Research on Von Bertalanffy model and its application for predicting recoverable reserves and production of oilfield

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Abstract. Water drive type curves are simple, convenient and practical in reservoir engineering, which is widely used in reservoir production performance analysis. However, the commonly used water drive type curve can only draw one curve between water cut and recoverable reserves recovery degree, which greatly limits the application range of water drive curve. At the same time, the fitting of straight line section of water drive type curve is often affected by artificial selection, and the error is large. In order to better describe and reflect the rising law of water cut in different type reservoirs, the new water drive type curve is established, and the relationships between water cut and cumulative production, water cut and recovery degree of recoverable reserves are derived. Through MATLAB software programming and multi-parameter fitting method, the parameters in the new water drive type curve can be easily calculated. The $f_c-R^*$ curve shape of the new model includes convex curve, concave curve and S-shape curve. The new water drive type curve is applied to two actual reservoirs. The new curve fits well with the low water cut stage of the Baolang oilfield, and the error is small, which can be used to predict the future production performance of the oilfield. At the same time, the new water drive type curve is consistent with the change trend of the whole production stage of the Saertu oilfield, and the error is small, too. which can be used for the prediction of the whole production period and high water cut stage of the reservoir. The new model expands the application range of water drive type curve and provides reference for production performance prediction of other similar oilfields.

Key words: Water drive type curve, Water flooding reservoir, Multi-parameter fitting, Recovery degree, Water cut.

1. Introduction

Water drive type curve describes the relationship between water cut and recovery degree, which is an important method for reservoir production performance analysis. So far, more than 50 kinds of water drive type curves have been proposed(Li CL2005; Zou CY2010; Chen YQ2011; Zhou P2012; Liu F2013; Wang XL2015; Fan HJ2016; Liu P2017;). However, these water drive type curves can only
correspond to one relationship between water cut and recoverable reserves recovery degree, and the calculation results of different water drive type curves are quite different, so the application effect is not ideal. For example, the recoverable reserves predicted by water drive characteristic curves of type A, type B, type C and type D are from large to small: type B > type A > type C > type D. Sometimes, the predicted recoverable reserves of type B are even about 3 times of type D (Li WC2009), which seriously affects the application of water drive type curve in reservoir production performance analysis. On the basis of previous studies about water drive type curve, the new generalized water drive type curve equation is established. The new curve is "a cluster of curves", which is not only suitable for reservoirs of low, medium and high water cut stage, but also can reflect different types of water cut rising laws of different oilfields, expanding the application scope of water drive type curve.

2. Establishment of the new model
Two kinds of water drive type curves expressions are shown as follows (Yu QT1997):

\[
\log_{10} \left( \frac{W_p}{N_p} \right) = a + bN_p
\]

\[
\log_{10} \left( \frac{L_p}{N_p} \right) = a + bN_p
\]

Because \( L_p = W_p + N_p \), Eq.(2) can be rewritten as:

\[
\log_{10} \left( \frac{W_p + N_p}{N_p} \right) = a + bN_p
\]

Combining Eq.(1) and Eq.(2), we can get the following results:

\[
\log_{10} \left( \frac{W_p + nN_p}{N_p} \right) = a + bN_p
\]

When \( n=0 \), Eq.(4) is transformed into Eq.(1), and when \( n=1 \), Eq.(4) is transformed into Eq.(2). It can be seen that when \( n \) is taken as different values, a series of water drive type curves can be obtained. Therefore, through choosing the different \( n \), the better water drive type curve can be found for fitting the actual production data.

3. Derivation of the new model
Eq.(4) can be rewritten as follows:

\[
\frac{W_p + nN_p}{N_p} = 10^{a+bn_p}
\]

Based on the Eq.(5), the following formula can be obtained:

\[
\frac{W_p}{N_p} = 10^{a+bn_p} - n
\]
Eq.(6) is the newly established water drive type curve, the cumulative water production and oil production of the new water drive type curve are nonlinear equations. Through selecting different \( n \), a series of water drive type curves can be obtained.

