A graceful multiversal link of particle physics to cosmology

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In this paper we work out a multiverse scenario whose physical characteristics enable us to advance the following conjecture: whereas the physics of particles and fields is confined to live in the realm of the whole multiverse formed by finite-time single universes, that for our observable universe must be confined just in one of the infinite number of universes of the multiverse when such a universe is consistently referred to an infinite cosmic time. If this conjecture is adopted then some current fundamental problems that appear when one tries to make compatible particle physics and cosmology- such as that for the cosmological constant, the arrow of time and the existence of a finite proper size of the event horizon- can be solved.

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I. INTRODUCTION.

The perception that the very big and the very small are both governed by the same physical laws is an ancient conception that has rendered extremely fruitful results such as the Galileo’s mechanics or the Newton’s universal gravitation law. That conception actually is a very popular one and is felt by many people at all cultural levels. However, the current status of particle physics and cosmology seems to inexorably avoid such a common treatment. In fact, by the time being, in spite of the the success of the inflationary paradigm, they look irreconcilable indeed [1]. There are several main points for that discrepancy. In particular, the known problem of the cosmological constant [2] by which the predicted value of the quantum-field vacuum energy density and that for cosmology are currently separated by many orders of magnitude, the feature that whereas fundamental physical theories are time-symmetric, cosmology contains an intrinsic arrow of time [3], and the existence of a future event horizon with finite proper size in accelerating cosmology which is mathematically and physically incompatible with any fundamental theory, such as string theory or quantum gravity, based on the introduction of an S-matrix requiring the propagation between points infinitely space-separated [4]. Actually, that can be regarded to be one of the greatest problems of all theoretical physics.

In this paper we take the above difficulties as being fundamental and essentially inescapable, provided that one keep oneself and the physics of particles and cosmology with in the realm of a single universe like the one which we live in. Really, what we are going to argue is that all these difficulties simply vanish whenever we take for cosmology a multiversal conception in such a way that whereas fundamental physics lives in any of the finite-time universes of the multiverse, our observed cosmology can only be described in one single universe endowed with an infinite cosmic time, isolated out from the multiverse and whose physical characteristics are precisely and consistently relatable to those of the original whole multiverse.

There already are a plethora of multiverse models, essentially including those coming from quantum mechanics [5], those described in inflationary theory [6], those which come about in string or M theories [7], and those which are just based on classical general relativity [8]. In order for trying to implement our main idea we should choose a classical multiverse model that is able to account for the current accelerating cosmic behaviour, leaving all quantum considerations to be built up later on. One of such multiverse models, which moreover, provides us with possibly the most general framework, is the recently suggested dark energy multiverse [9]. Throughout this paper we shall make all our considerations in the realm of such a model.

This idea of the multiverse and its relation with the physics in a single universe reminds us of Plato’s cave [10]. In this myth a group of humans chained in a cave with their back to the entrance with a bonfire between them and this entrance, thus they are condemned to see only the shadows of the world outside. So Plato explains the relation between the sensitive world, which we perceive, and the world of ideas, the real world. Independently of metaphysical considerations related to pure ideas and their hierarchy, we could adapt this philosophical myth to a physical myth, in such a way that the exterior of the cave is the true nature which we can measure only through its shadows. Thus the real world would play the role of the multiverse, where the particle physics are well defined, but the world that we are able to experience, the cosmological world, is only made of shadows projected from the real world. So the
II. DARK ENERGY MULTIVERSE.

In this section we shall briefly review the dark energy multiverse, a scenario based on accelerating cosmology which has recently been introduced in Ref. [9]. If we consider a cosmological model which is dominated by a quintessential fluid with constant equation of state \( p = w \rho = w \rho_0 (a(t)/a_0)^{-3(1+w)} \) (where \( a(t) \) is the scale factor for the universe, and \( p \) and \( \rho \) are respectively the pressure and energy density, with the subscript "0" denoting current value) plus a negative cosmological constant, \( \Lambda \), then the Friedmann equation can be written as

\[
H^2 = -\lambda + C a^{-2\beta},
\]

(2.1)

where \( \lambda = |\Lambda|/3, \quad C = 8\pi \rho_0/(3a_0^{-3\beta}) \) and \( \beta = 1 + w \). By integrating Eq. (2.1), we can obtain the time evolution of the cosmic scale factor, for \( \beta < 0 \),

\[
a(t) = a_0 [\cos(\alpha(t - t_0)) - b \sin(\alpha(t - t_0))]^{-2/3\beta},
\]

