Heavy baryon production with an instanton interaction

Sang-In Shim\(^1\), Atsushi Hosaka\(^1\), and Hyun-Chul Kim\(^2,3,\ast\)

\(^1\)Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka, 567-0047, Japan
\(^2\)Department of Physics, Inha University, Incheon 22212, Republic of Korea
\(^3\)School of Physics, Korea Institute for Advanced Study (KIAS), Seoul 02455, Republic of Korea

\ast E-mail: hchkim@inha.ac.kr

Received December 11, 2019; Revised March 17, 2020; Accepted March 17, 2020; Published May 13, 2020

---

We propose a new reaction mechanism for the study of strange and charmed baryon production. In this mechanism we consider the correlation of two quarks in baryons, so it can be called the two-quark process. As in the previously studied one-quark process, we find large production rates for charmed baryons in comparison with strange baryons. Moreover, the new mechanism causes the excitation of both the \(\rho\) mode and the \(\lambda\) mode. Using the wave functions for baryons from a quark model, we compute the production rates of various baryon states. We find that the production rates reflect the structure of the wave functions that imply the usefulness of the reactions for the study of baryon structures.

---

Subject Index D32

1. Introduction

A large part of the recent activities in hadron spectroscopy has been devoted to the study of hadrons containing heavy quarks (see Ref. [1] and references therein). This is largely motivated by a series of observations of new heavy hadrons [2–15] that were not expected in the conventional naive quark model [16,17]. In order to understand the production mechanism of these newly found heavy hadrons, including the exotic ones, we need to consider more sophisticated quark–gluon dynamics inside a heavy hadron.

However, one clear virtue of the heavy–light quark systems is the presence of the heavy quarks. Since the heavy quark has a very large mass, the kinetic energies of the heavy quarks inside a heavy hadron are suppressed by the inverse of the heavy quark mass, which makes the quark dynamics inside a heavy baryon simpler than inside a light baryon. For example, in a conventional heavy baryon, two light quarks govern dynamics inside it and can be viewed as a diquark. On the other hand, the heavy quark can be regarded as an almost static color source and makes the structure of the heavy baryon decompose easily into two excitation modes, namely the so-called \(\lambda\) and \(\rho\) modes. As shown in Fig. 1, the former mode describes the motion of the light diquark with respect to the heavy quark, and the latter explains the relative motion between the two light quarks.

The essential features of these modes were discussed a long time ago [18], but the experimental data at the time were not sufficient to examine the idea quantitatively. As modern accelerators and detectors have been developed to perform experiments with unprecedented precision, it is interesting to describe the production of heavy hadrons based on these two modes. Moreover, since the E50 experiment at J-PARC will soon measure the charmed baryon production in the reaction \(\pi^{-} + p \rightarrow D^{*+} + Y_{c}\) and will yield important information on the structure of various charmed baryons \(Y_{c}\) [19],

---

© The Author(s) 2020. Published by Oxford University Press on behalf of the Physical Society of Japan. This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted reuse, distribution, and reproduction in any medium, provided the original work is properly cited. Funded by SCOAP3
it is of great importance to study theoretically the heavy hadron reactions with these two different modes considered. Motivated by these discussions, we have started the study of the above production reactions [20–22].

In the present work we propose a new microscopic mechanism of hadronic production reactions and investigate how this new mechanism allows one to understand the baryon structures for strangeness and charm production.

Though the mass of the strange quark is much smaller than that of the charm quark, one can consider it effectively as a heavy object in some cases (but not always) [18,23]. In Ref. [18] the mass inversion of $\Lambda(1830)$ and $\Sigma(1775)$ was used to indicate that the strange quarks are heavier than the $u$ and $d$ quarks. In Ref. [23], in some cases it was shown that the mixing of the $\lambda-\rho$ modes is rather small even at the strange quark mass, which indicates that the strange quark is often effectively considered to be heavy. In a slightly different context it is also useful to know the cases of the Skyrme models where the bound state approaches describe the properties of the SU(3) hyperons and heavy baryons successfully, the strange quark being regarded as a heavy object [24–27] (see also a review in Ref. [28]). In this respect, we can still apply the method of the two modes to both the single strange hyperons and singly heavy baryons. It is also useful to consider the strangeness sector, because strange hadrons can be produced at J-PARC together charmed hadrons.

In this work we develop a two-quark microscopic process of baryon production: two constituent quarks in a baryon are internally involved in a production reaction of mesons and baryons by pion beams, in addition to the one-quark process that was already studied in previous work [20–22]. This new mechanism has the virtue that one can look into the reaction mechanisms in a microscopic way. Note that one-quark and two-quark processes are similar to one-step and two-step processes, which are often considered in calculations of nuclear reactions. For example, when a deuteron or a helium target is scattered off by mesons or photons and then is broken into new baryons, one has to take into account both the one-step and two-step processes [29]. Similarly, when charmed hadrons are produced, large momentum transfer is inevitable, which indicates that both the one-quark and two-quark processes will contribute to the production of charmed hadrons. In particular, the two-quark process makes it possible to excite both $\lambda$ and $\rho$ modes, while it is possible to excite only $\lambda$ modes in the one-quark process.

To formulate and compute the reaction matrix elements we employ a nonrelativistic quark model (from now on we refer to it simply as the quark model) for baryon wave functions and a simple
interaction which involves three quarks, one anti-quark in the projectile pion and two constituent quarks in the target proton. The baryon wave functions are constructed in the heavy quark basis, where the total baryon spin is formed by those of light degrees of freedom (brown muck) and the heavy quark [30]. In this way we can clearly see relations between baryon structures and production rates. This is indeed the main purpose of the present study. In contrast, to the best of our knowledge, an interaction that suitable for charm or strangeness production is not known. Therefore, we shall tentatively employ a three-quark interaction that is inspired by 't Hooft for three flavors [31,32]. This is an effective interaction induced by instanton dynamics [33–35], and has been applied to the study of meson properties, for instance [36–40], baryon spectrum [41–46], and heavy hadrons [47–49]. The instanton-induced interactions were also used phenomenologically in the description of proton–anti-proton annihilation [50]. In the present study we employ that interaction for $u,d,s$ and for $u,d,c$ quarks. Though its applicability to production reactions in all details is not clear, we argue that the most important formula that we will derive in Eq. (25) shares common features with the two-quark process.

This paper is organized as follows. In Sect. 2 we briefly introduce the general formalism of how one can introduce the 't Hooft-like interaction to describe microscopically strange and charmed baryon production. Then we derive a general formula for the two-quark process for the productions. In Sect. 3 we perform numerical calculations and show the results for forward-angle scattering. We will then discuss essential features of the production mechanism of strange and charmed baryons. More general discussions related to observables such as the angular dependence of the cross sections will appear elsewhere. The final section is devoted to summary and conclusions.

2. Formalism

Let us consider the reaction $\pi^- p \rightarrow MY_{s,c}$ as shown in Fig. 2, where $M$ denotes a $K^0$ or $D^-$ meson with an anti-strange quark or an anti-charm quark and $Y_{s,c}$ represents a heavy baryon with a strange or charm quark. Various kinematic variables are defined in Fig. 2. $\vec{p}_\pi$, $\vec{p}_M$, $\vec{P}_N$, and $\vec{P}_Y$ stand respectively for the momenta of the $\pi^-$, the proton ($p$), the meson, and the baryon.

In Fig. 3 we draw the quark-line representations for one-quark and two-quark processes in the left and right panels, respectively. In the one-quark process, an anti-quark in the pion annihilates with one quark in the proton, and an $s\bar{s}$ or $c\bar{c}$ pair is created, while in the two-quark process, an anti-quark in the pion interacts with two quarks in the proton. From these pictures, we see that the one-quark process excites only $\lambda$ modes, while the two-quark process excites both $\lambda$ and $\rho$ modes.

