Relation between Barrier Conductance and Coulomb Blockade

Peak Splitting for Tunnel-Coupled Quantum Dots

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Abstract

We study the relation between the barrier conductance and the Coulomb blockade peak splitting for two electrostatically equivalent dots connected by tunneling channels with bandwidths much larger than the dot charging energies. We note that this problem is equivalent to a well-known single-dot problem and present solutions for the relation between peak splitting and barrier conductance in both the weak and strong coupling limits. Results are in good qualitative agreement with the experimental findings of F. R. Waugh et al.

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I. INTRODUCTION

Turning on a tunnel junction to a quantum dot leads to progressive destruction of the single-dot Coulomb blockade. Experiments by Waugh et al. [2] and Molenkamp et al. [3] chronicle this eradication for two tunnel-coupled dots of equal and widely disparate charging energies, respectively. Inspired by the experimental results of Ref. [2], the present paper seeks to develop a simple model for the coherent tunneling of electrons between a pair of electrostatically identical quantum dots. (See Figure 1(a) for a schematic view of Waugh et al.'s double-dot structure.) The goal is to describe the evolution of the Coulomb blockade from that of two isolated dots to that of one composite dot in terms of parameters that determine the states of the isolated dots and the nature of the connection between them. In the limits relevant to the experimental situation in Ref. [2], we find that the most important dimensionless parameters are the number $N_{ch}$ of conducting channels between the two dots and the dimensionless interdot barrier conductance $g$ of each channel, which is measured when the Coulomb blockade has been removed. (The interdot barrier conductance was measured in Ref. [2] by de-energizing the external barrier potentials $V_{xi}$ that separate the dots from the leads. This conductance is to be distinguished from the conductance measured in the double-dot Coulomb blockade measurements, which will be referred to as the *Coulomb blockade conductance* or *double-dot conductance*.)

The problem of coupled quantum dots and more generally, of the effect of tunnel-couplings upon the Coulomb blockade has received much attention. I. M. Ruzin et al. [4] examined the Coulomb blockade structure of two non-identical dots in series via a standard activation-energy approach. C. A. Stafford and S. Das Sarma [13,15] as well as G. Klimeck et al. [14] have applied Hubbard-like models with and without interdot capacitances to determine the many-body wavefunctions for tunnel-coupling between a small array of single-dot eigenstates. Many investigators have studied the effect of tunneling upon the Coulomb blockade for metallic junctions, in which there are a large number of conducting channels [5-9,17]. Relatively few have considered junctions with only one or two
channels [16,18-20].

In Section II of this paper, we present a brief review of the experimental results that have motivated our investigation. In Section III, we define a tunneling model which is useful for calculations in the limit of weak coupling between the two dots. The strong-coupling limit is analyzed in Section IV, and a summary of our conclusions is presented in Section V.

II. MOTIVATION

The experiment of F. R. Waugh et al. [2] provides the primary motivation for this paper. These authors study the effect that varying the interdot potential barriers has upon the Coulomb blockade conductance peak structure for arrays of $n$ dots, where $n$ equals 2 or 3. For their Coulomb blockade measurements, they energize the confining gates ($V_{xi}$ in Figure 1(a)) so that the conductance between the dots and the external leads is much less than $2e^2/h$. Having tuned the dots to be electrostatically identical—i.e., to have common gate and total dot capacitances $C_g$ and $C_{\Sigma}$—they find that lowering the interdot barriers results in interpolation between the peak structure characteristic of the isolated individual dots and that characteristic of a single composite dot having capacitance $nC_{\Sigma}$: the initial isolated-dot peaks split into bunches of $n$ sub-peaks, and the splitting within the bunched sub-peaks increases until they are essentially equally distributed with $n$-times the periodicity of the original peaks. (See Figures 2(b) and 2(c).) For the double dot ($n = 2$), Waugh et al. also measure the conductance $G_b$ of the barrier between the two dots after the exterior walls of the double dot have been removed. In plots of the sub-peak splitting and barrier conductance as functions of the barrier gate voltage, they remark that the latter appears to be a displaced repetition of the former, the displacement being explicable as a result of the need to correct for the exterior walls’ influence upon the barrier.

