MarKov-Yukawa Transversality On Covariant Null Plane: Pion Form Factor, Gauge Invariance And Lorentz Completion

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Abstract

The Markov-Yukawa Transversality Principle (MYTP) on a 2-body Bethe-Salpeter kernel is formulated on a covariant Null Plane (NP) to reconstruct the 4D BS wave function for 2 fermion quarks in terms of 3D entities that satisfy a 3D BSE. This result is the null-plane counterpart of a 3D-4D interconnection for the 2-body BS wave functions found earlier by imposing MYTP covariantly in the instantaneous rest frame (termed CIA) of the composite. This formulation yields a 3D BSE which is formally identical to its Covariant Instantaneity form, thus fully preserving its spectral results, while ensuring full covariance. More importantly, the reconstructed 4D vertex functions in the covariant null-plane ensure that 4D quark-loops are now free from ill-defined time-like momentum integrations (which had plagued the earlier CIA vertex functions), while a simple prescription of ‘Lorentz completion’ in the new description yields a manifestly Lorentz-invariant result. This is illustrated for the pion and kaon form factors with full QED gauge-invariance, showing a $k^{-2}$ behaviour at large $k^2$, and ‘correct’ slopes at small $k^2$. This method is compared with the Kadychevsky-Karmanov light-front formalism.

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1 Introduction: Markov-Yukawa Transversality

For a relativistic 2-body problem, the historical issue of 3D reduction from a 4D BSE has been in the forefront of its physics from the outset [1-3]: Instantaneous approximation [1]; Quasi-potential approach [2]; variants of on-shellness of propagators [3]. In all these methods the starting BSE is 4D in all details, including its kernel, but the associated propagators are manipulated in various ways to reduce the 4D BSE to a 3D form as a fresh starting point, giving up its original 4D form, more in conformity with the Weinberg 3D spirit [4]. Kadychevsky [5] and Karmanov [6] have given it a more formal shape with on-shell propagators and spurious [6], making up a covariant light-front formulation which has been recently reviewed by Carbonell et al [7]. (This paper will be termed KK [7] in the text).

An alternative approach of more recent origin [8,9] is based on the Markov-Yukawa Transversality Principle (MYTP) [10] wherein a Lorentz-covariant 3D support is postulated at the outset to the pairwise BSE kernel \( K \) by demanding that it be a function of only \( \hat{q}_\mu = q_\mu - q.P P_\mu / P^2 \), implying \( \hat{q}.P \equiv 0 \); but the propagators are left untouched in their original 4D forms. This is somewhat complementary to the approaches [1-3] (propagators manipulated but kernel left untouched), so that the resulting equations [8-9] look unfamiliar vis-a-vis these [1-3], but it has the advantage of allowing a double track use of both 3D and 4D BSE forms via their interlinkage. Indeed what distinguishes the Covariant Instaneity Ansatz (CIA) [8] from the more familiar 3D reductions of the BSE [1-3] is its capacity for a 2-way connection: an exact 3D BSE reduction, and an equally exact reconstruction of the original 4D BSE form without extra charge [8]: the former to access the observed O(3)-like spectra [11], and the latter to give transition amplitudes as 4D quark loop integrals [8]. [In the corresponding approach of the Pervushin Group too [9], this twin feature was also present, but seemingly unnoticed]. In contrast the more orthodox methods [1-3] give only a one-way connection, viz., \( 4D \rightarrow 3D \) reduction, but not vice versa. This unique feature of MYTP [10], providing a 2-way 3D-4D interconnection in the BSE structure, has somehow remained unnoticed in the literature, despite a demonstration [8] of its existence.

1.1 Physics of MYTP on Null-plane

The MYTP [10] controlled BSE, termed 3D-4D-BSE in the following, of course needs supplementing by physical ingredients to define the BSE kernel, much as a Hamiltonian needs a properly defined ‘potential’. However its canvas is broad enough to accommodate a wide variety of kernels which must in turn be governed by independent physical principles. In this respect, the orthodox view (which we adopt) is to keep close to the traditional 4D BSE-cum-SDE methods [12] of Dynamical Breaking of Chiral Symmetry (\( DB\chi S \)) a la NJL [13] whose basic feature of chiral symmetry breaking survives a (space-time extended) 4-quark interaction mediated by vector exchange [14] as a generalized \( DB\chi S \) mechanism to generate a mass-function \( m(p) \) via Schwinger-Dyson equation (SDE), which accounts for the bulk of the constituent mass of \( ud \) quarks via Politzer additivity [15]. Indeed, the BSE-SDE formalism [12-15] can be simply adapted [16] to the hybrid 3D-4D-BSE form [8] which produces 3D spectra of both hadron types [17] under a common parametrization for the gluon propagator, with a self-consistent SDE determination [16] of the constituent mass.
One disadvantage of MYTP [10] for 3D-4D interconnection [8] achieved under covariant instantaneity (CIA) in the composite’s rest frame [8] is the ill-defined nature of 4D loop integrals which acquire time-like momentum components in the exponential/gaussian factors associated with the different vertex functions due to a ‘Lorentz-mismatch’ among the rest-frames of the participating hadronic composites. This is especially so for triangle loops and above, such as the pion form factor, while 2-quark loops [18] just escape this pathology. This problem was not explicitly encountered in the null-plane ansatz (NPA) [19] in an earlier study of 4D triangle loop integrals, but the NPA approach was criticized [20] on grounds of non-covariance. The CIA approach [8] which makes use of MYTP [10,9], was an attempt to rectify the Lorentz covariance defect, but the presence of time-like components in the gaussian factors inside triangle loop integrals, e.g., in the pion form factor [21], impeded further progress.

In this paper we wish to explore if it is possible to ensure formal covariance without having to encounter the time-like components in the (gaussian) wave functions inside the 4D loop integrals. This paper is an attempt to show that by extending the Transversality Principle [10] from the covariant rest frame of the (hadron) composite [8-10], to a covariantly defined null-plane (NP) both these features can be preserved, so that even the old-fashioned null-plane framework [19,22] for 3D-4D interconnection can be given a covariant ”look”. In this respect, we shall also compare the present ‘TP-NP’ method with other covariant NP approaches [23], especially KK [7], which has some obvious similarities with the earlier (old-fashioned) NPA formulation [22].

1.2 Scope of the Paper

This paper has a 4-fold objective: i) To formulate MYTP [10] on a covariant null-plane (NP), i.e., demand that the BSE kernel \( K \) for pairwise interaction is a function of relative momentum \( q \) transverse to the NP, just as in CIA [8] the Transversality is w.r.t. the composite 4-momentum \( P_\mu \); ii) to show that the reduced 3D BSE has formally the same structure as in the corresponding CIA [8] case, so that with the same parametrization for \( K \), the spectral predictions [17] remain unchanged; iii) to show that the reconstructed 4D wave function no longer suffers a ‘Lorentz-mismatch’ with other such functions involved in a loop integral (which, in CIA [8,21], is the main cause for the appearence of time-like components in the gaussian form factors); iv) to illustrate the detailed procedure by working out the pion-form factor under QED gauge invariance and a simple prescription of ‘Lorentz-completion’ to obtain an explicitly Lorentz- and gauge-invariant quantity which shows the desired \( k^{-2} \) behaviour at high \( k^2 \). For a better perspective, we shall also offer a critical comparison of this procedure with the covariant NP approach of KK [7].

