Gate-controlled spin polarized current in ferromagnetic single electron transistors

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We theoretically investigate spin dependent transport in ferromagnetic/normal metal/ferromagnetic single electron transistors by applying master equation calculations using a two dimensional space of states involving spin and charge degrees of freedom. When the magnetizations of ferromagnetic leads are in anti-parallel alignment, the spins accumulate in the island and a difference of chemical potentials of the two spins is built up. This shift in chemical potential acts as charge offset in the island and alternates the gate dependence of spin current. Taking advantage of this effect, one can control the polarization of current up to the polarization of lead by tuning gate voltages.

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Ferromagnetic single electron transistor (SET) has been an interesting system shown to exhibit novel phenomena with an interplay between spin and charge. Recently, Chen \textit{et al.} and Ono \textit{et al.} succeeded in fabricating small double junctions containing metal or superconductors sandwiched between two ferromagnets. In their experiments, enhanced tunneling magnetoresistance(TMR), magneto Coulomb oscillations and spin accumulations were observed. On the other hand, theories of ferromagnet/ferromagnet/ferromagnet(F/F/F) and ferromagnet/normal metal/ferromagnet(F/N/F) SET’s based on transition rate and master equation formalism were developed to derive bias-voltage and gate-voltage dependent TMR in both sequential and strong tunneling regimes. The pioneer experiment conducted by Johnson and Silsbee demonstrated the importance of spin accumulation effect on ferromagnet-normal metal systems. For F/N/F double junctions, the spin accumulation is predicted to occur when two ferromagnet leads are in anti-parallel alignment, which would lead to a new origin of TMR in contrast to that of F/F/F cases. In this study, we investigate the spin accumulation and related phenomena in F/N/F SET under the influence of gate charge.

The spin dependent transport in a ferromagnet is usually described by the relative difference of the majority and minority spins of conduction electrons, denoted as polarization $P$. Under the condition that spins do not flip, the transport current in F/N/F double tunnel junctions can be separated into two channels labeled as upspin and downspin which, throughout this article, are assumed to be contributed by majority and minority spins, respectively, in the source ferromagnet. For example, when lead magnetizations are in anti-parallel alignment, the upspin channel has a larger tunneling rate for the source junction than for the drain junction. In this case and in the steady condition, the upspin chemical potential in central electrode rises to balance the spin’s incoming and outgoing rates, and the chemical potential of the downspin would decrease by the same amount. This shift in spin chemical potential, denoted as $\Delta \mu_{\uparrow\downarrow}$, is predicted to be $\pm P eV_b/2$, in which $V_b$ is the applied bias voltage (see inset of Fig. 1) and $P$ is the polarization of the two leads. Therefore, the net spin in normal metal becomes nonzero and this effect is known as the spin accumulation (or spin imbalance).

A gate voltage $V_g$ is applied to turn on and off the charge transport in SET by tuning the electrostatic potential of the island. When SET is symmetrically biased with $V_g=0$, an energy cost of roughly the charging energy $E_C$ is required for adding or removing an excess charge in the island, and the charge transport is blocked provided that temperature is low, $k_B T \ll E_C$. When the gate voltage is tuned with two adjacent charge states becoming energetically degenerate, electrons can enter or leave the island without extra energy cost, producing sequential tunneling current. Ideally, the current is at a minimum and maximum, respectively, for $V_g=0$ and $e/2C_g$ and can be modulated periodically with a period of $\Delta V_g = \epsilon/C_g$; here $C_g$ is the island-to-gate capacitance. These two gate voltages are thus referred to as minimal and maximal gate voltages. Our study suggests that the shift in spin chemical potentials generated by spin accumulation produces an effective charge offsets.

To demonstrate this charge offset effect, we apply a modified master equation calculation which takes into account the spin dependent charge states of the island. In this framework, the states are described by two parameters, $Q_\uparrow$ and $Q_\downarrow$, denoting excess upspin charge and excess downspin charge, respectively. Comparing with spin-independent case, each primitive charge state $|Q\rangle$ is built up by spin charge states of $Q = Q_\uparrow + Q_\downarrow$. Generally speaking, these spin dependent charge states are in non-equilibrium with the presence of spin accumulation. However, under limit of short energy relaxation time, the occupation distribution of a particular spin would form an equilibrium Fermi distribution, i.e. $f(\epsilon - \mu_T(\downarrow))$. The numbers of net spin $N = (Q_\uparrow - Q_\downarrow)/e$ is related to the spin chemical potentials and the density of states of the island $\rho(\epsilon)$ as $N = \int d\epsilon \rho(\epsilon) (f(\epsilon - \mu_\uparrow) - f(\epsilon - \mu_\downarrow))$.

