$Z \to b\bar{b}$ versus Dynamical Electroweak Symmetry Breaking Involving the Top Quark

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Abstract

In models of dynamical electroweak symmetry breaking which sensitively involve the third generation, such as top quark condensation, the effects of the new dynamics can show up experimentally in $Z \to b\bar{b}$. We compare the sensitivity of $Z \to b\bar{b}$ and top quark production at the Tevatron to models of the new physics. $Z \to b\bar{b}$ is a relatively more sensitive probe to new strongly coupled $U(1)$ gauge bosons, while it is generally less sensitive a probe to new physics involving color octet gauge bosons as is top quark production itself. Nonetheless, to accomodate a significant excess in $Z \to b\bar{b}$ requires choosing model parameters that may be ruled out within run I(b) at the Tevatron.

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1. Introduction

Is the third generation special? It contains the only fermion with a mass of order the weak scale, the top quark. As such, the third generation, through the top quark, may be singled out to play a key role in electroweak symmetry or horizontal symmetry breaking physics. At very least, it is a privileged spectator to that dynamics.

The possibility of a heavy top quark was anticipated within the context of the infrared–fixed point of the Higgs–Yukawa coupling in the standard model, \[1\], and its SUSY generalization, which appears to be phenomenologically compelling \[2\]. These ideas are, moreover, connected to dynamical symmetry breaking through top quark condensation \[3\]. Some authors, attempting to accommodate the heavy top quark in technicolor schemes, have been led to extended technicolor involving the top quark into strong dynamics, as well \[4\]. In a gauge version of top condensation in the standard model, new strong physics at the $\sim 1$ TeV scale is proposed as an imbedding of QCD: $SU(3) \rightarrow SU(3)_1 \times SU(3)_2$ \[5\], where the color assignments are generationally sensitive. Thus the effects of new dynamics may show up experimentally in various channels. Anomalous top production rates and distributions at the Tevatron might be expected because the topgluon production mode interferes with the single gluon mode in $q\bar{q}$ annihilation \[6\]. These effects however, can potentially be dramatic, showing up as an anomaly in the production cross-section immediately, or they can be subtle, requiring many thousands of top quark pairs to become manifest. In technicolor schemes similar effects can happen through pseudo–Nambu-Goldstone modes that are produced via glue-glue collisions. \[7\]. These latter effects are distinguished by
their general kinematic structures, such as angular distributions observable at the Tevatron and the top production rates may differ dramatically at the LHC from QCD. It is important to realize that thus far the standard model is tested only on energy scales ranging from $\sim 0$ GeV to the $Z$-pole. Radiative corrections largely test the running of coupling constants on these scales and the few instances where violation of decoupling occurs in fermion loops, such as the $S$ and $U$ parameters, and the $T$ (or equivalently the $\rho$) parameter. Top quark production represents the first time the standard model has been examined on a new scale of $\sim 500$ GeV, and indeed the Tevatron sensitivity to new physics extends up to $\sim 1$ TeV. The emergence of new physics is certainly not prohibited at these scales.

Dynamical symmetry breaking schemes, such as the topcolor model, necessarily involve the $b$-quark into the strong dynamics at the $\sim 1$ TeV scale, or at very least, $b_L$, since the $SU(2)_L$ group places $(t_L, b_L)$ into a common doublet. New dynamical effects may become manifest in anomalous $b\bar{b}$ production at high mass at the Tevatron through a single “topgluon” interfering with a single gluon. Moreover, sensitive studies of the $b$ quark in electroweak production modes may reveal the new dynamics through radiative corrections at the Tevatron through $q\bar{q} \rightarrow W \rightarrow \bar{t}b$, or at the NLC through $e^+e^- \rightarrow \gamma(Z) \rightarrow \bar{b}b$, at high $Q^2$, where the $tbW$ or $bb\gamma(Z)$ vertex may receive large corrections.

