Multi-Slice Fusion for Sparse-View and Limited-Angle 4D CT Reconstruction

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Abstract—Inverse problems spanning four or more dimensions such as space, time and other independent parameters have become increasingly important. State-of-the-art 4D reconstruction methods use model based iterative reconstruction (MBIR), but depend critically on the quality of the prior modeling. Recently, plug-and-play (PnP) methods have been shown to be an effective way to incorporate advanced prior models using state-of-the-art denoising algorithms. However, state-of-the-art denoisers such as BM4D and deep convolutional neural networks (CNNs) are primarily available for 2D or 3D images and extending them to higher dimensions is difficult due to algorithmic complexity and the increased difficulty of effective training.

In this paper, we present multi-slice fusion, a novel algorithm for 4D reconstruction, based on the fusion of multiple low-dimensional denoisers. Our approach uses multi-agent consensus equilibrium (MACE), an extension of plug-and-play, as a framework for integrating the multiple lower-dimensional models. We apply our method to 4D cone-beam X-ray CT reconstruction for non destructive evaluation (NDE) of samples that are dynamically moving during acquisition. We implement multi-slice fusion on distributed, heterogeneous clusters in order to reconstruct large 4D volumes in reasonable time and demonstrate the inherent parallelizable nature of the algorithm. We present simulated and real experimental results on sparse-view and limited-angle CT data to demonstrate that multi-slice fusion can substantially improve the quality of reconstructions relative to traditional methods, while also being practical to implement and train.

Index Terms—Inverse problems, 4D tomography, Model based reconstruction, Plug-and-play, Deep neural networks

I. INTRODUCTION

Improvements in imaging sensors and computing power have made it possible to solve increasingly difficult reconstruction problems. In particular, the dimensionality of reconstruction problems has increased from the traditional 2D and 3D problems representing space to more difficult 4D or even 5D problems representing space-time and, for example, heart or respiratory phase [1]–[6].

These higher-dimensional reconstruction problems pose surprisingly difficult challenges computationally and perhaps more importantly, in terms of algorithmic design and training due to the curse of dimensionality [7]. However, the high dimensionality of the reconstruction also presents important opportunities to improve reconstruction quality by exploiting the regularity in the high-dimensional space. In particular, for time-resolved imaging, we can exploit the regularity of the image to reconstruct each frame with fewer measurements and thereby increase temporal resolution. In the case of 4D CT, the contributions of [2], [8], [9] have increased the temporal resolution by an order of magnitude by exploiting the space-time regularity of objects being imaged. These approaches use model-based iterative reconstruction (MBIR) [10], [11] to enforce regularity in 4D using simple space-time prior models. More recently, deep learning based post-processing for 4D reconstruction has been proposed as a method to improve reconstructed image quality [12].

Recently, it has been demonstrated that plug-and-play (PnP) priors [13]–[16] can dramatically improve reconstruction quality by enabling the use of state-of-the-art denoisers as prior models in MBIR. So PnP has great potential to improve reconstruction quality in 4D CT imaging problems. However, state-of-the-art denoisers such as deep convolutional neural networks (CNN) and BM4D are primarily available for 2D and sometimes 3D images, and it is difficult to extend them to higher dimensions [7], [17], [18]. In particular, extending CNNs to 4D requires very computationally and memory intensive 4D convolution applied to 5D feature tensor structures. This problem is further compounded by the lack of GPU accelerated routines for 4D convolution from major Deep-
Learning frameworks such as Tensorflow, Keras, PyTorch\footnote{Currently only 1D, 2D, and 3D convolutions are supported with GPU acceleration}.

Furthermore, 4D CNNs require 4D ground truth data to train the PnP denoisers, which might be difficult or impossible to obtain.

In this paper, we present a novel 4D X-ray CT reconstruction algorithm that combines multiple low-dimensional CNN denoisers to implement a highly effective 4D prior model. Our approach, multi-slice fusion, integrates the multiple low-dimensional priors using multi-agent consensus equilibrium (MACE)\footnote{MACE is an extension of the PnP framework that formulates the inversion problem using an equilibrium equation—as opposed to an optimization—and allows for the use of multiple prior models and agents.}. MACE is an extension of the PnP framework which integrates the multiple low-dimensional priors using multi-agent consensus equilibrium (MACE)\footnote{MACE is an extension of the PnP framework that formulates the inversion problem using an equilibrium equation—as opposed to an optimization—and allows for the use of multiple prior models and agents.}. MACE is an extension of the PnP framework that formulates the inversion problem using an equilibrium equation—as opposed to an optimization—and allows for the use of multiple prior models and agents.

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In section IV, we use the MACE framework to introduce multi-slice fusion. In section V, we describe our training pipeline for training the CNN denoisers. In section VI, we describe our distributed implementation of multi-slice fusion on heterogeneous clusters. Finally, in section VII, we present results on sparse-view and limited-angle 4D CT using both simulated and real data.

