Two-phase flow in porous media: power-law scaling of effective permeability

Morten Grøva, Alex Hansen
Department of Physics, NTNU, NO-7491 Trondheim, Norway
E-mail: Morten.Grova@ntnu.no, Alex.Hansen@ntnu.no

Abstract.
A recent experiment has reported power-law scaling of effective permeability of two-phase flow with respect to capillary number for a two-dimensional model porous medium. In this paper, we consider the simultaneous flow of two phases through a porous medium under steady-state conditions, fixed total flow-rate and saturation, using a two-dimensional network simulator. We obtain power-law exponents for the scaling of effective permeability with respect to capillary number. The simulations are performed both for viscosity matched fluids and for a high viscosity ratio resembling that of air and water. Good power-law behaviour is found for both cases. Different exponents are found, depending on saturation.

1. Introduction
Two immiscible fluids (such as oil and water) sharing a common pore space within a porous medium (such as sandstone) self-organize under flowing conditions [1, 2]. Viscous and capillary forces are thought to dominate this process. Some transients are known to display power-law behaviour, and have attracted significant attention for several decades.

Two-phase flow in porous media is not only important as an interesting application and testing-ground for theoretical models. With global oil reservoirs dwindling, the problem of recovering the 20 to 60 percent of oil which remains at the end of traditional recovery is of great importance. An improved understanding of the connection between pore-level dynamics (where the menisci sit) and the reservoir scale is crucial to the field of enhanced oil recovery.

This system therefore provides a unique challenge for physicists and others with an interest in complex systems and self-organized criticality [3]. Two models have been successful in capturing extreme limits of the parameter space for displacement processes: diffusion-limited aggregation [4] describes displacement of a more-viscous fluid by a less-viscous fluid (or gas) when capillary forces can be neglected while invasion percolation [5] describes displacement of a wetting fluid by a non-wetting fluid (or gas) when viscous forces can be neglected.

The displacement of one phase by another in the limits of high viscosity ratio or strongly dominant capillary forces leads to power-law scaling for, e.g., pressure-burst size distributions [6]. Also, the displacement structures have fractal dimensions [7, 8, 9]. For this reason we believe that in these limits the internal self-organization of fluid-fluid interfaces (menisci) are the result of critical behaviour giving rise to scale-free structures.

It is important to use a parameter space which is large enough to capture the relevant physics, but not larger. Viscosity ratio M and capillary number Ca are generally thought to be the two
most significant parameters. Saturation $S$ or fractional flow $F$, of which one should be kept fixed to achieve steady-state, is a third parameter. In this context, steady-state implies that macroscopic observables only fluctuate around well-defined, reproducible average values. In our simulations we control saturation, and find that fractional flow eventually reaches an average value, around which it fluctuates, and this average value does not depend on initial conditions or the realization of the porous medium disorder. If fractional flow is controlled, we expect the saturation to fluctuate around some well defined average when steady-state is achieved.

Depending on the applied flow-rate and saturation, both phases or only one of the phases may be mobile. In this work we consider the case where both phases are mobile under conditions of fixed saturation. We call this the regime of free fractional flow. The transition where one of the phases becomes immobilized will be referred to as the freezing transition. If we are below the freezing transition, such that only one phase would be mobile under conditions of fixed saturation, but both phases are forced to flow due to, e.g., simultaneous injection of both phases, we have a regime of forced fractional flow.

Together with $F$ (or $S$ if $F$ is fixed), the effective permeability $\kappa_{\text{eff}}$ is the main macroscopic quantity of interest. When these two are known, relative permeabilities are easily calculated.

Some earlier numerical work has explored steady-state properties. Pioneering work was done by Constantinides and Payatakes [10]. A similar model to the one used here has been described and discussed for simulations focusing on both transients and steady-state [11, 12]. The need for a statistical mechanics approach to describe steady-state has been suggested by work finding a relationship between the applied pressure-drop and fractional flow [13, 14] and first order phase-transition-like behaviour in the freezing transition [15]. An approach to a thermodynamics of two-phase flow in porous media has been sketched [16].

