GENETIC ALGORITHM FOR OBSTACLE LOCATION-ALLOCATION PROBLEMS WITH CUSTOMER PRIORITIES

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Abstract. In this paper we propose a metaheuristic approach to solve a customer priority based location-allocation problem in presence of obstacles and location-dependent supplier capacities. In many network optimization problems presence of obstacles prohibits feasibility of a regular network design. This includes a wide range of applications including disaster relief and pandemic disease containment problems in healthcare management. We focus on this application since fast and efficient allocation of suppliers to demand nodes is a critical process that impacts the results of the containment strategy. In this study, we propose an integrated mixed-integer program with location-based capacity decisions that considers customer priorities in the network design. We propose an efficient multi-stage genetic algorithm that solves the problem in continuous space. The computational findings show the best allocation strategies derived from proposed algorithms.

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1. **Introduction.** Location-allocation problems have been rigorously investigated in a variety of applications in the literature, and numerous configurations have been proposed. However, with computational advancements in recent years many new applications arise every day that require relaxation of previous assumptions or addition of more realistic factors to better simulate the actual problem. Capacitated Obstacle location-allocation problems (COLAP) refers to a subset of location-allocation problems that considers existence of unavailable or prohibited regions in the feasible space from either locating a facility, creating a path, or both. In this problem, resources (suppliers) are located in the accessible areas in the location phase of the problem, and consequently they are assigned to the demand points (customers) based on certain criteria. In this problem, paths that connect suppliers to the customers must avoid crossing the infeasible regions (also referred to as obstacles, interchangeably). Applications of COLAP can be found in facility location in presence of lakes, mountains, or other natural barriers, and optimizing allocation of emergency service units to multiple destinations in cases such as natural disasters, or disease outbreaks.

Examples of emergency and disaster management are, unfortunately, not rare. Only in year 2016 two earthquakes in Japan caused $31 billion in losses, and floods in China during summer of 2016 caused $20 billion in damage. In North America, the single event Hurricane Matthew killed hundreds of people in Haiti and produced $10 billion in damage. In United States alone there were 103 disaster declarations with different magnitudes in year 2016, according to Federal Emergency Management Agency (FEMA). Besides the natural disasters, we are encountering many disease outbreaks around the world every year. In year 2016 there were 136 disease outbreaks with variety of impacts recorded by World Health Organization (WHO). The Ebola outbreak of West Africa in 2014 caused over 28 thousand suspected cases and more than 14 thousand deaths. Every disease outbreak effects populations and economies in multiple dimensions, and therefore it is necessary to develop efficient algorithms to combat the disaster and minimize the negative impact of it in human populations, infrastructures, and natural resources as fast and as efficient as possible.

COLAP can be utilized in cases were obstacles have high impact on the decision maker’s choices, and consequently lead to alternative solutions that deviate from the optimal solution. For example, when a natural disaster happens, there are certain areas that are not accessible (obstacles), and disaster management units require fast access to the demand points avoiding those obstacles. In this case a classical location-allocation problem is unable to provide optimal solutions. In cases of pandemic disease outbreaks that often occur in obsolete locations, medical units are dispatched to accessible regions and are allocated to demand points, however, considering danger zones and avoiding certain regions are necessary for location and allocation of the medical units.

[1] provided an excellent classification of different types of non-emergency and emergency healthcare facility location problems. They emphasized that attention to disaster management (humanitarian logistics) applications, and investigation of continuous location models in healthcare applications are necessary. This paper addresses the problem of location-based capacitated location-allocation in a continuous space, and in existence of obstacles considering customer priorities. In our study, we uniquely consider two distinguishing factors: a) location based capacities
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of suppliers, and b) customer priorities. Customer priorities are particularly crucial in cases of disaster management.

It is very important that demand heterogeneity, i.e., differences in requirements, and priorities among the demand points, are considered during the location-allocation phase. For example, in a pandemic disease outbreak, there are variety of aid units with different specialties such as first aid, special care, or laboratories, and therefore, there are asymmetric priorities in proximity of suppliers to demand nodes. On the other hand, presence of obstacles can impact the accessible capacity in the nearby areas. Consequently locating a supplier in nearby regions may negatively impact the maximum capacity that a supplier could provide. Unlike other studies in the literature, that focus purely on the network of paths and distances between supplier/customer nodes avoiding obstacles, we consider location dependent capacities for suppliers, and customer priorities, for the location-allocation problem and show the impact of these in our computational study.

This study is organized as follows. Section 2 summarizes relevant literature in obstacle location problems. Section 3 presents our proposed mixed integer programming formulation of the problem. In Section 4 we demonstrate our genetic algorithm approach to solve the obstacle location-allocation problem. Section 5 demonstrates the results of computational study. We conclude this paper in Section 6 with final remarks and future research directions.

2. Literature review. There is a wide variety of studies that are dedicated to the location-allocation problems. Many articles focus on theoretical aspects and development of computationally effective algorithms due to the high complexity of the problem setting, and others investigate applications of theory and development of special cases and relaxation of basic assumptions in the problem settings to better reflect the reality. [24] provided a review of literature in facility location models in the context of supply chain management and discusses the importance of integration of location decisions with other decisions relevant to the design of a supply chain network. [4] provided a review of common formulations for static and dynamic facility location problems. They highlighted the need for disaster management applications as one of the trends and prospect topics in facility location decisions. [29] reviewed papers related to the median and plant location models and center and covering models. They highlighted site dependent fixed cost, and also site-dependent capacity acquisition costs. Multi period capacity problems and location dependent capacity models are also discussed.

Establishment of location-allocation problems also binds closely with set packing, set covering, and set partitioning studies in the literature. Multiple studies summarize the work that have been done in this field. [16] delivered a summary on simple plant location problems and report computational techniques, spanning from heuristics to exact methods and deliver an analyses of approximate algorithms. [28] reported differences in formulating approaches in location problems based on objectives, and classification of network problems. In this section we briefly review location-allocation studies in the literature and subsequently present the latest network problems in presence of obstacles in Section 2.1.

