Patterns of interactions at grade 5 classroom in learning the topic of statistics viewed from cognitive load theory

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Abstract. The nature of interactions that occurs among teacher, students, learning sources, and learning environment creates different settings to enhance learning. Any setting created by a teacher is affected by 3 (three) types of cognitive load: intrinsic cognitive load, extraneous cognitive load, and germane cognitive load. This study is qualitative in nature, aims to analyse the patterns of interaction that are constituted in mathematics instructions by taking into account the cognitive load theory. The subjects of this study are 21 fifth-grade students who learn mathematics in small groups and whole-class interactive lessons. The data were collected through classroom observations which were videotaped, while field notes were also taken. The data analysis revealed that students engaged in productive interaction and inquiry while they were learning mathematics in small groups or in whole class setting, in which there was a different type of cognitive load that dominantly affecting the learning processes at each setting. During learning mathematics in whole class setting, the most frequently found interaction patterns were to discuss and compare solution based on self-developed models, followed by expressing opinions. This is consistent with the principles of mathematics learning, which gives students wide opportunities to construct mathematical knowledge through individual learning, learning in small groups as well as learning in whole class settings. It means that by participating in interactive learning, the students are habitually engaged in productive interactions and high level of mathematical thinking.

Keywords: interaction, mathematics learning, cognitive load theory

1. Introduction
As a consequence of the implementation of curriculum 2013, which is better known as ‘K-13’ in Indonesia, mathematics instruction should stress the fundamental goals of education which is to provide students with creativity, critical thinking skills, communication and collaboration, which are called as the 'Four Cs' [1], [2]. As an effort to focus on providing 4C skills, mathematics learning should elaborate aspects of critical thinking and problem solving, communication and collaboration, as well as creativity and innovation [3]. Parallel to this point of view, it is stated that learning mathematics is viewed as a social endeavour in which mathematics classroom functions as a community where thinking, talking, agreeing, and disagreeing are encouraged [4] and [5]. The teacher should provide students with powerful instructional materials which can promote the students to work together as well as to justify and to explain their solutions. The primary goal is to extend one’s own thinking as well as that of others.

K-13 which has been implemented in Indonesia since 2013, places a strong emphasis on meaningful process of learning. Consequently, students are expected to construct viable arguments and express their opinion as well as critique the reasoning of others. In mathematics classroom, meaningful collaboration relies on purposeful instructional moves from the teacher, as well as a clear
understanding of the lesson materials that are placed on students. It means that a teacher should prepare the learning materials properly, and form the small group purposively in order to promote students’ interaction and to manage cognitive load that go together with learning. It often happens that, if group members are not heterogeneous or students are not in their zone of proximal development, the students’ collaboration will not work optimally [6]. Therefore, in this study the researcher formed groups of students heterogeneously, and uncovered the actual zone or actual development level of students before collecting the data.

Reviewing Vygotsky’s theory [7] about Zone of Proximal Development (henceforth, ZPD) it is stated that there are two levels of development, namely, the actual development level and the potential development level [8]. The actual development level refers to the individual’s ability to perform certain activities independently, while in the potential level the individual can perform activities when help or support is given. The difference between the two levels is the ZPD which Vygotsky defines as “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance of and in collaboration with more capable peers.” In fact, Vygotsky’s work put emphasis on the importance of having students work with one another in addition to receiving instruction from adults.

The use of small group work in mathematics classes has both academic and social benefits to students ([9], [10], [11], [12]. Although there are many ways that group work can and has been used in mathematics classrooms, one very typical way that some teachers utilize this instructional practice is for students to work on an often rich and open-ended task in small groups, often for a substantial period of time within a single lesson. Despite its promise, the use of group work poses particular instructional challenges. For example, (a) Students may not work cooperatively; (b) Academically heterogeneous groups may be difficult to manage; and (c) The teacher’s role in monitoring and supervising group work may be confusing and uncomfortable. Furthermore, (d) The teacher must be mindful of the ‘groupworthiness’ of the task assigned [10], (e) Create classroom norms to facilitate positive group functioning, and ensure that classroom management is kept under control while monitoring accountability and participation of all members of each group.

