Detection of invisible and crucial events: from seismic fluctuations to the war against terrorism

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We argue that the recent discovery of the non-Poissonian statistics of the seismic main-shocks is a special case of a more general approach to the detection of the distribution of the time increments between one crucial but invisible event and the next. We make the conjecture that the proposed approach can be applied to the analysis of terrorist network with significant benefits for the Intelligence Community.

I. INTRODUCTION

The main aim of this paper is to prove the efficiency of a new method for the detection of crucial events that might have useful applications to the war against terrorism. This has to do with the search for rare but significant events, a theme of research that has been made of extreme importance by the tragedy of September 11. This method is applied here to defining the statistics of seismic main-shocks, as done in an earlier publication\textsuperscript{1}. However, the emphasis here is more on the conceptual issues behind the interesting results obtained in Ref. \textsuperscript{1} than on their geophysical significance. In fact, the discussion of these conceptual issues aims at supporting the conjecture that the method has a wider range of validity. We shall help the reader to understand this general discussion with a dynamic model, originally proposed in Ref. \textsuperscript{2}. We point out that this model was proposed for purposes different from geophysical applications. However, it is a case where the crucial events to detect are under our control, thereby making it possible for us to check the accuracy of the method of detection of invisible and crucial events that we propose here for a more general purpose, including the war against terrorism. Furthermore, for this model an analytical treatment has been recently found\textsuperscript{3}, supporting the claims that we make in this paper for the accuracy of the method of detection. For the reader’s convenience, the results on the seismic fluctuations are suitably reviewed, and discussed in the light of the more general perspective of this paper. We also review the model for seismic fluctuations, proposed in the earlier work of Ref. \textsuperscript{1}. This model shares with the model of Ref.\textsuperscript{2} the property that the crucial events are imbedded in a sea of secondary events, but it allows us to reveal with accuracy the statistics of the crucial events for different mathematical reasons.

II. CRUCIAL EVENTS, MEMORY AND PREDICTABILITY

The analysis shown in action on the seismic fluctuations, should serve the more general purpose of detecting the statistical properties of crucial events that are invisible. By crucial events we mean events causing other events, which would be predictable if the time occurrence of their causes were known. By invisible and crucial events we mean crucial events embedded in a sea of many other events, either caused by the invisible events or by environmental fluctuations. These secondary events play a camouflage action that makes it difficult to detect the crucial events with accuracy. We discuss first the property that the crucial events must have, regardless of whether they are visible or not. We want also to address the delicate issue of the extent to which the crucial events are predictable and the extent to which they are random, this being a problem that has caused much confusion in the past. Let us consider the following dynamic model. A particle moves in the interval \( I \equiv [0, 1] \). Its trajectory \( x(t) \) is governed by the equation:

\[
\frac{dx}{dt} = \alpha x^z. \tag{1}
\]

The parameter \( \alpha \) is a positive number, which has to be kept much smaller than 1, if the integration time step is 1. This fundamental equation serves the purpose of generating non-Poisson statistics. In fact, as we shall see, \( z = 1 \)
generates Poisson statistics, while the wide dominion of non-Poisson statistics is given by $z > 1$. When the particle reaches the border $x = 1$ it is injected back to a new initial condition $x(0) > 0$, with uniform probability. Now, let us imagine that the times of sojourn within the interval are recorded via direct observation. For tutorial purposes, we assume these random events to be visible. We shall get the time series

$$\{\tau\} \equiv \tau(1), \tau(2), \ldots$$

(2)

Let us consider the initial condition $x(0)$. The time spent by the particle, moving from this initial condition, within the interval $I$, before reaching the border is

$$\tau = \frac{1}{\alpha} \left[ \frac{1}{1 - z} - \frac{x(0)^{1-z}}{1 - z} \right],$$

(3)

as predicted by the time integration of Eq. (1). The connection between the waiting-time distribution and the injection process into the initial condition is given by

$$\psi(\tau) d\tau = p(x(0)) dx(0).$$

(4)

In the case of a uniform injection, $p(x(0)) = 1$, inserting Eq. (3) into Eq. (4) yields after some algebra

$$\psi(\tau) = (\mu - 1) \frac{T^{\mu-1}}{(T + \tau)^\mu},$$

(5)

where

$$\mu = \frac{z}{z - 1}$$

(6)

and

$$T = \frac{1}{\alpha(z - 1)}.$$ 

(7)

