A non-universal aspect in the temporal occurrence of earthquakes

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\textit{New Journal of Physics} \textbf{12} (2010) 063010 (10pp)
Received 3 March 2010
Published 7 June 2010
Online at http://www.njp.org/
doi:10.1088/1367-2630/12/6/063010

\textbf{Abstract.} It has been emphasized that the temporal occurrence of earthquakes in various spatial areas and over ranges of magnitude may be described by a unique distribution of inter-earthquake intervals under suitable rescaling, implying the presence of a universal mechanism governing seismicity. Nevertheless, it is possible that some features in the fine temporal patterns of event occurrences differ between spatial regions, reflecting different conditions that cause earthquakes, such as relative motion of tectonic plates sharing a boundary. By abstracting the non-Poissonian feature from non-stationary sequences using a metric of local variation of event intervals $L_v$, we find a wide range of non-Poissonian burstiness present in the temporal event occurrences in different spatial areas. Firstly, the degree of bursty features in the occurrence of earthquakes depends on spatial location; earthquakes tend to be bursty in areas where they are less frequent. Secondly, systematic regional differences remain even if the overall correlation between burstiness and the rate of event occurrence is eliminated. Thirdly, the degree of burstiness is particularly high on divergent tectonic boundaries compared to convergent and transform boundaries. In this way, temporal patterns of event occurrences bear witness to the circumstances underlying event generation.

\textbf{Online supplementary data available from } stacks.iop.org/NJP/12/063010/mmedia

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1. Introduction

The large-scale motion of tectonic plates induces stress, causing an abrupt discharge of energy or an earthquake, which may be described as a point event occurring in space and time. Although it is difficult to foretell the time, place and magnitude of individual earthquakes that cause immeasurable damage, it may be possible to capture the characteristics of a temporal sequence of all earthquakes occurring in a given region by clarifying the deviation from a simple random Poisson process [1–7]. It has recently been argued that the inter-earthquake interval distribution for different spatial areas and the magnitude range may be described by a distribution of an identical shape under suitable rescaling [8–14]. The similarity in the temporal structure of the recurrence of earthquakes implies that earthquakes are generated by an underlying universal mechanism.

Nevertheless, plate boundaries that generate earthquakes are inhomogeneous and are classified into different types based on the relative motion of adjacent plates: divergent, convergent and transform [15]. It is possible that the fine temporal patterns of event occurrences differ between spatial regions and reflect the internal conditions causing events, which are represented by the type of tectonic boundary.

Herein, we closely examine the temporal patterns of earthquakes occurring in various regions on the Earth by abstracting the non-Poissonian feature from non-stationary sequences. This can be realized by rescaling the time coordinate with the instantaneous rate to diminish rate fluctuation or by using a metric of local variation of event intervals \( L_v \). \( L_v \) was originally devised to analyze irregularity in event occurrence and has revealed that neuronal firing patterns are greatly correlated with the functional category of the cortical areas [16, 17]. Thus, we applied the metric of local variation \( L_v \) to earthquake sequences to determine whether there are systematic regional variations in the temporal patterns of event occurrences.
2. Materials and methods

2.1. Sampling seismological data

Our analysis is based on the global catalog of NEIC-PDE, which includes information about the time, spatial coordinates and magnitude of earthquakes. We adopted data from 1 January 1973 to 31 December 2009, during which a total of 530,481 earthquakes were recorded. To avoid problems arising from missing data for smaller earthquakes, we considered only events with a magnitude above the threshold $M = 4.5$, above which the cumulative number of earthquakes obeys the Gutenberg–Richter law \cite{18}. In the present analysis, every earthquake of a magnitude higher than the threshold was regarded as a unit event, regardless of whether it would be classified as a mainshock or an aftershock \cite{8–10}.

We divided the Earth’s surface into regional pieces with identical areas. The longitude angle was evenly divided into 72 intervals ($5^\circ$ for each cell), while the latitude was variably divided so that each cell has an area of 250,000 km$^2$. Fractions adjoining the North or South Pole were ignored because few earthquakes occur at the poles.

We collected earthquake events occurring in each cell, while ignoring spatial information, such as location within a cell and the depth of the hypocenter. Thus, events in each cell were solely characterized by the occurrence times $t_i$ ($i = 0, 1, 2, \ldots, n$). The temporal occurrence was analyzed by applying the metric $L_v$ to the sequence of inter-event intervals, $I_i = t_i - t_{i-1}$ ($i = 1, 2, \ldots, n$). To avoid large fluctuations in the evaluation, cells containing fewer than 50 intervals were excluded from the analysis. The areas adopted under these conditions were assembled along the boundaries of tectonic plates \cite{15}.

