The Impact of Plasma Instabilities on the Spectra of TeV Blazars

RAFAEL ALVES BATISTA, ANDREY SAVELIEV, AND ELISABETE M. DE GOUVEIA DAL PINO

1Instituto de Astronomia, Geofísica e Ciências Atmosféricas, Universidade de São Paulo, 05508-090 São Paulo-SP, Brazil
2Institute of Physics, Mathematics and Information Technology, Immanuel Kant Baltic Federal University, 236041 Kaliningrad, Russia
3Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University, 119991 Moscow, Russia

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ABSTRACT

Relativistic jets from blazars are known to be sources of very-high-energy gamma rays (VHEGRs). During their propagation in the intergalactic space, VHEGRs interact with pervasive cosmological photon fields such as the extragalactic background light (EBL) and the cosmic microwave background (CMB), producing electron-positron pairs. These pairs can upscatter CMB/EBL photons to high energies via inverse Compton scattering, thereby continuing the cascade process. This is often used to set limits on intergalactic magnetic fields (IGMFs). However, the picture may change if plasma instabilities, arising due to the interaction of the pairs with the intergalactic medium (IGM), cool down the electrons/positrons faster than inverse Compton scattering. As a consequence, the kinetic energy lost by the pairs to the IGM could cause a hardening in the observed gamma-ray spectrum at energies below \(\sim 100\) GeV. Here we study several types and models of instabilities and assess their impact for interpreting observations of distant blazars. Our results suggest that plasma instabilities can describe the spectra of some blazars and mimic the effects of IGMFs, depending on parameters such as intrinsic spectrum of the object, the density and temperature of the IGM, and the luminosity of the beam.

Keywords: plasma instabilities, gamma rays, electromagnetic cascades, intergalactic medium, intergalactic magnetic fields

1. INTRODUCTION

High-energy (i.e. GeV and TeV) and very-high-energy gamma rays (VHEGRs) have become important tools to probe astrophysical and cosmological phenomena. This includes studies of our own galaxy (Weekes et al. 1989; Strong et al. 2004; Aharoonian et al. 2006), BL Lac objects (Quinn et al. 1996), and gamma-ray bursts (Gehrels & Meszaros 2012). Cosmological scales can also be probed with high-energy gamma rays, enabling constraints on the opacity of the Universe (Taylor et al. 2011), and heating of the intergalactic medium (IGM) (Chang et al. 2012), among others. The plethora of information that can be obtained with this messenger is possible mainly thanks to experimental efforts such as the High-Energy Stereoscopic System (H.E.S.S.) (Hofmann 2000), the Very Energetic Radiation Imaging Telescope Array System (VERITAS) (Weekes et al. 2002), the Fermi Large Area Telescope (Fermi-LAT) (Atwood et al. 2009), and the Major Atmospheric Gamma Imaging Cherenkov (MAGIC) (Rico 2017). Future air-Cherenkov telescopes such as the upcoming Cherenkov Telescope Array (CTA) (CTA Consortium 2019) will provide further insights into the high-energy Universe.

Of particular interest are gamma rays observations of TeV blazars, i.e. active galactic nuclei (AGNs) whose jets are nearly parallel to the line of sight of emission. The standard model for the propagation of TeV gamma rays from the source to the observer is well-understood (Neronov & Semikoz 2009). The emitted VHEGRs interact with background photons from the extragalactic background light (EBL) and cosmic microwave background (CMB), with a mean free path of approximately 100 Mpc. Note that the EBL, in particular infrared photons, are the most relevant ones for the energies considered here. This

Corresponding author: Andrey Saveliev
andr.alexiev@desy.de, rafael.ab@usp.br, dalpino@iag.usp.br
interaction results in the production of electron-positron pairs. These electrons\(^1\) then proceed by scattering off (mostly) CMB photons via inverse Compton (IC) scattering, producing new gamma rays and hence creating an electromagnetic cascade.

Intergalactic magnetic fields (IGMFs) can deflect the charged particles produced in the cascade, i.e., electrons and positrons. Hence, gamma-ray observations should, in principle, provide information about IGMFs. Possible signatures are halos around point-like sources (Neronov & Semikoz 2007, 2009; Dolag et al. 2009; Ackermann et al. 2018; Chen et al. 2018), or spiral patterns in case of helical fields (Tashiro et al. 2014; Alves Batista et al. 2016a; Duplessis & Vachaspati 2017), as well as time delays (Neronov & Semikoz 2009; Murase et al. 2008; Fitoussi et al. 2017), and a suppression of the observed GeV flux (d’Avezac et al. 2007; Neronov & Vovk 2010; Taylor et al. 2011; Kempf et al. 2016).

While seemingly simple, the underlying mechanism of electromagnetic cascades is controversial. As first pointed out by Broderick et al. (2012), the pairs may interact with the IGM due to emerging plasma instabilities in the beam, cooling down before IC scattering can occur. This is a possible explanation for the absence of photons at GeV energies, observed in the spectra of some blazars. Yet, it should be noted that this interpretation, too, is disputed. While it is true that instabilities should occur when a relativistic beam traverses a background plasma, it is presently virtually impossible to directly simulate these scenarios, such that one has to rely on extrapolations which, however, are rather uncertain.

In this paper we study the spectra of some specific blazars for different plasma instability models. We first (section 2) provide a brief description of the types of instabilities considered including a discussion of the relevant physical parameters and conditions, accompanied by a number of models found in the literature. Then, in section 3, we describe the implementation of the instabilities in our Monte Carlo code. In section 4 we study the effect of some relevant quantities on the expected gamma-ray fluxes for one specific scenario, while section 5 gives an account of the predicted fluxes of gamma rays for a selected list of blazars. In section 6 we discuss our results, and present our concluding remarks in section 7.

