Robustness and Predictivity of 4 TeV Unification

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Abstract

The stability of the predictions of two of the standard model parameters, $\alpha_3(M_Z)$ and $\sin^2 \theta(M_Z)$, in a $M_U \sim 4$ TeV unification model is examined. It is concluded that varying the unification scale between $M_U \simeq 2.5$ TeV and $M_U \simeq 5$ TeV leaves robust all predictions within reasonable bounds. Choosing $M_U = 3.8 \pm 0.4$ TeV gives, at lowest order, accurate predictions at $M_Z$. The impact of threshold effects on unification depends on the spectrum of states beyond the standard model.
One of the principal motivations for extending the standard model is the GUT gauge hierarchy between the weak scale and the grand unification or GUT scale. A related concern, not addressed here, is the Planck hierarchy between the weak scale and the Planck scale; the model we consider has flat spacetime, vanishing Newton’s constant and infinite Planck scale.

The most popular solution of the GUT hierarchy is low-energy supersymmetry \[1-4\] where the three gauge couplings \(\alpha_i(\mu)\) (\(i = 1,2,3\)) run logarithmically from \(\mu = M_Z \sim 91\) GeV, where they are known, up to \(M_{\text{GUT}} \sim 2 \times 10^{16}\) GeV, where they coincide with impressive accuracy.

In a recently-proposed model \[5\], grand unification occurs differently. The three couplings run from \(\mu = M_Z\) up to a lower unification scale \(M_U \sim 4\) TeV, at which scale the theory is embedded in a larger gauge group \(G \equiv SU(3)^{12}\). The \(SU(3)\) gauge couplings \(\alpha_j(\mu)\) (\(j=1-12\)) are all equal at \(\mu = M_U\). The embedding of the standard model gauge group in the larger gauge group \(G\) provides a group-theoretical explanation for the different values of \(\alpha_i(M_U)\).

This low-scale unification model also has a top-down inspiration from string theory through the AdS/CFT correspondence \[6-8\] arising from consideration of a Type IIB superstring in \(d = 10\) dimensional spacetime compactified on \(AdS_5 \times S^5\). Using a finite group \(\Gamma = Z_{12}\) in an abelian orbifold \(AdS_5 \times S^5 / \Gamma\) gives a quiver gauge theory \[9\] with gauge group \(SU(N)^{12}\) either with no supersymmetry \(\mathcal{N} = 0\) \[5\] or with \(\mathcal{N} = 1\) supersymmetry \[10\].

Several issues were left open in \[5\]: robustness of the predictions under variations of the scale \(M_U\) (conversely, the accuracy of the predictions at \(\mu = M_Z\)); the size of flavor-changing effects, and the consistency of the additional states around \(M \sim M_U\) with constraints imposed by precision low-energy data. In this article we shall address all of these issues.
Robustness of Predictions to Variation in \( M_U \)

The calculations of [5] were done in the one-loop approximation to the renormalization group equations without threshold effects. Because the couplings remain weak this can be self-consistent provided the masses of the new states in the model are sufficiently close to \( M_U \). Other corrections due to non-perturbative effects, and the effects of large extra dimensions, are outside of the scope of this paper. In one sense the robustness of this TeV-scale unification is almost self-evident, in that it follows from the weakness of the coupling constants in the evolution from \( M_Z \) to \( M_U \). That is, in order to define the theory at \( M_U \), one must combine the effects of threshold corrections (due to \( O(\alpha(M_U)) \) mass splittings) and potential corrections from redefinitions of the coupling constants and the unification scale. We can then impose the coupling constant relations at \( M_U \) as renormalization conditions and this is valid to the extent that higher order corrections do not destabilize the vacuum state.

We shall approach the comparison with data in two different but almost equivalent ways. The first is “bottom-up”, where we use as input the requirement that the values of \( \alpha_3(\mu)/\alpha_2(\mu) \) and \( \sin^2 \theta(\mu) \) are expected to be \( 5/2 \) and \( 1/4 \), respectively, at \( \mu = M_U \). Using the experimental ranges allowed for \( \sin^2 \theta(M_Z) = 0.23113 \pm 0.00015, \alpha_3(M_Z) = 0.1172 \pm 0.0020 \) and \( \alpha^{-1}_{em}(M_Z) = 127.934 \pm 0.027 \) from [11] we have plotted in Figure 1 the values of \( \sin^2 \theta(M_U) \) (vertical axis) and \( \alpha_3(M_U)/\alpha_2(M_U) \) (horizontal axis) for a range of \( M_U \) between 1.5 TeV and 8 TeV. Allowing a maximum discrepancy of \( \pm 1\% \) in \( \sin^2 \theta(M_U) \) and \( \pm 4\% \) in \( \alpha_3(M_U)/\alpha_2(M_U) \) as reasonable estimates of corrections, we deduce that the unification scale \( M_U \) may vary between 2.5 TeV and 5 TeV. Thus the theory is robust in the sense that uncertainty in the renormalization group equations does not effect the existence of unification.
Accuracy of Predictions at $\mu = M_Z$

