Unification of Weak and Hypercharge Interactions at the TeV Scale

Lawrence J. Hall and Yasunori Nomura

Department of Physics, University of California, Berkeley, CA 94720, USA
Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

Abstract

A realistic $SU(3)_C \times SU(3)_W$ unified theory is constructed with a TeV sized extra dimension compactified on the orbifold $S_1/Z_2$, leaving only the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ unbroken in the low energy 4D theory. The Higgs doublets are zero modes of bulk $SU(3)_W$ triplets and serve to normalize the hypercharge generator, apparently giving a tree-level prediction for the weak mixing angle: $\sin^2 \theta = 1/4$. The orbifold boundary conditions imply a restricted set of $SU(3)_W$ gauge transformations: at an orbifold fixed point only the transformations of $SU(2)_L \times U(1)_Y$ are operative. This allows quarks to be located at this fixed point, overcoming the longstanding problem of how to incorporate matter in a unified $SU(3)_W$ theory. However, in general this local, explicit breaking of $SU(3)_W$ symmetry, necessary for including quarks into the theory, destroys the tree-level prediction for the weak mixing angle. This apparent contradiction is reconciled by making the volume of the extra dimension large, diluting the effects of the local $SU(3)_W$ violation. In the case that the electroweak theory is strongly coupled at the cutoff scale of the effective theory, radiative corrections to the weak mixing angle can be reliably computed, and used to predict the scale of compactification: $1 - 2$ TeV without supersymmetry, and in the region of $3 - 6$ TeV for a supersymmetric theory. The experimental signature of electroweak unification into $SU(3)_W$ is a set of “weak partners” of mass $1/2R$, which are all electrically charged and are expected to be accessible at LHC. These include weak doublets of gauge particles of electric charge $(+++, +)$, and a charged scalar. When pair produced, they yield events containing multiple charged leptons, missing large transverse energy and possibly Higgs and electroweak gauge bosons.
1 Introduction

The quest for unification of the forces of nature has been a dominant theme of particle physics for the last 30 years. The weak force acts only at short distances, and must apparently have a very different underlying origin from the electromagnetic force. The triumph of the standard electroweak theory is to provide a common picture for these forces [1]. However, while both the weak and electromagnetic forces are understood to arise from the interaction of spin 1 gauge particles, the standard model does not unify these interactions. The weak and electromagnetic forces originate from two separate gauge forces: the weak force based on the group $SU(2)_L$ and the hypercharge force based on the gauge group $U(1)_Y$.

The most striking success in the unification of the gauge forces of nature occurs in grand unified theories based on $SU(5)$ or $SO(10)$ symmetries [2, 3]. These groups contain both the electroweak group, $SU(2)_L \times U(1)_Y$, and the group, $SU(3)_C$, of the QCD gauge interaction of the strong force, so that there is a single coupling constant for all three interactions. Furthermore, the quarks and leptons of a single generation are unified into one ($SO(10)$) or two ($SU(5)$) representations of the unified gauge symmetry. Such embeddings of the quarks and leptons force a unique normalization for hypercharge, so that the relative strengths of the weak and hypercharge forces is determined, leading to a tree-level prediction for the weak mixing angle of $\sin^2 \theta = 3/8$. Including radiative corrections [4], a successful result follows only if weak scale supersymmetry is incorporated into the theory [5, 6], in which case the scale at which the three forces are unified is found to be of order $10^{16}$ GeV. This successful prediction has led to a dominant paradigm for physics beyond the standard model: supersymmetry at the TeV scale above which there is large energy desert, with no new physics appearing until $10^{16}$ GeV. Given this large energy, and uncertainties in the nature of the grand unified theory, it has not been possible to devise definitive experimental tests for this picture of force unification. The new gauge bosons and scalars may simply be too heavy to give observable signals.

The first attempt to unify any of the gauge forces of nature came before grand unification. It was an attempt at electroweak unification, with the weak and hypercharge groups $SU(2)_L \times U(1)_Y$ unified into $SU(3)_W$ [7]. The theory possessed a hierarchy of symmetry breaking, with the unified symmetry breaking, $SU(3)_W \rightarrow SU(2)_L \times U(1)_Y$, occurring at a much larger mass scale than the scale at which electroweak symmetry breaks to electromagnetism $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$. Embedding a lepton doublet $l = (\nu, e)_L$ and a lepton singlet $e = \bar{e}_R$ into a fundamental representation of $SU(3)_W$, leads to the tree-level prediction of $\sin^2 \theta = 1/4$. While this is within 10% of the present experimental value of 0.231, this approach to gauge unification was not pursued because it met immediate and insuperable obstacles [7, 8]. The most devastating difficulty is that quarks cannot be accommodated in the theory. The quark doublet $q = (u, d)_L$ has too small a hypercharge quantum number to appear in any $SU(3)_W$. 


multiplet. Furthermore, the lepton assignment has gauge anomalies, requiring the introduction of additional light charged fermions. One possibility is to build theories based on the electroweak group $SU(3)_W \times U(1)$, but this clearly does not unify the forces and does not predict the weak mixing angle. There appears to be a fundamental inconsistency between electroweak unification into $SU(3)$ and the observed quark and lepton quantum numbers.

