Uncertainty of angular displacement measurement with a MEMS gyroscope integrated in a smartphone

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Abstract. Low-cost inertial sensors have recently gained popularity and are now widely used in electronic devices such as smartphones and tablets. In this paper we present the results of a set of experiments aiming to assess the angular displacement measurement errors of a gyroscope integrated in a smartphone of a recent model. The goal is to verify whether these sensors could substitute dedicated electronic inclinometers for the measurement of angular displacement. We estimated a maximum error of 0.3° (sum of expanded uncertainty and maximum absolute bias) for the roll and pitch axes, for a measurement time without referencing up to 1 h.

1. Introduction
Inertial navigation systems composed of tridimensional acceleration sensors (accelerometers) and tridimensional rotation rate sensors (gyroscopes) are typically used for the assessment of position and orientation in scenarios with poor GPS signal availability [1,2]. More recently, low cost inertial sensors based on MEMS (microelectromechanical systems) technology are also widely applied in consumer electronics such as smartphones, tablets and video games. An application for this type of sensor is the determination of the orientation of a smartphone, and thus switch between landscape and portrait display.

Low-cost MEMS inertial sensors are currently a popular research topic. Several researchers around the world are deriving new applications for this kind of sensors, developing solutions to improve the characterization (calibration) process and investigating the performance of the sensors [3,4,5,6,7,8,9]. However, there is a lack of proposals for the estimation of the final measurement uncertainty of these sensors. Furthermore, no performance investigation of sensors integrated in a smartphone could be identified. In this paper we will address the performance investigation of a gyroscope sensor integrated in a smartphone from a measurement uncertainty approach, with focus on angular displacement measurements.

Gyroscopes provide an output signal proportional to the rotation rate of the object. The angular displacement of the object can be obtained by integrating this signal [1,2]. Based on the results of initial experiments, we formulated the hypothesis that the accuracy of the orientation data provided by a gyroscope integrated in a smartphone of a recent model may be sufficient for applications where one would typically use a dedicated electronic inclinometer. A typical application is the determination of the inclination of a surface in relation to a given reference system. As target we defined a maximum error of 0.1°. This is a typical value for electronic inclinometers available in the market such as the digital protractor Mitutoyo 950-315 [10].
Some advantages of the application of gyroscope sensors integrated in a smartphone are the low cost and the possibility of wireless data transfer by means of WiFi, GPRS or Bluetooth.

In this paper we present the results of a set of experiments conducted to evaluate the overall metrological performance and estimate the maximum error of a MEMS gyroscope, integrated in a high-end smartphone, for the measurement of angular displacement in the pitch and roll axes (figure 1) for low-dynamic applications. We did not evaluate the yaw axis due to the lack of an appropriate reference system.

![Figure 1. Definition of the gyroscope angles](image)

The maximum error was estimated for the measurement range between -20° and 20° in each axis. Maximum error \( E_{\text{max}} \) is defined by equation (1), where \( U_{\text{max}} \) is the maximum expanded measurement uncertainty for a coverage probability of 95.45% and \( |b|_{\text{max}} \) is the maximum absolute bias. The measurement uncertainty was estimated according to the “Guide to the Expression of Uncertainty in Measurement - GUM” [11].

\[
E_{\text{max}} = U_{\text{max}} + |b|_{\text{max}}
\]  

(1)

2. Equipment and software

In our experiments we used an Apple iPhone 5 smartphone (model MD297DN/A) running iOS 7.0.4. The measurement data was transferred to a laptop via WiFi network using the OSC (Open Sound Control) protocol. On the smartphone side we used the GyroSC application, which is commercially available [12]. This application allows the transmission of the raw data of several sensors of the smartphone (accelerometer, gyroscope, magnetometer etc.) with a sampling rate of 40 Hz.

The reception and storage of the data on the laptop was realized with an application developed by us in Processing programming language [13].

For the determination of the bias curve and the repeatability of the sensors we used a calibrated sinus table and a set of calibrated gauge blocks. The sinus table presents a distance between cylinder axis of 250 mm and a leveling height of 12.5 mm. Gauge-blocks of nominal length 12.5 mm, 15 mm, 20 mm, 30 mm, 40 mm, 50 mm, 75 mm and 100 mm were used, resulting in inclinations in respect to the reference surface of the sinus table of 0.00°, 0.57°, 1.72°, 2.87°, 4.01°, 6.32°, 8.63°, 14.48° and 20.49°.

