Noncommutative $AdS_2$ II: The Correspondence Principle

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Using the exact solutions to the field equation for a massive scalar field on noncommutative $AdS_2$, we apply the AdS/CFT correspondence principle to obtain an exact result for the associated two-point function on the conformal boundary. The answer satisfies conformal invariance and has the correct commutative limit and massless limit.

1. Introduction

The conjectured $AdS/CFT$ correspondence principle has played a central role in theoretical physics in the past two decades. It posits a strong/weak duality between gravity in the bulk of an asymptotically anti-de Sitter ($AdS$) space and a conformal field theory ($CFT$) located at the so-called conformal boundary. For obvious reasons many practical applications of the correspondence principle utilize classical, or weak, gravity in the bulk, with the intention of exploring the strong coupling regime of the boundary theory. Since a fully consistent quantum gravity treatment of the bulk remains out of reach, it is a nontrivial task explore other domains of the correspondence. On the other hand, the incorporation of some quantum gravity effects in the bulk is possible. This has been the motivation for our recent works.

As remarked in, the inclusion of some quantum gravity effects might be achieved by replacing the $AdS$ bulk by its noncommutative analogue, $ncAdS_2$. While, in general, the introduction of noncommutativ-
ity destroys the isometries of a manifold, and is not unique, this is not the case for $AdS_2$. Therefore, $ncAdS_2$ can serve as a toy model for the introduction of quantum gravity effects in the bulk (barring the known difficulties of the $AdS_2/CFT_1$ correspondence that already exist in the commutative case$^5$). Moreover, in$^2$ it was shown i) that the star product for $ncAdS_2$, when acting on functions having a well behaved boundary limit, reduces to the point-wise product in this limit, and ii) that noncommutative corrections to isometry generators, i.e., Killing vectors, vanish near the boundary. In other words, $ncAdS_2$ is an asymptotically $AdS$ space and, so according to the correspondence principle, a $CFT$ should be present at its boundary.

The on-shell field theory action in the bulk $S|\text{on-shell}$ plays a central role in the explicit construction of the $AdS/CFT$ correspondence. It generates the $n$-point connected correlation functions for operators on the boundary$^2$. As a first step, it is therefore necessary to obtain the solutions to field theories in the bulk, which in our case is noncommutative. Exact solutions on $ncAdS_2$ were obtained for free massless scalar and spinor fields in$^5$ and massive scalar fields in$^6$. These exact solutions will therefore lead to exact expressions for the corresponding two-point function on the boundary. This was shown in$^5$ for the case of massless fields. It is the purpose of this article to obtain the boundary two-point function resulting from the exact solutions to the massive field equation found in$^6$

The generating function $S|\text{on-shell}$ is expressed in terms of the boundary values $\phi_0$ of the solutions for the bulk fields, and the prescription for computing the $n$-point connected correlation function for operators $\mathcal{O}$ located at non-coincidental points $x_i$ on the boundary is given by

$$<\mathcal{O}(x_1)\cdots\mathcal{O}(x_n)> = \frac{\delta^n S|\text{on-shell}}{\delta \phi_0(x_1)\cdots\delta \phi_0(x_n)} \bigg|_{\phi_0=0}. \quad (1)$$

For the case of a 2-dimensional bulk theory, both $\mathcal{O}$ and $\phi_0$ are functions of only one coordinate, the time $t$. Conformal invariance severely restricts the form of the $n$-point correlators; For $n=2$ one has

$$<\mathcal{O}(t)\mathcal{O}(t')> = C_{\Delta\pm} \frac{1}{|t-t'|^{2\Delta\pm}}, \quad (2)$$

where $\Delta\pm$ is the conformal dimension and $C_{\Delta\pm}$ is a constant which can be computed using (1). We shall compute the two-point function that results from massive scalar fields on $ncAdS_2$ and show that it has the form given in (2). The result for the overall factor $C_{\Delta\pm}$ differs from that of the

$^a$See for example$^7,8$
commutative theory. By taking various limits we can compare with the previous results.

The outline for the remainder of this article is as follows: We first review the calculations for \( C_{\Delta} \) in the commutative case in section 2, and then apply the analogous procedure to find the constant factor in the non-commutative case in section 3. Some concluding remarks are made in section 4.

