Galactopause Formation and Gas Precipitation during Strong Galactic Outflows

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Abstract

Using X-ray constrained $\beta$-models for the radial distribution of gas in the outskirts of galaxies, we analyze the termination of galactic winds and the formation and evolution of halo clouds by thermal instability. At low mass-loss rates ($M_{\text{w}}$) galactic winds are trapped within the halo, but they burst into the intergalactic medium during intermittent strong outflows with $M_{\text{w}} > 10 M_\odot$ yr$^{-1}$. We develop analytic models of halo clouds as they cool radiatively over condensation timescales $t_c \approx (390$ Myr$)(T_\text{g}/n_{\text{H}})(Z/Z_\odot)^{-1}$ for hydrogen number densities $n_{\text{H}} \approx (10^{-4}$ cm$^{-3})n_{\text{gas}}$, gas temperatures $T \approx (10^6$ K$)M_{\text{w}}$, and metallicities $(Z/Z_\odot)$. Halo gas can form kiloparsec-scale clouds out to galactocentric distances $r \approx 30–65$ kpc, when efficient radiative cooling from $10^6$ K down to $T \approx 10^4$ K occurs at $Z \geq 0.3 Z_\odot$ on timescales less than 1 Gyr. After condensing to column densities $N_{\text{H}} \geq 4 \times 10^{16}$ cm$^{-2}$, these clouds lose hydrostatic pressure support and fall inward on dynamical timescales of $\sim 200$ Myr. Our baseline analysis will be followed by numerical calculations to understand the governing principles of halo cloud formation and transport of gas to the galactic disk.

Unified Astronomy Thesaurus concepts: Galactic winds (572); High-velocity clouds (735); Interstellar medium (847)

1. Introduction

The circumgalactic medium (CGM) and intergalactic medium (IGM) help regulate the evolution of our Galaxy through the infall of gas that fuels continued star formation in the disk. Progress in understanding these processes was stimulated by UV spectroscopic studies (Stocke et al. 2013; Tumlinson et al. 2013; Keeney et al. 2017) with the Cosmic Origins Spectrograph (Green et al. 2012) on the Hubble Space Telescope. Quasar absorption lines originating in the halos of intervening galaxies discovered large metal-enriched gas reservoirs at impact parameters extending to radial distances of 100–150 kpc. These gas clouds are unlikely to be at rest with respect to hot gas at the virial temperature of the extended halo, as they are influenced by gravitational forces and intermittent pressure from galactic outflows. After losing hydrostatic support, halo gas falls inward as "precipitation" to the galactic disk (Voit et al. 2019) and observed as high-velocity clouds (HVCs) in the low halo (Wakker et al. 1999). The estimated infall rate of HVCs onto the disk (Shull et al. 2009; Putman et al. 2012; Fox et al. 2019) makes a significant contribution to the observed star formation rate (SFR) of 1–3 $M_\odot$ yr$^{-1}$ (Chomiuk & Povich 2011).

A key issue is characterizing the density perturbations that trigger condensations, which cool, condense, and fall to the disk. These perturbations may arise from gas compression from galactic outflows and passing satellite galaxies. Stalled galactic winds may produce a "galactopause" (Shull 2014), a stagnation point for outflows marking a boundary between the CGM and IGM. As we will show, the existence of a galactopause depends sensitively on the radial density structure of the halo gas. Halos with density profiles decreasing more rapidly with radius than $n(r) \propto r^{-2}$ ($\beta > 2/3$ in the standard $\beta$-model) have insufficient pressure to confine the outflows.

In this paper, we present analytic formulations of thermal and dynamical processes in the extended halos of galaxies. We explore the structure of a possible galactopause, at which gas outflows stall against the pressure of the CGM and IGM. Accumulation of gas at this interface defines a sphere of chemical enrichment of the CGM. During periods of starburst activity, galactic outflows can burst through the CGM and inject chemically enriched gas into the IGM. In Section 2, we use gas-density models ($\beta$-models) to describe the CGM confining pressure and the extent of galactic winds. The wind-CGM interface may furnish seeds for cloud formation, depending on the amount of compression and its range of influence. In Section 3, we develop analytic descriptions of cloud formation by radiative cooling, thermal instability, cloud precipitation, and cloud interactions with the wind. Section 4 concludes by describing effects on the long-term behavior of the CGM and implications for future research.

2. Spatial Extent of Galaxy Halos

In this section, we discuss the radial extent of galaxies. As discussed previously (Shull 2014), galaxies may end by gravity or by gas outflows. For gravitational estimates, we consider the virial radius ($R_{\text{vir}}$) computed from the turnaround and gravitational collapse of overdense perturbations in an expanding universe and the gravitational radius ($r_g$) computed from the potential energy of spherical systems. We then explore the confinement of galactic outflows by the pressure of hot gas in the extended halo. The CGM can stall galactic winds and produce a galactopause, much like the heliopause that terminates the solar wind, and it marks the boundary with the interstellar medium.