3. Mathematical model of the measurement process

The first step for the estimation of the uncertainty of a measurement result is to define a mathematical model that describes the measurement process. We assumed a simple linear model as described by equation (2). In this equation \( \theta_{\text{ind}} \) is the displacement indicated by the sensor, \( \theta \) is the actual angular displacement and \( \delta_{\text{rep}} \), \( \delta_{\text{drift}} \), and \( \delta_{\text{temp}} \), are errors caused by the natural repeatability of the measurement process, by the drift of the signal related to the integration of the rotation rate signal and errors caused by thermal effects, respectively. The term \( \delta_{\text{ref}} \) represents errors related to the uncertainty of the reference measurement system used to assess the bias (sinus table and gauge blocks). The bias is represented by the term \( b \).

\[ \theta = \theta_{\text{ind}} + \delta_{\text{rep}} + \delta_{\text{drift}} + \delta_{\text{temp}} + \delta_{\text{ref}} + b \]
4. Experimental quantification of the measurement errors

In this section we describe the procedure and results of the experiments we performed aiming to quantify the terms $\delta_{\text{rep}}$, $\delta_{\text{drift}}$, $\delta_{\text{temp}}$ and $b$ of equation (2) as well as to assess the overall metrological performance of the sensor.

4.1. Bias and repeatability

The smartphone was fastened on a metallic block with low geometric errors (flatness, perpendicularity and parallelism). The positioning of the equipment on the sinus table was realized with the aid of a precision square. This setup is presented in figure 2.

For the evaluation of bias and repeatability, the smartphone was positioned in four different orientations with respect to the inclination generated by the sinus table, as shown in figure 3. This procedure allowed the realization of both positive and negative angles. Furthermore, the correction of the misalignment between the sensor and the sinus table was possible using the measurement data of these four positions. Three measurement cycles were performed in each position. Each measurement cycle was concluded in about 3 min.

The displacement of the sinus table was done smoothly in order to reduce the influence of dynamic effects.
The repeatability was evaluated only for the points 0º and 20.49º at positions 1 and 3. At both points we made 20 consecutive observations, moving the table after each measurement.

The bias and repeatability experiments where performed in a temperature controlled metrology laboratory in which the temperature is kept in the range between 19.5ºC and 20.5ºC. For all measurements we adopted the average of 100 consecutive values as indication. These values were captured after stabilization of the signal.

The bias curves for the pitch and roll axes are displayed in figures 4 and 5. The dashed line represents the bias without correction of the misalignment of the sensor. The bias was calculated according to equation (3). 

\[ \theta_{\text{ind}}, i = \theta_{\text{ref}}, i \cdot (C_{\text{align}} - 1) \]

\[ C_{\text{align}, \text{pitch}} = \frac{1}{2} \left[ \frac{\bar{\theta}_{\text{pitch}} - \theta_{\text{ref}, \text{pitch}}}{\theta_{\text{pitch}}} \right]_{0-0,49} + \frac{\bar{\theta}_{\text{pitch}} - \theta_{\text{ref}, \text{pitch}}}{\theta_{\text{pitch}}} \right]_{0-0,49} \]

\[ C_{\text{align}, \text{roll}} = \frac{1}{2} \left[ \frac{\bar{\theta}_{\text{roll}} - \theta_{\text{ref}, \text{roll}}}{\theta_{\text{roll}}} \right]_{0-0,49} + \frac{\bar{\theta}_{\text{roll}} - \theta_{\text{ref}, \text{roll}}}{\theta_{\text{roll}}} \right]_{0-0,49} \]

\[ b_i = \theta_i + \theta_{\text{ref}, i} \cdot (C_{\text{align}} - 1) \]

The maximum absolute bias are \(|b| = 0.14º\) for the pitch axis and \(|b| = 0.16º\) for the roll axis.

Figure 4. Bias curve - pitch axis
Figure 5. Bias curve - roll axis

The results for the repeatability can be found in table 1. The sample standard deviation $s$ was calculated based on the results of 20 consecutive observations.

| Axis  | $\theta$ [$^\circ$] | $s$ [$^\circ$] |
|-------|---------------------|----------------|
| Pitch | 0,000               | 0,015          |
|       | 20,487              | 0,034          |
| Roll  | 0,000               | 0,016          |
|       | 20,487              | 0,033          |

4.2. Signal drift

As already mentioned, the raw output signal of a gyroscope is proportional to the rotation rate of the object. The angular displacement is obtained by integrating this signal. Consequently, a high accuracy measurement of angular displacement is only possible if the bias of the rotation rate signal is stable in time and thus can be corrected.

