Dynamic Power Allocation in MIMO Fading Systems Without Channel Distribution Information

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Abstract—This paper considers dynamic power allocation in MIMO fading systems with unknown channel state distributions. First, the case of instantaneous but possibly inaccurate channel state information at the transmitter (CSIT) is treated. By extending the drift-plus-penalty method, a dynamic power allocation policy is developed and shown to approach optimality with an $O(\delta)$ gap, where $\delta$ is the error of CSIT, regardless of the channel state distribution and without requiring knowledge of this distribution. Next, the case of delayed and inaccurate channel state information is considered. Optimal utility is fundamentally different in this case, and a different online algorithm is developed that is based on convex projections. The proposed algorithm for this delayed-CSIT case is also shown to have an $O(\delta)$ optimality gap, where $\delta$ is again the error of CSIT.

I. INTRODUCTION

During the past decade, the multiple-input multiple-output (MIMO) technique has been recognized as one of the most important techniques for increasing the capabilities of wireless communication systems. In the wireless fading channel, where the channel changes over time, the problem of power allocation is to determine the transmit covariance of the transmitter to maximize the ergodic capacity subject to both long term and short term power constraints. It is often reasonable to assume that instantaneous channel state information (CSI) is available at the receiver through training. Most works on power allocation in MIMO fading systems also assume that statistical information about the channel state, referred to as channel distribution information (CDI), is available at the transmitter. Under the assumption of perfect instantaneous channel state information at the receiver (CSIR) and perfect channel distribution information at the transmitter (CDIT), prior work on power allocation in MIMO fading systems can be grouped into two categories:

- Perfect instantaneous channel state information at the transmitter (ideal-CSIT): In the ideal case of perfect CSIT, optimal power allocation is known to be a water-filling solution [2]. Computation of water-levels involves a one-dimensional integral equation for fading channels with i.i.d. Rayleigh entries or a multi-dimensional integral equation for general fading channels [3]. The involved multi-dimensional integral equation is in general intractable and can only be approximately solved with numerical algorithms with huge complexity.
- No CSIT: If CSIT is unavailable, the optimal power allocation is in general still open. If the channel matrix has i.i.d. Rayleigh entries, then the optimal power allocation is known to be the identity transmit covariance scaled to satisfy the power constraint [2]. The optimal power allocation in MIMO fading channels with correlated Rayleigh entries is obtained in [4], [5]. The power allocation in MIMO fading channels is further considered in [6] under a more general channel correlation model.

This prior work relies on accurate CDIT and/or on restrictive channel distribution assumptions. It can be difficult to accurately estimate the CDI, especially when there are complicated correlations. Solutions that base decisions on perfect CDI can be suboptimal due to mismatches. This paper designs algorithms that do not require prior knowledge of the channel distribution, yet perform arbitrarily close to the optimal value that can be achieved by having this knowledge. Further, the convergence time is computed and shown to be significantly smaller than the time required to accurately estimate the channel distribution.

In time-division duplex (TDD) systems with symmetric wireless channels, the CSI can be measured directly at the transmitter using the unlink channel. However, in frequency-division duplex (FDD) scenarios and other scenarios without channel symmetry, the CSI must be measured at the receiver, quantized, and reported back to the transmitter with a time delay.

Depending on the measurement delay in TDD systems or the overall channel acquisition delay in FDD systems, the CSIT can be instantaneous or delayed. In general, the CSIT can also be inaccurate due to the measurement, quantization or feedback error. This paper first considers the instantaneous (but possibly inaccurate) CSIT case and develops an algorithm that does not require CDIT. This algorithm can achieve a utility within $O(\delta)$ of the best utility that can be achieved even with perfect CDIT and perfect CSIT, where $\delta$ is the CSIT error. This shows that instantaneous but accurate CSIT (with $\delta \approx 0$) is almost as good as having both perfect CDIT and accurate instantaneous CSIT.

Next, the case of delayed (but possibly inaccurate) CSIT is considered and a fundamentally different algorithm is developed for that case. The latter algorithm again does not use CDIT, but achieves a utility within $O(\delta)$ of the best utility that can be achieved even with perfect CDIT, where $\delta$ is the CSIT...
error. This shows that delayed but accurate CSIT (with \(\delta \approx 0\)) is almost as good as having perfect CDIT.

A. Related work and our contributions

In the instantaneous (and possibly inaccurate) CSIT case, the proposed dynamic power allocation policy extends the general drift-plus-penalty algorithm for stochastic network optimization \[7, 8\] to deal with inaccurate observations of the system states. In this MIMO context, the current paper shows the algorithm provides strong sample path and convergence time guarantees. The dynamic of the drift-plus-penalty algorithm is similar to that of the stochastic dual subgradient algorithm, although the optimality analysis and performance bounds are different. The stochastic dual subgradient algorithm has been applied in optimization of the wireless fading channel without CDI, e.g., downlink power scheduling in single antenna cellular systems \[9\], power allocation in single antenna broadcast OFDM channels \[10\], scheduling and resource allocation in random access channels \[11\], power allocation in multi-carrier MIMO networks \[12\].

In the delayed (and possibly inaccurate) CSIT case, the situation is similar to the scenario of online convex optimization \[13\] except that we are unable to observe true history reward functions due to channel error. The proposed dynamic power allocation policy can be viewed as an online algorithm with inaccurate history information. The current paper analyzes the performance loss due to CSI quantization error and provides strong sample path and convergence time guarantees of this algorithm. The analysis in this MIMO context can be extended to more general online convex optimization with inaccurate history information. Online optimization has been applied in power allocation in wireless fading channels without CDIT and with delayed and accurate CSIT, e.g., suboptimal online power allocation in single antenna single user channels \[14\], suboptimal online power allocation in single antenna multiple user channels \[15\]. A close related recent work is \[16\], where online power allocation in MIMO systems is considered. The online algorithm in \[16\] is different from our algorithm and follows a matrix exponential learning scheme. In contrast, our online algorithm involves a projection at each slot and a closed-form solution of this projection is derived in this paper. Work \[16\] also considers the effect of imperfect CSIT by assuming CSIT is unbiased, i.e., expected CSIT error conditional on observed previous CSIT is zero. This assumption of imperfect CSIT is suitable to model the CSIT measurement error or feedback error but can not capture the CSI quantization error. In contrast, the current paper only requires that CSIT error is bounded.

II. SIGNAL MODEL AND PROBLEM FORMULATIONS

A. Signal model

Consider a point-to-point MIMO fading channel that operates in slotted time with normalized time slots \(t \in \{0, 1, 2, \ldots\}\). There are \(N_R\) antennas at the receiver and \(N_T\) antennas at the transmitter. The channel can be modeled as

\[
y(t) = H(t)x(t) + z(t)
\]

where \(t \in \{0, 1, 2, \ldots\}\) is the time index, \(z(t) \in \mathbb{C}^{N_R}\) is the additive noise vector, \(x(t) \in \mathbb{C}^{N_T}\) is the transmitted signal vector, \(H(t) \in \mathbb{C}^{N_R \times N_T}\) is the channel matrix, and \(y(t) \in \mathbb{C}^{N_R}\) is the received signal vector. Assume that noise vectors \(z(t)\) are independent and identically distributed (i.i.d.) over time slots \(t\) and are normalized circularly symmetric complex Gaussian random vectors with \(E[z(t)z^H(t)] = I_{N_R}\), where \(I_{N_R}\) is an \(N_R \times N_R\) identity matrix. Assume that channel matrices \(H(t)\) are i.i.d. across time \(t\) and have a fixed but arbitrary probability distribution, possibly one with correlations between entries of the matrix. Assume there is a constant \(B > 0\) such that \(\|H\|_F \leq B\) with probability one, where \(\| \cdot \|_F\) denotes the Frobenius norm. Recall that the Frobenius norm of a complex \(m \times n\) matrix \(A = (a_{ij})\) is

\[
\|A\|_F = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2} = \sqrt{\text{tr}(A^H A)} \tag{1}
\]

where \(A^H\) is the Hermitian transpose of \(A\) and \(\text{tr}(\cdot)\) is the trace operator.

Assume that the receiver can track channel matrices \(H(t)\) exactly through training. In symmetric TDD scenarios, the transmitter can estimate \(H(t)\) by using the symmetric uplink channel. In more general scenarios, the channel matrix \(H(t)\) is measured at the receiver at each slot \(t\), a quantized version is created as a function of \(H(t)\), and this quantized version is reported back to the transmitter with a certain amount of delay.

In general, there are two possibilities when we consider the availabilities of CSIT:

- **Instantaneous CSIT Case**: At each time slot \(t\), an approximate version \(\tilde{H}(t)\) for the true channel \(H(t)\) is known at the transmitter. This usually happens in TDD systems or FDD systems where the measurement, quantization and feedback delay is negligible with respect to the channel coherence time.
- **Delayed CSIT Case**: At each time slot \(t\), due to the delay of CSIT acquisition, the transmitter only knows \(\tilde{H}(t-1)\), which is an approximate version of channel \(H(t-1)\), and has no idea of the current channel \(H(t)\). Since channels are i.i.d. over slots, this delayed information is independent of the current (and unknown) \(H(t)\). Remarkably, it turns out that the outdated information is still useful.

In both cases, we assume the CSIT error is bounded, i.e., there exists \(\delta > 0\) such that \(\|H(t) - H(t)\|_F \leq \delta\) for all \(t\).

1. If the size of the identity matrix is clear, we often simply write \(I\).
2. A bounded Frobenius norm always holds in the physical world because the channel attenuates the signal. Particular models such as Rayleigh and Rician fading violate this assumption in order to have simpler distribution functions.
3. In general, the dynamic power allocation developed in this paper can be extended to deal with arbitrary CSIT acquisition delay as discussed in Section IV-D. For the simplicity of presentations, we assume the CSIT acquisition delay is always one slot in this paper.
B. Optimal power allocation with perfect CDIT

If the channel matrix is fixed at $\mathbf{H}$ and the transmit covariance is fixed at $\mathbf{Q}$, the MIMO capacity is given by [2]:

$$\log \det(\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^H)$$

where $\det(\cdot)$ denotes the determinant operator of matrices. If $\mathbf{H}$ is random then the average capacity, formally called the ergodic capacity [17], is given by $\mathbb{E}_\mathbf{H}[\log \det(\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^H)]$.

We consider two types of power constraints at the transmitter:

An average power constraint $\mathbb{E}[\text{tr}(\mathbf{Q})] \leq \bar{P}$ and an instantaneous power constraint $\text{tr}(\mathbf{Q}) \leq P$. If accurate instantaneous CSIT is available, the problem is to choose $\mathbf{Q}$ as a (possibly random) function of the observed $\mathbf{H}$ to maximize the ergodic capacity subject to power constraints:

$$\max_{\mathbf{Q} \mathbf{H}} \mathbb{E}[\log \det(\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^H)]$$

s.t. $\mathbb{E}[\text{tr}(\mathbf{Q})] \leq \bar{P}$,

$$\mathbf{Q} \mathbf{H} \in \mathcal{Q}, \forall \mathbf{H},$$

where $\mathcal{Q}$ is a set that enforces the instantaneous power constraint:

$$\mathcal{Q} = \left\{ \mathbf{Q} \in \mathbb{S}_{++}^{N_T} : \text{tr}(\mathbf{Q}) \leq P \right\}$$

where $\mathbb{S}_{++}^{N_T}$ denotes the $N_T \times N_T$ positive semidefinite matrix space. To avoid trivialities it is assumed that $P \geq \bar{P}$. In [2], we use notation $\mathbf{Q} (\mathbf{H})$ to emphasize that $\mathbf{Q}$ can depend on $\mathbf{H}$, i.e., adaptive to channel realizations.

