On some peculiarity in behavior of plasma with an arbitrary degree of degeneracy of electron gas in thin layer

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Abstract. In this study we considered the response of electron plasma with an arbitrary degree of degeneration to an alternating electromagnetic field. The electromagnetic field directed perpendicularly to the boundary of the plasma layer. We present an analytic solution of the boundary problem. The kinetic Boltzmann-Vlasov equation with the Bhatnagar–Gross–Krook collision integral and the Maxwell equation for the electric field are used. We consider the mirror boundary conditions for the reflections of electrons from the layer boundary. The boundary problem may be reduced to a one-dimensional problem with a single velocity. Separation of variables allow reducing the problem equations to a characteristic system of equations. The eigensolutions of the initial system, which correspond to the continuous spectrum, are Van Kampen mode. The eigensolutions corresponding to the adjoint and discrete spectra are Drude and Debye modes. We can construct the general solution. The Debye mode determines the plasma screening of the electric field. The behavior of this mode was analyzed. It depends on the parameters of the problem. In case of sufficiently high degrees of the electron gas degeneracy, the range of the Debye mode existence has a substantially nontrivial character, in which the ranges of existence and absence of this mode alternate with increasing electric field frequency.

1. Introduction
Vlasov was the first to tackle the problem of plasma fluctuations [1]. Landau [2] analytically solved the problem of the behavior of a collisionless plasma in the half-space in an external electric field perpendicular to the surface where the electrons reflect from the boundary according to the mirror law. Hence, the problem of plasma fluctuations is naturally called the Vlasov–Landau problem. The problem of fluctuations of a degenerate electron plasma in a metal layer was first analytically solved in [3]. Fluctuations of a Maxwellian plasma in the half-space were considered in [4]. This paper is a continuation of [3] and [4]. We generalize these results for the general case of a plasma with an arbitrary degree of degeneracy of the electron gas.

The problem with a diffusion boundary condition for a collision plasma in the half-space was first considered in [5], where general problems of the solvability of this problem with diffusion boundary conditions and the structure of the discrete spectrum depending on the parameters of the problem were studied.
Nevertheless, there was no detailed investigation of the general solution there because the solution is very complicated. A complete solution of the problem of fluctuations of the degenerate plasma was given in [6, 7].

Some related problems appear in studying of the reflection of an electromagnetic field from surfaces. They were examined in [8, 9], where the method of integral transformations was used. A general asymptotic analysis of the behavior of an electric field far from the surface was performed in [10, 11]. Special importance of analyzing the behavior of the field near the plasma resonance was emphasized in [10]. It was found in [11] that the behavior of the electromagnetic field in this case for the mirror and diffusion reflection of electrons on the surface differs crucially.

This problem is of great importance in plasma theory (see, e.g., [12, 13]) and currently remains to be studied from different standpoints [14].

The goal is to find an analytic solution. Reducing the problem to a one-dimensional problem with a single effective velocity is a crucial step toward an analytic solution. This idea is not innovative. It goes back to Landau [2], who introduced the main idea: to consider a lateral external electric field directed along the axis perpendicular to surface (x-axis). After this assumption, the problem becomes one-dimensional (along the x axis) with a single effective velocity.

This velocity is the projection $v_x$ of the electron velocity $v$ on the x axis. The initial three-dimensional problem (with respect to time and velocities) with such a decomposition does not lose generality, because this decomposition is only given by the direction of the external electric field.

When passing to dimensionless variables and parameters, a small parameter naturally appears in the equations, a variation of the dimensionless (chemical) potential due to the presence of the external electric field. Using the small-parameter method, we can linearize the problem with respect to the absolute Fermi--Dirac electron distribution. Using the conservation law for the number of particles, we can finally solve the problem as a one-dimensional boundary problem with a single effective velocity.

The separation of variables reduces the equations of the problem to the characteristic system of equations containing the spectral parameter. In the space of generalized functions, there exists a continuous family of eigensolutions of the initial system corresponding to the continuous spectrum. These are Van Kampen modes. This family corresponds to the only possible solution in the form of an integral along the continuous spectrum of the eigensolutions multiplied by an unknown function called the coefficient of the continuous spectrum.

