Phenomenology of spin zero mesons and glueballs

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We discuss the phenomenology of scalar and pseudoscalar mesons, emphasizing those which do not carry manifest flavor quantum numbers. Many of the properties of these mesons are still not fully understood. Some of them probably do not have the usual two-quark (quark-antiquark) structure, but may be mixed with glueball states or other exotics. We construct or discuss simple models for these mesons and point out which measurements can shed light on their composition.
I. INTRODUCTION

Of all the low-mass mesons (masses below 2 GeV), the properties of the ones with spin zero (both scalar and pseudoscalar) are the least understood as a class, especially those which do not carry manifest flavor quantum numbers. We call these latter mesons “flavorless,” although some of them may have hidden flavor. They are isoscalar mesons which are self-conjugate (under charge conjugation). Some of these mesons may be usual two-quark (quark-antiquark) states, but others may be four-quark states (two quarks, two antiquarks), glueballs (composed only of gluons), or hybrids (composed of a quark-antiquark pair plus glue). Of the mesons with manifest flavor, some may be two-quark states, but others may be four-quark or hybrid states. Probably many of the observed spin-0 mesons are actually mixed states, and that is what makes their structure so hard to determine. We call attention to three recent reviews of meson spectroscopy [1], which discuss problems in distinguishing between those mesons which are ordinary two-quark states and those which are not. Many of the known experimental properties of the spin-0 mesons are summarized in the tables of the Particle Data Group [2].

Most of the light vector and tensor mesons (containing only $u$, $d$, or $s$ quarks) are “ideally” mixed, which means that $SU(3)$ is broken in such a way that the two physical flavorless states with isospin zero have the quark composition $u\bar{u} + d\bar{d}$ and $s\bar{s}$. A possible reason why light flavorless spin-0 mesons are not ideally mixed is that instantons contribute to mixing them with glueball states, as has been discussed by a number of authors, including Shuryak [3], Blask et al. [4], and Zakharov et al. [5]. The interaction induced by instantons is apparently short range on the scale of hadron size [6], and so should be more important in the pseudoscalar sector (in which the quark-antiquark pair is in an $S$ wave) than in the scalar sector (in which the quark-antiquark pair is in a $P$ wave). Another possible reason for non-ideal mixing of spin-0 mesons, whether carrying manifest flavor or not, is mixing with four-quark and hybrid states.

In this paper we discuss the phenomenology of both scalar and pseudoscalar mesons, emphasizing those mesons which are flavorless and can mix with glueballs. Some of the issues discussed are not new, but they are presented in such a way as to allow a direct comparison with models once new data become available. We also try to make clear where new measurements would be particularly welcome and necessary for testing theoretical schemes. Thus, throughout the paper the emphasis is on the phenomenological application of a large number of theoretical ideas which in the previous literature appear rather sparsely. As a consequence, our paper contains not only new results but some old results presented in a rather new form.

Concerning the pseudoscalar states, our main approach is not qualitatively different from some previous ones [7], although many details are different. In some instances we simplify previous models; this simplification enables us to make a large number of predictions that can be tested in the new generation of experiments designed to search for glueballs.

In the scalar sector, we make a large number of new experimentally testable predictions plus give an extensive resumé of old results which, to the best of our knowledge, have never been collected together previously. Among other things, our ideas should allow future experiments to clear up most of the puzzles concerning the low-lying $0^{++}$ resonances and to disentangle the various associated ambiguities. We also point out the limitations of present theoretical schemes and cast our results in such a form as to make clear where experiments in progress will in the near future be able to shed light on existing uncertainties.

Altogether, the principal new features, aside from details, are: (i) We treat essentially all the light flavorless spin-0 mesons. (ii) We emphasize models which have enough simplifying features to enable us to make testable predictions. (iii) We point out where the present theory is inadequate and what experimental information is necessary to clarify the situation.

The issues on pseudoscalar and on scalar glueball candidates can sometimes be sharply distinguished from each other. Nevertheless, we have chosen to present our results on scalars and pseudoscalars in a single paper so as to present a view on spin-zero glueball searches which is as unified as possible.
II. THE PSEUDOSCALAR SECTOR

A. Overview

In this section we discuss flavorless pseudoscalar mesons with positive $C$ parity. We point out where our understanding is good, where it falls short, and where experiments can improve the situation. We need especially to improve our understanding of glueballs. Although, according to QCD, glueballs should exist, theoretical calculations [8–22] of pseudoscalar glueball masses vary over a large mass interval, as can be seen from Table I.

The number of gluons in a glueball is not necessarily a conserved quantity in flux tube and string models and in lattice QCD calculations, but in some models the low-mass glueballs are composed either of two gluons (digluonium) or three gluons (trigluonium). The results of Table I are divided into flux tube and lattice predictions and model predictions for digluonium and trigluonium states. In this paper we do not need to specify whether glueballs are digluonium or trigluonium states. However, it is plausible that the glueballs we discuss are digluonium states, because the lowest digluonium is usually calculated to be less massive than the lowest trigluonium state. For a further discussion of trigluonium, see Anselmino et al. [23] and references therein.

B. Simplified $\eta$, $\eta'$, $\iota$ mixing

Physicists have achieved remarkable success in using quark potential models motivated by QCD to predict the masses and other properties of light, as well as heavy, mesons as quark-antiquark bound states. See, e.g., the paper of Godfrey and Isgur [24] and references therein. However, without additional ad hoc parameters, such models fail to give anywhere near the correct masses of the $\eta$ and $\eta'$ mesons.

The problem of the $\eta$ and $\eta'$ mesons has been with us a long time, and may arise in large part because of instanton effects and also in part because of conventional quark-antiquark annihilation diagrams. Both these contributions are likely to lead to $\eta$ and $\eta'$ mesons which contain not only quark-antiquark pairs but also a gluonic component [25]. However, some experiments by the DM2 [26] and MARK III [27] collaborations indicate that the glueball content of the $\eta$ is either absent or small. A good candidate for a state which is largely glue is the $\eta(1440)$, but this state is probably mixed with two-quark states.

A scheme that has been suggested to discuss these states follows an idea which was first advocated by Pinsky [28] who argued that doubly disconnected diagrams may be important in the discussion of $J/\psi$ decays. As a development of this idea, the MARK III [29] and DM2 collaborations [26] suggested a scheme with disconnected diagrams in which neither the $\eta$ nor the $\eta'$ has any gluonic component. The problem, however, is that disconnected diagrams are quite difficult to estimate, so that additional parameters have to be introduced.

In what follows, we take an alternative viewpoint which includes a glueball component in the $\eta'$ wave function, partly to avoid the parameters of the disconnected diagrams, partly because there are theoretical reasons why the $\eta'$ should contain an appreciable admixture of glue [3], and partly because of arguments [7] that just singlet-octet mixing is insufficient to explain the experimental data.

We shall, however, take advantage of the already mentioned indication [26, 27] that no gluonic component seems to be present in the $\eta$ to suggest a simpler mixing scheme than usual, i.e., one in which (i) only the $\eta$, $\eta'$, and $\eta(1440)$ are mixed, and (ii) the $\eta$ is a pure two-quark state. Hereafter, we call the $\eta(1440)$ by its former name $\iota$ for short.

In the past, many different models of mixing between the $\eta$, the $\eta'$, the $\iota$, and sometimes the $\eta_c$ and the $\eta(1295)$ have been proposed [7, 30–36]. Although our model is less, rather than more general than some previous schemes, it has the advantage that it contains fewer free parameters than other models, and so has more predictive power. The simplifications of our model allow us to fix the mixing parameters using only the two-photon decay widths of the $\eta$ and $\eta'$ and to make many testable predictions. It will be for experiment to decide whether our model is adequate.
There is probably more than one flavorless pseudoscalar state with mass in the region 1400 to 1500 MeV. Although the Particle Data Group [2] lists in its summary table only the \( \eta(1440) \) in this region, the meson full listing contains a discussion which points out that the \( \eta(1440) \) probably consists of two states with quantum numbers \( J^G = 0^+, \ J^{PC} = 0^{-+} \). One of these states may be at about 1410 MeV, and the other may be at about 1490 MeV, as can be seen from the full listings of the Particle Data Group [2]; see also M. G. Rath et al. [37]. (We note, however, that one experiment [38] finds some evidence for two pseudoscalar states around 1410 MeV.) Of the states around 1410 and 1490, one is probably a quark-antiquark excitation of the \( \eta \) and the other is a good candidate to have a large admixture of glue. The predominately glueball state whichever it is, is the one we call \( \iota \). We keep the discussion rather general with respect to the iota mass, presenting results for the different mass values 1410, 1440, and 1480 MeV.

We show below that our mixing scheme, with no glue in the \( \eta \) but with a gluonic component in the \( \eta' \) and \( \iota \), is able to account for many experimental observations. The success of our scheme does not mean that the contributions of disconnected diagrams in \( J/\psi \) decays are negligible, but only that they do not have the large effect required by Refs. [26,29]. A possible, quite recent, further indication of unusual behavior of the \( \eta' \) comes from a comparison of the \( \eta' \) production rate with the predictions of the HERWIG [39] and JETSET [40] Monte Carlo programs, as carried out by ALEPH [41]. Numerical simulations with purely quarkonic components for both the \( \eta \) and the \( \eta' \), overestimate this rate. A similar situation occurs in the analyses of the ARGUS data [42] on \( \eta' \) production in the nonresonant continuum of \( e^+e^- \) around \( \sqrt{s} \approx 10 \) GeV and of the decays of the \( \Upsilon \) resonances. Here again the LUND [43] and the UCLA [44] models considerably overestimate the experimental rates.

