Heterogeneous Coexistence of Cognitive Radio Networks in TV White Space

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Abstract

Wireless standards (e.g., IEEE 802.11af and 802.22) have been developed for enabling opportunistic access in TV white space (TVWS) using cognitive radio (CR) technology. When heterogeneous CR networks that are based on different wireless standards operate in the same TVWS, coexistence issues can potentially cause major problems. Enabling collaborative coexistence via direct coordination between heterogeneous CR networks is very challenging, due to incompatible MAC/PHY designs of coexisting networks, requirement of an over-the-air common control channel for inter-network communications, and time synchronization across devices from different networks. Moreover, such a coexistence scheme would require competing networks or service providers to exchange sensitive control information that may raise conflict of interest issues and customer privacy concerns. In this paper, we present an architecture for enabling collaborative coexistence of heterogeneous CR networks over TVWS, called Symbiotic Heterogeneous coexistence ARchitecturE (SHARE). By mimicking the symbiotic relationships between heterogeneous organisms in a stable ecosystem, SHARE establishes an indirect coordination mechanism between heterogeneous CR networks via a mediator system, which avoids the drawbacks of direct coordination. SHARE includes two spectrum sharing algorithms whose designs were inspired by well-known models and theories from theoretical ecology, viz, the interspecific competition model and the ideal free distribution model.

I. INTRODUCTION

The TV “white space” (TVWS) has the potential of providing significant bandwidth in frequencies that have very favorable propagation characteristics (i.e., long transmission ranges and
superior capability of penetrating objects) [1]. In the U.S., United Kingdom, and other countries, changes in the regulatory rules have been made or are being amended to open up the TVWS for opportunistic operations of unlicensed (or secondary) users on a non-interference basis to licensed users (a.k.a. incumbent or primary users) [7]. Industry and research stakeholders have initialized standardization efforts to enable the utilization of TVWS by leveraging cognitive radio (CR) technology. These efforts include IEEE 802.22 Wireless Regional Area Networks (WRAN) [15], IEEE 802.11af (WiFi over TVWS) [14], ECMA 392 (WPAN over TVWS) [6], etc. All of these standards rely on CR technology to overcome the challenging interference issues between incumbent and secondary networks as well as between secondary networks. In this paper, we simply use the term “CR network” to denote a CR-enabled wireless network operating over TVWS.

The coexistence of secondary wireless networks in TVWS can be broadly classified into two categories [2]: heterogeneous coexistence and homogeneous coexistence (a.k.a. self coexistence). The former refers to the coexistence of networks that employ different wireless technologies (e.g., WiFi and Bluetooth) and the latter refers to the coexistence of networks that employ the same wireless technology (e.g., neighboring 802.22 networks). There are two types of coexistence schemes: non-collaborative and collaborative coexistence schemes.

- A non-collaborative coexistence scheme is the only feasible approach when there are no means of coordination between the coexisting networks. In the existing literature, such an approach has been used to address the heterogeneous coexistence of WiFi and ZigBee networks [13], [31] as well as the homogeneous coexistence of uncoordinated WiFi deployments [21] and femto cell deployments [26], [29].
- A collaborative coexistence scheme can be employed when coexisting networks can directly coordinate their operations, and examples of such an approach include coexistence schemes for cellular networks [10], [24] and 802.22 networks [5], [9] [17], [25]. The recently formed IEEE 802.19.1 task group (TG) was chartered with the task of developing standardized methods, which are radio access technology-independent, for enabling coexistence among dissimilar or independently operated wireless networks [16]. This standard is currently being developed, and it has yet to prescribe solid solutions.

As described below, existing coexistence schemes—both non-collaborative and collaborative—cannot adequately address the problems posed by the coexistence of heterogeneous CR networks
Non-collaborative coexistence schemes are simpler and cheaper to deploy, but not as effective as collaborative schemes. Moreover, non-collaborative schemes cannot facilitate the coexistence among networks with incompatible MAC strategies (e.g., coexistence between contention-based and reservation-based MAC protocols) and cannot adequately address the hidden node problem.

Collaborative strategies are stricken with a number of very difficult technical and policy problems. First, coexisting networks would need to exchange spectrum sharing control information over a common control channel, and the realization of such a channel may require a broad standardization effort across secondary systems that would be costly. Second, even if an effective means of inter-network communications exists, implementation of collaborative strategies would rely on time synchronization across devices from different networks. Achieving synchronization over a potentially large number of coexisting TVWS networks may not be feasible. Third, collaborative approaches would require the coexisting networks to exchange potentially sensitive information—such as traffic load, bandwidth requirements, and network characteristics—to negotiate partitioning of the spectrum. Exchanging such information between competing wireless networks or service providers could potentially raise conflict-of-interest issues and customer privacy concerns. Hence, it is difficult to find a global or centralized decision maker to allocate spectrum for all competing networks.

In this paper, we propose a coexistence framework, called the Symbiotic Heterogeneous coexistence ARchitectuRE (SHARE), for enabling collaborative coexistence among heterogeneous CR networks. As its name implies, the proposed framework was inspired by the inter-species relations that exist in biological ecosystems. A symbiotic relation is a term used in biology to describe the coexistence of different species that form relations via indirect coordination. SHARE exploits a mediator system (e.g., the 802.19.1 system) to establish the indirect coordination mechanism between coexisting networks.

