High-efficiency single-photon Fock state production by transitionless quantum driving

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Abstract

A single-photon source is one of the key devices for optical quantum information processing. Differing from the usual stimulated Raman adiabatic passage to obtain single-photon radiation, here we propose an approach to produce an optical Fock state on demand in the usual atom–cavity system by utilizing the technique of transitionless quantum driving. The present proposal effectively suppresses the unwanted but practically unavoidable nonadiabatic transitions in the previous adiabatic schemes. Therefore, the efficiency of Fock state production by the present technique could be significantly high, even in the presence of various atomic and cavity dissipations.

Keywords: high efficiency, single photon, transitionless quantum driving

(Some figures may appear in colour only in the online journal)

Formally, the Hamiltonian for such a TQD can be expressed as  \( \hat{H}(t) = \hat{H}_0(t) + \hat{H}_1(t) \) [10] with

\[
\hat{H}_1(t) = i\hbar \sum_n |\partial_t \lambda_n \rangle \langle \lambda_n |.
\]

As \( \hat{H}_1(t) \) is supplemented to \( \hat{H}_0(t) \), the dynamics of the system can be restricted along the instantaneous eigenstate \( |\lambda_n \rangle \) beyond the adiabatic limit. Therefore, this technique could be used to speed up the population passages for high-efficiency quantum state controls by introducing various classical pulses [11–15]. Here, with the full quantized atom–cavity interaction we discuss how to apply the TQD technique to achieve the high efficiency single-photon FSP with the usual double \( \Lambda \)-level atom interacting with a single-mode high-Q cavity. Although a similar configuration had been used to implement the production of a single photon by using the STIRAP technique (see, e.g. [16–18]), the present proposal possesses a manifest advantage: the population transfer for the FSP could be implemented fast (as it is beyond the...
adiabatic limit) and deterministically (as it does not yield any unwanted leakage from the driven states). Therefore, the desirable FSP can be achieved with a significantly high efficiency.

The configuration of the atom–cavity system for our proposal is depicted in figure 1. The atom consists of two excited states \(|e\rangle\) and \(|e_m\rangle\), and two ground states \(|g_1\rangle\) and \(|g_2\rangle\). The transitions \(|g_1\rangle \leftrightarrow |e\rangle\) and \(|g_1\rangle \leftrightarrow |e_m\rangle\) are coupled by classical pumps and cavity couplings. In a rotating frame, defined relative to the non-resonant transitions by the classical Hamiltonian \(H_{\text{eff}}\) (3), the energy levels are neglected as they do not affect the follow-up progress of STIRAP in the effective three-level system, and the ‘coefficients’ \(C_i\) (\(i = 1, 2, 3, 4\)). In the far-off-resonant case, the upper atomic state \(|e_m\rangle\) can be effectively eliminated [17], i.e. \(C_1 = 0\). As a consequence, an effective Raman atom–photon coupling \(\Omega_1 = \Omega_{mS} \Delta_m\) is delivered [18], and then the above double-\(L\)-atom–cavity system is reduced to an effective three-level \(L\) one (see figure 1). Therefore, an effective Hamiltonian is delivered in the atomic space \(|g_1, e, g_2\rangle\) (4).

During the derivation, all the Stark shifts of the relevant energy levels are neglected as they do not affect the following progress for the FSP. Formally, the above Hamiltonian (4) can be written as \(H_{\text{eff}}(t) = \hat{H}_0(t) + \hat{H}_1'(t)\), wherein \(\hat{H}_0(t) = D|e\rangle\langle e| + \Omega_0 S^1_+ + g S^3_+ + |g_m\rangle\langle g_m| + \text{h.c.}\) describes the usual progress of STIRAP in the effective three-level \(L\) atom–cavity system, and \(\hat{H}_1'(t) = i\Omega g|g_2\rangle\langle e| + \text{h.c.}\) illustrates the effective Raman atom–photon coupling. Obviously, the auxiliary Hamiltonian \(\hat{H}_1'(t)\) affects the efficiency of single-photon generation.

