Deducing $L_{\text{fermion}}$ at $M_{\text{GUT}}$ using Low Energy Data

or

Towards a Theory of Fermion Masses

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1 Introduction

In the last 20 years we have accumulated an enormous amount of data on elementary particles and their interactions. This data serves two purposes: to fix the phenomenological parameters of the Standard Model [SM] and to verify that the SM is an excellent description of nature. It is our goal to understand the origin of these many arbitrary parameters. In this talk we consider a supersymmetric [SUSY] SO(10) grand unified theory [GUT]. We present a straightforward procedure, incorporating a general operator analysis, which allows us to use low energy data to determine the fermionic sector of the theory at the GUT scale \(^1\). In what follows we first review the status quo, i.e. the low energy data [LED]. We then review the evidence for SUSY GUTs, and focus on the virtues of the particular group SO(10). Following a discussion of our dynamical principles, we present a general operator analysis for $L_{\text{fermion}}$, i.e. the fermionic sector of the GUT theory. We argue that all fermion masses and mixing angles can be described with a minimum of 5 arbitrary parameters in the Yukawa sector of the theory at the GUT scale, $M_G$. Including the parameter $\tan \beta$, the ratio of Higgs vevs present in any SUSY theory, we thus obtain a 6 parameter description of fermion masses and mixing angles, leading to 8 predictions. Finally, we present preliminary results. These preliminary results are encouraging and eminently testable. Notwithstanding, the power of our analysis is in the paradigm whereby the use of LED, hand in hand with symmetry conditions, allows us determine the fermion sector of the theory at $M_G$.

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2 Status Quo

There are 18 arbitrary parameters in the SM with 13 of these in the fermion sector of the theory. Let us briefly review the state of our knowledge of these 18 parameters since they play a central role in what follows. In Table 1 we list the 18 parameters of the SM along with their experimental and/or theoretical uncertainties.

Table 1. The 18 parameters of the Standard Model.

| Parameter | Uncertainty | Comments |
|-----------|-------------|----------|
| $\alpha, \sin^2 \theta_W$ | < 1/2 % | high accuracy |
| $\alpha_s(M_Z)$ | $\sim$ 10% | less certain – scale dependent |
| $m_e, m_\mu, m_\tau$ | < 1/3% | high accuracy |
| $m_c, m_b$ | $\sim$ 4% | less certain |
| $m_u/m_d, m_s/m_d$ | ? | chiral Lagrangian – ambiguous |
| $(m_u + m_d)/2$ | ? | QCD sum rules |
| $|V_{cd}| \approx |V_{us}|$ | $\sim$ 1.5% | fair accuracy |
| $|V_{cb}|, |V_{ub}/V_{cb}|$ | $\sim$ 20% | poorly known |

| $m_t, m_H$ | $m_t = 150 + 19 + 15 - 24 - 20$ | $60 < m_H < 1000$ in SM |
| $m_t, m_H$ | $m_t = 131 + 23 + 5 - 28 - 5$ | $60 < m_H < 150$ in MSSM |
| $m_t$ | $m_t > 108$ GeV | Fermilab data |
| $M_Z$ | < .025% | high accuracy |
| J (Jarlskog invariant) | $\sim$ 30% | uncertainty in $B_K$ |

Some of these parameters are known to high accuracy, these include $\alpha, \sin^2 \theta_W, m_e, m_\mu, m_\tau, M_Z$. A few are known with fair accuracy, $|V_{cd}|, m_c, m_b$. Finally, the following are poorly known: $\alpha_s(M_Z), m_u, m_d, m_s, |V_{cb}|, |V_{ub}/V_{cb}|, m_t, m_H$ and the Jarlskog invariant measure of CP violation, J.

Our knowledge of $\alpha_s(M_Z)$ suffers predominantly from theoretical uncertainties associated with renormalization scale ambiguities. The experimentally allowed range is now $\alpha_s(M_Z) = .118 \pm .007$, where the errors probably underestimate the theoretical uncertainties.

