Bosonic Open Strings in a Tachyonic Background Field

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Abstract

We study a bosonic open string coupled to a tachyonic background field $T[X]$ and find that the tachyon field can effectively be replaced by a configuration of D-branes placed at either the zeros or the critical points of $T[X]$, depending on our choice of boundary conditions. This dual picture of the open string tachyon is explored in detail for the explicit case when the tachyonic field is quadratic.

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1 Introduction and motivation

Recent interest in the construction of non-BPS branes [1] has led to the investigation of relationships between Dp-branes in which the tachyonic field figures prominently. In particular, it has been established that a D-brane can be thought of as the solitonic solution of a tachyonic field living on a higher dimensional brane-antibrane system [1]. Sen showed that a $D(p-1)$-brane can be identified with the tachyonic kink on a $Dp$-$\bar{D}p$ system, or alternatively, with the tachyonic lump on a single Dp-brane [2]; the latter picture is supported by calculations in string bosonic field theory [3]. Subsequently, in [5], these lumps were interpreted in terms of relevant boundary perturbations of the world-sheet CFT. To illustrate some of the salient features of this description, we consider a model in which a bosonic open string couples to a tachyonic background field $T[X]$ (in flat Minkowski space-time), and look for the consistent open string configurations which may exist.

The paper is organized as follows: We start in Section 2 with an arbitrary tachyonic background field and a diffeomorphism invariant world-sheet action, analyzing all possible situations in which a string interpretation can be obtained. World-sheet parity plays a critical role in the analysis. We find that locally, D-branes can be associated with either tachyonic kinks or lumps. To make these ideas explicit, we focus on a quadratic tachyonic profile in Section 3. This is an instructive example as it illustrates the main features under discussion, yet is a model which can be solved explicitly. In Section 4, we synthesize the results of our analysis. The Appendix contains a brief outline of how one might deal with a more general profile.

2 A General Analysis

Consider a bosonic open string moving in a flat background with metric $\eta_{\mu\nu} = (-1, 1, \ldots, 1)$ and coupling to a tachyonic profile $T[X]$. The diffeomorphism invariant action is:

$$S = -\frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma \sqrt{-h} \delta_{\alpha\beta} \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu} \eta_{\mu\nu} + \int_{\partial\Sigma} d\tau \sqrt{-h_{00}} T[X].$$  \hspace{1cm} (2.1)

This model is Weyl invariant in the bulk but not at the boundary

$$h_{\alpha\beta} \rightarrow \Lambda(\sigma) h_{\alpha\beta}, \quad \Lambda|_{\partial\Sigma} = 0.$$ \hspace{1cm} (2.2)

Despite the fact that the boundary term does not preserve Weyl invariance, we still have enough symmetries to impose conformal gauge, since the metric dependence in the boundary

\hspace{1cm} \footnote{The analysis can be extended to a tachyonic field depending on more than one spatial directions.}
term can be completely gauged away using only diffeomorphism invariance. In conformal gauge the constraints are given by:

\[ T_{00} = \frac{1}{2}(\dot{X}^2 + X'^2) + 2\pi\alpha'[\delta(\sigma)T[X] + \delta(\pi - \sigma)T[X]] = 0, \quad (2.3) \]

\[ T_{01} = T_{10} = \dot{X} \cdot X' = 0, \quad T_{11} = \frac{1}{2}(\dot{X}^2 + X'^2) = 0. \quad (2.4) \]

Consistency of the model forces us to require that along with the standard Virasoro conditions \((\dot{X} \pm X')^2 = 0\), the condition \([\delta(\sigma)T[X] + \delta(\pi - \sigma)T[X]] = 0\) also be fulfilled classically, restoring Weyl invariance in the original action. Re-writing this as \(T[X]|_{\sigma=0,\pi} = 0\), we see that it is equivalent to imposing Dirichlet boundary conditions on \(X^i\) in such a way that string end-points are confined to zeros of \(T[X]\).

