Recent Advances in Studies of Current Noise

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Summary. This is a brief review of recent activities in the field of current noise intended for newcomers. We first briefly discuss main properties of shot noise in nanostructures, and then turn to recent developments, concentrating on issues related to experimental progress: non-symmetrized cumulants and quantum noise; counting statistics; super-Poissonian noise; current noise and interferometry.

1 Introduction

Current noise in the last decade proved to be an efficient means of investigation of nanostructures. Currently, it is a broad field, with over a hundred groups, experimental as well as theoretical, and is actively developing. This short review presents a brief introduction to the field, concentrating on recent developments. There is a large amount of literature available. General introduction to noise in solids can be found in the broad scope book by Kogan [1]. An extensive review on shot noise was written by Büttiker and the author [2]. A collection of shorter review articles intended to summarize the main directions of the field was published as proceedings of the NATO ARW held in Delft in 2002 [3]. These publications cover the field comprehensively, and there is no need to repeat all the material. This article is intended for researchers wishing to enter the field. We will only give a brief introduction to the subject of shot noise, turning then to recent developments of the field. The size of this article makes it impossible to describe all research. We will therefore self-impose the following limitations. First, we only consider papers published in 2000 and later — everything before that date can be found in Ref. [2]. Second, the choice of topics is mainly related to the experimental breakthroughs. Many papers of excellent quality will have to stay outside the framework of this article. Additionally, on purpose we do not discuss here very important issues of entanglement and the theory of measurement: In our opinion, they are better discussed in connection with properties of qubits, and we do not have enough space here for a comprehensive review of the field.

Now we give a very brief overview of results well established in the field. Current through any nanostructure fluctuates in time. There are at least two reasons for these fluctuations: (i) thermal fluctuations of occupation numbers in the reservoirs;
(ii) randomness of transmission and reflection of electrons. At equilibrium, only
the former are important, and one has Nyquist-Johnson noise. We define the noise
spectral power,

$$S(\omega) = \langle \delta \hat{I}(t) \delta \hat{I}(t') + \delta \hat{I}(t') \delta \hat{I}(t) \rangle_\omega,$$

where $\delta \hat{I}(t) \equiv \hat{I}(t) - \langle \hat{I} \rangle$, $\hat{I}$ is the current operator, and the averaging is both
quantum-mechanical and statistical over the states in the reservoirs. Nyquist noise
$S(0) = 4Gk_B T$, with $G$ being the conductance of a nanostructure, just follows from
the fluctuation-dissipation theorem.

At zero temperature, only fluctuations due to the randomness of scattering are
important. They are known as shot noise and can be expressed \cite{4,5,6,7} in terms of
the transmission eigenvalues $\{T_p\}$ of the nanostructure, where $p$ label the transport
channels,

$$S(0) = 2se^3 |V| \sum_p T_p (1 - T_p),$$

and $2s$ is the number of spin projections. Fully open ($T_p = 1$) and fully closed
($T_p = 0$) channels do not produce any noise, since scattering is not random: electrons
are either fully reflected or fully transmitted.

To appreciate Eq. (2), we need a reference point. The latter was provided as
early as 1918 by Schottky. Consider the Poisson process: electrons enter a reservoir
in a random and uncorrelated fashion. In other words, the current is expressed as
$I(t) = e \sum \delta(t - t_n)$, where $t_n$ are random uncorrelated quantities, with the average
interval $\tau$ between arrivals of consequent electrons. The average current is $I = e/\tau$,
while the current noise for this process is $S(0) = 2eI$ and does not depend on
frequency. This Poisson value $S_P = 2eI$ gives us the reference point. Taking into
account Landauer formula for conductance,

$$G = \frac{2e^2}{2\pi h} \sum_p T_p ,$$

we see that the actual noise \cite{2} is always suppressed with respect to $S_P$. This sup-
pression is characterized by the Fano factor $F \equiv S(0)/S_P$, which can vary between
zero and one.

Note that Nyquist and shot noises are in fact the two limiting cases of the same
phenomenon. One can express this noise in terms of the transmission eigenvalues.
There are other types of noise, which are always or often present in nanostructures,
and which are not related to transmission properties. The most common example is
low-frequency noise, proportional to the square of the applied voltage and inversely
proportional to the frequency. The origin of this noise is not universal and usually is
attributed to slow motion of impurities in the substrate. Such transport-unrelated
noises are not considered here.

Let us now mention basic properties of shot noise. More details can be found in
Ref. \cite{2} and references cited there.

- For basic types of nanostructures, the Fano factor assumes universal values:
  $F = 1$ for a tunnel barrier, $F = 1/2$ for a symmetric double barrier, $F =
  1/3$ for a diffusive wire, $F = 1/4$ for a symmetric chaotic cavity, $F = 0$ for a
ballistic conductor (for instance, a quantum point contact in the plateau regime).
These results have been derived theoretically by various means and confirmed
experimentally.
• These results are classical; quantum mechanics only enters for calculation of transmission eigenvalues and in quantum (Fermi) statistics of electrons. For this reason, many results can be reproduced by purely classical methods, based on Boltzmann or rate equations with Langevin random forces.

• Notion of noise can be generalized to multi-terminal conductors. Current correlations calculated at different terminals are always negative. This follows from the fact that electrons obey Fermi statistics.