II. Problem Formulation

In 4D X-ray CT imaging, a dynamic object is rotated and several 2D projections (radiographs) of the object are measured for different angles as illustrated in Figure 2. The problem is then to reconstruct the 4D array of X-ray attenuation coefficients from these measurements, where three dimensions correspond to the spatial dimensions and the fourth dimension corresponds to time.

Let $N_t$ be the number of time-points, and $N_s$ be the number of voxels at each time-point of the 4D volume. For each time-point $n \in \{1, \ldots, N_t\}$, define $y_n \in \mathbb{R}^{M_n}$ to be the vector of sinogram measurements at time $n$, and $x_n \in \mathbb{R}^{N_s}$ to be the vectorized 3D volume of X-ray attenuation coefficients for that time-point. Let us stack all the measurements to form a measurement vector $y = [y_1^\top, \ldots, y_{N_t}^\top]^\top \in \mathbb{R}^M$ where $M = \sum_{n=1}^{N_t} M_n$ is the total number of measurements. Similarly, let us stack the 3D volumes at each time-point to form a vectorized 4D volume $x = [x_1^\top, \ldots, x_{N_t N_s}^\top]^\top \in \mathbb{R}^N$, where $N = N_t N_s$ is the total number of voxels in the 4D volume. The 4D reconstruction problem then becomes the task of recovering the 4D volume of attenuation coefficients, $x$, from the series of sinogram measurements, $y$.

In the traditional maximum a posteriori (MAP) approach, the reconstruction is given by

$$x^* = \arg\min_x \{l(x) + \beta h(x)\}, \tag{1}$$

where $l(x)$ is the data fidelity term and $h(x)$ is the 4D prior model.
where \( l(x) \) is the data-fidelity or log-likelihood term, \( h(x) \) is the 4D regularizer or prior model, and the unitless parameter \( \beta \) controls the level of regularization in the reconstruction. The data-fidelity term, \( l(x) \), can be written in a separable fashion as

\[
l(x) = \frac{1}{2} \sum_{n=1}^{N_t} \| y_n - A_n x_n \|_{\Lambda_n}^2 ,
\]

where \( A_n \) is the system matrix, and \( \Lambda_n \) is the weight matrix \([3]\) for time-point \( n \). The values in the diagonal weight matrix, \( \Lambda_n \), account for the non-uniform noise variance for each measurement.

If the prior model, \( h(x) \), can be expressed analytically like a 4D Markov random field (MRF) as in \([2], [4]\), then the expression in equation \((1)\) can be minimized iteratively to reconstruct the image. However, in practice, it can be difficult to represent an advanced prior model in the form of a tractable cost function \( h(x) \) that can be minimized. Consequently, PnP algorithms have been created as a method for representing prior models as denoising operations \([13], [14]\). More recently, PnP methods have been generalized to the multi-agent consensus equilibrium (MACE) framework as a way to integrate multiple models in a principled manner \([4], [19], [23]\).

### III. MACE Model Fusion

In this section, we use the multi-agent consensus equilibrium (MACE) framework to fuse the data-fidelity term and multiple denoisers; these multiple denoisers form a single prior model for reconstruction. This allows us to construct a 4D prior model using low-dimensional CNN denoisers (described in Section \([IV]\)).

To introduce the concept of consensus equilibrium, let us first consider a variation of the optimization problem in equation \((1)\) with \( K \) regularizers \( h_k(x), \ k = 1, \ldots, K \). The modified optimization problem can thus be written as

\[
x^* = \arg \min_x \left\{ l(x) + \frac{\beta}{K} \sum_{k=1}^{K} h_k(x) \right\} ,
\]

where the normalization by \( K \) is done to make the regularization strength independent of the number of regularizers.

Now we transform the optimization problem of equation \((3)\) to an equivalent consensus equilibrium formulation. However, in order to do this, we must introduce additional notation. First, we define the proximal maps of each term in equation \((3)\). We define \( L(x) : \mathbb{R}^N \rightarrow \mathbb{R}^N \) to be the proximal map of \( l(x) \) as

\[
L(x) = \arg \min_{z \in \mathbb{R}^N} \left\{ l(z) + \frac{1}{2\sigma^2} \| x - z \|_2^2 \right\} ,
\]

for some \( \sigma > 0 \). Similarly, we define \( H_k(x) : \mathbb{R}^N \rightarrow \mathbb{R}^N \) to be the the proximal map of each \( h_k(x) \), \( k = 1, \ldots, K \) as

\[
H_k(x) = \arg \min_{z \in \mathbb{R}^N} \left\{ \frac{1}{2\sigma^2} \| x - z \|_2^2 + h_k(z) \right\} .
\]

