Conceptualization in solving a geometric-function problem: an effective and efficient process

F Alghadari\(^1\)*, Y Yuni\(^1\), and A Wulandari\(^1\)

\(^1\)STKIP Kusuma Negara, Jalan Raya Bogor KM.24, Jakarta 13770, Indonesia

*Email: alghar6450@gmail.com

Abstract. There has been more than one the solving process of the problem sometimes. It has been named an open-ended context when related to an approach, and in this study, purposed to use a geometric-function problem for analyzing each process that was experienced a student when they were conceptualizing the completion. Three students purposively who was a sample is registered by the senior high school in a science program at the region of West Jakarta, Indonesia. Student solved the problem in this explorative study, there were two choices of completion ways, technical or conceptual, but they used the procedural technique operational. Two conceptualizing process in solving the problem, it was constructing the algebraic representation of the function and organizing concepts. But, there is only one between the other where was effective and efficient of the process based on the perspective of the management theory. The first process was efficient by recognizing the function in algebraic and graphic representation. But, the second process affected the solution ineffective because all three students have been miss controlling the domain function concept even though it is a resource. So, an effective and efficient for learning is when students involving more conceptual knowledge.

1. Introduction

More advanced education level, especially for mathematics subject, so concepts in learning are more complex. For example, the big two of mathematical concepts is about geometry and function. There is a reason why this paper involves its concept. It is about function concept can be treated as a unifying chapter in mathematics curricula [1]. These statements showed stead in the relation among mathematical concept. The base thing is that there is a conceptual bond between function and its geometry-shape in spite of a student frequently not realize its relation [1–3]. Indeed for learning at a low level of school is not stated by explicitly that the one of mathematics chapter is to be one of all or both of the concept, it was learned in the different chapter sometimes, so it is not close the probably that there was a student when they were in high school was also thought by toward the two concepts for using in context separately. It is because they did not generate yet the scheme network both of them for putting together in knowledge structure connection, which is constructed in cognitive space [3]. Knowledge is constructed by each person, for completing concept collection, so it becomes a scheme [2]. This is illustrating that constructed knowledge like filling or completing a puzzle. To do it, there is a cognitive process when unifying one concept with the other. As an example, an activity to construct and get the knowledge is experienced in learning like to solve a mathematical problem [4,5]. Furthermore, the knowledge base and system-based knowledge are doing through conceptualization [6,7].

Knowledge and experiences determine a student steps in solving a problem [5,8]. There is a cause that able to solve a problem and have meaningful experiences after involvement in scheme construction challenges in cognitive space [5]. We refer to the definition which was explained in Dossey [4] that a problem is not a routine so the solution will not be easy to achieve because there are intellectual
challenges. The same thing was expressed by Widodo, Darhim & Ikhwanudin [9], which states mathematical problems are mathematical problems involving non-routine and students who have the desire to solve the problem. Next, mathematical problem meant contains conceptual obstacles, and there is no algorithm or a typical symbolic procedure for immediately find a solution. Thus, solving a problem is getting knowledge by the ways to solve the conceptual obstacle. Because of there is no a procedure, so it will open an opportunity to people who find a completion, an identical or not, that a process in approach either there is a different or not the same for all. In other words, there is a problem which is the completion approach more than one, and for the problem is named by an open-ended problem, so it is able for inviting a different thinking process [5,10]. Because of in above was mentioned that there are two mathematical concepts involved, geometry and function, so the problem in this study is a geometric-function problem. We defined the problem based on conceptual of completion approach, which can be looked from the two concepts; it is stated as a function problem [11], or geometric problem [12]. So, for problem-solving, there is a different cognitive process from two or more a different head in an activity to solve the same one problem [4]. This also shows that there is a kind process in solving phase in a problem-solving activity.

Yeo [10] has been distinguished as an activity and process terminology in problem-solving, and process sequence, which was meant by him is an activity. For the capture, that there are four of phase in problem-solving, it is understanding, devise a plan, carrying out the plan, and looking back [4,10]. These four can be stated as a process, and it is acted in solving a problem which was sets of a process, so it is expressed by activity [10]. In this case, the phase of understanding is recalling the knowledge that has been understood while reading the information contained in the problem [5,13], then at the planning phase but have not applied what is understood. Thus, all the four is not as simple as the mentioned process for each phase, because it can be broken down to be more specific process [4]. For example at the phase of understanding, in this phase and lead up to a solution, so it is necessary several processes such as interpreting, summarizing or concluding, and according to Radmehr & Drake [8,13] that its process is the part of cognitive process dimension.

