Harmonic Component Analysis of Mode Shapes for Packeted Bladed Disk

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Abstract. Vibration characteristics of packeted bladed disk is complicated. To study the vibration characteristics of the governing stage bladed disk of 660mw steam turbine, cyclic symmetry method is used to calculate the mode shapes. Fourier transformation of vibration distribution in the tangential and axial direction along the circumferential of the bladed disk is applied to obtain the harmonic components. It is found that the Fourier decomposition of each blade displacement contains not only unique component, but also other components. For integrally shrouded blades, the resonance of the m nodal diameter mode can be obtained only when the corresponding natural frequency satisfies the condition \( n \omega_n = k \Omega = mn \), where \( n \) is the rotational speed and \( m \) is the harmonic number. For packeted bladed disk, there are additional harmonics that resonance occurs. The harmonic components are related to the nodal diameter number when the number of blades groups and the total number of blades are given. The diagrammatic method is applied to predict the wavenumbers present in modulated eigenfunctions.

1. Introduction

Vibrations have great damage to gas and steam turbomachine blading. To improve the vibration behavior of the turbomachine blading, connection of the blades into groups or packets is widely used in steam turbines, especially the governing stage of control stage. The blades are grouped by mechanical coupling of shrouds and lashing wires. A particular benefit of blade grouping is the possibility of minimizing the vibration response of certain modes to specific harmonic excitation orders [1]. Grouped bladed disk would also reduce the sensitivity of a single blade to mistuning, since the blade would be limited by adjacent blades [2]. For integrally shrouded blades, the resonance of the \( n \) nodal diameter mode can be obtained only when the corresponding natural frequency satisfies the condition when the corresponding natural frequency is \( \omega_n = m \Omega \), where \( \Omega \) is the angular velocity of the force [3]. Wildheim [4] made an in-depth study on the structural excitation of rotationally symmetric structure, the response of blade groups on a flexible disk to a rotating concentrated force is studied, a resonance condition in an \( n \) nodal diameter mode can be obtained as long as the natural frequency is given by \( \omega_n = (lN \pm m)\Omega \) (where \( N \) is the number of substructures, and \( l=0,1,2 \)). In 1984, Ewins and Imregun [5] numerically simulated a specific 30-bladed disk assembly to establish the feature of the natural modes of packeted bladed disks and proposed a utility modal interference diagram to analysis the wavenumbers in packeted bladed disk. Wagner and Griffin [6-7] developed the diagram method which displayed the resonance into a Mode-excitation orthogonality diagram for a simple blade...
groups. Kim [8] studied the spatial modulation of repetitive modes characteristic in a rotationally periodic structure, he investigated spatial modulation when a structure deviates from axisymmetry by experiment, and derived an algebraic relation and a diagrammatic method which can deduce the harmonics present in modulated features when the number of nodal diameters in the basic mode and the number of equidistant model features were given.

In this paper, the three-dimensional finite element model of the governing stage bladed disk of 660MW steam turbine is established. Equation is presented to show the different resonance conditions between integrally shrouded blades and packeted blades. The bladed disk has a rotating disc and 44 blades, the blades are connected by riveting shroud. By changing the packeted configuration, the natural frequencies and vibration modes are calculated by cyclic symmetric method and the harmonics included in the nodal diameter mode are analyzed. The harmonics are listed in the modal interference diagram.

2. Analysis Description

For the actual turbine machinery, when the blades are rotating, blade group will withstand the periodical change of force due to the fluctuation of pressure. The periodic force can be decomposed into many harmonic. Consider the $k$ harmonic, the excitation force can be represented as

$$P_k(\theta, t) = p_k \sin(k(\omega t + \theta))$$  \hspace{1cm} (1)

For integrally shrouded blades, the vibration displacement of $m$ nodal diameter modes can be represented as

$$X_m(\theta, t) = -A_m \cos(\omega_m t + m\theta)$$  \hspace{1cm} (2)

The condition of resonance of the bladed disk is that the exciting forces are able to do positive work and then transmit energy to the blades. The work done by exciting forces on vibrational blades in one period can be represented as

$$W = \frac{N}{2\pi} \int_0^{2\pi} \int_0^{\tau} P_k(\theta, t) \frac{\partial}{\partial t} X_m(\theta, t) dt d\theta$$

$$= \frac{N}{2\pi} \int_0^{2\pi} \int_0^{\tau} p_k \sin(k(\omega t + \theta)) \cdot A_m \omega_m \sin(\omega_m t + m\theta) dt d\theta$$

$$= \begin{cases} 2\pi^2 P_k A_m & \text{(for } m = k \text{ and } \omega_m = k\omega) \\ 0 & \text{(for } m \neq k \text{ or } \omega_m \neq k\omega) \end{cases}$$  \hspace{1cm} (4)

So we can get the resonance condition of integrally shrouded blades is:

$$\omega_m = \omega = k = m$$  \hspace{1cm} (5)

For packeted bladed disk, the vibration of $m$ nodal diameter is

$$X_m(\theta, t) = -A_m \cos(\omega_m t + L\theta)$$  \hspace{1cm} (6)