Figure 3 also shows the momentum fractions carried by various quarks: the momenta of the initial- and final-state baryons consist of the momenta of the three quarks inside the baryons, $\vec{P}_N = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$, $\vec{P}_Y = \vec{p}'_1 + \vec{p}'_2 + \vec{p}'_3$, where $\vec{p}_i$ and $\vec{p}'_i = \vec{p}_i + \vec{q}_i$ ($i = 1, 2, 3$) are the quark momenta inside the baryons and $\vec{q}_i$ is the transferred momentum from the initial pion to the $i$th quark in the heavy baryon. In the two-quark process the momentum transfer $\vec{q}$ is shared by two quarks ($2, 3$), so
Fig. 3. One-quark and two-quark processes for heavy baryon production. Quark-line representations for one-quark (left) and two-quark (right) processes. The thin lines between the initial and final particles represent light quarks, and the thick lines correspond to the heavy quarks. \( \vec{p}_Y \) and \( \vec{Y}_c \) denote the momenta of the initial proton and the final-state heavy baryons. The momentum \( \vec{q} \) stands for the transferred momentum from the initial pion to the heavy baryon. The momenta \( \vec{p}_i \) and \( \vec{p}'_i \) \((i = 1, 2, 3)\) designate the quark momenta inside the initial- and final-state baryons, respectively.

that \( \vec{q} = \vec{P}_Y - \vec{P}_p = \vec{q}_2 + \vec{q}_3 \) becomes the transferred momentum from the pion to the heavy baryon. Since the one-quark process has been studied previously [20–22], we will focus on the two-quark process in the following sections.

2.1. Three-quark interaction

In this subsection we discuss briefly several features of the 't Hooft-like interaction, which will be useful for discussions of various production rates. An advantage of this interaction is that the reaction occurs at one place (single step), which makes the computation of matrix elements easy. The 't Hooft-like interaction is for three quarks with three flavors, \( N_f = 3 \), which arises from the instanton dynamics of quantum chromodynamics [31–35]. In general, it is a nonlocal interaction in which the dynamical quark mass is momentum dependent. Moreover, the \( 2N_f \) quark–quark interaction considers only the light flavors, i.e. the up, down, and strange quarks. When one includes heavy quarks together with the light quarks, one has to derive the heavy–light quark interactions from the instanton vacuum again. Though there are some theoretical works on heavy–light quark interactions from the instanton vacuum [47–49], its applicability is not sufficiently matured. Thus, in the present work we will consider a simplified version of the 't Hooft-like interaction including strange or charm quarks. Actually, it is also possible to transform this simplified one into the form of a heavy–light quark interaction similar to that of Refs. [47–49]. We will also take a local form of the 't Hooft-like interaction.

We start from the 't Hooft-like six-quark interaction defined by [31,32]

\[
\mathcal{L}_{tH} = c \det[\bar{q}_i(1 + \gamma_5)q_j] + H.c. = c \begin{vmatrix}
\bar{u}(1 + \gamma_5)u & \bar{u}(1 + \gamma_5)d & \bar{u}(1 + \gamma_5)s \\
\bar{d}(1 + \gamma_5)u & \bar{d}(1 + \gamma_5)d & \bar{d}(1 + \gamma_5)s \\
\bar{s}(1 + \gamma_5)u & \bar{s}(1 + \gamma_5)d & \bar{s}(1 + \gamma_5)s
\end{vmatrix} + H.c.,
\]

where \( c \) is an interaction strength. In general, it is difficult to predict the absolute magnitudes of the reaction cross sections. Therefore, we treat the strength \( c \) as a free parameter. On the other hand, we can discuss at least the ratios of various cross sections rather than their absolute values, so we will focus on the ratios in the present paper.

It is convenient to rewrite Eq. (1) by using the Fierz transformation to rearrange six quarks by observing the following: the \( \bar{u} \) field annihilates the \( \bar{u} \) state in the incoming \( \pi^- \), the \( s \) field creates
the $s$ state in the produced $K$ meson, the $d$ fields annihilate the corresponding quarks in the proton, and the $\bar{d}$ and $s$ fields create the corresponding ones in the strange baryon. Thus, the 't Hooft-like interaction can be reexpressed as

\[
L_{\text{tH}} = 4 \det[\bar{q}_L q_R] + \text{H.c.}
\]

\[
= 4 \left\{ \left( 1 + \frac{1}{N_c} \right) \left( \bar{u}_L s_R \right) \left[ \left( \bar{d}_L u_R \right) (\bar{s}_L d_R) - \left( \bar{d}_L d_R \right) (\bar{s}_L u_R) \right] + \frac{1}{8N_c} \left( \bar{u}_L \sigma^{\mu \nu} s_R \right) \left[ \left( \bar{d}_L \sigma_{\mu \nu} u_R \right) (\bar{s}_L d_R) + \left( \bar{d}_L u_R \right) (\bar{s}_L \sigma_{\mu \nu} d_R) \right] - \left( \bar{d}_L \sigma_{\mu \nu} d_R \right) (\bar{s}_L u_R) - \left( \bar{d}_L d_R \right) (\bar{s}_L \sigma_{\mu \nu} u_R) \right] + \left( \bar{d}_L \lambda^i_2 s_R \right) \left[ \left( \bar{d}_L \lambda^i_2 u_R \right) (\bar{s}_L d_R) + \left( \bar{d}_L u_R \right) (\bar{s}_L \lambda^i_2 d_R) - \left( \bar{d}_L \lambda^i_2 d_R \right) (\bar{s}_L u_R) \right] \right\} + \text{H.c.,} \tag{2}
\]

where $\bar{q}_L$ and $q_R$ denote the left- and right-handed quark fields, $q_R = (1 + \gamma_5)q_L/2$ and $\bar{q}_L = \bar{q}_L(1 + \gamma_5)/2$, and $\lambda^i$ are the SU(3) Gell-Mann matrices defined in color space. Since the mesons and baryons in the initial and final states should be color singlets, the terms with $\lambda^i$ in Eq. (2) do not contribute to the present reaction. Considering suitable leading-order terms in the $1/N_c$ expansion, we need only the following terms:

\[
L_{\text{tH}} \rightarrow 4 \left( \bar{u}_L u_R \right) (\bar{s}_L d_R) - \left( \bar{d}_L d_R \right) (\bar{s}_L u_R) + \text{H.c.}
\]

\[
\equiv (\bar{u}s) \left[ (\bar{d}u)(\bar{s}d) + (\bar{d}\gamma_5 u)(\bar{s}\gamma_5 d) - (\bar{d}d)(\bar{s}u) - (\bar{d}\gamma_5 d)(\bar{s}\gamma_5 u) \right] + (\bar{u}\gamma_5 s) \left[ (\bar{d}u)(\bar{s}\gamma_5 d) + (\bar{d}\gamma_5 u)(\bar{s}d) - (\bar{d}d)(\bar{s}\gamma_5 u) - (\bar{d}\gamma_5 d)(\bar{s}u) \right]. \tag{3}
\]

In this expression, only the terms in the second line are relevant, because the meson matrix elements of $(\bar{u}s)$ in the first line vanish in the production reaction of a pseudoscalar meson due to parity conservation. For baryon matrix elements in the nonrelativistic quark model, we need expressions in terms of two component spinors. We have explicitly computed the $(\bar{u}s)$ term of Eq. (3) and found that the relevant operators are reduced to the identity operators. This can be verified by neglecting the Fermi motion of the quarks confined in baryons and for forward-angle scattering, which is the dominant component of the reactions that we study in this paper. Therefore, the operator that we need is written as

\[
L_{\text{tH}} \rightarrow (\bar{u}s) \left[ (d^\dagger u)(s^\dagger d) - (d^\dagger d)(s^\dagger u) \right] \equiv O^M \cdot O^B, \tag{4}
\]

where $O^M \sim (\bar{u}s)$ acts on the meson transition, $\pi \to K$, whereas $O^B$ acts on the baryon transition, $p \to Y$, and $u, d, d^\dagger, s^\dagger$ are two-component spinors for the quarks in a baryon.
2.2. Baryon wave functions

As mentioned previously, we employ the baryon wave functions taken from the quark model. In the limit of infinitely heavy quark mass ($m_Q \to \infty$), the spin of the heavy quark $s_Q$ is conserved, which leads to the conservation of the light quark spin $j$. This is known as heavy quark spin symmetry. Thus, we construct the baryon wave functions that are the simultaneous eigenstates of $j$ and $s_Q$ to describe the baryon with one heavy (strange or charm) quark (for more explanation, refer to Refs. [23,30]). In the quark model, a baryon wave function is given as a product of the orbital, spin, flavor, and color parts as follows:

$$|\Psi\rangle = |\text{orbit}\rangle \otimes |\text{spin}\rangle \otimes |\text{flavor}\rangle \otimes |\text{color}\rangle. \quad (5)$$

Since the color part is always antisymmetric, the rest of the baryon wavefunction should be taken to be totally symmetric. Note that the interaction Lagrangian in Eq. (4) is given as a color singlet and a scalar in spin space.

