Thermodynamics Phase Transition of Regular Hayward Black Hole Surrounded by Quintessence

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In this work, we investigate thermodynamics of regular Hayward black hole surrounded by quintessence. Using the metric of the black hole surrounded by quintessence and the new approach of the holographic principle, we derive the expression of Unruh Verlinde temperature. Hawking temperature and specific heat were derived using the first law of black holes thermodynamics. The plots of the behaviors of these two last quantities show that, the parameter of regular Hayward black hole $\beta$ induces a decreasing of Hawking temperature of the black hole, and that decrease is accentuated when increasing the magnitude of $\beta$ and the normalization factor related to the density of quintessence ($\alpha$). For the lower entropies (small black holes), the black hole passes from the unstable to the stable phase by a first order thermodynamics phase transition. When increasing the entropy (large black holes), a second phase transition occurs. This new phase transition is a second-order thermodynamics phase transition and brings the black hole to unstable state. It results that, When increasing of magnitude of $\beta$, the phase transition points are shifted to the higher entropies. Moreover, the phenomena of phase transitions are preserved by adding the quintessence. Furthermore, when increasing the normalization factor of quintessence, the first order transition point is shifted to higher entropies while the discontinuity point (second-order thermodynamics phase transition point) is shifted to lower entropies.

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I. INTRODUCTION

The solution to Einstein’s field equations discovered by Schwarzschild in 1916 [1, 2], has been the starting point for black hole physics. Following this solution, many scientists have investigated more and more in this field and found other solutions describing charged and rotating black holes [3, 4, 5], as well the perturbed black holes [6, 7]. However, a major problem namely the singularity which is the famous point where all the physical quantities which describe the space-time curvature become infinite characterizes these solutions. This problem of black hole singularity is an acknowledged difficulty in general relativity [8, 9]. To solve this problem, scientists decide to construct the solutions of Einstein’s equation where the metric and the curvature invariants are all regular everywhere [10]. The first of these solutions which does not contain a singularity at the center has been constructed by Bardeen in 1968 [11, 12, 13, 14, 15]. After Bardeen solutions, regular black holes are more investigated and different other solutions like Hayward black hole [16], and modified Hayward black hole [17, 20, 21] were constructed.

Nowadays, through the high-precision observational data it is well known that the accelerating expansion of the universe is due to the mysterious form of energy called dark energy [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46]. This dark energy exists in different forms among which we can cite cosmological constant [31, 32], phantom [33, 34, 35, 36], quintessence [37, 38, 39, 40, 41, 42, 43], quintom [41, 42, 43]. The different forms of dark energy differ from each other by their equation of state $p = \rho\omega_q$ where $\omega_q$ is called state parameter. More information on these different candidates of dark energy are given in previous works [22, 23, 24, 25, 26, 27, 28, 29, 30]. Quintessence is a dark energy candidate for which the state parameter is in the range of $-1 \leq \omega_q \leq -1/3$ [35, 44, 45, 46, 47, 48]. The contradictory effects of a black hole and dark energy on the future of the universe, have pushed scientists to study the consequences of their co-existence in our universe. Studying the influence of dark energy on the behaviors of a black hole is actually investigated by more scientists [35, 44, 45, 46, 47, 48].

Since the work of Hawking and Bekenstein in 1974, black hole is presented as thermodynamic objet [49, 50]. That question of thermodynamics is investigated in the literature using different thermodynamics quantities such as Hawking temperature [6, 44, 47, 48], Tolman temperature [51, 52]. Bekenstein-
Hawking entropy, specific heat \[46, 47, 56, 57\], free energy \[54, 55\], and Gibbs free energy \[56\]. One of the well-known thermodynamics characteristics of Schwarzschild black hole is that its specific heat is always negative. For this the Schwarzschild black hole is always unstable in absence of any perturbation field \[6, 44, 47\]. But in the presence of a perturbation field like dark energy, it is proved that the specific heat is not always negative. So, it exists some region where it can be positive \[6, 44, 57–60\]. This behavior of specific heat is at the origin of the first and second order thermodynamic phase transition phenomena inside the black hole \[6, 44\]. The question to the thermodynamics phase transition is more studied today using different methods: the first law of black hole thermodynamics \[4, 47\], Van Der Waals phase transition \[61\], Geometrothermodynamics \[62\] just to cite a few.

Recently Mehdipour et al have investigated thermodynamics and phase transition of Hayward solutions \[63\]. More recently, Saleh et al have studied the effects of quintessence on the thermodynamics and Phase Transition from Regular Bardeen Black Hole \[64\]. In this paper, we aim at investigating the effects of quintessence on the thermodynamic behavior of Hayward black hole.

