Unexpected symmetries in classical moduli spaces

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ABSTRACT

We analyze the structure of the moduli space of a supersymmetric $SU(5)$ chiral gauge theory with 2 matter fields in the $10$ representation, and 2 fields in the $\bar{5}$ representation. Inspection of the exact Kähler potential of the classical moduli space shows that the symmetry group of the moduli space is larger than the global symmetry group of the underlying gauge theory. As a consequence, the gauge theory has classical inequivalent vacua which yield identical low energy theories.
1 Introduction

Depending on the matter content, supersymmetric gauge theories can have large vacuum degeneracies \cite{1}. In the absence of a superpotential, classical vacua are associated with vacuum expectation values for which the D–terms of the scalar potential vanish. In Wess–Zumino gauge, the D–flat directions contain those points in the vector space of scalar components of the chiral superfields that satisfy the condition

$$D^a = \sum_i \phi_i^\dagger T^a \phi^i = 0,$$

(1)

where the sum is over all matter multiplets, $\phi^i$ is the scalar component of the superfield $\Phi^i$, and $T^a$ are the generators of the gauge group in the appropriate representation. In case all matter transforms under (anti)–fundamental representations of the gauge group, it is relatively simple to construct solutions to Eq. (1), but for theories with matter in tensor representations, the solutions may be rather complex. No standard methods to find the most general solution are available in the latter case. Ref. \cite{2} gives an overview of the efforts to parametrize flat directions in various models.

The D-flatness condition Eq. (1) is covariant under the gauge group $G$ and invariant under the global symmetry group $H_G$ of the gauge theory. The manifold of flat directions is therefore covered with $G \otimes H_G$ orbits. Points in the manifold that lie on the same $G \otimes H_G$ orbit are physically equivalent. The analysis of the flat directions is therefore simplified considerably when the redundancy due to gauge and global symmetry transformations is removed.

To this end, the $G$ orbits in the flat direction manifold can be labeled by a finite set of basic holomorphic gauge invariant polynomials $X_n(\phi^i)$ \cite{3,4,5}. Any holomorphic gauge invariant polynomial in the fields $\phi^i$ can be written in terms of products and sums of the basic invariants $X_n$, by virtue of the decomposition rules for the products of representations of the fields $\phi^i$. For some theories the invariants $X_n$ are algebraically independent; for others, relations exist among them.

The invariants $X_n$ form the coordinates of the moduli space. (the flat direction manifold modulo gauge transformations) The $H_G$ orbits in the moduli space can be labeled by the finite set \{I_x\} – the basic, $H_G$ invariant, Hermitian polynomials in terms of $X_n$ and $X_n^\dagger$. The moduli space is, in fact, a Kähler manifold. Its Kähler potential, induced by the Kähler potential of
the gauge theory, is defined by $K_M(I_x(X_n^+, X_n)) = \phi_i^\dagger \phi^i$ for every point in the flat direction manifold.

When the $X_n$ are promoted to chiral superfields, the moduli space becomes equivalent to a supersymmetric chiral sigma model \cite{4, 6}. This sigma model describes the low energy limit of the underlying gauge theory if the gauge symmetry is completely broken – the effective, classical theory describing the low energy limit of the gauge theory built on the classical vacuum with expectation values $< \phi^i >$ is equivalent to the sigma model when it is expanded around the expectation values $< X_n >= X_n(< \phi^i >)$. This effective theory, which describes the interactions of the massless degrees of freedom, can also be obtained directly by integrating out the massive vector multiplets in the gauge theory.

Non-perturbative effects can change the classical picture of the moduli space dramatically \cite{2, 7, 8}. In some cases, a dynamically generated superpotential lifts the moduli space; in other cases, classical constraints among the moduli fields are modified; and in still other cases the structure of the moduli space remains unchanged. Holomorphy and symmetries severely constrain the form of dynamically generated superpotentials. Unfortunately, modifications to the Kähler potential are less well understood.

In some situations, however, corrections to the classical Kähler potential are small. This is for example the case in models with calculable dynamical supersymmetry breaking, where the vacuum expectation values of the scalar fields are much larger than the dynamical scale of the gauge theory.

By construction, the Kähler potential of the classical moduli space is invariant under global symmetry group $H_G$ of the underlying gauge theory. By a detailed analysis of the moduli space of a chiral $SU(5)$ gauge theory with two antisymmetric tensors and two anti-fundamentals, we will show that the symmetry group $H_M$ of the moduli space can be larger than $H_G$. When a superpotential is added and non-perturbative effects are taken into account, this $SU(5)$ theory is one of the classic models with calculable dynamical supersymmetry breaking \cite{2, 3, 10, 11, 12, 13}.