Introducing a quark potential of harmonic oscillator type for confinement, we can decompose the orbital wavefunction into those of the center of mass (CM) $\vec{X}$ and of the internal coordinates $\vec{\rho}$, $\vec{\lambda}$, as

$$|\Psi_N(\vec{x}_1, \vec{x}_2, \vec{x}_3)\rangle = e^{i\vec{p}_N \cdot \vec{X}} \psi^0_\rho(\vec{\rho}) \psi^\lambda_\lambda(\vec{\lambda}),$$
$$|\Psi_Y(\vec{x}_1, \vec{x}_2, \vec{x}_3)\rangle = e^{i\vec{p}_Y \cdot \vec{X}} \psi^\rho_\rho(\vec{\rho}) \psi^{l\lambda}_{n\lambda}(\vec{\lambda}), \quad (6)$$

where $\vec{X}$, $\vec{\rho}$, and $\vec{\lambda}$ are related to $\vec{x}_1$, $\vec{x}_2$, and $\vec{x}_3$, respectively, as

$$\vec{X} = \frac{1}{2m_q + m_Q}(m_q(\vec{x}_1 + \vec{x}_3) + m_Q\vec{x}_3),$$
$$\vec{\rho} = \vec{x}_2 - \vec{x}_1,$$
$$\vec{\lambda} = \frac{1}{2}(\vec{x}_1 + \vec{x}_2) - \vec{x}_3. \quad (7)$$

Here, the light quarks are labeled by 1 and 2, and the heavy quark by 3. Assuming isospin symmetry, we can express the quark masses as $m_1 = m_2 = m_q < m_3 = m_Q$. The internal wavefunctions $\psi^\rho_\rho(\vec{\rho})$ and $\psi^{l\lambda}_{n\lambda}(\vec{\lambda})$ are typically written as

$$\psi_{nlm}(\vec{r}) = R_{nl}(r) Y_{lm}(\hat{r}), \quad (8)$$

where $Y_{lm}(\hat{r})$ denote the spherical harmonics and $R_{nl}(r)$ the radial wavefunctions, which are given explicitly in Appendix A. The wavefunction $\psi_0(\vec{r})$ represents the ground state with $n = l = m = 0$, and $\psi_{nlm}(\vec{r})$ the ground state and excited states with quantum numbers $n$, $l$, $m$. From now on, $\psi_{nlm}(\vec{r})$ will be written compactly by $\psi_l(\vec{r})$, because we will consider only the excitations of $l$ in the present work. In a more realistic model containing the linear confining potential with the spin–spin interaction, the $\lambda$ and $\rho$ modes are mixed with each other. However, in Ref. [23] it was shown that some baryon states are dominated by either the $\lambda$ or the $\rho$ mode. Moreover, once we know the properties of the $\lambda$ and $\rho$ modes separately, the realistic cases of their mixing can be estimated. Because of these reasons in the present study we consider various matrix elements for the $\lambda$ and $\rho$ modes separately.
The flavor (isospin) parts of the heavy baryons will be expressed by $D_I^I Q$. For $I = 0$, 

$$D_0^0 Q = \frac{1}{\sqrt{2}} (|ud| - |du|) Q,$$  

and for $I = 1$ and $I_z = 0$, 

$$D_1^1 Q = \frac{1}{\sqrt{2}} (|ud| + |du|) Q,$$  

where $Q$ stands for a heavy quark.

Similarly, the spin part of the diquark can be expressed by $d_{s_z}^s$, 

$$d_0^0 = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle),$$  

$$d_1^1_{s_z} = \begin{cases} 
|\uparrow\uparrow\rangle, & s_z = 1, \\
\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), & s_z = 0, \\
|\downarrow\downarrow\rangle, & s_z = -1, 
\end{cases}$$  

where $s$ designates the spin angular momentum of the diquark and $s_z$ corresponds to its $z$th component. The spin part of a heavy quark is denoted by $\chi_Q$.

By using these expressions, the baryon wavefunctions of $\Lambda_Q$ and $\Sigma_Q$ with total spin $J$ can be written as 

$$|\Lambda_Q(J, J_z)\rangle = [\psi_0^{\rho} (\vec{\rho}) \psi_0^{\lambda} (\vec{\lambda}), d_1^1_{J_z} D_1^1 Q],$$  

$$|\Sigma_Q(J, J_z)\rangle = [\psi_0^{\rho} (\vec{\rho}) \psi_0^{\lambda} (\vec{\lambda}), d_0^0, \chi_0^J Q],$$  

where $l_1, l_2, l_3$ represents angular momentum coupling of $l_1 + l_2 = l_3$ with the Clebsch–Gordan coefficients included properly, and the color and CM parts of the wavefunctions not included.

The SU(6) proton wavefunction with $J_z = 1/2$ is given as 

$$|p(1/2, 1/2)\rangle = \psi_0^{\rho} (\vec{\rho}) \psi_0^{\lambda} (\vec{\lambda}) \frac{1}{\sqrt{2}} (\chi_{1/2}^{\rho,\lambda} + \chi_{1/2}^{\rho,\lambda}),$$  

where the spin and isospin wavefunctions, $\chi_{1/2}^{\rho,\lambda}$ and $\phi^{\rho,\lambda}$, are given respectively by 

$$\chi_{1/2}^{\rho} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle),$$  

$$\chi_{1/2}^{\lambda} = -\frac{1}{\sqrt{6}}(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\uparrow\rangle),$$  

and 

$$\phi^{\rho} = \frac{1}{\sqrt{2}} (|ud\rangle - |du\rangle),$$  

$$\phi^{\lambda} = -\frac{1}{\sqrt{6}} (|ud\rangle + |du\rangle - 2|uu\rangle).$$
2.3. Transition amplitudes

The transition amplitude for the reaction $\pi^- p \rightarrow MY$ is written as a factorized form,

$$
\int d^4x \langle YM | \mathcal{L}_{M1} | N \pi^- \rangle \sim \langle M|O^M|\pi^- \rangle \langle Y|O^B|N \rangle 2\pi \delta(E_Y + E_M - E_N - E_\pi),
$$

(20)

where the baryon part is the only relevant one in the following discussion. In the two-quark process, the operator $O^B$ is a two-body operator and is written as

$$
\sum_{ij} O^B(i,j),
$$

(21)

where $i,j = 1, 2, 3$ denote the quark numbers. Fixing the number of the heavy quark as 3, we have only two terms:

$$
\sum_{ij} O^B(i,j) \rightarrow O^B(1,3) + O^B(2,3).
$$

(22)

