Direct Measurement of Thermalization to Rayleigh-Jeans Distribution in Optical Beam Self-Cleaning

Hamed Pourbeyram\textsuperscript{1*}, Pavel Sidorenko\textsuperscript{1}, Fan Wu\textsuperscript{2}, Logan Wright\textsuperscript{1}, Demetrios Christodoulides\textsuperscript{2}, and Frank Wise\textsuperscript{1}

\textsuperscript{1} School of Applied and Engineering Physics, Cornell University, Ithaca, NY 14853, USA
\textsuperscript{2} CREOL/College of Optics and Photonics, University of Central Florida, Orlando, Florida 32816, USA

Abstract

An equilibrium thermodynamic framework that describes highly-multimode optical processes was introduced recently. This theory predicts the outcomes of complex linear and nonlinear multimode interactions based on thermodynamic principles and provides analytic results for the distributions of mode occupancies in optical systems. Here, we present direct experimental measurements of irreversible thermalization of initial distributions of mode occupancies to the predicted equilibrium Rayleigh-Jeans distribution in nonlinear wave propagation in multimode optical fiber. Kerr self-cleaning of femtosecond light pulses is shown to yield the Rayleigh-Jeans distribution, which confirms that the process maximizes entropy. Analytic theory, numerical simulations, and experimental results agree.

1. Introduction and motivation

There is currently much interest in optical systems that support multiple modes of the electromagnetic field, such as cavities, waveguides, and random media. These systems support a variety of interesting phenomena and their investigation presents serious theoretical challenges. In a highly-multimode or -multidimensional nonlinear system, the presence of a large number ($M$) of modes requires accounting for $M^2$ ways that pairs of modes can interact along with $M^4$ four-wave mixing products, for example. As a result, analysis of such systems is complex and time-consuming, or even impossible. This situation has motivated the development of thermodynamic approaches to understanding highly-multimode systems. Condensation-like effects have been studied in lasers, which are driven and dissipative systems with feedback, and thus are far from equilibrium \cite{1,2,3,4}. Here we are interested in weakly-nonlinear and conservative systems, where the evolution to an equilibrium distribution is governed by a Hamiltonian. A primary example of such a system is nonlinear wave propagation through passive optical fiber.

In general, “thermalization” refers to the evolution of a highly-nonequilibrium initial population distribution to a thermal-equilibrium distribution. Nonequilibrium kinetic formulations based on optical wave turbulence theories \cite{5} have been applied to the modeling of thermalization and condensation-like phenomena involving classical waves in several settings. These include supercontinuum generation \cite{6,7} and nonlinear propagation in photorefractive crystals \cite{8} and multimode fibers \cite{9} as well as fiber lasers \cite{3,4}. Quite recently, an equilibrium thermodynamic theory capable of describing such phenomena in highly-multimode systems was put forward \cite{10}. Given the spectrum and initial occupation of the eigenstates of a system, this formalism determines the equilibrium distribution from three conserved quantities: the internal energy, the optical power, and the number of modes. For an optical fiber with known propagation constants $\beta_m$, total number
of modes $M$ (which should be large but finite), and mode field amplitudes $c_m$, the equilibrium state of the system is simply given by the Rayleigh-Jeans (RJ) distribution $|c_m|^2 = -T/(\beta_m + \mu)$, where $T$ and $\mu$ are the optical temperature and chemical potential obtained from the initial mode occupancies. The thermodynamic theory can be applied to predict and control the evolution of multimode fields in a variety of interesting waveguide and cavity structures. Given its central role in thermodynamic approaches to multimode optical systems, direct experimental observations of the RJ distribution would seem to be critical to establishment of such approaches. Despite significant experimental progress, clear-cut demonstrations of condensation of classical optical waves are just beginning to appear [11].

Among several new nonlinear phenomena that have been observed in multimode fibers in the past few years, so-called beam self-cleaning has attracted particular interest [12,13,14]. In this process, a highly-multimode and speckled beam is launched into graded-index multimode fiber. When the input power exceeds 10-50 kW (depending on the pulse duration and initial mode occupancies), the beam self-organizes into a bell-shaped intensity profile that approximates the fundamental mode of the fiber, accompanied by some energy in higher-order modes of the fiber. Although studies have shown that beam cleaning is the result of energy transferring from higher-to lower-order modes of the fiber, and numerical simulations that include only the electronic Kerr nonlinearity, diffraction, and the waveguide profile can exhibit the effect [12], a detailed understanding is still debated [15,16,17,18,19,20]. The observation of an apparent increase in the brightness in a conservative system naturally raises questions about the evolution of the entropy. Turbulent wave-mixing theoretically leads to condensation of power into lower-order modes (as well as transfer of power to higher-order modes), which can model the beam-cleaning process [9,17,19]. A primary conclusion of these analyses is that nonlinearity and structural disorder are responsible for the irreversible thermalization to the RJ spectrum. The connections between optical physics and fluid flow broaden the interest in multimode fiber optics and illustrate the universal nature of pattern formation in diverse settings.

