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Efficient compensation of inter-channel nonlinear effects via digital backward propagation in WDM optical transmission

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Abstract: An advanced split-step method is employed for the digital backward-propagation (DBP) method using the coupled nonlinear Schrödinger equations for the compensation of inter-channel nonlinearities. Compared to the conventional DBP, cross-phase modulation (XPM) can be efficiently compensated by including the effect of the inter-channel walk-off in the nonlinear step of the split-step method (SSM). While self-phase modulation (SPM) compensation is inefficient in WDM systems, XPM compensation is able to increase the transmission reach by a factor of 2.5 for 16-QAM-modulated signals. The advanced SSM significantly relaxes the step size requirements resulting in a factor of 4 reduction in computational load.

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1. Introduction

Recent trends in optical communication are focusing on high data-rates as well as spectrally efficient systems in order to cope with the demands for capacity growth. Higher bit-rates per channel involve the deployment of high-order modulation formats, requiring increased SNR and hence higher power per channel. Alternatively, higher spectral efficiency also demands tightly spaced wavelength-division multiplexed (WDM) channels to optimize the operational bandwidth of optical amplifiers. The above scenario clearly leads to increased nonlinearity in the form of intra- and inter-channel effects respectively. Therefore, the mitigation or compensation of fiber impairments which involve Kerr nonlinearity becomes crucial to increasing transmission capacity [1].

In particular, techniques capable of compensate the joint effect of chromatic dispersion (CD) and nonlinearity have contributed to approach the maximum achievable fiber capacity, which eventually becomes limited by only non-deterministic noise sources. Such comprehensive compensation of fiber impairments is based first, on the coherent detection of the optical signal [2] and second, on the implementation of digital backward propagation (DBP) by means of digital signal processing. By reversing the optical transmission in the digital domain, DBP enables the compensation of any deterministic impairment provided that the channel characteristics are known. Pioneering works on pre- and post- compensation via DBP can be found in [3–5]. First experimental demonstration of post-compensation using coherent detection in orthogonal WDM (OWDM) environment was reported in [6] and single channel experiments were carried out in [7,8]. Post-compensation of polarization-division multiplexed signals using vectorial backward propagation has been reported in [9] for single channel and in [10] for WDM. Simpler approaches based on lumped compensation have been also analyzed, being only effective in regimes with very low dispersion [11] or regimes with ad hoc dispersion management [12]. A review of fiber impairment compensation including DBP can be read in [13].

One of the challenges for DBP is its high complexity. In particular, full impairment compensation of inter-channel effects via DBP requires high oversampling as well as short step sizes when solving the $z$-reversed nonlinear Schrodinger equation (NLSE). In [14] is shown that among inter-channel nonlinear effects, four-wave mixing (FWM) can be neglected in DPB for dispersive channels by incurring small penalty. In addition in [15] is shown that compensation of XPM together with the partial compensation of FWM can be implemented in a channel-by-channel basis by using a set of enhanced coupled NLSEs which reduces computational cost by together increasing step size and reducing sampling requirements.

In this paper, we present an advanced method for XPM post-compensation via DBP. Based on the method proposed by Liebrich et al. [16], an advanced split-step method for DBP is presented. This method consists in the factorization of the walk-off effect within the...
nonlinear step of the SSM. Simulation results are obtained for different WDM systems as well as for different channel spacing. First, XPM compensation is compared with SPM and CD compensation showing a remarkable improvement in terms of performance and transmission reach. Second, a rigorous analysis of the computational cost is carried out comparing the conventional and advanced split-step methods for XPM compensation.

