I. INTRODUCTION

The ATLAS [1, 2] and CMS [3–5] collaboration at LHC have both discovered a new resonance of mass around 125 GeV that is found to be largely consistent with the observation of a Higgs boson. Several studies [6–59] done both before and after the discovery of the Higgs boson have examined how to determine the spin, parity and coupling of the Higgs boson. In gauge sector, decay modes such as $H \rightarrow \gamma\gamma$, $H \rightarrow ZZ$ and $H \rightarrow WW$ etc., where one (or both) of the $Z$’s and $W$’s are off-shell, are used to study the spin, parity and coupling of the Higgs boson. Observation of the decay mode $H \rightarrow \gamma\gamma$ establishes that the discovered resonance is necessarily a boson and the Landau-Yang theorem [65, 66] excludes the possibility of it having spin $J = 1$. Furthermore if Higgs boson is a eigenstate of charge conjugation, charge conjugation invariance along with observation of $H \rightarrow \gamma\gamma$ also enforce [10] that Higgs is a charge conjugation $C = +$ state. Recent measurements [70–73] have shown that the resonance favors Spin 0 over spin 2. Moreover Ref.[55] rules out pure pseudoscalar hypothesis i.e. $J^{PC} = 0^{-}$ at a 99.98 \% CL. However the discovered Higgs can still have small CP-odd admixture or higher derivative CP-even contribution to its coupling. Angular distributions and angular asymmetries of Higgs decay are essential to investigate whether the discovered resonance is a CP eigenstate or a resonance with mixed CP configuration. As these angular asymmetries are functions of the Higgs couplings, studying them will allow us to probe the nature of the Higgs coupling directly.

In this paper we will restrict ourselves to experimentally clean golden channel $H \rightarrow ZZ^{*} \rightarrow (\ell_1^+\ell_1^-)(\ell_2^+\ell_2^-)$, where $\ell_1, \ell_2$ are leptons $e$ or $\mu$. We consider Spin-0 Higgs boson $H$ with even parity but, include the possibility of a small CP-odd admixture and higher derivative CP-even contribution. We first calculate the differential decay rate for $H \rightarrow ZZ^{*} \rightarrow (\ell_1^+\ell_1^-)(\ell_2^+\ell_2^-)$ process in terms of invariant mass of the dileptons coming from the off shell $Z$ boson and angular distributions of the final state leptons. From these distributions, we construct angular asymmetries (observables) and utilize them to probe anomaly in $HZZ$ couplings. Similar asymmetries have also been discussed in Ref. [60–62]. As these observables are functions of $HZZ$ vertex factors, the values of the different observables differ for the various cases such as SM Higgs, CP-odd admixture, CP-even higher derivative contribution. Ref.[53, 55] have discussed how ratios of couplings can be measured at 8 TeV LHC run. Ref.[53] shows how using uniaxial distribution as input in likelihood analysis one can discriminate different spin possibilities. In this paper a study showing how using simple angular asymmetries one can study the CP property of $H$ in future LHC runs.

We benchmark these observables for SM Higgs and Higgs with a CP-odd admixture at 14 TeV 300 fb\(^{-1}\) LHC. We determine the ratios of the coupling constants and use them to discriminate possible CP-odd admixture from SM Higgs. We then perform the same analysis at 3000 fb\(^{-1}\) to distinguish SM, case with CP-odd admixture and CP-even higher derivative contribution. We also consider the scenario when higher derivative CP-even and and CP-
odd admixture are present in $HZZ$ couplings. We denote this scenario as ‘CP-even-odd’ case. In our analysis we have also included a complex phase for CP-odd admixture Higgs and show how to determine the phase using angular asymmetries. Furthermore we use these angular asymmetries and perform chi-square analysis to probe both CP-even and CP-odd anomalous contributions in the Higgs couplings.

Exotic models of Higgs can have momentum dependence in its couplings. It is thus essential to study the momentum dependence of the Higgs couplings to establish its SM nature. Angular asymmetries will provide necessary tools to investigate the momentum dependence of the Higgs couplings. In our work we develop necessary techniques and utilize them to probe the momentum dependence of $HZZ$ couplings. It is important to note this it will definitely require higher statistics. However the enhanced statistics at 14 TeV, LHC will enable us to study the momentum dependence of Higgs couplings in different momentum regions. Since the mass of $H$ does not allow both the $Z$ bosons to be on-shell, the invariant mass distribution of the dileptons from the off-shell $Z$ boson$(Z_2)$ will offer us a test for the momentum dependence of the $HZZ$ couplings. The $HZZ$ couplings in the most general case could be function of the invariant mass of the off-shell $Z$. The constancy of the ratios of the Higgs couplings can be measured by finding out the values of the ratios of couplings in different momentum regions for the invariant mass of the off-shell $Z$. In our analysis we have shown for SM how one can test the momentum dependence of the Higgs couplings at 14 TeV 300 fb$^{-1}$ LHC run.

In LHC we measure $\sigma \cdot BR$ and that does not allow us to measure the decay width of the Higgs, as a result we can only measure the ratios of Higgs couplings. However ILC$^6$ will be able to measure the inclusive cross section$(\sigma_{ZH})$ using recoil technique for the process $e^+ e^- \rightarrow Z H$. Hence inclusive cross section provide a direct measurement of $HZZ$ couplings at ILC by measuring partial width $\Gamma(H \rightarrow ZZ)$. After energy upgrade LHC will run at 33 TeV and enhanced statistics will enable us to measure Higgs couplings more precisely than that possible at 14 TeV. We discuss how much this energy upgrade will improve the measurement of $HZZ$ couplings and discuss what sensitivity can be achieved compared to that of an ILC$^6$ measurement.

The paper is organized as follows: In Section II we formalize the necessary tools for our analysis. Section III is divided into three subsection. First we benchmark our analysis for 300 fb$^{-1}$ in Subsection IIIA and then we benchmark our analysis for 3000 fb$^{-1}$ for SM Higgs, Higgs with CP-odd admixture, CP-even higher derivative contribution and CP-even-odd scenario in Subsection IIIB. In Subsection IIIC we develop the necessary technique to measure the momentum dependence of Higgs couplings. A qualitative comparison between future runs of LHC and ILC precision measurement has been made in Section IV. Finally we conclude in Section V.

