Reliability Analysis Deflector of Underwater Wall Detection Device Based on Finite Element and Response Surface Methodology

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Abstract. Before the diversion plugging in hydropower project, underwater wall detection device can complete the probe of visual detection and obstacle clearing in high-velocity flow. Deflector which can provide a stronger compress diversion tunnel sluice gate power is the most important part in underwater wall detection device, but the mechanical and uncertainty analysis aren’t enough. In this paper, Finite Element Method and Response Surface Methodology are used to analyze strength and deformation of the deflector, meanwhile, the limit state functions about the maximal bending stress and maximal deformation are developed. Then, though comparing the data with experimental data and the value computed by user formula, the error is less than 2% and the error is allowable in engineering design. Finally, reliability analysis the deflector is achieved with the First-Order Reliability Method. In a word, from the calculating results and engineering operation, we know that the propose method in this paper using finite element method and response surface methodology to develop the function related to the limit functions of deflector under uncertainty is practical and feasible.

Key Words: Deflector, Limit state function, Uncertainty, Finite element method, Response surface methodology

1. Introduction

In the process of hydropower project construction, the diversion tunnel sluice gates (see in Fig. 1) are seriously damaged due to long term erosion by high velocity silt laden flows. Before the diversion plugging, it is necessary to detect and understand the situation that steel water erosion and warpage of the diversion tunnel sluice gates. The long way would be to trial test by simple gate frame or underwater detect by frogman, but these methods have merits as well as their limitations and demerits. For example, though trial test by simple gate frame can detect obstacles, and we can’t get the enough information that the steel water erosion and warpage. And the same time, the flow in the diversion...
tunnel of the hydropower station is very high, profound depth and poor visibility due to water turbidity, frogman touch is unable to obtain the accurate data of diversion tunnel sluice gates and the frogman faces danger each time under turbulent flow conditions.

Figure 1. Diversion tunnel of hydropower project

With the demand of detecting and cleaning for diversion tunnel sluice gates, underwater wall detection device represents the direction of underwater vehicle has gained rapid development. Underwater wall detection device which can carry sonar detection instrument, underwater television and cleaning device, in order to complete the probe of visual detection and obstacle clearing in high-velocity flow with convenient use and highly reliable. This device is mainly used for corrosion detecting and checking shoal materials in sluice gates, to ensure that the gates can run reliability during the diversion plugging, and then successfully close sluice gates.

At the same time, uncertainties are ubiquitous in any stage of underwater wall detection device development, and they are considerable and cannot be controlled. Uncertainties impact the underwater wall detection performance significantly, and a small variation of system inputs may cause an extreme quality loss, even a catastrophic failure. They may come from many aspects, such as environment, manufacturing, incomplete knowledge and so on. Uncertainties can be classified in two general types: aleatory (stochastic or random) uncertainty and epistemic (subjective) uncertainty [1-2]. Aleatory uncertainty is related to inherent variability and is efficiently modeled using probability theory. Epistemic uncertainty describes subjectivity, ignorance or lack of information and it can be reduced with increased state of knowledge or collection of more data. They may be from many aspects, such as environment, manufacturing tolerance, or incomplete knowledge. Therefore, it is important to quantify and analyze the uncertainty during the design procedure of underwater wall detection device.

The rest of this paper is organized as follows. Section 2 provides the general layout of underwater wall detection device. The analyze strength and deformation of deflector use Finite Element Method in Section 3. The develop limit state functions of deflector use Response Surface Methodology in Section 4. Section 5 presents the reliability analysis of the deflector. Some conclusions are given in Section 6.

2. General layout of underwater wall detection device

Underwater wall detection device which is used for detecting and cleaning of diversion tunnel sluice gates hydropower project consists several independent parts, shown in Fig. 2.
In Fig. 2, the major structure can carry powerful magnet, steel wheel, guide plate, seal motor, cleaning device, guide plate, main control unit, underwater camera and color sonar, etc. The powerful magnet and steel wheel enable the underwater wall detection close to the diversion tunnel sluice wall at a certain distance. The seal motor and cleaning device are used for cleaning shoal materials in sluice gates. The underwater camera and color sonar can detect in the underwater environment for hours on end. The underwater part is connected by wirerope and armour cable to the surface control part.