To analyze the patterns of interaction in small group work, in this study the researcher prepare a lesson plan and students’ worksheet in which the teacher’s activities as well as those of the students are clearly managed. It should be noted that the definition of group work in this study aligns with the term cooperative group work, in which individual students accountable for learning collectively without competition; the teacher creates roles for students, facilitates small group work, intervenes the group work if only needed, and leads the whole class discussion in the stage of meaning negotiation, as well as explicitly teaches students the social skills to work together productively [10], [13]. Parallel to this view, it is observed that students benefit when teachers’ intervention during group work can focus on both the dynamics of group functioning (process-help interventions) and the mathematical content (product-help interventions) [14]. In accordance with these research results, in this study the researcher created a lesson plan that was expected to help the teacher in assisting the students either as process-help interventions or product-help interventions, in the material of arithmetic mean.

The aim of this study is to describe the patterns of interaction in mathematics instructions viewed from cognitive load theory. An interaction involves at least two participants. Thus, interaction is a mutually beneficial activity that requires the involvement of at least two people and which provides a mutually beneficial effect. The relationship between the two elements is said to be interactive if it is possible for an interaction to happen. Interaction can also take place in a person's mind, whether he/she is involved in the kind of 'private conversation' as voiced by Vygotsky, and when different schemes in the mind of the person are interacting to construct an understanding or response to a number of phenomena. This is the underlying form of 'interaction with self' in the process of problem solving and the construction of mathematical knowledge. It was argued that the quality of interaction between teachers and students is an example of an external influence that has been shown to contribute to student involvement in learning.
This study involves two types of interaction, namely vertical interaction and horizontal interaction. Vertical interactions include the interaction between teachers and students, which can occur in three ways: (a) between teacher and a student, (b) between teacher and small group of students, and (c) between teacher and all students in class. In horizontal interaction, students work and discuss together. This can happen to a small group or to an entire class. In both cases, in principle the discussion is led by the teacher. Therefore, it can be said that the term interaction includes various teaching activities.

Teachers, students and learning materials are the main elements that are directly involved in the teaching and learning process. Teachers facilitate the learning process by creating situations and conditions that stimulate students to be active in learning, while lesson material acts as an intermediary in the interaction between teachers and students. Based on the above opinions, the so-called interaction in this study is activities between a student and another student, between a student and other students in groups, between students and a teacher, and between students and teaching materials, as long as students assigned to solve problems in a mathematics instruction.

Some experts mention the factors affecting the interaction [15], which says that there are two fundamental factors influencing the interaction process, namely (1) the degree of difference in skills or in terms of knowledge and experience; (2) The level of intensity of interaction or cooperation. If the problem is interesting, students will discuss it or discuss it with great interest and enjoyment, both with friends and with teachers. However, students will only be able to think and discuss about the problem if they have the knowledge or experience.

Viewed from the relation between teacher and students, there are two types of interactions, namely, horizontal interaction and vertical interaction. Vertical interaction is interaction between teachers and students, while horizontal interaction is interaction among students in a naturally occurring relationship. There is another type of interaction [16], simultaneous interaction, which integrates vertical interaction and horizontal interaction. Vertical interaction occurs between teacher and one student, a group of students or the whole class, whereas horizontal interactions occur between students and students, or between students and groups of students [6]. In addition, she also states that the third type of interaction, i.e. simultaneous interaction, occurs between teachers and students, as well as between teachers, students or groups of students. In the division of the above three types of interactions, learning resources and learning environments are not included into the learning elements that participate in the learning interaction.

This research involves three types of interaction, namely vertical interaction, horizontal interaction and simultaneous interaction, with an additional element which is learning resource/material. This is consistent with the notion that teachers, students and learning materials or subject matter are the main elements involved directly in the teaching and learning process. Teachers facilitate the learning process by creating situations and conditions that stimulate students to be active in learning, while learning materials act as an intermediary in the interaction between teachers and students.

In horizontal interaction, students work and discuss the learning materials together, so that an interaction occurs between two students, or between students and groups of students, or between students with all students in the classroom. Based on the above explanation, the so-called interaction in this study is activities between two students, between a student and other students in groups, between students and teachers, and between students and teaching materials, as long as students assigned to construct knowledge or to solve mathematics problems.

