New physics effects in rare $Z$ decays

M. A. Pérez
Departamento de Física, Cinvestav,
Apartado Postal 14–740, 07000, México D. F. México.
mperez@fis.cinvestav.mx

G. Tavares–Velasco
Instituto de Física y Matemáticas,
Universidad Michoacana de San Nicolás de Hidalgo,
Apartado Postal 2–82, 58040, Morelia, Michoacán, México.
gtv@itzel.ifm.umich.mx

J. J. Toscano
Facultad de Ciencias Físico Matemáticas,
Benemérita Universidad Autónoma de Puebla,
Apartado Postal 1152, Puebla, Puebla, México.
jtoscano@fcfm.buap.mx

October 29, 2018

Abstract

Virtual effects induced by new physics in rare $Z$ decays are reviewed. Since the expected sensitivity of the giga–$Z$ linear collider is of the order of $10^{-8}$, we emphasize the importance of any new physics effect that gives a prediction above this limit. It is also pointed out that an improvement on the known experimental constraints on rare $Z$ decays will provide us with a critical test of the validity of the standard model at the loop level.

1 Introduction

Processes that are forbidden or highly suppressed constitute a natural framework to test any new physics lying beyond the standard model (SM). In particular, rare $Z$ decays have been studied extensively in order to yield information on new physics. The major decays of the $Z$ boson into fermion pairs are by now well established within an accuracy of one part in ten thousands. While the sensitivity of the measurement for the branching ratios of rare $Z$ decays reached at LEP–2 is about $10^{-5}$, future linear colliders (NLC, TESLA) will bring this sensitivity up to the $10^{-8}$ level. As a consequence, the interest in the study of rare $Z$ decays is expected to increase.

The general aim of the present paper is to review rare $Z$ decays that may be induced in the SM at the loop level or by any other mean from new degrees of freedom predicted by some extensions of the SM. Since the energy scale $\Lambda$ associated with new degrees of freedom should be large as compared to the electroweak scale, it is expected that their virtual effects may show up in some rare $Z$ decays where the conventional SM radiative corrections are suppressed. We will not consider thus $Z$ decays into four fermions as they are induced at the tree level by the SM $ZWW$ and $ZZh$ couplings. For the purpose of the present paper, we will classify rare $Z$ decays into the following groups:
1. Single–photon decays.
2. Two–photon decays.
3. Decays with photons and gluons.
4. Decays involving scalars.
5. Flavor changing decays, \( Z \to l_i^\pm l_j^\mp, q_i^\pm q_j^\mp \).

Our presentation will proceed by addressing each one of the above sets of channels in the following sections. We will be interested in comparing SM predictions with those obtained in some of its extensions. We will find that in most cases the language of effective theories is the most efficient tool to make an objective comparison between the known experimental bounds on rare \( Z \) decays and the various extension of the SM.

The expression for the total \( Z \) decay width \( \Gamma_Z \) is required in the calculation of the branching ratios of rare \( Z \) decays. According to our present knowledge of the \( Z \) properties \([2]\), \( \Gamma_Z \) is obtained to a very good approximation by summing over all the partial decay widths into fermion pairs

\[
\Gamma_Z = \sum_{f \not= t} \Gamma_{f \bar{f}},
\]

where the partial decay widths \( \Gamma_{f \bar{f}} \) are given in the SM, including QCD and electroweak radiative corrections, by the following expression \([5]\)

\[
\Gamma_{f \bar{f}} = N_f \Gamma_0 \frac{\sqrt{1 - 4\mu_f}}{1 + Re(\Pi^Z(m_Z^2))} \left( (1 + 2\mu_f)|g_V^{Zf}(m_Z^2)|^2 + (1 - 4\mu_f)|g_A^{Zf}(m_Z^2)|^2 \right) \times (1 + \delta_{QED}^f)(1 + \frac{N_f - 1}{2}\delta_{QCD}^f),
\]

where \( N_f = 1 \ (3), \) for \( f = l (q), \) is the color factor, \( \Gamma_0 = \alpha m_Z/3, \mu_f = m_f^2/m_Z^2, g_{V,A}^{Zf} \) are the effective coupling constants, \( \Pi^Z \) is the \( Z \) wave function renormalization contribution, and the QED and QCD corrections are given by \([5]\)

\[
\delta_{QED}^f = \frac{3\alpha Q_f^2}{4\pi},
\]

\[
\delta_{QCD}^f = \frac{\alpha_s(m_Z^2)}{\pi} + 1.405 \left( \frac{\alpha_s(m_Z^2)}{\pi} \right)^2 - 12.8 \left( \frac{\alpha_s(m_Z^2)}{\pi} \right)^3 - \frac{Q_f^2\alpha\alpha_s(m_Z^2)}{4\pi^2}.
\]

We will now proceed to discuss the most interesting rare \( Z \) decays.

2 Single–photon decays

The single–photon decays \( Z \to X + \gamma \), where \( X \) stands for any neutral, invisible state, has played a privileged role in our quest for new physics beyond the SM. The L3 and DELPHI Collaborations searched for energetic single–photon events near the \( Z \) pole at the CERN LEP collider and set the bound \([6]\)

\[
\text{BR}(Z \to \bar{\nu}\nu\gamma) \leq 10^{-6}.
\]

In the SM this decay is negligibly small and receives contributions from the Feynman diagrams shown in Fig. 2. It has been found \([7]\) that the main contribution comes from a \( U_e(1) \) gauge structure induced by the neutrino magnetic dipole transition (Fig. 2b–2c) and the box diagrams (Fig. 2d–2i). The calculation was performed in a nonlinear \( R_\xi \)–gauge, and the result obtained for the branching ratio is \([7]\)
Figure 1: Feynman diagrams contributing to the decay $Z \rightarrow \bar{\nu}\nu\gamma$ in the SM [7]. Crossed diagrams must be added.

$$BR_{SM}(Z \rightarrow \bar{\nu}\nu\gamma) = 7.16 \times 10^{-10},$$

which is about four orders of magnitude below the experimental limit [5] and thus it leaves open a window to search for new physics effects in single–photon decays of the $Z$ boson.