2.1. Gravitational Extent

In their study of collapsed halos in cosmological N-body simulations, Navarro et al. (1997) found that dark matter collapses into structures with cuspy cores and extended halos, fitted to radial (NFW) profiles of density, potential, and
enclosed mass,
\[ \rho(r) = \frac{\rho_0}{(r/r_e)[1 + (r/r_e)]^2} \]  
(1)

\[ \Phi(r) = -4\pi G \rho_0 r_e^2 \frac{\ln(1 + r/r_e)}{r/r_e} \]  
(2)

\[ M(r) = (4\pi \rho_0 r_e^3) \left[ \ln \left(1 + \frac{r}{r_e} \right) - \frac{r/r_e}{1 + r/r_e} \right]. \]  
(3)

Here, \( r_e = R_{\text{vir}}/c \), is a characteristic radius related to the virial radius by a concentration parameter \( c \). For extended self-gravitating systems without a sharp boundary, it is useful to define the “gravitational radius” (Binney & Tremaine 2008) as \( r_g = GM_c^2/|W| \), where the integrated gravitational potential energy is

\[ W = \frac{1}{2} \int d^3 x \, \rho(x) \, \Phi(x) = -4\pi G \int_0^\infty \rho(r) \, M(r) \, r \, dr. \]  
(4)

Although the NFW mass diverges logarithmically with radius, the gravitational radius can be found by integrating Equation (4) out to \( R_{\text{vir}} = cr_g \).

\[ W_{\text{vir}}^{(\text{NFW})} = \left( \frac{GM_c^2}{2r_g} \right) \left[ \frac{1}{1 + c} - \frac{c}{1 + c} \right]^2 \]  
(5)

\[ r_{r_g}^{(\text{NFW})} = \frac{GM_c^2}{|W|} = (2r_g) \left[ \frac{\ln(1 + c) - \frac{c}{1 + c}}{1 - \frac{\ln(1 + c)}{1 + c} - \frac{c}{1 + c}} \right]^2. \]  
(6)

The gravitational radius is similar to the “half-light radius,” \( r_h \approx 0.45(GM_c^2/|W|) \), defined by Spitzer (1969) for spherical stellar systems with various mass distributions. For most NFW halos, the ratio of gravitational radius to virial radius, \( (r_g/R_{\text{vir}}) \), is 0.5–0.7. For example, \( r_g = 3.44r_h = 0.69R_h \) for \( c = 5 \), and \( r_g = 10.8r_h = 0.54R_h \) for \( c = 20 \).

The virial radius is commonly used as an estimate of the collapsed region around halos of dark matter, including galaxies, groups, and clusters of galaxies. The original definition adopted critical mass overdensities \( \Delta_{\text{vir}} \approx 200 \) times the ambient density at collapse. That formulation was based on an outdated cosmological model with closure density in matter (\( \Omega_m = 1 \)). Using criteria for galaxy collapse and mass assembly in a ΛCDM universe, Shull (2014) defined the virial radius,

\[ R_{\text{vir}}(M_h, z_a) = (206 \text{ kpc}) h_{70}^{-2/3} M_h^{1/3} \times \left[ \frac{\Omega_m(z_a) \Delta_{\text{vir}}(z_a)}{200} \right]^{1/3} (1 + z_a)^{-1}, \]  
(7)

for a galaxy associated with total halo mass \( M_h = (10^{12} M_\odot) M_{12} \). The extra factor \((1 + z_a)^{-1}\) appears because virialization is assumed to occur at the assembly redshift \( z_a \) when the background density was higher by a factor of \((1 + z_a)\). This redshift was determined from cosmological collapse criteria (Lacey & Cole 1993; Sheth & Tormen 1999). As computed by Trenti et al. (2013), these virialization redshifts range from \( z_a \approx 1.35 \) for \( M_h = 10^{11} M_\odot \) to \( z_a \approx 0.81 \) for \( M_h = 10^{14} M_\odot \).

2.2. **Wind Confinement and Galactopause**

An estimate for the termination radius for a galactic wind in a biconical outflow into solid angle \( \Omega_w \) for constant CGM thermal pressure \( P_{\text{CGM}} = nkT_{\text{CGM}} \) is,

\[ R_{\text{term}} = \left( \frac{M_h V_w}{P_{\text{CGM}} \Omega_w} \right)^{0.45} \approx (138 \text{ kpc}) \left[ \frac{M_{10} V_{200}}{P_{40} \Omega_w^{0.45}} \right]^{0.45}. \]  
(8)