2.2. Measuring non-Poissonian burstiness

Numerous studies have investigated the non-Poissonian features of event sequences, such as earthquakes \cite{19–21}, neuronal spikes \cite{22–24} and human activity patterns \cite{25–27}, by observing the deviation of the inter-event interval histograms from an exponential distribution, which should be realized in an ideal Poisson random process. However, the argument based on histograms tends to be descriptive and qualitative. It is possible to quantify the deviation from Poissonian randomness using a metric such as the coefficient of variation $C_v$ \cite{28}. $C_v$ is defined as the ratio of the standard deviation of the intervals to the mean. A value of zero indicates a regular sequence in which inter-event intervals are constant, whereas unity indicates a Poisson random sequence. A negative deviation from unity denotes a tendency for a regular sequence. In contrast, a positive deviation indicates that the variation of intervals is greater than that of Poissonian randomness, reflecting the bursty feature of a sequence.

However, these analyses of raw intervals are vulnerable to fluctuations in the occurrence rate, which are inevitable in real data. Even for a temporally regular sequence with an instantaneous rate modulated greatly, such methods would conclude that the sequence is irregular, either by drawing a dispersed histogram or by indicating a high $C_v$ value due to the large deviation of raw intervals. Methods have been suggested to eliminate such non-stationarity by rescaling the time coordinate with the instantaneous rate \cite{29–32}. The idea of diminishing the effect of non-stationarity is common in detrended fluctuation analysis, which has been introduced to analyze DNA sequences \cite{33–36}.

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4 National Earthquake Information Center, Preliminary Determination of Epicenters catalog, http://earthquake.usgs.gov/earthquakes/eqarchives/epic_global.php

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Among the various methods for eliminating the non-stationarity [37, 38], we adopted a metric measuring the local variation of inter-event intervals $L_v$ [16, 17], defined as

$$L_v = \frac{3}{n-1} \sum_{i=1}^{n-1} \left( \frac{I_i - I_{i+1}}{I_i + I_{i+1}} \right)^2,$$

where $I_i$ and $I_{i+1}$ are the $i$th and $i+1$st consecutive intervals, respectively, and $n$ is the total number of intervals. It is the same as $C_v$, which $L_v$ adopts a value of zero for a regular sequence and a value of 1 for a Poisson sequence. Unlike $C_v$, however, non-stationarity is eliminated in $L_v$ by rescaling intervals with momentary frequency: the summand

$$\left( \frac{I_i - I_{i+1}}{I_i + I_{i+1}} \right)^2 = 1 - \frac{4I_i I_{i+1}}{(I_i + I_{i+1})^2}$$

may also be interpreted as the cross-correlation between consecutive intervals $I_i$ and $I_{i+1}$, each rescaled with the instantaneous rate $2/(I_i + I_{i+1})$.

3. Results

We found a wide range of non-Poissonian burstiness in the temporal occurrence of earthquakes at various spatial locations. Firstly, we discriminated between the degree of burstiness in event occurrences in terms of the metric $L_v$, which measures the local variation of inter-event intervals, and explored a map of the Earth to examine how different features of the temporal occurrence of earthquakes are distributed in various spatial locations. Secondly, we examined whether regional differences are present after the overall correlation between burstiness and the rate of event occurrence is removed. Thirdly, we tested if the temporal burstiness of earthquake occurrence is correlated to the type of tectonic boundary.

3.1. Difference in temporal patterns of earthquake occurrences

By applying the metric $L_v$ to temporal sequences of earthquakes, which have occurred at various local areas on the Earth’s surface, we found that non-Poissonian burstiness is present in the occurrence of earthquakes. Figure 1(a) demonstrates sample sequences exhibiting different values of $L_v$. Because a Poissonian random sequence indicates $L_v = 1$, values that significantly deviate from unity imply that earthquakes are not independent and random within a local area. A positive deviation from unity indicates that earthquakes tend to occur in bursts. Figure 1(b) also captures the bursty feature in the inter-event interval histogram; it positively deviates for the smaller and larger intervals from the exponential distribution, which should be realized in an ideal Poisson process. Here, the interval is rescaled with the instantaneous rate so that the time-local variance of the intervals is abstracted by eliminating non-stationary fluctuation. This distribution itself indicates that a bursty feature with a short interval is often followed by another short interval, but occasionally is followed by a very long interval. Note that $L_v$ metric also senses the correlation between consecutive intervals, which cannot be grasped from the distribution of individual intervals.