In the present note we discuss the sensitivity of the $Z \rightarrow b\bar{b}$ rate (measured at LEP) to new physics, such as the topcolor model. Our approach is somewhat parallel to that of Chivukula, et.al., in a discussion of technicolor schemes. This process is potentially sensitive to new physics since it is a non-universal radiative correction,
and may probe new forces acting in the final state at higher energies. While the ratio $R_b = \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$ is slightly high in comparison to standard model expectations, the discrepancy is at present only at a level of $2\sigma$. Moreover $b$-tagging and the various QCD contributions to $b$’s in such processes are nontrivial issues and a possible resolution of the rate excess puzzle in favor of a conventional effect is quite possible \[12\]. Nevertheless, we can inquire whether the present situation and future prospects are potentially sensitive to new dynamics of this sort, and to understand what limits on dynamical models may ultimately obtain with increasing precision in the $Z \rightarrow b\bar{b}$ rate. The models we consider are “straw person” dynamics that we are using to theoretically assess sensitivity to new physics, and we do not seek to explain the slight $2\sigma$ excess.

2. Phenomenology of $Z \rightarrow b\bar{b}$

Let us take the couplings of the $b$ quark to the $Z$ boson to be given phenomenologically by the expression:

$$Z^\mu (g_{eff}^L \tau_{\gamma \mu} b_L + g_{eff}^R \tau_{\gamma \mu} b_R).$$

(1)

We introduce two parameters $\kappa_L$ and $\kappa_R$ to describe the non-universal effects in the $Zb\bar{b}$ vertex. These parameters shift the standard model tree level couplings of the $g_{L,R}$ to effective couplings $g_{eff}^{L,R}$:

$$g_{eff}^L = g_L(1 + \kappa_L); \quad g_{eff}^R = g_R(1 + \kappa_R).$$

(2)

where:

$$g_L = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W; \quad g_R = \frac{1}{3} \sin^2 \theta_W.$$  

(3)
Defining $\delta \Gamma$ to be the purely non-universal correction of the new physics beyond the standard model to the $Zb\bar{b}$ width, $\Gamma_{b\bar{b}}$, we have
\[
\frac{\delta \Gamma}{\Gamma_{b\bar{b}}} \sim 2 \left( \frac{g^2_L \kappa_L + g^2_R \kappa_R}{g^2_L + g^2_R} \right). 
\] (4)

Since $g^2_L \gg g^2_R$ and the $\kappa_R$ is expected to be at most the same order of magnitude as $\kappa_L$, one has approximately
\[
\frac{\delta \Gamma}{\Gamma_{b\bar{b}}} \sim 2 \kappa_L. 
\] (5)

Then the $R_b$ becomes:
\[
R_b \sim R^{SM}_b \left( 1 + \frac{\delta \Gamma}{\Gamma_{b\bar{b}}} \right) \sim R^{SM}_b (1 + 2 \kappa_L), 
\] (6)

where the standard model value $R^{SM}_b = 0.2157 \pm 0.0004$, includes the large top quark contributions for $m_t = 174 \pm 11$ GeV and $m_H = 60 \sim 1000$ GeV. The experimental value of $R_b$ measured in aggregate at LEP is $R_b = 0.2192 \pm 0.0018$ [12], which is roughly within $2\sigma$ of the standard model prediction. New physics leading to a positive $\kappa_L$, such as a short range attractive force between $b$ and $\bar{b}$ could improve the situation, while a negative $\kappa_L$ at the $\sim 0.5\%$ level would imply a discrepancy worse than $\sim 3\sigma$ with experiment. Thus, one can put constraints on a new physics as emphasized by many authors (see the recent discussion of Chivukula [11]).

In general $\kappa_{L,R}$ can be viewed as functions of $q^2$, where $q$ is the 4–momentum of the Z boson, and at LEP, $q^2 = m^2_Z$. Expanding $\kappa_{L,R}$ in terms of $q^2$, we have
\[
\kappa_{L,R} \sim \kappa_{L,R}^0 + \kappa_{L,R}^1 \frac{q^2}{\Lambda^2}, 
\] (7)
where \( \kappa_{L,R}^1/\Lambda^2 = d\kappa_{L(R)}/dq^2|_{q^2=0} \), and \( \Lambda \) is the new physics scale. Gauge invariant operators describing \( \kappa_{L,R}^0 \) and \( \kappa_{L,R}^1 \) can always be constructed explicitly in a non-linear realization of \( SU_L(2) \times U_Y(1) \) [13].

