Global Sliding Mode Control for the Bank-to-Turn of Hypersonic Glide Vehicle

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Abstract. The technology of Bank-to-Turn has been recognized as an attractive direction due to their significance for the control of hypersonic glide vehicle. Strong coupling existing among pitch, yaw and roll channel was a great challenge for banking to turn, and thus a novel global sliding mode controller was designed for hypersonic glider in this paper. Considering the coupling among channels as interference, we can use invariance principle of sliding mode motion to realize the decoupling control. The global sliding mode control system could eliminate the stage of reaching, which can lead to the realization of whole systematic process decoupling control. When the global sliding mode factor was designed, a minimum norm pole assignment method of the sliding mode matrix was introduced to improve the robustness of the system. The method of continuity of symbolic function was adopted to overcome the chatter, which furtherly modify the transient performance of the system. The simulation results show that this method has good performance of three channel decoupling control and guidance command tracking. And it can meet the requirements of the dynamic performance of the system.

1. Introduction

Hypersonic vehicle has received more and more attentions of military powers because of its high speed, short reaction time, big combat radius, and good for hiding and strong penetration ability [1-6]. At present, the hypersonic vehicle has been a hot topic in the aerospace area [7, 8]. Especially the hypersonic boost-glide vehicle, whose technical bottleneck is easy to break, has become the first choice for the development of hypersonic weapon in military powers [9, 10].

Plane-symmetry configuration is always used in the hypersonic glide vehicle, which determines the application of the Bank-to-Turn control in the lateral maneuvers [11]. A typical flight trajectory of hypersonic glide vehicle is shown in Figure 1. However, when banking to turn, the aircraft has a bigger roll rate in the roll channel. And obvious kinematic and aerodynamic coupling [12] among pitch, yaw and roll channel are also existed. The design of the control system also will be more complex. In addition, due to the characteristics of hypersonic vehicle, coupling and model uncertainty problems are more prominent.
Considering coupling and model uncertainty problems the hypersonic glide vehicle has when banking to turn, the domestic and foreign scholars put forward the trajectory linearization method, dynamic inverse method and adaptive control method etc as solutions[13]. For example, in Ref [13], according to the singular perturbation theory, a attitude control system of the aircraft is designed by using the trajectory linearization method. The designed controller has good control performance, but with the increase of altitude, the flight control ability has declined. Ref [15] presents a design of a nonlinear adaptive dynamic inversion controller for a Generic Hypersonic Vehicle. In Ref [16], a decoupling controller based on nonlinear dynamic inversion and full state feedback is designed, but the design method of dynamic inverse control law requires accurate mathematical model, and the robustness of the system is poor. On the basis of multi input and multi output feature model, the robustness of the nonlinear strong coupling uncertainty system is verified by the adaptive control method in Ref [19], but in the present study, the measurement error of the attitude angle is not considered as well as the delay and error of the actuator, the research is limited to the feasibility of this method. Ref [20] introduces an adaptive control design which aims to improve tracking and stability of the generic hypersonic vehicle. In Ref [21], a robust controller was designed for the longitudinal dynamics of a hypersonic aircraft. Ref [23] and [24] show that sliding mode control is an effective robustness control method for strong coupling and model uncertainty problems. A sliding mode variable structure control law is designed for hypersonic glide vehicle in reference [25], but only when in the sliding mode motion, the system will be in the state of decoupling and resisting parameter distortion.

In order to overcome the shortcomings of the general sliding mode control design, this paper proposes a method of global sliding mode control to design a high speed glide vehicle with a bank-to-turn controller. By introducing a global sliding mode factor in the sliding hyperplane, the designed control law is allowed to make the system remain on the switching hyperplane from the beginning, which can achieve the whole process decoupling control of the system. In the design of the whole sliding mode factor, a minimum norm pole assignment method of the sliding mode matrix is introduced. In order to overcome the chattering of the system, the method of continuity symbolic function is adopted, which improves the transient performance of the system. Finally, the effectiveness of the proposed method is verified by the simulation of a hypersonic glide vehicle.

