Abstract

We improve two linear-time data reduction algorithms for the $d$-Hitting Set problem to work in linear space, thus obtaining the first algorithms for computing problem kernels of asymptotically optimal size $O(k^d)$ for $d$-Hitting Set in linear time and space. We experimentally compare the two algorithms to a classical data reduction algorithm of Weihe and evaluate their combinations.

Keywords: NP-hard problem, data reduction, parameterized complexity, computational experiments

1. Introduction

We study data reduction algorithms for the following combinatorial optimization problem:

**Problem 1** ($d$-Hitting Set, for constant $d \in \mathbb{N}$).

**Instance:** A hypergraph $H = (V, E)$ with vertex set $V = \{1, \ldots, n\}$, edge set $E \subseteq \{e \subseteq V : |e| \leq d\}$, and $k \in \mathbb{N}$.

**Question:** Is there a hitting set $S \subseteq V$ of cardinality at most $k$, that is, $\forall e \in E : e \cap S \neq \emptyset$?

Throughout this work, we denote $n := |V|$ and $m := |E|$.

The $d$-Hitting Set problem is one of Karp’s classical 21 NP-complete problems [21] and one of the most fundamental combinatorial optimization problems, arising in bioinformatics [23], data profiling [9], automatic reasoning [10, 16, 28], feature selection [19], radio frequency allocation [30], software engineering [26], and public transport optimization [8, 31].

Exact algorithms for NP-complete problems usually take time exponential in the input size. Thus, an important preprocessing step is data reduction, which has proven to quite tremendously shrink real-world instances of NP-hard problems [2, 7, 8, 31]. The main notion of data reduction with performance guarantees is problem kernelization [18], here stated for $d$-Hitting Set:

**Definition 2.** A kernelization maps any $d$-Hitting Set instance $(H_{in}, k_{in})$ to an instance $(H_{out}, k_{out})$ in polynomial time such that

1. $H_{in}$ has a hitting set of size $k_{in}$ if and only if $H_{out}$ has a hitting set of size $k_{out}$.
2. $|H_{out}| + k_{out} \leq g(k_{in})$ for a computable function $g : \mathbb{N} \to \mathbb{N}$.

One calls $(H_{out}, k_{out})$ the problem kernel and $g(k_{in})$ its size. In the kernelizations studied in our work, $k := k_{in} \leq k_{out}$.

There are two known $O(n + m)$-time kernelizations for $d$-Hitting Set [6, 15]. They yield problem kernels of size $O(k^d)$, whereas the existence of kernels of size $O(k^{d+\varepsilon})$ for any $\varepsilon > 0$ results in a collapse of the polynomial-time hierarchy [13].

Both algorithms output a subgraph of the input hypergraph and can thus be used to kernelize many constant-factor approximable problems [22]. Their weak point is a data structure that may require $O(nm)$ space [29, Section 3], which proved prohibitively large in experiments [5, Chapter 5]. On the other end, one can compute problem kernels of size $O(k^d \log k)$ in $O(k^d \log n)$ space and $O(k^d m)$ time [14] and problem kernels of size $O(k^d)$ in logarithmic space and $O(m^{k/2})$ time [15].

Our contributions and organization of this work. **Section 2** shows the two known linear-time $d$-Hitting Set kernelizations. **Section 3**, we show how to implement them in $O(n + m)$ time and space, thus enabling the computation of $d$-Hitting Set problem kernels of provably optimal size $O(k^d)$ in linear time and space. Only one of the two algorithms has been experimentally evaluated before [5, Chapter 5]. In **Section 4**, for the first time, we experimentally compare the two algorithms and also a well-known data reduction algorithm of Weihe [31], which runs in superlinear time, does not yield problem kernels, but proved very effective in practise. We will see that the kernelizations outperform the algorithm of Weihe [31] when good upper bounds on the solution size are available.

