Light Bosons of Electromagnetic Field and Breakdown of Relativistic Theory.

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Abstract

In our analysis, a quantisation scheme for local electromagnetic waves in vacuum is introduced by the model of nonideal Bose-gas consisting of Bose-light-particles (which are no photons) with spin one and a finite mass. This fact destroys the Relativistic Theory of Einstein as well as displays a wrong sound of so-called spontaneous breakdown of symmetry because the light boson can be moved by speed of wave in vacuum.

PACS:01.55. + b General physics
1. INTRODUCTION.

The field of prediction elementary particles is a very complex, and as yet not fully solved, Problem. The study of field theoretical models connected with spontaneous breakdown of symmetry (there is presence a non-zero vacuum expectation value) which occurs, if Lagrangian fully invariant under an internal Lee group were proposed by different authors [1-4]. Following initial works, done by the Freund and Namby [2], Goldstone [5] and Higgs [6], exhibit on display the bosons as yields of Lorentz invariant relativistic field theory. In this letter, we predict an existence of light-bosons of electromagnetic field. This reasoning destroys a concept of the non-zero vacuum, and in turn the existence of Freund and Namby scalar bosons, Goldstone massless bosons and Higgs massive bosons because predicted light-bosons of electromagnetic field can be moved by speed of electromagnetic wave in vacuum.

Theoretical description of quantization local electromagnetic field in vacuum within a model of Bose-gas of local electromagnetic waves, which are propagated by speed $c$ in vacuum, was first proposed by Dirac [7].

11. DIRAC THEORY

For beginning, we present the Maxwell equations in the zero-vacuum:

\[
\text{curl} \vec{H} - \frac{1}{c} \frac{d \vec{E}}{dt} = 0 \quad (1) \\
\text{curl} \vec{E} - \frac{1}{c} \frac{d \vec{H}}{dt} = 0 \\
\text{div} \vec{E} = 0 \quad (3) \\
\text{div} \vec{H} = 0 \quad (4)
\]

where $\vec{E} = \vec{E}(\vec{r}, t)$ and $\vec{H} = \vec{H}(\vec{r}, t)$ are, respectively, the local electric and magnetic fields presented in dependence of the coordinate $\vec{r}$ and current time $t$; $c$ is the velocity of wave in vacuum.

The Hamiltonian of radiation $\hat{H}_R$ is determined as:

\[
\hat{H}_R = \frac{1}{8\pi V} \int \left( E^2 + H^2 \right) dV \quad (5)
\]

In this respect, the Dirac proposed to examine a quantization scheme of electromagnetic field by introducing of the vector potential for local electromagnetic field $\vec{A}(\vec{r}, t)$:

\[
\vec{H} = \text{curl} \vec{A} \quad (6)
\]

and

\[
\vec{E} = -\frac{1}{c} \frac{d \vec{A}}{dt} \quad (7)
\]

which by inserting in (1)-(4), determines a wave-equation:

\[
\nabla^2 \vec{A} - \frac{1}{c^2} \frac{d^2 \vec{A}}{dt^2} = 0 \quad (8)
\]
with condition

\[ \text{div} \vec{A} = 0 \]  

(9)

where

\[
\vec{A}(\vec{r}, t) = \int \left( \vec{A}_k \exp(i\vec{k}\cdot\vec{r} + kct) + \vec{A}_k^+ \exp(-i\vec{k}\cdot\vec{r} + kct) \right) d^3k =
\]

(10)

where \( \vec{A}_k^+ \) and \( \vec{A}_k^- \) are, respectively, the Fourier components of vector potentials electromagnetic field which are considered as the vector Bose-operators “creation” and “annihilation” of a Bose-plane wave with spin one.

Obviously, we have an expression for component \( H_x \) on coordinate \( x \):

\[
H_x = (\text{curl} \vec{A})_x = \frac{dA_z}{dy} - \frac{dA_y}{dz} =
\]

(11)

Then,

\[
\vec{H} = \text{curl} \vec{A} = i \sum_k \vec{k} \times \left( \vec{A}_k \exp(i\vec{k}\cdot\vec{r} + kct) - \vec{A}_k^+ \exp(-i\vec{k}\cdot\vec{r} + kct) \right)
\]

(12)

To find \( H^2 \), we use of a supporting formulae from textbook [8]

\[
\begin{bmatrix} \vec{a} \times \vec{b} \end{bmatrix} \cdot \begin{bmatrix} \vec{c} \times \vec{d} \end{bmatrix} = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})
\]

which leads to following form for \( \frac{1}{8\pi V} \int H^2 dV \) at application (12) with using of the condition of transverse wave \( \vec{k} \cdot \vec{A}_k = 0 \) and

\[
\frac{1}{V} \int e^{i\vec{k}\vec{r}} dV = \delta_k
\]