The bias instability of gyroscopes is typically evaluated by means of the Allan variance ($\sigma_\alpha^2$) and its square root, the Allan deviation ($\sigma_\alpha$) [1,14]. A long sequence of raw data (rotation rate) is used for this analysis. The sensor is kept as stable as possible during the collection of this data sequence. This sequence is then divided in $n$ bins of equal length. Each of these bins corresponds to an integration time $\tau$. The analysis starts with a small integration time, e.g. $\tau = 1/f_{\text{eq}}$, where $f_{\text{eq}}$ is the frequency of the data acquisition. The average of the data $y(\tau)$, is then calculated for each bin $i$. The integration time is successively incremented, always in a way that the number of bins is an integer. This procedure is
repeated until nine bins are obtained. The Allan deviation $\sigma_a(\tau)$ is calculated for each integration time $\tau$ according to equation (6). The results for $\sigma_a(\tau)$ are plotted in a log-log diagram.

$$\sigma_a(\tau) = \sqrt{\frac{1}{2(n-1)} \sum_i (y(\tau)_{i+1} - y(\tau)_{i})^2}$$  \hspace{1cm} (6)$$

For short integration times, the Allan deviation is dominated by the signal noise. As the integration time increases, $\sigma_a(\tau)$ decreases due to the filtering effect. However, at some point $\sigma_a(\tau)$ starts to increase. This is related to the so-called rate random walk in the sensor, which is the inherent instability in the output of the sensor. The bias instability is defined at the minimum point of the $\sigma_a(\tau)$ curve [4].

We used a signal sequence of 10.5 h in our analysis. The raw signal for the pitch axis is shown in figure 6. The result for the roll axis was similar. The standard deviation of the signal along the acquisition time was 0.25º/s for pitch and 0.26º/s for roll.

![Figure 6. Raw signal - pitch](image)

The Allan deviation curve for the pitch axis is shown in figure 7. The curve for the roll axis is similar. The minimum point of the curve (bias instability) was $9.0\times10^{-4}$º/s (3.2º/h) for $\tau = 3543$ s for the pitch axis and $10.0\times10^{-4}$º/s (3.7º/h) for $\tau = 3402$ s for the roll axis. However, it is not clear if the acquisition time we adopted was sufficient to reach the inflexion point of the curve.

Both the noise level of the signal and the bias instability were better than expected, compared to other MEMS based sensors. For instance, Aydemir et al. measured a bias instability of 41º/h and a signal standard deviation of 0.69º/s for a dedicated inertial measurement system [3]. Another dedicated MEMS sensor features a specification of 20º/h for the bias instability [15].
Additionally, we evaluated the stability of the angle data provided by the sensor directly by monitoring this parameter over approximately 9.5 h, keeping the sensor stationary. The results are depicted in figure 8. An average drift of 0.036º/h was observed for the pitch axis and 0.008º/h for the roll axis. Although the level of noise is similar for both axes, the data of the roll axis presented randomly spaced peaks of around 0.1º. The cause of this effect could not be identified.

![Figure 7. Allan deviation – pitch](image)

![Figure 8. Stability of the angle data – pitch and roll](image)

4.3. Influence of temperature variation

Temperature variation due to changes in the environment and self-heating is reported to affect the stability of the rotation rate bias [1,8]. In this study we did not evaluate the influence of temperature variations under controlled conditions. However, since the average drift of the rotation rate signal was not significant in the data collected for the Allan deviation technique (see figure 6) we assume that the influence of temperature variations (at least those generated by self-heating) is negligible or internally compensated.
5. Measurement uncertainty budget
Since no significant influence of temperature variations could be identified, we simplified the model of equation (2), resulting in the final model of the measurement process (equation (7)).

\[ \theta_{\text{ind}} = \theta + \delta_{\text{rep}} + \delta_{\text{drift}} + \delta_{\text{ref}} + b \] (7)

The standard uncertainties \( u \) related to this model with the respective degrees of freedom \( \nu \) are estimated in sections 5.1 to 5.3. The sensitivity coefficient for all uncertainty sources is 1, as we are assuming that the model is linear and additive.

5.1. Standard uncertainty related to the natural repeatability of the sensor (\( u_{\text{rep}} \))
The standard uncertainty is equal to the maximum standard deviation obtained from the experiments described in section 4.1, divided by the square root of the number of observations (\( n' \)) expected to be performed in the regular use of the sensor. A Student’s t-distribution with \( \nu_{\text{rep}} = n-1=19 \) is assumed (\( n \) is the number of observations used to determine \( s_{\text{max}} \)). Assuming \( n'=1 \), \( u_{\text{rep}} \) is given by equation (8).

\[ u_{\text{rep}} = \frac{s_{\text{max}}}{\sqrt{n'}} = 0.034^\circ \] (8)

5.2. Standard uncertainty related to the signal drift (\( u_{\text{drift}} \))
The angle signal of a gyroscope will drift due to uncorrected bias and bias instability, calibration errors and due to noise in the rotation rate signal (angle random walk) [1]. Instead of taking each one of these causes separately into account, we opted to an empirical black box approach. The maximum observed average drift of the angle signal presented in figure 9 was used. This approach presents the advantage of being simple, but there is a risk of sub-estimating this source of uncertainty, since it takes only errors in account that are observed under static conditions.

