FEM simulation of deformation of self-stress rock mass with disjunction nearby a tunnel

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Abstract. In the work the constitutive equations which allow to describe the structurally inhomogeneous rock mass taking into account the property of rocks to accumulate and release elastic energy, are applied. The finite element algorithm and programs for solving plane boundary-value problems are developed. The authors solve a problem on deformation of rock mass with a disjunction surrounding a tunnel. It is shown that driving of horizontal tunnel nearby the disjunction can provoke the accumulated elastic energy release, which affects stress-strain state of rock mass around the tunnel

1. Introduction

One of the priority trends in the modern geomechanics is concerned with the studies into internal structure of soil and rock. In the mechanics of soil and granular material, the granular structure governs internal friction, cohesion and dilatancy. During deformation, such properties may give rise to an increase in the pressure applied to enclosing structures, as well as to localization of shears and generation of isolated slip surfaces [1–3]. In the mechanics of rocks, the occurrence of discontinuities and the block structure determine essential nonlinearity and anisotropy of rock mass behavior [4–7].

One of the important properties of rock mass is the capacity to accumulate energy of external forces in the form of internal self-balancing stresses. This property is connected with the hierarchical block structure of rocks, having numerous equilibrium states [8]. Some areas in rock mass can accumulate energy and later, under certain conditions, to release it, i.e. such areas function as energy sources. The energy release process can either be stable relaxation or unstable disaster [9–11].

There exist different approaches to taking into account the hierarchical structure and internal self-balancing stresses of rock mass. Among them there are, for instance, the mathematical apparatus and models with internal variables [12–15], the approach based on the methods of the non-Archimedean mathematical analysis [16], the discrete element method [17], etc. In this study we used the approach from [12] and the mathematical model from [15] to analyze numerical deformation of rock mass surrounding a horizontal tunnel

2. Mathematical model

We choose a two-scale model of self-stress rock mass [12, 15] (plane deformation). The microscale elastic particles (grains) occur at the points of a square lattice. The pore space is filled with a binding material having elastic characteristics other than the properties of the grains (Figure 1). In-between the particles plastic shears develop by the nonlinear law, including stages of strengthening, softening and residual strength. The diagram of microslip between grains, connecting tangential microstress $t_{12}$ and microshears $\varepsilon_{ij}^R$ (in the general case, it is assumed that $\varepsilon_{12}^R \neq \varepsilon_{21}^R$), is shown in Fig. 2 (piece-wise linear approximation).
The conditions for the slip between grains are approximated by the piecewise-linear diagram (Fig. 2), and for the increments in the microstresses and microshears are given by

$$\Delta \varepsilon_{12}^R = \Delta t_{12} / G_i^r, \quad \Delta \sigma_{23}^R = \Delta t_{12} / G_{2}^s. \quad (1)$$

The parameters $G_i^r, G_{2}^s$ in Eq. (1) are the preset moduli of slip for each family of contacts. The moduli $G_i^r$ are defined by the preset constants $\gamma_i, \gamma_i^{\max}, \tau_i^{\max}, \tau_i^{\text{res}}$, where $i = 1, 2$ is the number of the family of contacts $G_i^r = \begin{cases} G_i^0, & 0 \leq \gamma_i < \gamma_i^* \\ -G_i^0, & \gamma_i^* \leq \gamma_i < \gamma_i^{\max} \\ 0, & \gamma_i^{\max} \leq \gamma_i \end{cases}$

The constitutive relations at the macroscale of the model connect the increments of the macrostresses $\Delta \sigma_{ij}$ with the increments of the macrostrains $\Delta \varepsilon_{ij}$:

$$\begin{pmatrix} \Delta \varepsilon_{11} \\ \Delta \varepsilon_{22} \\ \Delta \varepsilon_{12} \end{pmatrix} = W^{-1} \left( (T + R)^{-1} + 2(T + P)^{-1} \right) W^{-1} \begin{pmatrix} \Delta \sigma_{11} \\ \Delta \sigma_{22} \\ \Delta \sigma_{12} \end{pmatrix} \quad (2)$$

The coefficients in Eq. (2) depend on the microproperties of grains (matrix $T$), pore-filling material (matrix $P$), moduli of contact slip between grains (matrix $R$) and on the angle $\alpha$ (see Fig. 1), governing the orientation of regular grain packing in the Cartesian coordinates (matrix $W = W(\alpha)$).

