CP violation in realistic String Models
with family universal anomalous $U(1)$

Alon E. Faraggi and Oscar Vives

Theoretical Physics Department,
University of Oxford, Oxford, OX1 3NP, United Kingdom

Abstract

The characteristic property of the $Z_2 \times Z_2$ orbifold compactification is the cyclic permutation symmetry between the three twisted sectors. We discuss how this property, which is retained in a class of realistic free fermionic string models, may be instrumental in allowing fermion mass hierarchy while ensuring sfermion mass degeneracy, irrespective of the dominant source of supersymmetry breaking. The cyclic symmetry is reflected in some models in the existence of a family universal anomalous $U(1)_A$. We analyze the FCNC and $CP$ violation effects in a model with a dominant $U(1)_A$ SUSY breaking. In this theories, new sources of FC are always suppressed with respect to the average sfermion masses. We make a phenomenological analysis of these effects and find that in most of the cases they are in qualitative agreement with the phenomenological limits. The most sensitive low-energy observables to these new FC sources are the $\varepsilon_K$ parameter, measuring indirect $CP$ violation in neutral kaon mixing and EDMs. These observables set important constraints on the structure of the sfermion mass matrices, but do not require a large fine-tuning of the initial parameters and can be satisfied in most realistic constructions.
1 Introduction

The flavour puzzle in supersymmetric extensions of the Standard Model poses an especially interesting problem. On the one hand, there is a clear need for flavour dependence to explain the fermion mass hierarchy. On the other hand, the absence of Flavour Changing Neutral Currents (FCNC) at an observable rate suggests the need for flavour independent symmetries, which force sfermion mass degeneracy. In addition one must insure that non–trivial phases which may appear in the supersymmetry breaking sector, do not produce Electric Dipole Moments (EDMs) that violate the experimental bounds [1, 2]. Furthermore, as emphasized most recently in ref. [2], even if the sfermion masses are degenerate and the supersymmetric phases are suppressed, non–universality in the soft trilinear A–terms may result in large EDMs, merely due to the existence of CP violation in the Cabbibo–Kobayashi-Maskawa (CKM) mixing matrix. In ref. [2] this was referred to as the “String CP problem”, to emphasize the fact that in string theories one generically expects non–universal trilinear terms and must necessarily provide for non-vanishing phases in the Yukawa matrices [2].

In the context of supersymmetric field theory models the problem is of course evaded by simply imposing that the relevant parameters have the needed properties to avoid conflict with the experimental data. The riddle however becomes more intricate in the context of superstring theories in which the soft SUSY breaking parameters are, in general, expected to be non–universal [3]. The problem is further exacerbated due to our ignorance of the precise mechanism responsible for supersymmetry breaking. Moreover, the SUSY breaking mechanism is, in general, expected to involve nonperturbative dynamics, on which we have very little calculational handle. Therefore, even if one scenario can offer some remedy to the superstring flavour and CP problems, there is no preference for any particular scenario, nor any reason for such a remedy to survive the nonperturbative dynamics. These arguments suggest that what is needed is to seek structures in string theory, or in particular string compactifications, that are independent of our ignorance of the details of the SUSY breaking scenario and of the nonperturbative dynamics.

In this paper we study the superstring flavour and CP problems in the context of realistic string models with a family universal anomalous $U(1)$. Supersymmetry breaking in this class of models was investigated in refs. [4, 5]. The supersymmetry breaking mechanism in these papers utilizes both an anomalous $U(1)$ gauge symmetry and an effective mass term $m \sim 1$TeV of some fields in the massless string spectrum. It was shown that non–renormalizable terms which contain hidden sector condensates, generate the required suppression of the relevant mass term $m$ compared to the Planck scale. While the non–vanishing $D$–term of the family universal $U(1)_A$ led to squark degeneracy, those of the family dependent $U(1)_s$, remarkably enough, were found to vanish for the solutions that were considered, owing to minimization of the potential. This result therefore suggests the possibility that, while the flavor $U(1)$ symmetries are responsible for generating the fermion mass hierarchy, they do not
necessarily spoil the sfermion mass degeneracy [5]. Analysis of fermion mass textures in the string models that we consider here was done in ref. [6]. It was argued in ref. [5] that the phenomenological constraints motivate a combined $U(1)_A$–Dilaton SUSY breaking scenario. It was then shown how the superstring FCNC problem, the analogous to the SUSY flavour problem in a superstring derived SUSY model, could be adequately resolved in this combined $U(1)_A$–Dilaton SUSY breaking scenario.

In this work we extend the discussion of ref. [5] and examine both the superstring flavour and CP problems in the context of realistic string models with combined family universal $U(1)_A$–Dilaton SUSY breaking scenario. Here we show that this combined scenario can naturally evade the string CP problem due to the suppression of the non-universal component of the trilinear soft SUSY breaking parameters. We then discuss the general structure of the string models that produce the family universal anomalous $U(1)$. The root cause of the flavour universality of $U(1)_A$ is the cyclic permutation symmetry that characterizes the $Z_2 \times Z_2$ orbifold compactification with standard embedding [6, 5], realized in the free fermionic models by the NAHE set [7, 8]. The cyclic permutation symmetry is the remarkable property of the free fermionic models, or more precisely of the $Z_2 \times Z_2$ orbifold, which may eventually prove to be instrumental in resolving the string flavour and CP problems in any SUSY breaking scenario. Namely, while in the present paper our focus is on the family universality of the anomalous $U(1)_A$, the cyclic permutation symmetry also exhibits itself in the universality of the untwisted moduli, and in other sectors of the models. It may therefore prove to be instrumental to the understanding of the sfermion mass degeneracy in different SUSY breaking scenarios. In this respect the $U(1)_A$ SUSY breaking scenario that is analyzed here and in ref. [6], merely offers a glimpse into the deeper structures that underly this class of superstring compactifications. In section (4) we discuss how the cyclic permutation symmetry may ensure sfermion universality irrespective of the dominant source of supersymmetry breaking. Additionally, we discuss how the spectrum in the realistic free fermionic models may divide in a way which allows fermion mass hierarchy while ensuring that the phenomenological constraints on the sfermion mixing and phases are satisfied.

2 Anomalous $U(1)_A$ SUSY breaking

In a large class of free fermionic string models with an anomalous $U(1)_A$ gauge symmetry, supersymmetry may be softly broken by the anomalous $D$-term. At low energies these models yield a MSSM gauge group with three generations and softly broken SUSY [6]. Furthermore, the existence of free fermionic string models that reproduce solely the MSSM states in the low energy, Standard Model charged, spectrum has also been demonstrated [5]. These remarkable properties render these string models as some of the most interesting constructions in string phenomenology. Issues like fermion mass textures and CP violation in these models are discussed in refs. [6, 8].
In this letter we analyze the presence of FCNC and CP violation in these models. For this purpose we study two different realizations of free fermionic string models. We refer to them as model I \cite{10} and model II \cite{11}.

