Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.
Fuzzy Electromagnetism Optimization (FEMO) and its application in biomedical image segmentation

Shouvik Chakraborty * , Kalyani Mali

Department of Computer Science & Engineering, University of Kalyani, India

Abstract

In this work, a new unsupervised classification approach is proposed for the biomedical image segmentation. The proposed method will be known as Fuzzy Electromagnetism Optimization (FEMO). As the name suggests, the proposed approach is based on the electromagnetism-like optimization (EMO) method. The EMO method is extended, modified, and combined with the modified type 2 fuzzy C-Means algorithm to improve its efficiency especially for biomedical image segmentation. The proposed FEMO method uses fuzzy membership and the electromagnetism-like optimization method to locate the optimal positions for the cluster centers. The proposed FEMO approach does not have any dependency on the initial selection of the cluster centers. Moreover, this method is suitable for the biomedical images of different modalities. This method is compared with some standard metaheuristics and evolutionary methods (e.g. Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Electromagnetism-like optimization (EMO), Ant Colony Optimization (ACO), etc.) based image segmentation approaches. Four different indices Davies–Bouldin, Xie–Beni, Dunn and β index are used for the comparison and evaluation purpose. For the GA, PSO, ACO, EMO and the proposed FEMO approach, the optimal average value of the Davies–Bouldin index is 1.833578359 (8 clusters), 1.669359475 (3 clusters), 1.623119284 (3 clusters), 1.647743907 (4 clusters) and 1.456889343 (3 clusters) respectively. It shows that the proposed approach can efficiently determine the optimal clusters. Moreover, the results of the other quantitative indices are quite promising for the proposed approach compared to the other approaches. The detailed comparison is performed in both qualitative and quantitative manner and it is found that the proposed method outperforms some of the existing methods concerning some standard evaluation parameters.

© 2020 Elsevier B.V. All rights reserved.

1. Introduction

Digital image processing has widespread applications in various domains like agriculture, security, medical domain, industrial
applications, etc. Image segmentation plays a vital role and is required as the preliminary step in most of the applications of digital image processing [1]. Image segmentation is considered as one of the challenging tasks in the field of computer vision. The main job is to automatically divide an image into its constituting regions based on the gray level intensity or the values of the Red, Green, and Blue channels (in case of a color image). The integration of the segments should produce the actual image. The segmented regions must possess some similarities. Similarities can be measured based on different features like color, texture, intensity, or something else. This similarity can also be computed using some mathematical functions. Biomedical image segmentation is an important step in computer-aided diagnostic processes. In general, biomedical images suffer from several problems like poor quality and correlation, ambiguous overlapping regions, noise, etc. and these problems makes the job more difficult for the physicians. Biomedical image segmentation is a useful tool to detect various abnormalities like abnormal tissue growth, infections, foreign objects, etc. [2,3] and it is also helpful for the precise and quick investigations of different parts of the body. Anatomical structures can be easily studied with the help of automated image segmentation. Pathologists, microbiologists, etc. can take advantage of the biomedical image segmentation methods in cell counting, detecting the presence of viruses, microbes, classification of diseases, etc. [4,5]. Therefore, automated image segmentation methods have a significant impact in the field of biomedical image analysis and can make significant progress in this domain which is necessary for providing accurate and timely treatment to the patients [4,6]. Biomedical image segmentation is one of the major processes to get information about the different internal organs of the body in a non-invasive manner and therefore segmentation plays a vital role in different medical applications like analysis and exploration of anatomical structures, simulation of different biological mechanisms, pathological investigations, disease analysis, and progression tracking, assessing the condition and the requirement of surgery and many more. With the advancement of technology, computer-aided diagnostics systems are getting improved day-by-day and advanced image segmentation approaches become an inevitable part of these systems. Moreover, different operational functionalities are highly dependent on the performance of the associated image segmentation procedure. From this discussion, it can be understood that the task of segmentation demands some sophisticated methods to identify various sections of an image. As discussed earlier, there are some inherent problems which are associated with the biomedical images, and in many occasions, it has been observed that the crisp segmentation approaches cannot efficiently and precisely determine the segmented regions from the biomedical images and hence, the prime goal of this research work is to develop a fuzzy method that can effectively and reliably determine the segments from the biomedical images of different modalities with the help of the EMO algorithm.

Image segmentation is one of the prime research areas which is being addressed by many researchers of various domains. A single solution is not enough to apply to all types of images. Some prominent issues of digital images like poor contrast, uneven intensity distribution, poor illumination, ambiguous boundaries, etc. make the problem difficult and hard to analyze [7–10]. Therefore, the crisp method may not be suitable always to segment such types of images [8]. The theory of the fuzzy sets can help in this issue [11]. Here, the basic concept is one point can belong to more than one cluster with some degree of membership [12]. The sum of all the membership values for a particular point must be 1 and this type of membership is known as the partial membership [13]. The degree of membership is decided by the fuzzy membership functions and based on this theory many difficult problems are solved. Image segmentation is no exception and exploits the concept of the fuzzy set theory to find various regions of an image. Using the concept of fuzzy set theory, the popular fuzzy C-means [14] clustering method is developed and it has a widespread application not only in image segmentation but, many other fields are also using this concept [6,15]. The major advantages of the fuzzy C-means clustering method are, this method can retain more information than the crisp clustering method and suitable for the overlapped datasets because a data point can belong to more than one cluster with some degree of membership. This method is robust and gives better performance than the K-means method for those images where the prominent overlapped regions can be observed. The generalized version of type 1 fuzzy set which is nothing but the type 2 fuzzy set, is adapted in this work. In normal fuzzy C-means clustering method, the cluster centers are dependent on the mean of all its constituting points. It can be easily understood that the final clustering outcome is heavily dependent on the selection of the initial cluster centers which is a major problem associated with this. This work designed a biomedical image segmentation method called Fuzzy Electromagnetism-like Optimization (FEMO) that uses the type 2 fuzzy set theory [16] and electromagnetism-like optimization [10]. Electromagnetism-like optimization can be used to solve global optimization problems. This method is based on electromagnetism and the behavior of the charged particles. This method mimics the attraction and the repulsion mechanism of the charged particles. The electromagnetism-like optimization method is used to solve different problems of various domains. In [17], authors proposed a solution to segment white blood cells from microscopic images using electromagnetism-like optimization methods. The circle detection problem is used to find white blood cells using the electromagnetism-like optimization method. The objective function is designed in such a way so that it can find the similarity between a probable circle and the shape of the white blood cells. The proposed method is tested with some problems of different levels of complexity and the experimental results prove its efficiency and the real-life applicability. A neural-fuzzy system optimization based on the EMO method is proposed in [18]. The proposed method is used to solve the nonlinear system identification and classification problems. In this work, the population size is dynamic and it is decided based on the similarity measure. The EMO method is combined with the gradient descent method to improve the convergence. Its performance and efficiency can be judged from the experimental results. The EMO algorithm is applied on the project scheduling problem in [19]. In this work, a resource-constrained environment is selected. Different types of instances are used to test this method and a consistently good performance can be achieved. This work also gives a future research direction to apply this method in other domains. The EMO method is hybridized with the migration strategy and applied...
to optimize the truss structures. This hybrid algorithm is used to update the positions of the particles and to reach the global optimum. The truss structure optimization problem is solved by the proposed method considering some frequency constraints. The experimental results show the practical applicability of this proposed algorithm. In [20], a constraint handling technique is proposed for the EMO algorithm. This method can be used to solve various practical engineering design problems. Each iteration of this method is supported using a derivative-free searching mechanism to improve the convergence. This method is tested and compared with some stochastic methods using some engineering problems. A modification of the local search method in the EMO algorithm is proposed in [21]. The proposed local search is implemented using the chaotic optimization procedure and this modified EMO is tested using some problems and the results are compared with the actual EMO process to prove its efficiency. The EMO algorithm is applied in [22] to produce isospectral non-uniform beams corresponding to a given uniform beam. The authors propose an error function to compute the separation between the spectra of the uniform and the non-uniform beam and this difference is minimized using the EMO method. Experimental results prove the presence of the isospectral non-uniform beams corresponding to a uniform beam as a local optimum. Moreover, structural damages are also detected in this work by finding the isospectral non-uniform beams corresponding to a damaged beam using the EMO method. A modification to the EMO algorithm is proposed in [23] to optimize the fractional-order PID. The proposed method is named the IEMGA i.e. ‘improved EM algorithm with genetic algorithm technique’. Here the different controller parameters are used as the charged particles. The local search procedure is implemented using the genetic algorithm and this improvement reduces the computational complexity of the overall system. The performance of the proposed method is illustrated using many examples. A binary version of the EMO method is proposed in [24] to determine the solution of the combinatorial optimization problems. This method is applied to solve the traveling salesman problem and the results are compared with some of the standard methods of the literature to prove the superiority. The EMO algorithm is used to optimize electromagnetic devices in [25]. A novel application of the EMO algorithm can be found in [26] where the EMO is used for the feature selection purposes. Here
one nearest neighbor (i.e. 1NN) is used with the EMO method for feature selection and classification. The main criterion for feature selection is the minimum rate of miss-classification. Various types of datasets are used to prove the efficiency of the proposed method. This method proves to be effective compared to some other standard methods in terms of both classification and feature selection and the practical application of this method is demonstrated using a real-life dataset of gestational diabetes mellitus. A modification of the multi-objective optimization method (MOEM) based on the EMO method is proposed in [27]. In this work, the proposed MOEM method is compared with some of the standard multi-objective optimization methods and the Pareto fronts
are compared and proved that the proposed method is a good competitor of existing multi-objective optimization algorithms. Apart from these approaches, a hybrid ant swarm optimization approach which is known as the immune ant swarm optimization is proposed in [28]. The ant swarm optimization is a hybridization of the ant colony optimization and particle swarm optimization. This approach preserves the convergence of the global search in PSO. This approach is the combination of the PSO, ACO and the immune approach which helps obtain better optimal rough reducts. 12 benchmark datasets are incorporated in this work for the experiments and evaluation purposes and the proposed approach generates some encouraging results. A grey wolf optimizer (GWO) based solution is designed for the path planning problem and for a Proportional–Integral (PI)-fuzzy controller tuning problem in [29]. The experiments are performed to evaluate the proposed approaches in a single multiagent environment with nRobotic platform. The experimental approaches prove the effectiveness and improvement in the performance of the GWO based solution for the path planning problem and tuning the Proportional–Integral (PI)-fuzzy controller.

