A survey of phenomenological constraints on supergravity models

JORGE L. LOPEZ
Department of Physics, Texas A&M University
College Station, TX 77843–4242, USA
E-mail: lopez@phys.tamu.edu

Abstract

We advocate the study of supergravity models as well motivated few-parameter low-energy supersymmetric models. In this context we survey a broad range of phenomenological constraints and future tests, including present and near-future Tevatron ($\tilde{q}, \tilde{g}, \chi^\pm_1, \tilde{t}_1$) and LEP ($h, \chi^{\pm}_1$) mass limits, collider indirect tests ($\Gamma^{\text{inv}}_Z, R_b, m_t$), rare processes ($b \to s\gamma$, $(g-2)_\mu$), proton decay, and dark matter (cosmology, direct and indirect detection).

1 Why supergravity models?

The most general "minimal" low-energy supersymmetric model, i.e., the Minimal Supersymmetric extension of the Standard Model (MSSM) has more than 20 free parameters, and therefore experimental tests and constraints are impractical to implement. One needs further theoretical input to test/constrain "sensible" models of low-energy supersymmetry. One promising avenue consists of invoking models for physics at very high energies (for a recent review see [1]):

- Grand unification, provides gauge and Yukawa coupling relations, as well as gaugino mass relations.
- Supergravity, allows the calculation of the soft supersymmetry breaking parameters in terms of the Kähler function ($K$) and the gauge kinetic function ($f$). These constraints lead to four-parameter models.
- Superstrings, provide specific forms for $K$ and $f$, and in principle reduce the number of free parameters to none.

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We can consider a “generic” supergravity model described by the MSSM matter content, gauge coupling unification at $\sim 10^{16}$ GeV, a universal gaugino mass ($m_{1/2}$), a universal scalar mass ($m_0$), a universal scalar coupling ($A$), and at low energies tan $\beta$. The radiative electroweak breaking mechanism allows the determination of $\mu$, $B$, and the parameter space is four dimensional (plus the sign of $\mu$). Further constraints can reduce the dimension of the parameter space, like Yukawa coupling unification which determines tan $\beta$ for a given value of $m_t$.

More “modern” features have been recently incorporated in the study of supergravity models, such as string unification at $\sim 10^{18}$ GeV, simple string models (e.g., $SU(5) \times U(1)$), and calculable values of the ratios $\xi_0 = m_0/m_{1/2}$ and $\xi_A = A/m_{1/2}$. In this case the four-parameter models become two-parameter models. Moreover, in specific string models one can also calculate the ratios of $\mu$ and $B$ to $m_{1/2}$, which lead to zero-parameter models! It is also found that the soft supersymmetry breaking terms are not necessarily universal.

At this point one can say that “generic” supergravity models provide a sensible “standard” parametrization for comparisons of various tests/constraints. However, “modern” ingredients have subtle effects in the sparticle spectrum, which sometimes produce new twists not present in “generic” models. In fact, string model-building will likely soon provide a new “standard” parametrization, with novel/testable effects.

2 Phenomenological constraints

We do not consider theoretical constraints (e.g., gauge/Yukawa unification, doublet-triplet splitting, fixed points, etc.). With few-parameter models in hand, one can run the whole gamut of tests and produce an array of experimental predictions. **Caution:** hidden assumptions make some constraints not as strong as others (e.g., detection of dark matter assumes there is enough in the halo to be detectable). Also, some constraints/tests are sensitive to a small subset of sparticle spectrum, whereas others test the “scale of supersymmetry”. As expected, the largest effects occur for the lightest sparticle masses, but the “reach” depends on the process and its experimental sensitivity. We consider the following **not exhaustive** list of constraints and tests:

- Collider mass bounds: LEP ($\chi^{\pm}_1, h, \tilde{l}, \tilde{t}_1$) and Tevatron ($\tilde{q}, \tilde{g}, \chi^{\pm}_1, \tilde{t}_1$).
- Collider indirect tests: LEP ($\Gamma^{\text{inv}}_Z$, $R_b$, global fits) and Tevatron ($t \to X$).
- Rare processes: CLEO ($b \to s\gamma$) and Brookhaven ($((g-2)_\mu$)
- Proton decay: SuperKamiokande ($p \to K^+\bar{\nu}$).
- Dark matter: Cosmology (age and structure formation), direct detection in cryogenic detectors, and indirect detection in neutrino telescopes.
2.1 Collider mass bounds