Light quark masses are determined using chiral Lagrangians and QCD sum rules. In the chiral Lagrangian approach, light fermion masses are characterized by their transformation properties under the $SU(3)_L \times SU(3)_R$ chiral symmetry. The mass matrix, $M$, transforms as $(3, \bar{3})$. However, as shown by Kaplan and Manohar, this is if we assume that neutrinos are massless. Incorporating neutrino masses leads to an additional 9 parameters, 3 masses and 6 mixing angles.
\[ M' = M + \kappa \text{det}(M)M^{-1} \] with \( \kappa \) an arbitrary constant, has the same transformation property. This leads to a large theoretical uncertainty in light quark masses.

The weak mixing angles \(|V_{ub}|\) and \(|V_{cb}|\) suffer from both large theoretical and/or experimental uncertainties. The theoretical uncertainties are evidenced by the different model dependent calculations of B decay. It is hoped that the application of heavy quark effective field theory to the exclusive semileptonic B decays will reduce these uncertainties, but this requires better statistics. The latest experimental result from \( B \rightarrow D^* l\nu \) is \[^5|V_{cb}| = 0.50 \pm 0.08 \pm 0.07\] where the first error is from statistics and the second is from extrapolation uncertainties. The ratio \(|V_{ub}/V_{cb}|\), determined from inclusive B decay, has large model dependences. Recently the experimental results from CLEO II changed significantly from previous measurements by both Argus and CLEO. The latest results give \[^50.38 \leq |V_{ub}/V_{cb}| \leq 0.97\]. This is a factor of two smaller than previous results and the change was significantly larger than any of the experimental errors for any particular model. We will just have to wait and see if this settles down with more data.

The top and Higgs masses are only known from the radiative effects they have on electroweak parameters. The latest data, analyzed by Langacker \[^2\], gives the results presented in Table 1, where MSSM denotes the minimal supersymmetric standard model. The difference between the SM and MSSM result is the requirement of a lighter Higgs boson in the MSSM.

Finally, J suffers from strong interaction uncertainties associated with the so-called bag constant, \( B_K \). The value of \( J \times B_K \) can be derived given the experimental value for \( \epsilon_K \), all quark masses, and the magnitudes of all Kobayashi-Maskawa [KM] elements. Thus, the uncertainty in J is limited by the uncertainty in \( B_K \) which is of order 30%.

I have briefly reviewed the status quo, since some of these parameters will be used as inputs to fix the fundamental parameters in the fermion mass matrices and the others test the theory. For example, the biggest uncertainties in our prediction of the top mass comes from the uncertainties in \( m_b \) and \( \alpha_s(M_Z) \). Reducing these uncertainties would make this prediction much tighter.

### 3 SUSY GUTs and the virtues of SO(10)

- **SUSY GUT**

  Given two parameters the fine structure constant, \( \alpha_G \), at the GUT scale, \( M_G \), one determines the three low energy parameters, \( \alpha, \sin^2 \theta_W \) and \( \alpha_s \), all evaluated at some renormalization scale, \( \mu \), which for convenience we might choose to be \( M_Z \). In actuality, we use the two best determined parameters, \( \alpha \) and \( \sin^2 \theta_W \), to fix \( \alpha_G \sim 1/25 

\[ M_G \sim 10^{16} \text{GeV} \] and predict (in a SUSY GUT) \( \alpha_s(M_Z) = 0.125 \pm 0.002 \pm 0.009 \) for a central value of \( m_t = 138 \text{GeV} \).\[^2\] The errors take into account uncertainties in \( m_{H^\pm}, m_t \)

\[^5\]In a non-SUSY GUT the prediction is \( \alpha_s(M_Z) \sim 0.07 \), which is inconsistent with the data.
and estimates of threshold corrections at both $M_G$ and the weak scale. This result is in remarkably good agreement with the data. For a heavier top, the central value for $\alpha_s$ decreases by several percent. This prediction assumes a supersymmetric desert, i.e. the only threshold between $M_Z$ and $M_G$ occurs below $\sim 1$ TeV due to the new spectrum of states encountered in the MSSM.