If the transmitter has no CSIT, the optimal power allocation problem is different, given as follows:

$$\max_{\mathbf{Q}} \mathbb{E}[\log \det(\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^H)]$$

s.t. $\mathbb{E}[\text{tr}(\mathbf{Q})] \leq \bar{P}$,

$$\mathbf{Q} \in \mathcal{Q},$$

where set $\mathcal{Q}$ is defined in (5). Again assume $\bar{P} \geq P$. Since the instantaneous CSIT is unavailable, the transmit covariance cannot adapt to $\mathbf{H}$. By the convexity of this problem and Jensen’s inequality, a randomized $\mathbf{Q}$ can be shown to be useless. It suffices to consider a constant $\mathbf{Q}$. Since $\bar{P} \geq P$, this implies the problem is equivalent to a problem that removes the constraint (7) and that changes the constraint (6) to:

$$\mathcal{Q} = \epsilon \mathcal{Q} = \left\{ \mathbf{Q} \in \mathbb{S}_{++}^{N_T} : \text{tr}(\mathbf{Q}) \leq \bar{P} \right\}$$

The problems (2)-(4) and (6)-(8) are fundamentally different and have different optimal objective function values. Optimality for these problems is defined by the channel distribution information (CDI). In this paper, the problems are solved via dynamic algorithms that do not require CDI. The algorithms are different for the two cases, and use different techniques.

C. Linear algebra and matrix derivatives

Recall that if $\mathbf{A} \in \mathbb{C}^{m \times n}$ and $\mathbf{B} \in \mathbb{C}^{n \times m}$ then $\text{tr}(\mathbf{A}\mathbf{B}) = \text{tr}((\mathbf{B}\mathbf{A}))$. This subsection presents additional basic facts about Frobenius norms (defined in (1)) and complex matrices.

Fact 1. For any $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{m \times n}$ and $\mathbf{C} \in \mathbb{C}^{n \times k}$ we have:

1) $\|\mathbf{A}\|_F = \|\mathbf{A}^H\|_F = \|\mathbf{A}^T\| = \| - \mathbf{A}\|_F$.
2) $\|\mathbf{A} + \mathbf{B}\|_F \leq \|\mathbf{A}\|_F + \|\mathbf{B}\|_F$.
3) $\|\mathbf{A}C\|_F \leq \|\mathbf{A}\|_F \|\mathbf{C}\|_F$.
4) $\text{tr}(\mathbf{A}^H\mathbf{B}) \leq \|\mathbf{A}\|_F \|\mathbf{B}\|_F$.

Fact 2. For any $\mathbf{A} \in \mathbb{S}_{++}^{n}$ we have $\|\mathbf{A}\|_F \leq \text{tr}(\mathbf{A})$.

Fact 3 ([18]). The function $f : \mathbb{S}_{++}^{n} \to \mathbb{R}$ defined by $f(\mathbf{Q}) = \log \det(\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^H)$ is concave and its gradient is given by $\nabla_Q f(\mathbf{Q}) = \mathbf{H}^H(\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^H)^{-1}\mathbf{H}$, $\forall \mathbf{Q} \in \mathbb{S}_{++}^{n}$.

The next fact is the complex matrix version of the first order condition for concave functions of real number variables, i.e., $f(y) \leq f(x) + f'(x)(y - x)$, $\forall x, y \in \text{dom} f$ if $f$ is concave. A brief proof is given in Appendix A for completeness.

Fact 4. Let function $f(\mathbf{Q}) : \mathbb{S}_{++}^{n} \to \mathbb{R}$ be a concave function and have gradient $\nabla_Q f(\mathbf{Q}) \in \mathbb{S}_{++}^{n}$ at point $\mathbf{Q}$. Then, $f(\mathbf{Q}) \leq f(\mathbf{Q}) + \text{tr}[(\nabla_Q f(\mathbf{Q}))^H(\mathbf{Q} - \mathbf{Q})]$, $\forall \mathbf{Q} \in \mathbb{S}_{++}^{n}$.

III. Instantaneous CSIT case

Consider the case of instantaneous but inaccurate CSIT where at each slot $t \in \{0, 1, 2, \ldots\}$ channel $\mathbf{H}(t)$ is unknown and only an approximate version $\hat{\mathbf{H}}(t)$ is known.

In this case, the problem (2)-(4) can be interpreted as a stochastic optimization problem where channel $\mathbf{H}(t)$ is the instantaneous system state and transmit covariance $\mathbf{Q}(t)$ is the control action at each time $t$. This is similar to the scenario of stochastic optimization/control with i.i.d. time-varying system states, where the decision maker needs to choose an action based on the observed instantaneous system state at each slot such that time average expected utility is maximized and the time average expected constraints are guaranteed. The drift-plus-penalty technique from [3] is a mature framework to solve stochastic optimization without any distribution information of the system states.

This is different from the conventional stochastic optimization considered by the drift-plus-penalty technique because at each slot $t$, the true “system state” $\mathbf{H}(t)$ is unavailable and only an approximate version $\hat{\mathbf{H}}(t)$ is known. Nevertheless, a modified version of the standard drift-plus-penalty algorithm is developed in Algorithm 1. Note that Algorithm 1 does not use channel distribution information (i.e., no CDI).

Algorithm 1 Dynamic power allocation with instantaneous CSIT

Let $V > 0$ be a constant parameter and $Z(0) = 0$. At each time $t \in \{0, 1, 2, \ldots\}$, observe $\hat{\mathbf{H}}(t)$ and $Z(t)$. Then do the following:

- Choose transmit covariance $\mathbf{Q}(t) \in \mathcal{Q}$ to solve:

$$\max_{\mathbf{Q} \in \mathcal{Q}} \{ V \log \det(\mathbf{I} + \hat{\mathbf{H}}(t)\mathbf{Q}\hat{\mathbf{H}}(t)) - Z(t)\text{tr}(\mathbf{Q}) \}$$

- Update $Z(t + 1) = \max[0, Z(t) + \text{tr}(\mathbf{Q}(t)) - \bar{P}]$.

For each slot $t \in \{0, 1, 2, \ldots\}$ define the reward $R(t)$:

$$R(t) = \log \det(\mathbf{I} + \mathbf{H}(t)\mathbf{Q}(t)\mathbf{H}^H(t)).$$

(9)
Define $R^{opt}$ as the optimal average utility in (2). The value $R^{opt}$ depends on the (unknown) distribution for $H(t)$. Fix $\epsilon > 0$ and define $V = (P + \tilde{P})^2/(2\epsilon)$. If $H(t) = H(t), \forall t$, regardless of the distribution of $H(t)$, the standard drift-plus-penalty technique (8) ensures:

$$\frac{1}{t} \sum_{\tau=0}^{t-1} E[R(\tau)] \geq R^{opt} - \epsilon, \forall t > 0 \quad (10)$$

$$\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[\text{tr}(Q(\tau))] \leq \tilde{P} \quad (11)$$

This holds for arbitrarily small values of $\epsilon > 0$, and so the algorithm comes arbitrarily close to optimality. However, the above is true only if $H(t) = H(t), \forall t$.

The development and analysis of Algorithm 4 extends the standard drift-plus-penalty technique in the following two aspects:

- At each time $t$, the standard drift-plus-penalty technique requires accurate “system state” $H(t)$ and can not deal with inaccurate “system state” $H(t)$. In contrast, Algorithm 4 works with inaccurate CSIT $H(t)$. The next subsections analyze the performance of Algorithm 4 and show that the performance degrades linearly with respect to CSIT error $\delta$. If $\delta = 0$, then (10) is recovered.

- Inequality (11) only treats infinite horizon time average expected power. The next subsections show that Algorithm 4 can guarantee $\frac{1}{t} \sum_{\tau=0}^{t-1} \text{tr}(Q(\tau)) \leq \tilde{P} + \frac{B^2(P + \tilde{P})^2}{2t}$ for all $t > 0$ and regardless of the CSIT error $\delta$. This sample path guarantee on average power is much stronger than (11).

The next subsections analyze the performance of Algorithm 4 and shows that the special structure of this MIMO problem produces a sample path guarantee that is significantly stronger than (11).

A. Transmit covariance updates in Algorithm 7

This subsection shows the $Q(t)$ selection in Algorithm 4 can be easily solved and has an (almost) closed-form solution. The convex program involved in the transmit covariance update of Algorithm 4 is in the form

$$\max_Q \quad \log \det(I + HQH^t) - \frac{Z}{V} \text{tr}(Q) \quad (12)$$

$$\text{s.t.} \quad \text{tr}(Q) \leq P \quad (13)$$

$$Q \in S^N_t \quad (14)$$

This convex program is similar to the conventional problem of transmit covariance design with a deterministic channel $H$, except that objective (12) has an additional penalty term $-(Z/V)\text{tr}(Q)$. It is well known that, without this penalty term, the solution is to diagonalize the channel matrix and allocate power over eigen-modes according to a water-filling technique [2]. The next lemma summarizes that the optimal solution to problem (12)-(14) has a similar structure.

Lemma 1. Consider the SVD $H^tH = U^t\Theta^tU$, where $U$ is a unitary matrix and $\Sigma$ is a diagonal matrix with non-negative entries $\sigma_1, \ldots, \sigma_{N_T}$. Then the optimal solution to (12)-(14) is given by $Q^* = U^t\Theta^tU$, where $\Theta^*$ is a diagonal matrix with entries $\theta_1^*, \ldots, \theta_{N_T}^*$ given by:

$$\theta_i^* = \max \left\{ 0, \frac{1}{\mu^* + Z/V} - \frac{1}{\sigma_i} \right\}, \quad \forall i \in \{1, \ldots, N_T\},$$

where $\mu^*$ is chosen such that $\sum_{i=1}^{N_T} \theta_i^* \leq P, \mu^* \geq 0$ and $\mu^* \left[ \sum_{i=1}^{N_T} \theta_i^* - P \right] = 0$. The exact $\mu^*$ can be determined with complexity $O(N_T \log N_T)$, described in Algorithm 2.

Proof: See Appendix B.

Algorithm 2 Algorithm to solve problem (12)-(14)

1) Check if $\sum_{i=1}^{N_T} \max \{0, \frac{1}{\sigma_i} - \frac{1}{\sigma_i} \} \leq P$ holds. If yes, let $\mu^* = 0$ and $\theta_i^* = \max \{0, \frac{1}{\sigma_i} - \frac{1}{\sigma_i} \}, \forall i \in \{1, 2, \ldots, N_T\}$ and terminate the algorithm; else, continue to the next step.

2) Sort all $\sigma_i, \in \{1, 2, \ldots, N_T\}$ in a decreasing order $\pi$ such that $\sigma_{\pi(1)} \geq \sigma_{\pi(2)} \geq \cdots \geq \sigma_{\pi(N_T)}$. Define $S_0 = 0$.

3) For $i = 1$ to $N_T$

   - Let $S_i = S_{i-1} + \frac{1}{\sigma_{\pi(i)}}$. Let $\mu^* = \frac{1}{S_i} - P - (Z/V)$.
   - If $\mu^* \geq 0$, $\frac{1}{S_i} - \frac{1}{\sigma_{\pi(i)}} > 0$ and $\frac{1}{S_i} - \frac{1}{\sigma_{\pi(i+1)}} \leq 0$, then terminate the loop; else, continue to the next iteration in the loop.

4) Let $\theta_i^* = \max \left\{ 0, \frac{1}{\sigma_i} - \frac{1}{\sigma_i} \right\}, \forall i \in \{1, 2, \ldots, N_T\}$ and terminate the algorithm.

The complexity of Algorithm 2 is dominated by the sorting of all $\sigma_i$ in step 2. Recall that the water-filling solution of power allocation in multiple parallel channels can also be found by an exact algorithm (see Section 6 in [19]), which is similar to Algorithm 2. The main difference is that Algorithm 2 has a first step to verify if $\mu^* = 0$. This is because unlike the power allocation in multiple parallel channels, where the optimal solution always uses full power, the optimal solution to problem (12)-(14) may not use full power for large $Z$ due to the penalty term $-(Z/V)\text{tr}(Q)$ in objective (12).

B. Deterministic bounds

Recall that $\|H(t)\|_F \leq B$ for all $t$, for some constant $B$. Thus, $\|H(t)\|_F^2 \leq (\|H(t)\|_F + \|H(t) - H(t)\|_F)^2 \leq (B + \delta)^2$.

Lemma 2. In Algorithm 7 if $Z(t) \geq V B^2$, then $Q(t) = 0$.