Indeed, the above description of new physics in terms of modified \( d=4 \) current couplings of the \( b \) quark to the \( Z \) boson is apropos the broken phase of the standard model. These effects would also be expected to arise from new effective contact terms that are \( d>4 \), \( SU(2)_L \times U(1) \) linearly invariant operators occuring at a high energy scale \( \Lambda \) above the breaking of electroweak symmetry. For example, if we organize the quarks into \( SU(2)_L \times SU(2)_R \) doublets as \( \psi_L = (t,b)_L \) and \( \psi_R = (t,b)_R \), then a complete basis of operators, not including the Higgs field (here we have a dynamical symmetry breaking in mind, and no Higgs field explicitly occuring at the scale \( \Lambda \) which directly mediate the process \( Z \to b\bar{b} \) can then be written:

\[
\begin{align*}
O^1_{L,R} &= (\bar{\psi}_{L,R} \gamma_\mu \tau^a \psi_{L,R})(D_\mu F^{\mu\nu})^a; \\
O^2_{L,R} &= (\bar{\psi}_{L,R} \gamma_\mu \tau^a \psi_{L,R})(F^{\mu\nu})^a; \\
O^3_{L,R} &= (\bar{\psi}_{L,R} \gamma_\mu \psi_{L,R})(\partial_\mu F^{\mu\nu}); \\
O^4_{L,R} &= (\bar{\psi}_{L,R} \gamma_\mu D_\mu \psi_{L,R})F^{\mu\nu};
\end{align*}
\]

where \( F^a_{\mu\nu} \) (\( F_{\mu\nu} \)) is the \( SU(2) \) (\( U(1) \)) field strength. Here we define:

\[
\tilde{D}_\mu = \vec{\partial}_\mu + ig_1 A^a_\mu \frac{\tau^a}{2} + ig_2 B_\mu \frac{Y}{2}; \quad D_\mu = \frac{1}{2} \left( \tilde{D}_\mu - \tilde{D}_\mu \right).
\]

There are many other operators that can be written, but all are reducible to this set, or operators of lower dimension, by use of equations of motion and algebraic identities. Operators such as \( \bar{\psi} \gamma_\mu D_\nu \psi \tilde{F}^{\mu\nu} \) are odd in CP and are not considered, while for example the even CP operator \( \bar{\psi} \gamma^5 \gamma_\mu D_\nu \psi \tilde{F}^{\mu\nu} \) is reducible to the above set using massless quark equations of motion together with various identities.
In ref. [14] a general list of contact terms is provided and the operators $O_{L,R}^1$ and $O_{L,R}^3$ are already reduced to four-fermion form by use of the equation of motion $(D^\mu F_{\mu\nu})^a = g_2 j_\nu^a$. In the broken phase of the theory we must modify the equation of motion to include the mass terms of the $W$ and $Z$, e.g., $(D^\mu F_{\mu\nu})^Z = M_Z^2 Z_{\nu} + g_2 j_\nu^Z$. Thus, in the broken phase $O_{L,R}^1$ and $O_{L,R}^3$ are related to a four-fermion contact term and a correction to the current couplings of the $b$ and $t$ quarks to the electroweak gauge bosons $\gamma$, $Z$, and $W$. Let us introduce the effective Lagrangian containing the contact terms:

$$L_{\text{eff}}(M_Z) = \frac{1}{\Lambda^2} \left( c_X^1 O_X^1 + c_X^2 O_X^2 + c_X^3 O_X^3 + c_X^4 O_X^4 + \ldots \right),$$

where we sum over $X = (L, R)$ in the above. Note that $L_{\text{eff}}$ is defined here at the scale $M_Z$ and not at the scale $\Lambda$. At the scale $M_Z$ it is generally necessary to introduce the Higgs field as well, which may be only an interpolating field for a composite state (see below). After electroweak symmetry breaking the amplitude involving the coupling of the $Z$ boson to the $b\bar{b}$ pair contained in $L_{\text{eff}}$ becomes:

$$L_{\text{eff}}(M_Z) \rightarrow -\frac{1}{\Lambda^2} \left[ \left( \frac{1}{2} \cos \theta_W c_X^1 + \sin \theta_W c_X^2 \right) M_Z^2 b_X \gamma_\mu b_X Z^\mu \\
+ \left( \frac{1}{2} \cos \theta_W c_X^3 + \sin \theta_W c_X^4 \right) b_X \gamma_\nu \partial_\mu b_X F_Z^{\mu\nu} \right],$$