It is worth devoting some comments to this result. Let us imagine that we convert the time series $\{\tau\}$ into a diffusion process with the same prescription as that adopted in the earlier work on the seismic fluctuations in Southern California. This means that the random walkers makes a jump ahead by a given quantity, equal to 1, for instance, at the times $t(1) = \tau_1$, $t(2) = \tau_1 + \tau_2$, $t(3) = \tau_1 + \tau_2 + \tau_3$, and so on. An ensemble of random walkers obeying the same prescription (see Ref. [4], for details on how to derive this ensemble from the single sequence $\{\tau\}$) undergoes a diffusion process that in the specific case $2 < \mu < 3$ yields a diffusion process. The probability distribution function, $p(x,t)$, for this process, in the time asymptotic limit is expected to obey the scaling condition

$$p(x,t) = \frac{1}{t^\delta} F\left(\frac{x - wt}{t^\delta}\right),$$

(8)

where $w$ is the mean velocity produced by the walking rule adopted and $\delta$ is the scaling index given by

$$\delta = \frac{1}{\mu - 1}.$$ 

(9)

$F(y)$ is an asymmetric function of $y$, whose detailed analytical form is discussed in Ref. [4]. Ref. [4] discusses other walking prescriptions, and physical conditions different from $2 < \mu < 3$ as well. For simplicity, in this paper we discuss only the earlier walking prescription and the case where $\mu < 3$, so as to create a strong departure from ordinary statistical mechanics, namely from the condition where the second moment of $\psi(\tau)$ is finite. We also set the condition $\mu > 2$ which keeps the system far from the condition of a diverging first moment. Why do we assign to $\psi(\tau)$ this condition of strong departure from ordinary statistical mechanics? We shall answer this important question after making the reader familiar with the intriguing issue of the memory emerging from the breakdown of the Poisson statistics. This is a poorly understood property, in spite of the fact that 32 years ago Bedeaux, Lindenberg and Shuler [5] wrote a clarifying paper on this subject. We have seen that our method of analysis rests on turning a time series into a diffusion process. If we imagine the one-dimensional axis on which this diffusion process is realized as a chain of infinite discrete sites, we can denote the state at time $t$ of the diffusing system through the vector $p(t)$, with $p_i(t)$ denoting the probability for the random walker to be at the $i$-th site at time $t$. Thus, it is legitimate to ask the important question of whether the knowledge of $p(t)$ allows us to determine $p(t')$ with $t' > t$. In other words,
the question is the following: does the information \( p(t) \) at a given time \( t \) allow us to predict the state of the system at a later time? We want to point out that the question refers to a set of random walkers, not to a single random walker, whose walk, at a time scale larger than the first moment of the waiting time distribution \( \psi(\tau) \), is certainly unpredictable.

A careful reading of Ref. 2 allows us to answer this question with this apparently striking statement: this is possible only in the Poisson case 2. In the non-Poisson case an infinitely extended memory emerges. The Poisson case is the only one where the state \( p(t) \), with \( t > 0 \), determines the time evolution of the system of interest. In all the other cases, the future time evolution of the system also depends on \( p(t') \), with \( t' < t \). In other words, the system time evolution retains memory of the initial condition \( p(0) \) forever. It is evident that the concept of crucial event implies a departure from the Poisson condition. In fact, the probability of occurrence of a main-shock is expected to have memory, this corresponds to the fact that the geophysical processes, responsible for the main-shocks, do not generate random fluctuations, but long-range correlation.