• The Fano factor is proportional to the electron charge. This concept can be generalized to the situation when current is carried by fermionic quasiparticles. For instance, transport between a normal metal and a superconductor for voltages below the superconducting gap is only possible by means of Andreev reflection, and is associated with the charge transfer in quanta of 2e. This gives $F = 2$. Another example is transport in a quantum Hall bar over a barrier, which is associated with the charge $e/q$ for the filling factor $\nu = p/q$. The Fano factor in this case becomes $F = 1/q$, which also has been measured experimentally. Generally, shot noise can be used to determine the quasiparticle charge.

• Effect of interactions of shot noise can be very different. Dephasing does not have any effect on noise, unless, of course, one discusses a phase-sensitive effect like Aharonov-Bohm oscillations. Electron-electron interactions result in heating, increasing the Fano factor, for instance, to the value $F = \sqrt{3}/4$ in diffusive wires instead of $F = 1/3$. Electron-phonon interactions suppress shot noise down to zero, since the energy is taken out of the system. This is why there is no shot noise in macroscopic systems. All these considerations assume that the ground state of a conductor is Fermi-liquid-like. If interactions lead to a formation of a new state, the situation can be very different.

• Both shot and Nyquist noises are white — frequency independent in a wide interval of frequency. For non-interacting electrons, the frequency dependence appears at the quantum scale $\hbar \omega \sim k_B T, e|V|$. For instance, at zero temperature shot noise has the following form,

$$S(\omega) = \frac{2se^2}{\pi \hbar} \begin{cases} \hbar |\omega| \sum_p T_p^2 + e|V| \sum_p T_p(1 - T_p), \hbar |\omega| < e|V|; \\ \hbar |\omega| > e|V|. \end{cases} \quad (3)$$

The part growing as $|\omega|$ is known as quantum noise. Other energy scales come from electron-electron interaction; in the most common case, the scale is just inverse $RC$-time — the time scale for classical charge relaxation.

Real life is fortunately more complicated than these simple rules, and this is why current noise is still a subject of active research. Below we consider a number of phenomena which go beyond these rules and are currently in the focus of attention. We specifically concentrate on four topics: non-symmetrized cumulants and quantum noise; counting statistics; super-Poissonian noise; and current noise and interferometry. For each of these subjects, we outline the main ideas, describe the experiments, and provide the full collection of references to the original papers.

Prior to that we would like to mention a number of experimental developments of fundamental importance over the last five years. They confirmed existed theoretical predictions, and generated a subsequent stream of literature, but due to the space limitations we can not discuss them in detail.

• Observation of shot noise suppression (cold and hot electrons) in chaotic cavities \cite{8}; crossover from classical ($F = 0$) to quantum ($F = 1/4$) shot noise in chaotic cavities by tuning the dwell-time \cite{9}.
• Doubling of the Fano factor at the interface between a normal metal and a superconductor [10]; also in the presence of finite-frequency field (photon-assisted effect) [11].

• Very clean observations of giant shot noise in SNS junctions due to multiple Andreev reflection (MAR) [12]; crossover from MAR regime to noise of the quasiparticle current [13]; shot noise in the regime of coherent and incoherent MAR in disordered SNS junctions [14]; MAR in superconductor – semiconductor – superconductor junctions [15].

• Noise in an array of quantum dots [16]; noise in an array of chaotic cavities formed by point contacts [17].

• Shot noise suppression in hopping conduction [18, 19].

• Noise for photon-assisted tunneling [20].

• Noise in quantum dots in the Coulomb blockade regime [21, 22].

2 Quantum noise

In Eq. (1), we defined noise power as a symmetric correlator. With this definition, $S(\omega)$ is always even in frequency. Indeed, a classical detector does not know anything about the order of the current operators and can not distinguish between positive and negative frequencies: It only measures a symmetric combination. Can we measure non-symmetrized noise,

$$S_q(\omega) = 2 \int dt e^{-i\omega(t-t')} \langle \hat{I}(t)\hat{I}(t') \rangle?$$

(4)

For such measurement, we obviously need a quantum detector. Let us illustrate the basic notions with an example of a detector which is a two-level system [23, 24], with the states $|a\rangle$ (energy $E_a$) and $|b\rangle$ (energy $E_b$). Interaction between the detector and the system is supposed to be weak and proportional to the current operator, $\hat{H} = \alpha |b\rangle\langle a| \hat{I}(t) + \text{h.c.}$ The transition rates between the two states of the detector follow from the Fermi golden rule,

$$\Gamma_{a\rightarrow b} = \frac{|\alpha|^2}{2\hbar^2} S_q \left( \frac{E_b - E_a}{\hbar} \right).$$

(5)

