Revision of Defeasible Logic Preferences

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Abstract. There are several contexts of non-monotonic reasoning where a priority between rules is established whose purpose is preventing conflicts. One formalism that has been widely employed for non-monotonic reasoning is the sceptical one known as Defeasible Logic. In Defeasible Logic the tool used for conflict resolution is a preference relation between rules, that establishes the priority among them. In this paper we investigate how to modify such a preference relation in a defeasible logic theory in order to change the conclusions of the theory itself. We argue that the approach we adopt is applicable to legal reasoning where users, in general, cannot change facts or rules, but can propose their preferences about the relative strength of the rules. We provide a comprehensive study of the possible combinatorial cases and we identify and analyse the cases where the revision process is successful. After this analysis, we identify three revision/update operators and study them against the AGM postulates for belief revision operators, to discover that only a part of these postulates are satisfied by the three operators.

Keywords. Knowledge representation, non-monotonic reasoning, sceptical logics, belief revision.

1 Introduction

A large number of real-life cases in legal reasoning, information security, digital forensics, and even engineering or medical diagnosis, exhibit the two following circumstances: (a) different persons have different preferences, and (b) decision making depends upon the order the rules are applied. When the decision mechanism is based on rules, and the rules are in conflict, then inconsistencies may be generated, and decision making may require preferences to solve/avoid conflicts. It may occur that, under certain circumstances, using a particular set of preferences to solve a conflict does not result in the desired/expected outcome. Accordingly, to revise the outcome we can revise the underlying preferences.

Non-monotonic reasoning has been advanced for common-sense reasoning and reasoning with partial and conflicting information. We can distinguish two types of non-monotonic reasoning: credulous and sceptical. In credulous non-monotonic reasoning, once a conflict arises, we independently explore the two branches of the conflict, while
in sceptical non-monotonic reasoning a conflict must be solved before proceeding with
the reasoning. Here we concentrate on sceptical non-monotonic reasoning.

Typically sceptical non-monotonic formalisms are equipped with techniques to ad-
dress conflicts, where a conflict is a combination of reasoning chains leading to a con-
tradiction. The most common device to handle conflicts is a preference or superiority
relation over the elements used by the formalism to reason. These elements can be for-
male, axioms, rules or arguments, and the preference relation states that one of such
elements is to be preferred to another one when both can be used.

In this research we focus on a specific rule-based non-monotonic formalism, Defeas-
sible Logic, but the motivation behind the particular technical development applies in
general to other rule-based non-monotonic formalisms equipped with a priority about
rules. Indeed, considering a rule based formalism, knowledge is partitioned in facts
(describing immutable propositions/statements about a case), rules (describing rela-
tionships between a set of premises and a conclusion), and a preference relation or
superiority relation (describing the relative strength of rules). A revision operation[1]
transforms a theory by changing some of its elements, be it the facts, rules, or supe-
riority relation. Revision based on change of facts corresponds to an update operation
[1], revision based on modification of rules has been investigated in [2], whilst to the
best of our knowledge, revision of non-monotonic theories based on modifications of
the underlying superiority relation has been neglected so far.

In this paper we study revision of defeasible theories operating on the superiority
relation. We begin by arguing that, while little attention has been dedicated to this topic,
it has natural correspondences to reasoning patterns in legal reasoning. After that, we
investigate the type of operations that are possible for this kind of revision.

Once we introduced the operation, it is only natural to follow by establishing which
properties they enjoy. The properties that are significant for revision operations in belief
revision in order to be rational have already been isolated in a systematic view proposed
by Alchourrón, Gärdenfors and Makinson [3], namely the AGM postulates.

The AGM postulates where designed with classical logic in mind. Classical logic
is monotonic, therefore if we add new information which is incompatible with the old
one, an inconsistency arises. In this scenario, the sole way to recover consistency is to
invoke a revision operator.

Conversely, due to the nature of non-monotonic reasoning, adding new “incompat-
able” information in a non-monotonic system usually does not generate a contradiction
within the theory, even if the result may not be conceptually satisfactory.

Given the difference in nature between classical logic and non-monotonic reason-
ing, it is of interest to investigate which AGM postulates apply to non-monotonic rea-
soning, to what extent, and in which form they apply.

In the recent years, a few works addressed the issue of belief revision in non-
monotonic logics where there is a general understanding that the AGM postulates are
not fully appropriate for non-monotonic reasoning. For example, [4] shows that belief
revision methodologies are not suitable to changes in specific significant non-monotonic

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1 In general we will use the term revision operation to denote any operation that changes a
theory. In Sections 4 and 5 the term will be understood in a specific technical sense.
theories, and that it is possible to revise such theories fully satisfying the AGM postulates, but then the outcome is utterly meaningless for their purposes.

Still, the matter whether and which postulates hold is far from being settled. For example, [5] proposed an approach to belief revision of logic programs under answer set semantics that is fully compliant with the base AGM postulates for revision. On the other hand, Delgrande [6, p. 568] asserts that a subset of postulates for belief revision is not appropriate for belief revision of non-monotonic theories (and thus is ignored in his work), while we will argue in Section 5 that the same postulates can be adopted (Alternativa: are meaningful) in our approach. This suggests that the suitability of AGM postulates to a belief revision approach for non-monotonic reasoning is still debatable.

The paper is organised as follows: In Section 2 we motivate that reasoning over preferences on rules and on how to modify the preferences is a natural reasoning pattern in legal reasoning. Then, in Section 3 we introduce Defeasible Logic, the formalism chosen for our investigation; in particular, we introduce new auxiliary proof tags to describe derivations in Defeasible Logic. The new proof tags do not modify the expressive power of the logic, but they identify patterns where instances of the superiority relation contribute to the derivation of a conclusion. Armed with this technical machinery, in Section 4 we start by proving that the problem of revising a theory changing the superiority relation is, in general, an NP-complete problem; secondly, we provide an exhaustive analysis of the cases and conditions under which revision operations modifying only the superiority relation are successful. Section 5 analyses the AGM postulates against the introduced operators. Section 6 overviews closely related approaches, and Section 7 concludes the paper with a summary of the achieved results, discussion of related works and quick hints for future developments.

2 Norms and Preferences in Legal Reasoning

It has been argued [7] that some aspects of legal reasoning can be captured by non-monotonic rule-based formalisms. The main intuition is that norms can be represented by rules, the evidence in cases by facts, and that the superiority relation is induced by legal principles determining how to solve conflicts between norms.

We take the stance that, typically in the legal reasoning domain, we do not have control over the rules (norms) or their modification, but have some control on how they can be used. An average citizen has no power to change the Law, and has no power on what norms are effective in the jurisdiction she is situated in. These powers instead are reserved to persons, entities and institutions specifically designated to do so, for example the parliament and, under some given constraints, also by judges (in Common Law juridical system, especially).

However, a citizen can argue that one norm instead of another applies in a specific case. This amounts to saying that one norm is to be preferred to the other in the case. *Prima-facie* conflicts appear in legal systems for a few main reasons, among which we can easily identify three major representatives: (1) norms from different sources, (2) norms emitted at different times, and (3) exceptions. These phenomena are well understood and have given rise to principles which existed for a long time in legal theory.
and been used to solve such issues. These principles are still used in many situations, such as an argument to drive constitutional judgement against a given norm or a given sentence. Here we list the three major legal principles, expressing preferences among rules to be applied [8].

Lex Superior This principle states that when there is a conflict between two norms from different sources, the norms originating from the legislative source higher in the legislative source hierarchy takes precedence over the other norm. This means that if there is a conflict between a federal law and a state law, the federal law prevails over the state law.

Lex Posterior According to this principle, a norm emitted after another norm takes precedence over the older norm.

Lex Specialis This principle states that when a norm is limited to a specific set of admissible circumstances, and under more general conditions another norm applies, the most specific norm prevails.

Besides the above principles a legislator can explicitly establish that one norm prevails over a conflicting norm.

The intuition behind these principles (and eventually others) is that when there are two conflicting norms, and the two norms are applicable in a specific case, we can apply one of these principles to create an instance of a superiority relation that discriminates between the two conflicting norms. However, there are further complications. What if several principles apply and these produce opposite preferences? Do the preferences lead to opposite outcomes of a case? These are examples of situations when revision of preferences is relevant. The following example illustrates this situation.

Charlie is an immigrant son of an Italian, and living in Italy, who is interested in joining the Italian Army, based on Law 91 of 1992. However, his application is rejected, based upon a constitutional norm (Article 51 of the Italian Constitution). The two norms Law 91 and Article 51 are in conflict, thus the Army’s decision is based on the lex superior principle. Charlie appeals against the decision in court. The facts of the case are undisputed, and so are the norms to be applied and their interpretation. Thus the only chance for Bob, Charlie’s lawyer, to overturn the decision is to argue that Law 91 overrides Article 51 of the Constitution. Thus Bob counter-argues appealing to the lex specialis principle since Law 91 of 1992 explicitly covers the case of a foreigner who applies for joining the Army for the purpose of obtaining citizenship.

The two arguments do not discuss about facts and rules that hold in the case. They disagree about which rule prevails over the other, Article 51 of the Constitution or Law 91. In particular, Bob’s argument can be seen as an argument where the relative strength of the two rules is reversed compared to the argument of the Army’s lawyer, and it can be used to revise the previous decision.

While the mechanism sketched above concerns the notion of strategic reasoning, where a discussant looks at the best argument to be used in a case to prove a given claim, in this case, that one rule prevails over another rule. However, the key aspect is that before embarking in this kind of arguments, one has to ensure that changing a preference leads to a different outcome of the claim of the case. It is not our aim to study how to justify preferences using argumentation. In this work, we investigate if it is possible to modify the extension of a theory (as represented by a defeasible theory)
only through changes on the superiority (preference) relation. Thus, we believe that our framework is foundational for argumentation of preferences. This means that one can first determine whether the outcome of a discussion can be turned in her favour only changing the superiority relation, and then to figure out which argument (if any) supports the preference.

In the current literature about formalisms apt to model normative and legal reasoning, a simple and efficient non-monotonic formalism which has been discussed in the community is Defeasible Logic. This system is described in detail in the next sections.

One of the strong aspects of Defeasible Logic is its characterisation in terms of argumentation semantics [9]. In other words, it is possible to relate it to general reasoning structure in non-monotonic reasoning which is based on the notion of admissible reasoning chain. An admissible reasoning chain is an argument in favour of a thesis. For these reasons, much research effort has been spent upon Defeasible Logic, and once formulated in a complete way it encompasses other (sceptical) formalisms proposed for legal reasoning [9,10,11].

Most interestingly, in Defeasible Logic we can reach positive conclusions as well as negative conclusions, thus it gives understanding to both accept a conclusion as well as reject a conclusion. This is particularly advantageous when trying to address the issues determined by reasoning conflicts.

It has been pointed out that the AGM framework for belief revision is not always appropriate for legal reasoning [4]. Moreover, it is not clear how to apply AGM to preference revision. Accordingly, this paper provides a comprehensive study of the conditions under which it is possible to revise a defeasible theory by changing the superiority relation of the theory.

### 3 Defeasible Logic

A defeasible theory consists of five different kinds of knowledge: facts, strict rules, defeasible rules, defeaters, and a superiority relation [12]. Examples of facts and rules below are standard in the literature of the field.

**Facts** denote simple pieces of information that are considered always to be true. For example, a fact is that “Sylvester is a cat”, formally $\text{cat}(\text{Sylvester})$. A **rule** $r$ consists of its **antecedent** $A(r)$ which is a finite set of literals, an **arrow**, and its **consequent** (or **head**) $C(r)$, which is a single literal. A **strict rule** is a rule in which whenever the premises are indisputable (e.g., facts), then so is the conclusion. For example,

$$\text{cat}(X) \rightarrow \text{mammal}(X)$$

means that “every cat is a mammal”. A **defeasible rule** is a rule that can be defeated by contrary evidence; for example, “cats typically eat birds”:

$$\text{cat}(X) \Rightarrow \text{eatBirds}(X).$$

The underlying idea is that if we know that something is a cat, then we may conclude that it eats birds, unless there is evidence proving otherwise. **Defeaters** are rules that cannot be used to draw any conclusion. Their only use is to prevent some conclusions,
i.e., to defeat defeasible rules by producing evidence to the contrary. An example is “if a cat has just fed itself, then it might not eat birds”:

\[ \text{justFed}(X) \Rightarrow \neg \text{eatBirds}(X). \]

The superiority relation among rules is used to define where one rule may override the conclusion of another one, e.g., given the defeasible rules

\[ r : \text{cat}(X) \Rightarrow \text{eatBirds}(X) \]
\[ r' : \text{domesticCat}(X) \Rightarrow \neg \text{eatBirds}(X) \]

which would contradict one another if Sylvester is both a cat and a domestic cat, they do not in fact contradict if we state that \( r' \) wins against \( r \), leading Sylvester not to eat birds.

Notice that in Defeasible Logic the superiority relation determines the relative strength of two conflicting rules, i.e., rules whose heads are complementary. The complementary of a literal \( q \) is denoted by \( \sim q \); if \( q \) is a positive literal \( p \), then \( \sim q \) is \( \neg p \), and if \( q \) is a negative literal \( \neg p \) then \( \sim q \) is \( p \).

Like in [12], we consider only a propositional version of this logic, and we do not take into account function symbols. Every expression with variables represents the finite set of its variable-free instances.