II. THE FORMALISM

In this section we first write down the $HZZ$ vertex, helicity amplitudes in transversity basis and finally derive the expression for angular distribution of $H \rightarrow ZZ^* \rightarrow 4\ell$ process assuming $H$ to be a spin 0 particle. In SM the process $H \rightarrow ZZ$ is characterised by the Lagrangian

$$L_{HZZ} = \frac{g M_Z}{2 \cos \theta_W} Z_{\mu} Z_{\mu} H$$  (1)

where $\theta_W$ is the Weinberg angle and $g$ is the electroweak coupling constant. However there may exist anomalous couplings of $H$ to $Z$ boson. These couplings can in general be CP-even or CP-odd and can be generated from the effective Lagrangians

$$L_c \sim -\frac{1}{4} Z^{\mu \nu} Z_{\mu \nu} H$$  (2)

and

$$L_o \sim -\frac{1}{4} \tilde{Z}^{\mu \nu} \tilde{Z}_{\mu \nu} H$$  (3)

respectively, where $Z_{\mu \nu}$ and $\tilde{Z}_{\mu \nu}$ are defined as $Z_{\mu \nu} = \partial_{\mu} Z_{\nu} - \partial_{\nu} Z_{\mu}$ and $\tilde{Z}_{\mu \nu} = \epsilon_{\mu \nu \rho \sigma} Z^{\rho \sigma}$ respectively.

Following these Lagrangians one can write down the most general $HZZ$ vertex as follows

$$V^{\mu \nu} = \frac{ig M_Z}{\cos \theta_W} \left( a g^{\mu \nu} + b (g_2^{\mu} g_2^{\nu} - q_1 \cdot q_2 g^{\mu \nu}) + ic e^{i\nu \rho \sigma} \epsilon_{\mu \rho \sigma} q_1 \cdot q_2 \right),$$  (4)

where $a$, $b$, $c$ are momentum dependent vertex factors and $q_1$, $q_2$ and $P$ are the four momenta of $Z(Z_1)$, $Z^*(Z_2)$ and $H$ respectively. Off-shellness of the $Z$ is denoted by the superscript ‘*’. In Standard Model at tree level the values of the vertex factors are $a = 1$ and $b = c = 0$ and they are constant. However a non zero $b$ and $c$ can arise from higher order correction. If the vertex factors $a$, $b$, $c$ show any deviations from SM values or exhibit a momentum dependence it would provide a hint about the non standard nature of the $HZZ$ couplings. The CP-odd admixture is characterised by the non zero value of $c$ of the form $c e^{i\delta}$, where $\delta$ is the CP violating phase associated with $c$.

The decay under consideration can be characterised by three helicity amplitudes $A_L$, $A_{\parallel}$ and $A_{\perp}$ defined as (see appendix) :

$$A_L = q_1 \cdot q_2 \left( a - b q_1 \cdot q_2 \right) + M_H^2 X^2 b,$$  (5)

$$A_{\parallel} = \sqrt{2q_1^2 q_2^2} \left( a - b q_1 \cdot q_2 \right),$$  (6)

$$A_{\perp} = \sqrt{2q_1^2 q_2^2} X M_H c,$$  (7)
where $\sqrt{q_1^2}$ and $\sqrt{q_2^2}$ are the invariant masses of the $Z_1$ and $Z_2$ respectively, with

$$X = \sqrt{\lambda(M_H^2, q_1^2, q_2^2)} \over 2M_H,$$ (8)

and

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xz - 2yz \, .$$ (9)

The helicity amplitudes have definite parity properties as $A_L, A_\parallel$ are CP-even and $A_\perp$ is CP-odd.

Having defined the helicity amplitudes in transversity basis the full angular distribution for $H \to Z_1 + Z_2 \to (\ell_1^- + \ell_1^+) + (\ell_2^- + \ell_2^+)$, can be written as[53]

$$\frac{8\pi}{\Gamma_H} \frac{d^4 \Gamma}{dq_2^2 \, d\cos \theta_1 \, d\cos \theta_2 \, d\phi} = 1 + \frac{|\mathcal{F}_\parallel|^2 - |\mathcal{F}_\perp|^2}{4} \cos 2\phi \left(1 - P_2(\cos \theta_1)\right) \left(1 - P_2(\cos \theta_2)\right) + \frac{1}{2} \text{Im}(\mathcal{F}_\parallel^* \mathcal{F}_\perp) \sin 2\phi \times \left(1 - P_2(\cos \theta_1)\right) \left(1 - P_2(\cos \theta_2)\right) + \frac{1}{2} \left(1 - 3|\mathcal{F}_L|^2\right) \left(P_2(\cos \theta_1) + P_2(\cos \theta_2)\right) + \frac{1}{4} \left(1 + 3|\mathcal{F}_L|^2\right) P_2(\cos \theta_1) P_2(\cos \theta_2) \right.$$

$$+ \frac{9}{8\sqrt{2}} \left(\text{Re}(\mathcal{F}_L \mathcal{F}_\parallel^*) \cos \phi + \text{Im}(\mathcal{F}_L \mathcal{F}_\perp^*) \sin \phi\right) \left(1 - |\mathcal{F}_L|^2\right) \cos \theta_1 \cos \theta_2 \right.$$ 

$$+ \frac{9}{8\sqrt{2}} \left(\text{Re}(\mathcal{F}_L \mathcal{F}_\perp^*) \cos \phi + \text{Im}(\mathcal{F}_L \mathcal{F}_\parallel^*) \sin \phi\right) \left(1 - |\mathcal{F}_L|^2\right) \cos \theta_1 \cos \theta_2 \right.$$ 

$$- \frac{9}{4}\eta^2 \left(1 - |\mathcal{F}_L|^2\right) \cos \theta_1 \cos \theta_2 + \sqrt{2} \left(\text{Re}(\mathcal{F}_L \mathcal{F}_\parallel^*) \cos \phi + \text{Im}(\mathcal{F}_L \mathcal{F}_\parallel^*) \sin \phi\right) \left(1 - |\mathcal{F}_L|^2\right) \cos \theta_1 \cos \theta_2),$$ (10)

where $\mathcal{F}_L, \mathcal{F}_\parallel, \mathcal{F}_\perp$ are the helicity fractions defined in the appendix along with $\Gamma_H$ and $\eta$. The angle $\theta_1(\theta_2)$ is the angle between three momenta of $\ell_1^- (\ell_2^-)$ in $Z_1 (Z_2)$ rest frame and the direction of three momenta of $Z_1 (Z_2)$ in $H$ rest frame. The angle $\phi$ is defined as the angle between the normals to the planes defined by $Z_1 \to \ell_1^- \ell_1^+$ and $Z_2 \to \ell_2^- \ell_2^+$ in $H$ rest frame as shown in Fig. 1. It should be noted that Eq.(10) is exact and no assumptions has been made apart from assuming that the leptons are massless.