Thus, if one measures the transition rate at zero frequency, the result yields asymmetric noise correlator at finite frequency. For $E_b > E_a$, the detector absorbs energy from the noise; otherwise, it emits energy. Thus, noise at positive/negative frequency correspond to absorption/emission, respectively. It is not symmetric, since transition rates are not the same for emission and absorption. For instance, at equilibrium these transition rates obey the detailed balance, $\Gamma_{a\rightarrow b} p_a = \Gamma_{b\rightarrow a} p_b$, where $p_a$ and $p_b$ are the occupation probabilities of the detector states, which obey Boltzmann distribution. We obtain

$$S_q(\omega)/S_q(-\omega) = \exp(-\hbar \omega/k_B T).$$

At zero temperature, $S(\omega) = 0$ for positive frequencies: There is no energy that detector can absorb from noise. For non-equilibrium noise, at zero temperature absorption is only possible if the energy provided by the external voltage is high enough,
\( h\omega < e|V| \). The result at zero frequency expressed in terms of conductance \( G \) and the Fano factor \( F \) of the nanostructure is

\[
S_q(\omega) = 2G \begin{cases} 
-2\hbar|\omega|, & h\omega < -e|V|; \\
(e|V| - h\omega) - (1 - F)(e|V| + h\omega), & -e|V| < h\omega < 0; \\
F(e|V| - h\omega), & 0 < h\omega < e|V|; \\
0, & e|V| < h\omega.
\end{cases}
\)

First measurement of non-symmetrized noise using a quantum detector was performed by Deblock et al.\(^{25}\) who used a Josephson junction (JJ) as a detector. If JJ is biased at a constant voltage, there is no dissipative (quasiparticle) current at voltages lower than \( 2\Delta/e \), where \( \Delta \) is the superconducting gap. Such current could come from MAR, however, if the transparency of the insulating layer between two superconductors is low, these can be neglected. Thus, the dissipative current is zero for \( eV < 2\Delta \) and linearly grows for higher voltages. However, if the junction is submitted to external radiation, the situation is different: an electron can absorb a phonon with the frequency \( \omega \). Provided the voltage is between \( (2\Delta - h\omega)/e \) and \( 2\Delta/e \), this absorption will result in the quasiparticle dc current. The amplitude of the current depends on the amplitude of the external radiation. In particular, if the external radiation originates from current noise produced by a nanostructure, the quasiparticle current linearly depends on the non-symmetrized spectral power of the current noise \(^{26}\)–\(^{29}\),

\[
I_{\text{pat}}(V) = \int_0^\infty \frac{d\omega}{2\pi} \left( \frac{e}{\hbar\omega} \right)^2 S_{qV}(-\omega)I_{qp}\left( V + \frac{h\omega}{e} \right), \quad eV < 2\Delta,
\]

where \( I_{qp}(V) \) is the quasiparticle current without external radiation, the photo-assisted current \( I_{\text{pat}}(V) \) is the dc current in the presence of external noise, and \( S_{qV}(-\omega) \) is the non-symmetrized voltage noise, which is related to the current noise via the impedance of the circuit. Sweeping the bias voltage \( V \) and measuring the current, one can restore the frequency dependence of the non-symmetrized correlator. A great advantage of such a detector is that the detector itself at \( eV < 2\Delta \) is noiseless, since there is no quasiparticle current.

To demonstrate that the possibility of detection of non-symmetric noise, Deblock et al.\(^{25}\) measured noise produced by a Cooper pair box (also known as superconducting charge qubit). This is a double junction superconducting structure with two close levels corresponding to the states with \( N \) and \( N+1 \) Cooper pairs in the box, respectively. All other states of the system lie far away from these two states and can be ignored. The splitting \( \epsilon \) between the states \( N \) and \( N+1 \) can be tuned with the gate voltage; the minimal value of this splitting is \( \epsilon = E_J \), with \( E_J \) being the Josephson energy, achieved for \( Q \equiv C_qV_G = e \), where \( C_q \) is the capacitance to the gate. In an ideal system, the current noise is determined from Eq. \(^5\): only one transition is possible, with the frequency \( \epsilon/h \), and thus the current noise has a delta peak around this frequency. In reality, one has to take into account that the levels are broadened by tunneling, and thus the noise sharply peaks around the frequency \( \omega_0 = \sqrt{\epsilon^2 + \Gamma^2}/h \), with \( \Gamma \) being the tunnel rate \(^{27}\). One speaks of the quasiparticle peak in noise, with \( Q < e \) and \( Q > e \) corresponding to emission and absorption, respectively. This is valid in the coherent regime \( E_J \gg \Gamma \); in the opposite incoherent regime, \( E_J \ll \Gamma \), one has a broad peak around zero frequency. The experimental observation of Ref. \(^{25}\) was that noise on the emission side of the quasiparticle peak
is much stronger than noise on the absorption side, thus confirming that the quantity measured is the non-symmetric current correlator.

A clear measurement of noise for a broad interval of frequencies which could demonstrate the crossover from zero noise at the absorption side to white shot noise at low frequencies and further to quantum noise at the emission side is still not available in the literature. However, there are further data demonstrating detection of quantum noise. In Ref. [26], one Josephson junction was used as a noise source, and another one as a detector. The source was biased at voltages above $2\Delta/e$ and thus produced white shot noise from the quasiparticle current. The detected noise was twice as low as the full expected shot noise of the quasiparticle current, which shows that the non-symmetric correlator was measured: only $S_q(-\omega)$, but not the contribution from $S_q(\omega)$. Another detector used in the experiments is a quantum dot [28] at low bias. Without external noise, current only flows when an electron level lies in the window between the chemical potentials of the reservoirs. As the function of the gate voltage, current has a peak. With the external noise, tunneling via excited states becomes possible, and additional peaks in the current appear. In the experiments, the magnitude of these additional peaks at the emission side was clearly stronger than the one at the emission side.

