Contact binary stars of the W UMa type as distance tracers

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Abstract

Contact binaries can be used for distance determinations of stellar systems. They are easy to discover and identify and are very abundant among solar-type stars, particularly for $M_V > +3$. The period–luminosity–colour (PLC) relations have similar properties to those for pulsating stars and can currently predict individual values of $M_V$ to about $\pm 0.25$ mag.

Key words: binaries: general, binaries: eclipsing, stars: statistics, delta Scuti

1. Introduction

Contact binary stars of the W UMa-type (also known as EW) are unique objects (Rucinski, 1993; Webbink, 2003). The luminosity, produced almost exclusively in the more massive component is efficiently distributed (by poorly understood processes) through the common envelope so that the surface brightness is practically identical over the whole visible surface of the binary (the gravity variations over the surface introduce a very minor modification). Thus, the effective temperature is everywhere – over the whole common surface – the same, in spite of usually very different masses hiding inside the common envelope. Mass ratios are known to span the whole wide range, from almost unity (V753 Mon, $q = 0.97$, Rucinski et al. 2000), to very small values, as small as $q = 0.066$ (SX Crv, Rucinski et al. 2001). The more common – but more difficult to detect – small mass-ratio systems are the best illustration to why the contact binaries are so unique. Simplifying: the primary component provides the luminosity, while both components provide the radiating area. Thus, it is practical to consider the primary mass, $M_1$, and the mass ratio, $q = M_2/M_1 \leq 1$, as the two main parameters. The third parameter, the orbital period, $P$, is related to the amount of the angular momentum in the system. The range of the primary masses is moderate and corresponds to Main Sequence spectral types from early A to early K and roughly maps into the orbital-period range of about 1.5 days to 0.22 days. Since the period distribution appears to continue the $1/P$ shape (as for other binaries) to about 0.45 days, and shows a mild bending down between 0.45 and 0.25 days, the most common are short period systems with periods within 0.25 – 0.5 days, down to the sharp and currently unexplained cutoff at 0.22 days (Rucinski 1992; Stepien et al. 2001).

The contact binaries are known to exist only within the Main Sequence. Somewhat similar light
curves of some giant or other more evolved binaries can be explained by semi-detached binaries or spotted stars. Very rare early-type contact systems on the upper Main Sequence, including a few currently known O-type contact systems, do not appear to be very different from the more typical solar-type W UMa binaries, but will not be considered here.

With their special properties, the contact binaries of the W UMa type form a distinct group of objects which are very easy to detect and identify due to: (1) rather large amplitudes of light variations reaching one magnitude, (2) short orbital periods, so that limited-duration monitoring is sufficient. Massive, systematic searches of the sky for stellar variability, such as the micro-lensing projects, have led to many detections of EW systems; e.g. in the first catalogue of OGLE-I, as much as two thirds of the one thousand detected eclipsing binaries were EW systems [Rucinski 1997]. Many EW systems have been detected during the galactic cluster searches, both in open and in globular clusters (see Rucinski 1998, 2000 for the references to the original works).

2. The $P\sqrt{\rho}$ relation for contact binaries

Contact binaries are not as convenient as detached binaries for distance determinations. Although described by fewer parameters than detached binaries (one potential in place of two independent radii), they have more complex geometry. As pointed by Dr. R. E. Wilson during this conference, a single detached, eclipsing, double-line spectroscopic binary can provide a distance determination without any calibration, basically using the “first-principles” approach. In this respect, the contact binaries are more like pulsating stars and – with still many uncertainties about their structure and adherence to the strict Roche model – do require empirical luminosity calibrations.

The rationale for the existence of a period – luminosity relation is based in the strong geometric constraints imposed by the common equipotential envelope which permit to consider an equivalent of the $Q = P\sqrt{\rho}$ relation for contact binary systems. To obtain this, one combines the total volume of the contact configuration, described by the Roche common equipotential, with the Kepler’s law: $a^3/P^2 = G(M_1 + M_2)$. The volume of the binary system, $V$, can be written in non-dimensional way as: $v = V/a^3$. Such a dimensionless value, $v = v(q, F)$, does depend slightly on the mass ratio $q$ and on the degree of contact, $0 \leq F \leq 1$ ($F = 0$ for the inner common equipotential surface with one point of contact at the Lagrangian point $L_1$; $F = 1$ for the outer surface opening to the space at $L_2$). Tables of $v(q, F)$ have been calculated by Mochnacki (1984, 1985). It is observed that most of contact binaries have $0 \leq F \leq 0.2$; however, this is a model-dependent result [Rucinski 1993].

