Survey on Multi-Output Learning

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Abstract—The aim of multi-output learning is to simultaneously predict multiple outputs given an input. It is an important learning problem for decision-making since making decisions in the real world often involves multiple complex factors and criteria. In recent times, an increasing number of research studies have focused on ways to predict multiple outputs at once. Such efforts have transpired in different forms according to the particular multi-output learning problem under study. Classic cases of multi-output learning include multi-label learning, multi-dimensional learning, multi-target regression, and others. From our survey of the topic, we were struck by a lack in studies that generalize the different forms of multi-output learning into a common framework. This article fills that gap with a comprehensive review and analysis of the multi-output learning paradigm. In particular, we characterize the four Vs of multi-output learning, i.e., volume, velocity, variety, and veracity, and the ways in which the four Vs both benefit and bring challenges to multi-output learning by taking inspiration from big data. We analyze the life cycle of output labeling, present the main mathematical definitions of multi-output learning, and examine the field’s key challenges and corresponding solutions as found in the literature. Several model evaluation metrics and popular data repositories are also discussed. Last but not least, we highlight some emerging challenges with multi-output learning from the perspective of the four Vs as potential research directions worthy of further studies.

Index Terms—Crowdsourcing, extreme classification, label distribution, multi-output learning, output label representation, structured output prediction.

I. INTRODUCTION

T

RADITIONAL supervised learning is one of the most well established and adopted machine learning paradigms. It offers fast and accurate predictions for today’s real-world smart systems and applications. The goal of traditional supervised learning is to learn a function that maps each of the given inputs to a corresponding known output. For prediction tasks, the output comes in the form of a single label. For regression tasks, it is a single value. Traditional supervised learning has been shown to be good at solving these simple single-output problems, classical examples being binary classification, such as filtering spam in an email system, or a regression problem where the daily energy consumption of a machine needs to be predicted based on temperature, wind speed, humidity levels, and so on.

However, the traditional supervised learning paradigm is not coping well with the increasing needs of today’s complex decision making. As a result, there is a pressing need for new machine learning paradigms. Here, multi-output learning has emerged as a solution. The aim is to simultaneously predict multiple outputs given a single input, which means it is possible to solve far more complex decision-making problems. Compared to traditional single-output learning, multi-output learning is multi-variate nature, and the outputs may have complex interactions that can only be handled by structured inference. In addition, the potentially diverse data types of outputs have led to various categories of machine learning problems and corresponding subfields of study. For example, binary output values relate to multi-label classification problems [1], [2]; nominal output values relate to multi-dimensional classification problems [3]; ordinal output values are studied in label ranking problems [4]; and real-valued outputs are considered in multi-target regression problems [5].

Together, all these problems constitute the multi-output paradigm, and the body of literature surrounding this field has grown rapidly. Several works have been presented that provide a comprehensive review of the emerging challenges and learning algorithms in each subfield. For instance, Zhang and Zhou [1] studied the emerging area of multi-label learning; Borchani et al. [5] summarized the increasing problems in multi-target regression; Vembu and Gärtner [4] presented a review on multi-label ranking. However, little attention has been paid to the global picture of multi-output learning and the importance of the output labels (see Section II). In addition, although the problems in each subfield seem distinctive due to the differences in their output structures (see Section III-A), they do share common traits (see Section III-B) and encounter common challenges brought...
by the characteristics of the output labels. In this article, we attempt to provide such a view.

A. Four Vs Challenges of Multiple Outputs

The popular four Vs, i.e., volume, velocity, variety, and veracity, have been well established as the main characteristics of big data. When scholars discuss the four Vs in multi-output learning scenarios, they are usually referring to input data; however, the four Vs can also be used to describe output labels. Moreover, these four Vs bring with them a set of challenges to multi-output learning processes, explained as follows.

1) Volume refers to explosive growth in output labels, which poses many challenges to multi-output learning. First, output label spaces can grow extremely large, which causes scalability issues. Second, the burden for label annotators is significantly increased, and still, there are often insufficient annotations in a data set to adequately train a model. In turn, this may lead to unseen outputs during testing. Third, the volume may pose label imbalance issues, especially if not all the generated labels in a data set have sufficient data instances (inputs).

2) Velocity refers to how rapidly output labels are acquired, which includes the phenomenon of concept drift [6]. Velocity can present challenges due to changes in output distributions, where the target outputs vary over time in unforeseen ways.

3) Variety refers to the heterogeneous nature of output labels. Output labels are gathered from multiple sources and are of various data formats with different structures. In particular, output labels with complex structures can create multiple challenges in multi-output learning, such as finding an appropriate method of modeling output dependencies, or how to design a multi-variate loss function, or how to design efficient algorithms.

4) Veracity refers to differences in the quality of the output labels. Issues such as noise, missing values, abnormalities, or incomplete data are all characteristics of veracity.

B. Purpose and Organization of This Survey

The goal of this article is to provide a comprehensive overview of the multi-output learning paradigm using the four Vs as a frame for the current and future challenges facing this field of study. Multi-output learning has attracted significant attention from many machine learning disciplines, such as part-of-speech (POS) sequence tagging, language translation and natural language processing, motion tracking and optical character recognition in computer vision, and document categorization and ranking in information retrieval. We expect this survey to deliver a complete picture of multi-output learning and a summation of the different problems being tackled across multiple communities. Ultimately, we hope to promote further development in multi-output learning and inspire researchers to pursue worthy and needed future research directions.

The remainder of this survey is structured as follows. Section II illustrates the life cycle of output labels to help understand the challenges presented by the four Vs. Section III provides an overview of the myriad output structures along with definitions for the common subproblems addressed in multi-output learning. This section also includes some brief details on the common metrics and publicly available data used when evaluating models. Section IV presents the challenges in multi-output learning presented by the four Vs and their corresponding representative works. Section V concludes the survey.

II. LIFE CYCLE OF OUTPUT LABELS

Output labels play an important role in multi-output learning tasks in that how well a model performs a task relies heavily on the quality of those labels. Fig. 1 depicts the three stages of a label’s life cycle: annotation, representation, and evaluation. A brief overview of each stage follows along with the underlying issues that could potentially harm the effectiveness of multi-output learning systems.

A. How Data Is Labeled

Label annotation requires a human to semantically annotate a piece of data and is a crucial step for training multi-output learning models. Data can be used directly with its basic annotations, or once labeled; they can be aggregated into sets for further analysis. Depending on the application and the task, label annotations come in various types. For example, the images for an image classification task should be labeled with tags or keywords, whereas a segmentation task would require each object in the images to be localized with a mask. A captioning task would require the images to be labeled with some textual descriptions, and so on.

Typically, creating large annotated data sets from scratch is time-consuming and labor-intensive no matter the annotation requirement. There are multiple ways to acquire labeled data. Social media provides a platform for researchers to search for labeled data sets, for example, Facebook and Flickr, which allow users to post pictures and comments with tags. Open-source collections, such as WordNet and Wikipedia, can also be useful sources of labeled data sets.

Beyond directly obtaining labeled data sets, crowdsourcing platforms, such as Amazon Mechanical Turk, help researchers solicit labels for unlabeled data sets by recruiting online workers. The annotation type depends on the modeling task, and due to the efficiency of crowdsourcing, this method has quickly become a popular way of obtaining labeled data sets.
ImageNet [7] is a popular data set that was labeled through a crowdsourcing platform. Its database of images is organized into a WordNet hierarchy, and it has been used to help researchers solve problems in a range of areas.

There are also many annotation tools that have been developed to annotate different types of data. LabelMe [8], a web-based tool, provides users with a convenient way to label every object in an image and also correct labels annotated by other users. BRAT [9] is also web-based but is specifically designed for natural language processing tasks, such as named-entity recognition and POS-tagging. TURKSENT [10] is an annotation tool to support sentiment analysis in social media posts.

B. Forms of Label Representations

There are many different types of label annotations for different tasks, such as tags, captions, and masks, and each type of annotation may have several representations, which are frequently represented as vectors. For example, the most common is the binary vector, whose size equals the vocabulary size of the tags. Annotated samples, e.g., samples with tags, are assigned with a value of 1 and the rest are given a 0. However, binary vectors are not optimal for more complex multi-output tasks because these representations do not preserve all useful information. Details such as the semantics or the inherent structure are lost. To tackle this issue, alternative representation methods have been developed. For instance, real-valued vectors of tags [11] indicate the strength and degree of the annotated tags using real values. Binary vectors of the associations between a tag’s attributes have been used to convey the characteristics of tags. Hierarchical label embedding vectors [12] capture the structure information in tags. Semantic word vectors, such as Word2Vec [13], can be used to represent the semantics and/or context of tags and text descriptions. What is key in real-world multi-output applications is to select the label representation that is most appropriate for the given task.

C. Label Evaluation and Challenges

Label evaluation is an essential step in guaranteeing the quality of labels and label representations. Thus, label evaluation plays a key role in the performance of multi-output tasks. Different models can be used to evaluate label quality, which to choose depends on the task. Generally, labels can be evaluated in three different respects: 1) whether the annotation is of good quality (Step A); 2) whether the chosen label representation represents the labels well (Step B); and 3) whether the provided label set adequately covers the data set (Label Set). After evaluation, a human expert is generally required to explore any underlying issues and provide feedback to improve different aspects of the labels if needed.

