Neutral meson tests of time-reversal symmetry invariance

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The laws of quantum physics can be studied under the mathematical operation $T$ that inverts the direction of time. Strong and electromagnetic forces are known to be invariant under temporal inversion, however the weak force is not. The $\text{BaBar}$ experiment recently exploited the quantum-correlated production of pairs of $B^\pm$ mesons to show that $T$ is a broken symmetry. Here we show that it is possible to perform a wide range of tests of quark flavour changing processes under $T$ in order to validate the Standard Model of particle physics covering $b$ to $u$, $d$, $s$, and $c$ transitions as well as $c$ to $u$, $d$ and $s$ transitions using entangled $B$ and $D$ pairs created in $T(4S)$ and $\psi(3770)$ decays. We also note that pseudoscalar decays to two spin one particle final states provide an additional set of CP filter bases to use for $T$ violation tests.

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Weak decays are known to violate the set of discrete symmetries charge conjugation ($C$), spatial invariance otherwise known as parity ($P$) [1], CP [2] which distinguishes between matter and antimatter, and time-reversal ($T$) [3,4]. Historically there have been a number of incorrect claims of testing $T$ violation through triple product asymmetries of four body decays of $K$, $B$, and $D$ decays. Such approaches are not able to test $T$ symmetry invariance as one is not able to experimentally identify an asymmetry of $T$ conjugate transitions such as that indicated in Eq. 1. One can find a recent review of triple product asymmetry measurements using the cor general nomenclature in Ref. [1]. In this paper we identify a new set of orthogonal CP filter bases that can be used for $T$ violation tests. Using these, along with $K_0^*/K_0$ basis filters identified in Ref. [6], we outline a programme of symmetry invariance tests in $B$ and $D$ decays. We also illustrate how these measurements relate to the SM weak force quark mixing mechanism given by Cabibbo-Kobayashi-Maskawa (CKM) matrix in the SM [10, 11], where the Kobayashi-Maskawa (KM) phase $\delta_{KM}$ is responsible for both $CP$ and $T$ violation in the SM.

Experimental evidence so far supports the hypothesis that the overall symmetry $CPT$ is conserved, for example see [12,14] for the results of tests using $B$ meson decays. The most significant hint for a non-conservation of $CPT$ comes from $\text{BaBar}$, however the $CPT$ violation search using sidereal time evolution of di-lepton decays only has a significance of $2.8 \sigma$ from expectations of being consistent with the Standard Model of particle physics (SM) [13]. For $CPT$ to be conserved, the level of $CP$ violation has to be balanced by ‘just enough’ $T$ violation so that these two violations cancel each other out to preserve $CPT$ symmetry. While this discussion may seem to be of academic interest it is worth recalling that $CPT$ is conserved in locally invariant field theories such as the SM [17]. A number of scenarios that have been proposed in order to work toward a theory of quantum gravity can naturally manifest $CPT$ violation. The details of how this might happen depend on the specifics of such models (see for example [18,20]), and as the weak force has a lack of respect for discrete symmetries one should be motivated to test the behaviour of this force with respect to $T$ (and other symmetries) in as many types of weak decay as possible to experimentally verify if the results are found to be consistent with expectations. Such a programme of measurements would parallel the time-dependent $CP$ violation measurements performed by $\text{BaBar}$, $\text{Belle}$, the Tevatron and LHC experiments since the start of this millennium.

The observable used to study $T$ symmetry invariance parallels that of a time-dependent or time-integrated $CP$ asymmetry: one constructs an asymmetry of $T$ conjugate processes from some state $|1\rangle$ to another $|2\rangle$ state. Under $T$ the time ordering of these two states is reversed, i.e.

$$A_T = \frac{P(|1\rangle \rightarrow |2\rangle) - P(|2\rangle \rightarrow |1\rangle)}{P(|1\rangle \rightarrow |2\rangle) + P(|2\rangle \rightarrow |1\rangle)},$$

and $T$ symmetry invariance is violated if $A_T \neq 0$. As strong and electromagnetic processes conserve $T$, one needs to identify weak transitions that satisfy Eq. 1. It is not trivial to identify measurable systems that satisfy this condition, and the real issue that remains is to identify the sets of interesting states, that can be used for such a test. The decay $|1\rangle \rightarrow |2\rangle$ requires a $T$ conjugate partner $|\rangle \rightarrow |1\rangle$) that is experimentally distinguishable.