Furthermore, there are kind process have occurred while in solving and its process is the realization of device a plan. Hanafi [14] explained the planning is a process to set a purpose and determine how to achieve it. Then, continued until the completion is obtained confidently toward the truth because it has passed the phase of looking back.

From the phases of problem-solving, we looked that the producing process for the completion is in carrying out the planning phase. Ward [15] has stated that problem-solving scope a process when conceptualizing the completion and move structurally until arise a finding that becomes knowledge. This conceptualization structure is important to be analyzed because it is especially to one of all steps in general problem-solving. The conceptualization is a terminology which is used to stating there is a process in carrying out the planning phase that informed about the structure of the process for solving a problem. In a study found, reported that the most frequently of an error from student have was wrong conceptualization [16]. Conceptualization has been defined as an ontological idea in the explicit formal specification of context, and a simplified abstract view of representation for the purpose [6]. Or, the process design for producing a concept which the applying to handle all the needed [17]. Here, the completion and solution were intended for all the needed [10]. But, not for each the needed was constructed form an effective and efficient process. The effective and efficient meant is to the purpose achieved managerially with involving resources optimally, and if it is efficient, so it tends to an effective [14]. An about the efficiency of the completion is when students show various procedures and their ability to be flexible in making choices, but in the report that was still much unknown effectiveness and efficiency of the strategy [11]. In the other study, reported that converting process among the representation was acted effectively if a student able to inverse of its conversion systematically [1]. Thus, in solving a problem, there is self-management by the solver who has fully controlled in the process.

Regarding the conceptualization process described above, there is knowledge of management theory, which is purposed inside it. Hanafi [14] defined management is a coordination of all resources was passed planning, organizing, directing, and controlling process so that to achieve the purpose effectively and efficiently. Furthermore, planning is to determine the ways of a purpose achievement, organizing is
to determine what to be work, directing is to the optimization process, and controlling is involving a monitoring process and ensure that the purpose was achieved. In the other hand, Dossey [4] and Radmehr & Drake [13] stated that there is kind of different factors which affected in problem-solving of students activity, it is resources, heuristics, control, and belief system. Next, explained that the resource meant is formal and informal knowledge scope about fact and routinely; heuristics refers to a strategy and technique for an approach of the problem; and the control is involving a used method by each for checking their solving process of the problem and observe the partial result to decide an action for the advanced of problem-solving, and determine how and when to use an available resource and heuristics.

By the citation above, so conceptualization process also operated a metacognitive knowledge [4], and has been stated in Radmehr & Drake [8] literature that there is a knowledge dimension, like metacognitive knowledge, apart from the cognitive process dimension. Moreover, there was a function of resources so by the solving process, managerially then get an effective and efficient purpose [14]. Therefore, the conceptualization process for the completion of a problem and optimization resources theoretically is an efficient process. Checking for the efficiency of processing in solving a problem is only can be assessed through representation from a student, so the abstract nature of mathematics underlined it the need for the inspection [1]. That can be monitored by observing students solve a problem accompanied with their explanation about all process which written [16]. The big purpose of this is to understand the condition of students learning or to all of them learn particular concept based on mostly in general of students thinking, it is because of education must be adjusted by how student ways to conceptualize mathematical concepts so become aware of the obstacles and misunderstandings that are experienced by them [1]. Moreover, all information in the citation above also explained the method to get data in this study. That is acted by the one purpose that this study is to analyze the effectiveness and efficiency of the conceptualization process, which was experienced by students when they solve a geometric-function problem.

2. Method
This is an explorative study at selected students purposively with they are in category high-level mathematical ability. For their, has been given a geometric-function problem, which is an open-ended context, using a test within three numbers. Firstly, we faced seven students from the one of national high school in West Jakarta for becomes a respondent. But, based on their performance, there were four students among their data was not a part in the analysis because of not fulfilling the need that the answer must show a complete process of the completion along with the interpretation based on their writing. Therefore, three completion from students would be analyzed and signed by the initial name of them; it is AN, NI, and PR. A problem on the test which was used in this study is to sketch a derivative function graph of the graph in Figure 1 that was adapted from Tobin [12], and the following is the graph in question.