1) Issues of Label Annotation: The aforementioned annotation methods, e.g., crowdsourcing, annotation tools, and social media, help researchers collect annotated data efficiently. However, without experts, these annotations methods are highly likely to result in the so-called noisy label problem, which includes both missing annotations and incorrect annotations. There are various reasons for noisy labels, for example, using crowdsourced workers that lack the required domain knowledge, social media users that include irrelevant tags with their image or post, or ambiguous text in a caption.

2) Issues of Label Representation: Output labels can also have internal structures, and often, this structure information is critical to the performance of the multi-output learning task at hand. Tag-based information retrieval [14] and image captioning [15] are two examples where structure is crucial. However, incorporating this information into representation as the labels is a nontrivial undertaking as the data are usually many and domain knowledge is required to define their structure. In addition, the output label space might contain ambiguity. For example, a bag-of-words (BOW) is traditionally used as a representation of a label space in natural language processing tasks, but BOW contains word sense ambiguity, as two different words may have the same meaning and one word might refer to multiple meanings.

3) Issues of the Label Set: Constructing a label set for data annotation requires a human expert with domain knowledge. Plus, it is common that the provided label set does not contain sufficient labels for the data, perhaps due to fast data growth or the low occurrence of some labels. Therefore, there are likely to be unseen labels in the test data, which leads to open-set [16], zero-shot [17], or concept drift [18] problems.

III. Multi-Output Learning

In contrast to traditional single-output learning, multi-output learning can concurrently predict multiple outputs. The outputs can be of various types and structures, and the problems that can be solved are diverse. A summary of the subfields that use multi-output learning along with their corresponding output types, structures, and applications is presented in Table I.

We begin this section with an introduction to some of the output structures in multi-output learning problems. The different problem definitions common to various subfields are provided next, along with the different constraints placed on the output space. We also discuss some special cases of these problems and give a brief overview of some of the evaluation metrics that are specific to multi-output learning. This section concludes with some insights into the evolution of output dimensions through an analysis of several commonly used data sets.

A. Myriads of Output Structures

The increasing demand for sophisticated decision-making tasks has led to new creations of outputs, some of which have complex structures. With social media, social networks, and various online services becoming ubiquitous, a wide range of output labels can be stored and then collected by researchers. Output labels can be anything; they could be text, images, audio, or video, or a combination as multimedia. For example, given a long document as input, the output might be a summary of the input in text format. Given some text fragments, the output might be an image with its contents described by the input text. Similarly, audio, such as music and videos, can be generated given different types of inputs. In addition to the different output types, there are also a number of different possible output structures. Here, we present several typical output structures given an image as an input using the example
| Subfield                        | Output Structure          | Application                                      | Discipline              |
|--------------------------------|---------------------------|--------------------------------------------------|-------------------------|
| Multi-label Learning           | Independent Binary Vector | Document Categorization [19]                      | Natural Language Processing |
|                                |                           | Semantic Scene Classification [20]                | Computer Vision         |
|                                |                           | Automatic Video Annotation [21]                   | Computer Vision         |
| Multi-target Regression        | Independent Real-valued Vector | Natural Gas Demand Forecasting [23]             | Energy Meteorology      |
|                                |                           | Drug Efficacy Prediction [24]                    | Medicine                |
| Label Distribution Learning    | Distribution              | Head Pose Estimation [25]                        | Computer Vision         |
|                                |                           | Facial Age Estimation [26]                       | Computer Vision         |
|                                |                           | Text Mining [27]                                 | Data Mining             |
| Label Ranking                  | Ranking                   | Text Categorization Ranking [28]                 | Information Retrieval   |
|                                |                           | Question Answering [29]                          | Information Retrieval   |
| Sequence Alignment Learning    | Sequence                  | Protein Function Prediction [31]                 | Bioinformatics          |
|                                |                           | Language Translation [32]                        | Natural Language Processing |
|                                |                           | Named Entity Recognition [33]                    | Natural Language Processing |
| Network Analysis               | Graph                     | Scene Graph [34]                                | Computer Vision         |
|                                | Tree                      | Natural Language Parsing [35]                    | Natural Language Processing |
|                                | Link                      | Link Prediction [36]                             | Data Mining             |
| Data Generation                | Image                     | Super-resolution Image Reconstruction [37]       | Computer Vision         |
|                                | Text                      | Language Generation                              | Natural Language Processing |
|                                | Audio                     | Music Generation [38]                            | Signal Processing       |
| Semantic Retrieval             | Independent Real-valued Vector | Content-based Image Retrieval [39]              | Computer Vision         |
|                                |                           | Microblog Retrieval [40]                         | Data Mining             |
|                                |                           | News Retrieval [41]                              | Data Mining             |
| Time-series Prediction         | Time Series               | DNA Microarray Data Analysis [42]                | Biomathematics          |
|                                |                           | Energy Consumption Forecasting [43]              | Energy Meteorology      |
|                                |                           | Video Surveillance [44]                          | Computer Vision         |

Fig. 2. Illustration of the myriads of output structures given an input image from a social network.

in Fig. 2 as a way to illustrate just how many output structures might be possible across all the different input types.

1) Independent Vector: An independent vector is a vector with separate dimensions (features), where each dimension represents a particular label that does not necessarily depend on other labels. Binary vectors can be used to represent a given piece of data as tags, attributes, BOW, bag-of-visual-words, hash codes, and so on. Real-valued vectors provide the weighted dimensions, where the real value represents the strength of the input data against the corresponding label. Applications include annotation or classification of text, images, or video with binary vectors [19]–[21] and demand or energy prediction with real-valued vectors [23]. An independent vector can be used to represent the tags of an image, as shown in Fig. 2(1), where all the tags “people,” “dinner,” “table,” and “wine” have equal weight.

2) Distribution: Unlike independent vectors, distributions provide information about the probability that a particular dimension will be associated with a particular data sample. In Fig. 2(2), the tag with the largest weight is “people” and is the main content of the image, while “dinner” and “table” have similar distributions. Applications for distribution outputs include head pose estimation [25], facial age estimation [26], and text mining [27].

3) Ranking: Outputs might also be in the form of a ranking, which shows the tags ordered from the most to least important. The results from a distribution learning model can be converted into a ranking, but a ranking model is not restricted to only distribution learning models. Text categorization [28], question answering [29], and visual object recognition [30] are applications where rankings are often used.

4) Text: Text can be in the form of keywords, sentences, paragraphs, or even documents. Fig. 2(4) illustrates an example of text output as a caption of the image—“People are having dinner.” Other applications for text outputs are document summarization [45] and paragraph generation [46].

5) Sequence: Sequence outputs refer to a series of elements selected from a label set. Each element is predicted depending on the input as well as the predicted output(s) from the preceding element. An output sequence often corresponds to an input sequence. For example, in speech recognition, we expect the output to be a sequence of text that corresponds to a given audio signal of speech [47]. In language translation, we expect the output to be a sentence transformed into the target language [32]. In the example shown in Fig. 2(5), the input is an image caption, i.e., text, and the outputs are POS tags for each word in the sequence.
6) Tree: Tree outputs are essentially the outputs in the form of a hierarchy. The outputs, usually labels, have an internal structure where each output has a label that belongs to, or is connected to, its ancestors in the tree. For example, in syntactic parsing [35], as shown in Fig. 2(6), each of the outputs for an input sentence is a POS tag and the entire output is a parsing tree; “people” is labeled as a noun N, but it is also a noun phrase NP as per the tree.

7) Image: Images are a special form of output that consists of multiple pixel values, where each pixel is predicted depending on the input and the pixels around it. Fig. 2(7) shows super-resolution construction [37] as one popular application where images are common outputs. Super-resolution construction means constructing a high-resolution image from a low-resolution image. Other image output applications include text-to-image synthesis [48], which generates images from natural language descriptions, and face generation [49].

8) Bounding Box: Bounding boxes as outputs are often used to find the exact locations of an object or objects appearing in an image. This is a common task in object recognition and object detection [30]. In Fig. 2(8), each of the faces is localized by a bounding box so that each person can be identified.

9) Link: Links as outputs usually represent the association between two nodes in a network [36]. Fig. 2(9) illustrates a task to predict whether two currently unlinked users will be friends in the future given a partitioned social network where the edges represent friendships between users.

10) Graph: Graphs are commonly used to model relationships between. They consist of a set of nodes and edges, where a node represents an object and an edge indicates a relationship between two objects. Scene graphs [50], for example, are often output as a way to describe the content of an image [34]. Fig. 2(10) shows that given an input image, the output is a graph definition where the nodes are the objects appearing in the image, i.e., “people,” “dinner,” “table,” and “wine,” and the edges are the relationships between these objects. Scene graphs are very useful as representations for tasks, such as image generation [51] and visual question answering [52].

11) Other Outputs: Beyond these few types, there are still many other types of output structures. For example, contour and polygon outputs are similar to bounding boxes and can be used as labels for object localization. In information retrieval, the output(s) could be of the list type, say, of data objects that are similar to the given query. In image segmentation, the outputs are usually segmentation masks of different objects. In signal processing, outputs might be audio of speech or music. In addition, some real-world applications may require more sophisticated output structures relating to multiple tasks. For example, one may require that objects be recognized and localized at the same time, such as in cosaliency, i.e., discovering the common saliency of multiple images [53], simultaneously segmenting similar objects given multiple images in cosegmentation [54], or detecting and identifying objects in multiple images in object codetection [55].