The key to identifying pairs of measurable $T$ conjugate states is highlighted in Ref. [8]. One studies the ensemble of entangled [or Einstein-Podolsky-Rosen (EPR) correlated [21,22] neutral meson pairs, and reconstructs an experimentally distinguishable double tagged $T$ conjugate pair of transitions. If one follows this procedure using decays into only one orthonormal basis it is possible to construct an asymmetry that is both $T$ and $CP$ violating (for example this parallels the concept of a Kabir asymmetry measured in kaon decays [22]). One can go further than this, as proposed by Bernabeu et al. [8] one can use any two different orthonormal basis pairs to classify the decays of the neutral meson pairs in terms of $T$ and $CP$ asymmetries that are different observables. For example natural filter basis choices include the dete
nation of the flavour of one of the $B$ decays via transitions to a flavour specific final state (e.g. a semi-leptonic decay of a neutral pseudoscalar meson), and another natural choice is that of a neutral meson decaying into a $CP$ tag final state. Bernabeu et al. proposed the comparison of four sets of processes which satisfy the above criteria. Here we write these combinations generally in terms of $B$ and $D$ mesons denoted by $P$, where $\pm$ subscripts refer to the $CP$ eigenvalue of the $CP$ filter mode, and the flavour filter mode is denoted by the decay of particle or antiparticle using the nomenclature of $P (\not P)$:

\[
\begin{align*}
\not P_0 & \to P_+ \quad \text{vs} \quad P_+ \to \not P_0, \\
\not P_0 & \to P_0 \quad \text{vs} \quad P_0 \to \not P_0, \\
\not P_0 & \to P_+ \quad \text{vs} \quad \not P_0 \to \not P_0, \\
\not P_0 & \to P_0 \quad \text{vs} \quad P_0 \to \not P_.
\end{align*}
\]

For example in the $\text{BaBar}$ measurement the pairing $\not P_0 \to P_-$ corresponds to an identified $\not P_0 \to (\ell^+ X$ and $P_0 \to \ell^- X$) is used to construct an approximately orthonormal $CP$ filter basis, where final states involving a $K_0^0$ and $\Lambda^0$ meson can be compared to perform a number of previously unforeseen measurements of $T$ violation in $B$ mesons at the $\psi(3770)$. This can be generalised as follows: the entangled states $K_{S,L}^{0,0}$, $\not K_{S,L}^{0,0}$, $A_{0,1}$, $A_{1,0}$ are experimentally more challenging to identify, resulting in lower signal efficiency (event yield) and lower purity than the $T$ conjugate filter with a $K_0^0$.}

We note that it is possible to identify other experimentally distinguishable $CP$ filter bases that, unlike $\{K_0^0, K_0^0, K_0^0, K_0^0\}$, are exact. These arise from the set of $B$ and $D$ decays to final states with two spin-one particles, for example pairs of vector ($V; J^P = 1^-)$ or axial vector ($A; J^P = 1^+$) particles. If one performs a transversity analysis of a $B \to VV$ final state, one can experimentally resolve $CP$ even and $CP$ odd parts as $A_0$ and $A_{odd}$ (even) and $A_{odd}$ (odd). Here $A_0$ is the longitudinal amplitude and $A_{odd}$ are components of the transverse amplitude. In terms of the helicity basis $A_0$ is helicity $\lambda$ zero and the transverse components are admixtures of $\lambda = \pm 1$. Typically the experimental differences between even and odd basis states $\{A_{0,1,0,1}\}$ of a $V, AV$ or $AA$ is smaller than that of the $\{K_0^0, K_0^0\}$ basis. One could also use a $CP$ admixture to perform a $T$ violation test, however one would have the added complication of having to distinguish between $CP$-even and $CP$-odd components of said admixture in order to identify the orthonormal $CP$ filter basis, and also have the additional complexity in the time-dependence. By using these additional $CP$ filter bases one will be able to perform a number of previously unforeseen measurements of $T$ violation in $B$ and $D$ systems.

The previous discussion is quite general, the thing that remains is to identify the set of pairs of decay channels that can be used to compute $A_T$, beyond those already studied. The measured asymmetry parameters $\Delta S_T^\pm$ or the underlying parameters $S_T^\pm_m$, corresponding to the $CP$ filter sinusoidal oscillation amplitudes, provide a set of measurements of angles of unitarity triangles in the SM. For example the $b \to c\bar{s}s$ measurement performed by $\text{BaBar}$ corresponds to four measurements of the angle $\beta$ (or $\phi_1$). This can be seen from the value of $\lambda = (q/p)^2 A//A$, where $q$ and $p$ are related to neutral meson mixing and

one can compute the asymmetry observable of Eq. (11) as a function of the proper time difference $\Delta t$ between the decay of the first meson $t_1$ and that of the second meson $t_2$. This relies on the quantum coherence of the wavefunction of the entangled state over macroscopic scales which, at least in the case of the $\Upsilon(4S)$, has been tested experimentally by $\text{Belle}$ [23].

