Neutrino Emission from Dense Matter, 
and Neutron Star Thermal Evolution

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Abstract

A brief review is given of neutrino emission processes in dense matter, with particular emphasis on recent developments. These include direct Urca processes for nucleons and hyperons, which can give rise to rapid energy loss from the stellar core without exotic matter, and the effect of band structure on neutrino bremsstrahlung from electrons in the crust, which results in much lower energy losses by this process than had previously been estimated.
1 Introduction

The problem of the thermal evolution of neutron stars is a complex one, and in investigating it theoretically one can identify a number of rather distinct aspects. The first set of questions concerns the problem of how thermal energy is lost from the star. One way in which this occurs is by emission, from matter in the stellar interior, of neutrinos, and possibly of other particles that can readily escape from the neutron star. These aspects of the problem are the ones we shall focus on in this article. Another way is by emission of thermal energy from the surface of the star. The energy radiated is transported to the surface by thermal conduction and radiative transport, the former process being the most important one at high densities, and the latter being the most important at low densities. Heat flow to the stellar surface is controlled by a bottleneck at relatively low densities, where neither thermal conduction nor radiative transport are particularly effective. These problems have been considered in detail by a number of authors, both in the absence of magnetic fields\[1\], and with the effects of the magnetic field included\[2\].

A second set of questions concerns mechanisms for heating the neutron star. Among these one may name friction between superfluid and normal components of the quantum fluids in the interior, if some of these are superfluid, and ohmic losses due to the electrical currents in the star that sustain the star’s magnetic field. Some of these processes will be considered in other contributions to this volume.

A third set of questions concerns the spectrum of radiation emitted from the stellar surface, which will be considered elsewhere in these proceedings\[3\]. This is of great importance for understanding observations of radiation from neutron stars\[4\], but fortunately the total energy radiated from the stellar surface is largely independent of processes in the last few photon mean free paths in the atmosphere, and therefore one can calculate the total energy radiated without knowing the physics that determines the details of the stellar spectrum, just as is the case in ordinary stars.

Ever since neutron stars became objects of serious study by physicists over a quarter of a century ago it has been realized that the observation of thermal radiation from neutron stars provides a way of probing the interior temperature of the star, and therefore has potential for giving valuable information about the nature of matter in
the stellar interior. Theoretical arguments alone are insufficient to establish exactly what state of matter exists in the cores of neutron stars, and therefore information that can be gleaned from astronomical observations could play an important role. As we have heard in other talks at this meeting[4, 5] techniques for making observations in the X-ray and soft ultraviolet ranges of the spectrum have now been developed to such a degree that this dream is close to being realized.

We begin by summarizing the theoretical situation as of 1990, when the previous workshop in this series on neutron stars and related phenomena was held in Agia Pelagia. If a neutron star were observed to cool slowly, on a timescale of order $1\. yr/T_9^4$, where $T_9$ is the core temperature in units of $10^9$ K, this would be a clear sign of the core being composed of normal matter (a mixture of neutrons, protons, and electrons, with possible admixtures of other constituents), while if the neutron star were observed to cool rapidly, on a timescale of order $1\. min/T_9^4$, this would indicate that matter in the core was in an exotic state, such as quark matter, or a Bose condensate of pions or kaons. The reason for the very different timescales is that it was argued that in ordinary matter, the dominant neutrino emission process is the so-called modified Urca process, first discussed by Chiu and Salpeter[6], in which the two reactions

$$n + n \rightarrow n + p + e^- + \bar{\nu}_e \quad \text{and} \quad n + p + e^- \rightarrow n + n + \nu_e$$

occur in equal numbers. These reactions are just the usual processes of neutron beta decay and electron capture on protons,

$$n \rightarrow p + e^- + \bar{\nu}_e \quad \text{and} \quad p + e^- \rightarrow n + \nu_e$$