Each of these proximal maps serve as agents in the MACE framework. We stack the agents together to form a stacked operator \( F : \mathbb{R}^{(K+1)N} \rightarrow \mathbb{R}^{(K+1)N} \) as

\[
F(W) = \begin{bmatrix}
L(W_0) \\
H_1(W_1) \\
\vdots \\
H_K(W_K)
\end{bmatrix} ,
\]

where \( W \in \mathbb{R}^{(K+1)N} \) is stacked representative variable. The consensus equilibrium is the vector \( W^* \in \mathbb{R}^{(K+1)N} \) that satisfies

\[
F(W^*) = G(W^*) ,
\]

where \( G \) is an averaging operator given as

\[
G(W) = \begin{bmatrix}
\overline{W} \\
\vdots \\
\overline{W}
\end{bmatrix} ,
\]

and the weighted average is defined as

\[
\overline{W} = \frac{1}{1 + \beta} W_0 + \frac{\beta}{1 + \beta} \left( \frac{1}{K} \sum_{k=1}^{K} W_k \right) .
\]

Notice the weighting scheme is chosen to balance the forward and prior models. The unitless parameter \( \beta \) is used to tune the weights given to the prior model and thus the regularization of the reconstruction. Equal weighing of the forward and prior models can be achieved using \( \beta = 1 \).

If \( W^* \) satisfies the consensus equilibrium condition of equation \((7)\), then it can be shown \([19]\) that \( W^* \) is the solution to the optimization problem in equation \((3)\). Thus if the agents in MACE are true proximal maps then the consensus equilibrium solves an equivalent optimization problem.

However, if the MACE agents are not true proximal maps, then there is no inherent optimization problem to be solved, but the MACE solution still exists. In this case, the MACE solution can be interpreted as the balance point between the forces of each agent as illustrated in Figure \([3]\). Each agent pulls the solution toward its manifold and at equilibrium the forces balance each other.

To see how we can incorporate deep neural network based prior models, first notice that equation \((5)\) can be interpreted...
as the MAP estimate for a Gaussian denoising problem with prior model $b_k$ and noise standard deviation $\sigma$. Thus we can replace each MACE operator, $H_k$, for each $k = 1, \ldots, K$ in equation (5) with a deep neural network trained to remove additive white Gaussian noise of standard deviation $\sigma$.

It is interesting to note that when $H_k$ is implemented with a deep neural network denoiser, then the agent $H_k$ is not, in general, a proximal map and there is no corresponding cost function $h_k$. We know this because for $H_k$ to be a proximal map, it must satisfy the condition that $\nabla H_k(x) = [\nabla H_k(x)]^T$ (see [13], [24]), which is equivalent to $H_k$ being a conservative vector function (see for example [25] Theorem 2.6, p. 527)). For a CNN, $\nabla H_k$ is a function of the trained weights, and in the general case, the condition will not be met unless the CNN architecture is specifically designed to enforce such a condition.

The consensus equilibrium equation [7] states the condition that the equilibrium solution must satisfy. However, the question remains of how to compute this equilibrium solution. Our approach to solving the consensus equilibrium equations is to first find an operator that has the equilibrium solution as a fixed point, and then use standard fixed point solvers. To do this, we first notice that the averaging operator has the property that $G(G(W)) = G(W)$. Intuitively, this is true because applying averaging twice is the same as applying it once. Using this fact, we see that

$$ (2G - I)(2G - I) = 4GG - 4G + I = I , \quad (10) $$

where $I$ is the identity mapping. We then rewrite equation (7) as

$$ (2G - I)(2G - I)W^* = GW^* \quad (2G - I)W^* = (2G - I)W^* \quad (2G - I)(2F - I)W^* = W^* . $$

So from this we see that the following fixed point relationship must hold for the consensus equilibrium solution.

$$ (2G - I)(2G - I)W^* = W^* , \quad (11) $$

and the consensus equilibrium solution $W^*$ is a fixed point of the mapping $T = (2G - I)(2F - I)$.

We can apply a variety of iterative fixed point algorithms to equation (11) to compute the equilibrium solution. These algorithms have varying convergence guarantees and convergence speeds [19]. One such algorithm is Mann iteration [19], [23], [26]. Mann iteration performs the following pseudo-code steps until convergence where $\leftarrow$ indicates assignment of a pseudo-code variable.

$$ W \leftarrow (1 - \rho)W + \rho TW , \quad (12) $$

where weighing parameter $\rho \in (0, 1)$ is used to control the speed of convergence. In particular, when $\rho = 0.5$, the Mann-iteration solver is equivalent to the consensus-ADMM algorithm [19], [27]. It can be shown that the Mann iteration converges to a fixed point of $T = (2G - I)(2F - I)$ if $T$ is a non-expansive mapping [19].

