The Zakharov-Kuznetsov Equation as a
Two-Dimensional Model for Nonlinear Rossby Waves

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Abstract
We study the dynamics of two-dimensional coherent structures in planetary atmospheres and oceans. We derive the Zakharov-Kuznetsov equation for large scale motion from the barotropic quasigeostrophic equation in a weakly nonlinear, long wave approximation. We consider coherent structures emerging out of an instability caused by a narrow jet-like meanflow. We use multiple scale analysis combined with asymptotic matching.

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1 Introduction
Planetary atmospheres and oceans are strongly turbulent media. However, highly ordered coherent structures arise in a process of self-organization, and dominate the dynamics on slow temporal and large spatial scales.

The spontaneous appearance of coherent structures is a characteristic of two-dimensional fluid flows. The basic underlying structure of these flows is linked to the existence of two quadratic, positive definite invariants, energy and enstrophy. In spectra of two-dimensional turbulence one observes two different cascades associated with these conserved quantities; a direct enstrophy cascade towards small spatial scales, and an indirect energy cascade towards larger spatial scales. It is the latter which gives rise to vortex merging leading to larger and larger vortices. In this Letter we view the problem as one of weakly nonlinear hydrodynamic stability rather than turbulence phenomenology.

Most vortices are monopolar, but dipole and even tripole vortices can also appear spontaneously. Monopole vortices are mainly created by shear flow instabilities whereas dipole vortices typically appear when some
additional forcing is applied to the flow.
The richness and complexity of two-dimensional flows and the simultaneous presence of motion on very
different temporal and spatial scales makes a direct analysis of the basic equations of motion very
difficult. The presence of rotation reinforces the two-dimensional character in accordance with the
Taylor-Proudman theorem, but rotation can also introduce baroclinic instability. The latter is a three-
dimensional feature and, thus supports a direct energy cascade towards small scales. Hence, the dynamics
is determined by competing two-dimensional and three-dimensional processes [22, 1]. To study vor-
tices in geophysical fluid dynamics the primitive equations are further reduced by approximations which
allow to focus on temporal and spatial length scales of vortices [28]. If additionally baroclinic pro-
cesses are excluded a further simplification can be made. The dynamically important variable is the so
called potential vorticity \( q \). The resulting quasigeostrophic barotropic vorticity equation

\[ \frac{D}{Dt} q = 0 \quad \text{where} \quad q = \frac{\Delta \psi + f(y)}{H} \]  

(1)
describes large scale motion on a slow time scale. Here \( \psi \) is the stream function, \( f(y) \) describes the
ambient rotation of the planet and \( H \) is the fluid depth. This equation was first derived by Charney [4], and then independently by Obukhov [26]. In the context of low-frequency drift waves in magnetized plasmas Equation (1) is known as the Hasegawa-Mima equation [11]. This equation has been the
mathematical starting point for much of the research done on coherent structures and vortices. It sup-
ports so called modons which are localized soliton-like coherent solutions. Exact modon solutions were
obtained by Larichev & Reznik [18] for a stationary double-vortex solution which is antisymmetric in
longitude. Extensions to more general solutions have been made [5], and the spherical geometry of
planets has been incorporated [30, 31, 25]. However, modons have the drawback that the potential vor-
ticity is not a smooth function of the stream function, but may be multivalued. Therefore interest has
grown in low-dimensional models, although a rigorous proof of existence of a low-dimensional attractor
in quasigeostrophic systems is still an unsolved problem. Strictly speaking, one can only define a “slowest
invariant manifold” [3], since the small-scale events, i.e. the high-frequency and high-wavenumber pro-
cesses, enlarge the Hausdorff dimension for the attractor without any convergence [34]. Nevertheless, in
order to understand better the particular mechanisms involved in the formation and dynamics of vortices
in geophysical fluid dynamics, it is useful to perform asymptotic techniques to derive reduced amplitude
equations of the basic quasigeostrophic equations in a multiple scale analysis and study the derived model
evolution-equations. The basic idea is that coherent vortices may be identified with solitary wave solu-
tions of generic nonlinear dispersive wave equations.