We introduce the notation

\[
|\eta_8\rangle = \frac{1}{\sqrt{6}}|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle, \quad |\eta_1\rangle = \frac{1}{\sqrt{3}}|u\bar{u} + d\bar{d} + s\bar{s}\rangle,
\]

\[
|q\bar{q}\rangle = \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle,
\]

and a pure glueball state \( |G\rangle \). We then consider the following admixtures:

\[
|\eta\rangle = \cos \alpha |\eta_8\rangle + \sin \alpha |\eta_1\rangle
\]

\[
= a_{11} |\eta_8\rangle + a_{12} |\eta_1\rangle
\]

\[
= X_\eta |q\bar{q}\rangle + Y_\eta |s\bar{s}\rangle
\]

\[
|\eta'\rangle = - \cos \beta \sin \alpha |\eta_8\rangle + \cos \beta \cos \alpha |\eta_1\rangle + \sin \beta |G\rangle
\]

\[
= a_{21} |\eta_8\rangle + a_{22} |\eta_1\rangle + a_{23} |G\rangle
\]

\[
= X_{\eta'} |q\bar{q}\rangle + Y_{\eta'} |s\bar{s}\rangle + Z_{\eta'} |G\rangle
\]

\[
|\iota\rangle = \sin \beta \sin \alpha |\eta_8\rangle - \sin \beta \cos \alpha |\eta_1\rangle + \cos \beta |G\rangle
\]

\[
= a_{31} |\eta_8\rangle + a_{32} |\eta_1\rangle + a_{33} |G\rangle
\]

\[
= X_\iota |q\bar{q}\rangle + Y_\iota |s\bar{s}\rangle + Z_\iota |G\rangle,
\]

where the coefficients of the various states are constant parameters to be determined. Hereafter, we use the symbols \( X_P, Y_P, \) and \( Z_P \) to refer to the coefficients of \( |q\bar{q}\rangle, |s\bar{s}\rangle, \) and \( |G\rangle \) respectively, when referring to any pseudoscalar meson \( P \). Our notation follows that of Caruso et al. [7]. In the case in which the glueball components of \( \eta \) and \( \eta' \) are both absent, the angle \( \alpha \) above is related to the angle \( \theta_P \) of the Particle Data Group [2] by \( \alpha = -\theta_P \).

In order to fix the two mixing angles \( \alpha \) and \( \beta \) (the third mixing angle that one normally has in three-particle mixing is absent because of our choice of a mixing scheme), we use the experimental data:

\[
R_\eta \equiv \frac{\Gamma(\eta \rightarrow \gamma\gamma)}{\Gamma(\pi^0 \rightarrow \gamma\gamma)} = \frac{1}{3} \left( \frac{m_\eta}{m_{\pi^0}} \right)^2 \frac{f_\eta}{f_{\pi^0}} a_{11} + 2\sqrt{2} a_{12} = 59.8,
\]

\[
R_{\eta'} \equiv \frac{\Gamma(\eta' \rightarrow \gamma\gamma)}{\Gamma(\pi^0 \rightarrow \gamma\gamma)} = \frac{1}{3} \left( \frac{m_{\eta'}}{m_{\pi^0}} \right)^2 \frac{f_{\eta'}}{f_{\pi^0}} a_{21} + 2\sqrt{2} a_{22} = 555.
\]
where \((m_i/m_{\pi^0})^3\) is a kinematical (phase space) factor and the \(f_i\) are decay constants.

In Eqs. (4), \(R_\eta\) and \(R_{\eta'}\) depend on \(\alpha\) and \(\beta\) through the quantities \(a_{ij}\). Following Caruso et al. [7], we introduce the quantity \(\tilde{R}_i:\)

\[
\tilde{R}_i = 3R_i(m_{\pi^0}/m_i)^3(f_i/f_\pi)^2, \quad i = \eta, \eta', \iota.
\]

(5)

Then we can get \(\alpha\) by eliminating the dependence on \(\beta\) from Eqs. (4), obtaining

\[
\sin(\alpha + \arcsin \frac{1}{3}) = \pm \frac{1}{3} \tilde{R}^{1/2}_\eta.
\]

(6)

The quantity \(\tilde{R}_\eta\) is known, and we get \(\alpha = 13.7^\circ\). Analogously, we can eliminate the \(\alpha\) dependence from Eqs. (4), getting

\[
\sin^2 \beta = \frac{9 - \tilde{R}_\eta - \tilde{R}_{\eta'}}{9 - \tilde{R}_\eta}
\]

(7)

Because the left side of Eq. (6) is a trigonometric function and the left side of Eq. (7) is a square of a trigonometric function, the first must be between \(-1\) and 1 and the second between 0 and 1. We can use these limits to obtain a bound on the ratios \(f_\eta/f_\pi\) and \(f_{\eta'}/f_\pi\), given by the inequality [7]:

\[
\frac{1}{3} R_{\eta'}(m_{\pi^0}/m_{\eta'})^3(f_{\eta'}/f_\pi)^2 + \frac{1}{3} R_\eta(m_{\pi^0}/m_\eta)^3(f_\eta/f_\pi)^2 \leq 1.
\]

(8)

Then, using the definition of \(R_i\), and setting either \(R_\eta\) or \(R_{\eta'}\) equal to zero, we obtain the following upper bounds

\[
f_\eta \leq 1.83 f_\pi, \quad f_{\eta'} \leq 1.39 f_\pi,
\]

(9)

where the the equality sign corresponds to \(\beta = 0\), namely to an unmixed glueball. We get from Eq. (7) \(\beta = 30.8^\circ\) when we set \(f_\pi = f_\eta = f_{\eta'}\). This assumption is justified, both from their approximate experimental equality [2] and from theoretical considerations. Theoretically, the approximate equality holds either in the Nambu–Jona-Lasinio model [45] or with a Wess-Zumino Lagrangian [46] as well as in lattice calculations [47].

We obtain from the previous mixing angles the results:

\[
X_\eta = 0.75, \quad Y_\eta = -0.66,
\]

\[
X_{\eta'} = 0.56, \quad Y_{\eta'} = 0.65, \quad Z_{\eta'} = 0.51,
\]

\[
X_i = -0.34, \quad Y_i = -0.39, \quad Z_i = 0.86.
\]

(10)

We note that the sign of \(Y_P\) depends on who does the analysis, but the sign is usually irrelevant because most experiments determine only the square. Both signs are found in the literature, and often only the modulus is given. The predictions given in Eq. (10) can be compared with experimental data. The results of the DM2 collaboration [26] are (we use their fit without disconnected diagrams):

\[
X_\eta = 0.732 \pm 0.039, \quad |Y_\eta| = 0.667 \pm 0.065,
\]

\[
X_{\eta'} = 0.335 \pm 0.063, \quad Y_{\eta'} = 0.623 \pm 0.061.
\]

(11)

The Mark III results (Perrier [27]) are:

\[
X^2_\eta + Y^2_\eta = 1.1 \pm 0.1, \quad X^2_{\eta'} + Y^2_{\eta'} = 0.65 \pm 0.1.
\]

(12)

Haber and Perrier [48] have reanalyzed Mark III data, obtaining:

\[
X_\eta = 0.63 \pm 0.05, \quad Y_\eta = 0.80 \pm 0.12
\]
\[ X_{\eta'} = 0.36 \pm 0.05, \quad Y_{\eta'} = 0.69 \pm 0.11. \] (13)

The Crystal Barrel [49] result is

\[ \left( \frac{X'_{\eta}}{X_{\eta}} \right)^2 = 0.585 \pm 0.008. \] (14)

Our predictions, given in Eqs. (10), are in fair agreement with the experimental results, given in Eqs. (11)–(14). Incidentally, the Crystal Barrel group, following DM2 [26], did their analysis assuming that neither the \( \eta \) nor \( \eta' \) contains any glueball content. They obtained a mixing angle \( \alpha = (17.3 \pm 1.8)^\circ \), which should be compared to our result \( \alpha = 13.7^\circ \).

In our mixing scheme we can also estimate the so-called box-anomaly contribution to \( \eta [\eta'] \to \gamma\pi^+\pi^- \). Following the model of Refs. [50,51], we estimate the contribution of the box anomaly by considering the effective amplitude \( M_P \), given by

\[ M_P = E_P(p_\pi+k_\gamma,p_\pi-k_\gamma)\epsilon_{\alpha\beta\mu\nu}\epsilon^{\alpha\beta\gamma\delta}p^\delta_{\pi'}p^\mu_{\pi''}, \] (15)

where \( (P = \pi^0, \eta, \eta') \). In (15), \( k_\gamma \) and \( p_\pi \) are, respectively, the four-momenta of the photon and of the two pions and \( \epsilon \) is the polarization of the photon. At low energies and in the approximation in which the decay constants for the octet and singlet pseudoscalar states are equal to \( f_\pi \), the functions \( E_P(p_\pi+k_\gamma,p_\pi-k_\gamma) \) reduce to the constants [50]

\[ E_\eta = \frac{-e}{4\pi^2\sqrt{3}f_\pi^3}(\cos \alpha + \sqrt{2}\sin \alpha). \] (16)

\[ E_{\eta'} = \frac{-e}{4\pi^2\sqrt{3}f_\pi^3}(-\sin \alpha + \sqrt{2}\cos \alpha)\cos \beta. \] (17)

With our mixing parameters, we find

\[ E_\eta = -7 \text{ (GeV)}^{-3}, \quad E_{\eta'} = -5.3 \text{ (GeV)}^{-3}. \] (18)

These values are compatible with the results [51]

\[ E_\eta = -5 \pm 1.5 \text{ (GeV)}^{-3}, \quad E_{\eta'} = -5.1 \pm 0.7 \text{ (GeV)}^{-3}, \] (19)

where both the box anomaly term (15) and the effect of the \( \rho \) resonance have been included as independent contributions to fit the data. The \( \rho \) contribution in Ref. [51] has been taken into account using a relativistic Breit-Wigner amplitude where the width has been parameterized by

\[ \Gamma_\rho(m) = \Gamma_\rho(m_\rho) \left[ \frac{q_\pi(m_\rho)}{q_\pi(m_\rho)} \right]^3 \left( \frac{m_\rho}{m} \right)^\lambda, \] (20)

where \( q_\pi \) is the pion center-of-mass momentum. It is evaluated with the assumption that either a physical \( \rho \) (\( q_\pi(m_\rho) \)) or virtual \( \rho \) (\( q_\pi(m) \)) decays. The parameters \( m_\rho, \Gamma_\rho(m_\rho) \) and \( \lambda \) have been determined from the process \( e^+e^- \to \pi^+\pi^- \) in the \( \rho \) region.

