The fraction of muon tracks in cosmic neutrinos

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Abstract. The study of the distinctive signatures of the ultra high energy events recently seen by IceCube \cite{1-4} can allow to single the neutrino origin out. The detection of tau neutrinos would be a clear way to prove that they come from cosmic distances, but at the highest energies currently seen, about 1 PeV, an experimental characterization of tau events is difficult. The study of the fraction of the muon tracks seems more promising. In fact, for any initial composition, because of the occurrence of flavor oscillations and despite their uncertainties, the fraction of muon tracks in the cosmic neutrinos is smaller than the one of atmospheric neutrinos, even hypothesizing an arbitrarily large contribution from charmed mesons. A good understanding of the detection efficiencies and the optimization of the analysis cuts, along with a reasonable increase in the statistics, should provide us with a significant test of the cosmic origin of these events.

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1 Introduction

The two events announced by the IceCube collaboration at Neutrino 2012 and recently described in [1] are of enormous interest [5–15] since they could be the first indication of a high energy cosmic neutrino population. It is unlikely that they are produced by atmospheric neutrinos since the atmospheric neutrino flux is extremely low at PeV energy. Moreover, these events are detected as showers in the IceCube detector and thus they have been attributed to electron neutrino (or perhaps to neutral current) interactions [1]; by contrast, the atmospheric neutrino flux is mostly composed by $\nu_\mu$ and $\bar{\nu}_\mu$.

The absence of more events above PeV could suggest that these events come from a new population of neutrinos that is not power law distributed. Finally, it is exciting to note that a further set of 26 events has been preliminary announced [2]. These events could fill the gap at lower energies, till the region of the spectrum where ordinary atmospheric neutrinos dominate. Note incidentally that a $2\sigma$ excess was already present in a muon sample $\sim 3$ times smaller than the present one, see Fig.4 of [3].

An interpretation of the IceCube observations in terms of cosmic neutrinos is very attractive and the key question becomes how to test this hypothesis. In this note, we argue that a viable possibility is to verify whether the fraction of muon neutrinos obeys the predictions of 3 flavor neutrino oscillations.\footnote{Three flavor neutrino oscillations have been widely discussed as a tool to measure the oscillation parameters, to learn on the original flavor composition of the neutrinos, to test the production mechanism or exotic scenarios, etc. [16–21]; our goal here is to discuss the less ambitious but preliminary and urgent question, concerning the origin of these events. The importance of the shower events has been emphasized in [22].}

2 Flavor composition of cosmic and atmospheric neutrinos

For neutrinos travelling over cosmic distances, the oscillation probabilities are constant [32] and are given by $P_{\ell\ell'} = \sum_{i=1,3} |U_{\ell i}|^2 |U_{\ell' i}|^2$ where $\ell, \ell' = e, \mu, \tau$ and $U$ is the neutrino mixing matrix. By using the latest determinations of the oscillation parameters [23], updated after the measurement of $\theta_{13}$, we obtain that the approximate numerical values of the probability matrix are:

$$P = \begin{pmatrix} 0.548 & 0.244 & 0.208 \\ 0.404 & 0.352 & 0.439 \end{pmatrix} \quad (2.1)$$

Let us consider the fraction of neutrinos with given flavor, namely

$$\xi_\ell = \frac{\Phi_\ell}{\Phi_e + \Phi_\mu + \Phi_\tau} \quad (2.2)$$

where $\Phi_\ell$ is the flux and $\ell = e, \mu$ or $\tau$. The blend of neutrinos (or antineutrinos) at the source, that we generically indicate as $(\xi_e^0 : \xi_\mu^0 : \xi_\tau^0)$, is transformed in a mixture $(\xi_e : \xi_\mu : \xi_\tau)$ of all neutrino types according to

$$\xi_\ell = \sum_{\ell'} P_{\ell\ell'} \xi_{\ell'}^0 \quad (2.3)$$
Let us assume, e.g., that neutrinos are produced by $\pi^+$ decay, deriving from collisions of high energy protons with $\gamma$ rays in the vicinity of their source. The original flavor composition is $(1 : 1 : 0)$ for neutrinos and $(0 : 1 : 0)$ for antineutrinos and, thus, we obtain $(\xi_e^0 : \xi_\mu^0 : \xi_\tau^0) = (1/3 : 2/3 : 0)$ where we summed the neutrinos and antineutrinos and we considered that $\sum \xi = 1$. After oscillations, this flavor composition becomes

$$ (\xi_e : \xi_\mu : \xi_\tau) = (0.35 : 0.35 : 0.30) \quad (2.4) $$

from which we see that a non-negligible fraction of tau neutrinos is expected and the muon neutrino fraction $\xi_\mu$ is reduced to values that are lower than 0.5.

