RENORMALIZATION GROUP AND TRIVIALITY
IN NONCOMPACT LATTICE QED WITH LIGHT FERMIONS.

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ABSTRACT

In the framework of noncompact lattice QED with light fermions, we derive the functional dependence of the average energy per plaquette on the bare parameters using block-spin Renormalization Group arguments and assuming that the renormalized coupling vanishes. Our numerical results for this quantity in $8^4$ and $10^4$ lattices show evidence for triviality in the weak coupling phase and point to a non vanishing value for the renormalized coupling constant in the strong coupling phase.
One of the most successful results of lattice regularization of gauge theories, combined with Monte Carlo simulations has been a deeper understanding of the non-perturbative dynamics of asymptotically free gauge theories, like QCD. Conversely, very little is known about non-asymptotically free theories.

As a matter of fact, the most important question for lattice regularization, i.e. the existence of a quantum continuum limit with non-trivial dynamics, has not (yet) be answered for the simplest and perturbatively most successful gauge theory, namely, Quantum Electrodynamics.

In recent years, many efforts have been devoted to the study of this problem, firstly using the compact regularization of the abelian model [1,2,3] and more recently within the non compact formulation.

The first numerical investigations of the non compact model, in the quenched approximation [4], have shown the existence of a continuous chiral transition at finite value of the coupling constant. This transition survives after the inclusion of dynamical fermions [5-8] so suggesting that the quantum continuum physics could be reached there.

Having found a candidate point for the continuum limit, two important questions should be answered: i) Is the theory defined by taking the limit at the chiral critical point non trivial, i.e. does this model possess a particle spectrum with non-trivial interactions in this limit? and ii) Assuming answer to i) is positive, has this limiting theory something to do with standard quantum electrodynamics?

Concerning the first point, there exist extensive numerical simulations performed by several groups [5-9,11-13]. The first indication of a power-law (as opposed to essential singularity [10]) scaling for the chiral condensate was suggested by A. Horowitz in [11]. On the other hand, the Illinois group found a good quantitative support for a non-mean-field power law scaling in the quenched model [12,13]. Their results ruled out the mean field scenario and illustrated the degree of difficulty required in extracting the critical indices in the full theory with dynamical fermions, where larger lattices and a precise determination of the critical coupling are necessary in order to compute critical exponents [9].

On the other hand, Göckeler et al. [14,15] computed the renormalized charge and fermion mass and found that the corresponding Callan-Symanzik $\beta$ function is consistent with the prediction of renormalized perturbation theory. Furthermore, they have not found lines of constant physics for the matter sector in the two parameters region they explored [15]; on the basis of their results they argue about the non-renormalizability of the theory.

As for point ii) above, Hands et al. [9] have shown that the vacuum in the broken phase of non compact QED is a monopole condensate with U(1) symmetry while the continuum model has no finite action monopoles, and the gauge group symmetry is $\mathbb{R}$. This means that the lattice model is qual-
itatively different from standard QED, belonging to a different universality class. This result also casts serious doubts on the validity of renormalization group flow calculations as those of refs. [14,15].

In this letter we report a study of the triviality problem of the quantum continuum limit of non compact Lattice QED. To this end, we introduce a new approach based on a characterization of the behaviour of the mean plaquette energy as a function of the bare parameters $\beta$ and $m$. The equation describing such a behaviour holds only if the continuum limit of the model consists of particles without electromagnetic interactions.

Consider the action of non compact lattice QED

$$S = \frac{1}{2} \sum_{x, \mu} \eta_\mu(x) \bar{\chi}(x) \{U_\mu(x) \chi(x + \mu) - U^*_\mu(x - \mu) \chi(x - \mu)\} + m \sum_x \bar{\chi}(x) \chi(x) + \frac{\beta}{2} \sum_{x, \mu<\nu} F_{\mu\nu}^2(x)$$

(1)

where $\beta = 1/e^2$ and we use staggered fermions coupled to the gauge fields $A_\mu(x)$ through the compact link variable $U_\mu(x)$.

A problem working in the non compact formulation comes from the fact that the partition function associated to action (1) is not well defined even in a lattice of finite size. In fact the gauge group integration, in contrast to the compact case, is divergent. The problem can be overcome by gauge fixing. We instead factorize the divergency in the density of states as follows.

Define the density of states at fixed non compact normalized energy $E$ in a lattice of volume $V$

$$N(E) = \int [dA_\mu(x)] \delta \left( \frac{1}{2} \sum_{x, \mu<\nu} F_{\mu\nu}^2(x) - 6VE \right)$$

(2)

$N(E)$ is divergent because of the infinite volume of gauge integration. However, this divergence can be factorized out and one can easily show that

$$N(E) = C_G (6VE)^{3V-1}$$

(3)

where $C_G$ is a divergent constant (the volume of the gauge group).

