SOME ASPECTS OF NUCLEAR DEEP INELASTIC SCATTERING

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Abstract

Nuclear deep inelastic scattering is considered in the framework of a model in which the current operator explicitly satisfies Poincare invariance and current conservation. The results considerably differ from the standard ones at small values of the Bjorken variable $x$. In particular, it is impossible to extract the neutron structure functions from the deuteron data at $x \lesssim 0.01$ and we predict that the behavior of the deuteron structure functions at low $x$ and large momentum transfer considerably differs from the behavior of the nucleon structure functions at such conditions. We also argue that for heavier nuclei the effect of the final state interaction is important even in the Bjorken limit.

1 Introduction

There exists a vast literature devoted to the problem of describing nuclear deep inelastic scattering (DIS). After the discovery of the original EMC effect the central point of the extensive discussion was whether this effect can be explained in the framework of conventional nuclear physics. The point of view advocated by many authors is that nuclear DIS can be described with a good accuracy if only the nucleon degrees of freedom in the nucleus wave function are taken into account but relativity of the internal motion of nucleons in the nucleus is rather important.

The first calculations in such a framework were carried out in Refs. and others; as pointed out in Ref. the important role in these calculations plays the "flux factor". The status of nuclear DIS at the end of 80th is
discussed in detail in Ref. \[4\]. Since that time the convolution formula for nuclear DIS was rederived by several authors using different considerations (see Refs. \[5\] and others). However, as argued in Ref. \[6\], the convolution formula can be obtained only if some simplifying assumptions are made, and in particular the off-shellness of the struck nucleon is neglected.

Let \(P'\) be the 4-momentum of the initial nucleus and \(q\) be the four-momentum of the virtual photon absorbed by this nucleus (for definiteness we shall speak about electromagnetic interactions but the same is valid for weak ones). We shall consider nuclear DIS only in the Bjorken limit when \(|q^2| \gg m^2\), where \(m\) is the nucleon mass, but \(x = -q^2 / 2(P'q)\) is not too close to 0 or 1.

The idea adopted in practically all works on nuclear DIS is that, by analogy with the well-known nonrelativistic calculations in atomic and nuclear physics, nuclear DIS in the Bjorken limit can be described in the impulse approximation (IA). By definition, the IA implies that the electromagnetic or weak current operator (CO) of the system under consideration is a sum of the CO’s for the constituents comprising this system. Therefore the IA in nuclear DIS implies that the nucleus CO is a sum of the CO’s for the nucleons comprising the given nucleus, and the IA in nucleon DIS implies that the CO is a sum of the quark CO’s.

In the nonrelativistic case we always work in such a representation of the generators of the Galilei group when only the Hamiltonian is interaction dependent while all other nine generators are free. This choice of the representation is reasonable since time flows equally in all reference frames. However, as shown by Dirac \[7\], in the relativistic case at least three of ten representation generators of the Poincare group are interaction dependent. If we wish to use the IA in the relativistic case then the question immediately arises in which representation of the Poincare group the CO can be taken as a sum of the constituent CO’s. Different representations of the Poincare group describe the same physics if these representations are unitarily equivalent, but the unitary operators realizing the equivalence are (generally speaking) interaction dependent. Therefore if the CO is a sum of the constituent CO’s in some representation, then (as pointed out by several authors) this property does not take place in other representations.

The system CO \(\hat{J}^\mu(x)\), where \(\mu = 0, 1, 2, 3\) and \(x\) is a point in Minkowski space, should satisfy the following necessary conditions.

First, let \(\hat{U}(a) = exp(i\hat{P}_u a^\mu)\) be the representation operator corresponding
to the displacement of the origin in spacetime translation of Minkowski space by the 4-vector $a$. Here $\hat{P} = (\hat{P}^0, \hat{\mathbf{P}})$ is the operator of the 4-momentum, $\hat{P}^0$ is the Hamiltonian, and $\hat{\mathbf{P}}$ is the operator of ordinary momentum. Let also $\hat{U}(l)$ be the representation operator corresponding to $l \in SL(2, C)$. Then $\hat{J}^\mu(x)$ must be the relativistic vector operator such that

$$\hat{U}(a)^{-1} \hat{J}^\mu(x) \hat{U}(a) = \hat{J}^\mu(x - a)$$ (1)

$$\hat{U}(l)^{-1} \hat{J}^\mu(x) \hat{U}(l) = L(l)^\mu_\nu \hat{J}^\nu(L(l)^{-1}x)$$ (2)

where $L(l)$ is the element of the Lorentz group corresponding to $l$ and a sum over repeated indices $\mu, \nu = 0, 1, 2, 3$ is assumed.

Second, if $\hat{J}^\mu(x)$ satisfies the continuity equation $\partial \hat{J}^\mu(x)/\partial x^\mu = 0$ then, as follows from Eq. (2), this equation can be written in the form

$$[\hat{J}^\mu(x), \hat{P}_\mu] = 0$$ (3)

Finally, the operator $\hat{J}^\mu(x)$ should also satisfy the cluster separability condition. Briefly speaking, this condition implies that if the interaction between any subsystems $\alpha_1, \ldots, \alpha_n$ comprising the system under consideration is turned off then $\hat{J}^\mu(x)$ must become a sum of the current operators $\hat{J}^\mu_{\alpha_i}(x)$ for the subsystems.

As pointed out by Dirac [7], any physical system can be described in different forms of relativistic dynamics. Let $\hat{M}^{\mu\nu}$ ($\hat{M}^{\mu\nu} = -\hat{M}^{\nu\mu}$) be the generators of the Lorentz group. We use $P$ and $M^{\mu\nu}$ to denote the 4-momentum operator and the generators of the Lorentz group in the case when all interactions are turned off. By definition, the description in the point form implies that the operators $\hat{U}(l)$ are the same as for noninteracting particles, i.e. $\hat{U}(l) = U(l)$ and $\hat{M}^{\mu\nu} = M^{\mu\nu}$, and thus interaction terms can be present only in the 4-momentum operators $\hat{P}$ (i.e. in the general case $\hat{P}^\mu \neq P^\mu$ for all $\mu$). The description in the instant form implies that the operators of ordinary momentum and angular momentum do not depend on interactions, i.e. $\hat{P} = P$, $\hat{M} = M$ ($\hat{M} = (\hat{M}^{23}, \hat{M}^{31}, \hat{M}^{12})$) and therefore interactions may be present only in $\hat{P}^0$ and the generators of the Lorentz boosts $\hat{N} = (\hat{M}^{01}, \hat{M}^{02}, \hat{M}^{03})$.

In the front form with the marked $z$ axis we introduce the + and - components of the 4-vectors as $p^+ = (p^0 + p^z)/\sqrt{2}$, $p^- = (p^0 - p^z)/\sqrt{2}$. Then we require that the operators $\hat{P}^+, \hat{P}^i, \hat{M}^{12}, \hat{M}^{+-}, \hat{M}^{+j}$ ($j = 1, 2$) are the same as the corresponding free operators and therefore interaction terms may be present only in the operators $\hat{M}^{-j}$ and $\hat{P}^-$. 

