LATTICE QCD WITH THE OVERLAP-DIRAC OPERATOR: ITS $\Lambda$ PARAMETER, AND ONE-LOOP RENORMALIZATION OF FERMIONIC CURRENTS

C. ALEXANDROU, E. FOLLANA, H. PANAGOPoulos
Department of Physics, University of Cyprus,
P.O.Box 20537, Nicosia CY-1678, Cyprus

E. VICARI
Dipartimento di Fisica dell’Università and I.N.F.N.,
Via Buonarroti 2, I-56127 Pisa, Italy

We compute the ratio between the scale $\Lambda_L$ associated with a lattice formulation of QCD using the overlap-Dirac operator, and $\Lambda_{\overline{MS}}$. To this end, the one-loop relation between the lattice coupling $g_0$ and the coupling renormalized in the $\overline{MS}$ scheme is calculated, using the lattice background field technique. We also compute the one-loop renormalization $Z_{\Gamma}$ of the two-quark operators $\bar{\psi}\Gamma\psi$, where $\Gamma$ denotes a generic Dirac matrix. Furthermore, we study the renormalization of quark bilinears which are more extended and have better chiral properties. Finally, we present improved estimates of $Z_{\Gamma}$, coming from cactus resummation and from mean field perturbation theory.

It has recently been shown that chiral symmetry can be realized in lattice QCD without fermion doubling, circumventing the Nielsen-Ninomiya theorem (for a list of references, see our publications [1,2]; for reviews see, e.g., Refs. [3,4]). This has been achieved by introducing an overlap-Dirac operator derived from the overlap formulation of chiral fermions. The simplest example, for a massless fermion, is given by the Neuberger-Dirac operator:

$$D_N = \frac{1}{a} \rho \left[ 1 + X(x)^\dagger X(x) \right]^{-1/2}, \quad X = D_W - \frac{1}{a} \rho, \quad (1)$$

$a$ is the lattice spacing, $\rho \in (0, 2)$ a parameter, $D_W$ the Wilson-Dirac operator

$$D_W = \frac{1}{2} \left[ \gamma_\mu \left( \nabla_\mu^* + \nabla_\mu \right) - a \nabla_\mu^* \nabla_\mu \right], \quad a \nabla_\mu \psi(x) = U(x, \mu) \psi(x + a\hat{\mu}) - \psi(x). \quad (2)$$

$D_N$ has a number of desirable features: The Ginsparg-Wilson relation:

$$\gamma_5 D_N + D_N \gamma_5 = a D_N \gamma_5 D_N,$$

protects the quark masses from additive renormalization, and implies renormalizability to all orders of perturbation theory. This relation also leads to the existence of an exact chiral symmetry of the lattice action, with chiral Ward identities which ensure the non-renormalization

$^a$Presented the talk
of vector and flavor non-singlet axial vector currents and the absence of mixing among operators in different chiral representations. Chiral symmetry results in leading scaling corrections to hadron masses which are $O(a^2)$, rather than $O(a)$. The axial anomaly is correctly reproduced by the fermion integral measure, which is non-invariant under flavour-singlet chiral transformations. $D_N$ avoids fermion doubling at the expense of not being strictly local: Localit y is recovered in a more general sense, i.e. allowing an exponential decay of the kernel of $D_N$ at a rate which scales with the lattice spacing and not with the physical quantities.

In what follows, we present perturbative calculations, in lattice QCD with the operator $D_N$, of several quantities which are needed to relate Monte Carlo data to physical observables. Lack of strict locality greatly complicates these calculations, as compared to the Wilson case. Technical details may be found in our publications [1,2].

To evaluate $\Lambda_L/\Lambda_{\overline{\text{MS}}}$ we need to calculate the one-loop relation between $g_0$ and the renormalized MS coupling $g$ at scale $\mu$: $g_0 = Z_g(g_0, a_\mu) g$. Writing: $Z_g(g_0, x)^2 = 1 + g_0^2 (2b_0 \ln x + l_0) + O(g_0^4)$, one has: $l_0 = 2b_0 \ln \left(\frac{\Lambda_L}{\Lambda_{\overline{\text{MS}}}}\right)$.

The algebra was performed using a symbolic manipulation package which we have developed in Mathematica. For the present purposes, this package was augmented to include the propagator and vertices of the overlap action.