The fact that $H_M$ is larger than $H_G$ has some interesting consequences. Classical vacua which are not related by gauge and global symmetry tran-

\footnotetext[1]{At the classical level, this means considering tree–diagrams with only massless degrees of freedom at the external lines, and contracting internal propagators of massive degrees of freedom in the limit $p^2/M^2 \to 0$.}
formations still give rise to the same effective theory in the low energy limit. Moreover, the unbroken symmetry group in the effective theory extends the unbroken symmetry group of the full gauge theory. We have verified that the extended symmetry is not a consequence of a custodial symmetry. Moreover, as we calculated the exact classical Kähler potential, it is not a consequence of a truncation either.

In Section 3, we present a detailed analysis of the moduli space of the $SU(5)$ theory. However, we first discuss a simple, well-known, vector-like theory with $SU(3)$ gauge symmetry \cite{4} in Section 2; this serves to illustrate our methods, and to emphasize the main point of this Letter by contrast – as the symmetry group of the moduli space of the $SU(3)$ model coincides with the global symmetry group of the underlying gauge theory.

## 2 Supersymmetric QCD with two flavors

We consider supersymmetric QCD with three colors and two flavors \cite{4}. The quark chiral superfields, which are denoted by $Q^i_a$, and $\bar{Q}^\alpha_i$, transform as $3$ and $\bar{3}$ under $SU(3)$. Here $i = 1, 2, 3$ is the color index, and $a = 1, 2$ and $\alpha = 1, 2$ are flavor indices. The global symmetry group $H_G$ of the theory – the relevant symmetry group at the classical level – is $SU(2)_Q \otimes SU(2)_\bar{Q} \otimes U(1)_Q \otimes U(1)_\bar{Q} \otimes U(1)_R$. Under $H_G$ the quark superfields transform as $Q^i_a \sim (2, 1, 1, 0, 0)$ and $\bar{Q}^\alpha_i \sim (1, 2, 0, 1, 0)$. The scalar components of the chiral superfields do not transform under $U(1)_R$, and therefore this factor can not be spontaneously broken by expectation values of the scalar fields.

The non-anomalous subgroup $H_{NA}$ of $H_G$ – the relevant symmetry group at the quantum level – is $SU(2)_Q \otimes SU(2)_\bar{Q} \otimes U(1)_B \otimes U(1)_{R'}$. Under $H_{NA}$ the quark superfields transform as $Q^i_a \sim (2, 1, 1, -1/2, 1/2)$ and $\bar{Q}^\alpha_i \sim (1, 2, -1, -1/2)$.

The flat directions of the theory are solutions to the equation

$$Q^i_a Q^j_a - Q^{ij}_a \bar{Q}^\alpha_i \bar{Q}^\alpha_j = c \delta^i_1. \quad (2)$$

Here $c$ is, a priori, an arbitrary real constant. However, it turns out there are only solutions for $c = 0$. Any solution to Eq. (2) can be obtained from the

\footnote{According to our conventions for R-symmetry, the charge of the scalar component of a chiral superfield is $R$, whereas the charge of the fermionic component is $R - 1$. The gaugino has charge 1.}
solution \( Q_1^1 = Q_1^1 = a, \ Q_2^2 = Q_2^2 = b, \) with all other components vanishing, by applying appropriate gauge and global symmetry transformations. For generic values of the real parameters \( a \) and \( b \), the gauge group is completely broken. Eight of the twelve chiral superfields are eaten to give mass to the vector multiplets. As a consequence, the number of moduli fields is four and the moduli space is eight–dimensional. The unbroken global symmetry group is \( U(1) \otimes U(1) \otimes U(1)_R \). The moduli space is therefore spanned by the two parameters \( a \) and \( b \), and six of the nine parameters of \( H_G \) transformations.

The basic holomorphic gauge invariants for this theory are \( M_\alpha^a = \bar{Q}_\alpha^i Q_i^a \), transforming as \((2,2,1,1,0)\) under \( H_G \). These four meson fields form the coordinates of the moduli space, and their vacuum expectation values can be written in the form \( M_1^1 = m_1, \ M_2^2 = m_2, \) and \( M_1^2 = M_2^1 = 0, \) by \( H_G \) transformations.

The basic Hermitian structures – invariant under the global symmetry transformations and constructed out of the meson fields – are \( I_1 = M_\alpha^a M_\alpha^a \) and \( I_2 = M_\alpha^a M_\beta^b M_\alpha^b M_\alpha^a, \) with the range of \( I_2 \) limited by the inequality \( 1/2I_1^2 \leq I_2 \leq I_1^2. \) The exact induced Kähler potential of the classical moduli space is defined as \( K_M(I_1(M^1, M), I_2(M^1, M)) \equiv Q_\alpha^i Q_\alpha^i + \bar{Q}_\alpha^i \bar{Q}_\alpha^i, \) and a simple calculation gives

\[
K_M = 2\sqrt{\frac{1}{2}I_1 + \frac{1}{2}\sqrt{2I_2 - I_1^2}} + 2\sqrt{\frac{1}{2}I_1 - \frac{1}{2}\sqrt{2I_2 - I_1^2}}. \tag{3}
\]

This Kähler potential is invariant under the global symmetry group \( H_G \) of the underlying gauge theory by construction. As is conventional, \( K_M \) is not invariant under any other symmetries, so that the symmetry group \( H_M \) of the moduli space is equal to \( H_G. \) As will become clear in the next section, however, even though \( H_M \) always contains \( H_G, \) it can in fact be larger.