In a recent study of beam-cleaning in multimode fiber, Baudin and co-workers inferred the condensate fraction and chemical potential from the near-field and far-field intensity profiles, which can be related to the power and the internal energy [11]. In order to mimic the behavior of a thermal distribution with varying temperature, experiments were done with fixed power and decreasing energy. Ensemble averaging of the profiles leads to condensate fractions that agree very well with the RJ distribution. The behavior of the entropy and heat capacity are also consistent with the observed condensation being the result of equilibrium thermodynamics of classical waves.

Both the kinetic [5] and thermodynamic [10] theories of multimode systems were developed to model the propagation of continuous-wave (CW) optical fields. However, experiments always employ pulsed fields to reach the required peak powers. The initial demonstrations of beam-cleaning performed with sub-nanosecond [12] and femtosecond [14] pulses were modeled reasonably well by numerical solutions of single-field and coupled-mode equations, respectively. In practice, models based on CW fields are often adequate for the propagation of pulses with durations in the nanosecond range, where chromatic and modal dispersions produce negligible effects. On the other hand, recent studies reveal complex intrapulse dynamics in beam cleaning with nanosecond pulses [21,22]. The experiments reported by Leventoux et al. [22] and Baudin et al. [11] are similar – nanosecond pulses at wavelength near 1 $\mu$m launched into graded-index fiber – which suggests that the thermodynamic approach accurately predicts the equilibrium state despite the apparent violation of the assumptions. More work to address the influence of possible spatiotemporal propagation effects on the thermalization process will be valuable.
Direct measurements of the distribution of modal occupancies are desirable for verifying the emergence of a thermal equilibrium distribution, and would also allow observation of the evolution from a non-equilibrium distribution to the thermal distribution. Such measurements require decomposing the field into modes, which is common for experiments on linear propagation [23], but still rare for studies of nonlinear propagation [24]. In this paper we present direct mode-resolved measurements of the distribution of modal occupancies that results from launching short-pulse multimode beams into graded-index fiber. Spectrally-resolved spatial mode analysis of the Kerr beam cleaning process demonstrates optical thermalization of the system to the equilibrium Rayleigh-Jeans distribution expected from the input field. The use of femtosecond pulses complements experiments performed with nanosecond pulses, and the results indicate that the thermodynamic theory can be applied even with such broadband pulses.

2. Experiments

The experimental arrangement (Fig. 1) is conceptually simple. A multimode mode-locked laser [25] generates frequency-chirped picosecond-duration pulses at 1040 nm that consist of multiple transverse modes locked together in time. These pulses are temporally compressed to their transform limit with a pair of diffraction gratings. The spatially-integrated pulse duration determined from autocorrelation measurements is about 200 fs. With pulse energy of about 20 nJ, the peak power can be as high as 70 kW. The pulses are launched into various lengths of a graded-index (GRIN) multimode fiber that has a 50-μm core diameter and zero-dispersion wavelength near 1300 nm. The fiber supports about 100 spatial eigenmodes (200 including polarization) at 1040 nm. Different combinations of transverse modes can be excited by controlling the position and angle of the lens that couples light into the fiber while maintaining around 80% coupling efficiency. Nonlinear interactions among the modes underlie the approach to equilibrium, and the peak power is used as a control parameter to observe the evolution to a thermal distribution. The field that exits the fiber is decomposed into a spectrally-resolved eigenmode basis.
The apparatus for determination of the electric field after propagation through the fiber is the heart of the experiment. Several techniques for measuring the three-dimensional electric field of ultrashort light pulses have been reported [26,27,28,29]. However, measurement of spatiotemporally-complex fields is still quite challenging, especially for fields with large space-time-bandwidth product. For characterization of the complex pulsed states of the multimode (or spatiotemporally) mode-locked laser, a technique with readily-adjustable space-time-bandwidth product is desirable. Following a technique for characterization of intense ultrashort pulses at a focus [30], we employ delay-scanned off-axis digital holography [31]. The pulsed field to be measured is split. One of the copies is filtered spatially to create a Gaussian spatial beam that is used as a spatiotemporal reference beam (Fig. 1), and the other (object) is imaged into the camera (Fig. 1). We verified that the spatially-filtered reference beam contains all wavelengths in the original field. Fringes caused by interference of the reference and object fields contain information about the phase of the latter. The phase relationship between frequency components can be obtained by measuring the frequency-resolved optical gating (FROG) trace of the reference field. The electric field $E(x, y, \tau)$ is Fourier-transformed to obtain $\hat{E}(x, y, \omega)$ for the mode decomposition [32,33,34]. At each frequency $\omega_i$, the complex field $\hat{E}(x, y, \omega_i)$ is projected on to the eigenmode basis of the fiber to get the occupancy of the modes. The relative uncertainty in the mode
occupancies is estimated to be 10%. As an example, the frequency-integrated modal decomposition of pulses from one particular state of the mode-locked laser is also shown in Fig. 1. The beam profile reconstructed from the 3D field modal decomposition is compared to the measured beam profile to assess the mode decomposition.