In any GUT, the number of fundamental parameters in the gauge sector of the theory decreases by one and only in a SUSY GUT does the resulting prediction agree with the low energy data [LED]. The powerful assumption of a SUSY desert has an equally important consequence. The LED becomes a window into physics at the GUT scale, i.e. measurements at the weak scale gives us information about the physics at $M_G$.

- **Virtues of SO(10)**

A single family of fermions fits into one irreducible representation — i.e. $16_i \supset \{u_i, d_i, e_i, \nu_i\}$ with $i = 1, 2, 3$ labelling the three families. We take the 3rd family to be the top family.

The two Higgs doublets required in the MSSM fit into one irreducible representation — i.e. $10 \supset \{H, H', H_3, H'_3\}$ where $H, H'$ are weak doublets necessary for weak symmetry breaking and giving masses to quarks and leptons and $H_3, H'_3$ are color triplet Higgs which must get mass of order $M_G$ to avoid rapid proton decay. We will return to this point shortly.

In order to accomplish the GUT scale symmetry breaking we must have additional representations including $\{45, 16, \bar{16}, \cdots\}$, where, for example, a $16$ and $\bar{16}$ vev can break SO(10) to SU(5) and the 45 vev can then break SU(5) to SU(3) $\times$ SU(2) $\times$ U(1). This may happen at the same scale, $M_G << M_P$, or at two separate scales, where the first occurs at a scale $v_{10}$ such that $v_5 = M_G << v_{10} << M_P$.

We have not considered other possible representations which may be relevant for GUT symmetry breaking, such as 54, 126, etc. We shall now assume that only the 45 plays a crucial role in the generation of fermion masses. It is thus necessary to elaborate the possible directions the 45 vev may point in the two dimensional space of U(1) subgroups of SO(10) which commute with SU(3) $\times$ SU(2) $\times$ U(1). Although there are only two orthogonal directions in this space, we nevertheless consider the following 4 possible symmetry breaking vevs —

\[
\langle 45 \rangle_X = v_{10} e^{i\alpha_X} X \\
\langle 45 \rangle_Y = v_5 e^{i\alpha_Y} Y
\]

\[
\langle 45 \rangle_{B-L} = v_5 e^{i\alpha_{B-L}} (B - L) \\
\langle 45 \rangle_{T_{3R}} = v_5 e^{i\alpha_{T_{3R}}} T_{3R}
\]  

where we have explicitly represented them as two groups of two orthogonal vevs. We consider all four since two of these vevs (one in each group) are well motivated.
X, in Eq. (1), is the U(1) direction which leaves SU(5) invariant. This is why we have taken the magnitude of the vev to be $v_{10}$, whereas all the others are taken to be $v_5$ since they do not commute with SU(5).

B − L, in Eq. (2), just measures baryon number minus lepton number. It can play a crucial role in splitting the weak doublet and color triplet Higgs multiplets, i.e. solving the hierarchy problem. The Higgs doublets carry zero B-L whereas the triplets have non-zero B-L. Thus if the Higgs in the 10 gets mass by coupling to this 45, only the color triplets will acquire mass at the scale $M_G$. Hence, this vev is expected to be a necessary ingredient in any complete SO(10) model which also solves the hierarchy problem.

4 Dynamic Principles

Let us now discuss the dynamical principles which guide us towards a theory of fermion masses.

0. At zeroth order, we work in the context of a SUSY GUT with the MSSM below $M_G$.

1. We use SO(10) as the GUT symmetry with three families of fermions \( \{16_i, \ i = 1, 2, 3\} \) and the minimal electroweak Higgs content in one 10. Using SO(10) symmetry relations allows us to reduce the number of fundamental parameters.

2. We will assume that there are family symmetries which enforce zeros of the mass matrix, although we will not specify these symmetries at this time. As we will make clear shortly, these symmetries will be realized at the level of the fundamental theory defined at $M_P$.