In the early days of the Higgs–boson hunting, when the mass limit was well below $m_Z$, the decay mode $Z \rightarrow h\gamma$ was taken as one of the best candidates to discover a light Higgs boson. Its branching ratio in the SM was found somewhat small [8], of the order of $10^{-4}$ for $2/3 m_Z < m_h < m_Z$, but it was widely studied in various extensions of the SM due to its sensitivity to gauge couplings and clean signature. For instance, while in Left–Right (LR) symmetric models this decay mode has essentially the same width as in the SM [9], one could expect an enhancement of about one order of magnitude in supersymmetric models [10] and in effective theories where the tree–level generated bosonic operators of dimension 8 become important [11].

The decays of the $Z$ boson into a single–photon plus a virtual Higgs boson are highly suppressed unless there is a resonant effect for $m_h < m_Z$. Such was the case for a very light axion $A$, with a mass $m_A \sim 1$ MeV, with the resonant sequence $Z \rightarrow A\gamma$, $A \rightarrow \gamma\gamma$ giving a spectacular event with three photons [12]. The $Z A\gamma$ and $A\gamma\gamma$ couplings arise from the effective interaction

$$\mathcal{L}_{\text{Eff.}} = \frac{1}{32\pi^2} A \left( g'^a F^a_{\mu\nu} \tilde{F}^a_{\mu\nu} + e^2 C_{a\gamma\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{2e^2}{s_W c_W} C_{a\gamma Z} Z_{\mu\nu} \tilde{F}^{\mu\nu} \right),$$

where $F^a_{\mu\nu}$, $F_{\mu\nu}$, $Z_{\mu\nu}$ are the field strengths of the gluon, photon and $Z$ boson, respectively. For a variant axion with a decay constant $F_A$ of the order of 10 GeV, this decay mode was estimated to be viable in the LEP–1 run [12]. However, the chance of having such a light axion seems to be ruled out [2].

The possibility of using the rare decay modes $Z \rightarrow h\gamma$, $hh\gamma$, $\bar{\nu}\nu\gamma$, and $\bar{\nu}h$ was also considered in order to test strongly–coupled standard (SCS) models with a light Higgs boson [13]. In this type of models, the $SU_L(2)$ gauge group is not spontaneously broken but instead confining. As a consequence, the left–handed quarks and leptons are fermion–boson bound states and the Higgs and intermediate vector bosons are boson–boson bound states. The above decay modes are induced at the loop level but they may be studied in a model independent way with an effective Lagrangian similar to (7) for the $Zh\gamma$, $Zh'h\gamma$, $Z\bar{\nu}\nu\gamma$, and $Z\nu\bar{\nu}$ couplings. The conclusion of this analysis indicated that the decay modes $Z \rightarrow h\gamma$, $\bar{\nu}\nu\gamma$ were specially suited to test a SCS model with a light Higgs boson [13]. However, this possibility has been excluded since the lower bound on the Higgs boson mass is well above the $Z$ boson mass [13].

In a similar way, single–photon decays were proposed to test the existence of other very light invisible particles predicted in some supersymmetric (SUSY) models: $Z \rightarrow G\bar{Z} \rightarrow GG\gamma$, $Z \rightarrow J\gamma$, $JJ\gamma$, where $G$ is a superlight gravitino $m_{\tilde{G}} \leq 10^{-1}$ eV, $\bar{Z}$ is the lightest neutralino [15], and $J$ is a (nearly) massless pseudoscalar Goldstone boson (a Majoron) which appears in SUSY models with spontaneous violation of $R$–parity [16], or with spontaneous lepton number violation [17]. In all these decay modes, the respective branching ratios were
in principle accessible to the LEP–1 sensitivity. Therefore, the negative search for single–photon events in $Z$ decays \[3\] can be translated into severe constraints on the parameters involved in these new physics effects. Even more, the decay mode into a neutralino and a gravitino is almost excluded by the current limits on the neutralino mass obtained at the Tevatron \[2\].

It was also realized long time ago that the production of single photons at LEP–1 energies constitute a process which is most sensitive to anomalous $ZZ\gamma$ couplings due to the large branching ratio for the $Z \rightarrow \bar{\nu}\nu\gamma$ mode and the absence of background from final state radiation \[18, 19\]. Since the L3 and DELPHI Collaborations found that the level of energetic single–photon events is consistent with what is expected in the SM \[20\], from the limit (5) it is possible to derive upper bounds on the $ZZ\gamma$ coupling. The self couplings of photons and the $Z$ boson constitute the most direct consequence of the $SU(2) \times U(1)$ gauge symmetry.

In the SM they vanish at tree level and one–loop effects are of the order of $10^{-10}$ \[21\]. These couplings have not been measured with good precision \[18, 19, 20\] and any deviation from the SM prediction may be thus associated with physics beyond the SM.

Single photon events coming from $Z$ decays are best analyzed with the machinery of the effective Lagrangian approach (ELA). The Feynman diagrams associated with the effective couplings which may induce the rare decay $Z \rightarrow \bar{\nu}\nu\gamma$ are depicted in Fig. 2. They correspond to the contributions generated by the effective couplings $ZZ^*\gamma$, $\bar{\nu}\nu^*\gamma$ and $\bar{\nu}\nu Z\gamma$. The upper limit \[\delta\] obtained by the L3 and DELPHI Collaborations can be translated in turn into constraints on these couplings. The $ZZ^*\gamma$ effective vertex can be parametrized in terms of four form factors $h_i^Z$. Two of them are CP–conserving and two are CP–violating \[21, 22\].

