Polariton localization and spectroscopic properties of disordered quantum emitters in spatially-extended microcavities

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Experiments have demonstrated that the strong light-matter coupling in polaritonic microcavities significantly enhance conductivity. The theoretical investigation is challenging, as one has to accurately describe both the continuum of light modes and energetically, spatially and orientational disorder of the quantum emitters. In this Letter, we exactly solve the continuum-mode disordered Tavis-Cummings model and predict the spectral properties and the structure of the eigenstates, i.e., the polaritons, and their localization length. Independent of the dimension, all polaritons are localized in space and decay exponentially with distance from the center. Yet, the localization length diverges rapidly as the energy moves away from the quantum emitter excitation energies, such that it by far exceeds the typical length of microcavities. This defines a ballistic and a localized transport regime. The localization length depends sensitively on the light-matter interaction strength and can increase by more than one order of magnitude when slightly increasing the interaction strength. Intriguingly, the localization length can exhibit a turnover as a function of disorder, which is in contrast to the monotonically decaying localization length for the celebrated Anderson localization. The analytical predictions are confirmed by numerically exact finite-size calculations and shed light on spectroscopic and transport measurements in cavities.

Introduction. The spatial confinement of the light field in microcavities gives rise to dispersive polaritons with outstanding spectroscopic properties $^{[1]}$ and establishes an alternative channel for charge and energy transport different from the short-range hopping. Recent experimental measurements of microcavities have found that conductivity can be enhanced by orders of magnitude $^{[2–6]}$. A thorough description is challenging because of the large number of light modes in the cavity and the energetic, spatial and orientational disorder.

Many theoretical models describe the light field by a single cavity mode, which is coupled to a macroscopic number of quantum emitters $^{[7–21]}$. Our recent investigation has predicted an intriguing turnover of the conductivity, relaxation and the linewidth as a function of disorder $^{[22]}$. However, in single-mode models an exciton can travel instantaneously between distant emitters and thus exceed the speed of light.

Since the photonic dispersion relation ensures the speed of light, the light fields should be described as a continuum of cavity modes. These type of models have been investigated since the early theoretical research on polaritons $^{[23,24]}$. References $^{[25–29]}$ have investigated the impact of disorder perturbatively and numerically, respectively. Yet, a fully microscopic and quantitative understanding of the light-matter dynamics for disordered quantum emitters is still lacking.

In this Letter, we solve the continuum-mode disordered Tavis-Cummings model non-perturbatively and analyze the polariton structure. The solution predicts that all polaritons are exponentially localized. Away from the center of the disorder distribution, the localization length rapidly diverges and exceeds by far the typical length of realistic microcavities. This defines two transport regimes in the energy spectrum: one regime of strongly localized polaritons, where transport is suppressed, and one regime of ballistic transport, where the polaritons exceed the cavity length. The continuous crossover from the localized to the delocalized regimes of the polaritons is in contrast to the mobility edge formed between the two distinct phases in the Anderson localization. Moreover, the localization length exhibits a turnover as a function of disorder, which has no analogue in the Anderson localization $^{[30,31]}$, but is reminiscent of noise-assisted transport $^{[32,33]}$.

Continuum-mode disordered Tavis-Cummings model. As shown in Fig. 1(a) and 1(b), we consider a microcavity which contains $N$ quantum emitters representing atoms, molecules, NV centers or particle-hole pairs in semiconductors. We adopt the system by a continuum-mode disordered Tavis-Cummings model, whose Hamiltonian is given as $\hat{H} = \hat{H}_M + \hat{H}_L + \hat{H}_{LM}$, where

$$
\hat{H}_M = \sum_{j=1}^{N} E_j \hat{B}_j^\dagger \hat{B}_j, \quad \hat{H}_L = \sum_{\kappa} \omega_{k} \hat{a}_{\kappa}^\dagger \hat{a}_{\kappa},
$$

where
The quantum emitters are described by the operators $\hat{a}_\kappa$ with $\kappa = (k, p)$, where $k$ denotes the spatial quantum numbers and $p$ labels the polarization. The light-matter interaction in Eq. (1) is given as $\hat{g}_{j, \kappa} = g_k \hat{d}_j \cdot \hat{E}_p \varphi_\kappa(r^{(j)})$ where $r^{(j)}$ is the position of the quantum emitter $j$, $\hat{E}_p$ is the light polarization, $\hat{d}_j$ is the dipole direction of emitter $j$, $g_k$ is the wavevector dependent light-matter interaction, and $\varphi_\kappa(r)$ is the photonic mode function.

