Investigation of frequency response of microwave active ring resonator based on ferrite film

M I Martynov, A A Nikitin, A B Ustinov, and B A Kalinikos

Department of Physical Electronics and Technology, St. Petersburg Electrotechnical University, St. Petersburg, 197376 Russia

E-Mail: ustinov_rus@yahoo.com

Abstract. The complex transmission coefficient of active ring resonators based on ferrite-film delay lines was investigated both theoretically and experimentally. Influence of the parameters of the delay line on the transmission coefficients was investigated. It was shown that the resonant frequencies of the ring depend on the ferrite film thickness and the distance between spin-wave antennae. These dependences give possibility to control the shape of the transmission coefficient that in combination with magnetic tuning provide flexibility for microwave applications.

During the past two decades the increased research interest to active ring systems is observed [1, 2]. Nowadays they are widely used for study nonlinear physical phenomena such as solitons, modulation instability, and chaos, as well as for signal processing and generation [3]. Tunable microwave active ring resonators or oscillators can be easily fabricated with the use of delay lines based on the thin ferrite films such as epitaxial yttrium iron garnet (YIG) films.

Distinguished features of the YIG-film devices are low insertion losses and magnetic field tunability. Performance characteristics of such devises are determined by the features of the spin wave (SW) dispersion. It makes the YIG films attractive for applications in miniature tunable microwave devices. In particular, a narrow band filters [4, 5], matched filters [6], stationary and chaotic signal generators [7, 8, 9, 10] based on active rings have been developed and studied experimentally.

The aim of this work is theoretical and experimental investigation of microwave properties of the spin-wave active ring resonators based on the YIG films.

Schematic diagram of the resonator is shown in the Fig. 1.a. This circuity consists of spin-wave delay line, variable attenuator, microwave amplifier and two directional couplers for input and output signals. All elements of the scheme are connected consequently in order to form the closed loop. The spin-wave delay line is used to introduce time delay in the ring. The wideband microwave amplifier and the variable attenuator provide adjustable amplification of the microwave signal to compensate the losses in the delay line. We will call this amplification as a ring gain and denote it by letter \( G \). We assume that the gain has no frequency dependence.

Consider now the formation of transmission coefficient of the active ring resonator. The transmission coefficient has multiresonance behavior at frequencies determined by the dispersion law in the delay line. For resonance frequencies phase shift of the microwave signal circulating in the ring is equal to \( \Delta \phi = 2 \pi n \), where \( n \) is a number of the signal circulations. Therefore, the resonance condition for SW wave numbers is \( k_n \left( f_{res} \right) = 2 \pi n / d \), where \( k \left( f \right) \) is a dispersion relation of the SW, \( d \) is the length of the delay line. Note that in the case of the spin wave delay line the delay of all
other electrical interconnections is negligibly small. The formation of the transmission coefficient of active ring resonator illustrated in the Fig. 1.b. As is shown, resonant wave numbers \( k_{\text{res}} \) define the resonance frequencies through the dispersion characteristic. Therefore, one of the advantages of the active ring resonators is the magnetic tunability.

For development of the theoretical model we assume that a monochromatic signal

\[
\hat{A}_{\text{in}}(\omega) = A_{\text{in}} e^{i \omega t}
\]

(1)

is supplied to the input port. This signal circulates in the ring, as was described above. The output signal is a superposition of an infinite number of the circulating waves. As a result, an expression for the output signal after \( m \) circulations has the following form

\[
\hat{A}_{\text{out}}(\omega) = A_{\text{in}} \left[ \sum_{n=1}^{\infty} e^{-i(k(\omega)-\alpha(\omega))nd} \cdot e^{im\omega} \right] e^{i \omega t}
\]

(2)

where \( k(\omega) \) is the wave number, \( \alpha(\omega) \) is the damping decrement, and \( g \) is the amplifier gain coefficient. In order to derive generalized theory the SW dispersion relation \( k_{\text{sw}}(\omega) \) and the damping decrement \( \alpha_{\text{sw}}(\omega) \) will be used on the final step.

