ABSTRACT

These are the proceedings of the workshop on “Lattice QCD, Chiral Perturbation Theory and Hadron Phenomenology” held at the European Centre for Theoretical Studies in Nuclear Physics and Related Areas from October 2 to 6, 2006. The workshop concentrated on bringing together researchers working in lattice QCD and chiral perturbation theory with the aim of improving our understanding how hadron properties can be calculated and analyzed from first principles. Included are a short contribution per talk.

* This workshop was funded by the ECT* and by the EU Integrated Infrastructure Initiative Hadron Physics Project under contract number RII3-CT-2004-506078. Work supported in part by DFG (SFB/TR 16 “Subnuclear Structure of Matter” and FOR 465 “Forscherguppe Gitter-Hadronen-Phänomenologie”) and by BMBF (research grant 06BN411).
1 Introduction

As a result of developments in lattice field theory and computer technology, the first full QCD calculations in the chiral regime are becoming available now. An essential tool for extracting hadronic quantities from lattice QCD is chiral perturbation theory. Chiral perturbation theory is the effective field theory of QCD and allows to analyze the quark mass dependence of observables in a model-independent manner. Since the technologies for large scale lattice calculations and for detailed chiral perturbation calculations are each so different and so demanding, the same theorists generally do not pursue both aspects. Hence, we considered it to be valuable to bring together practitioners of both aspects of lattice calculations, and deepen each others’ appreciation of the issues involved, thus opening a new era for confronting experiment with solutions of QCD from first principles.

Thus, such a meeting was organized at the ECT* (Trento) from October 2-6, 2006, with financial support from the ECT* as well as the I3HP networks N2 and N5 and by EURONS. The meeting had 36 participants whose names, institutes and email addresses are listed below. 35 of them presented results in presentations of various lengths. Ample time was left for discussions on the various talks each day. In addition, there was also a special discussion session “Lattice meets chiral perturbation theory” convened by Hartmut Wittig, in which the interplay between the various issues as seen from the two communities were addressed. A short description of the contents of each talk and a list of the most relevant references can be found below. We felt that this was more appropriate a framework than full-fledged proceedings. Most results are or will soon be published and available on the archives, so this way we can achieve speedy publication and avoid duplication of results in the archives.

Below follows first the program, then the list of participants followed by the abstracts of the talks. The summary of the aforementioned discussion session is given at the end of these proceedings. Most talks can also be obtained from the workshop website

http://www.ect.it/

We would like to thank the ECT*, in particular Serena Degli Avancini, for the excellent organization of the workshop and all participants for their valuable contributions. We believe that this was only the first workshop of this kind and look forward to similar meetings in the future.

Ulf-G. Meißner and Gerrit Schierholz
2 Program

Monday, October 2nd 2006

Morning session, chair: Evgeny Epelbaum

9:45 Ulf-G. Meißner / Opening Remarks
9:50 Luigi Scorzato (Trento) Dynamical simulations with twisted mass
10:25 Gerrit Schierholz (Hamburg) Lattice QCD simulations at small quark masses
11:00 Coffee
11:30 Christian Lang (Graz) Hadrons for chirally improved fermions
12:15 Meifeng Lin (New York) Dynamical domain wall fermion simulations and chiral perturbation theory
13:00 Lunch

Afternoon session, chair: Bugra Borasoy

14:30 Christopher Aubin (New York) Applications of staggered ChPT
15:15 Stephan Dürr (Bern) Are there alternatives to rooted staggered fermions?
15:50 Coffee
15:15 Sinya Aoki (Tsukuba) ChPT and lattice QCD with Wilson-type quarks
17:05 Discussions
18:00 End of Session

Tuesday, October 3rd, 2006

Morning Session, chair: Meinulf Göckeler

09:00 Oliver Bär (Berlin) Lattice QCD with mixed actions:
Overlap fermions on a twisted mass sea
09:35 Ulf-G. Meißner (Bonn) Thoughts on chiral extrapolations for excited states
10:10 Véronique Bernard (Strasbourg) Chiral extrapolations of baryon properties
10:45 Coffee
11:15 Matthias Schindler (Mainz) Some topics in manifestly Lorentz-invariant CHPT
11:50 Evgeny Epelbaum (Jülich) Chiral extrapolations in few-nucleon systems
12:25 Lunch

Afternoon session, chair: Christopher Aubin

14:30 Bugra Borasoy (Bonn) Nuclear lattice simulations
15:15 José A. Oller (Murcia) Chiral non-perturbative study of pseudoscalar self-energies
15:50 Coffee
16:20 Hermann Krebs (Jülich) Scalar two-loop diagrams on the lattice
16:55 Dina Alexandrou (Nicosia) $N$ and $N$ to $\Delta$ form factors in lattice QCD
17:40 Discussions
18:00 End of Session
Wednesday, October 4th, 2006

Morning Session, chair: Akaki Rusetsky

09:00 David Richards (Newport News) Hadron structure using DWF quarks on an asqtad sea
09:45 Wolfram Schroers (Zeuthen) Distribution amplitudes from the lattice
10:20 Dirk Brömmel (Hamburg) Pion structure from lattice QCD
10:55 Coffee
11:55 Dirk Pleiter (Zeuthen) Nucleon structure functions and form factors
12:10 Enno Scholz (Upton) $B_K$ from dynamical domain wall fermion simulations
12:45 Lunch

Afternoon Session, chair: Meifeng Lin

14:30 Hartmut Wittig (Mainz) Low-energy constants for $\Delta S = 1$ transitions
15:05 Akaki Rusetsky (Bonn) The $\Delta$-resonance in a finite volume
15:40 Coffee
16:10 Meinulf Göckeler (Regensburg) Lattice QCD on smaller and larger volumes
16:55 Tobias Gail (Garching) Utilizing covariant BChPT for chiral extrapolations
17:30 Discussions
18:00 End of Session

Thursday, October 5th, 2006

Morning Session, chair: Oliver Bär

9:00 Christoph Haefeli (Bern) Aspects of CHPT at large $m_s$
9:45 Poul Damgaard (Copenhagen) A chiral two-matrix theory and the chiral Lagrangian
10:30 Coffee
11:00 Jack Laiho (Batavia) The $B \to D^*\ell\nu$ form factor from lattice QCD
11:45 Michele Della Morte (Geneva) Heavy quark effective theory on the lattice:
12:25 Lunch

Afternoon Session, chair: Keh-Fei Liu

14:30 Wolfgang Bietenholz (Belin) Overlap hypercube quarks in the $p$- and the $\epsilon$-regime
15:05 Discussion: Lattice meets chiral perturbation theory‡
16:35 End of Session
Friday, October 6th, 2006

*Morning Session, chair: Christian Lang*

09:00 Keh-Fei Liu (Kentucky)  \( \sigma(600) \) as a tetraquark mesonium and the pattern of scalar mesons

09:35 Marina Dorati (Pavia)  Moments of generalized parton distribution functions

10:10 Andreas Schäfer (Regensburg)  ChPT for generalized parton distributions (GPDs)

10:20  \textit{Coffee}

11:15 Volker Weinberg (Zeuthen)  The QCD vacuum as seen by overlap fermions

11:50 Pietro Faccioli (Trento)  The chiral regime of QCD in the instanton liquid model

12:25 Christian Lang (Graz)  Farewell

12:30  \textit{Lunch and End of Workshop}

† no abstract was provided
‡ summary given at the end of these mini-proceedings
# Participants and their email

| Name               | Institution                        | Email                          |
|--------------------|------------------------------------|--------------------------------|
| C. Alexandrou      | Univ. of Cyprus                    | alexand@ucy.ac.cy              |
| S. Aoki            | Univ. Tsukuba                      | saoki@het.ph.tsukuba.ac.jp     |
| C. Aubin           | Columbia Univ.                     | caubin@phys.columbia.edu       |
| O. Bär             | Humboldt Univ. Berlin              | obaer@physik.hu-berlin.de      |
| V. Bernard         | ULP Strasbourg                      | bernard@lpt6.u-strasbg.fr     |
| W. Bietenholz      | Humboldt Univ.                     | bietenho@physik.hu-berlin.de   |
| B. Borasoy         | Univ. Bonn                         | borasoy@itkp.uni-bonn.de       |
| D. Brömmel         | DESY                               | dirk.brommel@desy.de           |
| P. H. Damgaard     | Niels Bohr Institute                | phdamg@nbi.dk                  |
| M. Della Morte     | CERN                               | dellamor@mail.cern.ch          |
| M. Dorati          | Univ. Pavia                        | marina.dorati@pv.infn.it       |
| S. Dürr            | Bern Univ.                         | durr@itp.unibe.ch              |
| E. Epelbaum        | FZ Jülich & Univ. Bonn             | epelbaum@fz-juelich.de         |
| P. Faccioli        | Univ. Trento                       | faccioli@science.unitn.it      |
| T. Gail            | TU München                         | tgail@ph.tum.de                |
| M. Göckeler        | Univ. Regensburg                    | meinulf.goekeler@physik.uni-regensburg.de |
| C. Haefeli         | Bern Univ.                         | haefeli@itp.unibe.ch           |
| K. Koller          | LMU München                         | koller@lrz.uni-muenchen.de     |
| H. Krebs           | FZ Jülich & Univ. Bonn             | hkrebs@itkp.uni-bonn.de        |
| J. Laiho           | Fermilab                           | jlaiho@fnal.gov                |
| C. B. Lang         | Univ. Graz                          | christian.lang@uni-graz.at     |
| M. Lin             | Columbia Univ.                     | mflin@physics.columbia.edu     |
| K.-F. Liu          | Kentucky Univ.                     | liu@pa.uky.edu                 |
| U.-G. Meißner      | Univ. Bonn & FZ Jülich             | meissner@itkp.uni-bonn.de      |
| J. Oller           | Univ. Murcia                       | oller@um.es                    |
| D. Pleiter         | DESY                               | Dirk.Pleiter@desy.de           |
| D. Richards        | Jefferson Lab                      | dgr@jlab.org                   |
| A. Rusetsky        | Univ. Bonn                         | rusetsky@itkp.uni-bonn.de      |
| A. Schäfer         | Univ. Regensburg                    | andreas.schafer@physik.uni-regensburg.de |
| G. Schierholz      | DESY                               | Gerrit.Schierholz@desy.de      |
| M. Schindler       | Univ. Mainz                        | schindle@kph.uni-mainz.de      |
| E. E. Scholz       | Brookhaven National Lab            | scholze@quark.phy.bnl.gov      |
| W. Schroers        | DESY                               | Wolfram.Schroers@desy.de       |
| L. Scorzato        | ECT Trento                         | scorzato@ect.it                |
| V. Weinberg        | DESY                               | Volker.Weinberg@desy.de        |
| H. Wittig          | Univ. Mainz                        | wittig@kph.uni-mainz.de        |
Results from Lattice QCD simulations with a twisted mass

Luigi Scorzato¹, on behalf of the ETM Collaboration

¹ECT*, strada delle tabarelle 286. 38050 - Villazzano (TN). Italy

We report on first results of an ongoing effort to simulate lattice QCD with two degenerate flavours of quarks by means of the twisted mass formulation tuned to maximal twist. We obtain pseudo-scalar masses below 300 MeV on volumes with spatial size larger than 2 fm at values of the lattice spacing similar or smaller than 0.1 fm. [1]. We present first comparison with ChPT including Finite Size Effects. With the help of Wilson-ChPT we give evidence that $O(a^2)$ lattice artifacts are under control. Additionally, exploratory results for the case of $N_f = 2 + 1 + 1$ flavours are discussed.

