PHENOMENOLOGICAL DETERMINATION OF THE BEAUTY MESON DECAY PARAMETER $f_B$ AND THE CP-VIOLATING ANGLE $\delta$

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Abstract

We fit the $\mathcal{CKM}$-matrix to all recent data with the following free parameters: three mixing angles, the CP-violating angle $\delta$ in the Maiani parametrisation, the top quark mass $m_t$, and the product $f_B\mathcal{B}^{1/2}_{B_0}$, where $f_B$ is the $B$-meson decay parameter and $\mathcal{B}_{B_0}$ is the bag parameter. Our fits span a contiguous region in the $(f_B\mathcal{B}^{1/2}_{B_0}, \cos \delta)$-plane, limited by $0.117 \lesssim f_B\mathcal{B}^{1/2}_{B_0} / \text{GeV} \lesssim 0.231$ and $-0.95 \lesssim \cos \delta \lesssim 0.70$. The parameters $f_B\mathcal{B}^{1/2}_{B_0}$ and $\cos \delta$ are strongly positively correlated.
When $B^0 - \bar{B}^0$ mixing was first discovered [1] this offered a way to estimate the mass of the top quark, since the box amplitude responsible for mixing is dominated by the top exchange. The $B^0 - \bar{B}^0$ mixing actually determines only the product $f_B B^{1/2} m_t^2 F(m_t^2)$, where $f_B$ is the unknown pseudoscalar decay constant of the $B^0$, $B_{ps}$ is the unknown bag parameter, and $F(m_t^2)$ is a known smooth function of the top mass (given below). To estimate $m_t$ from this product some theoretical input [2] of the QCD quantity $f_B B^{1/2}$ was needed. The mixing data combined with other charged current input then predicted a top mass value $m_t \gtrsim 100$ GeV [3],[4],[5].

The situation has now changed when the first experimental determination of $m_t$ is available. This allows one to reverse the problem, using the charged current data and $m_t$ to obtain a phenomenological and less model-dependent estimate of $f_B B^{1/2}$. This information may then be confronted with the predictions of theoretical models and lattice calculations, which differ quite substantially from each other, ranging from about 115 MeV [2] to 300 MeV [3].

In this paper we update our previous analysis of the CKM matrix [3] by taking into account the new CDF result for $m_t$ [7], and by using the most recent data for other relevant observables: the CKM matrix elements from various charged current processes, the CP-violation parameter $|\epsilon|$ from the neutral kaon system, and the $B^0 - \bar{B}^0$ mixing parameters $\chi_{d,s}$. The input data are collected in Table 1. For $m_t$ we have used a value obtained by combining the CDF result $m_t = 174 \pm 10^{+13}_{-12}$ GeV [7] with the indirect value from the fits of the LEP data, the deep inelastic neutrino-nucleon scattering data and the W mass measurement, $m_t = 164^{+16}_{-17} +^{13}_{-21}$ GeV [8].

The formalism used to relate the CKM matrix elements to various experimental input is well known (c.f. e.g. [3, 9]). What is new in our approach compared with our previous analysis [3] is a quite trivial change in the CP-violating parameter: we use the cosine of the angle $\delta$ (in the Maiani parametrisation) rather than the sine. This choice has the advantage of exhibiting explicitly that the allowed region in the $(f_B B^{1/2}, \cos \delta)$-space is contiguous, in contrast to previous analyses which found separate solutions in the first and the second quadrant of $\delta$.

The quantities measured in $B^0 - \bar{B}^0$ mixing experiments are the probability fractions

$$\chi_{d,s} = \frac{P(B_{d,s} \to \overline{B}_{d,s})}{P(B_{d,s} \to B_{d,s}) + P(B_{d,s} \to \overline{B}_{d,s})},$$

(1)
which can be expressed in the form $\chi_{d,s} = x^2_{d,s}(1 + x^2_{d,s})^{-1/2}$, where

$$x_q = \frac{G^2_F m_W^2}{6\pi^2} \tau_{B_q} m_{B_q} (f_{B_q} B_{B_q}) \eta_B \frac{m_i^2}{m_W^2} F \left( \frac{m_i^2}{m_W^2} \right) |V_{tq}^* V_{tb}|^2 (q = d, s). \quad (2)$$

