Confinement/deconfinement phase transition in SU(3) Yang-Mills theory in view of dual superconductivity

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Abstract

In the preceding works, we have given a non-Abelian dual superconductivity picture for quark confinement, and demonstrated the numerical evidences on the lattice. In this talk, we discuss the confinement and deconfinement phase transition at finite temperature in view of the dual superconductivity. We investigate chromomagnetic monopole currents induced by chromoelectric flux in both confinement and deconfinement phase by the numerical simulations on a lattice at finite temperature, and discuss the role of the chromomagnetic monopole in the confinement/deconfinement phase transition.

Keywords: quark confinement, dual superconductivity, dual Meissner effect, phase transition
I. INTRODUCTION

The dual superconductivity is a promising mechanism for quark confinement. In order to establish this picture, we have to show evidences of the dual version of the superconductivity. For this purpose, we have presented a new formulation of the Yang-Mills theory and shown the numerical evidences on a lattice: the non-Abelian magnetic monopole dominantly reproduces the string tension in the linear potential in SU(3) Yang-Mills theory, and the SU(3) Yang-Mills vacuum is the type I dual superconductor profiled by the chromoelectric flux tube and the magnetic monopole current induced around it, which is a novel feature obtained by our simulations.

In this talk, we further study the confinement and deconfinement phase transition at finite temperature in view of the dual superconductivity. We introduce a new formulation of the Yang-Mills theory on a lattice, and investigate confinement/deconfinement phase transition at finite temperature by using the new variable which extracts the dominant mode of the quark as well as original Yang-Mills fields. We first measure the space-averaged Polyakov-loop for each configuration and the Polyakov-loop average to investigate the role of the new variable at finite temperature. We then measure chromo fluxes and induced magnetic-monopole currents induced by a pair of quark and antiquark source to investigate the dual Meissner effect. We will demonstrate confinement/deconfinement phase transition in view of the non-Abelian dual superconductivity picture.

II. GAUGE-LINK DECOMPOSITION

We introduce a new formulation of the lattice Yang-Mills theory in the minimal option, which extracts the dominant mode of the quark confinement for SU(3) Yang-Mills theory, since we consider the quark confinement in the fundamental representation. Let \( U_{x,\mu} = X_{x,\mu} V_{x,\mu} \) be a decomposition of the Yang-Mills link variable \( U_{x,\mu} \), where \( V_{x,\mu} \) could be the dominant mode for quark confinement, and \( X_{x,\mu} \) the remainder part. The Yang-Mills field and the decomposed new variables are transformed by full SU(3) gauge transformation \( \Omega_x \) such that

\[
\begin{align*}
U_{x,\mu} &\rightarrow U'_{x,\nu} = \Omega_x U_{x,\mu} \Omega_x^\dagger, \\
V_{x,\mu} &\rightarrow V'_{x,\nu} = \Omega_x V_{x,\mu} \Omega_x^\dagger, \\
X_{x,\mu} &\rightarrow X'_{x,\nu} = \Omega_x X_{x,\mu} \Omega_x^\dagger.
\end{align*}
\]

The decomposition is given by solving the defining equation:

\[
\begin{align*}
D_\mu^\nu [V] h_x &= \frac{1}{\epsilon} [V_{x,\mu} h_{x+\mu} - h_x V_{x,\mu}] = 0, \\
g_x &= e^{i2\pi q/3} \exp(-ia^0_x h_x - i \sum_{j=1}^3 a_x^{(j)} u_x^{(j)}) = 1,
\end{align*}
\]

where \( h_x \) is an introduced color field \( h_x = \xi_x (\lambda^8/2) \xi_\alpha^\dagger \in [SU(3)/U(2)] \) with \( \lambda^8 \) being the Gell-Mann matrix and \( \xi_x \) an SU(3) group element. The variable \( g_x \) is an undetermined parameter from Eq.(2a), \( u_x^{(j)} \)'s are \( su(2) \)-Lie algebra valued, and \( q_2 \) has an integer value 0, 1, 2. These defining equations can be solved exactly, and the solution is given by

\[
\begin{align*}
X_{x,\mu} &= \hat{L}_{x,\mu} \det(\hat{L}_{x,\mu})^{-1/3} g_x^{-1}, \\
V_{x,\mu} &= X_{x,\mu}^\dagger U_{x,\mu} = g_x \hat{L}_{x,\mu} U_{x,\mu}, \\
\hat{L}_{x,\mu} &= \left( L_{x,\mu} L_{x,\mu}^\dagger \right)^{-1/2} L_{x,\mu}, \\
L_{x,\mu} &= \frac{5}{3} + \frac{2}{\sqrt{3}} (h_x + U_{x,\mu} h_{x+\mu} U_{x,\mu}^\dagger) + 8 h_x U_{x,\mu} h_{x+\mu} U_{x,\mu}^\dagger.
\end{align*}
\]
The decomposition is uniquely obtained as the solution \( \{ \overline{h}_x \} \) of Eqs. (2), if color fields \( \{ h_x \} \) are obtained. To determine the configuration of color fields, we use the reduction condition to formulate the new theory written by new variables \( \{ X_{x,\mu}, V_{x,\mu} \} \) which is equipollent to the original Yang-Mills theory. Here, we use the reduction functional:

\[
F_{\text{red}}[h_x] = \sum_{x,\mu} \text{tr} \left\{ (D^\mu_{\nu}[U_{x,\mu}][h_x])^\dagger (D^\mu_{\nu}[U_{x,\mu}][h_x]) \right\},
\]

and then color fields \( \{ h_x \} \) are obtained by minimizing the functional (4).

