Modeing and Robust Continuous Power Allocation Strategy with Imperfect Channel State Information in Cognitive Radio Networks

Zhong Chen*, Jun Cai, Feng Zhu, Rui Guo, Gengtian Niu and Yanjie Liu

The 28th Research Institute of China Electronics Technology Group Corporation, Nanjing, 210007, China

*Corresponding author email: sky2007cz@163.com

Abstract. A robust continuous power allocation strategy for secondary users (SUs) with imperfect channel state information (CSI) in the cognitive radio networks is proposed. Modeling the error of CSI by normbounded distribution, we address the optimization of the transmit power allocation at SU under the constraints of average transmit power at SU and average interference power at the primary user. Simulation results show that the proposed robust tools can improve the system performance of the SU.

Keywords: Cognitive radio (CR); Continuous power allocation; Spectrum sensing; Achievable rate; Outage probability.

1. Introduction

Cognitive radio (CR) has drawn a lot of research interest recently for it is a possible solution for the spectrum scarcity problem [1]. Currently, some spectrum access methods have been proposed: underlay [2], opportunistic spectrum access and sensing-based spectrum sharing. Specially, sensing-based spectrum sharing approach can get the maximum secondary rate under certain interference compared to the underlay and opportunistic spectrum access. This approach consists of a sensing period where a SU can sense to decide that the PU is on or not, and a transmission period where the SU can use the channel using a big transmit power under the condition that PU is not on, otherwise, using a low power [3], [4]. Notice that, the all methods use constant discrete powers. However, this kind of methods are not optimal, since it we can make the power to be a continuous one. In most of the previous work, the channel state information (CSI) is assumed to be constant and known at SU, however, it is hard to get the parameters. So in this paper, we consider a robust continuous power allocation method and study it on the scenario of imperfect CSI. After obtaining the sensing information of the PU, the SU doesn’t make decision on the whether PU is on or not but directly decides the continuous power due to the sensing information. The paper aims to make the achievable rate at SU while controlling interference level to PU and the average power at SU.
2. System Model

2.1. System Model

We study a simplified CR as Fig. 1. Let $\gamma_1, \gamma_2, \gamma_3$ and $g$ be the channel gain between the primary transmitter (PT) and the secondary transmitter (ST), PT and the secondary receiver (SR), PT and the primary receiver (PR), ST and PR, and ST and SR. In the sensing time $\tau$, the ST gets the information, and in the transmission time $T-\tau$, the ST uses the channel with an optimal power continuous due to the accumulated energy.

During the sensing slot, the receiving information of the $j$th signal $r_j$, can be written as

$$r_j = \begin{cases} n_j, & H_0, \\ \gamma_j s_j + n_j, & H_1, \end{cases}$$

(1)

where $H_0$ and $H_1$ denotes the idle and present state of PU separately; $s_j$ denotes the $j$th signal sent by PT assuming to follow a Gaussian distribution $s_j \sim N(0, P_p)$; and $n_j \sim N(0, N_0)$ denotes the noise. Denote $x$ as

$$x = \sum_{j=1}^{s_f} |p_j|^2,$$

(2)

where $s_f$ is the sampling rate at ST. So the probability density functions (pdf) of $x$ due to $H_0$ and $H_1$ are

$$f(x|H_0) = \frac{x^{s_f-1} e^{-x/2N_0}}{\Gamma\left(\frac{s_f}{2}\right)\left(2N_0\right)^{s_f/2}},$$

(3)

$$f(x|H_1) = \frac{x^{s_f-1} e^{-x/2N_0 + 2\gamma_p^2 P_p}}{\Gamma\left(\frac{s_f}{2}\right)\left(2N_0 + 2\gamma_p^2 P_p\right)^{s_f/2}}$$

and $\Gamma(\cdot)$ is the gamma function.

