Characterizing Convection in Stellar Atmospheres

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Abstract. We perform 3D radiative hydrodynamic simulations to study the properties of convection in the superadiabatic layer of stars. The simulations show differences in both the stratification and turbulent quantities for different types of stars. We extract turbulent pressure and eddy sizes, as well as the \( T - \tau \) relation for different stars and find that they are sensitive to the energy flux and gravity. We also show that contrary to what is usually assumed in the field of stellar atmospheres, the structure and gas dynamics of simulations of turbulent atmospheres cannot be parameterized with \( T_{\text{eff}} \) and \( \log(g) \) alone.

1. Introduction
Large eddy simulations (LES) have been successfully applied to stellar atmospheres to investigate the effect of convection on stellar structure and atmosphere stratification. LES can produce accurate turbulent gas dynamics in regions of efficient and inefficient convection, and treat overshoot self consistently. Statistics from the simulated turbulence can be parameterized and applied to 1D envelope or stellar models. For example, Kupka & Robinson (2007) demonstrate that some effects of convection can be described by averaging the skewness of the vertical velocity.

Some progress has been made in parameterizing the turbulent dynamics in the superadiabatic layer (SAL) in this way. For example, Ludwig et al. (1999) and Freytag et al. (1999) have undertaken extensive efforts to map the mixing efficiency (or effective \( \alpha \) for MLT) using 2D HRD simulations, and constructing 1D envelope models by matching the specific entropy of the models and simulations. Trampedach et al. (1999) has also attempted to extract a mixing length parameter from simulations by matching averaged 3D simulation stratifications to 1D envelope models and modifying the turbulent pressure.

Although all attempts to date to parameterize convection have been carried out in the \( \log(g) \) and \( \log(T_{\text{eff}}) \) plane, it has been pointed out by other researchers (e.g., Ludwig et al, 1999) that these traditional atmospheric parameters are not necessarily suitable for the study of convection in the SAL. In this work, we examine the parameter space over which the turbulent dynamics of convection in the SAL are described.

2. Modelling Stellar Convection
A 3D simulation is characterized by its surface gravity, effective temperature and chemical composition. We get the surface gravity, stellar flux and initial stratification for each simulation from a 1D stellar evolution model. The simulations are evolved until they have thermally relaxed to a new equilibrium state. The simulation domain is located at the top of the convection zone, with the top and bottom of the domain located at approximately 3 and 8 pressure scale heights.
above and below the photosphere, respectively. The domain is small enough that curvature and radial variation in gravity can be safely ignored. The vertical walls are periodic and the horizontal walls are slip free and impenetrable (closed box). Radiative transfer is treated with the 3D Eddington approximation (Unno & Spiegel, 1966) in the optically thin region. Properties of the simulations are largely insensitive to the boundary conditions. Kupka (2005) has found good agreement between simulations of the solar SAL computed with different codes with varied boundary conditions, radiative transfer models and resolution.

3. Standard Solar Models
We present a comparison of solar simulations to demonstrate that the initial stratification near the surface taken from the 1D models does not determine the relaxed simulated state. The solar simulations begin from two standard solar models with slightly different input physics. One model is constructed with the Eddington $T-\tau$ relation, and the other with the semi-empirical Krishna Swamy $T-\tau$ (Krishna Swamy, 1966). The mixing length parameter $\alpha$ is adjusted to compensate for the different surface conditions and produce the correct solar radius in both models. The mixing length parameter is 1.83 and 2.14 for the Eddington and Krishna Swamy standard solar models, respectively.

Figure 1 shows the superadiabatic gradient ($\nabla - \nabla_{ad}$) of the two solar models and their corresponding simulations. The two solar models only differ near the surface, and are the same deeper in the stellar interior. Differences can be seen in both the peak value and position of the SAL because of the different $T-\tau$ relations and mixing length parameters. The simulations are not affected by the treatment of convection or the atmosphere in the initial 1D model, and produce the same thermally relaxed stratifications through the entire simulated domain.

