Pions in isospin asymmetric nuclei

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Abstract

Using a pair of the lightest mirror nuclei, \(^3\)He and \(^3\)H, we study the effect of the medium modification of pion fields on the flavor non-singlet structure function. The change of the pion fields leads to an enhancement of the flavor asymmetry of the antiquark distributions in a nucleus.

Keywords: Non-singlet structure function, Flavor asymmetry, Pions, Gottfried sum, Mirror nuclei

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The partonic distribution functions of the nucleon, in particular the flavor dependence of the antiquark distributions, are of considerable interest \[4\]. Within the framework of perturbative QCD (pQCD) the light quark sea is expected to be flavor symmetric. However, the experimental data \[2\] contradict this idea, revealing an excess of $\bar{d}$ over $\bar{u}$ in the free proton. This inconsistency indicates that non-perturbative effects should be responsible for the flavor asymmetry in the light sea quark distributions. For example, some flavor asymmetry was anticipated before the measurements on the basis of the chiral structure of the nucleon \[1\].

The physical proton has a relatively large $\pi^+$-neutron Fock component which naturally leads to a surplus of $\bar{d}$ \[1\]. It is known that this Fock component offers the main contribution to the $\bar{d}$ excess and that the contributions of the other mesons and $\Delta$ isobars have opposite signs and tend to cancel each other \[1\]. An alternative explanation for an excess of $\bar{d}$ over $\bar{u}$ involved the Pauli exclusion principle, given that there are two valence $u$ quarks in the proton and one valence $d$ \[4\]. Perturbative estimates failed to support this \[4\], with the first non-perturbative explanation of the origin of such an effect in terms of the vacuum structure of the proton given by Signal and Thomas \[6\]. Estimates of this effect within chiral quark models have also been given in Refs. \[7–9\]. It may well be that the experimentally observed excess involves contributions from both of these effects \[10\].

One way to learn more about the non-perturbative structure of the nucleon is to study the non-singlet difference between the proton ($p$) and neutron ($n$) structure functions, for nucleons bound in a pair of mirror nuclei \[1\]. In this case any discrepancy between theoretical predictions and observed data will indicate a modification of the non-perturbative mechanism giving rise to the flavor asymmetry in the free proton, in the nuclear medium. In particular, such a discrepancy would be a sensitive probe to study pions in nuclei. In this paper we examine the effect of changes in the pion cloud on the non-singlet combination of nuclear structure functions, using the lightest pair of mirror nuclei, $^3$He and $^3$H.

How is the pion field modified in a nucleus? To study it we concentrate here on only Fock states consisting of a "bare" nucleon and pion, and ignore nuclear binding, Fermi motion and shadowing/antishadowing effects for the moment. Under these assumptions, the structure functions of the proton and neutron in the nucleus $A$ are given by \[4\]

$$F_2^{p/A} = z_{p/A} \tilde{F}_2^p + f_{\pi^0 p/p/A} \otimes \tilde{F}_2^p + f_{\pi^+ p/p/A} \otimes F_2^{\pi^+} + f_{\pi^0 p/p/A} \otimes \tilde{F}_2^0 + f_{\pi^+ n/p/A} \otimes \tilde{F}_2^0 + f_{\pi^0 n/p/A} \otimes F_2^{\pi^+},$$

$$F_2^{n/A} = z_{n/A} \tilde{F}_2^n + f_{\pi^0 n/n/A} \otimes \tilde{F}_2^n + f_{\pi^+ n/n/A} \otimes F_2^{\pi^+} + f_{\pi^0 n/n/A} \otimes \tilde{F}_2^0 + f_{\pi^- p/n/A} \otimes F_2^\pi^- + f_{\pi^- p/n/A} \otimes F_2^\pi^-, (2)$$

where $\tilde{F}_2^{p(n)}$ is the structure function of a 'bare' proton (neutron). The probability to find the 'bare' proton (neutron) in the physical proton (neutron) in $A$ is denoted by the normalization constant, $z_{p(n)/A}$. The shorthand notation, $f_{MB/N/A} \otimes F_2^K$, stands for the convolution of the (light-cone) momentum distribution of the pion $M (= \pi^-, \pi^0, \pi^+)$ per $N (= p, n), f_{MB/N/A}(y)$ ($B = p, n$), and the structure function of $K$, $F_2^K(x)$ ($K = B, M$) \[4\]:

