Meeting the Constraint of Neutrino–Higgsino Mixing in Gravity Unified Theories

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Abstract

In Gravity Unified Theories all operators that are consistent with the local gauge and discrete symmetries are expected to arise in the effective low–energy theory. Given the absence of multiplets like 126 of $SO(10)$ in string models, and assuming that $B - L$ is violated spontaneously to generate light neutrino masses via a seesaw mechanism, it is observed that string theory solutions generically face the problem of producing an excessive $\nu_L - \tilde{H}$ mixing mass at the GUT scale, which is some nineteen orders of magnitude larger than the experimental bound of 1 MeV. The suppression of $\nu_L - \tilde{H}$ mixing, like proton longevity, thus provides one of the most severe constraints on the validity of any string theory solution. We examine this problem in a class of superstring derived models. We find a family of solutions within this class for which the symmetries of the models and an allowed pattern of VEVs, surprisingly, succeed in adequately suppressing the neutrino–Higgsino mixing terms. At the same time they produce the terms required to generate small neutrino masses via a seesaw mechanism.

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While supersymmetry appears to be a key ingredient for higher unification, it is known that it poses two generic problems pertaining to non-conservation of baryon and lepton numbers. First and the better discussed is the problem of rapid proton decay, which arise through $d = 4$ and color–triplet mediated and/or gravity induced $d = 5$ operators [1]. A problem of a similar magnitude, which is however not so well emphasized in the literature is the one of excessive neutrino–Higgsino mixing, which could arise effectively through bilinear operators and could induce unacceptably large masses for the neutrinos. Using standard notation the relevant $(B, L)$ violating operators, which may arise in the superpotential, while respecting the Standard Model gauge symmetries, are as follows:

\[ W = (\xi M')LH_2 + \left[ \eta_1 \bar{U}D + \eta_2 QL\bar{D} + \eta_3 LL\bar{E} \right] \]
\[ + \left[ \lambda_1 QQQL + \lambda_2 \bar{U}\bar{U}\bar{D}\bar{E} + \lambda_3 LLH_2H_2 \right] / M \]  

(1)

Here, generation, $SU(2)_L$ and $SU(3)_C$ indices are suppressed. $M$ and $M'$ denote characteristic mass scales. Experimental lower limits on proton life–time ($\tau_p > 10^{32}$ yrs) turn out to impose the constraints [2]: $\eta_1\eta_2 \leq 10^{-24}$ and $\lambda_{1,2}/M \leq 10^{-25}$ GeV$^{-1}$. Now, Eq. (1) would also give rise to a $\nu - \tilde{H}_2$ mixing mass $\delta m_{\nu\tilde{H}} = \xi M'$, which in conjunction with a Majorana mass for $\tilde{H}_2$ of order $v_{wk} \sim 100$ GeV, would induce Majorana masses for $\nu'_L$'s of order $(\xi M')^2/v_{wk}$ if $\xi M' << v_{wk}$. Astroparticle physics restrictions on masses of stable neutrinos show that $m_{\nu_L} \leq 10$ eV, which implies that $\delta m_{\nu\tilde{H}} = \xi M' \leq 1$ MeV$^2$.

Supersymmetry thus confronts us with two major puzzles: (I) Why is the magnitude of the $d = 4$ and $d = 5$ operators so extraordinarily small, \textit{i.e.} why $\eta_1\eta_2 \leq 10^{-24}$ as opposed to being $O(1)$, and $\lambda_{1,2}/M \leq 10^{-25}$ GeV$^{-1}$, as opposed to being $O(1\text{TeV}^{-1})$ ? and (II) why is the neutrino–Higgsino mixing mass so small -- $< 1$ MeV or even 10 KeV, as opposed to being given, for example, by the string scale or the $(B - L)$ breaking scale which is expected to be superheavy ($\geq 10^{12}$ GeV, see below)?

*imposing $m(\nu_e) < 10^{-3}$ eV, for MSW type solutions for the solar neutrinos puzzle, yields $\delta m_{\nu\tilde{H}} = \xi M' < 10$ KeV.
We would like to stress that these problems are not technical but rather fundamental. In the context of point–particle quantum field theories one can impose discrete and global symmetries, which forbid the undesired couplings. However, these type of symmetries are in general expected to be violated by quantum gravity effects, unless they are obtained from broken gauge symmetries. Furthermore, the need to impose such ad hoc symmetries clearly indicates a structure which goes beyond conventional Grand Unified Theories. Indeed, it has been observed that a class of string models, possessing three families, not only provide a natural doublet–triplet splitting mechanism [3], but also possess the desired gauge symmetries, beyond SUSY GUTS, which safeguard proton stability from all potential dangers [4, 5], including those which may arise from higher dimensional operators and the color triplets in the infinite tower of string states.

The purpose of this note is to probe into the second puzzle and to explore whether the strong suppression of $\nu_L - \tilde{H}$ can be understood in the context of string derived models in a manner similar to the understanding of proton stability.