The Kähler potential of the moduli space is derived in terms of the scalar components of the superfields. However, when the the moduli fields \( M_\alpha^a \) are promoted to superfields, a supersymmetric sigma model ensues. The low energy limit of the classical gauge theory constructed on the vacuum with expectation values \( < Q_\alpha^i > \) and \( < \bar{Q}_\alpha^i > \) is equivalent to the sigma model with vacuum expectation values \( < M_\alpha^a > \equiv < Q_\alpha^i > < Q_\alpha^i >. \)

The \( H_G \) orbits that cover the moduli space can be labeled by \( \{a, b\}, \) or \( \{m_1, m_2\}, \) or \( \{I_1, I_2\}. \) Points in the moduli space that lie on the same orbit yield physically equivalent classical vacua. The orbits, in turn, can be
grouped into strata. Different orbits that belong to the same stratum yield vacua that are physically inequivalent, but qualitatively similar. Such vacua yield the same symmetry breaking pattern and the same degeneracies in the mass spectrum, but the masses are quantitatively different. The strata can be categorized as follows:

i) For generic $H_G$ orbits, labeled by generic values of $\{I_1, I_2\}$, the little group is $U(1) \otimes U(1) \otimes U(1)_R$. The gauge symmetry is completely broken.

ii) For orbits with $I_2 = 1/2 I_1^2$, $(b = \pm a; \, m_1 = \pm m_2)$ the little group is $SU(2) \otimes U(1) \otimes U(1)_R$. The gauge symmetry is completely broken.

iii) For orbits with $I_2 = I_1^2$, $(b = 0; \, m_2 = 0)$ the little group is $U(1) \otimes U(1) \otimes U(1)_R$. In the sigma model, the metric derived from the Kähler potential $K_M$ is singular. Moreover, in the gauge theory the gauge group is broken to $SU(2)$ and therefore, the low energy theory should include the massless gauge multiplets.

iv) When $I_1 = 0$, none of the gauge and global symmetries are broken.

The classical picture of the moduli space is altered dramatically by non-perturbative effects. The non-anomalous global symmetry group $H_{NA}$ of the gauge theory allows a unique, non-perturbative superpotential $[4]$ of the form

$$W_{np} = \frac{\Lambda^7}{\bar{Q}^a_i Q^a_i \bar{Q}^b_j Q^b_j \epsilon_{\alpha \beta} \epsilon^{ab}}. \quad (4)$$

Explicit instanton calculations in the semi-classical approximation $[14, 15]$ show that such an effective superpotential is indeed generated and that $\Lambda$ is the dynamical scale of the gauge theory. The F-term contributions to the scalar potential completely lift the D–flat directions. The scalar potential does not have a minimum, tends to zero only at infinity, and renders the theory unstable. However, the scalar potential is stabilized if a mass term of the form

$$W_m = m^a_\alpha \bar{Q}^a_i Q^i_\alpha \quad (5)$$

is added to the superpotential. If the scale of the masses $m^a_\alpha$ is much smaller than the dynamical scale $\Lambda$, then the vacuum expectation values of the scalar
fields are much larger than $\Lambda$ and the theory is weakly coupled. It is in this limit that the classical Kähler potential is relevant. The theory below the dynamical scale can be described in terms of the moduli fields $M_\alpha^a$, with Kähler potential $K_M$ and superpotential

$$W = \frac{\Lambda^7}{M_\alpha^a M_\beta^b \epsilon_{\alpha\beta\epsilon}^{ab}} + m_\alpha^a M_\alpha^a.$$  \hspace{1cm} (6)

The vacuum energy vanishes, and supersymmetry is not broken in this theory.

3 \hspace{0.5cm} Chiral $SU(5)$ theory

The chiral supersymmetric $SU(5)$ gauge theory we discuss in this section contains two matter fields transforming under the 10 representation of $SU(5)$, and two fields transforming under the $\bar{5}$ representation. These matter fields are denoted by the two index anti-symmetric tensors $T^{ij}_a$, and $\bar{F}^\alpha_i$, where $i, j = 1, \ldots, 5$ are gauge indices, and $a = 1, 2$ and $\alpha = 1, 2$ are flavor indices. With this matter content, the theory is anomaly free and asymptotically free.

