Finite-Time $H_\infty$ Control for Itô-Type Nonlinear Time-Delay Stochastic Systems

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ABSTRACT The finite-time $H_\infty$ control problem for an Itô-type stochastic system with nonlinear perturbation and time delay is investigated. First, the finite-time $H_\infty$ control problem for a nonlinear time-delay stochastic system is presented taking into consideration both the transient performance and the capability to attenuate the disturbance of a closed-loop system in a given finite-time interval. Second, using the Lyapunov–Krasoviskii functional method and the matrix inequality technique, some sufficient conditions for the existence of finite-time $H_\infty$ state feedback controller and dynamic-output feedback controller for nonlinear time-delay stochastic systems are obtained. These conditions guarantee the mean-square finite-time bounded-ness of the closed-loop systems and determine the $H_\infty$ control performance index. Third, this problem is transformed into an optimization problem with matrix inequality constraints, and the corresponding algorithms are given to optimize the $H_\infty$ performance index and obtain the maximum time-delay. Finally, a numerical example is used to illustrate the effectiveness of the proposed method.

INDEX TERMS Stochastic systems, nonlinear perturbations, finite-time stability, $H_\infty$ control, time-delay.

I. INTRODUCTION

In recent years, control problems of stochastic nonlinear systems have been receiving increased attention because of their extensive applications in many practical systems, such as liquid-level systems [1], chemical reactor systems [2], [3], and industrial and economic systems [4]. In addition, many excellent results have been published. For example, [5] proposed three stochastic nonlinear control schemes to study the global stabilization of stochastic nonlinear systems. In [6], finite-time stability for stochastic nonlinear systems was considered and a general Lyapunov theorem of stochastic finite-time stability was proved. A finite-time tracking problem of switched stochastic nonlinear uncertain systems was studied in [7]. Some other nice results can be referred to [8]–[10]. In considering the influence of time-delays on the system, much of the focus has been on a general model of stochastic nonlinear time-delay systems. To date, numerous results on these systems have been obtained. For instance, [11] studied the stability of a class of nonlinear uncertain stochastic time-delay systems, and a sufficient delay-dependent criterion was established by constructing a new Lyapunov–Krasovskii function. The output feedback adaptive tracking control problem for a class of stochastic nonlinear time-delay systems was studied in [12], and an observer-based adaptive neural quantization tracking control scheme was proposed. For other excellent results, the reader is referred to [13]–[15] and references therein.

At present, most of the results of stochastic systems are based on the asymptotic stability in the Lyapunov sense, which only concerns the asymptotic behavior of the system in the limit of infinite time. However, the transient behavior is also significant in many practical systems. For example, a large transient voltage can destroy the normal operation of a power system [16]. To deal with this problem, the concept of finite-time stability was proposed, and many interesting results have been published, such as the finite-time stability of stochastic discrete-time-varying systems in [17], stochastic Markov jump systems in [18]–[22], stochastic time-delay systems in [23], and T-S fuzzy systems in [24]. In contrast,
$H_\infty$ control is one of the most crucial robust control methods because an external interference can be suppressed; many results have been reported. For instance, [25] and [26] are devoted to the robust $H_\infty$ control problem for uncertain stochastic nonlinear systems with time-varying delays and stochastic nonlinear uncertain T-S fuzzy systems with time-delay. Robust $H_\infty$ filtering and control for a class of linear systems with fractional stochastic noise were studied in [27]. Other nice results are to be found in [28]–[31]. Taking advantage of finite-time stability and $H_\infty$ control these systems were investigated in our study. The main contributions from this study are: (i) a precise statement of the finite-time $H_\infty$ control problem for Itô-type stochastic nonlinear systems with time-delay that considers both the transient performance and the capability of attenuating the disturbances in the closed-loop systems in a given finite-time interval; (ii) two new conditions developed from matrix inequalities concerning the sufficiency for the existence of two $H_\infty$ controllers one providing state feedback and the other dynamic-output feedback; and (iii) the establishment of two algorithms that solve the gain parameter settings of the two controllers and that optimize the $H_\infty$ performance index and maximum time-delay, simultaneously.

The rest of this paper is organized as follows. Section II gives some preliminaries, definitions, and lemmas. In Section III, we provide the conditions of sufficiency for the existence of the finite time $H_\infty$ controllers for Itô-type stochastic nonlinear time-delay systems. Section IV provides the two algorithms that solve the theorems. Section V presents a numerical example to demonstrate the effectiveness of the proposed method. In the last section, our conclusions are stated.

Notation: $A^\top$ denotes the transpose of matrix $A$; $tr(A)$ denotes the trace of matrix $A$; $A>0$ $(A\geq 0)$ signifies that $A$ is a positive definite (positive semi-definite) matrix; $\lambda_{\text{max}}(A)$ $(\lambda_{\text{min}}(A))$ denotes the maximum (minimum) eigenvalue of matrix $A$; $I_{n\times n}$ denotes the $n$-dimensional identity matrix; $\mathcal{R}^n$ denotes an $n$-dimensional Euclidean space; $E$ represents the mathematical expectation of a random process; an asterisk “$*$” in a matrix marks elements to be obtained by the symmetry of the matrix.

