Analysis of the load on the taxi system in a medium-sized city

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Abstract. In this research, the carriage of taxi drivers is considered by means of the queue theory. The study of the taxi system found out that the number of servers was too large. This results in low efficiency of taxi cars. The purpose of this study is to analyse the workload of the working servers in terms of their number within one day. An application was created in Matlab environment, which solves the problem and allows to evaluate the effect of this solution. The result of this research recognizes the need for reducing the number of working taxi vehicles by half or keeping their number but reducing their load factor variation between 11% and 19%.

1. Introduction

According to Germany’s Taxi Association there are too many taxis on the roads. It means driver’s income is reduced in the hunt for fares. Lower revenues mean slower system development, poorer quality of transport service and, most importantly, reduced transportation safety due to poor car maintenance because of lack of finances. This problem exists in other countries as well, as different cities and companies apply different solutions. For example, Berlin plans a minimum wage for the taxi sector – raising the prospect of fare hikes [1].

Many researchers came across issues to find optimal solution - such as leveraging computers and updated computing systems [2]. Previous attempts, to solve these issues, have been made also with other mathematic methods. The most famous way previously used to name the problem was the "traveling salesman problem," in which mathematicians would minimize the total distance a salesman would travel if he had to visit a certain number of locations to stop in a day of sales. The problems described also apply to medium-sized Bulgarian cities such as Rousse.

Another problem with the large number of taxis and shared car services is the generation of too much traffic in cities. There are a total of 2800000 vehicles registered in Bulgaria, which in each group are roughly divided as follows [3]: light cars – 2200000; lightweight cargo vans to 3.5 tons – 50000; buses for urban transport – 6000.

The large number of passenger cars in the country cause problems related to road traffic, which necessitates the study of the characteristics of traffic flow and, above all, traffic intensity.

In connection with Bulgaria’s membership in the European Union and the integration of its transport infrastructure, considerable efforts are being made to introduce European standards and laws for modern, environmentally friendly and safe transport. Insufficient investment in building new infrastructure and maintaining existing infrastructure requires timely planning for its future development [4, 5].
Road transport in Bulgaria is developed at fast pace due to its high velocity and great flexibility. The same applies to urban passenger transport and, in particular, to taxi transport. Its advantages over mass city transit of passengers are:

- possibility to carry out a free route;
- possibility to use 24 hours a day;
- the possibility of transporting “from door to door” [5].

One of the main features of traffic flow is traffic intensity. It is expressed in the number of cars passing through a section of the road per unit of time. This intensity changes throughout the day, week, month and year. The intensity of traffic in a given city depends on the amount of vehicles in motion, the links between transport and pedestrian flows, the density of the street network and the city’s territory. The intensity of traffic and the parameters of taxi transport strongly influence the number of taxi drivers operating in the particular settlement.

As of January 2019 in the Municipality of Ruse are registered 151 companies with a permit for carrying out taxi activity. The total number of taxi cars operating on the territory of the Municipality of Rousse is 588 [6]. The average number of taxi cars in a company is 3.89, and the number of taxi cars per 1000 people is 4.15. For Rousse, the number of companies providing radio communication for management of taxis is 4, shown in table 1.

Table 1. Taxi companies providing radio communication for taxi management as of 30.05.2019.

| Number | Firm                          |
|--------|-------------------------------|
| 1      | Taxi 2222 – to4nite           |
| 2      | 8806 – Taxi 6                 |
| 3      | Orion Taxi 8800               |
| 4      | Milanov Taxi 8111             |

Table 2. Registered taxis in five European countries and Bulgaria.

| Inhabitants, thousands | Bulgaria | Holland | Switzerland | Great Britain | Belgium | France |
|------------------------|----------|---------|-------------|---------------|---------|--------|
| Number of taxi cars    | 30560    | 19291   | 14558       | 70000         | 4000    | 44000  |
| Number of taxi cars per 1000 people | 4.15  | 1.2  | 1.67 | 1.2 | 0.4 | 0.7 |

On average, for the five European countries, the number of taxi cars per 1000 people is 1.03, while for Bulgaria this index is 4.17.

In the table 3 is a number of taxis per thousand people for five European capitals with an average of 2.22. The table shows that for Rousse this indicator is 1.8 times higher than the average. From the information presented, it seems that the number of taxi cars is much higher than the average for the big European cities. What is needed to achieve efficiency in the taxi system is to establish the optimal number of taxi cars for meeting the needs for this type of transport in society and a transport scheme tailored to the infrastructure of the respective settlement.

