Fibre-reinforced cementitious composite: parameter identification using Ohno shear beam test

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Abstract. Computational-experimental methodology based on artificial neural networks used to identify the material parameters of fibre-reinforced cementitious composite is presented and applied for Ohno shear beam test. The aim is to provide techniques for an advanced assessment of the mechanical fracture properties of these materials, and the subsequent numerical simulation of components/structures made from them. The paper describes the development of computational and material models utilized for efficient material parameter determination with regards to a studied composite. The data is used in inverse analysis based on artificial neural networks together with sensitivity analysis which plays an important role in the process. Developed software tool FRCID-S is also briefly described.

1. Introduction

A key aspect when performing nonlinear fracture mechanics modelling of structures made of fibre-reinforced composite (FRC) is the knowledge about the tensile softening model and its parameters, as well as the corresponding fracture energy which dissipates during the cracking process, as was emphasized e.g. in [1, 2]. The aim of the performed research is to deepen existing knowledge about the behaviour of the studied FRC material especially in relation to its resistance to crack propagation. There are several recent applications of artificial neural networks (ANN) for FRC or general engineered cementitious materials. For example, in [3], selected properties of engineered cementitious composites were predicted using an ANN which had been trained using data collected from the literature. Another practical example can be found in [4], where an ANN was employed for the prediction of tensile and compressive strengths at 28 days of hardening. The utilization of the ANN in this paper is quite different. Here, selected material parameters of FRC are identified based on inverse analysis and the virtual simulation of a fracture test. The ANN is used here as a surrogate model describing the inverse relationship between structural response parameters and material parameters.

The research consists of several parts: First, a suitable constitutive law for FRC cementitious composites was developed within ATENA nonlinear fracture mechanics software [5]. The computational model was verified using experimental data of Ohno shear beam test [6]. Second, sensitivity analysis was performed. It showed the importance of individual material parameters related to the response of specimens tested under laboratory conditions in specific shear test configuration. With the help of the sensitivity analysis results, software for the identification of material parameters was developed. This program, which is called FRCID-S, implements the inverse analysis method based on an artificial neural network (ANN) [7, 8] in combination with efficient statistical simulation.
2. **Shear test**

Identification of material parameters is performed by the inverse analysis method with the use of laboratory test results. Here, the focus is on the shear test of the so-called Ohno beam with dimensions and load as shown in figure 1. The inner part of the Ohno beam (marked in grey in the figure) is subjected to pure shear stress (the bending moment is zero in the middle). Flexural reinforcement is placed in the outer parts of the beam to prevent bending failure. Average shear stress is calculated as:

\[
\tau_{av} = \frac{2}{3} \tau_{max} = \frac{2}{3} \frac{F}{2A} = \frac{F}{3A},
\]

where \(F\) is the magnitude of the force load and \(A\) is the cross-sectional area of the beam at the point of the largest shear stress (the inner part of the beam). The shear strain corresponds to the oblique direction indicated by the green arrow in figure 1. Strain was determined in the laboratory using the Optotrak system, see [6] for details.

![Figure 1. A schematic view of Ohno beam subjected to shear test.](image1)

When creating and setting the computational model of the Ohno beam tested in shear, the result of the laboratory test in the form of a shear stress vs. shear strain diagram was used, see figure 2. A DRECC composite containing 7% by volume of Dramix steel fibres 6 mm long and 0.15 mm in diameter was tested. The ratio of water to cement in the mix was 0.27. It also contained the following additives – silica fume (ratio to cement weight was 0.20), coupling agent (0.0025) and super-plasticizer (0.03). Specimen was moist cured for 24 hours before demoulding, and subsequently cured in water for four weeks. The age at testing was 6 weeks. DRECC composite shows relatively high tensile and compressive strength and average ductility. The value of the strength at the formation of the first crack was 4.9 MPa, the corresponding relative deformation 0.17%, the value of the strength at maximum load was 9.89 MPa, the maximum deformation was 0.7%. Further details on the material used, its properties and details of laboratory testing can be found in [6].