with the addition of an extra bystander neutron. They produce neutrino-antineutrino pairs, but leave the composition of the matter constant on average. The rate at which modified Urca reactions occur per unit volume varies as $T^7$, a result that can easily be understood from phase space arguments. At temperature $T$, the energy available to a neutrino is of order $k_B T$, and therefore, since the neutrinos are nondegenerate and can freely escape from the star, the phase space volume accessible is a sphere in neutrino momentum space, of radius $\sim T$. Consequently the phase space factor is
proportional to $T^3$. The other particles participating in the modified Urca reaction are degenerate fermions, and therefore the number of states (either initial or final) is proportional to the volume of a shell of states with a width in energy $k_B T$ about the Fermi surface, and therefore each of these fermions gives a factor $T$ to the phase space factor. However, the energies of the particles are constrained by the condition of energy conservation, and therefore the energies of only 4 of the 5 degenerate fermions are independent. As a result, the degenerate fermions give a factor $T^4$ to the phase space factor. Combining this with the factor for the neutrino phase space, we see that the rate of the modified Urca process should vary as $T^7$. For neutron star thermal evolution, the relevant quantity is the rate of energy emission, $\dot{E}$ which, since the neutrino energy is $\sim k_B T$, varies as $T^8$. The thermal energy, $E_{\text{th}}$ of degenerate fermions varies as $T^2$ at low temperatures, and consequently the cooling time,

$$\tau = \frac{E_{\text{th}}}{\dot{E}}$$  \hspace{1cm} (3)

which is determined locally, varies as $T^{-6}$, as given above.

In the case of exotic states it is possible for processes of the type

$$f_1 \rightarrow f_2 + e^- + \bar{\nu}_e \ \text{and} \ f_2 + e^- \rightarrow f_1 + \nu_e$$ \hspace{1cm} (4)

to occur. Here $f_1$ and $f_2$ are two fermions, whose character depends on the specific exotic state considered. From arguments analogous to those given above for the modified Urca process, one can see that the rate of neutrino energy loss varies as $T^6$, and therefore the characteristic cooling time varies as $T^{-4}$, in agreement with the result stated earlier. For a discussion of the relevant reactions for specific exotic states, we refer to Ref. [7].

Over the past few years it has become clear that there are a number of possibilities for processes of the type (4) to occur in dense matter, even in the absence of exotic states. These so-called “direct Urca processes” will be discussed in Sec. 2.

The arguments we have made for the temperature dependence of reactions that produce neutrinos depend on the spectrum of elementary excitations being smooth in the vicinity of the Fermi surface. Should matter become superfluid, or superconducting, by pairing of particles, as in metallic superconductors, gaps can open up at the Fermi surface, and these will suppress neutrino emission at temperatures less than or
of the order of the transition temperature to the paired state. If the effects of superfluidity in the core of the star are sufficiently pronounced, neutrino emission from the crust of the neutron star could be the dominant energy loss mechanism. Recent work on what was generally regarded as the dominant neutrino production mechanism in the crust, bremsstrahlung of neutrino pairs by electrons, has shown that at temperatures of order $10^{10} \text{K}$ this process is much less effective than previously estimated\cite{8}. The reason for this suppression is the band structure of electron states resulting from the motion of the electrons in the periodic lattice of nuclei, an effect we shall describe in the Sec. 3.

In Sec. 4 we shall address a number of unsolved problems. These include the reliability of estimates of the modified Urca process, and some processes that can occur in neutron star crusts as a consequence of the aspherical nuclear shapes described elsewhere in this volume\cite{9}. Concluding remarks are made in Sec. 5.

2 Direct Urca Processes

One of the simplest possible ways of generating neutrinos and antineutrinos in dense matter is the pair of processes (4), and we begin by investigating the their kinematics. In degenerate matter, the condition of energy conservation, together with the requirement that final states for fermions must be unoccupied, lead to the conclusion that particles participating in reactions must lie within an energy $\sim k_B T$ of their respective Fermi surfaces. The momenta of the degenerate fermions must therefore be close to their Fermi momenta, and the neutrino momentum is of order $k_B T/c$, which at low temperatures may be neglected compared with the Fermi momenta. The condition for momentum conservation therefore amounts to the requirement that it be possible to construct a triangle from the three Fermi momenta. In the canonical view, neutron star matter at around nuclear density consists mainly of neutrons with a small fraction of protons, together with an equal number of electrons to ensure charge neutrality. The density of a particle of species $i$ is proportional to $p_i^3$, where $p_i$ is its Fermi momentum, and therefore the Fermi momenta of protons and electrons are equal if there are no other charged constituents. If the proton fraction is of order 1%, as was estimated to be the case in the 1960’s, when microscopic mechanisms for neutrino production in dense matter were first investigated, one can
see that $p_p/p_n = p_e/p_n \simeq 1/5$. Thus one can see that it is impossible to satisfy the triangle inequality, and the direct Urca process could not occur in dense matter. On the basis of this argument, Chiu and Salpeter were led to investigate other possibilities, in particular the modified Urca process mentioned above, which acquired the status of the “standard” process for neutron star cooling.