Some experts state that cognitive load affects the process of constructing new knowledge. The cognitive load theory was introduced in the mid-1980s by John Sweller, an Australian educational psychologist. He refers to the information processing theory introduced by George Miller in the 1950s, that human working memory has a very limited duration and capacity (7 ± 2) or even just (4 ± 1) elements [17]. This number will be reduced to only two or three if multiple manipulations are performed with those elements. He also stated that learning is an alteration of long-term memory, which in humans has an unlimited practical capacity. Such capacity is used to store knowledge in the form of a "scheme," which means as cognitive constructs that incorporate multiple elements of information into a single element that has a particular function. A key aspect of cognitive load theory
is the attention given to human cognitive architecture - the characteristics and relationship between long-term memory and memory, and how the load on the cognitive system affects learning.

The purpose of cognitive load theory is to predict learning outcomes by taking into account the abilities and limitations of human cognition. The cognitive load theory is inspired by the idea that effective learning scenario design should be based on knowledge of the workings of the human mind. The cognitive load during learning is determined by the working memory of cognitive activities designed to achieve the learning objectives. The consideration of how cognitive load affects learning is one of the core objectives of cognitive load theory. The cognitive learning theory emphasizes the way a person uses his mind to learn, remember, and use the knowledge that has been acquired and stored in his mind effectively.

With regard to cognitive load theory, it is stated that cognitive load in working memory can be caused by three sources, which are (1) intrinsic cognitive load, (2) extrinsic or extraneous cognitive load, and (3) constructive (germane) cognitive load [18]. If cognitive load works beyond the memory capacity, then the cognitive load will be higher. In other words, if the total working memory load is excessive, the chances of a meaningful change in long-term memory will decrease. Intrinsic cognitive load is determined by the degree of complexity of the information or material being studied, while extraneous cognitive load is determined by the presentation technique of the material [19]. Intrinsic cognitive load cannot be manipulated because it has become the character of the interactivity of elements in the material. It means that intrinsic cognitive load is fixed. In contrast, extraneous cognitive load can be manipulated. Good material presentation techniques, which do not complicate understanding, will decrease extraneous cognitive load.

Understanding of a material can easily occur if there is sufficient prerequisite knowledge, which can be summoned from long-term memory. If this prerequisite knowledge can automatically be presented in the working memory, then the extraneous cognitive load will be minimal. The more knowledge that can be used automatically, the lower the cognitive load in working memory. In this case, the working memory capacity is increasing. Materials that have intrinsically heavy loads, if presented well, then the cognitive processes in the working memory will run smoothly. Conversely, although intrinsic cognitive load of a material, including mild if poorly presented, e.g. too much or random, then the cognitive process of working memory will run slowly.

Extraneous cognitive load is essential when intrinsic cognitive load is high, since both types of cognitive loads are additive. The total cognitive load should not exceed the working memory capacity. As a result, the instructional design intended to reduce cognitive loads will be effective when the interactivity is high. When the interactivity of the element is low, the design intended to reduce the load on working memory has little effect, even no effect [21]. In general, extraneous cognitive load can be generated by one or more of the following causes: (1) To find the solution steps using the procedure (rather than directly learning the procedure which is the solution of the lesson) (2) Students need to build new references of information (instead of using cognitive resources to build new representations) 3. Learning that introduces new elements which involves too much information in working memory, to be incorporated into long-term memory structures, (4) Separate representation (in space and time) of subjects that require students to discover and to process.

The third type of cognitive load experienced during the learning process is germane cognitive load which describes the mental effort that a student invests in processing information. In other words, the teacher designs materials and activities with the aim of improving germane cognitive load. If the students' intrinsic motivation to study a particular subject is low, then the teacher may use an approach to increase extrinsic motivation, such as through study incentives. A teacher can plan learning materials aimed at reducing extraneous cognitive load, i.e. processing tasks that have no relevance of information, and improving germane cognitive load, i.e. the amount of mental effort that students invest in processing the subject matter. This balance is very important when intrinsic cognitive load is high. If extraneous cognitive load is reduced, germane cognitive load will automatically increase, as students will devote the same effort to learning regardless of the effectiveness of the lesson. One strategy to improve germane cognitive load is to increase variability or contextual disturbance.
Encouraging students to explain for themselves the reasons for choosing problem solving can also lead to germane cognitive load. Germane cognitive load will only be effective if students are at appropriate difficulty levels for their age. An exaggerated explanation or overload from a teacher will lead to extraneous cognitive load, not germane cognitive load [22].

