CP violation in $\eta$ muonic decays

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In this study, we investigate the imprints of $CP$ violation in certain $\eta$ muonic decays that could arise within the Standard Model effective field theory. In particular, we study the sensitivities that could be reached at REDTOP, a proposed $\eta$ facility. After estimating the bounds that the neutron EDM places, we find still viable to discover signals of $CP$ violation via the muons' polarization in $\eta \rightarrow \mu^+\mu^-$ decays at REDTOP, with a single effective operator as its plausible source.

1 Introduction

In this article, we investigate the signal of $CP$-violation in a set of $\eta$ muonic decays. In doing so, we assume the signal as arising from heavy physics, so that the Standard Model effective field theory (SMEFT) can be applied. As a result, two different scenarios arise: that of $CP$-violating purely hadronic operators ($CP_H$) and that of $CP$-violating quark-lepton ones ($CP_{HL}$). We provide the necessary amplitudes for Monte Carlo (MC) generators and evaluate the impact of such operators in terms of certain asymmetries, using as a benchmark a proposed $\eta$ factory with the ability of measuring muon polarization: REDTOP [1]. After estimating the impact of our operators on the neutron dipole moment (nEDM), we find that $CP$-violating quark-lepton interactions could be at the reach of REDTOP, while evading nEDM bounds. Dealing with muons, these bounds are complementary to those of the electron case which have been put recently after the ACME Collaboration results [2] in Ref. [3].

The article is organized as follows: in Section 2, we discuss the two $CP$ violating scenarios arising from the SMEFT operators and their connection from quark to hadron degrees of freedom (e.g. with $\eta$-physics). Then, in Section 3, we compute $\eta \rightarrow \{\mu^+\mu^-, \gamma\mu^+\mu^-, \mu^+\mu^-e^+e^-\}$ decays, accounting for muon polarization effects in dilepton and Dalitz cases. We provide necessary expressions for MC generators and compute different asymmetries that could be generated, providing the sensitivities at reach at REDTOP. Finally, in Section 4, we evaluate the impact of both $CP$-violating scenarios on the nEDM, which put powerful constraints.

2 The $CP$-violating scenarios

In our study, we assume that the $CP$ violating new-physics effects are heavy enough to be described through the SMEFT. Following the operator basis in Ref. [4], we can find $CP$-violation
of relevance in three different categories: those acting on the hadronic part only, which we include in the \( CP_H \) category, those mixing quark and leptons, which we include in the \( CP_{HL} \) one, and those affecting lepton-photon interactions (\( O_{\text{WEB}} \)), which we checked to be negligible and discard\(^1\) for brevity.\(^2\) Regarding the \( CP_H \) category, since we are interested in decays connecting \( \eta \) to muons, these will result at low-energies in a \( CP \)-violating shift of the \( \eta \gamma^+\gamma^- \) coupling, a scenario extensively discussed in the literature in the context of light pseudoscalar mesons \([8–10]\). In general, one has

\[
i\mathcal{M}^{\mu\bar{\mu}} = i e^2 \left( \epsilon^{\mu\nu\rho\sigma} q_1 q_2 e F_{\eta\gamma^+\gamma^-}(q_1^2, q_2^2) + \left[ \epsilon^{\mu\nu} (q_1 \cdot q_2) - q_2^\mu q_1^\nu \right] F_{\eta\gamma^+\gamma^-}^{CP}(q_1^2, q_2^2) \right.
+ \left[ \epsilon^{\mu\nu} q_2^\rho q_1^\sigma - q_1^\rho q_2^\sigma \right. - \left. q_2^\sigma q_1^\rho + (q_1 \cdot q_2) q_1^\mu q_2^\nu \right] F_{\eta\gamma^+\gamma^-}^{CP}(q_1^2, q_2^2) \right),
\]

where \( \epsilon^{0123} = +1 \), \( F_{\eta\gamma^+\gamma^-} \) is the standard transition form factor (TFF), and the latter two are \( CP \)-violating ones. For some details on our TFF description, we refer to Appendix A. Accounting for the hadronization details linking the SMEFT \( CP_H \) operators to \( F_{\eta\gamma^+\gamma^-}^{CP}(q_1^2, q_2^2) \) in a quantitative manner is a formidable task; however, this is enough to our purposes as we shall see. Coming back to the \( CP_{HL} \) category, the relevant operators here are

\[
\mathcal{O}_{\text{lequ}}^{(1)} = \frac{c_{\text{lequ}}^{(1)prst}}{v^2} (\bar{q}_2^p e_+)(q_2^s q_1^r u_t) \quad \text{and} \quad \mathcal{O}_{\text{ledq}}^{(1)} = \frac{c_{\text{ledq}}^{(1)prst}}{v^2} (\bar{d}_2^p e_+)(q_1^s q_2^r d_t),
\]

where \( v^2 \simeq \sqrt{2}G_F \), and \( \{p, r, s, t\} \) label flavor. Concerning the \( \eta \), these contain \( CP \)-violating interactions of the kind \( \mathcal{L} = -3 \mathcal{C}(\eta \bar{e} e^\prime) \), with\(^3\)

\[
\mathcal{C} \equiv \frac{\text{Im} \mathcal{C}_Q}{2v^2} \langle \Omega | \bar{q}^s \gamma^5 q^f | \eta \rangle = \text{Im}(1.57(c_{\text{lequ}}^{(1)pr11} + c_{\text{ledq}}^{pr11}) - 2.37c_{\text{ledq}}^{pr22}) \times 10^{-6}.
\]

### 3 Muonic decays and asymmetries

Having discussed the relevant hadronic matrix elements, we are prepared to discuss the different muonic decays, which in the first two cases involve the muon polarization—a property that can be measured at REDTOP\(^1\).

#### 3.1 The golden channel: \( \eta \rightarrow \mu^+\mu^- \)

In general, there are two structures governing the \( \eta \rightarrow \mu^+\mu^- \) decay amplitude, namely \([13, 14]\) \((g_P \text{ and } g_S \text{ are dimensionless})\):

\[
i\mathcal{M} = i \left[ g_P (\bar{u}i\gamma^5 v) + g_S (\bar{u}v) \right],
\]

where the first(second) term is \( CP \) even(odd). In the SM, \( g_P = -2m_\mu a^2 F_{\gamma\gamma\gamma} A \), where \( A \equiv A(m_\eta^2) \) is defined in terms of a loop integral, see Refs. \([15, 16]\) and references therein.\(^4\) The polarized

\(^1\)Particularly, the \( \mu \text{EDM} \) \([5]\) put tight constraints on these operators \([6]\).

\(^2\)We could have \( CP \) violation in the pure leptonic side via the \( \mathcal{O}_4 \) operator \([6, 7]\). However, this is irrelevant in what we find the best channel, \( \eta \rightarrow \mu^+\mu^- \), and should be negligible in other cases—see discussions later.

\(^3\)This involves the FKS scheme \([11]\) for describing the \( \eta - \eta' \) mixing (and parameters from Ref. \([12]\)), implying \( \langle \Omega | 2m_\eta \bar{q}^s \gamma^5 q^f | \eta(P) \rangle = F^{\eta}_{\gamma}(a^\lambda + F^{\eta}_{\gamma}(2m_\mu^2 - m_\eta^2) \text{tr}(a^\lambda) \). Moreover, we employ the quark masses in PDG \([5]\), at the scale \( \mu = 2 \text{ GeV}: m_u = m_d \equiv m = 3.5 \text{ MeV} \) and \( m_s = 96 \text{ MeV} \).