2. Commutative case

The analysis of the correspondence principle is more conveniently performed for the Euclidean version of AdS, or \( E\text{AdS} \), due to the absence of propagating states. Here we specialize to \( E\text{AdS}_2 \), closely following the work in\(^{13,14}\)

The field equation for a scalar field \( \Phi \) with mass \( m \) on \( E\text{AdS}_2 \) is

\[
\Delta \Phi = m^2 \Phi ,
\]

with \( \Delta \) denoting the appropriate Laplacian. \( E\text{AdS}_2 \) is conveniently parametrized by Fefferman-Graham coordinates \((z, t)\). They span the half-plane, \( z > 0 \), \(-\infty < t < \infty \), with the conformal boundary located at \( z \rightarrow 0 \). In terms of these coordinates the metric tensor and Laplacian are given, respectively, by

\[
ds^2 = \frac{dz^2 + dt^2}{z^2} ,
\]

and

\[
\Delta = z^2 (\partial_z^2 + \partial_t^2) .
\]

The field equation \(^{8}\) results from the standard scalar field action

\[
S = \frac{1}{2} \int_{R \times R_+} dt dz \left( (\partial_z \Phi)^2 + (\partial_t \Phi)^2 + \frac{m^2}{z^2} \Phi^2 \right) = -\frac{1}{2} \int_{R \times R_+} dt dz \frac{\Phi(\Delta - m^2) \Phi}{z^2} - \frac{1}{2} \int_{R} dt \Phi \partial_z \Phi |_{z=0} .
\]

Only the boundary term survives when evaluating the action on-shell.

General solutions to \(^{3}\) that are well behaved away from the conformal boundary have the form

\[
\Phi(z, t) = \frac{1}{2\pi} \int d\omega e^{i\omega t} \sqrt{z} a(\omega) \ K_{\nu}(|\omega||z) , \quad \nu = \sqrt{m^2 + \frac{1}{4}} ,
\]

\(^{9}\)For treatments of the correspondence principle relying on Lorentzian signature see\(^{10,11,12}\)

\(^{1}\)Some care is needed to define the boundary limit. One should first assume that the boundary is located at some \( z = \epsilon \), and then take the limit \( \epsilon \rightarrow 0 \).
where $K_\nu(x)$ denotes the modified Bessel function. To determine the behavior of the solutions near the conformal boundary we can use

$$K_\nu(x) \to \frac{1}{2} \left( 2\nu \Gamma(\nu)x^{-\nu} + 2^{-2\nu}\Gamma(-\nu)x^\nu \right) \left( 1 + O(x^2) \right), \quad \text{as } x \to 0. \quad (8)$$

Then for $0 < \nu < 1$ (corresponding to $-\frac{1}{4} < m^2 < \frac{3}{4}$), $\Phi(z, t)$ tends towards

$$\frac{1}{4\pi} \int d\omega e^{i\omega t} a(\omega) \left( \left( \frac{2}{|\omega|} \right)^\nu \Gamma(\nu) z^{\Delta_-} + \left( \frac{|\omega|}{2} \right)^\nu \Gamma(-\nu) z^{\Delta_+} \right), \quad \text{as } z = \epsilon \to 0, \quad (9)$$

where $\Delta_\pm = \frac{1}{4} \pm \nu$, with $\Delta_+$ to be identified with the conformal dimension. So upon keeping just the leading term,

$$\Phi(\epsilon, t) \to \phi_\epsilon(t) = \epsilon^{\Delta_-} \phi_0(t), \quad \text{as } \epsilon \to 0. \quad (10)$$

Here we shall assume Dirichlet boundary conditions. $\phi_0(t)$ is regarded as the source field on the boundary, which determines the solution for $\Phi$ in the bulk. This can be made explicit by writing the coefficients $a(\omega)$ of the solution (7) in terms of the Fourier coefficients $\phi(\omega)$ of the boundary field $\phi_\epsilon(t) = \frac{1}{2\pi} \int d\omega e^{i\omega t} \phi(\omega)$,

$$\phi(\omega) = \sqrt{\epsilon} a(\omega) K_\nu \left( |\omega| \epsilon \right). \quad (11)$$

The solution for $\Phi(z, t)$ in the bulk can thereby be expressed in terms of these Fourier coefficients

$$\Phi(z, t) = \frac{1}{2\pi} \int d\omega e^{i\omega t} \phi(\omega) \sqrt{z} K_\nu \left( |\omega| z \right) \sqrt{\epsilon} K_\nu \left( |\omega| \epsilon \right). \quad (12)$$