This radius marks the location where the wind ram pressure, \( P_w = \rho_w V_w^2 \), in a steady-state mass-outflow rate, \( M_a = \Omega_a r_w^2 \rho_w V_w \), stagnates against the thermal pressure. The mass-loss rate is expressed in units of \( 10 M_\odot \text{ yr}^{-1} \) with wind speed \( V_w = (200 \text{ km s}^{-1}) V_{200} \). We scale the solid-angle coverage of the outflow to \( (\Omega_w/4\pi) \) and the confining thermal pressure to \( P_{\text{CGM}}/k = (40 \text{ cm}^{-3} \text{ K}) P_{40} \). The total particle number density is \( n = n_{H1} + n_{He} + n_e \approx 2.24 n_{H1} \) for fully ionized gas (\( \text{H}^+, \text{He}^{+2}, \ldots \)) with \( n_e = n_{H1} + 2 n_{He} = 1.165 n_{H1} \). We adopt \( Y_p = 0.2477 \) for the He/H ratio by mass and \( \gamma = 0.0823 \) by number (Peimbert et al. 2007), comparable to estimates of the primordial value, \( Y_p = 0.2449 \pm 0.0040 \) (Cyburt et al. 2016). Typical galactic outflows have \( M_a = 1-10 M_\odot \text{ yr}^{-1}, V_w = 100-300 \text{ km s}^{-1} \), and \( \Omega_w/4\pi = 0.2-0.4 \) (Veilleux et al. 2005). Inferred gas pressures in the CGM range from \( P/k = 10-50 \text{ cm}^{-3} \text{ K} \) (Keeney et al. 2017), appropriate for halo virial temperature \( T_{\text{CGM}} \approx 2 \times 10^6 \text{ K} \) and hydrogen number density \( n_{H1} \approx 10^{-2} \text{ cm}^{-3} \) at \( r \sim 100 \text{ kpc} \). The outflow rate can be related to the SFR by a mass-loading factor \( \beta_m = M/SFR \) inferred from optical and X-ray data to lie in the range \( \beta_m = 1-3 \) (Strickland & Heckman 2009).

For these parameters, we estimate that most winds terminate at \( R_{\text{term}} \approx 100-200 \text{ kpc} \) during stages of moderate star formation. Some of the gas in the winds escapes to the IGM, and a portion is recycled back to the disk on a freefall timescale \( t_f \approx 1 \text{ Gyr} \). Stronger winds concentrated into biconical outflows with \( \Delta_{\text{vir}} \approx 0.3 \times 4\pi \) shift the termination radius to much larger radii, where these assumptions break down. The estimate in Equation (8) is also unreliable because it assumes a constant confining pressure from a hot CGM. More realistic estimates for wind termination must account for the decrease in density with radius observed in the outskirts of galaxies (Bregman et al. 2018).

For a better analysis of wind termination and its sensitivity to the CGM, we employ the “β model” (Cavaliere & Fusco-Femiano 1976; Sarazin 1986) often used to estimate the density in the outskirts of galaxies and groups. For the Milky Way halo, these models are constrained by X-ray emission lines and absorption lines (e.g., O VII and O VIII). X-ray surface brightness profiles have also been fitted for some external galaxies, with considerable uncertainty in their outer regions. Following the approximation adopted by Miller & Bregman (2013), we write the number density as

\[ n(r) = n_0 \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^{-3/2} \approx \left[ n_0 \left( \frac{r}{r_c} \right)^{3/2} \right] r^{-3/2}. \]  
(9)

The latter approximation is valid in the limit of \( r \gg r_c \), where \( r_c \) is the core radius and \( n_0 \) is a fiducial number density.\(^1\)

\(^1\) The β-models developed by Miller & Bregman (2013, 2015) refer to \( n(r) \) as “the gas density,” presumably the electron density \( n_e(r) \), as noted explicitly in Voit (2019). We assume fully ionized gas with \( \text{He}/H = 0.0823 \), so that \( n_e = 1.165 n_{H1} \). The total particle density is then \( n = 2.24 n_{H1} = 1.929 n_{H1} \).
Miller & Bregman (2013) found that a spherically symmetric $\beta$ model works just as well as a variation that considers the geometry of the galaxy, and we use it for simplicity. The galactopause lies well outside of the core radius, so that the wind typically has a termination shock in the CGM. For specific calculations, we adopt the parameters of Bregman et al. (2018), with index $\beta = 0.51 \pm 0.02$ and normalization $[n_0 r_c^{3.5}] = (2.82 \pm 0.33) \times 10^{-2} \text{ cm}^{-3} \text{ kpc}^{3.5}$, where $r_c$ is in units of kiloparsecs. The wind outflow sets against CGM thermal pressure in a halo described by a $\beta$-model when

$$1.93 \frac{[n_0 r_c^{3.5}]}{r_o^{3.5}} kT = \frac{M_w V_w}{\Omega_w r_o^2}.$$  \hspace{1cm} (10)