We will show that the above conclusions are generic since they are essentially independent on the original composition of the neutrino flux. In fact, assuming the oscillation probabilities reported in Eq. (2.1), the arrival neutrino flavor fractions are given by:

$$ \begin{align*}
\xi_e &= 0.244 + 0.304 \xi_e^0 - 0.035 \xi_\tau^0 \\
\xi_\mu &= 0.404 - 0.160 \xi_e^0 - 0.052 \xi_\tau^0 \\
\xi_\tau &= 0.352 - 0.144 \xi_e^0 + 0.087 \xi_\tau^0
\end{align*} \quad (2.5) $$

where we considered that, whichever is the neutrino production mechanism, $\xi_{\ell j}^0 \equiv 1 - \xi_{e j}^0 - \xi_{\tau j}^0$. The dark green areas in Fig. 1 show the flavor ratios predicted by Eqs. (2.5) for an arbitrary value of the initial electron neutrino fraction $\xi_e^0$ when $\xi_{\tau j}^0$ is varied in the physical range $0 \leq \xi_{\tau j}^0 \leq 1 - \xi_{e j}^0$. In general terms, we have that, for any fixed $\xi_e^0$, the fraction $\xi_\ell$ of cosmic neutrinos with flavor $\ell = e, \mu, \tau$ that reach us is comprised between:

$$ P_{\mu \ell} + (P_{e \ell} - P_{\mu \ell}) \xi_e^0 \quad \text{and} \quad P_{\tau \ell} + (P_{e \ell} - P_{\tau \ell}) \xi_e^0 \quad (2.6) $$

where the first limit corresponds to the assumption $\xi_e^0 = 0$ while the second is obtained by assuming $\xi_\mu^0 = 0$ (i.e., $\xi_e^0 = 1 - \xi_\tau^0$); this range is narrow since $P_{\mu \ell} \sim P_{\tau \ell}$ as it is seen from Eq. (2.1). Also the dependence of $\xi_\mu$ and $\xi_\tau$ on $\xi_e^0$ is not very strong, due to partial cancellations between $P_{\mu \ell}$, $P_{\tau \ell}$ and $P_{e \ell}$. This holds true even considering the cases when the initial fraction of electrons $\xi_e^0$ and of tau $\xi_\tau^0$ are energy dependent.

Our considerations are not affected by the uncertainties in neutrino oscillation parameters. To quantify their relevance, we constructed the likelihood distributions of $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$, and $\sin^2 \theta_{23}$ from the $\chi^2$ profiles given by [23] (assuming negligible correlations). For the CP-violating phase $\delta$, in order to be extremely conservative, we took a flat distribution between $[0, 2\pi]$. We then determined the probability distributions of $\xi_e$, $\xi_\mu$ and $\xi_\tau$ by MonteCarlo extraction of the oscillation parameters. In Fig. 2 we show the result obtained for the specific case of cosmic neutrinos produced by charged pions. The muon neutrino fraction $\xi_\mu$ derived assuming the best-fit values for oscillations parameters...
in [23] is $\xi_\mu = 0.35$. By propagating the uncertainties, we obtain a $\xi_\mu$ probability distribution which has a finite width and a non trivial structure resulting from the non linear dependence of the $P_{\ell\ell'}$ from the oscillation parameters (see [24, 25] and [26] for a discussion of this aspect). However, the conclusion that the expected muonic neutrino fraction at Earth is reduced to values that are lower than 0.5 is not spoiled. In fact, the 90% C.L. allowed range is

$$\xi_\mu = 0.325 - 0.37,$$

as it is obtained by integrating out symmetrically the 5% on both sides of the distribution. A general appraisal of the relevance of oscillation parameter uncertainties is finally obtained from the light green areas in Fig. 1 that show the spread of the limits in Eq. (2.6) when 90% C.L. allowed ranges are considered.