**Related work.** There are several kernelizations for $d$-Hitting Set [1, 3, 4, 12, 14, 17, 22, 24, 25]. The first linear-time kernelization was shown by van Bevern [5, 6]. The second, due to Fafianie and Kratsch [15], is simpler and has smaller constant factors: the problem kernel of van Bevern [6] has at most $d! \cdot d^{d+1} \cdot (k + 1)^d$ edges, whereas the problem kernel of Fafianie and Kratsch [15] has at most $(k + 1)^d$ edges. Both kernelizations work in $O(d \cdot n + 2^d d \log d \cdot m)$ time. Dell and van Melkebeek [13] showed that the existence of a problem kernel with $O(k^{d+\varepsilon})$ edges for any $\varepsilon > 0$ for $d$-Hitting Set implies a collapse of the polynomial-time hierarchy. Therefore, we do not expect polynomial-size problem kernels for $d$-Hitting Set if $d$ is not constant.

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Algorithm FK: Algorithm of Fafianie and Kratsch [15].

Input: Hypergraph $(V_{in}, E_{in}), k \in \mathbb{N}$.
Output: Problem kernel $((V_{out}, E_{out}), k)$ with $|E_{out}| \leq (k + 1)^d$.
// Initially, $V \subseteq V :$ supersets$[s] = \text{false}.$
1. $E_{out} \leftarrow \emptyset$
2. foreach $e \in E_{in}$ do
3. \hspace{1em} if $V \subseteq e :$ supersets$[s] < (k + 1)^d$ then
4. \hspace{2em} $E_{out} \leftarrow E_{out} \cup \{e\}$
5. \hspace{1em} foreach $s \subseteq e$ do supersets$[s] \leftarrow \text{supersets}[s] + 1$
6. $V_{out} \leftarrow \bigcup_{e \in E_{out}} e$
7. return $((V_{out}, E_{out}), k)$

Algorithm Bev: Algorithm of van Bevern [6].

Input: Hypergraph $(V_{in}, E_{in}), k \in \mathbb{N}$.
Output: Kernel $((V_{out}, E_{out}), k)$ with $|E_{out}| \leq d!d^4(k + 1)^d$.
// Initially, $V \subseteq V :$ petals$[s] = 0, \text{used}[s][v] = \text{false}.$
1. $E_{out} \leftarrow \emptyset$
2. foreach $e \in E_{in}$ do
3. \hspace{1em} if $V \subseteq e :$ petals$[s] \leq k$ then
4. \hspace{2em} $E_{out} \leftarrow E_{out} \cup \{e\}$
5. \hspace{1em} foreach $s \subseteq e$ do petals$[s] \leftarrow \text{petals}[s] + 1$
6. \hspace{1em} if $\forall v \in e :$ used$[s][v] = \text{false}$ then
7. \hspace{2em} petals$[s] \leftarrow \text{petals}[s] + 1$
8. \hspace{1em} foreach $v \in e \subseteq s$ do used$[s][v] \leftarrow \text{true}$
9. $V_{out} \leftarrow \bigcup_{e \in E_{out}} e$
10. return $((V_{out}, E_{out}), k)$

2. Known linear-time algorithms

There are two known linear-time kernelizations for $d$-Hitting Set: FK and Bev. Both iterate over each input edge $e \in E_{in}$ once and decide whether to add $e$ to the output edge set $E_{out}$:

- **FK** does not add $e$ to $E_{out}$ if $e$ contains a subset $s$ that is already contained in $(k + 1)^d$ edges of $E_{out}$.

- **Bev** does not add $e$ to $E_{out}$ if it finds that $e$ contains a subset $s$ being the pairwise intersection of $k + 1$ edges of $E_{out}$.

In both cases, any hitting set of size $k$ for $(V_{out}, E_{out})$ has to intersect $s$, and thus $e$ [6, 15]. To check these conditions, FK and Bev look up the values supersets$[s], \text{used}[s], \text{petals}[s]$ for each of the at most $2^d$ subsets $s \subseteq e$. Access to these values can be organized in $O(d)$ time using a trie that can be initialized in $O(dn + 2^d \cdot m)$ time [5, Lemma 5.3]. After initialization, FK and Bev work in $O(2^d \cdot m)$ and $O(2^d \cdot m)$ time, respectively. The culprit is that the trie can take $\Theta(nm)$ space [29, Section 3].