2. Mathematical model of hypersonic glide vehicle with sliding mode control
The pitch, yaw and roll channel equation with BTT control can be expressed as[26]:

$$
\dot{\alpha} = \omega_z - a_{14}\alpha - a_{15}\delta_x \\
\dot{\beta} = \omega_x + a_{24}\alpha - b_{11}\beta - b_{12}\delta_y \\
\dot{\omega}_z = -b_{14}\beta - b_{15}\omega_x - b_{16}\omega_y - b_{17}\delta_x - b_{18}\delta_y \\
\dot{\omega}_x = -b_{24}\beta + b_{25}\omega_x - b_{26}\omega_y - b_{27}\delta_x - b_{28}\delta_y \\
\dot{\omega}_y = -a_{34}\alpha - a_{35}\omega_x - a_{36}\omega_y - a_{37}\delta_x - a_{38}\delta_y
$$

(1)
Where, $\alpha, \beta$ are attack and sideslip angle of the system, $\omega_x, \omega_y, \omega_z$ are roll, yaw and pitch rate, $\delta_x, \delta_y, \delta_z$ are roll, yaw and pitch rudder angle, $\alpha_0$ is constant balance angle of attack. $\alpha, \omega$ is the kinematic coupling term and $-b_{14}, \beta, -b_{24}, \delta, -b_{34}, \delta_y, -b_{32}, \delta_z$ are aerodynamic coupling terms. The coefficients of these coupling terms are comparatively large and cannot be ignored easily. The calculation formulas of the dynamic coefficient of the formula (1) are as followed:

$$ a_{22} = -m^2 \alpha_q S L / (J V) , \quad a_{24} = -m^2 \alpha_q S / J_z , $$
$$ a_{25} = -m^2 \alpha_q S / J_z , \quad a_{34} = -C_x \alpha S / (mV) , $$
$$ a_{35} = -C_x \alpha S / (mV) , \quad b_{14} = -m^2 \delta_q S L / (J V) , $$
$$ b_{18} = -m^2 \delta_q S L / J_x , \quad b_{15} = -m^2 \delta_y q S / J , $$
$$ b_{34} = -m^2 q S L / (J V) , \quad b_{32} = -m^2 q S L / J_y , $$
$$ b_{23} = -m^2 q S L / J_z , \quad b_{24} = -m^2 q S L / J_x , $$
$$ b_{27} = -m^2 q S L / (J V) , \quad b_{24} = -m^2 q S L / J_x , $$

Where $m$ is the mass of the vehicle, $J_x, J_y, J_z$ are the moment of inertia, $q$ is the dynamic pressure, $V$ is the speed, $S$ is the reference area, and $L$ is the reference length.

Write the roll angle of the vehicle as $\gamma$, then

$$ \dot{\gamma} = \omega_x $$

(2)

Let’s select state variable as $Z = [\alpha, \beta, \gamma, \omega_x, \omega_y, \omega_z]^T$, considering the formula (1) and (2), the mathematical model of the state equation of a hypersonic glide vehicle can be derived as follows:

$$ \dot{Z} = A_x \cdot Z + B_x \cdot U $$

(3)

Where

$$ A_x = \begin{bmatrix}
-a_{34} & 0 & 0 & 0 & 0 & 1 \\
0 & -b_{34} & 0 & \alpha_0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & -b_{34} & 0 & -b_{32} & 0 & 0 \\
0 & -b_{24} & 0 & b_{21} & -b_{22} & 0 \\
-a_{24} & 0 & 0 & 0 & 0 & -a_{22}
\end{bmatrix} $$

$$ B_x = \begin{bmatrix}
0 & 0 & -a_{35} \\
0 & -b_{37} & 0 \\
0 & 0 & 0 \\
-b_{38} & -b_{32} & 0 \\
-b_{23} & -b_{27} & 0 \\
0 & 0 & -a_{25}
\end{bmatrix}, \quad U = \begin{bmatrix}
\delta_x \\
\delta_y \\
\delta_z
\end{bmatrix} $$