\(^4\) In particular, and neglecting uncertainties, we take \( A(m_\eta^2) = -1.26 - 5.47i \) \([15]\).
In order to define the asymmetries and estimate their size (in vacuum), we need to supplement this

\[ |\mathcal{M}(\lambda\mathbf{n}, \bar{\lambda}\mathbf{n})|^2 = \frac{m^2}{2} \left[ |g_P|^2 (1 - \lambda\bar{\lambda}[\mathbf{n} \cdot \mathbf{\bar{n}}]) + |g_S|^2 \beta^2_\mu (1 - \lambda\bar{\lambda}[n_z n_z - n_T \cdot \bar{n}_T]) \right. \\
+ \left. 2 \left[ \lambda\bar{\lambda} \text{Re}(g_P g_S^*)(\mathbf{n} \times \mathbf{n}) \cdot \beta_\mu + \text{Im}(g_P g_S^*) \beta_\mu \cdot (\lambda\mathbf{n} - \bar{\lambda}\mathbf{n}) \right] \right], \tag{6} \]

where—hereinafter—\( n(\bar{n}) \) will refer to the polarization axis for the \( \mu^+ (\mu^-) \) (in its rest frame), the \( \hat{z} \)-axis will point along \( \mu^+ \) direction, and \( \beta_\mu \) will refer to the \( \mu^+ \) velocity in dimuon rest-frame—here coinciding with the \( \eta \) one. This would suffice to produce a MC for polarized decays.\(^6\) Still, in order to define the asymmetries and estimate their size (in vacuum), we need to suplement this with the polarized muon decay in Appendix B, that leads to\(^7\)

\[ \frac{d\Gamma}{\Gamma_{\gamma\gamma}} = 2\beta_\mu \left( \frac{\alpha m_\mu}{\pi m_\eta} \right)^2 \left[ |\mathcal{A}|^2 (1 + b\{\beta \cdot \bar{\beta}\}) + 2\beta^2_\mu \bar{g}_S \{ b\bar{b}\{\bar{\beta} \times \bar{\beta}\} \cdot \hat{z}\} \text{Re} \mathcal{A} \\
- (b\beta_z + b\bar{\beta}_z) \text{Im} \mathcal{A} \right] + \bar{g}_S^2 (1 + b\{\beta \bar{z}_z - \beta_T \cdot \bar{T}_T\}) \right] d_{e^+}, \tag{7} \]

where \( \bar{g}_S = -g_S/(2m_\mu \alpha^2 F_{\gamma\gamma}) \) and \( d_{e^\pm} = d\Omega_{dxd\bar{d}}d\bar{x}d\bar{x} (4\pi)^{-2} n(x) n(\bar{x}) \) refers to the \( e^+ e^- \) differential spectra, which is normalized to \( \text{BR}(\mu \to e\nu\nu) \approx 1 \). The unbarred (barred) variables are kept for the \( e^+ (e^-) \) and \( b \equiv b(x) \), with \( n(x) \) and \( b(x) \) defined below Eq. (57). If integrating over \( d_{e^\pm} \), the term in braces vanish, recovering the standard result for \( \bar{g}_S^2 \to 0 \).

For the \( CP_{\eta\eta} \) scenario, \( g_S = -C \), and from Eq. (4), \( \bar{g}_S = (0.510 (c_{\text{equ}}^{(1)} + c_{\text{equ}}^{(2)}) - 0.771 c_{\text{equ}}^{(2222)}) \). For the \( CP \) one, the contribution is generated at the loop level and parallels the SM calculation. Defining \( q(l) = p_{\mu^-} \pm p_{\mu^+} \), we find

\[ \bar{g}_S = \frac{i F_{\gamma\gamma}^{CP}}{\pi^2 q^2 \beta_\mu^2} \int d^4k \left( F_{\gamma\gamma}^{CP} (k^2, (q - k)^2) w_1 + F_{\gamma\gamma}^{CP} (k^2, (q - k)^2) w_2 \right) \frac{2}{k^2(q - k)^2(p_{\mu^-} - k)^2 - m_\mu^2}, \tag{8} \]

\[ w_1 = k \cdot (q - k) l^2 - (k \cdot l)(k^2 + (q - k)^2), \quad w_2 = k^2(q - k)^2(l^2 + 2k \cdot l). \]

Using the form factors description in Appendix A, we find \( \bar{g}_S = (-0.87 - 5.5i)\epsilon_1 + 0.66\epsilon_2 \), where large hadronic uncertainties are implied.

To test our \( CP \)-violating scenarios, we define the following asymmetries

\[ A_L \equiv \frac{N(c_\phi > 0) - N(c_\phi < 0)}{N(\text{all})} = \frac{\beta_\mu}{3} \text{Im} \mathcal{A} \bar{g}_S, \tag{9} \]

\[ A_T \equiv \frac{N(s_{\phi-\bar{\phi}} > 0) - N(s_{\phi-\bar{\phi}} < 0)}{N(\text{all})} = \frac{\pi \beta_\mu}{36} \text{Re} \mathcal{A} \bar{g}_S, \tag{10} \]

where the barred version is the \( A_L \) asymmetry for the \( e^- \). As a result, we find

\[ A_L^H = 0.11\epsilon_1 - 0.04\epsilon_2, \quad A_L^L = -\text{Im} \left( 2.7 (c_{\text{equ}}^{(1)} + c_{\text{equ}}^{(2)}) - 4.1 \rho \right) \times 10^{-2}, \]

\[ A_T^H = -0.07\epsilon_1 - 0.002\epsilon_2, \quad A_T^L = -\text{Im} \left( 1.6 (c_{\text{equ}}^{(1)} + c_{\text{equ}}^{(2)}) - 2.5 \rho \right) \times 10^{-3}. \]

\(^5\)For that, we use the polarized spin projectors \( u(p, \lambda\mathbf{n}) u(p, \lambda\mathbf{n}) = \frac{1}{2} (1 + \gamma^\lambda\gamma^\phi) (\gamma^\lambda + (\lambda\mathbf{n}) \cdot (\gamma^\lambda \mathbf{n})) \). Particularly, \( n\alpha^\prime(0, \mathbf{n}) \to (\gamma\beta_n, n_T, n_T, \gamma_n) \).

\(^6\)In a real experiment the muon trajectory, its polarization, and subsequent—polarized—decay are accounted through Geant4 [17].

\(^7\)We use, as it is standard, \( \Gamma_{\gamma\gamma} = |F_{\gamma\gamma}|^2 m_\mu^3 \alpha \pi/4 \), to normalize the result. Although we give \( \bar{g}_S \) terms, in the following we will only consider the interference with SM terms.
for the $CP_H$ and $CP_{HL}$ scenarios, respectively. From BR($\eta \rightarrow \mu^+\mu^-$) = $5.8 \times 10^{-6}$, and expected $\eta$ mesons at REDTOP ($2 \times 10^{12}$) [1], we obtain that the SM background for the asymmetry, at the a 1$\sigma$ level, is of order of $N^{-1/2}$ = $3 \times 10^{-4}$. As a result, we find the following sensitivities: $\epsilon_{1(2)} \sim 10^{-3(2)}$ and $c_{22st} \sim 10^{-2}$.

### 3.2 The Dalitz decay: $\eta \rightarrow \gamma\mu^+\mu^-$

In the following we introduce the—polarized—Dalitz decays: for simplicity we do not consider the most general amplitude, but the LO SM result and its interference with our $CP$-violating amplitudes. Concerning the SM, the LO amplitude arises from the diagram in Fig. 1 (left), whose amplitude reads

$$i\mathcal{M} = ie^3\epsilon_{\mu\nu\rho\sigma} k^{\nu} q^{\rho} \tilde{\varepsilon}^{\mu*} (\bar{u}\gamma^{\rho} v) F_{\eta\gamma\gamma^*} (q^2) q^{-2}. \quad (11)$$

Employing the phase space description in terms of the dilepton invariant mass ($q^2 = (p_+ + p_-)^2 \equiv s \equiv x_\mu m_\eta^2$) and polar angle ($\theta$) (see Fig. 1 [right]), the differential decay width can be expressed as

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{m_\eta}{64} (1 - x_\mu)|\mathcal{M}|^2 dx_\mu dy = \Gamma_{\gamma\gamma} \frac{1}{2\pi} \frac{1 - x_\mu}{m_\eta^2} |\mathcal{M}|^2 dx_\mu dy. \quad (12)$$

The LO SM result for the polarized Dalitz decay results in

$$|\mathcal{M}(\lambda n, \bar{\lambda} \bar{n})|^2 = \frac{1}{4} e^6 |F_{\eta\gamma\gamma^*} (s)|^2 (m_\eta^2 - s)^2 \left[ 2 - \beta_\mu^2 \sin^2 \theta + \lambda \bar{\lambda} \left( \beta_\mu^2 \sin^2 \theta (n_z \bar{n}_z - n_T \cdot \bar{n}_T) + 2 (n_z \bar{n}_y \cos \theta + n_y \bar{n}_y \sin^2 \theta) - 2 \sqrt{1 - \beta_\mu^2 \sin^2 \theta} \cos \theta (n_z \bar{n}_y + n_y \bar{n}_n) \right) \right]. \quad (13)$$

with similar conventions as in the previous section (note that we choose, again, the $\mu^+$ to mark the $\hat{z}$ direction and the $\gamma$ to have an additional component along the $\hat{y}$ directions—see Fig. 1 [right]).