A defeasible theory \( D \) is a triple \( (F, R, >) \), where \( F \) is a finite consistent set of literals called facts, \( R \) is a finite set of rules, and \( > \) is an acyclic superiority relation on \( R \); given two rules \( r \) and \( s \), we will use the infix notation \( r > s \) to mean that \( (r, s) \in > \). The set of all strict rules in \( R \) is denoted by \( R_s \), and the set of strict and defeasible rules by \( R_{sd} \). We name \( R[q] \) the rule set in \( R \) with head \( q \). A conclusion of \( D \) is a tagged literal and can have one of the following forms:

1. \(+\Delta q \), which means that \( q \) is definitely provable in \( D \), i.e., there is a definite proof for \( q \), that is a proof using facts, and strict rules only;
2. \(-\Delta q \), which means that \( q \) is definitely not provable in \( D \) (i.e., a definite proof for \( q \) does not exist);
3. \(+\partial q \), which means that \( q \) is defeasibly provable in \( D \);
4. \(-\partial q \), which means that \( q \) is not defeasibly provable, or refuted in \( D \).

A proof (or derivation) is a finite sequence \( P = (P(1), \ldots, P(i)) \) of tagged literals where for each \( n, 0 \leq n \leq i \) the following conditions (proof conditions) are satisfied and \( P(1..i) \) denotes the initial part of the sequence of length \( i \).

\[ +\Delta : \text{if } P(n+1) = +\Delta q \text{ then} \]
\[ (1) q \in F \text{ or} \]
\[ (2) \exists r \in R_s[q] \forall a \in A(r) : +\Delta a \in P(1..n). \]

The negative proof conditions for \( \Delta \) are the strong negation of the positive counterpart: this is closely related to the function that simplifies a formula by moving all negations to an inner most position in the resulting formula, and replaces the positive tags with the respective negative tags, and the other way around [1314].
\[ -\Delta: \text{If } P(n+1) = -\Delta p \text{ then}
\]
\begin{enumerate}
\item \( q \notin F \) and
\item \( \forall r \in R_{L}[q] \exists a \in A(r) : -\Delta a \in P(1..n). \)
\end{enumerate}

The proof conditions just given are meant to represent forward chaining of facts and
strict rules \((+\Delta)\), and that it is not possible to obtain a conclusion just by using forward
chaining of facts and strict rules \((-\Delta)\).

The proof conditions for \((\pm \partial)\) are as follows:

\[ +\partial: \text{If } P(n+1) = +\partial q \text{ then either}
\]
\begin{enumerate}
\item \(+\Delta q \in P(1..n)\), or
\item \(\Delta \sim q \in P(1..n)\) and
  \begin{enumerate}
  \item \(\forall r \in R_{\Delta}[q] \exists a \in A(r) : +\partial a \in P(1..n)\) and
  \item \(\forall s \in R_{\sim}[q] \text{ either}
    \begin{enumerate}
    \item \(\forall a \in A(s) : -\partial a \in P(1..n)\), or
    \item \(\exists r \in R_{\Delta}[q] \text{ such that}
      \begin{enumerate}
      \item \(\forall a \in A(t) : +\partial a \in P(1..n)\) and \(t > s\).
      \end{enumerate}
      \end{enumerate}
  \end{enumerate}
\end{enumerate}
\]

\[ -\partial: \text{If } P(n+1) = -\partial q \text{ then}
\]
\begin{enumerate}
\item \(+\Delta q \notin P(1..n)\) and either
\item \(\Delta \sim q \in P(1..n)\), or
\item \(\forall r \in R_{\Delta}[q] \exists a \in A(r) : -\partial a \in P(1..n)\) or
  \begin{enumerate}
  \item \(\forall s \in R_{\sim}[q] \text{ such that}
    \begin{enumerate}
    \item \(\forall a \in A(s) : +\partial a \in P(1..n)\) and
    \item \(\forall t \in R_{\Delta}[q] \text{ either}
      \begin{enumerate}
      \item \(\exists a \in A(t) : -\partial a \in P(1..n)\), or \(t > s\).
      \end{enumerate}
      \end{enumerate}
  \end{enumerate}
\end{enumerate}
\]

The main idea of the conditions for a defeasible proof \((+\partial)\) is that there is an ap-
plicable rule \(\text{i.e., a rule where all the antecedents are defeasibly proved}\) and every rule
for the opposite conclusion is either discarded \(\text{i.e., one of the antecedents is not defea-
sibly provable}\) or defeated by a stronger applicable rule for the conclusion we want to
prove. The conditions for the negative proof tags \(\text{e.g., } -\partial\) show that any systematic
attempt to defeasibly prove the conclusion fails. The conditions for \(+\Delta\) and \(-\Delta\), and
\(+\partial\) and \(-\partial\) are related by the Principle of Strong Negation introduced in \([13]\). The key
idea behind this principle is that conclusions labelled with a negative proof tag are the
outcome of a constructive proof that the corresponding positive conclusion cannot be
obtained \(\text{and the other way around}\). The principle states that the inference conditions
for a pair of proof tags \(+\#\) and \(-\#\) are the strong negation of the other, where the strong
negation of a condition corresponds essentially to the function that simplifies a formula
by moving all negations to an innermost position in the resulting formula \(\text{for the full}
details see \([13]\)\).

As usual, given a proof tag \#, a literal \(p\) and a theory \(D\), we use \(D \vdash \pm \# p\) to denote
that there is a proof \(P\) in \(D\) where for some line \(i\), \(P(i) = \pm \# p\). Alternatively, we say
that \(\pm \# p\) holds in \(D\), or simply \(\pm \# p\) holds when the theory is clear from the context.

The set of positive and negative conclusion is called extension. Formally,
Definition 1 Given a defeasible theory $D$, the defeasible extension of $D$ is defined as

$$E(D) = (+\partial, -\partial),$$

where $\pm \partial = \{l : l$ appears in $D$ and $D \vdash \pm \partial l\}$.

Due to the nature of the revision operators discussed in this paper, the extension does not contain strict conclusions since the only way to modify them is to operate on the set of strict rules (i.e., addition or removal). Similarly, the extension will not include information about the proof tags introduced below. Such proof tags are useful to identify structures in proofs and where to operate in the theory, but they do not specify what is defeasibly provable, or not.

The inference mechanism of Defeasible Logic does not allow us to derive inconsistencies unless the monotonic part of the logic is inconsistent, as clarified by the following definition.

Definition 2 A defeasible theory $D$ is inconsistent iff there exists a literal $p$ such that $((D \vdash +\partial p$ and $D \vdash +\partial \neg p)$ iff $(D \vdash +\Delta p$ and $D \vdash +\Delta \neg p))$.

In this paper, we do not make use of strict rules, nor defeaters\(^2\), since every revision changes only the priority among defeasible rules (the only rules that act in our framework), but we need to introduce eight new types of auxiliary tagged literals, whose meaning is clarified in Example 1. As it will be clear in the remainder, they will be significantly useful in simplifying the categorisation process, and consequently, the revision calculus.

Example 1. Let $D$ be the following defeasible theory:

$$F = \emptyset$$

$$R = \{r_1 : \Rightarrow a, r_7 : \Rightarrow b, r_2 : a \Rightarrow c, r_8 : \Rightarrow \neg c, r_3 : c \Rightarrow d, r_9 : \Rightarrow \neg b, r_4 : \Rightarrow \neg a, r_{10} : \Rightarrow e, r_5 : \Rightarrow \neg d, r_{11} : \Rightarrow f\}$$

$$> = \{(r_1, r_4), (r_5, r_3)\}.$$

To improve readability, from now on we use the following graphical notation to represent a theory like the previous one:

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\(^2\) The restriction does not result in any loss of generality: (1) the superiority relation does not play any role in proving definite conclusions, and (2) for defeasible conclusions \cite{[12]} proves that it is always possible to remove (a) strict rules from the superiority relation and (b) defeaters from the theory to obtain an equivalent theory without defeaters and where the strict rules are not involved in the superiority relation. A consequence of this assumption is that the theories we work with in this paper are consistent.
\[ \Rightarrow r_1 \enspace a \quad \Rightarrow r_2 \quad c \quad \Rightarrow r_3 \quad d \quad \land \quad \Rightarrow r_4 \quad \neg a \quad \Rightarrow r_5 \quad \neg d \quad \Rightarrow r_6 \quad p \quad \lor \quad \Rightarrow r_7 \quad b \quad \Rightarrow r_8 \quad \neg c \quad \Rightarrow r_9 \quad \neg b \quad \Rightarrow r_{10} \quad e \quad \Rightarrow r_{11} \quad f \]

where the \( \land \) and \( \lor \) symbols in the graphical representation of a theory are not conjunctions and disjunctions but they represent the superiority relation \( > \). In the example, \( \lor \) means that \( r_1 > r_4 \) and \( \land \) that \( r_5 > r_3 \).

A conclusion in a defeasible proof can now take one or more of the following forms:

1. \( +\Sigma q \), which means there is a reasoning chain supporting \( q \):
   - \( r_1, r_2, r_3 \) form a chain supporting literal \( d \) (\(+\Sigma d\)).
2. \( -\Sigma q \), which means there is no reasoning chain supporting \( q \):
   - Since there are no rules for literal \( \neg p \), then we have \(-\Sigma \neg p\).
3. \( +\sigma q \), which means there exists a reasoning chain supporting \( q \) that is not defeated by any applicable reasoning chain attacking it:
   - \( r_1, r_2 \) and \( r_7, r_8 \) are two undefeated chains for \( c \) and \( \neg c \), respectively; Thus, we have \(+\sigma c, +\sigma \neg c\).
4. \( -\sigma q \), which means that every reasoning chain supporting \( q \) is attacked by an applicable reasoning chain:
   - Every chain for \( d \) is defeated (\(-\sigma d\), notice that there exists only one in this case).
5. \( +\omega q \), which means there exists a reasoning chain supporting \( q \) that defeasibly proves all its antecedents:
   - In the chain \( r_1, r_2, r_3 \), only rule \( r_3 \) is defeated, hence \(+\omega d\) holds.
6. \( -\omega q \), which means that in every reasoning chain supporting \( q \), at least one of its antecedents is not defeasibly provable:
   - Since \(+\partial b\) does not hold, we can conclude \(-\omega \neg c\).
7. \( +\phi q \), which means that there exists a reasoning chain that defeasibly proves \( q \) made of elements such that there does not exist any rule for the opposite conclusion:
   - There are no rules for \( \neg e \), thus \(+\phi e\) holds.
8. \( -\phi q \), which means that for every reasoning chain supporting \( q \) there exists an element such that a rule for the opposite conclusion could fire:
   - \( r_4 \) supports \( \neg a \), hence we have \(-\phi a\).

The tagged literals are formally defined by the following proof conditions. Again, the negative counterparts are obtained by the principle of strong negation. An important consequence of using this principle to formulate the conditions for asserting tagged literals is that for any literal \( p \) and any proof tag \#, it is not possible to have both \(+\#p\) and \(-\#p\) (the interested reader is referred to [13][14]).

Such proof tags identify structures of rules and demonstrations that are significant for the revision operations when we change the superiority relation. For example, \(+\Sigma p\) means that we could use Modus Ponens (or forward chaining) for deriving \(+\partial p\).
The definitions of \( \pm \Sigma \) formalise the concept of \textit{chain} leading to a given literal. With respect of the analysis on how to change a theory by only acting on the superiority relation, if there does not exist any chain leading to a literal \( p \) (i.e., \( -\Sigma p \) holds), then no modification of the theory is possible to prove \( p \).

\[ +\Sigma: \text{If } P(n+1) = +\Sigma q \text{ then} \]
\[ (1) \ +\Delta q \in P(1..n) \text{ or} \]
\[ (2) \ \exists r \in R_{\text{ad}}[q] \text{ such that } \forall a \in A(r) : +\Sigma a \in P(1..n) \]

\[ -\Sigma: \text{If } P(n+1) = -\Sigma q \text{ then} \]
\[ (1) \ +\Delta q \notin P(1..n) \text{ and} \]
\[ (2) \ \forall r \in R_{\text{ad}}[q] : \exists a \in A(r) \text{ such that } -\Sigma a \in P(1..n) \]

\[ +\sigma: \text{If } P(n+1) = +\sigma q \text{ then} \]
\[ (1) \ +\Delta q \in P(1..n) \text{ or} \]
\[ (2) \ \exists r \in R_{\text{ad}}[q] \text{ such that} \]
\[ (2.1) \ \forall a \in A(r) : +\sigma a \in P(1..n) \text{ and} \]
\[ (2.2) \ \forall s \in R[\sim q] \exists a \in A(s) \text{ such that} \]
\[ -\sigma a \in P(1..n) \text{ or } s \notin r. \]

\[ -\sigma: \text{If } P(n+1) = -\sigma q \text{ then} \]
\[ (1) \ +\Delta q \notin P(1..n) \text{ and} \]
\[ (2) \ \forall r \in R_{\text{ad}}[q] : \]
\[ (2.1) \ \exists a \in A(r) \text{ such that } -\sigma a \in P(1..n) \text{ or} \]
\[ (2.2) \ \exists s \in R[\sim q] \text{ such that} \]
\[ \forall a \in A(s) : +\sigma a \in P(1..n) \text{ and } s > r. \]

Notice that the definitions given above for \( \pm \sigma \) are weak forms of the notion of support proposed in [13] for the definition of an ambiguity propagating variant of Defeasible Logic, in the sense that these definitions are less selective than the ones of [15]. The undefeated chain that allows to state \( +\sigma p \) may be a good candidate for the revision process in order to defeasibly prove \( p \).

\[ +\omega: \text{If } P(n+1) = +\omega q \text{ then} \]
\[ (1) \ +\Delta q \in P(1..n) \text{ or} \]
\[ (2) \ \exists r \in R_{\text{ad}}[q] \text{ such that } \forall a \in A(r) : +\omega a \in P(1..n). \]

\[ -\omega: \text{If } P(n+1) = -\omega q \text{ then} \]
\[ (1) \ +\Delta q \notin P(1..n) \text{ and} \]
\[ (2) \ \forall r \in R_{\text{ad}}[q] : \exists a \in A(r) \text{ such that } -\omega a \in P(1..n). \]

The chain that allows to state \( +\omega p \) represents a defeasible proof for \( p \) that can only fail on the last derivation step. Thus, possible modifications can focus on this last step instead of considering the whole chain.

\[ +\varphi: \text{If } P(n+1) = +\varphi q \text{ then} \]
\[ (1) \ +\Delta q \in P(1..n) \text{ or} \]
\[ (2) \ \exists r \in R_{\text{ad}}[q] \text{ such that} \]
\[ (2.1) \ \forall a \in A(r) : +\varphi a \in P(1..n) \text{ and} \]
\[ (2.2) \ \forall s \in R[\sim q] : \exists a \in A(s) \text{ such that } -\Sigma a \in P(1..n). \]
−φ: If \( P(n + 1) = −φq \) then
(1) \( +Δq \notin P(1..n) \) and
(2) \( ∀r ∈ R[φ][q] : \)
(2.1) \( ∃a ∈ A(r) \) such that \( −φa ∈ P(1..n) \) or
(2.2) \( ∃s ∈ R[¬q] \) such that \( ∀a ∈ A(s) : +Σa ∈ P(1..n) \).

