ICMOC

Finite Element Modeling and Analysis of Functionally Graded (FG) Composite Shell Structures
Koteswara Rao D*, Blessington P. J, Tarapada Roy

Abstract
This article deals with the finite element modeling and analysis of functionally graded (FG) shell structures under different loading such as thermal and mechanical. Free vibration analysis of functionally graded (FG) spherical shell structure has also been presented. In order to study the influences of important parameters on the responses of FG shell structures, different types of shells have been considered. The responses obtained for FG shells are compared with the homogeneous shells of pure ceramic (Al₂O₃) and pure metal (steel) shells and it has been observed that the responses of the FGM shells are in between the responses of the homogeneous shells. Based on the analysis, some important results are presented and discussed for thick as well as thin shells.

© 2012 Published by Elsevier Ltd. Selection and/or peer-review under responsibility of Noorul Islam Centre for Higher Education.
Open access under CC BY-NC-ND license.
Keywords: FG shell structures; FE analysis; thermal and mechanical loading; free vibration analysis

1. Introduction
Laminated composites have various advantages over conventional materials such as high specific stiffness and lightweight. Their major drawback is the weakness of interfaces between adjacent layers, known as delamination phenomena. When laminated composites are subjected to extreme temperatures, the effects of interlaminar stresses become even more severe that lead to failure of composite structure due to delamination. These problems can be reduced if the sudden change of material properties of the laminated composite structure is prevented. Functionally graded material (FGM) is a kind of material in which the individual material composition varies continuously along certain directions.
in a controllable way. There are many areas of application for FGM. The concept is to make a composite material by varying the microstructure from one material to another material with a specific gradient. Many works have been reported in the broad area of FGM structures, some of the important works in the direction of finite element analysis of FG structures have been presented in the following paragraphs.

Lakshminarayana et al. [1] performed a significant research on the analysis of FGM composite structures. Gunes et al. [2] investigated the thermal response of FGM plates. Kashtalyan [3] addressed a three-dimensional elasticity solution for the bending of functionally graded rectangular plates. Zhong and Shang [4] studied the three-dimensional exact analysis of a simply supported functionally graded piezoelectric plate. Charles et al. [5] developed a new finite element to evaluate the global response of a structure composed of laminated beams which contain de-laminations. Bambole et al. [6] developed a FE formulation based on the minimum energy principle for the analysis of thick/thin laminated composite beams. Ray and Sachade [7] derived an exact solution for functionally graded plates integrated with layers of piezoelectric reinforced composite. Croce and Venini [8] developed a finite element for the analysis of shear-deformable FGM Midlin plates. Zenkour [9, 10] investigated the functionally graded sandwich of ceramic-metal panels. Kargarnovin et al. [11] introduced the limit analysis of FGM circular plates subjected to arbitrary rotational symmetric loads. Bouchafa and Benzaïr [12] presented an analytical modeling of thermal residual stresses in exponential functionally graded material system. Anastasia and Muliana [13] introduced a micromechanical model for predicting the effective thermo-visco-elastic behaviors of a functionally graded material (FGM). Senthil et al. [14] developed an exact solution for the thermo-elastic deformations of functionally graded thick rectangular plates. Shen [15] introduced the post buckling of simply supported and clamped cylindrical panels subject to axial compression and temperature. Bilgili [16] used the shear strains in a non-homogeneous rubber like slab subjected to a thermal gradient in the thickness direction. Liew et al [17] analyzed the problem of thermal stresses in FGM hollow cylindrical shells. Yang et al. [18] reported the bending response of thick FGM plates subjected to lateral pressure and uniform temperature using higher-order shear deformation theory combined with a first-order perturbation technique. Vidal and Polit [19] developed a new three-node multi-layered beam finite element for analysis of thermo-mechanical behaviour of sandwich and laminated beams. Senthil et al. [20] addressed three-dimensional analysis of transient thermal stresses in functionally graded plates and developed an analytical solution for three-dimensional of thermo-mechanical deformations of a simply supported functionally graded (FG) rectangular plate subjected to time-dependent thermal loads on its top and/or bottom surfaces. Reddy and Cheng [21] introduced three-dimensional solutions of smart functionally graded plates. From the exhaustive literature review, it has been observed that, while a number of works are available in the form of beam and plate finite elements analysis of FG structures, not many works are available in the form of analysis of FG shell structures. In the present work, thus, an attempt has been made for the analysis of FG shell structures in order to study the responses of various FG shell structures for both deep as well as shallow shells.