The operator has flavor dependence as in Eq. (4), while the spin part becomes trivial because it is a scalar. Therefore, the baryon matrix element is given by

$$
\langle Y|O^B|N \rangle
\begin{align*}
&= \int d^3x_1 d^3x_2 d^3x_3 \Psi_Y^+(\vec{x}_1,\vec{x}_2,\vec{x}_3) \langle \text{spin}|\otimes|Y \text{ flavor}| \left[ O^B(1,3) + O^B(2,3) \right] \\
&\times \langle \text{spin}|\otimes|N \text{ flavor}| \Psi_N (\vec{x}_1,\vec{x}_2,\vec{x}_3) \\
&= \frac{C_Y}{2} \int d^3x_1 d^3x_2 d^3x_3 \Psi_Y^+(\vec{x}_1,\vec{x}_2,\vec{x}_3) \left[ e^{i\vec{q} \cdot \vec{x}_1} \delta^{(3)}(\vec{x}_1 - \vec{x}_3) + (1 \leftrightarrow 2) \right] \Psi_N (\vec{x}_1,\vec{x}_2,\vec{x}_3).
\end{align*}
$$

(23)

Note that we have carried out the calculation in the coordinate space of three quarks, $x_1, x_2, x_3$. The two-quark operator $O(i,j)$ acts on the $i$th and $j$th quarks. In the second equality, the delta function indicates that the interaction occurs at a single point. The spin–isospin factor $C_Y$ arises from the Clebsch–Gordan coefficients in the computations of spin and flavor matrix elements. The factor $1/2$ was introduced for convenience.

Using the identity

$$
e^{i\vec{q} \cdot \vec{x}_1} \delta^{(3)}(\vec{x}_1 - \vec{x}_3) = \int d^3q_1 d^3q_3 e^{i\vec{q}_1 \cdot \vec{x}_1} e^{i\vec{q}_3 \cdot \vec{x}_3} \delta^{(3)}(\vec{q} - \vec{q}_1 - \vec{q}_3),
$$

(24)

one can rewrite the transition amplitude as

$$
\langle Y|O^B|N \rangle
\begin{align*}
&= \frac{C_Y}{2} \int d^3q_1 d^3q_3 \delta^{(3)}(\vec{q} - \vec{q}_1 - \vec{q}_3) \\
&\quad \times \int d^3x_1 d^3x_2 d^3x_3 \Psi_Y^+(\vec{x}_1,\vec{x}_2,\vec{x}_3) e^{i\vec{q}_1 \cdot \vec{x}_1} e^{i\vec{q}_3 \cdot \vec{x}_3} \Psi_N (\vec{x}_1,\vec{x}_2,\vec{x}_3) \\
&\quad + \frac{C_Y}{2} \int d^3q_2 d^3q_3 \delta^{(3)}(\vec{q} - \vec{q}_2 - \vec{q}_3) \\
&\quad \times \int d^3x_1 d^3x_2 d^3x_3 \Psi_Y^+(\vec{x}_1,\vec{x}_2,\vec{x}_3) e^{i\vec{q}_2 \cdot \vec{x}_2} e^{i\vec{q}_3 \cdot \vec{x}_3} \Psi_N (\vec{x}_1,\vec{x}_2,\vec{x}_3)
\end{align*}
$$

8/18
\[
\delta^{(3)}(\vec{p}_Y - \vec{P}_N - \vec{q}) \frac{C_Y}{2} \int d^3q_1 d^3q_3 \delta^{(3)}(\vec{q} - \vec{q}_1 - \vec{q}_3) \int d^3\rho e^{i\vec{q}_1 \cdot \vec{\rho}} \psi_{\rho}^* (\vec{\rho}) \psi_{\rho} (\vec{\rho}) \\
\times \int d^3\lambda e^{i\vec{q}_3 \cdot \vec{\lambda}} \psi_{\lambda}^{* \prime} (\vec{\lambda}) \psi_{\lambda} (\vec{\lambda}) \\
+ (1 \leftrightarrow 2, \vec{\rho} \rightarrow -\vec{\rho}),
\]  
(25)

where \(\vec{q}_\rho = \frac{1}{2} \vec{q}_1\) and \(\vec{q}_\lambda = \vec{q}_1 + \vec{q}_{\text{eff}}\), with the effective momentum transfer defined as

\[
\vec{q}_{\text{eff}} \equiv \frac{m_d}{m_d + m_q} \vec{P}_N - \frac{m_d}{m_d + m_Q} \vec{P}_Y.
\]

Having performed the integration over \(q_1\) and \(q_3\), we obtain the matrix elements for the productions of the ground-state heavy baryon as

\[
\langle Y(l_\rho = l_\rho = 0) | \mathcal{O}^B | N \rangle = C_Y I_{g.s.} (2\pi)^3 \delta^{(3)}(\vec{P}_Y - \vec{P}_p - \vec{q}),
\]

(27)

where \(I_{g.s.}\) is defined by

\[
I_{g.s.} = \int d^3k \int d^3\rho e^{i\vec{k} \cdot \vec{\rho}} \psi_{\rho}^* (\vec{\rho}) \psi_{\rho} (\vec{\rho}) \int d^3\lambda e^{i(\vec{k} + \vec{q}_{\text{eff}}) \cdot \vec{\lambda}} \psi_{\lambda}^{* \prime} (\vec{\lambda}) \psi_{\lambda} (\vec{\lambda})
\]

\[
= \left( \frac{16\pi \alpha_\rho^2 \alpha_{\lambda\lambda}^2}{B^2} \right)^{3/2} e^{-q_{\text{eff}}^2/(4B^2)}.
\]

(28)

Here, \(B^2\) is defined by

\[
B^2 \equiv \frac{8\alpha_\rho^2 + \alpha_{\lambda\lambda}^2 + \alpha_{\lambda}^2}{2},
\]

(29)

where \(m_d\) denotes the effective mass of a diquark, and \(\alpha_\rho\), \(\alpha_{\lambda\lambda}\), and \(\alpha_{\lambda}\) (given in Appendix A) are the oscillator parameters for the \(\rho\) modes and the initial- and final-state \(\lambda\) modes, respectively. Except for the delta function, the matrix elements given in Eq. (27) depend on \(q_{\text{eff}}\) instead of \(q\) because the recoil effect occurs by the difference between the masses of particles in initial and final states. In Eq. (28), we have seen that the Gaussian form factor \(1/(1 + q_{\text{eff}}^2/(4B^2))\) arises as a consequence of the use of the harmonic oscillator wavefunctions. In a realistic situation, a dipole type, \(1/(1 + q_{\text{eff}}^2/(4B^2))\), would be preferable. Here in our discussions, however, we mostly treat the relative strengths of various transitions, where the form factors are almost canceled out and the actual form of a form factor does not affect the conclusion of the present work, as discussed below.

For the excited baryons in forward-angle scattering, the matrix elements are written as

\[
\langle Y(l_\rho = 1, l_\rho = 0) | \mathcal{O}^B | N \rangle = C_Y I_{l_\rho = 1} (2\pi)^3 \delta^{(3)}(\vec{P}_Y - \vec{P}_p - \vec{q}),
\]

(30)

\[
\langle Y(l_\rho = 0, l_\rho = 1) | \mathcal{O}^B | N \rangle = C_Y I_{l_\rho = 0} (2\pi)^3 \delta^{(3)}(\vec{P}_Y - \vec{P}_p - \vec{q}),
\]

(31)

where \(I_{l_\rho = 1}\) and \(I_{l_\rho = 0}\) are defined by

\[
I_{l_\rho = 1} = \int d^3k \int d^3\rho e^{i\vec{k} \cdot \vec{\rho}} \psi_{\rho}^* (\vec{\rho}) \psi_{\rho} (\vec{\rho}) \int d^3\lambda e^{i(\vec{k} + \vec{q}_{\text{eff}}) \cdot \vec{\lambda}} \psi_{\lambda}^{* \prime} (\vec{\lambda}) \psi_{\lambda} (\vec{\lambda})
\]

\[
= i \sqrt{2\alpha_{\lambda\lambda}} |q_{\text{eff}}| \left( \frac{16\pi \alpha_\rho^2 \alpha_{\lambda\lambda}^2}{B^2} \right)^{3/2} e^{-q_{\text{eff}}^2/(4B^2)},
\]