Then we find the eigensolutions corresponding to the adjoint and discrete spectra by solving the dispersion equation; these are the respective Drude and Debye modes. We then compose the general solution of the boundary problem as a decomposition into the eigensolutions corresponding to the continuous, adjoint, and discrete spectra. This decomposition is a sum of the integral over the continuous spectrum of the eigensolutions multiplied by some function of the spectral parameter and the eigensolutions corresponding to the adjoint and discrete spectra multiplied by unknown variables. These are the coefficients corresponding to the adjoint and continuous spectra.

The decomposition coefficients of the solution of the continuous, adjoint, and discrete spectra can be found from the boundary conditions. This allows to obtain the decomposition of the distribution function and the electric field in explicit form.

2. Statement of the problem and the main equations
Let the Fermi--Dirac plasma with arbitrary degree of degeneracy be located in a layer $|x| < L$. We consider the external field to be sufficiently weak such that we can use the linear approximation [12]. We use the $\tau$--model Vlasov--Boltzmann equation and the Maxwell equation for the electric field:

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial r} + eE \frac{\partial f}{\partial p} = \nu (f_{eq} - f),$$

$$\text{div } E = 4\pi \rho, \quad \rho = e \int (f - f_0) d\Omega, \quad d\Omega = \frac{(2s + 1) d^3 p}{(2\pi \hbar)^3},$$

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We can reduce equations (1) and (2) to one-dimensional equations with a single effective velocity. This step is necessary for obtaining an analytic solution. The idea, as previously mentioned, was proposed by Landau [2], who proposed considering a longitudinal external electric field directed along the x axis.

Therefore, we consider a longitudinal external electric field outside the plasma perpendicular to the plasma boundary and changing by the law $E_{\text{ext}}(t) = E_0 e^{-\omega_0 t}(1,0,0)$. The self-consistent electric field inside the plasma is denoted by $E(x,t) = E(x) e^{-\omega_0 t}(1,0,0)$. It is easy to verify that $H = -(ic / \omega) \text{rot} E = 0$ for a chosen configuration of the external electric field. Therefore, the magnetic field has no role in equation (1).

The electric field causes changes of the chemical potential $\mu(x,t) = \mu + \delta \mu(x,t) e^{-\omega_0 t}$ where $\mu = \text{const}$ is the value of the chemical potential in the absence of an external electric field on the plasma boundary. We introduce a dimensionless (adjusted) chemical potential of the electron gas $\alpha = \mu / kT$. For the adjusted chemical potential, the latter equality has the form $\alpha(x,t) = \alpha + \delta \alpha(x,t) e^{-\omega_0 t}$.

We assume that the value $\delta \alpha(x,t) = \delta \alpha(x) e^{-\omega_0 t}$, which is the perturbation of the adjusted chemical potential, is a small parameter, i.e., $|\delta \alpha(x,t)| = |\delta \alpha(x)| \ll 1$. Physically, this inequality means that the perturbation of the chemical potential is much smaller than the electron heat energy. We use the method of consecutive approximations, taking $\delta \alpha(x) \ll 1$.

We linearize equations (1) and (2) with respect to the absolute Fermi–Dirac distribution function $f_0$. We also pass to dimensionless variables and parameters. We have to change $E, x, v, f, \varepsilon$ for dimensionless parameters. As a result of passing to dimensionless parameters and functions, we obtain the system of equations:

$$
\mu \frac{\partial H}{\partial x_1} + w_0 H(x_1, \mu) = \mu e(x_1) + \int_{-\infty}^{+\infty} k(\mu', \alpha) H(x_1, \mu') d\mu' \tag{3}
$$

$$
\frac{de(x_1)}{dx_1} = \kappa^2(\alpha) \int_{-\infty}^{+\infty} k(\mu', \alpha) H(x_1, \mu') d\mu' \tag{4}
$$

Function $H(x_1, \mu)$ has only one dimensionless velocity variable $\mu$.

In the case of the mirror reflection of electrons from the plasma boundary, we have the boundary conditions for the electron distribution functions on the boundary of a layer of size $2L$.

$$
\begin{align*}
&f(\pm L, v_x, v_y, v_z, t) = f(\pm L, -v_x, v_y, v_z, t), \quad -\infty < v_x < +\infty, \\
&H(l, \mu) = H(-l, -\mu), \quad H(-l, \mu) = H(l, -\mu), \quad \mu > 0,
\end{align*}
$$

where $l = L / \lambda$ is the size of the layer in terms of the electron free path.

