Modelling Characteristics of Ferromagnetic Cores with the Influence of Temperature

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Abstract. The paper is devoted to modelling characteristics of ferromagnetic cores with the use of SPICE software. Some disadvantages of the selected literature models of such cores are discussed. A modified model of ferromagnetic cores taking into account the influence of temperature on the magnetizing characteristics and the core losses is proposed. The form of the elaborated model is presented and discussed. The correctness of this model is verified by comparing the calculated and the measured characteristics of the selected ferromagnetic cores.

1. Introduction
Temperature is one of the most important parameters affecting the properties of electronic components [1 - 4]. In the case of magnetic elements – choking-coils and transformers, the shape of their characteristics is mostly determined by the properties of the ferromagnetic core contained in these components [5 - 8]. As it is known from the literature, e.g. [5, 7, 9] the temperature significantly affects the value of the core saturation flux density and the permeability of this core. When the core temperature exceeds the Curie temperature, the core loses the ferromagnetic properties [5, 10, 11]. It results in a significant decrease in inductance of the choking-coil or in a loss of the voltage feedback between the windings of the transformer.

The models of electronic components play a significant role in the designing process and the analysis of electronic circuits. The accuracy of these models and a set of phenomena taken into account in them determine the validity of the calculations results. Currently, SPICE is the standard program for the analysis of electronic circuits and electric power engineering [6, 12]. Many literature positions concentrate on modelling magnetic elements but they do not include the influence of temperature on the characteristics of the core [12]. Well-known in this area is the classical Jiles – Atherton model which was also implemented in SPICE [10, 13].

As it results from [14, 15], the Jiles–Atherton model has many defects which cause that the modelling magnetization curve does not reflect the character compared with the characteristic of real magnetic cores. Therefore, to describe new ferromagnetic models, the equations from the paper [13] are used. In the model from the paper [13] the Langevin’s function is used to describe the magnetizing curve, whereas in the paper [16] the use of the Brillouin’s function to this end is proposed.

In the paper [17] the problem of reliable estimation of the values of parameters of the Jiles-Atherton’s model, the inaccurate image in this model of more complicated cycles of magnetizing and omission of eddy currents, are pointed out. This problem was described also in other works, for example in the paper [18], where the analysis was performed for the parameters values of the model obtained on the basis of the algorithm proposed in the paper [19]. The authors of the cited papers show
that for smaller hysteresis loops, where the difference between the performance of calculations and measurements increases, courses B(t) and H(t) become unsymmetrical and the hysteresis loop does not close. This results from an increase of the magnetic flux density B in the core and a decrease in the magnetic force H, which has no physical justification.

On the other hand, in the literature models of a ferromagnetic core, in the form of a macromodel for SPICE, the thermal properties of cores are taken into account. Such macromodels are shown e.g. in [7, 20 - 23]. Unfortunately, models from the papers [21 - 23] do not include the hysteresis of the core characteristics. Yet the model proposed in [7] contains a lot of errors, which results in the fact that in the core magnetization curve, obtained with the use of this model unjustified high values of the coercive magnetic field and the saturation flux density are observed. The model from [20] correctly describes the influence of temperature on the properties of the ferrite core, but a high complexity of that model prevents its practical application in the analysis of power electronic circuits.

This paper, which is an extended version of the paper [24], proposes a modified model of ferromagnetic core dedicated for SPICE. At the formulation of this model, the classic Jiles-Atherton model from [13], supplemented with the temperature dependence of the parameters of this model taken from [23] and the description of power losses in the core taken from [7] are used.

In the second section the modified ferromagnetic model is presented, whereas the third section contains the results of measurements and calculations of the cores characteristics.

2. Description of the model
Due to the SPICE specification, the elaborated by the authors model is implemented as a subcircuit. The network representation of this model is shown in figure 1. In this model the input signal is put between the terminal H and the ground. This signal represents the magnetic force H. The flux density corresponds to the voltage on the terminal B. The power losses in the core are equal to the voltage at the terminal \( P_{\text{loss}} \).

