Study on the Critical Shear Stress of Cohesive Sediments

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Abstract. Since the measurements of bulk property parameters of cohesive sediments can be done easily, it is useful to relate the critical shear stress of cohesive sediments to some of these parameters. In this paper, data from two experiments are reanalysed and the dependence of the critical shear stress on bulk density, clay solid content and clay volume content is highlighted. The results from Sharif demonstrate the critical shear stress as a function of bulk density and clay solid content, while that from Kothyari and Jain indicate that the critical shear stress depends on both the clay solid content and clay volume content of cohesive sediments in the presence of gravel and air. Moreover, it is concluded a critical clay solid content exists around 18\%, beyond which the critical shear stress increases rapidly with the increase of bulk density.

1. Introduction

A great number of works have been done on the critical shear stress for initiating movement on noncohesive bed by researchers, such as Shields [1], Miller et al. [2], Wiberg and Smith [3], and You [4]. In contrast, there is little information on the critical shear stress of cohesive sediments. The move behavior and mechanism of cohesive sediments are related to many factors, including physical parameters, chemical parameters and biological parameters [5].

A significant amount of attempts have been made to relate the critical shear stress of cohesive sediments to bulk property parameters. Villarte and Paulic (1986) found the critical shear stress as a function of bulk density [6]. Kusuda et al. (1984), Buscall et al. (1988), and Mehta and Lee (1994) investigated the dependence of the critical shear stress on the solid volume fraction [7–9]. Otsubo and Muraoka (1988), and Mignot and Hamm (1990) found relations that depended on yield stress of the sediments [10, 11]. In the systematic study of cohesive sediments, Sharif (2003) developed three theoretical models for predicting the critical shear stress of mixed soils with low clay content and high clay content, and pure cohesive soils [12]. The dependence of the critical shear stress on plasticity index, which is a function of the clay content, the type of mineral and the effect pore water chemistry, was experimentally investigated by Jacobs [13]. Xu et al. (2015) presented an empirical formula for the critical velocity of coastal muds with different densities, and found the shear stress for incipient motion increases exponentially with mud density [14]. However, a good understanding of the critical shear stress for initiating motion on cohesive bed is still lacking. This work aims at investigating the critical shear stress of cohesive sediments and its relation to bulk property parameters, based on the results of two laboratory experiments.
2. Concepts
There are two important concepts that are vital for reanalyzing the experimental data hereafter: the size fraction comprising silt (% 0.004-0.063mm) and clay (% < 0.004mm) is known as mud or fines; only the clay particles contribute to the cohesiveness of sediments [15].

3. Results and Conclusions
Data from two experiments are reanalyzed

3.1 Reanalysis on the “Critical Shera Stress and Erosion of Cohesive Soils”
In the original paper, the critical shear stress for sediments with different components are reported. Figure 1 presents the plot of the critical shear stress $\tau_c$ versus the bulk density $\rho$ for different clay contents by dry weight, $P_{cl}$. It is seen that for $P_{cl} \leq 2.7\%$, no detectable effect of $P_{cl}$ on $\tau_c$ is observed, i.e., $\tau_c = 0.23 \text{ N/m}^2$, whereas an abrupt increase in $\tau_c$ is obtained when more clay is added to sand. For any given value of $P_{cl} \geq 8.1\%$, the relation between the critical shear stress and bulk density has the following power form:

$$\tau_c = a \rho^b$$  \hspace{1cm} (1)

where $\tau_c = \text{critical shear stress (m/s}^2\text{)}$, $\rho = \text{bulk density (g/m}^3\text{)}$; and $a, b = \text{coefficients that depend upon the composition of soils and are summarized in Table 1}$. Generally, value of $a$ decreases with increasing $b$ for all tested soils except for the mixture, in which the clay content and sand content by dry weight is 27% and 0%, respectively. This phenomenon indicates potentially a different movement mechanism for the mixture and may be attributed to the increase of clay content and the absence of sand.

