The Ages of Pre-main-sequence Stars

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ABSTRACT

The position of pre-main-sequence or protostars in the Hertzsprung-Russell diagram is often used to determine their mass and age by comparison with pre-main-sequence evolution tracks. On the assumption that the stellar models are accurate, we demonstrate that, if the metallicity is known, the mass obtained is a good estimate. However, the age determination can be very misleading because it is significantly (generally different by a factor of two to five) dependent on the accretion rate and, for ages less than about $10^6$ yr, the initial state of the star. We present a number of accreting protostellar tracks that can be used to determine age if the initial conditions can be determined and the underlying accretion rate has been constant in the past. Because of the balance established between the Kelvin-Helmholtz, contraction timescale and the accretion timescale a pre-main-sequence star remembers its accretion history. Knowledge of the current accretion rate, together with an H–R-diagram position gives information about the rate of accretion in the past but does not necessarily improve any age estimate. We do not claim that ages obtained by comparison with these particular accreting tracks are likely to be any more reliable than those from comparisons with non-accreting tracks. Instead we stress the unreliability of any such comparisons and use the disparities between various tracks to estimate the likely errors in age and mass estimates. We also show how a set of coeval accreting objects do not appear coeval when compared with non-accreting tracks. Instead accreting pre-main-sequence stars of around a solar mass are likely to appear older than those of either smaller or larger mass.

Key words: stars: evolution – stars: pre-main-sequence – stars: formation – accretion, accretion discs

1 INTRODUCTION

The placement of an observed pre-main-sequence star in the theoretical Hertzsprung-Russell diagram ($\log L$ against $\log T_{\text{eff}}$) is notoriously difficult because of its sensitivity to distance and reddening (see for example Gullbring et al. 1998). In addition any contribution to the light from the accretion disc itself must be subtracted and obscuration by circumstellar material accounted for (see for example Hillenbrand 1997). Here, by examining where theory predicts a particular object ought to lie at a given age, we investigate what properties of a pre-main-sequence star can be determined if these difficulties can be overcome.

The process by which stars form from their constituent interstellar material is as relevant to all branches of astrophysics, from planets to cosmology, as their subsequent evolution. However our understanding and, without doubt, our predictive power lag well behind. This is partly because stars which are in the process of formation are more difficult to observe. The relative rapidity of the star formation process means that there are no nearby pre-main-sequence stars and the fact that they form in denser regions of the interstellar medium favours observations at wavelengths longer than optical. It is only recently that such objects have begun to be observed in statistically significant numbers (Cohen & Kuhi 1979). From a theoretical point of view, difficulties arise because much of the process is dynamical and so does not lend itself well to the one-dimensional models normally employed in stellar evolution. On the other hand we can model the hydrostatic inner regions using the methods normally employed in stellar evolution and so, with appropriate boundary conditions, approximate a forming star. Indeed this kind of pre-main-sequence theory can be said to have begun along side stellar evolution itself with the work of Henyey, Leleiver & Levee (1955), restricted to radiative solutions, and Hayashi (1961), with convection. These pioneers were able to describe how a spherical cloud of gas,
already in hydrostatic equilibrium, contracts down to the main sequence as it releases its own gravitational energy.

The question of how the initial hydrostatic sphere forms is further complicated by two major effects. First, dynamical processes must be important in addition to thermal and nuclear and, second, these can no longer be expected to be spherically or even oblate-spheroidally symmetric. Larson (1969) modelled the spherically symmetric collapse of a gas cloud that is not yet in hydrostatic equilibrium. He showed how the central regions collapse first to form a hydrostatic core on to which the rest of the cloud accretes. But this core cannot behave like Hayashi’s pre-main-sequence stars because its surface is no longer exposed to space and the boundary conditions are different. Larson introduced shock conditions at the surface of a near hydrostatic core. Building heavily on this, Stahler, Shu and Taam (1980a, b, 1981) were able to follow the evolution of the accreting core. Such a core, shrouded in its own accreting envelope, remains invisible as long as it accretes. Stahler et al. assumed that accretion and obscuration cease at the same time when the surrounding material is somehow blown away. Their stars then descend the classic Hayashi tracks until they develop radiative envelopes and move on to the corresponding Henyey track. However, if the accreting material does not obscure the entire stellar surface, we are able to see the star whilst it is still accreting. It is this latter situation that we model in this work. It is likely to arise because material accreting from far off will have too much angular momentum to fall radially on to the central core. Instead it will form an accretion disc in a plane perpendicular to the angular momentum axis and fall on to the core only as viscosity allows a small amount of material to carry the angular momentum outwards. If the disc reaches to the stellar surface then the material will accrete only in an equatorial band. If, on the other hand, the central core possesses a magnetic field strong enough to disrupt the inner parts of the disc, matter might finally flow in along field lines accreting at relatively small magnetic poles or thin accretion curtains. Similar processes are known to operate in magnetic cataclysmic variables (Warner 1995 for a review). In any of these cases most of the stellar surface is left exposed and free to radiate like a normal star.

