A Harmony Search Method for the Estimation of the Optimum Number of Wind Turbines in a Wind Farm

Christos A. Christodoulou 1, Vasiliki Vita 1, George-Calin Seritan 2 and Lambros Ekonomou 1,*

1 Department of Electrical and Electronic Engineering Educators, A.S.P.E.T.E.—School of Pedagogical and Technological Education, GR-14121 Athens, Greece; christ_fth@uth.gr (C.A.C.); vassvita@aspete.gr (V.V.)

2 Faculty of Electrical Engineering, University “Politehnica” of Bucharest, Spaiul Independentei 313, RO-060042 Bucharest, Romania; george.seritan@upb.ro

* Correspondence: leekonomou@aspete.gr; Tel.: +30-6972702218

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Abstract: During the last decades, renewable energy production has significantly increased in an effort to produce clean energy that will not affect the environment. Governments around the world are focusing on reducing greenhouse gas emissions by increasing the utilization of renewable energy sources in the power chain. Wind farms and wind generators are the main renewable technology that are used worldwide. The main scope of wind farm designers is the achievement of the maximum possible power, restraining the installation cost that is related to the use of a specific number of wind turbines for specific power production, and considering the area of land to be occupied. A harmony search method is presented in this paper for the determination of the optimum number of wind turbines in a wind farm and the total electric power produced. The method is applied for comparison purposes on data from previously published methodologies proving its accuracy and effectiveness. The harmony research method can be used in the studies of wind farm designers aiming to reduce installation costs.

Keywords: harmony research method; installation costs; optimization; wind farm

1. Introduction

During the last decades, various factors related to industrial and economic development and needs have resulted in an increase of the electrical energy consumption, giving rise to issues related to greenhouse gas emissions. The global concern for the environment and the need to oppose the factors that result in planet-warming are key drivers for an enhanced environmentally friendly electrical energy system. In order to face the above consequences and respond to these challenges, renewable energy sources have been widely developed, given their environmentally friendly characteristics and advantages. Indeed, nowadays energy production by renewable energy sources constitutes a very significant part of the worldwide energy production with wind generators the leading technology to be used for clean energy production among all the different renewable energy sources that have been presented in the last decades. However, renewable energy, especially originating from intermittent and variable sources pose additional challenges that must be examined during the design of new power plants. Considering the various forms of renewable energy sources (solar, wind, tidal power, biomass, etc.), wind power offers several benefits, due to its abundance and cost-competitiveness. In this context, wind energy and wind parks are a dominant technology, expanded all over the world, which ensures compliance with the objectives for sustainable renewable energy supply. The appropriate number of wind turbines and their positioning within the wind farm, in an effort to maximize the efficiency and the energy production and to reduce at the same time the installation costs, remain critical issues to be
further investigated. Note that several factors (wind speed and direction, morphology of the wind farm area, equipment costs, etc.) determine the number and the placement position of the wind turbines.

Various studies have been performed proposing different techniques for addressing the above issue. In [1] genetic algorithms have been applied to determine the optimal placement of the wind turbines, to maximize the production capacity, and limit the number of turbines and the acreage of land occupied by each wind farm. In [2] a Monte Carlo procedure is implemented to extract the optimal number and placement of wind turbines, while in [3] a combinatorial optimization model is applied, that provides adequate results. In [4] the Powell optimization method and in [5] artificial neural networks are used to decide about the optimal number of wind turbines and the total produced power. Both methodologies’ outcomes have been compared with those produced by earlier methodologies proving their accuracy. The binary particle swarm optimization with time-varying acceleration coefficients has been applied in [6] for the optimal placement of wind turbines by extracting the maximum power in a wind farm in a cost-efficient way. In [7] an innovative control strategy approach for the optimal design of a wind farm has been presented, while in [8] a hybrid methodology of a probabilistic variable decomposed multi-objective evolutionary algorithm and a gradient-based non-dominated normalized normal constraint method is proposed. Finally, in [9] a methodology based on the generation of pseudo-random numbers is presented in order to maximize the power production of a wind farm and to reduce the wake effect due to front-end turbines.

In the current paper, a different methodology is implemented to optimize the installation of the wind turbines in a specific wind farm in an effort to maximize its power production. The proposed approach, based on harmony search method, is applied for comparison purposes on data from previous published methodologies proving its accuracy and effectiveness.

2. Modeling

2.1. Wind Characteristics

Real recorded data about wind speed and direction are necessary in an effort to achieve an accurate modeling of the wind characteristics. Equation (1) represents the probability density of occurrence of wind speed $p(v)$, based on a two-parameter Weibull distribution [10]:

$$p(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} \exp \left( -\left( \frac{v}{c} \right)^k \right)$$

where $v$ is the wind speed and $c$ and $k$ are the scale and shape parameters, respectively.