In this respect, we possess

\[
\frac{1}{8\pi V} \int H^2 dV = -\frac{1}{8\pi} \sum_{k,E_1} \delta_{E_k + E_1} \vec{k} \vec{k}^* \left( \vec{A}_k - \vec{A}_k^+ \right) \left( \vec{A}_{E_1}^* - \vec{A}_{E_1}^+ \right) =
\]

\[
= \frac{1}{8\pi} \sum_{k} \vec{k}^2 \left( \vec{A}_k - \vec{A}_k^+ \right) \left( \vec{A}_{-\vec{k}}^* - \vec{A}_{-\vec{k}}^+ \right)
\]

(13)

We now calculate the part of the Hamiltonian radiation in (5)

\[
\frac{1}{8\pi V} \int E^2 dV = \frac{1}{8\pi c^2 V} \int \left( \frac{d\vec{A}}{dt} \right)^2 dV
\]

(14)

At calculation of value \( \frac{d\vec{A}}{dt} \), we use of a suggestion proposed by Dirac which implies a consideration of current time \( t = 0 \) [1]:

3
\[
\frac{d\vec{A}}{dt} = i e \sum_{\vec{k}} k \left( \vec{A}_{\vec{k}} - \vec{A}_{\vec{k}}^* \right) e^{i\vec{k}\cdot\vec{r}}
\]

(15)

Inserting value of \(\frac{d\vec{A}}{dt}\) from Eq.(15) into Eq.(14), we find the Hamiltonian of radiation \(\hat{H}_R\) by following form:

\[
\frac{1}{8\pi c^2 V} \int \left( \frac{d\vec{A}}{dt} \right)^2 dV = \frac{1}{8\pi} \sum_{\vec{k},\vec{k}_1} \delta_{\vec{k}+\vec{k}_1} |k| \cdot |k_1| \left( \vec{A}_{\vec{k}} - \vec{A}_{\vec{k}}^* \right) \left( \vec{A}_{\vec{k}_1} - \vec{A}_{\vec{k}_1}^* \right) = \\
= \frac{1}{8\pi} \sum_{\vec{k}} k^2 \left( \vec{A}_{\vec{k}} - \vec{A}_{\vec{k}}^* \right) \left( \vec{A}_{\vec{k}_1} - \vec{A}_{\vec{k}_1}^* \right)
\]

(16)

Thus, by using of results (13) and (16), we obtain the Dirac Hamiltonian

\[
\hat{H}_R = \frac{1}{8\pi V} \int E^2 dV + \frac{1}{8\pi V} \int H^2 dV = 0
\]

(17)

which is not able to describe the Plank photon gas. This fact allows us to suggest that it needs to find a new solution of Maxwell equations, which could provide the description of the Plank photon gas.

111. QUANTIZED MAXWELL EQUATIONS

To solve a problem connected with a quantization electromagnetic field, we propose the quantized equations of Maxwell. For beginning, we search the solution of (1)-(4) by following way:

\[
\vec{E} = -\frac{\alpha}{c} \cdot \frac{d\vec{H}_0}{dt} + \beta \cdot \vec{E}_0
\]

(18)

and

\[
\vec{H} = \frac{\alpha}{c} \cdot \frac{d\vec{E}_0}{dt} + \beta \vec{H}_0
\]

(19)

where \(\alpha\) and \(\beta\) are the constants which we obtain in the bellow by using of a physical property of electromagnetic field; \(\vec{E}_0 = \vec{E}_0(\vec{r},t)\) and \(\vec{H}_0 = \vec{H}_0(\vec{r},t)\) are, respectively, determined as the vectors second quantization wave functions for one Bose particle of electromagnetic field with spin one and mass \(m\). In this context, we claim that the vectors of local electric \(\vec{E}_0\) and magnetic \(\vec{H}_0\) fields, presented by equations (18) and (19), satisfy to the equations of Maxwell in vacuum which here describe the states of the Bose particles:

\[
curl\vec{H}_0 - \frac{1}{c} \frac{d\vec{E}_0}{dt} = 0
\]

(20)

\[
curl\vec{E}_0 + \frac{1}{c} \frac{d\vec{H}_0}{dt} = 0
\]

(21)

\[
div\vec{E}_0 = 0
\]

(22)

\[
div\vec{H}_0 = 0
\]