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As follows from the above formulas and definitions there is no form of dynamics in which the IA is valid. Let us note however that as follows from Eq. (1)

\[ \hat{J}^\mu(x) = \exp(i\hat{P}x)\hat{J}^\mu(0)\exp(-i\hat{P}x) \]  

(4)

Therefore, if the operator \( \hat{P} \) is known, the problem of constructing \( \hat{J}^\mu(x) \) can be reduced to that of constructing the operator \( \hat{J}^\mu(0) \) with the correct properties.

In turn, as follows from Eq. (2), Lorentz invariance of the CO implies

\[ [\hat{M}^{\mu\nu}, \hat{J}^\rho(0)] = -i(\eta^{\mu\rho}\hat{J}^\nu(0) - \eta^{\nu\rho}\hat{J}^\mu(0)) \]

(5)

where \( \eta^{\mu\nu} \) is the metric tensor in Minkowski space.

While the IA for the operator \( \hat{J}^\mu(x) \) is incompatible with Eqs. (1) and (2), the question arises whether this approximation can be valid for the operator \( \hat{J}^\mu(0) \). By definition, the operator \( \hat{J}^\mu(0) \) in the IA is given by

\[ \hat{J}^\mu(0) = J^\mu(0) = \sum_{i=1}^{N} J_i^\mu(0) \]

(6)

where \( J_i^\mu(0) \) is the CO for the i-th nucleon at \( x = 0 \). In this case cluster separability is satisfied automatically, though in the general case this condition is rather restrictive [8, 9].

As noted above, in the instant and front forms some of the operators \( \hat{M}^{\mu\nu} \) are interaction dependent. Since the free operators \( M^{\mu\nu} \) satisfy Eq. (2) if \( \hat{J}^\mu(0) \) is given by Eq. (4), Eqs. (3) and (4) will take place in these forms only if the interaction terms in \( \hat{M}^{\mu\nu} \) commute with \( J^\rho(0) \). However neither in systems with a fixed number of particles, nor in quantum field theory there is no reason for such a commutation [8, 9]. Let us also note that, as shown by many authors (see, for example, Ref. [11]), the parton model is a consequence of the IA in the front form, and therefore Eq. (3) is not satisfied in this case (see Ref. [12] for a detailed discussion). However in the point form the operators \( \hat{M}^{\mu\nu} \) are free and therefore in the point form the IA for the operator \( \hat{J}^\mu(0) \) is compatible with Lorentz invariance.

Let us now consider the constraints imposed on \( \hat{J}^\mu(0) \) by the continuity equation (3). Let \( m_A \) be the mass of the initial state \( |i\rangle \) and \( G' \) be its four-velocity such that \( P' = m_A G' \). Consider the transition of the nucleus to the final state \( |f\rangle \) with the mass \( M'' \) and four-velocity \( G'' \) such that the total
momentum in the final state is equal to $P'' = M''G''$. Since the Lorentz boost operators in the point form are free, there is no problem in boosting the wave function (WF) from one reference frame to another. As shown in Ref. [9], the current operator is fully defined by its matrix elements in the reference frame where $G'' + G' = 0$. The coordinate axes in this frame can be chosen in such a way that $G'_{\perp} = 0$ (where we use the subscript $\perp$ to denote the projection of the three-dimensional vector onto the plane $xy$) and the $z$ axis is directed along the momentum of the initial nucleus. Then $G^0 = G'^0$, $G'^z = -G''z$ and Eq. (3) for $x = 0$ can be written in the form

$$
(M''G' - m_A G'^+)(f|\hat{J}^-(0)|i) = -(M''G' + m_A G'^-)(f|\hat{J}^+(0)|i)
$$

Let us suppose that the four-velocity operator $\hat{G}$ does not contain interaction terms. Then the first consequence of Eq. (7) is that the $\perp$ components of $\hat{J}^\mu(0)$ are not constrained by the continuity equation, and therefore we can take the IA for these components. Since $(M'')^2 = (P' + q)^2$ and $P' + q = M''G''$, it is easy to show that

$$
(M'')^2 = m_A^2 - \frac{q^2(1-x)}{x} 
$$

and in the reference frame under consideration

$$
(G'^z)^2 = \frac{|q^2|^{1/2}}{4m_A|x(1-x)|^{1/2}}, \quad q^0 = (M'' - m_A)G'^0, \quad q = -(M'' + m_A)G'
$$

Therefore, as follows from Eqs. (8) and (9),

$$
\langle f|\hat{J}^+(0)|i\rangle = \frac{m_A x}{M''} \langle f|\hat{J}^-(0)|i\rangle \quad (10)
$$

We conclude that in the reference frame under consideration the matrix elements of the operator $\hat{J}^+(0)$ in the Bjorken limit are negligible in comparison with the matrix elements of the operator $\hat{J}^-(0)$. Therefore a possible prescription is to take $\hat{J}^-(0)$ in the IA and then the matrix elements of the operator $\hat{J}^+(0)$ can be defined (if necessary) from Eq. (10). However this procedure is not unique.

Many authors realize the idea of the IA not on the operator level but in the framework of the covariant approach based on Feynman diagrams. Namely the IA is associated with the disconnected (or one-leg) diagrams, i.e.
the diagrams in which the virtual photon interacts with only one nucleon while the other nucleons are spectators. The problem whether the contribution of the connected diagrams is indeed negligible in the Bjorken limit was investigated by several authors (see, for example, Refs. [13, 14] and references therein). The main motivation is that if $\tilde{p}$ is the four-momentum of the struck constituent in the disconnected diagram then $|\tilde{p}^2| \ll |q^2|$ while for all particles in the connected diagrams the off-shellness is of order $|q^2|$. However, even if some connected diagram can be reliably estimated, the problem exists what is the sum of all such diagrams. The above considerations show that the very notion of the IA is not covariant since even for the operator $\hat{J}^\mu(0)$ the IA can be consistent only at some special choice of the representation of the Poincare group. 

In view of the above discussion it seems reasonable to consider nuclear DIS in the Bjorken limit if the nucleus dynamics is described in the point form and the operator $\hat{J}^\mu(0)$ is given by Eq. (5). In Sec. 2 we describe the nucleus WF in the point form, and in Sec. 3 standard formulas for DIS are written down. The main result of the paper is given in Sec. 4 where we derive in detail the relations between the deuteron and nucleon structure functions. In Sec. 5 the case of heavier nuclei is briefly considered, and Sec. 6 is discussion.