B. Problem Definition of Multi-Output Learning

Multi-output learning maps each input (instance) to multiple outputs. Assume that $X = \mathbb{R}^d$ is a $d$-dimensional input space, and $Y = \mathbb{R}^m$ is an $m$-dimensional output label space. The aim of multi-output learning is to learn a function $f : X \rightarrow Y$ from the training set $D = \{(x_i, y_i)\}_{1 \leq i \leq n}$. For each training example $(x_i, y_i)$, $x_i \in X$ is a $d$-dimensional feature vector, and $y_i \in Y$ is the corresponding output associated with $x_i$. The general definition of multi-output learning is given as follows: finding a function $F : X \times Y \rightarrow R$ based on the training sample of input–output pairs, where $F(x, y)$ is a compatibility function that evaluates how compatible the input $x$ and the output $y$ are. Then, given an unseen instance $x$ at the test state, the output is predicted to be the one with the largest compatibility score, namely, $f(x) = \tilde{y} = \arg \max_{y \in Y} F(x, y)$ [56].

This definition provides a general framework for multi-output learning problems. Although different multi-output learning subfields vary in their output structures, they can be defined within this framework given certain constraints on the output label space $Y$.

We selected several popular subfields and present the constraints of their output space in Sections III-B1–III-B9. Note that multi-output learning is not restricted to these particular scenarios; they are just examples for illustration.

1) Multi-Label Learning: The task of multi-label learning is to learn a function $f(\cdot)$ that predicts the proper label sets for unseen instances [1]. In this task, each instance is associated with a set of class labels/tags and is represented by a sparse binary label vector. A value of +1 denotes that the instance is labeled and 1 means unlabeled. Thus, $y_i \in Y = \{-1, +1\}^m$. Given an unseen instance $x \in X$, the learned multi-label classification function $f(\cdot)$ outputs $f(x) \in Y$, where the labels in the output vector with a value of +1 are used as the predicted labels for $x$.

2) Multi-Target Regression: The aim of multi-target regression is to simultaneously predict multiple real-valued output variables for one instance [5], [57]. Here, multiple labels are associated with each instance, represented by a real-valued vector, where the values represent how strongly the instance corresponds to a label. Therefore, we have the constraint of $y_i \in Y = \mathbb{R}^m$. Given an unseen instance $x \in X$, the learned multi-target regression function $f(\cdot)$ predicts a real-valued vector $f(x) \in Y$ as the output.

3) Label Distribution Learning: Label distribution learning determines the relative importance of each label in the multi-label learning problem [58]. This is opposed to multi-label learning, which simply learns to predict a set of labels. However, as illustrated in Fig. 2, the idea of label distribution learning is to predict multiple labels with a degree value that represents how well each label describes the instance. Therefore, the sum of the degree values for each instance is 1. Thus, the output space for label distribution learning satisfies $y_i = (y_j^1, y_j^2, \ldots, y_j^m) \in Y = \mathbb{R}^m$ with the constraints $y_j^i \in [0, 1], 1 \leq j \leq m$ and $\sum_{j=1}^{m} y_j^i = 1$.

4) Label Ranking: The goal of label ranking is to map instances to a total order over a finite set of predefined
labels [4]. In label ranking, each instance is associated with the rankings of multiple labels. Therefore, the outputs of the problem are the total order of all the labels for each instance. Let \( L = \{ \lambda_1, \lambda_2, \ldots, \lambda_m \} \) denotes the predefined label set. A ranking can be represented as a permutation \( \pi \) on \( \{1, 2, \ldots, m\} \), such that \( \pi(j) = \pi(\lambda_j) \) is the position of the label \( \lambda_j \) in the ranking. Therefore, given an unseen instance \( x \in X \), the learned label ranking function \( f(\cdot) \) predicts a permutation \( f(x) = (y_1^{\pi(1)}, y_1^{\pi(2)}, \ldots, y_1^{\pi(m)}) \in Y \) as the output.

5) Sequence Alignment Learning: Sequence alignment learning aims to identify the regions of relationships between two or more sequences. The outputs in this task are a sequence of multiple labels for the input instance. The output vector has the constraint \( y_i \in Y = \{0, 1, \ldots, c\}^m \), where \( c \) denotes the total number of labels. In sequence alignment learning, \( m \) may vary depending on the input. Given an unseen instance \( x \in X \), the learned sequence alignment function \( f(\cdot) \) outputs \( f(x) \in Y \), where all of the predicted labels in the output vector form the predicted sequence for \( x \).

6) Network Analysis: Network analysis explores the relationships and interactions between objects and entities in a network structure, and link prediction is a common task within this subfield. Let \( G = (V, E) \) denote the graph of a network. \( V \) is the set of nodes, which represent objects, and \( E \) is the set of edges, which represents the relationships between objects. Given a snapshot of a network, the goal of link prediction is to infer whether a connection exists between two nodes. The output vector \( y_t \in Y = \{-1, +1\}^m \) is a binary vector whose value represents whether there will be an edge \( e = (u, v) \) between any pair of nodes \( u, v \in V \) and \( e \notin E \). \( m \) is the number of node pairs that do not appear in the current graph \( G \), and each dimension in \( y_t \) represents a pair of nodes that are not currently connected.

7) Data Generation: Data generation is a subfield of multi-output learning that aims to create and then output structured data of a certain distribution. Deep generative models are usually used to generate the data, which may be in the form of text, images, or audio. The multiple output labels in the problem become the different words in the vocabulary, the pixel values, the audio tones, and so on. Take image test data set. Let \( T = \{ (x_i, y_i) \}_{i=1}^N \) be the test data set, \( f(\cdot) \) be the multi-output learning model, and \( \hat{y}_t = f(x_t) \) be the predicted output of \( f(\cdot) \) for the testing example \( x_t \). In addition, let \( \hat{y}_t \) and \( y_t \) denote the set of labels corresponding to \( y_t \) and \( \hat{y}_t \), respectively. \( I \) is an indicator function, where \( I(g) = 1 \) if \( g \) is true, and 0 otherwise.

8) Semantic Retrieval: Semantic retrieval means finding the meanings within some given information. Here, we consider semantic retrieval in a setting where each input instance has semantic labels that can be used to help retrieval [59]. Thus, each instance representation comprises semantic labels as the output \( y_i \in Y = \mathbb{R}^m \). Given an unseen instance \( x \in X \) as the query, the learned retrieval function \( f(\cdot) \) predicts a real-valued vector \( f(x) \in Y \) as the intermediate output result. The intermediate output vector can then be used to retrieve a list of similar data instances from the database by using a proper distance-based retrieval method.

9) Time-Series Prediction: The goal in time-series prediction is to predict the future values in a series based on previous observations [60]. The inputs are a series of data vectors for a period of time, and the output is a data vector for a future timestamp. Let \( t \) denote the time index. The output vector at time \( t \) is represented as \( y_t \in Y = \mathbb{R}^m \). Therefore, the outputs within a period of time from \( t = 0 \) to \( t = T \) are \( y_t = (y_t^0, y_t^1, \ldots, y_t^j) \). Given the previously observed values, the learned time-series function outputs predicted consecutive future values.

C. Special Cases of Multi-Output Learning

1) Multi-Class Classification: Multi-class classification can be categorized as a traditional single-output learning paradigm if the output class is represented as either an integer encoding or a one-hot vector.

2) Fine-Grained Classification: Fine-grained classification is a challenging multi-classification task where the categories may only have subtle visual differences [61]. Although the output of fine-grained classification shares the same vector representation as multi-class classification, the vectors have different internal structures. Also, in its label hierarchy, labels with the same parents tend to be more closely related than labels with different parents.

3) Multi-Task Learning: The aim of multi-task learning (MTL) is the subfield that aims to improve generalization performance by learning multiple related tasks simultaneously [62], [63]. Each task in the problem outputs one single label or value. This can be thought of as part of the multi-output learning paradigm in that learning multiple tasks is similar to learning multiple outputs. MTL leverages the relatedness between tasks to improve the performance of learning models. One major difference between MTL and multi-output learning is that in MTL, different tasks might be trained on different training sets or features, while in multi-output learning, the output variables usually share the same training data or features.

D. Model Evaluation Metrics

In this section, we present the conventional evaluation metrics used to assess the multi-output learning models with a test data set. Let \( T = \{ (x_i, y_i) \}_{i=1}^N \) be the test data set, \( f(\cdot) \) be the multi-output learning model, and \( \hat{y}_i = f(x_i) \) be the predicted output of \( f(\cdot) \) for the testing example \( x_i \). In addition, let \( \hat{y}_i \) and \( y_i \) denote the set of labels corresponding to \( y_i \) and \( \hat{y}_i \), respectively. \( I \) is an indicator function, where \( I(g) = 1 \) if \( g \) is true, and 0 otherwise.

