Universal Quantum Entanglement between an Oscillator and Continuous Fields

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Quantum entanglement has been actively sought for in optomechanical and electromechanical systems. The simplest such system is a mechanical oscillator interacting with a coherent beam, while the oscillator also suffers from thermal decoherence. For this system, we show that quantum entanglement is always present between the oscillator and continuous outgoing fields — even when the environmental temperature is high and the oscillator is highly classical. Such universal entanglement is also shown to be able to survive more than one oscillation cycle if characteristic frequency of the optomechanical interaction is larger than that of the thermal noise. Furthermore, we derive the effective optical mode that is maximally entangled with the oscillator, which will be useful for future quantum computing and encoding information into mechanical degrees of freedom.

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Introduction. Entanglement, as one of the most fascinating features of quantum mechanics, lies in the heart of quantum computing and many quantum communication protocols [1]. Great efforts have been devoted to theoretical and experimental investigations of quantum entanglements in different systems with discrete or continuous variables. Due to advancements in fabricating low-loss optical elements and high-Q mechanical resonators, the quantum entanglement in optomechanical systems has recently aroused great interests, especially when many table-top experiments demonstrated significant cooling of mechanical degrees of freedom via active feedback or passive damping (self-cooling) [2, 3, 4, 5, 6, 7], which unveils the possibility of achieving quantum ground state of macroscopic objects [11, 12, 13]. This not only paves the way for high-precision measurements but also incorporating mechanical degrees of freedom as possible medium for storing and retrieving quantum information.

Theoretical analysis shows that by coupling oscillators to a Fabry-Perot cavity, one can create stationary (Einstein-Podolsky-Rosen) EPR-type quantum entanglement between optical modes and an oscillator [14] or even between two macroscopic oscillators [15, 16]. In Ref. [17], it was shown that entanglement between two oscillators can also be created by conditioning on continuous measurements of the common and differential optical modes in a laser interferometer. Interestingly, such entanglement does not depend on the environmental temperature \( T \) explicitly but rather scales as the ratio between characteristic interaction frequency \( \Omega_q \) (equivalent to optical power), and characteristic thermal-noise frequency \( \Omega_F \) (equivalent to \( T \)). In contrast, \( T \) enters explicitly and entanglement generally vanishes at high temperature in cases considered in Refs. [14, 15, 16]. This discrepancy arises from the following facts: (i) The cavity in Ref. [17] is tuned and thus always stable, while in Refs. [14, 15, 16], stability requirements of the system set an upper limit on \( \Omega_q \); (ii) More importantly, due to finite transmission of the cavity, information leaks into the environment. Therefore, even regardless of thermal heat bath, the reduced system consisting of cavity modes and the oscillator is not in a pure state. In Ref. [17], however, there can be either no cavity or cavity with very broad bandwidth, the outgoing fields containing information of oscillator motion are all registered by photodetector.

This motivates us to consider that the entanglement scaling obtained in Ref. [17] can be inherent in the simplest system — an oscillator interacting with a coherent beam, which models the essential process in all above-mentioned optomechanical systems. The model and its spacetime diagram are shown schematically in Fig. 1. Similar system was analyzed previously by Pirandola et al. [18]. They used narrow-detection-band approximation to introduce sideband modes, which maps outgoing fields into two effective degrees of freedom. In the situation here, sideband modes are not well-defined, because...
the interaction turns off at \( t = 0 \) and only half-space \([-\infty, 0)\) is involved. Instead, we will directly evaluate the entanglement between the oscillator and outgoing fields \( \hat{b} \) (infinite degrees of freedom) using the positivity of partial transpose (PPT) criterion \([13, 20, 21, 22, 23, 24, 25, 26]\). Only in weak-interaction and low-thermal-noise limit (\( \Omega_q, \Omega_F \ll \omega_m \)) can we make correspondences between our results and those obtained in Ref. \([18]\).