2.1 The Nucleon Process

It is interesting to enquire just how large a proton fraction is necessary for the direct Urca process to proceed. If protons and electrons are the only charged constituents, it is easy to see that, since the electron and proton Fermi momenta are equal, the smallest proton Fermi momentum for which one can construct a triangle from the Fermi momenta is one half of the Fermi momentum. Thus the proton to neutron ratio is 1/8, and the ratio of protons to nucleons is 1/9, or just over 11%. This calculation was made as long ago as 1981 by Boguta\cite{10} in a paper that went largely unnoticed by the neutron-star community, and the same arguments were made a decade later in Ref.\cite{11}, where the rate of the process was also calculated for proton fractions in excess of the threshold value. If one allows for muons as well as electrons, the threshold proton fraction is slightly higher: in particular, when the electron chemical potential is much larger than the muon rest mass energy, it is $\simeq 0.148$.

While such proton concentrations were regarded as unrealistically high in the mid 1960’s, the situation is less clear today. Using methods that describe the nucleon-nucleon interaction by a potential, Wiringa, Fiks, and Fabrocini\cite{12} find proton fractions that lie below the threshold value, although for some interactions the proton fraction is rather close to the threshold value. It is important to note that proton fractions are not well determined theoretically, because they depend on, among other things, the isospin dependence of the three-body interaction, which is not well characterized. Models of dense matter based on mean field theory (see, e.g., Ref.\cite{13}) tend to give larger proton fractions than do the potential models, and the proton fraction in many cases exceeds the threshold value. To determine whether or not the direct Urca process for nucleons is a realistic possibility in dense matter, more reliable estimates of proton fractions are required.

Should the kinematical constraints for the process \cite{2} be satisfied, the cooling
time (3), will be given by

\[
\frac{1}{\tau_{\text{Urca}}} = \frac{457}{3360\pi} \frac{G_F^2 \cos^2 \theta_C (1 + 3g_A^2) m_n c}{\hbar e^0} \frac{\mu_e (k_B T)^4}{p_n},
\]

where the thermal energy, \(E_{th}\), has been approximated by that for neutrons alone.\(^7\)

Here, \(G_F\) is the weak interaction coupling constant, \(\theta_C\) the Cabibbo angle, \(g_A\) the axial vector coupling constant, and \(m_n\) the neutron rest mass.

One interesting possibility that has been explored recently is that the appearance of a Bose condensate of kaons would lead to an increase of the proton fraction, which, if it were sufficiently large, could result in the kinematic condition for the direct Urca process for nucleons being satisfied.\(^14\) If a \(K^-\) condensate appears, it is less costly energetically to add negative charge than it is in the absence of the condensate. Since bulk matter is electrically neutral, this implies that in equilibrium there will be more positive charge, i.e. protons, than there would be in the absence of the condensate. A competing effect is that when a condensate appears, the electron fraction tends to decrease, because some of the negative charge resides in the condensate, but for most models of dense matter that have been investigated to date, this effect is more than outweighed by the increase in the proton fraction. Should the proton concentration exceed the threshold value for the direct Urca process, the cooling time in the presence of a kaon condensate would be \(\tau_{K_{\text{Urca}}} = \sec^2(\theta/2) \tau_{\text{Urca}}\), where \(\theta\) is angle that measures the amplitude of the condensate. Even if the proton concentration is below the direct Urca threshold, alternative processes are possible, such as (4) with \(f_1\) and \(f_2\) both representing neutrons, modified due to the presence of a condensate. However, since such processes do not conserve strangeness, the corresponding cooling time, \(\tau = 4 \cot^2 \theta_C \csc^2 \theta \tau_{\text{Urca}}\), is typically much greater than \(\tau_{K_{\text{Urca}}}\).