3. Only the third generation obtains mass via a dimension 4 operator. The fermionic sector of the Lagrangian thus contains the term $\mathcal{L}_f \supset A O_{33} \equiv A \ 16_3 \ 10 \ 16_3$. This term gives mass to t, b and $\tau$. It results in the symmetry relation $\lambda_t = \lambda_b$ = $\lambda_\tau$ \equiv A at $M_G$. This relation has been studied before by Ananthanarayan, Lazarides and Shafi [ALS] \(^6\) and using $m_b$ and $m_\tau$ as input it leads to reasonable results for $m_t$ and $\tan \beta$.

4. All other masses come from operators with dimension $> 4$. As a consequence, the family hierarchy will be related to the ratio of scales above $M_G$. We will show shortly how to understand the higher dimension operators in terms of an effective field theory at $M_G$, obtained by integrating out states with mass $> M_G$.

5. [Predictivity requirement] We demand the minimal set of effective fermion mass operators at $M_G$ consistent with the LED.
Let us now consider the general operator basis for fermion masses. Let $L_{\text{fermion}}$ include operators of the form

$$O_{ij} = 16_i \cdot \cdot \cdot n \cdot 10 \cdot \cdot \cdot m \cdot 16_j$$

where

$$\cdot \cdot \cdot n = \frac{M^k_G \cdot 45_{k+1} \cdot \cdot \cdot 45_n}{M^t_P \cdot 45_{X}^{n-t}}$$

and the 45 vevs in the numerator can be in any of the 4 directions, X, Y, B – L, T3R discussed earlier.

We said that such operators can be considered as the result of integrating out states with mass $> M_G$. For example, you can convince yourself that an operator of the form $O_{22} = 16_2 \cdot 10 \frac{45_y \cdot M_G}{45_2} 16_2$ is generated by the tree graph of Figure 1, assuming $v_{10} \gg v_5 \sim M_G$. Note it is at this level in the fundamental theory at $M_P$ that additional family symmetries are needed to enforce zeros in the mass matrix. It is also trivial to evaluate the Clebsch-Gordon coefficients associated with any particular operator since the matrices $X, Y, B – L, T3R$ are diagonal. Their eigenvalues on the fermion states are given in Table 2.

### 5 Operator analysis

Our goal is to find the minimal set of fermion mass operators consistent with the LED. With any given operator set one can evaluate the fermion mass matrices for up and down quarks and charged leptons. One obtains relations between mixing angles and ratios of fermion masses which can be compared with the data. It is easy to show, however, without any detailed calculations that the minimal operator set consistent with the LED is given by

$$L_{\text{fermion}} \supset O_{33} + O_{23} + O_{22} + O_{12}$$

or

$$O_{33} + O_{23} + O'_{23} + O_{12}$$

It is clear that at least 3 operators are needed to give non-vanishing mass to all charged fermions, i.e. $det(m_a) \neq 0$ for $a = u, d, e$. That the operators must be in
Table 2. Quantum numbers of the four 45 vevs on fermion states.
Note, if $u$ denotes a left-handed up quark, then ${\bar u}$ denotes a left-handed charge conjugate up quark.

|       | X | Y | B − L | $T_{3R}$ |
|-------|---|---|-------|----------|
| $u$   | 1 | 1/3 | 1     | 0        |
| ${\bar u}$ | 1 | -4/3 | -1     | -1/2     |
| $d$   | 1 | 1/3 | 1     | 0        |
| ${\bar d}$ | -3 | 2/3 | -1     | 1/2      |
| $e$   | -3 | -1 | -3     | 0        |
| ${\bar e}$ | 1 | 2 | 3     | 1/2      |
| $\nu$ | -3 | -1 | -3     | 0        |
| ${\bar \nu}$ | 5 | 0 | 3     | -1/2     |