The $Z \rightarrow \bar{\nu}\nu\gamma$ decay width receives the following contribution from these vertices \[21\]

\[
\Gamma_{\alpha\beta\gamma}(p, q, k) = \frac{ie}{m_Z^2} \left[ h_1^Z (k^\alpha g^{\mu\beta} - k^\beta g^{\mu\alpha}) + \frac{h_2^Z}{m_Z^2} q^\alpha (q \cdot k g^{\beta\mu} - k^\beta q^\mu) \right. \\
+ \left. h_3^Z \epsilon^{\alpha\beta\mu\nu} k_{\nu} + \frac{h_4^Z}{m_Z^2} q^\alpha \epsilon^{\beta\mu\nu\rho} p_\nu q_\rho \right] (q^2 - m_Z^2). \tag{8}
\]

The $Z \rightarrow \bar{\nu}\nu\gamma$ decay width receives the following contribution from these vertices \[21\]

\[
BR(Z \rightarrow \bar{\nu}\nu\gamma) = \frac{2g^2}{c_W s_W} 2.912 \times 10^{-5} (|h_1^Z|^2 + |h_3^Z|^2). \tag{9}
\]

In obtaining this expression, only the CP–conserving terms were considered as the CP–violating ones are expected to be strongly suppressed. The experimental bound \[\delta\] induces then the limits

\[
|h_{1,3}^Z| < 0.38, \tag{10}
\]
which agrees with previous bounds obtained from scattering experiments [20].

The LEP–1 bound [15] on the $Z \to \nu\nu\gamma$ decay has also been used to put a direct limit on the magnetic moment of the $\tau$ neutrino [13, 21, 22, 23, 24]. The study of the neutrino electromagnetic properties have renewed interest since they may play a key role in elucidating the solar neutrino puzzle [26]: it can be explained by a large neutrino magnetic moment in the range $10^{-12} \mu_B$, where $\mu_B$ stands for the Bohr magneton $\mu_B = e/(2m_e)$. In the simplest extension of the SM with massive neutrinos, one–loop radiative corrections generate a magnetic moment proportional to the neutrino mass [27]

$$\mu_\nu = \frac{3G_F e m_\nu}{8\sqrt{2}\pi} = 3.2 \times 10^{-19} \left( \frac{m_\nu}{1 \text{ eV}} \right),$$

which seems to be too small if one uses neutrino masses compatible with the mass square differences needed by atmospheric [28], solar [29], and the LSND data [30].

The transition magnetic moments of Dirac neutrinos can be parametrized through the effective interaction [29]

$$\mathcal{L}_{\nu_i\nu_j\gamma} = \frac{1}{2} \mu_{\nu_i\nu_j} \bar{\nu}_i \gamma \nu_j F^{\mu\nu},$$

where $\mu_{\nu_i\nu_j} = \mu_{\nu_i\nu_j}$ will correspond to the $\nu_i$ (diagonal) magnetic moment. Within the effective Lagrangian framework, the effective $\bar{\nu}\nu\gamma$ interaction will induce a contribution given by the Feynman diagram shown in Fig. 2d. The LEP–1 limit [15] gives the following bound on the $\nu_\tau$ magnetic moment [22, 25]

$$\mu_{\nu_\tau} < 2.62 \times 10^{-6} \mu_B.$$ (13)

This bound is in good agreement with that found by the L3 and DELPHI Collaborations [6]. It compares favorably with the limits $\mu_{\nu_\tau} < 4 \times 10^{-6} \mu_B$ [31] and $\mu_{\nu_\tau} < 2.7 \times 10^{-6} \mu_B$ [22] obtained from low–energy experiments and from the invisible width of the $Z$ boson, respectively. However, all these bounds are still weaker than the experimental bounds $\mu_{\nu_\mu} < 1.1 \times 10^{-10} \mu_B$, $\mu_{\nu_e} < 7.4 \times 10^{-9} \mu_B$ [32], and $\mu_{\nu_\tau} < 5.4 \times 10^{-7} \mu_B$ [34], or the most stringent bound $(0.2 - 0.8) \times 10^{-11} \mu_B$ obtained from chirality flip in the 1987 Supernova explosion and valid for the three neutrino flavors [35].

The LEP–1 limit [15] on $Z \to \nu\nu\gamma$ can also be used to get bounds on the effective coupling $Z\nu\nu\gamma$, which is generated by the dimension–six operators [13, 22]

$$\mathcal{L}_{E ff.} = \frac{\alpha_1}{\Lambda^2} \bar{\nu}_L \gamma^{\alpha\mu} D^\alpha D^\mu W^i_\mu + \frac{\alpha_2}{\Lambda^2} \bar{\nu}_L \gamma^{\alpha\mu} D^\alpha D^\mu B_{i\mu},$$

where $\bar{\nu}_L$ is the left–handed doublet, $W^i_\mu$ and $B_{i\mu}$ are the $SU_L(2)$ and $U_Y(1)$ strength tensors, respectively, and $D^\mu$ is the covariant derivative. The bound obtained for the coefficients of these operators is given by $\epsilon_8 < 0.165$ [22], with the following definition for this dimensionless coupling

$$\epsilon_8 = (\alpha_1 + \alpha_2) \left( \frac{v}{\Lambda^2} \right).$$ (15)

In the SM, the dimension–six operators given in [14] may be induced by the diagrams shown in Fig. 2d. However, this contribution is negligibly small, of the order of $10^{-8}$ [7]. In the strongly–coupled standard model they are induced by similar box diagrams with excited vector bosons or leptoquarks [13]. In this case, the limit $\epsilon_8 < 0.165$ can be used to set bounds on the masses or couplings of these new degrees of freedom. However, this calculation has not been done to our knowledge.