2 Nucleus wave function in the point form of dynamics

In local quantum field theory the number of particles is not a conserving physical quantity, and one might think that if the nucleus is described relativistically then it cannot be considered as a system of a fixed number of nucleons. However there are grounds to believe that in few nucleon systems at low energies the main part of the relativistic corrections are due to relativistic kinematics rather than the excitation of new degrees of freedom (see, for example, the discussion in Refs. [15, 16, 17]). Therefore a good approximation to the description of relativistic effects is relativistic quantum mechanics, i.e. the relativistic theory of systems with a fixed number of particles. In this theory it is possible to explicitly construct all the representation generators of the Poincare group in such a way that cluster separability is
A rather strange feature of the present approaches to DIS is that while nucleon DIS is usually considered in the infinite momentum frame (IMF), nucleus DIS is usually considered in the reference frame where the initial nucleus is at rest. The matter is that in usual approaches the WF of a composite system essentially depends on the reference frame (this is clear, for example, in the instant form where the Lorentz boost generators are interaction dependent). Therefore it is desirable to work in the reference frame in which the WF has the simplest form. It is believed that such a frame for the nucleon is IMF (where the nucleon is a system of almost-free partons) while for the nucleus this is the rest frame where the nucleus can be approximately described as a nonrelativistic system of nucleons (see the discussion in Ref. 12)). However, as we already noted, in the point form there is no problem in boosting the WF from one reference frame to another.

The Hilbert space for the system of $A$ (free or interacting) nucleons is the space of functions $\varphi(p_{1\perp}, p_1^+, \sigma_1, \ldots p_{A\perp}, p_A^+, \sigma_A)$ where $p_i$ and $\sigma_i = \pm 1/2$ ($i = 1, \ldots A$) are the four-momentum and the spin projection on the $z$ axis for the $i$-th nucleon,

$$
\sum_{\sigma_1 \ldots \sigma_A} \int |\varphi(p_{1\perp}, p_1^+, \sigma_1, \ldots p_{A\perp}, p_A^+, \sigma_A)|^2 \prod_{i=1}^A d\rho(p_{i\perp}, p_i^+) < \infty \tag{11}
$$

and

$$
d\rho(p_{\perp}, p^+) = \frac{d^2p_\perp dp^+}{2(2\pi)^3 p^+} \tag{12}
$$

We define $P = p_1 + \ldots + p_A$, $M_0 = |P| = (P^2)^{1/2}$, and $G = PM_0^{-1}$. Let $\beta(G) \equiv \beta(G_{\perp}, G^+) \in SL(2, C)$ be the matrix with the components

$$
\beta_{11} = \beta_{22}^{-1} = 2^{1/4}(G^+)^{1/2}, \quad \beta_{12} = 0, \quad \beta_{21} = (G^x + iG^y)\beta_{22} \tag{13}
$$

and

$$
k_i = L[\beta(G)]^{-1}p_i \quad (i = 1, \ldots A) \tag{14}
$$

The four-vectors $p_i$ have the components $(\omega(p_i), p_i)$, and the four-vectors $k_i$ have the components $(\omega(k_i), k_i)$ where $\omega(k) = (m^2 + k^2)^{1/2}$. In turn, only $A - 1$ vectors $k_i$ are independent since, as follows from Eqs. (13) and (14), $k_1 + \ldots + k_A = 0$. Therefore $L[\beta(G)]$ has the meaning of the boost,
and $k_i$ are the momenta in the c.m. frame. It is easy to show that $M_0 = \omega(k_1) + \ldots + \omega(k_A)$.

It follows from direct calculations that

$$A \prod_{i=1}^A d\rho(p_{i\perp}, p_{i+}^+) = d\rho(G_{\perp}, G^+)d\rho(int),$$

$$d\rho(int) = 2(2\pi)^3 M_0^3 \delta^{(3)}(k_1 + \ldots + k_A) \prod_{i=1}^A d\rho(k_{i\perp}, k_{i+}^+) \quad (15)$$

We also define the ”internal” space $H_{int}$ as the space of functions $\chi(k_1, \sigma_1, \ldots k_A, \sigma_A)$ such that

$$||\chi||^2 = \sum_{\sigma_1 \ldots \sigma_A} \int |\chi(k_1, \sigma_1, \ldots k_A, \sigma_A)|^2 d\rho(int) < \infty \quad (16)$$

Then the space of functions satisfying Eq. (11) can be realized as the space of functions $\varphi(G_{\perp}, G^+)$ with the range in $H_{int}$ and such that

$$\int ||\varphi(G_{\perp}, G^+)||^2 d\rho(G_{\perp}, G^+) < \infty \quad (17)$$

For the system of $A$ interacting particles the four-velocity operator $\hat{G}$ is not generally speaking free, but is unitarily equivalent to $G$ [18]: $\hat{G} = BGB^{-1}$ where $B$ is the Sokolov packing operator. If $A = 2$ then it is possible to choose $B = 1$ (see the discussion in [23]), but for $A \geq 3$ the operator $B$ in the point form is necessarily nontrivial [18]. The mass operator $\hat{M}$ in the general case acts not only through the variables of the space $H_{int}$, but also through $G$, but the operator $B$ can be chosen in such a way that the operator $\tilde{M} = B^{-1}\hat{M}B$ acts only through the variables of the space $H_{int}$ [18, 17, 9]. If the nucleus is in the bound state then its internal WF $\chi \in H_{int}$ is the eigenfunction of the operator $\tilde{M}$ with the eigenvalue $m_A$: $\tilde{M}\chi = m_A\chi$.

Taking into account the normalization of free states in the scattering theory, the WF of the free one-nucleon state with the four-momentum $p^\nu$ and the spin projection $\sigma^\nu$ can be written in the form

$$|p^\nu, \sigma^\nu\rangle = 2(2\pi)^3 p^+ \delta^{(2)}(p_{\perp} - p_{\perp}^\nu) \delta(p^+ - p^+\nu) \delta_{\sigma\sigma^\nu} \quad (18)$$

where $\delta_{\sigma\sigma^\nu}$ is the Cronecker symbol.
Analogously if the initial nucleus has the four-velocity $G'$, its WF has the form

$$|G', \chi\rangle = B \frac{2}{m_A} (2\pi)^3 G' \delta^{(2)} (G_\perp - G'_\perp) \delta(G^+ - G'^+) \chi$$  \hspace{1cm} (19)$$

where the internal WF is normalized as $||\chi|| = 1$.

3 General formulas for deep inelastic scattering

It is well known that the DIS cross-section is defined by the tensor

$$W^{\mu\nu} = \frac{1}{4\pi} \sum_X (2\pi)^4 \delta^{(4)}(P' + q - P_X) \langle i| \hat{J}^\mu(0) |X\rangle \langle X| \hat{J}^\nu(0) |i\rangle$$  \hspace{1cm} (20)$$

where a sum is taken over all possible final states $|X\rangle$, and $P_X$ is the 4-momentum of the state $|X\rangle$. As follows from relativistic invariance and current conservation, the tensor $W^{\mu\nu}$ can be written in the form

$$W^{\mu\nu}(P', q) = F_1(x, q^2) \left( \frac{q^\mu q^\nu}{q^2} - \eta^{\mu\nu} \right) +$$

$$\frac{F_2(x, q^2)}{(P' q)} \left( P'^\mu - \frac{q^\mu (P' q)}{q^2} \right) \left( P'^\nu - \frac{q^\nu (P' q)}{q^2} \right) +$$

$$\frac{i g_1(x, q^2)}{(P' q)} e^{\nu\rho\lambda} q_\rho S_\lambda - i \frac{F_3(x, q^2)}{2(P' q)} e^{\nu\rho\lambda} P'_\rho q_\lambda$$  \hspace{1cm} (21)$$

Here $e^{\mu\nu\rho\lambda}$ is the absolutely antisymmetric tensor ($e^{0123} = 1$) and $S_\lambda$ is the relativistic spin operator. We do not write down terms which vanish in the Bjorken limit, depend on the tensor polarization etc. The term with $F_3$ is present only in the case of the neutrino (antineutrino) - nucleus scattering.