On the other hand it can be shown, following ref. [3], that the partition function can be written as an integral over the normalized non compact energy $E$

$$Z = \int dE N(E) e^{-\beta V E} e^{-S_{eff}(E,m)}$$

(4)
where

\[ e^{-S_{\text{eff}}^F(E,m)} = \frac{\int [dA_\mu(x)] det\Delta(m, A_\mu(x)) \delta(\frac{1}{2} \sum_{x,\mu<\nu} F_{\mu\nu}^2(x) - 6VE)}{\int [dA_\mu(x)] \delta(\frac{1}{2} \sum_{x,\mu<\nu} F_{\mu\nu}^2(x) - 6VE)} \]  

(5)

From eq. (3),(4),(5) we can derive an effective action for the full theory in the thermodynamical limit \( V \to \infty \) as

\[ S_{\text{eff}}(E,V,\beta,m) = -\frac{3}{2} V \ln E + 6\beta V E + S_{\text{eff}}^F(E,m) \]  

(6)

Now let us write the partition function associated to (1) as a integral over the plaquette variables \( F_{\mu\nu}^2 \) in the following way

\[ Z = \int [dA_\mu(n)][d\tilde{\chi}(n)][d\chi(n)][dE_{\mu\nu}(n)] \prod \delta(F_{\mu\nu}^2(n) - E_{\mu\nu}(n)) e^{-S} = \]

\[ \int [dE_{\mu\nu}(n)]N(E_{\mu\nu}(n))e^{-S(E_{\mu\nu}(n))} \]  

(7)

where

\[ e^{-S(E_{\mu\nu}(n))} = \]

\[ \frac{\int [dA_\mu(n)][d\tilde{\chi}(n)][d\chi(n)] \prod \delta(F_{\mu\nu}^2(n) - E_{\mu\nu}(n)) e^{-S}}{\int [dA_\mu(n)][d\tilde{\chi}(n)][d\chi(n)] \prod \delta(F_{\mu\nu}^2(n) - E_{\mu\nu}(n))} \]  

(8)

and the denominator in (8) is just the density of states \( N(E_{\mu\nu}(n)) \).

Next, imagine we apply linear block-spin renormalization group transformations to the theory described by the effective action \( S(E_{\mu\nu}(n)) - \ln N(E_{\mu\nu}(n)) \). Our spin variable is the plaquette variable \( E_{\mu\nu}(n) \) which takes values from 0 to \( \infty \) and blocking is performed at each \( \mu\nu \) plane. We generate in this way a series of effective actions which are equivalent at large distances since we are integrating out all the short distance details. If the theory is trivial i.e., if all renormalized couplings vanish, the only relevant parameter at the end of this procedure will be the coefficient of the kinetic term \( E_{\mu\nu}(n) \). Then, the renormalized action \( S_R(E_{\mu\nu}(n)) \) will be, apart from the density of states contribution, of the form

\[ S_R(E_{\mu\nu}(n)) = \frac{1}{2} \tilde{\beta}(m,\beta) \sum_{n,\mu<\nu} E_{\mu\nu}(n) + h(m,\beta) \]  

(9)

where \( \tilde{\beta}(m,\beta) \) and \( h(m,\beta) \) are unknown renormalized constants. Defining \( E_R = \frac{1}{6V} \sum_{n,\mu<\nu} E_{\mu\nu}(n) \) we get
\[ S_{R}^{\text{eff}}(E_R) = -\frac{3}{2} V \ln E_R + 6\beta V E_R + h(m, \beta) \] (10)

This action and action (6) can differ only by a multiplicative factor \( X(m, \beta) \) in the mean energy \( E \) since we have obtained (10) by means of linear block-spin transformations plus a final linear global transformation. Therefore triviality means that action (6), apart from the logarithmic term coming from the density of states, must be a linear function of the mean energy \( E \) or equivalently, that \( S_{\text{eff}}^{F}(E, m) \) in (6) is a linear function of \( E \).

We would like to remark at this point that the connection between actions (6) and (10) can be established owing to the use of linear block-spin Renormalization Group transformations, so that we can obtain eq. (10) from (6) through a linear change of variables. Using non linear transformations or transformations in other kind of variables, we could identify the partition functions but we would not be able to establish any connection between the corresponding effective actions.

Due to the fact that \( S_{\text{eff}}^{F}(E, m) \) is a linear function of \( E \), we get that all effects of dynamical fermions can be reduced to a redefinition of the coupling constant \( \beta \). Therefore, the mean plaquette energy can be written as

\[ E(m, \beta) = \frac{1}{4(\beta + h_1(m))} \] (11)

The linearity of the effective action (6) as well as equation (11), which should hold around the critical point if the theory is trivial, can be compared with data obtained by numerical simulations.

Following a method that we have recently proposed [3], we calculated the mean plaquette energy using the fermionic effective action (5). We have obtained the fermionic effective action in 27 values of \( E \) in the range \( 0.5 - 1.7 \), allowing us to calculate thermodynamical quantities as a function of \( \beta \) in the range \( 0.14 \leq \beta \leq 0.40 \); with these values we go deeply inside the strong coupling and Coulomb phase respectively.

The largest part of the simulations has been performed on a 8\( ^4 \) lattice, but from an analysis of the scaling properties of fermionic effective action in lattices from 4\( ^4 \) to 10\( ^4 \), we can exclude significant finite volume effects on the mean plaquette energy of the full theory, already in the 8\( ^4 \) lattice. For a detailed report of these simulations see [16].