Where $L = l \cdot p \pm m$, $p$ is the number of the packets and $l=0,1,2$

The resonance condition becomes

$$\omega_m = \omega = k = l \cdot p \pm m$$  \hspace{1cm} (7)

Different from integrally shrouded blades, blades resonance when $\omega_m = \omega = k = m$, there are additional harmonics that resonance occurs for packeted bladed disk so long as $\omega_m = \omega = k = l \cdot p \pm m$. The harmonic components are related to the nodal diameter number when the count of blades groups and the total count of blades are given.
3. Harmonic Component of a Packeted Bladed Disk

Figure 1 is the three dimensional element model of the governing stage bladed disk of the 660MW steam turbine. The control stage adopts bowed static blade cascades with convergent tip endwall and integral shroud rotating blades. The rotating blades are continuously coupled connections by rivetting shroud.

![Finite element model of the packeted bladed disc.](image)

The first series of calculations made were for 11 packets with 4 blades in each packet. The natural frequencies and mode shapes are calculated via the FE model by cyclic symmetry method. The mode shape of one nodal diameter is shown in figure 2 and the axial displacement of blades is shown in figure 3.

![Mode shape of the one nodal diameter vibration.](image)

![Blade displacement of the one nodal diameter mode.](image)

By discrete Fourier transform, the wavenumber included in the mode shapes is obtained as shown in figure 4.
Figure 4. Wavenumber concluded in the one nodal diameter mode.

The vibration of bladed disk is complicated and the nodal diameter contains many other frequencies. By harmonic analysis the mode shape precisely harmonic become contaminated with additional wavenumbers include 10, 12 as well as 21 wavenumbers. Figure 5 is the mode shape of the 10 harmonic.

Figure 5. Mode shape of the ten harmonic contaminated in the one nodal diameter mode.

In the similar vein of figure 6, the mode shape of five nodal diameter mode is shown in axial direction, the mode is distorted primarily by additional wavenumbers include 6, 16 and 17 harmonics shown in figure 7.

Figure 6. Calculated mode shape of the five nodal diameter vibration.
In short, for doublet modes of rotationally periodic features, the main feature of the repeat pairs is the wavenumber contamination.

4. Various Packeting Configurations
To study the harmonics of different packeting configuration, the modes and natural frequencies of several alternative configurations are calculated. The modal interference diagram for 11 packets with 4 blades in each packet is shown in figure 8 to predict the vibration characteristics. In the diagram, \( M \) means the maximum number of nodal diameters and \( p \) means the packets number of the assembly. Ewins [5] introduced the diagrams by drawing 45 degree lines emanating from each integer-multiple of \( p \) on the nodal diameter axe. The horizontal direction indicates harmonic content, and vertical direction indicates nodal diameter.

For a bladed disk system where the blades number is \( N \) and packets number is \( n \), the maximum horizontal and vertical value will be \( \frac{N}{2} \) (or \( \frac{(N-1)}{2} \) when \( N \) is odd). In this example, the model has 44 blades, so the maximum horizontal and vertical designations are twenty-two. From the diagram, we can easily find out the harmonics in each nodal diameter vibration. For example, the five nodal diameter mode becomes contaminated with 6, 16 and 17 harmonics.

In figure 9, the assembly has 22 packets with 2 blades in each packet. The harmonic of nodal diameter vibration can be expressed as \( L = l \cdot 22 \pm m \). Different from the result for 11 packets with 4 blades in each packet, the five nodal diameter mode is contaminated with the 17 harmonics.
Figure 9. Twenty-two packets of two blades.

Figure 10 is the modal interference diagram of continuously shrouded blades. The diagram indicates the harmonic contents of any mode. For integrally shrouded blades disc, there is only one harmonic per mode, but for the model of 11 packets, the one nodal diameter mode owns 1, 10, 12 and 21 harmonics contents as shown in figure 8. It means that the one nodal diameter mode of the 11 packets model will also have a response to the 10, 12 and 21 harmonics of exciting force, though the magnitude of the harmonics are different.

Figure 10. Continuously shrouded.

5. Conclusions

For integrally shrouded blades, mode shapes are precisely harmonic, resonance for the $m$ nodal diameters mode is obtained only when the corresponding natural frequency meet the condition $\omega_n = kn = mn$, where $n$ is the angular velocity and $m$ is the harmonic number. For packeted bladed disk, the resonance condition becomes $\omega_n / \omega = k = l \cdot p \pm m$, where $p$ is the number of packets and $l=0, 1, 2$. There are additional harmonics that resonance occurs.

The checkerboard diagram is applied to the governing stage bladed disk of 660MW steam turbine. From the diagram, we can easily predict harmonics of a given mode.

Nomenclature

$k$ number of harmonic of the exciting force
$L$ possible harmonics of packeted blades
$l$ constant
$m$ number of nodal diameters of the mode pattern
$N$ total number of blades on the disk
$p$ number of packets in the disk
$P_k$ excitation force in the blade
$p_k$ excitation force of the harmonic
$X_m$ axial displacement of the blade
$\omega$ rotating speed of the blade
$\omega_m$ natural frequency of m nodal diameter mode

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