where $F_Z^{\mu\nu}$ is the usual $U(1)$ field strength composed of a $Z$ field. In the first term of the rhs of eq.(11) we have used the equation of motion of the gauge field in the broken phase, and this term modifies the $Z \rightarrow b\bar{b}$ current coupling. Integrating the second term by parts yields zero in the limit of vanishing $m_b$, where we use the equation of motion for the process $Z \rightarrow b\bar{b}$. Thus, at this stage all of the relevant physics effects
are absorbed into the definitions of $\kappa_{L,R}^1$:

$$\kappa_{L,R}^1 = -\frac{1}{g_Z g_{L,R}} \left[ \frac{1}{2} c_{L,R}^1 \cos \theta_W + c_{L,R}^3 \sin \theta_W \right],$$

(12)

where $g_Z = e / \sin \theta_W \cos \theta_W$.

The momentum independent term $\kappa_{L,R}^0$ can be generated when the standard model Higgs doublet field $\phi$ is explicitly included. For example, an operator in the following form

$$O_{L,R}^5 = i \bar{\psi}_{L,R} \gamma^\mu \psi_{L,R} \phi^\dagger D_\mu \phi + h.c.$$

(13)

will generate a non-vanishing $\kappa_{L,R}^0$ in the broken phase of the standard model. Such a term can also be induced in models in which the top quark condenses due to new strong interaction physics affecting the third generation. We will presently turn to two possible aspects of that case.

3. Top Condensation Models

In the top quark condensation models, the third generation and $t_R$ states at a minimum participate in a new strong interaction. In this case top and bottom quarks can be viewed as forming the field theoretic bound state Higgs doublet $\phi \sim \Psi_{Lt_R}$ (and perhaps others, such as, $\rho$-like vector meson). Here one would expect that the top quark loop shown in Fig.(1) will generate the constant piece $\kappa_{L,R}^0$ corresponding physically to the Higgs field containing operator of eq.(13). We will presently make an estimate the contributions to $\kappa_{L,R}^0$ when there are contact terms representing the new strong dynamics. Here, we will limit ourselves to the use of a momentum independent top quark (constituent) mass up to a cut-off $\Lambda$, and follow the method of ref. [13] in
the calculation of the top loop correction to $Zb\bar{b}$ vertex in the chiral lagrangian.

We begin by considering the general strength of the induced corrections to $Z \to b\bar{b}$ from contact terms involving the third generation. Let us assume the general form of the contact term to be of the color singlet $s$-channel form as in \cite{6}. The relevant part of the effective Lagrangian we will take to be given in the broken phase by:

$$L' = -\frac{1}{\Lambda^2} \bar{b} \gamma_\mu b \, \bar{t} \gamma^\mu (g_V - g_A \gamma_5) t + ...,$$

(14)

where $g_V$, $g_A$, are parameters (we follow \cite{6} for comparison of normalizations and we will define $g_A \sim 4\pi \times (0.11)$ below). To compute $\kappa^0_{L,R}$ we consider Fig.(1), and we see that we are effectively computing the top contribution to the $Z$ boson self-energy, $\Pi_{33}$, attached to the $b\bar{b}$ vector current. We then have from Fig.(1):

$$\kappa^0_{L,R} = \frac{g_A}{g_{L,R}} \frac{N_c}{8\pi^2} \frac{m_t^2}{\Lambda^2} \ln \left( \frac{\Lambda^2}{m_t^2} \right),$$