In the case of the terrorist network, we conjecture that the crucial events, having either ideological or religious origin, and so historical roots, are driven by non-Markovian master equations: This implies that the irrelevant degrees of freedom, playing the role of a thermal bath, are not equivalent to white noise, as it would be in the case of Poisson statistics 3. The choice of the condition of \( 2 < \mu < 3 \) makes the resulting diffusion process depart dramatically from the Gaussian state, thereby assigning to the crucial events, either main-shocks or main secret events of the terrorist network, a condition of striking departure from ordinary statistical mechanics. In other words, we conjecture that the crucial events, which, although invisible, influence cascades of secondary events, are located in a basin of attraction of anomalous rather than normal statistics, an assumption that fits the traditional wisdom of the researchers in the field of complexity. It has to be pointed out that from a formal point of view a condition of infinite memory is realized by \( \mu < \infty \), without necessarily implying \( \mu < 3 \). However, the condition \( \mu > 3 \) would not make the crucial event generate a visibly anomalous diffusion, and an even more sensitive procedure should be planned, to discover the existence of this kind of crucial events. Thus, the condition \( \mu < 3 \), which, as earlier pointed out, seems to be a plausible property of complex systems, corresponds to a case where the procedure illustrated in this paper, is already adequate, in the present form, to reveal their existence. Before ending this Section, we must clarify a problem that is a frequent source of confusion. The infinite memory associated with non-Poisson statistics might be mistaken as a way to make prediction.

We have to point out that the infinite memory is a concept referring to probabilities, or to a set of walkers. The concept of Gibbs ensemble, although fundamental for statistical mechanics, is based on the assumption that many identical copies of the system are available to us. Actually, we study only single systems. For instance, the predictability of earthquakes, implies that knowing that a crucial event occurred at time \( t = t_1 \), we can predict that the next will take place at a time \( t = t_2 > t_1 \). The time distance \( t_2 - t_1 \) cannot be predicted, if the events under study are crucial in the sense earlier defined. However, if Eq. (1) were a reliable model for the process under study, one might conjecture that a specific observation of the geophysical motion is equivalent to informing us about the new initial condition, after the back injection taking place at \( t = t_1 \). The instant of the back injection is the genuinely random event. The laminar motion ensuing this random event is deterministic and, consequently, compatible with predictability, at least in principle. Why do we leave room for randomness, in the moment of selection of the new initial condition? This is equivalent to associating crucial events to randomness, and a thorough discussion of this issue beyond the limits of this paper. If we adopt the usual view that randomness is an expression of our ignorance about the infinitely many and irrelevant degrees of freedom in a system, this choice is equivalent to a drastic simplification of the problem under study. Going beyond that would be equivalent to predict the occurrence time of main-shocks, in the case of seismic fluctuations, and of terrorist actions, in the case of the war to terrorism. For the time being, our purpose is much less ambitious.

III. MEMORY BEYOND MEMORY

Notice that the title of the paper of Ref. 2, memory beyond memory, is probably incomprehensible to all those who do not know the fundamental work of Ref. 2. On the basis of the results of Ref. 2 we can explain the meaning of this title. The majority of events under observation in Ref. 2 are not crucial events. The crucial events, which are rare, are imbedded in a sea of secondary events, also called pseudo-events. These secondary events are influenced by the crucial events and play a camouflage role that makes the really crucial events invisible. As a consequence of being secondary, the pseudo-events have memory of the crucial events influencing them.

It is worth remarking that in the case of seismic events the crucial events are the main shocks and the secondary events are the Omori swarms of aftershocks. We make an important conjecture: the case of the terrorist network rests on the picture of the passive supporters of terrorist activities. These supporters generate events that, although secondary, are dependent on the main terrorist events, of which they bear memory. This is the memory of the second
kind, the memory of first kind being, as pointed out in the earlier Section, the memory corresponding to the non-Poisson statistics of the crucial events. The terrorists trigger events, with either religious or ideological memory, and these crucial events influence secondary events, the action of passive supporters, which are characterized by memory beyond memory. Although, the term memory beyond memory was originally coined for the completely different purpose of helping the search for the key physiological processes behind heart-beating \( \mathcal{I} \), we find it to be especially adequate to describe a procedure of statistical analysis aiming at helping the Intelligence Community in the war against terrorism. For all these reasons, it is convenient to review a model that was originally proposed to illustrate the origin of memory of the second type \( \mathcal{I} \). The model consists of two particles. The first particle is the visible one. We maintain the same dynamic rule as that established by Eq. \( \mathcal{I} \), though now we change the role of this particle from the generator of crucial events to the generator of pseudo-events. This means that we keep using the visible particle to generate events, and consequently the time series to study, with a different back injection rule, though, for the purpose of generating events that are not random. To do that, following Ref. \( \mathcal{I} \), we introduce a second particle to generate events, and consequently the time series to study, with a different back injection rule, though, for the purpose of generating events that are not random. To do that, following Ref. \( \mathcal{I} \), we introduce a second particle ruled by an equation of the same kind as Eq. \( \mathcal{L} \). However, while Eq. \( \mathcal{L} \) refers to events that we can monitor, now the second particle refers to hidden events. Therefore we refer to this particle as the invisible particle. In conclusion, we describe this model by means of the following set of equations:

