On the General Randić index of polymeric networks modelled by generalized Sierpiński graphs

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Abstract

The General Randić index \(R_\alpha\) of a simple graph \(G\) is defined as

\[
R_\alpha(G) = \sum_{v_i \sim v_j} (\delta_i \delta_j)^\alpha,
\]

where \(\delta_i\) denotes the degree of the vertex \(v_i\). Rodríguez-Velázquez and Tomás-Andreu \[MATCH Commun. Math. Comput. Chem. 74 (1) (2015) 145–160\] obtained closed formulae for the Randić index \(R_{-1/2}\) of Sierpiński-type polymeric networks, where the base graph is a complete graph, a triangle-free \(\delta\)-regular graph or a bipartite \((\delta_1, \delta_2)\)-semirregular graph. In the present article we obtain closed formulae for the general Randić index \(R_\alpha\) of Sierpiński-type polymeric networks, where the base graph is arbitrary.

1 Introduction

Around the middle of the last century theoretical chemists proposed the use of topological indices to obtain information on the dependence of various properties of organic substances on molecular structure. In this sense, a large number of various topological indices was proposed and considered in the chemical literature \[30\]. We highlight the article \[2\] where Camarda and Maranas addressed the design of polymers with optimal levels of macroscopic properties through the use of topological indices. Specifically, in the above mentioned article two zeroth-order and two first-order connectivity indices were employed for the first time as descriptors in structure-property correlations in
an optimization study. Based on these descriptors, a set of new correlations for heat capacity, cohesive energy, glass transition temperature, refractive index, and dielectric constant were proposed.

The molecular structure-descriptor, introduced in 1975 by Milan Randić in [24], is defined as

\[ R(G) = \sum_{v_i, v_j \in E} \frac{1}{\sqrt{d(v_i)d(v_j)}}, \]

where \( d(v_i) \) represents the degree of the vertex \( v_i \) in \( G \). Nowadays, \( R(G) \) is referred to as the Randić index of \( G = (V, E) \). This graph topological index, sometimes referred to as connectivity index, has been successfully related to a variety of physical, chemical, and pharmacological properties of organic molecules and became one of the most popular molecular-structure descriptors [25]. After the publication of the first paper [24], mathematical properties and generalizations of \( R(G) \) were extensively studied, for instance, see [4, 7, 11, 19–23, 26–28, 32] and the references cited therein.

The Randić index was generalized by Gutman and Lepović in [8] as

\[ R_\alpha(G) = \sum_{v_i, v_j \in E} (d(v_i)d(v_j))^\alpha, \; \alpha \neq 0. \]

Obviously, the standard Randić index is obtained when \( \alpha = -1/2 \). In the chemical literature the quantity

\[ R_1(\Gamma) = \sum_{v_i, v_j \in E} d(v_i)d(v_j). \]

is called the second Zagreb index [3].

Some topological indices have been studied also for the case of polymeric networks. For instance, we cite the article [31], where the authors gave the explicitly formula of the \( k \)-connectivity index of an infinite class of dendrimer nanostars.

Over the past three decades, polymer networks has emerged as a coherent subject area [1, 12, 13, 28, 29]. While the basic works on polymer modelling started from linear polymeric systems, in recent years the attention has focused more and more on complex underlying geometries including fractals generalized networks. It is well-known that, in comparison with those linear polymers, the properties of polymer networks depend to a much larger extent on methods and condition of preparation, i.e., properties depend not only on the chemical structure of the individual polymer chains, but on how those chains are joined together to form a network [29]. In this article we consider a model of polymer networks based on generalized Sierpiński graphs, which was previously studied in [28].

To begin with, we need some notation and terminology. Let \( G = (V, E) \) be a non-empty graph of order \( n \) and vertex set \( V = \{1, 2, ..., n\} \). We denote by \( \{1, 2, ..., n\}^t \) the set of words of size \( t \) on alphabet \( \{1, 2, ..., n\} \). The letters of a word \( u \) of length \( t \) are denoted by \( u_1u_2...u_t \). The concatenation of two words \( u \) and \( v \) is denoted by \( uv \). Klavari and Milutinović introduced in [14] the graph \( S(K_n, t) \) whose vertex set is \( \{1, 2, ..., n\}^t \), where \( \{u, v\} \) is an edge if and only if there exists \( i \in \{1, ..., t\} \) such that:

(i) \( u_j = v_j \), if \( j < i \); (ii) \( u_i \neq v_i \); (iii) \( u_j = v_i \) and \( v_j = u_i \) if \( j > i \).
When \( n = 3 \), those graphs are exactly Tower of Hanoi graphs. Later, those graphs have been called Sierpiński graphs in [15] and they were studied by now from numerous points of view. The reader is invited to read, for instance, the following recent papers [5, 9, 10, 15–17] and references therein.

This construction was generalized in [6] for any graph \( G = (V, E) \), by defining the generalized Sierpiński graph, \( S(G, t) \), as the graph with vertex set \( \{1, 2, \ldots, n\}^t \) and edge set defined as follows. \( \{u, v\} \) is an edge if and only if there exists \( i \in \{1, \ldots, t\} \) such that:

1. \( u_j = v_j \), if \( j < i \);
2. \( u_i \neq v_i \) and \( \{u_i, v_i\} \in E \);
3. \( u_j = v_i \) and \( v_j = u_i \), if \( j > i \).