In a recent proposal it has been demonstrated that the intriguing tree-level prediction of $\sin^2 \theta = 1/4$ can be preserved even when the weak and hypercharge forces are not unified, provided they are embedded in some larger semi-simple group that includes a new $SU(3)'$ gauge interaction [9]. For example, at high energies assume the gauge forces of nature are based on the expanded group $SU(4)_C \times SU(2)'_L \times SU(2)'_R \times SU(3)'$, with quarks and leptons transforming in the usual way under the Pati-Salam group, but not transforming under $SU(3)'$. The Pati-Salam group is broken in the usual way to $SU(3)_C \times SU(2)'_L \times U(1)'_Y$, and a further symmetry breaking $SU(3)' \times SU(2)'_R \times U(1)'_Y \rightarrow SU(2)_L \times U(1)_Y$ occurs in such a way that the electroweak symmetry group, $SU(2)_L \times U(1)_Y$, is embedded partly in $SU(3)'$ and partly in $SU(2)'_R \times U(1)'_Y$. The relation $\sin^2 \theta = 1/4$ now emerges in the limit that the coupling constants for $SU(2)'_L$ and $SU(2)'_R$ are taken much larger than the coupling for $SU(3)'$.

In this paper we do not follow this approach of adding a new $SU(3)'$ gauge interaction which does not couple to quarks and leptons. Rather, we return to the original idea of unifying the weak and hypercharge interactions into a single $SU(3)_W$ gauge force [7], so that the standard model gauge group is embedded in $SU(3)_C \times SU(3)_W$, which becomes the symmetry group of nature above the TeV scale. We use tools developed in Refs. [10, 11]. Consider a higher dimensional theory with gauge group $G$ compactified on an orbifold, with different gauge fields having different boundary conditions. This results in a theory with a restricted set of gauge transformations; in particular, at orbifold fixed points the operative gauge symmetry is $H$, a subgroup of $G$. These points of reduced symmetry are very interesting. They can support brane fields in any multiplets of $H$, whether these are parts of $G$ multiplets or not. Similarly, the zero modes of bulk fields do not fill out complete $G$ multiplets. At first sight, the presence of such points destroys the gauge coupling relation coming from $G$, since gauge kinetic terms that are not $G$ universal can be placed at the fixed points. However, this point defect breaking of gauge coupling unification is a small effect if the volume of the bulk is large compared to the cutoff of the effective higher dimensional theory. Said differently, the brane kinetic energy operators have higher mass dimension, and are therefore less relevant in the infrared, than the bulk kinetic energy operators. The larger the energy interval from the compactification scale up to the cutoff of the effective theory, the more accurate gauge coupling unification becomes. This powerful tool implies that, in higher dimensional field theories, it is possible to have incomplete multiplets of the gauge group while still maintaining tree-level gauge coupling unification. When applied to grand unification, it allows the construction of completely realistic theories above
the supersymmetric desert, which we call Kaluza-Klein (KK) grand unification. The minimal
$SU(5)$ theory in 5D has automatic doublet-triplet splitting, no proton decay from dimension
four or five operators, no unwanted mass relations for light generations, and a prediction for
the QCD coupling of $\alpha_s = 0.118 \pm 0.005$ from gauge coupling unification $[1]$. It is this new tool that allows us to overcome the obstacles to building an electroweak theory
based on $SU(3)_W$. The quarks and leptons do not need to fill out complete $SU(3)_W$ multiplets
if they live at orbifold fixed points. Because the breaking of $SU(3)_W$ is localized, if the bulk
has a large volume the tree-level prediction, $\sin^2 \theta = 1/4$, is maintained to high accuracy.
The normalization of the hypercharge generator within the $SU(3)_W$ group is determined by
the Higgs field which is a bulk field transforming as a triplet of $SU(3)_W$. Including radiative
 corrections to $\sin^2 \theta$, the compactification scale can be computed to the factor of three level,
and is found to be about 2 TeV, so that the new charged gauge bosons of $SU(3)_W$ and the new
charged Higgs scalar are likely to be accessible to the LHC.

2 The Basic Idea

In this section we present the basic idea of our scheme. We consider an $SU(3)_W$ gauge theory
in 5D, compactified on an $S^1/Z_3$ orbifold. Under the compactification, the gauge fields $A_\mu =
\{A_\mu, A_5\}$ ($\mu = 0, \cdots, 3$) obey the following boundary conditions:

\begin{align}
A_\mu(x^\mu, y) &= Z A_\mu(x^\mu, -y) Z^{-1} = T A_\mu(x^\mu, y + 2\pi R) T^{-1}, \\
A_5(x^\mu, y) &= -Z A_5(x^\mu, -y) Z^{-1} = T A_5(x^\mu, y + 2\pi R) T^{-1},
\end{align}

where $Z$ and $T$ are $3 \times 3$ matrices, and $A_M \equiv A_M^a T^a$ are also represented as $3 \times 3$ matrices. To reduce the gauge group to $SU(2)_L \times U(1)_Y$ at low energies, we have two independent choices:
(i) $\{Z, T\} = \{\text{diag}(1, 1, 1), \text{diag}(1, 1, -1)\}$ and (ii) $\{Z, T\} = \{\text{diag}(1, 1, -1), \text{diag}(1, 1, 1)\}$. In
the former case we have only $SU(2)_L \times U(1)_Y$ gauge fields, $A_\mu^{EW}$, as massless fields, but in
the latter case we also have off-diagonal pieces of the fifth component of the gauge fields, $A_5^X$.
Although it is possible to construct realistic theories based on the second choice, we defer the
discussion of this possibility to section 4 and here adopt the first one. Then, the KK tower for
the gauge fields is given as follows: the standard-model $SU(2)_L \times U(1)_Y$ gauge bosons $A_\mu^{EW}$
with masses $n/R$, joined at $n \neq 0$ levels by $A_5^{EW}$, and the broken $SU(3)_W$ vectors $A_\mu^X$, joined
by $A_5^X$, with masses $(n + 1/2)/R$, where $n = 0, 1, \cdots$.