The definition of \( +φ \) ensures that it is not possible to have a counter-argument for a reasoning chain, i.e., a proof, for a literal tagged with it. In particular, we can not have a direct attack, nor an attack to one of its arguments. Therefore, no modification on the superiority relation is possible to reject a literal tagged with \( +φ \).

Given the above definitions, it is straightforward to derive the implication chains reported below in Figure [1(a)–(b)] using techniques presented in [16].

One could be tempted to say that \( +σ \) implies \( +ω \) (and symmetrically, \( −ω \) implies \( −σ \)). This is not the case. Indeed, if we consider theory \( D \) of Example 1 we have: (i.) \( +ωd \) and \( −σc \), (ii.) \( +σe \) and \( −ωc \).

To better explain how the new proof tags behave, we report in Table 1 the set of all conclusions. For each literal, we only report the proof tag which is minimal with respect to the orderings given in Figure 1. For example, \( +∂a \) means that we prove \( +ωa, +σa, +Σa \), but \( −φa \). If no tag is reported, then it is not possible to derive the literal with any tags with respect to the ordering given in Figure 1.

| a  | b  | c  | d  | e  | f  | p  |
|----|----|----|----|----|----|----|
| +  | +  | +  | +  | +  | +  | -  |
|    | +φ| +σ| +Δ| +ω| +φ| +Δ|
| -φ| -σ| -Δ| -ω| -φ| -σ| -Δ|

Table 1: Conclusions for literals in Example 1.
We report now some theoretical results: these are useful during the revision process described in Section 4. Proposition 3 highlights the fact that, given a theory \( D \) and literal \( q \), if \( D \vdash +\varphi q \) then there are no chains for the complementary. Notice that, in general, the opposite does not hold (as for literal \( p \) in Example 1).

**Proposition 3** Given a consistent defeasible theory \( D \), if \( D \vdash +\varphi p \) for a literal \( p \) (with \( p \not\in F \)), then \( D \vdash -\Sigma \sim p \).

**Proof.** The proof straightly follows from the definition of \( +\varphi \), condition (2.2). This condition must hold for each element in the chain, as well as for \( p \).

The next proposition formally states the following idea: given a defeasibly proved literal \( p \) and a chain leading to \( \sim p \) with all the antecedents defeasibly proved, then such a chain is defeated by a priority rule at the last proof step (by the rule proving \( p \)).

**Proposition 4** Given a consistent defeasible theory \( D \), if \( D \vdash +\partial p \) and \( D \vdash +\omega \sim p \) for a literal \( p \) (with \( p \not\in F \)), then \( D \vdash -\sigma \sim p \).

**Proof.** By definition of \( +\partial \), we have that the condition below holds for \( p \). In fact condition (2.3.2) has to be true since we know condition (2.3.1) is not, because

\[
\begin{align*}
D \vdash +\partial p & \text{ implies } \exists r \in R[p], \forall a \in A(r) : +\partial a \\
D \vdash +\omega \sim p & \text{ implies } \exists s \in R[\sim p], \forall a \in A(s) : +\partial a \\
& \text{ thus } \exists r \in R[p], \forall a \in A(t) : +\partial a \text{ and } t > s.
\end{align*}
\]

This is condition (2.2) for \( -\sigma \sim p \). Moreover, since all the premises of \( \sim p \) are defeasibly proved by hypothesis and we have proved that the chain is defeated, then it has to loose on the last proof step.

We now capture the concept of a derivation based on a contradiction. To do so, we begin by defining what the meaning of dependency between literals is; afterwards, we look at how the notion of \( \partial \)-unreachability defines literals whose derivation is based upon an inconsistency.

**Definition 5** Let \( a \) and \( b \) be two literals. Then \( a \) depends on \( b \) iff (1) \( b = a \) or (2) \( \forall r \in R[a], \text{ either (2.1) } b \in A(r), \text{ or (2.2) } \exists c \text{ such that } c \in A(r), \text{ and } c \text{ depends on } b \).

The following result shows that a defeasibly proved literal also implies the provability of all literals it depends on. In other words, the property of dependency given above propagates backwards the defeasible provability of literals.
Proposition 6  Given a defeasible theory $D$, if $D \vdash +p$ and $p$ depends on $q$, then $D \vdash +q$.

Proof. The proof is by case inspection of Definition 5. If clause (1) holds, the claim trivially follows. For the other cases, the proof is by induction on the degree of dependency between literals. A literal $a$ depends on $b$ with degree 1 if $a$ depends on $b$ and there exists a rule $r$, with $C(r) = a$ and $b \in A(r)$. A literal $a$ depends on $b$ with degree $n + 1$ if $a$ depends on $b$ and there is a literal $c$ such that $a$ depends on $c$ with degree 1 and $c$ depends on $b$ with degree $n$.

For the inductive base (i.e., $p$ depends on $q$ with degree 1), $+p$ means that there is a rule for $p$ with every antecedent defeasibly proved. Thus, $D \vdash +q$.

For the inductive step, suppose that the property holds up to degree $n$ and $p$ depends on $q$ with degree $n + 1$. By definition, there exists a literal $c$ such that $p$ depends on $c$ with degree 1, thus $D \vdash +c$ (given $D \vdash +p$ by hypothesis) and $c$ depends on $q$ with degree $n$. Thus, by inductive hypothesis, $D \vdash +q$.

The next definition identifies literals only depending on contradictions. For example, consider the theory with the following rule:

$$a, \neg a, b \Rightarrow_r p.$$ 

For deriving $+p$ we need both $+a$ and $+\neg a$, and this is possible only in the case that the theory is inconsistent. However, we have also to cater for situations where the dependency is not direct, for example in theories like

$$a \Rightarrow_{r_1} b \quad \neg a, b \Rightarrow_{r_2} p.$$ 

Definition 7  A literal $p$ is $\partial$-unreachable iff $\forall r \in R[p]$, either (1) $\exists l, \exists a, b \in A(r)$ such that (1.1) $a$ depends on $l$, and (1.2) $b$ depends on $\neg l$, or (2) $\exists d \in A(r)$ such that $d$ is $\partial$-unreachable. Otherwise, we define $p$ to be $\partial$-reachable.

The result below formalises the relationship between $\partial$-unreachable literals and inconsistent theories.

Proposition 8  Given a theory $D$, let $p$ be a $\partial$-unreachable literal. If $D \vdash +p$, then $D$ is inconsistent.

Proof. The proof is by induction on the number of $\partial$-unreachable literals in a derivation.

For the base case, $p$ is the only $\partial$-unreachable literal in its derivation. Given that $D \vdash +p$, there is a rule for $p$ such that all its antecedents are provable. By Definition 7 for every rule for $p$ there are two antecedents $a$ and $b$ depending on a literal $l$ and its complement, respectively. Thus, we have both $+a$ and $+b$. By Proposition 6, we have $D \vdash +l$ and $D \vdash +\neg l$, then $D$ is inconsistent.

For the inductive step, we assume that the property holds up to $n$ $\partial$-unreachable literals, and $p$ is the $(n + 1)^{th}$ $\partial$-unreachable literal. Beside the case we examined in the inductive base, we have to consider that the antecedent of a rule contains a $\partial$-unreachable literal $d$ and $D \vdash +d$. Thus, $d$ falls under the inductive hypothesis, therefore $D$ is inconsistent.
The following proposition states that if there is a $\partial$-reachable literal $p$ with at least one supporting chain, then it is always possible to defeasibly prove $p$. In other words, the problem of modifying the superiority relation to pass from $-\partial p$ to $+\partial p$ (or from $+\partial \neg p$ to $+\partial p$) has always a solution, provided that there exists a non-contradictory support.

**Proposition 9** Given a consistent defeasible theory $D = (F, R, \succ)$ and a $\partial$-reachable literal $p$ with $D \vdash +\Sigma p$, there exists a theory $D' = (F, R, \succ')$ such that $D' \vdash +\partial p$.

**Proof.** Proposition [8] shows that a $\partial$-unreachable literal is provable only when the theory is inconsistent, which is against the hypothesis of the proposition.

Suppose that $D \vdash +\Sigma p$ for a theory $D$. Then, there is at least one reasoning chain $C$ supporting $p$. Among all the possible superiority relations based on $F$ and $R$, there is a superiority relation $\succ'$ where every rule $r : A \Rightarrow c$ in $C$ is superior to any rule for $\neg c$. Thus, theory $D' = (F, R, \succ')$ is such that $D' \vdash +\partial p$.

To illustrate why both conditions of Proposition 9 are required to guarantee that the canonical case whose outcome is $+\partial p$ after the revision operation, let us consider a theory with the following rules:

$$
\Rightarrow r_1 \ a \\
\Rightarrow r_2 \neg a \\
a, \neg a \Rightarrow r_3 p.
$$

In this case we have both $+\Sigma a$ and $+\Sigma \neg a$, therefore we can build a reasoning chain to $p$, but $p$ itself is $\partial$-unreachable because it depends on a contradiction. Thus, there is no way to change the previous theory to prove $p$.

We end this section proposing an example to translate a real-life case into our logic. This will also help in giving an intuitive revision mechanism that shows how argumentation in legal reasoning is easily mapped in changing the superiority relation of a defeasible theory.

**Example 2.** A couple can have offspring but, since both mother and father are affected of cystic fibrosis, they know that every their child will be most likely affected by the same genetic anomaly. Since they want their offspring to be healthy, they request for medically assisted reproduction techniques. Their case is disputed first in an Italian Court where the judge has to establish which between Art. 4 of Italian Legislative Act 40/2004 ($r_0$ and $r_1$) and the standard common medical practice ($r_3$) in force in 15 countries of the EU prevails.

The couple is indeed able to produce embryos and they cannot be considered as sterile ($r_2$). This makes both Art. 4 and the standard common medical practice to be applicable to their case. The judge argues in favour of $r_1$ based on *lex superior* and refuses their request: this principle applies since Art. 4 Act 40/2004 is a legal rule while $r_3$ has a juridical validity but it is not a proper legal rule.

---

3 Art. 4 of Italian Legislative Act 40/2004: “The recourse to medically assisted reproduction techniques is allowed only [...] in the cases of sterility.”
The couple appeals to the European Court for Human Rights. The Court establishes that not permitting the medical techniques would demote the goal of family health promoted by Article 8 of the Convention. In our example, \( r_3 \) promotes the goal of family health \((r_5)\), and thus we invert the priority between \( r_1 \) and \( r_3 \) based both on \textit{lex superior} and \textit{lex specialis} \((\geq \{ (r_3, r_1) \})\). In here, the \textit{lex superior} principle applies because \( r_3 \) is an european directive, while the \textit{lex specialis} principle applies because \( r_3 \) covers a more specific case than \( r_0 \).

4 Changing defeasible preferences

We now analyse the processes of revision in a defeasible theory, when no changes to rules and facts are allowed. Henceforth, when no confusion arises, every time we speak about a (revision) transformation we refer to a (revision) transformation acting only on the superiority relation.

A good starting point for our investigation is to focus on the corresponding decision problem (i.e., answering the question \textit{if} it is possible to modify the state of a literal in a defeasible theory by changing the relative strength of rules) and characterise it in a formal way (so as to be able also to answer the question \textit{when}). In particular, in Subsection 4.1 we show that the decision problem at hand is computationally hard in general, while in the remaining of the section we partition the decision problem in three sub-cases which correspond to the possible ways in which we can modify the (provability) state of a literal.

4.1 NP-Completeness

First, we introduce some additional terminology. Definition 10 constructs a defeasible theory starting from a fixed set of rules and an empty set of facts. This formulation limits the revision problem to preference changes, notwithstanding the particular instance of the superiority relation.

