Coupled-channel calculation of bound and resonant spectra of $^9\Lambda$Be and $^{13}\Lambda$C hypernuclei.

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Abstract

A Multi-Channel Algebraic Scattering (MCAS) approach has been used to analyze the spectra of two hypernuclear systems, $^9\Lambda$Be and $^{13}\Lambda$C. The splitting of the two odd-parity excited levels ($\frac{1}{2}^-$ and $\frac{3}{2}^-$) at 11 MeV excitation in $^{13}\Lambda$C is driven mainly by the weak $\Lambda$-nucleus spin-orbit force, but the splittings of the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ levels in both $^9\Lambda$Be and $^{13}\Lambda$C have a different origin. These cases appear to be dominated by coupling to the collective $2^+$ states of the core nuclei. Using simple phenomenological potentials as input to the MCAS method, the observed splitting and level ordering in $^{9}\Lambda$Be is reproduced with the addition of a weak spin-spin interaction acting between the hyperon and the spin of the excited target. With no such spin-spin interaction, the level ordering in $^{9}\Lambda$Be is inverted with respect to that currently observed. In both hypernuclei, our calculations suggest that there are additional low-lying resonant states in the $\Lambda$-nucleus continua.

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I. INTRODUCTION

The study of hypernuclei opens up a new dimension for probing the structure and reactions of nuclei, with the presence of the strangeness degree of freedom. One can carry out nuclear physics with a “tagged” fermion which is distinguishable from the others resulting in various exotic hyperon-effects, and consider new systems such as excess-neutron systems with an hyperon tag.

Hypernuclei have been the subject of quite extensive experimental studies and their excitation spectra are now accessible with high-quality measurements involving reactions such as $(\pi^+, K^+)$, $(K^-, \pi^-)$ and $(e,e'K^+)$ \cite{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, especially with the new generation of germanium-based detectors for high-precision $\gamma$-ray spectroscopy. The advances in $\Lambda$-hypernuclear spectroscopy have been reviewed recently \cite{11, 12}. In those reviews, important aspects of the role of the hyperon in nuclear systems have been highlighted. Effects such as the “shrinkage” of the hypernuclear size \cite{13}, the (weak) role of spin-orbit splitting in light hypernuclei, and the connection of the hypernuclear excitation spectra with the in-medium hyperon-nucleon interaction were noted; the last with regard to its specific spin structure in particular.

Most theoretical studies of hypernuclei structure have sought to use experimental data to define properties of the underlying $\Lambda$-nucleon interaction, with the many-nucleon properties specified by shell-model states \cite{14, 15} or by mean-field theories \cite{16}. Recently, a mean-field model \cite{17} assessed the level of collectivity in hypernuclei through particle-hole RPA-like calculations, with the indication that the role of compound collectivity in hypernuclei is much less important than in ordinary nuclei. While this gives support to mean-field approaches in the description of hypernuclear spectra, further exploration of the role of ordinary nuclear collectivity due to the core dynamics coupled to the single-hyperon motion is warranted.

Additionally, and specifically for the two hypernuclear systems, $^9\Lambda$Be and $^{13}\Lambda$C, that we consider, cluster-model calculations have been made \cite{18}. In this context, the two hypernuclei were considered as a three-body (\$^{\alpha\alpha\Lambda}$) and a four-body (\$^{\alpha\alpha\alpha\Lambda}$) system, respectively. That study sought to define characteristics of the $\alpha\Lambda$ interaction and, by de-convolution, of the $\Lambda$-nucleon one. Of particular concern was the $\Lambda$-nucleon spin-orbit term. With $^9\Lambda$Be, Hiyama et al. \cite{18} took the ground state to be an $s_{1/2}$-wave $\Lambda$ coupled to the $0^+$ ground state of $^8\Lambda$Be. They then evaluated energies of a doublet $(3/2^+, 5/2^+)$ considered dominantly composed of an $s_{1/2}$-wave $\Lambda$ coupled to the $2^+_1$ state of $^8\Lambda$Be. Splittings of between 0.08 and 0.2 MeV were found using diverse parameter sets. From their cluster model calculation of $^9\Lambda$Be, Filikhin, Gal, and Suslov \cite{19} found that the $p$-wave $\Lambda$-$\alpha$ interaction played a role in defining the binding energy of $^9\Lambda$Be as well as in determining the excitation energies of the positive-parity doublet. They suggest that the residual mismatch to experimental values may be attributed to three-body forces.

In this paper we explore the possible description of the levels of both hypernuclei $^9\Lambda$Be and $^{13}\Lambda$C in terms of a phenomenological $\Lambda$-light mass nucleus interaction which explicitly couples the hyperon to the collective low-lying states of the ordinary nuclear core. The phenomenological character of an appropriate $\Lambda$-light mass nucleus interaction was established in recent reviews of hypernuclear theory and experiments \cite{11, 12, 20}. Essentially it has a central depth of $\sim 30$ MeV (with a Woods-Saxon form) and a spin-orbit attribute considerably weaker than that of a nucleon-nucleus interaction.

In this work we do not investigate the microscopic origin of such model interaction. That would require a folding made to deduce $\Lambda$-nucleus (optical) potentials from some specific $\Lambda$-
nucleon effective force model. This would be similar to the nucleon-nucleus optical-potential model constructed by $g$-folding [21], which combines detailed nuclear structure information with $NN$ effective interactions that have central, two-body spin-orbit, and tensor components, with the results being a very non-local, energy and medium dependent optical potential that has central and spin-orbit character.