In rectangular microcavities, the photonic modes $\varphi_k(r) \propto \prod_i \sin[(Q_k)_i r_i]$ are characterized by wavevectors $(Q_k)_i = \pi \cdot k_i / L_i$, where $L_i$ is the length of the cavity in direction $i = 1, 2, 3$, and integer $k_i > 0$. We partition the wavevector as $Q_k = q_k + \tilde{q}_k$, where $q_k$ and $\tilde{q}_k$ are the projections in the extended and confined dimensions of the cavity, respectively. The energies of the photonic modes are $\omega_\kappa = \sqrt{c^2 q_k^2 + E_C^2}$, where $c$ is the speed of light and $E_C = c^2 \tilde{q}_k^2$ is the confinement energy. In the numerical calculations, we consider a cavity that is extended in one dimension, and neglect the spatial and orientational disorder of the quantum emitters, such that $r_j = L/N \cdot j$, $\hat{d}_j \propto \hat{E}_p$, and $g_k = g$.

**Exact solution in the high-density limit.** The Heisenberg equations of $\hat{B}_j$ and $a_\kappa$ are transformed into the Laplace space defined by $f(z) = \int_0^\infty dt e^{zt} \hat{f}(t)$ for arbitrary operators $\hat{f}(t)$. We find that the coupling between different cavity modes $\kappa_1, \kappa_2$ scales as $\hat{a}_{\kappa_1}(z) \propto N^{-1/2} \hat{a}_{\kappa_2}(z)$ and thus vanishes in the high-density limit [32]. In other words, one can treat the system as a superposition of uncoupled single-mode systems, which has been investigated in detail in Refs. [22, 33, 34]. The solution of the Heisenberg operators in this limit becomes

$$
\hat{a}_\kappa(z) = \frac{\hat{a}_\kappa^{(0)}}{(z + i\omega_\kappa(z))} - i \sum_j \frac{g_{j, \kappa} \hat{B}_j^{(0)}}{(z + i\omega_\kappa(z)) (z + iE_j)},
$$

$$
\hat{B}_j(z) = \frac{\hat{B}_j^{(0)}}{(z + iE_j)} + i \sum_{\kappa} \frac{g_{j, \kappa} \hat{a}_\kappa^{(0)}}{(z + i\omega_\kappa(z)) (z + iE_j)},
$$

where $\hat{a}_\kappa^{(0)}$ and $\hat{B}_j^{(0)}$ denote the initial conditions of the time evolution. We have defined the renormalized photon energy

$$
\omega_\kappa(z) = \omega_\kappa + \sum_j \frac{|g_{j, \kappa}|^2}{(z + iE_j)} \omega_\kappa + \sum_j \frac{|g_{j, \kappa}|^2}{(z + iE_j)} \Pi(z),
$$

where the $z$ dependence reflects a retardation effect. We have expressed the disorder average in terms of the quantum emitter density $\rho = N/V$ and the self energy
II(\omega) = \chi_P \int dE \frac{P(E)\rho(E)}{|\omega - \epsilon|},
where \chi_P depends on the orientational disorder [22]. Using Eq. (2), we can construct arbitrary retarded Green’s functions such as \(G_{\kappa}^{(L)}(\omega) = i \langle \hat{a}_\kappa(z)\hat{a}_\kappa(0)^\dagger \rangle\) or \(G_{\kappa,k'}^{(M)}(\omega) = i \langle \hat{B}_\kappa(z)\hat{B}_{k'}(0)^\dagger \rangle\). The solution in Eq. (2) is equivalent to the single-mode system when the sum over \(\kappa\) is neglected. The simple superposition of all \(\kappa\) modes reflects the mode decoupling in the high-density limit \(\rho \to \infty\), for which the matter system becomes homogeneous such that \(\kappa\) is an integral of motion.

