A genuinely natural information measure

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The theoretical measuring of information was famously initiated by Shannon in his mathematical theory of communication, in which he proposed a now widely used quantity, the entropy, measured in \textit{bits}. Yet, in the same paper, Shannon also chose to measure the information in continuous systems in \textit{nats}, which differ from bits by the use of the natural rather than the binary logarithm.

We point out that there is nothing natural about the choice of logarithm basis, rather it is arbitrary. We remedy this problematic state of affairs by proposing a genuinely natural measure of information, which we dub \textit{gnats}. We show that gnats have many advantages in information theory, and propose to adopt the underlying methodology throughout science, arts and everyday life.

Nuts and bolts of information theory. The ongoing digital revolution is fond of considering Claude Shannon as its godfather, his information theory\textsuperscript{1} providing the theoretical underpinning for the pervasive digitization of every kind of data, which today makes the representation and indeed measurement of information in \textit{bits} seem almost inevitable, if not entirely natural. Indeed, in the same founding paper, Shannon started the habit of measuring the information, and in particular the capacity, of continuous-variable systems in \textit{nats}. This parallel use of two conflicting conventions has persisted in information theory, and has been embedded into the grain of its various sub-disciplines.

Today, after more than seventy years of information theory, which saw it become one of the most successful tools of all engineering branches occupied with the development of information processing and communication devices, we can state that this was an unfortunate move, as it imported an annoying ambiguity into the very foundations of this otherwise awe-inspiring theoretical framework. The detractors who wish to deny the severity of this well-known problem will inevitably point to the mathematical fact that bits and nats are distinguished by the choice of binary and natural logarithm, respectively, when defining the entropy. Indeed, for the simplest case of a uniform distribution on $N$ objects,

\begin{align}
S &= \log N \text{ \textit{[bits],}} \quad (1) \\
S &= \ln N \text{ \textit{[nats],}} \quad (2)
\end{align}

are the two commonly employed formulas. They differ by a factor of $\ln 2$, which is well-known to be impossible to apply correctly (does it divide or multiply?).

In addition, it is often claimed that the difference between Eqs. (1) and (2) is merely down to a choice of units (of information, presumably). But that is evidently nonsense, as entropy is, by definition, a unitless quantity. In other words, writing the entropy as a number, one cannot intrinsically deduce this elusive “unit” from it, nor is there any way of attaching a unit in the usual sense that would make it unambiguous. The usual way out is to attach the words “bits” or “nats” to indicate the calculation by which the result was obtained.

We are thus faced with the truly fundamental problem of information science, which has been left unattended from the beginning, but cannot be ignored any longer: to devise a universal measure of information, avoiding the ultimately arbitrary choice of the basis of a logarithm. Obviously, despite all its success in information technology, the binary logarithm (basis 2) cannot claim any more fundamental significance than the common one (basis 10); as for the logarithm to basis $e$, it may be called “natural”, but it is not any more natural then the previous two when it comes to calculating entropies.

In the present work, we finally address this issue at the very foundations of the field. Our approach shall be a natural one, looking for guidance in the physical world. While the conclusion is that henceforth we ought to abandon formulas like Eqs. (1) and (2), it is hoped that this slight discomfort (surely overcome quickly with habit) will provide a sounder basis for information science.

From bits to gnats. Our quest is to find a genuinely natural information measure, which directly suggests to consider gnats, to obtain a reasonable acronym.

While gnats have been studied in biology for centuries (see Fig. 1), they have never been considered in connection with information technology. Indeed, the only role insects have played hitherto in computer science was restricted to \textit{bugs}, which is rather unspecific on the one hand, and on the other hand seems to originally refer to a completely different order (\textit{lepidoptera} rather than \textit{diptera}).

However, once the idea is presented, the advantages of gnats in information technology are blindingly obvious:

- gnats are everywhere, and occur in unlimited supply, thus guaranteeing global connectivity and sustainability;
- because of their omnipresence, both data storage and calibration are easily solved problems;
- gnats are discrete by design, yet entirely natural;
- because of their eponymous attention span, information systems based on gnats will to good approximation be memoryless, thus rendering obsolete

\begin{figure}
\centering
\includegraphics[width=\textwidth]{gnat.png}
\caption{Gnats in the information technology landscape.}
\end{figure}
much of one-shot information theory, and restoring the wonderful world of iid.

Methodology. Unfortunately we have to have this section, but that does not mean we have to waste any time on it.

In the few words necessary, our method of discovery is morphological, lexical, and strictly asemantic. In these regards it follows the paradigm (a dozen indeed) of Shannon’s approach to information [1]. It is however influenced by certain aspects of gnomonology and numerology, but the reader does not have to be an expert in the latter to understand the derivation or the result. Let it be enough to point out that the number seven, and hence the seventh letter of the Latin (English) alphabet, is of fundamental importance.

Calibration and conversion. As gnats occur naturally, their comparison with bits (and for that matter, nats) is a question of empirical observation, see Fig. 1. Initial rough measurements indicate that 1 gnat equals $956828.3$ bits (663222.8 nats); these values are obtained from the entropy of a simple black hole formed by an average gnat. The identification is justified by the fact that throwing it into a black hole is the only reliable way known of definitely getting rid of a gnat. As is well-known, the black hole entropy depends quadratically on the mass,

$$1 \text{ gnat} \equiv S_{\text{BH}}(\globe) = \frac{4\pi GM_{\perp}^2}{\hbar c}, \quad (3)$$

and so high-precision measurements of the gnat mass $M_{\perp}$ will be vital for the future development of natural information theory.

For the moment, a good rule of thumb is that 1 gnat is roughly $10^9$ bits, meaning that we can in practice continue to use bits in information theory, keeping in mind that 1 bit is roughly 1 µgnat (micro-gnat). Evidently, in due course bits and nats will be defined in terms of gnats, as soon as a sufficiently reliable standard for the latter has been established.

One might ask why we do not use the Planck mass $m_P = \sqrt{\frac{\hbar c}{G}}$, which is only two orders of magnitude smaller than $M_{\perp}$, and is supposedly a universal unit of mass. Well, nice try; but it won’t work.

Discussion. Our proposal to base information theory on gnats has been shown to unite several advantages: gnats appear naturally in discrete units, they are fungible, being for all practical purposes infinite in number, and omnipresent, meaning that global calibration is easy. This gives a wholly new foundation (see Fig. 2) to information theory, which we expect to have profound repercussions.

Our novel approach points to a more fundamental principle, which has the potential to revolutionise all gnatural sciences, but also medicine [2], psychology [3], and economics [4], all the way to literature [5] and psychological self-help [6]. The possibilities are endless [7] (see Fig. 3).
Not all conceptual problems are solved by our proposal, and it leaves in particular a number of open questions, not least in relation to quantum physics and quantum information theory. First, black hole evaporation and holography are waiting to be reformulated in terms of the new information measure. In fact, we conjecture that the interior of black holes has a consistent description entirely in terms of gnats; consequently, when a black hole evaporates, it should do so by releasing all its gnats. Secondly, in quantum information theory there are other types of information beyond the bit, most importantly qubits and ebits, cf. \cite{8}. It remains an open question how to quantise gnats, since “qnats” is not very convincing. Among the proposals currently being evaluated are knats and knots.

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