Precipitation Extremes and Water Vapor

Relationships in Current Climate and Implications for Climate Change

J. David Neelin1 · Cristian Martinez-Villalobos2,3 · Samuel N. Stechmann4 · Fiaz Ahmed1 · Gang Chen1 · Jesse M. Norris1 · Yi-Hung Kuo1 · Geert Lenderink5

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Abstract

Purpose of Review: Review our current understanding of how precipitation is related to its thermodynamic environment, i.e., the water vapor and temperature in the surroundings, and implications for changes in extremes in a warmer climate.

Recent Findings: Multiple research threads have i) sought empirical relationships that govern onset of strong convective precipitation, or that might identify how precipitation extremes scale with changes in temperature; ii) examined how such extremes change with water vapor in global and regional climate models under warming scenarios; iii) identified fundamental processes that set the characteristic shapes of precipitation distributions.

Summary: While water vapor increases tend to be governed by the Clausius-Clapeyron relationship to temperature, precipitation extreme changes are more complex and can increase more rapidly, particularly in the tropics. Progress may be aided by bringing separate research threads together and by casting theory in terms of a full explanation of the precipitation probability distribution.

Keywords Rainfall · Climate change · Deep convection · Extreme events · Precipitation probability · Stochastic model

Introduction

Examination of climate change impacts on the probability of strong precipitation events has been an ongoing effort since the late 1980s (Noda and Tokioka 1989) and much work since then (e.g., Meehl et al. 2000; Allen and Ingram 2002; Trenberth et al. 2003; Tebaldi et al. 2006; Min et al. 2011; Chou et al. 2012; O’Gorman 2012; Wuebbles et al. 2014; Sillmann et al. 2013; Pendergrass and Hartmann 2014; Myhre et al. 2019; Papalexiou and Montanari 2019; Tabari 2020) including reviews by Schneider et al. (2010), Trenberth (2011), O’Gorman (2015) and Donat et al. (2020). However, confidence in projections of precipitation change is affected by limitations in simulations of various aspects of precipitation in current climate (e.g., Biasutti et al. 2006; Qian et al. 2015; Lintner et al. 2017; Hagos et al. 2021; Biasutti et al. 2018), by differences in the projection of changes in extreme precipitation among models, especially in the tropics (Pendergrass et al. 2019), by sensitivity to model parameters (e.g., Knight et al. 2007; Sanderson 2011; Covey et al. 2013; Bernstein and Neelin 2016; Qian et al. 2018), and by limited understanding of the interaction between the large-scale flow and small-scale convective precipitation (Tomassini 2020). Narrowing uncertainties in simulated precipitation probability distribution changes becomes all the more important as procedures for event attribution (Haustein et al. 2016; Eden et al. 2016; van der Wiel et al. 2017; van Oldenborgh et al. 2017; Emanuel...
2017; Risser and Wehner 2017; Pall et al. 2017; Wang et al. 2018) potentially inform decisions regarding whether
and how to rebuild after extreme events. Such procedures
provide estimates of the extent to which probabilities of
equaling or exceeding a given event size have changed
due to anthropogenic warming, typically using ensembles
of current climate simulations compared to simulations
approximating conditions that would have occurred in
absence of anthropogenic emissions.

It has been common to ask whether precipitation scales
with the Clausius-Clapeyron (CC) relationship of saturation
water vapor to temperature, roughly 7% per degree Celsius
of large-scale warming or whether it increases slower (Eden
et al. 2016), or faster than this, the latter termed “super-
CC” scaling (Pall et al. 2007; Lenderink and Van Meijgaard
2008; Sugiyama et al. 2010; Loriaux et al. 2013; Prein et al.
2017; Wasko and Sharma 2014; Lenderink et al. 2017; Pfahl
et al. 2017; Pendergrass 2018). Various simple balances
based on approximations to the moisture budget or energy
balance, respectively, have been put forward, as discussed
in the Section “Moisture Equation and Thermodynamic
Equation”, but these do not attempt to explain the under-
lying distributions of precipitation or vertical velocity.
Diagnostic statements based on these budgets often divide
changes in moisture convergence into those associated with
changes in moisture, commonly termed the “thermo-
dynamic” contribution, and those associate with changes
in convergence, termed the “dynamic” contribution, with
the latter being viewed as separate and not easily explained.

At the same time, there have been advances in under-
standing the relationship of precipitation—particularly that
associated with deep convection—to its temperature and
moisture environment. Part of this literature has shown that
essential features of the precipitation distribution can be
explained relatively simply in terms of the onset of precip-
itation in relationship to variations affecting this thermody-
namic environment. These results imply that the thermo-
dynamic and dynamic contributions affect the precipitation
jointly. Indeed, for the tropical case, we argue that the ther-
mospheric and dynamic components are involved in a
feedback such that it may be more productive for the field
to move away from this artificial separation.

Key Balances

Moisture Equation and Thermodynamic Equation

The vertically integrated moisture equation may be written:
\[
\langle \partial_t q \rangle + \langle \nabla \cdot v q \rangle = E - P,
\]
where \( q \) is water vapor mixing ratio, \( t \) is time, \( P \) is
precipitation, \( E \) is evaporation, \( v \) is the horizontal wind
vector and \( \langle x \rangle = \int_0^{P_s} x dp/g \) denotes a mass weighted
integral in pressure coordinates, with \( P_s \) surface pressure,
and \( g \) the gravitational acceleration. Equivalently, (1a) can
be written as
\[
\langle \partial_t q \rangle + \langle \nabla \cdot v q \rangle + \langle \omega \partial_p q \rangle = E - P,
\]
where the transport term has been rewritten using \( \langle \nabla \cdot v q \rangle = \langle \nabla \cdot v q \rangle + \langle \omega \partial_p q \rangle \), where \( \omega \) is vertical pressure velocity, with
the vertical transport term being equivalent to the moisture
integrated with the horizontal convergence.