Definition 10 \textit{Given a set of defeasible rules }\( R \), a defeasible theory }\( D \) \textit{is based on }\( R \) \textit{iff}

\[
D = (\emptyset, R, >)
\]
The aim of Definition 11 is to specify the possible relationships between a literal and all theories based on a set of rules $R$.

**Definition 11**

Given a set of defeasible rules $R$, a literal $p$ is

1. **$>_R$-tautological** iff for all theories $D$ based on $R$, $D \vdash +\partial p$.
2. **$>_R$-non-tautological** iff there exists a theory $D$ based on $R$ such that $D \not\vdash +\partial p$.
3. **$>_R$-refutable** iff there exists a theory $D$ based on $R$ such that $D \vdash -\partial p$.
4. **$>_R$-irrefutable** iff for all theories $D$ based on $R$, $D \not\vdash -\partial p$.

The notion of $>_R$-irrefutable represents the negative counterpart of $>_R$-tautological; the same holds for $>_R$-refutable and $>_R$-non-tautological.

If $p$ is $>_R$-tautological, then, in every theory based on the set of rules $R$, an instance of the superiority relation such that $p$ is defeasibly refuted does not exist. Accordingly, if a literal is $>_R$-tautological, then we cannot revise it.

On the contrary, if an instance of the superiority relation where $p$ is no longer provable exists, then $p$ is $>_R$-refutable.

To prove the NP-completeness of the problem of establishing if it is possible to revise a theory modifying only the superiority relation, we reduce the 3-SAT problem – a known NP-complete problem – to our decision problem. In particular, we are going to map a 3-SAT formula to a defeasible theory and we check whether the literal corresponding to the 3-SAT formula is tautological. Definition 12 exhibits the reduction adopted.

**Definition 12**

Given a 3-SAT formula $\Gamma = \bigwedge_{i=1}^{n} C_i$ such that $C_i = \bigvee_{j=1}^{3} a_{ij}$, we define the $\Gamma$-transformation as the operation that maps $\Gamma$ into the following set of defeasible rules

$$R_{\Gamma} = \{ r_{ij} : a_{ij} \Rightarrow a_{ij} \}
\quad r_{ij} : a_{ij} \Rightarrow c_i
\quad r_{\sim i} : \Rightarrow \sim c_i
\quad r_i : c_i \Rightarrow p \}.$$  

The above definition clearly shows that the mapping is polynomial in the number of literals appearing in the 3-SAT formula $\Gamma$.

The second step of the proof construction is to ensure that the proposed mapping always allows the revision problem to give a correct answer (either positive, or negative) for every 3-SAT formula. Proposition 14 and Lemma 15 guarantee that any theory obtained by $\Gamma$-transformation provides an answer. These results are also intended to establish relationships between the notions of tautological and refutable given in Definition 11.

**Definition 13**

A defeasible theory $D$ is **decisive** iff for every literal $p$ in $D$ either $D \vdash +\partial p$, or $D \vdash -\partial p$.

**Proposition 14**

Given a defeasible theory $D$, if the atom dependency graph of $D$ is acyclic, then $D$ is decisive.
Proof. For a detailed definition of atom dependency graph and a complete proof of the claim, the interested reader should refer to [17].

Lemma 15 Any defeasible theory $D$ based on $R_{\Gamma}$ of Definition 12 (for any $\Gamma$) is decisive.

Proof. It is easy to verify by case inspection that the atom dependency graph is acyclic.

We are now ready to introduce the main result of NP-completeness. First of all, we have to prove that the revision problem is in NP. Second, we show that it is NP-hard by exploiting the formulation of the 3-SAT problem and the transformation proposed in Definition 12.

Theorem 16 The problem of determining the revision of a defeasible literal by changing the preference relation is NP-complete.

Proof. The proof that the problem is in NP is straightforward. Given a defeasible theory $D = (F, R, >)$ and a literal $p$ to be revised, an oracle guesses a revision (in terms of a new preference relation $>'$ applied to $D$) and checks if the state of $p$ has changed based on the extensions of $D$ and $D' = (F, R, >')$. The complexity of this check is bound to the computation of the extensions of $D$ and $D'$, which [18] proves to be linear in the order of the theory.

To prove the NP-hardness, given a 3-SAT formula $\Gamma = \bigwedge_{i=1}^{n} C_i$ such that $C_i = \bigvee_{j=1}^{3} a_{ij}$, a defeasible theory $D$ based on the set of defeasible rules $R_{\Gamma}$ obtained by $\Gamma$-transformation, and a literal $p$ in $D$, we show that:

1. if $p$ is $>_{R_{\Gamma}}$-tautological, then $\Gamma$ is not satisfiable;
2. if $p$ is $>_{R_{\Gamma}}$-non-tautological, then $\Gamma$ is satisfiable.

(1) Lemma 15 allows us to reformulate the contrapositive using $>_{R_{\Gamma}}$-refutable. If $\Gamma$ is satisfiable, then there exists an interpretation $I$ such that

$$I \models \Gamma \iff I \models \bigwedge_{i=1}^{n} C_i \iff I \models C_1 \text{ and } \ldots \text{ and } I \models C_n \iff I \models \bigvee_{j=1}^{3} a_{1j} \text{ and } \ldots \text{ and } I \models \bigvee_{j=1}^{3} a_{nj}.$$ 

Thus, for each $i$, there exists $j$ such that $I \models a_{ij}$.

We build a defeasible theory $D' = (\emptyset, R_{\Gamma}, >')$ as follows. If there exists a literal $b_k^i$ such that $b_k^i = \sim a_{ij}$, then $(a_{ij}, r_{kj})$ is in $>'$. It follows that, by construction, $D'$ proves $+\partial a_{ij}$. This means that every rule $r_{ij}$ is applicable and it is not weaker than the corresponding rule $r_{\sim,ij}$. Hence, we have $-\partial \sim c_i$, for all $i$. Consequently, each rule $r_i$ for $p$ is discarded and we conclude $-\partial p$. Accordingly, $p$ is $>_{R_{\Gamma}}$-refutable.
Again, by Lemma 15, every theory based on \( R_\Gamma \) is decisive. Thus, \( p \) is \( \triangleright R_\Gamma \)-refutable. Accordingly, there exists a theory \( D' = (\emptyset, R_\Gamma, \triangleright') \) such that \( D' \vdash \neg \partial p \). Given that \( R_\Gamma[p] \neq \emptyset \) and \( R_\Gamma[\sim p] = \emptyset \) by construction, every rule for \( p \) is discarded. Namely, we have \( \neg \partial \sim c_i \), for all \( i \).

Each rule \( r_{\sim i} \) is vacuously applicable. Hence, in order to have \( \neg \partial \sim c_i \), there must exist a rule \( r_{ij} \) that is applicable. Therefore, for each \( i \) there is at least one \( j \) such that \( +\partial a_i^j \).

We build an interpretation \( I \) as follows:\footnote{We use the standard notation where \( I(a) = 1 \) iff \( a \) is evaluated to \text{True} in \( I \), and \( I(a) = 0 \) otherwise.}

\[
I(a_i^j) = 1 \text{ iff } D \vdash +\partial a_i^j.
\]

Since for each \( 1 \leq i \leq n \), we have \( I(a_i^j) = 1 \) for at least one \( j \), then also \( I \models C_i \) for all \( i \), and we conclude that \( I \models \Gamma \).

In addition, Theorem 16 specifies that there are situations where it is not possible to revise a literal only using the superiority relation. For example, if we take a 3-SAT formula which is a tautology, the \( \Gamma \)-transformation generates a theory that cannot be revised only using the superiority relation. Thus, we can formulate the following result.

**Corollary 17** There are theories and literals for which a revision modifying only the superiority relation is not possible.

### 4.2 Revision in Legal Domain

Similarly to what we did in Section 2, we now motivate the types of changes we are going to study by appealing again to the legal domain. When two lawyers dispute a case, there are four situations in which each of them can be if she revises the superiority relation employed by the other one.

(a) The revision process supports the argument of reasonable doubt. Someone proves that the rules imply a given conclusion. If the preference is revised then we can derive that this is not the case, showing that the conclusion was not beyond reasonable doubt.

(b) The revision process beats the argument of beyond reasonable doubt. Analogously to situation (a), someone proves that the rules do not imply a given conclusion. If the preference is revised, then we can derive that this is indeed the case.

(c) The revision process supports the argument of proof of innocence/guilt. Someone proves that the rules imply a given conclusion. If the preference is revised, then we can derive that the opposite holds.

(d) The revision process cannot support a given thesis.

Revising a defeasible theory by changing only the priority among its rules means studying how an hypothetic revision operator works in the three cases reported below:
(1) how to obtain $-\partial p$, starting from $+\partial p$;
(2) how to obtain $+\partial \sim p$, starting from $+\partial p$;
(3) how to obtain $+\partial p$, starting from $-\partial p$.

We name these three revisions canonical. We provide an exhaustive analysis, based on
the definitions above, in the next subsections.

Situation (a) corresponds to canonical case (1). Situation (b) corresponds to canonical case (3). Situation (c) corresponds to canonical case (2).

Situation (d) includes several contexts that are deemed as sub-cases of the previous ones: in particular, it captures cases where indeed, the revision process based the
superiority relation is impossible, namely:

- in the first canonical case, when literal $p$ is $>_R$-tautological (by Definition [11]);
- in the second canonical case, when literal $p$ is $>_R$-tautological, or a reasoning
  chain supporting the complementary does not exist (i.e., condition $-\Sigma \sim p$ holds);
- in the third canonical case, when literal $p$ is $\partial$-unreachable (as stated in Propo-
  sition [8]), or a reasoning chain supporting it does not exist (i.e., condition $-\Sigma p$
  holds).

Notice that literals provable with tag $+\varphi$ are special cases of tautological literals (cf.
Definition [11]). As such, this kind of literals leads the revision process to be unsuccessful
for the first and the second canonical case. A possible legal scenario is when one of the
parties argues in favour of a thesis in a defeasible way and the counter-part cannot
discredit it, or cannot exhibit a proof for the opposite, independently of the changes in
the superiority relation. The next proposition formally captures the above intuitions.

**Proposition 18** Given a consistent defeasible theory $D = (F, R, >)$, if $D \vdash +\varphi p$ for a
literal $p$, then there does not exist a theory $D' = (F, R, >')$ such that (1) $D' \vdash +\partial \sim p$, or
(2) $D' \vdash -\partial p$.

**Proof.** (1) Given any theory, to obtain a defeasible proof of a literal $q$, there must exist
at least one reasoning chain supporting $q$, i.e., $+\Sigma q$ must hold. This is in contradiction
with Proposition [3], which states that if $+\varphi \sim q$ holds, also $-\Sigma q$ does.

(2) By definition of $+\varphi p$, there exists a reasoning chain that defeasibly proves $p$
made of elements such that there does not exist any rule for the opposite conclusion.
Thus, no attack to this chain is possible, and condition (2.3.1) of $+\partial$ always holds for
each element of this chain (we recall that $-\Sigma l$ implies $-\partial l$ for any literal $l$).

In the rest of the section we are going to describe three types of revision of prefer-
ences. For each case we identify the conditions under which such revisions are possible.
Therefore, all revision cases studied below will not consider tautological literals as well
as $\partial$-unreachable literals, assuming that the underlying theories are consistent.

Given a defeasible theory $D$ a literal $p$ can be proved (i.e., $+\partial p$) or refuted (i.e.,
$-\partial p$). The three canonical cases cover the situations where: we pass from a theory
proving $p$ to a theory refuting $p$ (without necessarily proving the opposite, $\sim p$); we
pass from a theory refuting $p$ to a theory proving $p$; and from a theory proving $p$ to a
theory proving its opposite ($\sim p$), and then consequently refuting $p$. Notice that these
three cases are the only ones meaningful involving provable and refutable literals. In Section 5 we are going to argue that our canonical cases can be understood as expansion, revision and contraction of the AGM belief revision framework. Combinatorially, one could consider another case, where \( p \) is refuted and we want to obtain a theory where we refute \( \neg p \). However, the meaning of this operation is not clear to us, and it is partially subsumed by our third canonical case (given that \(+\partial p\) implies \(-\partial\neg p\)).

We are now ready to go onto the systematic analysis of the combinations arising from the above defined model. We list the cases by tagging each macroscopic case by the name Canonical case and the combinations depending upon the analytical schema introduced above by the name Instance. The instances show the combination of proof tags where a canonical revision is possible, as well as how to operate on the theory to perform the revision. Where necessary, a general reasoning chain supporting a literal \( p \) will be denoted as \( P_p \).

4.3 First canonical case: from \(+\partial p\) to \(-\partial p\)

For the sake of clarity, Figure 2 gives a tree-based graphical representation of all analysed instances: for example, the leftmost leaf (labeled as \(+\sigma\neg p\)) represents the instance where conditions are \(+\sigma\neg p\), \(+\omega\neg p\), \(-\varphi p\) and \(+\Sigma\neg p\). The scheme will be reprised also in the two remaining canonical cases, with the appropriate graphical modifications for the particular case.

![Diagram](from+∂p to -∂p: revision cases)

**Instance** \(-\Sigma\neg p\land +\partial p\): this case is not reported in Figure 2 but nonetheless it represents a case worth considering. This means there is no supporting chains for \( \neg p \), so we cannot operate on them. Given \(-\varphi p\), there exists at least one of its premises that could be defeated by a rule leading to the opposite conclusion. Thus, in order to obtain \(-\partial p\), we have to revise the theory allowing at least one of such rules to be able to fire (to defeat, or at least to have the same power of a rule which actually proves one of the antecedents in the chain supporting \( p \)).