Once more, we include the muon decay to estimate the asymmetries, obtaining

$$\frac{d\Gamma}{\Gamma_{\gamma\gamma}} = \frac{1}{4\pi} \frac{\alpha}{s} |\tilde{F}_{\eta\gamma\gamma^*} (s)|^2 (1 - x_\mu)^3 ds dy dz \left[ 2 - \beta_\mu^2 \sin^2 \theta - b b \left( \beta_\mu^2 \sin^2 \theta (\beta_z \bar{\beta}_z - \beta_T \cdot \bar{\beta}_T) + 2 (\beta_z \bar{\beta}_z \cos^2 \theta + \beta_y \bar{\beta}_y \sin^2 \theta) - 2 \sqrt{1 - \beta_\mu^2 \sin^2 \theta} \cos \theta (\beta_z \bar{\beta}_y + \bar{\beta}_z \beta_y) \right) \right]. \quad (14)$$

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*The $Z$ boson contribution is discussed in Appendix D and do not affect the results here.

*The parameter $y = \beta_\mu \cos \theta$ and $\beta_\mu^2 = 1 - 4m_\mu^2/s$.

* $F_{\eta\gamma\gamma^*} (s)$ stands for the normalized transition form factor.
Integrating over $d_{e^z}$, the terms in braces vanishes and we obtain the standard result [18, 19]. Concerning our $CP$-violating scenarios, we give here the main results and relegate intermediate steps to Appendix C. The final result reads

$$\frac{d\Gamma_{C\Phi}}{\Gamma_{\gamma\gamma}} = \frac{\alpha}{\pi} \frac{\text{Im} \tilde{F}_{\gamma\gamma}^*(s) \tilde{F}_{\gamma\gamma}^{C\Phi 1*}(s)}{s} (1 - x_\mu)^3 ds dy d_{e^z}$$

$$\times \left[ \sqrt{1 - \beta_{e^z}^2} \sin \theta(b\beta_y - \bar{b}\beta_y) - \cos \theta(b\beta_y - \bar{b}\beta_y) \right], \quad \text{(15)}$$

where $\alpha_{R,I}$ is obtained from the results in Appendix C upon $\alpha_{R,I} \to \alpha_{R,I}/m_\eta^2$ and, for $\alpha_R$, $\lambda n(\bar{\lambda} n) \to b\beta(\bar{b}\beta)$ while for $\alpha_I$, $\lambda n(\bar{\lambda} n) \to -b\beta(\bar{b}\beta)$. In the following, we introduce two additional asymmetries besides those in Section 3.1

$$A_{L\gamma} = \frac{N(s_\phi > 0) - N(s_\phi < 0)}{N(\text{all})}, \quad A_{TL} = \frac{N(c_\phi c_\bar{b} > 0) - N(c_\phi c_\bar{b} < 0)}{N(\text{all})}. \quad \text{(17)}$$

While for our SM result these vanish, we find numerically\(^\text{11}\)

$$A_L^H = 0 \quad A_L^{HL} = -4 \text{Im}(1.1(c_{\text{equ}}^{(1)221} + c_{\text{edq}}^{221}) - 2c_{\text{edq}}^{222}) \times 10^{-7}, \quad \text{(18)}$$

$$A_L^{H\gamma} = -0.002 \epsilon_1 \quad A_L^{H\gamma L} = 5 \text{Im}(1.1(c_{\text{equ}}^{(1)221} + c_{\text{edq}}^{221}) - 1.7c_{\text{edq}}^{222}) \times 10^{-6}, \quad \text{(19)}$$

$$A_L^{H T L} = 2 \text{Im}(1.1(c_{\text{equ}}^{(1)221} + c_{\text{edq}}^{221}) - 1.7c_{\text{edq}}^{222}) \times 10^{-5}, \quad \text{(20)}$$

$$A_L^H = 0 \quad A_L^{HL} = -5 \text{Im}(1.1(c_{\text{equ}}^{(1)221} + c_{\text{edq}}^{221}) - 1.7c_{\text{edq}}^{222}) \times 10^{-6}. \quad \text{(21)}$$

Using $\text{BR}(\eta \to \mu^+\mu^-\gamma) = 3.1 \times 10^{-4}$ [5], we obtain that the SM background for the asymmetry, at the a $1\sigma$ level, is of order of $10^{-5}$, finding the following sensitivities: $\epsilon_1 \sim 10^{-2}$ and $c_{\text{edq}}^{224} \sim 1$.

Also, in Appendix D we find that the Z-boson parity asymmetry does not show up at this level of precision.

### 3.3 Classical channel: $\eta \to \mu^+\mu^-e^+e^-$

The double Dalitz decay has been the standard way to test $CP$-violation of pseudoscalar mesons since polarization experiments are not required [8–10]. In this study, we restrict here to the $\eta \to \mu^+\mu^-e^+e^-$ decay\(^\text{12}\) and study the interference terms alone. Concerning SM results, notation, etc., we refer to Ref. [20]. Regarding the $CP_H$ interaction, we recover the results in [20] with the addition of the the second form factor that was omitted there

$$\frac{d\Gamma_{C\Phi_H}}{\Gamma_{\gamma\gamma}} = \alpha^2 \frac{32\pi^3}{3} \text{Re} \tilde{F}_{\gamma^+\gamma^-}^{CP_H} \lambda^2 \left[ \gamma_{12} \gamma_{34} \sqrt{1 - \lambda_1^2 - \lambda_2^2} \left( 2\tilde{F}_{\gamma^+\gamma^-}^{CP_H} - z m_\eta^2 \tilde{F}_{\gamma^+\gamma^-}^{CP} \right) - \lambda_1^2 \lambda_2 \lambda_3 \lambda_4 \cos \phi \left( 2z \tilde{F}_{\gamma^+\gamma^-}^{CP_H} - w^2 m_\eta^2 \tilde{F}_{\gamma^+\gamma^-}^{CP} \right) \sin \phi \right] d\Phi. \quad \text{(22)}$$

\(^\text{11}\)For analytic results in terms of phase-space integrals, see Appendix C. In these results we use the form factors in Appendix A and assume the $CP$-violating form factors to be real.

\(^\text{12}\)The SMEFT operators involving electrons are tightly constrained as we shall see and we neglect them, while the purely muonic channel has a BR that is two orders of magnitude below, so we restrict to this channel.
Concerning the $CP_{\text{HL}}$ scenario, there are four different contributions. Those arising from the effective operators coupling to muons are

$$iM_1 = -i \frac{e^2 C}{s_{34}(p_{134}^2 - m_\mu^2)} \left[ \bar{u}_1 \gamma^\mu (\phi_{134} + m_\mu)v_2 \right] \left[ \bar{u}_3 \gamma_{\mu\nu} v_4 \right],$$

$$iM_2 = i \frac{e^2 C}{s_{34}(p_{234}^2 - m_\mu^2)} \left[ \bar{u}_1 \gamma^\mu (\phi_{234} - m_\mu)v_2 \right] \left[ \bar{u}_3 \gamma_{\mu\nu} v_4 \right],$$

while, if considering the coupling to electrons, the remaining two would be obtained upon $1(2) \rightarrow 3(4)$ and $\mu \rightarrow e$ exchange. Its contribution to the differential asymmetry reads

$$d\Gamma_{CP_{\text{HL}}} \over \Gamma_{\gamma\gamma} = \frac{\alpha}{16\pi^4} \text{Re} \frac{F_{\gamma \gamma}}{s_{12} s_{34}} \lambda^2 \sin \phi \left[ \frac{M_{\mu\nu} C 2x_{34} y_{12} y_{34} (1 + \delta) - (1 - \delta)\Xi}{s_{34}[(1 - \delta)^2 - \lambda^2 y_{12}^2]} \right] \times \sqrt{w^2(\lambda_{12}^2 - y_{12}^2(\lambda_{34}^2 - y_{34}^2))} \ d\phi.$$  

As said, in these decays a polarization analysis is not required to test for $CP$-violation; this is related to the lepton plane angular asymmetries. Defining

$$A_{\phi/2} = \frac{N(s_\phi c_\phi > 0) - N(s_\phi c_\phi < 0)}{N(\text{all})},$$

we obtain

$$A_{\phi/2}^H = -0.2\epsilon_1 + 0.0003\epsilon_2, \quad A_{\phi/2}^{HL} = -\text{Im}(1.3(c_{\text{equ}}^{(1)2211} + c_{\text{equ}}^{2211}) - 1.9 c_{\text{equ}}^{2222}) \times 10^{-5},$$

From [20], $\text{BR}(\eta \rightarrow \mu^+ \mu^- e^+ e^-) = 2.3 \times 10^{-6}$, and expected $\eta$ mesons at REDTOP, the $1\sigma$ SM background is $5 \times 10^{-4}$. Consequently, we are sensitive to $\epsilon_1 \sim 10^{-3}$ and $c_{\phi} \sim 40$.

