Massive States in Chiral Perturbation Theory

S. Mallik
Saha Institute of Nuclear Physics
1/AF, Bidhannagar, Calcutta 700064, India

Abstract

It is shown that the chiral nonanalytic terms generated by $\Delta_{33}$ resonance in the nucleon self-energy is reproduced in chiral perturbation theory by perturbing appropriate local operators contained in the pion-nucleon effective Lagrangian itself.
1 Introduction

The notion of spontaneously broken chiral symmetry [1] as applied to strong interactions has a long history [2]. Through an important series of investigations, it has developed into the chiral perturbation theory ($\chi PT$) based on the symmetries of QCD. It has now led to a rather detailed and quantitative understanding of hadron physics in the low energy region, not accessible to a conventional perturbative treatment [3].

Consider QCD with u and d quarks only. Its effective Lagrangian in $\chi PT$ consists of a series of terms in powers of symmetry breaking parameters (masses) and derivatives (momenta), built out of the (Goldstone) pion fields incorporating the symmetries. It, however, brings in a number of effective coupling constants which, unless estimated from experiment, tends to limit the predictive power of this approach [4]. Unitarity is restored up to a certain order by including loop diagrams with lower order terms in $L$ as vertices. These loop diagrams give rise to the nonanalytic contributions, which are proportional to odd powers or logarithm of the pion mass. They arise from the infrared properties of Feynman diagrams in the chiral limit.

The baryon fields can also be included in the framework of the effective field theory [5,6]. Thus the effective Lagrangian for the pion-nucleon system contains the pion and the nucleon fields. Predictions of $\chi PT$ for this system can be confronted with rich experimental data.

A question arises as to how the effective Lagrangian takes into account the contributions (in particular, its nonanalytic pieces) of more massive particles whose fields are not present in it. In a dispersion theoretic framework such particles can be present as intermediate states. But in the $\chi PT$ approach these contributions are implicitly contained in the terms of the effective Lagrangian together with the loops they generate. In other words, the method not only accounts for higher order interactions of particles whose fields are present in the effective Lagrangian but also the interactions with the massive states.

The $\Delta_{33}(1232)$ resonance in the $\pi N$ system is a good example to discuss the role of higher mass states in the effective field theory. Here we calculate the contribution of $\Delta_{33}$ exchange to the nucleon self-energy loop and extract the leading nonanalytic piece in it. The calculation is conveniently carried out in the heavy nucleon formalism. We then identify the lower order terms in $L$ which when treated as vertices in self-energy loop, reproduce the same
nonanalytic piece.

Of course, the nonanalytic piece is identified in the chiral limit where the pion mass $m_\pi \to 0$. But in the real world the difference $\delta$ between the $\Delta$ and $N$ masses is comparable to $m_\pi$. In such cases one may have to sum the contributions from an entire series of terms of the effective Lagrangian, which, in effect, amounts to having nonlocal terms in $\mathcal{L}$. It may then be useful to introduce an independent field for the excited state (here $\Delta_{33}$) to avoid this nonlocality.

In Sec.2 we review the $\chi PT$ in the heavy nucleon formalism. In Sec.3 we calculate the self-energy diagram in this formalism to extract the nonanalytic piece. We then identify the local operators in $\mathcal{L}$ which in loops just reproduce this result. Our concluding remarks are contained in Sec.4.

2 Heavy Nucleon $\chi PT$

Here we use the heavy nucleon formalism of Georgi [7], as applied by Jenkins and Manohar [8,9] to construct the effective chiral Lagrangian. It extends the usual power counting of $\chi PT$ in presence of nucleon. Here the nucleon momentum is written as

$$P^\mu = mv^\mu + p^\mu.$$  

(1)

If the nucleon mass $m$ is large, its velocity $v^\mu$ remains (almost) unchanged by scattering with fixed (small) momentum transfer $p^\mu$. One introduces the nucleon field $b(x)$ with definite velocity $v^\mu$ related to the original field $B(x)$ by

$$b(x) = e^{imv_H x} B(x).$$  

(2)

The reduced field satisfies the modified Dirac equation

$$i\partial b(x) = 0.$$  

(3)

without a mass term. The $\partial^\mu$ produces a $p^\mu$ rather than $P^\mu$. The Dirac structure of the theory simplifies considerably [8]; the bilinear covariants can now be simply written in terms of $v^\mu$ and the spin-vector $S^\mu = \frac{i}{2} \sigma^{\mu\nu} \gamma_5 v_\nu$, which is the Pauli-Lubanski 4-vector in this formalism.

