Discontinuous Shear Thickening (DST) fluids show a remarkable effect where the suspension behaves like a thin liquid at low shear rates, but when sheared faster, resistance to flow increases discontinuously with shear rate [1, 2]. This effect has been observed in a large variety of concentrated suspensions of hard, frictional, non-attractive particles [2, 3]. DST fluids also show strong impact response such as the ability of a person to run on the surface of a pool filled with a suspension of cornstarch and water [2, 4]. Current models of shear thickening based on lubrication hydrodynamics [5, 6], dilatancy [7–10], and inertial displacement [11] fail to explain a person’s ability to run on the surface and impact response in general. In our experiments we discover a transition at a critical impact velocity in DST fluids above which fronts of solid-like regions are generated in the fluid. The collision of these fronts with a solid boundary leads to a shear thickening transition which has not been previously predicted or reported. The large stresses generated above the shear thickening transition are enough to hold up a person’s weight, and have potential applications for protective materials [12, 13].

Since these strong impact response phenomena happen in cornstarch and water suspensions they have been assumed by the scientific community to be the result of shear thickening [3]. Indeed, qualitative observations show that it is possible to run on the surface of cornstarch and water suspensions at higher velocity but one sinks at a lower velocity or after stopping [2, 4] (See Supplementary Video 1). Qualitative observations also show that this effect happens only at high packing fractions similar to DST [14]. Current models for DST can explain some aspects of steady state shear response, for example successfully predicting that cornstarch and water can support stresses of up to \( \sim 10^3 \) Pa [3]. However, a simple estimate shows that the stress on a person’s foot from their weight is \( \sim 4 \cdot 10^4 \) Pa. Thus, DST models would predict that a person would always sink in cornstarch and water suspensions. Other shear thickening models predict even lower stresses [2, 5].

Recently a model, referred to as the ‘added mass’ model, has been developed for impact response in which a dynamically jammed region forms ahead of the impactor in the fluid with a front which propagates away from the impactor [11]. This front is characterized by a sharp velocity gradient [13, 10] and a packing fraction gradient in which the material behind the front has a higher packing fraction [10]. It was shown that the front velocity and width diverge at a critical packing fraction similar to the case of DST [16]. It is presumed that this dynamically jammed region holds together like a solid [11]. However, evidence for the solid like nature of this region is lacking. In this model the impact response of the suspension comes from an added mass effect in which suspension mass is added to the dynamically jammed region which slows down to conserve momentum [11]. The added mass model requires an impact velocity \( V_I \gtrsim 6 \) m/s to generate enough stress hold up a person’s weight [11]. The minimum velocity required goes up for fluid height \( H \lesssim 0.3 \) m as there will be insufficient mass to add before the front reaches the boundary [23]. However there are observations of walking on the surface of cornstarch and water for \( V_I = 0.3 \) m/s and \( H = 0.1 \) m (See Supplementary Video). Thus the model fails to explain these striking observations. The regime of small \( H \) is particularly important for impact protection applications where thin layers of protective material are desired [12, 13].

The purpose of this letter is to explain the strong impact response of DST fluids. We hypothesize that if a dynamically jammed region spans from the impactor to the boundary then it could temporarily support a load like a solid, perhaps strong enough to support a person running on the surface. To address this we perform impact experiments with an impactor of diameter \( d \) driven at constant velocity \( V_I \) far enough to see fronts interacting with the boundary, in contrast to experiments which probe the bulk [11, 16], but not so close to the boundary (\( \sim 3 \) mm) to be in the regime studied in Ref. [17]. Our experiments are at \( V_I \) faster than quasistatic, so that fronts exist, but at speeds slow enough that inertial effects are negligible [18, 19] (including added mass [17] and high Mach number [20, 21]). This velocity regime is where steady state DST occurs, but surprisingly quantitative impact experiments have not been reported in this regime.

In order to visualize the front we first performed qualitative experiments in a quasi two-dimensional setup (see Methods). The image clearly shows the emergence of a front, roughly semi-circular in shape beneath the impactor shown in Fig. 1a (See Supplementary Video 2). Cracks form and always remain behind the propagating front indicating that the material behind the front is solid-like in character. Close inspection shows that the lighter region behind the front is the result of voids opening up between the particles and the wall, an expected result of dilation. Dilation from particle contacts is known to trigger DST in steady state shear [2]. This image illustrates solid-like properties of the dynamically jammed region as proposed in Ref. [11].