Table 3. The density of the taxi network to 10.10.2019.

| Inhabitants | Rousse | Stockholm | London | Bruxel | Copenhagen | Paris |
|-------------|--------|-----------|--------|--------|------------|-------|
| Taxi cars   | 150000 | 1600000   | 700000 | 960000 | 1400000    | 600000 |
| Number of taxi cars per thousand people | 3.89  | 2.8  | 2.5  | 1.3  | 2.0  | 2.5  |
The purpose of this study is to find a proper solution for the large number of taxis in a medium-sized Bulgarian city.

2. Methodology of the research

The chosen method of examining the organization of the taxi system is the mathematical apparatus of a queue theory system. This theory has been chosen because it best describes the process of taxi transportation in its entirety.

To describe the operation mode of a taxi company by means of the queue theory, it is necessary to know the characteristics of the input stream of orders treated as a stochastic process, the service intensity, the maximum length of the queue and the number of service units [7, 8].

For the incoming flow of cargo it could be made the following assumptions for the ordinal flow and flow without consequences. Regarding the stationarity / non-stationarity of the flow – for sufficiently long periods of time – 1 month, 6 months, 1 year, etc. it is possible to assume the steady-state of the incoming stream, i.e. with some provisionallyty. The probability of availability of a certain number of cars in a given, sufficiently long interval depends only on the length of that interval. Generally, at random times, flow $\lambda$ is non-stationary $\lambda = \lambda(t)$. This non-stationarity stands out well over a 1-day working day (24 hours). The incoming and outgoing stream is modelled [9-11].

For the intensity of service $\mu$, it is known by the company manager that the time for servicing one order is a relatively constant quantity and is 8.5 minutes in the time intervals between 07:00-10:00 h and 16:00-19:30 h, while for the rest of the day it is 5.5 min. The number of serving channels (servers), over the day, is variable and changes between 26 and 120 (26 $\leq n \leq 120$). It is also known that at the moment when all servers are busy, the received order is rejected. To test the operation of the system, it is necessary to find the probability that the system will have $k$-number of vehicles at time $t$ when $n$-servers are operating, i.e.

$$P_k(t) = \frac{\lambda}{k!} \mu^k e^{-\lambda t}, k=0, n, t \in [0, T], 26 \leq n(t) \leq 120,$$

where for one period $T$ is taken a full working day $T = 0$ of 24 hours. The beginning of the working day $t = 0$ coincides with the astronomical start of the day (0 hours and 0 min), the end of the working day $t = T = 24$, the end of the astronomical day (24 hours and 0 min).

For the queue theory model, the following can be summarized: a non-stationary flow of densities $\lambda(t)$ flows into a queue theory with $n$-number service channels. Request processing time is a random variable with periodicity. This is appropriate given the daily fluctuations. Approximate squares [14] was used to approximate the average of the received queries over a 24-hour period (figure 1). The model should be as simple as possible but reflect the most characteristic behaviour of the actual stream. For a model, the following trigonometric line (7-point Fourier row) is selected, linear to the quoted odds:

$$\lambda(t) = a_0 + a_1 \cos \left(\frac{2\pi t}{24}\right) + b_1 \sin \left(\frac{2\pi t}{24}\right) + a_2 \cos \left(\frac{4\pi t}{24}\right) + b_2 \sin \left(\frac{4\pi t}{24}\right) + a_3 \cos \left(\frac{6\pi t}{24}\right) + b_3 \sin \left(\frac{6\pi t}{24}\right).$$
3. Result of the research

The data for the received requests (by hour) for 24 hours over a week are given in Table 4.

| Date/hour | Number of cars in the day |
|-----------|---------------------------|
| 2.4.2019  | 41 30 18 23 24 59 102 137 127 119 108 121 108 107 118 101 72 76 65 55 49 275 |
| 3.4.2019  | 34 35 20 19 20 27 60 112 142 132 120 126 96 108 110 94 102 118 95 89 81 68 66 50 284 |
| 4.4.2019  | 40 29 21 20 26 31 59 118 136 137 127 106 105 114 93 115 90 108 94 101 81 72 59 51 290 |
| 5.4.2019  | 40 31 19 19 22 29 63 119 133 134 118 108 109 113 100 127 123 118 117 121 102 98 89 84 292 |
| 6.4.2019  | 76 53 44 39 32 42 46 72 78 99 106 94 92 96 92 77 86 99 100 113 107 105 95 100 276 |
| 7.4.2019  | 93 76 58 46 38 49 50 51 57 73 78 78 63 92 80 80 79 82 85 76 73 59 63 68 246 |
| 8.4.2019  | 40 26 17 20 16 35 73 130 157 146 146 116 100 127 107 116 109 111 95 80 65 60 61 67 285 |