**Spectroscopy.** The wavevector-resolved light and matter local density of states (LDOSs) are defined by \(\nu_{X,\kappa}(\omega) = -\frac{1}{\pi} \text{Im} G_{\kappa,k}^{(X)}(-i\omega)\) with \(X = L\) and \(X = M\), respectively, where \(G_{\kappa,k}^{(X)}(\omega) = \sum_{j,j'} \varphi_k(\mathbf{r}_j)\varphi_k(\mathbf{r}_{j'}) \left\langle \hat{d}_j \hat{P}_E \right\rangle \left\langle \hat{d}_j \hat{P}_E \right\rangle G_{j,j'}^{(M)}(\omega)\). The light and matter LDOSs can be measured spectroscopically [22]. Using Eq. (2), we find that

\[
\nu_{L,\kappa}(\omega) = -\frac{1}{\pi} \text{Im} \left[ \frac{i}{(-i\omega + i\omega_{\kappa}(-i\omega))} \right],
\nu_{M,\kappa}(\omega) = -\frac{1}{\pi} \text{Im} \left[ i\Pi(-i\omega) - \frac{\nu_{M,\kappa}(\omega)}{(-i\omega + i\omega_{\kappa}(\omega))} \right],
\]

which relate the contribution of a light or matter state with quantum number \(\kappa\) to the total density of states \(\nu(\omega) = \sum_{\kappa} (\nu_{L,\kappa}(\omega) + \nu_{M,\kappa}(\omega))\).

In Fig. 1(c), we investigate the light and matter LDOSs for \(E_C = 0.4\text{eV}\). The LDOSs for \(E_C = 1.0\text{eV}\) and \(E_C = 1.3\text{eV}\) can be found in the Supplementary Information. The dashed lines depict the lower and upper polaritons for a vanishing disorder \(\sigma = 0\). Close to \(\omega = \omega_{\kappa},\) where both dispersions would cross for \(g = 0\), the lower and upper polaritons exhibit a Rabi splitting of \(\Omega \approx 2g\sqrt{\rho}\). The light and matter LDOSs closely follow the photonic dispersion curves of the disorder-free systems (dashed). The photon LDOS accumulates close to the photon dispersion \(\omega_{\kappa}\), but also around \(E_M\) close to the polarization anticrossing, where light and matter are strongly mixed. The matter LDOS accumulates around \(E_M\), where it resembles the original disorder distribution. Along \(\omega_{\kappa}\) and away \(E_M\), the matter LDOS is one order of magnitude smaller than the light LDOS. Because of level repulsion, the matter LDOS is suppressed for energies \(\omega_{\kappa}\) at the anticrossing (purple arrow), which resembles the electromagnetically induced transparency and related effects [22] [55] [57]. As each photon mode interacts with an disordered ensemble, the level repulsion is smeared out in the light LDOS.

The photon and matter weights of a specific eigenstate \(|j\rangle\) with energy \(\omega\) is given as \(W^{(X)}(\omega) = \langle j | \hat{P}^{(X)} | j \rangle = \sum_{\kappa} \nu_{X,\kappa}(\omega)/\nu(\omega),\) where \(\hat{P}^{(X)}\) is the light (matter) projection operator. The numerical calculation in Fig. 1(d,e,f) verifies the analytical solution for various \(E_C\): (i) For \(E_C = 0.4\text{eV} < E_M\), the photon weight vanishes around the resonance condition \(\omega \approx 1\text{eV}\), as the emitters by far outnumber the photon modes in this energy region. The photon weight increases monotonically with increasing distance from the resonance condition. (ii) For \(E_C = 1.0\text{eV} = E_M\), we observe that the photon weight does not monotonically increase with distance from \(E_M\). The peak around \(\omega \approx 0.9\text{eV}\) is a consequence of the polarization formation, causing the light field to be pushed down energetically. (iii) For \(E_C = 1.3\text{eV} > E_M\), light and matter are energetically separated such that the mutual influence is rather weak.

**Polariton localization.** In Fig. 2 we depict the
disorder-averaged wave functions as a function of position and wavevector. The light and matter contributions are plotted in red and blue, respectively. The numerical calculations are compared to analytical calculations of the Green’s function $|G_r(-i\omega)|^2$ with $r = |r_j - r_{ji}|$ and $|G(\mathbf{q},-i\omega)|^2$, which are given as

$$G_r(-i\omega) \propto \sum_k \frac{g_{j,k} g_{j,i,k}}{(-i\omega + i\omega_k(z))} \rightarrow \frac{dq}{2\pi} G_q(-i\omega)e^{i\mathbf{r} \cdot \mathbf{q}},$$

$$G_q(-i\omega) = \frac{g_q^2}{-i\omega + i\omega_q + g_q^2 \rho \Pi(-i\omega)}.$$  (5)

The light and matter contributions are normalized such that they match $W^{(L)}(\omega)$ and $W^{(M)}(\omega)$, respectively. We note that the eigenstate wave function and the Green’s function are not equivalent, yet, the agreement of analytical and numerical results observed in Fig. 2 is striking.