### 3 Two–photon decays

Since the decay $Z \to \gamma\gamma$ is forbidden by Bose symmetry and angular momentum conservation [36], two–photon events may arise from the decay of the $Z$ boson into a photon pair plus a neutrino pair $Z \to \nu\nu\gamma\gamma$, which has been studied in order to constrain new physics effects. The L3 and OPAL Collaborations have looked for events with a photon pair of large invariant mass accompanied by a lepton pair and have put the bound [37]...
The respective decay width for this mode has not been computed in the SM to our knowledge. It is expected to be suppressed with respect to the $Z \to \bar{\nu}\nu\gamma$ decay width [7] by an additional $\alpha$ factor. In the effective Lagrangian formalism, the two–photon decay mode is generated by the Feynman diagrams shown in Fig. 3 [25, 38]. Besides the contributions induced by the neutrino magnetic dipole transition $\nu\bar{\nu}\gamma$ given in Fig. 3a–3c, it is necessary to include the contributions associated with the neutrino–two–photon interaction $\bar{\nu}\nu\gamma\gamma$ (Fig. 3d–3e) and the quartic gauge boson coupling $ZZ\gamma\gamma$ (Fig. 3f).

Neutrino–two–photon interactions may have direct implications on some astrophysical processes such as the cooling of stars by a high annihilation rate of photons into neutrinos [39]. This interaction can be parametrized with the following effective Lagrangian [40]

$$\mathcal{L}_{\text{Eff.}}^{\bar{\nu}\nu\gamma\gamma} = \frac{1}{4\Lambda} \bar{\nu}_i \left( \alpha^{ij}_L P_L + \alpha^{ij}_R P_R \right) \nu_j F^{\mu\nu} F_{\mu\nu},$$

(17)

where $\alpha^{ij}_L$ are dimensionless coupling constants. This interaction induces the following contribution to the $Z \to \bar{\nu}\nu\gamma\gamma$ branching ratio [25]

$$BR(Z \to \bar{\nu}\nu\gamma\gamma) = 1.092 \times 10^3 \sum_i \sum_j \left( |\alpha^{ij}_L|^2 + |\alpha^{ij}_R|^2 \right) \frac{1}{\Lambda^6},$$

(18)

where the sum runs over all neutrino species and $\Lambda$ should be expressed in GeV. The experimental bound given in (16) induces the limit

$$\frac{1}{\Lambda^6} \sum_i \sum_j \left( |\alpha^{ij}_L|^2 + |\alpha^{ij}_R|^2 \right) \leq 2.85 \times 10^{-9},$$

(19)

which in turn can be translated into a lower bound on the lifetime of the neutrino double radiative decay $\nu_i \to \nu_j\gamma\gamma$ [25]

$$\tau_{\nu_i} \geq 1.79 \times 10^{12} \left[ \frac{1\text{ MeV}}{m_{\nu_j}} \right]^7 \text{s.}$$

(20)
This limit is one order of magnitude weaker than that obtained from the analysis of the Primakoff effect on the process $\nu + N \to \nu + N$ in the presence of the external field of a nucleus $N$. Nevertheless, as occurred with the bounds obtained from the $Z \to \bar{\nu}\nu\gamma$ mode, the advantage of the bound on the neutrino–two–photon interaction is that it is a model–independent result and relies on very few assumptions.

The limit (11) on the decay width of $Z \to \bar{\nu}\nu\gamma\gamma$ can also be used to constrain the quartic neutral gauge boson (QNGB) coupling shown in Fig. 3. An interesting feature of the QNGB couplings that involve at least one photon field is that they are induced by effective operators which are not related to the triple neutral gauge boson couplings $V_i V_j V_k$. As a consequence, the known constraints on the latter do not apply to the former and it is thus necessary to study them in an independent way. The lowest dimension operators that induce the $ZZ\gamma\gamma$ coupling have dimension six (eight) in the nonlinear (linear) realization of effective Lagrangians. New physics effects could become more evident thus in the nonlinear scenario. In such case, there are fourteen dimension–six operators that induce the $ZZ\gamma\gamma$ effective coupling. However, in the unitary gauge there are only two independent Lorentz structures for this coupling

$$\mathcal{L}_{\text{Eff}}^{ZZ\gamma\gamma} = -\frac{e^2}{16\Lambda^2 c_W^2} a_0 F_{\mu\nu} F^{\mu\nu} Z^\alpha Z_\alpha - \frac{e^2}{16\Lambda^2 c_W^2} a_c F_{\mu\nu} F^{\mu\alpha} Z^\nu Z_\alpha,$$

which in turn give the following contribution to the decay width

$$\Gamma(Z \to \bar{\nu}\nu\gamma\gamma) = \left(\frac{1\text{ GeV}}{\Lambda}\right)^4 \left(N_{0c} |a_0 + a_c|^2 + N_c |a_c|^2\right) \text{ GeV},$$

with $N_{0c} \approx 3.46 \times 10^{-6}$ and $N_c \approx 10.31 \times 10^{-6}$. If one assumes that either $a_0$ or $a_c$ is dominant, then the following bounds are obtained

$$\frac{|a_0|}{\Lambda^2} \leq 0.106 \text{ GeV}^{-2} \quad \text{if} \quad a_0 \gg a_c,$$

$$\frac{|a_c|}{\Lambda^2} \leq 0.215 \text{ GeV}^{-2} \quad \text{if} \quad a_c \gg a_0.$$

These limits are weaker by about one order of magnitude than those obtained at LEP–2 from $Z\gamma\gamma$ and $WW\gamma$ production.