In model I, all scalar fields charged under $U(1)_A$ acquire a soft mass given by,

$$m_{f_i}^2 D_A = g^2 \left( Q_A \langle D_A \rangle + Q_\chi \langle D_\chi \rangle \right).$$

(1)

where $U(1)_\chi$ is a non-anomalous, family universal $U(1)$ that gets a nonvanishing D-term in the model. In this case $\langle D_A \rangle = 2 \langle D_\chi \rangle = \sqrt{3/5} m^2/g^2$, and the charges are all family universal, although they vary for different members of the same family. Using the charges in \cite{10} we obtain,

$$m_{Q_L}^2 = m_{d_R}^2 = 3 m_{u_R}^2 = 3 m_{e_R}^2 = 3 m_{\bar{e}_R}^2 = m^2/4,$$

(2)

where $m$ leads to SUSY breaking through the superpotential mass term

$$W \propto m \Phi \bar{\Phi} + \ldots$$

(3)

In Ref. \cite{5} this effective mass term in the superpotential corresponds to a $m \Phi_{45} \bar{\Phi}_{45}$ term, with $m \sim O(1 \text{ TeV}) \ll M_{Pl}$ arising from higher dimensional operators and hidden sector matter condensates. Here all the MSSM fields have family universal charges under $U(1)_A$ and $U(1)_\chi$, and hence all of them receive an equal mass from the D-term. However, it is clear that the D-terms cannot contribute to gaugino masses and trilinear terms. Hence, in this scenario, these soft terms can only appear through higher dimensional operators. Gaugino masses are generated \cite{12} from,

$$\lambda_g \int d^2 \theta \frac{\Phi \bar{\Phi}}{M_{Pl}^2} W_a W_a \longrightarrow \lambda_g \frac{\langle F_\Phi \rangle \langle \Phi \rangle}{M_{Pl}^2} = \lambda_g \frac{\sqrt{15} \xi}{5 M_{Pl}^2} \equiv \lambda_g \epsilon m/5$$

(4)

with $\xi$ the Fayet-Iliopoulos term, that in this model is equal to,

$$\xi = \frac{g^2 \langle \text{Tr} Q_A \rangle}{192 \pi^2} M_{Pl}^2 = \frac{15 g^2}{16 \sqrt{15} \pi^2} M_{Pl}^2,$$

(5)

and therefore we can estimate $\epsilon = 0.095 \times g^2$. As $g^2$ is the unified coupling at the string scale and $\alpha_{GUT} = g^2/4\pi \simeq 1/25$, we have $\epsilon \simeq 1/20$. Similarly trilinear couplings come from superpotential terms with their flavour structure determined by a flavour symmetry as discussed in section 4 and therefore sufficiently similar to the Yukawa matrices. Still, we take these flavour structures as free in the following,

$$W = \lambda' Q H_u u^c \frac{\Phi \bar{\Phi}}{M_{Pl}^2} \longrightarrow \lambda' \frac{\langle F_\Phi \rangle \langle \Phi \rangle}{M_{Pl}^2} = \lambda' \epsilon m/5.$$  

(6)

Here the couplings $\lambda_g$, $\lambda'$... respect the string symmetries and may be suppressed and/or arise at 1 loop level. For a scalar mass $m = 1\text{ TeV}$, this implies that the
ghuino mass at the electroweak scale is \( m_{\tilde{g}}(M_Z) \simeq 2.8 \, m_\lambda(M_{\text{GUT}}) = 30 \lambda \text{GeV} \). Clearly too small for the observed experimental limits. Hence we need an additional contribution to the gaugino masses. To obtain an adequate gaugino mass we can consider a combined anomalous \( U(1)-\text{Dilaton} \) scenario. Here, scalar masses would get a dominant contribution from \( U(1)_A \) while the dilaton would generate the main contribution to gaugino masses. These dilaton contributions give rise also to sfermion masses and trilinear couplings although these contributions are family universal [13],

\[
m_{1/2} = \pm \sqrt{3} m_{3/2}^S, \quad m^2(\tilde{f}_i) = (m_{3/2}^S)^2, \quad A_{ijk} = -m_{1/2}
\]
in terms of the gravitino mass, \( m_{3/2}^S \), that arises from the dilaton \( F \)-term. Notice that from here, the dilaton contributions to the trilinear terms and to the gaugino masses have exactly the same phase. Therefore, taking into account that in our scheme, dilaton provides always the great bulk of the gaugino mass, dilaton contribution to trilinear couplings play no role in \( CP \) violation. Moreover, it is important to remember that flavour universal contributions to the soft breaking terms, even with the two new SUSY phases (the \( \mu \) and \( A \) phases), can only contribute sizeably in \( CP \) violation in EDMs and \( b \to s\gamma \), but never in \( CP \) violation in kaon mixing (\( \varepsilon_K \) and \( \varepsilon'/\varepsilon \)) or in the \( B^0 \) \( CP \) asymmetries [14, 15].

Still, we must take into account that sfermion masses may also receive nonuniversal contributions from the Kähler potential [12],

\[
(m_f^2)_K \simeq \frac{\lambda |\langle F_\Phi \rangle|^2}{M_{\text{Pl}}^2} \simeq \frac{\lambda m^2 \sqrt{15} \xi}{5 M_{\text{Pl}}^2} = \lambda m^2 \epsilon/5
\]

Nevertheless, due to the \( \epsilon \) suppression, these nonuniversal contributions are always small and, as we will see in the next section, generate reasonably small FCNC and \( CP \) violation effects.

Similarly in model II the three generation sfermion masses are,

\[
[m_{f_i}^2]_{D_A} = g^2 \left( Q_A^i \langle D_A \rangle \right) = m^2/4,
\]

where in this case \( Q_A^i \) is not only family universal, but also intra–family universal, with \( Q_A^i = 1/\sqrt{12} \), and \( \langle D_A \rangle = \sqrt{3}/2 \ m^2/g^2 \). Trilinear terms are obtained via higher order terms similar to model I,

\[
W = \lambda' QH_u u^c \frac{\Phi \Phi}{M_{\text{Pl}}^2} \rightarrow \lambda' \frac{\langle F_\Phi \rangle \langle \Phi \rangle}{M_{\text{Pl}}^2} = \lambda' m^2/2,
\]

and in this case \( \epsilon' = \sqrt{3} \xi / M_{\text{Pl}}^2 = g^2 3/(8 \pi^2) \simeq 1/50 \). Remarkably \( \epsilon'/2 = \epsilon/5 \) and the relative suppression of trilinear terms with respect to \( m \) is exactly the same as in model I. Nonuniversal sfermion masses from the Kähler potential are now,

\[
(m_f^2)_K \simeq \lambda \frac{|\langle F_\Phi \rangle|^2}{M_{\text{Pl}}^2} \simeq \frac{\lambda m^2 \sqrt{3} \xi}{2 M_{\text{Pl}}^2} = \lambda m^2 \epsilon'/2,
\]
and again the relative suppression with respect to \( m \) is equal to model I. This means that, at least in the down quark sector, the analysis of FCNC and \( CP \) violation will be analogous in both model I and II, as \( \epsilon/5 = \epsilon'/2 \).