After some preliminaries on the definition of a covariant null-plane, Sec.2 employs MYTP [10] for the BS kernel \( K \) for spinless quarks to define its structure on such a null-plane, by close analogy to the CIA formulation [8], and outlines the derivation of the 3D BSE, as well as an explicit reconstruction of the 4D wave function in terms of 3D ingredients, in which the 3-momentum is \( \bar{q} = (q_L, q_3) \), where the third component emerges as a \( P \)-dependent one lying in the null plane (NP). Sec.3 is a self-contained derivation of the 3D BSE for a fermion pair, in which the full structure of the non-perturbative gluonic kernel [16] is redefined in the NP language, so as to bring out a strong similarity to the corresponding CIA equation [24], to justify identical predictions on the spectroscopy front [17]. With this covariant NP-oriented 3D-4D formulation, Sec.4
outlines the calculation of the P-meson form factor for unequal mass kinematics in a fully gauge invariant manner, including correction terms arising from QED gauge invariance, and illustrating the techniques of ‘Lorentz-completion’ to obtain an explicitly Lorentz invariant quantity. As a check on the consistency of the formalism, the expected \( k^{-2} \) behaviour of the pion form factor at high \( k^2 \) is realized. Sec.5 concludes with a short discussion, a summary of this method in retrospect, plus a comparison with KK [7] as a prototype for other NP approaches [23]. Some calculational details on the triangle-loop integral for the P-meson form factor are given in Appendix A.

## 2 3D-4D BSE Formalism on Covariant Null-Plane

As a preliminary to defining a 3D support to the BS kernel on the null-plane (NP), on the lines of CIA [8], a covariant NP orientation may be represented by the 4-vector \( n_\mu \), as well as its dual \( n_\mu \), obeying the normalizations \( n^2 = n^2 = 0 \) and \( n.\bar{n} = 1 \). In the standard NP scheme (in euclidean notation), these quantities are \( n = (001; -i)/\sqrt{2} \) and \( \bar{n} = (001; i)/\sqrt{2} \), while the two other perpendicular directions are collectively denoted by the subscript \( \perp \) on the concerned momenta. We shall try to maintain the \( n \)-dependence of various momenta to ensure explicit covariance; and to keep track of the old-fashioned NP notation \( p_{\perp} = p_0 \pm p_3 \), our covariant notation is normalized to the latter as \( p_+ = n.p/\sqrt{2} \); \( p_- = -\bar{n}.p/\sqrt{2} \), while the perpendicular components continue to be denoted by \( p_{\perp} \) in both notations.

For the various quantities (masses, momenta, etc) we shall stick to the notation of [8] without explanation, except when new features arise. For the relative momentum \( q = \vec{m}_2p_1 - \vec{m}_1p_2 \), the fourth component to be eliminated for obtaining a 3D equation, is proportional to \( q_n \equiv \bar{n}.q \), as the NP analogue [22] of \( P.qP/P^2 \) in CIA [8], where \( P = p_1 + p_2 \) is the total 4-momentum of the hadron. However the quantity \( q - q_n.n \) is still only \( q_{\perp} \), since its square is \( q^2 - 2n.q\bar{n}.q \), as befits \( q_{\perp}^2 \) (readily checked against the ‘special’ NP frame). We still need a third component \( p_3 \), for which a first guess is \( zP \), where \( z = n.q/n.P \). And for calculational convenience we shall need to (temporarily) invoke the ‘collinear frame’ which amounts to \( P_{\perp}q_{\perp} = 0 \), a restriction which will be removed later by a simple prescription of ‘Lorentz completion’. Unfortunately the definition \( \hat{q}_{\mu} = (q_{\perp} ; zP_{\mu}) \) does not quite fit the bill for a covariant 3-vector, since a short calculation shows again that \( \hat{q}_{\perp}^2 = q_{\perp}^2 \). The correct definition turns out to be \( q_{3\mu} = zP_n.n_{\mu} \), where \( P.n = P.\bar{n} \), giving \( \hat{q}_{\perp}^2 = q_{\perp}^2 + z^2M^2 \), as required. We now collect the following definitions/results:

\[
\begin{align*}
q_{\perp} &= q - q_n.n; \quad \hat{q} = q_{\perp} + zP_n.n; \quad z = q.n/P.n; \quad P^2 = -M^2; \\
q_n, P_n &= \bar{n}(q, P); \hat{q}.n = q.n; \quad \hat{q}.\bar{n} = 0; \quad P_{\perp}q_{\perp} = 0; \\
P.q &= P_n.q.n + P.nq_n; \quad P.\hat{q} = P_n.q.n; \quad \hat{q}_{\perp}^2 = q_{\perp}^2 + M^2x^2
\end{align*}
\]  

### 2.1 3D-4D BSE on Cov. NP : Spinless Quarks

We now proceed to derive the reduced 3D BSE (wave-fn \( \phi \)) from the 4D BSE with spinless quarks (wave-fn \( \Phi \)) when its kernel \( K \) is decreed to be independent of the component \( q_n \), i.e., \( K = K(\hat{q}, \hat{q}') \), with \( \hat{q} = (q_{\perp}, P_n.n) \), in accordance with the TP [10] condition imposed on the null-plane (NP). The 4D BSE with such a kernel is, c.f., [8,9]:

\[
i(2\pi)^4\Phi(q) = \Delta_1^{-1}\Delta_2^{-1}\int d^4q'K(\hat{q}, \hat{q}')\Phi(q')
\]  

(2.2)
where $\Delta_i = p_i^2 + m_i^2$; $m_i$ = quark mass; $(P^2 = -M^2)$; and $d^4q = dq_1 dq_2 dq_3 dq_4$. Now define a 3D wave-fn $\phi(q) = \int dq_n \Phi(q)$, and use this result on the RHS of (2.2) to give

$$i(2\pi)^4 \Phi(q) = \Delta_1^{-1} \Delta_2^{-1} \int d^4q' K(q, q') \phi(q')$$

(2.3)

Now integrate both sides of eq.(2.3) w.r.t. $dq_n$ to give a 3D BSE in the variable $\hat{q}$:

$$(2\pi)^3 D_n(\hat{q}) \phi(\hat{q}) = \int d^2q \int dq_3 K(\hat{q}, \hat{q}') \phi(\hat{q}')$$

(2.4)

where the function $D_n(\hat{q})$, is defined, analogously to CIA [8], by

$$\int dq_n \Delta_1^{-1} \Delta_2^{-1} = 2\pi i D_n^{-1}(\hat{q})$$

(2.5)

and may be obtained by standard NP techniques [22] (Chaps 5-7) as follows. In the $q_n$ plane, the poles of $\Delta_{1,2}$ lie on opposite sides of the real axis, so that only one pole will contribute at a time. Taking the $\Delta_2$-pole, which gives

$$2q_n = -\sqrt{2} q_m = \frac{m_2^2 + (q_1 - m_2 P)^2}{m_2 P.n - q.n}$$

(2.6)

the residue of $\Delta_1$ works out, after a routine simplification, to just $2P.q = 2P.nq_n + 2P_nq.n$, after using the collinearity condition $P_1.q_1 = 0$ from (2.1). And when the value (2.6) of $q_n$ is substituted in (2.5), one obtains (with $P_n P.n = -M^2/2$):

$$D_n(\hat{q}) = 2P.n(q^2 - \frac{\lambda(M^2, m_1^2, m_2^2)}{4M^2}); \quad \hat{q}^2 = q_1^2 + M^2 x^2; \quad x = q.n/P.n$$

(2.7)

Now a comparison of (2.2) with (2.4) relates the 4D and 3D wave-fns:

$$2\pi i \Phi(q) = D_n(\hat{q}) \Delta_n^{-1} \Delta_2^{-1} \phi(\hat{q})$$

(2.8)

which is valid near the bound state pole. The BS vertex function now becomes $\Gamma = D_n \times \phi/(2\pi i)$, analogously to the CIA result [8]. This result, though dependent on the NP orientation, is nevertheless formally covariant, and meets the ‘covariance’ criticism [20] of the earlier NPA formulation [19,22], where an identical result for $D_+ \times \phi$ had been found. For fermion quarks, the $q \bar{q}$ wave function $\Psi$ has formally the same structure as (2.8), except that $D_F(p_i) = -\Delta(p_i)^{-1}$ is replaced by $S_F(p_i)$, and the vertex function $\Gamma$ has an extra factor $\gamma_5$ for pseudoscalar, $i\gamma_\mu$ for vector, etc meson [8,24].