Waugh et al. use a $T = 0$ “capacitive charging model” to interpret their data. In this model, electrons on the dots are treated as charged particles with no kinetic energy that occupy each dot in integer amounts. In the absence of coupling, the energy is given by the
sum of the potential energies of the individual dots. For two dots with common capacitances $C_\Sigma$ and $C_g$, the expression for the energy has a particularly simple form:

$$E = \frac{U}{2} \sum_{i=1}^{2} (n_i - \phi_i)^2,$$

(1)

where $U$ is the charging energy for each individual dot, $U = e^2/C_\Sigma$; $n_i$ is the number of electrons on the $i$th dot; and $\phi_i$ is the gate voltage parameter that determines the energy-minimizing value of $n_i$. For common gate voltages and gate-to-dot capacitances, we have the relations $\phi_i \equiv C_{gi}V_{gi}/e = C_gV_g/e \equiv \phi$. Figure 1(a) should help put these parameters in context.

For each set of integer occupation numbers $(n_1, n_2)$, the capacitive charging model with $\phi_i = \phi$ gives an energy $E_{(n_1, n_2)}$ that is a parabolic function of the common gate voltage parameter $\phi$. (See Figure 2.) All the parabolas are identical in shape, their only distinguishing features being the locations of their minima. The lowest-energy parabola $E_{N_{tot}}(\phi)$ for a given value of $N_{tot} = \sum_{i=1}^{2} n_i$ has $n_1 = n_2 = N_{tot}/2$ for $N_{tot}$ even and $n_1 = n_2 \pm 1 = N_{tot}/2 \pm 1/2$ for $N_{tot}$ odd. In the former even case, the minima all lie on the line $E = 0$. In the latter odd case, the minima are displaced upward, sitting along $E = U/4$. For all parabolas, the $\phi$-coordinate of the minimum is $N_{tot}/2$.

A prominent peak in the double-dot conductance occurs at values of $\phi$ such that the lowest-energy parabolas corresponding to consecutive values of $N_{tot}$ cross—in other words, at values of $\phi$ for which $E_{N_{tot}}(\phi) = E_{N_{tot}+1}(\phi)$ for some integer $N_{tot}$. For the model of Equation (1), this occurs whenever $\phi = m + \frac{1}{2}$, where $m$ is an integer. (One such crossing point is marked by the black dot in Figure 2.)

In a model in which coupling between the dots is included, the lowest-energy parabolas for odd $N_{tot}$ are shifted downward relative to the lowest-energy even-$N_{tot}$ parabolas by an “interaction energy” $E_{int}$. This downward shift splits each of the initial crossing points into a pair of crossing points symmetric about the position of the initial degeneracy, from which they are separated by a distance proportional to $E_{int}$. As a result, each of the initial conductance peaks is similarly split into two sub-peaks with separation proportional to $E_{int}$. 

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The sub-peak splitting reaches its saturation value when $E_{\text{int}} = U/4$—i.e., when the lowest-energy even and odd parabolas have the same minimum energy. At this point, the relevant crossing points occur for $\phi = \frac{1}{2}(m + \frac{1}{2})$. The corresponding conductance peaks are once again equally spaced, but their period is now that characteristic of a single dot with capacitance $2C_\Sigma$.

Thus, in the capacitive charging model, the problem of explaining the peak splitting reduces to the problem of describing the shift in the ground state energy of a double dot containing a fixed total number of particles. Waugh [2] has shown that introduction of a capacitive coupling $C_{\text{int}}$ between the two dots would allow one to obtain a picture in qualitative agreement with the experimental results: as the interdot capacitance goes to infinity, $E_{\text{int}}$ converges to $U/4$. However, the magnitude of the interdot coupling necessary to fit the experimental data is much larger than what one would expect from an electrostatic interaction between two adjacent dots having a narrow tunneling channel between them. Consequently, the use of the interdot capacitance $C_{\text{int}}$ must be regarded as simply a reparametrization of the problem which replaces one unknown, $E_{\text{int}}$, with another unknown, $C_{\text{int}}$. What we really want is a theory which produces at least qualitative agreement with experiment and expresses $E_{\text{int}}$ in terms of simple measurable quantities. Waugh et al. provide one candidate: the conductance $G_b$ of the barrier between the two dots. The remainder of this paper is devoted to developing a theory of the relation between $E_{\text{int}}$ and the dimensionless conductance per tunneling channel $g = \frac{G_b}{N_{\text{ch}}G_0}$, where $N_{\text{ch}}$ is the number of independent interdot tunneling channels (assumed to have identical conductances) and $G_0$ is the conductance quantum $e^2/h$. In the experiment of Ref. [2], there is no applied magnetic field and the dots are connected by a narrow constriction allowing only a single transverse orbital mode with double spin degeneracy. As a result, in this experimental case, $N_{\text{ch}}$ equals 2.