3. Implementing FK and Bev in linear space

3.1. FK in linear space

To implement FK in linear space, we apply a series of preprocessing steps. First, by iterating over $E_{in}$ once, simultaneously incrementing a counter, we assign to each edge $e \in E_{in}$ a unique index $e_{id} \in \{1, \ldots, m\}$, in $O(m)$ time giving a set $E_{id} := \{(e, e_{id}) : e \in E_{in}\}$. Then, we compute a unique index $s_{id} \in \{1, \ldots, 2^d m\}$ for each $s \subseteq e \in E_{in}$ and a size-$m$ array $A$ satisfying

$$A[e_{id}] = \{(s, s_{id}) : s \subseteq e\} \text{ for each } (e, e_{id}) \in E_{id}.$$ 

Instead of iterating over each $e \in E_{in}$, each $s \subseteq e$, and looking up supersets$[s]$ in a trie, this allows FK to iterate over each $(e, e_{id}) \in E_{id}$, each $(s, s_{id}) \in A[e_{id}]$, and look up supersets$'[s_{id}]$ in an array of length $2^d m$ in $O(1)$ time.

**Theorem 3.** FK can be run in $O(nd + 2^d \cdot m)$ time and space.

**Proof.** To prove the theorem, we show how to compute the array $A[\cdot]$ and a unique index $s_{id}$ for each $s \subseteq e \in E_{in}$ in linear space and time. The tricky bit is that $s$ may be a subset of several edges in $E_{in}$, yet its index $s_{id}$ must be unique. Thus, we use the following canonical encoding of edges: for any subset $s \subseteq V = \{1, \ldots, n\}$ of size at most $d$, $\text{l.e.s.} \in (V \cup [\square])^d$ is a $d$-tuple containing the elements of $s$ in increasing order and padded with $\square$ at the end. For example, for $d = 4, \text{l.e.s.} \in (1, 2, 3, \square)$. Obviously, i.e., for any edge $e \in E_{in}$ is computable in $O(d \log d)$ time. The preprocessing for FK now consists of three steps.

1. Compute a list $L = \{(s, s_{id}) : s \subseteq e, (e, e_{id}) \in E_{id}\}$ by first computing i.e. for each $e \in E_{in}$ in $O(m \cdot d \log d)$ time and space and then enumerating all substrings of i.e. for each $(e, e_{id}) \in E_{id}$ in $O(2^d \cdot m)$ time and space.

2. Sort $L$ by lexicographically non-decreasing i.e., where we assume $n \leq \square$. Since the i.s. are $d$-tuples over $\{1, \ldots, \square, n\}$, this works in $O(dn + \log |L|) = O(nd + 2^d \cdot m)$ time and space using radix sort [11, Section 8.3]. Note that all pairs $(s, j, e_{id})$ belonging to the same subset $s$ now occur consecutively in $L$.

3. Initialize a size-$m$ array $A[\cdot]$ of empty lists and $s_{id} \leftarrow 1$. Iterate over $L$ as follows. For the current pair $(s, j, e_{id})$, add $(s, s_{id})$ to $A[e_{id}]$. If there is a next pair $(s', j', e_{id}')$ on $L$ and $s, s' \neq \square$, then increment $s_{id} \leftarrow s_{id} + 1$ and continue.

This concludes the computation of the $s_{id}$ and the array $A[\cdot]$. The running time and space bottleneck is step 2. After this preprocessing, FK can be implemented to run in $O(2^d \cdot m)$ time using an array supersets$'[\cdot]$ of size $O(2^d \cdot m)$.