A few comments are now in order on a comparison of the KK [7] vs the present covariant formulation of NP dynamics [19,22]. Both are ‘covariantly’ dependent on the orientation $n_\mu$ of the NP, i.e., have certain $n$-dependent 3-scalars, in addition to genuine 4-scalars. Note also the formal identity of (2.4) (and (2.7)) with KK’s eq.(3.48) [7]. Our $n_\mu$ is KK’s $\omega_\mu$, but we have an independent 4-vector $\hat{n}_\mu$ which has a dual interplay with $n_\mu$ in the above formulation, but without a counterpart in KK [7]. Secondly our manifestly covariant 4D formulation needs no 3-vector like $\mathbf{n}$ [7], nor explicit Lorentz transformations. As to the ‘angular condition’ [25] discussed in KK, we have not (in our Cov. NP formalism) had to make any special effort to satisfy this, since the very appearance of the ‘effective’ 3-vector $\hat{q}_\mu$ in the 3D BSE in a rotationally invariant manner is an automatic guarantee
(in the sense of the ‘proof of the pudding’) of the satisfaction of this condition [25] without further assumptions.

Our 3D-4D (hybrid) BSE formulation (which allows for off-shell momenta) has no further need for spurions which KK [7] require to make up for energy-momentum balance since all their physical momenta are on-shell. Nor are fresh Feynman techniques with spurions [7] needed, since normal 4D techniques apply directly to the 3D-4D formalism per se [19,22]. Finally, to rid the physical amplitudes of \( n_\mu \)-dependent terms in the external (hadron) momenta, after integration over the internal loop momenta, we shall use a simple technique of ‘Lorentz-completion’ (to be illustrated in Sec.4 for the pion form factor calculation) as an alternative to other NP prescriptions [7,23] to remove \( n \)-dependent terms.

A more succinct comparison with other null-plane approaches concerns the inverse process of reconstruction of the 4D hadron-quark vertex, eq.(2.8), which does not seem to have a counterpart in these, e.g., KK [7], or the Wilson group [23], which are basically 3D oriented. Thus the nearest analogue of this in KK [7] is to express the 3D NP wave function in terms of the 4D BS wave function (see eq.(3.58) of KK [7]), but not vice versa. This illustrates the problem of “loss of Hilbert space information” inherent in such a process of reconstruction, as discussed recently in the context of the \( q\bar{q}q \) problem [26]. The TP [10] is a big help in this regard: For the two-body \( q\bar{q} \) case, the inversion is exact [8] (this is a sort of degenerate situation). However for 3 or more ‘bodies’, even TP [10] has its limitations, since some additional assumptions are needed to fill the information gap; (see [26] for more details).

### 3 3D Fermion BSE on Covariant Null Plane

In this Section we shall collect under one head the covariant counterparts of the main features of an earlier CIA [8,16] and (old-fashioned) NP [22] 3D formulations, and indicate their ramifications on Spectroscopy. We stress at the outset that the Transversality Principle (MYTP), when applied to the Covariant null-plane (CNP), gives formally the same structure of the 3D BSE as the application of MYTP gives under CIA [8,16], or even under the old-fashioned non-covariant NPA [22].

The 4D BSE for fermionic quarks under a gluonic (vector-type) interaction kernel with 3D support has the standard form [24]:

\[
 i(2\pi)^4 \Psi(P,q) = S F_1(p_1) S F_2(p_2) \int d^4q' K(\hat{q},\hat{q}') \Psi(P, q'); \quad K = F_{12} i\gamma^{(1)}_\mu i\gamma^{(2)}_\mu V(\hat{q}, \hat{q}') \tag{3.1}
\]

where \( F_{12} \) is the color factor \( \lambda_1 \lambda_2/4 \) and the \( V \)- function expresses the scalar structure of the gluon propagator in the perturbative (o.g.e.) plus non-perturbative regimes, whose full structure (as employed in actual calculations) [22,17] is collected as under, using the simplified notations \( k \) for \( q - q' \), and \( V(\hat{k}) \) for the \( V \) fn:

\[
 V(\hat{k}) = 4\pi \alpha_s/\hat{k}^2 + \frac{3}{4} \omega_{q\bar{q}}^2 \int d\mathbf{r} [r^2(1 + 4A_0 \hat{m}_1 \hat{m}_2 M_>^2 r^2)^{-1/2} - C_0/\omega_{q\bar{q}}^2] e^{i\hat{k}\cdot\mathbf{r}}; \tag{3.2}
\]

\[
 \omega_{q\bar{q}}^2 = 4M_> \hat{m}_1 \hat{m}_2 \omega^2 \alpha_s(M_>^2); \quad \alpha_s(Q^2) = \frac{6\pi}{33 - 2n_f} \ln(M_>\Lambda)^{-1}; \tag{3.3}
\]

\[
 \hat{m}_{1,2} = [1 \pm (m_1^2 - m_2^2)/M^2]/2; \quad M_> = \text{Max}(M, m_1 + m_2); \quad C_0 = 0.27; \quad A_0 = 0.0283 \tag{3.4}
\]
And the values of the basic constants (all in $MeV$) are \[22\]

\[
\omega_0 = 158; \quad m_{ud} = 265; \quad m_s = 415; \quad m_c = 1530; \quad m_b = 4900. \quad (5.5)
\]

The BSE form (3.1) is however not the most convenient one for wider applications in practice, since the Dirac matrices entail several coupled integral equations. Indeed, as noted long ago \[24\], a considerable simplification is effected by expressing them in ‘Gordon-reduced’ form, (permissible on the quark mass shells, or better on the surface $P.q = 0$), a step which may be regarded as a fresh starting point of our dynamics, in the sense of an ‘analytic continuation’ of the $\gamma$-matrices to ‘off-shell’ regions (i.e., away from the surface $P.q = 0$). Admittedly this constitutes a conscious departure from the original BSE structure (3.1), but such technical modifications are not unknown in the BS literature \[27\] in the interest of greater manoeuvrability, without giving up the essentials, in view of the ”effective” (and not fundamental) nature of the BS kernel.

The ‘Gordon-reduced’ BSE form of (3.1) is given by \[24\]

\[
\Delta_1 \Delta_2 \Phi(P, q) = -i(2\pi)^{-4} F_{12} \int d^4 q' V^{(1)}(2) V^{(2)}(1) \Phi(P, q') \Phi(P, q); \quad (3.6)
\]

where the connection between the $\Psi$- and $\Phi$-functions is

\[
\Psi(P, q) = (m_1 - i \gamma(1).p_1)(m_2 + i \gamma(2).p_2)\Phi(P, q); \quad p_{1,2} = \hat{m}_{1,2} P \pm q \quad (3.7)
\]

\[
V^{(1,2)}_{\mu} = \pm 2m_{1,2}\gamma^{(1,2)}_{\mu}; \quad V^{(i)}_{\mu} = p_{\mu} + p'_{\mu} + i\sigma^{(i)}_{\mu\nu}(p_{\nu} - p'_{\nu}) \quad (3.8)
\]

Now to implement the Transversality Condition \[8-10\] for the entire kernel of eq.(3.6), all time-like components $q_n, \hat{q}_n$ in the product $V^{(1)} V^{(2)}$, as defined in eq.(2.1), must first be dropped. Substituting from (3.8) and simplifying gives

\[
(p_1 + p'_1)(p_2 + p'_2) = 4\hat{m}_1\hat{m}_2P^2 - (\hat{q} + \hat{q}')^2 - 2(\hat{m}_1 - \hat{m}_2)P.(q + q') + "spin-Terms"; \quad (3.9)
\]

"SpinTerms" = \[-i(2\hat{m}_1 P + \hat{q} + \hat{q}')[\sigma_{\mu\nu}]_{\mu}^{(2)}\hat{k}_\nu + i(2\hat{m}_2 P - \hat{q} - \hat{q}')[\sigma_{\mu\nu}]_{\mu}^{(1)}\hat{k}_\nu + [\sigma_{\mu\nu}]_{\mu}^{(1)}[\sigma_{\mu\nu}]_{\nu}^{(2)} \quad (3.10)
\]