![Figure 1. A function graph](image)

Figure 1 is the three models of a function graph, which is each of (a), (b), and (c) will be constructed by all three students is the derivative function graph. The process sequences of the completion and the
interpretation based on those process are data which is collected to be analyzed the resources, or mathematical concepts were applied. Then, by the base of management process theory that there are several steps which are noticed like planning, organizing, directing, and monitoring at the resources [14], so arise an effective and efficient completion which is stated by comparative technique.

3. Result and Discussion

Based on data analyze, knowledge and applying concept ability from each student becomes a primary resource for conceptualizing process efficiently and the fact that implemented concept has been students action partially.

3.1. Mathematical concept and conceptualization process by AN

Solving problem in Figure 1(a), as planning to construct an algebraic function formula of the graph, AN determined the concept of the exponential function graph as a classification of figure model based on recognizable shape. But, the model has not been appropriate yet then for its function model was implemented a geometry transformation to make a shape of the model is fit, and the end is generating an algebraic function formula. Next, the student-directed the last model of function by implementing the limit concept of tend to infinity for defining an example of the graph, so that is obtained algebraic representation, which is involving its asymptote. The differentiation algebraic function procedure also directed AN to find the algebraic description of the derivative function before its graph is sketched. Anyway, there is no an any action by AN for critiquing because of the graphic 2(a), like the Figure below, which was formed by him has been passed a negative of x-axis so that the domain concept is not in monitoring and that is affecting effectiveness the completion.

Figure 2. Sketch of derivative function by AN

Figure 2 above is three models of graphic for the completion of the problem in figure 1, and each of the Figure corresponds to the alphabet. Now, is the description solving process for problem 1(b) so that AN until sketching Figure 2(b). It is clear that there is a different mathematical concept knowledge...
would be implemented to the problem of Figure 1(b). The conceptual idea of the completion still the same. Here, two coordinate points in the Figure as an object to be conceptualized with an elliptical equation. The model shaped and fourth-degree of power in ellipse equation as the concept to produce algebraic representation until he wrote the equation and implemented an explicit function form and implicit differentiation before sketch the graph 2(b). Now, is the part of graphical construction Figure 2(c). For the purpose, AN organized the point of intersection between the graph and the y-axis, periodical function $\cos x$, and $-x$ has been added in the algebraic representation because of the graph more decrease. Not because of we didn't agree with the completion of AN for the last two, but for Figure 2(b) that there is a sketch an inappropriate at the domain of function because actual it should be equal zero. While for Figure 2(c), AN didn’t show the existence of $a$ and $b$, which has been contained in a problem. We must acknowledge that there was a solving process which is not wrong, in spite of in last of a part that the student has been missing process of monitoring an involved concept, it is a domain of the function, so the conceptualization process by AN is not viewed overall as an effective in applying resources.

3.2. Mathematical concept and conceptualization process by NI

For construction algebraic formula of the function graph in Figure 1(a), NI exploited model shape of a rational function; it is by variable $x$ at the denominator and a number as the numerator. However, using the concept was not appropriate yet with a graphic shape of the problem. By geometry transformation, so for the rational function and then it is translated. As the impact, there was adding a real number on the algebraic formula of the first rational function. However, unfortunately of NI because the adding its number was not defined detail as a specific number for definite domain. Moreover, the rational function which was formulated in the fact that it does not pass the origin of the coordinate system so that there is something different with the derivative function graph which is wanted as the right answer. This is talking about the effectiveness of the conceptualization. The consequence is an algebraic representation formed from its process; then it was differentiated and sketched like Figure 3(a), it does not exactly produce the completion in the correct answer.