The global symmetry group $H_G$ of the theory is $SU(2)_T \otimes SU(2)_{\bar{F}} \otimes U(1)_T \otimes U(1)_{\bar{F}} \otimes U(1)_R$. Under $H_G$, the matter fields transform as $T_a \sim (1, 2, 1, 0, 0)$ and $\bar{F}^\alpha \sim (2, 1, 0, 1, 0)$. The scalar components of the chiral superfields do not transform under $U(1)_R$. Their vacuum expectation values therefore do not break this symmetry. Under the non-anomalous subgroup of $H_G$, $SU(2)_T \otimes SU(2)_{\bar{F}} \otimes U(1)_A \otimes U(1)_{R'}$, the matter fields transform as $T_a \sim (1, 2, 1, 1)$ and $\bar{F}^\alpha \sim (2, 1, -3, -4)$.

The D-flat directions of the theory are solutions to the equation

$$T^{a}_{ij} T^{\alpha}_{ik} - \bar{F}^{\alpha}_{\alpha} \bar{F}^\alpha_j = c \delta^k_j,$$  \hspace{1cm} (7)

where $c$ is an arbitrary real constant. In Refs. [2, 3, 4, 5], some incomplete families of solutions to Eq. (7) were presented. Here, we give the most general solution which of course includes the previously–found families. Any solution to Eq. (7) can be obtained from a four–parameter solution through gauge and global symmetry transformations. This four–parameter solution takes the form, $T^{12}_{2} = a$, $T^{34}_{3} = b$, $\bar{F}^{1}_{1} = c$, $\bar{F}^{2}_{2} = d$, and

$$T^{13}_{1} = \frac{c}{b} \sqrt{a^2 - c^2} \sqrt{\frac{b^2}{a^2 - c^2} + \frac{d^2}{a^2},}$$
\[ T_1^{45} = \frac{a}{b} \sqrt{a^2 - c^2} \sqrt{\frac{b^2}{a^2 - c^2} + \frac{d^2}{a^2}}, \]
\[ T_2^{23} = \frac{c}{\sqrt{a^2 - c^2}} \sqrt{b^2 - (a^2 - c^2)}, \]
\[ T_2^{25} = -\frac{cd}{a}, \]
\[ T_2^{45} = \frac{d}{ba} \sqrt{a^2 - c^2} \sqrt{b^2 - (a^2 - c^2)}, \]
\[ \bar{F}_3^1 = -\frac{a}{\sqrt{a^2 - c^2}} \sqrt{b^2 - (a^2 - c^2)}, \]
\[ \bar{F}_2^2 = \frac{c}{b} \sqrt{b^2 - (a^2 - c^2)} \sqrt{\frac{b^2}{a^2 - c^2} + \frac{d^2}{a^2}}, \]
\[ \bar{F}_4^2 = -\sqrt{a^2 - c^2} \sqrt{\frac{b^2}{a^2 - c^2} + \frac{d^2}{a^2}}. \] (8)

All other components vanish, and \( a, b, c \) and \( d \) are real parameters. For generic values of \( \{a, b, c, d\} \), the gauge symmetry is completely broken. Therefore, twenty-four of the thirty chiral superfields are eaten to give masses to the vector multiplets, leaving six moduli fields to function as coordinates for the twelve–dimensional moduli space. The global symmetry group \( H_G \) is broken to \( U(1) \). In terms of the fundamental fields, the moduli space is spanned by the four parameters \( \{a, b, c, d\} \) of the solution given in Eq. (8), and eight of the nine parameters of \( H_G \) transformations. The basic holomorphic gauge invariants for this theory are given by

\[ X_a = \epsilon_{\alpha\beta} F_1^\alpha F_2^\beta T_a^{ij}, \]
\[ J_a^\alpha = \epsilon_{ijklm} F_1^\alpha T_a^{ijklmn} T_k T_l T_m T_n \epsilon^{bc}. \] (9)

Under \( H_G \), these holomorphic gauge invariants transform as \( X_a \sim (1, 2, 1, 2, 0) \) and \( J_a^\alpha \sim (2, 2, 3, 1, 0) \). By suitable \( H_G \) transformations, the vacuum expectation values of the basic holomorphic gauge invariants can be written as \( X_1 = x_1, \ X_2 = x_2, \ J_1^1 = j_1, \ J_2^2 = j_2 \) and \( J_1^2 = J_2^1 = 0 \), with \( x_1, \ x_2, \ j_1 \) and \( j_2 \) real parameters. In fact, the expectation values of the holomorphic gauge invariants for the four–parameter solution, given in Eq. (8), already have this form:

\[ X_1 = 2 \frac{ad}{b} (a^2 - c^2) \left( \frac{b^2}{a^2 - c^2} + \frac{d^2}{a^2} \right), \]
The holomorphic invariants $X_a$ and $J_a^\alpha$ provide the coordinates for the moduli space. A completely $H_G$ invariant description of the moduli space can be given in terms of the four Hermitian invariants

\[ I_1 = X_a^\dagger X_a, \]
\[ I_2 = J_a^\alpha J_a^\dagger, \]
\[ I_3 = X_a^\dagger J_\beta^b X_b J_\beta^a, \]
\[ I_4 = J_a^\alpha J_\beta^b J_\alpha^\dagger J_\beta^a, \]

where the range of $I_4$ is limited to $1/2I_2^2 \leq I_4 \leq I_2^2$, and the range of $I_3$ is limited by $(2I_3 - I_1 I_2)^2 \leq (2I_4 - I_2^2) I_1^2$. The moduli space is thus covered by $H_G$ orbits, labeled by \{a, b, c, d\}, or \{x_t, x_2, j_1, j_2\}, or \{I_1, I_2, I_3, I_4\}. In our previous work [13], the exact Kähler potential of the classical moduli space was derived. Invariance under $H_G$ dictates that the Kähler potential has the functional form

\[ K_M(X^\dagger, X, J^\dagger, J) = K_M(I_1, I_2, I_3, I_4). \] (12)

