Masses of open charm and bottom tetraquark states in a relativized quark model

Qi-Fang Liu\textsuperscript{1} and Yu-Bing Dong\textsuperscript{1,2,*}

\textsuperscript{1}Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
\textsuperscript{2}Theoretical Physics Center for Science Facilities (TPCSF), CAS, Beijing 100049, China

We study the masses of open charm and bottom tetraquark states within the diquark-antidiquark scenario in the relativized quark model proposed by Godfrey and Isgur. The diquark and antidiquark masses are firstly solved by relativized quark potential, and then treated as the usual antiquark and quark, respectively. The masses of tetraquark states are obtained by solving the Schrödinger-type equation between the new diquark and antidiquark. We find the masses of $q\bar{q}q\bar{q}$ tetraquark configuration are much higher than that of $X(5568)$. This conclusion disfavors the possibility of $X(5568)$ as a tetraquark state within the diquark-antidiquark scenario. Further experimental searches are needed to clarify the nature of the signal observed by D0 collaboration.

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I. INTRODUCTION

Recently, the D0 Collaboration reported the evidence of a narrow structure $X(5568)$ in the $X(5568) \to B_s^0\pi^+$ decay process based on the $pp$ collision data at $\sqrt{s} = 1.96$ TeV [1]. Given the final states, the $X(5568)^+$ should have four different quark flavors $sud\bar{d}$. Its mass and width are $5567.8 \pm 2.9^{+0.7}_{-1.9}$ MeV and $21.9 \pm 6.4^{+2.0}_{-2.5}$ MeV, respectively. Assuming the final $B_s^0\pi^+$ in $S$ wave, the quantum number is $I(J^P) = 1(0^+)$. There is the structure decays through the chains of $B_s^0\pi^+$, $B^0\gamma$, where the soft photon is not detected. In the later situation, the quantum number of the new state would be $I(J^P) = 1(1^+)$, and the mass is shifted by adding the mass difference $m(B_s^0) - m(B_s^0) = 48.6^{+1.8}_{-1.6}$ MeV. Whereafter, the LHCb Collaboration analysed the $B_s^0\pi^+$ invariant mass distribution of $pp$ collision data at $\sqrt{s} = 7$ and 8 TeV, however, they found no significant excess corresponding to the claimed $X(5568)$ state [2]. Also, the CMS Collaboration failed to observe the $X(5568)$ structure in the $B_s^0\pi^+$ invariant mass distribution [3].

The experimental efforts have immediately attracted great interests and many theoretical studies on the $X(5568)$ with different interpretations. Considering the mass, production, strong decay, and decay constant, many investigations regard the $X(5568)$ as a tetraquark state within the framework of QCD sum rule and simple quark model [4,20], which explanation is suggested by D0 Collaboration. The structures under $B_s\pi$, $B\bar{K}$ and $B^*\bar{K}$ molecular pictures, dynamically generated states, and hybridized tetraquarks are also proposed [21–30]. Moreover, some works discuss the charmed partners of $X(5568)$ [31–33]. In addition, the non-resonance interpretations of the $X(5568)$ structure also exist. In Refs. [34,35], the authors suggest that $X(5568)$ may be resulted from the near threshold kinematic effects. The quite large production rate of $X(5568)$ cannot be understood by the general hadronization mechanism [36]. It should be mentioned that the comprehensive discussions and reviews on $X(5568)$ are performed in Refs. [37–39].

Before the observation of $X(5568)$, there have been some studies on open charm and bottom tetraquark states, which mainly focus on their mass spectra [40,51]. Those calculations include diquark-antidiquark picture, compact tetraquark, mixture of quark-antiquark and four quark components, and molecular scenario, which intends to reveal the nature of $D_s^*(2317)$. Some calculations indicate the $D_s^*(2317)$ can be treated as a tetraquark state [40,41], while others give much higher masses of the four quark components than that of $D_s^*(2317)$ as well as the $DK$ threshold [42–46]. For the molecular scenario, most works indicate that a weakly bounded $DK$ state can be obtained [47,49], while others suggest the attachment between these two pseudoscalar mesons is not strong enough to form bound state [50]. In the open bottom sector, higher masses are given in diquark-antidiquark pictures [46], and it is found that the $BK$ system can be weakly bounded [48,51].