Rearranging the two sides of Eq.(5) and taking the derivation to the time \( t \), where \( \frac{dW_p}{dt} = Q_w \),

\[
\frac{dN_p}{dt} = Q_o:
\]

\[
Q_w + nQ_o = Q_o10^{a+bN_w}
\]

(7)

Taking the derivation of \( Q_o \) on both sides of the Eq.(7).

\[
\frac{f_w}{1-f_w} = 10^{a+bN_w} - n
\]

(8)

Where \( f_w = \frac{Q_w}{Q_w + Q_o} \), based on the Eq.(8), the relationship between cumulative oil production and water cut can be obtained as follows:

\[
N_p = \frac{\log \left( \frac{f_w+1}{1-f_w+n} \right) - a}{b}
\]

(9)

When the limit water cut is 98%, the recoverable reserves \( N_R \) is:

\[
N_R = \frac{\log (49+n) - a}{b}
\]

(10)

The recovery degree of recoverable reserves \( R^\ast \) is:

\[
R^\ast = \frac{N_p}{N_R} = \frac{\log \left( \frac{f_w+1}{1-f_w+n} \right) - a}{\log (49+n) - a}
\]

(11)

The relationship between water cut and recovery degree of recoverable reserves is as follows:

\[
f_w = \frac{10^R \left[ \log (49+n) - a \right] + a}{1 + 10^R \left[ \log (49+n) - a \right] + a}
\]

(12)

Eq.(9) and Eq.(12) are the relationship expressions of \( N_p \sim f_w, f_w \sim R^\ast \) corresponding to Eq.(4).

4. Discussion and solution of the new model

(1) In order to calculate the value of \( n \), suppose that \( f_w = 0 \), \( N_P = 0 \) (Wang NT2006,2007,2008, Zhou P2012, Guo F2015, Gao WJ2020). During the actual reservoir production process, when the water cut
is 0, the cumulative oil production is not necessarily 0, such as the pure oil production stage (Deng S 2017); at the same time, when the cumulative oil production is close to 0, the water cut may not be approximately 0. Therefore, the original simplified curve has some limitations, and the new model removes the limitation of this condition, and the calculation results are more in line with the actual situation. (2) The calculation of parameters in the new model is based on the actual data of oil field, and the values of parameters $a$, $b$ and $n$ in the new model are calculated by using multi parameter fitting method of MATLAB software programming, which avoids the error caused by artificial trial and error method, and the calculation result is more accurate. (3) It can be seen from Fig. 1 and Fig. 2 that for different $n$, the shape of water drive type curve is different. Given the different $n$ and $a$ values, the curve shape of $f_w-R^*$ changes from concave to "S" and then to convex. At the same time, the $R^*=0$, that is, $N_P=0, f_w$ is not necessarily equal to 0. For some high water cut wells with water breakthrough as soon as they are put into production, the new curve can better reflect the actual production of the oilfield. Therefore, the new water drive type curve reflects different water cut rising rules of different reservoirs.

![Fig. 1 n=10, the relationship $f_w-R^*$ with different values of $a$](image1)

![Fig. 2 n=-40, the relationship $f_w-R^*$ with different values of $a$](image2)

5. Case application

5.1. Baolang Oilfield
Baolang oilfield is a low porosity and low permeability reservoir, and The oilfield put into water flooding after 3 years of development, belongs to a typical water flooding oilfield(Table 1).

Substituting the data in Table 1 into Eq.(6), through multi parameter fitting calculation, the relate
parameters are $a=-2.30395$, $b=0.01102$, $n=-0.406563$, and the correlation coefficient is 0.8926. The equation of water drive type curve in Baolang oilfield is obtained as follows:

$$\frac{W_p}{N_p} = 10^{-2.30395+0.01102N_p} + 0.406563 \quad (13)$$

Using the similar methods to substitute the data in Table 1 into Eq.(9), the relate parameters are $a=-2.346$, $b=0.0166$, $n=-0.008524$, and the correlation coefficient is 0.96262, then the cumulative oil production of Baolang oilfield with different water cut is:

$$N_p = \log \left( \frac{f_w - 0.008524}{1-f_w} \right) + 2.346 \quad (14)$$

Based on the cumulative oil production, the annual oil production of the oilfield can be calculated. Taking the limit water cut as 98%, the recoverable reserve is $243.14 \times 10^4$ t in Baolang oilfield.

Substituting the data in Table 1 into Eq.(12). Similarly, through multi parameter fitting calculation, $a=-1.846812$, $n=0.019863$, and the correlation coefficient is 0.95907, then the water cut equation with different recoverable reserves recovery degree in Baolang oilfield is:

$$f_w = \frac{10^{3.537184R-1.846812}}{1+10^{3.537184R-1.846812}} \quad (15)$$

It can be seen from Fig.3 and Fig.4 that the error is small between the theoretical prediction value of the new equation and the actual value, and Eq.(14) and Eq.(15) can be respectively used to predict the relationship between cumulative oil production and water cut, between water cut and recovery degree of recoverable reserves in Baolang oilfield.