By defining the mean density of the whole configuration, $\rho = (M_1 + M_2)/V$, then eliminating $a^3$, and by expressing the quantities in familiar units, one obtains the “pulsation equation”, $P\sqrt{\rho} = Q(q, F) = \sqrt{0.079/v(q, F)}$, with $P$ in days and $\rho$ in g/cm$^3$ (Figure 2). The similarity of contact binaries to pulsating stars is not accidental as the underlying physical time scale, of the pulsation or of the orbital revolution, is the same familiar dynamical time scale.

3. The basis of the luminosity calibration

One attempts to derives a luminosity calibration which would not depend on any spectroscopic...
data. Instead of the size of the orbit, $a$, we want to use the orbital period, $P$. Similarly as for the non-dimensional volume in the previous section, one can define the non-dimensional surface of the contact binary, $s$, so that the total area, $S$ can be written as $S = sa^2$. Again, $s = s(q, F)$, but this dependence is weak (the tables are in Mochnacki 1984, 1985). The luminosity of the system can be written quite generally, as: $L \propto ST^4 \propto sa^2 T^4$.

We remove the unknown $a$ using the Kepler’s law and, in its place, introduce the orbital period, $P$, which is determinable with an accuracy by many orders of magnitude higher than any other quantity. One can rewrite all these as one equation for the absolute magnitude $M_V$:

$$M_V = -10 \log T_{\text{eff}} + B.C.(T_{\text{eff}}) - 10/3 \log P - 5/3 \log M_1 - 5/3 \log (1 + q) - 2.5 \log s(q, F) + \text{const}$$

Note that the two first terms contain the $T_{\text{eff}}$ dependence, then there is the “infinitely well-known” $P$–term and then there are terms dependent on $M_1$, $q$ and $F$. As always, when photometric data are available, $T_{\text{eff}}$ can be estimated from the colour index or from the spectral type, but the last terms are not known and require spectroscopic data. We “sweep them under the rug” at this point to utilize the simplest possible calibration equation:

$$M_V = C_1 \ast \text{colour} + C_2 \log P + C_3$$

In this period–luminosity–colour relation (PLC), colour is any of the available colour indices such as $B-V$ or $V-I$; for each choice of the index, another set of the coefficients $C_i$ must be determined. Lumping together all $M_1$, $q$ and $F$ dependencies is not as artificial as it may seem as several hidden correlations exists between these parameters; some of these correlations are in fact of substantial importance for the theory of the structure of these binaries (Rucinski 1993).

4. Existing PLC calibrations

After a very early attempt (Rucinski 1974), the two main works dealing with the subject were Rucinski (1994) and Rucinski & Duerbeck (1997) (=RD97), see also Rucinski (1990). Several small improvements were discussed later in the context of the massive variable star searches, e.g. Rucinski (1998, 2001). While Rucinski (1994, 1998) were based mostly on the open cluster data (EW systems appear at a relative frequency of some $1/500 - 1/1000$ in old open clusters), RD97 was based on the Hipparcos data. The Hipparcos database remains the best source and has recently been analyzed in great detail for the complete sample of systems with $V < 7.5$ mag (Rucinski 2002). The currently best calibrations utilizing de-reddened $B - V$ and $V - I$ colour indices are:

$$M_V = -4.44 \log P + 3.02 (B - V) + 0.12$$
$$M_V = -4.43 \log P + 3.63 (V - I) - 0.31$$

The colour–magnitude diagram (CMD) for the RD97 sample of contact binaries with Hipparcos parallaxes is shown in Figure 2. The thin continuous and broken contours delineate the density of stars in the general Hipparcos database; one can see the Main Sequence, the Red Giant branch and the Red Clump. Each binary system appears as one point corresponding to the combined brightness of both components at light maximum; this is in concordance with the fact that there is just one (common) radiating area which radiates the energy produced (mostly) in one, more massive star. The expected dependence on the mass ratio is shown in the left corner of the figure. For $q = 1$ two identical components provide equal amounts of luminosity so that the shift is upwards by 0.75 mag; for $q < 1$ the large radiating area, together with the diminished luminosity, result in lowering of the effective temperature. For the most typical cases with $q$ around 0.3 to 0.5, there is almost no change of the luminosity relative to the single star case, but the reddening can reach up to 0.17 in $B-V$.

As we can see in the figure, the combined shift to the right and possibly evolution within the Main Sequence have resulted in the EW systems following the upper border of the MS for single stars.