$$f_{MB/N/A} \otimes F_2^B(x) = \int_0^{1-x} dy f_{MB/N/A}(y) F_2^B \left( \frac{x}{1-y} \right),$$

$$f_{MB/N/A} \otimes F_2^M(x) = \int_x^{1} dy f_{MB/N/A}(y) F_2^M \left( \frac{x}{y} \right).$$

The nuclear structure function is then simply given by $F_2^{A}(x) = ZF_2^{p/A}(x) + NF_2^{n/A}(x)$, where $Z$ and $N$ are the numbers of protons and neutrons, respectively.
We consider a pair of mirror nuclei: \( A = Z + N \ (Z > N) \) (proton rich) and \( A' = Z' + N' \ (N' > Z') \) (neutron rich). In a nucleus we can expect a significant difference in the positive and negative pion light-cone momentum distributions. Recently, Korpa and Dieperink have calculated the pion fields in asymmetric nuclear matter [13], with a result which is consistent with the Drell-Yan experiment of Alde et al. [14]. Their result suggests that the difference in the distributions basically comes from two factors. One is the Pauli blocking of the nucleon in the final state; in the proton rich nucleus \( A \) which we consider here, the emission of \( \pi^- \) (from a neutron, creating a proton in the final state) is more suppressed than \( \pi^+ \) emission. The other effect is the dressing of the pion propagator in matter [15], where the particle-hole self-energy dominates. Korpa and Dieperink find that the delta-hole contribution is minor and that the neutral pion field is not much altered in the nuclear medium [13]. In summary, their analysis suggests that in the proton rich nucleus \( A \) the \( \pi^+ (\pi^-) \) field is enhanced (reduced) as compared with that in the free nucleon, while the \( \pi^0 \) field is not changed a great deal.

In a nucleus the Coulomb interaction may affect the shape of the pion momentum distribution and, of course, one cannot use isospin symmetry:

\[
f_{\pi^+ n/p/A} \neq f_{\pi^- p/n/A'} \quad \text{and} \quad f_{\pi^- p/n/A} \neq f_{\pi^+ n/p/A'}.
\]

However, for the reasons discussed above, we suppose that the \( \pi^0 \) distribution is not changed much in matter:

\[
f_{\pi^0 n/p/A} = f_{\pi^0 n/p/A'} = f_{\pi^0 n/n/A} = f_{\pi^0 n/n/A'} \equiv f_{\pi^0 N}.
\]

Here \( f_{\pi^0 N} \) is the \( \pi^0 \) distribution in the free nucleon, which is given by the Sullivan process [1]

\[
f_{\pi^0 N}(y) = \frac{g^2}{16\pi^2y(1-y)^2} \int_0^\infty dk_i^2 \frac{F_{\pi N}^2(s)}{(M_N^2 - s)^2(k_i^2 + y^2M_N^2)},
\]

with \( g(= 13) \) the \( \pi - N \) coupling constant, \( k_i^2 \) the transverse momentum squared of the pion and

\[
s = \frac{m^2_\pi + k_i^2}{y} + \frac{M_N^2 + k_i^2}{1-y}.
\]

The free nucleon mass is denoted \( M_N \) (0.94 GeV) and \( m_\pi \) (0.138 GeV) is the pion mass. The form factor, \( F_{\pi N}(s) \), is given by [1]

\[
F_{\pi N}(s) = \exp \left[ \frac{M_N^2 - s}{2\Lambda^2} \right],
\]

with \( \Lambda \) the cut off parameter.