Many studies consider Euclidean space when calculating distances between nodes. This classification of problems are generally known as Weber Problems. The multiple-facility capacitated location-allocation is also referred to as multiple-facility Weber problem named after Alfred Weber (1868-1958). This problem locates m facilities
and allocates them to n customers and minimizes the total traveling costs. [17] presented an exact method for the multi-source Weber problem. [22] introduced an un-capacitated probabilistic approach to formulate location-allocation model. Solution methods for this class of problems spans a wide range from exact algorithms to heuristics and metaheuristics approaches. Next, we introduce studies with exact and approximate methodologies in the literature. [19] gathered the most important capacity location problems and classifies them into p-center, p-median, fixed-cost, set covering, and anti-set covering problems. In an interesting chapter of this work relevant to our study [32] proposed a facility location model for locating two-dimensional structures such as line segments or circles instead of isolated points on the feasible space.

Research is abundant with exact methods to formulate this class of problems. [34] offered a nonlinear mixed-integer programming problem to find location of shelters in a region threatened by a hurricane that minimizes total congestion-related evacuation times. [18] presented a capacitated facility location problem where demands are stochastic. The problem has been optimally solved using branch and cut method. [7] considered a capacitated mathematical model with probabilistic customer locations and certain demands. A simplex algorithm and a stochastic simulation have been devised to solve the model. [38] applied simple deformation and linear estimation in their proposed model and offer an e-optimality method and constraint generation algorithm in their solution strategy. [13] suggested p-facilities multiple-criteria location problem and design a median network with positive and negative weights. The objective is to obtain a set of Pareto optimal locations with a set of independent objective values. [25] combined DEA and location-allocation problem while using robust optimization technique.

There are also a number of studies in the field of disaster operations planning. [27] presented a location-allocation model to integrate decisions about the localization of the refuges and the assignment of the affected homes to the selected refuges so that the maximum distance that a victim should travel to a refuge is limited and the risks associated with that relocation is diminished. They considered the preferences of refuges on some basic requirements as the quality of service of the refuges. [21] analyzed a disaster relief problem with a 2 stage modeling approach. The first stage problem is to determine how many and which facilities to locate and how to allocate demand to them so as to minimize total demand-weighted distance between the subset of considered demand locations and their assigned facilities. The second stage problem is to determine the total distance from all true demand points to their nearest facilities once the uncertainty has been resolved and the set of unverified social data requests is partitioned into true and false requests.

Heuristic and metaheuristic methodologies have also been investigated for location-allocation problems. [5] presented an algorithm with the ability of variable neighborhood search for obtaining optimal results for un-capacitated multi-source Weber problem. [12] consider a genetic algorithm as an alternative approach for generating optimal or approximately optimal solutions for location problems. [31] applied contingent programming model and its solution algorithm for supply chain network design. [39] proposed a stochastic model for solving capacitated location-allocation problem with contingent demands. [33] presented a maximal covering location model and a genetic algorithm in their methodology.
[10] modeled a single source capacitated facility location problem and offered an iterated tabu search heuristic that combines tabu search with perturbation operators to avoid local optimal solutions. [11] studied a traditional multi-facility location problem and compares three common heuristics, a random restart, a two-opt procedure, and genetic algorithm. They find that the genetic algorithm provides the best solutions. [30] developed a GA-based approach to address the un-capacitated continuous location-allocation problem. [35] proposed a genetic algorithm for the capacitated single allocation p-Hub median problem.

2.1. Obstacle location-allocation problems. Many of the location-allocation models with continuous space study ideal cases without considering obstacles. This sub-set of location-allocation problems are mostly pertinent to circuit and networks design, autonomous mobile robot tracking systems, natural disaster respond, and disease containment strategies. [14] for the first time addressed a special case of Weber problem with forbidden regions. Their model employs Euclidean distances and forbidden regions are presented as disjoint circular areas and a single convex polygon. They proposed two solution methodologies, a constrained optimization method and a discrete grid search. They used a controlling fuzzy logic approach to efficiently solve their proposed formulation. [20] considered rectilinear distances, and convert the problem into an equivalent network representation. [3] implemented a simulated annealing approach for Euclidean distance problem with multiple polygonal forbidden regions. In a similar study presented by [6] a series of relaxations and an iterative algorithm is proposed to find a local optimal solution.

In a study by [15] several forbidden regions are presented in form of polygons that limit the passage among supply and demand nodes. The proposed objective function aims to reduce distances between location of a newly introduced facility and existing facilities. [23] proposed 1-median problem with convex polygonal forbidden regions and found global optimal solutions using a branch and bound algorithm. They consider a median one-facility location problem with convex polygon forbidden regions. A new facility is located such that sum of weighted distances are minimized. The solution process employ small and big squares algorithm based on Branch and Bound approach. [36] presented a genetic algorithm with fuzzy logic operators to solve an obstacle location-allocation problem.

[8] developed an efficient solution method for obstacle location allocation problem. They emphasized the complex nature of this problem which is modeled with MIP structure to minimize the total distance. [9] studied an extension of location-allocation model with service capacity constraints and propose a genetic algorithms (GAs) and evolutionary strategy (ES) to minimize total distance. [26] addressed hospital location problems in the areas prone to natural disasters. They consider minimization of mean travel distance for patients to hospitals, and maximization of the system’s effectiveness. [37] proposed two models for disaster facility location problems. The first model is a deterministic model that incorporates distance-dependent damages to disaster response facilities and population centers. The second model is a stochastic model considering the damage intensity as a random variable. They showed that their model significantly reduces the expected cost of providing supplies.

