The Role of Temperature in the occurrence of some Zeno Phenomena

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Temperature can be responsible for strengthening effective couplings between quantum states, determining a hierarchy of interactions, and making it possible to establish such dynamical regimes known as Zeno dynamics, wherein a strong coupling can hinder the effects of a weak one. The relevant physical mechanisms which connect the structure of a thermal state with the appearance of special dynamical regimes are analyzed in depth.

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I. INTRODUCTION

In the last decades a lot of attention has been devoted to the role of temperature in quantum information and in quantum mechanics in general. In particular, the detrimental effects of an environment at finite or even high temperature have been studied in connection with many aspects of quantum mechanics. Nevertheless, many studies aimed at singling out robustness of quantum features even in the presence of high temperature, which for instance is the case of thermal entanglement. Recently, the connection between temperature and Quantum Zeno phenomena has also been explored.

The concept of quantum Zeno effect (QZE) — according to its original definition QZE consists in the inhibition of the dynamics of a physical system due to repeated “pulsed” measurements [1], which has been demonstrated in various systems [2–8] — has been progressively generalized. The first generalization consists in considering continuous measurements meant as dissipative processes [9], showing that a strong decay can be responsible for a dynamical decoupling that weakens the effects of other interactions [9–11]. Another important generalization is related to the knowledge that even an additional coherent coupling can be responsible for diminishing the effects of other interactions, hence realizing the so-called Zeno Dynamics [9–11] and eventually leading to the concept of Zeno subspaces [12]. Also pulsed interactions have been proven to be responsible for the formation of dynamically invariant subspaces [13–15].

Quantum Zeno effect has been exploited to develop applications in various fields, such as quantum information and nanotechnologies [16–18]. Moreover, its importance in connection with fundamental concepts of quantum mechanics has been widely discussed [19]. Nevertheless, Zeno phenomena are not confined to the realm of quantum mechanics, but, according to Peres, similar occurrences are possible even in classical mechanics [20].

The connection between Zeno phenomena and temperature has been investigated by various authors. In 2002 Ruseckas analyzed the effects of thermalization of a measurement apparatus on the appearance and intensity of the quantum Zeno effect, showing that an higher temperature can amplify the power of pulsed measurements in inducing Zeno phenomena [21]. In 2006 Maniscalco et al. have studied the crossover between QZE and AZE — Anti-Zeno effect is the acceleration of the dynamics induced by measurements, usually occurring for short-but-not-too-short time intervals between subsequent measurements — in connection with the temperature of the bath the system under scrutiny is interacting with, showing that the QZE-AZE border can be significantly affected by temperature [22]. Recently, Scala et al. have analyzed Landau-Zener (LZ) transitions in the presence of environmental effects, even in the case of a bath at finite or high temperature [23]. The analysis, developed through a master equation approach beyond the secular approximation, has shown that temperature can be responsible for a dynamical decoupling, which eventually leads to a Zeno effect. A subsequent study [24] developed in connection with a different physical situation — a simplified version of the previous problem, wherein a time-independent Hamiltonian has been considered in place of the LZ time-dependent coupling scheme — has shown that in some situations temperature can clearly contribute to induce a Zeno dynamics, weakening the effects of coherent couplings.

Starting from these results, one could wonder whether in those cases wherein temperature can influence more than one interactions it is possible to have temperature-dependent hierarchies of couplings, driving the appearance of Zeno behaviors. Indeed, for instance, if two or more interactions are strengthened by temperature, occurrence of Zeno phenomena could be obstructed, because of the impossibility of finding a coupling which is much stronger than others; in other cases temperature can enhance inhibition of the dynamics. In this paper we analyze the role of temperature when it affects more than one interactions, discussing appearance and disappearance of temperature-induced Zeno dynamics, and studying in depth the underlying physical mechanisms.

The paper is organized as follows. In section II we analyze the dynamics of a three-level system resonantly interacting with two harmonic oscillators initially prepared in a thermal state. In section III we extend the scheme by considering some detuning effects and show

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an interesting competition between detuning and temperature in determining Zeno regimes. In section IV we give some preliminary results that pave the way to the analysis of the case where many oscillators are involved in the coupling scheme. Finally, in section VI we give some conclusive remarks.