2. XPM post-compensation using backward propagation

In a coherent detection system, a full reconstruction of the optical field can be achieved by beating the received field with a co-polarized local oscillator [2]. The reconstructed field will be used as the input for backward propagation in order to compensate the transmission impairments. Let \( E_m(t, z) \) be the complex envelope of the received \( m \)-th-channel field where \( m = \{1, 2, \ldots, N\} \) and \( N \) is the number of channels. By considering only incoherent nonlinear effects, i.e., SPM and XPM, the backward propagation of the WDM channels through the optical fiber is described by the following coupled nonlinear NLSEs [15,17].

\[
\frac{\partial E_m}{\partial z} + \frac{\alpha}{2} E_m + K_{1m} \frac{\partial E_m}{\partial t} + K_{2m} \frac{\partial^2 E_m}{\partial t^2} + K_{3m} \frac{\partial^3 E_m}{\partial t^3} + i\gamma \left( |E_m|^2 + 2 \sum_{q \neq m} |E_q|^2 \right) E_m = 0, \tag{1}
\]

where, \( K_{nm} (\beta_{2,3}) \) is the \( n \)-th-order dispersion parameter at frequency \( \omega_m \) [15]. Parameters \( \beta_{2,3} \) are the second and third-order dispersion whereas \( \alpha, \gamma \) and \( t \) are the absorption coefficient, nonlinear parameter and retarded time frame respectively. The system of coupled equations in Eq. (1) describes the backward evolution of the baseband WDM channels where dispersion, SPM and XPM are compensated.

The above system of equations is solved in the digital domain by the well-known Split-Step Method (SSM) [17,18]. This method relies on decoupling the linear and nonlinear contributions in Eq. (1) over a sufficiently short distance. In each short segment of size \( h \) (hereafter step size), linear and nonlinear parts are solved independently or in an uncoupled fashion. In order for this method to be accurate, the step size has to be short enough to ensure: (i) The solution of the linear part from \( z \) to \( z + h \) is not perturbed by the variations of the optical fields due to nonlinear effects and (ii) The solution of the nonlinear part from \( z \) to \( z + h \) is not perturbed by the variations of the optical fields due to linear effects. Under these conditions, the step size will be limited by the fastest of the above variations. We use uniform step distribution for DBP.

Typically, the linear step is solved in the frequency domain using efficient algorithms for both the direct and inverse Fourier Transforms.

\[
E_m(t, z + h) = F^{-1} \left[ F \left[ E_m(t, z) \right] H_m(\omega, h) \right], \tag{2}
\]

where the multi-channel linear transfer function is given by,

\[
H_m(\omega, h) = \exp \left[ i \beta_3 \left( \frac{(\omega - \omega_0 - 2\Delta\omega)^2}{2} + i \frac{\beta_3}{6} (\omega - \omega_0 - 2\Delta\omega)^3 \right) h \right]. \tag{3}
\]

The above approximation is valid provided that the spectral change induced by nonlinearity is weak along the step length. Fourier domain filtering requires block-by-block computation which can be efficiently implemented by the overlap-and-add or the overlap-and-save methods [19,20]. For the nonlinear step, i.e., by neglecting the linear terms in Eq. (1), the solution is given by,

\[
E_m(t, z + h) = E_m(t, z) \exp(i \phi_{m,\text{XPM}} + i \phi_{m,\text{XPM}}), \tag{4}
\]

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where,

\[
\phi_{m,SPM}(t,z+h) = \gamma \int_{z}^{z+h} |E_m(t,\hat{z})|^2 e^{i\alpha} d\hat{z}
\]

(5)

\[
\phi_{m,XPM}(t,z+h) = 2\gamma \int_{z}^{z+h} \left( \sum_{q\neq m} |E_q(t,\hat{z})|^2 \right) e^{i\alpha} d\hat{z}.
\]

(6)

In the typical split-step method, the above integrals are approximated by,

\[
\phi_{m,SPM}(t,z+h) = \gamma h_{\text{eff}} |E_m(t,z)|^2,
\]

(7)

\[
\phi_{m,XPM}(t,z+h) = 2\gamma h_{\text{eff}} \sum_{q\neq m} |E_q(t,z)|^2,
\]