Proof: Suppose the SVD of $H^t(t)H(t)$ is given by $H^t(t)H(t) = U^t\Sigma^tU$, where diagonal matrix $\Sigma$ has non-negative diagonal entries $\sigma_1, \ldots, \sigma_{N_T}$. Note that $\sigma_i \leq \text{tr}(H^t(t)H(t)) \leq \|H(t)\|_F^2 \leq (B + \delta)^2$ where (a) follows from $\text{tr}(H^t(t)H(t)) = \sum_{i=1}^{N_T} \sigma_i$; and (b) follows from Fact 1. By Lemma 1, $Q(t) = U^t\Theta^tU$, where $\Theta^*$ is a diagonal matrix with entries $\theta_1^*, \ldots, \theta_{N_T}^*$ given by $\theta_i^* = \max \{0, \frac{1}{\mu^* + Z/V} - \frac{1}{\sigma_i} \}, \forall i \in \{1, 2, \ldots, N_T\}$, where $\mu^* \geq 0$. 

Since $\sigma_t \leq (B + \delta)^2, \forall i \in \{1, 2, \ldots, N_T\}$, we know that if $Z(t) \geq V(B + \delta)^2$, then $\frac{1}{\mu} Z(t) \geq \frac{1}{\sigma_t} \leq 0$ for all $\mu \geq 0$ and hence $\theta_t^* = 0, \forall i \in \{1, 2, \ldots, N_T\}$. ■

Lemma 3. Let $Z(t)$ be yielded by Algorithm 7. For all slots $t \in \{0, 1, 2, \ldots\}$, we have $Z(t) \leq V(B + \delta)^2 + (P - \bar{P})$.

Proof: By Lemma 2, $Z(t)$ can not increase if $Z(t) \geq V B^2$. If $Z(t) \leq V(B + \delta)^2$, then $Z(t + 1)$ is at most $V(B + \delta)^2 + (P - \bar{P})$ by the update equation of $Z(t + 1)$ and the instantaneous power constraint. ■

C. Performance of Algorithm 7

Let $Q^*(H)$ be an optimal solution to problem (2)-(4). Note that CDI is required to solve problem (2)-(4); $Q^*(H)$ is a mapping from channel states to transmit covariances; and $R^{opt} = E[\log \det(I + H Q^*(H) H^H)]$.

To settle the notation, we use $Q^*(h)$ to denote $Q^*(H(t))$, i.e., the transmit covariance at time $t$ selected by the optimal solution to problem (2)-(4) with perfect CDI and perfect instantaneous CSIT. The next lemma is the key to extend the standard drift-plus-penalty technique to deal with stochastic optimization with inaccurate “system state”.

Lemma 4. Let $Q(t)$ be yielded by Algorithm 7. At each time $t$, we have

$$V \log \det(I + H(t) Q(t) H^t(t)) - Z(t) \operatorname{tr}(Q(t)) \geq 2V P \sqrt{N_T}(2B + \delta)\delta.$$

Proof: See Appendix C. ■

Define a Lyapunov function $L(t) = \frac{1}{2} Z^2(t)$. The Lyapunov drift is given by $\Delta(t) = L(t + 1) - L(t)$. Combining Lemma 4 with the analysis method in the standard drift-plus-penalty technique yields the following lemma.

Lemma 5. Let $Q(t)$ be yielded by Algorithm 7. At each time $t$, we have $E[R(t)] \geq R^{opt} + \frac{1}{2} V \log \det(I + H(t) Q(t) H^t(t)) - Z(t) \operatorname{tr}(Q(t)) - 2P \sqrt{N_T}(2B + \delta)\delta$.

Proof: See Appendix D. ■

Now we are ready to present the main theorem on the performance of Algorithm 7.

Theorem 1. Fix $\epsilon > 0$ and define $V = (P + \bar{P})^2/(2\epsilon)$. Under Algorithm 7 we have for all $t > 0$:

$$\frac{1}{t} \sum_{\tau=0}^{t-1} E[R(\tau)] \geq R^{opt} - \epsilon - \phi(\delta)$$

where $\phi(\delta) = 2P \sqrt{N_T}(2B + \delta)\delta$ satisfying $\phi(\delta) \rightarrow 0$ as $\delta \rightarrow 0$, i.e., $\phi(\delta) \in O(\delta)$. In particular, the average expected utility is within $\epsilon + \phi(\delta)$ of $R^{opt}$ and the sample path time average power is within $\epsilon$ of its required constraint $\bar{P}$ whenever $t \geq \Omega(1/\epsilon^2)$.

Proof: Fix $t > 0$.

• **Proof of the first inequality:** For all slots $\tau \in \{0, 1, \ldots, t-1\}$, Lemma 5 guarantees that

$$E[R(\tau)] \geq R^{opt} + \frac{1}{2} V \log \det(I + H(t) Q(t) H^t(t)) - Z(t) \operatorname{tr}(Q(t)) - 2P \sqrt{N_T}(2B + \delta)\delta$$

Summing over $\tau \in \{0, 1, \ldots, t-1\}$ and dividing by $t$ gives:

$$\frac{1}{t} \sum_{\tau=0}^{t-1} E[R(\tau)] \geq R^{opt} + \frac{1}{2} V \sum_{\tau=0}^{t-1} \log \det(I + H(t) Q(t) H^t(t)) - \sum_{\tau=0}^{t-1} Z(t) \operatorname{tr}(Q(t)) - 2P \sqrt{N_T}(2B + \delta)\delta$$

where (a) follows from the definition that $Z(t) = \frac{1}{2} (Z(t) + 1)^2 - \frac{1}{2} Z^2(t)$; (b) follows by simplifying the telescope sum; (c) follows from $Z(0) = 0$ and $Z(t) \geq 0$; and (d) follows by substituting $V = \frac{(P + \bar{P})^2}{2\epsilon}$.

• **Proof of the second inequality:** For all slots $\tau \in \{0, 1, \ldots, t-1\}$, the update for $Z(\tau)$ satisfies:

$$Z(\tau + 1) = \max[0, Z(\tau) + \operatorname{tr}(Q(\tau)) - \bar{P}] \geq Z(\tau) + \operatorname{tr}(Q(\tau)) - \bar{P}$$

Rearranging terms gives:

$$\operatorname{tr}(Q(\tau)) - \bar{P} \leq Z(\tau + 1) - Z(\tau)$$

Summing over $\tau \in \{0, 1, \ldots, t-1\}$ and dividing by $t$ gives:

$$\frac{1}{t} \sum_{\tau=0}^{t-1} \operatorname{tr}(Q(\tau)) - \bar{P} \leq \frac{Z(t) - Z(0)}{t}$$

where (a) holds because $Z(0) = 0$ and $Z(t) \leq V(B + \delta)^2 + (P - \bar{P})$ by Lemma 3 and (b) follows by substituting $V = \frac{(P + \bar{P})^2}{2\epsilon}$.

Theorem 7 provides a sample path guarantee on average power, which is much stronger than the guarantee in (11). It
also shows that convergence time to reach an \( \epsilon + O(\delta) \) approximate solution is \( O(1/\epsilon^2) \). Typically, this is dramatically more efficient than the convergence time required to obtain even a coarse estimate of the joint distribution for the entries of \( \mathbf{H}(t) \). Indeed, if each channel entry \( h_{ij} \) were quantized into \( 1/\xi \) distinct levels, there would be \( (1/\xi)^{N_f N_r} \) different possible (quantized) matrix realizations. Waiting for \( (1/\xi)^{N_f N_r} \) slots would at best allow each of these realizations to appear once, which is still not enough for accurate estimation of the probabilities associated with each realization. Fortunately, Theorem 1 shows that such estimation is not needed.

**D. Discussion**

Note that even if we assume the distribution of \( \mathbf{H}(t) \) is known and \( \mathbf{Q}^*(\mathbf{H}) \) can be computed by solving problem (2)-(4), the power allocation policy \( \mathbf{Q}^*(\mathbf{H}) \) in general can not achieve \( R^{\text{opt}} \) and may even violate the average power constraints when only the approximate versions \( \tilde{\mathbf{H}}(t) \) are known. For example, consider a MIMO fading system with \( N_f \) feedback channels and shows that the performance in Theorem 1 based on inaccurate instantaneous CSIT is infeasible. In contrast, Algorithm 1 can attain the optimal power policy \( \tilde{\mathbf{H}}(t) \) at each slot \( t \), the average power constraint is violated and hence the power allocation scheme is infeasible. In contrast, Algorithm 1 can attain the performance in Theorem 1 based on inaccurate instantaneous CSIT and no CDIT.

**IV. DELAYED CSIT CASE**

Consider the case of delayed and inaccurate CSIT. At the beginning of each slot \( t \in \{0, 1, 2, \ldots \} \), channel \( \mathbf{H}(t) \) is known and only quantized channels of previous slots \( \tilde{\mathbf{H}}(\tau), \tau \in \{0, 1, \ldots, t - 1\} \) are known.

This is similar to the scenario of online optimization where the decision maker selects \( x(\tau) \in \mathcal{X} \) at each slot \( \tau \) to maximize an unknown reward function \( f(\tau) \) based on the information of previous reward functions \( f(\tau), \tau \in \{0, 1, \ldots, t - 1\} \). The goal is to minimize average regret \( \frac{1}{t} \sum_{\tau=0}^{t-1} f(x(\tau)) \) and take the best known average regret of online optimization with Lipschitz continuous and convex reward functions is \( O(\sqrt{t}) \) in [13].

The situation in the current paper is different from conventional online optimization because at each slot \( t \), the rewards of previous slots, i.e., \( R(\tau) = \log \det(\mathbf{I} + \mathbf{H}(\tau)\mathbf{Q}(\tau)\mathbf{H}^H(\tau)) \), \( \tau \in \{0, 1, \ldots, t - 1\} \), are still unknown due to the fact that the reported channels \( \tilde{\mathbf{H}}(\tau) \) are the approximate versions. Nevertheless, an online algorithm without using CDIT is developed in Algorithm 3.

Define \( \mathbf{Q}^* \in \tilde{\mathcal{Q}} \) as an optimal solution to problem (6)-(8), which depends on the (unknown) distribution for \( \mathbf{H}(t) \). Define

\[
R^{\text{opt}}(t) = \log \det(\mathbf{I} + \mathbf{H}(t)\mathbf{Q}^*\mathbf{H}^H(t))
\]

as the utility at slot \( t \) attained by \( \mathbf{Q}^* \).

**Algorithm 3 Dynamic Power Allocation with Delayed CSIT**

Let \( \gamma \geq 0 \) be a constant parameter and \( \mathbf{Q}(0) \in \mathcal{Q} \) be arbitrary. At each time \( t \in \{1, 2, \ldots \} \), observe \( \mathbf{H}(t - 1) \) and do the following:

- Let \( \tilde{\mathbf{D}}(t - 1) = \mathbf{H}(t - 1)(\mathbf{I}_{N_f} + \tilde{\mathbf{H}}(t - 1)\mathbf{Q}(t - 1)\tilde{\mathbf{H}}^H(t - 1))^{-1} \tilde{\mathbf{H}}(t - 1) \). Choose transmit covariance

\[
\mathbf{Q}(t) = \mathcal{P}_{\mathcal{Q}}[\mathbf{Q}(t - 1) + \gamma \tilde{\mathbf{D}}(t - 1)],
\]

where \( \mathcal{P}_{\mathcal{Q}}[\cdot] \) is the projection onto convex set \( \tilde{\mathcal{Q}} = \{ \mathbf{Q} \in \mathbb{S}_{+}^{N_f} : \text{tr}(\mathbf{Q}) \leq P \} \).

The next subsections analyze the performance of Algorithm 3 with inaccurate channels and shows that the performance degrades linearly with respect to the quantization error \( \delta \). If \( \delta = 0 \), then (15) and (16) are recovered.

**A. Transmit Covariance Updates in Algorithm 3**

This subsection shows the \( \mathbf{Q}(t) \) selection in Algorithm 3 can be easily solved and has an (almost) closed-form solution.