Since the tachyonic field does not couple to the bulk, the equation of motion is unchanged and we have, as usual, \(\partial_\alpha \partial^\alpha X^\mu = 0\). However, the boundary term does change and we can now impose either Dirichlet or "modified Neumann" boundary conditions

\[ \delta X^\mu|_{0,\pi} = 0, \quad (\eta_{\mu\nu}X'^\nu - 2\pi\alpha'\partial_\mu T[X])|_{\sigma=0,\pi} = 0. \quad (2.5) \]

Consider a tachyonic field which depends on coordinates \(X^i\) \((i = p+1, \ldots, d-1)\). For the sake of simplicity, we impose Neumann boundary conditions on directions \(X^n\), \((n = 0, 1, \ldots, p)\) transverse to the profile. Along \(X^i\) however, we have Dirichlet boundary conditions and the following mode expansion

\[ X^i(\tau, \sigma) = q^i + b^i_0\sigma + \sum_{n \neq 0} e^{-in\tau} b^i_n \sin n\sigma \quad (2.6) \]

The positions of the Dp-branes are given by \(X^i|_0 = q^i\) and \(X^i|_\pi = q^i + b^i_0\pi\), where \(q^i\) and \(b^i_0\) satisfy \(T[q] = 0 = T[q + b_0\pi] = 0\). Hence, zeros of a tachyonic profile can be thought of as positions of Dp-branes which span the p directions transverse to the tachyon field. This is rather remarkable statement as it enables us to 'replace' a tachyon field with a number of D-p branes in the appropriate positions, an interpretation supported by results obtained in the closed string channel.

To illustrate this point we consider a tachyonic field depending on only one spatial direction, and proceed to construct the bosonic boundary state \(\langle B_X \rangle\). Open and closed string channels are related through the following correspondence:

\[
\begin{align*}
\partial_\tau X^n|_{\sigma=0} = 0 & \quad \Rightarrow \quad \partial_\tau X^n|_{\tau=0}B_X = 0 \\
T[X]|_{\sigma=0} = 0 & \quad \Rightarrow \quad T[X]|_{\tau=0}B_X = 0.
\end{align*}
\quad (2.7)
\]
One can solve these conditions and construct the boundary state

\[ |B_X\rangle = N \sum_{q_n} \frac{1}{T'[q_n]} \delta(q - q_n) \left( \prod_{n=1}^{\infty} e^{-\frac{1}{2}q_n S a_n} \right) |0\rangle_a |0\rangle_{\tilde{a}} |p = 0\rangle. \quad (2.8) \]

Here, \( N \) is a normalization constant (given in [6], eq.(6.180)) and we have used the following identity

\[ \delta(f(x)) = \sum_n \frac{\delta(x - x_n)}{|f'(x_n)|} \]

where \( f(x) \in C^\infty \) and \( x_n \) are simple zeros of \( f(x) \). Thus at every simple zero of \( T[X] \) we can place one D p-brane.

While the positions of the branes are dictated by zeros of the field, the number of D p-branes at a particular position is determined by the order of the zero. As a simple illustration of the above statement, consider \( T[X] \sim (X - X_0 + \epsilon)(X - X_0 - \epsilon) \). There is now one D p-brane each localised at \( (X_0 - \epsilon) \) and \( (X_0 + \epsilon) \), with a U(1) gauge theory on its world-volume. As \( \epsilon \to 0 \) the Dp branes approach each other, meeting at \( X_0 \), so we end up with 2 coincident branes and a U(2) world-volume gauge theory. Reversing this logic, we can separate 2 coincident branes and thus break gauge symmetry from \( U(2) \) to \( U(1) \times U(1) \). The masses of the Higgs and W-bosons will be determined by the form of the tachyon field, or more precisely, the distance between its zeros.

Now consider the modified Neumann boundary conditions in (2.5). In general they break conformal invariance, ruling out the possibility of a string interpretation. However one can follow the logic proposed by Harvey et.al [3] and think of the boundary interaction as a relevant operator which induces a world-sheet renormalization group flow such that the string picture is recovered at the IR and UV fixed points.