The voltage source \( E_4 \) represents magnetization on the initial magnetization curve described by:

\[
M_a = \frac{B_{S0} \cdot (1 + \alpha_{BS} \cdot (T_R - T_0))}{\mu_0} - H_S \cdot F\left(\frac{H + \alpha \cdot M_a}{A}\right)
\]  

(1)

where \( B_{S0} \) denotes the saturation magnetic flux density on the reference temperature \( T_0 \) at the magnetic field \( H_S \), \( \alpha_{BS} \) – the temperature coefficient of the saturation magnetic flux density changes, \( T_R \) – the core temperature, \( \mu_0 \) – the permeability of free air, \( \alpha \) (the magnetic field coefficient) and \( A \) (the hysteresis shape parameter). They all are the Jiles–Atherton model parameters. The \( F(x) \) function replaces the Langevin function in the classical Jiles–Atherton model and has the following form:

\[
F(x) = \frac{x}{\sqrt{x^2 + 1}} \left[ 1 - 0.9 \cdot \exp\left(-\frac{|x|}{2.5}\right) - 0.1 \cdot \exp\left(-\frac{|x|}{25}\right) \right]
\]

(2)

The current of the voltage source \( V_{dM_a} \) is equal to the time derivative \( dM_a/dt \).

The voltage source \( E_5 \) represents the magnetic flux density expressed by the formula

\[
B = \mu_0 \cdot \left( H + M \cdot C_R \cdot x \right)
\]

(3)

where

\[
x = \begin{cases}
1 & \text{for } T_R < T_C \\
1 - (T_R - T_C) \cdot 0.06 & \text{for } T_C < T_R < T_C + 17 \\
0 & \text{for } T_R > T_C + 17
\end{cases}
\]

(4)

\( T_C \) denotes the Curie temperature.

The current of the voltage source \( V_{dB} \) is equal to the derivative \( dB/dt \).
The voltage on the controlled voltage source $E_1$ is equal to the magnetic field strength $H$ and the current of the voltage source $V_{dB}$ is equal to the derivative $dH/dt$. The voltage at $M$ node denotes magnetization $M$ in the core. The current on the controlled current source $G_1$ is described by the formula:

$$G_1 = - \left( M_a - M \cdot C_2 \right) \cdot \text{sgn}\left( \frac{dH}{dt} \right) \cdot \frac{dH}{dt} - \frac{C}{C_1 \cdot (1 + C)} \cdot \frac{dM_a}{dt} + M \cdot C_2$$

(5)

where $\text{sgn}(x)$ denotes the sign of the $x$ argument, $C$ is the elastic deformation domain wall coefficient, $H_{CO}$ – the coercive field at $T_0$ temperature and $\alpha_{HC}$ – the temperature coefficient of the coercive field changes. The value of the C parameter is equal to the output voltage of the controlled voltage source $E_C$ described by

$$C = \frac{\mu - 1}{\frac{B_{R0}}{\mu_0 \cdot H_{CO}} \cdot \left[ 1 + \alpha_{HC} \cdot (T_R - T_0) \right] - \mu}$$

(6)

where $B_{R0}$ denotes the residual flux density at the reference temperature $T_0$, $\alpha_{RC}$ – the temperature coefficient of the residual flux density, $\mu$ is the core relative permeability described by

$$\mu = \mu_{ip} \cdot \left[ 1 + \alpha_{ip} \cdot (T_R - T_0) \right] \cdot x + 1.01$$

(7)

In the dependence (7) - $\mu_{ip}$ denotes the initial core permeability at temperature $T_0$ and $\alpha_{ip}$ is the temperature coefficient of the initial permeability changes. In turn, the parameter $A$ is equal to the

Figure 1. Circuit representation of a new model of ferromagnetic core.
the controlled voltage source $E_{A1}$ expressed by the formula

$$A = \left( \frac{H_X - C \cdot M_X}{(1+C) \cdot (\mu - 1)} \right) \cdot \frac{1 + \alpha_{\delta B} \cdot (T_B - T_0)}{M_{S0} \cdot (M_{S0} - M_X) - \frac{3 \cdot M_X}{M_{S0}}}$$

(8)

The values of the magnetic field $H_X$ and magnetization $M_X$ appearing in this formula are the coordinates of the point laying on the initial magnetization curve at the reference temperature and $M_{S0}$ corresponding to the saturation value of magnetization for the magnetic field equal to the value $H_S$ at $T_0$ temperature.