### 2.2 Processes for Hyperons and Isobars

Dense matter may well contain particles other than the ones that we have discussed up to now. These include hyperons and resonances, which, while they decay in the lab, can exist stably in dense matter because possible states for the decay products are blocked by the Pauli principle. As was pointed out in Ref.\(^{13}\), such particles could also participate in direct Urca processes. Possible particles include \(\Sigma^-\), and \(\Lambda\).
hyperons, and $\Delta$ isobars, and possible pairs of Urca processes include

$$\Sigma^- \rightarrow n + e^- + \bar{\nu}_e \quad \text{and} \quad n + e^- \rightarrow \Sigma^- + \nu_e, \quad (6)$$

$$\Lambda \rightarrow p + e^- + \bar{\nu}_e \quad \text{and} \quad p + e^- \rightarrow \Lambda + \nu_e, \quad (7)$$

$$\Sigma^- \rightarrow \Lambda + e^- + \bar{\nu}_e \quad \text{and} \quad \Lambda + e^- \rightarrow \Sigma^- + \nu_e, \quad (8)$$

and

$$\Delta^- \rightarrow n + e^- + \bar{\nu}_e \quad \text{and} \quad n + e^- \rightarrow \Delta^- + \nu_e. \quad (9)$$

For these processes to occur, the Fermi momenta of the participating particles must satisfy the triangle inequalities, just as in the case of the nucleon process. One interesting conclusion is that the process (7) can take place for quite low concentrations of $\Lambda$ hyperons: if matter consisted only of neutrons, protons, electrons, and $\Lambda$ hyperons, the threshold concentration of $\Lambda$ hyperons would be zero, and for more realistic models of dense matter, the threshold fraction of $\Lambda$ hyperons to nucleons is of order one part in a thousand. Again, in order to evaluate whether any of these hyperon and isobar processes are realistic possibilities, better estimates of the composition of dense matter are needed. Should the processes be allowed kinematically, the characteristic cooling time is similar to the nucleon direct Urca time, Eq. 5, increased by a factor $\cot^2 \theta_C$ when the process involves a strangeness change, as reactions (6) and (7) do.

### 3 Neutrino Pair Bremsstrahlung by Electrons

The basic process is emission of a neutrino-antineutrino pair accompanying scattering of an electron by ions in the crust of a neutron star. The rate of the process has previously been considered in many articles [16], all of which share the common feature that the electron-nucleus interaction is treated in first-order perturbation theory. These estimates suggested that, depending on what is assumed about the neutron star model, energy emission by neutrino bremsstrahlung in the crust could be competitive with the modified Urca process in the core if the core were made of
normal matter, while the crust bremsstrahlung process could dominate energy loss if
the nucleons in the core underwent a transition to a superfluid and/or superconducting
state.

The basic process is shown in Fig. 1a, where the double lines represent an
electron moving in the Coulomb potential of the nuclei, and Fig. 1b shows the
diagrams that contribute in the usual approximation in which the electron-nucleus
interaction is treated perturbatively. The dependence of the rate of energy emission
on temperature can be found as we did earlier for the modified Urca process. Phase
space gives a factor of $T^3$ for each of the neutrino and antineutrino, a factor of $T^2$
each of the incoming and outgoing electrons, and a factor of $1/T$ to take into account
the fact that, because energy must be conserved in the process, one of the particle
energies is not an independent variable. On the basis of phase space alone, one would
thus expect the energy emission rate, which contains an extra factor $T$ for the energy
of the neutrino and antineutrino, to vary as $T^8$, just as the modified Urca process does. However, in the contribution to the matrix element corresponding to the perturbation
theory diagrams in Fig. 1b there is an intermediate energy denominator where the
electron is off the energy shell by an amount proportional to the momentum of the
neutrino pair, $\sim T$. Thus in the calculation of the rate, the square of the matrix
element gives an extra factor $T^{-2}$, and the rate of energy emission is proportional to
$T^6$.