To quantify the stresses and front propagation of the dynamically jammed region, we use a three-dimensional...
FIG. 1: Measurements of the dynamically jammed region. (a) A quasi two-dimensional impact showing the generation of a front ahead of the impactor. Fractures in the region behind the propagating front suggest it is solid-like in nature. (b) Schematic of the three-dimensional experiment. This allows for simultaneous measurement of forces and imaging of the top, bottom and side surfaces. (c) A representative stress $\sigma$ vs. depth $z$ plot at $V_f = 20 \text{ m/s}$, $H = 4.2 \text{ cm}$ and $d = 12.7 \text{ mm}$, showing a delay $z_1$ before stress is measured. The dashed line is a fit to a constant slope.

setup to avoid the perturbing effect of sidewalls. Since Fig. 1a indicates that visible structural changes accompany front propagation, we perform optical imaging of the top, bottom and side surfaces as shown in Fig. 1b, while simultaneously measuring the stress $\sigma$ and depth $z$ of the impactor (See Methods).

In Fig. 1(c) we show a representative graph of the measured stress $\sigma(z)$. Although the impactor hits the suspension surface at depth $z = 0$ the graph shows clearly that there is no stress $\sigma$ within our resolution of 1600 Pa until a depth $z_1$ is reached. After this delay the stress increases beyond $10^5 \text{ Pa}$, enough to hold a person’s weight.

Current models for impact are insufficient to generate the large stresses or the delay that we observe. The maximum stress for the parameters in Fig. 1(c) due to buoyancy is $\sigma = \rho g z \sim 400 \text{ Pa}$ [19], where $\rho$ is the suspension density and $g$ is the acceleration due to gravity. The stress due to lubrication forces in the hydrocluster model [24], commonly used for shear thickening, has an upper bound of $\sigma \sim 20 \text{ Pa}$ [2]. For inertial displacements, $\sigma \sim \rho V_f^2 \sim 0.5 \text{ Pa}$ [18, 19]. Using the ‘added-mass’ model [11], we get a stress of $\sigma \sim 500 \text{ Pa}$ [29]. None of these models allow for a delay in stress response. Hence current models are insufficient to predict the behaviour shown in Fig. 1(c).

We hypothesize that the delays before the sharp increases in stress observed in Fig. 1(c) are due to the time it takes for the fronts to reach the opposite (bottom) boundary. To test this hypothesis, we divide the $\sigma(z)$ curve into distinct regimes. Shortly after the onset of stress at $z_1$, a region of constant slope of stress $\sigma$ with $z$ is generally observed, indicated by the straight line in Fig. 1(c). We define $z_2$ as the depth where the slope increases above this constant value [30].

If $z_1$ and $z_2$ are delays before fronts collide with the boundary, then this would correspond to front propagation velocities given by $V_{F_i} = H/T_i$, where $i = 1, 2$ and the delay time $T_i = z_i/V_f$. We plot the front speeds $V_{F_i}$ for a range of fluid depths $H$ in Fig. 2(a) and 2(b). The collapse of the data confirm that the delay is due to a front travel time. This data does not collapse when plotted in terms of distance from the bottom surface which would correspond to a structure of constant size in front of the impactor [17, 23], nor does the second front collapse with a scaling corresponding to a reflection of front 1 from the bottom surface. We fit each $V_{F_i}$ to a power law, given by $V_{F_1} \sim V_f^{1.47 \pm 0.11}$ and $V_{F_2} \sim V_f^{1.07 \pm 0.03}$. Our measured exponents are inconsistent with previously known scalings for inertial and granular models of front propagation under impact [11, 16, 26, 27].

In order to probe the flow structure we perform particle tracking at the bottom surface (see Methods). We find a delay time before any tracer particle displacement is observed followed by a sharp onset of radial displacement away from the impactor shown in Fig. 2(c). The flow onset times are consistent with $T_1$ within a relative error of 8 milliseconds between the force measurements and imaging. This shows that the onset of flow occurs when Front 1 reaches the bottom. Since there is no lag between the flow onset and $T_1$, this shows that the time for the force to travel back to the force sensor is less than the uncertainty in time measurement. The correlation with the flow indicates that this front is the same as observed in Ref. [15].

We also observe structural change on the bottom surface, appearing as a ring shaped pattern shown in the left lower inset to Fig. 2(c). The ring appears after a delay time measured as a sharp change in light intensity in the camera. This delay time is consistent with $T_2$ within a relative error of 8 milliseconds. The correspondence between stress, structure and particle flow is also seen for other $H$ and $d$ (not shown). The consistency of tim-
The data are for $d = 12$ mm. The state diagram reveals a critical point at $V_C$.

