Using of divergent problems based on teacher scaffolding levels to grow of advanced mathematical thinking of senior high school students

E Pujisatuti*, A Suyitno and Sugiman

Department of Mathematics, Universitas Negeri Semarang, Indonesia

*Corresponding author: emipujunnes@gmail.com

Abstract. Senior High School (SHS) students need to have advanced mathematical thinking. Its growth can be measured through divergent problem solutions based on the teacher's scaffolding levels. The problems are: (1) How to identify the levels of scaffolding by the teacher to foster advanced mathematical thinking? (2) How to get the results of the analysis to foster of advanced mathematical thinking of SHS students through divergent problem solutions based on the teacher scaffolding levels? The research method was a qualitative approach. The subject of this study was to take 5-6 students of SHS 1 of Ungaran. Research activities: (1) identifying of scaffolding leveling by teachers to foster the advanced mathematical thinking, (2) analyze the growth of advanced mathematical thinking of students of SMAN 1 Ungaran through divergent problem solutions based on the teacher scaffolding levels, (3) conducting of solution analysis of divergent problems based on teacher scaffolding levels; Interview; data analysis; and triangulation. The results: (1) Identified the level of scaffolding by the teacher how to foster advanced mathematical thinking. (2) Obtained the analysis results to foster of advanced mathematical thinking of high school students through divergent problem solutions based on teacher scaffolding leveling. Recommended advice: Teachers need to train their students to be able to work on divergent mathematical problems and provide tiered scaffolding.

1. Introduction

This article was written based on Basic Research, which focuses on mathematics learning which currently should be emphasized on students' mathematical thinking processes. This is because through searching of students' mathematical thinking processes, the teacher can know students' thinking abilities well, so the teacher can use appropriate strategies and approaches, which are based on basic research results in order to achieve optimal mathematical learning outcomes. This fundamental discovery needs to be initiated by the research team of Mathematics Education lecturers of UNNES. The advanced mathematical thinking ability is an important factor that determines student mastery of concepts and principles in advanced mathematics so it needs serious attention from the teacher, [1] and [2].

The ability of advanced mathematical thinking is not the same for every student because it is influenced by various factors, including due to differences in the level of academic ability of students themselves. The teacher guidance to students, namely scaffolding that is appropriate to the level of student ability needs to be done, [3] and [4]. Divergent problems are appropriate tools to train students...
to use all the potential mathematical thinking. In addition, through the stages of divergent problem solutions, the mathematical thinking process of students can be traced.

Divergent Problems are problems that have divergent characteristics, the meaning that the problem does not have a single correct answer or the problem has a non-single algorithm. According to [5] and [6], divergent problems also be interpreted as problems that are not routine. In addition, [7] wrote that teachers need to provide divergent problems or divergent problems whose solutions require mathematical thinking.

Examples of divergent problems that have several algorithms, as example the problem of Linear Equation Systems in Three Variables, can be solved by Elimination and Substitution. In Quadratic Equations, the roots can be searched with the abc Formula, Factoring, or Completing Squares. In order for High School students to be able to find several algorithms to get the correct answer like the example above, then the students need to develop Advanced Mathematical Thinking abilities.

Advanced Mathematical Thinking Ability is the ability of a person to be able to think logically and systematically in dealing with various problems both in mathematics and in solving life's problems. [8] stated that this ability is related to mathematical power, namely the ability of a person to be able to connect facts and evidence so that it is possible to arrive at an appropriate conclusion.

According to [9] and [10], the growth of Advanced Mathematical Thinking in students is very important, because it can be used as a means to develop critical and creative thinking skills related to reasoning and problem solving skills for students in school and if students are already involved in the community. This is consistent with the notion that the ability to think mathematically is one determinant of one's intelligence as a prerequisite for the development of critical and creative thinking skills. To grow Advanced Mathematical Thinking of High School students, teachers need to provide and train the students to get accustomed to working on open ended problems or divergent problems, those are problems that have some of the correct answer or have non-single problem solving algorithms.

Not all High School students have high mathematical abilities. Therefore, to form and grow the ability of Advanced Mathematical Thinking requires scaffolding of teachers who are gradual, programmed, and measured. According to [11], the growth of Advanced Mathematical Thinking can be described by the following scheme.

![Figure 1. Scheme to foster Advanced Mathematical Thinking](image_url)

In this scheme, it means that to foster Advanced Mathematical Thinking, compression can be in the form of teacher treatment and guidance so that there is a balance between mathematical concepts or principles that must be mastered by students with the processes that must be carried out by the teacher and students. By referring to the thought of [12] and [13], the leveling of teacher scaffolding to foster the ability of Advanced Mathematical Thinking through these basic research findings, are as follows.
1) Level 1: The teacher introduces math problems and solutions to students and then the students learn or practice the solving problems based on instructions or examples from the teacher.