1) Classification-Based Metrics: Classification-based metrics evaluate the performance of multi-output learning with respect to classification problems, such as multi-label classification, semantic retrieval, image annotation, and label ranking. The outputs are usually in discrete values. The conventional classification metrics fall into three groups: example-based, label-based, and ranking-based.

a) Example-based metrics: Example-based metrics [64] evaluate the performance of multi-output learning models with respect to each data instance. Performance is first evaluated
on each test instance separately, and then, the mean of all the individual results is used to reflect the overall performance of the model. The evaluation for multi-output classification tasks works under the same mechanism as binary classification (single output) tasks: the classic metrics for binary classification can be extended to evaluate multi-output classification models [1]. The commonly used metrics are exact match ratio, accuracy, precision, recall, \( F_1 \) score and hamming loss.

b) **Label-based metrics:** Label-based metrics evaluate performance with respect to each output label. These metrics aggregate the contributions of all the labels to arrive at an averaged evaluation of the model. There are two techniques for obtaining label-based metrics: macroaveraging and microaveraging. Macroaveraging-based approaches compute the metrics for each label independently and then average over all the labels with equal weights. By contrast, microaveraging-based approaches give equal weight to every data sample. Let \( TP_l, FP_l, TN_l, \) and \( FN_l \) denote the number of true positives, true negatives, false positives, and false negatives, for each label, respectively. Let \( B \) be a binary evaluation metric (accuracy, precision, recall, or \( F_1 \) score) for a particular label. The macroapproach and microapproach are therefore defined as follows.

**Macroaveraging:**
\[
B_{\text{macro}} = \frac{1}{m} \sum_{l=1}^{m} B(TP_l, FP_l, TN_l, FN_l).
\]

**Microaveraging:**
\[
B_{\text{micro}} = B \left( \frac{1}{m} \sum_{l=1}^{m} TP_l, \frac{1}{m} \sum_{l=1}^{m} FP_l, \frac{1}{m} \sum_{l=1}^{m} TN_l, \frac{1}{m} \sum_{l=1}^{m} FN_l \right).
\]


c) **Ranking-based metrics:** Ranking-based metrics evaluate the performance in terms of the ordering of the output labels.  

**One-error** is the number of times the top-ranked label is not in the true label set. This approach only considers the most confident predicted label of the model. An averaged one-error over all data instances is computed as
\[
\text{One-error} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}(\arg\min_{\lambda \in \mathcal{L}} \pi_i(\lambda) \notin Y_i)
\]
where \( \mathbb{I} \) is an indicator function, \( \mathcal{L} \) denotes the label set, and \( \pi_i(\lambda) \) is the predicted rank of label \( \lambda \) for the test instance \( x_i \). The smaller the one-error, the better the performance.

**Ranking loss** indicates the average proportion of incorrectly ordered label pairs
\[
\text{Ranking Loss} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{|Y_i||\overline{Y}_i|} |E|
\]
where
\[
E = (\lambda_a, \lambda_b) : \pi_i(\lambda_a) > \pi_i(\lambda_b), (\lambda_a, \lambda_b) \in Y_i \times Y_i \setminus \mathcal{L}, \overline{Y}_i = \mathcal{L} \setminus Y_i.
\]
The smaller the ranking loss, the better the performance of the model.

**Average precision (AP)** is the proportion of the labels ranked above a particular label in the true label set as an average over all the true labels. The larger the value, the better the performance of the model. The averaged AP over all test data instances is defined as follows:
\[
AP = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{|Y_i|} \sum_{\lambda \in Y_i} \frac{1}{|Y_i|} \sum_{\lambda' \in Y_i} \mathcal{I}(\pi_i(\lambda') \leq \pi_i(\lambda))
\]

**Discussion:** The metrics listed earlier are those commonly used with classification-based multi-output learning problems, but the choice of metrics varies according to the different considerations of each application. Take image annotation for example. If the aim of the task is to annotate each image correctly, example-based metrics are optimal for evaluating performance. However, if the objective is keyword-based image retrieval, the macroaveraging metric is preferable [64]. Furthermore, some metrics are more suited to special cases of multi-output learning problems. For instance, for imbalanced learning tasks, geometric mean [65] for some classification-based metrics, e.g., the errors, accuracy, and \( F_1 \)-scores, are more convincing to be used for evaluation. The minimum sensitivity [66] can help determine the classes that hinder the performance in the imbalanced setting. We do not discuss these metrics in detail as they are not the focus here.

2) **Regression-Based Metrics:** Unsurprisingly, regression-based metrics evaluate multi-output learning performance with regression problems, e.g., object localization or image generation. The outputs are usually real values. The commonly used regression-based metrics are mean absolute error (MAE), mean squared error (MSE), average correlation coefficient (ACC) and intersection over union (IoU). Details for these metrics can be found in the Supplementary Materials.

3) **New Metrics:** Data generation is an emerging subfield of multi-out learning that uses generative models to output structured data with certain distributions. Based on the particulars of the task at hand, a model’s performance is usually evaluated in two respects: 1) whether the generated data actually follows the desired real data distribution and 2) the quality of the generated samples. Metrics such as average log-likelihood [67], coverage metric [68], maximum mean discrepancy (MMD) [69], and geometry score [70] are frequently used to assess the veracity of the distribution. Metrics that quantify the quality of the generated data remain challenging. The commonly used are inception scores (IS) [71], mode score (MS) [72], Fréchet inception distance (FID) [73], and kernel inception distance (KID) [74]. Precision, recall, and \( F_1 \) score score are also employed in GANs to quantify the degree of overfitting in the model [75].

E. Multi-Output Learning Data Sets

Most of the data sets used to experiment with multi-output learning problems have either been constructed or become popular because they reflect and, therefore, test a challenge that needs to be overcome. We have presented these data sets according to the challenges reflected in the four Vs. Table II lists the data sets, including their multi-output characteristics,
The challenge can be tested, the application domain, plus the data set name, source, and descriptive statistics.

The large-scale data sets, i.e., the data sets that can be used to test volume, are extremely large. The enormity of their corresponding statistics illustrates the pressing need to overcome the challenges caused by this particular V among the 4.

Many studies that have focused on change in output distribution, e.g., concept drift/velocity, rely on synthetic streaming data or static databases in their experiments. We have also included some of the more popular real-world and/or dynamic databases that are used to experiment with these tasks. As shown in the table, the data sets come from various application domains, demonstrating the importance of this challenge.

The data sets designed to test complex multi-output learning problems contain a mix of different output structures. For example, the image data sets listed in the table includes both labels and bounding boxes for the objects. These data sets can be used to test a variety of data.

Finally, we come to veracity. Many efforts to detail with noisy labels evaluate their methods by beginning with a clean data set to which artificial noise is then added. This helps researchers control and test different levels of noise. We have also listed several popular real-world data sets with some unknown level of errors in the annotation.

IV. CHALLENGES OF MULTI-OUTPUT LEARNING AND REPRESENTATIVE WORKS

The pressing need for the complex prediction output and the explosive growth of output labels pose several challenges to multi-output learning and have exposed the inadequacies

| Multi-output Characteristic | Challenge | Application Domain | Dataset Name | Statistics | Source |
|----------------------------|-----------|--------------------|--------------|------------|--------|
| Volume                     | Extreme Output Dimension | Review Text | AmazonCat-13K | 13,330 | [76] |
|                           |           | Review Text | AmazonCat-14K | 14,588 | [77], [78] |
|                           |           | Text       | Wiki10-31 | 30,936 | [79], [80] |
|                           |           | Social Bookmarking | Deliicioo-300K | 205,443 | [79], [81] |
|                           |           | Text       | WikiLHSTC-355K | 325,056 | [82], [83] |
|                           |           | Product Network | Amazon-670K | 670,091 | [76], [79] |
|                           |           | Text       | Adl-1M | 1,082,898 | [82] |
|                           |           | Product Network | Amazon-3M | 2,812,231 | [77], [78] |
| Unseen Outputs | Extreme Class Imbalance | Scene Image | WIDER-Attribute | 1.25 | [84] |
|                           |           | Face Image | Celeb Paces Attributes | 1.43 | [85] |
|                           |           | Clothing Image | DeepFashion | 1.73 | [86] |
|                           |           | Clothing Image | X-Domain | 1.4,162 | [87] |
|                           |           | Scene Image | Attribute Pascal abd Yahoo | 20 / 72 | [88] |
|                           |           | Animal Image | Animal with Attributes | 80 / 10 | [88] |
|                           |           | Music       | MagTag5K | 107 / 79 | [90] |
|                           |           | Bird Image  | Caltech-UCSD Birds 200 | 130 / 70 | [91] |
|                           |           | Scene Image | MIT Attribute | 645 / 72 | [10] |
|                           |           | Health      | MIMIC II | 3,228 / 355 | [92] |
|                           |           | Health      | MIMIC III | 4,403 / 178 | [93] |
| Velocity                  | Change of Output Distribution | Text       | Reuters | 365 days | [94] |
|                           |           | Route       | ECMLPKDD 15 | Taxi trajectory Prediction | 365 days | [95] |
|                           |           | Route       | epllnability | 30 days | [96] |
|                           |           | Electricity | Portuguese Electricity Consumption | 365 days | [97] |
|                           |           | Traffic Video | MIT Traffic Data Set | 30 minutes | [84] |
|                           |           | Surveillance Video | VIREA! Video | 8.5 hours | [98] |
| Variety                   | Complex Structures | Image       | LabelMe | Label, Bounding Box | [8] |
|                           |           | Image       | ImageNet | Label, Bounding Box | [71] |
|                           |           | Image       | PASCAL VOC | Label, Bounding Box | [99] |
|                           |           | Image       | CIPAR100 | Hierarchical Label | [100] |
|                           |           | Lexical Database | WordNet | Hierarchy | [101] |
|                           |           | Wikipedia Network | Wikipedia | Graph, Link | [102] |
|                           |           | Blog Network | BlogCatalog | Graph, Link | [103] |
|                           |           | Author Collaboration Network | arXiv-AstroPh | Link | [104] |
|                           |           | Author Collaboration Network | arXiv-GrQc | Link | [104] |
|                           |           | Text        | ConLL-2000 Shared Task | Text Chunky | [105] |
|                           |           | Text        | Wall Street Journal (WSJ) corpus | POS, Tag, Parsing Tree | - |
|                           |           | Text        | European Languages | Europarl corpus | Sequence | [32] |
| Veracity                  | Noisy Output Labels | Dog Image  | AMT | 7,354 | [106] |
|                           |           | Food Image  | Food101N | 310K | [107] |
|                           |           | Clothing Image | Clothing1M | 1M | [108] |
|                           |           | Web Image   | WebVision | 2.4M | [109] |
|                           |           | Image and Video | YFCC100M | 100M | [110] |

