GUT Model Hierarchies from Intersecting Branes

Christos Kokorelis 1

Departamento de Física Teórica C-XI and Instituto de Física Teórica C-XVI, Universidad Autónoma de Madrid, Cantoblanco, 28049, Madrid, Spain

ABSTRACT

By employing D6-branes intersecting at angles in $D = 4$ type I strings, we construct the first examples of three generation string GUT models (PS-A class), that contain at low energy exactly the standard model spectrum with no extra matter and/or extra gauge group factors. They are based on the group $SU(4)_C \times SU(2)_L \times SU(2)_R$. The models are non-supersymmetric, even though SUSY is unbroken in the bulk. Baryon number is gauged and its anomalies are cancelled through a generalized Green-Schwarz mechanism. We also discuss models (PS-B class) which at low energy have the standard model augmented by an anomaly free $U(1)$ symmetry and show that multibrane wrappings correspond to a trivial redefinition of the surviving global $U(1)$ at low energies. There are no colour triplet couplings to mediate proton decay and proton is stable. The models are compatible with a low string scale of energy less than 650 GeV and are directly testable at present or future accelerators as they predict the existence of light left handed weak fermion doublets at energies between 90 and 246 GeV. The neutrinos get a mass through an unconventional see-saw mechanism. The mass relation $m_e = m_d$ at the GUT scale is recovered. Imposing supersymmetry at particular intersections generates non-zero Majorana masses for right handed neutrinos as well providing the necessary singlets needed to break the surviving anomaly free $U(1)$, thus suggesting a gauge symmetry breaking method that can be applied in general left-right symmetric models.

1Christos.Kokorelis@uam.es
1 Introduction

While string theory remains the only candidate for a consistent theory of fundamental interactions it still has to solve some major problems like explaining the hierarchy of scale and particle masses after supersymmetry breaking. These phenomenological issues have by far been explored in the context of construction of semirealistic supersymmetric models of weakly coupled heterotic string theories [1]. Leaving aside the weakly coupled heterotic string, $N = 1$ four-dimensional orientifold models [2] represent a particular class of consistent string solutions which explore the physics of strongly coupled heterotic strings. Semirealistic model building has been explored in the context of $N = 1$ supersymmetric (SUSY) four-dimensional orientifolds [3]. The main futures of the models constructed include an extended gauge group which includes the standard model or extensions of it, with a variety of exotic matter.

Recently some new constructions have appeared in a type I string vacuum background which use intersecting branes [4] and give four dimensional non-supersymmetric models. These are the kind of models that we will be examining in this work. The question that someone might address at this point is why we have to use non-SUSY models while in heterotic string compactifications we examined SUSY one’s? The reason for doing so is mainly phenomenological. In $N = 1$ (orbifold) compactifications of the heterotic string the string scale was of the order of $10^{18}$ GeV something that was in clear disagreement with the observed unification of gauge coupling constants in the MSSM of $10^{16}$ GeV. In these models the observed discrepancy between the two high scales was attributed to the presence of the $N = 1$ string threshold corrections to the gauge coupling constants [5]. On the contrary in type I models the string scale is a free parameter. Moreover, recent results suggest the string scale in type I models can be in the TeV range [6]. The latter result suggests that non-SUSY models with a string scale in the TeV region is a viable possibility.

Because in the open string models of [4] background fluxes were used, following past ideas about the use of magnetic fields in open strings [7], in a D9 brane type I background with background fluxes \(^1\) it was possible to break supersymmetry on the brane and to get chiral fermions with an even number of generations [4]. The fermions on those models appear in the intersections between branes [8], [9].

After introducing a quantized background NS-NS B field [10, 11, 12], that makes the tori tilted, it was then possible to get semirealistic models with three generations [13].

\(^1\)In the T-dual language these backgrounds are represented by D6 branes wrapping 3-cycles on a dual torus and intersecting each other at certain angles.
We also note that these backgrounds are T-dual to models with magnetic deformations [14].

Additional non-SUSY constructions in the context of intersecting branes, from IIB orientifolds, consisting of getting at low energy the standard model spectrum with extra matter and additional chiral fermions were derived in [15]. The construction involves D(3+n) branes wrapping on the compact space $T^{2n} \times (T^{2(3-n)}/Z_N)$, for $n = 1, 2, 3$ and intersecting at angles in the $T^{2n}$.

Furthermore, an important step was taken in [16], by showing how to construct the standard model (SM) spectrum together with right handed neutrinos in a systematic way. The authors considered, as a starting point, IIA theory compactified on $T^6$ assigned with an orientifold product $\Omega \times R$, where $\Omega$ is the worldsheet parity operator and $R$ is the reflection operator with respect to one of the axis of each tori. In this case, the four stack D6-branes contain Minkowski space and each of the three remaining dimensions is wrapped up on a different $T^2$ torus. In this construction the proton is stable since the baryon number is a gauged $U(1)$ global symmetry. A special feature of these models is that the neutrinos can only get Dirac mass. These models have been generalized to models with five stack of D6-branes at [17]. For a discussion of non-SUSY SM in the context of D3-branes on orbifold singularities see [18]. A different attempt to construct non-SUSY GUT models in the context of intersecting branes was made in [19]. However, there were some problems with the phenomenology of the $SU(5)$ GUT model presented, as some of the Yukawa couplings were excluded and the standard electroweak Higgs scalar was not realized, while proton decay problems appeared. Also SUSY constructions in the context of intersecting branes were considered in [20]. Nevertheless, despite the fact that much progress has been made, constructing string models with interesting phenomenology is still a difficult task.

The purpose of this paper is to present the first three generation string models that are based on a grand unified gauge group, and contain at low energy exactly the standard model spectrum, namely $SU(3)_C \times SU(2)_L \times U(1)_Y$, without any extra chiral fermions and/or extra gauge group factors. The four-dimensional models are non-supersymmetric intersecting brane constructions and are based on the Pati-Salam (PS) $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge group [21]. The basic structure behind the models includes D6-branes intersecting each other at non-trivial angles, in an orientifolded factorized six-torus, where $O_6$ orientifold planes are on top of D6-branes.

The proposed models have some distinctive features:

- The models (characterized as belonging to the PS-A class) start with a gauge
group at the string scale $U(4) \times U(2) \times U(2) \times U(1)$. At the scale of symmetry breaking of the left-right symmetry, $M_{GUT}$, the initial symmetry group breaks to the the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$ augmented with an extra anomaly free $U(1)$ symmetry. The additional $U(1)$ symmetry breaks by the vev of charged singlet scalars, e.g. $s^H_L$, to the SM itself at a scale set by its vev. The singlets responsible for breaking the $U(1)$ symmetry are obtained by demanding certain open string sectors of the non-SUSY model to respect $N = 1$ supersymmetry.

- Neutrinos gets a mass of the right order, consistent with the LSND oscillation experiments [22], from a see-saw mechanism of the Frogatt-Nielsen type [23]. The structure of Yukawa couplings involved in the see-saw mechanism [24] supports the smalleness of neutrino masses thus generating a hierarchy in consistency with neutrino oscillation experiments.

- Proton is stable due to the fact that baryon number is an unbroken gauged global symmetry surviving at low energies and no colour triplet couplings that could mediate proton decay exist. Thus a gauged baryon number provides a natural explanation for proton stability. As in the models of [16] the baryon number associated $U(1)$ gauge boson becomes massive through its couplings to Green-Schwarz mechanism. That has an an immediate effect that baryon number is surviving as a global symmetry to low energies providing for a natural explanation for proton stability in general brane-world scenarios.

- The model uses small Higgs representations in the adjoint to break the PS symmetry, instead of using large Higgs representations, e.g. 126 like in the standard $SO(10)$ models.

- The bidoublet Higgs fields $h$ responsible for electroweak symmetry breaking do not get charged under the global $U(1)$ and thus lepton number is not broken at the standard model.

We should note that in the past three generation four dimensional string vacua that include the $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge group together with extra matter and additional non-abelian gauge group structure have been discussed both in the context of supersymmetric vacua coming from orientifolds of type IIB [25] and from non-supersymmetric brane-antibrane pair configurations [26]. For some other proposals for realistic D-brane model building, based not on a particular string construction, see
[27] for the standard model, [28] for the PS model or for the standard model in a non-compact set-up [29].

The paper is organized as follows. In chapter two we describe the general characteristics of the models, with particular emphasis on how to calculate the fermionic spectrum from intersecting branes, as well providing the multi-parameter solutions to the RR tadpole cancellation conditions. We discuss two kinds of models, characterized in this work as belonging to the PS-A, PS-B classes of models. In addition we discuss how the PS-A classes of models accommodate singlet fields. The latter fields are necessary in order to break the surviving $U(1)$ symmetry and getting just the SM at low energy. In chapter 3 we examine the cancellation of $U(1)$ anomalies via a generalized Green-Schwarz (GS) mechanism finding the general solution for the non-anomalous $U(1)$ which remains light. We also discuss arguments related to multi-wrapping branes and show that they correspond to a trivial redefinition of the global non-anomalous $U(1)$ surviving the GS mechanism. In chapter 4 we discuss the Higgs sector of the model involving the appearance of Higgs scalar responsible for breaking the PS $SU(4) \otimes SU(2)_R$ symmetry at the intermediate grand unified scale $M_{GUT}$ and the electroweak breaking Higgs scalars. We also discuss how the imposition of supersymmetry in particular sectors of the classes of models succeeds to break the PS-A class to the SM itself at low energies, even though there is not a similar effect for the PS-B class of models. In chapter 5 we examine the problem of neutrino masses. We also show that for the PS-A class of models all additional fermions beyond those of SM become massive and disappear from the low energy spectrum. In this section, we describe in detail how the presence of supersymmetry in particular sectors of the theory realizes the particular couplings taking part in the see-saw mechanism. We also discuss bounds for the string scale and right handed neutrino masses that follow from the Yukawa couplings of the models. Chapter 6 contains our conclusions. Finally, Appendices I, II include the conditions for the absence of tachyonic modes in the spectrum of the PS-A, PS-B class of models presented, while in Appendix III we provide an equivalent structure to PS-B class of models presented in the main body of this article together with its tadpole solutions.
2 The models and the rules of computing the spectrum

In the present work, we are going to look for a three family non-supersymmetric model that is based on the left-right symmetric $SU(4)_C \times SU(2)_L \times SU(2)_R$ Pati-Salam model with the right phenomenological properties and discuss in more detail its phenomenology. It will come from D6-branes wrapping on 3-cycles of toroidal orientifolds of type I in four dimensions. We will present a simultaneous discussion of the two classes of PS models, PS-A and PS-B, so unless otherwise stated the discussions will hold for both classes of models. Let at this point describe the general futures of the non-supersymmetric $SU(4)_C \times SU(2)_L \times SU(2)_R$ model. Important characteristic of all vacua coming from these type I constructions is the replication of massless fermion spectrum by an equal number of massive particles in the same representations and with the same quantum numbers.

The quark and lepton fields appear in three complete generations and are accommodated into the following representations:

\[
F_L = (4, \bar{2}, 1) = q(3, \bar{2}, \frac{1}{6}) + l(1, \bar{2}, -\frac{1}{2}) \equiv (u, d, l),
\]

\[
F_R = (\bar{4}, 1, 2) = \bar{u}^c(\bar{3}, 1, -\frac{2}{3}) + \bar{d}^c(\bar{3}, 1, \frac{1}{3}) + \bar{e}^c(1, 1, 1) + N^c(1, 1, 0) \equiv (\bar{u}^c, \bar{d}^c, \bar{e}^c),
\]

where the quantum numbers on the right hand side of (2.1) are with respect to the decomposition of the $SU(4)_C \times SU(2)_L \times SU(2)_R$ under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group and $l = (\nu, e)$ is the standard left handed lepton doublet, $l^c = (N^c, e^c)$ are the right handed leptons. Note that the assignment of the accommodation of the quarks and leptons into the representations $F_L + F_R$ is the one appearing in the spinorial decomposition of the 16 representation of $SO(10)$ under the PS gauge group.

A set of useful fermions appear also in the model

\[
\chi_L = (1, \bar{2}, 1), \quad \chi_R = (1, 1, \bar{2}).
\]

These fermions are a general prediction of left-right symmetric theories. As we comment later the existence of these representations in the model follows from RR tadpole cancellation conditions.

The symmetry breaking of the left-right PS symmetry at the $M_{GUT}$ breaking scale \(^2\)

\(^2\)In principle this scale can be lower than the string scale.
proceeds through the representations of the set of Higgs fields,

\[ H_1 = (\bar{4}, 1, 2), \quad H_2 = (4, 1, 2), \]  

(2.3)

where, e.g.

\[ H_1 = (\bar{4}, 1, 2) = u_H(3, 1, \frac{2}{3}) + d_H(3, 1, -\frac{1}{3}) + e_H(1, 1, -1) + \nu_H(1, 1, 0). \]  

(2.4)

The electroweak symmetry is delivered through bi-doublet Higgs fields \( h_i \), \( i = 1, 2 \), field in the representations

\[ h_1 = (1, 2, 2), \quad h_2 = (1, \bar{2}, 2). \]  

(2.5)

Also present are the massive scalar superpartners \(^3\) of the quarks, leptons and antiparticles

\[ \tilde{F}_H^H = (\bar{4}, 1, 2) = u_H^c(3, 1, -\frac{4}{6}) + d_H^c(3, 1, \frac{1}{3}) + e_H^c(1, 1, 1) + N_H^c(1, 1, 0) \equiv (u_H^c, d_H^c, l_H^c). \]  

(2.6)

Also, only for the PS-A class models, a number of charged exotic fermion fields appear

\[ 12(6, 1, 1), \quad 6(\bar{6}, 1, 1), \quad 6(\bar{10}, 1, 1), \quad 24(1, 1, 1, 1) \]  

(2.7)

as well as the singlets

\[ 24(1, 1, 1)^H \]  

(2.8)

Next, we describe the construction of the PS classes of models. It is based on type I string with D9-branes compactified on a six-dimensional orientifolded torus \( T^6 \), where internal background gauge fluxes on the branes are turned on. By performing a T-duality transformation on the \( x^4, x^5, x^6 \), directions the D9-branes with fluxes are translated into D6-branes intersecting at angles. The branes are not parallel to the orientifold planes. We assume that the D6\(_a\)-branes are wrapping 1-cycles \( (n^i_a, m^i_a) \) along each of the \( T^2 \) torus of the factorized \( T^6 \) torus, namely \( T^6 = T^2 \times T^2 \times T^2 \).