\[
\begin{align*}
\frac{dx_{\text{vis}}}{dt} &= \alpha x_{\text{vis}}^2, \\
\frac{dx_{\text{invis}}}{dt} &= \beta x_{\text{invis}}^\gamma.
\end{align*}
\]

We assume that the dynamics of the visible particle are much faster than the dynamics of the invisible particle. Thus, the visible particle gets to the border and is injected back many times before the occurrence of the leading, or crucial, event. The crucial event occurs when the invisible particle reaches the border and is injected back randomly to a new initial condition in the interval \( I \). Before the occurrence of this crucial event, the visible particle has been injected back following a very simple deterministic prescription. In the earlier work of Ref. \( \mathcal{I} \) to check the efficiency of our method of analysis we have made the assumption that the initial condition is always the same, and it is changed randomly only when the invisible particle is injected back.

At this stage, we wonder if it is possible to distinguish the crucial events from the surrounding pseudo events; in particular we wonder if a statistical method of analysis exists that detects the waiting time distribution of the crucial events. The answer is positive, and can be found in the paper of Ref. \( \mathcal{I} \). First of all, we have to convert the time series into a diffusion process. According to the prescriptions of Ref. \( \mathcal{I} \), we evaluate the Shannon entropy of this diffusion process. This is why this technique of analysis is called Diffusion Entropy (DE) method. As is well known, the distance \( x \) travelled by the walkers is related to time by the relation \( x \propto t^\delta \), where \( \delta \) is termed diffusion index. If the diffusion process is not the sum of uncorrelated fluctuations, the scaling parameter \( \delta \) departs from the prescription of ordinary statistical mechanics, namely, from \( \delta = 0.5 \). The DE method is an efficient way to determine \( \delta \).

In the case where the visible events are not correlated, and the walking rule of Section II is adopted, the scaling \( \delta \), determined by means of the DE method, and the power index \( \mu \) of the waiting time distribution \( \psi(\tau) \) of Eq. \( \mathcal{I} \), are related by means of Eq. \( \mathcal{I} \). The violation of this crucial condition suggests that the events under observation are not genuine events, but rather pseudo-events, bearing, as a consequence of that, memory of the second type. Actually, the way to proceed is as follows. We evaluate numerically the waiting time distribution \( \psi_{exp}(\tau) \), by running the two-walkers model. The observation of visible events determines the waiting time distribution

\[
\psi_{exp}(\tau) = (\mu' - 1) \frac{T^\mu' - 1}{(T + \tau)^\mu'},
\]

We do not address here the interesting problem of establishing \( \mu' \) as a function of the parameter of the two-walkers model. This is not crucial for the discussion of this paper. Let us limit ourselves to noticing that \( \mu' > \mu \). The numerical results of Ref. \( \mathcal{I} \) show that \( \delta \) does not have anything to do with \( 1/(\mu' - 1) \). These numerical results rather prove the attractive fact that Eq. \( \mathcal{I} \) applies, but with \( \mu \) denoting the power coefficient of \( \psi(\tau^{[m]}) \), and \( \tau^{[m]} \) the time distance between two consecutive crucial events (the subfix \( m \) here stands for main events, in analogy with the definition used in \( \mathcal{I} \)). In other words, the scaling coefficient \( \delta \), detected by means of the DE method, reveals an important statistical property of crucial and invisible events. Let us summarize the procedure that we propose to detect the statistical properties of invisible and crucial events. First of all, we adopt an experimental view, and we derive from the real sequence under study the waiting time distribution \( \psi_{exp}(\tau) \), referring to the time distance between two consecutive events. If we find that the waiting time distribution is not exponential, we have a first indication of complexity. If the distribution is an inverse power law, we record the power law index, \( \mu' \). Then we use the DE method to measure the scaling parameter \( \delta \). The condition \( \delta = 1/(\mu' - 1) \) is a plausible indication that we are observing a time sequence of significant events. If \( \delta \) significantly departs from \( 1/(\mu' - 1) \), there are good reasons
to believe that