A comprehensive study of such exposed protostellar cores was made by Mercer-Smith, Cameron and Epstein (1984). Because more than the usual insight is needed to elucidate what they actually did this work has largely been forgotten. However it turns out that their accreting tracks qualitatively differ from ours and so it is important to identify exactly why this is. Their calculations begin with a hydrostatic core of 0.0015 $M_\odot$ of apparently 2 $R_\odot$ based on the dynamical collapse calculations of Winkler and Newman (1980). We shall argue in section 3 that, at this mass, the core is still embedded in a rapidly collapsing cloud and that something like 0.1 $M_\odot$ and 3 $R_\odot$ gives a more realistic representation of the central core when non-spherical accretion begins in earnest. But we shall also show that the subsequent evolution is not overly sensitive to this initial state. More significantly, Mercer-Smith et al. require that at least one quarter of the accretion luminosity be radiated uniformly over the whole stellar surface, while we claim that it can all be radiated locally in a disc boundary layer or localised shocks. This, coupled with their extreme accretion rates of typically $10^{-5} M_\odot \text{yr}^{-1}$, most probably accounts for the huge discrepancy between their and our tracks, manifested by the fact that their standard model is at a much higher effective temperature for a given mass than ours. As a consequence, our models evolve smoothly even if the accretion rate is abruptly changed while theirs relax to a normal Hayashi track rapidly over about 100 yr when accretion is halted.

A careful analysis of the effects of accretion on stellar structure has been made by Siess & Forestini (1996) varying a number of the physical properties of the accreted material relative to the stellar surface from angular momentum content to internal energy and find that reasonable values of these parameters have little affect on the stellar structure. Siess, Forestini & Bertout (1997) then went on to use their formalism to follow a small number of evolutionary sequences. They confirmed the lack of sensitivity to their various parameters except for the dependence on the fraction, $\alpha$, of the accretion boundary-layer energy released below the stellar photosphere. Large values of this parameter are similar to Mercer-Smith et al.’s formalism while our models correspond to $\alpha = 0$. Siess et al.’s models with $\alpha = 0.01$ are indeed very similar to our tracks when the accretion history is comparable.

We present several evolution tracks for pre-main-sequence stars accreting from various initial conditions to quantify the accuracy to which age can be determined. Most of the tracks are for solar metallicity of $Z = 0.02$. However measurements of metallicity in Orion’s star forming regions, although highly uncertain, indicate that $Z = 0.001$ may be more appropriate (Rubin et al. 1997). Such low metallicity is also typical of star forming regions in the Large Magellanic Cloud. We therefore discuss a set of low-metallicity tracks which demonstrate how both mass and age determinations from colour–magnitude diagrams depend critically on a knowledge of metallicity. Because the stellar mass function dictates that the bulk of stars have final masses in the low side we restrict our presentation here to accreting objects of less than 2 $M_\odot$. We can expect almost all stars in the star-forming regions with which we may wish to compare properties to lie below this mass. Also, as stressed by Palla and Stahler (1993), the contraction timescales for massive stars are short compared with accretion timescales so that the accreting tracks will tend to follow the zero-age main-sequence and the effective pre-main-sequence life of massive stars is dominated by their early, low-mass evolution. Indeed at higher masses the accretion timescale becomes long compared with the nuclear timescale and it is difficult to separate pre- and post-main-sequence evolution for some stars.

We find that masses can be fairly well established if the metallicity is known but that ages are very dependent on the accretion history and the initial state of the star particularly below $5 \times 10^6$ yr. However before we can begin to discuss age determination we must first establish to what this age is relative.

2 THE ZERO AGE

It is very often unclear how to define the zero-age point for a forming star and of course it is rather uninformative to quote an age $t$ without explaining exactly what we mean by $t = 0$. For evolved stars the zero-age main-sequence
Figure 1. A Hertzsprung–Russell diagram showing constant-mass pre-main-sequence tracks of solar metallicity (Z = 0.02) stars in the range 0.1 to 2.0 M⊙. Isochrones of ages ranging from 10^3 to 10^8 yr are drawn across the tracks. The models were begun at radii large enough that these isochrones are not affected by small displacements of this starting point. The zero-age main and deuterium-burning sequences appear as dots logarithmically spaced in mass.

(ZAMS) provides a convenient starting point from which we can both begin the evolution and measure the age of the star. The ZAMS must then be defined. Historically a star was started in a state of hydrostatic and thermal equilibrium with a uniform initial composition. In reality a star never actually passes through this zero-age state because some nuclear burning takes place while a newly formed star is still contracting to the main sequence. In practice, because the thermal-evolution timescale of pre-main-sequence stars is several orders of magnitude shorter than the post-main-sequence nuclear timescale, very little of the initial hydrogen is burnt and a uniform hydrogen abundance throughout the star is a reasonable approximation. This is not so for the catalytic elements of the CNO cycle in sufficiently massive stars because these elements are driven towards equilibrium during pre-main-sequence evolution. Even so, it is possible to define a zero-age main sequence (see for example Tout et al. 1996) that roughly corresponds to the minimum luminosity attained as a star evolves from a pre- to post-main-sequence phase. Nor is the assumption of uniform abundance true for the elements involved in the pp chain, notably deuterium and He^3. However, on the zero-age main sequence, the pp chain is complete in transforming hydrogen to He^4 at the stellar centres and so the abundances of D and He^3 are in equilibrium for given temperatures and number densities and subsequently need not be followed explicitly. On the other hand deuterium burning is a major source of energy in pre-main-sequence stars and is important throughout this work.