In SHARE, the heterogeneous coexistence problem is addressed in two ways. First, we propose an ecology-inspired spectrum allocation algorithm inspired by an interspecific resource competition model. This algorithm enables a CR network to calculate the amount of spectrum that it should appropriate without direct negotiation with competing networks. Second, we propose a foraging-based channel selection algorithm, inspired by the ideal free distribution model in the optimal foraging theory, that enables each CR network to select the most appropriate TVWS channels. Note that these algorithms do not require coexisting networks to engage in direct
negotiation. Our analytical and simulation results show that SHARE guarantees weighted-fairness in partitioning spectrum and improves spectrum utilization.

The rest of this paper is organized as follows. We provide background knowledge of the mediator system, and theoretical ecology in Section II. In Section III we give an overview of SHARE. We present the two SHARE algorithms and provide analytical results in Sections IV and V, respectively. In Section VI, we evaluate the performance of SHARE using the simulation. We conclude the paper in Section VII.

II. TECHNICAL BACKGROUND

As stated previously, SHARE employs a mediator system to establish an indirect coordination mechanism between CR networks. Due to the conflict-of-interest issues and customer privacy concerns between competing networks, the mediator is not the global decision maker, and it only forwards sanitized data to the coexisting networks. Using the forwarded information, each CR network makes coexistence decisions autonomously using the two algorithms proposed in this paper.

A. The Mediator System

The IEEE 802.19.1 system is a good candidate to serve as the mediator. The IEEE 802.19.1 system [16] defines a set of logical entities and a set of standardized interfaces for enabling coordination between heterogeneous CR networks. In Figure 1, we show the architecture of an 802.19.1 system which includes three entities in the grey box: (1) the coexistence manager (CM) acts as the local decision maker of the coexistence process; (2) the coexistence database and information server (CDIS) provides coexistence-related control information to the CMs, and (3) the coexistence enabler (CE) enables communications between the 802.19.1 system and the TV band device (TVBD) network. The TVWS database indicates the list of channels used by incumbent users and their locations, and it is connected to the 802.19.1 system via backhaul connections.

B. Coexistence-related Constructs in Ecology

In this subsection, we review the models and constructs in theoretical ecology that inspired the design of SHARE.
1) Interspecific competition model: In ecology, interspecific competition is a distributed form of competition in which individuals of different species compete for the same resource in an ecosystem without direct interactions between them [27]. The impact of interspecific competition on populations have been formalized in a mathematical model called the Lotka-Volterra (L-V) competition model [18], [28]. In this model, the impact on population dynamics of species $i$ can be calculated separately by a differential equation given below:

$$\frac{dN_i}{dt} = r_i N_i \left( 1 - \frac{N_i + \sum_{j \neq i} \alpha_{ij} N_j}{K_i} \right).$$

(1)

In this equation, $N_i$ is the population size of species $i$, $K_i$ is the carrying capacity (which is the maximum population of species $i$ if it is the only species present in the environment), $r_i$ is the intrinsic rate of increase, and $\alpha_{ij}$ is the competition coefficient which represents the impact of species $j$’s population growth on the population dynamics of species $i$.

2) Ideal free distribution (IFD): In the optimal foraging theory (OFT), animals forage in such a way as to maximize their net energy intake per unit time, and the ideal free distribution (IFD) was introduced in [8] as an OFT model. IFD has been used to analyze how animals distribute themselves across different patches of resources. Suppose there are a number of disjoint patches of resource (e.g., food) to be allocated to animals in a given environment. These patches are indexed by $i = 0, ..., p - 1$. Let $x_i$ denote the amount of animals in the $i$-th patch. The total population of animals in the environment is $\rho = \sum_{i \in [0, p-1]} x_i$.

Let $u_i$ be the suitability of the $i$-th patch, which quantifies the patch’s attractiveness to the animals.

$$u_i = \frac{a_i}{x_i};$$

(2)

where $a_i$ represents the nutrients per second in patch $i$. 

Fig. 1. IEEE 802.19.1 system architecture.
The fitness of an animal in a patch is typically assumed to be equal to the suitability of the patch. Let \( \phi_i \) be the fitness of an animal in the \( i \)-th patch, and thus \( \phi_i = u_i \).

The IFD is a sequential allocation process where more (less) animals are distributed in patches with higher (lower) suitability. The IFD’s equilibrium point is achieved when each animal simultaneously maximizes its own fitness by moving into the patch with the highest suitability. At the equilibrium point, the suitability of all patches and the fitness of all animals equalize.

The “input matching rule” is used to characterize the equilibrium point of an IFD process [23], and is prescribed as follows: animals are distributed such that for all \( i \in [0, p - 1] \),

\[
\frac{x_i}{\sum_{j=0}^{p-1} x_j} = \frac{a_i}{\sum_{j=0}^{p-1} a_j}.
\]

III. OVERVIEW OF SHARE

In this section, we present the system model, underlying assumptions and the architecture of SHARE.

A. System Model

We assume \( n \) heterogeneous CR networks are co-located, and they coexist in the same TVWS that includes \( N \) TVWS channels with identical bandwidth. Let \( K \) denote this set of CR networks, and all of these networks in \( K \) are registered with the mediator system. Every CR network is composed of multiple TVBDs and a CR-enabled base station (BS) (or access point). The TVWS channels are labeled with indices \( 0, 1, ..., N - 1 \).

Number of TVWS channels. In this paper, we focus on the case when the number of channels \( N \) is no less than the number of co-located heterogeneous networks, \( n \). That is, every network is allowed to exclusively occupy at least one channel. Here, we assume the 6 MHz TVWS channel.

