Bayesian approach to single-tree detection in airborne laser scanning – use of training data for prior and likelihood modeling

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Abstract. This paper introduces a novel computational approach to handling remote sensing data from forests. More specifically, we consider the problem of detecting an unknown number of trees in a forest plot based on airborne laser scanning (ALS) data. In addition to detecting the locations of individual trees, their heights and canopy shapes are estimated. This detection-estimation problem is treated in the Bayesian inversion framework. We use simplified, rotationally symmetric models for the tree canopies to model the reflections of laser beams from the canopies. To account for the associated modeling errors, we use training data consisting of ALS data and field measurements to build a likelihood function which models statistically the propagation of a laser beam in the presence of a canopy. The training data is utilized also for constructing empirical prior models for the canopy height/shape parameters. As a Bayesian point estimate, we consider the maximum a posteriori estimate. The proposed approach is tested with ALS measurement data from boreal forest, and validated with field measurements.

1. Introduction
Airborne laser scanning (ALS) is a widely used tool for remote sensing of forest [5, 7, 9]. The applications include, e.g., the forest inventory and ecology. ALS is usually performed using an aeroplane, but it is also possible to use unmanned aerial vehicles, such as drones. ALS is based on light detection and ranging (LiDAR) technology, where laser beams are directed towards ground within a device-specific scan angle; these beams reflect from surfaces of objects they encounter, such as tree crowns and ground (see Figure 1). The times of the pulses reflected back from the tree crowns/ground are recorded and transformed into distance information, which together with GPS coordinates of the aeroplane and beam direction allow for calculating accurate coordinates of the reflection point. The ALS point clouds thus include accurate (yet discrete) information on the locations of surfaces above ground, and provide versatile information on the structure of the forest.

Because canopy surfaces are not solid but formed by separate leaves, a reflected laser wave can contain more than one intensity peaks, which are registered as separate reflections from the
same scanning direction. The collected ALS data points are thus identified as only, first of many, intermediate and last echoes. Figure 2 shows an example of ALS-based point cloud data from a boreal forest. Point clouds corresponding to only echoes and first of many echoes are shown separately, and approximate (field measured) locations of trees are drawn in the same figures. The figure clearly indicates a qualitative difference between the spatial distributions of the two point clouds: Most of the only echoes are reflected from the ground or from tops (centers) of tree crowns, while the first of many echoes mostly result from reflections from the canopies but not from their centers.

The interpretation of ALS data can be divided into two categories: the area-based approaches [5, 7, 9] and single-tree detection [5, 2, 7, 9, 1, 3, 8]. While in the widely used area-based approach, the plot-level statistics of forest attributes are estimated directly from the statistics of ALS data using regression-type methods, the single-tree detection aims at deriving the plot-level statistics from individual tree-level information. Potentially, the latter approach—which accounts for the spatial distribution of the ALS point clouds—could improve the accuracy of the ALS-based forest inventory, and enable deriving more versatile information (such as distinguishing tree species) from the data. This, however, requires development of the computational methods used in the analysis of the ALS data.

In this paper, the problem of individual tree detection is cast in the framework of Bayesian inverse problems. As in a recent work [2], simplified, rotationally symmetric models for the tree canopies are used for modeling the reflections of laser beams from the canopies. However, while in [2], an ad hoc, additive noise model was used for constructing a Gaussian likelihood model for the ALS observations, the aim of the present work is to study whether a feasible likelihood model could be constructed systematically by analyzing ALS data from a set of training plots. The training data is utilized also for constructing empirical prior models for the canopy height/shape parameters. The feasibility of the approach is tested with ALS measurement data from boreal forest, and validated with field measurements.