The projection operator involved in Algorithm 3 by definition is

\[
\min_{\mathbf{Q}} \frac{1}{2} \| \mathbf{Q} - \mathbf{X} \|_F^2 \quad \text{s.t.} \quad \text{tr}(\mathbf{Q}) \leq P \quad \mathbf{Q} \in \mathbb{S}^{N_f}_{+}
\]

where \( \mathbf{X} = \mathbf{Q}(t - 1) + \gamma \tilde{\mathbf{D}}(t - 1) \) is an Hermitian matrix at each time \( t \).

Without constraint \( \text{tr}(\mathbf{Q}) \leq P \), the projection of Hermitian matrix \( \mathbf{X} \) onto the positive semidefinite cone \( \mathbb{S}^{N_f}_{+} \) is simply taking the eigenvalue expansion of \( \mathbf{X} \) and dropping terms associated with negative eigenvalues (see Section 8.1.1. in [29]).

Work [21] considered the projection onto the intersection of the positive semidefinite cone \( \mathbb{S}^{N_f}_{+} \) and an affine subspace given by \( \{ \mathbf{Q} : \text{tr}(\mathbf{A}_{ij}\mathbf{Q}) = b_{ij}, i \in \{1, 2, \ldots, p\}, \text{tr}(\mathbf{B}_{ij}\mathbf{Q}) \leq d_{ij}, j \in \{1, 2, \ldots, m\} \} \) and developed the dual-based iterative numerical algorithm to calculate the projection. Problem (17)-(19) is a special case, where the affine subspace is given by \( \text{tr}(\mathbf{Q}) \leq P \), of the projection considered in [21]. Instead of solving problem (17)-(19) using numerical algorithms, the next lemma summarizes that problem (17)-(19) has an (almost) closed-form solution.
Lemma 6. Consider SVD $X = U^H\Sigma U$, where $U$ is a unitary matrix and $\Sigma$ is a diagonal matrix with entries $\sigma_1, \ldots, \sigma_N$. Then the optimal solution to problem \((17)-(19)\) is given by $Q^* = U^H\Theta^*U$, where $\Theta^*$ is a diagonal matrix with entries $\theta_1^*, \ldots, \theta_N^*$ given by,
\[
\theta_i^* = \max[0, \sigma_i - \mu^*], \forall i \in \{1, 2, \ldots, N\},
\]
where $\mu^*$ is chosen such that $\sum_{i=1}^N \theta_i^* \leq \bar{P}$, $\mu^* \geq 0$ and $\mu^* \leq \frac{1}{\sum_{i=1}^N \theta_i} = \bar{P}$. The exact $\mu^*$ can be determined with complexity $O(N_T \log N_T)$, described in Algorithm 2.

**Proof:** See Appendix [2] for details.

---

**Algorithm 4** Algorithm to solve problem \((17)-(19)\)

1. Check if $\sum_{i=1}^{N_T} \max[0, \sigma_i] \leq \bar{P}$ holds. If yes, let $\mu^* = 0$ and $\theta_i^* = \max[0, \sigma_i]$, $\forall i \in \{1, 2, \ldots, N_T\}$ and terminate the algorithm; else, continue to the next step.

2. Sort all $\sigma_i \in \{1, 2, \ldots, N_T\}$ in a decreasing order $\sigma$ such that $\sigma(1) \geq \sigma(2) \geq \cdots \geq \sigma(N_T)$. Define $S_0 = 0$.

3. For $i = 1$ to $N_T$:
   - Let $S_i = S_{i-1} + \sigma_i$. Let $\mu^* = S_i - \bar{P}$.
   - If $\mu^* \geq 0$, $\sigma(1) - \mu^* > 0$ and $\sigma(i+1) - \mu^* < 0$, then terminate the loop; else, continue to the next iteration in the loop.

4. Let $\theta_i^* = \max[0, \sigma_i - \mu^*], \forall i \in \{1, 2, \ldots, N_T\}$ and terminate the algorithm.

---

**B. Property of $\bar{D}(t-1)$**

Define $D(t-1) = H^H(t-1)(I_{N_R} + H(t-1)Q(t-1)H^H(t-1))^{-1}H(t-1)$, which is the gradient of $R(t-1)$ at point $Q(t-1)$ and is unknown to the transmitter due to the unavailability of $H(t-1)$. The next lemma relates $\bar{D}(t-1)$ and $D(t-1)$.

**Lemma 7.** For all slots $t \in \{1, 2, \ldots\}$, we have

1. $\|D(t-1)\|_F \leq \sqrt{N_R}B^2$.
2. $\|\bar{D}(t-1) - D(t-1)\|_F \leq \psi(\delta)$, where $\psi(\delta) = (\sqrt{N_R}B + \sqrt{N_R}(B + \delta) + (B + \delta)^2 B^2P(2B + \delta))\delta$, satisfying $\psi(\delta) \rightarrow 0$ as $\delta \rightarrow 0$, i.e., $\psi(\delta) \in O(\delta)$.
3. $\|\bar{D}(t-1)\|_F \leq \psi(\delta) + \sqrt{N_R}B^2$

**Proof:** See Appendix [5]

---

**C. Performance of Algorithm [5]**

**Theorem 2.** Fix $\epsilon > 0$ and define $\gamma = \epsilon$. Under Algorithm [5] we have for all $t > 0$:

\[
\frac{1}{t} \sum_{\tau=0}^{t-1} R(\tau) \geq \frac{1}{t} \sum_{\tau=0}^{t-1} R^{opt}(\tau) - \frac{\bar{P}}{c\epsilon} \left( \frac{\psi(\delta)}{} + \frac{\sqrt{N_R}B^2}{2} \right) - 2\psi(\delta)\bar{P}
\]

\[
\frac{1}{t} \sum_{\tau=0}^{t-1} tr(Q(\tau)) \leq \bar{P}
\]

where $\psi(\delta)$ is the constant defined in Lemma [7]. In particular, the sample path time average utility is within $\epsilon + 2\psi(\delta)\bar{P}$ of the optimal average utility for problem \((9)-(13)\) whenever $t \geq \Omega(1/\epsilon^2)$.

**Proof:** The second inequality trivially follows from the fact that $Q(t) \in \bar{Q}, \forall t \in \{0, 1, \ldots\}$. It remains to prove the first inequality. This proof extends the regret analysis of conventional online convex optimization [13] by considering inexact gradient $\bar{D}(t-1)$.

For all slots $t \in \{1, 2, \ldots\}$, the transmit covariance update in Algorithm [3] satisfies:

\[
\|Q(t) - Q^*\|^2_F \\
\leq \|P_Q[Q(t-1) + \gamma \bar{D}(t-1)] - Q^*\|^2_F
\]

\[
\leq \|Q(t-1) - Q^*\|^2_F + 2\gamma tr(D^H(t-1)(Q(t-1) - Q^*)) + \gamma^2 \|\bar{D}(t-1)\|^2_F
\]

\[
\leq \|Q(t-1) - Q^*\|^2_F + 2\gamma tr(D^H(t-1)(Q(t-1) - Q^*)) + \gamma^2 \|\bar{D}(t-1) - D(t-1)\|^2_F(\bar{Q}(t-1) - Q^*)
\]

\[
\leq \|Q(t) - Q^*\|^2_F + \gamma^2 \|\bar{D}(t-1)\|^2_F,
\]

where (a) follows from the non-expansive property of projections onto convex sets. Define $\Delta(t) = \|Q(t) - Q^*\|^2_F - \|Q(t) - Q^*\|^2_F$. Rearranging terms in the last equation and dividing by factor $2\gamma$ implies

\[
tr(D^H(t-1)(Q(t-1) - Q^*)) \\
\leq \frac{1}{2\gamma} \Delta(t) - \frac{\gamma}{2} \|\bar{D}(t-1)\|^2_F \\
- tr((\bar{D}(t-1) - D(t-1))H(\bar{Q}(t-1) - Q^*))
\]

Define $f(Q) = \log det(I + H(t-1)QH^H(t-1))$. By Fact [5] $f(\cdot)$ is concave over $\bar{Q}$. Note that $D(t-1) = \nabla_Q f(Q(t-1))$. By Fact [3] and $Q^* \in \bar{Q}$. By Fact [3] we have

\[
f(Q(t-1)) - f(Q^*) \geq tr(D^H(t-1)(Q(t-1) - Q^*)
\]

\[
\geq \frac{1}{2\gamma} \Delta(t) - \frac{\gamma}{2} \|\bar{D}(t-1)\|^2_F
- tr((\bar{D}(t-1) - D(t-1))H(\bar{Q}(t-1) - Q^*))
\]

Note that $f(Q(t-1)) = R(t-1)$ and $f(Q^*) = R^{opt}(t-1)$. Combining (20) and (21) yields

\[
R(t-1) - R^{opt}(t-1)
\geq \frac{1}{2\gamma} \Delta(t) - \frac{\gamma}{2} \|\bar{D}(t-1)\|^2_F
- tr((\bar{D}(t-1) - D(t-1))H(\bar{Q}(t-1) - Q^*))
\]

\[
\geq \frac{1}{2\gamma} \Delta(t) - \frac{\gamma}{2} \|\bar{D}(t-1)\|^2_F
- \|\bar{D}(t-1) - D(t-1)\|_F \|Q(t-1) - Q^*\|_F
\]

\[
\geq \frac{1}{2\gamma} \Delta(t) - \frac{\gamma}{2} \|\psi(\delta) + \sqrt{N_R}B^2\|^2_F - 2\psi(\delta)\bar{P}
\]

where (a) follows from Fact [5] and (b) follows from Lemma [7] and the fact that $\|Q(t-1) - Q^*\|_F \leq \|\bar{Q}(t-1)\|_F + \|Q^*\|_F \leq \|Q(t-1)\| + tr(Q^*) \leq 2\bar{P}$, which is implied by Fact [5].
Lemma 8. \( R(\tau) \geq R^{opt}(\tau) - \frac{1}{\gamma} \sum_{\tau=0}^{\tau-1} \Delta(\tau) - \frac{\sqrt{N}R^2}{2} - 2\psi(\delta)\bar{P} \) \( \text{for all } \tau \in \{0,1,\ldots\} \).

Proof: The second inequality still follows from the fact that \( Q(t) \in [0,\bar{Q}] \), \( \forall t \in \{0,1,\ldots\} \). It remains to prove the first inequality. With \( \gamma(t) = \frac{1}{\tau} \), equation (22) in the proof of Theorem 2 becomes \( R(\tau) - R^{opt}(\tau) \geq \frac{1}{\tau} \sum_{\tau=0}^{\tau-1} \Delta(\tau) - \frac{1}{\sqrt{\tau}}(\psi(\delta) + \sqrt{N}R^2) - 2\psi(\delta)\bar{P} \) for all \( \tau \in \{0,1,\ldots\} \). Fix \( t > 0 \).

Summing over \( \tau \in \{0,1,\ldots,t-1\} \) and dividing by factor \( t \) yields that for all \( t > 0 \):

\[
\frac{1}{t} \sum_{\tau=0}^{\tau-1} R(\tau) - \frac{1}{t} \sum_{\tau=0}^{\tau-1} R^{opt}(\tau) \geq \frac{1}{2t} \sum_{\tau=0}^{\tau-1} \Delta(\tau) - \frac{1}{\sqrt{\tau}}(\psi(\delta) + \sqrt{N}R^2) - 2\psi(\delta)\bar{P}
\]

where the last inequality follows because \( \|Q(0) - Q^*\|_F \leq \|Q(\tau) - Q^*\|_F \leq \|Q(0) - Q^*\|_F + \|Q^* - Q^*\|_F \leq \text{tr}(Q(0)) + \text{tr}(Q^*) \leq 2\bar{P} \) and \( \|Q(t) - Q^*\|_F \geq 0 \).

Similar to the instantaneous CSIT case, Theorem 2 also isolates the effects of delay and channel inaccuracy. The observation is that the effect of CSIT delay vanishes as Algorithm 3 runs for a sufficiently long time. In some sense, delayed but accurate CSIT is almost as good as perfect CDIT. In contrast, the effect of CSIT error does not vanish as Algorithm 3 runs for a sufficiently long time. The performance degradation due to channel inaccuracy scales linearly with respect to the channel error since \( \psi(\delta) \in O(\delta) \). Intuitively, this is reasonable since the power allocation based on inaccurate CSIT is actually optimizing another different MIMO system.