2) Level 2: The teacher introduces some mathematical divergent problems and solutions to students and then students learn or practice the solving of divergent problems based on instructions or examples from the teacher.

3) Level 3: The teacher gives some divergent mathematical problems to students and then students learn or practice the solving of divergent problems with instructions or guidance from the teacher as needed.

4) Level 4: The teacher gives divergent mathematical problems to students and then the students solve or find the solutions to the divergent problems without guidance or guidance from the teacher.

The relation of this basic research with the scheme is utilizing the development of science and technology in the field of mathematics education to find the process of advanced mathematical thinking growth of High School students through divergent problem solutions based on the teacher scaffolding level. Based on the description above, the article that examines the process of growth in advanced mathematical thinking of High School students through divergent problem solutions based on teacher scaffolding is feasible and needs to be implemented.

Problems that examined in this article: (1) How to identify the level of scaffolding by the teacher to foster of students' advanced mathematical thinking? (2) How is the analysis of fundamental thinking to develop advanced mathematical thinking of High School students through divergent problem solutions based on the teacher scaffolding level?

The purposes of writing this article: (1) To identify the level of scaffolding by the teacher to foster of students' advanced mathematical thinking. (2) To get the results of the analysis of fundamental thoughts to foster students' advanced mathematical thinking of High School students through divergent problem solutions based on the teacher scaffolding level.

2. Research Method

2.1 Research Approaches and Subjects
This research was used a research method with a qualitative approach. The subject of this study was to take 6 students of SMAN 1 Ungaran Regency of Semarang during this research, which was appointed by the Research Team and the SMAN 1 Ungaran.

2.2 Research Location and Time
The research location is in Grade XI of SMAN 1 Ungaran Regency of Semarang. The time of the study was 4 months from when the research was conducted.

2.3 Research Data and Data Sources
The source of the qualitative research data in this first year is students in Grade XI of SMAN 1 Ungaran Regency of Semarang in the 2019/2020 Academic Year who were selected as research subjects. The data is the results of student work which will be analyzed and traced to reveal the growth of the advanced mathematical thinking of High School students through divergent problem solutions based on the teacher scaffolding level.

2.4 Data Collection Techniques
In this qualitative study, the data collection instrument was the researcher himself. On the other hand, researchers have limitations in remembering and determining the data that must be collected. Therefore, researchers need tools in the form of interview guidelines, observation guidelines, study
document record notes so that the collection of research data can be directed and focused on the problem to be solved.

2.5 Data Validity Techniques
In qualitative research, the data collection instruments are the researchers themselves. The data collected is tested for validity in order to obtain truly objective data. There are several data validity testing techniques such as extension of the research chain, increased research persistence, review of research subjects, and triangulation/comparing findings.

2.6 Data Analysis and Interpretation Techniques
Data analysis in this study uses the rules of [15]. Data analysis includes: data reduction, data presentation, data interpretation, conclusions and verification.

2.7 Category of the Advanced Mathematical Thinking based on Scaffolding

| No | Scaffolding Leveling Component to Finds Divergent Problem Solutions | AMT Growth Category |
|----|---------------------------------------------------------------|-------------------|
| 1. | Students reach level 4                                         | Very Good         |
| 2. | Students reach level 3                                         | Good              |
| 3. | Students reach level 2                                         | Average           |
| 4. | Students reach level 1                                         | Low               |
| 5. | Student is Failed                                              | Very Low          |

2.8 Indicators of Success.
Indicators of the success of the basic research were as follows:
1) Successfully identified leveling scaffolding by the teacher to foster the advanced mathematical thinking of High School students.
2) Successfully obtained analytical results in preparing for the growth of the advanced mathematical thinking of High School students through divergent problem solutions based on the teacher scaffolding level.

3. Research Results and Discussion

3.1 Research Results
In accordance with the problems and objectives achieved, the research results were as follows.

3.1.1 Identification of scaffolding levels. To identify the level of scaffolding by the teacher to foster the advanced mathematical thinking, it is as follows:
1) Level 1: The teacher gives the subject matter in accordance with the contents of the lesson plan, then gives math problems as an exercise and the solution to the students. Students learn or practice the solving problems based on instructions or examples from the teacher. The teacher must act as a resource and facilitator.
2) Level 2: The teacher begins to give the examples of the divergent mathematical problems and through the method of discussion and question & answer, the teacher gives the solution to the students and then the students learn or practice on their own to solve the divergent problems based on instructions or examples given by the teacher. Mathematical thinking begins to be implanted in students' thinking. The teacher is still the main resource.
3) Level 3: The teacher gives the divergent mathematical divergent problems to students and then students learn or practice the solving divergent problems themselves with instructions or guidance from the teacher proportionally. Teacher assistance is individual to students who need it. Mathematical thinking begins to grow.