Similarly, the vertically integrated thermodynamic
energy equation (temperature equation) is, in an approxima-
tion sufficient for present purposes,
\[
\langle \partial_t c_p T \rangle + \langle \nabla \cdot c_p T \rangle + \langle \omega \partial_p s \rangle = \langle Q_c \rangle + F_s
\]
with \( T \) the temperature, \( s = c_p T + \phi \) the dry static energy,
\( c_p \) the heat capacity at constant pressure, \( \phi \) the geopotential,
\( F_s \) the net flux of longwave and shortwave radiation plus
 sensible heat into the column. The convective heating \( Q_c \)
includes latent heat release by condensation and freezing
processes and vertical transport by subgrid scale convective
motions, and can here be approximated as \( \langle Q_c \rangle = L P \),
with \( L \) the net latent heat of condensation released per unit
of moisture loss by precipitation. Condensate loss to the
column by lateral transport is sometimes represented by a
precipitation efficiency (Muller and Takayabu 2020), which
is particularly relevant at smaller scales or high rates of
moisture convergence, and ideally should be modeled by
including a condensate equation. Freezing processes are not
explicitly addressed here for brevity, but can be included
using a frozen moist static energy (Wing et al. 2007, 2014).

The leading balance for the moisture equation under
heavily precipitating conditions, especially in the tropics,
can be written equivalently as either
\[
\langle q C \rangle \approx P \text{ or } \langle \omega \partial_p q \rangle \approx P
\]
defining convergence \( C = -\nabla \cdot v \), and using the continuity equation
\( C = \partial_p \omega \), assuming that \( \omega \) is small at upper and lower limits of integration. With
moisture being relatively small in the upper troposphere
these approximately correspond to a balance between low-
level moisture convergence and precipitation. Likewise,
the leading balance for the thermodynamic equation under
similar circumstances can be written equivalently as either
\[
-(\langle x \rangle C) \approx L P \text{ or } \langle \omega \partial_p s \rangle \approx L P.
\]
and thermodynamic equations separately, including the time derivative of moisture, can be important. In the midlatitudes where horizontal advection is important, \((4)\) may be refined with the quasi-geostrophic omega equation (O’Gorman 2015; Nie et al. 2018) which includes the effects of vorticity and temperature advection on vertical velocity. It is further worth underlining that a dominant source of variations leading to the probability density function (pdf) of precipitation arises from the variations in convergence.

**“Thermodynamic” and Dynamical Contributions and Issues**

For changes in mean precipitation, defining \(\Delta x\) to be the difference in time mean under a global warming scenario relative to historical climatology, denoted with \(\bar{x}\), it has been common to diagnose the changes in the moisture budget as:

\[
\langle \Delta q \bar{C} \rangle + \langle \bar{q} \Delta C \rangle + \ldots = \Delta P - \Delta E
\]

where “...” denotes additional terms such as changes in \(v \cdot \nabla q\) and transients (e.g., Seager et al. 2014), and the second-order covariance term \(\langle \Delta q \Delta C \rangle\) is ignored. The first and second terms in \((5)\) have come to be referred to as the “thermodynamic” and “dynamic” contributions, respectively (Chou and Neelin 2004; Emori and Brown 2005; Held and Soden 2006; Oueslati et al. 2019; O’Gorman et al. 2021), where the thermodynamic terminology for related analysis of clouds (Bony et al. 2004) quickly supplanted “direct moisture effect” (Chou and Neelin 2004). This breakdown between the two components highlights that the rich-get-richer or wet-get-wetter effect (i.e., regions where historically precipitation exceeds evaporation will receive enhanced precipitation, while regions where historically evaporation exceeds precipitation will become even drier) arises from the thermodynamic term, but can be regionally increased or offset by the dynamic term.

A corresponding division can be defined for precipitation quantiles (Emori and Brown 2005; Chen et al. 2019; Norris et al. 2019a), with \(x_i\) denoting \(x\) conditioned on the \(i^{th}\) quantile of \(P\) in historical or future climatology and \(\Delta x_i\) denoting the change of \(x\) at the \(i^{th}\) quantile of \(P\) in a warmer climate. We use percentiles or return time in the historical climatology to refer to these quantiles where convenient. The leading balances for precipitation \(P_i\) at a given quantile are

\[
\langle \Delta q_i C_i \rangle + \langle q_i \Delta C_i \rangle \approx \Delta P_i
\]

with the first and second terms on the LHS again termed thermodynamic and dynamic. The first term represents additional moisture in a warmer climate subject to the present-day circulation when precipitation extremes occur, while the second term represents the future changes to circulation when extremes occur acting on present-day moisture. However, each grid point is typically analyzed in isolation (Emori and Brown 2005; Pfahl et al. 2017; Tandon et al. 2018; Norris et al. 2019a), so that, unlike for the mean budget \((5)\), \(C_i\) does not represent a closed circulation.

The thermodynamic term, \(\langle \Delta q_i C_i \rangle\), is well predicted by the increase in temperature corresponding to the given quantile of \(P\), with about 7% increase of moisture at each vertical level per K warming, as per the Clausius-Clapeyron relation (Chen et al. 2019; Norris et al. 2019a). The dynamic term, \(\langle q_i \Delta C_i \rangle\), could in theory result from an amplification/dampening of the circulation or a change in the vertical structure of the circulation. In the CESM Large Ensemble, the component relating to vertical structure is small, so that the dynamic term approximately results from a rescaling of the circulation when precipitation occurs (Chen et al. 2019; Norris et al. 2019a).