The paper is organized as follows. In Sec. II we express the metric of the Hayward black hole with quintessence. In Sec. III we derive the expression of the Unruh-Verlinde temperature and show the effects of quintessence on the temperature of regular Hayward black hole. In Sec. IV we derive the expression of the Hawking temperature and study its behavior with respect to parameters \(a\) and \(\beta\). In Sec. V we establish the expression of the specific heat of the black hole, afterward we show the effects occurring due to the presence of quintessence. Sec. VI is reserved to the conclusion.

II. EXPRESSION OF METRIC OF HAYWARD BLACK HOLE SURROUNDING BY QUINTESSENCE

The metric of Hayward regular black hole in given by \[12, 13\]

\[
dS^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\theta^2 + r^2 \sin^2 \theta d\varphi^2, \tag{1}
\]

where

\[
f(r) = 1 - \frac{2M(r)}{r}, \tag{2}
\]

with \(M(r) = \frac{mr^3}{r + \beta}, \beta^2 = ml^2, l = \frac{1}{A}\), \(m\) is the black hole mass and \(A\) is cosmological constant.

Kiselev proposed a new solution of the metric for static and spherically symmetric space-times in presence of quintessence \[47\]. This new solution depends on the state parameter of quintessence \(\omega_q\). Through this work, it results that the expression of metric function of such black hole surrounded by quintessence is obtained by adding the term \(-\frac{a}{r^{3\omega_q + 1}}\) to that of the black hole free from quintessence \[47, 65\]. The parameter \(\omega_q\) in that term is the state parameter of quintessence dark energy and \(a\) is the positive normalization factor related to the density of quintessence.

With these considerations, the Hayward black hole metric surrounded by quintessence would be deduced from the metric of regular Hayward black hole by adding the term \(-\frac{a}{r^{3\omega_q + 1}}\) on Eq.(3) as follows.

\[
dS^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\theta^2 + r^2 \sin^2 \theta d\varphi^2, \tag{3}
\]

with

\[
f(r) = 1 - \frac{2M(r)}{r} - \frac{a}{r^{3\omega_q + 1}}. \tag{4}
\]

Using this solution, we are going to investigate the thermodynamics behaviors of Hayward black hole surrounded by quintessence dark energy.

III. UNRUH-VERLINDE TEMPERATURE OF HAYWARD BLACK HOLE WITH QUINTESSENCE

Considering a static background with a global time like Killing vector \(\xi^\alpha\), Verlinde derived the expressions of the potential \((\phi)\) and temperature \((T)\) in the form \[2, 7, 67\].

\[
\phi = \frac{1}{2}\log \left( -g^{\alpha\beta} \xi_\alpha \xi_\beta \right). \tag{5}
\]

\[
T = \frac{\hbar}{2\pi} e^\phi n^\alpha \nabla_\alpha \phi, \tag{6}
\]

where a red-shift factor \(e^\phi\) is inserted because the temperature \(T\) is measured with respect to the reference point at infinity. \(e^\phi\) is supposed to be equal to unity at the infinity \((\phi = 0\ at\ r = \infty)\), if the space-time is asymptotically flat. \(n\) is the number of bits on screen. The temperature defined in Eq.(6) is called Unruh-Verlinde temperature.

For static and spherically symmetric black hole, these expressions are reduced to \[2\]:

\[
\phi = \frac{1}{2}\log(f(r)), \tag{7}
\]

\[
T = \frac{\hbar}{4\pi} | f(r)' |. \tag{8}
\]

Substituting Eq.(8) and Eq.(11) in Eq.(8), the expression of Unruh-Verlinde temperature for the regular Hayward black hole surrounded by quintessence is

\[
T = \hbar \frac{mr(r^3 - 4\beta^2)}{2\pi(r^3 + 2\beta^2)^2} + \hbar \frac{(3\omega_q + 1)a}{4\pi r^3 r(3\omega_q + 2)}. \tag{9}
\]

In the absence of quintessence \((a = 0)\), this expression of Unruh-Verlinde temperature is reduced to

\[
T = \hbar \frac{mr(r^3 - 4\beta^2)}{2\pi(r^3 + 2\beta^2)^2}. \tag{10}
\]
In the presence of quintessence \((a \neq 0)\), for \(\beta = 0\), the expression of the Unruh-Verlinde temperature become

\[
T = \frac{\hbar}{2\pi r^2} + \hbar \frac{(3\omega_q + 1)a}{4\pi r^3(3\omega_q + 2)}. \tag{11}
\]

This expression corresponds to the Unruh-Verlinde temperature of the Schwarzschild black hole surrounded by quintessence \(^{44}\). Setting \(a = 0\), this expression become

\[
T = \frac{\hbar}{2\pi r^2}, \tag{12}
\]

which is the Unruh-Verlinde temperature obtained by Y. X. Liu et al for Schwarzschild black hole without any perturbation field \(^{2}\).