References

[1] K. Jansen and C. Urbach [ETM Collaboration], arXiv:hep-lat/0610015
Lattice QCD Simulations at Small Quark Masses

Gerrit Schierholz

Deutsches Elektronen-Synchrotron DESY
D-22603 Hamburg, Germany
John von Neumann-Institut für Computing NIC
Deutsches Elektronen-Synchrotron DESY
D-15738 Zeuthen, Germany

– For the QCDSF Collaboration –

Due to improvement in algorithms and computer performance unquenched simulations of Wilson-type fermions with lighter quark masses are now possible [1], [2]. This enables us to make contact with chiral perturbation theory (ChPT) and the real world.

We report results for the pseudoscalar decay constants $f_{PS}^{AB}$ at pion masses down to $O(300)$ MeV, using $N_f = 2$ nonperturbatively $O(a)$ improved Wilson fermions. For calculational details see [3]. The renormalzation constants and improvement coefficients of the axial vector current are computed nonperturbatively as well.

We first investigate to see if we are entering a regime, where chiral logarithms are becoming visible, and look at the ratio of nondegenerate, partially quenched decay constants [4]

$$R = \frac{f_{PS}^{VS}}{\sqrt{f_{PS}^{VV} f_{PS}^{SS}}}$$

($V$: valence, $S$: sea quark). Our data displays chiral logarithms of about the expected size. Next we fit our data to the predictions of $NLO$ ChPT. We obtain $\alpha_4 \approx -0.58$, $\alpha_5 \approx -0.45$. While $\alpha_4$ is in reasonable agreement with other phenomenological estimates, $\alpha_5$ is not. Using $r_0 = 0.5$ fm [$r_0 = 0.467$ fm] to set the scale, we find $f_{K^+}/f_{\pi^+} = 1.24$ [$f_{K^+}/f_{\pi^+} = 1.22$], to be compared with the experimental value 1.223, and $f_{\pi^+} = 76(4)(2)$ MeV [$f_{\pi^+} = 81(4)(2)$ MeV]. The first error is statistical, while the second error is due to the error in $r_0/a$.

References

[1] M. Gökeler et al., [hep-lat/0610066].
[2] M. Gökeler et al., hep/lat/0610071.
[3] M. Gökeler et al., Phys. Rev. D57 (1998) 5562 [hep/lat/9707021].
[4] S.R. Sharpe, Phys. Rev. D56 (1997) 7052, Erratum ibid. D62 (2000) 099901.
Hadrons for Chirally Improved Fermions

T. Burch\textsuperscript{1}, C. Gattringer\textsuperscript{2}, L. Y. Glozman\textsuperscript{2}, C. Hagen\textsuperscript{1},
D. Hierl\textsuperscript{1}, \textbf{C. B. Lang}\textsuperscript{2} and Andreas Schäfer\textsuperscript{1}

(BGR [Bern-Graz-Regensburg] Collaboration)

\textsuperscript{1}Inst. f. Theoretische Physik, Universität Regensburg, D-93040 Regensburg,
\textsuperscript{2}Inst. f. Physik, FB Theoretische Physik, Karl-Franzens-Universit"at Graz,
A-8010 Graz,Austria

We present results for masses of excited mesons \textsuperscript{1} and baryons \textsuperscript{2} from a quenched calculation with Chirally Improved quarks at pion masses down to 350 MeV. Our analysis of the correlators is based on the variational method. It was shown \textsuperscript{3} that in this approach ghost-channels may be identified safely. The key features are the use of a matrix of correlators from various source and sink operators and a basis which includes Jacobi smeared quark sources with different spatial widths, thereby improving overlap with states exhibiting radial excitations. In order to provide a large basis set for spanning the physical states, we also use interpolators with different Dirac structures. Our spectroscopy results for a wide range of ground state and excited hadrons are discussed.

The figures show the results of straightforward extrapolations to the chiral region (mesons and positive as well as negative parity baryons). The horizontal bars represent the experimental numbers (where known). Filled symbols are used for those states where no corresponding state is listed in the particle data summary.

References

\textsuperscript{1} T. Burch, C. Gattringer, L. Ya. Glozman, C. Hagen, C. B. Lang, and Andreas Schäfer, Phys. Rev. D \textbf{73} (2006) 094505, [arXiv: hep-lat/0601026].

\textsuperscript{2} T. Burch, C. Gattringer, L. Ya. Glozman, C. Hagen, D. Hierl, C. B. Lang, and A. Schäfer, Phys. Rev. D \textbf{74} (2006) 014504, [arXiv: hep-lat/0604019].

\textsuperscript{3} Tommy Burch, Christof Gattringer, Leonid Ya. Glozman, Christian Hagen, and C. B. Lang, Phys. Rev. D\textbf{73} (2006) 017502, [arXiv: hep-lat/0511054].
Dynamical domain wall fermion simulations and the chiral perturbation theory

Meifeng Lin
Department of Physics, Columbia University, New York, NY 10027, USA

The domain wall fermion formulation has the advantage of possessing exact flavor symmetry, and chiral symmetry is only mildly broken by a controllable amount. This allows us to compare the numerical results from the domain wall fermion simulations directly to the predictions of the continuum chiral perturbation theory \[1\], up to corrections from the residual chiral symmetry breaking, quantified as \(m_{\text{res}}\) \[2\], and possible \(\mathcal{O}(a)\) lattice artifacts. From a field theoretic point of view, the residual mass \(m_{\text{res}}\) serves as an additive mass renormalization to the input quark mass, and the chiral extrapolations are much simplified compared to the Wilson or Staggered fermion formulation. For domain wall fermions, the contamination from higher-dimension operators as a result of the \(\mathcal{O}(a)\) lattice artifacts is also exponentially suppressed by the span of the fifth dimension \[3\]. And the Symanzik effective action can be written as

\[
S_{\text{eff}} = \int d^4x [\bar{\psi}(x) (i\gamma_{\mu} D^\mu - m_q) \psi(x)] + ae^{-\alpha L} c_{\text{def}} \bar{\psi}(x) \sigma^{\mu\nu} F_{\mu\nu} \psi(x) \tag{1}
\]

where \(m_q = m_f + m_{\text{res}}\). Guided by this equation, I show that to the next-to-leading order, including the \(\mathcal{O}(a)\) chiral symmetry breaking term in the chiral expansions introduces only one new parameter \[3\].

I also present numerical results for the pseudoscalar masses \(M_{PS}\) and decay constants \(f_{PS}\) from a series of 2+1 flavor dynamical domain wall fermion simulations carried out by the RBC and UKQCD collaborations \[4\]. When the Goldstone pion mass is above 400 MeV, the numerical results are not consistent with the NLO chiral perturbation theory in that simultaneous fits of \(M_{PS}^2\) and \(f_{PS}\) fail badly. However, for pion masses below 400 MeV, our preliminary data shows reasonable consistency with the predictions of NLO ChPT.

References

[1] J. Gasser and H. Leutwyler, Nucl. Phys. B 250 (1985) 465.
[2] T. Blum et al., Phys. Rev. D 69 (2004) 074502 arXiv:hep-lat/0007038.
[3] M. Lin, PoS LAT2006 (2006) 185 arXiv:hep-lat/0610052.
[4] C. Allton et al., PoS LAT2006 (2006) 096
Applications of Staggered Chiral Perturbation Theory

C. Aubin

Dept. of Physics, Columbia University,
New York, NY USA

One can account for the chiral symmetry violations that arise from staggered fermions on the lattice [1]. Given current lattice simulations performed by the MILC Collaboration, the $\mathcal{O}(a^2)$ taste-violating errors are of the same order as the light quark masses. Some results using staggered chiral perturbation theory (S$\chi$PT) when applied to light meson physics have been presented in Ref. [2].

This can be extended to include heavy quark effective theory with S$\chi$PT to understand the chiral behavior of lattice data for heavy-light quarks [3]. From this formalism, several results have been obtained in Ref. [4], most notably a prediction of the $D$ decay constant before they were measured by CLEO-c [5].

Applying these techniques to the kaon mixing parameter $B_K$ is also possible [6], however it becomes difficult to apply to lattice data due to the large number of free parameters. An option is to used a “mixed action” approach, where one uses domain-wall valence quarks and staggered sea quarks [7]. This technique is a slight modification of the continuum form for $B_K$ and is a promising method with which to extract this quantity.

References

[1] C. Bernard, Phys. Rev. D 65, 054031 (2002); C. Aubin and C. Bernard, Phys. Rev. D 68, 034014 (2003); Phys. Rev. D 68, 074011 (2003).

[2] C. Aubin et al., Phys. Rev. D 70, 094505 (2004); C. Aubin et al. [Fermilab Lattice, MILC, and HPQCD Collaborations], Phys. Rev. D 70, 031504 (2004); C. Aubin et al. [MILC Collaboration], Phys. Rev. D 70, 114501 (2004); C. Bernard et al. [MILC Collaboration], arXiv:hep-lat/0609053.

[3] C. Aubin and C. Bernard, Phys. Rev. D 73, 014515 (2006), J. Laiho and R. S. Van de Water, Phys. Rev. D 73, 054501 (2006).

[4] C. Aubin et al. [Fermilab Lattice and MILC Collaborations], Phys. Rev. Lett. 94, 011601 (2005); M. Okamoto et al., Nucl. Phys. Proc. Suppl. 140, 461 (2005); C. Aubin et al., Phys. Rev. Lett. 95, 122002 (2005).

[5] M. Artuso et al. [CLEO Collaboration], Phys. Rev. Lett. 95, 251801 (2005).

[6] Ruth S. Van de Water and S. R. Sharpe, Phys. Rev. D 73, 014003 (2006).

[7] C. Aubin, J. Laiho and Ruth S. Van de Water, arXiv:hep-lat/0609009.
Chiral Perturbation Theory and Lattice QCD with Wilson-type Quarks

Sinya Aoki

Graduate School of Pure and Applied Sciences, University of Tsukuba, Ten-oh-dai 1-1-1, Tsukuba, Ibaraki 305-8571, Japan

I explain a necessity of introducing the lattice spacing effect into the chiral perturbation theory to fit lattice data obtained with Wilson-type quarks such as pion masses and decay constants. This formulation is called the Wilson chiral perturbation theory (WChPT) [1]. The WChPT has been applied to the analysis of the twisted-mass QCD [2,3,6,8]. The twisted-mass QCD becomes automatically $O(a)$ improved as long as the twisted-mass is set to the maximal value (the maximal twist). In actual numerical simulations, however, several different definitions of the maximal twist have been employed, and different definitions show quite different chiral behaviours. I show that these differences among several definition of the maximal twist can be understood by the WChPT, and that pion mass and decay constant in quenched twisted-mass QCD can be fitted as a function of quark mass by the WChPT very well. The WChPT has been extended to $N_f = 2 + 1$ flavor QCD for pseudo-scalar meson masses [4] and for vector meson masses [5]. These formula have been applied to $N_f = 2 + 1$ flavor QCD, in order to fit data. I show some examples of some chiral fits [7].

References

[1] S. Aoki, Phys. Rev. D68 (2003) 054508
[2] S. Aoki, O. Bär, Phys. Rev. D70 (2004) 116011
[3] S. Aoki, O. Bär, PoS LAT2005 (2005) 046
[4] S. Aoki, O. Bär, T. Ishikawa, S. Takeda, Phys. Rev. D73 (2006) 014511
[5] S. Aoki, O. Bär, S. Takeda, Phys. Rev. D73 (2006) 094501
[6] S. Aoki, O. Bär, Phys. Rev. D74 (2006) 034511
[7] T. Ishikawa, S. Aoki, et al., PoS LAT2006 181
[8] S. Aoki, O. Bär, PoS LAT2006 165
Lattice QCD with mixed actions:
Overlap fermions on a twisted mass sea

Oliver Bär¹, K. Jansen², S. Schaefer², L. Scorzato³, A. Shindler²

¹Institute of Physics, Humboldt University,
Newtonstrasse 15, 12489 Berlin, Germany

²NIC, DESY,
Platanenallee 6, 15738 Zeuthen, Germany

³ECT*
Strada delle Tabarelle 286, 38050 Villazzano (TN), Italy

We present first results of a mixed action project [1]. We analyze gauge configurations generated by the ETM collaboration (for recent summaries see [2],[3]). Two flavors of dynamical twisted mass fermions are taken into account with $m_{PS} \approx 300$ MeV. Neuberger’s overlap Dirac operator [4] is used for the valence sector. For the quark mass matching we tune the charged valence pion mass such that it equals the charged sea pion mass.