We employ the MCAS approach to particle-nucleus structure and scattering [22] since it emphasises the couplings of single-particle dynamics with low-lying collective excitations of the ordinary nuclear core. The MCAS approach is holistic in that it considers the system of a hyperon and a nucleus as a dynamic coupling of that hyperon to the whole nucleus, allowing complete freedom to the channel-coupling dynamics of the selected states. That this dynamics may play a significant role in defining hypernuclear states has been conjectured recently by Hashimoto and Tamura [11], who noted that when a hyperon is added to a nucleus, nuclear properties of the compound vary from those of that when an extra nucleon is considered in place of the hyperon. A quote from that article on this aspect reads: “Nuclear properties such as shape, size, symmetry, cluster and shell structures, and collective motions may be altered”.

The role of MCAS studies to date, has been to analyze bound and resonant spectra to support and interpret experimental investigations. When the effects of the Pauli principle were incorporated in nucleon-nucleus dynamics, the method has been shown to describe, consistently, the bound and resonant spectra of normal (zero-strangeness) light-mass nuclei [22, 23, 24, 25]. Starting with the properties of spectra of non-strange nuclei, we consider the modifications to the Hamiltonians in MCAS that are required to describe hyperon-nucleus dynamics. In particular, we analyze low-lying level structures of two $p$-shell $\Lambda$ hypernuclei with regards to the structure of the hypernuclear doublet levels. We consider splittings that have been measured recently and, as well, find level structures that are just above the $\Lambda$-nucleus scattering threshold. Perhaps, they may be observed in future experiments.

We discuss, in brief, the MCAS method in the next section, while the results for $^9\Lambda$Be and $^{13}\Lambda$C are presented and discussed in Sections III and IV respectively. Conclusions are given in Section V.

II. THE MCAS APPROACH.

We describe the interaction Hamiltonian for the hyperon-core system within a phenomenological potential model which couples ground state and low-lying core excitations

$$V_{cc'}(r) = \sum_n V_n \mathcal{O}_n f_n(r, R, \theta_{r,R}).$$  \hspace{1cm} (1)

We assume that these low-lying excitations are of rotational collective nature generated by a permanent quadrupole deformation of the core subsystem. The deformed nuclear surface is then described by the radial angular dependence, $R = R_0 [1 + \beta_2 P_2(\theta)]$, and leads to a second-order expansion (in the $\beta_2$ parameter) of the nuclear-interaction functions $f_n$

$$f_n(r, R, \theta) = f_n^{(0)}(r) - \beta_2 R_0 P_2(\theta) \frac{d}{dr} f_n^{(0)}(r) + \frac{\beta_2^2 R_0^2}{2 \sqrt{\pi}} \left( P_0 - \frac{2\sqrt{5}}{7} P_2(\theta) + \frac{2}{7} P_4(\theta) \right) \frac{d^2}{dr^2} f_n^{(0)}(r).$$  \hspace{1cm} (2)
In the calculations we have made, phenomenological Woods-Saxon functions and their derivatives have been used. Also we have used just three types of operators, $O_n$, namely central $n = 0$, spin-orbit, $n = \ell s$, and spin-spin, $n = sI$. We have not considered other possible terms in the hyperon-nucleus interaction since the amount of data available for hypernuclei is too limited as yet to allow for the determination of other small components. For this and other details in the general MCAS scheme we refer to Ref. [22]. The potential we employ generates a coupled-channel description of the dynamics, which couples the ground-state to the first $2^+$ state, as well as to possible higher excitations of the core such as the second-order $0^+$, $2^+$, and $4^+$ levels.

In the MCAS approach, sturmian functions are chosen as a basis with which coupled-channel interaction potentials can be expanded. Each element of the interaction matrix then becomes a sum of separable interactions. The analytic properties of the $S$-matrix from a separable Schrödinger potential in momentum space allow a full algebraic solution of the multichannel scattering problem. With MCAS then, one starts with a coupled-channel system that describes nucleon-core dynamics with explicit inclusion of the low-lying excitations of the core. The Λ-nucleus system is analogous as the Λ has spin $\frac{1}{2}$ and a similar mass to the nucleon. However, the Pauli principle, which is so important when dealing with the nucleon-nucleus systems [26], is not relevant in an MCAS treatment of the Λ-nucleus ones.

With the MCAS method, for each allowed scattering spin-parity, $J^\pi$, one solves Lippmann-Schwinger integral equations in momentum space, i.e.

$$T_{cc'}(p, q; E) = V_{cc'}(p, q) + \frac{2\mu}{\hbar^2} \sum_{\text{open}} \int_0^\infty V_{cc''}(p, x) \frac{x^2}{k_{c''}^2 - x^2 + i\epsilon} T_{c''c'}(x, q; E) \, dx - \sum_{\text{closed}} \int_0^\infty V_{cc''}(p, x) \frac{x^2}{\hbar^2_{c''} + x^2} T_{c''c'}(x, q; E) \, dx,$$

(3)

where the index $c$ denotes the set of quantum numbers that identify each channel uniquely. To use this approach, one must specify input potential matrices $V_{cc''}(p, q)$. The open and closed channels contributions have channel-wave numbers $k_c$ and $\hbar c$ which are defined for $E > \mathcal{E}_c$ and $E < \mathcal{E}_c$ respectively. $\mathcal{E}_c$ are the excitation energies of the associated target state in channel $c$, and $\mu$ is the reduced mass. Solutions of Eq. (3) are found using expansions of the potential matrix elements in (finite) sums of energy-independent separable terms,

$$V_{cc'}(p, q) \sim \sum_{n=1}^N \chi_{cn}(p) \ \eta_n^{-1} \ \chi_{c'n}(q).$$

(4)

The method involves expansion form factors, $\chi_{cn}(q)$, that are built using sturmian functions associated with the actual (coordinate space) model interaction potential matrices, $V_{cc'}(r)$, we take to describe the coupled-channel problem. The eigenvalues of the sturmian equations for all channels are the set of $\eta_n$. When they are listed in order of decreasing size, each term in the expansion, Eq. (4), scales similarly permitting truncation of the series. That makes the matrix problem one of finite dimension.