In Fig. 1 we report the effective fermionic action (5) for vanishing fermion mass as a function of \( E \). Two different regimes, corresponding to two different phases, can be seen from this figure.

Coulomb phase (\( \beta > 0.206 \)) which is dominated in the thermodynamical limit by energies (\( E \leq 1.0 \)), is characterized by an effective action linear in \( E \), meaning that the effect of the inclusion of fermionic degrees of freedom merely reduces to a shift in the coupling constant, indicating
triviality. In fact, if we try to fit the plaquette energy data in this phase with a functional form like (11), we obtain a very good fit for $h_1 = 0.0409$ (see Fig. 2). The fact that (11) is able to reproduce in such a good way the numerical data is again a strong indication of triviality in this phase.

Completely different is the situation in the strong coupling broken phase ($E > 1.0$). Indeed, the behavior of the plaquette energy for $\beta < 0.206$ deviates from the fit (11), this indicating the existence of a phase transition at $\beta_c \simeq 0.206$. For $\beta < \beta_c$ we tried to fit our data with a function like (11). However, we have found that we need to give a $\beta$ dependence to $h_1(m)$, as can be seen in Fig. 3. From the two fits for $h_1(m)$ in this Figure, we get $\beta_c = 0.206(5)$.

Our results on the effective fermionic action reported in Fig.1 can be very well understood if a second order phase transition occurs. Indeed, applying the saddle point technique to the computation of the partition function (4) it can be shown that a discontinuity in the specific heat implies a discontinuity of the second energy derivative of the effective fermionic action at the energy critical value. Furthermore and as following from the main content of this paper, a non vanishing value for the renormalized coupling is directly related to a non linear energy dependence of the effective fermionic action. Therefore a second order phase transition should produce a discontinuity of the renormalized coupling at the critical point.

Fitting the points in Fig.1 by two polynomials, one for $E < 1.018$ and the other for $E > 1.018$ (continuous line in the Figure), being $E_c = 1.018$ the mean energy at $\beta = 0.206$ $m = 0$, we get very good fits with a gap of $0.38(2)$ in the second energy derivative normalized by the lattice volume. As a result of the fits we also find that the first energy derivative of the effective fermionic action is continuous at $E = E_c$ and second and higher order energy derivatives vanish for $E < E_c$ inside the errors of the fits, these results being very stable when we increase the degree of the polynomial fits. The observed approximate scaling of the effective fermionic action with the lattice volume when we go from the $6^4$ to the $10^4$ lattice [16] implies that finite size effects does not affect our results in a significant way. In any case, the important qualitative finding is that the second energy derivative of effective fermionic action is always discontinuous at the critical energy.

In conclusion, our numerical analysis shows the existence of a phase transition at $\beta_c = 0.206(5)$, $N_f = 4$, $\beta_c = 0.226(5)$, $N_f = 2$, in agreement within errors with the critical value obtained from the behaviour of the chiral condensate [9, 16]. The behavior of the effective fermionic action and mean plaquette energy in the broken phase, strongly suggests a non vanishing value for the renormalized coupling constant in this phase, even when we approach the critical point.

Does this result implies the existence of a non trivial (non gaussian) fixed point? In the general formulation of the Renormalization Group approach it is generally assumed that any point at or near the critical
surface is in the attraction domain of some fixed point, even though singular behaviour can not be excluded by general arguments [17]. Excluding such a singular behavior, our numerical results strongly indicate that the fixed point is non gaussian.

The important question now is: is the quantum theory described by this fixed point renormalizable? The results reported in [15] about non perturbative renormalizability of the model show that there are no lines of constant physics in the ($\beta$, $m$) plane. However, this result does not imply necessarily non renormalizability since it could be that the two dimensional parameter space is too small. In fact, in the two parameters action (1) we have neglected coupling terms such as four Fermi interactions and monopole contributions which can be generated in the renormalization procedure and whose associated couplings could become relevant for the continuum limit in the strong coupling phase, as suggested firstly in [18] and also by the results of ref. [9] (this was also the possibility left open in [15]). If this is the real scenario, our numerical results in the broken phase should be regarded as a strong indication for a non trivial continuum limit.

On the contrary, when the transition is approached from the Coulomb phase, as more appropriate for the definition of continuum QED, the theory is non interacting. This behaviour is not totally unexpected, since we know from perturbative QED the existence of Landau pole problem.

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FIGURE CAPTIONS

1) Effective fermionic action (5), versus $E$ at $m = 0.0$ and four flavors, obtained through microcanonical simulation. Statistical errors are invisible at this scale.

2) Mean plaquette energy $E(m = 0, \beta)$ versus $\beta$. Solid line is a fit, equation (11), with $h_1 = 0.0409$. Errors are of the order of symbols size.

3) $h_1(m)$ versus $\beta$ at $m = 0.0$. In the weak coupling phase ($\beta > 0.206$), $h_1(m)$ is well fitted by a horizontal line. For $\beta < \beta_c$ equation (11) does not hold and we need to give a $\beta$ dependence to $h_1(m)$. The solid line in the strong coupling phase is a polynomial fit.