In Fig. 2(a), several wave functions in position space show an exponential decay with distance from the center, while other wave functions are seemingly translationally invariant. As explained immediately, all polaritons are exponentially localized; yet, the localization length diverges as the energy moves away from the center of the quantum emitter distribution. Figure 2(b) depicts the wavevector-resolved wave functions. Overall, we observe that the widths of the wave functions in position and wavevector space are related by the Heisenberg uncertainty principle. In contrast to the photon contribution, which converges to zero for large wavevectors $q$, the matter contribution converges to a finite value for energies close to $E_M$. This is reflected by spatial fluctuations of the matter wave function in position space in Fig. 2(a), that are absent for the light wave function.

From Eq. 3, we can determine the localization length $\zeta$, which is defined via the exponential decay $|\Psi(\Delta r)| \propto \exp(-\Delta r/\zeta)$ with distance $\Delta r$ from the center of the disorder-averaged wave function, using a theorem in complex analysis [33]. Specifically, the Green’s function decays as $G_{\Delta r} \propto e^{-\alpha \Delta r}$, where $\alpha$ is the largest value such that $G(q-i\alpha')$ is analytic for all $|\alpha'| < \alpha$. $G(q)$ in Eq. 2 has two types of non-analyticities, namely the roots of the denominator and branch cuts along the imaginary axis $\pm i\omega \in [\sqrt{E}/c, i\infty]$ due to the root in $\omega_k$. Consequently, the localization length $\zeta = 1/\alpha$ becomes $\zeta = \max(2/\sqrt{E}, \zeta_R)$ with

$$\zeta_R^{-1} = \frac{2}{c} \Im \sqrt{-(i\omega - g^2 \rho \Pi(-i\omega))^2 - E_C^2},$$  (6)

where $\zeta_R^{-1}$ is the imaginary part of the pole of $G(q)$, and a constant light-matter coupling $g_q = g$ is assumed. We emphasize that the result Eq. (6) holds also for higher dimensions as long as the dispersion $\omega_k$ and the light-matter coupling $g_q$ are isotropic. The localization length is identical to the absorption length and can be determined by measuring the transmission of the cavity parallel to the mirrors as a function of cavity length $L$ in Fig. 1(a).

In Fig. 1(g,h,i), we compare the analytical expression for $\zeta$ with the numerical evaluation, which confirms the validity of the analytical approach. For large energies $\omega$, we observe that the analytical prediction diverges, while the numerical result converges to a constant value. This discrepancy is a consequence of the finite cavity size used in the numerical calculation, while the analytical calculation formally assumes an infinite cavity. Minor deviations can be traced back to the numerical averaging procedure. For realistic parameters and energies $\omega \approx E_M$, the value $2\sqrt{E}/c$ has a minor influence on the localization properties of the polaritons, as for a large $E_C$, it is significantly smaller than $\zeta_R$, while for small $E_C$, the influence of the branch cut on the Fourier transformation in Eq. 5 is negligible and the wave function is still mainly determined by the pole of the Green’s function.

Analysis. In Fig. 1 we observe a correlation between the photon weight and the localization length. As the interaction between the quantum emitters is mediated via photons, the localization length is larger when photons can travel further without being scattered by quantum emitters. A low scattering probability is reflected by a large photon weight in the formation of eigenstates.

In Fig. 3 we analyze the localization length $\zeta$ as a function of $g/\sqrt{\rho}$ and $\sigma$ for $E_C = 0.4$ eV and $E_C = 1.0$ eV. In Fig. 3(a) for small $g/\sqrt{\rho}$, we observe a clear linear dependence with slope $-2$ for all energies $\omega$. This can be explained by photon scattering, which consist of absorption ($\propto g\rho$) and re-emission ($\propto g$). Moreover, the localization length is bounded by $\zeta \geq 2c/E_C$ for large $g/\sqrt{\rho}$. Interestingly, the localization length for $\omega = 1.2E_M$ exhibits a dip for large $g/\sqrt{\rho}$, as the matter LDOS is strongly deformed and accumulates around $\omega = 1.2E_M$ causing to an enhanced photon scattering. Similar effects could be also observed for other energies, which are however masked by the lower bound at $2c/E_C$. 

