Supersymmetry:
From the Fermi Scale to the Planck Scale

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Abstract
The physics of supersymmetry is reviewed from the perspective of physics at ever increasing energies. Starting from the minimal supersymmetric extension of the Standard Model at the electroweak scale, we proceed to higher energies seeking to understand the origin of the many model parameters. Supersymmetric grand unification, supergravity, and superstrings are introduced sequentially, and their contribution to the sought explanations is discussed. Typical low-energy supersymmetric models are also presented, along with their possible experimental consequences via direct and indirect processes at high-energy physics experimental facilities.

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1 Introduction

1.1 The Standard Model and beyond

With the commissioning of Fermilab’s Tevatron proton-antiproton collider in 1988 and CERN’s LEP electron-positron collider in 1989, the Standard Model of the strong and electroweak interactions has received overwhelming experimental support, with some predictions checked to accuracies as high as 0.1%. Among the several experimental measurements, perhaps the most impressive ones have been the precise determination of the masses of the $W$ ($M_W = 80.41 \pm 0.18\text{ GeV}$) and $Z$ ($M_Z = 91.1884 \pm 0.0022\text{ GeV}$) gauge bosons which mediate the electroweak interactions, and the related weak mixing angle ($\sin^2\theta_W = 0.23186 \pm 0.00034$), as well as the number of light neutrino species ($N_\nu = 2.991 \pm 0.016$). This body of data agrees with the predictions of the Standard Model in basically every instance, and helps “clean up” the mass range below $\frac{1}{2}M_Z$, where no new physics has been observed. Such degree of purity of the Standard Model below the electroweak scale needs to be naturally accommodated by any of its proposed extensions.

The most recent confirmation of the Standard Model has been the discovery at the Tevatron (1995) of the long sought-for top quark by the CDF and D0 Collaborations. The mass of the top quark, which early on had been theorized to be as low as 20 GeV, has experimentally turned out to be some ten times larger. The Standard Model, in fact, does not predict the mass of the top quark, or the mass of any other quark or lepton, or the quark mixing angles. Similarly there is no explanation for the observed number of fermion families (three), the quantization of the electric charge, the magnitude of the weak mixing angle, the dynamical origin of the electroweak symmetry breaking, the mass of the Higgs boson, etc. The Standard Model also lacks some appealing features, such as neutrino masses, unified strong and electroweak symmetry and gravity, matter instability (proton decay), and a cold dark matter candidate. Finally, the Standard Model has some subtle problems when extrapolated to very high energies: the electromagnetic (QED) and top-quark Yukawa couplings encounter a Landau pole ($i.e.$, they become very large) at sufficiently high energies. These various shortcomings of the Standard Model require theoretical explanation, although they do not detract from the fact that the Standard Model is an excellent effective theory for energies $\lesssim \mathcal{O}(100\text{ GeV})$.

In this review we will argue that the Standard Model parameters and features are most clearly understood when the energy scale of the interactions is extrapolated to higher values ($Q \gg M_Z$). This extrapolation is postulated to uncover larger symmetries which correlate various model parameters. These larger theories are (much) more constrained than the Standard Model, and therefore have (much) greater predictive power.

The basic underlying assumptions that we make in considering different scales is that physical quantities at different mass scales ($e.g.$, the electroweak scale $Q \sim M_Z \sim 10^2\text{ GeV}$ and the Planck scale $Q \sim M_{Pl} \sim 10^{19}\text{ GeV}$) are connected in a calculable and natural way. In the realm of quantum field theory, specific relations
between physical quantities at different mass scales are required, as dictated by the renormalization group invariance of the theory. These renormalization group equations (RGEs) depend on the nature of the theory, and can be derived explicitly in all cases of interest. Starting from a weakly interacting theory at low mass scales (the Standard Model), these equations can be used to evolve the model parameters to larger mass scales. Our calculability assumption implies that the model should remain perturbative (i.e., weakly interacting) all the way up to the highest mass scales to be considered. The RGEs themselves do not guarantee this, as for example in the case of the QED gauge coupling (the fine-structure constant $\alpha$), which becomes large at very high mass scales. The calculability assumption is satisfied in this case by the existence of the Planck scale, which effectively cuts off the growth of $\alpha$.

Our naturalness assumption is also not guaranteed by the RGEs. Very heavy particles can creep into the low-energy world through their appearance in self-energy loop diagrams of fundamental scalar particles (like the Higgs boson). These diagrams have a quadratic dependence on the high-energy cutoff (a quadratic divergence), and once renormalized yield a finite contribution to the scalar mass shift proportional to, e.g., $(M_{Pl}/M_Z)^2 \sim 10^{34}$. These extremely large mass shifts can be compensated by an equally extreme and unnatural fine-tuning of the renormalized model parameters. This is the gauge hierarchy problem, which pervades any attempt at extrapolation to very high mass scales of theories with fundamental scalar particles, and violates our naturalness assumption. Two solutions to this problem have been proposed: either there are no fundamental scalar particles, or the high-energy behavior of the scalar self-energy diagrams is somehow alleviated. The first solution leads to the ideas of technicolor and compositeness, which have as a principal drawback their general lack of calculability, and therefore violate our first assumption. (Nonetheless, models based on these ideas are regularly considered, although with limited phenomenological success.)

The second solution to the gauge hierarchy problem is based on the idea of Supersymmetry. In a theory with fundamental scalars, supersymmetry tames the quadratic divergences by predicting the existence of a “superpartner” to each particle, with the same mass and gauge quantum numbers, but with spin differing by half-a-unit. The new fermionic superpartners of scalars and gauge bosons contribute to the Higgs-boson self-energy loop diagrams and, because of their different spin-statistics but same mass and gauge quantum numbers, lead to an automatic cancellation of the quadratic divergences. The scalar mass shifts vanish altogether in the limit of exact supersymmetry. However, supersymmetry cannot be an exact symmetry of Nature, since otherwise light superpartners of the quarks and leptons would have been observed. The breaking of supersymmetry manifests itself via mass splittings between superpartners. Such mass splittings contribute to the Higgs-boson mass shifts, and should not exceed $\sim O(100 \text{ GeV})$ if the gauge hierarchy problem is not to be reintroduced, otherwise our naturalness assumption would again be violated.

Further theoretical motivation for supersymmetry is found in the form of supergravity, which provides an effective description of quantum gravity at energies below the Planck scale. Also, spacetime (four-dimensional) supersymmetry is a typi-
cal prediction of string theory, whereas world-sheet (two-dimensional) supersymmetry is required in order for particles with half-integer spin to exist. Phenomenologically, supersymmetry solves the gauge hierarchy problem and gives meaning to grand unification, which agrees well with the low-energy measurements of the Standard Model gauge couplings. Supersymmetry also explains dynamically the breaking of the electroweak symmetry via radiative corrections, and predicts the existence of a light Higgs boson ($m_h \lesssim 150 \text{ GeV}$). Supersymmetric models typically provide a good candidate for cosmological (cold) dark matter: the lightest supersymmetric particle (LSP) – the neutralino. Experimentally, supersymmetric models predict the existence of many ($\gtrsim 30$) new elementary particles – the superpartners of the Standard Model particles – which should be accessible at present and near-future high-energy accelerators via distinct signatures, such as missing energy and low-background multi-leptons signals.

### 1.2 Supersymmetry and unification

Our first step away from the Standard Model entails the introduction of low-energy supersymmetry, whereby each particle in the Standard Model is accompanied by a superpartner. As such this step does not appear to accomplish much in our quest for explanation of the many Standard Model parameters. On the contrary, the number of parameters is greatly increased by the addition of more than 30 parameters needed to describe the new particle masses. Because of the nature of supersymmetry one also needs to enlarge the Higgs sector from one to two Higgs doublets. However, since supersymmetry “commutes” with the gauge interactions, no new unknown couplings are introduced. The resulting model is referred to as the Minimal Supersymmetric Standard Model (MSSM). Low-energy supersymmetry does provide a sometimes overlooked benefit in the Higgs sector. In the Standard Model the mass of the Higgs boson derives from a quartic coupling in the scalar potential, and is thus unconstrained, although considerations of a weakly interacting Higgs sector suggest that $M_H \lesssim (500 - 1000) \text{ GeV}$ [4]. In supersymmetry the quartic coupling is not an independent parameter, but rather a simple function of the gauge couplings, which are known and are not large. This prediction implies an upper bound on the lightest Higgs boson in the MSSM of $m_h \lesssim 130 \text{ GeV}$. That is, the mass of the (lightest) Higgs boson derives from the electroweak interactions, in contrast with the Standard Model, where a new sector of the theory needs to be postulated.

To make real gains in explaining the Standard Model features, we extrapolate the MSSM to higher energies. This procedure entails “running” or scaling up the model parameters (gauge and Yukawa couplings and mass parameters) by means of the RGs, hoping to uncover simple relations among the parameters at sufficiently high mass scales. Since this scaling is logarithmic, significant changes in the magnitude of the parameters requires order-of-magnitude changes in the mass scales. Using

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\[ \text{The discussion that starts here and continues through to the end of this section outlines the contents of this review and the logic that underlies it, and is graphically summarized in the form of a flow chart or road map in Fig. 1.} \]
The route to few-parameter supersymmetric models

Figure 1: Supersymmetry: From the Fermi Scale to the Planck Scale. The explicit numbers (20,4,2,0) indicate the number of new parameters required to describe the corresponding model.
the precise LEP measurements of the Standard Model gauge couplings, it has been shown that this exercise yields a convergence of the gauge couplings towards a single value at the scale $Q = M_{\text{GUT}} \sim 10^{16}$ GeV. This phenomenon is not observed in the Standard Model, which evolves differently because of its different particle content. The convergence of the gauge couplings in the MSSM is very suggestive of a larger symmetry being encountered at such large mass scales: a grand unified theory (GUT). In fact, this result has been taken by many as indirect evidence for the existence of (low-energy) supersymmetry. However, the MSSM alone does not lead to true unification since, without the new larger symmetry, the gauge couplings would diverge when extrapolated past $Q = M_{\text{GUT}}$.

If a GUT is indeed present above $Q = M_{\text{GUT}}$, then one can rephrase the unification of the gauge couplings (a bottom-up result) as a prediction for the weak mixing angle ($\sin^2 \theta_W$) and the strong coupling ($\alpha_3$) at the electroweak scale (a top-down result). The larger gauge symmetry is accompanied by new gauge bosons (which conspire to yield a single unified gauge coupling above $M_{\text{GUT}}$) and new Higgs bosons to effect the gauge symmetry breaking down to the Standard Model gauge group. Typical unified groups include $SU(5)$, $SO(10)$, and “flipped” $SU(5) \times U(1)$. The new GUT degrees of freedom have various observable consequences. One of these is the violation of baryon number, which leads to the decay of protons with a lifetime $\tau_p \gtrsim 10^{32}$ y through various channels such as $p \to e^+\pi^0$ and $p \to K^+\bar{\nu}$. The unified symmetry implies that the Standard Model particles are grouped into larger representations, and thus their properties are correlated. These correlations lead, for example, to an explanation of the charge quantization observed in the Standard Model, and to relations between the Yukawa couplings ($e.g.$, $\lambda_b = \lambda_\tau$). Another interesting consequence of the existence of a large mass scale and of our calculability assumption, is the need to restrict the Standard Model top-quark Yukawa coupling ($\lambda_t \lesssim 1$) so that it does not blow up below the unification scale. This constraint implies $m_t \lesssim 200$ GeV.

1.3 Supergravity and superstrings

Despite their successes, supersymmetric GUTs provide only relations among the Standard Model parameters, with no possibility of first-principles calculations of their actual values. (Of course, GUTs predict new phenomena beyond the Standard Model so that they can themselves be in principle tested.) Among the uncalculated parameters we have the many supersymmetric particle masses. These, in fact, are crucial to the unification program, as only if they are in the $O(100$ GeV) $- O(1$ TeV) range does one achieve unification. The needed breaking of supersymmetry is phenomenologically viable only if supersymmetry is a local symmetry (as the gauge symmetries are). Local supersymmetry necessarily involves gravitational interactions, and thus the name supergravity. In a supergravity theory the supersymmetry-breaking parameters are calculable in terms of just two functions: the Kähler function and the gauge kinetic function. Thus a great synthesis of the unknown parameters can be attained if these functions are known or can be somehow parametrized. In fact, one of the sim-
plest possible choices for the Kähler function gives universal scalar masses, i.e., the masses of the scalar superpartners (squarks, sleptons, etc) are all equal at the Planck scale. (Renormalization group evolution down to low energies breaks this degeneracy.) These “minimal” models can be described in terms of just four parameters, and have received a great deal of attention in the literature ever since their inception, and especially since the advent of LEP.

A special and interesting class of supergravity models aims at solving two thorny problems in supersymmetry: the vanishing of the cosmological constant and the ultimate determination of the scale of the supersymmetric spectrum. In these no-scale supergravity models, the tree-level cosmological constant vanishes, and the scalar potential possesses a flat direction along which the scale of the supersymmetric spectrum (the gravitino mass $m_{3/2}$) is undetermined. At the electroweak scale this flat direction is “bent” and the gravitino naturally acquires a mass of electroweak size. This mechanism satisfies automatically our naturalness assumption.

Another consequence of supergravity is the radiative electroweak symmetry breaking mechanism. The Higgs mass-squared parameter in the scalar potential, starting from a positive value at the Planck scale, decreases in magnitude as the scale is lowered, eventually vanishing and turning negative, signalling the dynamical breaking of the electroweak symmetry. In contrast, in the Standard Model the negative Higgs mass-squared parameter is put in by hand. The radiative breaking mechanism relies on the running of the mass parameters and assures that the mass-squared of charged and colored particles remain positive. Even more interesting is the fact that this phenomenon is possible only in the presence of a not-too-light top quark ($m_t \gtrsim 60$ GeV).