(2.2)

with \( \alpha = \frac{3|\beta|}{2} \lambda^{1/2} \) and \( b = \left( \frac{C}{\lambda} a_0^{-3\beta} - 1 \right)^{1/2} = \left( \frac{8\pi}{3\lambda} \rho_0 - 1 \right)^{1/2} \). Therefore, an infinite number of big rip-like singularities will occur in this model at times given by

\[
t_{br_m} = t_0 + \frac{2}{3|\beta| \lambda^{1/2}} \arctg \left[ \left( \frac{8\pi \rho_0}{3\lambda} - 1 \right)^{-1/2} \right] + \frac{2m\pi}{3|\beta| \lambda^{1/2}},
\]

(2.3)

in which \( m \) is any natural number. It is easy to check that, for \( m = 0 \), expanding the above expression for \( \lambda << 1 \), we recover the occurrence time of the big rip for a quintessence model of phantom energy without any cosmological constant, that is,

\[
t_{br} = t_0 + \frac{1}{|\beta| (6\pi \rho_0)^{1/2}}.
\]

(2.4)

Even though we have used a negative cosmological constant, the fact that one can define an overall positive vacuum energy density has led us to a true cosmic model. It is worth noticing that in order to make all possible physical regions in the above solution physically meaningful, the parameter of the equation of state should be discretized, \( |\beta| = \frac{1}{3n} \), with \( n = 1, 2, 3, \ldots \), so guaranteeing the scale factor to be always positive.

Due to the singular character of the big rips, in the absence of any wormhole-type connection between single universes, the regions between two such singularities are causally disconnected and each of these regions can be interpreted as a different spacetime (see Ref [9]), in fact a different universe within the whole infinite multiverse. Classically and in the absence of observable matter, all of these universes in the multiverse are physically indistinguishable; all starting with an infinite size, which will then steadily decrease until a minimum nonzero value,

\[
a_{\min} = a_0 \left( \frac{8\pi \rho_0}{3\lambda} \right)^{-1/3|\beta|} > 0.
\]

(2.5)

After that moment the universe acceleratingly expands again to infinity.
We finally want to remark that even though all the single universes in the multiverse are classically identical, it could well be that one might envisage the contents of the set of universes like given realizations of the quantum superposition in a quantum-mechanical treatment, somehow paralleling the Everett’s many-world interpretation [11].

It is also worth noticing that a multiverse scenario with essentially the same structure as the one discussed above can be considered as well in the Randall-Sundrum brane with positive tension, \( \mu > 0 \), provided \( 2\mu > \lambda \). In fact, if the brane is filled with phantom energy with equation of state \( p = \omega p \) (\( \omega = -1 - \alpha/3, \alpha > 0 \)) and a negative cosmological constant, \( \Lambda < 0 \), such that \( \lambda = -\Lambda/3 > 0 \), then from the Friedmann equations for the brane [12],

\[
H^2 = \rho \left( 1 + \frac{\rho}{2\mu} \right)
\]

(2.6)

\[
\dot{\rho} = -3H (\rho + p),
\]

(2.7)

we can obtain

\[
\rho = -\lambda + Da^\alpha,
\]

(2.8)

with \( D \) a constant, and for \( 2\mu > \lambda \)

\[
a(t)^\alpha = \frac{\lambda(2\mu - \lambda)}{D \left[ \frac{2\mu}{2\mu \cos^2 \left( \frac{\alpha \sqrt{(2\mu - \lambda)}}{2\sqrt{2\mu}} t \right)} - \lambda \right]].
\]

(2.9)

It can be now immediately seen that there will be an infinite number of big rip singularities at

\[
t_{br} = \frac{2\sqrt{2\mu}}{\alpha \sqrt{\lambda(2\lambda - \lambda)}} \left( \pm \arccos \sqrt{\frac{\lambda}{2\mu}} + 2\pi m \right),
\]

(2.10)

or

\[
t_{br} = \frac{2\sqrt{2\mu}}{\alpha \sqrt{\lambda(2\lambda - \lambda)}} \left( \mp \arccos \sqrt{\frac{\lambda}{2\mu}} + 2\pi (2m \pm 1) \right).
\]