**Instance** \(+\omega\neg p\land +\sigma\neg p\): as stated in Proposition 4 this branch represents an impossible case for any consistent defeasible theory.

**Instance** \(+\omega\neg p\land -\sigma\neg p\): by the straightforward implication of Proposition 4 the chain supporting \( \neg p \) fails on the last proof step, defeated by priorities for rules which defeasibly prove \( p \). Thus, we have only to erase these priorities.
Instance $-\omega \sim p \land +\sigma \sim p$: since a chain $P_{-p}$ exists, and is never defeated (condition $-\omega \sim p$ only illustrates that such a chain fails before the last proof step), a revision process does not have to operate on a chain supporting $p$. We have to strengthen $P_{-p}$ by changing many priorities in order to let a rule in $P_{-p}$ obtain at least the same strength of such a rule in $P_p$. In this process, we do not remove any priority among elements in $P_p$, but only add priorities to let a rule in $P_{-p}$ win.

Instance $-\omega \sim p \land -\sigma \sim p$: the reasoning chain $P_{-p}$ supporting $\sim p$ is defeated, but not necessarily by a chain $P_p$ proving $p$. The case is analogous to the aforementioned instance, but (1) we probably have to act not only on $P_{-p}$, but also on $P_p$, and (2) not only introduce priorities, but erase or invert them. This case represents the most generic situation, where less information is given: a revision is possible, but we do not know a priori where to change the theory.

4.4 Second canonical case: from $+\partial p$ to $+\partial \sim p$

By following the cases depicted in Figure 2, we explain how a revision operator should work by changing the root label to “$+\partial p$ to $+\partial \sim p$” and starting from the same premises ($-\varphi p \land +\Sigma \sim p$). Once more, our revision tree does not take into account tags $\pm \varphi$ for the same reasons explained at the beginning of Section 4.

Instance $+\omega \sim p \land +\sigma \sim p$: as stated in Proposition 4 this branch represents an impossible case for any consistent defeasible theory.

Instance $+\omega \sim p \land -\sigma \sim p$: Proposition 4 states that the chain supporting $\sim p$ is defeated on the last proof step. This, combined with $-\sigma \sim p$, implies that the last step is defeated by a priority for the rule which defeasibly proves $p$. In fact, there may exist more than one defeated chain for $\sim p$ on the last proof step, as well as more than one chain which proves $p$. We propose two different approaches. We name $P$ the set of chains defeasibly proving $p$, $P_{ls} \subseteq P$ the set of chains that defeasibly prove $p$ for which there is a priority that applies at the last proof step (against a chain that proves $\sim p$), and $N$ the set of chains for which the premises hold:

1. We choose a chain in $N$. We invert the priority for every chain in $P_{ls}$ that wins at the last proof step. We introduce a new priority for making it win against any remaining chain in $P$.

2. In this approach we have two neatly distinguished cases:
   (a) $|P_{ls}| > |N|$: for every chain in $N$ we invert the priorities on the last proof step. For every remaining chain in $P$, we add a priority between the defeasible rule used in the last proof step of a chain in $N$ and the rule used in the last proof step of a chain in $P$ (possibly different for each chain in $N$) such that the chain in $P$ is defeated.
   (b) $|N| > |P_{ls}|$: we choose a number $|P_{ls}|$ of chains in $N$ and invert the priority on the step where they are defeated. If at the end of this step there are still chains in $P$ that defeasibly prove $p$, we go on with the method used for the case 2(a), focusing on the subset of chains in $N$ modified during the first step.
These two approaches rely on different underlying ideas. In the first case we want a unique winning chain. This makes the revision procedure faster than the second method, since we do not have to choose a different chain to manipulate every time. Moreover, the number of changes made with the first approach is equal to that of the second one in the worst case scenario (in general, it revises the theory with the minimum number of changes).

The strength of the second method relies on the concept of team defeater: we do not give power to a single element, but to a team of rules. In the first method the entire revision process must be repeated once the winning chain is defeated, while in the second method if one of the winning chains is defeated, we have only to apply the revision process on it, and not on all the other winning chains.

Let us consider the following example:

\[ \Rightarrow r_1 \ p \quad \Rightarrow r_2 \ p \]
\[ \vee \quad \vee \]
\[ \Rightarrow r_3 \ \neg p \quad \Rightarrow r_4 \ \neg p \]

If the chain for \( \neg p \) with rule \( r_4 \) is chosen as the winning chain, the first approach would give \( \{ r_1 > r_3, r_4 > r_1, r_4 > r_2 \} \) as an output, erasing one priority and introducing two, while the second approach would generate the following priority set: \( \{ r_3 > r_1, r_4 > \} \), erasing two priorities, and introducing two. If a new rule \( r_i \) defeats \( r_4 \), it is easy to see that in the first case we have to entirely revise the theory (for example, let \( r_3 \) win against \( r_1 \) and \( r_2 \)), while in the second case we have only to introduce \( r_3 > r_2 \).

Instance \( -\omega \sim p \land +\sigma \sim p \): there exists an undefeated chain supporting \( \sim p \). To revise the theory, we have to choose one of them and, starting from \( \sim p \), go back in the chain to the ambiguity point (where \( P(i) = +\partial p_i \land P(i+1) = -\partial p_{i+1} \) holds), and strengthen the chain by adding a priority where a rule leading to an antecedent in the chain for \( \sim p \) and a rule for the opposite have the same strength.

Instance \( -\omega \sim p \land -\sigma \sim p \): every chain supporting \( \sim p \) is defeated at least one time. A plausible solution could be to go back in the chain searching for the point where \( P(i) = +\sigma p_i \) and \( P(i+1) = -\sigma p_{i+1} \). But this is not enough to guarantee the chain to win. Let us consider the following example.

**Example 3.** Let \( D \) be a theory having the following rules:

\[ \begin{align*}
+\partial/\neg \partial & \quad +\sigma/-\sigma \\
\Rightarrow r_1 \ a & \Rightarrow r_2 \ b \quad \Rightarrow r_3 \ c = r_4 \ p \\
& \land \\
\Rightarrow r_5 \ a & = r_6 \ b
\end{align*} \]

It is easy to see that the sole condition of \( r_3 \) winning over \( r_6 \) is not sufficient: we have also to introduce a priority between \( r_1 \) and \( r_5 \). Thus, we have to act exactly as in the aforementioned case, with the solely difference that every time a rule in the chain supporting \( \sim p \) is defeated, the priority rule has to be inverted.
4.5 Third canonical case: from $-\partial p$ to $+\partial p$

For a proper analysis of this case, condition $-\partial \sim p$ must hold since otherwise the case is analogous of the previous revision from $+\partial q$ to $+\partial \sim q$. Also, we do not take into consideration the case where $-\Sigma p$ holds: if there are no chains leading to $p$, then no revision to obtain $+\partial p$ is possible. The cases studied in this subsection are reported in Figure 3.

\[\begin{align*}
\text{From } -\partial p & \text{ to } +\partial p \\
(-\partial \sim p \land +\Sigma p) & \\
+\omega p \land +\sigma p & \quad -\omega p \\
(-\omega \sim p) & \\
+\sigma p & \quad -\sigma p
\end{align*}\]

Fig. 3: From $-\partial p$ to $+\partial p$: revision cases.

Notice that $+\omega p$ and $-\sigma p$ cannot hold at the same time: if all the premises for $p$ are proved, then the chain fails on the last step, i.e., it has to be defeated by a firing rule for $\sim p$. This would defeasibly prove $\sim p$, but this cannot happen since we have stated that $-\partial \sim p$ holds. Furthermore, $-\omega p$ implies that $-\omega \sim p$ holds as well, since if it is not the case, we would have either $+\omega p$, or $+\partial \sim p$, which are both against the hypothesis.

**Instance** $+\omega p \land +\sigma p$: we choose one of the chains where condition $+\omega p \land +\sigma p$ holds, and introduce as many priorities as the number of chains where $+\omega \sim p$ holds.

**Instance** $-\omega p \land +\sigma p$: this case is analogous to instance $-\omega \sim p \land +\sigma \sim p$ of canonical case from $+\partial p$ to $+\partial \sim p$.

**Instance** $-\omega p \land -\sigma p$: this case is analogous to instance $-\omega \sim p \land -\sigma \sim p$ of canonical case from $+\partial p$ to $+\partial \sim p$.

We remark that conditions $\pm \sigma \sim p$ are not useful for the revision process, since they do not give information whether the changes will affect chains for $\sim p$, or not. Example 4 shows that, even if $+\sigma \sim p$ holds, two distinct revisions exist: the first involves the chain for $\sim p$ (introducing $r_1 > r_3$), the second does not (introducing $r_5 > r_6$).

**Example 4.** Let $D$ be a theory having the following rules:

\[\Rightarrow r_1 \quad a \quad \Rightarrow r_2 \quad p\]
\[\Rightarrow r_3 \quad \neg a \quad \Rightarrow r_4 \quad \neg p\]
\[\Rightarrow r_5 \quad b \quad \Rightarrow r_6 \quad p\]
\[\Rightarrow r_6 \quad \neg b\]

An analogous situation is proposed for $-\sigma \sim p$ in Example 5.
Example 5. Let $D$ be a theory having the following rules:

\begin{align*}
\Rightarrow r_1 & \quad \Rightarrow r_3 \\ \Rightarrow r_3 & \quad \neg a \Rightarrow r_1 \\
\neg a \wedge & \quad \Rightarrow r_5 \\
\Rightarrow r_6 & \quad \neg b \\
\Rightarrow r_7 & \quad c \Rightarrow r_6 \\
\Rightarrow r_8 & \quad p \\
\Rightarrow r_9 & \quad \neg c
\end{align*}

In here, two revisions exist: one introducing $r_1 > r_3$, and the other one which introduces $r_7 > r_9$.

Notice that in all the canonical cases, the revision mechanism guarantees that no cycle can be introduced. We can formulate this result, which is a straightforward consequence of the case analysis presented here.

Theorem 19 *Revising a superiority relation generates a superiority relation.*

5 AGM postulates analysis

The aim of this section is to study the canonical cases described in Section 4 from the point of view of the AGM approach. In particular, we try to relate the canonical cases with the types of theory change analysed by Alchourrón, Gärdenfors and Makinson in their seminal work [3] as far as possible. Afterwards, we focus on understanding the meaning of the various AGM postulates in terms of the changes we proposed. This allows us then to identify which of the AGM postulates are satisfied by our canonical cases.

This research issue is motivated, as introduced in Section 1, by the fact that the AGM postulates analysis in non-monotonic formalisms is still controversial, and thus open to discussion.

We recall that Delgrande proposed an approach to belief revision of logic programs under answer set semantics that is fully compliant with the base AGM postulates for revision [5]. He also claims in a later work [6, p. 568] that the third and fourth postulates for belief revision are not appropriate for belief revision of non-monotonic theories, and thus are ignored in his work. However, we are going to argue that these two postulates can be adopted in our approach, which suggests that the question whether the AGM postulates are suitable for non-monotonic reasoning is still open.

In the remainder we assume that the reader is familiar with the terminology used in the AGM framework, in particular with the notions of belief, belief set, and theory [5].

To adjust the AGM framework in the perspective of preference revision, we first rephrase the concept of extension into that of belief set corresponding to a defeasible theory.

\footnote{Notice that in our framework the hypothesis of completeness of a theory does not hold in general, as it could be the case that in a defeasible theory neither $+\partial p$, nor $+\partial \neg p$ is derivable.}
Definition 20 Let $D = (F, R, >)$ be a defeasible theory. Then

$$BS(D) = BS^+\partial(D) \cup BS^-\partial(D)$$

is the belief set of $D$, where

$$BS^+\partial(D) = \{ p | p \text{ is a literal appearing in } D \text{ and } D \vdash +\partial p \},$$

$$BS^-\partial(D) = \{ p | p \text{ is a literal appearing in } D \text{ and } D \vdash -\partial p \}.$$ 

We also state that when a literal $p$ is believed, $p \in K$ in AGM notation, then $p \in BS^+\partial(D)$. Conversely, if a literal is not believed, i.e., $p \notin K$, then $p \in BS^-\partial(D)$. Intuitively, the idea is that if we prove $+\partial p$ then we believe in $p$, and if we prove $-\partial p$ then we do not believe in $p$.

In the remainder of the section, we relate the AGM operators of contraction, expansion, and revision, and then reframe the corresponding postulates of AGM in the terminology of defeasible theories (in Subsections 5.1–5.2).

**Belief contraction** is the process of rationally removing from a belief set $K$ a certain belief $\psi$ previously in the set. From the point of view of Defeasible Logic, by Definition 20, a defeasible theory $D = (F, R, >)$ where $D \vdash +\partial p$ (i.e., $p \in BS^+\partial(D)$) is modified such that $-\partial p$ holds in the contracted theory (denoted by $D^\sim_C$) after the process (i.e., $p \in BS^-\partial(D^\sim_C)$). For the above reasoning, it seems reasonable to argue that the process of belief contraction as formalised in AGM approach corresponds to our first canonical case, i.e., from $+\partial p$ to $-\partial p$. If we consider a set of literals $C = \{ p_1, \ldots, p_n \}$, we define the contracted theory $D^\sim_C$ as the theory where for each $p_i \in C$, $p_i \in BS^-\partial(D^\sim_C)$.