The coefficients $a_0$, $a_1$, $b_1$, $a_2$, $b_2$, $a_3$, $b_3$ calculated by the LSM, as well as their confidence intervals are given in Table 5. All coefficients are statistically significant (the confidence intervals do not contain 0). The determinant coefficient is $R^2 = 0.9067$, also statistically significant, meaning that the chosen model is adequate.

| Value of the coefficients in the model | Trust confidence interval guaranteed by probability $\gamma = 0.95$ |
|--------------------------------------|------------------------------------------------------------|
| $a_0$ = 111.6                        | (105.3; 117.8)                                             |
| $a_1$ = −29.73                       | (−38.57; −20.89)                                           |
| $b_1$ = −10.64                       | (−19.48; −1.803)                                           |
| $a_2$ = −8.927                       | (−17.77; −0.0879)                                          |
| $b_2$ = −36.8                        | (−45.64; −27.96)                                           |
| $a_3$ = 21.13                        | (12.29; 29.97)                                             |
| $b_3$ = −4.83                        | (−9.66; −0.009)                                            |

For the service $\mu(t)$ speed, it is known that during the more busy daytime, 07:00-10:00 h and 16:00-19:30 h, the service of one request takes about 8.5 minutes. For one hour, average service speed is $120/17 = 7.0588$ queries per hour, and during the rest of the day, queries are served for 5.5 minutes; the speed is $120/11 = 10.909$ requests per hour.

$$\mu(t) = \begin{cases} 
\frac{120}{11}, & 0 \leq t < 7; \ 10 \leq t < 16; \ 19.5 \leq t < 24 \\
\frac{120}{17}, & 7 \leq t < 10; \ 16 \leq t < 19.5 
\end{cases}$$

(3)
Statistical data over the course of one week for the number of servers \( t \) operating overnight are given in Table 6.

| Date/Hour | 00-01 | 01-02 | 02-03 | 03-04 | 04-05 | 05-06 | 06-07 | 07-08 | 08-09 | 09-10 | 10-11 | 11-12 | 12-13 | 13-14 | 14-15 | 15-16 | 16-17 | 17-18 | 18-19 | 19-20 | 20-21 | 21-22 | 22-23 | 23-24 | Total applications |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|------------------|
| 2.4.2019  | 71    | 54    | 27    | 35    | 39    | 40    | 85    | 184   | 216   | 176   | 157   | 137   | 119   | 157   | 116   | 122   | 133   | 145   | 122   | 90    | 106   | 104   | 85    | 75    | 2595             |
| 3.4.2019  | 51    | 46    | 23    | 27    | 39    | 37    | 94    | 183   | 229   | 176   | 157   | 137   | 119   | 157   | 116   | 122   | 133   | 145   | 122   | 90    | 106   | 104   | 85    | 75    | 2595             |
| 4.4.2019  | 71    | 44    | 28    | 30    | 27    | 43    | 83    | 203   | 221   | 173   | 157   | 137   | 119   | 157   | 116   | 122   | 133   | 145   | 122   | 90    | 106   | 104   | 85    | 75    | 2595             |
| 5.4.2019  | 61    | 48    | 31    | 48    | 24    | 43    | 83    | 212   | 211   | 175   | 158   | 137   | 119   | 157   | 116   | 122   | 133   | 145   | 122   | 90    | 106   | 104   | 85    | 75    | 2595             |
| 6.4.2019  | 137   | 82    | 65    | 45    | 52    | 81    | 66    | 117   | 108   | 120   | 132   | 105   | 113   | 110   | 120   | 137   | 159   | 132   | 167   | 162   | 136   | 171   | 120   | 148   | 2536             |
| 7.4.2019  | 230   | 132   | 71    | 69    | 47    | 93    | 89    | 105   | 89    | 108   | 110   | 99    | 71    | 117   | 92    | 119   | 97    | 111   | 103   | 118   | 104   | 127   | 91    | 2579            |
| 8.4.2019  | 68    | 54    | 22    | 42    | 25    | 72    | 118   | 290   | 267   | 196   | 181   | 128   | 109   | 148   | 128   | 108   | 128   | 124   | 138   | 117   | 100   | 97    | 81    | 110   | 2842             |