The $\alpha$ parameter is described by the formula

$$\alpha = 3 \cdot \frac{A}{M_{S0} \cdot [1 + \alpha_{\delta B} \cdot (T_B - T_0)] - (\mu - 1) \cdot (1 + C)}$$

(9)

Modelling the core losses $P_{loss}$ by the controlled voltage source $E_P$ is described by the formula from the paper [4] extended with the influence of temperature

$$P_{loss} = \left( \frac{B_{\text{pp}}}{2} \right)^{\beta - \alpha} \cdot \left( 1 + \alpha_{\delta B} \cdot (T_B - T_m) \right) \cdot \left( \frac{P_{V_0}}{T} \right) \int_0^T \left| dB \right|^\alpha \, dt$$

(10)

In equation (10), $\alpha_P$ denotes the square coefficient of temperature changes of power losses, $T_m$ – the temperature for minimum losses, $T$ is a cycle of induction of the magnetic field and $B_{\text{pp}}$ is the peak to peak value of magnetic induction designated in the circuit consisting of the controlled voltage sources $E_{11}$ and $E_{DB1}$, diode $D_{11}$, resistors $R_{11}$ and $R_{21}$ and two capacitors $C_{11}$ and $C_{21}$.

3. Results of calculations and measurements

In order to examine the accuracy of the elaborated model the magnetization characteristics of the selected ferromagnetic cores in the selected temperatures are calculated and compared with the measured characteristics. Additionally, the dependence of the power losses on temperature, amplitude of flux density and frequency were determined. As an example, figure 2 shows the results of calculations (lines) and measurements (points) of the magnetization curve for the ferrite core made of material N27 by EPCOS [9] performed at the frequency $f = 10$ kHz, whereas figure 3 presents calculated (lines) and catalogue (points) dependences of the core loss on temperature for the same material at the frequency $f = 25$ kHz. The values of the model parameters, obtained using the algorithm described in [25], are shown in table 1.

![Figure 2](image-url)
As it is visible, a good agreement between calculated and measured characteristics was obtained. The calculated and measured values of the saturation magnetic flux density, the coercive field and the residual flux density differ each from other no more than 5%. This confirms the correctness of the proposed model. In figure 2, it can be seen that an increase in temperature causes a decrease in the saturation flux density and the narrowing of the hysteresis loop. In turn, figure 3 shows that the core losses are an increasing function of the amplitude of the magnetic flux density. It is worth noticing that for the specified value of the amplitude of the magnetic flux density, the core loses achieve the minimum value at the temperature equal to about 80°C.

### Table 1. Values of model parameters for the core made of material N27.

| parameter | value |
|-----------|-------|
| \( B_{50} \) [T] | 0.485 |
| \( \alpha_{BS} \) [1/K] | -1.65 \times 10^{-3} |
| \( H_S \) [A/m] | 1200 |
| \( T_0 \) [K] | 300 |
| \( T_C \) [K] | 490 |
| \( H_{CB} \) [A/m] | 40 |
| \( \alpha_{HC} \) [1/K] | -3.33 \times 10^{-3} |
| \( P_{VO} \) [W/T^3/s^-4] | 1200 |

### Figure 3. Calculated (lines) and catalogue (points) dependences of a core loss on temperature for material N27.

In figure 4 the calculated and measured dependences of the lossiness of the ferrite core made from material F-867 [26] on the amplitude of the magnitude of the magnetic flux density at the selected values of the temperature and the frequency are shown. The parameters values of the model of this core are collected in table 2.