II. ZENO DYNAMICS INDUCED BY TEMPERATURE

Let us consider a three-state system interacting with two harmonic oscillators initially prepared in a thermal state. The interaction with the oscillators can induce transitions from the upper state to the intermediate one and from the intermediate to the lowest. Both interactions are assumed to be resonant (see Fig. 1).

From general statements for the appearance of Zeno dynamics [11–13], if the strength of the coupling between \( |2\rangle \) and \( |3\rangle \) is much greater than the strength of the other coupling, i.e. if \( |g_{23}| \gg |g_{12}| \), then transitions from \( |1\rangle \) to \( |2\rangle \) — and vice versa — are hindered. Therefore, if the atomic system is initially prepared in the state \( |1\rangle \) then the system does not evolve significantly.

We will show that this behavior occurs when temperature is high even if the condition \( |g_{23}| \gg |g_{12}| \) is not fulfilled, provided the frequency of the oscillator assisting \( 2 \rightarrow 3 \) transitions is much smaller than the frequency of the oscillator which couples \( |1\rangle \) and \( |2\rangle \).

A. The Hamiltonian Model

Consider the following Hamiltonian model (\( \hbar = 1 \)):

\[
\hat{H} = \sum_k \omega_k |k\rangle \langle k| + \omega_a \hat{a}^\dag \hat{a} + \omega_b \hat{b}^\dag \hat{b} \\
+ g_{12} (|1\rangle \langle 2| \hat{a} + |2\rangle \langle 1| \hat{a}^\dag) + g_{23} (|2\rangle \langle 3| \hat{b} + |3\rangle \langle 2| \hat{b}^\dag).
\]

(1)

Assuming complete resonance (\( \omega_1 - \omega_2 = \omega_a \) and \( \omega_2 - \omega_3 = \omega_b \), it clearly corresponds to the coupling scheme represented in Fig. 1.

The structure of the Hamiltonian is such that the total number of excitations,

\[
\hat{N} = 2 |1\rangle \langle 1| + |2\rangle \langle 2| + \hat{a}^\dag \hat{a} + \hat{b}^\dag \hat{b},
\]

(2)

is a constant of the motion. Therefore the Hilbert space is partitioned into invariant three-dimensional subspaces, each spanned by a set \( |1, n_a, n_b\rangle \), \( |2, n_a + 1, n_b\rangle \) and \( |3, n_a + 1, n_b + 1\rangle \), and each one corresponding to a restriction of the Hamiltonian operator which has the following form:

\[
H_{n_a,n_b} = (n_a \omega_a + n_b \omega_b + \omega_1) \hat{I}_3 \\
+ \left( \begin{array}{ccc} 0 & g_{12} \sqrt{n_a + 1} & 0 \\ g_{12} \sqrt{n_a + 1} & 0 & g_{23} \sqrt{n_b + 1} \\ 0 & g_{23} \sqrt{n_b + 1} & 0 \end{array} \right).
\]

(3)

where \( \hat{I}_3 \) is the identity of the \( 3 \times 3 \) matrix space. The ‘restricted’ operator \( H_{n_a,n_b} \) generates the ‘restricted’ unitary evolution \( U_{n_a,n_b} = \exp(-i t H_{n_a,n_b}) \).

We now prove that when \( g_{23} \sqrt{n_b + 1} \gg g_{12} \sqrt{n_a + 1} \) the survival probability of the state \( |1, n_a, n_b\rangle \) is preserved. Preliminarily, consider that one of the three eigenenergies associated to \( H_{n_a,n_b} \) is \( n_a \omega_a + n_b \omega_b + \omega_1 \) — according to the analysis in the appendix A the matrix in the second line has a vanishing eigenvalue — and that the corresponding eigenstate is

\[
|n_a \omega_a + n_b \omega_b + \omega_1\rangle = \frac{g_{23} \sqrt{n_b + 1}}{g} |1, n_a, n_b\rangle \\
- \frac{g_{12} \sqrt{n_b + 1}}{g} |3, n_a + 1, n_b + 1\rangle,
\]

(4)

with \( g = [g_{12}^2(n_a + 1) + g_{23}^2(n_b + 1)]^{1/2} \).