Notice that if \( \{u, v\} \) is an edge of \( S(G, t) \), there is an edge \( \{x, y\} \) of \( G \) and a word \( w \) such that \( u = wxyy\ldots y \) and \( v = wyyx\ldots x \). In general, \( S(G, t) \) can be constructed recursively from \( G \) with the following process: \( S(G, 1) = G \) and, for \( t \geq 2 \), we copy \( n \) times \( S(G, t - 1) \) and add the letter \( x \) at the beginning of each label of the vertices belonging to the copy of \( S(G, t - 1) \) corresponding to \( x \). Then for every edge \( \{x, y\} \) of \( G \), add an edge between vertex \( xyy\ldots y \) and vertex \( yxx\ldots x \). See, for instance, Figure 1 that shows a graph \( G \) and the Sierpiński graph \( S(G, 2) \). Besides Figure 2 shows the Sierpiński graph \( S(G, 3) \). Vertices of the form \( xx\ldots x \) are called extreme vertices. Notice that for any graph \( G \) of order \( n \) and any integer \( t \geq 2 \), \( S(G, t) \) has \( n \) extreme vertices and, if \( x \) has degree \( d(x) \) in \( G \), then the extreme vertex \( xx\ldots x \) of \( S(G, t) \) also has degree \( d(x) \). Moreover, the degrees of two vertices \( yxx\ldots x \) and \( xyy\ldots y \), which connect two copies of \( S(G, t - 1) \), are equal to \( d(x) + 1 \) and \( d(y) + 1 \), respectively.

![Figure 1: A graph G and the Sierpiński graph S(G, 2).](image)

We denote by \( P_r \) the path graph of order \( r \). Notice that for \( G = K_2 \) we obtain \( S(K_2, 2) = P_4 \) and, in general, \( S(K_2, t) = P_{2^t} \), which is the simplest possible polymer model presented by the ideal chain. Also, the graphs \( S(K_n, t) \) were used in [1, 12, 13] to analyse the scaling behaviour of experimentally accessible dynamical relaxation forms for polymers modelled through finite Sierpiński-type graphs, which we denote here by \( P(K_n, t) \). Using the approach developed in [12, 13] to construct \( P(K_n, t) \), now we
define the polymeric Sierpiński graphs $P(G, t) = (V^*, E^*)$ introduced in [28], where $G$ is a connected graph of order $n$ and $t$ is a positive integer. For $i \in \{1, ..., t\}$ we define the sets $A_i = \{a_{i_1}, ..., a_{i_{n-1}}\}$ and we denote $S(G, i) = (V_i, E_i)$ and $V_i = \{v_{i_1}, ..., v_{i_{n-1}}\}$. Then, the vertex set of $P(G, t)$ is

$$V^* = \bigcup_{i=1}^{t} (A_i \cup V_i)$$

and the edge set of $P(G, t)$ is

$$E^* = \left( \bigcup_{i=1}^{n} (E_i \cup B_i) \right) \cup \left( \bigcup_{i=1}^{t-1} C_i \right),$$

where $C_i = \{v_{i_j}, a_{i_{j+1}} : j = 1, ..., n_i\}$, $B_i = \bigcup_{j=1}^{i-1} W_j$, and $W_j$ is formed by the edges obtained by connecting $a_{i_j}$ to every vertex belonging to the $j$-th copy of $G$ in

Figure 2: The Sierpiński graph $S(G, 3)$ for the graph $G$ of Figure 1.
In other words, we construct \( P(G, t) \) as follows: The iterative construction starts from one vertex, \( a_1 \), and one copy of \( G = S(G, 1) \). So, we obtain \( P(G, 1) \) by connecting \( a_1 \) to every vertex of \( S(G, 1) \). To obtain \( P(G, 2) \) we take \( P(G, 1), A_2 \) and \( S(G, 2) \). Then we connect each element \( a_{2j} \in A_2 \) to \( v_1 \) and we also connect \( a_{2j} \) to every vertex in the \( j \)-th copy of \( G \) in \( S(G, 2) \). Analogously, for the construction of \( P(G, t) \) we take \( P(G, t - 1), A_t \) and \( S(G, t) \). Then, we connect each element \( a_{tj} \in A_t \) to \( v_1 \) and we also connect \( a_{tj} \) to every vertex in the \( j \)-th copy of \( G \) in \( S(G, t) \). Notice that \( P(K_3, 2) = S(K_4, 2), S(K_3, 2) = P(K_2, 2) \), while for \( t \geq 3 \), \( P(K_n, t) \neq S(K_{n+1}, t) \). Figure 3 shows a sketch of a polymeric Sierpiński graph \( P(G, 2) \).

Figure 3: Sketch of a polymeric Sierpiński graph \( P(G, 2) \), where a small ellipse represents a copy of \( G \) and the dashed lines connecting a vertex \( a_{ik} \) and a small ellipse mean that each vertex of the \( k \)-th copy of \( G \) in \( S(G, 2) \) is connected to \( a_{ik} \).

To the best of our knowledge, [28] is the first published paper studying the generalized Sierpiński graphs. In that article, the authors obtained closed formulae for the Randić index \( R_{1/2} \) of \( S(G, t) \) and \( P(G, t) \), where the base graph \( G \) is a complete graph, a triangle-free \( \delta \)-regular graph or a bipartite \((\delta_1, \delta_2)\)-semirregular graph. The present article is a continuation of [28] where we study the general Randić index \( R_\alpha \) of Sierpiński-type polymeric networks. In particular, we obtain closed formulae for the general Randić index \( R_\alpha \) of \( S(G, t) \) and \( P(G, t) \), where the base graph \( G, t \) and \( \alpha \) are arbitrary.

2 Computing the General Randić index of \( S(G, t) \)

From now on, given a graph \( G = (V, E) \) and an edge \( \{x, y\} \in E \), the number of copies of \( \{x, y\} \) in \( S(G, t) \), where \( x \) has degree \( \deg(x) \in \{d(x), d(x) + 1\} \) and \( y \) has degree
deg(y) ∈ \{d(y), d(y) + 1\} will be denoted by \(f_{S(G,t)}(\deg(x), \deg(y))\). Also, the set of neighbours that \(x \in V\) has in \(G\) will be denoted by \(N(x)\), i.e., \(N(x) = \{z \in V : \{x,z\} \in E\}\).