2. PLASMA INSTABILITIES IN THE IGM

In this section we present instabilities that might occur due to the interaction between the beam and the plasma.

2.1. Types of instabilities

The two-stream instability, first described in Bohm & Gross (1949), appears if the wave vector of the electrostatic perturbation is parallel to the flow for a range of wave numbers \(0 < k \leq \omega_{pl}/v_{beam}\), where \(\omega_{pl}\) is the plasma frequency (Bret 2009). In a hot beam the speed of some particles equals the phase velocity of the wave, hence being in resonance with it. Due to the form of the velocity distribution, more energy is transferred from the beam particles to the wave mode than vice versa, thus resulting in an instability. For a cold beam plasma the mechanism is slightly different, but still results in growing wave modes since the hot beam the speed of some particles equals the phase velocity of the wave, hence being in resonance with it. Due to the form of the velocity distribution, more energy is transferred from the beam particles to the wave mode than vice versa, thus resulting in an instability. For a cold beam plasma the mechanism is slightly different, but still results in growing wave modes since the speed of some particles equals the phase velocity of the wave, hence being in resonance with it. Due to the form of the velocity distribution, more energy is transferred from the beam particles to the wave mode than vice versa, thus resulting in an instability.

The oblique instability is, in principle, very similar to the two-stream instability described above. The main difference is the fact that in this case the wave vector is not parallel to the beam; instead, the two are at an angle \(\theta\) to each other. It can then be shown (Nakar et al. 2011) that in the astrophysical setting the dispersion relation is given by

\[
\frac{1}{m_{beam}} m_{IGM} \Psi (\Gamma, \gamma, k, \omega) - \left( \omega - C \frac{k \cos \theta n_{beam}}{\omega_{pl} n_{IGM}} \right)^{-1},
\]

where \(k\) is the wave vector, \(C\) is the fractional charge of the beam, \(\Gamma\) is the shock Lorentz factor and \(\Psi\) is a function of the given quantities. For \(\theta = 0\) this equation can be solved analytically and gives the growth time for the two-stream instability discussed.

\(^1\) Henceforth we refer collectively to electrons and positrons simply as ‘electrons’, unless stated otherwise.
above. To find the growth time for the oblique instability at an arbitrary angle $\theta$ one has, in general, to resort to numerical simulations. As has been pointed out by Broderick et al. (2012), these two growth rates can differ significantly due to the fact that the relevant quantity for the beam–wave interaction is the projected velocity of the beam onto the wave vector, which for large $\theta$ is rather small, such that it is easier to create deflections and, subsequently, instabilities.

The modulation instability in its simplest form can be explained as the result of ions in a turbulent medium scattering off oscillations caused by the beam. For this reason, they are transferred to smaller wave numbers, shifting the wave energy to higher (even superluminal) phase speeds (Galeev et al. 1977). This takes place if the energy density of the electrostatic fluctuations $\epsilon_e$, given by (Schlickeiser et al. 2002)

$$\epsilon_e = n_{\text{beam}} m_e c^2 (\gamma - 1)$$

becomes larger than the critical value (Galeev et al. 1975; Schlickeiser et al. 2013)

$$\epsilon_{\text{crit}} = \frac{5}{3} n_{\text{IGM}} \left( k_B T_{\text{IGM}} \right)^2 \frac{m_e c^2}{\epsilon_e}.$$  

If this condition is fulfilled, one can find the growth rate for the modulation instability by analyzing the dispersion relation

$$1 = \frac{\omega_{\text{pl}}^2}{\omega^2} + \frac{\omega^2 \sin^2 \theta \left( 1 - \frac{\gamma^2 - 1}{\gamma^2} \cos^2 \theta \right)}{\left( \omega - kc \cos \theta \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \right)^2},$$

It should be noted here that formally the modulation instability is nothing but an oscillating two-stream instability (Papadopoulos 1975; Schamel et al. 1976; Schlickeiser et al. 2012).

The Weibel instability, named after the author of the seminal work on the topic, Weibel (1959), is the result of the interaction between several counter-streaming beams (Fried 1959). In the simplest form, following Medvedev & Loeb (1999), we can consider two beams in opposite directions with a vanishing net current. Small anisotropies in velocity cause fluctuations of the magnetic field perpendicular to the direction of the beam. For example, without loss of generality, if we take the particles to move along the $x$-axis, then the magnetic fluctuation can be represented in the form

$$\mathbf{B} = B \cos (ky) \mathbf{e}_z.$$  

The resulting Lorentz force deflects the electrons in the beams in such a way that two current sheets are produced. Such large-scale currents induce larger magnetic fields which, in turn, increase the force, thus enhancing the effect and consequently producing the instability. The instability growth time is then given by

$$T = \frac{c}{\omega_{\text{pl}} v_y}.$$  

Following this interpretation, we can regard the Weibel Instability as the generalization of the two-stream instability with magnetic fields. Note that while the former does not contain oscillatory modes, they are present in the latter instability, which represents a fundamental difference between the two. The instability ceases to grow once the magnetic fields are strong enough to disrupt the currents or once the kinetic regime commences, due to large transverse velocities (Broderick et al. 2012).