To appreciate the nature of the problem, we note that the $LH_2$–term, as also the $d = 4$ operator in Eq. (1) (though not the $d = 5$), would of course be forbidden if one imposes a multiplicative R–parity symmetry: $R = (-1)^{3(B-L)}$, which would have a natural origin through a gauged $(B-L)$ symmetry, as in $SO(10)$. The difficulty is that $SO(10)$ is expected to be violated spontaneously at some heavy intermediate scale, especially if the neutrinos acquire light masses through a seesaw mechanism, which assigns heavy Majorana masses to $\nu_R$s – perhaps of order $10^{12}$GeV. Once $(B - L)$ is violated by the VEV of a scalar field, effective $LH_2$ term and $d = 4$ proton decay operators can in general be induced through higher dimensional operators ($d \geq 4$ and $d \geq 5$ respectively) by utilizing the VEV of such a field. Typically, the strength of such induced terms would be too large (see below), unless symmetries beyond $(B - L)$ provide the desired protection. That such additional symmetries arise naturally in the context of a desirable class of three generation superstring models which suppress adequately the $d = 4$ and $d = 5$ proton decay operators was noted in Ref. [5]. Here we examine whether string symmetries, beyond $(B - L)$ can adequately suppress the
effective $LH_2$-term.

It is worth noting at this stage that if $(B-L)$ is violated by the VEV of the 126 of $SO(10)$ (or equivalently $(10,3,1)$ of $SU(4) \times SU(2)_L \times SU(2)_R$), then effectively $R$-parity = $(-1)^{3(B-L)}$ would still be preserved because this VEV violate $(B-L)$ by two units \[3\]. In this case, the effective $LH_2$-term cannot be induced even if $(B-L)$ is broken. Recent works show, however, that 126 and very likely $(10,3,1)$ as well, are hard – perhaps impossible – to obtain in string theories \[4\]. We will proceed by assuming that this constraint holds.

In the absence of 126 of Higgs, $(B-L)$ can still be broken and $\nu_R^i$s can acquire heavy Majorana masses, quite simply by utilizing the VEVs of the sneutrino-like fields $\tilde{N}$ and $\tilde{N}_L$, which belong to the $16_H$ and $\overline{10}_H$ of $SO(10)$ respectively. The subscript “$H$” signifies that the corresponding 16 is Higgs-like, which in general need not, and very likely does not, coincide with the familiar chiral 16’s. Such vector-like representations (i.e. pairs of 16 and $\overline{10}$, some of which may not be Higgs-like), together with unpaired chiral 16’s, do in fact arise generically in a large class of string derived models (see e.g. Ref. \[5, 6, 7\]). In this case, an effective operator of the form $16 \cdot 16 \cdot \overline{10}_H \overline{10}_H/M$, which is allowed by $SO(10)$, would induce a Majorana mass $(\tilde{\nu}_R C^{-1} \tilde{\nu}_R^T)(\tilde{N}_L \tilde{N}_L^T)/M + h.c.$ of magnitude $M_R \sim 10^{12.5}$ GeV, as desired, for $\langle \tilde{N}_L \rangle \sim 10^{15.5}$ GeV and $M = M_{st} \sim 10^{18}$ GeV. With the sneutrino acquiring a VEV however, there is a danger that an effective $d = 4$ operator of the form $16_i 16_H 10_H$, which is allowed by $SO(10)$ can lead to neutrino–Higgsino (i.e. $\nu_L - \tilde{H}_2$) mixing through the coupling

$$LH_2 \langle \tilde{N} \rangle$$

which is unacceptably large, if $\langle \tilde{N} \rangle \sim 10^{15.5}$ GeV.

As mentioned before in the context of point particle field theory models, the problem can be resolved simply by imposing a discrete symmetry which forbids the Yukawa couplings of $16_H$ to the observed chiral families $(16)_i$ (this is in fact the reason why $16_H$ should not be identified with any of the chiral families, since the later must have large enough Yukawa couplings with each other to generate observed fermion masses and mixings).
In any theory linked with gravity, like superstring theory, however, one can not impose such a discrete symmetry by hand. Rather, one must examine whether such a symmetry emerges from within the underlying theory. Furthermore, even if the cubic level Yukawa coupling $16_i 16_H 10_H$ is absent, it may in general be induced through gravity induced higher dimensional operators, by utilizing VEVs of relevant fields. One must thus examine whether intrinsic symmetries of the underlying theory – e.g. a superstring theory – together with an allowed pattern of VEVs of the massless fields, which emerge from such a theory, can provide the needed enormous suppression for $\nu_L - \tilde{H}$ mixing. *In this sense, the neutrino–Higgsino mixing, like proton stability, imposes a highly nontrivial constraint on all theories which incorporate gravity, especially if multiplets like 126 are not available to break $B - L$.*

In the following, we examine how the problem can be resolved in the context of some string derived solutions. First we note that for the free fermionic constructions yielding either the flipped $SU(5) \times U(1)$ \[8\] or the $SO(6) \times SO(4)$ \[9\] models, there appear in addition to the three chiral families, additional massless vector–like pairs as components of 16 and $\mathbf{16}$ (for $SU(5) \times U(1)$) or half of 16 and $\mathbf{16}$ (for $SO(6) \times SO(4)$) containing additional right–handed neutrinos – i.e. $\tilde{N}_R$ and $N'_L$, respectively. These additional vector–like pairs typically combine utilizing VEVs of singlets to become superheavy ($\sim 10^{15} - 10^{15.5}\text{GeV}$) while their sneutrino components $\tilde{N}$ and $\tilde{N}'$ acquire VEVs of the order of the GUT or string scales and thereby break the extended symmetries. As alluded to above, in these models excessive neutrino–Higgsino mixing will in general appear through $16_i 16_H 10_H$ coupling\[10\] which may be either primary or induced via higher dimensional operators, unless suitable discrete, or continuous symmetries, forbid such couplings, as well as the higher dimensional operators to a sufficient degree. It is of great interest, and of crucial importance, to examine whether the string derived models of the type mentioned above, for which the sneutrino–like fields must acquire large VEVs, posses the desired symmetries to sufficiently suppress the $\nu_L - \tilde{H}$ mixing.