1http://manikvarma.org/downloads/XC/XMLRepository.html
of many learning models that exist to date. In this section, we discuss each of these challenges and review several representative works on how they cope with these emerging phenomena. Furthermore, given the success of artificial neural networks (ANNs), we also present several state-of-the-art examples of multi-output learning using an ANN for each challenge.

A. Volume—Extreme Output Dimensions

Large-scale data sets are ubiquitous in real-world applications. A data set is defined to be large-scale if it meets one of three criteria: it has a large number of data instances, the input feature space has high dimensionality, or the output space has high dimensionality. Many studies have sought to solve the scalability issues caused by a large number of data instances, e.g., the instance selection method in [212], or with high-dimensional feature spaces, such as the feature selection method in [213]. However, the issues associated with high output dimensions have received much less attention.

Consider, for example, that if the label for each dimension of an $m$-dimensional output vector can be selected from a label set with $c$ different labels, then the number of output outcomes is $c^m$. Hence, these ultrahigh-output dimensions/labels result in an extremely large output space and, in turn, high computation costs. Therefore, it is crucial to design multi-output learning models that can handle the immense and ongoing growth in outputs.

An analysis of the current state-of-the-art research on ultrahigh-output dimensions revealed some interesting insights. Our analysis was based on the data sets used in studies of multiple disciplines, such as machine learning, computer vision, natural language processing, information retrieval, and data mining. We specifically focused on articles in three top journals and three top international conferences: the IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE (TPAMI), the IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS (TNNLS), the Journal of Machine Learning Research (JMLR), the International Conference on Machine Learning (ICML), the Conference on Neural Information Processing Systems (NIPS), and the Conference on Knowledge Discovery and Data Mining (KDD). Figs. 3 and 4 summarize our review. From Figs. 3 and 4, it is evident that the output dimensionality of the understudied algorithms has continued to increase over time. In addition, the latest articles to address this issue in all selected titles are now dealing with more than a million output dimensions and, in some cases, are approaching billions of outputs. Moreover, the statistics for the conferences with shorter time-lags to publication demonstrate just how rapidly output dimensionality is increasing. From this analysis, we conclude that the explosion in output dimensionality is driving many developments in multi-output learning algorithms.

The studies we reviewed tend to fall into two categories: qualitative and quantitative approaches. The qualitative approaches generally involve generative models, while the quantitative models generally involve discriminative models. The main difference between the two models is that generative models focus on learning the joint probability $P(x, y)$ of the inputs $x$ and the label $y$, while the discriminative models focus on the posterior $P(y|x)$. Note that in a generative model, $P(x, y)$ can be used to generate some data $x$, where, in this case, $x$ is the generated output in this particular case.

1) Qualitative Approaches/Generative Models: The aim of image synthesis [48], [214] is to synthesize new images from textual image descriptions of the image. Some pioneering researchers have synthesized images using a GAN with the image distribution as multiple outputs [67]. However, in real life, GANs can only generate low-resolution images. However, since the first attempts at this foray, there has been a progress in scaling up GANs to generate high-resolution images with sensible outputs. For example, Reed et al. [48] proposed a GAN architecture that generates visually plausible $64 \times 64$ pixel images given text descriptions. In a follow-up study, they presented GAWWN [214], which scales the synthesized image up to $128 \times 128$ resolution by leveraging additional annotations. Subsequently, StackGAN [215] was proposed, which is capable of generating photo-realistic images at a $256 \times 256$ resolution from text descriptions. HDGAN [216] is the current state of the art in image synthesis. It models high-resolution images in an end-to-end fashion at $512 \times 512$ pixels. Inevitably, the future will see further increases in resolution.

MaskGAN [217] uses GAN to generate text (i.e., meaningful word sequences). The label set size accords with the vocabulary size. The output dimension is the length of the word sequence that is generated, which, technically, can be unlimited. However, MaskGAN only handles sentence-level text generation. Document- and book-level text generations are still challenging.

2) Quantitative Approaches/Discriminative Models: Like instance and feature selection methods that reduce the number of input instances and, in turn, reduce input dimensionality, it is natural to design models that similarly reduce output dimensionality. Embedding methods can be used to compress a space by projecting the original space onto a lower dimensional space, with the expected information preserved, such as label correlations and neighborhood structure. Popular methods, such as random projections or canonical correlation analysis projections [218]–[221], can be adopted to reduce the dimensions of the output label space. As a result, these modeling tasks can be performed on a compressed output label space, and then, the predicted compressed label can be projected back onto the original high-dimensional label space. Recently, several embedding methods have been proposed for extreme output dimensions. Mineiro and Karampatziakis [222] proposed a novel randomized embedding for extremely large output spaces. AnnexML [169] is another novel embedding method for graphs that captures graph structures in the embedding space. The embeddings are constructed from the $k$-nearest neighbors ($k$NNs) of the label vectors, and the predictions are made efficiently through an approximate nearest neighbor search method. Two popular ANN methods for handling extreme output dimensions are fastText learn tree [223] and XML-CNN [224]. FastText learn tree [223] jointly learns the data representation and the tree structure, and the learned
Fig. 3. Output dimension trends from articles published in the journals TPAMI, TNNLS, and JMLR since 2013 [111]–[165].

Fig. 4. Output dimension trends from articles published in the conferences ICML, NIPS, and KDD since 2013 [79], [82], [166]–[211].

tree structure is then used for efficient hierarchical prediction. XML-CNN is a CNN-based model that incorporates a dynamic max-pooling scheme to capture fine-grained features from regions of the input document. A hidden bottleneck layer is used to reduce the model size.

B. Variety—Complex Structures

With the increasing abundance of labels, there is a pressing need to understand their inherent structures. Complex output structures can lead to multiple challenges in multi-output learning. For instance, it is common for strong correlations and complex dependencies to exist between labels. Therefore, appropriately modeling output dependencies in the label representation is critical but nontrivial in multi-output learning. In addition, designing a multi-variate loss function and proposing an efficient algorithm to alleviate the high complexity caused by complex structures are also challenging.

1) Appropriate Modeling of Output Dependencies: The simplest method of multi-output learning is to decompose the learning problem into $m$ independent single-output problems with each corresponding to a single value in the output space. A representative approach is a binary relevance (BR) [225], which independently learns binary classifiers for all the labels in the output space. Given an unseen instance $x$, BR predicts the output labels by predicting each of the binary classifiers and then aggregating the predicted labels. However, such independent models do not consider the dependencies between outputs. A set of predicted output labels might be assigned to the testing instance even though these labels never co-occur in the training set. Hence, it is crucial to model the output dependencies appropriately to obtain better performance for multi-output tasks.

Many classic learning methods have been proposed to model multiple outputs with interdependencies. These include label powersets (LPs) [226], classifier chains (CCs) [227], [228], structured support vector machine (SSVM) [56], conditional random fields (CRFs) [229], and so on. LPs model the output dependencies by treating each different combination of labels in the output space as a single label, which transforms the problem into one of learning multiple single-label classifiers. The number of single-label classifiers to be trained is the number of label combinations, which grows exponentially with the number of labels. Therefore, LP has the drawback of high computation cost when training with a large number of output labels. Random k-labelsets [230], an ensemble of LP classifiers, is a variant of LP that alleviates the computational complexity problem by training each LP classifier on a different random subset of labels.
CC improves BR by taking the output correlations into account. It links all the binary classifiers from BR into a chain via a modified feature space. Given the jth label, the instance \( x_i \) is augmented with the first, second, ..., \( (j-1) \)th label, i.e., \( (x_i, l_1, l_2, \ldots, l_{j-1}) \), as the input to train the jth classifier. Given an unseen instance, CC predicts the output using the first classifier and then augments the instance with the prediction from the first classifier as the input to the second classifier for predicting the next output. CC processes values in this way from the first classifier to the last and, thus, preserves the output correlations. However, a different order of chains leads to different results. ECC [227], an ensemble of CC, was proposed to solve this problem. It trains the classifiers over a set of random ordering chains and averages the results. Probabilistic CCs (PCCs) [231] provide a probabilistic interpretation of CC by estimating the joint distribution of the output labels to help solve the harder labels. Easy and hard labels and uses the predictions for easy labels to reduce the degradation in performance caused by ambiguous data, such as images, text, and audio [67].