II. PRELIMINARIES

Consider an Itô-type stochastic nonlinear system with time delay described by

$$
\begin{align*}
\dot{x}(t) &= (A_{11}x(t) + A_{12}x(t - \tau) + B_{11}u(t) + B_{12}y(t))dt + (A_{21}x(t) + A_{22}x(t - \tau) + H_1(x(t)))dw(t),
\end{align*}
$$

where $x(t)$ denotes the state of the system, $y(t)$ the measurement output, $u(t)$ the control input, $z(t)$ the control output, $\phi(t)$ the initial state output, and $w(t)$ a one-dimensional standard Wiener process defined on probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$. $\mathcal{F}_t$ stands for the smallest $\sigma$-algebra generated by $w(s)$, with $0\leq s \leq t$, i.e., $\mathcal{F}_t = \sigma\{w(s)\mid 0 \leq s \leq t\}$. $A_{11}, A_{12}, A_{21}, A_{22}, B_{11}, B_{12}, C_1, C_2$ are constant matrices with appropriate dimensions. $\tau > 0$ denotes the time-delay, $\nu(t)$ an external disturbance that satisfies

$$
\Psi(\nu(t)) = \|v(t)|E\int_0^T v'(s)v(s)ds < d^2. \tag{2}
$$

The nonlinear terms $H_0(x(t))$ and $H_1(x(t))$ satisfy

$$
\|H_i(x)\| \leq \varepsilon \|x\| \quad i = 0, 1, \tag{3}
$$

where $\varepsilon > 0$.

Next, a new definition of the mean-square finite-time boundedness for the system (1) is given.

Definition 1: For given $0 < c_1 < c_2$, $R > 0$, $T > 0$, system (1) $(u(t) \equiv 0)$ is said to be the mean-square finite-time bounded with respect to $(c_1, c_2, T, R, d^2)$ if

$$
\sup_{-\tau \leq t \leq 0} x'(t_0)R x(t_0) \leq c_1^2 \Rightarrow E\left[x'(t)R x(t)\right] < c_2^2,
$$

for all $t \in [0, T]$ and $\nu(t) \in \Psi$.

Remark 1: The mean-square finite-time boundedness reflects the transient performance of the system in a fixed time interval. That is, the average energy of the system does not exceed a given upper bound in the prescribed time-interval. The transient performance is also important in many practical systems. For example, a large transient voltage can destroy the normal operation of the power system.

Next, some lemmas for obtaining the main results are introduced.

Lemma 1 (Gronwall Inequality): [35] Let $f(t)$ be a nonnegative function; if it satisfies

$$
f(t) \leq a + b \int_0^t f(s)ds \quad 0 \leq t \leq T,
$$

for all $t \in [0, T]$ and $\nu(t) \in \Psi$. 

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for some constants \( a \geq 0 \) and \( b \geq 0 \), then the following inequality is established:

\[
f(t) \leq a \exp(bt) \quad 0 \leq t \leq T.
\]

**Lemma 2** [36]: For given \( x \in \mathbb{R}^n, y \in \mathbb{R}^m, N \in \mathbb{R}^{n \times m} \) and \( \rho > 0 \), then we have

\[
2x'Ny \leq \rho x'x + \frac{1}{\rho} y'N'y.
\]

**Lemma 3** [37]: Let \( V(t, x) \in C^{1,2}(R_+ ,\mathbb{R}^n) \) be a scalar function, and \( V(t, x) > 0 \); for the following stochastic system

\[
dx(t) = a(x)dt + b(x)dw(t),
\]

the Itô formula of \( V(t, x) \) is given as follows:

\[
dV(t, x) = LV(t, x)dt + \frac{\partial V'(t, x)}{\partial x}b(x)dw(t),
\]

where

\[
LV(t, x) = \frac{\partial V(t, x)}{\partial t} + \frac{\partial V'(t, x)}{\partial x}a(x) + \frac{1}{2}b'(x)\frac{\partial^2 V(t, x)}{\partial x^2}b(x).
\]

**III. MAIN RESULTS**

The design of the state feedback finite-time \( H_\infty \) controller and that of the dynamic-output feedback finite-time \( H_\infty \) controller are described next.

**A. STATE FEEDBACK FINITE-TIME \( H_\infty \) CONTROL**

Consider the following state feedback controller

\[
u(t) = Kx(t),
\]

where \( K \) is the controller gain matrix to be solved.

A closed-loop system is obtained by substituting (4) into system (1) giving

\[
\begin{align*}
dx(t) &= (\tilde{A}_{11}x(t) + A_{12}x(t - \tau) + B_{12}v(t) + H_0\xi(t))dt + (A_{21}x(t) + A_{22}x(t - \tau) + H_1\xi(t))dw(t), \\
y(t) &= C_1x(t), \\
z(t) &= C_2x(t), \\
x(t) &= \phi(t) \in L^2(w, \mathcal{F}_0, \mathbb{C}([-\tau, 0], \mathbb{R}^n)), \quad t \in [-\tau, 0],
\end{align*}
\]

where \( \tilde{A}_{11} = A_{11} + B_{11}K \).

Next, the problem concerning the state feedback finite-time \( H_\infty \) controller is described.

**Definition 2**: For given scalars \( 0 < c_1 < c_2, T > 0, d > 0, \gamma > 0 \), and a matrix \( R > 0 \), if there exists a state feedback controller (4) such that:

(i) the closed-loop system (5) is mean-square finite-time bounded with respect to \( (c_1, c_2, T, R, d^2) \);

(ii) for any non-zero disturbance \( v(t) \), the control output \( z(t) \) satisfies the following inequality with zero initial condition,

\[
E \int_0^t \zeta^2(s)z(s)ds < \gamma^2 E \int_0^t \nu^2(s)v(s)ds, \quad \forall t \in [0, T].
\]

then (4) is said to be a state feedback finite-time \( H_\infty \) controller for system (1).