D. Extensions

1) T-Slot Delayed and Inaccurate CSIT: Thus far, we have assumed that CSIT is always delayed by one slot. In fact, if CSIT is delayed by \( T \) slots, we can modify the update of transmit covariances in Algorithm 3 as \( Q(t) = \mathcal{P}_D(Q(t-T) + \gamma \mathcal{D}(t-T)) \). A T-slot version of Theorem 2 can be similarly proven.

2) Algorithm 3 with Time Varying \( \gamma \): Algorithm 3 can be extended to have time varying step size \( \gamma(t) = \frac{1}{\tau} \) at time \( t \). The next lemma shows that such an algorithm can approach an \( \epsilon + 2\psi(\delta)\bar{P} \) approximate solution with convergence time \( O(1/\epsilon^2) \).

Lemma 8. Fix \( \epsilon > 0 \). If we modify Algorithm 3 by using \( \gamma(t) = \frac{1}{\tau} \) as the step size \( \gamma \) at each time \( t \), then we have for all \( t > 0 \):

\[
\frac{1}{t} \sum_{\tau=0}^{\tau-1} R(\tau) \geq \frac{1}{t} \sum_{\tau=0}^{\tau-1} R^{opt}(\tau) - \frac{\bar{P}}{\sqrt{t}} - \frac{1}{\sqrt{t}}(\psi(\delta) + \sqrt{N}R^2)^2 - 2\psi(\delta)\bar{P}
\]

and

\[
\frac{1}{t} \sum_{\tau=0}^{\tau-1} \text{tr}(Q(\tau)) \leq \bar{P}
\]

An advantage of time varying step sizes is the performance automatically gets improved as the algorithm runs and there is no need to restart the algorithm with a different constant step size if a better performance is demanded.

V. RATE ALLOCATION FOR DYNAMIC POWER ALLOCATION

To achieve the ergodic capacity characterized either by (2) or by (6)–(8), we also need to consider the rate allocation associated with the power allocation scheme, namely, we need to decide how many data are delivered at each slot. In the case when the accurate instantaneous CSIT is available, the transmitter can simply deliver \( \log \det(I + H(t)Q(t)H(t)^H) \) amount of data at slot \( t \) once \( Q(t) \) is decided. However, in the cases when the instantaneous CSIT is inaccurate or only delayed CSIT is available, the transmitter does not know the associated instantaneous channel capacity without knowing \( H(t) \). These cases belong to the representative communication scenarios where channels are unknown to the transmitter and rateless codes are usually used as a solution. To send \( N \) bits of source data, the rateless code keeps sending encoded information bits without knowing instantaneous channel capacity such that the
receiver can decode all $N$ bits as long as the accumulated channel capacity for sufficiently many slots is larger than $N$. Many practical rateless codes for scalar or MIMO fading channels have been designed in 22, 23, 24.

This section provides an information theoretical rate allocation policy based on rateless codes that can be combined with the dynamic power allocation algorithms developed in this paper.

The rate allocation scheme is as follows: Let $N$ be a large number. Encode $N$ bit source data with a capacity achieving code for a channel with capacity no less than $N$. At slot 0, deliver the above encoded data with transmit covariance $Q(0)$ given by Algorithm 1 or Algorithm 3. The receiver knows channel $H(0)$, calculates the channel capacity $R(0) = \log \det(I + H(0)Q(0)H^H(0))$; and reports back the scalar $R(0)$ to the transmitter. At slot 1, the transmitter removes the first $R(0)$ bits from the $N$ source data, encodes the remaining $N - R(0)$ bits with a capacity achieving code for a channel with capacity no less than $N - R(0)$; and delivers the encoded data with transmit covariance $Q(1)$ given by Algorithm 1 or Algorithm 3. The receiver knows channel $H(1)$, calculates the channel capacity $R(1) = \log \det(I + H(1)Q(1)H^H(1))$; and reports back the scalar $R(1)$ to the transmitter. Repeat the above process until slot $T$ such that $\sum_{t=0}^{T-1} R(t) > N$.

For the decoding, the receiver can decode all the $N$ bits in a reverse order using the idea of successive decoding 25. At slot $T-1$, since $N - \sum_{t=0}^{T-2} R(t) < R(T-1)$, that is, $N - \sum_{t=0}^{T-2} R(t) < R(T-1)$ source data are delivered over a channel with capacity $R(T-1)$, the receiver can decode all delivered data $(N - \sum_{t=0}^{T-2} R(t)$ bits) with zero error. Note that $N - \sum_{t=0}^{T-2} R(t) = R(T-2) + N - \sum_{t=0}^{T-2} R(t)$ bits are delivered at slot $T-2$ over a channel with capacity $R(T-2)$. The receiver subtracts the $N - \sum_{t=0}^{T-2} R(t)$ bits that are already decoded such that only $R(T-2)$ bits remain to be decoded. Thus, the $(T-2)$ bits can be decoded with zero error. Repeat this process until all $N$ bits are decoded.

Using the above rate allocation and decoding strategy, $N$ bits are delivered and decoded within $T - 1$ slots during which the sum capacity is $\sum_{t=1}^{T-1} R(t)$. When $N$ is large enough, the rate loss $\sum_{t=1}^{T-1} R(t) - N$ is negligible. This rate allocation scheme does not require $H(t)$ and only requires a scalar $R(t-1)$ feedback available at the transmitter at each slot $t$. (Note that if $R(t)$ is available at the transmitter with delay larger than one, we can extend the above rate allocation scheme in a way similar to what we did in Section IV-D)

VI. SIMULATIONS
A. A Simple MIMO System with Two Channel Realizations

Consider a $2 \times 2$ MIMO system with two equally likely channel realizations:

$$H_1 = \begin{bmatrix} 1.3131e^{1.9590\pi} & 2.3880e^{0.7104\pi} \\ 2.5567e^{1.2529\pi} & 2.8380e^{0.3845\pi} \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 1.4781e^{0.9674\pi} & 1.5291e^{0.1396\pi} \\ 0.0601e^{0.9849\pi} & 0.1842e^{1.9126\pi} \end{bmatrix}$$

This simple scenario is considered as a test case because, when there are only two possible channels with known channel probabilities, it is easy to design an offline baseline algorithm that computes optimal transmission probabilities. The goal is to show that the proposed algorithms (which do not have channel distribution information) come close to this baseline. The proposed algorithms can be implemented just as easily in cases when there are an infinite number of possible channel state matrices, rather than just two. However, in that case it is difficult to compare to the baseline algorithm because offline computation of the required baseline probabilities is difficult. 3

The power constraints are $P = 2$ and $P = 3$. If CSIT has error, $H_1$ and $H_2$ are observed as $\tilde{H}_1$ and $\tilde{H}_2$, respectively. We consider two CSIT error cases. CSIT Error Case 1: $\tilde{H}_1 = \begin{bmatrix} 1.3131e^{2\pi} & 2.3880e^{0.775\pi} \\ 2.5567e^{1.5\pi} & 2.8380e^{0.5\pi} \end{bmatrix}$ and $\tilde{H}_2 = \begin{bmatrix} 1.4781e^{1\pi} & 1.5291e^{0.25\pi} \\ 0.0601e^{1\pi} & 0.1842e^{2\pi} \end{bmatrix}$, where the magnitudes are accurate but the phases are rounded to nearest $\pi/4$ phase; CSIT Error Case 2: $\tilde{H}_1 = \begin{bmatrix} 1.3e^{2\pi} & 2.4e^{0.5\pi} \\ 2.6e^{1.5\pi} & 2.8e^{0.5\pi} \end{bmatrix}$ and $\tilde{H}_2 = \begin{bmatrix} 1.5e^{1\pi} & 1.5e^{0\pi} \\ 0.2e^{2\pi} \end{bmatrix}$, where the magnitudes are rounded to the first digit after the decimal point and the phases are rounded to the nearest $\pi/2$ phase.

In the instantaneous CSIT case, consider Baseline 1 where the optimal solution $Q^*(H)$ to problem (2)-(4) is calculated by assuming the knowledge that $H_1$ and $H_2$ appears with equal probabilities and $Q(t) = Q^*(H(t))$ is used at each time $t$.

Figure 1 compares the performance of Algorithm 4 (with $V = 100$) under various CSIT accuracy conditions and Baseline 1. It can be seen that Algorithm 4 has a performance close to that attained by the optimal power allocation problem (2)-(4) requiring channel distribution information. (Note that a larger $V$ can gives a even closer performance with a larger convergence time.) It can also be observed that the performance of Algorithm 4 becomes worse as CSIT error gets larger.

In the delayed CSIT case, consider Baseline 2 where the optimal solution $Q^*$ to problem (6)-(8) is calculated by assuming the knowledge that that $H_1$ and $H_2$ appears with equal probabilities; and $Q(t) = Q^*$ is used at each time $t$.

Figure 2 compares the performance of Algorithm 3 (with $\gamma = 0.01$) under various CSIT accuracy conditions and Baseline 2. Note that the average power is not drawn since all schemes guarantee the average power constraints are satisfied for all $t$. It can be seen that Algorithm 3 has a performance close to that attained by the optimal power allocation to problem (6)-(8) requiring channel distribution information. (Note that a smaller $\gamma$ can gives a even closer performance with a larger convergence time.) It can also be observed that

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4This is known as the curse of dimensionality for stochastic optimization due to the large sample size. That is, even with perfect CDIT, problem (2)-(4) and problem (6)-(8) can be numerically hard to solve when the sample size of $H$ is large. In contrast, the dynamic algorithms proposed in this paper can deal with problems even with an infinite number of samples and the performance guarantees are independent of the sample size.
the performance of Algorithm 1 becomes worse as CSIT error gets larger.

B. A MIMO system with continuous channel realizations

This section considers a $2 \times 2$ MIMO system with continuous channel realizations. Each entry in $\mathbf{H}(t)$ is equal to $u + iv$ where $u$ is a complex number whose real part and complex part are standard normal and $v$ is uniform over $[0, 0.5]$. In this case, even if the channel distribution information is perfectly known, problem (2)-(4) and problem (6)-(8) are infinite dimensional problem and is extremely hard to solve. In practice, to solve the stochastic optimization, people usually approximate the continuous distribution by a discrete distribution with a reasonable number of realizations and solve the approximated optimization that is a large scale deterministic optimization problem. (Baselines 3 and 4 considered below are essentially using this idea.)

In the instantaneous CSIT case, consider Baseline 3 where we spend 100 slots to record 100 accurate CSIT observations $\mathcal{H} = \{\mathbf{H}_1, \ldots, \mathbf{H}_{100}\}$ and obtain an empirical probability of the channel ($\mathcal{I}$), obtain the optimal solution $\mathbf{Q}^*(\mathbf{H})$, $\mathbf{H} \in \mathcal{H}$ to (2)-(4) using the empirical probability; choose $\mathbf{Q}^*(\mathbf{H})$ where $\mathbf{H} = \arg\min_{\mathbf{H} \in \mathcal{H}} \| \mathbf{H} - \mathbf{H}(t) \|_F$ at each time $t$.

Figure 3 compares the performance of Algorithm 1 (with $V = 100$) and Baseline 3; and shows that Algorithm 1 has a better performance than Baseline 3, which allocates power based on estimated channel distribution.

VII. Conclusion

This paper considers dynamic power allocation in MIMO fading systems without CDIT. Two different dynamic power policies are proposed to deal with the cases of instantaneous CSIT and delayed CSIT, respectively. In both cases, the proposed dynamic power policies can achieve $O(\delta)$ sub-optimality, where $\delta$ is the CSIT error.

APPENDIX A

Proof of Fact 4 in Section II.C

Recall that a function is concave if and only if it is concave when restricted to any line along its domain (see page 67 in [20]). For any $\mathbf{Q}, \mathbf{Q} \in \mathbb{S}_+^n$, define $g(t) = f(\mathbf{Q} + t(\mathbf{Q} - \mathbf{Q}))$. Thus, $g(t)$ is concave over $[0, 1]$; $g(0) = f(\mathbf{Q})$; and $g(1) = f(\mathbf{Q})$. Note that $g'(t) = \text{tr}[(\nabla f(\mathbf{Q} + t(\mathbf{Q} - \mathbf{Q})))]^\mathbf{H}(\mathbf{Q} - \mathbf{Q})$.