III. TUNNELING MODEL FOR THE DOUBLE-DOT COUPLING
A. Definition of the Model

Our goal can be stated a bit more precisely. For a general tunnel-coupling between two dots involving any number $N_{ch}$ of identical, independent channels and dimensionless channel conductance $g$, our aim is to express the fractional energy shift $f \equiv \frac{E_{int}}{U/4}$ as a function of $g$ and $N_{ch}$ plus any other parameters that might be found to be important. In order to derive an equation for $f$, we first choose a double-dot Hamiltonian. Since we hope to explain the evolution of the double-dot Coulomb blockade through the barrier conductance alone, we will ignore—for the moment at least—electrostatic coupling of the dots. Interaction between the dots will occur solely via tunneling through the barrier between them. Such tunnel Hamiltonians have been found useful from the beginnings of Coulomb blockade theory [11], and the model we will use is a double-dot version of the Hamiltonian used, for example, by Averin and Likharev to investigate the conductance oscillations of small metal-to-metal tunnel junctions [12]. In particular, we have the Hamiltonian $H = H_0 + H_T$, where

$$H_0 = K + V,$$
$$K = \sum_{i=1}^{2} \sum_{\sigma} \sum_{k} \epsilon_{ik\sigma} \hat{n}_{ik\sigma},$$
$$V = \frac{U}{2} \sum_{i=1}^{2} (\hat{n}_i - \phi_i)^2,$$
$$H_T = \sum_{\sigma} \sum_{k_1 k_2} \left( t_{k_1 k_2} c_{2k_2\sigma}^\dagger c_{1k_1\sigma} + h.c. \right).$$

(2)

In these equations, $i$ is the dot index, $\sigma$ is the channel index (which could signify different spin channels), and $k$ is the index for all internal degrees of freedom not included in the channel index. In addition, $\hat{n}_i = \sum_{k\sigma} \hat{n}_{ik\sigma}$ is the number operator for the $i$th dot, and $t_{k_1 k_2}$ is the tunneling matrix element between a dot 1 wavefunction indexed by $k_1$ and the dot 2 wavefunction lying in the same channel and indexed by $k_2$. The gate voltage parameter $\phi_i$ has the same meaning as in Equation (1). $\epsilon_{ik\sigma}$ is the kinetic energy of the single-particle eigenstate of the $i$th dot having the indicated degrees of freedom. For simplicity, we will take these energies to be independent of dot and channel: $\epsilon_{ik\sigma} = \epsilon_k$. 
The next step in focusing upon a model Hamiltonian is to choose a form for $t_{k_1k_2}$. Quite generally, $t_{k_1k_2}$ will be nonzero only when both $k_1$ and $k_2$ lie within some wavevector shell that maximally spans the space between the theory’s low and high momentum cutoffs. The size of the wavevector shell depends on details of the barrier: for a channel with an abrupt delta-function barrier, the shell spans the largest possible energy range; for a slowly developing adiabatic barrier, the shell width will be small on the scale of a Fermi wavevector. Important questions are how many states lie within this shell—i.e., how large is the width $W$ of the corresponding energy shell compared to the average level spacing $\delta$ between different states in the same channel (hereafter referred to as “the average level spacing” or just “the level spacing”)—and for a given $k_1$, for how many $k_2$ is $t_{k_1k_2}$ nonzero. Thin-shell models with “one-to-one” hopping elements (i.e., for which $t_{k_1k_2} = 0$ unless $k_1 = k_2$) have been applied to the coupled dot problem with some success [13–15], particularly for level spacings $\delta$ which are on the order of the charging energy $U$. For the nearly micron-sized dots used by Waugh et al. [2], however, $U$ is approximately 150 $\mu$eV and $\delta$ is on the order of 10 $\mu$eV, so we expect that a tunnel-coupling sufficient to destroy the isolated-dot Coulomb blockade will involve a large number of single-dot eigenstates. Consequently, we consider here a thick-shell model which is the antithesis of the injective thin-shell model. Working in a regime where $W \gg U \gg \delta$, we use a tunneling matrix element $t$ that is independent of $k_1$ and $k_2$ within the shell:

$$t_{k_1k_2} = t : \quad \forall k_1, k_2 \text{ s.t. } \epsilon_0 < \epsilon_{k_1}, \epsilon_{k_2} < \epsilon_0 + W.$$ 