Based on the above description, the understanding of the three types of cognitive load in relation to learning activities can be explained as follows: (1) Intrinsic cognitive load is an activity attached to the content of the lesson. This is primarily determined by the learning objectives, as well as the knowledge and skills associated with the learning objectives (2) Extraneous cognitive load is an activity that is not relevant to the learning objectives imposed by the lesson designer (3) Germane cognitive load is activity occurring in the learning activity that relies on learning objectives through the acquisition of schemes. Therefore, effective learning lies in the optimization of cognitive loads in the limited capacity of students’ working memory. If limited capacity will be overloaded, the acquisition scheme will be disrupted, resulting in lower performance [23]. This is a basic principle of cognitive load theory that effective learning can be achieved by managing intrinsic cognitive load, reducing extraneous cognitive load and increasing germane cognitive load, all within the limited management available for working memory.

2. Method
This study is descriptive-qualitative in nature. As the participants of this study were 21 grade 5th students of an elementary school in Surabaya. The teacher who conducted mathematics instructions in this study was Mrs. Mar, one of senior teachers in this school. The researcher prepared a lesson plan on the materials of statistics which consisted of reading the data presented in tabular form, writing the meaning of median data, and determining the median of a group of data. Besides, the researchers also provided student worksheets, and administered assessments of student learning.

The data collection method in this study involved video-taping the mathematics instructions with three cameras. One camera focused on the teacher, one camera focused on a group of students, while another camera focused on the whole classroom. This three-camera technique rendered this classroom data set an excellent resource for studying how was the teaching and learning process going on, and what were the teacher activities and the students activities in classroom discussion in order for the students to construct the learning materials. Besides, by affording visual and auditory access to both teacher and students concurrently, it became possible to conduct a detailed examination of how were the patterns of interactions in the mathematics classroom viewed from cognitive load theory.

Then, the researcher conducted a qualitative analysis which consists of 3 (three) stages which are data reduction, data presentation, and conclusion [24]. Data reduction is a form of analysis that sharpens, classifies, directs, discards the unnecessary, and organizes the data in such a way that the final conclusion can be drawn. One of the ways used by researchers in data reduction is coding the words or phrases in order to describe and identify meaning or data patterns. The next step is presenting the data. The data that can be obtained by the researcher are presented in the form of words or sentences. The presentation of the data is done with the transcript of the interview, which is a detailed and complete description of what is seen and heard through the recording. The third step in analyzing the data is to draw conclusion. The conclusion of the research is drawn continuously while in the field, but new conclusions can be obtained when all data have been collected and all processes of data analysis have been done.

3. Results and Discussion
In this study the researcher observes the teacher activities in implementing the prepared lesson plan to conduct mathematics instructions, and then analyse the patterns of interaction by taking into account the cognitive load theory. One of the instructional tools used in this study is student worksheet. Basically, by giving student worksheet to students, the teacher encompasses three types of cognitive loads at once, which are managing intrinsic load, decreasing extraneous load, and increasing germane load, because the function of student worksheet is to help student grasping the mathematical concept
to be learned easier. This also means the teacher simplifies the level of the material complexity being studied. In other words, teachers manage intrinsic cognitive load. At the same time, the teacher uses good material presentation techniques, which make it easier for students to understand the material, thereby increasing the intrinsic load, and lowering the extraneous load. The following is an example of contextual problem in a worksheet used in this study.