For precipitation event accumulations, i.e., precipitation integrated from beginning to end of a rain event, a similar budget can be written that also involves duration, although changes in the latter tend not to be of leading importance (Norris et al. 2019b) in diagnostics of the Community Earth System Model (CESM) Large Ensemble.

This approximation neglects the conditionally averaged terms such as \(\langle \partial_t q \rangle_i\), which is small in the tropics and shows some cancellation with \(\langle v \cdot \nabla q \rangle_i\) for midlatitude weather systems (Chen et al. 2019; Norris et al. 2019a). In other words, extreme precipitation is maintained not by the water vapor being reduced in the column but by moisture transport. The covariance between \(q_i\) and \(C_i\) at each percentile is also ignored (i.e., \(\langle q C \rangle_i \approx \langle q_i C_i \rangle\)), with the vertical structure of \(C_i\) varying with percentile.

Figure 1 shows an example of this decomposition for changes in 10-year event accumulation size between 1990–2005 and 2071–2080 from the CESM large ensemble under the scenario for anthropogenic emissions Representative Concentration Pathway RCP8.5 (Meinshausen et al. 2011). While the contribution from moisture changes is substantial both within and outside the tropics, the enhancement by convergence changes is particularly large in certain regions of the tropics. At longer return times (not shown), both contributions are larger, and the dynamical contribution tends to be further enhanced (Norris et al. 2019b). Similar spatial structures of thermodynamic and dynamic components, conditioned on precipitation extremes, have been calculated by other methods (Emori and Brown 2005; Held and Soden 2006; Pfahl et al. 2017; Tandon et al. 2018). In midlatitude regions where precipitating events are often associated with Atmospheric Rivers (ARs) (Zhu and Newell 1998; Gimeno et al. 2014; Waliser and Guan 2017; Ralph et al. 2017; Valenzuela and Garreaud 2019), projected changes to ARs likewise depend on changes to convergence as well as changes in moisture (Ma et al. 2019; Payne et al. 2020).
Fig. 1 Contributions to changes in 10-year event accumulation size (mm) over global land due to changes in moisture and in convergence, respectively, a.k.a. thermodynamic and dynamic contributions from the CESM large ensemble between 1990–2005 and 2071–2080 under RCP8.5, adapted from (Norris et al. 2019b). Note the substantial enhancement by convergence changes in certain regions in the tropics. Missing values are where fewer than 80% of bootstrap replications agree on the sign of the change.

A closely related diagnostic of the dynamic and thermodynamic contributions has been advocated based on approximations to the energy budget. If the stratification \((-\partial p_s)\) is assumed to be approximately moist adiabatic in convective regions, it can roughly be replaced by \(\partial p q_{sat}\) in (4) (Muller et al. 2011; Muller and Takayabu 2020) with the saturation specific humidity \(q_{sat}\) evaluated along moist adiabats. Given that departures from moist adiabatic structure are noted in observations even in heavily precipitating conditions (Holloway and Neelin 2009, 2010; Schiro et al. 2016), one might ask what the role is for this approximation compared to simply using the moisture equation leading balance (3) (e.g., Chou et al. 2012). Indeed, for some purposes the two formulations behave similarly (Muller and Takayabu 2020). Two useful aspects of making this approximation are that (i) the dry stability \((-\partial p_s)\) tends to vary less than the moisture in the large-scale convective environment, and (ii) it provides an indication of how the stratification may be expected to change with warming, partially compensating increases in moisture, without having to assume constant relative humidity. However, it is worth underlining that the overall stability for moist motions also depends on the vertical structure and vertical extent of the vertical velocity in (4), both of which may change.

If we expand (4) for quantiles, an expression parallel to (6)—and with similar approximations—for the thermodynamic equation is:

\[-(\Delta s_i C_i) - (s_i \Delta C_i) \approx L \Delta P_i\]  \hspace{1cm} (7)

For midlatitudes, this thermodynamic balance may be combined with vorticity and temperature advection in the quasi-geostrophic omega equation which can give similar dynamic amplification from increases in convective heating (Nie et al. 2018) with competition among changes in moisture, dry stratification in depth of heating (Li and O’Gorman 2020) and nonlinearity of the heating response (Nie et al. 2020). While the thermodynamic budget (7) has not yet been quantified in the same ways as (6), it allows us to make a simple but important point. Combining (6)–(7) to eliminate \(P\) yields a balance governing the dynamical changes:

\[(s_i + L q_i \Delta C_i) \approx -(\Delta s_i + L \Delta q_i) C_i + \ldots\]  \hspace{1cm} (8)

where “+...” is a reminder that terms considered small relative to \(P\) in (6) and (7) are not necessarily negligible here. The terms in \(\Delta q_i\) and \(\Delta s_i\) on the rhs of (8) must be balanced by the \(\Delta C_i\) term on the lhs. In other words, the interaction of the thermodynamic equation with the moisture equation can imply that changes in the dynamical terms must arise, i.e., there can be a thermodynamic requirement that changes in convergence occur. The substantial dynamical feedback at long return times in the tropics, or under midlatitude convective conditions where these balances apply, should thus be expected as seen in Fig. 1. In this sense, the term “thermodynamic” for changes associated only with moisture is a misnomer, and the division between thermodynamic and dynamic effects is artificial. We hypothesize that progress in better understanding intense precipitation changes, especially in the tropics, will require theory that combines both of these effects, especially since both combine naturally in the theory of how precipitation probability distributions arise, outlined in the Section “How Precipitation Distributions Arise: Variations Across a Moisture/Buoyancy Threshold”.