Supergravity theories provide an effective description of quantum gravity, valid at scales $Q < M_{Pl}$. As such they are non-renormalizable and contain an infinite number of higher-dimensional operators suppressed by powers of the Planck mass. To make further progress in our quest for understanding the parameters of the Standard Model (or the MSSM), we need to compute the Kähler function and gauge kinetic function, which determine the spectrum of supersymmetric particles. We also need to compute the Yukawa couplings which give rise to the Standard Model fermion masses and quark mixing angles. The “prototheory” that we seek must in fact have no free parameters. It turns out that the solution to our problems is offered by string theory, wherein elementary particles are replaced by tiny strings of dimension $\ell_{Pl} \sim 10^{-33}$ cm. In string theory all physical parameters are in principle calculable in terms of the Planck mass, or ratios of the Planck mass to other dynamically determined mass scales (such as the vacuum expectation value of the dilaton field, which determines the string gauge coupling). As is well-known, at the present stage of its development string theory has a very large number of equivalent ground states, or “string models”. Each of these models has a continuously connected family associated with it, parametrized by fields with flat potentials called moduli. Because of the present inability to pinpoint the string vacuum (if it is indeed unique), it is widely perceived that string theory can make no predictions. This perception is incorrect. For one thing, in string theory one knows that the gauge couplings become unified at the string scale $M_{string} \sim 10^{18}$ GeV,
irrespective of the existence of a unified gauge group at this scale. Also, if one
focuses on any given string model, in principle all model parameters are explicitly
calculable, in particular the supergravity Kähler and gauge kinetic functions and the
Yukawa couplings. Thus, in specific string models one should be able to calculate
all of the Standard Model and MSSM parameters. Consequences of the unparalelled
calculability of string models include that the supersymmetry-breaking parameters
should typically be not universal, and that the top-quark mass should be large $m_t \sim (150 - 200) \text{ GeV}$.

1.4 Observable supersymmetry

Another feature of many supersymmetric models is the existence of a discrete $R$-parity
symmetry which implies that real supersymmetric particles are always produced in
pairs. That is, they cannot appear as intermediate (virtual) states in tree-level pro-
cesses involving only external Standard Model particles (e.g., in $e^+e^- \to \mu^+\mu^-$).
They can first appear in such processes at the loop level, and therefore their indirect
effects are naturally suppressed. $R$-parity also implies the existence of a new stable
particle: the lightest supersymmetric particle (LSP). This particle has important cos-
mological consequences, since in large regions of parameter space it can account for
the all-pervading (cold) dark matter in the Universe.

Supersymmetric particles are being actively searched for at several experimen-
tal facilities. At the Tevatron the strongly-interacting gluinos and squarks have been
the long-time target in the $p\bar{p}$ collisions, but the kinematical reach of the collider has
been nearly reached. This has prompted the search for particles with feebler interac-
tion strengths, but which could still be produced because of their lighter masses, such
as the weakly-interacting gauginos. A proposed high-luminosity upgrade of the Teva-
tron would be particularly sensitive to the gaugino signal. At LEP 1 (1989–1995),
the large number of $Z$ bosons produced ($\sim 20$ million) has ruled out the existence of
new particles with unsuppressed couplings to the $Z$ and with masses below $\sim \frac{1}{2}M_Z$.
This result has constrained the masses of most supersymmetric particles. The search
for the Higgs boson at LEP 1 ($m_H \sim 65 \text{ GeV}$) has also constrained the Higgs bosons
of supersymmetric models. The LEP energy upgrade (LEP 2) should push these lim-
its even further starting in 1996. More realistically, CERN’s Large Hadron Collider
(LHC) (ca. 2004) should provide definitive evidence for low-energy supersymmetry
or, if no supersymmetric particles are observed, render it unappealing. Once super-
symmetry is established, a dedicated electron-positron linear collider would be ideal
for what has been called “sparticle spectroscopy.”

Indirect searches for supersymmetric particles are a useful complement to di-
rect searches. The recently observed $b \to s\gamma$ loop process at Cornell’s electron-
positron storage ring by the CLEO Collaboration, puts important constraints on the
supersymmetric parameter space. The upcoming anomalous magnetic moment of the
muon ($g-2$) experiment at Brookhaven should also prove to be a stringent test of
supersymmetric models. Finally, as the presumed main component of the galactic
dark matter halo, the LSP is being searched for in various direct and indirect ways.
1.5 Disclaimer

Because of the intended nature of this review many details have been left out, and particular emphasis has been placed on developments that have occurred over the last decade. The reader is encouraged to consult the standard accounts [3], as well as a number of recent detailed reviews on the subjects of supersymmetric model-building [4], supersymmetry phenomenology [7], and supersymmetric dark matter [8]. I should also point out that due to lack of space and expertise, the more formal topics of supersymmetric theories have been left uncovered. A particularly glaring omission pertains to the large and relatively recent body of literature on exact results in supersymmetric gauge theories and the rapidly evolving topic of superstring dualities; for recent reviews see Refs. [9] and [10] respectively.

2 Low-energy Supersymmetry

2.1 Supermultiplets

As discussed in Sec. 1 (see Fig. 1), our first step away from the Standard Model consists of introducing low-energy supersymmetry. Supersymmetry is a space-time symmetry, as are the well-known Lorentz and Poincaré symmetries. Under the Poincaré group particles are labelled by their mass and their spin. The mass can be any positive real number or zero, whereas the spin must be integer or half-integer. These space-time symmetries are distinct from internal symmetries (such as gauge symmetries), which do not change the mass or spin of a particle but can change its internal quantum numbers (such as electric or color charge). Conversely, the space-time symmetries do not change the internal quantum numbers of a particle. One says that space-time symmetries “commute” with internal symmetries. Supersymmetry, as a space-time symmetry, changes the spin of a particle (but not its mass) and leaves all its internal quantum numbers unchanged. There are various kinds of supersymmetric theories, depending on the number of different supersymmetry generators. Each of these generators can change the spin of a particle by half-a-unit. In building realistic models one only considers \( N = 1 \) supersymmetry, i.e., models with a single supersymmetry generator. The phenomenological problem of extended \( (N \geq 2) \) supersymmetries is that fermions with different chiralities get related by supersymmetry, a result which is incompatible with the left-handed nature of the electroweak interactions. Two kinds of supermultiplets are mostly used in building supersymmetric models: the chiral supermultiplet contains a chiral spin-\( \frac{1}{2} \) fermion and a spin-0 scalar, whereas the vector supermultiplet contains a spin-1 vector boson and a Majorana spin-\( \frac{1}{2} \) fermion. These supermultiplets can accommodate all of the Standard Model particles and their superpartners.
2.2 The Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model (MSSM) \([1]\) is generally considered to be the closest one can get to the Standard Model, if one allows for the possibility of low-energy supersymmetry. This model is very general but has little predictive power, with more than 30 parameters required to fully describe it. Nonetheless, it should contain any model obtained from the more constrained theories that we describe in the following sections. In the MSSM, each Standard Model particle is paired with a superpartner. The fermions (quarks and leptons) belong in chiral supermultiplets together with the spin-0 sfermions (squarks and sleptons), whereas the gauge bosons (photon, gluon, W, Z) belong in vector multiplets together with the Majorana spin-\(\frac{1}{2}\) gauginos (photino, gluino, wino, zino). Finally, the Higgs boson is paired with a spin-\(\frac{1}{2}\) Higgsino in a chiral multiplet. The novelty is that two Higgs doublets are required in supersymmetric models, as we discuss shortly. The MSSM particle content and notation are collected in Table 1.

Since supersymmetry commutes with the gauge symmetry of the Standard Model, the MSSM is still an \(SU(3)_C \times SU(2)_L \times U(1)_Y\) gauge theory. That is, the gauge interactions of the superpartners are the same as those of the ordinary particles. For instance, the left-handed \(SU(2)_L\) fermion doublet \((e^\nu e)_L\) is partnered with the scalar doublet \((\tilde{e}\tilde{\nu}_e)_L\). The Feynman rules for the superpartners allow the same interactions, with the same strength, although one must take into account the different spinor nature of the particles.

An important assumption usually made in the MSSM is the imposition of a discrete “R-parity” symmetry, that assigns a charge of \(-1\) to the superparticles, and a charge of 0 to the regular particles (and the Higgs bosons). This symmetry restricts the possible interactions in the theory: at every interaction vertex there must be an even number (0, 2, 4) of superparticles. One important consequence of this assumption is the elimination of possible interactions, allowed by the gauge symmetry, that would lead to very fast proton decay (via dimension-four operators). Another important phenomenological consequence is that superparticles cannot appear as intermediate states in tree-level processes involving regular external particles only, thus “protecting” the Standard Model predictions to lowest order. Since superparticles are known to be not-too-light, their first tree-level appearance requires colliders with sufficient energy to pair produce them. A related consequence is that there should exist a lightest supersymmetric particle (LSP), which is absolutely stable, and which should appear copiously in any reaction that produces superparticles. Astrophysical considerations restrict the possible LSP choices to neutral and colorless particles \([12]\), i.e., the lightest neutralino (\(\chi\)) or the sneutrino. For various theoretical and phenomenological reasons \([3]\), it is the neutralino which is usually associated with the LSP. Supersymmetric models without R-parity have also been considered in the literature \([13]\).
Table 1: Field content and notation in the Minimal Supersymmetric Standard Model (MSSM). Arrows indicate fields that mix due to the Yukawa interactions, and the corresponding physical fields that result.

| Quarks (spin-$\frac{1}{2}$) | Squarks (spin-0) |
|-----------------------------|------------------|
| \( u_L \) \( u_R \) \( d_R \) | \( \tilde{u}_R \) \( \tilde{d}_R \) |
| \( c_L \) \( c_R \) \( s_R \) | \( \tilde{c}_R \) \( \tilde{s}_R \) |
| \( t_L \) \( t_R \) \( b_R \) | \( \tilde{t}_R \) \( \tilde{b}_R \) → \( \tilde{t}_{1,2} \), \( \tilde{b}_{1,2} \) |

| Leptons (spin-$\frac{1}{2}$) | Sleptons (spin-0) |
|-------------------------------|------------------|
| \( e_L \) \( e_R \) | \( \tilde{e}_R \) |
| \( \mu_L \) \( \mu_R \) | \( \tilde{\mu}_R \) |
| \( \tau_L \) \( \tau_R \) | \( \tilde{\tau}_R \) → \( \tilde{\tau}_{1,2} \) |

| Gauge bosons (spin-1) | Gauginos (spin-$\frac{1}{2}$) |
|----------------------|-------------------------------|
| \( g \) \( \gamma \) \( Z \) \( W^{\pm} \) | \( \tilde{g} \) \( \tilde{\gamma} \) \( \tilde{Z} \) \( \tilde{W}^{\pm} \) |

| Higgs bosons (spin-0) | Higgsinos (spin-$\frac{1}{2}$) |
|----------------------|-------------------------------|
| \( h, H, A \) \( H^{\pm} \) | \( \tilde{H}^{0}_{1,2} \) \( \tilde{H}^{\pm} \) → \( \chi^{\pm}_{1,2} \) \( \{ \tilde{\gamma}, \tilde{Z}, \tilde{H}^{0}_{1,2} \} \) |

2.3 Yukawa and scalar interactions

The Yukawa interactions and the scalar potential in supersymmetry are more constrained than in the Standard Model. The Yukawa interactions derive from an object called the superpotential \( W \), which is a cubic function of the chiral superfields, with only one chirality allowed for all superfields present. One says that \( W \) is a holomorphic function of its arguments. The superpotential in the MSSM (restricted by R-parity) contains the Yukawa couplings giving rise to the fermion masses, \( i.e., \)

\[
W = \lambda^{ij}_u Q_i u^c_i H_2 + \lambda^{ij}_d Q_i d^c_i H_1 + \lambda^{ij}_e L_i e^c_i H_1 + \mu H_1 H_2 .
\] (1)

\[\text{This property has played a key role in recent developments concerning exact results in supersymmetric gauge theories} \[.\]
Note that in the MSSM two Higgs doublets are required to provide all the needed Yukawa couplings since, in contrast with the Standard Model, the superpotential does not allow conjugate fields. (Next-to-minimal supersymmetric models typically include an additional Higgs singlet field in the low-energy spectrum [14].) The sum of the squares of the two Higgs vacuum expectation values (vevs) is constrained by the usual Higgs vev in the Standard Model. However, their ratio is undetermined, and it is denoted by the parameter \( \tan \beta = \frac{\langle H_2 \rangle}{\langle H_1 \rangle} \).

The scalar potential in the MSSM contains both supersymmetry-conserving and supersymmetry-breaking terms. The supersymmetry-conserving terms come from two sources, schematically

\[
V_{\text{susy}} \sim \sum_i (g \phi_i^* \phi_i)^2 + \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 ,
\]

where the sum runs over all scalar fields \( \phi_i \) in the model. The first contribution (the “D-terms”) provide the gauge and Higgs boson masses and interactions. These terms are analogous to their Standard Model counterparts, with one important exception. In the MSSM the Standard Model quartic Higgs coupling (\( \lambda \)) is predicted to be \( \lambda \sim g^2 \). The second contribution (the “F-terms”) provide the quartic scalar interactions among the squarks, sleptons, and Higgs bosons, and provide an additional contribution to the Higgs boson mass matrix from the “\( \mu \) term” in Eq. (1).

The supersymmetry-breaking contributions to the scalar potential

\[
V_{\text{soft}} \sim \sum_i m_i^2 |\phi|^2 + AW_3 + BW_2 ,
\]

provide masses to the squarks, sleptons, and Higgs bosons, as well as a set of trilinear (\( AW_3 \)) and bilinear (\( BW_2 \)) interactions mimicking the trilinear (\( W_3 \)) and bilinear (\( W_2 \)) terms in the superpotential. The theory also includes supersymmetry-breaking masses for the gauginos: \( m_{\tilde{g}} \) for the gluino, \( M_2 \) for the wino, and \( M_1 \) for the bino.