Figure 1. An example of contextual problem in the material of arithmetic mean used in this study

As can be seen in figure 1, the worksheet contains mathematical tasks that are appropriately challenging and enhance students’ learning. The mathematical tasks investigate important mathematical ideas and relevance for students, as can be seen in the implicit notion of arithmetic mean in the problem. The students are also asked to count, to compare and then to analyse in their effort to find the answer of the problem. In other words, it can be said that the mathematical activities and the problems posed encourage investigation, promote reasoning, and require students to provide justifications for their thinking. Ultimately, the mathematical tasks in the worksheet are worthy of students’ work in group and emphasize important mathematical concepts.

We have a specific reason why we use the material of statistics in this study. Statistical knowledge is indispensable not only for individuals to learn the content, but it also has social impact. Therefore, the material of statistics functions as either a scientific knowledge or as a practice of scientific production. There are four steps of solving problem that are used as a basis to analyse the pattern of interaction in this study, namely, understanding the contextual problem, solving the problem

Please look at Table 1 and Table 2 below. What is the space for each student?

Table 1. Area of classroom for each grade fifth student

| Classes | Classroom area (m²) | Number of students | Area of classroom for each student |
|---------|---------------------|--------------------|-----------------------------------|
| Class 5A | 43.3 | 10 | |
| Class 5B | 70.0 | 25 | |
| Class 5C | 80.1 | 25 | |
| Class 5D | 64.2 | 23 | |
| Class 5E | 61.1 | 24 | |
| Class 5F | 61.8 | 24 | |
| Class 5G | 62.4 | 22 | |

Table 2. Area of classroom for each grade sixth student

| Classes | Classroom area (m²) | Number of students | Area of classroom for each student |
|---------|---------------------|--------------------|-----------------------------------|
| Class 6A | 60.7 | 24 | |
| Class 6B | 70.0 | 17 | |
| Class 6C | 62.3 | 26 | |
| Class 6D | 70.5 | 17 | |
| Class 6E | 64.1 | 26 | |
| Class 6F | 92.0 | 16 | |

1. What is the average space for each grade 5 student? ___________________
2. What is the average space for each grade 6 student? ___________________
3. What is the average space for each grade 5 and sixth grade students? ____________
4. If there are 3 new students in a class, will the class become more crowded or less crowded? Adding a new student to a class means the number of students in the class ____________, so the area or space for each student will be _________________.
individually, comparing and discussing answers, as well as drawing a conclusion or writing a summary. Description of interaction patterns on each stage will be given in the following paragraphs.

At the introductory stage, the teacher prepares students to study, for example by asking students to put unused items into lockers, so that only stationery and notebooks are remaining on the table. This activity is included into environmental provision [25]. At this level students are supported for independent learning, so the teacher needs to prepare the learning environment and condition so as the students are ready to learn. In addition, before interacting with students, teachers prepare student worksheets for activities in small groups and make classroom arrangements.

Other activities undertaken by a teacher in the preliminary stage is to conduct apperception or associate the material to be studied with relevant material ever studied by the students. For example, the teacher asks students of the material learned in the last mathematics lesson which consist of modus, median, and various graphs or diagrams to present data. By knowing the materials that have been studied by the students, it means that the teacher succeeds in determining the student's actual zone which will be developed maximally in learning.

In the second phase, the interaction patterns that occurred are as follows. (1) The teacher distributes the teaching materials in the form of student worksheet; (2) The students are required to self-study the material within a certain time (5 minutes); (3) When the students encounter difficulties, they are asked to communicate with a friend who is sitting beside (or closed to) him/her before asking the teacher; (4) If the students already understand the problem, then he/she is asked to solve the problem individually; and then (5) The teacher gives scaffolding to generate the emergence of interaction. These patterns of interaction are in harmony with Piaget's individual constructivism theory which gives students widespread opportunity to construct their own knowledge, since knowledge is not passively accepted through the sense device or by means of communication, but must be constructed by the student himself as the cognizing subject. Therefore, the form of interaction in the second stage can be illustrated as follows.

![Figure 2. Interaction pattern in the phase of solving contextual problem individually](image)

In the third step, the teacher divided the students into five small groups whose members were heterogeneous. This is parallel to the findings of studies by Galton and his colleagues which suggest that students are more likely to succeed in undertaking cognitive tasks when they work in pairs or small groups. The small groups consisted of 4 (four) or 5 (five) students which was in line with the recommended size for the pursuit of cooperative and collaborative tasks, with the tasks involving enrichment and incremental learning. After small-group work, the teacher also implemented whole class learning that was expected to provide a context for the wider transmission of knowledge.