2.1.1 LEPI

As is well known, all sparticles that couple to the $Z$ with unsuppressed strength must have masses $\gtrsim \frac{1}{2} M_Z$. This is the case for $\chi_{1}^{\pm}, \tilde{t}_\text{R}, \tilde{\nu}, \tilde{t}_1$. The Higgs boson is produced in the process $e^+e^- \rightarrow Z \rightarrow Z^*h, \ h \rightarrow 2j$, where $\sigma_{\text{SUSY}} = \sin^2(\alpha - \beta)\sigma_{\text{SM}}$, and $\sin^2(\alpha - \beta)_{\text{SUGRA}} \approx 1$. The latter is a consequence of the supersymmetric decoupling phenomenon which is transmitted to the Higgs sector via the radiative electroweak breaking mechanism [2]. Also, for LEPI accessible mass scales, $B(H \rightarrow j j)^{\text{SM}} \approx B(h \rightarrow j j)^{\text{SUSY}}$. These results imply $m_h \gtrsim 62$ GeV. (In supergravity models $m_A > m_h$, and thus the $hA$ mode is not accessible at LEPI.)

2.1.2 LEPII

- Gauginos: The mass relation $m_{\chi_{1}^{\pm}} \approx m_{\chi_{1}^0} \approx 2m_{\tilde{t}_1}$ holds to various degrees of approximation in this class of models. Charginos would be explored via $e^+e^- \rightarrow \chi_{1}^{\pm}\chi_{1}^- \rightarrow 1l + 2j$ (“mixed” mode) and, if the branching ratios are not suppressed (this does happen though), the reach should be $m_{\chi_{1}^{\pm}} \approx \frac{1}{2}\sqrt{s}$, as Fig. 1 shows [3]. The mode $e^+e^- \rightarrow \chi_{1}^{0}\chi_{2}^{0}, \chi_{2}^{0}\chi_{2}^0, \chi_{2}^{0} \rightarrow \chi_{1}^{0} + 2j$ has a smaller cross section but a larger kinematical reach $m_{\chi_{1}^{\pm}} \approx \frac{1}{3}\sqrt{s}$ [4], and should be rather background free.

- sleptons: The processes of interest are $e^+e^- \rightarrow \tilde{e}_{R,L}^\pm \tilde{\nu}_{R,L}, \tilde{\mu}_{R,L}^\pm \tilde{\nu}_{R,L}, \tilde{\tau}_{1,2} \rightarrow 2l$, where $\tilde{e}_{R,L}, \tilde{\mu}_{R,L}, \tilde{\tau}_{1}$ are lightest and decay nearly $\sim 100\%$ to $\tilde{t}_{1}^{0}$. The irreducible background $\sigma(WW \rightarrow 2l) = 0.9 \text{ pb}$ limits the reach somewhat, as Fig. 1 shows [3].

- Higgs: $e^+e^- \rightarrow Zh, h \rightarrow \tilde{b}\tilde{b}$, the novelty is that $h \rightarrow \chi_{1}^{0}\chi_{1}^0$ may spoil the $bb$ signal (for $m_h > 2m_{\chi_{1}} \approx m_{\chi_{1}^{\pm}}$) [3]. The reach is estimated to be $m_h \approx \sqrt{s} - 95$ [3].

2.1.3 Tevatron

- Missing $E_T$: $p\bar{p} \rightarrow \tilde{g}\bar{g}, \tilde{q}\bar{q} \rightarrow E_T + l's + j's$ is near the “kinematical” limit. The latest bounds are $m_\tilde{q} \sim m_{\tilde{g}} \sim 200 \text{ GeV}$ [3]. $\text{Caution:}$ limits apply only to the specific choice of supersymmetry parameters used in the analysis.

- dileptons/trileptons: estimated sensitivities for Run-IB/Main-Injector [4, 8]:

| process | $100 \text{ pb}^{-1}$ | $1 \text{ fb}^{-1}$ |
|---------|-----------------|-----------------|
| $p\bar{p} \rightarrow \chi_{1}^{0}\chi_{1}^0 \rightarrow 3l$ | 0.4 pb | 0.07 pb |
| $p\bar{p} \rightarrow \chi_{1}^{+}\chi_{1}^- \rightarrow 2l$ | 1 pb | 0.3 pb |

For light sleptons, $B(\chi_{1}^{\pm} \rightarrow l^\pm)$ may be enhanced but at the same time $B(\chi_{1}^{0} \rightarrow 2l)$ may be suppressed [8]. When either $\chi_{2}^{0} \rightarrow \chi_{1}^{0}Z, \chi_{1}^{0}h$ are allowed, the trilepton signal becomes unobservable. As Fig. 2 shows [10], the two signals can have an important complementary role when the neutralino branching fraction is suppressed. In $100 \text{ pb}^{-1}$ (1 $\text{ fb}^{-1}$) the reach in chargino masses using both signals can be as high as 100 (150) GeV.