Adopting $\beta = 0.51$ and scaling the mass-outflow rate of strong winds as $M_w = (10 M_\odot \text{ yr}^{-1})M_{10}$, we find that the galactopause (GP) occurs at radius

$$R_{GP} = \left[ \frac{M_w V_w / \Omega_w kT}{1.93(n_0 r_c^{3.5})[3.086 \times 10^{13} \text{ cm}^2]} \right]^{1/(2-3\beta)} \approx (277 \text{ kpc}) \left[ \frac{M_{10} V_{200}}{T_6(\Omega_w / 4\pi)} \right]^{2.13}.$$  \hspace{1cm} (11)

We could also include the small decline in temperature with radius expected for halos near hydrostatic equilibrium with lower circular velocities in their outer regions. In hydrostatic equilibrium, $d\ln P / d\ln r = -2T_6 / T$, where the “potential temperature” $T_\beta \equiv (\mu c_2^2)^{1/2}$ assuming thermal energy of $kT/2$ for each of the two kinetic-energy components of circular velocity $V_c$. At the solar distance in the Milky Way, $V_c \approx 230 \text{ km s}^{-1}$ and $T_\beta \approx 1.9 \times 10^6 \text{ K}$ for mean particle mass $\mu = 0.592 m_H$. The circular velocity peaks at $r = 2.163 r_c$ for NFW halos (0.216$R_{200}$ for $c = 10$) and decreases slowly at larger distances into the halo. Models that adopt a constant ratio of cooling time to freefall time (Voit 2019) find shallow temperature profiles with $T(r) \propto r^{-0.1}$.

The expression above illustrates the sensitivity of the potential galactopause to the radial density profile. Because of the close competition between decreasing wind ram pressure ($P_{\text{ram}} \propto r^{-2}$) and CGM thermal pressure ($P_{\text{CGM}} \propto r^{-1.53}$), the galactopause radius scales nonlinearly with parameters of the outflow strength (mass-loss rate, wind velocity, and solid angle of the outflow). Because each of these quantities varies among galaxies, and will change over time in any individual galaxy, the wind termination moves inward and outward in radius. For example, a doubling of the outflow strength, either $M_w$ or $V_w$, would increase $R_{GP}$ by over a factor of 4, resulting in the wind bursting out of the CGM. Similarly, a decrease in the outflow strength would trap the wind, with a termination shock at $r < 100 \text{ kpc}$.

In Equation (11), the wind mass-loss rate ($M_w$) was expressed in units of $10 M_\odot \text{ yr}^{-1}$ flowing into total solid angle $\Omega_w$. The CGM temperature was scaled to $T_{\text{CGM}} = (10^6 \text{ K}) T_6$. These winds will have sufficient momentum to break through the CGM when $R_{GP} \gg R_{\text{CGM}}$ at

$$M_w \geq (10.4 M_\odot \text{ yr}^{-1}) \left( \frac{R_{\text{CGM}}}{300 \text{ kpc}} \right)^{0.47} \left( \frac{T_6}{V_{200}} \right)^{0.47}.$$  \hspace{1cm} (12)

This break-through criterion is sensitive to $\beta$ through the radial gradient of $n(r) \propto r^{-3\beta}$, particularly for values of $\beta > 0.5$.

Previous studies of O VII absorption in the Milky Way halo found $\beta = 0.56^{+0.11}_{-0.12}$ (Miller & Bregman 2013), $\beta = 0.53 \pm 0.03$ (Hodges-Kluck et al. 2016), and $\beta = 0.51 \pm 0.02$ (Bregman et al. 2018). Fits to O VII and O VIII emission lines in the Milky Way halo gave $\beta = 0.50 \pm 0.03$ (Miller & Bregman 2015). These density profiles are all sufficiently steep ($r^{-1.5}$ to $r^{-1.7}$) that strong outflows will break out of the CGM. However, X-ray observations in the outskirts of galaxies remain uncertain.

3. Halo Cloud Evolution with Radiative Cooling

3.1. Thermal Instability and Cloud Compression

We now discuss cloud evolution during the cooling phase and the possible effects of stripping and reassembly of the detached parcels of gas. Thermal instability followed by precipitation (Voit et al. 2019) is a plausible method of forming cool gas clouds in galaxy halos at a speed governed by the radiative cooling rate per unit volume, $n_e n_H \Lambda(T)$. Over the temperature range $5.0 < \log T < 6.5$, we approximate the cooling function (Gnat & Ferland 2012) as $\Lambda(T) \approx \Lambda_0 (T/T_0)^{-0.7}$, with $\Lambda_0 = (2 \times 10^{-22} \text{ erg cm}^{-3} \text{s}^{-1})(Z/Z_\odot)$ at fiducial temperature $T_0 = (10^6 \text{ K}) T_6$. The linear scaling of $\Lambda$ with metallicity is valid for $0.1 < (Z/Z_\odot) < 1$. For fully ionized gas in the galactic halo, this formula leads to an initial cooling time and cooling length for typical conditions at radial distances $r \approx 50 \text{ kpc}$.