3 Counting statistics

Shot noise originates from random nature of electron transfers. One can, at least in principle, count these transfers in real time, and from the results of the measurements deduce average current and current noise. We have seen that shot noise contains some information about scattering properties of the nanostructure which cannot be obtained from the conductance. Higher current moments can also be deduced from the same measurement and may contain even more information. They are described with the notion of full counting statistics (FCS).

Let us proceed with a bit of the probability theory. Suppose we make a measurement counting some random events — for instance, electron transfers through a barrier — during a certain time interval $\Delta t$. The number of events $N$ measured during the time interval is a random number, characterized by the probability $P_N$ that precisely $N$ events will be observed in a measurement. If one repeats identical measurements $M_{\text{tot}}$ times and counts the number of measurements $M_N$ that give the count $N$, the ratio $M_N/M_{\text{tot}}$ gives the probability $P_N$ in the limit $M_{\text{N}} \gg 1$. This probability distribution is normalized: $\sum_N P_N = 1$. Once we know it, we can estimate the average of any function $f_N$,

$$\langle f \rangle = \sum_N f_N P_N.$$

The description of the statistics with the distribution function $P_N$ is not always the most convenient one. The problem is that if we measure first during the time interval $\Delta t_1$ (distribution function $P_1$) and then during $\Delta t_2$ ($P_2$), the distribution function for the total interval $\Delta t = \Delta t_1 + \Delta t_2$ is a convolution (provided the two intervals are independent),
\[ P_N^{\text{tot}} = \sum_{M=0}^{N} P_{1,M} P_{2,N-M}. \]

Most conveniently, this is expressed in terms of characteristic function of a probability distribution,
\[ A(\chi) = \langle e^{i\chi N} \rangle = \sum_N P_N e^{i\chi N}. \]

For independent events, the characteristic function of the total distribution is just a product of characteristic functions of each type of events, \( A^{\text{tot}}(\chi) = A_1(\chi) A_2(\chi). \) The function \( \ln A(\chi) \) is thus proportional to the duration of the measurement \( \Delta t. \) Differentiating this function \( k \) times with respect to \( i\chi \) and setting subsequently \( \chi = 0, \) we generate the \( k \)th cumulant of the distribution. Thus, the first derivative produces the average \( N, \) and the second derivative reproduces the variance.

For electron transfers in nanostructures, it is customary to consider statistics of charge \( Q = eN \) transmitted from the left to the right during the time interval \( \Delta t. \) We assume that this measurement time is long enough, so that \( Q \gg e \) and the laws of statistics apply. On average, \( \langle Q \rangle = \langle I \rangle \Delta t. \) Second cumulant gives the shot noise at zero frequency,
\[ \langle\langle Q^2 \rangle\rangle = \langle Q^2 \rangle - \langle Q \rangle^2 = \Delta t S(0)/2. \]

The characteristic function of the transmitted charge can be expressed in terms of transmission eigenvalues of the nanostructure [29, 30, 31],
\[
\ln A(\chi) = 2s \Delta t \int \frac{dE}{2\pi \hbar} \sum_p \ln \left\{ 1 + T_p \left( e^{i\chi} - 1 \right) f_L(E) \left[ 1 - f_R(E) \right] \right\}.
\]

The logarithm of characteristic function is a sum over transport channels, this suggests that electron transfers in different channels and over different energy intervals are independent. Differentiating this expression once and twice over the counting field \( \chi, \) we reproduce Landauer formula and the expression for the current noise. At zero temperature, Eq. (7) becomes
\[
\ln A(\chi) = \pm \frac{2s eV \Delta t}{2\pi \hbar} \sum_p \ln \left[ 1 + T_p \left( e^{\pm i\chi} - 1 \right) \right], \quad (8)
\]
where the upper and lower signs refer to the case of positive and negative voltages, respectively. Let us for simplicity consider \( V > 0. \) We define the number of attempts \( N_{\text{at}} = 2s \Delta t eV/(2\pi \hbar) \) and assume it to be integer. Eq. (8) for one transport channel corresponds to the binomial distribution,
\[
P_{N}^{(p)} = \binom{N_{\text{at}}}{N} T_p^N (1 - T_p)^{N_{\text{at}} - N}. \quad (9)
\]
This is just the probability that out of \( N_{\text{at}} \) electrons arriving to the barrier \( N \) pass through, and others, \( N_{\text{at}} - N, \) are reflected back. For more than one channel, the binomial distribution does not hold any more: One obtains a convolution of binomial distributions corresponding to each channel. If all transmission eigenvalues are small, Eq. (9) yields the Poisson distribution, corresponding to the notion of independent electron transfers.
One can now averaging Eq. (8) over various distributions of transmission eigenvalues to produce the full counting statistics. In this way, FCS for a double barrier [32], diffusive wires [30, 33], and chaotic cavities [34] was produced.