Finally, another possible mechanism is non-linear Landau damping. As the name suggests, the effect is highly non-linear and may be described as induced scattering by thermal ions, such that the frequency and wave vector of a given plasma (Langmuir) wave are transformed to different values (Brejzman & Ryutov 1974; Chang et al. 2014). The corresponding kinetic equation, which describes the wave transformation $(\mathbf{k}, \omega) \to (\mathbf{k}', \omega')$, is given by

$$\frac{d\epsilon_k}{df} = \frac{\epsilon_k}{T_k} - \frac{\epsilon_k \omega_{\text{pl}}}{8(2\pi)^{3/2} m_e c^2} \int \frac{(\mathbf{k} \cdot \mathbf{k}') \phi(\mathbf{k}, \mathbf{k}') \epsilon_{k'} d\mathbf{k}'}{k'^2},$$

where $\epsilon_k$ is the spectral energy density and $\phi(\mathbf{k}, \mathbf{k}')$ is the overlap integral given in Kaplan & Tsytovich (1968). Here, we have $1/T_k = 1/T_{\text{inst}} + 1/T_{LD}$, where $T_{\text{inst}}$ denotes the instability growth time, and $T_{LD}$ denotes the growth time for linear Landau damping.
2.2. Physical Parameters

In this section we discuss the IGM and beam parameters which have an influence on the growth of plasma instabilities. First, one should consider the number density of the IGM ($n_{\text{IGM}}$), which evolves as

$$n_{\text{IGM}} = n_{\text{IGM},0} (1 + z)^3,$$

where $z$ is the redshift at which the number density is calculated, and $n_{\text{IGM},0}$ is the IGM density at present time. Our fiducial value for $n_{\text{IGM},0}$ is $n_{\text{IGM},0} = 10^{-7}$ cm$^{-3}$ (Broderick et al. 2012). It should be noted that this is a rather simple model, not taking into account density fluctuations; these may be included if we multiply equation 10 by $(1 + \delta)$, where $\delta$ is the overdensity.

The temperature of the IGM ($T_{\text{IGM}}$) has many uncertainties associated with it. Depending on whether one considers cosmic voids or the intracluster medium, the value of $T_{\text{IGM}}$ ranges from $\sim 10^{3}$ K (Hui & Gnedin 1997) in voids, to $\sim 10^{9}$ K in clusters (Reimer & Böttcher 2013; Peruch et al. 2011; Burns et al. 2010). In order to account for this range, we adopt $T_{\text{IGM}} = [10^{3}, 10^{4}, 10^{5}, 10^{6}]$ K, with $T_{\text{IGM}} = 10^{4}$ K being our fiducial value for the simulations presented below. This choice is fully justified if we consider that the volume filling factor of voids exceed by orders of magnitude that of clusters. This range also encompasses the values considered by Broderick et al. (2012), Chang et al. (2012), and Miniati & Elyiv (2013).

When it comes to the source parameters, the first important one is the total isotropic-equivalent luminosity ($\mathcal{L}$). For TeV sources this value lies in the range $10^{41}$ erg s$^{-1}$ and $10^{47}$ erg s$^{-1}$ (Broderick et al. 2012). We adopt the fiducial value of $\mathcal{L} = 10^{45}$ erg s$^{-1}$.

Another important quantity for the calculation of plasma instabilities is the beam plasma number density ($n_{\text{beam}}$). As this is a dynamic quantity, it is usually only possible to estimate it. In this work we follow Broderick et al. (2012), using an upper limit for $n_{\text{beam}}$, which comes from analytical estimates of the cascade development,

$$n_{\text{beam}} \simeq 7.4 \times 10^{-22} \text{ cm}^{-3} \left( \frac{\mathcal{L}}{10^{45} \text{ erg s}^{-1}} \right) \left( \frac{E_{\gamma}}{10^{12} \text{ eV}} \right) \left( \frac{1+z}{2} \right)^{3\zeta-4},$$

wherein $\zeta = 4.5$ for $z < 1$ is a parameter that can be inferred from the analysis of the local star formation rate (Kneiske et al. 2004).

2.3. Models

Here we present and discuss the different models which we use to obtain blazar spectra in chronological order. In general, in the literature on this topic we find two different fundamental time scales. The first is the instability growth rate, which we are going to label $\tau_i$. The second is the energy loss time of the electron/positron due to plasma effects, $\tau_i$, where in both cases $i$ refers to the model considered (see below).

In order to obtain the blazar spectra, the relevant quantity is the electron energy loss time $\tau_i$, which may be obtained even if only $\tau_i$ is given by using the estimate of the lower limit for $\tau_i$, which reads (Gronghard 1975; Pavan et al. 2011) $\tau_i = 100\tau_i$.

2.3.1. Model A

One of the first works on this topic and the one on which we mostly base our model A, is Broderick et al. (2012), later on expanded and further analyzed by Chang et al. (2012, 2014, 2016); Broderick et al. (2014); Shalaby et al. (2017, 2018). The authors distinguish between two regimes referred to as the ‘warm’ and ‘cold’ plasma cases depending on whether the beam density, $n_{\text{beam}}$, is below (warm) or above (cold) the critical value $n_{\text{crit},\text{A}}$:

$$n_{\text{crit},\text{A}} = 1.6 \times 10^{-19} \text{ cm}^{-3} \left( \frac{E_{\gamma}}{10^{12} \text{ eV}} \right)^{-2} \left( \frac{n_{\text{IGM}}}{10^{-7} \text{ cm}^{-3}} \right).$$