Figure 3 above is each of a derivative function graph model, which was constructed by NI for the completion of the problem in Figure 1. At the following is a description of the solving process of problem 1(b). NI’s conception toward the Figure in the problem was an arc that has been intersected x-axis more curved than which has been intersected y-axis. Therefore, the degree of polynomial concept was organized at the equation that has been conceived like the algebraic formula of a circle, but an arc which is more curved to be defined for a variable to the power of two, and the other is the power of four, so that was obtained an algebraic representation of the graph 1(b). By those representations, continued with the conversion process to be an explicit function which has been differentiated toward $x$, and then it has been sketched on a Cartesius coordinate like in Figure 3(b). And after this is the description for the problem 1(c) which was interpreted by NI. He directed the construction of the algebraic function formula to the graph of trigonometry $\cos 2x$ that has been added to $-x/a$ by the reason that the graph had been more decreasing. All its design was being organized so that an algebraic representation was formed, which then it has been differentiating, and the result has been sketched like the graph in Figure 3(c). But, NI has not seen $a$ and $b$ in Figure 1(c) a thing as useful because of his completion in a formula of trigonometry function context. However, it is not different from the previous student because NI has also experienced the monitoring disorientation at a domain concept. All three graphic which has been sketched involve the curve in the differential result out of the interval and based on the problem that it is cannot be justified, although for the other interval is suitable with the correct answer. Thus, it is also influencing the effectiveness of the completion.
3.3. Mathematical concept and conceptualization process by PR

For problem 1(a), PR applied the exponential function formula as the effort to convert a graphic representation to algebraic representation. An exploration of several models of graphic shape to find suitability with the graph of the problem, his action by geometry transformation like a reflection toward each of two axis of Cartesius coordinate and continued to a translation. The last phase after an algebraic function formula has been obtained from the translation result and then generalized to use the $e$ number. The problem for this alphabet is finished after the function was a differentiation toward $x$ and sketched like Figure 4. Figure 4 is the three model of derivative function graph for each a function graph of the problem in Figure 1. At the following description is the conceptualization process an algebraic representation of Figure 1(b). PR defined the general form of the graph like circle equation, with the variable $x$ and $y$ for each to the power of four, with a denominator respectively for $x$ and $y$ is $a$ which is the abscissa an intersection point the curve and $x$-axis and $b$ which is the ordinate an intersection point the curve and $y$-axis. After the value of $a$ and $b$ has been substituted by the abscissa and the ordinate on the equation, and that obtained equation was converted to be an explicit function form to be differentiated and then continued with making those sketch, so that look like in Figure 4(b).

Furthermore, there is a different from two other students, PR planned an algebraic function formula of graph 1(c) by polynomial concept to the power of four because the curve had had three stationary points. By organizing $a$ and $b$ in the Figure and the inflection point concept into his planning, PR produced a derivative function of algebraic formula directly without necessary to construct it in a general form of the fourth degree of the polynomial. Anyway, equally is like which has occurred with the two other students that there is informed in the problem but not utilized. The solving process for this last one number in PR's action used a calculus concept as the resource that is closely related with a geometric-function problem so that the problem space and managerial which has been acted by him is different from two other students. Related to that condition, Ward [15] stated that find the new of problem space or the representation occurred because of failure to find the operators (algorithm or heuristics) which
produces progress representation. But, all three completion has been made by him as shown as the equal disorientation because of not consideration to domain function concept.

Figure 4. Sketch of derivative function by PR

3.4. Procedural technique to conceptualize for efficiency conversion process

Findings in this study have shown that student’s performance in the completion is to select a recognized function in an algebraic and graphic form. Not only a student, because the expert has also still there is that focused on the formula repertoire in arranging a graph with paper and pencil [11]. This condition is a contrast with the one result of the study that their respondent was more select to manipulate a concept in the graphic shape and translate among representations [1], and actually that it is a solving process that was explained in Tobin [12]. However, what is a process becomes an option of each student in solving is about a scheme of concept in their cognitive structure. The conceptualization process completion of the problem showed about the difference between algorithm and heuristics, and ideas about sub-goals [11,15]. The different meant is the relation with two dimensions in knowledge and cognitive process [8,13]. Apart from what is a mathematical concept applied by all three students in their completion, they have concepted in the way to construct an algebraic representation of the function, so that there is a conversion process from the graphic image in the problem presented, and its consequence is more involved procedural knowledge. Its fact corresponds to a citation which stated that there is a student’s wish to find an equation of the function that is only given by graphic shape before making a derivative function graph [2].