3.2. Bev in linear space

To implement Bev in linear time and space, we replace the trie accesses petals$[s]$ and used$[s]$ for each $s \subseteq e \in E_{in}$ by array accesses petals$'[s_{id}]$ and used$'[s_{id}]$, as we did for FK. However, while petals$[s]$ is a counter that translates into a counter petals$'[s_{id}]$, used$[s]$ is a size-$n$ array indexed by vertices. Holding such an array in used$'[s_{id}]$ would again use $\Omega(nm)$ space. Instead, we organize used$'[s_{id}]$ as follows. Let $V' := \bigcup_{e \in E_{in}} (e \setminus s)$ for each $s \subseteq e \in E_{in}$. Then, we can use a $\bigcup_{e \in E_{in}} (e \setminus s)$ for each $s \subseteq e \in E_{in}$. The conclusion of the $s_{id}$ and the array $A[\cdot]$. The running time and space bottleneck is step 2. After this preprocessing, FK can be implemented to run in $O(2^d \cdot m)$ time using an array supersets$'[\cdot]$ of size $O(2^d \cdot m)$.
We compute unique indices \( v^t_{id} \in [1, \ldots, |V^t|) \) for the vertices \( v \in V^t \) for each \( s \subseteq e \in E_{in} \), unique indices \( s^t_{id} \in [1, \ldots, 2^{|d|}] \) of the subsets \( s \subseteq e \) of each \( e \in E_{in} \), an array \( B \) satisfying
\[
B[e_{id}] = ((s, s_{id}, v^t_{id}) : s \subseteq e)
\]
for each \((e, e_{id}) \in E_{id}\), and an array \( B \) of arrays satisfying
\[
C[e_{id}][s^t_{id}] = \{v^t_{id} : v \in e \setminus s\}
\]
for each \((e, e_{id}) \in E_{id}, (s, s_{id}, v^t_{id}) \in B[e_{id}]\).

Bev can then be implemented using arrays petals[] and used[] of size \( 2^m \) each, where for each \( s_{id} \), used[] is an array of size \( |V^t| \): instead of iterating over each \( e \in E_{in}, s \subseteq e \), each \( v \in e \setminus s \), and looking up petals[] and used[v] in tries, Bev can iterate over each \((e, e_{id}) \in E_{id}, (s, s_{id}, v^t_{id}) \in B[e_{id}]\), and look up petals[] and used[] in tries. These are simple array accesses, each working in constant time.

**Theorem 4.** Bev can be run in \( O(nd + 2^d \cdot m) \) time and space.

**Proof.** We describe how to compute the indices \( v^t_{id}, s^t_{id} \) and the arrays \( B \) and \( C \) in linear time and space. First, the indices \( s_{id} \) and array \( A[l] \) are computed as in Theorem 3 in \( O(nd + 2^d \cdot m) \) time and space. For Bev, we use three additional preprocessing steps.

1. Initialize a size-\( m \) array \( B[l] \). For each \((e, e_{id}) \in E_{id}\), compute \( B[e_{id}] \) from \( A[e_{id}] \) by iterating over each \((s, s_{id}) \in A[e_{id}]\), simultaneously incrementing a counter \( s^t_{id} \) from 0 to \( 2^{|d|} \). This works in time \( O(2^d \cdot m) \) and space.

2. Iterating over each \((e, e_{id}) \in E_{id}\) and each \((s, s_{id}, v^t_{id}) \in B[e_{id}]\), in \( O(2^d \cdot m) \) time, generate a list
\[
L := \{(s_{id}, v, s^t_{id}, e_{id}) \mid v \notin e \setminus s, s \subseteq e, e \in E_{in}\}.
\]

Sort the list by lexicographically non-decreasing \((s_{id}, v)\). Since these are pairs of numbers in \([1, \ldots, 2^m] \cup [1, \ldots, n]\), this works in \( O(n + 2^m + |L|) = O(n + 2^m) \) time using radix sort [11, Section 8.3]. Thereafter, all quadruples belonging to the same \( s_{id} \) occur consecutively in \( L \). Also, for each fixed \( s_{id} \), all quadruples belonging to \( s_{id} \) and the same \( v \) occur consecutively in \( L \).

3. Initialize a size-\( m \) array \( C[l] \), and for each \((e, e_{id}) \in E_{id}\), a size-\( 2^{|d|} \) array \( C[e_{id}][v^t_{id}] \) of empty lists. Iterate over each \((s, s_{id}, v^t_{id}, e_{id}) \in L\). If there is no predecessor on \( L \) or the predecessor \((s^t_{id}, v^t_{id}, s^t_{id}, e_{id}) \) satisfies \( s_{id} \neq s^t_{id} \), then initialize \( v^t_{id} \leftarrow 1 \). If \( s_{id} = s^t_{id} \) but \( v \neq v^t \), then increment \( v^t_{id} \leftarrow v^t_{id} + 1 \). Add \( v^t_{id} \) to \( C[e_{id}][v^t_{id}] \) and continue.

