Evidence for Spin-Charge Separation in the
Two-Dimensional $t$-$J$ Model

W. O. Putikka

National High Magnetic Field Laboratory, Florida State University, Tallahassee, FL 32306

R. L. Glenister and R. R. P. Singh

Department of Physics, University of California, Davis, California 95616

Hirokazu Tsunetsugu

Theoretische Physik, Eidgenössische Technische Hochschule, 8093 Zürich, Switzerland and Interdisziplinäres Projektzentrum für Supercomputing, ETH-Zentrum, 8092 Zürich, Switzerland

(September 20, 1993)

Abstract

We have calculated high temperature expansions for the momentum distribution function $n_k$ and the equal time spin and density correlation functions $S(q)$ and $N(q)$ of the two-dimensional $t$-$J$ model. On extrapolation to low temperatures we find that $n_k$ has a step-like feature at $k_F$ and $S(q)$ has $2k_F$ as a characteristic wavevector, whereas $N(q)$ has $2k^{SF}_F$ as a characteristic wavevector. Here $k_F$ and $k^{SF}_F$ are the Fermi wavevectors of the nearest-neighbor square lattice tight-binding and spinless fermion models, respectively. By comparison to the known results for one dimension this suggests spin-charge separation in the two-dimensional $t$-$J$ model.

74.65.+n, 74.70.Vy
The study of strongly correlated electrons in two-dimensions (2D) is currently one of
the most interesting and controversial topics in condensed matter physics, particularly with
regard to high temperature superconductivity (HTSC) \[1\]. Anderson \[2\] has put forward
the idea that the ground state of strongly correlated electron systems in 2D is a Luttinger
liquid analogous to the case in one dimension. In 1D, a Luttinger liquid has spin and charge
degrees of freedom with different velocities and wavevectors, behaving at low energies as
independent elementary excitations, a situation which has become known as spin-charge
separation \[2\].

Determining whether or not spin-charge separation can also occur in 2D has proven
quite difficult. We have calculated high temperature expansions for equal time correlation
functions (ETCF) of the 2D $t$-$J$ model \[3\] to investigate this possibility. We find two distinct
characteristic wavevectors for the spin and charge degrees of freedom, $2k_F$ and $2k_F^S$ defined
below. This shows that the spin and charge degrees of freedom have different distributions
in the Brillouin zone and provides evidence for spin-charge separation in this model.

We consider the 2D $t$-$J$ model on a square lattice, where the Hamiltonian is

$$H_{tJ} = -t \sum_{<ij>,\sigma} (c_{i\sigma}^{\dagger}c_{j\sigma} + \text{h.c.}) + J \sum_{<ij>} S_i \cdot S_j,$$

with the constraint of no double occupancy. The constraint represents the strong correlations
between the electrons and is difficult to treat by conventional many-body techniques.

We have studied three ETCF of this model using the high temperature series expansion
method. These are the single spin momentum distribution function, $n_k$, and the equal time
spin and density correlation functions, $S(q)$ and $N(q)$, defined by the relations

$$n_k = \sum_r e^{i k \cdot r} \langle c_{0\sigma}^{\dagger}c_{r\sigma} \rangle,$$

$$S(q) = \sum_r e^{i q \cdot r} \langle S_0^z S_r^z \rangle,$$

$$N(q) = \sum_r e^{i q \cdot r} \langle \Delta n_0 \Delta n_r \rangle,$$

where the angular brackets refer to thermal averaging in the grand canonical ensemble,
$S_r^z = \frac{1}{2} \sum_{\alpha\beta} c_{r\alpha}^{\dagger} \sigma_{\alpha\beta}^{z} c_{r\beta}$ and $\Delta n_r = \sum_\sigma c_{r\sigma}^{\dagger} c_{r\sigma} - n$. Here $n$ is the average density of electrons.
The expansions are calculated for \( n_k \) to eighth order and \( S(q) \) and \( N(q) \), to tenth order in the reciprocal temperature \( T^{-1} \), first at fixed fugacity, and then by a change of variables at fixed \( n \). The series are extrapolated at fixed \( n \) by Padé approximants to determine the low \( T \) properties.