\[1\] here we are using $SO(10)$ notation for convenience only. The corresponding coupling in terms of the subgroups $SU(5) \times U(1)$ and $SO(6) \times SO(4)$ can be written unambiguously
We next turn our attention to the class of superstring–derived standard–like models, obtained in ref. \cite{11, 13}. A priori, motivations for exploring this class of solutions are that – (a) they exhibit qualitatively the right texture for fermion masses and mixing, and (b) they possess extra symmetries, beyond conventional GUTs, which safeguard proton stability from all potential dangers. These symmetries also turn out to be helpful in suppressing $\nu_L - \tilde{H}$ mixing operators. Furthermore, unlike the models of Refs. \cite{8, 9}, the standard–like models possess fields carrying half–integer $B - L$ (see table 3). Thus, they may well break $B - L$ by utilizing VEVs of such fields rather than those of sneutrino–like fields. In this case, $\nu_L - \tilde{H}$ mixing can still occur, but only by utilizing products of VEVs of such fields and of composites. As a result, the mixing necessarily involves higher dimensional operators, which are suppressed. However, it is still far from clear as to whether $\nu_L - \tilde{H}$ mixing can be suppressed adequately in these models. This is what we examine next.

First we need to recall some salient features of the standard–like models. This class of models is constructed by using the four dimensional free fermionic construction \cite{12}. The standard–like models are constructed by a set of eight boundary condition basis vectors $\{1, S, b_1, b_2, b_3, \alpha, \beta, \gamma\}$ \cite{11}. The gauge symmetry of this class of models at the string scale, after application of all GSO projections, has the form:

$$G = \left[ SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{T_{3R}} \right] \times \left[ G_M = \prod_{i=1}^{6} U(1)_i \right] \times G_H \quad (3)$$

The first bracket denotes the part of the observable gauge symmetry which is a subgroup of $SO(10)$. The $U(1)_i$ denote six horizontal symmetry charges which act non–trivially on the three chiral families as well as the hidden matter states. In the models of Ref. \cite{11}, $G_H = SU(5)_H \times SU(3)_H \times U(1)_H^2$, which denotes the gauge symmetry of the hidden sector.

Below we examine in detail the model of Ref. \cite{13}. The full massless spectrum of this model is given in Ref. \cite{13}. A partial list which is relevant for our purposes is given in tables 1–3. It includes the following states:

(I) There are three chiral families of quarks and leptons, each with sixteen components, including $\bar{\nu}_R$, which arise from the twisted sectors $b_1$, $b_2$ and $b_3$. These
transform as 16’s of $SO(10)$ and are neutral under $G_H$.

(II) the Neveu–Schwarz (NS) sector corresponding to the untwisted sector of the orbifold model produces, in addition to the gravity multiplets, three pairs of electroweak scalar doublets $\{h_1, h_2, h_3, \bar{h}_1, \bar{h}_2, \bar{h}_3\}$, three pairs of $SO(10)$ – singlets with $U(1)$ charge $\{\Phi_{12}, \Phi_{13}, \Phi_{12}, \Phi_{23}, \Phi_{13}\}$, and three scalars that are singlets of the entire four dimensional gauge group, $\{\xi_1, \xi_2, \xi_3\}$.

(III) the sector $S + b_1 + b_2 + \alpha + \beta$ produces one additional pair of electroweak doublets $\{h_{45}, \bar{h}_{45}\}$, one pair of color triplets $\{D_{45}, \bar{D}_{45}\}$ and seven pairs of $SO(10)$ singlets with $U(1)$ charge $\{\Phi_{45}, \bar{\Phi}_{45}, \Phi_{1,2,3}^+, \bar{\Phi}_{1,2,3}^+\}$.

(IV) In addition to the states mentioned above, which transform solely under the observable gauge group and $U(1)_i$, the sectors $b_j + 2\gamma + (I = 1 + b_1 + b_2 + b_3)$ produce hidden–sector multiplets $\{T_i, \bar{T}_i, V_i, \bar{V}_i\}_{i=1,2,3}$ which are $SO(10)$ singlets but are non–neutral under $U(1)_i$ and the hidden $G_H$. The $T_i(\bar{T}_i)$ are $5(\bar{5})$ and $V_i(\bar{V}_i)$ are $3(\bar{3})$ of $SU(5)_H$ and $SU(3)_H$ gauge groups, respectively. These states are listed in table 2.

(V) The vectors in some combinations of $(b_1, b_2, b_3, \alpha, \beta) \pm \gamma + (I)$ produce additional states which are either singlets of $SU(3) \times SU(2) \times U(1)_Y \times SU(5)_H \times SU(3)_H$ or in vector–like representation of this group. The relevant states of this class $\{H_{17} - H_{26}\}$ are listed in table 3. As we will show below the states of class (V) are crucial for the resolution of the neutrino–Higgsino problem in the superstring standard–like models.