The evaluation of the on-shell action requires an analogous expression for $\partial_z \Phi(z, t)$. For this we can use the identity $\frac{d}{dz} K_\nu(x) = \frac{\nu}{2} K_\nu(x) - K_{\nu+1}(x)$ to write

$$\sqrt{z} \frac{d}{dz} \left( \sqrt{\epsilon} K_\nu \left( |\omega| z \right) \right) = \left( \frac{1}{2} + \nu \right) K_\nu \left( |\omega| z \right) - |\omega| z K_{\nu+1} \left( |\omega| z \right),$$

and hence

$$\partial_z \Phi(z, t) = \frac{1}{2\pi} \int d\omega e^{i\omega t} \phi(\omega) \left( \frac{1}{2} + \nu \right) K_\nu \left( |\omega| z \right) - |\omega| z K_{\nu+1} \left( |\omega| z \right) \sqrt{\epsilon} K_\nu \left( |\omega| \epsilon \right). \quad (13)$$

Substitution of (12) and (13) into the boundary term in (6) gives the following result for the on-shell action $S|_{\text{on-shell}}$:

$$-\frac{1}{4\pi} \int dt \int dt' \int d\omega e^{i\omega(t-t')} \phi_\epsilon(t) \phi_\epsilon(t') \frac{1}{\epsilon} \left( \frac{1}{2} + \nu - |\omega| \epsilon \frac{K_{\nu+1} \left( |\omega| \epsilon \right)}{K_\nu \left( |\omega| \epsilon \right)} \right) \bigg|_{\epsilon \to 0}. \quad (14)$$

For other boundary conditions, see for example...
From (8), the asymptotic limit of the ratio of Bessel functions is
\[ \frac{xK_{\nu+1}(x)}{K_\nu(x)} \to \frac{2^{\nu+1}\Gamma(\nu+1) + 2^{-\nu-1}\Gamma(-\nu-1)x^{2\nu+2}}{(2\nu\Gamma(\nu) + 2^{-\nu}\Gamma(-\nu)x^{2\nu})} \] as \( x \to 0 \),
which to leading order is
\[ 2\nu \left( 1 - \frac{\Gamma(-\nu)}{\Gamma(\nu)} \left( \frac{x}{2} \right)^{2\nu} \right) \]
for \( 0 < \nu < 1 \). Upon applying this result, along with the integral
\[ \frac{1}{2\pi} \int d\omega e^{-i\omega t} |\omega|^\rho = \frac{2\rho\Gamma(\frac{\rho+1}{2})}{\sqrt{\pi}\Gamma(-\frac{\rho}{2})} \] \( \rho \neq -1, -3, \ldots \),\( ^{(13)} \)
one gets
\[ S_{\text{on-shell}} = -\frac{\nu\Gamma(-\nu)}{2^{2\nu+1}\pi\Gamma(\nu)} \int dt \int dt' \phi_0(t)\phi_0(t') e^{2\nu-1} \int d\omega e^{i\omega(t-t')} |\omega|^{2\nu} \bigg|_{\epsilon \to 0} \]
\[ = -\frac{\nu}{\sqrt{\pi}} \frac{\Gamma(\nu + \frac{1}{2})}{\Gamma(\nu)} \int dt \int dt' \phi_0(t)\phi_0(t') e^{2\nu-1} \frac{1}{|t-t'|^{2\nu+1}} \bigg|_{\epsilon \to 0} \] \( ^{(16)} \)
In terms of the \( \epsilon \)-independent source field \( \phi_0(t) \) in (10) the result is
\[ S_{\text{on-shell}} = -\frac{\nu}{\sqrt{\pi}} \frac{\Gamma(\nu + \frac{1}{2})}{\Gamma(\nu)} \int dt \int dt' \phi_0(t)\phi_0(t') \bigg|_{\epsilon \to 0} \] \( ^{(17)} \)
The two-point function for the massive scalar (2) is now easily recovered from (11) with the resulting factor \( C_{\Delta_+} \) given by
\[ C_{\Delta_+} = -\frac{2\nu}{\sqrt{\pi}} \frac{\Gamma(\nu + 1)}{\Gamma(\nu)} \] \( ^{(18)} \)
3. Non-commutative case