We briefly review the construction of the effective Lagrangian [8,9]. One introduces the matrix

$$U(x) = e^{i\phi(x)}, \quad \phi(x) = \tau^a \phi^a(x)/f_\pi, \quad a = 1, 2, 3,$$

(4)
where $\phi^a$ are the pion fields, $f_\pi$ the pion decay constant and $\tau^a$ are the Pauli matrices. Under $SU(2)_L \times SU(2)_R, U \rightarrow V_L U V_R^\dagger$, where $V_L$ and $V_R$ are global SU(2) transformations. The square root of $U$ is denoted by $u$, $U = u^2$. It transforms as $u \rightarrow V_L u R^\dagger = R u V_R^\dagger$, defining $R$ implicitly. The reduced nucleon field transforms as $b(x) \rightarrow Rb(x)$.

With $u(x)$ one can build two vector fields. One is the gauge field $V_\mu = \frac{1}{2}(u \partial_\mu u^\dagger + u^\dagger \partial_\mu u)$ and the other is the axial vector field $A_\mu = \frac{1}{2}(u \partial_\mu u^\dagger - u^\dagger \partial_\mu u)$. The covariant derivative of the reduced nucleon field $b(x)$ is then $D_\mu b = \partial_\mu b + V_\mu b$.

The lowest order effective Lagrangian is given by

$$\mathcal{L}^{(1)} = \bar{b} v \cdot D b + 2g_{\pi N} \bar{b} S \cdot A b \quad (5)$$

At the next higher order an independent set of operators consists of [10]

\[
\begin{align*}
O_1 &= \bar{b} D^2 b, \\
O_2 &= \bar{b} (v \cdot D)^2 b, \\
O_3 &= \bar{b} A^2 b = \frac{1}{16} \bar{b} b \partial_\mu \phi^a \partial^\mu \phi^a + \cdots, \\
O_4 &= \bar{b} (v \cdot A)^2 b = \frac{1}{16} \bar{b} b (v \cdot \partial \phi^a)(v \cdot \partial \phi^a) + \cdots, \\
O_5 &= \bar{b} v \cdot A s \cdot D b, \\
O_6 &= \bar{b} s \cdot D v \cdot A b, \\
O_7 &= \epsilon_{\mu \nu \lambda \sigma} v^\lambda \bar{b} S^\sigma A^\mu A^\nu b, \\
O_8 &= \bar{b} (u M u + u^\dagger M u^\dagger) b, \\
\end{align*}
\]

$M$ being the quark mass matrix,

$$M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

In the above enumeration each operator represents a series of terms in powers of the meson field and the derivative. We have noted above for later use that only $O_3$ and $O_4$ contain terms which are quadratic in both these quantities and multiplied by $\bar{b} b$ (not $\bar{br}^a b$).

3 Self-energy loop with $\Delta_{33}$
We now calculate the contribution of the $\Delta_{33}$ intermediate state to the nucleon self-energy. From (5) we get the nucleon propagator as

$$i \frac{p \cdot v + i\eta}{p \cdot v + i\eta}$$

(7)

The $\Delta_{33}$ is described by a spinor-vector field $\psi_\mu(x)$. Again the nucleon mass is extracted as in (2) to define a reduced field $\chi_\mu(x)$, which satisfies the free equation of motion,

$$(iv \cdot \partial - \delta)\chi_\mu = 0, \quad \delta = m_\Delta - m$$

(8)

The original subsidiary condition $\gamma^\mu \psi_\mu = 0$ eliminating extra components translates for $\chi_\mu$ into

$$v^\mu \chi_\mu = 0, \quad S^\mu \chi_\mu = 0$$

(9)

Taking these conditions into account, the $\Delta_{33}$ propagator is given by

$$iP_{\mu\nu} = \frac{iP_{\mu\nu}}{p \cdot v - \delta + i\eta},$$

(10)

where

$$P_{\mu\nu} = \frac{2}{3}(v_\mu v_\nu - g_{\mu\nu} - i\epsilon_{\mu\nu\lambda\sigma} v^\lambda S^\sigma)$$

The $\pi N\Delta$ interaction is given by

$$\mathcal{L}_I = ig(\bar{\chi}^{\mu,a}\partial_\mu \phi^a - \bar{\phi}^{a}\partial_\mu \chi^{\mu,a})$$

(11)

The nucleon self-energy to second order (Fig.1a) at the nucleon pole ($p \cdot v = 0$) is

$$\Sigma = -3g^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{v \cdot k - \delta - k^2 - m_\pi^2} k^\mu k^\nu \frac{2}{3}(v_\mu v_\nu - g_{\mu\nu})$$

(12)