Given two vertices \(u, v \in V\), the number of triangles of \(G\) containing \(v\) and \(v\) will be denoted by \(\tau(u,v)\) and the number of triangles of \(G\) will be denoted by \(\tau(G)^t\). Note that \(\sum_{\{u,v\} \in E(G)} \tau(u,v) = 3\tau(G)\). The complexity of counting the number of triangles of a graph \(G\) is polynomial regarding to \(|V|\). The trivial approach of counting the number of triangles is to check for every triple \(x, y, z \in V\) if \(x, y, z\) forms a triangle. This procedure gives us the algorithmic complexity of \(O(n^3)\). However, as was shown in [18], this algorithmic complexity can be improved.

Notice that for any pair of adjacent vertices \(u, v \in V\) we have \(|N(u) \cap N(v)| = \tau(u,v)|N(u) \cup N(v)| = |N(u) - N(v)| = d(u) - \tau(u,v)|N(u)| = d(u) - \tau(u,v)\). Given a graph of order \(n\), from now on we will use the function \(\psi(t) = 1 + n + n^2 + \cdots + n^{t-1} = \frac{n^t - 1}{n - 1}\).

**Lemma 1.** For any integer \(t \geq 2\) and any edge \(\{x, y\}\) of a graph \(G\) of order \(n\),

(i) \(f_{S(G,t)}(d(x), d(y)) = n^{t-2}(n - d(x) - d(y) + \tau(x,y))\).
(ii) \(f_{S(G,t)}(d(x), d(y) + 1) = n^{t-2}(d(y) - \tau(x,y)) - \psi(t - 2)d(x)\).
(iii) \(f_{S(G,t)}(d(x) + 1, d(y)) = n^{t-2}(d(x) - \tau(x,y)) - \psi(t - 2)d(y)\).
(iv) \(f_{S(G,t)}(d(x) + 1, d(y) + 1) = n^{t-2}(\tau(x,y) + 1) + \psi(t - 2)(d(x) + d(y) + 1)\).

**Proof.** There are two different possibilities for the degree of any vertex \(zx\) of \(S(G,2)\), namely \(d(x)\) and \(d(x) + 1\), i.e., \(zx\) has degree \(d(x) + 1\) for all \(z \in N(x)\) and \(zx\) has degree \(d(x)\) for all \(z \not\in N(x)\). Therefore,

\[
f_{S(G,2)}(d(x), d(y)) = |V - (N(x) \cup N(y))| = n - d(x) - d(y) + \tau(x,y),
\]
\[
f_{S(G,2)}(d(x), d(y) + 1) = |N(y) - N(x)| = d(y) - \tau(x,y),
\]
\[
f_{S(G,2)}(d(x) + 1, d(y)) = |N(x) - N(y)| = d(x) - \tau(x,y),
\]
\[
f_{S(G,2)}(d(x) + 1, d(y) + 1) = |N(x) \cap N(y)| + 1 = \tau(x,y) + 1
\]

For any \(t \geq 3\), \(w \in V^{t-1}\) and \(z \in V\), the degree of \(zw\) in \(S(G,t)\) coincides with the degree of \(w\) in \(S(G,t - 1)\), except when \(w = j \cdots j\) is an extreme vertex and \(z \in N(j)\), in which case the degree of \(zw = zj \cdots j\) in \(S(G,t)\) is \(d(j) + 1\) while the degree of \(w = j \cdots j\) in \(S(G,t - 1)\) is \(d(j)\). Hence, we deduce the following:

\[
f_{S(G,t)}(d(x), d(y)) = n f_{S(G,t-1)}(d(x), d(y))
\]
\[
= n^{t-2} (n - d(x) - d(y) + \tau(x,y)),
\]

\[1\]A triangle in a graph is a set of three vertices whose induced subgraph is isomorphic to \(K_3\).
\[ f_{S(G,t)}(d(x),d(y)+1) = n f_{S(G,t-1)}(d(x),d(y)+1) - d(x) \]
\[ = n^{t-2} (d(y) - \tau(x,y)) - \sum_{i=0}^{t-3} n^i d(x) \]
\[ = n^{t-2} (d(y) - \tau(x,y)) - \psi(t-2)d(x), \]

\[ f_{S(G,t)}(d(x)+1,d(y)) = n f_{S(G,t-1)}(d(x)+1,d(y)) - d(y) \]
\[ = n^{t-2} (d(x) - \tau(x,y)) - \sum_{i=0}^{t-3} n^i d(y) \]
\[ = n^{t-2} (d(x) - \tau(x,y)) - \psi(t-2)d(y), \]

and finally,
\[ f_{S(G,t)}(d(x)+1,d(y)+1) = n f_{S(G,t-1)}(d(x)+1,d(y)+1) + d(x) + d(y) + 1 \]
\[ = n^{t-2} (\tau(x,y) + 1) + \sum_{i=0}^{t-3} n^i (d(x) + d(y) + 1) \]
\[ = n^{t-2} (\tau(x,y) + 1) + \psi(t-2)(d(x) + d(y) + 1). \]

Therefore, the result follows. \(\square\)

The main result of this section is Theorem 2 which provides a formula for the general Randić index of \(S(G,t)\), where the base graph \(G\) and \(\alpha\) are arbitrary, and \(t\) is an integer greater than one.

