Research Article

A Model for Shear Strength of FRP Bar Reinforced Concrete Beams without Stirrups

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The shear failure of a reinforced concrete beam generally occurs when the principal tensile stress near the neutral axis is equal to or greater than the tension strength of concrete. In order to set up a model for shear strength for FRP bar reinforced concrete beams without stirrups by the mechanical method, this paper equivalently transformed the FRP bar reinforced concrete rectangular beam with cracks as one composed of ideal elastic material to facilitate the analysis and proposed a new and more reasonable model of shear strength for FRP bar reinforced concrete beams without stirrups. Then, an experimental database including 235 FRP bar reinforced beams without stirrups was compiled to verify the validity of the proposed model. It was found that the values from the proposed model are in better agreement with the experimental results of shear strength of FRP bar reinforced concrete beams without stirrups in comparison with the models in codes.

1. Introduction

Fiber-reinforced polymer (FRP) bars have been considered as an advantageous alternative to replace steel bars for reinforced concrete structures due to the high-tension strength, durability, and good fatigue properties [1]. Since the modulus of elasticity, surface characteristics, tensile strength of FRP bars, and the bonding properties between FRP bars and concrete are different from those of steel bars, the use of FRP bars as reinforcement may cause different properties of FRP bar reinforced concrete members compared with the steel bar reinforced concrete members. It has been proved by several experimental investigations [2–4] that the shear capacity of FRP bar reinforced concrete beams without stirrups is different from that of steel bar reinforced members. Hence, the shear strength prediction models for steel bar reinforced concrete beams without stirrups cannot be directly applied to those for FRP bar reinforced members.

The shear design equations for shear strength of FRP bar reinforced concrete beams without stirrups were provided by many design codes or guidelines, including ACI 440.1R-15 [5], JSCE-97 [6], CAN/CSA S806-12 [7], GB50608 [8], and BISE-99 [9], as summarized in Table 1. It can be easily found that the calculation models are mainly based upon the empirical or semiempirical method or obtained from the modification of the existing design equations for concrete members reinforced with steel bars. Some of the models are expressed as a function of the square roots of concrete cylinder compression strength ($\sqrt{f_c'}$) [5,8], whilst the others are functions of the cubic roots of concrete cylinder compression strength ($\sqrt[3]{f_c'}$) [6, 7, 9], but none of them is a function of the cubic roots of the square of concrete cylinder compression strength ($\sqrt{f_c'^2}$). The shear strength calculation models provided by ACI 440.1R-15, JSCE-97, GB50608, and BISE-99 were reported to be conservative for the FRP bar reinforced concrete slender beams without stirrups, whilst the CAN/CSA S806-12 model did not well consider the effects of $\rho_f$ nor $E_f$ [10], and thus led to excessive
Table 1: Shear strength calculating models in codes.

| Code             | Calculating model                                                                 |
|------------------|-----------------------------------------------------------------------------------|
| ACI440.1R-15     | $V_{c,\text{ACI}} = 0.4\sqrt{f_c b (k d), k = \sqrt{2\rho f_n + (\rho f_t)^2} - \rho f_n, n_f = (E_f/E_c)}$ |
| JSCE-97          | $V_{c,\text{JSCE}} = \beta_d f_{\text{vcd}} b d, \beta_d = (1000/d)^{1/4} \leq 1.5$ |
| CAN/CSA S806-12  | $V_{c,\text{CAN/CSA S806}} = 0.11\sqrt{f_c b d \leq V_{c,\text{S806}} = 0.05 k_m k_s k_a \sqrt{f_c b d < 0.22\sqrt{f_c b d}}$ |
| GB50608-10       | $V_{c,\text{GB}} = 0.86 f_c b (k d), k = \sqrt{2\rho f_n + (\rho f_t)^2} - \rho f_n, n_f = (E_f/E_c)}$ |
| BISE-99          | $V_{c,\text{BISE}} = 0.79 (100\rho f_c b d)^{1/3} (400/d)^{1/4} (f_{\text{cd}}/25)^{1/3} b d$ |

reinforcement and also increasing the overall cost of construction [11].

