Calculation of high-frequency conductivity and Hall constant of a thin conductive layer in the view of equal specularity coefficients of its surfaces

O V Savenko¹, I A Kuznetsova², A A Yushkanov³

¹,²Department of Microelectronics and General Physics, P G Demidov Yaroslavl State University, Yaroslavl 150003, Russia
³Department of Theoretical Physics, Moscow State Regional University, Moscow 105005, Russia

Abstract. The kinetic task about high-frequency conductivity and Hall constant of a thin conductive layer placed in the transverse stationary magnetic field and longitudinal alternative electric field is solved. The ratio between the layer thickness and the mean free path of charge carriers is supposed to be arbitrary. The skin effect isn’t taken into account. We consider the diffuse-specular charge carrier reflection mechanism from layer boundaries. Specularity coefficients of the upper and lower layer surfaces are presumed to be equal. The limited cases of the degenerated and non-degenerated electron gas are examined. The dependences of conductivity and Hall constant on the electric field frequency, magnetic field induction, layer thickness and specularity coefficient of layer surfaces are studied.

1. Introduction

Electrical, optical and galvanomagnetic properties of conductive objects with the characteristic linear dimension that is comparable to and less than charge carrier mean free path differ essentially from the properties of macroscopic samples. At room temperature the values of charge carrier mean free path $\lambda$ for typical semiconductors range between 50 and 1000 nm, and de Broglie wavelength $\lambda_B$ is proportional to 10 nm [1, 2]. For metals with high conductivity $\lambda = 10 \div 100$ nm, and de Broglie wavelength is proportional to the interatomic distance $\lambda_B \approx 0.3$ nm [1, 2]. Present technologies permit to produce materials with the linear dimension of the order to a few dozen nanometers. Therefore, the situation, when we must take into consideration the classical size effects and may neglect quantum effects for describing of sample electric properties, is realizable in practice.

The first known work devoted to calculation of static conductivity of a thin metal film placed in the perpendicular magnetic field in the view of the diffuse and diffuse-specular electron surface reflection was published by Sondheimer in 1950 [3] and republished later in 2001 [3]. In next works [4, 5] the Sondheimer task was complicated to the cases of arbitrary magnetic field direction [4], non-spherical Fermi surface and different specularity coefficients of layer surfaces [5].

The theoretical investigations devoted to these subjects are continued in present time. In the works [6, 7] the tasks about high-frequency conductivity of a thin metal film in the view of different surface specularity coefficients for the cases of a homogeneous [6] and nonhomogeneous [7] electric field was
considered. In the works [8, 9] the theoretical models of high-frequency conductivity and Hall constant of a thin metal film in the view of diffuse [8] and diffuse-specular [9] electron surface scattering mechanisms for equal film surface specularity coefficients was built.

2. Task statement
We consider a thin conductive layer, the material of which is metal or $n$- or $p$-type semiconductor. The layer is placed in the transverse constant magnetic field with induction $B$. The alternative electric voltage with the frequency $\omega$ is applied to the ends of the layer. The electric and magnetic fields are supposed to be homogeneous. The skin effect is neglected (we assume, that $a < \delta$, where $a$ is the layer thickness, $\delta$ is the skin layer depth). Quantum effects aren’t taken into account, because $a >> \lambda_B$,
where $\lambda_B$ is de Broglie wavelength of charge carriers.

The time-periodic electric field

$$E = E_0 \exp(-i \omega t)$$

induces the deviation of charge carrier distribution function $f_i$ from Fermi distribution function $f_0$:

$$f(z, \nu, t) = f_0(\nu) + f_1(z, \nu, t) = f_0(\nu) + f_1(z, \nu) \exp(-i \omega t),$$

where $\nu = m\nu^2 / 2$ is the electron (hole) kinetic energy in the case of spherically-symmetric energy band, $\nu$ and $m$ are the electron (hole) velocity and effective mass respectively.

The distribution function $f_i$ is determined by solving of the kinetic Boltzmann equation in the relaxation time $\tau$ approximation and linear approximation at the external field:

$$\frac{\partial f_1}{\partial z} + \frac{\nu}{v_z} f_1 + \frac{eB}{mv_z} \left( \nu_y \frac{\partial f_1}{\partial \nu_x} - \nu_x \frac{\partial f_1}{\partial \nu_y} \right) + \frac{e}{mv_z} \left( E_x \frac{\partial f_0}{\partial \nu_x} + E_y \frac{\partial f_0}{\partial \nu_y} \right) = 0.$$  

Here $\nu = \tau^{-1} - i \omega$ is the complex scattering frequency.