As the quantities we calculate are independent of the phase of $t$, we guiltlessly choose $t$ to be real. This model is roughly equivalent to one in which each dot is represented by a tight-binding lattice with intersite hopping elements of order $W/\delta$ and where interdot tunneling occurs via a tunneling Hamiltonian with a single site-to-site connection. Choosing these tunneling sites to be at the origins $0_1$ and $0_2$ of the respective lattices, we may write

$$H_T = \sum_{\sigma}(Tc^\dagger_{0_2\sigma}c_{1\sigma} + h.c.),$$
where \( T \equiv N_W t \) and \( N_W = W/\delta \) is the number of orbital states per channel in each dot within the bandwidth \( W \). (The equivalent lattice model should include second and further neighbor hopping so that the density of states is approximately constant between \( \epsilon_0 \) and \( \epsilon_0 + W \). The lattice constant is chosen by requiring that the product of \( N_W \) and the area of a unit cell equals the area of a single dot.)

As the Fermi energy \( \epsilon_F \) must be somewhere between \( \epsilon_0 \) and \( \epsilon_0 + W \), the meaning of \( \epsilon_0 \) depends on the width of the band. For a maximally thick shell, \( \epsilon_0 \) lies at the bottom of the conduction band, and \( W \) is an ultraviolet cutoff, chosen to be of order twice the Fermi energy. Alternatively, when the barrier between the dots has a broader spatial extent, the energy shell sits more narrowly about the Fermi energy, and the width \( W \) is on the order of the energy difference needed to produce a factor-of-two change in the magnitude of the transmission amplitude for an incident particle. We define a dimensionless filling parameter

\[
F \equiv \frac{\epsilon_F - \epsilon_0}{W},
\]

which gives the position of the Fermi level within the bandwidth \( W \). Provided that \((1 - F)W\) and \(FW\) are both large compared to \( U \), our final results should be independent of the precise values of \( W \) or \( F \).

The model we have constructed is basically the two-dot version of that used by L. I. Glazman and K. A. Matveev [16] and H. Grabert [17] to study the charge fluctuations of a single metal particle connected via point-tunnel junctions to conducting leads. (See Figure 1(b).) Indeed, the similarity between the two-dot and one-dot problems is even more fundamental than this observation indicates. Consider again the double-dot potential energy \( V \). By transforming to the analog of center-of-mass coordinates, one generates the following form:

\[
V = \frac{U}{4}(\hat{N}_{tot} - \Phi_{tot})^2 + U(\hat{n} - \rho/2)^2,
\]

where \( \hat{N}_{tot} = \sum_{i=1}^{2} \hat{n}_i \), \( \Phi_{tot} = \sum_{i=1}^{2} \phi_i \), \( \hat{n} = \frac{\hat{n}_2 - \hat{n}_1}{2} \), \( \rho = \phi_2 - \phi_1 \). The rationale for the normalizations for \( \hat{n} \) and \( \rho \) will soon be made apparent. In the meantime, note that for our
Hamiltonian, $N_{\text{tot}}$ is a constant of motion. Thus, for given $N_{\text{tot}}$, $\Phi_{\text{tot}}$, and $U$, we can drop the first term and insert in the Hamiltonian a reduced potential energy:

$$V_{\text{red}} = U(\hat{n} - \rho/2)^2.$$  \hspace{1cm} (4)

Now restrict $N_{\text{tot}}$ to be even. Then, $\hat{n}$ has integer expectation values in all the unperturbed double-dot eigenstates. The Hamiltonian is exactly that of a single dot tunnel-coupled to an ideal lead. The dot has number operator $\hat{n}$, charging energy $2U$, and gate voltage parameter $\rho/2$. In the absence of tunneling and with the level spacing in both dots much less than the charging energy $U$, the ground state is an eigenstate of $\hat{n}$ that minimizes the reduced potential energy, which in the future we consider equivalent to “the potential energy.” For $\rho = 0$, the minimum potential energy is zero and is achieved when the eigenvalue $n$ of $\hat{n}$ is zero—i.e., when there are an equal number of particles in the two dots. All other values of $n$ give higher potential energies. For $\rho = 1$, on the other hand, the minimum potential energy is $U/4$, and $n = 0$ and $n = 1$ give degenerate minima.