### 4 Bounds from neutron dipole moment

The interaction of a charged fermion with the electromagnetic current ($j^\mu$) can be expressed as $\langle \ell(p')|j^\mu|\ell(p)\rangle = Q_\ell \bar{u}_\ell \Gamma^\mu(q) u_\ell$, where [31]^{13}

$$\Gamma^\mu = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_\ell} F_2(q^2) - \frac{\sigma^{\mu\nu} q_\nu}{2m_\ell} \gamma^5 F_E(q^2) + (q^2 \gamma^\mu - q q^\mu) \gamma^5 F_A(q^2)$$

with $q(\ell) = p' \mp p$. At low energies, $F_2$ and $F_E$ generate magnetic and electric dipole moments, respectively. Particularly, in their non-relativistic limit[^14],

$$e\Gamma^\mu A^{\text{NR}}_\mu = -\mu \sigma \cdot B - d \sigma \cdot E, \quad \mu = \frac{e\hbar}{2m_\ell} (1 + F_2(0)), \quad d = \frac{e\hbar}{2m_\ell} F_E(0).$$

[^13]: For a neutral fermion, such the neutron, we take $Q_n = 1$, while $F_1(0) \equiv 0$.

[^14]: Usually $\mu$ is given in units of $e\hbar/2m_\ell$ and $F_2(0)$ yields the anomalous magnetic moment. The electric dipole moment commonly refers to $d$ in cm units, such that involves $(\hbar e [\text{GeV}]^2)/(2m_\ell [\text{GeV}])F_E(0)$. Also, we take $\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m_\mu)\psi$ and $D_\mu = \partial_\mu - ieQ_\ell A_\mu$, so that $iM = ie\bar{u}_\mu \Gamma^\mu u_\mu$. With these definitions, the dipole moments can be also obtained from the effective lagrangians $\mathcal{L} = Q_\ell \bar{\psi} \sigma^{\mu\nu}(\mu + i\gamma^5 d)\psi F_{\mu\nu}$.  

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Figure 2: Contributions to the nucleon EDM via a $CP$-violating $\eta$ coupling to $\gamma\gamma$.

Being suppressed in the SM, EDMs put severe constraints on $CP$-violating new physics scenarios [6]. In addition, heavy atoms and molecules dipole moments put strong constraints for contact $CP$-violating electron-quark $D = 6$ operators [32], the reason for which we did not consider the electronic, but the muonic case—see also in this respect the recent implications from ACME Coll. [2] electron EDM result in Ref. [3]. In the sections below, it will be useful to employ projectors (in analogy to Refs. [33] for the magnetic moment) for $F$ which, in $D = 4$ dimensions read\(^\text{15}\)

\[
F_E(q^2) = \text{tr} \frac{-im\gamma^5}{q^2(q^2 - 4m^2)}(p' + m)\Gamma^\mu(p + m), \quad F_E(0) = \frac{i}{12m^2} \text{tr}(p + m)\gamma_\rho\gamma^5(p + m). \quad (32)
\]

In the following we discuss the bounds that nEDM puts on our new physics scenarios, for which we employ the projector in the $q \to 0$ limit, where dipole moments are defined.

### 4.1 nEDM bounds on $CP_H$ scenario

As stated in Section 2, there might be a number of effective operators belonging to this case—each of them contributing differently to the nEDM and posing an individual challenge. However, for our purpose—as we shall see—it will suffice to account that the $CP$-violating $\eta$ TFF will generate a nEDM via the diagrams in Fig. 2, which amplitudes read\(^\text{16}\)

\[
\begin{align*}
    iM_1 &= ie \frac{-e^2 g_{NN}}{2F_\eta} \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}_p \Gamma_\nu(-k)(p' + k + m_N)(k + q)\gamma^5u_p}{k^2((k + q)^2 - m_N^2)(p' + k)^2 - m^2_N}, \\
    iM_2 &= ie \frac{-e^2 g_{NN}}{2F_\eta} \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}_p (k + q)\gamma^5(p - k + m_N)\Gamma_\nu(-k)u_p}{k^2((k + q)^2 - m_N^2)(p - k)^2 - m^2_N}.
\end{align*} \tag{33, 34}
\]

Regarding the $\Gamma_\nu$ vertex [see Eq. (30)], we take $F_{1,2}$ on-shell form factors, which is rather similar to the methodology in [36]. Of course, this contribution is rather model dependent and there will be additional ones, but should be enough to provide an order of magnitude estimate. Using the

\(^{15}\Gamma^\mu = -q_\mu \Gamma^{\mu\nu}, \text{ where } \Gamma^{\mu\nu} \equiv \lim_{q \to 0} \partial_{q\mu} \Gamma_{\nu}.
\]

\(^{16}\text{For the } NN\eta \text{ coupling we take the results in Ref. [36], where this was given by } \mathcal{L} \supset \frac{g_{NN}}{2F_\eta} \bar{N} \gamma^\mu \gamma^5 N \partial_\mu \eta \text{ with } g_{NN} = 0.673 \text{ and } F_\eta = 1.37F_\pi.\)
projector technique, we obtain

\[
F_E(0) = \frac{g_{\eta NN} \alpha}{6F_P} \frac{16\pi^2}{i} \int \frac{d^4k}{2\pi^4} F_{\eta \gamma \gamma^*}(k^2,0) \left[ \frac{k^2[2m_N^2 - 3(k \cdot p)] - 2(k \cdot p)^2}{k^2(k^2 - m_\eta^2)((p + k)^2 - m_\eta^2)} F_1(k^2) \right.
\]

\[
+ \frac{4(k \cdot p)^3 + 2k^2(k \cdot p)[(k \cdot p) - 2m_N^2] + m_N^2 k^4 F_2(k^2)}{k^2(k^2 - m_\eta^2)((p + k)^2 - m_\eta^2)} \left. \frac{2m_N^2}{m_N^2} \right] ;
\]

\[
\tag{35}
\]

\[
= \epsilon_1 F_{\eta \gamma \gamma} g_{\eta NN} \frac{\alpha}{6F_\eta} \int_0^\infty dK^2 \frac{K^2}{K^2 + m_\eta^2} \tilde{F}_{\eta \gamma \gamma^*}(-K^2,0)(1 - \beta)
\]

\[
\times \left( F_2(-K^2) - \frac{3K^2}{16m_N^2}(3 - \beta) - F_1(-K^2) \left[ 1 + (1 + \beta)^{-1} \right] \right) ,
\]

\[
\tag{36}
\]

where $\beta = (1 + \frac{4m_N^2}{K^2})^{1/2}$ and in the second line we have used the Gegenbauer polynomials technique [35] and employed the form factor description in Appendix A. For the numerical evaluation, we adopt the electromagnetic form factors parametrization in Ref. [38], obtaining $d_E^p = 9.6 \times 10^{-19} \epsilon_1$ and $d_E^n = -6.2 \times 10^{-20} \epsilon_1$, in units of $e$ cm. Accounting that $d_E^{p(n)} = 2.1(0.30) \times 10^{-25}$ this places bounds which are orders of magnitude beyond the experimental sensitivities previously discussed. Of course, this offers no bounds on $\epsilon_2$ and an alternative would be to set $F_{\eta \gamma \gamma}^{CP(1,2)}(0,0) \to 0$—this is, a vanishing coupling to real photons. A tuning such that would be suspicious however without a dynamical origin and we conclude that CP-violating physics in the context of CP-violating $\eta \gamma \gamma^*$ interactions is out of any experiment so far.

4.2 nEDM bounds on CP_{HL} scenario

Here, there is no mechanism inducing a dipole moment at one loop, which can be related to the fact that the Green function $\langle 0 | T \{ V^\mu(x) S(P)(0) \} | 0 \rangle$ vanishes in QED+QCD due to charge conjugation. The first contribution appears at two-loops and requires renormalization, which is sketched at the quark level in Fig. 3. This involves the following operators

\[
O^{(3)}_{\ell equ} = \ldots \to -i \frac{\text{Im} c^{(3)}_{\text{equ}}}{2v^2} \left[ (\bar{e}\sigma^{\mu\nu}\gamma^5 e)(\bar{u}\sigma_{\mu\nu}u) + (\bar{e}\sigma^{\mu\nu}e)(\bar{u}\sigma_{\mu\nu}\gamma^5 u) \right] ,
\]

\[
\tag{37}
\]

\[
O_{qB(W)} = \ldots \to i \frac{\text{Im} c_{q\gamma}}{v} (\bar{q}\gamma^{\mu\nu}\gamma^5 q) F_{\mu\nu} \quad c_{u(d)\gamma} = \frac{c_{u(d)B_{Cw}} \pm c_{u(d)W^8w}}{\sqrt{2}} . \]

\[
\tag{38}
\]
Figure 4: The generic contribution to a nucleon (N) EDM (additional counterterms need to be included as well). We approximate as a low-energy (finite) contribution saturated via an intermediate nucleon (N) state contribution at low energies (reversed diagrams implied) and a high-energy contribution, including counterterms, which mimics a contact term.