Because of the required two Higgs-boson doublets, the Higgs boson spectrum in the MSSM is much richer than in the Standard Model: two neutral scalars (\( h, H \)), a neutral pseudoscalar (\( A \)), and a charged scalar (\( H^\pm \)). One can show that in the MSSM the \( h \) Higgs boson is always light: \( m_h^{\text{tree}} < M_Z |\cos 2\beta| \). The upper limit is approached when the supersymmetry breaking contributions in \( V_{\text{soft}} \) are large, in which case the remaining Higgs bosons become very heavy and decouple. Moreover, the interactions of the light Higgs boson become indistinguishable from the Higgs boson in the Standard Model. One of the most phenomenologically relevant realizations of the last several years is that one-loop corrections to the Higgs boson mass in supersymmetric models are enhanced by a heavy top quark [15].

\[
(m_h^2)^{\text{one-loop}} \sim (m_h^2)^{\text{tree}} + c(m_t^4/m_Z^2) \ln(m_t^2/m_Z^2) .
\]

For \( m_t \sim 180 \text{ GeV} \), the upper limit becomes \( m_h \lesssim 130 \text{ GeV} \), greatly affecting the discovery potential for the Higgs boson at LEP.
2.4 Experimental limits

Experimental limits on the MSSM model parameters had been rather mild before the advent of the Tevatron and LEP. At the Tevatron the strongly-interacting squarks and gluino can be pair produced \((p\bar{p} \rightarrow \tilde{g}\tilde{g}, \tilde{q}\tilde{q}, \tilde{g}\tilde{q})\) with sizeable cross sections. After production these particles decay in a “cascade” until the LSP is reached. Since the LSP is “invisible”, it leads to an imbalance in the transverse-momentum total sum, or in practice to a “missing transverse energy” \((\text{missing } E_T)\) signature. The latest experimental limits from the Tevatron indicate that \(m_{\tilde{q}} > \sim 175 \text{ GeV}, \ m_{\tilde{g}} > \sim 175 \text{ GeV}; \ m_{\tilde{q}} \approx m_{\tilde{g}} > \sim 230 \text{ GeV}\). (5)

LEP 1 limits on superparticle masses are generally close to \(\frac{1}{2}M_Z\) for all pair-produced particles with unsuppressed couplings to the \(Z\) boson [17], this includes the squarks, sleptons, and charginos. The neutralinos, which are admixtures of the photino, zino, and neutral Higgsinos, can couple to the \(Z\) boson only through their Higgsino components. In general, the masses and compositions of the neutralinos depend on the choices for \(\mu, M_1, M_2, \) and \(\tan \beta\). The LEP 1 limits on neutralinos therefore depend on these three parameters [18]. In experimental analyses of the MSSM, a “GUT assumption” is often made, which relates the masses of the gauginos to one another

\[
M_1 = \frac{5}{3} \tan^2 \theta_W M_2, \quad M_2 = \left(\frac{\alpha_2}{\alpha_3}\right) m_{\tilde{g}} .
\]

(6)

As we discuss below, this result in fact follows in many GUTs, but in the MSSM it just serves as a simplifying assumption. Assuming this relation one can use the experimental limits on \(m_{\tilde{g}}\) to bound \(M_1, M_2\) and therefore the lightest neutralino mass: \(m_{\chi^0_1} > \sim 20 \text{ GeV}\) [18]. If the GUT assumption is not made, very light neutralinos are still allowed, provided that they be mostly higgsino admixtures (this requires \(\mu\) to be small too) [18]. The Higgs bosons have also been searched for at LEP 1, most notably the lightest one via \(e^+e^- \rightarrow Z \rightarrow Z^*h \rightarrow f\bar{f}h\). The production cross section differs from the Standard Model one only by an overall factor of \(\sin^2(\alpha - \beta)\), where \(\alpha\) is the Higgs mixing angle. For arbitrary values of this factor, and assuming the dominant Standard Model \(h \rightarrow b\bar{b}\) decay, LEP 1 has determined that \(m_h > \sim 40 \text{ GeV}\) [20]. The corresponding limit in the Standard Model is \(m_{h_{\text{SM}}} > 65 \text{ GeV}\) [20]. This limit is actually applicable in the MSSM if the superparticle masses are large, since in this limit the lightest supersymmetric Higgs boson becomes indistinguishable from the Standard Model Higgs boson.

2.5 Shortcomings

The MSSM has many parameters, and therefore a wide range of possible experimental predictions, \(i.e.,\) very little predictive power. In addition to this obvious shortcoming, there are more subtle hints that make the need to go beyond the MSSM pressing. In the Standard Model, flavor-changing neutral currents (FCNC) are absent at tree-level and are sufficiently suppressed in one-loop processes (such as \(K - \bar{K}\) mixing
and $\mu \rightarrow e\gamma$) because of the small light-fermion masses (e.g., $(m^2_c - m^2_u)/M^2_W \ll 1$). Such processes receive new one-loop contributions in the MSSM, with the analogous requirement being $(m^2_l - m^2_\tilde{u})/m^2_{\tilde{q}} \ll 1$, and similarly for the sleptons. In the same vein, new supersymmetric phases contribute to the dipole moment of the neutron, and are experimentally required to be rather suppressed ($\phi_{\text{susy}} \lesssim 10^{-3}$) unless the supersymmetric spectrum is rather heavy [21]. Such stringent constraints must be imposed by hand on the MSSM parameter space.

3 Supersymmetric Grand Unification

3.1 Running gauge couplings

The most basic prediction of a grand unified theory (GUT) that is to encompass the Standard Model, is that the low-energy gauge couplings should converge to a single value at a sufficiently high mass scale ($M_{\text{GUT}}$). Above this scale new degrees of freedom are excited, and the gauge couplings do not diverge again, but continue as a single unified coupling. From the low-energy point of view (the bottom-up approach), the convergence test must be passed. From the high-energy point of view (the top-down approach), there is a distinct prediction for, e.g., $\sin^2 \theta_W$ which, when evolved down to low-energies, should agree with the experimental measurement. As early as 1987 [22], the experimental determinations of the low-energy gauge couplings were precise enough to indicate that, even though both the Standard Model and the MSSM passed the convergence test, the MSSM did so more persuasively – in retrospect, the Standard Model had actually failed, but it managed to hide behind the experimental uncertainties. When the LEP data became available starting in 1989, the failure of the Standard Model as a unified theory became clear, as did the success of the MSSM [23]. The reason for the different outcomes in the Standard Model versus the MSSM is not the gauge symmetry, but rather the supersymmetry and the spectrum of light particles. Because of this ambiguity, schemes have been devised to “fix” the running of the gauge couplings in the Standard Model by adding ad-hoc intermediate-scale particles [24]. These schemes may be logically viable but, without supersymmetry, are incapable of solving the gauge hierarchy problem.

The running of the gauge couplings in unified models is familiar from our previous experience with the running coupling “constant” in QCD. The basic quantity of interest is the beta function, which quantifies the rate of change of the gauge coupling with the logarithm of the mass scale ($Q$), and depends on the numbers of light particles in the given model (at a given scale). Writing this differential equation in terms of the inverses of the “structure constants” $\alpha_i = g^2_i/4\pi$, to next-to-leading order one obtains

$$\frac{d\alpha_i^{-1}}{dt} = -\frac{b_i}{2\pi} - \frac{1}{8\pi^2} \sum_{j=1}^{3} b_{ij} \alpha_j ,$$

where $i = Y, 2, 3$ and $t = \ln(Q/M_{\text{GUT}})$. In this form it is evident that (to lowest order) the convergence of the gauge couplings reduces to the meeting of three lines.
$[\alpha^{-1}_i(t)]$ on a plane. In the Standard Model and in the MSSM, the one-loop ($b_i$) beta functions are given by [23]

\begin{align*}
\beta_i^{\text{SM}} &= \left( \frac{41}{10}, -\frac{19}{6}, -7 \right); \\
\beta_i^{\text{MSSM}} &= \left( \frac{23}{5}, 1, -3 \right). 
\end{align*}

(8)

The relative increase of the beta functions in the MSSM versus the SM is expected because of the larger number of fermions in the MSSM (i.e., the gauginos). The MSSM (SM) beta functions are valid for mass scales above (below) the masses of all the particles in the MSSM. In solving the RGEs in Eq. (7) between the scales $Q = M_{\text{GUT}}$ and $Q = M_Z$, in first approximation one assumes that all MSSM particles have the same mass (e.g., $M_Z$), and that one-loop beta functions suffice. In the next level of approximation one includes two-loop corrections to the beta functions ($b_{ij}$), and accounts for the many “thresholds” at which the supersymmetric particles decouple. This is a complicated exercise which has been performed in several different schemes [25, 26]. The original motivation for these calculational refinements was to gain some information about the spectrum of supersymmetric particle masses by fitting it to get the “best” convergence at $M_{\text{GUT}}$. This exercise proved to be futile because of the very nature of the unification process, that itself involves thresholds of much less understood particles near the GUT scale [25].

The gauge coupling RGEs in Eq. (7) are supplemented with the initial conditions

\begin{align*}
\alpha_e^{-1}(M_Z) &= 127.9 \pm 0.1, \\
\alpha_3(M_Z) &= 0.118 \pm 0.006, \\
\sin^2 \theta_W(M_Z) &= 0.23186 \pm 0.00034.
\end{align*}

(9) (10) (11)

The $U(1)_Y$ and $SU(2)_L$ gauge couplings are related to these by \footnote{The factor of $\frac{5}{3}$ in the definition of $\alpha_1$ follows from a rescaling of the Standard Model hypercharges so that, once embedded in a simple gauge group, all generators are equally normalized. From $Q = T_3 + Y = T_3 + cY', g_1 = cg_1'$, one determines $c$ such that $\text{Tr} T_3^2 = \text{Tr} Y'^2$ over any representation of the GUT group. In $SU(5)$ $c = \sqrt{5/3}$, and thus $\alpha_1 \propto g_1^2 \propto \frac{5}{3} g_2^2$.} $\alpha_1 = \frac{5}{3} (\alpha_e / \cos^2 \theta_W)$ and $\alpha_2 = (\alpha_e / \sin^2 \theta_W)$. In Fig. 2 we show a lowest-order comparison of the convergence of the gauge couplings in the MSSM versus the SM.

### 3.2 SU(5) GUTs

The convergence of the gauge couplings, as observed from the bottom-up approach, suggests that at scales $Q = M_{\text{GUT}} \sim 10^{16}$ GeV, a new structure emerges, that of a unified theory. Let us take $SU(5)$ as a prototype grand unified theory [24], and discuss various features that are qualitatively common to all unified theories. Later we compare $SU(5)$ to other unified groups, such as $SO(10)$ and $SU(5) \times U(1)$. In $SU(5)$, the Standard Model fermions of each generation are accommodated in the $\overline{5}$ and 10 representations:

\begin{align*}
\overline{5} &= (\mathbf{3}, 1) + (1, 2) = \{d^c, L\}, \\
10 &= (\mathbf{3}, 2) + (\mathbf{3}, 1) + (1, 1) = \{Q, u^c, e^c\}.
\end{align*}

(12)
This combination is anomaly-free. Because the right-handed down quark ($d^c$) and the lepton doublet ($L$) belong to the same $SU(5)$ representation, and the $SU(5)$ group generators are traceless (in particular the electromagnetic charge), the charge quantization relation $3q_{d^c} + q_e = 0$ follows for each generation. The gauge bosons of $SU(5)$ belong to the adjoint ($24$) representation,

$$24 = (8,1) + (1,3) + (1,1) + (3,2) + (3,2) = \{ g, W^\pm, W^0, B, X^{\pm 4/3}, Y^{\pm 1/3} \}$$

and contain the 12 Standard Model gauge bosons plus 12 new, heavy ($\sim M_{GUT}$), charged, colored gauge bosons denoted by $X, Y$. The breaking of the unified symmetry down to the Standard Model gauge group is effected by Higgs bosons in the $24$ representation, whose single neutral component [the $(1,1)$ in Eq. (13)] gets a vev. The real parts of the Higgs fields in the $(3,2) + (\bar{3},2)$ representations are eaten by the $X,Y$ gauge bosons, whereas the imaginary parts acquire their same mass ($M_V$). Further Higgs bosons belong to the $(8,1), (1,3), \text{ and } (1,1)$ representations with masses $M_2, M_\Sigma, ^1\!^1M_\Sigma$. The Higgs-boson doublet in the Standard Model is promoted to a Higgs pentaplet $H = \{ H_2, H_3 \}$, which contains a new, colored Higgs triplet field with mass $M_{H_3}$. The mass of the Higgs triplet is constrained by proton decay limits (see below) to be no smaller than the GUT scale, i.e., $M_{H_2}/M_{H_3} \sim M_Z/M_{GUT} \ll 1$. This doublet-triplet splitting problem is resolved in minimal $SU(5)$ by resorting to a severe fine tuning [11]. An alternative $SU(5)$ GUT model, the missing doublet model (MDM) naturally solves this problem [28, 29], although at the expense of introducing new $50, \overline{50}$ Higgs multiplets and a $75$ to break the gauge symmetry. Further alternative mechanisms to address this problem have also been suggested.
Since the three mass parameters \((M_V, M_\Sigma, M_{H_3})\) in minimal \(SU(5)\) (or even more parameters in the MDM) are generally not all equal, one encounters a “heavy threshold” behavior near \(M_{GUT}\), which smears the (lowest-order) concept of a single unification point.

From the GUT (or top-down) approach, one obtains an expression for the weak mixing angle: \(\sin^2 \theta_W(M_{GUT}) = 3/8\). The test of the unified theory is the prediction for \(\sin^2 \theta_W\) at the electroweak scale; to lowest order:

\[
\sin^2 \theta_W(M_Z)_{\text{GUTs}} = \frac{1}{6} + \frac{\alpha}{9 \alpha_3} \approx 0.203 ;
\]

\[
\sin^2 \theta_W(M_Z)_{\text{SUSYGUTs}} = \frac{1}{5} + \frac{7 \alpha}{15 \alpha_3} \approx 0.230 .
\]

The difference between these two predictions, even though small, is far greater than the present uncertainty in the experimental determination of \(\sin^2 \theta_W(M_Z)\) \([11]\). In the state-of-the-art calculations one inputs the precisely measured value of \(\sin^2 \theta_W\), and obtains a prediction for the not-as-well-measured strong coupling \(\alpha_3(M_Z)\). This is done by taking into account a variety of subleading effects, such as two-loop contributions to the beta functions, effect of Yukawa couplings, effect of finite (non–leading-logarithmic) corrections, light supersymmetric particle thresholds, heavy particle thresholds, different models of \(SU(5)\) GUT physics, sophisticated decoupling methods beyond the step-function approximation, non-renormalizable Planck-scale operators, universal and non-universal supersymmetry-breaking masses, etc. \([25, 26, 31]\). Interestingly enough, it has been recently realized that a more accurate treatment of the various threshold effects leads to an increase in the predicted value of \(\alpha_3(M_Z)\) \([26]\). In fact, it has been suggested that minimal \(SU(5)\) cannot reproduce any of the known values of \(\alpha_3(M_Z)\) \([32]\) for a discussion of this separate controversy), whereas the MDM can because of its richer heavy threshold structure \([29]\). In either case, the effect of GUT particles near the Planck scale and non-renormalizable Planck-scale operators has been emphasized as a means to resolve this impasse \([31]\).