For FRP bar reinforced concrete beams without stirrups and shear span-to-depth ratio of more than 2.7, a number of studies have shown that several parameters have the impacts on the shear strength ($V_c$) [12–15], such as concrete cylinder compression strength ($f_{\text{ct}}$), beam width ($b$), effective depth ($d$), reinforcement ratio ($\rho$), and modulus of elasticity ($E_f$) of the longitudinal FRP bar. The effect of the concrete compression strength on the shear strength of FRP bar reinforced concrete beams has been studied by many researchers. Experimental results [2, 16] indicated that the shear strength of FRP bar reinforced concrete beams increases with the increase of the concrete compression strength. It has been known that, in a beam without the external axial forces, the principal tensile stress arises from the interaction of normal and shear stresses. When the principal tensile stress from this interaction exceeds the tensile strength of concrete, the crack occurs and the beam is finally damaged [17]. Thus, the shear strength $V_c$ of FRP bar reinforced concrete beams is a function of $f_{\text{ct}}$. The concrete tension strength can be evaluated through the $\sqrt{f_{\text{ct}}^2}$ in Eurocode 2 [18]; therefore, $V_c$ can be expressed as a function of $\sqrt{f_{\text{ct}}^2}$ to show the significant effect of $f_{\text{ct}}$ on $V_c$.

In this paper, the primary objective is to propose a new and more accurate shear strength model expressed as a function of $\sqrt{f_{\text{ct}}^2}$ for FRP bar reinforced concrete beams without stirrups by the theoretical method. A database of 235 FRP bar reinforced concrete beams without stirrups was collected to verify the validity of the proposed model. The efficiency of the proposed model and the ACI 440.1R-15, CAN/CSA S806-12, GB50608-2010, JSCE-97, and BISE-99 models were evaluated by comparing the predictions with experimental results in the database.

2. Proposed Model

According to the mechanics theory, the shear failure of a reinforced concrete beam generally occurs when the principal tensile stress near the neutral axis is equal to or greater than the tension strength of concrete. For FRP bar reinforced concrete beams without stirrups, the shear strength in shear bending section of the beam is mainly provided by the uncracked concrete and the aggregate interlock of the cracked concrete. Based on the reasoning similar to the original model by Tureyen and Frosch [17], an actual FRP bar reinforced concrete rectangular beam with cracks shown in Figure 1(a) can be equivalently transmitted to that shown in Figure 1(b), which is composed of an ideal elastic material to facilitate the analysis and modeling of the shear strength. In the ideal elastic material beam, the elastic modulus and tension strength of the ideal elastic material are equal to $E_c$ and $f_{\text{ct}}$ of concrete. The effect of the longitudinal FRP bars and the aggregate interlock of the cracked concrete on the shear strength can be considered by the tension zone of the equivalent rectangular beam with the ideal elastic material, and the depth of concrete tension zone is equal to that of concrete compression zone $c$, as shown in Figure 2.

The depth of the compression zone of ideal elastic material beam can be calculated, as follows, according to ACI 440.1R-15 [5]:

$$c = k d,$$

where $k = \sqrt{2\rho f_n + (\rho f_t)^2} - \rho f_n, n_f = E_f/E_c$. For a rectangular section beam, as shown in Figure 2, the maximum shear stress occurs at the neutral axis and can be calculated by the material mechanics as follows:

$$\tau_{\text{max}} = \frac{3V}{2b(2c)}.$$

The normal stress at the neutral axis is

$$\sigma = 0.$$  

The principal stress shown in Figure 3 can be determined by the normal stress and shear stress as follows:

$$\sigma_1 = \sigma + \sqrt{\frac{\sigma^2}{2} + \tau^2}.$$
Figure 1: Transition of the FRP bar reinforced concrete beam to the ideal elastic material beam. (a) FRP reinforced beam. (b) Equal ideal elastic material beam.

Figure 2: Stress distribution in the equivalent rectangular beam composed with ideal elastic material. 

Figure 3: Principal stress.
Substituting the values of $\tau$ and $\sigma$ in equations (2)–(3) into equation (4), the principal tensile stress $\sigma_1$ at the neutral axis can be rewritten as follows:

$$\sigma_1 = \frac{3V}{2b(2c)}$$

(5)

It is assumed that the shear failure occurs when the principal tensile stress $\sigma_1$ is equal to or greater than the tension strength of concrete:

$$\sigma_1 \leq f_t.$$  

(6)

From equations (5) and (6), the shear strength of the FRP bar reinforced concrete beam without stirrups can be predicted as follows:

$$V_c = \frac{4}{3}f_c bc.$$  

(7)