Note that each Mann iteration update in equation (12) involves performing the minimization in equation (4). This nested iteration is computationally expensive and leads to slow convergence. Instead of minimizing equation (4) till convergence, we initialize with the result of the previous Mann iteration and perform only three iterations of iterative coordinate descent (ICD). We denote this partial update operator as $\tilde{L}(W_0, X_0)$ where $X_0$ is the initial condition to the iterative update. The corresponding new $F$ operator approximation is then given by

$$ \tilde{F}(W; X) = \begin{bmatrix} \tilde{L}(W_0; X_0) \\ H_1(W_1) \\ \vdots \\ H_K(W_K) \end{bmatrix} . \quad (13) $$

Algorithm 1 shows a simplified Mann iteration using partial updates. We perform algebraic manipulation of the traditional Mann iterations [23], [26] in order to obtain the simplified but equivalent Algorithm 1. It can be shown that partial update Mann iteration also converges [23], [25] to the fixed point in equation (11). We used a zero initialization, $x(0) = 0$, in all our experiments and continue the partial update Mann iteration until the differences between state vectors $X_k$ become smaller than a fixed threshold.

**Algorithm 1:** Partial update Mann iteration for computing the MACE solution

**Input:** Initial Reconstruction: $x(0) \in \mathbb{R}^N$

**Output:** Final Reconstruction: $x^*$

1. $X \leftarrow W \leftarrow \begin{bmatrix} x(0) \\ \vdots \\ x(0) \end{bmatrix}$
2. **while not converged do**
   3. $X \leftarrow \tilde{F}(W; X)$
   4. $Z \leftarrow G(2X - W)$
   5. $W \leftarrow W + 2\rho(Z - X)$
   6. $x^* \leftarrow X_0$

IV. MULTI-SLICE FUSION USING MACE

We use four MACE agents to implement multi-slice fusion. We set $K = 3$ and use the names $H_{x,y,t}$, $H_{y,z,t}$, $H_{z,x,t}$ to denote...
the denoising agents $H_1$, $H_2$, $H_3$ in equation (3). The agent $L$ enforces fidelity to the measurement while each of the denoisers $H_{xy,t}$, $H_{yz,t}$, $H_{zx,t}$ enforces regularity of the image in orthogonal image planes. MACE imposes a consensus between the operators $L$, $H_{xy,t}$, $H_{yz,t}$, $H_{zx,t}$ to achieve a balanced reconstruction that lies at the intersection of the solution space of the measurement model and each of the prior models. The MACE stacked operator $F$ encompassing all four agents can be written as

$$F(W) = \begin{bmatrix} L(W_0) \\ H_{xy}(W_1) \\ H_{yz}(W_2) \\ H_{zx}(W_3) \end{bmatrix}. \quad (14)$$

Here the representative variable $W \in \mathbb{R}^{4N}$ is formed by stacking four vectorized 4D volumes.

The three denoisers $H_{xy,t}$, $H_{yz,t}$, and $H_{zx,t}$ share the same architecture and trained model but are applied along different planes of the 4D space. The model is shown in Figure 4. We have modified a typical CNN architecture to input information from a third dimension. The channel dimension of a convolution layer is typically used to input multiple vectorized 4D volumes. We re-purpose the channel dimension to input five adjacent 2D slices of the noisy image to the network and output the denoised center slice. The other slices are being denoised by shifting the 5-slice moving window. We call this 2.5D since the receptive field along the convolution dimensions is large but in the channel dimension is small. It has been shown that this type of 2.5D processing is a computationally efficient way of performing effective 3D denoising with CNNs. We use the notation $H_{xy,t}$ to denote a CNN space-time denoiser that performs convolution in the xy-plane and uses the convolution channels to input slices from neighboring time-points. The denoisers $H_{yz,t}$ and $H_{zx,t}$ are analogous to $H_{xy,t}$ but are applied along the yz and zx-plane, respectively. This orientation of the three denoisers ensures that

1) The spatial dimensions x, y, z are treated equivalently. This ensures the regularization to be uniform across all spatial dimensions;
2) The regularization along spatial dimensions and the time dimension are separate. This is important for constructing images with varying amount of motion per time-point;
3) Each dimension in x, y, z, and t is considered at least once. This ensures that model fusion using MACE incorporates information along all four dimensions.

Since the three denoising operators $H_{xy,t}$, $H_{yz,t}$, and $H_{zx,t}$ process the 4D volume “slice by slice”, they can be implemented in parallel on large scale parallel computers. Details on distributed implementation are described in section VI.

V. TRAINING OF CNN DENOISERS

All three prior model agents $H_{xy,t}$, $H_{yz,t}$, and $H_{zx,t}$ in multi-slice fusion share the same 2.5D model shown in Figure 4 but are applied along different planes. Thus we only need to train a single 2.5D CNN and need 3D data to train it.