**Scaling Arguments**

There are variants in the way the term “scaling” is used for precipitation, but in essence they posit that the precipitation
probability distribution, or some aspect of it, under the warmer climate is similar to that under historical conditions once precipitation is scaled by some factor. This can be argued from leading balances in either the moisture (3) (Pall et al. 2007; O’Gorman and Schneider 2009, 2009; Trenberth 2011; Chou et al. 2012) or thermodynamic (4) (Muller et al. 2011), assuming that the short timescale variations that lead to the distribution come primarily from convergence and change little in the warmer climate, while the large-scale moisture or stratification increases by a factor \((1 + \alpha)\), with \(\gamma\) given by Clausius-Clapeyron. Chou et al. (2012) posit that the pdf 
\[
\frac{\text{prob}(P)}{\langle q_i \Delta C_i \rangle} \quad \text{for reference. Red solid and dashed curves: 95th percentile hourly precipitation intensity from current and future climate simulations, respectively, with the Rossby Center regional climate model over the Netherlands, with thin red curve showing CC-based prediction, shifting the current climate curve by the projected warming and multiplying by 1.07 (adapted from Zhang et al. 2017)
\]

Another step in this argument is whether the variations in meteorological conditions that lead to these diagrams in current climate can potentially be interpreted as characterizing the way they will scale with temperature under global warming (Fowler et al. 2021). An analysis of this by Zhang et al. (Zhang et al. 2017) is summarized by the red curves in Fig. 2. A hydrostatic regional climate model reproduces apparent super-CC scaling in current climate, but under a greenhouse change scenario, the change of the curve does not follow the current climate curve, but rather the curve shifts in a manner that is more consistent with CC. A similar result was earlier obtained (Sahany et al. 2014) for the way that curves characterizing the onset of convection shift under warming.

Despite these caveats, there is considerable evidence in model simulations for regions of super-CC scaling (Attema et al. 2014; Lenderink et al. 2019; Singleton and Toumi 2013; Prein et al. 2017; Lochbihler et al. 2019). Also, convection-permitting climate model simulations reveal shifts in scaling curves exceeding the CC rate, in particular for the most extreme events and for scenarios that have
relatively small lapse rate changes (Lenderink et al. 2021). Maps of fractional change in precipitation at a given high percentile in model simulations of global warming provide one means of visualizing the scaling as a function of region. Commonly there are some super-CC regions, i.e., where this fractional change is larger than expected from CC scaling, especially within the tropics (Pall et al. 2007; Sugiyama et al. 2010; Hoegh-Guldberg et al. 2018), as further discussed in the Section “How Precipitation Distributions Change: Changes in Extremes”. Indications of a band of super-CC scaling within the tropics may also be seen in observed annual maximum precipitation (Westra et al. 2013), although highly uneven spatial coverage places caveats on this.

The key balances of the Section “Thermodynamic” and Dynamical Contributions and Issues” suggest a reason for this. The thermodynamic contribution tends to obey CC scaling, so departures from this trend to occur where the dynamical contribution is substantial. The moist static energy balance (8) suggests that there are conditions under which this must occur, and furthermore be proportional to changes in moisture and stratification. If we approximate $\langle s_i \Delta C_i \rangle \approx -M_{si} \Delta C_i^*, \langle L_{qi} \Delta C_i \rangle \approx M_{qi} \Delta C_i^*$, with $\Delta C_i^*$ characterizing the low-level convergence and the change in vertical structure being small, (6)–(7) yield

$$\Delta P_i \approx \left( \langle \Delta q_i C_i \rangle + \langle \Delta s_i L_i^{-1} + \Delta q_i \rangle (M_{qi}/M_i) \right) \text{“thermodynamic”}$$

with $M_{qi}$ a gross moisture stratification for the changes, and $M_i = M_{qi} - M_{gi}$ a gross moist stability (which is typically smaller). The first term is the effect commonly termed “thermodynamic”, which yields CC scaling if $\Delta q_i$ changes with constant relative humidity. The second term represents the result of the feedback between convective heating and convergence changes required by simultaneously satisfying thermodynamic and moisture balances. Recall that it is commonly termed “dynamic” because it entails changes in the flow; the form here clarifies that the vertical velocity changes are proportional to moisture and stability changes that initiate the feedback. It tends to amplify the precipitation changes if the moisture change effect in the feedback term is larger than the compensation by stratification increases, and thus tends to yield changes that differ from CC scaling. The importance of feedbacks from changes in convective heating on changes in vertical motion have been noted in multiple studies (Pall et al. 2007; Pendergrass and Gerber 2016; Lenderink et al. 2017; Berg et al. 2013; Singleton and Toumi 2013; Nie et al. 2018) to varying degrees (Abbott et al. 2020). We underline that (9) is a statement of this, phrased in terms of the large-scale moist stability of the thermodynamic environment. There have been slow advances in theory and diagnostics of gross moist stability over the years (Neelin and Held 1987, 2000, 2007; Yu et al. 1998; Back and Bretherton 2006; Raymond et al. 2007; Raymond et al. 2009; Muller and O’Gorman 2011; Inoue and Back 2015, 2017), but (9) suggests a need for further advances. It makes the basic point that super-CC scaling of changes is quite reasonable to expect under conditions that can be widespread in the tropics.