Because we can define a ZAMS reasonably uniquely a good way to measure pre-main-sequence ages would be backwards from the ZAMS. However this is not acceptable if one wishes to measure the time elapsed since the birth of a star, where relatively small changes in age lead to large excursions in the H–R diagram. A similar problem is encountered with the upper parts of the red giant, and particularly asymptotic giant, branch but in these cases we regard absolute age as relatively useless preferring such quantities as degenerate core mass as a measure of the evolutionary state (Tout et al. 1997).

The concept of a stellar birthline in the H–R diagram was introduced by Stahler (1983) as the locus of points at which stars forming from a spherically accreting cloud would first become visible. In the model of Stahler, Shu and Taam (1980a,b, 1981) this occurs when deuterium ignites in the protostellar core and some ensuing wind blows away the remainder of the accreting cloud which has, up to this point, shrouded the star itself from view. With such a theory, a perfect place to fix the zero age of a pre-main-sequence star would be the onset of deuterium burning. Deuterium burning provides pressure support for the star for a time comparable with the Kelvin–Helmholtz timescale $\tau_{KH}$, on which it contracts once deuterium is exhausted. This timescale is similar to the entire time taken to contract to this point from any initial state so that by the time a star begins to contract again, below the deuterium burning sequence, it is already relatively old and has a reasonably well defined age. Here we concern ourselves with accretion through a disc. In this case most of the stellar photosphere is exposed while accretion is still taking place. Nor do we assume that accretion ceases at the onset of deuterium burning. Under such circumstances there is no reason why stars should not appear above Stahler’s birthline and it is no longer possible to define a birthline as a locus of maximum luminosity at which pre-main-sequence stars appear. However, the interruption of contraction when deuterium ignites means that we are much more likely to see stars on and below the deuterium burning sequence than above it. By definition Stahler’s birthline is more or less coincident with the deuterium burning sequence and this explains the consistency of observations with the idea of a birthline. Unfortunately this apparent birthline is not the place where stars are born and so an age measured from a zero-age deuterium burning sequence is too young by an unknown amount which is normally at least as much as the deuterium burning lifetime.

D’Antona and Mazzitelli (1994) take another approach which is to begin evolution at a point in the H–R diagram of sufficiently high luminosity, or equivalently at sufficiently large radius on a Hayashi track, that $\tau_{KH}$ is much less than some acceptable error in the age at any later time. This error might be chosen to be about 100 yr. Such a definition leads to a well-defined age at any point on a track corresponding to a constant mass. For comparison, figure 1 shows such a set of pre-main-sequence tracks for $M = 0.1, 0.2, 0.5, 1$ and $2 M_\odot$ and isochrones fitted to 50 models in this range. We describe our models in detail in the following sections but note that, because we use very similar physics, they do not differ greatly from those of D’Antona and Mazzitelli.

However stars do continue to accrete long after their photospheres are exposed and they can be placed in an H–R diagram. A star of about 1 $M_\odot$ is most unlikely to have reached this mass while $\tau_{KH}$ was still small or indeed even before deuterium exhaustion. For this reason we take the zero-age point of each of our tracks to be a point at which the protostellar core has the mass and radius of a typical self-gravitating fragment of a protostellar cloud and model the subsequent evolution with ongoing accretion. We then investigate how changing these initial conditions alters the subsequent isochrones in the H–R diagram to get an idea of how well we can constrain the age of an observed pre-main-sequence star relative to its birth as a self-gravitating accreting body.

3 THE STELLAR MODELS

We construct our stellar models using the most recent version of the Eggleton evolution program (Eggleton 1971, 1972, 1973). The equation of state, which includes molecular hydrogen, pressure ionization and coulomb interactions, is discussed by Pols et al. (1995). The initial composition is taken to be uniform with a hydrogen abundance $X = 0.7,$
helium \( Y = 0.28 \), deuterium \( X_D = 3.5 \times 10^{-5} \) and metals \( Z = 0.02 \) with the meteoritic mixture determined by Anders and Grevesse (1989). Hydrogen burning is allowed by the pp chain and the CNO cycles. Deuterium burning is explicitly included at temperatures too low for the pp chain. Once the pp chain is active hydrogen is assumed to burn to helium via deuterium and helium in equilibrium. The burning of helium is not explicitly followed. Opacity tables are those calculated by Iglesias, Rogers and Wilson (1992) and Alexander and Ferguson (1994). An Eddington approximation (Woolley and Stibbs 1953) is used for the surface boundary conditions at an optical depth of \( \tau = 2/3 \). This means that low-temperature atmospheres, in which convection extends out as far as \( \tau \approx 0.01 \) (Baraffe et al. 1995), are not modelled perfectly. However the effect of this approximation on observable quantities is not significant in this work (see for example Kroupa and Tout 1997).