The concept of FCS can be generalized to different situations, where Eq. (7) does not apply any more. By now, full counting statistics is a field by itself. Various methods have been developed to calculate higher cumulants, which include semi-classical approach based on the cascade of Langevin equations, both for dif- fusive wires [35] and chaotic cavities [36], semi-classical circuit theory based on Keldysh Green’s function technique [37, 38, 39], semi-classical stochastic path integral method [40], Keldysh sigma-model [41, 42], direct numerical simulation [43], and analytical [44] and numerical [45] treatment of exclusion models. Results on FCS for normal-superconductor interfaces in various situations [46, 37, 47], for charge transfer between superconductors, both for applied constant phase [48] or applied constant voltage [40, 60, 61], for quantum to classical crossover in chaotic cavities [49, 50, 51], for FCS in double quantum dots [54], in multi-terminal circuits, including superconducting elements [52, 53], quantum dots in the Coulomb blockade regime [54, 60], interacting diffusive conductors [41, 57, 58], for frequency dependence of the higher cumulants [59, 60, 61], and for the time-dependent current [52, 55] are available. There is one issue we would like to mention here. For FCS in the charge transfer between superconductors, both for applied constant phase [48] or applied constant voltage [40, 60, 61], for quantum to classical crossover in chaotic cavities [49, 50, 51], for FCS in double quantum dots [54], in multi-terminal circuits, including superconducting elements [52, 53], quantum dots in the Coulomb blockade regime [54, 60], interacting diffusive conductors [41, 57, 58], for frequency dependence of the higher cumulants [59, 60, 61], and for the time-dependent current [52, 55] are available. There is one issue we would like to mention here. For FCS in the charge transfer between two superconductors with the fixed phase difference, the probabilities $P_N$ can sometimes assume negative values [48]. The reason is that phase and charge are canonically conjugated variables and can not be measured simultaneously. No such problem exists for voltage-biased junctions. Thus, in this case the quantities $P_N$ can not be interpreted as probabilities. However, Ref. [48] suggested a scheme that makes a measurement of $P_N$ possible, even if they are negative. The set of the quantities $P_N$ contains full information about the charge transfer in the system even in this situation.

Let us now specifically consider the third cumulant of transmitted charge. It has a very important property: At equilibrium ($V = 0$) the characteristic function of Eq. (7) becomes even in $\chi$, and therefore all the odd cumulants disappear. There is no “Nyquist third cumulant”. For this reason, one does not have to measure at very low temperatures to extract the information of transmission eigenvalues. On the other hand, noise measurement is already more complicated than the average current measurement since it requires to collect more measurement results to achieve decent accuracy. The direct measurement of the third cumulant is even more challenging.

From Eq. (8) we get the third cumulant in the “shot noise” regime $eV \gg k_B T$,

$$\langle \langle Q^3 \rangle \rangle = e^3 \frac{\partial^3}{\partial (i\chi)^3} \ln A(\chi) = e^2 V G_Q \Delta t \sum_p T_p (1 - T_p) (1 - 2T_p).$$  

(10)

For a tunnel barrier $T_p \ll 1$ we get $\langle \langle Q^3 \rangle \rangle = e^2 \langle I \rangle \Delta t$, which can be derived directly from the Poisson distribution. In a diffusive wire, the averaging over the distribution function of transmission eigenvalues yields $\langle \langle Q^3 \rangle \rangle = e^2 \langle I \rangle \Delta t / 15$. The third cumulant can be either positive or negative, the open channels with $T_p > 1/2$ favour negative sign. One can also derive the full voltage dependence, including the regime $eV \ll k_B T$, from Eq. (4).

The first measurement of a higher cumulant of current noise was performed by Reulet, Senzier, and Prober [54], who studied the third cumulant of the voltage drop $S_1^3$ across a tunnel junction biased by a constant current. The result is plotted in
Figure 1. Measurements of the third cumulant [64]. The top panel represents comparison of the results with the theory. The agreement is only achieved if the environmental fluctuations are taken into account, see Eq. (11). Copyright (2003) by American Physical Society.

Figure 1 against the average voltage drop over the junction (the resistance of the circuit $R_D$ is found independently from the noise measurements). One now has to compare experimental measurements with the theoretical prediction (10). Naively, third cumulant of the voltage is obtained from the third cumulant of the current $S^I_3$ by multiplication with $R^3_D$. This operation produces a dashed line shown in Figure 1 as “voltage bias result” — in strong contradiction with experimental data. The reason for this disagreement was discovered in Refs. [65]. It is known that, similarly to the average voltage $V = R_D I$, voltage fluctuations can be obtained from the current fluctuations by multiplication with $R^2_D$, $S_V = R^2_D S_I$. However, for third and higher cumulants the situation is more complicated, since voltage fluctuations generated at the sample can perturb other parts of the electric circuit and generate there current fluctuations, which affect current fluctuations at the sample. This strongly modifies the relation between current and voltage third cumulants. In the simplest situation, when the environment itself is noiseless (“non-invasive measurement”) one has

$$S_{V,3} = -R^3_D S^I_3 + 3 R^2_D S^I S^I dS^I / dV.$$  (11)
The difference between higher order cumulants is even more dramatic. To characterize voltage fluctuations, one defines the phase \( \phi = \int_0^{\Delta t} \delta V(t) dt \) and studies full counting statistics for the phase. It turns out that the phase has Pascal distribution rather than binomial distribution which one finds for the transmitted charge; in particular, for the tunnel junction, the distribution is chi square rather than Poisson one \[65, 67\].

