Electroweak Strings, Zero Modes and Baryon Number
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The Dirac equations for leptons and quarks in the background of an electroweak $Z$–string have zero mode solutions. If two loops of electroweak string are linked, the zero modes on one of the loops interacts with the other loop via an Aharonov-Bohm interaction. The effects of this interaction are briefly discussed and it is shown that the fermions induce a baryon number on linked loops of $Z$–string.

1. Introduction

Topological defects have received widespread attention in the particle physics community since the discovery of string and monopole solutions in the 70’s in certain toy field theories. These discoveries have also provided impetus to considerable work in cosmology and astrophysics since the early universe is precisely the arena in which defects are likely to play a major role.

Over the last two decades, it has been gradually recognised that the standard model of the electroweak interactions also contains classical solutions that closely resemble topological defects. There are two kinds of strings in the standard model - the $W$– and $Z$– strings. Furthermore, the $Z$–string terminates on magnetic monopoles. This recognition immediately makes the entire two decades long study of topological defects relevant to the world of particle physics and opens up some surprises too.

One of the recent surprises is that linked loops of $Z$–string carry baryon number. This can be shown by a simple exercise. The baryon number current satisfies the well-known anomaly equation which can be integrated to yield the change in the baryon number between two different times. For field configurations in which all the gauge fields except for the $Z$ gauge field are zero, we get

\[ \Delta B = N_F \frac{\alpha^2}{16\pi^2} \cos(2\theta_W) \Delta \int d^3x \, \vec{Z} \cdot (\vec{\nabla} \times \vec{Z}) \]  

where, $\alpha = \sqrt{g^2 + g'^2}$ and $N_F$ is the number of families. When two loops of string that are linked once decay into a final vacuum state with zero baryon number, the integration can be done and yields the baryon number of the initial string configuration:

\[ B = 2N_F \cos(2\theta_W) . \]  

This calculation of the baryon number of two linked loops is an indirect calculation since no direct reference has been made to the fermions that actually carry the baryon number. (Indirectly, the fermions have come in because the anomalous baryon current conservation equation has included the effects of the fermions.) In recent and ongoing work with Jaume Garriga, we have directly considered the effects of leptons and quarks in the singly linked string background. In this way we have recovered eqn. 2 and also found some other surprises. These are:

(i) The ground state of the fermions in the background of singly linked string loops is lower than that if the loops are not linked.

(ii) In the ground state, the strings carry non-vanishing electromagnetic currents but zero net charge.

Our treatment of this problem enables us to evaluate any other charge of the linked string configuration that we may wish to calculate though some quantities will require a numerical evaluation. I now outline the calculation; details may be found in our paper.

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2. Fermions on linked strings

There are three basic facts that need to be put together to understand how the leptons and quarks contribute to say the baryon number of linked strings. The first fact is that the fermions interact non-trivially with the strings and have zero energy states in the background of the string. These zero modes were first discovered by Jackiw and Rebbi [9] and constructed for the Z-string by Earnshaw and Perkins [10]. The second fact is that the leptons and quarks in the standard model have an Aharanov-Bohm interaction with the Z-string. As a result of this interaction, the zero modes on linked loops have a dispersion relation which is different from the dispersion relation for zero modes on unlinked loops. The third fact is that an adiabatic change in the dispersion relation can lead to the production of particles and alteration can lead to the production of certain charges. Alternatively, the ground states of the two configurations (linked and unlinked strings) can differ in their energy and charges. The change in the fermion energy eigenvalues as the background configuration is changed is known as “spectral flow” and has been studied in a variety of circumstances. The early work most relevant for us is the work on Schwinger electrodynamics by Manton [11].

In the background of a Z-string, the zero modes satisfy the dispersion relation:

\[ E = \pm p \]  \hspace{1cm} (3)

where, the electron and d quark travel in one direction (− sign) and the neutrino and u quark in the opposite direction (+ sign). (Similarly we can deal with the other fermion families.) If two loops are linked, the dispersion relation is modified to

\[ E = \pm (p - qZ) \]  \hspace{1cm} (4)

where, \( q \) is the \( Z \)-charge on the lepton or quark in units of the charge on the Higgs field and \( Z = n/a \) where, \( n \) is the number of strings threading a loop and \( a \) is the radius of the loop which we assume to be circular and much larger than the thickness of the string. (In the following we choose units such that \( a = 1 \). Then, \( Z = n \).) Given this dispersion relation, we can find the energy of the Dirac sea. Naively, the energy is infinite but we can use zeta function regularization to find the energy contribution due to one fermion

\[ E = -\frac{1}{24} + \frac{1}{2}(p_F - qn \pm \frac{1}{2})^2 \]  \hspace{1cm} (5)

where \( p_F \) labels the Fermi momentum (highest filled state) and is an integer and the sign depends on the fermion in question. The expression for \( E \) contains the Casimir energy contribution since the loop is closed. This is the same for linked or unlinked strings and does not interest us. In the second term, the \( qn \) piece is due to the linkage. If the loops are unlinked, this term is absent and the ground state energy for unlinked loops is

\[ E[n = 0] = -\frac{1}{24} + \frac{1}{8} = \frac{1}{12} \]  \hspace{1cm} (6)

As a function of \( qn \), this energy is a maximum. The minimum energy can be attained if \( qn \) is half integral because then we can fill the states (that is, choose \( p_F \)) such that the second term in (5) is zero and the energy \( E = -1/24 \). For the leptons and quarks occurring in the standard model, \( q \) is not an integer and so the ground state energy of linked strings is lower than that of unlinked strings. But since the charges are related to \( \sin^2 \theta_W \), they are probably irrational and the energy functional can never take on its minimum possible value.

We can evaluate the charges on the linked or unlinked loops in a similar manner. Each energy eigenstate that is filled carries a certain charge and adding up the charges in the Dirac sea means that we have to find

\[ Q = c \sum_{p=-\infty}^{p_F-qZ} 1 \]  \hspace{1cm} (7)

where each fermion carries a charge \( c \). Once again this sum is divergent. But we can make sense of the sum by regularizing it using zeta function regularization. We then have:

\[ Q = c \sum_{p=-\infty}^{p_F-qZ} (p - qZ)^0 = \pm c[p_F - qZ \pm \frac{1}{2}] \]  \hspace{1cm} (8)

where the signs need to be chosen according to the fermion whose charge we are evaluating. Note
that we have chosen to write \( 1 = (p - qZ)^0 \) rather than \( 1 = p^0 \) since \( p - qZ \) is the gauge invariant combination. (This valuable trick was used by Manton in Ref. [11].)

Now we can sum over all fermions with the appropriate charges \( q \) and \( c \) when the Fermi levels \( p_F \) are evaluated by minimizing the energy. We also have to choose the states to be colour singlets. When we put all this arithmetic together we find the electric charge

\[
Q_A = 0
\]  

and the baryonic charge

\[
Q_B = 2N_F \cos(2\theta_W) \, .
\]

This agrees with the indirect calculation described in the introduction.

3. Conclusions

The linking of loops of string is detected by the fermions that live on the strings even though the loops of string themselves are well separated. This is possible due to the Aharonov-Bohm interaction of the fermions with the strings and leads to non-trivial charges and currents on linked strings.

It is known that in general string knots are characterized by an infinite sequence of numbers and the link invariant is only the first of this sequence. What we have found is that the fermions living on the strings are sensitive to this first knot invariant. It would be amusing if a situation could be devised in which the fermions not only experience the linkage but also get affected by the higher knot invariants.

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