Evaluation of Cross-Section and Wing Length in Free Vibration Analysis of Aircraft Wings

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Abstract
This study presents the numerical free vibration analysis of aircraft wings created using different airfoil cross sections such as NACA 0009, NACA 2424, and NACA 4415. Aircraft wings were made of different lengths. Numerical frequency analyses were conducted Taguchi L9 orthogonal array with two control factors including three levels and so nine numerical modal analyses were performed. Airfoil cross sections and lengths of aircraft wings were used as the first and the second control factors. To detect the control factors with optimal levels, analysis of signal-to-noise (S/N) ratio was employed. In addition, analysis of variance (ANOVA) at the 95 % confidence level was implemented to carry out percent contributions of airfoil cross sections and lengths of aircraft wings on free vibration. As can be summarized from this study, the maximum free vibration behavior was obtained by using NACA 2424 wing profiles with a length of 5 meters. Also, the most dominant control factors were found to be airfoil type with 85.21 % effect and wing length with 12.87 % effect, according to ANOVA.

Keywords: Airfoil, Free vibration, ANSYS, Taguchi Method

1. Introduction
Aircraft wings are generally modelled using airfoil cross sections. Each airfoil has different profile and thus aircraft wings created using various airfoil cross section can show differences on free vibration characteristics. A lot of aircraft wings and blades were usually designed based on National Advisory Committee for Aeronautics (NACA). There are many studies including the aircraft wings, NACA cross sections. Bayraktar and Demirtaş [1] investigated the free vibration of a NACA 4415 airfoil profiled wing as a cantilever beam using theoretical and numerical methods for different modes. Eken [2] presented the model analysis of composite aircraft wings created in accordance with thin-walled beams which have NACA airfoil sections. Tenguria et al. [3] analyzed the free...
vibration characteristic for blade of horizontal axis wind turbine and they also used NACA 634-221 model. Alabaş et al. [4] evaluated the impact of cavity structure on the NACA 0018 wing profile using computational fluid dynamics. Bakirci et al. [5] analyzed the aerodynamic performances of NACA 23012 and NREL S809 wing sections using computational fluid dynamics based on ANSYS software. Doğru et al. [6] examined increasing the aerodynamic performance of the NACA 4412 using the replaceable wing profile during flight. Rubel et al. [7] presented aerodynamics analysis of NACA 0015 airfoil using numerical and experimental methods. Durhasan [8] presented the numerical analysis of the impact of flow suction at trailing side in accordance with aerodynamic performance of NACA 0015 airfoil using ANSYS software. Oktay and Kanat [9] examined the aerodynamic influences of a suction channel on NACA 4412 wing using ANSYS software. As can be seen above mentioned open literature, there are many studies about NACA sections and airfoils. However, there is no study about investigation of airfoil sections and aircraft wings made of metal on numerical free vibration analysis for the first mode using ANSYS Parametric Design Language (APDL) software accordingly to Taguchi L9 orthogonal array which has two control factors and their levels were indicated in Table 1 [16]. Each control factor was considered as airfoil types and cross sections of these airfoil were determined as NACA 0009, NACA 2424, and NACA 4415. Airfoil data were taken from Airfoil Tools [17]. The second control factor was employed as wing lengths. The control factors and their levels were indicated in Table 2.

In the statistical analysis, Taguchi method was used based on L9 orthogonal array. The array consists of two control factors. Each control factor includes three different levels. The first control factor was considered as airfoil types and cross sections of these airfoil were determined as NACA 0009, NACA 2424, and NACA 4415. Airfoil data were taken from Airfoil Tools [17]. The second control factor was employed as wing lengths. The control factors and their levels were indicated in Table 2.

### Table 2. Airfoil cross section and wing length at different levels

| Control Factors  | Symbol | Unit  | Level 1     | Level 2     | Level 3     |
|------------------|--------|-------|-------------|-------------|-------------|
| Airfoil Cross Section | A      |       | NACA 0009 [17] | NACA 2424 [17] | NACA 4415 [17] |
| Wing Length      | B      | meter | 5.0         | 5.5         | 6.0         |

In order to obtain the maximum frequency data of aircraft wings at the first mode, analysis of signal-to-noise (S/N) ratio and variance analyses in this study were implemented to determine the effects of control factors and their percent contributions on natural frequency responses.