Equation (21) can also be written for DIS on the nucleon. In this case we use $\tilde{W}^{\mu\nu}(p', \tilde{q})$ to denote the hadronic tensor, $p'$ to denote the four-momentum of the initial nucleon, $\tilde{q}$ to denote the four-momentum of the virtual photon or W-boson, and $\tilde{x}$ to denote the Bjorken variable $-\tilde{q}^2 / 2p'\tilde{q}$.

We shall always assume that the nucleus under consideration has the lowest mass among all states with the baryon number equal to $A$. Then, as follows from Eq. (8), if $q^2 < 0$ then the structure functions are not equal to
zero only if \( x \in (0, 1] \). Analogously, in nucleon DIS the structure functions are not equal to zero only if

\[
q^2 \frac{1 - \tilde{x}}{\tilde{x}} \leq 0
\]  

(22)

If \( q^2 < 0 \) then \( \tilde{x} \in (0, 1] \), but this condition is also satisfied for \( q^2 > 0 \) if \( \tilde{x} < 0 \) or \( \tilde{x} > 1 \).

Let us consider the nucleon DIS in the reference frame where \( p'_z > 0, p_\perp = \tilde{q}_\perp = 0 \). Then, as follows from Eq. (21),

\[
\tilde{W}^{--} = \frac{F_N^N(\tilde{x}, \tilde{q}^2)\tilde{q}^-}{4\tilde{x}^2p'++}, \quad \tilde{W}^{++} = \frac{F_N^N(\tilde{x}, \tilde{q}^2)p'--}{4\tilde{q}^-}
\]  

(23)

where \( F_N^N = F_2^N - 2\tilde{x}F_1^N \) is the longitudinal nucleon structure function (the index \( N \) takes the values \( p \) and \( n \) for the proton and neutron respectively).

Another consequence of Eq. (21) is that if the initial nucleon is fully polarized along the \( z \) axis, then the tensor \( \tilde{W}^{jl} \) for \( j, l = 1, 2 \) has the form

\[
\tilde{W}^{jl}(p', \tilde{q}) = F_1^N(\tilde{x}, \tilde{q}^2)\delta_{jl} - \frac{3}{2}\epsilon_{jl}F_3^N(\tilde{x}, \tilde{q}^2) \pm \epsilon_{jl}g_1^N(\tilde{x}, \tilde{q}^2)
\]  

(24)

where \( \epsilon_{12} = -\epsilon_{21} = 1, \epsilon_{11} = \epsilon_{22} = 0 \), and \( \pm \) corresponds to the \( z \) projection of the spin equal to \( \pm 1/2 \).

### 4 Deep inelastic scattering on the deuteron

We assume that particle 1 in the deuteron is the proton and particle 2 is the neutron. Then, as follows from Eqs. (13) and (16), the norm of the internal deuteron WF is given by

\[
||\chi||^2 = \sum_{\sigma_p, \sigma_n} \int |\chi(k, \sigma_p, \sigma_n)|^2 \frac{M_0(k)^2d^3k}{2(2\pi)^3\omega(k)^2}
\]  

(25)

where \( k \equiv k_1 \) and \( M_0(k) = 2\omega(k) \). As follows from the minimal relativity principle \([24, 25, 13, 26]\) (see also the discussion in Ref. [17]), the relativistic WF \( \chi \) can be taken as in the standard nuclear physics. Therefore we can take \( \chi \) in the form

\[
\chi(k, \sigma_p, \sigma_n) = \frac{1}{\sqrt{2M_0(k)}[\varphi_0(k)\delta_{jl} - \frac{1}{\sqrt{2}}(\delta_{jl} - 3\frac{k_jk_l}{k^2})\varphi_2(k)]\epsilon_l(\tau_j\tau_2)_{\sigma_p\sigma_n}
\]  

(26)
where $k = |k|$, $e$ is the polarization vector of the deuteron, $\tau_i$ are the Pauli matrices and a sum over repeated indices $j, l = 1, 2, 3$ is assumed. The functions $\varphi_0(k)$ and $\varphi_2(k)$ are the amplitudes of the $S$ and $D$ states in the deuteron, and the normalization is chosen such that

$$
\int [\varphi_0(k)^2 + \varphi_2(k)^2] \frac{d^3k}{(2\pi)^3\omega(k)} = 1
$$

(27)

As noted in Sec. 2, one can choose $B = 1$ in the deuteron case and therefore the IA is compatible with the continuity equation (see Sec. 1). The calculations in the parton model and in the model where the IA is used in the point form \cite{11, 12} show that if the IA is valid then the constituents comprising the system under consideration interact with the virtual photon incoherently. Therefore it is reasonable to assume that the final state interaction between the spectator nucleon and the particles produced in the DIS on the struck nucleon can be neglected if $|q^2|$ is very large.

With all these assumptions we can write Eq. (20) in the form

$$
W^{\mu\nu}(P', q) = \frac{1}{4\pi} \sum_{\sigma_p} \sum_{\sigma_n} \int (2\pi)^4 \delta^4(P' + q - p_n - P_\tilde{X}) d\rho(p_{n\perp}, p_n^+) \cdot
$$

$$
\langle G'\chi | J^{\mu}_p(0) | p_n, \sigma_n; \tilde{X} \rangle \langle p_n, \sigma_n; \tilde{X} | J^{\nu}_p(0) | G'\chi \rangle + (...)$$

(28)

and one should bear in mind that this expression is valid for those $\mu, \nu$ for which the deuteron CO can be taken in the IA. Here $\tilde{X}$ are all possible final states in the DIS on the proton, $P_\tilde{X}$ is the four-momentum of the state $\tilde{X}$, $J^{\mu}_p(0)$ is the proton CO, the state $|p_n, \sigma_n; \tilde{X} \rangle$ is the tensor product of the states $|p_n, \sigma_n \rangle$ and $|\tilde{X} \rangle$ and $(...)$ is the contribution from the DIS on the neutron which can be written analogously.