What is the symmetry of this system? Since we are considering an effective field theory
below the cutoff scale, it makes sense to consider field theoretic symmetries using the classical
spacetime picture. We then find that the gauge symmetry of the system is $SU(3)_W$ but with
the gauge transformation parameters obeying the same boundary conditions as the gauge fields:

$$\xi(x^\mu, y) = Z \xi(x^\mu, -y) Z^{-1} = T \xi(x^\mu, y + 2\pi R) T^{-1},$$

(3)
Figure 1: The structure of the fifth dimension. Solid and dotted lines represent the profiles of gauge transformation parameters $\xi^{\text{EW}}$ and $\xi^X$, respectively. The gauge symmetry $SU(3)_W$ is reduced to $SU(2)_L \times U(1)_Y$ on the $y = \pi R$ brane, so that we can introduce quark and lepton fields on this brane. The Higgs field $\phi$ is located in the bulk, fixing $\sin^2 \theta = 1/4$ at tree level.

which we refer to as restricted gauge symmetry [10]. This means that while the $SU(2)_L \times U(1)_Y$ gauge parameters, $\xi^{\text{EW}}$, have profiles $\cos[ny/R]$ in the extra dimension, $SU(3)_W/(SU(2)_L \times U(1)_Y)$ ones, $\xi^X$, have different profiles $\cos[(n + 1/2)y/R]$, as depicted in Fig. [1]. The crucial observation is that $\xi^X$ always vanish at $y = \pi R$, and hence the gauge symmetry is reduced to $SU(2)_L \times U(1)_Y$ on this point. In particular, we can introduce any representation of $SU(2)_L \times U(1)_Y$ on the $y = \pi R$ brane, even if it does not arise from any $SU(3)_W$ representation. This allows us to introduce quark and lepton fields, $q(2, \alpha/6)$, $u(1, -2\alpha/3)$, $d(1, \alpha/3)$, $l(2, -\alpha/2)$, and $e(1, \alpha)$, on this brane, where the numbers in parentheses represent $SU(2)_L \times U(1)_Y$ quantum numbers. At this stage, the overall normalization of these $U(1)_Y$ charges, described by the parameter $\alpha$, is arbitrary, since it is not related to $SU(2)_L$ (or $SU(3)_W$) by the operation of $\xi^X$.

Does the explicit breaking of $SU(3)_W$ at $y = \pi R$ destroy the electroweak gauge coupling unification that originates from $SU(3)_W$ symmetry? Generically, the answer is yes. We can write down gauge kinetic operators on the $y = \pi R$ brane with non-unified coefficients for $SU(2)_L$ and $U(1)_Y$ gauge fields. However, since this $SU(3)_W$ breaking is point-like in the extra dimension, we can reduce its effect by making the volume of the extra dimension large, recovering the unified relation [10]. Specifically, the most general form for the gauge kinetic
energy is given by
\[ S = \int d^4x \, dy \left[ \frac{1}{g_5^2} F^2 + \delta(y - \pi R) \frac{1}{g_i^2} F_i^2 \right], \tag{4} \]
where the first term is an \( SU(3)_W \)-invariant bulk gauge kinetic energy, while the second term represents non-unified kinetic operators located on the \( y = \pi R \) brane (\( i \) represents \( SU(2)_L \) and \( U(1)_Y \)). Here, we have omitted an \( SU(3)_W \)-invariant term on the \( y = 0 \) brane, since it is irrelevant for the discussion below. The zero-mode gauge couplings are obtained by integrating over the extra dimension:
\[ \frac{1}{g_i^2} = \frac{\pi R}{g_5^2} + \frac{1}{g_i^2}, \tag{5} \]
where \( g_1 = \sqrt{3} g' \) and \( g_2 = g \). Now, suppose the bulk and brane kinetic terms have “comparable” strength at the cutoff scale \( M_s \). This implies that \( g_5 \approx \tilde{g}_i \) in units of \( M_s \), so we write \( \tilde{g}_i^2 = g_5^2 M_s/c_i \) and get
\[ \frac{1}{g_i^2} = \frac{\pi R}{g_5^2} \left( 1 + \frac{c_i}{\pi R M_s} \right), \tag{6} \]
where \( c_i \) are non-universal coefficients of order unity. We find that the \( SU(3)_W \)-violating effect from the \( y = \pi R \) brane is suppressed by the volume of the extra dimension, \( \pi R M_s \). Therefore, by making the extra dimension large, we can reconcile two seemingly contradicting ideas: having fields that do not fit into any \( SU(3)_W \) representation and keeping the \( SU(3)_W \) relations for the gauge couplings!