(23)
In this context, by using of (20), we can rewrite (19) as

$$\hat{H} = \frac{\alpha}{c} \frac{d\vec{E}_0}{dt} + \beta \hat{H}_0$$

(24)

By presentation of new terms $E_0$ and $H_0$, the Hamiltonian of radiation $\hat{H}_R$ in (5) takes a following form:

$$\hat{H}_R = \frac{1}{8\pi V} \int \left( E_0^2 + H_0^2 \right) dV = \frac{1}{8\pi V} \int \left[ \left( \frac{\alpha}{c} \frac{d\vec{E}_0}{dt} + \beta \vec{H}_0 \right)^2 \right] dV = \hat{H}_c + \hat{H}_h$$

(25)

where

$$\hat{H}_c = \frac{1}{8\pi V} \int \left[ \left( \frac{\alpha}{c} \frac{d\vec{E}_0}{dt} \right)^2 + \beta^2 \vec{E}_0^2 \right] dV$$

(26)

$$\hat{H}_h = \frac{1}{8\pi V} \int \left[ \left( \frac{\alpha}{c} \frac{d\vec{H}_0}{dt} \right)^2 + \beta^2 \vec{H}_0^2 \right] dV$$

(27)

Obviously, the equations (20)-(23) lead to a following wave-equation:

$$\nabla^2 \vec{E}_0 - \frac{1}{c^2} \frac{d^2 \vec{E}_0}{dt^2} = 0$$

(28)

and

$$\nabla^2 \vec{H}_0 - \frac{1}{c^2} \frac{d^2 \vec{H}_0}{dt^2} = 0$$

(29)

which in turn have following solutions:

$$\vec{E}_0 = \frac{1}{V} \sum_k \left( \vec{E}_k e^{j(k \vec{r} + kct)} + \vec{E}_k^+ e^{-j(k \vec{r} + kct)} \right)$$

(30)

$$\vec{H}_0 = \frac{1}{V} \sum_k \left( \vec{H}_k e^{j(k \vec{r} + kct)} + \vec{H}_k^+ e^{-j(k \vec{r} + kct)} \right)$$

(31)

where $\vec{E}_k^+$, $\vec{H}_k^+$ and $\vec{E}_k$, $\vec{H}_k$ are, respectively, the second quantization vectors wave functions, which are represented as the vector Bose-operators ”creation” and ”annihilation” of the Bose-particles of electric and magnetic waves with spin one.

We now insert a value of $\vec{E}_0$ from (30) into (26), and then:

$$\frac{1}{8\pi V} \int \left( \frac{\alpha}{c} \frac{d\vec{E}_0}{dt} \right)^2 dV = -\frac{\alpha^2}{8\pi} \sum_{k,k_1} \delta_{k+k_1} |k| \cdot |k_1| \left( \vec{E}_k - \vec{E}_k^+ \right) \left( \vec{E}_{-k} - \vec{E}_{-k}^+ \right) =$$

$$= -\frac{\alpha^2}{8\pi} \sum_k k^2 \left( \vec{E}_k - \vec{E}_k^+ \right) \left( \vec{E}_{-k} - \vec{E}_{-k}^+ \right)$$

(32)

Consequently, within introducing of assumption that the term with square wave vector $k^2$ describes the kinetic energy of free Bose-particles of electromagnetic field with mass $m$ by definition $\frac{\hbar^2 k^2}{4\pi} = \frac{\hbar^2 k^2}{2m}$, we find a constant $\alpha = \frac{\hbar}{\sqrt{2m}}$. Then, we posses:
\[
\frac{1}{8\pi V} \int \left[ \left( \frac{\alpha dE_0}{c \, dt} \right) \right]^2 dV = \sum_k \frac{\hbar^2 k^2}{2m} \hat{E}_k^+ \hat{E}_k^- - \sum_k \frac{\hbar^2 k^2}{4m} \left( \hat{E}_k^+ \hat{E}_{-k}^- + \hat{E}_{-k}^+ \hat{E}_k^- \right) - \sum_k \frac{\hbar^2 k^2}{4m}
\]

(33)

As we see the first term in right part of (33) represents as the kinetic energy of the Bose gas consisting of the Bose-particles of electromagnetic field but the second term in right part of (33) describes the term of the interaction between particles. In this context, the part in \( \hat{H}_e \) in (26) takes a following form:

\[
\frac{1}{8\pi V} \int \beta^2 E_0^2 dV = \frac{\beta^2}{8\pi} \sum_{k,k_1} \delta(k + k_1) \left( \hat{E}_{k}^+ + \hat{E}_{-k}^+ \right) \left( \hat{E}_{k_1}^- + \hat{E}_{-k_1}^- \right) = \frac{\beta^2}{8\pi} \sum_k \left( \hat{E}_{k}^+ + \hat{E}_{-k}^+ \right) \left( \hat{E}_{-k}^- + \hat{E}_{k}^- \right) = \frac{\beta^2}{8\pi} \sum_k \left( 2\hat{E}_{k}^+ \hat{E}_{k}^- + \hat{E}_{k}^+ \hat{E}_{-k}^- + \hat{E}_{-k}^+ \hat{E}_{k}^- \right) + \frac{\beta^2}{8\pi} \sum_k 1
\]