We now turn to the mass of the glueball. The masses of the pure states in terms of the physical-state masses are:

\[ m_{\eta_8} = \langle \eta_8 | H | \eta_8 \rangle = a_{11}^2 m_\eta + a_{22}^2 m_{\eta'} + a_{33}^2 m_\iota, \]

\[ m_{\eta_1} = \langle \eta_1 | H | \eta_1 \rangle = a_{12}^2 m_\eta + a_{22}^2 m_{\eta'} + a_{32}^2 m_\iota, \]

\[ m_{G} = \langle G | H | G \rangle = a_{23}^2 m_{\eta'} + a_{33}^2 m_\iota, \] (21)

\[ m_{\eta'} = \langle \eta' | H | \eta' \rangle = a_{11}^2 m_{\eta'} + a_{22}^2 m_\eta + a_{33}^2 m_\iota, \]

\[ m_\iota = \langle \iota | H | \iota \rangle = a_{12}^2 m_{\eta'} + a_{22}^2 m_\eta + a_{32}^2 m_\iota. \]
where \( H \) is the Hamiltonian. Using the last of Eqs. (21) and the mixing parameters (10) we get for \( m_s = 1410, 1440, 1480 \) MeV,

\[
m_G = 1302, \quad 1324, \quad 1354 \text{ MeV}
\]

respectively. These results are compatible with some other theoretical predictions (see Table I).

Our analysis has allowed us to derive theoretical expectations for the relative admixture of \( u\bar{u} + d\bar{d}, s\bar{s} \) and \( G \) for \( \eta, \eta' \) and \( \iota \). Therefore, an accurate measurement of the mass of the \( \iota \) gives us the mass of the glueball state \( |G\rangle \), which can be compared to calculations with lattice gauge theory or phenomenological models.

**C. Comparison of the quark composition of the \( \eta \) and \( \eta' \) with data**

In order to check our mixing scheme we compare the ratios between some decay widths of the \( \eta \) and of the \( \eta' \) evaluated in our model with the experimental data. This comparison also enables us to tell which future measurements will be useful.

In the following, we use known decay widths to evaluate unknown widths, taking into account

\[
\text{In order to check our mixing scheme we compare the ratios between some decay widths of the }
\]

\[
\text{in good agreement with the experimental result } 
\]

\[
\text{Similarly, from the observed } \omega \text{ decay branching fraction } B(\omega \rightarrow \pi^0 \gamma) = (0.085 \pm 0.005) \text{ we obtain the result}
\]

\[
\Gamma(\rho \rightarrow \eta \gamma) = \frac{\left(\frac{m_\rho^2 - m_\eta^2}{m_\rho^2 - m_\omega^2}\right) m_\omega}{m_\rho} X_\eta^2 \Gamma(\omega \rightarrow \pi^0 \gamma) = (5.0 \pm 0.3) \times 10^{-2} \text{MeV},
\]

in good agreement with the experimental result \( \Gamma(\rho \rightarrow \eta \gamma) = (5.7 \pm 1.4) \times 10^{-2} \text{ MeV} \). Furthermore, we have

\[
\Gamma(\phi \rightarrow \eta \gamma) = \frac{\left(\frac{m_\phi^2 - m_\eta^2}{m_\rho^2 - m_\omega^2}\right) m_\omega}{m_\phi} \left[ 9 \frac{4}{9} Y_\eta^2 \left(\frac{m_\omega}{m_s}\right)^2 \Gamma(\omega \rightarrow \pi^0 \gamma) \right],
\]

where we have taken into account \( SU_F(3) \) (\( F \) for flavor) symmetry breaking [53], by assuming the strange-quark magnetic moment to be smaller than the \( u \)-quark moment by a factor \( m_\omega/m_s \). If we assume \( m_u/m_s = 3/5 \), then \( \Gamma(\phi \rightarrow \eta \gamma) = 0.044 \text{ MeV} \). This assumption gives sort of an upper bound on \( \Gamma(\phi \rightarrow \eta \gamma) \) because at the \( \phi \) energy the quark masses should be between the constituent and the current masses [55]. The experimental result is \( \Gamma(\phi \rightarrow \eta \gamma) = 0.057 \pm 0.027 \) MeV, compatible with our bound.

Next, we have

\[
\Gamma(\eta' \rightarrow \rho \gamma) = 3 \frac{\left(\frac{m_\rho^2 - m_\eta'^2}{m_\rho^2 - m_\omega^2}\right) m_\omega}{m_\rho} X_\eta'^2 \Gamma(\omega \rightarrow \pi^0 \gamma) = 0.062 \pm 0.004 \text{ MeV},
\]

\[
\text{Using the known branching fraction } B(\phi \rightarrow \eta \gamma) = 0.0128 \pm 0.0006, \text{ we obtain:}
\]

\[
B(\phi \rightarrow \eta' \gamma) = \left(\frac{m_\eta' - m_\phi}{m_\rho - m_\omega}\right)^3 \left(\frac{Y_\eta'}{Y_\eta}\right)^2 B(\phi \rightarrow \eta \gamma) = (5.6 \pm 0.3) \times 10^{-5},
\]

a result which is compatible with the experimental bound \( B(\phi \rightarrow \eta' \gamma) \leq 4.1 \times 10^{-4} \).

In this case there will appear factors proportional to certain parameters of the mixing scheme for which we can use our previous results (10).

\[
\text{Similarly, from the observed } \omega \text{ decay branching fraction } B(\omega \rightarrow \pi^0 \gamma) = (0.085 \pm 0.005) \text{ we obtain the result}
\]

\[
\Gamma(\rho \rightarrow \eta \gamma) = \frac{\left(\frac{m_\rho^2 - m_\eta^2}{m_\rho^2 - m_\omega^2}\right) m_\omega}{m_\rho} X_\eta^2 \Gamma(\omega \rightarrow \pi^0 \gamma) = (5.0 \pm 0.3) \times 10^{-2} \text{MeV},
\]
again, compatible with the experimental finding $\Gamma(\eta' \to \rho\gamma) = 0.059 \pm 0.003$ MeV.

We can calculate another set of branching ratios in the charmed sector, assuming the dominance of spectator quark diagrams [53]. Because of this assumption, the following results can at the best be approximations to the real situation. However, we should at least get the right order of magnitude.

We parameterize the partial width for a decay into final states with orbital angular momentum $l$ as [53]

$$\Gamma \approx \tilde{\Gamma}(k/2M)^{2l+1},$$

where $\tilde{\Gamma}$ is the partial width with kinematic factors taken out, $k$ denotes the center-of-mass three-momentum, and $M$ is the mass of the decaying particle. For an $S$ wave, using the experimental value $B(D_s^+ \to \eta\pi^+) = 0.015 \pm 0.004$, we get from Eq. (27):

$$B(D_s^+ \to \eta\pi^+) = \left[ (m_{D_s^+}^2 - (m_\eta + m_\pi)^2)(m_{D_s^+}^2 - (m_\eta' - m_\pi)^2) \right]^{\frac{1}{2}} \left[ (m_{D_s^+}^2 - (m_\eta + m_\pi)^2)(m_{D_s^+}^2 - (m_\eta' - m_\pi)^2) \right]^{\frac{1}{2}} \left( \frac{Y_\eta}{Y_\eta'} \right)^2 B(D_s^+ \to \eta\pi^+)$$

$$= (1.2 \pm 0.3)\%,$$

to be compared with the experimental result $(3.7 \pm 1.2)\%$. In the same hypothesis we get the predictions:

$$B(D_s^+ \to \eta\pi^+) = (2.4 \pm 0.6) \times 10^{-3} \quad \text{or} \quad B(D_s^+ \to \eta\pi^+) = (2.6 \pm 0.7) \times 10^{-3}$$

respectively for $m_\eta = 1480$ MeV and $m_\eta' = 1410$ MeV.

Another interesting set of comparisons with experimental data can be obtained from the decays of the $J/\psi$ into a vector and a pseudoscalar meson. The phase space factor is given by the modulus of the ratio of the center-of-mass three-momenta of the final particles raised to third power. We compare processes which are expressed in terms of the same amplitudes [26, 48, 56] and we work in the approximation in which the contributions of doubly disconnected (or doubly OZI suppressed) diagrams are neglected. We get

$$B(J/\psi \to \omega \eta) = X_\eta^2 \left| \frac{k_\omega}{k_\pi} \right|^3 B(J/\psi \to \rho^0 \pi^0) = (2.09 \pm 0.25) \times 10^{-3},$$

in fair agreement with the observed value $(1.58 \pm 0.16) \times 10^{-3}$. We also obtain

$$B(J/\psi \to \omega \eta') = X_\eta^2 \left| \frac{k_\omega}{k_\pi} \right|^3 B(J/\psi \to \rho^0 \pi^0) = (0.89 \pm 0.11) \times 10^{-3},$$

which grossly overestimates the observed value $(1.67 \pm 0.25) \times 10^{-4}$. It should, however, be pointed out that in the case of $J/\psi \to \omega \eta'$, the procedure used (i.e. the neglect of the doubly OZI suppressed diagrams) leads us to overestimate the branching ratio (31). As it turns out, in the presence of a gluonic component of the $\eta'$, one finds that the neglected diagrams would in fact lower our result; see Jousset et al. [26].

For the ratio of $J/\psi$ decays into $\phi \eta$ and $\phi \eta'$ we find

$$\frac{B(J/\psi \to \phi \eta)}{B(J/\psi \to \phi \eta')} = \left( \frac{Y_\phi}{Y_\phi'} \right)^2 \left| \frac{k_\phi}{k_\phi'} \right|^3 = 1.42,$$

which approximately agrees with the observed value $1.97 \pm 0.45$. The already mentioned neglect of doubly OZI suppressed diagrams appears not to be as important as in the previous case. Nevertheless, including such diagrams somewhat improves the agreement with the data as this leads to an increase of the ratio (32). We also get

$$B(J/\psi \to \rho^0 \eta) = X_\eta^2 \left| \frac{k_\rho}{k_\pi} \right|^3 B(J/\psi \to \omega \rho^0) = (2.12 \pm 0.30) \times 10^{-4},$$

respectively for $m_\eta = 1480$ MeV and $m_\eta' = 1410$ MeV.
in good agreement with the observed value \((1.93 \pm 0.23) \times 10^{-4}\). Furthermore, we get

\[
B(J/\psi \rightarrow \rho^0 \eta') = X^2 \left[ \frac{k_{\eta'}^3}{k_{\rho^0}} \right] B(J/\psi \rightarrow \omega \pi^0) = (0.91 \pm 0.14) \times 10^{-4},
\]

in good agreement with the observed value \((1.05 \pm 0.18) \times 10^{-4}\).