**Theorem 2.** For any graph \(G\) of order \(n \geq 2\) and any integer \(t \geq 2\),
\[
R_\alpha(S(G,t)) = \sum_{\{x,y\} \in E} W_{\{x,y\}},
\]
where
\[
W_{\{x,y\}} = n^{t-2} (n - d(x) - d(y) + \tau(x,y)) d(x)^\alpha d(y)^\alpha + \\
+ (n^{t-2} (d(y) - \tau(x,y)) - \psi(t-2)d(x)) d(x)^\alpha (d(y) + 1)^\alpha + \\
+ (n^{t-2} (d(x) - \tau(x,y)) - \psi(t-2)d(y)) (d(x) + 1)^\alpha d(y)^\alpha + \\
+ (n^{t-2} (\tau(x,y) + 1) + \psi(t-2)(d(x) + d(y) + 1)) (d(x) + 1)^\alpha (d(y) + 1)^\alpha.
\]

**Proof.** Since any copy of a vertex \(z \in V\) in \(S(G,t)\) has degree \(d(z)\) or \(d(z) + 1\), the general Randić index of \(S(G,t)\) can be expressed as
\[
R_\alpha(S(G,t)) = \sum_{\{x,y\} \in E} \sum_{i=0}^{1} \sum_{j=0}^{1} f_{S(G,t)}(d(x)+i,d(y)+j) (d(x)+i)^\alpha (d(y)+j)^\alpha.
\]

Hence, by Lemma 1 the result immediately follows. \(\square\)
The remaining results of this section are directly derived from Theorem 2.

**Corollary 3.** For any \( \delta \)-regular graph \( G \) of order \( n \) and any integer \( t \geq 2 \),

\[
R_\alpha(S(G,t)) = \left( \frac{n^{t-1}\delta}{2}(n-2\delta) + 3n^{t-2}\tau(G) \right) \delta^{2\alpha} \\
+ \left( (n^{t-1} + \psi(t-1)) \delta^2 - 6n^{t-2}\tau(G) \right) \delta^\alpha (\delta + 1)^\alpha \\
+ \left( \frac{n\delta}{2}\psi(t-1) + n\delta^2\psi(t-2) + 3n^{t-2}\tau(G) \right) (\delta + 1)^{2\alpha}.
\]

A complete graph \( K_n \) of order \( n \geq 2 \) is \((n-1)\)-regular and it has \( \binom{n}{3} \) triangles. Therefore, the next result follows.

**Remark 4.** For any integers \( t, n \geq 2 \),

\[
R_\alpha(S(K_n,t)) = n^{\alpha+1}(n-1)^{\alpha+1} + \frac{n^{2\alpha+t+1} - 2n^{2(\alpha+1)} + n^{2\alpha+1}}{2}
\]

**Remark 5.** For any integers \( t \geq 2 \) and \( n \geq 4 \),

\[
R_\alpha(S(C_n,t)) = 4^\alpha n^{t-1}(n-4) + 4 \cdot 6^\alpha (n^{t-1} - n\psi(t-2)) + 9^\alpha n (\psi(t-1) + 4\psi(t-2)).
\]

**Corollary 6.** Let \( G = (U_1 \cup U_2, E) \) be a bipartite \((\delta_1, \delta_2)\)-semiregular graph of order \( n = n_1 + n_2 \), where \( |U_1| = n_1 \) and \( |U_2| = n_2 \). Then for any integer \( t \geq 2 \),

\[
R_\alpha(S(G,t)) = n_1n^{t-2}\delta_1^{\alpha+1}\delta_2^\alpha(n - \delta_1 - \delta_2) + n_1\delta_1^{\alpha+1}(\delta_2 + 1)^\alpha\left( \delta_2n^{t-2} - \delta_1\psi(t-2) \right) \\
+ n_2(\delta_1 + 1)^\alpha\delta_2^\alpha\left( \delta_1n^{t-2} - \delta_2\psi(t-2) \right) \\
+ n_1\delta_1(\delta_1 + 1)^\alpha(\delta_2 + 1)^\alpha\left( n^{t-2} + (\delta_1 + \delta_2 + 1)\psi(t-2) \right).
\]

Chemical trees are trees that have no vertex with degree greater than 4. The graph \( S(K_{1,3},2) \) is an example of a chemical tree. Notice that for any \( t \geq 2 \), the Sierpiński graph \( S(K_{1,3},t) \), is a chemical tree.

As a particular case of Corollary 6 we obtain the following result.

**Remark 7.** For any integers \( r, t \geq 2 \),

\[
R_\alpha(S(K_{1,r},t)) = (r + 1)^\alpha ((r + 1)^{t-1}(r - 1) + 1) + 2^\alpha r^{\alpha+1} + \\
+ (2(r + 1))^\alpha (2(r + 1)^{t-1} - r - 2).
\]

**Corollary 8.** Let \( t, n \) be integers such that \( t \geq 2 \) and \( n \geq 3 \). Then

\[
R_\alpha(S(P_2,t)) = 2^{\alpha+1} + (2^t - 3)2^{2\alpha}
\]

and

\[
R_\alpha(S(P_n,t)) = 2^{\alpha}n^{t-2}(n - 3)\left( 2^{\alpha}n - 2^{\alpha+2} + 2 \right) + \\
+ 3^{\alpha} \left[ 2^{\alpha+2}(n - 3)\left( n^{t-2} - \psi(t-2) \right) + 4n^{t-2} - 2\psi(t-2) \right] + \\
+ 2^{2\alpha+1}\left( n^{t-2} - 2\psi(t-2) \right) + \\
+ 3^{\alpha} \left[ 3^{\alpha}(n - 3)\left( n^{t-2} + 5\psi(t-2) \right) + 2^{\alpha+1}\left( n^{t-2} + 4\psi(t-2) \right) \right].
\]
In order to continue presenting our results, we need to introduce a definition. Given a graph $G$ on the vertex set $V$, we define the parameter

$$M_{\alpha}(G) = \sum_{x \in V} \delta(x)^{\alpha}. $$

Note that $M_1(G)$ is equal to twice the number of edges of $G$ and $M_2(G)$ is the first Zagreb index. Considering that for any graph $G$ of maximum degree $\Delta(G)$ and minimum degree $\delta(G)$, and any vertex $x \in V(G)$ we have that $d(x)^\alpha + (\delta(G) + 1)^\alpha - \Delta(G)^\alpha \leq (d(x) + 1)^\alpha \leq d(x)^\alpha + (\Delta(G) + 1)^\alpha - \delta(G)^\alpha$ and replacing in Theorem 2, we deduce the following bounds.