(34)

Consequently, the operator \( \hat{H}_e \) is presented by a following form:

\[
\hat{H}_e = \sum_k \left( \frac{\hbar^2 k^2}{2m} + \frac{\beta^2}{4\pi} \right) \hat{E}_k^+ \hat{E}_k^- + \frac{1}{2V} \sum_k \hat{U}_k \left( \hat{E}_k^+ \hat{E}_{-k}^- + \hat{E}_{-k}^+ \hat{E}_k^- \right)
\]

(35)

where \( \hat{U}_k = -\frac{\hbar^2 k^2 V}{2m} + \frac{\beta^2 V}{4\pi} \), in the second term in right side of (35) describes the interaction between the Bose-particles. We claim that the inter-particle interaction \( \hat{U}_k \) represents as a repulsive \( \hat{U}_R > 0 \) in the space of wave vector \( \vec{k} \).

This assumption leads to the condition for wave numbers \( k \leq k_0 = \frac{\beta}{\hbar} \sqrt{\frac{\pi}{2\pi}} \) where \( k_0 \) is the boundary maximal wave number which provides that the existence of the interaction energy \( \hat{U}_R \) between two light bosons in the coordinate space (the form of \( \hat{U}_R \) will be presented in section V):

\[
\hat{U}_R = \frac{1}{V} \sum_k \hat{U}_k \cdot e^{i\vec{k}\vec{r}}
\]

(36)

Obviously, the sum in (36) diverges, within introducing the concept of a boundary wave number \( k_0 \) for electromagnetic field. The light bosons with wave number exceeding the boundary wave number \( k_0 \) do not exist.

In analogy manner, we can find

\[
\hat{H}_h = \sum_k \left( \frac{\hbar^2 k^2}{2m} + \frac{\beta^2}{4\pi} \right) \hat{H}_k^+ \hat{H}_k^- + \frac{1}{2V} \sum_k \hat{U}_k \left( \hat{H}_k^+ \hat{H}_{-k}^- + \hat{H}_{-k}^+ \hat{H}_k^- \right)
\]

(37)

Thus, the Hamiltonian radiation \( \hat{H}_R \) is determined for the Bose gas consisting of the Bose-particles with wave numbers \( k \leq k_0 \):

\[
\hat{H}_R = \hat{H}_e + \hat{H}_h
\]

(38)
where $\hat{H}_e$ and $\hat{H}_h$ are presented by formulas (35) and (37).

To evaluate an energy levels of the operators $\hat{H}_R$ in (38) within diagonal form, we again apply new linear transformation of vector Bose-operator which is a similar to one for scalar Bose-operator presented by Bogoliubov [9]:

$$\vec{E}_k = \vec{H}_k = \frac{\vec{H}_k + M_k \vec{h}_k}{\sqrt{1 - M_k^2}}$$  \hspace{1cm} (39)

where $M_k$ is the real symmetrical functions of a wave vector $k$.
which transforms a form of operator Hamiltonian $\hat{H}_R$ by following way:

$$\hat{H}_R = 2 \sum_{k \leq k_0} \eta_k \vec{h}_k \vec{h}_k$$  \hspace{1cm} (40)

Hence, we infer that the Bose-operators $\vec{h}_k$ and $\vec{h}_k$ are, respectively, the vector operators "creation" and "annihilation" of free photons with energy

$$\eta_k = \sqrt{\left(\frac{\hbar^2 k^2}{2m} + \frac{\beta^2}{4\pi}\right)^2 - \left(\frac{\hbar^2 k^2}{2m} - \frac{\beta^2}{4\pi}\right)^2} = \frac{\hbar k \beta}{\sqrt{2m\pi}} = \hbar c$$  \hspace{1cm} (41)

where $\vec{h}_k \vec{h}_k$ is the scalar operator of the number photons occupying the wave vector $k$; $c$ is the velocity of photon which defines $\beta = c/\sqrt{2m\pi}$ because $c = \sqrt{2me}\sqrt{\pi}$ in (41). In this respect, the maximal wave number equals to $k_0 = \beta \sqrt{2m} = \frac{mc}{\hbar}$.