We now repeat the procedure for the non-commutative case. The field equation for a massive scalar field \( \hat{\Phi} \) on non-commutative \( EAdS_2 \) is
\[ \Delta \hat{\Phi} = m^2 \hat{\Phi} \] \( ^{(19)} \)
where \( \Delta \) is the noncommutative version of the Laplacian [3]. As in [2] one can express \( \Delta \) in terms of operators \( \hat{X}^a, \ a = 1, 2, 3 \), which satisfy the \( su(1,1) \) algebra
\[ [\hat{X}^a, \hat{X}^b] = i\epsilon^{abc} \hat{X}_c \] \( ^{(20)} \)
The singular term
\[ -\frac{1}{4\pi} \frac{1}{2 - \nu} \int dt \int dt' \int d\omega e^{i\omega(t-t')} \frac{\phi_0(t)\phi_0(t')}{\epsilon^{2\nu}} \bigg|_{\epsilon \to 0} \]
can be ignored since the integration in \( \omega \) leads to a delta function and we are interested in \( t \neq t' \).
and have $\hat{X}^a \hat{X}_a = -1$. The indices are raised and lowered with the metric \( \text{diag}(-1,1,1) \), \( e^{abc} \) is totally antisymmetric, with \( e^{012} = 1 \), and \( \alpha \) is the noncommutative parameter, analogous to Planck’s constant, which should correspond to the quantum gravity scale. The Laplacian is given by

\[
\frac{\alpha^2}{2} \hat{\Delta} \hat{\Phi} = \hat{X}_a \hat{\Phi} \hat{X}^a + \hat{\Phi}.
\]

(21)

The field equation (19) results from the variation of \( \hat{\Phi} \) in the following action

\[
\hat{S} = -\frac{1}{2\alpha^2} \text{Tr} \left\{ [\hat{X}^a, \hat{\Phi}] [\hat{X}_a, \hat{\Phi}] - (\alpha m)^2 \hat{\Phi}^2 \right\}.
\]

(22)

The trace in (22) can be understood as an integration over symbols of operators, and the integration domain can again be taken to be the half-plane spanned by commuting coordinates \( z \) and \( t \). As in (6) the action can be split up into two terms, one of which vanishes on-shell and the other is a boundary term. Moreover, as was shown in, the boundary term for the noncommutative theory is identical to that of the commutative theory. (A conformal boundary limit can be defined for noncommutative EAdS in terms of the representation theory for the \( su(1,1) \) algebra \( ^2 \) Thus the on-shell action once again takes the form

\[
S|_{\text{on-shell}} = -\frac{1}{2} \int_R dt \hat{\Phi} \partial_z \hat{\Phi} \bigg|_{z \to 0},
\]

(23)

where here \( \hat{\Phi} \) actually denotes the symbol of the field.

As shown in, has exact solutions, which are given in terms of generalized Legendre functions. Upon restricting to fields that are well-behaved in the bulk, and integrating over all frequencies \( \omega \), one has

\[
\hat{\Phi} = \frac{1}{2\pi} \int d\omega a(\omega) e^{i\omega t/2} P_{\Delta-} \left( \frac{2\hat{r}}{|\omega| \kappa} \right) e^{i\omega t/2},
\]

(24)

where \( \hat{\epsilon} \) and \( \hat{r} \) are noncommutative operators, which can be used to construct \( \hat{X}^a \) (see \( ^2 \)), and \( \kappa = \sqrt{1 + \frac{\alpha^2}{4}} \). \( \hat{\epsilon} \) and \( \hat{r} \) satisfy the commutator \( [\hat{\epsilon}, \hat{r}] = -i\alpha \).

As in the commutative case, the solution can be expressed in terms of boundary fields. The symbol of \( \hat{r}^{-1} \) tends to zero in this limit, and the algebra of functions of \( \hat{r} \) and \( \hat{\epsilon} \) is effectively commutative near the boundary. Therefore the symbol of the solution (24) has the conformal boundary limit:

\[
\phi_\epsilon(t) = \frac{1}{2\pi} \int d\omega e^{i\omega t} a(\omega) P_{\Delta-} \left( \frac{2}{|\omega| \alpha \epsilon} \right), \quad \text{as } \epsilon \to 0.
\]

(25)
From this tends to $\epsilon^{\Delta - \phi_0(t)}$ as $\epsilon \to 0$ for $\nu > 0$, just as in the commutative case \([11]\). We shall again assume Dirichlet boundary conditions with $\phi_0(t)$ regarded as the source field. Re-expressing the solution \((24)\) in terms of the Fourier transform $\phi(\omega)$ of the boundary field $\phi_0(t)$,

$$
\phi(\omega) = a(\omega) P^{-\frac{\Delta}{\alpha}} \left( \frac{2}{|\omega|\alpha \epsilon} \right),
$$

we get

$$
\hat{\Phi} = \frac{1}{2\pi} \int d\omega \phi(\omega) e^{i\omega t/2} \frac{P^{-\frac{\Delta}{\alpha}} \left( \frac{2\epsilon}{|\omega|\alpha} \right)}{P^{-\frac{\Delta}{\alpha}} \left( \frac{2}{|\omega|\alpha \epsilon} \right)} e^{i|\omega|t/2}.
$$