![Figure 1. Variation of $\tau_c$ with $\rho$ for different $P_{cl}$.](image)

| Mud solid content (%) | Clay solid content (%) | $a$    | $b$       | $R^2$ |
|-----------------------|------------------------|--------|-----------|-------|
| 30                    | 8.2                    | 0.053  | 4.879     | 0.89  |
| 40                    | 11.0                   | 0.037  | 6.395     | 0.83  |
| 50                    | 13.7                   | 0.014  | 9.042     | 0.93  |
| 60                    | 16.4                   | 0.014  | 9.531     | 0.83  |
| 70                    | 19.2                   | 0.001  | 17.225    | 0.86  |
| 80                    | 21.9                   | 0.002  | 17.319    | 0.83  |
| 100                   | 27.4                   | 0.055  | 9.190     | 0.71  |
The critical shear stress, calculated by equation (1) with parameters in Table 1, is plotted against the clay content by dry weight, $P_{cl}$, for different bulk densities $\rho$ in Figure 2. By comparing the lines for different $\rho$, it is seen that for any given value of $P_{cl}$, $\tau_c$ is systematically higher for a larger $\rho$ and the difference becomes large for a large $P_{cl}$. It may be also observed that an essentially linear relationship exists between $\tau_c$ and $P_{cl}$ for any given value of $\rho$ if $P_{cl} \leq 16.4\%$. This is in agreement with the experimental results of Torfs who used kaolinite and montmorillonite as cohesive fraction in the soils [16], as indicated in Figure 3. Figure 2 also shows an abrupt increase in $\tau_c$ occurs if $P_{cl} > 16.4\%$--19.2%. The results are consistent with the observation of Panagiotopoulos et al [17], who noticed the increment of the critical shear stress is greater if the mud content exceeds 30%, corresponding to approximate 17.7\% of clay content by dry weight ($\leq 4\mu m$ by weight).

3.2. Reanalysis on the “Influence of Cohesion on the Incipient Motion Condition of Sediment Mixtures”

The writer has developed a new threshold criterion for initial movement of sediments that are consisted of fine gravel mixed with clay in proportions varying from 10\% to 50\% by weight, and fine gravel with fine sand mixed in equal proportion, with clay proportions varying from 10\% to 50\% by weight. In the paper, a dimensional method was adopted to study the variations that may mainly influence the critical shear stress of sediments. Since the initial movement of sediments including clay particles is so complicated that whether the threshold criterion from a dimensional method can work well needs to be further verified. Therefore, an attempt is adopted to relate the critical shear stress to bulk property parameters of the sediment mixtures.

As a representative example, the experimental data for fine gravel mixed with clay is reanalysed. The critical shear stress $\tau_c$ is plotted against the clay volume content $\Phi_{cl}$ for different clay contents by dry weight $P_{cl}$ in Figure 4. It is shown that for a given value of $P_{cl}$, $\tau_c$ increases linearly for increasing $\Phi_{cl}$, and the $\tau_c$ vs. $\Phi_{cl}$ functional relationships for different $P_{cl}$ can be depicted by a series of parallel lines. Therefore, based on the experimental results, the critical shear stress depends on the clay volume content and clay solid content as (see Figure. 4)
\[ \tau_c = 0.147\phi_d - 0.169P_{cl} + 2.192 \]  \hfill (2)

Here, \( \phi_d = \) clay volume content. The comparison between the measured critical shear stress and the calculated value from equation (2) is plotted in Figure 5. It is obvious that the simply equation (2) predicts the critical shear stress with a maximum error of \( \pm 15\% \) for about 90\% of data.

**Figure 4.** Variation of \( \tau_c \) with \( \phi_d \) for different \( P_{cl} \).

**Figure 5.** Comparison of measured values of \( \tau_c \) with those computed by equation (2).

**Figure 6.** Variation of \( \tau_c \) with \( \rho \) for clay-gravel-sand mixtures.

The critical shear stress is plotted against bulk density for clay-gravel-sand mixtures in Figure 6. It can be seen that \( \tau_c \) increases with the increase of \( \rho \). Meanwhile, it could be also found that the slope is larger for \( P_{cl} \geq 16.8\% \), which is in accordance with characteristics of the critical shear stress in Figure 2. These results indicate that a critical clay content by dry weight may exist in the range of 16.4\%–19.2\% with an average value of 18\%, beyond which the critical shear stress increases rapidly with the increase of bulk density. According to the original paper, the initial movement of sediment is noticed in the form of thin flakes or chunks of varying sizes for \( P_{cl} > 16.8\% \), which indicates a cohesive behaviour happens here. It isn’t in agreement with the opinions of Van Ledden et al. [15] who...
considered the transition from non-cohesive to cohesive behavior at a clay solid content 5-10%. It is probably due to the presence of gravel or air in the sediments in Kothyari’s experiments [18].

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