Broderick et al. (2012) come to the conclusion that in both cases the oblique instability is the dominant one, however resulting in different expressions which can be summarized as:

$$\tau_A = \begin{cases} 
7.1 \times 10^7 \text{ s} \left( \frac{E_{\gamma}}{10^{12} \text{ eV}} \right)^{-1} \left( \frac{n_{\text{beam}}}{10^{-22} \text{ cm}^{-3}} \right)^{-1} \left( \frac{n_{\text{IGM}}}{10^{-7} \text{ cm}^{-3}} \right)^{\frac{1}{4}} & n_{\text{beam}} \leq n_{\text{crit},\text{A}} \text{ (warm beam plasma)}, \\
5.1 \times 10^5 \text{ s} \left( \frac{E_{\gamma}}{10^{12} \text{ eV}} \right)^{\frac{1}{2}} \left( \frac{n_{\text{beam}}}{10^{-22} \text{ cm}^{-3}} \right)^{\frac{1}{4}} \left( \frac{n_{\text{IGM}}}{10^{-7} \text{ cm}^{-3}} \right)^{-\frac{1}{2}} & n_{\text{beam}} > n_{\text{crit},\text{A}} \text{ (cold beam plasma)},
\end{cases}$$

2.3.2. Model B
Table 1. Tabulated values for $\xi_B(D)$ from equation 14.

| $\log_{10}(D/{\text{Mpc}})$ | -0.05 | 0.14 | 0.35 | 0.59 | 0.79 | 1.17 | 1.40 | 1.57 | 1.77 | 1.99 | 2.20 | 2.41 | 2.60 | 2.80 | 3.00 |
|-----------------------------|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| $\log_{10}\xi_B(D)$       | 3.28  | 3.46 | 3.14 | 3.08 | 3.05 | 3.00 | 2.84 | 2.56 | 2.40 | 2.55 | 2.36 | 2.08 | 1.73 | 1.55 | 1.05 | 0.75 |

Miniati & Elyiv (2013) consider the most relevant effect to be the interplay between Langmuir waves and non-linear Landau damping. They conclude that due to that the effect of plasma oscillations on energy losses of the electron-positron beams is mostly negligible. Nevertheless, for the sake of completeness, we consider it here and compare it with the other models.

The authors obtained the energy loss time, $\tau_B$, through a combination of Monte Carlo simulations to determine the beam properties as it propagates, and analytic solutions of appropriate equations, to obtain the instability growth rate. We can write

$$\tau_B = \tau_C \xi_B(D) (1 + z)^2 = 3.9 \times 10^{13} \text{s} \left(1 + z\right)^{-2} \left(\frac{E_e}{10^{12} \text{eV}}\right)^{-1} \xi_B(D),$$

where $E_e$ is the electron/positron energy, $D$ is the co-moving distance to the source, and $\xi_B(D)$ is tabulated as shown in table 1.

2.3.3. Model C

This model is based on analytic calculations developed by Schlickeiser et al. (2012, 2013). In particular, in Schlickeiser et al. (2012) the authors find that the different effects contribute to the suppression of the electromagnetic cascade at different regimes. In the ‘strong blazar’ regime the beam plasma density is lower than a certain critical value $n_{\text{crit},C}$, resulting in a dominance of the modulation instability. If $n_{\text{beam}} > n_{\text{crit},C}$, the ‘weak blazar’ case, the modulation instability does not set in; in this case, non-Linear Landau damping becomes relevant, such that energy is deposited in electrostatic and electromagnetic fluctuations.

The energy loss time is

$$\tau_C = \begin{cases} 
5.2 \times 10^{14} \text{s} \left(\frac{E_e}{10^{12} \text{eV}}\right)^{\frac{3}{2}} \left(\frac{n_{\text{beam}}}{10^{-22} \text{cm}^{-3}}\right)^{\frac{1}{2}} \left(\frac{n_{\text{IGM}}}{10^{-7} \text{cm}^{-3}}\right)^{-\frac{1}{2}} \left(\frac{T_{\text{IGM}}}{10^4 \text{K}}\right)^{-\frac{1}{2}} 
& n_{\text{beam}} \leq n_{\text{crit},C} \text{ (weak blazar)}, \\
8.3 \times 10^6 \text{s} \left(\frac{E_e}{10^{12} \text{eV}}\right)^{\frac{1}{2}} \left(\frac{n_{\text{beam}}}{10^{-22} \text{cm}^{-3}}\right)^{-\frac{1}{2}} \left(\frac{n_{\text{IGM}}}{10^{-7} \text{cm}^{-3}}\right)^{-\frac{1}{2}} \xi_C(n_{\text{IGM}}, T_{\text{IGM}}) 
& n_{\text{beam}} > n_{\text{crit},C} \text{ (strong blazar)},
\end{cases}$$

where $\xi_C$ is

$$\xi_C(n_{\text{IGM}}, T_{\text{IGM}}) = 1 + \frac{5}{4} \ln \left(\frac{T_{\text{IGM}}}{10^4 \text{K}}\right) - \frac{1}{4} \ln \left(\frac{n_{\text{IGM}}}{10^7 \text{cm}^{-3}}\right),$$

and $n_{\text{crit},C}$ is given by

$$n_{\text{crit},C} = 2.5 \times 10^{-25} \text{cm}^{-3} \left(\frac{E_e}{10^{12} \text{eV}}\right)^{-1} \left(\frac{n_{\text{IGM}}}{10^{-7} \text{cm}^{-3}}\right) \left(\frac{T}{10^4 \text{K}}\right)^2.$$  