2. Materials
We consider managed boreal forest plots as were also investigated in [2, 6, 8], i.e. Eastern Finland latitude 62°31’N, longitude 30°10’E. The ALS data were collected by using an Optech Airborne Laser Terrain Mapper Gemini laser scanning system in 2009. The scanning height was approximately 720 m above ground level and the scan angle was 26°. Multiple echoes were recorded (multipulse mode) and pulse repetition frequency was 125 kHz. In this paper, we consider first of many echoes and only echoes, as illustrated in Figure 2. The densities of data points were approximately 8/m² for each type of echoes. The sizes of forest plots vary from 20
Figure 2. An illustration of real ALS data. The ALS observations consist of coordinates of points where the laser beams intersect with the surfaces of trees or ground. The only echoes are illustrated in the top row and first of many echoes in the second row. The left column shows side views of the two point clouds and in the right column, the observations are shown from top. The black circles illustrate the field-measured locations of the trees – the centers of the circles correspond to trunk locations on the ground and the radii of the circles approximate the radii of the canopies.

m × 20 m up to 30 m × 30 m consisting of Scots pines, Norway spruces and deciduous trees. For validation purposes, the locations and sizes of the trees were also measured manually from field. The field measurement data includes tree locations, tree heights, diameters at breast height and tree species and it was collected during 2010. In the present work, the field measurement data is used also as a training data, for constructing the likelihood and prior models for Bayesian inference. Note however, that different plots were selected for training the model and validating the solutions of the inverse problems.
3. Computational methods
In this section, a novel approach to detecting individual trees and estimating their heights and canopy shapes is proposed. First, in Section 3.1, we describe the parametrization of trees and a computational model that approximates the formation of ALS point clouds. In Section 3.2, statistics of the uncertainty of the ALS observations are estimated based on a set of training data, and an approximative likelihood model is written. In Section 3.3, the likelihood function is combined with a field measurement-based prior model for the model unknowns (canopy shape parameters) to form the posterior density for the unknowns. As an estimate for the individual tree parameters, the maximum a posteriori estimate is considered.

3.1. Parametrization and observation model
The parametrization of individual trees is illustrated in Figure 3. We denote the horizontal location of a tree trunk on the ground by \((x, y)\) and the height of the tree by \(h\). The canopy of the tree is approximated as a rotationally symmetric object and the vertical profile of the crown radius is modeled as

\[
R(h_s) = c_r \sin(h_s)^{a_t},
\]

where \(c_r\) denotes the radius of the tree crown at the lower limit of the living crown \(s_h\),

\[
h_s = \left(\pi h_v\right) / (2c_h), \quad h_v \in [0, c_h], \quad c_h = h - s_h, \quad \text{and} \quad a_t \text{ is a species-specific shape parameter; for pines, birch and spruces, } a_t = 0.3755, 0.2463 \text{ and } 0.3825, \text{ respectively, see [2].}
\]

In this paper, we aim at estimating the parameters \(x, y, h\) and \(c_r\) based on ALS data. In principle, it would be possible to consider also the lower limit of the living crown \(s_h\) (or alternatively, the crown height \(c_h\)) as an additional unknown in the model. However, the sensitivity of the ALS measurements to the ratio of \(s_h\) and \(c_h\) is very low, and hence we write a species-specific, deterministic model for the dependence of \(c_h\) and the tree height \(h\):

\[
c_h = \alpha_t h + \beta_t.
\]

Here, \(\alpha_t\) and \(\beta_t\) are fitting parameters based on analysis of field data. This approximation can be used for rewriting variable \(h_s\) as \(h_s = (\pi c_r) / (2(a_t h + \beta_t))\), which allows for expressing the vertical profile of the tree crown (Equation (1)) as the function of the tree height \(h\).

Figure 3. Illustration of a rotationally symmetric approximation of the tree crown and parametrization. The primary unknowns in the inverse problem are marked with red font: the horizontal location of a tree trunk on the ground \((x, y)\), tree height \(h\) and \(c_r\), the radius of the tree crown at the lower limit of the crown \(s_h\).

In addition to tree location, size and shape parameters \(x, y, h\) and \(c_r\), the tree species \(t\) is considered as an unknown in the model. Here, \(t\) is a discrete variable which has four possible
realizations: \( t \in \{ \text{pine, spruce, deciduous, none} \}. \) Here, the last value ("none") signifies a case where the corresponding tree model is to be deleted – this option is needed, because the number of trees is unknown. We denote the vector consisting of all model unknowns by \( \theta \):

\[
\theta = [\theta_1, \ldots, \theta_M]^T,
\]

where \( \theta_n = (x_n, y_n, h_n, c_{rn}, t_n), n \in \{1, \ldots, M\} \) is the index of a tree, and \( M \) is an estimate for the number of trees.