In order to build a PS model with minimal Higgs structure we consider four stacks of D6-branes giving rise to their world-volume to an initial gauge group \( U(4) \times U(2) \times U(2) \times U(1) \) at the string scale. In addition, we consider the addition of NS B-flux, such that the tori are not orthogonal, avoiding in this way an even number of families, and leading to effective tilted wrapping numbers,

\[ (n^i, m = \tilde{m}^i + n^i/2); \quad n, \quad \tilde{m} \in \mathbb{Z}. \]  

(2.9)

\(^3\)These fields are replicas of the fermion fields appearing in the intersection \( ac \) of table one.
that allows semi-integer values for the m-numbers.

Because of the $\Omega R$ symmetry, where $\Omega$ is the worldvolume parity and $R$ is the reflection on the T-dualized coordinates,

$$T(\Omega R)T^{-1} = \Omega R,$$

(2.10)

each D6$_a$-brane 1-cycle, must have its $\Omega R$ partner $(n^i_a, -m^i_a)$.

Chiral fermions are obtained by stretched open strings between intersecting D6-branes [9]. The chiral spectrum of the models is obtained after solving simultaneously the intersection constraints coming from the existence of the different sectors together with the RR tadpole cancellation conditions.

There are a number of different sectors, which should be taken into account when computing the chiral spectrum. We denote the action of $\Omega R$ on a sector $\alpha, \beta$, by $\alpha \star, \beta \star$, respectively. The possible sectors are:

- The $\alpha \beta + \beta \alpha$ sector: involves open strings stretching between the D6$_\alpha$ and D6$_\beta$ branes. Under the $\Omega R$ symmetry this sector is mapped to its image, $\alpha^* \beta^* + \beta^* \alpha^*$ sector. The number, $I_{\alpha \beta}$, of chiral fermions in this sector, transforms in the bifundamental representation $(N_\alpha, \bar{N}_\alpha)$ of $U(N_\alpha) \times U(N_\beta)$, and reads

$$I_{\alpha \beta} = (n^1_\alpha m^1_\beta - m^1_\alpha n^1_\beta)(n^2_\alpha m^2_\beta - m^2_\alpha n^2_\beta)(n^3_\alpha m^3_\beta - m^3_\alpha n^3_\beta),$$

(2.11)

where $I_{\alpha \beta}$ is the intersection number of the wrapped cycles. Note that the sign of $I_{\alpha \beta}$ denotes the chirality of the fermion and with $I_{\alpha \beta} > 0$ we denote left handed fermions. Negative multiplicity denotes opposite chirality.

- The $\alpha \alpha$ sector: it involves open strings stretching on a single stack of D6$_\alpha$ branes. Under the $\Omega R$ symmetry this sector is mapped to its image $\alpha^* \alpha^*$. This sector contain $N = 4$ super Yang Yills and if it exists SO(N), SP(N) groups appear. This sector is of no importance to us as we are interested in unitary groups.

- The $\alpha \beta^* + \beta^* \alpha$ sector: It involves chiral fermions transforming into the $(N_\alpha, N_\beta)$ representation with multiplicity given by

$$I_{\alpha \beta^*} = -(n^1_\alpha m^1_\beta + m^1_\alpha n^1_\beta)(n^2_\alpha m^2_\beta + m^2_\alpha n^2_\beta)(n^3_\alpha m^3_\beta + m^3_\alpha n^3_\beta).$$

(2.12)

Under the $\Omega R$ symmetry transforms to itself.

- the $\alpha \alpha^*$ sector: under the $\Omega R$ symmetry is transformed to itself. From this sector the invariant intersections will give $8m^1_\alpha m^2_\alpha m^3_\alpha$ fermions in the antisymmetric representation and the non-invariant intersections that come in pairs provide us with
4m_α^1 m_α^2 m_α^3 (n_α^1 n_α^2 n_α^3 - 1) additional fermions in the symmetric and antisymmetric representation of the $U(N_α)$ gauge group.

Any vacuum derived from the previous intersection number constraints of the chiral spectrum is subject to constraints coming from RR tadpole cancellation conditions [4]. That requires cancellation of D6-branes charges \(^4\), wrapping on three cycles with homology \([Π_a]\) and O6-plane 7-form charges wrapping on 3-cycles with homology \([Π_{O6}]\). In formal terms, the RR tadpole cancellation conditions in terms of cancellations of RR charges in homology, read:

$$\sum_a N_a [Π_a] + \sum_α N_α^* [Π_α^*] - 32 [Π_{O6}] = 0.$$  \tag{2.13}

Explicitly, the RR tadpole conditions read:

$$\sum_a N_a n_α^1 n_α^2 n_α^3 = 16,$$

$$\sum_a N_a m_α^1 m_α^2 m_α^3 = 0,$$

$$\sum_a N_a m_α^1 n_α^2 m_α^3 = 0,$$

$$\sum_a N_a n_α^1 m_α^2 m_α^3 = 0. \tag{2.14}$$

That ensures absence of non-abelian gauge anomalies. A comment is in order. It is important to notice that the RR tadpole cancellation condition can be understood as a constraint that demands that for each gauge group the number of fundamentals to be equal to the number of bifundamentals. As a general rule to D-brane model building, by considering \(N_a\) stacks of D-brane configurations with \(N_a, a = 1, \cdots, N\), paralleled branes, the gauge group appearing is in the form $U(N_1) \times U(N_2) \times \cdots \times U(N_a)$. Effectively, each $U(N_i)$ factor will give rise to an $SU(N_i)$ charged under the associated $U(1_i)$ gauge group factor that appears in the decomposition $SU(N_a) \times U(1_a)$. A type I brane configuration with the unique minimal PS particle content such that intersection numbers, tadpole conditions and various phenomenological requirements including the absence of exotic representations are accommodated, can be obtained by considering four stacks of branes yielding an initial $U(4)_a \times U(2)_b \times U(2)_c \times U(1)_d$ gauge group equivalent to an $SU(4)_a \times SU(2)_b \times SU(2)_b \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d$. Thus, in the first instance, we can identify, without loss of generality, $SU(4)_a$ as the $SU(4)_c$ colour group that its breaking could induce the usual $SU(3)$ colour group of strong interactions, the $SU(2)_b$ with $SU(2)_L$ of weak interactions and $SU(2)_c$ with $SU(2)_R$.

\(^4\)Taken together with their orientifold images \((n_α^1, -m_α^1)\) wrapping on three cycles of homology class \([Π_α^*]\).
The complete accommodation of the fermion structure of the model under study can be seen in table one.

| Fields | Intersection | $SU(4)C \times SU(2)_L \times SU(2)_R$ | $Q_a$ | $Q_b$ | $Q_c$ | $Q_d$ |
|--------|--------------|--------------------------------------|-------|-------|-------|-------|
| $F_L$  | $I_{ab^*} = 3$ | $3 \times (4,2,1)$ | 1     | 1     | 0     | 0     |
| $F_R$  | $I_{ac} = -3$ | $3 \times (\overline{3},1,2)$ | -1    | 0     | 1     | 0     |
| $\chi_L$ | $I_{bd} = -12$ | $12 \times (1,\overline{3},1)$ | 0     | -1    | 0     | 1     |
| $\chi_R$ | $I_{cd^*} = -12$ | $12 \times (1,1,\overline{3})$ | 0     | 0     | -1    | -1    |
| $\omega_L$ | $I_{aa^*}$ | $12\beta^2 \bar{\epsilon} \times (6,1,1)$ | $2\bar{\epsilon}$ | 0     | 0     | 0     |
| $s_R$  | $I_{dd^*}$ | $24\beta^2 \bar{\epsilon} \times (1,1,1)$ | 0     | 0     | 0     | $-2\bar{\epsilon}$ |

Table 1: Fermionic spectrum of the $SU(4)C \times SU(2)_L \times SU(2)_R$, type I models together with $U(1)$ charges. The spectrum appearing in the full table is of PS-A models. The top part corresponds to PS-B models. Note that the representation context in the bottom part is considered by assuming $\bar{\epsilon} = 1$. In the general case $\bar{\epsilon} = \pm 1$. If $\bar{\epsilon} = -1$ then the conjugate fields should be considered, e.g. if $\bar{\epsilon} = -1$, the $\omega_L$ field should transform as $(\overline{6},1,1)_{(-2,0,0,0)}$.

We note a number of interesting comments:

a) Two main directions towards model building classes of PS-models will be emphasized in this work. We can either choose to include sectors $\alpha \alpha^*$ in the model, we call this class of models type PS-A or not to include them, we call this class of models type PS-B. In the former case, PS-A, the surviving gauge group at low energies is exactly the SM. We get, at low energies, just the fermionic content of the SM spectrum with all particles having the correct hypercharge assignment. The fermionic spectrum of PS-A models is given by the full spectrum appearing in table (1). The tadpole solutions in this case appear in table (3). In the latter case, PS-B classes of models, the gauge group at low energies is the SM augmented by an extra anomaly free $U(1)$. The tadpole solutions for PS-B models appear in table (4). The fermionic particle content of PS-B models appear in the four top rows of table (1).

Also, in order to realize certain couplings we will impose that some intersections will preserve some supersymmetry. In both PS-A, PS-B models, some massive fields will be “pulled out” from the massive spectrum and become massless. For example, in order to realize a Majorana mass term for the right handed neutrinos for both PS-A, PS-B models we will demand that the sector $ac$ preserves $N = 1$ SUSY. That will have
as an immediate effect to "pull out" from the massive mode spectrum the $F^H_R$ particles.

b) The intersection numbers, in table one, of the fermions $F_L + \bar{F}_R$ are chosen such that $I_{ac} = -3, I_{ab} = 3$. Here, $-3$ denotes opposite chirality to that of a left handed fermion. The choice of additional fermion fields $(1, \bar{2}, 1), (1, 1, \bar{2})$ is imposed to us by the RR tadpole cancellation conditions that are equivalent to $SU(N_a)$ gauge anomaly cancellation, in this case of $SU(2)_L, SU(2)_R$ gauge anomalies,

$$\sum_i I_{ia} N_a = 0, \ a = L, R. \quad (2.15)$$

c) The PS-A, PS-B classes of models lack representations of scalar sextets $(6, 1, 1)$ fields, that appear in attempts to construct realistic 4D $N = 1$ PS heterotic models from the fermionic formulation [30] or in D-brane inspired models [28], even through examples of heterotic fermionic models where those representations are lacking exist [32]. Those representations were imposed earlier in attempts to produce a realistic PS model $^5$ as a recipe for saving the models from proton decay. Fast proton decay was avoided by making the mediating $d_H$ triplets of (2.4) superheavy and of order of the $SU(2)_R$ breaking scale via their couplings to the sextets. In the models we examine in this work, baryon number is a gauged global symmetry, so that proton is stable. Thus there is no need to introduce sextets to save the models from fact proton decay as proton is stable anyhow.

Also in this case, there is no problem of having $d_H$ becoming light enough and causing catastrophic proton decay, as the only way this could happen, is through the existense of the $d_H$ coupling to sextets to quarks and leptons. But such a coupling is forbidden by the symmetries of the models by construction.

Also, the PS-B model class has some shortcomings. The weak and right doublets $\chi_L, \chi_R$ respectively survive massless at low energies of order $M_Z$. Both massless particles are unwelcome as they are not observed at energies of order $M_Z$. Nevertheless this case is interesting as a number of useful conclusions could be derived from the study of those models.

To be convinced that scalar sextet fields cannot exist in intersecting PS-B type I D-brane models let us imagine that they do existed $^6$. Then it may then be easily seen that with four stacks of branes, they would have to be $^7$ in the form:

$^5$See the first reference of [30].

$^6$Introducing scalar sextets in this case would demand imposing $N = 1$ SUSY in this sector such that the full $N = 1$ sextet hypermultiplet would be massless.

$^7$An alternative equivalent choice of $(6, 1, 1)_{(1,0,0,0,-1)}, (\bar{6}, 1, 1)_{(-1,0,0,1)}$ would demand $I_{ad} = 1, I_{ad} = -1$ which is impossible anyway to accommodate. Even by using a PS-B model with five stacks
This choice is consistent with the cancellation of mixed anomalies of $U(1)$'s with the non-abelian gauge group factors. However, this choice demands

$$I_{ad^*} = -1 \text{ for } (6, 1, 1); \quad I_{ad^*} = 1 \text{ for } (6, 1, 1).$$

(2.17)

Obviously, it is not possible to accommodate simultaneously the two different intersection numbers in (2.17), ruling out the problematic representations (2.16).

For PS-A classes of models, there are no shortcomings. The theory breaks just to the standard model $SU(3) \times SU(2) \times U(1)_Y$ at low energies. The complete spectrum of the model appears in table (4). The tadpole solutions of PS-A models are presented in table (3).

d) The mixed anomalies $A_{ij}$ of the four surplus $U(1)$'s with the non-abelian gauge groups $SU(N_a)$ of the theory cancel through a generalized GS mechanism [31, 34], involving close string modes couplings to worldsheet gauge fields. Two combinations of the $U(1)$'s are anomalous and become massive through their couplings to RR fields, their orthogonal non-anomalous combinations survives, combining to a single $U(1)$ that remains massless.

e) For PS-A models the constraint

$$\prod_{i=1}^{3} m^i = 0.$$  

(2.18)

is not imposed and thus leads to the appearance of the non-trivial chiral fermion content from the $aa^*$, $dd^*$ sectors with corresponding fermions $\omega_L, y_R, z_R, s_Z$. After breaking the PS left-right symmetry at $M_{GUT}$, the surviving gauge symmetry is that of the SM augmented by an anomaly free $U(1)$ symmetry surviving the Green-Schwarz mechanism. To break the latter $U(1)$ symmetry we will impose that the $dd^*$ sector respects $N = 1$ SUSY. Thus singlets scalars will appear, that are superpartners of $s_L$ fermions.