Next we introduce the Higgs field as a bulk scalar field, \( \phi \), transforming as a triplet under \( SU(3)_W \). We choose boundary conditions so that the \( SU(2)_L \) doublet component of \( \phi \) remains massless at tree level:
\[ \phi(x^\mu, y) = Z \phi(x^\mu, -y) = T \phi(x^\mu, y + 2\pi R), \tag{7} \]
which gives a KK tower of masses \( n/R \) for the doublet component, \( \phi_D \), and a tower of masses \( (n+1/2)/R \) for the singlet component, \( \phi_S \). We identify the zero mode of \( \phi_D \) as the Higgs doublet, \( h \), of the standard model. This identification fixes the normalization of \( U(1)_Y \). Normalizing \( U(1)_Y \) charges so that the Higgs doublet has \( h(2, -1/2) \), the \( SU(2)_L \) and \( U(1)_Y \) gauge couplings, \( g \) and \( g' \), are related by \( g = \sqrt{3} g' \), giving \( \sin^2 \theta = 1/4 \), at tree level. In order that the usual Yukawa couplings, \( \mathcal{L}_4 = quh^\dagger + qdh + leh \), can be introduced on the \( y = \pi R \) brane, the overall scale for the quark and lepton \( U(1)_Y \) charges must be chosen to be \( \alpha = 1 \). Note that the Yukawa couplings respect only the \( SU(2)_L \times U(1)_Y \) invariance and not the full \( SU(3)_W \) symmetry. The \( SU(3)_C \) gauge fields are introduced either in the 5D bulk or on the \( y = \pi R \) brane. Below the compactification scale \( M_c = 1/R \), the theory reduces to the standard model with \( \sin^2 \theta \approx 1/4 \). Radiative corrections to \( \sin^2 \theta \) are discussed in the next section.
3 Calculable Framework

In the previous section, we have described a basic mechanism of introducing quarks and leptons while preserving the $SU(3)_W$ relation among the gauge couplings. However, the correction to this relation from brane kinetic operators depends on the size of the extra dimension, which is an unknown free parameter. Furthermore, although unlikely, it is a logical possibility that the coefficients of the brane operators are anomalously large, destroying the gauge coupling relation. In Ref. [11], we have introduced a framework which removes these concerns and allows a reliable calculation for gauge coupling unification. The crucial new ingredient is the assumption that the gauge interaction is strongly coupled at the cutoff scale $M_s$. This assumption gives the largest possible volume for the extra dimension, and hence minimizes unknown contributions from tree-level brane operators. It also determines the size of the leading radiative correction coming from the energy interval between $M_c$ and $M_s$.

To see how this works explicitly, let us first consider the effective action at the scale $M_s$. No matter what physics occurs above $M_s$, the restricted gauge symmetry ensures that the 5D bulk is $SU(3)_W$ symmetric and all $SU(3)_W$-violating effects appear only on the $y = \pi R$ brane. Therefore, the gauge kinetic energy must take the form of Eq. (4). Now, since the theory is assumed to be strongly coupled at $M_s$, both bulk and brane gauge couplings are reliably estimated as $g^2 \simeq 16\pi^3/M_s$ and $\tilde{g}^2 \simeq 16\pi^2$ using naive dimensional analysis. Inserting these estimates into Eq. (5), we find that the bulk term contributes to $1/g^2_i$ an amount $M_s R/16\pi^2$ while the brane terms contribute an amount $1/16\pi^2$. From this, we learn two things. First, since the 4D gauge couplings are $O(1)$, the volume of the extra dimension must be large, $M_s R = O(100)$. Second, since the non-universal contribution is suppressed to the percent level, the resulting uncertainty in the calculation of gauge coupling unification is small, $\delta \sin^2 \theta / \sin^2 \theta \simeq 0.4\%$.

Having obtained gauge coupling unification at tree level at $M_s$, we turn to the quantum effects below $M_s$ that result from the $y = \pi R$ brane. Below $M_s$, the 4D gauge couplings run by power law. However, since the leading power-law piece comes from the evolution of the bulk term, it is $SU(3)_W$ symmetric. Therefore, the relative running between $g$ and $g'$, which is relevant for gauge coupling unification, entirely comes from the evolutions of the gauge kinetic terms localized on the $y = \pi R$ brane. Since these evolutions are logarithmic, they can be reliably computed in the effective theory. Furthermore, they contribute to $1/g^2_i$ an amount

---

One way of estimating these couplings is to consider loop diagrams in the equivalent 4D KK theory. In the 4D picture, the bulk term gives gauge kinetic terms with KK momentum conservation, while the brane ones give terms with KK momentum violation. After diagonalizing these kinetic terms, the gauge couplings among KK towers are obtained. Requiring that contributions from all loop diagrams become comparable at the scale $M_s$ (i.e. the theory is strongly coupled at $M_s$), we obtain the result in the text, neglecting group theoretical factors of order unity.
(1/16\pi^2) \ln(M_s/M_c) and dominate the unknown tree-level correction of order 1/16\pi^2, by a factor of \ln(M_s/M_c) \simeq \ln(100) \simeq 5. Including the radiative correction below \( M_c \), we obtain the 4D gauge couplings at the weak scale:

\[ \frac{1}{g_i^2}(m_Z) \simeq \frac{1}{g_i^2} + \frac{b_i}{4\pi^2} \left[ \left( \frac{M_s}{M_c} \right) - 1 \right] + \frac{\tilde{b}_i}{8\pi^2} \ln \frac{M_s}{M_c'} + \frac{b_i'}{8\pi^2} \ln \frac{M'_c}{m_Z}, \]  

(8)

where \( b \) and \( \tilde{b}_i \) are the \( \beta \)-function coefficients above \( M_c \), and \( b_i' \) those below \( M_c \); \( g_* \) is the unified gauge coupling at \( M_s \). Here, we have matched the logarithmic contribution in higher dimensions to that in 4D at the scale \( M'_c = M_c/\pi \), which represents the length scale of extra dimensions \[12\]. Eliminating \( 1/g_*^2 \), we obtain the expression for \( \sin^2 \theta \) at \( m_Z \):

\[ \sin^2 \theta \simeq \frac{1}{4} - \frac{3}{8\pi} \alpha_{em} \left[ (\tilde{b}_1 - \tilde{b}_2) \ln \frac{M_s}{M_c'} + (b_1' - b_2') \ln \frac{M'_c}{m_Z} \right], \]  