Little experimental work has been done under steady-state conditions. Rather, the focus has been on transients: flooding a sample by one of the phases until this is the only phase that flows. A notable exception is the work done by the Payatakes group [17, 18], which were etched-glass experiments performed under steady-state conditions. To our knowledge they did not explore scaling of effective permeability.

Progress was made in a recent experiment by Tallakstad et al [19, 20] where a power-law was found for the scaling of effective permeability with respect to capillary number,

$$\kappa_{\text{eff}} \sim \text{Ca}^\beta.$$  \hfill (1)

In their set-up, they find $\beta = 0.54 \pm 0.08$.

This scaling indicates that self-organized criticality persists over a wide range of parameters, i.e., the system does not only approach a critical point in some limit of the parameter space. In the experiment there was a large viscosity ratio due to one phase being air and the other a mixture of water and glycerol. Also, fractional flow was fixed by simultaneous injection of the two phases. Seven of fifteen injection syringes contained the non-wetting phase (air), so $F = 7/15$.

Two immediate questions present themselves. Does power-law scaling also happen in the absence of a large viscosity ratio? And what happens when we control saturation instead of fractional flow? Some progress has been achieved recently [21].

This work explores the dependency of effective permeability on saturation, viscosity ratio and capillary number, using a disordered transport network as a model for the porous medium.

2. The network simulator
An impression of the model is provided by Fig. 1.

A porous medium can be considered to be a transport network, consisting of tubes connected at pores, which in a network may be identified as links and nodes respectively. The network
considered in this work is a square lattice inclined at 45° relative to the main direction of flow. Boundary conditions are bi-periodic (the network may be mapped onto a torus). Disorder is introduced by assigning random radii to tubes, from a flat distribution between 10-40% of the tube length. All tubes are assumed to be equally long, with length $\ell$.

For our model, we consider the nodes to have no properties other than that of connecting links, while links are assigned a volume and radius $r$. Links also need to model the narrowing geometry of pore necks. For this purpose, links are assigned an hourglass-shape with respect to the capillary pressure of menisci.

A modified version of the Young-Laplace relation then gives the capillary pressure $p_c$ as

$$p_c(x) = \frac{2\gamma}{r} \left(1 - \cos\left(\frac{2\pi x}{\ell}\right)\right),$$

where $x$ is the position of the meniscus in the interval $[0, \ell]$ and $\gamma$ is surface tension.

The Hagen-Poiseulle relation for cylindrical tubes gives us the flow rate of a tube for a given pressure drop,

$$q = -\frac{\pi r^4}{8\ell\mu} \left(\Delta p + \sum p_c\right),$$

where $\mu$ is the viscosity of the fluid contained in the tube and $\Delta p$ is the difference in pressure of the two nodes connected by the tube. The sum runs over all menisci within the tube. If the viscosities of the two fluids are different, the viscosity of the above equation is replaced by the volume-weighted effective viscosity, averaged over the volume of the tube.

A constant flow-rate $Q$ is achieved by solving the pressure field for two applied pressure-drops $\Delta P$ and extrapolating linearly to the desired $Q$ using the relationship

$$Q = a\Delta P + b,$$

where $a$ is the single-phase mobility and $b$ is a flow-rate induced by the capillary pressures. This approach is valid due to the flow in each tube being a linear function of $\Delta p$. 

Figure 1. Two snapshots of the spatial distribution of non-wetting bubbles (red). The overall direction of flow is upwards. The initial condition was a horizontal band of non-wetting fluid. Bi-periodic boundary conditions are used. Capillary fingering and imbibition is seen along the middle and bottom mixing front, respectively. System size is $256 \times 256$. Ca = $10^{-4}$ and $S = 0.5$. 
The effective permeability is given by

$$\kappa_{\text{eff}} = \frac{Q}{A\Delta P},$$

(5)

which is unity if there are no macroscopic effects due to capillaries, and smaller than unity if the flow-rate is smaller than it would be without the effect of capillaries.

The effective pressure drop $P_{\text{eff}}$ is the pressure drop normalized by the pressure drop that would be required in the absence of capillaries, and is nothing but the reciprocal of $\kappa_{\text{eff}}$.

