A reliability evaluation method for RV reducer by combining multi-fidelity model and Bayesian updating technology

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Abstract. RV (rotating vector) reducer is a two-stage transmission mechanism composed of involute planetary gear transmission and cycloid gear transmission with small tooth difference. It is widely used in the transmission mechanism of industrial robots and CNC machine tools. Nonlinear dynamics of gear system is always an important and difficult problem, the randomness of manufacturing and assembly makes the dynamic response become more complex. In this paper, the stochastic dynamic response analysis and dynamic reliability estimation method is proposed for RV reducer. In this method, firstly, considering the uncertainties of manufacturing error and assembly error, the dynamic transmission performance is analyzed respectively using physical prototype test, virtual prototype simulation and theoretical model. Secondly, multifidly model and Bayesian theory are combined to estimate the statistical distribution and distribution parameters of the dynamic transmission error. Finally, the reliability of RV reducer is estimated, and the effects of uncertainties from manufacturing error and assembly error on the transmission accuracy are discussed.

1. Introduction
RV reducer is widely used in the transmission mechanism of industrial robots and CNC machine tools, with the advantages of smaller volume, larger torsional stiffness, larger transmission ratio and higher load-carrying capacities. RV reducer has become one of the three core technologies of industrial robots, to improve the performance of RV reducer, plenty of research work has been done focusing on dynamic response estimation and design optimization of RV reducer, such as load distribution analysis [1-3], dynamics analysis [4-7], design optimization [7-11], torsional stiffness calculation [12-13], reliability estimation [14-16].

As the key parts of industrial robots, RV reducer’s transmission performance directly affects the kinematic accuracy and pose precision of the end effector. Therefore, it is important to study the transmission performance of RV reducer. Transmission error of RV reducer refers to the difference between the actual output angle and theory output angles. Static transmission error of no load is widely used as a basic indicator to evaluate the performance of RV reducer, therefore, a lot of researches have been done focusing on the static transmission error analysis for RV reducer. Blanche [17] proposed a geometric analysis method to accurately calculate the tooth clearance caused by tooth profile modification of cycloid gear, further discussed the rotation error of cycloid gear drive quantitatively, and found that both the backlash and the
torque ripple were inherent periodic functions of the input crank angle. Teruaki [18] established an equivalent error model to analyze the transmission error of RV reducer based on mass-spring model ignoring the elastic deformation, and the effects of manufacturing errors, assembly clearances on transmission errors were analyzed. Tan [19] established a kinematics model for RV reducer, studied the influence of cycloid gear with different tooth profile to the transmission performance of RV reducer. In addition, transmission error test systems were established to estimate the transmission accuracy of RV reducer [20].

In practical application, RV reducers used in industrial robots are working under various load conditions, therefore, the dynamic performance study of RV reducer under various loads is an important research topic. He [21] and Shan [22] established a dynamical transmission analytical model for RV reducer based on Teruaki’s equivalent error model, and calculated the dynamic transmission error by numerical method. As Teruaki’s equivalent error model did not consider the elastic deformation, to improve the analysis accuracy, Li [23] proposed a meshing stiffness calculation method considering the influence of tooth profile modification and eccentricity error, and discussed the influence of these factors on the loaded transmission error. Li [24] established a kinematics model of RV reducer using Adams software, and based on the orthogonal test method, the influence of various factors on the transmission error of RV gear reducer was analyzed.

Engineering applications show that the randomness of manufacturing errors, assembly clearances and operating conditions have significant influence to the transmission performance and service life of RV reducer. Therefore, the uncertainty of influencing factors should be considered in the transmission accuracy analysis. Lin [25] presented a method for kinetic error analysis and tolerance design of cycloid gear reducers considering the uncertainties of manufacturing error and tolerance. Han [26] adopted Sobol method to analyze the global sensitivity of transmission accuracy for RV reducer by considering nonlinear factors and uncertainty factors (such as manufacturing error, assembly error, bearing clearance, etc.).