$$\mu = 1 + \frac{1}{\delta} \tag{13}$$

is a reliable indicator of the complexity of invisible and crucial events. It is important to note that this important conclusion is supported by the analytical treatment of Ref. [3]. The authors of Ref. [3] shows that in the long-time limit the memory of the pseudo-events is lost, and the process under study becomes equivalent to a Lévy flight, corresponding to the power index $\mu$ of the crucial events.

### IV. THE OMORI’S LAW AS A SOURCE OF PSEUDO-EVENTS

In this and in the next Section we review the work of Ref. [1] for the main purpose of proving that the results of this paper are a realization of the method for the search of invisible and crucial events illustrated in Section III. In Fig. 1 we report the sketch of the typical earthquakes frequency vs time in the catalog that we shall consider in the next Section. By $\tau_i = t_{i+1} - t_i$ we indicate the time interval between an earthquake and the next. Each peak of frequency (cluster) in figure includes the time location of a main-shock. The time interval between one peak and the next is reported in figure and is denoted by the symbol $\tau^{[m]}$, where the superscript $m$ stands for main-shock, since the main-shocks are the main event in the case of seismic fluctuations. According to the definition of crucial events given in Section I, we must make the assumption that two different $\tau^{[m]}$s are not correlated, i.e. that the correlation function is:

$$\langle \tau_i^{[m]} \tau_j^{[m]} \rangle = \left\langle \left( \tau^{[m]} \right)^2 \right\rangle \delta_{i,j}. \tag{14}$$

Note that with the symbol $\tau^{[m]}$ we denotes distances between two consecutive crucial events. Thus the corresponding waiting time distribution is equivalent to that distribution of Eq. (5).

The experimental determination of this distribution would imply the adoption of a way to identify the main-shocks. Although the geologists might suggest reliable criteria for their identification, for instance through the magnitude, with the use of our method we can determine their statistical properties without identifying them. Thus, the symbols $\tau^{[m]}$ denote distances between consecutive events that are assumed to be invisible. One of the models adopted to describe the time distribution of earthquakes is the Generalized Poisson (GP) model [7, 8, 9, 10, 11]. Basically the GP model assumes that the earthquakes are grouped into temporal clusters of events and these clusters are not long-range correlated: in fact the clusters are distributed at random in time and therefore the time intervals between one cluster and the next one follow a Poisson distribution. On the other hand, the *intra-cluster earthquakes are correlated* in time as it is expressed by the Omori’s law [12, 13], an empirical law stating that the main-shock, i.e. the highest magnitude earthquake of the cluster, occurring at time $t_0$ is followed by a swarm of correlated earthquakes (after-shocks) whose number (or frequency) $n(t)$ decays in time as a power law, $n(t) \propto (t - t_0)^{-p}$, with the exponent $p$ being very close to 1. The Omori’s law implies [14] that the distribution of the time intervals between one earthquake and the next, denoted by $\tau$, is a power law $\psi(\tau) \propto \tau^{-\delta}$. This property has been recently studied by the authors of Ref. [14] by means of a unified scaling law $\psi_{L,M}(\tau)$, the probability of having a time interval $\tau$ between two seismic events with a magnitude larger than $M$ and occurring within a spatial distance $L$. This has the effect of taking into account also space and extending the correlation within a finite time range $\tau^*$, beyond which the authors of Ref. [14] recover Poisson statistics. Let us discuss the GP model in the light of the general remarks of Sections II and III. The Poisson assumption about the distribution of time distances between one main-shock and the next is equivalent to assigning no memory to the geophysical process responsible for the main-shock. This conflicts with our definition of crucial events and with our conviction that the crucial events, as unpredictable as the time duration of a laminar region is, cannot be determined by erratic bath fluctuations. The GP model, if supported by the statistical analysis of data, would imply that our definition is not correct, and that crucial events can be generated also from within ordinary statistical mechanics. This would conflict also with the tenets of complexity, which seem to connect cooperation and inverse power law relaxation. In Section V we shall prove that the GP model must be dismissed. In fact, using the method of statistical analysis reviewed in this paper, it is shown [3] that the asymptotic scaling generated by the GP scaling would be $\delta = 0.5$, which in fact corresponds to the prescription of Section II, when $\mu > 3$. It is worth recalling that the Poisson condition sets the exponential decay of $\psi(\tau)$, and thus $\mu = \infty$. The statistical analysis of real data, discussed in Section V, will prove that $\delta = 0.94$, that the GP model is incorrect, and that we cannot rule out the possibility that the main-shocks fulfill our definition of crucial events.
FIG. 1: We report a schematic figure illustrating the typical earthquakes frequency vs time. In correspondence to each main-shock we observe a frequency peak determined by the after-shock swarm. The peaks decay according to the Omori’s law, see text. The horizontal dotted arrows indicate the time intervals $\tau_i$ between two consecutive main-shocks. The DE method gives information on the distribution of these time intervals.