2.3.4. Model D

This model is based on Sironi & Giannios (2014), though it is quite similar to model A. The authors distinguish two cases, the ‘cold beam’ and the ‘hot beam’, depending on whether the beam plasma density $n_{\text{beam}}$ is above or below value $n_{\text{crit},D}$, respectively. For both cases they find that the oblique instability is the most relevant one, resulting in the energy loss time

$$\tau_D = \begin{cases} 
1.4 \times 10^7 \text{s} \left(\frac{E_e}{10^{12} \text{eV}}\right)^{-1} \left(\frac{n_{\text{beam}}}{10^{-22} \text{cm}^{-3}}\right)^{-1} \left(\frac{n_{\text{IGM}}}{10^{-7} \text{cm}^{-3}}\right)^{\frac{1}{2}} 
& n_{\text{beam}} \leq n_{\text{crit},D} \text{ (hot beam plasma)}, \\
9.6 \times 10^5 \text{s} \left(\frac{E_e}{10^{12} \text{eV}}\right)^{\frac{1}{2}} \left(\frac{n_{\text{beam}}}{10^{-22} \text{cm}^{-3}}\right)^{-\frac{1}{2}} \left(\frac{n_{\text{IGM}}}{10^{-7} \text{cm}^{-3}}\right)^{-\frac{1}{2}} 
& n_{\text{beam}} > n_{\text{crit},D} \text{ (cold beam plasma)},
\end{cases}$$

where $n_{\text{crit},D}$ is given by

$$n_{\text{crit},D} = 8.0 \times 10^{-20} \text{cm}^{-3} \left(\frac{E_e}{10^{12} \text{eV}}\right)^{-2} \left(\frac{n_{\text{IGM}}}{10^{-7} \text{cm}^{-3}}\right).$$
2.3.5. Model E

The final model we adopt is mostly taken from Vafin et al. (2018), who find that the modulation instability dominates the energy losses. Through a thorough analysis, they arrive at

\[ \tau_E = 1.9 \times 10^{11} \text{ s} \left( \frac{E_e}{10^{12} \text{ eV}} \right)^{\frac{2}{3}} \left( \frac{n_{\text{beam}}}{10^{-22} \text{ cm}^{-3}} \right)^{\frac{1}{3}} \left( \frac{n_{\text{IGM}}}{10^{-7} \text{ cm}^{-3}} \right)^{-\frac{1}{3}} \left( \frac{T_{\text{IGM}}}{10^4 \text{ K}} \right)^{-1}. \]  

(20)

3. SIMULATION SETUP

For this analysis we employ the Monte Carlo code CRPropa 3 (Alves Batista et al. 2016b). It enables the propagation of high-energy particles in the intergalactic space. It includes the relevant interactions for the development of electromagnetic cascades, namely inverse Compton scattering, pair production, and adiabatic losses due to the expansion of the Universe. We consider as target background fields the CMB and choose EBL model by Gilmore et al. (2012). This choice quantitatively affects the shape of the simulated spectra, but it is virtually irrelevant for the qualitative discussion that follows.

We took advantage of the modular structure of CRPropa code to develop an extension\(^2\) dedicated to calculate the effects of plasma instabilities in an approximate fashion. To this end, we convert the energy loss times given by equations 13, 14, 15, 18, and 20 to the energy loss per distance, \( P(E, x, z) \), which in the ultrarelativistic limit can be written as

\[ P(E_e, x, z) \equiv -\frac{dE_e}{dx}(E_e, x, z) = \frac{E_e}{c\tau(E_e, x, z)}, \]  

(21)

where \( x \) is the trajectory length propagated by the particle and \( c \) is the speed of light.

We should stress that the values of \( \tau \) for models B, C, and E are the actual energy loss times of the beams, as in our treatment; for models A and D, they correspond to the maximum instability growth time. As has been argued in the corresponding publications, the actual energy loss (and hence the value of \( P \)) can be significantly smaller. Therefore, all values of \( \tau \) presented here should be interpreted as lower limits; this should be kept in mind for the analysis carried out in sections 4 and 5.

To calculate the energy loss function \( P \) for the different models, we combine the respective values for \( \tau \) from section 2.3 with the model of the beam density \( n_{\text{beam}} \) from equation 11 (with \( \zeta = 4.5 \)), as we are only interested in blazars with \( z \ll 1.0 \) and the value of \( n_{\text{IGM}} \) from (10).

In figure 1 we compare the interaction rates for the different models, for a typical combination of parameters. Note that the energy dependence across the different models differ considerably.

As a benchmark scenario, we consider the case of a source located at \( z \approx 0.14 \), which is approximately the redshift of the extreme blazar 1ES 0229+200, commonly used in studies of IGMFs (Neronov & Vovk 2010; Vovk et al. 2012; Arlen et al. 2014; Yang & Dai 2015; Yan et al. 2019). We model the spectrum injected by the blazar as a power law of the form

\[ \frac{dN}{dE} \propto E^{-\alpha} \exp \left( -\frac{E}{E_{\text{max}}} \right). \]

(22)
Figure 2. Gamma-ray fluxes observed at Earth for the benchmark scenario and the different models, assuming $n_{\text{IGM,0}} = 10^{-7}$ cm$^{-3}$ and $\mathcal{L} = 10^{45}$ erg/s. The scenario with no plasma instabilities is represented as a dashed line.

Unless stated otherwise, we adopt the parameters from Vovk et al. (2012): $\alpha = 1.2$ and $E_{\text{max}} = 5 \times 10^{12}$ eV. We use the EBL model by Gilmore et al. (2012), although this choice should not significantly change the interpretation of the results.

4. SIMULATION RESULTS

Now we present the results of the simulations. Throughout this section, we present the results for our benchmark scenario, fixing $T_{\text{IGM,0}} = 10^4$ K, $n_{\text{IGM,0}} = 10^{-7}$ cm$^{-3}$, and $\mathcal{L} = 10^{45}$ erg/s and varying one of the parameters at a time. As stated in section 2.2, we also fix $\alpha = 1.2$ and $E_{\text{max}} = 5$ TeV.