To the best of our knowledge, none of the studies in the literature consider the impact of obstacles on capacities, or the importance of customer priorities on the supplier-customer allocation process. In this paper we formulate the COLAP as a
mixed-integer mathematical program with Euclidean distances that reflect location-based capacities for suppliers, and customer priorities for supplier allocation process. We developed an efficient genetic algorithm to solve the proposed formulation for fast and near-optimal solutions. The details of the mathematical formulation and the solution methodology are presented in Sections 3 and 4.

3. Mixed-integer mathematical program for COLAP. In this section we first present a generic mixed-integer mathematical representation of capacitated obstacle location-allocation problem with location-based capacities and customer priorities, in a Euclidean space pane. The generic formulation presumes prior knowledge on the location of the suppliers, given the limitations of integer programming to formulate continuous space location problem. We then propose a special case mixed-integer formulation where demand nodes are candidate points for supplier locations. Later in the computational results section we examine performance of this formulation relative to the proposed meta-heuristic approach.

We assume presence of forbidden areas (obstacles) with two restrictions: a) Obstacles are infeasible for location of suppliers, and b) obstacles prohibit passage of any rout among pairs of supplier-customer nodes. The proposed model seeks to locate m suppliers on the pane and allocate them to n customers (demand nodes) respecting the two limiting conditions. We also add two important factors that add to the complexity of the problem. The first factor reflects the dependency of the supplier capacities on the locations in the feasible region. In certain areas, mostly closer to the obstacles, proximity to the customers comes at the expense of reduction in capacities. The second factor we add to our formulation is the heterogeneity of the supplier-customer priority. In our proposed model we offer an asymmetric preference in proximity of suppliers and customers. We elaborate on the incorporation of these two factors into the model in the following.

3.1. Model assumptions. Marginal Areas: When locating suppliers in presence of forbidden areas, it is often not desirable to position the suppliers within a certain margin from the obstacle limits. Suppose polygon $P_k$ (concave or convex) is a representation of an obstacle on the plane. We define regular capacity for a given supplier as $s_i$. If supplier $i$ is located in certain distance of obstacle $P_k$ the capacity of the supplier would be equal to $s_{ik}$. Normally, the capacity inside marginal areas is lower than capacity outside the margins. Marginal areas may be in shape of a polygon surrounding the obstacle. In this study we assume a certain radius from the centroid of the obstacle as the margins. Figure 1 depicts four suppliers located inside and outside marginal area of obstacle $P_1$.

Connecting Paths: The connecting paths among nodes, which in our case are suppliers and customers, may not pass through the obstacles. However, the paths can cross the marginal areas, or move along the edges of an obstacle. Figure 2 demonstrates a case where supplier $i$ and customer $j$ are being connected.

Customer Priorities: In this study we consider a priority in proximity of each supplier/customer pair on the plane. These priorities or preferences are independent from the amounts of customer demands and capacity of suppliers. Existence of such priority is important since often times capacity allocation is not determined simply by the proximity of a supplier to a customer. For instance, if we interpret these priorities as customer preferences, allow customers to have free authority with regards to their procurement strategy. In another instance, in case of an epidemic disease control, there is a need for proximity of certain medical units to
specific impacted areas due to heterogeneity of the impact. We define Matrix $H$ as presented in the following in form of $m \times n$ matrix with $m$ suppliers and $n$ demand nodes. Each matrix element represents a weight in proximity of supplier/customer pairs.

$$H = \begin{bmatrix} h_{11} & \ldots & h_{1n} \\ \vdots & \ddots & \vdots \\ h_{m1} & \ldots & h_{mn} \end{bmatrix}$$

Other Model Assumptions: We assume that each customer can be serviced by more than one distribution center. We also consider Euclidean distances in order to calculate the length of alternative connecting paths from suppliers to customers.

3.2. **Set of parameters and variables.** In order to formulate the problem, indices, parameters, and decision variables are defined as below.

**Sets and Indices:**
- $I = \{1, 2, \ldots, m\}, i \in I$: Set of suppliers
- $J = \{1, 2, \ldots, n\}, j \in J$: Set of customers
- $K = \{1, 2, \ldots, k\}, k \in K$: Set of obstacles
- $P_k(A_k)$: Obstacle polygon $k$, with set of vertices and centroid coordinates $A_k$
- $Q \in \mathbb{R}^2$: Location pane

**Parameters:**
- $s_i$: Capacity of supplier $i$ outside of marginal areas
- $s'_{ik}$: Capacity of supplier $i$ inside marginal area of obstacle $k$
- $c_j$: Demand of customer $j$
- $S_i(u_i, v_i) \in Q$: Coordinates of supplier $i$ on the location pane
• \( C_j(u_j, v_j) \in Q \): Coordinates of customer \( j \) on the location pane  
• \( D_{ij} \): Shortest path distance between supplier \( i \) and customer \( j \) considering obstacles  
• \( L_{ik} \): Euclidean distance from supplier \( i \) to obstacle \( k \)'s centroid  
• \( h_{ij} \in H \): Priority weight for supplier/customer proximity  
• \( r_k \): Marginal radius from obstacle \( k \)'s centroid  

Decision variables:  
• \( z_{ij} \): Binary variable, 1 if supplier \( i \) is allocated to customer \( j \), 0 otherwise  
• \( x_{ij} \): Amount of capacity allocation from supplier \( i \) to customer \( j \)  
• \( p_{ik} \): Binary variable, 1 if supplier \( i \) is located inside marginal area of obstacle \( k \)  

3.3. General MIP model for COLAP. In this section we present a general mixed-integer mathematical formulation for location-based obstacle location-allocation problem. Assuming that we have the a priori knowledge on the location of the suppliers in the Euclidean space \( Q \), the proposed mixed-integer program resolves at the same time the location-based capacities for suppliers in supplier locating phase, and customer priorities in capacity allocation phase. Our proposed model for Capacitated Obstacle Location-Allocation Problem (COLAP) is as follows.

