Dynamic Scaling at the Zero-field 2D Superconducting Transition

S. M. Ammirata,1 Mark Friesen,2 Stephen W. Pierson,3 LeRoy A. Gorham,3 Jeffrey C. Hunnicutt,3 M. L. Trawick,1 and C. D. Keener1

1Department of Physics, Ohio State University, Columbus, Ohio 43210
2Physics Department, Purdue University, West Lafayette, IN 47907-1396
3Department of Physics, Worcester Polytechnic Institute (WPI), Worcester, MA 01609-2280

(March 24, 2022)

Zero-field current-voltage (I-V) characteristics of a thin (“two-dimensional”) Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ crystal are reported and analyzed in two ways. The “conventional” approach yields ambiguous results while a dynamical scaling analysis offers new insights into the Kosterlitz-Thouless-Berezinskii transition. The scaling theory predicts that the universal jump of the I-V exponent $\alpha$ should be between $z+1$ and $1$. A value of $z \approx 5.6$ is obtained for the dynamical critical exponent, and is corroborated by data from other 2D superconductors. A simple dynamical model is presented to account for the results.

Perhaps no transition is better known than the Kosterlitz-Thouless-Berezinskii (KTB) transition, which occurs in systems in the universality class of the two-dimensional (2D) XY model and which is marked by the unbinding of topological excitations known as vortex pairs. Since the original theory was derived in the early 1970’s, the KTB transition has been mentioned in systems as diverse as superfluids, superconductors, 2D lattices, ferromagnets, and liquid crystals.

Evidence for Kosterlitz-Thouless-Berezinskii (KTB) critical behavior in two-dimensional superconductors in zero-field was first reported in 1981 and has since been widely studied in thin films, 2D Josephson Junction arrays, and more recently in high-temperature superconductor (HTSC) ultrathin films. Evidence for KTB behavior (but not necessarily a transition) also abounds for multilayers and HTSC single crystals.

Two main approaches can be used to investigate electronic transport phenomena in superconductors near the KTB transition. In the first, more “conventional” approach, there are two principal signatures of KTB behavior. First, the ohmic resistance should have a unique temperature dependence above the superconducting transition temperature, $T_{KT B}$: $R(T) \propto \exp[-2\sqrt{b/(T/T_{KT B} - 1)}]$, where $b$ is a non-universal constant. Second, current-voltage (I-V) isotherms should be described by a power law, $V \propto I^\alpha$, at low currents. Below $T_{KT B}$, $\alpha$ decreases linearly with increasing temperature, making a “universal jump” from three to one at $T = T_{KT B}$. Then remains ohmic ($\alpha = 1$) for all $T > T_{KT B}$.

There are several difficulties with this approach. For example, the jump in $\alpha$ is strictly defined only in the $I \rightarrow 0$ limit, making it difficult to detect experimentally. Indeed, many of the original papers do not observe or report universal jumps. Furthermore, it cannot address the eventual upturn of ohmic isotherms, observed near $T_{KT B}$. (See Fig. 1 below, and Refs. 1, 4, 5, 13.) Similar behavior is also observed in finite field near the superconducting transition in bulk superconductors, where it is attributed to the competition between thermal and current-induced effects. In the second approach, in which one applies the techniques developed for such bulk systems, it is possible to resolve both of these issues.

The second approach treats both low and high current behaviors, via dynamic scaling. The main ideas were first discussed for 3D XY and glassy critical phenomena in bulk superconductors. In this analysis, dynamic phenomena near the critical point involves a correlation time scale $\tau \propto \xi^z$, which like the correlation length $\xi$, diverges at $T_{KT B}$. ($z$ is the dynamic critical exponent.) The scaling approach has two distinct advantages over the conventional theory. First, it extends the transport theory to finite currents, and is able to describe the observed upturn of ohmic isotherms. Second, it admits dynamics which are non-diffusive. This issue has recently received much attention in bulk HTSC’s near the 3D XY critical point, where unexpected (non-diffusive) dynamics have been encountered.