The form of the ETCF for tight-binding (TB) (non-interacting, spin-half electrons with nearest neighbor hopping on a square lattice) and spinless fermions (SF) (physically, SF are fully spin-polarized TB electrons, freezing out the spin degrees of freedom and doubling the number of occupied \( k \) states) in 2D are helpful in understanding the \( t-J \) model results presented below. They are given by

\[
N(q) = n - g \int \frac{dk}{(2\pi)^2} n_k n_{k+q},
\]

where \( g = 2 \) for TB or \( g = 1 \) for SF, and \( 4S(q) = N(q) \) for TB. From the form of this equation we can see that at \( T = 0 \) and \( n \leq 1 \) the ETCF will saturate at \( n \) when \( n_k \) and \( n_{k+q} \) no longer overlap (note that for SF with \( n > 0.5 \), \( N(q) \) saturates at \( 1 - n \) when the hole Fermi surfaces no longer overlap). The Kohn anomaly at \( 2k_F \) or \( 2k_{SF}^F \) is due to the existence of a sharp Fermi surface.

In Fig. 1 we compare \( N(q) \) of the 2D \( t-J \) model to \( N(q) \) of SF at the same density for \( T/J = 0.5, J/t = 0.5 \) and a range of \( n \). The similarities are remarkable throughout the Brillouin zone, with the differences near the \( \Gamma \) point due to the \( t-J \) model having a larger compressibility than SF. To focus the discussion below we now limit ourselves to two sets of parameters outside of the phase separated or ferromagnetic regions of the 2D \( t-J \) model. We fix \( J/t = 0.5 \) and consider \( n = 0.75 \) and \( n = 0.20 \). The results for \( n_k \), \( S(q) \) and \( N(q) \) along \( q_{FM} = (0,0) \rightarrow (\pi,\pi) \) are shown in Fig. 1. We see that \( n_k \approx 1/2 \) at \( k_{F \Gamma M} \), the Fermi momentum of the TB model at the same density and \( S(q) \) is enhanced over its TB value and either flattens out or has a peak at \( q \approx 2k_{F \Gamma M} \). However, the most anomalous curves are for \( N(q) \). They are suppressed from their TB values and flatten out at \( q \approx 2k_{SF \Gamma M} \), the Fermi momentum of SF at the same density. We observe no feature at \( q = 2k_F \), though \( N(q) \) flattens out more gradually for \( n = 0.20 \) than for \( n = 0.75 \).
For comparison, we recall the behaviors of $n_k$, $S(q)$ and $N(q)$ for the $U/t \to \infty$ Hubbard model ($J/t \to 0$ $t$-$J$ model) in 1D. From the work of Ogata and Shiba [10] we know that $n_k$ has a power law singularity at $k_F$ with $n_{k_F} = 1/2$. Also $S(q)$ has a peak at $2k_F$ and $N(q)$ has $2k_F^{SF} = 4k_F$ as a characteristic wavevector. For arbitrary $U/t$, $N(q)$ also has a feature at $2k_F$ due to a mixture of spin and charge excitations, but the $2k_F^{SF}$ feature is due to charge alone [11] and shows that the charge degrees of freedom truly reside at $k_F^{SF}$. In addition to the singularity at $k_F$, $n_k$ has a singularity at $3k_F$, but with a very small step size.

Our calculated 2D ETCF show behaviors very similar to their counterparts in 1D. The characteristic wavevectors for $S(q)$ and $N(q)$ are $2k_F$ and $2k_F^{SF}$, respectively, which we believe implies low energy spin degrees of freedom near $k_F$ and low energy charge degrees of freedom near $k_F^{SF}$. Note that in 2D $k_F$ and $k_F^{SF}$ are *incommensurate* wavevectors; the charge degrees of freedom do not occur at a harmonic of $k_F$, but at an independent wavevector. In Fig. 3 we show $k_F$ and $k_F^{SF}$ for the whole Brillouin zone at $n = 0.75$ with representative nesting wavevectors. For weak coupling calculations of the 2D Hubbard model [12] and Gutzwiller projected free electrons [13] the picture is quite different. In these cases while $S(q)$ is enhanced and $N(q)$ is suppressed, they both have $2k_F$ as a characteristic wavevector which is not what we find for the 2D $t$-$J$ model. The behavior of $n_k$ is also similar to 1D. The step in $n_k$ at $n = 0.20$ is comparable in size and shape to the TB model at the same $T$, but at $n = 0.75$ the step is much weaker and too smeared out to be explained by thermal broadening alone [4]. We have not seen any evidence for a singularity at $3k_F$ in 2D. This could be due to the relatively high temperature $T/J = 1.0$ in our calculation or possibly the angular averaging in 2D which is not present in 1D.