![Figure 2. Experimental diagram of average shear stress vs. shear strain obtained from the Ohno beam test.](image2)
3. Computational model

The nonlinear numerical simulation of the shear test (figure 1) is performed using the finite element method (FEM) based program ATENA [5]. The constitutive relationship at the material point plays the most important role in nonlinear calculation and determines to what extent the computer model captures the actual behaviour of the structure. Since fibre-reinforced concrete is a complex heterogeneous material with a strongly nonlinear response, the nonlinear material model "3DNonlinearCementitious2User" is used to realistically calculate the response of this composite, capturing all important aspects of its behaviour in tension, compression, and shear.

The concrete tensile damage model is based on nonlinear fracture mechanics combined with the crack band model and the concept of smeared cracks. The main material parameters here are the tensile strength and the shape of the softening function (curve characterizing the crack opening versus the residual tensile stress). The real discrete crack is replaced in the model by a band of localized strains. The relative strain corresponding to the crack width is related to the finite element size. The softening function in the material law for the smeared crack model must therefore be determined individually for each finite element so that the prescribed relationship for the crack width is maintained. Only such a model based on an energy formulation ensures the objectivity of the solution and independence from the finite element network [5].

The proposed function to capture the tensile softening phase of the studied composite is shown in figure 3a. Its shape tries to capture the behaviour of the composite according to the modelled test. The shape of the function is described by other three parameters $T_1–T_3$ in addition to the tensile strength, as can be seen in figure 3a. The response of the material to shear loading and failure is described by two functions, which characterize the reduction of the shear stiffness (see figure 3b) and the reduction of the shear strength of the material (see figure 3c) due to cracks. Bilinear functions were chosen as the basic model with the parameters $S_1, S_2, R_1, R_2$ defining the breakpoint of the shear softening function and shear strength reduction function (figure 3b,c).

The developed FEM model of the Ohno shear beam is shown in figure 4. The forces are applied in accordance with the real test at the load locations. The nonlinear solution is initially implemented using a modified Newton-Raphson iterative method, which is later changed to the Arc-length method in order to more accurately determine the ultimate capacity of the model.

Figure 3. Proposed functions for the studied composite: a) tensile softening, b) shear stiffness reduction, c) shear strength reduction.

Figure 4. FEM computational model of Ohno shear beam.
4. Sensitivity analysis
A parametric study was performed to understand the influence of the individual input parameters of the material model on the response of the beam in shear. The aim was to identify the so-called dominant parameters, to identify parameters that have no effect on the response in the test and thus exclude them from the inverse analysis, to determine the realistic ranges of the individual parameters and to propose a set of suitable response parameters to be used as input to the inverse analysis and to allow unambiguous identification of the material parameters of interest. The results of the sensitivity analysis yielded the following conclusions relevant to the identification presented below:

- Tensile strength is one of the dominant parameters. An increase in its value leads to an increase in the overall resistance of the specimen. As the value increases, the initial linear phase of the diagram also increases.
- The parameter $T_1$ related to the activation of fibres in the cement matrix shows a similar effect as the tensile strength, however, the length of the linear part of the diagram does not change with increasing value – the point where the change in stiffness due to microcracks occurs is maintained.
- Parameters $T_2$ and $T_3$ do not affect the overall load capacity of the specimen but have some influence on the value of the maximum relative deformation achieved. This effect is more pronounced for $T_2$, while for $T_3$ a reasonable maximum limit is soon reached.
- Of the shear parameters $S_1$, $S_2$, $R_1$ and $R_2$, the only parameter with some degree of influence on the response shape is the $S_1$ parameter related to shear stiffness reduction. With a change in its value, a slight change in the stiffness of the final part of the diagram occurs when shear failure of the specimen occurs.
- The modulus of elasticity $E$ has an expected effect on the stiffness of the whole specimen and thus on the initial slope of the whole response diagram.