Unlike the $D_s^*(2317)$, the $X(5568)$ cannot be regarded as a conventional meson or the mixture of quark-antiquark and four quark components due to its four distinct quark flavors. Most of the molecular and dynamically generated states interpretations cannot give the right mass and therefore can be excluded [22,28,37,47,48,51]. However, the authors argue that the $BK$ and $B\pi$ interactions can generate the $X(5568)$ dynamically when the next-leading order Lagrangian is included [29]. The tetraquark explanation is supported by QCD sum rule and simple quark model [5–15], but disfavored by the relativistic calculation and general discussions [37,38,46]. Hence, it is natural to study the open charm and bottom tetraquark masses within a more realistic potential model, which is helpful to disentangle this conflict.

In this work, we apply the relativized quark model to calculate the masses of diquark and tetraquark states. The relativized quark model, proposed by Godfrey and Isgur, has been extensively used to predict the properties of the conventional mesons [52,60]. It has been concluded that this model gives a unified description of the light mesons, heavy-light mesons and heavy quarkonium, and therefore, it is suitable to deal with the $X(5568)$ state, in which both light-light and heavy-light systems are included. Moreover, the relativistic effects...
are also considered in the model, which may be essential for the light quarks. We perform a calculation in the diquark-antidiquark picture following the route proposed by Ebert, Faustov, and Galkin [44, 61, 67]. The corresponding diquark and antidiquark masses are estimated with the relativized potential firstly, and then treated as the usual antiquark and quark, respectively. The masses of the tetraquark states are, therefore, obtained by solving the Schrödinger-type equation between the diquark and antidiquark. In Ref. [68], Capstick and Isgur adopted the same relativized potential to evaluate the baryon spectra within three body calculations. The computational schedule is very complicated, and can be hardly extended to study the four quark systems. We prefer to use the diquark-antidiquark picture to estimate the open charm and bottom tetraquark states in the present work. We find that our results give much higher masses of $sqb$ tetraquark configuration. It disfavors the assumption of $X(5568)$ as a tetraquark state within the diquark-antidiquark scenario.

This paper is organized as follows. The relativized quark model is briefly introduced and the masses of diquarks are calculated in Sec. II. The masses of tetraquark states and discussions are presented in Sec. III. Finally, we give a short summary in the last section.

II. MASSES OF DIQUARKS

The Hamiltonian between quark and antiquark in the relativized quark model can be expressed as

$$
\hat{H} = H_0 + \hat{V}(p, r),
$$

(1)

$$
H_0 = (p^2 + m_0^2)^{1/2} + (p^2 + m_0^2)^{1/2},
$$

(2)

$$
\hat{V}(p, r) = \hat{H}_{12}^{conf} + \hat{H}_{12}^{cont} + \hat{H}_{12}^{cont} + \hat{H}_{12}^{so},
$$

(3)

where the $\hat{H}_{12}^{conf}$ includes the spin-independent linear confinement and Coulomb-like interaction, the $\hat{H}_{12}^{cont}$ is the color contact term, the $\hat{H}_{12}^{cont}$ is the color tensor interaction, and $\hat{H}_{12}^{so}$ is the spin-orbit term. $\hat{H}$ denotes an operator that has taken account of the relativistic effects according to the relativized scheme. The explicit forms of these interactions and the details of this relativization procedure can be found in Ref. [52]. For the quark-quark interaction in a diquark, the relation $V_{qq}(p, r) = \hat{V}_{qq}(p, r)/2$ is employed since we only consider the 3 type diquark in color space. All the model parameters used in our calculations are taken from Ref. [52]. It should be emphasized that these parameters can describe the low lying meson and baryon spectra well even comparing with the recently experimental data. While for the higher excited states, since the discussions of their assignments do not agree with each other theoretically, and they can be hardly employed for fitting the new parameters. Since only the ground states of the tetraquarks are calculated in the present work, where the parameters about the spin-orbit and tensor forces are not employed, we believe that it is enough to describe the ground tetraquark states with the original parameters.