As the boundary condition to the equation (3) we use the model of charge carrier diffuse-specular reflection from layer surfaces. The specularity coefficients of upper and lower layer boundaries are supposed to be equal.

$$\begin{cases} 
  f_i(\nu_z, 0) = q \cdot f_i(-\nu_z, 0); \\
  f_i(-\nu_z, a) = q \cdot f_i(\nu_z, a).
\end{cases}$$

If we know the non-equilibrium distribution function, we can calculate the current density, conductivity and Hall constant:

$$j = 2e \left( \frac{m}{h} \right)^3 \int \nu f_i d^3 \nu,$$

$$\sigma = \frac{j_x}{E_x}, \quad A_H = \frac{E_x}{Bj_x}.$$  

By conducting the mathematical calculation series, we obtain the following expressions of the conductivity and Hall constant:

$$\sigma(x_0, y_0, \beta_0, q, U_{\mu}) = \sigma_0 \Sigma(x_0, y_0, \beta_0, q, U_{\mu});$$

$$A_H(x_0, y_0, \beta_0, q, U_{\mu}) = A_{H,0} R_H(x_0, y_0, \beta_0, q, U_{\mu});$$

$$2$$
\[ \Sigma(x_0, y_0, \beta_0, q, U_\mu) = x_0 \frac{(z_0 - b_1)^2 + (\beta_0 - b_2)^2}{(z_0 - b_1)(z_0^2 - \beta_0^2) - 2z_0\beta_0(b_2 - \beta_0)}; \]

\[ R_H(x_0, y_0, \beta_0, q, U_\mu) = \frac{1}{\beta_0} \frac{1}{(z_0 - b_1)^2 + (\beta_0 - b_2)^2} \frac{U_0^2}{U_\mu^2} \frac{A_{1,2}}{A_0} d\gamma dU; \]

\[ b_{1,2} = \frac{1 - q}{I_0 \tilde{\nu}_1} \int_0^\infty \left( \frac{1}{\gamma^3} - \frac{1}{\gamma^5} \right) U^2 \exp\left( U - U_\mu \right) \frac{A_{1,2}}{A_0} d\gamma dU; \]

where \( \sigma_0 = ne^2 \tau / m \) is the static conductivity, \( A_{H,0} = 1/(en) \) is Hall constant in the classic case.

We introduce dimensionless parameters:

\[ z_0 = \frac{v a}{\nu_1} - i \frac{a \omega}{\nu_1} = x_0 - iy_0; \quad \beta_0 = \frac{e a B}{m \nu_1}; \quad U = \frac{m \nu_1^2}{2k_f T}; \quad U_\mu = \frac{\mu}{k_f T}; \quad \tilde{\nu}_1 = \sqrt{\frac{m \nu_1^2}{2k_f T}}. \]

The parameters \( x_0, y_0, z_0 \) and \( \beta_0 \) are dimensionlessed to characteristic charge carrier velocity introduces by the following view:

\[ m \nu_1^2 = \frac{2}{3} \int \nu^2 f_0 \frac{2d^3(m\nu_1)}{h^3}. \]

In the case of degenerated electron gas \( \nu_1 \rightarrow \nu_F \) where \( \nu_F \) is Fermi velocity. In the case of non-degenerated Fermi gas \( \nu_1 \rightarrow \nu_F = \sqrt{5k_f T / m} \). Here \( \nu_F \) is proportional to the averaged thermal velocity of charge carriers.

3. Limited cases

3.1. The case of degenerated electron gas

Consider the case of degenerated electron gas, i.e. \( \exp(U_\mu) \gg 1 \). The expressions of the dimensionless conductivity and Hall constant have the view (9) – (10) with the following designations:

\[ b_{1,2} = \frac{3}{2}(1-q) \int_0^\infty \left( \frac{1}{\gamma^3} - \frac{1}{\gamma^5} \right) \frac{A_{1,2}}{A_0} d\gamma, \quad p_1 = z_0\gamma, \quad p_2 = \beta_0\gamma. \]
3.2. The case of non-degenerated electron gas
Consider the case of non-degenerated electron gas, i.e. when the condition $\exp(U_\mu) << 1$ is satisfied. The expressions of the dimensionless conductivity and Hall constant are determined by the formulae (9) – (10) with the following designations:

$$b_{1,2} = \sqrt{\frac{8}{5\pi}} (1-q) \int_{0}^{\infty} w \exp(-w^2) \frac{A_{1,2}}{A_0} dw, \quad p_1 = \frac{z_0}{w \sqrt{2}}, \quad p_2 = \frac{\beta_0}{w \sqrt{2}}, \quad w = \frac{\sqrt{U}}{\gamma}. \quad (15)$$

4. Results analysis
In figure 1 the dependences of layer dimensionless conductivity module (a) and argument (b) on the dimensionless electric field frequency are represented. The solid and dashed curves are built for the cases of a metal and semiconductor layer respectively. With frequency increasing the conductivity module decreases. This is due to the fact that charge carriers haven’t time to respond to external electric field oscillations and behave as the complexity of charge carriers not contributing to layer conductivity. The conductivity argument grows and seeks to $\pi/2$, i.e. the conductivity becomes a purely imaginary value at the high-frequency region.

