We discuss some recent developments made in the computation of the baryon asymmetry generated at the electroweak scale. We emphasize that the local number density asymmetries of the particles involved in supersymmetric electroweak baryogenesis must be described by a set of Quantum Boltzmann Equations. These diffusion equations automatically and self-consistently incorporate the CP-violating sources which fuel baryogenesis and manifest “memory” effects which are typical of the quantum transport theory and are not present in the classical approach.

1 Prologo

Because of the presence of unsuppressed baryon number violating processes at high temperatures, the Standard Model (SM) of weak interactions fulfills all the requirements for a successful generation of the baryon number at the electroweak scale. The baryon number violating processes also impose severe constraints on models where the baryon asymmetry is created at energy scales much higher than the electroweak scale. Unfortunately, the electroweak phase transition is too weak in the SM. This means that the baryon asymmetry generated during the transition would subsequently be erased by unsuppressed sphaleron transitions in the broken phase.

The most promising and well-motivated framework for electroweak baryogenesis beyond the SM seems to be supersymmetry (SUSY). Electroweak baryogenesis in the framework of the Minimal Supersymmetric Standard Model (MSSM) has attracted much attention in the past years, with particular emphasis on the strength of the phase transition \(1\) and the mechanism of baryon number generation \(2\).

Recent analytical \(3\) and lattice computations \(4\) have also revealed that the phase transition can be sufficiently strongly first order if the ratio of the vacuum expectation values of the two neutral Higgses \(\tan \beta\) is smaller than \(\sim 4\).

\(a\) Talk given at the XXXIIIrd Rencontres de Moriond Conference, *Fundamental Parameters in Cosmology*, Les Arcs, France, January 17-24, 1998.

\(b\) On leave of absence from Theoretical Physics Dept., University of Oxford, Oxford, U.K.
Moreover, taking into account all the experimental bounds as well as those coming from the requirement of avoiding dangerous color breaking minima, the lightest Higgs boson should be lighter than about 105 GeV, while the right-handed stop mass might be close to the present experimental bound and should be smaller than, or of the order of, the top quark mass.

Moreover, the MSSM contains additional sources of CP-violation besides the CKM matrix phase. These new phases are essential for the generation of the baryon number since large CP-violating sources may be locally induced by the passage of the bubble wall separating the broken from the unbroken phase during the electroweak phase transition. Baryogenesis is fuelled when transport properties allow the CP-violating charges to efficiently diffuse in front of the advancing bubble wall where anomalous electroweak baryon violating processes are not suppressed. The new phases appear in the soft supersymmetry breaking parameters associated to the stop mixing angle and to the gaugino and neutralino mass matrices; large values of the stop mixing angle are, however, strongly restricted in order to preserve a sufficiently strong first order electroweak phase transition. Therefore, an acceptable baryon asymmetry from the stop sector may only be generated through a delicate balance between the values of the different soft supersymmetry breaking parameters contributing to the stop mixing parameter, and their associated CP-violating phases. As a result, the contribution to the final baryon asymmetry from the stop sector turns out to be negligible. On the other hand, charginos and neutralinos may be responsible for the observed baryon asymmetry if the phase of the parameter $\mu$ is large enough. Yet, this is true within the MSSM. If the strength of the electroweak phase transition is enhanced by the presence of some new degrees of freedom beyond the ones contained in the MSSM, e.g. some extra standard model gauge singlets, light stops (predominantly the right-handed ones) and charginos/neutralinos are expected to give quantitatively the same contribution to the final baryon asymmetry.

2 The old wisdom

In this talk we will mainly concern ourselves with the recent developments done in the computation of the baryon asymmetry. Since it not our intention to get into technical details, we will only try to give the feeling of what is going on.

Let us first mention what was done in the old days.