V. DATA AND RESULTS

The catalog we have studied covers the period 1976-2002 in the region of Southern California spanning 20° N -45° N latitude and 100° W -125° W longitude [15]. This region is crossed by the most seismogenetic part of the San Andrea fault, which accommodates by displacement the primarily strike-slip motion between the North America and the Pacific plates, producing velocities up to 47 mm/yr [16]. The total number of recorded earthquakes in the catalog is 383687 and includes the June 28 1992 Landers earthquake (M = 7.3), the January 17 1994 Northridge earthquake (M = 6.7), and the October 16 1999 Hector Mine earthquake (M = 7.1). Geophysical observations point out that these large earthquakes have triggered a widespread increase of seismic events at remote distances in space and in time [17, 18]. The coupling of the sources of stress change (i.e. large earthquakes occurrence) and seismicity triggering mechanisms is a primary target of geophysical investigations, and, as shown below, is revealed by the DE analysis. In Fig. 2 we report the results of the DE method. The analysis was performed by setting $\xi(t) = 1$ when an earthquake occurs at time $t$ (independently of whether it is a main-shock or an after-shock), and $\xi(t) = 0$ if no earthquake happens. By means of the full circles we denote the entropy $S(t)$ as a function of time when all the seismic events of the catalog are considered (independently of their magnitude $M$). After a short transient, the function $S(t)$ is characterized by a linear dependence on $\ln t$. A fit in the linear region gives a value of the scaling parameter $\delta = 0.94 \pm 0.01$ at 95% of confidence level. We next consider only the earthquakes with magnitude larger than a fixed value $\bar{M} = 2.3, 4$. We see that, regardless of the value of the threshold $\bar{M}$ adopted, the function $S(t)$ is characterized by the same long-time behavior with the same slope. This indicates that we are observing a property of the time location of large earthquakes. This leads us to conclude that the time intervals between two large events fit the distribution of Eq.(5), with the value of $\mu$ related to $\delta$ through Eq.(9), $\mu = 2.06 \pm 0.01$. Our conclusion is also supported by other two numerical analyses based on different prescriptions to construct the diffusion process. The former rests on assuming $\xi(t)$ equal to the magnitude $M$ of the earthquake, at each time when an earthquake occurs. The latter sets with equal probability either $\xi(t) = 1$ or $\xi(t) = -1$ when an earthquake occurs [4]. Both the methods give the same exponent $\mu = 2.06 \pm 0.01$. In conclusion, the statistical analysis of real data rules out the GP model, which would conflict with our definition of crucial events. Thus, there is still room for the main-shocks to fit our definition of crucial events. The authors of Ref. [1] prove that, under the stationary condition, they do. Are these crucial events also invisible? This is a question of fundamental importance for the war against terrorism. The
answer is that the main-shocks are not quite invisible. For professional geologists it is possible to identify all of them. However, our method of analysis works, regardless of whether the crucial events are invisible or not. In fact, Fig. 2 (a) shows that the asymptotic properties of the entropy indicator are independent of the threshold $M$ adopted. Since the magnitude is a distinctive property of the main-shocks, we conclude that the results of our analysis do not imply that the main-shocks are identified. This is the reason of our conviction that our method can be successfully used even when the crucial events are quite invisible.