### 4 Decays with photons and gluons

As already mentioned, the decay of the $Z$ boson into two massless vector particles ($Z \to \gamma\gamma$, $gg$) is forbidden by the Landau–Yang theorem, while the decay $Z \to \gamma g$ is also forbidden by color conservation. On the other hand, the rare decays $Z \to \gamma\gamma\gamma$, $gg\gamma$, $\gamma g g$ can be induced in the SM only at the loop level: the coupling of the $Z$ boson to three gauge vector particles requires an effective interaction of dimension higher than four with three tensor fields $F_{\mu\nu}$ or $F_{\mu\nu}$. In the case of the three–photon decay mode, the fermion and the vector boson contributions are of the order of $10^{-10}$ and $10^{-11}$, respectively, whereas the charged scalar boson contribution is about four orders lower. The branching ratios for decays involving gluons are somewhat higher and $BR(Z \to ggg) \sim 1.8 \times 10^{-5}$ and $BR(Z \to \gamma gg) \sim 4.9 \times 10^{-6}$. In spite of the smallness of these branching ratios, there has been some interest in estimating new physics effects in the rare $Z$ decay modes involving three vector gauge bosons. Furthermore, since the decay amplitude for the three–photon mode is proportional to the cubic of the electric charge of the particles circulating in the box diagrams, it might be possible that the contribution of particles with charge greater than unity, such as doubly charged ones, may induce a dramatic enhancement similar to that expected in $\gamma\gamma$ collisions.

In the ELA there are two independent operators of dimension eight, which are $U(1)$ invariant and CP conserving, inducing the $Z\gamma\gamma\gamma$ coupling:

$$\mathcal{L}_{\text{Eff}}^{Z\gamma\gamma\gamma} = G_1 F^\alpha F^{\alpha\nu} \partial_\rho F_{\mu\nu} F_{\rho\sigma} Z_\sigma + G_2 F^{\alpha\beta} F_{\mu}^{\rho\sigma} \partial_\alpha F_{\nu}^{\rho\sigma} Z_\beta,$$
This interaction induces a decay width given by

\[ \Gamma(Z \to \gamma\gamma) = \frac{m_Z^3}{55960\pi^3} \left( 2G_1^2 + 3G_2^2 - 3G_1G_2 \right). \] (25)

Of course, this general result can be used to get bounds on the \( G_1 \) and \( G_2 \) coupling constants once we have a sensible limit on this decay mode.

The possibility of inducing the two–gauge–bosons decay modes \( Z \to \gamma\gamma, gg \) has been explored in a background magnetic field \([52]\). In principle, these decay modes should be equivalent to the \( Z \to \gamma\gamma \) and \( \gamma gg \) decay channels in vacuum. Accordingly, the respective branching ratios come out of the same order of magnitude \([52]\): \( BR(Z \to \gamma\gamma) \sim 10^{-11}(B/B_o)^2, BR(Z \to gg) \sim 10^{-10}(B/B_o)^2 \), where \( B \) is the strength of the background magnetic field and \( B_o = m_t^2/e \).

## 5 Decays involving scalars

Until now, the only missing ingredient of the SM is the Higgs scalar boson. Even more, in many beyond–the–SM extensions there is the prediction of more than one Higgs boson \([53]\). For instance, the simplest extension of the SM is comprised by two Higgs scalar doublets and predicts five scalar bosons: two CP–even scalar bosons \( h \) and \( H \), one CP–odd scalar boson \( A \), and one pair of charged scalar bosons \( H^\pm \). There has been thus a great deal of interest in studying Higgs boson production from \( Z \) decays. Currently, the nonobservation of \( e^-e^+ \to Z^* \to Zh \) has set the bound \( m_h > 114.7 \) GeV on the SM Higgs boson mass \([14]\). However, it has been argued that there are some theories in which the \( ZZh \) coupling may be largely suppressed, thereby weakening the above bound to a great extent \([54, 55]\). It is thus possible that a light Higgs boson, with a mass \( m_h < m_Z/2 \), may have escaped detection so far via Higgs boson radiation off a \( Z \) boson at LEP–2. On the other hand, it has also been pointed out that the \( ZZhh \) coupling happens to be model independent and unsuppressed \([55]\). In this scenario, there is the chance that some rare decays of the \( Z \) boson into two or three Higgs bosons may be kinematically allowed and at the reach of future colliders \([55, 56]\). Since Bose symmetry forbids the \( Z \to hh \) decay, other modes have to be studied. Among them, the decay modes \( Z \to h_5\lambda_0\lambda_0 \) and \( Z \to h_5^0h_5^0h_5^0 \), with \( h_5^0 \) a very light scalar boson and \( \lambda_0 \) a massless Majoron, have been studied in the framework of a doublet Majoron model \([56]\). It was found that the respective branching ratio may reach the level of \( 10^{-7} \), which is also valid for models with a more exotic Higgs sector. The existence of a very light Higgs scalar would kinematically allow also the rare \( Z \to hhff \) decay, with \( f \) a light fermion, which may occur with a branching ratio of the order of \( 10^{-7} \). This decay mode would be observable especially in models in which the scalar boson \( h \) decays invisibly, such as in some Majoron models \([55]\).