FIG. 3. Localization length as a function of light-matter interaction [(a), (c)] and disorder [(b), (d)]. Parameters are the same as in Fig. 1.
The observations in Fig. 3(c) for $E_C = 1.0 \text{ eV}$ and large $\omega = 1.0, 1.1, 1.2 \text{ eV}$ are qualitatively similar to panel (a). The localization length behaves very differently for small $\omega$, where the photonic modes are absent for $g = 0$ as $\omega_C > E_C$. As for these energies eigenstates can be only formed with virtual (i.e., non-resonant) photon modes, the localization length is very small for small $g\sqrt{\beta}$. Interestingly, the localization length increases over more than one order of magnitude for $g\sqrt{\beta} \approx 0.3 \text{ eV}$ and $\omega = 0.8 \text{ eV}$ because of the peak in the photon weight for small energies $\omega \approx 0.8 \text{ eV}$ in Fig. 3(c).

Analyzing the localization length as a function of disorder in Fig. 3(b), we observe a turnover as a function of $\sigma$. This is in contrast to the Anderson localization, where the localization length monotonically decreases with disorder. Recent work has revealed a turnover of the conductance as a function of disorder in a single-mode Tavis-Cummings model [9, 12, 22]. It has been shown that the turnover can be explained by the overlap of the light LDOS and the quantum emitter energy disorder distribution $P(E)$ [22]. This interpretation can also be employed here. For small $\sigma$, the disorder distribution is strongly centered around $E_M$. With increasing $\sigma$, the disorder distribution increases for $\omega \neq E_M$, such that more quantum emitters can resonantly scatter the photons with energy $\omega$, which reduces the localization length. For a large disorder, the quantum emitter energies spread over a large energy region, such that there are only few quantum emitters in resonance with the photon modes close to $\omega$, which enhances the localization length. As the Gauss distribution becomes very flat close to the center for large $\sigma$, the localization length becomes independent of $\omega$ for large $\sigma$. For $\omega = 1.0 \text{ eV}$, the localization does not have a turnover, as the disorder distribution $P(\omega \approx E_M)$ decreases monotonically for increasing $\sigma$. The turnovers can be also observed for $E_C = 1.0 \text{ eV}$ in Fig. 3(d) for large $\omega = 1.1 \text{ eV}, 1.2 \text{ eV}$, while overall the dependence on $\sigma$ is more complicated because of the significant influence of the square root dispersion relation of $\omega_C$ close to $q = 0$.

Conclusions. We have solved the disordered continuum-mode Tavis-Cummings model in the experimentally relevant high-density limit and used this solution to analyze its spectral and localization properties. We found that all eigenstates are exponentially localized in space. The localization length in Eq. (6) is the inverse of the imaginary part of the pole of the retarded Green’s function. Under special conditions (large coupling $g$, small $E_C$), the localization length is determined by the confinement energy $E_C$. For energies away from the center of the disorder distribution and above $E_C$, the localization length rapidly diverges such that it exceeds the length of typical microcavities, which defines a ballistic transport regime and a localized regime.

The localization length depends crucially on the light-matter coupling and the disorder magnitude. Noteworthy, the localization length can increase by more than one order of magnitude with a slight increase of the light-matter interaction [cf. Fig. 3(c)]. The localization length can exhibit a turnover as a function of disorder, which is in direct contrast to the monotonically decreasing localization length known from the Anderson localization, but is reminiscent to noise-assisted quantum tunneling [40, 41, 43, 44]. Arising from the overlap of the light LDOS and the energy disorder distribution, this turnover is induced by the same mechanism as the transport turnover previously investigated in the single-mode disordered Tavis-Cummings model [22].