Let us introduce the function

$$
\Phi(p_p, p_n, \sigma_p, \sigma_n) = \frac{2}{m_d} (2\pi)^3 G^{\prime+} \delta(2) (G_{\perp} - G^{\prime}_{\perp}) \delta(G^{+} - G^{\prime+}) \chi(k, \sigma_p, \sigma_n)
$$

(29)

where $m_d$ is the mass of the deuteron. Then as follows from Eqs. (12), (18) and (13)

$$
|G', \chi \rangle = \sum_{\sigma_p, \sigma_n} \int \Phi(p_p, p_n, \sigma_p, \sigma_n) |p_p, \sigma_p \rangle |p_n, \sigma_n \rangle d\rho(p_{p\perp}, p_p^+) d\rho(p_{n\perp}, p_n^+)
$$

(30)

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As follows from Eqs. (18) and (30), Eq. (28) can be written in the form

\[
W_{\mu\nu}(P', q) = \frac{1}{4\pi} \sum_{\sigma_p^p, \sigma_n} \sum_{X} \int (2\pi)^4 \delta(4) (P' + q - p_n - P_X) \cdot \\
\Phi(p_p, p_n, \sigma_p, \sigma_n) \Phi(p_p', p_n, \sigma_p', \sigma_n) \langle p_p, \sigma_p | J^\mu_p(0) | X \rangle \langle X | J^\nu_p(0) | p_p', \sigma_p' \rangle \cdot \\
d\rho(p_{p\perp}, p_p^+ \perp) d\rho(p_{p\perp}', p_{p'}^+ \perp) d\rho(p_{n\perp}, p_n^+ \perp) + (\ldots) \quad (31)
\]

Now we use the following relation which can be verified by a direct calculation [18]: if \( p_n \) is fixed and \( G' = (p_p + p_n)/|p_p + p_n| \), where the modulus of the four-vector is understood in the Lorentz metric, then

\[
d\rho(p_{p\perp}, p_p^+ \perp) = \frac{|p_p + p_n|^4}{(p_p, p_p + p_n)} d\rho(G \perp, G^+) \quad (32)
\]

Since \( |p_p + p_n| = M_0(k) \), \( (p_p, p_p + p_n) = \omega(k)M_0(k) \), it follows from Eqs. (29), (31) and (32) that

\[
W_{\mu\nu}(P', q) = \frac{1}{4\pi} \sum_{\sigma_p^p, \sigma_n} \sum_{X} \int (2\pi)^4 \delta(4) (P' + q - p_n - P_X) \cdot \\
\frac{M_0(k)^6}{m_0^2\omega(k)^2} \chi(k, \sigma_p, \sigma_n)^* \chi(k, \sigma_p', \sigma_n) \langle p_p, \sigma_p | J^\mu_p(0) | X \rangle \cdot \\
\langle X | J^\nu_p(0) | p_p, \sigma_p' \rangle d\rho(p_{n\perp}, p_n^+ \perp) + (\ldots) \quad (33)
\]

where the four-vector \( p_p \) is a function of \( p_n \) and \( G' \) defined by the condition \( (p_p + p_n)/|p_p + p_n| = G' \), and \( k \) is the spatial part of the four-vector \( k_1 \) defined by Eq. (14) with \( G = G' \).

The next step is to change the variables from \( p_{n\perp}, p_n^+ \) to \( k \). For this purpose we can use Eq. (12) and write

\[
d\rho(p_{n\perp}, p_n^+ \perp) = d\rho(p_{n\perp}, p_n^+ \perp) \int 2(2\pi)^3 p_p^+ \delta(2) (p_{\perp} - p_{\perp}') \delta(p^+ - p^+ \perp) d\rho(p_{p\perp}, p_{p'}^+ \perp) \quad (34)
\]

Then using Eqs. (15) and (32) we get

\[
d\rho(p_{n\perp}, p_n^+ \perp) = \frac{\omega(k) d\rho(int)}{M_0(k)^3} \quad (35)
\]

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Therefore using again Eq. (33) we can write Eq. (36) in the form

\[ W_{\mu\nu}(P', q) = \frac{1}{4\pi} \sum_{\sigma_p \sigma'_p, \sigma_n} \sum_{\tilde{X}} \int (2\pi)^4 \delta^4(P' + q - p_n - P) \cdot \]

\[ \frac{M_0(k)^3}{m_d^2 \omega(k)} \chi(k, \sigma_p, \sigma_n)^* \chi(k, \sigma'_p, \sigma_n) \langle p_p, \sigma_p | J_\mu(0) | \tilde{X} \rangle \cdot \]

\[ \langle \tilde{X} | J_\nu'(0) | p_p, \sigma'_p \rangle d\rho(int) + (...) \]  

(36)

where \( p_p \) and \( p_n \) are the four-vectors depending on \( G' \) and \( k \) as (see Eq. (14))

\[ p_p = \beta(G') k_p \quad p_n = \beta(G') k_n \]  

(37)

The purpose of exposing the detailed derivation of Eq. (36) from Eq. (28) is to stress that this derivation does not use any additional assumptions, i.e. Eq. (36) is an unambiguous consequence of Eq. (28).

Let us suppose that the deuteron is fully polarized along the positive direction of the \( z \) axis. In this case \( e_j e^*_l \) = \( (\delta_{jl}/3) - (i\epsilon_{jl}/2) \) where we do not include the contribution of the tensor polarization. Then a direct calculation using Eqs. (15), (20), (26) and (36) gives

\[ W_{\mu\nu}(P', q) = \int \left\{ [\varphi_0(k)^2 + \varphi_2(k)^2][\tilde{W}_{\mu\nu}^{1p}(p_p, \tilde{q}) + \tilde{W}_{\nu\mu}^{1p}(p_p, \tilde{q})] + \right. \]

\[ \left[ \varphi_0(k) - \varphi_2(k) \sqrt{2} \right][\varphi_0(k) + \varphi_2(k) \sqrt{2} \left( \frac{3k^2}{k^2} - 1 \right)] [\tilde{W}_{\mu\nu}^{1p}(p_p, \tilde{q}) - \tilde{W}_{\nu\mu}^{1p}(p_p, \tilde{q})] \right\} \cdot \]

\[ \frac{M_0(k)^3 d^3k}{2(2\pi)^3 \omega(k)^2 m_d^2} + (...) \]  

(38)

where \( \tilde{q} = q - [M_0(k) - m_d]G' \), and therefore, as follows from Eq. (9), the four-vector \( \tilde{q} \) has the following components in the reference frame considered in Sec. 1:

\[ \tilde{q}^0 = (M'' - M_0(k))G'^0, \quad \tilde{q} = -(M'' + M_0(k))G' \]  

(39)

It is clear from Eq. (38) that the nucleon structure functions entering into this expression depend on \( \tilde{x} = -\tilde{q}^2/2(p_p \tilde{q}) \). As follows from Eqs. (37) and (39), in the Bjorken limit

\[ \tilde{x} = \frac{M_0(k) - m_d(1 - x)}{\omega(k) + k^2}, \quad \tilde{q}^2 = -\frac{|\tilde{q}|^2}{m_d x} [M_0(k) - m_d(1 - x)] \]  

(40)

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If $x > 1$ then it follows from Eq. (40) that $\tilde{q}^2 < 0$, $\tilde{x} > 1$. If $x < 0$ and $M_0(k) - m_d(1 - x) > 0$ then $\tilde{q}^2 > 0$, $\tilde{x} > 0$. Finally, if $x < 0$ and $M_0(k) - m_d(1 - x) < 0$ then $\tilde{q}^2 < 0$, $\tilde{x} < 0$. In all these cases Eq. (22) is not satisfied and therefore the nucleon structure functions are equal to zero. We conclude that the deuteron structure functions automatically satisfy the condition that they are equal to zero if $x < 0$ and $x > 1$, as it should be.