In underwater wall detection device, the most important part is deflector, which can provide a stronger drive power that can compress the underwater wall detection device to diversion tunnel sluice gate. So far, there is limited work related to mechanical analysis of the deflector. Uncertainties are ubiquitous in any stage of deflector development, and they are considerable and cannot be controlled. Uncertainties impact the deflector performance significantly, and a small variation of system inputs may cause an extreme quality loss, even a catastrophic failure. Therefore, it is important to quantify and analyze the uncertainty during the design procedure of deflector.

In this paper, Finite Element Method (FEM) and Response Surface Methodology (RSM) is used to analyze strength and deformation of deflector and the limit state functions related to the maximal bending stress and maximal deformation of deflector are developed [3]. Based on them, reliability analysis of the deflector is achieved with the First-Order Reliability Method [4-5].

3. Stress and strain analysis of deflector
This thesis regards deflector of the underwater wall detection device as the research object, and the underwater part inside in groove wall of diversion tunnel sluice gate is given in Fig. 3.
**Figure 3.** The underwater part inside in the groove wall model
On the major structure, there are four deflectors. The high-velocity flow generates force on the deflectors is shown in Fig. 4.

![Underwater wall detection device](image)

**Figure 4.** Stress of the deflectors
As shown in Fig. 4, on the overhead view of the deflectors, the mathematical formulation to get $F_x$ and $F_y$ are given

$$
\begin{align*}
F_x &= \sum F_{x_{f-j}} + \sum F_{x_{b-j}} = \frac{1}{2} \rho \cdot (\sum s) \cdot v^2 \left[ c_f \cdot (\sin \theta_f)^3 + c_b \cdot (\sin \theta_b)^3 \right] \\
F_y &= \sum F_{y_{f-j}} + \sum F_{y_{b-j}} = \frac{1}{2} \rho \cdot c \cdot (\sum s) \cdot v^2 \left[ c_f \cdot (\sin \theta_f)^2 \cdot \cos \theta_f + c_b \cdot (\sin \theta_b)^2 \cdot \cos \theta_b \right] \\
s_j &= s_j \\
c_x &= c_f \cdot [0.3 \cdot \sin(\theta_b - \theta_a) + 0.7]
\end{align*}
$$

In Eq. (3.1), $F_x$ is horizontal component of the pressure of high-velocity flow on deflectors, $F_y$ is vertical component of the pressure of high-velocity flow on deflectors, $\rho$ is water density, $s$ area of the deflectors, $c$ is drag coefficient of water, $\theta$ is angle adjustment between deflector and major structure, $v$ is flow velocity.

When, $\rho=1000\text{kg/m}^3$, $s=4\times0.62\text{mx}0.31\text{m}$, $c=1.2$, $\theta_b-\theta\geq0^\circ$, $\theta_b-\theta\leq15^\circ$, $\theta\leq60^\circ$, $v=18\text{kn}$ (9.3m/s), the curves $F_x$, $F_y$ with $\theta_b$ and $\theta_f$ are given in Fig. 5. And the same time, the curves $F_x$, $F_y$ with $\theta_f$ when the maximum value of $F_y-F_x$ are shown in Fig. 6.

![Plot of $F_x$, $F_y$ with $\theta_b$ and $\theta_f$](image)
Figure 6. Plot of $F_x$, $F_y$ with $\theta_l$ (when $F_y - F_x$ = maximum value)

From Fig. 5 and Fig. 6, we can find that the value of $F_y - F_x$ is reaching the maximum ($F_y$ = 14092N, $F_x$ = 7922N) when $\theta_l$ is 28.5º, $\theta_b$ is 30.4º.

In this paper, we design the deflector that the material using Q235 ($\sigma_s$=225MPa, $\sigma_b$=225MPa) and the three basic sizes is $0.62m(b) \times 0.31m(a) \times 0.015m(h)$. In order to have sufficient strength and stiffness under the impact of the high-velocity flow, we used two stiffeners to enhance the structure on the back of deflector (see Fig. 7). Then, Finite Element Method is used to analyze the strength and deformation of deflector when $x_1$=100mm, $x_2$=60mm, $x_3$=5mm (see Fig. 8 and Fig. 9).

Figure 7. Structure of the deflector

Figure 8. Stress analysis with FEM
Figure 9. Deformation analysis with FEM

Form Fig. 8 and Fig. 9, we can find that the maximal bending stress of the deflector (σ=212.71MPa) appeared at the hinge and the maximal deformation of the deflector (δ=1.1434mm) appeared at the middle of the main structure. After several cycles of analysis, the calculation model and the actual structure will arrive at the basic line, and also the accuracy of relative theories are confirmed basically.