For type PS-B models, in order to cancel the appearance of exotic representations in the model appearing from the general $DD^*$ sectors, in antisymmetric and symmetric representations of the $U(N_a)$ group, we require that (2.18) constraint holds.

of branes, or more, e.g. an $U(4) \times U(2) \times U(2) \times U(1) \times U(1)$, it will be impossible to accommodate sextet fields like those in (2.16) for similar reasons.

The same constraint was working perfectly at the level of building just the standard model at low energies, starting from stacks of branes that are not based on a non-GUT group at the string scale. For example see [16] for the four-stack D6 SM and [17] for the five stack D6 SM.
Note that the choice of fermion fields for PS-B models in table (4) is absolutely minimal, as a different choice of the set of fields with three stacks of branes, does not have a tadpole solution as long as we demand (2.18).

f) Demanding $I_{ab} = 3$, $I_{ac} = -3$, it implies that the third tori should be tilted. By looking at the intersection numbers of table one, we conclude that the b-brane should be parallel to the c-brane and the a-brane should be parallel to the d-brane as there is an absence of intersection numbers for those branes. The complete list of intersection numbers for PS-B class is listed in table two.

| $I_{ab} = 3$ | $I_{ac} = -3$ | $I_{bd} = -12$ | $I_{cd} = -12$ | $I_{ad} = 0$ | $I_{bc} = 0$ |
|-------------|--------------|----------------|---------------|------------|----------|
| $I_{ab} = 0$ | $I_{ac} = 0$ | $I_{bd} = 0$   | $I_{cd} = 0$  | $I_{ad} = 0$ | $I_{bc} = 0$ |

Table 2: List of intersection constraints for the $SU(4)_C \times SU(2)_L \times SU(2)_R$ type I PS-B classes of models.

The cancellation of the RR crosscap tadpole constraints is solved from parametric sets of solutions. For PS-A and PS-B classes of models they are given in tables (3) and (4) respectively.

- **Tadpoles for PS-A classes of models**

For the PS-A classes of models, giving just the SM at low energies, the choice of wrapping numbers appearing in table (3) satisfies all tadpole conditions but the third of eqn’s (2.14). The latter becomes

$$2n_a^2 + n_d^2 + \frac{1}{\beta_2}(m_b^1 - m_c^1) = 0,$$

which may be solved by either

$$n_d^2 = -2n_a^2, \quad m_b^1 = m_c^1$$  \hspace{1cm} (2.20)

or

$$2n_a^2 = \frac{m_c^1}{\beta_2}, \quad n_d^2 = -\frac{m_b^1}{\beta_2},$$

or

$$2n_a^2 = -\frac{m_b^1}{\beta_2}, \quad n_d^2 = \frac{m_c^1}{\beta_2}$$

(2.22)

Choosing for example the solutions (2.20) we are effectively making the tadpole solutions of table (3) to depend on one integer $n_a^2$ and the tilted wrapping number.
$m_b^1$, the phase parameters $\epsilon = \pm 1$, $\bar{\epsilon} = \pm 1$ and the NS-background parameter $\beta_i = 1 - b_i$, which is associated to the parametrization of the NS B-field with $b_i = 0, 1/2$. In this case, an example of wrappings satisfying all tadpoles is given by the choise of wrappings

$$n_a^2 = -1, \ m_b^1 = m_c^1 = 1, \ \beta_1 = 1, \ \beta_2 = 1/2, \ \bar{\epsilon} = -1. \ \ (2.23)$$

$$N_a = 4 \ (0, \epsilon)(-1, 3/2)(-1, -1/2)$$
$$N_b = 2 \ (-1, \ \epsilon)(2, 0)(-1, -1/2)$$
$$N_c = 2 \ (1, \ \epsilon)(2, 0)(-1, 1/2)$$
$$N_d = 1 \ (0, \ \epsilon)(2, 3)(2, -1) \ \ (2.24)$$

However, as we will argue later the choise (2.21), or (2.22) is more na tural, as the choise (2.20) gives that the number of electroweak Higgs present in the models is zero, an unnatural choise.

**Tadpoles for PS-B classes of models**

The solution to the tadpole constraints depend on four integer parameters $n_a^2$, $n_b^1$, $n_c^1$, the phase parameter $\epsilon = \pm 1$, the parameter $\rho = 1, 1/3$ and the NS-background parameter $\beta = 1 - b_i$, which is associated to the parametrization of the NS B-field by $b_i = 0, 1/2$, and the condition $\alpha \gamma = 4$. The latter condition effectively gives the set of values

$$\alpha \gamma = [(\pm 1, \pm 4), (\pm 2, \pm 2)], \ \ (2.25)$$

where by underline we denote permutation of entries.

We note that the presented two different classes of solutions to the tadpoles, are distinguished by the fixed positive or negative entry $m$-wrapping in the colour a-brane.

In the rest of this section we will be examining the tadpole solutions of the models described in table (4). The choises of wrapping numbers of table (4) satisfy all the tadpole constraints. The first tadpole condition in (2.14) reads $^9$

$$\frac{4n_a^2}{\rho \beta_1} + 2 \frac{n_b^1}{\rho \beta_2} + 2 \frac{n_c^1}{\rho \beta_2} + \frac{\alpha n_d^2}{\rho \beta_1} + N_D n_1 n_2 n_3 = 16. \ \ (2.26)$$

$^9$We have added an arbitrary number of $N_D$ branes which do not contribute to the rest of the tadpoles and intersection numbers. This is always an allowed choise. We chosen not to exhibit the rest of the tadpoles as they involve the identity $0 = 0$. Also we have chosen $\bar{\epsilon} = 1$. 

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Table 3: Tadpole solutions for PS-A type models with D6-branes wrapping numbers giving rise to the fermionic spectrum of the type I model, with the SM, \( SU(3)_C \times SU(2)_L \times U(1)_Y \), gauge group at low energies. The wrappings depend on two integer parameters, \( n_{a}^2, n_{d}^2 \), the NS-background \( \beta_i \) and the phase parameters \( \epsilon = \tilde{\epsilon} = \pm 1 \). Also there is an additional dependence on the two wrapping numbers, integer of half integer, \( m_{1b}, m_{1c} \).

To see clearly the cancellation of tadpoles, we have to choose a consistent numerical set of wrapping numbers, e.g

\[ \rho = \epsilon = 1, \; n_a^2 = 0, \; n_b^1 = 0, \; n_c^1 = 1, \; n_d^2 = -1, \; \beta_2 = 1, \; \beta_1 = 1, \; \alpha = 2, \; \gamma = 2. \]

(2.27)

With the above choice, all tadpole conditions are satisfied but the first, which gives

\[ N_Dn_1n_2n_3 = 16, \]

(2.28)

The latter can be satisfied with the addition of eight D6-branes with wrapping numbers \((1,0)(1,0)(2,0)\), effectively giving to the models the structure

\[
\begin{align*}
N_a &= 4 \quad (1,0)(0,-1)(1,3/2) \\
N_b &= 2 \quad (0,1)(1,0)(1,3/2) \\
N_c &= 2 \quad (1,1)(1,0)(1,-3/2) \\
N_d &= 1 \quad (2,0)(-1,-2)(1,-3/2) \\
N_D &= 8 \quad (1,0)(1,0)(2,0)
\end{align*}
\]

(2.29)

Alternatively, we can choose

\[ \rho = \epsilon = 1, \; n_a^2 = 0, \; n_b^1 = 0, \; n_c^1 = 1, \; n_d^2 = 1, \; \beta_2 = 1, \; \beta_1 = 1, \; \alpha = 2, \; \gamma = 2. \]

(2.30)
Table 4: Tadpole solutions of PS-B type models with D6-branes wrapping numbers giving rise to the fermionic spectrum of type I model, with an $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)$ gauge group at low energies, the extra $U(1)$ being anomaly free. The parameter $\rho$ takes the values 1, 1/3, while there is an additional dependence on four integer parameters, $n^2_a$, $n^2_d$, $n^1_b$, $n^1_c$, the NS-background $\beta_i$, $i = 1, 2$, and the phase parameters $\epsilon = \pm 1$, $\tilde{\epsilon} = \pm 1$. Note the condition $\alpha \gamma = 4$ and the positive wrapping number entry on the 3rd tori of the colour a-brane.

With the above choice, all tadpole conditions are satisfied but the first, which gives

$$4 + N_D n_1 n_2 n_3 = 16,$$

(2.31)

The latter can be satisfied with the addition of six D6-branes with wrapping numbers $(1,0)(1,0)(2,0)$, effectively giving the model the structure

$$N_a = 4 \quad (1,0)(0,-1)(1,3/2)$$
$$N_b = 2 \quad (0,1)(1,0)(1,3/2)$$
$$N_c = 2 \quad (1,1)(1,0)(1,-3/2)$$
$$N_d = 1 \quad (2,0)(1,-2)(1,-3/2)$$
$$N_D = 6 \quad (1,0)(1,0)(2,0)$$

(2.32)

Note that it appears that the wrapping number $(2,0)$ along the first tori gives
rise to an additional $U(1)$ at low energies. However, as we will explain in the next section, this is an artifact of the procedure as its presence can be absorbed into the surviving, the GS mechanism, massless anomaly free $U(1)$ field, by a proper field redefinition.

f) the hypercharge operator for PS-A, PS-B classes of models is defined as a linear combination of the three diagonal generators of the $SU(4)$, $SU(2)_L$, $SU(2)_R$ groups:

$$Y = \frac{1}{2} T_{3R} + \frac{1}{2} T_{B-L}, \quad T_{3R} = \text{diag}(1, -1), \quad T_{B-L} = \text{diag}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1).$$

(2.33)

Explicitly,

$$Q = Y + \frac{1}{2} T_{3L}. \quad (2.34)$$

$$Q = Y + \frac{1}{2} T_{3L}. \quad (2.35)$$

3 Cancellation of U(1) Anomalies

The mixed anomalies $A_{ij}$ of the four $U(1)$’s with the non-Abelian gauge groups are given by

$$A_{ij} = \frac{1}{2}(I_{ij} - I_{ji})N_i. \quad (3.1)$$

Moreover, analyzing the mixed anomalies of the extra $U(1)$’s with the non-abelian gauge groups $SU(4)_c$, $SU(2)_R$, $SU(2)_L$ we can see that there are two anomaly free combinations $Q_b - Q_c$, $Q_a - Q_d$. Note that gravitational anomalies cancel since D6-branes never intersect O6-planes. In the orientifolded type I torus models gauge anomaly cancellation [34] proceeds through a generalized GS mechanism [16] that makes use of the 10-dimensional RR gauge fields $C_2$ and $C_6$ and gives at four dimensions the couplings to gauge fields

$$N_a m_a^1 m_a^2 m_a^3 \int_{M_4} B_2^o \wedge F_a ; \quad n_b^1 n_b^2 n_b^3 \int_{M_4} C_6^o \wedge F_b \wedge F_b,$$

(3.2)

$$N_a n_J^1 n_K^2 n_J^3 \int_{M_4} B_2^I \wedge F_a ; \quad n_b^I n_b^j n_b^K \int_{M_4} C_6^I \wedge F_b \wedge F_b,$$

(3.3)

where $C_2 \equiv B_2^o$ and $B_2^I \equiv \int_{(T^2)^I \times (T^2)^K} C_6$ with $I = 1, 2, 3$ and $I \neq J \neq K$. Notice the four dimensional duals of $B_2^o$, $B_2^I$:

$$C_6^o \equiv \int_{(T^2)^I \times (T^2)^J \times (T^2)^K} C_6 ; \quad C_6^I \equiv \int_{(T^2)^I} C_2,$$

(3.4)

where $dC_6^o = -\star dB_2^o$, $dC_6^I = -\star dB_2^I$. 

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The triangle anomalies (3.1) cancel from the existence of the string amplitude involved in the GS mechanism [31] in four dimensions [34]. The latter amplitude, where the $U(1)_a$ gauge field couples to one of the propagating $B_2$ fields, coupled to dual scalars, that couple in turn to two $SU(N)$ gauge bosons, is proportional [16] to

$$-N_a m_a^1 m_a^2 m_b^1 n_b^1 n_b^2 n_b^3 - N_a \sum_I n_a^I n_b^J n_a^K m_a^I m_b^K , I \neq J, K$$

(3.5)

The study of $U(1)$ anomalies in the models is performed separately for PS-A, PS-B models. We distinguish two cases:

- **PS-A models**

  For this class of models the RR couplings $B^I_2$ of (3.3), appear into three terms (we set for simplicity $\tilde{\epsilon} = 1$):

  $$B^3_2 \wedge \left( \frac{\epsilon \tilde{\epsilon}}{\beta_2} \right) (F^b + F^c),$$

  $$B^1_2 \wedge \left( \epsilon \tilde{\epsilon} [4n_a^2 F^a - 2n_a^2 F^d + \frac{2m_1^a}{\beta_2} F^b + \frac{2m_1^a}{\beta_2} F^c] \right),$$

  $$B^2_2 \wedge (6\beta_2 \tilde{\epsilon}) (F^a + F^d).$$

(3.6)

As can be seen from (3.6) two anomalous combinations of $U(1)$'s, e.g. $F^a + F^d$, $F^b + F^c$ become massive through their couplings to RR fields $B^a_2, B^3_2$. Also there are two non-anomalous $U(1)$'s, the combinations of $Q^b - Q^c$, $Q^a - Q^d$. A third non-anomalous combination of $U(1)$'s is made massive by its coupling to $B^1_2$.

