Input-output relations at dispersing and absorbing planar multilayers for the quantized electromagnetic field containing evanescent components

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By using the Green-function concept of quantization of the electromagnetic field in dispersing and absorbing media, the quantized field in the presence of a dispersing and absorbing dielectric multilayer plate is studied. Three-dimensional input-output relations are derived for both amplitude operators in the k-space and the field operators in the coordinate space. The conditions are discussed, under which the input-output relations can be expressed in terms of bosonic operators. The theory applies to both (effectively) free fields and fields, created by active atomic sources inside and/or outside the plate, including also evanescent-field components.

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I. INTRODUCTION

Research on quantum statistical properties of light has been a subject of intense investigations and discussions. In view of the very wide-spread potential applications, the influence of material bodies on the quantum features of light must be thoroughly investigated. Typical quantum effects that are closely related to the change of the quantum vacuum due to the presence of material bodies are the Casimir effect [1] (for a review, see [2]) and the Purcell effect [3] (see also [4]). Though there has been a large body of work on the problem of quantization of the electromagnetic field in dielectric media (for a review, see [5]), most work has been concentrated on nonabsorbing media. Roughly speaking, there have been two routes to treat radiation in absorbing media. In the first, attention has been restricted to equilibrium field correlation functions that are calculated by employing the dissipation-fluctuation theorem [6]. In the second, explicit field quantization has been performed (for a review, see [7]).

A consistent quantization scheme for the electromagnetic field in bulk material has been given by Huttner and Barnett [8]. Using the mesoscopic Hopfield model [9] for a dielectric, they have diagonalized the bilinear Hamiltonian of the system that consists of the electromagnetic field, a harmonic-oscillator polarization field, and an infinite set of harmonic-oscillator reservoir fields responsible for absorption. By starting directly with the phenomenological Maxwell equations for the macroscopic electromagnetic field, the scheme can naturally be extended to arbitrary inhomogeneous media [10] characterized by a space- and frequency-dependent complex permittivity that satisfies the Kramers-Kronig relations. The theory is based on a source-quantity representation of the electromagnetic field, where the field operators are expressed in terms of a continuous set of fundamental Bose fields via the Green tensor of the classical problem (for details, see also [7]).

The Green-tensor formalism is well suited to study the behavior of the quantized electromagnetic field in the presence of dispersing and absorbing bodies. In particular, it has successfully been applied to the study of input-output relations [11, 12], the spontaneous emission [13–15], the resonant energy exchange [16], the resonant dipole-dipole interaction [17], and the Casimir effect [18], and also various geometries (including multilayer structures) have been considered. It is worth noting that the theory not only takes into account material absorption in a consistent way but it also includes automatically evanescent-field contributions generated, e.g., by radiating atoms that are very close to the bodies under consideration.

For the particular case of a dispersing and absorbing dielectric structure in vacuum, without active light sources at any finite distance from the structure (and without active light sources inside the structure), a rather involved hybrid quantization scheme has been proposed [19] and used to study the problem of three-dimensional input-output relations [20, 21]. In the scheme, the whole space is divided into two regions, namely a dielectric scattering region and a vacuum region that surrounds the scattering region. The quantization in the vacuum region is performed on the basis of a mode decomposition of the electromagnetic-field operators in a similar way as in vacuum quantum electrodynamics, whereas the contribution of the dielectric scattering region is taken into account by applying the Green tensor formalism [10]. The argument is [21] that the Green tensor formalism alone is not complete, because it does not explicitly treat the boundary of the medium with free space and thus does not describe explicitly the scattering and emission processes which are usually experimentally investigated. This is of course not the case as the above mentioned applications [11–16, 18] and the following study of the three-dimensional input-output relations at multilayer plates clearly show. Moreover, when there are active light sources at finite distances from the scattering region, then the incoming field contains both propagating and evanescent components. The latter ones, which are typically observed for small distances, are not included in the hybrid quantization scheme [19] and the input-output relations derived...
from it [21]. However, they are automatically included in the Green tensor formalism [10].

In the following we consider the problem of the three-dimensional input-output relations for the electromagnetic-field operators at the boundaries of a dielectric planar multilayer structure in more detail, by applying the Green tensor formalism and extending our previous one-dimensional analysis [11] to three dimensions. To be quite general, we allow (i) for the presence of active light sources at arbitrary positions inside and/or outside the multilayer plate and thus (ii) for both propagating-field components and evanescent-field components. In particular, in cavity QED the active sources are typically inside the plate, and the outgoing fields are not only determined with the incoming fields and the noise fields (unavoidably associated with material absorption) but also with the fields generated by the sources inside the plate. On the contrary, in optical near-field microscopy the active (probe) sources are typically outside the plate, but near its surface.

Employing the well-known three-dimensional Green tensor for a multilayer plate [22], we introduce amplitude operators for the input and output fields as well as for the fields inside the plate and derive input-output relations both in the two-dimensional Fourier space and in the coordinate space. Finally, the problem of introduction of bosonic input and output operators in the two-dimensional Fourier space is considered in detail.

The paper is organized as follows. In Sec. II the quantization scheme is briefly summarized and the Green tensor for a multilayer plate is introduced. In Sec. III the generally valid input-output relations are derived, and the problem of formulating input-output relations for bosonic field operators is studied. A summary and some concluding remarks are given in Sec. IV followed by an appendix, in which relevant commutation relations are derived.

II. BASIC EQUATIONS

To study optical fields interacting with active sources in the presence of dispersing and absorbing (linear) dielectric bodies, we first note that on a length scale that is large compared with interatomic distances in the bodies, the effect of the bodies can be described within the frame of macroscopic Maxwell equations in terms of a spatially varying permittivity which is a complex function of frequency. This concept, which is widely used in classical optics also applies in quantum optics.

A. Field quantization in media

Let us assume that the active light sources are neutral atoms and consider an arbitrarily inhomogeneous medium characterized by a permittivity

\[ \varepsilon(r, \omega) = \varepsilon'(r, \omega) + i\varepsilon''(r, \omega), \quad (1) \]

where the real part \( \varepsilon'(r, \omega) \) and the imaginary part \( \varepsilon''(r, \omega) \) are necessarily related to each other through the Kramers-Kronig relations, due to the causality principle. The motion of the atoms and the medium-assisted electromagnetic field is then governed by the multipolar-coupling Hamiltonian [7]

\[
\hat{H} = \int d^3r \int_0^\infty d\omega \hbar \omega \hat{f}^\dagger(r, \omega) \cdot \hat{f}(r, \omega) + \sum_{A, \alpha} \frac{1}{2m_\alpha} \left\{ \hat{p}_\alpha + q_\alpha \int_0^1 d\lambda \lambda (\hat{r}_\alpha - r_A) \times \hat{B} [r_A + \lambda (\hat{r}_\alpha - r_A)] \right\}^2 \\
+ \frac{1}{2\varepsilon_0} \sum_{A, A'} \int d^3 r \hat{P}_A(r) \cdot \hat{P}_{A'}(r) - \sum_A \int d^3 r \hat{P}_A(r) \cdot \hat{E}(r), \quad (2)
\]

where \( A \) numbers the atoms, and \( \alpha_A \) numbers the charged particles inside the \( A \)-th atom. Further, \( r_{\alpha_A} \) and \( \hat{p}_{\alpha_A} \) are respectively the operators of coordinates and canonical momenta of the particles, and

\[
\hat{P}_A(r) = \sum_{\alpha_A} q_\alpha (\hat{r}_\alpha - r_A) \int_0^1 d\lambda \delta [r - r_A - \lambda (\hat{r}_\alpha - r_A)] \\
(3)
\]

is the operator of the polarization of the \( A \)-th atom at position \( r_A \). The \( \hat{f}(r, \omega) \) [and \( \hat{f}^\dagger(r, \omega) \)] are bosonic field operators that play the role of fundamental variables of the electromagnetic field and the medium, including a reservoir necessarily associated with material absorption.