In the subspace \(|g_1, 0\rangle, |e, 0\rangle, |g_2, 1\rangle\), one can easily check that the instantaneous eigenstates \(|\lambda\rangle\) of the Hamiltonian \(\hat{H}_0(t)\) read, \(|\lambda_0\rangle = \cos \theta|g_1, 0\rangle + \sin \theta|g_2, 1\rangle\), \(|\lambda_+\rangle = \sin \theta \cos \phi|g_1, 0\rangle + \cos \theta \sin \phi|g_2, 1\rangle\), and \(|\lambda_-\rangle = \sin \theta \sin \phi|g_1, 0\rangle + \cos \theta \cos \phi|g_2, 1\rangle\). Here, the mixing angles \(\theta\) and \(\phi\) are defined by \(\tan \theta = \Omega g\Omega_0\) and \(\tan 2\phi = 2\Omega / \Delta\), whereas \(\Omega = \sqrt{\Omega_0^2 + g^2}\). In the previous STIRAP technique, the single-photon FSP is realized by the dark-state manipulation, i.e. the system evolves along the \(|\lambda_0\rangle\)-path via adiabatically adjusting the mixed angle \(\theta\). However, when the adiabatic condition is not satisfied exactly, the non-adiabatic transitions from the driven dark state to the bright

**Figure 1.** Effective three-level \(L\) atomic configuration formed by adiabatically eliminating the auxiliary level \(|e_m\rangle\).
states $|\lambda\rangle$ will take place [19]. As a consequence, the above manipulation induces unwanted errors. In our proposal, once the effective Raman atom–photon coupling satisfies the condition

$$\frac{1}{\kappa T} = \Omega^2 \text{gg}$$

from equation (1), a shortcut to STIRAP is found and the undesired non-adiabatic transition can be suppressed [10]. Then, the population from $|g_1, 0\rangle$ to $|g_2, 1\rangle$, i.e. the expected single-photon emission can be achieved beyond the adiabatic limit.

Specifically, for the pulse sequence shown in figure 2(a) with the detuning $\Delta = 1/T$ and the two drivings: $\Omega_R = \Omega_0 \exp[-(t - \tau_p)^2/T^2]$ and $g = \Omega_0 \exp[-(t + \tau_s)^2/T^2]$ (with $\tau_p = 0.5T$ and $\tau_s = 0.5T$), figure 2(b) (with $T = 1 \mu s$) shows that the usual STIRAP could not be perfectly realized, i.e. the final population of the target state $|g_2, 1\rangle$ cannot reach unit (e.g. only about 73.5%). However, when an auxiliary driving $\Omega_1$ is applied additionally, then the desired TQD is implemented and complete population passage from the initial state $|g_1, 0\rangle$ to the target one $|g_2, 1\rangle$ is achieved. This is numerically verified in figure 2(c), which indicates the single-photon Fock state of the cavity mode is generated perfectly. The above argument clearly shows that the TQD, rather than the usual STIRAP, provides a high efficient approach to deliver a single-photon emission.

We now compared the robustness of the above STIRAP- and TQD-based single-photon FSP in the presence of the dissipations. In fact, atomic spontaneous emissions from excited state and photon absorption by the cavity mirrors are the main dissipation for the single-photon generations [20, 21]. In the presence of dissipation, the dynamics of the above effective $\Lambda$ atom–cavity system is alternatively described by the following master equation [7, 8]:

$$\frac{\partial \hat{\rho}}{\partial t} = -i\left(\hat{H}_{\text{eff}} - \text{h.c.}\right) + \kappa \hat{a} \hat{a}^\dagger + \Gamma \sum_{j=1,2} S_j \hat{S}_j^j. \quad (5)$$

with $\hat{H}_{\text{eff}} = \hat{H}_{\text{eff}}(t) - i\Gamma |e\rangle\langle e|/2 - i\kappa \hat{a}^\dagger \hat{a} / 2$. Above, $\hat{\rho}$ is the reduced density operator, $\Gamma$ is the atomic spontaneous emission rate, and $\kappa$ the cavity dissipation. By solving the above master equation, the time-dependent mean photon number $n = \langle \hat{a}^\dagger \hat{a} \rangle$ in the cavity, and the Mandel $Q$ parameter during the TQD and STIRAP; without dissipation (e), and the presence of dissipation (f). Here, $\Gamma = 5/T$ and $\kappa = 0.05/T$.

