Neutrino Mass Matrix
in Terms of Up-Quark Masses

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Abstract

We demonstrate that under the "symmetric zero texture" with minimal Majorana mass matrix, neutrino masses and mixing angles are expressed in terms of up-quark masses, \(m_t, m_c, m_u\). This provides interesting relations among neutrino mixing angles and up-type quark masses. Especially we predict \(|U_{e3}| \leq 0.11\) even if we include the small mixing effects coming from charged lepton side. Also absolute masses of three neutrinos are predicted almost uniquely. This is quite in contrast to the case where bi-large mixings come from the charged lepton sector with non-symmetric charged lepton mass matrix.
1 Introduction

Recent results from KamLAND [1] have established the Large Mixing Angle (LMA) solution [2]. Combined with the observations by Super-Kamiokande [3, 4] and SNO [5], this confirms that $V_{\text{MNS}}$ has two large mixing angles [6, 7, 8]:

$$\sin^2 2\theta_{23} > 0.83 \ (99\% \ C.L.), \quad \tan^2 \theta_{12} = 0.86 \leq \sin^2 2\theta_{12} \leq 1, \quad (1.1)$$

with the mass squared differences

$$\Delta m_{32}^2 \sim 2.5 \times 10^{-3} \text{ eV}^2, \quad \Delta m_{21}^2 \sim 7 \times 10^{-5} \text{ eV}^2. \quad (1.2)$$

Now the question is why such a large difference can exist between the quark and lepton sectors. Within grand unified theories (GUTs), the Yukawa couplings of quarks and leptons to Higgs field are related each other. Can GUT predict two large mixing angles from some symmetry principle?

The neutrino mixing angles are expressed in terms of MNS matrix [9]:

$$V_{\text{MNS}} = U_l^\dagger U_\nu, \quad (1.3)$$

where $U_l$ and $U_\nu$ diagonalizes $M_l$ and $M_\nu$, respectively,

$$U_l^T M_l U_l = \text{diag}(m_e, m_\mu, m_\tau), \quad U_\nu^T M_\nu U_\nu = \text{diag}(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}), \quad (1.4)$$

where $M_\nu$ is calculated from neutrino right-handed Majorana mass matrix ($M_R$) and Dirac mass matrix ($M_{\nu_D}$):

$$M_\nu = M_{\nu_D}^T M_R^{-1} M_{\nu_D}. \quad (1.5)$$

We call ”up-road” (“down-road”) option when such a large mixing angle comes from $M_\nu$ ($M_l$) side. In GUT framework, then, the up- (down-) quark mass matrix will play an important role. Here we assume the up-road option and make a semi-empirical analysis by adopting the so-called symmetric four zero texture. We shall show how we can reproduce bi-large mixing angles.

2 Symmetric Texture

First we make a comment on the so-called symmetric texture which has been extensively investigated by many authors [10]. In general symmetric texture, $M_l$ is hierarchical mass matrix and never gives large mixing angles since down quark mass matrix, $M_d$ is hierarchical. Thus symmetric textures dictate only up-road option. Can then hierarchical $M_u$ be consistent with $M_\nu$ while $M_{\nu_D}$ is hierarchical? We shall show first that large mixing angles does not arise from $M_\nu$.
if we restrict ourselves to symmetric texture with \( U(1) \) family structure. As we know well, the most popular mechanism which explains hierarchical structure of masses may be the so-called Froggatt-Nielsen mechanism [11] using anomalous \( U(1) \) family quantum number.