Integrating Eq.(10) with respect to any two out of the three angles $\theta_1, \theta_2, \phi$ one finds three uniaxial distributions for the process $H \to ZZ \to (\ell_1^- \ell_1^+) (\ell_2^- \ell_2^+)$ as:

$$\frac{1}{\Gamma_H} \frac{d^4 \Gamma}{dq_2^2 \, d\cos \theta_1} = 1 + T_2 P_2(\cos \theta_1) - T_1 \cos \theta_1,$$ (11)

$$\frac{1}{\Gamma_H} \frac{d^4 \Gamma}{dq_2^2 \, d\cos \theta_2} = 1 + T_2 P_2(\cos \theta_2) + T_1 \cos \theta_2,$$ (12)
The observables \( T_1, T_2, U_1, U_2, V_1 \) and \( V_2 \) are defined as:

\[
T_1 = \frac{3}{2} \eta \text{Re}(\mathcal{F}_L \mathcal{F}_L^\perp),
\]

\[
T_2 = \frac{1}{4} (1 - 3 |\mathcal{F}_L|^2),
\]

\[
U_1 = -\frac{9n^2}{32\sqrt{2}} \eta^2 \text{Re}(\mathcal{F}_L \mathcal{F}_L^*),
\]

\[
U_2 = \frac{1}{4} (|\mathcal{F}_L|^2 - |\mathcal{F}_L|^2),
\]

\[
V_1 = -\frac{9\pi^2}{32\sqrt{2}} \eta^2 \text{Im}(\mathcal{F}_L \mathcal{F}_L^+),
\]

\[
V_2 = \frac{1}{2} \text{Im}(\mathcal{F}_L \mathcal{F}_L^+),
\]

and are functions of \( q^2 \). Moreover \( P_1(\cos \theta_{1,2}) \) and \( P_2(\cos \theta_{1,2}) \) are first and second degree Legendre Polynomials respectively. It should be noted that the observables \( T_1, T_2, U_1, U_2, V_1 \) and \( V_2 \) are coefficients of orthogonal functions \( P_2(\cos \theta_{1,2}), P_1(\cos \theta_{1,2}), \cos 2\phi, \cos \phi, \sin 2\phi, \sin \phi \) respectively and can be extracted individually. These observables are functions of the helicity fractions \( \mathcal{F}_L, \mathcal{F}_\| \) and \( \mathcal{F}_\perp \), hence they are functions of vertex factors \( a, b \) and \( c \). Measurement of these observables will enable us to probe the vertex factors \( a, b \) and \( c \). Moreover in SM \( T_2, U_2 \) and \( V_1 \) all are zero as \( \mathcal{F}_L \) and \( \mathcal{F}_\| \) are non zero. In SM at tree level \( c = 0 \) which enforces \( \mathcal{F}_\perp = 0 \). As \( T_1, V_2 \) and \( V_1 \) all are functions of \( \mathcal{F}_\perp = 0 \), in SM they are all zero. A CP-odd admixture is characterised by non zero value of \( c \) and hence non vanishing values of \( T_1, V_2 \) and \( V_1 \). Measurements of \( T_1, V_2 \) and \( V_1 \) allow us to probe CP violating phase in \( HZZ \) couplings. If there exist any CP-even higher derivative contribution in \( HZZ \) couplings, the vertex factor \( b \) becomes non zero, hence the observables \( T_2, U_2 \) and \( U_1 \) will have different values than that of SM. In SM at tree level \( b = 0 \) but at one loop level the value of \( b \) will be non zero. Measurement of \( b \) will allow us to probe triple-Higgs vertex which arises at one loop level and provide the first verification of Higgs self-coupling.

This observables can be extracted using following angular asymmetries

\[
T_1 = \left( \int_{-1}^{+1} d\cos \theta_1 \left( \frac{1}{\Gamma} \frac{\text{d}^2 \Gamma}{\text{d}q_2^2 \text{d}\cos \theta_1} \right) \right) = \left( -\int_{-1}^{+1} \frac{\text{d} \cos \theta_2}{\Gamma} \left( \frac{1}{\Gamma} \frac{\text{d}^2 \Gamma}{\text{d}q_2^2 \text{d} \cos \theta_2} \right) \right),
\]

\[
T_2 = \frac{4}{3} \left( \int_{-1}^{+1} d\cos \theta_{1,2} \left( \frac{1}{\Gamma} \frac{\text{d}^2 \Gamma}{\text{d}q_2^2 \text{d} \cos \theta_{1,2}} \right) \right),
\]

\[
U_1 = \frac{1}{4} \left( -\int_{-\pi}^{+\pi} d\phi \left( \frac{2\pi}{\Gamma} \frac{\text{d}^2 \Gamma}{\text{d}q_2^2 \text{d} \phi} \right) \right),
\]

\[
U_2 = \frac{\pi}{2\Gamma} \left( \int_{-\pi}^{+\pi} d\phi \left( \frac{2\pi}{\Gamma} \frac{\text{d}^2 \Gamma}{\text{d}q_2^2 \text{d} \phi} \right) \right),
\]

\[
V_1 = \frac{1}{4} \left( -\int_{-\pi}^{+\pi} d\phi \left( \frac{2\pi}{\Gamma} \frac{\text{d}^2 \Gamma}{\text{d}q_2^2 \text{d} \phi} \right) \right),
\]

\[
V_2 = \frac{1}{4} \left( \int_{-\pi}^{+\pi} d\phi \left( \frac{2\pi}{\Gamma} \frac{\text{d}^2 \Gamma}{\text{d}q_2^2 \text{d} \phi} \right) \right).
\]

III. NUMERICAL STUDY

In this section we demonstrate how angular analysis can be used to benchmark different CP scenarios of \( H \). We have generated events with MADEVENT5 [75] event generator interfaced with PYTHIA6.4 [76] and Delphes 3 [77]. The vertex, Eq. (4) is parametrized by UFO format of MadGraph5 using HiggsCharacterisation model [78]. The events are generated by \( pp \) collisions via \( gg \to H \) and \( gg \to H + 1 \text{jet} \), for center of mass en-
The branching ratios and cross sections have been taken from Higgs working Group webpage. The need for Fermi antisymmetrisation of the identical low both the $Z$-bosons to be on-shell, this in turn breaks the derivative contribution and CP-even-odd scenario. The branching ratios and cross sections have been taken from Higgs working Group webpage.