For 2D superconductors, the I-V curves should scale as $V = I\xi^{-\alpha}(\xi/T)$, where $\chi_{\pm}(x)$ is the scaling function for temperatures above (below) $T_{KT B}$. The asymptotic behaviors of $\chi_{\pm}(x)$ can be deduced from the conventional KTB theory: $\lim_{x \rightarrow 0} \chi_+(x) = \text{const.}$ (ohmic limit); $\lim_{x \rightarrow 0} \chi_-(x) \propto \exp[-a \ln(1/x)]$ (thermally activated vortex pair unbinding); $\lim_{x \rightarrow \infty} \chi_{\pm}(x) \propto x^z$ (critical isotherm). The universal jump is expressed as the difference in (log-log) slopes between the two asymptotic limits of $\chi_{\pm}(x)$.
to 2D superconductors. In the former, the dynamic universality class was not explicitly studied while in the latter, the issue was treated only peripherally.

In this Letter, we apply both approaches to $I$-$V$ curves from a thin crystal of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ (BSCCO). It is found that the conventional approach gives rather ambiguous results. However, by applying dynamical scaling, with $z$ as a fitting parameter, the data can be made to collapse, obtaining $z \approx 5.6 \pm 0.3$ (not $z = 2$). To check the universality of our results, we also apply the scaling analysis to other $I$-$V$ data sets obtained from the literature, including both HTSC and conventional 2D superconductors, with an average result of $z \approx 5.7$.

BSCCO crystals were prepared using a standard self flux method. A single crystal was selected and cleaved using a Scotch tape technique. Thin 1500 Å crystals were obtained with atomically smooth surface areas of at least $(50 \text{ micron})^2$. Thicknesses were measured using an atomic force microscope. Four silver pads were photolithographically patterned on top of the crystal. The crystals were postannealed at 600°C in 1.5 scfh flowing $\text{O}_2$ for 45 minutes to achieve optimum oxygenation. Gold wires were ultrasonically bonded to the silver pads where they extended beyond the crystal.

$R(T)$ and $I$-$V$ characteristics were measured using a standard four probe inline geometry and a low frequency lock-in technique (16.9 Hz). A temperature controller was used to insure a temperature stability of better than 10 mK. Although current was injected only through the top crystal surface, the thickness of the sample assured uniform current distribution. Data was taken on three different samples. Data presented here is from a crystal of dimensions 0.2mm x 0.5mm x (1500 ± 500 Å). For thicker samples, KTB behavior could not be observed.

Fig. 1 shows zero field ($H \lesssim 100$ mGauss) $I$-$V$ isotherms, corresponding to 100 mK steps, from $T = 80.1$ K (left side) to 76.9 K (right side). The curves display features typical of 2D samples. The high temperature ($> T_{KTB}$) isotherms exhibit positive curvature as they cross over from ohmic to non-ohmic behavior, while the low temperature ($< T_{KTB}$) isotherms exhibit negative curvature. The critical isotherm separates the two regions at $T = T_{KTB}$, and describes a strict power law: $V \propto I^{z+1}$. In Fig. 1 and for other published 2D isotherms (e.g., Refs. 14, 16), identification of the critical isotherm shows that $z$ is clearly larger than 2.

We first perform a conventional analysis. The ohmic resistance data $R(T)$ is fit to the KTB prediction, with apparent success, obtaining $T_{KTB} = 78.87$ K and $b = 1.3$. The $I$-$V$ exponent $\alpha(T)$ is then determined by fitting $V \propto I^{\alpha}$ to the lower portion (2 to 10 nV) of each isotherm. In Fig. 1, high temperature (low current) ohmic behavior, expected to persist for all $T > T_{KTB}$, is obscured by noise already for $T < 79.7$ K. Such experimental limitations lead to significant errors in $\alpha(T)$ near and below $T_{KTB}$, thus smearing out the universal jump until it becomes unrecognizable, as in Fig. 1 (inset). These same difficulties affect $R(T)$ measurements, reducing the accuracy of the obtained fitting parameters.