Given the statistics, the number of working servers is approximated as a whole (Figure 2). The length of time intervals in which the number of servers is more dynamic is shorter (a shorter time interval of 1 hour cannot be taken due to the specificity of the statistical measurements), those in which the dynamics is smaller have longer intervals.

\[
n(t) = 46, \quad 0 \leq t < 1
\]
\[
34, \quad 1 \leq t < 2
\]
\[
26, \quad 2 \leq t < 5
\]
\[
46, \quad 5 \leq t < 6
\]
\[
80, \quad 6 \leq t < 7
\]
\[
120, \quad 7 \leq t < 10
\]
\[
104, \quad 10 \leq t < 18
\]
\[
91, \quad 18 \leq t < 21
\]
\[
68, \quad 21 \leq t < 24
\]

The computational features associated with system (1) after the following:
- a large-scale system (generally);
- the system is of the “rigid system” type (rigid system of equations);
- the number of equations is variable at the various integration intervals.

This can be summarized as: a system with varying number of solid, differential equations of large size. To overcome these difficulties, special numerical methods have been developed. A Matlab [14] program was developed to solve a system (2), using the built-in “solver ode15s”, implementing the Gir method. When entering \( \lambda(t), \mu(t), n \) the application returns a numerical solution to \( P(t) \). Artificially, the precision is greater than that implied for the solver – from \( 10^{-6} \) to \( 10^{-9} \) for absolute error and \( 10^{-3} \) to
10^{-5} for a relative error. The integration interval [0, T] is broken down into subfields \( t_0, t_1, \ldots, t_{i-1}, t_i, t_{i+1}, \ldots, t_N \) so that in each subinterval \([t_i, t_{i+1}],[\mu(t)\) and \(n(t)\) are constants.

The initial state of the system \( P_0(t_0)\) is unknown. It is known that these types of processes are persistent and after a long period of time they enter into regular mode of operation. Therefore, a random state may be taken arbitrarily. The values \( P_0(t)\), at the end of the time interval \([t_{i-1}, t_i]\), become initial values for the subinterval \([t_i, t_{i+1}]\). If there are more equations for a given time interval than the previous interval, then zero values are assumed for the initial conditions of these equations. System integration in the \([0, T]\) interval is required to be done not once but a sufficient number of times, and after each integration the values at the end of the period become initial values for the next integration. In this way, the probability functions \( P_0(t)\) begin to bend to their regular values. After multiple integration, it was found out that only after 5-6 periods the functions \( P_0(t)\) entered into regular mode (for two adjacent periods, they remain the same). Accuracy is also increased here, with integration being done over 20 periods, with the difference of all \( P_0(t)\) in the last and penultimate periods being less than \(10^{-8}\) for each \( t\). Table 7 lists the time intervals \([t_{i-1}, t_i]\), the values of the function \(\mu\), and the number of service channels \(n\) in the given interval [14].

| A time interval within 24 hours \([t_{i-1}, t_i]\) | \([0, 1]\) | \([1, 2]\) | \([2, 5]\) | \([5, 6]\) | \([6, 7]\) | \([7, 10]\) | \([10, 16]\) | \([16, 18]\) | \([18, 19.5]\) | \([19.5, 21]\) | \([21, 24]\) |
|---------------------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Average Service Rate \(\mu\)   | 120/11   | 120/11   | 120/11   | 120/11   | 120/11   | 120/17   | 120/11   | 120/17   | 120/17   | 120/11   | 120/11   |
| Average number of serving servers \(n\) | 46       | 34       | 26       | 46       | 80       | 120      | 104      | 104      | 91       | 91       | 68       |

Table 7. Distribution of servers at time intervals, the values of the function \(\mu\), and the number of service channels \(n\) in the given interval.

After integrating a system (2) and calculating the \(P_0(t)\), it is interesting to see how the average number of servers are occupied and how the load factor of a server changes.