We divide the pion distribution into two pieces:

\[
f_{\pi^+ n/p(A')} (y) = f_{\pi^+ n/p} (y) + \delta f_{\pi^+ /A(A')} (y), \] (10)
\[
f_{\pi^- p/n(A')} (y) = f_{\pi^- p/n} (y) + \delta f_{\pi^- /A(A')} (y), \] (11)

where \( f_{\pi^+ n/p(\pi^- p/n)} \) is the momentum distribution of \( \pi^+ (\pi^-) \) in the free proton (neutron). The nuclear many-body effects on the pion field in \( A(A') \) are expressed by \( \delta f_{M/A(A')} \).
The normalization constants in Eqs. (1) and (2) can be related to those for the free nucleon. For example, using Eqs. (3) and (4) we find

\[ z_{p/A} = 1 - \langle f_{\pi^0N}^p \rangle - \langle f_{\pi^+N/p}^p \rangle - \langle \delta f_{\pi^+/A} \rangle \equiv z_N - \langle \delta f_{\pi^+/A} \rangle, \tag{12} \]

where \( z_N \) is the normalization constant for the free nucleon. Finally, the nucleon structure functions in those nuclei become:

\[ F_{2/p/A}^p = F_{2/n}^n \equiv \langle \delta f_{\pi^+/A} \rangle \tilde{F}_{2/n}^p + \delta f_{\pi^+/A} \otimes \tilde{F}_{2/n}^p + \tilde{F}_{2/n}^p, \tag{13} \]

\[ F_{2/n/A}^n = F_{2/n}^n \equiv \langle \delta f_{\pi^-/A} \rangle \tilde{F}_{2/n}^n + \delta f_{\pi^-/A} \otimes \tilde{F}_{2/n}^n + \tilde{F}_{2/n}^n, \tag{14} \]

\[ F_{2/p/A'}^p = F_{2/n}^n \equiv \langle \delta f_{\pi^+/A'} \rangle \tilde{F}_{2/n}^p + \delta f_{\pi^+/A'} \otimes \tilde{F}_{2/n}^p + \tilde{F}_{2/n}^p, \tag{15} \]

\[ F_{2/n/A'}^n = F_{2/n}^n \equiv \langle \delta f_{\pi^-/A'} \rangle \tilde{F}_{2/n}^n + \delta f_{\pi^-/A'} \otimes \tilde{F}_{2/n}^n + \tilde{F}_{2/n}^n, \tag{16} \]

where \( F_{p(n)}^n \) is the free proton (neutron) structure function.

Now we study the lightest mirror nuclei: \( A = ^3\text{He} \) and \( A' = ^3\text{H} \). In \( ^3\text{He} \), the \( \pi^+ \) meson is generated from the proton and the final state is given by \( ^3\text{He} = 2p + n \rightarrow p + 2n + \pi^+ \), where \( \pi^+ \) feels a repulsive force from the Coulomb interaction with the single \( p \). (We neglect two pion emissions in the final states, such as \( 2p + n \rightarrow 3n + 2\pi^+ \), for which the probability is expected to be very small.) On the other hand, the \( \pi^- \) meson is produced by the neutron, and the final state is \( 3p + \pi^- \), where the \( \pi^- \) feels a strong attractive force due to the \( 3p - \pi^- \) interaction. Thus, we expect that the Coulomb force between \( 3p \) and \( \pi^- \) is about three times stronger than that between \( p \) and \( \pi^- \) in the former case. In \( ^3\text{H} \), the \( \pi^- \) feels an attractive force in the final state \( 2p + n + \pi^- \), and the Coulomb force is twice as large as that for the \( \pi^+ \) in \( ^3\text{He} \). On the other hand, the \( \pi^+ \) in \( ^3\text{H} \) does not feel any Coulomb force because the final state consists of \( 3n + \pi^+ \).