The proposed FEMO method incorporates the concept of type 2 fuzzy sets with EMO method and it is applied in biomedical image segmentation. As discussed earlier, the proposed algorithm helps to determine the segments without having any sensitivity to the selection of the initial cluster centers. The number of the parameter for the EMO method is quite reasonable compared to many other metaheuristic algorithms. The EMO method can reach the global optima even after getting stuck in a local optima [30,31]. The attractive region of the population is thoroughly examined by the EMO method which is a major advantage of this method [30]. So, the advantages of both type 2 fuzzy set and the EMO method are exploited in this method to get better segmentation results and this is the basic motivation behind this work. Some biomedical images with different modalities are successfully segmented by the proposed method. The efficiency of the proposed FEMO method is analyzed by both visual and numerical methods. Some well-known cluster validity indices are used for this purpose.

Various real-life problems involve multiple-variable based non-linear functions. Moreover, these functions may contain some attributes like multimodality, discontinuity, etc. Stochastic search approaches are proved to be efficient in solving these problems [32]. It is worth mentioning in this context that the discrete optimization problems are harder to solve compared to the
continuous optimization problems [33,34]. Typically, population-based optimization approaches begin by taking a short sample from the possible solution space. The value of the fitness function is determined for each solution and the domain of preference is determined based on the fitness function. The probable candidate domains are further exploited by certain mechanisms like reproduction, mutation, and crossover in the case of genetic algorithms. The electromagnetic-like optimization approach is inspired by the attraction–repulsion mechanisms of the electromagnetically charged particle. In this approach, each data point is considered as the charged particle. Apart from performing the global search, the EMO approach incorporates a local search procedure to exploit the neighborhood of the best solution discovered. It allows an efficient way to search for the total solution space. The performance of the EMO method is evaluated and compared in [35]. It can be observed that the performance of the EMO method is quite better compared to some other standard approaches. It inspires us to design a new approach based on the EMO method and applied on the biomedical images and the obtained results are quite promising. The incorporation of type 2 fuzzy system helps to efficiently model various real-life scenarios and the associated uncertainties. Therefore, the proposed FEMO approach is helpful to realistically model the problem of biomedical image segmentation because biomedical images often suffer from poor correlation, overlapping regions without proper boundary line, poor contrast etc. There are several metaheuristic approaches that can be found in the literature which is dedicated to certain types of applications [13,36,37]. In general, there are no concrete and universal algorithm or method that can determine the suitable algorithms for a specific approach. The decision can only be made on the basis of the quantitative and qualitative experiments [38–43].

The proposed FEMO approach is used for the biomedical image segmentation purposes and four existing approaches like genetic algorithm (GA) [44], particle swarm optimization (PSO) [45], electromagnetism-like optimization (EMO) [17], ant colony optimization (ACO) [46] are used to evaluate and demonstrate the comparative performance using the both qualitative and quantitative manner. In this work, the segmented outcomes are not compared with any manual delineations. However, the proposed work can perform the job of segmentation without having any knowledge about the ground truth data. To demonstrate the effective performance of the proposed approach further and to make the results more reliable, some standard test images are
also incorporated in the revised article. The proposed approach and the other four approaches are applied to these standard test images and the value of the four validation indices are reported for the different number of clusters. The segmented outputs are also reported which are useful for the visual investigations. The obtained results are found to be quite promising and establish the efficiency of the proposed FEMO approach in both a qualitative and quantitative manner. The overall average values of the four quantitative validation metrics are reported at the bottom of every table to get a glimpse of the overall performance of the proposed system compared to other approaches.

The remaining article is organized as follows. Sections 2 and 3 briefly illustrate the concept of electromagnetism-like optimization method and the type 2 fuzzy set respectively and gives a hint of how these two methods can be unitedly applied for the image segmentation purpose. Section 4 describes the proposed method. Section 5 shows the obtained results and their interpretation. Section 6 presents some of the limitations of the proposed approach. Section 7 concludes this article and Section 8 discusses some important and interesting points and also gives some future directions on it.

2. The electromagnetism-like optimization (EMO) method

Electromagnetism-like optimization method is first proposed in [30] and it can be used for the global optimization purposes. This method mimics the attraction–repulsion mechanism of the charged particles as per the theory of electromagnetism. The basic electromagnetism-like optimization method is described in Algorithm 1. This method works in four distinct phases. In the first phase of the algorithm, some points are randomly selected from the search space to initialize the population. In the second phase, this method looks for the local optimum. This step requires a local search procedure that can be achieved using different methods like simulated annealing [47], hill-climbing [48], gradient descent method [49] etc. the third stage uses superposition theory of the electromagnetism [50] to calculate the exerted force by each point. The amount of charge and the distance between two points are the necessary parameters which are used to compute the force. This concept is demonstrated in Fig. 1. Here, $\vec{F}_{ca}$ is the force which is experienced by $q_a$ and exerted by $q_c$. Here, $q_a$ is repulsed by the $q_c$. In EMO method, it is possible when, the fitness value of $q_a$ is better than the $q_c$. $\vec{F}_{ba}$ is the force
which is experienced by $q_a$ and exerted by $q_b$. In this case, $q_a$ is attracted by the $q_b$. From perspective of the EMO method, this phenomenon indicates that the fitness value of $q_b$ is better than $q_a$. The resultant force, which is experienced by $q_a$ can be represented as $\vec{F}_a = \vec{F}_{ca} + \vec{F}_{ba}$. So, in the EMO process, each point is treated as the charged particles with attraction or repulsion forces with respect to other particles. Here, the amount of charge represents the fitness value for that particle. A particle with better fitness value can attract other particles and a particle with worse fitness value can repel other particles. So, in the third phase, the cumulative force experienced by a particle due to the attraction and repulsion of other particles is computed in this phase. In the fourth phase, the particle is moved towards the direction of the force. From the second to the fourth stage is repeated while the termination criteria are satisfied. This process can be terminated if the maximum number of iterations is reached. This process can also be terminated if the state of the points does not change in two or more than two successive iterations.

The EMO algorithm can solve nonlinear optimization problems (which have a form like Eq. (1) with a box constraint) and can reach the global optimum.

$$\max f(t), \quad t = (t_1, t_2, t_3, \ldots, t_n) \in \mathbb{R}^n \text{ subject to } t \in T$$  \hspace{1cm} (1)

Here, $f: \mathbb{R}^n \rightarrow \mathbb{R}$ denotes a non-linear function and $T = \{ t \in \mathbb{R}^n | lb_k \leq t_k \leq ub_k, \ k = 1, 2, 3, \ldots, n \}$ is a bounded region and the bounds are the two constraints, upper bound and lower bound, and these are represented by $lb_k$ and $ub_k$ respectively.

3. The type-2 fuzzy set and clustering

The fuzzy C-means (FCM) clustering method is one of the most popular and frequently used clustering approaches in the literature [51]. Unlike crisp clustering methods, it allows a data point to be part of more than one cluster with some membership values. The membership value for a point decides the level or a degree of a point that belongs to a certain cluster. This value is known as the degree of membership. The summation of all the membership values for a point must be 1. FCM method has a widespread application and frequently adapted to solve various problems in the field of computer vision and pattern recognition. Fuzzy clustering approaches are useful and can efficiently solve various problems which are may not be solvable by the crisp clustering approaches [52]. The objective function which is optimized by the fuzzy C-means clustering method is given in Eq. (2). The centers of the clusters are computed by minimizing this function.
Algorithm 1: The electromagnetism-like optimization (EMO) method

Inputs: The size of the initial population $S$. The maximum number of iterations $gItrCount$. The maximum number of iterations for the local search $lItrCount$. The local search parameter $\lambda \in [0,1]$.