FIG. 2: Collision of fronts with the boundary. The propagation velocities $V_F$ for fronts 1 and 2 in panels (a) and (b) respectively. The dashed lines show fits to power laws. The collapses confirm that the delays observed in Fig. 1c are due to fronts propagating to the bottom boundary. (c) The delay times $T_1(♦)$ and $T_2(●)$ before increases in stress and the delay times before particle tracer motion(■) and the onset of ring structure(▼) at the bottom. The data are for $H = 4.2$ cm and $d = 12.7$ mm. The lower inset shows the ring structure. The consistency of timings shows the observed stresses coincide with the structural changes at the boundary. The upper right inset shows a schematic of the dynamically jammed region spanning the system, as suggested by the structural changes observed at the top and bottom boundary.

We observe dilation at the top surface similar to previous observations [22], while no structural changes are observed on the side surfaces. These observations suggest a columnar structure spanning from the impactor down to the bottom surface, shown by a schematic in the upper right inset in Fig. 2(c). This dynamically jammed region spanning between solid boundaries can then support a load like a solid. With this localized structure the stress is not limited by the free surface as in steady state DST where the stresses are more homogeneous [9].

Using these experimental observations we can now put together a state diagram, shown in Fig. 3, illustrating the combination of $V_I$ and $z$ for which the dynamically jammed region spans to the boundary. This is done by using the data in Fig. 2 and combining the equations $T_i = z_i/V_I$ and $V_F = H/T_i$ to get $z_i/H = V_I/V_F$. At high impact velocities $V_I$ and large depths $z$ one or more fronts reach the boundary. We observe that when the impactor velocity $V_I$ is below a critical velocity $V_C$ no fronts are detected. The extrapolation of the fits for $V_F$ in Fig. 3 gives two transitions where $V_F = V_I$, below which the respective fronts would be unphysical. In Fig. 3 we see that $V_F$ and $V_C$ intersect at the maximum depth $z = H$, above which would also be unphysical and which shows that the extrapolation is consistent with the observations. We also performed experiments at lower impactor velocities $V_I$ and larger fluid heights $H$ than the data shown in Fig. 2(c), but did not observe any collision of the fronts with the boundary. This can now be explained by the state diagram which demonstrates that not only is there a critical velocity for the existence of fronts but whether a front collides with the boundary depends on both $z/H$ and $V_I$.

Another feature of the state diagram is that it allows one to delineate the region where added mass effects are found [11, 15]. As the added mass effect requires fronts it only occurs for $V_I > V_C$. Since it is a bulk effect it may only happen at depths $z/H$ before any fronts reach
the boundary. Hence these conditions bound the region where added mass effects occur, shown in Fig. 3. While the effect of added mass can be comparable to boundary effects for higher \( V_I \) and/or larger \( H \) [11], in the parameter space shown in Fig. 3 this contribution to stress \( (\sigma \lesssim 500 \text{ Pa}) \) is \( 10^3 \) times smaller than the stresses observed when fronts collide with the boundary.

To show how the fronts reaching the boundary lead to shear thickening we show \( \sigma(V_I) \) in Fig. 4(a). The stress is computed at a constant depth \( z = 3 \text{ cm} \). At lower velocities the measured stress can be attributed to buoyancy which is \( \rho gz \sim 400 \text{ Pa} \). A discontinuous shear thickening behaviour is observed, with a sharp increase in stress above a transition velocity. The large stresses above the transition are sufficient to hold up a person’s weight, and the magnitude of stress is \( 10^2 \) times higher than steady state DST which demonstrates that this phenomena is distinct from steady state DST.

To understand the origin of this shear thickening transition we show a state diagram in Fig. 4(b) similar to Fig. 3, but corresponding to the data shown in Fig. 4(a). The intersection of the line demarcating front-boundary collision with \( z = 3 \text{ cm} \) gives a transition velocity which is indicated by the vertical line in Fig. 4. This vertical line coincides with the onset of the shear thickening transition in Fig. 4(a), showing that the shear thickening transition is a result of the fronts reaching the boundary. The existence of the shear thickening transition is independent of the \( z \) at which the stress is computed, although there is a shift of the transition velocity for a different \( z \) as suggested by Fig. 3.

The critical velocity \( V_C \) at \( z = H \) is analogous to a first order critical point (like a gas-liquid phase transition) in the sense that \( \sigma(V_I) \) is discontinuous across the transition but this discontinuity vanishes as \( V_I \to V_C \) and \( z \to H \). In that limit there is no room to compress the dynamically jammed region after collision with the opposite boundary. This is a state transition rather than an equilibrium thermodynamic phase transition since the system is highly dissipative.