Here the function $F$ is defined by

$$F(x) = \frac{1}{4} + 9 \frac{1}{4(1-x)} - 3 \frac{1}{2(1-x)^2} - \frac{3}{2} \frac{x^2 \ln x}{(1-x)^3}. \quad (3)$$

The hard QCD correction factor $\eta_{B_q}$ depends quite strongly on the top mass. For $m_t = 174$ GeV one finds from $\eta_B \simeq 0.49$. The experimental averages for the $B_d^0$ and $B_s^0$ lifetimes are $\tau_{B_d^0} = 1.48 \pm 0.10 \text{ ps}$, $\tau_{B_s^0} = 1.26_{-0.17}^{+0.22} \text{ ps}. \quad (4)$

For the masses of the neutral beauty mesons we will use

$$m_{B_d} = 5.2790 \pm 0.0020 \text{ GeV},$$

$$m_{B_s} = 5.3732 \pm 0.0042 \text{ GeV}. \quad (5)$$

The only unknown parameters, apart from the CKM matrix elements, in the decay constant $f_{B_q}$ and the bag parameter $B_{B_q}$. Combining the most recent results of ARGUS and CLEO gives for $\chi_d$ the value

$$\chi_d = 0.152 \pm 0.030. \quad (6)$$

In the experiments where both $B_d B_d$ and $B_s B_s$ are produced one measures the sum $\chi = f_d \chi_d + f_s \chi_s$ where $f_d$ and $f_s$ are the abundances of $B_d$ and $B_s$ in the $b$-quark jet. A recent average of all existing results is given by Danilov $\chi = 0.121 \pm 0.010. \quad (7)$

In applying this quantity we use $f_d = 0.375$ and $f_s = 0.15$.

Furthermore, we shall assume a fixed SU(3) breaking ratio from lattice calculations $f_{B_d}^2 B_{B_d}^0 / f_{B_s}^2 B_{B_s}^0 = 1.19, \quad (8)$

to which the fit is quite insensitive.
The theoretical expression for the CP-violation parameter $\epsilon$ depends on the poorly known bag factor $B_K$, whereas $\epsilon$ itself is well measured ($|\epsilon| = (2.26 \pm 0.02) \times 10^{-3}$ [13]). Thus the proper procedure is to use a constraint for $B_K$ expressed in terms of a constant $\epsilon$ without error. For $B_K$ we use the value

$$B_K = 0.73 \pm 0.13$$ (9)

which covers the values from different lattice and $1/N$ expansion evaluations quoted by ref. [10]. The theoretical expression for $|\epsilon|$ is obtained essentially as the imaginary part of the box amplitude for the neutral kaon mixing [18].

Experimental results [19, 20] on the parameter $|\epsilon'|$, describing CP violation in $K^0 \to \pi\pi$ decays, are controversial, and the experimental accuracy of this quantity is poor. Moreover, some terms in the theoretical expression are still imprecisely known. We shall therefore not include $|\epsilon'|$ in our analysis.

There are 7 free parameters and 14 constraints in the fit: the three mixing angles in the CKM matrix, the CP-phase $\delta$, the parameter $f_B B_{B^0}^{1/2}$, and the top quark mass $m_t$ (the seventh parameter is a quark ratio $\kappa$ entering two constraints on $|V_{cs}|$). The constraints can be fitted excellently with $\chi^2 = 8.6$ for 7 degrees of freedom. There is obviously no reason to increase the “theoretical errors” further on various input parameters, as some people would advocate, because that would just make the fit too good.

The conventional definition of errors on the parameters is always to increment the best fit $\chi^2$ by 1. However, since we are mainly interested in the simultaneous 68.3 \% confidence region for the two parameters $f_B B_{B^0}^{1/2}$ and $\cos \delta$, one should increment the best fit $\chi^2$ by 2.3. This then gives the contour in Fig. 1. The errors on the mixing angles are of less interest, these parameters being rather unphysical, and the fit error on $m_t$ is essentially equal to the experimental error.