We apply this procedure to our theory, defining coordinates \( \phi^\mu = X^\mu / \sqrt{2\pi\alpha'} \) which enable us to canonically normalize the scalar in two dimensions. In conformal gauge, boundary interactions have the following form

\[ \pm \lambda \int d\tau V(\phi)|_0 \pm \lambda \int d\tau V(\phi)|_\pi \]

(2.10)

where \( \lambda \) is a dimensionful coupling constant\(^5\) and \( V(\phi) \) is a dimensionless potential given by \( \lambda V(\phi) = T(\sqrt{2\pi\alpha'} \phi) \). A priori there are no restrictions on the signs in front of boundary couplings in (2.10) so we have four possibilities : \((+, +), (+, -), (-, +) \) and \((-,-)\). Each choice corresponds to a different action and hence a different theory. We use these signs to label the theories. Boundary conditions for theories \((+, +)\) and \((+, -)\) are

\[ (\eta_{ij} \phi^{ij} + \lambda \partial_i V(\phi))|_0 = 0, \quad (\eta_{ij} \phi^{ij} \mp \lambda \partial_i V(\phi))|_\pi = 0 \]

(2.11)

\(^5\)Generically there may be many coupling constants, however for the time being we restrict ourselves to the case where there is only one
respectively, whereas for theories \((-, -)\) and \((-, +)\) we find
\[
(\eta_{ij}\phi'^j - \lambda \partial_i V(\phi))|_{\sigma = 0} = 0, \quad (\eta_{ij}\phi'^j \pm \lambda \partial_i V(\phi))|_{\pi} = 0
\]
Theories \((+, +)\) and \((-, -)\) are invariant under worldsheet parity \(\Omega (\sigma \rightarrow \pi - \sigma)\) while theories \((+, -)\) and \((-, +)\) are interchanged. Since \(\Omega\) is a global symmetry of open string theory and is expected to be preserved, we restrict future analysis to the parity invariant theories only.

The parameter \(\lambda\) has dimension of mass, so one can naively consider \(\lambda V(\phi)\) as a relevant boundary perturbation\(^6\). Thus, in each theory, we have some massless scalars in the bulk and a relevant boundary perturbation which breaks conformal invariance explicitly. At the fixed points we find the following boundary conditions
\[
\phi''|_{\sigma = 0, \pi} = 0 \text{ at } \lambda = 0, \quad \partial_{\sigma} V(\phi)|_{\sigma = 0, \pi} = 0 \text{ at } \lambda = \infty.
\]
Again, we can interpret the latter as Dirichlet boundary conditions. So the relevant boundary perturbation induces a RG flow from the UV fixed point with Neumann boundary conditions, to the IR fixed point where we have D p-branes localized at the critical points of \(V\); these could be maxima or minima depending on the theories under consideration. In the Dirichlet case, we require that \(V(\phi) = 0\) at the endpoints of the string, as this is the only way to restore Weyl invariance in the theory. For the modified Neumann case this condition is no longer needed, since Weyl invariance is restored anyway at the IR and UV fixed points.

To summarize then, we find that Dirichlet conditions along \(X^i\) lead to Dp-branes localized at zeros of \(T[X]\) and resembling tachyonic kinks, whereas modified Neumann boundary conditions lead to Dp-branes which are localized at the maxima/minima of \(T[X]\) and are reminiscent of tachyonic lumps.

Since Dp-branes can be localized either at minima or at maxima of \(V(\phi)\) in the IR, it is important that perturbation theory be well defined for both cases. Let us argue that this is true for a boundary potential. For the sake of simplicity we drop space-time indices. The Minkowski world-sheet action for the \((+, +)\) theory can then be expressed as follows:
\[
S = \frac{1}{2} \int d^2\sigma [\dot{\phi}^2 - (\phi')^2] + \lambda \int d\tau V(\phi)|_0 + \lambda \int d\tau V(\phi)|_\pi,
\]
In order to use the path integral formalism we make the Wick rotation \(\tau = it\). Assuming that Taylor expansion is valid around the extrema \(\phi_0\) and \(\phi_1\) where the ends of the string are localized, we have
\[
V(\phi) = \sum_{n=0}^{\infty} \frac{1}{n!} V^{(n)}(\phi_0)(\phi - \phi_0)^n \text{ for } \sigma = 0 \quad \text{and} \quad V(\phi) = \sum_{n=0}^{\infty} \frac{1}{n!} V^{(n)}(\phi_1)(\phi - \phi_1)^n \text{ for } \sigma = \pi.
\]
\(^6\)In general one should be careful in determining whether an operator is relevant or not; a relevant operator should have conformal weight less than one.
The Euclidean action can then be written as

