Abstract

The number of infections and the number of fatalities in the 2019 novel coronavirus epidemics follows a remarkably regular trend. Since the end of January, the ratio of fatalities per infection is about 2% and remarkably stable. Here we show that, since January 20, the number of fatalities increases quadratically and not exponentially. At present, no departure from this behavior can be seen, allowing tentative predictions to be made for the next 1–2 months.

1 Introduction

Daily news reports suggest a steady, nearly exponential increase in the number of fatalities and infections in the 2019 novel coronavirus (2019-nCoV) outbreak\textsuperscript{1,2}. This wealth of quantitative data must be contrasted against the qualitative judgments of what may appear like a sudden increase in the number of cases from one day to the next. In the popular press, there are also speculations about a large number of unreported cases, casting doubt on the usefulness of the reported numbers. Although this is justified in view of the length of the incubation period of about 6 days in this case\textsuperscript{3}, it affects all data uniformly. Instead of such subjective descriptions, it could be more valuable and informative to quote, for example, the current $e$-folding time and the perhaps the time scale of its change. Ideally, however, it would be desirable to quantify the parameters of the underlying differential equation describing the growth of fatalities and infections.

Significant effort is currently also going into quantitative modeling of the 2019-nCoV epidemic. Particularly noteworthy in this connection is the time delay model\textsuperscript{4} the prediction model\textsuperscript{5} and a model involving the basic reproduction number\textsuperscript{6}. We feel, however, that at the current time, more detailed diagnostics is needed to monitor the change in the epidemics. This is the purpose of the present paper.

2 Results

The daily news reports provide an easily accessible source of information that appears to reveal a remarkably regular trend; see the 2019-nCoV situation reports of the World Health Organization\textsuperscript{7} and the DEVEX\textsuperscript{8} and worldometer\textsuperscript{9} data bases. In Table 1 we list the number of fatalities, infections, and the ratio of fatalities per infection.

A graphic representation of the increase in the number of fatalities and infections is given in Figure 1. It is tempting to fit the data with an exponential growth,\textsuperscript{10} $n(t) = n_0 \exp(t/\tau)$, but it is clear that the instanta-
Figure 1: Number of fatalities (black solid line) and infections, scaled by 0.022 (red dashed line), and the fit described later in the text (blue dotted line) in a semi-logarithmic representation. Negative days count the days in January backward.

Figure 2: Instantaneous e-folding time $\tau(t) = \epsilon(t - t_0)$ for the numbers of fatalities (black solid line) and infections, scaled by 0.022 (red dashed line), and the linear fit described in the text (blue dotted line).

Figure 3: Similar to Figure 1 but in a double-logarithmic representation. The straight solid blue line denotes the fit.

e-fulding time $\tau$ would need to increase gradually for achieving a reasonable fit both at early and late times.

In fact, a possible fit to the data is provided by a model in which the e-folding time is allowed to change linearly in time, i.e., $n(t) = n_0 \exp[t/\tau(t)]$, where $\tau(t) = \epsilon(t - t_0)$ models a linear increase and $\epsilon$ is a constant factor. In Figure 2 we plot $\tau(t) = (d \ln n/dt)^{-1}$ along with the fit. One can see that $\tau$ increases by one day every two days, i.e., $\epsilon = 1/2$. This finding will turn out to be significant.

Let us now discuss how the growth with a variable e-folding time could be described in terms of a population dynamics model. At the level of an ordinary differential equation, a strictly exponential growth corresponds to the equation $dn/dt = n/\tau$. If we now assume $\tau(t)$ to increase linearly in time, as stated above, we have

$$\frac{dn}{dt} = \frac{n}{\epsilon(t - t_0)}$$  \hspace{1cm} (1)

with the solution

$$n(t) = \left[(t - t_0)/\tau\right]^{1/\epsilon},$$  \hspace{1cm} (2)

where $\tau$ is an integration constant. Given that $\epsilon = 1/2$, the growth in Equation (2) is quadratic and we can state the final equations in the following explicit form:

$$n_{\text{fatal}}(t) = \left[(t - \text{Jan 20})/0.7\text{ days}\right]^2,$$  \hspace{1cm} (3)

and, because $0.7 \times \sqrt{0.022} \approx 0.1$, we have

$$n_{\text{infect}}(t) = \left[(t - \text{Jan 20})/0.1\text{ days}\right]^2.$$  \hspace{1cm} (4)

3 Conclusions

The present work has demonstrated that for the 2019-nCoV epidemic, the available data are accurate enough to distinguish between an early exponential growth, as was found for the 2009 A/H1N1 influenza pandemic in Mexico City and the quadratic growth found here. It was expected that the growth would not continue to be exponential, and that it would gradually level off in response to changes in the population behavior and interventions as found in the 2014-15 Ebola epidemic in West Africa. However, that the growth turns out to be quadratic to high accuracy is rather surprising and has not previously been predicted by...
any of the recently developed models of the 2019-nCoV epidemic. It is also a remarkable that the fit isolates January 20 as a crucial time in the development of the outbreak. At that time, the actual death toll was just three and the number of confirmed infections just a little over 200; see Table 1. The 2% fraction of fatalities per infection has not yet been established.

It is rare that an epidemic provides us with such clean data as in the present case of the 2019-nCoV outbreak. At the moment, the quadratic growth of the epidemic does not show any sign of a decline, and so Equation (3) predicts a continued increase and a death toll of about 10,000 by April 1. This number is still significantly less than 0.1% of the population of Wuhan, but it may be hoped that control interventions prevent this from happening.

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I declare no competing interests.

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Appendix: Peripheral spreading of the 2019 novel coronavirus

The following material contains an explanation of the quadratic growth behavior in terms of what can be called peripheral growth. Contrary to the usual exponential growth, a quadratic growth is the result of control interventions. It can be explained by spreading on the periphery of a bulk structure, which can be geometrical or sociological nature.

Introduction
In spite of the drastic confinement efforts, the 2019 novel coronavirus (2019-nCoV) epidemic continues to claim lives at a high rate. As is well known in population dynamics, because of such control efforts, the number of fatalities follows a subexponential growth. Ideas to explain the resulting growth of $N$ in terms of a linear increase in the characteristic time scale are not fully satisfactory; see the main part of the paper. They predict an algebraic increase with time $t$, i.e., $N \propto t^{\gamma}$, but they do not rigorously constrain the value of the exponent $\gamma$, which must instead be established empirically (see the main part of the paper).

The model
Here we propose a model where the continued confinement efforts prevent the spreading of the bulk of the infected population, but they cannot prevent spreading on its periphery; see Figure 1 for a sketch. The rate $\frac{dN}{dt}$, with which $N$ increases with time, is therefore equal to the number of infected people on the periphery divided by a characteristic spreading time $\tau$. We therefore arrive at the following simple differential equations:

$$\frac{dN}{dt} = \frac{n}{\tau}, \quad (5)$$

where $n \approx 2\sqrt{\pi N}$ is the number of people in the periphery. Here, the prefactor depends on the geometry and we would
have \( n = 4\sqrt{N} \) for a rectangular geometry. We may therefore set \( n = \alpha\sqrt{N} \), where \( \alpha \approx 3.5 \) for a circular geometry. Inserting this into Equation (5) yields

\[
\frac{dN}{dt} = \frac{\alpha}{\tau} \sqrt{N},
\]

with the solution

\[
N(t) = (\alpha t/2\tau)^2.
\]