Multimode pulsed fields that have significant overlap with the fundamental mode were launched into the fiber. The characteristic (chromatic) dispersion length $L_{DS} \sim 0.5$ m, which is comparable to the fiber lengths. Modal dispersion (differential group delay between modes of the fiber) is normal for the lower-order modes of this fiber and its effects are small compared to the effects of chromatic dispersion. The characteristic nonlinear length (the length over which a nonlinear phase $\phi_{NL} \sim 1$ is accumulated by the pulse) $L_{NL} \sim 15$ cm for the highest-energy pulses launched into the fiber. Nonlinear spectral broadening enhances the effects of dispersion in the latter stages of propagation, and as a result the pulse will broaden by about a factor of two and acquire a corresponding frequency chirp as it traverses the fiber. Because the pulse accumulates some chirp as it propagates, different frequencies are mapped to different local time positions in the pulse. The mode decomposition is performed for each frequency in the spectrum, which gives information about the spatial profile at the corresponding times. However, any variation of the decomposition with frequency is below the fluctuations in the measurements. We take this as an indication that any spatiotemporal evolution is not dramatic, but further work will be needed to reach a stronger conclusion.

A closed (Hamiltonian) system will reach a unique thermodynamic equilibrium state for given values of the temperature and chemical potential, which are determined by the initial modal populations. We performed several measurements to verify that the fiber has negligible dissipation, as required for the thermodynamic theory to be valid. Short lengths of fiber were chosen for the experiments, to increase the peak power at which stimulated Raman scattering is observed. For given fiber length, as the input peak power is increased, the output spectrum broadens due to self-phase modulation (Fig. 2), but remains symmetric about the center frequency. The total nonlinear phase shift accumulated in propagation is $\phi_{NL} \sim \pi$. We measure the Hamiltonian two different ways: by direct evaluation of $H = \sum \beta_i |c_i|^2$ from mode-resolved measurements of the output beam, and by measuring the second moments of the near- and far-field beam profiles, which can be related to the Hamiltonian in the case of graded-index fiber [19]. Both of these measures are invariant over the range of input powers employed in the experiments (Fig. 2). Conservation of the Hamiltonian implies that energy transfers from intermediate modes to both lower- and higher-order modes as the peak power increases, so the average mode number is approximately constant [19]. Finally, with fiber lengths between 0.5 and 1.5 m and bend radii about 0.4 m, linear mode coupling is negligible. Nonlinear mode coupling is responsible for energy redistribution among the modes, as required by the optical thermodynamic theory.
Figure 2. Conservation of internal energy and average mode number. a) power spectrum versus input peak power. b) spectra at minimum and maximum powers. c) Hamiltonian inferred from near- and far-field beam profiles (red) and Hamiltonian measured by modal occupation (black).