We assume that material is accreted from a disc on to a thin equatorial region of the star so that normal photospheric boundary conditions are appropriate over most of its surface. This would also be true even if the inner edge of the disc is magnetically disrupted and the material funnelled to a few spots or narrow accretion curtains whose areas represent a relatively small fraction of the stellar surface. Because our models are one-dimensional we must apply these same boundary conditions over the whole surface. Similarly we must assume that accreted material is rapidly mixed over this same complete surface so that, on accretion of mass \( \delta M \), we can add a spherical shell of mass \( \delta M \) with composition equal to the initial, or ambient, composition. We note that the photospheric boundary conditions effectively fix the thermodynamic state of the accreted material to those conditions over the radiating stellar surface. This is equivalent to the assumption that boundary layer shocks or, in the case of magnetically funnelled accretion, shocks at or just above the stellar surface remove any excess entropy from the accreting material and so is not unduly restrictive. Ideally we would like to treat this problem in two-dimensions. We could then apply different boundary conditions over the equatorial band or polar spots where accretion is actually taking place. With current computational power and techniques such models may not be too far off (Tout, Cannon and Pichon, private communication).

4 INITIAL CONDITIONS

We wish to take as an initial model a typical protostellar core of mass \( M_0 \) that is self gravitating within a cloud and that has reached hydrostatic but not yet thermal equilibrium out to a radius \( R_0 \). Additional material beyond \( R_0 \) may be gravitationally bound to the star but not yet accreted. We assume that the core is spherically symmetric out to \( R_0 \) and that beyond this radius material sinks on to a disc from which it is accreted in a thin equatorial band or other relatively small part of the stellar photosphere.

The technique we use to construct the initial model is fairly standard. We take a uniform composition zero-age main-sequence model of mass \( M_0 \) and add in an artificial energy generation rate \( \epsilon_c \) per unit mass uniformly throughout the star. Initially \( \epsilon_c \) is negligible but we gradually increase it so that the star is slowly driven back up its Hayashi track. In a sense \( \epsilon_c \) mimics the thermal luminosity that would be released if the star were contracting down the Hayashi track. These objects, however, are in thermal equilibrium. We continue to increase \( \epsilon_c \) until the radius of the object is considerably more than \( R_0 \). At this point we may add or subtract mass freely, while maintaining hydrostatic and thermal equilibrium, and so vary \( M_0 \). In this way we can reach masses below the hydrogen burning limit that would not have a zero-age main-sequence state of their own. We then switch off the artificial energy generation and allow the star to contract down its Hayashi track supported by the usual gravitational energy release. When \( R = R_0 \) we have our initial model.

We choose a protostellar core of \( M_0 = 0.1 M_\odot \) and \( R_0 = 3 R_\odot \) as our standard initial model. This choice of the initial mass and radius of the pre-main-sequence star is necessarily somewhat arbitrary because it depends on the pre-collapse conditions and the dynamics of the collapse process. We choose a mass and radius taken to represent a young star at the end of the collapse and spherical infall phase of evolution at a time when it first becomes optically visible. This will include the initial protostellar core that forms from the collapse phase plus any mass that is accreted on to this core before the infall becomes significantly aspherical. This happens when the infalling material has enough angular momentum to force it to collapse towards a disc rather than be accreted directly by the protostellar core. Any further accretion from this point will be through this circumstellar disc.

The gravitational collapse of a molecular cloud forms a first protostellar core when the density becomes large enough to trap the escaping IR radiation (e.g. Larson 1969). This sets a minimum mass for opacity limited fragmentation at about \( 0.01 M_\odot \) (Low & Lynden-Bell 1976, Rees 1976). This minimum mass will be increased by the material from further out with low angular momentum (along the rotation axis) plus the matter that has its angular momentum removed/redistributed by gravitational torques (e.g. Larson 1984) in the disc on timescales short compared with the free-fall time. The accretion of low-angular momentum matter probably increases the protostar’s mass by a factor of 3 (for initially uniform density collapse). The accreted disc material can be estimated as that with dynamical times significantly less than the original free-fall time. Thus, material within 20 – 50 au should be accreted within \( 10^3 \) years, which corresponds to disc sizes several times larger than the first core. For initially solid body rotation and uniform cloud density this translates to a mass at least three times larger. We thus estimate our initial mass as about \( 0.1 M_\odot \). This compares well to the mass of a protostellar core from a spherical collapse with a fraction of a free-fall time (Winkler & Newman 1980) and to the mass within 50 au in a collapse including rotation (Boss 1987, see also Lin & Pringle 1990). An initial mass of 0.1 \( M_\odot \) is also comparable to the observed lower limit for stellar masses and still allows for significant mass increase through subsequent accretion.

The choice of the initial stellar radius is perhaps more constrained. Estimates of this radius depend on the dynamics of the collapse, but are generally in the range of 2.5 to 3 \( R_\odot \) (Stahler 1988, Winkler & Newman 1980). We have chosen the value of 3 \( R_\odot \) as given by Winkler & Newman (1980) for an accreting protostar of 0.1 to greater than 0.5 \( M_\odot \). We
investigate variations in \( R_0 \) and \( M_0 \) and find that the precise choice is not terribly critical anyway.