Alternatively, to test of predictivity we fix the unification values at $M_U$ of $\sin^2 \theta(M_U) = 1/4$ and $\alpha_3(M_U)/\alpha_2(M_U) = 5/2$ and compute the resultant predictions at the scale $\mu = M_Z$. The results are shown for $\sin^2 \theta(M_Z)$ in Fig. 2 with the allowed range $[11] \alpha_3(M_Z) = 0.1172 \pm 0.0020$. The precise data on $\sin^2(M_Z)$ are indicated in Fig. 2 demonstrating that the model makes correct predictions for $\sin^2 \theta(M_Z)$. Similarly, in Fig 3, there is a plot of the prediction for $\alpha_3(M_Z)$ versus $M_U$ with $\sin^2 \theta(M_Z)$ held within the allowed empirical range. The two quantities plotted in Figs 2 and 3 are consistent for similar ranges of $M_U$: both $\sin^2 \theta(M_Z)$ and $\alpha_3(M_Z)$ are within the empirical limits if $M_U = 3.8 \pm 0.4$ TeV.
The model has many additional gauge bosons at the unification scale, including neutral $Z'$s and charged $W'$s, which could mediate flavor-changing processes on which there are strong empirical upper limits. The lower bound on a $Z'$ coupling like the standard $Z$ is $M(Z') < 1.5 \text{ TeV}$ [11] which is below the $M_U$ values considered here; however, the couplings of the other $SU(3)$ gauge groups associated with $SU(3)_W$ have a coupling generically stronger by a factor 4 requiring that $M(Z'') < 6 \text{ TeV}$ and hence a real danger of too-strong FCNC. This is, in our view, the tightest constraint on the viability of such conformality models. Full analysis requires commitment to a specific identification of quark flavors in the quiver diagram.

Since there are many new states predicted at the unification scale $\sim 4 \text{ TeV}$, there is, in addition, a potential of being ruled out by other precision low energy data, as conveniently studied in terms of the parameters $S$ and $T$ introduced in [12], designed to measure departure from the predictions of the standard model. Concerning $T$, if the new $SU(2)$ doublets are mass-degenerate and hence do not violate a custodial $SU(2)$ symmetry, they do not contribute $T$. This provides a constraint on the spectrum of new states. According to [12], a multiplet of degenerate heavy chiral fermions gives a contribution to $S$:

$$S = C \sum_i (t_{3L}(i) - t_{3R}(i))^2 / 3\pi$$

(1)

where $t_{3L,R}$ is the third component of weak isospin of the left- and right- handed component of fermion $i$ and $C$ is the number of colors. In the present model, the additional fermions are non-chiral and fall into vector-like multiplets and so do not contribute to $S$. Provided that the extra isospin multiplets at the unification scale $M_U$ are sufficiently mass-degenerate, therefore, there is no conflict of chiral fermions with precision data at low energy.

For contribution of new gauge bosons, we refer to the analysis in [13]. In the limit where the bilepton gauge bosons are degenerate $M_{++} = M_+$ the contribution to $S$ vanishes except for the subtlety of the pinch contribution. From the formula presented in [13] we find $(S|_P$ is the pinch contribution):
The first term in Eq.(2) is explicitly:

\[ S_0 = -16\pi \text{Re} \frac{\Pi^Y(m_Z^2) - \Pi^Y(0)}{m_Z^2} \]

\[ = \frac{9}{4\pi} \left[ \ln \frac{M_{++}}{M_+^2} + \frac{2}{m_Z^2} \left( M_{++}^2 \bar{F}_0(m_Z^2, M_{++}, M_{++}) - M_+^2 \bar{F}_0(m_Z^2, M_+, M_+) \right) \right. \]

\[ + \frac{4}{3} \left( \bar{F}_0(m_Z^2, M_{++}, M_{++}) - \bar{F}_0(m_Z^2, M_+, M_+) \right) \]

\[ - 2 \left( \bar{F}_3(m_Z^2, M_{++}, M_{++}) - \bar{F}_3(m_Z^2, M_+, M_+) \right) \] \hspace{1cm} (3)

in which \( \bar{F}_{0,3} \) are given by:

\[ \bar{F}_0(s, M, m) = \int_0^1 dx \ln \left( (1-x)M^2 + xm^2 - x(1-x)s \right) - \ln Mm \]

\[ = \frac{2}{s} \sqrt{(M+m)^2 - s} \sqrt{s - (M-m)^2} \tan \sqrt{\frac{s - (M-m)^2}{(M+m)^2 - s}} + \frac{M^2 - m^2}{s} \ln \frac{M}{m} - 2, \] \hspace{1cm} (4)