The bandwidth requirement. We define the bandwidth requirement of a CR network as the number of TVWS channels that it needs to satisfy the QoS requirements of its traffic load. Let \( R_i \) denote the bandwidth requirement of network \( i \).

The mediator-based indirect coordination. SHARE establishes a mediator-based indirect coordination mechanism between coexisting CR networks. There is no direct coordination between the coexisting networks, and they have to interact with each other by exchanging control information through a third-party mediator. Specifically, SHARE utilizes a CDIS (which is one
of the components of an 802.19.1 system) as a mediator. Note that CDIS is not a global or centralized decision maker, but rather it is an information directory server with simple data processing capabilities.

**Exchange of sanitized information.** The mediator helps address conflict-of-interest issues and customer privacy concerns, which may arise when coexisting networks operated by competing service providers are required to exchange sensitive traffic information in order to carry out coexistence mechanisms. If needed, the mediator sanitizes the sensitive information received from the coexisting networks and then returns the sanitized information back to them. The coexisting networks execute their coordinated coexistence mechanisms using the sanitized data.

**B. The Two Tasks of Spectrum Sharing**

In a spectrum sharing process, a CR network has to perform two tasks: (1) figure out how much spectrum it can appropriate given its bandwidth requirement; and (2) select the best segment of spectrum to utilize. The first task is called *spectrum share allocation*, and the second task is called *channel selection*.

1) Ecology-inspired spectrum share allocation: Suppose a TVWS channel is the minimum unit amount of spectrum allocation. Let $A_i$ denote the *amount of spectrum* allocated to network $i$. Since $N \geq n$, every CR network is assumed to exclusively occupy at least one channel, and thus $A_i \in [1, N - n + 1]$ for network $i \in K$.

Equivalently, we can rewrite $A_i$ as $A_i = 1 + S_i$, where $S_i \in [0, N - n]$ is the amount of spectrum that is *dynamically* allocated to network $i$ during the spectrum sharing process. We refer to $S_i$ as the *spectrum share* of network $i$. Given $n$ competing networks in $K$,
• When $N = n$ (the trivial case), every CR network acquires only one channel, and the dynamically allocated spectrum share is zero.
• When $N > n$, every CR network may acquire more than one channel, and the sum of the spectrum share values of all networks is equal to $(N - n)$.

Our objective is that the spectrum share allocation process will eventually reach a state of equilibrium, where the spectrum share of each network is proportional to its reported bandwidth requirement.

Spectrum share allocation among the coexisting networks through direct coordination may not be possible (due to a lack of infrastructure), too costly, or may be shunned by the competing network operators because they do not want to provide their network traffic information. Instead of direct coordination, the SHARE framework adopts an indirect coordination mechanism, which is inspired by an interspecific competition model from theoretical ecology.

In ecology, the population dynamics of a species in the interspecific resource competition process can be captured by the L-V competition model. In the context of CR network coexistence, we build a weighted competition model to help a CR network to determine the dynamics of its spectrum share, given its bandwidth requirement. To complete this task, the mediator exchanges two types of control information with every CR network: (1) network $i$ reports the current value of $S_i$ to the mediator; and (2) the mediator replies back to network $i$ with the sanitized data, i.e., sum of spectrum share values of all other coexisting networks, i.e., $\sum_{j \neq i} S_j$.

2) Foraging-based channel selection: To maximally fulfill its allocated spectrum share, each CR network is allowed to select up to $\lfloor S_i \rfloor$ channels. Let $C_i$ denote the set of channels selected by network $i$, where $|C_i| \leq \lfloor S_i \rfloor$. Without explicitly knowing others’ selection, it is possible that multiple CR networks select the same channel, thereby causing inter-network interference.

In the optimal foraging theory, an animal is free to move to the patch of resource that has the maximum suitability such that the animal’s fitness can be maximized. Similarly, SHARE guides a CR network to always select the channel that has the maximum attractiveness (i.e., the minimum interference).

In the channel selection process, two types of control information are exchanged: (1) CR network $i$ sends $C_i$ to the mediator; and (2) the mediator calculates the suitability of every channel, and sends the values back to network $i$. Network $i$ uses this information in its channel selection. The definition of suitability will be given in Section V.
IV. AN ECOLOGY-INSPIRED SPECTRUM SHARE ALLOCATION ALGORITHM

A. Weighted-fair Spectrum Share Allocation Problem

Suppose a set $\mathcal{K}$ of $n$ co-located CR networks have individual bandwidth requirements $R_1, R_2, \ldots, R_n$, and operate over the same TVWS. The first objective for coexisting CR networks is to split the TVWS into $n$ pieces of spectrum shares that are proportional to their individual bandwidth requirements, without sharing individual’s bandwidth requirement with each other.

If network $j$ achieves a spectrum share of $(N-n)R_j/\sum_i R_i$, we say that this spectrum share allocation process is \textit{weighted-fair}.

Let $S(\mathcal{K}) = [S_1, S_2, \ldots, S_n]$ denote the spectrum share vector for $\mathcal{K}$ over the TVWS. We define the \textit{fairness index}, $F(S(\mathcal{K}))$, for networks in $\mathcal{K}$ as follows:

$$F(S(\mathcal{K})) = \frac{(\sum_{i\in\mathcal{K}} S_i)^2}{\sum_{i\in\mathcal{K}} R_i \cdot \sum_{i\in\mathcal{K}} R_i \left( \frac{S_i}{R_i} \right)^2}. \quad (4)$$

The maximum value of $F(S(\mathcal{K}))$ is one (the best or weighted-fair case), where the allocated spectrum share value of a network is proportional to its bandwidth requirement.