These no-tunneling distinctions between zero and $U/4$ and between nondegeneracy and double degeneracy are quite familiar: they characterized the previously discussed even and odd double-dot ground states ($\rho = 0$ for both). Indeed, what we called the “even double-dot ground state” is precisely the “$N_{\text{tot}}$ even, $\rho = 0$ ground state.” The “odd double-dot ground state” is not exactly the same as the “$N_{\text{tot}}$ even, $\rho = 1$ ground state”; there is no getting around the fact that one case has one more (or less) particle than the other. However, in terms of their ground state energies, the difference between the two will be down by a factor of $FN_W$ or $(1 - F)N_W$. For a wide shell somewhere in the vicinity of half-filling, both $FN_W$ and $(1 - F)N_W$ are much greater than one, and the above difference is negligible. Calculation of $E_{\text{int}}$ with $\phi_1 = \phi_2$ for the double dot is therefore effectively equivalent to calculating the relative shifts of the $\rho = 0$ and $\rho = 1$ ground states of a single dot tunnel-coupled to a bulk two-dimensional electron gas. More generally, the value of $\rho$ can be arbitrary, and the difference in the ground-state energies of the double-dot system for even $N_{\text{tot}}$ and odd $N_{\text{tot}}$ is related to the difference in the ground-state energies of the
single-dot system for gate voltage parameters ρ and 1 + ρ.

We have mapped our double-dot problem onto a more general single-dot problem for which we can calculate the relative downward shift of the ρ ≠ 0 ground state to the ρ = 0 ground state. As the minimum potential energy is periodic in ρ with period two and is also even in ρ, we need only calculate up to ρ = 1. Dividing by the zero-tunneling energy difference of the two ground states, we find that our emended aim is to calculate

\[
 f_\rho \equiv \left( \frac{E_{\text{int}}(\rho)}{U\rho^2/4} \right) = \Psi_\rho(g, N_{\text{ch}}, u, N_W, F),
\]

where 0 < ρ ≤ 1, u = U/W, N_W = W/δ, and \(E_{\text{int}}(\rho)\) is the ground-state energy relative to the ground-state energy for ρ = 0.

### B. Barrier Conductance in the Weak-Coupling Limit

Before we can derive our equation for \(f_\rho\) in terms of \(g\), we must find a formula for the barrier conductance. Measurement of the barrier conductance \(G_b\) with the exterior gates turned off can be modeled by calculating the tunnel junction conductance for \(U = 0\). As mentioned before, we assume the different conducting channels to be identical yet independent—their individual conductances are the same and they do not interfere with one another. These assumptions are certainly reasonable for the two spin channels in the experiment of Ref. [2].

Using the Lippmann-Schwinger equation with \(H_T\) inserted for the scattering potential [10], one can solve for the perturbed electron eigenfunctions. The Heisenberg equation of motion for \(\hat{n}_1\) can then be used to solve for the particle flow from dot 1 to dot 2 for a given voltage bias. Solving the resulting expression for the linear conductance gives the following equation for the dimensionless conductance per channel:

\[
 g = \frac{G_b}{N_{\text{ch}}G_0} = \frac{4\alpha}{|1 + \chi\alpha|^2},
\]