2. Modeling of FGM properties

Different types of gradations laws are available in the literature. In the present work power law gradation has been considered in order to calculate the material properties of FG structures. FGM consisting of two constituent materials has been considered. The top surface is assumed to be rich in material-2 (ceramic) and bottom surface is assumed to be rich in material-1 (metal). The region between the two surfaces consists of a combination of the two materials with continuously varying mixing ratios of two materials. Volume fraction following power law distribution can be written as [21].
\[ V_c(z) = \left( \frac{z+h}{2h} \right)^m \]
\[ V_c(z) = 1 - V_{mat}(z) \]
\[ P(z) = P_{mat} + (P_c - P_{mat})V_c(z) \]  

(1)

Where \( z \) is the distance from mid-surface and \( m \) is the power law index, a positive real number. \( P_{mat} \) is the material property of metal (steel) surface and \( P_c \) is material property of ceramic (\( Al_2O_3 \)) surface. \( V_c(z) \) and \( V_{mat}(z) \) are the volume fractions of the ceramic and metal surface. In the present analysis, the material properties are varying though the thickness from bottom to top. The material properties such as young’s modulus, thermal expansion, conductivity and density are varying in thickness direction only and poisons ratios is constant throughout the thickness as shown in Fig 2. The material properties used for modeling and analysis of FG structures is presented in the Table 1.

### Table 1. Thermo-elastic properties for metallic (Steel) and ceramic (Al2O3) phases

| Material   | Thermal expansion coefficient \( \alpha (c^{-1}) \) | Poisons’ ratio \( \mu \) | Young’s modulus \( E (GPa) \) | Density(kg/m\(^3\)) | Conductivity\( (k \) (W/mK)) |
|------------|-----------------------------------------------------|--------------------------|--------------------------------|----------------------|--------------------------|
| AL\(2O_3\) | \(6.9 \times 10^{-6} \)                           | 0.25                     | 390                            | 3.89\times 10^{3}   | 25                       |
| Steel      | \(14\times 10^{-6} \)                             | 0.25                     | 210                            | 7.85\times 10^{3}   | 40                       |

3. **Modeling of FG Structures using ANSYS**

The material properties of the FGM change throughout the thickness; the numerical model has been divided into various layers in order to make the changes in properties. Each layer has the finite portion of the thickness and treated like isotropic material. Material properties have been calculated at the mid-plane of each of this layer by using the power law gradation. The thickness of FG beam has been discretised into several layers as shown in Fig 1. Similarly, shell structures have also been discretised through the thickness into several layers in order to model FG shell structures. The finite element (FE) modelling has been carried out using ANSYS. An eight nodded layered shell element (SHELL99) has been used for modelling of FG structures. The element has six degrees of freedom at each node. Translations are in the nodal \( x \), \( y \), and \( z \) directions and rotations about the nodal \( x \), \( y \), and \( z \)-axis. In this paper finite element modelling of functionally graded beam as well as different shell structures have been done. A typical FG shell is shown in Fig 2.

![Fig.1. Thickness discretization of beam](image1)

![Fig.2. FG shell structure](image2)
4. Results and Discussions

Based on the FE analysis as discussed in the section 2, two types of FG structures (viz. beam and shell) have been analyzed using ANSYS. Results obtained of different analysis (such as static, free vibration and thermal) have been presented in the following subsections.

4.1. Static Analysis of FG Beam

For validation of present analysis, a FG cantilever beam has been considered. The three layered system i.e steel-FGM-ceramic has been analyzed. The FG cantilever beam of 0.5 m length subjected to transverse distributed load (100 kN) on upper surface has been taken. Width is taken as 0.001m. The top most material is ceramic which has a thickness of 0.005m and bottom layer is metal of thickness 0.005m. In between these layers there is a FGM layer of 0.01m and the FGM layer is divided into five layers i.e. total number of layers of the model is seven. Eight noded SHELL99 element is used for FE modeling of the FG beam and the material properties are calculated by using the power law. Material properties used for this analysis are listed in the Table 1. Four numbers of elements are taken in width and twenty numbers of elements are taken in length. The beam is subjected to a uniform distributed loading (q =100 kN/m). Fig 3 shows the results i.e. depth wise stress distribution in the beam using the ANSYS.

