Analytical Model of a Single Link of Elastic Optical Networks

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ABSTRACT This article discusses and evaluates a model of resources to which multiservice traffic is offered. The initial assumption for the model is that calls of particular traffic classes are always serviced in neighboring allocation units of the resources under investigation. An elastic optical network, in which an allocation management mechanism that allocates an appropriate number of neighboring spectrum units depending on the required bitrate related to a given service class is employed, is chosen as a good example of this type of resource. The article proposes new occupancy distributions for these resources, both at the so-called microstate and macrostate levels, with particular attention given to the way the transitions between neighboring states of the Markov process that approximates the service process in the systems under investigation are modeled. The presented model is an approximate model, therefore the analytical results of the modeling are validated and verified on the basis of comparisons with the results of the simulations of a number of selected network systems.

INDEX TERMS Multiservice resources, elastic optical network, frequency slot unit, Markov process, occupancy distribution, blocking probability.

I. INTRODUCTION
There has been a constant and dramatic increase in the amount of data transferred over the network, which has been clearly observed over the recent years. All of this data transfer requires a good network speed and very low congestion from the sender’s as well as the receiver’s end, and consequently not only a larger number of network resources but also the most optimum allocation of these resources to differentiated traffic demands. Currently, the main solution in telecommunication being able to fulfill all the requirements addressed above is an elastic optical network (EON). In EONs, to make better use of the available resources and to allocate them to individual demands in an as optimal way as possible, a mechanism of allocation of an appropriate number of spectrum units depending on the required bitrate related to a given service class has been widely introduced. The spectrum units are referred to as frequency slot units (FSUs) [1].

The number of FSUs available in a single link is influenced by the applied modulation technique, distance and the quality of the connection path [1], [2], [3], [4], [5], [6], [7]. Having at our disposal a given number of available FSUs and the possibility of controlling a gap between channels, we are able to minimize the phenomenon of misalignment between the bitrate of a data stream related to a specific service (traffic class) and the optical layer of a backbone optical network [2], [8], [9].

The possibility of allocating a certain number of FSUs to a single traffic demand (a flow, a connection) is, still, however, limited to some extent: the FSUs (slots) that are required for the execution of a given connection (that requires a defined number of slots) has to be neighboring ones. This is one of the constraints defined in the routing and spectrum assignment (RSA) problem and it has a direct influence on the performance evaluation and on analytical analysis of the EONs [10], [11], [12], [13], [14].

The multiservice mathematical models of full-availability resources that have been used recently do not take into
consideration the necessity of the neighboring location of allocation units necessary for new calls to be executed, e.g. [15] and [16]. In [17], the so-called virtual resource model is presented that allows a connection path composed of several optical links of EONs to be set up. This model assumes that to execute each of the connections, neighboring FSUs in each link that belongs to the selected connection path are to be used. The model [17], however, has not been designed for analysis of single links in EONs. [18] proposes the first, approximated model of multiserive resources for EONs, in which neighboring FSUs are always used to set up connections. This model is based on the assumption that in a given state of the service process, the occupied FSUs that belong to already serviced connections are allocated randomly and do not take into consideration their immediate neighborhood. For thus located occupied FSUs, the probability of the occurrence of free (unoccupied) FSUs located next to one another is then determined, in the number defined by a new connection of a given class. This model is characterized by fair accuracy of the occupancy distribution for those mixtures of offered multiservice traffic in which the variances (divergencies) between the demands of individual classes – measured in the number of demanded FSUs – are just slight. Unfortunately, as the number of differences in the number of FSUs demanded by individual call classes increases, the accuracy of the model significantly decreases.

In [19], the classic approach of the matrix solution to the state equations of the multidimensional Markov process was used to construct the occupancy distribution in EON links, allocating each request in the neighboring FSUs. For this purpose, the states of the process and the transition intensity matrix between states were properly defined. The model proposed in [19] is characterized by high computational complexity, appropriate for algorithms for solving Markov processes with a very large number of states. The article considered two ways of allocating calls in links: Random-Fit, i.e., random allocation of calls (sequences next to each other FSUs) in a link, and First-Fit, where calls are allocated “next to each other”, starting with FSUs with the smallest indexes [20], [21]. The results concerning the comparison of the blocking probability in links with the Random-Fit and First-Fit type allocation indicate no significant – from an engineering point of view – differences between these allocation methods. Blocking probabilities for First-Fit and Random-Fit are slightly different, with Random-Fit being slightly higher. This result confirms and generalizes, the results known from research on telephone [22], [23] and multi-service network [24] resources, where the differences between the blocking probability for network systems with random call allocation (corresponding to Random-Fit allocation) and sequential call allocation (corresponding to First-Fit allocation) are small and irrelevant from an engineering point of view. Among other works on occupancy distribution calculation in EONs, it is worth noting the work [25], in which the mixture of call streams (up to three) offered to the EON link was approximated by the Markov-Modulated Poisson Process (MMPP) [26]. In [27], [28], and [29] the blocking probability in optical links, which were offered with two call classes, was considered. The small number of offered call classes is a significant limitation of the proposals [25], [27], [28], [29]. On the other hand, [30] analyzed the problem of improving the traffic capacity of optical links by introducing the so-called defragmentation mechanism, i.e., such a change in allocated FSUs that will enable the servicing of new calls.