$$t_{\text{cool}} = \frac{3n kT}{n_e n_H \Lambda(T)} \approx (630 \text{ Myr}) T_6^{7/(Z/Z_\odot)} n_e^{-1}.$$  \hspace{1cm} (13)

$$\ell_{\text{cool}} = c_s(t) t_{\text{cool}} \approx (98 \text{ kpc}) T_6^{2/(Z/Z_\odot)} n_e^{-1}.$$  \hspace{1cm} (14)

We evaluated these quantities for hydrogen number density $n_H = (10^{-4} \text{ cm}^{-3}) n_e$, with $n_{10} = 0.0823 n_H$, $n_e = 1.165 n_{10}$, $n = 2.247 n_{10}$, $\mu = 0.592 n_{10}$, and adiabatic sound speed $c_s = (5 kT / 3 \mu)^{1/2} \approx (152 \text{ km s}^{-1}) T_6^{1/2}$. This illustrates the dramatic decrease of $\ell_{\text{cool}}$ when hot halo gas is triggered into rapid cooling at constant temperature. As the gas cools below the peak of the cooling function, at $T \approx 10^5 \text{ K}$, the timescales drop to much smaller values, $\ell_{\text{cool}} \approx 10 \text{ Myr}$ and $\ell_{\text{cool}} \approx 1 \text{ kpc}$. The nonlinear dependence of these parameters on temperature produces rapid cloud evolution.

To describe the basic effects of cooling on cloud compression, we develop an analytic model of a cooling spherical cloud of radius $R$ and temperature $T$. We make a few simplifying assumptions: mass conservation ($nR^3 =$ constant), pressure equilibrium ($nT =$ constant), and enthalpy change with compression and radiative cooling. The cloud compresses with $n \sim T^{-1} \sim R^{-3}$, and the gas radiates away energy at constant pressure, with a cooling function $\Lambda(T) = \Lambda_0 (T/T_0)^{-0.7}$ and change in enthalpy of cloud volume $dH = 4 \pi R^3 / 3$.

$$\frac{dH}{dt} = -n_e n_H \Lambda(T) V_c = \frac{d}{dt} \left[ \frac{5}{2} n kT \times \frac{4 \pi}{3} R^3 \right].$$  \hspace{1cm} (15)

Approximating $R^{-7.1} \approx R^{-7}$ leads to a simple differential equation,

$$R^7 \frac{dR}{dt} = -\left[ \frac{\Lambda_0 n^2 R^8}{14.38 P_0} \right].$$  \hspace{1cm} (16)

with analytic solutions,

$$R(t) = R_0 \left[ 1 - \left( t / t_c \right) \right]^{1/8}.$$  \hspace{1cm} (17)
galactic mass is dominated by a spherically symmetric DM halo, with \( g(r) = GM(r)/r^2 \) for \( r \gg r_{\text{disk}} \). For an isothermal medium with \( T = 2 \times 10^5 \) K, ambient pressure differences are due to density gradients in the CGM. We approximate the total particle density as \( n(r) = n_0(r/r_0)^{-3\beta} \), with \( r_0 = 50 \) kpc and \( \beta = 0.51 \). We assume that the cloud lies near the polar axis, with \( n(z) = n(r), g(z) \approx g(r), \) and \( \Delta P = KT(\Delta n) \).

\[
\Delta n = \left| \frac{dn}{dr} \right|, \quad \Delta z = \left( \frac{3\beta n_0}{r_0} \right) \frac{r}{r_0}^{-3\beta+1} \Delta z. \tag{22}
\]

For a flat rotation curve, we approximate the enclosed mass \( M(r) = M_0(r/r_0) \) with \( r_0 = 50 \) kpc. Recent estimates for the Milky Way (Callingham et al. 2019) find a virial mass \( M_{\text{vir}} = 1.17 \times 10^{12} M_\odot \) and \( M(r \leq 20 \) kpc) = 0.12M_{\text{vir}}. For an NFW model with \( M_{\text{vir}} = (4\pi/3)\lambda[\ln(1 + c) - c/(1 + c)] \) and \( c = 10 \), the enclosed mass within 50 kpc is \( M_0 = 3.89 \times 10^{11} M_\odot \), comparable to the value \( 4 \times 10^{11} M_\odot \) found by Deason et al. (2012). With mean cloud pressure \( P_0 = 2.247n_0kT_0 \) at \( r_0 = 50 \) kpc and slab thickness \( \Delta z = 1 \) kpc, the minimum column density for precipitation is

\[
N_{\text{H}} > \frac{3\beta n_0kT_0}{1.33m_{\text{H}}g(r)} \left( \frac{r}{r_0} \right)^{-2.13} \left( \Delta z \right)^{-1.33} \left( \frac{P_0/k}{40 \text{ cm}^{-3} \text{ K}} \right). \tag{23}
\]

This column density is similar to that in the lowest column density HVCs (Collins et al. 2003), which can extend up to \( N_{\text{H}} \sim 10^{20} \) cm\(^{-2}\) (Wakker et al. 1999). Photoionization models of HVCs with \( R = 0.5–2 \) kpc and \( n_{\text{el}} = 10^{-3} \) to \( 10^{-2} \) cm\(^{-3}\) typically have total hydrogen column densities of \( 10^{18}–20 \) cm\(^{-2}\).