For the diquark, the $qq$ locates in $S$ wave. The spin-parities of the diquark are $J^P = 0^+$ and $J^P = 1^+$, named as the scalar diquark and axial diquark, respectively. We use the Gaussian expansion method to solve the Hamiltonian (1) with $\hat{V}_{qq}(p, r)$ potential [69]. The obtained masses of these diquarks are listed in Table 1.

III. MASSES OF TETRAQUARK STATES

Table 1: The masses of the scalar and axial vector diquarks. $S$ and $A$ denote the scalar and axial vector diquarks, respectively. The bracket and brace correspond to symmetric and antisymmetric quark contents in flavor, respectively.

| Quark content | Diquark type | Mass(MeV) |
|---------------|-------------|-----------|
| [u, d]        | S           | 691       |
| [u, d]        | A           | 840       |
| [u, s]        | S           | 886       |
| [u, s]        | A           | 992       |
| [s, s]        | A           | 1135      |
| [c, q]        | S           | 2099      |
| [c, q]        | A           | 2138      |
| [c, s]        | S           | 2230      |
| [c, s]        | A           | 2264      |
| [b, q]        | S           | 5451      |
| [b, q]        | A           | 5465      |
| [b, s]        | S           | 5572      |
| [b, s]        | A           | 5585      |
in which the $J^P = 0^+ \ S \ S$ type is the lowest one. If the spin-spin interaction is treated perturbatively, it can provide the fine splitting with coefficients of -2, -1, and 2 for the $0^+$, $1^+$, and $2^+$ $A\bar{A}$ states, respectively, and no fine structure exists for the $S \ S$ tetraquark. Although the mass of $A$ type diquark is higher than that of $S$ type, the larger fine splitting induced by the spin-spin interaction can reduce $J^P = 0^+ \ A \bar{A}$ state to be the lowest one. The lowest mass of $J^P = 0^+ \ sqbq$ state, in our calculation, is 6150 MeV, which is much larger than the mass of $X(5568)$ state. The lowest mass of $J^P = 1^+ \ sqbq$ state is 6210 MeV, which is also larger than $m(X(5568)) + m(B_1^\pm) - m(B_0)$. We see that for both the $0^+$ and $1^+$ cases, the predicted masses are much larger than the experimental data. Hence, our calculated results disfavor the possibility of $X(5568)$ as a tetraquark state within the diquark-antidiquark scenario.

We can also obtain a rather low tetraquark mass of 5672 MeV.

When the finite size of diquark is considered, the one gluon exchange interaction between the diquark and antidiquark becomes weaker as well as the spin-spin interaction. The masses of the tetraquarks will increase, while the fine splitting becomes smaller. This situation is the same as the case adopted by Ebert, Faustov, and Galkin, where the mass of $J^P = 0^+ \ sqbq$ type tetraquark is the lowest and the fine splitting is small \[46]. In the $F(r) = 0$ limit, only the linear confining interaction remains, and the three $A\bar{A}$ type tetraquark states degenerate. Of course, the $X(5568)$ cannot be described as a tetraquark state even the finite size and form factor of the diquark are taken into account.

In present work, we predict many open charm and bottom tetraquark states within the diquark-antidiquark scenario by solving the Schrödinger-type equation. It should be noted that only the mass spectra cannot ensure the existences of these states, and their production mechanisms and decay behaviors should also be investigated simultaneously. The strong decay behaviors are more essential, since the much broader structures cannot form or be detected. In fact, we predict the lowest $sqbq$ state is 6150 MeV, which is much higher than the $B_\pi$ and $B K$ thresholds. Due to the large phase space, the predicted tetraquark states may fall apart immediately. Further studies on these tetraquark states are needed both theoretically and

