Simulation of positron energy spectra generated by channeling radiation of GeV electrons in a tungsten single crystal

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Abstract. Positron production based on the generation of channeling radiation by relativistic electrons channeled along the (110) crystallographic plane of a W crystal and the subsequent conversion of radiation into $e^+e^-$-pairs in an amorphous tungsten target is described. Electron dechanneling is considered by solving of the Fokker-Planck equation. The continuous potential of the channeling plane is calculated using the Doyle–Turner approximation to the atomic scattering factor taking into account thermal vibrations of the crystal atoms. The trajectories, velocities and accelerations of planar channeled electrons are obtained by solving the classical equation of motion. In the framework of classical electrodynamics, the spectral-energy distribution of radiation is obtained from the Fourier transforms of realistic electron trajectories, velocities and accelerations within the W crystal. The calculations of channeling radiation and dechanneling are carried out by means of our Mathematica codes. The conversion of radiation into $e^+e^-$-pairs and the energy distributions of produced positrons are simulated using the GEANT4 package.

1. Introduction

Channeling radiation (CR) is generated by relativistic charged particles which traverse a single crystal parallel to a major crystallographic plane or axis [1]. Intense $\gamma$-radiation directed into a narrow cone can be obtained by channeling of high-energy (GeV) electrons and may be applied for the creation of positron beams. In conventional positron sources, $e^-e^+$-pairs are produced by conversion of ordinary bremsstrahlung (BS) generated in relatively thick amorphous targets which simultaneously serve as radiation converters. It has, however, been shown that aligned crystal targets are more effective radiators than amorphous ones. The enhancement of the positron yield due to CR has been confirmed experimentally at CERN [2-4] and KEK [5, 6]. The shower process of pair production in a single W crystal as well as combined target assemblies consisting of a crystalline radiator (W, Si, Ge, diamond) and an amorphous converter have been studied in [7-9].

For optimization of the type and thickness of the crystal radiator of a non-conventional positron source, consideration of dechanneling in thick crystals is essential. Applying the method explained in [10], this work aims at calculation of dechanneling of electrons planar channeled in a W single crystal by solving the Fokker-Planck equation. The dynamics of the particle distribution density is investigated in dependence on the electron energy and the initial scattering distribution. Radiation spectra of CR are obtained for thick W crystals using the theory already described in [11-13]. The
conversion of the calculated radiation into $e^{-}e^{+}$-pairs in an amorphous W target has been simulated by means of the GEANT4 package. Positron energy spectra resulting from the conversion of CR generated in the crystalline W radiator are presented.

2. Basics of channeling
The shape of the continuous periodic transverse potential, $U(x)$, depends on the structure and lattice parameters of the used crystal. Our calculations are based on the Doyle–Turner approach [14] already applied in [11-13]. Figure 1 shows the continuous potential calculated for electrons channeled along the (110) plane of a W single crystal. For convenience, it was shifted with its minimum to zero. The equation of motion of a relativistic particle in the one-dimensional planar potential $U(x)$ reads

$$m \gamma \ddot{x}(t) = F = -\frac{\partial U(x)}{\partial x},$$

(1)

where $m$ is the particle rest mass and $\gamma$ is the Lorentz factor. The initial conditions for solving of equation (1) are (i) the point of incidence of the particle into the crystal, $x(0) = x_0$, and (ii) its transverse momentum, $p_x(0) = p \theta_0$, where $\theta_0$ is the angle of incidence with respect to the (110) plane. The initial transverse energy is given by

$$E_x = \frac{p^2 \theta_0^2}{2m} + V(x_0).$$

(2)

Figure1. Continuous potential for electrons channeled in the (110) plane of a W single crystal.

3. Solution of the Fokker-Planck equation
3.1. Basic background
The general expression of the Fokker-Planck equation which describes the evolution of the probability density $F(z, E_\perp)$ in time (or with the traversed distance $z$ in the crystal, respectively) reads [10]

$$\frac{\partial F(z, E_\perp)}{\partial z} = \frac{\partial^2}{\partial E_\perp^2} \left[ D^2(E_\perp) F(z, E_\perp) \right] - \frac{\partial}{\partial E_\perp} \left[ D^1(E_\perp) F(z, E_\perp) \right]$$

(3)

where the drift coefficient $D^1(E_\perp) = \frac{\langle \Delta E_\perp \rangle}{\Delta z} |_T$ describes the mean increase of the transverse energy and the diffusion is given by $D^2(E_\perp) = \frac{1}{2} \frac{\langle (\Delta E_\perp)^2 \rangle}{\Delta z} |_T$. These coefficients are averaged over one oscillation period of the electron within the continuous potential. Using the Kitagava-Ohtsuki approximation [15], drift and diffusion coefficients are calculated by means of the integrals

$$D^1(E_\perp) = \kappa \int_{-X_d}^{X_d} \exp(-x^2/2\delta^2) \frac{\exp(-x^2/2\delta^2)}{\sqrt{2(E_\perp - U(x))/\delta}} dx$$