At this point we should list the couplings of the dual scalars $C^I$ of $B^I_2$ required to cancel the mixed anomalies of the four $U(1)$'s with the non-abelian gauge groups $SU(N_a)$. They are given by

$$C^a \wedge \left( \frac{\tilde{\epsilon}}{\beta_2} \right) [- (F^b \wedge F^b) + (F^c \wedge F^c)],$$

$$C^3 \wedge (3\tilde{\epsilon} \beta_2) [(F^a \wedge F^a) - 4(F^d \wedge F^d)],$$

$$C^2 \wedge \frac{n_a^2 \tilde{\epsilon}}{2} (F^a \wedge F^a) + \frac{m_1^a \tilde{\epsilon}}{2\beta_2} (F^b \wedge F^b) - \frac{m_1^a \tilde{\epsilon}}{2\beta_2} (F^c \wedge F^c) - 2n_a^2 \tilde{\epsilon} (F^d \wedge F^d).$$

(3.7)

Note that the combination of $U(1)$'s which survives massless to low energies is uniquely given by

$$Q_l = \kappa ((Q_a - Q_d) + (Q_b - Q_c)),$$

(3.8)

where $\kappa$ an arbitrary number.

- **PS-B models**
If we take into account the phenomenological requirements of eqn. (2.18) the RR couplings $B_I^2$ of (3.3), appear into three terms $^{10}$:

$$B_2^1 \wedge \left( \frac{2\epsilon \beta_1}{\beta_2 \rho} \right) (F^b + F^c),$$

$$B_2^2 \wedge \left( \frac{-4\epsilon \beta_2}{\beta_1 \rho} \right) (F^a + F^d),$$

$$B_2^3 \wedge \left( \frac{3\rho}{\beta_2} \right) \left( \frac{2n_a^2 \beta_2 F^a}{\beta_1} + n_b^1 F^b - n_c^1 F^c - \frac{\beta_2 \alpha n_d^2}{2\beta_1} F^d \right).$$

(3.9)

At this point we should list the couplings of the dual scalars $C_I$ of $B_I^2$ required to cancel the mixed anomalies of the four $U(1)$’s with the non-abelian gauge groups $SU(N_a)$. They are given by

$$C^1 \wedge \left[ \left( \frac{-3\epsilon \beta_2 \rho}{2\beta_1} \right) (F^a \wedge F^a - 4F^d \wedge F^d) \right],$$

$$C^2 \wedge \left( \frac{3\beta_2 \rho \epsilon}{2\beta_2} \right) [(F^b \wedge F^b) - (F^c \wedge F^c)],$$

$$C^\alpha \wedge \left( \frac{n_a^2}{\rho \beta_1} (F^a \wedge F^a) + \frac{n_b^1}{\rho \beta_2} (F^b \wedge F^b) + \frac{n_c^1}{\rho \beta_2} (F^c \wedge F^c) + \frac{\alpha n_d^2}{\rho \beta_1} (F^d \wedge F^d) \right),$$

(3.10)

Notice that the RR scalar $B_2^0$ does not couple to any field $F^3$ as we have imposed the condition (2.18) which prevents the appearance of any exotic matter.

Looking at (3.9) we conclude that there are two anomalous $U(1)$’s, $Q^b + Q^c$, $Q^a + Q^d$, which become massive through their couplings to the RR 2-form fields $B_2^1, B_2^2$ and two non-anomalous free combinations $Q^b - Q^c$, $Q^a - Q^d$. Note that the mixed anomalies $A_{ij}$ are cancelled by the GS mechanism set by the couplings (3.9, 3.10). In addition, the combination of the $U(1)$’s which remains light at low energies, and is orthogonal to the massive $U(1)$’s coupled to the RR fields $B_2^3, B_2^2, B_2^1$ is

$$n_b^1 + n_c^1 \neq 0, \quad Q_l = \frac{1}{(n_b^1 + n_c^1)} (Q_b - Q_c) - \frac{\beta_1}{\beta_2 (2n_a^2 + \alpha n_d^2)} (Q_a - Q_d).$$

(3.11)

Making the choise of wrapping numbers (2.27), the surviving massless non-anomalous $U(1)$ reads

$$Q_l = (Q_b - Q_c) + (Q_a - Q_d).$$

(3.12)

Instead, if we make the choise (2.30) the surviving massless non-anomalous $U(1)$ reads

$$Q_l = (Q_b - Q_c) - (Q_a - Q_d).$$

(3.13)

$^{10}$We set for simplicity $\epsilon = 1.$
Both choices of global $U(1)$’s are consistent with electroweak data in the sense that they don’t break the lepton number. That happens because the bidoublet Higgs fields $h_1, h_2$ don’t get charged. A similar effect holds for (3.8).

A comment is in order. The interpretation of the presence of the $(2,0)$ wrapping number, in (2.29), (2.32) found in the d-brane of the first tori is subtle in principle. That happens since D-brane gauge theory analysis indicates that it should be interpreted either as one brane wrapping twice around the cycle $(1,0)$ or as two branes wrapping once around the cycle $(1,0)$ giving rise to two $U(1)$’s, $Q_d^1, Q_d^2$, where the two $U(1)$’s correspond to the combinations

$$Q_d^{(1)} = (Q_d^1 + Q_d^2); \quad Q_d^{(2)} = (Q_d^1 - Q_d^2).$$

The two $U(1)$’s listed in (3.14) correspond to open strings stretching between the first wrapping of the d-brane, namely $Q_d^{(1)}$, and the first wrapping of the extra brane, namely $Q_d^{(2)}$.

However, for the string GUT model which starts at the string scale with four $U(1)$’s, it is only tadpole cancellation that introduces an additional $U(1)$, from “multiwrapping”. The additional $U(1)$ was not needed at the gauge theory level, as cancellation of the mixed $U(1)$ gauge anomalies was already consistent without the need of adding an extra $U(1)$. Clearly, at the level of the effective action we shouldn’t have found any additional $U(1)$’s beyond those, four, already present at the string scale.

Let us now redefine the massless non-anomalous $U(1)$ as

$$Q_l \rightarrow Q_l = (Q_b - Q_c) + \left(Q_a - \frac{1}{2}(Q_d^{(1)} + Q_d^{(2)})\right),$$

$$Q_l \rightarrow Q_l = (Q_b - Q_c) + \left(Q_a - \frac{1}{2}(Q_d^1 + Q_d^2 + Q_d^1 - Q_d^2)\right),$$

where is is clear that we have identify $Q_d = Q_d^1$. Let us rewrite the charges of the fermion fields of table one, as

\begin{align*}
&(4,2,1)_{[1,1,0,0,0]}, \quad (\bar{4},1,2)_{[-1,0,1,0,0]}, \\
&(1,2,1)_{[0,-1,0,1,0]}, \quad (1,1,\bar{2})_{[0,0,-1,1,0]},
\end{align*}

where by underline we indicate a simultaneous permutation of the fourth, fifth entries for all fermion fields. Thus no additional charges are introduced for the fields beyond the already present. It is now clearly seen that the additional $U(1)$, from “multiwrapping” corresponds just to a field redefinition of the surviving global $U(1)$ at low energies.

\textsuperscript{11}Note that there is no NSNS b-field in the first torus.

\textsuperscript{12}Note that there was no stringy Higgs effect present that could introduce additional gauge bosons.
and hence at the level of the effective action at low energy has no physical effect. In fact, at the level the cancellation of the mixed global $U(1)$ gauge anomalies its time either $Q_d^{(1)}$ or $Q_d^{(2)}$ get charged.

Let us close this section by noticing that the non-anomalous massless $U(1)$ which is free from gauge and gravitational anomalies can be written in three more different ways. We enumerate them here for consistency. They read:

$$\beta_1 \neq 0, \quad Q_I = \frac{1}{\beta_1} (Q_b - Q_c) - \frac{n_1^b + n_1^c}{\beta_2 (2n_2^a - \alpha n_2^d)} (Q_a - Q_d), \quad (3.18)$$

$$\beta_2^2 \left( 2n_2^a + \frac{\alpha n_2^d}{2} \right) \neq 0, \quad Q_I = \beta_2^2 \left( 2n_2^a + \frac{\alpha n_2^d}{2} \right) (Q_b - Q_c) - \frac{(n_2^b + n_2^c)}{(\beta_1)} (Q_a - Q_d), \quad (3.19)$$

$$\frac{\beta_2 (2n_2^a + \frac{\alpha n_2^d}{2})}{\beta_1} \neq 0, \quad Q_I = \frac{\beta_2 (2n_2^a + \frac{\alpha n_2^d}{2})}{(\beta_1)} (Q_b - Q_c) - (n_2^b + n_2^c) (Q_a - Q_d). \quad (3.20)$$

### 4 Higgs sector, global symmetries, proton stability,

$N = 1$ SUSY on intersections and neutrino masses

#### 4.1 Stability of the configurations and Higgs sector

We have so far seen the appearance in the R-sector of $I_{ab}$ massless fermions in the D-brane intersections transforming under bifundamental representations $N_a, \bar{N}_b$. In intersecting brane words, besides the actual presence of massless fermions at each intersection, we have evident the presence of an equal number of massive bosons, in the NS-sector, in the same representations as the massless fermions [15]. Their mass is of order of the string scale and it should be taken into account when examining phenomenological applications related to the renormalization group equations. However, it is possible that some of those massive bosons may become tachyonic $^{13}$, especially when their mass, that depends on the angles between the branes, is such that is decreases the world volume of the 3-cycles involved in the recombination process of joining the two branes into a single one [36]. Denoting the twist vector by $(\vartheta_1, \vartheta_2, \vartheta_3, 0)$, in the NS open string sector the lowest lying states are given by $^{14}$

| State | Mass |
|-------|------|
| $(-1 + \vartheta_1, \vartheta_2, \vartheta_3, 0)$ | $\alpha' M^2 = \frac{1}{2} (-\vartheta_1 + \vartheta_2 + \vartheta_3)$ |
| $(\vartheta_1, -1 + \vartheta_2, \vartheta_3, 0)$ | $\alpha' M^2 = \frac{1}{2} (\vartheta_1 - \vartheta_2 + \vartheta_3)$ |
| $(\vartheta_1, \vartheta_2, -1 + \vartheta_3, 0)$ | $\alpha' M^2 = \frac{1}{2} (\vartheta_1 + \vartheta_2 - \vartheta_3)$ |
| $(-1 + \vartheta_1, -1 + \vartheta_2, -1 + \vartheta_3, 0)$ | $\alpha' M^2 = 1 - \frac{1}{2} (\vartheta_1 + \vartheta_2 + \vartheta_3)$ |

$^{13}$For consequences when these set of fields may become massless see [35].

$^{14}$we assume $0 \leq \vartheta_i \leq 1$.
Exactly at the point, where one of these masses may become massless we have preservation of $\mathcal{N} = 1$ locally. The angles at the four different intersections can be expressed in terms of the parameters of the tadpole solutions.

We note that in the study of Higgs sector, we will deal separately with the definition of the angle structure for the PS-A, PS-B types of PS models. However, where it applies we will list the similarities.

• **Angle structure and Higgs fields for PS-A classes of models**

The angles at the different intersections can be expressed in terms of the tadpole solution parameters. We define the angles:

$$
\theta_1 = \frac{1}{\pi} \cot^{-1} \frac{R_1^{(1)}}{m_b R_2^{(1)}}; \quad \theta_2 = \frac{1}{\pi} \cot^{-1} \frac{n_a^2 R_1^{(2)}}{3 \beta_2 R_2^{(2)}}; \quad \theta_3 = \frac{1}{\pi} \cot^{-1} \frac{2 R_1^{(3)}}{R_2^{(3)}},
$$

$$
\tilde{\theta}_2 = \frac{1}{\pi} \cot^{-1} \frac{n_a^2 R_1^{(1)}}{3 \beta_2 R_2^{(1)}}; \quad \tilde{\theta}_1 = \frac{1}{\pi} \cot^{-1} \frac{R_1^{(1)}}{m_c R_2^{(1)}},
$$

where $R_{i,j}^{(k)}$, $i = 1, 2$ are the compactification radii for the three $j = 1, 2, 3$ tori, namely projections of the radii onto the cartesian axis $X^{(i)}$ directions when the NS flux B field, $b^k$, $k = 1, 2$ is turned on.

At each of the four non-trivial intersections we have the presence of four states $t_i$, $i = 1, \cdots, 4$, associated to the states (4.1). Hence we have a total of sixteen different scalars in the model. The setup is seen clearly if we look at figure one. These scalars are generally massive but for some values of their angles could become tachyonic (or massless).

Also, if we demand that the scalars associated with (4.1) and PS-A models may not be tachyonic, we obtain a total of twelve conditions for the PS-A type models with a D6-brane at angles configuration to be stable. They are given in Appendix I. We don’t consider the scalars from the $aa^*$, $dd^*$ intersections. For these sectors we will require later that they preserve $\mathcal{N} = 1$ SUSY. As a result all scalars in these sectors may become massive for both PS-A, PS-B models.