For the purpose of creating a stochastic model for the inverse analysis and subsequent identification, the results show that due to the relatively unambiguous linear dependence of the elastic modulus $E$ on the value of the initial stiffness of the test specimen and its easy determination independently of the other parameters, it can be excluded from the identification by neural networks and thus reduce the complexity of the inverse problem. Parameters that have no effect on the resulting shear response can be excluded from the identification and considered as default values in the calculation. Four parameters of the material model remain to be identified – tensile strength $f_t$, tensile parameters $T_1$, $T_2$ and shear parameter $S_1$.

Based on the interpretation of the results in terms of the selection of a set of appropriate response parameters for the inverse analysis, it appears appropriate to use the following input parameters for the neural network: force value at shear strain of 0.002, force value at shear strain of 0.005, maximum force value from the entire test, and shear strain value corresponding to the maximum force.

5. Methodology and software implementation
An inverse analysis method based on artificial neural network (ANN) is used to identify the parameters of the studied composite. The theoretical details and identification procedure is beyond the scope of this paper and can be found in papers [7, 8, 9]. The basic cornerstone of the method is an artificial neural network that transforms the measured data obtained from fracture experiments into the desired material parameters. The parameters of the network that determine its behaviour, such as synapses and biases, must first be set in a phase called network training. Network training is performed with the help of a training set, which is a set of known ordered input-output pairs. The individual elements of the training set are obtained using a virtual stochastic simulation of the test using the ATENA model described above. The first step is to create a stochastic model for the identified material parameters, the set of which was generated from the above sensitivity analysis. The parameters are assigned a probability distribution and the means (initial rough parameter estimates), and standard deviations are defined. A suitable distribution is a uniform distribution, which allows the boundaries of each parameter to be defined and leads to a uniform distribution of realizations over the range of their values. From
the defined variables, random realizations of all parameters are generated using the Latin hypercube sampling simulation method. Here, 30 simulations were used. Note that the importance of training sample preparation was previously emphasized and tested by Tong and Liu [10], including the LHS scheme. In spite of the fact that these authors concluded that number-theoretic methods appear to be the most efficient, the LHS scheme also provided very good results. Moreover, our focus on LHS is also determined by the general applicability of this small-sample simulation technique for practical statistical, sensitivity and reliability analyses in many fields of engineering.

A set of numerical FEM analyses is then performed with the generated realizations to provide a random response of the test specimen. With a suitably chosen stochastic model, the obtained responses well represent the range of experiments performed and contribute to good convergence in network training and subsequent identification. The random response in the form of average shear stress vs. shear strain diagrams are obtained. Appropriate response parameters are determined using sensitivity analysis, in this case the shear stresses at shear strains of 0.002 and 0.005, the shear stresses corresponding to the maximum load capacity and the shear strain corresponding to the maximum load capacity.

The random realizations of the material parameters together with the random response parameters of the structure are used as the training set for the neural network whose parameters are optimized. Once the network is properly trained to respond to the input information (random response) with the corresponding output (material parameters), it can be simulated with the experimental response as its input, see ANN in figure 5. The output of the simulation is the set of identified material parameters. With the obtained parameters, a validation numerical simulation can be performed, which should lead to the best possible agreement with the experimental measurements.

![Figure 5. FRCID-S software – parameter identification and export.](image)

The above-described method of parameter identification, which combines nonlinear simulations with the training of an ANN, is relatively time consuming and of high complexity. Therefore, the whole procedure was implemented in FraMePID-3PB software [8] and successfully used for the material parameter identification of plain concrete. The software has now been modified and updated for fibre-reinforced concrete tested in shear, including the tensile softening function and two shear functions, as
described in the section on the computational model. A screenshot of the first version of FRCID-S software is shown in figure 5.

