Ground-state electric quadrupole moment of $^{31}\text{Al}$

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Ground-state electric quadrupole moment of $^{31}\text{Al}$ ($I^\pi =5/2^+$, $T_{1/2} = 644(25)$ ms) has been measured by means of the $\beta$-NMR spectroscopy using a spin-polarized $^{31}\text{Al}$ beam produced in the projectile fragmentation reaction. The obtained $Q$ moment, $|Q_{\text{exp}}(^{31}\text{Al})| = 112(32)$ emu, are in agreement with conventional shell model calculations within the sd valence space. Previous result on the magnetic moment also supports the validity of the sd model in this isotope, and thus it is concluded that $^{31}\text{Al}$ is located outside of the island of inversion.

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The ground states of Ne, Na, and Mg isotopes with neutron numbers around the magic number $N = 20$ have been known to show anomalously tight bindings since 1970’s [1, 2]. Later, spectroscopic studies have revealed that the first exited $2^+$ levels are lowered [2, 4] and their $B(E2)$ values are enhanced [2] sizably in these isotopes, and the possibility of deformation has been proposed. Theoretical analyses [6] discussed the importance of $2\hbar$-$2\hbar$ excitations from the $sd$ shell to the upper $pf$ shell, and concluded it plausible that an inversion of amplitudes between the $sd$ normal and $pf$ intruder configurations would lead to deformation of the ground states. The region of nuclei where such a phenomenon occurs is called the island of inversion. In elucidating the underlying mechanism for the inversion, the measurements of the electromagnetic moments have played an important role. For example in a series of neutron-rich Na isotopes, it has been found that, once entering the island of inversion, the ground-state magnetic dipole moment $\mu$ and electric quadrupole moment $Q$ [7, 8] show clear deviations from the conventional shell-model predictions [9], indicating that $\mu$ and $Q$ are sensitive to changes in the nuclear configuration [10]. Also in the recent study of Mg isotopes, anomalous ground-state properties have been revealed through the $\mu$-moment measurements [11, 12].

In the present work, the ground-state $Q$ moment of $^{31}\text{Al}$ ($I^\pi =5/2^+$, $T_{1/2} = 644(25)$ ms) has been measured by means of the $\beta$-ray detected nuclear magnetic resonance (\(\beta\)-NMR) spectroscopy [13] applied on a projectile fragment $^{31}\text{Al}$ implanted in an $\alpha$-$\text{Al}_2\text{O}_3$ (corundum) single crystal in which a non-zero electric field gradient acts. A spin-polarized radioactive-isotope beam (RIB) of $^{31}\text{Al}$ was obtained from the projectile fragmentation reaction [14]. Since the neutron-rich aluminum isotopes are located in the neighborhood of the island of inversion, their electromagnetic moments would signify the possible onset of evolution in the nuclear structure that ultimately leads to the inversion phenomenon. So far, the $\mu$ moments of $^{31-34}\text{Al}$ [15, 16] and $^{30, 32}\text{Al}$ [17] have been reported. The obtained values of $\mu$ for $^{30-32}\text{Al}$ seem to stay within the conventional sd-model predictions [9, 18], indicating that their structures are suitably described within the normal sd model space. Those of $^{33, 34}\text{Al}$ having neutron numbers $N = 20$ and 21, on the other hand, seem to indicate deviations from the $0h\omega$ shell-model predictions [16, 19]. Since the island of inversion is considered to involve the nuclear deformation, the $Q$ moment would be a more suitable probe. It has been found in a recent measurement that $^{32}\text{Al}$ has a very small $Q$ moment $|Q_{\text{exp}}(^{32}\text{Al})| = 24(2)$ emu [20] characteristic of a simple $(\pi d_{3/2}^1 \otimes \nu d_{3/2}^1)_{J=1}^+$ configuration, indicating a spherical shape. The $Q$ moment of $^{31}\text{Al}$ is important in elucidating how the nuclear shape evolves along the aluminum isotopes toward and beyond the $^{32}\text{Al}$ nuclide.