(32)
In order to evaluate the production rates, we also need the meson matrix elements \( \langle M | O^M | \pi^- \rangle \). This also depends on the properties of the mesons involved. However, considering the fact that the meson states in both the initial and final states are the same, and assuming that the results depend mildly on meson form factors, we are able to ignore the matrix elements \( \langle M | O^M | \pi^- \rangle \) for the study of the relative production rates of various baryons. Thus, the differential cross sections are computed by

\[
R = \frac{1}{\text{Flux}} \times |t_f|^2 \times \text{Phase space} \\
\sim \frac{1}{\text{Flux}} \times |C_Y I_l|^2 \times \text{Phase space},
\]

where \( t_f \) denotes the transition amplitudes from the proton state \( (i \sim p) \) to various heavy baryon states \( (f \sim Y_s \text{ or } Y_c) \). In the CM frame, this can be written as

\[
R(Y(J_p, J_z)) \sim \frac{1}{4|p_f|\sqrt{s}} |C_Y|^2 |I_l|^2 \frac{\vec{p}_f}{4\pi \sqrt{s}},
\]

where \( s \) denotes the Mandelstam variable \( s = (\vec{p}_\pi + \vec{P}_N)^2 = (\vec{p}_M + \vec{P}_Y)^2 \).

We note that the main formulae that we have derived, from Eq. (25) to Eq. (33), are for the \( \tilde{\alpha}L^{\text{TMt}} \) t Hooft-like interaction which is unity in spin space in the non-relativistic approximation, namely, \( O^B \sim 1 \) in Eq. (23). These formulae still hold for other types of interactions with the operator \( O^B \) suitably changed. If it has spin dependence, its effect is included in the spin–isospin factor \( C_Y \).

3. Results and discussions

3.1. Kinematic conditions

We are now in a position to present the numerical results and discuss them. Since this is the first work on the two-quark process in heavy baryon production, we will consider only the case of forward-angle scattering for simplicity. The angular dependence and other observables will be studied in future works. To demonstrate the production rates, we first fix the momentum of the pion at \( k^{\text{Lab}}_\pi = 5 \text{ GeV} \) for strange baryons and \( k^{\text{Lab}}_\pi = 20 \text{ GeV} \) for charmed baryons. These values of the momenta will provide sufficient energies to create the \( s\bar{s} \) or \( c\bar{c} \) pairs. In the two-quark process, the momentum transfer \( \vec{q} \) is shared by the heavy quark and the diquark in the heavy baryon, which may excite both \( \lambda \) and \( \rho \) modes. This contrasts with the one-quark process where only one quark receives the momentum transfer and therefore possible excitation occurs only in the \( \lambda \) modes.

We need numerical values of baryon masses with proper assignment of the corresponding states to compute the cross sections. Actually, baryon masses in the quark model do not always agree with the experimental data. For example, the mass of \( \Lambda(1405) \) cannot be easily described by the quark model. So, we take the masses of baryons from the Particle Data Group when available [51]. Otherwise, they are taken from the values of the quark models [23]. By using these masses, we compute various matrix elements for the transitions up to \( p \)-wave excitations. The results are shown in Table 1, where
Table 1. Baryon masses $M$ in units of MeV, the spin–isospin coefficients for the heavy baryons $C_Y$, and the relative magnitudes of the differential cross sections $R(Y)$ that are normalized by that of the ground state, $\Lambda(1/2^+)$. $Y_s$ and $Y_c$ denote the strange and charmed baryons, respectively, and $j$ stands for the brown muck spin.

| $l$ = 0 | $\Lambda \left( \frac{1}{2}^- \right)$ | $\Sigma \left( \frac{1}{2}^- \right)$ | $\Sigma \left( \frac{1}{2}^+ \right)$ |
|---------|----------------|----------------|----------------|
| $M$ [MeV] | 1116 | 193 | 1385 |
| | 2286 | 2453 | 2518 |
| $|C_Y|^2$ | 1 | 3 | 0 |
| $R(Y_s)$ | 1 | 3.2 | 0 |
| $R(Y_c)$ | 1 | 2.9 | 0 |

$\Sigma = \left( \frac{1}{2}^+ \right)$, $\Sigma = \left( \frac{3}{2}^+ \right)$

we also list the masses of the excited states, spin–isospin factors $|C_Y|^2$, and relative magnitudes of differential cross sections $R(Y)$ defined in Eq. (36), which are normalized by that of the ground state, $\Lambda(1/2^+)$. $Y_s$ and $Y_c$ denote the strange and charmed baryons, respectively, and $j$ stands for the brown muck spin, which is the sum of the intrinsic spin and the orbital angular momentum of a diquark. In the following subsections we will discuss the results in Table 1 one by one.

3.2. Production rates of ground and excited states

We first discuss the difference between the production rates of the strange and charmed baryons. In Table 1, we list the results of the production rates for both the strange and charmed baryons. As shown clearly in Table 1, the ground more strange baryons are produced than excited ones, whereas the production rates of the excited charmed baryons are comparable with those of the ground ones. In Ref. [20] we see a similar tendency. This can be understood by the dependence of the transition amplitudes on the momentum transfer. Using the wavefunctions in the basis of the harmonic oscillator, we are able to derive the matrix elements analytically with Gaussian form factors depending on $q_{\text{eff}}^2$, which are given in Eqs. (27), (30), and (31). The momentum transfer $|\vec{q}_{\text{eff}}|$ is given as a function of the initial and final momenta, which depends on the total mass of the hadrons in the final states. The squared effective momentum transfer $q_{\text{eff}}^2$ governs the production of the heavy baryons. For example, the production rates of the lowest-lying heavy baryons decrease as $q_{\text{eff}}^2$ increases. This implies that in the case of the production of the ground-state heavy baryons, the Gaussian form factor $e^{-q_{\text{eff}}^2/(4B^2)}$ mainly governs the production mechanism. On the other hand, when it comes to the production rates...
Fig. 4. $|\vec{q}_{\text{eff}}|$ dependences of the transition amplitudes with two-quark and one-quark processes. The left panel is for the effects of the two-quark process with $B \simeq 1$ GeV, whereas the right panel is for the contributions of the one-quark process with $A \simeq 0.5$ GeV. The solid curves, the long-dashed ones, and the short-dashed ones represent the contributions to the ground state ($l = 0$), the $P$-wave, and $D$-wave excited state, respectively. The gray shaded areas show the regions of the typical momentum transfers for strange and charmed baryon productions, Region 1 and Region 2, respectively.

of the excited states, the $q_{\text{eff}}^2$ dependence is very different from the case of the ground-state heavy baryons. In addition to the Gaussian form factor, there exist other factors that are proportional to the $l$th power of $|\vec{q}_{\text{eff}}|$, where $l$ denotes the orbital angular momentum of the baryon in the final state. Thus, both the production rates for the $\rho$ and $\lambda$ modes are enhanced up to the maximum point as $q_{\text{eff}}^2$ increases, and then start to fall off as $q_{\text{eff}}^2$ further increases.

To understand this feature more explicitly, let us examine various transition amplitudes as functions of the momentum transfer $|\vec{q}_{\text{eff}}|$. In the left panel of Fig. 4, we show the normalized amplitudes for the transitions to $l = 0$ (ground state) and 1, 2 ($\lambda$ modes) baryons as functions of $|\vec{q}_{\text{eff}}|$ with Clebsh–Gordan coefficients removed,$^1$

$$I_0 = e^{-q_{\text{eff}}^2/(4B^2)},$$

$$I_1 = \frac{1}{\sqrt{2}} \left( \frac{\alpha_{\lambda}}{B} \right) |\vec{q}_{\text{eff}}/B| e^{-q_{\text{eff}}^2/(4B^2)},$$

$$I_2 = \frac{1}{2\sqrt{3}} \left( \frac{\alpha_{\lambda}}{B} \right)^2 |\vec{q}_{\text{eff}}/B|^2 e^{-q_{\text{eff}}^2/(4B^2)}. $$

For the strangeness production, the typical momentum transfer is shown by Region 1, where the ground state is the most abundantly produced, while for the charm production, Region 2 shows that the production rates of excited states become closer to that of the ground state.