3.3. Cloud Infall and Stripping

When a thermally unstable cloud with sufficient column density loses pressure support, it begins falling inward, a process referred to as “precipitation” (Voit et al. 2019). For standard dark-matter halos containing a CGM with hot gas, we can estimate the cloud dynamics with a simple model of a 1 kpc cloud with total particle density \( n_{\text{el}} = 10^{-3} \) cm\(^{-3}\) at initial distance \( r_0 = (50 \) kpc)\( r_0 \) from the galactic center on the polar axis. In freefall, its inward acceleration is \( g(r) \approx -GM(r)/r^2 \), neglecting hydrostatic pressure gradients and cloud drag forces (which we consider later). We can estimate the cloud infall velocity in the halo, over the region with a flat rotation curve, where \( M(r) = M_0(r/r_0) \), using the first integral of motion with initial conditions \( \dot{r} = 0 \) and \( r = r_0 \) at \( t = 0 \).

\[
\frac{1}{2}r^2 = -\int_{r_0}^r GM(r)/r^2 \, dr = -\int_{r_0}^r GM_0/r_0 \, dr. \tag{24}
\]

The cloud infall velocity has a weak dependence on radius,

\[
\dot{r} = -\left( \frac{2GM_0}{r_0} \right)^{1/2} \sqrt{\ln(r_0/r)}. \tag{25}
\]
With the substitution \( r = r_0 \exp(-u^2) \), the drag-free infall can be described as
\[
 t(r) = \left( \frac{r_0^3}{2GM_0} \right)^{1/2} \int_0^{u(r)} \exp(-u^2) \, du \\
= \left( \frac{\pi r_0^3}{8GM_0} \right)^{1/2} \operatorname{erf} \left[ u(r) \right],
\] (26)
where \( u(r) = (\ln(r_0/r))/2 \) is the dimensionless argument of the Gaussian error function. We define a dynamical infall time in the dark-matter halo, from \( r_0 \) to \( r \ll r_0 \), where the error function approaches 1. For the Milky Way, with \( M_0 = 3.89 \times 10^{11} M_{\odot} \), interior to 50 kpc, the infall time and characteristic velocity are,
\[
 t_{in} \approx \left( \frac{\pi r_0^3}{8GM_0} \right)^{1/2} \approx (190 \text{ Myr}) r_0^{3/2} \text{ and } \\
(2GM_0/r_0)^{1/2} \approx (260 \text{ km s}^{-1}) r_0^{1/2}.
\] (27)
As noted previously, the density perturbations from thermal instability in diffuse halo gas do not remain stationary relative to the hot halo gas. Clouds in the halo are acted upon by gravitational forces and ram-pressure stripping; during periods of active star formation they can be buffeted by galactic winds. The clouds typically achieve velocities of 100–300 km s\(^{-1}\) relative to hot halo gas, often assumed to be in hydrostatic equilibrium. For cloud infall through a static halo with no wind, stripping begins when the ram pressure \( \rho_{\text{halo}} v_r^2 \) exceeds the internal cloud pressure, \( n_{\text{cl}} kT_{\text{cl}} \). We assume gas mass density \( \rho_{\text{halo}} = 1.33 m_\text{H} n_\text{H}(r) \) with \( n_\text{H}(r) = n_{\text{H,0}} (r/r_\odot)^{-\beta} \). From Equation (25), cloud stripping starts when
\[
(r_0/r)^{3/2} \ln(r_0/r) \geq \left[ \frac{P_{\text{cl}}}{1.33 m_\text{H} n_{\text{H,0}} (2GM_0/r_0)^{1/2}} \right].
\] (28)
For a condensing cloud with \( P_{\text{cl}}/k \approx 225 \text{ cm}^{-3} \text{ K} \) \( n_{\text{H,0}} = 10^{-3} \text{ cm}^{-3} \), \( T = 10^4 \text{ K} \) in a halo with \( n_{\text{H,0}} = 10^{-4} \text{ cm}^{-3} \) and \( \beta = 0.51 \), the right-hand side of Equation (28) equals 0.209. Stripping begins when the cloud falls to a fraction \( (r_0/r) \approx 0.85 \) of its initial radius.