In order to check the partial quenching effects in our mixed action theory we compute the scalar correlator. As expected, the correlator is negative [5]. Mixed action theories can also be studied with ChPT methods [6],[7],[8]. We fit our data to the LO ChPT result for the scalar correlator of Ref. [9] and find good qualitative agreement.

References

[1] O. Bär, K. Jansen, S. Schaefer, L. Scorzato and A. Shindler, arXiv:hep-lat/0609039.
[2] ETM Collaboration, A. Shindler, ICHEP proceedings (2006).
[3] K. Jansen and C. Urbach [ETM Collaboration], arXiv:hep-lat/0610015.
[4] H. Neuberger, Phys. Lett. B417 (1998) 141–144.
[5] S. Prelovsek, C. Dawson, T. Izubuchi, K. Orginos, and A. Soni, Phys. Rev. D70 (2004) 094503.
[6] O. Bär, G. Rupak, and N. Shoresh, Phys. Rev. D67 (2003) 114505.
[7] O. Bär, C. Bernard, G. Rupak, and N. Shoresh, Phys. Rev. D72 (2005) 054502.
[8] O. Bär, G. Rupak, and N. Shoresh, Phys. Rev. D70 (2004) 034508.
[9] M. Golterman, T. Izubuchi, and Y. Shamir, Phys. Rev. D71 (2005) 114508.
Thoughts on chiral extrapolations for excited hadrons

Ulf-G. Meißner$^{1,2}$

$^1$HISKP (Th), Universität Bonn, D-53115 Bonn, Germany

$^2$IKP (Th), Forschungszentrum Jülich, D-52425 Jülich, Germany

An understanding of QCD without its excitation spectrum is incomplete. In this talk I review work on chiral extrapolations for excited hadron states - I concentrate on the $\rho(770)$ and the Roper $N^*(1440)$. Both these fields are considered as explicit degrees of freedom in the pertinent chiral effective Lagrangian. The new (large) scale related to their mass introduces complications in the power counting, how to treat this in case of the real part of the rho-meson self-energy is discussed in [1] and the even more complicated nucleon-Roper-pion system is considered in [2] by extending the standard infrared regularization scale. The quark mass expansion of the $\rho$ looks very similar to the one of the nucleon, analyzing e.g. the old CP-PACS data, we find $650 \text{ MeV} \leq M_0^\rho \leq 800 \text{ MeV}$ for the chiral limit mass. For the Roper, the quark mass expansion comes out very close to the one of the nucleon. With coupling constants of natural size, no sharp decrease of $m_{\text{Roper}}(M_\pi)$ for small pion masses is observed. Of course, these considerations need to be sharpened. For the $\rho$, one has to analyze the more recent data and in case of the Roper, an extension to include the $\Delta \pi$ and $N(\pi\pi)_S$ channels is called for. The $\Delta(1232)$ resonance is discussed in Bernard’s talk [3] and for dynamically resonances I refer to Oller’s talk [4].

References

[1] P. C. Bruns and U.-G. Meißner, Eur. Phys. J. C 40 (2005) 97 [arXiv:hep-ph/0411223].

[2] B. Borasoy, P. C. Bruns, U.-G. Meißner and R. Lewis, Phys. Lett. B 641 (2006) 294 [arXiv:hep-lat/0608001].

[3] V. Bernard, these proceedings.

[4] J. A. Oller, these proceedings.
Chiral extrapolation of baryon properties

V. Bernard

Laboratoire de Physique Théorique, 67085 Strasbourg, France

Enormous progresses have been made in lattice calculation in the last years. However these calculations still involve rather large pion masses. It is thus necessary to perform chiral extrapolations in order to make contact to the physical world. This can be done in a model independent manner using chiral perturbation theory. For discussion on the different regularization used in baryon CHPT see [1]. In this talk I reported on chiral extrapolation of some baryon properties, namely the mass [2] and axial vector coupling [3] of the nucleon and the ∆ mass [4] essentially in the continuum limit. In the nucleon case two loop calculations are by now available (only the $O(p^5)$ terms have been determined in the case of the mass). In these calculations only 7 combinations of LEC’s from the $\pi N$ Lagrangian up to dimension 4 appear, 5 of these being rather well determined from $\pi N$ scattering and the Goldberger Treiman discrepancy. Taking into account the error bars on these LECs and studying the convergence of the series one finds that the chiral extrapolation of these two quantities can be trusted up to $\sim 350$ MeV. Taking explicitly into account the ∆ degree of freedom does not improve on the result [5]. Discussion on finite volume effects can be found for exemple in [6]. In the case of the ∆ mass fits to the lattice data in the small scale expansion seems to indicate that the symmetry breaker LEC $a_1$ is far from its SU(6) value leading to a $\pi \Delta$ sigma term smaller than the $\pi N$ one. Further study is needed to confirm this result. The conclusion of these studies is that it is not uselfull to fit lattice data up to rather large pion masses but that it is much more important to perform simultaneous systematic chiral extrapolation with a consistent set of LEC’s and to carefully evaluate the theoretical uncertainties.

References

[1] M. Schindler, these proceedings; T. Gail, these proceedings.
[2] J. A. Mc Govern and M. C. Birse, hep-ph/0608002; V. Bernard, T.-H. Hemmert and U.-G. Meißner, Nucl. Phys. A 732 (2004) 149.
[3] V. Bernard and U.-G. Meißner, Phys. Lett. B 639 (2006) 278.
[4] V. Bernard, T.R. Hemmert and U.-G. Meißner, Phys. Lett. B 622 (2005) 141.
[5] M. Procura, B.U. Musch, T. Wollenweber, T.R. Hemmert and W. Weise, Phys. Rev. D 73 (2006) 114510.
[6] G. Colangelo, A. Fuhrer, C. Haefeli, Nucl. Phys. Proc. Suppl. 153 (2006) 41; A. Ali Khan et al. Nucl. Phys. B 689 (2004) 175.
Some topics in manifestly Lorentz-invariant ChPT

D. Djukanovic, J. Gegelia, M. R. Schindler and S. Scherer

Institut für Kernphysik, Johannes Gutenberg-Universität
55099 Mainz, Germany

We present three topics in manifestly Lorentz-invariant baryon chiral perturbation theory (BChPT).

The higher-derivative formulation of ChPT [1] presents an alternative regularization scheme that can be used in the vacuum, one- and few nucleon sector of ChPT. By performing field transformations in the canonical Lagrangian, additional higher-derivative terms are introduced in the lowest-order Lagrangian. These terms improve the ultraviolet behavior of propagators while preserving all symmetries. The new parameters can be interpreted as smooth cutoffs, whose values can be chosen freely.

To investigate the applicability of chiral expansions [2] we compare the sum of an infinite number of terms contributing to the nucleon self energy starting at the two-loop level with the leading non-analytic term $\sim M^3$. For pion masses $M > 550$ MeV the contributions from the sum of higher-order terms is larger than the leading-order contribution. This shows that for $M > 550$ MeV the power counting is no longer valid.

We also present the results of the first complete two-loop calculation of the nucleon mass [3] using the reformulated infrared renormalization [4] and compare our result to the heavy baryon result at order $O(q^3)$ [5]. While at order $O(q^5)$ only low energy constants appear that have previously been determined, the terms at order $O(q^n)$ contain combinations of numerically unknown constants.

References

[1] D. Djukanovic, M. R. Schindler, J. Gegelia and S. Scherer, Phys. Rev. D 72, 045002 (2005)
[2] D. Djukanovic, J. Gegelia and S. Scherer, arXiv:hep-ph/0604164
[3] M. R. Schindler, D. Djukanovic, J. Gegelia and S. Scherer, in preparation
[4] M. R. Schindler, J. Gegelia and S. Scherer, Phys. Lett. B 586, 258 (2004)
[5] J. A. McGovern and M. C. Birse, Phys. Lett. B 446, 300 (1999)
Chiral Extrapolations in Few–Nucleon Systems

Evgeny Epelbaum$^{1,2}$

$^{1}$Institut für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich, Germany
$^{2}$HISKP (Theorie), Universität Bonn, Nußallee 14-16, 53115 Bonn, Germany

The dependence of the low-energy dynamics of QCD on the variation of the up and down quark masses can be naturally studied within the framework of chiral effective field theory. While the situation in the Goldstone boson and single-nucleon sectors is rather straightforward due to the validity of perturbation theory, it is highly non-trivial in the few-nucleon sector where non-perturbative methods are required to understand e.g. the properties of the shallow bound states. The quark mass dependence of various few-nucleon observables is not only of academic interest, but also relevant for interpolating the results from lattice gauge theory $^{[1]}$, $^{[2]}$ and imposing bounds on the time-dependence of fundamental couplings.

We have studied the quark mass dependence of the nucleon-nucleon (NN) force and various two-nucleon observables at next-to-leading order in the chiral expansion $^{[3]}$, see also $^{[4]}$ for a similar work. We found that both NN S-wave scattering lengths become smaller in magnitude (i.e. more natural) in the chiral limit. The deuteron is less bound when the quark mass is increased and becomes unbound for $M_\pi$ larger than $\sim 200$ MeV. The theoretical uncertainty of our results is rather large and mainly caused by the uncertainty in the determination of the LEC $\bar{d}_{16}$ and by the unknown LECs associated with the leading $M_\pi$-dependent NN contact interactions. We have also studied the infrared renormalization group limit cycle in the three-nucleon system $^{[5]}$, whose existence for certain values of $m_u$ and $m_d$ was conjectured in $^{[6]}$.

References

$^{[1]}$ M. Fukugita et al., Phys. Rev. D52, 3003 (1995), [hep-lat/9501024].

$^{[2]}$ S.R. Beane et al., Phys. Rev. Lett. 97, 012001 (2006), [hep-lat/0602010].

$^{[3]}$ E. Epelbaum, U.-G. Meißner, and W. Glöckle, Nucl. Phys. A714, 535 (2003), [nucl-th/0207089, nucl-th/0208040].

$^{[4]}$ S.R. Beane and M.J. Savage, Nucl. Phys. A713, 148 (2003), [hep-ph/0206113, nucl-th/0208021].

$^{[5]}$ E. Epelbaum et al., Eur. Phys. J. C48, 169 (2006), [hep-ph/0602225].

$^{[6]}$ E. Braaten and H.W. Hammer, Phys. Rev. Lett. 91, 102002 (2003), [nucl-th/0303038].
Nuclear Lattice Simulations

B. Borasoy

Helmholtz-Institut für Strahlen- und Kernphysik (Theorie)
Universität Bonn, Nußallee 14-16, D-53115 Bonn, Germany

Nuclear lattice simulations are a novel approach to nuclear physics based on numerical simulations of chiral effective field theory on the lattice. Within this scheme the chiral effective Lagrangian is formulated on the lattice and the corresponding path integral is evaluated numerically by Monte Carlo sampling.