2) Multivariate Loss Functions: Various loss functions were defined to compute the difference between the groundtruth and the predicted output. Different loss functions present different errors given the same data set, and they greatly affect the performance of the model.

\[
L_{0/1}(y, y') = \mathbb{I}(y \neq y')
\]  

where \( \mathbb{I} \) is the indicator function. In general, 0/1 loss refers to the number of misclassified training examples. However, it is very restrictive and does not consider label dependence. Therefore, it is not suitable for large numbers of outputs or for outputs with complex structures. In addition, it is nonconvex and nondifferentiable, so it is difficult to minimize the loss using standard convex optimization methods. In practice, one typically uses a surrogate loss, which is a convex upper bound of the task loss. However, a surrogate loss in multi-output learning usually loses consistency when generalizing single-output methods to deal with multiple outputs [238]. Several works on subfields of multi-output learning study the consistency of different surrogate functions and show that they are consistent under some sufficient conditions [239], [240]. Yet, this is still a challenging aspect of multi-output learning. More exploration of the theoretical consistency of different problems is required.

In the following, we describe four popular surrogate losses: hinge loss, negative log loss, perceptron loss, and softmargin loss.

- **Hinge loss** is one of the most widely used surrogate losses and is usually used in structured SVMs [241]. It pushes the score of the correct outputs to be greater than that of the prediction

\[
L_{\text{Hinge}}(x, y, w) = \max\{\Delta(y, y') + w^T\Phi(x, y') - w^T\Phi(x, y), 0\} - w^T\Phi(x, y).
\]  

The margin, \( \Delta(y, y') \), has different definitions based on the output structures and task. For example, for sequence learning or outputs with equal weights, \( \Delta(y, y') \) can be simply defined as the Hamming loss \( \sum_{j=1}^{m} \mathbb{I}(y_{(j)} \neq y'_{(j)}) \). For taxonomic classification with the hierarchical output structure, \( \Delta(y, y') \) can be defined as the tree distance between \( y \) and \( y' \) [19]. For ranking, \( \Delta(y, y') \) can be defined as the mean AP of a ranking \( y' \) compared to the optimal \( y \) [242]. In syntactic parsing, \( \Delta(y, y') \) is defined as the number of labeled spans, where \( y \) and \( y' \) do not agree [35]. Nondecomposable losses, such as the F1 measure, AP, or IOU, can also be defined as a margin.

- **Negative log loss** is commonly used in CRFs [229]. Note that minimizing negative log loss is the same as maximizing the log probability of the data

\[
L_{\text{NegativeLog}}(x, y, w) = \log \sum_{y' \in \mathcal{Y}} \exp[w^T\Phi(x, y')] - w^T\Phi(x, y).
\]
**Perceptron loss** is usually adopted in structured perceptron tasks [243] and is the same as hinge loss without the margin

\[
L_{\text{Perceptron}}(\mathbf{x}, \mathbf{y}, \mathbf{w}) = \max_{y' \in \mathcal{Y}} [\mathbf{w}^T \Phi(\mathbf{x}, y') - \mathbf{w}^T \Phi(\mathbf{x}, \mathbf{y})].
\]  
(5)

**Softmax-margin loss** is one of the most popular loss functions in multi-output learning models, such as SSVMs [244] and CRFs [245]

\[
L_{\text{SoftmaxMargin}}(\mathbf{x}, \mathbf{y}, \mathbf{w}) = \log \sum_{y' \in \mathcal{Y}} \exp[\Delta(\mathbf{y}, y') + \mathbf{w}^T \Phi(\mathbf{x}, y')] - \mathbf{w}^T \Phi(\mathbf{x}, \mathbf{y})
\]  
(6)

**Squared loss** is a popular and convenient loss function that quadratically penalizes the difference between the ground truth and the prediction. It is commonly used in traditional single-output learning and can be easily extended to multi-output learning by summing the squared differences over all the outputs

\[
L_{\text{Squared}}(\mathbf{y}, \mathbf{y}') = (\mathbf{y} - \mathbf{y}')^2.
\]  
(7)

In multi-output learning, it is usually used with continuous-valued outputs or continuous intermediate results before converting them into discrete-valued outputs. It is also commonly used in neural networks and boosting.

3) **Efficient Algorithms:** Complex output structures significantly increase the burden on algorithms to formulate a model. Large-scale outputs, complex output dependencies, and/or complex loss functions can all be problematic. Therefore, several algorithms have been proposed specifically to tackle these challenges efficiently. Many leverage classic machine learning models so as to speed up the algorithms and alleviate the burden of complexity. The four most widely used classic models are based on kNN, decision trees, k-means, and hashing.

1) kNN-based methods are simple yet powerful machine learning models. Predictions are made based on the closest k instances to the test instance vector in terms of the Euclidean distance. LMMO-kNN [246] is an SSVM-based model involving an exponential number of constraints with respect to the number of labels. This model imposes kNN constraints instantiated by the label vectors from neighboring examples to significantly reduce the training time and make rapid predictions.

2) Decision tree-based methods [247], [248] learn a tree from the training data with a hierarchical output label space. They recursively partition the nodes until each leaf contains a small number of labels. Each novel data point is passed down the tree until it reaches a leaf. This method usually achieves a logarithmic time prediction.

3) k-means based methods, such as SLEEC [79], cluster the training data using k-means clustering. SLEEC learns a separate embedding per cluster and performs classification for a novel instance within its cluster alone. This significantly reduces the prediction time.

4) Hashing methods, such as cohashing [249], [250] and DBPC [251], reduce the prediction time by using hashing on the input or the intermediate embedding space. Cohashing learns an embedding space to preserve semantic similarity structures between inputs and outputs. Compact binary representations are then generated for the learned embeddings for prediction efficiency. DBPC jointly learns a deep latent Hamming space and binary prototypes while capturing the latent nonlinear structures of the data with an ANN. The learned Hamming space and binary prototypes significantly decrease the prediction complexity and reduce memory/storage costs.

C. **Volume—Extreme Class Imbalances**

Real-world multi-output applications rarely provide data with an equal number of training instances for all labels/classes. Too many instances in one class over another mean the data is imbalanced, this is, common in many applications. Therefore, traditional models learned from such data tend to favor majority classes more. For example, in face generation, a trained model tends to generate the faces of famous people because there are so many more images of celebrities than other people. Though class imbalance problems have been studied extensively in the context of binary classification, this issue still remains a challenge in multi-output learning, especially with extreme imbalances.

Many studies on multi-output learning either create a balanced data set or ignore the problems introduced by imbalanced data. A natural way to balance class distributions is to resample the data set. There are two main resampling techniques: undersampling and oversampling [252]. Undersampling methods downsize the majority classes. The NearMiss family of methods [253] are representative works of this category. The oversampling methods, such as SMOTE and its variants [254], adopt oversampling technique on minority classes to handle the imbalanced class learning problem. However, all these resampling methods are mainly designed for single output learning problems. There are other techniques to handle class imbalance in multi-output learning tasks with ANN.

For example, Dong et al. [255] combined incremental rectification of mini-batches with a deep neural network. Then, a hard sample mining strategy minimizes the dominant effect of the majority classes by discovering the boundaries of sparsely sampled minority classes. Both of the methods in [256] and [257] leveraged adversarial training to mitigate imbalance by using a reweighting technique so that majority classes tend to have a similar impact as minority classes.

D. **Volume—Unseen Outputs**

Traditional multi-output learning assumes that the output set in testing is the same as the one in training, i.e., the output labels of a testing instance have already appeared during training. However, this may not be true in real-world applications. For example, a new emerging living species cannot be detected using a learned classifier based on existing living animals. Similarly, it is infeasible to recognize the actions or events in a real-time video if no such actions or events with the same labels appeared in the training video set, nor could a coarse animal classifier provide details of the species of a detected animal, such as whether a dog is a labrador or a shepherd.
Depending on the complexity of the learning task, label annotation is usually very costly. In addition, the enormous growth in the number labels not only leads to high-dimensional output space as a result of computation inefficiency but also makes supervised learning tasks challenging due to unseen output labels during testing.

1) Zero-Shot Multi-Label Classification: Multi-label classification is a typical multi-output learning problem. Multi-label classification problems can have various inputs, such as text, images, and video, depending on the application. The output for each input instance is usually a binary label vector, indicating what labels are associated with the input. Multi-label classification problems learn a mapping from the input to the output. However, as the label space increases, it is common to find unseen output labels during testing, where no such labels have appeared in the training set. To study such cases, the zero-shot multi-class classification problem was first proposed in [17] and [260] and most leverage the predefined semantic information, such as attributes [11] and word representations [13]. This technique was then extended to zero-shot multi-label classification to assign multiple unseen labels to an instance. Similarly, zero-shot multi-label learning leverages the knowledge of the seen and unseen labels and models the relationships between the input features, label representations, and labels. For example, Gaure et al. [259] leverage the co-occurrence statistics of seen and unseen labels and model the label matrix and co-occurrence matrix jointly using a generative model. Rios and Kavuluru [260] and Lee et al. [261] incorporate knowledge graphs of the label relationships with neural networks.

