Neutrino-production of single pions: dipole description

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The light-cone distribution amplitudes for the axial current are derived within the instanton vacuum model (IVM), which incorporates nonperturbative effects including spontaneous chiral symmetry breaking. This allows to extend applicability of the dipole approach, usually used in the perturbative domain, down to $Q^2 \to 0$, where partially conserved axial current (PCAC) imposes a relation between the neutrino-production cross section and the one induced by pions. A dramatic breakdown of the Adler relation (AR) for diffractive neutrino-production of pions, caused by absorptive corrections, was revealed recently in [1]. Indeed, comparing with the cross section predicted by the dipole phenomenology at $Q^2 \to 0$ on a proton target we confirmed the sizable deviation from the value given by the AR, as was estimated in [1] within a simplified two-channel model. The dipole approach also confirms that in the black-disc limit, where the absorptive corrections maximize, the diffractive cross section ceases, on the contrary to the expectation based on PCAC.

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I. INTRODUCTION

Due to its $V$-$A$ shape the neutrino-hadron interactions possess a very rich structure. However, because of the smallness of the cross-sections until recently the experimental data have been scarce, mostly being limited to the total cross-sections. With the launch of the new high-statistics experiments like MINERνA at Fermilab [2], now the neutrino-hadron interactions may be studied with a better precision. The $V$-$A$ structure of the neutrino-quark vertices enables us to study simultaneously $\langle VV \rangle$, $\langle AA \rangle$ and $\langle VA \rangle$ correlators in the same process.

The properties of the vector current have been well studied in the processes of deep inelastic scattering (DIS) of leptons on protons and nuclei, deeply virtual Compton scattering (DVCS), real Compton scattering (RCS), and vector meson production. The standard approach in the description of such processes is based on the large-$Q^2$ factorization of the cross section into a process-dependent hard part, which is evaluated in pQCD, and a universal target-dependent soft part. The latter is extracted from fits to experimental data. Factorization, however, is not valid at small photon virtualities, where one can rely on the dispersion relation, or on the assumption of generalized vector meson dominance (GVMD) [3–6]. Such a description, however, involves a lot of ad-hoc modeling.

An alternative phenomenology for high-energy QCD processes is based on the color dipole approach [7]. One assumes that before interaction, the projectile (virtual $W$- or $Z$-boson in case of the neutrino scattering) fluctuates into a quark-antiquark dipole. After the dipole is formed, it scatters in the field of the target and then fluctuates back to the final hadron [7]. Recently the color dipole approach has been successfully applied to the description of different reactions with vector currents (see [8–27] and references therein).

For the axial current the situation is more complicated, especially at small $Q^2$, because the chiral symmetry breaking generates the near-massless pseudo-goldstone mesons (pions). Straightforward extension of the vector dominance model to the axial current leads to the so called Piketty-Stodolsky paradox [28, 29], which appears because axial meson dominance is broken by a large contribution of multipion singularities in the dispersion relation [29, 30]. The dipole description is free of this problem, because in this model there is no explicit hadronic degrees of freedom and the interaction occurs via dipole scattering.

According to the Adler theorem [31, 32], based on the hypothesis of partial conservation of axial current (PCAC), the neutrino-proton interactions cross-section at zero $Q^2$...
is proportional to the cross-section of the pure hadronic process, where the heavy intermediate boson is replaced by a pion,

$$\nu \frac{d\sigma_{\nu p \rightarrow F}}{d\nu dQ^2} \bigg|_{Q^2=0} = \frac{G_F^2}{2\pi^2} f_{\pi}^2 (1 - y) \sigma_{\pi p \rightarrow F}(\nu), \quad (1)$$

where $F$ denotes the final hadronic state; $y = \nu/E_{\nu}$; $E_{\nu}$ is the energy of the neutrino; and $\nu$ is the energy of the heavy boson $W$, or $Z$, in the target rest frame. The chiral symmetry is vital and should be embedded into any dynamical model which is used for calculation of the cross section at small $Q^2$.

In what follows we entirely neglect the lepton mass (accurate for neutral currents or electrons), which can be easily incorporated [28], but is dropped for the sake of simplicity. If one interpreted the Adler relation (AR) (Eq. (1)) in terms of pion pole dominance, one would arrive at a vanishing cross section. Indeed, the pion pole term in the amplitude contains a factor $q_\mu$, which multiplied by the conserved lepton current $l_\mu$ terminates this contribution [1, 28, 29, 33]. Other, heavier meson states provide a final contribution, but have to conspire in a way that all together they act like a small contribution [1, 28, 33].

Such a fine tuning looks miraculous if one has no clue of the underlying dynamics. Similar paradox is known for the neutrino-production of pions due to conservation of lepton current. This is why the color transparency effect has not been understood within the GVMD, but was revealed in the color dipole representation [2].

Similarly, in order to test the mysterious relation between the contribution of heavy hadronic fluctuations and pion, one should switch to the dipole representation and employ models for the distribution amplitudes (DA) of the axial current which have built-in chiral symmetry. Recently, we used the DA of the vector current calculated in the IVM for the evaluation of several processes [11, 14]. In this paper we apply the IVM to construct the DAs for the axial current and pion and use them to calculate the neutrino-production cross sections. Since the IVM includes spontaneous chiral symmetry breaking, the $qq$ DAs of axial current and pion should automatically satisfy PCAC, and in the small-$Q^2$ limit reproduce the Adler relation (1).

Notice that the color dipole description is valid only at high-energies or at small $x_B \ll 1$, where the contribution of quark exchange (reggeons) is suppressed as $1/\sqrt{\nu}$. For moderate energies another mechanisms, such as e.g. formation of resonances in the direct channel [36, 37], and/or reggeon exchange in the crossed channel may be important [38, 39]. In this paper we do not consider those corrections, but concentrate on the well developed small-x dipole phenomenology.

Experimentally, the neutrino-production of hadrons on protons and nuclei has been studied in the recent experiments K2K [40, 42], MiniBoone [43, 44], NuTeV [45-47] (see also review [29] and [30, 48–51] for references to earlier neutrino experiments). For high energy neutrino scattering, there are data from the early bubble chamber experiments [33, 52] with energies up to 100 GeV, though with low statistics and only for the total (integrated) cross-sections. Currently, with the launch of the high statistics experiment Minerva at Fermilab [2, 27], the precision of measurements should be considerably improved, and data for the differential cross-sections at high energies will become available.

In this paper we consider a particular process – diffractive single pion production on a proton target. As was demonstrated in [1] and below, this process provides a most sensitive way to test PCAC in high energy neutrino interactions. Besides, it generates an important background to the measurements of neutrino oscillations [53-55], and is also important for the neutrino astronomy of astrophysical and cosmological sources.

The paper is organized as follows. In Section VI we present the color dipole formalism. In Section IV we perform calculations of the DAs of the axial current and pion. In Section V we calculate the overlap of the DAs for the axial current and of the pion and found it to be proportional to $q_\mu$, what terminates this contribution to the neutrino-production of pions due to conservation of lepton current. In Section VI we present the numerical results and summarize the observations in Section VII.
II. DIFFRACTIVE PRODUCTION OF PIONS

The cross section of diffractive neutrino-production of a pion on a proton, $\nu p \to l\pi p$, has the form,

$$\frac{d^3\sigma_{\nu p \to l\pi p}}{d^2p d\Omega} = \frac{G_F^2 L_{\mu\nu} (W_{\mu}^A)^* W_{\nu}^A}{32\pi^3 m_N^2 E^2_\pi \sqrt{1 + Q^2/\nu^2}}$$

(2)

where $m_N$ is the nucleon mass; $L_{\mu\nu}$ is the lepton tensor; and $W_{\mu}^A$ is the amplitude of pion production by the axial current on the proton target. In the color dipole model this amplitude has the form

$$W_{\mu}^A (s, \Delta, Q^2) = \int_0^1 d\beta_1 d\beta_2 d^2r_1 d^2r_2 \Psi^\pi (\beta_2, r_2) \times A^d (\beta_1, \vec{r}_1; \beta_2, \vec{r}_2; \Delta) \Psi_0^A (\beta_1, \vec{r}_1)$$

(3)

where $\Psi^\pi$ and $\Psi_0^A$ are the distribution amplitudes (DAs) of the pion and axial current respectively, and $A^d(...)$ is the dipole scattering amplitude. The axial current DA $\Psi_0^A$ contains a pion pole, whose contribution to the amplitude is proportional to $q_0$, because the pion is spinless. This factor terminates the pion pole because of conservation of

the lepton current. As we assumed, the lepton is massless, otherwise the pion pole contribution is not zero and leads to corrections of the order of $O \left( \frac{m_{\pi}^2}{m_N^2 + Q^2} \right)$.