It can be evaluated as in Ref [9]. The $k$ integration in the dimensional regularization scheme results in two terms. The one proportional to $v_\mu v_\nu$ goes to zero on contracting with $(v_\mu v_\nu - g_{\mu\nu})$, while the other proportional to $g_{\mu\nu}$ gives

$$\Sigma = \frac{i \pi^{d/2}}{(2\pi)^d} \Gamma(1 - d/2) 6g^2 \int_0^\infty \frac{d\lambda}{(\lambda^2 + 2\lambda\delta + m_\pi^2)^{1-d/2}}$$

(13)
The integral may be evaluated by integrating by parts to get finally

\[ \Sigma = \frac{ig^2}{8\pi^2} \mu^{-2\epsilon} \left[ \left( -\frac{1}{\epsilon} + \gamma - 1 - \ln 4\pi \right) \delta(2\delta^2 - 3m_{\pi}^2) - \frac{2}{3} \delta(5\delta^2 - 6m_{\pi}^2) \right] 
+ \delta(2\delta^2 - 3m_{\pi}^2) \ln \frac{m_{\pi}^2}{\mu^2} - 2(\delta^2 - m_{\pi}^2)^{3/2} \ln \left( \frac{\delta - \sqrt{\delta^2 - m_{\pi}^2}}{\delta + \sqrt{\delta^2 - m_{\pi}^2}} \right) \] (14)

where \( \mu \) is the renormalization scale, \( \epsilon = 2 - d/2 \) and \( \gamma = 0.5772 \). The renormalization can be carried out in the standard way. Here we are interested in the leading nonanalytic piece in the chiral limit, which is left untouched by any renormalisation prescription. On expanding the last term in (14) for \( m_{\pi} \ll \delta \), we get the leading nonanalytic contribution to \( \Sigma \) as

\[ \Sigma_{\text{nonanalytic}} = -\frac{i3g^2}{32\pi^2} \frac{m_{\pi}^4}{\delta} \ln m_{\pi}^2 \] (15)

Let us now consider the situation as the chiral limit is approached. Then the relevant internal momenta are of order \( m_{\pi} \), so that \( v \cdot p \ll \delta \). We may then expand the \( \Delta_{33} \) propagator and retain the leading term in the integrand of (12). We thus get the tadpole diagram (Fig.1b) due to insertion of a local operator generated by the \( \Delta_{33} \) intermediate state. The corresponding self-energy is given by

\[ \Sigma_{\text{tadpole}} = -\frac{3g^2}{\delta} \int \frac{d^4k}{2\pi^4} \frac{1}{k^2 - m_{\pi}^2} k^\mu k^\nu \left( \frac{2}{3} v_\mu v_\nu - g_\mu \nu \right) \] (16)

\[ = \frac{i3g^2}{32\pi^2} \frac{m_{\pi}^4}{\delta} \mu^{-2\epsilon} \left( \frac{1}{\epsilon} - \gamma + \frac{3}{2} + \ln 4\pi - \ln \frac{m_{\pi}^2}{\mu^2} \right) \] (17)

which coincides with (15) for the chiral logarithm obtained from the Feynman diagram with \( \Delta_{33} \) intermediate state. Further, looking at the list (6) of \( O(k^2) \) operators, the one in (16) is easily identified as

\[ \frac{32g^2}{\delta}(O_4 - O_3) \]

4 Conclusion
We demonstrate, within the context of the nucleon self-energy, that non-analytic terms in the chiral limit produced by loops involving higher mass states can be reproduced exactly by perturbing appropriate local operators contained in the effective chiral Lagrangian itself. This result should be true in general, as the manipulations involved here are of the same kind as required in establishing the short distance expansion [11]. It follows that the chiral properties of $\pi N$ system can be adequately described by an effective chiral Lagrangian involving pion and nucleon fields alone.

Our work disproves conclusively a claim made in a recent work [12] that the leading nonanalytic piece in the chiral limit generated by $\Delta_{33}$ exchange in $\pi N \sigma$-term [13] cannot be reproduced by a phenomenological Lagrangian for this system. What these authors fail to note that all the relevant pieces of this Lagrangian must be included in the loop calculation; otherwise unitarity will be violated.

There is, however, a question of quantitative importance of the nonanalytic pieces in the real world. As mentioned in the introduction, if an excited state is low enough, the $\chi PT$ involving the basic fields only may not be very convenient. Indeed this has been argued to be the case for $\Delta_{33}$ itself [9]. In that case, one may include the field corresponding to such a state in constructing the effective Lagrangian. But to be consistent one must calculate all quantities within the framework of $\chi PT$, as was pointed out long ago in Ref.[14,15].

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Figure Caption

Fig.1. Nucleon self-energy diagrams. The thin and thick solid lines and the dashed line represent the nucleon, the \( \Delta_{33} \) resonance and the pion respectively. (a) \( \Delta_{33} \) exchange. (b) insertion of local operators (represented by the dot) stated in the text.