**Belief revision** is the process of rationally deleting a certain belief $\psi$ from a belief set $K$ and adding its opposite. From the point of view of Defeasible Logic, by Definition 20, a defeasible theory $D = (F, R, >)$ where $D \vdash +\partial \neg p$ (i.e., $\neg p \in BS^+\partial(D)$ and $p \in BS^-\partial(D)$) is modified such that $+\partial p$ holds in the revised theory (denoted by $D^\sim_v$) after the process (i.e., $p \in BS^+\partial(D^\sim_v)$). Remember that in Defeasible Logic $\neg p$ now belongs to $BS^-\partial(D^\sim_v)$. For the above reasoning, it seems reasonable to argue that the process of belief revision as formalised in AGM approach corresponds to our second canonical case, i.e., from $+\partial \neg p$ to $+\partial p$.

**Belief expansion** is the process of adding a certain belief $\psi$ to a belief set $K$. It is possible to consider two interpretations of the expansion process: the first where we simply force the belief in, the second where a belief is added if the opposite is not believed. Our third canonical case, i.e., from $-\partial p$ to $+\partial p$, follows the second strategy. Therefore, from the point of view of Defeasible Logic, by Definition 20, this process describes the case of an initial defeasible theory $D = (F, R, >)$ where $D \vdash -\partial p$ and $D \vdash -\partial \sim p$ (i.e., $p, \neg p \in BS^-\partial(D)$) hold being modified such that $+\partial p$ holds in the expanded theory (denoted by $D^\sim_e$) after the process (i.e., $p \in BS^+\partial(D^\sim_e)$). Remember

\[\text{Notice that it is possible that a literal } p \text{ and its complement do not belong to } BS(D). \text{ For example, consider the theory consisting only of } p \Rightarrow p \text{ and } \neg p \Rightarrow \neg p. \text{ In this theory none of } \pm \partial p \text{ and } \pm \partial \neg p \text{ is provable.}\]
that in Defeasible Logic \( \sim p \) still belongs to \( BS^{-\bar{d}}(D^+_p) \). If we consider a set of literals \( C = \{p_1, \ldots, p_n\} \), we define the expanded theory \( D^+_C \) as the theory where for each \( p_i \in C \), \( p_i \in BS^{+\bar{d}}(D^+_C) \).

5.1 Preference contraction

Throughout this subsection, we assume that \( D \vdash p \) for a literal \( p \) in \( D \).

The first postulate in AGM belief contraction states that when a belief set is contracted by a sentence \( p \), the outcome should be logically closed. In Defeasible Logic, we distinguish between a theory (i.e., a set of rules), and its extension (i.e., its set of conclusions). In general, given an extension in Defeasible Logic, there are multiple (possibly not equivalent) theories that generate the extension. This means that in AGM there is no difference to contract a theory or its base, while it is not the case in Defeasible Logic.

\((K^{-1})\) \( D^+_p \) is a theory.

As preference contraction acts only on the superiority relation, to ensure that a contraction operation satisfies the postulate, we only have to check if the operation itself does not create a cycle in the superiority relation. This is guaranteed by the following proposition.

**Proposition 21** Given a defeasible theory \( D = (F, R, >) \), if \( D' = (F, R, >') \) is obtained from \( D \) by erasing preference tuples from \( > \), then \( >' \) is acyclic.

**Proof.** By contradiction, let us suppose that there is a cycle in \( >' \). Since, by hypothesis, \( >' \) is obtained from \( > \) by simply removing preference tuples, then each element of \( >' \) is an element of \( > \) and the cycle in \( >' \) is also in \( > \), against the hypothesis.

The idea of the second AGM postulate for belief contraction is that a contraction removes beliefs, thus a contraction operation always produces a belief set smaller than the original. AGM focuses only on “positive” beliefs. However, in our framework we have two possible types of defeasible conclusions (as it turns out also by Definition 20), thus we have to check the relationships between the elements of the defeasible belief sets before and after the operation. In particular, since we remove a belief, then the set of formulae believed should be smaller after the contraction; this also means that the set of formulae we do not believe is increased by the formula we contract. As a consequence, the second postulate must be rewritten taking into account the two parts.

\((K^{-2})\) \( BS^{+\bar{d}}(D^+_p) \subseteq BS^{+\bar{d}}(D) \) and \( BS^{-\bar{d}}(D^+_p) \supseteq BS^{-\bar{d}}(D) \).

This postulate cannot be adopted in our framework because it contradicts the sceptical non-monotonic nature of Defeasible Logic. To see this, suppose that we know \( a \) and we have the rules \( a \Rightarrow p \) and \( a \Rightarrow \neg p \). Then \( a \) is sceptically provable, and \( p \) is not. But if we decide to contract \( a \), then \( p \) becomes defeasibly provable, thus we have \( p \in BS^{-\bar{d}}(D) \) but
\( p \in BS^{+\delta}(D_p) \) Notice that this behaviour is not confined to the specifics of Defeasible Logic, but holds in any sceptical non-monotonic formalism.

The third postulate of AGM considers the case when a belief \( \psi \) is not in the initial belief set. As we have already discussed, AGM focuses on a classical notion of consequence relation, thus if \( \psi \) is not a consequence of the theory, then there is no reason to change anything at all. In Defeasible Logic, this corresponds to not being able to prove \( p \). Accordingly, we can state that \( p \in BS^{-\delta}(D) \).

(\( K^\sim-3 \)) If \( p \in BS^{-\delta}(D) \) then \( BS(D_p) = BS(D) \).

Since we want to obtain a theory where \( -\partial p \) holds and by hypothesis \( p \in BS^{-\delta}(D) \), then the postulate trivially holds.

The fourth AGM postulate states that the only literals that are immutable in the contraction process are tautologies. Defeasible Logic does not have logical connectives, thus it is not possible to have tautologies in the classical sense. Nevertheless, the concept of tautology is that of a statement that cannot ever be refuted, i.e., it is true in every interpretation. In classical logic, an interpretation is an assignment of truth values to the propositional atoms, while in Defeasible Logic corresponds to consider a particular set of propositional atoms as factual knowledge. In the context of this paper, where we assume that the set of facts cannot be changed, the closest thing to an interpretation is an assignment of the superiority relation. We give the formulation of the success postulate for contraction using the contrapositive.

(\( K^\sim-4 \)) If \( p \in BS(D_p) \) then \( D \vdash +\Delta p \).

The concept of strict derivation embodied by \( +\Delta \) cannot fully capture the notion of tautology as a non-refutable statement, since the proof tag \( +\varphi \) indeed denotes the presence of a supporting chain made of elements for which there are no rules for the opposite, and so de facto a non-refutable argument obtained from defeasible rules.

Thus, it seems reasonable to reformulate the success postulate for contraction as follows.

(\( K^\sim-4' \)) If \( p \in BS(D_p) \) then \( D \vdash +\varphi p \).

Even this version of the postulate does not hold in Defeasible Logic. Indeed, there exist situations where there is a proof for \( p \) and it is not possible to change the theory in order to make \( p \) no longer provable, even if there are opposite literals of some elements for every chain supporting \( p \). A simple situation is to take a tautologous 3-SAT formula and to generate its \( \Gamma \)-transformation (see Definition [12]). There are literals in the theory obtained that cannot be contracted. However, there are more cases.

\(^7\) In general, making a literal \( p \) no longer defeasibly provable does not imply that \( -\partial p \) holds after the revision process. For example, consider the theory \( \Rightarrow r, p \) and \( -p \Rightarrow s \). The only way to prevent \( +\partial p \) is to impose \( s > r \), but in the resulting theory none of \( +\partial p \) and \( -\partial p \) holds (same for \( -p \)). Notice that in this case the conditions for our canonical cases to succeed do not hold.
Example 6. Let $D$ be a defeasible theory with the following set of rules:

$\Rightarrow r_1 \ l \Rightarrow r_2 \neg a$
$\Rightarrow r_3 \ a \Rightarrow r_4 \ p$
$\Rightarrow r_5 \ b \Rightarrow r_6 \ p$
$\Rightarrow r_7 \ l \Rightarrow r_8 \neg b$.

To contract $p$, we must block both the chains proving $p$. But, in order to do so, we should have that $D \vdash +\partial l$ as well as $D \vdash +\partial \neg l$. This is not possible since $D$ is consistent.

Unfortunately, the rule pattern shown in Example 6 is not a sufficient condition to reframe the postulate $(K^-4')$. Indeed, as Example 7 shows, it is possible to find counter-examples where $p$ can be contracted, as well as counter-examples to counter-examples (we refer to Example 8 where, by extending the theory of Example 7 with rules $\{r_{19}, \ldots, r_{25}\}$, the contraction of $p$ becomes, again, not possible.

Example 7. Let $D$ be a defeasible theory with the following set of rules:

$\Rightarrow r_1 \ a \Rightarrow r_2 \ p$
$\Rightarrow r_3 \ b \Rightarrow r_4 \ p$
$\Rightarrow r_5 \ c \Rightarrow r_6 \ p$
$\Rightarrow r_7 \ l \Rightarrow r_8 \neg a$
$\Rightarrow r_9 \ \neg l \Rightarrow r_{10} \neg b$
$\Rightarrow r_{11} \ m \Rightarrow r_{12} \neg b$
$\Rightarrow r_{13} \ \neg m \Rightarrow r_{14} \neg c$
$\Rightarrow r_{15} \ n \Rightarrow r_{16} \neg c$
$\Rightarrow r_{17} \ \neg n \Rightarrow r_{18} \neg a$.

To contract $p$, we must block derivations of $+\partial a$, $+\partial b$ and $+\partial c$. This can be obtained by adding the following tuples to the superiority relation: $(r_7, r_9)$, $(r_{11}, r_{13})$ and $(r_{15}, r_{17})$.

Example 8.

$\Rightarrow r_{19} \ e \Rightarrow r_{20} \ p$
$\Rightarrow r_{21} \ f \Rightarrow r_{22} \ p$
$\neg n \Rightarrow r_{23} \ \neg e$
$\neg n \Rightarrow r_{24} \ \neg f$
$\neg m \Rightarrow r_{25} \ \neg f$.

To contract $p$, we must now block derivations also of $+\partial e$, and $+\partial f$. Derivation of $e$ can be blocked only if we prove the antecedent of $r_{23}$, that is $n$ (the derivation of $c$ is blocked as well). This implies that the derivation of $f$ is blocked only if $+\partial \neg m$ holds (the only antecedent of rule $r_{25}$). We can now operate only on the provability of either $l$, or $\neg l$. In both cases, one between $a$ or $b$ cannot be refuted.

In Subsection 4.1 we have shown that, in general, revising a defeasible theory using only the superiority relation is an NP-complete problem. This suggests that there might not be a simple condition, based on proof tags, that can be computed in polynomial time and also guarantees a successful contraction.
The fifth AGM postulate states that contracting, and then expanding by the same belief $\psi$ will give back at least the initial theory.

\[(K^-5)\] If $p \in BS^+(D)$ then $BS(D) \subseteq BS((D_p^-)^+)$. 

This postulate cannot be adopted since, once the contracted theory has been obtained, the backward step does not uniquely correspond to expanding the obtained theory by the same literal, as the following example shows.

**Example 9.** Let $D$ be a defeasible theory having the following rules:

\[
\begin{align*}
\Rightarrow r_1 &\quad a \quad \Rightarrow r_2 & \quad p \\
&\quad \lor \\
&\quad \Rightarrow r_3 & \quad \neg a \\
&\quad \Rightarrow r_4 & \quad b \\
&\quad \Rightarrow r_5 & \quad p \\
&\quad \Rightarrow r_6 & \quad \neg b
\end{align*}
\]

If we contract $D$ by $p$, in the contracted theory the preference $r_1 > r_3$ is no longer present. If we now expand $D_p^-$, one solution is the initial theory, but also the theory containing the preference $r_4 > r_6$ is another valid solution.

Nevertheless, if all operations in the contraction process can be traced, then we can easily backtrack and obtain the initial theory, satisfying the postulate.

The sixth AGM postulate, also known as the postulate of the irrelevancy of syntax, states that if two beliefs $\psi$ and $\chi$ are logically equivalent, then contracting by $\psi$ and contracting by $\chi$ produce the same result.

\[(K^-6)\] If $\vdash p \equiv q$ then $BS(D_p^-) = BS(D_q^-)$. 

In the framework of Defeasible Logic, the language is restricted to literals, thus two elements $p$ and $q$ are equivalent only if they represent the same literal. For this reason, the sixth postulate straightforwardly follows.

The seventh and the eighth postulate are best understood if seen in combination. They essentially relate two individual contractions with respect to a pair of sentences $\psi$ and $\chi$, with the contraction of their conjunction $\psi \land \chi$. As already stated, in Defeasible Logic there are no logical connectives, and a conjunction of literals is equivalent to the set of the same literals; the same reasoning used to introduce postulate $(K^-2)$ applies here. Thus, the two postulates can be rewritten as

\[(K^-7)\] $BS^{+\delta}(D_p^-) \cap BS^{+\delta}(D_q^-) \subseteq BS^{+\delta}(D_{p,q}^-)$ and $BS^{-\delta}(D_p^-) \cap BS^{-\delta}(D_q^-) \supseteq BS^{-\delta}(D_{p,q}^-)$.

\[(K^-8)\] If $p \in BS^{-\delta}(D_{p,q}^-)$ then $BS^{+\delta}(D_{p,q}^-) \subseteq BS^{+\delta}(D_p^-)$ and $BS^{-\delta}(D_{p,q}^-) \subseteq BS^{-\delta}(D_p^-)$.