Now, given a positive number \( \varepsilon \), provided

\[
\frac{g_{23} \sqrt{n_b + 1}}{g_{12} \sqrt{n_a + 1}} \geq \chi \varepsilon \equiv \sqrt{\frac{1 + (1 - \varepsilon)^{1/4}}{1 - (1 - \varepsilon)^{1/4}}},
\]

(5)

one easily gets that if the initial state is \( |\psi(0)\rangle = |1, n_a, n_b\rangle \) then the population of such initial state is kept close to unity through all the evolution:

\[
P_{1,n_a,n_b}(t) \geq \sqrt{1 - \varepsilon}, \quad \forall t \geq 0,
\]

(6)

which can be proven on the basis of the analysis in appendix A.

This result can be thought of as the occurrence of a Zeno dynamics, since the \( 2 \rightarrow 3 \) coupling hinders the dynamics induced by the \( 1 \rightarrow 2 \) interaction.
B. Inhibition of Time Evolution

Let us now consider the system in the following initial state:

$$\rho(0) = |1\rangle \langle 1| \otimes \rho_a \otimes \rho_b,$$

where

$$\rho_k = N_k \sum_n \exp(-\omega_k n/k_B T) |n\rangle \langle n|$$  \hspace{1cm} (8a)

$$N_k = [1 - \exp(-\omega_k/k_B T)],$$  \hspace{1cm} (8b)

with $k = a, b$, are the thermal states of the two oscillators.

The probability to find the three-state system in the initial state $|1\rangle$ after a time $t$, irrespectively of the number of excitations of the harmonic oscillators, is given by:

$$P_1(t) = N_a N_b \sum_{n_a} \sum_{n_b} \exp\left(-\frac{\omega_a n_a + \omega_b n_b}{k_B T}\right)$$

$$\times \left|(1, n_a, n_b) \tilde{U}_{n_a n_b}(t) |1, n_a, n_b\rangle^2 \right|^2.$$  \hspace{1cm} (9)

Condition in Eq.[5] can be rewritten as

$$n_b \geq \tilde{n}_b(n_a, \varepsilon),$$

with

$$\tilde{n}_b(n_a, \varepsilon) = \frac{g_{12}^2}{g_{23}^2} \chi_{a}^2 n_a + \chi_{b}^2 \frac{g_{12}^2}{g_{23}^2} - \frac{g_{23}^2}{g_{23}}$$

$$\equiv \alpha_{e} n_a + \beta_{e}.$$  \hspace{1cm} (11)

Since all the terms in the right-hand side of Eq.(9) are nonnegative, one has

$$P_1(t) \geq N_a N_b \sum_{n_a} \sum_{n_b} \exp\left(-\frac{\omega_a n_a + \omega_b n_b}{k_B T}\right)$$

$$\times \left|(1, n_a, n_b) \tilde{U}_{n_a n_b}(t) |1, n_a, n_b\rangle^2 \right| \geq \sqrt{1 - \varepsilon}$$

for any $n_b^*$, and then one has:

$$P_1(t) \geq N_a N_b \sum_{n_a} \sum_{n_b^*} \exp\left[-\frac{(\beta_{e} + 1)\omega_b/(k_B T)}{k_B T}\right] \frac{1 - \exp[-(\omega_a + \alpha_{e}\omega_b)/(k_B T)]}{1 - \exp[-(\alpha_{e}\omega_b)/(k_B T)]} \sqrt{1 - \varepsilon},$$

where the symbol $[[x]]$ indicates the first integer number larger than or equal to $x$, and the relations $x + 1 \geq [[x]]$ and $\sum_{n=0}^{\infty} e^{-nx} = e^{-s_2}/(1 - e^{-s_2})$ have been used.

For any given value of $\varepsilon$, positive and smaller than unity, in the limit of very high temperature, one can assume $\exp[-(\beta_{e} + 1)\omega_b/(k_B T)] \approx 1$ and $1 - \exp[x/(k_B T)] \approx x/(k_B T)$, so that one gets

$$P_1(t) \gtrsim \frac{\sqrt{1 - \varepsilon}}{1 + \eta \chi_{e}},$$

with

$$\eta = \frac{g_{12}^2}{g_{23}^2} \frac{\omega_b}{\omega_a}.$$  \hspace{1cm} (15)