For the electric field, the boundary condition has the form

$$
e(l) = 1, \quad e(-l) = 1 \tag{6}
$$

The boundary problem for plasma fluctuations in a conductive medium layer is thus completely formulated and consists in finding a solution of equations (3) and (4) such that boundary conditions (5) and (6) are satisfied.

3. Eigensolutions of the continuous spectrum

First, we seek a general solution of system of equations (3), (4). Separation of variables by the general Fourier method leads to the substitution

$$
H_\eta(x_1, \mu) = e^{-w_0 x_1 / \eta} \Phi_1(\eta, \mu) + e^{w_0 x_1 / \eta} \Phi_2(\eta, \mu),
$$

$$
e_\eta(x_1) = \left[ e^{-w_0 x_1 / \eta} + e^{w_0 x_1 / \eta} \right] E(\eta), \tag{7}
$$
where \( \eta \) is the spectral parameter (or separation) parameter, which is usually complex. Substituting (7) in (3) and (4), we obtain the characteristic system of equations, transform them and obtain finally the following expressions

\[
\Phi_1(\eta, \mu) = \frac{E(\eta)}{w_0} (\mu \eta - \eta^2 \nu) + \frac{1}{\eta - \nu} + g_1(\eta) \delta(\eta - \nu),
\]

\[
\Phi_2(\eta, \mu) = \frac{E(\eta)}{w_0} (\mu \eta + \eta^2 \nu) + \frac{1}{\eta + \nu} + g_2(\eta) \delta(\eta + \nu),
\]

Solutions (8) and (9) are called the eigenfunctions of the characteristic equation. Functions \( g_1(\eta) \) and \( g_2(\eta) \) depend on the dispersion function.

\[
\Lambda(z) = \Lambda(z, \Omega, \epsilon) = 1 + \frac{z}{w_0 \eta^2} \int_{-\infty}^{\infty} \frac{\eta^2 - \mu'}{\mu' - z} k(\mu', \alpha) d\mu'
\]

Family (8), (9) of the eigensolutions of equations (3) and (4) corresponds to the continuous spectrum. This family of the continuous spectrum is often called the Van Kampen mode.

4. Eigensolutions of adjoint and discrete spectra

We have to find the roots of the dispersion equation

\[
\frac{\Lambda(z)}{z} = 0.
\]

To that end, we expand the function \( \Lambda_\alpha(z, \alpha) \) in a Laurent series in the neighborhood of infinity:

\[
\Lambda(z) = \Lambda_\infty + \frac{\Lambda_{-2}}{z^2} + \frac{\Lambda_{-4}}{z^4} + \cdots
\]

Solution \( \Lambda_\infty \) is independent of the chemical potential. It is natural to call it a Drude mode. It describes the volume conductance of the metal plasma considered by Drude (see, e.g., [17]).

By definition, a discrete spectrum of the characteristic system of equations is a set of finite complex zeroes of the dispersion equation that do not belong to the real line (the section of the dispersion function).

By expansion (10), in a neighborhood of infinity, there are two zeros \( \pm \eta_0 \) of the dispersion function.

\[
\pm \eta_0 \approx \frac{\eta_1^2 k_3(\nu) - k_4(\nu)}{\eta_1^2 (w_0 - 1) + k_2(\nu)} = \frac{\epsilon(\epsilon - i\Omega) s_2^2(\epsilon) - s_0(\epsilon) s_4(\epsilon)}{s_0(\epsilon) s_2(\epsilon)(1 - \Omega^2 + i\epsilon\Omega)}
\]

Solution (11) is naturally called a Debye mode (it is a plasma mode). In the low-frequency case, it describes the famous Debye screening [17].

From (11), we can see that near the plasma resonance (for \( \Omega \approx 1 \), i.e., for \( \omega \approx \omega_n \), the absolute value of the zero \( \eta_0 \) is not bounded from above if \( \epsilon \uplus 1 \).

The set of physically meaningful parameters \( (\Omega, \epsilon) \) fills the quarter-plane \( \Omega \geq 0, \epsilon \geq 0 \). The case \( \Omega = 0 \) (or \( \omega = 0 \)) corresponds to a stationary external electric field, and the case \( \epsilon = 0 \) (or \( \nu = 0 \)) corresponds to the case of a collisionless plasma.