Let us introduce the internal variable $\xi = \left[ \omega(k) + k_z \right] / M_0(k)$. Then it is easy to show that $\xi \in (0, 1)$ and

$$M_0(k) \equiv M_0(k_\perp, \xi) = \frac{m^2 + k_z^2}{\xi(1 - \xi)}, \quad k_z = (\xi - \frac{1}{2})M_0(k_{\perp}, \xi),$$

Therefore the sets $(k_{\perp}, k_z)$ and $(k_{\perp}, \xi)$ have a one-to-one relation to each other, and the normalization condition (27) can be written in the form

$$\int d^2k_{\perp} \int_0^1 \frac{d\xi}{\omega(k)} \frac{d^2k_{\perp} d\xi}{2\xi(1 - \xi)^3} \left[ \varphi_0(k)^2 + \varphi_2(k)^2 \right] = 1$$

where $k = (k_{\perp}^2 + k_z^2)^{1/2}$ is the function of $k_{\perp}, \xi$. However not all $\xi$’s contribute to Eq. (38) since, as follows from Eq. (40), $\tilde{x} \leq 1$ only if $\xi \in [\xi_{\text{min}}, 1]$ where

$$\xi_{\text{min}} \equiv \xi_{\text{min}}(k_{\perp}, x) = \frac{m^2 + k_z^2}{m^2 + k_{\perp}^2 + m_d(1 - x)^2}$$

Now everything is ready to write down the explicit expressions for the deuteron structure functions in terms of the nucleon ones. Since $p_p^+ \gg |k_{\perp}|$ in the Bjorken limit, we can use Eq. (24). Let us recall that Eq. (38) is valid for $\mu, \nu = 1, 2$ since the $\perp$ components of the CO are not constrained by the continuity equation. Therefore, as follows from Eqs. (24), (38) and (41)

$$F_i^d(x, q^2) = \int d^2k_{\perp} \int_{\xi_{\text{min}}}^1 \frac{d\xi}{(2\pi)^3 \xi(1 - \xi)^3} \left[ \frac{M_0(k_{\perp}, \xi)^2}{m_d} \right] \left[ \varphi_0(k)^2 + \varphi_2(k)^2 \right][F_i^p(\tilde{x}, \tilde{q}^2) + F_i^n(\tilde{x}, \tilde{q}^2)], \quad (i = 1, 3)$$

$$g_i^d(x, q^2) = \int d^2k_{\perp} \int_{\xi_{\text{min}}}^1 \frac{d\xi}{(2\pi)^3 \xi(1 - \xi)^3} \left[ \frac{M_0(k_{\perp}, \xi)^2}{m_d} \right] \left[ \varphi_0(k) - \frac{\varphi_2(k)}{\sqrt{2}} \right] \left[ \varphi_0(k) + \frac{\varphi_2(k)}{\sqrt{2}} \left( \frac{3k_{\perp}^2}{k_z^2} - 1 \right)[g_i^p(\tilde{x}, \tilde{q}^2) + g_i^n(\tilde{x}, \tilde{q}^2)] \right]$$
where $\bar{x}$ and $\bar{q}^2$ are given by Eq. (40) and $M_0(k)$ and $k$ are the functions of $k_\perp, \xi$ defined by Eq. (41).

Let us now consider the longitudinal structure function. If the minus component of the CO is taken in the IA then, as follows from Eqs. (21), (23), (37), (38) and (41)

$$
F_d^L(x, q^2) = \int d^2 k_\perp \int_{\xi_{\text{min}}}^1 \frac{d\xi}{(2\pi)^3} \frac{M_0(k_\perp, \xi)}{m_d} \left(\frac{x}{\bar{x}}\right)^2 \cdot \left[\varphi_0(k)^2 + \varphi_2(k)^2\right] \left[F_p^L(\bar{x}, \bar{q}^2) + F_n^L(\bar{x}, \bar{q}^2)\right]
$$

(46)

Here we have taken into account that, as follows from Eqs. (9) and (39), $q^- = \bar{q}^-$ in the Bjorken limit. An analogous derivation in the case when the plus component of the CO is taken in the IA gives

$$
F_p^L(x, q^2) = \int d^2 k_\perp \int_{\xi_{\text{min}}}^1 \frac{d\xi}{(2\pi)^3} \frac{M_0(k_\perp, \xi)}{m_d} \left[\varphi_0(k)^2 + \varphi_2(k)^2\right] \left[F_p^L(\bar{x}, \bar{q}^2) + F_n^L(\bar{x}, \bar{q}^2)\right]
$$

(47)

5 Structure functions of heavier nuclei

If the IA is valid, the nucleons absorb the virtual photon incoherently and the final state interaction between the spectator nucleons and the particles produced in the DIS on the struck nucleon can be neglected then as follows from Eq. (20)

$$
W_{\mu\nu}(P', q) = \frac{1}{4\pi} \sum_{i=1}^A \sum_{Y_i, \bar{X}_i} (2\pi)^4 \delta^{(4)}(P' + q - P_{Y_i} - P_{\bar{X}_i}) \cdot 
$$

$$
\langle G' \chi | J_i^\mu(0) | Y_i; \bar{X}_i \rangle \langle Y_i; \bar{X}_i | J_i^\nu(0) | G' \chi \rangle
$$

(48)

Here $\bar{X}_i$ are all possible final states in the DIS on the $i$-th nucleon, $Y_i$ are all possible states of the final system of the nucleons $1, ... i - 1, i + 1, ... A$, $P_{\bar{X}_i}$ and $P_{Y_i}$ are the corresponding four-momenta and $J_i^\mu(0)$ is the CO of the $i$-th nucleon.

The calculation of the nucleus structure functions from Eq. (48) is much more complicated than the calculation of the deuteron structure functions from Eq. (28). The reasons of the complications are as follows. Firstly,
as noted in Sec. 2, the operator $B$ is not equal to unity if $A \geq 3$ (the explicit expressions for $B$ were written only for $A = 3$ though in principle this expression can be written for any $A$ [8, 7]). Secondly, $P_{Y_i}$ is not equal to a sum of the four-momenta of the nucleons 1, ...$i - 1$, $i + 1$, ...$A$ since there is no ground to neglect the final state interaction in the system (1, ...$i - 1$, $i + 1$, ...$A$) and for the same reason $Y_i$ is not an antisymmetrized tensor product of the one-nucleon states. Finally, the relativistic WF $\chi$ has never been calculated.

If we assume that the nucleus is the nonrelativistic system with a good accuracy then we can take for $\chi$ its nonrelativistic expression and neglect the operator $B - 1$. Let us also assume that in the nonrelativistic approximation the final state interaction in the system (1, ...$i - 1$, $i + 1$, ...$A$) can be neglected. Once these assumptions are made, the nucleus structure functions can be calculated from Eq. (48) by analogy with the calculation of the deuteron structure functions in the preceding section.