4.1. Plasma instability models

We first compare all the models for our benchmark scenario. This is shown in figure 2. Note that the slope for all models but B are rather similar in the energy range of $\sim 10 - 300$ GeV. This is expected because the spectral index of injection ($\alpha = 1.2$ for the benchmark scenario) should be retrieved if virtually all secondary electrons are removed from the cascade. If plasma instabilities are not as efficient as we considered, then the picture changes and intermediate cases are expected. If identified, this slope would be a phenomenological signature of a very efficient process quenching the cascade; plasma instabilities would then be a candidate explanation. Departures from this behaviour may occur for different values of $n_{\text{IGM,0}}$, $T_{\text{IGM,0}}$, and $\mathcal{L}$, as will be discussed later. Nevertheless, for $30 \lesssim E/\text{GeV} \lesssim 100$ all models A, C, D, and E led to a slope of $E^{-1.2}$ at energies $\lesssim 100$ GeV in most cases studied, the exception being model C for some particular combinations of parameters.

We confirm the results by Miniati & Elyiv (2013), who argue that plasma effects do not lead to noticeable changes in the observed gamma-ray flux. This is shown in figure 2 and can be confirmed by analyzing figure 1, which shows that interaction rates for this model are orders of magnitude below those for IC scattering at the energy range of interest ($E \lesssim 1$ PeV). For this reason, we omit plots for model B when studying the effects of temperature, IGM density, beam luminosity, and spectral parameters.

4.2. Temperature of the IGM

We study the effects of the temperature on the spectrum for our benchmark scenario. We vary only the temperature, while keeping all other quantities ($n_{\text{IGM,0}} = 10^{-7}$ cm$^{-3}$ and $\mathcal{L} = 10^{45}$ erg/s) fixed.

As shown in figure 3, the temperature dependence is more prominent for models C and E, while models A and D are virtually temperature-independent. This follows immediately from equations 13, 15, 18, and 20 as there are no explicit temperature dependencies on equations 13 and 18.

At $T_{\text{IGM,0}} = 10^4$ K the spectrum for model C presents a change in slope at energies $\lesssim 3$ GeV. This may be an observational signature of this model, and if observed in multiple blazars, would allow us to draw conclusions about the temperature or correlated parameters.

4.3. Density of the IGM

Now we discuss how the density of the IGM affects the gamma-ray spectrum. We fix $T_{\text{IGM,0}} = 10^4$ K and $\mathcal{L} = 10^{45}$ erg/s, varying the density only. The predicted gamma-ray spectra for our benchmark object are shown in figure 4.
Models A, D, and E are unremarkable, in the sense that the cascade is completely suppressed by the plasma instabilities, as shown in figure 4. Model C presents clear spectral signatures of the IGM density. There is a bump at $E \sim 5$ GeV for $n_{\text{IGM},0} = 10^{-7}$ cm$^{-3}$. The onset of this feature seems to start at higher energies as $n_{\text{IGM},0}$ increases. With our simulations we cannot confirm if this bump is shifted to lower energies, as our minimum energy is set to 1 GeV. However, if we extrapolate the analysis of the scenario with $n_{\text{IGM},0} = 10^{-6}$ cm$^{-3}$ it is not unreasonable to expect this to be the case.

4.4. Beam luminosity

We proceed our study of the relevant parameters by investigating how the luminosity of the blazar beam affects the shape of the gamma-ray spectrum. Once again, we fix all other variables and vary solely the luminosity for $T_{\text{IGM,0}} = 10^4$ K and $n_{\text{IGM,0}} = 10^{-7}$ cm$^{-3}$. The results are shown in figure 5.

As in the case of the temperature and the IGM density, spectral features arise for model C and, to a lesser degree, in models A and D. As the beam luminosity decreases, an increase in the flux at $E \sim 3$ GeV can be seen. The onset of this flux enhancement with respect to the full suppression of the cascade becomes visible at increasingly higher energies as the beam becomes dimmer.

The aforementioned spectral signatures are not surprising. In fact, equations 13, 15, 18, and 20 all depend on the beam density, as can be seen in equation 11.

The vast majority of blazars have isotropic-equivalent luminosities $L \lesssim 10^{45.5}$ erg/s. Therefore, the cooling due to plasma instabilities may not be as severe as for the case of very luminous objects.

4.5. Spectral parameters

We now investigate the effects of the spectral parameters ($E_{\text{max}}$ and $\alpha$) on the shape of the predicted gamma-ray spectra. To this end, we use our benchmark scenario fixing the following parameters: $T_{\text{IGM,0}} = 10^4$ K, $n_{\text{IGM,0}} = 10^{-7}$ cm$^{-3}$, and $L = 10^{45}$ erg/s.

\footnote{The lower energy cutoff of 1 GeV was chosen to optimise the time required to run the simulations, while covering a range of energies that would allow for meaningful conclusions to be drawn.}
As previously mentioned, for model B there are no significant spectral changes due to plasma instabilities. For the other models the spectral modifications are similar to each other, so that we choose only one model to illustrate this discussion: model A.

The energy-dependent quenching factor is defined as the ratio between \( j_{pl} \), the spectrum obtained in the presence of plasma instabilities, and \( j_0 \), the corresponding spectra in their absence, where \( j \equiv dN/dE \). This is plotted as a function of the energy in figure 6, for different spectral indices.

We find that the blazar spectral index plays a major role in the development of the cascades. Harder spectra (\( \alpha \gtrsim 1.5 \)) lead to quenching factors of \( \sim 100 \) at \( E \sim 1 \) GeV for \( E_{\text{max}} \lesssim 10 \) TeV, whereas for \( \alpha \gtrsim 2 \) the quenching factors are \( \lesssim 2 \). The maximal energy also significantly affects the spectral shape. The higher the \( E_{\text{max}} \), the larger the quenching factors for a fixed spectral index.

These results are expected. The contribution of events with higher energies is larger in the case of harder spectra (lower \( \alpha \)) than for softer spectra. Similarly, as \( E_{\text{max}} \) increases, so does the quenching factor.