The question arises whether there are more complex zeroes of the dispersion function except \( \pm \eta_0 \). As was done in [18], we can show that the number of zeros of the dispersion function is equal to twice the index of the function \( G(r) = \Lambda^+(r)/\Lambda^-(r) \) on the real positive half-axis, \( N = 2\kappa(G), \kappa(\alpha) = ind_{l(0, +\infty)} G(\tau) \). As in [18], we can show that from equations

\[ ReG(\mu, \Omega, \epsilon, \alpha) = 0, \ \Im G(\mu, \Omega, \epsilon, \alpha) = 0, \ 0 \leq \mu \leq +\infty, \]

we can find a curve \( L(\alpha) \) separating the regions \( D^+(\alpha) \) and \( D^-(\alpha) \) (see Fig. 1). If \( (\Omega, \epsilon) \in D^+(\alpha) \), then the number of zeros of the dispersion function is two of these zeros \( \pm \eta_0 \), and if \( (\Omega, \epsilon) \in D^-(\alpha) \), then the dispersion function does not have zeros. We note that a method for investigating the boundary regime was developed for \( (\Omega, \epsilon) \in L(\alpha) \) in [19]. The curve \( L(\alpha) \) is defined by the parametric equations:

\[ L(\alpha): \Omega = \sqrt{L_1(\mu, \alpha)}, \ \epsilon = \sqrt{L_2(\mu, \alpha)}, \ 0 \leq \mu \leq +\infty. \]
Here
\[
L_1(\mu, \alpha) = \frac{s_0(\alpha)}{s_2(\alpha)} \mu^2 \left[ \lambda_0(\mu, \alpha) \left( 1 + \lambda_0(\mu, \alpha) \right) + s^2(\mu, \alpha) \right]^2,
\]
\[
L_2(\mu, \alpha) = \frac{s_0(\alpha)}{s_2(\alpha)} \frac{\mu^2 s^2(\mu, \alpha)}{(-\bar{\lambda}_0(\mu, \alpha)) \left[ (1 + \bar{\lambda}_0(\mu, \alpha))^2 + s^2(\mu, \alpha) \right]}.
\]

Here we use notation according to [18].

**Figure 1.** The case \( \alpha = 0 \): the curve \( L(\alpha) \) separates the regions of index one \( D^+(\alpha) \) and index zero \( D^-(\alpha) \). If \( (\Omega, \varepsilon) \in D^+(\alpha) \), then \( \text{N}=2 \), and if \( (\Omega, \varepsilon) \in D^-(\alpha) \), then \( \text{N}=0 \).

Figure 1 corresponds to the case, when degeneracy degree of plasma has moderate values. More general case will be considered later.

5. **Mirror reflection of electrons from the plasma boundary**

We consider a boundary value problem consisting of equations (3) and (4), condition (5) for the mirror reflection of electrons from the plasma boundary, and conditions (6) on the electric field. We show that this problem has a solution that can be represented as an expansion in the eigenfunctions of the characteristic system.

After a chain of routine transformations and the application of the Sokhotsky formulas we obtain the result.

The structure of the electric field generally has the form
\[
e(x_1) = \frac{\Lambda_1}{\Lambda_0} \eta_0 + \frac{2\Lambda_1}{\Lambda_0 w_0} \cosh \left( \frac{w_0 x_1}{\eta_0} \right) \cosh \left( \frac{w_0 l}{\eta_0} \right) + \frac{\Lambda_1}{\Lambda_0 w_0} \int_{-\infty}^{\infty} \frac{\eta^2 k(\eta, \alpha)}{\cosh \left( \frac{w_0 l}{\eta} \right)} d\eta.
\] (12)

We can construct the distribution function in the form
\[
H(x_1, \mu) = \frac{\Lambda_1}{\Lambda_0 w_0} + \frac{\Lambda_1}{\Lambda_0 w_0} \frac{w_0 (\eta_0^2 - \tilde{\eta}_0^2) \Lambda(\eta_0) \cosh \left( \frac{w_0 l}{\eta_0} \right)}{\cosh \left( \frac{w_0 x_1}{\eta_0} \right)}
\]

\[
\times \left[ e^{-\frac{w_0 x_1}{\eta_0} \mu} - e^{-\frac{w_0 x_1}{\eta_0} \mu} \frac{\eta_0 - \tilde{\eta}_0}{\eta_0 + \tilde{\eta}_0} \right] \left[ e^{-\frac{w_0 x_1}{\eta_0} \mu} + e^{-\frac{w_0 x_1}{\eta_0} \mu} \frac{\eta_0 + \tilde{\eta}_0}{\eta_0 - \tilde{\eta}_0} \right]
\]

\[
+ \frac{\Lambda_1}{\Lambda_0 w_0^2 n_1^2} \int_{-\infty}^{\infty} e^{-\frac{\eta}{\eta_0} F_1(\eta, \mu) \eta^2} k(\eta, \alpha) \cosh \left( \frac{w_0 l}{\eta} \right) d\eta.
\] (13)