(8)

where, \(h_{\text{eff}} = \exp(\alpha h) - 1/\alpha\) is the effective step size. In general, the above approximations are valid provided the envelopes are not significantly modified along the step size \(h\) due to dispersive effects. In particular, for the SPM phase shift, Eq. (7) is valid provided the intra-channel pulse broadening is small along the step size. The length scale of intra-channel pulse broadening is given by \(L_m = 1/|\beta_m B|^2\) where \(B\) is the baud-rate. With regard to the nonlinear phase shift due to XPM, the step size has to satisfy a stricter condition for the approximation in Eq. (8) to hold. Here, to properly evaluate the phase shift induced by channel \(q\) over channel \(m\), the dispersive walk-off induced by a different group delay has to be tracked. This delay between channels \(m\) and \(q\) occurs in a length scale given by the walk-off length, \(L_{\text{wo}} = 1/|\beta_m B \Delta \omega_{mq}|\) [14,17] where \(\Delta \omega_{mq}\) is the frequency difference. Typically, \(N\) is large and \(\min(L_m) << L_{\text{wo}}\) which makes XPM the limiting effect for the step size [14]. By grouping SPM and XPM contributions, the total nonlinear phase shift can be expressed as follows,

\[
\phi_q(t,z+h) = \sum_{q} |E_q(t,z)|^2 W_{mq}(h),
\]

(9)

where,

\[
W_{mq}(h) = \begin{cases} 
\gamma h_{\text{eff}} & \text{for } q = m \\
2\gamma h_{\text{eff}} & \text{for } q \neq m.
\end{cases}
\]

(10)

One way to relax the step size requirements for XPM compensation is to separate the effects of pulse broadening and walk-off in the nonlinear phase-shift calculation. For that, let us rewrite Eq. (6) by including the time delay caused by the dispersive walk-off, that is,

\[
\phi_{m,XPM}(t,z+h) = 2\gamma \int_{z}^{z+h} \left( \sum_{q\neq m} |E_q(t-d_{mq},\hat{z})|^2 \right) e^{i\alpha} d\hat{z},
\]

(11)

where \(d_{mq} = \beta_m (\omega_m - \omega_q)\) is the walk-off parameter between channels \(m\) and \(q\). By Fourier transforming Eq. (11), the following expression is obtained for the XPM phase shift, now in the frequency domain,

\[
\phi_{m,XPM}(\omega,z+h) = 2\gamma \int_{z}^{z+h} \sum_{q\neq m} F\left( |E_q(t,\hat{z})|^2 \right) e^{-i\omega d_{mq}} d\hat{z},
\]

(12)

where \(F\) represents the Fourier Transform. Now, the above integral can be approximated by,
\[
\phi_{\text{m,XPM}}(\omega, z + h) = 2\gamma \sum_{q=\pm \infty} F \left[ |E_0(t, \hat{z})|^2 \right] \exp\left( \frac{\alpha h + id_{mq} \omega h}{\alpha + id_{mq} \omega} \right)
\]

(13)

provided that the individual power spectra do not change significantly over the step size, which is now limited by the minimum of the intra-channel dispersion length and the XPM nonlinear length [14].

Under this formulation, the total SPM + XPM nonlinear phase shift can be expressed as follows,

\[
\phi_m(t, z + h) = F^{-1} \left[ \sum_{q} F \left( |E_q(t, z)|^2 \right) W_{mq}(\omega, h) \right].
\]

(14)

where \( W_{mq}(\omega, h) \) is now given by,

\[
W_{mq}(\omega, h) = \begin{cases} 
\frac{\gamma h_{\text{eff}}}{2} e^{-\alpha h/2} & \text{for } q = m \\
\frac{\gamma}{2} \frac{e^{\frac{(\alpha + id_{mq} \omega)h}{2}} - 1}{\alpha + id_{mq} \omega} & \text{for } q \neq m.
\end{cases}
\]

(15)

By comparing Eq. (9) and Eq. (14) it is clear that the latter approach, which requires an additional direct and inverse Fourier transform, presents an increased complexity per step. However, by factorizing the walk-off effect, the step size can be substantially increased in typical WDM scenarios, where the walk-off length is much shorter than both the nonlinear length and the intra-channel dispersion length. Therefore, remarkable savings in computation are expected.