• **Angle structure and Higgs fields for PS-B classes of models**

Let us define the angles :

$$
\theta_1 = \frac{1}{\pi} \cot^{-1} \frac{n_b^1 R_1^{(1)}}{\beta_1 R_2^{(1)}}; \quad \theta_2 = \frac{1}{\pi} \cot^{-1} \frac{n_a^2 R_1^{(2)}}{\beta_2 R_2^{(2)}}; \quad \theta_3 = \frac{1}{\pi} \cot^{-1} \frac{2 R_1^{(3)}}{3 \beta R_2^{(3)}},
$$

$$
\tilde{\theta}_1 = \frac{1}{\pi} \cot^{-1} \frac{n_b^1 R_1^{(1)}}{\beta_1 R_2^{(1)}}; \quad \tilde{\theta}_2 = \frac{1}{\pi} \cot^{-1} \frac{n_a^2 R_1^{(2)}}{4 \beta_2 R_2^{(2)}}; \quad \tilde{\theta}_3 = \frac{1}{\pi} \cot^{-1} \frac{2 R_1^{(3)}}{3 \beta R_2^{(3)}},
$$

where $R_{i,j}^{(k)}$ are the compactification radii for the three $i = 1, 2, 3$ tori, namely projections of the radii onto the $X^{(i)}_{1,2}$ directions when the NS flux B field, $b^i$, is turned on.
Figure 1: Assignment of angles between D6-branes on a type I PS-A class of models based on the initial gauge group $U(4)_C \times U(2)_L \times U(2)_R$. The angles between branes are shown on a product of $T^2 \times T^2 \times T^2$. We have chosen $\beta_1 = 1$, $m_b^1, m_c^1, n_a^2 > 0$, $\epsilon = \bar{\epsilon} = 1$. These models break to low energies to exactly the SM.

At each of the four non-trivial intersections we have the presence of four states $t_i, i = 1, \ldots, 4$, associated to the states (4.1). Hence we have a total of sixteen different scalars in the model. The setup is seen clearly if we look at figure two.

In addition, some interesting relations between the different scalar fields hold e.g for PS-B models:

$$
m_{cd}(t_2) + m_{cd}(t_3) = m_{ac}(t_2) + m_{ac}(t_3)
$$

$$
m_{ab}(t_1) + m_{ab}(t_3) = m_{ac}(t_1) + m_{ac}(t_3)
$$

$$
m_{cd}(t_2) + m_{bd}(t_3) = m_{cd}(t_3) + m_{bd}(t_2)
$$

$$
m_{ab}(t_2) + m_{ab}(t_3) = m_{bd}(t_2) + m_{bd}(t_3)
$$

(4.4)

Demanding that the scalars associated with (4.1) in PS-B models may not be tachyonic, we obtain a total of twelve conditions for a D6-brane at angles configuration to be stable. They are given in Appendix II.

Let us now turn our discussion to the Higgs sector of PS-A, PS-B models. In general there are two different Higgs fields that may be used to break the PS symmetry. We
Figure 2: Assignment of angles between D6-branes on a type I PS-B class of models based on the initial gauge group $U(4)_C \times U(2)_L \times U(2)_R$. The angles between branes are shown on a product of $T^2 \times T^2 \times T^2$. We have chosen $\rho = \beta_1 = 1$, $n_b^1, n_c^1, n_a^2, n_d^2 > 0$, $\epsilon = 1$. These models break to low energy to the SM augmented by an anomaly free $U(1)$ symmetry.
remind that they were given in (2.3). The question is if $H_1$, $H_2$ are present in the spectrum of PS-A, PS-B models. The following discussion unless otherwise stated it will apply for both classes of models. In general, tachyonic scalars stretching between two different branes $\tilde{a}, \tilde{b}$, can be used as Higgs scalars as they can become non-tachyonic by varying the distance between the branes. Looking at the $I_{ac^*}$ intersection we can answer positively to our question since there are scalar doublets $H^\pm$ localized. They come from open strings stretching between the $U(4)$ $a$-brane and $U(2)_{Rc^*}$-brane.

| Intersection | PS breaking Higgs | $Q_a$ | $Q_b$ | $Q_c$ | $Q_d$ |
|-------------|-----------------|------|------|------|------|
| $ac^*$      | $H_1$           | 1    | 0    | 1    | 0    |
| $ac^*$      | $H_2$           | -1   | 0    | -1   | 0    |

Table 5: Higgs fields responsible for the breaking of $SU(4) \times SU(2)_R$ symmetry of the $SU(4)_C \times SU(2)_L \times SU(2)_R$ type I model with D6-branes intersecting at angles. These Higgs are responsible for giving masses to the right handed neutrinos in a single family.

The $H^\pm$’s come from the NS sector and correspond to the states $^{15}$

\[
\begin{align*}
\text{State} & \quad \text{Mass}^2 \\
(-1 + \vartheta_1, \vartheta_2, 0, 0) & \quad \alpha'(\text{Mass})^2_{H^+_1} = \frac{Z_3}{4\pi^2} + \frac{1}{2}(\vartheta_2 - \vartheta_1) \\
(\vartheta_1, -1 + \vartheta_2, 0, 0) & \quad \alpha'(\text{Mass})^2_{H^-_1} = \frac{Z_3}{4\pi^2} + \frac{1}{2}(\vartheta_1 - \vartheta_2)
\end{align*}
\]

(4.5)

where $Z_3$ is the distance$^2$ in transverse space along the third torus, $\vartheta_1, \vartheta_2$ are the (relative) angles between the $a$, $c^*$-branes in the first and second complex planes respectively. The presence of scalar doublets $H^\pm$ can be seen as coming from the field theory mass matrix

\[
(H_1^* \ H_2) \left( M^2 \right) \left( \begin{array}{c} H_1 \\ H_2^* \end{array} \right) + h.c.
\]

(4.6)

where

\[
M^2 = M_s^2 \left( \begin{array}{cc}
Z_3^{(ac^*)} (4\pi^2)^{-1} & \frac{1}{2}|\vartheta_1^{(ac^*)} - \vartheta_3^{(ac^*)}| \\
\frac{1}{2}|\vartheta_1^{(ac^*)} - \vartheta_3^{(ac^*)}| & Z_3^{(ac^*)} (4\pi^2)^{-1}
\end{array} \right),
\]

(4.7)

The fields $H_1$ and $H_2$ are thus defined as

\[
H^\pm = \frac{1}{2}(H_1^* \pm H_2)
\]

(4.8)

$^{15}$a similar set of states was used in [16] to provide the model with electroweak Higgs scalars.
where their charges are given in table (5). Hence the effective potential which corresponds to the spectrum of the PS symmetry breaking Higgs scalars is given by

$$V_{Higgs} = m_H^2 (|H_1|^2 + |H_2|^2) + (m_B^2 H_1 H_2 + h.c)$$  \hspace{1cm} (4.9)

where

$$m_H^2 = \frac{Z_3^{(ac^c)}}{4\pi^2\alpha'}; \quad m_B^2 = \frac{1}{2\alpha'}|\varphi_1^{(ac^c)} - \varphi_2^{(ac^c)}|$$  \hspace{1cm} (4.10)

The precise values of $m_H^2$, $m_B^2$, for PS-A models, are:

$$m_H^2 \text{ PS-A} = (\xi_a + \xi_c)^2 \frac{1}{2\alpha'}; \quad m_B^2 \text{ PS-A} = \frac{1}{2\alpha'} \left|\tilde{\theta}_1 - \theta_2\right|$$  \hspace{1cm} (4.11)

where $\xi_a(\xi_c)$ is the distance between the orientifold plane and the $a(c)$ branes and $\tilde{\theta}_1$, $\theta_2$ were defined in (4.2). In terms of those data for PS-A models we found:

$$m_B^2 \text{ PS-A} = \frac{1}{2}|m_{\chi_R}(t_2) + m_{\chi_R}(t_3) - m_{\chi_L}(t_1) - m_{\chi_L}(t_3)|$$

$$= \frac{1}{2}|m_{\chi_R}(t_2) + m_{\chi_R}(t_3) - m_{\chi_L}(t_1) - m_{\chi_L}(t_3)|$$  \hspace{1cm} (4.12)

For PS-B models,

$$m_H^2 \text{ PS-B} = (\xi_a + \xi_c)^2 \frac{1}{2\alpha'}; \quad m_B^2 \text{ PS-B} = \frac{1}{2\alpha'} \left|\tilde{\theta}_1 - \theta_2\right|$$  \hspace{1cm} (4.13)

where $\xi_a(\xi_c)$ is the distance between the orientifold plane and the $a(c)$ branes and $\tilde{\theta}_1$, $\theta_2$ were defined in (4.3).

The $m_B^2$ mass can be expressed in terms of the scalar masses (4.1) present, using the relations (4.4). Explicitly we found:

$$m_B^2 \text{ PS-A} = \frac{1}{2}|m_{\chi_R}(t_2) - m_{\chi_R}(t_1)|$$  \hspace{1cm} (4.14)

$$m_B^2 \text{ PS-B} = \frac{1}{2}|m_{\chi_R}(t_2) - m_{\chi_R}(t_1)|$$

$$= \frac{1}{2}|m_{\chi_R}(t_2) + m_{\chi_R}(t_3) - m_{\chi_L}(t_1) - m_{\chi_L}(t_3)|$$

$$= \frac{1}{2}|m_{\chi_R}(t_2) + m_{\chi_R}(t_3) - m_{\chi_L}(t_1) - m_{\chi_L}(t_3)|$$

$$= \frac{1}{2}|m_{\chi_R}(t_2) + m_{\chi_R}(t_3) - m_{\chi_L}(t_1) - m_{\chi_L}(t_3)|$$  \hspace{1cm} (4.15)
Intersection | Higgs | $Q_a$ | $Q_b$ | $Q_c$ | $Q_d$
---|---|---|---|---|---
$bc^*$ | $h_1 = (1, 2, 2)$ | 0 | 1 | 1 | 0
$bc^*$ | $h_2 = (1, \bar{2}, \bar{2})$ | 0 | $-1$ | $-1$ | 0

Table 6: Higgs fields present in the intersection $bc^*$ of the $SU(4)_C \times SU(2)_L \times SU(2)_R$ type I model with D6-branes intersecting at angles. These Higgs give masses to the quarks and leptons in a single family and are responsible for electroweak symmetry breaking.

For PS-A, PS-B models the number of Higgs present is equal to the intersection number product between the $a^*$, $c^*$-branes in the first and second complex planes,

$$n_{H^\pm}^{PS-A} I_{ac^*} = |3\epsilon^2| = 3.$$  \hspace{1cm} (4.16)

$$n_{H^\pm}^{PS-B} I_{ac^*} = |\epsilon^2| = 1.$$  \hspace{1cm} (4.17)

A comment is in order. For PS-A models the number of PS Higgs is three. That means that we have three intersections and to each one we have a Higgs particle which is a linear combination of the Higgs $H_1$ and $H_2$. For PS-B models the number of scalar doublets present is one, thus the Higgs responsible for breaking the PS symmetry will be a linear combination of the $H_1$, $H_2$.

There are, however, more Higgs present. In the $bc^*$ intersection we have present some of the most useful Higgs fields of the model. They will be used later to give mass to the quarks and leptons of the model. They appear in the representations $(1, 2, 2)$, $(1, \bar{2}, \bar{2})$ and from now on we will we denote them as $h_1$, $h_2$.

In the NS sector the lightest scalar states $h^\pm$ originate from open strings stretching between the $bc^*$ branes

| State | Mass $^2$ |
|-------|-----------|
| $(-1 + \theta_1, 0, 0, 0)$ | $\alpha'(\text{Mass})^2 = \frac{Z_{bc^*}^2}{4\pi^2} - \frac{1}{2}(\theta_1)$ |
| $(\theta_1, -1, 0, 0)$ | $\alpha'(\text{Mass})^2 = \frac{Z_{bc^*}^2}{4\pi^2} + \frac{1}{2}(\theta_1)$ |

where $Z_{bc^*}^2$ is the relative distance in transverse space along the second and third torus from the orientifold plane, $\theta_1$, is the (relative)angle between the $b^*$, $c^*$-branes in the first complex plane.

The presence of scalar doublets $h^\pm$ can be seen as coming from the field theory mass matrix
\[ (h_1^* h_2) \left( \mathbf{M}^2 \right) \begin{pmatrix} h_1 \\ h_2^* \end{pmatrix} + \text{h.c.} \]  

(4.19)

where

\[ \mathbf{M}^2 = M_s^2 \begin{pmatrix} Z_{23}^{(b^c)} (4\pi^2)^{-1} & \frac{1}{2} |\psi_1^{(b^c)} - \psi_3^{(bc^*)}| \\ \frac{1}{2} |\psi_1^{(b^c)} - \psi_3^{(bc^*)}| & Z_{23}^{(b^c)} (4\pi^2)^{-1} \end{pmatrix}, \]  

(4.20)

The fields \( h_1 \) and \( h_2 \) are thus defined as

\[ h^\pm = \frac{1}{2} (h_1^* \pm h_2). \]  

(4.21)

The effective potential which corresponds to the spectrum of electroweak Higgs \( h_1, h_2 \) is given by

\[ V_{Higgs}^{b^c} = \overline{m}_H^2 (|h_1|^2 + |h_2|^2) + (\overline{m}_B^2 h_1 h_2 + \text{h.c}) \]  

(4.22)

where

\[ \overline{m}_H^2 = \frac{Z_{23}^{(b^c)}}{4\pi^2 \alpha'}; \overline{m}_B^2 = \frac{1}{2\alpha'} |\psi_1^{(b^c)}|. \]  

(4.23)

The precise values of for PS-A classes of models \( \overline{m}_H^2, \overline{m}_B^2 \) are

\[ \overline{m}_H^{PS-A} = \frac{\bar{\chi}_b^{(2)} + \bar{\chi}_c^{(2)}}{\alpha'}; \overline{m}_B^{PS-A} = \frac{1}{2\alpha'} |\bar{\theta}_1 + \theta_1 - 1| \]  

(4.24)

where \( \bar{\theta}_1, \theta_1 \) were defined in (4.2). Also \( \bar{\chi}_b, \bar{\chi}_c^* \) are the distances of the \( b, c^* \) branes from the orientifold plane in the second tori and \( \bar{\xi}_b, \bar{\xi}_c^* \) are the distances of the \( b, c^* \) branes from the orientifold plane in the third tori. Notice that the \( b, c^* \) branes are parallel along the second and third tori.