and

\[ \bar{F}_3(s, M, m) = \int_0^1 dx x(1-x) \ln \left( (1-x)M^2 + xm^2 - x(1-x)s \right) - \frac{1}{6} \ln Mm \]

\[ = \frac{1}{6} \left[ 1 + \frac{M^2 + m^2}{s} - \frac{2(M^2 - m^2)^2}{s^2} \right] \bar{F}_0(s, M, m) \]

\[ - \frac{1}{6} \left( 1 - \frac{2(M^2 + m^2)}{s} \right) \frac{M^2 - m^2}{s} \ln \frac{M}{m} + \frac{1}{18} - \frac{(M^2 - m^2)^2}{3s^2}. \] \hspace{1cm} (5)

The second term in Eq.(2) is:

\[ S|_P = \frac{1}{\pi} \left[ 3 \ln \frac{M_{++}^2}{M_+^2} + 2(1 + 2\sin^2 \theta_W) \bar{F}_0(m_Z^2, M_{++}, M_{++}) \right. \]

\[ - (1 - 4\sin^2 \theta_W) \bar{F}_0(m_Z^2, M_+, M_+) \] \hspace{1cm} (6)

From these equations, we find that the contributions of gauge bosons to \( S \) are suppressed by \( (M_Z/M_U)^2 \sim 10^{-4} \) and so even for many such new gauge bosons the contribution to \( S \) is acceptably small provided the \( SU(2) \) doublets are adequately degenerate.
Threshold Effects

In the above analysis we have assumed all the new states beyond the standard model are essentially mass degenerate at $M_U$. More realistically, a subset of the new states may lie below $M_U$ and consequently affect the running of the couplings $\alpha_{3c,2L,Y}$ because of changes in the corresponding renormalization group $\beta-$functions.

For the chiral fermions there are 48 bifundamental representations under $SU(3)^{12}$, some of which are in the light sector of the standard model, but most of which are heavy. We may label them by their transformation properties under $SU(3)_C \times SU(3)_W \times SU(3)_H$ and they are shown in Table 1. In the normalization [4,14] of the $\beta-$function a factor $g^3/(16\pi^2)$ has been absorbed so that in the three-family minimal standard model (MSM) with one Higgs doublet: $\beta_{3C}^{MSM} = -7, \beta_{2L}^{MSM} = -19/6$ and $\beta_Y^{MSM} = 41/6$. All the $\Delta\beta$ entries in Table 1 are necessarily positive. Note that $Y = (2/\sqrt{3})(T_{8W} - T_{8H})$ with $T_8 = \text{diag}(1/\sqrt{12}, 1, -2)$.

| FERMION MULTIPLET | $\Delta\beta_C$ | $\Delta\beta_{2L}$ | $\Delta\beta_Y$ |
|-------------------|---------|----------|---------|
| CC: $(3, \bar{3})_C$ | 2       | 0        | 0       |
| CW: $5(3C, 3W) + 2(\bar{3}C, 3W)$ | 7       | 7        | 28/3    |
|                  | $2(3C, 3W) + 2(\bar{3}C, 3W)$ | 4       | 4        | 16/3    |
| CH: $2(3C, \bar{3}H) + 5(3C, 3H)$ | 7       | 0        | 28/3    |
|                  | $2(3C, 3H) + 2(\bar{3}C, 3H)$ | 4       | 0        | 16/3    |
| WW: $9(3, \bar{3})_W$ | 0       | 9        | 12      |
| HH: $9(3, \bar{3})_H$ | 0       | 0        | 12      |
| WH: $9(3W, 3H) + 6(\bar{3}W, 3H)$ | 0       | 15       | 40      |
|                  | $6(3W, 3H) + 6(\bar{3}W, 3H)$ | 0       | 12       | 32      |

TABLE 1
For the CW, CH, WH multiplets, the vector-like part in the second row, like all the CC, WW, HH multiplets naturally acquire a mass $\sim M_U$. Of the remaining chiral pieces, 45 of the 81 states are light being chiral under the standard model gauge group and the remaining 36 also acquire mass $\sim M_U$ under the symmetry breaking $3^{12} \to 3_C 3_W 3_H \to 3_C 2_L 1_Y$ since they are vector-like under $3_C 2_L 1_Y$. Threshold effects occur when some of the heavy states lie below $M_U$. We shall illustrate below, by examples, the magnitude of such effects.