**Remark 2**: The definition considers both the attenuation level of the disturbance and the mean-square finite-time boundedness, which is widely applied in practical systems. For example, in the solar power supply system, if the load power is too large or the external interference is strong, the normal operation of the system will deteriorate.

The sufficient conditions for the existence of the state feedback finite-time \( H_\infty \) controller (4) are given below. For this purpose, an important lemma is first stated and proved.

**Lemma 4**: For given scalars \( 0 < c_1 < c_2, T > 0, d > 0, \gamma > 0 \), and a matrix \( R > 0 \), if there are scalars \( \alpha \geq 0, \gamma > 0, \beta_1 > 0, \beta_2 > 0, \beta_3 > 0, \tau > 0, \varepsilon > 0 \), and two symmetric positive definite matrices \( P_1, Q_1 \) such that

\[
\begin{align*}
\Gamma_1 &= \begin{bmatrix} 1 & A_{12} & A_{21}^T & A_{22}^T \\ * & -Q_1 & 2A_{22}^T & 0 \\ * & * & -\gamma^2I \end{bmatrix} \leq 0, \\
\beta_1I &< P_1 < \beta_2I, \\
0 &< Q_1 < \beta_3I, \\
d^2\gamma^2 + c_1^2(\beta_2 + \tau\beta_3) &< c_2^2\beta_1e^{-\alpha T},
\end{align*}
\]

hold, where \( \Gamma_1 = 4\varepsilon^2\beta_2I + \tilde{P}_1 + \tilde{Q}_1 + \tilde{A}_{11}^T\tilde{P}_1 + \tilde{P}_1\tilde{A}_{11} + 2\tilde{A}_{12}^T\tilde{P}_1\tilde{A}_{21} - \alpha\tilde{P}_1 + C_2^TC_2, \tilde{P}_1 = R^2P_1R^2, \tilde{Q}_1 = R^2Q_1R^2 \), then system (5) is mean-square finite-time bounded with respect to \( (c_1, c_2, T, R, d^2) \) and satisfies (6).

**Proof**: The proof is divided into two parts. First, the closed-loop system (5) is proved to be mean-square finite-time bounded.

Note that

\[
\begin{bmatrix} C_2^TC_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \geq 0,
\]

Therefore, condition (7) implies

\[
\begin{align*}
\Gamma_2 &= \begin{bmatrix} 1 & A_{12} & A_{21}^T & A_{22}^T \\ * & -Q_1 & 2A_{22}^T & 0 \\ * & * & -\gamma^2I \end{bmatrix} \leq 0,
\end{align*}
\]

where \( \Gamma_2 = 4\varepsilon^2\beta_2I + \tilde{P}_1 + \tilde{Q}_1 + \tilde{A}_{11}^T\tilde{P}_1 + \tilde{P}_1\tilde{A}_{11} + 2\tilde{A}_{12}^T\tilde{P}_1\tilde{A}_{21} - \alpha\tilde{P}_1. \)

From (3) and (8), two inequalities are obtained,

\[
H_0^0(\delta)xP_1H_0(\delta) \leq \varepsilon^2\beta_2 \| x \|^2, \\
H_1^0(\delta)xP_1H_1(\delta) \leq \varepsilon^2\beta_2 \| x \|^2.
\]

Introducing quadratic function \( V(x(t)) = x'(t)\tilde{P}_1x(t) + \int_{t-\tau}^t x'(s)\tilde{Q}_1x(s)ds \) and the differential generation operator of the system (5) \( L_1V(x(t)) \), then, according to the Itô formula, we obtain

\[
L_1V(x(t)) = (\tilde{A}_{11}x(t) + A_{12}x(t - \tau) + B_{12}v(t) + H_0\xi(t))' \\
\times \frac{\partial V(x(t))}{\partial x} + \frac{1}{2}[(A_{21}x(t) + A_{22}x(t - \tau) + H_1\xi(t))'] \\
\times \frac{\partial^2 V(x(t))}{\partial x^2} - [A_{21}x(t) + A_{22}x(t - \tau) + H_1\xi(t)];
\]

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that is,
\[
L_1 V(x(t)) = x'(t) \left[ \tilde{Q}_1 + A_2' \tilde{P}_1 A_{21} + \tilde{P}_1 A_{11} + \tilde{A}'_1 \tilde{P}_1 \right] x(t) + x'(t) \left[ \tilde{Q}_1 + A_2' \tilde{P}_1 A_{21} \right] x(t) + x'(t) \left[ \tilde{P}_1 A_{12} + A_2' \tilde{P}_1 A_{22} \right] x(t) + x'(t) \tilde{P}_1 B_{12}(t) + 2H'_{12}(x) + 2H'_{12}(x) x(t) + 2H_{12}(x) \tilde{P}_1 A_{21} x(t) + H'_1 x(t) + x'(t) B_{12}(t). \tag{16}
\]

From lemma 2, we have
\[
2H'_{0}(x) \tilde{P}_1 x(t) + 2H'_{1}(x) \tilde{P}_1 A_{21} x(t) + 2H'_{1}(x) \tilde{P}_1 A_{22} x(t) + 2H_{1}(x) \tilde{P}_1 A_{21} x(t) + H'_1 \tilde{P}_1 A_{21} x(t).
\tag{17}
\]

Based on (13), (14), (16), and (17), we see that
\[
L_1 V(x(t)) \leq \begin{bmatrix} x'(t) \\ v(t) \end{bmatrix} \begin{bmatrix} \gamma_3 & \tilde{P}_1 A_{12} + A_2' \tilde{P}_1 A_{22} \\ - \tilde{Q}_1 + A_2' \tilde{P}_1 A_{22} & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} \Gamma_3 & \tilde{P}_1 A_{12} + A_2' \tilde{P}_1 A_{22} \\ - \tilde{Q}_1 + A_2' \tilde{P}_1 A_{22} & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}, \tag{18}
\]

where \( \Gamma_3 = 4e^2 \beta_2 J + \tilde{P}_1 + \tilde{Q}_1 + \tilde{A}'_1 \tilde{P}_1 + \tilde{P}_1 A_{11} + 2A_2' \tilde{P}_1 A_{21} \).