(2.11)

Thus, one can construct a multiversal scenario fully analogous to the one discussed above for brane worlds with \( \mu > \lambda/2 \). This will be no longer the case however if \( \lambda \geq 2\mu \). In fact, when \( \lambda > 2\mu \), we get the solution

\[
a(t)^\alpha = \frac{\lambda(2\mu - \lambda)}{D \left[ \frac{2\mu}{2\mu \sinh^2 \left( \frac{\alpha \sqrt{(2\mu - \lambda)}}{2\sqrt{2\mu}} t \right)} + \lambda \right]],
\]

(2.12)

which does not show any big rip singularities and therefore cannot be cut off in an infinite set of independent universes, and if \( \lambda = 2\mu \) such a solution reduces to

\[
a(t)^\alpha = \frac{4\lambda}{D (4 - \lambda \alpha^2 t^2)}
\]

(2.13)

that describes a single universe which starts and dies at big rip singularities taking place at

\[
t_{br} = \pm \frac{2}{\alpha \sqrt{\lambda}}.
\]

(2.14)

Thus, the last two cases could not lead to any multiverse scenarios.

In the rest of the paper we shall restrict ourselves to the braneless multiverse case (a generalized version of which is dealt with in Appendix A), leaving the treatment of the brane multiverse to be dealt with elsewhere. We only advance here that the brane world topological defect splits in an infinite set of single defects out from which just one may develop the physical properties that we can observe in our universe.
III. THE OBSERVABLE SINGLE UNIVERSE.

Singling out a universe out from the whole multiverse should imply the consideration of observers in that single universe that would in principle interpret their spacetime as the unique, full spacetime for which, in the absence of past or future singularities or crunches, the time ought to be infinite. Since every single universe in the multiverse is defined for a finite time interval, our first task must be to re-scale the finite time interval of one such single spacetimes so that it became infinite.

Thus, out from Eq. (2.2) we first consider a single finite-time universe whose scale factor is expressed as

$$a(\tau) = a_{\min} \cos^{2/n}(\tau),$$

where $$-\pi/2 \leq \tau \leq \pi/2$$ with $$\tau = \frac{3|\beta|}{2} \lambda^{1/2} (t - t_0) + \arctan \left( \frac{H_0}{\lambda^{1/4}} \right)$$, so that $$a(\tau)$$ reaches its minimum value at $$\tau = 0$$.

We can then refer the scale factor to an infinite time interval by re-defining the time $$\tau$$ so that the new infinite time is given by $$T = \tan(\tau)$$, with $$-\infty \leq T \leq \infty$$. As expressed in terms of time $$T$$ the scale factor would become

$$a(T) = \frac{a_{\min}}{\cos^{2/(3|\beta|)}(\arctan T)} = a_{\min}(1 + T^2)^{1/(3|\beta|)}.$$  \hspace{1cm} (3.2)

In order to obtain a more familiar expression for the scale factor, it is convenient to re-define again $$T$$ in terms of another infinite time given by

$$\eta = \frac{1}{\lambda^{1/2}} \arccosh(1 + T^2),$$  \hspace{1cm} (3.3)

so that

$$a(\eta) = a_{\min} \cosh^{1/(3|\beta|)}(\lambda^{1/2} \eta),$$ \hspace{1cm} (3.4)

with $$-\infty \leq \eta \leq \infty$$ again.

Thus, we have been able to derive an expression for the scale factor which somehow resembles that for a de Sitter space. Even Eq. (3.4) reduces to the scale factor for a de Sitter space if we specialize to the case $$n = 1$$, i.e. $$|\beta| = 1/3$$. However, in order to see how our model adjust to the available observational data, one need to use the current value of the hyperbolic cosinus in Eq. (3.4), whose expression in terms of $$\eta$$ becomes

$$\cosh(\lambda^{1/2} \eta) = (H_0^2 + \lambda)^{1/2} \eta$$

Following the first procedure and $$\cosh(\lambda^{1/2} \eta) = \left(\frac{\lambda - (3|\beta|)^2 H_0^2}{\lambda}\right)^{-1/2}$$, using the second one. The ultimate reason for such a discrepancy should reside indeed in the feature that the Friedmann equation (2.1) describes the whole multiverse, not every single isolated universe in it. Actually, the most general Friedmann equation which is compatible with a generic functional form for the scale factor like in Eq. (3.4) can be checked to be