In order to evaluate Eqs. (13)-(16), we need to estimate the distributions, \( \delta f_{M/A}(y) \), in \( ^3\text{He} \) and \( ^3\text{H} \), individually. The Coulomb force may change the shape of the pion momentum distribution. As discussed above, since it acts on \( \pi^- \) as an attractive force in the nucleus, the wave function of the pion in coordinate space shrinks. This means that the pion gets a (relatively) higher momentum and the shape of the distribution in momentum space should shift toward larger \( y \). For the \( \pi^+ \) the distribution should be modified the opposite way, because it feels a repulsive force. To calculate these effects quantitatively requires very complicated many-body calculations, including Coulomb forces. This is extremely difficult in order to make a first estimate of the effects one might expect the following simple scaling assumption to be reasonable. That is, the change in the pion distributions are assumed to be given by

\[ \delta f_{\pi^+/A}(y) = 2\alpha_{\pi^+/A}(1 + \beta_{\pi^+/A})f_{\pi^0N}(1 + \beta_{\pi^+/A})y \equiv 2\alpha_{\pi^+/A}f_{\pi^+/A}(y), \tag{17} \]

\[ \delta f_{\pi^-/A}(y) = 2\alpha_{\pi^-/A}(1 - \beta_{\pi^-/A})f_{\pi^0N}(1 - \beta_{\pi^-/A})y \equiv 2\alpha_{\pi^-/A}f_{\pi^-/A}(y), \tag{18} \]

\[ \delta f_{\pi^+/A'}(y) = 2\alpha_{\pi^+/A'}f_{\pi^0N}(y), \tag{19} \]

\[ \delta f_{\pi^-/A'}(y) = 2\alpha_{\pi^-/A'}(1 - \beta_{\pi^-/A'})f_{\pi^0N}(1 - \beta_{\pi^-/A'})y \equiv 2\alpha_{\pi^-/A'}f_{\pi^-/A'}(y), \tag{20} \]

where \( \alpha_{M/A(A')} \) represents a change caused by the strong interaction in a nucleus (for example, Pauli blocking, correlations of random phase approximation (RPA), etc [13]) and \( \beta_{M/A(A')} \) describes a shift of the distribution because of the Coulomb force. Note
that the π + in $^3\text{H}$ does not feel any Coulomb force. As pointed out above, we expect that $\beta_{\pi^+/A} : \beta_{\pi^-/A} : \beta_{\pi^-/A'} = 1 : 3 : 2$ from the point of view of the strength of the Coulomb force acting on the pion. We therefore choose $3\beta_{\pi^+/A} = \beta_{\pi^-/A} = \frac{3}{2}\beta_{\pi^-/A'} = \beta > 0$ (β is assumed to be small). The new function, $f_{M/A(A')}$, is normalized as

$$\int_0^1 dy f_{M/A(A')}^*(y) = \langle f_{\pi^0N} \rangle,$$

where we ignored a tiny quantity stemming from $\int_0^1 \beta_{M/A(A')} dy f_{\pi^0N}(y)$.

Next, we suppose that the average number of pions per nucleon in a nucleus is equal to that in the free nucleon – experimental indications are that the pion field is not much enhanced in a nucleus [2][3]. The requirement of pion number conservation reduces the number of parameters “α” in Eqs. (17)-(20). We find that in $^3\text{He}$, $2\alpha_{\pi^+/A} + \alpha_{\pi^-/A} = 0$, while in $^3\text{H}$, $\alpha_{\pi^+/A} + 2\alpha_{\pi^-/A} = 0$. Thus, we set $2\alpha_{\pi^+/A} = -\alpha_{\pi^-/A} = \alpha_A > 0$ (the π − field is suppressed in $^3\text{He}$) and $2\alpha_{\pi^-/A} = -\alpha_{\pi^+/A} = \alpha_{A'} > 0$ (the π + field is suppressed in $^3\text{H}$). Furthermore, since $\alpha_{A(A')}$, describes the change of the pion field because of the strong interaction, we can set $\alpha_A = \alpha_{A'} = \alpha$ (isospin is a good symmetry in this case). This leaves just two parameters, α and β.