### 3.3. Two- vs. one-quark processes

Here we briefly discuss the difference in the momentum dependences of transition amplitudes in the two-quark and one-quark processes. The amplitudes corresponding to Eqs. (37)–(39) for the one-quark process are obtained by replacing the parameter $B$ by $A$, where $A^2 = (\alpha_{\lambda}^2 + \alpha_{\lambda}^2)/2$ [20]. Because of the exponential form factor $\exp(-q_{\text{eff}}^2/(4A^2$ or $4B^2$)), the relation $B \sim 2A$ implies that

---

$^1$ These definitions are different from those in Ref. [20] by $\sqrt{2}$, and $A$ is replaced by $B$. 

12/18
when the momentum transfer becomes large the two-quark process dominates over the one-quark process. This is explained physically by the fact that the momentum transfer is shared by two quarks rather than by one quark. By comparing the two panels of Fig. 4, where the right panel is for the result of the one-quark process, this feature is observed. For large momentum transfer $q_{\text{eff}} > 2.5$ GeV, the transition amplitude becomes negligibly small in the right panel while it is still considerable in the left one. So, the two-quark process is dominant over the one-quark process as $q_{\text{eff}}$ increases.

As listed in Table 1 and shown in Fig. 4, the production rates of various excited states of the two-quark process are not as large as of those of the one-quark process [20]. A reason is that the transition amplitudes for the two-quark process are more broadly distributed to both the $\lambda$ and $\rho$ modes, while the one-quark process contributes mainly to the $\lambda$ modes.

### 3.4. Transitions to $\lambda$ and $\rho$ modes of $\Lambda$ and $\Sigma$ baryons

In order to discuss the relations between production rates and spin structures, we want to examine the production rates of the $\lambda$ and $\rho$ modes of $\Lambda$ and $\Sigma$ baryons. Table 2 reorganizes the relevant differential cross sections $R(Y)$ taken from Table 1 and roughly estimated ratios in each group. Here, $s$ and $j$ denote respectively the spin of the light diquarks and the spin of the brown muck, which are just the coupled angular momentum of the diquark spin and its orbital angular momentum. If we scrutinize the results listed in Table 2, we can observe a systematic property in the $\lambda$- and $\rho$-mode production. Namely, the ratio of the $\Lambda$ baryons of the $\lambda$ modes is $1 : 2$, and it is same as that of the $\Sigma$ baryons of the $\rho$ modes; also, that of the $\Lambda$ baryons of the $\rho$ modes, $1 : 2 : 1 : 5 : 0$, coincides with that of the $\Sigma$ baryons of the $\lambda$ modes. Considering the values of $s$ and $j$, we find that the excited $\Lambda$ baryons in the $\lambda$ mode have similar spin structures which have same quantum numbers, $s, j$, and $J^P$, to those of the excited $\Sigma$ baryons in the $\rho$ mode. Similarly, the excited $\Sigma$ baryons in the $\lambda$ mode correspond to the excited $\Lambda$ baryons in the $\rho$ mode by the spin content. The explicit forms of the wavefunctions can be found by using Eqs. (13) and (14). Thus, the identity of a baryon either in the $\lambda$ mode or in the $\rho$ mode is determined by the study of the production rates.

### 3.5. Restriction on the spin due to the instanton interaction

We want to mention that in the present work the spin flip of the quark does not occur during the process of baryon production, because the leading terms in the $1/N_c$ expansion of the 't Hooft-like

| $l_{\rho} = 1$ | $\Lambda \left( \frac{1}{2} \right)$ | $\Lambda \left( \frac{3}{2} \right)$ | $\Sigma \left( \frac{1}{2} \right)$ | $\Sigma \left( \frac{3}{2} \right)$ | $\Sigma \left( \frac{5}{2} \right)$ |
|---------------|----------------|----------------|----------------|----------------|----------------|
| $s = 0$       | $\Lambda \left( \frac{1}{2} \right) \Sigma \left( \frac{1}{2} \right)$ | $\Lambda \left( \frac{3}{2} \right) \Sigma \left( \frac{1}{2} \right)$ | $\Lambda \left( \frac{3}{2} \right) \Sigma \left( \frac{3}{2} \right)$ | $\Lambda \left( \frac{3}{2} \right) \Sigma \left( \frac{5}{2} \right)$ |
| $j = 1$       | $\Lambda \left( \frac{1}{2} \right) \Sigma \left( \frac{1}{2} \right)$ | $\Lambda \left( \frac{3}{2} \right) \Sigma \left( \frac{1}{2} \right)$ | $\Lambda \left( \frac{3}{2} \right) \Sigma \left( \frac{3}{2} \right)$ | $\Lambda \left( \frac{3}{2} \right) \Sigma \left( \frac{5}{2} \right)$ |
| $\mathcal{R}(Y)$ | 0.10 | 0.20 | 0.12 | 0.23 | 0.12 | 0.58 |
| Ratio | 1 | 2 | 1 | 2 | 1 | 5 | 0 |

| $l_{\rho} = 1$ | $\Sigma \left( \frac{1}{2} \right)$ | $\Sigma \left( \frac{3}{2} \right)$ | $\Lambda \left( \frac{1}{2} \right)$ | $\Lambda \left( \frac{3}{2} \right)$ | $\Lambda \left( \frac{5}{2} \right)$ |
|---------------|----------------|----------------|----------------|----------------|----------------|
| $s = 0$       | $\Sigma \left( \frac{1}{2} \right) \Lambda \left( \frac{1}{2} \right)$ | $\Sigma \left( \frac{1}{2} \right) \Lambda \left( \frac{3}{2} \right)$ | $\Sigma \left( \frac{3}{2} \right) \Lambda \left( \frac{1}{2} \right)$ | $\Sigma \left( \frac{3}{2} \right) \Lambda \left( \frac{3}{2} \right)$ | $\Sigma \left( \frac{5}{2} \right) \Lambda \left( \frac{5}{2} \right)$ |
| $j = 1$       | $\Sigma \left( \frac{1}{2} \right) \Lambda \left( \frac{1}{2} \right)$ | $\Sigma \left( \frac{1}{2} \right) \Lambda \left( \frac{3}{2} \right)$ | $\Sigma \left( \frac{3}{2} \right) \Lambda \left( \frac{1}{2} \right)$ | $\Sigma \left( \frac{3}{2} \right) \Lambda \left( \frac{3}{2} \right)$ | $\Sigma \left( \frac{5}{2} \right) \Lambda \left( \frac{5}{2} \right)$ |
| $\mathcal{R}(Y)$ | 0.016 | 0.032 | 0.017 | 0.039 | 0.018 | 0.10 | 0 |
| Ratio | 1 | 2 | 1 | 2 | 1 | 5 | 0 |
interaction are spin independent. This restricts the transition processes by certain conditions. As already shown in Table 1, the excited hyperons $\Sigma(\frac{3}{2}^+)$, $\Sigma(\frac{5}{2}^-)$, and $\Lambda(\frac{5}{2}^-)$ are not allowed to be produced off the proton. The absence of spin-flip interactions keeps the intrinsic spins of the quarks intact, which implies that the excitations of the orbital angular momenta cannot produce the abovementioned excited hyperons. The intrinsic spins of the quarks inside a proton can be flipped only by the vector or tensor interactions in the course of the production processes. Thus, we need to consider the vector or tensor interactions that make the intrinsic spins flipped. We leave this as future work.