$$H^2 = C_n a^{-3|\beta_n|} + \lambda_n,$$ \hspace{1cm} (3.5)

in which $$C_n = 8\pi \rho_0 a_0^{3|\beta_n|}/3 < 0$$, $$\beta_n = 1 + w_n > 0$$ and $$\lambda_n = \Lambda_n/3 > 0$$, with $$\rho_0$$, $$w_n$$ and $$\Lambda_n$$ being general values to be specified later on. Eq. (3.5) admits the solution

$$a(t_c) = a_{\min} \cosh^{3|\beta_n|/2} \left( \frac{3\beta_n}{2} \lambda_n^{1/2} t_c \right),$$ \hspace{1cm} (3.6)

where $$a_{\min} = a_0 \left( \frac{8\pi |\rho_0|}{3\Lambda_n} \right)^{1/3|\beta_n|}.$$

Equalizing finally Eqs. (3.4) and (3.6) we get

$$\beta_n = 2|\beta| = \frac{2}{3n},$$ \hspace{1cm} (3.7)

and

$$\left( \frac{8\pi |\rho_0|}{3\Lambda_n} \right)^{1/3n} = \left( \frac{8\pi \rho_0}{3\lambda} \right)^{-1/3n}.$$ \hspace{1cm} (3.8)
and

\[(\lambda)^{1/2} = \frac{3\beta n}{2}(\lambda_n)^{1/2}t_c.\]  

(3.9)

From Eqs. (3.7) and (3.8), using the definition of \(\lambda\) and \(\lambda_n\), we have

\[\left(8\pi\right)^3 \frac{|\rho_0|}{\Lambda_n} = \frac{|\Lambda|^2}{\rho_0}.\]  

(3.10)

Now, the above two distinct procedures to check consistency of the model produce the same expression for the hyperbolic cosine as referred to current time \(t_c\),

\[\cosh^2 \left(\frac{3\beta n}{2} \lambda^{1/2} t_c\right) = \frac{\lambda}{\lambda - H_0^2},\]  

(3.11)

so implying that the new time \(t_c\) makes a well-defined choice for the cosmic time.

Besides the above argument, full consistency of the model requires that it produces a suitable acceleration; more precisely, we need also that the expression for \(\kappa = -q_0\) derived from the Friedmann equation for a flat geometry is the same as that is obtained by directly applying the definition of \(q_0\) in the present model, and that the predicted value of \(\kappa\) be compatible with the one which is expected for an accelerating universe, that is to say [13], \(\kappa\) should be slightly greater than unity [23]. In fact, if we have \(\Omega_T = \Omega_n + \Omega_{\Lambda_n} = 1\), with \(\Omega = \rho/\rho_{\text{crit}}, \rho_{\Lambda_n} = \Lambda_n/\rho_n\) and \(w_{\Lambda_n} = -1\), and from the second Friedmann equation

\[\frac{\ddot{a}}{a} = -4\pi(3p_T + \rho_T),\]  

(3.12)

we can get

\[\frac{\ddot{a}_0}{a_0} = H_0^2 \left(\frac{3\beta}{2} |\Omega_n| + 1\right),\]  

(3.13)

and whence

\[\kappa = \frac{3\beta}{2} |\Omega_n| + 1 > 1,\]  

(3.14)

which should in fact be just slightly greater than unity as \(\beta\) and \(|\Omega_n|\) are both very small as we will see later on.

Now, directly differentiating the scale factor given by Eq. (3.6) and using Eq. (3.11) we finally obtain

\[\kappa = \frac{3\beta}{2} \left(\frac{\lambda}{H_0^2} - 1\right) + 1,\]  

(3.15)

which can readily be seen to be the same as (3.14).