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I. HEISENBERG EQUATIONS OF MOTION

Here, we solve of the Heisenberg equations of motion of the continuum-mode disordered Tavis-Cummings model

\[ H_M + \hat{H}_L + \hat{H}_{LM}, \tag{7} \]

where

\[ \hat{H}_M = \sum_{j=1}^{N} E_j \hat{B}_j^\dagger \hat{B}_j, \quad \hat{H}_L = \sum_{\kappa} \omega_{k} \hat{a}_{\kappa}^\dagger \hat{a}_{\kappa}, \]

\[ \hat{H}_{LM} = \sum_{j=1}^{N} \sum_{\kappa} g_{j,\kappa} \hat{B}_j^\dagger \hat{a}_{\kappa} + \text{H.c.} \tag{8} \]

We recall that \( j \) and \( \kappa \) label the quantum emitters and the photonic modes respectively. The label \( \kappa = (k, p) \) includes both the spatial mode index \( k \) and the polarization \( p \) of the electromagnetic field.

The analytical solution for the system operators \( \hat{B}_j \) and \( \hat{a}_{\kappa} \) can be obtained in Laplace space. The solution can be used to construct arbitrary retarded Green’s functions. The Heisenberg equations read as

\[ \partial_t \hat{B}_j = -i E_j \hat{B}_j - i \sum_{k} g_{j,\kappa} \hat{a}_{\kappa}, \quad \tag{9} \]

\[ \partial_t \hat{a}_{\kappa} = -i \omega_{\kappa} \hat{a}_{\kappa} - i \sum_{k} g_{j,\kappa} \hat{B}_j. \tag{10} \]

Transforming into the Laplace space defined by \( \tilde{f}(z) = \int_0^\infty dt e^{-z t} \tilde{f}(t) \) for arbitrary operators \( \tilde{f}(t) \), the equations of motions become

\[ z \hat{B}_j - \hat{B}_j(0) = -i E_j \hat{B}_j - i \sum_{k} g_{j,\kappa} \hat{a}_{\kappa}, \tag{11} \]

\[ z \hat{a}_{\kappa} - \hat{a}_{\kappa}(0) = -i \omega_{\kappa} \hat{a}_{\kappa} - i \sum_{j=1}^{N} g_{j,\kappa}^* \hat{B}_j. \tag{12} \]

In general, these set of coupled linear equations can not be solved analytically for a large number of photonic modes. However, we can find an exact solution in the thermodynamic limit \( N \to \infty \). To see this, we first resolve Eq. [11] and obtain

\[ \hat{B}_j = \frac{\hat{B}_j(0)}{(z + i E_j)} - i \sum_{\kappa} g_{j,\kappa} \hat{a}_{\kappa} \frac{(z + i \omega_{\kappa})}{(z + i E_j)}. \]

Inserting this into Eq. [12] and resolving for \( \hat{a}_{\kappa} \), we find

\[ \hat{a}_{\kappa} = \frac{\hat{a}_{\kappa}(0)}{(z + i \omega_{\kappa}(z))} - i \sum_{j=1}^{N} \frac{g_{j,\kappa}^* \hat{B}_j(0)}{(z + i \omega_{\kappa}(z))(z + i E_j)} \tag{13} \]

where we have defined

\[ \omega_{\kappa}(z) = \omega_{\kappa} - i \sum_{j=1}^{N} \frac{|g_{j,\kappa}|^2}{(z + i E_j)} = \omega_{\kappa} - \tilde{\Pi}(z). \tag{14} \]

The second term in Eq. (13) represents an all-to-all coupling of the photonic modes, which cannot be solved analytically. Fortunately, this term vanishes in the thermodynamic limit, as we explain in the following. To this end, we explicitly consider the factor

\[ A = \sum_{j=1}^{N} g_{j,\kappa} g_{j,\kappa_1} = \sum_{j=1}^{N} \frac{g_{k}^2 g_{j} \varphi_{k,\kappa}(r_j)\varphi_{k_1,\kappa_1}(r_j)}{(z + i E_j)} \left( \hat{d}_j \cdot \hat{E}_p \right) \left( \hat{d}_j \cdot \hat{E}_{p_1} \right) \]

\[ = \frac{g^2}{V} \sum_{j=1}^{N} (x_j + iy_j). \tag{15} \]

In the second equality, we have inserted the parameterization \( g_{j,\kappa} = g_k \hat{d}_j \cdot \hat{E}_p \varphi_{k,\kappa}(r_j) \), where \( r_j \) is the position of the quantum emitter \( j \), \( \hat{E}_p \) is the light polarization (i.e., \( \hat{E}_p = 1 \)), \( \hat{d}_j \) is the dipole direction of emitter \( j \) (i.e., \( \hat{d}_j = 1 \)), \( g_k \) is the wavevector-dependent light-matter interaction, and \( \varphi_{k,\kappa}(r_j) \) is the photonic mode function. In the third line, we have introduced \( g \) as a typical measure for \( g_k \). To explain the scaling of \( A \), we have excluded the cavity volume \( V \) normalizing the photonic mode functions \( \varphi_{k,\kappa}(r) \propto 1/\sqrt{V} \).

