QCD chiral symmetry restoration with a large number of quarks in a model with a confining propagator and dynamically massive gluons

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Considering a QCD chiral symmetry breaking model where the gap equation contains an effective confining propagator and a dressed gluon propagator with a dynamically generated mass, we verify that the chiral symmetry is restored for a large number of quarks $n_f \approx 11 - 13$. We discuss the uncertainty in the results, that is related to the determination of the string tension ($K_F$), appearing in the confining propagator, and the effective gluon mass ($m_g$) at large $n_f$. 
Chiral symmetry breaking (csb) is one of the main QCD characteristics. It is well understood from the phenomenological point of view; where a quark condensate $\langle \bar{q}q \rangle$ has a vacuum expectation value at the scale of few hundred MeV, and a dynamical quark mass is generated at the same scale. Therefore, the chiral symmetry is broken and the (almost)massless (pseudo)Goldstone pions are generated. The knowledge of the csb mechanism in strongly interacting gauge theories will be complete when we know how this symmetry is broken or restored as we vary the number of fermions, the fermionic representations, the temperature or the chemical potential, and how this symmetry is related to confinement.

In the last years lattice simulations have contributed considerably to the understanding of a possible connection between csb and confinement, where center vortices play a fundamental role. In the $SU(2)$ case the artificial center vortices removal also implies a recovery of the chiral symmetry [1–3]. The relation between vortices and csb is discussed at length in Ref.[4], and follows old proposals that confinement and csb are intimately connected [5–7]. There are also lattice results indicating no direct one-to-one correspondence between confinement and csb in QCD [8]. Although these results appear to be in conflict, they may not be fully contradictory according to the ideas first delineated by Cornwall in Ref. [9], where it was observed that confinement is necessary for csb when quarks are in the fundamental representation, but the symmetry breaking happens through one-dressed-gluon exchange if we consider quarks in the adjoint representation. This means that we may have competing mechanisms as proposed in the model of Ref.[10], which was further studied in Refs.[11–13].

The csb model of Ref.[10] describe a gap equation that contains an effective confining propagator and a dressed gluon propagator with a dynamically generated mass. To grasp the idea behind the model we can first discuss what happens with the one-dressed massive gluon exchange. The effects of dynamical gluon mass generation have been discussed in Ref.[9], and implies a frozen coupling constant given by

$$g^2(k^2) = \frac{1}{b\ln[(k^2 + 4m_g^2)/\Lambda^2]},$$  \hspace{1cm} (1)

where $b = (11N - 4n_fT(R))/48\pi^2$ for the $SU(N)$ group with $n_f$ flavors, and $T(R)$ is connected to the quadratic Casimir eigenvalue $C_2(R)$ for fermions in one specific representation $(R)$ of the gauge group. The infrared value of this coupling obtained through the functional Schrödinger equation [9] and through phenomenological analysis [14] is small.
This fact together with a dynamically massive gluon propagator that roughly behaves as $1/[k^2 + m^2]$ in the infrared (IR), cause a damping in the gap equation erasing the possibility to generate a phenomenologically acceptable dynamical mass for quarks in the fundamental representation \cite{9,15,16}. These results have been obtained based on the Schwinger-Dyson calculations for the gluon propagator \cite{17,20}, which are in agreement with lattice simulations for the gluon propagator \cite{21}. Note that the study of the gap equation with quarks in the adjoint representation does lead to csb due to the larger Casimir operator appearing in the gluon exchange \cite{9,11}.

The difficulties to produce csb at the desired level, as discussed above, led to new approaches to study csb in the context of Schwinger-Dyson equations (SDE). One of them considers the one-dressed gluon exchange making use of a gluon propagator described by the lattice data, which is less damped at intermediate momenta than the one obtained with the SDE, and with a larger value for the quark-gluon vertex due to a possible enhancement of the quark-ghost scattering kernel \cite{22}. The other is the model of Ref.\cite{10} that we addressed before, where csb for quarks in the fundamental representation is essentially triggered by a confining propagator, and the main purpose of this work is to determine how the chiral symmetry is recovered in this model when we increase the number of fermions.