The link between the multichannel $T$- and the scattering ($S$-) matrices involves the nu-
merical integration of a Green’s function matrix,

\[
(G_0)_{nn'} = \frac{2\mu}{\hbar^2} \left[ \sum_{c=1}^{\text{open}} \int_0^\infty \chi cn(x) \frac{x^2}{k_c^2 - x^2 + i\epsilon} \chi cn'(x) \, dx - \sum_{c=1}^{\text{closed}} \int_0^\infty \chi cn(x) \frac{x^2}{k_c^2 + x^2} \chi cn'(x) \, dx \right],
\]

with a diagonal eigenvalue matrix, \((\eta)_{nn'} = \eta_n \delta_{nn'}\). The bound states of the compound system are defined by the zeros of the matrix determinant for energy \(E < 0\). They link to the zeros of \(\{|\eta - G_0|\}\) when all channels in Eq. (5) are closed.

Elastic scattering observables are determined by the on-shell properties \((k_1 = k_1' = k)\) of the scattering matrices. For the elastic scattering of spin \(\frac{1}{2}\) particles from spin zero targets, \(c = c' = 1\), and the \(S\)-matrix, \(S_{11} \equiv S_{\ell} = S_{\ell}^{(+)}\), is

\[
S_{11} = 1 - i\pi \frac{2\mu}{\hbar^2} k \sum_{nn'} \chi_{1n}(k) \frac{1}{\sqrt{\eta_n}} \left[ \left( 1 - \eta^{-\frac{1}{2}} G_0 \eta^{-\frac{1}{2}} \right)^{-1} \right]_{nn'} \frac{1}{\sqrt{\eta_{n'}}} \chi_{1n'}(k).
\]

(6)

Diagonalizing the complex-symmetric matrix,

\[
\sum_{n' = 1}^{N} \eta_{n'}^{-\frac{1}{2}} G_0_{nn'} \eta_{n'}^{-\frac{1}{2}} \tilde{Q}_{n'r} = \zeta_r \tilde{Q}_{nr},
\]

establishes the evolution of the complex eigenvalues \(\zeta_r\) with respect to energy, and that defines the resonance attributes. It can be shown \[22\] that

\[
\left[ \left( 1 - \eta^{-\frac{1}{2}} G_0 \eta^{-\frac{1}{2}} \right)^{-1} \right]_{nn'} = \sum_{r=1}^{N} \tilde{Q}_{nr} \frac{1}{1 - \zeta_r} \tilde{Q}_{n'r}.
\]

(8)

Resonant behaviour occurs when one of the complex \(\zeta_r\) eigenvalues passes close to the point \((1,0)\) in the Gauss plane, since the elastic channel \(S\)-matrix has a pole structure at the corresponding energy.

III. THE \(^9\Lambda\)BE SYSTEM

Before considering the \(^9\Lambda\)Be hypernucleus, we consider first the ordinary (non-strange) \(^9\)Be nucleus. To use the MCAS scheme, we view \(^9\)Be as an extended two-body system. We presume the neutron-\(^8\)Be core system involves coupling to the first and second states of \(^8\)Be (a \(2^+\) at 3.03 MeV and a \(4^+\) at 11.35 MeV, respectively). Though those two states are resonances in their own right, in calculation we take them to be discrete and to couple via a rotational model prescription as has been used heretofore \[22, 23, 24\].

In this study we consider only the positive-parity states of the compound nucleus, and employ a most simple Hamiltonian interaction. A more extensive analysis of this system for states of both parities has been made recently \[27\]. The effect of the alpha-decay process of the two excited levels of \(^8\)Be was included in the coupled-channel dynamics. While we refer to that work for full details, it is sufficient here to note that the decay-widths of the two excited \(^8\)Be states had an important effect on the widths of the compound levels, but did not change significantly their energy centroids.

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TABLE I: Parameter values of the $n-^8\text{Be}$ interaction.

| Parameter | Value |
|-----------|-------|
| $V_0$ (MeV) | $-43.2$ |
| $R_0$ (fm) | $2.7$ |
| $V_{ls}$ (MeV) | $10.0$ |
| $a$ (fm) | $0.65$ |
| $\beta_2$ | $0.7$ |

The neutron-core coupled-channel Hamiltonian we use for the positive-parity states is defined by the parameter values listed under the heading $n+^8\text{Be}$ in Table I. We do not present extensive aspects since we seek only the bulk features of the $n-^8\text{Be}$ system to construct a phenomenologically plausible interaction for the $\Lambda-^8\text{Be}$ system.

Since the deeply bound $s$ states are already occupied by the core nucleons, an appropriate OPP term has been included in the nucleon-core Hamiltonian [26]. However, when considering the hyperon-nucleus system, these OPP terms are not used since the hyperon is distinguishable from the nucleons and access to the deeply-bound orbits is not hindered by Pauli exclusion. Some $^9\text{Be}$ states that result from the MCAS evaluation are listed in Table II. Clearly the bulk features of the positive-parity spectrum can be reproduced with

TABLE II: The low lying positive-parity spectra of $^9\text{Be}$ compared with the results of an MCAS evaluation of $n+^8\text{Be}$ (in MeV).

| $J^\pi$ | Exp. [28] | Theory |
|---------|-----------|--------|
| $\frac{3}{2}^+$ | 5.0946 | 4.82 |
| $\frac{1}{2}^+$ | 3.0386 | 3.02 |
| $\frac{5}{2}^+$ | 1.3836 | 1.06 |
| $\frac{3}{2}^+$ | 0.0186 | -0.05 |

a remarkably simple coupled-channel Hamiltonian.