5. APPLICATION TO SELECTED BLAZARS

In this section we discuss the consequences of plasma instabilities for VHEGR spectra of a few blazars. We select a few extreme BL Lacertae objects known to produce multi-TeV gamma rays. This list is not meant to be complete, being merely a sample of objects commonly used in studies aiming to constrain IGMFs with gamma rays. We list these object in table 2.

Note that the parameters \( \alpha \) and \( E_{\text{max}} \), presented in table 2, are used only to illustrate the effects of the plasma instabilities on the gamma-ray flux, and for a qualitative discussion. It is beyond the scope of this work to perform a fit of the model to the data.

In figure 7 we compute the expected gamma-ray fluxes considering the effect of plasma instabilities. As discussed in section 4, the predictions for model B are virtually compatible with a negligible action of the instabilities and are ignored in this section. We consider the range of parameters discussed in the section 4 and represent these uncertainties as bands.

From figure 7 we can draw our first conclusion, that the cooling due to the instabilities can cause a substantial hardening of the spectrum at \( E \lesssim 100 \) GeV. The spectral signatures of this effect are similar to those of IGMFs.
We note that the spectral suppression due to the plasma instability depends on the intrinsic spectrum of the source. For instance, this effect is small for objects like 1ES 1218+304 and 1ES 1312-423, whose (fitted) spectral indices are $\alpha \approx 1.9$. On the other hand, for 1ES 0229+200, whose spectral index is $\alpha \approx 1.2$ (Taylor et al. 2011), the predicted flux varies by several orders of magnitude at $E \lesssim 100$ GeV. For this reason, the spectra of most of the objects shown in figure 7 could change considerably, improving the agreement with the observations.

For 1ES 0229+200, Costamante et al. (2018) found that $\alpha = 1.5$ and $E_{\text{max}} = 12$ TeV adjusts the data satisfactorily. On the one hand, the increase in the spectral index would decrease the width of the bands and mitigate the effects of plasma instabilities; on the other hand, the increase in the maximal energy from 5 TeV to 12 TeV may increase this effect as $\alpha$ and $E_{\text{max}}$ are degenerate parameters.
We have considered blazars whose variabilities are small over the period of time of observation, with exception of PKS 2155-304. This object presents a state of enhanced emission during a flaring episode whose contribution is excluded from the analysis, as done by Abramowski et al. (2014).

The uncertainty due to the choice of the EBL model (Gilmore et al. (2012) in this work) does affect our results quantitatively. Qualitatively, the same conclusions as the ones presented here hold. In this case, the breadth of the band could be enlarged or reduced.

The luminosity of the beam emitted by the object can affect the observed gamma-ray flux, as shown in figure 5. This is particularly important for model C. Nevertheless, all objects listed here have very similar luminosities, of the order of $L \sim 10^{43.5} - 10^{45}$ erg/s (see e.g. Broderick et al. (2012)).

One could think that for the combinations of $\alpha$ and $E_{\text{max}}$ adopted, it would be possible to constrain some plasma instability models using the plots from figure 7. However, our calculations are meant to maximise the effects of the instabilities. A change in their growth time, for instance, could render the plasma cooling much more inefficient.

6. DISCUSSION

The relevance of the plasma instabilities on the development of electromagnetic cascades is a rather controversial issue. Many authors have constrained IGMFs neglecting the possible existence of the instabilities.

In their seminal work, Broderick et al. (2012) claimed that the effects of plasma instabilities place stringent bounds on the strength of IGMFs, $B \lesssim 10^{-12}$ G. This limit was derived by comparing the growth time of the instability with the Larmor radius described by the electrons. This argument holds approximately, and other factors such as the magnetic power spectrum may play an important role in determining the propagation regime of the electrons. For instance, if the coherence length of the IGMF were much smaller than the cooling length of the charged component of the cascade, then plasma instabilities could not arise as the initial beam would be disrupted to the point where it ceases to be collimated, since the particles do not move in a specific direction, but rather propagate via (magnetic) diffusion.
Figure 7. Simulated gamma-ray fluxes predicted for the blazars listed in table 2. The different models of plasma instabilities including a range of uncertainties in the parameters $L$, $T_{\text{in}}, \eta_{\text{ISM}}$ (coloured bands). The solid black line represents the case without the instabilities. The parameters corresponding to the intrinsic spectrum of the objects are also presented in table 2, along with the references for the data points. The dashed line represents the intrinsic source spectrum.
An important issue raised by Durrer & Neronov (2013) is that Broderick et al. (2012), in their original work, have limited their analysis to the the linear approximation, neglecting the backreaction of the beam perturbations on the growth rate. As a consequence, oblique modes of Langmuir waves would be suppressed due to non-linear Landau damping, stabilizing the beam and thus minimizing the role plasma instabilities would play on the development of the cascade. This argument explains why plasma effects on the gamma-ray spectrum are not visible for model B, and are in line with the findings of Miniati & Elyiv (2013). The same arguments are made by Schlickeiser et al. (2012, 2013), though the range of parameters for which it holds is different, explaining why in this scenario (model C) the cascade is quenched by the instabilities for most combination of parameters.

Yan et al. (2019) claimed that even if plasma instabilities are considered, in particular a fast-growing oblique instability, IGMFs could still be reliably constrained using electromagnetic cascades. In this case, the IGMF strength would change by a factor $\sim 10$. At first, these results do not seem to agree with ours. Nevertheless, since the fluxes we obtained are just the limiting cases, for lower efficiencies of the energy loss due to plasma instabilities, we also can obtain higher fluxes which might be in agreement with the results in Yan et al. (2019). More studies using three-dimensional simulations are required to draw further conclusions.