**Precipitation Relationship to the Thermodynamic Environment**

A more quantitative route to modeling and understanding precipitation extremes must relate precipitation to its thermodynamic environment, in a manner more similar to a parameterization of precipitation. An empirical relationship linking tropical oceanic precipitation to its environmental moisture content has been noted at both daily (Bretherton et al. 2004) and instantaneous timescales (Peters and Neelin 2006) and explorations of this continue (e.g., Rushley et al. 2018; Wolding et al. 2020). For regions dominated by deep convection, this empirical relationship can be understood by computing the buoyancy of entraining plumes (Holloway and Neelin 2009; Schiro et al. 2016), which are consistent with observations provided the plume is allowed to entrain environmental air through a deep lower tropospheric layer.

The precipitation-moisture relationship shows dependence on column temperature (Neelin et al. 2009; Kuo et al. 2020), land surface (Ahmed and Schumacher 2017) and the land-sea interface (Bergemann and Jakob 2016). Figure 3a shows one way of quantifying this. Column water vapor (CWV) is used as a measure of moisture since satellite microwave retrievals are widely available over oceans (e.g., Hilburn and Wentz 2008). Temperature is measured by the column-integrated saturation value $\tilde{q}_{sat}$ from reanalysis (Kanamitsu et al. 2002) (other bulk measures of tropospheric temperature work similarly). For the range of temperatures relevant to the tropics, the dependence on moisture can be collapsed to a common form for all temperatures, when the water vapor is taken relative to a critical value $w_c$. This value characterizes the rapid increase of precipitation associated with the onset of convective conditional instability, with the difference $(CWV - w_c)$ acting like a crude measure of buoyancy, albeit with errors due to the presence of more detailed vertical structure than these bulk variables can capture. Percentiles of precipitation (from TRMM precipitation radar, coarse-grained to the CWV grid as in (Kuo et al. 2020) as a function of $(CWV - w_c)$ behave similarly for each temperature. The 50th percentile behaves similarly to the conditional average, exhibiting a rapid pickup above the critical value; higher percentiles have a corresponding rapid pickup in parallel with this.

Much of the moisture-temperature dependence is explainable if the moisture and temperature variations are
mapped onto a measure of cloud buoyancy (Ahmed and Neelin 2018; Schiro et al. 2018; Adames et al. 2021). The resulting precipitation-buoyancy relationship is valid across a wide range of environments, and is therefore a generalized version of the precipitation-moisture relationship—essentially acting like an empirical precipitation parameterization. Figure 3b depicts this buoyancy relationship, where tropical oceanic precipitation is conditionally averaged by a measure of lower tropospheric buoyancy ($B_L$). The probability density functions (pdfs) of $B_L$ are sharply peaked near the value beyond which the conditional precipitation increases linearly.

Figure 3 suggests a simple empirical precipitation parameterization in which a threshold buoyancy value ($B_c$) separates the non-precipitating and precipitating regimes, and the linear precipitating regime is characterized by its slope, $\alpha$:

$$P = \begin{cases} \alpha (B_L - B_c), & \text{if } B_L > B_c \\ 0, & \text{if } B_L \leq B_c. \end{cases}$$

(10)

The use of $B_L$ provides a framework to understand the sensitivity of precipitation to changes in its thermodynamic environment. Following Ahmed et al. (2020), we can decompose $B_L$ into two terms:

$$B_L = \text{CAPE}_L - \text{SUBSAT}_L.$$

(11)

In (11), $\text{CAPE}_L$ is akin to the commonly used convectively available potential energy (CAPE) and depends on the difference between the boundary layer moist enthalpy and the free-tropospheric temperature, acting like a lower tropospheric static stability. The decreases in cloud buoyancy due to entrainment of dry air are captured by $\text{SUBSAT}_L$. The empirical buoyancy framework therefore quantifies the joint influence (Louf et al. 2019; Powell 2019; Tian and Kuang 2019) of moisture and CAPE on precipitation.

The buoyancy framework can be used to study precipitation distributions, provided one has knowledge of $B_L$ evolution. The problem can be simplified by assuming that CAPE$_L$ remains fixed, so that $B_L$ variations can be tracked purely by moisture variations. Under this assumption, the buoyancy threshold for precipitation, $B_c$, is transformed into a moisture threshold, $q_c$. This simple—but empirically rooted—precipitation parameterization can then be coupled to equally simple evolution equations for the environmental thermodynamics. Surprisingly, this has implications for probabilities of precipitation extremes, as outlined in the following section.

How Precipitation Distributions Arise: Variations Across a Moisture/Buoyancy Threshold

Denoting the size of a precipitation event as $S$, measured in units of mm, a key quantity of interest is the pdf $p_S(S)$ of event sizes. The event size pdf can be measured from

![Fig. 3 Two ways of quantifying the relationship of precipitation to its moisture-temperature environment. (a) Precipitation percentiles as a function of column water vapor (CWV) relative to a critical value $w_c$ that depends on temperature, here measured by the column-integrated saturation humidity $\tilde{q}_{sat}$. CWV-$w_c(\tilde{q}_{sat})$ acts as a rough proxy for buoyancy of convective plumes, yielding a rapid pickup in precipitation percentiles or conditional average in the vicinity of the critical value that is similar for all temperatures (Kuo et al. 2018). (b) TRMM 3B42 precipitation conditionally averaged by an empirical estimator of plume buoyancy $B_L$ for four different tropical ocean basins (left axis) and the pdfs of $B_L$ (right axis).](current-climate-change-reports)
observational data or climate model data (Peters et al. 2002; Neelin et al. 2008; Peters et al. 2010; Deluca and Corral 2014; Neelin et al. 2017; Martinez-Villalobos and Neelin 2018), and has been seen to take the form

\[ p_n(S) \propto S^{-\tau} \exp[-S/S_L], \tag{12} \]

which includes a power law range due to \( S^{-\tau} \), and an exponential decay \( \exp[-S/S_L] \) that becomes significant for the largest events beyond a large cutoff size \( S_L \) (Stechmann and Neelin 2011, 2014; Neelin et al. 2017). See Fig. 4 for an illustration of the pdf from observational data.