(9)

where \( \alpha_{em} \equiv e^2/4\pi \simeq 1/128 \) represents the fine structure constant at \( m_Z \). Since the strong coupling requirement determines \( M_s/M'_c \simeq 16\pi^3 \), we can use this equation to estimate the compactification scale from the observed value of \( \sin^2 \theta \). Assuming that the tree-level spectrum is not much changed by radiative corrections, we obtain the \( \beta \)-function coefficients above \( M'_c \) as \( (\tilde{b}_1, \tilde{b}_2) = (9/4, 1/4) \), following the prescription given in Ref. \[11\]. (The gauge, Higgs and matter fields contribute \( 0, -23/6 \), \( 1/36, 1/12 \) and \( 20/9, 4 \), respectively). Using the standard-model \( \beta \)-functions below \( M'_c \), \( (b_1', b_2') = (41/18, -19/6) \), the compactification scale is estimated to be \( 1/R \simeq 1 - 2 \) TeV. It is interesting to note that both logarithmic runnings above and below \( M_c \) reduce the value of \( \sin^2 \theta \) from \( 1/4 \) with comparable contributions. Thus, even if we do not assume strong coupling at \( M_s \), we expect the compactification scale to be around a TeV, as long as unknown contributions from tree-level brane operators are sufficiently small.

Since the TeV scale extra dimension suggests \( M_s \approx 100 \) TeV, the present model needs a fine tuning to get \( \langle k \rangle \equiv v \approx 175 \) GeV. The required fine tuning is of order \( v^2/M_s^2 \approx 10^{-6} \). However, this unpleasant feature is avoided by making the theory supersymmetric. It is straightforward to supersymmetrize the model of the previous section. The SU(3)_W gauge field is now a 5D gauge supermultiplet, which consists of a 5D vector field, \( A_H \), two gauginos, \( \lambda \) and \( \lambda' \), and a real scalar \( \sigma \). Using the 4D \( N = 1 \) superfield language, \( V(A_\mu, \lambda) \) and \( \Sigma((\sigma + iA_5)/\sqrt{2}, \lambda') \), the boundary conditions are given by Eqs. \[1, 2\] with \( A_\mu \rightarrow V \) and \( A_5 \rightarrow \Sigma \). The matter fields on the \( y = \pi R \) brane become chiral superfields: \( Q, U, D, L \) and \( E \). Since the Yukawa couplings on the brane must be supersymmetric, we need two Higgs multiplets. Thus we introduce two Higgs hypermultiplets, \( \{\Phi, \Phi^c\} \) in the bulk, where \( \Phi(3), \Phi^c(\bar{3}^c) \), \( \bar{\Phi}(\bar{3}^c) \), and \( \Phi^c(3) \) are 4D \( N = 1 \) chiral superfields with SU(3)_W transformations given in the parentheses. For \( \Phi \) and \( \bar{\Phi} \), the boundary conditions are given by Eq. \[7\] with \( \phi \rightarrow \Phi, \bar{\Phi} \); for the conjugate fields, the boundary conditions are Eq. \[7\] with \( Z \rightarrow -Z \) and \( \phi \rightarrow \Phi^c, \bar{\Phi}^c \). These
boundary conditions yield zero modes for \(SU(2)_L\) doublets of \(\Phi\) and \(\bar{\Phi}\), which we identify with the two Higgs doublets of the minimal supersymmetric standard model (MSSM), \(H_u \equiv \bar{\Phi}_{D,0}\) and \(H_d \equiv \Phi_{D,0}\). The Yukawa couplings, \([QU\Phi_D + QD\Phi_D + LE\Phi_D]_{\theta^2}\), are introduced on the \(y = \pi R\) brane, and the QCD gauge interaction is introduced either in the 5D bulk or on the \(y = \pi R\) brane. Below \(M_c\), the theory reduces to the MSSM with \(\sin^2 \theta \approx 1/4\).

As in the case of the non-supersymmetric theory, we can reliably estimate the compactification scale by requiring that the theory is strongly coupled at \(M_s\). Assuming that all superparticle masses are around \(1/R\), we can use the standard-model \(\beta\)-functions below \(M'_c\). The \(\beta\)-functions coefficients above \(M'_c\) are given by \((\tilde{b}_1, \tilde{b}_2) = (10/3, 2)\). (The gauge, Higgs and matter fields contribute \((0, -4), (0, 0)\) and \((10/3, 6)\), respectively). Then, using Eq. (9), we obtain \(1/R \approx 3\) TeV from the observed value of \(\sin^2 \theta\). This estimate has a considerable uncertainty coming from the actual superparticle spectrum; for example, in the extreme case that all superpartners are at \(M_Z\), we find \(1/R \approx 30\) TeV, using MSSM beta functions for \(b'_i\).