### 2. Materials and Methods

Aircraft wings are generally produced using high strength lightweight materials. Metal and composite materials can be shown as some of these materials. Aluminum material has the high strength and low weight [15]. In the study, aircraft wings were modelled using Aluminum (Al). The properties such as Young’s module (E) and density (ρ) for metal were presented in Table 1.

| Material          | E (N/m²) | ν | ρ (kg/m³) |
|-------------------|----------|--|-----------|
| Aluminum (Al)     | 70 x 10⁹ | 0.3 | 2707      |

In the statistical analysis, Taguchi method was used based on L9 orthogonal array. The array consists of two control factors. Each control factor includes three different levels. The first control factor was considered as airfoil types and cross sections of these airfoil were determined as NACA 0009, NACA 2424, and NACA 4415. Airfoil data were taken from Airfoil Tools [17]. The second control factor was employed as wing lengths. The control factors and their levels were indicated in Table 2.

### Table 1. Material Properties [16]

3. **Numerical Analysis**

In modelling, three different cross sections of airfoils such as NACA 0009, NACA 2424, and NACA 4415 were used. Cross sections of airfoils were generated using numerical code of X, and Y coordinate locations for two dimensions. 2-D aircraft wings were created by SolidWorks software using airfoil sections. After that, 3-D structure was modelled using SolidWorks software. Airfoil cross sections were indicated in Figure 1 [17].
The airfoil data were taken from Airfoil Tools [17]. In modelling, the chord length was evaluated as 1 in meter. Wing lengths were used to be 5, 5.5 and 6 in meter. Aircraft wings were considered as cantilever beams which is clamped at one end and free at the other end. Each wing was modelled using (Al) metal material. In finite element analysis, ANSYS APDL software was used for free vibration analysis. As element type, 3-D 20-Node Structural Solid named SOLID186 that displays quadratic displacement behavior was employed and it is described by twenty nodes with three degrees of freedom each node: translations for the nodal x, y, and z directions [19]. Block Lanczos was employed as extraction method. Problem dimensionality was decided as 3-D. UX, UY, and UZ were utilized as degrees of freedom. Thus, UX = UY = UZ was determined to be 0 for clamped edge while UX = UY = UZ was claimed to be zero for free end. The cantilever aircraft wing was illustrated in Figure 2a while the structural solid geometry, node positions, and the coordinate system of element for SOLID186 element type were presented in Figure 2b [19].

4. Verification Analysis

In order to verify the frequency results of aircraft wings using finite element software ANSYS, obtained result for NACA4415 profiled wing in this study was compared with a study [1]. In analysis, lengths of chord and wing were determined to be 1 m and 5 m, respectively. Aluminium Alloy 6061 was assumed to be the material type for wing. Elasticity module and density for Aluminium Alloy 6061 in a study [1] were used 69x10^9 Pa and 2700 kg/m^3, respectively. According to the material properties, comparison of numerical and theoretical frequency results was tabulated in Table 3.

| Mode | Theoretical Result [1] | ANSYS APDL | Error ratio | ANSYS Workbench [1] | ANSYS APDL | Error ratio |
|------|------------------------|------------|-------------|----------------------|------------|-------------|
| 1st  | 4.301895 Hz            | 4.2355 Hz  | 1.5676 %    | 4.2446 Hz            | 4.2355 Hz  | 0.2148 %    |

It can be seen from Table 3 that there is a good agreement between numerical and theoretical results at the first mode of free vibration analysis. Also, the frequency results obtained from ANSYS APDL software are smaller than free vibration data found from theoretical approach and ANSYS Workbench software. Error ratios were found to be 1.5676 % for theoretical approach, 0.2148 % for ANSYS Workbench.

5. Results and Discussions

The scheme of performing finite element analysis was selected and the numerical analyses were conducted to evaluate the impacts of control
factors such as cross section and wing length of aircraft wings on free vibration characteristics. According to L9 orthogonal array based on Taguchi method, finite element results and their S/N ratio values were tabulated in Table 4.

**Table 4. Results of numerical frequency and S/N ratio**

| Tests | Designations | Control Factors | Frequency \( \lambda \) (Hz) | S/N ratio \( \eta \) (dB) |
|-------|--------------|----------------|-----------------------------|--------------------------|
| 1     | A1B1         | NACA0009       | 5.0                         | 2.45075                  | 7.7860                   |
| 2     | A1B2         | NACA0009       | 5.5                         | 2.02682                  | 6.1363                   |
| 3     | A1B3         | NACA0009       | 6.0                         | 1.70755                  | 4.6475                   |
| 4     | A2B1         | NACA2424       | 5.0                         | 6.61109                  | 16.4055                  |
| 5     | A2B2         | NACA2424       | 5.5                         | 5.46260                  | 14.7480                  |
| 6     | A2B3         | NACA2424       | 6.0                         | 4.58834                  | 13.2331                  |
| 7     | A3B1         | NACA4415       | 5.0                         | 4.26057                  | 12.5894                  |
| 8     | A3B2         | NACA4415       | 5.5                         | 3.51913                  | 10.9287                  |
| 9     | A3B3         | NACA4415       | 6.0                         | 2.95569                  | 9.4132                   |