Although the electrostatic energy of each charge state is spin independent, the tunneling rate is spin dependent.
in two ways: first, the effective tunneling resistances for the majority and minority spin tunneling processes are multiplied respectively by \(2/(1-P)\) and \(2/(1+P)\); second, the spin chemical potential shift’s presence also modifies transition rates by changing the numbers of possible tunneling processes. Consequently, the master equation for each spin charge state, together with certain spin flipping transitions, reads

\[
\frac{dp_{ij}}{dt} = \sum_{l=S,D} \left\{ \Gamma_l^i(i,j|i\uparrow,j) p_{i\uparrow,l+1,j} + \Gamma_l^i(i,j|i,j) p_{i,j,l+1} \right\} - \sum_{l=S,D} \left\{ \Gamma_l^i(i\uparrow,j|i,j) + \Gamma_l^i(i,j\uparrow|i,j) \right\} p_{ij} + \left( \frac{dp_{ij}}{dt} \right)_{sf},
\]

in which \(i = Q_1/e\) and \(j = Q_2/e\) denote the numbers of upspins and downspins respectively, and \(\Gamma_l^i(i',j'|i,j)\) is the tunneling rate for spin direction \(s(=\uparrow, \downarrow)\) in junction \(l\) (S for source and D for drain) from states |\(i,j\rangle\) to |\(i',j'\rangle\), and \(p_{ij}\) is the probability that the island is in state |\(i,j\rangle\). In this equation, only sequential tunneling process is considered. By introducing an energy-independent spin relaxation time \(\tau_s\), the spin flipping transitions can be explicitly written as:

\[
\left( \frac{dp_{ij}}{dt} \right)_{sf} = \frac{1}{\tau_s} \left\{ \frac{U/-\delta}{1-\exp(-\beta U)} p_{i+1,j-1} + \frac{-U/-\delta}{1-\exp(\beta U)} p_{i-1,j+1} \right\} = \frac{1}{\tau_s} \left( \frac{U}{\delta} \right) 1 + \exp (-\beta U) \right) p_{ij}
\]

The first and second terms describe respectively the increase of probability due to up-to-down and down-to-up flip processes, and the third term is the decrease arising from the opposite processes. Here we assume that the up-to-down spin flipping rate is proportional to \(f(\varepsilon - \mu_\uparrow)(1-f(\varepsilon - \mu_\downarrow))\) for electron with energy \(\varepsilon\). \(U = \mu_\uparrow - \mu_\downarrow = (i-j)\delta\) is the chemical potential difference of upspins and downspins, and \(\delta \sim 1/\rho\) is the energy level spacing, a constant for normal metal. For positive \(U\), up-down spin flip is favorable, while for negative \(U\), down-to-up spin flip dominates. Under these conditions the probabilities of major spin states with large \(|U|\) are greatly reduced while suppressing the spin accumulation.

Equation (1) can be solved under the stationary condition given by \(dp_{ij}/dt = 0\) as described in the spin independent case. Through a particular distribution of \(p_{ij}\), one can obtain the amount of spin accumulation, quantified as average chemical potential difference, \(\overline{\varepsilon} = \sum_{i,j}(i-j)\delta p_{ij}\), and the spin current for spin \(s\) tunneling through junction \(l\),

\[
I_s = e \sum_{ij} \left[ \Gamma_l^i(i+1,j|i,j) - \Gamma_l^i(i-1,j|i,j) \right] p_{ij}.
\]

If there is no spin flipping processes, the spin is conserved and the spin current passing through the source and drain junctions is the same. If the spin flips too quickly so as to completely destroy the spin accumulation, then the ratios between the two spin currents, \(I_\uparrow/I_\downarrow\), for source and drain junctions, will be the same as polarization of source and drain electrodes, respectively.