where \(\alpha = (\pi T/W)^2 = (\pi t/\delta)^2\) and \(\chi = (1 + \frac{i}{\pi} \log(\frac{F}{1-F}))^2\). H. O. Frota and K. Flensberg have derived this equation for half-filling \((F = 0.5, \chi = 1)\) via a Green’s function-Kubo formula approach [18].
The calculated conductance $G_b$ exhibits rather curious behavior: it first rises to a maximum of $N_{ch} G_0$ corresponding to $N_{ch}$ fully open channels (It does this even when $Im[\chi] \neq 0$), and then falls asymptotically to zero as $(T/W = t/\delta) \to \infty$. As Frota and Flensberg note [18], the asymptotic damping of the conductance results from the fact that formation of bonding and anti-bonding states at the tunnel junction makes the cost of passing through prohibitively high. The limit of $(T/W = t/\delta) \to \infty$ is in some sense unphysical: we do not expect a point-to-point hopping coefficient $T$ to significantly exceed the tunneling shell width; nor do we expect the tunneling matrix element $t$ to be much greater than the average level spacing. Nevertheless, the apparent absence of any good reason to truncate the theory at a particular value of $t$ leaves us with a model that is at best unwieldy in the limit of strong coupling. To get the correct limiting behavior for strong coupling, it is more convenient to use a different approach, suitable for perturbation about the $g = 1$ limit. This will be described in Section IV.

C. Relative Energy Shift of Even and Odd States in the Weak-Coupling Limit

In the meantime, the site-to-site tunneling model is still useful in the weak coupling regime. So we plod ahead, calculating via standard Rayleigh-Schrödinger perturbation theory the second order shift in the ground state energy for $\rho \neq 0$ minus that for $\rho = 0$. The $\rho = 1$ shift will be taken to equal the limit of the general $0 \leq \rho < 1$ shift as $\rho \to 1$. It might be objected—correctly— that this limit fails properly to account for the degeneracy of the ground state at $\rho = 1$. Such a failing is pardonable, however, for the contributions that are left out are all smaller by a factor of $FN_W$ or $(1 - F)N_W$ from those which are retained. Since we assume that $t/\delta$ is finite, $F$ is of order $\frac{1}{2}$, and $N_W$ is large, the omitted terms are negligible.

For $N_W \gg 1$, the perturbation theory sums can be approximated as integrals. Observing that $u \equiv \frac{U}{W} \ll 1$, we divide the difference between the second order shifts by $U \rho^2 / 4$ to get the leading approximation to $f_\rho$:
\[ f^{(1)}_\rho = 4N_{\text{ch}} \frac{\rho^2}{g^2} \left[ (1 - \rho) \log(1 - \rho) + (1 + \rho) \log(1 + \rho) + O(u\rho^2) \right]/\rho^2. \]  

(7)

The second-order term indicates a significant feature of \( f_\rho \): it is even in \( \rho \). This property has been noted by H. Grabert \[17\] and results from the fact that at any order of perturbation theory, every tunneling process contributing to the energy shift has a twin with the roles of dots 1 and 2 interchanged. In any intermediate virtual state with eigenvalue \( n \) for \( \hat{n} \), the potential energy is greater than that for the unperturbed ground state by \( \delta V(\rho) = U(n(n - \rho)) \).

Therefore, when dots 1 and 2 are interchanged, \( \delta V(\rho) \rightarrow \delta V(-\rho) \) for all the intermediate states. If we represent one of the twin terms by \( \Delta(\rho) \), the other is \( \Delta(-\rho) \), and we see that \( f_\rho \) is constructed of sums that are even in \( \rho \).

Using the second-order (in \( t/\delta \)) parts of \( g \) and \( f_\rho \), we can now write a first-order equation for \( f_\rho \) in terms of \( g \):

\[ f^{(1)}_\rho = \frac{N_{\text{ch}} g}{\pi^2} \left[ (1 - \rho) \log(1 - \rho) + (1 + \rho) \log(1 + \rho) + O(u\rho^2) \right]/\rho^2, \]

(8)

a result consistent with the large-\( N_{\text{ch}} \) calculation of the effective capacitance of a single dot at \( \rho = 0 \) \[5\]. Setting \( \rho = 1 \) to calculate the relative shifts of the original even and odd states, we find

\[ f^{(1)} = \frac{2 \log 2}{\pi^2} N_{\text{ch}} g + O(ug,g^2), \tag{9} \]

where we have used the fact that \( f \) as originally defined without the subscript is equivalent in our limits to \( f_{\rho=1} \). The above equation indicates that the suggestion that the plot of \( f \) is a displaced version of the plot for \( g \) is not precisely correct. In particular, for \( g \ll 1 \) and \( N_{\text{ch}} = 2 \), Equation (9) gives a slope of approximately 0.28 for \( f(g) \), rather than unity. Thus, in this regime, the fractional splitting \( f \) of the double-dot conductance peaks should lag \( g \), the dimensionless barrier conductance per channel.