3. Results

Typical results obtained with a 50-cm segment of fiber will be described. The beam profiles and modal decompositions observed at the output of the fiber are shown in Fig. 3 for the indicated range of input peak powers. It is worth mentioning that for a given mode distribution, the spatial intensity profile may appear less “speckled” with ultrashort pulses than with nanosecond pulses, owing to some averaging over the corresponding broadband spectrum. For each value of the input power, the output field is projected onto the modes of the fiber. Only energy transfer between mode groups will affect the entropy, so the sum of the energies of the modes in a group is divided by the degeneracy of the group to produce the plots of mode occupancy versus propagation constant. Because linear mode coupling is negligible, the distribution in the bottom panel of Fig. 3 is a good approximation to the launched field. As the peak power is increased above ~10 kW, the spatial profile becomes an intense bell-shaped lobe with approximately the diameter of the fundamental mode of the fiber, accompanied by a low-intensity background. The corresponding distributions of modal occupancy clearly reflect the transfer of energy from intermediate modes to the fundamental mode.
Figure 3. Near-field beam profiles and modal decompositions measured at the indicated peak powers. The thermal Rayleigh-Jeans distribution is plotted as the red line. The propagation constant of the highest guided mode has been subtracted from the propagation constants to facilitate comparison with theoretical distributions [10].

From the initial distribution (recorded with input power of 1 kW) the optical temperature $T = 0.4$ and chemical potential $\mu = -73 \text{ mm}^{-1}$ are obtained. These values can be interpreted intuitively: with $|\mu|$ greater than all of the propagation constants of the system and positive temperature, the thermal distribution is expected to have maximum occupancy of the fundamental mode, with the occupancy decreasing monotonically in higher-order modes. The measured modal distributions clearly evolve from the initial nonequilibrium distribution to a thermal distribution with increasing power. The equilibrium Rayleigh-Jeans distribution with the values of $T$ and $\mu$ above is plotted along with the distribution of modal occupancies observed at the highest power in Fig. 3, and the agreement is very good.

Numerical simulations based on coupled nonlinear Schrödinger equations for the modes were performed. The first 55 modes (first 10 mode groups) of the fiber were included in the calculations, which corresponds to propagation constants larger than $\beta = 20 \text{ mm}^{-1}$. Neglecting higher-order modes was necessary to keep the computation times reasonable; even with parallel code the execution times exceed several hours for multiple modes with all relevant short-pulse effects included (details of the simulations are in the Supplementary Material). We expect the neglect of higher-order modes to be a reasonable approximation given the low observed occupancy of those modes, and their uncertainties, in the experiments. In the numerical simulations 200-fs pulses were
launched into 50 cm of fiber with the experimental parameters. The measured low-power modal distribution (with random phases) was taken as the input field. After an initial exchange of energy between low-order modes, the occupancy of the fundamental mode increases and it is the dominant mode after about 15 cm of propagation (Fig. 4(a)). Beyond 20 cm, there is little energy exchanged among the modes. The numerical results agree reasonably well with the measured modal distributions as power is varied, and in particular the distribution that corresponds to the highest experimental power agrees with both the experimental distribution and the equilibrium Rayleigh-Jeans distribution (Fig. 4(b)). Simulations performed with a variety of different input distributions but the same values of chemical potential and temperature evolve to the same equilibrium state (data not shown).

Fig. 4. Results of numerical simulations. a) simulation corresponding to experiment of Fig. 3. b) comparison of experimental and numerical results with the analytic theory.

Once a modal distribution reaches equilibrium, it should not change with further evolution. In optical thermodynamics a measure of the evolution of a multimode system is the accumulated nonlinear phase shift. For the experiments summarized in Fig. 3, the system evolves until \( \phi_{NL} \sim \pi \). Experiments on beam-cleaning were performed with longer fibers and/or higher peak powers, to verify that the process is irreversible, as required for a thermalization process. For peak power as high as 70 kW launched into 1.5 m of fiber, the field accumulates \( \phi_{NL} \sim 4\pi \) so \( L \sim 12L_{NL} \). As in the experiments of Fig. 3, the occupation of the fundamental mode of the fiber reaches the equilibrium value after propagation through \( 3L_{NL} \) and retains that value (Fig. 5(a)). The normalized mean-square error (NMSE) between the measured and Rayleigh-Jeans distributions (Fig. 5(b)) decreases until \( L = 3L_{NL} \) and is approximately constant beyond that point. A final signature of thermalization is equipartition of power among modes within a mode group. The mode-resolved measurements do exhibit a trend toward equalization of the mode occupancies within a group as the field evolves. We quantify the variation of mode occupancies by calculating the mean coefficient of variation (CV) for the first 6 mode groups.
with \( \mu_g \) the mean occupancy of the mode group and \( \sigma_g \) the standard deviation of the occupancies of the mode group. The mean CV decreases from as high as 0.32 for the launched distribution of modes in Fig. 3 to ~0.15 at the highest nonlinear phase shifts (Fig. 5(c)). Thus, the power within the modes in a group is tending to equalize. A similar trend is observed for the experiment of Fig. 3 but the mean CV is still decreasing at the highest power that can be used without stimulated Raman scattering.