During periods of active star formation, galactic winds sweep over newly condensed halo clouds. The gas dynamics of cloud-wind interaction has been studied numerically by many authors (Stone & Norman 1992; Klein et al. 1994; Mac Low et al. 1994; Orlando et al. 2005; Silva et al. 2010). In the wind-cloud encounter, we assume that the momentum imparted to the cloud depends on the fraction of wind mass striking the geometric cross section, with radius \( R = (500 \text{ pc}) R_{500} \). We adopt a density contrast \( \Delta_{\text{cl}} = n_{\text{cl}}/n_{\text{halo}} = 100 \Delta_{200} \) between the cloud and ambient halo gas, of density \( n_{\text{halo}} = (10^{-4} \text{ cm}^{-3}) n_{-4} \) at \( r = 50 \text{ kpc} \). When a wind with \( V_w = (200 \text{ km s}^{-1}) V_{200} \) impacts the cloud, it sends a slow shock of velocity \( V_s = V_{200} \Delta_{200}^{-1/2} \approx (20 \text{ km s}^{-1}) V_{200} \Delta_{100}^{-1/2} \) through the cloud, transiting on a crossing time,
\[
 t_c \approx \left( 2R/V_s \right) = (49 \text{ Myr}) R_{500} V_{200}^{-1} \Delta_{100}^{1/2}.
\] (29)
Most simulations find that clouds are shredded on a time scale of \( \sim 2 t_c \). For a cloud of mass \( m_{\text{cl}} \), the intercepted mass is the product of wind mass flux, cloud cross section, and cloud-crossing time,
\[
\Delta m = (\dot{M}/\Omega_r) (\pi R^2) (2R/v_r) \\
\approx (1.2 \times 10^4 M_\odot) (4\pi/\Omega_r) R_{500}^3 M_{10} V_{200}^{-1} \Delta_{100}^{1/2}. \tag{30}
\]
\[ m_{\text{cl}} = (4\pi R^3/3) (1.33 m_\text{H} n_{\text{H,0}} \Delta_{\text{cl}}) \approx (1.7 \times 10^5 M_\odot) R_{500}^3 n_{-4} \Delta_{100}. \tag{31}
\]
Comparing \( \Delta m \) to \( m_{\text{cl}} \), we see that halo clouds can intercepts a moderate amount of wind material. The fractional mass gain is independent of cloud radius \( R \), but it depends on outflow parameters and CGM conditions at \( r = 30–100 \text{ kpc} \).
\[
(\Delta m/m_{\text{cl}}) \approx (0.072) (4\pi/\Omega_r) M_{10} V_{200}^{-1} n_{-4} r_{500}^2 \Delta_{100}^{-1/2}. \tag{32}
\]
Momentum conservation during the wind-cloud encounter results in a shift in velocity,
\[
\Delta V_r = \left( \Delta m/m_{\text{cl}} \right) v_w \approx (14 \text{ km s}^{-1}) \times (4\pi/\Omega_r) M_{10} V_{200}^{-1} n_{-4} r_{500}^2 \Delta_{100}^{-1/2}. \tag{33}
\]
Because of the range in outflow strengths (\( \dot{M} \) and \( \Omega_r \)) and CGM density, this velocity impulse varies widely, from a few km s\(^{-1}\) to much larger values. Particularly for clouds at \( r < 50 \text{ kpc} \) swept by biconical outflows with \( (\Omega/4\pi) \approx 0.3 \), winds can significantly affect the structure of the cloud, sometimes shredding it. However, some parcels of stripped gas may reaccrete onto the cloud, if they experience less drag than the main cloud that precedes it. This is akin to the “Peloton effect” well known in bicycle racing and bird flying formation.

4. Summary and Discussion
The purpose of this paper was to investigate the formation and evolution of clouds in the galactic halo as they cool, condense, and lose hydrostatic pressure support from the hot gas. The CGM is not a passive region between the IGM and the galactic disk, but an environment conducive to forming clouds by thermal instability. We developed a variety of analytic models to estimate the radial extent of large-scale galactic outflows and their influence on halo cloud formation and infall to galactic disks. Recent observations suggest that the CGM is enriched with heavy elements, consistent with injection of disk gas into the halo through galactic winds during periods of active star formation. After new clouds form and condense, they fall inward on 200 Myr timescales, providing material for continued star formation in the disk. Cooling and compression of the clouds occurs on longer timescales, \( t_c \approx (390 \text{ Myr}) (T_{0}/n_{-4}) (Z/Z_{\odot})^{-1} \), possibly triggered by external compression events. After condensing to column densities \( n_{\text{H}} \geq 4 \times 10^{16} \text{ cm}^{-2} \), these clouds lose hydrostatic pressure support and fall inward.