Once we have checked the above consistency criteria, let us consider the physics of the resulting cosmological model for one single universe in the multiverse. Actually, after starting with a cosmic model equipped with a negative cosmological constant plus a vacuum phantom fluid characterized by a positive energy density and \(\beta < 0\), so that the total vacuum energy density was positive, we have finally singled out an observable universe which still has a positive total vacuum energy density but now distributed as a sum of a positive cosmological constant and a negative dynamical part. The latter part corresponds to the so-called dual of dark energy, or in short, dual dark energy [14]. It is generally defined as a fluid having negative energy density and positive pressure, with \(\beta_n\) taking on values from 0 to 2/3. Besides the important property that the total energy density of the model is definite positive, such a negative dynamic density-energy component violates most energy conditions [15] and only may be allowed to exist provided that it is very small, in a quantum-mechanical context. This condition will be shown to be fulfilled in sec. IV and, since the quantum inequality condition [16] that any existing negative energy should be always accompanied by an overcompensating amount of positive energy (here given by the cosmological constant terms) is also satisfied, it appears that the resulting cosmic model fulfills all observational requirements and can be taken to provide us with a consistent and realistic scenario to deal with current cosmology. In fact, such a scenario is again somehow similar to a de Sitter framework and appears to also have a cosmological horizon. Because \(a(t_c) = a_{\text{min}} \cosh \left(\frac{3\beta n}{2} \lambda_n^{1/2} t_c\right)\), where \(n = 1, 2, 3, \ldots\) and the case for \(n = 1\) corresponds to de Sitter universe, the acceleration predicted by these models goes generally beyond that of a de Sitter space, without giving rise to any future singularity of the big rip type, a case which is certainly compatible with nowadays observational data.
In the Introduction it was pointed out that in spite of belonging to a long and fruitful tradition the idea that the very large and the very small are both governed by essentially the same laws has reached a turning point during the last decades from which one only finds failures in its application to current particle physics and cosmology. Moreover, it is not just that such laws are not similar or the same, but that these two branches of physics appear to be actually incompatible. In this paper we distinguish three main situations where the discrepancies are most apparent: the so-called problem of the cosmological constant, the existence of an arrow of time in cosmology and the current cosmological prediction that there exists a future event horizon whose proper size is finite. In what follows we shall argue that the headaches produced by these three apparently basic difficulties all vanish in the multiverse framework considered in this paper. We actually content that the above three shortcomings are nothing but artifacts coming from considering as the universe what really is nothing but a part of the whole physical reality. That is to say, whereas the fundamental physics resides and is well-defined in the whole realm of the multiverse (or just in one of its finite-time universes if all the multiversal components are identical), what we usually take as cosmology is defined just for one of the infinite independent spacetimes which the multiverse is made of when it is referred to a suitable infinite cosmic time.

In order to satisfy the observational data it is necessary that \( H_0^2 \sim \lambda_n \sim 10^{-52}\text{m}^{-2} \) [17]. Furthermore, the Friedmann equation for the single infinite-time universe, Eq. 3.5, imposes that \( \lambda_n > H_0^2 \) as \( C_n \) is definite negative. It follows that the absolute value of \( \rho_n \) must be very small and therefore negative energies could only appear in our observable universe when they are small enough. On the other hand, if we consider that the absolute value of the constant vacuum energy density in the multiverse corresponds to the Planck value, i.e. \( \rho_\Lambda^P \sim 10^{109}\text{erg/cm}^3 \), then \( \lambda \sim 10^{62}\text{m}^{-2} \). Taking into account relation (5.8) we then have

\[
\rho_0 \sim \frac{10^{35}\text{m}^{-3}}{|\rho_{n0}|^{1/2}}.
\]

In addition, in order to have a positive definite Hubble parameter also in the multiverse it is required that \( \rho_0 > 3\lambda/(8\pi) \sim 10^{61}\text{m}^{-2} \). From these considerations, it follows that

\[
|\rho_{n0}| \sim \frac{10^{70}}{\rho_0} < 10^{-52}\text{m}^{-2}.
\]

Because this should be always satisfied, as we have pointed out above, we can finally conclude that in the present scenario it is natural to have a cosmological constant in the multiverse with a value compatible with high energy physics and simultaneously a much smaller value for that constant of the order of those predicted in current cosmology in our observable universe.