Also shown in Fig. 4 is the pdf for a related quantity, the daily precipitation, i.e., the amount of precipitation \( P \) that falls in one day, whose pdf has a less-steep, approximately power law range. The power law range is slightly less steep in the midlatitude compared to the tropical case (in part due to 1-h vs 1-m data), but the pdf still shows essentially the same functional form. There is a long history of approximating rainfall distributions for daily average intensities (or similar averaging interval) as a gamma distribution

\[ p_P(P) \propto P^{-\tau_P} \exp[-P/P_L], \quad \tau_P < 1 \tag{13} \]

(Barger and Thom 1949; Thom 1958; Groisman et al. 1999) including for projections of changes under warming (Wilby and Wigley 2002; Watterson and Dix 2003) (noting that some related distributions have also been used (Papalexiou and Koutsoyiannis 2013; O’Gorman 2014; Kirchmeier-Young et al. 2016; Moustakis et al. 2021)). We are now in a position to explain the physics behind this, beginning with an explanation of the event accumulation size distribution, which is simpler because it involves only the precipitating regime.

Why does the event size pdf take the form of (12)? A theory can be described using some of the intuition from previous sections (Stechmann and Neelin 2011, 2014; Hottovy and Stechmann 2015b, Neelin et al. 2017). Two important ingredients are (1) the threshold provided by the rapid onset in (10) and Fig. 3, and (2) variations across that threshold. To see this, consider the evolution equation for vertically integrated moisture from (1a), and write it in approximate form as

\[ dq = -Pdt + \tilde{\delta}^{1/2} d\tilde{W}_t, \tag{14} \]

where the moisture convergence \( \langle \nabla \cdot \mathbf{v}_q \rangle \) and evaporation \( E \) have been approximated by white noise. The parameterization of \( P \) could be as in (10), which indicates the first important ingredient: a threshold in buoyancy and therefore in moisture. The second important ingredient is that the evolution in (14) governs the variations in both non-precipitating and precipitating regimes.

To arrive at the event size pdf in (12), consider the evolution of moisture, according to (14), at the start of a precipitation event. Prior to the start of the event, \( P = 0 \) in (14). At the start of the event, \( P \) turns on, and one can replace time with a running accumulation \( \tilde{S} \), i.e., the amount of water rained out up to time \( t \) within an event:

\[ \tilde{S}(t) = \int_{t_0}^t P(t')dt', \quad \text{i.e., } d\tilde{S} = Pdt. \tag{15} \]

The size \( S \) of an event is simply the value of the running accumulation \( \tilde{S} \) when the event terminates. Since \( d\tilde{S} = Pdt \), one can rewrite (14) as

\[ dq = -d\tilde{S} + D^{1/2} d\tilde{W}_S. \tag{16} \]

The event size \( S \) can now be viewed as the “time” elapsed in waiting for moisture to decrease below a threshold value, similar to the threshold in (10), at which point the precipitation terminates. This is a first-passage-time problem for the stochastic differential equation in (16), and it can be solved analytically for the event size pdf \( p_n(S) \), leading to the form shown in (12) (Gardiner 2009; Stechmann and Neelin 2014). Intuitively, the two parts of the pdf—the power law \( S^{-\tau} \) and the exponential decay \( \exp[-S/S_L] \)—arise from the two parts of the stochastic evolution in (14). In particular, the stochastic variability from \( D^{1/2} d\tilde{W}_t \) leads to the power law by the random crossing of a threshold, and after enough time has elapsed, the precipitation loss \( -Pdt \) becomes more important and eventually cuts off the power law. These two processes are of equal importance at the cutoff scale, \( S_L \).

While physically accumulations (from event onset to termination) provide a more fundamental connection between the moisture budget and precipitation pdfs, in practice precipitation aggregated over fixed time intervals (e.g., daily precipitation) is the main object of the research community interest. An important distinction between event accumulation and daily precipitation is that the accumulation only depends on dynamics occurring while precipitating whereas daily precipitation mixes dynamics occurring at wet and dry times. This distinction has an important imprint in the resulting accumulation and daily precipitation pdfs. Figure 4 shows the typical shape of these pdfs for one location in the tropics (Fig. 4a) and one location in midlatitudes (Fig. 4b). In both cases pdfs display a power law range for low and moderate values and a cutoff scale where the probability drops much faster, in agreement with the stochastic prototype for accumulations. The main difference is the gentler power law exponent \( \tau_P \) for daily precipitation \( (\tau_P < \tau) \), which can be explained using the stochastic prototype for accumulations as a starting point. Under suitable conditions for the length of accumulation events with respect to the averaging interval (e.g., 1 day for daily precipitation), daily precipitation is approximately the summation of individual accumulation events within a day. This reduces daily precipitation power law exponent
because days with multiple accumulation events contribute to a larger probability of higher daily precipitation amounts at the expense of low and moderate amounts, flattening the power law range (Martinez-Villalobos and Neelin 2019). The daily precipitation cutoff scale \( P_L \) is set by the underlying accumulation cutoff scale \( S_L \) (confirmed in observations over the USA, see Martinez-Villalobos and Neelin 2018), implying that daily precipitation extremes are approximately controlled by the same balances (fluctuations in moisture convergence vs moisture loss by precipitation) as accumulation extremes (Martinez-Villalobos and Neelin 2019).