For the nucleon, the CP-violating contribution to the electromagnetic vertex is

\[
\bar{u}_p \Gamma_{\mu} u_p = e^2 \sum_i \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \left[ \frac{1}{i} \int e^{ikz} \langle N_{p'} | T \left\{ j_\nu(z) (q \Gamma_1(q)) \right\} | N_p \rangle \right] \times \left[ \frac{1}{i} \int e^{-i(q-x+k-y)} \langle 0 | T \left\{ j_\mu(x) j_\nu(y) (i\tilde{\Gamma}_1) \right\}(0) | 0 \rangle \right] \\
\approx e^2 \sum_i \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \Pi_{NVT,i}^\rho(-k,k+q) \Pi_{\mu VVT,i}^\nu(k,q) g_{\nu\rho}, \tag{39}
\]

where for \( O_{\ell\text{equ}(\text{leda})} \), we have the combinations \( \sum_i \{ \Gamma_1, \tilde{\Gamma}_1 \} = -\frac{c_{\ell\text{equ}(\text{leda})}}{2} \{ i\gamma^5, 1 \} \pm \{ i\gamma^5, 1 \} \).\(^{17}\) In the following, we will simplify the calculation to get an order of magnitude estimate as follows: in the low energy region (which we take below 2 GeV), we will assume the hadronic blob to be dominated by an intermediate neutron state, as shown in Fig. 4. Above, we employ the operator product expansion (OPE) for large (euclidean) \( k \).

Concerning the low-energy part, we have two hadronic elements to be computed. For \( \Gamma = P \), we approximate such an interaction via an intermediate pseudo-Goldstone boson state (\( \pi^0, \eta, \eta' \))—similar to the \( CP_H \) scenario. Regarding \( \Gamma = S \), we approximate it via the scalar form factor (see Appendix E), while for the electromagnetic form factors we use again Ref. [38]. Regarding the \( \Pi_{\mu VVT(P)}^\nu(k,q) \), we provide them in the vanishing \( q \rightarrow 0 \) limit

\[
\Pi_{VVP}^{\mu\nu}(0,k) = \epsilon^{\mu\nu\rho\sigma} \frac{-8m_\ell}{K^2 \beta_\ell} \ln \left( \frac{\beta_\ell + 1}{\beta_\ell - 1} \right), \tag{40}
\]

\[
\Pi_{VVS}^{\mu\nu}(0,k) = (g^{\mu\nu}(k \cdot q) - k^\mu q^\nu) \frac{i}{16\pi^2} \frac{8m_\ell}{K^2 \beta_\ell} \left[ \frac{1 + \beta_\ell^2}{2\beta_\ell} \ln \left( \frac{\beta_\ell + 1}{\beta_\ell - 1} \right) - 1 \right], \tag{41}
\]

up to \( O(q^2) \) corrections, where \( \beta_\ell^2 = 1 + 4m_\ell^2K^{-2} \) and \( K^2 = -k^2 \). We obtain\(^{18}\)

\[
F_E^{Nq}(0) = \text{Im} c_{\ell\text{equ}(\text{leda})} \frac{G_F m_\mu}{\pi} \frac{1}{6\sqrt{2}\pi^2} \int_0^\infty dK \left[ m_N H_S \pm \frac{q_{NN} h_0^q}{F_P} H_P \right]. \tag{42}
\]

\(^{17}\)Note in particular that potential additional diagrams with a single photon attached to the lepton line will be related again to \( 0 | T \{ V'(x) S(P) \} | 0 \rangle = 0 \).

\(^{18}\)From Ref. [36], we have \( g_{aNN} = g_A = 1.27 \), \( g_{(\eta')(NN)} = 0.67(1.17) \) and \( F_{n(\eta')} = 1.37(1.16)F_s \) with \( F_s = 92 \text{ MeV} \). Concerning the pseudoscalar amtrix elements, and following Ref. [11], \( h_n^p = -h_n^s = F_n m_n^2 m_\ell^{-1} = 0.48 \text{ GeV}^2 \), \( h_n^{\eta'} = F_n^{\eta'} m_0^2 m_\ell^{-1} = 0.40(0.35) \text{ GeV}^2 \) and \( h_0^{\eta'} = F_0^{\eta'} (2m_K - m_0^2) m_\ell^{-1} = -0.42(0.53) \text{ GeV}^2 \).
with \( h'_p = \langle 0 | \bar{q} i \gamma^5 q | P \rangle \), where \( q = \{u, d, s\} \) is a flavor index, and the functions

\[
H_S(K) = \frac{K}{m_N^2 \beta} \left( \frac{2 \beta^2_N}{1 + \beta_N} - 1 \right) \ln \left( \frac{\beta + 1 + \beta}{\beta + 1} \right) \frac{F^N_{S P}(-K^2)}{F^N_{S P}(-K^2) + F^N_{P 2}(-K^2)},
\]

\[
H_P(K) = \frac{K \beta^{-1}}{K^2 + m_P^2} \left( \frac{1 + \beta^2}{2 \beta} \ln \left( \frac{\beta + 1 + \beta}{\beta + 1} \right) - 1 \right) \left[ F^N_{S P}(-K^2)(1 + (1 + \beta_N)^{-1}) \right.
\]
\[+ F^N_{P 2}(-K^2) \frac{3K^2}{16m_N^2}(3 - \beta_N). \]  

Numerically we find\(^{19}\)

\[
d_E^p = \Im(2.65c^{(1)2211}_{\text{equ}} - 2.75c^{2211}_{\text{eqdq}} - 0.18c^{2222}_{\text{eqdq}}) \times 10^{-23},
\]

\[
d_E^n = \Im(-1.12(32)c^{(1)2211}_{\text{equ}} + 0.34(34)c^{2211}_{\text{eqdq}} + 0.08(1)c^{2222}_{\text{eqdq}}) \times 10^{-23}. \]  

For the high-energy region the calculation parallels that of the quark level, the differences being the scale. Assuming that the theory was renormalized at a scale close to the electroweak one and assuming the resulting dipole moment negligible there, the result can be estimated by the large logs. To find these, we opt to use a cutoff regularization (\( \Lambda \)) for the quark diagram level leading to

\[
F_E^p(0) = \frac{\alpha G_F m_e m_q}{\pi} \frac{1}{6\sqrt{2}\pi^2} Q_q \int_0^\infty dK \frac{K}{m_q^2 \beta} \left[ \left( \beta_q - \frac{2}{1 + \beta_q} \right) \left( \frac{1 + \beta^2_q}{2 \beta_q} \ln \left( \frac{\beta_q + 1 + \beta_q}{\beta_q + 1} \right) - 1 \right) \right.
\]
\[\pm \left( \frac{2 \beta_q^2}{1 + \beta_q} - 1 \right) \ln \left( \frac{\beta_q + 1 + \beta_q}{\beta_q + 1} \right) \right] \Im c_{\text{equ}(dq)} \]
\[\to \frac{\alpha G_F m_e m_q}{\pi} \frac{1}{6\sqrt{2}\pi^2} \left\{ \Im c^{(1)}_{\text{equ}} (\ln^2 \Lambda - \ln \Lambda^2), \Im c_{\text{eqdq}} \ln \Lambda \right\}. \]  

As a check, the \( \ln^2 \Lambda \) terms reproduce the expectation from the one-loop RG equations.\(^{20}\) Moreover, we find good agreement for the \( \ln \Lambda \) term (which represents the leading log for \( \sigma_{\text{eqdq}} \)) comparing to the results of the recent Ref. [7].\(^{21}\) From the neutron matrix elements \( g_E^p \equiv \langle n | \bar{q} \sigma^{\mu\nu} \gamma^5 q | n \rangle \) obtained from lattice QCD in Ref. [40] at \( \mu = 2 \text{ GeV} \), and using the renormalization scale \( \mu_0 = 100 \text{ GeV} \) we obtain

\[
d_E^n = \Im(-0.59c^{(1)2211}_{\text{equ}} + 0.15c^{2211}_{\text{eqdq}} + 0.001c^{2222}_{\text{eqdq}}) \times 10^{-23}, \]  

an order of magnitude below the low-energy contribution. Summarizing, and assuming uncorrelated Wilson coefficients, we find that nEDM puts the following constraints

\[
\Im c^{(1)2211}_{\text{equ}} < 0.002, \quad \Im c^{2211}_{\text{eqdq}} < 0.007, \quad \Im c^{2222}_{\text{eqdq}} < 0.04. \]  

Once more, we emphasize that large uncertainties are implied and that results should be taken as an order of magnitude estimate. As a conclusion, we find that \( \eta \to \mu^+ \mu^- \) decays are the only ones that might show \( CP \)-violating signatures for \( c^{2222}_{\text{eqdq}} \approx 10^{-2} \).