Output: Optimal result(s)

1: $initPop()$: Initialize the population
2: $fitness \leftarrow calcFitness()$: compute the fitness value for each particle
3: $localSearch(lItrCount, \lambda)$: perform the local search
4: $force \leftarrow calcForce( fitness )$
5: $moveParticles( force )$: move the particles depending on the exerted force
6: Repeat step 3 to 4 until the termination criteria is satisfied.
This dissimilarity function is used by the traditional type 1 FCM clustering method as the objective function. The cluster centers are updated using Eq. (4).

\[ J_q = \sum_{i=1}^{P} \sum_{j=1}^{M} \mu_{ij}^q \| x_i - c_j \|^2 , \quad \text{where } 1 \leq q < \infty \]  

Eq. (3) expresses the \( \mu_{ij} \) and it denotes the degree of membership of point \( x_i \) to the \( j \)th cluster. As discussed earlier, for a certain point, the summation of \( \mu_{ij} \) must be 1 i.e. \( \sum_{j=1}^{M} \mu_{ij} = 1 \) for \( j = 1, 2, 3, \ldots, n \). \( x_i \) is the \( k \) dimensional data point \( c_j \) represents the \( k \) dimensional cluster center. \( P \) is the number of data points and \( M \) is the number of clusters.

\[ \mu_{ij} = \frac{1}{\sum_{l=1}^{c} \left( \frac{\| x_i - c_l \|}{\| x_i - q_l \|} \right)^{\frac{2}{q-1}}} \]  

\[ c_j = \frac{\sum_{i=1}^{P} \mu_{ij}^q x_i}{\sum_{i=1}^{P} \mu_{ij}^q} \]  

This error function used to compute the degree of membership. It is a squared function and used by the traditional type 1 fuzzy clustering system. The fuzzy type 1 clustering system suffers from some inherent problems. The first problem is the noise. Fuzzy type 1 clustering system is heavily affected by noise and therefore not suitable for the applications in biomedical image analysis [53]. Sometimes, relative membership values creates some difficulties and restricts it from being applied in various problems [54]. Motivated from this, the type 2 fuzzy system is adapted and combined with the EMO algorithm [55]. From the above discussions, one thing is very clear that if the membership value for a certain point for a cluster is very high i.e. close to 1 then, the uncertainty of that point is reduced because the membership value 1 indicates the complete involvement of a point to a particular cluster. Similarly, the uncertainty of a point increases with the lesser value of the membership i.e. closer to 0. So, the points with lesser uncertainty (i.e. with higher membership value) have more chance of participation and, the points with higher uncertainty (i.e. with lesser membership value) have a lesser chance of participation. This concept is exploited by the type 2 fuzzy systems and the advantages are given below [55]:

a. The points with the lesser degree of the membership (i.e. with higher uncertainty) have a comparatively lesser
effect on a cluster center than a point with a higher degree of membership (i.e., with lesser uncertainty). Therefore, if the degree of membership for a certain point tends to 0 then the effect of that point on the cluster center almost vanishes. This concept is very useful and well suited to solve various real-life problems.

b. As discussed earlier, noise handling is a major challenge in type 1 fuzzy systems. Type 2 fuzzy systems can handle the noise more effectively than the type 1 fuzzy systems. More realistic and stable cluster centers can be achieved with the help of type 2 fuzzy clustering systems. Incorporating the type 2 fuzzy system in the fuzzy C-means clustering method requires the concrete definition of the membership function. The membership values can be considered as the weights and can be computed using Eq. (3) where the value of $\mu_j$ is the degree of membership which is obtained using Eq. (3).

$$\gamma_{ij} = \mu_{ij} - \frac{1 - \mu_{ij}}{2}$$

From Eq. (4), it is evident that the type 2 fuzzy clustering system is a function of type 1 fuzzy clustering system. Now, the fuzzy type 2 membership $\gamma_{ij}$ can be replaced in Eq. (4) and the modified equation to update the cluster centers can be written as Eq. (6).

$$\tilde{c}_j = \frac{\sum_{i=1}^{p} \gamma_{ij} x_i}{\sum_{i=1}^{p} \gamma_{ij}}$$

Type 2 fuzzy clustering system is simple to implement and well suited to model various practical problems. This method is adopted in this work because it can produce better segmentation results due to its efficient noise handling capability [55]. The type 2 fuzzy C-means based clustering method is illustrated in algorithm 2. This procedure starts by initializing the degree of membership for the input data points. Initially, the degree of membership for each point is initialized randomly and the total sum of the membership values for a certain point must be 1. A threshold value is supplied to the algorithm which is useful to check whether the procedure can be stopped or not. Actually, if the difference in the fitness value of the objective function in the $i$th iteration and the $(i+1)$th iteration is greater than or equal to the threshold value then, the process repeated otherwise the process can be terminated. Therefore, this threshold value can be used as a termination criterion and the quality control
parameter for this clustering method. This algorithm improves
the uncertainty handling capability of type 1 fuzzy clustering
system apart from the increased efficiency from the perspective
of noise handling.

4. The proposed FEMO method

The proposed method obtains the optimal clustering out-
come by combining type 2 fuzzy C-means clustering with the
electromagnetism-like optimization (FEMO) method and the
name for this proposed method is given accordingly. The overall
flow of the proposed system is depicted in Fig. 2. According
to algorithm 1, the FEMO method starts by initializing the cluster
centers in a random manner. The cluster centers are denoted as
$CC_i$ and $l_{\text{low}} \leq CC_i \leq l_{\text{high}}, i = 1, 2, 3, \ldots, n$. Here, $l_{\text{low}}$ and
$l_{\text{high}}$ are the lowest and the highest intensities of the image under
consideration and $n$ is the number of cluster centers. Eq. (7) can
be helpful to initialize the cluster centers. Here, random $(0, 1)$
produce a uniformly distributed random number between 0 and
1.

$$CC_i = l_{\text{low}} + \text{random} (0, 1) (l_{\text{high}} - l_{\text{low}})$$

Table 1

| Important parameters and their values in the proposed FEMO method. |
|---------------------------------------------------------------|
| Name of the parameter                | Value   |
| Number of populations $n$            | 20      |
| gIterCount                          | 300     |
| lIterCount                          | 40      |
| The local search parameter $\lambda$| 0.005   |

The objective function which is to be optimized by the FEMO
method is given in Eq. (8).

$$J_k = \arg \min_{CC} \sum_{s=1}^{n} \sum_{t \in CC_j} y_{st}^2 * D_{st}$$

(8)

Here, $y_{st}$ is the value of the membership that denotes the degree
of membership of the $t$th pixel to the $s$th cluster in fuzzy type 2
system and it can be computed using Eq. (5) and $D_{st}$ denotes the
distance between them.

The proposed FEMO approach randomly initializes the cluster
centers and guide them so that it can reach the global optima.
The optimization procedure guides the whole system to achieve
Algorithm 2: Type 2 fuzzy C-means based clustering

Input: The data points to be clustered and a threshold value \( th \).

Output: Clustered data points

1: \( Y \leftarrow \text{initMembership}(\ ) \) random initialization of the membership values or the degree of memberships for the data points and for a certain point, the sum of all the membership values must be 1.

2: The cluster centers can be located using equation 6.

3: Compute the fitness value using the objective function which is given in equation 2.

4: Check if \( \text{improvement} \geq th \) then
   a. Determine the value of \( Y_y \) using 5
   b. Goto step 2 and repeat again
end if

5: Return the clustered data points

Table 2
The description of the ten biomedical images and five standard test images which are selected for the testing purpose.

| Image Id | Image | Source | Description |
|----------|-------|--------|-------------|
| IM01     | CT Scan image (grayscale) | [56] | The CT Scan image (axial view) of the chest region. This image is collected from a 43 years old COVID-19 positive male patient from Italy. (Credit to R. Bonacini, G. Besutti, P. Pattacini Radiology IRCCS Reggio Emilia; Director Pierpaolo Pattacini). |
| IM02     | False color CT Scan image of the heart and the lungs region (axial view). Both heart and lungs are in normal state. |
| IM03     | AT2 weighted MRI image (axial view) of the head region of a 20 years old female patient. The patient was suffering from Vague headaches. (Case courtesy Associate Professor Dr. Frank Gaillard) |
| IM04     | MRI color image of the head region (axial view) |
| IM05     | X-ray image of the chest region (PA view). This image is collected from a 70 years old COVID-19 positive male patient from Riccione, Italy. The patient was suffering from fever, cough, breathing difficulties. (Case courtesy Dr Domenico Nicoletti, Radiopaedia.org, rID: 74724) |
| IM06     | X-ray image of the chest region (Lateral view). It is collected from the same source as described in IM05. |
| IM07     | The mammogram of the right breast in MLO view. This image is collected from a 50 years old female patient.(Case courtesy of Dr Garth Kruger, Radiopaedia.org, rID: 21555) |
| IM08     | Ultrasonic image (grayscale) | [62] | The case is collected from a 30 years old female patient of the pelvic region(Case of Assoc Prof Craig Hacking, Radiopaedia.org, rID: 43471) |
| IM09     | Ultrasonic image (color) | [62] | Ultrasonic image of the outer of the left kidney. |
| IM10     | Positron Emission Tomography image (PET) (color) | [64] | Positron Emission Tomography of the head region. |
| IM11     | Airplane | [65] | Standard test image |
| IM12     | Baboon | [65] | Standard test image |
| IM13     | Barbara | [65] | Standard test image |
| IM14     | Tulips | [65] | Standard test image |
| IM15     | Cameraman | [65] | Standard test image |

the optimal value of the objective function which is given in Eq. (8). Now, to update the cluster centers, Eq. (6) is not required here. The cluster centers will be updated by the FEMO process. Here, the EMO method is used to guide the cluster centers to reach the globally optimum state by optimizing Eq. (8). The FEMO process produces the optimal set of clusters that represents the segmented regions of the image. The clusters are formed by assigning the nearest points corresponding to a cluster center. Type 2 fuzzy set is helpful in better convergence of this method in presence of noise [66].

After the initialization is over then, the fitness value for each member of the population is computed using Eq. (8) and the member with the best fitness values is stored in the \( \xi_{\text{best}} \). The initialization process is elaborated and can be easily understood from algorithm 3.