As far as the CP–odd Higgs boson \( A \) is concerned, the current bounds on its mass are model dependent and a light CP–odd scalar is still not ruled out in some specific models \([57, 58]\). Even more, some SM extensions, such as the minimal composite Higgs model \([59]\) or the next–to–minimal supersymmetric standard model \([60]\), do predict a very light CP–odd scalar. Even if such a light particle happens to exist, the rare decay \( Z \to hA \) \([60]\) would not be kinematically allowed for a CP–even Higgs boson whose mass is close to the current lower bound \( m_h > 114.7 \) GeV. Nevertheless, it is still feasible to look for a light \( A \) as the product of other rare \( Z \) decays such as \( Z \to AA \ell \ell \) \([61]\). A situation resembling that discussed above for the CP–even scalar \( h \) arises for the CP–odd scalar: while the \( ZZA \) coupling is absent at the tree level, the \( ZZAA \) coupling is fixed by the gauge invariance of the theory and it is not suppressed. As a consequence, the decay mode \( Z \to AA \ell \ell \) may be feasible for a light CP–odd scalar, with the lepton pair arising from a virtual \( Z \) boson, even though with a small branching ratio of the order of \( 10^{-8} \) \([61]\).

If \( A \) is very light, the \( Z \to AAA \) decay mode would be kinematically allowed \([61, 62, 63]\). In the two–Higgs–doublet model (THDM), this decay may proceed at the tree level by the exchange of the CP even Higgs bosons \( h \) and \( H \) as depicted in Fig. 4. The three level induced \( \phi AA \) coupling, with \( \phi = h \ (H) \), can be written as \([62]\)

\[ \mathcal{L}_{eff}^{\phi AA} = \lambda \phi AA, \] (26)

where \( \lambda \) lies in the Fermi scale in several specific models \([62]\). The contribution of a CP–even Higgs boson \( h \) to the rare \( Z \to AAA \) decay is
and indirectly from the analysis of the bounds on low–energy observables [2, 66].

be produced from $Z$ decays according to the current bounds on $m_{H^\pm}$, obtained from direct searches at LEP–2 and indirectly from the analysis of the bounds on low–energy observables [2, 66].

| Figure 4: Feynman diagrams contributing to the decay $Z \to AAA$ at the tree and one–loop level in the THDM. Charged fermions circulate in the loop. |

\[ \mathcal{M}(Z \to AAA) = \frac{2g}{\cos \theta_W} \lambda \sum_{i=1}^{3} \frac{\epsilon(k_i, \lambda) \cdot k_i}{(k_i - k_1)^2 - m_h^2}. \]  

Assuming a typical value of $\lambda \sim 100$ GeV, it was found that $BR(Z \to AAA) \sim 10^{-5}$ when $m_h \sim m_Z$ and $m_A \ll m_Z$. This branching fraction drops suddenly as $m_h$ becomes heavier than $m_Z$ and $m_A$ approaches $m_Z/3$, the upper limit allowed by the kinematics of the process. There is also the possibility of a large contribution to $Z \to AAA$ coming from loop diagrams [57, 62]. At the one–loop level (Fig. 4b), this decay is induced by fermion loops whose contribution to the decay width is proportional to $m_f^3 C_f^3$, where $C_f$ is the strength of the $Aff$ coupling. It might be that $C_f$ were so large that this enhancement factor would overcome the natural suppression factor coming from the loop. In this case the dominant contributions are those from the $b$ and $t$ quarks. The effective $Aqq$ coupling can be written as

\[ \mathcal{L}_{\text{Eff.}}^{Aqq} = - \frac{q}{2m_W} \sum_q m_q C_q \bar{u}_q \gamma^5 u_q A, \]  

In THDMs type I, $C_q = \tan \beta \left( \cot \beta \right)$ for up (down) quarks, whereas in THDMs type II $C_q = \cot \beta$ for any quark. The one–loop contribution was roughly estimated in Ref. [62] and the exact calculation was presented in the appendix of Ref. [57]. In the $m_A \to 0$ limit, it was found that

\[ BR(Z \to AAA) = 1.3 \times 10^{-18} C_t^6 + 2.47 \times 10^{-17} C_b^6 + 7.63 \times 10^{-18} C_t^3 C_b^3. \]  

It turns out that the $b$ contribution is larger than that of the $t$ quark as long as $C_b > C_t$. However, because of unitarity, $C_b$ cannot be arbitrarily large. Furthermore, requiring the validity of perturbation theory yields the bound $C_b < 120$. In this limit, $BR(Z \to AAA) \sim 10^{-5}$ [57]. To assess the possibility of observing this decay mode, a more realistic analysis is indeed required. In particular, the parameters of the THDM should be constrained from the current low–energy data on several observables, such as the $\rho$ parameter, $BR(b \to s\gamma)$, $R_b$, $A_b$, $BR(\Psi \to A\gamma)$ and $(g - 2)$ of $\mu$. Such an analysis was presented in Refs. [57, 58] and [64]. It was found that the low–energy data still leave open a small window for the existence of a CP–odd scalar $A$ with a mass as light as $m_A < 0.2$ GeV [57]. Unfortunately, the remaining parameters of the model are tightly constrained. In this scenario, the triple pseudoscalar decay $Z \to AAA$ may occur with a branching ratio of the order of $10^{-8}$ [57, 62, 63], arising mainly from the three–level contribution. The signature of this decay would be spectacular: each one of the three CP–odd scalars $A$ will decay predominately into a photon pair, which in turn will be registered in the detectors of high energy colliders as a single photon when the momentum of $A$ is much larger than its mass [63].