The present article proposes therefore a new model that can help increase the accuracy of the results for mixtures of call streams that are characterized by large differences in demanded FSUs. In the model proposed — unlike in the model [18] — the initial assumption is that, in a given state of the service process, the occupied FSUs that belong to already serviced connections are arranged in a way that takes into consideration their neighborhood (mutual location). For thus arranged busy FSUs, the probability of finding free FSUs that are located next to one another, in the number defined by a new connection of a given class, is then determined. Additionally, a single EON link is considered, an example of which can be a direct connection between two different EON elements as well as a link between different levels in the Data Center structure.

According to the authors’ knowledge, apart from [18] and the model presented in this article, there are no works on models taking into account the occupation of neighboring FSUs and enabling the determination of different characteristics (occupancy distribution, blocking probability) in a single EON network link.

The article is structured as follows. Section II discusses the general occupancy distribution in state-dependent resources at the level of the so-called microstates, i.e. those states of the service process that are defined by the set of serviced calls of individual traffic classes. Section III proposes a method to determine conditional transition probabilities for individual microstates that belong to the service process of calls in those resources in which the groups of neighboring FSUs are always chosen to service calls. Section IV describes the general occupancy distribution in state-dependent resources at the level of the so-called macrostates, i.e. the states of the service process by the total number of occupied FSUs. Section V proposes a method for the determination of conditional transition probabilities for individual macrostates in the service process of calls in those resources in which always the neighboring FSUs are chosen to service calls. The proposed method is based on the choice of an appropriate representative microstate, for each of the macrostates, whose conditional transition probability approximates the conditional transition probability of a given macrostate. In addition, Section V proposes an algorithm for the determination of the occupancy distribution and conditional transition probabilities at the macrostate level in resources in which the neighboring FSUs are always chosen for service calls. In Section VI, the results of the analytical modeling and the simulation data for a selected number of resources that allocate neighboring serviced FSUs of particular calls are compared...
II. OCCUPANCY DISTRIBUTION AT THE MICRO-STATE LEVEL

Let us denote by $R_{SEQ}$, as in [18], the resources in which connections can be established exclusively in the neighboring FSUs. The neighboring FSUs required for a new connection to be set up are chosen randomly from among all possible sets of free FSUs. The resources $R_{SEQ}$ are offered a finite number of Poisson call classes. Each call of a given class demands an appropriate integer number of FSUs to execute a connection. The service time has exponential nature.

Let us adopt the following notation for System $R_{SEQ}$:

$M$ – the number of all call classes that are offered in the system under consideration,

$A_i$ – intensity of offered traffic of class $i$:

$$A_i = \lambda_i/\mu_i,$$  \hspace{0.5cm} (1)

$\lambda_i$ – intensity of calls of class $i$ ($1 \leq i \leq M$),

$\mu_i$ – service intensity for calls of class $i$,

$t_i$ – the number of FSUs required for a connection of class $i$ to be set up,

$V$ – system capacity expressed in FSUs.

Let us define the so-called microstate of the service process in the considered system [31], [32]: $X = \{x_1, x_2, \ldots, x_M\}$, where $x_i$ is the number of calls of class $i$ serviced in the microstate $X$. For each microstate $X$ the following parameters can be determined:

$x$– the number of all calls serviced in microstate $X$:

$$x = \sum_{i=1}^{M} x_i,$$  \hspace{0.5cm} (2)

$n_X$ – the number of all occupied FSUs in microstate $X$:

$$n_X = \sum_{i=1}^{M} x_it_i,$$  \hspace{0.5cm} (3)

$w_X$ – the number of all free FSUs in microstate $X$:

$$w_X = V - n_X.$$  \hspace{0.5cm} (4)

The introduction of the condition of the allocation of a new connection in neighboring FSUs implies that the full-availability resources (i.e. those in which a new call will always be serviced if only the system has the demanded number of free FSUs) will become state-dependent resources (i.e. the resources in which a new call will be serviced exclusively when the system has the demanded number of neighboring free FSUs). To determine the occupancy distribution at the macrostate level in state-dependent resources, it is possible to construct a model in which the necessary assumption is that the service process in a state-dependent system is a reversible process, eg. [33] and [15]. The property of reversibility leads to a description of the Markovian process on the basis of local balance equations between the neighboring states. Such equation for the microstates $X = \{x_1, x_2, \ldots, x_i, \ldots, x_M\}$ and $X - 1_i = \{x_1, x_2, \ldots, x_i - 1, \ldots, x_M\}$, where $1_i$ denotes a single call of class $i$, can be written as follows (Fig. 1):

$$x_i\mu_i [p(X)]_V = \lambda_i\sigma_i(X - 1_i) [p(X - 1_i)]_V,$$  \hspace{0.5cm} (5)

where:

$[p(X)]_V$ – the probability of microstate $X$,

$\sigma_i(X)$ – conditional transition probability for a call stream of class $i$ between neighboring microstates $X$ and $X + 1_i$.