The problem with the experiment \[64\] is that the measured third cumulant was dominated by the shot noise contribution (second term in Eq. 11), and the contribution \( S_{I3} \) only became important at high temperatures, when the voltage dependence of the noise is weak. To avoid this, several groups performed measurements of the transmitted charge in real time. Following an earlier theoretical suggestion of Ref. \[68\], Bomze et al \[69\] measured the third current cumulant of a tunnel junction by amplifying and analyzing in real time voltage fluctuations on a detector — a resistor with the conductance much higher than that of the sample. The measurements were performed at 4K and demonstrated the dependence \( S_{I3} = e^2 \langle I \rangle \), expected from the Poisson distribution.

Real-time measurements are easier in the Coulomb blockade regime in quantum dots, since the electron spends a relatively long time in the system, electrons enter the dot one by one, and individual tunnel processes are easier to resolve. The earlier measurements \[70\] used a single-electron transistor electrostatically coupled to the quantum dot as a detector, and observed real-time detection of single electron tunneling; however, the measurement precision was not enough to extract full counting statistics. Subsequently, more precise real-time detector measurements were performed in quantum dots \[71\] and in superconducting junction arrays \[72\], still without extracting the full counting statistics. Then Gustavsson et al used charge detection with a quantum point contact electrostatically coupled to the quantum dot: When an electron enters the dot, it increases the height of the potential barrier in the quantum point contact. If the detector is tuned close to the step between the plateaus, this increase would block electron passage through the junction. Thus, measurement of the current through the detector gives the real-time information on the occupation of the quantum dot. They plotted the histogram of the transmitted charge during the measurement time \( \Delta t \) (half a second in their experiment) and analyzed the FCS. Very recently Fujisawa et al \[74\] reported measurements of FCS in a double quantum dot, also with a quantum point contact as a detector. They collected enough statistics to restore the rates for all possible tunnel processes, checked the detailed balance relation between the rates, produced the occupation probabilities by solving the master equations, and compared the results with the observed FCS.

All these recent experimental advances concentrate on the situations when the FCS from the point of view of the theory is trivial — Poisson distribution in tunnel junctions, and only two possible charge states in quantum dots — and so far serve rather to demonstrate that FCS can be measured. Measurements of less trivial effects, for instance, of tails of the distribution function of the transmitted charge, are still to come.
4 Super-Poissonian noise

It follows from Eq. (2) that shot noise in the system of non-interacting electrons is always sub-Poissonian: the Fano factor $F$ is less than one. This means that every time super-Poissonian noise is measured the reason must be looked for in interactions (typically electron-electron interactions). However, this statement is too general, and, as many too general statements, useless. Let us look in more details at different situations which can produce super-Poissonian noise.

As we mentioned, shot noise measures charge quantum transferred across the nanostructure. An example we already mentioned is an interface between a normal metal and a superconductor. For voltages below the superconducting gap transport is only possible by Andreev reflection, and the corresponding charge quantum is $2e$. The Fano factor for such system can be up to $F = 2$. Another example is transport in SNS systems, which proceeds via multiple Andreev reflections. Such process is associated with transfer of $\Delta/eV$ charge quanta, which provides super-Poissonian Fano factors.

![Fig. 2. Tunneling via localized states.](image)

Fig. 2. Tunneling via localized states. [75]. Noise is shown at the right panel; solid line represent Poisson value. Copyright (2003) by American Physical Society.

Let us give another example illustrating the same mechanism. Safonov et al [75] studied noise in tunneling via localized states. They discovered that the Fano factor strongly depends on the gate voltage, sometimes achieving values above one (Figure 2). To explain these results, Ref. [75] suggested the following model. Imagine the transport occurs through two localized states ("impurities") in parallel: $R$ and $M$. $M$ is coupled to the leads much weaker than $R$, so that its contribution to the current and noise is negligible. If $R$ and $M$ were independent, the Fano factor would vary between $1/2$ and 1, depending on the asymmetry of the two barriers separating the impurity from the reservoirs. However, things change if the two impurities are coupled electrostatically. Due to this coupling, the occupation of $M$ affects the position of electron levels in $R$ and can, for instance, shift a level off the resonance, blocking the transport through $R$. In this case, if $M$ is occupied, current through $R$ is blocked, and if $M$ is empty, current through $M$ proceeds in an ordinary way. Thus, if tunnel rates for $M$ and $R$ are of the order of $\Gamma_R$ and $\Gamma_M$, respectively,
transport through the system proceeds in bunches of $\Gamma_R/\Gamma_M \gg 1$ electrons. Thus, the Fano factor can achieve large super-Poisson values. Since such a two-impurity configuration is not expected to be typical, after impurity averaging, shot noise is considerably reduced, which explains Fano factors slightly above one observed in the experiment.