- Light top-squark: The process $p\bar{p} \rightarrow \tilde{t}_1\tilde{t}_1 \rightarrow (\chi_{1}^{\pm}b)(\chi_{1}^{\pm}\bar{b}) \rightarrow 2l + 2j$ has an estimated sensitivity of $m_{\tilde{t}_1} \sim (100 - 130) \text{ GeV}$ (in 100 $\text{ pb}^{-1}$) [11].
Figure 1: Chargino and slepton production at LEP II in one-parameter $SU(5) \times U(1)$ supergravity. The dashed lines indicate the expected experimental sensitivity.

Figure 2: Dilepton and trilepton production at the Tevatron in one-parameter $SU(5) \times U(1)$ supergravity. The upper (lower) dashed line represents the estimated sensitivity for $100 \text{ pb}^{-1}$ ($1 \text{ fb}^{-1}$). Note the complementary nature of the two signals.
2.2 Collider indirect tests

- Determination of $B(Z \rightarrow \chi_1^0 \chi_2^0) \lesssim 10^{-4}$ at LEP constrains the $\chi_1^0, \chi_2^0$ masses. Using the relation $m_{\chi_1^\pm} \approx m_{\chi_2^0} \approx 2m_{\chi_1^0}$, we find that $m_{\chi_1^\pm} \gtrsim 50 \text{ GeV}$ is required, unless $\tan \beta \lesssim 2$, in which case the $Z\chi_1^0\chi_2^0$ coupling is very small.

- Global fits to the electroweak data yield $m_t = 160^{+11}_{-12,-12}$ GeV in the context of supersymmetric models with a light Higgs boson ($m_h = (60-150)$ GeV gives the second error). This program may constrain the supersymmetric sector too [12].

- The averaged LEP measurement of $R_b = \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$ is about $2\sigma$ higher than the Standard Model prediction [13]. Moreover, supergravity-like contributions to $R_{b_{\text{susy}}}$ make the total prediction for $R_b$ only slightly closer to the LEP result [14]. However, a glance at the measurements that are being averaged shows that this apparent discrepancy may eventually disappear.

- Measurement of $B(t \rightarrow \tilde{t}_1 \chi_1^0)$ at the Tevatron would not be as good as direct $\tilde{t}_1$ detection [3].

2.3 Rare processes

2.3.1 $b \rightarrow s\gamma$

This process has been observed at CLEO: $B(b \rightarrow s\gamma) = (1 - 4) \times 10^{-4}$ at 95% CL. There are various dominant contributions written schematically as

$$B(b \rightarrow s\gamma)_{\text{SUSY}} \propto \left[ -|A_{\text{SM}}| - |A_{H^\pm}| \pm |A_{\chi^\pm}| - |C| \right]^2$$

Because of the chargino contribution, $B(b \rightarrow s\gamma)_{\text{SUSY}}$ can be larger or smaller than the Standard Model prediction. QCD corrections are large, and a complete two-loop calculation is required to settle the scale ($Q$) uncertainty, which can be estimated by varying the scale $Q : \frac{1}{2}m_b \rightarrow 2m_b$. Taking the theoretical calculations at face value, large values of $\tan \beta$ appear disfavored, at least for one sign of $\mu$ [15].

2.3.2 $(g - 2)_\mu$

The last measurement of $a_\mu = \frac{1}{2}(g - 2)_\mu$, i.e., $a_\mu^{\text{exp}} = 1165923 (8.5) \times 10^{-9}$ dates back to 1970. The latest Standard Model prediction is $a_\mu^{\text{SM}} = 1165919.20 (1.76) \times 10^{-9}$. Thus, at the 95% CL we can tolerate a new contribution in the range $-13.2 \times 10^{-9} < a_\mu^{\text{susy}} < 20.8 \times 10^{-9}$. In 1996 the new Brookhaven E821 experiment will start running with an expected sensitivity of $0.4 \times 10^{-9}$, which is adequate to observe the electroweak contribution ($\sim 2 \times 10^{-9}$). A difficulty in this test is that the hadronic uncertainty in the Standard Model prediction is of a similar magnitude, although experiments at Novosibirsk should reduce this uncertainty considerably. Such impasse should be of no consequence to tests of supersymmetric models, since their predictions can easily exceed the present allowed interval [16].
2.4 Proton decay