Most of the recent work on $^9\Lambda\text{Be}$ concentrates on the ground level and on the splitting of the two first excited levels $\frac{5}{2}^+$ and $\frac{3}{2}^+$ [12, 20, 29, 30]. These states are all of positive parity and therefore herein we also consider only the positive-parity spectrum. However, we should mention for completeness that earlier theoretical works extensively dealt with predictions on low-energy negative-parity states. Such states were first predicted by Dalitz and Gal, using shell model [31, 32]. Amongst these, specific hypernuclear states termed “supersymmetric states” have been considered and later described as “genuine hypernuclear states” in Refs. [33, 34], with the $\Lambda$ hyperon moving on a $p$-wave orbit parallel to the symmetric $\alpha-\alpha$ axis. Based on comparison with the predicted spectra, a few peaks at about 6 and 10 MeV excitation in the E336 experiment [35] were interpreted as generated by those specific states.

To initiate study of the $\Lambda-^8\text{Be}$ system we applied scalings to the $n-^8\text{Be}$ Hamiltonian adjusting the parameter values by reducing the central potential by 30%, reducing the interaction radius by 15%, and reducing the spin-orbit strength by an order of magnitude. Small adjustments were then made to give best results. The final values of the coupled-channel potential parameters are given in Table III. In this Table, we consider two sets of parameters, one with the onset of a small spin-spin component of the interaction (‘Case 2’). Our choice of the spin-spin interaction strength was made to be comparable with that
TABLE III: Strengths of the $^{\Lambda-8}$Be interaction with $R_0 = 2.3$ fm., $a = 0.65$ fm., and $\beta_2 = 0.7$

|                | Case 1 | Case 2 |
|----------------|--------|--------|
| $V_0$ (MeV)    | $-26.4$| $-26.4$|
| $V_{ls}$ (MeV) | 0.35   | 0.35   |
| $V_{st}$ (MeV) | 0.0    | $-0.1$ |

of the spin-orbit interaction and by the knowledge that the hypernuclear state splittings are themselves small. There are small spin-spin, spin-orbit, and tensor interactions in most phenomenological $\Lambda$-nucleon interactions (see Table 6 in Ref. [14] for an example of their effects). Such components albeit stronger, are present in nucleon-nucleon forces, the folding of which with microscopic models of structure [21] lead to nucleon-nucleus central and spin-orbit interactions. So one should allow for such terms in effective $\Lambda$-nucleus potentials. Furthermore, and in parallel with the phenomenological interactions required in MCAS studies of nucleon-nucleus systems [22]-[27], we expect there to be ($\Lambda$-nucleus) spin-spin terms. Given the small strengths of both spin-orbit and spin-spin interactions in the $\Lambda$-nucleon potentials, we anticipate the both will also be small in the $\Lambda$-nucleus interactions. Herein our purpose is to identify the effects of such an interaction rather than ascertaining what would be an optimum value. For the study of the $\Lambda-^{8}$Be system, values of 0 and -0.1 for this term suffice. We do study effects of variation in this term with our study of the $\Lambda-^{12}$C system, though only over the range between ±0.1.

In Table [IV] we give the spectrum calculated with the MCAS approach and the two sets of parameters given in Table [III]. We report the experimental binding energy of $^{9}_\Lambda$Be as determined from emulsion data [36], but have also indicated in brackets the additional, newer value obtained by the E336 experiment at KEK (see Ref. [11]). It has been already observed by Hashimoto and Tamura [11] that, while the emulsion data for $^{7}_\Lambda$Li and $^{13}_\Lambda$C agreed well with the KEK-experiment results, there was significant disagreement between those regarding the $^{9}_\Lambda$Be ground-state binding energy, with the reason for this disagreement not known. Also, we consider for comparison shell-model results. These results [15] were obtained by considering not only the $\Lambda N$ and $\Sigma N$ interactions but also the $\Lambda \Sigma$ coupling, which was shown to make an important contribution to the spacing of the $1^+$ and $0^+$ states in $^{4}_\Lambda$H and $^{4}_\Lambda$He. The $\Lambda$ was assumed to be in the $0s$ orbit while the nucleons were assumed to be in the $0p$-shell. Comparison with the bound-state spectrum obtained from MCAS is quite good.

It is interesting to observe that we get the correct size of this fine splitting between these two states with a simple phenomenological model consisting only of a central and a spin-orbit potential (‘Case 1’ in Table [III]). Indeed, assuming no $sI$ coupling, the magnitude of the splitting between the $^{3}\frac{3}{2}^+$ and $^{5}\frac{5}{2}^+$ states is very small, but consistent with the separation value recently measured [37]. But it is predicted that the $^{3}\frac{3}{2}^+$ state to be the lower, at variance with the recent analysis [29], where the $^{3}\frac{3}{2}^+$ state was assessed to be the less bound of the pair.