Most research has been done in the framework of the Korteweg-de Vries equation [21, 33, 27, 11, 19,
20, 28, 6, 7] or in the framework of the Boussinesq equation [12, 13]. While these models were helpful
in describing and identifying mechanisms for atmospheric blocking, cyclogenesis, meandering of oceanic
streams and the persistence of the Great Red Spot in the Jovian atmosphere, they are all one-dimensional
models with their obvious limitations.

In this Letter we will extend weakly nonlinear, long wave multiple scale analysis to two dimensions and derive the Zakharov-Kuznetsov equation

\[ A_T + \Delta A_X - \mu A A_X - \xi A_{XXX} - \zeta A_{XY} = 0. \]

The Zakharov-Kuznetsov (ZK) equation \[35\] is one of two well-studied canonical two-dimensional extensions of the Korteweg-de Vries equation \[17\]; the other being the Kadomtsev-Petviashvili (KP) equation \[19\]. In contrast to the KP-equation, the ZK-equation has so far never been derived in a geophysical fluid dynamics context. For a derivation of the KP equations for internal waves, see \[8\]. Whereas the KP-equation is valid in isotropic situations, the ZK-equation is valid in anisotropic settings which is exactly the case for rotating fluids where the differential longitudinal dependence of the rotation rate causes anisotropy between the meridional and the longitudinal directions. Moreover, in contrast to the KP-equation the ZK-equation supports stable lump solitary waves. This makes the ZK-equation a very attractive model equation for the study of vortices in geophysical flows.

The Letter is organized as follows. In Section 2 we set up the barotropic vorticity equation and the mean flow configurations under consideration. In the beginning of Section 3 we will give a simple heuristic scaling argument based on the linearized barotropic vorticity equation to motivate why the ZK-equation is the generic two-dimensional nonlinear wave equation. In the remainder of Section 3 we will derive the ZK-equation in an asymptotic multiple scale analysis. Section 4 concludes the Letter with a discussion and an outlook on further research.

2 Barotropic Quasigeostrophic Equation

We shall use a non-dimensional coordinate system, based on a typical horizontal length scale \(L_0\), a typical vertical scale \(H_0\), and typical Coriolis parameter \(f_0\). A typical velocity \(\bar{U}\) is taken to be the maximum of the mean current velocity and the time scale is given by \(\bar{U}/L_0\). If we separate the meridional meanflow \(U\) from the perturbation pressure fields \(p\) and use the Boussinesq approximation, we obtain the following equation for the non-dimensional perturbation pressure field \[28\]

\[
\left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) q + \psi_x Q_y + J(\psi, q) = 0, \tag{2}
\]

where

\[
q = \nabla^2 \psi - F\psi,
\]

\[
Q_y = \beta - \nu_y + FU,
\]
with Froude number $F$ and the Jacobian defined by $J(a,b) = a_x b_y - a_y b_x$. We investigate a channel flow with a storm track superimposed on a constant meanflow $U_m$ confined at $y = \pm L$ (see Fig.1). The storm tracks may have a critical layer where $U(y) = 0$. Important is, as we will see, the non-vanishing slope at at least one boundary of the localized storm track. The boundary conditions are $\psi = \text{const} \text{ at } y = \pm \infty$, and we require that the jet forms a transport barrier to the flow.

![Figure 1: Sketch of a typical mean flow.](image)

## 3 Nonlinear Wave Equation

### 3.1 Linear Dispersion Relation

Before we consider the weakly nonlinear, long wave approximation, we motivate our approach by looking at the linearization of equation (2) which yields

$$\left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) q + \frac{\partial \psi}{\partial x} \frac{\partial Q}{\partial y} = 0.$$  

(3)

In terms of $\psi = a_0 \exp i(k(x - ct) + ly)$ we obtain the dispersion relation

$$c = U - \frac{Q_y}{k^2 + l^2 + F},$$

provided that the meanflow $U(y)$ is constant. If we focus on dynamics on the spatial and temporal time scales $X = \epsilon x, Y = \epsilon y$, i.e. small $k, l$, and $T = \epsilon^3 t$ this is suggestive of the coupled Zakharov-Kuznetsov equation

$$A_T + \Delta A_X - \mu AA_X - \xi A_{XXX} - \zeta A_{XY} = 0.$$  

(4)
The reason why we expect an equation of the Zakharov-Kuznetsov type instead of the usual Kadomtsev-Petviashvilli type mostly encountered in fluid systems is the anisotropic character of \( \beta \)-effect.