The teacher's step of asking the students to work with his/her friend sitting closed to him/her is a right step, and can be done smoothly. This is also supported by opinions on cognitive load theory that a teacher should focus on the learning material so that the teacher can manage intrinsic cognitive load as well as improve germane cognitive load [26] and [17]. It is possible that both students who are working in pair encounter difficulty in understanding the contextual problem. If it happens, these students may ask some help from the teacher. The patterns of interaction in this stage can be described as follows.
ZPD (Develop students' actual abilities).
Managing Intrinsic CL
Increasing Germane CL.

T = Teacher
S = Student

Figure 3. Interaction pattern in which two students sitting nearby try to solve the contextual problem, but it turns out to be difficult, so they can ask for help from the teacher.

If there are two students working together but still encounter difficulty in solving the problem, the teacher asks them to work with two or three other students sitting opposite them (the chairs in the classroom are arranged so that the students sit facing each other). Therefore, there are two possible forms of interaction, i.e. student interaction in groups, in a free interaction model, meaning that no one becomes the leader of the discussion; or controlled interaction, i.e. one student leads a discussion within the group.

Figure 4. Interaction pattern where four students sit facing each other try to solve the contextual problem.

In such interaction pattern, students experience to express opinions, listen to the opinions of other members of the groups, solve problems and examine the results of problem solving. This is in line with the suggestion that students need experience with individual problem solving, because sharing opinions or skills with other students by discussing or showing results is not enough for students to learn.

In the third stage of mathematics learning which is the stage of comparing and discussing answers, the interactions patterns are as follows, (1) The teacher transforms the learning into discussion with classroom setting, so the interaction in this step is a multi-direction interaction that occurs between groups, and between the teacher and groups, (2) Students are asked to discuss in small groups consisting of 4 (four) persons, (3) Student groups are asked to present the results of their discussion in front of the class, and (4) The teacher leads the class discussion at the stage of negotiation of meaning. With such an interaction pattern, students interact with other students in one group, before they interact with the teacher. If small groups have difficulty working on student worksheet or solving contextual problems, they can ask the teacher. It is at this point that the teacher gives appropriate scaffolding. This form of interaction is in accordance with the opinion which says that classical learning plays an important role in mathematics instruction, and that learning mathematics is a social activity [27], [28].
In the fourth stage of mathematics learning in this study, the teacher motivates students to use their reasoning, and to analyze arguments. There is an interaction between students and learning materials or between students and teacher. The teacher directs students to draw conclusions about the concepts or procedures learned, and then the students write a summary about the mathematics concepts that have been learned. If it is viewed from cognitive load theory, it means that the teacher manages the intrinsic cognitive load, as well as increases germane cognitive load. This also means that the teacher trains students to think critically, in addition to direct students to draw conclusions about the concepts or procedures being studied. The teacher’s activities in this stage are in line with the recommendation on the need to equip students with 4C skills, which students desperately need to live in the 21st century.

4. Conclusion
In the new curriculum (K-13), learning mathematics is viewed as a social endeavour in which mathematics classroom functions as a community where thinking, talking, agreeing, and disagreeing are encouraged. It is parallel to the recommendation that encourages educators to prepare the students with the skills needed to live in the 21st century which are called ‘Four Cs’ (creativity, critical thinking skills, communication and collaboration).

Based on the data which were collected, the dominant student activity is to discuss, compare and discuss answers based on self-developed models, followed by expressing opinions. This is consistent with the principles of mathematics learning, which gives students wide opportunities to construct mathematical knowledge through individual learning, learning in small groups as well as learning in whole class settings [29], [30], [31], [32], [27], [28], [33].