The amplitude $A^d (\beta_1, \vec{r}_1; \beta_2, \vec{r}_2; \Delta)$ in (3) depends on the initial and final quark transverse separations $\vec{r}_{1,2}$, fractional light-cone momenta $\beta_{1,2}$, and transverse momentum transfer $\Delta$. This is a universal function dependent only on the target but not on the initial and final states. In addition to the axial current contribution, in (3) there should be the contribution of the vector current. This contribution involves a poorly known helicity flip dipole amplitude $A_d$, which vanishes at high energies as $1/\nu$. Besides, at small $Q^2$, the vector current contribution is suppressed by a factor $Q^2$. At high energies, in the small angle approximation, $\Delta/\sqrt{s} \ll 1$, and the quark separation and fractional momenta $\beta$ are preserved, so

$$A^d (\beta_1, \vec{r}_1; \beta_2, \vec{r}_2; Q^2, x, \Delta) \approx \delta (\beta_1 - \beta_2) \delta (\vec{r}_1 - \vec{r}_2) \times (\epsilon + i) \Im m f_{qq}^N (\vec{r}, \Delta, \beta, x)$$

where $\epsilon$ is the ratio of the real to imaginary parts, and for the imaginary part of the elastic dipole amplitude we employ the model developed in [11, 57–59].

The phenomenological functions $\sigma_0 (x), R_2^0 (x)$ and $B (x)$ are fitted to DIS and $p$-electroproduction data. We rely here on the Bjorken variable $x = Q^2/2(pq)$, which has the meaning of fractional light-cone momentum of the parton only at large $Q^2$. At low $Q^2$ important for the axial current, one should switch to an energy dependent parametrization, as is explained in Section V.

For the forward scattering, $\Delta \to 0$, the imaginary part of the amplitude [4] reduces to the saturated parameterization of the dipole cross-section proposed by Golec-Biernat and Wüsthoff (GBW) [8],

$$\Im m f_{qq}^N (\vec{r}, \Delta, x) = \frac{\sigma_0 (x)}{4} \exp \left[ - \left( \frac{B (x)}{2} + \frac{R_2^0 (x)}{16} \right) \Delta^2 \right] \left( e^{-i\beta\vec{r}} \Delta + e^{(1-\beta)\vec{r}} \Delta - 2 e^{i(\frac{1}{2} - \beta)\vec{r}} \Delta e^{-\frac{\vec{r}^2}{R_2^0 (x)}} \right),$$

(5)

and

$$\sigma_d (r, x) = \Im m f_{qq}^N (\vec{r}, \Delta = 0, \beta, x) = \sigma_0 (x) \left[ 1 - \exp \left( -\frac{r^2}{R_2^0 (x)} \right) \right].$$

(6)

Generally speaking, the amplitude $f_{qq}^N (...)$ involves nonperturbative physics, but its asymptotic behavior at small $r$ is controlled by pQCD [5]:

$$f_{qq}^N (r, r \to 0) \propto r^2,$$

up to slowly varying factors $\sim \ln (r)$ [7].

Calculation of the differential cross section also involves the real part of scattering amplitude, whose relation to the
imaginary part is quite straightforward. According to [60], if \( \lim_{s \to \infty} \left( \frac{Im f}{s^\alpha} \right) \) is finite, then the real and imaginary parts of the forward amplitude are related as

\[
Re f(\Delta = 0) = s^\alpha \tan \left[ \frac{\pi}{2} \left( \alpha - 1 + \frac{\partial}{\partial \ln s} \right) \right] \frac{Im f(\Delta = 0)}{s^\alpha}.
\]

(7)

In the model under consideration the imaginary part of the forward dipole amplitude indeed has a power dependence on energy, \( Im f(\Delta = 0; s) \sim s^{\alpha - 1} \), where \( \alpha \) is the intercept of the effective Pomeron trajectory. Then Eq. (7) simplifies to

\[
\frac{Re A}{3m A} = \tan \left( \frac{\pi}{2} (\alpha - 1) \right) = \epsilon.
\]

(8)

This fixes the phase of the forward scattering amplitude, which we retain for nonzero momentum transfer, assuming similar \( \Delta \)-dependences for the real and imaginary parts.

### III. DISTRIBUTION AMPALITUDES AND THE INSTANTON VACUUM MODEL

In this section we define the DAs and give a brief description of the instanton model used for their evaluations (see [61] [62] and references therein).

#### A. Instanton vacuum model

The central object of the model is the effective action for light quarks in the instanton vacuum, which in the leading order in \( N_c \) has the form [62] [63]

\[
S = \int d^4x \left( \frac{N}{V} \ln \lambda + 2 \Phi^2(x) \right) - \hat{\psi} \left( \hat{\not{\!p}} + \hat{\not{\!v}} + \hat{\not{\!a}} \gamma_5 - m \not{\!v} \right) \psi
\]

(9)

where \( \Gamma_m \) is one of the matrices, \( \Gamma_m = 1, i\gamma^\tau, i\gamma^\tau \gamma_5; \psi \) and \( \Phi \) are the fields of constituent quarks and mesons respectively; \( N/V \) is the density of the instanton gas; \( m \approx 5 \) MeV is the current quark mass; \( \hat{v} \equiv v_\mu \gamma^\mu \) is the external vector current corresponding to the photon. \( L \) is the gauge factor defined as,

\[
L(x,z) = P \exp \left( i \int \frac{d\zeta}{z} (v_\mu (\zeta) + a_\mu (\zeta) \gamma_5) \right).
\]

(10)

It provides the gauge invariance of the action, and \( f(p) \) is the Fourier transform of the zero-mode profile in the single-instanton background. In this paper we used for evaluations the dipole-type parameterization [62]

\[
f(p) = \frac{L^2}{L^2 - p^2}
\]

with \( L \sim 850 \) MeV.

In the leading order in \( N_c \), we have the same Feynman rules as in the perturbative theory, but with a momentum-dependent quark mass \( \mu(p) \) in the quark propagator

\[
S(p) = \frac{1}{\not{p} - \mu(p) + i\delta}.
\]

(12)

The mass of the constituent quark has the form

\[
\mu(p) = m + M f^2(p),
\]

where \( m \approx 5 \) MeV is the current quark mass, \( M \approx 350 \) MeV is the dynamical mass generated by the interaction with the instanton vacuum background. Due to the presence of the instantons the vector current - quark coupling is also modified,

\[
\hat{v} \equiv v_\mu \gamma^\mu \rightarrow \hat{V} = \hat{v} + \hat{V}^{\text{nonl}},
\]

\[
\hat{a} \equiv a_\mu \gamma^\mu \rightarrow \hat{A} = \hat{a} + \hat{A}^{\text{nonl}},
\]

In addition to the vertices of the perturbative QCD, the model contains the nonlocal terms with higher-order couplings of currents to mesons. The exact expressions for the nonlocal terms \( V^{\text{nonl}}, A^{\text{nonl}} \) depend on the choice of the path in [10], so one can find different results in the literature [64] [67]. However, for the longitudinal parts of the axial and vector currents important here, this ambiguity cancels out and couplings have the form

\[
\hat{V}^{\text{nonl}} = v_\mu \left[ iM_1 \frac{p_1^\mu}{p_1^2 - p_1^2} \left( f(p_2) - f(p_1) \right)^2 \right],
\]

\[
\hat{A}^{\text{nonl}} = a_\mu \left[ iM_1 \frac{p_1^\mu + p_2^\mu}{p_1^2 - p_2^2} \left( f(p_2) - f(p_1) \right)^2 \right],
\]

where \( p_1, p_2 \) are the momenta of the initial and final quarks.

