An inverse problem for a system of nonlinear parabolic equations

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Abstract. The inverse problem for a system of nonlinear parabolic equations is considered in the present paper. Namely, it is required to restore the initial condition by a given time-average value of the solution to the system of the nonlinear parabolic equations. An exact in the order error estimate of the optimal method for solving the inverse problem through the error estimate for the corresponding linear problem is obtained. A stable approximate solution to the unstable nonlinear problem under study is constructed by means of the projection regularization method which consists of using the representation of the approximate solution as a partial sum of the Fourier series. An exact in the order estimate for the error of the projection regularization method is obtained on one of the standard correctness classes. As a consequence, it is proved the optimality of the projection regularization method. As an example of a nonlinear system of parabolic equations, which has important practical applications, a spatially distributed model of blood coagulation is considered.

1. Introductory remarks

The paper considers an inverse problem for a system of nonlinear parabolic equations. It is required to determine the initial condition under which the time-average value of the solution on the segment $[0, T]$ at each point is the closest to the given value.

Mathematical models based on the systems of nonlinear equations of parabolic type are one of the main types of models that are used in biology and medicine. Their systematic investigation was initiated by Kolmogorov, Petrovsky, Piskunov, who revealed waves in a medium where diffusion processes and a nonlinear increase in the amount of substance compete with each other.

The problem under study is unstable, therefore, for its numerical solution, it is necessary to use stable methods of approximate solving. Moreover, to obtain adequate numerical results, it is necessary to develop methods for estimating the accuracy of approximate solutions.

The paper uses the classical spectral technique to obtain error estimates of the optimal method for the inverse problem under consideration. The obtained estimates make it possible to study the optimality of methods for the approximate solving the inverse problem.

To construct a stable approximate solution of the inverse problem, the projection regularization method and the method based on solving an auxiliary problem with a small parameter in the nonlocal condition are used.

The projection regularization method consists of representing the approximate solution as a finite segment of the Fourier series corresponding to the exact solution of the problem.

An error estimate is obtained for the projection regularization method on one of the standard correctness classes, and optimality in the order of this method is proved.
The numerical examples are provided to validate the theoretical results.

To complete the introductory remarks, we will describe the structure of the paper.

In section 2 we formulate the mixed problem for a system of parabolic equations, that is we formulate the forward problem corresponding to the inverse problem under study.

In section 3 we give an example of a parabolic system that arises as a mathematical model of blood coagulation. Parabolic systems are commonly used in modeling biological and medical processes, that is a significant reason to study mathematical issues connected with such systems.

Section 4 is aimed at obtaining the representation of the solutions to the mixed problem for a system of parabolic equations in the linear and nonlinear cases. The representations of the solutions through the Fourier series is the basic step for studying the inverse problem and constructing stable approximate methods for the inverse problem.

In Section 5 we formulate the inverse problem for a nonlinear parabolic system and define the uniform regularization set which describes the necessary a priori information about the exact solution to the inverse problem.

In Section 6 we state the projection regularization method and establish the error estimate for the approximate solution.

In Section 7 we provide a numerical example to validate the theoretical results.

2. The direct problem for a system of nonlinear parabolic equations

Consider the following mixed problem for a system of parabolic equations.

Let the functions $p, q \in C([0,T]; W^1_2[0; L]) \cap C^2((0,T); L_2[0; L])$

are the solutions to the system of nonlinear parabolic equations

\begin{align}
\frac{\partial p}{\partial t} &= D \frac{\partial^2 p}{\partial x^2} + ap + f(p, q), \\
\frac{\partial q}{\partial t} &= D \frac{\partial^2 q}{\partial x^2} + bp + g(p, q).
\end{align}

The boundary conditions

$$
\frac{\partial p}{\partial x} (0, t) = \frac{\partial p}{\partial x} (L, t) = 0,
$$

$$
\frac{\partial q}{\partial x} (0, t) = \frac{\partial q}{\partial x} (L, t) = 0,
$$

and the initial conditions

$$
p(x, 0) = \varphi(x); \quad q(x, 0) = \psi(x)
$$

are also satisfied.