\[-iS = -\frac{1}{2} \int d^2 \sigma \phi \Box \phi + \lambda V(\phi_0) \beta + \lambda V(\phi_1) \beta - \frac{\lambda}{2} \sum_{n=1}^{\infty} \frac{(n-2)}{n!} V^{(n)}(\phi_0) \int dt (\phi - \phi_0)^n|_0 \]

\[-\frac{\lambda}{2} \phi_0 \sum_{n=1}^{\infty} \frac{1}{(n-1)!} V^{(n)}(\phi_0) \int dt (\phi - \phi_0)^{n-1}|_0 - ... \quad (2.16)\]

where ... denotes a similar contribution from the boundary at \(\pi\). To avoid IR singularities we introduce finite temperature \(\beta = 1/T\) and consider the theory on an annulus.

Dropping \(V(\phi_0)\) and \(V(\phi_1)\) and taking into account that \(V'(\phi_0) = 0\) and \(V'(\phi_1) = 0\) we find that second derivatives appear in the following terms

\[-\frac{\lambda \beta}{2} \left[ \phi_0^2 V^{(2)}(\phi_0) + \phi_1^2 V^{(2)}(\phi_1) \right] + \frac{\lambda}{2} \left[ \phi_0 V^{(2)}(\phi_0) \int dt \phi|_0 + \phi_1 V^{(2)}(\phi_1) \int dt \phi|_\pi \right]. \quad (2.17)\]

If both ends of the 'string' lie at the same critical point, \(\phi_0 = \phi_1\), the field \(\phi\) can be shifted in such way that \(V^{(2)}\) term drops out completely (contrary to what happens with a bulk potential) so locally there is no way to distinguish between maxima and minima or to prefer one over the other

If we consider the case when the 'string' is stretched between two different critical points, \(\phi_0 \neq \phi_1\), second derivatives can be important. In particular, the sign of \(\phi_0^2 V^{(2)}(\phi_0) + \phi_1^2 V^{(2)}(\phi_1)\) should be negative to prevent the exponential from blowing up as \(\lambda \to \infty\). In other words, perturbations around two minima will lead to trouble, perturbations around a maximum and a minimum will be subject to constraints but perturbations around two maxima will always be well defined in (+, +) theory. A similar analysis can be carried out for each of the theories. In (+, −) theory for example, we would find that perturbations around a minimum-maximum configuration would always be safe.

As an alternative to the above argument, (atleast for the case of a quadratic potential), we can study the worldsheet RG flow. Demanding that this flow be free of singularities we reproduce exactly the same results as above regarding 'safe' sectors of the theory. We return to this point in the Appendix.

### 3 The Quadratic Approximation

Let us assume that \(T[X]\) is sufficiently well behaved to be expanded in a Taylor series

\[T[X] = T_0 + T_{1i} X^i + \frac{1}{2} T_{2ij} X^i X^j + \frac{1}{3} T_{3ijk} X^i X^j X^k + ..., \quad (3.18)\]

\(^7\) Ofcourse the sign of higher derivatives might still be important for consistency of the theory.
where \( X^i \) are spatial directions \((i = p + 1, \ldots, d - 1)\). The general solution of equation of motion has the following form

\[
X^\mu(\tau, \sigma) = q^\mu + a_0^\mu \tau + b_0^\mu \sigma + \sum_{n \neq 0} e^{-in\tau} (ia_n^\mu \cos n\sigma + b_n^\mu \sin n\sigma).
\] (3.19)

For transverse directions \( X^m \) \((m = 0, 1, \ldots, p)\) we have the standard solution with Neumann conditions. For \( X^i \) along the tachyon profile, we can express \( b_n^i \) as a function of \( a_n^i \). In particular, for the zero modes we have

\[
\eta_{ij} b_0^j - 2\pi \alpha' \partial_j T[q] = 0, \quad \eta_{ij} b_0^j - 2\pi \alpha' \partial_j T[q + b_0 \tau] = 0 \tag{3.20}
\]

As \( \lambda \to \infty \), the string ends become localized at extrema of \( T[X] \). There are no momentum modes \( a_0^i \) since translational symmetry is broken by the profile.