![Graph showing the Shannon entropy $S(t)$ of the diffusion process as a function of time (in minutes), in a logarithmic time scale. From top to bottom, the curves refer to all seismic events without considering the magnitude $M$ and to events with magnitude greater than $\bar{M} = 2, 3, 4$ respectively. The straight lines are plotted to guide the eye and have the slope $\delta = 0.94$.

(b) We plot the Shannon entropy $S(t)$ of the diffusion process as a function of time, in a logarithmic time scale for the GP and LR model. We plot also two straight lines with slopes $\delta = 0.5$ and $\delta = 0.94$, see text for further details.

VI. GENERALIZED POISSON (GP) AND LONG-RANGE (LR) MODEL

We now illustrate how the DE method works on two artificial earthquakes time series: the first generated by means of the GP model, and the second generated by a new model, the Long-Range (LR) model, that we propose as a better model to reproduce the properties of the catalog considered. In the LR model the earthquakes are grouped into temporal clusters, and, as in the GP model, the number of earthquakes in a cluster follows the Pareto law, i.e. a power law distribution with exponent equal to 2.5 [10]. The events within the same cluster are distributed according to the Omori’s law: the interval $\tau$ follows a power law with exponent $p = 1$. However, in the LR model the time distance $\tau^{[m]}$ between one cluster and the next follows a power law with exponent $\mu = 2.06$, rather than a Poisson prescription as in the GP. Notice that this value of $\mu$ is close to the border between stationary and non-stationary condition [4]. The two sequences have the same time length. We choose the number of clusters in order to have the same total number of earthquakes as in the real data [19]. The result of the DE on the artificial sequences is reported in Fig. 2(b). The GP model is characterized by a long-time behavior that, as expected, fits very well the prescription of ordinary statistical mechanics, with $\delta = 0.5$. The LR model yields the quite different scaling $\delta = 0.94$. It is also clear that the LR model yields a behavior qualitatively similar to that produced by the real data of Fig. 2 (a) as well as the same scaling parameter $\delta = 0.94$, while the GP fail reproducing both properties. Note that from the reasons why the DE method reveals the genuine statistical properties of the crucial events in this case, are different from
those justifying the efficiency of the method in the case of the model of Section III. In this case, the true reason seems to be that for large distances between one main-shock and next the pseudo-events tend to concentrate immediately after the last main-shock with a so low density immediately before the occurrence of the next as to create a condition where the number of pseudo-events is essentially independent of the length of the laminar region. In the case of the model of Section III, on the contrary, the number of pseudo-events is proportional to the length of the laminar region.

VII. CONCLUSIONS

It is the time for us to balance the detection of crucial and invisible events in general, with possible applications to the war against terrorism. Our definition of crucial events implies a strong departure from Poisson statistics. In the recent literature there is a general agreement about the fact that complex networks, including the terrorist network, are scale-free systems. The authors of another paper of these Proceedings show that there exists a connection between the scale-free condition and non-Poisson statistics. This seems to support the definition of crucial event adopted in this paper. On the other hand, there exists an interesting connection with the Small Words theory illustrated by Latora and Marchiori, a paper of these Proceedings for the same purpose. It is worth mentioning that a local version of the DE method can be applied to the timely detection of toxicants. In conclusion, the present paper belongs to a set of contributions to these Proceedings, which might bear benefits to a program of research to combat terrorism. As to the detection of invisible and crucial events for specific purpose of the war to terrorism, the procedure to follow depends on the data to analyze, and on the forms, under which they will be made available to the investigators. Nevertheless, with the present paper, we are convinced that at least the first few steps of the search for crucial and invisible events, take a clear shape. It seems to be evident what the first step of this procedure will be the evaluation of \( \psi_{\exp}(\tau) \), and of the corresponding power index, denoted by the symbol \( \mu' \) in this paper. The second step will be the evaluation of \( \delta \), by means of the DE method, and the comparison of \( \delta \) with \( 1/(\mu' - 1) \). If the two values do not coincide, and the difference is larger than the statistical error, this has to be thought of as a plausible indication that there are invisible and crucial events involved. Then, we shall have to decide whether or not recourse can be done to the simple prescription of Eq.(13) to establish the degree of complexity of the invisible crucial events. This will require further research work determined by the specific nature of the data to analyze.

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