Let us consider an example of 2-family model in which we have the same \( U(1) \) charges to left-handed up-type fermions and the right-handed fermions, \( x_1, x_2 \ (x_1 > x_2) \), respectively. Then we get the form \( M_\nu \) from Eq. (1.5) with general forms of \( M_{\nu D} \) and \( M_R \),

\[
M_{\nu D} \sim \begin{pmatrix}
\lambda^{2x_1} & \lambda^{x_1+x_2} \\
\lambda^{x_1+x_2} & \lambda^{2x_2}
\end{pmatrix}, \quad M_R^{-1} = \begin{pmatrix}
a & c \\
c & b
\end{pmatrix}
\]

\[
\rightarrow M_\nu \sim (a\lambda^{2x_1} + 2c\lambda^{x_1+x_2} + b\lambda^{2x_2}) \begin{pmatrix}
\lambda^{2x_1} & \lambda^{x_1+x_2} \\
\lambda^{x_1+x_2} & \lambda^{2x_2}
\end{pmatrix}.
\tag{2.6}
\]

This indicates that \( M_\nu \) is always proportional to the hierarchical matrix, \( M_{\nu D} \). Hence, unless the dominant terms are canceled accidentally by making fine tuning, it is impossible to get large mixing angles.

On the contrary, the above argument is no more valid if we choose the texture zero matrix in Eq. (2.6). Actually if we take zero texture, we obtain large mixing angle;

\[
M_{\nu D} \sim \begin{pmatrix}
\lambda^{2x_1} & 0 \\
0 & \lambda^{2x_2}
\end{pmatrix}, \quad M_R \sim \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix} \rightarrow M_\nu \sim \lambda^{2x_1+2x_j} \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}.
\tag{2.7}
\]

The above example shows that \( M_\nu \) is no more proportional to \( M_u \) (see more general discussion in a separate paper [12]).

3 GUT with Symmetric Texture

The informations of \( M_d \) and \( M_u \) are well established and popular. A simple example of quark mass matrices is symmetric ”zero texture” [10]. Let us take the following forms of \( M_u \) and \( M_d \) which reproduce the observed quark and charged lepton masses as well as CKM mixing angles [13]. Then we get their relations in \( SO(10) \) GUT;

\[
M_U : \quad M_u \simeq \begin{pmatrix}
0 & \sqrt{m_u m_c} & 0 \\
\sqrt{m_u m_c} & m_c & \sqrt{m_u m_t} \\
0 & \sqrt{m_u m_t} & m_t
\end{pmatrix} \leftrightarrow M_{\nu D},
\tag{3.8}
\]

\[\rightarrow \quad \begin{pmatrix}
\lambda^{2x_1} & 0 \\
0 & \lambda^{2x_2}
\end{pmatrix}, \quad M_R \sim \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix} \rightarrow M_\nu \sim \lambda^{2x_1+2x_j} \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}.
\]

The above example shows that \( M_\nu \) is no more proportional to \( M_u \) (see more general discussion in a separate paper [12]).
Thus once we fix the representation of Higgs field in each matrix element, $M_l$ and $M_{\nu_D}$ are uniquely determined from $M_d$ and $M_u$, respectively.

$$M_D : M_d \simeq \begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & m_s & \sqrt{m_d m_b} \\ 0 & \sqrt{m_d m_b} & m_b \end{pmatrix} \leftrightarrow M_l. \quad (3.9)$$

Here let us adopt a simple assumption that each elements of $M_U$ and $M_D$ is dominated by the contribution either from 10 or 126 of $SO(10)$ representation. There are 16 options for the Higgs configuration of $M_U$ (see Table I). We show that the following option of $M_U$, together with $M_D$ (Georgi-Jarlskog type [14]) and the most economical form of $M_R$;

$$M_U = \begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix}, \quad M_D = \begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 10 \\ 0 & 10 & 10 \end{pmatrix},$$

$$M_R = \begin{pmatrix} 0 & r M_R & 0 \\ r M_R & 0 & 0 \\ 0 & 0 & M_R \end{pmatrix}. \quad (3.10)$$

can reproduce all the masses and mixing angles of neutrinos consistently with present experiments.