The shell model calculations [15] gave the observed splitting and order of the three bound states of $^{9}_\Lambda$Be and from them it was noted that the $\Lambda$-nucleon spin-orbit interaction was primarily responsible for the $^{3}\frac{3}{2}^+ -^{5}\frac{5}{2}^+$ states splitting. Other components in the chosen interaction (spin-spin, tensor, etc.) contributed values that essentially cancelled. But, it is to be noted...
TABLE IV: The positive-parity spectrum, in MeV, of $^9\bar{\Lambda}$Be. The columns labelled ‘Case 1’ and ‘Case 2’ refer, respectively, to calculations made without and with the spin-spin term in the potential. The spin-spin strength, $V_{sI}$, was $-0.1$ MeV. The widths of resonant states are given in brackets. Comparison is also made with the results of a shell model calculation [15], where the ground state binding energy has been set to the measured value for comparison.

| $J^+$  | Exp. [30] | Case 1 | Case 2 | Shell Model [15] |
|--------|-----------|--------|--------|------------------|
| $^2_2^+$ | $-$       | 4.791 (4.1 keV) | 4.92 (4.9 keV) |                  |
| $^4_2^+$ | $-$       | 4.788 (4.4 keV) | 4.68 (3.8 keV) |                  |
| $^5_2^+$ | $-3.64$   | $-3.70$ | $-3.63$ | $-3.66$          |
| $^3_2^+$ | $-3.69$   | $-3.65$ | $-3.70$ | $-3.71$          |
| $^4_2^+$ | $-6.71 (-5.99^\dagger)$ | $-6.73$ | $-6.73$ | $-6.71$          |

$^\dagger$ This value result was found from a 1998 experiment [11] (see the discussion in the text).

that the Λ-nucleon force is phenomenological. So it is the parameter set chosen for this force that gives that splitting and state order obtained with the shell-model calculation.

To achieve the correct level ordering with MCAS, requires the introduction of a small spin-spin contribution in the Λ-$^8$Be phenomenological interaction (‘Case 2’). This need is emphasised by considering the variation with deformation of the binding energies of the two positive-parity states in question. This is shown in Fig. 11 for the two parameter sets, ‘Case 1’ (bottom panel) and ‘Case 2’ (top panel). With no spin-spin interaction strength (Case 1), the splitting of the states is small but with the $^3_2^+$ state always the more bound of the pair. In contrast are the results when the spin-spin interaction strength is finite though small as shown in the top panel of the figure. Now the $^5_2^+$ state is always the more bound of the pair. The actual size of the splitting varies slightly with the deformation but more so with the value of $V_{sI}$.

At 4.7 MeV above the scattering threshold, we predict two additional positive-parity states (resonances). These are formed by coupling the $4^+$ state in $^8$Be with a Λ in the $s_{1/2}$ state. The widths of these resonances shown in brackets in Table IV were calculated assuming that the $^8$Be-core $4^+$ state has zero width. If we consider the α-decay probability of this $4^+$ level, then the widths reported in the table can be expected to increase quite significantly [27].

IV. THE $^{13}_\Lambda$C SYSTEM

The bound and scattering properties of the $^{13}_\Lambda$C (and $^{13}_\Lambda$N) systems have been studied extensively using the MCAS approach [22, 23, 38]. Good reproduction of the low-lying spectra, sub-threshold and resonant states, and of the elastic scattering cross-section and polarization data have been obtained with a relatively simple model Hamiltonian. But such only resulted if the incoming nucleon (on $^{12}_C$) was blocked from access to the phase-space already occupied by closed shells (in this case, the $s_{1/2}$ and $p_{3/2}$ levels). With MCAS, that is achieved by using the Orthogonalizing Pseudo-Potential (OPP) method [26, 39].

There is no such requirement in using MCAS to study the Λ-$^{12}_C$ system. However, the depth of the Λ-nucleus optical potential is about $\frac{2}{3}$ that of the standard nucleon-nucleus
The spin-orbit strength also is much smaller, an order of magnitude smaller, than that of the corresponding nucleon-nucleus system. Additionally, the potential radius is \( \sim 15\% \) smaller than used for the \( n + 12 \) system. Starting from that set of parameter values, with but small adjustments, the coupled-channel potential interaction used in MCAS describes the known spectrum. The resultant final potential parameters we have used are listed in Table V. Again, the parameters identified as ‘Case 1’ were those we found with the spin-spin interaction strength set to zero, while those defined as ‘Case 2’ had the small spin-spin interaction strength listed. The parities of the channel interactions are designated by \( \pi = \pm 1 \).

To date, four state energies have been measured and they are listed in Table VI in the column labelled ‘Exp’. A splitting between the \( \frac{3}{2}^+ \) and \( \frac{5}{2}^+ \) states is not resolved as yet. The theoretical spectra, however, contain a richer structure as shown by the results listed under the ‘Case 1’ and ‘Case 2’ columns in the table. Both model calculations predict a \( \frac{1}{2}^+ \) bound state at 4.12 MeV below threshold. As the spin-spin interaction has no effect upon

FIG. 1: The binding energies of the \( J^\pi = \frac{3}{2}^+ \) (solid curves) and \( J^\pi = \frac{5}{2}^+ \) (dashed curves) states in \(^9\Lambda\)Be with variation of the deformation parameter. The horizontal lines denote the experimental values of the two states.
TABLE V: Strengths of the $\Lambda^{-12}$C interaction with $R_0 = 2.6$ fm., $a = 0.6$ fm., and $\beta_2 = -0.52$

| Case 1 | Case 2 |
|--------|--------|
| $\pi = -1$ | $\pi = +1$ | $\pi = -1$ | $\pi = +1$ |
| $V_0$ (MeV) | $-28.9$ | $-30.4$ | $-28.9$ | $-30.4$ |
| $V_\ell$ (MeV) | $0.35$ | $0.35$ | $0.35$ | $0.35$ |
| $V_s I$ (MeV) | $0.0$ | $0.0$ | $-0.1$ | $-0.1$ |