In general, the symmetric version of the split-step method must be used in order to improve the algorithm efficiency [17,18]. Here, the nonlinear phase shift is calculated by using the value of the optical field in the mid-segment. In this case a correction factor has to be added to the filter, \( W_{mq} \) [16]. By performing a simple change of variable in Eq. (12), Eq. (15) now becomes,

\[
W_{mq}(\omega, h) = \begin{cases} 
\frac{\gamma h_{\text{eff}}}{2} e^{-\alpha h/2} & \text{for } q = m \\
2\gamma \frac{e^{\frac{(\alpha + id_{mq} \omega)h}{2}} - 1}{\alpha + id_{mq} \omega} e^{-\alpha h/2} & \text{for } q \neq m.
\end{cases}
\]

(16)

Figure 1 depicts a schematic for the implementation of the split-step method,
3. Simulation Results and discussion

A 100 Gb/s per channel (single polarization) 16QAM WDM system has been simulated using the VPI TransmissionMaker. The transmission system consists of ten spans of NZ-DSF fiber with a length of 100 km per span, a dispersion parameter of \( D = 4.4 \) ps/km/nm and a dispersion slope of \( D_s = 0.045 \) ps/km/nm\(^2\). The loss is 0.2 dB/km and the nonlinear coefficient is, \( \gamma = 1.46 \) 1/W/km. Fiber loss is compensated per span using Erbium-doped fiber amplifiers with a noise figure of 5 dB. Three WDM systems with 12, 24 and 36 channels spaced at 50 GHz have been simulated to evaluate the impact of the channel count on the post-compensation algorithm. Likewise a 12 channel system spaced at 100 GHz has been also simulated to assess the dependence on channel spacing. Forward transmission is simulated in VPI by solving the total scalar NLSE [15] where the entire WDM band is automatically up-sampled to properly account for third order nonlinear effects. The step-size used by VPI is chosen to keep the nonlinear phase-shift below 0.05 degrees. Raised-cosine filters are used for demultiplexing.

After propagation and coherent detection, each channel is sampled at 2 samples/symbol and backward-propagated using Eq. (1). Four different cases will be analyzed: Dispersion compensation only, compensation of SPM (by neglecting XPM in Eq. (1)), compensation of SPM + XPM using the conventional method and compensation of SPM + XPM compensation using the advanced method.

Figure 2 shows the baseline results after backward propagation when different effects are compensated. These results are obtained for a step size sufficiently short, where the Q-factor behaves asymptotically as shown in Fig. (4). Values in Fig. 2 are the Q-factor averages of the WDM channels. Each channel carries 1024 16-QAM symbols. The Q-factor is obtained from the constellation by averaging the standard deviations of the 16 constellation clusters. The 16-QAM Q-factor calculation has been tested with direct error counting of transmission over an additive white Gaussian noise (AWGN) channel in comparison with a Gaussian model [21] which predicts the following relation between the Q-factor and the symbol error rate:

\[
Q_{\text{SER}} = 2 \times \text{erfc} \left( \frac{10^{(y/20)}}{3} \right),
\]

(e.g., a Q-factor of 7.6 dB for a SER of \( 10^{-3} \)).
As shown in Fig. (2), the compensation of XPM significantly improves the performance with respect to dispersion compensation only. Regardless of the channel count, both optimum power and maximum $Q$-factors are increased by approximately 5 dB when inter-channel effects are compensated. In general terms, results show that compensation of SPM in a WDM environment is ineffective. This is because inter-channel effects are sufficiently strong to modify the optical fields in such a way that individual channel compensation is inefficient. This effect can be seen by comparing different channel counts. The larger the channel count, the larger the XPM impact and hence the smaller the improvement achieved by SPM compensation as it is shown in Figs. 2(A) and 2(C). Similarly, the effect can be appreciated when comparing different channel spacing, where the improvement for SPM compensation grows as the spacing grows due to the reduced impact of XPM. Therefore, it is important to remark that even for channels spaced at four times the baud rate, inter-channel effects have a strong impact on backward propagation which causes the poor performance of single-channel compensation.

Figure 2(A) also shows the improved performance of 100 GHz channel spacing as compared to 50 GHz channel spacing. Two effects explain such behavior. First, increased walk-off for the 100 GHz case mitigates the overall impact of XPM which, in turn, reduces the non-deterministic ASE-signal nonlinearities [22], which cannot be compensated using DPB. Second, larger spacing mitigates four-wave mixing (FWM) through phase mismatch, which is not included in Eq. (1) and hence, not compensated for. In [4,14], it is shown that compensation of FWM is only relevant in DSF fibers where WDM channels maintain a high degree of phase matching and high FWM conversion efficiency. In addition, in [14,15] it is
shown that FWM compensation is very costly in terms of computation as well as cumbersome in its physical-layer implementation, where phase-locked local oscillators must be used.