Finally, as far as the charged Higgs scalar is concerned, it seems that there is no way that this particle can be produced from $Z$ decays according to the current bounds on $m_{H^\pm}$, obtained from direct searches at LEP–2 and indirectly from the analysis of the bounds on low–energy observables [2, 66].
6 Flavor changing decays

Since lepton flavor violation (LFV) is forbidden in the SM, the rare $Z \to l_i^\pm l_j^\mp$ decays, with $l_i = e, \mu$ and $\tau$, have been widely studied as the detection of any effect of this kind would serve as an indisputable evidence of new physics. Furthermore, the possibility that neutrinos have nonzero mass \([28, 29, 30]\), which in turn would have been widely studied as the detection of any effect of this kind would serve as an indisputable evidence of new physics effects induced by other SM extensions. The current experimental bounds on the LFV $Z$ decays were obtained at the CERN LEP–1 collider \([2]\):

$$
\begin{align*}
BR(Z \to e^\mp \mu^\pm) &< 1.7 \times 10^{-6}, \\
BR(Z \to e^\mp \tau^\pm) &< 9.8 \times 10^{-6}, \\
BR(Z \to \mu^\mp \tau^\pm) &< 1.2 \times 10^{-5}.
\end{align*}
$$

(30)

A plenty of work has been done in the past to analyze LFV $Z$ decays, which have been approached in two different ways: model–independent analyses and predictions from specific extensions of the SM. In the former case, the starting point is the effective Lagrangian which leads to the most general structure for the $Zl_il_j$ effective vertex \([68, 69]\):

$$
M^{Zl_il_j} = \frac{ig}{2c_W} \bar{u}(p_i) \left( \gamma_\mu \left( F_{1L}^i P_L + F_{1R}^{ij} P_R \right) + \frac{i}{m_Z} F_{3R}^{ij} P_R \sigma_{\mu\nu} k^\nu \right) v(p_j) Z^\mu,
$$

(31)

We have dropped the $k_\mu$ term from Eq. (31) as it does not contribute when the $Z$ boson is on its mass–shell. In the ELA, the monopole and dipole moment contributions can be generated by the following effective operators \([69]\):

$$
\begin{align*}
O^{ij}_{\phi} &= i \left( \phi^\dagger D_{\mu} \phi \right) \left( \bar{\ell}_{Ri} \gamma^\mu \ell_{Rj} \right), \\
O^{(1)ij}_{\phi L} &= i \left( \phi^\dagger D_{\mu} \phi \right) \left( \bar{L}_i \gamma^\mu L_j \right), \\
O^{(3)ij}_{\phi L} &= i \left( \phi^\dagger \tau^a D_{\mu} \phi \right) \left( \bar{L}_i \tau^a \gamma^\mu L_j \right),
\end{align*}
$$

(32)

$$
\begin{align*}
O^{ij}_{\phi W} &= g \left( \bar{L}_i \sigma_{\mu\nu} W^{\mu\nu} \ell_{Rj} \right) \phi, \\
O^{ij}_{\phi B} &= g' \left( \bar{L}_i \sigma_{\mu\nu} B^{\mu\nu} \ell_{Rj} \right) \phi.
\end{align*}
$$

(33)

where $L_i$ and $\ell_{Ri}$ stand for the left–handed doublet and right–handed singlet of the $SU(2) \times U(1)$ gauge group, respectively. The monopole moment structures $F_{1L,R}$ arise from the operators \([32]\), which in turn are generated at the tree level in the underlying theory, while the dipole moment structure $F_{3R}$ is induced by the operators \([33]\). The latter can arise only at the one–loop level in the underlying theory \([70]\) and their contribution has an additional suppression factor of the order of $(4\pi)^{-1}$. It is thus a good approximation to consider only the contribution arising from the operators \([32]\):

$$
BR(Z \to l_i^\pm l_j^\mp) = \frac{\alpha}{3s^2_{2W}} \left( \frac{m_Z}{\Gamma_Z} \right) \left( \left| F_{1L}^{ij} \right|^2 + \left| F_{1R}^{ij} \right|^2 \right).
$$

(34)

The effective coupling given in Eq. (31) induces also a contribution to the LFV decays $l_i \to l_j l_k l_k$. Using the known experimental limits on $BR(l_i \to l_j l_k l_k)$ \([2]\), the following bounds on LFV $Z$ decays are thus obtained \([68, 69]\):
where the quantities given in the right-hand side of (35) correspond to the limits obtained from unitarity-inspired arguments [85], while the ones in parenthesis are obtained from the analysis performed in the effective Lagrangian approach [69]. Along this line, it is interesting to note that the analysis of $\mu \rightarrow e$ conversion in a nuclear field leads to indirect bounds on the branching ratios of the LFV decay $Z \rightarrow l_i^\pm l_j^\mp$, which are in agreement with those shown in Eq. [65] [71].

LFV decay modes of the Z boson have also been studied in a wide variety of extensions of the SM, from which we would like to mention some of the more representative: left–right symmetric models [72], SUSY [73], left–right supersymmetric models [74], the Zee model [75], theories with a heavy $Z'$ boson [76], the SM enlarged with massive neutrinos, [77], universal top color assisted technicolor models [77], and the general two Higgs doublet model (THDM–III) [78]. For a more comprehensive list of the literature dealing with this topic, the reader is referred to Ref. [79]. It turns out that one of the more promising scenarios for LFV Z decays is that favored by the type–III THDM, in which the $Z$ favored by the type–III THDM, in which the $Z$ exchange of a virtual Higgs boson with tree-level LFV couplings $Zqq'$ is possible.

The study of virtual effects induced by new physics in rare $Z$ decays provides an important opportunity to probe the presence of interactions beyond the SM. In the present review we have appreciated that there is

$$
\begin{align*}
BR (Z \rightarrow \mu^\pm e^\pm) & \leq 5.0 \times 10^{-13} (1.04 \times 10^{-12}), \\
BR (Z \rightarrow \tau^\pm e^\pm) & \leq 3.0 \times 10^{-6} (1.7 \times 10^{-5}), \\
BR (Z \rightarrow \tau^\pm \mu^\pm) & \leq 3.0 \times 10^{-6} (1.0 \times 10^{-5}),
\end{align*}
$$

where the quantities given in the right-hand side of (35) correspond to the limits obtained from unitarity-inspired arguments [85], while the ones in parenthesis are obtained from the analysis performed in the effective Lagrangian approach [69]. Along this line, it is interesting to note that the analysis of $\mu \rightarrow e$ conversion in a nuclear field leads to indirect bounds on the branching ratios of the LFV decay $Z \rightarrow l_i^\pm l_j^\mp$, which are in agreement with those shown in Eq. [65] [71].