2.2. Imperfect CSI Model

In practice, the perfect CSI of $h$ and $g$ are difficult to get at ST for little information sharing among the CR nodes, or quantization of the channel estimation. We assume that the ST can only...
obtain the imperfect CSI to decide its transmit power. Specifically, we have the following uncertainly model for the channel

\[
\begin{align*}
\hat{h} &= h + \Delta h, \\
\hat{g} &= g + \Delta g,
\end{align*}
\]

where \(\hat{h}\) and \(\hat{g}\) are imperfect estimation of actual channel \(h\) and \(g\) which can be obtained at the ST, \(\Delta h\) and \(\Delta g\) are noises. The channel uncertainties \(\Delta h\) and \(\Delta g\) can be modelled with norm-bounded error (NBE). Correspondingly, the errors are modelled as

\[
\begin{align*}
Vh &\sim N(0, \sigma_h^2), \\
Vg &\sim N(0, \sigma_g^2),
\end{align*}
\]

Where \(\sigma_h^2\) and \(\sigma_g^2\) represent the uncertainty bounds indicating the CSI quality.

### 3. Optimization of Achievable Rate at SU

#### 3.1. Continuous Power Allocation Methods

During the transmission time, in the conventional methods, the ST decides the whether PU is on or not. Energy decision rule is used, i.e., ST compares \(x\) to a parameter \(\rho\), that is \(H_i\) if \(x > \rho\), else \(H_0\). Then SU employs two different powers defined as \(P_0\) and \(P_1\) due to the two states, i.e.,

\[
P(x) = \begin{cases} 
  P_0, & \text{if } H_0, \\
  P_1, & \text{if } H_1.
\end{cases}
\]

In this paper, we propose that, the transmit power is continuous with \(x\), i.e.,

\[
P(x) = P\left(\sum_{j=1}^{r} |r_j|^2\right).
\]

In (6), we notice that, information is lost for the detection transformation that is \(H_i\) if \(x > \rho\), else \(H_0\). However, joint from (2) and (7), we can conclude that no information is lost for the transformation \(\sum_{j=1}^{r} |r_j|^2\) and the proposed method is the optimal one according to the signal processing rules.

#### 3.2. Problem Formulation

For given receiving energy \(x\) and channel gain \(g\), the instantaneous achievable rates at SR conditioned on \(H_0\) and \(H_1\) can be represented as

\[
\begin{align*}
\log_2 \left( 1 + \frac{P(x) g^2}{N_0} \right), & \quad H_0 \\
\log_2 \left( 1 + \frac{P(x) g^2}{N_0 + \gamma^2 g^2} \right), & \quad H_1
\end{align*}
\]

respectively. And the achievable rate for SU is modelled as

\[
R = \frac{T - \tau}{T} E_{s,g} \left\{ \log_2 \left( 1 + \frac{P(x) g^2}{N_0} \right) f(H_0|x) + \log_2 \left( 1 + \frac{P(x) g^2}{N_0 + \gamma^2 g^2} \right) f(H_1|x) \right\},
\]

(9)
where \( f(H_0|x) \) and \( f(H_1|x) \) are posterior probabilities written as
\[
f(H_i|x) = \frac{f(x|H_i)P(H_i)}{f(x|H_0)P(H_0) + f(x|H_1)P(H_1)}, \quad i = 0, 1,
\]
and \( P(H_0), P(H_1) \) are idle and present probabilities of PU respectively. \( \frac{T - \tau}{T} \) in (9) means that, the rate of SU is obtained in the transmission slot \( T - \tau \).

The average transmit power constraint can be written as
\[
\frac{T - \tau}{T} E_x \{ P(x) \} \leq \bar{P}.
\]
(11)

The interference power constraint at PU can be written as
\[
\frac{T - \tau}{T} E_{x,h} \{ h^2 P(x) f(H_1|x) \} \leq T.
\]
(12)

Finally, the problem that maximizes the rate of SU due to the two constraints above can be written as
\[
\max_{\tau, P(x)} R \\
s.t. (11), (12), 0 \leq \tau \leq T, P(x) \geq 0, \forall_x.
\]
(13)

3.3. Solutions

Due to the nonconvexity of problem (13) over \( \tau \), the optimal sensing slot cannot be got under the convex optimization methods. But we can use the one-dimensional exhaustive search in \([0, T]\) to get optimal \( \tau \). So next, we mainly solve the problem of \( P(x) \).