This is identical to eq.(7.1.9) of Ref.[22], via the formal correspondence $\hat{q}_\mu \Rightarrow q_\mu + q.nP.n_{\mu}/P.n$, where the covariance is now explicit. The 3D reduction of eq.(3.4) now goes through exactly as in sec.2.1, so that without further ado, the full structure of the 3D BSE can be literally taken over from Ref.[22]-Chap 7 (derived under non-covariant NPA). In particular, for harmonic confinement, obtained by dropping the $A_0$ term in the ‘potential’ $U(r)$ of (3.4) (a very good approximation for light ($ud$) quarks), the 3D BSE works out as

\[
D_n\phi(\hat{q}) = \frac{P_n}{M}\omega_n^2 \hat{D}(\hat{q})\phi(\hat{q}); \quad (3.11)
\]

\[
\hat{D}(\hat{q}) = 4\hat{m}_1\hat{m}_2M^2(\nabla^2 + C_0/\omega_0^2) + 4\hat{q}^2\nabla^2 + 8\hat{q}\nabla + 18 - 8J.S + (4C_0/\omega_0^2)\hat{q}^2 \quad (3.12)
\]

Note that the covariance is again manifest, since both sides are proportional to $P_n$, as checked from eq.(2.7) for $D_n$. Since one of the objects of this exercise is to strengthen the mathematical foundations of results [22,17] that have already made successful contact with data, we may simply refer to [22,17] for the details of spectroscopic, etc predictions. For the sake of algebraic completeness however, we record the (gaussian) parameter $\beta$ of
the 3D wave function $\phi(\hat{q}) = \exp(-\hat{q}^2/2\beta^2)$, which is the solution of (3.11) for a ground state hadron [22,16-18]:

$$\beta^4 = \frac{8\hat{m}_1^2\hat{m}_2^2M_2^2\omega_0^2\alpha(M_0^2)}{[1 - 8C_0\hat{m}_1\hat{m}_2\alpha_s(M_0^2)]} < \sigma >^2 = 1 + 24A_0(\hat{m}_1\hat{m}_2M_0^2)^2/\beta^2 \quad (3.13)$$

Note that $\beta$ is independent of the null-plane orientation $n_\mu$. (For an L-excited hadron wave function, see [19,22]). The full 4D BS wave function $\Psi(P, q)$ in a 4x4 matrix form [22] is then reconstructed from (3.6-7) as [19,22,8]

$$\Psi(P, q) = S_F(p_1)\Gamma(\hat{q})\gamma_D S_F(-p_2); \quad \Gamma(\hat{q}) = N_n(P)D_n(\hat{q})\phi(\hat{q})/2i\pi \quad (3.14)$$

where $\gamma_D$ is a Dirac matrix which equals $\gamma_5$ for a P-meson, $i\gamma_\mu$ for a V-meson, $i\gamma_\mu\gamma_5$ for an A-meson, etc. $N_n(P)$ represents the hadron normalization which is derived in Sec.4.

We end this section with the remark that the TP [10] approach on the covariant null plane has yielded a wave function whose form resembles one of the varieties obtained in the old-fashioned NPA treatment [19,22] that had been termed the “on-shell” variety. However, two other varieties encountered in the old-fashioned treatment, viz., “off-shell” and “half-off-shell”, have not found a natural counterpart in this manifestly covariant treatment.

4 P-Meson E.M. Form Factor for Unequal masses

The pion form factor has through the ages been a good laboratory for subjecting theoretical models and ideas on strong interactions to observational test. Among the crucial parameters are the squared radius $<r_{\text{exp}}^2> = 0.43 \pm .014 fm^2$ [28a], and the scaled form factor at high $k^2$, viz., $k^2F(k^2) \approx 0.5 \pm 0.1 GeV^2$ [28b] which represent important check points for theoretical candidates such as QCD-sum rules [29], Finite Energy sum rules [30], perturbative QCD [31], covariant null-plane approaches [7, 23], 4D SDE-BSE methods [12], including Euclidean SDE [32], etc. An important issue in this regard concerns the interface of perturbative and non-perturbative QCD regimes. Indeed it has been suggested on the basis of certain model studies in terms of light-front wave functions with specific combinations of longitudinal and transverse components [33], that non-perturbative effects probably persist up to the highest $k^2$, as deduced from the relative importance of the transverse component vis-a-vis the longitudinal, for all $k^2$.

While such a conclusion [33] sounds apriori reasonable, the demonstration of any intimate connection between the transverse and longitudinal components of the (light front) wave function is the task of a more detailed dynamical theory (such as an effective Lagrangian) than can be captured by such ad hoc but intuitive ansatze [33]. From this angle a dynamical model such as the covariant NP formalism developed in Secs 2-3 (pre-calibrated to spectroscopic details [17]) would appear to be a more promising candidate, like [7,23,29-32], for addressing such dynamical issues. Indeed the pion e.m. form factor was already worked out in the old NPA formalism [19], with very reasonable results, but we outline below a fresh, self-contained ‘covariant’ derivation valid for unequal masses of the quark constituens, one in which issues of gauge invariance as well as of ‘Lorentz-completion’ will receive particular attention, with a view to check (among other things) on the expected $k^{-2}$ behaviour at high $k^2$, and the ‘slope’ at low $k^2$. 

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4.1 4D Triangle Loop Integral for $F(k^2)$

Using the two diagrams (figs.1a and 1b) of ref.[19], and in the same notation, the Feynman amplitude for the $h \rightarrow h' + \gamma$ transition contributed by fig.1a (quark 2 as spectator is given by [19]

$$2\bar{P}_\mu F(k^2) = 4(2\pi)^4 N_n(P) N_n(P') e\bar{m}_1 \int d^4 T^{(1)}_\mu \frac{D_n(\bar{q})\phi(\bar{q}) D_n(q')\phi(q')}{\Delta_1\Delta'_1\Delta_2} + [1 \Rightarrow 2]; \quad (4.1)$$

$$4T^{(1)}_\mu = Tr[\gamma_5(m_1 - i\gamma.p_1)i\gamma_\mu(m_1 - i\gamma.p'_1)\gamma_5(m_2 + i\gamma.p_2)]; \quad \Delta_1 = m_1^2 + p_1^2; \quad (4.2)$$

$$p_{1,2} = \hat{m}_{1,2} P \pm q; \quad p'_{1,2} = \hat{m}_{1,2} P' \pm q' \quad p_2 = p'_2; \quad P - P' = p_1 - p'_1 = k; \quad 2\bar{P} = P + P'. \quad (4.3)$$

After evaluating the traces and simplifying via (4.2-3), $T_\mu$ becomes

$$T^{(1)}_\mu = (p_{2\mu} - \bar{P}_\mu)[\delta m^2 - M^2 - \Delta_2] - k^2 p_{2\mu}/2 + (\Delta_1 - \Delta'_1)k_\mu/4 \quad (4.4)$$