![FIG. 1: The predicted mass spectrum of the $sqbq$ tetraquarks.](image)

| $J^P$ | Diquark content | Open charm (MeV) | Open bottom (MeV) |
|-------|----------------|----------------|------------------|
| $0^+$ | $SS$           | 2729           | 6063             |
| $1^+$ | $SA$           | 2838           | 6077             |
| $1^+$ | $AS$           | 2767           | 6164             |
| $0^+$ | $A\bar{A}$     | 2575           | 6046             |
| $1^+$ | $A\bar{A}$     | 2747           | 6118             |
| $2^+$ | $A\bar{A}$     | 2969           | 6226             |
| $0^+$ | $csbq$         | 2873           | 6196             |
| $1^+$ | $cA$           | 2957           | 6210             |
| $1^+$ | $A\bar{A}$     | 2911           | 6274             |
| $0^+$ | $A\bar{A}$     | 2692           | 6150             |
| $1^+$ | $A\bar{A}$     | 2866           | 6226             |
| $2^+$ | $A\bar{A}$     | 3087           | 6337             |

TABLE II: The masses of tetraquark states with diquark-antidiquark in ground $1^S$ state. A dash denotes this state do not exist.

is well consistent with the 6150 MeV ($A\bar{A}$ case) in our present calculation.
experimentally.

IV. SUMMARY

In this work, we study the masses of open charm and bottom tetraquark states in the diquark-antidiquark pictures using the relativized quark model proposed by Godfrey and Isgur. The diquark and antidiquark masses are obtained with the relativized potential, which is the half of $q\bar{q}$ interaction. Then, the diquark and antidiquark are regarded as the usual antiquark and quark, respectively. The form factor, simulating the diquark (antiquark) internal structure, is neglected in our calculations. This assumption means the diquark is treated as point-like or the distance between diquark and antidiquark is large enough.

The masses of the tetraquark states are obtained by solving the Schrödinger-type equation between diquark and antidiquark. We find the masses of $s\bar{d}b\bar{q}$ tetraquark configuration are much higher than that of $X(5568)$, which disfavors the possibility of $X(5568)$ as a tetraquark state within the diquark-antidiquark scenario. The effects induced by the form factor and the finite size of the diquark are qualitatively analyzed. We expect that further experimental information can reveal the nature of the signal observed by D0 collaboration.