Any cosmological models, including the one which gives rise to the multiverse, possesses an intrinsic arrow of time, that is a privileged direction along which the time only flows towards the future. If the physics of particles and fields is time symmetric and should be described in the realm of the whole multiverse (or just in one of its finite-time universes if all the multiversal components are identical), then one must have a deep physical reason that makes the microscopic behaviour to appear as time symmetric. Such a physical reason can be found if we consider the existence of wormholes in the realm of the multiverse. Since these wormholes accrete dark energy [18] they can actually grow so big that the whole spacetime of any of the universes making the multiverse is engulfed by the wormhole immediately before (after) the universe reaches (leaves) the future (past) big rip singularity. Such a gigantic process has been denoted as big trip and has hitherto been considered to just predict unwanted catastrophic cataclysms in the future of a hypothetical universe filled with phantom energy [18]. Nevertheless, when considered in the context of our multiversal model, the phenomenon of the big trip may provide unexpected benefits. If fact, it can be shown (see Appendix B) that an observer in one of the finite-time universes that make the infinite multiverse will see big trips to crop up in his (her) future and his (her) past. Thus, such as it was pointed out in Ref. [14], the mouths of the wormholes in the past and future may be moving, and so travelling in time, in such a way that they can be inserted into each other during each big trip time so that the given universe can freely journey from future to past and vice versa, so destroying any arrow of time of that universe and rendering the time fully time symmetric in the whole multiverse. In Appendix B it is also shown that any big trip phenomena are prevented to take place in a single universe with infinite time, and therefore a reason is found why current cosmology keeps an arrow of time.

The cropping up of wormholes with distinct sizes in the neighborhood of the big rips [19] of the multiverse helps us to furthermore consider a future event horizon for any observer in the multiverse, defined to have a proper size given by

\[
R_H = a(t) \int_t^\infty \frac{dt'}{a(t')}.
\]
Inserting then the expression for the scale factor given by Eq. (2.2) one can easily check [20] that $R_h$ becomes infinite for any observer in the multiverse, so making mathematically and physically fully consistent the consideration of any fundamental theory based on the definition of an S-matrix in the context of the multiverse. Since a single universe with infinite time resembles a de Sitter space and hence contains a future event horizon with finite proper size, it is not possible to define such fundamental theories in what we now consider as cosmology.

Based on the above considerations we can conjecture that the whole physical reality consists of a multiverse whose structure can by instance be described by the model reviewed in Sec. II, which is a natural framework to consistently describe all time-symmetric particle physics and fields, being what we call our own universe just one among the infinite number of independent, identical universes that form up the multiverse, whenever it is referred to a consistently extended to infinity cosmic time. Such a singled component is perceived by observers in it as a space which currently expands in an super-accelerated fashion along an infinite cosmic irreversible time in a similar manner to as de Sitter space does. If that conjectured description is adopted, then the known incompatibilities between particle physics and cosmology fade out. The price to be paid is having a cosmological model for a fluid characterized by a dynamics negative energy density (which is nevertheless over-compensated by a positive cosmological constant in accordance with the requirements of quantum theory) which is defined on a physical domain where the quantization rules should be expected to be not the same as those used in the whole multiverse (or just in one of its finite-time universes if all the multiversal components are identical) for particles and fields.

V. CONCLUSIONS AND FURTHER COMMENTS.

We have tried to solve the incompatibilities between particle physics and cosmology by resorting to the Copernican principle that every founded scientific advance must necessarily be accompanied by lowering of the human role in nature in the following sense. The universe which we live in is nothing but one among the infinite number of universes in a whole multiversal scenario. If we adscribe to such a spacetime the observable properties of our own universe as referred to an infinite cosmic time, then we derived in this paper a cosmic model in which we can provide with some tentative solutions to several key problems that arise when one tries to make compatible particle physics with current cosmology.

Starting with a multiverse model recently suggested, we have singled out a universe whose scale factor has been derived in terms of an infinite cosmic time. That universe is interpreted as being ours own and is characterized by a total positive vacuum energy density which is made of two parts, a dynamical one which is small and negative and a positive cosmological constant. We have then been able to establish precise mathematical relations between the cosmological parameters of the multiverse and those of the observable universe which actually restore compatibility between cosmology and particle physics if the latter is taken to be defined in the original multiverse (or just in one of its finite-time universes if all the multiversal components are identical). Thus, we have shown that: (i) the ratio of values of the cosmological constant for particle physics and cosmology derived in this way is precisely what has been considered as the basis to formulate the so-called problem of the cosmological constant, (ii) because for an observer in any of the finite-time universes of the multiverse there are two big rip phenomena, one in the past and other in the future, all the physics in the multiverse will be time symmetry, but as no such phenomena may take place in the single infinite-time universe, there will be an arrow of time in our observable universe, and (iii) the proper size of the event horizon of the multiverse is seen to be infinity, i.e., there is no future event horizon in the multiverse, and therefore any fundamental physics description can be consistently carried out. All the above results amount to our main conjecture: If we assume that particle physics lives in the multiverse (or just in one of its finite-time universes if all the multiversal components are identical) where it is well defined, then our cosmology results from the observation referred to an infinite cosmic time of just one of the infinite number of universes that form up the multiverse, with the known incompatibilities between particle physics and cosmology turning out to be nothing but artifacts arising from our attempt to interpret our own universe as containing everything.