Other types of student activities tend to be evenly distributed. This indicates that students perform a variety of activities, indicating that they are actively interacting with teachers, with fellow students, as well as with teaching materials throughout the learning process. The most dominant student activity is to discuss and compare students’ answers based on self-developed models. When viewed from the standpoint of cognitive load theory, this is consistent with teachers' attempts to manage intrinsic cognitive loads, because students make their own models so that they can make course material easier to learn. In addition, by asking students to discuss and compare their solutions, the teacher attempts to reduce extraneous cognitive load, and increase germane cognitive load. As a result, there is sufficient space in the students’ memory to make it easier to construct the material learned [35], [26], [21], [36], [37].

In order to help students summarize and understand their thinking as well as the thinking of others, the teacher provide opportunities for students to “turn and talk” about their solutions. Teacher should also facilitate the sharing of strategies with all students in the classroom to promote student reflection on the different strategies. This gives students practice in constructing arguments, providing justifications, and critiquing the thinking of others. In summary, how successfully a teacher facilitates collaborative work will drives how mathematically rigorous the work is for students. Equally important is that students know how to listen to the thinking of others, and pose questions, as a way of deepening their mathematical understanding.

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References

[1] National Education Association NEA 2011. Preparing 21st century students for a global society: an educator’s guide to the “four Cs”. (Washington: National Education)

[2] As’ari A.R. 2016. Menjawab tantangan pengembangan 4C’s melalui pengembangan kurikulum dan pembelajaran matematika Proc. Sem. Nas. Pendidikan Matematika (Malang: Pascasarjana Universitas Negeri Malang) pp 1-7

[3] Partnership for 21st Century Skills 2011 21st Century Skills Map: Math (Washington: P21.org)

[4] National Council of Teachers of Mathematics 2000 Principles and standards for school mathematics (Reston, VA: NCTM)

[5] Nathan MJ and Knuth EJ 2003 A study of whole classroom mathematical discourse and teacher change Cognition and Instruction vol 27 no 2 pp 175–207

[6] Setianingsih R, Sa’dijah C, As’ari AR and Muksar M 2017 Investigating fifth-grade students’ construction of mathematical knowledge through classroom discussion IEJME — Mathematics Education vol 12 no 4 Look Academic Publishers pp 383-396

[7] Vygotsky L 1978 Mind in Society: The development of higher psychological processes (Cambridge, MA: Harvard University Press)

[8] William, M and Burden, R 1997 Psychology for language teachers (Cambridge, UK: Cambridge University Press)

[9] Boaler J 2008 Promoting ‘relational equity’ and high mathematics achievement through an innovative mixed ability approach British Educ. Res. J. vol 34 no 2 (Taylor & Francis) pp 167-194 retrieved from https://dx.doi.org/10.1080/01411920701532145

[10] Cohen EG 1994 Designing group work: strategies for the heterogeneous classroom (New York, NY: Teachers College Press) retrieved from https://bookgoogleco

[11] Slavin RE 1980 Cooperative learning Rev. of Educ. Res. vol 50 (Baltimore, MD: Center for Social Organization of Schools, John Hopkins University) pp 315-342

[12] Yackel E, Cobb P, and Wood T 1991 Small-group interactions as a source of learning opportunities in second-grade mathematics J. for Res. in Math. Educ. vol 22 no 5 retrieved from http://mathforum.org

[13] Sharan S and Sharan Y 1992 Expanding cooperative learning through group investigation (Colchester, VT: Teachers College Press)

[14] Dekker R and Elshout-Mohr M 2004 Teacher interventions aimed at mathematical level raising during collaborative learning Educ. Stud. in Math. vol 56 no 1 pp 39-65 Retrieved from http://www.jstor.org/stable/4150263

[15] Granott N and Gardner H 1994 When minds meet: interaction, coincidence and development in domains of activity RJ Sternberg & RK Wagner Eds, Mind in Contexts Interactionist Perspectives on Human Intelligence (Cambridge, MA: Cambridge University Press) pp 133-152

[16] Nelissen JMC 2002 Interactie: een vakpsychologische analyse 2 R Keijzer and W Uittenbogaard red Interactie in het reken-wiskundeonderwijs Freudenthal Instituut, Universiteit Utrecht retrieved from https://www.yumpu.com/nl/document/view/20990576/interactie-freudenthal-instituut-universiteit-utrecht