From the above considerations one can easily see that the perturbation expan-
sion is really an expansion in terms of the strength of the electron-ion interaction
compared with the energy denominator $\sim T$. The strength of the Coulomb inter-
action with the static lattice is of order $|V_K|$, where $V_K$ is the electron-ion matrix
element, which depends on the reciprocal lattice wavevector, $K$, describing the scat-
tering process. The crystal potential gives rise to electronic band structure with
splittings between bands whose magnitude is $2|V_K|$. For point-like nuclei, the split-
ting for the lowest reciprocal lattice vector is $\sim 0.018(Z/60)^{2/3}\mu_e$, where $\mu_e$ is the
electron chemical potential, and $Z$ is the atomic number of the nucleus. Since $Z$ is
estimated to be close to 60 and $\mu_e$ is $\sim 75$ MeV at the highest densities at which
spherical nuclei exist in the crust, band splittings can be as large as 1 MeV. This
shows that perturbation theory is inapplicable at temperatures below about $10^{10}$ K,
since the dimensionless parameter in the perturbation series is then larger than unity.
Recently we have calculated the rate of the bremsstrahlung process without making the perturbation expansion. The rate of the process is calculated from the diagram shown in Fig. 1a, with the weak-interaction matrix elements being ones for band states, where the electron-ion interaction is included to arbitrarily high order. An analytical result has been obtained in the low temperature limit, in which \( k_B T \ll |V_K| \):

\[
\dot{E}_< = \frac{2G_F^2}{3\pi^2\hbar^2} \frac{C_A^2 + C_V^2}{2} \frac{\mu_e(k_B T)^7}{v_\perp} \sum_K \frac{(1 - v_\perp)^{1/2}}{v_\perp^2 (1 + v_\perp)} |V_K|^4 e^{-\frac{1}{k_B T} \frac{2|V_K|}{v_\perp}}. \tag{10}
\]

Here, \( C_A = 1/2, \) \( C_V = 1/2 + 2 \sin^2 \theta_W \) in terms of the weak mixing angle \( \theta_W \), and \( v_\perp = \sqrt{1 - (K/(2k_F))^2} \), with \( k_F \simeq \mu_e/(\hbar c) \) being the Fermi wavenumber. The total emission rate from all types of neutrinos is obtained by replacing \( C_A^2 + C_V^2 \) by \( C_A^2 + C_V^2 + 2((1 - C_A)^2 + (1 - C_V)^2) \). This shows that the emission rate falls exponentially at temperatures below the band splitting for the particular reciprocal lattice vector considered.

We have not yet performed detailed numerical integrations to evaluate the neutrino emission rate when band splittings are comparable to the thermal energy, but we have constructed a simple formula which interpolates between the perturbation theory result, \( \dot{E}_> \), which is valid at high temperatures, and Eq.(10), which holds at low temperatures. Results for conditions appropriate for the highest density at which nuclei are roughly spherical are shown in Fig. 2, where we compare the energy emission rate according to the interpolation formula with \( \dot{E}_> \). We see that at temperatures below about \( 10^{10} \)K, the reduction in the neutrino emissivity is more than a factor 2, and below \( 10^9 \)K it is more than an order of magnitude.

What these results demonstrate is that for perturbation theory calculations of the rate of the bremsstrahlung process to be applicable it is necessary that band splittings be less the thermal energy, and not just less than the Fermi energy, as one might expect at first sight. This circumstance is a consequence of the small energy denominators that arise in the perturbation expansion.
4 Open Questions

The rate of the modified Urca process has been estimated by a number of authors, and the one that is generally quoted is that by Friman and Maxwell[17]. A perturbation theory diagram for a typical process is shown in Fig. 3a. Some years ago Voskresenski˘ı and Senatorov[18] pointed out two effects that had not been taken into account in earlier estimates. The first is that the nucleon-nucleon interaction is modified by the presence of the dense medium. In calculations of the rate of the modified Urca process, the long range part of the interaction, due to exchange of a single pion, dominates the rate. The pion field is modified by the medium, which in turn changes the long-range part of the nucleon-nucleon interaction. A second effect is that the exchanged pion, rather than one of the nucleons, can undergo a weak interaction process, as shown in Fig. 3b. The latter is an example of what is usually referred to as an “exchange current effect”. The estimates of Ref.[18] suggest that these two effects could lead to an enhancement of the rate of the modified Urca process by as much as several orders of magnitude. However, the results are very dependent on the specific assumptions made, and more detailed estimates should be made.