Equation (2) gives the probability of the wind speed to be lower than or equal to a value $v$:

$$P(v) = 1 - \exp \left( -\left( \frac{v}{c} \right)^k \right)$$

2.2. Wake Model

Considering that the generated power by the wind turbines depends on the wind speed, Equation (3) gives the wind speed, $v$, downstream the turbine, according to the Jensen’s wake decay model [11]:

$$v = v_o \left[ 1 - \frac{2f}{1 + a \left( \frac{x}{r_1} \right)^2} \right]$$

where

$v_o$, the mean free stream wind speed,
$x$, the distance downstream of the turbine,
$a$, the entrainment constant, as shown in Figure 1.
$r_1$, the downstream rotor radius, given by the Equation (4)
\[ r_1 = r_r \sqrt{\frac{1-f}{1-2f}} \]  

(4)

where

\( r_r \), the rotor radius,
\( f \), is the axial induction factor, calculated according to the Equation (5)

\[ f = \frac{0.5}{\ln \left( \frac{z}{z_o} \right)} \]  

(5)

where,

\( z \), the hub height of the wind turbine,
\( z_o \), the surface roughness of the site.

\[ A_0 \]
\[ \vdots \]
\[ A(x) \]

\[ v \]
\[ v_0 \]
\[ v_\theta \]
\[ v_i \]

\[ \alpha \]
\[ \mu \]
\[ 1 \]
\[ r_0 \]
\[ r_i = r_0 + \alpha x \]

Figure 1. Schematic of the wind decrease downstream the wind turbine.

The velocity downstream of \( N \) turbines is given by Equation (6):

\[ v = v_0 \left[ 1 - \sqrt{\sum_{i=1}^{N} \left( 1 - \frac{v_i}{v_0} \right)^2} \right] \]  

(6)

2.3. Objective Function

The generated power by a wind turbine is determined by the wind speed, according to Equation (7) [12]:

\[ P_i = \eta \left( 0.5 \cdot \rho \cdot A \cdot v_i^3 \right) \]  

(7)

where,

\( \eta \), the efficiency factor of the wind turbine,
\( \rho \), the air density,
\( A \), the swept area of the rotor.

The power curve of a wind turbine obeys to the above equation, between the wind speed at which the operation of the turbine is starting and the rated capacity; for higher wind speeds, the expected power generation is limited. Equation (8) gives the total generated power of the wind farm as the sum of the power of each wind turbine [12]:

\[ P_{\text{wind farm}} = \sum_{i=1}^{N} P_i \]  

(8)
Optimization of the wind farm layout intends to attain the minimum cost per unit of produced energy. This can be achieved when the wind farm generated energy reaches its maximum. The objective function that represents the above statement is given by the following equations [13]:

$$e(K) = \frac{\cos t}{P_{el, wf}}$$

(9)

where

$$\cos t = N(K) \left( \frac{2}{3} + \frac{1}{3} e^{-0.00174 \cdot N(K)^2} \right)$$

(10)

$$P_{el, wf} = \sum_{i=1}^{N(K)} \eta \left( 0.5 \cdot \rho \cdot A \cdot v_i^3 \right)$$

(11)

$K = \{x_i\} = 1, \ldots, d$ is a binary vector, where $k_i = 1$ indicates the $i$th grid where a wind turbine is placed, and $k_i = 0$ otherwise.

### 3. Optimization Method

The harmony search (HS) method is a meta-heuristic algorithm that imitates the improvisation procedure of music players, presenting several benefits compared with the conventional optimization techniques. Indeed, HS has advanced flexibility, since it does not require derivative information and setting of the initial values of the variables. Added to these, the HS method takes into consideration all the existing vectors during the generation of a new one, in contrast with a genetic algorithm, which takes into consideration only the two-parent vectors. The improved harmony search (IHS) method combines the advantages of the HS with advanced mathematical techniques, in an effort to upgrade fine-tuning features and the convergence rate of HS [14–22].

Considering an objective function $f(x)$, $x \in X_i$, $i = 1, 2, \ldots, n$, where $x_i$, the set of each decision variable $x_i$, $n$, the number of decision variables, $X_i$, the set of the possible range of values for each decision variable, the HM algorithm consists of the following steps [14,21,22]:

**Step 1:** Initialization of the problem under study and determination of the parameters.

**Step 2:** Initialization of the HM, given as (for a $n$-dimension problem):

$$HM = \begin{bmatrix}
    x_1^1 & \cdots & x_n^1 \\
    \vdots & \ddots & \vdots \\
    x_1^{HMS} & \cdots & x_n^{HMS}
\end{bmatrix}$$

(12)

where

$x_{j=1,2,\ldots,HMS}$, randomly generated solutions

$HMS$, the harmony memory size.

**Step 3:** Generation of a new solution from the HM based on:

- the harmony memory considering rate (HMCR) that generates the solution, and,
- the pitch adjusting rate (PAR) that mutates the solution.