Effectiveness of the converting that was developed from a source to target is actually to reveal a cognitive difficulty that arises from the need to do flexible and competent translation back and forth between different kinds of mathematical representations [1]. Therefore, the derivative completion of a problem which is much to imply procedural understanding then basically that it is a sign of students did not understand a material of derivative conceptually, so the student fills it by procedural understanding as an essential one [3]. Its several statement correspondence to the other one of finding in this study that all three students were missing one of the relevant basic concepts in their solving process and its concept.
is the domain of a function. But, all about the finding, its discussion is loaded after this section. Paralleling the present findings with solving problem context and management theory, by cited to two skill in Hanafi [14] literature, it is a conceptual and technical, about a different emphasis for each of the two, has shown the level of solver ability in processing the completion. Next, by conceptual ability so the conceptualization of the completion is organized based on related mathematical concepts which are looked comprehensively and dependency among concept organizations, while the technical ability is the completion was concepted based on a mathematical concept which is processed operationally. Thus, in general, that students concepted the completion of a geometric-function problem in this study had operationalized concepts by procedural technique.

Actually, the problem can be looked at the result from searching for different representation of the probably one set representation [15], ways of look at this which are meant for a conceptual or technical skill [14], and selecting to solve a problem by an approach is caused there is an emphasis factor in students learning experiences before [7,11]. However, confirmed in a citation that stated about students ability in solving a function problem had been affected by characteristics task in the school which tends to direct for using standard procedures, in difficulties experiences with function concepts when using a graphical representation in problem-solving, representational flexibility for doing a conceptual and procedural interaction [1]. Further, the learning emphasis to an algebraic representation and disregard graphical representation is the one situation which caused most students solve a differential problems easily but do not understanding the correct concept about derivative graphic representation and the correlation between algebraic and graphic representation, so that the basic for students learn a mathematical object complexity in calculus is to use and translate among different representation [2]. Anyhow, it is no why because all three students able to bridge a conversion process between algebraic and graphics function representation, it is by the way concepted a construct a relation between graphics model and its algebraic function formula used an application of mathematical concept, so this is also the different finding with the citation that had been reported [1]. Because of students able to convert graphics to algebraic representation, then its conceptualization process is the efficient one of all other processes.

3.5. Missing in Domain Concept Affected toward An Effective Process

Based on three student’s completion, was true that all of them always passed two next processes after the algebraic representation of the graph had found, they have determined the derivative function of algebraic representation and then sketched its derivative function graph. However, basically that these two processes have been shown by them and it has not been a problem in the conceptualization process. Procedurally and correspondingly with all three students approach, a monitoring process in management theory context should have been acted after an algebraic representation was going constructed, and before the representation of function have been operated on the differential concept. Certainly, the construction problem algebraic representation finished, and it is one of some of the conceptualization process. Despite successful in solving a problem is a sign a coherent of conceptual understanding which was constructed by definition correctly and flexibility in using multiple representation [1], but if one of all idea has been ignored, so it was the sign that the derivative concept may be not absolutely understood rationally then they probably don’t really understand what is the conceptual meaning of the derivative concept [3,7,16]. For evaluating that what students understand a mathematical idea or not, is by checking the representational model which was students choose for use in the construction of concept, that is total of cognitive structures related to the concept [1,7]. Therefore, this analysis study does not discuss two processes which were shown by students that it is not a problem. However, there is more important as an investigation finding toward representation models, that is because of all three students has been miss monitoring control on involved one basic concept, and it is a domain of the function. As the impact is on the efficiency of the derivative function graph that was sketched by them for the answer [16]. Dossey [4] stated that controlling is very important to check the missing link in the solution development where it will determine the next step of a phase in problem-solving.

At before was explained that the mathematical concept related to the gradient of the function had presented a challenge for students, and learning calculus indeed needed a mathematical understanding significantly for shaping a concept image in detail and connected [7]. The other one finding in this study, concept image and internal conceptual system of student still fragmented and affected to one process
becomes not effective, so the solution is considered. Therefore, one of this conceptual process, that is organizing concepts, and because of one concept has been not applied so result in the solution is not efficient. Based on some findings, then we conclude how about the conceptualization process of the completion, especially to a geometric-function problem that an effective and efficient process for a student’s learning is when they more involve a conceptual knowledge. This study finding is a detail of checking result a student’s actual thinking process, and despite the scope is limited, so there is a need to study further, but its checking had become one recommendation in the study by Yeo [10]. Furthermore, despite the information processing approach in conceptualizing the completion problem was only developed for the sequence problem which is in limited context coverage, but the principles can be applied for the other situation widely [15], and this exploration study is the part of a research for learning approach especially derivative concept to students.