Then, we explored the distribution of temporal occurrences of earthquakes among spatial locations on the map. Figure 1(c) clearly shows that areas with similar $L_v$ values tend to cluster on the map, indicating that the degree of burstiness in the occurrence of earthquakes is similar among spatially close areas.
Figure 1. Sequences of earthquakes depicting different temporal patterns of occurrences and their locations on the map. (a) Event sequences recorded from various areas may exhibit different values of Lv. Sequences of \( n = 50 \) inter-event intervals assuming \( \text{Lv} \sim 1.8, 1.4 \text{ and } 1.0 \) (±0.1) exhibit bursty, semi-bursty and quasi-Poissonian random temporal patterns, respectively. (b) Histograms of intervals rescaled with the instantaneous rate, which is estimated from the adjacent 11 intervals (five in front, the present one and five in the rear). Sequences from individual areas are classified into three types (Lv > 1.6, 1.6 > Lv > 1.2, 1.2 > Lv). The dashed line represents the exponential distribution, which should be realized for an ideal Poisson random process. (c) Color of the cells where each cell measured 250 000 km\(^2\) and had more than 50 earthquakes (>4.5 M) from 1 January 1973 to 31 December 2009 indicates the distribution of Lv values on the Earth’s surface.

3.2. Spatial distributions of temporal characteristics in event occurrences

In addition to the spatial distribution of burstiness demonstrated in figure 1(c), the frequency of earthquakes can be used as a barometer to measure seismic activity. Most earthquakes occur in the basin of the Pacific Ocean, which is referred to as the ‘circum-Pacific seismic
Figure 2. Distribution of seismic characteristics on the Earth. (a) Rate of earthquake occurrence, which is measured by the number of events per year, is represented by the depth of shading. (b) The values of Lv and Rate of event occurrence for the cells, represented in a scattergram. The solid line and two dashed lines, respectively, represent a regression line $-a \log(\text{Rate}) + b$ and the upper (lower) 10 percentiles for possible deviation. (c) Spatial distributions of the upper and lower outliers are shown in red and blue, respectively, whereas the intermediate groups are depicted in gray.

belt’, as demonstrated in figure 2(a). Comparing figures 1(c) and 2(a) demonstrates that more active areas tend to exhibit Lv values close to unity. Figure 2(b) shows the overall correlation between burstiness and occurrence rate by plotting the values of the local variation Lv versus the log(Rate) of event occurrence. The negative regression line, $Lv = -a \log(\text{Rate}) + b$, indicates that events tend to be less correlated in areas with frequent earthquakes, but individual events are more correlated, suggesting that one event triggers others in areas with less frequent earthquakes.

There is room for interpreting the spatial clustering of similar burstiness in figure 1(c) as being due to the similarity in seismic activity in figure 2(a), although their causal relationship is left open. Factors that are independent of the overall trend may be expressed by the deviation from the regression line in figure 2(b). We selected data whose Lv values deviated significantly from the regression line. The upper and lower 10 percentiles for each rate of occurrence were determined by repeatedly simulating a renewal process where a given number of intervals
were derived from the gamma distribution, which may reproduce any specified mean value of $L_V$ [16, 17]. The fractions of real data outlying the upper and the lower 10 percentiles were 26.1 and 25.6%, respectively. This observation implies that additional factors make the individual burstiness deviate further from the overall trend. The map in figure 2(c) shows the original locations of the outliers and demonstrates a strong tendency for the outliers to assemble in different parts of tectonic boundaries.

The spatial distributions of the larger and the smaller burstiness, which are defined by the raw $L_V$ (figure 1(c)), and those defined by the deviation $D$ from the regression line (figure 2(c)) are roughly the same, but they have some differences: the raw $L_V$ is positively deviated in most divergent plate boundaries, while the deviation of $L_V$ from the regression line is positive in most oceanic plate boundaries. The main difference lies in the seismic belt connecting Japan and New Zealand, in which the raw $L_V$ and the deviation $D$ from the regression are negatively and positively deviated, respectively.

3.3. Relating earthquake burstiness to the type of tectonic boundary

Tectonic boundaries are classified into three types: divergent, convergent and transform, according to the relative motion of adjacent plates [15]. Thus, we examined whether the temporal patterns of event occurrences differ between boundary types. For this purpose, individual cells on the Earth’s surface were categorized into three groups according to the type of the nearest tectonic boundary (see supplementary data (available from stacks.iop.org/NJP/12/063010/mmedia)). Each boundary type is a different color in the scattergrams of $L_V$ and Rate in figure 3(a). We applied the Student’s $t$-test to each pair among the three groups to determine whether the means of the $L_V$ values statistically differed. Figure 3(b) shows histograms of the $L_V$ values for these three boundary types. In terms of the difference in the means of $L_V$’s, the divergent boundary significantly differed from the convergent ($t = 8.9$, $n_1 = 150$, $n_2 = 178$, $p < 0.0001$) and transform boundaries ($t = 5.1$, $n_1 = 150$, $n_2 = 47$, $p < 0.0001$), but the convergent and transform boundaries were indistinguishable ($t = 1.8$, $n_1 = 178$, $n_2 = 47$, $p > 0.05$).