The parameter $\sigma_i(X)$ is the measure of state dependence in a given Markovian process and defines this part of the call stream of class $i$ that transfers the process to older states. Taking into consideration (1) and multiplying both sides of Equation (5) by $t_i$, we get:

$$x_it_i [p(X)]_V = A_i\sigma_i(X - 1_i) [p(X - 1_i)]_V. $$  \hspace{0.5cm} (6)

Since by assumption the process under consideration is a reversible process, we can therefore sum up both sides of Equation (6) over all call streams between state $X$ and the states that are younger states. As a result, we obtain the following (Fig. 1):

$$[p(X)]_V \sum_{i=0}^{M} x_it_i = \sum_{i=0}^{M} A_i\sigma_i(X - 1_i) [p(X - 1_i)]_V.$$  \hspace{0.5cm} (7)

In accordance with (3), the sum of the left side of Equation (7) determines the number of all occupied FSUs in the microstate $X$. Therefore, eventually the occupancy distribution in the state dependent system at the microstate level can be written as follows:

$$n_X [p(X)]_V = \sum_{i=0}^{M} A_i\sigma_i(X - 1_i) [p(X - 1_i)]_V.$$  \hspace{0.5cm} (8)

The blocking probability $E_i$ for calls of class $i$ is the sum of all blocking states:

$$E_i = \sum_X (1 - \sigma_i(X)) [p(X)]_V.$$  \hspace{0.5cm} (9)
The difference in (9) is the conditional blocking probability for calls of class $i$ in microstate $X$. To determine the occupancy distribution (8) and the blocking probability (9), it is necessary to determine the value of the parameter $\sigma_i(X)$. The method for its determination will be presented in Section III.

### III. CONDITIONAL TRANSITION PROBABILITY FOR MICROSTATES

For a given connection of class $i$, the resources in $R_{SEQ}$ are occupied exclusively in the neighboring FSUs. The set of all $t_i$ neighboring FSUs that constitute a connection of class $i$, is chosen randomly from among all possible sets of free FSUs with the number of slots not lower than $t_i$. Let us consider the arrangement of busy and free FSUs in $R_{SEQ}$. If we assign a binary value of “1” to a free FSU and the binary value “0” to an occupied FSU, then the occupancy of resources can be presented as a binary sequence with the length $V$. Therefore, the microstate $X$ corresponds to the binary string in which there are $w_X$ “ones” interleaved by $x$ sets of “zeros” in which the total number of “zeros” is $n_X$. Let us call each set of “zeros” the interleaving (dividing) set, whereas each set of “ones” the access set. Fig. 2 shows a number of examples of the resources with the capacity of $V = 15$ FSUs. These resources service 3 classes of calls with demands equal to: $t_1 = 1$, $t_2 = 2$, $t_3 = 3$, respectively. Therefore Fig. 2a defines the microstate $X = \{1, 1, 1\}$. In this microstate, there are $n_X = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 3 = 6$ occupied FSUs (in the figure the shadowed “zeros”) and $w_X = V - n_X = 15 - 6 = 9$ free FSUs (“ones”). Note that in one of a number of possible allocations of the connections that correspond to microstate $X$ presented in the figure we have 3 interleaving (dividing) sets and 4 access sets. The arrival of a new call of the third class ($t_3 = 3$) is then tantamount to finding the access set that has at least three free FSUs. Note that from the point of view of the search of an appropriate access set, the number of busy FSUs in the interleaving sets is irrelevant. Hence, we will treat each serviced call as a dividing (interleaving) set to which one “zero” can be assigned (Fig. 2b). The allocation of occupied resources in $R_{SEQ}$ in state $X = \{1, 1, 1\}$ presented in Fig. 2 is just one of a number of possible allocations. The three other exemplary allocations are shown in Fig. 3. Having adopted the assumption that each interleaving set is composed of one “zero” (i.e. one fictitious FSU), we can obtain some fictitious resources with the capacity $V_X$:

$$V_X = V - n_X + x,$$

where $x$ and $n_X$ are defined by Formulas (2) and (3). Note that the parameter $w_X$, i.e. the number of free FSUs in the fictitious resources with the capacity $V_X$ is identical to the number of free FSUs in the real resources. Figure 4 shows selected allocations (arrangements) of calls and free FSUs in fictitious resources that correspond to respective allocations (arrangements) from Fig. 3 ($V_X = 15 - 6 + 3 = 12$).

### A. THE MAXIMUM AND THE MINIMUM NUMBER OF ACCESS SETS

Figures 3 and 4 show that allocations that are related to a given microstate can differ in the number of access and interleaving sets. Further on in our considerations, we will assume that all possible arrangements of occupied resources are equally probable for a given microstate. It is easy to see then that if in a given microstate $X$ there are $x$ calls that are being serviced, the maximum number of access sets $k_{X,max}$ is equal to:

$$k_{X,max} = x + 1. \quad (11)$$

In the considered microstate $X = \{1, 1, 1\}$, shown in Figures 3 and 4, the maximum number of access sets is equal to $k_{X,max} = 4$ (Fig. 3b, Fig. 4b).