One characteristic feature of this class of models, is that, barring the three chiral 16’s there are no additional vector–like $16 + \overline{16}$ pairs. As a result, elementary fields with the quantum numbers of $N'_L \in \overline{16}$ do not exist in this class of models. Nevertheless, VEVs of products of certain condensates, which are expected to form through the hidden sector force and certain fields belonging to the set (V) can provide the desired quantum numbers of sneutrino like fields – i.e $\bar{N}_R \in 16$ and $N'_L \in \overline{16}$, as, for example, in the combinations shown below:

\[
\langle H_{19} \bar{T}_i \rangle \langle H_{23} \rangle \rightarrow (B - L = -1, T_{3R} = 1/2) \sim N'_L \in \overline{16}
\]

\[
\langle H_{20} T_i \rangle \langle H_{26} \rangle \rightarrow (B - L = +1, T_{3R} = -1/2) \sim \bar{N}_R \in 16
\]
Note $H_{19}$ and $T_i$ transform as 5, and $H_{20}$ and $T_i$ transform as 5, of $SU(5)_H$, respectively. Thus, $H_{19}(H_{20})$ can pair up with $\bar{T}_i(T_i)$ to make condensates at the scale $\Lambda_H$, where $SU(5)_H$ force becomes strong. In this model an effective seesaw mechanism [14, 15] is implemented by combining the familiar Dirac masses of the neutrinos which arise through electroweak–symmetry breaking scale, with superheavy mass terms which mix $\bar{\nu}_R^i$ with the singlet $\phi$ fields in sets (II) and (III) [16]. The details of how the seesaw mechanism arises in the superstring model are given in Ref. [16]. Here we briefly sketch the main features. Subject to the relevant symmetries, the mixing terms arise in order $N = 6$, 7 and 8 for $\bar{\nu}_{2R}$, $\bar{\nu}_{3R}$ and $\bar{\nu}_{1R}$ respectively by utilizing VEVs of fields having the quantum numbers of $N'_L$ (see Eq. (4)). For example, for $N = 6$, the mixing arises through the operators $\bar{\nu}_{2R}\Phi^+_2\langle H_{19}\bar{T}_2\rangle\langle H_{25}\rangle\langle \Phi_{45}\rangle$ while for $N = 7$ and 8, the relevant terms are given by $\bar{\nu}_{3R}\Phi^-_3\langle H_{19}\bar{T}_3\rangle\langle H_{25}\rangle\langle \Phi_{45}\rangle\langle \Phi_{13}\rangle$ and $\bar{\nu}_{1R}\xi_2\langle H_{19}\bar{T}_1\rangle\langle H_{25}\rangle\langle \Phi_{45}\rangle\langle \Phi_{13}\rangle$. Thus, the neutrino mass matrix for a given family takes the form

$$
\begin{pmatrix}
\nu_i & \nu_j^C & \phi_m \\
(km_u)_{ij} & 0 & (km_u)_{ij} \\
0 & M_X & O(M_\phi)
\end{pmatrix}
\begin{pmatrix}
\nu_i \\
\nu_j^C \\
\phi_m
\end{pmatrix},
$$

with $m_\chi \sim (\Delta_H^M)^2 (\phi^M)_n M$ and $M_\phi \sim (\Delta_H^4)^m (\phi^M) M$. $n$ and $m$ are the orders at which the terms for a given neutrino flavor are obtained. The mass eigenstates are mainly $\nu$, $\nu_j^C$ and $\phi$ with a small mixing and with the eigenvalues

$$
m_{\nu_j} \sim m_\phi \left(\frac{km_u}{m_\chi}\right)^2 \quad m_{\nu_j^C}, m_\phi \sim m_\chi
$$

Given that there are VEVs with the quantum numbers of $N'_L$ and $\bar{N}_R$ belonging to the $\overline{16}$ and 16 respectively (see Eq. (4)), the latter can lead to the $\nu_L\bar{H}$ mixing through higher dimensional operators. In the model of ref. [13] we find that, subject to the constraints of the string–derived symmetries and the string selection rules [17], the relevant terms arise only at $N = 6$, which we list below:

$$
L_1\bar{h}_1H_{26}H_{20}T_1\Phi_{13},
L_1\bar{h}_2H_{24}H_{20}T_1\xi_1,
$$

8
\[ L_1 \bar{h}_{45} H_{26} H_{20} T_1 \Phi_{45}, \]
\[ L_2 \bar{h}_1 H_{26} H_{20} T_2 \Phi_{13}, \]
\[ L_2 \bar{h}_2 H_{24} H_{20} T_3 \xi_1, \]
\[ L_2 \bar{h}_{18} H_{20} T_2 \Phi_{45}, \]
\[ L_2 \bar{h}_{45} H_{26} H_{20} T_2 \Phi_{45}, \]
\[ L_3 \bar{h}_{45} H_{17} H_{17} V_3 \Phi_1^-, \]
\[ L_3 \bar{h}_2 H_{24} H_{20} T_3 \xi_2, \]
\[ L_3 \bar{h}_{18} H_{20} T_3 \Phi_{45}. \]  

(6)

It may be verified using tables 1–3 that all the terms listed above conserve the full
gauge symmetry listed in Eq. (3), including all the \( U(1)'s \). The gauge singlet fields\( \xi_{1,2} \) appear in these terms to produce non–vanishing Ising model correlators [17].