In light of (12) and (18), we have
\[
L_1 V(x(t)) < \alpha V(x(t)) + \gamma^2 v'(t)v(t). \tag{19}
\]

Integrating from 0 to \( t \) and taking the mathematical expectation on both sides of (19), we have
\[
EV(x(t)) < EV(x(0)) + \alpha \int_0^t EV(x(s))ds + \gamma^2 E \int_0^t v'(s)v(s)ds. \tag{20}
\]

From lemma 1, we see that
\[
EV(x(t)) < EV(x(0))e^{\alpha t} + \gamma^2 e^{\alpha t} E \int_0^t v'(s)v(s)ds, \tag{21}
\]

and because
\[
EV(x(t)) = E[x'(t) \tilde{P}_1 x(t) + \int_{t-\tau}^t x'(s) \tilde{Q}_1 x(s)ds] = E[x'(t)R_1^2 P_1 R_1^2 x(t)] + \int_{t-\tau}^t x'(s) R_1^2 Q_1 R_1^2 x(s)ds \geq \lambda_{\min}(P_1) E[x'(t)R_x(t)], \tag{22}
\]

\[
V(x(0))e^{\alpha t} = x'(0) \tilde{P}_1 x(0)e^{\alpha t} + e^{\alpha t} \int_0^t x'(s) \tilde{Q}_1 x(s)ds = x'(0)R_1^2 P_1 R_1^2 x(0)e^{\alpha t} + e^{\alpha t} \int_0^t x'(s) R_1^2 Q_1 R_1^2 x(s)ds \leq \lambda\max(P_1) e^{\alpha t} + \lambda\max(Q_1) e^{\alpha t}, \tag{23}
\]

\[
y^2 e^{\alpha T} E \int_0^t v'(s)v(s)ds < y^2 e^{\alpha T} E \int_0^t v(t)v(t)dt \leq e^{\alpha T} \left( \frac{\gamma^2 (\lambda\max(P_1) + \lambda\max(Q_1) e^{\alpha T})}{\lambda_{\min}(P_1)} + \frac{d^2 \gamma^2}{\lambda_{\min}(P_1)} \right), \tag{25}
\]

and exploiting (8) and (9), (25) becomes
\[
E[x'(t)R_x(t)] \leq e^{\alpha T} \left( \frac{\gamma^2 (\beta_2 + \gamma^2 \beta_3) + d^2 \gamma^2}{\beta_1} \right).
\]

With condition (10), we obtain \( E[x'(t)R_x(t)] < c_2^2 \) for \( t \in [0, T] \); that is, the closed-loop system (5) is mean-square finite-time bounded.

Next, we prove that the control output \( z(t) \) satisfies (6) for any non-zero disturbance \( v(t) \) imposing to the zero initial condition. From (7) and (18), we have
\[
L_1 V(x(t)) < \alpha V(x(t)) + \gamma^2 v'(t)v(t) - \gamma' z'(t)z(t). \tag{26}
\]

Multiplying both sides of (26) by \( e^{-\alpha t} \), we have
\[
e^{-\alpha t} L_1 V(x(t)) < \alpha e^{-\alpha t} V(x(t)) + e^{-\alpha t}[\gamma^2 v'(t)v(t) - \gamma' z'(t)z(t)]. \tag{27}
\]

From lemma 3, we have
\[
L_1 [e^{-\alpha t} V(x(t))] = -\alpha e^{-\alpha t} V(x(t)) + e^{-\alpha t} L_1 V(x(t)). \tag{28}
\]

Combining (27) and (28), we find
\[
L_1 [e^{-\alpha t} V(x(t))] < e^{-\alpha t}[\gamma^2 v'(t)v(t) - \gamma' z'(t)z(t)]. \tag{29}
\]

Integrating from 0 to \( t \), taking the mathematical expectation on both sides of (29), and imposing the zero initial condition, we have
\[
e^{-\alpha t} EV(x(t)) \leq [E \int_0^t e^{-\alpha s} (\gamma^2 v'(s)v(s) - \gamma' z(s)z(s))ds] \]
\[
< \gamma^2 E \int_0^t e^{-\alpha s} v'(s)v(s)ds - E \int_0^t e^{-\alpha s} z'(s)z(s)ds. \tag{30}
\]

Because
\[
e^{-\alpha t} EV(x(t)) > 0, \tag{31}
\]

then (30) implies that
\[
E \int_0^t z'(s)z(s)ds < \gamma^2 E \int_0^t v'(s)v(s)ds, \forall t \in [0, T].
\]

and the proof is complete. \( \square \)
Based on Lemma 4, we next derive the following Theorem 1.