VI. DISTRIBUTED RECONSTRUCTION

The computational structure of multi-slice fusion is well-suited to a highly distributed implementation. The main computational bottleneck in Algorithm 1 is the $F$ operator. Fortunately, $F$ is a parallel operator and thus its individual components $L$, $H_{xy,t}$, $H_{yz,t}$, and $H_{zx,t}$ can be executed in parallel. The operators $L$, $H_{xy,t}$, $H_{yz,t}$, and $H_{zx,t}$ can themselves be parallelized internally as well. The distributed implementation of multi-slice fusion is illustrated in Figure 6.

The CNN denoisers $H_{xy,t}$, $H_{yz,t}$, and $H_{zx,t}$ are 2.5D denoisers that denoise the 4D volume by processing it slice by slice and thus can be trivially parallelized leading to a large number of concurrent operations. The concurrent operations for all three denoisers are distributed among multiple GPUs due to the availability of optimized GPU routines in Tensorflow. In our experiments we used a GPU cluster with three Nvidia Tesla P100 GPUs to compute the CNN denoising operators.

The cone-beam inversion operator, $L$, can also be computed slice by slice due to the separable structure in equations (4) and (2). This leads to a large number of concurrent operations which are distributed among multiple CPU nodes. Additional multi-threaded parallelism is also implemented similar to [3].

VII. EXPERIMENTAL RESULTS

We present experimental results on two simulated and two real 4D X-ray CT data for NDE applications to demonstrate
the improved reconstruction quality of our method. The four experimental cases are outlined below

1) **Simulated Data 360°**: Sparse-view results on simulated data with a set of sparse views ranging over 360° at each reconstructed time-point;

2) **Simulated Data 90°**: Sparse-view limited-angle results on simulated data with a set of sparse views ranging over 90° at each reconstructed time-point;

3) **Real Data 360°**: Sparse-view results on real data with a set of sparse views ranging over 360° at each reconstructed time-point;

4) **Real Data 90°**: Sparse-view limited-angle results on real data with a set of sparse views ranging over 90° at each reconstructed time-point.

The selection of the rotation range per time-point is arbitrary and can be chosen after the measurements have been taken. For example, a full rotation with 400 views can be used as a single time-point or as four time-points with 100 views each. The four time-points per rotation can provide extra temporal resolution, however, they require a more difficult reconstruction with incomplete information.

We compare multi-slice fusion with several other methods outlined below

- FBP: Conventional 3D filtered back projection reconstruction.
- MBIR+4D-MRF: MBIR reconstruction using 4D Markov random field prior [2] with 26 spatial and 2 temporal neighbors;
- MBIR+$H_{xyz}$: MBIR using the CNN $H_{xyz}$ as a PnP prior;
- MBIR+$H_{zy,t}$: MBIR using the CNN $H_{zy,t}$ as a PnP prior;
- MBIR+$H_{xz,t}$: MBIR using the CNN $H_{xz,t}$ as a PnP prior.

We used two CPU cluster nodes, each with 20 Kaby Lake CPU cores to compute the cone-beam inversion and three GPU nodes each with a Nvidia Tesla P100 GPU to compute the CNN denoisers. The 2.5D CNN denoiser model was trained using a low-noise 3D CT reconstruction of a bottle and screw cap made from different plastics. The object is representative of a variety of Non-Destructive Evaluation (NDE) problems in which objects to be imaged are constructed from a relatively small number of distinct materials. The standard deviation of the additive white Gaussian noise added during training was 0.1.

### A. Simulated Data 360°

| Magnification | Number of Views per Time-point | Rotation per Time-point | Cropped Detector Array | Voxel Size | Reconstruction Size (x,y,z,t) |
|---------------|-------------------------------|-------------------------|------------------------|------------|-----------------------------|
| 5.57          | 75                            | 360°                    | 240 x 28 , (0.95 mm)$^2$ | (0.17 mm)$^3$ | 240 x 240 x 28 x 8         |

In this section we present results on simulated data to evaluate our method in a sparse-view setting. Each time-point is reconstructed from a sparse set of views spanning 360°. We take a low-noise CT reconstruction of a bottle and screw cap and denoise it further using BM4D [18] to generate a clean 3D volume to be used as a 3D phantom. We then vertically translate the 3D phantom by one pixel per time-point to generate a 4D phantom. We forward project the phantom using the system matrix $A$ to simulate noisy sinogram data and use that to reconstruct the object. The experimental specifications are summarized in Table I.

Figure 7 compares reconstructions using multi-slice fusion with several other methods. Each image is a slice through the reconstructed object for one time-point along the spatial xy-plane. Both FBP and MBIR+4D-MRF suffer from high noise and jagged edges and fail to recover the small hole in the bottom of the image. MBIR+$H_{zy,t}$ and MBIR+$H_{xz,t}$ suffer from horizontal and vertical streaks, respectively, since the denoisers were applied in those planes. MBIR+$H_{zy,t}$ does not suffer from streaks in the figure since we are viewing a slice along the xy-plane, but it suffers from other artifacts. MBIR+$H_{x,t}$ cannot reconstruct the small hole in the bottom of the image since the xy-plane does not contain sufficient information.