5 STANDARD MODELS

From the standard initial conditions \( M_0 = 0.1 \, M_\odot \) and \( R_0 = 3 \, R_\odot \), we evolve a set of thirteen pre-main-sequence stars accreting at constant accretion rates ranging from \( 10^{-9} \) to \( 10^{-8} \) in steps of \( 10^{-9.5} \) and then to \( 10^{-5.5} \, M_\odot \, yr^{-1} \) in steps of \( 10^{-5.25} \) to a final mass of \( 2 \, M_\odot \). Thick lines are isochrones of \( 10^4 \) to \( 10^8 \) yr or join points of equal mass from \( 0.1 \) to \( 2.0 \, M_\odot \). The zero-age main and deuterium-burning sequences appear as dots logarithmically spaced in mass.

Figure 2. A Hertzsprung–Russell diagram showing, as thin lines, tracks followed by pre-main-sequence stars of initial mass \( 0.1 \, M_\odot \) and radius \( 3 \, R_\odot \) evolved with constant accretion rates ranging from \( 10^{-9} \) to \( 10^{-8} \) in steps of \( 10^{-9.5} \) and then to \( 10^{-5.5} \, M_\odot \, yr^{-1} \) in steps of \( 10^{-5.25} \) to a final mass of \( 2 \, M_\odot \). Thick lines are isochrones of \( 10^4 \) to \( 10^8 \) yr or join points of equal mass from \( 0.1 \) to \( 2.0 \, M_\odot \). The zero-age main and deuterium-burning sequences appear as dots logarithmically spaced in mass.

Figure 3. The isochrones and equal-mass loci from figure 2—solid lines—overlaid with the pre-main-sequence tracks and isochrones of figure 2—dotted lines. The zero-age main and deuterium-burning sequences appear as dots logarithmically spaced in mass.

In figure 2 we overlay both the isochrones and the equal-mass loci on the pre-main-sequence tracks and isochrones of figure 2 for comparison. During the Hayashi contraction the accreting equal-mass loci follow very closely the tracks of non-accreting pre-main-sequence stars so that determination of a pre-main-sequence star’s precise position in the Hertzsprung–Russell diagram gives an equally precise estimate of its mass, irrespective of age and accretion rate. However, once the radiative core forms, an error in the mass determination is introduced. By comparing the change in the slope of the equal-mass loci with the non-accreting tracks we see that accretion delays the effects of the establishment of the radiative core. We note that, up to the point at which the radiative core forms, the fully convective structure has meant that stars evolve essentially homologously. For this reason the approximations used by Hartmann et al. (1997) have remained valid and we expect their tracks to be a good representation. Once this homology is lost their equation (6) is no longer valid nor is their assumption that luminosity is a
The tracks would begin to deviate radically and it becomes much more important to follow the full evolution as we do here.

Below $0.2 M_\odot$ and ages greater than $10^6$ yr the isochrones are quite similar too but they deviate drastically for larger masses with the accreting stars appearing significantly older than they are by factors of two or more according to the non-accreting pre-main-sequence isochrones. This is because, for a given Kelvin-Helmholtz timescale, a lower-mass Hayashi-track star is smaller in radius so that, as mass is added and the star moves to higher temperature it always remains smaller and appears older than a star that originally formed with its current mass. As an example, a pre-main-sequence star accreting at $10^{-6.73} M_\odot$ yr$^{-1}$ would appear to be over $10^7$ yr old at $1 M_\odot$ when it is only $5 \times 10^6$ yr old, while one accreting at $10^{-6} M_\odot$ yr$^{-1}$ at $1 M_\odot$ would appear to be $5 \times 10^6$ yr old when it is only $2 \times 10^6$ yr old. Thus, because of the delayed appearance of a radiative core, more massive stars begin to appear younger again. For instance, our star accreting at $10^{-7} M_\odot$ yr$^{-1}$ would appear to remain at $10^7$ yr from about 1.3 to 1.74 $M_\odot$, during which time it ages from 1.3 to 1.65 $10^7$ yr. At ages of less than $10^6$ years the position in the Hertzsprung-Russell diagram depends far more on the starting point than on whether or not the star is accreting but generally the age will be overestimated by up to the estimate itself.

6 CHANGING $R_0$ AND $M_0$

To illustrate the sensitivity of the evolutionary tracks, and the corresponding mass and age determinations, to our choice of initial model we consider two alternative starting points. First, keeping $M_0 = 0.1 M_\odot$, we reduce $R_0$ until the initial protostellar core is just beginning to ignite deuterium. This occurs at about $R_0 = 1.1 R_\odot$. Second we reduce $M_0$ to $0.05 M_\odot$ and change $R_0$ to 2.4 $R_\odot$ so as to keep the same mean density. The evolutionary tracks followed by pre-main-sequence stars accreting at the same accretion rates, in the range $10^{-9}$ to $10^{-5.5} M_\odot$ yr$^{-1}$, together with isochrones and equal-mass loci are plotted in figures 5 and 6 respectively. A non-accreting pre-main-sequence track for $0.05 M_\odot$ in figure 6 is seen to run asymptotically down the main sequence. Because its mass is below the hydrogen burning limit of about $0.08 M_\odot$, this downward progress to lower luminosities and temperatures will not be halted and this star will end up as a degenerate brown dwarf.