The benefit of XPM compensation in terms of performance is clear from results in Fig. 2. However, it is important to evaluate the benefits of DBP in terms of transmission reach. Figure 3 shows results comparing CD compensation and XPM compensation. For a similar $Q$-value (equivalent to an approximate SER of $10^{-3}$) XPM compensation leads to an increase in the transmission reach by a factor of 2.5.

![Fig. 3. Performance comparison of CD and XPM compensation for different transmission lengths. Results are obtained for the 24 channel WDM system.](image)

Figure 4 compares the step size requirements for XPM compensation using the conventional and the advanced split-step methods introduced in the previous section.

![Fig. 4. Step size for the advanced and conventional implementation of the split-step method: (A) 12, (B) 24 and (C) 36 channels respectively. For the 12 channels case, results are shown for channel spacing of 50 GHz (solid) and channel spacing of 100 GHz (dotted).](image)
The results correspond to the respective optimum powers (i.e., power value at the best Q-factor) obtained from Fig. 2 whereas vertical markers indicate the operational step size. This value is obtained by cubic interpolation of the simulation results and by choosing the step size value corresponding to a $Q$-value penalty of 0.25 dB with respect to the plateau value. The operational step size is chosen as a compromise between performance and computational load. The advantage of the walk-off factorization is clear in terms of step size. When comparing the advanced- and conventional-SSM, the step size is substantially increased when both channel count and channel spacing are increased. The step size can be increased by 11, 17 and 19 times for the 12, 24 and 36 channel cases respectively. In addition, an increase of 13 times is achieved for the 12 channel case when the channel spacing is increased from 50 to 100 GHz.

Focusing on the advanced-SSM results, Fig. 4(A) also shows a step size reduction as the channel spacing increases. Such reduction comes from the increased optimum power observed in Fig. 2 for the 100 GHz with respect to the 50 GHz. Since the dispersive walk-off has no effect on the step size for the advanced-SSM, the latter becomes limited by the nonlinear phase-shift per step, which in turn, depends on the power. To illustrate this effect, Fig. 5 shows the optimum step size as a function of the launching power per channel (recall that the results in Fig. 4 were obtained for the optimum power values).

Figure 5 clearly shows how the step size decreases with the optical power. Most importantly, results are similar for both channel spacing for the same power per channel, proving that the advanced-SSM step size is independent of the walk-off length, which depends on the channel spacing. The discrepancy observed at high power values for the 50 GHz case is because of the low Q-factor. Here, the high ASE-induced nonlinear noise saturates the optimum step size due to non-deterministic effects.

Together with the step size, it is important to compare the computation requirements for each method. For simplicity only the number of complex multiplications will be considered, neglecting the number of additions. Furthermore, considerations regarding the numeric representation (fixed point/floating point) will be ignored. By recalling the schematic diagram in Fig. 1, the following number of operations is involved in backward propagation for a block-length of $M$ samples:

- Intensity operator: $M$
- Filtering: $2(M + P) \log_2 (M + P) + (M + P)$
- Exponential operator (4th order Taylor expansion): $6M$
In general, the filter implementation in the frequency domain is done by the overlap-and-
add method. This method requires an overhead of \( P \) samples with a length larger than filter
length (group delay). Moreover, FFT/IFFT algorithms operate more efficiently if \( M + P \) is a
power of 2. The exponential operation can be implemented using Taylor expansion. The
4th order Taylor expansion of the exponential is:
\[
\exp(x) \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}
\]
where \( y = x \times x \). Hence, it requires 6 multiplications by saving the square \( y = x \times x \) in
memory. Computation of the exponential operator is sometimes done using a look-up table.