Postulates $(K^-7)$ and $(K^-8)$ do not hold for the same reason formulated for postulate $(K^-2)$. The following example shows the truth of the statement for both of them.
Example 10. Let $D$ be a defeasible theory having the following rules:

\[
\begin{align*}
\Rightarrow r_3 & \neg a \\
\land \quad & \\
\Rightarrow r_1 & a \Rightarrow r_2 c \\
\lor \quad & \\
\Rightarrow r_3 & \neg c \quad \Rightarrow r_4 p \\
\lor \quad & \\
\Rightarrow r_5 & \neg d \\
\land \quad & \\
\Rightarrow r_7 & b \Rightarrow r_8 d \\
\lor \quad & \\
\Rightarrow r_9 & \neg b
\end{align*}
\]

In this theory, we have $BS^+\partial(D) = \{a, b, c, d, \neg p\}$, and $BS^-\partial(D) = \{\neg a, \neg b, \neg c, \neg d, p\}$.

Let us contract $D$ by literal $a$ and by literal $b$ (where the contractions are minimal with respect to the changes in the superiority relation) obtaining:

\[
\begin{align*}
BS^+(D_a^-) &= \{b, \neg c, d, \neg p\} \\
BS^+(D_b^-) &= \{a, c, \neg d, \neg p\} \\
BS^-(D_a^-) &= \{a, \neg a, \neg b, c, \neg d, p\} \\
BS^-(D_b^-) &= \{\neg a, b, \neg b, \neg c, d, p\}.
\end{align*}
\]

The respective intersections are:

\[
\begin{align*}
BS^+(D_a^-) \cap BS^+(D_b^-) &= \{\neg p\} \\
BS^-(D_a^-) \cap BS^-(D_b^-) &= \{\neg a, \neg b, p\}.
\end{align*}
\]

We can now contract $a$ and $b$ simultaneously, and obtain

\[
\begin{align*}
BS^+(D_{ab}^-) &= \{\neg c, \neg d, p\} \\
BS^-(D_{ab}^-) &= \{a, \neg a, b, \neg b, c, d, \neg p\}
\end{align*}
\]

proving our claim.

Throughout postulates $(K^-1)$ to $(K^-8)$ we took care of the transition effects of the contraction process, due to the specific nature of positive and negative beliefs in Defeasible Logic. However, for each postulate this specificity has no effect. In fact, what can be claimed for contractions in $BS^+\partial$ extends to $BS^-\partial$, and vice versa.

For the sake of completeness, we apply the same care to expansion and revision cases further on. As it will be clear at the end of each analysis, analogous conclusions about the redundancy are derived.

5.2 Preference revision

Throughout this subsection, we assume that $D \vdash +\partial \neg p$ and $D \vdash +\Sigma p$ for a literal $p$ in $D$. 
The first AGM postulate for revision states that the revision process has to preserve the logical closure of the initial theory.

\[(K \ast 1) \ D_p^* \text{ is a theory.}\]

The reasoning is the same made for the first postulate for contraction and expansion, and it assures that also \((K \ast 1)\) is satisfied in our framework.

The second AGM postulate for revision captures the most general interpretation of theory change; the new information \(\psi\) is always included in the new belief set, even if \(\psi\) is self-inconsistent, or contradicts some belief of the initial theory. Henceforth, the complete reliability of the new information is always assumed.

\[(K \ast 2) \ p \in BS^{+\partial} (D_p^*).\]

As by definition of our second canonical case, literal \(p\) is forced to be defeasibly proved after the process, provided that preconditions \(+\partial \sim p\) and \(+\Sigma p\) hold, the postulate is clearly satisfied.

The third and the fourth postulates of AGM revision explain the relationship between the revision and the expansion processes. The quintessential meaning is that they are independent by operators implementing them.

\[(K \ast 3) \ BS^{+\partial} (D_p^*) \subseteq BS^{+\partial} (D_p^\Sigma).\]

\[(K \ast 4) \text{ If } \sim p \in BS^{-\partial} (D) \text{ then } BS^{+\partial} (D_p^-) \subseteq BS^{+\partial} (D_p^\Sigma).\]

Both the first two canonical cases, starting from an initial theory and considering a literal \(p\), operate to obtain a final theory where \(+\partial p\) holds. What we have to care about, however, are the preconditions for which these two canonical cases apply. The third postulate essentially states that every belief which can be derived revising a theory by a belief \(\psi\) can be also obtained expanding the same initial theory with respect to the same belief. This statement is perfectly allowed in our framework; the case where both revision and expansion can apply is when \(+\partial \sim p\) (and hence \(-\partial p\)) holds in the initial theory; in this situation, the two processes behave in the same manner, i.e., they calculates the same extensions. However, if we regard at proper expansion, i.e., when condition \(-\partial \sim p\) holds, then it is easy to see that the preconditions for expansion and revision are mutually exclusive and can not be applied at the same time.

The fifth AGM postulate states that the result of a revision by a belief \(\psi\) is the absurd belief set iff the new information is in itself inconsistent.

\[(K \ast 5) \text{ If } p \text{ is consistent then } BS^{+\partial} (D_p^*) \text{ is also consistent.}\]

Since by definition a literal \(p\) is always consistent, and the extension of a consistent theory is also consistent, the postulate is trivially satisfied.
The sixth AGM postulate for revision follows the same idea of \((K \ast 6)\): the syntax of the new information has no effect on the revision process, all that matters is its content. Again, it has a natural counterpart in Defeasible Logic.

\[(K \ast 6)\] If \(\vdash p \equiv q\) then \(BS^{+\partial}(D_p^*) = BS^{+\partial}(D_q^*)\).

The reasoning is the same exploited in the counterpart postulate for contraction, and the postulate is straightforward.

The seventh and the eighth postulate of AGM revision cope with the revision process with respect to conjunctions of literals. In the classical AGM framework, the principle of minimal change takes an important role in the formulation of these postulates. The revision with both \(\psi\) and \(\chi\) should correspond to a revision of the theory with \(\psi\) followed by an expansion by \(\chi\), provided that \(\chi\) does not contradict the beliefs in the theory revised by \(\psi\).

\[(K \ast 7)\] \(BS^{+\partial}(D_{p,q}^*) \subseteq BS^{+\partial}((D_p^*)_{q}^+)\) and \(BS^{-\partial}((D_p^*)_{q}^+) \subseteq BS^{-\partial}(D_{p,q}^*)\).

\[(K \ast 8)\] If \(\neg q \in BS^{-\partial}(D_p^*)\) then \(BS^{+\partial}((D_p^*)_{q}^+) \subseteq BS^{+\partial}(D_p^*)\) and \(BS^{-\partial}(D_{p,q}^*) \subseteq BS^{-\partial}((D_p^*)_{q}^+))\).

Again, since the sceptical nature of Defeasible Logic, these postulates cannot be satisfied. The following example gives a specific case that falsifies them.

**Example 11.** Let \(D\) be a defeasible theory having the following rules:

\[
\begin{align*}
\rightarrow r_1 & \quad \neg a \\
\rightarrow r_2 & \quad a \quad \Rightarrow r_3 \quad p \\
& \quad \Rightarrow r_4 \quad \neg p \\
\rightarrow r_5 & \quad b \\
\rightarrow r_6 & \quad \neg b \\
& \quad b \Rightarrow r_7 \quad p \\
& \quad b \quad \Rightarrow r_8 \quad q \\
& \quad \Rightarrow r_9 \quad \neg q \\
\rightarrow r_{10} & \quad c \quad \Rightarrow r_{11} \quad q
\end{align*}
\]

Given theory \(D\), we have \(BS^{+\partial}(D) = \{\neg p\}\), while all other literals are in \(BS^{-\partial}(D)\). Revising \(D\) for \(p\) and \(q\), one possible theory is \(D_{p,q}^*\), obtained operating through the provability of literal \(b\) and adding the following superiorities: \(r_5 > r_6, r_7 > r_4,\) and \(r_8 > r_9\). The resulting \(BS^{+\partial}(D_{p,q}^*), BS^{-\partial}(D_{p,q}^*)\) are respectively \(\{b, c, p, q\}\), and \(\{a, \neg a, \neg b, \neg p, \neg q\}\).

Now, let us consider the revision just by \(p\). A possible solution is \(D_{p}^*\) such that \(BS^{+\partial}(D_p^*) = \{a, c, p\}\), and \(BS^{-\partial}(D_p^*) = \{a, b, \neg b, \neg p, \neg q\}\). In this case the revision process acts on the provability of literal \(a\), adding \(r_2 > r_1\), and \(r_3 > r_4\).

If we expand \(D_p^*\) by \(q\), one possible resulting theory is \((D_{p,q}^*)_q\) (the expansion process now operates on the provability of literal \(c\) adding \(r_{11} > r_9\)) where \(BS^{+\partial}((D_{p,q}^*)_q) = \{a, c, p, q\}\), and \(BS^{-\partial}((D_{p,q}^*)_q) = \{a, b, \neg b, \neg p, \neg q\}\). The intersection of \(D_{p,q}^*\) and \((D_{p,q}^*)_q\) is not empty, but neither theory is contained in the other.
5.3 Preference expansion

Throughout this subsection, we assume that for a literal $p$ in $D$ both $D \vdash -\partial p$ and $D \vdash -\partial \neg p$. Moreover, $+\Sigma p$ holds.

The first AGM postulate for expansion states that if a theory is expanded with a belief $\psi$, then the resulting theory is the logical closure of the initial theory.

$(K + 1)$ $D^+_p$ is a theory.

The same idea for postulate $(K - 1)$ can be exploited, thus the postulate is clearly satisfied.

The second AGM postulate for expansion assures that a belief $\psi$ for which the expansion is performed always belongs to the belief set of the resulting theory.

$(K + 2)$ $p \in BS^{+\partial} (D^+_p)$.

By the hypotheses given at the beginning of this subsection, the postulate trivially holds since the expansion process forces literal $p$ to be defeasibly proved.

The joint formulation of the third and the fourth AGM postulates for expansion states that if a belief is already present in the initial belief set, then an expansion process lets the theory unchanged.

$(K + 3)$ $BS^{+\partial} (T) \subseteq BS^{+\partial} (T^+_p)$ and $BS^{-\partial} (T^+_p) \subseteq BS^{-\partial} (T)$.

$(K + 4)$ If $p \in BS^{+\partial} (T)$ then $BS^{+\partial} (T^+_p) \subseteq BS^{+\partial} (T)$ and $BS^{-\partial} (T) \subseteq BS^{-\partial} (T^+_p)$.

Since we aim at obtaining a theory where $+\partial p$ holds, and by hypothesis $p \in BS^{+\partial} (T)$, the postulates seen together trivially hold given that, by definition, its premise does not hold.

The fifth AGM postulate states that if a belief set is contained in another one, then the expansion of both sets wrt the same belief preserves the relation of inclusion.

$(K + 5)$ If $BS^{+\partial} (D) \subseteq BS^{+\partial} (D')$ then $BS^{+\partial} (D^+_p) \subseteq BS^{+\partial} (D^+_p)$.

Also in this case, because of the sceptical non-monotonic nature of Defeasible Logic, this postulate can not be satisfied, as already pointed out in the explanation of Postulate $(K - 2)$.

Non-monotonic formalisms derive conclusions that are tagged. The specific nature of this tagging is that it makes the notion of minimality for a set of conclusions useless. We can consider minimality only for one given tag, and not for all tags (this is particularly obvious for formalisms where tags of formulae are interdependently defined). Thus, the
idea of “smallest resulting set” is meaningless in non-monotonic systems. The sixth AGM postulate assures the minimality of the expanded belief set.

\[(K + 6)\] Given a theory \(D\) and a belief \(p\),

\[
BS(D_p^+) \text{ is the smallest belief set satisfying } (K + 1) - (K + 5).
\]

In the perspective of non-monotonic reasoning, the operation of expanding a defeasible theory to prove a literal \(p\) (only changing the preference relation) necessarily falsifies some other literals, previously provable in the initial theory.

5.4 Preference identities

In AGM framework a process that defines revision in terms of expansion is available, suggested by Isaac Levi [19]. The idea is that to revise a theory \(D\) by a belief \(\psi\) we may firstly contract \(D\) by \(\neg \psi\) in order to remove any information that may contradict \(\psi\), and then expand the resulting theory with \(\psi\). This is know as the Levi identity, which can be rewritten using our terminology as:

\[(LI)\] \(BS(D_p^+) = BS((D_{\neg p})_p^+).\)

The following example shows that the Levi identity does not hold in our framework.

**Example 12.** Let \(D\) be a defeasible theory having the following rules:

\[
\begin{align*}
\Rightarrow r_1 & \quad a \\ \Rightarrow r_2 & \quad p \\
\Rightarrow r_5 & \quad \neg a \\ \Rightarrow r_4 & \quad \neg p \\ \Rightarrow r_6 & \quad b \\ \Rightarrow r_7 & \quad \neg b
\end{align*}
\]

If we revise \(D\) by \(p\), a possible solution is \(D_p^+\) such that \(BS(D_p^+) = \{a, p\}\), and \(BS^{-\delta}(D_p^+) = \{\neg a, b, \neg b, \neg p\}\). Now, contracting \(D\) by \(\neg p\) can lead to \(D_{\neg p}\) with \(BS^{-\delta}(D_{\neg p}) = \{b\}\), and \(BS^{-\delta}(D_{\neg p}) = \{a, \neg a, \neg b, p, \neg p\}\). If we expand \(D_{\neg p}\) by \(p\), we obtain \((D_{\neg p})_p^+\) with \(BS^{-\delta}((D_{\neg p})_p^+) = \{b, p\}\), and \(BS^{-\delta}((D_{\neg p})_p^+) = \{a, \neg a, \neg b, \neg p\}\).

