Radiation from Relativistic Electrons in a Thin Crystal Target as Complementary X-Ray and Gamma Source at Synchrotron Light Facilities

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Abstract. Concept of using crystal targets as complementary X-ray and gamma ray sources at synchrotron light facilities is considered. Calculations of channeling radiation, parametric X-radiation and bremsstrahlung from relativistic electrons in the crystal targets are performed for parameters of synchrotrons operating in Moscow (Kurchatov Center for Synchrotron Radiation, KCSR) and Karlsruhe (ANKA).

1. Introduction
The relativistic electrons penetrating through a thin crystal target emit several kinds of radiation (in X-ray and gamma region): coherent and ordinary bremsstrahlung (CBS and BS), transition radiation (TR), parametric X-radiation (PXR), channeling radiation (CR).

The BS and TR produced in internal amorphous thin target have been considered recently as principal EUV and hard X-ray generation mechanisms in the table-top synchrotron storage rings [1–2], operating at beam energy 6–20 MeV.

Similar concept probably can be extended to more high energy storage rings, e.g. operating at 2.5 GeV:
- Kurchatov Center for Synchrotron Radiation in Moscow (KCSR) [3],
- ANKA at FZ Karlsruhe [4].

The crystal target can be used both as internal target in the main storage ring and photon production target in the booster synchrotrons or injectors (80-100 MeV linac at KCSR and 53 MeV microtron at ANKA).

So, the goal of the present work is to perform calculations of CR, PXR and BS for beam energies indicated above to optimize the production of photons in the thin Si, C, Ge crystal targets.

2. Theory
Calculation of channeling radiation (CR) from relativistic electrons under (111) planar channeling in a Si and LiF crystal is performed using both classical and quantum methods. The quantum approach is used when the number of allowed transverse energy levels of channeled electron is not large. The classical treatment can be applied for the greater energies of electrons. We developed the new computer code for solving these problems. It is called “Basic Channeling with Mathematica (BCM)”
and will include a set of packages to make symbolic calculation of the planar channeling phenomena [5].

2.1. Classical description of CR

In the classical description, for every individual electron trajectory $x(t)$ ($x(t)$ is the electron transverse motion coordinate perpendicular to the channeling planes), the CR intensity spectrum is calculated using [6]:

$$
\frac{dW}{d\omega d\theta} = \frac{e^2 \omega}{c^2 T^2} \sum_{l=1}^{\infty} \Theta[1-\eta] \left( \eta^2 - \eta + \frac{1}{2} \right) |\hat{\chi}_{\beta}|^2 ;
$$

(1)

$$
\eta = \frac{T \omega}{4\pi\gamma^2 \omega} = \frac{2\pi}{T} \hat{x}_\beta = \int_{-T/2}^{T/2} \hat{x} e^{i\omega t} dt.
$$

Here, $\Theta$ is the Heaviside theta function, $\gamma$ is the relativistic factor, $T$ is oscillation period of electron in the planar channel, which depends on the initial transverse energy, $\hat{x}_\beta$ is the Fourier component of electron velocity, $l$ is the harmonic number. To obtain electron trajectory $x(t)$ we numerically solved classical equation of motion of relativistic electron. The periodic planar potential used in the equation of motion was obtained as in our previous works [7-8]. Equation (1) is obtained assuming: $\nu_e \ll c << \nu_\parallel c << \gamma$, $\nu_\parallel$ is the velocity of transverse motion, $\nu_e$ is the average velocity of electron along channeling planes. The Fourier components of velocity $\hat{x}_\beta$ have been defined numerically using the calculated trajectories.

2.2. Quantum description CR

In the quantum description, the spectral distribution of CR intensity is described by the formula [9]

$$
\frac{dW}{d\omega d\theta} = \frac{\hbar \omega}{d}\frac{dW_d}{d\omega} = \sum_{j} \sum_{f,j} dM_{j}\omega |\sum_{j} \sum_{f,j} M_{j}\omega| 1 - 2 \frac{\omega}{\omega_m} + 2 \left( \frac{\omega}{\omega_m} \right)^2 P_l(k_j, k_\alpha),
$$

(2)

$\alpha$ is the fine-structure constant, $M_{j}$ are the transition matrix elements, and $\omega_m = \hbar \Omega_f (1 - \beta_\parallel)$, $\beta_\parallel = \nu_\parallel c$, $\hbar \Omega_f = E_j(k_j) - E_j(k_\alpha)$, $E_j(k)$ is the transverse energy of electron, $k_j$ is transverse wave vector of electron in allowed band, and $P_l(k_j, k_\alpha)$ is the initial population of $l^{th}$ band:

$$
P_l(k_j, k_\alpha) = \frac{1}{d_p} \left| \int \exp(-ip\theta \partial / \hbar) \phi_l(k_j, x) dx \right|^2 .
$$

(3)

Here, $p$ is the incident electron momentum, $d_p$ is the interplanar spacing. The wavefunctions $\phi_l(k_j,x)$, which describe the transverse motion, have been calculated for planar periodic potential from the Schrodinger equation with relativistic mass $m^* = \gamma m_e$ and using the Bloch boundary conditions. The matrix elements $M_{j}$ were calculated without use of dipole approximation.