In two exploratory studies we have employed this framework to calculate properties of the lightest nuclei: the deuteron, triton and Helium-4. In [1] the triton was investigated in the SU(4) limit of the pionless theory including the three-nucleon force. By comparing with continuum results we demonstrated that the nuclear lattice formalism can be used to study few-body nucleon physics. A first measurement of the three-nucleon force on the lattice in the Wigner symmetry limit has been provided. Moreover, we have shown that many-body simulations of cold dilute nuclear matter in the Wigner symmetry limit should be possible without a sign problem. This work is thus also of relevance for future many-body simulations with arbitrary numbers of nucleons including three-body effects.

The pionful theory is considered in [2] where we calculate the binding energies and sizes of triton and $^4$He. A Monte Carlo algorithm with pions and auxiliary fields is constructed which reproduces both the lowest order $S$-wave contact interactions and instantaneous one-pion exchange between the nucleons. Due to the approximate Wigner symmetry of the actual interactions we find only minor sign and phase oscillations for light nuclei. The importance of higher order interactions is also investigated.

Although our investigations are still at the exploratory level, lattice simulations of chiral effective field theory appear to be a promising tool to investigate few- and many-body nuclear physics with a clear theoretical connection to QCD and a systematic chiral expansion.

References

[1] B. Borasoy, H. Krebs, D. Lee and U.-G. Meißner, Nucl. Phys. A 768 (2006) 179 [arXiv:nucl-th/0510047].

[2] B. Borasoy, E. Epelbaum, H. Krebs, D. Lee and U.-G. Meißner, (in preparation).
In this talk we report about our recent work [1]. We perform a non-perturbative chiral study of the masses of the lightest pseudoscalar mesons. The pseudoscalar self-energies are calculated by the evaluation of the scalar self-energy loops with full S-wave meson-meson amplitudes taken from Unitary Chiral Perturbation Theory (UCHPT) [2]. These amplitudes, among other features, contain the lightest nonet of scalar resonances $\sigma$, $f_0(980)$, $a_0(980)$ and $\kappa$ and the heavier ones up to 1.5 GeV. The self-energy loops are regularized by a proper subtraction of the infinities within a dispersion relation formulation of the scattering amplitudes. Values for the bare masses of pions and kaons are obtained as well as an estimate of the mass of the $\eta_8$. We then match to the self-energies from standard Chiral Perturbation Theory (CHPT) to $O(p^4)$ and resum higher orders from our calculated scalar self-energies. The dependence of the self-energies on the quark masses allows a determination of the ratio of the strange quark mass upon the mean of the lightest quark masses, $m_s/\hat{m}$, in terms of the $O(p^4)$ CHPT low energy constant combinations $2L_8^s - L_5^s$ and $2L_6^r - L_4^r$. In this way, we give a range for the values of these low energy counterterms and for $3L_7^r + L_8^r$, once the $\eta$ meson mass is invoked. The low energy constants are further constraint by performing a fit to the recent MILC lattice data on the pseudoscalar masses. An excellent reproduction of the MILC data is obtained, at the level of 1% of relative error in the pseudoscalar masses, and $m_s/\hat{m} = 25.6 \pm 2.5$ results. This value is consistent with $24.4 \pm 1.5$ from CHPT and phenomenology and more marginally with the value $27.4 \pm 0.5$ obtained from pure perturbative chiral extrapolations of the MILC lattice data to physical values of the lightest quark masses. We also show that our estimation of the $O(p^6)$ and higher chiral orders, indicates that the SU(3) CHPT series on pseudoscalar self-energies seems to stabilize and behave much better once the $O(p^6)$ CHPT contributions to the pseudoscalar masses are included, despite that the $O(p^4)$ contributions are typically smaller than the $O(p^6)$ ones.

References

[1] J. A. Oller and L. Roca, “Non-perturbative study of the light pseudoscalar masses in chiral dynamics,” arXiv:hep-ph/0608290.

[2] J. A. Oller and E. Oset, Phys. Rev. D 60, 074023 (1999) arXiv:hep-ph/9809337.
Scalar two-loop diagrams on the lattice

Hermann Krebs

HISKP, Universität Bonn and IKP, Forschungszentrum Jülich

Numerical simulations on the lattice are always performed in finite volume with a finite lattice spacing. In order to understand both the infinite volume and the continuum limit non-trivial extrapolations are needed. In some cases lattice perturbation theory provides a powerful tool in getting an analytical control over nonperturbative simulations.

Lattice perturbation theory itself is a challenging field. Since Lorentz-invariance is lost on the lattice many tools developed for continuum perturbation theory such as Feynman parameters are not applicable on the lattice. Especially for multi-loop calculations where conventional methods often lead to loss in precision new techniques are highly desirable. A novel powerful technique in this respect was introduced by Lüscher and Weisz [1]. Instead of integration over the first Brillouin-zone in momentum space they suggested to calculate infinite sums in coordinate space. In this case extensive knowledge of the free lattice propagators is required. Recursion relations for the massless propagator were derived which allowed to express the Green function at any lattice site as a linear combination of two constants which can be determined to very high precision.

We extended this technique to the massive case and could give the small mass expansion of the massive scalar propagator [2]. In the BPHZ subtraction scheme any scalar two loop diagram can be expressed as a sum of a regular and a singular part which converge and diverge in the continuum limit, respectively. The singular part is always of the following sunset type:

\[
\int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \prod_{\mu} \frac{k^{2\mu} \cdot \hat{q}^{2\mu} \cdot (k + q)^{2\mu}}{(k^2 + m^2)^{\alpha}[q^2 + m^2]^\beta[(k + q)^2 + m^2]^\gamma}, \quad \hat{k}_\mu = 2 \sin \left( \frac{k_\mu}{2} \right). \tag{1}
\]

We developed a method for deriving the small mass expansion of any singular part [3]. The coefficients of this expansion can be calculated numerically to very high precision such that even discretization effects can also be studied within our method. The proposed method for the small mass expansion is universal and can be applied once the particle propagator is known, e.g., it can easily be extended to Wilson fermions.

References

[1] M. Lüscher, P. Weisz, Nucl. Phys. B 445 (1995) 429.
[2] B. Borasoy, H. Krebs, Phys. Rev. D 72 (2005) 056003.
[3] B. Borasoy, H. Krebs, Nucl. Phys. B 748 (2006) 1.
Nucleon and N to ∆ form factors in Lattice QCD

C. Alexandrou\(^1\), Th. Leontiou\(^1\), J. W. Negele\(^2\) and A. Tsapalis\(^3\)

\(^1\)Dep. of Physics, University of Cyprus, CY-1678 Nicosia, Cyprus
\(^2\)Dep. of Physics, M.I.T, Cambridge, Massachusetts 02139, U.S.A.,
\(^3\)I.A.S.A., University of Athens, Athens, Greece.

The isovector nucleon form factors are evaluated in lattice QCD in the quenched theory and using two degenerate flavors of dynamical Wilson fermions for a range of pion masses between 690 and 380 MeV, using the nucleon mass to set the lattice spacing \(a\). We find that the isovector electric form factor, \(G_E\), at the physical limit deviates more from experiment than the magnetic form factor, \(G_M\) \(^1\). Lattice results show a weaker \(q^2\)-dependence for the isovector ratio \(\mu G_E/G_M\) as compared to the results obtained in recent polarization experiments \(^2\). The electromagnetic and axial N to ∆ transition form factors are evaluated for quenched and two dynamical flavor of Wilson fermions as well as in a hybrid scheme where we use MILC configurations and domain wall fermions over a range of pion masses between 690 and 360 MeV. We find that lattice results for the dominant magnetic dipole form factor in \(\gamma^* N \rightarrow \Delta\), when linearly extrapolated in \(m^2_{\pi}\) to the physical point, yield larger values than experiment \(^3\). Results for the ratios of electric and coulomb quadrupole amplitudes to the magnetic dipole, EMR and CMR, are non-zero and negative as in experiment albeit with statistical errors that are still too large in the unquenched case \(^4\) to allow assessment of pion cloud contributions. Our results for the four N to ∆ axial form factors show that \(C^A_3 \sim 0\), \(C^A_4\) is small and \(C^A_5\) and \(C^A_6\) are the dominant form factors. We evaluate the ratio \(C^A_5/C^A_3\), which is the analog of \(g_A/g_V\), as a function of \(Q^2\). This ratio provides the leading contribution to the the parity violating asymmetry \(^5\) to be measured by the G0 experiment at JLab \(^6\). The off-diagonal Goldberger-Treiman relation is examined by evaluating, at the same lattice parameters, \(g_{\pi N\Delta}\) and \(f_\pi\).

References

[1] C. Alexandrou \textit{et al.}, Phys. Rev. D \textbf{74} (2006) 034508.
[2] C. Hyde-Wright and K. de Jager, Ann. Rev. Nucl. Part. Sci. \textbf{54}, (2004) 217.
[3] C. Alexandrou \textit{et al.}, Phys. Rev. Lett. \textbf{94}, 021601 (2005).
[4] C. Alexandrou \textit{et al.}, PoSLat2005 (2006) 091.
[5] C. Alexandrou \textit{et al.}, [hep-lat/0607030].
[6] S. P. Wells, PAVI 2002, Mainz, Germany, June 5-8, 2002.
Hadron Structure using DWF Quarks on an Asqtad Sea

David Richards, for Lattice Hadron Physics Collaboration

Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, VA 23606, USA.

Moments of unpolarized, helicity, and transversity distributions, electromagnetic form factors, and generalized form factors of the nucleon are presented from a preliminary analysis of lattice results using pion masses down to 359 MeV[1]. We employ a hybrid approach, in which improved, staggered quarks are used for the generation of the gauge configurations, whilst domain-wall fermions, with their desirable chiral properties, are used for the valence quarks.

The nucleon axial-vector charge, a benchmark quantity of QCD, is particularly robust under chiral extrapolation; the consistency of the hybrid calculation, both with other lattice calculations, and with experiment at the physical pion mass, is encouraging[2]. Lattice moments of structure functions and GPDs likewise require extrapolation to the physical quark masses; a long-standing puzzle has been the flat behaviour of the flavour-non-singlet momentum fraction, \( \langle x \rangle \), of the nucleon, at a value considerably higher than the experimental value. An approach in which we apply \( \chi \)PT, with low-energy constants \( g_A \) and \( f_\pi \) given by their lattice values at each quark mass, allows a two-parameter extrapolation in \( m_{\pi}^{\text{lat}} / f_{\pi}^{\text{lat}} \) to yield a value for \( \langle x \rangle \), and other benchmark quantities, at the physical quark masses that are consistent with experiment. This development encourages to now exploit the predictive power of these calculations.

The low-\( Q^2 \) behaviour of the nucleon form factors describes the distribution of charge and magnetism within a nucleon. The slope of the \( F_1 \) form factor is related to the rms charge radius; the chiral extrapolation of the isovector charge radius likewise yields values consistent with experiment[3]. Generalized Parton Distributions provide new insight to hadron structure. For example, the total angular momentum carried by the quarks is related to a combination of moments \( J_q = \frac{1}{2}(A_{20}^{u+d} + B_{20}^{u+d})[4]. \) Combined with measurements of quark spins, we find the total orbital angular momentum carried by quarks is small, though that carried by individual flavours is substantial.

References

[1] R. G. Edwards et al., arXiv:hep-lat/0610007.
[2] R. G. Edwards et al. [LHPC Collaboration], Phys. Rev. Lett. 96, 052001 (2006).
[3] G. V. Dunne, A. W. Thomas and S. V. Wright, Phys. Lett. B 531, 77 (2002).
[4] X. D. Ji, Phys. Rev. Lett. 78, 610 (1997).