Based on the mentioned researches, there are three main methods for the transmission error analysis of RV reducer, (1) transmission error test by physical prototype; (2) transmission process simulation based on virtual prototype, such as multi-body dynamics simulation based on ADAMS; (3) transmission error theoretical analysis model based on transmission principle, such as equivalent error model. Different research methods have different advantages and disadvantages: physical prototype test can reflect the practical operation situation better, but usually has longer periods, higher cost and smaller data. Multi-body dynamics simulation technology can solve these problems, however, the establishment of complex multi-body dynamics model is still a very complex work. The theoretical analysis model can improve the analysis efficiency with lower calculation accuracy. Therefore, this paper proposes a method by integrating the three kinds of models to analyze the dynamic transmission accuracy. Multi-fidelity techniques have been widely developed to deal with data from multiple models. With a long and successful history in computational science and engineering, several multi-fidelity frameworks have been proposed, such as multi-fidelity Monte Carlo [27], recursive difference estimator [28] and approximate control variate estimators [29] etc. In this paper, based on the properties of three transmission error models, a multi-fidelity model based on Bayesian updating technology is proposed to analyze the uncertainty of loaded dynamic transmission error.

The rest of this paper is structured as follows: Section 2 introduces the structure and operation principle of RV reducer. In Section 3, transmission error analysis results from physical prototype test, multi-body dynamics simulation and theoretical analysis model are presented. Section 4 presents the stochastic characteristics of transmission error by combining the three kinds of data, and the reliability estimation results are discussed. In Section 5, some conclusions are drawn.

2. Structure and operation principle of RV reducer

2.1 Transmission error analysis

RV reducer is composed of the first stage involute cylindrical gear planetary reducer and the
second stage cycloidal-pin wheel reducer [30]. It is a closed differential gear mechanism, as shown in figure 1. In the first stage, the driving sun gear is connected with the input shaft, and the involute center gear rotates driving \( n_c \) planetary gears arranged in \( 2\pi / n_c \) around the center gear axis and rotating in a counter clockwise direction. The \( n_c \) crankshafts are connected with the planetary gears and rotate at the same speed. In the second stage, cycloidal gears with a phase difference of \( 2\pi / n_c \) are hinged on the \( n_c \) crankshafts and mesh with the fixed pin-wheel when its axis revolves around the axis of the pin-wheel. The output mechanism is driven by \( n_c \) pairs of crankshaft support bearings installed on it, and transmits the rotation vector on the cycloid gear with a speed ratio of 1:1.

![Figure 1](image)

**Figure 1.** The transmission principle of the RV reducer.

Taking RV-20E reducer as an example, the design parameters of RV-20E reducer are presented in table 1.

| Description                             | Symbol | Value |
|-----------------------------------------|--------|-------|
| Number of input gear teeth              | \( Z_s \) | 20    |
| Number of planetary gear teeth          | \( Z_{sp} \) | 60    |
| Module of planetary gear                | \( m_{sp} \) | 0.6   |
| Meshing pressure angle of planetary gear| \( \alpha_{sp} \) | 20°   |
| Basic circle radius of the planetary gear/mm | \( R_{sp} \) | 36    |
| The central distance between the sun wheel and the central wheel/mm | \( R_{sc} \) | 48    |
| Number of cycloid gear teeth            | \( Z_c \) | 39    |
| Number of pin teeth                     | \( Z_p \) | 40    |
| Radius of pin wheel/mm                  | \( R_p \) | 46    |
| Eccentric distance of cycloid gear/mm   | \( R_e \) | 0.85  |
| Center circle radius of the cycloid gear/mm | \( R_d \) | 39    |

During one rotation cycle of input shaft, the transmission error of the RV reducer is defined as the relative difference between actual rotation angle and theoretical rotation angle of output shaft when the input axis rotates at any angle. In this paper, \( \theta_c \) is defined as rotation angle of input gear shaft, \( \theta_{ca} \) is defined as actual rotation angle of output disk, and \( \theta_c \) is the theoretical rotation angle of output shaft, \( \rho \) is the transmission ratio of the reducer. Then transmission error \( \varepsilon \) can be expressed as equation (1):

\[
\varepsilon = \theta_{ca} - \theta_c = \theta_{ca} - \theta_c / \rho
\]  

(1)
2.2 The uncertainty influencing factors of transmission error

The main causes of transmission error are elastic deformation under load, manufacturing error, assembly error, tooth profile modification of gear parts. The transmission error is also different when the reducer operates at different angles, so the input shaft angle $\theta_s$ is a uncertainty influencing factor. Therefore, this paper mainly studied the three influencing factors and they are (a) Component error (include machining error, assembly error, tooth profile modification of gears); (b) Theoretical output angle $\theta_c$; (c) Load $M$.

In this study, the uncertainty factors of the second stage are considered and the description of uncertainty is shown in table 2. In table 2, the eccentricity error caused by manufacturing or assembly is represented by $E\beta$, where $E$ is the magnitude of error, and $\beta$ is phase angle to describe the direction of error.