The minimum number of access sets occurs in the allocation in which all occupied FSUs in a given microstate are arranged next to one another (just as it is shown in Figs. 3c and 4c). We can write therefore:

$$k_{X,min} = \begin{cases} 1 & \text{for } w_X \geq 1, \\ 0 & \text{for } w_X = 0. \end{cases} \quad (12)$$
B. THE NUMBER OF ALLOCATIONS IN ACCESS SETS
To determine the number of allocations in the access sets we can apply the reasoning used in [18]. Let us assume that in a given microstate \( X \) there are \( k \) access sets. Let us determine the number of possible arrangements of \( w_X \) elements in \( k \) sets with the maximum capacity \( t_i \). The assumption that the maximum capacity of the access set is \( t_i \) free FSUs means consideration of only those arrangements in which the maximum length of an access set is \( t_i \) “ones”. Note that the assumption on the existence of \( k \) access sets requires at least one of the “ones” allocated in each set of this type.

Let us denote the number of possible arrangements of \( w_X \) elements in \( k \) sets with the maximum capacity \( t_i \) by \( F(w_X, k, t_i, 1) \), with the additional assumption that in each set at least one element will be allocated. The parameter \( F(w_X, k, t_i, 1) \) can be determined by Formula [34]:

\[
F(w_X, k, t_i, 1) = \sum_{j=0}^{\lfloor w_X/k \rfloor} (-1)^j {k \choose j}(w_X - j t_i - 1)_{k - 1}. \tag{13}
\]

Formula (13) determines the number of allocations in the access sets with the maximum length of \( t_i \) “ones”. These allocations also include the arrangement in which all \( k \) access sets do not include \( t_i \) elements. Therefore the number of allocations that certainly include \( t_i \) “ones” in at least one access set is:

\[
R_{X,1}(k, t_i) = F(w_X, k, t_i, 1) - F(w_X, k, t_i - 1, 1), \tag{14}
\]

where \( F(w_X, k, t_i - 1, 1) \) is the number of the allocations of \( w_X \) elements in \( k \) access sets, determined with the assumption that the maximum length of an access set is \( t_i - 1 \) ones:

\[
F(w_X, k, t_i - 1, 1) = \sum_{j=0}^{\lfloor w_X/k \rfloor} (-1)^j {k \choose j}(w_X - j (t_i - 1) - 1)_{k - 1}. \tag{15}
\]

C. THE NUMBER OF ALLOCATIONS OF ACCESS SETS
For the required number of \( x \) interleaving sets (of serviced calls in microstate \( X \)), there is a number of different possible allocations of \( k \) access sets. For example, for the resources \( V = 15 \) FSUs, we can consider the microstate \( X = \{1, 1, 1\} \), where \( t_1 = 1, t_2 = 2, t_3 = 3 \). The fictitious capacity for this microstate is equal to \( V_X = 15 - 6 + 3 = 12 \) FSUs. Our further assumption is that we consider \( k = 3 \) access sets. For \( x = 3 \), it is possible to find 4 possible allocations of 3 access sets vs. the interleaving (dividing) sets. Let us denote the access set by \( d \) and the interleaving set by “0”. A possible allocation of the access sets and interleaving sets is the following: \( d00d00, 0d0d0d, d0d0d0, d0d0d0d \).

The number of possible allocations of access sets relative to the interleaving sets can be determined on the basis of the following reasoning: the number of the interleaving sets is equal to \( x \). In such a case, the number of potential allocations of access sets is \( x + 1 \) and it is just that number that the \( k \) access sets can be chosen from. Therefore:

\[
R_{X,0}(k, t_i) = \sum_{s=0}^{x} \binom{x+1}{k}. \tag{16}
\]

Now it is possible to determine the total number of allocations in access sets that takes into consideration their arrangement (location) relative to the dividing sets:

\[
R_X(k, t_i) = R_{X,0}(k, t_i) R_{X,1}(k, t_i). \tag{17}
\]

Eventually, on the basis of (14) and (16) we get:

\[
R_X(k, t_i) = \binom{x+1}{k} \left[ F(w_X, k, t_i, 1) - F(w_X, k, t_i - 1, 1) \right]. \tag{18}
\]

D. THE POSSIBLE NUMBER OF ALLOCATIONS IN MACROSTATE \( X \)
The number of possible allocations \( R_X(k_{X,\text{max}}) \) of unoccupied FSUs (the “ones”) in a given microstate \( X \) corresponds and is equal to unconditional allocations of \( w_X \) elements in \( k_{X,\text{max}} \) access sets. Therefore:

\[
R_X(k_{X,\text{max}}) = \binom{k_{X,\text{max}} + w_X - 1}{w_X}. \tag{19}
\]