The green regions in Fig. 1 should be compared with expectations for the atmospheric neutrino component. At high energies, atmospheric electron neutrinos are rare since the muons produced in the atmosphere have no time to decay (see e.g., [27, 28]) and oscillations are not effective.\(^2\) As a consequence, the conventional production mechanism through charged pions gives a flavor composition $\sim (0 : 1 : 0)$. The production of heavy (charmed) mesons introduces an additional component, the so-called ‘prompt’ atmospheric neutrinos, with a flavor composition $(1/2 : 1/2 : 0)$: see e.g., [29]. The prompt neutrino component contains electron neutrinos, thus it can account for the observed shower events, and the electron-to-muon neutrino ratio is close to the one expected from cosmic sources. The number of prompt neutrino events is estimated to be small and marginally observable with the present statistics; however, in view of the large theoretical uncertainty, it is worthwhile to consider a conservative attitude, and to assume that the atmospheric flavor composition is between the two extreme situations represented by charged pions and charmed mesons decay. Such a flavor composition of the atmospheric neutrino events is represented in Fig.1 by the red horizontal bands and in Fig.2 by the red vertical band.

From this conservative standpoint, the observation of shower events of ultra-high energies is not sufficient to discriminate between cosmic and atmospheric origin of the events—as it is seen in the left panel of Fig.1—even if the mere existence of showers at PeV shows that there is something more than the conventional atmospheric neutrinos [22]. The crucial information is instead provided by the observations of tau neutrinos of ultra-high energies (absent in the conventional atmospheric neutrinos) and/or by the determination of the muon neutrino fraction $\xi_\mu$ around PeV, as we discuss in the following.

\(^2\)Let us consider neutrinos with energies between 10 TeV and 10 PeV. The phases of 3 flavor oscillations, $\Delta m^2 L c^3/(4 E \hbar)$, are smaller than $1/300$ for atmospheric neutrinos ($L < 2 R_\oplus$) and larger than 400 already for neutrinos coming from Proxima Centauri ($L = 4.2$ ly): therefore, for the energies of interest, vacuum oscillations are fully developed for cosmic neutrinos and absent for atmospheric neutrinos.
\section{Testing the cosmic origin}

We discuss a couple of tests to address the cosmic origin of high energy events observed in neutrino telescopes. The most direct evidence could be provided from the observation of tau neutrinos, by seeing some manifestation of the two vertices where tau is produced and where it decays. However, the efficiencies for reliable identification of taus are not very large, especially for energies of 1 PeV or lower. In fact, the separation among the strings is about 120 m. If we ask that the two vertices are separated by this distance, and equate it to the theoretical expression \( E/(mc^2) \times (e t_{1/2}) \) (where \( E \), \( m \) and \( t_{1/2} \) are the energy, mass and half-life of the tau), we get that the energy should be \( E = 3.5 \) PeV, see e.g. \cite{16}. This is a region of energies where no neutrino has been seen yet. If this were due to a cut in the spectrum then the search for tau events would be challenging. In this case, we should rely on fluctuations, very refined analyses and/or new setup, before we can firmly claim the presence of a tau event. (For more discussion, see the paper of IceCube concerning the search of tau neutrino, in the range of energies between 0.3 and 200 PeV \cite{30}.)

An alternative test, that should be possible below PeV (allowing us to increase the statistics) is to show that the muon neutrino fraction \( \xi_\mu \) of the observed population does not fall in the range (0.5, 1) expected for atmospheric neutrinos. This relies on the possibility to identify \( \nu_\mu \) charged current interactions in the detector by looking at the presence of muon tracks. The rate \( R_T \) of track events produced by a flux \( \Phi_\mu \) of muon neutrinos is estimated as:

\[
R_T = N_{\text{tar}} \varepsilon_T \int dE_\nu \sigma_{CC}(E_\nu) \Phi_\mu(E_\nu)
\]  \hspace{1cm} (3.1)