Let $\mathcal{I}_i$ denote the set of shared control information known by network $i$, and it is easy to see that $R_i \in \mathcal{I}_i$. However, $R_j \notin \mathcal{I}_i$ because competing networks $i$ and $j$ have conflict of interest issues and customer privacy concerns.

Then, we formulate a \textit{weighted-fair spectrum sharing allocation} problem where heterogeneous CR networks dynamically determine their spectrum share values.

**Problem 1.** Given a set of $n$ co-located CR networks, $\mathcal{K}$, operating over $N$ TVWS channels, one has to solve the following problem to find the spectrum share vector for $\mathcal{K}$:

$$\text{Maximize} \quad F(S(\mathcal{K}))$$

$$\text{subject to} \quad \frac{S_i}{S_j} = \frac{R_i}{R_j}, R_j \notin \mathcal{I}_i, \forall i, j \in \mathcal{K}.$$

The first constraint $\frac{S_i}{S_j} = \frac{R_i}{R_j}$ guarantees the weighted fairness, and the second constraint implies that a network $i$ has no idea about any other network $j$’s bandwidth requirement.

\footnote{The vector is a row vector or a $1 \times n$ matrix.}
B. A Weighted-fair Spectrum Competition Model

1) The stable equilibrium of the L-V competition model: The L-V competition model provides a method for defining a state of “stable equilibrium” and finding the sufficient conditions for achieving it. Consider the interspecific competition process described by equation (1), when \( K_i = K_j \) and \( \alpha_{ij} = \alpha_{ji} \) for any two species \( i \) and \( j \), the sufficient condition for stable equilibrium is \( \alpha_{ij} < 1 \).

2) The basic spectrum competition model: In Table I, we identify a number of analogies between a biological ecosystem and a CR network system. Based on equation (1) and the analogies, we can easily obtain a basic spectrum competition model as follows.

\[
\frac{dS_i}{dt} = r S_i \left( 1 - \frac{S_i + \alpha \sum_{j \neq i} S_j}{N - n} \right),
\]

where \( S_i \) is the spectrum share for network \( i \), and \( r \) is an intrinsic rate of increase. In equation (5), the carrying capacity is equal to the sum of spectrum share values of all CR networks, i.e., \( N - n \). A competition coefficient \( \alpha < 1 \) will guarantee the stable equilibrium—i.e., all the competing networks will have the same spectrum share value.

Next, we will show how to extend the basic competition model to be a weighted-fair spectrum competition model that complies with the weighted-fairness requirement (i.e., \( \frac{S_i}{S_j} = \frac{R_i}{R_j} \) for any two networks \( i \) and \( j \)) at the stable equilibrium of CR network coexistence.

3) The weighted-fair spectrum competition model: The basic spectrum competition model guarantees a stable equilibrium where all the competing networks have the same spectrum share value. However, solutions to Problem 1 must satisfy the requirement of weighted fairness, which implies that the competing networks’ spectrum share values are proportional to their bandwidth requirements. For example, if network \( i \) has a bandwidth requirement that is twice of that of network \( j \), then network \( i \)’s allocated spectrum share should be twice of the allocated spectrum share of network \( j \).

To support the weighted-fairness in spectrum share allocation, we construct a weighted-fair spectrum competition model by introducing the concept of “sub-species”. We model a CR network as a number of sub-species, and the network with a higher bandwidth requirement would have a greater number of sub-species than a network with a lower bandwidth requirement.

We use the bandwidth requirement \( R_i \) as the number of sub-species of network \( i \). Let \( S_{i,k} \) denote the spectrum share allocated to the sub-species \( k \) of network \( i \), where \( k \in [1, R_i] \). In
the weighted competition model, every sub-species \( k \) of network \( i \) calculates the change in its spectrum share according to the following equation.

\[
\delta_{i,k} = \frac{dS_{i,k}}{dt} = rS_{i,k} \left( 1 - \frac{S_{i,k} + \alpha \sum_{\kappa \neq k} S_{i,\kappa} + \alpha \sum_{j \neq i} S_j}{N - n} \right).
\]

(6)

Then, network \( i \) obtains its spectrum share value by combining the spectrum share values of all its sub-species, i.e., \( S_i = \sum_k S_{i,k} \).

In SHARE, every network \( i \) periodically sends its spectrum share value \( S_i \) to the mediator, and then the mediator sends back the sanitized data \( \beta_i = \sum_{j \neq i} S_j \) to network \( i \). The spectrum share allocation process terminates when \( \delta_{i,k} = 0 \) for all \( i \) and \( k \). Note that the sanitized data \( \beta_i \) is useful to address the conflict of interests and privacy issues between competing networks. That is, even though \( \beta_i \) is known to network \( i \), it is unable to figure out any other network \( j \)'s bandwidth requirement, i.e., \( R_j \notin I_i, \forall j \neq i \). The use of sanitized data coincides with the second constraint of Problem 1.

The pseudo-code in Algorithm 1 illustrates the spectrum share allocation process, and the detailed steps are described below.

1) A CR network \( i \) (which is viewed as a species) starts its spectrum share allocation process by creating a number of \( R_i \) sub-species.

2) At the beginning of every iteration, every sub-species calculates the change rate of its spectrum share (i.e., \( \frac{dS_{i,k}}{dt} \)) using the sanitized data \( \beta_i \) obtained from the mediator.