21
distribution amplitudes from the lattice

v. m. braun\textsuperscript{1}, m. göckeler\textsuperscript{1}, r. horsley\textsuperscript{2}, h. perl\textsuperscript{3}, d. pleiter\textsuperscript{4}, p. e. l. rakow\textsuperscript{5}, g. schierholz\textsuperscript{36}, a. schiller\textsuperscript{3}, w. schroers\textsuperscript{4}, h. stuben\textsuperscript{7}, and j. m. zanotti\textsuperscript{2}

qcdfs/ukqcd collaboration

\textsuperscript{1}institut für theoretische physik, universität regensburg, 93040 regensburg, germany

\textsuperscript{2}school of physics, university of edinburgh, edinburgh eh9 3jz, uk

\textsuperscript{3}institut für theoretische physik, universität leipzig, 04109 leipzig, germany

\textsuperscript{4}john von neumann-institut für computing nic / desy, 15738 zeuthen, germany

\textsuperscript{5}theoretical physics division, department of mathematical sciences, university of liverpool, liverpool l69 3bx, uk

\textsuperscript{6}deutsches elektronen-synchrotron desy, 22603 hamburg, germany

\textsuperscript{7}konrad-zuse-zentrum für informationstechnik berlin, 14195 berlin, germany

hadronic wave functions are of crucial importance when describing exclusive and semi-exclusive reactions \cite{1}. for detailed references and applications consult \cite{2}. distribution amplitudes (dAs) are related to the hadron’s bethe-salpeter wave function, $\phi(x, k_\perp)$, by an integral over transverse momenta. for the leading twist meson-dAs we have

$$\phi(x, \mu^2) = Z_2(\mu^2) \int_{|k_\perp|<\mu} d^2k \phi(x, k_\perp), \quad (1)$$

where $x$ is the quark longitudinal momentum fraction, $Z_2$ the renormalization factor (in the light-cone gauge) for the quark-field operators in the wave-function, and $\mu$ denotes the renormalization scale. in this presentation we quote all numbers with a scale $\mu^2 = 4 \text{ GeV}^2$ in the $\overline{\text{ms}}$-scheme.

It is convenient to rescale $\xi = 2x - 1$. it is common to expand dAs into their gegenbauer moments and quote the expansion coefficients, $a_i$, at a given
renormalization scale as a parameterization of DAs,

\[ \phi(\xi, \mu^2) = \frac{3}{4}(1 - \xi^2) \left( 1 + \sum_{n=1}^{\infty} a_n(\mu^2)C_n^{3/2}(\xi) \right). \]  

The zeroth moment is normalized to unity, \( \int_{-1}^{1} d\xi \phi(\xi, \mu^2) = 1 \), at any energy scale \( \mu^2 \). Taking the \( u \)- and \( d \)-quarks to be degenerate, \( G \)-parity implies that the pion DA is an even function of \( \xi \) and hence all odd moments vanish, i.e., \( a_{2n+1}^\pi = 0 \).

Recently, we have computed the first moments of meson distribution amplitudes \[2\] on the lattice. Independently, a calculation of the first moment of the kaon distribution amplitude has appeared which uses a different discretization scheme and different working points \[3\]. We find that their results are compatible with ours.

This presentation discusses the calculation of the first non-vanishing moment of the pion DA, \( a_2^\pi \), and the first two moments of the kaon DA, \( a_1^K \) and \( a_2^K \). We compare our results to previous estimates from sum rules and experiment and discuss the phenomenological implications of our lattice computation.

We obtain \( a_2^\pi(4 \text{ GeV}^2) = 0.201(114) \) which disfavors the commonly adopted “asymptotic” model with \( a_{2n}^\pi = 0 \). This result is also compatible with model estimates, cf. e.g. \[4\].

We also find \( a_2^K(4 \text{ GeV}^2) = 0.175(18)(47) \) which provides an important test of different and competing kaon models employed in the literature, see \[3\].

Finally, we find \( a_1^K(4 \text{ GeV}^2) = 0.0453(9)(29) \) which not only confirms the sign, but also the order of magnitude for the asymmetry of the kaon wave function. This key finding is of major importance both for the understanding of \( B \) physics and of weak decays \[5\].

References

[1] S. J. Brodsky and G. P. Lepage, *Adv. Ser. Direct. High Energy Phys.* 5 (1989) 93.

[2] V. M. Braun *et al.,* [arXiv:hep-lat/0606012]; V. M. Braun *et al.,* [arXiv:hep-lat/0610055]

[3] P. A. Boyle, M. A. Donnellan, J. M. Flynn, A. Jüttner, J. Noaki, C. T. Sachrajda and R. J. Tweedie [UKQCD Collaboration], [arXiv:hep-lat/0607018]; A. Jüttner *et al.,* [arXiv:hep-lat/0610025]; A. Jüttner *et al.,* in preparation.

[4] A. P. Bakulev, S. V. Mikhailov and N. G. Stefanis, *Phys. Lett. B* 578, 91 (2004); A. P. Bakulev, S. V. Mikhailov and N. G. Stefanis, *Phys. Rev. D* 67, 074012 (2003).

[5] P. Ball, V. M. Braun and A. Lenz, *JHEP* 0605, 004 (2006).
Pion structure from lattice QCD

D. Brömmel1,2, M. Diehl1, M. Göckeler2, Ph. Hägler3, R. Horsley4, Y. Nakamura5, D. Pleiter5, P.E.L. Rakow6, A. Schäfer2, G. Schierholz1,5, H. Stüben7 and J.M. Zanotti4

1Deutsches Elektronen-Synchrotron DESY, Hamburg, 2Universität Regensburg, 3Technische Universität München, 4University of Edinburgh, 5John von Neumann-Institut für Computing NIC/ DESY, Zeuthen, 6University of Liverpool, 7Konrad-Zuse-Institut für Informatik ZIB, Berlin

We calculate moments of generalised parton distributions (GPDs) of the pion. This is done using a simulation of non-perturbatively improved dynamical Wilson fermions with two flavours with Wilson glue. The configurations have been generated within the QCDSF/UKQCD/DIK collaborations and cover a range of pion masses down to \( \sim 340 \text{ MeV} \). We show partly preliminary results for the lowest (two) moment(s) of the tensor (vector) GPD \( H_{q,T}^{q,\pi} \), defined as

\[
\frac{2P^\mu}{n \cdot P} H_{q,T}^{q,\pi} = \int \frac{d\lambda}{2\pi} e^{i\lambda n \cdot P x} \langle \pi(p') | \bar{q}(-\frac{\lambda}{2} n) \gamma^\mu U q(\frac{\lambda}{2} n) | \pi(p) \rangle + \text{higher twist} \ , \tag{1}
\]

\[
\frac{P^{[\mu \Delta \nu]}}{m_\pi n \cdot P} H_{T,\pi}^{q,\pi} = \int \frac{d\lambda}{2\pi} e^{i\lambda n \cdot P x} \langle \pi(p') | \bar{q}(-\frac{\lambda}{2} n) i\sigma^{\mu \nu} U q(\frac{\lambda}{2} n) | \pi(p) \rangle + \text{higher twist} \ . \tag{2}
\]

The moments of these GPDs are parametrised by generalised form factors. We focus on the lowest moment of the vector GPD Eq. (1) which is the pion form factor \( F_\pi \). Details of the calculation and the lattice setup and further references can be found in [1],[2]. The pion form factor can be very well described by a monopole ansatz. The squared monopole masses obtained from this ansatz are extrapolated to the physical point linearly against square of the pion mass. From that we obtain a charge radius of \( \langle r^2 \rangle = 0.440(19) \text{ fm}^2 \) in good agreement with experiment. We also perform a study of the finite size and scaling effects [2]. The second moment of the vector GPD corresponds to the momentum fraction of the quarks inside the pion. A preliminary result indicates that quarks and antiquarks inside the pion carry \( \sim 50\% \) of the momentum fraction. The calculation of the pion tensor GPD Eq. (2) has not been attempted before. This quantity is experimentally unknown and thus provides genuine new insights to the internal spin structure of the pion [3]. Our preliminary data so far indicate a rich structure with a non-vanishing tensor GPD also in the forward limit.

References

[1] D. Brömmel et al., Proc. Sci. LAT2005, 360 (2005), [hep-lat/0509133]
[2] D. Brömmel et al., [hep-lat/0608021]
[3] M. Diehl and Ph. Hägler, Eur. Phys. J. C 44 (2005) 87, [hep-ph/0504175]
Nucleon Structure Functions and Form Factors

Dirk Pleiter

For the QCDSF collaboration

Deutsches Elektronen-Synchrotron DESY, 15738 Zeuthen, Germany

In this talk we have reported on recent results for the moments of the unpolarised nucleon structure functions \[1\] and the electromagnetic form factors \[2\]. The calculations have been done using dynamical gauge field configurations with \(N_f = 2\) flavours of non-perturbatively \(O(a)\)-improved Wilson fermions, which have been generated by the QCDSF, DIK and UKQCD collaborations. On the different gauge field ensembles the pseudo-scalar meson mass is in the range of 340 to 1170 MeV. The lattice spacings vary between 0.07 and 0.11 fm, i.e. our lattice are relatively fine. The spatial extension of our lattice is 1.5 fm for the heavy sea quark masses and up to 2.6 fm for the very light quark mass. The renormalisation constants have been determined non-perturbatively.

By comparing our results for different lattice spacings and different volumes we find that discretisation and finite volume effects are relatively small. The systematic error which seems most difficult to control stems from the extrapolation of the lattice results to the physical mass of the up/down quarks. When using a naive ansatz linear in the quark mass, which describes our data actually very well, we find a significant discrepancy with the experimental values. Using the quenched approximation we previously obtained surprisingly similar results \[3\],\[4\].

For the moments of the structure functions as well as for the form factor radii and anomalous magnetic moments chiral effective theories have been used to investigate the quark mass dependence. These calculations suggest a rather strong quark mass dependence at very small quark masses. The relevant range of quark masses is only starting to become accessible for lattice simulations.

References

[1] M. Göckeler et al. [QCDSF Collaboration], Nucl. Phys. Proc. Suppl. 140 (2005) 399 \texttt{arXiv:hep-lat/0409162}.

[2] M. Göckeler et al. [QCDSF Collaboration], PoS(LAT2006)120 \texttt{arXiv:hep-lat/0610118}.

[3] M. Göckeler et al. [QCDSF Collaboration], Phys. Rev. D 71 (2005) 114511 \texttt{arXiv:hep-ph/0410187}.

[4] M. Göckeler et al. [QCDSF Collaboration], Phys. Rev. D 71 (2005) 034508 \texttt{arXiv:hep-lat/0303019}.
The kaon bag-parameter, $B_K = \langle K^0|\bar{s}\gamma_\mu(1-\gamma_5)d\bar{\gamma}_\mu(1-\gamma_5)d|K^0\rangle/(8/3 f_K^2 m_K^2)$, defined as the ratio of the $\Delta S = 2$ matrix element to its value obtained in the vacuum saturation approximation is a measure of indirect CP-violation and provides an important non-perturbative input to the Cabbibo-Kobayashi-Maskawa matrix fit, cf. e.g. [1].