By following the steps shown in Fig. 1, the total number of multiplications per sample per
channel for each XPM compensation method is given by,
\[
OP_{XPM-a} = n_a \left[ 4(M_a + P_a) \log_2 (M_a + P_a) + (N+1)(M_a + P_a) + 8M_a \right] / M_a,
\]
\[
OP_{XPM-c} = n_c \left[ 2(M_c + P_c) \log_2 (M_c + P_c) + (M_c + P_c) + 9M_c \right] / M_c,
\]
where sub-indices \( a/c \) stand for advanced/conventional split-step methods and \( N \) is the
number of WDM channels. Recall that for the conventional case, the XPM module reduces to
one multiplication per channel since the XPM phase shift is equal for all the channels when
implemented as in [15]. The total number of steps for DBP \( n_{DBP} \) can be approximated by:
\[
\tau = 2\pi |\beta| (N-1) \Delta f \times h, \quad \text{which assuming a sampling rate} \ S, \quad \text{yields a filter length (overhead}
\text{for the overlap-and-add method) of:} \quad P = 2\pi |\beta| (N-1) \Delta f \times h \times S. \quad \text{Therefore, depending of}
\text{the step size} \ h \text{ the required overhead} \ P \text{ will determine the block length for each case. By}
\text{sampling at a frequency of} \ S = 50 \text{ GHz, Table 1 summarizes the number of computations}
\text{including SPM compensation. The number of operations for SPM is also given by Eq. (18)
\text{(when additions are neglected) for the corresponding step size and block-length for SPM. In}
\text{this case,} \ P = 2\pi |\beta| B \times h \times S.}

| Spacing [GHz] | 12 channels | 24 channels | 36 channels |
|---------------|-------------|-------------|-------------|
| \( h \) [km]  | 100         | 25.0        | 100         |
| \( n \) [steps/span] | 1          | 1           | 1           |
| \( P \) [samples] | 55         | 55          | 55          |
| \( M \) [samples] | 2^4        | 2^4         | 2^6         |
| \( OP \) \times 100 | 2.2        | 2.2         | 2.2         |

Table 1 shows the number of multiplications required for each method and each system.
The block size \( P+M \) has been chosen to minimize the number of operations. A factor of
around 4 in computation savings is obtained for each WDM system by using the advanced
XPM compensation scheme. The number of operation grows with the channel count due to
the increased complexity of the XPM module in Fig. (1), where the number of multiplications
scale with the number of channels. With regard to channel spacing, savings are increased for the 100 GHz spacing with respect to the 50 GHz case.

The results summarized in Table 1 have been obtained for the respective optimum power values. Therefore, the results correspond to the maximum achievable $Q$-factor for each method and WDM system. It is also interesting to obtain the computational cost per dB of $Q$-factor improvement. Figure 6 shows the number of operations as a function of the $Q$-factor improvement with respect to the CD compensation case. Each point corresponds to a certain value of the channel power as shown in the figure. The OP values are obtained from Eq. (17) for a $M + P$ value of $2^n$ and the corresponding optimum step sizes for each power value. Note that in a rigorous way, the block-length should be optimized for each power value since the filter length scales with the step size.

![Figure 6](image)

Fig. 6. Computation results as a function of the $Q$-improvement provided by the advanced SSM for XPM compensation. The power values per channel are also shown for each point. Results are shown for the 50 GHz spacing case.

Figure 6 shows the required number of operation for a target $Q$-improvement. The minimum improvement shown in the figure corresponds to the case where 1 step/span is used. Here, almost a 1.8 dB improvement is obtained which outperforms the 0.6 dB improvement for one step/span SPM compensation. In this case, XPM compensation increases the number of operations by a factor of 2, which comes from the extra FFT/IFFT operations required for the advanced SSM.

3. Conclusion

An efficient implementation of the split-step method for the compensation of XPM via digital backward propagation has been reported. From the physical point of view, the results demonstrate that compensation of intra-channel effects becomes inefficient in WDM systems even with large channel spacing (up to 4 times the baud-rate). Also, XPM compensation significantly improves the system performance resulting in a 5 dB improvement in the $Q$-factor as well as in a 2.5 times longer transmission distance. From a computation complexity point of view, XPM compensation can be efficiently implemented using an advanced split-step method. This method is based on the factorization of the dispersive walk-off effect between channels, which leads to a remarkable increase in DPB step size. The advanced SSM yields a reduction of the computational load by a factor of 4 with respect to the conventional SSM.