The last term in (4.4) is non-gauge invariant, but it does not survive the integration in (4.1), since the coefficient of $k_\mu$, viz., $\Delta_1 - \Delta'_1$ is antisymmetric in $p_1$ and $p'_1$, while the rest of the integrand in (4.1) is symmetric in these two variables. Next, to bring out the proportionality of the integral (4.1) to $\bar{P}_\mu$, it is necessary to resolve $p_2$ into the mutually perpendicular components $p_{2\perp}$, $(p_{2\perp}k/k^2)k$ and $(p_{2\perp}/\bar{P})\bar{P}$, of which the first two will again not survive the integration, the first due to the angular integration, and the second due to the antisymmetry of $k = p_1 - p'_1$ in $p_1$ and $p'_1$, just as in the last term of (4.4). The third term is explicitly proportional to $\bar{P}_\mu$, and is of course gauge invariant since $\bar{P}k = 0$. (This fact had been anticipated while writing the LHS of (4.4)). Now with the help of the results

$$p_2.\bar{P} = -\hat{m}_2 M^2 - \Delta_1/4 - \Delta'_1/4; \quad 2\hat{m}_2 = 1 - (m_1^2 - m_2^2)/M^2; \quad \bar{P}^2 = -M^2 - k^2/4, \quad (4.5)$$

it is a simple matter to integrate (4.1), on the lines of Sec.2, noting that terms proportional to $\Delta_1\Delta_2$ and $\Delta'_1\Delta_2$ will give zero, while the non-vanishing terms will get contributions only from the residues of the $\Delta_2$-pole, eq.(2.5). Before collecting the various pieces, note that the 3D gaussian wave functions $\phi, \phi'$, as well as the 3D denominator functions $D_n, D'_n$, do not depend on the time-like components of the internal 4-momenta inside the gaussian $\phi$-functions under the CIA approach [8], that had plagued an earlier study of the pion form factor [21], and had to be abandoned). To proceed further, it is now convenient to define the quantity $\bar{q}.n = p_{2n} - \hat{m}_2 \bar{P}.n$ to simplify the $\phi$- and $D_n$-functions. To that end define the symbols:

$$\begin{align*}
(q, q') &= \bar{q} \pm \hat{m}_2 k/2; \quad z_2 = \bar{q}.n/\bar{P}.n; \quad \hat{k} = k.n/\bar{P}.n; \quad (\theta_k, \eta_k) = 1 \pm \hat{k}^2/4 \quad (4.6)
\end{align*}$$

and note the following results of pole integration w.r.t. $p_{2n}$ [22]:

$$\int dp_{2n} \frac{1}{\Delta_2} \frac{1}{\Delta'_1} \frac{1}{(\Delta_1\Delta'_1)} = 2i\pi [1/D_n; 1/D'_n; 2p_{2n}/(D_n D'_n)] \quad (4.7)$$

The details of further calculation of the form factor are given in Appendix A. An essential result is the normalizer $N_n(P)$ of the hadron, obtained by setting $k_\mu = 0$, and demanding
that $F(0) = 1$. The reduced normalizer $N_H = N_n(P)P.n/M$, which is Lorentz-invariant, is given via eq.(A.9) by:

$$N_H^2 = 2M(2\pi)^3 \int d^3\hat{q} e^{-\hat{q}^2/\beta^2} \left[ (1 + \delta m^2/M^2)(\hat{q}^2 - \lambda/4M^2) + 2\hat{m}_1\hat{m}_2(M^2 - \delta m^2) \right]$$  \hspace{1cm} (4.8)

where the internal momentum $\hat{q} = (q_\perp, Mz_2)$ is formally a 3-vector, in conformity with the ‘angular condition’ [25]. The corresponding expression for the form factor is (see Appendix A):

$$F(k^2) = 2MN_H^2(2\pi)^3 e^{\frac{-(M\hat{m}_2\hat{k}/\beta)^2}{4\theta_k}} \left[ \pi \beta^2 \frac{\eta_k}{\sqrt{\theta_k}} \hat{m}_1 G(\hat{k}) \right] + [1 \Rightarrow 2]$$ \hspace{1cm} (4.9)

where $G(\hat{k})$ is defined by eqs.(A.12-13) of Appendix A.

4.2 ‘Lorentz Completion’ for $F(k^2)$

The expression (4.9) for $F(k^2)$ still depends on the null-plane orientation $n_\mu$ via the dimensionless quantity $\hat{k} = k.n/P.n$ which while having simple Lorentz transformation properties, is nevertheless not Lorentz invariant by itself. To make it explicitly Lorentz invariant, we shall employ a simple method of ‘Lorentz completion’ which is merely an extension of the ‘collinearity trick’ employed at the quark level, viz., $P_\perp.q_\perp = 0$; see eq.(2.1). Note that this collinearity ansatz has already become redundant at the level of the Normalizer $N_H$, eq.(4.8), which owes its Lorentz invariance to the integrating out of the null-plane dependent quantity $z_2$ in (4.8). This is of course because $N_H$ depends only on one 4-momentum (that of a single hadron), so that the collinearity ansatz is exactly valid. However the form factor $F(k^2)$ depends on two independent 4-momenta $P, P'$, for which the collinearity assumption is non-trivial, since the existence of the perpendicular components cannot be wished away! Actually the quark-level assumption $P_\perp.q_\perp = 0$ has, so to say, got transferred, via the $\hat{q}$-integration in eq.(4.9), to the hadron level, as evidenced from the $\hat{k}$-dependence of $F(k^2)$; therefore an obvious logical inference is to suppose this $\hat{k}$-dependence to be the result of the collinearity ansatz $P_\perp.P'_\perp = 0$ at the hadron level. Now, under the collinearity condition, one has

$$P.P' = P_\perp.P'_\perp + P.nP'.\tilde{n} + P'.nP.\tilde{n} = P.nP'_n + P'.nP_n; \hspace{1cm} P.\tilde{n} \equiv P_n.$$ \hspace{1cm} (4.10)

Therefore ‘Lorentz completion’ (the opposite of the collinearity ansatz) merely amounts to reversing the direction of the above equation by supplying the (zero term) $P_\perp.P'_\perp$ to a 3-scalar product to render it a 4-scalar! Indeed the process is quite unique for 3-point functions such as the form factor under study, although for more involved cases (e.g., 4-point functions), further assumptions may be needed.

In the present case, the prescription of Lorentz completion is relatively simple, being already contained in eq.(4.10). Thus since $P, P' = \tilde{P} \pm k/2$, a simple application of (4.10) gives

$$k.nk_n = +k^2; \hspace{1cm} \tilde{P}.n\tilde{P} = -M^2 - k^2/4; \hspace{1cm} \hat{k}^2 = \frac{4k^2}{4M^2 + k^2} = 4\theta_k - 4 = 4 - 4\eta_k$$ \hspace{1cm} (4.11)

This simple prescription for $\hat{k}$ automatically ensures the 4D (Lorentz) invariance of $F(k^2)$ at the hadron level. (It may be instructive to compare this to the KK [7] prescription
of ‘recognizing’ the $n$-dependent terms (unphysical) of $F(k^2)$ and then dropping them. For more involved amplitudes (e.g., 4-point functions) too, this prescription works fairly unambiguously, if their diagrams can be analyzed in terms of more elementary 3-point vertices (which is often possible). We hasten to add however that strictly speaking, a ‘Lorentz completion’ goes beyond the original premises of restricting the (pairwise $qq$) interaction to the covariant null-plane (in accordance with the Transversality Principle [8-10]), but such ‘analytic continuations’ are not unwarranted. Indeed a careful scrutiny of the KK [7] prescription for implementing the angular condition [25] would reveal the introduction of ‘derivative’ terms, implying a tacit enlargement of the Hilbert space beyond the null-plane (see Chap 2 of [7]).

### 4.3 QED Gauge Correction to $F(k^2)$

While the ‘kinematic’ gauge invariance of $F(k^2)$ has already been ensured in the main Sec.4 above, there are additional contributions to the triangle loops - figs.1a and 1b of [19] - obtained by inserting the photon lines at each of the two vertex blobs instead of on the quark lines themselves. These terms arise from the demands of QED gauge invariance, as pointed out by Kisslinger and Li (KL) [34] in the context of two-point functions, and are simulated by inserting exponential phase integrals with the e.m. currents. However, this method (which works ideally for point interactions) is not amenable to extended (momentum-dependent) vertex functions, and an alternative strategy is needed, as described in the context of the e.m. self-energy problem of the baryon [35]. We briefly recapitulate the steps.