We stress that equations (12) and (13) hold for \( (\Omega, \varepsilon) \in D^+(\alpha) \). In the case \( (\Omega, \varepsilon) \in D^-(\alpha) \), the zero \( \eta_0 \) of the dispersion function does not exist. Hence, we can assume that \( \eta_0 = 0 \) in this case. The second summand in (12) and (13) then disappears, and these equations simplify.
6. Analysis of the Debye mode for plasma with high degree of degeneracy

So the electric field depends only on one coordinate. As we have demonstrated earlier, any solution can be represented as a decomposition of the field strength on their own decisions. In dimensionless form we may write down

\[ e(x) = e_{as} + e_d(x) + e_c(x). \]

Terms \( e_{as}, e_d(x), e_c(x) \) are Drude, Debye, and van Kampen modes respectively. Note that the Drude mode does not depend on the \( x \) coordinate. Earlier it has been proved that Debye mode is not always present in the decomposition of its solutions. It is only present when the dispersion function has a root at these parameters (the external electric field frequency and the collision frequency in the plasma). Our goal is to determine the existence regions of Debye mode in the space of these parameters for plasma with high degree of degeneracy. To determine the existence of a root, the authors used the principle of the argument. This principle in the application to the conditions of this problem leads to the next result. The complex plane was divided into regions, where the root and exists and where it does not exist. The boundary of this region is determined by the parametric curve \( L(\alpha) \).

Let us conduct a numerical analysis of these expressions. Figure 2 presents parametric curves for several variants of partially degenerate plasma. The ratio of the degeneration of plasma \( \alpha \) changes from -5 to 5. We use the following notation in the figure: \( D^+ \) – region of existence of the root and \( D^- \) is the area where there is no root. It is seen that graphs with \( \alpha = -3 \) and \( \alpha = -5 \) are practically identical. Also the graph for Maxwell plasma, where \( \alpha = -\infty \), is also close to graphs 1 and 2.

**Figure 2.** The dependence of the location of the curve \( L \) from the value of the coefficient \( \alpha \).

Further study of the behavior of the curve during the growth of \( \alpha \) showed an interesting feature. We can observe the appearance of region in which the root (and therefore Debye mode) appears and then disappears. Figure 3 shows the curve \( L \) corresponding to the ratio of the degeneration of plasma equal to \( \alpha = 30 \). It is seen that for value \( \varepsilon = 1.5 \) with increasing frequency of the external field we get the following nontrivial result: Debye mode firstly exists (for low frequencies), then she disappears, then appears again, then vanishes again (for high frequencies). We have the intermittency effect.

**Figure 3.** Curve \( L \) corresponds to value \( \alpha = 30 \).
At $\varepsilon\approx1.8$ the graph will always touch the axis $\varepsilon$. When $\alpha \to +\infty$ (plasma becomes fully degenerate) the width of the loops on the graph tends to zero.

7. Conclusion
We have analytically solved the classic problem of fluctuations of the electron plasma in a thin layer with an arbitrary degree of degeneracy of the electron gas in an external varying electric field. We found the electron distribution function and the screened electric field inside the plasma in explicit form. The method implemented here can also be used to solve the boundary value problems for Vlasov–Poisson systems of equations (see, e.g., [19]).

We again stress that in order to solve a complicated problem like the plasma fluctuation problem analytically, we must first reduce it to a one-dimensional problem with a single velocity. For this, we direct the strength vector of the external electric field along the axis orthogonal to the surface of the layer with the plasma. In the considered Landau–Vlasov problem, this is the $x$ axis, and the field has the form $E_{\text{ext}}(t) = E_0 e^{-i\omega t}(1,0,0)$.

It generates the self-consistent field $E(x) e^{-i\omega t}(1,0,0)$ inside the plasma. We point out another special feature of the problem. Nonhomogeneous boundary value problem in the theory of functions of a complex variable does not require factoring the coefficient (or solving the corresponding homogeneous problem). In fact, because the boundary conditions are symmetric, problem turns out to be defined on the same cut to which the dispersion function of the problem belongs.

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