The confining propagator used in Refs.\cite{10,13} is giving by

\begin{equation}
D_{\mu\nu}^{\text{eff}}(k) \equiv \delta^{\mu\nu}D_{\text{eff}}(k); \quad D_{\text{eff}}(k) = \frac{8\pi K_F}{(k^2 + m^2)^2},
\end{equation}

where $m$, which is related with the dynamical quark mass $M$, not only cures the IR singularities of the $1/k^4$ propagator, but also contributes with a negative term to the effective Hamiltonian, which is crucial to generate the massless pions associated to the csb. This entropic quality of this propagator has been stressed in Ref.\cite{23}.

In QCD we expect that quarks interact through a linear potential proportional to the string tension $K_F$, at least up to a certain distance \cite{24}. This behavior is not observed if we perform the Fourier transform of the time-time component of the dynamically massive gluon propagator obtained in lattice simulations or in SDE solutions, however this is a property of Eq.(2). As far as we know there is no evidence that the dynamically massive gluon propagator may generate such linear potential.

A confinement scenario, fully described in \cite{17}, claims that dynamical mass generation in QCD lead to an effective theory where the gluons acquire an effective mass, and consequently
this theory has vortex solutions which are responsible for confinement. This scenario is consistent with the lattice simulations, where center vortices seem to be necessary for csb [13]. In this way vortices appear in the effective dynamically massive theory, and not in the QCD Lagrangian and consequently in the SDE. This is the main reason for the introduction, by hand, of the effective propagator shown in Eq. (2) into the gap equation.

The complete gap equation that we shall consider is

\[
M(p^2) = \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{32\pi K_F}{(p-k)^2 + m^2} + \frac{3C_2 g^2 (p-k)}{(p-k)^2 + m_g^2 k^2} \right\} \frac{M(k^2)}{k^2 + M^2(k^2)},
\]

where we consider a simple fit for the running gluon mass discussed in [25]

\[
m_g^2(k^2) = \frac{m_g^4}{k^2 + m_g^2}.
\]

It can be proven that the entropic parameter (\(m\)) of Eqs. (2) and (3) has to be proportional to the string tension. For instance, in [10] the condition to generate massless pions is \(M^2 = 9K_F/4\pi^2\), and it was also verified that naturally \(m \approx M\). In Ref. [11] the bifurcation condition for the complete gap equation was studied and we can assume

\[
m^2 = \kappa K_F,
\]

where \(\kappa \approx 0.18\) imply in reasonable values of the dynamical quark mass (200MeV < \(M\) < 300MeV) for \(n_f = 2\).

There is another important reason to consider the complete gap equation model of Eq. (3) with a confining propagator and one-dressed-gluon exchange. We have observed that for quarks in the fundamental representation approximately 95% of the dynamical quarks mass is generated by the confining propagator and 5% by the dynamically massive gluon exchange, while for quarks in the adjoint representation the result is exactly the opposite [11, 13]. This is what we meant in the beginning by competing mechanisms, and may explain the apparently contradictory lattice results quoted in the second paragraph, where csb is related to confinement due to vortices, but no one-to-one relation between csb and confinement is found in other simulations.

The model that we discuss in this work has also a clear difference with the scenario in which all the symmetry breaking is generated by gluon exchange, i.e. when the gap equation has only the dressed-gluon exchange and a more sophisticated vertex function. If we just
assume naive Casimir scaling for the interaction in the SDE, it is clear that csb for quarks in different representations will be increased accordingly to the value of this operator, which will appear in all Green’s functions implicit in the gap equation, and this fact may be tested by lattice simulations. For instance, if the csb of quarks in the fundamental representation seems to be enhanced in the one-dressed gluon exchange approach, the csb for quarks in the adjoint representation will be enhanced even more. To further strength the qualities of the model of Ref.[10], we comment in the sequence how it can explain the difference between the chiral transition of fundamental and adjoint quarks [13], without appealing for an enhancement of the quark-gluon vertex and without the use of the lattice gluon propagator.