In the next phase, the local information about a particle \( \zeta^i \) is gathered using a maximum of \( \text{litrCount} \) number of iterations. The local search parameter \( \lambda \in [0, 1] \) and the \( \text{litrCount} \) is used to perform this local search procedure. The local search method begins by finding the maximum possible step length using the local
search parameter $\lambda$. The information about a cluster center i.e. a particle $\zeta_i$ is temporarily stored in a point $\kappa$. $\kappa$ is moved along the direction of a selected random number. If better point can be achieved within the InteCount number of iterations then, the point $\zeta_i$ will be replaced by $\kappa$ and the search can be terminated. The $\zeta_{best}$ is updated with the current best fitness value which is obtained at the end of this process. The local search method is illustrated in algorithm 4. After the local search procedure, the resultant force is computed for each particle. The force exerted by a particle is determined with the help of the superposition principle [67]. According to this theorem, the exerted force by a particle due to another particle is directly proportional to the product of the number of charges and inversely proportional to the amount of distance between them. Here, the charge of a particle is mimicked by the fitness value of the corresponding solution i.e. the fitness value of a point is used to compute the charge of a particle. The value of the charge for a particle determines the attraction and the repulsion power of that particle. The charge $ch_i$ for the $i$th particle can be computed using Eq. (9).

$$ch_i = e^{-d \frac{calcFitness(\zeta_i) - calcFitness(\zeta_{best})}{\sum_{i=1}^{n} calcFitness(\zeta_i) - calcFitness(\zeta_{best})}}$$, \quad \forall i (9)$$

Therefore, a higher charge of a point indicates better fitness value and vice-versa. The dimension $d$ is multiplied to make this equation computational friendly in higher dimension and to avoid overflow problems. One important point that can be observed from Eq. (9) is that, no sign is associated with it. The force exerted by a point $p$, $\bar{F}_p$, due to all other points can be expressed by Eq. (10). A point with better fitness value compared to the other points attracts others and a point with lesser fitness value compared to the other points repels others. The attraction and the repulsion mechanism between two points is implemented by comparing the fitness values of both the points. The point $\zeta_{best}$ is the point with the best fitness value (in this work, it is point with minimum fitness value) and therefore, it will attract all other points towards it. Here, the distance is computed using the Euclidean distance measure. The direction of the movement of a point is highly dependent on the points which are close to each other. Algorithm 5 will demonstrate this concept in detail.

$$\bar{F}_p = \sum_{q \neq p}^{n} \begin{cases} (\zeta_i - \zeta_p) \frac{ch_i ch_q}{\|\zeta_i - \zeta_p\|^2} & \text{if } calcFitness(\zeta_i) < calcFitness(\zeta_p) \\ (\zeta_i - \zeta_p) \frac{ch_i ch_q}{\|\zeta_i - \zeta_p\|^2} & \text{if } calcFitness(\zeta_i) \geq calcFitness(\zeta_p) \end{cases}, \quad \forall p (10)$$
In the next phase, the particles are moved according to their exerted force. A step length is computed in a random manner using Eq. (11). A uniformly distributed random value \( \epsilon \) (between 0 and 1) is generated to compute the length of the steps. From Eq. (11), it can be clearly observed that the exerted force by a particle is normalized. \( \vec{v} \) is a vector that contains the possible and allowed dimension wise movements towards \( I_{\text{high}} \) or \( I_{\text{low}} \). The point with the best fitness value \( \zeta_{\text{best}} \) is not moved and simply transferred to the next iteration and so on. Algorithm 6 illustrates this move procedure.

This movement procedure helps to adjust the cluster centers without explicitly using the equation to update the cluster centers. This procedure can be terminated if it reaches the optimal clustering outcome. However, it is not practical because the optimal clustering result is not possible to know beforehand. Therefore, the maximum number of iterations \( g_{\text{itr Count}} \) can be used to terminate this process. Another possible termination criterion can be the optimum value remains constant in two or more successive iterations. It is an important point to be considered because to get the optimal results, the algorithm must not be stopped before the global optimum. Moreover, to reduce the overhead, unnecessary iterations are not at all desirable. The algorithm for the FEMO method is given in algorithm 7 and the flow diagram is given in Fig. 3.

One important point worth mentioning here is that no histogram-based operations are required here to perform the segmentation. The necessary parameters and their values that are used in this method is reported in Table 1. The efficiency and the real-life applicability of the proposed FEMO method is tested both visually and quantitatively. Some standard and well-known cluster validation measures are used for the experiment. The detailed results of the simulation are given in the next section.

The FEMO approach begins by initializing the cluster centers randomly using Eq. (7). This equation helps to initialize the cluster centers within the range of the gray-level intensity values. The initial solution space is guided further by the proposed FEMO approach to reach the global peak. After generating the points, the matrix \( Y \) is prepared that contains the values of the degree of memberships using Eq. (5). It is obviously the type 2 fuzzy membership value. The values of the control parameters are used to control the overall functionality of the proposed system. The fitness is calculated using the objective function which is given in Eq. (8) and it is stored in a variable \( \text{fitness} \). The definition of the
| Algorithm  | Number of Clusters |
|------------|--------------------|
| GA         | 3                  |
| PSO        | 4                  |
| ACO        | 6                  |
| EMO        | 8                  |
| FEMO (Proposed) |             |

Fig. 15. Airplane image (color) (Fig. 4(k)-I011) obtained using different methods and for different no. of clusters.

**Algorithm 3: initPop(n, d):** Initialize the population

**Input:** Size of the population \( n \) and the dimension of each point \( d \).

**Output:** Initial population and the best fitness value of this population \( \zeta^{best} \).

1: Repeat the following for \( i = 1 : n \) times
2: \hspace{1em} Repeat the following for \( j = 1 : d \) times
3: \hspace{2em} \( \zeta^{i}_{j} = I_{low} + random(0,1)(I_{high} - I_{low}) \)
4: \hspace{1em} end for
5: \hspace{1em} end for
6: Compute the fitness of each point \( fitness(i) \leftarrow calcFitness(\zeta^{i}) \)
7: \hspace{1em} end for
8: \( \zeta^{best} \leftarrow \arg \min \{ fitness(i), \forall i \} \)
The objective function certainly varies depending on the underlying problem definition. One important point worth mentioning here is that the proposed approach can handle only one objective at a time. The cntIteration is a counter that takes care of the number of iterations and the total process is repeated for gItrCount number of times. A local search is performed localSearch(IltrCount, λ) to exploit the best solutions achieved so far. It is helpful in effectively reaching the global optimum. The maximum number of iterations for the local search is denoted by IltrCount and the local search parameter is denoted by λ ∈ [0, 1]. After performing the local search, the force exerted by each particle is computed using Eq. (10). Depending on the fitness values, two types of forces can be computed i.e. attraction or repulsion. Depending on these computed forces the particles are moved further. After getting the new positions of the particles, the cluster centers are updated and the value of type 2 fuzzy membership is also updated. The same process repeats until the maximum number of iterations are achieved.

The proposed FEMO approach is the modification of the actual EMO approach where the type 2 fuzzy system is incorporated. The proposed system has a strong theoretical background and can be applied in different scenarios. The proposed systems can efficiently handle the uncertainties with the help of the fuzzy type 2 system. This work is applied to the proposed FEMO approach for image segmentation purposes, to be more precise, for the biomedical image segmentation purpose. But from the discussion of the proposed approach, it can be understood that the proposed FEMO approach covers the total application domain of the actual EMO procedure. Moreover, it will be interesting to apply the proposed approach with or without modifications to some similar problems where the traditional crisp clustering approaches and their allied methods fail. Moreover, images with different application domains (e.g. satellite images) can also be segmented with some little modifications to the proposed approach.

5. Results of the simulation

In this section, the experimental results are discussed in detail. A comparative study is presented along with the detailed interpretation of the obtained results.
Algorithm 5: `calcForce(fitness)`: calculate force

**Input:** fitness of the points `fitness`.

**Output:** Computed force $\tilde{F}$

1: Repeat for $i = 1 : n$ times
2: \hspace{1em} Compute $ch'$ using equation 9
3: \hspace{1em} $\tilde{F}_i \leftarrow 0$
4: \hspace{2em} end for
5: Repeat for $i = 1 : n$ times
6: \hspace{2em} If $fitness(j) < fitness(i)$ then
7: \hspace{3em} $\tilde{F}_i \leftarrow \tilde{F}_i + \left( \xi' - \xi_i \right) \frac{ch'ch_i}{\| \xi' - \xi_i \|^2}$ \hspace{1em} // Attraction force
8: \hspace{2em} Otherwise
9: \hspace{3em} $\tilde{F}_i \leftarrow \tilde{F}_i - \left( \xi' - \xi_i \right) \frac{ch'ch_i}{\| \xi' - \xi_i \|^2}$ \hspace{1em} // Repulsion force
10: \hspace{2em} end if
11: \hspace{2em} end for
12: end for

Algorithm 6: `moveParticles(force)`: move particles depending on their exerted force

**Input:** Exerted force by each particle `force`.

**Output:** Moved particles

1: Repeat for $i = 1 : n$ times
2: \hspace{1em} Check if $i \neq best$ then
3: \hspace{2em} $\varepsilon \leftarrow random(0,1)$
4: \hspace{2em} $\tilde{F}_i \leftarrow \tilde{F}_i$
5: \hspace{2em} end if
6: \hspace{1em} end for
7: \hspace{2em} Repeat for $j = 1 : d$ times
8: \hspace{3em} Check if $\tilde{F}_i^j > 0$ then
9: \hspace{4em} $\xi'_j \leftarrow \xi'_j + \varepsilon \cdot F_i^j \cdot (I_{high}^i - \xi'_j)$
10: \hspace{4em} Otherwise
11: \hspace{5em} $\xi'_j \leftarrow \xi'_j + \varepsilon \cdot F_i^j \cdot (\xi'_j - I_{low}^i)$
12: \hspace{4em} end if
13: \hspace{3em} end for
14: end for
5.1. Description of the dataset

As discussed earlier, the proposed FEMO method is applied to the biomedical images for segmentation purposes. To test the effectiveness and the real-life applicability of the proposed method, biomedical images of different modalities are considered to perform the experiments. Both grayscale and the color images are segmented using the proposed biomedical image segmentation method. The images are consolidated from various sources. Table 2 gives an overview of the selected images along with their sources. 10 biomedical images are considered from different sources with modalities including both color and grayscale types to illustrate the versatility of the proposed FEMO method. Although the proposed approach is dedicated to the biomedical image data but, for the sake of completeness and to show the efficiency and the effectiveness of the proposed approach the experiments are carried out on some other standard image data. The segmentation output of these images is reported in Figs. 15 through 19. The corresponding quantitative results are reported in Tables 3 to 6. Fig. 4 shows the original images under consideration.