Our best fit yields the parameters and conventional 1$\sigma$ errors

$$\sin \theta_{12} = 0.2203 \pm 0.0008,$$
$$\sin \theta_{23} = 0.048 \pm 0.003,$$
$$\sin \theta_{13} = 0.0046 \pm 0.0008,$$
$$m_t = 174.5 \pm 12.5 \text{ GeV},$$
$$f_B B_{B^0}^{1/2} = 149^{+23}_{-21} \text{ MeV},$$
$$\cos \delta = -0.46^{+0.55}_{-0.35}. $$ (10)
The elements of the CKM matrix obtain the following values:

\[
V = \begin{pmatrix}
0.9754 & 0.2202 & -0.0021 + 0.0041 i \\
-0.2199 + 0.0002 i & 0.9743 - 0.0000 i & 0.0480 \\
0.0126 + 0.0040 i & -0.0464 + 0.0009 i & 0.9988
\end{pmatrix}
\] (11)

Fig. 1 shows that \(f_{B^0}B_{B^0}^{1/2}\) and \(\cos \delta\) are very strongly positively correlated, and still poorly determined. The 1σ contour in the \((f_{B^0}B_{B^0}^{1/2}, \cos \delta)\)-plane, is limited to \(0.117 < f_{B^0}B_{B^0}^{1/2}/\text{GeV} < 0.231\) and \(-0.95 < \cos \delta < 0.70\).

Recent lattice calculations of \(f_{B^0}\) yield the values \(180 \pm 50\) MeV \[21\], \(187 \pm 37\) MeV \[22\] and \(200 \pm 40\) MeV \[23\]. This is in good agreement with the contour in Fig. 1. Some earlier estimates, e.g. \[1\], with higher values of \(f_B\) are instead excluded. Perhaps one could expect the next improvement in precision to come from the lattice calculations; this would then permit to determine \(\cos \delta\) well.

On the other hand, if the situation regarding \(|\epsilon'|/|\epsilon|\) were clarified theoretically and well measured experimentally, that would help to pin down \(\cos \delta\), and in consequence \(f_{B^0}B_{B^0}^{1/2}\) could be determined more precisely.

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TABLE CAPTION

FIGURE CAPTION

Figure 1. The 68.3 % confidence level contour for $f_B B_{B^0}^{1/2}$ and $\cos \delta$ and the best fit point.

Table 1. The experimental data used in the analysis and their best fit values.
| Quantity                  | Value     | Best fit | Reference |
|--------------------------|-----------|----------|-----------|
| $|V_{ud}|$                  | 0.9753±0.0002 | 0.9754   | [24]      |
| $|V_{us}|$                  | 0.2188±0.0016 | 0.2202   | [25]      |
| $|V_{cd}|$                  | 0.202±0.010$^1$ | 0.2199   | [20, 27, 28]|
| $|V_{cs}|$                  | 1.07±0.14$^1$ | 0.9743   | [29, 30]  |
| $|V_{cb}|$                  | 0.041±0.006 | 0.0482   | [31]      |
| $|V_{cd}/V_{cs}|^2$       | 0.057±0.016$^2$ | 0.0509   | [32]      |
| $|V_{ub}|/|V_{cb}|$        | 0.080±0.025$^{1,3}$ | 0.0961   | [31, 33]  |
| $\kappa|V_{cs}|^2$          | 0.43±0.06$^1$ | 0.4471   | [20, 27, 28]|
| $\kappa|V_{cs}|^2/|V_{cd}|^2$ | 9.6±1.2$^1$ | 9.247    | [20, 27, 28]|
| $|V_{ts}|/|V_{cb}|$        | 1.09±0.36   | 0.9659   | [9]       |
| $\chi$                   | 0.121±0.010 | 0.1291   | [12]      |
| $\chi_d$                 | 0.152±0.030 | 0.1273   | [14]      |
| $B_K$                     | 0.73±0.13$^3$ | 0.7552   | [16]      |
| $m_t$                     | 171±13 GeV$^1$ | 174.6 GeV| [8, 17]   |

1) Our average.
2) Asymmetric error; only the fit-side error is used.
3) Includes error due to spread of different theoretical models.

**TABLE 1.**
This figure "fig1-1.png" is available in "png" format from:

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