**IV. CONNECTION TO THE STRONG-COUPLING LIMIT**

If we blithely extended our perturbative equation for \( f \) to the limit \( g \rightarrow 1 \), the large-\( N_{\text{ch}} \) \( f \) would greatly overshoot its mark and the one or two-channel \( f \) would fall substantially
short. The real issue is not, however, how badly such a naive extrapolation fails, but whether we can interpolate between these weak-tunneling results and those which can be calculated for the strong-tunneling limit. For the large-$N_{ch}$ limit, a reasonable interpolation between the solutions for weak and strong coupling has already been found \cite{1,2,5}. The situation is less clear when $N_{ch}$ equals one or two. Flensberg and Matveev \cite{19,20} have proposed a useful Lüttiger-liquid approach in which the nearly transparent link between a dot and an electrode is modeled as a one-dimensional channel with a slightly reflective potential barrier. Convergence to the single composite-dot limit is achieved naturally and neatly, and $E_{int}$ is calculated perturbatively in $r$, where $r$ is the reflection amplitude, and $g = 1 - |r|^2$. Translating Matveev’s calculations of the leading term for $(1 - g) \ll 1$ into our language, we find that for $N_{ch} = 1$ (i.e., assuming spin polarization),

\[
    f = 1 - \frac{8e\gamma}{\pi^2} \sqrt{1 - g},
\]

where $\gamma \approx 0.577$ is the Euler-Mascheroni constant. For the case relevant to the experiment of Ref. \cite{2}, $N_{ch} = 2$, Matveev’s calculation gives

\[
    f = 1 + \frac{16e\gamma}{\pi^3} (1 - g) \log(1 - g).
\]

Except for the logarithmic factor in the second formula, these equations are of the form suggested by the scaling analysis of Flensberg \cite{19}, which predicts effective charging energies behaving as $(1 - g)^{N_{ch}/2}$. Matveev’s initial two-channel solution is, in fact, linear in $(1 - g)$ but diverges logarithmically as $U/\delta \to \infty$. A higher-order analysis to eliminate the divergence replaces the logarithm having argument $U/\delta$ with one having argument $(1 - g)^{-1}$ \cite{20}.

In Figure 3, we show the $f$-versus-$g$ plots given by the weak and strong coupling formulas (4), (10), and (11) for $N_{ch} = 1$ and $N_{ch} = 2$. In each case, a plausible interpolation between the weak and strong coupling limits is given by a dashed curve. The experimental data of Ref. \cite{2} are in reasonable agreement with the dashed curve for $N_{ch} = 2$. Nevertheless, from a theoretical standpoint, it is clear that, unlike the calculations for $N_{ch} \gg 1$, for $N_{ch} = 2$ the order of calculation completed so far does not really allow confident interpolation between

the weak and strong coupling limits. Calculation of higher orders in perturbation theory should improve the matching, but such computations are made difficult by the fact that the correlations induced by the strong Coulomb interaction make normal Green’s functions methods inapplicable [9]. Different time orders must be treated separately, and as appears to occur quite generally in Coulomb-blockade problems [21], the number of diagrams grows pathologically with the order in perturbation theory. Nevertheless, calculation of the \( g^2 \)-term in the weak-tunneling limit is within reach, and this term may suffice to give a more reliable interpolation between the weak and strong coupling regimes.

V. CONCLUSION

Following the work of Waugh et al. [2], we have investigated the relation between the barrier conductance and the Coulomb blockade for two electrostatically equivalent dots connected by one or more identical tunneling channels. We propose to write the fractional peak splitting \( f \) of the Coulomb blockade conductance peaks as a function of the number of channels \( N_{ch} \) and the dimensionless barrier conductance per channel \( g \), assuming that the energy level spacing \( \delta \) is small compared to the Coulomb blockade energy \( U \) and that \( U \) is small compared to the bandwidth \( W \) of states over which the amplitudes for transmission through the barrier are roughly constant. Using a “uniform thick-shell model” for the tunneling term in the Hamiltonian, we solve for this function to leading order in the limit of weak interdot coupling. We find that the peak splitting should evolve substantially more slowly than the barrier conductance in this limit. Having noted that the two-dot problem can be mapped onto a better known one-dot problem, we adapt previous strong-coupling results to obtain the asymptotic form of the double-dot peak-splitting in the limit \( g \to 1 \). For the case of \( N_{ch} = 2 \), which is pertinent to the experimental results of Ref. [2], the limiting forms for the strong and weak coupling do not match up well enough to allow a reliable quantitative interpolation between the two limits. Nevertheless, a visually plausible interpolating curve is in good qualitative agreement with existing experimental data.
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After this manuscript was essentially completed, the authors received a preprint by K. A. Matveev, L. I. Glazman, and H. U. Baranger [22] in which similar ideas were independently developed.
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FIGURES