In 1D the statistics of the excitations play no role, but in 2D they are important. Our data give no direct evidence on the statistics of the excitations in 2D, but we can formulate a hypothesis as to what they might be [4]. If we think of a single electron as being composed of an elementary spin degree of freedom and an elementary charge degree of freedom, we would expect one of them to be fermionic and the other bosonic to give a fermionic electron [14]. Since $n_k$ shows a step at $k_F$ and $S(q)$ has $2k_F$ as a characteristic wavevector we assign
the spin degrees of freedom as fermionic and the charge degrees of freedom as bosonic, but note that the charge degrees of freedom are not free bosons, but hard core bosons (HCB) to enforce the constraint of no double occupancy. This can be seen in Fig. 1 for \( n = 0.5 \) where the data points near \( (\pi, \pi) \) are already more rounded than SF at \( T/J = 0.5 \). Further evidence for this point of view can be gained from the work of Long and Zotos \cite{15} and Sorella, Parola and Tosatti \cite{16}.

We have also estimated the behavior of the HCB \( N(q) \) by a flux phase mean field calculation. In 2D, HCB on a square lattice with nearest neighbor hopping can be exactly mapped into SF by attaching a quantum of magnetic flux \( \phi_0 \) to each particle \cite{17}. If density fluctuations are not large we can replace the attached flux tubes by a uniform magnetic field, \( B_0 = n\phi_0 \), which will couple to the orbital motion of the particles. This corresponds to a SF model with a site dependent phase (the uniform flux phase). Using this flux phase mean field approximation we have calculated \( N(q) \), with the results at \( T = 0 \) shown in Fig. 2. The global features show a rounded flattening out of \( N(q) \) at \( 2k_{\pi}^SF \) and general agreement with SF and the \( t-J \) model. For small \( q \) the approximation we are using gives \( N(q) \propto q^2 \), but by general hydrodynamic arguments we know that for \( T = 0 \) if the system has a finite, non-zero compressibility the \( q \to 0 \) limit should be linear. The quadratic dependence is due to the “Fermi energy” of the flux phase sitting in an energy gap \cite{18}. Therefore the \( q^2 \)-dependence is an artifact due to our mean field approximation and should become linear after including fluctuations, which we will discuss in a future paper.

More information on the interactions between the spin and charge degrees of freedom could be obtained by considering the 2D \( t-J \) model with a non-zero spin polarization. If the spin and charge are coupled we would expect both \( S(q) \) and \( N(q) \) to change. However, if the spin and charge degrees of freedom are truly separate the characteristic \( q \)-vector of \( N(q) \) should not be affected by a non-zero spin polarization \cite{13} but \( S(q) \) would now have transverse and longitudinal components with features at wavevectors that depend on the number of up and down spins. This has been observed by Ogata, Sugiyama and Shiba \cite{20} for the 1D \( U \to \infty \) Hubbard model.
Having elementary degrees of freedom at $k_F$ and $k_{SF}^F$ should have experimental con-
sequences for the copper oxide planes in HTSC. Neutron scattering experiments [21] on
La$_{2-x}$Sr$_x$CuO$_4$ show four incommensurate peaks centered around $(\pi, \pi)$ that move with dop-
ing. This can be understood in terms of the nesting properties of the weak coupling Fermi
surface [22] where our results also put the spin degrees of freedom. The energy integrated
weight of angle resolved photoemission is a direct measure of $n_k$ and photoemission has also
been interpreted as supporting a large Fermi surface [23]. But the transport measurements
[24] are not so easy to understand in this picture.

Reconciling experiments which show a large electron-like contour in $k$-space with a den-
sity of $n$ carriers, with transport measurements, which show a much smaller hole density of
$1-n$ carriers, is one of the most puzzling problems of the copper oxides [24]. Our results for
the 2D $t$-$J$ model show one way this might occur [2,25]. One expects the transport experi-
ments to couple most strongly to charge. For $1-n \ll 1$, the charge degrees of freedom at
$k_{SF}^F$ have a small hole-like locus in momentum space centered around $(\pi, \pi)$ as shown in Fig.
3. With this the transport measurements are satisfied. At the same time the spin degrees
of freedom give a large, weak coupling Fermi surface also shown in Fig. 3, which is seen in
neutron scattering and photoemission experiments. We wish to emphasize that experiments
which could probe $N(q)$ directly may prove to be very interesting for HTSC materials.