4) Level 4: The teacher gives the divergent mathematical problems to students and then students solve or find solutions to the divergent problems without guidance from the teacher. Students are trained to learn independently, in order to grow the advanced mathematical thinking.

3.1.2. **Growing the advanced mathematical thinking of high school students.** The following describes the results of the analysis to get the fundamental thinking in revealing the readiness of the teacher to develop of advanced mathematical thinking of High School students through divergent problem solutions based on the teacher scaffolding level.

1) To foster the advanced mathematical thinking for High School students, teachers need to have the ability to create the divergent problems based on the teacher's scaffolding level.

2) In the initial stages, the teacher has difficulty in compiling divergent problems and solutions. With the Focus Group Discussion (FGD) between the Research Team and the teacher, then the teacher finally has the ability to create divergent problems and solutions.

3) The level of teacher scaffolding in fostering the advanced mathematical thinking for High School students, the teacher still encounters a problem at Level 3, that was when there was a need for individual teacher assistance to students who need it. Teachers need to design and share their activities carefully.

4) In the end, it was identified that High School mathematics teachers were seen to be able to foster the advanced mathematical thinking of High School students through 4 level scaffolding by the teacher.

3.2 Discussion

The focus of this Basic Research is on mathematics learning which currently should be emphasized on students' mathematical thinking processes, especially in entering the 21st century. Through tracking the mathematical thinking process of High School students, High School teachers can know and recognize their students' thinking abilities well, so they can use strategies and an appropriate approach, based on basic research results in order to achieve optimal of mathematical learning outcomes for students. The students' mastery of concepts and principles in advanced mathematics needs serious attention from the teacher. This advanced mathematical thinking ability is an important factor that determines the success of High School students in learning mathematics.

On the other hand, the growth of this advanced mathematical thinking ability is not the same for every student. The reason are several factors, for example due to differences in the level of academic ability of students themselves, ways of learning students who are not right, or the teacher has not given the right treatment to foster of advanced mathematical thinking. Therefore, the guidance of teacher to students, namely scaffolding in accordance with the level of student ability needs to be done. This is consistent with the opinion of [3] and [4] about the role of teachers in fostering the advanced mathematical thinking.

In developing of advanced mathematical thinking of High School students, divergent problems are appropriate tools to train the students to use all the potential their mathematical thinking. However, in the initial stages, teacher assistance or scaffolding is very much needed. Thus, the ability of teachers to be able to create divergent problems, make solutions, and provide assistance to students through systematic and measurable stages needs to be mastered by the teacher. The teacher must be able to explore students' mathematical thinking processes when the teacher tries to foster of advanced mathematical thinking.

The mathematical thinking process here is defined as a way to respond or think mathematically about information or when students solve problems. As written in the background of this study, the mathematical thinking process is a thought process that is classified in three stages when students
work on mathematical problems, namely: (1) able to foster understanding and understanding of information or elements that are known and the elements in question, (2) find a formula or algorithm to solve the problem correctly, and (3) draw conclusions or be able to find the final results as asked.

Based on the description above, this study has succeeded in identifying the ability of teachers to prepare for growing the advanced mathematical thinking of High School students through divergent problem solutions based on teacher scaffolding leveling. It also managed to analyze the levels of teachers in providing scaffolding so that High School students are able to work on divergent problems. The second year of this study, will be analyzed in depth the growth of advanced mathematical thinking of high school students through the divergent problem solutions based on the teacher scaffolding level.

4. Conclusions

Based on the results and discussion, we can conclude that (1) to identify the level of scaffolding by the teacher to foster the advanced mathematical thinking, there are 4 Levels namely: Level 1, the teacher gives the subject matter; the teacher gives some mathematical problems as an exercise, and the teacher provides solutions as examples to students. Level 2. The teacher starts giving examples of divergent problems and solutions. Level 3. The teacher gives the divergent problems to students and the students learn or practice the solving divergent problems themselves with instructions from the teacher. Level 4. the teacher gives the divergent problems to the students and students look for the divergent problem solution without guidance from the teacher; and (2) Teachers need to have the teacher's readiness to grow the advanced mathematical thinking of High School students through the divergent problem solutions based on the teacher's scaffolding level. The teacher's readiness: (1) has the ability to create the divergent problems based on the teacher's scaffolding level; (2) The teacher must be able to apply the level of the teacher scaffolding to foster the advanced mathematical thinking of High School students.

Based on that conclusions, we suggest to (1) Teachers need to really be able to make the divergent problems based on the level of scaffolding before the teacher steps in to develop the advanced mathematical thinking of High School students; and The teacher must really understand and be able to apply the 4 levels in fostering the students' advanced mathematical thinking.

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