To gain an understanding about this phenomena, here, we perform a simulation using device parameters similar to those in experiments et al. \(R_S = R_D = 400k\Omega\), \(C_S = C_D = 300aF\), \(C_\uparrow = 0.8aF\), \(P_S = P_D = 0.4\). Because the resistances are much higher than quantum resistance \(R_Q\), the contribution due to higher order tunneling processes is negligible and only sequential tunneling process is included. We consider \(IV_\uparrow\) characteristics and current-gate voltage dependences \((IV_\uparrow)\) with both parallel and anti-parallel alignment of leads under the no spin-flipping condition at a temperature of \(k_B T/E_C = 0.1\). In the parallel configuration, no particular feature is found because the ratio between two spin currents is simply the polarization 0.4, and total current is the same as that of the spin independent case. When the leads are in anti-parallel alignment, the calculation provides much more interesting results. The total current is smaller than that of the parallel case, and the high bias differential resistance is increased by a factor of \(1/(1-P^2)\) and shows a generic \(E/N/F\) TMR effect. The differential TMR as a function of bias voltage also exemplifies expected oscillatory behaviors.

The \(IV_\uparrow\) characteristics shown in Fig. 3a exhibit particularly different behaviors than from the parallel case. The upspin and downspin currents are only the same at \(V_0 = 0\) and \(V_0 = e/2C_\uparrow\). A closer inspection reveals that the peaks of two \(IV_\uparrow\) curves with opposite spins shift with increasing bias voltage. This effect can be explained when we consider two separated spin transport channels. When the leads are in anti-parallel alignment, the source and drain resistances for a particular spin channel may differ by several times. This results in a step-like structure, called Coulomb staircase, in the \(IV_\uparrow\) characteristics, and a distorted saw-tooth-like \(IV_\uparrow\) modulation. The Coulomb staircase effect can explain the TMR oscillation and the asymmetric gate dependence of spin currents. However, for a more rigorous study, it is necessary to include the spin entanglement term in the Hamiltonian, \(2E_C Q_1 Q_\downarrow/e^2\). In fact, our calculations suggest that the results of the two methods differ especially at high bias voltages where both \(Q_1\) and \(Q_\downarrow\) are large.
From the view point of spin accumulation, the raised upspin (lowered downspin) chemical potential effectively gives rise to a positive (negative) charge offset. At low bias voltage regime ($V_b < 2E_C/e$), the electrical current increases due to suppression of Coulomb blockade. The spin accumulation is, in turn, enhanced by the increased current, and consequently there is a rise in both upspin chemical potential and the effective charge offset. Since within $0 < V_b < e/2C_g$ region, the charge offset is an ascending function of $V_g$, and the upspin current increases more rapidly than that of the zero spin accumulation. On the other hand, the increment of downspin current is less effective, because the downspin chemical potential decreases as the current polarizations reach a maximum value of $\pm$.

Therefore, at low bias regime, the $IV_g$ for upspin and downspin are tilted respectively toward lower and higher $V_g$ directions, and form saw-tooth-like $IV_g$ patterns. At bias voltages far beyond threshold ($V_b \gg 2E_C/e$), the current is not much affected by $V_g$ and the spin chemical potential is less sensitive to $V_g$. The shift in spin chemical potential (relative to the no spin accumulation case) $\Delta \mu_{\pm}$ increases (decreases) with $V_b$. At $V_b = 6E_C/e$, $\Delta \mu_{\pm}$ is about $\pm EC$, corresponding to a charge offset of about $\pm e$. Consequently, as shown in Fig. 3, the upspin and downspin $IV_g$ characteristics shift by one period in respect with each other and differ from $IV_g$ characteristics at low bias voltages by half period. The total current is shown in Fig. 4, allowing a comparison with the experiments.