**Theorem 9.** For any triangle free graph $G$ of order $n \geq 2$, maximum degree $\Delta$ and minimum degree $\delta$, and any integer $t \geq 2$,

$$\beta_L(S(G,t)) \leq R_{\alpha}(S(G,t)) \leq \beta_U(S(G,t)),$$

where

$$\beta_L(S(G,t)) = n^{t-2}(n-\Delta)R_{\alpha}(G) + 2 \left(n^{t-2}\Delta - \delta \psi(t-2)\right)(R_{\alpha}(G) + M_{\alpha+1}(G)[(\Delta + 1)^\alpha - \delta^\alpha]) + (n^{t-2} + (2\Delta + 1)\psi(t-2))(R_{\alpha}(G) + 2M_{\alpha+1}(G)[(\Delta + 1)^\alpha - \delta^\alpha]) + (n^{t-2} + (2\delta + 1)\psi(t-2)) \frac{M_1(G)}{2} [(\delta + 1)^\alpha - \Delta^\alpha]^2,$$

and

$$\beta_U(S(G,t)) = n^{t-2}(n-\delta)R_{\alpha}(G) + 2 \left(n^{t-2}\Delta - \delta \psi(t-2)\right)(R_{\alpha}(G) + M_{\alpha+1}(G)[(\Delta + 1)^\alpha - \delta^\alpha]) + (n^{t-2} + (2\Delta + 1)\psi(t-2))(R_{\alpha}(G) + 2M_{\alpha+1}(G)[(\Delta + 1)^\alpha - \delta^\alpha]) + (n^{t-2} + (2\Delta + 1)\psi(t-2)) \frac{M_1(G)}{2} [(\Delta + 1)^\alpha - \delta^\alpha]^2.$$

Moreover, $\beta_L(S(G,t)) = R_{\alpha}(S(G,t)) = \beta_U(S(G,t))$ if and only if $G$ is a triangle free regular graph.

### 3 Computing the General Randić index of $P(G, t)$

The main result of this section is Theorem 14 which provides a formula for the general Randić index of $P(G, t)$, where the base graph $G$ and $\alpha$ are arbitrary, and $t$ is an integer greater than one.

From now on, given a graph $G$ and a vertex $x \in V(G)$, we refer to the degree of $x$ in $G$ as $d_G(x)$. If there is no ambiguity, we will simply write $d(x)$.

**Remark 10.** For any graph $G$ of order $n \geq 2$,

$$R_{\alpha}(P(G,1)) = n^{\alpha} \sum_{x \in V} (d(x) + 1)^\alpha + \sum_{\{x,y\} \in E} (d(x) + 1)^\alpha (d(y) + 1)^\alpha.$$

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Hence, for every vertex \( x \) and as a consequence, \( d_{P(G,1)}(x) = d_G(x) + 1 \). For the remaining edges \( \{x, y\} \) we have that \( d_{P(G,1)}(x) = d_G(x) + 1 \) and \( d_{P(G,1)}(y) = d_G(y) + 1 \). Therefore, the result follows.

**Corollary 11.** For any \( \delta \)-regular graph \( G \) of order \( n \geq 2 \),

\[
R_\alpha(P(G, 1)) = n^{\alpha+1}(\delta + 1)^\alpha + \frac{n\delta(\delta + 1)^{2\alpha}}{2}.
\]

In particular, for complete graphs,

\[
R_\alpha(P(K_n, 1)) = \frac{n^{2\alpha+1}(n + 1)}{2}.
\]

**Corollary 12.** Let \( G = (U_1 \cup U_2, E) \) be a bipartite \((\delta_1, \delta_2)\)-semiregular graph of order \( n = n_1 + n_2 \), where \( |U_1| = n_1 \) and \( |U_2| = n_2 \). Then,

\[
R_\alpha(P(G, 1)) = n^{\alpha}(n_1(\delta_1 + 1)^\alpha + n_2(\delta_2 + 1)^\alpha) + n_1\delta_1((\delta_1 + 1)(\delta_2 + 1))^\alpha.
\]

Given a graph \( G \) and a vertex \( x \in V \), the number of copies of \( x \) having degree \( \deg(x) \in \{d(x), d(x) + 1\} \) in \( S(G, t) \) will be denoted by \( g_{S(G,t)}(\deg(x)) \).

**Lemma 13.** For any graph \( G \) of order \( n \), any vertex \( x \in V \) and any integer \( t \geq 2 \),

(i) \( g_{S(G,t)}(d(x)) = n^{t-1} - d(x)\psi(t - 1) \).

(ii) \( g_{S(G,t)}(d(x) + 1) = d(x)\psi(t - 1) \).