### Table 2. Uncertainty factors of transmission error.

| Sources of error and uncertainty | Uncertainty factors | Quantization of uncertainty |
|----------------------------------|---------------------|-----------------------------|
| Pin wheel radial error          | $R_h \sim N(0,0.002)$ |
| Pin wheel pitch error           | $P_h \sim N(0,0.002)$ |
| Radial error of tooth profile of cycloid gear | $R_{dA} \sim N(0,0.002)$ |
| Pitch error of tooth profile of cycloid gear | $P_{dA} \sim N(0,0.002)$ |
| Radius error of pin              | $\delta_m \sim N(0,0.001)$ |
| Eccentricity error of crank bore on the cycloid gear | $E_{\theta_p} \sim N(0,0.0167^\circ)$ |
| Eccentricity error of the CAM on the crankshaft | $\beta_{\theta_p} \sim U(0,2\pi)$ |
| Eccentricity error of the crankshaft hole on the output disk | $E_{\theta_c} \sim N(0,0.003)$ |
| Installation error of output disk | $\beta_{\theta_c} \sim U(0,2\pi)$ |
| Input speed                      | $E_{\theta_s} \sim N(0,0.015)$ |
| Load                             | $\beta_{\theta_s} \sim U(0,2\pi)$ |
| Load on output shaft             | $M \in [0,M_{max}]$ |

2.3 Transmission reliability estimation

In this paper, the transmission accuracy reliability of RV reducer is defined as the probability that the transmission error exceeds the error limit when the input shaft of the reducer rotates at any angle. Set the limit of transmission error as $\varepsilon_{lim}$ and set the transmission error as $\varepsilon$. Then the transmission accuracy reliability of RV reducer can be calculated as equation (2):

$$R = P( |\varepsilon| \leq \varepsilon_{lim}, 0 \leq \theta_s \leq 2\pi )$$

(2)

And the transmission accuracy reliability under random loading conditions can be written as equation (3).

$$R = P( |\varepsilon| < \varepsilon_{lim} \mid M = M_{max}, 0 \leq \theta_s \leq 2\pi )$$

(3)

3. Transmission error analysis

In this section, the transmission error is analyzed by physical prototype test, multi-body dynamics simulation based on ADAMS and equivalent error model respectively. The
characteristics of each method is listed in table 3.

**Table 3.** Description of model performance.

| Data sources                                      | Characteristics of data                                      |
|--------------------------------------------------|-------------------------------------------------------------|
| Physical prototype test                          | ● Higher accuracy                                           |
|                                                  | ● Test data are less                                        |
|                                                  | ● Higher cost                                              |
| Multi-body dynamics simulation based on ADAMS    | ● The data accuracy is close to the real results            |
|                                                  | ● Mid-sized amount of data                                  |
|                                                  | ● The establishment and solving of dynamic equation are complex |
| Equivalent error model                           | ● Low analysis accuracy                                     |
|                                                  | ● Low cost                                                  |
|                                                  | ● Many test data                                            |

(1) The physical prototype test data is collected under no-load condition. Therefore, the test data cannot reflect the influence of different load conditions on transmission error and the influence of manufacturing error and assembly error on transmission error.

(2) The transmission error data from multi-body dynamic simulation based on ADAMS is collected under different load conditions, which can simply reflect the influence of the randomness of manufacturing, assembly and load conditions.

(3) The transmission error data based on the equivalent error model is also collected under no-load condition. It can better reflect the influence of randomness of manufacturing and assembly, and cannot reflect the influence of load conditions.

The characteristics response of the three kinds of data are presented in table 4, where "×" indicates that the data cannot reflect the influence of such uncertain factors on transmission error, and "√" indicates that it can.

**Table 4.** The ability of data to respond to uncertain sexual factors.

| Data source                              | Theoretical output angle | Load situation | Manufacturing error |
|------------------------------------------|--------------------------|----------------|---------------------|
| Physical prototype test                  | √                        | ×              | ×                   |
| Multi-body dynamics simulation           | √                        | ×              | ×                   |
| Equivalent error model                   | √                        | ×              | √                   |

Because all the three kinds of data can describe the performance of transmission accuracy of RV reducer under no-load condition, it is more accurate to integrate these three kinds of data to describe the transmission error under no-load. Bayesian method is a statistical inference method based on prior information, sample information and general information. In reliability analysis, Bayesian method can deal with multi-sources, multi-levels and multi-types reliability data comprehensively. From a mathematical point of view, Bayesian method can be described intuitively as equation (4) [31-32]:

\[
p(\lambda|X) = \frac{L(X|\lambda)\pi(\lambda)}{\int_{\lambda} L(X|\lambda)\pi(\lambda)d\lambda}
\]

Where, \(\pi(\lambda)\) is the prior distribution of the parameter \(\lambda\), \(L(X|\lambda)\) is the likelihood function, \(p(\lambda|X)\) is the posterior distribution of \(\lambda\) by integrating information of \(X\).