E. THE PROBABILITY OF SETTING UP A CONNECTION OF CLASS \( i \) IN MICROSTATE \( X \)
Let us consider the probability of the event \( h_X(k, t_i) \) such that for the required number of access sets \( k \) in microstate \( X \) a sequence of unoccupied FSUs can be found (i.e. a sequence of the “ones”) with the length \( t_i \). This probability can be determined on the basis of (18) and (19):

\[
h_X(k, t_i) = \frac{R_X(k, t_i)}{R_X(k_{X,\text{max}})}. \tag{20}
\]

The probability \( p_X(k, \min t_i) \) that it is possible to find a sequence of “ones” with the length \( t_i \) or more in a given microstate \( X \) is the probability of setting up a connection of class \( i \) in microstate \( X \) that has \( k \) access sets:

\[
p_X(k, \min t_i) = \sum_{s=\min t_i}^{w_X} h_X(k, s) = \sum_{s=\min t_i}^{w_X} \frac{R_X(k, s)}{R_X(k_{X,\text{max}})}. \tag{21}
\]

Now, by taking into consideration all possibilities of the occurrence of an appropriate number of access sets in this microstate, it is possible to determine the probability of setting up a connection of class \( i \) in microstate \( X \) that can be treated as the conditional transition probability \( \sigma_i(X) \) in the distribution (8):

\[
\sigma_i(X) = \sum_{k=\min k_{X,\text{min}}}^{k_{X,\text{max}}} p_X(k, \min t_i) = \sum_{k=\min k_{X,\text{min}}}^{k_{X,\text{max}}} \sum_{s=\min t_i}^{w_X} h_X(k, s). \tag{22}
\]

The parameters \( k_{X,\text{min}}, k_{X,\text{max}} \) in (22) can be determined on the basis of (11) and (12). The probability \( h_X(k, s) \) can be calculated from (20).
To determine the occupancy distribution (8), it is possible to use the algorithm for the calculations of the distribution at the macrostate level in a state-independent system [35]. The appropriate algorithm for the distribution (8) in a state-dependent system is presented in Appendix A in [35].

### IV. OCCUPANCY DISTRIBUTION AT THE MACROSTATE LEVEL

The macrostate \( \{ n \} \) is defined by the total number \( n \) occupied FSUs and by not taking into consideration the division of FSUs between particular call classes. Hence, the probability of the macrostate \( \{ n \} \), i.e. the probability \( [P(n)]_V \) of the occupancy of \( n \) FSUs in a state-dependent system with the capacity \( V \) is the sum of the probabilities of all microstates \( X \), for which \( n_X = n \):

\[
[P(n)]_V = \sum_{X \in \Omega(n)} [p(X)]_V ,
\]

(23)

where

\[
\Omega(n) = \{ X : n_X = n \} .
\]

(24)

The occupancy distribution \( [P(n)]_V \) at the macrostate level can be approximated by the following recursive dependence [15], [32]:

\[
n[P(n)]_V = \sum_{i=1}^{V} A_i t_i \sigma_i(n-t_i ) [P(n-t_i)]_V ,
\]

(25)

where \( \sigma_i(n) \) is the conditional transition probability between neighboring macrostates and determines the influence of state dependability on call streams that come from the macrostate \( \{ n \} \).

The approximation (25) results from the assumption that the service process in a state dependent system at the macrostate level is a reversible process and from the assumption that the values of transition probabilities at the microstate and macrostate levels are identical [15], [32]:

\[
\bigvee_{n_X=n} \sigma_i(n) = \sigma_i(X).
\]

(26)

Distribution (25) is characterized by high accuracy and efficiently approximates a large number of systems and mechanisms currently used in practice, for example resources with reservation [36], [37], [38], switching networks [31], [39], [40], non-full-availability resources [32], [33], [41], [42], resources with limited availability [34], [43], systems with threshold compression [44], [45], [46], [47], [48], [49], multi-service, non-full-availability queueing systems [17], overflow systems [50], [51], [52] and a large number of others.

The blocking probability for calls of class \( i \) in state-dependent resources can be determined by the following formula:

\[
E_i = \sum_{n=1}^{V} [1 - \sigma_i(n)][P(n)]_V ,
\]

(27)

where \( 1 - \sigma_i(n) \) determines the conditional blocking probability in state \( n \) for calls of class \( i \).

Distribution (25) also allows the parameter \( y_i(n) \), i.e. the average number of calls of class \( i \) in macrostate \( \{ n \} \), to be determined, e.g. [15]:

\[
y_i(n) = \frac{A_i \sigma_i(n-t_i ) [P(n-t_i)]_V }{[P(n)]_V } .
\]

(28)

The application of the distribution (25) to analyse the system \( R_{SEQ} \) requires the probabilities \( \sigma_i(n) \) to be known. These probabilities will be discussed in Section V.