The superstring model under consideration contains an anomalous \( U(1) \) symmetry which induces a Fayet–Iliopoulos D–term and destabilizes the vacuum [18]. To preserve supersymmetry at the Planck scale, one must satisfy the \( D \) and \( F \) constraints arising from the superpotential by giving VEVs to a some scalar fields in
the massless string spectrum. Depending on the choice of these VEVs at the string
scale, the dangerous terms may indeed be generated. Thus, our task is to find a
solution to the \( F \) and \( D \) constraints for which the neutrino–Higgsino mixing terms
vanish while the seesaw neutrino mass terms, as well as other desirable phenomeno-
logical properties are retained. Note that owing to differing quantum numbers (e.g.
\( B − L \) and \( T_3_{_R} \)) of \( \bar{\nu}_R \) and \( \nu_L \), the fields needed for \( \bar{\nu}_R \phi \)–mixing are distinct from
those which would induce \( \nu_L − \bar{H} \) mixing. For example, the former needs VEVs like
those of \{ \( H_{23}, H_{25}, \Phi_{45}, \Phi_{13}, \Phi_1^−, \xi_2 \} \) and the condensate \( \langle H_{19} T_i \rangle \) while the latter needs
VEVs like those of \{ \( H_{24}, H_{20}, H_{18}, \Phi_{45}, \Phi_{13}, \xi_1 \} \) and the condensate \( \langle H_{20} T_i \rangle \). Thus, if
a suitable subset of the latter VEVs were zero, while the former are non–zero, one
could avoid \( \nu_L − \bar{H} \) mixing (at least up to N=6 terms), while allowing for the desired
\( \bar{\nu}_R \phi \)–mixings. The catch is that one must, of course, ensure that he desired pattern
of VEVs is consistent with the \( F \) and \( D \)–flat conditions, which is highly non–trivial.
This is what we proceed to do in the following.
The cubic level superpotential and the anomalous as well as the anomaly free, \( U(1) \) combinations are given in ref. [13]. As an example, we find a solution to the \( F \) and \( D \) cubic level flatness constraints with the following set of fields

\[
\{ \bar{V}_2, V_3, H_{18}, H_{23}, H_{25}, \Phi_{45}, \bar{\Phi}_1, \phi_2, \phi_3, \phi_23, \phi_{13}, \xi_1 \},
\]

having non–zero VEVs and all other fields have vanishing VEV. With this set of fields the general solution is

\[
\begin{align*}
|\langle H_{23} \rangle|^2 &= |\langle H_{18} \rangle|^2 - |\langle \Phi_{23} \rangle|^2 - \frac{1}{6} |\langle V_3 \rangle|^2 \\
|\langle H_{25} \rangle|^2 &= |\langle \Phi_{23} \rangle|^2 + \frac{1}{6} |\langle V_3 \rangle|^2 \\
|\langle \Phi_{45} \rangle|^2 &= 3 \frac{g^2}{16 \pi^2} \frac{1}{2 \alpha'} + |\langle H_{18} \rangle|^2 - \frac{1}{10} |\langle V_3 \rangle|^2 \\
|\langle \phi_{13} \rangle|^2 &= \frac{g^2}{16 \pi^2} \frac{1}{2 \alpha'} - \frac{1}{5} |\langle V_3 \rangle|^2 \\
|\langle \phi_2^+ \rangle|^2 &= \frac{g^2}{16 \pi^2} \frac{1}{2 \alpha'} - \frac{8}{15} |\langle V_3 \rangle|^2 \\
|\langle \phi_3^- \rangle|^2 &= \frac{g^2}{16 \pi^2} \frac{1}{2 \alpha'} - \frac{1}{30} |\langle V_3 \rangle|^2 \\
|\langle \bar{\phi}_1 \rangle|^2 &= \frac{g^2}{16 \pi^2} \frac{1}{2 \alpha'} - \frac{8}{15} |\langle V_3 \rangle|^2 \\
|\langle V_2 \rangle|^2 &= |\langle \phi_2 \rangle|^2 \\
\langle \xi_1 \rangle &= - \frac{\langle \Phi_{23} \rangle \langle H_{25} \rangle}{\langle H_{23} \rangle}
\end{align*}
\]

In this solution the VEVs of three fields, \( \{ V_3, \phi_{23}, H_{18} \} \) remain as free parameters, which are restricted to give a positive definite solution for the set of \( D \)–term equations. Fixing those VEVs the above solution gives a qualitatively realistic fermion mass texture [19]. As noted above, the model yields altogether four pairs of electroweak Higgs–like doublets \( \{ h_1, h_2, h_3, h_{45} \} \) and \( \{ \bar{h}_1, \bar{h}_2, h_3, h_{45} \} \) (see sets II and III). It has been shown [21] that only one pair – i.e. \( \bar{h}_1 \) or \( \bar{h}_2 \) and \( h_{45} \), can remain light, while others acquire superheavy or intermediate masses. Assuming that the light electroweak doublets consist of \( \bar{h}_1 \) and \( h_{45} \) we observe that with this solution all the neutrino–Higgsino mixing terms in Eq. (8) vanish while the seesaw terms in the neutrino mass matrix are preserved. We checked with the aid of a computer program
that with this solution for the pattern of VEVs (Eq. (7)), surprisingly, the $\nu_L - \tilde{H}$ mixing terms are not induced even up to $N = 14$. One is led to suspect that very likely a discrete symmetry is left unbroken which prevents $\nu_L - \tilde{H}$ mixing to all orders.