The CMD in Fig. 2 contains also the lines of equal orbital period, shown as slanted dotted lines. One can directly see that correlation between the colour and the period exists in that bluer, brighter
systems have longer orbital periods. The “period–
colour” relation was discovered by Eggen (1967).
On one hand, it complicates the matters by intro-
ducing an internal correlation between the quanti-
ties in the \( M_V \) calibration so that they are not re-
ally “orthogonal”. On the other hand, it sometimes
helps in weeding out variable stars which are of
entirely different type in that the data points can-
not appear above and to the left of a short-period
blue envelope; they can only shift down to redder
colour indices and to the right due to the evolution
\( \text{(Rucinski, 2002)} \).

5. Uncertainties of the calibrations

There are several sources of uncertainties in the
calibrations. The trigonometric parallaxes from
the Hipparcos mission introduce errors typically
< 0.25 mag, but for a few systems in RD97 as
large as 0.5 mag. While such large errors can be
accounted for by an appropriate weighting, they
lower the quality of the calibration. In addition,
some of the binaries are unrecognized triple sys-
tems with an associated offset in brightness, loss in
accuracy and even entirely false data for the paral-
laxes. As for all calibrations involving de-reddened
colour indices, the \( M_V \) estimates will depend on
how well this procedure is actually done. The

simplification of the calibration to the colour and
log \( P \) dependence, as described in Section 3, also
worsens the fit, but – by far – the main source of
efforts is in the photospheric spots on individual
binaries. These binaries are very active and almost
always show spots. A brightness calibration must
simply assume some sort of an average for \( M_V \).

When a simple mean weighted error is consid-
ered, then the deviations in RD97 are character-
ized by \( \sigma M_V \simeq 0.35 \) mag. This is however a pes-
simistic estimate because the Monte–Carlo simu-
lations indicate that within the main range of the
applicability, the typical errors are \( \sigma M_V \simeq 0.25 \)
mag (RD97).

6. Metallicity dependence

As with any calibration for pulsating stars, one
must establish the metallicity dependence of the
PLC relation. At first, when limited material was
available, it seemed that such a dependence did
exist \( \text{(Rucinski, 1995)} \), but – when much more ex-
tensive data for many globular clusters were added – a metallicity term \( \propto [Fe/H] \) became no longer
needed \( \text{(Rucinski, 2000)} \).
The deviations from the $M_V$ calibrations, given in Section 4, are shown as the function of [Fe/H] in Fig. 3. They are plotted for the whole wide range of accessible metallicities ranging within $-2.2 < [Fe/H] < +0.3$. The negative values of [Fe/H] are provided by globular clusters of various ages, while the only positive metallicity is for the rich, old, open cluster NGC 6791 (Worthley & Jowett, 2003). There is no discernible trend in the figure so that the solar-neighbourhood calibrations work well for contact systems of different metallicities, at least at the level of uncertainty shown in the figure ($\sigma = 0.28$ and 0.27 for calibrations based on the $B-V$ (filled circles) and $V-I$ (open circles), respectively). These estimates of the uncertainties may be exaggerated, because many additional factors tend to increase the scatter; these could be errors in the assumed reddening and absorption corrections, the large measurement errors for faint stars in distant, crowded clusters and the uncertainties of the assumed distance moduli, $m - M$.

In spite of what has been said above about lack of an explicit dependence of $M_V$ on [Fe/H], the contact binaries with other metallicity content are different than the disk population ones; it is only that at the current level of accuracy there exists no need to introduce the metallicity corrections in $M_V$. Figure 4 shows the CMD for systems in globular clusters with available $B-V$ or $V-I$ (Rucinski, 2000). The $B-V$ panel shows also, as crosses, the same Hipparcos (RD97) systems as in the figure in Section 4. It is clear that the data points for globular-cluster binaries are shifted left and below the disk-population Main Sequence: the Population II contact binaries are bluer and smaller i.e. have shorter orbital periods. This latter property is clearly seen in the period distribution for globular cluster systems in Rucinski (2000). Thus, the shift in the CMD is mostly horizontal in $M_V$. This is a preliminary result which must be checked again as the data become more accurate.

7. Misclassification and spatial density

There are two final questions to answer: How many EW systems can one expect relative to other
stars? Is there any danger of misclassification and taking other variable stars for contact binaries or vice versa?