The result of the calculation is as follows. We introduce the quantities $\tilde{q} = q - (M_0 - m_A)G'$ and $\tilde{x} = -\tilde{q}^2/2(p_1 \tilde{q})$. Then in the Bjorken limit

$$\tilde{x} = \frac{M_0 - m_A(1 - x)}{\omega_1(k_1) + k_1^z}, \quad \tilde{q}^2 = -\frac{|q|^2}{m_Ax}[M_0 - m_A(1 - x)]$$

If $Z$ is the number of protons in the nucleus then

$$F_i^A(x, q^2) = \sum_{\sigma_1, \ldots, \sigma_A} \int \frac{M_0^3}{m_A^2 \omega_1(k_1)}[ZF_i^p(\tilde{x}, \tilde{q}^2) + (A - Z)F_i^n(\tilde{x}, \tilde{q}^2)] \cdot |\chi(k_1, \sigma_1, \ldots, k_A, \sigma_A)|^2 d\rho(int) \quad (i = 1, 3)$$

and, if the minus component of the CO is taken in the IA then

$$F_L^A(x, q^2) = \sum_{\sigma_1, \ldots, \sigma_A} \int \frac{M_0^3}{m_A^2 \omega_1(k_1)[\omega_1(k_1) + k_1^z]}(\frac{x}{\tilde{x}})^2[ZF_L^p(\tilde{x}, \tilde{q}^2) + (A - Z)F_L^n(\tilde{x}, \tilde{q}^2)] |\chi(k_1, \sigma_1, \ldots, k_A, \sigma_A)|^2 d\rho(int)$$

It is also possible to obtain the expressions for the spin dependent structure functions but we shall not dwell on this problem.

If we introduce the quantity $\xi = [\omega_1(k_1) + k_1^z]/M_0$ then it is easy to show that the integration over $\xi$ in Eqs. (50) and (51) is in fact performed not
from 0 to 1 but from $\xi_{\text{min}}$ to 1 where

$$\xi_{\text{min}} \equiv \xi_{\text{min}}(k_{1\perp}, M_1, x) = \frac{1}{2}(1 - \alpha - \beta + [(1 - \alpha - \beta)^2 + 4\alpha]^{1/2}),$$

$$\alpha = \frac{m^2 + k_{1\perp}^2}{M_1^2 - m^2}, \quad \beta = \frac{m_A^2 (1 - x)^2}{M_1^2 - m^2}$$

and $M_1$ is the free mass of the system $(2, \ldots A)$.

The validity of the assumptions used in the derivation of Eqs. (50) and (51) is questionable. Indeed, we neglected the quantities $B - 1$ and $p_1 + \ldots p_{n-1} + p_{i+1} + \ldots p_A - P_Y$ since they are small in the nonrelativistic approximation. However in this approximation it is also possible to neglect the difference between $M_0$ and $m_A$. We shall discuss this question in the next section.

6 Discussion

It is usually believed that DIS with very large momentum transfer can be described in the IA. As shown in Sec. [1], this approximation is compatible with Lorentz invariance only in the point form of dynamics. Then, assuming that the deuteron is the relativistic system of the proton and neutron, it is possible to obtain the unambiguous expressions for the deuteron structure functions $F_1^d(x, q^2)$, $F_3^d(x, q^2)$ and $g_1^d(x, q^2)$ (see Eqs. (44) and (45)) in terms of the corresponding nucleon structure functions. At the same time, the result for the deuteron longitudinal structure function depends on which component of the CO in the $+\perp$ plane is taken in the IA.

The main difference between our result and the results of many authors (see, for example, Refs. [2, 3, 4, 27, 5, 6]) is that in our approach the nucleon structure functions entering into the integrals defining the corresponding deuteron structure functions depend on $\tilde{x}$ and $\tilde{q}^2$ given by Eq. (50) while the standard expressions do not contain terms with $M_0(k) - m_d$:

$$\tilde{x} = \frac{m_d x}{\omega(k) + k^z}, \quad \tilde{q}^2 = q^2$$

These expressions have the well-known partonic interpretation since in this interpretation the effect of binding is considered as a higher twist effect. In particular, the expression for $\tilde{x}$ is compatible with the flux factor in Refs.
Equation (53) can be also obtained in the form of dynamics \cite{27,11,12}, but, as noted in Sec. 1, the CO in this form does not properly commute with the Lorentz boost generators. Let us stress that in our approach the particles are always on-shell and the nucleon structure functions in Eqs. (44-47) refer only to real nucleons. Therefore the difference between Eqs. (40) and (53) can be regarded as an analog of off-shellness in our approach.

The experimental data are usually given in terms of not the Bjorken variable $x$ but in terms of $x_{\text{eff}} = m_A x/m$. Therefore the results (40) and (53) are practically the same when $m x_{\text{eff}} \gg M_0(k) - m_d$ and considerably differ each other when $m x_{\text{eff}} < \sim M_0(k) - m_d$. The quantity $M_0(k) - m_d$ can be written as $T + |\epsilon_d|$ where $T$ is the kinetic energy of the nucleons in the deuteron and $\epsilon_d = -2.23\text{MeV}$ is the deuteron binding energy. According to the well-known phenomenological models of the nucleon-nucleon potential, the average value of $T$ is of order $20\text{MeV}$. Therefore our result coincides with the standard one at $x_{\text{eff}} \gg 0.02$ and considerably differs from it at $x_{\text{eff}} < \sim 0.02$.