Thus, the quantized Maxwell equations have following forms:

$$\vec{E} = -\frac{\hbar \sqrt{2\pi}}{e\sqrt{m}} \cdot \frac{d\vec{H}_0}{dt} + c\sqrt{2m\pi} \cdot \vec{E}_0$$  \hspace{1cm} (42)

and

$$\vec{H} = \frac{\hbar \sqrt{2\pi}}{e\sqrt{m}} \cdot \frac{d\vec{E}_0}{dt} + c\sqrt{2m\pi} \vec{H}_0$$  \hspace{1cm} (43)

Our investigation showed that the boson of electromagnetic field has a certainly finite mass $m$. To find the later we states that the source of the photon modes are been the chemical elements which may consider as an ion+electron system which are like to the Hydrogen atom. Due to changing of a electron of its energetic level, by going from high level to low one, leads to an appearance of a photon with energy is determined by a distance between energetic states. The ionization energy of the Hydrogen atom $E_I = \frac{m_e e^4}{2\hbar^2 c^2}$ (where $m_e$ and $e$ are the mass and charge of electron) defines the energy of the radiated photon by maximal wave-number $k_0$. Therefore, we may suggest that $\frac{m_e e^4}{2\hbar^2 c^2} = \hbar k_0 c$ where $k_0 = \frac{m_e}{\hbar}$. This fact discovers a new fundamental constant, which represents as a mass of the light boson:

$$m = \frac{m_e e^4}{2\hbar^2 c^2} = 2.4 \cdot 10^{-35} kg$$

**IV. CONCLUSION.**

In conclusion, we can note that four fundamental particles exist in the nature:

1. the electron with mass $m_e = 9 \cdot 10^{-31} kg$; 2. the proton with mass $m_p =$
1.6 \cdot 10^{-27} \text{kg}; 3. the neutron with \( m_n = 1.6 \cdot 10^{-27} \text{kg} \); 4. the light boson with mass \( m = 2.4 \cdot 10^{-35} \text{kg} \).

Now, we present the form of the interaction energy \( \hat{U}_\vec{r} \) between two light bosons in the coordinate space in (41), at

\[
\hat{U}_\vec{r} = -\frac{\hbar^2 k^2 V}{2m} + \frac{mc^2 V}{2} > 0
\]

Our calculation shows that

\[
\hat{U}_\vec{r} = \frac{1}{V} \sum_{k \leq k_0} \hat{U}_k \cdot e^{ik\vec{r}} = 4\pi \int_0^{k_0} k^2 \hat{U}_k \frac{\sin(kr)}{kr} dk = \\
= \frac{2\pi Vmc^2}{r^3}[\sin\left(\frac{mcr}{\hbar}\right) + \frac{mcr}{\hbar} \cos\left(\frac{mcr}{\hbar}\right)] - \\
- \frac{2\pi \hbar^2}{mcr^3}\left[\left(\frac{3m^2c^2r^2}{\hbar^2} - 6\right)\sin\left(\frac{mcr}{\hbar}\right) + \frac{m^3c^3r^3}{\hbar^4} - \frac{6mcr}{\hbar}\right] - \\
- \left(\frac{m^3c^3r^3}{\hbar^4} - \frac{6mcr}{\hbar}\right)\cos\left(\frac{mcr}{\hbar}\right)
\]

(44)

The existence of a boundary wave number \( k_0 = \frac{mc}{\hbar} \) for electromagnetic field is connected with the characteristic length of the interaction \( \hat{U}_\vec{r} \) between two light bosons in the coordinate space that is a minimal distance \( d = \frac{\hbar}{mc} = 2.6 \cdot 10^{-8} \text{m} \) between two neighboring light bosons into the electromagnetic field. We may state herein that the total number of light bosons in volume \( V \) is determined as

\[
\frac{V}{N} = \frac{4\pi d^3}{3}
\]

from which \( \frac{N}{V} = 1.4 \cdot 10^{22} \text{m}^{-3} \).

The existence of light-boson with speed \( v = c \) in clear vacuum confirms a wrong sound of theories, proposed by Namby, Goldstone and Higgs, which are based on the existence of so-called vacuons [10]. On other hand, we see that the maximal momentum of light boson equals to \( p_0 = mc \) which implies that the maximal speed of light boson is a velocity of light \( v = c \) in vacuum. As result of the Relativistic Theory the relativistic mass \( M \) is presented as

\[
M = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

which equals to infinity \( M = \infty \), at \( v = c \). This fact shows a wrong sound of Einstein Theory.
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