Overall Mean (\( T_\lambda \))

3.7314

Numerical free vibration results based on Taguchi L9 orthogonal array carried out using finite element software ANSYS for the first mode were given in Figure 3 as visually. Figure 3 shows that the most affected ends of the aircraft wings on the modal analysis are free ends. These findings are in agreement with many studies [10, 13].
5.1 Examination of Optimal Levels

The numerical analyses were performed in accordance with the parametric approach of Taguchi method in order to determine the effects of individual control parameters based on chosen quality characteristic. The average data and S/N ratio values of the free vibration characteristics for each control factor at various levels were calculated from numerical data. Response table for S/N ratio and means for cantilever boundary conditions were presented in Table 5.

Table 5. Response table

| Level | S/N ratio in dB | Means in Hz |
|-------|----------------|-------------|
|       | Airfoil Cross Section | Wing Length | Airfoil Cross Section | Wing Length |
| 1     | 6.190           | 12.260      | 2.062           | 4.441       |
| 2     | 14.796          | 10.604      | 5.554           | 3.670       |
| 3     | 10.977          | 9.098       | 3.578           | 3.084       |
| Delta | 8.606           | 3.162       | 3.492           | 1.357       |
| Rank  | 1               | 2           | 1               | 2           |

According to Table 4, the control factors having the optimum levels were found as airfoil type at the second level and length of aircraft wing at the first level. In other words, the maximum free vibration data was obtained by using NACA2424 wing profiles with a length of 5 meters. The S/N data is calculated to find the significant variables and to quantify effects on free vibration responses. The main impacts of control factors for S/N ratio values were plotted. The curves for the natural frequencies were used for analyzing the parametric influences on the free vibration characteristics. Main effects plot for S/N ratios was presented in Figure 4.

![Figure 4. Main effects plot for S/N ratios](image-url)

As can be seen from Figure 4, increase from the first level to the second level for airfoil type causes the increase of the free vibration characteristic while decrease from the second level to third level leads
to the decrease of the free vibration. Also, increase of wing lengths leads to the decrease of the free vibration.

5.3 Analysis of Variance

Table 6. ANOVA result for natural frequency

| Source | DF | Seq SS | Adj SS | Adj MS | F  | P   | % Effect |
|--------|----|--------|--------|--------|----|-----|----------|
| A      | 2  | 18.3995| 18.3995| 9.1998 | 88.9400 | 0.0000 | 85.21    |
| B      | 2  | 2.7792 | 2.7792 | 1.3896 | 13.4300 | 0.0170 | 12.87    |
| Error  | 4  | 0.4138 | 0.4138 | 0.1034 |           |       | 1.92     |
| Total  | 8  | 21.5924|        |        |         |       | 100      |

It was found that airfoil type and lengths of aircraft wings are the significant control factors due to P < 0.05, for 95 % confidence level. Also, the most dominant control factors on free vibration characteristic were noticed to be airfoil type having 85.21 % effect and wing length having 12.87 % effect.

5.2 Estimation of Optimal Frequency

Prediction of the optimal fundamental frequency data of aircraft wings along with its respective confidence intervals was estimated and thus optimal result of fundamental free vibration characteristic according to the first mode was estimated using the optimum levels of the strongest variable such as NACA 2424 profiled wing for airfoil cross section and wing length with 5 meter. These data were presented in Table 5. The predicted mean for fundamental frequency can be computed based on Equation 2 [18].

\[
\mu_\lambda = \bar{A}_2 + \bar{B}_1 - \bar{T}_\lambda \tag{2}
\]

in here, \(\bar{A}_2\) and \(\bar{B}_1\) present the average value of fundamental frequency for airfoil cross section at the second level and wing length at the first level, respectively. In Table 5, these data were given to be 5.554 Hz and 4.441 Hz, respectively. Also, \(\bar{T}_\lambda = 3.7314\) indicates the overall mean of the first mode free vibration results regarding Taguchi’s L9 orthogonal array. Based on the average data, predicted optimum natural frequency value \((\mu_\lambda)\) is solved as 6.2636 Hz. The 95 % confidence intervals (CI) of confirmation fundamental frequency analysis (CI\(_{CA}\)) and population (CI\(_{POP}\)) were computed using Equation 3 and Equation 4 [18].