When designing the control system of a hypersonic glide vehicle, we hope the roll angle $\gamma$ of the roll channel can quickly track the roll angle command signal and the pitch channel can also track the overload signal $n_y$ fast, in the meantime keep the sideslip angle as 0. Because on every feature point of the flight process, the overload command signal $n_y$ and the angle of attack command signal $\alpha_\gamma$ have the one-to-one corresponding relation, the purpose of the controller design can also be viewed as the angle of attack $\alpha$ tracking angle of attack command signal $\alpha_\gamma$, the roll angle tracking roll angle command signal $\gamma$, and at the same time keeping the sideslip angle command signal as 0.
The global sliding mode control is to adjust the state of the controlled system from nonzero to zero, which achieves the function of the state regulator. Therefore, in order to transform the regulator problem into the tracking problem, the error model is introduced.

\[ e_a = \alpha - \alpha_e, \quad e_\beta = \beta - \beta_e, \quad e_\gamma = \gamma - \gamma_e \]  

Then

\[ \dot{e}_a = \alpha - \alpha_e, \quad \dot{e}_\beta = \beta - \beta_e, \quad \dot{e}_\gamma = \gamma - \gamma_e \]  

Note state variable 

\[ \mathbf{Z}_e = [e_a, e_\beta, e_\gamma, \omega_\alpha, \omega_\beta, \omega_\gamma]^T \] . Considering the formula (3) and (5), we can obtain:

\[ \dot{\mathbf{Z}}_e = \mathbf{A}_e \cdot \mathbf{Z}_e + \mathbf{B}_e \cdot \mathbf{U} + \mathbf{D}_e \]  

Where

\[ \mathbf{D}_e = [-a_1 e_a - b_4 \beta_e - \gamma_e, -b_1 \beta_e, -b_2 \beta_e, -a_2 \alpha_e] \]  

The formula (6) represents the error model of the controlled object, which keep the angle of attack, sideslip angle and roll angle in expectations, that is, control the error to 0. Further, in order to facilitate the design of variable structure controller, the error model of the formula (6) is needed to be transformed into a simple standard type of variable structural control by linear transformation.

\[ \mathbf{X} = \mathbf{P}^{-1} \cdot \mathbf{Z_e} \]  

Then

\[ \dot{\mathbf{X}} = \mathbf{P}^{-1} \mathbf{A}_e \cdot \mathbf{P} \cdot \mathbf{X} + \mathbf{P}^{-1} \mathbf{B}_e \cdot \mathbf{U} + \mathbf{P}^{-1} \mathbf{D}_e \]  

Note \( \mathbf{P}^{-1} \mathbf{A}_e = \mathbf{A}_e \), \( \mathbf{P}^{-1} \mathbf{B}_e = \mathbf{B}_e \), \( \mathbf{P}^{-1} \mathbf{D}_e = \mathbf{D}_e \), then formula (8) can be written as followed:

\[ \dot{\mathbf{X}} = \mathbf{A} \cdot \mathbf{X} + \mathbf{B} \cdot \mathbf{U} + \mathbf{D} \]  

Where \( \mathbf{B} = [0 \quad \mathbf{B}_2]^T \), \( \mathbf{U} \) is a \( m \times 1 \) matrix and \( \mathbf{B}_2 \) is a \( m \times m \) matrix. The equation can be written in the following form:

\[ \begin{bmatrix} \dot{\mathbf{X}}_1 \\ \dot{\mathbf{X}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{B}_2 \end{bmatrix} \cdot \mathbf{U} + \mathbf{D} \]  

The mathematical model of the hypersonic gliding vehicle with sliding mode variable structure control is obtained.

3. **Design of global sliding mode controller for hypersonic glide vehicle**

The formula (9) presents the state equation of the multivariable system without considering parameter perturbation. If the parameter perturbation is taken into account, the general multivariable uncertain system can be expressed as

\[ \dot{\mathbf{X}}(t) = (\mathbf{A} + \Delta \mathbf{A}) \cdot \mathbf{X}(t) + (\mathbf{B} + \Delta \mathbf{B}) \cdot \mathbf{U}(t) + \mathbf{D} \cdot f(t) \]  

Where, \( \mathbf{X} \in \mathbb{R}^n \) is the state variable, \( \mathbf{U} \in \mathbb{R}^m \) is the control vector and \( f(t) \in \mathbb{R}^l \) is the external disturbance; \( \mathbf{A} \), \( \mathbf{B} \) and \( \mathbf{D} \) have the corresponding dimension. \( \Delta \mathbf{A} \) and \( \Delta \mathbf{B} \) are the parameter perturbation matrix.