In this paper, a dynamic transmission error evaluation method under various loads for RV reducer based on multi-fidelity model is proposed. The physical prototype transmission error test model, multi-body dynamic simulation model based on ADAMS, and the dynamic transmission error analysis model based on equivalent error model are established respectively.

3.1 Data from physical prototype test
The physical prototype transmission error test data in reference [33] shown in figure 2 is employed to analyze the transmission error of RV reducer rotating one circle under no-load condition. And the test equipment is provided in figure 3.
3.2 Data from virtual prototype simulation

In this section, dynamic transmission error data under different loads are obtained by multi-body dynamic simulation. The steps of dynamic simulation under different loads in ADAMS is shown in figure 4.

![Diagram showing the steps of dynamic simulation process in ADAMS](image)

**Figure 4.** Dynamic transmission process using ADAMS.

The simplified 3D model of RV-20E considering manufacturing error and simulation in ADAMS is displayed in figure 5, the constraints and forces between each part in multi-body dynamic simulation model are presented in table 5.

**Table 5.** Constraints and forces between components.

| Transmission parts               | Constraint types                     |
|----------------------------------|--------------------------------------|
| Input gear shaft and earth       | Revolute, Rotational motion          |
| Input gear shaft and planetary gear | Gear                                    |
| Planetary gear and crankshaft    | Fixed                                 |
| Crankshaft and cycloid gear      | Elastic bushing force                 |
| Cycloid gear and pin             | Impact (solid to solid)              |
| Output disk and earth            | Revolute, Load moment                |
| Pin tooth and earth              | Fixed                                 |

The rotating angle of input gear shaft \( \theta_1 \) can be determined by equation (5).

\[
\theta_1 = \omega t \tag{5}
\]

The output shaft conversion value of RV-20E without load rotation torque is shown in figure 5.
6. When dynamic transmission error simulation or dynamic transmission error calculation is carried out, rotational torque without load should be added to the output shaft.

**Figure 5.** Multi-body dynamic simulation of RV reducer based on ADAMS.

**Figure 6.** Rotating torque without load (Convert to output disk).

**Figure 7.** Simulation results of transmission error under different loads.
The simulation results of transmission error considering the influence of stochastic load are presented in Figure 7 and Figure 8 provides the simulation results of transmission error considering the effects of randomness of manufacturing error.

3.3 Data from theoretical analytical model

3.3.1 Equivalent error model

The ‘Equivalent error model’ is an analytical model proposed by Teruaki [18] to solve the transmission error problem under no load state. In this model, each part of the mechanism was set as rigid body, the influence of manufacturing error, small displacements, clearance and assembly error for each part are transformed to compression of a spring connect along the direction of action line between each part, which was defined as "equivalent error".

3.3.2 Establishment of dynamic equivalent model of transmission error

The equivalent error model of RV-20E as shown in Figure 9, three coordinate systems of this equivalent model are defined as follows: one stationary coordinate system \( O(X, Y) \) with the center of the sun gear as the origin. And two movable coordinate systems \( O_j(\eta_j, \xi_j) \) which take the center of mass of the \( j \)th cycloid gear \( O_j \) as origin, and the eccentric direction of cycloid gear \( \eta_j \) is taken as the positive direction. \( \phi_i = 2\pi(i-1)/2, (i=1,2) \) denotes the phase angle of the \( i \)th crankshaft, \( \psi_j = 2\pi(i-1)/2, (i=1,2) \) is the phase angle of the theoretical barycenter of \( j \)th cycloid gear.

Based on the coordinate system, the rotation angle error of planetary gear \( \Delta\theta_p \), output disk \( \Delta\theta_c \), rotation error \( \Delta\theta_0 \) and revolution error \( \Delta\theta_{doj} \) of cycloid gear can be calculated as
equation (6):

$$\begin{align*}
\Delta \theta_p &= \theta_p - \theta_p \\
\Delta \theta_c &= \theta_c - \theta_c \\
\Delta \theta_c &= \theta_c - \theta_c \\
\Delta \theta_q &= \theta_q - \theta_q \\
\Delta \theta_q &= \theta_q - \theta_q \\
\end{align*}$$

(6)

where $\theta_p$ denotes the actual rotational angle of crankshaft, and $\theta_p$ represents the theoretical rotation angle of crankshaft, $\theta_q$ and $\theta_q$ denote the actual rotation and revolution angle of cycloid gear respectively.