By doing so, 100 slots are wasted without sending any data. The 100 slots are not counted in the simulation. If they are counted, Algorithm 1's performance advantage over Baseline 3 is even bigger. The delayed CSIT case is similar.
by the chain rule of derivatives when the inner product in complex matrix space \( \mathbb{C}^{n \times n} \) is defined as \( \langle A, B \rangle = \text{tr}(A^H B) \), \( \forall A, B \in \mathbb{C}^{n \times n} \). By the first-order condition of concave function \( g(t) \), we have \( g(1) \leq g'(0)(1 - 0) \). Note that \( g'(0) = \text{tr}([\nabla g(f)]^H (Q - Q)) \). Thus, we have \( f(\hat{Q}) \leq f(Q) + \text{tr}([\nabla g(f)]^H (Q - Q)) \).}

### Appendix B

#### PROOF OF LEMMA 1

The proof method is an extension of Section 3.2 in [2], which gives the structure of the optimal transmit covariance in deterministic MIMO channels.

Note that \( \log \det(I + HQH^H) = \text{tr}(A^H B) \), \( \forall A, B \in \mathbb{C}^{n \times n} \). By the elementary identity \( \text{det}(I + AB) = \text{det}(I + BA) \), \( \forall A \in \mathbb{C}^{m \times n} \). and \( B \in \mathbb{C}^{n \times m} \). and (b) follows from the fact that \( H^H H = U H \Sigma U^H \). Define \( \hat{Q} = U Q U^H \), which is semidefinite positive if and only if \( Q \). Note that \( \text{tr}(\hat{Q}) = \text{tr}(U Q U^H) = \text{tr}(Q) \) by the fact that \( \text{tr}(A^H B) = \text{tr}(BA) \), \( \forall A \in \mathbb{C}^{m \times n} \), \( B \in \mathbb{C}^{n \times m} \). Thus, problem \((22)-(24)\) is equivalent to

\[
\max_{\hat{Q}} \quad \log \det(I + \Sigma^{1/2} \hat{Q} \Sigma^{1/2}) - \frac{1}{V} \text{tr}(\hat{Q}) \tag{23}
\]

\[
\text{s.t.} \quad \hat{Q} \in S_+^{N_T} \tag{24}
\]

#### Fact 5 (Hadamard’s Inequality, Theorem 7.8.1 in [26]). For all \( A \in S_+^n \), \( \det(A) \leq \prod_{i=1}^n A_{ii} \) with equality if \( A \) is diagonal.

The next claim can be proven using Hadamard’s inequality.

#### Claim 1. Problem \((23)-(25)\) has a diagonal optimal solution.

**Proof:** Suppose problem \((23)-(25)\) has a non-diagonal optimal solution given by matrix \( \hat{Q} \). Consider a diagonal matrix \( Q \) whose entries are identical to the diagonal entries of \( \hat{Q} \). Note that \( \text{tr}(\hat{Q}) = \text{tr}(Q) \). To show \( Q \) is a solution no worse than \( \hat{Q} \), it suffices to show that \( \log \det(I + \Sigma^{1/2} \hat{Q} \Sigma^{1/2}) \geq \log \det(I + \Sigma^{1/2} Q \Sigma^{1/2}) \). This is true because \( \det(I + \Sigma^{1/2} \hat{Q} \Sigma^{1/2}) = \prod_{i=1}^{N_T} (1 + Q_{ii} \sigma_i) = \prod_{i=1}^{N_T} (1 + \hat{Q}_{ii} \sigma_i) \geq \det(I + \Sigma^{1/2} Q \Sigma^{1/2}) \), where the last inequality follows from Hadamard’s inequality. Thus, \( \hat{Q} \) is a solution no worse than \( Q \) and hence optimal.

By Claim 1, we can consider \( \hat{Q} = \Theta = \text{diag}(\theta_1, \theta_2, \ldots, \theta_{N_T}) \) and problem \((23)-(25)\) is equivalent to

\[
\max_{\theta_i} \quad \sum_{i=1}^{N_T} \log(1 + \theta_i \sigma_i) - \frac{Z_{\text{max}}}{V} \sum_{i=1}^{N_T} \theta_i \tag{26}
\]

\[
\text{s.t.} \quad \theta_i \leq P \tag{27}
\]

If \( \theta_i > 0 \), \( \forall i \in \{1, 2, \ldots, N_T\} \).

Note that problem \((26)-(28)\) satisfies Slater’s condition. So the optimal solution to problem \((26)-(28)\) is characterized by KKT conditions \([20]\). The remaining part is similar to the derivation of the water-filling solution of power allocation in parallel channels, e.g., the proof of Example 5.2 in [20].

Introducing Lagrange multipliers \( \mu \in \mathbb{R}^+ \) for inequality constraint \( \sum_{i=1}^{N_T} \theta_i \leq P \) and \( \nu = [\nu_1, \ldots, \nu_{N_T}]^T \in \mathbb{R}^+ \) for inequality constraints \( \theta_i \geq 0, i \in \{1, 2, \ldots, N_T\} \). Let \( \theta^* = [\theta^*_1, \ldots, \theta^*_N_T]^T \) and \( (\mu^*, \nu) \) be any primal and dual optimal points with zero duality gap. By the KKT conditions, we have \( -\frac{\sigma_{ii}}{1+\theta_{ii} \sigma_i} + Z/V + \mu^* - \nu_i^* = 0, \forall i \in \{1, 2, \ldots, N_T\}; \sum_{i=1}^{N_T} \theta_i^* \leq P; \theta^*_i \geq 0, \forall i \in \{1, 2, \ldots, N_T\}; \nu_i^* \geq 0, \forall i \in \{1, 2, \ldots, N_T\} \).

Eliminating \( \nu_i^*, \forall i \in \{1, 2, \ldots, N_T\} \) in all equations yields \( \mu^* + Z/V \geq \frac{\sigma_{ii}}{1+\theta_{ii} \sigma_i} \), \( \forall i \in \{1, 2, \ldots, N_T\}; \sum_{i=1}^{N_T} \theta_i^* \leq P; \theta^*_i \geq 0, \forall i \in \{1, 2, \ldots, N_T\}; \nu_i^* \geq 0, \forall i \in \{1, 2, \ldots, N_T\} \).

For all \( i \in \{1, 2, \ldots, N_T\} \), we consider \( \mu^* + Z/V < \sigma_i \) and \( \mu^* + Z/V \geq \sigma_i \) separately.

1) If \( \mu^* + Z/V < \sigma_i \), then \( \mu^* + Z/V \geq \frac{\sigma_{ii}}{1+\theta_{ii} \sigma_i} \) holds only when \( \theta_{ii}^* > 0 \), which by \( (\mu^* + Z/V - \frac{\sigma_{ii}}{1+\theta_{ii} \sigma_i}) \theta^*_i \) implies that \( \mu^* + Z/V - \frac{\sigma_{ii}}{1+\theta_{ii} \sigma_i} = 0 \), i.e., \( \theta_{ii}^* = \frac{1}{\mu^* + Z/V - \sigma_{ii}} \).

2) If \( \mu^* + Z/V \geq \sigma_i \), then \( \theta_{ii}^* \) is impossible, because \( \theta_{ii}^* > 0 \) implies that \( \mu^* + Z/V - \frac{\sigma_{ii}}{1+\theta_{ii} \sigma_i} > 0 \), which together with \( \theta_{ii}^* > 0 \) contradict the slackness condition \( (\mu^* + Z/V - \frac{\sigma_{ii}}{1+\theta_{ii} \sigma_i}) \theta^*_i = 0 \).

Thus, if \( \mu^* + Z/V \geq \sigma_i \), we must have \( \theta_{ii}^* = 0 \).
Otherwise, we know \( \mu^* > 0 \). By the slackness condition, we must have \( \sum_{i=1}^{N_T} \theta_i = \sum_{i=1}^{N_T} \max \left( 0, \frac{1}{\mu^* - Z/V - 1/\sigma_i} \right) = P \). To find \( \mu^* > 0 \) such that \( \sum_{i=1}^{N_T} \max \left( 0, \frac{1}{\mu^* - Z/V - 1/\sigma_i} \right) = P \), we could apply a bisection search by noting that all \( \theta_i \) are decreasing with respect to \( \mu^* \).

Another algorithm of finding \( \mu^* \) is inspired by the observation that if \( \sigma_j \geq \sigma_k, \forall j, k \in \{1, 2, \ldots, N_T\} \), then \( \theta_j \geq \theta_k \). Thus, we first sort all \( \sigma_i \) in a decreasing order, say \( \pi \) is the permutation such that \( \sigma_{\pi(1)} \geq \sigma_{\pi(2)} \geq \cdots \geq \sigma_{\pi(N_T)} \); and then sequentially check if \( i \in \{1, 2, \ldots, N_T\} \) is the index such that \( \sigma_{\pi(i)} - \mu^* > 0 \) and \( \sigma_{\pi(i+1)} - \mu^* \leq 0 \). To check this, we first assume \( i \) is indeed such an index and solve the equation \( \sum_{j=1}^{i} \left[ \frac{1}{\mu^* - 1/\sigma_j} \right] = P \) to obtain \( \mu^* \). (Note that in Algorithm A.2 to avoid recalculating the partial sum for each \( i \), we introduce the parameter \( S_i = \sum_{j=1}^{i} \frac{1}{\pi_{\sigma(j)}} \) and update \( S_i \) incrementally. By doing this, the complexity of each iteration in the loop is only \( O(1) \). Then verify the assumption by checking if \( \frac{1}{\mu^* - 1/\sigma_j} \geq 0 \) and \( \frac{1}{\mu^* - 1/\sigma_j} \leq 0 \). This algorithm is described in Algorithm A.2.

**Appendix C: Proof of Lemma 4**

To prove this lemma, the following facts are useful.

Fact 6. For all \( X \in S_n^+ \), we have \( \| (I + X)^{-1} \|_F \leq \sqrt{n} \).

Proof: Since \( X \in S_n^+ \), matrix \( X \) has SVD \( X = U^H \Sigma U \), where \( U \) is unitary and \( \Sigma \) is diagonal with non-negative entries \( \sigma_1, \ldots, \sigma_n \) \( \). Then \( Y = (I + X)^{-1} = U^H \text{diag}(\frac{1}{1+\sigma_1}, \ldots, \frac{1}{1+\sigma_n}) U \) is Hermitian. Thus, \( \| (I + X)^{-1} \|_F = \sqrt{\text{tr}(Y^2)} = \sqrt{\sum_{i=1}^n (\frac{1}{1+\sigma_i})^2} \leq \sqrt{n} \).

Fact 7. For any \( H, \tilde{H} \in \mathbb{C}^{N_T \times N_T} \) with \( \| H \|_F \leq \delta \) and \( \| \tilde{H} - H \|_F \leq \delta \), we have \( \| H^H H - \tilde{H}^H \tilde{H} \|_F \leq (2 \delta + \delta^2) \).

Proof:
\[
\| H^H H - \tilde{H}^H \tilde{H} \|_F \\
\leq \| H^H H - H^H \tilde{H} + H^H \tilde{H} - \tilde{H}^H \tilde{H} \|_F \\
\leq \| H^H \|_F \| H - \tilde{H} \|_F + \| \tilde{H}^H - \tilde{H}^H \|_F \\
\leq \| H^H \|_F \| H - \tilde{H} \|_F + \| H^H - \tilde{H}^H \|_F (\| H - \tilde{H} \|_F + \| H \|_F) \\
\leq 2 \delta (2 \delta + \delta^2)
\]

where (a) and (c) follow from the triangular inequality of the Frobenius norm; and (b) follows from Fact 4.

Fix \( Z(t) \) and \( V \). Denote \( \phi(Q, H) = V \log \det(I + HQH^H) - Z(t) \text{tr}(Q) \).