Climate models are known to exhibit errors in the current climate simulation of precipitation distributions, including too-frequent occurrences of low intensity rain (Hagos et al. 2021), and considerable differences at high percentiles (Pendergrass and Hartmann 2014; Goldenson et al. 2021; Norris et al. 2021; Fiedler et al. 2020). While marginal improvements are noted in Coupled Model Intercomparison Project Phase 6 (CMIP6) over earlier CMIP5 models in simulating precipitation extremes, inter-model spread and biases persist in the latest generation of models (Chen et al. 2020; Ha et al. 2020; Kim et al. 2020; Scoccimarro and Gualdi 2020; Wehner et al. 2020; Wehner 2020; Zhu et al. 2020; Chen et al. 2021; Li et al. 2021). How can the new connection of the precipitation intensity pdf to the thermodynamic environment inform this discussion? First, it suggests that the low-to-medium range intensity errors could be associated with a deficiency of variability, or an insufficiently sharp onset of precipitation as a function of moisture and temperature—both of which can occur in models. Second, it suggests that both of these factors affect the physical precipitation scale \( P_L \), governing the medium-to-high intensity range. This is consistent with the impact of resolution increases (Roberts et al. 2018; Wehner et al. 2014), increasing \( P_L \) while maintaining the shape of the medium-high intensity range, and of stochastic convective parameterization (Wang et al. 2021) which improves both ranges. In CMIP6 models (Martinez-Villalobos and Neelin 2021), the observed shape of the medium-high intensity range, and the spatial and seasonal changes in \( P_L \) are proportional to observed, yielding encouraging indications that fractional changes in the extreme events distribution may be reliable despite inter-model differences in the absolute value of this key parameter.

How Precipitation Distributions Change: Changes in Extremes

Physical insight from the stochastic prototype points to a single precipitation scale for accumulation \( (S_L) \) and daily precipitation \( (P_L) \) that encapsulate dynamical and thermodynamical effects for extremes. Because both thermodynamic and dynamic effects enter into the same cutoff scale \( (S_L, P_L) \), this scale provides a useful test, in a single quantity, of both thermodynamic and dynamic effects. In comparison to the common use of percentiles, one advantage of using the cutoff scale \( P_L \) for extreme precipitation change assessment is that it is a more fundamental quantity, whose physics and connection to the moisture budget are relatively well understood, as outlined in the Section “How Precipitation Distributions Arise: Variations Across a Moisture/Buoyancy Threshold”.

\[ \text{Fig. 4} \text{ Example of accumulation and daily precipitation pdfs in (a) a location in the tropics (Manus Island, 2° 3' S, 147° 25' E) and (b) mid-latitudes (Hartford Airport, CT, USA, 41° 56' N, 287° 19' E). Manus Island pdfs calculated from 1-min data quantized at 0.1 mm intervals. Hartford Airport pdfs calculated from 1-h data quantized at 0.254 mm intervals. Blue and red circles denote the location of the accumulation cutoff \( S_L \) and daily precipitation cutoff \( P_L \), respectively. Adapted from Martinez-Villalobos and Neelin (2019).} \]
The scale $P_L$ is also less affected than percentiles by the low-to-moderate range of the distribution for which models can exhibit substantial deficiencies (Pendergrass and Hartmann 2014; Hagos et al. 2021). We thus illustrate changes from example CMIP6 models in terms of this view of the distribution, and use it as a lens to review prior results.

For daily precipitation, Fig. 5a shows the pdf for an example region. Increases in $P_L$ with warming tend to stretch the pdf: the medium-large intensity regime where the pdf drops steeply in Fig. 5a shifts accordingly. This results in large increases in the probability of extremes, which is exacerbated with event rareness, as noted in various forms in multiple studies (e.g., Kunkel et al. 2013; Fischer and Knutti 2016; Pendergrass 2018; Myhre et al. 2019; Li et al. 2019; Kirchmeier-Young and Zhang 2020), including assessments based on CMIP6 models (Li et al. 2020; Akinsanola et al. 2020; Gupta et al. 2020; Ge et al. 2021; Dong et al. 2021). This is illustrated in Fig. 5b, which
shows the risk ratio (ratio between future and historical exceedances, conditioned on wet day occurrence) in two different regions in CESM2. In the US Northeast region (40°–48° N, 280°–293° E) the model shows a 10-fold increase in the probability of daily precipitation larger than about 100 mm (note logarithmic x-axis). A similar shape of the risk ratio occurs in the Southeast Asian region (15°–30° N, 90°–120° E) with increases in probability of almost 20 times for daily precipitation larger than 300 mm, consistent with the analysis in (Ge et al. 2021) over a similar region. Models’ representation of this characteristic risk ratio shape is in agreement with theoretical findings (Martinez-Villalobos and Neelin 2019, Fig. 8), and mirror observed accumulation and daily precipitation risk ratios in the USA (Martinez-Villalobos and Neelin 2018) and China (Chang et al. 2020), as well as accumulation risk ratios in CESM1 (Neelin et al. 2017).