5.2. Cluster validity indices

Visual inspection of the segmented images is not sufficient the judge the quality of the segmented regions and in most of the scenarios, the actual interpretation of the segmented output remains incomplete without the quantitative analysis. Moreover, to compare the proposed work, only the segmented images are not sufficient and some numerical results are required. In this work, four well-known cluster validity indices are used to prove the efficiency of the proposed method as well as for the comparison purpose. The cluster validity indices are discussed below.

A. Davies–Bouldin index: It is the ratio of the sum of the intra-cluster distance and the inter-cluster distance [68]. To achieve a good clustering result, the inter-cluster distance must be high and the intra-cluster distance should be low and therefore, a lower value of the Davies–Bouldin index DBInd is desirable and indicates a good clustering result. Eq. (11) can be used to compute the value of this index. Here, denotes the total count of the clusters. $p \neq l$ and the range of $p$ is $1 \leq p \leq c$.

$$DBInd = \frac{1}{c} \sum_{l=1}^{c} \max_{1 \leq q \leq m} \left( \frac{d_{iw}(a_{lp}) + d_{iw}(a_{lq})}{d_{ib}(a_{lp}, a_{lq})} \right)$$

B. Xie–Beni index: It is one of the most popular indices to judge the quality of the fuzzy clustering [69]. The value of the Xie–Beni Index XBind can be computed using Eq. (12). A smaller value of the XBind indicates good clustering outcome.

$$XBind = \frac{\sum_{q=1}^{c} \sum_{p=1}^{n} U_{qp}^2 \| V_q - X_p \|^2}{d_{min} \| V_q - V_p \|^2}$$

C. Dunn index: It is an important cluster validity index where the denominator contains the mean of the distances among all pairs of clusters and the numerator contains the distance between two clusters [70]. The value of the Dunn index can be computed using Eq. (13). Here, $d(c_i, c_j)$ is the distance between two clusters $c_i$ and $c_j$. $\Delta_q$ denotes the mean of the distances among all pairs of clusters. The higher value of the Dunn index indicates better result.

$$DL_n = \min_{1 \leq l \leq m} \left( \min_{1 \leq s \leq m, j \neq l} \left( \frac{d(c_i, c_j)}{\max_{1 \leq q \leq m} \Delta_q} \right) \right)$$
Fig. 16. Baboon image (color) (Fig. 4(l)-I₀12) obtained using different methods and for different no. of clusters.

### 5.3. Results

The experiments are performed using MatLab R2014a (windows version) with an Intel i3 processor (1.8 GHz Clock Speed) and 4 GB of RAM in the Windows 7 environment. As discussed earlier, both qualitative and quantitative analysis is performed to test and compare the proposed algorithm with some other standard methods like genetic algorithm (GA) [44], particle swarm optimization (PSO) [45], electromagnetism-like optimization (EMO) [17], ant colony optimization (ACO) [46]. The proposed algorithm is tested and compared for the different numbers of clusters. Figs. 5–19 illustrates the comparative results of the segmentation by applying the proposed FEMO method and the other methods. The images are obtained by optimizing the Davies–Bouldin index by the various optimization methods.

### 5.4. Analysis of the convergence rate

Study of the convergence is one of the important parameters which is needed to be discussed to complete the analysis of an optimization procedure. In this work, the study of the convergence is performed using the first image i.e. with image id I₀01. The Davies–Bouldin index is used to perform this study and the results are given in Fig. 20. The number of iterations vs. the value of the Davies–Bouldin index is plotted in each convergence curves which are given in Fig. 15. Different algorithms and for each of them, different numbers of clusters are compared. The graphical analysis is evident that the proposed FEMO method is effective and can be used to reach the global optimum in a reasonable amount of iterations. Moreover, the proposed FEMO

| Algorithm | Number of Clusters |
|-----------|--------------------|
| GA | 3 |
| PSO | 4 |
| ACO | 6 |
| EMO | 8 |

**D. β index:** The segmentation result can be evaluated by this index and it is defined in Eq. (14). The β index can be expressed as the ratio of the total variation to the inter-cluster variation [71]. Here, $cntP_s$ represents the count of the pixels in the $s$th cluster. $I_t$ is the intensity of the $t$th pixel which belongs to the $s$th cluster. The mean of the intensity values in a cluster $s$ is represented by $\hat{x}_s$. A well segmented set of clusters produces higher value of the $β$ index.

$$β = \frac{\sum_{s=1}^{C} \sum_{t=1}^{cntP_s} (I_t - \hat{x}_s)^2}{\sum_{s=1}^{C} \sum_{t=1}^{cntP_s} (I_t - \hat{x}_t)^2}$$

\(^{(14)}\)
method outperforms many standard methods in terms of the convergence.

The proposed approach is incorporating the type 2 fuzzy system with the EMO approach. The type 2 fuzzy system is helpful to model the uncertainties. The impact of the points which do not belong to a certain cluster creates a significant impact on finding the global optima. The proposed work addresses this problem. The proposed work can significantly diminish the impact of the points with very low membership values. The points with the lesser degree of the membership (i.e. with higher uncertainty) have a comparatively lesser effect on a cluster center than a point with a higher degree of membership (i.e. with lesser uncertainty). It helps to model various real-life problems efficiently and to reach global optima. The graphical analysis of the convergence is presented in Fig. 20 where the convergence curves for different approaches and the different number of clusters are presented that will provide a visual clue to analyze the convergence of the proposed work as well as some other works in the context of the biomedical image analysis.

6. Limitations of the proposed approach

The proposed approach is efficient enough to segment the biomedical images with different modalities but, the results will be more comprehensive if some more results are reported. The determination of the number of clusters is a challenging task and automated determination of the exact number of clusters can make the system robust but the proposed approach is not efficient enough to determine the number of clusters automatically which can be considered as the major drawback. The proposed approach is useful to optimize a single objective at any time. So the proposed approach cannot handle multiple objectives unless enhanced further. Besides some of these limitations, the proposed approach is efficient enough to segment the real-life biomedical images and the qualitative and quantitative results are helpful to show the efficiency of the proposed approach.

7. Conclusion

In this work, a new method of biomedical image segmentation is proposed where the advantages of the fuzzy type 2 clustering system and the EMO method is exploited to perform
Table 3
Quantitative analysis and comparison of different algorithms using Davies–Bouldin index (the acceptable values are highlighted).

| Image Id | Algorithm | No. of clusters | 3     | 4     | 6     | 8     |
|----------|-----------|----------------|-------|-------|-------|-------|
|          |            |                |       | 4     |       |       |
| 001      | GA         | 1.3914          | 1.60013 | 1.7985 | 1.4193 |       |
|          | PSO        | 1.9666          | 1.8196 | 1.8225 | 1.9007 |       |
|          | ACO        | 1.4182          | 1.3256 | 1.1093 | 1.7155 |       |
|          | EMO        | 1.7419          | 1.6385 | 1.4755 | 1.3027 |       |
|          | FEMO (Proposed) | 1.2748 | 1.3829 | **0.9936** | 1.1074 |       |
| 002      | GA         | 1.7845          | 2.3639 | 2.1007 | 1.8674 |       |
|          | PSO        | 2.0361          | **1.6784** | 1.7102 | 1.6974 |       |
|          | ACO        | 2.6337          | 1.9637 | 1.9069 | **1.8527** |       |
|          | EMO        | 1.7843          | 1.7486 | 2.0374 | **1.7109** |       |
|          | FEMO (Proposed) | 2.0396 | **1.6801** | 1.7023 | 1.9963 |       |
| 003      | GA         | 1.9011          | 0.8693 | **0.8687** | 1.3329 |       |
|          | PSO        | 0.9368          | **0.8875** | 0.9963 | 1.2736 |       |
|          | ACO        | 1.4032          | **1.1019** | 1.2986 | 1.4378 |       |
|          | EMO        | 1.3019          | **1.0014** | 1.3986 | 1.0137 |       |
|          | FEMO (Proposed) | 1.2013 | **0.9963** | 1.0113 | 0.9997 |       |
| 004      | GA         | 2.0003          | 2.0416 | **1.7485** | 1.8639 |       |
|          | PSO        | 1.9015          | 1.7851 | **1.6032** | 1.7663 |       |
|          | ACO        | 1.8056          | 1.7741 | **1.4398** | 1.6327 |       |
|          | EMO        | 1.9306          | **1.4916** | 1.7480 | 1.8023 |       |
|          | FEMO (Proposed) | 2.0136 | 1.4863 | **1.3918** | 1.6309 |       |
| 005      | GA         | 1.9608          | 1.8867 | **1.7486** | 1.8817 |       |
|          | PSO        | 1.8607          | 1.9005 | **1.8865** | 1.9036 |       |
|          | ACO        | 1.9306          | 1.8990 | **1.6031** | 1.7448 |       |
|          | EMO        | 2.0003          | 2.0126 | 1.9306 | **1.8803** |       |
|          | FEMO (Proposed) | 1.8805 | 1.6329 | **1.5589** | 1.7813 |       |
| 006      | GA         | 0.9906          | 1.0325 | **0.8923** | 1.2036 |       |
|          | PSO        | 1.4205          | 1.3062 | **1.0031** | 2.0003 |       |
|          | ACO        | 0.9940          | **0.8709** | 1.0231 | 1.2031 |       |
|          | EMO        | 0.9756          | 0.9806 | **0.8903** | 1.0032 |       |
|          | FEMO (Proposed) | 1.0596 | **0.7863** | 0.6696 | 1.4829 |       |