A complex transmission coefficient of the resonator circuit was found as a ratio of the output and input complex amplitudes, i.e.

\[
\hat{H}(\omega) = \frac{\hat{A}_{\text{out}}(\omega)}{\hat{A}_{\text{in}}(\omega)} ,
\]

(3)

In general case it can be written in the following form
\[ H(\omega) = \sqrt{H_p(\omega)} \exp[i\Phi(\omega)] , \]

where \( H_p(\omega) \) is the power transmission coefficient and \( \Phi(\omega) \) is the phase-frequency characteristic of the active ring resonator. After summation and simple mathematical manipulations we obtain

\[ H_p = \frac{1}{\cosh(G - \alpha(\omega)d) - \cos(k(\omega)d)} , \]

Expression for the phase-frequency characteristic is

\[ \Phi = \arctan \left[ \frac{-\sin(k(\omega)d)}{e^{G - \alpha(\omega)d} - \cos(k(\omega)d)} \right] \pm R\pi , \]

where \( R = 0, 1, 2, \ldots \)

It is important to note, that derived relations for the power transmission coefficient and the phase-frequency characteristic can be used for investigation of active ring resonators based on various delay lines. In the case of spin wave delay line the SW dispersion relation \( k_{sw}(\omega) \) and the damping decrement \( \alpha_{sw}(\omega) \) should be substituted in the Eq. 5 and 6, respectively.

The SW wave-number \( k_{sw}(\omega) \) is calculated using an appropriate dispersion equation. For the surface spin waves used in this work the wave-number is calculated as \[ k_{sw}(\omega) = \frac{-1 - 4(\omega^2 - \omega_n(\omega_n + \omega_H))}{\omega_n^2}/2L , \]

where \( L \) is a thickness of the YIG film, \( \omega_n = \mu_0 H_0, \omega_H = \mu_0 M_0, \mu_0 = 4\pi \times 10^{-7} \text{ H} \cdot \text{m}^{-1} \) is a magnetic field, \( M_0 \) is an external magnetic field, and \( \omega_n \) is a saturation magnetization. The spatial damping decrement for the surface spin waves is calculated as

\[ \alpha_{sw}(\omega) = 2\pi|g|\omega\Delta H \cdot L^{-1}\left(\left(\omega_n + \omega_H/2\right)^2 - \omega^2\right)^{-1} , \]

where \( \Delta H \) is a half of the ferromagnetic resonance line-width.

Let us turn now to investigation of an experimental prototype of the active ring resonator. Its main elements were a microwave amplifier, the original microwave spin-wave delay line, and variable attenuator in order to adjust the gain in the ring.

The spin-wave delay lines are utilized a single-crystal YIG films grown on a gadolinium gallium garnet (GGG) substrate by liquid phase epitaxy. The films had a width of 2 mm, a length of 40 mm, various saturation magnetization \( M_0 \) of 1750, 1660, and 1780 G, and various thickness \( L \) of 5.7, 9.64, and 13.6 \( \mu \)m, respectively. Two 50 \( \mu \)m wide and 2 mm long short circuited microstrip antennae were used for excitation and reception of spin waves in the delay line. Original experimental prototype allows to investigate an influence of the distance \( d \) between the antennae on the transmission coefficient. The measurements were carried out at \( d \) equal 3, 6, and 9 mm. Antennae were fed by microstrip transmission lines of 50 Ohm characteristic impedance. The microstrip lines were fabricated with conventional photolithography on grounded alumina substrates with thickness of 500 \( \mu \)m. The YIG film stripes were positioned over the antennae with YIG film side in contact with the antennae.

The SW delay lines are placed between the poles of permanent magnet. The bias magnetic field \( H_0 \) in a range from 1226 Oe to 1330 Oe is applied in the plane of the YIG film and parallel to the antennae. This orientation provides excitation of surface spin waves in the YIG films.

Consider now the principle of operation of the SW active ring resonator. A microwave signal supplied to the input directional coupler begins to circulate in the ring. The signal losses \( \Lambda \) in the YIG delay line are compensated by the microwave amplifier. The signal amplification is controlled by a...
variable attenuator and is characterized by the ring gain $G$. The active ring resonator could operate as a passive microwave filter for $G$ less than $\Lambda$ or as a microwave oscillator, otherwise. A value of $G$, for which the ring breaks up to the auto-oscillation regime, may be used as a reference point. In the work, this $G$ value is taken as a reference zero point.