The dynamic transmission error of RV-20E are calculated as equations (7)-(10).

1) Dynamic equations of input gear shaft:

$$\begin{align*}
m_\alpha \ddot{x}_i + \sum_{i=1}^{n} (F_{qsp} \cos A_i) + F_{xi} &= 0 \\
m_\alpha \ddot{y}_i + \sum_{i=1}^{n} (F_{qsp} \sin A_i) + F_{yi} &= 0
\end{align*}$$

(7)

3) Dynamic equations of planetary gear ($i = 1, 2$):

$$\begin{align*}
m_\alpha [\ddot{x}_p - r_\alpha \omega^2 \cos(\theta_p + \phi) - r_\alpha \dot{\omega} \sin(\theta_p + \phi) - 2\omega \dot{y}_p] \\
- F_{\alpha p} \cos A - \sum_{j=1}^{2} (F_{chj} - F_{chj}) &= 0 \\
m_\alpha [\ddot{y}_p - r_\alpha \omega^2 \sin(\theta_p + \phi) - r_\alpha \dot{\omega} \cos(\theta_p + \phi) - 2\omega \dot{x}_p] \\
- F_{\alpha p} \sin A - \sum_{j=1}^{2} (F_{chj} - F_{chj}) &= 0 \\
\sum_{j=1}^{2} (F_{chj} \sin(\theta_p + \psi_j) + F_{chj} \cos(\theta_p + \psi_j)) - F_{\alpha p} r_{\alpha p} + J_{\alpha p} \ddot{\theta}_{\alpha p} &= 0
\end{align*}$$

(8)

4) Dynamic equations of cycloid gear ($j = 1, 2$):

$$\begin{align*}
m_\alpha [\ddot{x}_q \cos(\theta_p + \psi_j) - r_\alpha \omega^2 \cos(\theta_p + \psi_j) - \dot{r}_\alpha \dot{\omega} \sin(\theta_p + \psi_j)] \\
- 2\omega \dot{\theta}_{\alpha q} \sin(\theta_p + \psi_j) + \sum_{k=1}^{2} F_{dk} \sin A_i &= 0 \\
- \sum_{i=1}^{n} (F_{chj} \sin(\theta_p + \psi_j) + F_{chj} \cos(\theta_p + \psi_j)) &= 0 \\
m_\alpha [\ddot{y}_q \sin(\theta_p + \psi_j) - r_\alpha \omega^2 \sin(\theta_p + \psi_j) - \dot{r}_\alpha \dot{\omega} \cos(\theta_p + \psi_j)] \\
- 2\omega \dot{\theta}_{\alpha q} \cos(\theta_p + \psi_j) + \sum_{k=1}^{2} F_{dk} \sin A_i &= 0 \\
- \sum_{i=1}^{n} (-F_{chj} \cos(\theta_p + \psi_j) + F_{chj} \sin(\theta_p + \psi_j)) &= 0 \\
J_{\alpha q} \ddot{\theta}_{\alpha q} - r_{\alpha q} \sum_{i=1}^{n} (F_{chj} \sin(\theta_p + \psi_j) + F_{chj} \cos(\theta_p + \psi_j)) &= 0
\end{align*}$$

(9)

2) Dynamic equations of output disk:

$$\begin{align*}
m_\alpha \ddot{x}_q - \sum_{i=1}^{n} F_{kx} + F_{kx} &= 0 \\
m_\alpha \ddot{y}_q - \sum_{i=1}^{n} F_{kx} + F_{kx} &= 0 \\
J_{kx} \ddot{\theta}_{kx} - r_{kp} \sum_{j=1}^{2} (F_{kchj} \sin(\theta_p + \phi) - F_{kchj} \cos(\theta_p + \phi)) &= M
\end{align*}$$

(10)