### III. LATTICE RESULT

We generate the Yang-Mills gauge field configurations (link variables) \( \{ U_{x,\mu} \} \) for the standard Wilson action. We prepare data sets for finite temperature on the lattice of size \( L^3 \times N_T \) at finite temperature by using the pseudo heat bath algorithm. For the fixed spatial size \( L = 24 \) and the temporal size \( N_T = 6 \): the temperature varies by changing the inverse gauge coupling constant \( \beta = 2N_c/g^2 \) \( (N_c = 3) \): \( \beta = 5.8, 5.9, 6.0, 6.1, 6.2, 6.3 \). In our simulations, we use the cold start to obtain the real-valued Polyakov loop average \( \langle P \rangle \) at high temperature, see Fig. 1. We thermalized 8000 sweeps, and we used 500 configurations for measurements.

We perform the decomposition of the gauge link variable \( U_{x,\mu} = X_{x,\mu}V_{x,\mu} \) by using the formula (3) given in the previous section, after the color-field configuration \( \{ h_x \} \) is obtained by solving the reduction condition of minimizing the functional (4) for each set of the gauge field configurations \( \{ U_{x,\mu} \} \). In the measurement of the Polyakov loop average and the Wilson loop average defined below, we apply the APE smearing technique to reduce noises.

#### A. Polyakov-loop average in the confinement/deconfinement transition

First, we measure the Polyakov-loop average which is a conventional order parameter for detecting the confinement and deconfinement phase transition in the pure Yang-Mills theory. We define the space-averaged Polyakov loop (i.e., the value of the Polyakov loop which is averaged over the space volume) for a set of the original gauge field configurations \( \{ U_{x,\mu} \} \) and the restricted
gauge field (V-field) configurations \{V_{x,\mu}\}:

\[ P_U := L^{-3} \sum_{\vec{x}} \text{tr} \left( \prod_{t=1}^{N_T} U_{(\vec{x},t),4} \right), \quad P_V := L^{-3} \sum_{\vec{x}} \text{tr} \left( \prod_{t=1}^{N_T} V_{(\vec{x},t),4} \right). \]  

(5)

Left and center panel of Fig. 1 show the plots of \(P_U\) (left panel) and \(P_V\) (center panel) on the complex plane measured from the original gauge field configurations and the restricted gauge field configurations respectively. Notice that the Polyakov loop average is in general complex-valued for the SU(3) group.

Then, we measure the Polyakov-loop average \(\langle P_U \rangle\) and \(\langle P_V \rangle\) obtained by averaging the space-averaged Polyakov loop over the total sets of the original gauge field configurations and the restricted gauge field configurations respectively. Note that the symbol \(\langle O \rangle\) denotes the average of the operator \(O\) over the space and the ensemble of the configurations. Right panel of Fig. 1 shows the result for the Polyakov loop average for various temperature (\(\beta\)). We find that the behavior of \(\langle P_U \rangle\) and \(\langle P_V \rangle\) give the same critical temperature for the phase transition separating the low-temperature confined phase characterized by the vanishing Polyakov loop average \(\langle P_U \rangle = \langle P_V \rangle = 0\) from the high-temperature deconfined phase characterized by the non-vanishing Polyakov loop average \(\langle P_U \rangle \neq 0\) and \(\langle P_V \rangle \neq 0\).

B. Chromo-flux tube at finite temperature

We proceed to investigate the non-Abelian dual Meissner effect at finite temperature. For this purpose, we measure the chromo-flux at finite temperature created by a quark-antiquark pair which is represented by the maximally extended Wilson loop \(W\) defined in Fig. 2. The chromo-field strength, i.e., the field strength of the chromo flux at the position \(P\) is measured by using a plaquette variable \(U_p\) as the probe operator for the field strength. See Fig. 2. We use the same gauge-invariant correlation function as that used at zero temperature \(\rho_{p}\):

\[ \rho_{p} := \frac{\langle \text{tr} (W L U_p L^d) \rangle}{\langle \text{tr} (W) \rangle} - \frac{1}{N_c} \frac{\langle \text{tr} (U_p) \text{tr} (W) \rangle}{\langle \text{tr} (W) \rangle}, \]  