This procedure is called improvisation. The HMCR determines the selection from the previous values stored in HM, since (1-HMCR) corresponds to the probability of randomly selecting one value from the possible range of values. The components provided by the memory consideration are further
examined to decide whether it should be pitch-adjusted. This process uses the PAR parameter that is the rate of pitch adjustment according to the following:

\[
\text{pitch adjusting decision for } x'_i \leftarrow \begin{cases} 
\text{Yes with probability PAR} \\
\text{No with probability } (1 - \text{PAR}) 
\end{cases}
\] (13)

In the case that the pitch adjustment decision for \( x'_i \) is YES, then it is replaced according to:

\[
x'_i \leftarrow x'_i \pm \text{rand()} \times bw
\] (14)

where

- \( bw \), an arbitrary distance bandwidth
- \( \text{rand()} \), a random number between 0 and 1.

Step 4: Update HM. The worst member of the HM is compared with the solution of Step 2; if it yields a better convergence, it will replace the already existing in the HM.

Step 5: Steps 2 and 3 are repeated until the stopping criterion is fulfilled.

The conventional HS algorithm uses a fixed value for both PAR and \( bw \). In the HS method, the values of PAR and \( bw \) that have been computed in Step 1, are constant during new generations. The main disadvantage is the number of iterations that the algorithm needs in order to extract the best possible solution. In an effort to enhance the performance of the HS algorithm and be discharged by the drawbacks lies with fixed values of PAR and \( bw \), the IHS algorithm uses variables PAR and \( bw \) computed in Step 3. PAR and \( bw \) alter in a dynamic way with generation number and are expressed as follows:

\[
\text{PAR}(gn) = \text{PAR}_{\text{min}} + \frac{(\text{PAR}_{\text{max}} - \text{PAR}_{\text{min}})}{NI} \cdot gn
\] (15)

where

- \( \text{PAR} \), the pitch adjusting rate for each generation
- \( NI \), the number of solution vector generations
- \( gn \), the generation number

and

\[
\text{bw}(gn) = \text{bw}_{\text{max}} \exp(c \cdot gn)
\] (16)

\[
c = \frac{\ln(\frac{\text{bw}_{\text{min}}}{\text{bw}_{\text{max}}})}{NI}
\] (17)

where

- \( \text{bw}(gn) \), bandwidth for each generation.

Figure 2 depicts a block diagram of the HS algorithm, where the basic steps of the methodology are shown. Note, that this paper does not suggest a new methodology, but it applies the HS method for an optimization problem in an effort to obtain reliable and efficient results for the optimal design of a wind farm.
4. System under Study and Results

The HS method for the determination of the optimum number of wind turbines to be installed is applied for the data of Table 1. The same data have been used in previous works and they have been chosen for comparison purposes [1,2,5,13].

| Wind Speed | 12 m/s |
|------------|--------|
| wind direction | 0 |
| tower height | 60 m |
| rotor diameter | 40 m |
| wind farm area | 2 m × 2 m |

Table 2 depicts the results of the harmony search method in comparison with the outcomes of other methodologies [1,2,5,13], in an effort to confirm the efficiency of the used methodology. The criterion that indicates the efficiency of the obtained outcomes is the fitness, i.e., the ratio between the cost and the total power ratio. The harmony search method provides the lower fitness value compared with the other applied methodologies in the previous studies, rendering its application more efficient to estimate the appropriate number of the wind turbines to be installed, taking also into consideration the electric power production.

| Method | Number of Wind Turbines | Total Produced Power (kW/Year) | Fitness Value |
|--------|-------------------------|-------------------------------|---------------|
| Harmony Search Method | 31 | 16934 | 0.0013090 |
| Artificial Neural Network [3] | 29 | 14298 | 0.0014760 |
| Genetic Algorithm [13] | 26 | 12352 | 0.0016197 |
| Genetic Algorithm [1] | 30 | 14310 | 0.0015436 |
| Monte Carlo [2] | 32 | 16395 | 0.0014107 |
5. Conclusions

The current paper addresses the optimization of wind farms layout, i.e., the appropriate installation of the wind turbines in a specific area in an effort to achieve maximum generated power at a non-extreme cost. Indeed, it is of great importance to attain the maximum energy production, so that the cost per energy unit is minimized, considering that the number of wind turbines and their location have a prominent effect on the expected power generation. For this reason, the wind turbines must be strategically placed in order to obtain as much energy as possible from the wind, considering also the wake effect. The determination of the optimal position of the wind turbines, in order to avoid their installation in inefficient positions, can be accomplished by implementing several techniques.

To this direction, in this work, the HS method is applied to estimate the optimum number of wind turbines in a wind farm and the produced electric power. The optimization of the wind park layout design offers significant cost savings and ensures the efficient electric power generation. Criterion of the optimization procedure is to minimize the per unit of produced energy cost. The HS method provides accurate results, compared with outcomes of other methodologies (artificial neural networks, genetic algorithms, Monte Carlo) published in the technical literature, indicating the appropriateness and the effectiveness of the implemented algorithm. Indeed, considering the fitness value as the ratio between the cost and the total generated electric power, the HS results have the smallest value compared with results obtained from the other three cases. The applied methodology and the extracted findings can be a useful tool for the designers of wind farm installations for the proper planning and development of an efficient wind park, to attain the maximum power production in a moderate cost. Future work includes the examination of additional factors, such as the internal grid electrical losses, and the use of different wake models.

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