Therefore, in the limit of high temperature the survival probability becomes higher and higher for any $t$, approaching unity in the limit $\eta \rightarrow 0$. In fact, for $\eta < 1$, by choosing $\varepsilon = \sqrt{\eta}$, since $\chi_{e} \approx 8/\varepsilon$ for very small $\varepsilon$, one gets $P_1(t) \gtrsim 1 - (17/2)\sqrt{\eta} \rightarrow 1$. The limit $\eta \rightarrow 0$ can be reached in different ways: two possibilities are assuming $|g_{12}| \ll |g_{23}|$ or assuming $\omega_b/\omega_a \ll 1$. On this basis, one can assert that in each of the two limits, $|g_{12}/g_{23}| \rightarrow 0$ or $\omega_b/\omega_a \rightarrow 0$, it turns out that $P_1(t) \rightarrow 1$ for any $t$, provided the temperature is high enough.

It is worth emphasizing that in order to achieve the limit in (14) one must legitimate truncation of the series expansions of the exponential involved, which is possible under the assumption that the following conditions are fulfilled:

$$\frac{(\beta_{e} + 1)\omega_b}{(k_B T)} \ll 1,$$

$$\frac{\omega_a + \alpha_{e}\omega_b}{(k_B T)} \ll 1.$$  \hspace{1cm} (16a)

These two inequalities define the limit of high temperature for any fixed value of $\varepsilon$.

In Fig.2 are shown the evolutions of the survival probability for different temperatures and for different ratios of the frequencies of the two oscillators. It is evident that the higher the temperature the higher the survival probability at any time. Moreover, the smaller the ratio $\omega_b/\omega_a$ the higher the asymptotic value of $P_1(t)$.

It is worth noting the difference with the case analyzed in the previous paper on Ženo subspaces induced by tem-
FIG. 3: (Color online) Survival probability of the atomic state $|1\rangle$ at low and high temperature. The relevant quantities are $\omega_a/\omega_a = 10$, $g_{12}/\omega_a = 1$, $g_{23}/g_{12} = 6$; temperature: $T/\omega_a = 0.1$ (red solid line), $T/\omega_a = 250$ (black dashed line).

FIG. 2: (Color online) The survival probability of the atomic state $|1\rangle$ as a function of time, for different values of the relevant parameters. (a) $\omega_a/\omega_b = 10$, $g_{12}/\omega_b = g_{23}/\omega_b = 1$, for different values of the temperature: $T/\omega_b = 0.1$ (red solid line), $T/\omega_b = 1$ (green dotted line), $T/\omega_b = 50$ (blue dashed line), $T/\omega_b = 250$ (black bold line). (b) $\omega_a/\omega_b = 50$, $g_{12}/\omega_b = g_{23}/\omega_b = 1$, for different values of the temperature: $T/\omega_b = 0.1$ (red solid line), $T/\omega_b = 1$ (green dotted line), $T/\omega_b = 250$ (blue dashed line), $T/\omega_b = 1250$ (black bold line).

The temperature $T$. In fact, in that case the limit of the survival probability for $T \to \infty$ is unity, while in the present case the survival probability does not tend toward unity, unless assuming $\omega_b/\omega_a \to 0$ (or $|g_{12}|/|g_{23}| \to 0$). This difference can be explained analyzing the physical mechanism on the basis of these Zeno-like behaviors induced by temperature. Indeed, the key points are two: first, in each invariant subspace the strength of the coupling with an oscillator grows up as the number of excitations increases; second, the higher the temperature the more the subspaces with high excitation numbers are populated. Now, in the case of Ref. [24] the interaction $1-2$ is not assisted by a harmonic oscillator and therefore it is not strengthened by temperature as the interaction $2-3$ is. As a consequence, an higher temperature makes the ratio between the two coupling strengths tend toward zero in most of the subspaces. On the contrary, in the present case, both couplings, $1-2$ and $2-3$, are effectively enhanced by temperature, and one needs $\omega_b \ll \omega_a$ in order to make the coupling $2-3$ prevail in the majority of the subspaces. Indeed, $\omega_b \ll \omega_a$ means that excited states of the oscillator $b$ are more populated than the corresponding states of oscillator $a$, at the same temperature. If this condition (or the alternative one, i.e., $|g_{12}|/|g_{23}| \ll 1$) is not properly fulfilled, occurrence of Zeno dynamics is attenuated.