Following the analysis presented in Ref.[73] data are selected using single-lepton or di-lepton triggers. For the single-muon trigger the transverse momentum, $p_T$, threshold is 25 GeV, while for single-electron trigger the transverse energy, $E_T$, threshold is 25 GeV. Di-muon triggers are selected using two ways. For asymmetric di-lepton trigger the thresholds are either $p_T1 = 18$ GeV and $p_T2 = 8$ GeV. Threshold for symmetric di-muon triggers are $p_T = 13$ GeV for both the muons. For the di-electron trigger the thresholds are $E_T = 12$ GeV for both electrons. There are two electron-muon triggers used with 12 or 24 GeV $E_T$ electron thresholds, differed by the electron identification requirements, and muon threshold $p_T = 8$ GeV.

Each electron (muon) must satisfy $E_T > 7$ GeV ($p_T > 6$ GeV) and be measured in the pseudo-rapidity range $|\eta| < 2.47$ ($|\eta| < 2.7$). We have selected the leptons in two sequential $p_T$ ordered way.

i) Case-I : $p_T$ of at least two leptons in a quadruplet must satisfy $p_T > 20$ GeV,

ii) Case-II : $p_T$ of at least three leptons in a quadruplet must satisfy $p_T > 20$ GeV.

The leptons are required to be separated from each other by $\Delta R > 0.1$ if they are of the same flavour and $\Delta R > 0.2$ otherwise. Each event is required to have the triggering lepton(s) correctly matched to one or two of the selected leptons.

Furthermore we also impose the invariant mass cuts on the $m_{Z1}(\sqrt{q_1^2})$, $m_{Z2}(\sqrt{q_2^2})$ and $m_{4\ell}$ described in Table I. $m_{Z1}$ is the invariant mass of the pair of opposite sign same flavor leptons closest to $m_Z$ while $m_{Z2}$ is the other combination. The two columns of Table I demonstrate the effect of $p_T$ ordering in event selection.

Now integrating Eq. (11),(12), (13) over $q_2^2$ we get three integrated distributions as follows

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta_1} = \frac{1}{2} - T_1 \cos \theta_1 + T_2 P_2(\cos \theta_1),$$  

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta_2} = \frac{1}{2} + T_1 \cos \theta_2 + T_2 P_2(\cos \theta_2),$$  

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\phi} = \frac{1}{2\pi} + U_1 \cos \phi + U_2 \cos 2\phi$$

$$+ V_1 \sin \phi + V_2 \sin 2\phi,$$

where $T_1$, $T_2$, $U_1$, $U_2$, $V_1$ and $V_2$ are observables integrated over $m_{4\ell}(q_2^2)$ and $m_{12}$.

The normalized distributions, $\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta_1}$ vs $\cos \theta_1$, $\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta_2}$ vs $\cos \theta_2$ and $\frac{1}{\Gamma} \frac{d\Gamma}{d\phi}$ vs $\phi$ for SM are shown in Fig. 2. Fig. 3 and Fig. 4 respectively for simulated data. It should be noted that the angular coverage for $\cos \theta_1$ or $\cos \theta_2$ covers the full range from $-1$ to $+1$ and coverage for $\phi$ from 0 to $2\pi$ are still retained even after using actual detector scenarios. The cut flow analysis of Case-I is followed by the analysis of SM Higgs and Higgs with CP-odd admixture at 300 fb$^{-1}$. At 3000 fb$^{-1}$ since the statistics is higher, we will use stronger cut based analysis i.e. sequential cut flow analysis of Case-II for benchmarking SM Higgs and Higgs with different CP configuration. Moreover it should be noted that we have used the same cut based analysis for CP-odd admixture, CP-even higher derivative contribution and CP-even-odd scenario. The cross section for each benchmark scenarios are within the current experimental allowed region.

![FIG. 2. The normalized distribution $\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta_1}$ vs $\cos \theta_1$ for SM Higgs events.](image-url)
The simulated data are binned in \( \cos \theta \), \( \phi \) and fitted using Eq.(26), Eq.(27) and Eq.(28) to obtain the angular asymmetries \( t_1, t_2, u_1, u_2, v_1, v_2 \) and their errors which correspond to the angular asymmetries \( T_1, T_2, U_1, U_2, V_1 \) and \( V_2 \) respectively. Once the values of the integrated observables \( t_1, t_2, u_1, u_2, v_1, v_2 \) and their respective errors are found, the \( \chi^2 \) formula:

\[
\chi^2 = \frac{(\Delta t_2)^2}{(\Delta t_2)^2} + \frac{(\cos \delta \Delta T_1 - t_1)^2}{(\Delta t_1)^2} + \frac{(\Delta u_1 - u_1)^2}{(\Delta u_1)^2} + \frac{(\Delta u_2 - u_2)^2}{(\Delta u_2)^2} + \frac{(\sin \delta \Delta V_2 - v_2)^2}{(\Delta v_2)^2} + \frac{(\sin \delta \Delta V_1 - v_1)^2}{(\Delta v_1)^2}
\]

will find the \( b/a, c/a \) and the phase \( \delta \). The errors in \( b/a, c/a \) and phase \( \delta \) can also be calculated using the error matrix

\[
\left( \frac{\partial^2 \chi^2}{\partial \alpha_i \partial \alpha_j} \right)_{\alpha_i}
\]

where \( \alpha_i, \alpha_j = b/a, c/a, \delta \). To find the best fit values we have used Mathematica 9[69].

A. Study of angular asymmetries of Higgs at 14 TeV and 300 fb\(^{-1}\)

We start benchmarking angular observables for SM Higgs and Higgs with CP-odd admixture in this section. The fit values for observables are tabulated along with the best fit values of \( b/a, c/a \) and phase \( \delta \) of the CP-odd coupling. We also obtain 1\( \sigma \) and 2\( \sigma \) contours for \( b/a \) vs \( c/a \) and \( \delta \) vs \( c/a \) for the case of CP-odd admixture. This will provide the precision at which one can rule out anomalous contributions in \( HZZ \) couplings, establishing the SM nature of \( H \) at 14 TeV 300 fb\(^{-1}\) LHC run.

1. SM Higgs

SM Higgs events are generated with \( a = 1, b = 0 \), \( c = 0 \). The fit values of the observables for the SM Higgs are tabulated in Table II.