\(^{19}\)We checked that the integral saturates at 2 GeV. The errors for the neutron are shown to illustrate the impact on the scalar form factor model only—dominated by the \( \sigma_{\pi N} \) term.

\(^{20}\)With our conventions, \( d^{(1)}_{\pi N} \supset -Q \langle \bar{Q} \rangle^{(1)}_{\text{equ}} \) and \( d^{(3)}_{\pi N} \supset -m_N \langle \bar{Q} \rangle^{(3)}_{\text{equ}} \) in agreement with Ref. [39].

\(^{21}\)In particular, with Eq. (2.35) in [7] one takes \( e \leftrightarrow d \) and \( N_c = 1 \). One also needs to take care of the sign conventions—which are essentially related to our opposite choice for the covariant derivative.
5 Conclusions and Outlook

In this study, we have examined different imprints of CP violation arising from the SMEFT in different muonic \( \eta \) decays, which are effectively encoded via CP-violating transition form factors or contact \( \eta \)-lepton interactions. With the mind in REDTOP experiment—a proposed \( \eta \) factory with the ability to measure muon polarization—we have estimated the sensitivities that can be reached for both scenarios. After computing the implication of these scenarios on the nEDM, we found that only \( \eta \)-lepton interactions—particularly the \( O_{\ell edq}^{2222} \) operator—might leave an imprint via the muons’ polarization in \( \eta \to \mu^+\mu^- \) decay.\(^{22}\) This is complementary then to first generation (electron) bounds from heavy atoms and molecules EDMs. Still, there would be possible ways to improve this study, but which were beyond the scope of the present work and we sketch below.

Regarding SMEFT operators

A possible extension would be an improved determination of nEDM bounds on \( O_{\ell equ,\ell edq}^{(3)} \) operators. There are different lines that could be pursued: considering non-vanishing \( O_{\ell equ}^{(3)} \) and \( O_{uW,uB,dW,dB} \) operators and employ the full RG equations [6]; computing the full two-loop calculation; improving the hadronic model (with a serious estimate of uncertainties).

Also, one could estimate the impact on the same operators for the \( \ell = \tau \) case. Here the large-logs will become as important as hadronic effects, as they are \( \propto m_\ell \), but the hadronic model will likely require to be improved up to higher scales.

Very differently, it might be interesting to check the induced \( O_{11st}^{\ell qu (dq)} \) operators that might appear at two loops from \( O_{22st}^{\ell qu (dq)} \) and to check whether these might allow to improve the bounds derived here.

Also, once could study the \( O_{\ell e} \) operator. As said, this does not produce an effect at LO in dilepton decays. In Dalitz decays, would be analogous (up to \( i \) factor) to the \( Z \)-boson contribution, which we found negligible. For double Dalitz decays it might appear as a loop contribution, so we expect this small, with lepton EDMs likely setting strong bounds [7].

Regarding additional decays

We did not discuss here \( \eta \to \mu^+\mu^-\pi^+\pi^- \) decays, especially in the \( CP_H \) scenario. Yet these have larger BR than the leptonic one, the nEDM contribution would be very similar (for the \( CP_H \) scenario) to that in Section 4.1 up to an \( \alpha^{-1}K^2 \) factor,\(^{23}\) which results in stronger bounds. For the \( CP_{HL} \) scenario, on turn, we expect too small asymmetries as in the leptonic case. Overall, we do not expect—in principle—any CP violation in these decays.

Finally, we did not discuss polarizations in the \( \eta \to \pi^0\mu^+\mu^- \) decay, that might be interesting to analyze [1], but are beyond the scope of this study.

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\(^{22}\) Being this a potential channel to look for CP violation, one might wonder about its \( \eta' \) counterpart. An analogous computation shows \( A^f_L = -\text{Im}(1.4(c_{\ell equ}^{(1)2211} + c_{\ell edq}^{2211} + 2.9c_{\ell edq}^{2222}) \times 10^{-2}) \). Since BR(\( \eta' \to \mu^+\mu^- \)) \( \simeq 1.4 \times 10^{-7} \) [15], this cannot place stronger bounds.

\(^{23}\) The \( \pi^\pm\pi^- \) state is essentially the low-energy manifestation of the vector isovector current, which would results in a similar diagram modulo photon propagator and form factors.
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A The form factors parametrization

Here we describe the parametrizations employed for the TFFs appearing in Eq. (1). Regarding the standard—CP conserving—one, \( F_{\eta \gamma^* \gamma^*} (q_1^2, q_2^2) \), we adopt the procedure described in Ref. [16, 21] and stick to the simplest parametrization that implements precisely the low-energy behavior and respects the high-energy one [21]

\[
F_{\eta \gamma^* \gamma^*} (q_1^2, q_2^2) = F_{\eta \gamma \gamma} \frac{\Lambda^2}{\Lambda^2 - q_1^2 - q_2^2},
\]

where, \( F_{\eta \gamma \gamma} = 0.2738 \, \text{GeV}^{-1} \) and \( \Lambda = 0.724 \, \text{GeV} \) [21, 22], except when imaginary parts are relevant, which we postpone to the end of this section. For the CP-violating form factors there is of course no theoretical knowledge as they are speculative and its microscopic origin unknown. In the following, we assume the high-energy behavior from Ref. [23], implying that \( P_1 (-Q_1^2, -Q_2^2) \) and \( -\bar{Q}^2 P_2 (-Q_1^2, -Q_2^2) \) behave as \( \bar{Q}^{-2} \), where \( 2\bar{Q}^2 = Q_1^2 + Q_2^2 \). Thereby we choose

\[
F_{\eta \gamma^* \gamma^*} = \frac{\epsilon_1 \Lambda^2 F_{\eta \gamma \gamma}}{\Lambda^2 - q_1^2 - q_2^2}, \quad F_{\eta \gamma^* \gamma^*} = \frac{-2\epsilon_2 \Lambda^2 F_{\eta \gamma \gamma}}{(\Lambda^2 - q_1^2 - q_2^2)(\Lambda_H^2 - q_1^2 - q_2^2)}.
\]

We take the same value for \( \Lambda \) as before and introduce \( \Lambda_H = 1.5 \, \text{GeV} \) inspired by heavier resonances (results are rather stable upon varying these masses).

Where only imaginary parts enter the asymmetry, we employ the TFF described in Ref. [15], where \( F_{P \gamma \gamma} (s) = F_{P \gamma \gamma} [c_{P \rho} G_{\rho} (s) + c_{P \omega} G_{\omega} (s) + c_{P \phi} G_{\phi} (s)] \) and the intermediate \( \pi \pi \) rescattering is modeled as in Refs. [24, 25] through

\[
G_{\rho} (s) = \frac{M_{\rho}^2}{M_{\rho}^2 - s + \frac{s M_{\rho}^2}{9 6 \pi^2 F_{\rho}^2} \left( \ln \left( \frac{m_{\pi}^2}{\mu^2} \right) + \frac{8 m_{\pi}^2}{s} - \frac{5}{3} - \sigma (s)^3 \ln \left( \frac{\sigma (s) - 1}{\sigma (s) + 1} \right) \right)}
\]

with \( c_{\eta (\rho, \omega, \phi)} = \{9, 1, -2\}/8 \), which is—effectively—similar to Ref. [26].