#### B. Axial current distribution amplitudes

The distribution amplitudes of the axial current are defined via 3-point correlators

\[
\Psi_\beta \sim \int d^4x e^{-iqx} \left< 0 \mid \hat{\psi}(y) \Gamma \psi(x) J_5^\beta(\xi) \mid 0 \right>,
\]

(13)
where $J_5^\mu(\xi)$ is the axial isovector current and $\Gamma$ is one of the Dirac matrices. Due to the spontaneous chiral symmetry breaking and existence of the near-massless pions the hadronic structure of the axial current differs from that of the vector current. In particular, the axial current may fluctuate into the pion state before production of the $\bar{q}q$ pair. Thus the correlator (13) has two contributions, schematically shown in the Figure 1.

\[ \psi(x) = \psi_\mu \gamma_\mu \psi(x) \]

FIG. 1: The distribution amplitude has two contributions, with intermediate heavy axial states and with a pion, labeled with (bulk) and (pion) respectively.

One term corresponds to the combined contribution of the intermediate heavy states ($a_1$ meson, $3\pi$, etc.), another one corresponds to the axial current fluctuating into a pion. Due to the built-in chiral symmetry, the two contributions are connected by PCAC, so for the full DA we have

\[ \Psi_\mu = \Psi_\mu^{(\text{bulk})} + \Psi_\mu^{(\text{pion})} = \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2 - m_\pi^2} \right) \Psi_\nu^{(\text{bulk})}. \]

This form of the DA reflects the relation between the pion pole and the bulk of heavy states contribution imposed by PCAC, which has been discussed above. In what follows we concentrate on the part of the amplitude presented in the dispersion relation by the bulk of heavy states excluding the pion pole (left pane in the Figure 1), tacitly assuming that the full distribution amplitudes are determined using (14).

For the part of the axial current presented by the bulk of heavy states, the DAs may be defined similar to the distribution amplitudes of the axial meson [68]:

\[
\langle 0 | \bar{\psi}(y) \gamma_\mu \gamma_5 \psi(x) | A(q) \rangle = i f_A^2 \int_0^1 d\beta e^{i \beta p \cdot y + \bar{\beta} p \cdot x} \times \left[ p_\mu \frac{e^{(\lambda)} \cdot z}{p \cdot z} \Phi_\parallel(\beta) + e^{(\lambda=\perp)} \gamma_\perp(\beta) - \frac{1}{2} \frac{e^{(\lambda)} \cdot z}{p \cdot z} f_A^2 \tilde{g}_\parallel(\beta) \right],
\]

\[
\langle 0 | \bar{\psi}(y) \gamma_\mu \psi(x) | A(q) \rangle = -i f_A^2 \varepsilon_{\mu\nu\rho\sigma} e^{(\nu)} p_\nu z_\sigma \int_0^1 d\beta e^{i(\beta p \cdot y + \bar{\beta} p \cdot x)} \frac{g^{(\nu)}(\beta)}{4},
\]

\[
\langle 0 | \bar{\psi}(y) \sigma_{\mu\nu} \gamma_5 \psi(x) | A(q) \rangle = f_A \int_0^1 d\beta e^{i(\beta p \cdot y + \bar{\beta} p \cdot x)} \left( e^{(\mu=\perp)} p_\nu - e^{(\nu=\perp)} p_\mu \right) \Phi_\perp(\beta) + \frac{e^{(\lambda)} \cdot z}{(p \cdot z)^2} f_A^2 (p_\mu z_\nu - p_\nu z_\mu) h^{(\parallel)}(\beta) + \frac{1}{2} \left( e^{(\lambda)} z_\nu - e^{(\nu)} z_\mu \right) \frac{f_A^2}{p \cdot z} h_\parallel(\beta),
\]

\[
\langle 0 | \bar{\psi}(y) \gamma_5 \psi(x) | A(q) \rangle = f_A^3 e^{(\lambda)} \cdot z \int_0^1 d\beta e^{i(\beta p \cdot y + \bar{\beta} p \cdot x)} \frac{h^{(\parallel)}(\beta)}{2}.
\]

where $q$ is the momentum carried by the axial current; $\beta$ is the fractional light-cone momentum; $\bar{\beta} = 1 - \beta$,
\( e^{(\lambda)} = e^{(\lambda)}(q) \) is the polarization vector of the axial meson with polarization vector \( \lambda \); \( x = x - y \); \( p_{\mu} \) is the “positive direction” vector on the light-cone; \( n_{\mu} \) is the “negative direction” vector on the light-cone. Light cone vectors \( p, n \) are chosen in such a way that the vector \( q \) does not have transverse components. The normalization constant \( f_A \) is a dimensional parameter introduced in order to make the distribution amplitudes dimensionless. Its value is fixed from the condition

\[
\int_0^1 d\beta \phi_{||}(\beta) = 1. \tag{19}
\]

We defined an “effective” axial state \(|A^{(\lambda)}(q)\rangle\) as

\[
|A^{(\lambda)}(q)\rangle = \int d^4 x e^{-iq \cdot x} e^{(\lambda)}(q) J_3^\lambda(x)|0\rangle. \tag{20}
\]

The DAs have the following twists: \( \Phi_{||}(\beta), \Phi_{\perp}(\beta) \) are twist-2; \( g_\perp, g_\parallel, h_\parallel, h_\perp \) are of twist-3; \( g_3, h_3 \) are of twist-4. All the wave functions in \([15,16]\) are chiral even; all the wave functions in \([17,18]\) are chiral odd.

### C. Pion distribution amplitudes

A spinless pion has only four independent DAs defined as \([69]\):

\[
\langle 0 | \bar{\psi}(y) \gamma_\mu \gamma_5 \psi(x) | \pi(q) \rangle = i f_\pi \int_0^1 d\beta e^{i(\beta p \cdot y + \beta p \cdot x)}
\times \left( p_{\mu} \phi_{2,\pi}(\beta) + \frac{1}{2} \frac{z_\mu}{(p \cdot z)} \psi_{4,\pi}(\beta) \right) \tag{21}
\]

\[
\langle 0 | \bar{\psi}(y) \gamma_5 \psi(x) | \pi(q) \rangle = -i f_\pi m_\pi^2 \left( \frac{m_u + m_d}{m_u + m_d} \right) \times \int_0^1 d\beta e^{i(\beta p \cdot y + \beta p \cdot x)} \phi_{3,\pi}^{(p)}(\beta), \tag{22}
\]

\[
\langle 0 | \bar{\psi}(y) \sigma_{\mu\nu} \gamma_5 \psi(x) | \pi(q) \rangle = -i \frac{f_\pi}{3} \frac{m_\pi^2}{m_u + m_d} \int_0^1 d\beta e^{i(\beta p \cdot x)}
\times \left( \frac{p_{\mu} z_\nu - p_{\nu} z_\mu}{p \cdot z} \right) \phi_{\mu,\nu}^{(\sigma)}(\beta) \tag{23}
\]

Twist counting is the following: \( \phi_{2,\pi} \) is a single twist-2 function (it was evaluated earlier in \([71,72]\)), \( \phi_{3,\pi}^{(p)} \) and \( \phi_{3,\pi}^{(\sigma)} \) are twist-3, \( \psi_{4,\pi} \) is the twist-4 DA.