The commutation relations for the operators \( \hat{f}(r, \omega) \) and \( \hat{f}^\dagger(r, \omega) \) are

\[
[\hat{f}_\mu(r, \omega), \hat{f}_{\mu'}^\dagger(r', \omega')] = \delta_{\mu\mu'} \delta(\omega - \omega') \delta^{(3)}(r - r'), \quad (4)
\]

\[
[\hat{f}_\mu(r, \omega), \hat{f}_{\mu'}(r', \omega')] = 0, \quad (5)
\]

where the Greek letters label the Cartesian coordinates \( x, y, z \). In Eq. (2), the operators \( \hat{B}(r) \) and \( \hat{E}(r) \) of the medium-assisted electromagnetic field are expressed in terms of the fundamental variables as follows:

\[
\hat{B}(r) = \int_0^\infty d\omega \hat{B}(r, \omega) + H.c., \quad (6)
\]
\[ \hat{E}(\mathbf{r}) = \int_0^\infty d\omega \hat{E}(\mathbf{r}, \omega) + \text{H.c.,} \quad (7) \]

\[ \hat{B}(\mathbf{r}, \omega) = (i\omega)^{-1} \nabla \times \hat{E}(\mathbf{r}, \omega), \quad (8) \]

\[ \hat{E}(\mathbf{r}, \omega) = i\mu_0 \sqrt{\frac{\hbar c}{\pi}} \omega^2 \times \int d^3 r' \sqrt{\varepsilon''(r', \omega)} G(\mathbf{r}, \mathbf{r}', \omega) \cdot \hat{f}(\mathbf{r}', \omega), \quad (9) \]

where the integration should be performed over all space, and \( G(\mathbf{r}, \mathbf{r}', \omega) \) is the classical Green tensor, which can be found from the equation

\[ \nabla \times \nabla \times G(\mathbf{r}, \mathbf{r}', \omega) - \frac{\omega^2}{c^2} \varepsilon(\mathbf{r}, \omega) G(\mathbf{r}, \mathbf{r}', \omega) = \delta^{(3)}(\mathbf{r} - \mathbf{r}') \quad (10) \]

together with appropriate boundary conditions at infinity. Equations (6 - 9) can be considered as generalization of the familiar mode decomposition that would apply if dispersion and absorption could be disregarded. Instead of dealing with equations of motion for mode operators, equations of motion for the fields \( \hat{f}(\mathbf{r}, \omega) \) must be treated.

It should be pointed out that since the real part \( \varepsilon'(\mathbf{r}, \omega) \) and the imaginary part \( \varepsilon''(\mathbf{r}, \omega) \) of the permittivity are related to each other through the Kramers-Kronig relations, \( \varepsilon''(\mathbf{r}, \omega) \) cannot vanish identically for really existing media. Clearly, \( \varepsilon''(\mathbf{r}', \omega) \) can be very small, so that \( \sqrt{\varepsilon''(\mathbf{r}', \omega)} G(\mathbf{r}, \mathbf{r}', \omega) \) in Eq. (9) is very small in certain areas of space (\( r' \)). However, this statement says nothing about the magnitude of the total integral over the coordinate \( r' \), because of the following integral relation for the classical Green tensor

\[ \int d^3 r' \frac{\omega^2}{c^2} \varepsilon''(r', \omega) G_{\mu\nu'}(\mathbf{r}, \mathbf{r}', \omega) G_{\mu'\nu''}(\mathbf{r}'', \mathbf{r}', \omega) = \text{Im} \left[ G_{\mu\nu''}(\mathbf{r}, \mathbf{r}'', \omega) \right], \quad (11) \]

where we have adopted the convention of summation over repeated vector-component indices. In particular, this relation enables one to include also vacuum-like areas in the consideration. Thus, all the calculations are to be performed by assuming a permittivity close to unity with a small but finite imaginary part in those areas, and at the end the permittivity may be set equal to unity. In practice, experimental realization of (macroscopic) vacuum areas is of course fictional.

### B. Multilayered planar structures

As mentioned, the quantization scheme is valid for an arbitrary space dependence of the permittivity. Here, we consider a multilayered planar structure (Fig. 1), whose permittivity is defined in a stepwise fashion (the \( z \)-direction is chosen to be perpendicular to the layers):

\[ \varepsilon(\omega, z) = \sum_{j=0}^{n} \lambda_j(z) \varepsilon_j(\omega), \quad (12) \]

**FIG. 1:** Scheme of the multilayer dielectric plate. The hatched regions indicate the presence of active light sources.

where

\[ \lambda_j(z) = \begin{cases} 1, & \text{if } z \in j\text{th layer,} \\ 0, & \text{otherwise,} \end{cases} \quad (13) \]

and \( \varepsilon_j(\omega) \) is the (complex) permittivity of the \( j \)-th layer. In the above, the index \( j \) labels the region on the left of the plate \(( j = 0)\), the region on the right of the plate \(( j = n)\), and the layers of the plate \(( j = 1, \ldots, n - 1)\).

For simplicity, we express the \( z \)-coordinate dependence in shifted coordinate systems, introduced in each layer separately, so that the range of the \( z \)-coordinate is taken to be \(-\infty < z < 0\) for the region on the left of the plate \(( j = 0)\), \( 0 < z < \infty \) for the region on the right of the plate \(( j = n)\), and \( 0 < z < d_j \) for the \( j \)-th layer of the plate with thickness \( d_j \) \(( j = 1, \ldots, n - 1)\).

Exploiting the translational symmetry in the \((xy)\)-plane, we may represent the Green tensor as a two-dimensional Fourier integral

\[ G^{(jj')}(\mathbf{r}, \mathbf{r}', \omega) = \frac{1}{(2\pi)^2} \int d^2 k e^{i\mathbf{k} \cdot (\mathbf{r}' - \mathbf{r})} G^{(jj')}(z, z', \mathbf{k}, \omega), \quad (14) \]

where \( \mathbf{r} = (x, y) \), and \( \mathbf{k} = (k_x, k_y) \) is the wave vector parallel to the layers. The notations \( G^{(jj')}(\mathbf{r}, \mathbf{r}', \omega) \) and \( G^{(jj')}(z, z', \mathbf{k}, \omega) \) indicate that \( z \) varies in the \( j \)-th layer and \( z' \) in the \( j' \)-th layer. Inserting Eq. (14) into Eq. (9), we may write the electric-field operator \( \hat{E}^{(j)}(\mathbf{r}, \omega) \) as a two-fold Fourier transform

\[ \hat{E}^{(j)}(\mathbf{r}, \omega) = \frac{1}{(2\pi)^2} \int d^2 k e^{i\mathbf{k} \cdot \mathbf{r}} \hat{E}^{(j)}(z, \mathbf{k}, \omega), \quad (15) \]

where

\[ \hat{E}^{(j)}(z, \mathbf{k}, \omega) = i\omega \mu_0 \]

\[ \times \int_{j=0}^{d_j} \int_{j'}^{d_j z'} G^{(jj')}(z, z', \mathbf{k}, \omega) \cdot \hat{f}^{(j')}(z', \mathbf{k}, \omega) \quad (16) \]
operators are the Fourier transforms of the original bosonic field operators $\hat{f}^{(j)}(r,\omega)$,

$$\hat{f}^{(j)}(r,\omega) = \frac{1}{(2\pi)^{3/2}} \int d^2k e^{i\mathbf{k}\mathbf{r}} \hat{f}^{(j)}(z,\mathbf{k},\omega).$$