the [33, 23 and 12] slots is not as obvious but is not difficult to show. It is then easy to show that 4 operators are required in order to have CP violation. This is because, with only 3 SO(10) invariant operators, we can redefine the phases of the three 16s of fermions to remove the three arbitrary phases. With one more operator, there is one additional phase which cannot be removed. A corollary of this observation is that this minimal operator set results in just 5 arbitrary parameters in the Yukawa matrices of all fermions, 4 magnitudes and one phase. This is the minimal parameter set which can be obtained without solving the remaining problems of the fermion mass hierarchy, one overall real mixing angle and a CP violating phase. We should point out however that the problem of understanding the fermion mass hierarchy and mixing has been rephrased as the problem of understanding the hierarchy of scales above $M_G$. Moreover given any particular operator set which fits the low energy data we would be obliged at some later time to construct the complete GUT theory, including symmetries which forbid additional operators and a consistent description of symmetry breaking scales. We leave this problem for future analysis.

For now we shall describe the detailed analysis of the “22” texture. Models with “22” texture give the following Yukawa matrices at $M_G$ –

$$\lambda_a = \begin{pmatrix} 0 & z_a C & 0 \\ z'_a C & y_a E e^{i\phi} & x_a B \\ 0 & x'_a B & A \end{pmatrix} \quad (5)$$

†As of this writing, we have found no models with “23” texture which fit the LED.
with the subscript $a = \{u, d, e\}$. The constants $x_a, x'_a, y_a, z_a, z'_a$ are Clebsches which can be determined once the 3 operators ( $O_{23}, O_{22}, O_{12}$) are specified. Recall, we have taken $O_{33} = A 16_3 10 16_3$, which is why the Clebsch in the 33 term is independent of $a$. Finally, combining the Yukawa matrices with the Higgs vevs to find the fermion mass matrices we have 6 arbitrary parameters given by $A, B, C, E, \phi$ and $\tan \beta$ describing 14 observables. We thus obtain 8 predictions. We shall use the best known parameters, $e, \mu, \tau, c, b, |V_{cd}|$, as input to fix the 6 unknowns. We then predict the values of $u, d, s, t, \tan \beta, |V_{cb}|, |V_{ub}|$ and $J$.

We now show how (within the context of our dynamic principles) we can use the LED to guide us towards the theory of fermion masses at $M_G$. We search for all operators ( $O_{23}, O_{22}, O_{12}$) with dimension $\leq 10$ which fit the data. Using a coarse-grained analysis, it is easy to show that just 9 models (with one caveat) may fit the data. We now describe this analysis.

5.1 3rd generation fit — $O_{33}$

Using the values of $\alpha$ and $\sin^2 \theta_W$ evaluated at $M_Z$, we obtain $\alpha_G, M_G$ and $\alpha_s(M_Z)$. There is a theoretical uncertainty in this prediction due to unknown threshold corrections at both $M_G$ and the weak scale. There is also a 10% experimental uncertainty in $\alpha_s(M_Z)$. Using arbitrary threshold corrections we can obtain any experimentally allowed value of $\alpha_s$. We thus allow for all values of $\alpha_s(M_Z) = .12 \pm .01$ self-consistently by introducing arbitrary threshold corrections.

The analysis for the third generation follows —

$$m_b = \frac{v}{\sqrt{2}} A \cos \beta \left( \frac{\eta_b}{S_b} \right)$$  \hspace{1cm} (6)  
$$m_\tau = \frac{v}{\sqrt{2}} A \cos \beta \left( \frac{\eta_\tau}{S_\tau} \right)$$  \hspace{1cm} (7)  
$$m_t = \frac{v}{\sqrt{2}} A \sin \beta \left( \frac{1}{S_t} \right)$$  \hspace{1cm} (8)

where $v = 246GeV$ and the terms in parentheses are renormalization group factors which are implicit functions of both $A$ and $\alpha_s(M_Z)$. The numerator takes into account renormalization from $M_G$ to $m_b$ or $m_\tau$, and the denominator takes into account the running from $M_G$ to $M_Z$. We use two loop renormalization group equations.