According to the system described, we chose the sliding hyper plane as:

\[ S(X,t) = \mathbf{C} \cdot \mathbf{X}(t) - C(E(t))X(0) \]  

Where,

\[ \mathbf{E}(t) = \begin{bmatrix} \mathbf{E}_1(t) \\ \mathbf{E}_2(t) \end{bmatrix} \]

\[ \mathbf{E}_i(t) = \text{diag}\left[\exp(-\beta_1 t), \cdots, \exp(-\beta_{n_i} t)\right] \]

\[ \text{Re}(\beta_i) > 0, i = 1, \cdots, n \] is the sliding mode parameter to be designed.

3.1. **Design of modal parameter matrix C**

Here we use pole assignment method to design matrix \( C \).

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The system that has become a simple standard type of variable structure control, can be written as follows:

\[
\begin{align*}
\dot{X}_1 &= A_1 X_1 + A_{22} X_2 + D_1 \\
\dot{X}_2 &= A_{21} X_1 + A_{22} X_2 + B U + D_2 \\
S &= C_1 X_1 + C_2 X_2 - C_1 E_1(t) X_1(0) - C_2 E_2(t) X_2(0)
\end{align*}
\]  \hspace{1cm} (13)

Where \( S = 0 \), we can get:

\[
X_2 = -K X_1 + K E_1(t) X_1(0) + E_2(t) X_2(0)
\]  \hspace{1cm} (14)

Where \( K = C_2^T C_i \). Substitute \( X_2 \) into the differential equation about \( X_1 \) in equation (13):

\[
\dot{X}_1 = (A_1 - A_{22} K) X_1
\]  \hspace{1cm} (15)

Because \((A, B)\) is completely controllable, \((A_1, A_2)\) is also completely controllable. Choose the right \( K \), to make sure that the characteristic root \( \lambda_i \) of \((A_1 - A_{22} K)\) is the expected characteristic root, so that the system has good dynamic quality in the sliding phase. Then,

\[
C = C_2 \begin{bmatrix} K & I_m \end{bmatrix}
\]  \hspace{1cm} (16)

Formula (16) is the sliding mode parameter matrix we need. Where \( C_2 \) is usually taken as a unit matrix.

Because it is a multiple input multiple output system, there are many solutions can configure the system to poles where required. In order to avoid the norm of matrix \( C \) is too large, resulting in transient control value of variable structure control become too large, referring to [27], a minimum norm pole assignment method for a sliding mode matrix is introduced, so that multiple solution problem in pole assignment can be transformed into a least squares problem:

\[
\min_{\lambda \in (A_1 - A_{22} K) \in \mathbb{C}^n} \left( \max_{j=1}^n \sum_{i=1}^m K_{ij} \right)
\]  \hspace{1cm} (17)

### 3.2. Design of sliding mode motion parameters

The system is in a global sliding mode motion state, that is, the system always has \( S = 0 \), then

\[
C_1(X_1 - E_1(t) X_1(0)) + C_2(X_2 - E_2(t) X_2(0)) = 0
\]  \hspace{1cm} (18)

From formula (15), we can know that the sliding mode motion of the system is expected to be

\[
X_1 = \bar{E}_1(t) X_1(0)
\]  \hspace{1cm} (19)

So choose \(-\beta_1\) as the desired pole, that is

\[
\beta_1 = -\lambda_i(A_1 - A_{22} K), \quad (i = 1, \ldots, n-m)
\]  \hspace{1cm} (20)

Considering formula (18) and (19), the desired motion characteristics of the system state \( X_2 \) can be

\[
X_2 = \bar{E}_2(t) X_2(0)
\]  \hspace{1cm} (21)

Similarly, we choose \( \beta_i \) and \( i = (n-m+1, \ldots, n) \) according to the requirements of the dynamic performance of the system, so as to make \( X_2 \) achieve the desired motion characteristics.