The symbols in equation (7)-(10) are explained in table 6.
### Table 6. Description of the symbols in equation (7)-(10)

| Symbols  | Description                                                                 |
|----------|------------------------------------------------------------------------------|
| $A_i$    | The direction angle of the meshing line                                      |
| $m_i$    | The mass of the input gear                                                  |
| $m_{cg}$ | The mass of the cycloid gear                                                 |
| $m_{dp}$ | The mass of the output disk and planet carrier                              |
| $J_{op}$ | The rotational inertia of planetary gear & crankshaft                        |
| $J_{op}$ | The rotational inertia of output disk                                        |
| $J_{op}$ | The rotational inertia of cycloid gear                                       |
| $\omega_d$ | The theoretical angular velocity of output disk                             |
| $\omega_p$ | The theoretical angular velocity of planetary gear & crankshaft             |
| $x_s$    | The deviation of the sun gear in the $X$ direction                           |
| $y_s$    | The deviation of the sun gear in the $Y$ direction                           |
| $x_{p_s}$ | The deviation of the planetary gear in the $X$ direction                     |
| $y_{p_s}$ | The deviation of the planetary gear in the $Y$ direction                     |
| $x_{ca}$ | The deviation of the output disk in the $X$ direction                        |
| $y_{ca}$ | The deviation of the output disk in the $Y$ direction                        |
| $\eta_{b_s}$ | The deviation of the cycloid gear in the $\eta_s$-direction                  |
| $\alpha_{ji}$ | The included angle between the connecting line between the needle tooth and the cycloid wheel node and the $\eta_j$-axis |
| $F_{X_s}$ | The force of the support spring in $X$ direction of sun gear                 |
| $F_{Y_s}$ | The force of the support spring in $Y$ direction of sun gear                 |
| $F_{sp}$ | The force of the spring between planetary gear and sun gear                  |
| $F_{X_{ca}}$ | The force of the spring in $X$ direction between crankshaft and cycloidal gear |
| $F_{Y_{ca}}$ | The force of the spring in $Y$ direction between crankshaft and cycloidal gear |
| $F_{X_{d_1,\eta}}$ | The force of the spring in $X$ direction between crankshaft and output disk |
| $F_{Y_{d_1,\eta}}$ | The force of the spring in $Y$ direction between crankshaft and output disk |
| $F_{d_2}$ | The force of the spring between pin and cycloidal gear                       |
| $F_{X_{ca}}$ | The force of the spring in $X$ direction between output disk and pin wheel housing |
| $F_{Y_{ca}}$ | The force of the spring in $Y$ direction between output disk and pin wheel housing |

### 3.3.3 Equation solution

Equation (7)-(10) are a set of differential equations with 17 degrees of freedom, which can be written as the matrix form at:

$$m\ddot{x} + c\dot{x} + kx = \mathbf{F}$$  \hspace{1cm} (11)

Where, $m$, $c$, and $k$ are mass matrix, damping matrix and stiffness matrix respectively. And $x$ is the vector composed of small displacements and angle errors of each part, and $x = [x_s, y_s, x_{p_1}, y_{p_1}, \eta_{d_1}, \eta_{d_2}, \Delta\theta_{p_1}, \Delta\theta_{p_2}, \Delta\theta_{d_1}, \Delta\theta_{d_2}, \Delta\theta_{d_3}, \Delta\theta_{d_4}, x_{ca}, y_{ca}, \Delta\theta_{a}]$. $\mathbf{F}$ is a generalized force vector which determined by manufacturing and assembly errors, load and spring strength. The dynamic transmission accuracy model which is shown in equation (11) is a nonlinear model with multiple degrees of freedom, and Newmark method is employed to
solve this complex model which is shown in table 7.

Table 7. Dynamic equivalent model of transmission error Solving by Newmark method.

| Step 1 | Initial calculation |
|--------|---------------------|
| • Provide cycloid gear structure parameters, input torque and rotation speed |
| • Given initial time \( t \), time step \( \Delta t \), and related parameters \( \gamma \) and \( \beta \) |
| (1) Establish equivalent error model considering clearance, gear modification and eccentricity as shown in figure 9. |
| (2) Calculate the meshing stiffness matrix \( k \), mass matrix \( m \) and damping matrix \( c \). |
| (3) Provide the initial dynamic transmission error, velocity and acceleration as \( x_s, x_l, x_a \) respectively. |
| (4) Calculate the related constants: |
| \( \alpha_0 = 1 / \beta \Delta t^2 \), \( \alpha_1 = \gamma / \beta \Delta t \), \( \alpha_2 = 1 / 2 \beta - 1 \), \( \alpha_3 = \gamma / \beta - 1 \), \( \alpha_4 = \alpha_3 / 2(\gamma / \beta - 2) \), \( \alpha_5 = \alpha_4 / (1 - \gamma) \), \( \alpha_6 = \gamma \Delta t \) |
| (5) Update stiffness matrix |
| \( \bar{k} = k + \alpha_0 m + \alpha_4 c \) |