\[
\begin{align*}
\text{CI}_{CA} &= \left( F_{\alpha;1;n_2} V_{\text{error}} \left[ \frac{1}{n_{\text{eff}}} + \frac{1}{R} \right] \right)^{1/2} \\
\text{CI}_{POP} &= \left( F_{\alpha;1;n_2}^* V_{\text{error}}^* \right)^{1/2} / n_{\text{eff}} \\
n_{\text{eff}} &= \frac{N}{(1 + T_{\text{DOF}})} \tag{5}
\end{align*}
\]

in here, \(T_{\text{DOF}} = 4\) expresses the sum of number of degrees of freedom for important control factors such as airfoil cross section and wing length in Table 5. \(N = 9\) presents the sum of number of the numerical results in Table 3. Thus, \(n_{\text{eff}} = 1.8\) was calculated. \(V_{\text{error}} = 0.1034\) states the error value of variance. \(\alpha = 0.05\) indicates the risk and \(n^2 = 4\) states the error data based on the degree of freedom in ANOVA. Therefore \(F_{0.05;1;4}\) is found to be 7.71 [18] for F-table regarding the 95 % confidence interval \((\alpha=0.05)\). \(R = 1\) denotes the sample size of numerical analysis. CI\(_{CA}\) and CI\(_{POP}\) are found to be \(\pm 1.1136\) and \(\pm 0.6655\), respectively. The predictable confidence interval for confirmation numerical fundamental frequency analyses [18] is as below:

\[
\text{Mean} \mu_\lambda - \text{CI}_{CA} < \mu_\lambda < \text{CI}_{CA} + \text{Mean} \mu_\lambda
\]

The population at the 95 % confidence interval [18] is as below:

\[
\text{Mean} \mu_\lambda - \text{CI}_{POP} < \mu_\lambda < \text{CI}_{POP} + \text{Mean} \mu_\lambda
\]

ANSYS APDL and predicted frequency results at the first mode were listed for estimated confidence intervals at the 95 % confidence level in Table 7.
Table 7. ANSYS and predicted results

| Optimal Designation | ANSYS Result | Predicted Result | Estimated Confidence Intervals at the 95 % Confidence Level |
|---------------------|--------------|------------------|----------------------------------------------------------------|
| A₂B₁                | 6.6111 Hz    | 6.2636 Hz        | 5.1500 < μₗ < 7.3772 for CIₖₐ < 5.5981 < μₗ < 6.9291 for CIₙ₉₉ |

In order to detect the residuals between ANSYS and predicted results, predicted frequency data based on L9 orthogonal array were calculated using average data for each level of each control factor. Results obtained were exhibited in Figure 5.

Figure 5. ANSYS and predicted results

It can be seen from Figure 5 that the maximum residual result between designations was obtained for A₂B₁ whereas the minimum residual result was calculated for A₂B₂.

6. Conclusion

In this study, free vibration behavior of aircraft wings with different airfoil cross section and wing length at the first mode was investigated using Taguchi’s L9 orthogonal array with two control factors involving three levels. In the analysis, three airfoil cross sections such as NACA 0009, NACA 2424, and NACA 4415 were used. Free vibration analyses were achieved utilizing finite element software ANSYS. In order to detect the airfoil cross section and wing length at the optimal level, the analysis of signal-to-noise (S/N) ratio was employed. The most powerful control factors and their contribution ratios were found using analysis of variance (ANOVA). According to this study, the results summarized were as bellow:

- The maximum free vibration behavior was obtained by using NACA2424 wing profiles with a length of 5 meters.
- The increase of wing length causes a decrease on free vibration of aircraft wings.
- ANOVA result shows that airfoil cross section and wing length has the significant effect on free vibration analysis of aircraft wings due to P < 0.05 value. Also, the most dominant control factors on free vibration characteristic were detected to be airfoil type having 85.21 % effect and wing length having 12.87 % effect.
- The most affected ends of aircraft wings on the free vibration analysis were analyzed as free ends.
- Estimated optimum fundamental vibration results of aircraft wings at the 95 % confidence level were investigated as 5.1500 < μₗ < 7.3772 for CIₖₐ and 5.5981 < μₗ < 6.9291 for CIₙ₉₉.
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Ethical Approval

Not applicable.

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