The full expressions for the DAs \([15-18]\) and \([21-23]\) are given in Appendices \([A1, A2]\) respectively.

The DAs for the vector current are presented in Appendix \([A3]\). However, the vector current does not contribute to the color dipole amplitudes of pion production. Although it contains nonzero components, their overlap with the pion DAs is zero. The vector part vanishes because the color dipole amplitude does not flip helicity. Within the VDM approximation such components may be expressed via the \( pN \rightarrow \pi N \) scattering amplitudes, which exist only due to quark-antiquark (Reggeon) exchange in the cross channel. This is beyond the employed dipole phenomenology corresponding to gluonic (Pomeran) exchanges.

### IV. DISAPPEARANCE OF THE PION POLE IN THE DIPOLE REPRESENTATION

As was emphasized above, the pion pole contribution to the pion production amplitude vanishes because of lepton current conservation (up to the lepton mass). This nontrivial observation of \([1,28,29,33]\) is in variance with the naive interpretation of the AR Eq. \([1]\), which relates diffractive neutrino-production of pions, \( \nu + p \rightarrow l + \pi + p \), with elastic pion-proton scattering, \( \pi + p \rightarrow \pi + p \). It is tempting to interpret this relation as pion pole dominance, i.e., neutrino fluctuates to a pion, which then interacts elastically with the proton target. If this were true, the amplitude should be maximized in the so called black disk limit, which correspond to unitarity saturation when the imaginary part of the partial elastic amplitude reaches the maximal value allowed by the unitarity relation.

On the other hand, if the pion pole does not contribute \([1,28,29,32]\) as is stressed above, all hadronic fluctuations of the neutrino contributing to \( \Psi^{(bulk)} \) are heavier than a pion, so all diffractive hadronic amplitudes of pion production are off-diagonal. Such amplitudes vanish in the black-disk limit, so the pion cannot be produced diffractively. The source of such a dramatic breakdown of PCAC was identified in \([1]\) as a result of strong absorptive corrections. Of course the deviation from the PCAC prediction, AR, on a proton or nuclear targets, which may be far from the unitarity bound, is not so dramatic, as was calculated in \([1]\).

In this section we present an explicit demonstration of disappearance of diffractive pion production in the black-disk limit relying on the dipole model. Namely in this regime all the partial elastic amplitudes \([5]\) reach the uni-
parity bound, so become independent of the dipole transverse separation \( \vec{r} \), and the equation \((30)\) simplifies to just an overlap of the initial (axial current) and final (pion) light-cone wave function. We intend to demonstrate that this overlap vanishes.

The amplitude of pion production in this regime has the form,

\[
F_{\mu}^{J_A \rightarrow \pi}(q, \Delta) = \sum_a \int d\beta d^2r \\
\times \bar{\psi}^{(a)}(\beta, \vec{r}; q - \Delta) \Psi^{(a)}(\mu, A; \beta, \vec{r}; q),
\]

where the index \( a \) numerates all the distribution amplitudes. This is suppressed, since transition from spin-1 to spin-0 requires helicity flip for one of the quarks in the quark-antiquark pair. Now we would like to demonstrate explicitly that such suppression indeed takes place in case of the perturbative QCD model.

The distribution amplitude of the meson state is defined as

\[
\Phi_M(\beta, \vec{r}; q) \sim \int d^4k \delta(\beta - k^+/q^+) e^{i\vec{k} \cdot \vec{r}} T \ [S(k)\Gamma_M S(k - q)\Gamma] =
\]

\[
= \int d^2k_\perp e^{i\vec{k}_\perp \cdot \vec{r}_\perp} \int dk^- (2k^+ - k^2 - m^2 + i0) \left(2(k^+ - q^+) (k^- - q^-) - (k_\perp + q_\perp)^2 - m^2 + i0\right),
\]

where the function \( f(k, q) \) depends on the spins of mesons and Dirac matrices and is not important for a moment.

where the separation between the quark and antiquark has a “minus” and transverse components \((z, r)\), and \( \Gamma \) is one of the Dirac matrices \((1, \gamma^5; \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu \nu})\) multiplied by the proper isospin factor (for isoscalar this is one, for isovector mesons it is \( i \vec{r} \) etc.). The exact expression on the right hand side depends on the matrix \( \Gamma \), spin of the meson and is usually given as a twist expansion over all possible Lorentz structures which may be constructed from \( \Gamma, q, \vec{r} \) and the polarization vector \( \epsilon(q) \).

![Diagram correspond to the distribution amplitude](image)

In the leading order of \( \alpha_s \) (which is justified for very large \( q^2 \)) the corresponding DA may be represented as a simple diagram shown schematically in Fig. 2. Then we have

\[
\Phi_M(\beta, \vec{r}; q) \sim \int d^4k \delta(\beta - k^+/q^+) e^{i\vec{k} \cdot \vec{r}} T \ [S(k)\Gamma_M S(k - q)\Gamma] =
\]

\[
= \theta(0 \leq \beta \leq 1) \int d^2k_\perp e^{i\vec{k}_\perp \cdot \vec{r}_\perp} \int dk^- \frac{f(k, q)}{2(\beta - 1)q^+ \left(2q^+q^- - \frac{k_\perp^2 + m^2}{\beta} - \frac{(k_\perp + q_\perp)^2 + m^2}{1 - \beta}\right)},
\]

Taking the integral over \( k^- \), we get

\[
\int d^4k \delta(\beta - k^+/q^+) e^{i\vec{k} \cdot \vec{r}} T \ [S(k)\Gamma_M S(k - q)\Gamma] =
\]

\[
= \theta(0 \leq \beta \leq 1) \int d^2k_\perp e^{i\vec{k}_\perp \cdot \vec{r}_\perp} \frac{f(k, q)}{2(\beta - 1)q^+ \left(2q^+q^- - \frac{k_\perp^2 + m^2}{\beta} - \frac{(k_\perp + q_\perp)^2 + m^2}{1 - \beta}\right)},
\]

Straightforward evaluation of the overlap of the two func-
tions \((\Phi_A, \Phi_x)\) is quite tedious, however we may significantly simplify the evaluations using completeness of the Dirac matrices, viz.

\[
\sum_n \Gamma^{(n)}_{\alpha \beta} \Gamma_{\alpha' \beta'}^{(n)} = \delta_{\alpha \alpha'} \delta_{\beta \beta'},
\]

so the product of numerators of the two DAs is now converted to the effective diagram shown in the Fig. 3.

\[
\begin{aligned}
F_{\mu - \pi} (q, \Delta) & \sim \sum_\Gamma \int d\beta d^2 r \Phi_\Gamma^{(\pi)} (\beta, r_\perp; q - \Delta) \Phi_\Gamma^{(a)} (\beta, r_\perp; q) \\
& \sim 4mN_c \int d^2 k_\perp \int d\beta \frac{q_\mu (4m^2 + \Delta^2) - \Delta_\mu (4m^2 + 4k \cdot q + q^2) + 2k_\mu (-2k \cdot q + \Delta^2 - q^2 + q \cdot \Delta)}{4 (\beta (1 - \beta) q^2 - (k_\perp^2 + m^2)) (\beta (1 - \beta) (q - \Delta)^2 - (k_\perp^2 + m^2))}.
\end{aligned}
\]

As we can see, the result is proportional to \(O(m) \sim O(m^2)\) and thus is suppressed in the chiral limit. We expect that the same result is valid for the nonperturbative DAs evaluated in the instanton vacuum model.