The prediction for the proton lifetime depends strongly on the GUT group. In the case of $SU(5)$, dimension-six operators are mediated by $X,Y$ gauge bosons and the largest mode is $p \to e^+\pi^0$, with $\tau_p \sim 3.3 \times 10^{35} (M_U/10^{16})^4$, which is basically unobservable, although it implies $M_U \gtrsim 10^{15}$ GeV. More important are the dimension-five operators, which need to be dressed by a chargino loop. The largest contribution comes from CKM mixing with the second generation, and the largest mode is $p \to \bar{\nu}_{\mu,\tau} K^+$. Schematically one has

$$\tau(p \to \bar{\nu}_{\mu,\tau} K^+) \sim \left| M_H \sin 2\beta \frac{1}{f} \frac{1}{1 + y^K} \right|^2$$

where $M_H$ is the Higgs triplet mass, $f$ is the one-loop dressing function ($f \sim m_{\tilde{q}}/m_{\tilde{q}}$, i.e., heavy squarks, light charginos are preferred), and $1 + y^K$ gives the ratio of 3rd/2nd generation contributions to the dressing. Requiring $m_{\tilde{q},\tilde{g}} \lesssim 1$ TeV, one finds for $M_H < 3M_U (10M_U)$: $\tan \beta \lesssim 6$ (10) and $\xi_0 \gtrsim 6$ (4) [17]. Moreover, the relic density cosmological constraint requires $m_{\tilde{q}} \lesssim 120$ GeV, since one needs to be near the $\chi \chi \to h, Z \to f \bar{f}$ annihilation poles [18].

2.5 Dark matter

2.5.1 Cosmology

Conservation of $R$-parity in supergravity models implies the existence of a stable lightest supersymmetric particle (LSP), which corresponds to the lightest neutralino ($\chi$). The relic abundance $\Omega_\chi h^2$ can be computed from the pair-annihilation amplitude, and depends on all supersymmetry parameters, with more accurate methods needed near $s$-channel poles. If one requires a Universe at least 10 Gyr old, one must demand $\Omega_\chi h^2 < 1$ (in a pure cold dark matter universe). One can also consider various structure formation models, such as the cold plus hot dark matter model (CHDM) with $\Omega_\chi \sim 0.7$ and $\Omega_\gamma \sim 0.3$, or the cosmological constant model (CC) with $\Omega_\gamma \sim 0.2$ and $\Omega_\Lambda \sim 0.8$. With a given value for the Hubble parameter, such as $h = 0.80 \pm 0.17$ from the Hubble Space Telescope, one can obtain ranges for $\Omega_\chi h^2$ which can be contrasted with the predictions of specific supergravity models (see Fig. 3 [3]).

2.5.2 Direct detection: cryogenic detectors

Neutralino scattering off nuclei in cryogenic detectors is a promising avenue to detect the LSP, especially in view of the new generation of detectors coming on line (the Ge detector at Stanford) or in development stage [19]. It is important to keep in mind that since $\Omega_{\text{halo}} \gtrsim 0.1$, one requires $\Omega_\chi h^2 \gtrsim 0.05$ to get the “full load” of LSPs, otherwise the neutralino halo density should be scaled down. In Fig. 3 we show typical rates ($R$) for the Ge detector [20], where the depletion of LSPs near the $Z$ and $h$ poles
Figure 3: Relic abundance of neutralinos in one-parameter $SU(5) \times U(1)$ supergravity. Needed amounts in two cosmological models (CHDM and CC) can discriminate between supergravity models. Also shown the direct detection rates in the Ge detector. The dashed lines indicate expected near- and long-term sensitivities.

is evident. We also note that experimentally excluded values for $B(b \to s\gamma)$ happen sometimes for interestingly large values of $R$ [21].

2.5.3 Indirect detection: Neutrino telescopes

Neutralinos can also be captured by the Sun or Earth and then pair annihilate producing eventually high-energy neutrinos which can be detected in a growing number of underground/water/ice detectors such as Kamiokande, MACRO, Dumand, Amanda, Nestor. This detection technique is competitive with the direct one [22].

3 Conclusions

Given a few-parameter supergravity model one can combine all the above constraints to determine the still-allowed region of parameter space to be further explored at the Tevatron, LEPII, and the LHC, or through the rare processes and indirect detection methods surveyed above [23, 3]. We conclude by remarking that supergravity models are well motivated few-parameter low-energy supersymmetric models, in which experimental constraints can be enforced and testable predictions can be worked out in detail. Experimentalists like them because they are “easy targets” to kill or severely damage. The procedure described above will need to be applied to the better motivated models for the soft breaking parameters which are just emerging from superstring model building. A lot of surprises may be in store.
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