The main parameter of our model is the relative strength of capillary and viscous forces, parametrized by the capillary number

$$Ca = \frac{\mu Q}{\gamma \phi A},$$

(6)

where $A$ is the cross-sectional area of the sample and $\phi$ is its porosity. If the viscosities of the two fluids are different, the viscosity of the above equation must be replaced by the volume-weighted effective viscosity, averaged over the entire system.

The viscosity ratio $M$ is defined by

$$M = \frac{\mu_{nw}}{\mu_{w}},$$

(7)

where the subscripts denote non-wetting and wetting viscosity.

Saturation $S$ is defined by the ratio of non-wetting to total volume,

$$S = \frac{V_{nw}}{V}. $$

(8)

Fractional flow $F$ is defined by the ratio of non-wetting to total flow,

$$F = \frac{Q_{nw}}{Q}. $$

(9)

The system is moved forward by solving the pressure field and time-stepping with the timestep chosen by the largest flow-rate within the system. Each configuration consists of a set of bubbles, each with two menisci. At all timesteps the Kirchhoff equations are satisfied by solving these for the pressure and flow field. This is done using a customized version of the conjugate gradient method with an incomplete cholesky preconditioner [22].

Fig. 2 provides an impression of the saturation, flow and pressure spatial distributions for snapshots of a displacement process. Fig. 3 provides snapshots of the saturation and flow spatial distribution for a steady-state configuration.

3. Bubble mechanics – coalescence and snap-off

Menisci are created and destroyed as they are transported across nodes in the network. For each timestep the total volume of non-wetting liquid entering each node is distributed to the links transporting fluid away from the nodes, weighted according to its flow-rate. Within each link, there is a piston-like movement of bubbles, with the exception of the case when a maximum number of bubbles is exceeded, in which case the two nearest bubbles are found and merged.

An alternative mechanism of preventing the total number of bubbles from growing indefinitely is to set a minimum distance between menisci. When two menisci come closer than this distance they are merged. In the results presented here a minimum distance of 10 percent of the tube length was used. The effect of varying this distance, and also of changing to the method of a maximum number of bubbles, varied from 2 to 8, has been explored. No effect was found with respect to $\kappa_{\text{eff}}$, but $F(S)$ and the Ca value for the freezing transition are sensitive to this detail.
Figure 2. Saturation, flow and pressure distribution for the configuration shown in Fig. 1a (left) and Fig. 1b (right). System size is $256 \times 256$. $Ca = 10^{-4}$ and $S = 0.5$. 
Certainly, the way in which bubbles are transported across nodes, created and destroyed is of significance generally. However, the significance is greatest for low capillary numbers. Also, when the number of bubbles becomes large the sensitivity to the bubble mechanics appears to be reduced. In particular, a large number of bubbles reduces the effect of coalescence. We consider an understanding of this simplified system to be of importance in order to improve our understanding of the more involved problem which includes dependence on the detailed bubble mechanics.
Table 1. The scaling exponent $\beta$ for different values of $M$ and $S$. Determined by best-fits to the data points of Fig. 4.

| $M$  | $S$  | $\beta$ |
|------|------|---------|
| $10^{-4}$ | 0.2  | 0.53    |
| $10^{-4}$ | 0.3  | 0.74    |
| $10^{-4}$ | 0.4  | 0.80    |

| $M$  | $S$  | $\beta$ |
|------|------|---------|
| 1    | 0.2  | 0.53    |
| 1    | 0.4  | 0.81    |

4. Simulation results

Flow-rates are chosen to explore the regime of free fractional flow, where both fluids move. For each run the saturation and flow-rate is kept fixed. System sizes of $L_x \times L_y = 256 \times 256$, measured in number of tubes, are easily accessible, however we find good enough statistics at smaller sizes. For the results of Figs. 4 and 5, $32 \times 32$ and $64 \times 64$ systems were used. Initial conditions consist of a random distribution of non-wetting bubbles which gives the desired saturation. The initialization conditions of these simulations are different than those used in [21], the significance of which will be discussed below. We run several individual realizations of disorder and initial conditions for each set of parameters, and present averaged results.

The freezing transition depends strongly on the effects of bubble snap-off, bubble coalescence and possibly film flow. Staying well above this transition simplifies the system considerably, as details of bubble mechanics and films are not expected to affect the qualitative picture. As we wish to stay above the freezing transition, and this transition occurs at quite large values of $Ca$ when $S$ is very large or small, or $M \gg 1$, the parameter space which we choose to explore becomes somewhat restricted.