According to Fig. 3, to see a discontinuous shear thickening transition requires crossing horizontally from left to right into the state where the dynamically jammed region spans to the boundary in order to go from the low stress to the high stress region. This transition can only happen if the depth \( z/H \) at which the front collides with the boundary is a decreasing function of \( V_I \). In the added mass model \( V_F \propto V_I^{1/6} \), which gives a state boundary at constant \( z/H \) and thus no discontinuous shear thickening transition or a critical velocity. On the other hand, any inertial mechanism can have a continuous (but not discontinuous) shear thickening transition when the inertial forces overcome buoyant or other forces, similar to steady state shear thickening [9, 18].

In conclusion, we demonstrate the existence of two distinct fronts in DST fluids under impact, whereas only one front has been observed previously [11, 12]. Our measurements show that when a front reaches the boundary creating a solid like region spanning the system, it is strong enough to support a person’s weight, thus allowing running or even walking on the surface. The first collision of a front with the boundary leads to a discontinuous shear thickening transition which is distinct from steady state DST in that the stresses are \( \sim 10^2 \) times higher. This shear thickening transition under impact not only requires a critical velocity to generate fronts but also has a depth dependence for the fronts to reach the boundary. Prior to these observations, a discontinuous shear thickening transition under impact had not been predicted by existing models nor observed at running velocities despite

![FIG. 4: Discontinuous shear thickening transition when a front reaches the boundary.](image-url)
popular belief that running on cornstarch suspensions is a result of DST. The relationship between the shear thickening observed in impact experiments and steady state DST remains to be explored in future work.

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I. MATERIALS AND METHODS

A. Quasi Two-Dimensional setup

This consists of a cell 238 mm wide, 155 mm high and 8 mm thick, shown in Fig. 1(a). The cell is filled with a suspension of cornstarch and water and the solid rectangular block is pushed into the fluid from above at a constant velocity $V_I = 100 \text{ mm/sec}$. The cell is made of transparent acrylic and is lit from behind such that light passes through the 8 mm thick cell. The suspension for the quasi two-dimensional experiments was prepared at a weight fraction of 56% and humidity of 46%.

B. Three Dimensional setup

The impact response is studied in a specially designed transparent acrylic container which has a square base of length 15 cm and filled to a height $H$ with the suspension shown in Fig. 1(b). Simultaneous force measurements and optical imaging of the top, bottom and side surfaces are performed with a high speed camera (Phantom M110) and mirrors. The entire structure containing the suspension is placed in an Instron E-1000, a dynamic materials tester that allows simultaneous force measurements. A cylindrical impactor of diameter $d$ is driven at constant velocity $V_I$ into the suspension from a starting position typically 5 mm above the suspension surface to a final position typically within 10% of the bottom of the suspension. The stress on the impactor is measured as $\sigma = 4F/\pi d^2$. The maximum variation on the velocity during a run is 11% with a position resolution of 1µm. The timing resolution on the force sensor (attached to the impactor) is 1 millisecond. Since the force measured by the sensor includes that required to accelerate the mass of the impactor, we calibrate the force sensor by using a control experiment in air to subtract out this inertial contribution to the measured force as a function of the acceleration. As a result of the inertial correction the reported force comes only from the impact with the suspension. The force sensor attached to the impactor has a resolution of 0.15 N with an additional uncertainty of 0.06 N from the inertial correction.

C. Sample Preparation

The initial suspension of cornstarch (Tate and Lyle) and tap water was prepared at a weight fraction of 55% (±0.5%) at a humidity of 15% and controlled temperature of 21 ± 1°C. We aim to get a consistent packing fraction and thus mechanical response from all suspensions. However, the packing fraction is sensitive to relative humidity which varied from 8 – 22% on different days. Hence when making new samples we perform inclined plane tests, adding either cornstarch or water until a target relaxation time of 3.5 s ± 0.5 s was reached which was the relaxation time of the initial suspension. This protocol gives a method of obtaining consistent packing fractions without a controlled humidity environment.

D. Particle Tracking

Particle tracking is done with iron-oxide particles 100µm in diameter as tracers settled on the bottom surface with a spatial resolution of 200µm.

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[28] This velocity is calculated using the model parameters for the jammed solid front in Ref. [11] and a time between steps of 0.3 s. To reach peak deceleration, the added mass must be comparable to the person’s mass. In pools of depth $H \lesssim 0.3m$ there is not enough fluid to provide this added mass.

[29] We observe added mass effects at $V_I \geq 200mm/s$ which reaches up to a maximum stress of $\sim 50kPa$ at a velocity of 600 mm/s, which we subtract from the measured stress to obtain the appropriate $z_1$ for front calculations.

[30] We perform smoothing of $\sigma(z)$ over a range of 10 milliseconds before taking a mid-point derivative. The delay depth $z_2$ is indicated by the point where the slope changed by 10% from the fit value. Performing a larger, e.x. 20 millisecond smoothing did not change $z_2$ within error.