Using $m_b = 4.25 \pm .1GeV$ and $m_\tau = 1.7841GeV$ (these are running masses $m(m)$) as input for a given value of $\alpha_s = .118$ (for example) we obtain $A$ from the relation

$$\frac{m_b}{m_\tau} = \left( \frac{\eta_b S_\tau}{\eta_\tau S_b} \right) (\alpha_s(M_Z), A).$$

Plugging this value of $A$ into the expression for $m_\tau$ we then obtain $\tan \beta$. We find $\tan \beta \sim 59$ for $m_b = 4.34GeV$. Now using the values of $A$ and $\tan \beta$ in the expression for $m_t$ we find $m_t(pole) \sim 188GeV$. In general, we find values of $\tan \beta = 54 \pm 5$
\( m_t = 185 \pm 15 \text{GeV} \). Both \( m_t \) and \( \tan \beta \) increase for increasing values of \( \alpha_s \) or decreasing values of \( m_b \). Thus \( m_t = 170 \text{GeV} \) and \( \tan \beta = 50 \) is obtained for \( \alpha_s(M_Z) = .110, m_b = 4.35 \text{GeV} \). Similar results have been obtained previously by ALS\(^6\). Note they find lower values of \( m_t \) since they allow for values of \( \alpha_s \) which are lower than those presently admissable by the data.

5.2 2nd generation — \( O_{22} \)

Let us now consider the 2nd generation. We have 4 relations which must be satisfied by the LED.

\[
|V_{cb}| \approx |x_u - x_d| \frac{B}{A} \sim 1/20 \tag{9}
\]

\[
\frac{m_{\mu}}{m_{\tau}} \approx \left| \frac{y_e}{A} e^{i\phi} - x_e x'_e \frac{B^2}{A^2} \right| \sim 1/17 \tag{10}
\]

\[
\frac{m_{s}}{m_{b}} \approx \left| \frac{y_d}{A} e^{i\phi} - x_d x'_d \frac{B^2}{A^2} \right| \sim 1/25 \tag{11}
\]

\[
\frac{m_{c}}{m_{t}} \approx \left| \frac{y_u}{A} e^{i\phi} - x_u x'_u \frac{B^2}{A^2} \right| \sim 10^{-2} \tag{12}
\]

We have written these equations using the parameters at \( M_G \), ignoring for the moment small renormalization group corrections. Using the first relation we see that the ratio \( B/A \sim 1/10 \), assuming Clebsch of order 1. The 2nd and 3rd relations thus require \( E/A \sim 1/10 \). The last relation then requires that the Clebsch, \( y_u << 1 \). Finally, the relation \( m_s = m_{\mu}/3 \) at \( M_G \), first suggested by Georgi-Jarlskog\(^7\), must be incorporated, since it is in good agreement with the LED. We thus conclude that the Clebsch, \( y_u \), (including RG corrections) should approximately be in the ratio\(^\parallel\)

\[
y_u : y_d : y_e = << 1/3 : 1 : 3.
\]

The Clebsch, \( y_a \), are derived from the operator \( O_{22} \). We have searched over all dimension 5 and 6 operators to find solutions to the above Clebsch ratios. We find 6 solutions —

\[
\begin{align*}
16_2 \ (45_X) & \ 10 \ \left( \frac{45_{B-L}}{45_X} \right) \ 16_2 \\
16_2 \ \frac{1}{45_X} & \ 10 \ (45_{B-L}) \ 16_2 \\
16_2 \ (45_X) & \ 10 \ (45_{B-L}) \ 16_2 \\
16_2 & \ 10 \ \left( \frac{45_{B-L}}{45_X} \right) \ 16_2
\end{align*}
\]

\(^\parallel\)If the Clebschs are not of order 1 as we assumed, then it is possible that the 4 relations may be satisfied with some fine-tuning and a completely different ratio of Clebschs. We have not pursued this possibility further. This is our one caveat.
However, note that all solutions give the same ratio of Clebschs —

\[ y_u : y_d : y_e = 0 : 1 : 3. \]  \(\text{(14)}\)