### 3 \( CP \) violation and FCNC

As we have seen in the previous section, the free fermionic string models of Refs. [10, 11] have a clear hierarchy between diagonal and off-diagonal elements,

\[
\begin{align*}
\left[ m^2(\tilde{f}_i) \simeq \frac{m^2}{4} + m^2_S \right] & > \left[ (m^2_{\tilde{f}})_{i \neq j} \simeq \lambda_{ij} \frac{m^2}{5} \epsilon \right]
\end{align*}
\]

(12)

with \( m^2_S \) the universal dilaton contribution to scalar masses. This implies that, in general, the SUSY flavour problem as well as some aspects of the SUSY \( CP \) problem are largely reduced in these scenarios. However, even with this suppression of off-diagonal elements, the FCNC and \( CP \) violation observables are still extremely sensitive to some entries in the sfermion mass matrices. In the following, we analyze the possible contributions to FCNC and \( CP \) violation observables in the quark sector.

In first place, we assume that the structure of the sfermion mass matrices at the Planck scale is generically given in Eq. (12). The off–diagonal elements are roughly a factor \( \epsilon \) smaller than the diagonal ones. The next step is to use the MSSM Renormalization Group Equations (RGE) to evolve the soft breaking parameters from the Planck scale to the electroweak scale. In fact, for simplicity, we identify the Planck scale and the GUT scale and use the MSSM RGE up to a scale of \( 2 \times 10^{16} \) GeV. In this RGE evolution the main effects are those associated with the strong coupling, the top quark Yukawa coupling, and possibly the bottom quark and tau lepton Yukawa couplings in the large \( \tan \beta \) regime. We can take the basis where up–quarks Yukawa couplings are diagonal, \( v^2_{Yu} = M_u \), and the off-diagonality in the down–quarks mass matrix is simply given by the usual CKM mixing matrix, \( v^1_{Yd} = K^\dagger_{CKM} \cdot M_d \). Under these conditions, it is clear that we can neglect small effects associated with Yukawa elements other than \( Y_{tt} \) and \( Y_{bb} \). It is easy to see from the general MSSM RGE equations [16] that this implies that off-diagonal elements in the doublet or singlet sfermion mass matrices are basically unchanged, while the diagonal elements receive a flavour universal contribution except for a small difference in the third generation masses (see for instance Tables I and IV in [13]),

\[
m^2_{D_{i(L,R)}}(M_W) \simeq 6 \cdot m^2_{1/2} + m^2_{D_{i(L,R)}}
\]

(13)

From this point of view it is clear that these effects reduce the FCNC and \( CP \) violation problems in the model. For instance, taking both the gaugino mass and the \( U(1)_A \) contribution to sfermion masses roughly of the same order at the GUT scale, \( m_{1/2} \simeq m/2 \), and replacing in Eq. (13) the different contributions from Eqs. (2) and (7),
the average sfermion mass, \(m_{\tilde{q}}^2\), at \(M_W\) is given by \(7.3 \cdot m_{1/2}^2 \simeq 7.3 \cdot m^2/4\). At the same time, as explained above, off-diagonal terms are not largely modified, \((m_{\tilde{q}}^2)_{i\neq j} \simeq \lambda_{ij} m^2 \epsilon/5\). Hence the low energy FC effects are reduced nearly an order of magnitude by the above RGE factor. Nevertheless, we must keep in mind that in this model the sfermion and gaugino masses are in principle unrelated because they come from different sources. On the other hand, the experimental constraints on sfermion and gaugino masses set for both a common lower limit of roughly 100 GeV at the GUT scale.

The RG evolution of the trilinear couplings is also similarly dominated by gluino contributions and the third generation Yukawa couplings. Therefore, in the basis of diagonal up quark Yukawa couplings, we have again that diagonal elements receive a large gaugino contribution at \(M_W\), while the \((t, t)\) (and possibly \((b, b)\)) element of the trilinear coupling, \(Y_{ij}^A = A_{ij} Y_{ij}\), is reduced due to the top (and bottom) Yukawa coupling (see Table V in [15]). Once again off-diagonal elements in this basis are basically unchanged. In particular, the values of \(A_{tb}\) depend on the value of \(\tan \beta\). For instance, for low \(\tan \beta \simeq 5\) we have,

\[
A_t(M_W) \simeq 0.24 \cdot A_t^0 - 2 \cdot m_{1/2} \\
A_b(M_W) \simeq 1 \cdot A_t^0 - 3.4 \cdot m_{1/2},
\]

while for \(\tan \beta \simeq 30\),

\[
A_t(M_W) \simeq 0.25 \cdot A_t^0 - 2 \cdot m_{1/2} \\
A_b(M_W) \simeq 0.74 \cdot A_t^0 - 2.9 \cdot m_{1/2},
\]

whereas, the flavour diagonal elements for the first two generations do not change strongly with \(\tan \beta\),

\[
A_u(M_W) \simeq 0.6 \cdot A_u^0 - 2.9 \cdot m_{1/2} \\
A_d(M_W) \simeq 1 \cdot A_d^0 - 3.6 \cdot m_{1/2}
\]

The main feature of these RG evolution is the alignment of the trilinear couplings and the gaugino masses. This alignment of the phases has important effects in \(CP\) violation observables.

To explore FCNC and \(CP\)-violation effects in the absence of a completely defined flavour structure as in this case, it is convenient to use the so-called Mass Insertion (MI) approximation [19, 20]. This approximation is defined in the SCKM basis where fermion and sfermion matrices are rotated in parallel to the basis where fermion masses are diagonal, such that neutral gaugino couplings are flavour diagonal and the flavour change is produced by non-diagonal sfermion propagators. These propagators can be expanded as a series in terms of,

\[
(\delta A)_{ij} = (m_{\tilde{A}}^2)_{ij} / m_{\tilde{q}}^2,
\]

with \(A = L, R, LR\) corresponding to the left–handed sfermion mass matrix, right–handed sfermion mass matrix, or in the left–right mixing sfermion mass matrices,
respectively. These $\delta$ parameters are the so-called Mass Insertions, and low energy observables place constraints on the allowed size of these MI [20, 21, 22].