Next we plot a cross-section through the object for multi-slice fusion, MBIR+4D-MRF, FBP, and the phantom in Figure 8. Multi-slice fusion results in the most accurate reconstruction of the gap between materials.

Finally we report the peak signal to noise ratio (PSNR) and the structural similarity index measure (SSIM) [29] with respect to the phantom for each method in Table I to objectively measure image quality. We define the PSNR for a given 4D reconstruction $x$ with a phantom $x^0$ as

$$\text{PSNR}(x) = 20 \log_{10} \left( \frac{\text{Range}(x^0)}{\text{RMSE}(x, x^0)} \right),$$

(15)

where range is computed from the 0.1st and 99.9th percentiles of the phantom. As can be seen from Table I, multi-slice fusion results in the highest PSNR and SSIM scores.
Fig. 7. Comparison of different methods for simulated data $360^\circ$. Each image is a slice through the reconstructed object for one time-point along the spatial xy-plane. Both FBP and MBIR+4D-MRF suffer from high noise and jagged edges and fail to recover the small hole in the bottom of the image. MBIR+$H_{yx,t}$ and MBIR+$H_{zx,t}$ suffer from horizontal and vertical streaks respectively since the denoisers were applied in those planes. MBIR+$H_{xy,t}$ cannot reconstruct the small hole in the bottom of the image since the xy-plane does not contain sufficient information.

| Method               | PSNR(dB) | SSIM  |
|----------------------|----------|-------|
| FBP                  | 19.69    | 0.609 |
| MBIR+4D-MRF          | 25.84    | 0.787 |
| Multi-slice fusion   | **29.07**| **0.943** |
| MBIR+$H_{xy,t}$      | 29.03    | 0.922 |
| MBIR+$H_{yz,t}$      | 28.04    | 0.932 |
| MBIR+$H_{zx,t}$      | 28.31    | 0.926 |

**TABLE II**

Quantitative Evaluation for simulated data $360^\circ$. Multi-slice fusion has the highest PSNR and SSIM metric among all the methods.

B. Simulated Data $90^\circ$

| Magnification | 5.57 |
|---------------|------|
| Number of Views per Time-point | 36   |
| Rotation per Time-point | $90^\circ$ |
| Cropped Detector Array | $240 \times 28$, $(0.95 \text{ mm})^2$ |
| Voxel Size     | $(0.17 \text{ mm})^3$ |
| Reconstruction Size (x,y,z,t) | $240 \times 240 \times 28 \times 8$ |

**TABLE III**

Experimental specifications for simulated data $90^\circ$

In this section we present results on simulated data to evaluate our method in a sparse-view and limited-angle setting. Each time-point is reconstructed from a sparse set of views spanning $90^\circ$. We take a low-noise CT reconstruction of a bottle and screw cap and denoise it further using BM4D [18] to generate a clean 3D volume to be used as a phantom. We then vertically translate the 3D phantom by one pixel per time-point to generate a 4D phantom. We forward project the phantom to generate sinogram data and use that to reconstruct the object. The experimental specifications are summarized in Table [III]

Figure 9 shows a comparison of different methods for simulated data with $90^\circ$ rotation of object per time-point. Limited angular information at each frame leads to severe artifacts in reconstructions with FBP. The simple 4D prior in MBIR+4D-MRF is unable to properly merge information from consecutive time-points leading to a blurred reconstruction. The CNN based 4D prior in multi-slice fusion is able to enforce a spatio-temporal smoothness on the image and thus the reconstructions do not suffer from artifacts due to limited angular information per time-point.

Table [IV] shows peak signal to noise ratio (PSNR) and structural similarity index measure (SSIM) with respect to the phantom for each method. Multi-slice fusion results in the highest PSNR and SSIM scores.

| Method               | PSNR(dB) | SSIM  |
|----------------------|----------|-------|
| FBP                  | 10.86    | 0.467 |
| MBIR+4D-MRF          | 14.25    | 0.742 |
| Multi-slice fusion   | **19.44**| **0.875** |

**TABLE IV**

Quantitative Evaluation for simulated data $90^\circ$. Multi-slice fusion has the highest PSNR and SSIM metric among all the methods.
C. Real Data 360°: Vial Compression

| Scanner Model      | North Star Imaging X50 |
|--------------------|------------------------|
| Voltage            | 140 kV                 |
| Current            | 500 µA                 |
| Exposure           | 20 ms                  |
| Source-Detector Distance | 839 mm            |
| Magnification      | 5.57                  |
| Number of Views per Time-point | 150            |
| Rotation per Time-point | 360°               |
| Cropped Detector Array | 731 × 91, (0.25 mm)² |
| Voxel Size         | (0.0456 mm)³         |
| Reconstruction Size (x,y,z,t) | 731 × 731 × 91 × 16 |

TABLE V
EXPERIMENTAL SPECIFICATIONS FOR REAL DATA 360°: VIAL COMPRESSION

In this section we present results on real data to evaluate our method in a sparse-view setting. The data is from a dynamic cone-beam X-ray scan of a glass vial, with elastomeric stopper and aluminum crimp-seal, using a North Star Imaging X50 X-ray CT system. The experimental specifications are summarized in Table V.