The Levi identity does not hold as our revision procedure concerns the reasons why one belief is obtained and not only whether we have one belief. Thus when there are multiple reasons to justify one belief, it is possible to contract the theory in multiple ways, and similarly to expand it in multiple ways, and the changes for the contractions are not necessarily the ‘opposite’ of those for contraction.

As Levi Identity relates the revision process in terms of expansion, Harper proposed a method to obtain the contraction using revision [20]; the underlying idea is that a theory \(D\) contracted by a belief \(\psi\) is equivalent to the theory containing only the information that remain unchanged during the process of revising \(D\) by \(\neg \psi\). In our terms, the Harper Identity can be rewritten as

\[(HI)\] \(BS(D_{\neg p}^-) = BS(D_{\neg p}) \cap BS(D)\).

Harper Identity does not hold for the operations we defined in this paper. Example [13] provides a counter-example to it.
Example 13. Let $D$ be a defeasible theory having the following rules:

$$\Rightarrow r_1 \ p \ \Rightarrow r_2 \ q$$

$$\lor \Rightarrow r_3 \ \neg q$$

$$\land \Rightarrow r_4 \ \neg p \Rightarrow r_5 \ q$$

The initial belief set is $BS^+\partial (D) = \{p, q\}$ and $BS^-\partial (D) = \{\neg p, \neg q\}$. If we contract $D$ by $p$, we obtain a theory $D^\neg p$ such that $BS^+\partial (D^\neg p) = \{\neg q\}$ and $BS^-\partial (D^\neg p)$ contains all the other literals. Instead, if we revise the initial theory with $\neg p$ the theory $D^{*}\neg p$ where $BS^+\partial (D^{*}\neg p) = \{\neg p, q\}$ and $BS^-\partial (D^{*}\neg p) = \{p, \neg q\}$ is obtained. The intersections between the revised theory and the initial one are $BS^+\partial (D^{*}\neg p) \cap BS^+\partial (D) = \{q\}$ and $BS^-\partial (D^{*}\neg p) \cap BS^-\partial (D) = \{\neg q\}$.

Again, the main reason for the failure of the Harper Identity resides in the non-monotonic nature of Defeasible Logic, where, in general it is not possible to control the consequences of a given formula.

In this section we have provided an interpretation of the AGM postulates for expansion, contraction and revision in terms of our canonical cases and the operations that are possible when the changes operate only on the superiority relation.

We believe that the contribution of this section is multi-fold. First of all, the definition of our canonical cases offer a more precise formal understanding of the intuition of the various operations. Second, we reconstructed the postulates for the canonical cases and discuss how to adapt them. The last contribution of the analysis confirms the outcome of [2], showing that in general the postulates describing inclusion relationships between belief sets before and after a revision operation do not hold for Defeasible Logic, and it is unreasonable to expect that they hold for non-monotonic reasoning in general.

6 Related Work

As far as we are aware of, the work most closely related to ours is that of [21] where the authors study, given a theory, how to abduct preference relation to support the derivation of a specific conclusion. Therefore the problem they address is conceptually different from what we presented in this paper, given that we focus on modifying the superiority relation.

Notice that in non-monotonic reasoning, a revision is not necessarily triggered by inconsistencies. [2] investigates revision for Defeasible Logic and relationships with AGM postulates. While their ultimate aim is similar to that of the present paper – i.e.,

8 Notice that while the main analysis in this paper is specific to revision of the superiority relation of Defeasible Logic, the definition of the canonical cases does not depend on it, and it can be applied in a much broader context. For example the canonical case from $\partial p$ to $\partial \neg p$ can be understood as “how do we modify a theory such that before the revision a formula holds, and after the revision the opposite holds?”; similarly for the other canonical cases.
transforming a theory to make a previously provable (resp. non provable) statement, non provable (resp. provable) – the approach is different, and more akin to standard belief revision. More precisely, revision is achieved by introducing new exceptional rules. Furthermore, they discuss how to adapt the AGM postulates for non-monotonic reasoning.

Our work is motivated by legal reasoning, where preference revision is just one of the aspects of legal interpretation. [22,23] propose a Defeasible Logic framework to model extensive and restrictive legal interpretation. This is achieved by using revision mechanisms on constitutive rules, where the mechanism is defined to change the strength of existing constitutive rules. Based on the specific type of norm to modify, they propose a revision (contraction) operator which modifies the theory by adding (removing) facts, strict rules, or defeaters, raising the question whether extensive and restrictive interpretation can be modelled as preference revision operators. An important aspect of legal interpretation is finding the legal rules to be applied in a case: in this work we assumed that the relevant rules have already been discovered, and in case of conflicts, preference revision can be used to solve them.

Another work, related to revision of Defeasible Logic is that of [4], where the key idea is to model revision operators corresponding to typical changes in the legal domain, specifically, abrogation and annulment. They show that, typically, belief revision methodologies are not suitable to changes in theories intended for legal reasoning. Similarly, they show that it is possible to revise theories fully satisfying the AGM postulates, but then the outcome is totally meaningless from a legal point of view.

The connection between sceptical non-monotonic formalisms and argumentation is well known in literature; in [9], authors adapt Dung’s argumentation framework [24,25] to give an argumentation semantics for Defeasible Logic: first, they prove that Dung’s grounded semantics characterises the ambiguity propagating DL; then, they show that the ambiguity blocking DL is described with an alternative notion of Dung’s acceptability. The main effort was to establish close connections between defeasible reasoning and other formulations of non-monotonic reasoning.

Non-monotonic revision through argumentation was also investigated in [26,27] using Defeasible Logic Programming (DEL). They define an argument revision operator that inserts a new argument into a defeasible logic program in such a way that this argument ends up undefeated after the revision, thus warranting its conclusion, where a conclusion \( \alpha \) is warranted if there exists a non-defeated argument supporting it. Despite the meaning given in this work, their concept of defeaters denotes stronger counter-arguments to a given conclusion based on a set of preferences stating which argument prevails against one other.

Their work suffers from a main drawback: imposing preferences among arguments (i.e., whole reasoning chains in our framework), instead of single rules, can lead to a situation when an argument is warranted even if all its sub-arguments are defeated. DEL formalism is very similar to Defeasible Logic. Therefore, techniques proposed in this work can be easily accommodated to join the framework presented in [26,27].

Other works closely related to ours are [28,29,30,31]. They propose extensions of an argumentation framework, Defeasible Logic and Logic Programming, where the su-
priority relation is dynamically derived from arguments and rules in given theories. While the details are different for the various approaches, the underlying idea is the same. For example, in [29], it is possible to have rules of the form \( r : a \Rightarrow (s > t) \) where \( s \) and \( t \) are identifiers for rules. Accordingly, to assert that rule \( s \) is stronger than rule \( t \) we have to be able to prove \( +\partial a \) and that there are no applicable rules for \( \neg(s > t) \). In addition, the inference rules require that instances of the superiority relation are provable (e.g., \(+\partial(s > t)\)) instead of being simply given (as facts) in \( > \), i.e., \((s, t) \in >\). The main difference with these works is that we investigate general conditions under which it is possible to modify the superiority relation in order to change the conclusions of a theory, while they provide specific mechanisms to compute conclusions where the preference relations are inferred from the context. They do not study which are the possible ways to revise a theory. For example, if a literal is \( > \)-tautological, no matter how we derive instances of the preference relation, there is no way to prevent its derivation, or to derive its negation.

In the scenario where the preferences over rules are computed dynamically, one could argue that it might be possible to encode in the theory the possible ways in which the superiority relation would behave. The problem with this approach is the combinatorial explosion of the number of rules required, since one would have to consider rules with the form \( a_1, \ldots, a_n \Rightarrow (r_i > r_j) \) for all possible combinations of literals \( a_k \) in the theory, and also for all possible combinations of instances of \( > \). In both cases there is an exponential number of combinations. Among the works mentioned above, [28] is motivated, as us, by legal reasoning, and they use rules to encode the legal principles we shortly discussed in the introduction.

7 Conclusions and further work

Over the years Defeasible Logic has proved to be simple but effective practical non-monotonic formalism suitable for applications in many areas. Its sceptical nature allows to have defeasible proofs both for a belief and its opposite, and still be consistent since, at most, one of them can be finally proven. Since from its first formulation in [32], many theoretical aspects of Defeasible Logic have been studied: from its proof theory [12] to relationships to logic programming [33], from variants of the logic [16] to its semantics [9] and computational properties [18]. Furthermore, several efficient implementations have been developed [34][35][36]. Methods to revise, contract, or expand a defeasible theory were first proposed in [2], where the authors studied how to revise the belief set of a theory based on introduction of new rules. The resulting methodology was then compared to the AGM belief revision framework.

In this work we took a different approach: since, in many situations, a person cannot change the rules governing a system (a theory) but only the way each rule interact with the others, it seems straightforward to consider revision methodologies of Defeasible Logic where derivation rules are considered as “static” or “untouchable”, and the only way to change a theory with respect to a statement is to modify the relative strength of a rule with respect to another rule, i.e., to modify the superiority relation of the analysed theory.
Therefore, we presented in Section 3 the formalism adopted: eight different types of tagged literals were described to simplify the categorisation process and, consequently, the revision calculus. In Section 4 we introduced three canonical cases of possible revisions and systematically analysed every canonical instance. In both sections, we presented several theoretical results on conditions under which a revision process is possible.

Upon these theoretical basis, in Section 5 we proposed a systematic comparison between our framework and the AGM postulates. In there, the three canonical cases were compared to the AGM contraction, expansion, and revision: for each belief change operator, all the AGM postulates were rewritten using our terminology, and their validity was studied in our framework.

The work presented in this paper paves the way to several lines of further investigation to extend the proposed change methodologies.

The first extension we want to mention is that where we change the status of a sets of literals instead of a single literal. Studying conditions (supporting chains, proof tags, and so on) to understand when, and where, it is possible to change a theory by more than a single literal is not a trivial issue. Consider the following theory

Example 14. \( D = \)

\[
\Rightarrow r_1 \neg b \Rightarrow r_2 q \\
\Rightarrow r_3 a \Rightarrow r_4 b \Rightarrow r_5 p \\
\wedge \\
\Rightarrow r_6 \neg a.
\]

It is clear it is not possible to change the initial theory if we want to obtain both \(+\partial p\) and \(+\partial q\).

The second extension concerns how to limit the scope of the revision operators. Revision of preferences should not involve minimal defeasible rules. This constraint captures the idea that a rule that wins against all other rules is a basic juridical principle. A similar aspect is that under given circumstances the revision process should not, for at least a subset of “protected” pairs, violate the original preferential order. For instance, we should not revise those preferences that are unquestioned because derived by commonly accepted principles or explicitly expressed by the legislator, as discussed in the introduction.

As we have seen in Section 2 in the legal domain we can identify several sources for the preference relation. Preference handling in Defeasible Logic can gain much from typisation of preferences themselves. The notion of preference type and its algebraic structure has been studied previously and can be applied directly here \[37\]. Analogously, one of the possible directions of generalisation for the notion of preference is the notion of partial order, investigated at a combinatorial level by \[38\] and then studied from a computational viewpoint in \[39\].

The main aim of the paper was to identify conditions under which revision based on changes of the superiority relation was possible. Accordingly, the next important aspect of belief revision is to identify criteria of minimal change. It is possible to give alternative definitions of minimal revision. For example, one notion could be on the cardinality of instances of the superiority relation, while another one is to consider minimality with
respect to the conclusions derived from a theory. A few research questions naturally follow: ‘Are there conditions on a theory to guarantee that a revision is minimal?’, or ‘Is it possible to compare different minimality criteria?’.

We illustrate some of these issue with the help of the following example.

Example 15. Let $D$ be the following theory

\[
\Rightarrow_{r_1} a \Rightarrow_{r_2} b \Rightarrow_{r_3} p
\]
\[
\Rightarrow_{r_4} -a \Rightarrow_{r_5} -b
\]
\[
\Rightarrow_{r_6} c \Rightarrow_{r_7} d \Rightarrow_{r_8} e \Rightarrow_{r_9} p
\]
\[
\wedge
\]
\[
\Rightarrow_{r_{10}} -c
\]

Superiority relation $\succ'$ guarantees to change only one preference, but modify the extension of the former theory by five literals ($c, -c, d, e, \text{ and } p$), while $\succ''$, by adding two preferences, changes only three literals ($a, b$ and $p$).

Finally, the present work provides a further indication that the AGM postulates are not appropriate for belief revision of non-monotonic reasoning. Consequently, a natural question is whether there is a set of rational postulates for this kind of logics. We are sceptical about this endeavor; there are many different and often incompatible facets of non-monotonic reasoning, and a set of postulates might satisfy some particular non-monotonic features but not appropriate for others. For example, as we have seen in this paper, if we ignore monotonic conclusions (conclusions tagged with $\pm \Delta$), there are other cases where we cannot guarantee the success of the revision operation. On the other hand, [2] argues that the success postulate for revision holds if we are allowed to operate on rules instead of preferences. This example suggests that it might possible to find a set of postulates, but this would specific to a logic and specific types of operations. The quest for an alternative set of postulates for revision of non-monotonic theories is left for future research.

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