It is required to determine the average values of the functions $u(x, t)$ and $v(x, t)$ on the segment $[0, T]$, that is the functions

$$
A(x) = \int_0^T p(x, t) dt; \quad B(x) = \int_0^T q(x, t) dt
$$

are to be determined.

Here $f, g$ are continuous functions such that for all $U(x), V(x) \in L_2[0, L]$, for which $\|U\| \leq c$, $\|V\| \leq c$, $f(U, V), g(U, V) \in L_2[0, L]$ and the inequalities

$$
|f(U_1, V_1) - f(U_2, V_2)| \leq r(|U_1 - U_2| + |V_1 - V_2|),
$$

$$
|g(U_1, V_1) - g(U_2, V_2)| \leq r(|U_1 - U_2| + |V_1 - V_2|).
$$

are true.
3. Example: spatially distributed model of blood coagulation

Consider the system of parabolic equations

\[
\frac{\partial \theta}{\partial t} = D_1 \Delta \theta + \frac{\partial^2 \theta}{\partial x^2} - k_1 \theta - \gamma \theta \varphi, \tag{3}
\]

\[
\frac{\partial \varphi}{\partial t} = D_2 \Delta \varphi + \beta \theta \left(1 - \frac{\varphi}{\varphi_0}\right) \left(1 - \frac{\varphi}{\varphi_0}\right) - k_2 \varphi. \tag{4}
\]

Here \(\alpha, \beta, \gamma, k_1, k_2, \theta_0, \varphi_0\) are the positive constants.

The initial perturbation has the form

\[
\theta(x, 0) = \begin{cases} \theta_1, & 0 \leq |x| \leq l, \\ 0, & l \leq |x| \leq L. \end{cases} \tag{5}
\]

The equations of the system describe a change in the concentration of two metabolites - an activator of the blood coagulation process (thrombin) and an inhibitor [3].

Here \(\theta\) and \(\varphi\) are the concentrations of the activator and the inhibitor, respectively. It is known from experiments that \(D_1 \approx D_2\). Hence, the system (3)–(4) is obviously has the form (1)–(2).

4. The solution to the direct problem for a system of parabolic equations

4.1 The direct problem for the corresponding linear system

Consider the following system of linear parabolic equations corresponding to the system (1)-(2).

\[
\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2} + \alpha v, \tag{6}
\]

\[
\frac{\partial v}{\partial t} = D \frac{\partial^2 v}{\partial x^2} + b u. \tag{7}
\]

The boundary conditions

\[
\frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(L, t) = 0,
\]

\[
\frac{\partial v}{\partial x}(0, t) = \frac{\partial v}{\partial x}(L, t) = 0,
\]

and the initial conditions

\[
u(x, 0) = \varphi_0(x), \quad v(x, 0) = \psi_0(x)
\]

are also satisfied. The functions

\[
A_0(x) = \int_0^T u(x, t) dt, \quad B_0(x) = \int_0^T v(x, t) dt
\]

are to be determined.

Given the boundary conditions, we will look for a solution to the system in the form

\[
u(x, t) = \sum_{n=1}^\infty u_n(t) \cos \frac{n\pi x}{L}, \tag{8}
\]

\[
v(x, t) = \sum_{n=1}^\infty v_n(t) \cos \frac{n\pi x}{L}. \tag{9}
\]

For functions \(u_n(t), v_n(t)\) we obtain a system of linear ordinary differential equations

\[
\dot{u}_n(t) = -D \lambda_n^2 u_n(t) + \alpha v_n(t), \tag{10}
\]

\[
\dot{v}_n(t) = -D \lambda_n^2 v_n(t) + \beta u_n(t).
\]
We can find the solution of the system (10)-(11) in the form
\[ u_n(t) = c_n e^{-\mu_n t}; \quad v_n(t) = d_n e^{-\mu_n t}. \] (12)