Fig. 2 shows representative data on the frequency response of the active ring resonator measured for $G = -1.5$ dB, $L = 9.64 \, \mu m$, and $d = 6$ mm by the symbols. The well-pronounced spikes correspond to the resonant frequencies of the ring. Theoretical data for frequency response, shown by solid line in Fig. 2, well coincides with experimental characteristic.

![Figure 2](image1)

Figure 2. Power transmission coefficient for the active ring resonator based on the ferrite film with thickness of 9.64 µm and distance between antennae $d = 6$ mm.

From general point of view it is clear that transmission coefficient is defined by the phase shift of the signal in the ring $\varphi = k(\omega)d$. It can be considered also through delay time $\tau = d\varphi / d\omega$. Further, we will use the time delay in order to describe properties of the active ring resonators. As is clear only two parameters of the SW delay line define the time delay that are the distance between antennae $d$ and YIG film thickness $L$. Influence of the both parameters are analyzed below.

Solid lines in Fig. 3 show theoretical dependences of the resonant frequencies from the distance between the antennae. Symbols show resonant frequencies measured for distance $d$ of 3, 6, and 9 mm. As is seen increasing of the distance, otherwise increasing of the time delay, leads to decrease of a distance between adjacent resonant frequencies $\Delta f$ (Fig. 4). This feature is one of the main advantage of the spin wave active ring resonator that allows to change a shape of the transmission characteristic.

![Figure 3](image2)

Figure 3. Dependences of the resonant frequencies of the active ring resonator from the distance between SW antennae $d$ for the ferrite film thickness of 5.7 µm.
Second factor defining the resonance frequency position is the film thickness \( L \). As is seen from the dispersion equation (7), the spin-waves in the ferrite films with various thickness have the different dispersion laws. As a result, positions of the resonant frequencies and range between adjacent resonance peaks \( \Delta f \) are different. Theoretical dependences of \( \Delta f \) values on frequency in ferrite films with different thicknesses calculated with Eq. 5 are shown by solid lines in Fig. 5. Experimental points are shown by symbols. The distance between antennae \( d \) for Fig. 5 equals 6 mm. Fig. 4 and 5 show a constructive flexibility of SW active ring resonators for creation of matched filters.

![Figure 4](image-url)  
**Figure 4.** Dependences of the \( \Delta f \) from the frequency for the ferrite film with 5.7 \( \mu \)m thickness.

![Figure 5](image-url)  
**Figure 5.** Dependences of the \( \Delta f \) from the frequency for distance between antennae \( d = 6 \) mm.

Variation of the bias magnetic field from 1226 Oe to 1330 Oe provided the tuning range for lowest resonant frequency of 5.4-5.7 GHz. In this range shift of the resonant frequencies are described by the dependences of dispersion relation versus magnetic field. Quality factor of the single resonant peak depends on the ring gain and it reaches maximum at \( G = 0 \).

Experimental dependences of the quality factor of the most pronounced resonance peak on the distance for three values of the film thickness are shown in the Fig. 6. The maximum quality factor measured for 13.6 \( \mu \)m film at 9 mm between antennae reaches 25 000. A distance between antennae \( d \) makes a primary contribution to the delay time. Increase of the delay time provides higher quality factor. However, due to higher values of the ferromagnetic resonance line-width in the thinner film quality factor decreases with the thickness.

![Figure 6](image-url)  
**Figure 6.** Dependences of the quality factor on the distance between antennae.
In conclusion, the theoretical and experimental investigation of the active ring resonator based on SW delay line was carried out. Transmission coefficient and phase-frequency characteristic of the active ring resonator were derived. Theoretical dependences for frequency response well coincided with experimental results. The quality factor of the resonator was about $2 \times 10^4$. Therefore the active ring resonators are promising for microwave applications, in particular for comb-frequency filtering.

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