### 5.3 1st generation — \(\text{O}_{12}\)

We can now show that the operator \(\text{O}_{12}\) is unique. The first two generations satisfy the relations —

\[
\frac{m_u}{m_d} \approx \frac{z_u}{z_d} \frac{z'_u}{z'_d} \frac{m_e}{m_{\mu}} \frac{\eta_{\mu}}{\eta_{e} \eta_{s}}
\]  \(\text{(15)}\)

\[
\frac{m_u}{m_d} \approx \frac{z_u}{z_d} \frac{z'_u}{z'_d} \frac{m_s}{m_{c}} \tan^2 \beta \left( \frac{S_d}{S_u} \right)^2
\]  \(\text{(16)}\)

\[
|V_{cd}| = \left| \frac{z_d}{z'_d} \frac{m_d}{m_s} - \sqrt{\frac{z_u}{z'_u} \frac{m_u}{m_c}} e^{-i\phi} \right|
\]  \(\text{(17)}\)

The first relation is satisfied if

\[ z_d \approx z'_d. \]

This relation is satisfied if the Clebsch is derived from an SU(5) invariant vev, i.e. \(45_X\). Since \(\tan \beta\) is large, the second relation requires

\[
\frac{z_u}{z'_u} \approx \left(\frac{1}{3}\right)^6 \text{ or } 7.
\]

Finally, the last relation requires

\[ z_d \approx z'_d. \]

The unique operator which satisfies the above 3 relations is

\[
\text{O}_{12} = 16_2 \left( \frac{45_X}{M_P} \right)^3 10 \left( \frac{45_X}{M_P} \right)^3 16_2.
\]  \(\text{(18)}\)

### 5.4 \(\text{O}_{23}\)

We have now determined, using simple arguments, all but one of the operators. The charged fermion mass matrices are given by —

\[
U = \begin{pmatrix}
0 & C & 0 \\
C & 0 & x_u B \\
0 & x'_u B & A
\end{pmatrix}
\]  \(\text{(19)}\)
\[
D = \begin{pmatrix}
0 & -27 C & 0 \\
-27 C & E e^{i\phi} & x_d B \\
0 & x'_d B & A
\end{pmatrix}
\]
\quad (20)

\[
E = \begin{pmatrix}
0 & -27 C & 0 \\
-27 C & 3 E e^{i\phi} & x_e B \\
0 & x'_e B & A
\end{pmatrix}
\]
\quad (21)

With this form for the mass matrices, the KM element \( V_{cb} \) satisfies the relation —

\[
|V_{cb}| \approx \chi \sqrt{\frac{m_c}{m_t}} \sqrt{\frac{S_u}{S_t S}} \sim 0.058 \chi.
\]

where \( \chi \equiv \frac{|x_u - x_d|}{\sqrt{|x_u x'_d|}} \) and the last term results from using central values of the input parameters.

Experimentally, we have an upper bound on \( |V_{cb}| \leq 0.054 \). We thus require that the function of Clebschs, \( \chi \), satisfy \( \chi < 1 \).

We have searched over all dimension 5 and 6 operators for \( \chi < 1 \). We find 9 possible models with only 3 different values of \( \chi = \frac{2}{3}, \frac{5}{6}, \) or \( \frac{8}{9} \). The 9 models are listed below.