3) If the change rate of spectrum share is positive (or negative), a sub-species increases (or decreases) its spectrum share.

4) At the end of every iteration, send the new spectrum share value to the mediator, and update the value of \( \beta_i \) from it.

5) Last three steps are repeated until there is no sub-species with a non-zero change rate of spectrum share; that is \( \frac{dS_{i,k}}{dt} = 0 \) for every sub-species \( k \) of any network \( i \).

6) The allocated spectrum share for network \( i \) is \( \sum_k S_{i,k} \).
Algorithm 1 The Spectrum Share Allocation Algorithm.

Input: the competition coefficient $\alpha$, capacity $N - n$, intrinsic rate of increase $r$, the sanitized data $\beta_i$.

Output: the spectrum share, $S_i$, for network $i$.

1: Network $i$ generates a number of $R_i$ sub-species.
2: Update the value of $\beta_i$ from the mediator.
3: while $(\exists k \in [1, R_i], s.t. \delta_{i,k} \neq 0)$ do
4:       for $k = 1$ to $R_i$ do
5:           if $\delta_k \neq 0$ then
6:               $S_{i,k} = S_{i,k} + \delta_{i,k}$.
7:           end if
8:       end for
9:       Send $S_i = \sum_k S_{i,k}$ to the mediator, and update the value of $\beta_i$.
10: end while
11: $S_i = \sum_k S_{i,k}$.

C. Weighted-fairness and Stability

In this section, we show the properties of the equilibrium status achieved by Algorithm 1. We first prove that the spectrum share allocation algorithm satisfies the requirement of weighted-fairness defined in Problem 1.

Lemma 1. Given $n$ coexisting CR networks in $\mathcal{K}$, when $\alpha < 1$, the spectrum share allocation process of Algorithm 1 is weighted-fair in partitioning the TVWS consisting of $(N - n)$ channels.

Proof. Suppose network $i \in \mathcal{K}$ has a number of $R_i$ sub-species. The spectrum share allocation problem is equivalent to a problem where all sub-species compete for the resource using the L-V competition model. Since the sufficient condition for the equilibrium in the L-V competition model, $\alpha < 1$, is satisfied, the algorithm will terminate after a finite number of iterations, and all sub-species obtain the same spectrum share at the equilibrium point [11], [12], which is equal to $\frac{R_i}{\sum_{j \in \mathcal{K}} R_j}$. Hence, network $i$ with $R_i$ sub-species will obtain a spectrum share $R_i \frac{N-n}{\sum_{j \in \mathcal{K}} R_j}$, and thus $\frac{S_i}{S_{i'}} = \frac{R_i}{R_{i'}}$, $\forall i, i' \in \mathcal{K}$. 

Then we prove that the equilibrium point achieved by the weighted-fair competition model is
Theorem 1. Let $l = \sum_{i \in K} R_i$ represent the total number of sub-species in the system. The differential equations (6) describe an $l$-dimensional system where the equilibrium when $S_i = R_i \frac{N-n}{l}$ is stable.

Proof. Suppose CR networks in $K$ generate a total number of $l$ sub-species. For the sake of simplicity, we assign every sub-species an index from $\{1, ..., l\}$. Let $S^* = [s_1^*, ..., s_l^*]$ be the spectrum share vector at the equilibrium point for all sub-species in the system, where $s_i^*$ is the allocated spectrum share of sub-species $i$ at the equilibrium point. By Lemma 1, we have $s_i^* = \frac{N-n}{l}$, where $i \in [1, l]$, and equation (6) is equivalent to

$$
\frac{ds_i^*}{dt} = r s_i^* \left( 1 - s_i^* + \frac{\alpha \sum_{j \neq i, j \in [1, l]} s_j^*}{N-n} \right) = 0. \tag{7}
$$

That is, $s_i^* + \alpha \sum_{j \neq i, j \in [1, l]} s_j^* = N - n$.

We will prove the equilibrium $S^*$ is stable by linearizing the system equations at this equilibrium point. Let $S = [s_1, ..., s_l]$ be a spectrum share vector for all sub-species at a non-equilibrium point. We denote the differential equation at this point as

$$
G_i(S) = rs_i \left( 1 - s_i + \frac{\alpha \sum_{j \neq i, j \in [1, l]} s_j}{N-n} \right). \tag{8}
$$

Let $\Delta s_i = s_i - s_i^*$. By linearizing equation (8) at the equilibrium point, we obtain

$$
G_i(S) = G_i(s_1^*, ..., s_l^*) + \sum_{i \in [1, l]} \left( \frac{\partial G_i(S)}{\partial s_i} \right)_{s_1^*, ..., s_l^*} \Delta s_i
= - \left( \frac{r}{l} \right) \Delta s_i - \frac{r \alpha}{l} \sum_{j \neq i, j \in [1, l]} \Delta s_j. \tag{9}
$$

We derive the $l$ by $l$ Jacobian matrix for the above equation (9) as follows

$$
A = \begin{bmatrix}
-\frac{r}{l} & -\frac{r \alpha}{l} & -\frac{r \alpha}{l} & \ldots & -\frac{r \alpha}{l} \\
-\frac{r \alpha}{l} & -\frac{r}{l} & -\frac{r \alpha}{l} & \ldots & -\frac{r \alpha}{l} \\
& & & & \\
& & & & \\
-\frac{r \alpha}{l} & -\frac{r \alpha}{l} & \ldots & -\frac{r \alpha}{l} & -\frac{r}{l}
\end{bmatrix}.
$$
which is a symmetric matrix. This matrix has two eigenvalues \( \lambda = -r - \frac{(l-1)r \alpha}{l} \) and \( r(\alpha-1) \).