For further investigation of the effect of applied gate voltage on two spin currents, we define a quantity describing the polarization of the tunneling current: $P_I = (I_I - I_D)/(I_I + I_D)$. Fig. 4 shows bias and gate voltage dependence of $P_I$. Such dependence suggests the possibility of using a ferromagnetic SET as a gate-controlled current polarizer. Because of large $P_I$ values, the optimum operating regime is at low bias voltage. At $\epsilon V_g/C_g = \pm 0.15$ and $\epsilon V_b/E_C = 0.7$, the current polarizations reach a maximum value of $\pm 0.33$. One can also explore the temperature dependence of the polarization current. There are two ways that the effects of temperature can enter, both leading to the destruction of current polarization. One is thermal activated charge fluctuation and the other is decrease of spin flip time. The former is automatically included in the master equation calculation and its effect is shown in Fig. 4(a). At $T = 0$, the value of $P_I$ can be as large as the polarization of the lead itself, while at $k_B T \gtrsim 0.5E_C$, the gate charge effect becomes negligible. To evaluate the effect of the spin flip process, we assume an energy independent spin flipping time and an energy level spacing $\delta$ of 1$\mu$eV in the island, and perform the calculations using the same device parameters as above. Fig. 4(b) shows the current polarization for drain junction operating at $V_b = 0.7E_C/e$ as a function of gate voltage at $k_B T/E_C = 0.1$ under several spin flipping times. Clearly, when the spin flip time is short as compared with the tunneling time $\tau_I = e/I$ of approximately 10ns, the spin accumulation diminishes and $P_I = -0.4$, which is simply the polarization of the drain electrode. However, since the chemical potential is proportional to the island’s density of states, the required spin flip time would be shorter for nanometer-sized normal-metal islands in which the level spacing is of the order of $10^{-8} ~ 10^{-9}eV$, which is much smaller than the assumed value. It has been proposed that the criteria for spin accumulation is related to the tunneling resistance $R_t$ as $\tau_I\delta/h > R_t/R_Q[4]$. Our calculation results agree with this prediction.

The co-tunneling processes, which are thus far not included in our calculations, can also give induce effective spin flipping. In the spin independent case, co-tunneling is a second order process that preserves the charge state but also produces current. In the Coulomb blockade regime, where sequential tunneling is suppressed, the current is mainly due to co-tunneling. For spin co-tunneling, there are spin-conserved and spin non-conserved processes. The latter is that a spin enters the island and an opposite spin leaves. The forward and backward co-tunneling rates $\Gamma_{co}^\pm$, $\Gamma_{co}$ for F/N/F SET can be written as:

$$\Gamma_{co}^\pm = \Gamma_{co} = \frac{R_Q}{4\pi^2e^2R_{S,eff}R_{D,eff}} \int d\varepsilon \frac{\Delta E - \varepsilon}{1 - \exp(-\beta\varepsilon)} \left(\bar{M}\right)^2,$$

(4)

where

$$\bar{M} = \frac{1}{E_{S}^\pm + \varepsilon + i\gamma^\pm} + \frac{1}{E_{D}^\pm + \Delta E - \varepsilon + i\gamma^\mp}.$$

(5)

The $\Delta E$ is the energy difference between initial and final states. For spin conserved co-tunneling, $\Delta E = \pm eV$, whereas for up-to-down and down-to-up co-tunneling $\Delta E = \pm eV - U$ and $\Delta E = \pm eV + U$, respectively (‘+’ for forward and ‘-’ for backward). $E_{S}^\pm$ and $E_{D}^\pm$ are energy changes of the tunneling processes $Q \rightarrow Q \pm e$ for source and drain junctions. $R_{S,eff}$ and $R_{D,eff}$ are the effective tunneling resistances for the source and drain junctions. Note that for anti-parallel configuration, $R_{S,eff}R_{D,eff}$ product in Eq. 4 is $R_SR_D$ product multiplied by $4/(1 - P_S)(1 - P_D)$, $4/(1 + P_S)(1 + P_D)$ and $4/(1 - P_S)(1 - P_D)$ for up-to-up, down-to-down, up-to-down and down-to-up forward co-tunneling events, respectively. $\gamma^\pm$ are decay rates for the final charge states $Q \pm e$ of the two processes and are given by

$$\gamma^\pm = \Gamma_{co} \left(\frac{E_{S}^\pm}{R_{S,eff}} \coth \frac{\beta E_{S,eff}^\pm}{2} + \frac{E_{D}^\pm}{R_{D,eff}} \coth \frac{\beta E_{D,eff}^\pm}{2}\right).$$

(6)
To investigate the co-tunneling spin flipping, we use tunneling resistances of 40kΩ, which is closer to $R_Q$, and leave other parameters unchanged. In the Coulomb blockade regime, the co-tunneling spin flipping rate can be as large as $10^8$ Hz, which is comparable to the tunneling rate of $1/e \sim 10^9$Hz. However, different from the spin relaxation as discussed above, the spin flipping induced by co-tunneling for $|U| \ll eV_b$ does not have preferred direction. That is, it does not smear out the spin accumulation but rather only enhances spin fluctuation. Fig. 4 shows the spin fluctuation discussed above, the spin flipping induced by co-tunneling for the two spin currents appear at different gate voltages. At the text. In (a) the dotted and solid curves represent respectively up-spin and down-spin currents. Notice that the peaks and

\[
\delta N = \sqrt{U^2 - U^2}/\delta
\]
as a function of gate voltage at $V_b = 0.75E_C/e$ with and without consideration of co-tunneling processes.