(4)
\[ D^2(E_\perp) = \kappa \int_{x_d}^{X_0} \frac{2(E_\perp-U(x))\exp(-x^2/2u_1^2)}{\sqrt{2(E_\perp-U(x))/E}} \, dx \]  

where \( \kappa = \frac{E^2_s}{4\pi e^2 v_0 x_0 T(E_\perp) \sqrt{2m_0 u_1}} \) is a function defined by projectile and crystal parameters with \( E_s = 13.6 \) MeV, \( E \) is the total electron energy, \( v \) is the electron velocity, \( u_1 \) is the thermal vibration amplitude, \( X_0 \) is the characteristic radiation length of the crystal, \( d_p \) is the inter-planar distance, \( x_d \) is the point corresponding to the solution \( E_\perp = U(x) \) for \( E_\perp < U(d_p/2) \) and \( x_d = d_p/2 \) for \( E_\perp \geq U(d_p/2) \). The time parameter is defined by

\[ T(E_\perp) = 2\sqrt{\frac{E}{2e^2}} \int_{x_d}^{X_0} \frac{dx}{\sqrt{(E_\perp-U(x))}}. \]  

As initial conditions at \( z = 0 \), required for the numerical solution of equation (3), a uniform electron distribution across the transverse \( x \)-coordinate and a Gaussian scattering distribution tilted by an angle \( \theta_0 \) and with a standard deviation \( \sigma_y \) for the angular divergence are assumed. The probability density is then given by the integral

\[ F_0(E_\perp) = \frac{1}{4\pi \sigma_y \theta_0 \sqrt{U(d_p/2)}} \int_{x_d}^{X_0} \frac{dx}{\theta(E_\perp)} \left( \exp \left[ -\frac{(\theta(E_\perp)-\theta_0)^2}{2(\sigma_y \theta_0)^2} \right] + \exp \left[ -\frac{(-\theta(E_\perp)-\theta_0)^2}{2(\sigma_y \theta_0)^2} \right] \right) \]  

with \( \theta(E_\perp) = \frac{2(E_\perp-U(x))}{E} \) and the critical channeling angle \( \theta_c = \sqrt{2U(d_p/2)/E} \).

Figure 2. (a) Time parameter \( cT \), (b) drift coefficient \( D^1 \), (c) diffusion coefficient \( D^2 \), (d) initial probability density distribution \( F_0 \) at \( z=0 \), calculated with standard deviation \( \sigma_y = 0.5 \) c and \( \theta_0 = 0 \), all as function of transverse energy \( E_\perp \) for 2 GeV electrons channeled along the (110) plane of a W single crystal.

3.2. Numerical results

The time parameter \( cT \), drift coefficient \( D^1(E_\perp) \), diffusion coefficient \( D^2(E_\perp) \) and the initial probability density distribution \( F_0(E_\perp) \) are shown in figures 2(a)-(d) as functions of transverse energy \( E_\perp \) for electrons of energy 2 GeV channeled in the (110) plane of a W single crystal. Equation (3) has been solved numerically within the bounds \( 0 \leq z \leq 200 \) \( \mu m \) and \( 0 \leq E_\perp \leq 10U(d_p/2) \) using Mathematica 7. The solution is depicted in figure 3.

The fraction \( f_{ch}(z) = \int_0^{U(d_p/2)} F(z, E_\perp) \, dE_\perp \) of channeled electrons with transverse energies \( \theta \leq E_\perp \leq U(d_p/2) \) is illustrated in figure 4. The dechanneling length is obtained from
\begin{equation}
J(z,E_L) = -\frac{\partial}{\partial E_L}[D^2(E_L)F(z,E_L)] + D^1(E_L)F(z,E_L) = J_{\text{diff}} + J_{\text{drift}} \tag{8}
\end{equation}

\begin{equation}
L_d(z) = \frac{f_{ch}(z)}{J_{\text{drift}}(z,E_L=U/E)} \tag{9}
\end{equation}

where \( z \) follows from the condition \( f_{ch}(z) = \frac{1}{e} f_{ch}(0) \) \([10]\). For 2 GeV electrons channeled in the (110) plane of the W crystal, a dechanneling length of 5.56 \( \mu m \) has been obtained for \( z=15 \mu m \). The dechanneling length as function of the electron energy is shown in figure 5. It scales with the beam energy such as \( L_d = 2.78 \mu m \times E/GeV \).