If $0.02 \ll x_{\text{eff}} < 1$ and $x_{\text{eff}}$ is not too close to 1, the main contribution to the deuteron structure functions is given by the nonrelativistic region of the deuteron WF where $|k| \ll m$. Then, as easily follows from Eqs. (42-47)
\begin{align*}
F^d_i(x, q^2) &= 2[F^p_i(x_{\text{eff}}, q^2) + F^n_i(x_{\text{eff}}, q^2)] \quad (i = 1, 3), \\
g^d_i(x, q^2) &= 2[g^p_i(x_{\text{eff}}, q^2) + g^n_i(x_{\text{eff}}, q^2)](1 - \frac{3}{2}P_D), \\
F^d_L(x, q^2) &= F^p_L(x_{\text{eff}}, q^2) + F^n_L(x_{\text{eff}}, q^2)
\end{align*}
where $x_{\text{eff}} = 2x$ and $P_D$ is the probability of the $D$ state in the deuteron. In this case the result for $F^d_L$ does not depend on the choice of Eq. (43) or (47). As a consequence of Eq. (54), $F^d_2(x, q^2) = F^p_L(x_{\text{eff}}, q^2) + F^n_L(x_{\text{eff}}, q^2)$ what is the well-known result in the case when nuclear effects in the deuteron are neglected. Let us also note that if the main contribution to the first moments $\Gamma^d_1$, $\Gamma^p_1$ and $\Gamma^n_1$ of the structure function $g_1(x)$ for the deuteron, proton and neutron respectively is given by $0.01 \ll x < 1$ and $0.02 \ll x_{\text{eff}} < 1$ then we obtain from Eq. (54) the well-known relation $\Gamma^d_1 = (\Gamma^p_1 + \Gamma^n_1)(1 - 1.5P_D)$ used for extracting $\Gamma^n_1$ from the proton and neutron data to test the Bjorken sum rule \cite{28}. The results (54) are well-known and, as shown by several authors (see, for example, Ref. \cite{29}), the nuclear corrections to these expressions are small.
Let us now consider the deuteron structure functions at $x \lesssim 0.01$. In the literature the role of shadowing at such values of $x$ is widely discussed \cite{22,30,31}. One of the problems is the behavior of the shadowing contribution at very large $|q^2|$. The authors of Refs. \cite{22,30,6} consider the scaling part of this contribution (which does not depend on $|q^2|$) but, as noted in Ref. \cite{31}, the shadowing contribution at large $|q^2|$ is a logarithmically decreasing function. Therefore one might expect that at large $|q^2|$ the IA is dominant (on the other hand, in the electroproduction $|q^2|$ should be much smaller than $m_Z^2$). Anyway, the shadowing contribution does not depend on the nucleon structure functions. Therefore if we wish to extract the neutron structure functions from the proton and deuteron data we have to know the contribution of the IA. Meanwhile, as follows from Eq. (40), even if $x \ll 0.01$ the quantity $\tilde{x}$ is of order 0.02. The minimum of $\tilde{x}$ is approximately equal to 0.002 at $x = 0$, $k_\perp = 0$ and $k_z = |\epsilon_d|/2m \approx 1\text{MeV}/c$, but the contribution of very small $k$'s to the integrals (44-47) is also small. We conclude that there is no way to extract the neutron structure functions from the proton and deuteron data at $\tilde{x} \lesssim 0.01$.

The problem of extracting the neutron structure functions from the proton and deuteron data was considered by many authors (see, for example, Ref. \cite{32} and references cited therein). The authors of Ref. \cite{33} argue that the Gottfried sum \cite{34} is rather sensitive to the pion admixture in the deuteron while the Bjorken sum \cite{28} is not sensitive to such an admixture. Our conclusion implies that even in an idealized case when the contribution of the IA is known exactly, the Bjorken and Gottfried sum rules cannot be tested. If the contribution of the region $\tilde{x} \lesssim 0.01$ to these sum rules is small then our conclusion is of only academic interest. However the data of the SMC Collaboration on $g_1^p(\tilde{x})$ \cite{35} and the dramatic rise of $F_2^p(\tilde{x})$ discovered recently at HERA \cite{36} give grounds to think that the role of $\tilde{x} \lesssim 0.01$ in the Bjorken and Gottfried sum rules is not negligible.

Let us now consider the results of Sec. 5 for the structure functions of heavier nuclei. It is clear from Eq. (49) that these results agree with the standard ones if $mx_{eff} \gg M_0 - m_A$. We can write $M_0 - m_A = A(T + |\epsilon|)$ where $T$ is the kinetic energy per nucleon and $\epsilon$ is the binding per nucleon. For heavy nuclei $\epsilon \approx 8\text{MeV}$, and the reasonable value of $T$ is 20MeV. Therefore $(M_0 - m_A)/m$ becomes larger than unity at $A \gtrsim 40$. We see that though $(M_0 - m_A)/m$ is the term of order $v^2/c^2$, this quantity cannot be neglected for heavy nuclei. Moreover, since we cannot neglect $M_0 - m_A$, we also cannot
neglect the operator $B - 1$ and the final state interaction in the system $(1, ... i - 1, i + 1, ... A)$. We conclude that while the effect of the final state interaction is usually believed to be a higher twist effect, in our approach this effect is important even in the Bjorken limit.

The above discussion shows that only the results of Sec. 4 for the deuteron are reliable. Moreover, in our opinion the results for the deuteron structure functions in terms of the proton and neutron structure functions are much more reliable than the standard results for the nucleon structure functions in terms of the quark structure functions. The matter is that in the deuteron case the assumption that the system under consideration is the relativistic system with a fixed number of particles is reasonable, and it is reasonable to think that the final state interaction can be neglected in the Bjorken limit.

If the IA is dominant at high $|q^2|$ and small $x$ then the results of Sec. 4 make it possible to give the following predictions on the behavior of the deuteron structure functions at $x \ll 0.01$. The functions $F_3^d(x, q^2)$ and $g_1^d(x, q^2)$ are given by Eqs. (44), and (45) where, as follows from Eqs. (40), and (43),

\[ \tilde{x} = \frac{2 \omega(k) - m_d}{\omega(k) + k^z}, \quad \tilde{q}^2 = -\frac{|q^2|}{m_d x^2} [2 \omega(k) - m_d], \quad \xi_{\text{min}} = \frac{m^2 + k^2}{m^2 + k^2_x + m_d^2}, \quad (55) \]

Therefore the $x$ dependence of these structure functions is fully determined by the $\tilde{q}$ dependence of the nucleon structure functions while $\tilde{x}$ does not depend on $x$ at $x \ll 0.01$. In particular, if the $\tilde{q}$ dependence of the nucleon structure functions is weak (e.g. logarithmic), the $x$ dependence of these deuteron structure functions also is weak. At the same time, if the longitudinal structure function is given by Eq. (44) then the ratio $R$ of the total cross sections for longitudinally and transversely polarized virtual photons falls off as $\sim x^2$ and $F_2^d(x, q^2)$ falls off as $\sim x$, while if the longitudinal structure function is given by Eq. (47) (what is less probable) then $R$ increases with the decrease of $x$ as $\sim 1/x$ and the dependence $F_2^d(x, q^2)$ on $x$ is weak.

The most unusual feature of Eq. (55) is that $\tilde{q}^2$ strongly differs from $q^2$ while in the covariant approach based on Feynman diagrams (see the discussion in Sec. 1) $\tilde{q}^2$ is always equal to $q^2$.

At present the low $x$ region has been investigated by the NMC and E665 Collaborations [37] but the values of $|q^2|$ in these experiments were small at small $x$. However the measurements of the deuteron structure functions at small $x$ and large $|q^2|$ are planned at HERA.
We see that the decrease of $F_2^d(x, q^2)$ with the decrease of $x$ may be not only a consequence of shadowing but also a consequence of the IA. In any case one might think that in view of the results obtained at HERA \cite{36} the described behavior of the deuteron structure functions at low $x$ fully differs from the behavior of the nucleon structure functions at such $x$. A possible reason is as follows. The central point of the above description of the deuteron structure functions is the relation between $m_d x$ and $M_0(k) - m_d$. In nuclear physics the difference between free mass operator and the mass of the bound system is always positive. However in quark models this difference may be negative if confinement is the main interaction between quarks. In this case the above approach does not work, since it is obvious that if confinement is taken into account then the final state interaction cannot be neglected. The importance of taking into account the role of confinement in nucleon DIS was pointed out by several authors (see the recent paper \cite{38} and references therein) but it is usually believed that confinement is only a higher twist effect. However if the CO is taken in the point form then confinement may play an important role even in the Bjorken limit. We suppose to investigate this problem in future publications.

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