Using analogs of Eqs. (30), (32), and (33), we can now make predictions for the branching ratios for \(J/\psi \rightarrow \iota + V\) where \(V\) is a vector meson. Using \(m_\iota = 1410\) MeV, we get

\[
B(J/\psi \rightarrow \omega \iota) = \left( \frac{X_\omega}{X_\eta} \right)^2 \left[ \frac{k_{\omega}}{k_{\eta \phi}} \right]^3 \Gamma(J/\psi \rightarrow \omega \eta) = (1.5 \pm 0.2) \times 10^{-4}
\]

and

\[
B(J/\psi \rightarrow \rho^0 \iota) = \left( \frac{X_\rho}{X_\eta} \right)^2 \left[ \frac{k_{\rho \iota}}{k_{\eta \phi}} \right]^3 \Gamma(J/\psi \rightarrow \rho^0 \eta) = (1.9 \pm 0.2) \times 10^{-5}.
\]

We have two expressions for \(B(J/\psi \rightarrow \phi \iota)\), depending on whether we use the decay width into \(\phi \eta\) or \(\phi \eta'\) as input:

\[
B(J/\psi \rightarrow \phi \iota) = \left( \frac{Y_\phi}{Y_\eta} \right)^2 \left[ \frac{k_{\phi \iota}}{k_{\eta \phi}} \right]^3 \Gamma(J/\psi \rightarrow \phi \eta) = (0.86 \pm 0.09) \times 10^{-4}
\]

and

\[
B(J/\psi \rightarrow \phi \iota) = \left( \frac{Y_\phi}{Y_{\eta'}} \right)^2 \left[ \frac{k_{\phi \iota}}{k_{\eta' \phi}} \right]^3 \Gamma(J/\psi \rightarrow \phi \eta') = (6.0 \pm 0.7) \times 10^{-5}.
\]

The above results are only marginally dependent on which value one uses for the \(\iota\) mass. In fact, using \(m_\iota = 1480\) MeV we obtain

\[
B(J/\psi \rightarrow \omega \iota) = (1.4 \pm 0.1) \times 10^{-4},
\]

and

\[
B(J/\psi \rightarrow \rho^0 \iota) = (1.7 \pm 0.2) \times 10^{-5}.
\]

while the results for \(B(J/\psi \rightarrow \phi \iota)\) become:

\[
B(J/\psi \rightarrow \phi \iota) = (0.71 \pm 0.08) \times 10^{-4}
\]

and

\[
B(J/\psi \rightarrow \phi \iota) = (5.2 \pm 0.6) \times 10^{-5}.
\]

The only present experimental information, the upper limit \([2]\) \(B(J/\psi \rightarrow \phi \iota) < 2.5 \times 10^{-4}\), is consistent with our values given in Eqs. (37, 38, 41, 42).

Next, we give predictions for various \(\iota\) decays. Using the fact that the amplitude for a pseudoscalar meson \(P\) decaying into \(\rho^0 \gamma\) is proportional to \(X_P\) and that the kinematical factor is proportional to \([m_P - (m_P^2/m_P)]^3\) we obtain:

\[
\Gamma(\iota \rightarrow \rho^0 \gamma) = \left[ \frac{m_\iota - m_\rho^2/m_\iota}{m_\eta - m_\rho^2/m_\eta} \right]^3 \tan^2 \beta \Gamma(\eta' \rightarrow \rho^0 \gamma)
\]

\[
= 0.51 \pm 0.02, 0.58 \pm 0.03, 0.67 \pm 0.03 \text{ MeV}
\]

according to whether \(m_\iota = 1410, 1440,\) or \(1480\) MeV respectively. Similarly, from

\[
\Gamma(\iota \rightarrow \phi \gamma) = \left[ \frac{m_\iota - m_\phi^2/m_\iota}{m_\phi - m_\eta^2/m_\phi} \right]^3 \left( \frac{Y_\iota}{Y_{\eta'}} \right)^2 \Gamma(\phi \rightarrow \eta' \gamma),
\]

\[
= 0.51 \pm 0.02, 0.58 \pm 0.03, 0.67 \pm 0.03 \text{ MeV}
\]
we get the upper bound
\[ \Gamma(\iota \to \phi\gamma) < 0.12, \ 0.14, \ 0.18 \ \text{MeV} \]
for \( m_\iota = 1410, 1440, \) and \( 1480 \ \text{MeV} \) respectively. Using the analog of Eqs. (4) for the \( \iota \), we get
\[ \Gamma(\iota \to \gamma\gamma) = 3\left(\frac{m_\iota}{m_{\pi^0}}\right)^3 \sin^2 \beta \cos^2(\alpha + \arcsin(1/3))(\frac{f_{\pi^0}}{f_\iota})^2 \Gamma(\pi^0 \to \gamma\gamma) \]
\[ = 4.86(\frac{f_{\pi^0}}{f_\iota})^2, \ 5.17(\frac{f_{\pi^0}}{f_\iota})^2, \ 5.62(\frac{f_{\pi^0}}{f_\iota})^2 \ \text{keV} \]
respectively for \( m_\iota = 1410, 1440, 1480 \ \text{MeV} \).

The next thing we can do is to derive the branching ratio for \( J/\psi \to \eta\gamma \), assuming that the decay goes principally through the gluonic component. The branching ratio for the radiative decay of the \( J/\psi \) into the \( \eta \) \([B(J/\psi \to \eta\gamma) \approx 8.6 \times 10^{-4}] \) gives an estimate of the decay into a pure quarkonic state and consequently gives us an idea of the error made by treating the \( \iota \) as a pure gluonic state. With this assumption we find (for the three values of \( m_\iota \) specified above):
\[ B(J/\psi \to \eta\gamma) = \left[ \frac{m_{J/\psi}^2 - m_{\eta}^2}{m_{J/\psi}^2 - m_{\eta'}^2} \right] \cot^2 \beta B(J/\psi \to \eta'\gamma) \]
\[ = (8.3 \pm 0.6) \times 10^{-3}, \ (8.0 \pm 0.6) \times 10^{-3}, \ (7.6 \pm 0.6) \times 10^{-3}. \]
The experimental branching ratio for the \( J/\psi \) to decay radiatively into the \( \eta(1440) \) is \([2] \) \( (2.4 \pm 0.4) \times 10^{-3} \), a factor 3 smaller than our estimate. The above result, if taken at face value, would suggest some serious discrepancies with the assumption leading to Eq. (46), i.e. the assumption that the decays \( J/\psi \to \eta\gamma \) and \( J/\psi \to \eta'\gamma \) go mainly through the gluonic component. On the other hand, as we have already mentioned, there are probably several resonances coexisting in the \( 1400–1500 \ \text{MeV} \) region. The above discrepancy may thus be rather an indication of serious problems of experimental resolution.

The assumption that the \( J/\psi \) radiative decays go mainly through the gluonic component could also be tested in \( \psi(2S) \) and \( \Upsilon \) decays. From the analog of Eq. (46) we get
\[ B(\psi(2S) \to \eta\gamma) = 2.12B(\psi(2S) \to \eta'\gamma) \]
and
\[ B(\Upsilon \to \eta\gamma) = 2.71B(\Upsilon \to \eta'\gamma). \]
Only the following experimental upper bounds are known at present:
\[ B(\psi(2S) \to \eta'\gamma) < 1.1 \times 10^{-3}, \]
\[ B(\psi(2S) \to \eta\gamma), \ B(\iota \to \pi K\bar{K}\gamma) < 1.2 \times 10^{-4}, \]
\[ B(\Upsilon \to \eta\gamma) < 1.3 \times 10^{-3}, \]
\[ B(\Upsilon \to \eta'\gamma) < 8.2 \times 10^{-5}, \]
which are inadequate to test the validity of the assumption leading to Eqs. (47) and (48). New and better data are needed to settle the issue.

Next we give an estimate of the cross section \( \sigma(e^+e^- \to e^+e^-\iota) \) in the equivalent photon approximation, namely the process in which the pseudoscalar particle (the \( \iota \) in this case), is produced by the interaction of the two virtual photons emitted by electron and positron (see Fig. 1). The idea is pretty old \([57]\), and the resulting formula for \( \sigma \) is \([7, 58]\)
\[ \sigma(e^+e^- \to e^+e^-\iota) = 16\alpha^2 \left( \frac{1}{m_{\iota}^2} \right) \left[ \ln \left( \frac{E}{m_{\iota}} \right) - \frac{1}{2} \right]^2 f\left( \frac{m_{\iota}}{2E} \right) \Gamma(\iota \to \gamma\gamma), \]
where $E$ is the beam energy and $f(x)$ is given by

$$f(x) = (2 + x^2)^2 \ln(1/x) - (1 - x^2)(3 + x^2).$$

(54)

Using our previous estimates of $\Gamma(\eta \to \gamma \gamma)$, we obtain $\sigma$ as a function of $f_{\pi}/f_\eta$ and for various energies:

- $E = 2$ GeV, $\sigma = (68, 66, 64) (f_{\pi}/f_\eta)^2$ pb,
- $E = 3$ GeV, $\sigma = (124, 121, 118) (f_{\pi}/f_\eta)^2$ pb,
- $E = 10$ GeV, $\sigma = (388, 383, 378) (f_{\pi}/f_\eta)^2$ pb,
- $E = 50$ GeV, $\sigma = (976, 969, 963) (f_{\pi}/f_\eta)^2$ pb

for $\eta$ masses 1410, 1440, and 1480 MeV. The ratios are independent of the uncertainties related to $f_{\pi}/f_\eta$. We get

$$\frac{\sigma(E = 2 \text{ GeV})}{\sigma(E = 3 \text{ GeV})} = 0.548, 0.546, 0.542$$
$$\frac{\sigma(E = 2 \text{ GeV})}{\sigma(E = 10 \text{ GeV})} = 0.175, 0.172, 0.169$$
$$\frac{\sigma(E = 2 \text{ GeV})}{\sigma(E = 50 \text{ GeV})} = 0.069 0.068, 0.066$$
$$\frac{\sigma(E = 3 \text{ GeV})}{\sigma(E = 10 \text{ GeV})} = 0.320 0.316, 0.312$$
$$\frac{\sigma(E = 3 \text{ GeV})}{\sigma(E = 50 \text{ GeV})} = 0.127 0.125, 0.123$$
$$\frac{\sigma(E = 10 \text{ GeV})}{\sigma(E = 50 \text{ GeV})} = 0.400 0.396, 0.393.$$  

(56)

Good data on $\gamma \gamma$ decay and on the cross section (53) would help us obtain a good value of $f_{\pi}/f_\eta$.