$$I^{z+1}/[V T^z] = \varepsilon_\pm(I^z \xi^z/T^z)$$

where $\varepsilon_\pm(x) \equiv x/\chi_\pm(x^{1/z})$. Note that above $T_{KTB}$, the scaling variable can also be written as $x = I^z/R(T)T^z$. To proceed with the analysis, $\xi(T)$ must be specified. We will assume that the KTB form, $\xi(T) \propto \exp[\sqrt{b/(T/T_{KTB} - 1)}]$, provides the most efficient parameterization of the correlation length. We further assume that $\xi(T)$ is symmetric about $T = T_{KTB}$ (modulo some prefactor). The fitting parameters for Eq. 1 then become $z$ (universal), and $T_{KTB}$ and $b$ (nonuniversal). The following three requirements must be fulfilled self-consistently in our scaling procedure: (i) $V \propto I^{z+1}$, along the critical isotherm $T = T_{KTB}$; (ii) $R(T) \propto \xi^{-z}$, in the high temperature range ($\gtrsim 79.7$ K), where ohmic $R(T)$ can be obtained; and (iii) scaling collapse of the $I$-$V$ isotherms, according to Eq. 1. The first two points place tight constraints on the fitting parameters. Within these constraints, the parameters are tuned to satisfy the final point.

The outcome of such a fitting procedure for the data of Fig. 1 gives the following results: $T_{KTB} = 78.87$K, $b = 0.57$, and $z = 5.6 \pm 0.3$. The scaling collapse, shown in Fig. 1, is convincing in both the upper ($T < T_{KTB}$) and lower ($T > T_{KTB}$) branches. For many isotherms in Fig. 1, the noisy (low current) portions of the curves do not scale; this may be due to electromagnetic screening (finite size effects). However the same difficulty is not
incurred for the other data analyzed below. (See Fig. 3.)

The caution expressed earlier for the low temperature branch \[21\] does not seem to affect the scaling collapse. The inset of Fig. 2 shows the results of a similar two-parameter fit to the data, obtained by setting \( z = 2 \). The results are extremely poor, indicating that \( z > 2 \).

![Diagram](image)

**FIG. 2.** The \( I-V \) curves of Fig. 1 scaled with Eq. (1). [INSET: The same scaling for \( z = 2 \).]

To check our results, we have applied the same scaling procedure to other data from the literature. The first data set \[16\] is from a YBCO/PrBa\(_2\)Cu\(_4\)O\(_{7-\delta}\) multilayer in which the YBCO layers have a thickness of 24 \( \AA \) and are electrically isolated from one another by PrBa\(_2\)Cu\(_4\)O\(_{7-\delta}\) barrier layers. The scaling procedure now leads to the results \( T_{KTB} = 32.0 \) K, \( b = 39.3 \), and \( z = 5.6 \pm 0.3 \), with the \( I-V \) collapse shown in Fig. 2. Analysis of \( I-V \) data from a 12 A YBCO monolayer gives \( T_{KTB} = 18.5 \) K, \( b = 22.29 \), and \( z = 5.8 \pm 0.3 \) and also collapses the data. (See Fig. 3.) The final data set corresponds to a conventional 2D superconducting sample \[9\] (Hg-Xe alloy) and collapses for \( T_{KTB} = 3.04 \) K, \( b = 9.64 \), and \( z = 5.6 \pm 0.3 \) as shown in Fig. 3. The apparent agreement between these four diverse samples suggests that the mean result, \( z \approx 5.7 \), may indeed be universal for superconductors. However, it was not possible to probe the universality of the scaling functions, because of the non-universal parameter \( b \) appearing in \( \xi \).

The allure of the KTB transition lies in the simplicity and integrity of the pair-dissociation paradigm. It is therefore unsettling that the corresponding dynamic paradigm could fail, as signaled above through the observation of \( z \gtrsim 2 \). To better understand this situation, it is helpful to reconcile the intuitive (“conventional”) picture with the scaling description. The conventional paradigm intrinsically involves the presence of dissociated or “free” vortices, with density \( n_f \). In a small but finite current, free vortices are independently driven across the sample, inducing dissipation consistent with the Bardeen-Stephen theory: \( R \propto n_f \). In larger currents, non-ohmic effects may arise from steady state dissociation-recombination processes, although dissipation is still understood in terms of driven, non-interacting vortices. To make connection with the scaling approach, we compare the dimensional statement \( n_f \propto \xi^{-2} \), which cannot be affected by dynamical processes, with the 2D ansatz \[10\] \( R \propto \xi^{-z} \). This leads immediately to the relation \( R \propto n_f^{-2/z} \), which reproduces the conventional relation when \( z = 2 \). Using this relationship in the derivation of the conventional signatures, one finds \( R(T) \propto \text{exp}[-z b/(T/T_{KTB} - 1)] \) and that \( \alpha \) jumps from \( z + 1 \) to \( 1 \) at \( T_{KTB} \), consistent with the dynamic scaling results.