The vial is undergoing dynamic compression during the scan, to capture the mechanical response of the components as shown in Figure 14. Of particular interest is the moment when the aluminum seal is no longer in contact with the underside of the glass neck finish. This indicates the moment when the force applied exceeds that exerted by the rubber on the glass; this is known as the “residual seal force” [30].

During the scan, the vial was held in place by fixtures that were placed out of the field of view as shown in Figure 14. As the object rotated, the fixtures periodically intercepted the path of the X-rays resulting in corrupted measurements and consequently artifacts in the reconstruction. To mitigate this problem, we incorporate additional corrections that are described in Appendix A.

Figure 10 compares multi-slice fusion with several other methods. Each image is a slice through the reconstructed vial for one time-point along the spatial xy-plane. Both FBP and MBIR+4D-MRF suffer from obvious artifacts, higher noise and blurred edges. In contrast to that, the multi-slice fusion reconstruction has smooth and uniform textures while preserving edge definition. Figure 10 also illustrates the effect of model fusion by comparing multi-slice fusion with MBIR+H_{xy,t}, MBIR+H_{yz,t}, and MBIR+H_{zx,t}. MBIR+H_{xy,t} suffer from horizontal and vertical streaks respectively since the denoisers were applied in those planes. MBIR+H_{xy,t} does not suffer from streaks in the figure since we are viewing a slice along the xy-plane, but it suffers from other artifacts. MBIR+H_{xy,t} cannot reconstruct the outer ring since the slice displayed is at the edge of the aluminum seal and the xy-plane does not contain sufficient information. In contrast, multi-slice fusion can resolve the edges of the rings better than either of MBIR+H_{xy,t}, MBIR+H_{yz,t}, and MBIR+H_{zx,t} since it uses information from all the spatial coordinates.

Next, we plot a cross-section through the object for multi-slice fusion, MBIR+4D-MRF and FBP in Figure 11. For this, we choose a time-point where we know the aluminum and glass have separated spatially. The multi-slice fusion reconstruction has a steeper and more defined slope in the junction of aluminum and glass. This supports that multi-slice fusion is able to preserve fine details in spite of producing a smooth regularized image.

Finally in Figure 12 we plot a cross-section through the object with respect to time to show the improved space-time resolution of our method. We do this for FBP, MBIR+4D-MRF and multi-slice fusion. Multi-slice fusion results in improved space-time resolution of the separation of aluminum and glass.

D. Real Data 90°: Injector Pen

| Scanner Model      | North Star Imaging X50 |
|--------------------|------------------------|
| Voltage            | 165 kV                 |
| Current            | 550 µA                 |
| Exposure           | 12.5 ms                |
| Source-Detector Distance | 694 mm           |
| Magnification      | 2.83                  |
| Number of Views per Time-point | 144             |
| Rotation per Time-point | 90°               |
| Cropped Detector Array | 263 × 768, (0.254 mm)² |
| Voxel Size         | (0.089 mm)³          |
| Reconstruction Size (x,y,z,t) | 263 × 263 × 778 × 12 |

TABLE VI
EXPERIMENTAL SPECIFICATIONS FOR REAL DATA 90°: INJECTOR PEN
In this section we present results on real data to evaluate our method in a sparse-view and limited-angle setting. The data is from a dynamic cone-beam X-ray scan of an injector pen using a North Star Imaging X50 X-ray CT system. The experimental specifications are summarized in Table VI.

The injection device is initiated before the dynamic scan.
Fig. 10. Comparison of different methods for Real Data $360^\circ$: vial. Each image is a slice through the reconstructed vial for one time-point along the spatial $xy$-plane. Both FBP and MBIR+4D-MRF suffer from obvious windmill artifacts, higher noise and blurred edges. In contrast to that, the multi-slice fusion reconstruction has smooth and uniform textures while preserving edge definition. MBIR+$H_{yz,t}$ and MBIR+$H_{zx,t}$ suffer from horizontal and vertical streaks. MBIR+$H_{xy,t}$ cannot reconstruct the outer ring since the slice displayed is at the edge of the aluminum seal and the $xy$-plane does not contain sufficient information. Multi-slice fusion can resolve the edges of the rings better than either of MBIR+$H_{yz,t}$, MBIR+$H_{xy,t}$, and MBIR+$H_{zx,t}$ since it has information from all the spatial coordinates.

starts and completes a full injection during the duration of the scan. We are interested in observing the motion of a particular spring within the injector pen in order to determine whether it is working as expected. The spring exhibits a fast motion and as a result we need a high temporal resolution to observe the motion of the spring. To have sufficient temporal resolution we reconstruct one frame for every $90^\circ$ rotation of the object instead of the conventional $360^\circ$ rotation.