We should note here that even if α = 0 the Coulomb effect would modify the proton and neutron structure functions in the nucleus. Such a case could be described by replacing the pion distribution in the free nucleon structure function, $f_{MB/N}$, in Eqs. (13)-(16) with $f_{M/A(A')}^*$. However, we expect that by itself the Coulomb effect on the structure function should be quite small (see below Eq. (30) and Fig. 2). We therefore neglect the Coulomb effect on $F_2^n$ in $F_2^n/A(A')$. Equations (13)-(16) then give

$$\delta F_2^{p/3\text{He}} = F_2^{p/3\text{He}} - F_2^p = -\alpha\langle f_{\pi^0N} \rangle \tilde{F}_2^p + \alpha f_{\pi^+/3\text{He}}^* \otimes [\tilde{F}_2^p + F_2^{\pi^+}],$$

$$\delta F_2^{n/3\text{He}} = F_2^{n/3\text{He}} - F_2^n = 2\alpha\langle f_{\pi^0N} \rangle \tilde{F}_2^n - 2\alpha f_{\pi^-/3\text{He}}^* \otimes [\tilde{F}_2^n + F_2^{\pi^-}],$$

$$\delta F_2^{p/3\text{H}} = F_2^{p/3\text{H}} - F_2^p = 2\alpha\langle f_{\pi^0N} \rangle \tilde{F}_2^p - 2\alpha f_{\pi^0N} \otimes [\tilde{F}_2^p + F_2^{\pi^+}],$$

$$\delta F_2^{n/3\text{H}} = F_2^{n/3\text{H}} - F_2^n = -\alpha\langle f_{\pi^0N} \rangle \tilde{F}_2^n + \alpha f_{\pi^-/3\text{H}}^* \otimes [\tilde{F}_2^n + F_2^{\pi^-}],$$

where,

$$f_{\pi^+/3\text{He}}(y) = \left(1 + \frac{1}{3}\beta \right) f_{\pi^0N} \left((1 + \frac{1}{3}\beta) y \right),$$

$$f_{\pi^-/3\text{He}}(y) = (1 - \beta) f_{\pi^0N}((1 - \beta)y),$$

$$f_{\pi^-/3\text{H}}(y) = \left(1 - \frac{2}{3}\beta \right) f_{\pi^0N} \left((1 - \frac{2}{3}\beta) y \right).$$

In Fig. 1 the pion distributions provided by the scaling assumption are presented taking $\Lambda = 1$ GeV (see also below Eq. (31)). As an example, we choose $\beta = 0.1$, which means that, for instance, the wave function of the π− in $^3\text{He}$ shrinks by about 10% in coordinate space because of the Coulomb force. The negative pion distribution carries somewhat higher momentum, while the positive one shifts toward lower y, compared with $f_{\pi^0N}$. Taking the non-singlet combination of the structure functions of $^3\text{He}$ ($F_2^{3\text{He}}$) and $^3\text{H}$ ($F_2^{3\text{H}}$) we find
\[ F_2^{3He} - F_2^H = (F_2^p - F_2^n) - 4\alpha \langle f_{\pi^0N} \rangle \delta \tilde{F}_2^N \]
\[ -2\alpha [f_{\pi^-/3H}^{*} + f_{\pi^-/3He}^{*}] \otimes \tilde{F}_2^p + 2\alpha [f_{\pi^+/3He} + f_{\pi^0N}] \otimes \tilde{F}_2^n \]
\[ -2\alpha [f_{\pi^-/3H} \otimes F_2^{\pi^-} + f_{\pi^-/3He} \otimes F_2^{\pi^-} - f_{\pi^+/3He} \otimes F_2^{\pi^+} - f_{\pi^0N} \otimes F_2^{\pi^+}], \quad (29) \]

where
\[ \delta \tilde{F}_2^N(x) = \tilde{F}_2^p(x) - \tilde{F}_2^n(x) = \frac{1}{3} x[\tilde{u}_v(x) - \tilde{d}_v(x)], \quad (30) \]

with \( \tilde{u}_v(\tilde{d}_v) \) the valence \( u(d) \) distribution in the bare proton.