V. NUMERICAL RESULTS

The Bjorken variable \(x = Q^2 / (2p \cdot q)\), used at high \(Q^2\), is not appropriate at small \(Q^2\), where it does not have the meaning of a fractional quark momentum any more, and may be very small even at low energies. For the case \(Q^2 = 0\), where the AR holds, \(x\) defined in this way would be zero. Therefore, one should rely on the phenomenological dipole cross section which depends on energy, rather than \(x\). At small \(Q^2\) we employ the \(s\)-dependent parametrization of the dipole cross section \([7, 8]\), which is similar to the \(x\)-dependent GBW parametrization \([8]\), but is more suitable for soft processes

\[
\sigma_{q\bar{q}} (r, s) = \sigma_0 (s) \left(1 - e^{-r^2/R_0^2 (s)}\right),
\]

\[
R_0 (s) = 0.88 \text{fm} \times \left(\frac{s_0}{s}\right)^{0.14}.
\]

These parameters and the scale \(s_0 = 1000 \text{GeV}^2\) are fitted to data on DIS, real photoproduction and \(\pi p\) scattering.

\[
\text{FIG. 3: Diagram corresponding to overlap of the distribution amplitudes}
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These parameters and the scale \(s_0 = 1000 \text{GeV}^2\) are fitted to data on DIS, real photoproduction and \(\pi p\) scattering.
Calculation of the left and right-hand side of Eq. (1) clearly demonstrates that absorptive corrections affect them differently. Indeed, the amplitude in the left-hand side of (1) vanishes. At the same time, the pion-proton cross section Eq. (32) is dominated by the first term in (30) and reaches maximum in the Froissart regime.

Now we are in a position to evaluate the accuracy of the AR for diffractive neutrino-production of pions on protons. In Figure 4 we plotted the ratio of the cross-sections calculated with the color dipole model (left-hand side of (1)) and using the AR (right-hand side of (1)),

$$K_{AR}(s) = \frac{d\sigma_{dipole}}{dt \, d\nu \, dQ^2} / \frac{d\sigma_{AR}}{dt \, d\nu \, dQ^2} |_{Q^2 = 0, \nu = 0}$$

as was defined in [1].

As was expected, $K_{AR} < 1$ due to different structures of the absorptive corrections to diffractive pion production and elastic pion scattering cross-sections. The deviation of $K_{AR}$ from unity is significant, even somewhat larger than was estimated in [1]. The ratio is falling at high energies towards the Froissart limit, where it eventually vanishes when $R_0(s) \to 0$.

B. Predicted cross sections

Most of the data on neutrino-production of pions on protons have been available so far only at energies close to the resonance region [29]. Data at higher energies are scarce and have rather low statistics [33, 52]. Because the dipole formalism should not be trusted at low energies, we provide predictions for the energy range of the ongoing experiment Minerva at Fermilab [2, 27].

The $Q^2$ dependence of the diffractive cross section deserves special attention. It would be very steep at small $Q^2$, if the pion dominance were real. However, since the pion pole is terminated due to conservation of the lepton current, the $Q^2$ dependence is controlled by heavier singularities. In the approximation of an effective singularity at $Q^2 = -M^2$ [1] one should expect the dipole form $\propto (Q^2 + M^2)^{-2}$. Within the dispersion approach the effective mass scale $M$ is expected to be of the order of 1 GeV [1, 29, 30]. Within the dipole description the $Q^2$ dependence is controlled by the the IVM mass scale, which is of the order of 700 MeV.

In Fig. 5 we plot the forward diffractive neutrino cross section scaled by the factor $(Q^2 + M^2)^2$, where the parameter $M$ is adjusted in a way to provide a flat $Q^2$ dependence at $Q^2 < 2$ GeV$^2$.

Indeed, we found that at $M = 0.91$ GeV the scaled cross section is constant up to rather large $Q^2 \sim 3$ GeV$^2$, but substantially deviates from the dipole form at larger $Q^2$.

The $t$-dependence of the cross section is controlled by the employed model Eqs. (3) for impact parameter dependence of the dipole amplitude. The results for $t$-dependence of the invariant cross-section are shown in Fig. 6 for several fixed values of $E_\nu$ and $Q^2 = 4$ GeV$^2$. For this calculation we fixed $y = 0.5$.

The forward invariant cross-section Eq. (2) of diffractive neutrino-production of pions on protons is depicted in the Fig. 7 as function of $t$ at several fixed values of $y$ and $Q^2$.

These calculations performed in the dipole approach are controlled in Eq. (3) by the light-cone DAs of the axial current and pion, which we extended to the soft interaction...
regime basing on the instanton vacuum model. It worth reminding that neither s-channel resonances, nor reggeons are included in the parametrization (30) of the universal dipole cross section, so the results are trustable only at sufficiently high energy $\nu$.

Experimental data for neutrino-production cross section are usually presented as function of neutrino energy $\nu$ integrated over $\nu$. Unfortunately, in this form one cannot separate physics of low and high energies. Indeed, the integration over $\nu$ results in the finite contribution of small-$\nu$ region, which is dominated by s-channel resonances. This small-$\nu$ contribution is constant at any high neutrino energy $\nu$ and its magnitude is comparable with the diffractive part.

Usually in low statistics experiments one integrates the multi-dimensional distributions presenting the results as function of one variable. As such a variable we chose the c.m. energy of the diffraction process, $W = \sqrt{m_N^2 - Q^2 + 2m_N\nu}$. Then we calculate the $W$-distribution as,

$$ \frac{d\sigma}{dW} = 2W \int d\nu \, dt \, dQ^2 \delta (W^2 - (p + q)^2) \frac{d\sigma}{d\nu \, dt \, dQ^2}. $$

(34)

In the left pane of the Figure 8 we plotted $W$-dependence of the cross-section (34) for several fixed values of $E_{\nu}$. We see that the $W$-dependence significantly varies with $E_{\nu}$, therefore one should average the cross section Eq. (34) weighted with a realistic neutrino energy distribution.

$$ \langle \frac{d\sigma}{dW} \rangle = \int dE_{\nu} \rho (E_{\nu}) \frac{d\sigma}{dW}, $$

(35)

where the neutrino spectrum $\rho (E_{\nu})$ is normalized as

$$ \int dE_{\nu} \rho (E_{\nu}) = 1. $$

(36)

As an example, we performed calculations with the neutrino energy spectrum of the MINERvA experiment [2]. We considered three different $E_{\nu}$-distributions corresponding to low (LE), medium (ME) and high energy (HE) beam configurations. The results are depicted in the right pane of the Figure 8.

We also compared our results for the $W$-distribution of neutrino diffractive events with data of the WA21 experiment at CERN [52]. We performed averaging over neutrino energy with the spectrum $\rho (E_{\nu})$ given in [74]. The results are depicted by dashed curve in Fig. 9. Since the low energy region is affected by reggeons, which we have neglected so far, we added their contribution to the dipole cross section [52].
\[
\sigma_{\pi p}^{\text{tot}}(s) = [13.6s^{0.08} + 19.2s^{-0.45}] \text{mb} \quad (37)
\]

The result shown by solid curve describes the data much better. For comparison we plotted also the prediction based on the AR with the realistic pion-proton cross section Eq. (37).

VI. SUMMARY

We developed the dipole description for high-energy neutrino interaction, in particular at low \( Q^2 \), whether the PCAC hypothesis plays important role. This approach is alternative to the conventional one based on the dispersion relation for the \( Q^2 \) dependent amplitude axial current interaction. While the latter faces the problem of lacking experimental information on most of the diffractive diagonal and off-diagonal amplitudes, the dipole formalism is free of these difficulties. Besides, one can employ the universal dipole cross section (see Eq. (33)), well fixed by numerous data for interactions of the vector current in electromagnetic processes (DIS, photoproduction, etc.).

The important challenge of the dipole description is the construction of the current distribution amplitudes at small \( Q^2 \), where the nonperturbative effects are unavoidable. We calculated the distribution amplitudes for the axial current (Sect. III B) and for the pion (Sect. III C) on the same footing, within the instanton vacuum model (Sect. III A). The model possesses the chiral symmetry properties, what guarantees a correct, controlled by PCAC behavior at small \( Q^2 \).