TABLE VI: Spectra of $^{13}\Lambda$C with energies in MeV. Nomenclature is as for Table IV. The shell model results are those received from Millener [40].

| $J^\pi$ | Exp. [41] | Case 1 | Case 2 | Shell Model [40] |
|--------|-----------|--------|--------|-----------------|
| $-^-$  | +4.65 (0.21 MeV) | +4.66 (0.23 MeV) |
| $-^-$  | +4.64 (0.22 MeV) | +4.63 (0.21 MeV) |
| $-^-$  | +4.28 (1.0 keV) | +4.31 (1.0 keV) |
| $-^-$  | +4.17 (1.0 keV) | +4.14 (1.0 keV) |
| $-^-$  | +3.10 (0.1 keV) | +3.15 (0.1 keV) |
| $-^-$  | +3.05 (>0.1 keV) | +3.02 (>0.1 keV) |
| $-^-$  | -0.708 | -0.74 |
| $-^-$  | -0.86 | -0.89 |
| $-^+$  | -4.12 | -4.12 |
| $-^+$  | -6.81 | -7.177 |
| $-^+$  | -6.81 | -7.178 |
| $-^+$  | -11.69 | -11.68 |

its excitation energy, this state corresponds to an $s_{1/2}^{-}\Lambda$ coupled to the $0^+_{2}^{-}$ state at 7.65 MeV in $^{12}$C, which is an highly exotic state as it corresponds to the coupling of the hyperon to the superdeformed Hoyle state. This state is not predicted by the shell model [40], as the $0p$-shell model of the underlying structure of $^{12}$C cannot predict the Hoyle state. The ground state and low-lying spectrum from the shell model for $^{13}$C, however, agree generally well with the predictions from MCAS. The $1^+_{2}^{-}$ state we expect at 7.56 MeV excitation in $^{13}\Lambda$C has been placed much higher in excitation (12.2 MeV) in the cluster model evaluations [18]. That is due to the strong state dependence of the $\Lambda$-nucleus interaction, found by s-wave folding in that model [42]. We have still to wait for more detailed experimental investigations of that spectrum to decide whether it is a strong state dependence effect or simply the extremely weak natural excitation of the second $0^+_{2}^{-}$ from the ground state of $^{12}$C that explains the position of a second $1^+_{2}^{-}$ state in the spectrum of this hypernucleus. Note that a similar state has been predicted by MCAS calculations of $n+^{12}$C to be in the spectrum of the non-strange $^{13}$C. That state has spin-parity $J^\pi = \frac{1}{2}^-$, lies 2.68 MeV above the scattering threshold, and though unobserved, is partner to a mirror state [23] of such structure in $^{13}$N at 6.97 MeV above the $p+^{12}$C threshold. That is 1.3 MeV higher than where a $J^\pi = \frac{1}{2}^-$ state has been observed in $^{13}$N.

Additionally, a set of six odd-parity states of $^{13}\Lambda$C are predicted to be just a few MeV
above the scattering threshold. They are states formed by coupling of a $p_{3/2}^-$ and of a $p_{1/2}^-\Lambda$ with the $2^+_1$ state in $^{12}\text{C}$. Without deformation (and no spin-spin interaction) they would form a degenerate quartet ($\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$, and $\frac{7}{2}^-$) and a degenerate doublet ($\frac{3}{2}^-$ and $\frac{5}{2}^-$) of states. Deformation and spin-spin effects then break those degeneracies and mix states of like spin-parity. Since these six states are embedded in the $\Lambda-^{12}\text{C}$ continuum, they are resonances and their widths are listed (in brackets) in Table VI. These resonance states are very narrow save for the doublet of $\frac{3}{2}^-|_3$ and $\frac{1}{2}^-|_2$ resonances. We have also calculated the corresponding excitation function, assuming a low-energy $\Lambda-^{12}\text{C}$ elastic scattering process.

![Diagram of energy levels and cross sections](image)

**FIG. 2**: The spectra and total elastic cross sections for $\Lambda-^{12}\text{C}$ scattering resulting from MCAS calculations made using two sets of parameter values.

The spectra for $^{13}\Lambda\text{C}$ are depicted in Fig. 2. Below the $\Lambda-^{12}\text{C}$ threshold the discrete bound states are shown while above that threshold the theoretical cross sections are given for both the case of $V_{sl} = 0.0$ and $-0.1$ MeV. Of prime interest is the fine splitting of the bound $\frac{1}{2}^-$ and $\frac{3}{2}^-$ levels, observed respectively at 10.98 MeV and 10.83 MeV above the ground state [37]. We note that there is also a very small splitting expected between two bound levels, $\frac{5}{2}^+$ and $\frac{3}{2}^+$, at the excitation energy of $\sim$4.88 MeV. Our exploratory calculation suggests a splitting
of $\sim 160$ keV for both doublets. Experimentally, the splitting of the $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states was found to be $152 \pm 54 \pm 36$ keV. The splitting of the more bound positive-parity states has not been determined quantitatively to date.