**COLAP:** Minimize \[ \sum_{i \in I} \sum_{j \in J} x_{ij} \frac{D_{ij}}{h_{ij}} \]  
Subject to:  
1. \[ \sum_{i \in I} x_{ij} \geq c_j, \quad \forall j \in J \]  
2. \[ \sum_{j \in J} x_{ij} \leq s'_ik p_{ik} + s_i(1 - p_{ik}), \quad \forall i \in I \]  
3. \[ p_{ik} \geq 1 - \frac{L_{ik}}{r_k}, \quad \forall i \in I, k \in K \]  
4. \[ D_{ij} \in Q^2, \quad \forall i \in I, j \in J \]  
5. \[ x_{ij} \text{ integer}, \quad \forall i \in I, j \in J \]  
6. \[ p_{ij} \text{ binary}, \quad \forall i \in I, j \in J. \]  

Objective function 1 minimizes total cost of location-allocation problem considering customer priorities \( h_{ij} \). Constraint 2 is the demand satisfaction criterion. Constraint 3 ensures capacity conservation for suppliers. This constraint accounts for the location-based capacities. If supplier \( i \) is located inside the marginal area of obstacle \( k \), binary variable \( p_{ik} \) will automatically set capacity to \( s'_ik \). Constraint 4 determines whether supplier \( i \) is located inside marginal area of obstacle \( k \). Constraint 5 is the Euclidean distance of supplier \( i \) from customer \( j \), if we had prior knowledge on the location of supplier \( S_i(u_i, v_i) \) in pane \( Q \). Constraints 6 and 7 identify the decision variables.

**Remark 1.** Constrains 3 and 4 ensure capacity adjustment for suppliers based on their location. Note that if \( \frac{L_{ik}}{r_k} \) is strictly between 0 and 1 (i.e. \( 0 < \frac{L_{ik}}{r_k} < 1 \)), therefore inequality 4 will force binary variable \( p_{ik} \) to be 1, and consequently capacity of supplier \( i \) will be \( s'_ik \). If we have \( \frac{L_{ik}}{r_k} \geq 1 \), binary variable \( p_{ik} \) will be
unrestricted, and the model automatically chooses the best capacity for supplier \( i \) based on the demand allocation in Constraint 2.

We propose to solve this general model COLAP with a genetic algorithm. We further formulate a special case of COLAP as a set covering model that can solve a specific setting of COLAP where only demand nodes are the candidate points for supplier locations. We present this special case formulation in the following section.

3.4. Special case for COLAP. In this section we propose a special case formulation for the general COLAP. In this formulation the suppliers can be only located on the demand nodes. Please note that in the COLAP formulation suppliers can be located at any point on the feasible space. However, in the formulation presented in this section candidate nodes for supplier locations are limited to the demand nodes. The solution space of this formulation would be a subset of the general COLAP formulation.

This formulation is feasible to be solved with branch-and-bound method using commercial optimization solvers. We test the performance of the optimization problem under randomly generated instances and report the results along with the performance of a meta-heuristic approach on the same test bed. We introduce binary variable \( y_{ij} \) for locating supplier \( i \) on the demand point \( j \). Since the location of customers are known, binary variable \( p_{ik} \) will become a known parameter. The optimization formulation of Special case COLAP (SCOLAP) is as follows.

\[
\text{SCOLAP: Minimize } \sum_{i \in I} \sum_{j \in J} \frac{z_{ij} D_{ij}}{h_{ij}} \tag{8}
\]

Subject to:

\[
\sum_{i \in I} x_{ij} \geq c_j, \quad \forall j \in J \tag{9}
\]

\[
\sum_{j \in J} x_{ij} \leq \sum_{j \in J} y_{ij} (s^t_{ik} p_{ik} + s_i (1 - p_{ik})), \quad \forall i \in I \tag{10}
\]

\[
x_{ij} \leq M z_{ij}, \quad \forall i \in I, j \in J \tag{11}
\]

\[
D_{ij} \geq \sum_{j_2 \in J} y_{ij_2} D_{j_1 j_2}, \quad \forall i \in I, j_1 \in J \tag{12}
\]

\[
\sum_{j \in J} y_{ij} = 1, \quad \forall i \in I \tag{13}
\]

\[
x_{ij} \text{ integer, } \forall i \in I, j \in J \tag{14}
\]

\[
z_{ij}, y_{ij}, p_{ij} \text{ binary, } \forall i \in I, j \in J. \tag{15}
\]

The purpose of this reformulation is to be able to optimally test and solve this problem with optimization packages and compare the results with the genetic algorithm approach. This reformulation locates suppliers on demand nodes instead of assigning them to the Euclidean pane \( Q \). In terms of the objective function it is similar to conventional set covering formulations. The objective function 8 ensures the minimum distance to priority allocation in the network. Constraint 9 is the demand satisfaction inequality. Constraint 10 is the capacity conservation constraint, given that each supplier is only located on a single demand node. Constraint 11 ensures that a transportation happens if a supplier is assigned to a customer. Constraint 12 computes the distances of each supplier from all demand nodes. \( D_{j_1 j_2} \) specifies
the distance from customer $j_1$ to customer $j_2$. Constraint 13 ensures each supplier $i$ is located only on a single demand node. Constraints 10, 12, and 13 together determine that if supplier $i$ is located on demand node $j$, then the distances of the supplier $i$ from other demand nodes are equal to distances of demand node $j$ to the rest of the nodes.

Remark 2. As speedy and efficient allocation of suppliers to demand points carry far more importance in emergency management applications, similar to [9] and [26], our main objective is to minimize the distances traveled from facilities to the demand nodes. In this reformulation we use $z_{ij}$ in the objective function and include constraint 11, thus the SCOLAP formulation forms a classic set-covering problem. This objective minimizes the total distance traveled from the suppliers to the demand nodes considering the costumer priorities.

In the computational study section, we first solve the continuous relaxation of this formulation (R-SCOLAP) on our randomly generated instances, to ensure the feasibility of the problem. Further we test an optimization package to find the optimal solution of the instances.