C. Low vs High Temperature

In order to better understand the role of temperature, it could be useful to compare the behaviors of the survival probability at low and high temperature.

In fact, in the zero-temperature case, the only block involved in the dynamics is the one related to $n_a = n_b = 0$, and the inhibition of the dynamics of $|1,0,0\rangle$ is determined by the condition $|g_{23}| \gg |g_{12}|$, so that it is essentially due to a significant difference between the strengths of the two couplings. On the contrary, when the high-temperature limit is considered, the parameter determining the minimum value of the survival probability is $\eta$, as defined in Eq. (13), which involves both the coupling strengths and the frequencies of the oscillators. As already discussed before, a smaller frequency implies an higher degree of population of the excited levels of the oscillator, and eventually a stronger coupling in most of the Hilbert space.

On this basis, one finds that Zeno dynamics is possible even in the case $|g_{23}| < |g_{12}|$, provided $\omega_b \ll \omega_a$ and temperature high enough. On the contrary, temperature can produce the opposite effect, determining disappearance of Zeno dynamics. As an example, consider the case $|g_{23}| \gg |g_{12}|$, which implies Zeno dynamics at zero temperature, and $\omega_b \gg \omega_a g_{23}/g_{12}^2 \gg \omega_a$, which renders $\eta$ large enough to invalidate inhibition of the dynamics. In Fig. 3 it is shown a case wherein by increasing temperature one lowers the survival probability at any time.
have inhibition of the dynamics of the state needs that making it a quasi-eigenstate of the Hamiltonian — one in which in turn is non resonantly coupled to the lowest level (|3⟩).

III. DETUNING EFFECTS

Let us now consider again the Hamiltonian in Eq. (11) but without the resonance conditions (see Fig. 4). Generally speaking the presence of a detuning causes a diminishing of the interaction strength, so that the presence of a detuning in the 2 − 3 transition will make us analyze the competition between detuning and temperature in connection with the occurrence of Zeno dynamics. On the contrary, the presence of a detuning in the 1−2 transition would simply hinder the evolution of the initial atomic state (|1⟩) even in the absence of the second oscillator and relevant coupling. Therefore this analysis would be less interesting. Hence, for the sake of simplicity, let us assume that the transition 1 − 2 is resonant with the relevant oscillator (ωa = ω1 − ω2), while the transition 2 − 3 is assumed not to be resonant with the second oscillator: Δ = ωb − (ω2 − ω3) ≠ 0. The restriction of the Hamiltonian in the generic invariant subspace corresponding to na and nb is given by the following matrix:

\[ H_{n_a,n_b} = \left( \begin{array}{cc} n_a \omega_a + n_b \omega_b + \omega_1 \end{array} \right) \]

\[ + \left( \begin{array}{cc} 0 & g_{12} \sqrt{n_a + 1} \Delta \\ g_{12} \sqrt{n_a + 1} & 0 \\ 0 & g_{23} \sqrt{n_b + 1} \\ g_{23} \sqrt{n_b + 1} & 0 \end{array} \right) \].

(17)

According with the analysis in appendix A, in order to have inhibition of the dynamics of the state |1, na, nb⟩ — making it a quasi-eigenstate of the Hamiltonian — one needs that g23√nb + 1 is much grater than g12√na + 1 and greater than Δ. A large value of detuning can obstacle the appearance of Zeno dynamics, but larger values of temperature can establish again such a regime by populating more and more those subspaces wherein g23√nb + 1 is larger (and even much larger) than Δ.

In Fig. 4 it is shown the behavior of the population of the atomic state |1⟩ as a function of time but for different values of temperature and detuning. From Fig. 4, we learn that even in the presence of non vanishing detuning there is an increase of the survival probability when temperature becomes higher. Nevertheless, comparing the bold line in this figure with the one in Fig. 2b, we see that the asymptotic value of the population is slightly smaller in the presence of detuning, in spite of the fact that the temperature is the same.

In Fig. 5b it is shown that, for a fixed value of temperature, a diminishing of the detuning produces an increase of the asymptotic population, as expected.