The concrete tension strength can be evaluated by Eurocode 2 [18] as follows:

$$f_t = 0.3\sqrt{f_c^2}.$$  

(8)

Then, the equation for predicting the shear strength of the FRP bar reinforced concrete beam without stirrups can be changed as follows:

$$V_c^{prop} = 0.4\sqrt{f_c^2 bc}.$$  

(9)

3. Experimental Verification and Comparison

To demonstrate the validity of the proposed model, a database of 235 FRP bar reinforced concrete beams without stirrups was collected based on the following criteria: the specimens had rectangular cross-sections, simply supported, tested under one- or two-point loading, statically loaded, failed in shear, and the shear span-to-depth ratio ($a/d$) of specimens was greater than 2.7.

The parameters in this study included the concrete cylinder compression strength $(f_c')$, beam width $(b)$, effective depth $(d)$, reinforcement ratio $(\rho_f)$, and modulus of elasticity $(E_f)$ of the longitudinal FRP bars. The changing ranges of each parameter and the corresponding shear strengths of beams are given in Table 2, which are identical to the original data collected from the literatures.

3.1. Effect of Longitudinal FRP Bars. Based on the collected experimental data of 235 beams reinforced with FRP bar without stirrups in Table 1, the relation of the normalized shear strength $(V_c^{exp}/\sqrt{f_c'2bd})$ with $\rho_f n_f$ ($n_f = (E_f/E_c)$) is shown in Figure 4(a). It obviously indicates that $(V_c^{exp}/\sqrt{f_c'2bd})$ increases as $\rho_f n_f$ increases like the trendline, which illustrates that the shear strength of FRP bar reinforced concrete beams without stirrups is highly dependent on the reinforcement ratio $(\rho_f)$ and modulus of elasticity $(E_f)$ of the longitudinal FRP bars.

The relation between normalized shear strength ($V_c^{exp}/\sqrt{f_c'2bd}$) and $k$ is also shown in Figure 4(b). It can be clearly seen that $(V_c^{exp}/\sqrt{f_c'2bd})$ increases as $k$ increases. Through the regression of the experimental data, the relation between normalized shear strength ($V_c^{exp}/\sqrt{f_c'2bd}$) and $k$ was obtained as follows:

$$\frac{V_c^{exp}}{\sqrt{f_c'2bd}} = 0.4k.$$  

(10)

From the comparison of equation (10) with equation (9), it confirms that the effect of longitudinal FRP bars on the shear strength of FRP bar reinforced concrete beams without stirrups can be involved in equation (9) through the depth of the compression zone of the beam.

3.2. Effect of Concrete Compression Strength. The relation between normalized shear strength ($V_c^{exp}/0.4bf_c(kd)$) and $f_c'$ is shown in Figure 5. It can be observed that the normalized shear strength ($V_c^{exp}/0.4bf_c(kd)$) increases as the concrete compression strength increases, and the trendline between $V_c^{exp}/0.4bf_c(kd)$ and $\sqrt{f_c'}$ is in better agreement with the experimental data than that between ($V_c^{exp}/0.4bf_c(kd)$) and $\sqrt{f_c'}$. It proves that it is suitable by using $\sqrt{f_c'^2}$ in equation (9) to express the effect of $f_c'$ on the shear strength of FRP bar reinforced concrete beams without stirrups.

3.3. Comparison and Verification. To verify the efficiency that equation (9) captures the effects of the concrete compression strength and the longitudinal FRP bars on shear strength of the FRP bar reinforced concrete beams without stirrups, the values ($V_c^{exp}/V_c^{pred}$), which are the ratios of the experimental shear strength to prediction values according to the proposed model, and models recommended by ACI-440.1R-15, JSCE-97, CAN/CSA S806-12, GB50608-2010, and BISE-99 are shown in Figures 6 and 7 with $f_c'$ and $\rho_f E_f/E_c$, respectively. It is apparent that the values of ($V_c^{exp}/V_c^{pred}$), ($V_c^{exp}/V_c^{ACI}$), ($V_c^{exp}/V_c^{JSCE}$), ($V_c^{exp}/V_c^{S806}$), ($V_c^{exp}/V_c^{GB}$), and ($V_c^{exp}/V_c^{BISE}$) scatter in a range of 0.46–3.11, 0.82–5.54, 0.44–3.41, 0.24–3.13, 0.81–5.48, and 0.74–7.53, respectively, as the changing of $f_c'$ and $\rho_f E_f/E_c$. It can be seen that the most values of ($V_c^{exp}/V_c^{pred}$) are scattered around the line of $(V_c^{exp}/V_c^{pred}) = 1$. Additionally, there is a declining trend of $(V_c^{exp}/V_c^{BISE})$ with the increasing of $(f_c')$ and an increasing trend of $(V_c^{exp}/V_c^{S806})$ with the increasing of $\rho_f n_f$, which illustrates that the BISE-99 model does not well capture the influence of concrete compression strength, and the CAN/CSA S806-12 model does not well capture the effect of FRP bars.