4. An efficient genetic algorithm for COLAP. The special case MIP model presented in Section 3 can be solved with exact methods or decomposition algorithms such as branch and price and column generation. However, in a Euclidean space where $S_i(u_i, v_i)$ can accept unlimited continuous feasible candidates, decomposition algorithms fail to effectively generate large solution space at each iteration. In order to solve COLAP efficiently, we propose a genetic algorithm that produces variety of initially scattered solutions and show that this approach leads to fast convergence with near optimal solutions. In this section we introduce the details of our proposed Genetic algorithm for COLAP (GCOLAP).

4.1. Initialization. A chromosome in our model consist of the location of a member (supplier), and the value of the member which is calculated based on the generated allocation matrix, distribution amounts, and shortest paths. A set of chromosomes is selected as the preliminary population. The value of the chromosomes are determined through a performance assessment procedure. We employ a random format for initial population and a tournament selection approach for chromosome selection and offspring generation. The details of the procedures and our proposed operators are as follows.

4.1.1. Feasibility check.

Proposition 1. Consider points $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, and $P_3(x_3, y_3)$ in an orthonormal plane. Area of the triangle $\Delta P_1P_2P_3$ can be calculated as follows.

$$S_{\Delta P_1P_2P_3} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Proposition 2. Let $P_{k_1}, P_{k_2}, P_{k_3}, \ldots, P_{k_{p_k}}$ be vertices of a polygon $P$ and $D_i$ is a point in proximity of the polygon. We calculate $S_{Q_k}$ as the area of the polygon $P$ and $S_{\Delta D_i P_{k_r} P_{k_{r+1}}}$ as the area of triangle $\Delta D_i P_{k_r} P_{k_{r+1}}, r \in \{1, \ldots, p_k\}$.

(a) Point $D_i$ is outside polygon $P_{k_1}, P_{k_2}, P_{k_3}, \ldots, P_{k_{p_k}}$ if and only if:
4.1.2. Shortest path avoiding obstacles. In a Euclidean space-state a shortest path is a direct line between any pair of two nodes, however, in presence of obstacles, construction a shortest path network is not apparent. We aim to create the shortest communicative paths between all pairs of suppliers and demand nodes avoiding the forbidden areas. In many studies, the shortest path graph avoiding all obstacles are created based on explicit connection graph ([2]). This method could computationally harm the shortest path algorithm when the size of the problem grows. In this study we adopt a visibility graph theorem and an implicit method to find shortest path avoiding obstacles. In our methodology, we construct a visibility graph only when an obstacle is in the path between two points. Nodes on the visible network consist of the location points corresponding to suppliers, demand nodes, and all vertices of the obstacles. Edges are the line segments between all pairs of points which do not break through any obstacle. Figure 3 elaborates an example of visibility graph and shortest path based on a Dijkstra algorithm.

\[ S_{Q_k} < \sum_{r=1}^{p_k} S_{\Delta D, p_k}, p_{k+r+1} \]  
\[ (b) \text{Point } D_i \text{ is strictly inside polygon } P_{k_1}, P_{k_2}, P_{k_3}, \ldots, P_{k_{p_k}} \text{ if and only if:} \]

\[ S_{Q_k} = \sum_{r=1}^{p_k} S_{\Delta D, p_k}, p_{k+r+1}, \text{ and } S_{\Delta D, p_k}, p_{k+r+1} > 0, \forall r \in \{1, \ldots, p_k\} \]  
\[ (17) \]

\[ (c) \text{Point } D_i \text{ is on the edge of polygon } P_{k_1}, P_{k_2}, P_{k_3}, \ldots, P_{k_{p_k}} \text{ if and only if:} \]

\[ S_{Q_k} = \sum_{r=1}^{p_k} S_{\Delta D, p_k}, p_{k+r+1}, \text{ and } S_{\Delta D, p_k}, p_{k+r+1} = 0, \exists r \in \{1, \ldots, p_k\} \]  
\[ (18) \]

4.1.3. Allocation strategies. In our computational study we employed two allocation strategies for supplier/customer matrix. In our first approach, (GCOLAP1) suppliers are assigned to customers with a uniformly distributed random variable. Next, amount of allocation is randomly assigned considering distance and priority matrix. The fit value (objective function) for the location-allocation plan is then calculated and attached to the chromosome. In our second approach (GCOLAP2),
we dedicate a “luck” factor to each demand node, dividing the corresponding distance by the priority values. Using a Roulette Wheel Cycle approach, we proceed to assign and allocate suppliers to demand nodes based on their luck factor.

4.2. Crossover operator. We keep population of three chromosome types. Chromosome type one indicates the location of suppliers on the feasible map, chromosome type two dictates the allocation of suppliers to demand nodes, and type three chromosome indicates the shipments. Type one chromosome is depicted as follows. The chromosomes are the matrix of supplier coordinates.

\[
D_{2i} = \begin{bmatrix} x_1 & \cdots & x_m \\ y_1 & \cdots & y_m \end{bmatrix}
\]

For the crossover of type one chromosomes, we select two parents \( V_1 \) and \( V_2 \):

\[
\begin{align*}
V_1 &= [(x_1^1, y_1^1), (x_2^1, y_2^1), \ldots, (x_m^1, y_m^1)] \\
V_2 &= [(x_1^2, y_1^2), (x_2^2, y_2^2), \ldots, (x_m^2, y_m^2)]
\end{align*}
\]

Offspring is obtained by a uniform continuous convex combination function as follows:

\[
\begin{align*}
\bar{x}_i^1 &= a_i x_i^1 + (1 - a_i) x_i^2 \\
\bar{y}_i^1 &= a_i y_i^1 + (1 - a_i) y_i^2 \\
i &\in I
\end{align*}
\]