Since \( 0 < \alpha < 1 \), the two eigenvalues are negative. Based on the stability theory, the system is stable if all eigenvalues are negative. Hence, the differential equations shown by (6) describe an \( l \)-dimensional system and the equilibrium \( S^* = \{ s^*_i | s^*_i = \frac{N-n}{l}, \forall i \in [1, l] \} \) is stable.

**Convergence time.** Next, we analyze the time required for the proposed algorithm to converge to the stable equilibrium.

**Theorem 2.** Consider \( N \) networks that compete for the same spectrum band, then the time-to-convergence to the SHARE’s equilibrium is \( T_c = O(\ln(C/l)) \).

**Proof.** Similar to the proof of Theorem 1, there are a total number of \( l \) sub-species. Let \( A = \sum_{j \neq i, j \in [1,l]} s_j = (l-1)s_0 \), and equation (7) can be rewritten as

\[
\frac{ds_i}{dt} = rs_i \left( 1 - \frac{s_i + \alpha A}{C} \right) = 0.
\]

By integrating (10), we can obtain

\[
s_i(t) = \frac{s_0 e^{rt(1-\frac{\alpha A}{C})}(C - \alpha A)}{s_0(e^{rt(1-\frac{\alpha A}{C})} - 1) + (C - \alpha A)}.
\]

To calculate the time-to-convergence, we consider the time which is required to increase the spectrum share for network \( i \) from \( s_0 \) to \( s_i(t) = s^* = C/l \). By solving (11), the time \( T_c \) becomes:

\[
T_c = \frac{C}{r(C - \alpha A)} \ln \left( \frac{s_i(t)(C - \alpha A - s_0)}{s_0(C - \alpha A - s_i(t))} \right).
\]

The time of convergence of SHARE is \( O(\ln(C/l)) \), and it is exponentially fast.

V. A Foraging-based Channel Selection Algorithm

A. The Channel Selection Problem

Given the allocated spectrum share, network \( i \) is allowed to select up to \( M_i = \lceil S_i \rceil + 1 \) channels. We call \( M_i \) as the number of allocated channels of network \( i \), i.e., \( |C_i| \leq M_i \). Without direct coordination (e.g., sharing of control information such as \( C_i \) and \( C_j \)), it is possible that two networks \( i \) and \( j \) select the same channel, and the resulting inter-network interference will degrade the network performance.

To minimize the interference with other networks, every CR network in \( K \) tries to select channels with the highest quality (or the least interference) so as to maximally utilize its allocated spectrum share. This process is similar to the behaviors of animal in the IFD model: an animal
TABLE II
A mapping between the animal’s foraging behavior and the CR network’s channel selection process.

| Foraging                  | Channel selection |
|--------------------------|-------------------|
| A patch of resource      | A TVWS channel    |
| An animal                | A network agent   |
| Suitability of a patch   | Selectivity of a channel |

selects a patch of resources that has the highest suitability such that its own fitness can be maximized \(^{[20]}\), which leads to an evolutionary stable equilibrium. We have created analogies between the foraging behavior and the channel selection process\(^{[2]}\), as given in Table II.

Based on the mapping between channel selection and IFD processes, we consider the \(N\) channels as \(N\) disjoint patches of resource in an environment that are indexed by \(i = 0, \ldots, N - 1\). Since every network \(i\) is allowed to select up to \(M_i\) channels, a CR network will create a number of \(M_i\) network agents to complete the channel selection task.

Let \(y_i\) denote the amount of agents that selects channel \(i\). The total population of agents is \(P = \sum_{i \in [0, N-1]} y_i = \sum_{i \in K} M_i\). These agents are indexed by the mediator as 0, 1, ..., \(P - 1\).

Similar to the definition of a patch’s suitability (equation (2)), we define the selectivity of a channel \(h\) (i.e., channel \(h\)’s quality) as

\[
e_h = \frac{1}{y_h},
\]

where \(y_h\) is the number of agents that has selected channel \(h\). We equate the agent fitness of an agent that selects channel \(h\), \(f_h\), to the selectivity of channel \(h\). That is, \(f_h = e_h\).

Then, we define the system fitness as

\[
\Phi = \min\{f_0, f_1, \ldots, f_{P-1}\}.
\]

The maximum possible value for \(\Phi\) is one. When \(\Phi = 1\), every network agent exclusively occupies one channel, and every channel is selected by at most one network agent. In other words, network \(i\) occupies a number of \(M_i\) allocated channels, and its allocated spectrum share is maximally fulfilled. Then, we formulate the channel selection problem as follows.

\(^{[2]}\) Detailed analyses can be found in \([30]\).
Problem 2. Given a system of \( n \) coexisting CR networks, \( K \), one has to solve the following problem to maximize the system fitness.

Maximize \( \Phi \)

subject to \( C_j \notin I_i, \forall i, j \in K \).

The constraint \( C_j \notin I_i \) implies that in the channel selection process, there is no sharing of control information, \( C_i \) and \( C_j \), between any two networks \( i \) and \( j \).

B. The Channel Selection Strategy and Algorithm

Similar to the animal’s foraging behavior in an IFD process, a network agent under SHARE selects a channel that has the highest selectivity value, and the system tends to reach an equilibrium point where the minimum agent fitness is maximized.