To determine the coefficients, we obtain a system of linear algebraic equations
\[
\begin{cases}
    c_n (D \lambda_n^2 - \mu_n) - ad_n = 0 \\
    -bc_n + (D \lambda_n^2 - \mu_n) d_n = 0.
\end{cases}
\] (13)

The system (13) has a nonzero solution provided that
\[
\begin{vmatrix}
    D \lambda_n^2 - \mu_n & -a \\
    -b & D \lambda_n^2 - \mu_n
\end{vmatrix} = 0.
\] (14)

Hence,
\[ \mu_n = D \lambda_n^2 \pm \sqrt{ab}. \] (15)

Under the condition (15), the matrix of the system has the form
\[
\begin{pmatrix}
    -\sqrt{ab} & -a \\
    -b & -\sqrt{ab}
\end{pmatrix}
\] or
\[
\begin{pmatrix}
    \sqrt{ab} & -a \\
    -b & \sqrt{ab}
\end{pmatrix}
\]

Therefore, the solution of the system has the form
\[ c_n = \sqrt{ab} d_n; \quad c'_n = -\sqrt{ab} d'_n. \] (16)

Further, the solutions to the system of the linear parabolic equations have the form
\[
\begin{align*}
    u(x, t) &= \frac{a}{b} \sum_{n=1}^{\infty} (c_n e^{-\mu_n t} - c'_n c_n e^{-\mu_n t}) \cos \frac{\pi n x}{L}; \\
    v(x, t) &= \sum_{n=1}^{\infty} (c_n e^{-\mu_n t} + c'_n c_n e^{-\mu_n t}) \cos \frac{\pi n x}{L}.
\end{align*}
\] (17)

Here \( \mu_n = \left( \frac{\pi n}{L} \right)^2 + \sqrt{ab}; \mu'_n = \left( \frac{\pi n}{L} \right)^2 - \sqrt{ab}. \) Writing down the equalities (17) for \( t = 0 \), we obtain the system
\[
\begin{cases}
    c_n - c'_n = \frac{b}{a} \varphi_n \\
    c_n + c'_n = \psi_n.
\end{cases}
\] (18)

It follows from (18) that we can find the Fourier coefficients, namely
\[
\begin{cases}
    c_n = \frac{1}{2} \left( \frac{b}{a} \varphi_n + \psi_n \right) \\
    c'_n = \frac{1}{2} (\psi_n - \frac{b}{a} \varphi_n).
\end{cases}
\] (19)

To determine the coefficients of expansion in a Fourier series for the solution, we consider the expansion in a Fourier series of the given functions
\[ \varphi_0(x) = \sum_{n=1}^{\infty} \varphi_n \cos \frac{\pi n x}{L}; \]
Integrating the equalities (18) with respect to \( t \), we verify that

\[
A_0(x) = \sum_{n=1}^{\infty} \frac{c_n}{\mu_n} \left( 1 - e^{-\mu_n t} \right) - \frac{c'_n}{\mu_n} \left( 1 - e^{-\mu_n t} \right) \cos \frac{\pi n x}{L},
\]

\[
B_0(x) = \sum_{n=1}^{\infty} \frac{c_n}{\mu_n} \left( 1 - e^{-\mu_n t} \right) + \frac{c'_n}{\mu_n} \left( 1 - e^{-\mu_n t} \right) \cos \frac{\pi n x}{L}.
\]

Here \( c_n \) and \( c'_n \) are defined by (19).

Equalities (20) and analysis the convergence of the corresponding Fourier series, allow us to guarantee that the mixed problem for the system (6) - (7) has a unique solution for any functions \( \varphi(x) \), \( \psi(x) \).