Dynamics and Covariance Matrix. The Heisenberg equations for this optomechanical system are simply

\[
\dot{x}(t) = \hat{p}(t)/m, \\
\dot{\hat{p}}(t) = -2\gamma_m \hat{p}(t) - m\omega_m^2 \hat{x}(t) + \alpha \hat{a}_1(t) + \hat{\xi}_{\text{th}}(t), \\
\dot{\hat{a}}_1(t) = \hat{a}_2(t) + (\alpha/\hbar) \hat{x}(t).
\]

(1)

(2)

(3)

Here \( \hat{x} \) and \( \hat{p} \) are oscillator position and momentum; \( \hat{a}_1\) and \( \hat{b}_1 \) \((i = 1, 2)\) are quadratures of ingoing and outgoing optical fields, \( \hat{a}_1 \equiv (\hat{a} + \hat{a}^\dagger)/\sqrt{2} \) and \( \hat{a}_2 \equiv (\hat{a} - \hat{a}^\dagger)/(i\sqrt{2}) \) (the same for \( \hat{b}_1 \); (a) \( I_0 \hat{h} \omega(q)/c^2 \)^{1/2} \( \equiv (h m \Omega_q^2)^{1/2} \) is the optomechanical coupling strength, where \( \omega_0 \) and \( I_0 \) denote the laser frequency and optical power respectively and we have defined \( \Omega_q \); \( \alpha \hat{a}_1 \) is the radiation-pressure term. Since \( \dot{\hat{a}}_1(t), \hat{a}_1(t') = 0 \), the presence of thermal noise \( \hat{\xi}_{\text{th}} \) ensures the correct commutator between \( \hat{x}(t) \) and \( \hat{p}(t) \). The solution to oscillator position \( \hat{x}(t) \)

\[
\hat{x}(t) = \int^t_{-\infty} dt' G_x(t - t') [\alpha \hat{a}_1(t') + \hat{\xi}_{\text{th}}(t')] ,
\]

(4)

where Green’s function \( G_x(t) \equiv e^{-\gamma_m t} \sin(\omega_m t)/(m \omega_m) \). The radiation-pressure term \( \alpha \hat{a}_1 \) induces quantum correlations between the oscillator and the optical fields, but it is undermined by \( \hat{\xi}_{\text{th}} \). The question would be whether quantum entanglement exists or not after evolving the entire system from \( t = -\infty \) to \( 0 \). Since variables involved are Gaussian and linear dynamics will preserve Gaussianity, entanglement is completely encoded in the covariance matrix \( V \) of the optomechanical system. With optical fields labeled by continuous coordinate \( t \), elements of \( V \) involving optical degrees of freedom would be defined in the functional space \( \mathcal{L}^2[-\infty, 0] \). Specifically,

\[
V = \begin{bmatrix} A & C^T \\ B & C \end{bmatrix} ,
\]

(5)

Here \( A_{ij} = \langle \hat{X}_i \hat{X}_j \rangle_{\text{sym}} \) \((i, j = 1, 2)\) with vector \( \hat{X} \equiv [\hat{x}(0), \hat{p}(0)] \) and \( \langle \hat{X}_i \hat{X}_j \rangle_{\text{sym}} \equiv (\hat{X}_i \hat{X}_j + \hat{X}_j \hat{X}_i)/2 \) denoting symmetrized ensemble average; \( C_{ij} \) and \( B_{ij} \) should be viewed as vectors and operators in \( \mathcal{L}^2[-\infty, 0] \). In the coordinate representation, \( (t | C_{ij}) \equiv (\hat{X}_i \hat{b}_j(t') )_{\text{sym}} \) and \( (t | B_{ij} | t') \equiv (\hat{b}_i(t) \hat{b}_j(t'))_{\text{sym}} \), in which \( (|) \) denotes the scalar inner product in \( \mathcal{L}^2[-\infty, 0] \). PPT Criterion. According to Refs. \([23, 26]\), in order for one particle and a joint system of arbitrarily large \( N \) particles to be separable, a necessary and sufficient condition is that partially transposed density matrix \( \hat{\rho}^T_N \) (with respect to the first particle) should be positive semidefinite, i.e. \( \rho^T_N \geq 0 \). In the phase space of continuous Gaussian variables, this reduces to the Uncertainty Principle

\[
V_{pt} + (1/2) K \geq 0.
\]

(6)

Here commutator matrix \( K = \bigoplus_{k=1}^{N+1} 2 \sigma_y \) with \( \sigma_y \) denoting Pauli matrix. According to the Williamson theorem, there exists a symplectic transformation \( S \in S_{2(2N+2, R)} \) such that \( S^T \hat{V}_{pt} S = \bigoplus_{k=1}^{N+1} \text{Diag}[\lambda_k, \lambda_k] \). Using the fact that \( S^T V_{pt} S = K \), the above Uncertainty Principle reads \( \lambda_k \geq 1 \). If this fails to be the case, i.e. \( \exists \lambda_k < 1 \), the states are entangled. The amount of entanglement can be quantified by the logarithmic negativity \( E_N \)

\[
E_N \equiv \max[-\sum_k \ln \lambda_k, 0] \text{ for } k : \lambda_k < 1.
\]

