An LC-Circuit Model for Dynamics of In-Building Heat Transfer across Atrium Space

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Abstract. We develop a novel mathematical model to represent the dynamics of heat transfer across atrium space, that is, a large open space inside a building. This modeling is established by a combination of an equivalent electric-circuit equation of thermal mass and measurement data. A novel point of it is to introduce not only resistance and capacitance but also inductance to the circuit model, which is different from traditional models of thermal mass and induces the second-order equation of heat balance. We propose to use the so-called Koopman mode decomposition to estimate the values of circuit parameters from measurable time-series data. Effectiveness of the circuit model is established with a numerical simulation incorporated with geometry, structure, and time-series data on a practically-used building in Japan.

1. Introduction

In this paper, we propose an equivalent electric-circuit model for the dynamics of heat transfer across atrium space, that is, a large open space inside a building. For zonal-level heat transfer in buildings, equivalent circuit models of a thermal system are used in many studies [1, 2], in which each zone acts as a thermal capacitor, and thermal mass forms an RC network [3]. Our model in this paper has two novel ideas. The first one is that in order to represent the heat transfer between different zones not necessarily separated by walls, we use temperature gradient and thermal resistance, which is suitable to modeling of large open space. The second one is that we represent the reaction of thermal mass at each zone with not only thermal capacitance but also inductance, leading to a linear second-order equation of heat balance. The thermal mass in this paper refers to zone air, non-structural elements such as furniture and books, and structural elements such as walls. The introduction of inductance is related to the notion of thermal inductance [4] and makes it possible to represent heat transfer phenomena resulting from the atrium, for example, an oscillatory response of the zonal-level temperature [5] and a time lag of the influence of heat injection on the temperature. The time lag emerges due to the presence of large thermal mass [6], which the atrium inherently possesses. In this paper, we use the so-called Koopman Mode Decomposition (KMD) to estimate the values of circuit parameters from measurable time-series data. KMD is an established technique of time-series analysis based on discrete spectrum (eigenvalues) of the Koopman operator defined for nonlinear dynamical systems [7–9]. KMD enables us to estimate the parameters of the linear second-order equation directly from measurable nonlinear time-series data on temperature. Here, we show that the
developed model works effectively with a numerical simulation incorporated with geometry, structure, and time-series data on a practically-used building with atrium space in Japan. We note that the same atrium space was investigated with a partial differential equation with the effective diffusion coefficient, namely distributed-parameter model [5], which is different from our present approach based on a lumped-parameter model.

2. Modeling target in a commercial building

This paper focuses on a practically-used atrium and office space in the commercial building in order to delineate the modeling idea. This space is located in the main building of OMRON Healthcare Co., Ltd. in Kyoto, Japan.

First, we provide basic information on the target space based on literature [5, 10]. This building has 6 floors and atrium structure from 2nd floor to 6th floor at the center of the building. The offices of different floors are connected through the atrium not separated by walls. In this paper, the 5th and 6th floors are the target space for modeling. Figure 1 and 2 show the cross section and outline of the target space. The solid lines in Figure 1 and 2 stand for walls and the broken lines for the boundaries of the rooms. The eastern space of the atrium is used for the office space, and the western space for the laboratory. From this structure, we speculate that the heat transfer between the office rooms of different floors occurs through the atrium.

Next, we review the HVAC system in the target space. The office space has the two types of HVAC units for local and global air-conditioning, called task-units and ambient-units, respectively. 3 ambient-units are installed on each floor; 15 task-units are installed for 5th floor, and 14 task-units for 6th floor; see literature [5] for its details (locations of air supply etc.) In the atrium, no HVAC unit is installed on each floor.

3. Koopman mode decomposition

In this section, we introduce KMD applied to measurement data on temperature. KMD is a technique of nonlinear time-series analysis and extracts spatio-temporal modal oscillations with single frequencies directly from data [7, 8]. This paper uses the Arnoldi-type algorithm [8] to decompose given snapshots $X_n \in \mathbb{R}^M$ ($n = 0, \ldots, N - 1$) with constant time-step $\Delta t$ into the following finite series:

$$
X_n = \sum_{m=1}^{N-1} \tilde{\lambda}_m^n \tilde{V}_m \quad n = 1, \ldots, N - 2
$$

where $\tilde{\lambda}_m \in \mathbb{C}$ is called the $m$-th Koopman Eigenvalue (KE) and $\tilde{V}_m \in \mathbb{C}^M$ associated $m$-th Koopman Mode (KM). The snapshots $X_n$ are regarded in this paper as multi-channel time-series on the measublack temperature.
4. Proposed model