### 3.2 Weakly Nonlinear Model

We consider weakly nonlinear waves riding on a background meanflow. The meanflow consists of a constant part \( U_m \) and a strong but narrow jetstream (see Fig. 1). The narrow storm track is located on a short meridional scale \( y \). In the outer region the problem \( \psi \) can be reduced to the linear problem \( \psi \) with constant meanflow \( U_m \). In the interior the structure of the storm track does not allow for sinusoidal wave solutions but instead we will derive a nonlinear wave equation. In order for the nonlinear wave equation which is valid only in the inner region where the storm tracks are nonuniform, the inner solution has to be matched to the outer sinusoidal solution.

#### 3.2.1 Outer solution

In the outer region where the meanflow is uniform and constant, \( \psi \) reduces to the simple linear equation \( \psi = a(t, T, X) \sin(lY) + b(t, T, X) \cos(lY) \),

\[
\psi^{(\text{out})} = a(t, T, X) \sin(lY) + b(t, T, X) \cos(lY),
\]

where \( l \) is the meridional wave number and is determined by the dispersion relation of the linearized model \( \psi \).

#### 3.2.2 Inner solution

In the interior of the storm track on the small scale \( y \), the meanflow is not constant. We shall study weakly nonlinear long waves. We introduce the following scales,

\[
X = \epsilon x, \quad Y = \epsilon y, \quad T = \epsilon^3 t,
\]

\[
\psi(X, Y, T, y) = \epsilon^2 \psi^{(0)} + \epsilon^3 \psi^{(1)} + \epsilon^4 \psi^{(2)} + \cdots,
\]

where \( \epsilon \) is a small parameter, the inverse of which measures the large scales of the disturbance. Next, we rescale the parameters \( F \rightarrow \epsilon^2 F \) and \( \beta \rightarrow \epsilon^2 \beta \). The scaling of the Froude numbers implies that our model is valid for situations where the internal Rossby radius of deformation is of the order of the long horizontal scale. Further, the scaling of \( \beta \) implies that \( Q_y \approx -U_{yy} \) at the lowest order. The boundary conditions we use are \( \psi = \text{constant at } y = \pm \infty \) and \( U_y \psi_X = U \psi_{XY} \) at \( y = \pm L \) which simply states that there is no transport of fluid across the jet-stream.
Substituting this scaling into equation (2) yields,

\[
0 = (\varepsilon^3 \partial_T + \varepsilon U \partial_X)(\varepsilon^4 \partial_X \psi^{(0)}) + \varepsilon^2 \partial_{yy} \psi^{(0)} + 2\varepsilon^3 \partial_y \psi^{(0)} + \varepsilon^4 \partial_Y \psi^{(0)} - \varepsilon^4 F \psi^{(0)} \\
+ \varepsilon^3 \partial_{yy} \psi^{(1)} + \varepsilon^4 \partial_{yy} \psi^{(2)} + 2\varepsilon^4 \partial_y \psi^{(1)} \\
+ \varepsilon^3 \psi^{(0)}(\varepsilon^2 \beta - U_{yy} + \varepsilon^2 FU) + \varepsilon^4 \psi^{(1)}(-U_{yy}) + \varepsilon^5 \psi^{(2)}(-U_{yy}) \\
+ \varepsilon^5 (\psi^{(0)} \psi^{(0)} - \psi^{(0)} \psi^{(0)} YX).
\]

Counting the orders of \( \varepsilon \) we obtain to the lowest order, \( \mathcal{O}(\varepsilon^3) \)

\[
\mathcal{L} \psi^{(0)}_X = 0,
\]

where

\[
\mathcal{L} = \frac{\partial}{\partial y} [U \partial_y - U_y].
\]

We look for an amplitude equation, i.e., we want to write

\[
\psi^{(0)}(X,Y,T,y) = A(X,Y,T) \varphi^{(0)}(y)
\]

and seek an evolution equation for the slowly varying amplitude \( A(X,Y,T) \). We easily find

\[
\varphi^{(0)}(y) = U(y)(1 + \int_{-L}^{y} \frac{\alpha_0}{U^2(y')} dy'),
\]

where \( \alpha_0 \) is a constant of integration. If we allow for zero meanflow within the narrow jet region we need to impose \( \alpha_0 = 0 \). We summarize the solution of equation (9)

\[
\psi^{(0)}(X,Y,T,y) = A(X,Y,T)U(y).
\]

The meridional structure on the small scale \( y \) of \( \psi \) is entirely determined by the mean currents at the leading order.