While there are many possibilities for supersymmetry breaking, two schemes are particularly well suited to our theory. The first possibility is that of Scherk-Schwarz breaking of supersymmetry by boundary conditions in the fifth dimension [13]. In this case the gauginos and Higgsinos acquire tree level masses, while those of the quark and lepton superpartners arise at radiative level. The gauginos acquire mass \(\approx 1/R\), while squarks and sleptons are much lighter. Another natural possibility is that there is strong local breaking of supersymmetry on the \(y = 0\) brane [14]. This deforms the gaugino and Higgsino wavefunctions, giving them mass \(1/R\), while the squarks and sleptons again acquire radiative masses. In both these cases, with gauginos at \(1/R\), the constraint from the weak mixing angle leads to \(1/R \approx 6\) TeV. Both schemes outlined here solve the supersymmetric flavor problem in an inherently extra-dimensional way. The locality of the squarks and sleptons forces supersymmetry breaking to be communicated to them via the gauge interactions.

Well beneath \(M_c\), our theory reduces to the (supersymmetric) standard model. The lightest states which signal the presence of \(SU(3)_W\) electroweak unification are the lowest members of the \(T\) odd KK towers. These “weak partners” have mass \(M_c/2 \approx 500 - 3000\) GeV, and are therefore expected to be within the reach of LHC. In the non-supersymmetric case, there are two weak partners: the charged scalar \(\phi_S\) (the weak partner of the Higgs doublet) and an \(SU(2)_L\) doublet of massive vectors \(A^{X}_{\mu,5}\) (the weak partners of the electroweak gauge bosons). Radiative corrections will lift the degeneracy of these states, and if \(T\) is conserved the lightest weak partner (LWP) will be stable. Since all the weak partners are integrally charged under electromagnetism, this would be a striking signal. The radiative corrections to the mass of \(A^{X}_{\mu,5}\) are under control, because at short distances they are components of the 5D gauge field. However, the mass of the \(\phi_S\), like that of the Higgs, is highly divergent, so its physical mass may be far from \(M_c/2\).
While the bulk interactions necessarily preserve $T$, those on the branes need not. The profiles of $\phi_S$ and $A_\mu^X$ vanish on the $y = \pi R$ brane where the quarks and leptons are located, and hence can only have derivative interactions with matter. The two fermion interactions involve leptons but not quarks: $ll\partial_y \phi_S$ and $e^\dagger \sigma^\mu l \partial_y A_\mu^X$. While $A_5^X$ is non-zero on this brane, its quantum numbers do not allow any couplings to pairs of fermions. Thus, if $T$ violation is present, the LWP will decay to leptons and not quarks. The LWP decay modes are $\phi^+ \rightarrow l^+\nu$, $A^+ \rightarrow l^+\nu$ and $A^{++} \rightarrow l^+l^+$, where superscripts are electric charges, and $l^+$ is a charged lepton of any generation. At LHC all accessible weak partners will be pair produced via $s$-channel $\gamma,Z$ or $W^\pm$ exchange. The interaction $A_\mu^X \phi_S^\dagger \partial^\mu \phi_D$ allows cascade decays of the heavier of $\phi_S$ and $A_\mu^X$ to the lighter and $\phi_D$, so that the final state may also contain Higgs bosons, $W^\pm$ or $Z$. Similarly, the heavier of $A^+$ and $A^{++}$ can $\beta$ decay to the lighter via a $W^{\pm}$. The production of singly charged LWPs yields events containing $l^+l^-$, possibly of differing generation, with large amounts of missing transverse energy. Pair production of doubly charged LWPs produces events with four isolated charged leptons. Pair production of non-LWP states leads to cascade decays followed by the LWP decay. Hence these events will have additional Higgs or weak bosons relative to the LWP events.

In the supersymmetric case the phenomenology of the bosonic weak partners is not greatly changed, with similar striking events with multi charged leptons. However, the weak partner states now fill out multiplets of $N = 1$ supersymmetry. They are: a weak doublet vector multiplet $(A_\mu^X, \lambda^X)$, a weak doublet chiral multiplet $(A_5^X, \lambda^X)$, and chiral multiplets $\Phi_S(\phi_S, \psi_S)$ and $\bar{\Phi}_S(\bar{\phi}_S, \bar{\psi}_S)$ and their conjugates. The radiative corrections to the masses of all these weak partners is under control. There is the possibility of a brane mass of the form $\Phi_S \bar{\Phi}_S$, but this is also expected to be of order $M_c$.

4 Alternative Model

In this section we discuss an alternative model in our scheme of electroweak unified theories. In the previous sections, we have taken the boundary conditions in which $SU(3)_W$ is broken by the orbifold translation: $\{Z, T\} = \{\text{diag}(1,1,1), \text{diag}(1,1,-1)\}$. However, we could alternatively choose the boundary conditions which breaks $SU(3)_W$ by the orbifold reflection: $\{Z, T\} = \{\text{diag}(1,1,-1), \text{diag}(1,1,1)\}$. In this case, the zero mode sector contains the fifth component of the gauge fields that transforms as an adjoint of $SU(2)_L \times U(1)_Y$, $(2,0) + (1,0)$, in addition to the $SU(2)_L \times U(1)_Y$ gauge fields and the electroweak Higgs doublet. Since these extra fields are scalars in the 4D picture, they receive masses of order $(1/4\pi)(1/R)$ through radiative corrections, and the model can be phenomenologically viable. An interesting property of this setup is that the gauge symmetry structure is different from that in the previous models. Specifically, $SU(2)_L \times U(1)_Y$ and $SU(3)_W/(SU(2)_L \times U(1)_Y)$ gauge parameters, $\xi^{\text{EW}}$ and $\xi^X$, ...
have profiles \( \cos[ny/R] \) and \( \sin[ny/R] \), respectively, so that the gauge symmetry is reduced to \( SU(2)_L \times U(1)_Y \) on both \( y = 0 \) and \( y = \pi R \) branes. The Higgs field is located in the bulk as a triplet of \( SU(3)_W \), so that it determines \( \sin^2 \theta = 1/4 \) upon identifying the doublet component with \( h \). Then, we can put quarks, \( q, u, d \), and leptons, \( l, e \), on different branes; for example, quarks on \( y = 0 \) and leptons on \( y = \pi R \). This is interesting because it provides proton stability through the separation of quarks and leptons in the extra dimension [15].