Even if the mixing is induced at $N = 15$, however, we see that it would lead to a mixing mass $\delta m_{\nu_H} \sim (\Lambda_H/M_{st})^2(\langle \phi \rangle/M_{st})^{10}(\langle \phi \rangle)$, where $\Lambda_H$ denotes the confinement scale of the hidden $SU(5)_H$ and $\langle \phi \rangle$ is a weighted mean of the VEVs of fields listed in Eqs. (8–16). For plausible values of $\langle \phi \rangle/M_{st} \sim (1/10 - 1/20)$, $\Lambda_H \sim 10^{12} - 10^{13}$GeV and $M_{st} \sim 10^{18}$GeV we see that $\delta m_{\nu_H} \sim (10^{-10} - 10^{-12})(10^{-13})(5 \times 10^{16}$GeV) $\sim (1/2$KeV to $1/2$MeV). Thus we see that the symmetries of the string theory, together with an allowed pattern of VEVs, are powerful enough to provide a suppression of $\delta m_{\nu_H}$ by as much as even 23 orders. This sort of induced $\delta m_{\nu_H}$ is of course perfectly compatible with the observational limits of $\delta m_{\nu_H} < 1$MeV (or even 10KeV). As noted above, while $\delta m_{\nu_H}$ is so strongly suppressed, $\bar{\nu}_R \phi - $ mixing terms are still allowed and are given by $N = 6, 7$ and 8 terms for $i = 1, 2$ and 3, mentioned before. These lead to acceptable masses for the light $\nu_i$s. A rough estimate (using Eq. (5)) yields: $m(\nu_{\tau_L}) \sim 10^{-5}$eV $> m(\nu_{\mu_L}) \geq m(\nu_{e_L})$. In this case, if $\nu_L - \tilde{H}$ mixing mass is of order $10$KeV $- 1$MeV, its contribution to $\nu_L$ masses would dominate and could yield observable masses for the light neutrinos. The solution exhibited above thus provides an example that string models can indeed yield qualitatively realistic phenomenology, while suppressing the dangerous neutrino–Higgsino mixing terms.

We would like to point out the important function of the exotic states of class (V) in the solution to the neutrino–Higgsino mixing problem. These states arise from sectors which break the $SO(10)$ symmetry to $SU(3) \times SU(2) \times U(1)^2$ [20]. While they carry the standard charges under the standard model gauge group (and indeed are Standard Model singlets) they carry non–standard charges under the $U(1)_Z$ symmetry which is embedded in $SO(10)$ and is orthogonal to the Standard Model weak hypercharge. It is precisely due to this fact, as well as due to the charges of these states under the extra $U(1)$ symmetries, exhibited in Eq. (3), which provides the discrete symmetry that are needed to suppress the neutrino–Higgsino mixing terms.

To illustrate this point further, we would like to mention in passing an alternative
scenario. In the solution above, to obtain \( D_{U(1)_{Z'}} = 0 \) we used the fields, \( H_{23}, H_{25} \) and the field with the opposite \( U(1)_{Z'} \) charge, \( H_{18} \). The field \( H_{18} \) is the one which, combined with \( H_{20} \), gives the \( U(1)_{Z'} \) charge of \( N^c_L \) and may therefore induce the dangerous neutrino–Higgsino mixing terms. Suppose that instead of assigning a VEV to \( H_{18} \) we give a VEV to \( H_{20} \), or \( H_{13} \), which carry the same charge under \( U(1)_{Z'} \). These fields transform under the hidden non–Abelian \( SU(5)_H \) and \( SU(3)_H \) gauge groups, respectively, and would break either of those gauge groups to a subgroup. In order to form a term which is invariant under all the gauge symmetries, a nonrenormalizable neutrino–Higgsino mixing term must contain fields with the quantum numbers of \( H_{18} \) and \( \bar{T} \). Therefore, if we impose that all the fields with the quantum numbers similar to \( H_{18} \) have vanishing VEVs then there is a residual local discrete symmetry which forbids the neutrino–Higgsino mixing terms to all orders of non–renormalizable terms. The vanishing of the D–term equations for \( U(1)_{B-L} \) and \( U(1)_{T_{3R}} \) and all the other \( U(1) 's \) in the observable sector can still be satisfied with this choice of VEVs. In our superstring model we find, however, that in this case the D–term equation for one of the hidden \( U(1) \) gauge groups cannot vanish (see Eq (3)). We note, though, that this in itself may not be a disaster as the communication to the observable sector may produce sufficiently small supersymmetry breaking. Also the remaining freedom in the solution of the D–term equations, as exhibited in Eqs. (8-16) may allow the VEV of \( H_{20} \) to be suppressed relative to the other VEVs, thus allowing the actual scale of SUSY breaking in the hidden sector to be suppressed relative to \( M_{st} \). We leave the investigation of this possibility to future work. This scenario illustrates the crucial role of the exotic states with “fractional” \( U(1)_{Z'} \) charge in suppressing the dangerous neutrino–Higgsino terms.

In this paper we examined the problem of neutrino–Higgsino mixing in superstring derived models. We stress that such a mixing is expected to arise in any theory that aims at unification of gravity with the gauge interactions. This mixing must be suppressed by at least 19 orders of magnitude, compared to the GUT–scale, to conform with observations. Therefore, neutrino–Higgsino mixing, like proton stability, provides one of the most stringent constraints on the validity of any gravity
unified theory. Furthermore, it is seen that the inclusion of the expected gravitational
effects, via the inclusion of the higher order nonrenormalizable terms, affects in a cru-
cial way the phenomenology of the unified models. We illustrated, in a specific string
model, that string theory can indeed produce qualitatively realistic solutions, while
the dangerous neutrino–Higgsino mixing terms are sufficiently suppressed. In the so-
lutions that we examined the exotic “stringy” states play a crucial role. These exotic
states, of a uniquely stringy origin, may have additional profound phenomenological
implications which merit further investigation.