IV. ATTEMPT OF GENERALIZATION

In view of a possible study of interactions between a few-level system and its environment, the previous analysis could deserve a generalization to the case wherein many harmonic oscillators couple both transitions, 1−2 and 2−3. Since we are interested to the case of quite different Bohr frequencies, we can consider two independent
sets of oscillators. The relevant Hamiltonian is given by,

\[
\hat{H} = \sum_k \omega_k |n_k\rangle \langle n_k| + \sum_{k=1}^p \nu_k \hat{a}_k \hat{a}_k^\dagger + \sum_{l=1}^q \mu_l \hat{b}_l \hat{b}_l^\dagger + \sum_{k=1}^p g_k^{(12)} (|1\rangle \langle 2| \hat{a}_k + |2\rangle \langle 1| \hat{a}_k^\dagger) + \sum_{l=1}^q g_l^{(23)} (|2\rangle \langle 3| \hat{b}_l + |3\rangle \langle 2| \hat{b}_l^\dagger),
\]

which conserves the following quantity:

\[
\hat{N} = 2 |1\rangle \langle 1| + |2\rangle \langle 2| + \sum_{k=1}^p \hat{a}_k \hat{a}_k^\dagger + \sum_{l=1}^q \hat{b}_l \hat{b}_l^\dagger.
\]

Therefore the Hilbert space is partitioned into invariant subspaces \(\mathcal{H}_{\vec{n}_p, \vec{m}_q}\) of dimension \(\dim \mathcal{H}_{\vec{n}_p, \vec{m}_q} = 1 + p + pq\) and spanned by the following vectors: \(|1, \vec{n}_p, \vec{m}_q\rangle \equiv |1, n_1, ... n_p, m_1, ... m_q\rangle\); \((n_k + 1)^{-1/2} \hat{a}_k^\dagger |2, \vec{n}_p, \vec{m}_q\rangle\) with \(k = 1, ... p\); \((n_k + 1)^{-1/2} (m_l + 1)^{-1/2} \hat{b}_l^\dagger |3, \vec{n}_p, \vec{m}_q\rangle\), with \(k = 1, ... p\) and \(l = 1, ... q\).

The restriction of the Hamiltonian to each subspace has the following structure:

\[
B_{p \times q} = \begin{pmatrix}
\delta_0 & \alpha_1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\alpha_1 & \delta_1 & 0 & \cdots & 0 & \beta_{11} & \cdots & \beta_{1q} & 0 & \cdots & 0 & \cdots & 0 \\
0 & \beta_{11} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \beta_{2q} & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \beta_{p1} & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \beta_{pq} & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 & \cdots & 0 & 0 & \cdots & \cdots
\end{pmatrix},
\]

(20)

The matrix in Eq. (21) has the same structure of the matrix analyzed in the Appendix B of Ref. [24]. On the basis of that analysis, we claim that when the system starts form \(|1, n_1, \vec{m}_q\rangle\) and \(\sum_l \beta_{l}^2 \gg |\alpha|\) then the dynamics is hindered. Therefore, focusing our attention on the population of the atomic state \(|1\rangle\) irrespectively of the number of excitations of the oscillators, by reasoning as in the previous sections, we can say that the evolution of such population turns out to be hindered when the frequencies \(\mu_l\) are smaller — preferably much smaller — than \(\nu \equiv \nu_1\). Such a condition on the frequencies makes the previous assertion true even if, for example, \(|g_1^{(12)}|^2 \sim \sum_l |g_l^{(23)}|^2\).
V. DISCUSSION

The analysis developed in this paper shows in a clear way the role of the temperature in determining the occurrence of Zeno dynamics.

In particular, we have considered a few-level system prepared in an excited state and interacting with a set of harmonic oscillators initially prepared in thermal states at the same temperature. We have pointed out that an higher temperature implies a more significant role of the subspaces of the Hilbert space characterized by higher numbers of excitations, which in turn imply stronger effective couplings. The smaller the frequency of an oscillator the more its excited states are involved in the dynamics, for a given temperature. Therefore, the presence of a coupling between the lowest state $|3\rangle$ and the intermediate one $|2\rangle$ can significantly weaken the effects of the $1-2$ coupling, especially when the Bohr frequency of the $2-3$ transition is sufficiently small. This is a significant difference between high-temperature and low-temperature limit, since in the latter case the role of the frequencies is absent and only coupling strengths are important.