Figure 8 presents the correlations among the experimental shear strength $V_c^{exp}$ of all 235 specimens and $V_c^{ACI}$, $V_c^{JSCE}$, $V_c^{S806}$, $V_c^{GB}$, $V_c^{BISE}$, and $V_c^{pred}$ predicted by ACI 440.1R-15, JSCE-97, CAN/CSA S806-12, GB50608-2010, and BISE-99 models and the proposed model. A line with tolerance of 0% has been represented in the graph, which indicates that...
Table 2: Beam specimen details and shear strengths.

| Investigators          | Quantity of specimens | \(a/d\) | \(f'c\) (MPa) | \(b\) (mm) | \(d\) (mm) | \(E_f\) (GPa) | \(\rho_f\) (%) | \(V_c\) (kN) |
|------------------------|-----------------------|---------|---------------|-----------|-----------|-------------|-------------|------------|
| Ashour et al. [19, 20] | 14                    | 2.7–5.9 | 23–50.2       | 150–200   | 163–371   | 32–142      | 0.12–1.39   | 9–36.1     |
| Yost et al. [4, 21, 22]| 42                    | 4.1–6.5 | 36.3–81.4     | 65–279    | 141–225   | 41–139      | 0.33–2.56   | 8.8–51     |
| El-Sayed et al. [16, 23, 24] | 18                  | 3.1–6.5 | 40–63         | 250–1000  | 155–326   | 39–135      | 0.39–2.63   | 60–190     |
| Razagpur et al. [25]  | 2                     | 3.6–4.2 | 40.5          | 200       | 225       | 145         | 0.5         | 38.5–49.7  |
| Tariq and Newhook [26] | 12                    | 2.8–3.7 | 34.1–43.2     | 130–160   | 310–346   | 42–120      | 0.72–1.54   | 42.7–63.7  |
| Tureyen and Frosch [27]| 6                     | 3.4     | 39.7–42.6     | 457       | 360       | 38–47       | 0.96–1.92   | 94.7–177   |
| Deitz et al. [28]     | 5                     | 4.5–5.8 | 27–30.8       | 305       | 158       | 40          | 0.73        | 26.8–29.2  |
| Duranovic [29]        | 3                     | 3.7     | 32.9–38.1     | 150       | 210       | 45–130      | 1.31–1.36   | 26.2–62.2  |
| Swamy [30]            | 2                     | 3.2–4.1 | 38–39         | 154–305   | 192–222   | 34–42       | 0.36–1.55   | 19.5–26.7  |
| Suzuki et al. [31]    | 3                     | 3       | 34.3          | 150       | 250       | 105         | 1.51–3.02   | 40.5–46    |
| Alam and Hussein [32] | 2                     | 3.5     | 34.3–39.8     | 250       | 305–310   | 47–144      | 0.42–0.86   | 43.7–58.9  |
| Nakamura and Higai [33]| 2                    | 3       | 22.7–27.8     | 300       | 150       | 29          | 1.3–1.8     | 33–36      |
| Bentz et al. [34]     | 6                     | 3.3–4.1 | 35–46         | 450       | 188–937   | 37          | 0.51–2.54   | 54.5–232   |
| Ražan and Yu [35]     | 1                     | 3.1     | 40            | 450       | 970       | 40          | 0.46        | 136        |
| Wakui and Tottori [36]| 4                     | 3.2     | 46.6–46.9     | 200       | 325       | 58–192      | 0.7–0.9     | 87–118     |
| Nagaoka et al. [37]   | 2                     | 3.1     | 22.9–34.1     | 250       | 265       | 56          | 1.9         | 83–113     |
| Issa et al. [38]      | 6                     | 5.7–7   | 35.9          | 300       | 165–170   | 48–53       | 0.8–4.12    | 29.3–51.5  |
| Tomlinson and Fam [39] | 3                    | 4.1–4.5 | 56.5–60       | 150       | 245–270   | 70          | 0.39–0.85   | 20.9–29.2  |
| Guadagnini et al. [40] | 1                    | 3.3     | 42.8          | 150       | 223       | 45          | 1.28        | 27.2       |
| Kim and Jang [41]     | 22                    | 3.1–4.5 | 30–40.3       | 150–200   | 214–216   | 40–148      | 0.33–0.79   | 16.6–28.9  |
| Olivito and Zuccarello [42] | 20               | 5.6     | 20.4–27.2     | 150       | 180       | 115         | 0.87–1.45   | 16.6–29.9  |
| Matta et al. [43]     | 12                    | 3.1     | 29.5–59.7     | 114–457   | 146–883   | 41–49       | 0.12–0.28   | 17.9–220.7 |
| Weggian and Abdalla [44] | 6                    | 6.5–9.5 | 32.5          | 1000      | 105–155   | 42–147      | 0.23–0.96   | 23.5–127   |
| El Refai and Abed [45] | 5                     | 3.5     | 49            | 152       | 195–215   | 50          | 0.31–1.53   | 16.9–29.9  |
| Chang and Seo [46]    | 14                    | 5.8–8   | 30            | 1200      | 130–182   | 44–50       | 0.24–1.22   | 26.3–159   |
| Abdul-Salam et al. [47]| 16                   | 5.7–6.3 | 41.3–6.2      | 1000      | 134–150   | 41–148      | 0.51–3.78   | 94–213     |
| Ali et al. [48]       | 6                     | 3       | 13–33.5       | 130       | 200       | 52          | 0.3–0.91    | 12.7–23.6  |
| Total                 | 235                   | 2.7–9.5 | 13–86.2       | 65–1200   | 105–970   | 29–192      | 0.12–4.12   | 8.8–232    |