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| $F$ | SEC | $SU(3)_C \times SU(2)_L$ | $Q_C$ | $Q_L$ | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ | $Q_6$ | $SU(5)_H \times SU(3)_H$ | $Q_7$ | $Q_8$ |
|-----|-----|-----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----------------|-----|-----|
| $L_1$ | $b_1$ | (1,2) | $-\frac{3}{2}$ | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | (1,1) | 0 | 0 |
| $Q_1$ | | (3,2) | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | (1,1) | 0 | 0 |
| $d_1$ | | (3,1) | $-\frac{3}{2}$ | $-1$ | $\frac{1}{2}$ | 0 | 0 | $-\frac{1}{2}$ | 0 | 0 | (1,1) | 0 | 0 |
| $N_1$ | | (1,1) | $-\frac{3}{2}$ | $-1$ | $\frac{1}{2}$ | 0 | 0 | $-\frac{1}{2}$ | 0 | 0 | (1,1) | 0 | 0 |
| $u_1$ | | (3,1) | $-\frac{1}{2}$ | 1 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | (1,1) | 0 | 0 |
| $e_1$ | | (1,1) | $\frac{3}{2}$ | 1 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | (1,1) | 0 | 0 |
| $L_2$ | $b_2$ | (1,2) | $-\frac{3}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | (1,1) | 0 | 0 |
| $Q_2$ | | (3,2) | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | (1,1) | 0 | 0 |
| $d_2$ | | (3,1) | $-\frac{1}{2}$ | $-1$ | 0 | $\frac{1}{2}$ | 0 | 0 | $-\frac{1}{2}$ | 0 | (1,1) | 0 | 0 |
| $N_2$ | | (1,1) | $\frac{3}{2}$ | $-1$ | 0 | $\frac{1}{2}$ | 0 | 0 | $-\frac{1}{2}$ | 0 | (1,1) | 0 | 0 |
| $u_2$ | | (3,1) | $-\frac{1}{2}$ | 1 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | (1,1) | 0 | 0 |
| $e_2$ | | (1,1) | $\frac{3}{2}$ | 1 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | (1,1) | 0 | 0 |
| $L_3$ | $b_3$ | (1,2) | $-\frac{3}{2}$ | 0 | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | (1,1) | 0 | 0 |
| $Q_3$ | | (3,2) | $\frac{1}{2}$ | 0 | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | (1,1) | 0 | 0 |
| $d_3$ | | (3,1) | $-\frac{1}{2}$ | $-1$ | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $-\frac{1}{2}$ | (1,1) | 0 | 0 |
| $N_3$ | | (1,1) | $\frac{3}{2}$ | $-1$ | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $-\frac{1}{2}$ | (1,1) | 0 | 0 |
| $u_3$ | | (3,1) | $-\frac{1}{2}$ | 1 | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | (1,1) | 0 | 0 |
| $e_3$ | | (1,1) | $\frac{3}{2}$ | 1 | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | (1,1) | 0 | 0 |
| $h_1$ | NS | (1,2) | 0 | $-1$ | 1 | 0 | 0 | 0 | 0 | 0 | (1,1) | 0 | 0 |
| $h_2$ | | (1,2) | 0 | $-1$ | 0 | 1 | 0 | 0 | 0 | 0 | (1,1) | 0 | 0 |
| $h_3$ | | (1,2) | 0 | $-1$ | 0 | 0 | 1 | 0 | 0 | 0 | (1,1) | 0 | 0 |
| $\Phi_{12}$ | | (1,1) | 0 | 0 | 1 | $-1$ | 0 | 0 | 0 | 0 | (1,1) | 0 | 0 |
| $\Phi_{13}$ | | (1,1) | 0 | 0 | 1 | 0 | $-1$ | 0 | 0 | 0 | (1,1) | 0 | 0 |
| $\Phi_{23}$ | | (1,1) | 0 | 0 | 1 | $-1$ | 0 | 0 | 0 | 0 | (1,1) | 0 | 0 |
| $h_{45}$ | $b_1 + b_2 + \alpha + \beta$ | (1,2) | 0 | $-1$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 0 | 0 | 0 | (1,1) | 0 | 0 |
| $D_{45}$ | | (3,1) | $-1$ | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 0 | 0 | 0 | (1,1) | 0 | 0 |
| $\Phi_{45}$ | | (1,1) | 0 | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-1$ | 0 | 0 | 0 | (1,1) | 0 | 0 |
| $\Phi_{1}^{\pm}$ | | (1,1) | 0 | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | $\pm1$ | 0 | 0 | (1,1) | 0 | 0 |
| $\Phi_{2}^{\pm}$ | | (1,1) | 0 | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 0 | $\pm1$ | 0 | (1,1) | 0 | 0 |
| $\Phi_{3}^{\pm}$ | | (1,1) | 0 | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 0 | 0 | $\pm1$ | (1,1) | 0 | 0 |