(15)

where $N_c = 3$. We see that, depending on the sign of $g_A$, the $Z \to b\bar{b}$ width can be enhanced or decreased. In the case of a negative $\kappa_L$, requiring $\kappa_L < 0.5\%$ gives that $\Lambda > 2.0 \text{ TeV}$, for $|g_A| \sim 4\pi \times (0.11)$ \cite{6}. In the case of a positive $\kappa_L$, the $R_b$ can be made to be comparable to, and even larger than the experimental value for a decreasing $\Lambda$. Requiring that the theoretical prediction be consistent with the experimental data within $\sim 3\sigma$, one has $\Lambda \gtrsim 0.8 \text{ TeV}$. These limits on $\Lambda$ are comparable to, and slightly stronger than those derived from the top quark production cross-section \cite{4}. We present these results for positive $\kappa_L$ in Table I. In Table I we also assume the light fermions participate in four–fermion operators as in eq.(14) (as in \cite{6}) and compare the resulting corrections to the top production cross-section. Since this assumes that the new physics acts universally on the light quarks, $u$ and $d$, it is in a sense an upper
limit on the effects on top production. Thus, in this case, where the new physics occurs in the color singlet channel, we see that \textit{the constraint from }$Z \rightarrow b \bar{b}$\textit{ is slightly stronger than from the top quark production cross-section.}

Note that in the case of color–octet, \textit{s–channel}, operators,

$$L' = \frac{1}{\Lambda^2} \bar{b} \gamma_\mu \frac{\lambda^a}{2} b \gamma^\mu (g_V - g_A \gamma_5) \frac{\lambda^a}{2} t + \ldots,$$

(16)

there is no contribution from the top quark owing to the trace over colors. There is however, a contribution to the $q^2$ dependent term from operators of this kind when the full $SU(2)_L$ structure is considered, and it leads to a less significant contribution to $Z \rightarrow b \bar{b}$. To discuss this class of effects let us pass over to the topcolor model \textit{[5]} which contains new heavy color octet gauge bosons, and which leads to terms such as eq.(16) in the effective Lagrangian.

We now assume a minimal extension of the standard model such that at scales $\mu \gg \Lambda$, we have the gauge group $U(1) \times SU(2)_L \times SU(3)_1 \times SU(3)_2 \textit{[5]}$, with coupling constants (gauge fields) of $SU(3)_1 \times SU(3)_2$ respectively $h_1$ and $h_2$ ($A^2_{1\mu}$ and $A^2_{2\mu}$). We assign quark and lepton fields to anomaly free representations as follows:

$$(u, d)_L, \ (c, s)_L \rightarrow (2, 3, 1); \quad u_R, d_R, c_R, s_R \rightarrow (1, 3, 1);$$

$$(\nu_e, e)_L, \ (\nu_\mu, \mu)_L, \ (\nu_\tau, \tau)_L \rightarrow (2, 1, 1); \quad e_R, \mu_R, \tau_R, (\nu_{iR}) \rightarrow (1, 1, 1);$$

$$(t, b)_L \rightarrow (2, 1, 3); \quad t_R, b_R \rightarrow (1, 1, 3);$$

(17)

using the notation ($SU(2)_L, SU(3)_1, SU(3)_2$). We break the symmetry $SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_c$ at the scale $M$ by introducing a $(1, 3, \bar{3})$ scalar (Higgs) field $\Phi^a_0$ and
a VEV: \( \langle \Phi \rangle = \text{diag}(M, M, M) \). This breaks \( SU(3)_1 \times SU(3)_2 \) to the massless QCD gauge group \( SU(3)_c \) with gluons, \( A^A_{\mu} \) and a residual global \( SU(3)' \) with degenerate, massive colorons, \( B^A_{\mu} \). The gluon \( (A^A_{\mu}) \) and coloron \( (B^A_{\mu}) \) fields are then defined by:

\[
A^A_{1\mu} = \cos \theta A^A_{\mu} - \sin \theta B^A_{\mu}; \quad A^A_{2\mu} = \sin \theta A^A_{\mu} + \cos \theta B^A_{\mu};
\]

where:

\[
h_1 \cos \theta = g_3; \quad h_2 \sin \theta = g_3; \quad \tan \theta = h_1/h_2; \quad \frac{1}{g_3^2} = \frac{1}{h_1^2} + \frac{1}{h_2^2};
\]

and \( g_3 \) is the QCD coupling constant at \( M \). We envision \( h_2 \gg h_1 \) and thus \( \cot \theta \gg 1 \) e.g., to select the top quark direction for condensation. The mass of the degenerate octet of colorons is given by:

\[
M_B = \left( \sqrt{h_1^2 + h_2^2} \right) M = \left( \frac{g_3}{\sin \theta \cos \theta} \right) M.
\]