where \( N_{\text{tar}} \) is the number of target nuclei, \( \sigma_{CC} \) is the charged current cross section and \( \varepsilon_T \) is the efficiency of \( \nu_\mu \) events identification (which includes the detector volume reduction implied by analysis cuts). The rate \( R_{S,CC} \) of shower events produced by charged current interactions of electron and tau neutrinos is:

\[
R_{S,CC} = N_{\text{tar}} \varepsilon_S \int dE_\nu \sigma_{CC}(E_\nu) \left[ \Phi_e(E_\nu) + \Phi_\tau(E_\nu) \right]
\]  \hspace{1cm} (3.2)

where \( \varepsilon_S \) is the shower detection efficiency. The fraction of track events is, thus, equal to

\[
f = \frac{R_T}{R_T + R_{S,CC}} = \frac{\eta \xi_\mu}{1 - \xi_\mu (1 - \eta)}
\]  \hspace{1cm} (3.3)

where \( \eta = \varepsilon_T/\varepsilon_S \) is the ratio of efficiencies and we considered that charged current cross sections of different neutrino flavors are approximately equal. In the above relation, we implicitly assumed that showers and tracks probe the same portion of the neutrino spectrum. In other words the rates (3.1) and (3.2) are obtained by integrating the fluxes of the different neutrino flavors above the same energy threshold \( E \). This implies that the reconstruction of the initial state neutrino energy is good enough. For shower events, the energy reconstruction is accurate at the \( \sim 15\% \) level \cite{2}. The accuracy is much worse for a generic track event which may be produced by a \( \nu_\mu \) interacting outside the detector. For this reason, it is better to consider only contained vertex track events in which the interaction point of the \( \nu_\mu \) is visible. In this case, in fact, the \( \nu_\mu \) interaction with the nucleon produces an hadronic cascade that deposits its energy inside the sensitive volume and a muon whose energy can be reconstructed from the rate of catastrophic energy losses along the track produced in the detector \cite{31}. We note that the recent IceCube analysis \cite{4} includes only contained vertex events of highest energy, with the goal of reducing the background due to atmospheric muons and neutrinos.

Few sub-dominant effects can modify the simple estimate given above. First, we neglected neutral current contribution to shower event rate. This is expected to be small because the energy deposited in the detector, in the form of an hadronic shower, is a limited fraction of the initial neutrino energy. Most of the energy is, in fact, carried away by the final state neutrino. When \( E_\nu = 1 - 10 \) PeV, we

\(^3\)In principle, this requirement is not strictly necessary but it permits to obtain predictions which are essentially independent on the theoretical assumptions about the neutrino spectrum. The analysis thresholds, which can be eventually different for tracks and showers, have to be optimized with the detailed knowledge of the experimental apparatus.
have that $\langle E_{\text{had}} \rangle \approx (1/4) E_\nu$, indicating that, an event with 1 PeV hadronic energy is produced by neutrinos with average energy of about 4 PeV. By taking this into account, the rate $R_{S,NC}$ of neutral current events is estimated as:

$$R_{S,NC} = N_{\text{tar}} \xi S \int_{4E} dE_\nu \sigma_{NC}(E_\nu) \left[ \Phi_e(E_\nu) + \Phi_\mu(E_\nu) + \Phi_\tau(E_\nu) \right]$$

(3.4)

If we postulate conservatively that the flux diminishes as $\Phi \propto E_\nu^{-2}$ we obtain:

$$\frac{R_{S,NC}}{R_{S,CC}} \approx A_{NC} \frac{1}{1 - \xi_\mu}$$

(3.5)

with $A_{NC} = 0.17$.\(^4\) In the above relation, we have taken into account that $\sigma_{NC}/\sigma_{CC} \approx 0.38$ almost independently on energy, and that the cross sections $\sigma_{NC}$ and $\sigma_{CC}$ scale approximately as $E_\nu^{0.44}$\(^3\). As an additional remark, we note that in the case of $\nu_\tau$ CC-interactions, a fraction $\approx 20\%$ of the initial neutrino energy is not detectable since it is lost in neutrinos produced by tau decay\(^3\). By using the arguments above, it can be calculated that this corresponds to reducing the $\nu_\tau$ signal by a coefficient $A_\tau \approx 12\%$. If we take the above effects into account, Eq. (3.3) is slightly modified. We obtain:

$$f \equiv \frac{R_T}{R_T + R_{S,CC} + R_{S,NC}} = \frac{\eta \xi_\mu}{(1 + A_{NC}) - A_\tau \xi_\tau - \xi_\mu \xi_\mu (1 - \eta)}$$

(3.6)

that will be used in the next section to evaluate the perspectives of the proposed measure.