The Green tensor $G^{(jj')}^{(j,j')}(z,z',\mathbf{k},\omega)$ taken from Ref. [22] may be written as

$$G^{(jj')}^{(j,j')}(z,z',\mathbf{k},\omega) = -\mathbf{e}_z \frac{\delta_{j,j'}}{k_j} \mathbf{e}_z \delta(z-z') + g^{(jj')}^{(j,j')}(z,z',\mathbf{k},\omega),$$

where

$$g^{(jj')}^{(j,j')}(z,z',\mathbf{k},\omega) = \frac{i}{2} \sum_{q,p,s} \left[ \mathcal{E}_q^{(j)}(z,\mathbf{k},\omega) \Xi_{q}^{(j)} \mathcal{E}_q^{(j')}^{<(j',-\mathbf{k},\omega)} \Theta(j-j') + \mathcal{E}_q^{(j')}^{<(j',-\mathbf{k},\omega)} \Xi_{q}^{(j)} \mathcal{E}_q^{(j)}^{(j,-\mathbf{k},\omega)} \Theta(j'-j) \right]$$

$$-\mathcal{E}_q^{(j')}^{<(j',-\mathbf{k},\omega)} \Xi_{q}^{(j)} \mathcal{E}_q^{(j)}^{(j,-\mathbf{k},\omega)} \Theta(j'-j).$$

The `propagation constant' $\beta_j$ determines the propagation behavior in $z$-direction of the waves in the $j$-th region. Note that in case of vacuum the waves are propagating only for $\beta_j = \beta_j^0$ (i.e., $\omega/c \leq k$). They are evanescent for $\beta_j = i\beta_j^0$ (i.e., $\omega/c \geq k$).

### III. INPUT-OUTPUT RELATIONS

The equations of motion which follow from the Hamiltonian (2) in Sec. II A together with the Green tensor as given in Sec. II B can be used to study various effects of the atom-light interaction in the presence of dispersing and absorbing multilayered planar structures (see the examples mentioned in Sec. I). Here, we restrict our attention to the general relation between the fields outside and inside a multilayer plate regardless of the generation of the fields by the active light sources. The theory thus includes the general case, where sources are present in the both regions, outside and inside the plate.

If the incoming fields, incident on the two boundary planes of the plate (i.e., $\mathbf{z} = 0^-$ for $j = 0$ and $\mathbf{z} = 0^+$ for $j = n$; cf. Fig. 1), are known as well as the fields generated inside the plate, one can calculate the fields outgoing from the two boundary planes by means of input-output relations. Note that any two planes $z = z^{(0)} \leq 0^-$ and $z = z^{(n)} \geq 0^+$ for $j = 0$ and $j = n$, respectively, also can be used in principle. As we shall see, these input-output relations are valid for both passive and active devices, for arbitrary layer materials and arbitrary media surrounding the plate and for both propagating-field components and evanescent-field components.

#### A. Input-output relations in the k-space

Here and in the following we restrict our attention to the electric field, noting that the corresponding expressions for the magnetic field can readily be obtained by applying Eq. (8). Following the line suggested in Ref. [11] and substituting the Green tensor (19) for $j = 0$ and $j = n$ into Eq. (16), we may decompose the field operators...
\( \mathbf{E}^{(0)}(z, \mathbf{k}, \omega) \) and \( \mathbf{E}^{(n)}(z, \mathbf{k}, \omega) \) in the form of

\[
\mathbf{E}^{(0)}(z, \mathbf{k}, \omega) = \sum_{q=p,s} \left[ \mathbf{e}_{q+}^{(0)}(\mathbf{k}) \mathbf{E}_{q+}^{(0)}(z, \mathbf{k}, \omega) + \mathbf{e}_{q-}^{(0)}(\mathbf{k}) \mathbf{E}_{q-}^{(0)}(z, \mathbf{k}, \omega) \right],
\]

(29)

\[
\mathbf{E}^{(n)}(z, \mathbf{k}, \omega) = \sum_{q=p,s} \left[ \mathbf{e}_{q+}^{(n)}(\mathbf{k}) \mathbf{E}_{q+}^{(n)}(z, \mathbf{k}, \omega) + \mathbf{e}_{q-}^{(n)}(\mathbf{k}) \mathbf{E}_{q-}^{(n)}(z, \mathbf{k}, \omega) \right].
\]

(30)

Here, the operators

\[
\mathbf{E}_{q+}^{(0)}(z, \mathbf{k}, \omega) = \frac{-im_0}{2\beta_0} e^{i\beta_0 z} \left( \int_{-\infty}^{z} dz' e^{-i\beta_0 z'} j^{(0)}(z', \mathbf{k}, \omega) \cdot \mathbf{e}_{q+}^{(0)}(\mathbf{k}) \right),
\]

(31)

\[
\mathbf{E}_{q+}^{(n)}(z, \mathbf{k}, \omega) = \frac{-im_0}{2\beta_n} e^{i\beta_n z} \left( \int_{-\infty}^{z} dz' e^{i\beta_n z'} j^{(n)}(z', \mathbf{k}, \omega) \cdot \mathbf{e}_{q+}^{(n)}(\mathbf{k}) \right)
\]

(32)

respectively, play the role of input and output amplitude operators, where the input-output relations

\[
\begin{pmatrix}
\mathbf{E}_{q+}^{(0)}(\mathbf{k}, \omega) \\
\mathbf{E}_{q+}^{(n)}(\mathbf{k}, \omega)
\end{pmatrix} = \begin{pmatrix}
\mathbf{r}_{0/n}^q(\mathbf{k}, \omega) \\
\mathbf{t}_{0/n}^q(\mathbf{k}, \omega)
\end{pmatrix} \begin{pmatrix}
\mathbf{E}_{q+}^{(0)}(\mathbf{k}, \omega) \\
\mathbf{E}_{q+}^{(n)}(\mathbf{k}, \omega)
\end{pmatrix}
\]

(33)

\[
\begin{pmatrix}
\mathbf{E}_{q+}^{(0)}(\mathbf{k}, \omega) \\
\mathbf{E}_{q+}^{(n)}(\mathbf{k}, \omega)
\end{pmatrix} + \sum_{j=1}^{n-1} \begin{pmatrix}
\phi_{q+}^{(j)}(\mathbf{k}, \omega) \\
\phi_{q+}^{(n+)}(\mathbf{k}, \omega)
\end{pmatrix} \begin{pmatrix}
\mathbf{E}_{q+}^{(j)}(\mathbf{k}, \omega) \\
\mathbf{E}_{q+}^{(j+)}(\mathbf{k}, \omega)
\end{pmatrix}
\]

(34)

It should be pointed out that the first term on the right-hand side in Eq. (19), which gives rise to a local contribution to the electric field, has been omitted in Eqs. (29) and (30). Though this contribution is irrelevant for the incoming and outgoing fields, it must be included in the overall field operator in general, even if there are effectively no sources at the points of observations.
render it possible to easily calculate the input and output fields at any position outside the plate. Needless to say that other than the boundary planes \( z = 0^- \) and \( z = 0^+ \) the plate can be chosen as reference planes for formulating the input-output relations.