### Table 2. Values of model parameters for the core made of material F-867.

| parameter | value |
|-----------|-------|
| \( C_1 \) | 10^{-2} |
| \( C_2 \) | 1 |
| \( B_{250} \) [T] | 0.2 |
| \( \alpha_{BM} \) [1/K] | -3.33 \times 10^{-3} |
| \( M_S \) [A/m] | 2200 |
| \( \phi \) | 4.67 \times 10^{-3} |
| \( M_{X} \) [A/m] | 2.54 \times 10^{-5} |
| \( T_m \) [K] | 353 |

### Figure 4. Calculated (lines) and catalogue (points) dependences of a core loss on temperature for material F-867.
As it is visible in figure 4, the good agreement between the results of calculations and measurements is obtained. The lossiness $P_V$ is an increasing function of the amplitude of the magnetic flux density $B_m$ and the frequency $f$. The dependence $P_V(B_m)$ has a character of the power function with the constant exponent. Only within the range of $B_m$ values close to the values of the saturation magnetic flux density, an increase of the inclination of the considered characteristic is visible. It is worth noticing that in the considered range of changes of $B_m$ and $f$, the change in temperature value from 25°C to 100°C causes a decrease of the lossiness by even 70%.

| parameter | $B_{50}$ [T] | $\alpha_{BS}$ [1/K] | $H_S$ [A/m] | $T_0$ [K] | $T_C$ [K] | $H_{C0}$ [A/m] | $\alpha_{HC}$ [1/K] | $P_{V0}$ [W/T$^0$/s$^0$] |
|-----------|--------------|-----------------|-------------|----------|----------|----------------|-----------------|------------------|
| value     | 0.52         | -3.3x10$^{-4}$  | 1200        | 300      | 513      | 19.3          | -1.3x10$^{-4}$  | 2400             |
| parameter | $C_1$        | $C_2$           | $B_{50}$ [T] | $\alpha_{bm}$ [1/K] | $\mu_p$ | $\alpha_{b}$ [1/K] | $M_S$ [A/m] | $T_m$ [K] |
| value     | 10$^{-3}$    | 1               | 0.095       | -4x10$^{-5}$ | 1630    | 7x10$^{-6}$ | 3.38x10$^{-4}$ | 361              |
| parameter | $H_K$ [A/m]  | $M_{50}$ [A/m]  | $R_S$       | $C_K$    | $\beta$  | $\alpha$  | $\alpha_{p}$ [K$^{-1}$] |
| value     | 200          | 4.126x10$^{-5}$ | 10$^{-3}$   | 1        | 2.286    | 1.46     | 2.2x10$^{-4}$ |

4. Conclusions
In the paper the modified model of the ferromagnetic core is proposed. This model is based on the basic equation described by Jiles and Atherton. The modified model possesses a different analytical description of the initial magnetization curve where the submission of the exponential function instead of the classic Langevin’s function is used. In addition, the influence of temperature on the magnetizing characteristics of the core is taken into account. Moreover, this model contains the description of the core losses resulting from the core remagnetization. The presented model uses only 16 parameters, but most of them can be calculated from the core catalogue.

The results of calculations and measurements, presented in the previous sections, confirmed the correctness of the description of the magnetizing characteristics in the wide range of temperature changes and the dependence of the core losses on the amplitude of the flux density, frequency and temperature. The presented results of calculations and measurements refer to the ferrite cores. The authors performed analogical calculations for ferromagnetic cores made of different materials and they obtained the good agreement between calculated and measured characteristics. The calculated and measured characteristics of the investigated ferromagnetic cores have the same shape and the coordinates of the characteristic points laying on these characteristics not significantly differ from each other. The obtained results prove the correctness of the presented model.

5. References
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Acknowledgements
This project is financed from the funds of National Science Centre which were awarded on the basis of the decision number DEC-2011/01/B/ST7/06738.