At the next order, \( \mathcal{O}(\varepsilon^4) \), we obtain a linear inhomogeneous equation for \( \psi^{(1)} \),

\[
2U \psi^{(0)}_{yyX} + U \psi^{(1)}_{yyX} - U_{yy} \psi^{(1)}_X = 0,
\]

which can be written, using (9), as

\[
\mathcal{L} \psi^{(1)}_X = -2U \varphi^{(0)}_y A_{XY}.
\]

This equation is again solved by the method of variation of parameters and we obtain

\[
\psi^{(1)} = \varphi^{(1)} A_Y.
\]
with
\[ \varphi^{(1)} = U(y)(1 - y - L) + \int_{-L}^{y} \frac{\alpha_1}{U^2(y')} dy'. \]

We note that the higher order term \( \psi^{(1)} \) is slaved to the \( \psi^{(0)} \) term and the dynamics of the corresponding amplitude equation which will be derived shortly. For the same reasons as above we set \( \alpha_1 = 0 \) and obtain
\[ \psi^{(1)} = U(1 - y - L)A_Y . \]

The \( \mathcal{O}(\epsilon^5) \) terms give us an evolution equation for the amplitude \( A \). We obtain
\[
U\psi_X^{(2)} - U_{yy}\psi_X^{(2)} + 2U\psi_{yy}^{(1)X} \\
+ \{ \partial_T\psi^{(0)}_y + U\psi^{(0)}_{XX} + U\psi^{(0)}_{XY} - FU\psi^{(0)}_X \\
+ \psi^{(0)}_X\psi^{(0)}_{yy} - \psi^{(0)}_y\psi^{(0)}_{Xy} + \beta\psi^{(0)}_X + FU\psi^{(0)}_X \} = 0 ,
\]
which, using (8), can be written as
\[ \mathcal{L}\psi_X^{(2)} = -G , \] (11)
where
\[
G = \varphi^{(0)}_y A_T + U\varphi^{(0)}(A_{XX} + A_{YY}) - FU\varphi^{(0)}A_X \\
+ \varphi^{(0)}(\beta + FU)A_X + \left( \varphi^{(0)}_y\varphi^{(0)}_{yy} - \varphi^{(0)}_y\varphi^{(0)}_{yy} \right) AA_X \\
+ 2U\varphi^{(1)}_y A_{XX} , \] (12)

To assure boundedness of the solutions of (11) we have to require a solvability condition in form of a Fredholm alternative.

The homogeneous adjoint problem to equation (11) may be written as
\[ \mathcal{L}^\dagger \phi = 0 , \] (13)
with
\[ \mathcal{L}^\dagger = 2Uy\partial_y + U\partial_{yy} , \]
where we have used the boundary conditions \( U_y\psi_X = U\psi_{Xy} \) at \( y = \pm L \). The adjoint eigenvalue problem (13) has one trivial constant kernel mode \( \phi_1 = \text{const} \) and one nontrivial, namely
\[ \phi_2(y) = \int_0^y \frac{1}{U^2(y')} dy' . \] (14)

The nontrivial kernel mode \( \phi_2 \) has to be discarded because it does not satisfy the boundary condition. To see this note that the Fredholm alternative for the elliptic operator (4) together with the boundary
condition \( U_y \psi_X = U \psi_{XY} \) is equivalent to one for the operator \( \mathcal{L}' = \partial_y [U \partial_y \psi] - U_y \partial_y \psi - U_{yy} \partial_y \psi \) with the boundary condition \( \partial_y \psi = 0 \) at \( y = \pm L \). Also note that a non-zero kernel mode \( \phi_1 \) is only consistent with the boundary conditions if \( U(\pm L) \neq 0 \).

