4} \cdot C \cdot QR_{1/4} \cdot E_{in} \right]
\]  

(4)

where the factor \( 1/4 \) originates from the PMCs of Alice’s and Bob’s interferometers. Since \( S_i, L_i \), and \( C \) are unitary, the interference output power can be expressed as

\[
P_{out} = E_{out}^* \cdot E_{out}
= E_{in}^* \left[ e^{i(\alpha_a + \beta_b + \phi)} + e^{i(\alpha_a + \beta_b + \psi + \phi)} \right] Q R_{1/4} \cdot C \cdot Q R_{1/4} \left[ e^{i(\alpha_a + \beta_b + \phi)} + e^{i(\alpha_a + \beta_b + \psi + \phi)} \right] Q R_{1/4} \cdot C \cdot Q R_{1/4} E_{in}

= \frac{1}{2} \left[ 1 + \cos(\Delta a + \Delta b - \varphi) \right]
\]  

(5)
where $\Delta \alpha = \alpha_a - \alpha_b$, $\Delta \beta = \beta_a - \beta_b$. This means that the interference output $P_{\text{out}}$ is independent of any polarization perturbation in the whole QKD system, especially that caused by the quantum channel. In an ideal case, $\Delta \alpha$ and $\Delta \beta$ are invariable; hence, interference fringe is only modulated by the phase shifter in the interferometer. In the real case, the phase drifting of $\Delta \alpha$ and $\Delta \beta$ caused by the fluctuation of temperature or environmental vibration can be solved by active compensation, such as controlling the phase shifter in real time.

![Figure 3. Double Q–M interferometers on Alice’s and Bob’s side, respectively, connected with each other by optical fiber quantum channel. LD: laser, Cir: optical circulator, PMC: polarization maintaining coupler, PS: phase shifter, QWPR: quarter-wave plate reflector, SPD: avalanche diode single photon detector, $L_i$ ($i = a$ or $b$): the long arm operator of the Q–M interferometer on Alice’s or Bob’s side, $S_i$ ($i = a$ or $b$): the short arm operator of the Q–M interferometer on Alice’s or Bob’s side.](image)

3. Experimental Results

To demonstrate the theory above, a time-bin phase encoding intrinsic-stabilization QKD experimental setup is built. The schematic setup of the QKD system is shown in Figure 4a. In the setup, the transmitter Alice encodes the key information randomly into a phase basis $\{0, \pi\}$ and a time-bin basis, and sends quantum pulses to the receiver Bob. Both Alice and Bob have a Q–M interferometer as the phase encoder and decoder for phases $[0, \pi]$ respectively, as seen in the red dash boxes in Figure 4a, which is similar as that in Figure 3. The stability of the QKD system validates the effectiveness of the Q–M interferometer. The BS$_1$ and BS$_4$ are used for time-bin encoding and decoding as seen in the black dash boxes in Figure 4a. A variable optical attenuator (VOA) on Alice’s side is used to attenuate the transmitted light to a single photon level, the BS$_3$ on Bob’s side is used for passive basis selection, and a dense wavelength division multiplexer (DWDM) used on Bob’s side is for spectral filtering to reduce the scattered and background noise. The system works in a way of decoy-state BB84 protocol including vacuum and weak decoy states [2,23]. The geographic distribution of the quantum channel with a standard telecommunication optical fiber is shown in Figure 4b, which is a round-trip loop fiber with the fiber length of 60.6 km and the optical loss of 16.9 dB.

In the experiment, the photons are generated by four strongly attenuated 1549.32 nm distributed-feedback pulsed laser diodes with a pulse width of 500 ps and 100 MHz repetition rate. The average photon number per pulse is 0.6, including transmitted signal, decoy state, and vacuum state. The vacuum state is generated by not triggering the lasers. The ratio of signal, decoy, and vacuum state numbers is 6:1:1. Four avalanche diode single photon detectors are used on Bob’s side with a gate width of 1 ns.

With the setup described above, we measure the QBER and safe key rate to examine the performance of the system. As shown in Figure 5, the average QBER and safe key rate are 1.3% and 1.0 kbps during 10 h, respectively. Here, we deal with the data collected from detectors every second as a QBER point in Figure 5a. In Figure 5b, every point represents the average number of the safe keys in one minute. The experimental results indicate that the Q–M interferometer scheme can keep the QKD system working stably.
which is free of polarization disturbance caused by optical fiber quantum channel and optical devices.

4. Conclusions

In summary, we propose a new decoder based on the Q–M interferometer for QKD systems, which is free of polarization disturbance caused by optical fiber quantum channel and optical devices.
in the system. Physical and theoretical analysis has been presented. An experimental verification is also implemented by building a Q–M interferometer based time-bin phase encoding QKD system. The experimental result reveals a long-term low QBER with about 1.3% over 60.6 km standard telecommunication optical fiber. All of the components in the Q–M interferometer are conventional commercial passive optical components, and can be easily fabricated. Since the Q–M interferometer is without Faraday magneto-optic components, our scheme can be expected to realize optical integration, which will be shown in our subsequent work, and also be applicable in magnetic environments [21]. The theoretical analysis and the experimental results indicate that the Q–M interferometer based QKD system is immune to quantum channel disturbance, and will be a competitive scheme in practical QKD applications.

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