4 Option S

Now each matrix element of $M_{\nu_D}$ is determined by multiplying an appropriate Clebsch-Gordan (CG) coefficient, 1 or $-3$, and also $M_\nu$ are easily calculated from Eq. (1.5),

$$M_{\nu_D} = m_t \begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & d \end{pmatrix}, \quad M_\nu = \begin{pmatrix} 0 & \frac{a^2}{r} & 0 \\ \frac{2ab}{r} & 2b^2 + c^2 & c(\frac{a}{r} + 1) \\ 0 & c(\frac{a}{r} + 1) & d^2 \end{pmatrix} \frac{m_t^2}{m_R}. \quad (4.11)$$

Hence, because of the hierarchical structure of $M_u$, $a \ll b \sim c \ll 1$. In order to get large mixing angle $\theta_{23}$, the first term of 2-3 element of $M_\nu$ in Eq. (4.11) should dominate and get the same order of magnitude as $d^2$, namely, $r \sim \frac{ac}{d^2} \sim \sqrt{\frac{m_3^2 m_\nu}{m_t^2}} \sim 10^{-7}$. This indicates that the ratio of the the right-handed Majorana mass of 3rd generation to those of the first and second generations, is very small. Such kind of mechanism is well known as “seesaw enhancement” [15] [16] [17]. This tiny $r$ is very welcome [15]; the right-handed Majorana mass of the third generation must become of order of GUT scale while those of the first and
Table 1: Classification of the up-type quark mass matrices.

| Type | Texture1 | Texture2 | Texture3 | Texture4 |
|------|----------|----------|----------|----------|
| S    | \( \begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix} \) |
| A    | \( \begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 126 \\ 0 & 126 & 126 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 126 \\ 0 & 126 & 10 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 126 \\ 0 & 126 & 126 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 126 \\ 0 & 126 & 126 \end{pmatrix} \) |
| B    | \( \begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 126 \\ 0 & 126 & 126 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 126 \\ 0 & 126 & 10 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 126 \\ 0 & 126 & 126 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 126 \\ 0 & 126 & 126 \end{pmatrix} \) |
| C    | \( \begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 126 \\ 0 & 126 & 126 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 126 \\ 0 & 126 & 10 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 126 \\ 0 & 126 & 126 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 126 \\ 0 & 126 & 126 \end{pmatrix} \) |
| F    | \( \begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 10 \\ 0 & 10 & 126 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 10 \\ 0 & 10 & 126 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 10 \\ 0 & 10 & 126 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 10 \\ 0 & 10 & 126 \end{pmatrix} \) |

The second generations are of order \( 10^8 \) GeV. This is quite favorable for the GUT scenario to reproduce the bottom-tau mass ratio. With such a tiny \( r \), \( M_\nu \) is approximately written as,

\[
M_\nu = \begin{pmatrix} 0 & \frac{a^2}{r} & 0 \\ \frac{a^2}{r} & \frac{2ab}{d^2} & \frac{ac}{d^2} \\ 0 & \frac{ac}{d^2} & \frac{c}{d^2} \end{pmatrix} m_\nu^2 \begin{pmatrix} 0 & \beta & 0 \\ \beta & \alpha & h \\ 0 & h & 1 \end{pmatrix} \begin{pmatrix} d^2 m_\nu^2 \\ m_R \end{pmatrix}, \quad h = \frac{ac}{rd^2}, \quad \alpha = \frac{2ab}{rd^2}, \quad \beta = \frac{a^2}{rd^2},
\]

where \( \beta \ll \alpha \) and \( h \sim O(1) \).

First let us diagonalize the dominant term with respect to 2-3 submatrix of Eq. (4.12) with the rotation angle \( \theta_{23} \),

\[
\theta_{23} \begin{pmatrix} 0 & \beta \cos \theta_{23} & \beta \sin \theta_{23} \\ \beta \cos \theta_{23} & \lambda_2 & 0 \\ \beta \sin \theta_{23} & 0 & \lambda_{\nu_3} \end{pmatrix}, \quad \tan^2 2\theta_{23} = \frac{4h^2}{(1-\alpha)^2}, \quad (4.13)
\]