2) Zero-Shot Action Localization: Similar to zero-shot classification problems, localizing human actions in videos without any training video examples is a challenging task. Inspired by zero-shot image classification, many studies into zero-shot action classification predict unseen actions from disjunct training actions based on the prior knowledge of action-to-attribute mappings [262]–[264]. Such mappings are usually predefined, and the seen and unseen actions are linked through a description of the attributes. Thus, they can be used to generalize undefined actions but are unable to localize actions. More recently, some works are proposed to overcome the issue. Jain et al. [265] propose Objects2action without using any video data or action annotations. It leverages vast object annotations, images, and text descriptions that can be obtained from open-source collections, such as WordNet and ImageNet. Mettes and Snoek [266] have subsequently enhanced Objects2action by considering the relationships between actors and objects.

3) Open-Set Recognition: Traditional multi-output learning problems, including zero-shot multi-output learning, operate under a closed-set assumption, i.e., where all the testing classes are known at the time of training time either through the training samples or because they are predefined in a semantic label space. However, Scheirer et al. [16] proposed a concept called open-set recognition to describe a scenario where unknown classes appear in testing. Open-set recognition presents one-vs-set machine to classify the known classes as well as deal with the unknown classes. In later studies [267], [268], they extended this idea into multi-class settings by formulating a compact abating probability model. Bendale and Boul [269] adapted ANNs for open-set recognition by proposing a new model layer that estimates the probability of an input being an unknown class.

Fig. 5 illustrates the relationships between different levels of unseen outputs in multi-output learning. Open-set recognition is the most generalized problem of all. Few-shot and zero-shot learning have studied with different multi-output learning problems, such as multi-label learning and event localization. However, open-set recognition has only been studied in conjunctions with multi-class classification. Other problems in the context of multi-output learning are still unexplored.

E. Veracity—Noisy Output Labels

Almost all methods of label annotation lead to some amount of noise for various reasons. Associations may be weak, the text may be ambiguous, and crowdsourced workers may not be domain experts, so labels may be incorrect [270]. Therefore, it is usually necessary to handle noisy outputs, such as missing, corrupt, incorrect, and/or partial labels, in real-world tasks.

1) Missing Labels: Often human annotators annotate an image or document with prominent labels but miss some of the less emphasized labels. In addition, all the objects in an image may not be localized because there are, say, too many objects or the objects are too small. Social media, such as Instagram, allow users to tag uploaded images. However, the tags could relate to anything: the type of event, the person’s mood, and the weather. Plus, no user is likely to tag every object or every aspect of an image. Directly using such labeled data sets in traditional multi-output learning models cannot guarantee the performance of the given tasks. Therefore, handling missing labels is necessary in real-world applications.

In early studies, missing labels were handled by treating them as negative labels [271]–[273]. Then, modeling tasks are performed based on a fully labeled data set. However, this approach can introduce undesirable bias into the learning problem. Therefore, a more widely used method now is missing value imputation through matrix completion [186], [192], [274]. Most of these approaches are based on a low-rank assumption and, more recently, on label correlations, which improves learning performance [275], [276].

2) Incorrect Labels: Many labels in high-dimensional output space are noninformative or simply wrong [277]. This
is especially common with annotations from crowdsourcing platforms that hire nonexpert workers. Labeled data sets from social media networks are also often less than useful. A basic approach for handling incorrect labels is to simply remove those samples [278], [279]. That said, it is frequently difficult to detect that samples have been mislabeled. Therefore, designing multi-output learning algorithms that learn from noisy data sets is of great practical importance.

Existing multi-output learning methods handling noisy labels generally fall into two groups. The first group is based on building robust loss functions [280]–[282], which modify the labels in the loss function to alleviate the effect of noise. The second group models latent labels and learns the transition from the latent to the noisy labels [283]–[285].

Partial Labels: A special case of incorrect labels is partial labels [286]–[288], where each training instance is associated with a set of candidate labels but only one of them is correct. This is a common problem in real-world applications. For example, a photograph might contain many faces with captions listing who is in the photograph, but the names are not matched to the face. Many methods for learning partial labels have been developed to recover the ground-truth labels from a candidate set [289], [290]. However, most are based on the assumption of exactly one ground truth for each instance, which may not always hold true by different label annotation methods. With the use of multiple workers on the crowdsourcing platform to annotate a data set, the final annotations are usually gathered from the union set of the annotations of all the workers, where each instance might associate with both multiple relevant and irrelevant labels. Hence, Xie and Huang [291] developed a new learning framework, partial multi-label learning (PML), that relaxes this assumption by leveraging the data structure information to optimize the confidence weighted rank loss. Fig. 6 summarizes all the scenarios with noisy output labels, including multi-label learning, missing labels, incorrect labels, partial label learning, and PML.

F. Velocity—Changes in Output Distribution

Many real-world applications must deal with data streams, where data arrive continuously and possibly endlessly. In these cases, the output distributions can change over time or concept drift can occur. Streaming data are common in surveillance [98], driver route prediction [95], demand forecasting [97], and many other applications. Take visual tracking [292] in the surveillance video as an example, where the video stream is potentially endless. Data streams come in high velocity as the video keeps generating consecutive frames. The goal is to detect, identify, and locate events or objects in the video. Therefore, the learning model must adapt to possible concept drift while working with limited memory.

Existing multi-output learning methods model changes in output distribution by updating the learning system each time data streams arrive. The update method might be ensemble-based [293]–[297] or ANN-based methods [292], [298]. Other strategies to handle concept drift include the assumption of a fading effect on past data [296]; maintaining a change detector on predictive performance measurements, and recalibrating models accordingly [295], [299]; using stochastic gradient descent to update the network and accommodate new data streams with an ANN [292]. Notably, the kNN is one of the most classic frameworks in handling multi-output problems, but it cannot be successfully adapted to deal with the challenge of change of output distribution due to the inefficiency issue. Many online hashing and online quantization-based methods [300], [301] are proposed to improve the efficiency of kNN while accommodating the changing output distribution.

G. Other Challenges

Any two of the aforementioned challenges can be combined to form a more complex challenge. For example, noisy labels and unseen outputs can be combined to form an open-set noisy label problem [302]. In addition, the combination of noisy labels and extreme output dimensions are also worthy of study and further exploration [206]. Changes in output distribution together with noisy labels result in online time-series prediction problems with missing values [303], while changes in distributions combined with dynamic label sets (unseen outputs) lead to open-world recognition problems with incremental labels [304]. Changing output distribution with extreme class imbalances creates the common problem of streaming data with concept drift and class imbalances at the same time [18], [305]. Moreover, the combination of complex output structures with changing output distribution is also frequent in real-world applications [306].

H. Open Challenges

1) Output Label Interpretation: There are different ways to represent output labels, and each expresses label information from a specific perspective. Taking label tags as an output, for example, binary attributed output embeddings represent what attributes the input relates to. Hierarchical label output embedding conveys the hierarchical structure of the inputs. Semantic word output embeddings reflect the semantic relationships between the outputs. As one can see, each exhibits...
a certain level of human interpretability. Hence, an emerging approach to label embedding is to incorporate different label information from multiple perspectives and rich contexts to enhance interpretability [307]. This is a challenging undertaking because it is quite difficult to appropriately model the interdependencies between outputs in a way that humans can easily interpret and understand. For example, an image of a centaur is expected to be described with semantic labels, such as horse and person. Moreover, the image is expected to be described with attributes, such as head, arm, and tail. As such, appropriately modeling the relationships between input and outputs with rich interpretations of the labels is an open challenge that should be explored in the future studies.

2) Output Heterogeneity: As the demand for sophisticated decision-making increases, so does demand for outputs with more complex structures. Returning to the example of surveillance, people reidentification in traditional approaches usually consists of two steps: people detection and then reidentifying that person if they are input. These steps are essentially two separate tasks that need to be learned together if performance is to be enhanced. Several researchers have recently attempted this demanding challenge, i.e., building a model that can simultaneously learn multiple tasks with different outputs. Mousavian et al. [308] undertook joint people detection in tandem with reidentification, while Van Ranst et al. [309] tackled image segmentation with depth estimation. However, more exploration and investigation to overcome this challenge are needed. As an example, one worthy undertaking would be to answer the question: Can we simultaneously learn the representation of a new user in a social network as well as their potential links to existing users?

V. CONCLUSION

Multi-output learning has attracted significant attention over the last decade. This article provides a comprehensive review of the study of multi-output learning using the four Vs as a frame. We explore the characteristics of the multi-output learning paradigm beginning with the life cycle of the output labels. We emphasize the issues associated with each step of the learning process. In addition, we provide an overview of the types of outputs, the structures, selected problem definitions, common model evaluation metrics, and the popular data repositories used in experiments, with representative works referenced throughout. The article concludes with a discussion on the challenges caused by four Vs and some future research directions that are worthy of further study.

REFERENCES

[1] M.-L. Zhang and Z.-H. Zhou, “A review on multi-label learning algorithms,” *IEEE Trans. Knowl. Data Eng.*, vol. 26, no. 8, pp. 1819–1837, Aug. 2014.