In this section, we introduce the circuit-based modeling of heat transfer dynamics of in-building space with atrium. In the traditional models based on an equivalent electric circuit [1, 2], each room acts as a thermal capacitor, and thermal mass forms an RC network [3]. In this paper, we introduce multiple inductance elements to the circuit model in order to represent adequately the reaction of thermal mass. The introduction of inductance is related to the notion of thermal inductance [4] and makes it possible to represent an oscillatory response of the in-building temperature and a time lag of the influence of heat injection on the temperature. Also, we assume that the zonal heat transfer modeling is based on temperature gradient and thermal resistance. This assumption is expected to be valid if the fluid motion inside the space is well managed by an HVAC system. Figure 3 shows the proposed circuit model. For this we divide the target space into 4 zones—6th atrium, 5th atrium, 6th office, and 5th office—which are named zone1 to zone4. In this paper, we do not consider the influence of 4th floor for simplicity of the current work. Modeling of the entire building, which includes the influence of 4th floor, is in our future work. The value of resistance $R_{i,j}$ depends on the coupling of nodes (zones) in the circuit model in order to represent the position-dependence of heat transfer due to both the conduction and convection. The values of capacitance $C_i$ and inductance $L_i$ also depend on nodes, namely, zones with individual thermal masses which also have position-dependence. Thus, the heat balance equations at time $t$ are given by

$$T_i - \bar{T}_i = L_i \frac{d(Q_{\text{int},i} + Q_i + Q_{\text{sys},i})}{dt} - C_i \frac{d^2T_i}{dt^2}$$

$$Q_i = \sum_{j=1}^{N_{\text{zone}}} \frac{T_j - T_i}{R_{i,j}} + \frac{T_{\text{out}} - T_i}{R_{w}}$$

(2)

where $T_i$ stands for the temperature at zone $i$ that corresponds to a state variable of the circuit model. The constant parameter $\bar{T}_i$ is a voltage source in the circuit model and set at the time average of the measurable time-series data on temperature. $T_{\text{out}}$ is the outside temperature. The constant $R_w$ stands for the heat transmission coefficient of windows in the offices. $Q_{\text{int},i}$ stands for the internal thermal gain of zone $i$. $Q_i$ stands for the total sum of the amount of heat transfer to zone $i$. $N_{\text{zone}}$ is the total number of zones considered in this modeling. $Q_{\text{sys},i}$ stands for the energy provided by the HVAC system at zone $i$. $Q_{\text{sys},i}$ is determined as the product of temperature difference between supply-air temperature and zone temperature $T_i$, density $\rho$, specific heat $c_p$ at constant pressure, and supply air mass flow rate $U$ [11].

![Figure 3. Equivalent electric-circuit model of the heat transfer through atrium](image)
Our circuit model includes both the capacitance and inductance elements, and it thus represents a variety of dynamic responses of the zone temperature that are richer than the conventional model with capacitance only. In particular, our model can represent an oscillatory response of the temperature and a time lag of the dynamic response subject to a heat induction from the outside of the zone, both of which are not modeled with any RC network. These phenomena are important for our current modeling of the large open space including the atrium, as mentioned in the introductory section. Later, we will show effectiveness of our model with a numerical simulation based on practical data.

5. Estimation of parameters

In this section, we estimate the parameters of the circuit model introduced in the previous section. The estimation is based on stationary and transient components of the measublack time-series data on temperature. Here, we define the stationary and transient component as follows. The stationary component is the time average of the measublack time-series data. The transient component is the damped oscillatory pattern related to the heat transfer through the atrium, which is extracted from the measublack time-series data via KMD.

5.1. Measublack time-series data and modeling assumptions

We review measublack time-series data on temperature for the parameter estimation. All temperature data were sampled by 0.1°C every 10 min. The data included the temperature measured in the office spaces and near the atrium, the supply air temperature from the ambient units, and the outside temperature. Multiple temperature sensors were placed within each zone. The average of the temperature data from such multiple sensors was used as the zone air temperature \( T_i \). In the same way, the average of supply air temperature from multiple ambient-units for each floor was used as the temperature \( T_{sa} \) of ambient-units in the circuit model. However, the outlet temperature of task-units was not measurable. For simplicity of the current modeling, we set the temperature \( T_{sa} \) of task-units to be constant. Also, we assumed that internal heat gain \( Q_{int} \) was spatially uniform over the target space.

5.2. Estimation of \( R \) based on the stationary component

We estimated the value of the parameter \( R_{ij} \) which represents the thermal resistance using the time-series data measured from 8:00 am to 5:10 pm on 30th July 2014 (Wed.). We used an optimization method to estimate the value of \( R_{ij} \). Neglecting the terms including the time derivative in (2), we obtained the four linear constraint equations for the time-invariant, steady state. The optimal solution satisfying the above constraint was taken as the value of \( R_{ij} \). The estimated parameters are the following:

\[
R_{1,2} = 1.20 \times 10^{-4} \, ^\circ \text{C/W}, \quad R_{1,3} = 1.16 \times 10^{-4} \, ^\circ \text{C/W}, \quad R_{2,4} = 1.29 \times 10^{-4} \, ^\circ \text{C/W}.
\]

5.3. Estimation of \( C \) and \( L \) based on the transient component

Next, we estimated the values of the parameters \( C_i \) and \( L_i \) which represent the reaction of thermal mass using the time-series data measured from 8:00 am to 5:10 pm on 30th July 2014 (Wed.). We have the three steps for the estimation. First of all, we apply KMD to the measublack time-series data on the zone temperature, the temperature \( T_{sa} \) of ambient-units, and the outside temperature. Next, we extract a spatio-temporal oscillatory pattern related to the heat transfer through the atrium. Finally, we estimate the parameters by substituting the oscillatory pattern into the circuit model.