Figure 2 illustrates the nuclear and Coulomb effects on the non-singlet structure function of the \( A = 3 \) system, which is given by \( \delta F_2^{A=3} = (F_2^{3He} - F_2^H) - (F_2^p - F_2^n) \). For the numerical calculations we have chosen (at \( Q^2 = 4 \) GeV²) [10]

\[ x\tilde{u}_v(x) = 0.65452 \times x^{0.38}(1 - x)^{2.49}(1 + 10.5x), \quad (31) \]
\[ x\tilde{d}_v(x) = 0.028660 \times x^{0.07}(1 - x)^{4.63}(1 + 150x), \quad (32) \]

and \( F_2^{\pi^+}(x) = F_2^{\pi^-}(x) = 0.98863 \times x^{0.61}(1 - x)^{1.02} \) [7]. Clearly the effect of the Coulomb distortion is quite small in the region \( x > 10^{-4} \), even if we choose \( \beta = 0.2 \). (We have checked that the Coulomb effect on \( \delta F_2^{N/A(A')} \) individually is also small.)

The nuclear Gottfried integral, \( I_G^{A,A'}(z) \), is defined by [1]
\[ I_G^{A,A'}(z) = \frac{1}{Y} \int_z^A \frac{dx}{x} [F_2^A(x) - F_2^{A'}(x)] = I_G^N(z) + \delta I_G^{A,A'}(z), \quad (33) \]

with \( Y(=Z-N) \) the number of excess protons in \( A \) and \( I_G^N(z) \) the Gottfried integral for the free nucleon. The nuclear effect is described by the second term on the r.h.s of Eq.(33)
\[ \delta I_G^{A,A'}(z) = \int_z^A \frac{dx}{x} \delta F_2^{A,A'}(x), \quad (34) \]

where \( \delta F_2^{A,A'} = \frac{1}{Y}(F_2^A - F_2^{A'}) - (F_2^p - F_2^n) \). In the case of the \( A = 3 \) system \( \delta F_2^{A,A'} \) is given by \( \delta F_2^{A=3} \). As we have already discussed in Ref. [12], the Gottfried integral is generally divergent when the effect of charge symmetry breaking is included, even for the free proton and neutron [12].

If we set \( \beta = 0 \), \( \delta F_2^{A=3} \) reads
\[ \delta F_2^{A=3} = -4\alpha \langle f_{\pi^0N} \rangle \otimes \delta \tilde{F}_2^N. \quad (35) \]

Since the effects of the nuclear binding and shadowing are ignored in the calculation for the time being, the change in the Gottfried integral is convergent and it is given by
\[ \delta I_G^{3He,3H}(0) = -\frac{8}{3} \alpha \langle f_{\pi^0N} \rangle. \quad (36) \]

With the cut-off-mass \( \Lambda = 1 \) GeV in \( f_{\pi^0N} \), we find \( \langle f_{\pi^0N} \rangle = 0.083 \). The Gottfried integral for the free nucleon is then estimated to be \( I_G^N(0) = \frac{1}{3}(1 - 4\langle f_{\pi^0N} \rangle) = 0.223 \), which is consistent with the measured value \((0.235 \pm 0.026) \) [1]. We can see that the modification of the pion field enhances the flavor asymmetry in a pair of mirror nuclei and hence it reduces the Gottfried
integral. If $\alpha = 0.05(0.1)[0.2]$ (and $\beta = 0$), $\delta I_G^{3\text{He},3\text{H}}(0) = -0.0111(-0.0221)[-0.0443]$, corresponding to a reduction of the Gottfried sum by about $5(9)[20] \%$ from the free value.

Next we consider the nuclear binding and shadowing effects. (For recent reviews see Ref. [15].) In Ref. [12] we studied shadowing and anti-shadowing corrections to the flavor non-singlet structure function using the Gribov-Glauber multiple scattering formalism. We found that the non-singlet structure function is enhanced at small $x$ by nuclear shadowing, increasing the nuclear Gottfried integral ($z$ is chosen to be $10^{-4}$ in Eq.(33)) by between 15 and 41%. The enhancement of the non-singlet structure function is caused by the difference between the density distributions of $^3\text{He}$ and $^3\text{H}$. In the shadowing region the structure functions of $^3\text{He}$ and $^3\text{H}$ are given by [12]

$$F_2^{3\text{He}} = 2F_2^p + F_2^n - (2.5f_{3\text{He}} - g_{3\text{He}})F_2^p - 0.5f_{3\text{He}}F_2^n, \quad (37)$$