As a result, the offspring will be:

\[
\bar{v} = [ (\bar{x}_1^1, \bar{y}_1^1), (\bar{x}_2^1, \bar{y}_2^1), \ldots, (\bar{x}_m^1, \bar{y}_m^1) ]
\]

Type two chromosomes are the allocation matrix which identifies allocation pattern of supplier-demand nodes. For example a sample type 2 chromosome \( Z_{ij} \) is as follows.

\[
Z_{ij} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 1 \end{bmatrix}
\]

A one-point crossover operator is used for type two chromosomes. Figure 4 depicts a sample of type two chromosome crossover \( Z_{2,3}^1 \) and \( Z_{2,3}^2 \) are parent chromosomes.

\[
Z_{2,3}^1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad Z_{2,3}^2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}
\]

\[
Z_{2,3}^1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad Z_{2,3}^2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}
\]

**Figure 4.** Type two chromosome crossover

Type three chromosome specify the amount of allocation based on the supplier-demand allocation matrix, customer priority matrix, supplier capacities, and customer demand amounts. A type three chromosome is depicted by the matrix \( d_{ij} \). We used a continuous convex combination function as the crossover operator for type three chromosomes.
\[
d_{ij} = \begin{bmatrix}
d_{11} & d_{12} & \ldots & d_{1n} \\
d_{21} & d_{22} & \ldots & d_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
d_{m1} & d_{m2} & \ldots & d_{mn}
\end{bmatrix}
\]

4.3. **Mutation operator.** We mutate the supplier location, supplier-demand allocation, and amount of allocations. To assure diversity and avoidance from local optima, we found that random allocation amounts lead to higher diversity, better quality solutions, and faster convergence. To keep the diversity of the population at higher levels we incorporated an operator that introduces random members to each generation of crossover, and mutation population, in proportion to the population. Members of the populations are then ranked based on their fitness value.

4.4. **Termination.** We set the termination criterion based on the maximum number of iterations and the percentage fitness improvement of the incumbent solution, which is the member with the best fit at a generation.

5. **Computational study.** In this section we present the computational results of the relaxation of special case reformulation of the model (R-SCOLAP), optimization of the reduced mixed-integer formulation (SCOLAP), and results with two variations of our proposed genetic algorithm (GCOLAP1, GCOLPA2). The models are tested on difficult randomly generated instances. We first present a sample example from our test bed, and later in the section we summarize the results of the proposed algorithms.

5.1. **Numerical example.** Generated instances are comprised of random number of customers and suppliers, location of the customers on the pane, obstacle and the corners of the obstacle region, centroid point of the obstacle, the marginal radius around the obstacle, and customer priorities in terms of emergency in fulfillment importance. A sample from our test bed demonstrates the attributes of our instances. This sample is comprised of 17 demand nodes, and 3 suppliers. There is one obstacle in the location pane with a polygon shape and 5 corner points. No demand point is inside the obstacle. No supplier can be located inside the obstacle, and no connecting path can cross the obstacle. The marginal radius in this sample instance is 15. This enforces that if a supplier is located inside the marginal radius (but outside of the obstacle), the capacity of the supplier will be automatically restricted to a lower quantity. Table 1 summarizes the input variables of the example.

| Parameter                      | Value     |
|-------------------------------|-----------|
| Number of customers           | 17        |
| Number of distribution centers| 3         |
| Number of forbidden areas     | 1         |
| The radius of margin          | 15        |
| The coordinates of margin center | (50,30) |
| Number of corners of each of regions | 5       |

Table 1. Sample instance from randomly generated instances
Figure 5a depicts the network without considering any obstacles. We solved this network with our genetic algorithm. We also solve this example by considering the obstacle. Figure 5b depicts the new network considering the obstacle. The polygon in this figure represents the obstacle, and the circle identifies the marginal radius around the obstacle. As it is depicted in these two graphs, presence of the obstacle dramatically impacts the network structure. In the second allocation network with presence of the obstacle, customer priorities are also considered.

5.2. Assessing performance. In this section we assess the performance of two proposed genetic algorithms for solving COLAP. The convergence of two proposed models GCOLAP1 and GCOLAP2 are presented in Figures 6a-6b. GCOLAP1 converged after about 150 seconds by the objective value of 81.49. GCOLAP2 converged after about 1000 seconds with an amount of 131.92 as the objective function. We also solved the SCOLAP to compare the performance and objective functions. The optimal value of SCOLAP was achieved after 38.7 seconds at the value of 81.38.

In this example GCOLAP1 performs much faster and it also returns a very near optimal objective value with only 0.1% gap from the SCOLAP objective. GCOLAP2 returns an objective with 38.3% gap from SCOLAP. We will test all of the
algorithms on 20 instances from small to large size. The results of the computational study are reported in the following section.

5.3. **Computational results.** We coded the proposed genetic algorithms in Matlab 14 in a 3-core computer system with a 1.73 GHz processor and a 6 GB memory. We also coded the optimization and the relaxation problem (R-SCOLAP and SCOLAP) in AMPL and run on CPLEX optimization solver. The instances start with 2 suppliers and 8 demand nodes, up to 8 suppliers and 82 customers. We solve in instances to find the relaxation value to ensure feasibility. We run the optimization algorithm on the special case SCOLAP where suppliers are located on the demand nodes. We also solve the instances with our proposed genetic algorithms GCOLAP1 and GCOLAP2 on the Euclidean space. Results of the computations are summarized in Table 2.