By applying KMD to the measublack time-series data on temperature, we obtained the 55 pairs of KE and KM. The obtained KE and KM included conjugates with each other because the data \( X_n \in \mathbb{R}^M \) (\( n = 0, \ldots, N - 1, M = 7, \text{and} \, N = 56 \)) in this paper were real-valued.
In the following, we denote each pair of complex conjugate mode vectors by \( \{m, m + 1\} \). The oscillation period \( T_m \) determined from KE is defined as \( T_m := 2\pi \Delta t / | \text{Im} \ln \lambda_m | \). From the result, we regarded \( \{9, 10\} \) as an oscillatory pattern related to the transfer through the atrium. This is mainly because the oscillation period of mode pair \( \{9, 10\} \) is 1.87 hours and is close to the period of heat transfer response across the atrium, about 2 hours, suggested in literature \[5\]. Also, because the modulus of the KE is close to 1 and the corresponding KM has a large norm in the current decomposition, the oscillatory pattern \( \{9, 10\} \) is relatively dominant and sustained.

We now determine the parameters \( C_i \) and \( L_i \) based on KM \( \{9, 10\} \). In order to obtain the equations for the parameter estimation, it is requiblack to re-formulate the oscillatory pattern in continuous time. Then, substituting the continuous-time formulation into (2), we could obtain the independent 8 equations because the values of unknown parameters \( C_i \) and \( L_i \) were real. The number of the unknown parameters is 8. Therefore, we are able to determine the unknown parameters uniquely as follows:

\[
\begin{align*}
C_1 &= 3.81 \times 10^7 \text{ J} / \circ C, \\
C_2 &= 6.79 \times 10^6 \text{ J} / \circ C, \\
C_3 &= 6.89 \times 10^7 \text{ J} / \circ C, \\
C_4 &= 8.30 \times 10^7 \text{ J} / \circ C, \\
L_1 &= 4.36 \times 10^{-2} \circ C / \text{W}, \\
L_2 &= 8.70 \times 10^{-2} \circ C / \text{W}, \\
L_3 &= 1.78 \times 10^{-2} \circ C / \text{W}, \\
L_4 &= 1.41 \times 10^{-2} \circ C / \text{W}.
\end{align*}
\]

6. Simulation
This section presents a simulation of the zone temperatures for the target space in Figure 1 on 30th July 2014. The simulation is conducted by incorporating the circuit model with the measublack temperature data. We compare the simulated zone temperatures to the oscillatory pattern extracted from measublack nonlinear time-series data on temperature in the previous section (i.e., sum of \( T_i \) and \( 2 \text{Re}[\lambda_n V_n] \)). We used MATLAB solver \texttt{ode45} for the simulation. As the initial conditions, the sum of \( T_i \) and \( 2 \text{Re}[\lambda_n V_n] \) \((n = 0, 1)\) was provided. As the input, the time-series measublack data on the outside temperature \( T_{\text{out}} \) and on the temperature \( T_{\text{sa}} \) of ambient-units were also provided. Figure 4 shows the simulation result of the zone temperatures.

![Simulation result of the zone temperatures](image)

**Figure 4.** Simulation result of the zone temperatures
and the oscillatory pattern (sum of $\bar{T_i}$ and $2\text{Re}[\lambda_0^\alpha \bar{V}_0]$). We clearly see that the time-series simulated with the model and extracted from the measurements are consistent. This implies that the circuit model can well reproduce the oscillatory pattern related to the heat transfer through the atrium, which existence is suggested in literature [5].

7. Conclusion
In this paper, we proposed the circuit model for heat transfer dynamics of in-building space with atrium and estimated the model parameters from a well-black nonlinear timeseries data on temperature using KMD. A simulation of the circuit model was conducted with measurement data in a practically-used building. The simulation shows that the circuit model well reproduces the oscillatory pattern of the zone temperatures related to the heat transfer through the atrium. In this model, both the capacitance and inductance elements act as a resonance circuit equipped with an external source, namely, source of heat injection. The resonance frequency is determined with data on a spatio-temporal oscillatory pattern of the zone temperatures, which represents the heat transfer through the atrium and is extracted with KMD. Thus, the model enables us to represent the periodic oscillatory pattern of interest. In this sense, we conclude that the introduction of inductance element is crucial to the accurate simulation of heat transfer across the atrium space.

Our future work includes generalization of the present modeling, e.g. by applying it to the entire building which we did not study in this paper, and by investigating the physical relevance of inductance element for another building.

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