2.3. Parametric X-ray radiation

The parametric X-ray radiation is emitted in the direction satisfying the Bragg conditions, i.e. at large angle with respect to the direction of particle momentum. The angular distribution of emitted photons per unit crystal length is described by the formula obtained in Ref. [10], using 2-wave theory of X-Ray diffraction:

$$
\left( \frac{d^3N}{d\theta_d d\theta_i d\varphi} \right)_{\text{pXR}} = \frac{\alpha \omega_B}{4 \pi \omega^2} \left( \frac{\theta_s^2}{4(1 + W_{\alpha,i}^2}) + \frac{\theta_x^2}{4(1 + W_{\alpha,i}^2)} \right),
$$

(4)
\[ W_{\sigma} = \frac{1}{2|\chi_g|P_\sigma} \left[ \theta_{\xi}^2 + \theta_{\eta}^2 + \theta_{\text{kin}}^2 - \frac{|\chi_g|P_\sigma}{\theta_{\xi}^2 + \theta_{\eta}^2 + \theta_{\text{kin}}^2} \right], \quad (\sigma = ||, \perp). \]  

Here, \( \theta_x \) and \( \theta_y \) are the angles of emission measured from the direction satisfying the Bragg condition, \( \theta_B \) is the Bragg angle, \( \omega_B = |g|/(2\sin \theta_B) \) is the frequency of virtual phonons satisfying the Bragg diffraction condition, \( g \) is the reciprocal lattice vector, \( \theta_{\text{kin}}^2 = \gamma^2 - |\chi_0|, \chi_g \) is the Fourier component of the dielectric susceptibility \( \chi \) and \( \chi_0 = \varepsilon_0 - 1 \).

**Figure 1.** The CR spectra calculated for Si crystals using quantum approach (electron beam energy \( E = 53 \) MeV), and classical approach (electron beam energy \( E = 100 \) MeV and \( E = 500 \) MeV), and for different incident angles \( \theta_0 \) with respect to the (111) planes (\( \theta_C \) is the Lindhard angle).

2.4. Bremsstrahlung

The bremsstrahlung spectra were calculated using well known free software package GEANT4 [11]. This package provides the energy loss of electrons and positrons due to the radiation of photons in the field of a nucleus. In the amorphous target the production of a photon of energy \( \hbar \omega \) is occurred due to bremsstrahlung. The probability of this process is estimated according to the appropriate cross section. For calculation of electrons transportation through target Method Monte-Carlo is used.
3. Results and discussion
We calculated of CR and BS spectra and PXR angular distribution using the computer code mentioned above.

- It is seen that the CR spectrum strongly depends on electron beam energy and the beam alignment with respect to the crystallographic planes [10].
- The CR spectra consist of large number of peaks which correspond to transition between large numbers of transverse energy levels if we use quantum approach (Fig. 1-3). For classical approach CR spectra contain two maxima which correspond to electron motion in two potential wells (Fig. 1). CR spectrum strongly depends on the beam alignment with respect to the crystallographic axes or planes (Fig. 1, 2).
Figure 4. The CR spectra (solid lines) and the BS spectra (dashed lines) calculated for LiF crystals using quantum approach and electron beam energies $E = 53, 500, 2500$ MeV, for incident angles $\theta_0 = 0$ with respect to the (111) planes.

Figure 5. The angular distribution of PXR for Si and LiF crystals (electron beam energy 53 MeV and 500 MeV) for the angle of incidence with respect to the crystallographic planes $\theta_0 = 0$, in vicinity of the Bragg angle (emission angle) $\theta_0 = \pi/8$. The photon energy is for Si 5.17 keV, for LiF 6.97 keV.

- The intensity of BS does not depend on the energy of incident electron beam and shape of the spectra changes with increasing energy of primary electron beam, mainly due to appearance of hard part of radiation spectrum (Fig 3.).
- Intensity of CR in contrast to BS increases with increasing of the electron energy and BS becomes negligibly small for energy electron beam greater than 500 MeV.
As one can see from Fig. 3,4 the intensity of BS much less then intensity of CR for low energy photons so these two kind of radiation may be separate from each others.

PXR in contrast to CR is emitted at large (Bragg) angle with respect to reflecting crystallographic planes and consists of several spectral peaks with photon energies ranging from several keV up to several decades keV. PXR peak positions do not depend on the electron beam energy while the shape and width of angular distributions of PXR is very specific in vicinity of the Bragg angle (see Fig.5).

4. Conclusions
In this paper we have presented the results of calculation of PXR, CR and BS from relativistic electrons passing through the crystal. for electron energies typical for the KCSR and ANKA injector and booster synchrotrons. For calculation of CR and PXR a new computer code BCM 1.0 was used [5]. The results of calculations show that CR, PXR and BS from relativistic electrons penetrating through the crystal indicate on possibility of creation of X-ray and gamma-ray source complementary to synchrotron radiation at existing of synchrotron light facilities. The PXR and CR can be used for generation of the photons from keV to MeV regions while BS can be used for generation of more hard photons. The crystal target can be used in the booster synchrotrons or injectors during beam lifetime in the main storage ring (about 20 hours) or as internal target in the main storage ring at the end of beam life-time.

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