Fig. 3 The relationship between cumulative oil production and water cut in Baolang oilfield,
Fig. 4 The relationship between water cut and recovery degree of recoverable reserves in Baolang oilfield

Table 1. The actual production data of Baolang oilfield

| Time (a) | Cumulative oil production $N_P$ (10^4t) | Cumulative water production $W_P$ (10^4t) | Water cut $f_w$ | $R^*$  |
|----------|---------------------------------------|----------------------------------------|----------------|--------|
| 1        | 6.79                                  | 0.25                                   | 0.02           | 0.02   |
| 2        | 13.75                                 | 0.35                                   | 0.01           | 0.05   |
| 3        | 22.23                                 | 0.46                                   | 0.01           | 0.08   |
| 4        | 32.40                                 | 0.48                                   | 0.00           | 0.12   |
| 5        | 42.29                                 | 0.81                                   | 0.04           | 0.15   |
| 6        | 52.16                                 | 1.17                                   | 0.05           | 0.19   |
| 7        | 61.35                                 | 1.8                                    | 0.06           | 0.22   |
| 8        | 70.02                                 | 2.7                                    | 0.06           | 0.26   |
| 9        | 78.20                                 | 3.73                                   | 0.11           | 0.29   |
| 10       | 85.40                                 | 4.73                                   | 0.13           | 0.31   |
| 11       | 92.10                                 | 5.72                                   | 0.12           | 0.34   |
| 12       | 99.00                                 | 6.94                                   | 0.19           | 0.36   |
| 13       | 105.47                                | 8.27                                   | 0.20           | 0.38   |
| 14       | 112.16                                | 9.79                                   | 0.24           | 0.41   |
| 15       | 117.47                                | 11.52                                  | 0.27           | 0.43   |
| 16       | 121.82                                | 13.03                                  | 0.34           | 0.44   |
| 17       | 125.69                                | 15.16                                  | 0.36           | 0.46   |
| 18       | 129.41                                | 17.22                                  | 0.38           | 0.47   |
| 19       | 132.71                                | 19.28                                  | 0.42           | 0.48   |
| 20       | 135.56                                | 21.25                                  | 0.45           | 0.49   |
| 21       | 138.03                                | 23.44                                  | 0.48           | 0.5    |
| 22       | 140.31                                | 25.63                                  | 0.50           | 0.51   |

5.2. Saertu oilfield
Saertu oilfield in Daqing was put into development in 1969(Chen YQ 2007, Cui YH 2017,). It is produced by artificial water injection, and the reservoir effective permeability is 350mD, which is a typical high permeability reservoir.

Substituting the data in Table 2 into Eq.(6), Using the similar methods as above, the calculated relate parameters are $a=-0.770884$, $b=0.001148$, $n=9.450185$, and the correlation coefficient is 0.89714. The equation of water drive type curve in Saertu oilfield is as follows:
Using the similar methods to substitute the data in Table 2 into Eq.(9), through multi parameter fitting calculation, $a=-0.764065$, $b=0.00173$, $n=-0.000433$, and the correlation coefficient is 0.9908, then the cumulative oil production equation of Saertu oilfield with different water cut is:

$$
\frac{W_p}{N_p} = 10^{-0.770884+0.001148N_p} - 19.450185
$$

(16)

Taking the limit water cut as 98% and substituting it into the Eq.(17), the recoverable reserves are $1418.26 \times 10^4 t$.

Substituting the data in Table 2 into Eq.(12), similarly, the calculated relate parameters are $a=-0.761704$, $n=0.00754$, the correlation coefficient is 0.99075, then the water cut equation with different recoverable reserves recovery degree is as follows:

$$
f_w = \frac{10^{2.451907R^*-0.761704}}{1+10^{2.451907R^*-0.761704}}
$$

(18)