### 3.6. Production rates of $\Lambda$ and $\Sigma$ baryons

There is yet another interesting point in the present results: we find that the ground-state $\Sigma$ baryons are in general produced more abundantly than the corresponding $\Lambda$ ones. As shown in Table 1, we have obtained a ratio of $\Lambda(\frac{1}{2}^+)$ to $\Sigma(\frac{1}{2}^+)$ of around 1/3, while the previous study [20], in which the one-quark process was only taken into account for the production of heavy baryons and vector mesons, yielded opposite results, i.e. the corresponding ratio turns out around 30.

These ratios reflect the spin and isospin structures of the reaction mechanism due to the relevant operators and wave functions. In this regard, it is interesting to observe that the ratio 1/3 also holds for the transitions to excited states: the sums of the transitions to the $\lambda$ modes of $\Lambda$ and $\Sigma$ baryons, and those of the $\rho$ modes of $\Sigma$ and $\Lambda$ baryons. Note that the available experimental data show that the ratio between $\Lambda(\frac{1}{2}^+)$ and $\Sigma(\frac{1}{2}^+)$ production is around 3/2 [52]. This implies that both the one-quark and two-quark processes should be taken into account to describe the existing data of $\Lambda(\frac{1}{2}^+)$ and $\Sigma(\frac{1}{2}^+)$. The relative strength of one-quark and two-quark processes may be determined by an additional study of the one-quark process for the production of heavy baryons and pseudoscalar mesons, or it is also possible by that of the two-quark process for heavy baryons and vector mesons with the previous study [20] as well. It will be possible to carry out more detailed studies when the features of the different reaction mechanisms are understood better.

### 4. Summary and conclusions

In the present work we have investigated the production of strange and charmed baryons, including both one-quark and two-quark processes. While the one-quark process was already considered previously, the two-quark process was proposed in this work. By the two-quark process, we mean that the two quarks inside a baryon undergo an interaction with a quark inside a meson beam, so that a strange or charmed baryon is produced. Thus, we need to introduce the three-quark interaction involving both light and heavy quarks. In order to realize this three-quark interaction, we introduced a ’t Hooft-like interaction arising from the instanton vacuum. The six-quark operators in the ’t Hooft-like interaction were decomposed into the quark fields for the mesons and those for the baryons. To make the investigation simpler, we constructed the baryon wave functions based on the nonrelativistic quark model with the confining potential of harmonic oscillator type. The excitations of the baryons produced consist of two modes, the $\lambda$ mode and the $\rho$ mode. As shown in previous works, the one-quark process excites only the $\lambda$ mode. However, the two-quark process affects both the $\lambda$ and $\rho$ modes. Thus, the two-quark process allows one to scrutinize the production mechanism of the excited charmed baryons in a more microscopic way. In particular, when the momentum transfer becomes large, the two-quark process will come into more important play. However, since introducing three-quark interactions involves additional ambiguity from unknown parameters, we
mainly focused here on the ratios of the production cross sections between the strange and charmed baryons.

The main results are summarized as follows:

- The excited states are produced more for the charmed baryons than for the strange baryons (hyperons), which was also found in the previous work. This can be understood by examining the dependence of the transition amplitudes on the momentum transfer. The amplitudes show the additional dependence on the momentum transfer, which arises from the higher orbital angular momentum.

- The two-quark processes excite not only the $\lambda$ modes but also the $\rho$ modes, which is different from the one-quark processes.

- The production rates reflect the spin structure of baryons. For instance, the relative production rates of $\lambda$-mode $\Lambda$ baryons are similar to those of $\rho$-mode $\Sigma$ baryons, because they have similar spin structures. These relations can be used to identify newly found baryons with unknown spin structure.

- For the ground-state heavy baryons, $\Sigma$ baryons are produced more than $\Lambda$ baryons. The one-quark processes exaggerate the relative production rates of $\Sigma$ baryons in comparison with $\Lambda$, since the observed ground-state $\Sigma$ production rates are about half of those of the $\Lambda$ hyperons. This implies that both the one-quark and two-quark processes come into play to describe the production mechanism of the hyperons. Thus, the two-quark processes should be considered as much as the one-quark processes.

In the present work we studied the production of strange and charmed baryons in a qualitative manner. To further investigate the production mechanisms of those baryons, we have to investigate the following issues:

- The instanton-induced interactions provide scalar-type interaction in the leading order of the $1/N_c$ expansion. However, the inclusion of the $1/N_c$ corrections is inevitable to describe the spin-flipped processes. Moreover, it is of great importance to introduce vector or tensor interactions for the baryon production in high-energy processes, as the Regge theories already implied.

- The present study was mainly focused on forward-angle production. We need to cover the whole angle to investigate the production of strange and charmed baryons in a more quantitative way.

- The study of baryon production aims eventually at extracting information on the structures of the baryons concerned. Thus, it is of great interest to implement microscopically the effects of the diquark and multi-quark structure in the description of the baryon production.

All these issues will be discussed in forthcoming works.

Acknowledgements

We thank H. Noumi and K. Shirotori for useful discussions about experimental situations. This work is supported in part by Grant-in-Aid for Scientific Research no. JP17K05441 (C) and by Scientific Research on Innovative Areas no. 18H05407. The work of S.-I. S. is supported by the Rotary Yoneyama Memorial Foundation. The work of H.-Ch. K. is supported by the Basic Science Research Program through the National Research Foundation (NRF) of Korea funded by the Ministry of Education, Science and Technology (MIST) (nos. 2018R1A5A1025563 and NRF-2018R1A2B2001752).

Funding

Open Access funding: SCOAP³.
Appendix A. Radial part of baryon wave functions

The radial part of the baryon wave functions $R_{nl}(r)$ are given with the wavefunctions of three-dimensional harmonic oscillators as follows:

$$R_{00}(r) = \left(\frac{4\alpha^3}{\sqrt{\pi}}\right)^{1/2} e^{-\alpha r^2/2},$$
$$R_{01}(r) = \left(\frac{8\alpha^3}{3\sqrt{\pi}}\right)^{1/2} a r e^{-\alpha r^2/2},$$

(A.1)

where the oscillator parameter $\alpha$ is given as

$$\alpha_{\rho} = \left(\frac{3k}{4m_q}\right)^{1/4}$$

(A.2)

for the $\rho$-mode wavefunctions of baryons and

$$\alpha_{\lambda} = \left(\frac{4k}{3m_q}\right)^{1/4},$$

$$\alpha_{\lambda'} = \left(\frac{2(m_d + m_Q)k}{m_km_Q}\right)^{1/4}$$

(A.3)

for the $\lambda$-mode wavefunctions of the initial- and final-state baryons, respectively. Here, $k$ is the spring constant between quarks.