The precise values of for PS-B models \( \overline{m}_H^2, \overline{m}_B^2 \) are

\[ \overline{m}_H^{PS-B} = \frac{\chi_b^{(2)} + \chi_c^{(2)}}{\alpha'}; \overline{m}_B^{PS-B} = \frac{1}{2\alpha'} |\bar{\theta}_1 + \theta_1| \]  

(4.25)

where \( \chi_b, \chi_c^* \) are the distances of the \( b, c^* \) branes from the orientifold plane in the second tori and \( \xi_b, \xi_c^* \) are the distances of the \( b, c^* \) branes from the orientifold plane in the third tori. Notice that the \( b, c^* \) branes are parallel along the second and third tori. The angle \( \bar{\theta}_1 \), was defined in (4.3) and \( \overline{m}_B^2 \) can be expressed in terms of the scalar masses of (4.1) and (4.4). We found

\[ \overline{m}_B^{PS-B} = \frac{1}{2} |m_{FR}(t_2) + m_{FR}(t_3) + m_{\chi_L}(t_2) + m_{\chi_L}(t_3)| \]
\[ \begin{align*}
&= \frac{1}{2} |m_{F_R}^2(t_2) + m_{F_R}^2(t_3) + m_{F_L}^2(t_2) + m_{F_L}^2(t_3)| \\
&= \frac{1}{2} |m_{F_L}^2(t_2) + m_{F_L}^2(t_3) + m_{\chi_R}^2(t_2) + m_{\chi_R}^2(t_3)| \\
&= \frac{1}{2} |m_{\chi_L}^2(t_2) + m_{\chi_L}^2(t_3) + m_{\chi_R}^2(t_2) + m_{\chi_R}^2(t_3)| \\
&= |m_{\chi_R}^2(t_2) + m_{\chi_L}^2(t_3)| = |m_{\chi_R}^2(t_3) + m_{\chi_L}^2(t_2)|
\end{align*} \] (4.26)

The number of \( h_1, h_2 \) fields in the \( bc^* \) intersection is given by the intersection number of the \( b, c^* \) branes in the first \( 16 \) tori for both PS-A, PS-B models,

\[ n_{h^\pm}^{bc^*} \overset{PS-A}{=} |\epsilon(m_{c^1} - m_{b^1})|, \] (4.27)

\[ n_{h^\pm}^{bc^*} \overset{PS-B}{=} \beta_1 |n_{b^1}^{1} + n_{c^1}^{1}|. \] (4.28)

A comment is in order. Because the number of the electroweak bidoublets in the PS-A models depends on the difference \( |m_{b^1}^1 - m_{c^1}^1| \), it is more natural to solve the remaining tadpole constraint (2.19) e.g. by making the choice

\[ m_{b^1}^1 - m_{c^1}^1 = -(\beta_2)(2n_a^2 + n_d^2). \] (4.29)

Hence, e.g. by choosing \( n_a^2 = 1, n_d^2 = 2, \beta_2 = 1/2 \), we get the constraint

\[ |m_{b^1}^1 - m_{c^1}^1| = 2, \] (4.30)

effectively choosing two electroweak Higgs bidoublets present. Within this choice a consistent numerical set of wrappings will be, we choose \( \epsilon = \tilde{\epsilon} = 1, m_{b^1} = -3, m_{c^1} = -1 \)

\[ N_a = 4 \quad (0, \ 1)(1, \ 3/2)(1, \ 1/2) \]

\[ N_b = 2 \quad (-1, \ -3)(2, \ 0)(1, \ 1/2) \]

\[ N_c = 2 \quad (1, \ -1)(2, \ 0)(1, -1/2) \]

\[ N_d = 1 \quad (0, \ 1)(2, \ 3)(-2, \ 1) \] (4.31)

### 4.2 Imposing \( N = 1 \) SUSY on Intersections

In this section, we will demand that certain sectors respect \( N = 1 \) supersymmetry. The reasons for doing so will become absolutely clear in the next section. Up to this point

\[^{16}\text{Note that in this section we imposed from the start that the } h_1, h_2 \text{ Higgs are present}\]
the massless spectrum of the PS-A, PS-B classes of models is that already described in table (1). In order for $N = 1$ SUSY to be preserved at some intersection between two branes $\alpha, \beta$ we need to satisfy $\pm \vartheta^1_{ab} \pm \vartheta^2_{ab} \pm \vartheta^3_{ab}$ for some choice of signs, where $\vartheta^i_{\alpha,\beta}, \ i = 1, 2, 3$ are the relative angles of the branes $\alpha, \beta$ across the three 2-tori.

A Majorana mass term for neutrinos is absent for PS-A, PS-B models when their massless spectrum is only the one given in table (1). This problem will disappear once we impose SUSY on intersections. That will have as an effect the appearance of the massless scalar superpartners of the $\bar{F}_R$ fermions, the $\bar{F}_H^R$’s, allowing a dimension 5 Majorana mass term for $\nu_R, F_R F_R \bar{F}_R \bar{F}_H^R$.

- **PS-A models**

  We demand that the sectors $ac, dd^*$ respect $N = 1$ supersymmetry. The conditions for $N = 1$ SUSY on the sectors $ac, dd^*$ are respectively:

  \[
  \pm (\frac{\pi}{2} + \tilde{\vartheta}_1) \pm \vartheta_2 \pm 2\vartheta_3 = 0, \quad (4.32)
  \]

  \[
  \pm \pi \pm 2\tilde{\vartheta}_2 \pm 2\vartheta_3 = 0 \quad (4.33)
  \]

  These conditions can be solved by the choice, respectively,

  \[
  ac \rightarrow (\frac{\pi}{2} + \tilde{\vartheta}_1) + \vartheta_2 - 2\vartheta_3 = 0, \quad (4.34)
  \]

  \[
  dd^* \rightarrow -\pi + 2\tilde{\vartheta}_2 + 2\vartheta_3 = 0 \quad (4.35)
  \]

  and thus may be solved by the choice \footnote{We have set $U^{(i)} = R_i^{(i)}, i = 1, 2, 3$} 17

  \[
  -\tilde{\vartheta}_1 = \tilde{\vartheta}_2 = \vartheta_2 = \vartheta_3 = \frac{\pi}{4}, \quad (4.36)
  \]

  effectively giving us

  \[
  m_c^1 U^{(1)} = \frac{6\beta_2}{n_d^2} U^{(2)} = \frac{3\beta_2}{n_a^2} U^{(2)} = \frac{1}{2} U^{(3)} = \frac{\pi}{4}. \quad (4.37)
  \]

  The latter condition implies

  \[
  2n_a^2 = n_d^2. \quad (4.38)
  \]

  A set of wrapping numbers consistent with this constraint can be seen in (4.31).

  By imposing $N = 1$ SUSY on sectors $ac, dd^*$ a massless scalar partner appears in each sector. They are the massless scalar superpartner of the fermions $\bar{F}_R, s_L$, namely the $\bar{F}_H^R, s_L^H$ respectively. An additional feature of, see (4.37), SUSY on intersections is
that the complex structure moduli $U^i$ takes specific values, decreasing the degeneracy of moduli parameters in the theory.

A comment is in order. If we list $^{18}$ the vectors $r$ describing a SUSY where we defined

$$
r_0 = \pm \frac{1}{2} (+--)
n_1 = \pm \frac{1}{2} (++-)
n_2 = \pm \frac{1}{2} (--+)
n_3 = \pm \frac{1}{2} (---)$$  \hspace{1cm} (4.39)

then the different SUSY’s preserved by the branes $a, c, d, d^*$ with the orientifold plane can be shown in table (7). As the intersection kl between branes k and l will preserve the common supersymmetries that the branes k and l share with the orientifold plane it is manifest from table (4.39) that the sectors $ac, dd^*$ preserve $N = 1$ SUSY. Also notice that the $c$-brane preserves a $N = 2$ SUSY.

| Brane | $\theta^1_a$ | $\theta^2_a$ | $\theta^3_a$ | SUSY preserved |
|-------|--------------|--------------|--------------|----------------|
| $a$   | \( \frac{\pi}{2} \) | \( \frac{\pi}{2} \) | \( \frac{\pi}{2} \) | $r_2$           |
| $c$   | $-\frac{\pi}{2}$ | 0            | $-\frac{\pi}{2}$ | $r_1, r_2$     |
| $d$   | \( \frac{\pi}{2} \) | \( \frac{\pi}{2} \) | \( \frac{\pi}{2} \) | $r_1$           |
| $d^*$ | \( \frac{3\pi}{2} \) | \( \frac{7\pi}{2} \) | \( \frac{5\pi}{2} \) | $r_1$           |

Table 7: Angle content for branes participating supersymmetric sectors of PS-A models. The supersymmetry that is preserved by each brane with the O$_6$-plane is shown.

- **PS-B models**

In these models there is no $dd^*$ sector, so we impose $N = 1$ SUSY on sector $ac$ only. The condition for $N = 1$ SUSY reads

$$\pm \tilde{\vartheta}_1 \pm \vartheta_2 \pm (\vartheta_3 + \tilde{\vartheta}_3) = 0$$ \hspace{1cm} (4.40)

and is solved by

$$\tilde{\vartheta}_1 + \vartheta_2 - (\vartheta_3 + \tilde{\vartheta}_3) = 0$$ \hspace{1cm} (4.41)

with

$$\frac{U^{(1)}}{U^{(3)}} = \frac{3\rho^2 n_c^1}{2\beta_1}, \quad \frac{U^{(2)}}{U^{(3)}} = \frac{3\rho^2 n_c^2}{2\beta_2}.$$ \hspace{1cm} (4.42)

---

$^{18}$see the 1st reference of [35].
4.3 Global symmetries

Proton decay is one of the most important problems of grand unification theories. In the standard versions of left-right symmetric PS models this problem could be avoided as B-L is a gauged symmetry but the problem persists in baryon number violating operators of sixth order, contributing to proton decay. In our models, PS-A or PS-B, proton decay is absent as baryon number survives as a global symmetry to low energies. That provides for an explanation for the origin of proton stability in general brane-world scenarios.

Clearly $Q_a = 3B + L$ and the baryon $B$ is given by

$$B = \frac{Q_a + Q_{B-L}}{4}. \quad (4.43)$$

As in the usual Pati-Salam model if the neutral component of $H_1$ (resp. $H_2$), $\nu_H$, assumes a vev, e.g $<\nu^H>$, then the initial gauge symmetry, $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d$, can break to the standard model gauge group $SU(3) \times U(2) \times U(1)_Y$ augmented by the non-anomalous $U(1)$ symmetry $Q^I$. Let us examine if it would be possible to break the extra $U(1)$ by appropriate Higgsing:

- **PS-A models**

  In those models, by imposing SUSY on sector $dd^*$ we have the appearance of the scalar superpartner of $s_L$, the $\tilde{s}_L$ with the same multiplicity. A linear combination of the $24\beta^2$ singlets $\tilde{s}_L$ gets charged under the anomaly free $U(1)$ symmetry (3.8) and thus breaks the PS-A models to exactly the much wanted SM gauge group structure, $SU(3) \otimes SU(2) \otimes U(1)_Y$. Note that it is necessary on phenomenological grounds to break the extra non-anomalous $U(1)$ (3.8) that survives massless to low energy, as the surviving gauge symmetry should be only of the observable standard model. Its breaking may be welcome as it provides the low energy standard model fermions with a flavour symmetry. In the case of the non-anomalous $U(1)$ (3.8) we deal with these models all SM fermions are not charged under it. Note that the extra non-anomalous $U(1)$ has some important phenomenological properties. In particular it does not charge the PS symmetry breaking Higgs scalars $H_1, H_2$ thus avoiding the appearance of axions. Note that the only issue remaining is how we can give non-zero masses to all fermions of table (1) beyond those of SM.

- **PS-B models**

  In this case, even by imposing SUSY on intersections it is not possible to create the Higgs particle with the right $U(1)$ charges that could break the extra non-anomalous $U(1)$ symmetry to the SM itself.
A comment is in order. We note that the $F_R^H$ scalars coming from the $ac$ sector could be used as Higgs scalars that can break the PS left-right symmetry at the $M_{GUT}$ scale. In this case it is not necessary to use the $H^\pm$ scalars as PS breaking Higgses.

Also the analysis of the Higgs sector and neutrino couplings (that follows) are independent of the choices of extra $U(1)$'s, (3.8), (3.12), (3.13).

5 Neutrino couplings and masses

The analysis of neutrino masses that follows is valid for both PS-A, PS-B models. However, as we will see later in this subsection the class of PS-B models have some shortcomings, e.g. the fermions $\chi_L, \chi_R$ could not get a mass.

On the contrary, the class of PS-A models has some remarkable features. Namely, all extra fermions apart from SM one’s get a mass and disappear from the low energy spectrum. The only particles with light mass close to the electroweak scale are those of fermions $\chi_L$. We note that the fermions $\chi_L, \chi_R$ is a general prediction of general left-right symmetric models in intersecting brane models of type I strings and the mechanism of making them massive was unknown. Here, we find a way for giving them a mass in the context of PS models.

In intersecting brane worlds trilinear Yukawa couplings between the fermion states $F^i_L, \tilde{F}^j_R$ and the Higgs fields $H^k$ arise from the stretching of the worldsheet between the three D6-branes which cross at those intersections. Its general form for a six dimensional torus is in the leading order [15],

$$Y^{ijk} = e^{-\tilde{A}_{ijk}},$$

where $\tilde{A}_{ijk}$ is the worldsheet area connecting the three vertices. The areas of each of the two dimensional torus involved in this interaction is typically of order one in string units. To simplify matters we can without loss of generality assume that the areas of the second and third tori are close to zero. In this case, the area of the full Yukawa coupling (5.1) reduces to

$$Y^{ijk} = e^{-R_1 R_2 A_{ijk}},$$

where $R_1, R_2$ the radii and $A_{ijk}$ the area of the two dimensional tori in the first complex plane. For a dimension five interaction term, like those involved in the Majorana mass term for the right handed neutrinos the interaction term is in the form

$$Y^{lmni} = e^{-\tilde{A}_{lmni}},$$
where $\tilde{A}_{lmni}$ the worldsheet area connecting the four interaction vertices. Assuming that the areas of the second and third tori are close to zero, the four term coupling can be approximated as
\begin{equation}
Y^{ijk} = e^{-\frac{R_1 R_2}{\alpha'} A_{lmni}}, \quad (5.4)
\end{equation}
where the area of the $A_{lmni}$ may be of order one in string units.