**Proof.** There are two different possibilities for the degree of any copy of vertex \( x \) in \( S(G, t) \), namely \( d(x) \) and \( d(x) + 1 \). Every copy of vertex \( x \) having degree \( d(x) + 1 \) in \( S(G, t) \) is a vertex of the form \( z_1 \cdots z_kyx \cdots x \), where \( y \in N(x) \), \( 0 \leq k \leq t - 2 \) and \( z_i \in V(G) \) for \( 1 \leq i \leq k \). So,

\[
g_{S(G,t)}(d(x) + 1) = d(x)\sum_{i=0}^{t-2} n^i = d(x)\psi(t - 1).
\]

On the other hand, every copy of vertex \( x \) in \( S(G, t) \) is a vertex of the form \( z_1 \cdots z_{t-1}x \), where \( z_i \in V(G) \) for \( 1 \leq i \leq t - 1 \). Hence, we have \( n^{t-1} \) copies of vertex \( x \) in \( S(G, t) \), and as a consequence,

\[
g_{S(G,t)}(d(x)) = n^{t-1} - g_{S(G,t)}(d(x) + 1) = n^{t-1} - d(x)\psi(t - 1).
\]

Now, we propose a formula for the general Randić of \( P(G, t) \), where the base graph \( G \) and \( \alpha \) are arbitrary, and \( t \) is an integer greater than one.
Theorem 14. For any graph $G = (V, E)$ of order $n \geq 2$ and any integer $t \geq 2$,

$$R_\alpha(P(G, t)) = \sum_{i=1}^{7} \beta_i,$$

where

$$\beta_1 = n^\alpha \sum_{x \in V} (d(x) + 2)^\alpha, \quad \beta_2 = \sum_{(x, y) \in E} (d(x) + 2)^\alpha (d(y) + 2)^\alpha,$$

$$\beta_3 = (n+1)^\alpha n \psi(t-2) \sum_{x \in V} (d(x) + 2)^\alpha +$$

$$+ \frac{(n+1)^\alpha (t-2 - n \psi(t-2))}{(n-1)} \sum_{x \in V} d(x)(d(x) + 2)^\alpha +$$

$$+ \frac{(n+1)^\alpha (t-2 - n \psi(t-2))}{(1-n)} \sum_{x \in V} d(x)(d(x) + 3)^\alpha,$$

$$\beta_4 = \sum_{\{x, y\} \in E} W_{4\{x,y\}} \text{ with }$$

$$W_{4\{x,y\}} = (d(x) + 2)^\alpha (d(y) + 2)^\alpha (n - d(x) - d(y) + \tau(x, y)) \psi(t-2) +$$

$$+ (d(x) + 2)^\alpha (d(y) + 3)^\alpha (d(y) - \tau(x, y)) \psi(t-2) + d(x) \frac{t-2 - \psi(t-2)}{n-1} +$$

$$+ (d(x) + 3)^\alpha (d(y) + 2)^\alpha (d(x) - \tau(x, y)) \psi(t-2) + d(y) \frac{t-2 - \psi(t-2)}{n-1} +$$

$$+ (d(x) + 3)^\alpha (d(y) + 3)^\alpha (\tau(x, y) + 1) \psi(t-2) +$$

$$+ (d(x) + 3)^\alpha (d(y) + 3)^\alpha (d(x) + d(y) + 1) \frac{t-2 - \psi(t-2)}{1-n},$$

$$\beta_5 = (n+1)^\alpha \psi(t-1) \sum_{x \in V} (d(x) + 2)^\alpha +$$

$$+ \frac{(n+1)^\alpha (t-1 - \psi(t-1))}{(n-1)} \sum_{x \in V} d(x)(d(x) + 2)^\alpha +$$

$$+ \frac{(n+1)^\alpha (t-1 - \psi(t-1))}{(1-n)} \sum_{x \in V} d(x)(d(x) + 3)^\alpha,$$

$$\beta_6 = (n+1)^\alpha \sum_{x \in V} (d(x) + 1)^\alpha (n^{t-1} - d(x) \psi(t-1)) +$$

$$+ (n+1)^\alpha \psi(t-1) \sum_{x \in V} d(x)(d(x) + 2)^\alpha,$$
and $\beta_7 = \sum_{\{x,y\} \in E} W_{7 \{x,y\}}$ with

$$W_{7 \{x,y\}} = (d(x) + 1)^\alpha (d(y) + 1)^\alpha n^{i-2} (n - d(x) - d(y) + \tau(x,y)) +$$

$$+ (d(x) + 1)^\alpha (d(y) + 2)^\alpha (n^{i-2} (d(y) - \tau(x,y)) - \psi(t - 2)d(x)) +$$

$$+ (d(x) + 2)^\alpha (d(y) + 1)^\alpha (n^{i-2} (d(x) - \tau(x,y)) - \psi(t - 2)d(y)) +$$

$$+ (d(x) + 2)^\alpha (d(y) + 2)^\alpha (n^{i-2} (\tau(x,y) + 1) + \psi(t - 2) (d(x) + d(y) + 1)).$$

**Proof.** Let $d(u), d(v)$ be degrees of $u, v$ in $P(G, t)$, respectively. We differentiate the following cases for any edge $\{u, v\}$ of $P(G, t)$.

1. $u = a_{11}$ and $v \in V_1$. In this case, there are $n$ edges $\{u, v\}$ with $d(u) = n$ and $d_{P(G, t)}(v) = d_G(v) + 2$. Then the contribution of these edges to the General Randić index is equal to $\beta_1$.

2. $u, v \in V_1$. In this case, each edge $\{u, v\}$ has $d_{P(G, t)}(u) = d_G(u) + 2$ and $d_{P(G, t)}(v) = d_G(v) + 2$. So, the contribution of these edges to the General Randić index is equal to $\beta_2$.