### 3.3 Proton decay

Proton decay is perhaps the most characteristic experimental signature of unified theories. Several modes have been searched for, most notably \(p \to e^+\pi^0\) and \(p \to K^+\bar{\nu}_\mu\), with “partial lifetime” lower bounds of \(5.5 \times 10^{32}\) y and \(1.0 \times 10^{32}\) y respectively \([33]\). The forthcoming SuperKamiokande experiment should be able to probe partial lifetimes as high as \(10^{34}\) y. The \(p \to e^+\pi^0\) decay channel (see Fig. 3a) is mediated by the exchange of the \(X, Y\) gauge bosons (dimension-six operators), and yields a lifetime \(\tau_p \approx M^4_{GUT}/m^4_p\). From the experimental lower bound one concludes that \(M_{GUT} \gtrsim 10^{15}\) GeV, which is not in conflict SUSY GUTs predictions. However, this lower bound disagrees with the expectation in non-supersymmetric \(SU(5)\), if we were to imagine an approximate unification scale from Fig. 3. The \(p \to K^+\bar{\nu}_\mu\) decay channel (see Fig. 3b) arises in supersymmetric theories \([34]\), with the superpartner of
Figure 3: Typical diagrams contributing to proton decay in the minimal $SU(5)$ GUT model: (a) $p \rightarrow e^+\pi^0$ mediated by the exchange of GUT gauge bosons ($X, Y$); (b) $p \rightarrow K^+\bar{\nu}_\mu$ mediated by the exchange of GUT Higgs bosons ($H_3$) and light supersymmetric particles.

The Higgs triplet (the Higgsino) mediating the conversion of the quarks in the proton into squarks and sleptons. A loop is needed to produce the final state particles. This is a dimension-five operator, with quadratic dependence on the Higgs triplet mass $M_{H_3}$, and significant dependence on the supersymmetric spectrum \[ \tau_p \propto M_{H_3}^2 \sin^2 2\beta/|f|^2 \], where $f \sim m_{\chi_1^+}/m_\tilde{q}$ represents the one-loop dressing function. On dimensional grounds it would appear that dimension-five operators are much too large. However, various small factors (i.e., light-quark masses, one-loop coefficients, etc.) make it acceptable as long as $M_{H_3} \gtrsim M_{\text{GUT}}$ and some important restrictions on the supersymmetric parameter space are imposed, notably light charginos and heavy squarks.\[ \textit{[36]} \].
3.4 Yukawa unification

Because in unified models quarks and leptons are assembled into larger representations, one typically obtains relations among different Yukawa couplings at the unification scale. In minimal \( SU(5) \) one has two Yukawa coupling terms

\[
\begin{align*}
\lambda_u & \cdot \mathbf{10} \cdot \mathbf{10} \cdot H & \rightarrow & \lambda_u Qu^c H_2 \\
\lambda_{d,e} & \cdot \mathbf{10} \cdot \mathbf{5} \cdot \bar{H} & \rightarrow & \lambda_{d,e} (Qd^c H_1 + Le^c H_1)
\end{align*}
\]

where \( H_{1,2} \) are the Higgs doublets of the MSSM. The second relation indicates that at the GUT scale \( \lambda_b(M_{\text{GUT}}) = \lambda_{\tau}(M_{\text{GUT}}) \), and similarly for the lighter generations. At low energies one has \( m_b = \lambda_b(m_b)v_1, m_{\tau} = \lambda_{\tau}(m_{\tau})v_1, \) and \( m_t = \lambda_t(m_t)v_2 \). Since these parameters are interrelated, results of this analysis are usually presented as an allowed curve (smeared by the choices of \( m_b \) and \( \alpha_3 \)) in the \((m_t, \tan \beta)\) plane (see Fig. 4). As in the case of gauge coupling unification, various sub-leading effects have been included in the sophisticated analysis (light and heavy thresholds, possible corrections to the \( \lambda_b = \lambda_{\tau} \) relation, etc.) \[37\], \[38\]. The predicted value of \( m_b \) has always tended to come out uncomfortably high (i.e., \( m_b > 5 \) GeV). Recent studies suggest that heavy supersymmetric particles in minimal \( SU(5) \), or a richer GUT structure (the MDM) help bring \( m_b \) into the preferred range \[39\].

In the \( SO(10) \) GUT model \[10\], the Standard Model fermions are accommodated in a single representation per generation

\[
16 = 5 + 10 + 1,
\]

with the novelty of a new Standard Model singlet (1) which contains a right-handed neutrino. The Higgs pentaplets get unified into \( \mathbf{10} = H + \bar{H} \). Of the various features of \( SO(10) \), let us just cite two: a see-saw mechanism for generating small neutrino masses (typically \( m_\nu \sim m_u^2/M \), where \( M \) is the large Majorana mass for the right-handed neutrinos, and \( m_u \) is the up-quark mass matrix), and the complete unification of all Yukawa couplings (i.e., \( \lambda_t = \lambda_b = \lambda_{\tau} \)). (Further model-building aspects have been extensively discussed in Ref. \[14\].) The latter condition introduces an additional constraint in the \((m_t, \tan \beta)\) plane, relative to the \( SU(5) \) Yukawa unification case (see Fig. 4), and thus determines these two parameters, up to the dependence on \( m_b \) and \( \alpha_3 \) mentioned above. Typically \( \tan \beta \sim 50 \) is large and \( m_t \sim (150 - 190) \) GeV in agreement with experiment. However, the large value of \( \tan \beta \) makes it difficult to reconcile \( SO(10) \) GUTs with various phenomenological constraints \[42\].

3.5 Flipped SU(5)

Finally let us discuss the “flipped” \( SU(5) \times U(1) \) model \[13\], \[14\], where non-abelian gauge unification occurs (i.e., \( SU(2) \times SU(3) \subset SU(5) \)) but part of the hypercharge \( U(1)_Y \) appears in the external \( U(1) \) factor. This model is very appealing because of the many simplifying features that it possesses over the traditional GUT models discussed above \[15\], and because of the special role it plays in string model building (see
Typical SO(10)-like predictions

Figure 4: Typical allowed area in the \((m_t, \tan \beta)\) plane from the Yukawa unification constraints \(\lambda_b = \lambda_\tau \left[ \text{SU}(5)-\text{like} \right]\) and \(\lambda_t = \lambda_b = \lambda_\tau \left[ \text{SO}(10)-\text{like} \right]\).

Sec. 3. Gauge symmetry breaking down to the Standard Model gauge group occurs via vacuum expectation values of the \(H\) \((10)\) and \(\bar{H}\) \((\overline{10})\) Higgs representations. This is possible because of the “flipping” \(u \leftrightarrow d, u^c \leftrightarrow d^c, e \leftrightarrow \nu, e^c \leftrightarrow \nu^c\) involved in the assignment of the Standard Model particles to the \(\bar{f} = \{u^c, L\} \left(\overline{5}\right)\) and \(F = \{Q, d^c, \nu^c\}\) \((10)\) representations. Thus, \(H\) and \(\bar{H}\) contain one pair of neutral fields \(\nu_H^c, \nu_{\bar{H}}^c\), which get vevs along the flat direction \(\langle \nu_H^c \rangle = \langle \nu_{\bar{H}}^c \rangle\). The missing-partner mechanism, which can be rather cumbersome in traditional GUTs, is here effected by the couplings \(HHh\) \([(10)(10)(\overline{5})]\) and \(HH\bar{h}\) \([(\overline{10})(\overline{10})(\overline{5})]\). The resulting Higgs triplet matrix \([44]\)

\[
\begin{pmatrix}
\lambda_3 & \lambda_4 \langle \nu_H^c \rangle \\
0 & \lambda_5 \langle \nu_{\bar{H}}^c \rangle
\end{pmatrix},
\]

does not need a large non-zero \((22)\) entry because the additional components of the \(H\) and \(\bar{H}\) representations are eaten by the GUT gauge bosons to become massive.
or become GUT Higgsinos. This natural zero mass term for $d_H^c d_R^c$ implies that the dimension-five proton decay operators are negligible. Regarding see-saw neutrino masses, the right-handed neutrinos are contained in the $F$ (10) representations. The coupling $\lambda_1 F \tilde{f} h$ provides the up-quark masses and Dirac neutrino masses, and there are two possible sources of right-handed neutrino masses, leading to a neutrino mass hierarchy $m_{\nu_e,\mu,\tau} \approx m_{u,c,t}/M_U$ [46]. The right-handed neutrino is also used to great advantage in the generation of the baryon asymmetry of the Universe [47]. A recent development [48] concerns the running of the gauge couplings and the prediction for $\alpha_s(M_Z)$, which is naturally lowered compared to the minimal SU(5) case and falls within the experimentally allowed range.

4 Supergravity

As remarked in the Introduction, the much heralded convergence of the Standard Model gauge couplings at very high energies in the presence of low-energy supersymmetry makes sense only in the presence of a larger symmetry at the GUT scale (at least in the field theoretical approach), and when the supersymmetric particle masses are in the 100 GeV – 1 TeV range. It is remarkable that this range is consistent with present experimental lower bounds and with the naturalness upper bounds on sparticle masses. Taken for granted in this success of SUSY GUTs is that such sparticle masses may be obtainable in theories of supersymmetry breaking. As is well known, models with global supersymmetry lead to phenomenologically disastrous predictions, whereas local supersymmetry cures all these maladies nicely [5]. Local supersymmetry also entails the automatic incorporation of gravitational interactions, with the spin-3/2 gravitino field as the gauge field of local supersymmetric interactions. In analogy with the spontaneous breaking of gauge symmetries, the massless goldstino field gets eaten by the gravitino to acquire a mass ($m_{3/2}$). The presence of such a mass (or order) parameter signals the breaking of supersymmetry and sets the scale for the mass splittings of the supermultiplet partners. It is important to realize that supergravity, even though it incorporates gravitational interactions, it is nonetheless only an effective theory valid at scales below the Planck mass [49]. Such an effective field theory cannot handle quantum corrections. These are parametrized by an infinite number of non-renormalizable operators suppressed by powers of $(Q/M_P)$ and endowed with arbitrary numerical coefficients.

4.1 Basic functions

The rather complicated Lagrangian of supergravity theories can be described in terms of two functions: the Kähler function and the gauge kinetic function. The Kähler function,

$$G = K + \ln |W|^2,$$

(20)
depends in turn on the Kähler potential ($K$) and the usual superpotential ($W$). The Kähler potential is a function of all the matter fields and their complex conjugates.
(e.g., $K = \phi\phi^\dagger$), whereas the superpotential may depend on only a subset of the fields which must all have the same chirality (e.g., $W \ni \phi\phi\phi$; while $\phi\phi\phi^\dagger$ is not allowed). For phenomenological purposes, the Kähler function enters in the calculation of the scalar potential, in the normalization of the fields, and in the calculation of the gravitino mass. The scalar potential is given by

$$V = e^G \left( G^I G_I - 3 \right) + |D|^2,$$

(21)

where the sum (over $I$) runs over all scalar fields in the theory, $G_I = \partial_I G$, $G^I = G^{IJ} G_J$, and $G^{IJ}$ is the inverse of $G_{IJ}$. The “D-terms” ($|D|^2$) are analogous to those in the case of global supersymmetry in Eq. (1), whereas the “F-terms” are more complicated now. The kinetic term for the scalar fields, $G_{IJ} \partial^\mu \phi_I \partial^\nu \phi_J$ depends on $G_{IJ} = K_{IJ}$, and determines the proper normalization of the fields. The gravitino mass is given by

$$m^2_{3/2} = e^{(G)} = e^{(K)} \langle |W| \rangle^2,$$

(22)

in units of the reduced Planck mass $M = M_{Pl}/\sqrt{8\pi}$. Inserting the desired form for $G$ into Eq. (21) one can compute the scalar masses (e.g., the masses of the squarks, sleptons, etc.). The simple choice $K = \sum_i \phi_i \phi_i^\dagger$ gives the same (universal) mass ($m_0 = m_{3/2}$) to all scalar fields.

Derivatives of the gauge kinetic function ($f_{ab}$) determine the gaugino masses, which in the simplest models are also universal ($m_{1/2}$). Supersymmetry breaking manifests itself also in the all-scalar interactions, which are patterned after the Yukawa interactions in the superpotential, although each interaction term is accompanied by a new scalar coupling coefficient. The universality assumption entails that all the interactions of the same dimension possess the same supersymmetry breaking coefficient: the trilinear scalar coupling $A_0$, and the bilinear scalar coupling $B_0$. A supergravity theory with universal supersymmetry-breaking terms is then described in terms of four parameters

$$m_0, m_{1/2}, A_0, B_0. \quad (23)$$

In addition one has the parameters in the superpotential: the fermion Yukawa couplings and the Higgs mixing term $\mu$, which do not break supersymmetry. Specific examples of the functions $G$ and $f$ will be given in Sec. 5 when discussing the physics of superstring models, where these functions can be calculated from first principles, and where the simple result in Eq. (23) may not hold.