Another set of questions concerns neutrino emission in the crusts of neutron stars. In Sec.3 we considered bremsstrahlung of neutrino pairs by electrons moving in the static Coulomb field of the nuclei, and showed that it is suppressed exponentially at low temperatures. This indicates that the phonon-assisted process, in which lattice vibrations generate neutrino pairs, will be the dominant mechanism for bremsstrahlung of neutrino pairs by electrons, since it has a power-law temperature dependence. This process deserves to be reexamined. One may also ask whether, in the crust, there are neutrino generating processes in which nucleons can participate†.

One of the qualitatively new features of the states with non-spherical nuclei that are discussed in another contribution to this volume[19] is that both neutrons and protons have continuous energy spectra in the vicinity of the respective Fermi energies. As a consequence, processes involving nucleons are not suppressed exponentially at low temperatures as they are in finite nuclei, where the spacing between different

†Such processes have previously been considered for the bulk interior, where neutrino pair bremsstrahlung can be produced in nucleon-nucleon collisions[19, 17], but for normal matter it is estimated to be less effective than the modified Urca process.
single-particles states is finite. Leinson\cite{20} has pointed to the possibility of neutrino pairs being produced by scattering of nucleons from inhomogeneities in the nuclear medium. Leinson considered the case of matter at relatively high temperatures, and he assumed that the nuclear charge distribution corresponded to a collection of bubbles at random positions, and he concluded that the process could be a significant source of energy loss during the very early life of a neutron star.

5 Conclusion

As our discussion shows, there have been a number of advances in our understanding of neutrino emission from dense matter over the past few years. New processes have been discovered, and some old ones have been found to have rates significantly different from earlier estimates.

One new result is the discovery of a number of possible processes in dense matter that can give rise to rapid cooling without the need for an exotic state. Consequently one can no longer argue that rapid cooling is a unique signature of an exotic state of matter in the interior of a neutron star. Whether or not these new processes take place depends crucially on the composition of neutron star matter at supernuclear densities, and therefore it is important in future to attempt to narrow down the range of possible compositions of dense matter.

Bremsstrahlung of neutrino pairs in the crusts of neutron stars has been shown to be less important than previously estimated for two reasons: first, the rate of the process is suppressed by band-structure effects at temperatures below $\sim 10^{10}$ K, and second, the amount of matter in the crust of a neutron star is now estimated to be considerably less than was given by earlier calculations. With respect to the crust, one topic for future investigation is whether, in phases with nonspherical nuclei, the neutral current process proposed by Leinson, or its charged current analog\cite{21}, are quantitatively important for neutron star cooling.

The work of Voskresenskiï and Senatorov provides motivation for a thorough reinvestigation of the modified Urca process, with allowance for the knowledge of pion physics that has been accumulated over the last two decades. The modified Urca process is much more sensitive to strong interactions, since, unlike the direct Urca processes, its rate depends explicitly on the nucleon-nucleon interaction, and
not just implicitly, through the composition.

Kaon condensation, were it to occur, would have important consequences for neutron star cooling, and more generally for the composition of dense matter. Calculations that take into account more physics than has been included to date are needed in order to assess how realistic a possibility kaon condensation is. Among these effects are the possible effects of resonance states, such as \( \Lambda(1405) \), and higher-body contributions to kaon-nucleon interactions.

One topic that we have not touched on in detail in the text is superfluidity and superconductivity of nucleons. Among recent developments are new evaluations of gaps\(^{[22]}\), and detailed calculations of the suppression of the direct Urca rate\(^{[23]}\) for both isotropic and anisotropic gaps. Among other more speculative possibilities that we have not been able to cover in this brief review are cooling by emission of axions, which allows one to obtain bounds on axion properties\(^{[24]}\), and the direct Urca process occurring in phases consisting of coexisting quark matter and nuclear matter\(^{[25]}\).

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Figure Captions

FIG.1. a) The basic bremsstrahlung process. The double line is the propagator for a band electron. b) The process in first-order perturbation theory. The cross denotes an electron-lattice interaction, and the propagators are free ones.

FIG.2. Energy emission rate according to the interpolation formula compared with the high temperature limit, $\dot{E}_\gamma$, as a function of temperature.

FIG.3. a) Perturbation-theory diagram for a typical contribution to the modified Urca process. The wavy line represents a weak interaction, while the dashed line represents a strong interaction. b) Contribution to the modified Urca process in which exchanged pion emits leptons.