\( f(z,E_L)(s^{-1}) \)

**Figure 3.** Probability density for 2 GeV electrons channeled along the (110) plane of a W single crystal for a standard deviation of the scattering angle \( \sigma_{\gamma}=0.5\% \) and the angle of incidence \( \theta_0=0\). 

**Figure 4.** Electron dechanneling function \( f_{ch}(z) \) for (110) planar channeling of 2 GeV electrons in a W single crystal.

**Figure 5.** Dechanneling lengths as function of the electron energy for (110) planar channeling of electrons in a W single crystal.

### 4. Channeling radiation generated in a thick W single crystal

For planar channeling, the total energy radiated into the solid angle \( 1/\gamma \) per unit path length in the crystal reads

\[
\frac{dE}{\hbar \omega \Delta \xi} = \frac{e^2}{4\pi^2 \hbar c^2} \sum_{n=1}^{\infty} \Theta[1-\eta_n] \omega \tilde{\omega} \left( \eta_n^2 - \eta_n + \frac{1}{2} \right) |\tilde{x}_{\omega\xi}|^2 \tag{10}
\]

where \( \Theta[1-\eta_n] \) is the theta-function, \( \tilde{x}_{\omega\xi} \) is the Fourier component of the transverse velocity, and \( T \) is the period of electron oscillations. The CR intensity for thick crystals is given by
These equations are valid for a single particle. Averaging over all possible electron trajectories (or over the complete set of transverse energies $E_{\perp}$) reflects a uniform distribution overall points of incidence into the crystal. Figure 6 shows the dependence of the CR spectrum on the crystal thickness for 2 GeV electrons channeled in the (110) plane of a W single crystal.

5. Simulation of positron spectra

The scheme of the considered positron source (figure 7) consists of a 2 GeV electron beam, a W crystal radiator being aligned with its (110) plane parallel to the beam and an 8 mm thick amorphous W converter. Taking the calculated CR spectra as input data, positron energy distributions obtained at the converter exit have been simulated by means of the GEANT4 Monte Carlo package and are shown in figure 8.

Figure 6. Relative thickness dependence of integrated CR spectra of 2 GeV electrons channeled along the (110) plane of a W single crystal for a standard deviation of the scattering angle $\sigma_y = 100 \mu$rad and the angle of incidence $\theta_0 = 0$.

Figure 7. Scheme of a non-conventional positron source consisting of a crystalline radiator for generation of CR combined with an amorphous conversion target for pair production.

Figure 8. Positron energy distributions simulated for conversion of CR generated by 2 GeV electrons channeled in the (110) plane of a W crystal radiator of different thickness. $L$ is the calculated dechanneling length.
Here it is important to note that we considered only that part of positrons which is produced by the amount of CR generated in the crystalline W radiator ($\theta_s = 0$). Therefore, the results shown in figure 8 are for now qualitative because for quantitative statements and comparison with experimental data one has to account yet for realistic beam parameters (emittance), precise geometry of the source set up and additional (not small) contributions to the positron spectra originating from ordinary and coherent BS. Nevertheless, from figure 6 one can already conclude that CR leads to a weakening of the positron spectrum compared with BS. Furthermore, it can be seen that due to dechanneling the obtained CR yield does not increase linearly with increasing radiator thickness. Note also that the structures in the CR spectra of figure 6 occurring due to integration over the total solid angle $1/\gamma$ of radiation emission are smeared out in the respective positron spectra due to the partitioning of the photon energy between electron and positron of created $e^-e^+$-pairs.

A series of positron-production experiments with axially-oriented single-crystal W targets has been performed at 4 and 8 GeV electron energy at the KEKB facility [5, 6]. The thickness of the used W crystals amounted to 2.2 ÷ 14.2 mm. Our calculations showed that typical dechanneling lengths for W crystals are much smaller. Moreover, the dechanneling length decreases with increasing atomic number of crystal material. Therefore, main part of positrons obtained in the KEK experiments, was produced by BS. Theoretically, a diamond radiator would probably be a good alternative.

6. Conclusion
We presented simulations of positron energy spectra obtained by CR induced $e^-e^+$-pair production in an amorphous W converter. Planar CR is generated by ultra-relativistic electrons in a separate crystalline W radiator aligned with its crystallographic (110) plane to the direction of the electron beam. We for the first time calculated dechanneling of high-energy electrons planar channeled in a W crystal and deduced the beam-energy scaling of the dechanneling length. Since dechanneling has remarkable consequences on the CR yield, it should be taken into account for optimization of the radiator thickness of a hybrid positron source. For comparison with experimental data, our formalism will be adapted for simulation of realistic non-conventional positron sources where, in a next step, the effect of BS accompanying CR on the produced positron spectra will be investigated.

7. References
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