The usual QCD gluonic interactions are obtained for all quarks (including top and bottom) while the coloron interaction takes the form:

\[
\mathcal{L}' = - \left[ g_3 \cot \theta \left( \overline{t}_\mu \frac{\lambda^A}{2} t + \overline{b}_\mu \frac{\lambda^A}{2} b \right) + \text{additional terms} \right] B^{\mu A}.
\]

If dynamical symmetry breaking occurs, or if the coloron plays a role in inducing a large top quark mass through near critical coupling, then \( \cot \theta \) is roughly determined, e.g., the NJL result is \( h^2 = g_3^2 \cot^2 \theta \gtrsim 8\pi/3 \). The computation of the coloron radiative corrections to the \( Zb\overline{b} \) vertex is similar to the computation of the penguin operators of the nonleptonic weak interactions. The diagram of Fig.(2) gives the usual \( d = 4 \) renormalization effects that are absorbed into counterterms. The amplitude corresponding to the \( d = 6 \) operator component of the diagram is found to be:

\[
g_z g_3^2 \cot^2 \theta \left( \frac{N^2 - 1}{6N} \right) \ln \left( \frac{M_B^2}{M_Z^2} \right) \overline{b} \left( q^2 \gamma_\mu - q_\mu \gamma \right) (g_L b_L + g_R b_R),
\]
where $N = 3$. Note that this result can be obtained by pinching the $B$ propagator and using penguin anomalous dimensions from the renormalization group. This result clearly implies a nonvanishing $\kappa_{L,R}^1$. Setting $q^2 = M_Z^2$ and using current conservation with $m_b \approx 0$, we then have:

$$\frac{\delta \Gamma}{\Gamma_{b\bar{b}}} \sim \frac{g_3^2 M_B^2 \cot^2 \theta}{4 M_B^2 \pi^2} \left( \frac{N^2 - 1}{6N} \right) \ln \left( \frac{M_B^2}{M_Z^2} \right).$$  \hspace{1cm} (23)

In Table II, we present the results for $R_b$ for various values of the model’s parameters. The coloron radiative correction is just a strong short–range attractive force acting as a rescattering of the $b\bar{b}$ final state. It increases the $Z \rightarrow b\bar{b}$ width, which makes the theoretical prediction slightly more compatible with the LEP data. This is expected on general grounds from any new physics that yields an attractive interaction between the $b$ and $\bar{b}$ in the final state. When using the critical value $g_3^2 \cot^2 \theta = 8\pi/3$, the corrections to the $Zb\bar{b}$ in the topcolor model depends on only one parameter, the mass of the coloron, as indicated in Table II. We see that for small $M_B$ the effects can be large, but that for $M_B \gtrsim 600$ GeV only the strongest coupling can accomodate the present $R_b$ excess. This can be compared to the effects upon $t\bar{t}$ production at the Tevatron [6], where a 600 GeV coloron would produce about a four–fold increase in the top production cross–section relative to QCD, and is ruled out if the CDF top mass and cross-section are accepted [10]. The 800 GeV coloron produces a cross-section for $t\bar{t}$ production that is about a factor of 2 greater than QCD. Though the situation wrt top quark production is in flux presently, these issues should be settled by the completion of Run I(b). We see, however, that top production itself is a more sensitive probe than $Z \rightarrow b\bar{b}$ in the topcolor model.
4. Conclusions

In this paper we have explored the interplay between strong dynamics that may be associated with dynamical electroweak symmetry breaking and the large top quark mass, and the observable $R_b$ in the process $Z \to b\bar{b}$. This also involves the top production cross-section at the Tevatron, and may represent a first window on new physics beyond the standard model. For one, our toy models illustrate how fluid the situation is; in some cases the effects of top production can be small when the effects upon $Z \to b\bar{b}$ are large (as in the case of new strongly coupled $U(1)$ interactions), while in other situations top production is more sensitive than $Z \to b\bar{b}$ (as in the case of topcolor). We have found the interesting result that, if a large anomaly in the top quark production cross-section is seen by CDF or D0 at the level of $\gtrsim 50\%$, then we can rule out the effects of a new strong $U(1)$ interaction as the source because it would give rise to very large enhancements (or suppressions) of $Z \to b\bar{b}$, as we have seen above. This is shown in Table I. Certainly, if the reportedly large cross-section at CDF continues to hold, then this cannot be explained by a $U(1)$ boson leading to the contact term of eq.(14) since $R_b$ forces too large a value of $\Lambda$ for such terms to have any effect upon top production at this level.