![Diagram](image)

**FIG. 3.** The \( I-V \) curves of (a) Ref. \[16\] (b) Ref. \[15\] and (c) Ref. \[9\] scaled with Eq. (1). Data sets (b) and (c) have been shifted arbitrarily.

The description in terms of a general \( z \) now allows for other more sophisticated dissipation phenomena. In particular, the observed value of \( z \approx 5.7 \) cannot be mistaken to be diffusive, and suggests transport processes which are highly collaborative in nature. We speculate that for superconductors, driven vortices may never be “free” in the conventional sense. Rather, the dominant transport mechanism may involve **collaborative** dissociation, in which pairs of vortices can unbind only by exchanging members with neighboring pairs. In contrast with the conventional picture, which involves only the two members of the dissociating vortex pair, the collaborative dissociation scenario involves four “mobile” (as opposed to “free”) vortices: the dissociating pair, and the two neighboring vortices with which they recombine. Assuming a steady state recombination process analogous to the conventional one, \[8\] we now obtain \( n_f \sim n_f/\tau \propto n_f^2 \), or \( \tau \propto \xi^6 \), with a dynamic exponent of 6 rather than 2. We emphasize that the relatively large observed value of \( z \) should not be attributed to pinning effects, as the phenomenon is observed in a variety of superconductors and...
must be regarded as intrinsic.

Finally, we contrast the conclusions of this work with Ref. [13], where it was suggested that the upturn of isotherms near $T_{KT B}$, from ohmic to non-ohmic behavior, may be attributed to electromagnetic screening, which produces free vortices even below $T_{KT B}$. Such effects, which result from the 2D penetration depth becoming smaller than the sample dimensions, should occur in some samples of finite thickness, ultimately destroying the KTB transition. However, in analogy with bulk superconductors, the disputed behavior may instead reflect only the non-ohmic behavior associated with finite currents. In this scenario, the upturning ohmic curves would correspond to $T > T_{KT B}$, and the KTB transition would remain intact. The non-ohmic behavior can be understood as follows. On either side of the transition, external currents tend to dissociate large vortex pairs through (highly collaborative) activation processes. For $T > T_{KT B}$, many large pairs are already unbound, so that small currents, which tend to dissociate large pairs, have little effect. However, beyond some characteristic current scale ($J_0 \sim T/\xi$), current-induced dissociation becomes the dominant dissipation mechanism, producing non-ohmic effects. It is worth noting that the disputed I-V isotherms of Ref. [13] are found to scale nicely with the techniques used here. (See Fig. 3b.)

In summary, a conventional analysis of electronic transport data is contrasted with a dynamical scaling analysis for several types of 2D superconductors. The latter approach is designed to include non-ohmic effects associated with finite currents. The conventional analysis obtains inconclusive results while the dynamical scaling analysis yields a collapse of the I-V data. The results show that dynamical critical behavior in 2D superconductors may be more interesting than previously anticipated. The scaling generalization of the universal jump of the I-V exponent $\alpha$ is from a value of $z + 1$ to 1. Our analysis gives a dynamical exponent of $z \simeq 5.7$, rather than the expected value of $z = 2$, while a simple dynamical model predicts $z \simeq 6$. Our results may permit more direct to superconductors, where previous reports of anomalous vortex diffusion have been made, rather than to pure XY models, where there is no indication of non-diffusive dynamics. It remains to be seen how the present results are reflected in 2D Josephson junction arrays and granular films.