It is known that the chiral symmetry restoration at finite temperature in QCD with two quark flavors in the fundamental representation is connected to the deconfinement transition [26, 27]. Whereas, when quarks are in the adjoint representation, it has been found that the chiral transition happens at a temperature \( T_c \) higher than the deconfinement temperature \( T_d \) [28–30], where the ratio between these temperatures for adjoint quarks is giving by [28]: \( T_c/T_d \approx 7.7 \pm 2.1 \), and factors of the same order were found in [29, 30]. Working with Eq.(3) we have been able to explain this difference in [13], which is basically related to the different contributions of the two propagators present in the gap equation. In the realm of SDE we are not aware of other explanation of this difference in the chiral transition, what may be a signal of the model success. The main uncertainty in the calculation of Ref.[13] is the poor knowledge of the effective gluon mass for quarks in the adjoint representation.

The chiral symmetry restoration does not happen exclusively with the increase of the temperature. Recent lattice calculations are indicating that the chiral symmetry is also restored with the increase of the number of flavors [31, 32]. In a gap calculation considering only one-gluon dynamically massive exchange it was observed that the chiral symmetry is recovered when the number of quarks is \( n_q \approx 8 \) [33], and our intention is to verify this effect in the model of Ref.[10]. It is important to mention that our calculation makes use of the rainbow approximation, a simple fit of the phenomenological running of the dynamical gluon mass, the confining effective propagator with an entropic parameter that follows Eq.(5), and, of course, we work in the Landau gauge. The main problem to calculate how the solution of Eq.(3) varies with \( n_f \) (or \( n_q \)), as stated in the previous paragraph in the finite temperature case, is the poor knowledge that we have about the variation of \( K_F \) and \( m_g \) with the number
of quarks.

In order to study how the solutions of Eq.(3) vary as we change the number of flavors ($n_f$) we need to know what happens with $K_F$ and $m_g$ as $n_f$ is increased. The infrared value of the dynamical gluon mass was recently determined in lattice simulations for a small number of flavors [33], and the variation of the string tension with $n_f$, also for a small number of quarks, was discussed in Ref.[34]. Therefore it will be necessary to extrapolate the known $K_F$ and $m_g$ values for large $n_f$. This will be the main uncertainty introduced in the results that we shall present. According to Ref.[33] the values for the infrared dynamical gluon mass are 373(8), 427(9), 470(12) MeV respectively for $n_f = 0, 2, 4$ quarks. We plot in Fig.(1) the linear and exponential curves used to extrapolate these values, whose best fits are given by

$$m_g(n_f) = \frac{0.5}{(1 - 0.053 n_f)} \text{GeV},$$

(6)

$$m_g(n_f) = 0.5 e^{0.059 n_f} \text{GeV}.$$

The numerical results of Ref.[33] are used only to obtain the $m_g$ rate of increasing, since the lattice calculation contains further effects that are not contained into the SDE calculation that we are considering here, therefore the fits were normalized in order to match the phenomenologically acceptable value $m_g \approx 2\Lambda_{QCD} \approx 500\text{MeV}$ that has been obtained in Refs.[14, 17, 35], and the simple fit of the SDE solution given by Eq.(4).

![Dynamical gluon mass infrared value as a function of the number of flavors. The mass is extrapolated for large $n_f$ values according the fits of Eq.(6).](image)
To know how the string tension \((K_F)\) for quarks in the fundamental representation varies with the number of quarks we followed Ref.\[34\] and assumed \(K_F = 0.18(1), 0.17(2), 0.14(1), 0.12(3)\) GeV\(^2\) respectively for \(n_f = 0, 2, 3, 4\) flavors. We then performed two different fits (in units of GeV\(^2\)):

\[
\text{Gaussian} : \quad K_F(n_f) = 0.183 \exp \left[ -\frac{(n_f)^2}{0.1} \right],
\]

\[
\text{Linear} : \quad K_F(n_f) = 0.187 - 0.015n_f.
\]

These fits are shown in Fig.\(\text{(2)}\), where we may see that the string tension becomes quite small above \(n_f \geq 10\). It is not difficult to imagine how the csb will depend on the variation of \(K_F\) with the number of flavors. The string tension in the confining propagator plays the same role of the coupling constant in the one-gluon exchange gap equation, and for \(K_F\) below a certain critical value csb will be restored. In the Fig.\(\text{(3)}\) we show the behavior of the dynamical quark masses as a function of the momenta for different number of flavors, computed with the Gaussian fit for the string tension and the exponential fit for the dynamical gluon mass. As the string tension value decreases we have smaller dynamical masses. Fig.\(\text{(4)}\) contains the infrared values of the dynamical quark masses as a function of the number of quarks. These curves were computed with the complete gap equation considering the fits proposed for the string tension combined with the exponential fit for the dynamical gluon mass. We may verify that the chiral symmetry is restored for \(n_f\) values of the order of 11 or 13 where
FIG. 3. Dynamical quark masses as a function of the momenta for different number of flavors, computed with the Gaussian fit for the string tension and the exponential fit for the dynamical gluon mass.