**Theorem 1**: For given scalars \(0 < c_1 < c_2, T > 0, d > 0, \gamma > 0\) and a matrix \(R > 0\), if there are scalars \(\alpha \geq 0, \sigma > 0, \mu > 0, \tau > 0, \epsilon > 0\), two symmetric positive definite matrices \(U, W\), and a matrix \(M\) satisfying
\[
\begin{bmatrix}
\Psi_{11} & * & * & * \\
\Psi_{21} & \Psi_{22} & 0 & W \\
UA_{12} & 0 & -W & * \\
0 & 0 & \sqrt{3}UA_{22} & -U \\
UB_{12} & 0 & 0 & 0 & -\gamma^2 I
\end{bmatrix} < 0, \quad (32)
\]
then (36) holds.

From the inequality (8) and condition (7), we have
\[
\begin{bmatrix}
\Gamma_5 & \tilde{P}_1A_{12} + A_{12}'\tilde{P}_1A_{22} & \tilde{P}_1B_{12} \\
* & -\tilde{Q}_1 + 2A_{22}'\tilde{P}_1A_{22} & 0 \\
* & * & -\gamma^2 I
\end{bmatrix} < 0, \quad (36)
\]
where \(\Gamma_5 = 4\epsilon^2\beta_2^2I + \tilde{P}_1 + \tilde{Q}_1 + A_{12}'\tilde{P}_1 + K'\tilde{P}_1 + \tilde{P}_1A_{12} + \tilde{P}_1B_{12} + K + 2A_{22}'\tilde{P}_1A_{22} - \alpha\tilde{P}_1 + C_2C_2^T\).

Obviously, if the following inequality holds,
\[
\begin{bmatrix}
\Gamma_5 + A_{22}'\tilde{P}_1A_{22} & \tilde{P}_1A_{12} & \tilde{P}_1B_{12} \\
* & -\tilde{Q}_1 + 3A_{22}'\tilde{P}_1A_{22} & 0 \\
* & * & -\gamma^2 I
\end{bmatrix} < 0, \quad (37)
\]
then, using the Schur complement, we obtain (32) from (38).

Using the Schur complement, (41) is equivalent to (35). Summarizing the process, the proof is complete.

**Remark 3**: Note that inequalities given by (32) in Theorem 1 are not linear. However, once the \(\alpha\) and \(\epsilon\) are fixed, these inequalities can be treated as linear.

### B. Dynamic Output Feedback Finite-Time \(H_{\infty}\) Control

The previous subsection assumes that the state variables are available, which does not always hold in practice. In this case, one should estimate \(x(t)\) from the measurement output \(y(t)\). As usual, consider the following dynamic-output feedback controller
\[
\begin{align*}
\dot{d}(t) &= (A_f\hat{x}(t) + B_fy(t))dt, \\
u(t) &= C_f\hat{x}(t).
\end{align*}
\]
Substituting (42) into (1) and setting \(\eta(t)=[x'(t), \dot{x}'(t)]'\), the following augmented closed-loop system is obtained
\[
\begin{align*}
\dot{d}(t) &= (\bar{A}_{11}\eta(t) + \bar{A}_{12}\eta(t) - \tau) + \bar{B}_{12}\tilde{v}(t) \\
&\quad + \bar{H}_0(x(t))dt + (\bar{A}_{11}\eta(t) \\
&\quad + \bar{A}_{22}\eta(t) - \tau) + \bar{H}_1(x(t))du(t),
\end{align*}
\]
where \(\tilde{v}(t) = [v'(t), 0]'\), \(\bar{H}_0(x) = [H_0'(x), 0]'\), \(\bar{H}_1(x) = [H_1'(x), 0]'\), \(\bar{C}_1 = [C_1, 0]\), \(\bar{C}_2 = [C_2, 0]\),
\[
\begin{bmatrix}
\bar{\lambda}_{11} & \bar{B}_{11}C_f & \bar{A}_f \\
\bar{A}_1 & \bar{B}_1 & 0 \\
\bar{A}_{21} & \bar{B}_{21} & 0
\end{bmatrix},
\]
where the problem concerning the dynamic-output feedback finite-time \(H_{\infty}\) controller is stated next.

**Definition 3**: For given scalars \(0 < c_1 < c_2, T > 0, d > 0, \gamma > 0\), and a positive definite matrix \(\bar{R}\), if there exists a dynamic-output feedback controller (42) such that
(i) the closed-loop system (43) is mean-square finite-time bounded with respect to \((c_1, c_2, T, R, d^2)\), that is,
\[
sup_{-\tau \leq t \leq 0} \eta'(t)\bar{R}\eta(t_0) \leq c_1^2 \Rightarrow E[\eta'(t)\tilde{R}\eta(t)] < c_2^2,
\]
where \(\bar{R} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}\), \(\forall t \in [0, T]\), \(\forall \eta(t) \in \mathcal{W}\), and
(ii) for any non-zero disturbance \(v(t)\), the control output \(z(t)\) satisfies (6) with zero initial condition, then (42) is said to be a dynamic-output feedback finite-time \(H_{\infty}\) controller for system (1).

Next, a sufficient condition for the existence of the dynamic-output feedback finite-time \(H_{\infty}\) controller is given below. First, an important lemma is proved.