(7)

In the case here, \( N \) approaches \( \infty \). Besides, the partial transpose is equivalent to time reversal. Therefore \( V_{pt} = V |\hat{\rho}(0)\rangle - \langle \hat{\rho}(0)|. \) Normalizing \( \hat{x} \) and \( \hat{p} \) with respect to their zero-point values, the commutator reads \( [\hat{x}, \hat{p}] = 2i \). For the optical fields, we set \( \langle \hat{b}_1(t) \hat{b}_2(t') \rangle = 2i \delta(t - t') \), which gives the coordinate representation of \( K \).

According to Ref. \([24]\), \( \lambda_k \) can be obtained by solving

\[
V_{pt} V = (1/2) \lambda K V,
\]

(8)

where \( V \equiv [\alpha_0, \beta_0, (|\beta\rangle)^T \) with \( |f \rangle \) denoting the vector in \( \mathcal{L}^2[-\infty, 0] \). Due to uniqueness of \( [\alpha] \) and \( [\beta] \) in terms of \( \alpha_0 \) and \( \beta_0 \) for any \( \lambda < 1 \) (non-singular), Eq. \( 8 \) leads to the following characteristic equation

\[
\lambda \sigma_y + C^T (\lambda \sigma_y + B)^{-1} C = 0
\]

(9)

It can be shown that

\[
(\lambda \sigma_y + B)^{-1} = \begin{bmatrix} 1 + B_{11} M^{-1} B_{12} & -B_{11} M^{-1} \\ -B_{12} M^{-1} B_{11} & M^{-1} \end{bmatrix}
\]

(10)

where we have used the fact that \( B_{12} = B_{21} \) in \( \mathcal{L}^2[-\infty, 0] \). Have defined \( B_{11} \equiv B_{12} - i \lambda \) and \( M \equiv B_{22} - B_{11} B_{11} \). The integral operator \( M \) can be inverted via Wiener-Hopf method. Given any function \( |g \rangle \equiv M^{-1} |h \rangle \), in the frequency domain, it reads

\[
\hat{g}(\Omega) = \int_0^\infty dt e^{i \Omega t} M^{-1} |h \rangle = \{1/\tilde{\psi}_-\} [\tilde{h}/\tilde{\psi}_+\]...

(11)

Here \( |\rangle \) means taking the causal part of given function (with poles in lower-half complex plane) and factorization \( \tilde{\psi}_- \tilde{\psi}_- \equiv \lambda \alpha \langle \alpha^2 / h | (\tilde{G}_{xx} - \tilde{G}_{xx}^*) + \langle \alpha/h \rangle^2 S_{FF} \tilde{G}_{xx} \tilde{G}_{xx}^* \) \( (12) \) with \( \lambda \equiv 1 - \lambda^2 \) and \( \tilde{G}_{xx} \) denoting the Fourier transformation of \( G_x(t) \). In the above equation, \( \tilde{\psi}_- (|\psi_-\rangle \) and its inverse are analytic in upper-half (lower-half) complex plane, \( \psi_+ (|\psi_+\rangle \) and \( \tilde{\psi}_+ \) are analytic in upper-half (lower-half) complex plane. In deriving Eq. \( 12 \), we have used \( \langle \hat{a}_i(t) \hat{a}_j(t') \rangle_{\text{sym}} = \delta_{ij} \delta(t - t') \), and for thermal noise, Markovian approximation is applied and \( \langle \hat{\xi}_{\text{th}}(t) \hat{\xi}_{\text{th}}(t') \rangle_{\text{sym}} = S_{RR} \delta(t - t') \) with \( S_{RR} = 4 m \gamma_m k_B T \equiv 2 h m \Omega_q^2 \) and \( \Omega_F \) defining the characteristic frequency.
Finally, an implicit polynomial equation for the simplicistic eigenvalue $\lambda$ is derived from Eq. (4). As it turns out, there always exists one eigenvalue $\lambda$ that is smaller than one and it only depends on the ratio between $\Omega_q$ and $\Omega_F$, which clearly indicates the universality of the quantum entanglement. In Fig. 2, the corresponding logarithmic negativity (c.f. Eq. (6)) is shown as a function of $\Omega_q/\Omega_F$. For a high-Q oscillator $Q_m \equiv \omega_m/(2\gamma_m) \gg 1$, up to the leading order of $1/Q_m$, a very elegant expression for $E_N'$ is derived and it is

$$E_N' = (1/2) \ln[1 + (25/8)\Omega_q^2/\Omega_F^2].$$  (13)