the uncertainty is basically due to our poor knowledge of the string tension and the gluon mass at large $n_f$. Note that $m_g$ values are not so relevant for the determination of the critical $n_f$ value, as could be expected in face of the results presented in Ref. [11, 13]. This is the main reason why we only focused on one of the fits for $m_g$.

In the context of a csb model where the gap equation contains an effective confining propagator and a dressed gluon propagator with a dynamically generated mass, we verified that the chiral symmetry is restored for a large number of quarks. We first discussed the properties of the model, indicating that the introduction of the effective confining propagator is one way to introduce confinement by vortices, which cannot appear in the SDE when we consider only the exchange of dynamically massive gluons [10, 17]. We also discussed possible differences between this and other models to explain csb with dynamically massive gluons. Our results indicate that the chiral symmetry is restored for $n_f \approx 11 - 13$, in agreement with lattice results [31, 32]. The values of the string tension ($K_F$) and dynamical gluon masses ($m_g$) were extrapolated from small to large number of quarks. This extrapolation is the larger source of uncertainty in our calculation. The gap equation calculation is quite numerically
FIG. 4. Infrared values of the dynamical quark masses as a function of the number of quarks. The points corresponding to the central values of the infrared quark mass were computed with the complete gap equation considering different combination of fits for the string tension and dynamical gluon mass. Case 1 was obtained with the $K_F$-Gaussian-fit and Case 2 has the $K_F$-Linear-fit. The influence of the dynamical gluon mass is negligible compared to the uncertainty in the string tension value. The fits are given just to guide the eyes.

sensitive to the decrease of parameters that define the critical bifurcation behavior (factors like $K_F$ or $C_2 g^2$ in the one-gluon gap equation), but certainly the $K_F$ extrapolated values is at the origin of the range of $n_f$ critical values. It is important to have new QCD simulations of the string tension and dynamical gluon masses for a large number of flavors and for different fermionic representations, otherwise it will not be possible to perform precise gap equation calculations of the csb phase diagram as a function of $n_f$.

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mento de Pessoal de Nível Superior (CAPES).