First, we analyze the $L$ and $R$ mass insertions. Here we take the off-diagonal element as $(m_{\tilde{q}}^2/4)$ in Eq. (12), and this implies,

$$(\delta_{L,R})_{ij} = \frac{\lambda_{ij} m_{\tilde{q}}^2 \epsilon / 55a}{7.3a(m^2/4)} = \frac{\lambda_{ij}}{a} 5.5 \times 10^{-3},$$

for an average squark mass, $m_{\tilde{q}}^2 = 7.3 \cdot (m^2/4)a$. In principle we make no further assumptions and take $\lambda_{ij}/a \simeq 1$. Therefore we can see that in general we expect these off-diagonal contributions to be quite small. In fact, if we compare with the phenomenological bounds in [20, 21, 22], we can see that the only places where these contributions could generate large effects at low energy is in the kaon sector and possibly in rare leptonic decays as $\mu \rightarrow e\gamma$ [17, 18]. On the contrary, in the $B$ sector, where a large signal of $CP$ violation has been recently measured in the $B$ factories [23], the expected SUSY contributions are small. The experimental limits from the $B$ mass difference are [24, 21, 24],

$$\sqrt{|\text{Re}(\delta_L^d)_{13}|^2} \sqrt{|\text{Re}(\delta_R^d)_{13}|^2} \leq 9.8 \times 10^{-2}$$

with an average squark mass equal to the gluino mass at the electroweak scale and equal to 500 GeV. Notice that these limits vary with the ratio $m_{\tilde{g}}^2/m_{\tilde{q}}^2$ and scale with $m_{\tilde{q}}^2/(500 \text{ GeV})^2$. Comparing the expected MI in this model, Eq. (18), and the limits in Eq. (19), we can see that in the case where off-diagonality is only present in the $L$ or in the $R$ mass matrix the maximum possible contribution in our model can never give a sizeable contribution. In principle, the simultaneous presence of maximum $\delta_L$ and a $\delta_R$ could still be observable. Moreover, we have to take into account that due to the fact that the $CP$ violating phase in $B \bar{B}$ mixing is $O(1)$, these limits are also approximately valid for the imaginary part. In the presence of $L$ and $R$ MI simultaneously, it is still possible that these contributions can be observed in the $CP$ asymmetries of the $B$ system. Nevertheless, in general, we would expect that not all the off-diagonal entries are of the maximum size in Eq. (18). For instance, SUSY contributions would be smaller if $\lambda/a < 1$ or the average sfermion masses are somewhat larger. Therefore, it is difficult to envision a sizeable contribution in the $B$ system. Another observable in the $B$ system is the decay $b \rightarrow s\gamma$ but it is only sensitive to $LR$ MI and we discuss it below.

More interesting are the contributions in the kaon sector. The kaon mass difference as can be seen from the updated bounds in [24] can still receive a large contribution in the simultaneous presence of $L$ and $R$ MI. This is due to the fact that in this case, most of the kaon mass difference is already given in the SM. This fact
already constrains the model, although the experimental value of $\varepsilon_K$ gives rise to a much stronger constraint \cite{20, 21, 22}. As generically we expect both the real and imaginary parts of these MI of the same order we consider only the $\varepsilon_K$ bound,

$$\sqrt{\text{Im} \left( \delta^d_{L12} \right)^2}, \sqrt{\text{Im} \left( \delta^d_{R12} \right)^2} \leq 6.1 \times 10^{-3}$$

(20)

$$\sqrt{\text{Im} \left\{ \left( \delta^d_{L12} \right) \left( \delta^d_{R12} \right) \right\} } \leq 1.3 \times 10^{-4}$$

Once again, comparing these bounds with the naive estimate in Eq. (18), we can see here that a single MI in the right– or left–handed sector should be in qualitative agreement with the phenomenological limits. In fact, in a complete theory, we would also expect some relation among the flavour structures in the Yukawa couplings and the soft breaking masses. Then, in the basis of diagonal Yukawa couplings, off–diagonal entries in the sfermion mass matrices would be further suppressed with respect to the diagonal ones. Another necessary ingredient in this case is the presence of a non-vanishing $CP$ violating phase. Therefore the constraints on these matrix elements could be easier to satisfy if we assume that phases are $O(0.1)$, although there is no special reason for this. However, the simultaneous presence of large right and left MI can still be problematic for kaon phenomenology. The phenomenological limit in Eq (20) would imply a constraint in the off-diagonal terms in the sfermion mass matrix,

$$\sqrt{\text{Im} \left\{ \lambda^d_{L12} \lambda^d_{R12} \right\} } a \leq \sqrt{\lambda^d_{L12} \lambda^d_{R12}} \sin \alpha \leq 1.3 \times 10^{-4}$$

$$\frac{5.5 \times 10^{-3}}{\simeq 0.024}$$

(21)

with $\alpha = (\alpha_L + \alpha_R)/2$, the $CP$ phase of the $\lambda$–couplings in Eq. (8), in the SCKM basis. Hence, even in these models with suppressed nonuniversal contributions to the sfermion mass matrices, the phenomenological constraint from $\varepsilon_K$ has to be taken into account and restricts the allowed flavour structure from the Kähler potential. However, we want to emphasize that here, these constraints do not require a large fine-tuning of the initial parameters and could be satisfied in realistic constructions. In the next section we will discuss these problems in the framework of a realistic string model.

A second source of flavour change comes from the nonuniversality in the trilinear soft terms. These contributions give rise to chirality changing MI, $(\delta_{LR})_{ij}$. In this case the off-diagonal entries are more difficult to estimate because the physical trilinear couplings are related to the Yukawa matrices $Y^A_{ij} = A_{ij} Y_{ij}$ and this definition is strongly dependent on the Yukawa basis. Indeed, in many flavour models, this relation implies a strong suppression with light quark masses in the phenomenologically interesting transitions. In fact, in some string or supergravity inspired models, although non-universal, it has been argued that these trilinear terms can be written
as [23], \( A_{ij} = A_i + A_j \), and then the trilinear couplings are factorizable in matrix form,

\[
Y^A_{ij} = \text{Diag} \left( A^L_1, A^L_2, A^L_3 \right) \cdot Y + Y \cdot \text{Diag} \left( A^R_1, A^R_2, A^R_3 \right)
\]  

(22)

If the trilinear couplings possess this structure, and taking into account that off–diagonal elements are not largely affected by RGE evolution, we can estimate the LR off–diagonal mass insertions [17],

\[
(\delta_{LR})_{i\neq j} = \frac{1}{m_{\tilde{q}}} m_j ((A_2 - A_1)K_{ij}K^*_{j2} + (A_3 - A_1)K_{ij}K^*_{j3}) \simeq \frac{m_{\tilde{q}}}{7.3} \frac{\lambda'}{a} \frac{m_j}{500 \text{ GeV}} \times 6.7 \times 10^{-3},
\]

(23)

where \( m_j \) is the mass of the heaviest quark involved in the coupling and \( K_{ij} \) the matrix that diagonalizes the Yukawa couplings in this basis‡. Once more, to obtain the order of magnitude of this coupling we can assume \( \lambda'/a \simeq 1 \). These chirality changing MI give rise to large flavour changing and \( CP \) violation effects in several low energy observables. In particular, the most important observables are the \( b \to s\gamma \) transition§ and the direct \( CP \) violation in the kaon sector, \( \varepsilon'/\varepsilon [27, 28] \).