The running time for the preprocessing is dominated by \( O(nd + 2^d \cdot m) \) for computing array \( A[l] \) as in Theorem 3. The space used additionally to Theorem 3 is the array \( B[l] \) of overall size \( O(2^d \cdot m) \), and the size-\( m \) array \( C[l] \). Each entry of \( C[l] \) is an array of size at most \( 2^d \), whose entries are lists of length at most \( d \). Thus, \( C[l] \) takes at most \( O(2^d \cdot m) \) total space.

After preprocessing, Bev can be run in \( O(2^d \cdot m) \) time using arrays petals[] and used[] with \( 2^d \cdot m \) entries each. For each \( s \subseteq e \in E_{in} \), used[] is an array indexed by \([1, \ldots, |V^t|]) \), where
\[
\sum_{s \subseteq e \in E_{in}} |V^t| \leq \left| \{(s, e, v) \mid s \subseteq e \in E_{in}, v \in e \setminus s \} \right| \leq 2^d \cdot m.
\]

Thus, the total size of used[] is \( O(2^d \cdot m) \).

4. Experiments

In this section, we compare our linear-space variants of FK and Bev and the well-known data reduction algorithm Wei. Wei does not work in linear time, yet works independently of \( k \). We use experimental results to show whether our algorithms are faster than Wei.

**Algorithm Wei:** Algorithm due to Weihe [31]

**Input:** Hypergraph \((V_{out}, E_{out})\).

**Output:** Hypergraph \((V_{out}, E_{out})\) that has a hitting set of size \( k \) if and only if \((V_{in}, E_{in})\) has.

Exhaustively apply the following two data reduction:

1. If, for some vertex \( v \), all edges containing \( v \) also contain some vertex \( u \neq v \), then delete \( v \).
2. If there are two edges \( e \subseteq e' \), then delete \( e' \).

Return the result \((V_{out}, E_{out})\).

Section 4.1, describes our experimental setup. Section 4.2 presents time and memory measurements. We analyze the effect of data reduction on instances arising in public transportation optimization (Section 4.3), and data clustering (Section 4.4).

4.1. Experimental setup

All algorithms were implemented in C++ [2]. The source code of FK, Bev, and Wei is about 310 lines, 440 lines, and 180 lines, respectively. The experiments were conducted on a 3.60 GHz processor with 16 GB of RAM. The running time is measured with the standard C++ library ctime. The memory consumption is measured using the valgrind memory measurement tool.

The algorithms FK and Bev require an upper bound \( k \) on the minimum hitting set size as input. Unless stated otherwise, we compute \( k \) using a greedy approach: repeatedly pick a vertex with a maximum number of incident edges, add it to the hitting set, and remove all incident edges, until all edges are hit.

We analyze the data reduction effect of the individual algorithms as well as of their combination, applying a data reduction algorithm to the output of previous data reduction algorithms. Since the data reduction effect of the algorithms may depend on the processing order of the edges [5, Fig. 5.3], each algorithm is applied to a random permutation of edges. This excludes the possibility that instance generators or previous data reduction algorithms generate particularly “friendly” input orders.

The order of combining the algorithms is determined by their running times: since the running time of Wei is non-linear, it is applied last, so that it is run on a problem kernel with size independent of \( n + m \). Bev is slower than FK, so it is applied after FK.

We measure the data reduction effect comparing the number \(|E_{in}|\) of input edges to the number \(|E_{out}|\) of output edges.

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1 The source code is freely available at https://gitlab.com/PavelSmirnov/hs-lintimespace.
4.2. Time and memory measurements

In the following, we present measurements of the running time and memory usage by FK and Bev. Since these do not depend on the input structure (both algorithms iterate over each subset of each input edge and store information on them), we can safely measure them on random instances. The instances consist of \( n = 100 \) vertices and \( m \in [i \cdot 10^5 \mid i \in \{1, \ldots, 10\}] \) edges of size \( d \in \{1, \ldots, 5\} \), each chosen with equal probability.