(continued on next page)
### Table 3 (continued)

| Image Id | Algorithm | No. of clusters |
|----------|-----------|-----------------|
|          |           | 3   | 4   | 6   | 8   |
| 007      | GA        | 1.8509 | 1.6332 | 1.9608 | 2.0037 |
|          | PSO       | 2.0369 | 1.5026 | 1.6478 | 1.9806 |
|          | ACO       | 1.9936 | 2.0019 | 1.7206 | 1.8476 |
|          | EMO       | 1.2208 | 1.8845 | 2.0063 | 2.0196 |
|          | FEMO (Proposed) | 1.1101 | 1.8929 | 1.4063 | 1.7178 |
| 008      | GA        | 2.8603 | 2.3605 | 2.9063 | 2.8693 |
|          | PSO       | 1.9963 | 1.8569 | 1.4757 | 1.7486 |
|          | ACO       | 1.8677 | 1.5869 | 1.8984 | 1.7728 |
|          | EMO       | 1.9698 | 1.5862 | 1.4869 | 1.4983 |
|          | FEMO (Proposed) | 1.6689 | 1.7842 | 1.3099 | 1.9907 |
| 009      | GA        | 1.9586 | 1.9086 | 1.7774 | 1.8693 |
|          | PSO       | 1.3396 | 2.1039 | 1.2019 | 2.0007 |
|          | ACO       | 1.2096 | 1.7059 | 1.8894 | 1.7692 |
|          | EMO       | 1.8692 | 1.7689 | 1.2307 | 1.8869 |
|          | FEMO (Proposed) | 1.0112 | 1.4966 | 1.3389 | 1.8639 |
| 010      | GA        | 2.4636 | 2.0456 | 1.8459 | 1.9663 |
|          | PSO       | 1.7863 | 1.4905 | 1.5369 | 1.7486 |
|          | ACO       | 2.0304 | 2.0036 | 1.8894 | 1.7728 |
|          | EMO       | 1.9936 | 2.0019 | 1.7206 | 1.8476 |
|          | FEMO (Proposed) | 1.1101 | 1.8929 | 1.4063 | 1.7178 |
| 011      | GA        | 2.2725938 | 2.44187343 | 3.06517091 | 2.499568245 |
|          | PSO       | 2.29770688 | 1.49987752 | 1.13358178 | 1.910599643 |
|          | ACO       | 1.8677 | 1.7689 | 1.2307 | 1.8869 |
|          | EMO       | 1.9698 | 1.5862 | 1.4869 | 1.4983 |
|          | FEMO (Proposed) | 1.6689 | 1.7842 | 1.3099 | 1.9907 |
| 012      | GA        | 2.1900947 | 1.59404807 | 1.316004409 | 1.301301326 |
|          | PSO       | 1.01294032 | 1.87395824 | 3.36971456 | 2.051345671 |
|          | ACO       | 1.20563868 | 2.04365834 | 1.77442625 | 1.205231313 |
|          | EMO       | 2.4056391 | 1.56893483 | 1.42893112 | 1.42893112 |
|          | FEMO (Proposed) | 1.44713586 | 1.55978613 | 2.30040073 | 1.9035 |
| 013      | GA        | 2.18770688 | 2.44187343 | 3.06517091 | 2.499568245 |
|          | PSO       | 2.29770688 | 1.49987752 | 1.13358178 | 1.910599643 |
|          | ACO       | 1.8677 | 1.7689 | 1.2307 | 1.8869 |
|          | EMO       | 1.9698 | 1.5862 | 1.4869 | 1.4983 |
|          | FEMO (Proposed) | 1.6689 | 1.7842 | 1.3099 | 1.9907 |
| 014      | GA        | 2.1900947 | 1.59404807 | 1.316004409 | 1.301301326 |
|          | PSO       | 1.01294032 | 1.87395824 | 3.36971456 | 2.051345671 |
|          | ACO       | 1.20563868 | 2.04365834 | 1.77442625 | 1.205231313 |
|          | EMO       | 2.4056391 | 1.56893483 | 1.42893112 | 1.42893112 |
|          | FEMO (Proposed) | 1.44713586 | 1.55978613 | 2.30040073 | 1.9035 |
| 015      | GA        | 2.18770688 | 2.44187343 | 3.06517091 | 2.499568245 |
|          | PSO       | 2.29770688 | 1.49987752 | 1.13358178 | 1.910599643 |
|          | ACO       | 1.8677 | 1.7689 | 1.2307 | 1.8869 |
|          | EMO       | 1.9698 | 1.5862 | 1.4869 | 1.4983 |
|          | FEMO (Proposed) | 1.6689 | 1.7842 | 1.3099 | 1.9907 |

The proposed approach is already successfully demonstrated and applied to different types of biomedical images as well as on some of the standard test images. This approach is certainly useful for various real-life applications like extraction of the lung nodules, tumor identification, identification of different foreign bodies inside the body as well as inside cells, cell counting, continuous assessment of patients, easy interpretation of the biomedical images, quantification of the tissue volumes, designing of the treatment flow and many more.

8. Discussion and future scopes

The proposed approach produces promising segmentation outputs as well as better quantitative results. The overall average of all 15 images is appended at the bottom of each table to get an insight into the overall performance of the proposed system as well as the performance of the other systems. It can be
Fig. 19. Cameraman image (grayscale) (Fig. 4(o)-h15) obtained using different methods and for different no. of clusters.

Table 4
Quantitative analysis and comparison of different algorithms using Xie–Beni index (the acceptable values are highlighted).

| Image Id | Algorithm | 3   | 4   | 6   | 8   |
|----------|-----------|-----|-----|-----|-----|
| 001      | GA        | 1.9863 | 1.8637 | 1.2569 | 1.8963 |
|          | PSO       | 1.5967 | 1.7386 | 1.4963 | 1.5586 |
|          | ACO       | 1.3266 | 1.4586 | 1.3376 | 1.5278 |
|          | EMO       | 1.4386 | 1.5369 | 1.3369 | 1.4757 |
|          | FEMO (Proposed) | 1.6348 | 1.1106 | 1.1014 | 1.3966 |
| 002      | GA        | 2.6931 | 3.4415 | 2.3596 | 2.1758 |
|          | PSO       | 1.9986 | 2.3468 | 1.5536 | 2.0034 |
|          | ACO       | 2.3391 | 2.0135 | 2.3332 | 2.1903 |
|          | EMO       | 3.1963 | 2.0034 | 1.9638 | 2.1034 |
|          | FEMO (Proposed) | 2.4537 | 1.9936 | 1.8837 | 2.2034 |
| 003      | GA        | 4.1023 | 3.0069 | 2.7362 | 3.0344 |
|          | PSO       | 3.1692 | 3.9968 | 2.4396 | 2.4997 |
|          | ACO       | 3.9001 | 3.6832 | 3.4759 | 2.8779 |
|          | EMO       | 2.9047 | 2.9986 | 2.6937 | 2.7966 |
|          | FEMO (Proposed) | 2.7968 | 2.8869 | 2.4371 | 2.5009 |