**Lemma 5**: For given scalars \(0 < c_1 < c_2, T > 0, d > 0, \gamma > 0\), and a matrix \(R > 0\), if there are scalars \(\alpha \geq 0, \gamma > 0\),
\[
\begin{align*}
\sigma_R^{-1} &< U < \gamma R^{-1}, \\
0 &< W < \mu R^{-1}, \\
d^2\gamma^2 &- \gamma_2^2e^{-\alpha T} + c_1^2(\sigma^{-1} + \tau\mu) < 0.
\end{align*}
\]
\( \delta_1 > 0, \delta_2 > 0, \delta_3 > 0, \tau > 0, \varepsilon > 0 \) and two symmetric positive definite matrices \( P_2, Q_2 \) such that

\[
\begin{bmatrix}
\Gamma_6 & \tilde{P}_2 \tilde{A}_{12} + \tilde{A}_{22}' \tilde{P}_2 \tilde{A}_{22} \\
\star & -\tilde{Q}_2 + 2\tilde{A}_{22}' \tilde{P}_2 \tilde{A}_{22} \\
\star & \star
\end{bmatrix} < 0, \tag{44}
\]

\( \delta_1 I < P_2 < \delta_2 I, \tag{45} \)

\( 0 < Q_2 < \delta_3 I, \tag{46} \)

\( d^2 \gamma^2 + c_1^2 (\delta_2 + 2\delta_3) < c_2^2 d_1 e^{-\alpha T}, \tag{47} \)

hold, where \( \Gamma_6 = 4\varepsilon^2 \tilde{Q}_2 + \tilde{Q}_2 + \tilde{A}_{11}' \tilde{P}_2 + \tilde{P}_2 \tilde{A}_{11} + 2\tilde{A}_{21}' \tilde{P}_2 \tilde{A}_{21} - \alpha \tilde{P}_2 + \tilde{C}_2' \tilde{C}_2, \tilde{P}_2 = \tilde{R}_2^2 \tilde{P}_2 \tilde{R}_2^2, \tilde{Q}_2 = \tilde{R}_2^2 \tilde{Q}_2 \tilde{R}_2^2, \)

then system (43) is mean-square finite-time bounded with respect to \( (c_1, c_2, T, R, d^2) \) and satisfies (6).

**Proof:** The proof is divided into two parts. First, the closed-loop system (43) is proved to be mean-square finite-time bounded.

Note that

\[
\begin{bmatrix}
\tilde{C}_2' \tilde{C}_2 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} = \begin{bmatrix}
\tilde{C}_2' & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \geq 0. \tag{48}
\]

Therefore, condition (44) implies

\[
\begin{bmatrix}
\Gamma_7 & \tilde{P}_2 \tilde{A}_{12} + \tilde{A}_{21}' \tilde{P}_2 \tilde{A}_{22} + \tilde{P}_2 \tilde{B}_{12} \\
\star & -\tilde{Q}_2 + 2\tilde{A}_{22}' \tilde{P}_2 \tilde{A}_{22} \\
\star & \star
\end{bmatrix} < 0. \tag{49}
\]

where \( \Gamma_7 = 4\varepsilon^2 \tilde{Q}_2 + \tilde{P}_2 + \tilde{Q}_2 + \tilde{A}_{11}' \tilde{P}_2 + \tilde{P}_2 \tilde{A}_{11} + 2\tilde{A}_{21}' \tilde{P}_2 \tilde{A}_{21} - \alpha \tilde{P}_2. \)

From (3) and (45), we have

\[
\begin{align*}
\tilde{H}_0'(x) \tilde{P}_2 \tilde{H}_0(x) & \leq \varepsilon^2 \tilde{Q}_2 \| x \|^2 \leq \varepsilon^2 \tilde{Q}_2 \| \eta \|^2, \tag{50} \\
\tilde{H}_1'(x) \tilde{P}_2 \tilde{H}_1(x) & \leq \varepsilon^2 \tilde{Q}_2 \| x \|^2 \leq \varepsilon^2 \tilde{Q}_2 \| \eta \|^2. \tag{51}
\end{align*}
\]

Let \( V(t) = \eta'(t) \tilde{P}_2 \eta(t) + \int_{t-\tau}^{t} \eta'(s) \tilde{Q}_2 \eta(s) ds \), and \( L_2 V(t) \) denote the differential generation operator of system (43). According to the Itô formula, we have

\[
L_2 V(t) = [\tilde{A}_{11} \eta(t) + \tilde{A}_{12} \eta(t - \tau) + \tilde{B}_1 \tilde{v}(t)] \frac{\partial V}{\partial \eta} + \frac{1}{2} [\tilde{A}_{21} \eta(t) + \tilde{A}_{22} \eta(t - \tau) + \tilde{H}_1(x(t))] \frac{\partial^2 V}{\partial \eta^2} [\tilde{A}_{21} \eta(t) + \tilde{A}_{22} \eta(t - \tau) + \tilde{H}_1(x(t))]. \tag{52}
\]

that is,

\[
L_2 V(t) = \eta'(t) [\tilde{Q}_2 + \tilde{A}_{21}' \tilde{P}_2 \tilde{A}_{21} + \tilde{P}_2 \tilde{A}_{11} + \tilde{A}_{11}' \tilde{P}_2 + \tilde{A}_{22}' \tilde{P}_2 \tilde{A}_{22}] \eta(t) + \eta'(t - \tau) [\tilde{Q}_2 + \tilde{A}_{21}' \tilde{P}_2 \tilde{A}_{21}] \eta(t - \tau)
\]

\[
+ \eta'(t) [\tilde{Q}_2 + \tilde{A}_{22}' \tilde{P}_2 \tilde{A}_{22}] \eta(t - \tau) + \frac{1}{2} \int_{t-\tau}^{t} \tilde{H}_1'(x) \tilde{P}_2 \tilde{A}_{22} \eta(t - \tau) \frac{\partial^2 V}{\partial \eta^2} [\tilde{A}_{21} \eta(t) + \tilde{A}_{22} \eta(t - \tau) + \tilde{H}_1(x(t))]. \tag{53}
\]
Equations (58), (59), (60), and (61), lead straightforwardly to
\[ E[\eta'(t)\tilde{R}_{\eta}(t)] \leq e^{\alpha T} \left( \frac{c_1(\lambda_{\max}(P_2) + \tau \lambda_{\max}(Q_2))}{\lambda_{\min}(P_2)} d^2 \gamma^2 \right) + \frac{d^2 \gamma^2}{\lambda_{\min}(P_2)}. \] (62)