| Table 2. Summary of computational results on randomly generated instances |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| Instance | m | n | R-SCOLAP | SCOLAP | GCOLAP1 | GCOLAP2 | MIP gap | GA gap |
| 1 | 2 | 8 | 0.0435 | 19.41 | 25.44 | 26.96 | 0 | 0.31 |
| 2 | 2 | 10 | 0.0443 | 30.56 | 36.57 | 34.21 | 0 | 0.2 |
| 3 | 2 | 12 | 0.0424 | 33.26 | 39.18 | 47.41 | 0 | 0.18 |
| 4 | 2 | 15 | 0.0493 | 65.52 | 80.89 | 77.46 | 0 | 0.23 |
| 5 | 3 | 17 | 0.0443 | 65.5 | 87.34 | 105.68 | 0 | 0.33 |
| 6 | 3 | 20 | 0.0542 | 138.12 | 177.07 | 139.72 | 0.00009 | 0.28 |
| 7 | 3 | 25 | 0.0677 | 163.25 | 209.29 | 214.77 | 0.0001 | 0.28 |
| 8 | 3 | 30 | 0.0723 | 273.86 | 329.96 | 277.23 | 0.00254 | 0.2 |
| 9 | 4 | 35 | 0.0738 | - | 249.24 | 501.61 | - | - |
| 10 | 4 | 40 | 0.0881 | - | 387.83 | 500.64 | - | - |
| 11 | 4 | 44 | 0.0733 | - | 618.62 | 645.86 | - | - |
| 12 | 5 | 47 | 0.077 | - | 721.32 | 941.63 | - | - |
| 13 | 5 | 50 | 0.138 | - | 835.72 | 1122.76 | - | - |
| 14 | 5 | 55 | 0.0971 | - | 854.85 | 1025.11 | - | - |
| 15 | 6 | 60 | 0.1343 | - | 1542.82 | 1825.43 | - | - |
| 16 | 6 | 65 | 0.1067 | - | 1382.88 | 1738.93 | - | - |
| 17 | 7 | 70 | 0.1536 | - | 1699.78 | 2836.1 | - | - |
| 18 | 7 | 74 | 0.1206 | - | 2021.15 | 2590.2 | - | - |
| 19 | 8 | 78 | 0.1088 | - | 2145.16 | 3079.13 | - | - |
| 20 | 8 | 82 | 0.1156 | - | 3210.13 | 3450.84 | - | - |

Table 2 summarizes the computational results on a set of randomly generated instances. Small instances start with only 2 suppliers and 8 demand nodes. The largest instance is comprised of 8 suppliers and 82 demand nodes. Column 1 in Table 2 reports the instance number. Columns 2 and 3 report the number of suppliers and number of demand nodes, respectively. Column 4-7 demonstrate the output of each algorithm. R-SCOLAP is the relaxation of the SCOLAP solved by CPLEX. SCOLAP is the output of the CPLEX optimization software within 1 hour of CPU time. As you see CPLEX was unable to return an MIP solution within the time limit for instances 9-20. GCOLAP1 and GCOLAP2 are the outputs of our proposed genetic algorithms. Column 8 in Table 2 reports the MIP gap reported by the CPLEX solver, and finally column 9 in this table demonstrates the relative gap between GCOLAP1 and SCOLAP.

Our proposed genetic algorithms were able to successfully return a solution for all of the instances. The optimization solver was unable to successfully solve the large instances. GCOLAP1 constantly produced better quality solutions than GCOLAP2. We conclude that the convergence of the algorithm on our test bed with
using the unguided mutation operator was overall more effective than the administered approach. GCOLAP1 constantly produce near optimal solutions, with an average gap of about 25 percent from the optimal solution. This average gap is based on the first 9 instances where the optimization solver was able to return an objective value within the time limit. The MIP gap reported in Table 2 shows that the objective value reported by the solver was indeed the optimal solution.

6. Conclusion and future research. This research addresses the problem of obstacle location-allocation problems in presence of customer priorities. We propose a priority matrix where supplier/customer allocation has a weighting factor. This weight impacts the network structure and proximity of suppliers to demand nodes. We also consider location dependent capacities for suppliers, where proximity of the suppliers to the obstacles would negatively impact the capacity of the suppliers. We present a MIP formulation of the problem in the continuous space and formulate a special case of the problem as a set covering program where only the demand nodes are candidate points of supplier locations. We propose an efficient genetic algorithm and solve two variations of the meta-heuristic on a set of randomly generated instances. The optimization solver was unable to successfully solve all of the instances. The proposed meta-heuristic continuously was able to converge within a manageable computational effort on all small and large instances in our test bed.

For future research, we propose exploration on decomposition algorithms to solve the set covering reformulation of COLAP presented in this paper. Further investigation of meta-heuristic algorithms such as ant colony is proposed to be tested on the obstacle location problem with customer priorities.

REFERENCES

[1] A. Ahmadi-Javid, P. Seyedi and S. S. Syam, A survey of healthcare facility location, Computers and Operations Research, 79 (2017), 223–263.
[2] H. Alt and E. Welzl, Visibility graphs and obstacle-avoiding shortest paths, Mathematical Methods of Operations Research, 32 (1988), 145–164.
[3] Y. P. Aneja and M. Parlar, Algorithms for Weber facility location in the presence of forbidden regions and/or barriers to travel, Transportation Science, 28 (1994), 70–76.
[4] A. B. Arabani and R. Z. Farahani, Facility location dynamics: An overview of classifications and applications, Computers and Industrial Engineering, 62 (2012), 408–420.
[5] J. Brimberg, P. Hansen, N. Mladenovic and E. D. Taillard, Improvements and comparison of heuristics for solving the uncapacitated multisource Weber problem, Operations Research, 48 (2000), 444–460.
[6] S. E. Butt and T. M. Cavalier, An efficient algorithm for facility location in the presence of forbidden regions, European Journal of Operational Research, 90 (1996), 56–70.
[7] E. Durmaz, N. Aras and I. K. Altunel, Discrete approximation heuristics for the capacitated continuous location-allocation problem with probabilistic customer locations, Computers and Operations Research, 36 (2009), 2139–2148.
[8] D. Gong, M. Gen, W. Xu and G. Yamazaki, Hybrid evolutionary method for obstacle location-allocation problem, Computers and Industrial Engineering, 29 (1995), 525–530.
[9] D. J. Gong, M. Gen, G. Yamazaki and W. X. Xu, Hybrid evolutionary method for capacitated location-allocation problem, Computers and Industrial Engineering, 33 (1997), 577–580.
[10] S. C. Ho, An iterated tabu search heuristic for the single source capacitated facility location problem, Applied Soft Computing, 27 (2015), 169–178.
[11] C. R. Houck, J. A. Joines and M. G. Kay, Comparison of genetic algorithms, random restart and two-opt switching for solving large location-allocation problems, Computers and Operations Research, 23 (1996), 587–596.
[12] J. H. Jaramillo, J. Bhadury and R. Batta, On the use of genetic algorithms to solve location problems, Computers and Operations Research, 29 (2002), 761–779.
[13] J. Kalcsics, S. Nickel, M. A. Pozo, J. Puerto and A. M. Rodríguez-Chía, The multicriteria $p$-facility median location problem on networks, *European Journal of Operational Research*, 235 (2014), 484–493.