A critical parameter for wind termination in a galactopause is the radial pressure profile of the halo gas. Much of the outflow mixes with the CGM, leading to overdensities and nonlinear thermal instability. Observations show that galactic outflows are multiphased, spanning temperatures from \( 10^8 \text{ K} \) to \( 10^5 \text{ K} \) (Zhang 2018). Through thermal instability, cooling and compressed gas at the galactopause or in the diffuse CGM could provide seeds for cloud formation. The CGM should constantly form rapidly cooling gas at \( T \lesssim 10^5 \text{ K} \). The cooling parameters suggest that smaller clouds form more frequently at smaller galactic radii. Cloud formation should occur out to radial distances \( \sim 30–65 \text{ kpc} \), depending on how steeply
density, temperature, and metallicity fall off with radius. From our analytic models, the primary conclusions are as follows.

1. For radial density profiles, \( n(r) = n_0 (r_0/r)^{-3/2} \), with \( \beta \approx 0.5 \) and \( n_0 \approx 10^{-4} \text{ cm}^{-3} \) at \( r_0 \approx 50 \text{ kpc} \), typical outflows will stall at galactocentric distances \( r \approx 200–300 \text{ kpc} \). The galactopause depends on wind strength parameters \( (\mathcal{M}_w, V_w, \Omega_w) \). If \( \beta > 0.6 \), the winds will have no termination shock in the CGM. Strong outflows with sufficiently fast winds will escape the galactic halo and burst into the IGM.

2. In strong galactic outflows \( (\mathcal{M}_w > 10 \mathcal{M}_\odot \text{ yr}^{-1}) \) wind material will mix with the CGM and interact with existing clouds. Depending on the timing, these winds could trigger cloud cooling through compression.

3. The cooling time \( t_{\text{cool}} \propto T_6^{-1/2} \) and cooling length \( \ell_{\text{cool}} \propto T_6^{-2/7} \) have strong temperature dependences for \( 10^4 \text{ K} \) down to \( 10^3 \text{ K} \). Metal-enriched gas that is slightly cooler than the virial temperature will cool and produce kiloparsec-scale clouds in less than 1 Gyr.

4. Clouds can form by thermal instability out to radii \( r \approx 30–65 \text{ kpc} \) for halo density profiles with \( \beta \approx 0.5 \) and \( n_\text{H} \approx 10^{-4} \text{ cm}^{-3} \). Efficient radiative cooling with \( t_{\text{cool}} < 1 \text{ Gyr} \) should occur at metallicities \( Z > 0.3 Z_\odot \).

5. Newly formed clouds that condense to densities \( N_\text{H} > 4 \times 10^{16} \text{ cm}^{-2} \) will fall out of hydrostatic equilibrium and precipitate onto the disk of the galaxy on 200 Myr timescales. Ram-pressure stripping can disrupt infalling clouds, although the trailing fragments may reassemble, by experiencing lower drag forces than the leading cloud.

An important question is how long galactic winds last. Bergvall et al. (2016) found a median starburst age of 70 Myr in a sample of active star-forming galaxies from the Sloan Digital Sky Survey. Outflows from superbubbles are driven by supernovae in OB associations and last at least 40 Myr, the lifetime of the last star to explode in a coeval starburst. Some outflows last longer, owing to noncoeval star formation in OB associations (Shull & Saken 1995). Cloud-wind interaction times (\( \approx 50 \text{ Myr} \)) are comparable to outflow times, but much smaller than timescales for radiative cooling (400–600 Myr) and cloud infall (200 Myr). Disruption of smaller clouds occurs by ram-pressure stripping arising from large relative velocities developed during infall or during intermittent galactic outflows.

Our analytic models of the thermal evolution of cooling clouds made several simplifying approximations, such as maintaining spherical shape and constant mass. We neglected cloud disruption during their 200 Myr infall, compared to the 40–70 Myr durations of galactic outflows and cloud interaction. Cloud disruption through infall has been simulated numerically (e.g., Heitsch & Putman 2009; Armillotta et al. 2017) with a consensus that bigger clouds are disrupted more slowly than smaller clouds. In large clouds \( (R > 250 \text{ pc}) \) a significant fraction of the cloud will survive longer than 250 Myr for relative velocities 100–300 km s\(^{-1}\). The next steps in our investigation will be to quantify the rate at which gas is accreted onto the disk, using numerical techniques to derive the cloud evolution \( (R, n, T) \) with realistic cooling rates and hydrodynamics. This will enable us to test the assumptions of our analytic models. We will model the cloud infall, including gravitational and drag forces on the infalling cloud and mass loss from interactions with hot halo gas and galactic outflows. We will also investigate the possibility that stripped gas behind the infalling cloud experiences less drag and reaccrtes onto the cloud. These theoretical studies will explore the parameter-space of cloud evolution, for galactic halos of different densities, temperatures, and metallicities.

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