Defining

\[ A = 125 I_1, \]
\[ B = \frac{25}{9} \left( \left\lfloor \frac{1}{2} I_2 + \frac{1}{2} \sqrt{2I_4 - I_2^2} \right\rfloor + \left\lfloor \frac{1}{2} I_2 - \frac{1}{2} \sqrt{2I_4 - I_2^2} \right\rfloor \right), \] (13)

\[ ^3 \text{The Kähler potential of the moduli space } K_M(I_1, I_2, I_3, I_4) = 1/2T_{ij}^a T_{ij}^a + \tilde{F}_{ij}^a \tilde{F}_{ij}^a \text{ for all values of the parameters } \{a, b, c, d\} \text{ of the four–parameter solution to the D-flatness equation.} \]
and

\[ p = 2\sqrt{B} \cos\left(\frac{1}{3} \arccos \frac{A}{B^2}\right), \]  

(14)

the Kähler potential of the moduli space is given by

\[ K_M = \frac{3}{10} \left( p + \frac{B}{p} \right). \]  

(15)

The metric derived from this Kähler potential is singular if \( I_4 = I_2^2 \). Curiously, \( K_M \) does not depend on \( I_3 \). As a consequence – and this illustrates the central point of this Letter – the symmetry group \( H_M \) of the moduli space, \( SU(2)_X \otimes SU(2)_1 \otimes SU(2)_2 \otimes U(1)_X \otimes U(1)_J \otimes U(1)_R \), is larger than the global symmetry group \( H_G \) of the underlying gauge theory. The moduli fields transform under \( H_M \) as \( X_a \sim (2, 1, 1, 1, 0, 0) \) and \( J^a_1 \sim (1, 2, 2, 0, 1, 0) \). The \( U(1)_R \) factor in \( H_M \) is the same factor that appears in \( H_G \). Only fermions transform under this symmetry, and it does not play any role in the discussion below. We will therefore suppress this factor from here on.

Generic \( H_M \) orbits in the moduli space, labeled by \( \{I_1, I_2, I_4\} \), contain one-parameter families of \( H_G \) orbits, labeled by \( I_3 \). In particular, the points

\[ \begin{align*}
X_1 &= x \cos \phi, \\
X_2 &= x \sin \phi, \\
J^1_1 &= j_1, \\
J^2_1 &= 0, \\
J^1_2 &= 0, \\
J^2_2 &= j_2,
\end{align*} \]

(16)

for fixed values of \( \{x, j_1, j_2\} \), and varying \( \phi \), are equivalent in the moduli space, as \( \phi \) corresponds to the parameter of an \( SU(2)_X \) rotation. \( H_M \) orbits can therefore also be labeled by \( \{x, j_1, j_2\} \), and the \( H_G \) orbits contained in an \( H_M \) orbit can be labeled by \( \phi \).

When the moduli fields are promoted to superfields, a supersymmetric sigma model results. The low energy limit of the gauge theory built on the classical vacuum with expectation values \( < T^i_j > \) and \( < \tilde{F}^\alpha_i > \) is equivalent to the sigma model with vacuum expectation values \( < X_a > = \epsilon_{\alpha\beta} < F^\alpha_i > < F^\beta_j > < T^i_j > \) and \( < J^\alpha_a > = \epsilon_{ijklm} < F^\alpha_n > < T^i_j > < T^{kl} > < T^{mn} > \epsilon^{bc} \).
The extended symmetry of the moduli space has two important consequences. First, vacua of the gauge theory corresponding to fixed values of \( \{x, j_1, j_2\} \), but varying \( \phi \), are physically inequivalent. In particular, the mass of the vector multiplets are a function of \( \phi \). In fact, while for generic values of \( \phi \) gauge and global symmetries are completely broken, for the special values of \( \phi = 0 \) and \( \phi = \pi/2 \) there is a remaining global \( U(1) \) symmetry. However, all vacua of the sigma model with fixed values of \( \{x, j_1, j_2\} \) and arbitrary value of \( \phi \), either generic or special, are equivalent. Therefore, the low energy limit of the gauge theory, which is obtained by integrating out the massive vector multiplets in the limit \( p^2/M^2 \to 0 \), is identical for each value of \( \phi \). Physically inequivalent vacua of the gauge theory, with distinct mass spectra and possibly even distinct global symmetry breaking patterns, yield the same low energy theory.