In the case of the \( b \to s\gamma \) transition, the quark mass involved would be \( m_b \) which implies a further suppression of \( 6 \times 10^{-3} \). Hence, we have,

\[
(\delta^d_{LR})_{23} \simeq \frac{\lambda'}{a} 4 \times 10^{-5}
\]

(24)

while the phenomenological limit is,

\[
\left| (\delta^d_{LR})_{23} \right| \leq 1.6 \times 10^{-2}.
\]

(25)

Therefore, it is clear that these \( LR \) MI do not have observable effects in the \( b \to s\gamma \) transition.

The second observable which is sensitive to \( LR \) MI is \( \varepsilon'/\varepsilon \). In this case, if we have a factorizable structure in the trilinear terms, the fermion mass involved is \( m_s \), and the MI is,

\[
(\delta^d_{LR})_{12} \simeq \frac{\lambda'}{a} 2 \times 10^{-6}
\]

(26)

to be compared with the limit,

\[
\text{Im} (\delta^d_{LR})_{12} \leq 2.0 \times 10^{-5}
\]

(27)

‡It is important to keep in mind that these off–diagonal MI do not depend on \( \tan \beta \) and are directly proportional to fermion masses. Only the flavour diagonal MI have a contribution proportional to \( m_{\mu} \tan \beta \).

§This decay is specially constraining in the large \( \tan \beta \) regime in any MSSM, even in the absence of new flavour sources in the soft breaking terms [27].
Therefore, in this case, due to the suppression of trilinear terms, no large contributions are possible. More generally, even in the case where the structure of the trilinear terms is not factorizable we always have at least a suppression associated with the highest mass in the down sector, i.e. $m_b/(500 \text{ GeV})$. In that case, we would have a MI,

\[
(\delta_{LH}^d)_{12} \simeq \frac{\lambda'}{a} K_{13}^d K_{23}^{d*} \times 10^{-5}
\]

with $K_{ij}^d$ the matrix that diagonalizes the down Yukawa matrix. It is clear that the maximum possible value is $K_{13}^d K_{23}^{d*} = 1/2$, and in most reasonable models we would expect a much lower mixing, for instance of the order of CKM mixings $\sim \lambda_C^5 = 3.2 \times 10^{-4}$. Therefore, it seems highly improbable to have a sizeable contribution to this observable. This has to be compared with the situation in general nonuniversal models where this observable can easily receive a sizeable contribution \cite{27, 28}.

Next, we examine the contributions to electric dipole moments (EDMs). In supersymmetric theories the EDMs get new contributions at 1-loop from the $\mu$ and $A$ phases. In fact, since the dawn of the SUSY phenomenology era, it is well known that the experimental limits on the neutron EDM constrain $\phi_\mu$ and $\phi_A$ at $M_W$ to be roughly $\leq 10^{-2}$, unless sfermion masses are pushed above $O(1) \text{ TeV}$. However, as we have seen in Eqs. (14–16), if we evolve these constraints to the GUT scale, the bounds on the initial phase of the $A$ terms are largely reduced and finally are $\phi_A \leq 10^{-1}$. On the other hand, at 1-loop, the phase of the $\mu$ term $\phi_\mu$ is invariant under the RGEs and only through the $B$ phase evolution some scale dependence is introduced at $M_W$ in the basis where $B_\mu$ is real. These effects are small for suppressed $A$-terms. Unfortunately, the mechanism to generate a $\mu$ term of the order of the electroweak scale, as phenomenology requires, is strongly model dependent. In the following, we simply assume that the relative phase between gaugino masses and the $\mu$ term is $\phi_\mu = 0$. Therefore, we concentrate on the effects of trilinear and Yukawa phases \cite{28, 2}.

Recently, it has been pointed out \cite{2} that in string models where non-universal trilinear terms are usually expected, even for purely real trilinear terms, you can expect large SUSY contributions to EDMs simply from the phases in the Yukawa matrix. These effects were used to show the tight constraints on non-universality from EDM experiments \cite{2}. Moreover, when additional flavour structures in the soft breaking terms are present, new phases become observable and can give rise to large effects here, as they do in neutral kaon mixing \cite{29}. However, although these contributions must be taken into account in a generic model, they are not necessarily important in any non-universal MSSM. For instance, if the trilinear terms are factorizable, as in Eq. (22), or the Yukawa and trilinear matrices hermitian \cite{31}, this problem is not present. Moreover, in a realistic string model, as the one we are analyzing here, we show that the problem is also softened and in fact absent in most reasonable constructions.

\*Always in the basis where we take the gaugino masses as real.
From here on, we consider the effects of an $O(1)$ trilinear phase. Clearly this case includes also possible effects of Yukawa phases [2]. As we discussed above the $A$-terms in $U(1)_{A}$-Dilaton SUSY breaking models are suppressed relative to diagonal sfermion masses. First, we analyze the case of a factorizable structure in the trilinear couplings. Here, the contribution to the diagonal $LR$ mass insertion is \[ (\delta_{LR})_{ii} \approx \frac{m_{i} \lambda' m_{e} / 5}{a} \left( \frac{m_{i}}{500 \text{ GeV}} \right)^{7.4 \times 10^{-3}}, \] (29)

Again, if we are interested in the electron or neutron EDMs, the additional suppression from the light quark mass is enough to satisfy the experimental limits [20, 30] *

\[
|\text{Im} \left( \delta_{LR}^{d} \right)_{11} | \leq 3.0 \times 10^{-6}, \quad |\text{Im} \left( \delta_{LR}^{u} \right)_{11} | \leq 5.9 \times 10^{-6}, \quad (30) \\
|\text{Im} \left( \delta_{LR}^{l} \right)_{11} | \leq 3.7 \times 10^{-7}.
\]

Here limits in the squark sector assume an average sfermion mass of 500 GeV, while in the slepton sector an average mass of 100 GeV is assumed. The mass suppression is $m_{u}/500 \text{ GeV} \simeq 1 \times 10^{-5}$, $m_{d}/500 \text{ GeV} \simeq 2 \times 10^{-5}$ and $m_{e}/100 \text{ GeV} \simeq 5 \times 10^{-6}$. Therefore all the EDM bounds are easily satisfied in this case.