Much of the collider phenomenology of this theory is similar to that discussed above for our first theory. The weak partners again consist of a charged scalar, \( \phi_S \), and a heavy vector doublet, \( A^X_{\mu, 5} \), but now have a mass \( 1/R \). While all the states of this theory are \( T \) even, these weak partners are \( Z \) odd and potentially stable. There is, however, a crucial new ingredient: there is a doublet of scalars which are even under both \( Z \) and \( T \) and acquire a radiative mass of order \( (1/4\pi)(1/R) \). These states could be accessible to both the Fermilab collider and the LHC. They will be distinctive since they come with both ++ and + charges. If the brane operator \( ll\partial_y \phi_S \) is present, these \( A^X_5 \) scalars will decay via a virtual \( \phi_S \) to \( ll\phi_D \). Thus the signal collider events contain either 2 or 4 isolated charged leptons, together with two electroweak gauge or Higgs bosons.

### 5 Conclusions

We have proposed a unification of weak and hypercharge gauge forces at the TeV scale into a single \( SU(3)_W \) interaction. One motivation for such a unification is a tree-level prediction of the weak mixing angle, \( \sin^2 \theta = 1/4 \), but apparently there is a fatal difficulty: quarks do not fit into \( SU(3)_W \) multiplets. A new opportunity arises if \( SU(3)_W \) is realized as a gauge symmetry in 5D rather than in 4D. The additional dimension is compactified on \( S_1/Z_2 \), with boundary conditions inducing \( SU(3)_W \to SU(2)_L \times U(1)_Y \). This leads to a fixed point which does not respect the full \( SU(3)_W \) symmetry, but only its \( SU(2)_L \times U(1)_Y \) subgroup, allowing quarks, which do not have the right quantum numbers to appear in \( SU(3)_W \) multiplets, to be located on this fixed point. However, since \( SU(3)_W \) is explicitly broken at this fixed point, it is not clear that the unification of hypercharge and weak gauge couplings persists. We have argued that the local operators for gauge kinetic terms which violate \( SU(3)_W \) are higher dimensional and irrelevant, so that the tree level prediction is preserved only in the case that the bulk has a large volume. We have pursued the possibility that the electroweak gauge sector is strongly coupled at high energies, and used this assumption to predict the compactification scale: \( 1/R \approx 1 - 2 \) TeV without supersymmetry. The scale \( 1/R \) is much more uncertain in the supersymmetric case, due to the superpartner spectrum. In schemes for supersymmetry breaking which solve the supersymmetric flavor problem, we find \( 1/R \approx 3 - 6 \) TeV. Experimental signatures for \( SU(3)_W \) unification are provided by “weak partners” with mass \( 1/2R \approx 500 - 3000 \) GeV: a
charged scalar partner of the Higgs doublet, and a weak doublet of heavy gauge bosons. These states will be pair produced at LHC. If the orbifold translation symmetry is unbroken the lightest weak partner (LWP), which is charged, will be stable. If the translation symmetry is broken, the decays of the weak partners lead to characteristic events with several isolated charged leptons.

We have argued that there is a large energy interval above the TeV scale where the effective theory of nature is 5D with gauge group $SU(3)_C \times SU(3)_W$. If color propagates in the bulk, then a further unification of forces is possible at higher energies [10]. Both color and weak gauge couplings undergo power-law running with beta function coefficients given by $-3$ and $-2$ respectively. Since QCD is more asymptotically free, the couplings approach each other at high energies, and meet at $M_s \approx 4\pi^2 M_c (1/g_W^2(M_c) - 1/g_C^2(M_c)) \approx 100$ TeV. Although this calculation has very large uncertainties, coming from power-law sensitivity to unknown ultraviolet physics, it is encouraging that this suggested grand unification scale is close to the scale at which the 5D theory becomes strongly coupled, $M_s$. At this higher scale of order 100 TeV our effective 5D theory may be embedded into a more fundamental theory, which includes gravity. For gravity to be strong at this scale requires some additional very large dimensions in which only gravity propagates [17]. Although the 100 TeV scale is somewhat higher than originally proposed, it comfortably avoids flavor problems, as well as cosmological and astrophysical limits on the case of two very large extra dimensions.

After the completion of this paper, we received Ref. [18] which considers $SU(3)_W$ in 5D.