Note 1: If the concrete cylinder compression strength \(f'c\) and modulus of elasticity of concrete \(E_f\) are not provided by the investigator while the concrete cube compression strength \(f'c\) is only measured, it is assumed that \(f'c = 0.8f'c\) and \(E_f = 4735\sqrt{f'c}\). Note 2: if the data are provided for BS by the investigator, they are converted as follows: 1 ksi = 6.895 MPa, 1 in = 25.4 mm, and 1 kip = 4.448 kN.

Figure 4: Effect of longitudinal reinforcement on \((V_{exp}/0.4(kd))\).

Figure 5: Effect of concrete compression strength on \((V_{exp}/0.4b(kd))\).
Figure 6: Comparison of \( \frac{V_{\text{exp}}}{V_{\text{pred}}} \) with the change of \( f'_c \).

Figure 7: Comparison of \( \frac{V_{\text{exp}}}{V_{\text{pred}}} \) with the change of \( \rho_f E_f/E_c \).
the exact prediction \((V_{exp}^c/V_{pred}^c) = 1\) of the shear strength. It can be seen that the ACI 440.1R-15, JSCE-97, CAN/CSA S806-12, GB50608-2010, and BISE-99 models provide the conservative predictions for the shear strength of the most FRP bar reinforced concrete beams without stirrups compared with the experimental results. Moreover, the larger the shear strength of beam, the greater the conservative degree of predictions. The ACI 440.1R-15, GB50608-2010, and BISE-99 models provide the most conservative predictions. The values calculated by the proposed model in this study are more consistent with the experimental results.

To further demonstrate the efficiency of the proposed model and compare it with the ACI 440.1R-15, CAN/CSA S806-12, GB50608-2010, JSCE-97, and BISE-99 models for shear strength prediction of FRP bar reinforced concrete beams without stirrups, the mean, standard deviation (SD), and coefficient of variation (COV) of \((V_{exp}^c/V_{pred}^c)\) for all of the beams in the database are shown in Table 3. The mean values of \((V_{exp}^c/V_{pred}^c)\) are closer to 1 than that of \((V_{exp}^c/V_{ACI}^c)\), \((V_{exp}^c/V_{SCE}^c)\), \((V_{exp}^c/V_{S806}^c)\), \((V_{exp}^c/V_{GB}^c)\), and \((V_{exp}^c/V_{BISE}^c)\), while the SD values of \((V_{exp}^c/V_{pred}^c)\) are smaller than that of \((V_{exp}^c/V_{ACI}^c)\), \((V_{exp}^c/V_{SCE}^c)\), \((V_{exp}^c/V_{S806}^c)\), \((V_{exp}^c/V_{GB}^c)\), and \((V_{exp}^c/V_{BISE}^c)\). It confirms that the proposed model provides more accurate predictions for the shear strength of the FRP bar reinforced concrete beams than ACI 440.1R-15, CAN/CSA S806-12, GB50608-2010, JSCE-97, or BISE-99 models.