Fact 8. For any \( Q \in S^{N_T} \), let \( Q = L^H L \) be the Cholesky decomposition of semidefinite positive matrix \( Q \). Then \( \phi(Q, H) = \psi(L, T) = V \log \det(I + LTL^H) - Z(t) \text{tr}(LTL^H) \) with \( T = H^H H \). Moreover, if \( L \) is fixed, then \( h(L, T) \) is a concave with respect to \( T \) and has gradient \( \nabla_T \psi(L, T) = VL^H(I + LTL^H)^{-1} L \).

Proof: Note that
\[
V \log \det(I + HQH^H) - Z(t) \text{tr}(Q) \\
= V \log \det(I + LH^H LH^H) - Z(t) \text{tr}(L^H L) \\
= V \log \det(I + LH^H LH^H) - Z(t) \text{tr}(L^H L) \\
\leq V \log \det(I + LH^H L) - Z(t) \text{tr}(L^H L) \\
= \psi(L, T, T)
\]

where (a) follows by applying the elementary identity \( \det(I + AB) = \det(I + BA) \) for any \( A \in \mathbb{C}^{m \times n} \) and \( B \in \mathbb{C}^{n \times m} \); (b) follows by the definition \( T = H^H H \).

Note that if \( L \) is fixed, then \( Z(t) \text{tr}(LTL^H) \) is a constant. It follows from Fact 3 that \( \psi(L, T) \) is concave with respect to \( T \) and has gradient \( \nabla_T \psi(L, T) = VL^H(I + LTL^H)^{-1} L \).

The above fact essentially follows from the symmetry between \( Q \) and \( H^H H \) (or equivalently, the symmetry between \( L \) and \( H \)) in the function \( \log \det(I + HQH^H) \).

Fix \( t > 0 \). Let \( H(t), \tilde{H}(t) \in \mathbb{C}^{N_T \times N_T} \) be any matrices satisfying \( \| H(t) \|_F \leq B \) and \( \| H(t) - \tilde{H}(t) \|_F \leq \delta \). The main proof of this lemma can be decomposed into 3 steps:

To show that \( \phi(Q(t), H(t)) \geq \phi(Q(t), \tilde{H}(t)) - 2 P \sqrt{N_T}(2B + \delta) \): Let \( Q(t) = L^T(t) L(t) \) be an Cholesky decomposition. Define \( T(t) = H^H(t) H(t) \) and \( \tilde{T}(t) = T(t) \). By Fact 8, we have \( \psi(L(t), T(t)) = \phi(Q(t), H(t)) \) and \( \psi(L(t), \tilde{T}(t)) = \phi(Q(t), \tilde{H}(t)) \); and \( \psi \) is concave with respect to \( T \). By Fact 4, we have
\[
\nabla_T \psi(L(t), T(t)) = V L^H(I + LTL^H)^{-1} L
\]

To show that \( \phi(Q(t), H(t)) \geq \phi(Q^*(t), H(t)) - 2 P \sqrt{N_T}(2B + \delta) \): This step simply follows from the fact that Algorithm 1 chooses \( Q(t) \) to maximize \( \phi(Q, H(t)) = V \log \det(I + H(t)Q(t)H^H(t)) - Z(t) \text{tr}(Q) \) and hence \( Q(t) \) should be no worse than \( Q^*(t) \).

To show that \( \phi(Q^*(t), \tilde{H}(t)) \geq \phi(Q^*(t), H(t)) - 2 P \sqrt{N_T}(2B + \delta) \): This step is similar to step 1. Let \( Q^*(t) = M^H(t) M(t) \) be an Cholesky decomposition. Define \( \tilde{Q}(t) = H^H(t) \tilde{H}(t) \) and \( \tilde{T}(t) = \tilde{H}^H(t) \tilde{H}(t) \). By
Fact [8] we have \( \psi(M(t), T(t)) = \phi(Q^*(t), H(t)) \) and
\( \psi(M(t), \tilde{T}(t)) = \phi(Q^*(t), \tilde{H}(t)) \); and \( \psi \) is concave with respect to \( T \). By Fact [3] we have
\[
\psi(M(t), \tilde{T}(t)) - \psi(M(t), T(t)) - \frac{1}{2} \langle P + \tilde{P} \rangle^2
\]
where (a) follows from Lemma 4.

The remaining part is similar to the analysis of the standard drift-plus-penalty technique. Taking expectations on both sides and rearranging terms yields
\[
\mathbb{E}[R(t)] \geq \mathbb{V}R^\text{opt} - \mathbb{E}[\langle Z(t) \rangle \text{tr}(\mathbf{Q}^*(t) - \bar{P})] + \mathbb{E}[\Delta(t)] - \frac{1}{2} \langle P + \tilde{P} \rangle^2 - 2\mathbb{V}\mathbb{P}^{\delta} \delta
\]
and
\[
\mathbb{E}[\langle \mathbf{R}^\text{opt} \rangle] - \mathbb{E}[\langle \mathbf{R} \rangle] \geq \frac{1}{2} \langle P + \tilde{P} \rangle^2 - 2\mathbb{V}\mathbb{P}^{\delta} \delta
\]
where (a) follows by noting that \( \mathbb{E}[\langle Z(t) \rangle \text{tr}(\mathbf{Q}^*(t) - \bar{P})]\mathbb{E}[\langle Z(t) \rangle] \leq 0 \), while the inequality follows from the fact that \( \mathbf{Q}^*(t) \) only depends on \( H(t) \) and is hence independent of \( Z(t) \) (that depends on \( \hat{H}(0), \ldots, H(t - 1) \)); and the inequality follows from \( Z(t) \geq 0 \) and \( E[\text{tr}((\mathbf{Q}^*(t) - \bar{P}))] \leq 0 \) since \( \mathbf{Q}^* \) is the optimal solution to problem (17)-(19).

Dividing by factor \( V \) on both sides yields
\[
\mathbb{E}[\langle \mathbf{R}^\text{opt} \rangle] = \mathbb{V}R^\text{opt} - \frac{1}{2} \mathbb{V}\mathbb{E}[\langle \Delta(t) \rangle] - \frac{1}{2} \langle P + \tilde{P} \rangle^2 - 2\mathbb{V}^{\delta} \delta
\]

Appendix D

Proof of Lemma 5

Recall \( L(t) = \frac{1}{2}Z^2(t) \) and \( \Delta(t) = L(t + 1) - L(t) \).

Lemma 9. At each time \( t \in \{0, 1, 2, \ldots\} \), we have
\[
-\Delta(t) \geq -Z(t)(\text{tr}(\mathbf{Q}(t)) - \bar{P}) - \frac{1}{2}(P + \tilde{P})^2. \tag{29}
\]

Proof: Fix \( t \in \{0, 1, 2, \ldots\} \). Note that \( Z(t + 1) = \max(0, Z(t) + \text{tr}(\mathbf{Q}(t)) - \bar{P}) \) implies that
\[
Z^2(t + 1) \geq (Z(t) + \text{tr}(\mathbf{Q}(t)) - \bar{P})^2
\]
\[
\leq Z^2(t) + 2Z(t)\text{tr}(\mathbf{Q}(t) - \bar{P}) + (\text{tr}(\mathbf{Q}(t)) - \bar{P})^2
\]
\[
= Z^2(t) + 2Z(t)\text{tr}(\mathbf{Q}(t) - \bar{P}) + (P + \tilde{P})^2
\]
where (a) follows from the fact that \( |\text{tr}(\mathbf{Q}(t)) - \bar{P}| \leq |\text{tr}(\mathbf{Q}(t)) + \tilde{P} \leq P + \tilde{P} \) where the last inequality follows from the instantaneous power constraint.

Rearranging the terms and dividing by factor 2 yields the desired result. \( \square \)

Adding \( V \log \det(I + H(t)\mathbf{Q}(t)\mathbf{H}^\dagger(t)) \) on both sides of (29) yields
\[
-\Delta(t) + V \log \det(I + H(t)\mathbf{Q}(t)\mathbf{H}^\dagger(t))
\]
\[
\geq V \log \det(I + H(t)\mathbf{Q}(t)\mathbf{H}^\dagger(t)) - Z(t)(\text{tr}(\mathbf{Q}(t)) - \bar{P})
\]
\[
- \frac{1}{2}(P + \tilde{P})^2
\]

Claim 2. If \( \hat{\mathbf{Q}} \) is an optimal solution to the following convex program:
\[
\begin{align*}
\min & \quad \frac{1}{2}\|\mathbf{Q} - \mathbf{S}\|^2_F \\
\text{s.t.} & \quad \text{tr}(\mathbf{Q}) \leq \bar{P} \\
& \quad \mathbf{Q} \in \mathbb{S}_+^N
\end{align*}
\] (30)

then \( \hat{\mathbf{Q}} = \mathbf{U}^\dagger \hat{\mathbf{\Theta}} \mathbf{U} \) is an optimal solution to problem (17)-(19).

Proof: This claim can be proven by contradiction. Let \( \hat{\mathbf{\Theta}} \) be an optimal solution to convex program (30)-(32) and
define $\hat{\bf Q} = \bf U^t \hat{\Theta} \bf U$. Assume that there exists $\hat{\bf Q} \in \mathbb{S}^n_+$ such that $\bf Q \neq \hat{\bf Q}$ and is a solution to problem (17)-(19) that is strictly better than $\bf Q$. Consider $\hat{\Theta} = \bf U^t \hat{\bf Q} \bf U$ and reach a contradiction by showing $\hat{\Theta}$ is strictly better than $\Theta$ as follows:

Note that $\text{tr}(\hat{\Theta}) = \text{tr}(\bf U^t \hat{\bf Q} \bf U) = \text{tr}(\hat{\bf Q}) \leq \bar{P}$, where the last inequality follows from the assumption that $\bf Q$ is solution to problem (17)-(19). Also note that $\hat{\Theta} \in \mathbb{S}^N_+$ since $\hat{\bf Q} \in \mathbb{S}^n_+$. Thus, $\hat{\Theta}$ is feasible to problem (30)-(32).

Note that $||\hat{\Theta} - \Sigma||_F = (a) ||\bf U^t \hat{\bf Q} \bf U - \bf U^t \hat{\bf Q} \bf U||_F = (b) ||\hat{\bf Q} - \bf X||_F < ||\hat{\bf Q} - \bf X||_F = (c) ||\bf U^t \hat{\bf Q} \bf U - \bf U^t \bf X \bf U||_F = (d) ||\hat{\bf Q} - \bf X||_F$, where (a) and (d) follow from the fact that Frobenius norm is unitary invariant, (b) follows from the fact that $\hat{\Theta} = \bf U^t \hat{\bf Q} \bf U$ and $\bf X = \bf U^t \bf X \bf U$; (c) follows from the fact that $\bf Q$ is strictly better than $\hat{\bf Q}$; and (e) follows from the fact that $\hat{\bf Q} = \bf U^t \hat{\bf Q} \bf U$ and $\bf X = \bf U^t \bf X \bf U$. Thus, $\Theta$ is strictly better than $\hat{\Theta}$. A contradiction!

Claim 3. The optimal solution to problem (30)-(32) must be a diagonal matrix.

Proof: This claim can be proven by contradiction. Assume that problem (30)-(32) has an optimal solution $\Theta$ that is not diagonal. Since $\Theta$ is positive semidefinite, all the diagonal entries of $\Theta$ are non-negative. Define $\hat{\Theta}$ as a diagonal matrix whose $i$-th diagonal entry is equal to the $i$-th diagonal entry of $\Theta$ for all $i \in \{1, 2, \ldots, N_T\}$. Note that $\text{tr}(\Theta) = \text{tr}(\hat{\Theta}) \leq \bar{P}$ and $\hat{\Theta} \in \mathbb{S}^n_+$. Thus, it is feasible to problem (30)-(32). Note that $||\Theta - \Sigma||_F < ||\Theta - \Sigma||_F$ since $\Sigma$ is diagonal. Thus, $\Theta$ is a solution strictly better than $\hat{\Theta}$. A contradiction! So the optimal solution to problem (30)-(32) must be a diagonal matrix.