The forces appearing in grains cause deformation of the grains and inter-grain shearing, which results in the deformation of the pore structure. Compression of the pore space is balanced by the tangential forces at the inter-grain contacts, and, in this way, even at zero external stresses, the internal self-balancing stresses can be considerably high. The condition of compatibility of micro- and macroparameters make it possible to connect the macrostresses $\sigma_{ij}^*$ with the increments of the macrostrains $\Delta \varepsilon_{ij}$:

$$\sigma_{ij}^* = t_{ij} + 2m \cdot p_{ij}, \quad i, j = 1, 2 \quad (3)$$

where the dimensionless value $0 < m < 1$ acts as a parameter of areal porosity [12].

The conditions (3) mean that one and the same macrostresses $\sigma_{ij}^*$ can agree with the multitude different stress states of grains $t_{ij}$ and pore-filling material $p_{ij}$. In this regard, zero macrostresses do not imply the absence of the internal microstresses. The microstresses in grains and in pore-filling material may have different signs and may completely balance each other.

The constitutive relations (2) in combination with the Cauchy relations, connecting strains and displacements, is closed by the equations of equilibrium (in terms of increments):
The formulated quasi-static problem is numerically solved with a help of finite element method by steps of loading with regard to the changing properties of the medium in the course of deformation: 
\[ \sigma^{(k+1)} = \sigma^{(k)} + \Delta \sigma^{(k)} , \quad \varepsilon^{(k+1)} = \varepsilon^{(k)} + \Delta \varepsilon^{(k)} , \]
where the superscript \( k \) is the number of iteration.

The analysis of the type of the system (2), (4) shows that if the slope of the descending branch of the microslips at the inter-grain contact is less than a critical value (depends on the shear modulus and Poisson’s ratio of grains, \( \mu' \), \( \nu' \), and pore-filling material, \( \mu'' \), \( \nu'' \)), the system of equations is elliptical. In this case, deformation is a stable process of microstrength loss without jumps. The examples of the numerical calculations in the given range of parameters are discussed below.

3. Numerical modeling results

The computational domain is set in the neighborhood of a horizontal tunnel with an arch cross-section (Fig. 3). The initial macrostress state is chosen as the linear distribution: 
\[ \sigma_{22}^0 = -\gamma(H - x_2) , \quad \sigma_{11}^0 = \xi \sigma_{22}^0 , \quad \sigma_{12}^0 = 0 , \]
where \( \gamma \) is the specific weight of rocks, \( H \) is the depth of the tunnel occurrence, \( \xi \) is the lateral earth pressure coefficient. Alongside with the macrostresses, it is required to determine initial distribution of microstresses such that satisfy the relations (3). We select distribution of microstresses as function of the parameters \( q_i \):
\[ \varepsilon_{11}^0 = 0.5\sigma_{11}^0 , \quad \varepsilon_{22}^0 = 0.5\sigma_{22}^0 , \quad \varepsilon_{12}^0 = \pm q_i \tau_{ij}^{\text{max}} , \]
\[ p_{ij}^0 = (\sigma_{ij}^0 - \varepsilon_{ij}^0) / 2\mu , \quad i, j = 1, 2 \text{ and consider two different problems. In the first problem, it is set that in the whole computational domain the shear microstresses of grains and, accordingly, pore-filling material, are zero: } q_i = 0 , \quad i = 1, 2 . \]

In the diagram of shearing (see Fig. 2), this state corresponds to the point \( O \). The second problem includes a disjunction nearby the tunnel (grey-colored zone in Fig. 3), and it is assumed that the shear microstresses of grains in the disjunction make a certain portion of the critical value, e.g. \( q_i = 0.9 \), while \( q_i = 0 \), \( i = 1, 2 \) in the surrounding rock mass. This state in the shearing diagram conforms with the points \( A \) or \( B \) depending on the sign of the shear microstress (see Fig. 2). In this manner, one and the same initial macrostresses are set in the two problems, and the difference is the distribution of the microstresses of the structural elements.