Equation (35) represents the basic input-output relations for the amplitude operators in the Fourier space. The coefficients therein are determined only by the (complex) permittivities and thicknesses of the layers of the plate and the permittivities of the surrounding media. In particular, for \( \varepsilon''_{0,n} \rightarrow 0 \) (i.e., \( \varepsilon_{0,n} \rightarrow 1 \)) Eq. (35) immediately yields the input-output relations for the special case of the plate being surrounded by vacuum.

The input-output relations in the form of Eq. (35) are generally valid, independent of the mechanism of creation of the incoming fields and the fields inside the layers of the plate. It is worth noting that they take into account both propagating waves and evanescent waves. In particular in the case when the plate is surrounded by vacuum, then the input and output amplitude operators are associated with propagating waves (\( \omega/c > k \)) or evanescent waves (\( \omega/c < k \)) in \( z \)-direction.

The temporal evolution (in the Heisenberg picture) of the amplitude operators is determined by the time dependence of the basic variables \( f(r, \omega) \) or equivalently, \( f(z, k, \omega) \), which is governed by the Hamiltonian (2). In the special case when the plate is free of active atomic sources, then the fields inside the layers of the plate represent Langevin noise sources associated with material absorption. If absorption is disregarded and the plate is surrounded by vacuum that may contain active sources, then Eq. (35) solves a scattering problem of the type considered in Ref. [23], namely scattering of fields containing evanescent components. In general, absorption cannot be disregarded and both atomic sources and Langevin noise sources contribute to a field (e.g., in cavity QED), which can contain both propagating and evanescent components (e.g., in near-field scanning probe microscopy).

The input-output relations (35) enable one to calculate correlation functions of the output field amplitudes in terms of those of the input field amplitudes and the amplitudes of the fields inside the plate. The simplest case is the calculation of the expectation values of the field amplitudes. For example, in a typical scattering arrangement the active light sources are located outside the plate, so that the field inside the plate is the absorption-assisted random field whose thermal-equilibrium expectation value vanishes. Application of Eq. (35) thus leads to the expectation-value relations

\[
\langle \hat{E}_{q}^{(0)}(k, \omega) \rangle = r_{q}/n \langle \hat{E}_{q}^{(0)}(k, \omega) \rangle + i^{q}/n_{0} \langle \hat{E}_{q}^{(n)}(k, \omega) \rangle,
\]

\[
\langle \hat{E}_{q}^{(n)}(k, \omega) \rangle = r_{q}/n \langle \hat{E}_{q}^{(0)}(k, \omega) \rangle + i^{q}/n_{0} \langle \hat{E}_{q}^{(n)}(k, \omega) \rangle,
\]

which exactly correspond to standard results in classical optics. On the other hand, when the active sources are located (in a cavity-like system) inside the plate, we find that

\[
\langle \hat{E}_{q}^{(0)}(k, \omega) \rangle = \sum_{j=1}^{n-1} r_{j}/q_{j} e^{i\beta_{j} d_{j}} \times \left[ e^{-i\beta_{j} d_{j}} \langle \hat{E}_{q}^{(j)}(k, \omega) \rangle + r_{j}/q_{j} e^{i\beta_{j} d_{j}} \langle \hat{E}_{q}^{(j)}(k, \omega) \rangle \right],
\]

\[
\langle \hat{E}_{q}^{(n)}(k, \omega) \rangle = \sum_{j=1}^{n-1} r_{j}/q_{j} e^{i\beta_{j} d_{j}} \times \left[ \langle \hat{E}_{q}^{(j)}(k, \omega) \rangle + r_{j}/q_{j} \langle \hat{E}_{q}^{(j)}(k, \omega) \rangle \right].
\]

 Needless to say that when there are no active sources inside the plate, then \( \langle \hat{E}_{q}^{(n)}(k, \omega) \rangle = 0 \) \((j = 1, 2, \ldots, n - 1)\) is valid and thus \( \langle \hat{E}_{q}^{(n)}(k, \omega) \rangle = 0 \). Clearly, higher-order correlation functions of the outgoing amplitude operators do not vanish in this case in general. For example, the spectral intensity (in the \( k \)-space) of the radiation outgoing from a plane of a plate in thermal equilibrium at temperature \( T \) for chosen polarization, is proportional to \( w_{q}^{(0)}(k, \omega) \), where

\[
\langle \hat{E}_{q}^{(0,n)}(k, \omega) \rangle \langle \hat{E}_{q}^{(n)}(k', \omega') \rangle
\]

\[
= w_{q}^{(0)}(k, \omega) \delta(\omega - \omega') \delta(k - k').
\]

Applying Eq. (35) together with Eqs. (38) and (17) and making use of

\[
\langle \hat{f}_{\mu}^{(j)}(z, k, \omega) \hat{f}_{\mu'}^{(j)}(z', k', \omega') \rangle
\]

\[
= n(\omega, T) \delta_{\mu\mu'} \delta(\omega - \omega') \delta(k - k')
\]

\((j \neq 0, n)\), where

\[
n(\omega, T) = \left[ \exp \left( \frac{\hbar \omega}{k_{B} T} \right) - 1 \right]^{-1}
\]

\((k_{B}, \text{Boltzmann constant})\) is the well-known Bose-Einstein distribution function, we derive

\[
w_{q}^{(n)}(k, \omega) = n(\omega, T) \sum_{j=1}^{n-1} r_{j}^{q}/q_{j} \langle \hat{E}_{q}^{(j)}(k, \omega) \rangle
\]

\[
+ |r_{j}/q_{j}|^{2} \langle \hat{E}_{q}^{(j)}(k, \omega) \rangle + |r_{j}/q_{j}|^{2} \langle \hat{E}_{q}^{(j)}(k, \omega) \rangle + \text{c.c.}
\]

\((and \ w_{q}^{(0)}(k, \omega) \text{ accordingly})\), with the coefficients \( c_{q}^{(j)}(k, \omega) \) \((\lambda, \lambda' = \pm)\) being given in Eqs. (A.15) and (A.16) in the appendix.