In summary, we find that our mixing scheme, with a glueball component in the $\eta'$ but not in the $\eta$, gives reasonable agreement with the experiments in most cases. The agreement is poorest where the theoretical assumptions are the most questionable (dominance of spectator diagrams, gluonic dominance of some decays, etc.). Further investigations will clarify how well this simple scheme can explain the pseudoscalar sector. Before considering more complex mixing models it makes good sense to test further the scheme we have described.

III. THE SCALAR SECTOR

A. Problems with the $^3P_0$ nonet

Of the $L = 1$ meson nonets [mixed octets and singlets of flavor $SU(3)$], the scalar, with spin, parity, and charge-conjugation $J^{PC} = 0^{++}$ ($^3P_0$ states in the quark model) is the poorest known. Here, the value of $C$ refers only to the self-conjugate members. The $0^{++}$ resonances known at present [2] are listed in Table II. As we can see from this table, there are quite a few candidates for members of the scalar nonet. The difficulty is in the interpretation.

The two states $a_0(980)$ and $f_0(975)$ are especially difficult to interpret. From here on, we denote these states in the main text simply by $a_0$ and $f_0$. Let us consider some of the difficulties. Godfrey and Isgur [24] have calculated the masses of the $P$-wave mesons in their potential model. They find rather good agreement with the observed masses of the axial vector and tensor mesons, but not with the scalars. These authors suggest that the scalar states composed of $u$ and $d$ quarks, i.e., the $^3P_0$ states, have masses around 1090 MeV, over 100 MeV larger than the masses of the $a_0$
and the $f_0$, but about 200 MeV smaller than the mass of the other possible candidate for the role of $0^{++}$ isovector: the $a_0(1230)$. Thus, the low masses of the $f_0$ and the $a_0$ give us reason to doubt their being ordinary quarkonium states. There is an additional problem if the $f_0$ is assumed to be a quarkonium state: the near degeneracy in mass with the isovector state $a_0$ seems to require the $f_0$ to be dominantly a $u\bar{u} + d\bar{d}$ meson, while the strong branching ratio into $K\bar{K}$ suggests a large strange component.

Another, admittedly qualitative, argument which causes us to doubt that the $a_0$ and the $f_0$ are the isovector and isoscalar members (respectively) of the scalar nonet is the following. One expects the mass splittings between corresponding members of the different nonets to be comparable. Indeed, this is very roughly the case for the corresponding members of the other $L=1$ nonets: $^3P_0$, $^3P_1$ and $^3P_2$. In contrast, the mass splitting of the $a_0$ and of the $f_0$ from their corresponding (isovector and isoscalar) partners in the other nonets is much larger than the splitting between the $K^0(1410)$ (which is likely the strange meson of the $^3P_0$ scalar nonet) and the corresponding $K$ states of the other nonets.

Many different models have been proposed suggesting that the $a_0$ and the $f_0$ might be: i) $q\bar{q}q\bar{q}$ states [59, 60]; ii) $KK$ molecules [60, 61]; iii) members of a quarkonium nonet mixed with a $qq\bar{q}\bar{q}$ state [62], or, in the case of the isoscalar state, with a glueball [60]. The $f_0$ has been interpreted as a dilaton by Halyu [63]. It seems unlikely that the $a_0$ and the $f_0$ are hybrids, as their masses are much lower than those usually assigned to hybrids [13, 18, 22]. On the other hand, the $f_0(1240)$ and the $a_0(1320)$ might be hybrids [18, 22], as their masses are considerably higher than those of the $a_0$ and the $f_0$. Another suggestion connects the phenomenology of the lowest scalar particles with the excitation of the QCD vacuum [64]. In this case a special role is advocated for the light-quark $(u,d)$ condensate.

Still other $0^{++}$ states do not have a clear interpretation. Because of their peculiar decay patterns, a predominantly gluonic component has been attributed to both the $f_0(1590)$ [65] and the $f_0(1710)$ [66]. These states might arise from the mixing of a glueball with an $s\bar{s}$ state [67]. Also, Alde et al. [68] have suggested that the $f_0(1400)$ may have a gluongic component.

Predictions of the mass of a scalar glueball spread over a large interval [9, 11, 13–19, 63, 69–74], as we show in Table III. Recent lattice [11, 70] and flux tube calculations [18] put the scalar glueball mass around 1500 MeV (not far from the $f_0(1590)$). In principle, the calculations using lattice QCD should be the most reliable. However, the lattice calculations are made in the valence (or quenched) approximation, and it is not obvious that this approximation is a good one. Besides that, calculations in nonrelativistic potential models [15, 21], bag models [13, 19] and other lattice simulations [9, 16, 73] indicate that the scalar glueball mass is under or around 1000 MeV. Also, in a Bethe-Salpeter equation approach, Bhatnagar and Mitra [22] do not find a consistent solution for the scalar glueball, but find a tensor glueball mass satisfying $1200\text{MeV} \leq m_{2^{++}} \leq 1600\text{MeV}$. We have to conclude that, so far, theorists have not been able to calculate the scalar glueball mass with any reliability.

A low-mass predominantly glueball state might be identified with a broad scalar resonance around 750 MeV. Recent evidence for this resonance has been given by Svec et al. [75], but the state should not be regarded as well established. In this case an excited state should be in the region of the $f_0(1590)$ and of the $f_0(1710)$.

We mentioned previously that the $a_0$ and $f_0$ may be $KK$ molecules. This idea has been suggested in order to explain the approximate mass degeneracy of the $a_0$ and $f_0$ and the fact that they are so near to the $KK$ threshold. Also, the assumption that these mesons are quark-antiquark pairs leads to difficulties. For example, Close et al. [76], interpreting the $f_0$ state as consisting only of $u$ and $d$ quarks, find

$$\frac{\Gamma(f_0 \to \pi\pi)}{\Gamma(a_0 \to \pi\eta)} \approx 4,$$

in contrast with the observed rate 0.6. Furthermore, the total widths calculated for light-quark mesons in a potential model [24] are respectively $\approx 1000\text{MeV}$ for the $f_0$ and $400\text{MeV}$ for the $a_0$, compared to the observed widths of $47 \pm 9\text{MeV}$ and $57 \pm 11\text{MeV}$. In a flux tube model [77] the
\[ \Gamma(f_0 \to \pi\pi) \approx 400 \text{ MeV} \] 

(58)

has been also obtained, compared to the observed value 37 MeV, and

\[ \Gamma(a_0 \to \eta\pi) \approx 225 \text{ MeV} \] 

(59)

compared to the observed value of \( \approx 57 \text{ MeV} \). In other quarkonic models, however, the results may be considerably different. All the above observations might be compatible with both the \( a_0 \) and the \( f_0 \) being molecular states, arising from their diffuse wave functions.

Another indication that the \( a_0 \) and \( f_0 \) might be \( K\bar{K} \) molecules comes from phase shift analyses in an approach in which effective meson-meson potentials are used [78–80]. In these papers a comparison is made between the data and the predictions obtained with the potentials. The result is that molecular bound states in the appropriate mass range are expected, or, turning things around, that a molecular model of these particles is admissible.

Arguments against multiquark interpretations have also be given. A recent analysis [81] of the poles of the amplitude suggests poles on an unphysical Riemann sheet, a result which favors a resonance over a bound state hypothesis. Also, the photoproduction of the \( a_0 \) is in agreement with calculations in the \( \bar{q}q \) scheme, while a prediction for a multiquark state has not been given. Furthermore, for \( \bar{q}q\bar{q}q \) states, some mass predictions are larger than 1000 MeV [82].

New experimental data on the ratio of the inclusive reactions \( e^+e^- \to Z_0 \to f_2(1270)X \) and \( e^+e^- \to Z_0 \to f_0X \), where \( X \) is anything, have been given by the Delphi collaboration [41,83], which finds

\[ R_i = \frac{\sigma(f_2(1270))}{\sigma(f_0)} = 3^{+7}_{-1}. \] 

(60)

Furthermore, the HRS Collaboration [84] finds from \( e^+e^- \) annihilation at \( \sqrt{s} = 29 \text{ GeV} \) that \( R_i = 2 \pm 1 \), while the NA27 Collaboration [85] finds \( R_i = 4.1 \pm 1.5 \) from pp-interactions at \( \sqrt{s} = 27.5 \text{ GeV} \). All these data have large errors, but are not inconsistent with the value \( R_i = 5 \), predicted from spin-statistics arguments for the ratio of the tensor to scalar mesons.

Further evidence against a multiquark interpretation of the \( f_0 \) comes from analyzing the ARGUS data [42] on the partial widths of the decays of \( \Upsilon \) states into \( f_0 \) mesons plus anything \((X)\). The partial widths into \( f_0 \) are in fact comparable with the \( \rho_0(770) \) yields once the different spin structure of the two resonances is taken into account.