Obviously a more realistic model should require the introduction of some matter. It is easy to see that if one add a matter term to the Friedmann equation (2.1), this term would dominate only at small values of the scale factor, becoming negligible on large values of the scale factor. Thus, our multiverse scenario arising from the occurrence of an infinite number of big rip singularities at which the size of the universes blows up would still be valid.

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APPENDIX A

We derive here a two-parameter cosmic solution which shows the properties of a multiverse that is the generalized form of the case reviewed in Sec. II. Let us consider a universe filled with a negative cosmological constant which, for the sake of convenience, we denote now as \(-\lambda^2\), and phantom energy with density \(\rho_{ph} = E^2a^\epsilon\), with \(E^2\) a constant and \(\epsilon > 0\). Integrating then the Friedmann equation for a state equation parameter \(w = -1 - \epsilon/3\) we have

\[
a(t) = \left(\frac{\lambda}{E \cos \omega}\right)^{2/\epsilon}, \quad w = \frac{\epsilon}{2}(1 - W),
\]

in which \(W\) is an arbitrary constant. A generalized dark energy multiverse can then be derived by resorting to the following theorem [21]. Let \(a = a(t)\) be a spatially flat solution of the Friedmann equations for energy density and pressure as given by

\[
\rho = \frac{\dot{a}^2}{a^2}, \quad p = -\frac{2\ddot{a} + 3\dot{a}^2}{3a^2}.
\]

It follows then that the two-parameter function \(a_k = a_k(t; c_1, c_2)\), that is

\[
a_k = a\left(c_1 + c_2 \int \frac{dt}{a^{2k}}\right)^{1/k}
\]

will also be a solution of the Friedmann equations for new expressions of the energy density, \(\rho_k\), and pressure, \(p_k\), satisfying

\[
k^2\rho_k - \frac{3k}{2}(\rho_k + p_k) = k^2\rho - \frac{3k}{2}(\rho + p),
\]

This theorem implies that the quantity \(k [(k - 1)\dot{a}^2 + \ddot{a}] / (a^2)\) is invariant under the transformation \(a \rightarrow a_k\), with \(a_k\) as defined by Eq. (A3). Taking then \(k = (2m + 1)\epsilon/4, m = 0, 1, 2, ...,\) and using Eq. (A3), one can get

\[
a^{(m)} = \left(\frac{\lambda}{E \cos \omega}\right)^{2} \times \left[c_1 + c_2 \sum_{s=0}^{m} \frac{(-1)^s C^s_m(\sin \omega)^{2s+1}}{2s+1}\right]^{\frac{4}{2m+1-\epsilon}},
\]

which describes a dark energy multiverse with a two-parameter freedom. This is the wanted generalization of the dark energy multiverse described in Sec. II to which it reduces when we take \(c_1 = 1\) and \(c_2 = 0\).

APPENDIX B

We consider in this appendix the accretion of the cosmic fluid onto wormholes in both the multiverse and the single universe with an infinite time [18] and [22]. It is known that the wormhole mass rate for an asymptotic observer is expressed through

\[
\dot{m} = 4\pi m^2 Q|\beta|\rho.
\]

In first place, we study this process in the multiverse scenario where the above expression must be integrated taking into account Eq. (2.2) and leads to

\[
m(t) = m_0 \left[1 - \frac{8\pi Q\rho_0 m_0}{3\lambda^{1/2}} \sin(\alpha(t-t_0)) \cos(\alpha(t-t_0)) - b \sin(\alpha(t-t_0))\right]^{-1}.
\]