B Polarized muon decay

In the effective Fermi theory, and using polarized spinor sums, we find for the \( \mu^\pm \rightarrow e^\pm (k) \nu_{\mu} (q_1) \nu_e (q_2) \) decay amplitude

\[
|\mathcal{M} (\mu^\pm, \lambda \mathbf{n})|^2 = 64 G_F^2 k_{\alpha} (p_\beta \pm \lambda m_{\mu} n_\beta) q_1^\alpha q_2^\beta.
\]

\[24\] We checked that higher order approximants did not change significantly this result, so we stick to this for simplicity.
Including phase-space and integrating over the neutrino spectra (we employ the muon rest frame), the result above reads\textsuperscript{25-26}

\[
\frac{d\Omega(\mu^\pm, \lambda n)}{dxd\Omega} = \frac{m_{\mu}}{8\pi^2} W_{\mu\mu}^A G_F^2 \beta x^2 n(x, x_0) \left[ 1 \mp \lambda b(x, x_0) \beta \cdot n \right],
\]

\[
dBR(\mu^\pm, \lambda n) = \frac{d\Omega}{4\pi} 2x^2 \beta n(x, x_0) \left[ 1 \mp \lambda b(x, x_0) \beta \cdot n \right] dx,
\]

with \( n(x, x_0) = (3 - 2x - x_0^2/x) \) and \( n(x, x_0) b(x, x_0) = 2 - 2x - \sqrt{1 - x_0^2} \). Above, \( W_{\mu\mu} = (m_{\mu}^2 + m_e^2)/2m_{\mu} \) is the maximum positron energy, \( x = E_\mu/W_{\mu\mu} \) the reduced positron energy, \( x_0 = m_e/W_{\mu\mu} \) the minimum reduced positron energy and \( \beta = \sqrt{1 - x_0^2/x^2} \) has the usual meaning. Typically, the approximation \( m_e/m_{\mu} \to 0 \) is employed, that results in the simpler expression

\[
dBR(\mu^\pm, \lambda n) = \frac{d\Omega}{4\pi} n(x) \left[ 1 \mp \lambda b(x, x_0) \beta \cdot n \right] dx,
\]

with \( x = 2E_\mu/m_{\mu} \), \( n(x) = 2x^2(3 - 2x) \) and \( b(x) = (1 - 2x)/(3 - 2x) \).

### C Results in Dalitz decays

The resulting amplitudes for our CP-violating scenarios read

\[
i\mathcal{M}_{CP}^H = -ie^3q^{-2}F_{\eta\gamma\gamma^*}(q^2)\varepsilon_{\mu}(g_{\mu\nu}(k \cdot q) - q_{\mu}k_{\nu})(\bar{u}\gamma^\nu v),
\]

\[
i\mathcal{M}_{CP}^{HL} = -ieC\varepsilon_{\mu}\left[ \frac{\bar{u}(\gamma^\mu k + 2p^\mu) v}{2k \cdot p_+} + \frac{\bar{u}(\gamma^\mu k - 2p^\mu) v}{2k \cdot p_-}\right].
\]

Their interference with the SM amplitude in Eq. (11) (\( \text{Int}_X = 2\text{Re} \mathcal{M}_{SM}^X \mathcal{M}_{CP}^X \)) yield

\[
\text{Int}_{CPH} = -\frac{1}{4} e^6 2 \text{Im} F_{\eta\gamma\gamma^*}(s) F_{\eta\gamma\gamma^*}^{CP1}(s)(m_{\eta}^2 - s)^2 s^{-1}
\times \left[ \sqrt{1 - \beta^2} \sin \theta (\lambda n_\gamma + \bar{\lambda} n_\gamma) - \cos \theta (\lambda n_\beta + \bar{\lambda} n_\beta) \right],
\]

\[
\text{Int}_{CPHL} = -\frac{1}{4} e^4 4C \left[ \alpha_R \text{Re} F_{\eta\gamma\gamma^*}(s) - \alpha_I \text{Im} F_{\eta\gamma\gamma^*}(s) \right],
\]

where we introduced the following coefficients

\[
\alpha_R = \beta_\mu \sin \theta \left\{ n_x \left[ (m_{\eta}^2 - s) \left( \sqrt{s} n_\beta (\beta_\mu - \cos \theta) + 2m_\mu \sin \theta n_\gamma \right) + 2\beta_\mu n_\beta s^{3/2}\right] \right\} \lambda \bar{\lambda},
\]

\[
\alpha_I = 2m_\mu (m_{\eta}^2 - s) \left[ -\lambda n_\beta (\beta_\mu \sin^2 \theta + 2 \cos \theta) + \bar{\lambda} n_\beta (\beta_\mu \sin^2 \theta - 2 \cos \theta) \right] - \lambda \sqrt{s} \sin \theta n_\gamma \left[ (m_{\eta}^2 - s) (\beta_\mu \cos \theta - 2) + \beta_\mu^2 (m_{\eta}^2 - 3s) \right] + \bar{\lambda} \sqrt{s} \sin \theta n_\gamma \left[ (m_{\eta}^2 - s) (\beta_\mu \cos \theta + 2) - \beta_\mu^2 (m_{\eta}^2 - 3s) \right].
\]

\textsuperscript{25}In the second line, the result for integration over \( d\Omega dx \) has been employed, that introduces \( \epsilon = m_e^2 [m_e^2 (m_e^2 - m_{\mu}^2)^2 + 6m_{\mu}^6 + 2m_e^2 m_{\mu}^2 (1 + 6 \text{Im}(m_e/m_{\mu}))] (m_{\mu}^2 + m_e^2)^{-4} \).

\textsuperscript{26}The second line is, essentially, the SM result from Ref. [27] and is that implemented in \textsc{Geant4}, where the latter is also improved upon the inclusion of radiative corrections.
Finally, we give here the asymmetries in terms of the phase space

\begin{align}
A_{L\gamma}^H &= -\frac{1}{N} \int \frac{\text{Im } \bar{F}(s) \bar{P}_\gamma^+(s)}{12s} (1 - x_\mu)^3 \beta_\mu \sqrt{1 - \beta_\mu^2} \, ds, \\
A_{L\gamma}^{HL} &= \frac{\tilde{C}}{N} \int \frac{\sqrt{x_\mu} (1 - x_\mu)}{3s_\mu^2} \text{Im } F_{\eta \gamma \gamma}(s) \left[ 2(1 - x_\mu) - \beta_\mu^2 (1 - 3x_\mu) \right] \left( 1 - \sqrt{1 - \beta_\mu^2} \right) \, ds, \\
A_{L\gamma}^{H\gamma} &= -2 \frac{\tilde{C}}{N} \int \frac{(1 - x_\mu)^2}{3s_\mu^4 m_\mu^2} \text{Im } F_{\eta \gamma \gamma}(s) \left[ 2 + (\beta_\mu - \beta_\mu^{-1}) \ln \left( \frac{1 + \beta_\mu}{1 - \beta_\mu} \right) \right] \, ds, \\
A_{L\gamma}^{HL} &= \frac{\tilde{C}}{N} \int \frac{\sqrt{x_\mu} (1 - x_\mu^2)}{18s} \text{Re } F_{\eta \gamma \gamma}(s) \beta_\mu \left( 1 - \sqrt{1 - \beta_\mu^2} \right) \, ds, \\
A_{L\gamma}^{H\gamma} &= -\frac{\tilde{C}}{N} \int \frac{(1 - x_\mu)^2}{18s^3 m_\mu^4} \text{Re } F_{\eta \gamma \gamma}(s) \left[ 2 + (\beta_\mu - \beta_\mu^{-1}) \ln \left( \frac{1 + \beta_\mu}{1 - \beta_\mu} \right) \right] \, ds,
\end{align}

where we have introduced the common parameter \( \tilde{C} = C/(e^2 m_\mu F_{\eta \gamma \gamma}) \)\(^{27} \) and

\[ N = \frac{1}{3\pi} \int \frac{1}{s} |\bar{F}_{\eta \gamma \gamma}(s)|^2 (1 - x_\mu)^3 \beta_\mu (3 - \beta_\mu^2) ds. \] (69)

The latter is, up to an \( \alpha \) factor, the \( dyd_\epsilon \)-integrated version of Eq. (13).