### V. CONDITIONAL TRANSITION PROBABILITY FOR MACROSTATES

Section III presents the method for a determination of the conditional transition probability \( \sigma_i(X) \) for the resources \( R_{SEQ} \) at the macrostate level. In the present section we will use this method to determine the conditional transition probability in \( R_{SEQ} \) for the macrostates \( \sigma_i(n) \). The adopted assumption is based on a determination – for each macrostate \( \{ n \} \) – of a representative microstate \( X_r = \{ x_1, r, x_2, r, \ldots, x_r, r, \ldots, x_M, r \} \) whose conditional transition probability approximates the conditional transition probability at the macrostate level:

\[
\sigma_i(n) = \sigma_i(X_r). \quad (29)
\]

### A. CHOICE OF A REPRESENTATIVE MICROSTATE

The number of occupied (free) FSUs in the representative state \( X \) is obviously equal to the number of occupied (free) FSUs in the macrostate \( \{ n \} \):

\[
n_X = n, \quad w_X = w. \quad (30)
\]

Our further assumption is that the number of serviced calls of class \( i \) in the representative microstate can be determined by the average value of the number of calls of class \( i \) in macrostate \( \{ n \} \) (Formula (28)). Therefore:

\[
\bigvee_{1 \leq i \leq M} x_{i,r} = y_i(n). \quad (31)
\]

Now, on the basis of the formulas presented in Section III, it is possible to determine the conditional transition probability for the macrostate \( \{ n \} \). Let us rewrite then in the successive sub-sections the formulas from Section III with appropriate indexation, relevant to the representative microstate \( X_r \).

### B. THE MAXIMUM AND THE MINIMUM NUMBER OF ACCESS SETS

The maximum \( k_{X_r, \text{max}} \) and the minimum \( k_{X_r, \text{min}} \) number of access sets can be rewritten, on the basis of (11) and (12), as follows:

\[
k_{X_r, \text{max}} = x_r + 1, \quad (32)
\]

\[
k_{X_r, \text{min}} = \begin{cases} 1 & \text{for } w \geq 1, \\ 0 & \text{for } w = 0. \end{cases} \quad (33)
\]
If non-integer values of \( x_r \) are obtained, further calculations should be carried out in parallel for the values of \( \lfloor x_r \rfloor \) and \( \lceil x_r \rceil \), and only at the final stage will linear interpolation of the final result be required.

According to (2) and (31), the total number of serviced calls in the representative microstate \( X_r \) is equal to:

\[
x_r = \sum_{i=1}^{M} x_{i,r} = \sum_{i=1}^{M} y_i(n).
\]

Now, we can determine the fictitious capacity of the representative state. In compliance with (10):

\[
V_{X_r} = V - n + x_r.
\]

### C. The Number of allocations/arrangements in access sets

The number of allocations in \( k \) access sets with the maximum length \( t_i \) of “ones”, on the basis of (18), can be rewritten in the following way:

\[
R_{X_s}(k, t_i) = \left( \frac{x_r + 1}{k} \right) [F(w, k, t_i, 1) - F(w, k, t_i - 1, 1)],
\]

where, in compliance with (13) and (15), we have:

\[
F(w, k, t_i, 1) = \sum_{j=0}^{\lfloor \frac{w}{k} \rfloor} (-1)^j \binom{k}{j} \left( \frac{w-jt_i-1}{k-1} \right),
\]

\[
F(w, k, t_i - 1, 1) = \sum_{j=0}^{\lfloor \frac{w-1}{k} \rfloor} (-1)^j \binom{k}{j} \left( \frac{w-jt_i-1}{k-1} \right).
\]

### D. Possible number of allocations in the representative state \( X_r \)

The number of possible allocations \( R_{X_s}(k_{X_r, \text{max}}) \) of free FSUs (“ones”) in the representative microstate \( X_r \) corresponds to the unconditional allocations (arrangements) \( w \) elements in \( k_{X_r, \text{max}} \) access sets. Therefore, on the basis of (19):

\[
R_{X_s}(k_{X_r, \text{max}}) = \binom{k_{X_r, \text{max}} + w - 1}{w}.
\]

### E. Conditional transition probability in the macro-state \( n \)

The conditional transition probability in the macrostate \( n \) can be approximated by the conditional transition probability in the representative macrostate \( X_r \). Hence, on the basis of (22), we can write:

\[
\sigma_i(n) = \sigma_i(X_r) = \sum_{k=k_{X_r, \text{min}}}^{k_{X_r, \text{max}}} \sum_{s=n}^{w} \frac{R_{X_s}(k, s)}{R_{X_s}(k_{X_r, \text{max}})}.
\]

### F. The Algorithm to Determine the Occupancy Distribution and Conditional Transition Probabilities at the Macrostate Level in Resources \( R_{SEQ} \)

To determine the occupancy distribution in the resources \( R_{SEQ} \) on the basis of (25) it is necessary to know the conditional transition probabilities \( \sigma_i(n) = \sigma_i(X_r) \) that can be in turn determined from the parameters resulting from the occupancy distribution, e.g. the total number of serviced calls \( x_r \) in the representative microstate \( X_r \) (Formula (34)) can be calculated on the basis of the average number of calls \( y_i(n) \) of individual classes in the macrostate \( n \) (Formula (28)).

In order to determine then the occupancy distribution in the resources \( R_{SEQ} \) and, on its basis, the blocking probability for calls of individual traffic classes, the calculation algorithm presented below should be applied (Macro-State-Distribution methods).