The authors gratefully acknowledge conversations with S. E. Hebboul, O. T. Valls, B. I. Halperin, J. C. Garland, S. M. Girvin, M. Wallin, H. Gould, P. Muzikar, and C. J. Lobb. This work was supported by the Midwest Superconductivity Consortium through D.O.E. Contract No. DE-FG02-90ER45427 and by NSF Grant No. DMR 95-01272. Acknowledgement is made to the donors of The Petroleum Research Fund, administered by the ACS, for support of this research.

References

[1] J. M. Kosterlitz, J. Phys. C 7, 1046 (1974); J. M. Kosterlitz and D. J. Thouless, J. Phys. C 6, 1181 (1973); V. L. Berezinskii, Sov. Phys. JETP 32, 493 (1971).
[2] D. J. Bishop et al., Phys. Rev. Lett. 40, 1727 (1978).
[3] B. I. Halperin and D. R. Nelson, J. Low Temp. Phys. 36, 599 (1979).
[4] B. A. Huberman et al., Phys. Rev. Lett. 43, 950 (1979).
[5] F. G. Mertens et al., Phys. Rev. Lett. 59, 117 (1987).
[6] R. Geer et al., Phys. Rev. Lett. 63, 540 (1989).
[7] K. Epstein et al., Phys. Rev. Lett. 47, 534 (1981).
[8] A. F. Hebard et al., Phys. Rev. Lett. 50, 1603 (1983).
[9] A. M. Kadin et al., Phys. Rev. B 27, 6991 (1983).
[10] D. J. Resnick et al., Phys. Rev. Lett. 47, 1542 (1981).
[11] D. P. Norton et al., Phys. Rev. B 48, 6460 (1993).
[12] See e.g., D. H. Kim et al., Phys. Rev. B 40, 8834 (1989); S. N. Artemenko et al., JETP Lett. 49, 654 (1989); N.-C. Yeh and C. C. Tsuei, Phys. Rev. B 39, 9708 (1989).
[13] P. Minnhagen, Rev. Mod. Phys. 59, 1001 (1987).
[14] D. R. Nelson et al., Phys. Rev. Lett. 39, 1201 (1977).
[15] J. M. Repaci et al., Phys. Rev. B 54, R9674 (1996).
[16] S. Vadlamannati et al., Phys. Rev. B 44, 7094 (1991).
[17] See, e.g., Fig. 29 of G. Blatter et al., Rev. Mod. Phys. 66, 1162 (1994).
[18] D. S. Fisher et al., Phys. Rev. B 43, 130 (1991).
[19] J. C. Booth et al., Phys. Rev. Lett. 77, 4438 (1996); K. Moloni et al., Phys. Rev. Lett. 78, 3173 (1997), and to be published; D. Ginsberg et al., to be published. Related 3D Monte Carlo results include K. H. Lee and D. Stroud, Phys. Rev. B 46, 5699 (1992); H. Weber and H. J. Jensen, Phys. Rev. Lett. 78, 2620 (1997); J. Lidmar et al., (unpublished). We are not aware of any discussion of non-diffusive dynamics in 2D superconductors.
[20] S. A. Wolf et al., Phys. Rev. Lett. 42, 324 (1979).

In conventional theory, an additional temperature dependence enters the $T < T_{KT B}$ asymptotic form, which cannot be accommodated by standard scaling theory.

[21] L. Miu et al., Phys. Rev. B 52, 4553 (1995).
[22] M. L. Trawick et al., J. Low Temp. Phys. 105, (1996).
[23] C. D. Keener et al., submitted to Physica C (1997).

One advantage of this scaling form over the original is that it does not stretch out the $q$ scale, only the $z$ scale. In addition, the asymptotic behaviors become very simple.

Such symmetry has not been verified theoretically. One could introduce asymmetry about $T_{KT B}$ through the nonuniversal parameter $b \rightarrow b_\pm$, as seems most likely. However, this procedure increases the number of fitting parameters, and is not attempted here. The assumed symmetry prohibits using the Minnhagen form of the resistance.

[27] R. Théron et al., Phys. Rev. Lett. 71, 1246 (1993).