The results of an analysis of parameter variation on these splittings is presented in Fig. 3. In this figure with basic parameter values of $\beta_2 = -0.52$, $V_{sl} = -0.1$ MeV, and $V_{ts} = 0.35$ MeV, variation of the deformation parameter (with the other two parameters held fixed) gave the results depicted in the top panel. Likewise with the other two parameter values fixed, variations of the state energies with the strengths of the spin-spin and of the spin-orbit interactions are shown in the middle and bottom panels, respectively. We first notice that both splittings have only a weak dependence on the $\beta_2$ parameter value, however the actual binding energies of the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states vary by 600 keV over the range of $\beta_2$ chosen. The splitting of the two odd-parity states has essentially no dependence upon the spin-spin ($V_{sl}$) coupling, thus indicating that origin of the splitting of those negative-parity states is almost solely the result of spin-orbit effects. But the splitting of the positive-parity ($\frac{5}{2}^+$ and $\frac{3}{2}^+$) levels is dominated by the $V_{sl}$ coupling that links with the excitation of the $2^+_1$ (4.43 MeV) state in $^{12}$C. Indeed, as shown in the middle panel of Fig. 3, there is no splitting of the positive-parity doublet for $V_{sl} = 0$, with a linear trend for small non-zero values. But the $\frac{5}{2}^+$ state is more bound than the $\frac{3}{2}^+$ one for negative values of $V_{sl}$. Over the range of $V_{sl}$ strength shown, the splitting of the negative-parity states changes little, while that of the positive-parity states changes by 300 keV. Finally we show the effect of the spin-orbit interaction terms in the bottom panel. With variation of this strength, the positive-parity doublet splitting is essentially unchanged, as are the individual state binding energies. This is expected since the states involve coupling of an $s\frac{1}{2}$-Λ particle. On the other hand, the splitting of the $\frac{1}{2}^-\frac{3}{2}^-$ varies linearly with the strength of the spin-orbit force in the $\Lambda$-$^{12}$C interaction. These two states have also been found in shell model studies, taking them to be $p\frac{1}{2}$ and $p\frac{3}{2}$ Λ single-particle states coupled to the $0^+$ ground state of $^{12}$C. Their separation was thought to be a measure of the Λ-nucleon spin-orbit force. With that prescription, spin-spin and tensor components of the two-body force were found to be important. However, Millener [14, 40] notes that these states will have components involving coupling to the $2^+_1$ state in $^{12}$C, that the ground state of $^{12}$C has components that bring into play other attributes in the interactions with the hyperon, and that the $p$-wave Λ orbits are weakly bound.

These properties indicate that one can extract the Λ-nucleus spin-orbit strength from the splitting of the $\frac{1}{2}^-\frac{3}{2}^-$ states. Using the experimental information of a 152 keV splitting for the two odd-parity levels, we settle upon a Λ-nucleus spin-orbit strength of 0.35 MeV. We also conclude that the current knowledge of experimental spectra is insufficient to assess any importance of the Λ-nucleus $V_{sl}$ coupling in $^{13}$ΛC. For this reason, for the $\frac{5}{2}^+$, $\frac{3}{2}^+$ level splitting we considered the $^{9}\Lambda$Be system for which accurate experimental information on the splitting of such levels has been obtained [11]. That information leads to a significant constraint on the $V_{sl}$ coupling.

V. SUMMARY AND CONCLUSIONS

The MCAS approach to define scattering of two clusters and to describe the structure of the compound system formed of them requires a matrix of coupled-channel interactions. These describe the pairwise interactions between the clusters, in this case the hyperon and
the nucleus. This interaction matrix is related to the one we have found best describes the scattering and compound system of a neutron and that same nucleus. This approach to study hypernuclear spectroscopy is quite new and reveals effects of coupled channel dynamics in the spectroscopy which, to our knowledge, have not been considered before. As we consider what might be the hyperon-nucleus coupled-channel interaction within collective model prescriptions of the nucleus, of course there may be specifics that can be improved. But the results offer an explanation of aspects concerning the limited amount of assured levels for these systems alternate to the usual adjusted-interaction approaches.

Specifically, we have applied the MCAS approach to study the excitation spectra of light hypernuclei $^9_\Lambda$Be and $^{13}_\Lambda$C. The theoretical approach emphasises the single-particle motion of the hyperon in the mean-field of the ordinary nuclear core, which is however coupled to its own low-lying collective motions.

The phenomenological hyperon-core potential was constructed from the ordinary nucleon-core potential by removing Pauli-blocking effects, making a 10-15% shrinkage of the radius, using a $2/3$ reduction of the strength of the central potential, and reducing drastically the spin-
orbit potential. However, the coupling of the single hyperon with the collective motion of the core is rather important in defining the hypernuclear spectra. The deformation parameter is essentially that required in the associated nucleon-nucleus dynamics.

In light of the recently observed fine splitting of the excited hypernuclear spectra for these two nuclei we conclude that the $\frac{1}{2}^-$-$\frac{3}{2}^+$ splitting in $^{\Lambda}C$ is dominated by the $\Lambda$-core spin-orbit interaction. This splitting is largely insensitive to other factors such as the deformation of the core or other spin-dependent components of the $\Lambda$-core potential. On the other hand, the fine splittings of the $\frac{3}{2}^+ - \frac{5}{2}^+$ doublet in $^9\Lambda$Be and $^{13}\Lambda$C have a different dynamical origin. They originate from the coupling of the $s_\frac{1}{2}$-$\Lambda$ single-particle motion with the $2^+$ collective excitation of the core. As it involves an $s$-wave $\Lambda$-particle, it is independent of the spin-orbit interaction. While this splitting varies slightly with the deformation of the core, it is most sensitive to the coupling of the $s\frac{1}{2}$-$\Lambda$ single-particle motion with the $2^+$ collective excitation of the core through the $V_{sl}$ interaction. The recently measured structure of the fine ($\frac{3}{2}^+ - \frac{5}{2}^+$) splitting in $^9\Lambda$Be enabled us to determine the sign and strength of that spin-spin interaction. With the same strength for the spin-spin component taken for the $^{13}\Lambda$C system, a similar splitting of the $\frac{3}{2}^+ - \frac{5}{2}^+$ doublet results.