$$F_2^{3\text{H}} = F_2^p + 2F_2^n - (2.5f_{3\text{H}} - g_{3\text{H}})F_2^p - 0.5f_{3\text{H}}F_2^n, \quad (38)$$

where $f_{3\text{He}(3\text{H})}$ and $g_{3\text{He}(3\text{H})}$, respectively, describe the single and double rescattering processes in $^3\text{He}$ ($^3\text{H}$), which depend on the nuclear density distribution. It is also necessary to include anti-shadowing in order to produce the structure function around $x \sim 0.1$ (using the baryon number and momentum sum rules). Detailed discussions can be found in Ref. [12].

Replacing the structure functions of the free proton and neutron in Eqs.(37) and (38) by $F_2^{N/A(A')} \ (\text{given by Eqs.}(13)-(16))$, we can calculate the non-singlet structure function of $^3\text{He}$ and $^3\text{H}$, including both the modification of the pion fields and nuclear shadowing. (This means that the effect of the pion cloud modifies the nucleon sea quarks only.) Note that this simple replacement is an approximation made in order to see the effect of the change of the pion fields. To treat this problem rigorously it would be necessary to construct a model where pions are handled consistently and contribute to both shadowing and anti-shadowing [13], which goes beyond the scope of the present work.

In the calculation of nuclear shadowing, the ground-state wave functions of $^3\text{He}$ and $^3\text{H}$ are assumed to be given by gaussian functions in coordinate space [12], $|\Psi|^2 \propto \exp[-\vec{r}^2/(2b)]$, where the parameter $b$ determines the correct matter radius of the nucleus. We take $b = 40.59(30.06) \ \text{GeV}^{-2}$, which produces the matter radius of 1.769 (1.524) $\text{fm}$ for $^3\text{He}$ ($^3\text{H}$). In Ref. [12] the effective cross section, $\sigma_{\text{eff}}$, was used to describe the interaction between the hadronic components in the virtual photon and the nuclear target. Here we take two models for $\sigma_{\text{eff}}$: the first (case 1) from Frankfurt and Strikman [21]; and the second (case 2) the two-phase model of Ref. [21]. At large $x(>0.2)$ we used the structure functions of $^3\text{He}$ and $^3\text{H}$, obtained as a solution of the Faddeev equations for three body system [12,22]. Since the contribution of the nuclear binding and Fermi motion effects to the non-singlet combinations of the structure functions of $^3\text{He}$ and $^3\text{H}$ is small, we make an approximation that the binding and Fermi motion effect of Refs. [1,22] and the pion cloud effect, discussed in the present work, are not correlated and, hence, contribute additively to $\delta F_2^{A=3}$. We present our main results in Figs. 3 and 4 (the calculations were performed at $Q^2 = 4 \ \text{GeV}^2$). To treat parton densities in the free proton and neutron realistically we used the CTEQ5L parametrization [23]. Since we know that the Coulomb effect is small in the region $x > 10^{-4}$, we set $\beta = 0$ and show only the dependence of the flavor non-singlet structure function on $\alpha$.

As expected, the change of the pion fields leads to a considerable suppression of the non-singlet structure function in a nucleus. However, at very small $x$ (typically $\sim 10^{-3} - 10^{-4}$)
the reduction is not large compared with the enhancement caused by shadowing. (In the figures we present our results only for the range $x \in [10^{-2}, 0.8]$ in order to see the difference among the curves clearly. Note that the non-singlet structure function (divided by $x$) is of order $200 \sim 300$ at $x = 10^{-4}$.) When the shadowing is turned off the difference of the proton and neutron structure functions gives the main contribution to the non-singlet structure function of the nucleus. In this case the change of the pion fields gives a sizable contribution at very small $x$ (e.g., about 20% at $x = 10^{-3}$). On the contrary, when the shadowing is switched on the main contribution to the non-singlet structure function at small $x$ is given by a term proportional to $(F_2^p + F_2^n) \times (f_{3He} - f_{3H})$, which is much larger than the effect of the change of the pion fields – the pion effect is at most 5% at $x = 10^{-3}$.