Although the dipole approach does not involve explicitly the intermediate hadronic states, absence of the pion pole can be tested in the "black-disc" regime, where all the partial elastic amplitude saturate at the unitarity bound. Indeed, the direct calculation performed in Sect. IV confirmed that diffractive pion production ceases, what may happen only if the pion pole does not contribute.

The dipole description also offers an unbiased way to test the AR on a proton target. This relation is expected to be broken by absorptive corrections \([1]\), which are implicitly included in the phenomenological dipole cross section. We rely on the dipole cross section parametrized in the saturation form Eq. (30), well confirmed by data for electromagnetic processes \([8]\). We found a significant, about 40% deviation from the AR on a proton target (see Fig. 4).

A much stronger breakdown of the AR is expected for nuclei \([1]\), and we plan to evaluate those effects employing the techniques developed here.

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Appendix A: Distribution amplitudes in the instanton vacuum model

In this section we present the results of calculation of the DAs for the axial current and pion performed in the instanton vacuum model (IVM).
\begin{equation}
\langle 0 | \bar{\psi}(y) \gamma_\mu \gamma_5 \psi(x) | A(q) \rangle = i f_A^2 \int_0^1 du \ e^{i (u p \cdot \bar{y} + \bar{u} p \cdot x)} \times \left[ p_\mu \frac{e^{(\lambda) \cdot z}}{p \cdot z} \left( \Phi_\parallel(u) + e^{(\lambda=\perp)} g_{\perp}^{(a)}(u) - \frac{1}{2} \frac{e^{(\lambda) \cdot z}}{(p \cdot z)^2} f_A^2 g_3(u) \right) \right],
\end{equation}

(A1)

\begin{equation}
\langle 0 | \bar{\psi}(y) \gamma_\mu \psi(x) | A(q) \rangle = -i f_A^2 \epsilon_{\mu \nu \sigma \rho} e_\nu^\lambda (p_\rho z_\sigma) \int_0^1 du \ e^{i (u p \cdot \bar{y} + \bar{u} p \cdot x)} \frac{g_\perp^{(a)}(u)}{4},
\end{equation}

(A2)

\begin{equation}
\langle 0 | \bar{\psi}(y) \sigma_{\mu \nu} \gamma_5 \psi(x) | A(q) \rangle = f_A \int_0^1 du \ e^{i (u p \cdot \bar{y} + \bar{u} p \cdot x)} \left[ \left( \epsilon_\mu^{(\lambda=\perp)} p_\nu - \epsilon_\nu^{(\lambda=\perp)} p_\mu \right) \Phi_\perp(u) + \frac{e^{(\lambda) \cdot z}}{(p \cdot z)^2} f_A^2 (p_\mu \bar{z}_\nu - \bar{p}_\nu z_\mu) h_\parallel^{(t)}(u) + \frac{1}{2} \left( \epsilon_\mu^{(\lambda) \cdot z} - \epsilon_\nu^{(\lambda) \cdot z} \right) \frac{f_A^2}{p \cdot z} h_3(u) \right],
\end{equation}

(A3)

\begin{equation}
\langle 0 | \bar{\psi}(y) \gamma_5 \psi(x) | A(q) \rangle = f_A^3 e^{(\lambda) \cdot z} \int_0^1 du \ e^{i (u p \cdot \bar{y} + \bar{u} p \cdot x)} \frac{h_\parallel^{(p)}(u)}{2},
\end{equation}

(A4)

FIG. 8: Left: Cross-section of diffractive neutrino-production \(d\sigma/dW\) as function of \(W\), for fixed neutrino energies \(E_\nu\). Right: the same cross-section \(\langle d\sigma/dW \rangle\) weighted with the neutrino spectrum from Minerva [2], see Eq. (35) for exact definition.

1. Axial DAs

There are eight independent axial DAs defined in [16,18],

\begin{equation}
\langle 0 | \bar{\psi}(y) \gamma_\mu \gamma_5 \psi(x) | A(q) \rangle = i f_A^2 \int_0^1 du \ e^{i (u p \cdot \bar{y} + \bar{u} p \cdot x)} \times \left[ p_\mu \frac{e^{(\lambda) \cdot z}}{p \cdot z} \left( \Phi_\parallel(u) + e^{(\lambda=\perp)} g_{\perp}^{(a)}(u) - \frac{1}{2} \frac{e^{(\lambda) \cdot z}}{(p \cdot z)^2} f_A^2 g_3(u) \right) \right],
\end{equation}

(A1)

\begin{equation}
\langle 0 | \bar{\psi}(y) \gamma_\mu \psi(x) | A(q) \rangle = -i f_A^2 \epsilon_{\mu \nu \sigma \rho} e_\nu^\lambda (p_\rho z_\sigma) \int_0^1 du \ e^{i (u p \cdot \bar{y} + \bar{u} p \cdot x)} \frac{g_\perp^{(a)}(u)}{4},
\end{equation}

(A2)

\begin{equation}
\langle 0 | \bar{\psi}(y) \sigma_{\mu \nu} \gamma_5 \psi(x) | A(q) \rangle = f_A \int_0^1 du \ e^{i (u p \cdot \bar{y} + \bar{u} p \cdot x)} \left[ \left( \epsilon_\mu^{(\lambda=\perp)} p_\nu - \epsilon_\nu^{(\lambda=\perp)} p_\mu \right) \Phi_\perp(u) + \frac{e^{(\lambda) \cdot z}}{(p \cdot z)^2} f_A^2 (p_\mu \bar{z}_\nu - \bar{p}_\nu z_\mu) h_\parallel^{(t)}(u) + \frac{1}{2} \left( \epsilon_\mu^{(\lambda) \cdot z} - \epsilon_\nu^{(\lambda) \cdot z} \right) \frac{f_A^2}{p \cdot z} h_3(u) \right],
\end{equation}

(A3)

\begin{equation}
\langle 0 | \bar{\psi}(y) \gamma_5 \psi(x) | A(q) \rangle = f_A^3 e^{(\lambda) \cdot z} \int_0^1 du \ e^{i (u p \cdot \bar{y} + \bar{u} p \cdot x)} \frac{h_\parallel^{(p)}(u)}{2},
\end{equation}

(A4)
After tedious but straightforward calculations we arrive at,

$$
\Phi_{||} (u, r_{||}) = \frac{1}{2 f A} \int \frac{dz}{2\pi} e^{i(z-u-1/2)z} 0 \left\langle \bar{\psi} \left( -\frac{z}{2} n - \frac{r_{||}}{2} \right) \hat{n} \gamma_5 \psi \left( \frac{z}{2} n + \frac{r_{||}}{2} \right) A^{(\lambda)} (q) \right\rangle = \tag{A5}
$$

$$
= \frac{8 N_c}{f A} \int \frac{dl^- d^2l_{\perp}}{(2\pi)^4} e^{-i l^- \cdot r_{\perp}} \times \left[ \frac{\mu(l) \mu(l+q) + l_{\perp}^2 + \left( \frac{q}{2} - u^2 \right) q^2}{(l^2 + \mu^2(l)) \left( (l+q)^2 + \mu^2(l+q) \right)} + \frac{M (f(l+q) - f(l))^2 (2l^- - q^2 (u - \frac{1}{2})) (\mu(l) (u + \frac{1}{2}) - \mu(l+q) (u - \frac{1}{2}))}{((l+q)^2 - l^2) (l^2 + \mu^2(l)) \left( (l+q)^2 + \mu^2(l+q) \right)} \right] \bigg|_{l^+ = (u - \frac{1}{2}) q^+} \tag{A6}
$$