### 3.3. Design of global sliding mode variable structure control law

The task of variable structure control law is to keep the state in the sliding mode, that is the designed \( U(t) \) meet the accessibility condition.

\[
S^T \dot{S} < 0
\]  \hspace{1cm} (22)

The variable structure control law is constructed as follows:

\[
U(t) = -g(t)(CB)^{-1} \text{sgn}(S)
\]  \hspace{1cm} (23)

Where \( g(t) \) is the designed variable structure control coefficient. Substitute formula (23) into \( S^T \dot{S} \):
\[ S^T \dot{S} = -g(t)S^T \text{sgn}(S) - g(t)S^T CA\text{sgn}(CB)^{-1} \text{sgn}(S) \]
\[ + S^T [CA + C\Delta A + CDf + CHE(t)X_0] \]
\[ \begin{align*}
&\leq -g(t) \| S^T \| [1 - \psi_a \| C \| \| (CB)^{-1} \| ] + \\
&\| S^T \| [\| CA \| + \psi_a \| C \| ] \| X \| + \| CD \| \psi_f \\
&+ \beta_{\text{max}} \exp(-\beta_{\text{max}}t) \| CX(0) \| 
\end{align*} \]
\[ (24) \]

Where \( H = \text{diag} [\beta_1, \cdots, \beta_n] \), \( \psi_a, \psi_f, \psi_f \) are known positive constants, and meet \( \| \Delta A \| \leq \psi_a \), \( \| \Delta B \| \leq \psi_f \), \( f \| \leq \psi_f \). Simultaneous formula (22) and formula (24) we can get:

\[ g(t) > [1 - \psi_a \| C \| \| (CB)^{-1} \| ]^{-1} [\| CA \| \\
+ \psi_a \| C \| ] \| X \| + \| CD \| \psi_f \\
+ \beta_{\text{max}} \exp(-\beta_{\text{max}}t) \| CX(0) \| 
\]
\[ (25) \]

Formula (25) can be written as:

\[ g(t) = (1 - a_i)^{-1} [a_1 \| X \| + a_2 + a_3 \exp(-a_4t)] + \varepsilon \]
\[ (26) \]

Where, \( \varepsilon \) is a small positive number, the remaining coefficients are:

\[ \begin{align*}
a_1 &= \| CA \| + \psi_a \| C \| \\
a_2 &= \| CD \| \psi_f \\
a_3 &= \beta_{\text{max}} \| CX(0) \| \\
a_4 &= \psi_a \| C \| \| (CB)^{-1} \| 
\end{align*} \]
\[ (27) \]

Substitute \( g(t) \) from formula (26) into formula (23), so that the variable structure control law satisfying the condition of the sliding mode accessibility is obtained.

In addition, the flutter is a common phenomenon in sliding mode control system. In order to eliminate high frequency flutter, a simple and practical method is used, that is replace \( \text{sgn}(S) \) in formula (23) by \( M(s) \).

\[ M(S) = \begin{bmatrix} m(s_1) & \cdots & m(s_m) \end{bmatrix}^T \]
\[ (28) \]

Where, \( \delta_1 \) is a small positive constant.

4. Simulation results and analysis

Suppose a hypersonic glide vehicle is at 40 Km, Ma=6 and the cruise balance angle of attack \( a_0=5 \), the aircraft’s kinetic coefficients in the cruise state are as follows:

\[ \begin{align*}
a_{22} &= 0.0648 \quad a_{23} = -37.4079, \quad a_{23} = 36.5729 \\
a_{33} &= 0.0648 \quad a_{33} = 0.0056, \quad b_{11} = 0.0528 \\
b_{11} &= 100.237 \quad b_{12} = 0.0078, \quad b_{14} = 123.5126 \\
b_{12} &= 27.8024 \quad b_{22} = 0.0118, \quad b_{24} = 7.3013 \\
b_{22} &= 6.6308 \quad b_{33} = 0.0121, \quad b_{34} = 0.0011, \\
b_{32} &= 0.0008 \quad b_{32} = 6.2823 \end{align*} \]

Substitute these kinetic coefficients into formula (3), the mathematical model of a hypersonic glide vehicle in the form of state equation is obtained.