This example illustrates a general mechanism of super-Poissonian noise. If transport proceeds via two or more electrostatically coupled states, so that occupation of one of the states ($M$) may block the transport through the other one(s), $R$, charge is transported in quanta of $e\Gamma_R/\Gamma_M$. Super-Poissonian noise appears provided this number is greater than one, that is the states are coupled differently to the leads. This situation may occur in quantum dots in various regimes under the Coulomb blockade condition: One needs that the charging energy is greater than the separation between the energy levels relevant for transport (dynamical channel blockade). Theoretical predictions of super-Poissonian noise exist for sequential tunneling regime in quantum dots with ferromagnetic leads [76, 77, 78] (if both leads are partially polarized say spin-up, then spin-up electrons tend to tunnel in bunches, and spin-down electrons block the current for a long time), in a magnetic field [79], general dynamical channel blockade for sequential tunneling [80, 81, 82, 83], in double quantum dots [84], and in quantum dots where the level coupling is mediated by non-equilibrium plasmons in the leads [85]. Ref. [86] predicted super-Poissonian noise in the inelastic cotunneling regime.

Recently super-Poissonian noise was experimentally observed by Onac et al [22] in a carbon nanotube quantum dot (carbon nanotube crossed by two barriers). They measured noise across the Coulomb diamond and found Fano factors up to $F = 3$. Super-Poissonian noise was observed inside the diamonds and is therefore associated with inelastic cotunneling.

Another source of super-Poissonian noise is bistability. An example was provided by Refs. [87, 88], that studied current through quantum wells in the resonant tunneling regime. A similar behavior was observed in tunneling through a zero-dimensional state [89]. Due to interactions, in a certain interval of voltages these wells become bistable: One state with zero current and one state with finite current. For lower voltages, zero-current branch becomes unstable and ceases to exist; for higher voltages, finite-current branch does not exist. Noise in such system comes from the two sources: (i) “shot” noise — small current fluctuations around each of the branches; (ii) random jumps — random telegraph noise — between the branches. Close to the instability point, when the finite-current branch disappears, current fluctuations around this branch diverge and exceed the Poisson value [90, 92]. Full analysis of the noise also includes large fluctuations, resulting in jumps between the branches [91, 92]. Ref. [93] treats current statistics in a generic bistable system. Experimentally, a link between super-Poissonian noise and bistability is not established convincingly: For instance, a superlattice tunnel diode is a bistable system, but experiments did not discover any super-Poissonian noise enhancement [94].

Finally, we discuss yet another situation, when there are other degrees of freedom in the system which affect transport properties, in particular, noise. Actually, this situation is rather common: In any mesoscopic and macroscopic system, electrons interact with phonons. If this interaction is effective enough — the characteristic length of electron-phonon relaxation $L_{ph}$ is shorter than the size of the system — electrons relax to the equilibrium distribution, and the noise they produce is Nyquist noise — shot noise disappears. If we want to have anything non-trivial,
we need to consider non-equilibrium phonons. Such opportunity was recently provided by a new class of devices — nanoelectromechanical systems (NEMS), which couple electron motion to mechanical degrees of freedom. Currently, many species of NEMS were made and investigated, including shuttles — single-electron tunneling devices with movable central island, double-clamped suspended beams in the Coulomb blockade regime, or single-clamped suspended cantilevers. First, in NEMS one can discuss not only charge noise, but also momentum noise — random fluctuations of momentum transferred from electrons to the crystalline lattice [95, 96, 97]. Second, ordinary current noise is strongly modified by mechanical degrees of freedom, both in the shuttling regime [98, 99, 100, 101] and for single-electron tunneling [102, 103, 104, 105, 106, 107]; in particular, in both situations noise can achieve super-Poissonian values. It is not our intention to give here a comprehensive review of noise in NEMS, and we restrict ourself to just one particular situation: a single electron tunneling device weakly coupled to a single-mode underdamped harmonic mechanical oscillator[108, 109].

Qualitatively, the situation is as follows. Imagine that bias and gate voltages are tuned just outside a Coulomb blockade diamond, so that only two charge states are important for electrons: say \( n = 0 \) and \( n = 1 \). Non-zero average current means that the number of electrons in the single-electron tunneling device fluctuates randomly between zero and one. The coupling between electrons and vibrations of the oscillator is provided by a force \( F_0 \) which depends on the charge state and acts on the oscillator. In the regime we discuss this force is a random function that can assume two values, \( F_0 \) and \( F_1 \). The force generates mechanical oscillations, which in the underdamped case have a large amplitude and a frequency close to the eigenmode \( \omega_0 \) of the oscillator. The vibration produces feedback on the current since the tunnel rates depend on the position of the oscillator, due to the position dependence of the energy differences available for tunneling. It turns out that the quality factor of the oscillator \( Q \) is renormalized due to electron tunneling, but even after the renormalization one still is in the underdamped regime.