Table 1: Massless states which transform solely under the observable gauge group. $Q_C = 3/2(B - L)$ and $Q_L = 2T_{3R}$. In the NS and the $b_1 + b_2 + \alpha + \beta$ sectors contain also the conjugate states ($\bar{h}_1$, etc.). The NS sector contains additional three singlet states, $\xi_{1,2,3}$, which are neutral under all the $U(1)$ symmetries.
| $F$ | SEC | $SU(3)_C \times SU(2)_L$ | $Q_C$ | $Q_L$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ | $Q_6$ | $SU(5)_H \times SU(3)_H$ | $Q_7$ | $Q_8$ |
|-----|-----|----------------------|-------|-------|-------|-------|-------|-------|-------|----------------------|-------|-------|
| $V_1$ | $b_1 + 2\gamma+$ | (1, 1) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (1, 3) | $-\frac{1}{2}$ | $\frac{5}{2}$ |
| $\tilde{V}_1$ | (1, 1) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (1, 3) | $\frac{1}{2}$ | $-\frac{5}{2}$ |
| $T_1$ | (1, 1) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (5, 1) | $-\frac{1}{2}$ | $-\frac{3}{2}$ |
| $\tilde{T}_1$ | (1, 1) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (5, 1) | $\frac{1}{2}$ | $3\frac{1}{2}$ |
| $V_2$ | $b_2 + 2\gamma+$ | (1, 1) | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | (1, 3) | $-\frac{1}{2}$ | $\frac{5}{2}$ |
| $\tilde{V}_2$ | (1, 1) | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | (1, 3) | $\frac{1}{2}$ | $-\frac{5}{2}$ |
| $T_2$ | (1, 1) | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | (5, 1) | $-\frac{1}{2}$ | $-\frac{3}{2}$ |
| $\tilde{T}_2$ | (1, 1) | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | (5, 1) | $\frac{1}{2}$ | $3\frac{1}{2}$ |
| $V_3$ | $b_3 + \gamma+$ | (1, 1) | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 | (1, 3) | $-\frac{1}{2}$ | $\frac{5}{2}$ |
| $\tilde{V}_3$ | (1, 1) | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 | $\frac{1}{2}$ | (1, 3) | $\frac{1}{2}$ | $-\frac{5}{2}$ |
| $T_3$ | (1, 1) | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 | $\frac{1}{2}$ | (5, 1) | $-\frac{1}{2}$ | $-\frac{3}{2}$ |
| $\tilde{T}_3$ | (1, 1) | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 | $\frac{1}{2}$ | (5, 1) | $\frac{1}{2}$ | $3\frac{1}{2}$ |

Table 2: Massless states from the sectors $b_j + 2\gamma$. $Q_C = 3/2(B - L)$ and $Q_L = 2T_{3R}$. 
| $F$ | SEC | $SU(3)_C \times SU(2)_L$ | $Q_C$ | $Q_L$ | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ | $Q_6$ | $SU(5)_H \times SU(3)_H$ | $Q_7$ | $Q_8$ |
|-----|-----|------------------------|-------|-------|-------|-------|-------|-------|-------|-------|------------------------|-------|-------|
| $H_{13}$ | $b_1 + b_3+$ | (1,1) | $-\frac{3}{4}$ | $\frac{1}{2}$ | $-\frac{1}{4}$ | $-\frac{3}{4}$ | $0$ | $0$ | $0$ | $0$ | (1,3) | $\frac{3}{4}$ | $\frac{5}{4}$ |
| $H_{14}$ | $\alpha \pm \gamma+$ | (1,1) | $\frac{3}{4}$ | $-\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $0$ | $0$ | $0$ | $0$ | (1,3) | $-\frac{3}{4}$ | $-\frac{5}{4}$ |
| $H_{15}$ | (I) | (1,2) | $-\frac{3}{4}$ | $-\frac{1}{2}$ | $-\frac{1}{4}$ | $\frac{1}{4}$ | $-\frac{1}{4}$ | $0$ | $0$ | $0$ | (1,1) | $-\frac{1}{4}$ | $-\frac{15}{4}$ |
| $H_{16}$ | (I) | (1,2) | $\frac{3}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $0$ | $0$ | $0$ | (1,1) | $\frac{1}{4}$ | $\frac{15}{4}$ |
| $H_{17}$ | (1,1) | $\frac{1}{4}$ | $-\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $0$ | $0$ | $0$ | $0$ | (1,1) | $-\frac{1}{4}$ | $-\frac{15}{4}$ |
| $H_{18}$ | (1,1) | $\frac{3}{4}$ | $-\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $0$ | $0$ | $0$ | $0$ | (1,1) | $\frac{1}{4}$ | $\frac{15}{4}$ |
| $H_{19}$ | $b_2 + b_3+$ | (1,1) | $-\frac{3}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $-\frac{1}{4}$ | $\frac{3}{4}$ | $0$ | $0$ | $0$ | (5,1) | $-\frac{1}{4}$ | $\frac{9}{4}$ |
| $H_{20}$ | $\alpha \pm \gamma+$ | (1,1) | $\frac{3}{4}$ | $-\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $0$ | $0$ | $0$ | (5,1) | $\frac{1}{4}$ | $-\frac{9}{4}$ |
| $H_{21}$ | (I) | (3,1) | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $0$ | $0$ | $0$ | (1,1) | $-\frac{1}{4}$ | $-\frac{15}{4}$ |
| $H_{22}$ | (I) | (3,1) | $-\frac{1}{4}$ | $-\frac{1}{2}$ | $-\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $0$ | $0$ | $0$ | (1,1) | $\frac{1}{4}$ | $\frac{15}{4}$ |
| $H_{23}$ | (1,1) | $\frac{3}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $0$ | $0$ | $0$ | $0$ | (1,1) | $\frac{1}{4}$ | $\frac{15}{4}$ |
| $H_{24}$ | (1,1) | $\frac{3}{4}$ | $-\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $-\frac{1}{4}$ | $0$ | $0$ | $0$ | $0$ | (1,1) | $-\frac{1}{4}$ | $-\frac{15}{4}$ |
| $H_{25}$ | (1,1) | $-\frac{3}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $-\frac{1}{4}$ | $0$ | $0$ | $0$ | $0$ | (1,1) | $-\frac{1}{4}$ | $-\frac{15}{4}$ |
| $H_{26}$ | (1,1) | $\frac{3}{4}$ | $-\frac{1}{2}$ | $\frac{1}{4}$ | $-\frac{3}{4}$ | $\frac{1}{4}$ | $0$ | $0$ | $0$ | $0$ | (1,1) | $\frac{1}{4}$ | $\frac{15}{4}$ |

Table 3: The exotic massless states from the sectors $b_1 + b_3 + \alpha \pm \gamma + (I)$ and $b_2 + b_3 + \alpha \pm \gamma + (I)$. 