Finally, using the scattering properties embedded in the MCAS scheme, with the interaction potential so determined by the measured hypernuclear low-lying levels, we expect there to be resonant states in both hypernuclei at low excitation energies. The centroid energies and widths of those resonances we anticipate to be rather more sensitive to the deformation in these systems.

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[1] P. H. Pile et al., Phys. Rev. Lett. 66, 2585 (1991).
[2] T. Hasegawa et al., Phys. Rev. C 53, 1210 (1996).
[3] S. Ajimura et al., Nucl. Phys. A639, 93c (1998).
[4] H. Hotchi et al., Phys. Rev. C 64, 044302 (2001).
[5] H. Tamura, R. S. Hayano, H. Outa, and Y. Yamazaki, Prog. Theor. Phys. Suppl. 117, 1 (1994).
[6] M. Agnello et al., Phys. Lett. B622, 35 (2005).
[7] M. Agnello et al., Nucl. Phys. A754, 399c (2005).
[8] T. Miyoshi et al., Phys. Rev. Lett. 90, 232502 (2003).
[9] F. Garibaldi, Eur. Phys. J. A 24, sl, 91 (2005).
[10] M. Iodice et al., Phys. Rev. Lett. 99, 052501 (2007).
[11] O. Hashimoto and H. Tamura, Prog. Part. Nucl. Phys. 57, 564 (2006).
[12] H. Tamura et al., Nucl. Phys. **A804**, 73 (2008).
[13] K. Tanida et al., Phys. Rev. Lett. **86**, 1982 (2001).
[14] D. J. Millener, Nucl. Phys. **A691**, 93c (2001).
[15] D. J. Millener, Nucl. Phys. **A754**, 48c (2005).
[16] D. Vretenar, W. Pöschl, G. A. Lalazissis, and P. Ring, Phys. Rev. C **57**, R1060 (1998).
[17] M. Martini, V. D. Donno, C. Maieron, and G. Co, Nucl. Phys. **A813**, 212 (2008).
[18] E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, and Y. Yamamoto, Phys. Rev. Lett. **85**, 270 (2000).
[19] I. Filikhin, A. Gal, and V. M. Suslov, Nucl. Phys. **A743**, 194 (2004).
[20] D. J. Millener, Lect. Notes Phys. **724**, 31 (2007).
[21] K. Amos, P. J. Dortmans, H. V. von Geramb, S. Karataglidis, and J. Raynal, Adv. in Nucl. Phys. **25**, 275 (2000).
[22] K. Amos, L. Canton, G. Pisent, J. P. Svenne, and D. van der Knijff, Nucl. Phys. **A728**, 65 (2003).
[23] G. Pisent, J. P. Svenne, L. Canton, K. Amos, S. Karataglidis, and D. van der Knijff, Phys. Rev. C **72**, 014601 (2005).
[24] L. Canton, G. Pisent, J. P. Svenne, K. Amos, and S. Karataglidis, Phys. Rev. Lett. **96**, 072502 (2006).
[25] L. Canton, G. Pisent, K. Amos, S. Karataglidis, J. P. Svenne, and D. van der Knijff, Phys. Rev. C **74**, 064605 (2006).
[26] L. Canton, G. Pisent, J. P. Svenne, D. van der Knijff, K. Amos, and S. Karataglidis, Phys. Rev. Lett. **94**, 122503 (2005).
[27] P. Fraser, K. Amos, L. Canton, G. Pisent, S. Karataglidis, J. Svenne, and D. van der Knijff, Phys. Rev. Lett. **101**, 0242501 (2008).
[28] D. Tilley et al., Nucl. Phys. **A745**, 155 (2004).
[29] H. Tamura, Nucl. Phys. **A752**, 155c (2005).
[30] H. Akikawa et al., Phys. Rev. Lett. **88**, 082501 (2002).
[31] R. H. Dalitz and A. Gal, Phys. Rev. Lett. **36**, 362 (1976).
[32] E. H. Auerbach, A. J. Daltz, C. B. Dover, A. Gal, S. H. Kahana, L. Ludeking, and D. J. Millener, Ann. Phys. (NY) **148**, 381 (1983).
[33] T. Motoba, H. Bando, and K. Ikeda, Prog. Theor. Phys. **70**, 189 (1983).
[34] T. Yamada, K. Ikeda, H. Bando, and T. Motoba, Phys. Rev. C **38**, 854 (1988).
[35] O. Hashimoto et al., Nucl. Phys. **A639**, 93c (1998).
[36] M. K. Juric et al., Nucl. Phys. **B52**, 1 (1973).
[37] H. Tamura et al., Nucl. Phys. **A754**, 58c (2005).
[38] J. P. Svenne, K. Amos, S. Karataglidis, D. van der Knijff, L. Canton, and G. Pisent, Phys. Rev. C **73**, 027601 (2006).
[39] L. Canton, K. Amos, S. Karataglidis, G. Pisent, J. P. Svenne, and D. van der Knijff, Nucl. Phys. **A790**, 251c (2007).
[40] D. J. Millener, private communication (2009).
[41] H. Kohri et al., Phys. Rev. C **65**, 034607 (2002).
[42] T. Yamada, T. Motoba, K. Ikeda, and H. Bando, Prog. Theo. Phys. Japan (supp) **81**, 104 (1985).