[1] H. Reinhardt, O. Schröder, T. Tok and V. C. Zhukovsky, Phys. Rev. D 66, 085004 (2002).
[2] J. Gattnar, C. Gattringer, K. Langfeld, H. Reinhardt, A. Schafer, S. Solbrig and T. Tok, Nucl. Phys. B 716, 105 (2005).
[3] P. de Forcrand and M. D’Elia, Phys. Rev. Lett. 82, 4582 (1999); P. O. Bowman et al., Phys. Rev. D 78, 054509 (2008).
[4] M. Faber and R. Hollwieser, Acta Phys. Polon. Supp. 7, 457 (2014); R. Hollwieser, M. Faber, T. Schweigler and U. M. Heller, PoS Lattice 2013, 505 (2013); R. Hollwieser, T. Schweigler, M. Faber and U. M. Heller, Phys. Rev. D 88, 114505 (2013).
[5] A. Casher, Phys. Lett. B 83, 395 (1979).
[6] T. Banks and A. Casher, Nucl. Phys. B 169, 103 (1980).
[7] J. M. Cornwall, Phys. Rev. D 22, 1452 (1980).
[8] H. Suganuma, T. M. Doi and T. Iritani, arXiv:1405.1289, 1404.6494; PoS (QCD-TNT-III) 042 (2014); S. Gongyo, T. Iritani and H. Suganuma, Phys. Rev. D 86, 034510 (2012).
[9] J. M. Cornwall, Center vortices, the functional Schrödinger equation, and CSB, Invited talk at the conference “Approaches to Quantum Chromodynamics”, Oberwölz, Austria, September 2008, arXiv:0812.0359 [hep-ph].
[10] J. M. Cornwall, Phys. Rev. D 83, 076001 (2011)
[11] A. Doff, F. A. Machado and A. A. Natale, Annals Phys. 327, 1030 (2012).
[12] A. Doff, F. A. Machado and A. A. Natale, New J. Phys. 14, 103043 (2012).
[13] R. M. Capdevilla, A. Doff and A. A. Natale, Phys. Lett. B 728, 626 (2014).
[14] A. C. Aguilar, A. Mihara and A. A. Natale, Phys. Rev. D 65, 054011 (2002).
[15] B. Haeri and M. B. Haeri, Phys. Rev. D 43, 3732 (1991).
[16] A. A. Natale and P. S. Rodrigues da Silva, Phys. Lett. B 392, 444 (1997); Phys. Lett. B 390, 378 (1997).
[17] J. M. Cornwall, Phys. Rev. D 26, 1453 (1982).
[18] A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D 89, 085032 (2014); A. C. Aguilar, D. Binosi, D. Ibaez and J. Papavassiliou, Phys. Rev. D 89, 085008 (2014); A. C. Aguilar, D. Binosi and J. Papavassiliou, JHEP 1201, 050 (2012); A. C. Aguilar, D. Binosi and J.
Papavassiliou, Phys. Rev. D 84, 085026 (2011); A. C. Aguilar and J. Papavassiliou, Phys. Rev. D 81, 034003 (2010); A. C. Aguilar, D. Binosi, J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D 80, 085018 (2009); A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D 78, 025010 (2008); A. C. Aguilar and J. Papavassiliou, JHEP 0612, 012 (2006).

[19] D. Binosi and J. Papavassiliou, Phys. Rept. 479, 1 (2009).

[20] J. M. Cornwall, J. Papavassiliou and D. Binosi, “The Pinch Technique and its Applications to Non-Abelian Gauge Theories”, Cambridge University Press, 2011.

[21] A. Cucchieri and T. Mendes, PoS QCD-TNT 09, 031 (2009); Phys. Rev. Lett. 100, 241601 (2008); Phys. Rev. D 81, 016005 (2010); I. Bogolubsky, E. Ilgenfritz, M. Muller-Preussker and A. Sternbeck, Phys. Lett. B 676, 69 (2009).

[22] A. C. Aguilar and J. Papavassiliou, Phys. Rev. D 83, 014013 (2011).

[23] J. M. Cornwall, Mod. Phys. Lett. A 27, 1230011 (2012).

[24] G. S. Bali et al. [SESAM Collaboration], Phys. Rev. D 71, 114513 (2005).

[25] A. C. Aguilar and A. A. Natale, JHEP 0408, 057 (2004).

[26] A. Bazavov et al., Phys. Rev. D 80, 014504 (2009).

[27] Y. Aoki et al., JHEP 0906, 088 (2009).

[28] F. Karsch and M. Lutgemeier, Nucl. Phys. B 550, 449 (1999).

[29] J. Engels, S. Holtmann and T. Schulze, Nucl. Phys. B 724, 357 (2005).

[30] E. Bilgici, C. Gattringer, E.-M. Ilgenfritz and A. Maas, JHEP 0911, 035 (2009).

[31] Ph. de Forcrand, S. Kim and W. Unger, JHEP 1302, 051 (2013).

[32] E. T. Tomboulis, Phys. Rev. D 87, 034513 (2013).

[33] A. Ayala, A. Bashir, D. Binosi, M. Cristoforetti, and J. Rodriguez-Quintero, Phys. Rev. D 86, 074512 (2012).

[34] F. Karsch and E. Laermann, Rep. Prog. Phys. 56, 1347 (1993); F. Karsch, E. Laermann and A. Peikert, Nucl. Phys. B 605, 579 (2001); F. Karsch, Lect. Notes Phys. 583, 209 (2002).

[35] E. G. S. Luna, Phys. Lett. B 641, 171 (2006); E. G. S. Luna and A. A. Natale, Phys. Rev. D 73, 074019 (2006); A. A. Natale, PoS QCD-TNT 09, 031 (2009); F. Halzen, G. I. Krein and A. A. Natale, Phys. Rev. D 47, 295 (1993).