**TABLE II.** The values of the observables for the SM Higgs with respective errors.

| Observables | Values with errors |
|-------------|-------------------|
| \( t_2 \)   | \(-0.21 \pm 0.09\) |
| \( t_1 \)   | \((-1.6 \pm 7.16) \times 10^{-2}\) |
| \( u_2 \)   | \(0.32 \pm 0.40\) |
| \( u_1 \)   | \((0.93 \pm 4.36) \times 10^{-1}\) |
| \( v_2 \)   | \((-0.72 \pm 4.03) \times 10^{-1}\) |
| \( v_1 \)   | \((0.19 \pm 3.70) \times 10^{-1}\) |

The values of the observables \( t_2 \) and \( u_2 \) are large compared to other observables as discussed in the previous section, playing important role in the \( \chi^2 \) expression in Eq.(29). The observables \( t_1, v_2 \) and \( v_1 \) provide information about phase for anomalous couplings \( b \) and \( c \). The best fit values of \( b/a \) and \( c/a \) with their respective errors for the SM Higgs are given as follows:

\[
b/a = (0.50 \pm 0.96) \times 10^{-4} \text{ GeV}^{-2} \quad (31)
\]

\[
c/a = (0.68 \pm 2.27) \times 10^{-4} \text{ GeV}^{-2} \quad (32)
\]

The best fit values with 1\( \sigma \) and 2\( \sigma \) contours for \( b/a \) vs \( c/a \) are shown in Fig. 5.

2. Higgs with CP-odd admixture

The CP-odd admixture is characterised by a non zero value of \( c \) in Eq.(4). For CP-odd admixture case, Higgs events are generated using \( a = 0.7, b = 0 \) and \( c = (2.2 + 2.2i) \times 10^{-4} \). The values of the observables are given in Table III.

![Graph](image1)

**FIG. 3.** The normalized distribution \( \frac{1}{\Gamma} \frac{d \Gamma}{d \cos \theta_2} \) vs \( \cos \theta_2 \) for SM Higgs events.

![Graph](image2)

**FIG. 4.** The normalized distribution \( \frac{1}{\Gamma} \frac{d \Gamma}{d \phi} \) vs \( \phi \) for SM Higgs events.
TABLE III. The values of the observables for CP-odd admixture Higgs with respective errors at 300 fb\(^{-1}\).

| Observables | Values with errors              |
|-------------|---------------------------------|
| \(t_2\)     | \(-0.06 \pm 0.10\)              |
| \(t_1\)     | \(-0.11 \pm 0.08\)              |
| \(u_2\)     | \(-0.08 \pm 0.40\)              |
| \(u_1\)     | \((-0.80 \pm 4.04) \times 10^{-1}\) |
| \(v_2\)     | \((0.99 \pm 4.20) \times 10^{-1}\) |
| \(v_1\)     | \((0.44 \pm 4.21) \times 10^{-1}\) |

The value of \(t_2\) has now become smaller compared to the SM case as shown in Table II. Most importantly the non zero value of \(t_1\) arises due to the complex CP-odd anomalous coupling \(c\). This will play a significant role along with \(t_2\) and \(u_2\) in probing anomalous CP-odd admixture of \(HZZ\) couplings. The best fit values for \(b/a\), \(c/a\) and the phase \(\delta\) for CP-odd admixture are:

\[
\begin{align*}
b/a &= (1.50 \pm 1.09) \times 10^{-4} \text{ GeV}^{-2} \quad (33) \\
c/a &= (5.48 \pm 1.12) \times 10^{-4} \text{ GeV}^{-2} \quad (34) \\
\delta &= (0.29 \pm 2.14) \text{ in radian}. \quad (35)
\end{align*}
\]

Note that the error in \(\delta\) is still very large at this luminosity.

The best fit values with \(1\sigma\) and \(2\sigma\) contours for \(c/a\) vs \(b/a\) and \(\delta\) vs \(c/a\) are shown in Fig. 6 and Fig. 7 respectively.

B. Study of angular asymmetries of Higgs at 14 TeV 3000 fb\(^{-1}\)

High Luminosity LHC (HL-LHC) i.e 14 TeV 3000 fb\(^{-1}\) run before the energy upgrade will allow us to test CP structure of \(HZZ\) couplings even more precisely. For

![FIG. 5. \(c/a\) vs \(b/a\) \(1\sigma\) (green) and \(2\sigma\) (yellow) contours for the SM Higgs at 300 fb\(^{-1}\). The best fit values \((b/a,c/a)\) is shown by the block dot. The ‘*’ corresponds to \(b = c = 0\).](image1)

![FIG. 6. \(c/a\) vs \(b/a\) \(1\sigma\) (green) and \(2\sigma\) (yellow) contours for CP-odd admixture Higgs at 300 fb\(^{-1}\). The best fit value of \((b/a,c/a)\) is shown by the block dot. The values with which data are generated \((b/a = 0, c/a = 4.44 \times 10^{-4})\) is shown by the ‘*’. The cross-hair corresponds to \(b = c = 0\).](image2)

![FIG. 7. \(\delta\) vs \(c/a\) \(1\sigma\) (green) and \(2\sigma\) (yellow) contours for CP-odd admixture Higgs at 300 fb\(^{-1}\). The best fit values \((c/a,\delta)\) is shown by the block dot. The values with which data are generated is shown by the ‘*’.](image3)

3000 fb\(^{-1}\) also, we have followed the same cut based analysis that we have discussed earlier apart from a strong sequential \(p_T\) ordering i.e. \(p_T\) of at least three leptons in a quadruplet must satisfy \(p_T > 20\) GeV.

At 3000 fb\(^{-1}\) we revisit the benchmark cases of SM and CP-odd admixture along with two new analysis of CP-even higher derivative contribution and CP-even-odd scenario.

1. SM Higgs and CP-odd admixture Higgs

First we investigate CP-odd Higgs and SM Higgs and find out the values of angular observables along with their respective errors. For CP-odd admixture we have again taken \(a = 0.7, b = 0, c = (2.2+2.2\iota) \times 10^{-4}\) and SM Higgs
a = 1, b = 0, c = 0. The fit values of the observables $t_2$, $t_1$, $u_2$, $u_1$, $v_2$, $v_1$ for the SM and CP-odd admixture Higgs are tabulated in Table IV and Table V respectively. The errors have significantly reduced for all the observables and the fit values for ratios of couplings for the SM Higgs $b/a$, $c/a$ are given

\[
\begin{align*}
    b/a &= (0.43 \pm 0.55) \times 10^{-4} \text{ GeV}^{-2} \quad (36) \\
    c/a &= (1.08 \pm 1.17) \times 10^{-4} \text{ GeV}^{-2} \quad (37)
\end{align*}
\]

The 1σ and 2σ contours for $b/a$ vs $c/a$ are shown in Fig. 8.