In summary, we proposed theoretically a gate-controlled polarized current in ferromagnetic single electron transistors. Using modified master equation formalism, we calculate spin sequential tunneling rates and spin current when spin accumulation is present. The spin accumulation-induced chemical potential shift behaves like charge offsets, producing interesting effects to the $IV_g$ characteristics. When the gate voltage is tuned away from the maximal and minimal gate voltages, the current passing through the junctions is polarized. The thermal fluctuation and spin flipping processes are both shown to suppress the effects from charge offset. The co-tunneling event provides effective spin flipping processes, but it only enhances spin fluctuation. Taking advantage of the gate dependence of polarized current in a ferromagnetic SET, one can utilize this system as a tunable current polarizer.

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FIG. 1. Current-gate voltage dependences for (a) the spin currents from $eV_b = 0.5E_C$ (bottom) to $6.0E_C$ (top) and (b) total current from $eV_b = 0.5E_C$ (bottom) to $4.0E_C$ (top) with a step $0.5E_C$ for an F/N/F SET, whose parameters are described in the text. In (a) the dotted and solid curves represent respectively up-spin and down-spin currents. Notice that the peaks and valleys for the two spin currents appear at different gate voltages. At $eV_b = 4.5E_C$ the two currents shift half period while near $eV_b = 6.0E_C$, they are the same. In (b) $IV_g$ curves shift with bias voltages: at $eV_b = 6.0E_C$, it shifts by 0.5e. (c) The scheme of the F/N/F SET considered. The arrows indicate the magnetizations of electrodes.

FIG. 2. (a) Polarization of current dependences on bias voltage at $eV_g/C_g = 0.15$ (solid curve) and -0.25 (dotted curve). At $eV_b/E_C = 0.7$ and $eV_g/C_g = \pm 0.15$, $P_I$ reaches a maximum value of $\pm 0.33$ while at $eV_b/E_C = 4.0$ and $eV_g/C_g = \pm 0.25$, $P_I$ has another maximum, which is approximately 0.04 with an opposite direction. (b) Polarization as a function of gate voltage at $eV_b/E_C = 0.5$ to 4.0 with a step 0.5.
FIG. 3. (a) Gate voltage dependences of polarization at $eV_b/E_C=0.7$ for temperatures ranging between 0 and $0.5E_C/k_B$ with a step 0.05 under no spin flip assumption. At $T = 0$, $P_I$ cannot be defined for $C_gV_g/e < 0.33$ and $C_gV_g/e > 0.67$ since the current is zero within that range. The effect is more pronounced at lower temperatures. (b) The same dependence for the drain junction at $eV_b/E_C=0.7$ and $k_B T/E_C = 0.1$ with several spin flipping times $\tau_s$, from bottom to top: 10ns, 100ns, 1µs, 10µs, 100µs (solid curves), and $\infty$ (dotted curve). For $\tau_s = 10$ns, $P_I$ is fairly close to the $P$ value of the drain electrode, i.e. no spin accumulation. For $\tau_s = 100$µs, spin accumulation is almost the same as the non-flipping case and $P_I$ increases dramatically.

FIG. 4. Spin fluctuation $\delta N$ v.s. gate voltage $V_g$ in the Coulomb blockade regime, $V_b = 0.1$mV. The solid and dotted curves represent respectively the result for sequential tunneling with and without co-tunneling processes. The fluctuation is increased by about 20%. The inset shows the average spin number $N_{ave} = \langle U \rangle / \delta$ for the two cases.
Figure (a) shows plots of current density ($I$ in nA) versus $C_g V_g / e$ for different values of $C_g V_g / e$. Figure (b) displays a similar set of plots for $C_g V_g / e$. Figure (c) presents a schematic diagram of the system's circuit configuration, with components $C_S$, $R_S$, $C_D$, and $R_D$ labeled.
\[ \tau_s = 100 \mu s \]

\[ \tau_s = 10 \text{ns} \]

\[ k_B T = 0 \]

\[ k_B T = 0.5 E_C \]