This analysis is the prosecution of the study developed in Ref. [24] where the $1-2$ limit, since in the latter case the role of the frequencies between high-temperature and low-temperature transition is sufficiently small. This is a significant difference between high-temperature and low-temperature limit, since in the latter case the role of the frequencies is absent and only coupling strengths are important.

Another point that could deserve attention is the role of the few-level system and the bosonic part of the total system.

VI. ACKNOWLEDGEMENTS

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Appendix A

In this appendix we prove that a matrix of the following form (written in the basis $|A\rangle$, $|B\rangle$, $|C\rangle$):

$$M = \begin{pmatrix}
0 & \alpha_1 & 0 \\
\alpha_1^* & 0 & \alpha_2 \\
0 & \alpha_2^* & \Delta
\end{pmatrix}, \quad (A1)$$

has an eigenvalue whose corresponding eigenstate becomes closer to $|A\rangle$ as the modulus of the coupling constant $\alpha$ grows up.

Let us first consider the special situation $\Delta = 0$, in which case we can directly check that $\lambda = 0$ is an eigenvalue and that the corresponding eigenstate is

$$|0\rangle = \frac{\alpha_2}{\alpha} |A\rangle - \frac{\alpha_1^*}{\alpha} |C\rangle, \quad \alpha = \sqrt{\alpha_1^2 + \alpha_2^2}, \quad (A2)$$

from which one deduces that $|\langle A|0\rangle| = |\alpha_2/\alpha|$ tends toward unity in the limit $|\alpha_1| \ll |\alpha_2|$.

Concerning the general case $\Delta \neq 0$, the secular polynomial $P(\lambda) = \det(M - \lambda I_3)$ is:

$$P(\lambda) = -\lambda^3 + \Delta \lambda^2 + (|\alpha_1|^2 + |\alpha_2|^2)\lambda - |\alpha_1|^2 \Delta. \quad (A3)$$

It turns out that

$$P(0) = -|\alpha_1|^2 \Delta, \quad (A4a)$$

and

$$P\left(\frac{|\alpha_1|^2}{|\alpha_2|^2} \Delta\right) = \left(\frac{|\alpha_1|^4}{|\alpha_2|^4} \Delta + \left(\frac{|\alpha_1|^4}{|\alpha_2|^4} - \frac{|\alpha_1|^6}{|\alpha_2|^6}\right) \Delta^3\right), \quad (A4b)$$

which for $\Delta \neq 0$ and $|\alpha_1| < |\alpha_2|$ implies a change of sign. Therefore there is an eigenvalue $\eta$ of modulus $|\eta| < |\alpha_2^2 \Delta/\alpha_1|^2$. The relevant eigenvector is:

$$|\eta\rangle = P_\eta \left(|A\rangle + \frac{\eta}{\alpha_1} |B\rangle - \frac{\eta \alpha_2^*}{(\Delta - \eta) \alpha_1} |C\rangle\right), \quad (A5a)$$

with

$$P_\eta = \left[1 + \frac{|\eta|^2}{|\alpha_1|^2} + \frac{|\eta|^2 |\alpha_2|^2}{|\Delta - \eta|^2 |\alpha_1|^2}\right]^{-1/2}. \quad (A5b)$$

Now, assume $|\Delta| < |\alpha_2|$ and $|\alpha_1| \ll |\alpha_2|$ (which in turn implies $|\eta| \ll |\Delta|$ and then $|\Delta/(\Delta - \eta)| < 2$, one finds that $|\eta/\alpha_1| < |\alpha_1/\alpha_2| \times |\Delta/\alpha_2| < |\alpha_1/\alpha_2|$ and $|\eta^2 \alpha_2^2/[(\Delta - \eta)^2 \alpha_1^2]| < |\alpha_1/\alpha_2| \times |\Delta/(\Delta - \eta)|^2 < 2|\alpha_1/\alpha_2|^2$. Then, under the previous hypotheses one has $|\langle A|\eta\rangle| = P_\eta \rightarrow 1$.

We conclude reminding that (see appendix A of Ref.[24]) if the system is prepared in the initial state $|\psi(0)\rangle = |A\rangle$, then the survival probability has a non vanishing lower bound: $P_A(t) = |\langle A|\psi(t)\rangle|^2 \geq (2 |\langle A|\eta\rangle|^2 - 1)^2$, which of course is close to unity when $|\langle A|\eta\rangle| \approx 1$.

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