The way to an effective QED gauge invariance lies in the simple-minded substitution $p_i = e_i A(x_i)$ for each 4-momentum $p_i$ (in a mixed $p, x$ representation) occurring in the structure of the vertex function. This amounts to replacing each $q_\mu$ occurring in $\Gamma(q) = D(q)\phi(q)$, by $\hat{q}_\mu = -e_j A_\mu$, where $e_q = \hat{m}_2 e_1 - \hat{m}_1 e_2$, and keeping only first order terms in $A_\mu$ after due expansion. Now the first order correction to $q^2$ is $-e_q \hat{q}.A - e_q \hat{A}.\hat{q}$, which simplifies on substitution from eq.(2.1) to

$$-2e_q \hat{q}.A \equiv -2e_q A_\mu [\hat{q}_\mu - \hat{q}.\hat{n}\hat{n}_\mu + P.\hat{n}\hat{q}.\hat{n}_{\mu}/P.n]$$

(4.12)

The net result is a first order correction to $\Gamma(q)$ of amount $e_q j(q)A$ where

$$j(q)_\mu = -4M_\mu \hat{q}_\mu \phi(q) (1 - (q^2 - \lambda/4M^2)/2\beta^2)$$

(4.13)

The contribution to the P-meson form factor from this hadron-quark-photon vertex (4-point) now gives the QED gauge correction to the triangle loops, figs.(1a,1b) of [19], to the main term $F(k^2)$, eq.(4.1), of an amount which, after a simple trace evaluation (and anticipating the vanishing of all $\Delta$-terms remaining in the trace, as a result of contour integration over $q_\mu$) simplifies to $(\phi = \phi(q))$ etc.

$$F_1(k^2) = 4(2\pi)^4 N_\mu^2 e_q \hat{m}_1 M_\mu^2 \int d^4q (M^2 - \delta m^2)\phi\phi' [\frac{D^\prime q.\hat{P}}{\Delta_1\Delta_2 P.n} + \frac{D q.\hat{P}}{\Delta_1\Delta_2 P.n}] + [1 \Rightarrow 2];$$

(4.14)

In writing down this term, the proportionality of the current to $2\hat{P}_\mu$ has been incorporated on both sides, on identical lines to that of (4.1), using results from (4.2-4.7) as well as from Appendix A. Note that $e_q$ is antisymmetric in ‘1’ and ‘2’, signifying a change of
sign when the second term $[1 \Rightarrow 2]$ is added to the first. The term $\hat{q}.\hat{P}/P^2$ simplifies to $2q.n(1 - \hat{k}/2)/P.n$, after extracting the proportionality to $\hat{P}_n$. Next, after the pole integrations over $q_n, q'_n$ in accordance with (4.7), it is useful to club together the results of photon insertions on both blobs for either index ('1' or '2'); this step generates two independent combinations for the '1' terms (and similarly for '2' terms):

$$A_n = q.n(1 - \hat{k}/2); \quad B_n = q.n(1 - \hat{k}/2)(\hat{q}^2 - \lambda/4M^2)/2\beta^2$$  (4.15)

Collecting all these contributions the result of $q_n$-integration is

$$F_1(k^2) = 8(2\pi)^3N_H^2\epsilon q\hat{m}_1M_2^2\int d^3\hat{q}(M^2 - \delta m^2)\phi'[\frac{A_n + A'_n - B_n - B'_n}{\eta_k \times (P.n)^2}] + [1 \Rightarrow 2]$$  (4.16)

The rest of the calculation is routine and follows closely the steps of Appendix A for the (main) $F(k^2)$ term, including the translation $z_2 \rightarrow z_2 + \hat{m}_2\hat{k}/2\theta_k$, and is omitted for brevity. The final result for $F_1(k^2)$ is

$$F_1(k^2) = -\epsilon q\hat{m}_1\hat{m}_2(3\eta_k + \hat{k}^2)[\frac{(M^2 - \delta m^2)\eta_k\hat{k}^4(M_2\hat{m}_2)^2}{8G(0)\theta_k^2/\beta^2}] + [1 \Rightarrow 2]$$  (4.17)

where we have dropped some terms which vanish on including the $[1 \Rightarrow 2]$ terms, noting the $(1,2)$ antisymmetry of $\epsilon_q$.

### 4.4 Large and Small $k^2$ Limits of Form Factor

We close this section with the large and small $k^2$ limits of the form factors $F(k^2)$ and $F_1(k^2)$. For large $k^2$, eq.(4.11) gives $k = 2$, $\theta_k = 2$, and $\eta_k = 4M^2/k^2$, so that

$$F(k^2) = 2MN_H^2(2\pi)^3\hat{m}_1\frac{4M^2}{k^2}(\pi\beta^2/2)^{3/2}G(\text{inf})exp[-(M\hat{m}_2/\beta)^2/2] + [1 \Rightarrow 2]$$  (4.18)

where, from eqs.(A.11-12),

$$G(\text{inf}) = (1 + \delta m^2/M^2)(\beta^2 - \lambda/4M^2 + M^2\hat{m}_2^2) + (M^2 - \delta m^2)\hat{m}_2 - 2\hat{m}_2M^2$$  (4.19)

Similarly from eq.(4.14), the large $k^2$ limit of $F_1(k^2)$ is

$$F_1(k^2) = 2\sqrt{2}M^2k^{-2}\epsilon q\hat{m}_1\hat{m}_2(M^2 - \delta m^2)[\frac{M^2(\hat{m}_1 - \hat{m}_2)}{\beta^2G(0)}]$$  (4.20)

where we have taken account of the $(1,2)$ antisymmetry of $\epsilon_q$ in simplifying the effect of the $[1 \Rightarrow 2]$ term on the RHS. As a check, both $F(k^2)$ and $F_1(k^2)$ are seen to satisfy the ‘scaling’ requirement of a $k^{-2}$ variation for large $k^2$. This result can be traced to the input dynamics of the (non-perturbative) gluonic interaction, eq.(3.2), on the structure of the vertex function, eq.(3.14). Perturbative QCD of course gives a $k^{-2}$ behaviour [31]. The covariant NP approach of KK [7] also gives a similar behaviour, but extracted in a somewhat different way from the present ‘Lorentz completion’ treatment. Note that for the pion case the QED gauge correction term $F_1(k^2)$ gives zero contribution in the large $k^2$ limit.
For small $k^2$, on the other hand, we have from eq.(4.11)

$$\hat{k}^2 = k^2/M^2; \quad [\theta_k, \eta_k] = 1 \pm k^2/4M^2 \tag{4.21}$$

In this limit, the form factor, after substituting for $N_H$ from (4.8), and summing over the ‘1’ and ‘2’ terms, works out as

$$F(k^2) = (1 - 3k^2/8M^2)[1 - \hat{m}_1\hat{m}_2(k^2/(4\beta^2) - k^2\delta m^2/M^2) - 3k^2\beta^2(1 + \delta m^2/M^2)] \frac{1}{8M^2G(0)} \tag{4.22}$$

where $G(0)$ is formally given by eq.(A.10), except for the replacement of $\delta^2$ by $3\beta^2/2$. As a check, $F(k^2)$ is symmetrical in ‘1’ ‘2’, as well as satisfies the consistency condition $F(0) = 1$. Similarly the small $k^2$ value of $F_1(k^2)$, after taking account of the (1, 2) antisymmetry of $e_q$, is of minimum order $k^4$, so that it contributes neither to the normalization ($F_1(0) = 0$), nor to the P-meson radius.

For completeness we record some numerical results for large and small $k^2$ limits. For the pion case, in the large $k^2$ limit, eqs.(4.12-13) yield after a little simplification the simple result

$$F(k^2) = C/k^2; \quad C \equiv 2\sqrt{2} M_\pi^2 G(0)(\beta^2 + m_q^2)e^{-M_\pi^2/8\beta^2} \tag{4.23}$$

where $m_q = 265 MeV$ stands for $m_1 = m_2$; and $M_\pi = maxm_1 + m_1, M$. Substituting for $\beta^2 = 0.0603GeV^2$ [18] and $G(0) = 0.166 GeV^2$, yields the result $C = 0.35 GeV^2$, vs the expt value of $0.50 \pm 0.10$ [28b]. For comparison, we also list the perturbative QCD value [31] of $8\pi\alpha_s f_\pi^2 = 0.296 GeV^2$, with $f_\pi = 133 MeV$, and the argument $Q^2$ of $\alpha_s$ taken as $M_\pi^2$.