One can easily see that this expression vanish at the big rip times and that it diverges an infinite number of times at

\[
t_{*m} = t_0 + \frac{2}{3|\beta|\lambda^{1/2}} \arctan \left[\frac{1}{b + \xi}\right] + \frac{2m\pi}{3|\beta|\lambda^{1/2}},
\]

where \(\xi = \frac{8\pi Q\rho_0 m_0}{3\lambda^{1/2}}\) and \(m\) is an integer number. As in the big rip case, for \(m = 0\) and expanding Eq. (B3) for \(\lambda << 1\), one can recover the big trip time expression of a simple phantom quintessential model, that is

\[
t_{br} = t_0 + \frac{t_{br} - t_0}{1 + (\frac{8\pi Q}{3})^{1/2} Qm_0}.
\]
In order to find the divergences of the wormhole mass, as expressed through Eq. \(\text{[B3]}\), we must consider two cases, \(t_{*m} - t_0 < 0\) and \(t_{*m} - t_0 > 0\). In the first case, \(t_{*m} - t_0 < 0\), if we take into account Eqs. \(\text{[2.3]}\) and \(\text{[B3]}\), then one obtains

\[
t_{brm} - t_{*m} = \frac{2}{3|\beta|^1/2} \left[ \text{arctg} \left( \frac{1}{b} \right) - \text{arctg} \left( \frac{1}{b + \xi} \right) \right] > 0.
\]  

(B5)

Since the divergences take place before the consecutive big rip, this divergence will be a big trip (it can be seen that that is actually the case if \(t_{*m}, t_0 > 0\) and \(t_{*m} > t_0\), if \(t_{*m}, t_0 < 0\) and \(|t_0| > |t_{*m}|\) and if \(t_{*m} > 0\) and \(t_0 < 0\)). It can also be seen that the time interval between two consecutive big rips is the same as between two big trips.

In the second case, \(t_{*m} - t_0 < 0\), Eq. \(\text{[B3]}\) is converted into

\[
t_{brm} - t_{*m} = \frac{2}{3|\beta|^1/2} \left[ \text{arctg} \left( \frac{1}{b} \right) - \text{arctg} \left( \frac{1}{b + \xi} \right) - \pi \right] < 0,
\]  

(B6)

where, because \(t_0 < t_0\), we have considered the previous big rip. Then, an observer will see a big trip in his (her) past taking place after the big rip singularity which is the origin of his (her) universe (that is the case if \(t_{*m}, t_0 > 0\) and \(t_0 > t_{*m}\), if \(t_{*m}, t_0 < 0\) and \(|t_0| < |t_{*m}|\) and if \(t_{*m} > 0\) and \(t_0 > 0\)). In short, for every observer in every finite-time universe, there will be one big trip in the past and one big trip in the future.

On the other hand, if we consider an observer which lives in a single universe described in terms of a suitable infinite cosmic time, as we should in fact be, we must take into account Eq. \(\text{[3.0]}\) in order to integrate Eq. \(\text{[B1]}\). So we get

\[
m(t_c) = m_0 \left[ 1 + A m_0 H_0 - A m_0 \lambda_n^{1/2} \tanh \left( \frac{3\beta_n}{2} \lambda_n^{1/2} t_c \right) \right]^{-1}.
\]  

(B7)

We can define now the function \(F(t_c) = A m_0 \lambda_n^{1/2} \tanh \left( \frac{3\beta_n}{2} \lambda_n^{1/2} t_c \right)\). Because of the properties of the tanh, we can see that \(F(t_c)\) is a monotonous increasing function, \(F(t_c) > 0\) if \(t_c > 0\) and \(F(t_c) < 0\) if \(t_c < 0\), and \(-A m_0 \lambda_n^{1/2} < F(t_c) < A m_0 \lambda_n^{1/2}\). So, one has a zero in the denominator of Eq. \(\text{[B7]}\) where the wormhole mass would diverge if and only if \(A m_0 \lambda_n^{1/2} > 1 + A m_0 H_0\), i.e., \(m_0 > 1/[A(\lambda_n^{1/2} - H_0)]\). By mere inspection of the data in the last section, it can be readily seen that this value of \(m_0\) corresponds to a minimum throat of order \(b_{\text{min}} \sim 10^{26}\) (meters). However, if we assume that the Universe is expanding in size at the speed of light, then its radius would be 13.7 billion light years, and its diameter would be 27.4 billion light years, which in meters is of the order \(10^{26}\). Then, to have a wormhole within the universe which could produce a big trip phenomenon, the throat of this wormhole must be at least so big as the observable universe, i.e., the universe would be already contained within a wormhole. Since that situation is not possible, we can conclude that a big trip phenomenon cannot take place in our single observable universe.

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