**IV. HAWKING TEMPERATURE OF REGULAR HAYWARD BLACK HOLE WITH QUINTESSENCE**

At the event horizon, the metric function \(f(r)\) vanish and the radius of event horizon \(r_h\) is obtained by

\[
f(r_h) = 1 - \frac{2M(r_h)}{r_h} - \frac{c}{r_h^{3(\omega_q + 1)}} = 0. \tag{13}
\]

Using Eq.(13), we can obtain

\[
m = \frac{(r_h^{3(\omega_q + 1)} - a)(r_h^3 + 2\beta^2)}{2r_h^{3(\omega_q + 3)}}. \tag{14}
\]

Using the area law, the black hole entropy \((S)\) is expressed as

\[
S = \pi r_h^2. \tag{15}
\]

From Eq.(14) and Eq.(15), we obtain the expression of black hole mass as function of entropy

\[
m(S) = \frac{\pi}{2} \left( \frac{(S_h)_{\omega_q + 1}^2}{2} - a \right) \left( \frac{2}{\pi^2} \right)^{\frac{3}{2}(\omega_q + 1)} \frac{2S(S_h)_{\omega_q + 1}^{\frac{3}{2} \omega_q + 1}}{2}. \tag{16}
\]

Applying the first law of black hole thermodynamics at the event horizon, the Hawking temperature is given by

\[
T_H = \frac{\partial E}{\partial S}, \tag{17}
\]

where \(E\) is the internal energy of black hole. Ma et al showed that for the regular black hole, the internal energy is given by \(^{15, 64}\)

\[
\partial E = C(m, r_h) \delta m, \tag{18}
\]

where \(C(m, r_h) = \frac{M(r_h)}{m}\).

Substituting Eq.(18) and Eq.(16) in Eq.(17), the Hawking temperature for Hayward regular black hole surrounded by quintessence is obtained in the form

\[
T_H = \frac{1}{4S(S_h)_{\omega_q + 1}^2} \left( \frac{3a(\omega S \sqrt{S_h} \sqrt{S - 2\pi\omega S + 2\pi\omega}^2 + 2\pi\beta^2)(S_h)_{\omega_q + 1} - a}{\pi} \right) + \frac{S^2 - 4\pi^2\beta^2}{\pi} \sqrt{S}, \tag{19}
\]

For \(\beta = 0\), and \(a \neq 0\), the temperature of Eq.(19) become

\[
T_H = \frac{1}{4(\pi S)^{\frac{1}{2}}} \left( \pi S + 3\omega S^{\frac{3\omega_q + 3}{2}} S^{\frac{3}{2}\omega_q} \right). \tag{20}
\]

This is the expression of Hawking temperature for a Schwarzschild black hole surrounded by quintessence \(^{44}\). In the absence of quintessence \((a = 0)\) and for \(\beta = 0\), the expression of Eq.(19) is reduced to

\[
T_H = \frac{1}{4\sqrt{\pi S}} = \frac{1}{8\pi m}, \tag{21}
\]

which is identical to the expression obtained for the Schwarzschild black hole free from any external field \(^{59}\).

The behavior of the Hawking Temperature is explicitly plotted on Fig.(1) and Fig.(2).

![FIG. 1: Behavior of Hawking temperature versus entropy for different values \(\beta\), with \(\omega_q = 0\), \(a = 0\)](image-url)
V. SPECIFIC HEAT AND PHASE TRANSITION OF HAYWARD BLACK HOLE WITH QUINTESSENCE

Knowing the expression of the black hole temperature, the specific heat is deduced by the following relation [6, 44, 58]

\[ C = T_h \left( \frac{\partial S}{\partial T_h} \right). \]  

(22)

Substituting Eq.\,(19) in Eq.\,(22), the specific heat can then be expressed as

\[ C = -2S^2 \left( S^2 + 2\pi S \beta^2 \right) \frac{H(S, \beta)}{D(S, \beta)}, \]  

(23)

where

\[ H(S, \beta) = 3a \left( 2\pi^2 S^2 \omega_q + 1 + \omega_q S \sqrt{\pi S} \right) \left( \frac{\pi}{S} \right)^{\frac{3}{2}} \]  

\[ - 4\pi^2 S^2 \sqrt{\pi S} + S^2, \]  

(24)

and

\[ D(S, \beta) = S^4 - 4\pi S \beta^2 \left( 2\pi^2 + 5\pi^2 S^2 \right) + a \left( 3\omega_q S^4 (3\omega_q + 2) \right. \]  

\[ + 36\pi^3 S^4 (\omega_q + 1)(\omega_q + 2) \left( \frac{\pi}{S} \right)^{\frac{3\omega_q + 1}{2}} \]  

\[ + 6\pi^2 S^2 \beta^2 \left( 6\omega^2 + 7\omega_q + 5 \right) \left( \frac{\pi}{S} \right)^{\frac{3\omega_q}{2}} \right). \]  