At 3000 fb$^{-1}$ from Table V one can see that the errors in $t_1$ and $t_2$ are much reduced, making them very good observables for probing CP-odd admixture. The best fit values for $b/a$, $c/a$ and phase $\delta$ for CP-odd admixture are given as

\[
\begin{align*}
    b/a &= (0.40 \pm 0.54) \times 10^{-4} \text{ GeV}^{-2} \quad (38) \\
    c/a &= (3.99 \pm 0.64) \times 10^{-4} \text{ GeV}^{-2} \quad (39) \\
    \delta &= 0.45 \pm 1.08 \text{ in radian.} \quad (40)
\end{align*}
\]

It should be noted that the error in $\delta$ has become lower due to the fact that the error in $t_1$, which constraints the phase $\delta$, is much reduced.

### Table IV. The values of the observables for the SM Higgs with respective errors at 3000 fb$^{-1}$.

| Observables | Values with errors |
|-------------|--------------------|
| $t_2$       | $-0.20 \pm 0.04$   |
| $t_1$       | $(0.28 \pm 0.35) \times 10^{-1}$ |
| $u_2$       | $0.21 \pm 0.19$    |
| $u_1$       | $(0.46 \pm 2.06) \times 10^{-1}$ |
| $v_2$       | $-(-0.07 \pm 1.96) \times 10^{-1}$ |
| $v_1$       | $-(-0.18 \pm 1.84) \times 10^{-1}$ |

### Table V. The values of the observables for CP-odd admixture Higgs with respective errors at 3000 fb$^{-1}$

| Observables | Values with errors |
|-------------|--------------------|
| $t_2$       | $-0.11 \pm 0.04$   |
| $t_1$       | $-0.06 \pm 0.03$   |
| $u_2$       | $0.02 \pm 0.18$    |
| $u_1$       | $(-0.10 \pm 0.56) \times 10^{-1}$ |
| $v_2$       | $(0.72 \pm 1.84) \times 10^{-1}$ |
| $v_1$       | $(0.67 \pm 1.83) \times 10^{-1}$ |

The 1σ and 2σ contours for $b/a$ vs $c/a$ and $\delta$ vs $c/a$ are shown in Fig. 9 and Fig. 10 respectively.

![FIG. 9. c/a vs b/a 1σ (green) and 2σ (yellow) contours for CP-odd admixture Higgs at 3000 fb$^{-1}$. The best fit value of (b/a,c/a) is shown by the block dot. The '⋆' corresponds to b = c = 0.](image)

So far, we have discussed how using angular asymmetries one can probe HZZZ couplings of the SM Higgs and Higgs with mixed CP scenarios at 14 TeV for two different luminosity 300 fb$^{-1}$ and 3000 fb$^{-1}$. The values of the observables vary depending on the values of $a$, $b$ and $c$. The observables $T_1$, $V_1$ and $V_2$ are sensitive to CP-odd admixture and can be a good candidate to probe CP-odd admixture. Finally the best fit values for the ratios of the couplings, $b/a$ and $c/a$ are calculated using Eq.(29).

### 2. Higgs with CP-even higher derivative contribution and CP-even-odd scenario

In this subsection we benchmark the angular asymmetries for the cases:

1) Higgs with CP-even higher derivative contribution.
2) Higgs with both CP-odd and CP-even higher derivative contribution i.e. CP-even-odd scenario.

For CP-even higher derivative contribution we have taken $a = 0.80$, $b = 10^{-4}$, $c = 0$. However for the CP-even-odd scenario, we have taken $a = 0$, $b = 0.80$, $c = 10^{-4}$. The 1σ and 2σ contours for $b/a$ vs $c/a$ and $\delta$ vs $c/a$ are shown in Fig. 9 and Fig. 10 respectively.
odd scenario we have taken $a = 0.75$, $b = 8 \times 10^{-5}$, $c = (1 + 1i) \times 10^{-4}$. The values of the observables for CP-even higher derivative contribution are tabulated in Table VI. The best fit values for $b/a$ and $c/a$ with errors for CP-even higher derivative contribution are given as

$$b/a = (1.04 \pm 0.43) \times 10^{-4} \text{GeV}^{-2}$$

$$c/a = (0.34 \pm 1.07) \times 10^{-4} \text{GeV}^{-2}$$

The $1\sigma$ and $2\sigma$ contours for $b/a$ vs $c/a$ for Higgs with CP-even higher derivative contribution is shown in Fig. 11.

The values of observables for Higgs in CP-even-odd scenario are tabulated in Table VII. It should be noted that value of the observable $t_1$ is large due to non zero CP-odd admixture for the CP-even-odd scenario.

At 3000 fb$^{-1}$ the best fit values of $b/a, c/a, \delta$ for the CP-even-odd scenario are given as

$$b/a = (0.59 \pm 0.40) \times 10^{-4} \text{GeV}^{-2}$$

$$c/a = (2.10 \pm 0.87) \times 10^{-4} \text{GeV}^{-2}$$

$$\delta = 0.57 \pm 1.33 \text{ in radian.}$$

The $1\sigma$ and $2\sigma$ contours for $b/a$ vs $c/a$ and $\delta$ vs $c/a$ are shown in Fig. 12 and Fig. 13 respectively.

### C. Momentum dependence of Higgs couplings

The vertex factors $a$, $b$ and $c$ written in Eq. (4) are in general momentum dependent but in SM they have no momentum dependence. Thus it is essential to verify their momentum independence to establish $H$ as the SM Higgs. To achieve this, one has to measure the momentum dependence of $a$, $b$ and $c$ in different momentum regions. In LHC one only measures the ratios of couplings i.e. $b/a$ and $c/a$. However one can measure the values of $b/a$ and $c/a$ in different $q_2^2$($m_{34}$) regions. This will allow us to check the momentum dependence of $b/a$ and $c/a$ and finally $a$ at 14 TeV 300 fb$^{-1}$ LHC run.