with their eigenvalues,

\[
\lambda_{\nu_3} = \frac{\alpha + 1 + \sqrt{(\alpha - 1)^2 + 4h^2}}{2}, \quad \lambda_2 = \frac{\alpha + 1 - \sqrt{(\alpha - 1)^2 + 4h^2}}{2}. \quad (4.14)
\]
Second step is to rotate with respect to 1 and 2 components of Eq. (4.13):

\[ \theta_{12} \rightarrow \begin{pmatrix}
\lambda_{\nu_1} & 0 & \beta \sin \theta_{23} \cos \theta_{12} \\
0 & \lambda_{\nu_2} & \beta \sin \theta_{23} \sin \theta_{12} \\
\beta \sin \theta_{23} \cos \theta_{12} & \beta \sin \theta_{23} \sin \theta_{12} & \lambda_{\nu_3}
\end{pmatrix}, \]

\[ \tan^2 2\theta_{12} = \left( \frac{2\beta \cos \theta_{23}}{\lambda_2} \right)^2, \quad (4.15) \]

with eigenvalues,

\[ \lambda_{\nu_2} = \frac{\lambda_2 + \sqrt{\lambda_2^2 + 4\beta^2 \cos^2 \theta_{23}}}{2}, \quad \lambda_{\nu_1} = \frac{\lambda_2 - \sqrt{\lambda_2^2 + 4\beta^2 \cos^2 \theta_{23}}}{2}. \quad (4.16) \]

Finally the neutrino masses are given as,

\[ m_{\nu_3} \sim \lambda_{\nu_3} \frac{d^2 m_t^2}{m_R}, \quad m_{\nu_2} \sim \lambda_{\nu_2} \frac{d^2 m_t^2}{m_R}, \quad m_{\nu_1} \sim \lambda_{\nu_1} \frac{d^2 m_t^2}{m_R}. \quad (4.17) \]

## 5 Numerical Calculations

Using the up-quark masses at GUT scale within the error \[18\],

\[ m_u = 1.04^{+0.19}_{-0.20} \text{ MeV}, \quad m_c = 302^{+25}_{-27} \text{ MeV}, \quad m_t = 129^{+196}_{-40} \text{ GeV}, \quad (5.18) \]

the parameter range of \( \alpha = 2hb/c \) and \( \beta = ha/c \) are estimated from the following values;

\[ \frac{2b}{c} \rightarrow \frac{2m_c}{\sqrt{m_u m_t}} \sim 1.0 - 2.4, \quad \frac{a}{c} \rightarrow \sqrt{\frac{m_u}{m_t}} \sim 0.03 - 0.05. \quad (5.19) \]

In order to realize large mixing angle \( \theta_{23} \), the option in which \( \alpha \) is close to 1 is a better choice. On the other hand, in order to realize large mixing angle \( \theta_{12} \), \( \lambda_2 \) must become at least of the same order as \( 2\beta \), so the option in which \( \beta \) is relatively large would be a better choice. Thus the desired candidate for the options of Table 1 would be 1) The Higgs representations coupled with 2-3 and 2-2 elements of \( M_U \) must be same. 2) The Higgs representation coupled with 1-2 elements of \( M_U \) must be as large as possible. The option \( S \) may be the best candidates which satisfy the conditions (i) and (ii).

Leaving the detailed calculations to our full paper [12], we here show an example of the figures of our results in Fig. 1. The explicit forms of up-type mass matrix for the class \( S \) are seen in Eq. (3.9) with Eq. (3.10), which we expected in section 2. Those two types yield the same predictions except for
Figure 1: Calculated values of $\sin^2 2\theta_{23}$ versus $h$ and $\tan^2 \theta_{12}$ versus $h$ in the class $S$. The experimentally allowed regions are indicated by the horizontal lines.