Fig. 1 shows the measured running time and space consumption of FK and Bev. As expected, Bev is roughly three times slower than FK: FK has an initialization phase running in \( O(nd + 2^d \cdot m) \) time and then runs in \( O(2^d \cdot m) \) time, whereas Bev has two initialization steps, each running in \( O(nd + 2^d \cdot m) \) time, and thereafter itself runs in \( O(2^d \cdot m) \) time. Bev also uses three times more memory than FK. However, for comparison, the original implementation of Bev used 20 GB of memory for \( d = 4 \) on \( m = 10^6 \) hyperedges [5, Fig. 5.6], whereas our linear-space implementation uses 3 GB.

4.3. Transportation Networks

Wei is well known for its data reduction effect on the Station Cover problem in real-world transportation networks [31]. Table 1 shows the data reduction effect of FK, Bev, and Wei on \( d \)-Hitting Set instances modeling the Station Cover problem in European transportation networks from different cities (athens, petersburg, warsaw), rural areas (ancf, kvv, vrs, rnv, vbb), and countries (nl, luxembourg, switzerland, db), which were kindly made available to us by Bläsius et al. [8]. Wei obviously outperforms FK and Bev on this data. Moreover, FK and Bev work equally bad. Combinations of the algorithms did not yield any additional data reduction effect.

The reason for the bad performance of FK and Bev is the large hyperedge size \( d \), shown in Table 1. Indeed, FK and Bev are not applicable to the shown instances right away: it is infeasible to iterate over all \( 2^d \) subsets of an edge \( e \) of size \( d \) when \( d \) is large (say, 10). In such cases, we do not iterate over all subsets of \( e \), but only over subsets of intersections with other hyperedges.

4.4. Cluster Vertex Deletion

In this section, we analyze the data reduction effect of FK, Bev, and Wei on 3-Hitting Set instances arising from the Cluster Vertex Deletion problem: the task is to delete at most \( k \) vertices from a graph so that each connected component in the remaining graph is a clique [20]. A Cluster Vertex Deletion instance \((G, k)\) with \( G = (V, E) \) can be reduced to a 3-Hitting Set instance \((H, k)\) with \( H = (V, \{e \subseteq V : G[e] \text{ is a path on three vertices}\}) \) [20].

We applied FK, Bev, and Wei to Cluster Vertex Deletion instances arising when clustering real-world protein similarity graphs initially used by Rahmann et al. [27]. In fact, they used these graphs as instances for the weighted Cluster Editing problem, where one adds and deletes edges instead of deleting vertices. As suggested by Rahmann et al. [27], we create an edge between two proteins if their similarity score is positive. Since our problem is unweighted, we stripped the graphs off the weights. To be able to run Wei on the obtained instances, we considered only those 3-Hitting Set instances with at most \( 10^5 \) edges.

Fig. 2a shows that Wei works well on small instances, but is outperformed by FK and Bev on larger instances. Interestingly, pipelining any two algorithms (FK+Wei and Bev+Wei) significantly improves the data reduction effect compared to the single algorithms. In all cases, Bev slightly outperforms FK. Pipelining all three algorithms showed no significant improvement.

Noticeably, the data reduction effect of all algorithms shrinks with the size of the input hypergraph. Fig. 3 suggests that this is connected to the locality of the input hypergraphs, which Bläsius et al. [8] observed to heavily influence the effectiveness of Wei: the locality of a hypergraph \( H = (V, E) \) is \( 4 \cdot \#C_4/\#P_4 \), where \#C_4 is the number of induced cycles of length four and \#P_4 is the number of induced paths on four vertices in the bipartite incidence graph of \( H \). It is plausible that locality influences the effectiveness of FK and Bev: the greater the locality is, the more likely it is that edges have common subsets.

\[\text{Available at https://bio.informatik.uni-jena.de/data/ as biological_bielefeld.zip}\]
Table 1: Data reduction effect on transportation networks for the Station Cover problem. Shown are the number $|V|$ of vertices, number $|E_{\text{in}}|$ of edges, maximum edge size $d$, and average edge size $\bar{d}$ of the input instance.