**Thermal Decoherence.** To investigate how long such entanglement can survive under thermal decoherence, after turning off the optomechanical coupling at $t = 0$, the mechanical oscillator freely evolves for a finite duration $\tau_s$, driven only by thermal noise. Due to thermal decoherence, entanglement will gradually vanish. Mathematically, the simplicistic eigenvalue will become larger than unity when $\tau$ is larger than the survival time $\tau_s$. By replacing $[\hat{x}(0), \hat{p}(0)]$ with $[\hat{x}(\tau), \hat{p}(\tau)]$ and making similar analysis, up to the leading order of $1/Q_m$, $\tau_s$ satisfies a transcendental equation: $4\Omega_q^4 \theta_q^2 - (2 \Omega_q^2 + \Omega_F^2)^2 \sin^2 \theta_q - 25 \omega_m^4 = 0$, with $\theta_q = \omega_m \tau_s$. In the case of $\Omega_q < \Omega_F < \omega_m$, the oscillating term can be neglected, leading to

$$\theta_q = (5/2)(\omega_m/\Omega_F)^2 = 5 Q_m/(2 \bar{n}_{th} + 1),$$  (14)

where we have defined the thermal occupation number $\bar{n}_{th}$ through $k_B T/(\hbar \omega_m) = \bar{n}_{th} + (1/2)$. For strong interaction $\Omega_q \gg \Omega_F$, the transcendental equation can be solved numerically, showing that $\theta_q > 1$ is always valid.

**Maximally Entangled Mode.** To gain insights into this entanglement, we apply the techniques in Ref. [25] and decompose outgoing fields into independent single modes by convoluting them with some weight functions $f_i$, namely

$$\hat{O}_i \equiv (f_i|\hat{b}), \quad [\hat{O}_i, \hat{O}^+_j] = 2 \delta_{ij},$$  (15)

which requires $(f_i|f_j) = \delta_{ij}$. If we define $g_{11} \equiv \Re[f_i]$ and $g_{22} \equiv \Im[f_i]$, the single-mode quadratures will be

$$\hat{X}_i = (\hat{O}_i + \hat{O}^+_i)/\sqrt{2} = \int_0^\infty dt \, g_{11} \hat{b}_1 - g_{12} \hat{b}_2,$$  (16)

$$\hat{Y}_i = (\hat{O}_i - \hat{O}^+_i)/(i \sqrt{2}) = \int_0^\infty dt \, g_{22} \hat{b}_1 + g_{11} \hat{b}_2.$$  (17)

Different choices of weight function will generally give optical modes that have different strength of entanglement with the mechanical oscillator. The function of particular interest is the one that gives an effective optical mode maximally entangled with the oscillator. Using the fact that logarithmic negativity is an entanglement monotone, the optimal weight function can be derived from the following constrained variational equation:

$$\left(\frac{\partial E_N^{\text{ub}}}{\partial g_i} + \mu_i g_i = 0 \quad (i = 1, 2),\right.$$  (18)

where we have neglected unnecessary indices and $\mu_k$ is Lagrange multiplier due to the constraint $(f|f) = 1$ and $E_N^{\text{ub}}$ quantifies entanglement in the subsystem consisting of the oscillator and the effective optical mode $[\hat{x}(0), \hat{p}(0), \hat{X}, \hat{Y}]$. As it turns out, the optimal weight functions $g_{12}$ have the shape of decay oscillation with poles $\omega$ given by the following polynomial equation

$$[\omega - \omega_m]^2 + \gamma_m^2 [\omega + \omega_m]^2 + \gamma_m^2] + \chi = 0,$$  (19)

where parameter $\chi$ is a functional of $g_{12}$ and also depends on $\Omega_q$ and $\Omega_F$. Therefore, the weight functions are

$$g_k(t) = A_k e^{\gamma_q t} \cos(\omega_q t + \theta_k) \quad (k = 1, 2),$$  (20)

with $\gamma_q$ and $\omega_q$ being imaginary and real parts of $\omega$. Analytical solutions to parameters $A_1, \omega_q, \gamma_q$ and $\theta_q$ requires exact expression of $\chi$ in terms of $g_k$, $\Omega_q$, and $\Omega_F$, which is rather complicated. Instead, we numerically optimize those parameters to maximize $E_N^{\text{ub}}$.