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[1] V. M. Abazov et al. (D0 Collaboration), Evidence for a $B^0\pi^+$ state, Phys. Rev. Lett. 117, 022003 (2016).
[2] R. Aaij et al. (LHCb Collaboration), Search for structure in the $B^{0}\pi^+$ invariant mass spectrum, Phys. Rev. Lett. 117, 152003 (2016).
[3] CMS Collaboration (CMS Collaboration), Search for the $X(5568)$ state in $B^+_s\pi^-$ decays, CMS-PAS-BPH-16-002.
[4] R. F. Lebed and A. D. Polosa, $\chi_{c6}(3915)$ As the Lightest $c\bar{c}ss$ State, Phys. Rev. D 93, 094024 (2016).
[5] S. S. Agaev, K. Azizi and H. Sundu, Mass and decay constant of the newly observed exotic $X(5568)$ state, Phys. Rev. D 93, 074024 (2016).
[6] W. Wang and R. Zhu, Can $X(5568)$ be a tetraquark state?, Chin. Phys. C 40, 093101 (2016).
[7] Z. G. Wang, Analysis of the $X(5568)$ as scalar tetraquark state in the diquark-antidiquark model with QCD sum rules, Commun. Theor. Phys. 66, 335 (2016).
[8] C. M. Zanetti, M. Nielsen and K. P. Khemchandani, QCD sum rule study of a charged bottom-strange scalar meson, Phys. Rev. D 93, 096011 (2016).
[9] W. Chen, H. X. Chen, X. Liu, T. G. Steele and S. L. Zhu, Decoding the $X(5568)$ as a fully open-flavor $s\bar{u}d\bar{d}$ tetraquark state, Phys. Rev. Lett. 117, 02002 (2016).
[10] S. S. Agaev, K. Azizi and H. Sundu, Width of the exotic $X_{a}(5568)$ state through its strong decay to $B^{0}_{s}\pi^{+}$, Phys. Rev. D 93, 114007 (2016).
[11] Y. R. Liu, X. Liu and S. L. Zhu, $X(5568)$ and its partner states, Phys. Rev. D 93, 074023 (2016).
[12] J. M. Dias, K. P. Khemchandani, A. Martinez Torres, M. Nielsen and C. M. Zanetti, A QCD sum rule calculation of the $X'(5568)\rightarrow B^{0}_{s}\pi^{+}$ decay width, Phys. Lett. B 758, 235 (2016).
[13] Z. G. Wang, Analysis of the strong decay $X(5568)\rightarrow B^{0}_{s}\pi^{+}$ with QCD sum rules, Eur. Phys. J. C 76, 279 (2016).
[14] F. Stancu, $X(5568)$ as a $s\bar{u}d\bar{b}$ tetraquark in a simple quark model, J. Phys. G 43, 105001 (2016).
[15] L. Tang and C. F. Qiao, $X(5568)$ as Tetraquark State with Open Flavors and its Charmed Partners, arXiv:1603.04761.
[16] A. Ali, L. Maiani, A. D. Polosa and V. Riquer, $B^0_s$ decays into tetraquarks, Phys. Rev. D 94, 034036 (2016).
[17] S. S. Agaev, K. Azizi and H. Sundu, Application of the QCD light cone sum rule to tetraquarks: the strong vertices $X_{a}X_{a}p$ and $X_{a}X_{a}p$, Phys. Rev. D 93, 114036 (2016).
[18] F. Goerke, T. Gutsche, M. A. Ivanov, J. G. Korner, V. E. Lyubovskij and P. Santorelli, Four-quark structure of $Z(3900)$, $Z(4430)$ and $X_{b}(5568)$ states, arXiv:1608.04656.
[19] S. S. Agaev, K. Azizi, B. Barsbay and H. Sundu, Resonance $X(5568)$ as an exotic axial-vector state, arXiv:1608.04785.
[20] A. K. Agamaliev, T. M. Aliev and M. Savci, Magnetic moment of $X_{b}$ state with $J^{PC} = 1^{+}$ in light cone QCD sum rules, arXiv:1610.03830.
[21] C. J. Xiao and D. Y. Chen, Possible $B^{0}\bar{K}$ hadronic molecule state, arXiv:1603.00228.
[22] S. S. Agaev, K. Azizi and H. Sundu, Exploring $X(5568)$ as a meson molecule, Eur. Phys. J. Plus 131, 351 (2016).
[23] M. Albaladejo, J. Nieves, E. Oset, Z. F. Sun and X. Liu, Can $X(5568)$ be described as a $B_s\pi$, $B\bar{K}$ resonant state?, Phys. Lett. B 757, 515 (2016).
[24] X. Chen and J. Ping, Is the exotic $X(5568)$ a bound state?, Eur. Phys. J. C 76, 351 (2016).
[25] X. W. Kang and J. A. Oller, $P$-wave coupled-channel scattering of $B_s\pi$, $B\pi$, $B\bar{K}$ and the puzzling $X(5568)$, Phys. Rev. D 94, 054010 (2016).
[26] C. B. Lang, D. Mohler and S. Prelovsek, $B_s\pi$ scattering and search for $X(5568)$ with lattice QCD, arXiv:1607.03185.
[27] R. Chen and X. Liu, Is the newly reported $X(5568)$ a $B\bar{K}$ molecular state?, Phys. Rev. D 94, 034006 (2016).
[28] J. X. Lu, X. L. Ren and L. S. Geng, $B_s\pi \sim B\bar{K}$ interaction in finite volume and the $X(5568)$, arXiv:1607.06327.
[29] B. X. Sun, F. Y. Dong and J. R. Pang, Study of $X(5568)$ in a unitary coupled-channel approximation of $B\bar{K}$ and $B_s\pi$, arXiv:1609.04068.
[30] A. Esposito, A. Pilloni and A. D. Polosa, Hybridized Tetraquarks, Phys. Lett. B 758, 292 (2016).
[31] S. S. Agaev, K. Azizi and H. Sundu, Charmed partner of the $X(5568)$ with lattice QCD, arXiv:1609.04068.
exotic $X(5568)$ state and its properties, Phys. Rev. D 93, 094006 (2016).

[32] X. G. He and P. Ko, Flavor SU(3) properties of beauty tetraquark states with three different light quarks, Phys. Lett. B 761, 92 (2016).