\[
CV_{\text{mean}} = \frac{1}{6} \sum_{g=1}^{6} \frac{\sigma_g}{\mu_g}
\]

Fig. 5. Evidence of irreversibility of thermalization. Results of experiments performed with 1.5 m of fiber are presented as a function of propagation distance in units of the nonlinear length \( L_{\text{NL}} \), for which a nonlinear phase shift of 1 is accumulated. a) occupation of fundamental mode. Dashed red line is theoretical thermal occupation of fundamental mode. b) deviation from RJ distribution. c) equipartition of energy among modes in first 6 mode groups.

4. Discussion

The use of pulses with femtosecond duration and broad bandwidth in the work described here raises questions about the appropriateness of the wave-kinetic theory or thermodynamics to account for the experiments. One might expect spatiotemporal dynamics to play a major role in this situation [18,35]. Although the experimental conditions might not rigorously satisfy the assumptions, it appears that optical thermodynamics predicts the experimental results quite well. Direct (i.e., pump-probe) measurements of spatiotemporal dynamics on the femtosecond time scale will be challenging but should be possible. As mentioned above, dispersion does not play a
dramatic role in the short fibers used in experiments, and numerical simulations indicate that the beam cleaning can occur in a small fraction of the total fiber length (Fig. 4 and Ref. [14]). Although the “threshold” for beam-cleaning occurs at higher peak power for femtosecond pulses than nanosecond pulses, the product $\chi^{(3)}IL$ that is expected to govern perturbative nonlinear effects is not much different in the two cases. Combined with the lack of observed variation of the mode decomposition across the spectrum, these lead to the tentative conclusion that spatiotemporal effects are not playing a major role in the observed phenomena. Further investigation of possible spatiotemporal dynamics is warranted and should be fruitful.

In the absence of loss or gain, the flow of energy from intermediate to lower-order modes should be accompanied by a simultaneous flow of energy to higher-order modes [19]. The measurements of the Hamiltonian (Fig. 2) are consistent with this expectation. However, the flow of energy to higher-order modes is difficult to verify in the data of Fig. 3 owing to the limited dynamic range of the modal decomposition. Future studies will benefit from the development of mode-resolved measurements with enhanced capabilities. Recently-developed techniques based on physical mode-decomposition [23] may offer advantages here.

5. Conclusion

We have used off-axis digital holography to make direct measurements of the distribution of modal occupancies in the process of beam self-cleaning. With increasing nonlinear phase shift, the measured distribution evolves irreversibly to the predicted thermal-equilibrium Rayleigh-Jeans distribution. Our experiments done with femtosecond-duration pulses and negligible disorder complement prior work done with nanosecond-duration pulses and extend the limits of applicability of the thermodynamic theory. Numerical simulations agree with the predictions of the thermodynamic theory and experimental results. This first confirmation of the optical thermodynamics theory will have implications for a wide variety of multimode systems.

Acknowledgements

Portions of this work were supported by the National Science Foundation (ECCS-1609129, ECCS-1912742) and the office of Naval Research (N00014-13-1-0649, N00014-16-1-3027, N00014-20-1-2789).