The only information at present relates to our Galaxy. The EW binaries are very common in the Disk Population. The estimate based on the OGLE-I pencil–beam search volume gave the density as high as 1/130 among late-A to early-K MS stars, a number consistent with some old open clusters (Rucinski, 1998). However, this high estimate is not confirmed by a rigorous analysis of the Hipparcos, solar-neighbourhood sample limited to stars brighter than 7.5 magnitude (Rucinski, Hipparcos, solar-neighbourhood sample limited to stars brighter than 7.5 magnitude [Rucinski, 2002]; this analysis suggests the relative density of about 1/500 (this corresponds to the spatial density of $1/500 \, \text{pc}^{-3}$). It is at present not clear, if the high density obtained from the OGLE-I sample was due to the improperly accounted image blending or to a genuine increase of the contact-binary density in the central parts of the Thick Disk of our Galaxy.

The local volume around the Sun to 20 pc contains about 300 Main Sequence stars of relevant spectral types (Wielen et al., 1983). In this volume, we know only one EW system (44 Boo, also known as i Boo). This simple estimate agrees with the lower number of the relative density of occurrence of the EW systems. It is possible that the main reason for the discrepancy in the density estimates is the mean population age. The fractional density definitely increases over time. It may be $\leq 1/1000$ at 1 Gyr, about 1/500 at 5 Gyr and may reach 1/100 for ages above that. Note that the fractional density among blue stragglers in globular clusters appears to be as high as 1/45 (Rucinski, 2000).

The variable stars which can most likely be taken for contact binaries are short-period pulsating stars in the $\delta$ Sct instability strip. Dr. G. Handler (private communication) estimates that about 1/3 of stars within the $\delta$ Sct instability strip are actually pulsating. This is confirmed by the statistics for the solar neighbourhood: Within the appropriate range of the luminosity and colour, there are 32 stars within $< 20 \, \text{pc}$ and 8 are known to pulsate so that the factor 1/3 is confirmed within the Poisson fluctuation limit.

The horizontal branch, Population II, short-period pulsators, the RR Lyr variables, are very infrequent in the solar neighbourhood (the spatial density $6.2 \times 10^{-9} \, \text{pc}^{-3}$, hence 0.0002 within 20 pc around the Sun, Suntzeff et al., 1991). Also infrequent are the MS Population II Main Sequence pulsators, the SX Phe stars; their density can be roughly estimated as about 1/100 of the Population I $\delta$ Sct stars. All these variables occur within narrow ranges of luminosities and spectral types, whereas contact binaries can consist of stars of any spectral types throughout the whole Main Sequence down to about K2 – K4 (it is not clear why they do not exist below this limit).

Taking the estimates given above together, one finds that there are about 8× more $\delta$ Sct stars in the solar vicinity than EW binaries. However, there should be no great difficulty in distinguishing them apart as their light-variation amplitude and period distributions are quite different. $\delta$ Sct tend to have very small amplitudes, < 0.1 mag, although large-amplitude variables do occur, see Figure 5 based on Rodriguez & Breger, 2001. But the most distinguishing is the period distribution: The $\delta$ Sct stars have very short periods, < 0.15 days (broken line). The contact binaries have longer periods starting just above 0.2 days. Even if one assigns a wrong period to a EW system equal to $P/2$ (dotted), this period in most cases will turn out to be still too long for a typical $\delta$ Sct star.

8. Conclusions

W UMa-type (EW) contact binaries are very easy to find in massive stellar variability programs, even in short-duration ones, thanks to the short orbital periods and large amplitudes of photometric variations. The $M_V$ calibrations utilizing de-reddened colour indices $B - V$ or $V - I$ and the periods can predict individual values to about ±0.25 mag, so that about a thousand systems are needed to reduce the group uncertainty to the level of ±0.01. Such large numbers will be discovered in the nearby galaxies once the surveys pass the threshold of $M_V \simeq 3 - 5$ which, for the Local Group typically corresponds to $V > 23 - 25$. The current calibrations involve de-reddened colour indices and thus remain sensitive to the reddening corrections.
Fig. 5. The left panel shows the amplitude distribution for contact binaries (EW, continuous line) and for δ Sct stars. The right panel shows the period distributions for these two types of variables. The dotted histogram shows the expected distribution for contact binaries under assumption that all have been assigned incorrect periods equal to one half of the true orbital periods.

The widespread availability of the K-band data suggests development of calibrations based on the $V - K$ or $I - K$ indices; this is a matter of the current work. The existing $M_V$ calibrations do not seem to require a correction for metallicity, but existence of a small $[Fe/H]$ term is not excluded.

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