A third interesting observable is the muon EDM. Although the current experimental limits, $d_{\mu} < 1.05 \times 10^{-18} \text{ ecm}$, cannot provide a constraint on the $LR$ MI, a recent proposal has been made at BNL for a dedicated experiment to reach a sensitivity of $10^{-24} \text{ ecm}$ [32]. If we take this value as the future experimental limit this implies that the future limit on the MI will be,

\[
|\text{Im} \left( \delta_{LR}^{l} \right)_{22} | \leq 5.3 \times 10^{-5}, \quad (31)
\]

and again the mass suppression is $m_{\mu}/100 \text{ GeV} \simeq 1 \times 10^{-3}$, which gives a value roughly a factor 5 below the expected future bounds.

Even in the case where this factorizable structure is not present, it is clear that in the leptonic sector, or in the down quark sector, the minimal suppression is given by, $m_{\tau}/(100 \text{ GeV}) = 1.7 \times 10^{-2}$, $m_{b}/(500 \text{ GeV}) = 6 \times 10^{-3}$. However, there is little suppression for the top quark in the up sector, $m_{t}/(500 \text{ GeV}) = 0.35$. In any case, it is clear that this minimal suppression is not enough even including the suppression

\[ 12 \]

\[ \text{assuming that the } K \text{ matrices that diagonalize the fermion masses have dominant terms } K_{ii} \simeq 1 \text{ in the basis of diagonal soft masses} \]

\[ ** \text{Here we consider only the bounds on the quark electric dipole moments. However the quark chromoelectric dipole moments from Hg atomic experiments can be more restrictive [31] and give rise to stronger constraints.} \]
of $\epsilon/5$. In these cases we would need to take also into account the mixings of the third generation with the first and second generations,

$$(\delta_{LR}^d)_{ii} = \left| K_{i3} \right|^2 \frac{c_3 A m_3}{m_i^2} \simeq \left| K_{i3} \right|^2 \frac{c_3 m_3 \lambda' m e / 5}{7.3 a (m^2 / 4)} \simeq \frac{\lambda'}{a} \frac{c_3 \left| K_{i3} \right|^2 m_3}{500 \text{ GeV}} \ 7.4 \times 10^{-3}$$

$$(\delta_{LR}^u)_{ii} = \left| K_{i3} \right|^2 \frac{c_3 A m_3}{m_i^2} \simeq \left| K_{i3} \right|^2 \frac{c_3 m_3 \lambda' m e / 5}{1.7 a (m^2 / 4)} \simeq \frac{\lambda'}{a} \frac{c_3 \left| K_{i3} \right|^2 m_3}{100 \text{ GeV}} \ 1.5 \times 10^{-2}$$

with $c_i$ a coefficient taking into account the RGE effects on the third generation trilinear couplings, Eqs. (14–16). We take $c_t \simeq 0.25$ and $c_b \simeq c_\tau \simeq 1$. Therefore we would have a maximum contribution,

$$(\delta_{LR}^d)_{11} \simeq \frac{\lambda'}{a} \left| K_{13}^d \right|^2 \ 4.4 \times 10^{-5},$$

$$(\delta_{LR}^u)_{11} \simeq \frac{\lambda'}{a} \left| K_{13}^u \right|^2 \ 6.5 \times 10^{-4},$$

for squarks of 500 GeV. In the case of sleptons, we have,

$$(\delta_{LR}^e)_{11} \simeq (\delta_{LR}^e)_{22} \simeq \frac{\lambda'}{a} \left| K_{13}^e \right|^2 \ 2.5 \times 10^{-4}$$

with an average slepton mass of 100 GeV.

Clearly, it is very difficult to make a definite statement without a complete theory of flavour that provides the relative rotation matrices between quarks and squarks, $K$. At least a very reasonable assumption [17, 18, 29] is to take these matrices to be of the same order as the CKM mixing matrix. This was indeed the choice in [2]. In that case, $K_{13}^u \simeq K_{13}^d \simeq 10^{-2}$, and obviously the bounds are in this case easily satisfied due to the $\epsilon$ suppression of the trilinear terms in anomalous $U(1)$ models. In the leptonic sector, things are not that straightforward because of our ignorance of the leptonic mixings.$^{11}$ In this case, the limits on the electron EDM (and future limits on the muon EDM) would require a mixing matrix roughly of the same order as the CKM matrix, as found for instance in [33, 34] even with maximal neutrino mixings.

We conclude that EDM constraints are not violated by nonuniversality of the $A$-terms in most reasonable scenarios in $U(1)_A$–Dilaton SUSY breaking models.

4 String insights

In this section we discuss how the cyclic permutation symmetry may ensure sfermion universality irrespective of the dominant source of supersymmetry breaking. Additionally, we discuss how the spectrum in the realistic free fermionic models may divide

$^{11}$Notice that neutrino mixings are largely influenced by the Seesaw mechanism, see for instance [33, 34, 35].
in a way which allows fermion mass hierarchy while ensuring that the phenomenological constraints on the sfermion mixings and phases are satisfied. As we have seen in the previous section, even with the suppression of the nonuniversal contributions in anomalous $U(1)$ SUSY breaking models, there are some low energy observables, such as the EDMs and $\varepsilon_K$, that place further constraints on the flavour structure of the nonuniversal soft terms. In first instance, the nonuniversality in the trilinear terms may contribute to EDMs. The important property of family universal $U(1)_A$ SUSY breaking is the suppression of the trilinear $A$–terms, which have the general form

$$A_{\alpha\beta\gamma} = F^m \left[ \hat{K}_m + \partial_m \log Y_{\alpha\beta\gamma} - \partial_m \log(\hat{K}_\alpha \hat{K}_\beta \hat{K}_\gamma) \right].$$

(37)

Here, Latin indices refer to the hidden sector fields, while Greek indices refer to the observable fields; the Kähler potential is expanded in observable fields as $K = \hat{K} + \hat{K}_\alpha |C^\alpha|^2 + \cdots$, and $\hat{K}_m = \partial_m \hat{K}$. The sum in $m$ runs over all of the SUSY breaking fields. Combined with a non–vanishing dilaton $F$–term this SUSY breaking scenario produces viable gaugino and sfermion masses, while maintaining adequate family universality, which is required by the FCNC and CP phenomenological constraints.