References

1. C. Conti, M. Leonetti, A. Fratalocchi, L. Angelani, G. Ruocco, Phys. Rev. Lett. 101, 143901 (2008).
2. R. Weill, B. Fischer, O. Gat, Phys. Rev. Lett. 104, 173901 (2010).
3. E. Turitsyna, S. Smirnov, S. Sugavanam, N. Tarasov, X. Shu, S. Babin, E. Podivilov, D. Churkin, G. Falkovich, S. Turitsyn, Nat. Photon. 7, 783 (2013).
4. D. Churkin, I. Kolokolov, E. Podivilov, I. Vatnik, S. Vergeles, I. Terekhov, V. Lebedev, G. Falkovich, M. Nikulin, S. Babin, S. Turitsyn, Nat. Commun. 2, 6214 (2015).
5. A. Picozzi et al., Phys. Rep. 542, 1 (2014).
6. S. Coen, B. Barvieu, B. Kibler and A. Picozzi, “Towards a thermodynamic description of supercontinuum generation,” 2nd IEEE LEOS Winter Topicals, WTM 2009, 33, 197 (2009).
7. B. Barviau, B. Kibler, A. Kudlinski, A. Mussot, G. Millot and A. Picozzi, Opt. Express 17, 7392 (2009).
8. C. Sun, S. Jia, C. barsi, S. Rica, A. Picozzi, and J. W. Fleischer, Nat. Phys. 8, 470 (2012).
9. P. Ascheri, J. Garnier, C. Michel, V. Doya and A. Picozzi, Phys. Rev. A 83, 033838 (2011).
10. F. O. Wu, A. U. Hassan and D. N Christodoulides, Nat. Photon. 13, 776 (2019).
11. K. Baudin, A. Fusaro, K. Krupa, J. Garnier, S. Rica, G. Millot, A. Picozzi, “Classical Rayleigh-Jeans condensation of light waves: Observation and thermodynamic characterization,” Phys. Rev. Lett. 125, 244101 (2020).
12. K. Krupa et al., Nat. Photon. 11, 237 (2017).
13. G. Lopez-Galmiche et al., Opt. Lett. 41, 2553 (2016).
14. Z. Liu, L. G. Wright, D. N Christodoulides, F. W. Wise, Opt. Lett. 41, 3675 (2016).
15. K. Krupa, A. Tonello, A. Barthélémy, V. Couderc, B. Shalaby, A. Bendahmane, G. Millot, S. Wabnitz, Phys. Rev. Lett. 116, 183901 (2016).
16. L. G. Wright, Z. Liu, D. A. Nolan, M.-J. Li, D. N. Christodoulides, F. W. Wise, Nat. Photon. 10, 771 (2016).
17. A. Fusaro, J. Garnier, K. Krupa, G. Millot, A. Picozzi, Phys. Rev. Lett. 122, 123902 (2019).
18. J. Garnier, A. Fusaro, K. Baudin, C. Michel, K. Krupa, G. Millot, A. Picozzi, Phys. Rev. A 100, 053835 (2019).
19. E. Podivilov et al., Phys. Rev. Lett. 122, 103902 (2019).
20. M. Parto, F. Wu, P. Jung, K. Makris, D. Christodoulides, Opt. Lett. 44, 3936 (2019).
21. K. Krupa, A. Tonello, V. Couderc, A. Barthélémy, G. Millot, D. Modotto, and S. Wabnitz, Phys. Rev. A 97, 043836 (2018).
22. Y. Leventoux, G. Granger, K. Krupa, A. Tonello, G. Millot, M. Ferraro, F. Mangini, M. Zitelli, S. Wabnitz, S. Fevrier, and V. Couderc, “3D time-domain beam mapping for studying nonlinear dynamics in multimode optical fibers,” arXiv.
23. J. Carpenter, B. J. Eggleton, and J. Schroder, Opt. Lett. 41, 5580 (2016).
24. Z. Zhu, L. G. Wright, J. Carpenter, D. A. Nolan, M.-J. Li, D. N. Christodoulides, and F. W. Wise, “Nonlinear Pulse Propagation Experiments in Multimode Fibers with Mode-Resolved Control,” paper NpTh4G.1, Advanced Photonics 2018 Technical Digest (Optical Society of America, 2018).
25. L. G. Wright, D. N. Christodoulides and F. W. Wise, Science 358, 94 (2017).
26. J. Nicholson, A. Yablon, S. Ramachandran, and S. Ghalmi, Opt. Express 16, 7233 (2008).
27. R. Rokitski and S. Fainman, Opt. Express 11, 1497 (2003).
28. Z. Guang, M. Rhodes, M. Davis and R. J. Trebino, J. Opt. Soc. Am. B 11, 2736 (2014).
29. Z. Guang, M. Rhodes and R. J. Trebino, Appl. Opt. 56, 3319 (2017).
30. G. Pariente, V. Gallet, A. Borot, O. Gobert and F. Quéré, Nat. Photon. 10, 547 (2016).
31. M. K. Kim, J. Photonics Energy 1, 018005 (2010).
32. O. Shapira, A. F. Abouraddy, J. D. Joannopoulos and Y. Fink, Phys. Rev. Lett. 94, 143902 (2005).
33. H. Lü, P. Zhou, X. Wang and J. Jiang, Appl. Opt. 52, 2905 (2013).
34. M. Paurisse, L. Lévéque, M. Hanna, F. Druon and P. Georges, Opt. Express 20, 4074 (2012).
35. J. Laegsgaard, Opt. Lett. 43, 2700 (2018).