3. $u \in A_i$ and $v \in V_i$ for $2 \leq i \leq t - 1$. We assume that $v = wx$, where $w \in V_{i-1}$ and $x \in V$. In this case $d(u) = n + 1$ and, by Lemma 13, there are $g_{S_{G, i}}(d(x) + 1) = d(x)\psi(i - 1)$ edges $\{u, v\}$ where $v$ has degree $d(v) = d(x) + 3$ and there are $g_{S_{G, i}}(d(x)) = n^{i-1} - d(x)\psi(i - 1)$ edges $\{u, v\}$ where $d(v) = d(x) + 2$. Thus, the contribution of these edges to the General Randić index is equal to $\sum_{i=2}^{t-1} \sum_{x \in V} W'_{3 \{x\}}$, where,

$$W'_{3 \{x\}} = (n + 1)^\alpha \sum_{i=0}^{1} (d(x) + i + 2)^\alpha g_{S_{G, i}}(d(x) + i).$$

Since $\sum_{i=2}^{t-1} n^{i-1} = n\psi(t - 2)$ and $\sum_{i=2}^{t-1} \psi(i - 1) = \frac{t - 2 - n\psi(t - 2)}{1 - n}$, we obtain

$$\beta_3 = \sum_{i=2}^{t-1} \sum_{x \in V} W'_{3 \{x\}}.$$

4. $u, v \in V_i$, for $2 \leq i \leq t - 1$. We assume that $u = wx$ and $v = w'y$, where $w, w' \in V_{i-1}$ and $\{x, y\} \in E$. By Lemma 1, there are

$$f_{S_{G, i}}(d(x), d(y)) = n^{i-2} (n - d(x) - d(y) + \tau(x,y))$$

edges $\{u, v\}$ where $d(u) = d(x) + 2$ and $d(v) = d(y) + 2$,

$$f_{S_{G, i}}(d(x), d(y) + 1) = n^{i-2} (d(y) - \tau(x,y)) - \psi(i - 2)d(x)$$

edges $\{u, v\}$ where $d(u) = d(x) + 2$ and $d(v) = d(y) + 3$,

$$f_{S_{G, i}}(d(x) + 1, d(y)) = n^{i-2} (d(x) - \tau(x,y)) - \psi(i - 2)d(y)$$

edges $\{u, v\}$ where $d(u) = d(x) + 3$ and $d(v) = d(y) + 2$. 

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\( f_{S(G,i)}(d(x) + 1, d(y) + 1) = n^{i-2} \left( \tau(x, y) + 1 \right) + \psi(i-2) (d(x) + d(y) + 1) \) edges \{u, v\} where \( d(u) = d(x) + 3 \) and \( d(v) = d(y) + 3 \).

Hence, the contribution of these edges to the General Randić index is equal to
\[
\sum_{i=2}^{t-1} \sum_{\{x,y\} \in E} W'_{4\{x,y\}}, \quad \text{where,}
\]
\[
W'_{4\{x,y\}} = \sum_{i=0}^{1} \sum_{j=0}^{1} (d(x) + i + 2)^{\alpha} (d(y) + j + 2)^{\alpha} f_{S(G,i)}(d(x) + i, d(y) + j).
\]

Since \( \sum_{i=2}^{t-1} n^{i-2} = \psi(t-2) \) and \( \sum_{i=2}^{t-1} \psi(i-2) = \frac{t - 2 - \psi(t-2)}{1 - n} \), we obtain \( \beta_4 = \sum_{i=2}^{t-1} \sum_{\{x,y\} \in E} W'_{4\{x,y\}} = \sum_{\{x,y\} \in E} W_{4\{x,y\}} \).

5. \( u \in A_{i+1} \) and \( v \in V_i \) for \( 1 \leq i \leq t-1 \). We assume that \( v = wx \), where \( w \in V^{i-1} \) and \( x \in V \). In this case \( d(u) = n + 1 \) and, by Lemma 13, there are \( g_{S(G,0)}(d(x) + 1) = d(x) \psi(i-1) \) edges \( \{u, v\} \) where \( v \) has degree \( d(v) = d(x) + 3 \) and there are \( g_{S(G,i)}(d(x)) = n^{i-1} - d(x) \psi(i-1) \) edges \( \{u, v\} \) where \( d(v) = d(x) + 2 \). Hence, the contribution of these edges to the General Randić index is equal to \( \sum_{i=2}^{t-1} \sum_{x \in V} W'_{5\{x\}}, \quad \text{where,}
\]
\[
W'_{5\{x\}} = (n + 1)^\alpha \sum_{i=0}^{1} (d(x) + i + 2)^{\alpha} g_{S(G,i)}(d(x) + i).
\]

Since \( \sum_{i=1}^{t-1} n^{i-1} = \psi(t-1) \) and \( \sum_{i=1}^{t-1} \psi(i-1) = \frac{t - 1 - n \psi(t-1)}{1 - n} \), we obtain \( \beta_5 = \sum_{i=2}^{t-1} \sum_{x \in V} W'_{5\{x\}} \).

6. \( u \in A_t \) and \( v \in V_t \). We assume that \( v = wx \), where \( w \in V^{t-1} \) and \( x \in V \). As above \( d(u) = n + 1 \) and, by Lemma 13, there are \( g_{S(G,1)}(d(x) + 1) = d(x) \psi(t-1) \) edges \( \{u, v\} \) where \( v \) has degree \( d(v) = d(x) + 2 \) and there are \( g_{S(G,i)}(d(x)) = n^{t-1} - d(x) \psi(t-1) \) edges \( \{u, v\} \) where \( d(v) = d(x) + 1 \). Thus, the contribution of these edges to the General Randić index is equal to \( \beta_6 \).