4. Conclusion
Based on the result and finding in this study, there is two conceptualization process related to effectiveness and efficiency in solving the problem. It is constructing an algebraic representation of the function and organizing relevant basic concepts. The process which is in the first mention was acted by students efficiently by recognizing the function in algebraic and graphic form. In progress, student’s performance was operationalizing concepts by procedural technique. However, it is not the same thing with the process which is mentioned in the next, because the monitoring process in management theory context necessarily was acted after the algebraic representation has been constructed. But, each student has been miss controlling at the one of involving a basic concept, that is domain function, so it affected to be an inefficient solution. These findings are about the actual process of a student thinking toward the problem, which the principles can be applied in other situation, but containing a limited context coverage so the need is required for further study.

5. References
[1] Panourea A, Michael-chrysanthou P, Gagatsis A, Elia I and Philippou A 2016 A Structural Model Related to the Understanding of the Concept of Function: Definition and Problem Solving Int. J. Sci. Math. Educ. 15 723–40
[2] Borji V, Font V, Alamolhodaei H and Sánchez A 2018 Application of the Complementarities of Two Theories, APOS and OSA, for the Analysis of the University Students’ Understanding on the Graph of the Function and its Derivative 14 2301–15
[3] Sahin Z, Yenmez A A and Erbas A K 2015 Relational Understanding of the Derivative Concept through Mathematical Modeling: A Case Study Eurasia J. Math. Sci. Technol. Educ. 11 177–88
[4] Dossey J A 2017 Problem Solving from a Mathematical Standpoint Educational Research and Innovation: The Nature of Problem Solving Using Research to Inspire 21st Century Learning: Using Research to Inspire 21st Century Learning ed C Benô and F Joachim (Paris: OECD Publishing) pp 59–72
[5] Bartholomew S R and Strimel G J 2018 Factors influencing student success on open-ended design problems Int. J. Technol. Des. Educ. 28 753–70
[6] Guarino N, Oberle D and Staab S 2004 Handbook on Ontologies Handbook on Ontologies (Berlin: Springer Berlin Heidelberg) pp 1–17
[7] Nagle C, Moore-russo D, Viglietti J and Martin K 2013 Calculus Students’ and Instructors’ Conceptualizations of Slope: A Comparison Across Academic Levels Int. J. Sci. Math. Educ. 11 1491–515
[8] Radmehr F and Drake M 2018 An assessment-based model for exploring the solving of mathematical problems: Utilizing revised bloom’s taxonomy and facets of metacognition Stud. Educ. Eval. 59 41–51
[9] Widodo S A, Darhim D and Ikwanudin T 2018 Improving mathematical problem solving skills through visual media Journal of Physics: Conference Series vol 948
[10] Yeo J B W 2014 Mathematical Investigation Proficiency among Singapore Secondary School Students: An Exploratory Study Southeast Asian Math. Educ. J. 4 3–21
[11] Kop P M G M, Janssen F J J M, Drijvers P H M, Veenman M V J and Driel J H Van 2015 Identifying a framework for graphing formulas from expert strategies J. Math. Behav. 39 121–34
[12] Tobin P 2007 International Baccalaureate Mathematics Standard Level ed F Cirrito (Australia: IBID Press)
[13] Radmehr F and Drake M 2017 Revised Bloom’s taxonomy and integral calculus: unpacking the knowledge dimension Int. J. Math. Educ. Sci. Technol. 48 1206–24
[14] Hanafi M 2015 Manajemen (Tangerang Selatan: Universitas Terbuka Press)
[15] Ward T B 2012 Problem Solving Handbook of Organizational Creativity ed M D Mumford (USA: Academic Press) pp 169–88
[16] Veloo A and Krishnasamy H N 2015 Types of Student Errors in Mathematical Symbols, Graphs and Problem-Solving Asian Soc. Sci. 11 324–34
[17] Nagle C, Moore-russo D and Viglietti J 2013 Calculus students’ and instructors’ conceptualizations of slope: A comparison across academic levels Int. J. Sci. Math. Educ. 11 1491–1515