Integrating Eq. (11), (12), (13) over $q_2^2$ we get unangular distribution in $n$-th bin as follows

$$\frac{1}{\Gamma^n \cos \theta_1} \frac{d\Gamma}{d\cos \theta_1} = \frac{1}{2} - T_1^n \cos \theta_1 + T_2^n P_2(\cos \theta_1),$$

where $T_n$ and $P_2$ are Legendre polynomials.
FIG. 12. c/a vs b/a 1σ (green) and 2σ (yellow) contours for CP-even-odd scenario Higgs at 3000 fb\(^{-1}\). The best fit value of (b/a, c/a) is shown by the black dot. The values with which data are generated (b/a = 1.06 × 10\(^{-4}\), c/a = 1.89 × 10\(^{-4}\)) is shown by the ‘+’. The cross-hair corresponds to b = c = 0.

FIG. 13. δ vs c/a 1σ (green) and 2σ (yellow) contours for CP-even-odd scenario Higgs at 3000 fb\(^{-1}\). The best fit values (b/a, c/a) is shown by the black dot. The cross-hair corresponds to δ = c = 0. The value with which data are generated is shown by the ‘+’.

TABLE VIII. Three \(\sqrt{q_T^2} = m_{34}\) bins and corresponding number of events in each bins for momentum dependence measurements at 14 TeV 300 fb\(^{-1}\) LHC run.

| Bin No. | Bin range \(\sqrt{q_T^2} = m_{34}\) | Number of events |
|---------|------------------------------------|------------------|
| Bin 1   | 12.00 GeV < \(m_{34}\) < 29.00 GeV | 210              |
| Bin 2   | 29.00 GeV < \(m_{34}\) < 46.00 GeV | 187              |
| Bin 3   | 46.00 GeV < \(m_{34}\) < 80.00 GeV | 52               |

tex factor ‘a’ in these 3 bins, we rewrite the decay width in \(n\)-th bin \(\Gamma_n\) as follows

\[
\Gamma_n = a_n^2 \Gamma_n'(b/a, c/a). \quad (49)
\]

where \(a_n\) is the value of \(a\) in \(n\)-th bin. \(\Gamma_n'\) is obtained by dividing \(\Gamma_n\) by \(a^2\) and making it a function of \(b/a\) and \(c/a\) only. We can calculate the values of \(\Gamma_n'\) and its errors in different bins by substituting the values of \(b/a\) and \(c/a\) from Table IX.

TABLE IX. b/a and c/a with corresponding errors in 3 bins in GeV\(^{-2}\).

| Bin No. | b/a in 10\(^{-4}\)GeV\(^{-2}\) | c/a in 10\(^{-4}\)GeV\(^{-2}\) |
|---------|-------------------------------|-------------------------------|
| Bin 1   | 0.02 ± 0.44                   | 0.30 ± 0.57                   |
| Bin 2   | 0.39 ± 0.79                   | 0.60 ± 0.89                   |
| Bin 3   | 1.35 ± 2.09                   | 1.56 ± 2.24                   |

Fit values of \(b/a\) and \(c/a\) have relatively larger errors in last bin due to small number of events and both \(b/a\) and \(c/a\) are consistent with zero in each \(m_{34}\) bins.

For resonant production of Higgs we can factor out the production cross-section and \(\Gamma\) in 10\(^{-4}\)GeV\(^{-2}\). At 3000 fb\(^{-1}\) \(\Gamma_i, \Gamma_j\) and \(N_i, N_j\) are the value of decay widths and number of events in \(i\) and \(j\)-th bin respectively, then

\[
\frac{\Gamma_i}{\Gamma_j} = \frac{N_i}{N_j} \quad (50)
\]

will hold between any two bins. Furthermore putting \(\Gamma_i\) and \(\Gamma_j\) expressions as written in Eq. (49) into Eq.(50) one finds the following relationship

\[
r_{ij} = \frac{a_i}{a_j} = \sqrt{\frac{N_i \Gamma_j}{N_j \Gamma_i}} \quad (51)
\]

If \(a\) is independent of momentum the ratio \(r_{ij}\) will always be 1 for any two bins. We tabulate all possible \(r_{ij}\) in Table. X for this three bins with their corresponding errors. One can similarly perform the same analysis at 3000 fb\(^{-1}\). At 3000 fb\(^{-1}\) the errors in \(b/a\) and \(c/a\) will be reduced and with the enhanced statistics, one may in principle have enough events to increase the number of bins to check momentum dependence of Higgs couplings.
These Ratios are consistent with SM (i.e ratio $r_{ij} = 1$) as they should. The ratio $r_{13}$ and $r_{23}$ have larger errors due to small statistics. These results will be improved by the bin size optimization and with larger statistics.

### IV. COMPARISON OF PRECISION MEASUREMENT BETWEEN 33 TEV LHC AND ILC

After HL-LHC (High Luminosity LHC), LHC energy could be upgraded to run at a center of mass energy of $\sqrt{s} = 33$ TeV. In this section we compare the precision measurements of $H$ coupling between 33 TeV LHC and ILC. We have followed the same cut based analysis as 14 TeV 3000 $fb^{-1}$ machine. At 33 TeV 3000 $fb^{-1}$ the number of events are given in the cut flow Table XI. The best fit values of $b/a$ and $c/a$ for SM Higgs are

$$b/a = (2.64 \pm 1.60) \times 10^{-5} \text{ GeV}^{-2} \quad (52)$$

$$c/a = (1.07 \pm 6.25) \times 10^{-5} \text{ GeV}^{-2} \quad (53)$$

In our first attempt, we find that a precision of $10^{-5}$ is achievable at the 33 TeV 3000 $fb^{-1}$ LHC, for CP-odd admixture and CP-even higher derivative contribution. In future, a more detailed analysis may improve the result and decrease of errors in $b/a$ and $c/a$. Although the precision achieved is high, LHC will only measure the ratios of Higgs couplings even at 33 TeV.

ILC will provide an independent test in measuring $HZZ$ couplings subject to its proposed implementation and will also offer invaluable probe to such couplings. At LHC one measures $\sigma \cdot BR(H \rightarrow ZZ)$ however ILC will measure the branching ratio $BR(H \rightarrow ZZ)$ by measuring the inclusive cross section $\sigma_{ZH}$ for the process $e^+e^- \rightarrow ZH$. This inclusive measurement of cross section alone will probe $HZZ$ couplings to 1.3$\%$\cite{63}. Identifying a $Z$ boson in recoil against the Higgs boson one can find out the partial width $\Gamma(H \rightarrow ZZ)$

$$\Gamma_{total} = \frac{\Gamma(H \rightarrow ZZ)}{BR(H \rightarrow ZZ)} \quad (54)$$

For example the expected precision for CP-odd anomalous coupling $c$ at ILC\cite{64} is $7 \times 10^{-4}$ to $8 \times 10^{-6}$ which is roughly around the loop induced CP-odd contribution. We have shown that precision in the measurement of $c/a$ can be $6 \times 10^{-5}$ for 33 TeV LHC using angular asymmetries. However at ILC the measurement of Higgs decay width will allow us to extract the absolute values $a$, $b$, $c$ which is beyond the scope of LHC.