Step 2 Transmission performance updating at time \( t + \Delta t \)

(1) Load updating |
| \( \bar{F}_{i+\Delta t} = F_i + m(\alpha_1 x_i + \alpha_2 \dot{x}_i + \alpha_3 \ddot{x}_i) + c(\alpha_4 x_i + \alpha_5 \dot{x}_i + \alpha_6 \ddot{x}_i) \) |

(2) Transmission error updating |
| \( \bar{F}_{i+\Delta t} = \bar{k} \dot{x}_{i+\Delta t} \) |

(3) Velocity and acceleration calculation at time \( t + \Delta t \)
| \( \dot{x}_{i+\Delta t} = \alpha_0 (x_{i+\Delta t} - x_i) - \alpha_2 \dot{x}_i - \alpha_3 \ddot{x}_i \) |
| \( \ddot{x}_{i+\Delta t} = \alpha_1 x_i + \alpha_2 \dot{x}_i + \alpha_3 \ddot{x}_i \) |

Step 3 Let \( t = t + \Delta t \), repeat Step 2 until satisfy convergence condition

The contact stiffness between multiple parts involved in the equation is calculated according to the empirical formula of gear or bearing [34-35]. A set of size error input is given randomly, and the contact stiffness coefficient is substituted into the equation for calculation.

Finally, the dynamic transmission error can be calculated by solving equation (11).

![Figure 10. Transmission error of equivalent error model.](image_url)

4. Reliability evaluation of transmission accuracy of RV reducer based on multi-model data
In this section, let \( D_t \) denote the transmission error data set from physical prototype test, \( D_s = \{ D_{s1}, D_{s2} \} \) represent the dynamic transmission error data set, \( D_c \) denote the transmission error data from theoretical model calculation, the statistics of dynamic
transmission error are analyzed using Bayesian information fusion technology by integrating the three kinds of data.

4.1 Uncertainty analysis of unload dynamic transmission error based on Bayesian method

The basic steps of applying Bayesian methods in data fusion are presented as follows:

Step 1. Determine the distribution function of transmission error \( f(\varepsilon) \) with unknown distribution parameter \( \lambda \).

Step 2. Based on the available information calculate the prior distribution of \( \lambda \).

Step 3. Provide new data and establish the likelihood function \( L(D|\lambda) \).

Step 4. Update the posterior distribution of \( \lambda \).

Step 5. According to the posterior distribution of \( \lambda \) and distribution function of transmission error \( f(\varepsilon) \), evaluate the reliability of the transmission accuracy of RV reducer.

Assuming that the dynamic transmission error in no-load is \( \varepsilon_0 \), the statistics analysis of the dynamic transmission error in no-load based on Bayesian method can be estimated, the analysis steps are displayed in figure 11.

(1) Transmission data analysis

(a) \( D_T \): As shown in figure 2, the transmission error data of a RV reducer rotating for one cycle under no-load condition.

(b) \( D_{S0} \): According to the uncertainty information in table 3, 30 groups of error combinations for different parts are selected using design of experiments, further 30 corresponding RV-20E multi-body dynamics simulation models are established.

(c) \( D_C \): 10000 samplings are selected using random sampling method, and the transmission errors are analyzed by equivalent error model.

![Figure 11. Steps of uncertainty analysis of dynamic transmission error in no-load.](image-url)
Figure 12. Frequency statistics and fitting of probability density function.

(2) Initial distribution of transmission error

Interval frequency statistics are performed on data $D_c$ to obtain the probability density function of transmission error as shown in figure 12:

From figure 12, we can find that the probability distribution of transmission errors calculated by the equivalent error model is a skewed distribution. In this paper, the sum of two normal distribution functions as in equation (12) is used to describe this distribution, and the distribution parameters $\lambda = [a_1, a_2, \mu_1, \mu_2, \sigma_1, \sigma_2]$, in equation (12) are fitted using maximum likelihood estimate method.

$$f_c(e_i) = \frac{a_1}{\sqrt{2\pi \sigma_1}} \exp \left[ -\frac{(e_i - \mu_1)^2}{2\sigma_1^2} \right] + \frac{a_2}{\sqrt{2\pi \sigma_2}} \exp \left[ -\frac{(e_i - \mu_2)^2}{2\sigma_2^2} \right]$$  \hspace{1cm} (12)

(3) Solution of posterior distribution $P(\lambda | D_T)$

The normal-inverse gamma distribution is the joint conjugate prior distribution of the distribution parameter of normal distribution [36]. It is difficult to estimate 6 parameters for normal-inverse gamma distribution, therefore, as shown in figure 11, the joint uniform distribution $\pi(\lambda)$ of parameters $\lambda$ in 95% confidence interval $[\lambda_{l}, \lambda_{u}]$ when fitting $f_c(e_i)$ is used as the prior distribution to calculate $P(\lambda | D_T)$.