$$
\Phi_{\perp} (u, r_{\perp}) = \frac{1}{2 f A} \int \frac{dz}{2\pi} e^{i(z-u-1/2)z} 0 \left\langle \bar{\psi} \left( -\frac{z}{2} n \right) \sigma_{\nu \rho} \gamma_5 \left( e^{(\lambda=\perp)}_{\nu} (n_{\rho} - e^{(\lambda=\perp)}_{\rho}) n_{\nu} \right) \psi \left( \frac{z}{2} n \right) A^{(\lambda=\perp)} (q) \right\rangle = \tag{A7}
$$

$$
= \frac{4 N_c}{\pi f A} \int \frac{dl^- d^2l_{\perp}}{(2\pi)^4} e^{-i l^- \cdot r_{\perp}} \times \left[ \frac{\mu(l) \mu(l+q) + (u + \frac{1}{2}) \mu(l)}{(l^2 + \mu^2(l)) \left( (l+q)^2 + \mu^2(l+q) \right)} - \frac{l_{\perp}^2}{(l+q) - l^2} \frac{M (f(l+q) - f(l))^2}{(l^2 + \mu^2(l)) \left( (l+q)^2 + \mu^2(l+q) \right)} \right] \bigg|_{l^+ = (u - \frac{1}{2}) q^+}.
$$
\begin{align*}
g^{(u)}_\perp (u, \bar{r}_\perp) &= \frac{1}{f^2_A} \int \frac{dz}{2\pi} e^{i(u-1/2)z} \left< 0 \left| \bar{\psi} \left( -\frac{z}{2} n \right) e^{(\lambda=\perp)\gamma_5 \psi \left( \frac{z}{2} n \right)} \right| A^{(\lambda=\perp)}(q) \right> \\
 &= \frac{4N_c}{f^2_A} \int \frac{dl^+ d^2l_\perp}{(2\pi)^4} e^{-il^+ \cdot \bar{r}_\perp} \times \\
 &\quad \times \left[ \left( \mu(l) \mu(l+q) - l^2 - l \cdot q \right) + l^2_\perp \right. \\
 &\quad \left. \left[ (l^2 + \mu^2(l)) ((l+q)^2 + \mu^2(l+q)) - \frac{l^2_\perp}{(l+q)^2 - l^2} M (f(l+q) - f(l))^2 (\mu(l+q) - \mu(l)) \right] \right]_{l^+ = (u-\frac{1}{2})q^+} \\

g^{(v)}_\perp (u, \bar{r}_\perp) &= \frac{4N_c}{f^2_A} \text{Coefficient} \left( \int \frac{dz}{2\pi} e^{i(u-1/2)z} \left< 0 \left| \bar{\psi} \left( -\frac{z}{2} n \right) e^{(\lambda=\perp)\gamma_5 \psi \left( \frac{z}{2} n \right)} \right| A^{(\lambda=\perp)}(q) \right), \epsilon_{\mu\nu\rho\sigma} \epsilon^{(\lambda)} \rho p_{\mu\sigma} \right> \\
 &= \frac{32N_c}{f^2_A} \int \frac{dl^+ d^2l_\perp}{(2\pi)^4} e^{-il^+ \cdot \bar{r}_\perp} \times \\
 &\quad \times q \cdot l - q^2 \left( u - \frac{1}{2} \right) \\
 &\quad \times \left[ (l^2 + \mu^2(l)) ((l+q)^2 + \mu^2(l+q)) \right]_{l^+ = (u-\frac{1}{2})q^+} \\

g^{(v)}_\parallel (u, \bar{r}_\perp) &= \frac{1}{(f^2_A e^{i(\lambda)} \cdot n)} \int \frac{dz}{2\pi} e^{i(u-1/2)z} \left< 0 \left| \bar{\psi} \left( -\frac{z}{2} n \right) e^{(\lambda=\perp)\gamma_5 \psi \left( \frac{z}{2} n \right)} \right| A^{(\lambda=\perp)}(q) \right> \\
 &= -\frac{8N_c}{f^2_A} \int \frac{dl^+ d^2l_\perp}{(2\pi)^4} e^{-il^+ \cdot \bar{r}_\perp} \times \\
 &\quad \times \left[ \mu(l+q) \left( l^+ - (u - \frac{1}{2}) \frac{q_\perp^2}{q^2} \right) \right. \\
 &\quad \left. \left[ (l^2 + \mu^2(l)) ((l+q)^2 + \mu^2(l+q)) \right] \right]_{l^+ = (u-\frac{1}{2})q^+} \\

g^{(p)}_\parallel (u, \bar{r}_\perp) &= \frac{1}{(f^2_A e^{i(\lambda)} \cdot n)} \int \frac{dz}{2\pi} e^{i(u-1/2)z} \left< 0 \left| \bar{\psi} \left( -\frac{z}{2} n \right) \right| A^{(\lambda)}(q) \right> \\
 &= -\frac{8N_c}{f^2_A} \int \frac{dl^+ d^2l_\perp}{(2\pi)^4} e^{-il^+ \cdot \bar{r}_\perp} \times \\
 &\quad \times q \cdot l - q^2 \left( u - \frac{1}{2} \right) \\
 &\quad \times \left[ \left( \mu(l+q) \left( l^+ - (u - \frac{1}{2}) \frac{q_\perp^2}{q^2} \right) - \left( \mu(l) \left( l^+ + \frac{q_\perp^2}{q^2} \right) - \mu(l+q) l^- \right) \right) \right. \\
 &\quad \left. \left[ (l^2 + \mu^2(l)) ((l+q)^2 + \mu^2(l+q)) \right] \right]_{l^+ = (u-\frac{1}{2})q^+} \\

g^{(p)} (u, \bar{r}_\perp) &= -\frac{2}{f^2_A e^{i(\lambda)} \cdot n} \int \frac{dz}{2\pi} e^{i(u-1/2)z} \left< 0 \left| \bar{\psi} \left( -\frac{z}{2} n \right) \right| A^{(\lambda)}(q) \right> \\
 &= -\frac{8N_c}{f^2_A} \int \frac{dl^+ d^2l_\perp}{(2\pi)^4} e^{-il^+ \cdot \bar{r}_\perp} \times \\
 &\quad \times \left[ \left( 2l^- + l^2 - q^2 \left( \mu(l) \mu(l+q) + l^2_\perp \right) \right. \right. \\
 &\quad \left. \left. \left[ (l^2 + \mu^2(l)) ((l+q)^2 + \mu^2(l+q)) \right] \right] \right]_{l^+ = (u-\frac{1}{2})q^+} \\

g^{(a)}_\parallel (u, \bar{r}_\perp) &= \frac{1}{(f^2_A e^{i(\lambda)} \cdot n)} \int \frac{dz}{2\pi} e^{i(u-1/2)z} \left< 0 \left| \bar{\psi} \left( -\frac{z}{2} n \right) \right| A^{(\lambda)}(q) \right> \\
 &= -\frac{8N_c}{f^2_A} \int \frac{dl^+ d^2l_\perp}{(2\pi)^4} e^{-il^+ \cdot \bar{r}_\perp} \times \\
 &\quad \times \left[ \left( 2l^- + l^2 - q^2 \left( \mu(l) \mu(l+q) + l^2_\perp \right) \right. \right. \\
 &\quad \left. \left. \left[ (l^2 + \mu^2(l)) ((l+q)^2 + \mu^2(l+q)) \right] \right] \right]_{l^+ = (u-\frac{1}{2})q^+}
\[ h_3(u, r_\perp) = \frac{1}{f_\pi} \int \frac{dz}{2\pi} e^{i(u-1)z} \left\langle 0 \left| \bar{\psi} \left( \frac{z}{2} - \frac{r_\perp}{2} \right) \gamma_\mu \gamma_5 \psi \left( x \right) \right| \pi(q) \right\rangle = \frac{16N_c}{f_\pi} \int \frac{dl^+ d\ell_\perp}{(2\pi)^4} e^{-i\ell_\perp \cdot r_\perp} \times \frac{M (f(l+q) - f(l))^2}{\left( l^2 + \mu^2(l) \right) \left( (l+q)^2 + \mu^2(l+q) \right)} \right|_{l^+ = (u-\frac{1}{2})y^+} \]