The solvability condition is thus given by the trivial constant kernel mode

\[
\int_{-L}^{L} G \, dy = 0 .
\]  

(15)

On substituting the expressions (8) with \( \varphi^{(0)}(y) = U(y) \) and (12) we obtain the desired amplitude equation for \( A \),

\[
A_T + \Delta A_X - \mu A_X - \xi A_{XXX} - \zeta A_{XY} = 0 ,
\]  

(16)

where

\[
I = -[U_y]_{-L}^{L} ,
\]

\[
I \xi = \int_{-L}^{L} U^2 \, dy ,
\]

\[
I \zeta = [U^2(1 - y - L)]_{-L}^{L} ,
\]

\[
I \mu = -[U_y^2]_{-L}^{L} + [UU_{yy}]_{-L}^{L} ,
\]

\[
I \Delta = -\int_{-L}^{L} \beta U \, dy .
\]  

(17)

We note that due to the last term of (12) the Zakharov-Kuznetsov equation is inhomogeneous in the sense \( \xi \neq \zeta \). It is pertinent to mention that a nonzero \( \zeta \) requires a nonzero mean flow at at least one of the boundaries of the storm track. The coefficients of the nonlinear terms \( \mu \) require a non-vanishing slope at at least one boundary. The slope \( U_y \) and also \( U_{yy} \) at the boundaries of the jet \( y = \pm L \) may be determined from a given meanflow configuration by averaging over a very short region, say \( y/\epsilon \), where a sudden change of the constant mean flow \( U_m \) to the jet occurs.

3.3 Asymptotic Matching

At the lowest order the inner solution (9) with \( \alpha_0 = 0 \) and the outer solution (5) have to be matched. The outer solution has been derived on the large scale \( Y \) whereas the inner solution and its associated amplitude equation, the Zakharov-Kuznetsov equation (16), were derived on the short scale \( y \). Henceforth we need to require that the asymptotic limit of the outer solutions for \( Y \to 0 \) coincides with the asymptotic limit of the inner solution for \( y \to \infty \). The limit of the inner solution is \( \psi^{(in)} = A(X,Y,T)U_m \). The limit of the outer solution is \( \psi^{(out)} = b(X,Y,T) \). Hence we find \( b(X,Y,T) = A(X,Y,T)U_m \), which extends the dynamics of the Zakharov-Kuznetsov equations to the outer region.
4 Discussion

We have derived the nonlinear dispersive Zakharov-Kuznetsov equation from the quasigeostrophic barotropic vorticity equation. It is well known that the ZK-equation, although it is not integrable by means of the inverse scattering transform, supports a family of steady-shape stable lump solitary waves, moving at an arbitrary velocity. These may help to describe two-dimensional coherent structures such as atmospheric blocking events, long lived eddies in the ocean or coherent structures in the Jovian atmosphere such as the Great Red Spot. The model is from an analytical point of view easier to treat than the full barotropic quasigeostrophic equation and its solutions do not exhibit multivalued potential vorticity-stream function relationships as modons do.

Geophysical flow on large scales is widely accepted to be conservative. This allows for Hamiltonian descriptions of the flow on large scales. Our model also exhibits a Hamiltonian structure. Note that the momentum

\[ P = \int_{-\infty}^{\infty} A^2 \, dX \, dY \]

and the Hamiltonian with the Hamiltonian density

\[ \mathcal{H} = \frac{\xi}{2} A_X^2 + \frac{\zeta}{2} A_Y^2 - \mu A^3. \]

are conserved.

We have assumed a meridional meanflow \( U \) which consists of a constant part \( U_m \) and a narrow localized storm track. Note that the jet stream may also be a narrow interface between two regions of meanflow with opposite flow direction. Such persistent shear layers exist between the zones and belts in the Jovian atmosphere.

Analysis of the solutions of (16) is planned. Their stability has to be numerically tested within the Zakharov-Kuznetsov system. The ZK-equation has been derived using asymptotic techniques and is as such an asymptotic limit to the barotropic quasigeostrophic vorticity equation. However, it is not clear that the same is true for the solutions. The solutions of the Zakharov-Kuznetsov equations do not necessarily have to be asymptotically close to the solutions of the full quasigeostrophic system. this is due to the lack of a centre manifold as discussed in the introduction. In further work we will test the approximation of the solution numerically by taking solutions of the ZK-equation and testing their dynamics in the full quasigeostrophic system.

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