The likelihood function of $\lambda$ is:

$$L(\lambda | D_T) = \prod_{i=1}^{n} \left\{ \frac{a_1}{\sqrt{2\pi \sigma_1}} \exp \left[ -\frac{(e_{T,i} - \mu_1)^2}{2\sigma_1^2} \right] + \frac{a_2}{\sqrt{2\pi \sigma_2}} \exp \left[ -\frac{(e_{T,i} - \mu_2)^2}{2\sigma_2^2} \right] \right\}$$  \hspace{1cm} (13)

Then $P(\lambda | D_T)$ can be solved by substituting $L(\lambda | D_T)$ and $\pi(\lambda)$ into equation (4).

(4) Solution of the PDF of transmission error $f(e_0)$ under no-load

As shown in equation (14), take $P(\lambda | D_T)$ as the prior distribution and $D_{SI}$ as the sample data for the maximum posterior estimation, the value of $\hat{\lambda}_{MAP}$ can be obtained, and substitute it into equation (12), the probability density function of dynamic transmission error $f(e_0)$ under no-load can be calculated, the fusion result is shown in figure 13.

$$\hat{\lambda}_{MAP} = \arg\max_{\lambda} [L(\lambda | D_{SI}) P(\lambda | D_T)]$$  \hspace{1cm} (14)
Figure 13. Results of data fusion.

4.2 Dynamic transmission error analysis under various loads
Multi-body dynamics simulation data $D_{sm}$ is used to describes the performance of transmission error under stochastic load. The 30 groups of DOE models are used for simulation analysis at load $M = [20, 40, 60, 80, 100, 120, 140, 160, 180, 230, 280]$ Nm. The statistical characteristics of simulation data are shown in figures 14-15.

It can be seen from figure 17 that the effect of load on the variance of transmission error is small, therefore, this paper assumes that the variance of transmission error is constant, and the mean value varies with load. Let $\mu[\varepsilon(M)]$ denote the mean value of dynamic transmission error, the distribution of dynamic transmission error can be written in equation (15).

$$f[\varepsilon(M)] = \mu[\varepsilon(M)] + N(0, \sigma^2_0)$$

(15)

where, $\sigma^2_0$ is the variance of $f(\varepsilon_0)$.

The comparison between the mean value of the prediction model and the mean value of the simulation data is shown in figure 16, and it can be seen that all simulation values mainly fall within the 95% confidence interval, so this model can be used for fitting of $\mu[\varepsilon(M)]$.

The probability density function of transmission error can be formulated as equation (16)

$$f[\varepsilon(M)] = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left\{ -\frac{(\varepsilon - \mu[\varepsilon(M)])^2}{2\sigma^2_n} \right\}$$

(16)
Figure 16. The comparison between the mean value of the prediction model and the mean value of the simulation data.

4.3 Reliability estimation
Combining equation (3) and equation (16), the reliability of transmission accuracy can be calculated by equation (17)

\[
R(\varepsilon) = \int_{-\varepsilon}^{\varepsilon} f(\varepsilon[\max(M)])d\varepsilon
\] (17)

Take the transmission error limit \( \varepsilon_{\text{Lim}} \) as 60” and load \( M \) as 0~280(Nm) to calculate the reliability of transmission accuracy, the reliability in this working condition is 0.968.

5 Conclusions
In this paper, the main contributions include (1) the uncertainties from manufacturing, assembly and load are discussed, and their effects on the performance of RV reducer are estimated by physical prototype test, virtual prototype simulation and theoretical model respectively. (2) the characteristics of the three kinds of data are discussed, then based on Bayesian updating technology, the three multi-fidelity data is integrated, and a transmission accuracy reliability estimation model is proposed. Based on the estimation results, to improve the performance of RV reducer, the uncertainties should be considered, and reliability based design optimization and intelligent assemble for RV reducer will be focused in the further study.

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