Figure 2. Comparison of single-photon FSPs by the STIRAP and TQD techniques. (a) Exciting pulses, $g$ and $\Omega_R$, are applied to realize the STIRAP; the auxiliary driving $\Omega_1$ is additionally applied to implement the desired TQD with the parameter $\Omega_0 T = 2$. Time-dependent populations during dark-state ($|\lambda0\rangle$) evolution: (b) by the STIRAP, and (c) by the TQD, respectively. (d) Exciting pulses and the auxiliary driving with the parameter $\Omega_0 T = 5$. Time-dependent mean cavity photon number $n$ and the Mandel $Q$ parameter during the TQD and STIRAP; without dissipation (e), and the presence of dissipation (f).
Certainly, if the population transfer is strictly going along the dark-state evolution, i.e., the excited state \( |e, 0\rangle \) has never been populated, then the desired single-photon FSP will be realized perfectly. This requires the significantly large amplitude \( \Omega_0 \) of the drivings in the STIRAP. However, in the TQD we do not care how strong the amplitude \( \Omega_0 \) is, but just apply a proper auxiliary driving \( \Omega_{\text{aux}}(t) \) to implement the exact dark-state evolution. During such a perfect dark-state evolution, the excited state \( |e, 0\rangle \) has never been populated. This is why the TQD is more robust than the STIRAP against the dissipation.

To implement the technique of our TQD, the most crucial step is to realize the effective Raman atom–photon coupling between the states \( |g_1, 0\rangle \) and \( |g_2, 1\rangle \) by adiabatically eliminating the auxiliary state \( |e_m\rangle \). For simplicity, we assume that the pulses \( g_m \) and \( \Omega_m \) are designed as the same shape, e.g., \( g_m = \Omega_m = \alpha \exp\left[-\frac{(t^2 + \tau^2)}{T^2}\right] / \beta \) with \( \alpha^2 = 2\Delta_m T \) and \( \beta = \sqrt{\exp\left[-2(t + \tau)^2/T^2\right] + \exp\left[-2(t - \tau)^2/T^2\right]} \).

Then, the population dynamics of the double \( \Lambda \)-level atom–cavity system can be numerically solved and the relevant results are depicted in figure 3 for, e.g., \( \Delta_m T = 18 \). It is clear to see that the population dynamics of the state \( |e_m\rangle \) is very small and complete population transfer from state \( |g_1, 0\rangle \) to \( |g_1, 1\rangle \) is robustly realized. This means that the auxiliary state \( |e_m\rangle \) can be adiabatically eliminated really. Certainly, as \( \Delta_m T \geq 18 \), the condition \( C_4 = 0 \) still holds and thus we can still eliminate the state \( |e_m\rangle \). Furthermore, figure 3 gives direct evidence that the efficiency of the single-photon FSP by the TQD is much higher than that by the STIRAP.

In summary, we have proposed a robust technique to produce a single-photon Fock state on demand with a double \( \Lambda \)-level atom in a single-mode cavity. The auxiliary driving to implement the transitionless evolution is introduced by using an auxiliary level \( |e_m\rangle \). By effectively eliminating such a level, the effective Raman atom–photon coupling makes the system evolve exactly along the relevant dark-state path. In particular, we have also analyzed specifically the quality of the proposed TQD-based single-photon FSP by numerically solving the relevant master equation. The results showed that the proposal is still robust in the presence of dissipation of the considered atom–cavity system. Hopefully, arbitrary multi-photon Fock states can also be produced by designing the relevant transitionless quantum drivings. Given the fact that the STIRAP technique has taken an important role in atomic physics and quantum optics, the TQD-based FSPs proposed here should be implementable with the current cavity-QED experiments.

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![Figure 3](image-url)  
Figure 3. Single-photon FSP without adiabatically eliminating the level \( |e_m\rangle \). (a) Exciting pulses: \( g, \Omega_R, g_m \) and \( \Omega_m \), Here, we consider the simplified case \( g_m = \Omega_m \). (b) Time-dependent populations during the double \( \Lambda \)-level atom–cavity system.
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