**MSD Method:**

**Step 1:** Determination of the occupancy distribution \( [P(n)]_V \) on the basis of Formula (25) with the following assumption:

\[
\sigma_i(n) = 1. \quad (41)
\]

**Step 2:** Determination of the value \( y_i(n) \) on the basis of Formula (28) with the assumption (41).

**Step 3:** Determination of the number of serviced calls \( x_r \) for each macro-state \( n \) on the basis of Formula (34).

**Step 4:** Determination of the parameters \( k_{X_r, \text{max}}^{[x_r]} \) and \( k_{X_r, \text{min}}^{[x_r]} \) as well as \( k_{X_r, \text{max}}^{[x_r]} \) and \( k_{X_r, \text{min}}^{[x_r]} \) on the basis of Formulas (32) and (33), respectively. With respect to the fact that the values \( x_r \) are non-integer numbers, the calculations of the remaining parameters will be performed in a parallel way for \( \lfloor x_r \rfloor \) and \( \lceil x_r \rceil \).

**Step 5:** Determination of the number \( R_{X_s}^{[x_r]}(k, t_i) \) and \( R_{X_s}^{[x_r]}(k, t_i) \) of allocations in \( k \) access sets with the maximum length \( t_i \) of ”ones” on the basis of Formula (36).

**Step 6:** Determination of the number of possible allocations \( R_{X_s}^{[x_r]}(k_{X_r, \text{max}}^{[x_r]}) \) and \( R_{X_s}^{[x_r]}(k_{X_r, \text{min}}^{[x_r]}) \) of unoccupied FSUs (the ”ones”) on the basis of Formula (39).

**Step 7:** Determination of the conditional transition probabilities \( \sigma_i^{[x_r]}(n) \) and \( \sigma_i^{[x_r]}(n) \) in macrostate \( n \) on the basis of Formula (40).

**Step 8:** Determination of occupancy distributions \( [P(n)]_V^{[x_r]} \) and \( [P(n)]_V^{[x_r]} \) on the basis of Formula (25).

**Step 9:** Determination of the proper occupancy distribution \( [P(n)]_V \) in resources \( R_{SEQ} \) with the application of linear interpolation:

\[
\sum_{0 \leq n \leq V} \sum_{1 \leq i \leq M} [P(n)]_V = [P(n)]_V^{[x_r]} + \left\{ [P(n)]_V^{[x_r]} - [P(n)]_V^{[x_r]} \right\} \lfloor x_r \rfloor - [x_r]. \quad (42)
\]

**Step 10:** Determination of the blocking probability \( E_i \) for calls of individual traffic classes with the application...
of linear interpolation:

\[ E_i = \sum_{n=1}^{N} \left[ E_i^{[x]}(n) + x \left( E_i^{[x]}(n) \right) \right] \]

where

\[ E_i^{[x]}(n) = [1 - \sigma_i^{[x]}(n)] [P(n)]_{[x]} \]

\[ E_i^{[x]}(n) = [1 - \sigma_i^{[x]}(n)] [P(n)]_{[x]} \]

### VI. ACCURACY OF THE MODEL

The proposed model of resources in which only neighboring FSUs are to be occupied for a given connection is an approximate model. To estimate the accuracy of the proposed model and its applicability in engineering issues related to systems in EONs, the results of the analytical modelling were compared with the results of the simulations performed for a number of selected systems.

The choice of an appropriate number of demanded FSUs was made on the basis of the data included in Table 1.

#### TABLE 1. Number of FSUs in different connections depending on required bitrate and modulation format [6].

| Number of FSUs | Bitrate (Gbps) | Modulation format |
|---------------|---------------|------------------|
| 1             | 40            | 64-QAM           |
| 1             | 40            | 32-QAM           |
| 1             | 40            | 16-QAM           |
| 2             | 40            | QPSK             |
| 2             | 100           | 64-QAM           |
| 2             | 100           | 32-QAM           |
| 3             | 100           | 16-QAM           |
| 5             | 100           | QPSK             |
| 3             | 160           | 64-QAM           |
| 4             | 160           | 32-QAM           |
| 4             | 160           | 16-QAM           |
| 8             | 160           | QPSK             |
| 7             | 400           | 64-QAM           |
| 8             | 400           | 32-QAM           |
| 10            | 400           | 16-QAM           |
| 20            | 400           | QPSK             |
| 10            | 600           | 64-QAM           |
| 12            | 600           | 32-QAM           |
| 15            | 600           | 16-QAM           |
| 30            | 600           | QPSK             |

The research study was performed for the systems with the following capacity and structure of offered traffic:

- **System 1:**
  - Capacity of EON link: \( V = 40 \) FSUs,
  - Structure of offered traffic: \( M = 4, t_1 = 1 \) FSU, \( \mu_1^{-1} = 1 \), \( t_2 = 2 \) FSUs, \( \mu_2^{-1} = 1 \), \( t_3 = 3 \) FSUs, \( \mu_3^{-1} = 1 \), \( t_4 = 5 \) FSUs, \( \mu_4^{-1} = 1 \).