4. Model Pairs in the Gravity-Temperature Plane
In order to examine the quantities that can adequately characterize the turbulent structure, we prepare a set of stellar models and corresponding simulations at strategic positions in the HR diagram. The initial conditions for our set of simulations is comprised of three pairs of stellar models, with each pair having the same surface gravity and effective temperature but may have other stellar properties that are varied. Figure 2 shows the location of the three model pairs in the log($g$)-log($T_{eff}$) plane (details are in Table 1).

5. Parameters for Turbulent Atmospheres
To investigate whether the traditional atmospheric parameters of surface gravity and effective temperature can uniquely characterize turbulent atmospheric structure, we compare the
Figure 2. Locations in the log($g$) – log($T_{\text{eff}}$) plane of three pairs of models and simulations. Each pair comprises two models in different evolutionary stages and corresponding simulations. Model pairs #1 and #2 comprise a pre- and a post-main-sequence model, while both models in pair #3 are pre-main sequence. Properties of the simulations and models are compared in Figures 3 and 4.

| Models  | log($g$) | log($T_{\text{eff}}$) | Mass      | Radius     | $\alpha$ |
|---------|---------|----------------------|-----------|------------|----------|
| A & B   | 4.426   | 3.736                | 0.90 & 1.00 | 0.960 & 1.012 | 1.8 & 1.8 |
| C & D   | 4.340   | 3.720                | 0.80 & 1.00 | 1.001 & 1.119 | 1.8 & 1.2 |
| E & F   | 4.450   | 3.741                | 1.00 & 1.00 | 0.985 & 0.985 | 2.1 & 1.5 |

Table 1. Properties of stellar models used as initial conditions for the three simulation pairs.

superadiabatic excess of the three model and simulation pairs described in section 4.

Fig. 3 compares the model and simulated superadiabaticity for the three positions in the log($g$)-log($T_{\text{eff}}$) plane. The first panel shows the structures for models (A & B), which begin with the same structure in the SAL. The relaxed state of the simulations is different than the 1D models, but the simulations are in agreement with each other.

The second panel compares the second model pair (C & D) and their corresponding simulations. Unlike the first model pair, these two models begin with different stratifications below the photosphere, but are both Eddington atmospheres above the photosphere and have the same surface parameters of log($g$) and log($T_{\text{eff}}$). The simulations produce new stratifications that differ from the 1D models and are not in agreement with each other, despite sharing the same surface parameters. This is contrary to what is predicted in static atmospheres.

Although the second pair demonstrates that log($g$) and log($T_{\text{eff}}$) do not necessarily uniquely characterize a turbulent atmosphere, the differences in simulation structure could be attributed to the different stellar model parameters of mass and radius. The third model pair (E & F) was constructed with the same mass and radius to remove this ambiguity. The third panel of Fig. 3 compares the SAL of the third pair, and again, the stratification and temperature gradients of the 1D models are identical above the photosphere but differ below, while the 3D simulations have stratifications that differ both above and below the photosphere.

We have demonstrated, by comparing the temperature gradients of the models in the log($g$)-log($T_{\text{eff}}$) plane, that turbulent atmospheres can have different stratifications despite having the same traditional atmospheric surface parameters.

Fig. 4 compares the run of specific entropy in the SAL of the same set of 1D models and corresponding simulations. All model pairs have the same specific entropy above the photosphere, but may have different entropy profiles in the deeper regions which reflect differences in model evolutionary history and interior structure.

The first model pair had the same stratification throughout the simulated domain, and as Fig. 4 shows, the same run of specific entropy. The second and third model pairs both began with
identical structures above the photosphere but resulted in different simulated stratifications. In both cases the run of specific entropy below the photosphere was different in the models and simulations. We suggest that when combined with the specific entropy, the surface gravity and effective temperature can characterize the structure of a turbulent atmosphere.

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