![Fig.3. Depth wise stress variation in the beam](image)

Depth wise stress distributions obtained using ANSYS are also tabulated in Table 2. It has been observed from the Table 2 that the results obtained from present analysis are within acceptable limits and are in excellent agreement with the already published results [22].

| Thickness (m) | Stress distribution(Pa) ($\times10^8$) | Stress distribution(Pa) ($\times10^8$) |
|--------------|----------------------------------------|----------------------------------------|
|              | ANSYS                                  | Published [22]                         |
| 0.01         | 1.75                                   | 1.64                                   |
| 0.008        | 1.47                                   | 1.3                                    |
| 0.006        | 1.09                                   | 0.9                                    |
| 0.004        | 0.76                                   | 0.65                                   |
| 0.002        | 0.4                                    | 0.35                                   |
| -0.002       | -0.105                                 | -0.2                                   |
| -0.004       | -0.22                                  | -0.3                                   |
| -0.006       | -0.582                                 | -0.65                                  |
| -0.008       | -0.89                                  | -0.85                                  |
| -0.01        | -1.21                                  | -1.2                                   |
4.2. FE analysis of FG shell structure under different loading

A generalized shell structure is depicted in Fig 4. After validation, a simply supported spherical shell \((R_1 = R_2)\) on square based \((a = b)\) has been analysed under mechanical as well as thermal loading. Finite element (FE) analyses have been done using eight noded shell element (SHELL99). A \(10 \times 10\) FE mesh been used with total degrees of freedom = 2046, number of elements = 100 and number of nodes = 341. Thickness of the shell \((h)\) has been considered as 0.02m. The mechanical and thermal material properties used in the present study have been listed in the Table 1. The simply supported spherical shell has been subjected uniformly distributed load \((q = 100\) kN) as show in Fig.5.

Central displacements of the shell structures under uniformly distributed load for various \((R/a)\) ratios have been listed in Table 3. From this table it has been observed that the center displacements of FG shells are in between the center displacements of the homogeneous shells of ceramic and metal. Figs 6 to 8 show the displacement variation through the surface for ceramic, FG and metallic shells under mechanical loading. The shell model has also been analysed under thermal loading. In thermal analysis the shell top surface is subjected to 130\(^\circ\)C and the shell bottom surface is subjected to 30\(^\circ\)C. The displacements of the shell structures due to the applied temperature loading are tabulated in Table 4. Figs 9 to 11 show the displacement variation through the surface for ceramic, FG and metallic shells under thermal loading. Free vibration analysis of the spherical FG shell has also been carried out for thick \((a/h = 10)\) as well as thin shell \((a/h =100)\). The natural frequencies of thin and thick shells have been tabulated in Table.5. Figs 12 and 13 show the first and second modes of vibration for thick shell and Figs.14 and 15 show the first and second mode of vibration for thin shell.
Fig. 6. Displacement variation for pure ceramic shell

Fig. 7. Displacement variation for FG shell

Fig. 8. Displacements variation for pure metallic shells

Fig. 9. Displacement variation for pure ceramic shell

Fig. 10. Displacement variations for FG shell

Fig. 11. Displacement variation for pure metallic shell

Fig. 12. First mode of vibration for $a/h = 10$

Fig. 13. Second mode of vibration for $a/h = 10$

Fig. 14. First mode of vibration for $a/h = 100$

Fig. 15. Second mode of vibration for $a/h = 100$
Table 3. Center displacement of shell with various R/a ratios for thin shell (a/h=100)

| R/a | Center displacement of the shell (mm) | Pure ceramic | FG | Pure metal |
|-----|--------------------------------------|--------------|----|------------|
| 1   | -0.047                               | -0.061       | -0.0876 |
| 2   | -0.216                               | -0.28        | -0.56  |
| 5   | -1.43                                | -1.8         | -2.655 |
| 10  | -3.19                                | -5.04        | -7.039 |
| 20  | -5.969                               | -8.06        | -11.05 |
| 50  | -7.068                               | -9.618       | -13.126 |
| 100 | -7.262                               | -9.894       | -13.486 |

Table 4. Center displacements of shell with various R/a ratios for thin shell (a/h=100)

| R/a | Center displacement of the shell (mm) | Pure ceramic | FG | Pure metal |
|-----|--------------------------------------|--------------|----|------------|
| 2   | -2.394                               | -3.358       | -4.858 |
| 5   | -6.186                               | -8.778       | -12.552 |
| 10  | -8.209                               | -11.839      | -16.656 |

Table 5. Comparison of natural frequencies for thick and thin spherical shells

| a/h | Natural Frequencies (Hz) | 1   | 2   | 3   | 4   | 5   |
|-----|--------------------------|-----|-----|-----|-----|-----|
| 10  | 6340.2                   | 7980.9 | 8254.1 | 10297.0 | 10501.0 |
| 100 | 550.41                   | 563.13 | 563.92 | 575.34 | 583.97 |

5. Conclusions

An eight noded layered shell element (SHELL99) has been used for the modeling and analysis of functionally graded (FG) composite shell structures for deep as well as shallow shells. Important conclusions obtained from the present study have been presented here. Static analysis of the FG beam structure subjected to uniformly distributed load has been done and validated with the published results. Responses of FG spherical shell structure have been analyzed under mechanical and thermal loading and compared with the responses of the homogeneous shell structures, and it has been noted that the response of FG shells are between the responses of the homogeneous shells of the ceramic and metal. Free vibration analysis of thick and thin FG shell structures has been done and first and second mode shapes have been presented. It has also been observed that at the deep region the responses are changing rapidly and converging towards the plate region.