It should also be mentioned that the result

\[ R = \frac{\sigma(\pi^\pm p \to f_0X)}{\sigma(K^\pm p \to f_0X)} = 1.66 \pm 0.35 \] 

(61)

found by the OMEGA Collaboration [86], using \( \pi \) and \( K \) beams of 80 and 140 GeV, is in agreement with the value 2, predicted by valence quark counting for a light \( q\bar{q} \) state. (The value is reduced somewhat by contributions from sea quarks and gluons.) The ratio \( R \) is expected to fall further for a state rich in strange quarks (like a \( K\bar{K} \) molecule). In fact, the process \( \pi p \to s\bar{s}X \) is a peripheral one and involves at least two hard gluons in order to produce a colorless \( s\bar{s} \) state, while \( Kp \to s\bar{s}X \) requires only one hard gluon: in this case, one expects, therefore, in a first approximation \( R \sim \alpha_s \).

Close et al. [76] have suggested that a measurement of the ratio of the radiative \( \phi \) decays in these two states (see Table 4) might clarify the situation. However, the \( f_0 \) may be far from being ideally mixed and might have a big gluonic content, complicating the situation. In addition, the prediction given by these authors for the case of a large gluonic component of the \( f_0 \) is quite model dependent.

### B. Phenomenology of the scalar mesons

In the following, we suggest some possible ways to investigate the properties of the scalar mesons, especially the flavorless ones. We propose measurements which we believe can be carried
out in the next experiments (AGS P852, SuperLear, DAΦNE, etc.) and can help to discriminate between the large number of hypotheses about the nature of the scalars.

Interesting information about the composition of the low-lying scalar resonances comes from the ratio [87],

$$\frac{\Gamma(a_0 \to \eta \pi)}{\Gamma(K^0(1430) \to K\pi)} = \frac{m^{2}_{a_0} - m^{2}_{\eta}}{m_{K^0}^2 - m_{\eta}^2} \left[ \frac{m^{2}_{K^0} f^{2}_{K} k_{a_0} 2}{m_{a_0}^{2} f^{2}_{a} k_{a_0} 3} \left[ \frac{\sqrt{3}}{2} a_{11} + \frac{2}{\sqrt{3}} a_{12} \right] ^2 \right]$$

(62)

where $a_{11}$ and $a_{12}$ are the mixing parameters defined previously for the $\eta$ and $f_{K}/f_{\pi} = 1.17$. The above ratio is obtained in the PCAC hypothesis for a quarkonic state [87]. In the scheme proposed here, we get 0.12 for the r.h.s. of Eq. (62). This should be compared with the value of 0.09 in the model of Caruso et al. [7] and 0.11 in the model of Teshina and Oneda [87]. Furthermore, we find 0.072 in the limiting case when $\eta = \eta_8$ and 0.144 when $\eta = \eta_1$.

On the other hand, as a first rough approximation, we can assume

$$\Gamma(a_0 \to \eta \pi) \approx \Gamma(a_0) = 57 \pm 11 \text{ MeV},$$

(63)

where $\Gamma(a_0)$ is the total width of the $a_0$. Then, using the experimental value $\Gamma(K^0(1430) \to K\pi) = 267 \pm 47$ we get 0.21 $\pm$ 0.06 for the l.h.s. of Eq. (62). A word of caution is necessary: the decay $a_0 \to \eta \pi$ is the dominant one, but the experimental value of the branching ratio $a_0 \to K\bar{K}$ is still controversial [2]. Given these uncertainties, the above result, while showing consistency between theory and experiment, is insufficient for us to reach any firm conclusion. A better measurement of the ratio given in Eq. (62) is necessary: a result clearly outside the range 0.072-0.144 would rule out a $q\bar{q}$ interpretation of the $a_0$.

Next, we consider the decays of the $a_0$ and $f_0$ into two photons. For the $a_0$, the experimental result is [2]

$$\frac{\Gamma(a_0 \to \gamma\gamma) \Gamma(a_0 \to \eta\pi)}{\Gamma_{tot}(a_0)} = 0.24 \pm 0.08 \text{ keV}.$$  

(64)

If we use once more the approximation $\Gamma(a_0 \to \eta\pi) \approx \Gamma(a_0)$ we get

$$\Gamma(a_0 \to \eta\pi) \approx 0.24 \pm 0.08 \text{ keV}.$$  

(65)

Similarly, for the $f_0$, one has [2]

$$\Gamma(f_0 \to \gamma\gamma) = 0.56 \pm 0.11 \text{ keV}.$$  

(66)

The results in Eqs. (65) and (66) are not well understood theoretically. Barnes [88], assuming that the $a_0$ and $f_0$ are light-quark mesons and that all the $l = 1$ states have identical spatial wave functions, finds $\Gamma(a_0 \to \gamma\gamma) \approx 1.5$ keV and $\Gamma(f_0 \to \gamma\gamma) \approx 4.5$ keV, in disagreement with the results (65) and (66). Also a $K\bar{K}$ state is disfavored because it leads to [88] $\Gamma(a_0 \to \gamma\gamma) = 0.6$ keV. The experimental result (66) appears to be most compatible with a $qq\bar{q}\bar{q}$ hypothesis [89], for which the suggested width is 0.27 keV. In this case the decay into two photons (of either the $a_0$ or the $f_0$) through two colorless vector mesons, shown in the diagram of Fig. 2 a), gives a small result in consequence of the smallness of their recoupling coefficients as $qq\bar{q}\bar{q}$ members of the $0^+$ four-quark nonet. In this scenario, the decay proceeds mainly through the OZI-suppressed channels, where one gluon is exchanged (diagrams of Fig. 2 b), c)). Babcock and Rosner [90], however, find a very different result $\Gamma(a_0 \to \gamma\gamma) \approx 40$ eV for a quarkonium meson. The theoretical predictions for quarkonium states are very dispersed [88, 90–95], as can be seen from Table V.

Interestingly information can come from a measurement of the ratio $\Gamma(f_0 \to \gamma\gamma)/\Gamma(a_0 \to \gamma\gamma)$, which, in the case of $f_0$ and $a_0$ made of $q\bar{q}$ states, can be predicted quite reliably, as it depends only on the quark charges and $SU(6)$ Clebsch-Gordan coefficients [88]

$$\frac{\Gamma(f_0 \to \gamma\gamma)}{\Gamma(a_0 \to \gamma\gamma)} = \left[ \frac{(2/3)^2 + (-1/3)^2}{(2/3)^2 - (-1/3)^2} \right]^2 \approx 2.8.$$  

(67)
In contrast one expects, in a first approximation, this ratio to be \( \approx 1 \) for multiquark states (whatever their configuration may be). This follows because the two particles decay through the same kind of processes, namely, according to the diagrams of Fig. 2 b), c) in the case of a \( q\bar{q}\bar{q}q \) state, and according to the diagram of Fig. 2 a) in the case of a \( KK \) molecule, where the decay goes via the formation of the photons from the two vector meson components. The experimental decay ratio \( 2.3 \pm 0.9 \) seems to favor the \( q\bar{q} \) interpretation.

The ratio \( \Gamma(f_0 \to \gamma\gamma)/\Gamma(a_0 \to \gamma\gamma) \) can also be used to get a bound on the \( s\bar{s} \) and \( G \) components of the isoscalar particle, assuming that it is a mixture of \( q\bar{q} \), \( s\bar{s} \) and \( G \). We define \( X_s, Y_s \) and \( Z_s \) for a scalar meson \( S \) analogously to our definition for a pseudoscalar below Eq. (3). For a light-quark meson mixed with a glueball, we use the experimental value of the ratio

\[
\frac{\Gamma(f_0 \to \gamma\gamma)}{\Gamma(a_0 \to \gamma\gamma)} = \left[ \frac{(2/3)^2 + (-1/3)^2}{(2/3)^2 - (-1/3)^2} \right]^2 |X_{f_0}|^2 \approx 2.3
\]  

(68)

to find the bound \( 0.71 < |X_{f_0}| < 1 \). If, on the other hand, the decay is dominated by the \( s\bar{s} \) component we should have

\[
\frac{\Gamma(f_0 \to \gamma\gamma)}{\Gamma(a_0 \to \gamma\gamma)} = \left[ \frac{\sqrt{2}(-1/3)^2}{(2/3)^2 - (-1/3)^2} |Y_{f_0}| \right]^2 \approx 2.3.
\]

(69)

This excludes a purely strange meson–glueball mixing, since it would require \( |Y_{f_0}| \) to be \( > 2 \), violating the theoretical limit \( |Y_{f_0}| < 1 \).

Pursuing this line of thought further, we are led to conclude that the strange component of the \( f_0 \) cannot be too large; in general one has:

\[
\frac{\Gamma(f_0 \to \gamma\gamma)}{\Gamma(a_0 \to \gamma\gamma)} = \left[ \frac{(2/3)^2 + (-1/3)^2}{(2/3)^2 - (-1/3)^2} |X_{f_0}| \right]^2 + \left[ \frac{\sqrt{2}(-1/3)^2}{(2/3)^2 - (-1/3)^2} |Y_{f_0}| \right]^2 \approx 2.3.
\]

(70)

Note that the \( Z_{f_0} \) component is absent because it is decoupled from the two-photon channel in lowest order. We can extract some information from (70), however, if we use \( X_{f_0}^2 + Y_{f_0}^2 \approx 1 \) (i.e. we neglect the gluonic component). Then we get

\[
|Y_{f_0}| = 0.4 \pm 0.4
\]

(71)

A more precise measurement of the ratio \( \Gamma(f_0 \to \gamma\gamma)/\Gamma(a_0 \to \gamma\gamma) \) would be very valuable in order to provide a good test of the models.