The experiment was carried out using the RIKEN projectile fragment separator RIPS [21]. The arrangement of RIPS for producing the spin-polarized RIB is essentially the same as that described in Ref. [20]. A beam of $^{31}\text{Al}$ was obtained from the fragmentation of $^{40}\text{Ar}$ projectiles at $E = 95\text{A}$ MeV on a 0.37 g/cm$^2$-thick $^{93}\text{Nb}$ target. It has been revealed that a spin-polarized RIB is obtained in the projectile fragmentation reaction simply by selecting the angle and momentum of the outgoing fragments [14]. Thus, $^{31}\text{Al}$ fragments emitted at angles $\theta_{\text{lab.}} = (1.3 - 5.7)\degree$ from the primary beam direction were accepted by RIPS using a beam swinger installed upstream of the tar-
the spin-lattice relaxation of shown in Fig. 1. The α to the stopper. The layout of the β stopper chamber so that the c axis is oriented parallel to the B0 field. The stopper was kept in vacuum and cooled to a temperature \( T = (70 \sim 100) \) K to suppress the spin-lattice relaxation of \(^{31}\)Al during the β decay.

FIG. 1: Schematic layout of the β-NMR apparatus.

The \( Q \) moment interacts with an electric field gradient \( eq \) acting at the site of the implanted nucleus in a single crystal stopper. The \( eqQ \) interaction causes the energy shift in the individual Zeeman magnetic sublevels. Thus, the \( Q \) moment is determined from the measurement of the frequencies for the resonance transition between the Zeeman + quadrupole splitted sublevels, whose signal is detected as a change in the β-ray asymmetry (the \( β\)-NQR method). The β-rays emitted from the implanted nuclei were detected by plastic-scintillator telescopes located above and below the stopper, each consisting of three 1 mm-thick plastic scintillators. The up/down ratio \( R \) of the β-ray counts is written as

\[
R = \frac{a + v/c \cdot A_β P}{1 - v/c \cdot A_β P} \approx a(1 + 2A_β P),
\]

where \( a \) is a constant factor representing asymmetries in the counter solid angles and efficiencies, \( v/c \) the velocity of the β particle, \( A_β \) the asymmetry parameter, and \( P \) the \(^{31}\)Al nuclear spin polarization. Since we adopted a high-energy portion of the β-ray energy spectrum (\( v/c \sim 1 \)) in the analysis, the \( R \) ratio is approximated as in the second expression in Eq. (1). The adiabatic fast passage (AFP) technique \([22]\) was incorporated in order to pursue the reversal, but not the destruction, of the spin polarization. By taking a double ratio \( R/R_0 \) where \( R_0 \) is the value for \( R \) measured without the \( B_1 \) field, the resonance frequency is derived from the position of a peak or dip deviating from unity. An oscillating magnetic field \( B_1 \) in a direction perpendicular to the external field \( B_0 \) was applied with a pair of coils located outside a vacuum jacket, in which the \( α-Al_2O_3 \) stopper was placed. In a first order perturbation theory, the resonance frequency \( ν_{m,m+1} \) between magnetic sublevels \( m \) and \( m+1 \) of the nuclear spin \( I \) under the combined Zeeman and quadrupole interactions is given by

\[
ν_{m,m+1} = ν_L - ν_Q(3cos^2θ_{c-axis} - 1)(2m + 1)/4
\]

\[
ν_Q = \frac{3}{2I(2I - 1)} \cdot \frac{eqQ}{h}
\]

where \( ν_L \) denotes the Larmor frequency, \( eq \) the electric field gradient along the c-axis (the additional term arising from a deviation \( η \) from the axial symmetry of the field gradient tensor is omitted, since \( η \) is reported to be small \([23]\), \( θ_{c-axis} \) the angle between the c-axis and the \( B_0 \) field, and \( eqQ/h \) the quadrupole coupling constant. \( Q \) and \( h \) denote the \( Q \) moment and the Planck’s constant, respectively. Inserting \( I = 5/2 \) and \( θ_{c-axis} = 0 \) for the present \(^{31}\)Al experiment, Eq. (2) reads as