4.2 No-scale supergravity

An important unresolved problem in physics is the issue of the cosmological constant ($\Lambda$) or vacuum energy ($\Lambda^4$). Observationally, the vacuum energy is seen to be extremely small (at least in the present cosmological epoch): $\Lambda \lesssim 10^{-31} M_{Pl} \sim 10^{-3}$ eV. This rather small mass scale is unusual in particle physics, although light neutrino masses obtained from the see-saw mechanism tend to reproduce such values. Whether the cosmological constant is exactly zero or extremely small is a basic question which
particle physics and cosmology have been facing ever since Einstein first introduced it. It is reasonable to expect that such a small number is not the result of an incredible fine tuning, but rather the result of exact or approximate symmetries. This line of thought would then demand that the vacuum energy be vanishingly small at all times, and in particular at the very high mass scales involved in supergravity models.

There is a particular class of supergravity models where the vacuum energy $V_0 = \langle V \rangle$ actually vanishes (at least at tree-level) [56]. In these no-scale supergravity models [57, 58, 59], a judicious choice of Kähler potential ($K$) accomplishes this feat. The Kähler potential must depend on all fields in the theory, since otherwise their normalizations would be ill-defined. However, most fields have zero vacuum expectation values (vevs) and for purposes of calculating the vacuum energy they can be neglected. Moreover, it is customary to think of supergravity theories as having an observable sector and a hidden sector, with the two communicating with each other only through gravitational interactions. As we discuss shortly, the hidden sector is the one usually assumed to include the physics of supersymmetry breaking, which would happen dynamically when certain fields gain vacuum expectation values. In the simplest no-scale supergravity models the hidden sector is assumed to consist of a single field $T$, which has no superpotential interactions and has a Kähler potential

$$K = -3 \ln(T + \bar{T}).$$

From Eq. (21) it then follows that $G^TG_T \equiv 3$ for all values of $T$, and thus at the minimum (where all other fields have zero vevs) $V_0 = 0$. Furthermore, the value of $\langle T \rangle$ is undetermined, i.e., we have a flat direction. This behavior is illustrated in Fig. 1. One can also show that there is a (modular) symmetry under which the Kähler potential remains unchanged: $SU(1,1)/U(1)$ in this case. This symmetry is important in generalizations to more complicated no-scale models [60, 61, 62], but it does not guarantee the vanishing of the vacuum energy, which depends crucially on the explicit number ‘3’ in Eqs. (21) and (24). Note that the gravitino mass $m^2_{3/2} = \langle W \rangle^2/(T + \bar{T})^3$, and thus the scale of supersymmetry breaking, is also undetermined. This property can be exploited to implemented the no-scale mechanism [57, 64], whereby physics at the electroweak scale “bends” this flat direction and determines the preferred value of $\langle T \rangle$ and therefore of $m^2_{3/2}$. It is important to realize that for the no-scale mechanism to preserve the desired hierarchy ($m^2_{3/2} \ll M_{Pl}$), there should not appear terms in the scalar potential with dimensional coefficients containing large mass scales [58, 62, 63]. This is satisfied automatically at the tree level. At the one-loop level there appears a term $\propto \text{Str} \mathcal{M}^2 \Lambda^2$, with $\Lambda \sim M_{Pl}$. A consistency requirement is then that the spectrum satisfy the sum rule $\text{Str} \mathcal{M}^2 = 0$, after supersymmetry breaking [58, 62]. Further consistency constraints at higher loops have been considered also [65].

5A dynamical alternative to this static scenario is to assume that the cosmological constant varies with time [51]. In the long time elapsed since the Big Bang, $\Lambda$ would have managed to reduce itself to the very small values of interest today [52]. This scenario seems favored by non-critical string theory considerations [53, 54, 55].
Figure 5: An example of a scalar potential of the no-scale supergravity type, where the vacuum energy vanishes ($V_0 = 0$) along a flat direction.

4.3 Supersymmetry breaking

Above we have seen how supersymmetry breaking is parametrized in terms of the gravitino mass, with specific spectra obtained for given choices of $G$ and $f$. Equation (22) shows that in order to have supersymmetry breaking (i.e., $m_{3/2} \neq 0$), one must have $e^{\langle K \rangle} \neq 0$, which is satisfied in all cases of interest, and $\langle W \rangle \neq 0$. The latter then becomes the real pre-requisite for supersymmetry breaking. In fact, non-renormalization theorems show that if supersymmetry is not broken at the tree level, then it is not broken at any order in perturbation theory. In the latter case supersymmetry could still be broken by non-perturbative effects. Both approaches have been pursued in the literature.

Tree-level breaking is accomplished in certain superstring models [66], where $\langle W \rangle = c$, with $c$ some $\mathcal{O}(1)$ constant. In this case our simple no-scale model in Eq. (24) gives

$$m_{3/2}^{\text{tree}} \sim \frac{c}{(\kappa T + \kappa T)^{3/2}} M_{Pl},$$

where $\kappa \sim 1/M_{Pl}$. Since $c \sim 1$, $\kappa T$ must be large to produce a sufficiently small
value of $m_{3/2}$. One can show that this is equivalent to a decompactification limit, where some of the superstring internal dimensions become nearly macroscopic [67]. Such scenario would have manifold observable consequences at the next generation of high-energy particle accelerators [68].

A more ‘popular’ approach to supersymmetry breaking entails the inclusion of non-perturbative contributions in $W$. This scenario has been extensively explored mostly in the context of strongly-interacting (hidden sector) gauge field theories [63, 70], as opposed to the little understood string non-perturbative interactions. In this scenario one assumes that the hidden sector is composed of a gauge theory with or without matter particles. This sector of the theory is chosen such that as the mass scale is lowered, the gauge coupling increases (like in the usual QCD). The scale at which the gauge coupling blows up (using the one-loop approximation to the beta function $\beta$) is called the condensation scale

$$\Lambda = M e^{-8\pi^2/|\beta|g^2}, \quad (26)$$

where the gauge coupling takes the value $g$ at the string or Planck scale $M$. At the condensation scale the strongly-interacting dynamics typically lead to the condensation of gauginos ($\langle \lambda \lambda \rangle \neq 0$) and, if light matter is also present, to the formation of “meson” ($\langle H \bar{H} \rangle$) bound states. Below the condensation scale, the theory described in terms of these objects generates a non-perturbative superpotential with non-zero vev $\langle W_{np} \rangle \sim \Lambda^3$, which breaks supersymmetry if certain conditions are met [70]. Moreover, the gravitino mass (see Eq. (22)) is then exponentially suppressed relative to the Planck scale

$$m_{3/2}^{np} \sim \frac{e^{-24\pi^2/|\beta|g^2}}{(\kappa T + \bar{\kappa} T)^{3/2} M_{Pl}}, \quad (27)$$

and of the desired magnitude for the ‘natural’ values of $\kappa T \sim 1$ (i.e., those preferred by $T$-duality considerations). The resulting scalar potential may or may not generate a large vacuum energy, depending on the model details. It may also not lift the $T$-flat direction, which may be determined by the usual no-scale mechanism.

The supergravity models described above are quite general, and thus rather unpredictable. One lacks the ability to calculate from first principles the Kähler potential, the superpotential, and the gauge kinetic function, although simple assumptions seem to work well. To proceed we must resort to a theory where gravity can be consistently quantized. The only known example is string theory. String theory has further advantages, one of which is the ability to calculate the above unknown functions. At the same time, string supergravity is complicated by the appearance of moduli fields and new symmetries, which do away with the simple ansätze made in traditional supergravity.

5 Superstrings

The path that we have followed so far – in the direction of ever increasing mass scales – has not yet provided a means of calculating the many unknown parameters.
of the Standard Model or its supersymmetric extensions, even though a great deal of synthesis has been accomplished at the various stages, implying that fewer parameters are required as we uncover larger and larger symmetries. This path has also led to the inclusion of gravitational interactions in the picture of elementary particle physics, although only at an effective level below the Planck mass. Superstring theory is usually described as the only known theory where quantum gravitational corrections can be consistently computed. This is accomplished by a drastic change in our picture of the particle world, which is now viewed as consisting of a single type of Planck-sized string \((\ell_P \sim 10^{-33} \, \text{cm})\), with the various particles represented by different modes of string vibration. The finite size of the string provides a cut-off in the distance scale (it cannot be arbitrarily small), which softens the ultraviolet divergences of conventional quantum field theory. String perturbation theory is envisioned as an expansion on the topology (genus) of the two-dimensional world-sheet that describes string propagation: a (no-hole) sphere at tree level, a (one-hole) torus at one loop, a two-hole surface at two loops, etc. This topological expansion is much simplified compared with field theory, as in string theory there is a single “Feynman diagram” at each order in perturbation theory. One could not ask for a more unified theory: not only are the interactions unified, but so are the particles themselves. This theory possesses only one parameter: the Planck mass; the various unknown low-energy parameters must be calculable from first principles.

### 5.1 String model building

In practice string theory is only partially understood. This state-of-affairs is most evident in the very large number of solutions to the string equations (vacua or “string models”) that are known to exist. With our limited understanding, all these solutions appear equally acceptable from the theoretical point of view. In contrast, it is found that phenomenology is very discriminating, basically wiping out all known solutions, although some fare much better than others. String model-builders are then charged with finding the best possible string model. This task was started in 1984, when the first consistent string solutions were found by Green and Schwarz with the gauge group \(SO(32)\) in ten dimensions. Shortly thereafter the heterotic string was introduced. Most of the work since then has concentrated on exploring different compactification schemes to reduce the theory from ten dimensions down to four. Within a fixed compactification scheme (Calabi-Yau manifolds, orbifolds, free-fermionic constructions, etc.) one can build consistent models and study their phenomenological properties.

A common feature in string model-building is the need to provide as inputs parameters describing the two-dimensional world-sheet, which underlies the four-

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6There are in fact a few types of strings one can consider (e.g., Type I, Type II, heterotic), although recent understanding of superstring dualities appears to indicate that all these are different manifestations of the same underlying “M-theory”.

7Such simplifying properties of string perturbation theory have been successfully exploited to compute complicated many-particle diagrams in field theory.
dimensional world according to string theory. This indirect input method makes
string model-building less “intuitive” than conventional GUT model building, and
harder because of the severe consistency constraints that need to be satisfied. In
any given compactification scheme, after the two-dimensional inputs are provided,
well defined procedures can be applied to obtain the four-dimensional results. In the
case of free-fermionic strings, the procedure involves a large amount of rather simple
algebraic manipulations, making it amenable to automation. In a typical situation,
the resulting model consists of a gauge group with several gauge factors, and set of
massless (and massive) matter representations, such that all anomalies automatically
cancel. Each state in the model can be represented by a vertex operator which encompasses all the gauge degrees of freedom, as well as important two-dimensional quantum numbers which appear as global quantum numbers from the four-dimensional point of view. These “hidden” quantum numbers restrict the possible interactions among the fields beyond the usual gauge symmetry constraints, leading to otherwise unexplainable zero couplings (or “textures”). In this way it is possible to calculate explicitly the contributions to the superpotential (at cubic and higher orders) and the Kähler potential, in any given model. A subtlety in this process arises by the presence of special fields called moduli, which have no scalar potential – they parametrize flat directions. It is important to identify these fields because the Kähler potential can then be recast (through field redefinitions) in a more useful form, which makes manifest the presence of the moduli and their corresponding modular symmetries (see e.g., [61]).

One of the more basic model-building choices to be made is the level of the
Kac-Moody algebra of two-dimensional currents that underlies the four-dimensional
gauge group [80]. This level \( k \) is a positive integer, which for most of the history of
string model-building was chosen implicitly or explicitly to be unity \( k = 1 \). More
complicated constructs are required to build models with levels greater than one [81].
The choice of level of the gauge group has a dramatic phenomenological implication:
the smaller the level, the smaller the set of allowed massless representations in a
possible consistent model. This is a general property of string models, and becomes
most restrictive at level one \( k = 1 \) allowing only \( \mathbb{Z}, \mathbb{R} \) SO(2n): singlet, vector,
and spinor representations; SU(n): totally antisymmetric representations, as shown
in Table 2: \( E_6: 1, 27, 27; E_7: 1, 56; E_8: 1 \). Note that the traditional GUT-breaking
(adjoint) Higgs representations are not allowed at level one. They become allowed at level two or higher. In fact, it has been recently become topical to investigate
methods by which level-two (or higher) models can be built. These methods have
been developed in the context of free-fermionic models [84], and symmetric [85]
and asymmetric [86] orbifold constructions. As mentioned above, methods to build
level-one models are manifold [76, 77, 78, 79] and have been known for some time.

5.2 String unification

To GUT or not to GUT? [87] The traditional motivation for GUTs, \( i.e. \), their prediction of the unification of all gauge couplings, turns out to be automatic in string
models (up to factors of the level of the respective gauge groups) \[88\]. Thus, it is not obvious that a string-derived GUT is particularly compelling. In fact, such models require higher-level constructions, which so far have met with limited phenomenological success in the areas of the number of generations and the doublet-triplet splitting problem \[84, 85\]. In any event, string unification (to lowest order) is predicted to occur at the scale \( M_{\text{string}} \approx 5 \times g \times 10^{17} \text{GeV} \) \[89\], where \( g \) is the unified gauge coupling. Above this scale the spectrum of massive string particles is excited and the conventional field theory description fails. Nonetheless, it is possible to calculate the “threshold” effects of these particles \[89, 90, 92, 91\], which entail splittings among the various gauge couplings at \( M_{\text{string}} \), or equivalently, a shift in the effective unification scale.

An important question in string model-building is how to reconcile the string unification scale (\( M_{\text{string}} \)) with the simplest SUSY GUTs unification scale (\( M_{\text{LEP}} \sim 10^{16} \text{GeV} \)), which is some twenty times smaller. Such “discrepancy” may disappear once string models are better understood, although in the meantime a few solutions to bridge this gap have been proposed, such as adding new intermediate-scale “gap” particles \[93\] or allowing the string threshold corrections to decrease the effective string unification scale down to \( M_{\text{LEP}} \) \[94\]. The latter scenario appears now disfavored, as it requires large values of the moduli fields that parametrize the threshold corrections, which are hard to obtain in actual string models \[93, 94\], and still requires the addition of new particles beyond the MSSM \[94\]. A recent proposal in the context of flipped SU(5) takes advantage of several stringy features of the model and yields a natural scenario for string unification, along the lines of the “gap” particle scenario \[94\].