Denote the index of a laser beam in a data set corresponding to one forest plot by \( j \). To construct a computational model for the formation of an ALS observation, we approximate for simplicity that the ALS beams are vertically oriented. If the horizontal location \((x^j, y^j)\) of a beam reflection point intersects with the area under the crown of a single tree \( n \), then the height of the ALS reflection point \( z^j \) is modeled as the height of the \( n \)th tree crown surface at \((x^j, y^j)\): i.e.,

\[
z^j = g_j(\theta) = g_n(x^j, y^j, \theta)(x^j, y^j, \theta).
\] (4)

Here we have highlighted that the index of the tree, \( n \), that a beam coincides with depends on both the horizontal location \((x^j, y^j)\) and the parameter vector \( \theta \). Further, if the beam reflection point intersects with more than one tree, then the highest canopy surface point at the location \((x^j, y^j)\) is considered as the reflection point, i.e.,

\[
z^j = g_j(\theta) = \max_{m \in n(x^j, y^j, \theta)} \{ g_m(x^j, y^j, \theta) \},
\] (5)

and if the beam does not intersect with any tree crown, ALS measurement modeled as a ground reflection, i.e., \( z^j = g_j(\theta) = 0 \).

Combining all vertical coordinates \( z^j \) into a vector \( z = [z^1, \ldots, z^N] \), we can write the approximative computational model for the observations as

\[
z = g(\theta)
\] (6)

where \( g(\theta) = [g_1(\theta), \ldots, g_N(\theta)] \). In principle, the problem of estimating the tree parameters \( \theta \) could be considered as an ordinary non-linear least-squares (LS) problem induced by the model (6). However, this problem would be severely ill-posed, as its solutions are non-unique and unstable. Moreover, as it turns out in the next section, the modeling errors associated with the computational model (6) have non-trivial statistics (non-homogeneous, multimodal and non-zero mean), which would yield additional difficulty in the LS fitting of tree crown surfaces to the observed ALS data. In the following sections, tree crown fitting problem is considered in the framework of Bayesian inverse problems.

### 3.2. Construction of prior and likelihood models using training data

In this section, we write the prior probability density for the tree location/shape parameters \( \theta \) and the likelihood function of the ALS observations \( z \) on the basis of training data. That is, we model both \( \theta \) and \( z \) as random variables, and approximate their statistics using models derived from ALS and field measurements.

#### 3.2.1. Prior model

The field measurements consist of tree locations, tree species, tree heights and trunk diameters at breast height. As in article [2], we approximate the crown radii \( c_r \) from these measurements using models presented in [4]. Figure 4 (left) shows a scatter plot of the observed/approximated tree heights \( h \) and crown radii \( c_r \). In this plot, different colors represent different tree species (pine, spruce and deciduous). The figure indicates a strong correlation between parameters \( h \) and \( c_r \). Moreover, the differences between the scatter plots
corresponding to three tree species are relatively small. Based on this training data set, we model the parameters \( h \) and \( c_r \) as mutually correlated species-independent Gaussian random variables, and approximate their expectations and covariance matrix by sample means and covariance. The Gaussian joint prior probability density for the model parameters \( h \) and \( c_r \) is illustrated in Figure 4 (right).

Further, the tree location parameters \( x \) and \( y \) are modeled as mutually independent Gaussian random variables, and the tree species parameter \( t \) is modeled as a uniformly distributed discrete random variable, i.e., \( P(t) = \frac{1}{4}, \forall t \in \{\text{pine}, \text{spruce}, \text{deciduous}, \text{none}\} \), where \( P(\cdot) \) denotes the probability. Using the above approximations, the probability density (prior density) of \( \theta \) can be written in the form

\[
\pi(\theta) \propto \exp \left( -\frac{1}{2} (\tilde{\theta} - \tilde{\theta}_s)^T \Gamma^{-1}_{\tilde{\theta}} (\tilde{\theta} - \tilde{\theta}_s) \right)
\]

where \( \tilde{\theta} \) is a vector consisting of those model unknowns that are modeled as continuous random variables, that is, \( \tilde{\theta} = [\tilde{\theta}_1, \ldots, \tilde{\theta}_N]^T, \tilde{\theta}_n = (x_n, y_n, h_n, c_{rn}) \). Further, \( \tilde{\theta}_s \) and \( \Gamma_{\tilde{\theta}} \) are the expectation and covariance matrix of \( \tilde{\theta} \), respectively. Note here that since the prior probabilities of tree species are assumed to be equal and independent from \( x, y, h \) and \( c_r \), the prior density of \( \theta \) is simply \( \pi(\theta) = \frac{1}{4}\pi(\tilde{\theta}) \), i.e., \( P(t) \) only acts as a scaling factor in the prior density of \( \theta \).