7. \( u, v \in V_t \). We assume that \( u = wx \) and \( v = w'y \), where \( w, w' \in V^{t-1} \) and \( x, y \in V \). By Lemma 1, there are
\[
f_{S(G,i)}(d(x), d(y)) = n^{i-2} (n - d(x) - d(y) + \tau(x, y)) \) edges \( \{u, v\} \) where \( d(u) = d(x) + 1 \) and \( d(v) = d(y) + 1 \),
\[ f_{S(G,t)}(d(x),d(y)+1) = n^{t-2}(d(y) - \tau(x,y)) - \psi(t-2)d(x) \text{ edges } \{u,v\} \text{ where } \]
\[ d(u) = d(x) + 1 \text{ and } d(v) = d(y) + 2, \]
\[ f_{S(G,t)}(d(x)+1,d(y)) = n^{t-2}(d(x) - \tau(x,y)) - \psi(t-2)d(y) \text{ edges } \{u,v\} \text{ where } \]
\[ d(u) = d(x) + 2 \text{ and } d(v) = d(y) + 1, \]
\[ f_{S(G,t)}(d(x)+1,d(y)+1) = n^{t-2}(\tau(x,y)+1)+\psi(t-2)(d(x)+d(y)+1) \text{ edges } \]
\[ \{u,v\} \text{ where } d(u) = d(x) + 2 \text{ and } d(v) = d(y) + 2. \]

Hence, the contribution of these edges to the General Randić index is equal to
\[ \beta_7 = \sum_{\{x,y\} \in E} W_{7\{x,y\}}. \]

According to the seven cases above, the result follows. \(\square\)

Now, we will show some particular cases of Theorem 14.

**Corollary 15.** For any \(\delta\)-regular graph \(G\) of order \(n \geq 2\) and any integer \(t \geq 2\),
\[ R_\alpha(P(G,t)) = \sum_{i=1}^{7} \beta_i, \]
where
\[ \beta_1 = n^{\alpha+1}(\delta + 2)\alpha, \quad \beta_2 = \frac{n\delta(\delta + 2)^2\alpha}{2}, \]
\[ \beta_3 = n^2\psi(t-2)(n+1)^\alpha(\delta + 2)\alpha + \]
\[ + \frac{n\delta(n+1)^\alpha(\delta + 2)\alpha(t-2 - n\psi(t-2))}{(n-1)} + \]
\[ + \frac{n\delta(n+1)^\alpha(\delta + 3)\alpha(t-2 - n\psi(t-2))}{(1-n)}, \]
\[ \beta_4 = (\delta + 2)^2\alpha \psi(t-2) \left( \frac{n\delta(n-2\delta)}{2} + 3\tau(G) \right) + \]
\[ + (\delta + 2)^\alpha(\delta + 3)^\alpha \left( n\delta^2 - 6\tau(G) \right) \psi(t-2) + \frac{n\delta^2(t-2 - \psi(t-2))}{n-1} + \]
\[ + (\delta + 3)^{2\alpha} \left( 3\tau(G) + n\delta \right) \psi(t-2) + \frac{n\delta(2\delta + 1)(t-2 - \psi(t-2))}{2(1-n)}, \]
\[ \beta_5 = \frac{n(n+1)\alpha(\delta + 2)\alpha(\delta(t-1) + \psi(t-1)(n - \delta - 1))}{(n-1)} + \]
\[ + \frac{n\delta(n+1)^\alpha(\delta + 3)\alpha(t-1 - \psi(t-1))}{(1-n)}. \]
\[ \beta_6 = (n+1)^\alpha (\delta+1)^\alpha (n^t - n\delta \psi(t-1)) + n\delta(n+1)^\alpha (\delta + 2)^\alpha \psi(t-1), \]

and

\[ \beta_7 = (\delta+1)^{2\alpha} n^{t-2} \left( \frac{n\delta}{2} (n-2\delta) + 3\tau(G) \right) + \\
+ (\delta + 1)^\alpha (\delta + 2)^\alpha \left( \delta^2 (n^{t-1} - n\psi(t-2)) - 6n^{t-2}\tau(G) \right) + \\
+ (\delta + 2)^{2\alpha} \left( \frac{n\delta}{2} \psi(t-1) + n\delta^2 \psi(t-2) + 3n^{t-2}\tau(G) \right). \]

As we mentioned above, a complete graph \( K_n \) of order \( n \geq 2 \) is \((n-1)\)-regular and it has \( \binom{n}{3} \) triangles. Therefore, the next result is deduced by Theorem 15.

**Corollary 16.** For any integers \( n, t \geq 2 \),

\[ R_\alpha(P(K_n, t)) = \sum_{l=1}^{7} \beta_l, \]

where

\[ \beta_1 = n^{\alpha+1} (n+1)^\alpha, \quad \beta_2 = \frac{n(n-1)(n+1)^{2\alpha}}{2}, \]
\[ \beta_3 = n(t-2)(n+1)^{2\alpha} + n(n+1)^\alpha (n+2)^\alpha (2 + n\psi(t-2) - t), \]
\[ \beta_4 = (t-2)n(n-1)(n+1)^\alpha (n+2)^\alpha + \frac{(n+2)^{2\alpha}}{2} \left(n^3 \psi(t-2) + (t-2)(n-2n^2)\right), \]
\[ \beta_5 = (t-1)n(n+1)^{2\alpha} + n(n+1)^\alpha (n+2)^\alpha (\psi(t-1) - (t-1)), \]
\[ \beta_6 = n^{\alpha+1} (n+1)^\alpha + (n^t - n)(n+1)^{2\alpha}, \]
\[ \beta_7 = (n-1)n^{\alpha+1} (n+1)^\alpha + \frac{(n^{t+1} - 2n^2 + n)(n+1)^{2\alpha}}{2}. \]

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