Figure 3 shows the average number of customer-driven taxi cars for a 24-hour period. Two peaks are observed, the first one is between 7 and 10 hours, and the smaller is between 16-19 hours. The smallest peaks are in the range of 2-3 hours in the morning.

Figure 4 shows the load factor of a server (taxi) over the course of a day. It can be seen that even with the highest peaks, the coefficient is below 0.25, which means that at the busiest intervals, the server occupancy is below 25% of the employed at that particular moment. Average server occupancy is about 15%.

![Figure 3: Average number of occupied servers (taxi cars) over a 24-hour period.](image3.png)

![Figure 4: Load factor of one server (taxi) for a period of 24 hours.](image4.png)
There are two options for distributing the average number of servers in the considered time intervals. The first option is to keep the average number of working cars during the day so the load factor does not change, but it is intended to reduce the limits of that factor over the day. A distribution of servers in the time slots that narrows the boundaries of the load factor is shown in table 8.

Table 8. Distribution of servers at time intervals adjusting the boundaries of the load factor.

| A time interval within 24 hours $[t_{i-1}, t_i]$ | [0, 1] | [1, 2] | [2, 5] | [5, 6] | [6, 7] | [7, 10] | [10, 16] | [16, 18] | [18, 19.5] | [19.5, 21] | [21, 24] |
|------------------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Average Service Rate $\mu$                      | 120/11| 120/11| 120/11| 120/11| 120/11| 120/11| 120/11| 120/11| 120/11| 120/11| 120/11 |
| Average number of serving servers $n$           | 44    | 34    | 23    | 31    | 69    | 138   | 83    | 101   | 96    | 75    | 61     |

In this embodiment, the average load factor is again about 15%. Except for the larger but shorter peak at about 10.00 hours, the load factor limits are about 11% and 19%.

According to the second option the number of servers must be reduced in such a way that the average ratio of workload to increase to 40%, but the difference between the smallest / largest and average (40%) of the load factor, is about 10% within the day. A variant of such distribution of servers in the time slot is shown in table 9.

Table 9. Distribution of servers in the time slots increasing the average load factor.

| A time interval within 24 hours $[t_{i-1}, t_i]$ | [0, 1] | [1, 2] | [2, 5] | [5, 6] | [6, 7] | [7, 10] | [10, 16] | [16, 18] | [18, 19.5] | [19.5, 21] | [21, 24] |
|------------------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Average Service Rate $\mu$                      | 120/11| 120/11| 120/11| 120/11| 120/11| 120/11| 120/11| 120/11| 120/11| 120/11| 120/11 |
| Average number of serving servers $n$           | 22    | 14    | 8     | 14    | 24    | 58    | 27    | 48    | 42    | 30    | 29     |

The load factor, calculated from the distribution in table 9, is given in figure 5. From the graph of Figure 6 shows that the average load factor is about 49%. Except for the peak of about 10 hours, which is negligibly short and the peak of about 7 hours, which is slightly longer but relatively small, it can be seen that the daytime load factor is between 30% and 50%. In order to achieve this effect, the average number of servers is the day-to-day need to be reduced by a little more than twice.

![Figure 5](image1.png)  ![Figure 6](image2.png)

**Figure 5.** Load factor of one server (taxi) over a 24-hour period with distribution from table 8. Red lines – load factor limits without short peaks.

**Figure 6.** Load factor of one server (taxi) over a 24-hour period with allocation from table 9. Green lines – load factor limits without short peaks; Red line – average values of the load factor.
4. Conclusion
The input flow is modelled on a query stream by means of statistical inventory. The function describing
the average number of working servers in the day is created using statistical data.

The statistics given are calculated by the load factor of a server. This factor is too small and leads to
inactivity of the servers. In addition, with this server distribution during daytime, the probability range
is between 10% and 25%.

A redistribution of servers is proposed overnight, while retaining their number but shrinking the
range of load factor variation between 11% and 19%. This provides a relatively uniform workload for
servers over the day or the number of orders they serve.

A variant is proposed to reduce and appropriately allocate servers at different times of the day. In
this variant, the average server load is about 40%, with the range of variation being between 30% and
50%. The number of servers has been reduced slightly more than twice. This ensures a higher average
workload per server.

With a database available for a similar city in size, it is possible to carry out an analogous study of
the workload of the taxi-transport system, and determine the relevant conclusions. Based on the findings,
it is possible to offer variants for changing the number of working servers in 24 hours.

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