For selected values within the parameter space we find that the effective permeability $\kappa_{\text{eff}}$ appears to follow power-law scaling with respect to $Ca$ below a threshold value. Figs. 4 and 5 and table 1 collect the results of some simulations.

We find that power-law scaling persists for capillary numbers up to $Ca \approx 2 \cdot 10^{-2}$. Above this threshold it appears that all effects due to capillaries vanish, with both the effective permeability becoming unity and fractional flow becoming equal to saturation.

Below the threshold $\kappa_{\text{eff}}$ decreases with decreasing $Ca$. This is caused by the menisci whose capillary pressure opposes the applied pressure outnumbering the rest, at any given time. As individual bubbles, consisting of a pair of menisci, are transported through hourglass-shaped tubes, the sum of capillary pressures acting on a single bubble changes direction as it is transported across the narrowest point of the tube. If there were only a single capillary tube, or if the porous medium were modelled as a bundle of capillaries, symmetry and a constant flow-rate would necessitate a perfect cancelling out of capillary effects for long times under steady-state conditions. Hence, the decrease of permeability can only be understood as a collective effect, where the ability of the system to choose between several flow paths yields correlations in both the direction and magnitude of capillary pressures, which conspire to produce the observed power-law behaviour.

For an example saturation of $S = 0.7$, shown in Fig. 5a, we have $\kappa_{\text{eff}}(Ca) \sim Ca^{0.67}$. For the same saturation, the dependence of fractional flow $F$ on $Ca$ is given in Fig. 5b. We see that $F(Ca) = S$ above the threshold, while the non-wetting fluid sees a larger permeability than the wetting fluid below the threshold.

From Fig. 4 and table 1, we see that there is an abrupt change in $\beta$ when $S$ increases from 0.2 to 0.3. For $S = 0.3$ and above, $\beta$ is somewhere in the range of $0.67 - 0.81$. Below this threshold, we find $\beta = 0.53$, close to that of the experiment.
The observation of two distinct regimes with respect to a sudden change in scaling exponent is surprising. It can be seen quite clearly when we plot $P_{\text{eff}}(S)$ for low Ca, as in Fig. 6, that there are indeed two distinct regimes. The onset of this transition also coincides with a sharp increase in $F(Ca)$.

Close inspection of individual samples reveals that samples with $S \approx 0.2$ contain stable, immobile clusters, as opposed to samples where $S \geq 0.3$. In the former case some residue of the initial conditions remains even after steady-state is reached, while in the latter case any initialization-dependent clustering is destroyed as the pressure builds up beyond some point where immobile clusters destabilize.

In [21] simulations were initialized with a gradual increase of surface tension while the system was flowing. After the desired value was reached the system continued to flow until it reached steady-state. For $S \geq 0.3$ the two initialization procedures give the same results, but for $S \approx 0.2$ and below the difference is significant. The initialization used here leads to two regimes, a small-exponent and a large-exponent regime, while the smooth increase in surface tension results in only a single regime, the large-exponent regime.

We have found little sensitivity to the viscosity ratio $M$. Even if heterogeneities in the saturation distribution develops at low Ca, capillary forces increase in importance relative to viscous forces in the same limit, and it does not appear that a viscous contrast is ever sufficient to have a significant impact.

As previously noted, the experiment of Tallakstad et al uses constant fractional flow $F = 7/15$ instead of constant $S$. Also, they are at capillary numbers below the freezing transition $Ca < 10^{-5}$. By our terminology, they are studying the regime of forced fractional flow. If they did not force both phases to flow by simultaneous injection, only the wetting phase (glycerol–water) would flow and the clusters of air would be immobile. We believe that this picture is compatible with our small-exponent regime, which also contains stable immobile clusters. With this assumption, the exponents of experiment and simulation are in excellent agreement.