![Figure 4](image-url)

Figure 4. An illustration of mutually correlated parameters \( h \) and \( c_r \) for different tree species (left), and a contour plot of the fitted Gaussian prior density (right).

### 3.2.2. Likelihood model

Figure 2 shows the spatial distribution of first of many echoes and only echoes in one example plot. The figure also illustrates the field measured locations of trees in the plot: the black circles approximate the extents of the tree crowns. The figure reveals that the above approximation of the tree crowns being rotationally symmetric objects, the surfaces of which reflect the ALS pulses, is erroneous. Indeed, Figure 2 (top right) shows that for the only echoes, a large number of ground reflections (\( z \approx 0 \)) are obtained from under the tree canopies, and Figure 2 (bottom right) shows that some of the first of many echoes at high altitude (\( z = 5, \ldots, 20 \) m) have reflected from tree branches outside the approximate rotationally symmetric tree crown models. In this section, we analyze the ALS observations in the training set, and combine their statistics with the computation model of ALS observations written in Section 3.1 to formulate an approximative likelihood model for ALS.
From the field measured tree locations and sizes corresponding to each training plot, we get a realization of the parameter vector $\theta$. Based on these observations, we also divide the horizontal planes of the training plots into following three subsets:

- $I^1$: areas under rotationally symmetric tree crown models
- $I^2$: “buffer zones” $I^{2,1}$ and $I^{2,2}$ within 0, ..., 25 cm and 25, ..., 50 cm from $I^1$, respectively.
- $I^3$: The remaining areas, i.e., those areas which are not in the proximity of any tree.

These zones are illustrated in Figure 5. The figure also shows the subdivision of the tree crown area $I^1$ into areas of inner circles $I^{1,1}$ and outer rings $I^{1,2}$; these subsets are used in the analysis of only echoes. Here, we set the inner circle $I^{1,1}$ radius to be $c_r/4$. We analyze the statistics of the following variable $r^j$ calculated from the set of ALS observations $(x^j, y^j, z^j)$ in the set of training plots:

$$r^j = \begin{cases} \frac{z^j - g_j(\theta)}{h_{j,\theta}}, & (x^j, y^j) \in I^{1}(\theta) \\ \frac{z^j}{h_{j,\theta}}, & (x^j, y^j) \in I^{2}(\theta) \\ z^j & (x^j, y^j) \in I^{3}(\theta) \end{cases}$$

where $h_{j,\theta}$ denotes the height of the tree, the crown model surface of which is intersected by the $j$th laser beam (for $(x^j, y^j) \in I^{1}(\theta)$) or the buffer zone of which is intersected by the $j$th laser beam (for $(x^j, y^j) \in I^{2}(\theta)$). Thus, for an ALS data point $(x^j, y^j, z^j)$ corresponding to an observation on the area of a tree crown, we consider a scaled residual between the observed height of reflection $z^j$ and the height given by the computational model: $g_j(\theta)$. In the buffer zone $I^{2}(\theta)$, where $g_j(\theta) = 0$, the scaled residual is obtained as $\frac{z^j}{h_{j,\theta}}$, and in the ground area $I^{3}(\theta)$, residuals $r^j = z^j - g_j(\theta) = z^j - 0$ are considered.

![Figure 5](image)

**Figure 5.** Left: An illustration of zones $I^1$, $I^2$ and $I^3$ and (scaled) residuals $r^j$. ALS data is illustrated using symbol ‘*’ and the modeled observations with symbol ‘◦’. Right: For the only echoes, the tree crown zone $I^1$ is further divided to two sub-zones $I^{1,1}$ and $I^{1,2}$.