The isochrones and equal-mass loci are overlaid with those corresponding to our standard tracks in figures 6 and 7. In neither case are the equal-mass loci significantly affected. Thus the deviation in these loci from non-accreting pre-main-sequence tracks can be solely attributed to the accretion. On the other hand, it is inevitable that the isochrones are affected but, in both cases, the difference is small compared with the overall effect of accretion. In the case of reduced $R_0$ the difference is a constant offset of $10^6$ yr. This is just the time taken by a $0.1 M_\odot$ star to contract from 3 to 1.1 $R_\odot$. Thus ages of about $5 \times 10^6$ yr would be accurately estimated to within 20 per cent and those of about $10^7$ yr to within 10 per cent etc. This error represents the extreme if we can be sure that all stars are born before igniting deuterium. If $R_0$ were increased beyond 3 $R_\odot$ the initial thermal timescale would be so short that the time taken to contract to the point of deuterium ignition would not be noticeably different.

From figure 6 we can see that the deviations from our standard isochrones are even smaller when $M_0 = 0.05 M_\odot$. In this case, what is important is the time taken to accrete the additional $0.05 M_\odot$ and so the most slowly accreting tracks are most affected. Thus the $10^{-9} M_\odot$ yr$^{-1}$ track reaches 0.1 $M_\odot$ after $5 \times 10^6$ yr leading to a relatively large absolute difference in the $10^6$ yr isochrone at 0.1 $M_\odot$. However, as this difference is always relative to the accretion rate, it rapidly becomes insignificant as we go up in mass.

In both these cases it is important to note that the actual tracks followed for a given accretion rate become very similar to our standard ones with the relative time between two points on a track being the same. It is just the time taken to reach an equivalent point that alters the isochrones. We deduce that accretion rate has a much more significant effect on the position in the H–R diagram than do the initial conditions.

7 VARIABLE ACCRETION RATES

The models we have presented so far have accreted at a constant rate throughout their pre-main-sequence life. This is unlikely to be the case in reality. Even so we might hope that a star of a given mass and accretion rate might be found at the intersection of the appropriate accreting track and equal-mass locus of figure 2 irrespective of its accretion history. However we find that this is not the case because a pre-main-sequence star remembers its past. We illustrate the effect of variable accretion rates by considering three paths, beginning at the same point, that converge to an accretion rate of $10^{-7} M_\odot$ yr$^{-1}$ when the star’s mass reaches 0.5 $M_\odot$ and continue to accrete at that rate, constant thereafter. These tracks are plotted in figure 6. The first has the standard constant accretion rate of $10^{-7} M_\odot$ yr$^{-1}$. The second accretes at a rate that decreases linearly from $10^{-6} M_\odot$ yr$^{-1}$ at $t = 0$ to $10^{-7} M_\odot$ yr$^{-1}$ at 0.5 $M_\odot$ while the third has an accretion rate that increases linearly from nothing to $10^{-7} M_\odot$ yr$^{-1}$. At 0.5 $M_\odot$ we find that the stars are well separated in luminosity. This is to be expected because a star will take a Kelvin-Helmholtz timescale to adjust its structure to a new accretion rate. As discussed in section 5, it is this same timescale that is balanced with the accretion timescale that is directly responsible for the deviation of the tracks. This timescale balance will be maintained until nuclear burning becomes important, in this case on the zero-age main sequence. Consequently the stars are never given enough time to thermally relax and their early accretion history can be remembered throughout their pre-main-sequence evolution.

At $1 M_\odot$ the mass can still be estimated accurately by comparison with the standard accreting tracks of figure 6, but note that this differs from the mass that would be estimated if we were to compare with non-accreting tracks. At 0.5$M_\odot$ age estimates from non-accreting tracks would be between 30 and 60 per cent too old and at 1 $M_\odot$ between 2 and 3 times too old for each of these stars. This reflects the general nature and magnitude of the difference between non-accreting and our standard accreting pre-main-sequence stars, comparison with which would give a better estimate in

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each of these particular cases (within 10 per cent at 0.5 \( M_\odot \) and 20 per cent at 1 \( M_\odot \)).

We emphasize again that we cannot estimate the current accretion rate from the position in the H–R diagram but if we know this current rate then placement in an H–R diagram does give us information about the accretion history. This behaviour is in accord with equation (6) of Hartmann et al. (1997), with \( \alpha = 0 \). As long as \( \dot{M} = \dot{M} \) dominates \( \dot{R}/R \), this equation predicts \( R \) as a function of \( M \) and \( \dot{M} \) only. Our track with decreasing \( \dot{M} \) always has \( \dot{R}/R \) somewhat greater than \( \dot{M}/\dot{M} \) because it has reached \( M = 0.5 \, M_\odot \) with the two terms in balance at higher accretion rates.