Taking into account $(f|f) = 1$, $A_1$ and $A_2$ can be reduced to a single parameter $\zeta$, which is defined through

$$A_k^2 = \frac{4 \gamma_q (\gamma_q^2 + \omega_q^2) \cos^2 [\zeta + k(\pi/2)]}{\gamma_q^2 + \omega_q^2 + \gamma_q \omega_q \sin(2\theta_k)}.$$  (21)

From Eq. [19], $\omega_q^2 - \gamma_q^2 = \omega_m^2 - \gamma_m^2$. In addition, a local unitary transformation (rotation and squeezing) will not change the simplicistic eigenvalue. Without loss of generality, we can fix that $\theta_1 = \pi/2$ and $\theta_2 = 0$. Therefore, only two parameters $\omega_q$ and $\zeta$ need to be optimized.

In the special case of weak-interaction and low-thermal-noise limit ($\Omega_q, \Omega_F \ll \omega_m$), the optimal $\zeta_{\text{opt}}$ is equal to $\pi/4$, which indicates $A_1 \approx A_2 = 2 \sqrt{\gamma_m}$ for a high-Q oscillator. Besides, as shown in the upper panel of Fig. 3 the optimal $\omega_q^{\text{opt}} = \omega_m$, leading to

$$f(t) = 2 \sqrt{\gamma_m} e^{\gamma_m t} + \zeta (i \omega_m t + \phi_0).$$  (22)

Therefore, the optimal weight function has the same shape as Stokes and Anti-Stokes sideband modes. This is similar to what been obtained in Refs. 13, 25; However, due to causality, the weight function here is defined in $\ell^2[-\infty, 0]$ rather than $\ell^2[-\infty, \infty]$ which is essential for defining sideband modes.

In the case of strong interaction and high thermal noise ($\Omega_q, \Omega_F > \omega_m$), the optimal $\omega_q$ deviates from $\omega_m$ and depends on $\Omega_F$ and $\Omega_q$, as shown in the lower panel of Fig. 3.
More generally, the optimal $\zeta_{\text{opt}} = \pi/3$ and $\omega_{\eta_{\text{opt}}}^0$ can be fitted by $\omega_{\eta_{\text{opt}}}^0 \approx (0.64 \Omega_F^2 + 0.57 \Omega_T^2)^{1/2}$. Correspondingly, the logarithmic negativity can be approximated as

$$E_N^{\text{sub}} \approx \frac{1}{2} \ln [1 + (15. \Omega_F^2 / (13. \Omega_T^2 + \Omega_F^2))],$$

which again manifests universality of the entanglement. Therefore, as long as the optimal weight function is chosen, one can always recover quantum correlations between the oscillator and the outgoing fields.

In principle, by choosing a weight function orthogonal to the optimal one obtained above, one can derive next-order optimal mode. Repeating this procedure will generate a complete spectrum of effective optical modes ordered by $E_N^{\text{sub}}$, which is analogous to obtaining wavefunctions and corresponding energy levels with variational method. This not only helps to understand the full entanglement structure but also sheds light on experimental verifications of such universal entanglement. Rather than trying to recover the infinite-dimension covariance matrix in Eq. (5), we can apply right weight functions to extract different effective optical modes and form low-dimension sub-systems. Take sub-system consisting of the oscillator and the maximally entangled optical mode for instance, $4 \times 4$ covariance matrix can be determined by measuring correlations among different quadratures. This can be achieved by using a local oscillator with time-dependent phase, which allows to probe both mechanical quadratures and those of the effective optical mode. For example, a quadrature $\hat{Q}_c = \hat{X} \sin \Omega t + \hat{Y} \cos \Omega t$ can be measured with the following local oscillator light:

$$L(t) \propto L_1(t) \cos \omega_0 t + L_2(t) \sin \omega_0 t$$

with $L_1(t) = g_1(t) \cos \zeta + g_2(t) \sin \zeta$ and $L_2(t) = g_2(t) \cos \zeta - g_1(t) \sin \zeta$. Synthesis of multiple measurements will recover the covariance matrix that we need to verify the entanglement.

Conclusion. We have demonstrated that quantum entanglement exists universally in system with a mechanical oscillator coupled to continuous optical fields. The entanglement measure — logarithmic negativity displays an elegant scaling which depends on the ratio between characteristic interaction and thermal-noise frequency. Such scaling should also apply in electromechanical systems whose dynamics are similar to what we have considered.

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