Because of the energetic, spatial and orientational disorder, the real and imaginary parts \( x_j \) and \( y_j \) are samples of random variables \( X_j \) and \( Y_j \), respectively. According to the Gaussian law of large numbers, the means and the variances of the accumulated random variables scale as

\[ \langle \sum_{j=1}^{N} X_j \rangle \propto \left( \sum_{j=1}^{N} Y_j \right) \propto \delta_{\kappa_1,\kappa}, \tag{16} \]

\[ \text{Var} \sum_{j=1}^{N} X_j \propto \text{Var} \sum_{j=1}^{N} Y_j \propto N. \tag{17} \]

The means vanish except for \( \kappa_1 = \kappa \), while the variances scale linearly with \( N \). Consequently, for \( \kappa_1 \neq \kappa \) we find

\[ A \propto \frac{g^2}{\sqrt{N}/V} = \frac{g^2}{\rho} \sqrt{N}. \]

Thus, when approaching the high-density limit \( \rho \to \infty \) while keeping the Rabi frequency \( \Omega = 2 \mu g^2 \) constant, the terms \( A \to 0 \) vanish as

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**Supplementary Material**

\[ - \sum_{\kappa_1 \neq \kappa} \sum_{j=1}^{N} \frac{g_{j,\kappa}^* g_{j,\kappa_1} \hat{a}_{\kappa_1}}{(z + i \omega_{\kappa}(z))(z + i E_j)}, \tag{13} \]
FIG. 4. Wavevector-resolved light and matter LDOSs for $E_C = 0.4$ eV, $E_C = 1.0$ eV and $E_C = 1.3$ eV. Overall parameters are $L = 125 \mu$m, $N = 5000$, $E_M = 1.0$ eV, $\sigma = 0.05$ eV, and $g = 0.02$ eV. The photonic cutoff energy is $\omega_c = 100$ eV, such that 10000 photonic modes are included in the simulations.

$N \to \infty$.

Next, we evaluate the statistical average of $\tilde{\Pi}(z)$ in Eq. (14) in the thermodynamic limit

$$\tilde{\Pi}(z) = -i \sum_{j=1}^{N} \frac{|g_k|^2}{(z + i E_j)} |\varphi_k(r_j)|^2 \left( \hat{d}_j \cdot \hat{E}_p \right)^2,$$

$$\to |g_k|^2 \frac{N}{V} \int d\theta d\phi \left( \hat{d}(\theta, \phi) \cdot \hat{E}_p \right)^2 P_{\theta, \phi} \int \frac{P(E) dE}{(z + i E)},$$

$$= |g_k|^2 \rho \cdot \Pi(z),$$

(18)

where

$$\Pi(z) = \chi_P \int dE \frac{P(E)}{(z + i E)}$$

(19)

is the self energy given in the Letter. As the quantum emitter energies $E_j$, their position $r_j$, and their orientation $\hat{d}_j$ are statistically independent, the integration factorizes. $P(E)$ denotes the energetic disorder distribution. The integral over the position is trivially 1 because of the normalization of the mode functions. The orientational average contributes the factor

$$\chi_P = \int_0^\pi d\theta \int_0^{2\pi} d\phi \left( \hat{d}(\theta, \phi) \cdot \hat{E}_p \right)^2 P_{\theta, \phi},$$

(20)

where $P_{\theta, \phi}$ is the orientational disorder distribution. If all quantum emitters are aligned $\hat{d}_j \propto \hat{E}_p$, then $\chi_P = 1$. For an isotropic distribution, a straightforward calculation reveals that $\chi_P = 1/2$.