Table 2: Allowed massless representation in SU(n) gauge groups realized with level one Kac-Moody algebras.

| n  | Representation                                      |
|----|-----------------------------------------------------|
| 2  | 1,2                                                 |
| 3  | 1,3,3                                               |
| 4  | 1,4,6                                               |
| 5  | 1,5,5,10,10                                        |
| 6  | 1,6,6,15,15,20                                     |
| 7  | 1,7,7,21,21,35,35                                   |
| 8  | 1,8,8,28,28,56,56,70                               |
| 9  | 1,9,9,36,36,84,84                                   |
| 10–23 | 1,n,n,n(n−1)/2,2,n(n−1)/2                        |

[88] [84, 85] [93] [94] [96]
5.3 Dilaton and S-duality

A manifestation of the “no-parameter” character of string models is the value of the gauge coupling, which is determined dynamically by the vacuum expectation value of the dilaton field \( S \): \( g^2 = 1/\text{Re} \langle S \rangle \), with \( \langle S \rangle \) in Planck units. The dilaton is a modulus field, which has no potential at any order in perturbation theory: the gauge coupling slides along this flat direction. A nagging question in string theory is how to determine \( \langle S \rangle \). In the mechanism of supersymmetry breaking via gaugino condensation, field theory non-perturbative effects involve \( S \), but typically along a runaway direction (i.e., \( \langle S \rangle \to \infty \)). This problem may be solved by tuning two gaugino condensates such that their competing effects stabilize \( \langle S \rangle \) [97]. In practice, such models have proved difficult to construct in string model building. On the other hand, string non-perturbative effects are expected to play a major role in the determination of \( \langle S \rangle \).

The most significant progress on this question has come from the assumption that the dilaton obeys duality symmetries similar to those obeyed by the traditional moduli fields [98]. This “S-duality” entails specific forms for the \( S \)-dependence of the scalar potential, and typically predicts \( \langle S \rangle \sim 1 \): a very desirable result. This symmetry has far-reaching consequences, as it entails transformations such as \( S \to 1/S \), which connect the weakly-interacting to the strongly-interacting regimes of string theory. Recent work in this direction has led to the discovery of dualities (of \( S \) and \( T \) types) connecting strings to higher-dimensional objects called membranes, and to dualities among different kinds of strings (e.g., heterotic and Type II). This topic has become very active recently [10] and is likely to greatly illuminate our understanding of string theory, especially in its non-perturbative regime. For our purposes, we hope that the ultimate picture that emerges will still allow for a meaningful perturbative approach to string model building.

5.4 Realistic models

String models have been built using several different string formulations [72]. Originally Calabi-Yau compactification [76] was the preferred construction, resulting in models with gauge groups such as \( SU(3)^3 \) [99]. Later symmetric [77] and asymmetric [78] orbifold constructions were found to be more mathematically accessible, and models with the Standard Model gauge group were constructed [100]. A sizeable fraction of the string model-building effort has been carried out in yet another construction: the free-fermionic formulation [79], where models with the gauge groups \( SU(5) \times U(1) \) [101, 102], \( SU(4) \times SU(2) \times SU(2) \) [103], and the Standard Model [104] gauge group have been constructed. A large amount of effort has been devoted to the study of these models, where the superpotential has been determined at the cubic and non-renormalizable levels [105], and the Kähler potential has become available recently [61]. There is no room here to discuss the properties of these models in any detail. However, a few important properties can be mentioned, such as their level-one nature, which implies that no adjoint representations are required to break their unified gauge groups. One also has the unparalleled ability to calculate the couplings
in the superpotential, in particular the fermion Yukawa couplings. A typical prediction is \( \lambda \sim g \sim 1 \), which implies a quark mass in the range \( m_q \sim (150 - 200) \) GeV \[101, 104, 107\]. Such prediction agrees with experiment for the top quark, and thus one should ask not why the top quark is so heavy, but instead ask why the other quarks are so light. The remaining quarks may have suppressed Yukawa couplings, principally because of several stringy selection rules stemming from the “hidden” quantum numbers discussed above. These couplings would vanish at the cubic level but would arise at higher orders in superpotential interactions, suppressed by powers of \( M_{\text{string}}/M \sim 10^{-10} \). This desirable ratio \[101\] is generated in the presence of a seemingly anomalous \( U_A(1) \) factor in the gauge group, which forces the theory into a nearby vacuum where some scalar fields gain vacuum expectation values \[106\].

This mechanism to generate a hierarchical fermion mass spectrum has inspired recent attempts at constructing textured fermion mass matrices \[108\].

### 5.5 String supergravity

With the knowledge gained from strings, low-energy effective theories can be constructed in the form of standard supergravity theories, but with calculable forms for the Kähler potential, superpotential, and gauge kinetic function \[109, 90\]. This exercise has turned out to be more subtle than naively expected because of the duality symmetries that string models possess to all orders in perturbation theory. These symmetries are not so evident at lowest order in perturbation theory, but one can invoke general arguments and rewrite the tree-level results so that the duality symmetry is manifest. For instance, the lowest order form for the Kähler potential in a typical model is

\[
K = \phi \phi^\dagger + \frac{1}{2} \phi \phi^\dagger \phi \phi^\dagger + \cdots = -\ln(1 - \phi \phi^\dagger).
\] (28)

Direct calculation yields the first two terms in this expression, whereas the logarithm is the presumed all-orders result obtained from duality symmetry considerations. Duality symmetries also arise in the calculation of the superpotential, especially at the non-renormalizable level (see e.g., \[110\]). A more dramatic result is obtained in the case of the gauge kinetic function \( f \), which receives a universal tree-level contribution of the form \( f = kS \) (\( k \) is the level of the Kac-Moody algebra), whereas considerations of duality anomalies show that it receives readily-calculable one-loop corrections only \[111\]. Duality symmetries also have a “down” side, in that one needs to understand how they are broken, i.e., what is the expectation value of the moduli fields, as otherwise every observable remains undetermined.

I conclude this section by discussing a particular class of string models, those that respect the postulates of no-scale supergravity: (i) the (tree-level) vacuum energy vanishes, (ii) there is a flat direction along which the gravitino mass is undetermined, and (iii) the scalar potential does not depend quadratically on large mass scales (i.e., \( \text{Str} \mathcal{M}^2 = 0 \)). Traditional supergravity models with these properties have been discussed in Sec. 4, whereas there has been recent progress in studying string models with these properties \[60, 62, 63\]. (In fact, the no-scale supergravity structure was
identified early on as a generic property of string supergravities \cite{112}.) In string models these constraints are quite restrictive: the (tree-level) Kähler potential takes the form \( K = -\ln(S + S^\dagger) - 2\ln(T + T^\dagger) \), whereas the spectrum of the model needs to be correlated with the corresponding gauge group in a special way, if the third constraint \((\text{Str} \mathcal{M}^2 = 0)\) is to be satisfied \cite{53}. The problem becomes more subtle when one considers realistic models with anomalous \( U_A(1) \) factors in the gauge group, in which case it has been possible to construct the first semi-realistic string models where the third postulate is satisfied \cite{113}. Given the large number of string models, it appears sensible to apply reasonable constraints to reduce the number of possible realistic models. String no-scale supergravity is an interesting example of such endeavor.

The above discussion has focused on critical string theory, implicitly assuming a flat gravitational background. This need not be the case, and certainly was not the case during the early universe. Non-critical string theory is required to describe such situations. This subject is rather interesting, as it introduces a dependence on the dynamical time that parametrizes the approach to the flat background (e.g., the “cosmic” time elapsed since the Big Bang). A variety of possible observable consequences have been studied in a class of such models \cite{53, 54}, such as generic violation of CPT, the collapse of the wavefunction in quantum mechanics, the time-dependence of the fundamental constants, a new model of inflation, etc. \cite{114}.

6 Dynamics

Being able to construct supersymmetric models of particle physics at very high energies is the first step in making contact with experimental reality. One must also take into account the fact that experiments are performed at energies (\( \sim 1 \text{ TeV} \)) much lower than those at which the models are most naturally built (\( \sim 10^{16-18} \text{ GeV} \)). This means that the model parameters need to be “evolved” down through a large ratio of scales: \( 10^{16-18}/10^2 \sim 10^{14-16} \). The underlying quantum field theory, upon which our gauge theories are built, provides a precise prescription for such dynamical evolution through the use of the renormalization group equations (RGEs). These equations encode the scale dependence of the model parameters, which is necessary to maintain the renormalization group invariance of the theory as a whole. All model parameters (gauge and Yukawa couplings, scalar masses and couplings, etc.) participate in this set of coupled first-order linear differential equations, with one equation per parameter. The coefficients in these equations can be calculated order-by-order in perturbation theory, although in practice are known only to one- or two-loop order \cite{115}. This renormalization-group scaling takes into account the largest contributions to such evolution, those coming from the large logarithms \( \ln(M_U/M_Z) \). The best known evolution equations describe the running of the gauge couplings, as discussed in Sec. \cite{3}. 

6.1 Radiative electroweak symmetry breaking

In the process of RG evolution to lower energies one may encounter two phenomena: decoupling of particles and gauge symmetry breaking. When the running scale falls below the mass of a given particle, such particle is dropped from the subsequent evolution by means of some decoupling procedure. This procedure must be followed repeatedly in the 100 GeV–1 TeV range, where most of the supersymmetric particles decouple. A more drastic procedure must be followed when the breaking of a gauge symmetry is encountered, typically the electroweak symmetry at $\sim 100$ GeV. In fact, our dynamical picture would be incomplete if as we lower the running scale we did not observe signs that the electroweak symmetry is broken, i.e., that the Higgs mechanism is happening. In the context of supersymmetric unified theories, the Higgs mechanism occurs dynamically as the appropriate mass parameters in the supersymmetric Standard Model scalar potential evolve with scale, and eventually change sign near the electroweak scale. This radiative electroweak breaking phenomenon \[116\] depends crucially on supersymmetry, supersymmetry breaking, and the running of mass parameters down from a large mass scale.

To illustrate the concept, let us consider a typical RGE for a scalar mass $\tilde{m}$

$$\frac{d\tilde{m}^2}{dt} = \frac{1}{(4\pi)^2} \left\{ -\sum_i c_i g_i^2 M_i^2 + c_t \lambda_t^2 \left( \sum_i \tilde{m}_i^2 \right) \right\},$$

where $M_i$ are the gaugino masses, and the $c$ coefficients are given below for the various MSSM particles

$$\begin{array}{ccc}
H_1 & 0 & 0 & 0 \\
H_2 & 6 & 0 & 6 \\
\tilde{Q} & 0 & \frac{2}{3} & 6 \\
\tilde{U}^c & 0 & \frac{2}{3} & 0 \\
\tilde{D}^c & 0 & \frac{2}{3} & 0 \\
\tilde{L} & 0 & 0 & 6 \\
\tilde{E}^c & 0 & 0 & 0 \\
\end{array}$$

The result of running these RGEs is illustrated in Fig. \[31\] for the indicated values of the parameters. For $Q < Q_0$, $m_{H_2}^2 < 0$ whereas $m_{H_1}^2 > 0$. The sign change signals the breaking of the electroweak symmetry. Note that the top-quark Yukawa coupling ($\lambda_t$) plays a fundamental role in driving $m_{H_2}^2$ to negative values. This is only possible if $\lambda_t$ is large enough to counteract the effect of the gauge couplings, and thus requires the existence of a “heavy top quark.” Note that $m_{\tilde{Q}, \tilde{U}, \tilde{D}}^2$ remain positive because of the large $\alpha_3$ contribution ($\propto c_3$) to their running. For the same reason the sleptons ($\tilde{L}, \tilde{E}^c$) “run” much less. Thus, this mechanism breaks the electroweak symmetry but preserves the SU(3)$_C$ color and U(1)$_{em}$ electromagnetic gauge symmetries.
Figure 6: Running of the scalar masses in supergravity for typical values of the parameters. Note that for $Q < Q_0$ the electroweak symmetry is broken ($m_{H_2}^2 < 0$), but the SU(3)$_C$ color and U(1)$_{em}$ electromagnetic gauge symmetries remain unbroken.

### 6.2 Supersymmetry breaking scenarios

In considering the low-energy predictions for the sparticle spectrum in the context of supergravity models, several scenarios have arisen in the literature. These scenarios correspond to specific choices of the Kähler function and/or the dominant source of supersymmetry breaking. If one assumes that the Kähler potential is observable-sector blind, then universal supersymmetry-breaking mass parameters are obtained, and these are described by the four parameters in Eq. (23): $m_0, m_{1/2}, A_0, B_0$. In this context one can impose further conditions on the Kähler potential leading to the special case:

$$m_0 = A_0 = 0,$$

(sometimes referred to as “no-scale”, since the earliest no-scale supergravity models predicted such relations [58], although modern no-scale models usually depart from
them \[107\]. If supersymmetry breaking is dominated by the F-term of the dilaton field (a stringy effect), one obtains \[117, 118\]

\[m_0 = \frac{1}{\sqrt{3}} m_{1/2} \quad , \quad A_0 = -m_{1/2} \quad .\]

One can make further assumptions concerning the origin of the Higgs mixing parameter (\(\mu\)) \[119\], and obtain predictions for \(B_0\) \[117, 118\], although these are rather model dependent. In string-derived models the Kähler potential has non-trivial structure, which distinguishes between different fields via their modular weights or charges under modular symmetries. In the simplest models of this kind, the scalar masses take the form \[118\]

\[m_i^2 = m_3^2 (1 + n_i \cos^2 \theta) \quad ,\]

where \(n_i\) is the modular weight of the \(i\)-th field, and \(\tan \theta = \langle F_S \rangle / \langle F_T \rangle\) quantifies the amount of dilaton \(\langle F_S \rangle\) and moduli \(\langle F_T \rangle\) contributions to the supersymmetry-breaking F-term. In the dilaton scenario of Eq. (32): \(\theta \to \frac{\pi}{2}\). The most striking property of this result is the general lack of universality of the scalar masses \[117\]. The possible choices of \(n_i\) are model dependent, although always integer. In generic orbifold models these can vary quite a bit (\(-1\) to \(-5\)), while in \(Z_2 \times Z_2\) orbifolds they are always equal to \(-1\), implying universal scalar masses automatically. In explicit string-derived models the scalar masses can be calculated explicitly, with results not necessarily following the simple formula in Eq. (33). For instance, one finds models where some of the states have common scalar masses equal to \(m_{3/2}\), while the rest of the states have vanishing scalar masses \[107\]. In any event, it is clear that the more specific models one considers, the more one seems to depart from the naive assumption of universal scalar masses. On the other hand, these specific models are much more predictive than the generic ones and may be easily falsifiable.