We present a preliminary study of the kaon bag-parameter $B_K$ measured on $L^3 \times T \times L_s = 16^3 \times 32 \times 16$ lattices using $N_f = 2 + 1$ dynamical flavors of domain wall fermions, see also [2], [3], [4]. The mass of the heavier (single) flavor was tuned to match the physical strange quark mass, while three different values for the masses of the two degenerate light quarks have been used in the simulation. These correspond to pion masses in the range of 390 to 630 MeV. Whereas the applicability of chiral perturbation theory to extrapolate the pion masses and decay constants to the physical limit is at least questionable in this mass range (see [5], [6] for a discussion), we still may use the partially quenched chiral perturbation theory formulae given in [7] as a smooth fitting function for an extrapolation of our results concerning $B_K$. Ignoring small non-degenerate effects even allows for an interpolation over just a small mass range. For details we refer to [3] and our upcoming paper [4].
Low-energy constants for $\Delta S = 1$ transitions

L. Giusti$^1$, P. Hernández$^2$, M. Laine$^3$, C. Pena$^4$, P. Weisz$^4$, J. Wennekers$^5$ and H. Wittig$^6$

$^1$CERN, Department of Physics, TH Division, CH-1211 Geneva 23
$^2$Dpto. Física Teórica and IFIC, E-46071 Valencia
$^3$Faculty of Physics, University of Bielefeld, D-33501 Bielefeld
$^4$Max-Planck-Institut für Physik, Föhringer Ring 6, D-80805 Munich
$^5$DESY, Notkestraße 85, D-22603 Hamburg
$^6$Institut für Kernphysik, Universität Mainz, D-55099 Mainz

In order to investigate the origins of the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decays we have devised a strategy to determine the LECs of operators describing $\Delta S = 1$ transitions [1]. We keep an active charm quark, express $K \to \pi\pi$ amplitudes in terms of LECs and study their dependence on $m_c$. By employing Neuberger fermions, the severe problem of mixing with lower dimensional operators is completely avoided. The LECs are extracted by matching lattice QCD data for suitable ratios of correlation functions to the expressions of ChPT in finite volume [2]. The first step in our implementation of the strategy is focused on the case of a light and degenerate charm quark, i.e. the GIM limit ($m_c = m_u$).

In order to tame the extreme statistical fluctuations encountered in simulations near the chiral limit, noise reduction techniques (low-mode averaging) must be employed [3]. After renormalising ratios of correlators non-perturbatively [4], we perform joint fits to the corresponding ChPT expressions in both the $\epsilon$ and $p$-regimes. Our results in the GIM limit [5] indicate a previously unseen, significant enhancement of the $\Delta I = 1/2$ amplitude, which however, is still a factor 4 smaller than the experimental number. First attempts to analyse the direct influence of the charm quark in ChPT [6] will be supplemented by future numerical studies.

References

[1] L. Giusti et al., JHEP 0411 (2004) 016
[2] M. Laine and P. Hernández, hep-lat/0607027
[3] L. Giusti et al., JHEP 0404 (2004) 013
[4] P. Dimopoulos et al., hep-lat/0607028
[5] L. Giusti, P. Hernández, M. Laine, C. Pena, J. Wennekers and H. Wittig, hep-ph/0607220
[6] M. Laine and P. Hernández, JHEP 0409 (2004) 018
The $\Delta$-resonance in a finite volume

Veronique Bernard$^1$, Ulf-G. Meißner$^{2,3}$ and Akaki Rusetsky$^{2,4}$

$^1$Université Louis Pasteur, Laboratoire de Physique Théorique
3-5, rue de l’Université, F-67084 Strasbourg, France

$^2$Universität Bonn, Helmholtz-Institut für Strahlen- und Kernphysik
Nußallee 14-16, D-53115 Bonn, Germany

$^3$Forschungszentrum Jülich, Institut für Kernphysik (Theorie)
D-52425 Jülich, Germany

$^4$High Energy Physics Institute, Tbilisi State University
University st. 9, 380086 Tbilisi, Georgia

The aim of the investigation, carried out in the present work, is to study the feasibility of the extraction of the $\Delta$-resonance parameters from the lattice data at the physical value of the pion mass, i.e. in the case of unstable $\Delta$. To this end, we calculate the self-energy of the $\Delta$-resonance in a finite volume up to and including $O(p^3)$ in Chiral Perturbation Theory, using infrared regularization. The poles of the $\Delta$-propagator, obtained in these calculations, determine the energy spectrum of the correlator of two $\Delta$-fields in a finite Euclidian box and can be obtained in lattice QCD. The results of our calculations enable one to investigate the dependence of the energy levels both on the size $L$ of the box and on the pion mass $M_\pi$.

As previously argued in the literature (see, e.g. [1],[2],[3]), the presence of a narrow resonance in the particle spectrum manifests itself through a peculiar irregular behavior of the energy levels with respect to $L$ (avoided level crossing near the resonance energy). We have however verified by explicit calculations at $O(p^3)$ that, in the case of interest, the dependence of energy levels is rather smooth – the avoided level crossing is nearly washed out due to a large $\Delta$-width. Despite this fact, on the basis of the numerical study, we argue that the extraction of the parameters of $\Delta$ (namely, of the mass and the $\pi N \Delta$ coupling constant) from a measured $L$-dependence of the lowest energy levels is still a feasible task.

References

[1] M. Lüscher, Les Houches lectures (1988); Nucl. Phys. B 364 (1991) 237.
[2] U. Wiese, Nucl. Phys. B (Proc. Suppl.) 9 (1989) 609.
[3] T. DeGrand, Phys. Rev. D 43 (1991) 2296.
Lattice QCD on Smaller and Larger Volumes

Meinulf Göckeler

For the QCDSF collaboration

1Institute for Theoretical Physics, University of Regensburg
93040 Regensburg, Germany.

The QCDSF collaboration (together with the DESY-ITEP-Kanazawa (DIK) collaboration) is generating gauge field configurations with two degenerate flavours of nonperturbatively $O(a)$-improved Wilson fermions (clover fermions) and Wilson glue, including in the analysis also older ensembles from UKQCD. In this talk we have discussed our approach to the treatment of finite size effects. It is based on appropriate versions of chiral effective field theory and hence intimately connected with the quark mass dependence of the physical quantities considered. In particular we deal with data from our two finite size runs (three volumes each, with the other simulation parameters kept fixed). Ongoing simulations at smaller quark masses are expected to shed new light on the chiral behaviour (see, e.g., Refs. [1], [2]).

Among the quantities studied are the nucleon mass [3] and the axial coupling constant of the nucleon [4]. For the pion (or pseudoscalar) mass and the pion decay constant we make use of the recently proposed “resummed” Lüscher type formulae [5].

References

[1] M. Göckeler et al. [QCDSF-UKQCD Collaboration], arXiv:hep-lat/0610066.
[2] M. Göckeler et al. [QCDSF-UKQCD Collaboration], arXiv:hep-lat/0610071.
[3] A. Ali Khan et al. [QCDSF-UKQCD Collaboration], Nucl. Phys. B689 (2004) 175 [arXiv:hep-lat/0312030].
[4] A. Ali Khan et al. [QCDSF Collaboration], arXiv:hep-lat/0603028.
[5] G. Colangelo, S. Dürr and C. Haefeli, Nucl. Phys. B721 (2005) 136 arXiv:hep-lat/0503014.
Aspects of ChPT at large $m_s$

Juerg Gasser\textsuperscript{a}, Christoph Haefeli\textsuperscript{a}, Mikhail A. Ivanov\textsuperscript{b}, Martin Schmid\textsuperscript{a}

\textsuperscript{a} Institute for Theoretical Physics, University of Bern
Sidlerstr. 5, 3012 Bern, Switzerland

\textsuperscript{b} Laboratory of Theoretical Physics, Joint Institute for Nuclear Research
141980 Dubna, Moscow region, Russia

Chiral perturbation theory determines the low–energy structure of Green functions in QCD through a systematic expansion in powers of quark masses and of the external momenta \cite{1,2}. The method has been successfully applied for an expansion in powers of $m_u$ and $m_d$ at fixed $m_s, m_c, \ldots$ [chiral SU(2)$\times$SU(2)] as well as for an expansion in powers of $m_u, m_d$ and $m_s$ at fixed $m_c, m_b, m_t$ [chiral SU(3)$\times$SU(3)]. It makes use of an effective action who’s pertinent low–energy constants (LECs) encode the heavy degrees of freedom that are not explicitly contained in it. The LECs in SU(2)$\times$SU(2) depend on the strange quark mass, while the ones in SU(3)$\times$SU(3) are independent thereof.

If one limits the external momenta to values small compared to $m_s$ and treats $m_u$ and $m_d$ as small in comparison to $m_s$, the degrees of freedom of the $K$– and $\eta$– mesons freeze and one can work out explicitly the strange quark mass dependence of the LECs in SU(2)$\times$SU(2) from the effective action pertaining to SU(3)$\times$SU(3). For example, for the pion decay constant $F$ at $m_u = m_d = 0$, $m_s \neq 0$, one has at one loop \cite{2}

\[
F = F_0 \left( 1 - \frac{m_s B_0}{32 \pi^2 F_0^2} \ln \frac{m_s B_0}{\mu^2} - \frac{256 \pi^2 L_4^*}{\mu^2} \right) + O(m_s^2).
\]

Here, $F_0, B_0$ and $L_4^*$ are LECs from SU(3)$\times$SU(3), and the renormalization scale is denoted by $\mu$. Relations of this type generate constraints on the possible values of the low–energy constants: information on LECs in SU(2)$\times$SU(2) may be translated into information on LECs in SU(3)$\times$SU(3) and vice versa \cite{2}.

In my talk, I discussed recent efforts \cite{3} to establish the corresponding relations between the LECs to two–loop accuracy.

References

\cite{1} S. Weinberg, PhysicaA 96 (1979) 327.

\cite{2} J. Gasser and H. Leutwyler, Annals Phys. 158, 142 (1984);
J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985).

\cite{3} J. Gasser, C. Haefeli, M. A. Ivanov, M. Schmid, in preparation.
A Chiral Two-Matrix Theory and the Chiral Lagrangian

Poul H. Damgaard

Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Denmark

In the $\epsilon$-regime of QCD an efficient method for extracting the chiral condensate $\Sigma$ is based on the distributions of Dirac operator eigenvalues. Previously, the analogous technique for the pion decay constant $F_\pi$ was based on introducing a vector source (“imaginary isospin chemical potential”). Compact analytical expressions have been found for the quenched case [1] and that of two light dynamical quarks [2]. It is computationally difficult to go beyond these two cases.

One would like to find a Random Matrix Theory [3] that describes the inclusion of imaginary isospin chemical potential. Because it seeks the microscopic spectra of Dirac operators with different chemical potential for $u$ and $d$ type quarks, it turns out to be given by a Random Two-Matrix Theory [4]. We solve this theory for finite size $N$ of the matrices by means of biorthogonal polynomials. By deriving the relevant kernels we find explicit formulas for all $(n,k)$ mixed or unmixed spectral correlation functions of the two involved Dirac operators. In the microscopic scaling limit we compute the corresponding analytical expressions as well. These correspond to the leading expressions of the $\epsilon$-expansion based on the chiral Lagrangian. In the two cases where comparison to the chiral Lagrangian framework are possible [1],[2] we find exact agreement. A useful by-product is the corresponding analytical expressions for spectral correlation functions of Dirac operators with chemical potential, evaluated in ensembles with dynamical quarks of zero chemical potential. These expressions allow for a determination of the pion decay constant by means of ordinary lattice configurations.