Another difference between the experiment and our simulations is that the experiment uses a compressible gas instead of our incompressible non-wetting liquid. A close inspection of the experiment suggests that the effect of compressibility might not be important as this would introduce a sensitivity to absolute pressure which decreases along the direction of flow. Such a dependence is not observed.
Figure 6. Top row: $F$ and $P_{\text{eff}}$ as functions of $S$, averaged over several realizations of the simulation, for $M = 1$ (black squares) and $M = 10^{-4}$ (red circles). Bottom row: $P_{\text{eff}}$ as a function of $S$. The results of individual realizations are shown. This reveals the existence of two distinct regimes. All simulations have $\text{Ca} = 10^{-4}$.

5. Conclusion

Using a two-dimensional network simulator with fixed saturation we have explored a system which is similar, but not identical to the experiment of Tallakstad et al [19, 20]. This has been done for both viscosity matched fluids and for a viscosity ratio $M = 10^{-4}$ which is close to that of the experiment.

We have found power-laws both for viscosity matched fluids and for the high viscosity ratio. We find that there are several values for the exponent $\beta$, which depends on the non-wetting saturation $S$. The power-laws cover more than three decades in $\text{Ca}$. At the large $\text{Ca}$ limit of the law the permeability becomes the single-phase permeability of the system, i.e., effects due to capillaries cancel out and do not affect macroscopic observables. The power-law is lost when $\text{Ca}$ reaches a low value, beneath which $k_{\text{eff}}$ is independent of $\text{Ca}$. This transition occurs when one of the two phases becomes immobilized, which can be for $\text{Ca}$ as low as $\approx 10^{-5}$, depending on $S$ and $M$.

We have found that two regimes coexist within the parameter space of free fractional flow, a large-exponent well-mixed regime and a small-exponent regime with stable, immobile clusters and hysteresis. The experimental exponent indicates that forced fractional flow belongs to the latter regime.

We thank Santanu Sinha for valuable discussions.
References

[1] Dullien F 1992 *Porous Media: Fluid Transport and Pore Structure*, 2nd ed. (Academic Press, San Diego)
[2] Sahimi M 1995 *Flow and transport in porous media and fractured rock* (VCH Verlagsgesellschaft)
[3] Bak P, Tang C and Wiesenfeld K 1987 *Physical Review Letters* 59 381–384
[4] Witten T A and Sander L M 1981 *Physical Review Letters* 47 1400–1403
[5] Wilkinson D and Willemsen J F 1983 *Journal of Physics A: Mathematical and General* 16 3365
[6] Furuberg L, Feder J, Aharony A and Jøssang T 1988 *Physical Review Letters* 61 2117–2120
[7] Lenormand R, Touboul E and Zarcone C 1988 *Journal of Fluid Mechanics* 189 165–187
[8] Måloøy K J, Feder J and Jøssang T 1985 *Physical Review Letters* 55 2688–2691
[9] Chen J D and Wilkinson D 1985 *Physical Review Letters* 55 1892–1895
[10] Constantinides G and Payatakes A 1996 *AIChE Journal* 42 369–382
[11] Aker E, Måloøy K, Hansen A and Batrouni G 1998 *Transport in Porous Media* 32 163–186
[12] Knudsen H, Aker E and Hansen A 2002 *Transport in Porous Media* 47 99–121
[13] Knudsen H and Hansen A 2002 *Physical Review E* 65 56310
[14] Knudsen H and Hansen A 2004 *EPL (Europhysics Letters)* 65 200–206
[15] Knudsen H and Hansen A 2006 *The European Physical Journal B* 49 109–118
[16] Hansen A and Ramstad T 2009 *Computational Geosciences* 13 227–234
[17] Avraam D, Kolonis G, Roumeliotis T, Constantinides G and Payatakes A 1994 *Transport in Porous Media* 16 75–101
[18] Avraam D and Payatakes A 1995 *Journal of Fluid Mechanics* 293 207–236
[19] Tallakstad K, Knudsen H, Ramstad T, Lovoll G, Måloøy K, Toussaint R and Flekkøy E 2009 *Physical Review Letters* 102 74502
[20] Tallakstad K, Lovoll G, Knudsen H, Ramstad T, Flekkøy E and Måloøy K 2009 *Physical Review E* 80 36308
[21] Grøva M and Hansen A 2011 *preprint*
[22] Meijerink J and van der Vorst H 1977 *Mathematics of Computation* 31 148–162