| input instance | name   | $|V|$ | $|E_{\text{in}}|$ | $d$  | $\bar{d}$ | $|E_{\text{out}}|$ | $|E_{\text{out}}|/|E_{\text{in}}|$ |
|----------------|--------|------|------------------|-----|----------|----------------|------------------|
| db             | 514    | 586  | 41               | 13.76 | 174      | 0.30           | 20               | 0.03             |
| sncf           | 3789   | 974  | 29               | 7.61  | 666      | 0.68           | 287              | 0.29             |
| luxembourg     | 2496   | 343  | 54               | 19.64 | 293      | 0.85           | 59               | 0.17             |
| switzerland    | 25315  | 4797 | 68               | 10.60 | 4166     | 0.87           | 2002             | 0.42             |
| kvv            | 2401   | 306  | 60               | 15.98 | 273      | 0.89           | 126              | 0.41             |
| vbb            | 13424  | 1241 | 59               | 16.13 | 1141     | 0.92           | 554              | 0.45             |
| nl             | 31250  | 2804 | 107              | 16.13 | 2620     | 0.93           | 1138             | 0.41             |
| vrs            | 5542   | 521  | 65               | 20.96 | 499      | 0.96           | 143              | 0.27             |
| petersburg     | 5689   | 741  | 62               | 17.18 | 718      | 0.97           | 382              | 0.52             |
| rnv            | 1094   | 92   | 48               | 16.45 | 90       | 0.98           | 36               | 0.39             |
| warsaw         | 4303   | 343  | 57               | 23.06 | 339      | 0.99           | 199              | 0.58             |
| athens         | 5922   | 247  | 129              | 42.62 | 247      | 1.00           | 191              | 0.77             |

Figure 2: Data reduction effect on 3-Hitting Set instances arising from Cluster Vertex Deletion instances.

(a) Cluster Vertex Deletion on protein similarity graphs.

(b) Artificial Cluster Vertex Deletion instances. Bev performs similar to FK and is thus omitted in the plot.
We presented the first linear-time and linear-space kernelization algorithms for \( d \)-Hitting Set, improving the space requirements of the known kernelizations FK and Bev due to Fafianie and Kratsch \cite{15} and van Bevern \cite{5}, respectively.

We also conducted the first experimental evaluation of FK, significantly extended previous experimental results for Bev, and compared them to the well-known Wei data reduction algorithm. The experiments show that Wei is outperformed by FK and Bev on hypergraphs of small edge cardinality when one has good upper bounds on the hitting set size. In other cases, Wei outperforms FK and Bev, so that the algorithms complement each other. The data reduction effect of Wei can be strengthened by applying FK and Bev in advance. This seems advantageous anyway, since Wei does not run in linear time and can then work on a problem kernel, whose size is independent of the input hypergraph.

We have also seen that, although its worst-case kernel size bound is worse, the data reduction effect of Bev is slightly better than that of FK since it exploits upper bounds on the hitting set size more willingly: FK reduces edges only when the size of the input hypergraph is “close” to its worst case guarantee of \((k + 1)^d\).

\section{Conclusion}

To further study the effect of cascading data reduction algorithms, we randomly generate Cluster Vertex Deletion instances with small solutions. To this end, we fix the number \( n \) of vertices and a number \( c \) of cliques. We randomly partition the \( n \) vertices into \( c \) cliques (each partition is equiprobable). Then, \( k \) times, we randomly pick two vertices \( u \) and \( v \), add the edge \( \{u, v\} \) to the graph if there is no such edge, and remove it otherwise. Since any such “wrong” edge can be removed by removing one of its endpoints, the generated Cluster Vertex Deletion instance (and resulting 3-Hitting Set instance) will have a solution of size at most \( k \).

Fig. 2b shows the data reduction effect of FK and Wei on the generated instances for \( k = 10, c = 10 \), and \( n \in \{100, 100, \ldots, 1000\} \) (the graph for Bev is omitted since it nearly coincides with FK). Like with the biological data, the combination FK+Wei works better than the individual algorithms. Interestingly, this combination often leaves no more than \( k \) edges, thus essentially solving the problem, since one can construct a hitting set of size \( k \) by just choosing an arbitrary vertex in each edge.

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