The full Yukawa interaction for the chiral spectrum of the PS-A, PS-B models reads:
\begin{equation}
\lambda_1 F_L \tilde{F}_R h + \lambda_2 \frac{F_R F_R F^H_R F^H_R}{M_s}, \quad (5.5)
\end{equation}
where
\begin{equation}
\lambda_1 \equiv e^{-\frac{R_1 R_2 A_1}{\alpha'}}, \quad \lambda_2 \equiv e^{-\frac{R_1 R_2 A_2}{\alpha'}}. \quad (5.6)
\end{equation}
and the Majorana coupling involves the massless scalar $^{19}$ partners $F^H_R$ of the antiparticles $\tilde{F}_R$. This coupling is unconventional, in the sense that the $F^H_R$ is generated by imposing SUSY on an sector of a non-SUSY model. We note the presence of $N = 1$ SUSY at the sector $ac$. As can be seen by comparison with (2.6) the $F^H_R$ has a neutral direction that receives the vev $<H>$. There is no restriction for the vev of $F^H_R$ from first principles and can be anywhere between the scale of electroweak symmetry breaking and $M_s$.

The Yukawa term
\begin{equation}
F_L \tilde{F}_R h, \quad h = \{h_1, h_2\}, \quad (5.7)
\end{equation}
is responsible for the electroweak symmetry breaking. This term is responsible for giving Dirac masses to up quarks and neutrinos. In fact, we get
\begin{equation}
\lambda_1 F_L \tilde{F}_R h \rightarrow (\lambda_1 v)(u_i u^c_j + \nu_i N^c_j) + (\lambda_1 \bar{\nu}) \cdot (d_i d^c_j + e_i e^c_j), \quad (5.8)
\end{equation}
where we have assumed that
\begin{equation}
<h> = \begin{pmatrix} v & 0 \\ 0 & \bar{\nu} \end{pmatrix} \quad (5.9)
\end{equation}
We observe that the model gives non-zero tree level masses to the fields present. These mass relations may be retained at tree level only, since as the model has a non-supersymmetric fermion spectrum, it breaks supersymmetry on the brane, it will receive higher order corrections. It is interesting that from (5.9) we derive the GUT relation [33]
\begin{equation}
m_d = m_e. \quad (5.10)
\end{equation}

\(^{19}\)Of order of the string scale.
Table 8: Observe that the string scale cannot be at the TeV but lower. Restricting the masses, $\nu^2/M_s$, of left handed fermion doublets $\chi_L$ to values greater than 90 GeV and up to 246 GeV, “pushes” the string scale to values less than 650 GeV. The lower mass limit of $\chi_L$ pushes the $M_s$ to maximum value.

as well the “unnatural”

$$m_u = m_{Nc\nu}.$$  (5.11)

In the case of neutrino masses, the “unnatural” (5.11), associated to the $\nu - N^c$ mixing, is modified due to the presence of the Majorana term in (5.5) leading to a see-saw mixing type neutrino mass matrix in the form

$$(\nu \ N^c) \times \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \times \begin{pmatrix} \nu \\ N^c \end{pmatrix},$$  (5.12)

where

$$m = \lambda_1 \nu.$$  (5.13)

After diagonalization the neutrino mass matrix gives us two eigenvalues, the “heavy” eigenvalue

$$m_{\text{heavy}} \approx M = \lambda_2 \frac{<H>^2}{M_s},$$  (5.14)

corresponding to the interacting right handed neutrino and the “light” eigenvalue

$$m_{\text{light}} \approx \frac{m^2}{M} = \frac{\lambda_2^2}{\lambda_2} \times \nu^2 \frac{M_s}{<H>^2},$$  (5.15)

corresponding to the interacting left handed neutrino. Note that the neutrino mass matrix is of the type of an extended Frogatt-Nielsen mechanism [23] mixing light with heavy states.

Values of the parameters giving us values for neutrino masses between 0.1-10 eV, consistent with the observed neutrino mixing in neutrino oscillation measurements, are shown in table (9). The nature of the parameters involved in the Yukawa couplings (5.1), generate naturally the hierarchy between the neutrino masses in the models.

In fact the hierarchy of neutrino masses can be investigated further by examining several different scenarios associated with a light $\nu_L$ mass. As can be seen in table 9 there are two main options that are available to us:
Table 9: Choices of the neutrino mass parameters for the $SU(4)_c \times SU(2)_L \times SU(2)_R$ type I model, giving us hierarchical values of neutrino masses between 0.1-10 eV in consistency with oscillation experiments. The Majorana mass term for the right handed neutrinos involves a massive scalar superpartner with mass of order of the string scale. The top row shows the neutrino mass hierarchy when $\langle H \rangle = M_s$ while the bottom part when $\langle H \rangle < M_s$. The analysis is valid for PS-A, PS-B classes of models.

- $\langle H \rangle = |M_s|$
  
  A long as the equality is preserved a consistent hierarchy of neutrino masses is easily obtained. It is important to note that the string scale cannot be at the TeV but as we will show later it is constrained from the existence of the light doublets $\chi_L$, to be less than 650 GeV. For simplicity, in table (9) we examine values of $M_s$ less than 600 GeV. As long as $\langle H \rangle = |M_s|$, the value of the $\lambda_2$ coupling should take the value one. In this case, the area $A_2$ should tend to zero in order to have a non-zero value for the product radii $R_1 \cdot R_2$, e.g $R_1 \cdot R_2 \neq 0$.

- $\langle H \rangle < |M_s|$
  
  In this case the structure of the theory is enough to constrain the ratio of the areas $A_1$, $A_2$ involved in the couplings of the see-saw mechanism. Let us look for example at the top row of the lower half of the table (9). By substituting the values of $M_s$, $\langle H \rangle$, $m_{\nu_L}$, $m_{\nu_R}$ in (5.14), (5.15), we get the constraint equations

$$m_{\nu_R} \to R_1 R_2 A_2 = 0.05, \ m_{\nu_L} \to R_1 R_2 A_1 = 3.67$$  \hspace{1cm} (5.16)

effectively determining the value of the ratio $A_1/A_2 = 77$ independently of the value of the product moduli $R_1 R_2$. We note that because of the special nature

| $M_s$ GeV | $\lambda_2$ | $A_2$ | $\langle H \rangle$ GeV | $A_1$ | $R_1 R_2$ | $m_{\nu_R} \leq E$ GeV | $m_{\nu_L}$ eV |
|-----------|-----------|-------|-----------------|-----|-----------|----------------|-----------|
| 600       | $\to 1$   | $\to 0$| 600             | 0.7 | 8         | 600             | 0.1       |
| 600       | $\to 1$   | $\to 0$| 600             | 0.79| 8         | 600             | 1         |
| 600       | $\to 1$   | $\to 0$| 600             | 0.96| 6         | 600             | 10        |
| 500       | $\to 1$   | $\to 0$| 500             | 0.80| 8         | 500             | 1         |
| 500       | $\to 1$   | $\to 0$| 500             | 0.97| 6         | 500             | 10        |
| 550       | 0.906     | 77$A_1$| 500             | $\neq 0$ | $\neq 0$ | 453             | 1         |
| 550       | 0.906     | 125$A_1$| 500             | $\neq 0$ | $\neq 0$ | 453             | 10        |
| 550       | 0.906     | 142$A_1$| 500             | $\neq 0$ | $\neq 0$ | 453             | 0.1       |
of (5.14) it is possible given the values for the string and the PS breaking scale to determine the maximum values of \( \nu_R \)'s such that the product radii \( R_1R_2 \) is positive. A range of values for \( \nu_R \) masses is shown in table (10).

Notice that we have investigated the neutrino masses corresponding to the first generation. This result could be extended to cover all three generations.

Several comments are in order:

1. **PS-A models**

   Our main objective in this part is to show that all additional particles, appearing in table (1), beyond those of SM get a heavy mass and disappear from the low energy spectrum. The slight exception will be the light mass of \( \chi_L \) which is of order of the electroweak symmetry breaking scale.

   Lets us discuss this issue in more detail. The left handed fermions \( \chi_L \) receive a mass from the coupling

\[
(1, 2, 1)(1, 2, 1) e^{-A} \frac{<h_2><h_2><\tilde{F}_R^H><H_1><\tilde{s}_L^H>}{M_s^4} \sim 0 \frac{v^2}{M_s} (1, 2, 1)(1, 2, 1) \tag{5.17}
\]

explicitly, in representation form, given by

\[
(1, 2, 1)_{(0, 1, 0, -1)} (1, 2, 1)_{(0, 1, 0, -1)} <(1, 2, 2)_{(0, -1, -1, 0)}> <(1, 2, 2)_{(0, -1, -1, 0)}> \\
\times <(\bar{4}, 1, 2)_{(-1, 0, 1, 0)}> <(4, 1, 2)_{(1, 0, 1, 0)}> <(1, 0, 0, 2)> \tag{5.18}
\]

where we have included the leading contribution of the worksheet area connecting the seven vertices. In the following for simplicity reasons we will set the leading contribution of the different couplings to one (e.g. area tends to zero). Altogether, \( \chi_L \) receives a low mass of order \( v^2/M_s \). Because there are no experimentally observed charged fermions, as can be seen for \( e^+e^- \) interactions\(^{20}\), below 90 GeV, by lowering the string

\[^{20}\text{I thank Luis Ibáñez for this comment.}\]

| \( M_s \) (GeV) | \( \langle H \rangle \) (GeV) | \( M_{\nu_R} < E \) (GeV) |
|-----------------|-----------------|-----------------|
| 600.0           | 500.0           | 416.7           |
| 550.0           | 500.0           | 454.5           |
| 500.0           | 400.0           | 320.0           |
| 600.0           | 400.0           | 266.7           |
| 650.0           | 470.1           | 500.0           |
| 650.0           | 600.0           | 553.8           |

Table 10: Bounds on \( \nu_R \) for PS models, given the scales \( M_s, \langle H \rangle \).
scale below 1 TeV, in fact below 650 GeV, we can push the $SU(2)_L$ fermions $\chi_L$ in the range between 90 GeV and the scale of electroweak symmetry breaking. A range of values showing different values of the string scale in connection to $\chi_L$ masses is shown in table (8).

Also, the $\chi_R$ doublet fermions receive heavy masses in the following way. The mass term

$$\frac{(1, 1, 2)(1, 1, 2) < H_2 > < F^H_R > < s^H_L >}{M_s^2}$$

(5.19)
can be realized. In explicit representation form

$$(1, 1, 2)_{(0,0,1,1)} \times (1, 1, 2)_{(0,0,1,1)} < (\bar{4}, 1, \bar{2})_{(-1,0,-1,0)} > < (4, 1, \bar{2})_{(1,0,-1,0)} > < 1_{(0,0,0,2)} >$$

(5.20)

With vev’s $< H_2 > \sim < F^H_R > \sim M_s$, the mass of $\chi_R$ is of order $< s^H_L > / M_s$. We note that in principle the vev of $s^H_L$, setting the scale of breaking of the extra anomaly free $U(1)$ could be anywhere between $< v >$ and $M_s$. However, since $M_s$ is constrained to be less or equal to 650 GeV, given the proximity of the intermediate scale $s^H_L$ and the string scale, we could suppose for the rest of this work that $s^H_L \sim M_s$. However, in principle the vev of $s^H_L$ can be anywhere between 246 GeV and $M_s$, the latter up to 650 GeV.

The 10-plet fermions $z_R$ receive a heavy mass of order $M_s$ from the coupling

$$\frac{(10, 1, 1)(10, 1, 1) < F^H_R > < F^H_R > < H_2 > < H_2 >}{M_s^3}$$

(5.21)

where we have used the tensor product representations for $SU(4)$, $10 \otimes 10 = 20 + 35 + 45$, $20 \otimes \bar{4} = \bar{1}5 + 20$, $20 \otimes \bar{4} = \bar{6} + 10$, $10 \otimes \bar{4} = 4 + 36$, $4 \otimes \bar{4} = 1 + 15$. Explicitly, in representation form,

$$(10, 1, 1)_{(2,0,0,0)}(10, 1, 1)_{(2,0,0,0)} < (\bar{4}, 1, 2)_{(-1,0,1,0)} > < (4, 1, \bar{2})_{(-1,0,1,0)} >$$

$$\times < (\bar{4}, 1, 2)_{(-1,0,-1,0)} > < (4, 1, \bar{2})_{(-1,0,-1,0)} >$$

(5.22)

The 6-plet fermions, $\omega_L$, receive a mass term of order $M_s$ from the coupling, e.g. for $\omega_L$

$$\frac{(\bar{6}, 1, 1)(\bar{6}, 1, 1) < H_1 > < F^H_R > < H_1 > < F^H_R >}{M_s^3}$$

(5.23)

where we have made use of the $SU(4)$ tensor products $6 \otimes 6 = 1 + 15 + 20$, $4 \otimes 4 = 6 + 10$. Explicitly, in representation form,

$$(\bar{6}, 1, 1)_{(-2,0,0,0)} \times (\bar{6}, 1, 1)_{(-2,0,0,0)} < (4, 1, 2)_{(1,0,1,0)} > < (4, 1, 2)_{(1,0,1,0)} >$$

$$\times < (4, 1, \bar{2})_{(1,0,-1,0)} > < (4, 1, \bar{2})_{(1,0,-1,0)} >$$

(5.24)
Finally, the singlet fermions $s_L$ receive a mass of order $M_s$ from the coupling

$$\bar{s}_L s_L \frac{<s^H_L>}{M_s} <s^H_L>$$

Thus only the chiral fermion content of the SM fermions remains at $M_Z$.