In Figure 13 we show a volume rendering of the reconstructed spring for four time-points and reconstruction methods FBP, MBIR+4D-MRF, and multi-slice fusion. Limited angular information at each frame leads to severe artifacts.
in reconstructions with FBP. The simple prior model in MBIR+4D-MRF is unable to merge the information from multiple time-points properly and some artifacts remain. The CNN based 4D prior in multi-slice fusion is able to enforce a spatio-temporal smoothness on the image and thus the reconstructions do not suffer from artifacts due to limited angular information per time-point.

VIII. CONCLUSION

In this paper, we proposed a novel 4D X-ray CT reconstruction algorithm, multi-slice fusion, that combines multiple low-dimensional denoisers to form a 4D prior. Our method allows the formation of an advanced 4D prior using state-of-the-art CNN denoisers without needing to train on 4D data. Furthermore, it allows for multiple levels of parallelism, thus enabling reconstruction of large volumes in a reasonable time. Although we focused on 4D X-ray CT reconstruction for NDE applications, our method can be used for any reconstruction problem involving multiple dimensions.

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APPENDIX A

CORRECTION FOR FIXTURES OUTSIDE THE FIELD OF VIEW

Algorithm 2: Blind fixture correction

Inputs: Original Sinogram: \( y \),  
System Matrix: \( A \),  

Output: Corrected Sinogram: \( y^{\text{corr}} \)

1. \( x \leftarrow \text{recon}(y, A) \)
2. \( x^m \leftarrow \text{mask}(x) \)
3. \( e \leftarrow y - Ax^m \)
4. \( p \leftarrow \text{blur}(e) \)
5. \( c \leftarrow \arg \min_{c \in \mathbb{R}} \| e - cp \|^2 \)
6. \( y^{\text{corr}} \leftarrow y - cp \)

Here we describe our correction for fixtures placed out of the field of view of the scanner. As shown in Figure 14, the setup is held together by a fixture constructed of tubes and plates. The tubes were placed outside the field of view of the CT scanner, thus causing artifacts in the reconstruction. Our method performs a blind source separation of the projection of the object from that of the tubes. Our blind separation relies on the fact that the projection of the tubes is spatially smooth. This is true since the tubes themselves do not have sharp features and there is motion blur due to the large distance of the tubes from the rotation axis.

Algorithm 2 shows our correction algorithm for the fixtures. Figure 15 illustrates the algorithm pictorially. The initial
Fig. 13. Volume rendering of the reconstructed spring for four time-points. A $90^\circ$ limited set of views is used to reconstruct each time-point. Limited angular information at each frame leads to severe artifacts in reconstructions with FBP. The simple prior model in MBIR+4D-MRF is unable to merge the information from multi time-points properly and some artifacts remain. The CNN based 4D prior in multi-slice fusion is able to enforce a spatio-temporal smoothness on the image and thus the reconstructions do not suffer from artifacts due to limited angular information per time-point.

Fig. 14. Experimental setup for Real Data $360^\circ$: Vial Compression. The vial is undergoing dynamic compression during the scan, to capture the mechanical response of the components. The glass vial (center) and the actuator (top) are held together by a frame constructed of tubes and plates. The tubes were placed outside the field of view of the CT scanner, thus causing artifacts in the reconstruction. We describe a correction for this in Appendix A.

reconstruction $x$ suffers from artifacts within the image and at the edge of the field of view. We mask $x$ so that the majority of the artifacts at the edge of the field of view are masked but the object remains unchanged in $x^m$. Consequently the error sinogram $e = y - Ax^m$ primarily contains the projection of the tubes with some residual projection of the object. The blurring of $e$ filters out the residual object projection but preserves the spatially smooth projection of the tubes. The corrected measurements $y_{\text{corr}}$ are found after performing a least squares fit. The correction can be repeated in order to get an improved reconstruction $x$ and consequently an improved correction $y_{\text{corr}}$.

Figure 15 shows the sinogram and reconstruction both before and after performing the blind correction. Not only does the reconstruction after fixture correction remove the artifacts in the air region, but it also improves the image quality inside the object. It can be seen that the vertical stripes in the object in the $yz$ view of the reconstruction have been eliminated after performing the correction.

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Fig. 15. Pipeline of the blind fixture correction in Algorithm 2. The vertical stripes in the yz-plane of the reconstruction and the ring at the edge of the field of view in the xy-plane of the reconstruction have been rectified after performing the correction.