### B. Input-output relations in the \( \rho \)-space

The input-output relations (35) in the \( k \)-space can be transformed, according to Eq. (15), into the \( \rho \)-space in
a straightforward manner. Writing the electric-field operator \( \mathbf{E}^{(0)}(\mathbf{r}, \omega) \equiv \mathbf{E}^{(0)}(z, \rho, \omega) \) at the boundaries of the plate as

\[
\mathbf{E}^{(0)}(\rho, \omega) = \mathbf{E}^{(0)}(z, \rho, \omega) \big|_{z=0^-} = \mathbf{E}^{(0)}_{q_{in}}(\rho, \omega) + \mathbf{E}^{(0)}_{q_{out}}(\rho, \omega),
\]

(51)

\[
\mathbf{E}^{(n)}(\rho, \omega) = \mathbf{E}^{(n)}(z, \rho, \omega) \big|_{z=0^+} = \mathbf{E}^{(n)}_{q_{in}}(\rho, \omega) + \mathbf{E}^{(n)}_{q_{out}}(\rho, \omega),
\]

(52)

we derive

\[
\mathbf{E}^{(0)}_{\text{out}}(\rho, \omega) = \int d^2\rho' R_{0/n}(\rho, \rho', \omega) \cdot \mathbf{E}^{(0)}_{\text{in}}(\rho', \omega) + \int d^2\rho' T_{n/0}(\rho, \rho', \omega) \cdot \mathbf{E}^{(n)}_{\text{in}}(\rho', \omega) + \sum_{j=1}^{n-1} \int d^2\rho' \left\{ \Phi_{0+}^{(j)}(\rho, \rho', \omega) \cdot \mathbf{E}^{(j)}_{+}(\rho', \omega) + \Phi_{0-}^{(j)}(\rho, \rho', \omega) \cdot \mathbf{E}^{(j)}_{-}(\rho', \omega) \right\},
\]

(53)

\[
\mathbf{E}^{(n)}_{\text{out}}(\rho, \omega) = \int d^2\rho' R_{n/0}(\rho, \rho', \omega) \cdot \mathbf{E}^{(n)}_{\text{in}}(\rho', \omega) + \int d^2\rho' T_{n/0}(\rho, \rho', \omega) \cdot \mathbf{E}^{(0)}_{\text{in}}(\rho', \omega) + \sum_{j=1}^{n-1} \int d^2\rho' \left\{ \Phi_{n+}^{(j)}(\rho, \rho', \omega) \cdot \mathbf{E}^{(j)}_{+}(\rho', \omega) + \Phi_{n-}^{(j)}(\rho, \rho', \omega) \cdot \mathbf{E}^{(j)}_{-}(\rho', \omega) \right\},
\]

(54)

where the electric-field operators \( \mathbf{E}^{(0)}_{\text{in}}(\rho, \omega) \) and \( \mathbf{E}^{(j)}_{\pm}(\rho, \omega) \) are respectively related to the amplitude operators \( \tilde{E}^{(0)}_{q_{in}}(k, \omega) \) and \( \tilde{E}^{(j)}_{q_{\pm}}(k, \omega) \) \((j = 1, 2, \ldots, n - 1)\) as

\[
\mathbf{E}^{(0)}_{\text{in}}(\rho, \omega) = \frac{1}{(2\pi)^2} \int d^2k \sum_{q=p,s} e^{(0)}_{q}(k) \tilde{E}^{(0)}_{q_{\pm}}(k, \omega) e^{ik\rho},
\]

(55)

\[
\mathbf{E}^{(n)}_{\text{in}}(\rho, \omega) = \frac{1}{(2\pi)^2} \int d^2k \sum_{q=p,s} e^{(n)}_{q}(k) \tilde{E}^{(n)}_{q_{\pm}}(k, \omega) e^{ik\rho},
\]

(56)

\[
\mathbf{E}^{(j)}_{\pm}(\rho, \omega) = \frac{1}{(2\pi)^2} \int d^2k \sum_{q=p,s} e^{(j)}_{q}(k) \tilde{E}^{(j)}_{q_{\pm}}(k, \omega) e^{ik\rho},
\]

(57)

In the above, the tensor-value reflection and transmission integral kernels, respectively, read as

\[
R_{0/n}(\rho, \rho', \omega) = \frac{1}{(2\pi)^2} \int d^2k \sum_{q=p,s} e^{(0)}_{q}(k) e^{ik\rho} e^{ik\rho'},
\]

(58)

\[
R_{n/0}(\rho, \rho', \omega) = \frac{1}{(2\pi)^2} \int d^2k \sum_{q=p,s} e^{(n)}_{q}(k) e^{ik\rho} e^{ik\rho'},
\]

(59)

and

\[
T_{0/n}(\rho, \rho', \omega) = \frac{1}{(2\pi)^2} \int d^2k \sum_{q=p,s} e^{(n)}_{q}(k) e^{ik\rho} e^{ik\rho'},
\]

(60)

\[
T_{n/0}(\rho, \rho', \omega) = \frac{1}{(2\pi)^2} \int d^2k \sum_{q=p,s} e^{(0)}_{q}(k) e^{ik\rho} e^{ik\rho'},
\]

(61)

and the integral kernels that relate the outgoing fields to the intraplate fields are given by

\[
\Phi_{0+}^{(j)}(\rho, \rho', \omega) = \frac{1}{(2\pi)^2} \int d^2k \sum_{q=p,s} e^{(j)}_{q}(k) e^{ik\rho} e^{ik\rho'},
\]

(62)

\[
\Phi_{n+}^{(j)}(\rho, \rho', \omega) = \frac{1}{(2\pi)^2} \int d^2k \sum_{q=p,s} e^{(j)}_{q}(k) e^{ik\rho} e^{ik\rho'},
\]

(63)

Note that the integral kernels depend only on the difference \( \rho - \rho' \), because of the translational symmetry in the \( x \)- and \( y \)-directions. The input-output relations in the form of Eqs. (53), (54) can be considered as a special realization of the general relations given for a dielectric scattering region of arbitrary shape in Ref. [12].
C. Input-output relations in terms of bosonic operators in the k-space

1. General case

In many applications it may be advantageous to express the incoming and the outgoing field operators in terms of appropriately chosen bosonic operators. Replacing the basic commutation relations (4) and (5) and the orthogonality of polarization unit vectors as given by Eqs. (27) and (28), from Eqs. (31) and (32) together with Eq. (17) and (18) and Eqs. (36) and (37) we find that the input amplitude operators satisfy the commutation relations

\[
\left[\hat{E}^{(0,n)}_{q\text{in}}(k,\omega), \hat{E}^{(0,n)\dagger}_{q'\text{in}}(k',\omega')\right] = c^{(0)}_{q\text{in}}(k,\omega) \delta_{qq'} \delta(\omega' - \omega) \delta(k - k'),
\]

where

\[
c^{(0)}_{q\text{in}}(k,\omega) = \frac{\pi \hbar}{\epsilon_0} \left(\frac{\omega}{c}\right)^2 |\beta|^2 e^{(0)}_q(k) \cdot e^{(0)*}_q(k),
\]

\[
c^{(n)}_{q\text{in}}(k,\omega) = \frac{\pi \hbar}{\epsilon_0} \left(\frac{\omega}{c}\right)^2 |\beta|^2 e^{(n)}_q(k) \cdot e^{(n)*}_q(k).
\]

Similarly, the output amplitude operators and the intraplate amplitude operators can be shown to satisfy commutation relations of the same type, i.e.,

\[
\left[\hat{E}^{(0,n)}_{q\text{out}}(k,\omega), \hat{E}^{(0,n)\dagger}_{q'\text{out}}(k',\omega')\right] = c^{(0)}_{q\text{out}}(k,\omega) \delta_{qq'} \delta(\omega' - \omega) \delta(k - k'),
\]

\[
\left[\hat{E}^{(j)}_{q\lambda}(k,\omega), \hat{E}^{(j)\dagger}_{q'\lambda'}(k',\omega')\right] = c^{(j)}_{q\lambda}(k,\omega) \delta_{qq'} \delta(\omega' - \omega) \delta(k - k')
\]

for the coefficients \(c^{(0)}_{q\text{out}}(k,\omega)\) and \(c^{(j)}_{q\lambda}(k,\omega)\), respectively, see Eqs. (A.12), (A.13) and Eqs. (A.15), (A.16) in the appendix. Needless to say that input amplitude operators, that refer to different sides of the plate, commute, input amplitude operators and intraplate amplitude operators commute, and intraplate amplitude operators that refer to different layers also commute. Note that output amplitude operators that refer to different sides of the plate do not commute in general [see Eqs. (A.9) and (A.10) in the appendix], which is similar as in the one-dimensional situation (cf. [11]).