Let \( \lambda \) and \( \mu \) denote the non-negative dual variables due to (11) and (12), respectively. The, we can build the Lagrangian \( L(P(x), \lambda, \mu) \) of (13) as
\[
L(P(x), \lambda, \mu) = R + \lambda \left( \bar{P} - \frac{T - \tau}{T} E_x \{ P(x) \} \right) + \mu \left( \frac{T - \tau}{T} E_{x,h} \{ h^2 P(x) f(H_1|x) \} \right).
\]
(14)

Define the Lagrange dual function \( g(\lambda, \mu) \) corresponding to problem (13), then we can build the Lagrange dual optimization problem as
\[
\max_{\lambda \geq 0, \mu \geq 0} g(\lambda, \mu) @ \sup_{P(x) \geq 0} L(P(x), \lambda, \mu).
\]
(15)

Using the KKT roles, the optimal \( P(x) \) under certain \( \lambda \) and \( \mu \) are written as
\[
P(x) = \left[ \frac{A + \sqrt{\lambda}}{2} \right]^+,
\]
(16)

where \([x]^+ \) means \( \max(0, x)\).
\[ A = \frac{\log_2(e)}{\lambda + \mu \left( \hat{h}^2 + \sigma_g^2 \right) f(H_1|x)} - \frac{2N_0 + \gamma_\rho^2 P_p}{\hat{g}^2 + \sigma_g^2}, \]

\[ V = A^2 + \frac{4}{\hat{g}^2 + \sigma_g^2} \left\{ \log_2(e) \left[ f(H_0|x)(N_0 + \gamma_\rho^2 P_p) + f(H_1|x)N_0 \right] - \frac{N_0(N_0 + \gamma_\rho^2 P_p)}{\hat{g}^2 + \sigma_g^2} \right\}. \] (17)

### 4. Simulation Results

Figure 2 compares the \( P(x) \) versus \( x \). In this figure we can get that, \( P(x) \) is a continuous function of \( x \). If \( x \) is low, \( P(x) \) is bigger than the conventional methods, otherwise, and \( P(x) \) is lower.

![Figure 2. Allocated Power Vs the receiving energy.](image)

### 5. Conclusions

We propose a robust continuous power method at SU based on sensing with imperfect CSI in the CR networks. We address the optimization problem on the transmit power allocation at SU to make the maximum capability at SU for different constraints. Compared with the conventional CR strategy and non-robust one, the proposed method can get rate promotion for the secondary users.

### References

[1] S. Haykin, “Cognitive radio: brain-empowered wireless communications,” IEEE J. Sel. Areas Commun., vol. 23, no. 2, pp. 201-220, Feb. 2005.

[2] X. W. Gong, S. A. Vorobyov, and C. Tellambura, “Optimal bandwidth and power allocation for sum ergodic capacity under fading channels in cognitive radio networks,” IEEE Trans. Signal Process., vol. 59, no. 4, pp. 1814-1826, Apr. 2011.

[3] R. F. Fan, J. Hai, Q. Guo, and Z. Zhang, “Joint optimal cooperative sensing and resource allocation in multichannel cognitive radio networks,” IEEE Trans. Veh. Technol., vol. 60, no. 2, pp. 722-729, Feb. 2011.

[4] X. Kang, Y. C. Liang, H. K. Garg, L. Zhang, “Sensing-based spectrum sharing in cognitive radio networks,” IEEE Trans. Veh. Technol., vol. 58, no. 8, pp. 4649-4654, Oct. 2009.

[5] Y. Huang, Q. Li, W.-K. Ma, and S. Zhang, “Robust multicast beamforming for spectrum sharing-based cognitive radios,” IEEE Trans. Signal Process., vol. 60, no. 1, pp. 527-533, Jan. 2012.