Our results are shown in Figure 1 for different numbers $N_f$ of fermion flavours. Some particular cases of interest are (SU(3), $\rho=1$): $\Lambda_L/\Lambda_{\overline{\text{MS}}} = 0.034711$ ($N_f=0$), 0.025042 ($N_f=1$), 0.011273 ($N_f=3$), (cf. Wilson fermions: $\Lambda_L/\Lambda_{\overline{\text{MS}}} = 0.029412$ ($N_f=1$), 0.019618 ($N_f=3$)).

![Figure 1: $\Lambda_L/\Lambda_{\overline{\text{MS}}}$ in SU(3), as a function of $\rho$.](image1)

![Figure 2: The coefficients $b^L_s(\rho)$, $b^L_v(\rho)$, as a function of $\rho$.](image2)
We have furthermore computed, to one loop, the renormalization constants $Z_O$ of the local fermionic currents:

\[ O_i = \bar{\psi}(x) \Gamma_i \psi(x), \quad \Gamma_i = 1 \ (S), \gamma_5 \ (P), \gamma_\mu \ (V), \gamma_\mu \gamma_5 \ (A), \sigma_{\mu\nu} \gamma_5 \ (T), \]

and their extended (non-ultralocal), improved counterparts:

\[ O'_i = \bar{\psi} \Gamma_i (1 - aD_N/2) \psi, \quad O''_i = \bar{\psi} (1 - aD_N/2) \Gamma_i (1 - aD_N/2) \psi. \]

\( O'_i \) obey Ward identities leading to: $Z_{S'} = Z_{\bar{P}}$, $Z_{V'} = Z_{A'}$. $O''_i$ are free of $O(a)$ errors, not only in the spectrum, but also in generic matrix elements.

We have proved that: $Z_{O'_i} = Z_{O_i}$, and also: $Z_{O''_i} = Z_{O_i}$.

We calculated $Z_O = 1 + g^2 c_F \left[ (c_O - c) \ln a^2 \mu^2 + b_{O\bar{O}} + b_{O\bar{L}} + b_L \right]$ (see Ref. [2] for notation). $Z_O$ are independent of the gauge parameter and of the fermion mass. The results for $b_L$ and $b_{O\bar{L}}$ are shown in Figure 2; they do not depend on $N$ or $N_f$. As an example, $Z_O$ at $\rho = 1$ is:

\[ Z_{S,P} = 1 + g^2 c_F \left[ 3 (\ln a^2 \mu^2)/16\pi^2 + 0.204977 \right], \quad Z_{A,V} = 1 + g^2 c_F \left[ 0.198206 \right], \]

Finally, we have obtained improved estimates for $Z_O$, coming from a re-summation to all orders of “cactus” diagrams. These diagrams are often largely responsible for lattice artifacts. Our method is gauge invariant, and systematic in dressing higher loop contributions; applied to a number of cases of interest, it has yielded results remarkably close to nonperturbative estimates.

In particular, for $Z_{V,A}$ we find (at $g_0 = 1$, $\rho = 1$): $Z_{V,A} \simeq 1.35$, as compared to our undressed result: $Z_{V,A} = 1.26427$.

To conclude, some feasible future tasks, alongside with numerical simulation, are: Calculation of the $\beta$-function for the overlap-Dirac operator, running fermion masses, renormalization of 4-fermion operators ($Z_{\Delta S^2}$, etc.).

References

1. C. Alexandrou, H. Panagopoulos, E. Vicari, Nucl. Phys. B 571, 257 (2000).
2. C. Alexandrou, E. Follana, H. Panagopoulos, E. Vicari, Nucl. Phys. B 580, 394 (2000).
3. H. Neuberger, Nucl. Phys. B(PS) 83, 67 (2000).
4. M. Lüscher, Nucl. Phys. B(PS) 83, 34 (2000).
5. R. Narayanan and H. Neuberger, Nucl. Phys. B 443, 305 (1995).
6. H. Neuberger, Phys. Lett. B 417, 141 (1998); B 427, 353 (1998).
7. T. Reisz and H. J. Rothe, Nucl. Phys. B 575, 255 (2000).
8. M. Lüscher, Phys. Lett. B 428, 342 (1998).
9. P. Hernández, K. Jansen, M. Lüscher, Nucl. Phys. B 552, 363 (1999).
10. H. Panagopoulos and E. Vicari, Phys. Rev. D 58, 114501 (1998); D 59, 057503 (1999).