[2] C. Gong, D. Tao, J. Yang, and W. Liu, “Teaching-to-learn and learning-to-teach for multi-label propagation,” in *Proc. AAAI*, 2016, pp. 1610–1616.

[3] C. Bielza, G. Li, and P. Larrañaga, “Multi-dimensional classification with Bayesian networks,” *Int. J. Approx. Reasoning*, vol. 52, no. 6, pp. 705–727, 2011.

[4] S. Yemini and T. Gärnér, “Label ranking algorithms: A survey,” in *Preference Learning*. Berlin, Germany: Springer, 2010, pp. 45–64, doi: 10.1007/978-3-642-14125-6_3.

[5] H. Borochani, G. Varando, C. Bielza, and P. Larrañaga, “A survey on multi-output regression,” *Data Mining Knowl. Discovery*, vol. 5, no. 5, pp. 216–233, 2015.

[6] G. Widner and M. Kubat, “Learning in the presence of concept drift and hidden contexts,” *Mach. Learn.*, vol. 23, no. 1, pp. 69–101, 1996.

[7] O. Russakovsky et al., “ImageNet large scale visual recognition challenge,” *Int. J. Comput. Vis.*, vol. 115, no. 3, pp. 211–252, Dec. 2015.

[8] B. C. Russell, A. Torralba, K. P. Murphy, and W. T. Freeman, “LabelMe: A database and Web-based tool for image annotation,” *Int. J. Comput. Vis.*, vol. 77, nos. 1–3, pp. 157–173, 2008.

[9] P. Stenetorp, S. Pyysalo, G. Topić, T. Ohta, S. Ananiadou, and J. Tsujii, “BRAT: A Web-based tool for NLP-assisted text annotation,” in *Proc. EACL*, 2012, pp. 102–107.

[10] G. Eryiğit, F. S. Çetin, M. Yanik, T. Temel, and İ. Çiçekli, “TURKSENT: A sentiment annotation tool for social media,” in *Proc. 7th Linguistic Annotation Workshop Interoperability Discourse*, 2013, pp. 131–134.

[11] C. H. Lampert, H. Nickisch, and S. Harmeling, “Learning to detect unseen object classes by between-class attribute transfer,” in *Proc. CVPR*, 2009, pp. 951–958.

[12] M. Rohrbach, M. Stark, and B. Schiele, “Evaluating knowledge transfer and zero-shot learning in a large-scale setting,” in *Proc. CVPR*, 2011, pp. 1641–1648.

[13] T. Mikolov, I. Sutskever, K. Chen, G. S. Corrado, and J. Dean, “Distributed representations of words and phrases and their compositionality,” in *Proc. NIPS*, 2013, pp. 3111–3119.

[14] S. C. Deerwester et al., “Indexing by latent semantic analysis,” U.S. Patent 5 839 853 A, Jun. 13, 1989.

[15] K. Xu et al., “Show, attend and tell: Neural image caption generation with visual attention,” in *Proc. Int. Conf. Mach. Learn.*, 2015, pp. 2048–2057.

[16] W. J. Scheirer, A. de Rezende Rocha, A. Sapkota, and T. E. Boult, “Toward open set recognition,” *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 35, no. 7, pp. 1757–1772, Jul. 2013.

[17] M. Palatucci, D. Pomerleau, G. E. Hinton, and T. M. Mitchell, “Zero-shot learning with semantic output codes,” in *Proc. NIPS*, 2009, pp. 1410–1418.

[18] T. R. Hoens, R. Polikar, and N. V. Chawla, “Learning from streaming data with concept drift and imbalance: An overview,” *Proc. Artif. Intell.*, vol. 1, no. 1, pp. 89–101, 2012.

[19] L. Cai and T. Hofmann, “Hierarchical document categorization with support vector machines,” in *Proc. CIKM*, 2004, pp. 78–87.

[20] J. Xiao, J. Hays, K. A. Ehinger, A. Oliva, and A. Torralba, “SUN database: Large-scale scene recognition from abbey to zoo,” in *Proc. CVPR*, 2010, pp. 3485–3492.

[21] G.-J. Qi, X.-S. Hua, Y. Rui, J. Tang, T. Mei, and H.-J. Zhang, “Correlative multi-label video annotation,” in *Proc. 15th ACM Int. Conf. Multimedia*, 2007, pp. 17–26.

[22] S. Džeroski, D. Demšar, and J. Grbović, “Predicting chemical parameters of river water quality from biocinder data,” *Appl. Intell.*, vol. 13, no. 1, pp. 7–17, 2000.

[23] H. Aras and N. Aras, “Forecasting residential natural gas demand,” *Energy Sour.*, vol. 26, no. 5, pp. 463–472, 2004.

[24] H. Li, W. Zhang, Y. Chen, Y. Guo, G.-Z. Li, and X. Zhu, “A novel multi-target regression framework for time-series prediction of drug efficacy,” *Sci. Rep.*, vol. 7, Jan. 2017, Art. no. 40652.

[25] X. Geng and Y. Xia, “Head pose estimation based on multivariate label distribution,” in *Proc. CVPR*, 2014, pp. 1837–1842.

[26] X. Geng, K. Smith-Miles, and Z. Zhou, “Facial age estimation by learning from label distributions,” in *Proc. AAAI*, 2010, pp. 451–456.

[27] D. Zhou, X. Zhang, Y. Zhou, Q. Zhao, and X. Geng, “Emotion distribution learning from texts,” in *Proc. EMNLP*, 2016, pp. 638–647.

[28] K. Cramer and Y. Singer, “A family of additive online algorithms for category ranking,” *J. Mach. Learn. Res.*, vol. 3, pp. 1025–1058, Mar. 2003.

[29] J. Ko, E. Nyberg, and L. Si, “A probabilistic graphical model for joint answer ranking in question answering,” in *Proc. SIGIR*, 2007, pp. 343–350.

[30] S. S. Bucak, P. K. Mallapragada, R. Jin, and A. K. Jain, “Efficient multi-label ranking for multi-class learning: Application to object recognition,” in *Proc. ICCV*, 2009, pp. 2098–2105.

[31] Y. Liu, E. P. Xing, and J. Carbonell, “Predicting protein folds with support vector machines,” in *Proc. CIKM*, 2004, pp. 78–87.

[32] P. Koehn, “Europarl: A parallel corpus for statistical machine translation,” in *Proc. MT Summit*, vol. 5, 2005, pp. 79–86.
A. Milan, S. H. Rezatofighi, A. R. Dick, I. D. Reid, and K. Schindler, “New ensemble methods for evolving data streams,” in Proc. KDD, 2009, pp. 139–148.

Y. Kong and P. S. Yu, “An ensemble-based approach to fast classification of multi-label data streams,” in Proc. 7th Int. Conf. Collaborative Comput., Netw. Appl. Worksharing, 2011, pp. 95–104.

A. Büyükaçik, H. Bonab, and F. Can, “A novel online stacked ensemble for multi-label stream classification,” in Proc. CIKM, 2018, pp. 1063–1072.

A. Milan, S. H. Rezatofighi, A. R. Dick, I. D. Reid, and K. Schindler, “Online multi-target tracking using recurrent neural networks,” in Proc. AAAI, 2017, pp. 4225–4232.

J. Read, A. Bifet, G. Holmes, and B. Pfahringer, “Scalable and efficient multi-label classification for evolving data streams,” Mach. Learn., vol. 88, nos. 1–2, pp. 243–272, 2012.

L. Huang, Q. Yang, and W. Zheng, “Online hashing,” IEEE Trans. Neural Netw. Learn. Syst., vol. 29, no. 6, pp. 2309–2322, Jun. 2018.

D. Xu, I. W. Tsang, and Y. Zhang, “Online product quantization,” IEEE Trans. Knowl. Data Eng., vol. 30, no. 11, pp. 2185–2198, Nov. 2018.

Y. Wang et al., “Iterative learning with open-set noisy labels,” in Proc. IEEE Conf. Comput. Vis. Pattern Recognit., Jun. 2018, pp. 8688–8696.

O. Anava, E. Hazan, and A. Zeevi, “Online time series prediction with missing data,” in Proc. ICML, 2015, pp. 2191–2199.

A. Bendale and T. E. Boult, “Towards open world recognition,” in Proc. CVPR, 2015, pp. 1893–1902.

E. S. Xioufis, M. Spiliopoulos, G. Tsoumakas, and I. Vlahavas, “Dealing with concept drift and class imbalance in multi-label stream classification,” in Proc. IJCAI, 2011, pp. 1583–1588.

Z. Ren, M. Poertz, S. Liang, W. van Dolen, and M. de Rijke, “Hierarchical multi-label classification of social text streams,” in Proc. SIGIR, 2014, pp. 213–222.

Y. Shi, D. Xu, Y. Pan, I. W. Tsang, and S. Pan, “Label embedding with partial heterogeneous contexts,” in Proc. AAAI, 2019, pp. 4926–4933.

A. Mousavian, H. Pirsiavash, and J. Kosecka, “Joint semantic segmentation and depth estimation with deep convolutional networks,” in Proc. TVCG, 2016, pp. 611–619.

W. Van Ranst, F. De Smedt, J. Berte, and T. Goedemé, “Fast simultaneous people detection and re-identification in a single shot network,” in Proc. AVSS, 2018, pp. 1–6.