### 8 CHANGING METALLICITY

Finally we consider the effect of different metallicities. In general reducing the metallicity moves the zero-age main sequence to hotter effective temperatures and slightly higher luminosities (see for example Tout et al. 1996). This is due to decreased opacity when there are fewer metal atoms providing free electrons. This shift is reflected throughout the pre-main-sequence evolution. Figure 5 shows the same tracks and isochrones as figure 3 but for a metallicity of \( Z = 0.001 \). These models have an initial helium abundance of \( Y = 0.242 \) and hydrogen \( X = 0.757 \) to account for the less-processed interstellar medium from which such stars must be forming. In practice we should correspondingly increase the deuterium abundance too but, because this is very uncertain anyway, we leave it at \( X_D = 3.5 \times 10^{-5} \) so as not to convolute the differences between the two metallicities. Apart from the shift, the tracks are qualitatively similar except for the disappearance of the second hook just above the ZAMS in the more massive star tracks. At \( Z = 0.02 \) this is due to the CNO catalytic isotopes moving towards equilibrium in the stellar cores before hydrogen burning begins in earnest.

Figure 5 overlays these tracks and isochrones with those for \( Z = 0.02 \). We can see directly that an error of a factor of two or more would be made in the mass estimate and a factor of ten or so in the age if a pre-main-sequence star of metallicity \( Z = 0.001 \) were compared with models made for \( Z = 0.02 \). Clearly, if stars are indeed still forming at such low metallicities, it is very important to be sure of the precise value before making any comparisons. For a rough estimate of how the tracks move with metallicity we interpolate these two sets of tracks together with a similar set for \( Z = 0.01 \). We find the difference in mass

\[
\delta M = M(Z = 0.02) - M(Z)
\]

between tracks of metallicity \( Z \) and those of solar metallicity that pass through a given point \((L, T_{\text{eff}})\) in the Hertzsprung–Russell diagram to be

\[
\delta M \approx 0.164 \left( \log_{10} \frac{Z}{0.02} \right)^{-0.7} \left( \frac{L}{L_\odot} \right)^{0.25} \left( \frac{T_{\text{eff}}}{10^3.5 K} \right)^{0.6}
\]

### 9 CONCLUSIONS

If the metallicity of a star forming region is known then the masses on the Hayashi tracks can be fairly accurately determined. As noted by Siess et al. (1997) accretion delays the formation of a radiative core, which consequently begins further down the Hayashi track at a given mass. However the locus of equal mass points will subsequently move to higher luminosities than a non-accreting star of the same mass. Thus the mass determined by comparison with the Hayashi portion of any track can be either an underestimate or an overestimate. Figure 13 shows the relative error that might be made in estimating the mass over the region of interest in the H–R diagram. In the Hayashi region accretion generally leads to an underestimate of the age in a comparison with non-accreting tracks while it can lead to an underestimate during the Haynay phase. At any time these errors in age could be a factor of two or more. Figure 14 illustrates the error distribution for age estimates. In addition, not knowing the zero-age mass and radius of the star can lead to an absolute offset in age of up to about 10^6 yr so that any age estimate of less than about 2 \times 10^6 yr cannot be trusted. Ages larger than this can be significantly in error if accretion is taking place at an unknown rate but, once accretion has ceased, the age can be expected to correspond to non-accreting isochrones within a Kelvin–Helmholtz timescale. Thus the absolute error would be about equal to the age at which accretion became insignificant. In all other cases great care must be taken when estimating ages.

Apart from the initial comparatively small offsets, even if the initial conditions of a set of stars are known to be the same, relative ages are equally affected by accretion history. As a particular example, we may wish to decide whether two components of a pre-main-sequence binary star are coeval. If, as pre-main-sequence stars often do, one or both lies in the temperature range between 10^3.55 and 10^3.75 K where the error in age is likely to be more than a factor of two we

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**Figure 4.** As Fig. 3 but for pre-main-sequence stars of initial mass 0.1 \( M_\odot \) and radius such that the initial model is just about to cross the deuterium burning sequence.

**Figure 5.** The isochrones and equal-mass loci of figure 3 – solid lines – overlaid with our standard ones from figure 2 – dotted lines.

**Figure 15.** The estimated age and mass for each point along the 5 \times 10^6 yr fitted to our standard accreting tracks (Figure 2) when the luminosity and effective temperature are compared with the non-accreting tracks of Figure 1.

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can expect a significant difference in estimated age even in a coeval system. This binary example can be extended to star clusters. Accretion can lead to an apparent mass-dependent age-spread in otherwise coeval systems when non-accreting pre-main-sequence tracks are used to estimate ages. At any time, if all the stars in a cluster are coeval and began with the same initial core mass, the low-mass stars must have accreted less and hence have lower disc-accretion rates than those of higher mass which must have accreted more material. Thus accretion does not greatly affect the age determination of low-mass stars while higher-mass stars are more affected. Comparison with non-accreting tracks makes these appear older while on Hayashi tracks and younger on the Henyey tracks (see figure 14). Thus, intermediate-mass pre-main-sequence stars can look older than their low-mass counterparts (by up to a factor of five) while yet higher-mass stars can appear younger again. Figure 15 illustrates this point by plotting the estimated age against the estimated mass for all points along the $5 \times 10^6$ yr isochrone fitted to our standard accreting tracks. Though this is a particular case for a particular set of models this qualitative behaviour would be true of any coeval sample that is still undergoing accretion. Indeed mass-dependent ages have been recorded in several young stellar clusters where the lowest-mass stars appear youngest with increasing ages for the intermediate mass stars and lower ages again for the higher mass stars (Hillenbrand 1997; Carpenter et al. 1997). These mass-dependent ages may reflect ongoing disc-accretion rather than a dispersion in formation time and age determinations in these clusters should be reevaluated in the light of this work.