2. Pion DAs

For the pion there are four independent pion DAs defined in \cite{21, 23},

\[ \langle 0 | \bar{\psi} (y) \gamma_\mu \gamma_5 \psi (x) | \pi(q) \rangle = i f_\pi \int_0^1 du e^{i(up+y+\bar{u}p-x)} \times \left( p_\mu \phi_{2,\pi}(u) + \frac{z_\mu}{2 \cdot p \cdot z} \psi_{4,\pi}(u) \right) ; \quad (A8) \]

\[ \langle 0 | \bar{\psi} (y) \gamma_5 \psi (x) | \pi(q) \rangle = -i f_\pi \frac{m_\pi^2}{m_u + m_d} \times \int_0^1 du e^{i(u+p+y+\bar{u}p-x)} \phi_{3,\pi}^{(p)}(u) ; \quad (A9) \]

Eventually we arrive at the following structures in the pion DA,

\[ \phi_{2,\pi}(u, r_\perp) = \frac{1}{if_\pi} \int \frac{dz}{2\pi} e^{i(u-1)z} \left\langle 0 \left| \bar{\psi} \left( \frac{z}{2} - \frac{r_\perp}{2} \right) \gamma_5 \psi \left( \frac{z}{2} + \frac{r_\perp}{2} \right) \right| \pi(q) \right\rangle = \frac{8N_c}{f_\pi} \int \frac{dl^+ d\ell_\perp}{(2\pi)^4} e^{-i\ell_\perp \cdot r_\perp} M f(l) f(l+q) \frac{\mu(l) (u + \frac{1}{2}) - \mu(l+q) (u - \frac{1}{2})}{\left( l^2 + \mu^2(l) \right) \left( (l+q)^2 + \mu^2(l+q) \right)} \right|_{l^+ = (u-\frac{1}{2})y^+} \]

\[ \psi_{4,\pi}(u, r_\perp) = \frac{2}{if_\pi} \int \frac{dz}{2\pi} e^{i(u-1)z} \left\langle 0 \left| \bar{\psi} \left( \frac{z}{2} - \frac{r_\perp}{2} \right) \gamma_5 \psi \left( \frac{z}{2} + \frac{r_\perp}{2} \right) \right| \pi(q) \right\rangle = \frac{16N_c}{f_\pi} \int \frac{dl^+ d\ell_\perp}{(2\pi)^4} e^{-i\ell_\perp \cdot r_\perp} M f(l) f(l+q) \frac{\mu(l) (l+q) - \mu(l+q) l}{\left( l^2 + \mu^2(l) \right) \left( (l+q)^2 + \mu^2(l+q) \right)} \right|_{l^+ = (u-\frac{1}{2})y^+} \]

\[ \phi_{3,\pi}^{(p)}(u, r_\perp) = \frac{1}{if_\pi} \int \frac{dz}{2\pi} e^{i(u-1)z} \left\langle 0 \left| \bar{\psi} \left( \frac{z}{2} - \frac{r_\perp}{2} \right) \gamma_5 \psi \left( \frac{z}{2} + \frac{r_\perp}{2} \right) \right| \pi(q) \right\rangle = \frac{8N_c m_u + m_d}{f_\pi m^2} \int \frac{dl^+ d\ell_\perp}{(2\pi)^4} e^{-i\ell_\perp \cdot r_\perp} M f(l) f(l+q) \frac{\mu(l) (l+q) + l^2 + l \cdot q}{\left( l^2 + \mu^2(l) \right) \left( (l+q)^2 + \mu^2(l+q) \right)} \right|_{l^+ = (u-\frac{1}{2})y^+} \]
\[
\phi^{(e)}_{d,\rho}(u, \bar{r}_+) = \frac{3i}{2f_\pi} \frac{m_u + m_d}{m_\pi^2} \int \frac{dz}{2\pi} e^{i(u-1/2)z} \left( 0 \left| \bar{\psi} \left( -\frac{z}{2} n - \frac{\bar{r}_+}{2} \right) (p_\mu n_\rho - p_\rho n_\mu) \sigma_{\mu\nu} \gamma_\nu \psi \left( \frac{z}{2} n + \frac{\bar{r}_+}{2} \right) \right| \pi(q) \right) = \text{A14}
\]
\[
= -24N_c m_u + m_d \int \frac{dl^+ d^2 l_+}{(2\pi)^4} e^{-i\bar{r}_+ \cdot \bar{r}_+} \left[ M f(l)f(l+q) q_1 l_+ - \frac{q_2^2}{T} (u - \frac{1}{2}) \right] \left( l^2 + \mu^2(l) \right) (l + q)^2 + \mu^2(l + q)) \right] \bigg|_{l^+=u-\bar{q}^+}.
\]

The details of calculation of the distribution amplitudes will be presented elsewhere.

3. Vector current DAs

The vector current DAs were derived in [64],

\[
\langle 0 \left| \bar{\psi} (y) \gamma_\mu \gamma_5 \psi (x) \right| V(q) \rangle = e_q f_{3\gamma} f^{\gamma}_2(q) e_{\mu\rho\sigma} e^{(\lambda)}_{\nu} p_\rho z_\sigma \int_0^1 du e^{i(u p^y + \bar{u} p^x)} \langle \gamma_\nu (u, q^2) \rangle \tag{A15}
\]

\[
\langle 0 \left| \bar{\psi} (y) \gamma_\mu \gamma_5 \psi (x) \right| V(q) \rangle = e_q f_{3\gamma} f^{\gamma}_{\perp}(q) \int_0^1 du e^{i(u p^y + \bar{u} p^x)} \times \left[ p_\mu \left( e^{(\lambda)} \cdot n \right) f_{\perp\gamma}^{(v)}(q) + e^{(\lambda = \perp)}_{\nu} \psi^{(v)}_{\perp\gamma} (u, q^2) + n_\mu \left( e^{(\lambda)} \cdot n \right) h^{(v)}_{\perp\gamma} (u, q^2) \right] \tag{A16}
\]

\[
\langle 0 \left| \bar{\psi} (y) \sigma_{\mu\nu} \psi (x) \right| V(q) \rangle = -i e_q \langle \bar{q} q \rangle f_{\perp\gamma}^{(v)}(q^2) \int_0^1 du e^{i(u p^y + \bar{u} p^x)} \left[ \left( e^{(\lambda = \perp)}_{\mu} p_\nu - e^{(\lambda = \perp)}_{\nu} p_\mu \right) e^{(\lambda = \perp)} \psi^{(v)}_{\perp\gamma} (u, q^2) + \left( e^{(\lambda = \perp)}_{\nu} n_\mu - e^{(\lambda = \perp)}_{\mu} n_\nu \right) h^{(v)}_{\perp\gamma} (u, q^2) \right], \tag{A17}
\]

\[
\langle 0 \left| \bar{\psi} (y) \psi (x) \right| V(q) \rangle = f^1_\perp m^2_\perp e^{(\lambda)} \cdot z \int_0^1 du e^{i(u p^y + \bar{u} p^x)} D_T (u, q^2), \tag{A18}
\]

where the distribution amplitudes \( \phi_{\|}, \phi_{\perp\gamma} \) have twist 2, \( \psi_{\perp\gamma}^{(v)}, \psi_{\perp\gamma}^{(s)}, \psi_{\perp\gamma}^{(l)}; D_T \) has twist 3; and \( h^{\perp\gamma}_{\perp\gamma}, h^{\perp\gamma}_{\perp\gamma} \) have twist 4. The formfactors \( f^2_\perp(q), f^{\perp\gamma}_2(q), f^{\perp\gamma}_2(q^2) \) and normalization constants \( f_{3\gamma}, \chi_{\perp\gamma}, f^1_\perp \) are discussed in detail in [64].

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