- **PS-B models**

  While the neutrino sector of those models can give small masses to neutrinos, the main shortcoming of the models is that the fermion doublets $\chi_L, \chi_R$ remain massless down to the electroweak scale in contrast with the observed low energy phenomenology. Also the $U(1)$ symmetry (3.11) survives unbroken to low energies. Thus PS-B models are phenomenologically not interesting in this respect.

## 6 Conclusions

In this work, we have presented the first examples of four dimensional string grand unified models that can give at low energy exactly the observable standard model spectrum and gauge interactions. These models, characterized as PS-A class in this work, are based on the Pati-Salam gauge group $SU(4)_C \times SU(2)_L \times SU(2)_R$ and are derived from D6-branes intersecting at non-trivial angles in four dimensional type I compactifications on a six dimensional orientifolded torus. The models have their quarks and leptons accommodated in three generations, and possess some remarkable features. Among them we mention that the models give some answers as matter as it concerns one of the most difficult aspects of gauge hierarchy, apart from the hierarchy of scales, that is the smallness of neutrino masses.

In this case it is particularly easy for the theory to accommodate a neutrino mass hierarchy between 0.1-10 eV consistent with oscillation measurements.

Throughout the paper we distinguished the different PS GUT solutions according to if the tadpoles admit or not exotic, antisymmetric and symmetric, representations of $U(N_a)$ groups coming from brane-orientifold image brane, $\alpha\alpha^*$, sectors. In this way, PS-A models, that give exactly the SM at low energies, possess $\alpha\alpha^*$ sectors. On the contrary, PS-B models which don’t admit $\alpha\alpha^*$ sectors, failed to produce just the SM at low energies. However, some important conclusions were derived from the study of PS-B models. We got an interpretation of the appearance of multi-brane wrapping in intersecting branes. It appears that, since in the absence of a stringy Higgs effect no more additional $U(1)$’s may be introduced, the additional $U(1)$’s can be absorbed into a trivial field redefinition of the non-anomalous $U(1)$, surviving the Green-Schwarz
mechanism at low energies. Moreover, colour triplet Higgs couplings that could couple to quarks and leptons and cause a problem to proton decay are absent in all classes of models. Proton is stable as baryon number survives as global symmetry to low energies.

We should note that a hint of motivation from searching for Grand Unified models (GUTS), comes from the fact that very recently, there is evidence from neutrinoless beta-decay, even though not conclusive, for the existence of non-zero Majorana masses for neutrinos and lepton number violation [37].

Despite the fact that the models we examined are free of RR tadpoles and, if the angle stabilization conditions of Appendices I, II hold, free of tachyons, they will always have NSNS tadpoles that cannot all be removed. The closed string NSNS tadpoles can be removed by freezing the complex moduli to discrete values [19], or by redefining the background in terms of wrapped metrics [38]. However, a dilaton tadpole will always remain that could in principle reintroduce tadpoles in the next leading order. A different mechanism, involving different type I compactification backgrounds to the one used in this article, that could avoid global tadpoles was described in [39]. We note that for PS-A models the complex structure moduli\textsuperscript{21} can be fixed to discrete values, e.g. see (4.37).

One point that there was no obvious stringy solution with general orientifolded six-torus compactifications is that these models do not offer an apparent explanation for keeping the string scale low [6], e.g. to 1-100 TeV region. This aspect of the hierarchy that makes the Planck scale large, while keeping the string scale low, by varying the radii of the transverse directions [6] does not apply here, as there are no transverse torus directions simultaneously to all D6-branes [4]. A possible solution, even though such manifolds are not known, was suggested in [15], could involve cutting a ball, to a region away from the D6-branes, and gluing a throat connecting the T6 torus to a large volume manifold. However, in this work we suggested an alternative mechanism that keeps the string scale $M_s$ low. In particular the existence of the light weak doublets in the PS-A models with a mass of order up to 246 GeV, makes a definite prediction for a low string scale in the energy range less than 650 GeV. That effectively, makes the PS-A class of D6-brane models directly testable to present or feature accelerators.

The general structure of the GUT models with PS structure presented in this article contains at low energy the standard model augmented by a non-anomalous $U(1)$

\textsuperscript{21}The Kähler moduli could be fixed from its value at the string scale, using for example the product radii in (5.16) but that would mean too large fine tuning for our theory to be naturally existent.
symmetry. For the PS-A class this additional $U(1)$ was broken by extra singlets that were created after modifying certain non-SUSY sectors such that they preserve $N = 1$ supersymmetry. Thus it appears that the model has $N = 1$ SUSY sectors even though overall is a non-SUSY model. Furthermore, the broken, anomaly free $U(1)$ symmetry, charges the fermions of the standard model with an interesting flavour symmetry.

String models, similar to present, without the presence of exotic matter and/or additional gauge group content (from gravity mediating “hidden” sectors) a low energies, has appeared in [16, 17], where however, the authors were able to have just the standard model at low energies without using a grand unified structure.

Also, it will be interesting to extend the methods employed in this article, to other GUT groups. Summarizing, in the present work, we have shown that we can start from a realistic Pati-Salam structure at the string scale and derive the first GUT string examples with exactly the observable standard model at low energies.

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7 Appendix I

In the appendix we list the conditions, mentioned in subsection (4.1), under which the PS-A model D6-brane configurations of tadpole solutions of table (3), are tachyon free. Note that the conditions are expressed in terms of the angles defined in (4.2).

\[-\vartheta_1 + \vartheta_2 + 2\vartheta_3 \geq 0\]
\[-\left(\frac{\pi}{2} + \tilde{\vartheta}_1\right) + \vartheta_2 + 2\vartheta_3 \geq 0\]
\[\left(\frac{\pi}{2} - \vartheta_1\right) + \pi - \vartheta_2 + \pi - 2\vartheta_3 \geq 0\]
\[-\left(\frac{\pi}{2} - \tilde{\vartheta}_1\right) + \pi - \vartheta_2 + \pi - 2\vartheta_3 \geq 0\]
\[\vartheta_1 - \vartheta_2 + 2\vartheta_3 \geq 0\]
\[-\left(\frac{\pi}{2} - \tilde{\vartheta}_1\right) - \vartheta_2 + 2\vartheta_3 \geq 0\]
\[-\left(\frac{\pi}{2} + \vartheta_1\right) + (-\pi + \vartheta_2) + (\pi - 2\vartheta_3) \geq 0\]
\[\left(\frac{\pi}{2} + \tilde{\vartheta}_1\right) + (-\pi + \vartheta_2) + (\pi - 2\vartheta_3) \geq 0\]
\[\vartheta_1 + \vartheta_2 - 2\vartheta_3 \geq 0\]
\[\left(\frac{\pi}{2} - \tilde{\vartheta}_1\right) + \vartheta_2 - 2\vartheta_3 \geq 0\]
\[\left(-\frac{\pi}{2} + \vartheta_1\right) + (\pi - \vartheta_2) + (-\pi + 2\vartheta_3) \geq 0\]
\[\left(\frac{\pi}{2} + \vartheta_1\right) + (\pi - \vartheta_2) + (-\pi + 2\vartheta_3) \geq 0\]
8 Appendix II

In the appendix we list the conditions, mentioned in subsection (4.1), under which the PS-B model D6-brane configurations of tadpole solutions of table (1), are tachyon free. Note that the conditions are expressed in terms of the angles defined in (4.3) and furthermore we have take into account that \( \vartheta_3 = \tilde{\vartheta}_3 \).

\[
\begin{align*}
-\vartheta_1 + \vartheta_2 + 2\tilde{\vartheta}_3 & \geq 0 \\
-\tilde{\vartheta}_1 + \vartheta_2 + 2\tilde{\vartheta}_3 & \geq 0 \\
-\vartheta_1 + \tilde{\vartheta}_2 + 2\tilde{\vartheta}_3 & \geq 0 \\
-\tilde{\vartheta}_1 + \tilde{\vartheta}_2 + 2\tilde{\vartheta}_3 & \geq 0 \\
\vartheta_1 - \vartheta_2 + 2\tilde{\vartheta}_3 & \geq 0 \\
\tilde{\vartheta}_1 - \vartheta_2 + 2\tilde{\vartheta}_3 & \geq 0 \\
\vartheta_1 - \tilde{\vartheta}_2 + 2\tilde{\vartheta}_3 & \geq 0 \\
\tilde{\vartheta}_1 - \tilde{\vartheta}_2 + 2\tilde{\vartheta}_3 & \geq 0 \\
\vartheta_1 + \vartheta_2 - 2\tilde{\vartheta}_3 & \geq 0 \\
\tilde{\vartheta}_1 + \vartheta_2 - 2\tilde{\vartheta}_3 & \geq 0 \\
\vartheta_1 + \tilde{\vartheta}_2 - 2\tilde{\vartheta}_3 & \geq 0 \\
\tilde{\vartheta}_1 + \tilde{\vartheta}_2 - 2\tilde{\vartheta}_3 & \geq 0
\end{align*}
\] (8.1)
9 Appendix III

The PS-B models appearing in table one can be proved that can be equivalent to the ones created after assigning the alternative accommodation of fermions charges, of table 11, below:

| Intersection | $SU(4)_C \times SU(2)_L \times SU(2)_R$ | $Q_a$ | $Q_b$ | $Q_c$ | $Q_d$ |
|--------------|----------------------------------------|------|------|------|------|
| $I_{ab'} = 3$ | $3 \times (4, 2, 1)$ | 1 | 1 | 0 | 0 |
| $I_{ac} = -3$ | $3 \times (4, 1, 2)$ | $-1$ | 0 | 1 | 0 |
| $I_{bd'} = -12$ | $12 \times (1, 2, 1)$ | 0 | $-1$ | 0 | $-1$ |
| $I_{cd} = -12$ | $12 \times (1, 2, 1)$ | 0 | 0 | $-1$ | 1 |

Table 11: Alternative accommodation of chiral spectrum for the $SU(4)_C \times SU(2)_L \times SU(2)_R$ type I PS-B models, discussed in the main body of the paper, together with $U(1)$ charges.

For the accommodation of Pati-Salam models with alternative fermion charges listed in table 11, the full solutions to the tadpole constraints are given by the following tables:

| $N_i$ | $(n^1_i, m^1_i)$ | $(n^2_i, m^2_i)$ | $(n^3_i, m^3_i)$ |
|-------|-----------------|-----------------|-----------------|
| $N_a = 4$ | $(1/\beta_1, 0)$ | $(n^2_a, -\epsilon \beta_2)$ | $(1/\rho, 3\beta_2^2/2)$ |
| $N_b = 2$ | $(n^1_b, \epsilon \beta_1)$ | $(1/\beta_2, 0)$ | $(1/\rho, 3\beta_2^2/2)$ |
| $N_c = 2$ | $(n^1_c, \epsilon \beta_1)$ | $(1/\beta_2, 0)$ | $(1/\rho, -3\beta_2^2/2)$ |
| $N_d = 1$ | $(\alpha / \beta_1, 0)$ | $(n^2_d, \gamma \epsilon \beta_2)$ | $(1/\rho, 3\beta_2^2/2)$ |

Table 12: First class of solutions for alternative accommodation of fermion charges, of D6-branes wrapping numbers giving rise to the fermionic spectrum of the $SU(4)_C \times SU(2)_L \times SU(2)_R$ type I PS-B models of table (1). The parameter $\rho$ takes the values 1, 1/3, while there is an additional dependence on four integer parameters, $n^2_a, n^2_d, n^1_b, n^1_c$, the NS-background $\beta_i$ and the phase parameter $\epsilon = \pm 1$. Note the condition $\alpha \gamma = 4$ and the positive wrapping number entry on the 3rd tori of the a-brane.
Table 13: Second class of solutions, for alternative accommodation of fermion charges, of D6-branes wrapping numbers giving rise to the fermionic spectrum of the $SU(4)_C \times SU(2)_L \times SU(2)_R$ type I PS-B models of table (1). The parameter $\rho$ takes the values $1, 1/3$, while there is an additional dependence on four integer parameters, $n^2_a, n^2_d, n^1_b, n^1_c$, the NS-background $\beta_i$ and the phase parameter $\epsilon = \pm 1$. Note the condition $\alpha \gamma = 4$ and the positive wrapping number entry on the 3rd tori of the a-brane.

The surviving $U(1)$ anomalous in this case reads:

$$\tilde{Q}_l = (Q_b - Q_c) - (Q_a + Q_d),$$

where an identical set of wrapping number solutions to (2.30) has been chosen. The low energy theory is the standard model augmented by the global gauged $U(1)$ $\tilde{Q}_l$. 

| $N_i$ | $(n^1_i, m^1_i)$ | $(n^2_i, m^2_i)$ | $(n^3_i, m^3_i)$ |
|-------|-----------------|-----------------|-----------------|
| $N_a = 4$ | $(1/\beta_1, 0)$ | $(n^2_a, -\epsilon \beta_2)$ | $(1/\rho, -\frac{3\rho}{2})$ |
| $N_b = 2$ | $(n^1_b, \epsilon \beta_1)$ | $(1/\beta_2, 0)$ | $(-1/\rho, \frac{3\rho}{2})$ |
| $N_c = 2$ | $(n^1_c, \epsilon \beta_1)$ | $(1/\beta_2, 0)$ | $(-1/\rho, \frac{3\rho}{2})$ |
| $N_d = 1$ | $(\alpha/\beta_1, 0)$ | $(n^2_d, \gamma \epsilon \beta_2)$ | $(1/\rho, -\frac{3\rho}{2})$ |
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