Appendix B. Integrations with Gaussian integrals

To find the final expressions of Eqs. (28), (32), and (32), we use Gaussian integrals. Some parts of the derivations for $I_{g.s.}$ and $I_{\lambda=1}$ are given as follows:

$$I_{g.s.} = \int d^3 \kappa \int d^3 \rho \ e^{i\vec{\kappa} \cdot \vec{\rho}^*} \psi_0^\rho(\vec{\rho}) \psi_0^\rho(\vec{\rho}) \int d^3 \lambda \ e^{i(\vec{\kappa} + \vec{\eta}_{\text{eff}}) \cdot \vec{\lambda}} \psi_0^\lambda(\vec{\lambda}) \psi_0^\lambda(\vec{\lambda})$$

$$= \int d^3 \kappa \left(\frac{\alpha_{\rho}^2}{\pi}\right)^{3/2} \int d^3 \rho \exp[-\alpha_{\rho}^2 \rho^2 + i \frac{1}{2} \vec{\kappa} \cdot \vec{\rho}] \times \left(\frac{\alpha_{\lambda} \alpha_{\lambda'}^{'}}{\pi}\right)^{3/2} \int d^3 \lambda \exp[-\alpha_{\lambda}^2 \lambda^2 + i(\vec{\kappa} + \vec{\eta}_{\text{eff}}) \cdot \vec{\lambda}]$$

$$= \left(\frac{2\alpha_{\lambda} \alpha_{\lambda'}^{'}}{\alpha_{\lambda} + \alpha_{\lambda'}^{'}}\right)^{3/2} \int d^3 \kappa \exp[-\frac{\kappa^2}{16\alpha_{\rho}^2} \left(\frac{2\alpha_{\lambda} \alpha_{\lambda'}^{'}}{\alpha_{\lambda} + \alpha_{\lambda'}^{'}}\right)^{3/2} \exp[-\left(\vec{\kappa} + \vec{\eta}_{\text{eff}}\right)^2 \left(2\alpha_{\lambda}^2 + 2\alpha_{\lambda'}^2\right)]$$

$$= \left(\frac{16\pi \alpha_{\lambda}^2 \alpha_{\lambda'}^{'}}{B^2}\right)^{3/2} e^{-\frac{\kappa^2}{4B^2}},$$

(B.1)

$$I_{\lambda=1} = \int d^3 \kappa \int d^3 \rho \ e^{i\vec{\kappa} \cdot \vec{\rho}^*} \psi_0^\rho(\vec{\rho}) \psi_0^\rho(\vec{\rho}) \int d^3 \lambda e^{i(\vec{\kappa} + \vec{\eta}_{\text{eff}}) \cdot \vec{\lambda}} \psi_1^\lambda(\vec{\lambda}) \psi_0^\lambda(\vec{\lambda})$$

$$= \int d^3 \kappa \exp[-\frac{\kappa^2}{16\alpha_{\rho}^2} \sqrt{2} \alpha_{\lambda} \left(\frac{\alpha_{\lambda} \alpha_{\lambda'}^{'}}{\pi}\right)^{3/2} \int d^3 \lambda \exp[-\alpha_{\lambda}^2 \lambda^2 + i(\vec{\kappa} + \vec{\eta}_{\text{eff}}) \cdot \vec{\lambda}]$$

16/18
\[
\int d^3 \kappa \exp \left[ -\frac{\kappa^2}{16\alpha_0^2} \right] \sqrt{2\alpha_0} \left( \frac{2\alpha_0 \alpha' \alpha}{\alpha_0^2 + \alpha^2} \right)^{3/2} \left( \frac{\kappa z + (\eta_{\text{eff}})z}{\alpha_0^2 + \alpha^2} \right) \exp \left[ -\frac{(\kappa + \eta_{\text{eff}})^2}{2(\alpha_0^2 + \alpha^2)} \right]
\]

\[
= \frac{i\sqrt{2\alpha_0'}(\eta_{\text{eff}})z}{2\eta_{\text{eff}}^2} \left( \frac{16\pi \alpha_0^2 \alpha' \alpha}{\eta_{\text{eff}}^2} \right)^{3/2} e^{-\eta_{\text{eff}}^2/(4\eta^2)}.
\]

(B.2)

Here, the following integral formulae have been used for integrating over \( \rho, \lambda, \) and \( q_1 \):

\[
\int dr \, e^{-A r^2 + B \bar{r}} = \left( \frac{\pi}{A} \right)^{3/2} e^{\frac{B^2}{4A}},
\]

(B.3)

\[
\int dr \, r_i \, e^{-A r^2 + B \bar{r}} = \frac{B_1}{2A} \left( \frac{\pi}{A} \right)^{3/2} e^{\frac{B^2}{4A}}.
\]

(B.4)

References

[1] A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, Prog. Theor. Exp. Phys. 2016, 062001 (2016) [arXiv:1603.09229 [hep-ph]] [Search INSPIRE].

[2] S.-K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262001 (2003) [arXiv:hep-ex/0309032] [Search INSPIRE].

[3] D. Acosta et al. [CDF II Collaboration], Phys. Rev. Lett. 93, 072001 (2004) [arXiv:hep-ex/0312021] [Search INSPIRE].

[4] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 71, 071103 (2005) [arXiv:hep-ex/0406022] [Search INSPIRE].

[5] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 101, 232002 (2008).

[6] S. Chatrchyan et al. [CMS Collaboration], Phys. Rev. Lett. 108, 252002 (2012).

[7] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 109, 172003 (2012).

[8] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 110, 252001 (2013) [arXiv:1303.5949 [hep-ex]] [Search INSPIRE].

[9] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 115, 072001 (2015) [arXiv:1507.03414 [hep-ex]] [Search INSPIRE].

[10] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 117, 082002 (2016) [arXiv:1604.05708 [hep-ex]] [Search INSPIRE].

[11] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 117, 082003 (2016); 117, 109902 (2016) [addendum]; 118, 119901 (2017) [addendum] [arXiv:1606.06999 [hep-ex]] [Search INSPIRE].

[12] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 118, 022003 (2017) [arXiv:1606.07895 [hep-ex]] [Search INSPIRE].

[13] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 118, 182001 (2017).

[14] J. Yelton et al. [Belle Collaboration], Phys. Rev. D 97, 051102(R) (2018) [arXiv:1711.07927 [hep-ex]] [Search INSPIRE].

[15] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 122, 222001 (2019) [arXiv:1904.03947 [hep-ex]] [Search INSPIRE].

[16] M. Gell-Mann, Phys. Lett. 8, 214 (1964).

[17] G. Zweig, An SU(3) model for strong interaction symmetry and its breaking. Version 2, in Developments in the Quark Theory of Hadrons, eds. D. Lichtenberg and S. Rosen (1964), Vol. 1, pp. 22–101.

[18] L. A. Copley, N. Isgur, and G. Karl, Phys. Rev. D 20, 768 (1979); 23, 817 (1981) [erratum].
[19] Y. Morino, T. Nakano, H. Nourmi, K. Shirotori, Y. Sugaya, and T. Yamaga, Charmed baryon spectroscopy via the $(\pi^-, D^{**}^-)$ reaction, J-PARC PS0 proposal (2012). Available at http://www.j-parc.jp/researcher/Hadron/en/Proposal_e.html/#1301.

[20] S.-H. Kim, A. Hosaka, H.-Ch. Kim, H. Nourmi, and K. Shirotori, Prog. Theor. Exp. Phys. 2014, 103D01 (2014) [arXiv:1405.3445 [hep-ph]] [Search INSPIRE].

[21] S.-H. Kim, H.-Ch. Kim, and A. Hosaka, Phys. Lett. B 763, 358 (2016) [arXiv:1605.02919 [hep-ph]] [Search INSPIRE].

[22] T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, and K. Sadato, Phys. Rev. D 92, 114029 (2015) [arXiv:1510.01067 [hep-ph]] [Search INSPIRE].

[23] T. Ezoe and A. Hosaka, Phys. Rev. D 94, 034022 (2016) [arXiv:1605.01203 [nucl-th]] [Search INSPIRE].

[24] C. G. Callan and I. Klebanov, Nucl. Phys. B 262, 365 (1985).

[25] C. G. Callan, K. Hornbostel, and I. Klebanov, Phys. Lett. B 202, 269 (1988).

[26] H. Weigel, Lect. Notes Phys. 743, 113 (2008).

[27] J. Yamagata-Sekihara, T. Sekihara, and D. Jido, Prog. Theor. Exp. Phys. 2013, 043D02 (2013) [arXiv:1210.6108 [nucl-th]] [Search INSPIRE].

[28] S.-I. Shim and H.-Ch. Kim, Phys. Lett. B 772, 687 (2017) [arXiv:1704.03263 [hep-ph]] [Search INSPIRE].

[29] M. Musakhanov, EPJ Web Conf. 137, 03013 (2017) [arXiv:1703.07825 [hep-ph]] [Search INSPIRE].

[30] C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40, 100001 (2016).