Other interesting tests of models in two-photon decays come from a comparison of the previous widths with that of the \( f_0(1400) \). These tests are not precise because the \( f_0(1400) \) is so much heavier than the \( a_0 \) and the \( f_0 \). Only a rough measurement of the two-photon decay width of the \( f_0(1400) \) has been made so far [2]: \( 5.4 \pm 2.3 \) keV. Given that the \( f_0(1400) \) is probably an almost pure \( uu + dd \) state (it decays mostly into pions) and taking into account the different kinematical factors, we expect from Eq. (67)

\[
\frac{\Gamma(f_0(1400) \to \gamma\gamma)}{\Gamma(a_0 \to \gamma\gamma)} \approx 2.8 \left( \frac{1400}{980} \right)^3 = 8.2,
\]

(72)

and, similarly,

\[
\frac{\Gamma(f_0(1400) \to \gamma\gamma)}{\Gamma(f_0 \to \gamma\gamma)} \approx 3
\]

(73)

if these are all light-quark resonances. Experimentally, one finds for these ratios \( 22.5 \pm 12.2 \) and \( 9.6 \pm 4.5 \) respectively, which, although not inconsistent with the predictions of Eqs. (72) and (73), are not in support of them either. Thus, once again no definitive conclusion can be drawn without better measurements. A better determination of \( \Gamma(f_0(1400) \to \gamma\gamma) \) would be very useful. Also the
determination of the width of the $a_0(1320)$ into two photons would be interesting. A comparison of these two partial decay widths could clarify, through Eq. (67), whether these resonances both belong to the same meson nonet.

We come now to the analysis of a possible $s\bar{s}$ component in the $f_0$. With the assumption of dominance of the spectator quark diagrams [53], the decay $B(D_s^+ \to f_0 \pi^+) = (7.8 \pm 3.2) \times 10^{-3}$ indicates the presence of $s\bar{s}$. In the case of an $f_0$ made predominantly by $s\bar{s}$, we have

$$B(D_s^+ \to f_0 \pi^+) \gg B(D_s^+ \to a_0 \pi^+)$$  (74)

and

$$B(D_s^+ \to f_0 \pi^+) \gg B(D^+ \to f_0 \pi^+),$$  (75)

since the decays in which there is a strange quark only in the initial or in the final state are suppressed. By contrast, in the case of multiquark states, both the $f_0$ and the $a_0$ have similar composition. Then we expect, in first approximation,

$$B(D_s^+ \to f_0 \pi^+) \approx B(D_s^+ \to a_0 \pi^+)$$  (76)

and

$$B(D_s^+ \to f_0 \pi^+) > B(D^+ \to f_0 \pi^+),$$  (77)

where the second inequality is due to the phase space difference and to the breaking of $SU_F(3)$.

In the case that a light-quark component predominates, we expect:

$$B(D_s^+ \to f_0 \pi^+) \ll B(D^+ \to a_0 \pi^+) \approx B(D^+ \to f_0 \pi^+),$$  (78)

following again the rule that the decays in which there is a strange quark only in the initial or in the final state are suppressed. We also expect

$$B(D_s^+ \to f_0 \pi^+) \approx B(D_s^+ \to a_0 \pi^+)$$  (79)

since now both particles are composed of light quarks. Intermediate situations can, of course, occur in the case of mixing. We also note that a gluonic component should couple weakly to $D$ mesons (one can expect an analogous pattern for $B$ mesons).

The decays of $D_s^+$, $D^+$ and $D^0$ into $f_0K$ are all suppressed in a quark-antiquark channel, except for $D^0 \to \bar{K}^0 f_0$ if $f_0$ has a large light-quark component. Recently such a decay has been analyzed by the ARGUS collaboration [96], who find $B(D^0 \to \bar{K}^0 f_0) = (0.48 \pm 0.20)\%$. This branching ratio, while small, is comparable to the branching ratio of the $D^0$ into $\bar{K}^0$ plus the $f_0(1400)$, which is believed to be composed predominantly of light quarks. (The ARGUS result [96] is $B(D^0 \to \bar{K}^0 f_0(1400)) = (0.71 \pm 0.28)\%$.) Therefore, the observed branching ratio into $f_0$ suggests that its light-quark component should not be neglected.

A rough quantitative estimate of the strange component of the $f_0$ can also be obtained from the experimental value of the ratio

$$\frac{B(f_0 \to \pi\pi)}{B(f_0 \to K\bar{K})} \approx 3.56$$  (80)

in the limit in which $f_0 \to K\bar{K}$ proceeds through the strange component only. Taking into account threshold effects in the $K\bar{K}$ channel by integrating over a Breit–Wigner multiplied by the appropriate phase space factor, we get:

$$\frac{B(f_0 \to \pi\pi)}{B(f_0 \to K\bar{K})} = \frac{\int_{m_{f_0}}^{\infty} dE \left[\frac{2(E-E_0)}{2(E-E_0)^2 + 1}\right]}{\int_{2m_K}^{\infty} dE \sqrt{(E^2-4m_K^2)/(4E^2)}/[2(E-E_0)/\Gamma)^2 + 1]} \left|\frac{X_{f_0}}{Y_{f_0}}\right|^2 \approx 10\left|\frac{X_{f_0}}{Y_{f_0}}\right|^2.$$  (81)
If there is no gluonic contribution to the decay ratio (80), then
\[ X_{f_0}^2 + Y_{f_0}^2 = 1. \] (82)

If we substitute Eq. (82) into Eq. (81) and use Eq. (80), we obtain:
\[ |Y_{f_0}| = 0.86. \] (83)

Of course, the light-quark component can also decay into \( K\bar{K} \), and therefore the above value of \(|Y_{f_0}|\) has to be considered as an upper bound. A lower bound on \(|Y_{f_0}|\) can be obtained in the limit of unbroken \( SU_F(3) \); in such a case the \( uu + dd \) component will decay with \( 2/3 \) probability into \( \pi\pi \) and \( 1/3 \) into \( K\bar{K} \). We get, therefore, from Eq. (81):
\[ \frac{B(f_0 \to \pi\pi)}{B(f_0 \to K\bar{K})} = 10 \frac{2/3|X_{f_0}|^2}{|Y_{f_0}|^2 + 1/3|X_{f_0}|^2} \approx 3.56, \] (84)

which leads to \(|Y_{f_0}| = 0.78\).

If the \( f_0 \) were a pure gluonic state, its decays should be in accord with flavor democracy. This, with the usual assumption of an unbroken \( SU_F(3) \) hypothesis, leads to a much too large result:
\[ \frac{B(f_0 \to \pi\pi)}{B(f_0 \to K\bar{K})} \approx 10 \times \frac{4}{5} = 8. \] (85)

The experimental value given in Eq. (80) suggests that the \( f_0 \) does not have a large gluonic component.

As we have just seen, the above argument (although obtained with rough approximations) leads to the result that \( 0.78 \lesssim Y_{f_0} \lesssim 0.86 \), not in good agreement with the smaller value of \( Y_{f_0} \) obtained earlier [see Eq. (71)]. Granting that in both cases the arguments are very qualitative, it is clear that better experimental measurements will be necessary to clarify our picture of what is going on. Of course, a better theoretical understanding of the general situation is also desirable.

Another test of the strange component in the \( f_0 \) could come from the high energy behavior of \( \pi^+ p \to f_0 n \). We expect
\[ \frac{\sigma(\pi^+ p \to f_0 n)}{\sigma(\pi^+ p \to a_0 n)} \approx 1 \] (86)

in the case that \( a_0 \) and \( f_0 \) are both light-quark mesons, because both processes proceed through the exchange of the same Regge trajectory (either the \( \pi \) or the \( a_1 \) trajectory). Also for a multiquark system one can expect a ratio near one. In contrast, the ratio (86) could be OZI suppressed if the \( s\bar{s} \) component of the \( f_0 \) is important. Then the ratio (86) could be lowered by a factor \( \sim \alpha_s^2 \approx 0.06 \). If the gluonic component dominates, the ratio (86) could be \( \sqrt{OZI} \) suppressed and could be lowered by a factor \( \sim \alpha_s \approx 0.2 \). (See, however, Chanowitz [97] for a discussion of the \( \sqrt{OZI} \) suppression.) Yet another test of the \( s\bar{s} \) component of the \( f_0 \) comes from the comparison of \( \sigma(\pi^- p \to f_0 n) \) and \( \sigma(K^- p \to f_0 \Lambda[\Sigma^0]) \). If the \( f_0 \) is mainly \( s\bar{s} \), then the first channel is proportional to \( \alpha_s^2 \) and the second to \( \alpha_s \). It follows that at sufficiently high energy, we have
\[ \frac{\sigma(\pi^- p \to f_0 n)}{\sigma(K^- p \to f_0 \Lambda)} < 1. \] (87)

In contrast, if the \( f_0 \) is mainly a light-quark state, we expect:
\[ \frac{\sigma(\pi^- p \to f_0 n)}{\sigma(K^- p \to f_0 \Lambda)} > 1, \] (88)

since the first process proceeds through the exchange of the \( \pi \) Regge trajectory (\( \alpha_\pi(0) \approx 0 \)) and the second through the exchange of the \( K \) trajectory (\( \alpha_K(0) \approx -0.3 \)).
We next give some considerations on a possible gluon component of the $f_0$. The observed branching ratio of the $J/\psi$ into $\omega f_0$ is $(1.4 \pm 0.5) \times 10^{-4}$ and $(3.2 \pm 0.9) \times 10^{-4}$ into $\phi f_0$ (into $\rho a_0$ has been not observed yet). Both disagree with the hypothesis of the $f_0$ being predominantly $s\bar{s}$, which has as consequences:

$$B(J/\psi \to \omega f_0) \approx 0,$$

being a doubly OZI violating decay, and

$$B(J/\psi \to \rho a_0^0) \approx B(J/\psi \to \phi f_0)$$

(neglecting phase space and $SU_F(3)$ breaking effects) being both singly OZI forbidden decays.

The lack of the observation of the decay $J/\psi \to \rho a_0$ is also in disagreement with the molecular hypothesis according to which $[98]$

$$B(J/\psi \to \rho a_0^0) = \frac{1}{2} B(J/\psi \to \phi f_0) = B(J/\psi \to \omega f_0)$$

(91)

The $K\bar{K}$ molecule predictions should be relatively unaffected by the the $SU_F(3)$ breaking. By contrast, in the $q\bar{q}$ hypothesis $SU_F(3)$ breaking effects are expected to suppress $J/\psi \to \phi f_0$, analogously to what is observed for the tensor mesons, where $[99]$

$$B(J/\psi \to \rho a_0^2) \approx 5B(J/\psi \to \phi f_0^2).$$

Of course, different mixing schemes can produce quite different results, but if the decays of the $f_0$ proceed through a gluonic component this would explain the lack of observation of the $a_0$; the latter having no gluonic component, a much smaller $B(J/\psi \to \rho a_0^0)$ is expected. An unusually large gluonic component would also lead to (neglecting phase space differences)

$$B(J/\psi \to \omega f_0) \approx B(J/\psi \to \phi f_0).$$

(92)

if the $f_0$ is formed through its gluonic part.