FIG. 5. Wavevector-resolved light and matter LDOSs for $E_C = 0.4$ eV, $E_C = 1.0$ eV and $E_C = 1.3$ eV. Overall parameters are $L = 125 \mu$m, $N = 5000$, $E_M = 1.0$ eV, $\sigma = 0.05$ eV, and $g = 0.04$ eV. The photonic cutoff energy is $\omega_c = 100$ eV, such that 10000 photonic modes are included in the simulations.

FIG. 6. Photon weight (a-c), matter weight (d-f), and light-matter mixing (g-i). Parameters are the same as in Fig. 5.
II. LIGHT AND MATTER LOCAL DENSITY OF STATES

In Fig. 4, we analyze the light and matter LDOSs for the confinement energies $E_C = 1.0 \text{ eV}$ and $E_C = 1.3 \text{ eV}$. In both cases, the light and matter LDOSs look qualitatively similar to the LDOSs for $E_C = 0.4 \text{ eV}$ depicted in Fig. 1(c) in the Letter. Yet, we find that the light LDOSs for $E_C = 1.3 \text{ eV}$ has a significant smaller contribution close to the photon dispersion $\omega_\kappa$ than the ones for $E_C = 0.4 \text{ eV}$ and $E_C = 1.0 \text{ eV}$. In contrast, the matter LDOS for $E_C = 1.3 \text{ eV}$ has a significant smaller contribution close to the photon dispersion $\omega_\kappa$ than the ones for $E_C = 0.4 \text{ eV}$ and $E_C = 1.0 \text{ eV}$.

III. NUMERICAL CALCULATION OF THE LOCALIZATION LENGTH

As explained in the Letter, we define the localization length $\zeta$ via the exponential decay of the disorder-averaged wave function $|\Psi(\Delta r)|^2 \propto e^{-|\Delta r|/\zeta}$ as function of distance from its center $\Delta r = r - r_c$, where $r_c = \langle \Psi | \hat{X} | \Psi \rangle$ is the mean of position operator. The exponential decay of the wave functions in position space can be seen in Fig. 2 in the Letter. For an efficient evaluation, we use the position variance of the wave function, which reduces to $\text{Var}|\Psi\rangle \hat{X} = 2 \zeta^2$. For an exponential decay: Introducing the decay rate $\lambda = 1/\zeta$, the correctly normalized exponentially decaying wave function reads as

$$|\Psi(x)|^2 = \frac{\lambda}{2} e^{-\lambda |x|}. \quad (21)$$

Consequently, the variance of the position operator becomes

$$\text{Var}|\Psi\rangle \hat{X} = 2 \int_{-\infty}^{\infty} \frac{\lambda}{2} x^2 e^{-\lambda x} dx$$

$$= \frac{\lambda^2}{\lambda^2} \int_{0}^{\infty} e^{-\lambda x} dx$$

$$= \frac{\lambda}{\lambda^2} \frac{1}{2}$$

$$= 2\lambda \frac{1}{\lambda^2} dx = 2 \frac{1}{\lambda^2} = 2\zeta^2. \quad (22)$$

In the last step, we have inserted the localization length $\zeta = 1/\lambda$. The calculation of the localization length based on the variance is numerically more stable than a fitting procedure.

IV. BENCHMARK CALCULATION

In Figs. 6 and 7, we carry out more benchmark calculations to verify the analytical calculation of the photon weight and the localization length for more extreme parameters.

In Fig. 6, we depict the photon ($X = L$) and matter weights ($X = M$) $W^{(X)}(\omega) \equiv \langle j | \hat{P}^{(X)} | j \rangle = \sum_{\kappa} \nu_{X,\kappa}(\omega) / \nu(\omega)$, where $\hat{P}^{(X)}$ is the light (matter) projection operator. Moreover, we depict the light-matter mixing, which we define by $W^{(L)}(\omega) \cdot W^{(M)}(\omega)$. This quantity resembles the two polarization peaks known from single-mode models.

In Figs. 7(a) and 7(d), we verify the validity for a small confinement energy $E_C = 0.2 \text{ eV}$. In Figs. 7(b) and 7(e) we investigate a very large light-matter coupling $g\sqrt{\rho} = 0.5 \text{ eV}$. In Figs. 7(c) and 7(f), we consider a very large disorder $\sigma = 0.25 \text{ eV}$. All benchmark calculations convincingly verify the analytical approach based on the retarded Green’s function.