4. Develop limit state function of deflector

Form Fig. 7, the dependent variable of the develop limit state function is σ that is maximal bending stress or δ that is maximal deformation of deflector, and the vector of independent variables is \( X = [x_1, x_2, x_3] \) which \( x_1 \in [70\text{mm}, 110\text{mm}] \), \( x_2 \in [50\text{mm}, 90\text{mm}] \), \( x_3 \in [3\text{mm}, 12\text{mm}] \). In this paper, the limit state functions of deflector use Second Order Response Surface Methodology [6] are developed

\[
\begin{align*}
\sigma &= \beta_{10} + \beta_{11} \cdot x_1 + \beta_{12} \cdot x_2 + \beta_{13} \cdot x_3 + \beta_{14} \cdot x_1^2 + \beta_{15} \cdot x_1 \cdot x_2 + \beta_{16} \cdot x_1 \cdot x_3 + \beta_{17} \cdot x_2 \cdot x_3 + \beta_{18} \cdot x_1 \cdot x_2 \cdot x_3 \\
\delta &= \beta_{20} + \beta_{21} \cdot x_1 + \beta_{22} \cdot x_2 + \beta_{23} \cdot x_3 + \beta_{24} \cdot x_1^2 + \beta_{25} \cdot x_1 \cdot x_2 + \beta_{26} \cdot x_1 \cdot x_3 + \beta_{27} \cdot x_2 \cdot x_3 + \beta_{28} \cdot x_1 \cdot x_2 \cdot x_3
\end{align*}
\]

After vectoring the Eq. (4.1), the vector formula is given

\[
y = X_{101}^{101}\beta_{101}
\]

In Eq. (4.2), \( X_{101} = [1, x_1, x_2, x_3, x_1^2, x_2^2, x_1 \cdot x_2, x_1 \cdot x_3, x_2 \cdot x_3, x_1 \cdot x_2 \cdot x_3] \), \( \beta_{101} = [\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9, \beta_{10}] \). Then, we are trying to get initial response surface test data through Finite Element Method in a number of different \( x_1, x_2 \) and \( x_3 \) as described in Table 1.

Table 1. Initial response surface test data

| No. | \( x_1 (\text{mm}) \) | \( x_2 (\text{mm}) \) | \( x_3 (\text{mm}) \) | \( \sigma (\text{MPa}) \) | \( \delta (\text{mm}) \) |
|-----|-------------------|-------------------|-------------------|-----------------|-----------------|
| 1   | 70                | 50                | 3                 | 274.30          | 1.1507          |
| 2   | 70                | 50                | 12                | 265.07          | 1.8874          |
| 3   | 70                | 90                | 3                 | 152.34          | 0.8192          |
| 4   | 70                | 90                | 12                | 165.18          | 0.9439          |
| 5   | 110               | 50                | 3                 | 242.90          | 1.1174          |
| 6   | 110               | 50                | 12                | 274.25          | 1.5228          |
| 7   | 110               | 90                | 3                 | 161.43          | 0.8036          |
| 8   | 110               | 90                | 12                | 156.63          | 0.9680          |
| 9   | 100               | 60                | 5                 | 212.71          | 1.1434          |
| 10  | 90                | 70                | 8                 | 177.58          | 1.1076          |
| 11  | 70                | 50                | 3                 | 274.30          | 1.1507          |
| 12  | 70                | 50                | 12                | 265.07          | 1.8874          |

Combining Eq. (4.2) with Table 1, the formula of \( \beta_{101} \) is provided

\[
\beta_{101} = \left[ D_{12 \times 10} \right]^T \left[ D_{12 \times 10} \right]^{-1} \left[ D_{12 \times 10} \right]^T Y_{12 \times 1} \]

(4.3)
In Eq. (4.3), $D_{12t}=[X_{1t}; X_{2t}; X_{3t}; \cdots; X_{12t}]; Y_{12t}=[y_1; y_2; \cdots; y_{12}]$. Then, writing a computer program that get the calculation of $\beta$ and the limit state functions of $\delta$ and $\delta$ are given

\[
\begin{align*}
\sigma &= 1107.335 - 3.639 - 21.11 + 18.963 + 0.016 + 0.129 \\
-1.308 x_1 - 0.007 x_2 + 0.03 x_3 \cdot x_3 - 0.018 x_2 \cdot x_3 \\
\delta &= 2.764 + 0.0532 x_1 - 0.1332 x_1 + 0.2811 x_1 - 0.0003 x_1 + 0.0008 x_2^2 \\
-0.0081 x_1^2 + 0.0001 x_2 \cdot x_3 - 0.0004 x_3 \cdot x_3 - 0.0012 x_2 \cdot x_3
\end{align*}
\]

Finally, we will use the data that see in Table 2 to verify accuracy of the Eq. (4.4).