In conclusion, we have studied the equal time correlation functions of the 2D $t$-$J$ model
by high temperature series expansion methods. We find that the spin and charge ETCF
exhibit signatures of two different wavevectors: the characteristic wavevector for the spins
being $k_F$ and that for charge $k_{SF}^F$, the Fermi wavevectors for TB and SF respectively. In
comparison with the results for 1D this suggests spin-charge separation in this strongly
correlated 2D model.

WOP was supported by NSF Grant No. DMR-91-14553 and by the National High Mag-
netic Field Laboratory at Florida State University. RLG and RRPS were supported by NSF
Grant No. DMR-9017361. HT was supported by the Swiss National Science Foundation
Grant No. NFP-304030-032833. The authors thank T. M. Rice for many useful conver-
tions and a critical reading of the manuscript. The authors also thank N. E. Bonesteel, H. Fukuyama, M. Ogata, N. P. Ong, D. J. Scalapino, J. R. Schrieffer and G. Zimanyi for many useful discussions. WOP and RRPS thank the CMS group at Los Alamos for hospitality while this manuscript was being completed. Part of the computations were done on a Cray YMP at Cray Research, Inc.

Note added in proof—After completion of this work we received a preprint by Y. C. Chen and T. K. Lee [26] which arrives at results similar to ours.
REFERENCES

[1] For a recent review see T. M. Rice in *High Temperature Superconductivity, Proceedings of the Thirty-Ninth Scottish Universities Summer School in Physics*, ed. by D. P. Tunstall and W. Barford (Adam Hilger, Bristol, 1992).

[2] P. W. Anderson, *Science* **235**, 1196 (1987); ibid. **256**, 1526 (1992); ibid. **258**, 672 (1992); in *Frontiers and Borderlines in Many-Particle Physics, International School of Physics “Enrico Fermi”, Course CIV*, ed. by R. A. Broglia and J. R. Schrieffer (North-Holland, Amsterdam, 1987); *Phys. Rev. Lett.* **64**, 1839 (1990); ibid. **65**, 2306 (1990); ibid. **66**, 3226 (1998); ibid. **67**, 2092 (1991); ibid. **67**, 3844 (1991); ibid. **71**, 1220 (1993) *Physics Rep.* **184**, 195 (1989); *Phys. Rev. B* **42**, 2624 (1990); *J. Phys. Chem. Solids* **52**, 1313 (1991); *Prog. Theor. Phys. Supple.* **107**, 41 (1992); preprints; P. W. Anderson and Z. Zou, *Phys. Rev. Lett.* **60**, 132 (1998); P. W. Anderson and Y. Ren, in *High Temperature Superconductivity*, ed. by K. S. Bedell, *et al.* (Addison-Wesley, Redwood City, CA, 1990); preprints; M. Ogata and P. W. Anderson, *Phys. Rev. Lett.* **70**, 3087 (1993).

[3] F. C. Zhang and T. M. Rice, *Phys. Rev. B* **37**, 3759 (1988).

[4] R. R. P. Singh and R. L. Glenister, *Phys. Rev. B* **46**, 14313 (1992).

[5] R. R. P. Singh and R. L. Glenister, *Phys. Rev. B* **46**, 11871 (1992).

[6] W. Kohn, *Phys. Rev. Lett.* **2**, 393 (1959).

[7] W. O. Putikka, M. U. Luchini and T. M. Rice, *Phys. Rev. Lett.* **68**, 538 (1992); V. J. Emery, S. A. Kivelson and H. Q. Lin, *Phys. Rev. Lett.* **64**, 475 (1991).

[8] W. O. Putikka, M. U. Luchini and M. Ogata, *Phys. Rev. Lett.* **69**, 2288 (1992).

[9] W. Stephan and P. Horsch, *Phys. Rev. Lett.* **66**, 2258 (1991).

[10] M. Ogata and H. Shiba, *Phys. Rev. B* **41**, 2326 (1990).

[11] H. Frahm and V. E. Korepin, *Phys. Rev. B* **42**, 10553 (1990); N. Kawakami and S.-K.
Yang, Phys. Rev. Lett. 65, 2039 (1990).