FIG. 1. (a) Schematic diagram for the double dot. Negative potentials are applied to each of the gates to form the double-dot structure. The gate potentials $V_{g1}$ and $V_{g2}$ control the average numbers of electrons on the dots. These are the potentials that are varied to see the Coulomb blockade. $V_b$ controls the rate of tunneling between the dots. $V_{x1}$ and $V_{x2}$ control the rate of tunneling to the adjacent bulk 2D electron gas (2DEG) leads. For calculations of the double-dot energy shifts, tunneling to the leads is assumed negligible compared to tunneling between the two dots. In measuring the barrier conductance $G_b$, however, the potentials $V_{xi}$ are turned off so that each dot is strongly connected to its lead. The side-wall potentials $V_{s1}$ and $V_{s2}$ are fixed. (b) Schematic diagram for the single dot. $V_b$ now controls tunneling between the dot and the bulk 2DEG. $V_g$ determines the average number of electrons on the dot. For our purposes, $V_s$ and $V_x$ are constant, and tunneling to the bulk 2DEG through the barrier defined by $V_x$ is negligible compared to tunneling through the barrier defined by $V_b$. 

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FIG. 2. (a) Energy curves in the capacitive charging model for electrostatically identical dots with $V_{g1} = V_{g2}$. Energies are given in units of the charging energy $U$; the gate voltage is given in units of $\frac{e}{C_g}$. Each zero-coupling eigenstate with definite particle number $n_i$ on the $i$th dot gives rise to a parabola, labeled $(n_1, n_2)$, which shows the state’s energy as a function of the gate voltage. The zero of energy is chosen to coincide with the lowest energy possible for states with an even value for the total number of particles $N_{tot}$. The solid odd-$N_{tot}$ parabola gives the lowest-energy curve only when there is no interdot coupling. The dotted parabola is the shifted-down energy curve for odd $N_{tot}$ that results from finite coupling between the dots. The relevant degeneracy points are indicated by a black dot for zero coupling and white dots for finite coupling. (b) “Zero-coupling” conductance through the double dot as a function of the gate voltage. (For ease of viewing, peaks are depicted as symmetric with uniform finite widths and heights.) Conductance peaks are aligned with the zero-coupling degeneracy points such as the one shown in (a) and occur regularly with unit period. (c) Conductance through the dot for finite interdot coupling. Conductance peaks are aligned with the perturbed degeneracy points. Each zero-coupling peak has split into two separate peaks, equally distant from the zero-coupling peak position. Increasing the interdot coupling increases the separation between the paired peaks until the full set of peaks is again regularly distributed, with half the original period. (This figure for the capacitive charging model follows that of Ref. [2].)

FIG. 3. Graphs of the fractional Coulomb blockade conductance peak splitting $f$ as a function of the dimensionless conductance per channel $g$ in the weak and strong tunneling limits for (a) $N_{ch} = 1$ and (b) $N_{ch} = 2$. Possible interpolating functions are shown by dashed curves.
Figure 1

(a) $V_{x1}$ $V_{s1}$ $V_b$ $V_{s2}$ $V_{x2}$

(b) $V_x$ $V_s$ $V_b$ $V_g$ $V_b$
Figure 2

(a) Energy levels for different values of gate voltage:
- (0,0)
- (0,1)
- (1,0)
- (1,1)

(b) Conductance graph showing multiple peaks at different gate voltages.

(c) Expanded view of conductance peaks, showing the detailed structure.
Figure 3

(a) and (b) show the relationship between $f$ and $g$. The solid line represents one scenario, while the dashed line represents another. The graphs illustrate how $f$ changes with $g$ across different intervals.