\[
ν_{m,m+1} = ν_L - \frac{2m + 1}{2} ν_Q
\]

\[
= \begin{cases} 
ν_L + 2ν_Q & \text{for } (m,m+1) = (-5/2,-3/2); \text{ (frequency “a”)}
ν_L + ν_Q & \text{for } (-3/2,-1/2), \text{ (“b“)}
ν_L & \text{for } (-1/2,+1/2), \text{ (“c“)}
ν_L - ν_Q & \text{for } (+1/2,+3/2), \text{ (“d“)}
ν_L - 2ν_Q & \text{for } (+3/2,+5/2), \text{ (“e“)}
\end{cases}
\]
The $B_1$ field was applied in $I(2I+1) = 15$ steps, each tuned to one of the five transitions of Eq. (4), in a sequence $abedabcabcaba$ within the $B_1$ application period of 63 ms duration. Actually, in the individual step the frequency $\nu$ of the $B_1$ field was swept for the AFP method, over a frequency bin $\nu_{m,m+1}(\nu_Q^{\text{lower}}) \rightarrow \nu_{m,m+1}(\nu_Q^{\text{upper}})$ that corresponded to a $\nu_Q$ region of $5.3$ standard deviation from unity, clearly indicating the expected for a single value of $\nu_Q$. Details of the $B_1$ field sequence was executed for $\nu = 5850(12)$ kHz. This value was obtained from a $\beta$-NMR experiment on $^{31}$Al in a Si crystal, carried out prior to the present $Q$-moment measurement using the same apparatus and $B_0$ setting.

Thus, the $\beta$-ray count ratio $R/R_0$ is expected to differ from unity when the $B_1$ field sequence was executed for the $\nu_Q$ region that includes the true $\nu_Q$ value given by Eq. (5). Figure 2 shows the measured $R/R_0$ ratio for $^{31}$Al in $\alpha$-$\text{Al}_2\text{O}_3$ as a function of $\nu_Q$ (the $\beta$-NQR spectrum). The horizontal bar attached to the data point (filled circle) indicates the $\nu_Q$ region over which the $B_1$ field frequency was swept. The vertical bar represents the error in $R/R_0$ arising from the $\beta$-ray counting statistics. The $R/R_0$ value at the dip bottom shows a displacement of $5.3$ standard deviation from unity, clearly indicating the occurrence of the AFP spin reversal. The width of the dip, however, seems to be substantially broader than that expected for a single value of $eq$.

\[
\nu_Q = \frac{3}{20} \frac{eqQ}{h} \quad (5)
\]

The obtained NQR spectrum was fitted with a function

\[
F(\nu_Q) = a \int G_\sigma(\xi) \cdot \mathcal{F}_{\text{AFP}}(\nu_Q - \nu_Q^{(0)} - \xi)d\xi + b \quad (6)
\]

with four free parameters $\nu_Q^{(0)}$, $\sigma$, $a$, and $b$ to be determined through the fitting. The $F(\nu_Q)$ function is a Gaussian convolution of a theoretical shape function $\mathcal{F}_{\text{AFP}}(x)$ of a detuning $x$ (i.e., $f(x)$ of Eq. (4) in Ref. [24]). The width of $\mathcal{F}_{\text{AFP}}(x)$ was evaluated to be $98$ kHz (FWHM), representing the expected shape of $\beta$-NQR spectrum in the AFP mode for a single value of electric field gradient $eq$. The parameter $\nu_Q^{(0)}$ represents the position of the dip, from which the quadrupole coupling constant $eqQ/h$ will be deduced through Eq. (5). The parameter $\sigma$ is the width of the Gaussian function $G_\sigma(\xi) \equiv (\sqrt{2\pi}\sigma)^{-1} \exp(-\xi^2/2\sigma^2)$, representing extrabroadening effects that are not included in the function $\mathcal{F}_{\text{AFP}}(x)$. Extrabroadening may arise from distribution of $eq$ value due to lattice defects or impurities in the stopper crystal and misalignments in the $c$-axis orientation and $\nu_L$ setting. In the present analysis such effects are expressed as a finite value of $\sigma$. The extrabroadening was not included in the preliminary reports [24, 26].