[33] X. G. He, W. Wang and R. L. Zhu, Production of Charmed Tetraquarks from $B_c$ and $B_s$ decays. arXiv:1606.00097

[34] X. H. Liu and G. Li, Could the observation of $X(5568)$ be a result of the near threshold rescattering effects?, Eur. Phys. J. C 76, 455 (2016).

[35] Z. Yang, Q. Wang and U. G. Meißner, Where does the $X(5568)$ structure come from?, arXiv:1609.08807

[36] Y. Jin, S. Y. Li and S. Q. Li, New $B_c^0\pi^+$ and $D_s^+\pi^+$ states in high energy multiproduction process, Phys. Rev. D 94, 014023 (2016).

[37] T. J. Burns and E. S. Swanson, Interpreting the X(5568), Phys. Lett. B 760, 627 (2016).

[38] F. K. Guo, U. G. Meißner and B. S. Zou, How the X(5568) challenges our understanding of QCD, Commun. Theor. Phys. 65, 593 (2016).

[39] H. X. Chen, W. Chen, X. Liu, Y. R. Liu and S. L. Zhu, A review of the open charm and open bottom mesons, arXiv:1609.08928

[40] M. E. Bracco, A. Lozea, R. D. Matheus, F. S. Navarra and M. Nielsen, Disentangling two- and four-quark state pictures of the charmed scalar mesons, Phys. Lett. B 624, 217 (2005).

[41] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Diquark-antidiquarks with hidden or open charm and the nature of $X(3872)$, Phys. Rev. D 71, 014028 (2005).

[42] J. Vijande, F. Fernandez and A. Valcarce, Open-charm meson spectroscopy, Phys. Rev. D 73, 034002 (2006); Erratum: Phys. Rev. D 74, 059903 (2006).

[43] M. V. Carlucci, F. Giannuzzi, G. Nardulli, M. Pellicoro and S. Stramaglia, AdS-QCD quark-antiquark potential, meson spectrum and tetraquarks, Eur. Phys. J. C 57, 569 (2008).

[44] H. X. Zhang, W. L. Wang, Y.-B. Dai and Z. Y. Zhang, Chiral SU(3) quark model study of tetraquark states: $c\bar{n}\bar{b}\bar{s}$/$c\bar{s}\bar{b}\bar{s}$, Commun. Theor. Phys. 49, 414 (2008).

[45] S. M. Gerasyuta and V. I. Kochkin, Tetraquarks with charm in coupled-channel formalism, Phys. Rev. D 78, 116004 (2008).

[46] D. Ebert, R. N. Faustov and V. O. Galkin, Masses of tetraquarks with open charm and bottom, Phys. Lett. B 696, 241 (2011)

[47] E. E. Kolomeitsev and M. F. M. Lutz, On Heavy light meson resonances and chiral symmetry, Phys. Lett. B 582, 39 (2004).

[48] Y. J. Zhang, H. C. Chiang, P. N. Shen and B. S. Zou, Possible S-wave bound-states of two pseudoscalar mesons, Phys. Rev. D 74, 014013 (2006).

[49] Y. R. Liu, X. Liu and S. L. Zhu, Light Pseudoscalar Meson and Heavy Meson Scattering Lengths, Phys. Rev. D 79, 094026 (2009).

[50] D. Zhang, Q. Y. Zhao and Q. Y. Zhang, A Study of S-wave DK interactions in the chiral SU(3) quark model, Chin. Phys. Lett. 26, 091201 (2009).

[51] G. Q. Feng, Z. X. Xie and X. H. Guo, Possible $BR$ molecular structure of $B_{cs}^0(5725)$ in the Bethe-Salpeter approach, Phys. Rev. D 83, 016003 (2011).

[52] S. Godfrey and N. Isgur, Mesons in a Relativized Quark Model with Chromodynamics, Phys. Rev. D 32, 189 (1985).

[53] S. Godfrey and J. Napolitano, Light meson spectroscopy, Rev. Mod. Phys. 71, 1411 (1999).