We can estimate the nuclear Gottfried integral, $I_{He,3H}(10^{-4})$, defined by Eq.(33). If $\alpha$ is set to be 0 (no change in the pion fields) we find $I_{He,3H}(10^{-4}) = 0.2953(0.3395)$ for case 1 (2) – note that the CTEQ5L fit gives $I_N(10^{-4}) = 0.2403$. When $\alpha = 0.1(0.2)$ we obtain $I_{He,3H}(10^{-4}) = 0.2699(0.2444)$ for case 1, while it is $0.3142(0.2829)$ for case 2. Therefore, we expect that the change of the pion fields in the $A = 3$ system might lead to a reduction of the Gottfried integral by about 10%, compared with the value for the case where $\alpha = 0$.

We here give some comments:

(1) Using the scaling assumption the pion distributions are shifted in the present calculation. The momentum fraction carried by pions is thus different from that in the free nucleon. For example, if $0 < \beta \ll 1$, we find $\langle y f_{pH}^{1-\beta}(y) \rangle \approx (1 + \beta)\langle y f_{p0}(y) \rangle$. Thus, in $^3$He ($^3$H) the ratio of the momentum fraction carried by pions to that in the case where $\beta = 0$ increases by about $\beta/9$ ($4\beta/9$). The change of the pion momentum fraction would lead to changes of the momentum fractions carried by the nucleons and other mesons. In such a case one would have to construct a model where the momentum fractions are balanced. Note however, that when we set $\beta = 0$ there is no change of the pion momentum fraction in a nucleus by virtue of the scaling assumption and pion number conservation.

(2) We have treated the change of the pion fields in the nuclear medium semi-quantitatively. In principle, such a modification should be attributed to various correlation phenomena in a nucleus like Pauli blocking, exchange currents, short-range correlations etc. Melnitchouk and Thomas [24] have reanalysed the nuclear shadowing effect on the deuteron structure function, including meson exchange currents, to extract the neutron structure function, and studied the Gottfried sum rule for the free nucleon. It should be possible to do such calculations for the three body system in the future. For a pair of mirror nuclei larger than the three body system, it would become important to consider Fock states including multi-pions, $\Delta$ isobars, other mesons and so on. That is also a very intriguing problem.

In summary, we have estimated the effect of the medium modification of the pion fields on the flavor non-singlet structure function of the lightest mirror nuclei. We have found that the change of the pion fields produces a considerable suppression of the non-singlet structure function, and that the Gottfried integral is correspondingly reduced. In general, charge symmetry is broken and the Gottfried integral is divergent [12]. However, the $x$-dependence of the flavor non-singlet structure function of a pair of mirror nuclei would provide significant information on phenomena involving non-pQCD dynamics (such as the pion cloud) in the nuclear medium. Experiments on deep-inelastic scattering off various mirror nuclei should be possible in the future [25]. If we could vary the atomic number $A$ and the difference between the proton and neutron numbers $Y$ independently in measuring
the nuclear structure functions, it would stimulate a great deal of work which should lead to new information on the dynamics of nuclear systems.

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FIG. 1. Pion distributions ($\beta = 0.1$ and $\Lambda = 1$ GeV). The dotted, solid, dot-dashed and dashed curves are for $f_{\pi^0N}$, $f_{\pi^-/^{3}H}$, $f_{\pi^-/^{3}He}$ and $f_{\pi^+/^{3}He}$, respectively.
FIG. 2. $\frac{\delta F^A_2}{x}$ vs $x$ ($\alpha = 0.1$). The dotted, solid and dot-dashed curves show the results with $\beta = 0, 0.1, 0.2$, respectively.
FIG. 3. Non-singlet structure function (divided by $x$) for case 1 discussed in the text. The dotted curve shows the non-singlet structure function for the free nucleon, while the dashed curve presents the result for the $A = 3$ system without any change of the pion fields. The upper (lower) solid curve is for the full calculation with $\alpha = 0.1(0.2)$. 
FIG. 4. Non-singlet structure function (divided by $x$) for case 2 discussed in the text. The curves are labelled as in Fig. 3.