If the metallicity is not known the situation becomes even worse. For instance, as mentioned in the introduction, the metallicity of some extragalactic star forming regions, and possibly even Orion, may be as low as $Z = 0.001$. This would lead to an overestimate in mass by a factor of two or more and an overestimate in age by about a factor of ten if a comparison were inadvertently made with solar metallicity tracks.

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Figure 8. Three stars accreting at a constant rate of $10^{-7} \, M_\odot \, \text{yr}^{-1}$ from 0.5 $M_\odot$ upwards but reached by different routes from the same initial core of $M_0 = 0.1 \, M_\odot$ and $R_0 = 3 \, R_\odot$: solid line – constant $\dot{M} = 10^{-7} \, M_\odot \, \text{yr}^{-1}$; dashed line – linearly increasing from $\dot{M} = 0$ at $t = 0$ to $\dot{M} = 10^{-7} \, M_\odot \, \text{yr}^{-1}$ at $t = 8 \times 10^6 \, \text{yr}$ when $M = 0.5 \, M_\odot$; dotted line – linearly decreasing rate from $\dot{M} = 10^{-6} \, M_\odot \, \text{yr}^{-1}$ at $t = 0$ to $\dot{M} = 10^{-7} \, M_\odot \, \text{yr}^{-1}$ at $t = 7.3 \times 10^5 \, \text{yr}$ when $M = 0.5 \, M_\odot$. Open circles indicate masses of 0.5 and 1 $M_\odot$ for each of these tracks. Thin solid lines are our standard accreting tracks for $10^{-7.5}$, $10^{-6.5}$ and $10^{-6} \, M_\odot \, \text{yr}^{-1}$ and non-accreting tracks of 0.5 and 1 $M_\odot$ while the dashed line is the locus of 1 $M_\odot$ for the standard accreting tracks. Dots mark the zero-age sequences as elsewhere.

Figure 9. A Hertzsprung–Russell diagram showing constant-mass pre-main-sequence tracks of low metallicity ($Z = 0.001$) stars in the range 0.1 to 2.0 $M_\odot$. Isochrones of ages ranging from 10$^3$ to 10$^8$ yr are drawn across the tracks. The models were begun at radii large enough that these isochrones are not affected by small displacements of this starting point. The zero-age main and deuterium-burning sequences appear as dots logarithmically spaced in mass.

Figure 10. The pre-main-sequence tracks and isochrones for $Z = 0.001$ from figure 8 – solid lines – overlaid with those for solar metallicity from figure 1 – dotted lines.

Figure 11. A Hertzsprung–Russell diagram showing, as thin lines, tracks followed by pre-main-sequence stars of initial mass 0.1 $M_\odot$ and radius 3 $R_\odot$ and metallicity $Z = 0.001$ evolved with constant accretion rates ranging from $10^{-9}$ to $10^{-5.5} \, M_\odot \, \text{yr}^{-1}$. Thick lines are isochrones of 10$^5$ to 10$^7$ yr or join points of equal mass from 0.1 to 2.0 $M_\odot$. The zero-age main and deuterium-burning sequences appear as dots logarithmically spaced in mass.

Figure 12. The isochrones and equal-mass loci from figure 11 – solid lines – overlaid with the pre-main-sequence tracks and isochrones of figure 8 – dotted lines. The zero-age main and deuterium-burning sequences appear as dots logarithmically spaced in mass.

Figure 13. The distribution of the factor by which mass at a given point in the H–R diagram differs between our standard accreting tracks and non-accreting tracks. Over most of the diagram the difference is small being nowhere more than 50 per cent. At temperatures less than about 10$^{3.76}$ K the accreting mass is larger while at higher temperatures the accreting mass is smaller. Some non-accreting tracks and isochrones are overlayed and the shaded region is that for which we have both accreting and non-accreting tracks available. Twice as many accreting tracks as plotted in figure 2 were required to achieve the resolution of this figure.

Figure 14. The distribution of the factor by which age at a given point in the H–R diagram differs between our standard accreting tracks and non-accreting tracks. For temperatures less than about 10$^{3.76}$ K the accreting stars are younger (i.e. they appear older than they are when their age is estimated by comparison with non-accreting tracks) while at higher temperatures they are older. Some non-accreting tracks and isochrones are overlayed and the shaded region is that for which we have both accreting and non-accreting tracks available. Twice as many accreting tracks as plotted in figure 2 were required to achieve the resolution of this figure.
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