(6)

where \(L\) is the Wilson line connecting the source \(W\) and the probe \(U_p\) needed to obtain the gauge-invariant result. Note that \(\rho_{p}\) is sensitive to the field strength rather than the disconnected one. Notice that the setup in the right figure of Fig. 2 is different from the correlation function for a pair of the Polyakov loop \(P(z = 0)\) and the anti-Polyakov loop \(\bar{P}(z = R)\) where each Polyakov loop is
defined by the corresponding closed loop which is obtained by identifying the end points at $\tau = 0$ and $\tau = 1/T$ by the periodic boundary condition, and the probe $U_P$ is attached to one the Polyakov loop or both the Polyakov and anti-Polyakov loops. Such operator was recently used to measure the chromo-flux in the work [9]. It should be remarked that the Polyakov loop correlation function, $\langle P_U(\vec{x})P_U^*(\vec{y}) \rangle \simeq e^{-F_{q\bar{q}}/T}$, proportional to the partition function in the presence of a quark at $\vec{x}$ and an anti-quark at $\vec{y}$ is decomposed into the singlet and the adjoint combinations in the color space. Furthermore, the decomposed component is gauge-dependent and thus should be taken with care [7] [8]. In sharp contrast to this fact, the potential obtained from the Wilson loop is the color singlet in the gauge-independent way.

Figure 3 and 4 show the results of the measurement of chromo-field strength at different temperatures obtained from the data set for the original Yang-Mills field and the restricted field ($V$-field), respectively. In the low temperature confined phase $T < T_c$ (see left panels of Fig.3 and Fig.4), we observe that only the $E_z$ component of the chromoelectric flux tube, i.e. the flux in the direction connecting a quark and antiquark pair is non-vanishing, and that the other components take vanishing values. This is consistent with the result obtained by Cea et al. [9], although they use the different operator for measuring the flux. In the high temperature deconfined phase $T > T_c$ (see right panels of Fig.3 and Fig.4), we observe the non-vanishing $E_y$ component in the chromoelectric flux, which means no more squeezing of the chromoelectric flux tube. This shows the disappearance of the dual Meissner effect in the high temperature deconfined phase.

C. Magnetic–monopole current and dual Meissner effect at finite temperature

Finally, we investigate the dual Meissner effect by measuring the magnetic–monopole current $k$ induced around the chromo-flux tube created by the quark-antiquark pair. We use the the
magnetic-monopole current $k$ defined by

$$k_\mu(x) = \frac{1}{2} \epsilon_{\mu \nu \alpha \beta} (F[V]_{\alpha \beta}(x + \hat{\nu}) - F[V]_{\alpha \beta}(x)).$$

(7)

Note that the magnetic–monopole current (7) must vanish due to the Bianchi identity, if there exist no singularity in the gauge potential. We show that the magnetic–monopole current defined in this way can be the order parameter for the confinement/deconfinement phase transition, as suggested from the dual superconductivity hypothesis. Figure 5 shows the result of the measurements of the magnitude $\sqrt{k_\mu k_\mu}$ of the induced magnetic current $k_\mu$ obtained according to (7). We observe the appearance and disappearance of the magnetic monopole current in the low temperature phase and high temperature phase, respectively.

IV. SUMMARY AND OUTLOOK

Using a new formulation of the Yang-Mills theory on a lattice, we have examined the confinement/deconfinement phase transition and the (non-Abelian) dual superconductivity in the $SU(3)$ Yang-Mills theory at finite temperature. The reformulation enables one to extract the dominant mode for quark confinement. Indeed, we have extracted the restricted field ($V$-field) from the original Yang-Mills field which plays a dominant role in confining quark in the fundamental representation at finite temperature.

First, we have given the numerical evidences for the restricted field dominance in the Polyakov loop average $P$ in the sense that the Polyakov loop average $P_V$ written in terms of the restricted field $V$ gives the same critical temperature $T_c$ as that detected by the Polyakov loop average $P_U$ written in terms of the original gauge field $U$: $P = 0$ for $T < T_c$ and $P \neq 0$ for $T > T_c$.

However, the Polyakov loop average cannot be the direct signal of the dual Meissner effect or magnetic monopole condensation. Therefore, it is important to find an order parameter which enables one to detect the dual Meissner effect directly.

In view of these, we have measured the chromoelectric and chromomagnetic flux for both the original field and the restricted field. In the low–temperature confined phase $T < T_c$, we have obtained the numerical evidences of the dual Meissner effect in the $SU(3)$ Yang-Mills theory, i.e., the squeezing of the chromoelectric flux tube created by a quark-antiquark pair and the associated magnetic–monopole current induced around the flux tube. In the high–temperature deconfined
phase $T > T_c$, on the other hand, we have observed the disappearance of the dual Meissner effect, no more squeezing of the chromoelectric flux tube detected by non-vanishing $E_y$ component in the chromoelectric flux and the vanishing of the magnetic-monoopole current associated with the chromo-flux tube. These results are also obtained by the restricted field alone. Therefore, we have confirmed the restricted field dominance in the dual Meissner effect even at finite temperature. Thus, we have given the evidences that the confinement/deconfinement phase transition is caused by appearance/disappearance of the non-Abelian dual superconductivity.

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