Select a linear transformationa matrix \( P^{-1} \) as:
Substitute $P^{-1}$ into formula (8), so that the mathematical model of aircraft can be changed into a simple standard type of variable structure control.

Suppose the initial angle of attack $\alpha_0$ of the vehicle is 5°, the sideslip angle has an initial deviation of 1° and the roll angle is 0. Give the angle of attack command $\alpha_c=10^\circ$, the sideslip angle command $\beta_c=0$ and roll angle command $\gamma_c=45^\circ$.

A global sliding mode controller for hypersonic glide vehicle is designed in this paper. In order to ensure the control system step response rise time is less than 1 second, when using the pole assignment method, we can set the pole to {-5} and calculate the variable structure control law of the formula (23).

To validate the robustness of controller, the uncertainties of the model are considered as $|\Delta m| \leq 0.03, |\Delta S| \leq 0.03, |\Delta J_1| \leq 0.05, |\Delta J_2| \leq 0.05, |\Delta J_3| \leq 0.10, |\Delta m_1| \leq 0.10, |\Delta m_2| \leq 0.10, |\Delta q| \leq 0.05$. The simulation results as shown in Figure 2 to Figure 7:

![Figure 2](response-angle-of-attack.png)  
**Figure 2**  
Response of Angle of attack

![Figure 3](response-sideslip-angle.png)  
**Figure 3**  
Response of sideslip angle

![Figure 4](response-roll-angle.png)  
**Figure 4**  
Response of roll angle

![Figure 5](results-elevator-angle.png)  
**Figure 5**  
Results of elevator angle
The simulation results show that the angle of attack and roll angle can track angle and roll angle command signal well and the system keeps the sideslip angle as 0. The designed controller can be applied to the BTT control of hypersonic glide vehicle.

5. Conclusions
In response to the phenomenon of the high coupling of the hypersonic glide vehicle in BTT control, a global sliding mode variable structure controller is designed. The coupling among channels is considered as a kind of interference. By using the invariance principle of sliding mode motion state, we can realize decoupling control. At the same time, the global sliding mode control system can eliminate the stage of reaching, which can enable the system to be in a state of sliding mode motion at the beginning, and realize the whole process decoupling control of the system. The given simulation example in this paper illustrates the effectiveness of the proposed method.

6. References
[1] Charania, A. C., and Bradford, J. 2013. Mission effectiveness tradeoffs for future military space access vehicles. *Intervention*, 6(1), 1749-1774.
[2] Walker, S., Sherk, J., Shell, D., Schena, R., Bergmann, J., and Gladbach, J. 2013. The DARPA/AF Falcon Program: The Hypersonic Technology Vehicle #2 (HTV-2) Flight Demonstration Phase. *Aiaa International Space Planes and Hypersonic Systems and Technologies Conference*.
[3] Walker, S., and Rodgers, F. 2005. Falcon Hypersonic Technology Overview. *Aiaa/cira, International Space Planes and Hypersonics Systems and Technologies Conference*.
[4] AIAA. 2000. Hypersonic vehicle control surface development.
[5] Wiese, D. P., Annaswamy, A. M., Muse, J. A., Bolender, M. A., and Lavretsky, E. 2015. Adaptive output feedback based on closed-loop reference models for hypersonic vehicles. *Journal of Guidance Control & Dynamics*, 38(12), 1-12.
[6] Rehman, O. U., Fidan, B., and Petersen, I. 2013. Minimax LQR Control Design for a Hypersonic Flight Vehicle. *Aiaa/dlr/dgler International Space Planes and Hypersonic Systems and Technologies Conference*.
[7] Johnson, E. N., Calise, A. J., Curry, M. D., Mease, K. D., and Corban, J. E. 2015. Adaptive guidance and control for autonomous hypersonic vehicles. *Georgia Institute of Technology*, 29(3), 725-737.
[8] Fan, Y. H., Yan, P. P., Wang, F., and Xu, H. Y. 2016. Discrete sliding mode control for hypersonic cruise missile. *Discrete Dynamics in Nature & Society*, 2016(1), 1-9.
[9] Qu, C.Y. 2015, Analysis on the development of the boost glide weapon of United States, *Tactical Missile Technology*, 04:1-4+52.