The boundary values are set as a sequential reduction in stresses at the tunnel boundary:
\[ \Delta \sigma_n |_{\Gamma_0} = -\Delta d^k \cdot \sigma_n^0 , \quad \Delta \tau_n |_{\Gamma_0} = -\Delta d^k \cdot \tau_n^0 , \]
where \( \Delta \sigma_n , \Delta \tau_n \) are the increments in the normal and shear macrostress, respectively; \( \sigma_n^0 , \tau_n^0 \) – initial normal and shear macrostresses; \( \Gamma_0 \) is the boundary of the tunnel; \( 0 \leq \Delta d^k \leq 1 \) is the increment in the dimensionless loading parameter at a \( k \)-th step. The condition \( \sum_k \Delta d^k = 1 \) means complete relieving of the tunnel boundary. The external boundary \( \Gamma_1 \) is assumed immobile: \( \Delta u_i |_{\Gamma_1} = 0 , \quad i = 1, 2 . \)

The input dimensionless parameters (all values of the dimension of stresses are related to the value of the maximum shear microstress \( \tau_{ij}^{\text{max}} \) at the peak of the diagram in Fig. 2) are:
\[ \mu' = 3.75 \cdot 10^3 , \quad \nu' = 0.2 , \quad \mu'' = 0.625 \cdot 10^3 , \quad \nu'' = 0.3 , \quad \gamma_1^* = \gamma_2^* = 0.5 \cdot 10^3 , \quad \gamma_1^{**} = \gamma_2^{**} = 10^3 , \]
\[ \tau_1^{\text{max}} = \tau_2^{\text{max}} = 1 , \quad \tau_1^{\text{es}} = \tau_2^{\text{es}} = 0.5 , \quad \mu H = 33.75 , \quad \Delta X_1 = \Delta X_2 = 0 , \quad \xi = 0.42 , \quad m = 0.5 , \quad \alpha = \pi / 4 , \quad H / R = 100 . \]

The calculations are continued till complete relieving of the tunnel boundary
Figures 4 and 5 depict the results of the first problem solution without the fracture. Figure 4a illustrates evolution of plastic deformation zones (light-grey are the zones of local softening—the descending branch; dark-grey are the residual strength zones—horizontal branch in Fig. 2). Figures 5a and 5b show contour lines of the maximum tangential stress $\tau_0 = 0.5\sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2}$ and maximum shear $\gamma_0 = \sqrt{(\epsilon_{11} - \epsilon_{22})^2 + 4\epsilon_{12}^2}$, respectively. Apparently, plastic shears initiate from the natural stress raisors at the tunnel boundary and propagate in rock mass predominantly in the vertical direction (since the preset gravity stresses exceed the tectonic stresses).

Figures 4b and 6 present the results of the second problem solution with regard to the disjunction. It is seen (Figure 4b) that during deformation the material inside the disjunction zone relatively rapidly passes to plastic state and exerts influence on the stress state of surrounding rocks (Figure 6a and 6b). On the whole, under the same initial macrostresses, the release of the accumulated energy in a relatively narrow section of the disjunction in the course of the tunnel driving induces macroshear of the fracture edges and the related change in the macrostress state of surrounding rock mass. The influence of the disjunction will even more increase in case of unstable deformation, when the release of the accumulated elastic energy will have the uncontrollable dynamic behavior.

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4. Conclusions
The implemented approach allows the problems on deformation of structurally inhomogeneous geomaterials to be solved with regard to internal self-balancing stresses.

Driving of a tunnel nearby a disjunction can result in relief of internal self-balancing stresses, which affects stress state of surrounding rock mass.

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