A bound on the gluonic component of the $f_0$ comes from comparing

$$B(J/\psi \to \gamma f_0) B(f_0 \to \pi\pi) < 7 \times 10^{-5}$$

with

$$B(J/\psi \to \gamma f_0(1710)) B(f_0(1710) \to K\bar{K}) = (9.7 \pm 1.2) \times 10^{-4}. $$

(94)

(However, the large mass difference between the two final states should be matter of caution.) If the decay proceeds predominantly through a gluonic component, we have:

$$\frac{B(J/\psi \to \gamma f_0)}{B(J/\psi \to \gamma f_0(1710))} = \left| \frac{Z_{f_0}}{Z_{f_0(1710)}} \right|^2 \left| \frac{k_{f_0}}{k_{f_0(1710)}} \right|^3 \frac{2.18}{2} \frac{Z_{f_0}}{Z_{f_0(1710)}}^2,$$

(95)

where $|k_{f_0}/k_{f_0(1710)}|^3$ comes from phase space. The above implies

$$\left| \frac{Z_{f_0}}{Z_{f_0(1710)}} \right|^2 < 4 \times 10^{-2}.$$

(96)

For a purely gluonic $f_0(1710)$ we then have $|Z_{f_0}| < 0.2$. The data, however, are still quite doubtful; a precise determination of these branching ratios and of those of the $f_0(1590)$ and of the $f_0(1400)$ would give information on their gluonic component. However, even if such a component is negligible for one or all of these particles, these branching ratios would give a clue on their quarkonic composition. For instance, in the case $Z_A \approx Z_B \approx 0$, we have

$$\frac{B(J/\psi \to \gamma A)}{B(J/\psi \to \gamma B)} = \left| \frac{k_A}{k_B} \right|^3 \left| \frac{\sqrt{2} X_A + Y_A}{\sqrt{2} X_B + Y_B} \right|^2.$$

(97)
Then we find

$$\frac{B(J/\psi \to \gamma f_0)}{B(J/\psi \to \gamma f_0(1400))} = 1.45$$

(98)

if both particles are composed of light quarks. On the other hand, this ratio is $= 0.73$ if the more massive state is composed predominantly of light quarks and the lighter one has a predominantly strange-quark component, as some of the previous considerations seem to indicate. See e.g. Eq. (83) and also the discussion preceding Eq. (74)).

### IV. CONCLUSIONS

Despite a vast literature on the subject of spin-0 mesons, the subject is not really well understood either theoretically or experimentally. We would like to know the quark and glueball content of these mesons (and their four-quark and hybrid content as well), but we believe that it will take the next generation of experiments, together with additional theoretical work, before we have a reasonably accurate picture. It is for this reason that we have tried to gather together in one place not only information about the present status of the subject, but also a large number of suggestions about feasible measurements which should help to clarify the situation.

Concerning the pseudoscalar sector, we have presented a simple mixing scheme (involving the $\eta$, the $\eta'$ and the $\iota$) and proposed various ways to test the composition of these particles. We believe that no single measurement will solve the complicated puzzles offered by these mesons, and we have suggested a number of different measurements which should help us to arrive at a definite solution. Such measurements should be well within experimental reach during the next few years.

The present picture is even less clear for the scalar mesons, where many poorly known resonances are involved. Nevertheless, also in this case it appears that a number of measurements we have suggested could help considerably in clarifying the situation.

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Table I. Mass predictions for the lowest pseudoscalar glueball states. These are subdivided into lattice and flux tube results and results in the digluonium and trigluonium approximations.

| Reference        | Mass (MeV) |
|------------------|------------|
| (Lattice, Flux Tube) |            |
| Teper 82 [9]     | 1450       |
| Berg 83 [16]     | 2175       |
| Ishikawa 83 [14] | 1250 - 1660|
| Isgur 85 [18]    | 2790       |
| Michael 88 [10]  | 2400       |
| Michael 89 [11]  | 2464-3124  |
| Simonov 91 [8]   | 1640       |
| (Digluonium)     |            |
| Novikov 81 [12]  | 2000 - 2500|
| Chanowitz 83 [13]| 1240 - 1440|
| Cornwall 83 [15] | 1300 - 1400|
| Narison 84 [17]  | < 1900 ± 400|
| Geiger 90 [19]   | 1440       |
| (Trigluonium)    |            |
| Iwao 83, Iwao 81 [20] | 3310,3620 |
| de Castro 90 [21] | 3200 (1S)  |
| Bhatnagar 91 [22]| ≥ 1541     |
| Simonov 91 [8]   | 1800       |

Table II. Scalar resonances from the Review of Particle Properties [2]. A question mark indicates a resonance which is not well established.

| Name of the resonance | mass     | Γ         |
|-----------------------|----------|-----------|
|                       | (MeV)    | (MeV)     |
| $f_0(975)$            | 974.1 ± 2.5 | 47 ± 9   |
| $a_0(980)$            | 982.7 ± 2.0 | 57 ± 11  |
| $f_0(1240)$ ?         | 1240 ± 30  | 140 ± 30  |
| $a_0(1320)$ ?         | 1322 ± 30  | 130 ± 30  |
| $f_0(1400)$ ?         | ≈ 1400     | 150 - 400 |
| $K_0^*(1430)$         | 1429 ± 9   | 287 ± 31  |
| $f_0(1525)$ ?         | ≈ 1525     | ≈ 90      |
| $f_0(1590)$ ?         | 1587 ± 11  | 175 ± 19  |
| $f_0(1710)$ ?         | 1709 ± 5   | 146 ± 12  |
| $X(1740)$ ?           | 1744 ± 15  | < 80      |
| $K_0^*(1950)$ ?       | 1945 ± 30  | 201 ± 103 |
Table III. Mass predictions for the lowest scalar states. These are subdivided into lattice, flux tube and string results and results in the digluonium approximation.

| Reference      | Mass (MeV) |
|----------------|------------|
| (Lattice, Flux tube, String) |            |
| Teper 82 [9]   | 770        |
| Berg 83 [16]   | 750        |
| Ishikawa 83 [14] | 740 ± 90  |
| Isgur 85 [18]  | 1520       |
| Albanese 87 [73] | 650-820   |
| Michael 89 [11] | 1408-1672 |
| van Baal 89 [70] | 1370 ± 90 |
| Halyu 91 [63]  | ≈ 880      |
| Bitar 91 [74]  | 1200 ± 300 |
| Gliozzi 92 [71] | 1542      |
| (Digluonium)   |            |
| Chanowitz 83 [13] | 670-1560 |
| Cornwall 83 [15] | 1100-1200 |
| Narison 84 [17] | 1400      |
| Lanik 88 [69]  | 850-990    |
| de Castro 90 [21] | 993 (1S) |
| Geiger 90 [19] | 796-1082   |
| Boos 92 [72]   | 1400      |

Table IV. Predictions of Close et al. [76] for the absolute branching ratios of $\phi$ into $a_0(980)\gamma$ or into $f_0(975)\gamma$, and for the ratio of these for different compositions of the two scalar states.

| Scalar meson constitution | Absolute branching ratio | $\frac{\Gamma(\phi\rightarrow a_0\gamma)}{\Gamma(\phi\rightarrow f_0\gamma)}$ |
|--------------------------|--------------------------|---------------------------------|
| $K\bar{K}$ molecule     | $a_0 \approx f_0 \approx 4 \times 10^{-5}$ | ≈ 1 |
| $g\bar{q}qg$ $K\bar{K}$ bag | < $10^{-6}$ | 1 |
| $g\bar{q}g\bar{q}$ $D\bar{D}$ bag | < $10^{-6}$ | 9 |
| $g\bar{q}qg$ $(u\bar{u} + d\bar{d})(s\bar{s})$ bag | < $10^{-6}$ | – |
| $f_0$ glueball, $a_0$ quarkonium | $\lesssim 10^{-6}$ | ≈ 1 |
| $f_0$ and $a_0$ light quark $^3P_0$ | $\lesssim 10^{-6}$ | ≈ 1 |
| $f_0$ $s\bar{s}$ and $a_0$ light quark $^3P_0$ | $f_0 \lesssim 10^{-5}$ | ≈ 0 |
Table V. Predictions of the width into two photons for the $a_0(980)$ and the $f_0(975)$ in different models.

| Reference       | Model                        | $\Gamma(a_0 \rightarrow \gamma \gamma)$ (keV) | $\Gamma(f_0 \rightarrow \gamma \gamma)$ (keV) |
|-----------------|------------------------------|-----------------------------------------------|-----------------------------------------------|
| Babcock 76 [90] | Vector dominance             | 0.04                                          | 8                                            |
| Barnes 85 [88]  | $f_0$ Light quarks           | 1.5                                           | 4.5                                          |
| Berger 73 [91]  | Vector dominance             | 3.8                                           | –                                            |
| Bramon 71 [92]  | Reggeon exchange             | 50                                            | –                                            |
| Budnev 79 [93]  | Potential                    | 4.8                                           | 12.8$B^*$                                    |
|                 | (Harmonic oscillator)        |                                               |                                               |
| Eliezer 75 [94] | $a_0 \rightarrow \gamma \gamma$ through (2 pseudoscalars) | –                                             | 0.2                                          |
| Greenhut 78 [95]| Vector dominance             | 550 ± 270                                     | –                                            |

$^*$The quantity $B$ depends on the quark content of the $f_0$.

Figure captions

Fig. 1. The process $e^+e^- \rightarrow e^+e^- +$ pseudoscalar in the equivalent photon approximation. The symbol $P$ is for pseudoscalar meson.

Fig. 2. Possible mechanisms of decay of the $a_0(980)$ and $f_0(975)$ mesons in models in which these mesons contain two quarks and two antiquarks. The symbol $V$ is for vector meson.