A uniform distribution is assumed. The standard uncertainty is estimated according to equation (9), where \( t \) is the time, in hours, in which the sensor is used without submitting it to a referencing procedure.

\[ u_{\text{drift}} = \frac{\text{drift}_{\text{max}} \cdot t}{\sqrt{3}} = \left(0.021 \cdot t\right)^\circ \] (9)

5.3. Standard uncertainty related to the reference system (\( u_{\text{ref}} \))
The uncertainty of the angle generated by the sinus table depends on the uncertainty of the gauge blocks and the uncertainty of the geometric parameters of the sinus table – distance between cylinders and leveling height. The expanded uncertainty of the reference system, for a coverage probability of 95.45%, was estimated to be \( U_{\text{ref}} = 0.007^\circ \) (coverage probability of 95.45%, \( k = 2 \)). A normal distribution is assumed. The standard uncertainty is calculated according to equation (10).

\[ u_{\text{ref}} = \frac{U_{\text{ref}}}{2} = 0.004^\circ \] (10)

5.4. Combined standard uncertainty (\( u_c \)), effective degrees of freedom (\( \nu_{\text{eff}} \)), and expanded uncertainty (\( U \))
The combined standard uncertainty is given by equation (11).

\[ u_c = u_{\text{rep}}^2 + u_{\text{drift}}^2 + u_{\text{ref}}^2 \]

\[ u_c = \left(\sqrt{0.0012^2 + 0.00044 \cdot t^2}\right)^\circ \] (11)
The effective degrees of freedom $\nu_{\text{eff}}$ are estimated by the Welch-Satterthwaite equation (equation (12)). There is a direct correlation between $\nu_{\text{eff}}$ and the measurement time $t$. We therefore used a relatively small $t$ of 0.1 h, which will result in a smaller $\nu_{\text{eff}}$, a higher coverage factor $k$ and a higher expanded uncertainty $U$.

$$\nu_{\text{eff}} = \frac{u_c^4}{\sum u_i^4}$$

$$\nu_{\text{eff}} = 20$$

(12)

The expanded measurement uncertainty $U$ is given by equation (13). The coverage factor corresponding to $\nu = 20$ and a coverage probability of 95.45% corresponds to $k = 2.13$.

$$U = k \cdot u_c$$

$$U = 2.13 \cdot \left(\sqrt{0.0012 + 0.00044 \cdot t^2}\right) \circ$$

(13)

The maximum error is calculated according to equation 1, resulting in equation (14).

$$E_{\text{max}} = U_{\text{max}} + |b|_{\text{max}}$$

$$E_{\text{max}} = 2.13 \cdot \left(\sqrt{0.0012 + 0.00044 \cdot t^2}\right) \circ + 0.16 \circ$$

(14)

For a measurement time of $t = 1$ h, the maximum error, rounded to one decimal place, is $E_{\text{max}} = 0.3 \circ$.

6. Conclusions
In this paper we evaluated the performance on angular displacement measurements of a MEMS gyroscope integrated in a smartphone. We performed experiments to quantify bias and repeatability of the sensor within the range of -20º and 20º for both pitch and roll axes. The bias stability of the raw rotation rate signal was evaluated by means of the Allan deviation. The stability of the angle signal was also evaluated through the acquisition and analysis of a long data sequence. The measurement uncertainty was estimated according to GUM.

We estimated a maximum error of 0.3º, for a measurement time of 1 h, which is significantly higher than the specification of a typical dedicated inclinometer (e.g. $E_{\text{max}} = 0.1 \circ$). However, we still consider this result positive, since the sensor integrated in the smartphone was not designed for accurate angular displacement measurement, but primarily for the estimation of the orientation of the smartphone in space. The bias stability and noise level of the raw rotation rate signal were better than expected, compared to dedicated MEMS-based sensors.

We could not investigate the influence of temperature variations under controlled conditions, due to the lack of an appropriate thermal chamber. We decided to neglect this influence in our measurement uncertainty budget, because no significant drift of the rotation rate signal was detected during an acquisition time of about 10.5 h. However, this hypothesis can only be confirmed by a controlled experiment.

It is important to mention that we performed the experiments under rather favorable measurement conditions (e.g. low temperature variation, low dynamic measurement, short measurement time). It is clear that the actual measurement error can be significantly higher depending on the measurement conditions at the actual application.
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