### 4.2 The nonlinear direct problem.

Consider the mixed problem for the nonlinear system (1) - (2). Using the Fourier method we find the solution to the system (1) - (2) in the form

\[
p(x, t) = \sum_{n=1}^{\infty} p_n(t) \cos \frac{\pi n x}{L},
\]

\[
q(x, t) = \sum_{n=1}^{\infty} q_n(t) \cos \frac{\pi n x}{L}.
\]

We obtain the following system of nonlinear differential equations for the functions \( p_n(t) \), \( q_n(t) \):

\[
\begin{align*}
p_n'(t) &= -\lambda_n^2 D p_n(t) + a q_n(t) + f_n(p, q, t) \\
q_n'(t) &= -\lambda_n^2 D q_n(t) + b p_n(t) + g_n(p, q, t).
\end{align*}
\]

Here

\[
f_n(p, q, t) = \frac{2}{L} \int_0^L f(p(x, t), q(x, t)) \cos \frac{\pi n x}{L} \, dx,
\]

\[
g_n(p, q, t) = \frac{2}{L} \int_0^L g(p(x, t), q(x, t)) \cos \frac{\pi n x}{L} \, dx.
\]

Given the initial conditions, we verify that the system (23) is equivalent to the system of integral equations

\[
\begin{align*}
p_n'(t) &= u_n(t) + \int_0^t e^{-\lambda_n(t-\tau)} f_n(p, q, \tau) \, d\tau \\
q_n'(t) &= v_n(t) + \int_0^t e^{-\lambda_n(t-\tau)} g_n(p, q, \tau) \, d\tau.
\end{align*}
\]

Considering (25) we rewrite the equations (21) as follows

\[
\begin{align*}
p(x, t) &= u(x, t) + \int_0^t \int_0^L G(x, \xi, t, \tau) f(p(\xi, \tau), q(\xi, \tau)) \, d\xi \, d\tau \\
q(x, t) &= v(x, t) + \int_0^t \int_0^L G(x, \xi, t, \tau) g(p(\xi, \tau), q(\xi, \tau)) \, d\xi \, d\tau.
\end{align*}
\]

Here

\[
G(x, \xi, t, \tau) = \sum_{n=1}^{\infty} e^{-\lambda_n(t-\tau)} \cos \frac{\pi n \xi}{L} \cos \frac{\pi n x}{L}.
\]

Consider the linear space

\[X = \{ p = (p(x), q(x)), \ p, q \in L^2_p[0, L] \}\]

equipped with the norm

\[\| P \|_X = \| p \|_{L^2_p[0, L]} + \| q \|_{L^2_p[0, L]}\]
Denote $P = (p, q), U = (u, v), F = (f, g) \in X$, 
$$
\Phi(x) = (\varphi(x), \psi(x)).
$$

Rewrite the system (26) in the form
$$
P(x, t) = U(x, t) + \int_0^t \int_0^L G(x, \xi, t, \tau) F(P(\xi, \tau)) d\xi d\tau.
$$
(27)

The conditions on the functions $f$ and $g$ imply
$$
\| F(P_1) - F(P_2) \|_X \leq r \| P_1 - P_2 \|_X.
$$
The existence and uniqueness of the solution to the equation (26) for any initial data for sufficiently small $r$ can be proved in the standard way using the Banach fixed point theorem.

5. The inverse problem for a system of nonlinear parabolic equations

Consider the inverse problem for the nonlinear equation (27). It is required to determine the initial condition for which the solution to the equation (27) meets the equality
$$
\int_0^T P(x, t) dt = (A(x), B(x)) = \hat{A}(x).
$$

We will assume that for the given $\hat{A}(x)$ there exists an exact solution $\Phi(x)$ that belongs to the set
$$
M_R = \{ \Phi(x) : \Phi' \in X \leq R \},
$$
but we do not know the accurate values of $\hat{A}(x)$. We know only a $\delta$ - approximation $\hat{A}_\delta(x)$ and the error level $\delta > 0$ such that $\| \hat{A}_\delta(x) - \hat{A}(x) \|_X < \delta$.

The following theorem holds.