[10] Acton, J. M. 2015. Hypersonic boost-glide weapons. *Science & Global Security*, 23(3), 191-219.

[11] Williams, D. E., Friedland, B., and Madiwale, A. N. 1986. Modern control theory for design of autopilots for bank-to-turn missiles. *Journal of Guidance Control & Dynamics*, 10(4), 1130-1136.

[12] Li, Q. 2015, Study on Reentry Guidance and Control Method for Hypersonic Glide Vehicle, *Beijing Institute of Technology*.

[13] Chen, X.Q., Hou, Z. X., Liu, J.X. 2012, Analysis and controller design of bank-to-turn system for hypersonic gliding vehicle, *Journal of National University of Defense Technology*, 03:17-23.

[14] Xu, B., Fan, Y., and Zhang, S. 2015. Minimal-learning-parameter technique based adaptive neural control of hypersonic flight dynamics without back-stepping. *Neurocomputing*, 164(C), 201-209.

[15] Rollins, E., Valasek, J., Muse, J. A., and Bolender, M. A. 2013. Nonlinear Adaptive Dynamic Inversion Applied to a Generic Hypersonic Vehicle. *AIAA Guidance, Navigation, and Control*.

[16] An, X. Y., Wang, X. H. 2010, Analysis and controller design of bank-to-turn system for hypersonic gliding vehicle, *Journal of System Simulation*, S1: 107-110.

[17] Weerdt, E., Kampen, E. J., Gemert, D., Chu, Q. P., and Mulder, J. A. 2013. *Adaptive Nonlinear Dynamic Inversion for Spacecraft Attitude Control with Fuel Sloshing*. *AIAA Guidance, Navigation and Control Conference and Exhibit*.

[18] AIAA. 2007. Robust Nonlinear Dynamic Inversion Control for a Hypersonic Cruise Vehicle. *AIAA Guidance, Navigation and Control Conference and Exhibit*.

[19] Gong, Y. L., and Hong-Xin, W. U. 2010. Characteristic model-based adaptive attitude control for hypersonic vehicle. *Journal of Astronautics*.

[20] Creagh, M., Kearney, M., and Beasley, P. 2011. Adaptive Control for a Hypersonic Glider using Parameter Feedback from System Identification. *AIAA Guidance, Navigation, and Control Conference* (Vol.1, pp.532-549).

[21] Fan, Y., Yang, J., and Zhang, Y. 2007. Robust control of hypersonic aircraft. *Proceedings of SPIE - The International Society for Optical Engineering*, 6795, 67955A-67955A-6.

[22] Marrison, C. I., and Stengel, R. F. 1998. Design of robust control systems for a hypersonic aircraft. *Journal of Guidance Control and Dynamics*, 21(1), 58-63.

[23] Chang, Y., Wang, X., Chun, H. E., and Liu, X. 2015. Study on adaptive variable structure control law for hypersonic gliding vehicle. *Aerospace Control*.

[24] Li, Y. J., Zhang, K. 2005. Adaptive control theory and its application, *Northwestern Polytechnical University Press*, p 179.

[25] Chang, Y., Wang, X., Chun, H. E., and Liu, X. 2015. Study on adaptive variable structure control law for hypersonic gliding vehicle. *Aerospace Control*.

[26] Lin, D.F., Wang, H., Wang, J., Fan, J.F. 2012, Autopilot design and guidance law analysis of tactical missile, *Beijing Institute of Technology*, p10-40.

[27] Li, S., Zhang, X. G., Xiang, J. W. 2000, Pole assignment method of output feedback with minimized norm in vibration control system design, *Chinese Journal of Aeronautics*, 05:446-449.

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