Depending on the voltages, one can identify four types of the behavior of the system, which are best described in terms of the probability \( P(A) \) to have certain amplitude \( A \) of the mechanical oscillator. First, \( P(A) \) can be sharply peaked around zero (meaning only very small amplitudes have significant probability) or around certain finite value. In both cases, noise can be estimated as follows. From dimensional analysis, one obtains \( S(0) \sim I^2 \tau \), where \( \tau \) has the dimensions of time. In the ordinary situation, \( \tau \) is of order of the inverse tunnel rate \( \Gamma^{-1} \) (the only energy scale in the problem), and one restores the Poisson value of the shot noise. In our case, there is a longer time scale — the decay time \( Q/\omega_0 \gg \Gamma^{-1} \); thus, we have \( S(0) \sim eI\Gamma Q/\omega_0 \), and noise considerably exceeds the Poisson value. In two further cases, the distribution function \( P(A) \) has two peaks: either one at \( A = 0 \) and another one at a finite value of \( A \), or both peaks at finite values of \( A \). This means that only two values of the amplitude are possible. In both cases, on top of super-Poissonian noise for each peak, we have additional enhancement of noise due to random jumps between the states with different values of the amplitude.
5 Interference effects

Interference effects are at the core of quantum mechanics. They provide information on the phase of the wave function probing its wave nature. These experiments are difficult to perform with electrons in solids, however, we witness steady progress over the last decade, with a number of proposals and successful realizations constantly increasing. We will only discuss here the types of interferometers for which studies of noise are available.

Qualitatively the simplest species is an Aharonov-Bohm (AB) ring — a two-terminal structure where the transmission probability is a periodic function of the magnetic flux penetrating the ring, with the period of the flux quantum. Conductance and shot noise retain this periodic dependence. However, this system has just too few handles, the amplitude of AB oscillations depends essentially on the dynamical phases acquired by electrons moving along the arms of the ring, and the AB vanishes in a ring already with several transport channels.

Fig. 3. Mach-Zehnder interferometer with edge states. Edge states are shown by solid lines with arrows; additional tunnel directions in the beam splitters with dashed lines.

Recently, Ji et al. [110] performed an experiment with the electronic Mach-Zehnder interferometer (MZI), an analog of the corresponding optical device. The electronic MZI with edge states in the integer quantum Hall regime is shown in Figure 3. In the simplest version, it has one source and two detectors (the voltage $V$ is applied to the source relative to both detectors), the beam splitter A and the beam splitted B, both realized as quantum point contacts. The transport proceeds via the edge states; we assume that in the contact A there is no reflection, so that the edge state proceeds either to the upper or to the lower arm. In the beam splitter B, there is also no reflection, and an electron from either arm can proceed to one of the detectors, 1 or 2. An Aharonov-Bohm flux penetrates the ring. Both conductance between the source and any of the detectors, and current correlations between any reservoirs are flux-dependent. In the experiment, transmission through one of the point contacts could be changed, which creates an additional handle.

Surprisingly, the visibility — the ratio of the phase-dependent and phase-independent parts of the conductance — observed in the experiment, was lower than expected. This suppression of the visibility can be attributed to the loss of phase coherence, which partially destroys the interference. There could be two reasons for this loss of coherence: phase averaging (for instance, due to the energy dependence of the phase) and dephasing by environment, in particular, by electron-electron interactions. Conductance is affected by both mechanisms in the same way:
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It is proportional to the transmission probability from the source to the corresponding drain, averaged over the phase. The details depend on the type of the dephasing [111, 112]. Thus, conductance generally can not distinguish between these two reasons of phase coherence loss, if they have the same dependence of the dephasing time (usually it is proportional to the temperature). It turns out that the situation is different for noise [113, 114], as well as for higher cumulants of the transmitted charge [115]. One has to compare the applied voltage $eV$ with the inverse characteristic lifetime for the phase correlations induced by the environment $\hbar/\tau_c$. In this way, one identifies “fast” $eV\tau_c \ll \hbar$ and “slow” $eV\tau_c \gg \hbar$ environments. For slow environment, shot noise is merely obtained by averaging of the usual expression, $T(1 - T)$, over the phase. In this case, it does not provide any new information on the environment as compared to the conductance. For fast environments, the behavior of noise is generally different, and thus may provide an information on the source of loss of the phase coherence.

In Refs. [116, 117], a more complicated two-particle interferometer was proposed. It consists of four sources and four detectors, separated by four quantum point contacts. Transport is again only possible via the edge states in the integer quantum Hall regime. The setup is designed in such a way that an electron can be transmitted from any source to any detector via only one trajectory. Then, average current is not sensitive to interference, does not depend on the Aharonov-Bohm phase, and is determined by transmission probabilities of the contacts. In contrast, if we consider current cross-correlations at different detectors, the interference contribution originates from the interference of different trajectories. In this geometry, these pairs of trajectories are taken in such a way that together they enclose a loop and are thus sensitive to the Aharonov-Bohm flux. This is probably up to now the nicest illustration of the two-particle nature of current noise in solid state systems.

We last mention an Andreev interferometer — a normal metal connected by two arms with the same superconducting reservoir. Transport properties of this system are sensitive to magnetic flux enclosed by the arms. Reulet et al [118] investigated current noise in this system experimentally and theoretically, and found periodic dependence on the applied flux. Full counting statistics in an Andreev interferometer was studied in Ref. [119].

6 Acknowledgements

The author very much appreciates collaboration with his friends and colleagues on various issues related to current noise, in chronological order: Stijn van Langen, Eugene Sukhorukov, Henning Schomerus, Carlo Beenakker, Gabriele Campagnano, Oleg Jouravlev, Yuli Nazarov, Omar Usmani, Thomas Ludwig, Alexander Mirlin, and Yuval Gefen. He is especially grateful to Markus Büttiker, who introduced him to the field of shot noise.

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