As mentioned above, distinct $T$ violation measurements require the use of two pairs of orthonormal bases that can be experimentally identified. BaBar and Belle have had great success in experimentally separating particle and anti-particle states using flavor tagging algorithms designed to select flavour specific final states so one can simply follow existing methods for flavor filtering. The $CP$ filter pair basis that has been used until now involves neutral $B$ decays to final states including a charmonium meson ($c\bar{c}$) with a $K_0^0$ ($CP$ odd) or a $K_0^0$ ($CP$ even) meson. Here $\{K_0^0, K_0^0\} = (-, +)$ is used to construct an approximately orthonormal $CP$ filter basis, where final states involving a $K_0^0$ are experimentally more challenging to identify, resulting in lower signal efficiency (event yield) and lower purity than the $T$ conjugate filter with a $K_0^0$. 

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\( A(\overline{A}) \) amplitudes are the decay from the initial meson to the final state.

As usual one finds that

\[
S = \frac{2Im\lambda}{1 + |\lambda|^2} \quad \text{and} \quad C = \frac{1 - |\lambda|^2}{1 + |\lambda|^2},
\]

(4)

for each of the \( \pm \) and \( \eta_{CP} \) combinations. Hence \( |\Delta S^\pm| = 4Im\lambda/\sqrt{1 + |\lambda|^2} \) and \( |\Delta C^\pm| = 2(1 - |\lambda|^2)/\sqrt{1 + |\lambda|^2} \) as the deltas are combinations of the \( (S,C)^{\eta_{CP}} \) observables.

For \( b \rightarrow c\overline{s} \) decays, in the absence of direct \( CP \) violation, \( |\Delta S^\pm| = 2\sin 2\beta \). Thus we can identify the measured asymmetries with the underlying weak phases in the decay relating to angles of unitarity triangles in the SM.

For \( B^0(\overline{B}) \) mesons, decays to flavour specific final states such as \( X\ell^\pm \nu \) constitute the flavour tag decay part of the problem. The other decay is into a \( CP \) odd or even eigenstate which has an experimentally distinguishable \( T \) conjugate final state. For simplicity we mostly restrict further discussion to definite \( CP \) final states.

The measurements performed by \( B\bar{B} \)Ar used a set of charmonium plus \( K^0_s \) decays (\( c\overline{s}K^0_s \)) and their \( T \)-conjugated partners (\( c\overline{s}K^0_L \)) as \( CP \) filters which are paired with flavour tag filters described above. These \( c\overline{s}K^0 \) states are theoretically clean to interpret within the SM hence the importance of studying them. The asymmetry parameters measured are \( \Delta S^\pm_T \) and \( \Delta C^\pm_T \), where

\[
A_T \approx \frac{\Delta C^\pm_T}{2} \cos \Delta m \Delta t + \frac{\Delta S^\pm_T}{2} \sin \Delta m \Delta t.
\]

(5)

It is also possible to perform such a measurement with any pair of final states that exhibit time-dependent \( CP \) violation such as the \( T \) conjugate pairs \( B \rightarrow (\eta', \phi, \omega)K^0 \) and \( (\eta', \phi, \omega)K^0_L \). These can be used to provide alternative measurements of \( \beta \) to complement the existing tree level determination obtained by \( B\bar{B} \)Ar. \( CP \) tag filters such as the \( b \rightarrow s \) loop transitions \( \eta'K^0 \) and \( \eta'K^0_L \) are ideal choices for another \( T \) violation measurement as physics beyond the SM may affect the asymmetries via non-SM loop amplitudes contributing to the final state. It is important to test the level of \( T \) violation in tree processes against that found in loops as a priori the nature of new physics is unknown and may be manifest here. It is also possible to study \( T \) symmetry invariance in \( b \rightarrow u \) (tree dominated) transitions via modes such as \( B \rightarrow \rho \rho \) and its axial vector counterparts. Here the decay to \( \rho^0 \rho^0 \) is of most interest as there are indications of a sizeable \( CP \) odd component in that final state. This mode can be used to provide an alternative measurement of \( \alpha \) via \( T \) violation in interference between mixing and decay amplitudes. Similarly one can test \( b \rightarrow d \) (loop) transitions via \( B \rightarrow D^{*+}D^{*-} \) decays, which also measures \( \beta \). The loop dominated \( b \rightarrow s \) decay \( B^0 \rightarrow \phi K^* \) is of particular interest as a \( CP \) filter because there is a sizeable \( CP \)-odd component to compare with the dominant \( T \)-conjugated \( CP \)-even part of the decay. This set of experimental tests can be used to independently compare \( T \), \( CP \), and \( CPT \) symmetry violation/invariance for \( b \rightarrow u, c, d, \) and \( s \) transitions in order to verify if the SM holds up to scrutiny. The time-dependent \( CP \) asymmetry parameters have already been determined for these modes, so in a sense half of the job has been done by the \( B \) Factories. These experiments have successfully confirmed the real benefit of studying \( B \) decays is the miracle that \( CP \) violation is manifestly large for these systems.