**Note added:**

In the models discussed above, the normalization of the quark and lepton hypercharges relative to those of the Higgs, $\alpha$, is not determined by the theory; rather $\alpha = 1$ was imposed as a phenomenological requirement. Here we construct theories in which this charge quantization ($\alpha = 1$) is determined by the consistency of the theory. Since the leptons have the correct quantum numbers to fit into an $SU(3)_W$ triplet, they can appear on the $y = 0$ brane, or in the bulk as two triplets per generation having opposite $T$ quantum numbers [19]. In both cases, triplet leptons induce a brane localized $SU(3)_W$ gauge anomaly. We find that this gauge anomaly at $y = 0$ can be cancelled by adding a bulk Chern-Simons term with fixed normalization, having a coefficient which is constant in the bulk, $Z$ odd and $T$ even. This Chern-Simons term induces an anomaly of fixed size on the $y = \pi R$ brane. The $SU(3)_W/(SU(2)_L \times U(1)_Y)$ gauge fields have profiles which vanish at $y = \pi R$, so this anomaly is only for the

---

1. One possible grand unified theory is $SU(6)$ in 6D, compactified on $T_2/(Z_2 \times Z'_2)$, with $R_5 \gg R_6$. Boundary conditions in the $x_5$ direction break $SU(6) \rightarrow SU(5) \times U(1)$, while those in the $x_6$ direction break $SU(6) \rightarrow SU(3)_C \times SU(3)_W \times U(1)$. Below the grand unification scale of $1/R_6 \approx M_s \approx 100$ TeV, the effective 5D theory can be any of the models described in this paper (with an additional $U(1)$).

2. We assume identical gauge quantum numbers for each generation.
$SU(2)_L \times U(1)_Y$ gauge fields. It is cancelled by a unique choice for the normalization of the hypercharges of the quarks. Thus gauge invariance is recovered for the entire $SU(3)_W$ theory, and the overall normalization for both quark and lepton hypercharges is determined ($\alpha = 1$). In these theories, our calculations of $1/R$ are unchanged. The leptons fill complete $SU(3)_W$ multiplets, and do not affect relative running.

The phenomenology of the theory with bulk leptons is similar to that discussed in section 3. There are additional $T$ odd weak partner states of mass $1/2R$: $SU(2)_L$ doublet and singlet vector leptons $L$ and $E$. Brane interactions such as $L e \phi_D$ and $l E \phi_D$, which violate $T$, could also contribute to LWP decays. These operators could lead to single production of $L$ and $E$ in $e^+ e^-$ collisions. Since the leptons are bulk modes, their masses, arising from brane Yukawa couplings, are volume suppressed by $1/(M_s R)$ relative to quark masses; a trend observed in all three generations. In the supersymmetric case the sleptons receive a tree level mass and are heavier than the squarks, which only acquire a radiative mass.

The phenomenology of triplet leptons on the $y = 0$ brane is altered because the weak partner vector boson $A^X_{\mu}$ has $T$ violating interactions with the lepton current $e^l l$, and the Higgs doublet must originate from an $SU(3)_W$ sextet [13]. Hence the scalar weak partners are a weak triplet $\phi_T(---, -, 0)$ and a weak singlet $\phi_S^{'+}(++)$ which have $T$ violating couplings on the $y = 0$ brane of $ll\phi_T^+, ee\phi_S^{'+}$. The decays of the weak partners, as always, are to leptons and not to quarks. If light enough, the vector weak partners could be produced singly at future lepton colliders via $e^-_L e^-_R \rightarrow A^-_{X}, A^-_{X} W^-$, but not via $e^+ e^-$ annihilation.

Acknowledgements

Y.N. thanks the Miller Institute for Basic Research in Science for financial support. This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098, and in part by the National Science Foundation under grant PHY-00-98840.

References

[1] S. L. Glashow, Nucl. Phys. 22, 579 (1961); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in Proceedings of the VIIIth Nobel Symposium, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.
[2] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).
[3] H. Georgi, in *Particles and Fields*, edited by C. Carlson (AIP, New York, 1975), p. 575; H. Fritzsch and P. Minkowski, Annals Phys. **93**, 193 (1975).

[4] H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).

[5] S. Dimopoulos and H. Georgi, Nucl. Phys. B **193**, 150 (1981); N. Sakai, Z. Phys. C **11**, 153 (1981).

[6] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D **24**, 1681 (1981); L. E. Ibanez and G. G. Ross, Phys. Lett. B **105**, 439 (1981).

[7] S. Weinberg, Phys. Rev. D **5**, 1962 (1972).

[8] H. Georgi and S. L. Glashow, Phys. Rev. D **7**, 2457 (1973).

[9] S. Dimopoulos and D. E. Kaplan, arXiv:hep-ph/0201148.

[10] L. J. Hall and Y. Nomura, Phys. Rev. D **64**, 055003 (2001) [arXiv:hep-ph/0103128].

[11] L. J. Hall and Y. Nomura, arXiv:hep-ph/0111068.

[12] Y. Nomura, D. R. Smith and N. Weiner, Nucl. Phys. B **613**, 147 (2001) [arXiv:hep-ph/0104041].

[13] I. Antoniadis, Phys. Lett. B **246**, 377 (1990); A. Pomarol and M. Quiros, Phys. Lett. B **438**, 255 (1998) [arXiv:hep-ph/9806263]; R. Barbieri, L. J. Hall and Y. Nomura, Phys. Rev. D **63**, 105007 (2001) [arXiv:hep-ph/0011311]; arXiv:hep-ph/0106190.

[14] N. Arkani-Hamed, L. J. Hall, Y. Nomura, D. R. Smith and N. Weiner, Nucl. Phys. B **605**, 81 (2001) [arXiv:hep-ph/0102090].

[15] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D **61**, 033005 (2000) [arXiv:hep-ph/9903417].

[16] K. R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. B **436**, 55 (1998) [arXiv:hep-ph/9803460]; Nucl. Phys. B **537**, 47 (1999) [arXiv:hep-ph/9806292].

[17] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398].

[18] T. Li and W. Liao, arXiv:hep-ph/0202090.

[19] S. Dimopoulos, D. E. Kaplan and N. Weiner, arXiv:hep-ph/0202136.