[12] D. J. Scalapino, E. Loh, Jr., and J. E. Hirsch, Phys. Rev. B 34, 8190 (1986); *ibid.* 35, 6694 (1987); N. Bulut, D. J. Scalapino and S. R. White, Phys. Rev. B 47, 2742 (1993); D. J. Scalapino, private communication.

[13] H. Yokoyama and H. Shiba, *J. Phys. Soc. Jpn.* 59, 3669 (1990).

[14] We do not consider the possibility of anyons, which cannot be ruled out.

[15] M. W. Long and X. Zotos, Phys. Rev. B 48, 317 (1993).

[16] S. Sorella, A. Parola and E. Tosatti, in *Strongly Correlated Electron Systems II*, ed. by G. Baskaran, *et al.* (World Scientific, Singapore, 1991).

[17] E. Fradkin, Phys. Rev. Lett. 63, 322 (1989); Y. R. Wang, Phys. Rev. B 43, 3786 (1991).

[18] D. Hofstadter, Phys. Rev. B 14, 2239 (1976); Y. Hasegawa, P. Lederer, T. M. Rice and P. B. Wiegmann, Phys. Rev. Lett. 63, 907 (1989).

[19] The feature for $N(q)$ at $k^{SF}_F$ will sharpen up at low $T$ as the spin polarization saturates due to the system going over to interacting spinless fermions with a sharp Fermi surface at $T = 0$. But the characteristic wavevector $2k^{SF}_F$ should not change.

[20] M. Ogata, T. Sugiyama and H. Shiba, Phys. Rev. B 43, 8401 (1991); see also J. M. P. Carmelo, P. Horsch, D. K. Campbell and A. H. Castro Neto, Phys. Rev. B 48, 4200 (1993).

[21] T. E. Mason, G. Aeppli and H. A. Mook, Phys. Rev. Lett. 68, 1414 (1992).

[22] P. B. Littlewood, J. Zaanen, G. Aeppli and H. Monien, Phys. Rev. B 48, 487 (1993); Q. Si, Y. Zha, K. Levin and J. P. Lu, Phys. Rev. B 47, 9055 (1993).

[23] C. G. Olson, *et al.*, Phys. Rev. B 42, 381 (1990).

[24] For experimental reviews see *High Temperature Superconductivity*, ed. by K. S. Bedell, *et
al. (Addison-Wesley, Redwood City, CA, 1990); Physical Properties of High Temperature Superconductors, vol. 1-3, ed. by D. M. Ginsberg (World Scientific, Singapore, 1989, 1990, 1992).

[25] H. Fukuyama and Y. Hasegawa, Physica 148B, 204 (1987); H. Fukuyama in Superconducting Materials, ed. by S. Nakajima and H. Fukuyama (Japanese Journal of Applied Physics, Tokyo, 1988) p. 205.

[26] Y. C. Chen and T. K. Lee, preprint.
FIGURES

FIG. 1. Plot of $N(q)$ at $T/J = 0.5$ and $J/t = 0.5$ along the irreducible wedge for a range of $n$. The data points are the $t$-$J$ model and the solid lines are spinless fermions for the same temperature. The small vertical arrows are the $T = 0$ locations of nesting vectors for spinless fermions.

FIG. 2. Plots along the diagonal $\Gamma \rightarrow M$ at $n = 0.20$ (a) Single spin momentum distribution function. Data points: $t$-$J$ model, solid line: tight-binding model at $T/J = 1.0$; (b) Spin correlation function. Data points: $t$-$J$ model, dashed line: $T = 0$ tight-binding model; (c) Density correlation function. Data points: $t$-$J$ model, solid line: $T = 0$ flux phase mean field approximation for hard core bosons, dashed line: $T = 0$ spinless fermions and dotted line: $T = 0$ tight-binding model. The vertical dashed lines indicate the important wavevectors along this line in the Brillouin zone for tight-binding electrons and spinless fermions: nesting wavevectors $2k_{F,\Gamma M}$ and $2k_{SF,\Gamma M}^s$, or Fermi wavevectors $k_{F,\Gamma M}$ and $k_{SF,\Gamma M}^s$. (d) - (f) same as (a) - (c) with $n = 0.75$. $K_{\Gamma M} = (2\pi, 2\pi)$ is a reciprocal lattice vector.

FIG. 3. Fermi wavevectors for $n = 0.75$. Solid curve: tight-binding electrons, dashed curve: spinless fermions. The arrows are representative nesting wavevectors along $\Gamma M$ and $MX$. 