### V. CONCLUSION

We demonstrate that angular asymmetries will provide a strong and efficient tool to probe Higgs couplings in high luminosity future LHC runs. With the increased statistics at 14 TeV run, LHC will enter into precision era and angular analysis will offer a step by step methodology to study the Higgs couplings. Angular asymmetries can be utilized to probe the Higgs couplings and to disentangle its exact CP property in the next LHC run. We benchmark our observables for SM, CP-odd admixture, CP-even higher derivative contribution and finally CP-odd admixture with Higher derivative CP-even coupling. We perform the analysis for two different luminosities at 300 $fb^{-1}$ as well as 3000 $fb^{-1}$ and study the precision with which angular analysis probe $HZZ$ couplings. The study of the momentum dependence of the Higgs couplings would be a significant step forward in establishing its SM nature, since in the SM Higgs couplings do not have any momentum dependence. At 14 TeV LHC run with the improved statistics, we present how one can examine the momentum dependence of the Higgs couplings in different momentum regions. We have further

| Cuts                          | Number of events |
|-------------------------------|------------------|
| Selection cuts                | 6523             |
| $50 \text{ GeV} < m_{31} < 106 \text{ GeV}$ | 6362             |
| $12 \text{ GeV} < m_{34} < 115 \text{ GeV}$ | 5935             |
| $115 \text{ GeV} < m_{44} < 130 \text{ GeV}$ | 5852             |
discussed what precision LHC can achieve in the measurement of the Higgs couplings to Z boson by angular analysis at 33 TeV. A comparison has also been made between precision measurement of 33 TeV LHC and ILC. Angular analysis will be a powerful technique to decipher the CP properties of Higgs couplings at 14 TeV LHC run and will open up a new domain of precision measurement.

ACKNOWLEDGMENTS

Work of BB is supported by Department of Science and Technology, Government of INDIA under the Grant Agreement numbers IFA13-PH-75 (INSPIRE Faculty Award).

Appendix A

The amplitudes for the process $H$ with spin $J$(spin 0) decays to two $Z$(spin 1) boson with spin projections along $z$ axis $\lambda_1$ and $\lambda_2$ is[67, 68]

$$\mathcal{M}(J_z, \lambda_1, \lambda_2) = \left(\frac{2J+1}{4\pi}\right)^{\frac{1}{2}} \mathcal{D}^J_{J_z\lambda}(\Phi, \Theta, -\Phi) A_{\lambda_1, \lambda_2}, \tag{A1}$$

where $\mathcal{D}^J_{J_z\lambda}$ is the Wigner-D function, $\lambda = |\lambda_1 - \lambda_2|$ and $A_{\lambda_1, \lambda_2}$ are the helicity amplitudes with $\lambda_{1,2} \in \{\pm 1, 0\}$, $J = |J|$. Angular momentum conservation implies $|\lambda| = |\lambda_1 - \lambda_2| \leq J$ and the helicity amplitudes are related as $A_{\lambda_2, \lambda_1} = (-1)^J A_{\lambda_1, -\lambda_2}$. We have three orthogonal helicity amplitudes $A_0$, $A_{++}$ and $A_{--}$. However one can write three helicity amplitudes in transversity basis as:

$$A_L = A_0$$
$$A_\parallel = \frac{1}{\sqrt{2}}(A_{++} + A_{--})$$
$$A_\perp = \frac{1}{\sqrt{2}}(A_{++} - A_{--}).$$

Helicity fractions $\mathcal{F}_L$, $\mathcal{F}_\parallel$ and $\mathcal{F}_\perp$ are defined as

$$\mathcal{F}_\lambda = \frac{A_\lambda}{\sqrt{|A_L|^2 + |A_\parallel|^2 + |A_\perp|^2}},$$

Moreover $\eta = \frac{2\nu_\alpha}{v^2 + \Delta^2}$ with $v_e = 2I_{3\ell} - 4e\sin^2 \theta_W$ and $\alpha_e = 2I_{3\ell}$.

$\Gamma_\ell$ is defined as

$$\Gamma_\ell = \frac{d\Gamma}{dq_\ell^2} = N \left(|A_L|^2 + |A_\parallel|^2 + |A_\perp|^2\right), \tag{A5}$$

with $N = \frac{1}{2\pi^2} \alpha_e^2 \frac{\sin^2\theta_W}{\Gamma_{1\ell}} \frac{1}{M_{H_1} M_{Z_2}} \frac{1}{(q^2 - M^2_{H_1})^2 + M^2_{Z_2} M^2_{H_1}}$. $\Gamma_Z$ is the decay width of $Z$, $\text{Br}_{H\ell}$ is the branching fraction for the decay of $Z$ to two massless leptons. However we assumed narrow width approximation for on-shell $Z_1$ boson in Sec.II. In Sec.III we have implemented the cut flow table while integrating over $q^2_1$ and $q^2_2$ to find the expressions for $T_1$, $T_2$, $U_1$, $U_2$, $V_1$ and $V_2$.

The expressions for $T_1$, $T_2$, $U_1$, $U_2$, $V_1$ and $V_2$ are

$$\begin{align*}
T_1 &= 1.32 \times 10^{-9} y \\
T_2 &= -9.65 \times 10^{-9} - 4.00 \times 10^{-10} y^2 \\
U_1 &= -2.0 \times 10^{-9} - 6.33 \times 10^{-10} x + 7.11 \times 10^{-11} y^2 \\
U_2 &= 6.06 \times 10^{-9} + 4.35 \times 10^{-10} x + 7.97 \times 10^{-10} y^2 \\
V_1 &= 5.57 \times 10^{-8} + 2.61 \times 10^{-8} x + 3.98 \times 10^{-9} y^2 + 1.60 \times 10^{-10} y^2 \\
V_2 &= 5.57 \times 10^{-8} + 2.61 \times 10^{-8} x + 3.98 \times 10^{-9} y^2 + 1.60 \times 10^{-10} y^2 \\
\end{align*} \tag{A6\text{-}A11}$$

where $x = \frac{b}{a} \times (100\text{Gev})^2$ and $y = \frac{c}{a} \times (100\text{Gev})^2$

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