By the above two claims, it suffices to assume that the optimal solution to problem (17)-(19) has the structure $\bf Q = \bf U^t \hat{\Theta} \bf U$, where $\hat{\Theta}$ is a diagonal with non-negative entries $\theta_1, \ldots, \theta_{N_T}$. To solve problem (17)-(19), it suffices to consider the following convex program:

$$\min \frac{1}{2} \sum_{i=1}^{N_T} (\theta_i - \sigma_i)^2 \quad (33)$$

$$\text{s.t. } \begin{array}{c}
\sum_{i=1}^{N_T} \theta_i \leq \bar{P} \\
\theta_i \geq 0, \forall i \in \{1, 2, \ldots, N_T\}
\end{array} \quad (34)$$

Note that problem (33)-(35) satisfies Slater’s condition. So the optimal solution to problem (33)-(35) is characterized by KKT conditions [20]. Introducing Lagrange multipliers $\mu \in \mathbb{R}^n$ for the inequality constraint $\sum_{i=1}^{N_T} \theta_i \leq \bar{P}$ and $\nu = [\nu_1, \ldots, \nu_{N_T}]^T \in \mathbb{R}^{N_T}$ for inequality constraints $\theta_i \geq 0, \forall i \in \{1, 2, \ldots, n\}$. Let $\nu^* = [\nu_{i1}, \ldots, \nu_{iN_T}]^T$ and $\mu^* = [\mu_{i1}, \ldots, \mu_{iN_T}]^T$. By KKT conditions, we have $\theta_i - \sigma_i + \mu^* - \nu^* = 0, \forall i \in \{1, 2, \ldots, n\}; \sum_{i=1}^{N_T} \theta_i \leq \bar{P}; \mu^* \geq 0; \mu^* \left[\sum_{i=1}^{N_T} \theta_i - \bar{P}\right] = 0; \nu^* \geq 0, \forall i \in \{1, 2, \ldots, N_T\}; \nu_i^* \theta_i^* = 0, \forall i \in \{1, 2, \ldots, N_T\}$.

Eliminating $\nu_i^*, \forall i \in \{1, 2, \ldots, N_T\}$ in all equations yields $\mu^* \geq \sigma_i - \theta_i^* ; i \in \{1, 2, \ldots, N_T\}; \sum_{i=1}^{N_T} \theta_i^* \leq \bar{P}; \mu^* \geq 0; \mu^* \left[\sum_{i=1}^{N_T} \theta_i^* - \bar{P}\right] = 0; \theta_i^* \geq 0, \forall i \in \{1, 2, \ldots, N_T\}; \nu_i^* \theta_i^* = 0, \forall i \in \{1, 2, \ldots, N_T\}$. For all $i \in \{1, 2, \ldots, N_T\}$, we consider $\mu^* < \sigma_i$ and $\mu^* \geq \sigma_i$ separately:

1) If $\mu^* < \sigma_i$, then $\mu^* \geq \sigma_i - \theta_i^*$ holds only when $\theta_i^* > 0$, by which $(\theta_i^* - \sigma_i + \mu^*) \theta_i^* = 0$ implies that $\theta_i^* = \sigma_i - \mu^*$.

2) If $\mu^* \geq \sigma_i$, then $\theta_i^* > 0$ is impossible, because $\theta_i^* > 0$ implies that $\theta_i^* - \sigma_i + \mu^* > 0$, which together with $\theta_i^* > 0$ contradicts the slackness condition $(\theta_i^* - \sigma_i + \mu^*) \theta_i^* = 0$. Thus, if $\mu^* \geq \sigma_i$, we must have $\theta_i^* = 0$.

Summarizing both cases, we have $\theta_i^* = \max[0, \sigma_i - \mu^*], \forall i \in \{1, 2, \ldots, N_T\}$, where $\mu^*$ is chosen such that $\sum_{i=1}^{N_T} \theta_i^* \leq \bar{P}$, $\mu^* \geq 0$ and $\mu^* \left[\sum_{i=1}^{N_T} \theta_i^* - \bar{P}\right] = 0$.

To find such $\mu^*$, we first check if $\mu^* = 0$. If $\mu^* = 0$ is true, the slackness condition $\mu^* \left[\sum_{i=1}^{N_T} \theta_i^* - \bar{P}\right] = 0$ is guaranteed to hold and we need to further require $\sum_{i=1}^{N_T} \theta_i^* = \sum_{i=1}^{N_T} \max[0, \sigma_i - \mu^*] \leq \bar{P}$. Thus, if $\mu^* = 0$ and only if $\sum_{i=1}^{N_T} \max[0, \sigma_i - \mu^*] \leq \bar{P}$, then verify the assumption by noting that all $\theta_i^*$ are decreasing with respect to $\mu^*$.

Another algorithm of finding $\mu^*$ is inspired by the observation that if $\sigma_j \geq \sigma_k, \forall j, k \in \{1, 2, \ldots, N_T\}$, then $\theta_j^* \geq \theta_k^*$. Thus, we first sort all $\sigma_i$ in a decreasing order, say $\pi$ is the permutation such that $\sigma_{\pi(1)} \geq \sigma_{\pi(2)} \geq \cdots \geq \sigma_{\pi(N_T)}$ and then sequentially check if $i \in \{1, 2, \ldots, N_T\}$ is the index such that $\sigma_{\pi(i)} - \mu^* \geq 0$ and $\sigma_{\pi(i+1)} - \mu^* < 0$. To check this, we first assume $i$ is indeed such an index and solve the equation $\sum_{j=1}^{i} [\sigma_{\pi(j)} - \mu^*] = \bar{P}$ to obtain $\mu^*$; (Note that in Algorithm 4, we avoid recalculating the partial sum $\sum_{j=1}^{i} \sigma_{\pi(j)}$ for each $i$, we introduce the parameter $S_i = \sum_{j=1}^{i} \sigma_{\pi(j)}$ and update $S_i$ incrementally. By doing this, the complexity of each iteration in the loop is only $O(1)$.) then verify the assumption by checking if $\mu^* \geq 0, \sigma_{\pi(i)} - \mu^* \geq 0$ and $\sigma_{\pi(i+1)} - \mu^* \leq 0$. The algorithm is described in Algorithm 4 and has complexity $O(N_T \log(N_T))$. The overall complexity is dominated by the step of sorting all $\sigma_i$.

APPENDIX F
PROOF OF LEMMA 7

A. Proof of part 1:

The boundedness of $D(t-1)$ can be shown as follows. $\|D(t-1)\|_F = \|H^t(t-1)I_{N_R} + H(t-1)Q(t-1)H^t(t-1)\|_F = \|H(t-1)\|_F \cdot \|I_{N_R} + H(t-1)Q(t-1)H^t(t-1)\|_F \leq \sqrt{N_R^2 B^2}$, where (a) follows from Fact 1 and (b) follows from $\|H(t-1)\|_F \leq B$ and Fact 6.
B. Proof of part 2:

To simplify the notation, this part uses $H$, $	ilde{H}$ and $Q$ to represent $H(t-1)$, $\tilde{H}(t-1)$ and $Q(t-1)$, respectively. Note that

\[
\|D(t-1) - \tilde{D}(t-1)\|_F \\
= \|H^t(I_{N_R} + HQH^t)^{-1}H - \tilde{H}^t(I_{N_R} + \tilde{H}Q\tilde{H}^t)^{-1}\tilde{H}\|_F \\
\leq \|H^t(I_{N_R} + HQH^t)^{-1}H - \tilde{H}^t(I_{N_R} + H\tilde{H}^t)^{-1}\tilde{H}\|_F \\
+ \|\tilde{H}^t(I_{N_R} + H\tilde{H}^t)^{-1}\tilde{H} - H\tilde{H}^t(I_{N_R} + HQH^t)^{-1}\|_F \\
+ \|H\tilde{H}^t(I_{N_R} + HQH^t)^{-1} - (I_{N_R} + H\tilde{H}^t)^{-1}\|_F \\
\leq \|H^t(I_{N_R} + HQH^t)^{-1} - (I_{N_R} + H\tilde{H}^t)^{-1}\|_F \leq \frac{1}{\sqrt{N_R}} \|H - \tilde{H}\|_F.
\]

(36)

where both inequalities follow from Fact 1.

Since $\|H\|_F \leq B$ and $\|\tilde{H} - H\| \leq \delta$, by Fact 1 we have $\|H\|_F \leq B + \delta$. By Fact 3 we have $\|H^t(I_{N_R} + HQH^t)^{-1}\|_F \leq \sqrt{N_R}$. The following lemma from [22] will be useful to bound $\|H^t(I_{N_R} + HQH^t)^{-1} - (I_{N_R} + H\tilde{H}^t)^{-1}\|_F$ from above.

Lemma 10 (Lemma 6 in [22]). Let $F: D \subseteq \mathbb{C}^{m \times n} \rightarrow \mathbb{C}^{p \times q}$ be a complex matrix-valued function defined on a convex set $D$, assumed to be continuous on $D$ and differentiable on the interior of $D$, with Jacobian matrix $D_X F(X)$. Then, for any given $X,Y \in D$, there exists some $t \in (0,1)$ such that $\|F(Y) - F(X)\|_F \leq \|D_X F(tY + (1-t)X)\|_{2,mat} \|Y - X\|_F$, where $\|A\|_{2,mat}$ denotes the spectral norm of $A$, i.e., the largest singular value of $A$.

Lemma 10 is essentially a mean value theorem for complex matrix valued functions. The next corollary is the complex matrix version of elementary inequality $|\frac{1}{x+y} - \frac{1}{x}| \leq |x-y|, \forall x, y \geq 0$ and follows directly from Lemma 10.

Corollary 1. Consider $F: S^n_+ \rightarrow S^n_+$ defined via $F(X) = (I_n + X)^{-1}$. Then, $\|F(Y) - F(X)\|_F \leq \frac{1}{\sqrt{N_R}} \|Y - X\|_F, \forall X, Y \in S^n_+$.

Proof: By [28, 22], $dX^{-1} = -X^{-1}(dX)X^{-1}$. Thus, $d(I + X)^{-1} = -(I + X)^{-1}(dX)(I + X)^{-1}$. By identity vec$(ABC) = (C^T \otimes A)$vec$(B)$, where $\otimes$ denotes the Kronecker product, we have $d\text{vec}(F(X)) = -(I + X)^{-1}T \otimes (I + X)^{-1}d\text{vec}(X)$. Thus, $D_X F(X) = -(I + X)^{-1}T \otimes (I + X)^{-1}$. Note that for any $X \in S^n_+$, $\|(I + X)^{-1}T \otimes (I + X)^{-1}\|_{2,mat} \leq \|(I + X)^{-1}T \otimes (I + X)^{-1}\|_F \leq \|(I + X)^{-1}\|_F \leq n$, where $(a)$ follows from the fact that $\|A \otimes B\|_F = \|A\|_F \cdot \|B\|_F, \forall A \in \mathbb{C}^{m \times n}, B \in \mathbb{C}^{n \times l}$ (see Exercise 28, page 253 in [29]); and $(b)$ follows Fact 6. Applying Lemma 10 yields $\|F(Y) - F(X)\|_F \leq n \|Y - X\|_F, \forall X, Y \in S^n_+$.

Applying the above corollary yields

$\|(I_{N_R} + HQH^t)^{-1} - (I_{N_R} + \tilde{H}Q\tilde{H}^t)^{-1}\|_F \leq \sqrt{N_R} \|H - \tilde{H}\|_F \leq \sqrt{N_R} (B + \delta)

where (a) follows from Corollary 1 and (b) and (c) follows from Fact 1 and (d) follows from the fact that $\|H\|_F \leq B$ and $\|H - \tilde{H}\|_F \leq \delta$, $\|\tilde{H}\|_F \leq B + \delta$, and the fact that $\|Q\|_F \leq \sqrt{N_R}$, which is implied by Fact 2 and $Q \in \mathbb{Q}$.

Plugging equations $\|H\|_F \leq B + \delta$, $\|\tilde{H}\|_F \leq B + \delta$ into equation (36) yields $\|D(t-1) - \tilde{D}(t-1)\|_F \leq \sqrt{N_R}B + \sqrt{N_R}(B + \delta) + (B + \delta)^2N_RP(B + \delta)$

C. Proof of part 3:

This part follows from $\|D(t-1)\|_F \leq \|D(t-1) - \tilde{D}(t-1)\|_F + \|D(t-1)\|_F$.

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