\( \chi = 2/3 \)

\begin{align*}
1 & \quad 16_2 \ (45_Y) \ 10 \ \left( \frac{1}{45_X} \right) \ 16_3 \quad (23) \\
2 & \quad 16_2 \ (45_Y) \ 10 \ \left( \frac{45_{B-L}}{45_X} \right) \ 16_3 \quad (24) \\
3 & \quad 16_2 \ \left( \frac{45_Y}{45_X} \right) \ 10 \ \left( \frac{1}{45_X} \right) \ 16_3 \quad (25) \\
4 & \quad 16_2 \ \left( \frac{45_Y}{45_X} \right) \ 10 \ \left( \frac{45_{B-L}}{45_X} \right) \ 16_3 \quad (26)
\end{align*}

\( \chi = 5/6 \)

\begin{align*}
5 & \quad 16_2 \ (45_Y) \ 10 \ \left( \frac{45_Y}{45_X} \right) \ 16_3 \quad (27) \\
6 & \quad 16_2 \ \left( \frac{45_Y}{45_X} \right) \ 10 \ \left( \frac{45_Y}{45_X} \right) \ 16_3 \quad (28)
\end{align*}

\( \chi = 8/9 \)
This is as far as we can get with our coarse-grained search. We must now take the 9 distinct models and test the predictions. We are presently in the midst of a complete renormalization group analysis, obtaining predictions as a function of the input parameters, $e, \mu, \tau, c, b, |V_{cd}|$, and $\alpha_s(M_Z)$. For now we present some preliminary results.

6 Preliminary results

The results in Table 3 are for inputs $m_c = 1.23 GeV, m_b = 4.34 GeV, |V_{cd}| = .221$ and $\alpha_s(M_Z) = .118$ for 4 different models. The model numbers are defined in Eqs. (23 - 31)

Table 3. Results for models 3, 6, 8, and 9.

|        | 3    | 6    | 8    | 9    |
|--------|------|------|------|------|
| $m_d[MeV]$ | 6.8  | 6.9  | 6.9  | 7.3  |
| $m_u/m_d$  | .82  | .79  | .80  | .79  |
| $m_s/m_d$  | 24.8 | 24.1 | 24.9 | 22.4 |
| $m_t$      | 188  | 188  | 188  | 188  |
| $\tan \beta$ | 59   | 59   | 59   | 59   |
| $|V_{cb}|$  | .039 | .049 | .052 | .052 |
| $|V_{ub}/V_{cb}|$ | .065 | .064 | .066 | .068 |
| $J \times 10^5$ | 1.8  | 2.9  | 3.3  | 3.7  |

Note, $|V_{cb}|$ and $J$ are sensitive to the value of $\chi$ and so can distinguish between the 3 types of models. With better data, $|V_{cb}|$ will distinguish between these choices. At present we can use $\epsilon_K$, the CP violating parameter in $K$ decay, to distinguish these models through its dependence on $J$. We find that models with $\chi = 2/3$ tend to give too little CP violation. Finally, we see that $m_u/m_d$ and $m_s/m_d$ will also be useful in constraining the models. Complete results will be presented in an upcoming publication. 

7 Conclusion

We have presented a straightforward method for trying to understand the origin of fermion masses. There are four main ingredients —

- The assumption of a SUSY GUT with its SUSY DESERT implies that the Low Energy Data becomes a window into the physics at the GUT scale.

- The assumption of symmetries (SO(10) plus family symmetries) with a general operator analysis allows us to reduce the number of fundamental parameters in the theory.

- With the above assumptions, we obtain predictive theories of fermion masses. Given 6 inputs, $e, \mu, \tau, c, b$, $|V_{cd}|$, we find 8 predictions for $u, d, s, t, \tan \beta, |V_{cb}|, |V_{ub}|$ and $J$.

- This method allows us to use the LED to systematically find the theory of fermion masses at $M_G$.

A final note: there is no apriori reason to believe that this scheme should work, but if it does work, i.e. if our results agree with the LED, then perhaps we will have learned something about the physics at the GUT scale. For example, consider the set of possible models 1 – 9 in Eqs. (23 - 31). Each one has the vev $45X$ in the denominator. This can only occur if this vev gives mass $>> M_G$ to some fermion. When this fermion is integrated out in the effective theory at $M_G$, the vev then appears in the denominator of these higher dimension operators. Thus fermion masses and mixing angles at low energies may tell us about the hierarchy of symmetry breaking scales above $M_G$.

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