From the fitting analysis of the NQR spectrum, we obtained $\nu_Q^{(0)} = 274(18)$ kHz for the dip position. The resulting curve $F(\nu_Q)$ is shown in Fig. 2 by a dotted line. Although the parameter $\nu_Q^{(0)}$ is determined with a rather small uncertainty ($\delta\nu_Q^{(0)} = 18$ kHz) from the fitting procedure, the actual spectrum is dominated by a much larger broadening, $\sigma = 76$ kHz, whose origins are not well pinpointed. We therefore take into account the extrawidth $\sigma$ as an independent error, and assign an experimental error $\delta\nu_Q^{(0)} = 78$ kHz, as shown in Table I. As a result, we obtain a quadrupole coupling constant $|eqQ| = 3/20 \cdot |eqQ/h|$ = $274(78)$ kHz, or $|eqQ/h(\text{Al})| = 1.83(52)$ MHz. The $Q$ moment of $^{31}$Al is deduced from the relation $|Q(\text{Al})| = |Q(\text{Al})| \cdot (eqQ/h(\text{Al})) / (eqQ/h(\text{Al}))$, where $Q(\text{Al})$ and $eqQ/h(\text{Al})$ denote the $Q$ moment of $^{27}$Al and the quadrupole coupling constant of $^{27}$Al in $\alpha$-$\text{Al}_2\text{O}_3$, respectively. By inserting the recently reported $Q$ moment $Q(\text{Al}) = 146.6(10)$ emb [27] and quadrupole coupling constant $eqQ/h(\text{Al}) = 2389(2)$ kHz [28], the ground-state $Q$ moment $^{31}$Al is determined as $|Q_{\text{exp}}(\text{Al})| = 112(32)$ emb.

In Fig. 3 the experimentally known $Q$ moments for the neutron-rich aluminum isotopes including the present data are plotted as a function of the mass number $A$. Also, the results of shell-model calculations within the sd shell [4, 18] are shown by the solid line. The calculations reproduce the observed trend of the $Q$ moments in
TABLE I: Uncertainties taken into account for the determination of $|Q_{\text{exp}}(^{31}\text{Al})|$ moment. Those uncertainties were converted into the corresponding $\nu_\Omega$ frequency.

| Resonance $\nu_\Omega$ (kHz) |
|-----------------------------|
| $\nu_\Omega$ (Statistical error) | 274 (kHz) |
| (Fitting error) | 18 (kHz) |
| (Systematic errors) |
| Ambiguity from the resonance width | 76 (kHz) |
| Uncertainty of the electric field gradient | 2 (kHz) |
| Ambiguity of the $\theta_0$-axis-angle setting | 0.1 (kHz) |
| Total | 78 (kHz) |
| $Q_{\text{exp}}(^{31}\text{Al})$ | 112 $\pm$ 32 (emb) |

FIG. 3: The experimental (filled circle) and theoretical (solid and dotted lines) $Q$ moments of neutron-rich aluminum isotopes as a function of mass number, whose nuclear spins are also shown. Theoretical values are obtained from shell-model calculations within the $sd$ shell with the USD interaction, using the constant effective charges $e_p = 1.3$ and $e_n = 0.5$ (solid line) and isospin-dependent effective charges [25] (dotted line).

In summary, the ground-state $Q$ moment of $^{31}$Al has been determined by the $\beta$-NQR method, using the fragmentation-induced spin polarization. The obtained $Q$ for $^{31}$Al as well as known $|Q|$ values for other neutron-rich aluminum isotopes were found to be well explained by shell-model calculations within the $sd$ shell. Viewing also that the magnetic moment of $^{31}$Al recently determined [13] is explained with the same calculations, it is concluded that $^{31}$Al is located outside the island of inversion.

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