References

[1] P. H. Damgaard, U. M. Heller, K. Splittorff and B. Svetitsky, Phys. Rev. D 72 (2005) 091501 [hep-lat/0508029].
[2] P. H. Damgaard, U. M. Heller, K. Splittorff, B. Svetitsky and D. Toublan, Phys. Rev. D 73 (2006) 074023 [hep-lat/0602030]; Phys. Rev. D 73 (2006) 105016 [hep-th/0604054].
[3] E. V. Shuryak and J. J. M. Verbaarschot, Nucl. Phys. A 560 (1993) 306 [hep-th/9212088].
[4] G. Akemann, P. H. Damgaard, J. C. Osborn and K. Splittorff, “A new chiral two-matrix theory for Dirac spectra with imaginary chemical [hep-th/0609059].
The $B \rightarrow D^*\ell\nu$ form factor from lattice QCD

Jack Laiho$^1$ on behalf of the MILC and Fermilab collaborations

$^1$Theoretical Physics Department, Fermilab, Batavia, IL 60510

The CKM element $V_{cb}$ is important for the phenomenology of flavor physics in determining the apex of the unitarity triangle in the complex plane. It is possible to determine $|V_{cb}|$ from both inclusive and exclusive semileptonic $B$ decays, and they are both limited by theoretical uncertainties. The inclusive method makes use of the heavy quark expansion [1], [2], but is limited by the breakdown of local quark-hadron duality, the errors of which are difficult to estimate. The exclusive method requires reducing the uncertainty of the form factor $F_{B \rightarrow D^*}$, which has been calculated with lattice QCD in the quenched approximation using the double ratio method [3]. Given the phenomenological importance of this quantity we have revisited this calculation of $F_{B \rightarrow D^*}$ using the 2+1 flavor MILC lattices with improved light staggered quarks [4]. The lattice calculation was done on the MILC coarse lattices ($a \approx 0.125$ fm) where the light quarks were computed with the “asqtad” action. The heavy quarks were computed using the clover action with the Fermilab interpretation in terms of HQET [5]. We have looked at three light masses at the full QCD points, $m_{valence} = m_{sea}$. The lightest such mass was around $m_s/7$. The chiral perturbation theory (ChPT) for heavy light mesons with a Wilson-like heavy quark and a staggered light quark was worked out by Aubin and Bernard in [6], and the ChPT relevant for $B \rightarrow D^*$ was done in [7]. The plan for the future is to add statistics by running on multiple time sources, and to run on additional lattice spacings generated by MILC and the Fermilab lattice group in order to determine the lattice spacing dependence. We will incorporate the matching coefficients into the analysis, and of course, we will do a careful analysis of all the systematic errors in order to report a final number for the $B \rightarrow D^*\ell\nu$ form factor.

References

[1] P. Ball, M. Beneke, and V.M. Braun, Phys. Rev. D 52, 3929 (1995) [hep-ph/9503492].
[2] I. Bigi, M. Shifman, and N. Uraltsev, Annu. Rev. Nucl. Part. Sci. 47, 591 (1997) [hep-ph/9703290].
[3] S. Hashimoto, et. al., Phys. Rev. D 66, 014503 (2002) [hep-ph/0110253].
[4] C. Aubin et al., (MILC), Phys. Rev. D 70, 114501 (2004) [hep-ph/0408306].
[5] A. El-Khadra, A. Kronfeld and P. Mackenzie, Phys. Rev. D 55 3933 (1997) [hep-lat/9604004].
[6] C. Aubin and C. Bernard, (2004) [hep-lat/0409027].
[7] J. Laiho and R. Van de Water, Phys. Rev. D 73, 054501 (2006) [hep-lat/0512007].
Heavy Quark Effective Theory on the lattice: The b-quark mass including O(1/m_b)

M. Della Morte

1CERN, Physics Department, TH Division, CH-1211 Geneva 23, Switzerland

The b-quark is too heavy to be treated dynamically on the lattice. Indeed, in units of nowadays affordable resolutions \( am_b \gg 1 \). A theoretically attractive option is to use effective theories like Heavy Quark Effective Theory (HQET). Introduced in [1], it provides the correct asymptotic description of QCD correlation functions in the limit \( m_b \to \infty \). Subleading effects are described by higher dimensional operators whose coupling constants are formally \( O(1/m_b) \) to the appropriate power. The degrees of freedom in the effective theory are strongly coupled and therefore a non-perturbative approach is needed. In addition HQET has to be matched to the full theory, a step, which must as well be performed non-perturbatively beyond leading order in \( 1/m_b \).

A framework for non-perturbative HQET on the lattice has been introduced in [2]. We present here a specific application of that approach to the computation of the b-quark mass in the quenched approximation. The details can be found in [3]. In a first step we match HQET and QCD in a small volume (\( L = 0.4 \) fm). To make contact with phenomenology the HQET expressions of the relevant quantities are then evolved to large volume. The essential tools in this step are the step scaling functions, which we introduce to describe the effects of a change in the linear size \( L \) of the system by a factor two. Eventually the b-quark mass is fixed using the (spin averaged) \( B_s \) meson mass. We include the subleading, \( 1/m_b \), terms in each step. Our final result is \( \overline{m}_b(\overline{m}_b) = 4.347(48) \) GeV in the \( \overline{MS} \) scheme. We have implemented twelve different matching conditions for this computation. Although the contributions at a given order may differ in the twelve different cases, once the LO and the NLO terms are summed together all the estimates agree. That implicitly suggests that the \( 1/m_b^2 \) corrections to the quark mass are extremely small as expected. We plan to extend the method to the computation of the decay constant \( F_{B_s} \), which we have already computed in the static approximation [4] and to real (unquenched) QCD.

References

[1] E. Eichten and B. Hill, Phys. Lett. B234 (1990) 511.
[2] ALPHA Collaboration, J. Heitger and R. Sommer, JHEP 0402:022, 2004.
[3] ALPHA Collaboration, M. Della Morte, N. Garron, M. Papinutto and R. Sommer, hep-ph/0609294.
[4] ALPHA Collaboration, M. Della Morte et al., Phys. Lett. B581 (2004) 93; Erratum-ibid. B612 (2005) 313.
Overlap hypercube quarks in the $p$- and the $\epsilon$-regime

Wolfgang Bietenholz$^1$, Stanislav Shcheredin$^2$ and Jan Volkholz$^1$

$^1$ Institut für Physik, Humboldt Universität zu Berlin,
Newtonstr. 15, D-12489 Berlin, Germany

$^2$ Fakultät für Physik, Universität Bielefeld
D-33615 Bielefeld, Germany

The overlap hypercube fermion is a variant of a chirally symmetric lattice fermion, which has a much higher degree of locality than the standard overlap fermion. We applied this formulation in quenched QCD simulations with light quarks$^1$, which were performed with HLRN machines on a $12^3 \times 24$ lattice at $\beta = 5.85$ (corresponding to a lattice spacing of $a \simeq 0.123$ fm according to the Sommer scale). The topological susceptibility that we obtained supports the Witten-Veneziano scenario to explain the mass of $\eta'$.

In the $p$-regime we evaluated the masses of light pseudoscalar and vector mesons. Although we reach $m_\pi < 300$ MeV it is difficult to extrapolate the corresponding value for $F_\pi$ — measured from a matrix element — to the physical point. However, we found a stable axial current renormalisation constant $Z_A$ close to 1, in contrast to the results with the standard overlap operator.

The use of tiny bare quark masses in the same volume took us to the $\epsilon$-regime, where we evaluated the leading Low Energy Constants $\Sigma$ and $F_\pi$ with various methods$^1$. The densities of low lying Dirac eigenvalues were fitted to predictions by chiral Random Matrix Theory, which yields $\Sigma \approx (300 \text{ MeV})^3$. Next we compared the axial current correlators to formulae of quenched Chiral Perturbation Theory, which led to $F_\pi \approx 108$ MeV. On the other hand, a method using only the zero mode contributions to the pseudoscalar correlators gave a lower value, close to the phenomenological $F_\pi$. We conclude that these methods have the potential to determine Low Energy Constants of the chiral Lagrangian directly from the first principles of QCD, but precise values have to await the feasibility of simulations with dynamical, chiral quarks.

As a step into this direction, we simulated dynamical overlap hypercube fermions in the Schwinger model using a Hybrid Monte Carlo algorithm with a simplified force, where we tested algorithmic requirements like reversibility$^2$. The measurements confirm an extremely high level of locality. We also evaluated the chiral condensate based on the Dirac spectrum in the $\epsilon$-regime, and we found good agreement with low energy predictions at two very light fermion masses.

References

[1] W. Bietenholz and S. Shcheredin, *Nucl. Phys.* B 754 (2006) 17.
[2] J. Volkholz, W. Bietenholz and S. Shcheredin, [hep-lat/0609003](http://arxiv.org/abs/hep-lat/0609003).
\textbf{σ(600) as a Tetraquark Mesonium and the Pattern of Scalar Mesons}

Keh-Fei Liu

Dept. of Physics and Astronomy, University of Kentucky, Lexington, KY 40506, USA

Scalar mesons are not as well known as the pseudoscalar and vector mesons in terms of their $SU(3)$ classification, their particle content and their spectrum. It has been known that there are too many experimental states for the $q\bar{q}$ nonet, possibly by a factor of two. We shall use lattice QCD calculations and combine with phenomenology to help understand the nature of the scalar mesons, their particle content and their $SU(3)$ classification.

A recent lattice calculation [1] with the overlap fermion on $16^3 \times 28$ and $12^3 \times 28$ quenched lattices with the Iwasski gauge action at $a = 0.2$ fm has shown that the isovector scalar mesons with the $\bar{\psi}\psi$ interpolation field has an unusual behavior as it approaches the chiral limit. Below the strange quark mass, its mass becomes flat and is almost independent of the quark mass, in contrast to other hadrons. This calculation which reaches a pion mass as low as 180 MeV is consistent with earlier findings with heavier quark masses [1]. Its chiral limit extrapolation indicates that $a_0(1450)$ is the $q\bar{q}$ meson and the $a_0(980)$ below it is not seen with the $\bar{\psi}\psi$ interpolation field. It explains the experimental fact that $K_{0}^{*}(1430)$ is basically degenerate with $a_0(1450)$. From the near degeneracy of $a_0(1450)$, $K_{0}^{*}(1430)$, and $f_0(1500)$, we build a model for the mixing of $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ through the annihilation of $u\bar{u}, d\bar{d}$ and $s\bar{s}$ with a slight $SU(3)$ breaking in their mixing and decays [2]. It is found that ratios of pseudoscalar meson decays can be well described by this simple model. The interesting thing is that we found $f_0(1500)$ is almost a pure octet, $f_0(1710)$ an almost pure glueball, and $f_0(1370)$ is mainly a singlet with $\sim 10\%$ glueball content.

We also calculated tetraquark mesonium with $I=0$ $\pi\pi$ interpolation field and found a state around 550 MeV in addition to the $\pi\pi$ scattering states in the range where pion mass is lower than 250 MeV. Using the volume study of their spectral weight [3],[4], we verified that the lowest state around 300 MeV is the interacting $\pi\pi$ state and the state at $\sim 550$ MeV is a one-particle state which we believe is the $\sigma(600)$. Our present lattice calculation only confirms its existence. To get its precise mass and width, one needs to vary the lattice size to bring down the scattering state with one unit of lattice momentum to mix with it and observe how far the levels avoid each other.

Based on the above two lattice calculations, we speculate on the following pattern for scalar mesons: $a_0(980), f_0(980)$ and $\kappa(800)$ form a tetraquark mesonium octet with the $\sigma(600)$ as the singlet. On the other hand, $a_0(1450), K_0^*(1430)$, and
\( f_0(1500) \) form a \( q\bar{q} \) octet with \( f_0(1370) \) as its singlet and a almost pure glueball which is \( f_0(1710) \). This needs additional experimental and lattice confirmation.

This work is partially supported by the US DOE grant DE-FG05-84ER40154. I thank the organizers of the workshop on Lattice QCD, Chiral Perturbation Theory, and Hadron Phenomenology for giving me a chance to present this work.

References

[1] \( \chi \)QCD Collaboration, N. Mathur, A. Alexandru, Y. Chen, S.J. Dong, T. Draper, I. Horvath, F.X. Lee, K.F. Liu, S. Tamhankar, J.B. Zhang, [hep-ph/0607110].

[2] H.Y. Cheng, C.K. Chua, and K.F. Liu, [hep-ph/0607206].

[3] N. Mathur et. al., Phys. Lett. B605, 137 (2005), [hep-ph/0306199].