Once again we need to compute the derivative of the solution with respect to the radial coordinate. This, in general, will introduce operator ordering ambiguities. However, such ambiguities are not a concern for the computation of the on-shell action, since we only need the result in the conformal boundary limit where the coordinates effectively commute. So let us choose the radial derivative to be $i\alpha [\hat{t}, \hat{\Phi}]$. Then the symbol of $i\alpha [\hat{t}, \hat{\Phi}]$ in the boundary limit is $-z^2 \partial_z \hat{\Phi} |_{z \to 0}$. After applying the identity \[dP^\mu(x) = \frac{1}{1 - x^2} ((\rho + 1)x P^\mu(x) - (\mu + \rho + 1)P^\mu_{(\rho+1)}(x)),\]

we get the following result for $\partial_z \hat{\Phi} |_{z = \epsilon}$ as $\epsilon \to 0$

$$
-\frac{1}{4\pi} \int d\omega e^{i\omega t} \phi(\omega) \alpha |\omega| \left( (\Delta_- - 1) \left( \frac{2}{|\omega|\alpha \epsilon} \right) + (\Delta_+ + \frac{2\kappa}{\alpha}) \left( \frac{2}{|\omega|\alpha} \right) \right).
$$

The substitution of this result in the on-shell action \((25)\) yields

$$
S_{\text{on-shell}} = \frac{1}{8\pi} \int dt \int dt' \int d\omega e^{i\omega(t-t')} \phi_0(t)\phi_0(t') \times \alpha |\omega| \left( (\Delta_- - 1) \left( \frac{2}{|\omega|\alpha \epsilon} \right) + (\Delta_+ + \frac{2\kappa}{\alpha}) \left( \frac{2}{|\omega|\alpha} \right) \right) |_{\epsilon \to 0}.
$$

As with the commutative answer, the term that yields a delta function after integration in $\omega$ can be dropped. To evaluate the remaining term we need the asymptotic behavior of $yP^\mu_{\nu+\frac{1}{2}}(1/y) / P^\mu_{\nu-\frac{1}{2}}(1/y)$ as $y \to 0$, which is

$$
\frac{2\nu}{\nu - \mu + \frac{1}{2}} \left( 1 - \frac{\Gamma(-\nu)\Gamma(\nu - \mu + \frac{1}{2})}{2^{2\nu}\Gamma(\nu)\Gamma(-\nu - \mu + \frac{1}{2})} y^{2\nu} \right), \text{ for } 0 < \nu < 1.
$$
Applying this to (29) gives
\[ S|_{\text{on-shell}} = \frac{1}{4\pi} \int dt \int dt' \int d\omega e^{i\omega(t-t')} \phi_\epsilon(t) \phi_\epsilon(t') \frac{1}{\epsilon} \times \]
\[ \frac{2\nu(\Delta_+ + \frac{2\kappa}{\alpha} + \frac{1}{2})}{\nu + \frac{2\kappa}{\alpha} + \frac{1}{2}} \left( 1 - \frac{\Gamma(-\nu)\Gamma(\nu + \frac{2\kappa}{\alpha} + \frac{1}{2})}{2^{2\nu}\Gamma(\nu)\Gamma(-\nu + \frac{2\kappa}{\alpha} + \frac{1}{2})} \frac{|\omega|\alpha\epsilon}{2} \right)^{2\nu} \bigg|_{\epsilon \to 0}. \] (30)

The integral over \( \omega \) can be performed, again using (15), with the result for the on-shell action being
\[ -\nu \sqrt{\pi} \left( \frac{\nu}{\nu + \frac{2\kappa}{\alpha} + \frac{1}{2}} \right) \int dt \int dt' \phi_\epsilon(t) \phi_\epsilon(t') \frac{1}{\epsilon} \left( \frac{\alpha\epsilon}{2} \right)^{2\nu} \left| \frac{t - t'}{|t - t'|^{2\nu + 1}} \right|_{\epsilon \to 0}. \]