The modelling of the gamma-ray flux considering plasma instabilities is non-trivial in the presence of IGMFs. For instance, the Weibel instability is known to generate and amplify magnetic fields, as observed in laboratory experiments (Huntington et al. 2015). This may distort the measurement of IGMFs as the pairs may be sensitive to these fields.

Care should be taken when stating whether or not plasma instabilities quench the development of electromagnetic cascades in the IGM. Multiple kinds of instabilities can arise, and under certain conditions their cooling rate may be subdominant with respect to inverse Compton scattering. This, for instance, is the case for model C if the luminosity of the blazar beam is $< 10^{44}$ erg/s, or if the electron-positron pairs are created in a relatively cold region of the IGM ($T_{\text{IGM}} \sim 10^5$ K), or if the density of the IGM is high. Similarly, some dependence on the temperature can also be seen for model E, although in none of the cases studied the effects of this instability is negligible.

Caveats exist in our analysis. First, we consider that the cooling is completely effective, which may not be true if the instability grows slower than predicted. In this sense, our results should be taken as a limiting case wherein energy dissipation via plasma instabilities is fully efficient. Longer growth times would decrease the energy loss rates assumed, modifying the interaction rates shown in figure 1 such that inverse Compton scattering could become the dominant cooling process.

Our results suggest that the quenching factors are strongly dependent on the spectral index and maximal energy of the blazar. These parameters are determined by the underlying acceleration mechanism in the source, as well as its opacity. Extreme blazars are thought to have spectra $\alpha \gtrsim 1.5$ (Aharonian et al. 2006; Cerruti et al. 2015; Bonnoli et al. 2015). Our results from figure 7 suggest that the effects of plasma instabilities are relatively small for our choices of $\alpha$ and $E_{\text{max}}$, with the exception of the object 1ES 0229+200. Therefore, since $\alpha \gtrsim 1.5$ for all extreme most blazars, the cascade will be suppressed by instabilities for the lower values of $\alpha$; for $\alpha \gtrsim 1.8$, the quenching factor is relatively small and we conclude that, unless $E_{\text{max}}$ is high, then the spectral changes due to the instabilities would be small.

We have treated $n_{\text{IGM}}$ and $T_{\text{IGM}}$ as independent quantities. In reality, their evolution are correlated as the temperature affects the reionisation of the gas and consequently the density of the IGM plasma (McDonald et al. 2001). This relation may be useful to decrease the number of degrees of freedom when attempting to probe the IGM with VHEGRs.

The energy dissipated by plasma instabilities may be absorbed by either the IGM, thus causing beam cooling, or by the beam itself, leading to its disruption (Broderick et al. 2012). This difference is not important when it comes to interpreting the spectra of TeV-emitting blazars, but it is of fundamental importance for discussing other observational consequences of this phenomenon such as its effect on structure formation (Pfrommer et al. 2012) and on IGM heating (Chang et al. 2012).

Our analysis focuses solely on blazars. In the context of the AGN paradigm, blazars have their jets pointing directly at Earth. Small deviations from the line of sight are not unexpected. Moreover, the jets have characteristic opening angles of $\sim 5^\circ$. This misalignment of the jet emission, too, may affect the growth of instabilities, since the density of the beam decreases as the distance to the axis of the jet increases. While this effect is likely small, a careful modelling of the cascade and three-dimensional simulations would be required to confirm this picture for the specific case of misaligned blazars.

Only after $\sim 300$ years can the instabilities grow enough to be able to cool the electron-positron pairs. Thus, flaring objects may not exhibit the same kind of behaviour as steady sources. As a consequence, they may still be reliable probes of IGMFs.

The spectral changes stemming from plasma instabilities or IGMFs are not unique. Essey et al. (2010, 2011a,b) have suggested that ultra-high-energy cosmic rays (UHECRs) can induce electromagnetic cascades in the IGM, with similar observational signatures.

7. CONCLUSIONS AND OUTLOOK
We developed a numerical tool to estimate the cooling rate of electrons due to plasma instabilities caused by interactions between a beam of very energetic particles emitted by blazars. We compared different kinds of instability and various models found in the literature. We applied this tool to simulate the spectrum of several blazars.

We conclude that for typical IGM parameters and beam luminosities some types of plasma instabilities may cool electron-positron pairs faster than inverse Compton scattering. As a consequence, these instabilities may lead to a hardening of the spectrum of blazars at energies $\lesssim 100$ GeV. This effect resembles the suppression of the flux caused by IGMFs. Therefore, IGMF constraints may be compromised due to a possible dominance of plasma instability cooling over inverse Compton scattering during the development of electromagnetic cascades in the IGM. This result, however, depends on the hardness of the intrinsic spectrum of the blazar.

The existence of pervasive IGMFs cannot be excluded. Therefore, future studies combining both the effects of IGMFs and plasma instabilities are required to predict gamma-ray fluxes from blazars more accurately. Moreover, efforts towards simulating the growth of these instabilities using magnetohydrodynamical simulations would be needed to unambiguously determine whether or not they can quench electromagnetic cascades.

An uncontroversial window of opportunity for constraining IGMFs with electromagnetic cascades remains. The search for echoes associated with transient events, i.e., magnetically-induced delays in the arrival secondary gamma rays from primary VHEGR emission, are still excellent probes of IGMFs because the effects of plasma instabilities may be small. This is the case for some particular energetic events in AGNs. Gamma-ray bursts seem very promising candidates for this purpose.

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**Software:** CRPropa (Alves Batista et al. 2016b), Matplotlib (Hunter 2007), NumPy (Oliphant 2006)

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