### D Z boson contribution to Dalitz decay

In the SM there are parity-violating contributions arising from an intermediate \( Z \)-boson state in Dalitz decays that reads

\[ iM = -i \frac{eG_F}{\sqrt{2}} \epsilon_{\mu \rho \sigma \theta} q^\rho \epsilon^{\mu \nu} (\bar{u} \gamma^\nu [(1 - 4 \sin^2 \theta_w) + \gamma^5] v) F_{\eta \gamma Z}(q^2), \] (70)

where \( \sqrt{2}G_F = g^2/(4m_W^2) \). The term without the \( \gamma^5 \) is analogous to the \( \gamma^* \eta \gamma \) up to form factor details and the \( s^{-1} \) factor; the \( \gamma^5 \) one induces the a parity-violating interference with the QED contribution. Particularly, \(^{28} \)

\[ 2 \text{Re } M_{\eta \gamma}^{SM} M_{\eta \gamma}^{Z,P} = \frac{1}{4} \frac{e^4 G_F}{\sqrt{2}} (m_\eta^2 - s^2)^2 \beta_\mu \left\{ (n_z + \bar{n}_z)(1 + \cos^2 \theta) - (n_y + \bar{n}_y) \sin \theta \cos \theta \sqrt{1 - \beta_\mu^2} \right\} \text{Re } F_{P_{\eta \gamma}^*} F_{P_{\eta \gamma} Z}^* - \sin \theta \left\{ (n_x \bar{n}_y + n_y \bar{n}_x) \sin \theta \right. \]
\[ - (n_x \bar{n}_z + n_z \bar{n}_x) \sqrt{1 - \beta_\mu^2} \cos \theta \left\} \sin \theta \text{Im } F_{P_{\eta \gamma}^*} F_{P_{\eta \gamma} Z}^* \right\}. \] (71)

Again, the full decay width can be obtained to be

\[ \frac{d\Gamma}{\Gamma_{\gamma \gamma}} = \frac{G_F}{8\sqrt{2}\pi^2} (1 - x_\mu)^3 \beta_\mu dsd\epsilon \left[ \alpha_Z^R \text{Re } \bar{F}_{P_{\eta \gamma}^*} F_{P_{\eta \gamma} Z}^* + \alpha_I^Z \text{Im } \bar{F}_{P_{\eta \gamma}^*} F_{P_{\eta \gamma} Z}^* \right], \] (72)

\[ \alpha_Z^R = (\bar{b}\bar{b}_x - b\beta_x)(1 + \cos^2 \theta) - (\bar{b}\bar{b}_y - b\beta_y) \sin \theta \cos \theta \sqrt{1 - \beta_\mu^2}, \] (73)

\[ \alpha_I^Z = (\bar{b}\beta_x + \bar{b}_x \beta_y) \sin^2 \theta - (\beta_x \bar{b}_y + \bar{b}_x \beta_y) \sin \theta \cos \theta \sqrt{1 - \beta_\mu^2}. \] (74)

\(^{27} \)From Eq. (4), \( \tilde{C} = (1.142(c_\epsilon + c_d) - 1.726c_\epsilon) \times 10^{-4}. \)

\(^{28} \)Though at this step it cannot be compared to the results in Ref. [28] that uses a different frame, we compared intermediate steps to their expression in their Eq. (9). We found agreement up to a minus sign (we remark that they calculate the \( \mu^+ \) polarization, lacking the necessary terms to compare to our full polarization amplitude).
With these results at hand, one finds that Z-boson contribution results in a non-vanishing asymmetry

\[
A_L = -\tilde{A}_L = \frac{1}{N\alpha} \frac{G_F}{18\sqrt{2}\pi^2} \int ds (1 - x_\mu) \beta^2 \mu \text{Re} \tilde{F}_\gamma \tilde{F}_\gamma^\ast.
\]  

(75)

Regarding the form factor implementation, for the normalization we employ the \(\eta - \eta'\) mixing at NLO, analogous to those in Refs. [11, 12].

\[
F_{\eta'\gamma Z} = \frac{1}{4\pi^2\sqrt{3}} \frac{F^{0}_{\eta'} F^0_{\eta} + 2\sqrt{2} F^8_{\eta'} F^8_{\eta}}{F^{8}_{\eta'} F^{0}_{\eta'\gamma} - F^{0}_{\eta'} F^{8}_{\eta'}}.
\]  

(76)

where \(c^8_Z = \frac{1}{2}(1 + K^{ST}_2 - 4\sin^2\theta_w c^8_{\eta'})\), \(c^0_{\eta} = (1 + K^{QT}_2 + K_1 - 2\sin^2\theta_w c^0_{\eta'})\), \(K^{ST}_2 = K_2(5m_n^2 - 4m_K^2)\), and \(K^{QT}_2 = K_2(m_K^2 + m_\eta^2)/2\). Finally, for its \(q^2\)-dependence, we take a TFF analog to that in Appendix A with \(\Lambda = 0.57(7)\) GeV and \(0.90(4)\) GeV for the \(\eta\) and \(\eta'\). This results from the values that would be obtained from a BL-interpolation formula [29] and from a resonance saturation approach with weights given by mixing parameters similar to Refs. [11, 22, 30]. We find then \(A_L = 6(1) \times 10^{-7}\). This is then irrelevant for the expected REDTOP statistics and would require of the order of \(10^{16}\) produced \(\eta\).

E The nucleon scalar form factors

In this section, we introduce the nucleon scalar form factors \(F^{N,s}_S(q^2) \equiv \langle N_p | \bar{q}q | N_p \rangle\). At \(q^2 = 0\), these are related to the \(\sigma\)-terms. In the following we combine theoretical (if available) and lattice results from Ref. [41] and average (enlarging errors if necessary). Regarding the theoretical input, we take it from Ref. [42] for the isoscalar \(\sigma^N_{ud} = \langle N_p | \bar{m}(\bar{u}u + \bar{d}d) | N_p \rangle\), from Ref. [43] for the isovector \(\sigma^N_3 = \langle N_p | \bar{m}(\bar{u}u - \bar{d}d) | N_p \rangle\) and restrict to lattice results for the strange one \(\sigma^s_3 = \langle N_p | m_s\bar{s}s | N_p \rangle\) due to the large theoretical uncertainties. This results in

\[
\sigma^{p,n}_{ud} = 45(29), \sigma^p_3 = 11(4), \sigma^s_3 = 20(4), \sigma^{p,n}_s = 54(5),
\]  

(77)

in MeV units. Regarding the \(q^2\)-dependency, we use the half-width rule [44], which proved to provide excellent estimates for the form factors. Since at high energies \(F^{-Q^2} \propto Q^{-6}\) [45, 46], we use three resonances. Following [47], we employ for the isoscalar channel \(f_0(500)\), \(f_0(1370)\), \(f_0(1500)\), for the isovector \(a_0(980)\), \(a_0(1450)\), \(a_0(1950)\) and for the strange one \(f_0(980)\), \(f_0(1500)\), \(f_0(1710)\). In the following we provide the central value [see ...] for the required form factors (\(x = -q^2\))

\[
\begin{align*}
F^{p,u}_S(q^2) & = \frac{6.43}{(1 + \frac{q^2}{m_f^2})(1 + \frac{q^2}{m_{T3}^2})(1 + \frac{q^2}{m_{T4}^2})} \pm \frac{1.6}{(1 + \frac{q^2}{m_{T2}^2})(1 + \frac{q^2}{m_{T4}^2})(1 + \frac{q^2}{m_{T5}^2})} \\
F^{n,u}_S(q^2) & = \frac{6.43}{(1 + \frac{q^2}{m_f^2})(1 + \frac{q^2}{m_{T3}^2})(1 + \frac{q^2}{m_{T4}^2})} \pm \frac{2.8}{(1 + \frac{q^2}{m_{T2}^2})(1 + \frac{q^2}{m_{T4}^2})(1 + \frac{q^2}{m_{T5}^2})} \\
F^{N,s}_S(q^2) & = \frac{0.57}{(1 + \frac{q^2}{m_{T3}^2})(1 + \frac{q^2}{m_{T4}^2})(1 + \frac{q^2}{m_{T5}^2})}.
\end{align*}
\]

(78)  

(79)  

(80)

29With this, we obtain \(F_{\eta'\gamma Z^*}(0,0) = 0.074(6)\) GeV\(^{-1}\) and \(F_{\eta'\gamma Z^*}(0,0) = 0.19(1)\) GeV\(^{-1}\).

30This might be compared to Ref. [28] results. Checking intermediate steps, we confirmed their results except for an overall sign. Still, we find 2 orders of magnitude suppression. This is due to the relevant scale in the problem (\(2m_\mu\) rather than \(m_\eta\)) and the resolution power function \(b(x) < 1\).
Figure 5: Comparison of the half-width estimate for the—normalized—isovector form factor (orange band) to that in Ref. [48]. We also include the isovector (blue) and strange (purple).

where \( f_{1,2,3,4,5} = f_0(500,980,1370,1500,1710) \) and \( a_i = a_0(980,450,1950) \). For illustrational purposes, we compare in Fig. 5 the prediction for the (normalized) isoscalar form factor to that in Ref. [48], obtaining an excellent agreement (note that Ref. [48] provides a reasonable estimate up to energies around 0.5 GeV).

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