The baryon asymmetry has been usually computed using the following separate steps:

1) Look for those charges which are approximately conserved in the sym-
metric phase, so that they can efficiently diffuse in front of the bubble where baryon number violation is fast, and non-orthogonal to baryon number, so that the generation of a non-zero baryon charge is energetically favoured. Charges with these characteristics in the MSSM are the axial stop charge and the Higgsino charge, which may be produced from the interactions of squarks and charginos and/or neutralinos with the bubble wall, provided a source of CP-violation is present in these sectors;

2) Compute the CP-violating currents of the plasma locally induced by the passage of the bubble wall. The methods present in the literature properly incorporate the decoherence effects which may have a crucial impact on the generation of the CP-violating observable;

3) Write and solve a set of coupled differential diffusion equations for the local particle densities, including the CP-violating source terms derived from the computation of the current at step 2) and the particle number changing reactions. The solution to these equations gives a net baryon number which is produced in the symmetric phase and then transmitted into the interior of the bubbles of the broken phase, where it is not wiped out if the first transition is strong enough. It is important to notice that the CP-violating sources have been usually inserted into the diffusion equations by hand only after the CP-violating currents have been defined and computed. This procedure is certainly appropriate to describe the damping effects on the CP-violating observables originated by the plasma interactions, but it does not incorporate any relaxation time scale arising when diffusion and particle changing interactions are included (even though this approximation might be good if the diffusion time scales are larger than the damping time scales) and is theoretically not consistent. More important, since a certain degree of arbitrariness is present in the way the CP-violating sources may be defined, different CP-violating sources have been adopted for the stop and the Higgsino sectors in the literature. This is certainly not an academic question since different sources may lead to different numerical results for the final baryon asymmetry (especially if the sources are expressed in terms of a different number of derivatives of the Higgs bubble wall profile and, therefore, in terms of different powers of the bubble wall velocity $v_\omega$ and bubble wall width $L_\omega$).

3 The new wisdom

It is indisputable that a set of transport (diffusion) equations already incorporating the CP-violating sources in a self-consistent way may be obtained only by means of a more complete treatment of the problem. Non-equilibrium Quantum Field Theory provides us with the necessary tools to write down a set of
quantum Boltzmann equations (QBE’s) describing the local particle densities and automatically incorporating the CP-violating sources. The most appropriate extension of the field theory to deal with these issues is to generalize the time contour of integration to a closed time-path (CTP). The CTP formalism is a powerful Green’s function formulation for describing non-equilibrium phenomena in field theory and it leads to a complete non-equilibrium quantum kinetic theory approach.

3.1 Out-of-equilibrium field theory with a broad rush

The CTP formalism (often dubbed as in-in formalism) is a powerful Green’s function formulation for describing non-equilibrium phenomena in field theory. It allows to describe phase-transition phenomena and to obtain a self-consistent set of quantum Boltzmann equations. The formalism yields various quantum averages of operators evaluated in the in-state without specifying the out-state. On the contrary, the ordinary quantum field theory (often dubbed as in-out formalism) yields quantum averages of the operators evaluated with an in-state at one end and an out-state at the other.

The partition function in the in-in formalism for a complex scalar field is defined to be

\[
Z[J, J^\dagger] = \text{Tr} \left[ T \left( \exp \left[ i \int_C (J \phi + J^\dagger \phi^\dagger) \right] \right) \rho \right] \\
= \text{Tr} \left[ T_+ \left( \exp \left[ i \int \left( J_+ \phi_+ + J^\dagger_+ \phi^{\dagger}_+ \right) \right] \right) \right] \\
\times T_- \left( \exp \left[ -i \int \left( J_- \phi_- + J^{\dagger}_- \phi^{\dagger}_- \right) \right] \right) \rho, \tag{1}
\]

where the suffix \(C\) in the integral denotes that the time integration contour runs from minus infinity to plus infinity and then back to minus infinity again. The symbol \(\rho\) represents the initial density matrix and the fields are in the Heisenberg picture and defined on this closed time contour.

As with the Euclidean time formulation, scalar (fermionic) fields \(\phi\) are still periodic (anti-periodic) in time, but with \(\phi(t, \vec{x}) = \phi(t - i \beta, \vec{x}), \beta = 1/T\). The temperature appears due to boundary condition, but time is now explicitly present in the integration contour.