At the beginning of the channel selection process, every network \( i \) knows the number of its allocated channels \( M_i \), and the set of its occupied channels \( C_i = \emptyset \). A network starts a channel selection process by sending a request on behalf of one of its agents, and this process terminates until all of its agents have finished the channel selection task. The main procedures of the channel selection process are stated below, and the pseudo-code is given in Algorithm 2.

1) The mediator processes all received requests sequentially: for a received request from network \( i \), it calculates the channel selectivity values of all channels, and send these values in a response to network \( i \).

2) The agent of network \( i \) follows a greedy algorithm to select a channel \( h \) from \( [0, ..., N-1] \), which has the highest selectivity value, i.e.,

\[
    h = \arg \max_{h \in [0,N-1]} e_h.
\]

3) Network \( i \) sends the channel selection decision to the mediator, and channel \( h \) is added to \( C_i \).

4) The mediator recalculate the selectivity value of channel \( h \) based on the received channel selection decision and equation (12), and then continue to process the next request.

C. Evolutionary Stable Strategy

In this section, we use the evolutionary game theory to prove that the above channel selection strategy is indeed an evolutionary stable strategy (ESS). In a game-theoretic perspective, each
**Algorithm 2** The Foraging-based Channel Selection Algorithm.

**Input**: $M_i$, and $K$

**Output**: $C_i$.

1: Update $C_i = \emptyset$.

2: **while** $|C_i| < M_i$ **do**

3: Obtain the selectivity values of all channels, $e_h, \forall h \in [0, N - 1]$, from the mediator.

4: Select channel $h = \arg \max_{h \in [0, N - 1]} e_h$.

5: $C_i = C_i \cup \{h\}$.

6: Send the channel selection decision $h$ to the mediator that will recalculate $e_h = \frac{1}{y_h + 1}$.

7: **end while**

A network agent under SHARE’s channel selection strategy is viewed as an individual animal that makes choices among $N$ patches (i.e., $N$ channels) to maximize its fitness according to an IFD process.

Let $P_\mu$ denote a population of animals that take strategy $\mu$, and let $f(\mu, P_\nu)$ be the fitness of an animal that takes strategy $\mu$ in a population of animals that take strategy $\nu$. A strategy $\mu$ is an ESS if both of the following two conditions hold [19].

1) For all $\nu \neq \mu$, $f(\nu, P_\mu) \leq f(\mu, P_\mu)$.

2) For all $\nu \neq \mu$, if $f(\nu, P_\mu) = f(\mu, P_\mu)$, then $f(\nu, P_\omega) < f(\mu, P_\omega)$, where $P_\omega$ is a population formed from both strategies $\mu$ and $\nu$, and $\omega = q\nu + (1 - q)\mu$ for a small $q > 0$.

By the following theorem, we establish the relationship between the ESS and the proposed channel selection strategy.

**Theorem 3.** The channel selection strategy of SHARE is an evolutionary stable strategy.

**Proof.** First, we consider the channel selection process where network agents make choices among $N$ channels as an IFD process where animals distribute themselves across $N$ patches of resource.

Let $\pi$ represent the channel selection strategy of SHARE where an agent always chooses the channel with the highest selectivity to maximize its agent fitness.

Since $\sum_i (S_i + 1) = N$, the total population of agents, $P = \sum_i M_i \leq N$. Let $P_\pi$ represent the population with all agents playing strategy $\pi$ such that the IFD is achieved.
Under strategy $\vec{\mu}$, $y_h = 1$ or 0, and thus the selectivity of a channel $e_h = 1$ or $\infty$ respectively, $\forall h \in [0, N-1]$. As a result, the strategy is equivalent to a strategy $\vec{\mu}'$ where an agent always chooses a channel $h$ with $y_h = 0$. Hence,

$$f(\vec{\mu}', P_{\vec{\nu}}) = f_h = 1,$$

for an agent that selects channel $h$ in a population of agents using strategy $\vec{\mu}'$.

Suppose that the agent makes a unilateral deviation to strategy $\vec{\nu} \neq \vec{\mu}'$ that corresponds to choosing channel $g \neq h$, where $g \in [0, N-1]$. Since $\vec{\nu} \neq \vec{\mu}'$, $y_g \neq y_h = 0$, and then $y_g \geq 1$. Then $f(\vec{\nu}, P_{\vec{\mu}}) \leq \frac{1}{2} < f(\vec{\mu}', P_{\vec{\nu}}) = f(\vec{\mu}, P_{\vec{\nu}})$.

Since $f(\vec{\nu}, P_{\vec{\mu}}) < f(\vec{\mu}, P_{\vec{\nu}})$, the channel selection strategy of SHARE is an ESS.

The resulting point of the proposed channel selection strategy $y^* = [y_0^*, ..., y_{N-1}^*]$, such that $y_h = 0$ or 1 for all $h = 0, ..., N-1$, is a unique global maximum point that solves Problem 2. This represents that each network agent simultaneously chooses a channel with the highest selectivity (i.e., the least interference) to maximize its own fitness. Any number of simultaneous disturbances from this point will lead to a possible degradation in fitness of agents.