Figure 6 shows the histograms of $r^j$ corresponding to zone $I^1(\theta)$ for only echoes (top row) and first of only echoes (bottom). The histograms show that the residuals are heavily concentrated to negative values; i.e., most of the ALS beams are reflected below the modeled tree crown surfaces. Moreover, as anticipated, the statistics of the only echoes and first of only echoes differ from...
each other significantly. In particular, the only echoes feature peaks around \( r^j = -1 \), which corresponds to a ground observation in the location of a tree crown.

We model the (scaled) residuals \( r^j \) as random variables and approximate their probability densities at each zone \( I^3(i) \) by sums of exponential functions

\[
\pi^I_{r^j}(r^j|\theta) = f_i(r^j; \theta) = \beta_i \sum_{k=1}^{k_i} a_{i,k} \exp \left( -\frac{(r^j - b_{i,k})^2}{c_{i,k}} \right),
\]

(9)

where \( a_{i,k}, b_{i,k} \) and \( c_{i,k} \) are defined by the fitting \( f_i(r^j; \theta) \) to the histogram corresponding to \( r^j \) at zone \( I^i \), \( k_i \) is the number of exponential functions, and \( \beta_i \) is a scaling coefficient. Here, however, we note that for the only echoes in \( I^1 \), we manually decreased the histogram peaks corresponding to ground observations \( r^j = -1 \); this was made to diminish the errors caused by inaccuracies in the field measurements of tree locations. The fitted probability density functions corresponding to zone \( I^1 \) are illustrated in Figure 6.

\[\text{Figure 6.} \text{ Example of histograms of the residuals and fitted likelihood functions in the zone } I^1. \text{ Top: only echoes in } I^{1,1} \text{ (ring 1)}, \text{ only echoes in } I^{1,2} \text{ (ring 2) and bottom: first echoes in } I^1.\]

Next, we write the probability density of an observation \( z^j \) corresponding to a given set of parameters as \( \pi(z^j|\theta) = \pi^I_{r^j}(r^j|\theta) = f_i(r^j; \theta) \), where \( f_i(r^j; \theta) \) is the approximate sample-based probability density of \( r^j \), given by Equation (9). Assuming that observations \( z^j, z^\ell \) are mutually independent for all \( j \neq \ell \), the likelihood \( \pi(z|\theta) \) of the observation vector \( z = [z^1, \ldots, z^N]^T \) is

\[
\pi(z|\theta) = \prod_{j=1}^{N} \pi(z^j|\theta) = \prod_{j=1}^{N} f_i(x_j, y_j, \theta)(r^j; \theta).
\]

(10)
Here, we have highlighted that the zone index $i$ depends on the horizontal coordinates $(x^j, y^j)$ of the ALS observation point and the model parameters $\theta$.

### 3.3. Bayesian inversion

Combining the prior probability density in Equation (7) and the likelihood in Equation (10), the conditional density of $\theta$ given $z$, or the posterior density, can be written using Bayes’ formula as

\[
\pi(\theta | z) = \frac{\pi(z | \theta) \pi(\theta)}{\pi(z)} \\
\propto \pi(z | \theta) \pi(\theta) \\
\propto \prod_{j=1}^{N} f_i(x^j, y^j, \theta)(r^j; \theta) \exp\left(-\frac{1}{2}(\tilde{\theta} - \tilde{\theta}_*)^T \Gamma_{\theta}^{-1}(\tilde{\theta} - \tilde{\theta}_*)\right)
\]

and the maximum a posteriori (MAP) estimate gets the form

\[
\theta_{\text{MAP}} = \arg \max_{\theta} \pi(\theta | z) \\
= \arg \min_{\theta} \left\{ -\sum_{j=1}^{N} A_j(z_j, \theta) + \frac{1}{2}||L_\theta(\tilde{\theta} - \tilde{\theta}_*)||^2_{L_\theta} \right\}, \quad (12)
\]

where $A_j(z_j, \theta) = \log(f_i(x^j, y^j, \theta)(r^j; \theta))$ and $L_\theta^T L_\theta = \Gamma_{\theta}^{-1}$.