For non-equilibrium phenomena and as a consequence of the time contour, we must now identify field variables with arguments on the positive or negative directional branches of the time path. This doubling of field variables leads to six different real-time propagators on the contour. It is possible to employ fewer than six propagators since they are not independent, but using
six simplifies the notation. For a generic bosonic charged scalar field $\phi$ they are defined as

$$G^>_\phi(x, y) = -i \langle \phi(x)\phi^\dagger(y) \rangle,$$

$$G^<_\phi(x, y) = -i \langle \phi^\dagger(y)\phi(x) \rangle,$$

$$G^t_\phi(x, y) = \theta(x, y)G^>_\phi(x, y) + \theta(y, x)G^<_\phi(x, y),$$

$$G^{\bar{t}}_\phi(x, y) = \theta(y, x)G^>_\phi(x, y) + \theta(x, y)G^<_\phi(x, y),$$

$$G^r_\phi(x, y) = G^t_\phi - G^<_\phi = G^>\phi - G^{\bar{t}}_\phi,$$

$$G^a_\phi(x, y) = G^t_\phi - G^>_\phi = G^<_\phi - G^{\bar{t}}_\phi,$$

where the last two Green functions are the retarded and advanced Green functions respectively and $\theta(x, y) = \theta(t_x - t_y)$ is the step function. Analogous formulae hold for fermion fields. The reader is referred to 12 for more technical details.

For interacting systems whether in equilibrium or not, one must define and calculate self-energy functions. There are six of them: $\Sigma^t$, $\Sigma^{\bar{t}}$, $\Sigma^<$, $\Sigma^>$, $\Sigma^r$ and $\Sigma^a$. There are some relationships among them, such as

$$\Sigma^r = \Sigma^t - \Sigma^< = \Sigma^> - \Sigma^f,$$

$$\Sigma^a = \Sigma^t - \Sigma^> = \Sigma^< - \Sigma^f. \quad (3)$$

The self-energies are incorporated into the Green functions through the use of Dyson’s equations which are the starting point to get the Quantum Boltzmann equations. The latter look as follow

$$\frac{\partial n_\phi(X)}{\partial T} + \nabla_R \vec{j}_\phi(X) = - \int d^3r_3 \int_T^{-\infty} dt_3 \left[ \Sigma^>_\phi(X, x_3)G^<_\phi(x_3, X) - G^<_\phi(x_3, X)\Sigma^>|_{\phi}(X, x_3) + G^>_\phi(x_3, X)\Sigma^<|_{\phi}(x_3, X) \right]. \quad (4)$$

Here $n_\phi$ and $\vec{j}_\phi$ are the particle density and current asymmetry, respectively, and the right-hand side represents the “scattering” term. It contains all the information necessary to describe the temporal evolution of the particle density asymmetries: particle number changing reactions and CP-violating source terms, which will pop out from the corresponding self-energy $\Sigma_{CP}$. In supersymmetric baryogenesis one has to consider the Quantum Boltzmann equations for right-handed stops and higgsinos.

3.2 It is worth doing out-of-equilibrium field theory

One of the merits of the CTP formalism is to guide us towards a rigorous and self-consistent definition of the CP-violating sources within the quantum Boltzmann equations. On the contrary, previous treatments are characterized
by the following common feature: CP-violating currents were first derived and then converted into sources for the diffusion equations. More specifically, CP-violating sources $S$ associated to a generic charge density $j^0$ were constructed from the current $j^\mu$ by the definition $S = \partial_t j^0$. A rigorous computation of the CP-violating currents for the right-handed stop $\tilde{t}_R$ and higgsino $\tilde{H}$ local densities was performed in [5] by means of the CTP formalism. Since currents were proportional to the first time derivative of the Higgs profile, sources turned out to be proportional to the second time derivative of the Higgs profile. The results obtained in [5], however, indicate that the sources in the quantum diffusion equations are proportional to the first time derivative of the Higgs configuration. A comparison between the sources $S$ obtained in [5] and the currents $j^0$ given in ref. [5] indicate that they may be related as

$$S(T) \sim \frac{j^0(T)}{\tau}$$

and it may be interpreted as the time derivative of the current density accumulated at the time $T$ after the wall has deposited at a given specific point the current density $j^0$ each interval $\tau$

$$S(T) \sim \partial_0 \int_0^T dt \frac{j^0(t)}{\tau}.$$  

Here $\tau = \Gamma^{-1}$ is the thermalization time of the right-handed stops and higgsinos, respectively. The integral over time is peculiar of the quantum approach and it induces memory effects. This tells us that the source obtained self-consistently in the present work differs from the one adopted in [5] by a factor $\sim L_\omega \Gamma/v_\omega$ (in the rest frame of the advancing bubble wall). Since $L_\omega \Gamma/v_\omega \gtrsim 1$ for the Higgs derivative expansion to hold, this result is important as far as the numerical estimate of the final baryon number is concerned.