### 6.3 Mass relations

The coupled set of renormalization group equations mentioned above must in principle be solved numerically. However, under reasonable assumptions some of the equations can be solved analytically. For instance, RGEs for the running gauge couplings in Eq. (7) can be solved exactly to lowest order in the beta functions

\[
\frac{d\alpha_i^{-1}}{dt} = -\frac{b_i}{2\pi} \quad \Rightarrow \quad \alpha_i(Q) = \frac{\alpha_i(Q_0)}{1 - \frac{2\pi}{b_i}\alpha_i(Q_0) \ln(Q/Q_0)} .
\]

In a slightly more complicated manner one can also solve the RGEs for the first- and second-generation squark and slepton masses. In this case one neglects the Yukawa couplings of the corresponding quarks and leptons, as these are much smaller than those of their third generation counterparts. One obtains

\[
\tilde{m}_i^2 = m_{1/2}^2(c_i + \epsilon_0^2) - d_i \frac{\tan^2 \beta - 1}{\tan^2 \beta + 1} M_W^2 .
\]
Table 3: Values of the $c_i$ and $d_i$ coefficients in Eq. (35) [$\alpha_3(M_Z) = 0.118$] for the first- and second-generation sfermions (i.e., $c_{\tilde{e}_R} = c_{\tilde{\mu}_R}$, and so on) for the minimal SU(5) GUT model and for a string-inspired GUT (GUST) that unifies at the string scale. Also shown is $c_3 \equiv m_{\tilde{g}}/m_{1/2}$. 

| $c_i$    | GUT | GUST | $d_i$            |
|----------|-----|------|------------------|
| $c_{\tilde{e}_R}$ | .149 | .143 | $-\tan^2 \theta_W$ |
| $c_{\tilde{e}_L}$ | .512 | .402 | $\frac{1}{2} + \frac{1}{2} \tan^2 \theta_W$ |
| $c_{\tilde{\nu}}$ | .512 | .402 | $\frac{1}{2} + \frac{1}{2} \tan^2 \theta_W$ |
| $c_{\tilde{u}_L}$ | 6.28 | 3.91 | $\frac{1}{2} - \frac{1}{6} \tan^2 \theta_W$ |
| $c_{\tilde{u}_R}$ | 6.28 | 3.91 | $\frac{1}{2} - \frac{1}{6} \tan^2 \theta_W$ |
| $c_{\tilde{d}_R}$ | 5.87 | 3.60 | $\frac{2}{3} \tan^2 \theta_W$ |
| $c_{\tilde{d}_L}$ | 5.82 | 3.55 | $-\frac{1}{3} \tan^2 \theta_W$ |
| $c_{\tilde{g}}$ | 2.77 | 2.01 |                  |

where $\xi_0 = m_0/m_{1/2}$ and $d_i = (T_{3i} - Q) \tan^2 \theta_w + T_{3i}$ (e.g., $d_{\tilde{u}_L} = \frac{1}{2} - \frac{1}{6} \tan^2 \theta_w$, $d_{\tilde{e}_R} = -\tan^2 \theta_w$). The $c_i$ coefficients can be calculated numerically in terms of the low-energy gauge couplings, and are given in Table 3 for $\alpha_3(M_Z) = 0.118$ and two GUT choices: standard minimal SU(5) unification at the scale $\sim 10^{16}$ GeV, and a string-inspired unification at the scale $\sim 10^{18}$ GeV. In the latter case a minimal set of additional matter representations has been introduced to delay unification (a vector-like quark doublet $Q, Q^c$ and a vector-like quark singlet $D, D^c$) [93]. In the table we also give $c_3 = m_{\tilde{g}}/m_{1/2}$. The above approximation fails for the third generation sparticles, especially when $\tan \beta$ is large, since then the $b$ and $\tau$ Yukawa couplings are enhanced and can be as large as the top-quark Yukawa coupling; analytical expressions are however still obtainable [120].

In unified supergravity models with radiative electroweak breaking [121] there are many predicted masses in terms of a few input parameters, entailing several mass relations. A particularly important one concerns the masses of the squarks of the first two generations. From Eq. (35) we see that for the squark masses of current interest ($m_{\tilde{q}} \gg 200$ GeV), the second (“D-term”) contribution is small relative to the first one because the corresponding $c_i$ are large (see Table 3). Thus, all squark masses are nearly degenerate and one usually talks about an average squark mass. On the other hand, the top-squark masses are obtained by diagonalizing a $2 \times 2$ matrix with off-diagonal entries proportional to the top-quark mass (i.e., $m_{\tilde{t}_L}(A_t + \mu/\tan \beta)$), and thus the lightest eigenvalue ($\tilde{t}_1$) can easily be much lighter than all the other squarks. In contrast, one does not usually talk about an average slepton mass because the corresponding $c_i$ coefficients are much smaller (see Table 3), making the sleptons typically lighter (or even much lighter) than the squarks. However, the sleptons can
be as heavy as the squarks as long as the other relevant parameter ($\xi_0$) is large enough. This spectrum of squark and slepton masses could in principle be measured accurately (“sparticle spectroscopy” [122]) at a suitable facility, such as the planned $e^+e^-$ next linear collider (NLC) [123] or a recently proposed $\mu^+\mu^-$ collider [124].

In a unified theory one also gets a relation among the gaugino masses

$$\frac{M_1}{\alpha_1} = \frac{M_2}{\alpha_2} = \frac{m_{\tilde{g}}}{\alpha_3} = \frac{m_{\chi_1/2}}{\alpha_U},$$

(36)

where $M_1, M_2$ are the U(1) and SU(2) gaugino masses, and $\alpha_U$ is the gauge coupling at the unification scale. The first relation gives $M_1 = \frac{3}{5} \tan^2 \theta_W M_2$, whereas the second one gives $M_2 = (\alpha_2/\alpha_3)m_{\tilde{g}}$, which is referred to by experimentalists as the “GUT relation”. In fact, these relations allow one to connect experiments at hadron colliders ($m_{\tilde{g}}$) with experiments at $e^+e^-$ colliders ($m_{\chi_1/2}$), and has been used to set a lower limit on the lightest neutralino mass $m_{\chi_1} \gtrsim 20$ GeV [123]. Experimental verification or falsification of these mass relations will provide a direct window into the physics at the GUT scale, in particular the gauge group and the gauge kinetic function. Another kind of mass relation which follows in supergravity models concerns the chargino and neutralino masses [126, 127]:

$$m_{\chi_1} \approx m_{\chi_2} \approx 2m_{\chi_1}, \quad m_{\chi_{3,4}} \approx |\mu|.$$  

(37)

The degree of approximation implied by these mass relations varies somewhat from model to model. The origin of these mass relations can be traced back to the relatively large value of $|\mu|$ that follows from the radiative electroweak symmetry breaking mechanism.

### 6.4 Typical spectra

To be concrete, in Fig. 7 we show the masses of the first- and second-generation sleptons ($m_{\tilde{e}_R} = m_{\tilde{\mu}_R}$, $m_{\tilde{e}_L} = m_{\tilde{\mu}_L}$, $m_{\tilde{\nu}_e} = m_{\tilde{\nu}_\mu}$) as a function of the gluino mass for three choices of $\xi_0 = 0, \frac{1}{\sqrt{3}}, 1$; and for $\tan \beta = 1$ (straight lines) and $\tan \beta \gg 1$ (curved lines), as calculated from Eq. (35) using the numerical coefficients in Table 3 [127, 128]. This exercise is repeated in Fig. 8 for the string-inspired model which unifies at the string scale. Perhaps the most interesting feature of these figures is the implied lower bound on the gluino mass from the presently known lower bounds on the slepton masses from LEP 1 data ($m_{\ell} \gtrsim 45$ GeV). In particular the sneutrino mass is quite restrictive. These indirect lower bounds show that discovery of the gluino at the Tevatron could not have occurred so far, as the experimental sensitivity has just recently reached the 200 GeV range.\[^8\] In Fig. 9 we present the analogous plots for the squark masses. Note that the masses of these first- and second-generation squarks are nearly degenerate (as indicated above) with the main dependence embodied in the

\[^8\] It should be noted that a light gluino window ($m_{\tilde{g}} \sim$ few GeV) appears to still be allowed experimentally [30], although it may be theoretically disfavored [31].
Figure 7: Slepton masses as a function of the gluino mass in GUTs, for different choices of the $\xi_0 = m_0/m_{1/2}$ parameter. The straight lines correspond to $\tan \beta = 1$, while the curved lines correspond to $\tan \beta \gg 1$. 
Figure 8: Slepтон masses as a function of the gluino mass in GUSTs, for different choices of the $\xi_0 = m_0/m_{1/2}$ parameter. The straight lines correspond to $\tan \beta = 1$, while the curved lines correspond to $\tan \beta \gg 1$. 
Figure 9: Squark masses as a function of the gluino mass in GUTs and GUSTs, for different choices of the $\xi_0 = m_0/m_{1/2}$ parameter. The choices of $\tan \beta (1, \gg 1)$, for each value of $\xi_0$, span the whole allowed range. Note that for $\xi_0 \sim 1$, $m_{\tilde{q}} \approx m_{\tilde{g}}$. 
parameter $\xi_0$. Note also that for $\xi_0 \sim 1$, we obtain $m_{\tilde{q}} \approx m_{\tilde{g}}$, which is the region in the $(m_{\tilde{q}}, m_{\tilde{g}})$ plane of greatest experimental sensitivity. These figures also show that unless $\xi_0 \gg 1$, the slepton masses are expected to be much lighter than the squark masses. From Eqs. (36) and (37) one can show that

$$m_{\chi^\pm_1} \approx M_2 \approx 0.3 m_{\tilde{g}},$$

and therefore the weakly-interacting charginos and neutralinos are much lighter than the strongly interacting squarks and gluino. Moreover, if one imposes an upper limit on the squark and gluino masses of 1 TeV, the corresponding upper limit on the lighter chargino and neutralinos is under 300 GeV.

We should also comment on the Higgs-boson mass spectrum. Because of the constraints from radiative electroweak symmetry breaking [129], which effectively link the sparticle and Higgs sectors of the theory, as the supersymmetry-breaking scale is raised, the lightest Higgs boson mass ($m_h$) approaches its asymptotic value, as determined by the one-loop expression in Eq. (4). For $m_t < 180$ GeV one obtains $m_h \lesssim 130$ GeV. The remaining Higgs bosons ($A, H, H^\pm$) acquire a mass close to $|\mu|$, and decouple from the fermions and gauge bosons (their couplings are suppressed). Moreover, the couplings of the lightest Higgs boson ($h$) to fermions and gauge bosons approach those of the Standard Model Higgs boson in this limit. Therefore, it becomes rather difficult to distinguish between these flavors of Higgs bosons, except for new supersymmetric decays of $h$ into the lightest supersymmetric particle ($h \rightarrow \chi^0_1 \chi^0_1$), which will erode the preferred $h \rightarrow b\bar{b}$ mode when kinematically allowed.

7 Experimental Prospects

The most basic experimental predictions of supersymmetric models, i.e., the type of particles to be found and their coupling strengths, are to a great extent fixed simply by the presence of supersymmetry. However, a quantification of supersymmetry breaking is essential to determine the masses of the superparticles, and therefore their discovery windows at experimental facilities. Unless one is dealing with the straight MSSM, where all superparticle masses are to be taken as independent parameters (something that is usually not done in practice anyway), the various levels of theoretical input that we have discussed above lead to a vast number of correlated experimental predictions. The popular models based on universal soft-supersymmetry-breaking can be described in terms of only four parameters. More detailed models require even less parameters, and in principle no parameters.

One can search for supersymmetry directly at collider experiments such as Fermilab’s proton-antiproton Tevatron collider and its proposed upgrades, CERN’s electron-positron LEP collider, CERN’s proposed Large-Hadron-Collider (LHC), the proposed Next-Linear-Collider (NLC), the proposed First-Muon-Collider (FMC), etc. In the time frame of 1996–2006 one expects to see the completion of the Tevatron program (Runs II and III), the completion of the LEP program, the start of the LHC
program, and a definite timetable for the NLC (and perhaps even the FMC). One can also search for supersymmetry through indirect effects which may affect the expected predictions of certain Standard Model processes. Such precision measurements will be carried out at CLEO \((b \rightarrow s\gamma)\), Brookhaven \((g-2)\), the Tevatron and LHC (rare top-quark decays), B-factories (CP violation), proton decay experiments, KTeV (rare kaon decays), cryogenic dark matter detectors (direct detection of dark matter) and neutrino telescopes (indirect detection of dark matter), neutrino oscillation experiments (neutrino masses and mixings), etc.

7.1 Direct Searches

7.1.1 Hadron Colliders

Supersymmetric particles have been searched for since 1988 in \(p\bar{p}\) collisions at \(\sqrt{s} = 1.8\) TeV by the CDF and D0 Collaborations at the Tevatron (Run I), with a total integrated luminosity \(\sim 100\,\text{pb}^{-1}\) at the end of 1995. Early on the preferred signature was that of jets plus missing energy, as predicted to occur in the production and decay of the strongly-interacting gluino and squarks \((p\bar{p} \rightarrow q\bar{q}, g\bar{g}, q\bar{g})\). Such signature has not been seen, and lower bounds of \(m_{\tilde{q}}, m_{\tilde{g}} > 175\) GeV and \(m_{\tilde{q}} \approx m_{\tilde{g}} > 230\) GeV have been set \([16]\). More recently it has been realized that since the practical reach into squark and gluino masses has been nearly reached, one should also consider the production of weakly-interacting superparticles (charginos and neutralinos: \(p\bar{p} \rightarrow \chi^\pm_1\chi^0_2 + X\)), which have smaller cross sections, but that are typically expected to be much lighter than squarks and gluino. This endeavor has benefited greatly from the existence of a nearly background-free decay of the chargino and neutralino into three charged leptons \([132, 133, 134, 135]\). Chargino pair-production into dileptons has also been considered recently \([135]\). Preliminary results have since appeared \([136]\). With the full data set for Run I, it is expected that the Tevatron could probe chargino masses as high as \(\sim 100\) GeV, in some regions of parameter space. Trilepton and dilepton rates as a function of the chargino mass in a generic supergravity model \([135]\) are displayed in Fig. 10. It should be noted that there are regions of parameter space where the trilepton rate is negligible, due to the presence of “spoiler” modes that overwhelm the trilepton signal \([133]\). The Tevatron should also be able to set new lower bounds on light top squarks \([137]\); first experimental results have recently appeared \([138]\).