Second, for generic vacua, the global symmetry group of the gauge theory is broken to \( U(1)_R \). However, the symmetry group \( H_M \) of the moduli space is broken to \( U(1) \otimes U(1) \otimes U(1)_R \). Therefore, the low energy limit of the gauge theory has a larger symmetry group than expected from the global symmetry breaking pattern of the full gauge theory.

The three–parameter solution to the D-flatness condition Eq. (7), obtained by imposing the condition \( b^2 = a^2 - c^2 \) on the four–parameter solution given in Eq. (8), corresponds to arbitrary \( \{I_1, I_2, I_4\} \) and \( I_3 = 0 \), or, alternatively, arbitrary \( \{x, j_1, j_2\} \) and \( \phi = 0 \). This three–parameter solution, therefore, contains a representative point on all \( H_M \) orbits in the moduli space. However, it does not contain a representative point on all \( H_G \) orbits. Therefore, the corresponding classical vacua yield all physically inequivalent low energy theories, yet not all physically inequivalent classical gauge theories.

We will describe the moduli space in terms of strata of \( H_M \) and \( H_G \) orbits in turn. The first approach lends itself for the study of all inequivalent low energy theories, while the latter is more suitable for the study of all inequivalent classical gauge theories.

As explained before, \( H_M \) orbits are labeled by either \( \{I_1, I_2, I_4\} \) or \( \{x, j_1, j_2\} \).

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Note: Some unusual features emerge in the mass spectrum of the vector multiplets. For generic values of \( \{x, j_1, j_2, \phi\} \) both the gauge and global symmetries of the gauge theory are completely broken. However, the spectrum contains four degenerate pairs of masses, and one degenerate quintuplet. Moreover, even though the spectrum changes with \( \phi \) for fixed values of \( \{x, j_1, j_2\} \), the sum of the squares of the masses and the mass of the degenerate quintuplet remain independent of \( \phi \).
For generic orbits, labeled by generic values of \( \{x, j_1, j_2\} \), \( H_M \) is broken to \( U(1) \otimes U(1) \). One of the \( U(1) \) factors is a subgroup of \( SU(2)_X \otimes U(1)_X \); the other, a subgroup of \( SU(2)_1 \otimes SU(2)_2 \otimes U(1)_J \). The number of broken symmetry generators, nine, is larger than the number of moduli fields, six, and therefore some of the corresponding Goldstone bosons are non-doubled.

Apart from the generic stratum, there are strata for which the little group is larger. The strata can be classified as follows:

i) \( I_1 = 0, I_2 = 0; (x = 0, j_1 = 0, j_2 = 0) \) The metric is singular and there is no spontaneous symmetry breaking. Therefore, the little group is \( SU(2)_X \otimes SU(2)_1 \otimes SU(2)_2 \otimes U(1)_X \otimes U(1)_J \). The multiplets transform as \( (2,0,0,1,0) \) and \( (1,2,2,0,1) \).

ii) \( I_2 = 0; (j_1 = 0, j_2 = 0) \) The metric is singular, and the little group is \( U(1) \otimes SU(2)_1 \otimes SU(2)_2 \otimes U(1)_J \). The multiplets transform as \( (0,2,2,1) \), \( (0,1,1,0) \) and \( (1,1,1,0) \).

iii) \( I_1 = 0, I_4 = I_2^2; (x = 0, j_2 = 0) \) The metric is singular, and the little group is \( SU(2)_X \otimes U(1) \otimes U(1) \otimes U(1)_X \). The multiplets transform as \( (2,0,0,1) \), \( (1,0,-2,0) \), \( (1,-1,1,0) \), \( (1,-1,-1,0) \) and \( (1,0,0,0) \).

iv) \( I_1 = 0, I_4 = \frac{1}{2} I_2^2; (x = 0, j_1 = \pm j_2) \) The little group is \( SU(2)_X \otimes SU(2) \otimes U(1)_X \). The multiplets transform as \( (2,1,1) \), \( (1,3,0) \) and \( (1,1,0) \).

v) \( I_1 = 0; (x = 0) \) The little group is \( SU(2)_X \otimes U(1) \otimes U(1)_X \). Two multiplets transform as \( (1,0,0) \), while the remaining multiplets transform as \( (2,0,1) \), \( (1,-1,0) \) and \( (1,1,0) \).

vi) \( I_4 = I_2^2; (j_2 = 0) \) The metric is singular, and the little group is \( U(1) \otimes U(1) \otimes U(1) \). Two multiplets transform as \( (0,0,0) \), while the remaining multiplets transform as \( (0,0,-2) \), \( (1,0,0) \), \( (0,-1,1) \) and \( (0,-1,-1) \).