In this paper, the MAP estimate is computed by using a simple random search algorithm, where at each iteration step: First, a new value for the discrete tree species variable, $t_{\text{new}}^n$, corresponding to $n$th tree model is drawn randomly from the uniform distribution $P(t_n) = \frac{1}{4}, \forall t_n \in \{\text{pine, spruce, deciduous, none}\}$. If $t_{\text{new}}^n$ gets the value ‘none’, the $n$th tree is removed from the model, and in other cases, the tree species dependent parameters $a_t, \alpha_t$ and $\beta_t$ are chosen to correspond to $t = t_{\text{new}}^n$. Next, a Gaussian distributed random vector $\xi_n \in \mathbb{R}^4$ is added to the parameters $\tilde{\theta}_n$ corresponding to $n$th tree model. If the functional in Equation (12) decreases, the parameters are changed to $t_{\text{new}}^n$ and $\tilde{\theta}_n + \xi_n$, otherwise $t_n$ and $\tilde{\theta}_n$ remain unchanged. The initial estimate for the iteration is computed by setting an excessive number of tree models into the areas of point clouds, by applying a tree detection algorithm introduced in [7] two times: first to the original ALS data $z$, and subsequently to a reduced data set where the ALS data points within the areas of canopy models fitted in the first round are removed.

### 4. Results and discussion

The proposed computational method was tested with data from two plots. The results from Plot 1 are shown in Figure 7. The figure compares the field measured trees (top row) with the MAP estimates (second row). In bottom left of the figure, both the field measured and estimated tree locations are marked; this illustration reveals that the MAP estimate fails to detect only two of the trees (marked with red circles) and gives three falsely detected trees (three green circles without gray counterpart). Also a fourth tree without field measured counterpart is detected (black circle), but this tree model is on the perimeter of the plot and hence the field measurements have probably neglected the tree; this tree estimate is thus not accounted for in the analysis of the success of the method. To assess the feasibility of detection, we calculated a
success rate (SR) as in [2]:

\[
SR = 100 \cdot \frac{N_{\text{correct}} - N_{\text{false}}}{N_{\text{field}}},
\]

(13)

where \(N_{\text{correct}}\) refers to correctly detected trees, \(N_{\text{false}}\) refers to false positives (estimation gives tree but field measurement does not) and \(N_{\text{field}}\) is the number of field measured trees. For Plot 1, \(SR \approx 88\%\). Figure 7 (bottom right) shows the estimated tree heights vs. field measured heights corresponding to trees in Plot 1. This figure illustrates the feasibility of the tree height estimation; for all detected trees, the height estimates are in good correspondence with the field measured heights. The results corresponding to Plot 2 are shown in Figure 8. Again, trees are detected with a high success rate (\(SR \approx 90\%\)), and the height estimates are feasible.

5. Conclusions

In this paper, we studied the problem of single tree detection using on ALS data. We proposed a Bayesian approach, where the prior and likelihood models were constructed on the basis of training data consisting of ALS and field measurements. The approach was tested using real ALS data and verified with field measured tree locations and sizes. The results demonstrate the feasibility of the approach: In the selected test cases, the trees were detected at high rate, and the tree height estimates were feasible. The Bayesian inversion accompanied with training data-based statistical models holds potential for becoming a computational tool for the analysis of ALS data in single tree level, and allowing for more reliable and versatile information on the forest structure.

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Figure 7. Results for Plot 1. Top row: field measured trees (illustrated as rotationally symmetric tree crown shapes) and ALS data (colored dots). Middle row: MAP estimate for the trees and ALS data. Bottom left: field measured (gray circles) and estimated (green circles) tree locations; the blue plus signs ‘+’ indicate correctly detected trees and red circles ‘◦’ indicate missed trees. The black circle with a cross ‘×’ is an estimated tree near the boundary that is not taken into account when computing the success rate. Bottom right: Estimated tree heights vs. the field measured heights. SR ≈ 88%.
Figure 8. Results for Plot 2. Top row: field measured trees (illustrated as rotationally symmetric tree crown shapes) and ALS data (colored dots). Middle row: MAP estimate for the trees and ALS data. Bottom left: field measured (gray circles) and estimated (green circles) tree locations; the blue plus signs '+' indicate correctly detected trees and red circles 'o' indicate missed trees. The black circles with a cross 'x' are estimated trees near the boundary which are not taken into account when computing the success rate. Bottom right: Estimated tree heights vs. the field measured heights. SR ≈ 90%.