On the basis of (45), (46), and (62), we have
\[ E[\eta'(t)\tilde{R}_{\eta}(t)] \leq e^{\alpha T} \left( \frac{c_1(\lambda_{\max}(P_2) + \tau \lambda_{\max}(Q_2))}{\lambda_{\min}(P_2)} d^2 \gamma^2 \right). \]

From (47), \( E[\eta'(t)\tilde{R}_{\eta}(t)] < c_2^2 \) for \( t \in [0, T] \) obtains; that is, the closed-loop system (43) is mean-square finite-time bounded.

In the second part of the proof, the control output \( z(t) \) is proved to satisfy (6) for any non-zero disturbance \( v(t) \) under zero initial condition. From (44) and (55), we have
\[ L_2 V(\eta(t)) < \alpha V(\eta(t)) + \gamma^2 \tilde{V}(t)\tilde{v}(t) - \tilde{z}'(t)z(t). \] (63)

Multiplying both sides of (63) by \( e^{-\alpha t} \), we obtain
\[ e^{-\alpha t} L_2 V(\eta(t)) < \alpha e^{-\alpha t} V(\eta(t)) + e^{-\alpha t} \gamma^2 \tilde{v}(t)\tilde{v}(t) - \tilde{z}'(t)z(t). \]

Using lemma 3, we have
\[ L_2[e^{-\alpha t} V(\eta(t))] = -\alpha e^{-\alpha t} V(\eta(t)) + e^{-\alpha t} L_2 V(\eta(t)). \] (65)

From (64) and (65), one sees that
\[ L_2[e^{-\alpha t} V(\eta(t))] < e^{-\alpha t}[\gamma^2 \tilde{v}(t)\tilde{v}(t) - \tilde{z}'(t)z(t)]. \] (66)

Integrating from 0 to \( t \) and taking the mathematical expectation on both sides of (66) applying the zero initial condition, we have
\[ e^{-\alpha t} EV(\eta(t)) \leq E \int_0^t e^{-\alpha s} \gamma^2 \tilde{v}(s)\tilde{v}(s) - \tilde{z}'(s)z(s)ds \]
\[ < \gamma^2 E \int_0^t e^{-\alpha s} \tilde{v}(s)\tilde{v}(s)ds \\
- E \int_0^t e^{-\alpha s} \tilde{z}'(s)z(s)ds. \] (67)

Because
\[ e^{-\alpha t} EV(\eta(t)) > 0, \] (68)
then (67) implies
\[ E \int_0^t \tilde{z}'(s)z(s)ds < \gamma^2 E \int_0^t \tilde{v}'(s)\tilde{v}(s)ds, \quad \forall t \in [0, T]. \]

The proof is complete. \( \square \)

On the basis of the above analysis, we prove the following Theorem 2.

**Theorem 2:** For given scalars \( 0 < c_1 < c_2, T > 0, d > 0, \gamma > 0, \) and a matrix \( R > 0, \) if there are scalars \( \alpha \geq 0, \gamma > 0, \delta_1 > 0, \delta_2 > 0, \delta_3 > 0, \tau > 0, \epsilon > 0, \) three symmetric positive definite matrices \( P_{22}, Q_{11}, Q_{22}, \) and two matrices \( X, Y \) satisfying (45), (46), (47), and

where \( \Gamma_0 = 4e^2 \delta_2 I + \tilde{Q}_{11} + A_{11}' + A_{11} + 2A_{11}'A_{21} - \alpha I + C_1' C_2, \Gamma_{10} = 4e^2 \delta_2 I + \tilde{P}_{22} + \tilde{Q}_{22} + Y' + Y' - \alpha \tilde{P}_{22}, \) then (42) is the dynamic-output feedback finite-time \( H_{\infty} \) controller. In this case, \( A_f = \tilde{P}_{22}^{-1} Y' \) and \( B_f = \tilde{P}_{22}^{-1} X'. \)

**Proof:** Let
\[ \tilde{P}_2 = \begin{bmatrix} I & 0 \\ 0 & \tilde{P}_{22} \end{bmatrix}, \quad \tilde{Q}_2 = \begin{bmatrix} \tilde{Q}_{11} & 0 \\ 0 & \tilde{Q}_{22} \end{bmatrix}. \]

According to (44), we have
\[ \begin{bmatrix} \Gamma_9 & \Omega_1 & A_{12} + A_{21}' \tilde{A}_{22} \\ \Omega_1^{\dagger} & \Lambda_1 & B_{12} \\ * & * & 0 \end{bmatrix} \begin{bmatrix} 11 & 11 & 0 \\ 11 & 11 & 0 \\ * & * & 0 \end{bmatrix} \begin{bmatrix} 11 & 11 & 0 \\ 11 & 11 & 0 \\ * & * & 0 \end{bmatrix} < 0, \]

where \( \Omega_1 = C_1' B_f' \tilde{P}_{22} + B_{11} C_f, \Gamma_{11} = 4e^2 \delta_2 I + \tilde{P}_{22} + \tilde{Q}_{22} + A_{21}' \tilde{P}_{22} + \tilde{P}_{22} A_f - \alpha \tilde{P}_{22}. \)

Let \( X = B_f' \tilde{P}_{22}, \quad Y = A_f' \tilde{P}_{22} \); we obtain (69), shown at the bottom of the next page.