From Eqs. (64) and (67) it is seen that bosonic input and output operators can be introduced according to

\[
\hat{E}^{(0,n)}_{q\text{in,out}}(k,\omega) = \sum_{\lambda' = \pm} r_{q\lambda\lambda'}(k,\omega) \hat{a}^{(j)}_{q\lambda'}(k,\omega),
\]

\[
(j = 1, \ldots, n - 1)
\]

and choosing the coefficients \(r_{q\lambda\lambda'}(k,\omega)\) [Eqs. (A.20) and (A.21) in the appendix] in such a way that

\[
\left[\hat{a}^{(j)}_{q\lambda}(k,\omega), \hat{a}^{(j)\dagger}_{q'\lambda'}(k',\omega')\right] = \delta_{\lambda\lambda'} \delta_{qq'} \delta(\omega' - \omega) \delta(k - k').
\]

Substituting Eqs. (69) and (71) into Eq. (35), we may express the input-output relations in the k-space in terms of bosonic operators,

\[
\left(\tilde{a}^{(0)}_{q\text{in}}(k,\omega), \tilde{a}^{(n)}_{q\text{in}}(k,\omega)\right) = \left(\tilde{r}^q_{0/n}(k,\omega), \tilde{t}^q_{n/0}(k,\omega)\right) \left(\tilde{a}^{(0)}_{q\text{in}}(k,\omega), \tilde{a}^{(n)}_{q\text{in}}(k,\omega)\right) + \sum_{j=1}^{n-1} \left(\tilde{c}^{(j)}_{q0+}(k,\omega), \tilde{c}^{(j)}_{q0-}(k,\omega)\right) \left(\tilde{c}^{(j)}_{q1+}(k,\omega), \tilde{c}^{(j)}_{q1-}(k,\omega)\right),
\]

where the coefficients [modified in comparison with Eq. (35)] read

\[
\tilde{r}^q_{0/n} = \sqrt{c^{(0)}_{q\text{out}}} \tilde{r}^q_{0/n}, \quad \tilde{t}^q_{n/0} = \sqrt{c^{(n)}_{q\text{out}}} \tilde{t}^q_{n/0},
\]

\[
\tilde{r}^q_{0/n} = \sqrt{c^{(0)}_{q\text{out}}} \tilde{r}^q_{0/n}, \quad \tilde{t}^q_{n/0} = \sqrt{c^{(n)}_{q\text{out}}} \tilde{t}^q_{n/0}.
\]
and

\[ \tilde{\phi}^{(j)}_{q,0,n,\lambda} = (\epsilon^{(0)}_{q,0})^{-1/2} \sum_{\lambda = \pm} \phi^{(j)}_{q,0,n,\lambda} \tau^{(j)}_{q,0,n,\lambda} \cdot \] (76)

Note that for \( k = 0 \) (i.e., perpendicular incidence of light on the plate), the input and output relations (73) exactly agree with previous results [11] obtained within a one-dimensional treatment.

2. Plate surrounded by vacuum

Let us turn to the limiting case that the space outside the plate – except for possible active atomic sources – may be regarded as being vacuum, i.e.,

\[ \varepsilon''_{0,n}(\omega) \to 0, \quad \varepsilon'_{0,n}(\omega) \to 1, \] (77)

\[ \beta^{'0,n}(\omega, k) = \begin{cases} 0 & \text{if } \omega/c \leq k, \\ \sqrt{\omega^2/c^2 - k^2} & \text{if } \omega/c > k. \end{cases} \] (78)

where the transformation matrix connecting the bosonic output operators with the bosonic input operators is exactly the same as that for the corresponding amplitude operators in Eq. (35), and the matrix equation

\[ \begin{pmatrix} r^q_{0/n} & t^q_{0/n} \\ r^q_{n/0} & t^q_{n/0} \end{pmatrix} \begin{pmatrix} \tilde{\phi}^{(0)}_{q,0} \tilde{\phi}^{(n)}_{q,0} \end{pmatrix} + \sum_{j=1}^{n-1} \begin{pmatrix} \gamma^{(j)}_{q,0+} & \gamma^{(j)}_{q,0-} \\ \gamma^{(j)}_{q,n+} & \gamma^{(j)}_{q,n-} \end{pmatrix} \begin{pmatrix} \tilde{\phi}^{(j)}_{q,0} \tilde{\phi}^{(j)}_{q,n} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \] (82)

is valid. In this case, the field operators \( \tilde{\mathbf{E}}^{(0)}(z, k, \omega) \) and \( \tilde{\mathbf{E}}^{(n)}(z, k, \omega) \) at the boundaries of the multilayer plate [see Eqs. (29), (30), (36), and (37)] can be represented as

\[ \tilde{\mathbf{E}}^{(0)}(0^-, k, \omega) = -\frac{\omega}{c} \sqrt{\frac{\pi \hbar}{\beta_0 \varepsilon_0}} \sum_{q,p,s} \left[ e^{(0)}_{q+}(k) \tilde{\phi}^{(0)}_{q,0} + e^{(0)}_{q-}(k) \tilde{\phi}^{(0)}_{q,0} \right], \] (83)

\[ \tilde{\mathbf{E}}^{(n)}(0^+, k, \omega) = -\frac{\omega}{c} \sqrt{\frac{\pi \hbar}{\beta_n \varepsilon_0}} \sum_{q,p,s} \left[ e^{(n)}_{q+}(k) \tilde{\phi}^{(n)}_{q,0} + e^{(n)}_{q+}(k) \tilde{\phi}^{(n)}_{q,0} \right] \] (84)

For the propagating-field components observed for \( \omega/c > k \), the coefficients \( \epsilon^{(0)}_{q,in} \) and \( \epsilon^{(0)}_{q,0} \), respectively, read

\[ \epsilon^{(0)}_{q,in} = \epsilon^{(0)}_{q,0} = \frac{\pi \hbar}{\varepsilon_0} \left( \frac{\omega}{c} \right)^2 \frac{1}{\beta_0} \] (79)

and

\[ \epsilon^{(0)}_{q,0} = \epsilon^{(0)}_{q,0}, \] (80)

as it can be seen from Eqs. (65) and (66) and Eqs. (A.23) and (A.24) in the appendix. Hence, bosonic input and output operators can be defined according to Eq. (69). Moreover, the output amplitude operators that refer to different sides of the plate commute [Eq. (A.22) in the appendix], which implies that the associated bosonic operators commute as well. Making use of Eqs. (79) and (80), we see that the modified reflection and transmission coefficients (74) and (75), respectively, become identical with the original ones. For \( \omega/c > k \), Eq. (73) thus reduces to

\( \beta_0 = \beta_n = \sqrt{\omega^2/c^2 - k^2}, \omega/c > k \). Note that in Eqs. (83) and (84) is nothing said about the location of the active
light sources. Clearly, if there are no active light sources outside the plate, then \( \hat{E}^{(0)}(z, k, \omega) \) and \( \hat{E}^{(n)}(z, k, \omega) \) evolve effectively freely there [cf. Eqs. (33) and (34)].