There are 36 bifundamental scalars under $SU(3)^{12}$. These transform under the $SU(3)_C \times SU(3)_W \times SU(3)_H$ subgroup and contribute to the $\Delta \beta_{3C,2L,Y}$ as shown in Table 2.

| SCALAR MULTIPLET | $\Delta \beta_C$ | $\Delta \beta_{2L}$ | $\Delta \beta_Y$ |
|------------------|------------------|-------------------|------------------|
| CC: $(3,3)_C$    | 1                | 0                 | 0                |
| CW: $4(3_C,3_W) + (3_C,3_W)$ | 5/2  | 5/2               | 10/3             |
| CH: $(3_C,3_H) + 4(3_C,3_H)$ | 5/2  | 0                 | 10/3             |
| WW: $4(3,\bar{3})_W$ | 0     | 2                 | 8/3              |
| HH: $4(3,3)_H$    | 0                | 0                 | 8/3              |
| WH: $10(3_W,3_H) + 7(3_W,3_H)$ | 0     | 16                | 128/3            |

**TABLE 2**

All of the scalar representations are real under $3_C 2_L 1_Y$, indeed under $SU(3)^{12}$, so all will naturally acquire a mass $\sim M_U$. One $SU(2)_L$ doublet from the WH row of Table 2 must, however, remain light as the standard Higgs doublet; this is the hierarchy problem.

Threshold effects are generally larger for fermions than for scalars, as seen from Table 1 and 2. Let us therefore illustrate how fermion masses below $M_U$ can effect the unification of $\alpha_{3C}$, $\alpha_{2L}$ and $\alpha_Y$.

Without any threshold corrections, the consistent unification of the three couplings, $\alpha^{-1}_{3C,2L,Y}$ is illustrated in Fig. 4.
Whether this unification survives threshold effects depends on the spectrum. We illustrate this by Fig. 5-7. Fig. 5 shows all the vector like CH fermions at 2 TeV; Fig. 6 shows all the vector-like WH fermions at 2 TeV. In both cases, unification fails. Fig. 7 shows all the vector-like CW fermions at 2 TeV; here, the unification is consistent at a higher scale $M_u \sim 5$ TeV. In all cases, $\alpha_{3C}(M_Z)$ and $\sin^2 \theta(M_Z)$ are at their experimental values.

Thus threshold effects are very significant because of the large number of extra states and may spoil unification. When $\Delta \beta_{3C}, \Delta \beta_{2L}$ and $\Delta \beta_Y$ are comparable, unification can remain consistent. Similar results are obtained for threshold effects from the scalar multiplets in Table 2.
Discussion

The plots we have presented clarify the accuracy of the predictions of this TeV unification scheme for the precision values accurately measured at the Z-pole. The predictivity is as accurate for $\sin^2 \theta$ as it is for supersymmetric GUT models [1–4]. There is, in addition, an accurate prediction for $\alpha_3$ which is used merely as input in SusyGUT models.

At the same time, the accuracy of the predictions remains robust if we allow the unification scale to vary from about 2.5 TeV to 5 TeV.

Threshold effects are large in some cases and may spoil unification, which depend on the spectrum of new states.

In conclusion, since this model ameliorates the GUT hierarchy problem and naturally accommodates three families, it provides a viable alternative to the widely-studied GUT models which unify by logarithmic evolution of couplings up to much higher scales.

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Figure Captions

Fig. 1.
Plot of $\sin^2 \theta(M_U)$ versus $\alpha_3(M_U)/\alpha_2(M_U)$ for various choices of $M_U$.

Fig. 2.
Plot of $\sin^2 \theta(M_Z)$ versus $M_U$ in TeV, assuming $\sin^2 \theta(M_U) = 1/4$ and $\alpha_3/\alpha_2(M_U) = 5/2$.

Fig. 3.
Plot of $\alpha_3(M_Z)$ versus $M_U$ in TeV, assuming $\sin^2 \theta(M_U) = 1/4$ and $\alpha_3/\alpha_2(M_U) = 5/2$.

Fig. 4.
Plot of $\alpha_3^{-1}, (2/5)\alpha_2^{-1}, (2/15)\alpha_Y^{-1}$ versus $E$(TeV) with no threshold effects.

Fig. 5.
Plot of $\alpha_3^{-1}, (2/5)\alpha_2^{-1}, (2/15)\alpha_Y^{-1}$ versus $E$(TeV) with all the vector-like CH fermions at 2 TeV.

Fig. 6.
Plot of $\alpha_3^{-1}, (2/5)\alpha_2^{-1}, (2/15)\alpha_Y^{-1}$ versus $E$(TeV) with all the vector-like WH fermions at 2 TeV.

Fig. 7.
Plot of $\alpha_3^{-1}, (2/5)\alpha_2^{-1}, (2/15)\alpha_Y^{-1}$ versus $E$(TeV) with all the vector-like CW fermions at 2 TeV.
FIG. 1.
$\sin^2 \theta (M_Z)$

$M_U (\text{TeV})$

FIG. 2.

$\alpha_3 (M_Z)$

$M_U (\text{TeV})$

FIG. 3.
FIG. 4.

FIG. 5.
FIG. 6.

FIG. 7.