[54] S. Godfrey and K. Moats, Bottomonium Mesons and Strategies for their Observation, Phys. Rev. D 92, 054034 (2015).

[55] J. Ferretti, G. Galatà and E. Santopinto, Interpretation of the $X(3872)$ as a charmonium state plus an extra component due to the coupling to the meson-meson continuum, Phys. Rev. C 88, 015207 (2013).

[56] J. Ferretti and E. Santopinto, Higher mass bottomonia, Phys. Rev. D 90, 094022 (2014).

[57] Q. F. Lü and M. Li, Understanding the charmed states recently observed by the LHCb and BaBar Collaborations in the quark model, Phys. Rev. D 90, 054024 (2014).

[58] J. Ferretti and E. Santopinto, Open-flavor strong decays of open-charm and open-bottom mesons in the $^3P_0$ pair-creation model, arXiv:1506.04413

[59] S. Godfrey and K. Moats, Properties of Excited Charm and Charm-Strange Mesons, Phys. Rev. D 93, 034035 (2016).

[60] Q. T. Song, D. Y. Chen, X. Liu and T. Matsuji, Charmed-strange mesons revisited: mass spectra and strong decays, Phys. Rev. D 91, 054031 (2015).

[61] D. Ebert, R. N. Faustov and V. O. Galkin, Masses of heavy tetraquarks in the relativistic quark model, Phys. Lett. B 634, 214 (2006).

[62] D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha, Masses of tetraquarks with two heavy quarks in the relativistic quark model, Phys. Rev. D 76, 114015 (2007).

[63] D. Ebert, R. N. Faustov and V. O. Galkin, Excited heavy tetraquarks with hidden charm, Eur. Phys. J. C 58, 399 (2008).

[64] D. Ebert, R. N. Faustov and V. O. Galkin, Masses of light tetraquarks and scalar mesons in the relativistic quark model, Eur. Phys. J. C 60, 273 (2009).

[65] M. Monemzadeh, N. Tazimi and P. Sadeghi, Tetraquarks as diquark-antidiquark bound systems, Phys. Lett. B 741, 124 (2015).

[66] M. R. Hadizadeh and A. Khaledi-Nasab, Heavy tetraquarks in the diquark-antidiquark picture, Phys. Lett. B 753, 8 (2016).

[67] Q. F. Lü and Y. B. Dong, $X(4140), X(4274), X(4500),$ and $X(4700)$ in the relativized quark model, Phys. Rev. D 94, 074007 (2016).

[68] S. Capstick and N. Isgur, Baryons in a Relativized Quark Model with Chromodynamics, Phys. Rev. D 34, 2809 (1986).

[69] E. Hiyama, Y. Kino and M. Kamimura, Gaussian expansion method for few-body systems, Prog. Part. Nucl. Phys. 51, 223 (2003).

[70] E. Santopinto, An Interacting quark-diquark model of baryons, Phys. Rev. C 72, 022201 (2005).

[71] J. Ferretti, A. Vassallo and E. Santopinto, Relativistic quark-diquark model of baryons, Phys. Rev. C 83, 065204 (2011).

[72] E. Santopinto and J. Ferretti, Strange and nonstrange baryon spectra in the relativistic interacting quark-diquark model with a Grey and Radicati-inspired exchange interaction, Phys. Rev. C 92, 025202 (2015).

[73] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, The $Z(4430)$ and a New Paradigm for Spin Interactions in Tetraquarks, Phys. Rev. D 89, 114010 (2014).

[74] L. Maiani, A. D. Polosa and V. Riquer, The New Pentaquarks in the Diquark Model, Phys. Lett. B 749, 289 (2015).

[75] S. J. Brodsky, D. S. Hwang and R. F. Lebed, Dynamical Picture for the Formation and Decay of the Exotic XYZ Mesons, Phys. Rev. Lett. 113, 112001 (2014).

[76] R. F. Lebed, The Pentaquark Candidates in the Dynamical Diquark Picture, Phys. Lett. B 749, 454 (2015).

[77] H. X. Chen, W. Chen, X. Liu and S. L. Zhu, The hidden-charm pentaquark and tetraquark states, Phys. Rept. 639, 1 (2016).