No matter what the explicit form of the tachyonic field may be globally, a quadratic approximation is always reasonable locally around the extrema. Since these are precisely the regions we are interested in, it is sufficient for our purposes to consider a tachyonic field which is quadratic in \( X^i \). For the sake of simplicity we consider the profile to depend on just one direction. One can always choose a special basis such that \( T[X] \) is brought to the form:

\[
T[X] = \pm \frac{t_i}{2} (X^i - x_i^0)(X^i - x_i^1),
\] (3.21)

where \( t_i \) are (positive) dimensionful parameters. The \( \pm \) signs correspond to perturbations around minima and maxima respectively. We want to study strings\(^8\) that stretch between the D-branes which arise in the IR, at maxima and minima of \( T[X] \).

We now solve for \( X^i \), fixing the ends of the 'string' to lie at critical points of the quadratic field \( T[X] \). In each given theory there arise four sectors, depending on whether \( \sigma = 0 \) and \( \pi \) lie at two minima, two maxima, a maximum and a minimum or vice versa. The mode expansion for \( X^i \) and the expressions for the Virasoro generators are in general \( \lambda \)-dependent; the string theory picture is recovered by taking the \( \lambda \to \infty \) limit.

We start with a detailed analysis of the theory labelled \((-,-)\). Boundary conditions are:

\[
X^i - 2\pi \alpha' \partial_i T[X]|_0 = 0 \quad \text{and} \quad X^i + 2\pi \alpha' \partial_i T[X]|_\pi = 0. \tag{3.22}
\]

Consider perturbations around a minimum and a maximum. In this sector boundary conditions become:

\[
X^i - \lambda_i(X^i - x^i)|_0 = 0 \quad \text{and} \quad X^i - \lambda_i(X^i - \tilde{x}^i)|_\pi = 0 \tag{3.23}
\]

\(^8\) Recall that conformal invariance is broken for arbitrary \( \lambda \), so a string theory interpretation is recovered only at the fixed points of the RG flow.
where \( \lambda_i = 2\pi \alpha' t_i, x^i = 1/2(X^i_0 + X^i_1) \) and \( \tilde{x}^i = 1/2(\tilde{X}^i_0 + \tilde{X}^i_1) \). and the mode expansion is

\[
X^i(\tau, \sigma) = q^i + b^i_0 \sigma + \sqrt{2\alpha'} \sum_{n>0} \frac{1}{\sqrt{n^2 + \lambda_i^2}}(\cos n\sigma + \frac{\lambda_i}{n} \sin n\sigma)[a^i_n e^{-in\tau} - (a^i_n)^* e^{in\tau}]
\]

(3.24)

where \( n \) is an integer. In the UV, \( b^i_0 = 0 \) and \( q^i \) is left undetermined. However, for \( \lambda_i \neq 0 \) we find

\[
b^i_0 = \frac{(\tilde{x}^i - x^i)}{\pi} \quad \text{and} \quad q^i = \frac{(\tilde{x}^i - x^i)}{\lambda_i \pi} + x^i.
\]

(3.25)

So as soon as \( \lambda_i \) becomes non-zero, \( b^i_0 \) is fixed. However, \( q^i \) has an explicit \( \lambda_i \) dependence, approaching \( x^i \) as \( \lambda_i \to \infty \). It is customary to include a factor of \( i \) in the mode expansion for a Neumann coordinate \( X^i \), but not for a Dirichlet coordinate. Hence, the way it stands, the above mode expansion is appropriate only for the UV; we can apply it to the IR by absorbing the factor of \( i \) in the transformation \( a^i_n \to ia^i_n \). Since this transformation is unitary, it preserves commutation relations.