The Tevatron is expected to be upgraded significantly with the commissioning of the Main Injector (1999), which will allow accumulation of integrated luminosities of a few \(\text{fb}^{-1}\). Supersymmetry searches will continue in this upgraded machine, with modest gains expected in the squark-gluino sector, but great improvements expected in the chargino-neutralino sector (see Fig. 10). Yet further into the future (2002) a high-luminosity Tevatron may be in operation (TeV33) \([139]\), entailing further exploration of the parameter space, mostly in the chargino-neutralino sector, and perhaps also in the Higgs sector \([140]\). (A doubling of the Tevatron energy (the DiTevatron) has also been considered \([134]\).) Around 2004 one expects the commissioning of the
Figure 10: The dilepton and trilepton rates at the Tevatron versus the chargino mass in a generic unified supergravity model with \(\tan\beta = 2, \xi_0 = 0, 1, 2, 5\) (as indicated), and \(A = 0\). The upper (lower) dashed lines represent estimated reaches with 100 pb\(^{-1}\) (1 fb\(^{-1}\)) of data.

LHC, where the searches for gluino and squarks in 14 TeV pp collisions will again take center stage. The LHC should reach easily into the TeV mass region for these particles \[141\]. Other supersymmetry searches at the LHC are more uncertain, given the extremely high collision rate and multiple-interaction environment. Detection of light Higgs bosons may also pose a problem \[142\]. In any event, the LHC is expected to be the definitive experiment for low-energy supersymmetry: either it will be observed there (or before) or it will be rendered rather unappealing as an extension of the Standard Model.
7.1.2 Lepton Colliders

Soon after its commissioning in 1989, LEP 1 data on the total width of the Z boson showed that new particles with unsuppressed couplings to the Z boson had to had masses larger than $\sim \frac{1}{2} M_Z$ [17]. This limit applies to most supersymmetric particles. Exceptions may include the lightest neutralinos [18] and the lightest top-squark, whose coupling to the Z may be suppressed in some regions of parameter space. LEP 1 also searched for the Standard Model Higgs boson via the process: $e^+e^- \rightarrow Z \rightarrow Z^*H$ and obtained the limit $M_H > \sim 65 \text{ GeV}$ [20]. This limit tends to apply to the lightest Higgs boson ($h$) in supergravity models, especially when the sparticle spectrum is in the few hundred GeV range [129]. Otherwise, the limit in the MSSM is weaker ($m_h > \sim 40 \text{ GeV}$, $m_A > \sim 20 \text{ GeV}$). The LEP program is undergoing an energy upgrade, with the near-term goal of reaching the threshold for production of $WW$ pairs in mid 1996, and an eventual goal of reaching a center-of-mass energy of 192 GeV by 1998. An intermediate-energy step (“LEP 1.5”) with $\sqrt{s} = 130 - 136 \text{ GeV}$, accumulated a $\sim 6 \text{ pb}^{-1}$ of data in November of 1995, and was able to increase the lower bound on the chargino mass up to 64 GeV (in most regions of parameter space) [143]. With the full-energy upgrade ($\sqrt{s} \sim 200 \text{ GeV}$), the LEP 2 program should be able to push the lower limits on sparticle masses to near the kinematical limit [144]. The Higgs boson will be searched for via the process $e^+e^- \rightarrow Z^* \rightarrow Zh$ [145], with an expected reach strongly dependent on $\sqrt{s}$ and the accumulated integrated luminosity (e.g., $m_h \sim \sqrt{s} - 100 \text{ GeV}$ for 500 pb$^{-1}$ of data) [146]. After sparticles are discovered and their masses approximately determined, the NLC should be able to follow a program of sparticle spectroscopy that will determine the spectrum of supersymmetric particles rather precisely [122, 123].

7.2 Indirect Searches

7.2.1 Precision measurements

Precision measurements at the Z pole have shown that the Standard Model works rather well; deviations from it should be naturally suppressed, as is the case of supersymmetry. Nonetheless, LEP 1 has left us with two puzzles: the measured value of $\alpha_s(M_Z)$ appears to be 10% higher [147] than that inferred from low-energy measurements [32], and the $R_b = \Gamma(Z \rightarrow bb)/\Gamma(Z \rightarrow \text{hadrons})$ observable appears inconsistent with the Standard Model prediction at the 3$\sigma$ level [2]. These discrepancies may disappear with a better understanding of the experimental procedures, or they may signal the presence of new physics beyond the Standard Model. Supersymmetry offers some hope to explain both of them, but only if superparticles are very light [148, 149]. LEP 2 searches will determine whether these particular regions of parameter space remain viable or not. In fact, LEP 1.5 searches imply that the $R_b$ anomaly is unlikely to be due to the presence of light supersymmetric particles [150].

The measurement of the one-loop FCNC decay $B(b \rightarrow s\gamma)$ by the CLEO Collaboration [151] agrees well with the Standard Model prediction and has con-
strained supersymmetric models in significant ways [152], as exemplified in Fig. 11. In particular, large values of tan\(\beta\) appear now disfavored. Future indirect test of supersymmetry include refinements of the \(B(b \to s\gamma)\) measurement (although the theoretical prediction may not have the corresponding precision [153]), and the new Brookhaven \((g-2)_{\mu}\) experiment [154], which is expected to constrain supersymmetric models in significant ways [155] (especially for large values of tan\(\beta\)) once it starts to take data in 1996.

### 7.2.2 CP, CPT, and LFV

Tests of CP violation are particularly relevant to supersymmetric models, as these models typically include new phases beyond that in the CKM matrix [156]. The
most stringent test has been that of $K$-$\bar{K}$ mixing ($\Delta M$ and $\epsilon_K$), which demands that squarks of the same electric charge (but different flavor) should be nearly degenerate in mass [157]. This requirement can be naturally accommodated in supergravity models with universal supersymmetry breaking masses at the unification scale, and provides an important restriction on novel string-based scenarios where universality is typically absent [154, 158]. Rare kaon decays (CP violating or not) are also sensitive probes of supersymmetry [158]. Lepton-flavor-violating (LFV) processes are expected in unified models, as this Standard Model symmetry is typically violated in GUT models [160]. String models generally predict the violation of CPT symmetry [161] because of their inherent lack of locality at very small distances. The $K$-$\bar{K}$ system [162] (and perhaps also the $D$ and $B$ systems [163]) appears to be particularly sensitive. The above kind of processes have been studied at Brookhaven, Fermilab, and CERN, and will continue in the future including new players such as KTeV [164], the SLAC B factory [165], DAFNE [166], etc.

7.2.3 Proton decay

Proton decay is an unambiguous test of new physics at the unification scale [35], and has in the past excluded altogether the original SU(5) model via the $p \rightarrow e^+\pi^0$ mode [33]. The minimal SU(5) supergravity model has also been challenged through the inherently supersymmetric $p \rightarrow \bar{\nu}K^+$ mode [36]. Future tests will commence as soon as SuperKamiokande goes online in 1996. The expected sensitivity should provide a definitive test of SU(5) GUTs (via $p \rightarrow \bar{\nu}K^+$) [36]. Moreover, the traditional $p \rightarrow e^+\pi^0$ mode may constrain flipped SU(5), should the strong coupling $\alpha_S(M_Z)$ settle near 0.110 [48].

7.2.4 Neutrino oscillations

Evidence for non-zero neutrino masses may constitute the first deviation from the Standard Model. Unified models have a natural mechanism for producing such small masses (the see-saw mechanism), and even provide detailed predictions for the pattern of neutrino masses and mixings [167]. There is a plethora of presently (Homestake, Kamiokande, SAGE, GALLEX) or soon-to-be operating (SNO, Superkamiokande, Borexino) solar neutrino detectors, atmospheric neutrino detectors (Kamiokande, IMB, Soudan II), and neutrino-oscillation experiments (LSND, CHORUS, NOMAD, E803), which are providing data that can be interpreted in this theoretical context. The data has so far been insufficient to declare the discovery of neutrino masses or neutrino oscillations, although the solar neutrino data appears to be most naturally explained by the MSW mechanism [168] of matter-enhanced oscillations in the Sun [169] with neutrino mass and mixing parameters that are readily obtained in GUTs. Atmospheric neutrino data and the Los Alamos (LSND) neutrino oscillation experiment [170] are more difficult to interpret in this theoretical context [171]. The theoretical situation should become more clear once enough data has been accumulated on each category of experiments to be able to assess their statistical significance.
7.2.5 Dark matter detectors

Supersymmetric models that respect \( R \) parity (a popular and well motivated assumption) possess a natural candidate for cold dark matter, the lightest supersymmetric particle, which must be neutral and colorless [12] and is usually assumed to be the lightest neutralino. (The sneutrino may happen to be the LSP, but this dark matter candidate is severely constrained.) Such stable particle would populate the galactic halo and could be detected directly or indirectly [8]. Direct detection experiments [172] rely on energy deposited in cryogenic detectors after a direct nucleon-neutralino collision in the detector. Indirect detection relies on the capture of neutralinos by the Sun or Earth [173], and their subsequent annihilation into (among other things) energetic neutrinos, which can be detected in underground or underwater facilities (“neutrino telescopes”) [174]. Both these detection mechanisms are quite promising [175] and a number of facilities of both kinds are currently in operation or will soon be operating, including the Stanford Germanium detector, Kamiokande and SuperKamiokande, MACRO, Amanda, Nestor, DUMAND, etc.

8 Conclusions

The Standard Model of elementary particle physics has been subjected to intense experimental scrutiny over the last twenty years. With the advent of the Tevatron and LEP colliders, these tests have reached unprecedented precision. Yet, in nearly all instances observations agree very well with theoretical expectations. This state-of-affairs indicates that whatever new physics may lie beyond the Standard Model, it will not be intertwined with it in any significant way. This is a very useful fact, as Standard Model processes will constitute the bulk of the events in searches for new physics at higher energies. However, because of the “purity” of the Standard Model, these background processes will be reliably calculated, and the signals for new physics will be more easily extractable. Such searches are underway and will reach new sensitivity levels in the near future, most notably with the LEP 2 energy upgrade, the Main Injector Tevatron upgrade, and in the long-term the Large Hadron Collider; and indirectly via searches for rare processes such as proton decay at SuperKamiokande and the anomalous magnetic moment of the muon at Brookhaven.

A very well motivated possibility for the type of new physics that one may encounter has been the subject of this review: supersymmetry. As we have discussed, supersymmetry is an underlying theme in the march towards ever increasing energy scales. In fact, it appears to be the only road that allows us to see the light at the end of the tunnel. There will always be alternatives, but they must all contend with the gauge hierarchy problem, and supersymmetry is the only known way of tackling it without giving up calculability. Interestingly enough, despite all of the impetus with which these ideas have been pursued, direct evidence for the existence of supersymmetry is yet to be found. However, I believe that supersymmetry will not run away from us for much longer. In fact, our best models today indicate that the
success of the Standard Model effectively pushes supersymmetry up to higher mass scales which are just now beginning to be explored experimentally.

On the other hand, supersymmetry has received a great deal of indirect evidence over the last several years. Most strikingly was the convergence of the precisely measured Standard Model gauge couplings at very high energies. This fact led to a revival of these ideas, including intense theoretical scrutiny of gauge and Yukawa coupling unification in unified supergravity models. It has also been claimed that the two known experimental “anomalies” in the Standard Model – the discrepancy between measurements of the strong coupling at LEP and their counterparts at low energies, and the discrepancy between the measurement of and the Standard Model prediction for $R_b$ – may reflect the presence of new physics and in particular the existence of light supersymmetric particles.

Another bit of indirect evidence comes from the discovery of the top quark with a mass $m_t \sim 200 \text{ GeV}$. There is only one known theory where quark masses can be calculated, namely string theory. In string models the Yukawa coupling that gives rise to the top-quark mass is naturally of the size needed to yield such “large” quark masses. Moreover, string models also explain the lightness of the remaining quark masses, as other Yukawa couplings tend to be suppressed by stringy selection rules. Thus, a large top-quark mass can only be understood in the context of string theory. Furthermore, a large top-quark mass can only be predicted in the presence of supersymmetry, as one needs to connect stringy predictions at the Planck scale with top-quark masses measured in the laboratory. String theory itself is undergoing a second revolution (the first one occurred in 1984 with the establishment of string theory itself) with many new relations being found among previously thought distinct types of string. Strings also appear to be intimately connected with higher-dimensional “membranes”, leading to the conjecture of a universal M-theory underlying all the different phases of strings. Hopefully these developments will shed light onto the unresolved problems of supersymmetry breaking and the determination of the vacuum. In fact, further progress in supersymmetry model building should come from string models, as these provide us with the ability to calculate.

Supersymmetry will continue to have an ever expanding role in the physics of the very early universe, and its present-day manifestations. The observed minute anisotropies in the cosmic microwave background radiation show an imprint left early on, which points towards the idea of inflation where GUT- and Planck-scale physics play a major role. It has also become clear that most of the matter in the universe is invisible. Moreover, conventional astrophysical explanations (red dwarfs, brown dwarfs, etc.) have been found to constitute only a small fraction of this “dark” matter. Supersymmetry comes in again by providing a natural candidate for such dark matter, which may in fact be detectable in the laboratory.

We should soon know whether supersymmetry is within our reach or not.
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