(25)

Setting the parameter \( \beta = 0 \), for \( a \neq 0 \), this expression of specific heat is reduced to

\[ C = -2S \frac{1 + 3a\omega_q \left( \frac{\pi}{S} \right)^{\frac{3\omega_q + 1}{2}}}{1 + 3a\omega_q \left( 3\omega_q + 2 \right) \left( \frac{\pi}{S} \right)^{\frac{3\omega_q + 1}{2}}} \]  

(26)

This expression of Eq.\,(26) represents the specific heat for the Schwarzschild black hole surrounded by quintessence [44].

For \( a = 0 \) and \( \beta \neq 0 \), the expression of the specific heat take the form

\[ C = -2S \frac{(S^2 + 2\pi^2 S \sqrt{\pi S}) \left( S^2 - 4\pi^2 \beta^2 S \sqrt{\pi S} \right)}{S^4 - 4\pi S \beta^2 \left( 2\pi^2 \beta^2 + 5\pi S \sqrt{\pi S} \right)}, \]  

(27)

this is the expression of specific heat for regular Hayward black hole.

Setting \( \beta = 0 \), Eq.\,(27) the specific heat is transformed to \( C = -2S \) which corresponds to the specific heat of Schwarzschild black hole free from any kind of perturbation [6, 44, 59]. The specific heat is negative showing that the black hole is thermodynamically unstable [6, 58].

We explicitly plot the behavior of the specific heat for different values of parameter \( \beta \), and quintessence normalization factor \( a \) versus black hole entropy. This is shown on Fig.\,(3), Fig.\,(5) and Fig.\,(6).

![Behavior of specific heat versus entropy](image_url)

FIG. 3: Behavior of specific heat versus entropy (lower entropies) for different values of \( \beta \), with \( \omega_q = 0, \ a = 0 \)

On Fig.\,(3) we see that, when \( \beta \neq 0 \) for the lower entropies it occurs a continuity point where the specific heat graph passes from negative to positive values. This
shows that it appears a first order thermodynamics phase transition in the black hole. It means that, the black hole passes from unstable to stable phase.

Fig. (4) represents the behavior of specific heat versus entropy for a fixed value of \( \beta \neq 0 \) in the presence of quintessence for the lower entropies. It results that, when increasing the magnitude of the normalization factor, the first order thermodynamics phase transition point is shifted to the higher entropies.

We remark on Fig. (5) that, for the higher entropies (large black holes) the parameter \( \beta \) induces a discontinuity point where the specific heat-entropy graph passes from positive values to negative values. We see that the black hole passes from stable phase to unstable phase. Moreover at that last phase, the specific heat remains low compared to that of Schwarzschild black hole. It results that the regular Hayward black hole is more unstable than Schwarzschild for higher entropies. Furthermore when increasing the magnitude of \( \beta \) the first and second order phase transition shifted to the higher entropies.

Fig. (6) shows that, for a fixed value of parameter \( \beta \neq 0 \), setting \( a \neq 0 \), the discontinuity point is always present. Moreover, we see that when increasing of the value of the normalization factor the discontinuity point is shifted to lower entropies.

Through these three figures (Fig. (4) and Fig. (6)), it appears that the quintessence accentuates the instability of regular Hayward black hole for a fixed value of \( \beta \neq 0 \).

VI. CONCLUSION

In this work, thermodynamics behavior of regular Hayward black hole surrounded by quintessence is investigated. Using the metric of regular Hayward Black hole surrounded by quintessence (Eq. (3) and Eq. (4)), and the new approach of holographic principle proposed by Verlinde we have expressed the Unruh Verlinde temperature. Then using the first law of black hole thermodynamics, we derived the Hawking temperature and specific heat for a regular Hayward black hole surrounded by quintessence. The behavior of Hawking temperature is plotted. It results that when increasing the quintessence parameter, that temperature respect black hole entropy is decreasing. The behavior of specific heat versus entropy is plotted for different values of \( \beta \) (see Fig. (4) and Fig. (5)). We see that, the parameter \( \beta \) is responsible for the first and second-order thermodynamic phase transition in black hole. Increasing the magnitude of \( \beta \) moves the phase transition points to the higher entropies. Through Fig. (6) and Fig. (4), it results that when increasing the magnitude of quintessence normalization factor, the thermodynamics phase transition points are shifted to higher and lower entropies for the first and second order thermodynamics phase transition points respectively. The quintessence accentuates the instability of regular
Hayward black hole, even for the higher magnitude of $\beta$. 

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