**Theorem 1.** Let $\hat{A}_1(x, t), \hat{A}_2(x, t)$ be the solutions of the equation (27) corresponding to the initial data $\Phi(x)$, let be $\hat{A}_1^0(x, t), \hat{A}_2^0(x, t)$ the solutions to the corresponding linear problem. Then the following inequalities hold
$$
e^{-rL} \| \hat{A}_1^0 - \hat{A}_2^0 \|_X \leq \| \hat{A}_1 - \hat{A}_2 \|_X \leq e^rL \| \hat{A}_1^0 - \hat{A}_2^0 \|_X.
$$

Let us denote $\Phi_\delta(x)$ the solution of the nonlinear problem corresponding to the initial conditions $\hat{A}_\delta(x)$. Denote
$$
\omega(R, \delta) = \sup \{ \| \Phi_1 - \Phi_2 \| : \Phi_1, \Phi_2 \in M_R, \| \hat{A}_1 - \hat{A}_2 \| \leq \delta \}
$$
the value of the continuity modulus for the nonlinear inverse problem, $\hat{\omega}(M, \delta)$ the value of the continuity modulus for the corresponding linear inverse problem.

The next theorem follows from Theorem 1 and the definition of the modulus of continuity.

**Theorem 2.** There exists $\delta_0 > 0$, such that for all $0 < \delta < \delta_0$ the inequalities are true
$$
\hat{\omega}(M, e^{-rL}\delta) \leq \omega(M, \delta) \leq \hat{\omega}(M, e^{rL}e^{rL}\delta).
$$

6. The projection regularization method.

To construct stable approximate solutions to the inverse problem for the system (1)-(2), we consider the expansion into the Fourier series of the given functions. Let us introduce the notations
$$
A(x) = \sum_{n=1}^{\infty} a_n \cos \frac{\pi nx}{L}, \quad B(x) = \sum_{n=1}^{\infty} b_n \cos \frac{\pi nx}{L};
A_N(x) = \sum_{n=1}^{N} a_n \cos \frac{\pi nx}{L}, \quad B_N(x) = \sum_{n=1}^{N} b_n \cos \frac{\pi nx}{L};
$$
$$
P_N(x, t) = \sum_{n=1}^{N} p_n(t) \cos \frac{\pi nx}{L}; \quad Q_N(x, t) = \sum_{n=1}^{N} q_n(t) \cos \frac{\pi nx}{L}.
$$
Here $p_N, q_N$ solve the system of equations

$$
\begin{aligned}
p'_n(t) &= -\lambda_n^2 Dp_n(t) + aq_n(t) + f_n(P_N, Q_N, t) \\
qu'_n(t) &= -\lambda_n^2 Dq_n(t) + aq_n(t) + g_n(P_N, Q_N, t) \\
\int_{0}^{T} p_n(t) dt &= a_n \\
\int_{0}^{T} q_n(t) dt &= b_n.
\end{aligned}
$$

(28)

We will consider the pair of functions

$$
\begin{aligned}
\phi_N^\delta(x, t) &= \sum_{n=1}^{N} p_n^\delta(0) \cos \frac{\pi n x}{L}; \\
\psi_N^\delta(x, t) &= \sum_{n=1}^{N} q_n^\delta(0) \cos \frac{\pi n x}{L}.
\end{aligned}
$$

that is the element

$$
\Phi_N^\delta = (\phi_N^\delta, \psi_N^\delta)
$$

of the space $X$ as an approximate solution to the nonlinear inverse problem under the choice of the relation $N = N(\delta)$ according to the balancing principle.

It is important to obtain the uniform estimate for the accuracy of the approximate solution on the set $M$, that is we have to estimate the value

$$
\Delta(N(\delta), \delta) = \sup \{ || \Phi_N^\delta - \Phi || ; \Phi \in M_R; \| A - A_\delta \| \leq \delta \}.
$$

Carrying out standard calculations, we obtain the estimate

$$
C_1 \sqrt{\delta} \leq \Delta(N(\delta), \delta) \leq C_2 \sqrt{\delta}
$$

(29)

for some constants $C_1, C_2$ and sufficiently small $\delta$.

The estimate (29) implies that the projection regularization method is optimal in the order on the set $M$.