While the combined $U(1)_A$–Dilaton SUSY breaking scenario is phenomenologically appealing, the difficulty lies in the fact that we do not have a dynamical reason to prefer this scenario over other SUSY breaking sources. The natural question to ask is what are the underlying string structures that yield the family universal $U(1)_A$ and whether this structures may also be preserved in other SUSY breaking scenarios. In this perspective the anomalous $U(1)_A$ charges merely provide us with a window to the properties of the underlying string compactification, which resulted in family universality [37]. In turn, we may expect these highlighted characteristics to be preserved in other sectors of the models, as well as in the nonperturbative regimes. Namely, in the class of models that we discuss, the family universality originates in the most robust structure of the underlying string compactification.

The class of models under consideration are the free fermionic heterotic–string models. The construction of these models has been amply discussed in the past as well as numerous phenomenological studies. We refer the interested reader to the original literature for the details [36]. Here we focus on the properties of the models that pertain to sfermion universality and the string CP problem.

The free fermionic models are specified in terms of a set of boundary condition basis vectors. The class of models that we discuss here are spanned by a set eight such basis vectors. The first five basis vectors \{1, S, b_1, b_2, b_3\}, in the models of interest here, consist of the so called NAHE set [4], which yields after the generalized GSO projections an $N = 1$ supersymmetric $SO(10) \times SO(6)^3 \times E_8$ gauge group, with 48 generations in the 16 representation of $SO(10)$. The remaining three boundary conditions basis vectors, typically denoted by \{\alpha, \beta, \gamma\}, break the $SO(10)$ symmetry to one of its subgroups and reduce the number of generations to three. One from each of the sectors $b_1$, $b_2$ and $b_3$. The importance of the NAHE set lies in its correspondence with $Z_2 \times Z_2$ orbifold compactification [5]. The three sectors $b_1$, $b_2$ and $b_3$ correspond
to the three twisted sectors of the $Z_2 \times Z_2$ orbifold, whereas the basis vectors $\{\alpha, \beta, \gamma\}$ correspond to Wilson lines in an orbifold formalism.

The characteristic property of the $Z_2 \times Z_2$ orbifold compactification, which is the origin for the emergence of a family universal anomalous $U(1)$, is the cyclic permutation symmetry between the three sectors $b_1$, $b_2$ and $b_3$ with respect to their left- and right-moving world-sheet charges. In general, the boundary condition basis vectors $\{\alpha, \beta, \gamma\}$ break the permutation symmetry between the three light generations. However, in string models of refs. [10] and [11] the permutation symmetry is maintained, which is the reason for the existence of a family universal anomalous $U(1)$ in these models. It is important to note that this universality structure of the $Z_2 \times Z_2$ orbifold compactification is also reflected in other sectors of the models, in particular for the untwisted moduli. Thus, even if SUSY breaking is dominated by the untwisted moduli sector, squark degeneracy is still expected.

As correctly emphasized in ref. [2] the difficulty in understanding the flavour and string CP problems lies in the simultaneous requirement to generate fermion mass hierarchy and sfermion mass degeneracy. From Eq. (37) it is seen that in order not to generate non-universal $A$-terms the fields that generate the Yukawa matrices should have a vanishing $F$-term and hence should not break supersymmetry. To understand how this situation may come about in the string models we recall how the fermion mass hierarchy is generated in these models. The most relevant feature for our purposes is the identification of the sectors and states that contribute to the generation of the fermion mass hierarchy and those that do not. Thus, it is those sectors and states of the second kind, namely those that do not participate in the generation of the fermion mass hierarchy, that may play the role in the SUSY breaking dynamics.

In addition to the three chiral generations arising from the twisted sectors $b_1$, $b_2$ and $b_3$ the untwisted sector of the models produces three pairs of Higgs doublets, $\{h_i, \bar{h}_i\}$ ($i = 1, 2, 3$). Typically the free fermionic string models contain one additional sector that produces electroweak Higgs doublets $\{h_{\alpha \beta}, \bar{h}_{\alpha \beta}\}$. This sector arises from a combination of the $\alpha$ and $\beta$ basis vector and is denoted as the $\alpha \beta$-sector. The sectors $b_{j+2\gamma}$ give rise to states in the $16_j$ representation of the hidden $SO(16)_H$ gauge group. These are decomposed under the final unbroken $SO(16)_H$ subgroup, which typically contains two unbroken non-Abelian gauge groups. Being, for example, $SU(3)_H$ and $SU(5)_H$ in the models of refs. [10] [11].

The fermion mass terms arise from $N^{th}$-order superpotential terms of the form $cg_i f_i h \phi^{N-3}$ or $cg_i f_j h \phi^{N-3}$, where $c$ is a calculable coefficient, $g$ is the gauge coupling at the unification scale, $f_i$, $f_j$ are the fermions from the sectors $b_1$, $b_2$ and $b_3$, $h$ and $\bar{h}$ are the light Higgs doublets, and $\phi^{N-3}$ represent a product of Standard Model singlet fields that get a VEV and produce a suppression factor $(\langle \phi \rangle / M)^{N-3}$ relative to the cubic level terms. Here $M \sim 10^{18}$GeV is a scale related to the heterotic-string unification scale. At the cubic level only the couplings $\{u_j Q_j + N_j L_j\} \bar{h}_j$ and $\{d_j Q_j + e_j L_j\} h_j$ are allowed. Note that each generation couples to a different Higgs...
pair, and that at this level the cyclic permutation symmetry is retained. As the anomalous \( U(1) \) Fayet–Iliopoulos term breaks supersymmetry near the Planck scale, we must assign VEVs to some Standard Model singlets, along flat \( F \) and \( D \) directions. In this process some of the nonrenormalizable terms become effective renormalizable operators. At the same time some of the Higgs doublet representations receive large mass. For specific solutions only two Higgs doublets remain massless down to the electroweak scale. In typical analysis these have consisted of \( \tilde{h}_1 \) and \( h_{\alpha\beta} \). Analysis of the nonrenormalizable terms up to order \( N = 8 \) reveals the following structure [6],

\[
M_U \sim \begin{pmatrix} \epsilon, a, b \\ \bar{a}, A, c \\ b, \tilde{c}, \lambda_t \end{pmatrix}; \quad M_D \sim \begin{pmatrix} \epsilon, d, e \\ \bar{d}, B, f \\ \tilde{c}, \tilde{f}, C \end{pmatrix}; \quad M_E \sim \begin{pmatrix} \epsilon, g, h \\ \bar{g}, \tilde{D}, i \\ \tilde{h}, \tilde{i}, E \end{pmatrix},
\]

where \( \epsilon \sim (\Lambda_{Z'}/M)^2 \approx 0 \). \( U(1)_{Z'} \) is the Abelian symmetry in \( SO(10) \), which is orthogonal to the Standard Model. The diagonal terms in capital letters represent leading terms that are suppressed by singlet VEVs [6], and \( \lambda_t = O(1) \). The mixing terms are generated by hidden sector states from the sectors \( b_j + 2\gamma \) and are represented by small letters. They are proportional to \( \langle TT \rangle/M^2 \), where \( \langle TT \rangle \) represents the VEVs of these hidden sector matter states. The states from the sector \( b_3 \) are identified with the lightest generation [6].