the Majorana mass scale. The type $S_1$ requires $m_R \sim 2 \times 10^{15}$ GeV and in the type $S_2$, we have $m_R \sim 10^{14}$ GeV, respectively. Thus more desirable one may be the type $S_1$ since it predicts more realistic bottom-tau ratio at low energy. Then the neutrino mass matrix is written as,

$$M_\nu \sim \begin{pmatrix} 0 & -3h \sqrt{\frac{m_c}{m_t}} & 0 \\ -3h \sqrt{\frac{m_c}{m_t}} & 2h \frac{m_{\nu}}{m_t} & h \\ 0 & h & 1 \end{pmatrix} \frac{9m_t^2}{m_R}.$$ (5.20)

From this, we obtain the following equations,

$$\tan^2 2\theta_{23} \simeq \frac{4h^2}{\left(1 - h \sqrt{\frac{2m}{m_um_t}}\right)^2}, \quad \tan^2 2\theta_{12} \simeq \frac{144h^2 m_{\nu} \cos^2 \theta_{23}}{m_t \left(1 - 2h + h \sqrt{\frac{2m}{m_um_t}}\right)^2}.$$ (5.21)

The neutrino masses are given by

$$m_{\nu_3} \simeq \lambda_{\nu_3} \cdot \frac{m_t^2}{m_R}, \quad m_{\nu_2} \simeq \lambda_{\nu_2} \cdot \frac{m_t^2}{m_R}, \quad m_{\nu_1} \simeq \lambda_{\nu_1} \cdot \frac{m_t^2}{m_R},$$ (5.22)

where the RGE factor ($\sim 1/3$) has been taken account in estimating lepton masses at low energy scale. Since $\lambda_{\nu_2} \ll \lambda_{\nu_3} \sim \mathcal{O}(1)$, this indeed yields $m_R \sim 10^{15}$ GeV, as many people require. We here list a set of typical values of neutrino masses and mixings at $h = 0.9$, $m_t \simeq 260$ GeV;

$$\sin^2 2\theta_{23} \sim 0.98 - 1, \quad \tan^2 \theta_{12} \sim 0.29 - 0.46,$$ (5.23)

$$m_{\nu_3} \sim 0.053 - 0.059 \text{ eV}, \quad m_{\nu_2} \sim 0.003 - 0.008 \text{ eV},$$ (5.24)

$$m_{\nu_1} \sim 0.0006 - 0.001 \text{ eV}.$$ (5.25)

Up to here we have estimated the contribution from $M_\nu$ side and this is fairly in a good approximation for estimating large mixing angles $\theta_{23}$ and $\theta_{12}$. Also
neutrino masses are determined from $M_\nu$ only. However, in estimating $|U_{e3}|$, we need careful estimation, since we cannot neglect the additional contribution from the charged lepton side in Eq. (1.3). The contribution from $M_\nu$ is as follows,

$$\sin \theta_{13} \simeq \frac{6h}{1 + 2h + h \frac{2m_e}{\sqrt{m_u m_t}} \sqrt{\frac{m_c}{m_t}}} \sin \theta_{23} \cos \theta_{12}. \quad (5.26)$$

Within the allowed range of $h$, the calculated value of $\theta_{13}$ from $M_\nu$ is almost of the same order as the one from $M_l$;

$$|\theta_{13}^\nu| \sim 0.037 - 0.038 \quad \leftrightarrow \quad \Delta \theta_{13}^l \sim \frac{\lambda}{3} \cdot U_{\mu 3} \sim 0.04, \quad (5.27)$$

where $\lambda \sim 0.2$ and the factor 3 comes from the Georgi-Jarlskog texture of Eq. (3.10). We have to combine those two contributions; unfortunately we do not yet have exact information of the relative phase. If the two terms act additively (negatively), we would have maximal (minimum) value. Still we can say that $|U_{e3}|$ becomes at most 0.11, which is within the experimental limit [19]. This would be one of the very important predictions of this model. In order to predict exact $|U_{e3}|$, the inclusion of CP phase of $M_\nu$ and $M_l$ is important, which is our next task. In conclusion we have seen that the up-road option can reproduce the present neutrino experimental data very well. However also down-road option may be also worthwhile to be investigated [20], in which case the Nature may show ”twisted family structure”. On the contrary in the case of up-road option it requires ”parallel family structure”.

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