Canonical quantization, together with the identity

\[
\pi \delta(\sigma - \sigma') = \sum_{n \neq 0} \frac{n^2}{n^2 + \lambda_i^2}(\cos n\sigma + \frac{\lambda_i}{n} \sin n\sigma)(\cos n\sigma' + \frac{\lambda_i}{n} \sin n\sigma'),
\]

(3.26)

results in the usual commutation relations between the modes

\[
[a^i_n, (a^j_m)^*] = \eta^{ij} n\delta_{n-m}, \quad [(a^i_n)^+, (a^j_m)^+] = [a^i_n, a^j_m] = 0.
\]

(3.27)

Hence the standard Fock space can be constructed for every value of \( \lambda_i \).

The transverse coordinates contribute to the Virasoro generators in the usual way, but the \( X^i \) contribution is modified so we write it out explicitly: The zero mode is

\[
L_0 = \bar{L}_0 = \frac{1}{4\alpha'} b^i_0 \dot{b}^i_0 + \frac{1}{2} \sum_{n \neq 0} a^i_n a^i_{-n}.
\]

(3.28)

As usual for bosonic string theory, the Hamiltonian is given by \( H = L_0 - 1 \). The Virasoro generators\(^9\) are given by:

\[
L_m = \frac{1}{\sqrt{2\alpha'}} \frac{\lambda_i - im}{\sqrt{m^2 + \lambda_i^2}} a^i_m \dot{b}^i_0 - \frac{1}{2} \sum_{n \neq m} \frac{(n + i\lambda_i)((m-n) + i\lambda_i)}{\sqrt{n^2 + \lambda_i^2} \sqrt{(m-n)^2 + \lambda_i^2}} a^i_n a^i_{m-n},
\]

(3.29)

\(^9\)The nontrivial \( n \) dependence in (3.24) comes from properly normalizing the eigenfunctions. This turns out to be crucial if we are to avoid trouble when imposing canonical commutation relations.

\(^10\)Since we are concerned with a Hermitian operator \( \partial^2_\sigma \) on an interval \([0, \pi]\) and our boundary conditions are of the required form, ours is a Sturm-Liouville problem. Properly normalized eigenvectors of \( \partial^2_\sigma \) hence span an orthonormal basis in \( L^2(0, \pi) \) and the expression (3.26) is simply the statement that this basis is complete.

\(^11\)These expressions are appropriate for \( \lambda_i \to \infty \). One can perform a unitary transformation to get the Virasoro generators which are appropriate for \( \lambda_i \to 0 \).
\[ L_m = -\frac{1}{\sqrt{2\alpha}} \frac{\lambda_i + im}{\sqrt{m^2 + \lambda_i^2}} a_m^{i\dagger} b_0^{i\dagger} - \frac{1}{2} \sum_{n \neq m} \frac{(n - i\lambda_i)((m - n) - i\lambda_i)}{n^2 + \lambda_i^2 \sqrt{(m - n)^2 + \lambda_i^2}} a_n^{i\dagger} a_{m-n}^{i\dagger}, \]  
(3.30)

and obey the relevant algebras

\[ [L_n, L_m] = (n - m)L_{n+m}, \quad [\bar{L}_n, \bar{L}_m] = (n - m)\bar{L}_{n+m}. \]  
(3.31)

However we see that \( L_m \neq L_m \) for \( m \neq 0 \), except at \( \lambda_i = 0 \) and \( \lambda_i = \infty \). This is a manifestation of the now familiar statement that Weyl invariance is broken for finite \( \lambda_i \) and is restored only at the fixed points.

We now consider perturbations around a maximum and a minimum. Boundary conditions in this sector are the same as those in the minimum-maximum sector, with the only difference is that \( \lambda_i \) is replaced by \( -\lambda_i \). Infact we find that all the results from our above discussion can be extended to this new sector of the theory, if we simply replace \( \lambda_i \) by \( -\lambda_i \) in the mode expansions and Virasoro generators. \( b_0^{i\dagger} \) is unaffected by this transformation as it has no \( \lambda_i \) dependence, but \( q^i \) is now given by

\[ q^i = \frac{(x^i - \tilde{x}^i)}{\lambda_i \pi} + x^i. \]  
(3.32)

Note that in the limit \( \lambda_i \to \infty \), \( q^i \) still approaches \( x^i \).