For \( b \rightarrow c, d, \) and \( s \) quark transitions \( \Delta S^\pm \) are measures of \( \mp 2\sin 2\beta \), and they are related to the unitarity triangle angle \( \alpha \) for \( b \rightarrow u \) transitions. In Table I we estimate, using available data. \( CP \) violation measurements, the precision on \( \sigma(\Delta S^\pm) \) that the current \( B \) Factories and Belle II can be expected to reach in some of the aforementioned modes. Belle II should be able to observe \( T \) violation at the SM rate in these \( CP \) filter basis channels.

| Filter basis pair | \( B \) Factories \( \eta'K^0_{S/L} \) | \( \phi K^* \) | \( \eta K^0_{S/L} \) | \( \omega K^0_{S/L} \) | \( D^*D^* \) |
|------------------|-----------------|-------------|-----------------|-----------------|--------------|
| \( \eta'K^0_{S/L} \) | 0.6 | 1.1 | 1.8 | 2.0 | 2.0 |
| \( \phi K^* \) | 0.08 | 0.13 | 0.17 | 0.22 | 0.29 |

TABLE I: Estimated sensitivities on \( \sigma(\Delta S^\pm) \) for the \( B \) Factories and Belle II.

It may be possible to extract information from \( B_s \) meson pairs collected at the \( T(5S) \), however it should be noted that current vertex detector technology is insufficient to resolve individual neutral meson oscillations in this system at an \( e^+e^- \) collider experiment such as Belle II, so one must rely on the \( \Delta \Gamma \) modulation of the neutral meson oscillation to obtain information related to \( \alpha_T \). \( B_s \) mesons are produced by hadronic collisions at the LHC, hence it would not be possible to perform a \( T \)-violation measurement using entangled pairs at the LHC experiments. However, one could perform a Kabir asymmetry test by studying flavour tagged \( B_s \) decays, which would at least provide a dual \( CP \) and \( T \) test until such time as one may perform the entanglement-based test. This would be a measurement of the difference in probabilities between \( B^0_s \rightarrow \eta K^0 \) and that of \( \eta K^0 \rightarrow B^0_s \), which could be done as a function of proper time using currently available techniques. ATLAS, CMS and LHCb have all demonstrated their capability to perform such a measurement through their studies of time-dependent \( CP \) asymmetries in \( B_s \rightarrow J/\psi \phi \).

Now we can turn to the issue of charm decays where we recently urged the experimental community to embark upon the systematic study of time-dependent \( CP \) asymmetries to parallel the work of the \( B \) Factories since 1999. This was in part motivated by the opportunity to make such measurements given the availability of data from LHCb and the promise of more to come from future flavour factories. Following our previous paper there have been tantalising indications of a non-zero direct \( CP \) asymmetry in \( D \rightarrow K^+K^- \) and \( D \rightarrow \pi^+\pi^- \).
decays from LHCb and CDF \cite{27, 28}, however current data are consistent with CP conservation \cite{29}. The underlying physics regarding the production of neutral D mesons in $e^+e^-$ collisions at the $\psi(3770)$ is a direct parallel of the production of B mesons at the $\Upsilon(4S)$. We note that as a consequence of this, one can re-use the measurement technique of Ref. \cite{8} adopted by BaBar and apply this to T violation searches in charm mesons, with the caveat that the lifetime difference matters for charm as $y = \Delta \Gamma/2\Gamma$ is non-zero. In analogy with our observations for B decays we note that one could test T at an asymmetric D factory running at the $\psi(3770)$ using a number of different final states. For example one can study T invariance for $c \to d$ and $c \to s$ transitions at leading order. While it is, in principle, possible to access $c \to u$ real and $c \to b$ virtual transitions from the second order, CKM suppressed, loop contributions, any results would be difficult to interpret in terms of the CKM matrix as one first needs to constrain the phase of charm mixing.