(continued on next page)
observed that in most situations the proposed method achieves the optimum values of the different indices. Also, the proposed approach shows a good convergence property with the Davies–Bouldin index. In this work, four different quantitative metrics are used among which Davies–Bouldin and Xie–Beni indices are expected to be minimized whereas Dunn and indices are expected to be maximized. The convergence curves are plotted against the Davies–Bouldin index. Typically, the biomedical

| Image Id | Algorithm | No. of clusters |
|----------|-----------|----------------|
| 004      | GA        | 1.9406         |
|          | PSO       | 2.0001         |
|          | ACO       | 2.3240         |
|          | EMO       | 1.9347         |
|          | FEMO (Proposed) | 1.8346 |
|          | GA        | 2.0397         |
|          | PSO       | 2.9638         |
|          | ACO       | 3.6015         |
|          | EMO       | 2.0936         |
|          | FEMO (Proposed) | 2.0037 |
| 005      | GA        | 1.7896         |
|          | PSO       | 0.8696         |
|          | ACO       | 1.9687         |
|          | EMO       | 1.6356         |
|          | FEMO (Proposed) | 0.9346 |
| 006      | GA        | 2.9637         |
|          | PSO       | 4.1906         |
|          | ACO       | 7.0946         |
|          | EMO       | 2.0316         |
|          | FEMO (Proposed) | 2.0031 |
| 008      | GA        | 1.9346         |
|          | PSO       | 1.4963         |
|          | ACO       | 1.7058         |
|          | EMO       | 3.0263         |
| 009      | GA        | 0.9963         |
|          | PSO       | 0.9976         |
|          | ACO       | 1.5031         |
|          | EMO       | 2.4023         |
|          | FEMO (Proposed) | 1.0126 |
| 010      | GA        | 1.0396         |
|          | PSO       | 1.9635         |
|          | ACO       | 3.1063         |
|          | EMO       | 1.4392         |
|          | FEMO (Proposed) | 1.8829 |
|          | GA        | 1.5562         |
|          | PSO       | 1.2613         |
|          | ACO       | 1.3868         |
|          | EMO       | 2.1888         |
|          | FEMO (Proposed) | 1.4319 |
| 011      | GA        | 1.5885         |
|          | PSO       | 2.6604         |
|          | ACO       | 1.9378         |
|          | EMO       | 1.2158         |
|          | FEMO (Proposed) | 1.0707 |
| 012      | GA        | 2.1723         |
|          | PSO       | 1.4694         |
|          | ACO       | 1.8361         |
|          | EMO       | 1.7054         |
|          | FEMO (Proposed) | 1.0035 |
| 013      | GA        | 1.2161         |
|          | PSO       | 1.7695         |
|          | ACO       | 1.4260         |
|          | EMO       | 1.7054         |
|          | FEMO (Proposed) | 1.0150 |
| 014      | GA        | 1.1099         |
|          | PSO       | 1.8117         |
|          | ACO       | 2.0662         |
|          | EMO       | 2.1786         |
|          | FEMO (Proposed) | 1.2197 |
| 015      | GA        | 1.9336         |
|          | PSO       | 2.3460         |
|          | ACO       | 2.2058         |
|          | EMO       | 2.0742         |
|          | FEMO (Proposed) | 1.6688 |

| Average  | GA        | 1.9336         |
|          | PSO       | 2.1460         |
|          | ACO       | 2.2058         |
|          | EMO       | 2.0742         |
|          | FEMO (Proposed) | 1.6688 |
Table 5
Quantitative analysis and comparison of different algorithms using Dunn index (the acceptable values are highlighted).

| Image Id | Algorithm | No. of clusters |
|----------|-----------|-----------------|
| 001      | GA        | 2.10684432      |
|          | PSO       | 4.09737247      |
|          | ACO       | 4.18330134      |
|          | EMO       | 2.94411463      |
|          | FEMO (Proposed) | 1.62773772 |
| 002      | GA        | 0.2677365       |
|          | PSO       | 1.3547746       |
|          | ACO       | 0.62490045      |
|          | EMO       | 0.8412856       |
|          | FEMO (Proposed) | 1.21635105 |
| 003      | GA        | 0.59273528      |
|          | PSO       | 0.2677365       |
|          | ACO       | 0.94482689      |
|          | EMO       | 1.7451169       |
|          | FEMO (Proposed) | 1.79089115 |
| 004      | GA        | 0.32102814      |
|          | PSO       | 0.40481052      |
|          | ACO       | 0.94482689      |
|          | EMO       | 1.7451169       |
|          | FEMO (Proposed) | 1.79089115 |
| 005      | GA        | 0.99832645      |
|          | PSO       | 0.54103151      |
|          | ACO       | 0.92769763      |
|          | EMO       | 1.29431165      |
|          | FEMO (Proposed) | 1.89290717 |
| 006      | GA        | 0.92588466      |
|          | PSO       | 0.64810528      |
|          | ACO       | 0.94482689      |
|          | EMO       | 1.29431165      |
|          | FEMO (Proposed) | 1.89290717 |
| 007      | GA        | 0.2488048      |
|          | PSO       | 2.20041125      |
|          | ACO       | 2.60217521      |
|          | EMO       | 2.2591123       |
|          | FEMO (Proposed) | 2.6822522     |
| 008      | GA        | 0.74537946      |
|          | PSO       | 3.51732955      |
|          | ACO       | 3.98345719      |
|          | EMO       | 5.7036333       |
|          | FEMO (Proposed) | 2.0857256     |
| 009      | GA        | 0.15338042      |
|          | PSO       | 2.20041125      |
|          | ACO       | 2.13070338      |
|          | EMO       | 2.5703633       |
|          | FEMO (Proposed) | 2.0857256     |
| 010      | GA        | 1.78282187      |
|          | PSO       | 2.25435492      |
|          | ACO       | 2.13070338      |
|          | EMO       | 2.17080214      |
|          | FEMO (Proposed) | 1.43919918 |
| 011      | GA        | 0.55622979      |
|          | PSO       | 1.26123657      |
|          | ACO       | 1.38880556      |
|          | EMO       | 2.18882664      |
|          | FEMO (Proposed) | 1.43919918 |
| 012      | GA        | 1.55855226      |
|          | PSO       | 2.06844017      |
|          | ACO       | 1.03787924      |
|          | EMO       | 1.21566022      |
|          | FEMO (Proposed) | 1.43079591 |
| 013      | GA        | 2.17231151      |
|          | PSO       | 0.46494438      |
|          | ACO       | 1.83618104      |
|          | EMO       | 1.7054192       |
|          | FEMO (Proposed) | 1.23335261    |
Table 5 (continued).

| Image Id | Algorithm | No. of clusters |
|----------|-----------|-----------------|
| 014      | GA        | 1.21610426      |
|          | PSO       | 1.76952746      |
|          | ACO       | 2.45396235      |
|          | EMO       | 2.814098457     |
|          | FEMO (Proposed) | 1.01508116 |
|          |           | 1.04945022      |
|          |           | 3.2373406       |
|          |           | 1.71508566      |
|          |           | 2.06480047      |
|          |           | 1.4152948       |
|          |           | 2.73561959      |
|          |           | 2.79780847      |
|          |           | 2.77900911      |
|          |           | 2.36404178      |
|          |           | 3.474456412     |
|          |           | 2.398212991     |
|          |           | 2.129794147     |
|          |           | 2.268506669     |
| 015      | GA        | 0.80693288      |
|          | PSO       | 1.81179581      |
|          | ACO       | 2.06625074      |
|          | EMO       | 2.17866432      |
|          | FEMO (Proposed) | 3.21971159 |
|          |           | 2.93229358      |
|          |           | 3.150539303     |
|          |           | 1.67891175      |
|          |           | 2.75567202      |
|          |           | 4.10878281      |
|          |           | 2.69453328      |
|          |           | 3.61384502      |
|          |           | 3.46823612      |
|          |           | 2.76789092      |
|          |           | 2.41636644      |
|          |           | 2.14954424      |
|          |           | 3.42263424      |
|          |           | 2.720478278     |
|          |           | 2.756816405     |
|          |           | 2.208487255     |
| Average  | GA        | 0.9635384       |
|          | PSO       | 1.8838942       |
|          | ACO       | 1.5460879       |
|          | EMO       | 1.9047452       |
|          | FEMO (Proposed) | 1.8116709 |
|          |           | 1.78725         |
|          |           | 1.5922767       |
|          |           | 2.187202        |
|          |           | 1.947281        |
|          |           | 2.0538313       |
|          |           | 2.549322        |
|          |           | 2.4373054       |
|          |           | 1.620324        |
|          |           | 2.0010333       |
|          |           | 1.739838        |
|          |           | 2.1213925       |

Fig. 20. Comparison of the convergence curves which are obtained using different methods and for the different number of clusters. The Davies–Bouldin index for the image $I_{001}$ is plotted in the Y-axis and the number of iterations is plotted in the X-axis of each curve. (a) Genetic Algorithm, (b) PSO, (c) ACO, (d) EMO, (e) FEMO (proposed).
Table 6
Quantitative analysis and comparison of different algorithms using β index (the acceptable values are highlighted).