6. Verification and application

Initial verification of the software and the implemented ANN was performed using a set of four randomly simulated shear responses of specimens made of the studied cement composite. Given the knowledge of the input set of material parameters, it was possible to directly compare their values with those obtained by identification (this is not possible for purely experimental data). A graphical comparison for the four tested specimens can be seen in figure 6, while a numerical comparison is given in table 1. Furthermore, a classical comparison was made in the form of the resulting responses of the specimens, i.e. shear stress–strain diagrams, see figure 7. The conclusions of the sensitivity analysis, i.e. the significantly greater dominance of the tensile strength $f_t$ and parameter $T_1$ of the tensile softening model compared to the other two identified parameters $T_2$ and $S_1$, were confirmed.

![Comparison of original and identified responses for 4 test samples.](image)

Table 1. Set of original and identified (in brackets) values of material model parameters for 4 test specimens.

| Specimen | $f_t$ (MPa) | $T_1$ (–) | $T_2$ (–) | $S_1$ (–) |
|----------|-------------|-----------|-----------|-----------|
| 1        | 5.728 (5.775) | 1.592 (1.616) | 2.83×10$^{-3}$ (2.76×10$^{-3}$) | 4.07×10$^{-1}$ (3.46×10$^{-1}$) |
| 2        | 5.971 (5.981) | 1.780 (1.774) | 4.62×10$^{-3}$ (3.76×10$^{-3}$) | 3.39×10$^{-1}$ (3.71×10$^{-1}$) |
| 3        | 6.292 (6.275) | 1.496 (1.484) | 3.84×10$^{-3}$ (3.67×10$^{-3}$) | 3.14×10$^{-1}$ (3.65×10$^{-1}$) |
| 4        | 6.531 (6.557) | 1.671 (1.680) | 2.34×10$^{-3}$ (2.47×10$^{-3}$) | 3.61×10$^{-1}$ (3.38×10$^{-1}$) |
Figure 7. Comparison of original and identified responses for 4 test samples.

The proposed material parameter identification method was applied together with the FRCID-S software to a selected specimen of the DRECC composite described in section 2. The result of the laboratory test in the form of shear stress–strain diagram is shown in figure 2. Compared to the simulated data in figure 7, the experimental curve shows a greater instability in the measurements, as is typical in real tests of fibre-reinforced concrete.

The input data in the form of the shear stress–strain diagram was loaded to the FRCID-S, processed and passed to the ANN for simulation, resulting in a set of corresponding material parameters, which are listed in table 2.

Table 2. A set of identified values of material model parameters for a sample of DRECC composite.

| Specimen | $f_i$ (MPa) | $T_1$ (–) | $T_2$ (–) | $S_1$ (–) |
|----------|------------|----------|----------|----------|
| DRECC    | 6.432      | 1.455    | $3.93 \times 10^{-3}$ | $3.79 \times 10^{-1}$ |

Verification of the obtained results was then carried out using numerical simulation of the test with the identified parameters. A comparison of the original experimental response with the response obtained by numerical calculation is shown in figure 8. It is evident from the comparison that the identified parameters define the tested composite very well and the resulting simulated response fits the experimental test well. Thus, the proposed ANN is a good surrogate model for the inverse problem of identifying the material parameters of the studied cement composite with Dramix steel fibres.
7. Conclusions

The paper describes a methodology and software tool which can be routinely used for the indirect determination of the mechanical fracture parameters of FRC. Experimental results of shear test beam were used for development of the user-friendly software consisting of a predefined and trained artificial neural network for the fast identification of material parameters. The whole concept is based on a combination of the statistical simulation method, finite element modelling and ANN.

Based on sensitivity analysis four parameters of the material model of FRC were identified as key parameters for inverse analysis – tensile strength \( f_t \), tensile parameters \( T_1 \), \( T_2 \) and shear parameter \( S_1 \). The results of identification and the subsequent performance of identified parameters on tested FRC samples proved the ANN’s efficiency and ability to identify material parameter values leading to the accurate simulation of the response of the studied composite with Dramix steel fibres.

Acknowledgment

This work has been supported by project “MUFRAS” No. 19-09491S, awarded by the Czech Science Foundation of the Czech Republic.

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