[14] I. N. Katz and L. Cooper, Facility location in the presence of forbidden regions: I. Formulation and the case of Euclidean distance with one forbidden circle, *European Journal of Operational Research*, 6 (1981), 166–173.

[15] K. Klamroth, A reduction result for location problems with polyhedral barriers, *European Journal of Operational Research*, 130 (2001), 486–497.

[16] J. Krarup and P. M. Pruzan, The simple plant location problem: Survey and synthesis, *European Journal of Operational Research*, 13 (1983), 36–81.

[17] R. E. Kuenne and R. M. Soland, Exact and approximate solutions to the multisource Weber problem, *Mathematical Programming*, 3 (1972), 193–209.

[18] G. Laporte, F. V. Louveaux and L. van Hamme, Exact solution to a location problem with stochastic demands, *Transportation Science*, 28 (1994), 95–103.

[19] G. Laporte, S. Nickel and F. S. da Gama, *Location Science*, Springer, Berlin, 2015.

[20] R. C. Larson and G. Sadiq, Facility locations with the Manhattan metric in the presence of barriers to travel, *Operations Research*, 31 (1983), 652–669.

[21] B. Li, I. Hernandez, A. B. Milburn and J. E. Ramirez-Marquez, Integrating uncertain user-generated demand data when locating facilities for disaster response commodity distribution, *Socio-Economic Planning Sciences*, 62 (2018), 84–103.

[22] R. Logendran and M. P. Terrell, Uncapacitated plant location-allocation problems with price sensitive stochastic demands, *Computers and Operations Research*, 15 (1988), 189–198.

[23] R. G. McGarvey and T. M. Cavalier, A global optimal approach to facility location in the presence of forbidden regions, *Computers and Industrial Engineering*, 45 (2003), 1–15.

[24] M. T. Melo, S. Nickel and F. Saldanha-da-Gama, Facility location and supply chain management – a review, *European journal of operational research*, 196 (2009), 401–412.

[25] S. M. Mousavi, S. T. A. Niaki, E. Mehdizadeh and M. R. Tavarroth, The capacitated multifacility location-allocation problem with probabilistic customer location and demand: Two hybrid metaheuristics algorithms, *International Journal of Systems Science*, 44 (2013), 1897–1912.

[26] J. A. Paul and R. Batta, Models for hospital location and capacity allocation for an area prone to natural disasters, *International Journal of Operational Research*, 3 (2008), 473–496.

[27] F. Pérez-Galarce, L. J. Canales, C. Vergara and A. Candia-Véjar, An optimization model for the location of disaster refuges, *Socio-Economic Planning Sciences*, 59 (2017), 56–66.

[28] C. S. ReVelle and H. A. Eiselt, Location analysis: A synthesis and survey, *European Journal of Operational Research*, 165 (2005), 1–19.

[29] C. S. ReVelle, H. A. Eiselt and M. S. Daskin, A bibliography for some fundamental problem categories in discrete location science, *European Journal of Operational Research*, 184 (2008), 817–848.

[30] S. Salhi and M. D. H. Gamal, A genetic algorithm based approach for the uncapacitated continuous location-allocation problem, *Annals of Operations Research*, 123 (2003), 203–222.

[31] T. Santos, S. Ahmed, M. Goetschalckx and A. Shapiro, A stochastic programming approach for supply chain network design under uncertainty, *European Journal of Operational Research*, 167 (2005), 96–115.

[32] A. Schöbel, *Location of Dimensional Facilities in a Continuous Space*, in: Laporte G., Nickel S., Saldanha da Gama F. (eds), Location Science, Berlin: Springer, 2015.

[33] S. R. Shariﬀ, N. H. Moin and M. Omar, Location allocation modeling for healthcare facility planning in Malaysia, *Computers and Industrial Engineering*, 62 (2012), 1000–1010.

[34] H. D. Sherali, T. B. Carter and A. G. Hobeika, A location-allocation model and algorithm for evacuation planning under hurricane/flood conditions, *Transportation Research Part B: Methodological*, 25 (1991), 439–452.

[35] Z. Stanimirović, A genetic algorithm approach for the capacitated single allocation $p$-hub median problem, *Computing and Informatics*, 29 (2012), 117–132.

[36] J. Taniguchi, X. Wang, M. Gen and T. Yokota, Hybrid genetic algorithm with fuzzy logic controller for obstacle location-allocation problem, *IEEE Transactions on Electronics, Information and Systems*, 124 (2004), 2027–2033.

[37] A. Verma and G. M. Gaukler, Pre-positioning disaster response facilities at safe locations: An evaluation of deterministic and stochastic modeling approaches, *Computers and Operations Research*, 62 (2015), 197–209.
[38] N. Vidyarthi and S. Jayaswal, Efficient solution of a class of locationallocation problems with stochastic demand and congestion, *Computers and Operations Research*, 48 (2014), 20–30.

[39] J. Zhou and B. D. Liu, New stochastic models for capacitated location-allocation problem, *Computers and Industrial Engineering*, 45 (2003), 111–125.

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