As we remarked earlier, perturbations around maxima-minima, or minima-maxima are related by a parity transformation. The Virasoro generators have the symmetry

\[ \lambda_i \leftrightarrow -\lambda_i, \quad \leftrightarrow \quad L_m \leftrightarrow \bar{L}_m. \]  
(3.33)

which can be interpreted as a flip in the orientation of the ’open string’ due to a change in the sign of the boundary conditions.

Now look at perturbations around two minima. Boundary conditions in this case are:

\[ X^{\prime i} - \lambda_i (X^i - \frac{1}{2}(X_0^i + X_1^i))|_0 = 0, \quad X^{\prime i} + \lambda_i (X^i - \frac{1}{2}(X_0^i + X_1^i))|_{\pi} = 0. \]  
(3.34)

The mode expansions, Virasoro generators \(^{12}\) and commutation relations are given by the same formal expressions as before, the only difference being that now \( n, q^i \) and \( b_0^{i\dagger} \) are functions of \( \lambda_i \). In particular, \( n \) is no longer an integer (except at the fixed points) and infact must satisfy the condition

\[ \tan n\pi = \frac{2\lambda_i n}{n^2 - \lambda_i^2}. \]  
(3.35)

\(^{12}\) Though we recover the standard Virasoro generators at fixed points \( \lambda_i = 0 \) and \( \lambda_i = \infty \), something slightly unusual happens for finite \( \lambda_i \). We can still define \( L_n \) and \( \bar{L}_n \) which obey the Virasoro algebra, however there are now non-trivial space-time dependent phases in the expressions for these generators. \(^{\ddagger}\)
We find that \( n(-\lambda_i) = -n(\lambda_i) \) in this parity invariant sector, so the statement \((3.33)\) does not hold here. In the UV, \( b_0^i = 0 \) and \( q^i \) is undetermined, whereas for \( \lambda_i \neq 0 \), we find

\[
q^i = \frac{\vec{x}^i + x^i(1 + \lambda_i \pi)}{(2 + \lambda_i \pi)} \quad \text{and} \quad b_0^i = \frac{\lambda_i(\vec{x}^i - x^i)}{(2 + \lambda_i \pi)}
\]

(3.36)

Though the form of the \( \lambda_i \) dependence in \( q^i \) is different to what it was in the two earlier cases, the asymptotic behaviour is the same, \( q^i \to x^i \) as \( \lambda_i \to \infty \). There is a drastic difference however, in \( b_0^i \) which now approaches \((x^i - \vec{x}^i)/\pi \) in the IR. Through this term, the Hamiltonian for a 'string' stretched between two minima of \( T[X] \) becomes \( \lambda \) dependent.

The boundary conditions for the maxima-maxima sector are related to those in the minima-minima sector by an inversion in the sign of \( \lambda_i \). In the UV we have the same result as before i.e \( b_0^i = 0 \) and \( q^i \) is undetermined. Along the flow however, we now have

\[
q^i(2 - \lambda_i \pi) = x^i(1 - \lambda_i \pi) + \vec{x}^i \quad \text{and} \quad b_0^i(2 - \lambda_i \pi) = \lambda_i(x^i - \vec{x}^i)
\]

(3.37)

which indicates a potential problem at \( \lambda_i = 2/\pi \). Inorder for the RG flow to be well defined, we must fix \( x^i = \vec{x}^i \), i.e, demand that \( q^i = x^i \) and \( b_0^i = 0 \) for all \( \lambda_i \). In other words, we are not able to obtain strings that interpolate between D-branes placed at two different maxima; the only consistent solutions that exist are for a string that begins and ends on the same brane.

The same approach can be used to study the case when \( T[X] \) depends on more than one direction and we have a set of couplings \((\lambda_{p+1}, \ldots, \lambda_{d-1})\). There are then \( 2^{d-p} \) different sectors related to each other through world-sheet parity \( \Omega \). Also, the nature of the critical points will be much more complicated.