**Table 2.** The actual production data of Saertu oilfield

| Time (a) | Cumulative oil production $N_p$ ($10^4 t$) | Cumulative water production $W_p$ ($10^4 t$) | Water cut $f_w$ | $R^*$ |
|---------|------------------------------------------|------------------------------------------|----------------|------|
| 1       | 5.66                                     | 0.10                                     | 0.00           | 0.15 |
| 2       | 14.36                                    | 0.22                                     | 0.01           | 0.15 |
| 3       | 30.34                                    | 1.73                                     | 0.02           | 0.16 |
| 4       | 64.86                                    | 6.12                                     | 0.05           | 0.18 |
| 5       | 128.10                                   | 15.70                                    | 0.09           | 0.22 |
| 6       | 196.27                                   | 28.99                                    | 0.14           | 0.27 |
| 7       | 259.40                                   | 53.15                                    | 0.18           | 0.33 |
| 8       | 331.97                                   | 103.78                                   | 0.23           | 0.39 |
| 9       | 402.78                                   | 176.00                                   | 0.28           | 0.46 |
| 10      | 475.19                                   | 268.77                                   | 0.33           | 0.53 |
| 11      | 548.78                                   | 378.76                                   | 0.39           | 0.61 |
| 12      | 622.48                                   | 508.57                                   | 0.44           | 0.67 |
| 13      | 689.66                                   | 671.11                                   | 0.49           | 0.73 |
| 14      | 752.08                                   | 876.57                                   | 0.53           | 0.78 |
| 15      | 807.46                                   | 1117.87                                  | 0.57           | 0.81 |
| 16      | 863.29                                   | 1417.85                                  | 0.61           | 0.84 |
| 17      | 916.31                                   | 1744.16                                  | 0.65           | 0.87 |
| 18      | 961.72                                   | 2079.29                                  | 0.68           | 0.89 |
| 19      | 1002.39                                  | 2418.27                                  | 0.71           | 0.90 |
| 20      | 1041.10                                  | 2771.89                                  | 0.73           | 0.92 |
| 21      | 1074.57                                  | 3115.38                                  | 0.76           | 0.93 |
| 22      | 1106.60                                  | 3471.50                                  | 0.78           | 0.93 |
| 23      | 1135.98                                  | 3842.26                                  | 0.80           | 0.94 |
| 24      | 1162.30                                  | 4228.46                                  | 0.82           | 0.95 |
| 25      | 1185.37                                  | 4584.76                                  | 0.83           | 0.95 |
Fig. 5 The relationship between cumulative oil production and water cut of Saertu oilfield

Fig. 6 The relationship between water cut and recovery degree of recoverable reserves in Saertu oilfield

It can be seen from Fig. 5 and Fig. 6 that the new theoretical model fits well with the actual reservoir development and production, and the error between the theoretical calculation value and the actual value in the whole development stage are all small, which can be used for the prediction of production performance parameters in high water cut stage.

Case 1 and case 2 show that the new water drive type curve has good fitting effect for reservoirs in low and high water cut stage, and can be used to predict the later production performance of oilfield. At the same time, the new water drive type curve can be applied to different types of reservoirs (low permeability and high permeability).

6. Conclusion
(1) On the basis of two kinds of commonly used water drive type curves, the new generalized water drive type curve is established. The new model is no longer a single curve, but a "cluster curve". It expands the application range of water drive curve. When $n$ is taken as 0 or 1, it can be simplified as original water drive curve. At the same time, the relationship between water cut and cumulative oil production, water cut and recovery degree of recoverable reserves is established.

(2) The limitation of $f_w = 0$ and $N_p = 0$ is removed from the new water drive type curve, which makes the calculation result more in line with the actual situation of oilfield. During the process of calculation, taking the oilfield actual production data, using the MATLAB software programming and multi parameter fitting method, the relevant parameters of water drive curve are calculated. Compared with the traditional water drive curve calculation, the error caused by artificial selection of data is avoided, and the prediction accuracy is improved.

(3) When $n$ is taken as different values, the $f_w-R^*$ shape of the new water drive type curve gradually
transits from concave type to "S" type and then to convex type, which expands the application scope of water drive curve.

(4) Case 1 Baolang oilfield is in the low water cut stage, and the predicted value of the new water drive type curve is in good agreement with the actual value. The water cut rising law and production performance of the reservoir in the middle and high water cut stage can be predicted according to the new equation. In case 2 Saertu oilfield, the new water drive type curve has more high degree of fitting with the whole production stage, which is in line with the actual production trend of the reservoir, and can be used for the performance prediction of the whole oilfield production and high water cut stage. It is proved that the new water drive type curve is suitable for different types of reservoirs at different water cut stages.

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7. Nomenclature
A, B, water drive type curve coefficients, constants; 
NR, recoverable reserves, 10^4 t; 
R*, the recovery degree of recoverable reserves; 
Np, cumulative oil production, 10^4 t; 
Lp, cumulative liquid production, 10^4 t; 
wp, cumulative water production, 10^4 t; 
Qo, Monthly oil production, 10^4 t; 
w, Monthly water production, 10^4 t; 
n, the introduction coefficient, constants; 
t, time, year; 
fw, water cut, f.

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