3.3 Memory effects

Baryogenesis is fuelled when transport properties allow the CP-violating charges to efficiently diffuse in front of the advancing bubble wall where anomalous electroweak baryon violating processes are not suppressed. However, the CP-violating processes of quantum interference, which build up CP-violating sources, must act in opposition to the incoherent nature of plasma physics responsible for the loss of quantistic interference. If the particles involved in the process of baryon number generation thermalize rapidly, CP-violating sources loose their coherence and are diminished. The CTP formalism properly describes the
quantum nature of CP-violation and tells us that CP-violating sources evaluated at some time $T$ are always proportional to an integral over the past history of the system. Therefore, it is fair to argue that these memory effects lead to “relaxation” times for the CP-violating sources which are typically longer than the ones dictated by the thermalization rates of the particles in the thermal bath. In fact, this observation is valid for all the processes described by the “scattering” term in the right-handed side of the quantum diffusion equations. The slowdown of the relaxation processes may help to keep the system out of equilibrium for longer times and therefore enhance the final baryon asymmetry. There are two more reasons why one should expect quantum relaxation times to be longer than the ones predicted by the classical approach. First, the decay of the Green’s functions as functions of the difference of the time arguments; secondly, the rather different oscillatory behaviour of the functions $G^>$ and $G^<$ present in the CTP formalism, for a given momentum, as functions of the time argument difference.

3.4 Resonance effects

In the limit of thick bubble walls, the CP-violating sources are characterized by resonance effects when the particles involved in the construction of the source are degenerate in mass. The interpretation of the resonance is rather straightforward if we think in terms of scatterings of the quasiparticles off the advancing low momentum bubble wall configuration. A similar effect has been found in ref. where the system was studied in the classical limit. The classical treatments should provide reasonable approximations to our formulae to those particles whose wavelength is short compared to $v_{\omega}/\Gamma$. On the other hand, formulae should not agree for small $\Gamma$ because our source is dominated by particles with long wavelength. In this regime, the classical approximation breaks down since it requires that the mean free path should be larger than the Compton wavelength of the underlying particle. This is relevant because quasiparticles with long wavelengths give a significant contribution to CP-violating sources.

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References

1. G.F. Giudice, *Phys. Rev.* D45, 3177 (1992).
2. S. Myint, *Phys. Lett.* B287, 325 (1992).
3. J.R. Espinosa, M. Quiros and F. Zwirner, *Phys. Lett.* B307, 106 (1993); A. Brignole, J.R. Espinosa, M. Quiros and F. Zwirner, *Phys. Lett.* B324, 181 (1994).
4. P. Huet and A.E. Nelson, *Phys. Rev.* D53, 4578 (1996).
5. M. Carena, M. Quiros, A. Riotto, I. Vilja and C.E.M. Wagner, *Nucl. Phys.* B503, 387 (1997); hep-ph/9702409.
6. A. Riotto, OUTP-97-43-P preprint, hep-ph/9709286.
7. J.M. Cline, M. Joyce and K. Kainulainen, McGill 97-26 preprint hep-ph/9708393.
8. M. Carena, M. Quiros and C.E.M. Wagner, *Phys. Lett.* B380, 81 (1996); D. Delepine, J.M. Gerard, R. Gonzalez Felipe and J. Weyers, *Phys. Lett.* B386, 183 (1996); J.R. Espinosa, *Nucl. Phys.* B475, 273 (1996); B. de Carlos and J.R. Espinosa, UPR-0737-T preprint, hep-ph/9703317.
9. M. Carena, M. Quiros and C.E.M. Wagner, CERN-TH/97-190, hep-ph/9710401.
10. M. Laine, *Nucl. Phys.* B481, 43 (1996); J.M. Cline and K. Kainulainen, *Nucl. Phys.* B482, 73 (1996).
11. K. Chou, Z. Su, B. Hao and L. Yu, *Phys. Rep.* 118, 1 (1985).
12. A. Riotto, CERN-TH-97-348 [hep-ph/9712221].