| Image Id | Algorithm | No. of clusters |
|----------|-----------|-----------------|
| 001      | GA        | 0.91524179      |
|          | PSO       | 0.90593466      |
|          | ACO       | 1.47670969      |
|          | EMO       | 2.82095422      |
|          | FEMO (Proposed) | 1.52238599 |
| 002      | GA        | 1.09853016      |
|          | PSO       | 2.47122669      |
|          | ACO       | 0.44665583      |
|          | EMO       | 1.45099991      |
|          | FEMO (Proposed) | 1.53693387 |
| 003      | GA        | 1.52168335      |
|          | PSO       | 1.41758502      |
|          | ACO       | 2.65010819      |
|          | EMO       | 1.50269677      |
|          | FEMO (Proposed) | 0.69408921 |
| 004      | GA        | 0.31689412      |
|          | PSO       | 2.72940603      |
|          | ACO       | 2.39855303      |
|          | EMO       | 1.98063279      |
|          | FEMO (Proposed) | 1.08031794 |
| 005      | GA        | 1.54918912      |
|          | PSO       | 1.17578502      |
|          | ACO       | 2.65010819      |
|          | EMO       | 1.50269677      |
|          | FEMO (Proposed) | 1.28657943 |
| 006      | GA        | 1.91867046      |
|          | PSO       | 2.72940603      |
|          | ACO       | 2.39855303      |
|          | EMO       | 1.98063279      |
|          | FEMO (Proposed) | 1.08031794 |
| 007      | GA        | 0.78766122      |
|          | PSO       | 1.90683966      |
|          | ACO       | 1.10712624      |
|          | EMO       | 3.24535456      |
|          | FEMO (Proposed) | 1.7773547      |
| 008      | GA        | 1.49193122      |
|          | PSO       | 1.90683966      |
|          | ACO       | 1.10712624      |
|          | EMO       | 3.24535456      |
|          | FEMO (Proposed) | 1.7773547      |
| 009      | GA        | 1.44033882      |
|          | PSO       | 1.90683966      |
|          | ACO       | 1.10712624      |
|          | EMO       | 3.24535456      |
|          | FEMO (Proposed) | 1.7773547      |
| 010      | GA        | 2.80969785      |
|          | PSO       | 3.00290809      |
|          | ACO       | 1.76221804      |
|          | EMO       | 2.92081599      |
|          | FEMO (Proposed) | 3.16463937 |
| 011      | GA        | 1.22596717      |
|          | PSO       | 0.83290282      |
|          | ACO       | 1.54944391      |
|          | EMO       | 1.99818848      |
|          | FEMO (Proposed) | 0.67989786 |
| 012      | GA        | 2.11798834      |
|          | PSO       | 1.9808905       |
|          | ACO       | 1.0597706       |
|          | EMO       | 1.99818848      |
|          | FEMO (Proposed) | 0.67989786 |
| 013      | GA        | 1.44033882      |
|          | PSO       | 0.8380722       |
|          | ACO       | 1.8518455       |
|          | EMO       | 1.62611906      |
|          | FEMO (Proposed) | 0.79865138 |

(continued on next page)

can effectively handle this problem and the local and the global search helps to explore the total solution space efficiently and hence the proposed approach is performing well compared to the other of its competitors.
The local search method in the EMO can also be replaced by other methods. In fact, in some works, the local search method is replaced by other algorithms (e.g. genetic algorithm). In this work, only the original method is considered because the proposed FEMO method is designed to incorporate the type 2 fuzzy system in the actual EMO method and this blend is used in the biomedical image segmentation. So, the effect of the modifications in the local search method in the FEMO algorithm can also be explored next. The step length for the movement of the particles is computed with a randomly generated value ε which belongs uniformly in the range 0 and 1 (refer Eq. (11)). Now, this step length can be replaced with some other distribution and the performance can be checked. From algorithm 6, it can be observed that there is no need to move the point with the best fitness value and therefore, the force exerted on this point is no longer needed to be computed.

CRediT authorship contribution statement

Shouvik Chakraborty: Solution formulation, Conceptualization, Methodology, Software development, Article writing, Editing, investigation. Kalyani Mali: Formal analysis, Resources, review, supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors would like to express their gratitude and thank the editors, anonymous reviewers, and referees for their valuable comments and suggestions which are helpful in further improvement of this research work.

References

[1] N. Dhanachandra, Y.J. Chanu, An image segmentation approach based on fuzzy c-means and dynamic particle swarm optimization algorithm, Multimedia Tools Appl. 2020 (2020) 1–20, http://dx.doi.org/10.1007/s11042-020-08695-8.

[2] S. Chakraborty, S. Chatterjee, A.S. Ashour, K. Mali, N. Dey, Intelligent computing in medical imaging: A study, in: N. Dey (Ed.), Adv. Appl. Metaheuristic Comput., IGI Global, 2017, pp. 143–163, http://dx.doi.org/10.4018/978-1-5225-4151-6.ch006.

[3] S. Hore, S. Chakraborty, S. Chatterjee, N. Dey, A.S. Ashour, L. Van Chung, G. Nguyen, D.-. Nhong Le, An integrated interactive technique for image segmentation using stack based seeded region growing and thresholding, Int. J. Electr. Comput. Eng. 6 (2016) 2773–2780, http://dx.doi.org/10.11591/ijece.v6i6.11801.

[4] S. Hore, S. Chakraborty, A.S. Ashour, N. Dey, A.S. Ashour, D. Sifaki-Pistolla, T. Bhattacharya, S.R.B. Chaudhuri, Finding contours of hippocampus brain cell using microscopic image analysis, J. Adv. Microsc. Res. 10 (2015) 93–103, http://dx.doi.org/10.1166/jamr.2015.1245.

[5] S. Chakraborty, S. Chatterjee, N. Dey, A.S. Ashour, A.S. Ashour, F. Shi, K. Mali, Modified cuckoo search algorithm in microscopic image segmentation of hippocampus, Microsc. Res. Tech. (2017) 1–22, http://dx.doi.org/10.1002/jemt.22900.

[6] S. Chakraborty, K. Mali, An overview of biomedical image analysis from the deep learning perspective, in: S. Chakraborty, K. Mali (Eds.), Appl. Adv. Mach. Intell., Comput. Vis. Object Recognit. Emerg. Res. Oppor., IGI Global, 2020, http://dx.doi.org/10.4018/978-1-7998-2736-8.ch008.

[7] K.S. Chuang, H.L. Tzeng, S. Chen, J. Wu, T.J. Chen, Fuzzy c-means clustering with spatial information for image segmentation, Comput. Med. Imaging Graph. 30 (2006) 9–15, http://dx.doi.org/10.1016/j.compmedimag.2005.10.001.

[8] S.K. Choy, K. Yuen, C. Yu, Fuzzy bit-plane-dependence image segmentation, Signal Process. 154 (2019) 30–44, http://dx.doi.org/10.1016/j.sigpro.2018.08.010.

[9] S. Chakraborty, A. Raman, S. Sen, K. Mali, S. Chatterjee, H. Hachimi, Contrast optimization using elitist metaheuristic optimization and gradient approximation for biomedical image enhancement, in: 2019 Amity Int. Conf. Artif. Intell., IEEE, 2019, pp. 712–717, http://dx.doi.org/10.1109/AICAL.2019.8701367.

[10] S. Chakraborty, K. Mali, S. Chatterjee, S. Banerjee, A. Sah, S. Pathak, S. Nath, D. Roy, Bio-medical image enhancement using hybrid metaheuristic coupled soft computing tools, in: 2017 IEEE 8th Annu. Ubiquitous Comput. Electron. Mob. Commun. Conf. UEMCON 2017, 2018, http://dx.doi.org/10.1109/UEMCON.2017.8249036.

[11] L. Zadeh, Fuzzy sets, Inf. Control 8 (1965) 338–353, http://dx.doi.org/10.1016/S0019-9958(65)90241-X.

[12] X. Lei, H. Ouyang, Image segmentation algorithm based on improved fuzzy clustering, Cluster Comput. 22 (2019) 13911–13921, http://dx.doi.org/10.1007/10586-018-2128-9.

[13] M. Zhang, W. Jiang, X. Zhou, Y. Xue, S. Chen, A hybrid biogeography-based optimization and fuzzy C-means algorithm for image segmentation, Soft Comput. 23 (2019) 2033–2046, http://dx.doi.org/10.1007/s00500-017-2916-9.

[14] Y. A Tolias, S.M. Panas, Image segmentation by a fuzzy clustering algorithm using adaptive spatially constrained membership functions, IEEE Trans. Syst. Man Cybern. A 28 (1998) 359–369, http://dx.doi.org/10.1109/SMCA.1998.693067.

[15] J.C. Bezdek, R. Ehrlich, W. Full, FCM: The fuzzy c-means clustering algorithm, Comput. Geosci. 10 (1984) 191–203, http://dx.doi.org/10.1016/0098-3004(84)90020-7.

[16] O. Castillo, P. Melin, J. Kacprzyk, W. Pedrycz, Type-2 Fuzzy Logic: Theory and Applications, Institute of Electrical and Electronics Engineers (IEEE), 2008, p. 145, http://dx.doi.org/10.1109/978-1-59726-071-8.

[17] E. Cuevas, D. Oliva, M. Díaz, D. Zaldívar, M. Pérez-Cisneros, G. Pajares, White blood cell segmentation by circle detection using electromagnetism-like optimization, Comput. Math. Methods Med. 2013 (2013) 395071, http://dx.doi.org/10.1155/2013/395071.

[18] C.H. Lee, C.T. Li, F.Y. Chang, A species-based improved electromagnetism-like mechanism algorithm for TSK-type interval-valued neural fuzzy system optimization, Fuzzy Sets and Systems 171 (2011) 22–43, http://dx.doi.org/10.1016/j.fss.2011.02.004.

[19] D. Debels, M. Vanhoucke, The electromagnetism meta-heuristic applied to the resource-constrained project scheduling problem, in: Lect. Notes
[68] D.L. Davies, D.W. Bouldin, A cluster separation measure, IEEE Trans. Pattern Anal. Mach. Intell. PAMI-1 (1979) 224–227, http://dx.doi.org/10.1109/TPAMI.1979.4766909.

[69] X.L. Xie, G. Beni, A validity measure for fuzzy clustering, IEEE Trans. Pattern Anal. Mach. Intell. 13 (1991) 841–847, http://dx.doi.org/10.1109/34.85677.

[70] J.C. Dunn, Well-separated clusters and optimal fuzzy partitions, J. Cybern. 4 (1974) 95–104, http://dx.doi.org/10.1108/01969727408546059.

[71] S.K. Pal, A. Ghosh, B.U. Shankar, Segmentation of remotely sensed images with fuzzy thresholding, and quantitative evaluation, Int. J. Remote Sens. 21 (2000) 2269–2300, http://dx.doi.org/10.1080/01431160050029567.