[17] Sweller J 1988 Cognitive load during problem solving: effects on learning Cog. Sci. vol 12 pp 257-285

[18] Nelissen JMC 2016 Transfer, a base for interaction and reflective thinking Curr. and Teach. vol 31 no 2 (Freudenthal Institute for Science and Mathematics Education, Utrecht University) pp 87-104 retrieved from http://dx.doi.org/10.7459/ct/31206

[19] Plaas FGWC, Renkl A and Sweller J 2006 Cognitive load theory and instructional design: recent developments Educ. Psych. vol 38 pp 1-4

[20] Chandler P and Sweller J 1991 Cognitive load theory and the format of instruction Cognition and Instruction vol 8 no 4 pp 293-332
[21] Sweller J 2004 Instructional design consequences of an analogy between evolution by natural selection and human cognitive architecture *Instruct. Sci.* vol 32 pp 9-31

[22] Kalyuga S, Ayres P, Chandler P and Sweller J 2003 The expertise reversal effect *Educ. Psy.* vol 38 pp 23-31

[23] Sweller J 2010 Element interactivity and intrinsic, extraneous, and germane cognitive load *Educ. Psy. Rev.* vol 22 pp 123-138

[24] Miles M B and Huberman AM 1994 *Qualitative data analysis: An expanded sourcebook* 2nd ed. (Thousand Oaks, CA: Sage Publications)

[25] Anghileri J 2006 Scaffolding practices that enhance mathematics learning. *J. of Math. Teach. Educ.* vol. 9 pp 33-52

[26] Paas F, Renkl A and Sweller J 2004 Cognitive load theory: instructional implications of the interaction between information structures and cognitive architecture *Instructional Science* vol 321 (Kluwer Academic Publishers the Netherlands) pp 1-8

[27] Van den Heuvel-Panhuizen M 2010 Reform under attack–forty years of working on better mathematics education thrown on the scrapheap? No way! *Shaping the future of mathematics education: Proc. of the 33rd Annual Conf. of the Math. Educ. (Research Group of Australasia)* Sparrow L, Kessane B and Hurst C Eds

[28] Van den Heuvel-Panhuizen M 2001 Realistic mathematics education as work in progress common sense in mathematics education *Proc. of 2001 the Netherlands and Taiwan Conf. on Math. Educ.* F L Lin Ed. pp 1-43

[29] Freudenthal H 1991 *Revisiting mathematics education* (Dordrecht, the Netherlands: Kluwer Academic Publishers)

[30] Gravemeijer KPE 2010 Realistics mathematics education theory as a guideline for problem-centered, interactive mathematics education *A Decade of PMRI in Indonesia* Eds Sembiring R; Hoogland K; Dolk M (Bandung, Utrecht) pp 41-50

[31] Gravemeijer KPE 1994 *Developing realistic mathematics education* (Utrecht, the Netherlands: Freudenthal Institute)

[32] Treffers A 1987 *Three dimensions: a model of goal and theory description in mathematics instruction* (Freudenthal Institute, Utrecht University: The Wiskobas Project)

[33] Nelissen JMC 1999 Thinking skills in realistic mathematics In Hamers JHM, Van Luit JEH and Csapo B. Eds *Teach. and Learn. Thinking Skills* (Lisse, the Netherlands: Swets & Zeitlinger)

[34] Nelissen, JMC 2002 Interactie: een vakpsychologische analyse In Keijzer R and Uittenbogaard W ed *Interactie in het reken-wiskundeonderwijs* (Freudenthal Instituut, Universiteit Utrecht) pp 11-40 retrieved from https://wwwyumpu.com/nl/document/view/20990576/interactie-freudenthal-instituut-universiteits-utrecht

[35] Bruning RH, Schraw GJ and Monica M 2011 *Cognitive psychology and instruction* 5th ed. (Boston, MA: Pearson)

[36] Chandler P and Sweller J 1991 Cognitive load theory and the format of instruction *Cog. and Instruct.* vol 8 no 4 pp 293-332

[37] Subanji 2013 *Pembelajaran matematika kreatif dan inovatif* (Malang: Penerbit Universitas Negeri Malang)