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Although FCNC effects in the quark sector have been extensively studied through the processes $b \rightarrow s\gamma$ [80], and $t \rightarrow c\gamma$ [81], rare FCNC Z decays are also good candidates to look for any new physics effects and have been the source of great interest recently. Although this class of transitions are only forbidden at the three level and can arise at the one–loop level, the GIM mechanism suppresses them effectively. In the SM, the one–loop induced FCNC $Zqq'$ coupling was calculated first in the context of the $K_L \rightarrow \mu\nu$ decay and in the limit of massless external quarks [80]. The calculation was later generalized to the case with massive internal and external up and down quarks [81]. Afterwards, the effects of a fourth fermion family [82] and the possibility of CP violation [83] in the $Z \rightarrow qq'$ decay were also examined. As for the dominant decay channel $Z \rightarrow bs^1$, the respective branching ratio is $BR(Z \rightarrow bs) \sim 3 \times 10^{-8}$ in the SM [84]. This decay mode has been studied also in several extensions of the SM: THDM type II [85] and type III [86]. SUSY models [87], and SUSY models with broken $R$–parity [88]. The predictions for $BR(Z \rightarrow bs)$ in these models happen to be very small and this rare $Z$ decay seems beyond the reach of the future colliders. However, quite recently, various scenarios have been considered in SUSY models with flavor violation in the scalar sector [89]. In this case it was found that $BR(Z \rightarrow bs)$ can reach $10^{-6}$ in SUSY models with mixing between the bottom and strange–type squarks and/or mixings between sleptons and Higgs fields for large tan $\beta$ values. A similar conclusion was reached in the context of topcolor–assisted technicolor models [90], where it was found that the contribution coming from top–pions can reach $BR(Z \rightarrow bs) \sim 10^{-5}$. For other works on the rare $Z \rightarrow qq'$ decay, we refer the reader to Ref. [91].

It is interesting to notice that any FCNC effect at the level of $BR(Z \rightarrow l_i l_j) \sim 10^{-10}$–$10^{-8}$ or $BR(Z \rightarrow bs) \sim 10^{-7}$–$10^{-6}$ would be at the reach of the expected sensitivity of the giga–Z linear collider [3]. While the prediction for LFV Z decays is expected to reach the giga–Z experimental upper limit [78] in models such as the THDM type III, with $BR(Z \rightarrow l_i^\pm l_j^\mp) \sim 10^{-11}$–$10^{-10}$, one can get at most $BR(Z \rightarrow bs) \sim 10^{-8}$ in the same model with the current experimental constraints, which in turn will be out of the reach of the giga–Z linear collider [88] [91].

7 Concluding remarks

The study of virtual effects induced by new physics in rare Z decays provides an important opportunity to probe the presence of interactions beyond the SM. In the present review we have appreciated that there is

1Unless stated otherwise, $Z \rightarrow bs$ stands for $Z \rightarrow bs + bs$. 

a complementary approach between the results obtained within the framework of radiative corrections to perturbatively calculable processes in the SM and transitions which are either suppressed or forbidden in the SM. The more promising situations arise when SM predictions are well below the expectations coming from new physics effects. A summary of the rare $Z$ decay modes considered in this article is presented in Table 1. We have included the existing experimental bounds, the respective SM predictions and the main new physics effects that may be tested with the expected sensitivity of the giga–$Z$ linear collider [3].

Table 1: Summary of rare $Z$ decay modes. References to specific results appear in brackets. $\Delta^{\pm\pm}$ stands for a doubly charged particle. We only show those new physics effects that appear to be at the reach of the giga–$Z$ linear collider.

| Decay mode | Experimental bound (BR) | SM prediction (BR) | New Physics effects |
|------------|-------------------------|--------------------|---------------------|
| $Z \to \bar{\nu}\nu\gamma$ | $1.0 \times 10^{-6}$ | $7.1 \times 10^{-10}$ | $ZZ\gamma$ |
| $Z \to \bar{\nu}\nu\gamma\gamma$ | $3.1 \times 10^{-8}$ | | $Z\bar{Z}\gamma\gamma$ |
| $Z \to \gamma\gamma\gamma$ | $1.3 \times 10^{-5}$ | $1.0 \times 10^{-10}$ | Light $A$ |
| $Z \to ggg$ | $1.8 \times 10^{-2}$ | $4.9 \times 10^{-6}$ | | |
| $Z \to AAA$ | | | | |
| $Z \to e^+\mu^+$ | $1.7 \times 10^{-6}$ | 0 | THDM–III |
| $Z \to e^+\tau^+$ | $9.8 \times 10^{-6}$ | 0 | |
| $Z \to \mu^+\tau^+$ | $1.2 \times 10^{-5}$ | 0 | |
| $Z \to bs$ | $3.0 \times 10^{-8}$ | | SUSY |

We would like to close by stating that even in case that no new physics effects were discovered in the planned giga–$Z$ linear collider, an improvement in the known experimental bounds on these processes will still provide a critical test of the validity of the SM at the loop level.

Acknowledgments

We would like to thank discussions with E. Ma, C.-P. Yuan and F. Larios. Support from CONACyT and SNI (Mexico) is also acknowledged. The work of G. T. V. is also supported by SEP-PROMEP.

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