References

[1] Lakshminarayana H.V, Boukhili R., Gauvin R. Impact response of laminated composite plates: prediction and verification. Composite structure 1994; 28:61–72.
[2] Recep Gunes, Murat Aydin. Elastic response of functionally graded circular plates under a drop-weight. Composite Structures 2010; 92: 2445–56.
[3] Kashtalyan M. Three-dimensional elasticity solution for bending of functionally graded rectangular plates. European journal of mechanics a/solids 2004; 23: pp.853–64.
[4] Zhong Z. and Shang E.T. Three-dimensional exact analysis of a simply supported functionally gradient piezoelectric plate. International journal of solids and structures 2003; 40: pp.5335–52.

[5] Charles H., Roche! Accorsi L.M. A new finite element for global modeling of delaminations in laminated beams. Finite Elements in Analysis and Design 1998; 31: pp.16-177.

[6] Bambole A.N., Desai Y.M. Hybrid-interface finite element for laminated composite and sandwich beams. Finite Elements in Analysis and Design 2007; 43: pp.1023 – 36.

[7] Ray M.C., Sachade H.M. Exact solutions for the functionally graded plates integrated with a layer of piezoelectric fiber-reinforced composite. ASME Journal of Applied Mechanics 2006; 73: pp.622-31.

[8] Croce D. and Venini P. Finite Elements for Functionally Graded Reissner-Mindlin Plates. Comput. Methods Appl. Mech. Eng 2004; 193: pp. 705–25.

[9] Zenkour A., Liew M. A Comprehensive Analysis of Functionally Graded Sandwich Plates: Part 1 Deflections and Stresses. Int. J. Solids Struct 2005; 42: pp. 5224–42.

[10] Zenkour A., Liew M. A Comprehensive Analysis of Functionally Graded Sandwich Plates: Part 2—Buckling and Free Vibration. Int. J. Solids Struct 2005; 42: pp. 5243–58.

[11] Kargarnovin M.H.; Faghidian S. A., and Arghavani. Limit Analysis of FGM Circular Plates Subjected to Arbitrary Rotational Symmetric Loads. World Academy of Science, Engineering and Technology 2007; 36: pp.133-38.

[12] Bouchafa A., Benzair A., Tounsi A., Draiche K., Mechab I., Abbas E., Bedia A. Analytical modelling of thermal residual stresses in exponential functionally graded material system. materials and design 2010; 31: pp.560–3.

[13] Anastasia H., Muliana, A micromechanical model for predicting thermal properties and thermo-viscoelastic responses of functionally graded materials. Journal of solids and structures 2009; 46: pp.1911–24.

[14] Senthil S. Vel and Batra RC. Exact solution for thermoelastic deformations of functionally graded thick rectangular plates. aiaa journal July 2002; 40, No. 7: pp.1421-33.

[15] Shen S H. and Leung A. Y. T. Postbuckling of Pressure-Loaded Functionally Graded Cylindrical Panels in Thermal Environments. J. Eng. Mech. 2003; 129: pp. 414–25.

[16] Bilgili E., Bernstein M., and Aratoomour H. Effect of Material Non-Homogeneity on the Inhomogeneous Shearing Deformations of a Gent Slab Subjected to a Temperature Gradient. Int. J. Non-Linear Mech. 2003; 38: pp. 1351–68.

[17] Liew K.M., Kitipornchai S., Zhang X. Z., and Lim C. W. Analysis of the Thermal Stress Behaviour of Functionally Graded Hollow Circular Cylinders. Int. J. Solids Struct 2003; 40: pp. 2355–80.

[18] Yang J., Liew K. M., and Kitipornchai S. Second-Order Statistics of the Elastic Buckling of Functionally Graded Rectangular Plates. Compos. Sci. Technol. 2005; 65: pp. 1165–75.

[19] Vidal P., Polit O. A thermo mechanical finite element for the analysis of rectangular laminated beams. Finite Elements in Analysis and Design 2006; 42.

[20] Senthil S., Vel, Batra R.C. Three-dimensional analysis of transient thermal stresses in functionally graded plates. International journal of solids and structures 2003; 40: pp.7181–96.

[21] Reddy J. N., Cheng Z.Q. Three-dimensional solutions of smart functionally graded plates. March 2001. 68: pp.234-41.

[22] Farhatnia F., Shariﬁ G.A., and Rasouli S. Numerical and Analytical Approach of Thermo-Mechanical Stresses in FGM Beams. Proceedings of the World Congress on Engineering 2009; 2.