4 Conclusions

In this paper, we look for consistent configurations of an open string in a tachyonic background. Starting with a perfectly generic \( T[X] \), we see that in the IR, D-branes should be localized either at zeros or extrema of the tachyonic field, depending on the boundary conditions we choose to impose. We then study the explicit case when \( T[X] \) is quadratic, as this is an exactly solvable model and is also a good approximation around the regions where D-branes will appear. Though the boundary conditions in the IR force us to localize D-branes at critical points of \( T[X] \), no distinction is made between maxim and minima so we consider both. However, proceeding along these lines, we find that there is a discrepancy when we put the picture together.
As we flow to the IR limit in \((-,-)\) theory, we find strings which stretch between maxima and minima, or two different minima but none that interpolate between two different maxima. Perhaps this is a signal that we allow D-branes to appear only at the minima of the tachyonic field since all possible strings which can exist in such a configuration are allowed, and the theory thus truncated becomes completely consistent. This coincides exactly with the picture proposed by Harvey et al\[5\] and supported by the g-theorem.

The discussion in the above paragraph carries over in totality to the \((+,-)\) theory, the only difference being that here there are no strings interpolating between two minima since the theories are equivalent under interchange of maxima and minima.

A Appendix

A quadratic approximation for $T[X]$ is, as we have argued, reasonable around critical points of the tachyonic field. However, the analysis in Section 3 does not fully exploit the possibilities of this approximation. We have restricted ourselves to maxima and minima which are identical in shape and size and are related simply by an inversion. This leads to the fact that second derivatives at all critical points have the same magnitude and differ only in sign. This would be the case for a sin or cos potential of course, but in general one could picture a more irregular form for $T[X]$ where the shapes of maxima and minima vary and consequently the second derivatives at the critical points may differ in their absolute values as well. We now analyse this situation, using the methods already outlined in Section 3.

Consider for example \((-,-)\) theory. Around the extrema where string ends are confined, $T[X]$ can be locally approximated by

$$
T[X] = \frac{t_i}{2} A(X^i - X_0^i)(X^i - X_1^i) \quad (\sigma = 0) \quad \text{and} \quad T[X]|_{\sigma = \pi} = \frac{t_i}{2} B(X^i - \tilde{X}_0^i)(X^i - \tilde{X}_1^i) \quad (\sigma = \pi)
$$

where $A$ and $B$ can be positive or negative corresponding to minima and maxima respectively. The boundary conditions become

$$
X^R + \lambda A(X^i - x^i) = 0 \quad \text{at} \quad \sigma = 0 \quad \text{and} \quad X^R + \lambda B(X^i - \tilde{x}^i) = 0 \quad \text{at} \quad \sigma = \pi. \quad (A.39)
$$

So $q^i$ and $b_0^i$ are given by

$$
q^i[A + B + \lambda AB\pi] = (\lambda AB\pi + A)x^i + b\tilde{x}^i \quad (A.40)
$$

and

$$
b_0^i[A + B + \lambda AB\pi] = \lambda AB(\tilde{x}^i - x^i). \quad (A.41)
$$
Hence there is a possible singularity in the flow when \([A + B + \lambda AB\pi] = 0\), unless we restrict both string ends to the same critical point. We do not however want to apply this restriction in general, since our aim is to obtain strings that stretch between two different D-branes in the IR. Keeping in mind that \(\lambda\) must always be positive, we see that the only case where there will never be a singularity, irrespective of the values of A and B, is when they are both positive.

Hence, in general for an irregularly shaped tachyonic background field, the only sector in which the renormalization group flow never develops a singularity is that which corresponds to perturbations around two minima. This extends the analysis of section 3 and strengthens the conclusions drawn in section 4. A similar analysis can be performed for the three remaining theories. In each case we find that there is just one sector which is always free from singularities; of course this sector differs from one theory to another. In each case though, it coincides exactly with the 'safe' sector that results from the path integral argument towards the end of Section 2.

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