In terms of the \( \epsilon \)-independent source fields \( \phi_0 \) this becomes
\[ S|_{\text{on-shell}} = -\left( \frac{\alpha}{2} \right)^{2\nu} \frac{\nu}{\sqrt{\pi}} \Gamma(\nu + \frac{1}{2}) \Gamma(\nu + \frac{2\kappa}{\alpha} + \frac{1}{2}) \frac{1}{\epsilon} \left( \frac{\alpha\epsilon}{2} \right)^{2\nu} \frac{1}{|t - t'|^{2\nu + 1}}. \]

The conformal answer for the two-point function (2) is then once again recovered after using (1). Now the overall factor is
\[ C^{nc}_{\Delta_+} = -\left( \frac{\alpha}{2} \right)^{2\nu} \frac{\Gamma(\nu + \frac{1}{2}) \Gamma(\nu + \frac{2\kappa}{\alpha} + \frac{1}{2})}{\sqrt{\pi} \Gamma(\nu) \Gamma(-\nu + \frac{2\kappa}{\alpha} + \frac{1}{2})}. \] (31)

The ratio of the noncommutative result to the commutative result (18) is given by the simple expression
\[ R^{(2)}(\alpha, \nu) = \left( \frac{\alpha}{2} \right)^{2\nu} \frac{\Gamma(\nu + \frac{2\kappa}{\alpha} + \frac{1}{2})}{\Gamma(\nu + \frac{1}{2}) \Gamma(-\nu + \frac{2\kappa}{\alpha} + \frac{1}{2})}. \] (32)

One can check various limits. The ratio smoothly goes to one in the commutative limit \( \alpha \to 0 \). It reduces to \( \kappa \) in the massless case, \( \nu = \frac{1}{2} \), which agrees with the result found in (18).\footnote{If one does a leading order expansion in \( \alpha^2 \) one gets
\[ R^{(2)}(\alpha, \nu) = 1 + \frac{\nu}{12} \left( \frac{13}{4} - \nu^2 \right) \alpha^2 + O(\alpha^4). \]}

This disagrees with the result found in (18) which is due to the fact that a different regularization scheme was used in that approach. The answers agree on the other hand, if one re-evaluates integrals in (18) with the regularization scheme used here. Both regularizations give the same result in the massless case. For a discussion of the different regularizations see the appendix in (18).

We also note that the result found here agrees with that found in (17) for the massless case, but it differs when \( \nu \neq \frac{1}{2} \).
4. Concluding Remarks

It had previously been conjectured that the boundary two-point function associated with field theory on a noncommutative $AdS_2$ bulk satisfies the constraints of conformal invariance, i.e. it has the form (2). This was previously shown to be true up to leading order in the noncommutativity parameter $\alpha$. In this article we proved that the conjecture is true to all orders in $\alpha$, at least for the scalar field. A similar proof should be possible for spinors. The result means that the noncommutativity of the bulk only affects the over-all normalization of the boundary two-point function.

It is a nontrivial step to see whether or not these results can be extended to the case of $n(>2)$-point correlators on the boundary. This will require analyzing a fully interacting field theory on the noncommutative bulk. If conformal invariance is satisfied for the $n(>2)$-point boundary correlators then, once again, only the over-all factor is effected by the noncommutative bulk. We would then have an expression for the ratio $R^{(n)}(\alpha, \nu)$ of the over-all factor for the noncommutative correlator with that of the commutative correlator. If the ratio is such that $\sqrt{R^{(n)}(\alpha, \nu)} = \sqrt{R^{(2)}(\alpha, \nu)}$, it would suggest that the only effect that bulk noncommutativity has on the boundary is to renormalize the conformal operators $O$. If instead this turns out not to be the case, we would conclude that the couplings between boundary operators pick up quantum gravity corrections as well. Only a preliminary investigation in this direction has been undertaken so far. A cubic interaction term was introduced to the massive scalar field on noncommutative $EAdS_2$ in the analysis there was quite nontrivial because only a perturbative solution (in $\alpha$) was available, and one needs to perturb in the other parameter, the coupling constant $\lambda$, as well. Although there were strong indications that the leading order correction to 3-point function satisfied the constraints of conformal invariance, we were not able to write down an explicit expression for the leading order correction, in either of the parameters. The fact that we now have an exact answer for the free theory, could be quite beneficial for obtaining the 3-point function, since then we would only have to perform an expansion in one parameter, i.e., $\lambda$. The only obstacle to proceeding with the analysis is the construction of a meaningful Green function on the noncommutative bulk. We hope to report on this construction in a future work.
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