Table 2. Results of calculation compared with experimental data

| No. | $x_1$(mm) | $x_2$(mm) | $x_3$(mm) | relative error of $\delta$ | relative error of $\delta$ |
|-----|-----------|-----------|-----------|--------------------------|--------------------------|
| 1   | 80        | 55        | 8         | 0.47%                    | 0.89%                    |
| 2   | 85        | 65        | 4         | 2.27%                    | 0.15%                    |
| 3   | 95        | 75        | 5         | 0.41%                    | 0.43%                    |
| 4   | 100       | 85        | 6         | 2.96%                    | 0.12%                    |
| 5   | 105       | 90        | 7         | 0.87%                    | 0.73%                    |

Form Table 2, the relative errors of $\delta$ and $\delta$ are no more than 2%, the result showed that the limit state functions are given using RSM proves credibility.

5. Reliability analysis of the deflector

In this paper, we use the First Order Reliability Method (FORM) to calculate the reliability. The computation procedure is given as follows.

Step one: Transform the original random variables from $X$-space to $U$-space by Rosenblatt transformation.

Step two: Search the MPP (Most Probable Point) in $U$-space and calculate the reliability index $\beta$. The MPP search algorithm uses a recursive formula and is based on the linearization of the performance function as follows

\[
\begin{align*}
\text{minimize} & \quad \beta = \|u_{MPP}(y)\| \\
\text{subject to} & \quad g(u, y) = 0 \\
& \quad y_l \leq y \leq y_u
\end{align*}
\]

where $u=(u_1, u_2, \cdots, u_n)$ stands for a set of random variables (in $u$-space) whose elements follow a standard normal distribution that transform from the original random variables $x=(x_1, x_2, \cdots, x_n)$ (in $x$-space).

Step three: Calculate reliability $R=\Phi(\beta)$.

Then, we would analyse the reliability of the deflector. And the information on the continuous random variables and parameters are provided in Table 3.

Table 3. Information of the variables and parameters

| No. | $\delta_s$ - yield strength (MPa) | $\delta_s$ - ultimate deformation (mm) | $x_1$ - stiffener spacing (mm) | $x_2$ - stiffener width (mm) | $x_3$ - stiffener thickness (mm) |
|-----|---------------------------------|--------------------------------------|-------------------------------|-------------------------------|------------------------------|
| 1   | $\delta_s \sim N(225, 25)$     | $\delta_s \sim N(1.5, 0.5)$          | $x_1 \sim N(x_1, 0.5)$       | $x_2 \sim N(x_2, 0.5)$       | $x_3 \sim N(x_3, 0.3)$       |

By using the proposed method, the reliability of deflector under uncertainty is formulated as follows
\[ \begin{align*}
R_o &= P_o \left\{ \sigma(x_1, x_2, x_3) - \sigma_o \leq 0 \right\} \\
R_d &= P_o \left\{ \delta(x_1, x_2, x_3) - \delta_o \leq 0 \right\}
\end{align*} \tag{5.2} \]

At the last, the \( R_o \) and \( R_d \) can be pursued by Eq. (5.2) when \( x_1=100 \text{mm}, x_2=65 \text{mm} \) and \( x_3=3 \text{mm} \) that the value of design results have been used, and the results \( R_o=0.9898 \) and \( R_d=0.9202 \) that greater than the desired reliability (\( R=0.9 \)). At the same time, form the calculating results and the engineering operation in hydropower station on the Jinsha River, we can find out that the deflector of underwater wall detection device design is practical and feasible.

6. Conclusions

In this paper, we used Finite Element and Response Surface Methodology to develop limit state functions of the deflector in underwater wall detection device, and then analysed the reliability of the deflector under uncertainty. According to the analysis results, we can research further on the deflector, and the use of the proposed model in this paper will dispense with the large number of repetitive works, to a large extent enhance the efficiency.

However, there exist some limitations. Firstly, when the number of variables in RSM is large, the proposed method of developing the limit state function will be in a very difficult position. Secondly, a lot of uncertainties should be refined in this limit state function. Finally, our future work will focus on optimal design of the deflector in underwater wall detection device.

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