For low $k^2$, eqs.(4.14-15) yield values of the pion and kaon radii, in accordance with the relation $<R^2> = -\nabla k^2 F(k^2)$ in the $k^2=0$ limit. Substitution of numerical values from (3.4-5) yields

$$R_K = 0.629 fm (vs : 0.53 - \text{expt}[28a]); \quad R_\pi = 0.661 fm (vs : 0.656 - \text{expt}[28a]) \tag{4.24}$$

5 Discussion, Summary and Conclusion

In this paper we have tried to give an exposition of the Markov-Yukawa Transversality Principle (MYTP) [8-10] as an alternative to the traditional methods of 3D BSE reduction [1-3], vis-a-vis intrinsically 3D light-front methods [4-7]. A basic difference between the two approaches lies in the former’s capacity to provide an interconnection between the 3D and 4D BS wave functions [8], via the facility of a reconstruction of the 4D form in terms of the 3D ingredients, while the latter gives at most a one-way connection, viz., the 3D form as an integral over the 4D BS amplitude [7], but not vice-versa. Thus MYTP can do with the normal 4D Feynman rules to govern the construction of various quark loop integrals [8,16] where off-shell momenta are allowed their full play, while the intrinsically 3D BSE/LF approaches [1-7] (with on-shell momenta) must rely upon essentially 3D Feynman diagrams (with their special rules [7,23]).

To push this obvious advantage of MYTP in facilitating a two-tier strategy, viz., i) the 3D BSE form for hadron spectroscopy, and ii) the reconstructed 4D BS vertex function for accessing various types of 4D quark loop integrals, (while retaining formal covariance), we have attempted in this paper to extend its scope from the initial covariant instantaneity
ansatz (CIA) [8] (in company with the Pervushin group [9]), to a much wider domain. Now the CIA [8] is explicitly Lorentz covariant, but the hadron-momentum dependence of each $Hq\bar{q}$ vertex function causes a ‘Lorentz mismatch’ among these in an arbitrary quark loop integral, and thus induces the appearance of time-like momentum components in the (gaussian) wave functions involved, which results in ‘poorly defined’ 4D integrals. (This pathology is reminiscent of a similar problem first encountered by Feynman et al in the famous FKR paper [36]). One just escapes this pathology in 2-quark loops [18], but it reappears in 3-quark loops [21] and above. A possible remedy lies in making the covariant 3D treatment less dependent (than in CIA) on the individual hadron momentum frames. It is precisely to meet this objective that the present approach of extending MYTP from the individual hadron frames to a more universal null-plane frame was conceived, so as to eliminate such unwanted (time-like) momentum components (responsible for the ill-defined loop integrals).

Looking back on this strategy, Sec 4 on the P-meson form factor already shows that the idea has worked, except for the null-plane (NP) orientation dependence. This is not a basically new result, for a simple-minded, conventional NP approach [19,22] to BS dynamics had already produced many results of this paper, both on spectroscopy [17] as well as on transition amplitudes [19], but had been criticized [20] on grounds of ‘non-covariance’. The present treatment with an explicit formulation of Markov-Yukawa Transversality (MYTP) [10-8] on a covariant null plane (CNP), hopefully, keeps both the advantages, since the 4D loop integrals, as Sec.4 shows on the form factor calculations, are not only perfectly well-defined, but even a good part of the $n_\mu$ dependence has got eliminated in the process of $\hat{q}$ integration, while the remaining NP orientation dependence has been transferred to the external (hadron) 4-momenta. In this regard the present approach is already in the company of a wider light-front community [7, 23] which has also to contend with some $n_\mu$ dependence. The solution we have offered to overcome this problem is a simple-minded prescription of ‘Lorentz completion’ wherein a ‘collinear frame’ ansatz $P_\perp q_\perp = 0$ is lifted on the external hadron momenta $P, P'$ etc, after doing the internal $\hat{q}$ integration, so as to yield an explicitly Lorentz-invariant result. The prescription, though different from KK [7], is nevertheless self-consistent, at least for 3-point hadron vertices, (and amenable to extension to higher-point vertices provided the latter can be expressed as a combination of simpler 3-point vertices).

In Sec 2 we have also offered a detailed comparison of the present method with the covariant Light Front (LF) approach of KK [7]. In particular, the angular condition [25] seems to be almost trivially satisfied as seen from the rotational invariance of the 3D BSE structure (Secs 2-3). And the manifest covariance of the present approach, with a second NP variable $\bar{n}_\mu$ which is a natural “dual” partner of $n_\mu$ (without a counterpart in other light-front approaches [7, 23]), has obviated the need for 3-vectors and/or Lorentz transformations [7] to meet the same ends.

To summarise, the result of invoking MYTP on a covariant NP has been two-fold: retaining the formal covariance of CIA [8,16], and avoiding the appearance of time-like components inside the loop integrals. Further, its general self-consistency is evidenced by its capacity to reproduce most of the results of the old-fashioned NP approach [19,22] on the one hand, and providing identical results to CIA [8,16] on the spectroscopy front [17] on the other. The results on the P-meson form factor $F(k^2)$ are also on expected lines, with ‘kinematical’ gauge invariance satisfied explicitly, and the QED gauge correction $F_1(k^2)$ showing identical features. Both terms exhibit the (expected) $k^{-2}$ behaviour at large $k^2,$
while for the (equal mass) pion case, the QED correction gives zero contribution. And although the predicted value 0.35 of the constant $C$ is somewhat less than the experimental value $0.50 \pm 0.1$ [28b], it is still not below the value $8\pi\alpha_s f_\pi^2 = 0.296$ predicted by the QCD limit [31]. At the opposite (small $k^2$) limit too, the e.m. radii of the pion and kaon are in fair accord with experiment [28a]. Again the QED gauge correction does not give any change up to order $k^2$. (We would like to add parenthetically that the old-fashioned NP treatment [19] had yielded a slightly better curve for the pion form factor, but this was due to the use of the “half-off-shell” form of the NP wave function [19], which however did not come out naturally from the present ‘covariant’ treatment).

We would like to end with some remarks on the crucial role of the Markov-Yukawa Transversality Principle [10] in providing a natural access to spectroscopy [11] as an integral part of any ‘dynamical equation based’ approach merely reiterates a philosophy initiated long ago by Feynman et al [36].

**Appendix A: Derivation of $F(k^2)$ and $N_H$ for P-meson**

In this Appendix we outline the main steps to the derivation of the P-meson form factor (4.9), as well as the Normalizer (4.8), given in Sec.4 of Text. Collecting the various pieces after $p_{2n}$-pole integration, gives for (4.1)

$$F(k^2) = 2(2\pi)^3 N_n(P)N_n(P') \hat{m}_1 \int d^2q_\perp dz_2 P.n g(z_2) e^{-q_\perp^2/\theta^2} [f(z_2)] + [1 \Rightarrow 2]; \quad (A.1)$$

$$f(z_2) = M^2 \eta_k^2 \theta_k z_2^2 - z_2 k^2 \hat{m}_2 + \theta_k \hat{m}_2 \hat{k}/4; \quad (A.2)$$

$$D_n + D'_n = 4\hat{P}.n[q_\perp^2 + M^2(z_2^2 - z_2 k^2 \hat{m}_2 + \hat{m}_2 \hat{k}/4)/\theta - \lambda/4M^2]; \quad (A.3)$$

$$g(z_2) = \frac{D_n + D'_n}{4} \frac{M^2 + \delta m^2}{M^2 + k^2/4} + h(z_2); \quad (A.4)$$

$$h(z_2) = 2\hat{P}.n(\hat{m}_2 - z_2)[M^2 - \delta m^2 + \hat{m}_2 M^2(\delta m^2 - M^2 - k^2/2)/(M^2 + k^2/4)] \quad (A.5)$$