We have also seen that, for the case of the topcolor model which would effectively generate new color octet contact terms, the effects in $Z \to b\bar{b}$ are not dramatic. For example, with $M_B \leq 600$ GeV, which may enhance top production too much, the discrepancy in $R_b$ with the standard model result can be reduced only from $\sim 2\sigma$ to $\sim 1\sigma$ in the limit of large coupling. $\kappa_L$ in the topcolor model is positive, and we cannot rule out the scheme with the present value of $R_b$ from the LEP experiments
even for critical coupling. For $M_B \gtrsim 800$ GeV the enhancements are within $1\sigma$ of the standard model. To explain an excess of $R_b$ at the $2\sigma$ level we require $M_B \sim 500$ GeV, and critical coupling. The effects of new physics from topcolor at this level are probably already ruled out, depending upon the mass of the top quark. Certainly, a more definitive situation will exist at the end of run I(b) at the Tevatron.

Unfortunately, we cannot presently argue that the experiments are any more than marginally inconsistent with the standard model. The standard model may end up prevailing in these observables. However, our toy models illustrate the importance of refining these measurements. Moreover, we see that establishing the validity of the standard model at the $Z$–pole is far from establishing its validity at $\sim 500$ GeV at which top production at the Tevatron occurs. In the long run the possibility of new physics beyond the standard model beginning to show up in these observables is real, and may represent our first excursion into the new territory of electroweak symmetry breaking.

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Table I: $R_b$ for various values of the cutoff $\Lambda$ in the color singlet contact term case. $m_t = 174$ GeV is assumed, and we take the standard model result to be 0.2157, and thus $R_b/R_{b,SM} = 1.0162 \pm 0.0083$. In the last column we give the ratio of the top pair production cross-section to the standard model result, where we also assume the light fermions participate in four–fermion operators as in [6]. Since this assumes that the new physics acts universally on the light quarks, $u$ and $d$, it is in a sense an upper limit on the effects on top production. We see that saturating the $R_b/R_{b,SM}$ excess leads to a $\lesssim 30\%$ excess in top production, while significantly larger top cross-sections are ruled out by the observed $R_b/R_{b,SM}$ if the physics is described by color singlet contact terms.
Table II: $R_b$ for various values of the parameters of the topcolor model, where we take the standard model result to be 0.2157 and thus $R_b/R_{b\,SM} = 1.0162 \pm 0.0083$. The value 1.0 for $k = N_c g_3^2 \cot^2 \theta/8\pi$ corresponds to NJL critical coupling. A 600 (800) GeV coloron would produce about a four–fold (two–fold) increase in the $t\overline{t}$ production cross–section relative to QCD (independently of $N_c g_3^2 \cot^2 \theta/8\pi$, for a top mass of 175 GeV) thus top production is a more sensitive probe of new physics than $Z \rightarrow b\overline{b}$ in the topcolor model.
Figure Captions

Figures 1. Vertex correction to $Zb\bar{b}$.

Figure 2. Radiative correction to the $Zb\bar{b}$ in the topcolor model.

Figure 3(a). $R_b/R_{b,SM}$ for various values of the parameters of the topcolor model, where the LEP result = 1.0162 ± 0.0083 is superimposed. The value $k = 1.0$ where $k = N_c g_3^2 \cot^2 \theta / 8\pi$ corresponds to NJL critical coupling, and we consider $k = 0.5$, $k = 1.0$ and $k = 2.0$.

Figure 3(b). The top production cross-section $\sigma_t$ normalized to the standard model result is given for the topcolor model [6]; we take $\sigma_t(SM) = 5.25 \text{ pb}$ for $m_t = 175 \text{ GeV}$, and we superimpose the CDF result [16]. The Figures 3(a) and 3(b) show that top cross-section measurements are more restrictive than $R_b$ for the topcolor model; thus, to fit the central value of the LEP $R_b$ requires an unacceptably large top cross-section in the model.
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