vii) \( I_4 = \frac{1}{2} I_2^2; (j_1 = \pm j_2) \) The little group is \( U(1) \otimes SU(2) \). Two multiplets transform as \( (0,1) \), while the remaining multiplets transform as \( (0,3) \) and \( (1,1) \).

viii) Generic \( I_1, I_2, I_4; (\text{generic} \ x_1, j_1, j_2) \) The little group is \( U(1) \otimes U(1) \). Three multiplets transform as \( (0,0) \), while the remaining multiplets transform as \( (1,0), (0,-1) \) and \( (0,1) \).
$H_G$ orbits can be labeled by \{${I_1, I_2, I_3, I_4}$\}, or \{${x_1, x_2, j_1, j_2}$\}, or \{${a, b, c, d}$\}. For each stratum, we indicate the subgroup of $H_G$ which remains unbroken, and also the remaining subgroup of the gauge group in case the gauge symmetry is not completely broken.

i) \(I_1 = 0, I_2 = 0; (x_1 = 0, x_2 = 0, j_1 = 0, j_2 = 0)\) The gauge and global symmetries remain unbroken.

ii) \(I_2 = 0; (x_2 = 0, j_1 = 0, j_2 = 0)\) The unbroken global symmetry group is $U(1) \otimes SU(2)_F \otimes U(1)$. The gauge symmetry is broken to $SU(3)$. The solution $T^{12}_1 = a, F^1_1 = a, F^2_2 = a$, with $x_1 = 2a^3$, contains representative points of the orbits in this stratum.

iii) \(I_1 = 0, I_4 = I^2_2; (x_1 = 0, x_2 = 0, j_2 = 0)\) The remaining global symmetry group is $U(1) \otimes U(1) \otimes U(1)$. The gauge symmetry is broken to $SU(2)$. The solution $T^{12}_1 = a, T^{45}_2 = F_1^1 = F_4^2 = a$, with $j_1 = 12a^4$ contains representative points of orbits in this stratum.

iv) \(I_1 = 0, I_4 = 1/2I^2_2; (x_1 = 0, x_2 = 0, j_1 = \pm j_2)\) The unbroken global symmetry group is $SU(2) \otimes U(1)$, and the gauge symmetry is completely broken. The solution $T^{12}_1 = T^{34}_1 = T^{15}_2 = T^{24}_2 = F_1^1 = F_4^2 = a$, with $j_1 = -j_2 = 12a^4$, contains representative points of orbits in this stratum.

v) \(I_1 = 0; (x_1 = 0, x_2 = 0)\) The remaining global symmetry group is $U(1) \otimes U(1)$, and the gauge symmetry is completely broken. The solution $T^{12}_1 = a, T^{34}_1 = T^{15}_2 = \sqrt{a^2 + b^2}, T^{24}_2 = b, F_1^1 = a$ and $F_2^2 = b$, with $j_1 = 12a^2(a^2 + b^2)$ and $j_2 = -12b^2(a^2 + b^2)$, contains representative points of orbits in this stratum. The solution presented in Ref. [9] also contains representative points of orbits in this stratum.

vi) \(I_3 = I_1I_2, I_4 = I^2_2; (x_2 = 0, j_2 = 0)\) The unbroken global symmetry group is $U(1) \otimes U(1)$, and the gauge symmetry broken to $SU(2)$. The solution $T^{12}_1 = a, T^{45}_1 = T^{33}_2 = b, F_1^1 = a$ and $F_2^2 = \sqrt{a^2 - b^2}$, with $x = 2a^2\sqrt{a^2 - b^2}$, $j_1 = 12a^2b^2$ and $j_2 = 0$, contains representative points of orbits in this stratum.

vii) \((2I_3 - I_1I_2)^2 = I^2_1(2I_4 - I^2_2); (x_2 = 0)\) The remaining global symmetry group is $U(1)$, and the gauge symmetry is completely broken. The flat
directions presented in Refs. [3, 11] contain representative points on the orbits in this stratum. As shown in Ref. [13], the classical vacuum of the $SU(5)$ model with calculable supersymmetry breaking lies on an orbit in this stratum with the property $j_1 = \pm j_2$. In terms of $H_G$ orbits, this additional condition does not lead to a larger little group.

\[ I_4^2 = I_2^2; \quad (j_2 = 0) \] The remaining global symmetry group is $U(1)$, and the gauge symmetry is broken to $SU(2)$.

\[ \text{ix}) \quad \text{Generic } I_1, I_2, I_3, I_4; \quad \text{(generic } x_1, x_2, j_1, j_2) \] Both global and gauge symmetries are completely broken.

Even though every $H_G$ orbit is contained in an $H_M$ orbit, not every stratum of $H_G$ orbits is completely contained in a stratum of $H_M$ orbits.