Additionally, we applied the same statistical test to the deviations of $L_V$ from the regression line $D = L_V - a \log(\text{Rate}) - b$. Figure 3(c) shows histograms of the deviations for these three boundary types and the same tendency remains: the divergent boundary significantly differed from the convergent ($t = 5.6$, $p < 0.0001$) and transform boundaries ($t = 3.6$, $p < 0.0001$), but the convergent and transform boundaries were indistinguishable ($t = 0.95$, $p > 0.05$).

Since a large seismic event causes a long-lasting effect that induces a number of aftershocks, it is possible that the occurrence of a mega earthquake alters the burstiness in a sequence of earthquakes. In order to examine whether $L_V$ is influenced by large earthquakes, we plotted the $L_V$ values against the magnitude of the largest earthquakes that have occurred in individual regional cells (see supplementary data (available from stacks.iop.org/NJP/12/063010/mmedia)). Interestingly, it turned out that all three types of tectonic boundaries exhibit negative dependence of $L_V$ on the value of the largest magnitude, implying that the burstiness is not enhanced but is rather weakened by the occurrence of mega earthquakes.

4. Discussion

The present analysis has revealed that non-Poissonian burstiness in temporal sequences of earthquakes depends on spatial location and the type of tectonic boundary. We confirmed the
Figure 3. Difference among temporal characteristics for the three types of boundaries. (a) Scattergrams of Lv and log(Rate) for three types of areas according to the nearest tectonic boundary: divergent, convergent or transform, which are in magenta, cyan and gray, respectively. (b) Distributions of the Lv values for three types are plotted in different colors. Dashed lines indicate their means. Mean Lv for divergent boundaries (magenta) significantly differs from that for either convergent boundary (cyan) ($p < 0.0001$) and transform boundary (gray) ($p < 0.0001$), while the convergent and transform boundaries are indistinguishable from the mean values of Lv ($p > 0.05$). (c) Distributions of the values of $D = \text{Lv} + a \log(\text{Rate}) - b$ for the three types are plotted in different colors. Mean $D$ for the divergent boundaries significantly differs from those of the convergent boundary ($p < 0.0001$) and the transform boundary ($p < 0.0001$), while the mean values of $D$ for the convergent and the transform boundaries are indistinguishable ($p > 0.05$).

The systematic dependency of the temporal burstiness on spatial area seems to contradict the previous belief that earthquake recurrence times are universal. This apparent discrepancy may have resulted from the different analysis methods. We examined the coefficient of variation $C_v$, which measures the ratio of the standard deviation of intervals to the mean, but $C_v$ does not efficiently detect differences among spatial areas (see supplementary data (available from stacks.iop.org/NJP/12/063010/mmedia)). Both $C_v$ analysis and the raw interval distribution analysis are designed to detect the global variability of intervals and are generally weak in detecting fine irregular patterns for a sequence whose instantaneous rate of event occurrence fluctuates with time. It should be noted that the imperfection of universality in the seismicity was previously reported by carefully analyzing the inter-event interval distribution of real data [39], or by analyzing the epidemic type aftershock sequences (ETAS) model proposed for representing consecutive induction of earthquakes [40, 41]. Thus, it is possible to detect
subtle difference in the interval distributions by elaborating analysis, but our analysis using Lv may capture the difference more vividly by detecting correlation in consecutive intervals and in practice rendered here the more detailed specific characteristics among different spatial areas.

The reason why Lv can efficiently detect structural differences hidden in event sequences is because the non-stationary fluctuation in the rate is eliminated by rescaling time with the instantaneous rate. This metric Lv works just like it has revealed the relation of neuronal firing patterns and functional categories of the cortical areas [16, 17]. These observations strongly suggest that carefully eliminating non-stationarity may reveal more information about the underlying mechanisms for evoking mechanical as well as biological signals [16, 17, 33, 42] or human activities [43, 44].

Acknowledgments
This study was supported in part by Grants-in-Aid for Scientific Research to SS from the MEXT, Japan (20300083 and 20020012) and the Global COE Program ‘The Next Generation of Physics, Spun from Universality and Emergence’. XZ was supported by the Bilateral International Exchange Program in the above-mentioned Global COE Program in the Department of Physics, Kyoto University. TO is supported by a JSPS Research Fellowship for Young Scientists.

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