VI. PERFORMANCE EVALUATION

In this section, we evaluate the performance of SHARE in two steps. We first investigate the stability of the equilibrium achieved by the weighted-fair spectrum share allocation scheme. Then, we compare the foraging-based channel selection scheme and the random channel selection strategy in terms of system fitness.
A. The Equilibrium in Spectrum Share Allocation

In the first set of simulations, we simulate two CR networks under SHARE that coexist over 20 channels. We fix the bandwidth requirements of two networks as $R_1 = 2$ and $R_2 = 3$, which implies that network 1 has two sub-species and network 2 has three in the spectrum share allocation process using the weighted-fair competition model. In a resource competition process based on the L-V competition model, the chosen parameter values, i.e., the competition coefficient $\alpha < 1$ and the intrinsic rate of increase $r < 2$ [22], also apply to the proposed model in this paper. The discussions on how to choose appropriate parameters values to achieve the fast convergence to the equilibrium can be found in [11], [22], and in this set of simulations we use $\alpha = 0.9$ and $r = 1.95$. Next, we show how the coexisting networks under SHARE achieve the equilibrium where the spectrum share of each network is proportional to its bandwidth requirement.
**Convergence to the equilibrium.** From Figure 2 we observe the dynamics of the spectrum share value of each network and each sub-species within a network. “Sub-species \((i, j)\)” in the figure legend represents the sub-species \(j\) within network \(i\). The system converges to the equilibrium state in finite time where all sub-species of every network are allocated the same spectrum share value. The aggregate spectrum share value for all sub-species in a network is proportional to the network’s given bandwidth requirement.

**Stability of the equilibrium.** To test the stability of the equilibrium point, we introduce two types of disturbance in bandwidth requirement by (1) silencing the sub-species \((2, 3)\) for a short time period, and (2) deleting the sub-species \((2, 3)\). Figure 3 shows the dynamics of spectrum share values when the disturbance is introduced: even the system is driven away by the change of bandwidth requirement from its current equilibrium status, it quickly converges to a new equilibrium point where the allocated spectrum share values are proportional to the new value of bandwidth requirements.

**The weighted fairness.** In this simulation, we vary the number of coexisting CR network, and in each simulation run, the bandwidth requirement, \(R_i\), for each network \(i\) is randomly chosen from the range \([1, 5]\). Then, we compare SHARE with a “fair” allocation scheme that splits the spectrum “equally” to \(n\) pieces of spectrum share and allocates them to \(n\) coexisting networks. Hence, in the fair allocation scheme, all networks get the same spectrum share value regardless of their bandwidth requirements. We measure the weighted fairness values using the fairness index defined in (4). Figure 4 clearly shows that SHARE allocates spectrum in a weighted-fair manner, and it has an advantage of guaranteeing the high weighted-fairness (close to one).

**B. The Channels Selection Strategy**

In this section, we assume the weighted-fair spectrum share allocation scheme, and compare four channel selection strategies: SHARE strategy, “random” strategy, and two hybrid strategies. A random strategy prescribes that every network selects a channel \(h\) randomly from the set of unoccupied channels \(\{0, 1, \ldots, N-1\}\). The first (or second) hybrid strategy is called “hybrid1” (or “hybrid2”), in which only one network (or half of networks) takes the random strategy, while the others follow the SHARE strategy. Besides the system fitness, we use a measure called collision probability to define the probability that the collision of channel selection decisions between two networks occurs (i.e., two networks simultaneously select the same channel).
**Number of coexisting networks.** First, we fix the number of allocated channels for every network $i$ as $M_i = 1$, and vary the number of coexisting networks. From simulation results in Figure 5, we observe that the system fitness of SHARE is close to one, and other strategies lead to a system fitness much lower than one. Similarly, the SHARE strategy avoids the collision of channel selection decisions of different networks, while other strategies fail to address this problem. For results of either the random or hybrid strategies, we also observe that the system fitness drops (or the collision probability increases) as the number of existing networks increases.

**Random bandwidth requirement values.** We simulate five coexisting networks, and let the bandwidth requirement (BR) for a network $i$ be a random variable that is uniformly distributed in the range of $[m - \sigma, m + \sigma]$, where $m = 4$ and $\sigma = 0, 1, 2, 3$. We call $m$ as the mean of BR, and $\sigma$ as the half range of BR’s change. In the trivial case when $\sigma = 0$, all networks have the same BR value.

According to the weighted-fair spectrum share allocation scheme, the increased value of $\sigma$ will introduce the discrepancy in the number of allocated channels to different coexisting networks. Note that two network agents that belong to the same network will not select the same channel. For a network that takes a random strategy, the increased number of agents will reduce the number of instances of collisions between channel selection decisions. Thus, the increase of $\sigma$ will help lower the collision probability, and thus improving the system fitness for random and hybrid strategies (see results of Figure 6).
Fig. 6. System fitness and collision probability, given random bandwidth requirements.

VII. CONCLUSIONS

Inspired by the symbiotic coexistence in ecology, in this paper we presented a framework called Symbiotic Heterogeneous coexistence ARchitecturE (SHARE), which enables collaborative coexistence among heterogeneous CR networks over TVWS. SHARE enables two heterogeneous CR networks to coexist in TVWS through a mediator-based indirect coordination mechanism between them, which avoids the drawbacks of the direct coordination mechanism. Based on the interspecific competition model and the ideal free distribution model in theoretical ecology, we proposed two SHARE algorithms for every coexisting CR network to autonomously complete the two spectrum sharing tasks: (1) dynamically determine its spectrum share that is proportional to its bandwidth requirement, and (2) select channels to improve the system fitness. Analytical and simulation results show that SHARE guarantees a stable equilibrium of coexisting networks in which the weighted-fairness is ensured and the system fitness is maximized.

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