For the evanescent-field components observed for \( \omega/c \leq k \), the coefficients \( c_{qin}^{(0)} \) [Eq. (65)] and \( c_{qin}^{(n)} \) [Eq. (66)] identically vanish, because of \( \beta_0 = \beta'_q = 0 \). Recalling Eq. (69), we see that bosonic input operators cannot be introduced for the evanescent-field components. Hence, it is impossible to extend the validity of Eq. (81) to the evanescent-field components. To treat them, one has therefore to go back to the generally valid input-output relations (35) for the amplitude operators. Clearly, when there are no active light sources at any finite distance from the plate, then the input field can be regarded as being effectively a propagating field [see Eqs. (31) and (32)], that is to say, a field that does not contain evanescent input components. However, the output field may contain evanescent output components resulting from the intraplate field.

**IV. SUMMARY AND CONCLUDING REMARKS**

Applying the Green-tensor formalism of the quantization of the electromagnetic field in the presence of inhomogeneous, causal, linear dielectric bodies, we have derived the most general three-dimensional quantum input-output relations for the electromagnetic-field operators at an arbitrary multilayer dielectric plate, and we have given the commutation relations needed. Taking into account (i) dispersion and absorption of the plate and the surrounding medium, (ii) possible active light sources inside and/or outside the plate, and (iii) both propagating-field components and evanescent-field components, the input-output relations connect the output field operators at the boundaries of the plate with the input field operators at the boundaries of the plate and the fields inside the layers of the plate. The outgoing fields outside the plate can then be obtained from the ones at the boundaries of the plate via quantum Langevin equations, with the Langevin noise sources being determined by the active and passive light sources in the respective half-space outside the plate.

We have given the input-output relations in terms of both algebraic relations for amplitude operators in the k-space and integral relations for field operators in the coordinate space. Further, we have shown that in the k-space it is possible, provided that the incoming fields contain effectively propagating components, to rewrite the input-output relations in order to obtain them in terms of relations between bosonic operators instead of amplitude operators. Therefore the input-output relations for the amplitude operators are more general than those for the bosonic operators.

Finally, we have studied the limiting case of the plate being surrounded by vacuum. In this case, the input-output relations in terms of bosonic operators only apply to the propagating-field components, for which \( \omega/c > k \) is valid. An application to the evanescent-field components \( (\omega/c \leq k) \) would run into contradictions. To treat them, one has therefore to go back to the amplitude operators.

The derived input-output relations generalize previous results obtained from a one-dimensional calculation [11]. In special case of a plate that is embedded in vacuum, without active light sources at any finite distance from the plate (and without active light inside the plate), they reduce to those given in Ref. [21], if evanescent-field components are disregarded.

In conclusion, the derived input-output relations are suited for studying the quantum statistical properties of electromagnetic fields in the presence of planar multilayered structures (such as cavity-like systems or photonic crystals), with special emphasis on absorption-assisted quantum decoherence, including evanescent-field effects.

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**APPENDIX: COMMUTATION RELATIONS**

Let us begin with the calculation of the commutators of the output amplitude operators \( \hat{E}^{(0,n)}_{q_{out}}(k, \omega) \). From Eq. (35) it follows that

\[
\hat{E}^{(0)}_{q_{out}}(k, \omega) = r^q_{0/n_n} \hat{E}^{(0)}_{q_{in}}(k, \omega) + r^q_{n/0_n} \hat{F}^{(n)}_{q}(k, \omega)
\]

(A.1)

and

\[
\hat{E}^{(n)}_{q_{out}}(k, \omega) = r^q_{0/n_n} \hat{E}^{(0)}_{q_{in}}(k, \omega) + r^q_{n/0_n} \hat{F}^{(n)}_{q}(k, \omega),
\]

(A.2)

where

\[
\hat{F}^{(0)}_{q}(k, \omega) = \sum_{j=1}^{n-1} \left[ \phi_{q}^{(j)} \hat{E}_{q}^{(j)}(k, \omega) + \phi_{q}^{(j)} \hat{E}_{q}^{(-j)}(k, \omega) \right]
\]

(A.3)

\[
\hat{F}^{(n)}_{q}(k, \omega) = \sum_{j=1}^{n-1} \left[ \phi_{q}^{(j)} \hat{E}_{q}^{(j)}(k, \omega) + \phi_{q}^{(j)} \hat{E}_{q}^{(-j)}(k, \omega) \right]
\]

(A.4)
Hence, the (relevant) commutators of the output amplitude operators that refer to different sides of the plate can be given by

\[
\left[ \hat{F}_{q_{\text{out}}}^{(0)}(k, \omega), \hat{F}_{q_{\text{out}}'}^{(n)}(k', \omega') \right] = \delta_{qq'} \delta(\omega - \omega') \delta(k - k')
\]

\[
= r_{q/n}^{2} r_{q'/n}^{*} \left[ \hat{E}_{q_{\text{in}}}^{(0)}(k, \omega), \hat{E}_{q_{\text{in}}'}^{(n)}(k', \omega') \right] + r_{q/n}^{2} r_{q'/n}^{*} \left[ \hat{E}_{q_{\text{in}}}^{(n)}(k, \omega), \hat{E}_{q_{\text{in}}'}^{(n)}(k', \omega') \right] + \left[ \hat{F}_{q}^{(0)}(k, \omega), \hat{F}_{q'}^{(n)}(k', \omega') \right],
\]

where Eqs. (64) – (66) have been used. Making use of Eqs. (17) and (18) and recalling the basic commutation relations (4) and (5), we derive

\[
\left[ \hat{F}_{q}^{(0)}(k, \omega), \hat{F}_{q'}^{(n)}(k', \omega') \right] = \delta_{qq'} \delta(\omega - \omega') \delta(k - k') \frac{4\pi \hbar}{\epsilon_{0}} \left( \frac{\omega}{c} \right)^{2} \epsilon_{q - \mu}(k) \epsilon_{q + \mu}^{*}(k) \sum_{j = 1}^{n - 1} \int_{[j]} dz g_{\mu\nu}^{(0j)}(0, z, k, \omega) \frac{\omega^{2}}{c^{2}} \epsilon_{\nu}^{*} g_{\mu\nu}^{(n)j}(0, z, k, \omega)
\]

\[
= \delta_{qq'} \delta(\omega - \omega') \delta(k - k') \frac{4\pi \hbar}{\epsilon_{0}} \left( \frac{\omega}{c} \right)^{2} \epsilon_{q - \mu}(k) \epsilon_{q + \mu}^{*}(k) \left\{ \sum_{j = 0}^{n} \int_{[j]} \int_{[j]} dz g_{\mu\nu}^{(0j)}(0, z, k, \omega) \frac{\omega^{2}}{c^{2}} \epsilon_{\nu}^{*} g_{\mu\nu}^{(n)j}(0, z, k, \omega) - \int_{-\infty}^{0} dz g_{\mu\nu}^{(0n)}(0, z, k, \omega) \frac{\omega^{2}}{c^{2}} \epsilon_{\nu}^{*} g_{\mu\nu}^{(nn)j}(0, z, k, \omega) \right\}. \quad \text{(A.5)}
\]