### Table 3: Mean, SD, and COV for ratio of \((V_{exp}^c/V_{pred}^c)\).

| Checks | \((V_{exp}^c/V_{ACI}^c)\) | \((V_{exp}^c/V_{SCE}^c)\) | \((V_{exp}^c/V_{S806}^c)\) | \((V_{exp}^c/V_{BISE}^c)\) | \((V_{exp}^c/V_{GB}^c)\) | \((V_{exp}^c/V_{Prop}^c)\) |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|
| Mean   | 1.96           | 1.36           | 1.23           | 2.28           | 1.92           | 1.07           |
| SD     | 0.65           | 0.43           | 0.47           | 0.96           | 0.64           | 0.37           |
| COV    | 0.33           | 0.31           | 0.38           | 0.42           | 0.33           | 0.34           |

### 4. Summary and Conclusions

This paper focused on the modeling of the shear strength prediction of FRP bar reinforced concrete beams without stirrups statically loaded. A simple and more accurate shear strength prediction model expressed as a function of \(\sqrt{f_c'}\) was proposed by theoretical analysis. An experimental database of 235 FRP bar reinforced concrete beams without stirrups was established to evaluate the efficiency of the proposed model by comparing the calculated shear strengths with the experimental results and with that of ACI 440.1R-15, CAN/CSA S806-12, GB50608-2010, JSCE-97, and BISE-99 models. The main conclusions of this research are summarized as follows:

1. An actual FRP bar reinforced concrete beam with cracks can be equivalently transmitted to a rectangular beam which composes of an ideal elastic material to propose the model of shear strength.
2. The effect of longitudinal FRP bars on the shear strength of FRP bar reinforced concrete beams without stirrups can be involved in the new model through the depth of the concrete compression zone.
3. It is more reasonable to be expressed as a function of \(\sqrt{f_c'}\) for considering the effect of concrete strength on the shear strength of FRP bar reinforced concrete beams without stirrups than that of \(\sqrt{f_c'}\) or \(\sqrt{f_c'}\).
(4) The calculating values by the proposed model are in better agreement with the experimental results of shear strength of FRP bar reinforced concrete beams without stirrups, comparing with the models in codes.

**Notations**

- $f'_c$: Cylinder compression strength of concrete, MPa
- $f_{cu}$: Cube compression strength of concrete, MPa
- $f_t$: Tension strength of concrete, MPa
- $\sigma$: Normal stress, MPa
- $\tau$: Shear stress, MPa
- $\sigma_t$: Principal tensile stress, MPa
- $a$: Shear span, mm
- $b$: Beam width, mm
- $c$: Distance from the extreme compression fiber to the neutral axis, mm
- $d$: Effective depth, mm
- $a/d$: Shear span-to-depth ratio
- $\rho$: Reinforcement ratio
- $A_f$: Area of longitudinal tension reinforcement, mm$^2$
- $E$: Modulus of elasticity of the FRP bars, GPa
- $E_s$: Modulus of elasticity of steel bars, GPa
- $E_c$: Modulus of elasticity of concrete, GPa
- $V$: Shear force on the cross-section, N
- $V_c$: Shear strength of the FRP bar reinforced concrete beams, N
- $V_{c,\text{exp}}$: Experimental shear strength, N
- $V_{c,\text{ACI}}$: Predicted shear strength by ACI440.1R-15 model, N
- $V_{c,\text{JSCE}}$: Predicted shear strength by JSCE-97 model, N
- $V_{c,\text{S806}}$: Predicted shear strength by CSA S806-12 model, N
- $V_{c,\text{BISE}}$: Predicted shear strength by BISE-99 model, N
- $V_{c,\text{GB50608}}$: Predicted shear strength by GB50608-2010 model, N
- $V_{c,\text{Prop}}$: Predicted shear strength by proposed model, N
- $V_{c,\text{pred}}$: Predicted shear strength, N.

**Data Availability**

All data included in this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest.

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