7. The method of nonlocal auxiliary conditions.

We use the following problem with a small parameter $\alpha > 0$ in auxiliary conditions with respect to $t$ to construct stable approximate solutions to the inverse problem. Let $(p^\delta_{\alpha}(x, t), q^\delta_{\alpha}(x, t))$ solve the system of parabolic equations

$$
\begin{aligned}
dp\delta_{\alpha}(x, t) &= D \frac{d^2 p^\delta_{\alpha}}{dx^2} + aq^\delta_{\alpha} + f\left(p^\delta_{\alpha}, q^\delta_{\alpha}\right), \\
\frac{dq^\delta_{\alpha}}{dt} &= D \frac{d^2 q^\delta_{\alpha}}{dx^2} + bp^\delta_{\alpha} + g(p^\delta_{\alpha}, q^\delta_{\alpha}).
\end{aligned}
$$

(30) (31)

The boundary conditions

$$
\begin{aligned}
\frac{dp^\delta_{\alpha}}{dx}(0, t) &= \frac{dp^\delta_{\alpha}}{dx}(L, t) = 0, \\
\frac{dq^\delta_{\alpha}}{dx}(0, t) &= \frac{dq^\delta_{\alpha}}{dx}(L, t) = 0,
\end{aligned}
$$

and the additional nonlocal conditions

$$
\begin{aligned}
A^\delta_{\alpha}(x) &= \int_{0}^{T} p^\delta_{\alpha}(x, t) dt + \alpha p^\delta_{\alpha}(x, 0); \\
B^\delta_{\alpha}(x) &= \int_{0}^{T} q^\delta_{\alpha}(x, t) dt + \alpha q^\delta_{\alpha}(x, 0).
\end{aligned}
$$
are also satisfied.

Involving the Banach fixed point theorem one can prove that the regularized problem (32)-(33) has a unique solution in the functional space under consideration.

Let \( (p_\delta(x,t), q_\delta(x,t)) \) be the solution to (30)-(31). We consider the functions
\[
p_\delta(x,0) = q_\delta(x); \quad q_\infty(x,0) = \psi_\infty(x)
\]
with the appropriate choice of the regularization parameter as approximate solutions to the inverse problem.

The error of the approximate solution defined by (30)-(31) can be estimated similar to the estimate for the projection regularization method. The estimate shows that the method is optimal in the order on the set \( M \).

8. The numerical results.

We solve the regularized problem (30)-(31) to obtain a stable approximate solution of the inverse problem. We use the following linear finite-difference scheme to approximate the parabolic equations:
\[
\frac{p_{l}^{n+1} - p_{l}^{n}}{\tau} = \frac{p_{l+1}^{n+1} - 2p_{l}^{n+1} + p_{l-1}^{n+1}}{h^2} + q_{l}^{n} + f(p_{l}^{n}).
\]
\[
\frac{q_{l}^{n+1} - q_{l}^{n}}{\tau} = \frac{q_{l+1}^{n+1} - 2q_{l}^{n+1} + q_{l-1}^{n+1}}{h^2} + p_{l}^{n} + g(q_{l}^{n}).
\]

The auxiliary nonlocal condition is also approximated by its discrete analog.

The choice of the regularization parameter is based on the balancing principle.

In the example, the exact solution of the inverse problem is the pair of piecewise continuous functions. The data for the model example was taken from [2] and the numerical results correspond to the results obtained in [2] quite closely. The initial data of the inverse problem were perturbed by a random error. In the examples on the Figure 1 below the value of the regularization parameter is \( \alpha = 1.3 \). The green line represents the plot of the exactly given function \( p(x) \), the yellow line represents the plot of the exactly given function \( q(x) \) (\( q(x) \equiv 0 \) in the example); the blue and red lines represent plots of \( p(x) \) and \( q(x) \) respectively for the approximate solution to the system. The values of the variable \( x \) are plotted on the horizontal axis, the values of \( p(x) \) and \( q(x) \) are plotted on the vertical axis.

The Figure 2 represents an approximate solution to the system with the value of the regularization parameter \( \alpha = 0.25 \).
The example shows that the choice of the regularization parameter plays an important role in obtaining reliable numerical results.

The proposed method was tested also for smooth initial data.

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