The integration over $q_\perp$ and $z_2$ are both routine, the latter with a translation $z_2 \rightarrow z_2 + \frac{1}{\theta} \hat{m}_2 \hat{k}/\theta_k$, to reduce the gaussian factor to the standard form. Note that, unlike the conventional (Weinberg) form [4] of light-front dynamics, the present 4D form which permits off-shellness of the internal momenta, does not restrict in principle the limits of $z_2$ integration. Thus after the translation, the odd-$z_2$ terms can be dropped, and $f(z_2)$ reduces to

$$f(z_2) = M^2 z_2^2 \theta_k/\eta_k^2 + (M \hat{m}_2 \hat{k})^2/4\theta_k \quad (A.6)$$

while the $g$-function is a sum of two pieces $g_1 + g_2$:

$$g_1 = \eta_k[q_\perp^2 + M^2 z_2^2/\theta_k + \frac{1}{4} M^2 \hat{m}_2 \hat{k}^2(1 + 3\hat{k}^2/4)/\theta_k^2 - \lambda/4M^2](1 + \delta m^2/M^2); \quad (A.7)$$
\[ g_2 = 2\eta_k (M^2 - \delta m^2)\hat{m}_2/\theta_k + 2(\delta m^2 - M^2 - k^2/2)\hat{m}_2^2\eta_k^2/\theta_k \]  

(A.8)

Before writing the final result for \( F(k^2) \), it is instructive at this stage to infer the normalizer \( N_H \) of the hadron, obtained by setting \( k_\mu = 0 \), and demanding that \( F(0) = 1 \). This gives after some routine steps:

\[
N_n(P)^{-2} = 2M(2\pi)^3(P.n/M)^2 \int d^3\hat{q}e^{-\hat{q}^2/\beta^2}G(0); \tag{A.9}
\]

G(0) = \[(1 + \delta m^2/M^2)(\hat{q}^2 - \lambda/4M^2) + 2\hat{m}_1\hat{m}_2(M^2 - \delta m^2)\] \tag{A.10}

where \( \hat{q} = (q_\perp, Mz_2) \) is effectively a 3-vector, in conformity with the requirements of the angular condition [7,23,25], which gives a formal meaning to its third component \( q_3 = Mq.n/P.n = Mz_2 \). The normalization factor \( N_n(P) \) is also seen to vary inversely as \( P.n \), while the multiplying integral is clearly independent of the NP-orientation \( n_\mu \).

To exhibit this \( P.n \) independence more explicitly, define a ‘reduced normalizer’ \( N_H \) which equals \( N_n(P) \times P.n/M \) and gives for \( N_H^{-2} \) the Lorentz-invariant result, eq.(4.8) of Text.

Now insert the result \( N_n(P) = MN_H/P.n \) on the RHS of (A.1), and note, via eq.(4.3), that

\[
M^2/\overline{P.n}P'.n = M^2/\overline{P.n}^2\eta_k; \quad \eta_k = 1 - \hat{k}^2/4. \tag{A.11}
\]

One now checks that the factors \( \overline{P.n} \) cancel out completely, and the evaluation of the gaussian integrals leads after a modest algebra to eq.(4.9) of Text, where \( G(\hat{k}) \), after collecting from eqs.(A.6-8), is given by

\[
G(\hat{k}) = (1 + \delta m^2/M^2)h(\hat{k}) + 2(M^2 - \delta m^2)\hat{m}_2/\theta_k + 2\hat{m}_2^2\eta_k\theta_k^{-1}(\delta m^2 - M^2 - k^2/2); \tag{A.12}
\]

\[
h(\hat{k}) = (1 + \eta_k^2/2\theta_k)\beta^2 - \lambda/4M^2 + (M\hat{m}_2\hat{k}/2\theta_k)^2(1 + 3\hat{k}^2); \quad \delta m = m_1 - m_2. \tag{A.13}
\]

REFERENCES

[1] M. Levy, Phys.Rev.88, 72 (1952); A.Klein, ibid 90, 1101 (1953).
[2] A. Logunov and A.N.Tavkhelidze, Nuovo Cimento 29, 380 (1963).
[3] R. Blankenbecler and R. Sugar, Phys.Rev.142, 105 (1966); F.Gross, ibid 1448.
[4] S. Weinberg, Phys.Rev.150, 1313 (1966)
[5] V. Kadychevsky, Nucl.Phys.B6, 125 (1968);
[6] V.A. Karmanov, Nucl.Phys.B166,378 (1980);
[7] Review: J.Carbonell et al, Phys.Rep.400, 215 (1998).
[8] A.N. Mitra and S. Bhatnagar, Int.J.Mod.Phys.A7, 121 (1992).
[9] Yu. L. Kalinowski et al(Pervushin Group), Phys.Lett.B231, 288 (1989).
[10] M.A. Markov, Sov.J.Phys.3, 452 (1940); H. Yukawa, Phys.Rev.77, 219 (1950).
[11] Particle Data Group, Phys.Rev.D54, July 1-Part I (1996).
[12] Review: C.D. Roberts et al, Prog.Part.Nucl.Phys.33, 471 (1994).
[13] Y. Nambu and G. Jona-Lasinio, Phys.Rev.122, 345 (1961).
[14] S.L. Adler and A.C. Davies, Nucl.Phys.B244, 469 (1984).
[15] H.D. Politzer, Nucl.Phys.B117, 397 (1976).
[16] A.N. Mitra and B.M. Sodermark, Int.J.Mod.Phys.A9, 915 (1994).
[17] a) A. Mittal et al, Phys.Rev.Lett.57, 290 (1986); b) K.K. Gupta et al, Phys.Rev.D42, 1604 (1990); c) A. Sharma et al, Phys.Rev.D50, 454 (1994).
[18] A.N.Mitra and K.-C. Yang, Phys.Rev.C51, 3404 (1995); A.N.Mitra, Int J Mod Phys A11, 5245 (1996).
[19] A.N.Mitra, A. Pagnamenta and N.N. Singh, Phys.Rev.Lett.D59, 2408 (1987); N.N. Singh and A.N. Mitra, Phys.Rev.38, 1454 (1988).
[20] C.R. Ji and S. Cotanch, Phys.Rev.Lett.64, 1484 (1990).
[21] S.R. Chaudhury et al, Delhi Univ. Preprint (1991) – Unpublished
[22] Review: S. Chakrabarty et al, Prog.Part.Nucl.Phys.22, 43-180 (1989).
[23] E.g., R.J. Perry, A. Harindranath and K. Wilson, Phys.Rev.Lett.65, 2959 (1990).
[24] A.N. Mitra, Zeits.f.Phys.C8, 25 (1981); A.N. Mitra and I. Santhanam, Few-Body Syst.12, 41 (1992).
[25] H. Leutwyler and J. Stern, Ann. Phys. (N.Y.) bf 112, 94 (1978).
[26] A.N. Mitra LANL hep-th/9803062; Intl.J.Mod.Phys.A14; in press.
[27] E.g., R. Barbieri and E. Rimitti, Nucl.Phys.B141, 413 (1978); G.P. Lepage, SLAC Preprint no. 212 (1978).
[28] (a) E.B. Daley et al, Phys.Rev.Lett.45, 232 (1980); (b) C. Bebec et al, Phys.Rev.D17, 1793 (1978); (c) A. Bramon, M. Greco in II Daphne Phys. Handbook 2 (1997) 451-466
[29] V. Chernyak and A. Zitnitsky, Phys.Rep.112C, 174 (1984).
[30] K.G. Chetyrkin et al INR Report no.P-0395, Moscow, 1985.
[31] G. Farrar and D. Jackson, Phys.Rev.Lett.43, 246 (1979); G.P. Lepage and S.J. Brodsky, Phys Rev.D22, 2157 (1980).
[32] M. Burkardt et al, Phys.Rev.Lett.78, 3059 (1997).
[33] N. Isgur and C.H.L. Smith, Nucl.Phys.B317, 526 (1989); M. Sawicki, Phys.Rev.D46, 474 (1992).
[34] L.S. Kisslinger and Z. Li, Phys.Rev.Lett.74, 2168 (1995).
[35] A. Sharma and A. N. Mitra, Intl.J.Mod.Phys.A14 1999; in press.
[36] R.P. Feynman et al, Phys.Rev.D3, 2706 (1971).