In the SM, the level of CP violation in charm is expected to be small so, unless T violation arises from other mechanisms, any experimental test of T symmetry invariance would require a high degree of systematic control (and careful design to minimise systematic uncertainties), and a large integrated luminosity of data collected via $e^+e^- \to \psi(3770) \to D^0\bar{D}^0[29]$. Given the sample sizes of experiments under discussion, with a few $ab^{-1}$, one would not expect to be sensitive to the level of T violation compatible with the SM — however that does mean that if one were to perform a T symmetry measurement and observe T violation, that would have to result from physics beyond the SM. Experimentally, the dilution effect present in determining flavour tag B states is absent for semi-leptonic flavour tagged neutral D mesons as there are only primary leptons produced in $D^0(\bar{D}^0) \to X_S K^\pm \nu$ decays, and as both D mesons are reconstructed with a well known initial set of conditions, there should be very little background present (at least in final states without a $K_L^0$ meson) with which one could study T symmetry invariance. The other experimental issue of relevance is that of detector resolution. Using vertex detector technology accessible today, with a center of mass boost factor $\beta\gamma$ relative to the laboratory frame of at least about 0.3-0.4, it should be possible for a $\tau$-charm factory to study T symmetry invariance in a wide range of final states including $h^+h^-K_{S,L}^0$. CP tag decays such as $D^0 \to K_{S,L}^0 \pi^0$ will be difficult to reconstruct, but should be experimentally accessible in terms of CP violation measurements, however the lack of charged particles originating from the D decay vertex means that the tag mode with $CP = +1$ would have to be reconstructed via $\pi^0$ Dalitz decays or photon conversion processes. However equivalent measurements with $D^0 \to K_{S,L}^0(\omega, \eta, \eta', \rho^0, \phi, f_0, a_0)$ are also possible, where these are expected to be less experimentally challenging than the $K_{S,L}^0\pi^0$ combination. Studies of $D^0 \to K_S^0K_L^0K_L^0$ and the $T$-conjugate $K_S^0K_L^0K_L^0$ mode could be used to explore the behaviour of this symmetry for W exchange amplitudes, assuming that one can reconstruct the decay vertex of the latter mode and isolate a clean signal with reasonable efficiency. These allow one to probe $T$ symmetry invariance in $c \to d$ and $c \to s$ transitions to complement the set of measurements in b quark transitions. A clean theoretical interpretation of such a measurement assumes negligible long distance contamination (i.e. strong force induced transitions) to the underlying weak structure of interest.

In summary, the recent observation of T violation in $b \to c$ transitions by BaBar raises an interesting (and old) question: what discrete symmetries are respected by the weak force? This paper introduces new CP filter bases that can be used to perform T symmetry invariance tests and outlines a number of measurements that can be made to test T. These cover all kinematically accessible quark transitions, i.e. for B decays one has $b \to u, d, s$ and $c$ transitions (to complement the existing $b \to c$ tests), and for D decays one can probe $c \to d$ and $c \to s$ transitions. While $c \to u$ transitions can be tested in principle, these are experimentally very challenging given the current state of the art. One expects sizeable T violation in the various B decay final states to balance the open form of the $bd$ Unitarity triangle. These are alternative measurements of the unitarity triangle angles $\alpha$ and $\beta$. Small effects (essentially zero within experimental precision that would be achievable with facilities under consideration today) are expected for D decays. The $b \to d$ and $s$ transition effects should be comparable in magnitude to those reported by BaBar for $b \to c$ transitions as they are a measure of the unitarity triangle angle $\beta$, whereas one expects a smaller value for $b \to u$ transitions which are a measure of $\alpha$ in the SM. By measuring this set of decays one would be able to constrain leading and higher order T violation contributions in the SM to complement the set of CP violation constraints reported by the B Factories since 1999. To do this one requires high statistics data samples from future $e^+e^-$ based (asymmetric energy) B and $\tau$-charm flavour factories operating at the $\Upsilon(4S)$ and $\psi(3770)$, respectively. The data samples recorded by BaBar and Belle provide a starting point for detailed exploration of T violation in B decays before Belle II starts taking data later this decade. Estimates of achievable sensitivities by those experiments are given. The full set of measurements indicated in this article would probe T violation in tree and loop transitions for both up and down type quarks. It is also worth remembering that one can probe (and over-constrain) the Kobayashi-Maskawa mechanism and the CKM matrix in terms of the set of discrete symmetries $T$, CP, and CPT. Hence one can systematically probe the behaviour of the weak force in terms of the T and CPT symmetries to complement almost five decades of study with regard to CP.
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