[4] \( \chi \)QCD Collaboration, N. Mathur et. al., Phys. Lett. B605, 137 (2005), [hep-ph/0306199].
Moments of Generalized Parton Distribution Functions

Marina Dorati\textsuperscript{1}, Thomas R. Hemmert\textsuperscript{2}

\textsuperscript{1}Dep. Theor. Phys., Univ. of Pavia, 27100 Pavia, Italy
\textsuperscript{2}Physik Dep. T39, TU München, D-85747 Garching

We apply the formalism of covariant Baryon Chiral Perturbation Theory (BChPT) to the analysis of the moments of the Generalized Parton Distributions (GPDs) of the nucleon. We concentrate on the isovector $n=1$ case and perform an $\mathcal{O}(p^4)$ calculation to obtain the Generalized Form Factors $A_{2,0}^v(q^2)$, $B_{2,0}^v(q^2)$ and $C_{2,0}^v(q^2)$. These results can be used for chiral extrapolations of lattice data for GPDs. Of particular interest is the forward limit, where $A_{2,0}^v(q^2 = 0)$ reduces to the averaged momentum fraction $\langle x \rangle_{u-d}$, a quantity known from phenomenology. Lattice data are also available for this moment. The figure shows two different chiral extrapolation functions. The dashed one indicates the influence of the well known leading chiral logarithm $[1]$, unable to reach the plateau observed in recent lattice simulations $[2]$. The full curve shows our $\mathcal{O}(p^4)$ covariant BChPT result in the modified Infrared Regularization scheme $[3]$. We conclude $[4]$ that a whole tower of quark mass dependent terms is required in order to understand $\langle x \rangle_{u-d}(m_\pi)$, in contrast to other recent claims $[5]$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure.png}
\caption{Chiral extrapolation functions.}
\end{figure}

References

[1] D. Arndt, M. Savage, Nucl.Phys. A697:429-439 (2002). J-W. Chen, X. Ji, Phys.Lett. B523:107-110 (2001).
[2] G. Schierholz et al., QCDSF collaboration, private communication.
[3] See contribution by T. Gail, these proceedings.
[4] M. Dorati, T.R. Hemmert, forthcoming.
[5] See e.g contribution by D. Richards, these proceedings.
ChPT for Generalized Parton Distributions (GPDs)

M. Diehl\textsuperscript{1}, A. Manashov\textsuperscript{2} and A. Schäfer\textsuperscript{2}

\textsuperscript{1}Deutsches Elektron-Synchrotron, DESY, 22603 Hamburg
\textsuperscript{2}Institute for Theoretical Physics, University of Regensburg, 93040 Regensburg

Generalized parton distributions provide a unified parametrization of hadron structure. They allow one to combine in the most efficient way the information obtained from various inclusive and exclusive reactions and permit to extract informations on the internal hadron structure which cannot be obtained directly by experiment. However, the extraction of GPDs from experiment is very non-trivial, especially to NLO and NNLO in $\alpha_s$, which is needed in view of the typical lowish $Q^2$. Therefore, lattice calculations of moments of GPDs on the lattice are especially important for GPD physics \cite{1}. The extrapolation of such results to physical quark masses requires a ChPT analysis of these moments.

We performed such an analysis, both for the pion GPDs \cite{2} and for the isosinglet nucleon GPDs \cite{3}. We proceed by first constructing the leading order ChPT equivalents of the operators appearing in the OPE analysis of moments of GPDs. We then construct the next-to-leading contributions and calculate the 1-loop corrections to the leading ones.

The results allow one to identify the leading analytic terms which appear and permit to relate some LECs to known physics.

References

\cite{1} M. Göckeler et al. [QCDSF Collaboration],
Phys. Rev. Lett. 92 (2004) 042002 [arXiv:hep-ph/0304249].
Phys. Lett. B 627 (2005) 113 [arXiv:hep-lat/0507001].
Ph. Hägler et al. [LHPC Collaboration],
Eur. Phys. J. A 24S1 (2005) 29 [arXiv:hep-ph/0410017].
Phys. Rev. Lett. 93 (2004) 112001 [arXiv:hep-lat/0312014].

\cite{2} M. Diehl, A. Manashov and A. Schäfer,
Phys. Lett. B 622 (2005) 69 [arXiv:hep-ph/0505269].

\cite{3} M. Diehl, A. Manashov and A. Schäfer,
Eur. Phys. J. A 29 (2006) 315 [arXiv:hep-ph/0608113].
The QCD vacuum as seen by overlap fermions

Volker Weinberg
Deutsches Elektronen-Synchrotron DESY, 15738 Zeuthen, Germany
– QCDSF / DIK Collaboration –

With the advent of overlap fermions, which are the cleanest known theoretical description of lattice fermions implementing chiral symmetry, it has recently become possible\(^1\) to analyze the vacuum structure of Yang-Mills theory and the nature of the chiral phase transition of QCD from first principles using a fermionic approach.

We report results based on \(\mathcal{O}(150)\) low lying overlap eigenmodes on quenched Lüscher-Weisz configurations on various \(T=0\) lattices that were obtained within the QCDSF-collaboration [1]. The analysis is also extended to dynamical finite temperature configurations generated by the DIK-collaboration [2] using the clover-improved Wilson action.

The distribution of the lowest modes in selected topological sectors and the spectral density is in good agreement with the finite-volume prediction of quenched chiral perturbation theory and Random Matrix Theory. In the vicinity of the finite-\(T\) phase transition the emergence of a gap in the spectrum is observed.

Truncating the spectral decomposition of both the overlap-based topological charge density operator and the gluonic field strength tensor allows for a removal of fluctuations at the scale of the cutoff. Comparing the UV-filtered operators, we observe a strong correlation between the peaks of the topological charge density, the regions of high (anti-)selfduality and the domains of strong local chirality of the lowest localized eigenmodes. In the chiral-symmetry restored phase, on the other hand, the signal of local chirality and (anti-)selfduality is strongly suppressed. In contrast to the singular low-dimensional structure of the untruncated topological charge density, the UV-filtering reveals a structure of the \(T=0\) vacuum much more reminiscent of semiclassical-like instanton pictures.

References

[1] Y. Koma et al., PoS LAT2005, 300 (2006); E.-M. Ilgenfritz et al., Nucl. Phys. Proc. Suppl. 153, 328 (2006); V. Weinberg et al., PoS LAT2006, 078 (2006); E.-M. Ilgenfritz et al., [hep-lat/0611007] (2006); E.-M. Ilgenfritz et al., in preparation.

[2] V.G. Bornyakov et al., Phys. Rev. D 71, 114504 (2005); V.G. Bornyakov et al., PoS LAT2005, 157 (2006); V. Weinberg et al., PoS LAT2005, 171 (2006); V. Weinberg et al., in preparation.

\(^1\)See Ref. [1] and references therein.
The Chiral Regime of QCD in the Instanton Liquid Model

Marco Cristoforetti, Pietro Faccioli, Marco C. Traini, and John W. Negele

1 Dipartimento di Fisica and I.N.F.N., Università degli Studi di Trento, Via Sommarive 15, Povo (Trento) 38050 Italy.

2 Center for Theoretical Physics Massachusetts Institute of Technology, NE25-4079, 77 Massachusetts Ave, Cambridge, MA 02139-4307, USA.

Contemporary lattice QCD calculations have highlighted the need to understand the dependence of QCD observables on the quark mass, in order to provide reliable extrapolation formulas. In addition to this motivation, studying the microscopic dynamical mechanisms involved in the transition into the chiral regime of QCD may shed light on the structure of the non-perturbative quark-quark interaction.

In this work we investigate the non-perturbative quark-gluon dynamics involved in such a transition in the context of the Interacting Instanton Liquid Model (IILM) [1]. By computing the nucleon and pion masses for a wide range of quark masses, we show that the IILM reproduces the existing lattice data for pion masses in the range $\simeq 500 - 600$ MeV. Fitting the nucleon masses, using covariant Baryon $\chi$PT at order $O(p^4)$, we obtain the chiral coefficients in good agreement phenomenology. In particular the parameters $c_1$ and $c_3$ match well the values extracted from the analysis of the existing $\pi N$ scattering data. On the other hand, the same chiral extrapolation performed using the available MILC data instead of the IILM data leads to chiral parameters $c_1$ and $c_3$ which are inconsistent with the experimental data. Such an inconsistency is probably due to the fact that the available MILC data have not been extrapolated to the continuum limit. We also show that the IILM agrees well also with Chiral Perturbation Theory in the small quark mass regime. For example, we found that the spectral density of the Dirac operator calculated in the IILM agrees with the behavior expected from chiral perturbation theory for two flavors. Hence, we can argue that the IILM can be used to study the transition into the chiral regime of QCD. We identify a characteristic quark energy scale, $m^* = 80$ MeV, which determines the boundary of the near-zero mode zone of the quark propagator in the instanton background. We argue that this scale governs the transition into the chiral regime and we discuss its physical interpretation.

References

[1] M. Cristoforetti, P. Faccioli, J.W. Negele and M. Traini, arXiv: [hep-ph/0605256].

40
Discussion: Lattice meets Chiral Perturbation Theory

Hartmut Wittig\(^1\) (convenor)

\(^1\)Institut für Kernphysik, Universität Mainz, D-55099 Mainz, Germany.

I summarise the main points of the discussion held at the workshop:

**What is the status of lattice determinations of LECs?**
Even “simple” LECs such as \(F_\pi\) are difficult to extract, signified by the large variation of results for this quantity. The compatibility of results from different simulations must be better understood, and an attempt to do this using recent data was presented \([1]\).

It was noted that the precision with which lattice determinations of LECs are quoted by some groups, is far too high, given the quality of the lattice data.

**Have we made contact between ChPT and lattice QCD?**
Often one finds that authors regard the presence of chiral logs in lattice data as the criterion for contact between lattice QCD and ChPT, since the coefficient of the chiral log is universal. It was pointed out \([2]\) that this is not necessarily true, since this coefficient acquires regularisation-dependent contributions if chiral symmetry is not respected.

In another contribution \([3]\) the two-loop analysis for the quark mass dependence of \(F_\pi\) \([4]\) was compared to lattice data by NPLQCD \([5]\) with pion masses as low as 300 MeV. While the two-loop formula describes the lattice data reasonably well, the claimed precision for the lattice determination of \(F_\pi\) seemed exaggerated.

**What do ChPT people need from the lattice?**
More lattice data at smaller pion masses (\(m_\pi < 300\) MeV) must become available for more reliable determinations of LECs. In mesonic ChPT, the \(O(p^4)\) LEC \(L_6\) is not well constrained by phenomenology and should be a target for lattice calculations. Also, there are few results for LECs in the baryonic sector. Here, the lattice should try to find ways to determine the so-called class II LECs, or symmetry breakers, since class I LECs having a dynamical origin (i.e. those multiplying terms with derivatives) are more easily determined from phenomenology \([6]\).

It was suggested that a compilation of the status of determining LECs be made available, possibly in the form of a web-page. Some information can also be found in \([6]\).

**What do lattice people need from lattice people?**
A standardised way of quoting results in order to facilitate comparison of results from several groups \([7]\). In particular, one standard choice of lattice scale (e.g.
$r_0$) should be employed in addition to any other scale that collaboration might favour otherwise.

References

[1] Luigi Scorzato, contribution to discussion; K. Jansen and C. Urbach [ETM Collaboration], hep-lat/0610015
[2] Oliver Bär, contribution to discussion
[3] Ulf-G. Meißner, contribution to discussion
[4] G. Colangelo, S. Dürr and Ch. Haefeli, Nucl. Phys. B721 (2005) 136
[5] S.R. Beane, P.F. Bedaque, K. Orginos and M.J. Savage [NPLQCD Collaboration], Phys. Rev. D73 (2006) 054503
[6] Ulf-G. Meißner, PoS LAT2005 (2006) 009, hep-lat/0509029
[7] Gerrit Schierholz, contribution to discussion