The important aspect for our purpose here is to identify in the string models the sectors and states that contribute to the products of fields \( \phi^n \) which induce the fermion mass hierarchies. The analysis of the nonrenormalizable terms that contribute to the fermion mass matrices was carried out in detail in ref. [6]. Inspection of the nonrenormalizable terms in [6] then reveals that fields that contribute to all the leading terms of the mass matrices in Eq. (38) arise from the following sectors

\{
\text{untwisted sector, } \alpha\beta - \text{sector, } b_j + 2\gamma \quad (j = 1, 2, 3)
\}

(39)

The mixing terms have the general underlying form \( 16, 16, 10, 16 \). The first two \( 16 \)s are observable chiral \( SO(10) \) representations and the last two are hidden \( SO(16) \) vectorial representations. The \( 10 \) are the vectorial \( SO(10) \) representations that produce the light Higgs multiplets. The hidden \( SO(16) \) gauge group is typically broken to two non–Abelian group factors. Being, for example, \( SU(5) \) and \( SU(3) \) in the models of refs. [10, 11]. The vectorial \( 16 \) representation is therefore broken into \( 5 \oplus 5 \) and \( 3 \oplus 3 \) of the hidden \( SU(5) \) and \( SU(3) \) group factors, respectively. Typically the matter states of one of this group factors are included in the products \( \phi^n \) that generate the fermion mass terms, whereas the matter states under the second group factor form matter condensates that may trigger supersymmetry breaking [6].

A very robust and model independent solution to the flavour and string CP problems is to require that the states that appear in the products \( \phi^n \) that generate the fermion mass terms do not have a non–vanishing \( F \)-term. In the class of models under study here we note from the discussion above that this simply means that none
of the states that arise from the sectors in Eq. (39) has a non-vanishing $F$-term. This is a very modest requirement. The fields that trigger supersymmetry breaking should arise from other sectors in the models. They may originate, for example, from the Wilsonian $SO(10)$ breaking sectors that contain combinations of the boundary condition basis vectors \{$\alpha, \beta, \gamma$\}. As we note here the states from these sectors do not appear in the products $\phi^n$ that induce the fermion mass hierarchies in (38).

As discussed in section (2), a further source of FCNC and CP violation is the nonuniversality in the sfermion masses that arises from the Kähler potential and affects $\varepsilon_K$. We comment here on the possible role of the cyclic permutation symmetry in suppressing non-universal contributions from the Kähler potential. Correspondingly to Eq. (37) the Kähler potential contribution to the sfermion masses is given by $m_{ij}^2 \sim F^a F_b \partial_a \partial^b K_{ij}$, where $\partial_a$ denotes $\partial_a / \partial \Phi^a$. Therefore, we may in general expect non-universal contributions through the dependence of the Kähler potential on the fields $\Phi^a$ with $F^a \neq 0$. In the case of the mixed universal $U(1)_A$–Dilaton SUSY breaking scenario, as discussed above, these non–universal contributions are suppressed relative to the dominant universal contributions. However, also in the case that $F$–term of some field dominates we may expect that the cyclic permutation symmetry might play a role in ensuring sfermion degeneracy. Namely, we may expect that the permutation symmetry is preserved in other sectors of the models, and not merely in the $U(1)_A$ charges. Examining the spectrum of the model of ref. [10] we note that additional sectors in the model obey the permutation symmetry. For example, the sectors $b_i + b_j + \alpha + \beta \pm \gamma + (I)$, $i, j = 1, 2, 3 \; i \neq j$ in table 2 of [10]. This suggests a similar situation to the the family universal anomalous $U(1)_A$. Namely, $U(1)_A$ is a combination of three generation dependent world–sheet currents. Its universality arises precisely because it is such a combination. Similarly, due to the permutation symmetry, we may envision the SUSY breaking fields to be a combination of the states from the sectors $b_i + b_j + \alpha + \beta \pm \gamma + (I)$ that preserves the permutation symmetry hence retain the universal contribution to the sfermion masses.

5 Conclusions

In this letter, we have analyzed the FCNC and CP violation effects in a model with a dominant family universal anomalous $U(1)$ source of Supersymmetry breaking. In this theories, new sources of FC are always suppressed with respect to the average sfermion masses. We made a phenomenological analysis of these effects and found that in most of the cases they are in qualitative agreement with the phenomenological limits. The most sensitive low-energy observables to these new FC sources are the $\varepsilon_K$

\footnote{Nevertheless, it has been shown in Ref. [38] that it may be difficult to generate CP violation without a contribution from the geometric moduli which generically would have non-vanishing F-terms}
parameter, measuring indirect CP violation in the neutral kaon mixing and EDMs. These observables set important constraints in the structure of the sfermion mass matrices. Nevertheless, these constraints do not require a large fine-tuning of the initial parameters and can be satisfied in most realistic constructions, with some mild assumptions about the flavour structure of the Kähler potential.

Additionally, we discussed in this paper how the fermion mass hierarchy may arise in the string models without inducing non–universal $A$–terms. This is achieved provided that the fields which induce the fermion mass hierarchy do not break supersymmetry and have a vanishing $F$–term. From the structure of the free fermionic models we noted that this requirement places a restriction on the sectors from which the SUSY breaking fields may arise.

Finally, the family universality of the anomalous $U(1)$ in the free fermionic models originates in the cyclic permutation symmetry of the $Z_2 \times Z_2$ orbifold compactification with respect to the twisted sectors. In the three generation models under consideration this cyclic permutation symmetry is retained with respect to the horizontal $U(1)$ charges of the chiral generations. The cyclic permutation symmetry between the twisted sectors is the characteristic property of the $Z_2 \times Z_2$ orbifold. As we discussed here, preservation of the cyclic permutation symmetry is reflected in other sectors of the models and may be instrumental to understand the sfermion mass degeneracy irrespective of the dominant source of supersymmetry breaking. Naturally, a complete solution to the sfermion mass degeneracy problem cannot be attained before a simultaneous detailed analysis of the fermion mass hierarchy is obtained. However, short of this ambitious and still distant goal, we note how the underlying structure of the realistic free fermionic models may be instrumental in allowing fermion mass hierarchy while ensuring sfermion mass degeneracy.

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