If the exponent of the approximately power law range does not change substantially, as illustrated in Fig. 5a, $P_L$ governs a simple rescaling of the entire distribution. For event accumulations, this holds to high accuracy (Peters et al. 2010; Neelin et al. 2017) because this distribution involves only the precipitating regime. For daily average precipitation, this low-to-medium range tends to adjust slightly if the typical number of events per day changes, an effect governed by the dynamics of the non-precipitating intervals (Martinez-Villalobos and Neelin 2019). Even so, $P_L$ governs the scaling of the large event range. Thus, maps of changes in daily precipitation cutoff scale $P_L$ (Fig. 5c,d) for 2075–2099 under SSP5-8.5 (O’Neill et al. 2016) relative to historical conditions (1990–2014) summarize scaling as a function of region in two CMIP6 models (CESM2 and GFDL-CM4). Increases in $P_L$ occur in most places with exception of some sub-tropical regions, as previously seen for high percentiles (Pall et al. 2007; Pfahl et al. 2017; Hoegh-Guldberg et al. 2018; Norris et al. 2019a). In midlatitudes, large areas tend to be roughly at CC scaling (alternating between slightly above and slightly below 7% K$^{-1}$), but ocean storm track regions tend to be consistently above CC. In sub-tropical dry regions there tend to be decreases or sub-CC scaling. However, in certain regions of the tropics, substantial areas of super-CC scaling by factors of 2 to exceeding 4 may be noted, although in the two examples shown, there is not good agreement on the specific spatial distribution of these. This summarizes, in a potentially more interpretable quantity, results seen in studies based on percentiles or other measures of extremes (Chen et al. 2021; Yin et al. 2021; Gupta et al. 2020).

Changes in organization of precipitation or in probability distributions that summarize aspects of convective organization have also been considered. One of the simplest behaviors is for area of clusters of contiguous precipitating points (Peters et al. 2009; Wood and Field 2011) or for precipitation integrated over these clusters, termed cluster power. The probability distribution of tropical cluster power has a power law range followed by an approximately exponential cutoff in observations, which is reasonably simulated by climate models (Quinn and Neelin 2017a; 2017b). The cutoff scale increases in a warming climate, yielding probability increases for the largest clusters (Quinn and Neelin 2017b) analogous to those described for the distribution of precipitation accumulations or intensity. While quantitative changes vary among models, the tropical cutoff scale increase can be super-CC (Quinn and Neelin 2017a). Explanations for the distribution shape and for the physics of the cutoff scale are slightly more complex than for precipitation distribution discussed in the Section “How Precipitation Distributions Arise: Variations Across a Moisture/Buoyancy Threshold” due to the involvement of interactions between neighboring grid cells (Hottovy and Stechmann 2015a; Ahmed and Neelin 2019) but it appears to similarly be a physical scale governed by the moist physics of a climate model. Other results involving clusters can be more complex. In Chang et al. (2016), spatiotemporal clusters at midlatitudes over the USA exhibit partial compensation for increased intensity by reduced storm size.

Other effects not taken into account in the simple CC paradigm include the role of intermittency (Schleiss 2018; Visser et al. 2020) and the duration of precipitating events (Stechmann and Neelin 2014; Wasko et al. 2015; Norris et al. 2019b).

### Discussion and Conclusions

There has been considerable progress in quantifying changes in precipitation distributions under warming as simulated in climate models. The default comparison has been to whether increases in high percentiles tend to scale proportional to expectations from Clausius-Clapeyron increases in moisture. There has been increasing recognition that departures from this scaling occur both in climate change simulations and within current observations. The sense that these must be associated with feedbacks from convective heating to the circulation is here summarized by showing that when the moisture and thermodynamic equations are considered together, such dynamical feedbacks should be expected unless specific conditions are met (Section “‘Thermodynamic’ and Dynamical Contributions and Issues”). In particular, for precipitation associated with convective conditions, which occur with large-scale moisture below saturation, such feedbacks are required by the combination of the moisture equation and thermodynamic equation. The common term “thermodynamic component” for changes associated with increased moisture alone can be severely misleading under these circumstances since
dynamical changes result from the need to satisfy thermodynamic balances.

In parallel with these developments, there has been progress in understanding the processes underlying precipitation probability distributions—including how they are shaped by their relationship to their thermodynamic environment in current climate. Dynamical insights from this can help clarify some of the factors at play in changes under global warming. In particular, these processes yield relatively simple basic postulates for the form of these changes.

As a pathway to further insight, it may be fruitful to bring together the various research threads described above. We have noted intriguing connections between the empirical exploration of super-CC scaling in the moisture-dependence of high percentiles and the onset of convection as a function of its thermodynamic environment, whether measured by water vapor and temperature, or by an empirical buoyancy variable (Section “Precipitation Relationship to the Thermodynamic Environment”). The role of the thermodynamic environment in dictating convective heating-convergence feedbacks through a gross moist stability may be valuable in understanding regions of super-CC scaling (the Sections “Thermodynamic” and Dynamical Contributions and Issues” and “Scaling Arguments”). Finally, the connection between key features of the precipitation probability distribution and fluctuations across the sharp onset threshold of precipitation given by the thermodynamic environment (the Section “How Precipitation Distributions Arise: Variations Across a Moisture/Buoyancy Threshold”) are only beginning to be exploited for the analysis of global climate models (the Section “How Precipitation Distributions Change: Changes in Extremes”). Quantities such as the precipitation scale—incorporating both dynamic and thermodynamic effects—that governs the medium-to-high intensity range for precipitation pdfs appear promising as tools for elucidating precipitation probability changes in global climate models.

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Declarations

Conflict of Interest The authors declare that they have no conflict of interest.

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