4 Perspectives

In conclusion, a possible test of the cosmic origin of the events observed by IceCube is simply to measure the fraction of muon tracks $f$ precisely enough. In fact, considering for simplicity the case of cosmic neutrinos with flavor composition $(\xi_e, \xi_\mu, \xi_\tau) = (1/3, 1/3, 1/3)$ and assuming $\eta = 1$, we obtain from Eq.(3.6):

$$f = \begin{cases} 
0.29 & \text{for cosmic neutrinos} \\
> 0.43 & \text{for atmospheric neutrinos}
\end{cases}$$

(4.1)

Note that the case $f = 0.43$ corresponds to assuming that the atmospheric neutrino flux is solely due to charm mesons decays (prompt contribution).

In order to illustrate the potential of this measure, simple statistical considerations are sufficient. Let us consider the normalized binomial distribution $B(n|f, N) = \binom{N}{n} f^n (1 - f)^{N-n}$ where the true frequency is $f$ and $N$ is the total number of events. The possibility that the frequency is $f' > f$ can be excluded at a certain confidence level CL requires that $N$ satisfies the condition

$$1 - \text{CL} > \sum_{n>f'N} B(n|f, N)$$

(4.2)

We assume the cosmic origin of the events ($f = 0.29$) and, by using the above expression, we conclude that the charm hypothesis ($f' = 0.43$) can be excluded as an alternative explanation when the total number of events is larger than:

$$N = 73, 125 \text{ at a CL}=99, 99.9\%,$$ \hspace{1cm} (4.3)

The assumption $\eta = 1$ corresponds to the optimal situation in which the shower and track detection efficiencies are equal. If contained vertex events are only considered, this assumption is, in principle, adequate since different neutrino flavors have the same interaction volume. However, the

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\(^4\) The coefficient $A_{NC}$ is reduced to 0.13 if the slope of the spectrum, $\Phi \propto E^{-\alpha}$, is equal to $\alpha = 2.2$, whereas $A_{NC} = 0.11$ if there is an exponential cut at energies 20 times more than the threshold (e.g., $E = 100$ TeV and $E_{\text{cut}} = 2$ PeV).
analysys cuts needed to reduce the background unavoidably modify the value of $\eta$. As an example, by imposing a fixed threshold for the energy released in the detector, a stronger selection of track events is performed. The muon range is, in fact, larger than the size of the apparatus, at $\sim 1$ PeV is $\sim 15$ km and, thus, muons deposit only a fraction of their energy in the detector. The value of $\eta$ is consequently reduced as it is also seen by comparing the effective areas for $\nu_e$, $\nu_\mu$ and $\nu_\tau$ detection in the recent IceCube analysis reported by [4]. To evaluate the importance of detection efficiencies, we repeat our analysis for $\eta = 0.5$. In this assumption, Eq. (3.6) gives $f = 0.17$ for the cosmic origin and $f' = 0.27$ for the charm hypothesis. To discriminate between the two options, the total number of events needs to be larger than:

$$N = 96, 164 \text{ at a CL}=99, 99.9\%,$$ respectively. \hfill (4.4)

In the above simplified considerations, we neglected the contamination of atmospheric neutrinos and few additional effects, such as e.g. the $\nu_\tau$ regeneration in the earth interior [35], etc., that could modify the expected value of $f$ and, thus, affect the estimate of the required statistics. However, the general picture seems promising. In summary, the existing statistics has to be increased by a small multiplicative factor ($\sim$ few) whose magnitude depends on the specific cuts adopted in the experimental data selection procedure. The IceCube collaboration has the data and the tools to optimize the cuts and perform a complete analysis: we hope that they will shed light on the origin of these exciting events.

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