As in the $SU(3)$ model discussed in Section 1, non-perturbative effects completely change the classical picture of the moduli space. A non-perturbative effective superpotential

\[ W_{np} = \frac{\Lambda^{11}}{J_a^\alpha J_b^\beta \epsilon_{\alpha\beta\epsilon_{ab}}}, \quad (17) \]

generated by instantons, lifts the vacuum degeneracy completely. However, instead of a mass term, which is not consistent with the chiral nature of the $SU(5)$ theory, a renormalizable Yukawa–type interaction in the superpotential can be introduced to stabilize the scalar potential. As described in Refs. [9, 10, 12, 13], if the coupling constant of this Yukawa term is sufficiently small, the theory below the dynamical scale of the gauge interactions is a supersymmetric sigma model, which has $X_a$ and $J_a^\alpha$ as coordinates, $K_M$ as Kähler potential, and

\[ W = \frac{\Lambda^{11}}{J_a^\alpha J_b^\beta \epsilon_{\alpha\beta\epsilon_{ab}}} + \lambda X_1 \quad (18) \]

as the superpotential. In contrast to the $SU(3)$ model, the vacuum energy does not vanish, and therefore supersymmetry is broken. The light mass spectrum, as calculated in Refs. [12, 13] displays some degeneracies which can not be explained by the symmetry breaking pattern of the global symmetry group of the gauge theory including the superpotential. However, as a consequence of the $H_M$ invariance of the Kähler potential, the symmetry
group of the sigma model extends the global symmetry group of the full gauge theory. In particular, the sigma model is invariant under $SU(2)_1 \otimes SU(2)_2$ transformations. The degeneracies in the light spectrum square with the breaking pattern of the extended symmetry group of the sigma model.

4 Conclusions

We have presented a detailed study of the classical moduli space of the $SU(5)$ gauge theory with two anti–symmetric tensors and two anti-fundamentals. We found that the symmetry group $H_M$ of the classical moduli space extends the global symmetry group $H_G$ of the gauge theory. We analyzed the moduli space in terms of orbits of both symmetry groups.

The extended symmetry of the moduli space has two main consequences. Physically inequivalent classical vacua of the gauge theory may have identical low energy limits, and the effective models that describe the massless degrees of freedom in the low energy limit have a symmetry group that is larger than the unbroken subgroup of $H_G$. Even though non–perturbative effects completely lift the classical moduli space, a remnant of the extended symmetry group of the Kähler potential is the origin of degeneracies in the mass spectrum of the calculable $SU(5)$ model with dynamical supersymmetry breaking.

The extended symmetry of the classical moduli space is traced to the fact that the Kähler potential does not depend on an Hermitian invariant consistent with the global symmetry group of the gauge theory. We calculated the mass spectrum of the gauge theory for vacua that are related by $H_M$ transformations but not by $H_G$ transformations, and we found that the mass spectrum of the massive vector multiplets differs. This assured us that the additional symmetry of the moduli space is not realized as a symmetry of the full gauge theory. In fact, the same evidence also eliminates the possibility that just the scalar potential is invariant under the extended symmetry.

As an aside, the degeneracies in the spectrum of the massive vector multiplets pose an intriguing question. In a generic point of the moduli space, all global and gauge symmetries are broken, and therefore no degeneracies are expected. However, the existence of a degenerate quintuplet hints at some kind of symmetry.

Returning to the question of the extended symmetry of the classical mod-
uli space, we cannot completely rule out the possibility that the full gauge theory, or the just the scalar potential, is invariant under some symmetry other than any of the extended symmetry transformations of the classical moduli space, maybe even a discrete symmetry, that we are unaware of. If such a symmetry exists and if it forbids the absent terms in the Kähler potential, then the extended symmetry of the classical moduli space that we have found would be coincidental.

If the latter scenario is not realized, it is possible to take the point of view that the classical moduli spaces of supersymmetric chiral gauge theories with matter in tensor representations have complicated structure, and that calculating their Kähler potential provides an apt tool to understand this structure. However, we find such a perspective somewhat unsatisfying and still feel that it is worthwhile to seek a fundamental principle that allows the determination of the symmetries of the classical moduli space without an explicit calculation of the Kähler potential.

Finally, we want to address the question whether the classical moduli spaces of other supersymmetric gauge theories have extended symmetries. Non-trivial flavor structure and matter transforming under non-fundamental representations of the gauge group seem to be prerequisites. However, with such matter content, the parametrization of generic flat directions often is prohibitively complicated, and an explicit calculation of the Kähler potential of the classical moduli space is impossible. This is particularly the case when the matter content is chosen so that the gauge symmetry is non-anomalous, although this does not seem to be required in a study of classical moduli spaces.

Looking at closely related models, the SU(5) model with one generation – one anti-symmetric tensor and one anti-fundamental – has no flat directions. The model with three generations has twenty-one moduli fields and its inequivalent classical vacua are labeled by twenty-four parameters. Parametrizing generic flat directions for this model is a forbidding task. Nevertheless, the structure of the classical moduli space is of interest: When non-perturbative effects are taken into account, the model is in an s-confining phase \([R]\), and the structure of its classical moduli space is conjectured to be unmodified.
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