In figure 2 the dependences of layer dimensionless conductivity module (a) and argument (b) on the dimensionless magnetic field induction are imaged. The solid and dashed curves are built for the cases of a metal and semiconductor layer respectively. In this figure we observe oscillations damped with magnetic field induction and layer surface specularity coefficient increasing. The oscillations of conductivity dependences on the magnetic field induction are less pronounced for the case of non-degenerated electron gas than those for the case of degenerated electron gas due to the charge carrier thermal velocity dispersion.

In figures 3 and 4 the dependences of layer dimensionless Hall constant module (a) and argument (b) on the dimensionless electric field frequency (figure 3) and magnetic field induction (figure 4) are built. The solid and dashed curves are built for the cases of a metal and semiconductor layer respectively. We observe the oscillations which are analogue to figure 2 and damped with electric field frequency and magnetic field induction increasing. When the electric field frequency is equal to the magnetic field induction we observe the resonant-like phenomenon for the Hall constant argument.

![Figure 1(a, b). The dependences of dimensionless conductivity $\Sigma$ module (a) and argument (b) for a metal (solid curves) and semiconductor (dashed curves) layer on the dimensionless electric field frequency $y_0$ at the values of the dimensionless magnetic field induction $\beta_0$ and layer thickness $x_0$ equal to 0.1.](image-url)
Figure 2(a, b). The dependences of dimensionless conductivity $\Sigma$ module (a) and argument (b) for a metal (solid curves) and semiconductor (dashed curves) layer on the dimensionless magnetic field induction $\beta_0$ at the values of the dimensionless electric field frequency $\gamma_0$ and layer thickness $x_0$ equal to 0.1.

Figure 3(a, b). The dependences of dimensionless Hall constant $R_{\text{HH}}$ module (a) and argument (b) for a metal (solid curves) and semiconductor (dashed curves) layer on the dimensionless electric field frequency $\gamma_0$ at the values of the dimensionless magnetic field induction $\beta_0$ and layer thickness $x_0$ equal to 0.1.
Figure 4(a, b). The dependences of dimensionless Hall constant $R_H$ module (a) and argument (b) for a metal (solid curves) and semiconductor (dashed curves) layer on the dimensionless magnetic field induction $\beta_0$ at the values of the dimensionless electric field frequency $\nu_0$ and layer thickness $x_0$ equal to 0.1.

5. Conclusions
The theoretical model of high-frequency conductivity and Hall constant of a thin conductive layer for the case of equal layer surface specularity coefficients is built. The behavior of conductivity and Hall constant dependences on the non-dimensional parameters: electric field frequency, magnetic field induction, layer thickness and layer surface specularity coefficient, is analyzed. It is detected the oscillations of the dependences of conductivity on the magnetic field induction, also those of the dependences of Hall constant on the electric field frequency and magnetic field induction. The conductivity and Hall constant oscillations for the case of a semiconductor layer are less pronounced than those for the case of a metal layer due to charge carrier thermal velocity dispersion. It is shown that Hall constant argument sharply grows when the dimensionless electric field frequency is equal to the dimensionless magnetic field induction.

References
[1] Anselm A I 1982 Introduction to Semiconductor Theory (Englewood Cliffs, NJ: Prentice Hall) p 616
[2] Lifshits I M, Azbel M Ya and Kaganov M I 1973 Electron Theory of Metals (New York: Plenum) p 415
[3] Sondheimer E H 2001 Adv. in Phys. 2001 50 499
[4] Gurevich V L 1959 Sov. Phys. JETP 35 464
[5] Grishin A M, Lutsishin P P, Ostroukhov Yu S and Panchenko O A 1979 Sov. Phys. JETP 49 673
[6] Utkin A I, Zavitaev E V and Yushkanov A A 2016 J Surf Inv 10 962
[7] Utkin A I and Yushkanov A A 2016 Russian Microelectronics 45 357
[8] Kuznetsova I A, Savenko O V and Yushkanov A A 2017 Tech Phys 62 1766
[9] Kuznetsova I A, Savenko O V and Yushkanov A A 2017 J Surf Inv 11 1159