- **System 2:**
  - Capacity of EON link: \( V = 320 \) FSUs,
  - Structure of offered traffic: \( M = 3, t_1 = 1 \) FSU, \( \mu_1^{-1} = 1 \), \( t_2 = 20 \) FSUs, \( \mu_2^{-1} = 1 \), \( t_3 = 30 \) FSUs, \( \mu_3^{-1} = 1 \).

- **System 3:**
  - System capacity: \( V = 320 \) FSUs,
  - Offered traffic: \( M = 3, t_1 = 10 \) FSUs, \( \mu_1^{-1} = 1 \), \( t_2 = 15 \) FSUs, \( \mu_2^{-1} = 1 \), \( t_3 = 30 \) FSUs, \( \mu_3^{-1} = 1 \).

- **System 4:**
  - System capacity: \( V = 640 \) FSUs,
  - Offered traffic: \( M = 4, t_1 = 12 \) FSUs, \( \mu_1^{-1} = 1 \), \( t_2 = 15 \) FSUs, \( \mu_2^{-1} = 1 \), \( t_3 = 20 \) FSUs, \( \mu_3^{-1} = 1 \), \( t_4 = 30 \) FSUs, \( \mu_4^{-1} = 1 \).

The results of the simulations (Figures 5-22) are presented in the graphs in the form of plotted points with confidence intervals calculated according to the Student’s t-distribution (with 95-percent confidence level) for 5 series of 1,000,000 runs, each with the least active class. In each of the cases, the confidence interval did not exceed 5% of the average value of the result of the simulation experiment. The results of the analytical calculations are shown in the graphs in the form of solid or dotted lines.

![FIGURE 5. Occupancy distribution in system 1 for \( \alpha = 0.7 \) Erl.](image)

The confidence intervals are determined on the basis of the following formula:

\[ \left( \bar{X} - t_\alpha \frac{\sigma}{\sqrt{d}}, \bar{X} + t_\alpha \frac{\sigma}{\sqrt{d}} \right) \]

where \( \bar{X} \) is the arithmetic average calculated from \( d \) results (simulation runs), \( t_\alpha \) is the value of the Student’s t-distribution for \( d - 1 \) degrees of freedom. The parameter \( \sigma \) that determines the standard deviation is then calculated after the following formula:

\[ \sigma^2 = \frac{1}{d-1} \sum_{s=1}^{d} x_s^2 - \frac{d}{d-1} \bar{X}^2, \]

where \( x_s \) is the result obtained in the \( s \)-th run of the simulation.
Figures 5-12 show a comparison of the values of the occupancy distribution obtained by different models (models [18] and [16], [53]) with the data obtained in the course of simulations. In the figures, the results marked as (FAG) were obtained using the full-availability group model [16], [53]. The results marked as (old model) were obtained using the model presented in [18]. On the other hand, the results marked as (new model) are the results obtained using the model presented in this article. The results (the values of the occupancy distribution) are presented in dependence of the state \( n \) of the occupancy of the system for different traffic values \( a \) offered to a single FSU of the system. In order to better present the level of accuracy of the model for low values, the results are presented using both linear scale and logarithmic scale.
logarithmic scale. It is observable that the model proposed in this article is characterized, for most of the cases, by higher accuracy than the model presented in [18].

The comparison of the values of the occupancy distribution obtained using the analytical model presented in the article with the data obtained in the simulation experiments is also shown in Figures 13-16 and 18-21. The presented results also validate the high accuracy of the new model.

In Figures 13 and 14 we can also observe zero values of the distribution for some occupancy states. This is because any combination of the number of requested FSUs by the calls of individual traffic classes does not lead to the occulation
of that number of FSUs on the link. This busy state is simply unattainable. Such results (corresponding to the data obtained from the simulation) are also obtained from analytical calculations. This confirms the high accuracy of the developed analytical model.

The model proposed in the article also makes it possible to determine the blocking probability for calls of individual traffic classes offered to the system. The values of the blocking probability are presented in dependence of traffic $a$ offered to a single FSU of the system. Again in this case, the new model is characterized by high accuracy (Figures 17 and 22).

VII. CONCLUSION
This article proposes a new model to determine the occupancy distribution for resources in the EONs. This model assumes the necessity to allocate connections in neighboring FSUs. The model is based on the general occupancy distribution (at the micro and macrostate) in a system with state-dependent service process. The parameter of the conditional transition probability for transitions between the neighboring microstates can be determined in a combinatorial way. In the service processes, defined at the microstate, this parameter can be approximated by the probability of the transition of an appropriately chosen representative microstate.

The results of a comparison of the analytical calculations with the data obtained in digital simulations and with the results given by the model [18] show high accuracy of the proposed solution, sufficient for engineering applications, and in consequence indicate its possible application in practical optimization issues related to EON networks. The occupancy
distribution, determined using the proposed model, can be further used in analytical models of nodes in EONs, which are examples of more complex structures with multiple paths and different path selection algorithms. The present authors intend to develop models of this type in the next stages of their research. We also intend to use the analytical approach of modeling complex Markov processes, proposed, inter alia, in [54], [55], and [56] to optimize EON structures.

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