It is not difficult to calculate the last two integrals in Eq. (A.6), by using the explicit expression (20) for \( g^{(jj')}_{\mu\nu}(z, z', k, \omega) \). To calculate the sum of integrals \( \sum_{j = 0}^{n} \int_{[j]} dz \ldots \), we employ the integral relation (11) for the classical Green tensor \( G(r, r', \omega) \), rewritten in terms of \( g^{(jj')}_{\mu\nu}(z, z', k, \omega) \),

\[
\sum_{j = 0}^{n} \int_{[j]} dz' \int_{[j']} dz'' g^{(jj'')}_{\mu\nu}(z, z', k, \omega) \frac{\omega^{2}}{c^{2}} \epsilon_{\nu}^{*} g^{(jj')}_{\mu\nu}(z', z'', k, \omega) = \frac{1}{2i} g^{(jj')}_{\mu\nu}(z, z', k, \omega)
\]

\[
- \frac{1}{2i} g^{(jj')}_{\mu\nu}(z, z', k, \omega) + g^{(jj')}_{\mu\nu}(z, z', k, \omega) e_{z \nu} \epsilon_{\nu}^{*} + e_{z \mu} \epsilon_{\nu}^{*} g^{(jj')}_{\mu\nu}(z', z, k, \omega) e_{z \nu}. \quad \text{(A.7)}
\]

After lengthy, but straightforward calculations we then derive, on using Eqs. (65) and (66),

\[
\left[ \hat{E}_{s_{\text{out}}}^{(0)}(k, \omega), \hat{E}_{s_{\text{out}}'}^{(n)}(k', \omega') \right] = \delta(\omega - \omega') \delta(k - k')
\]

\[
\times \frac{\pi \hbar}{\epsilon_{0}} \left( \frac{\omega}{c} \right)^{2} \left( \frac{i\beta_{0}^{n} t_{s}^{*}}{|\beta_{0}|^{2} t_{s}/n} - \frac{i\beta_{0}^{n} t_{s}^{*}}{|\beta_{n}|^{2} t_{s}/n} \right), \quad \text{(A.9)}
\]
\[
\left[ \hat{E}_{p,\text{out}}^{(0)}(k, \omega), \hat{E}_{p,\text{out}}^{(n)\dagger}(k', \omega') \right] = \frac{\pi \hbar}{\epsilon_0} \left( \frac{\omega}{c} \right)^2 \left\{ \frac{r^p_{n/0}}{\beta_n |k_n|^2} \left( k^2 \frac{k_0^2}{k_n^2} - |\beta_n|^2 \right) \\
+ \frac{r^p_{0/n}}{\beta_0 |k_0|^2} \left( k^2 \frac{k_0^2}{k_n^2} - |\beta_n|^2 \right) \right\} \delta(\omega - \omega') \delta(k - k').
\]

Next, let us consider the commutators of the output amplitude operators that refer to the same sides of the plate. Performing the same steps as before, we now arrive at
\[
\left[ \hat{E}_{s,\text{out}}^{(0)}(k, \omega), \hat{E}_{s,\text{out}}^{(n)\dagger}(k', \omega') \right] = 0,
\]

\[
\left[ \hat{E}_{p,\text{out}}^{(0)}(k, \omega), \hat{E}_{p,\text{out}}^{(n)\dagger}(k', \omega') \right] = \frac{\pi \hbar}{\epsilon_0} \left( \frac{\omega}{c} \right)^2 \left\{ \frac{r^p_{n/0}}{\beta_n |k_n|^2} \left( k^2 \frac{k_0^2}{k_n^2} - |\beta_n|^2 \right) \\
+ \frac{r^p_{0/n}}{\beta_0 |k_0|^2} \left( k^2 \frac{k_0^2}{k_n^2} - |\beta_n|^2 \right) \right\} \delta(\omega - \omega') \delta(k - k').
\]

The commutators \( \left[ \hat{E}_{q,\text{out}}^{(n)}(k, \omega), \hat{E}_{q,\text{out}}^{(n)\dagger}(k', \omega') \right] \) are obtained from Eqs. (A.11) – (A.13), by making the replacements \( \beta_0 \to \beta_n, k_0 \to k_n, e_{q,\text{out}}^{(n)\dagger} \to e_{q,\text{out}}^{(n)}, \) and \( r^0_{0/n} \to r^n_{0/n}. \)

Finally, it can easily be proved that the intraplate amplitude operators (38) satisfy the commutation relations (A.11) – (A.16), by making the replacements \( \beta_0 \to \beta_n, k_0 \to k_n, e_{q,\text{out}}^{(n)\dagger} \to e_{q,\text{out}}^{(n)}, \) and \( r^0_{0/n} \to r^n_{0/n}. \)

Using Eqs. (A.14) – (A.16), we find that the operators
\[
\delta_{q,\lambda}^{(j)}(k, \omega) = \frac{1}{\epsilon_{q,\lambda}^{(j)}(k, \omega)} \times \left[ e^{i\beta_j d_j} \hat{E}_{q,\text{out}}^{(j)}(k, \omega) \pm \hat{E}_{q,\text{out}}^{(j)}(k, \omega) \right],
\]

where
\[
\epsilon_{q,\lambda}^{(j)}(k, \omega) = \frac{2w}{\epsilon_{q,\lambda}^{(j)}(k, \omega)} \left( \frac{\pi \hbar}{\epsilon_0} \right)^{1/2} e^{-\beta_j' d_j/2} \times \{ \beta_j' \sinh(\beta_j' d_j) e^{(j)}(k) \cdot e^{(j)*}(k) \}^{1/2}
\]

satisfy bosonic commutation relations. From Eq. (A.17) it then follows that
\[
\hat{E}_{q,\lambda}^{(j)}(k, \omega) = \sum_{\lambda'=\pm} r_{q,\lambda,\lambda'}^{(j)}(k, \omega) \delta_{q,\lambda'}^{(j)}(k, \omega),
\]
we have (evanescent-field components). In the first case ($\omega/c > k$) for the bosonic output operators are only valid for propagating waves, not for evanescent ones. It should also be noted that the free-space light modes $U_{p,k}(z,\omega)$ used in Ref. [19] are introduced by explicitly requiring that $\omega/c > |K|$, where $K$ corresponds to $k$ in the present paper. Hence, these modes solely represent propagating waves.

Note that the commutation relations (42) in Ref. [21] for the bosonic output operators are only valid for propagating waves, not for evanescent ones. It should also be noted that the free-space light modes $U_{p,k}(z,\omega)$ used in Ref. [19] are introduced by explicitly requiring that $\omega/c > |K|$, where $K$ corresponds to $k$ in the present paper. Hence, these modes solely represent propagating waves.

\[
[\hat{E}_{q,\text{out}}^{(0)}(k,\omega),\hat{E}_{q',\text{out}}^{(n)\dagger}(k',\omega')] = \frac{\pi\hbar}{\epsilon_0} \left(\frac{\omega}{c}\right)^2 \delta_{qq'} \delta(k-k') \delta(\omega-\omega').
\]

and from Eqs. (A.11) – (A.13) we find

\[
[\hat{E}_{q,\text{out}}^{(0)}(k,\omega),\hat{E}_{q',\text{out}}^{(0)\dagger}(k',\omega')] = \frac{\pi\hbar}{\epsilon_0} \left(\frac{\omega}{c}\right)^2 2\delta_{qq'} \delta(k-k') \delta(\omega-\omega').
\]
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