This completes the proof. \( \square \)

### IV. NUMERICAL ALGORITHMS

In this section, two algorithms are presented that produced the results obtained above. One finds the minimum value of \( \gamma; \) the other finds the maximum value of \( \tau. \)

By analyzing (32)–(35) in Theorem 1, we find that if these equations have no feasible solutions when \( \alpha = 0, \) then they will have no feasible solutions for all \( \alpha > 0. \) The specifics of the algorithm are as follows.

**Algorithm 1**

Step 1: Set the values of \( c_1, c_2, T, R, d, \) and \( \tau. \)

Step 2: Using the linear search algorithm, if a series of \( \alpha_i (i = 1, \cdots, n) \) can be found that ensure inequalities (32)–(35) have feasible solutions, then move to Step 3; otherwise, move to Step 7.

Step 3: Let \( i = 1, \) then we take \( \alpha_i. \)

Step 4: Solve the following minimization problem:

\[ \min_{\gamma \in (32)–(35)} \gamma. \]

Step 5: Let \( i = i + 1, \) if \( i + 1 > n, \) then move to Step 6; otherwise, let \( \alpha_i = \alpha_{i-1}, \) and return to Step 4.

Step 6: There are solutions to this problem; print data and then stop.

Step 7: There is no solution to this problem; stop.

By analyzing (32)–(35) in Theorem 1, we find that if these equations have no feasible solutions when \( \alpha = 0 \) and \( \tau = 0, \) then they will have no feasible solutions for all \( \alpha > 0 \) and \( \tau > 0. \) The specific algorithm is as follows.
Algorithm 2

Step 1: Set the values of \( c_1, c_2, T, R, d \) and \( \gamma \).
Step 2: Take a series of \( \alpha_i (i = 1, \cdots, n) \) and a series of \( \tau_j (j = 1, \cdots, n) \).
Step 3: Let \( i = 1 \) and set \( \alpha_1 = 0 \).
Step 4: Let \( j = 1 \) and set \( \tau_1 = 0 \).
Step 5: If \( (\alpha_i, \tau_j) \) ensure (32)–(35) have feasible solutions, then store \( (\alpha_i, \tau_j) \) into \( (\Delta(i), \Delta(j)) \). Let \( \tau_j = \tau_{j+1} \) and move to Step 5; otherwise, move to Step 6.
Step 6: If \( i + 1 < n \), set \( \alpha_i = \alpha_{i+1} \), and with take \( \tau_j \), return to Step 5; otherwise, skip to Step 7.
Step 7: Stop.

V. NUMERICAL EXAMPLES

The matrix of coefficients of the system (1) is given as follows.

\[
\begin{align*}
A_{11} & = \begin{bmatrix} -30 & -0.1 & 0.2 \\ -10.38 & -30.16 & 0.1 \end{bmatrix}, \\
A_{12} & = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, \\
A_{21} & = \begin{bmatrix} -2.7 & 0.8 \\ 0.9 & -1.6 \end{bmatrix}, \\
A_{22} & = \begin{bmatrix} -0.2 & -0.5 \\ -0.3 & -1.4 \end{bmatrix}, \\
B_{11} & = \begin{bmatrix} -4 \\ 3 \end{bmatrix}, \\
B_{12} & = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}, \\
B_{12} & = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}, \\
C_1 & = \begin{bmatrix} 0 & 0 \end{bmatrix}, \\
C_2 & = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}.
\end{align*}
\]

Let \( c_1 = 1, c_2 = 4, T = 1, d = 1, \tau = 0.5, \varepsilon = 1.2, R = I \).

A. STATE FEEDBACK FINITE-TIME H\(_\infty\) CONTROL

To find the minimum value of \( \gamma \), the relationship between \( \gamma \) and \( \alpha \) is obtained using Algorithm 1 (see Fig. 1).

From Fig. 1, \( \gamma \) increases with increasing \( \alpha \), and the minimum value of \( \gamma = 0.0315 \) is obtained when \( \alpha = 0 \).

Setting \( \alpha = 0 \), according to Theorem 1, we obtain

\[
U = \begin{bmatrix} 0.5254 & 0.0079 \\ 0.0079 & 0.4524 \end{bmatrix}, \\
M = \begin{bmatrix} 0.7103 & 0.1159 \end{bmatrix}, \\
W = \begin{bmatrix} 5.4891 & -0.1883 \end{bmatrix}, \\
K = \begin{bmatrix} 1.3484 & 0.2326 \end{bmatrix}.
\]

Therefore, the state feedback controller is as follows:

\[
u(t) = 1.3484 \cdot x(t) - 0.2326 \cdot x(t).
\]

Setting \( \gamma = 0.1 \), then by Algorithm 2, the relationship between \( \gamma \) and \( \alpha \) so as to find the maximum value of \( \gamma \) was obtained (see Fig. 2).

From Fig. 2, \( \tau \) decreases with increasing \( \alpha \), and \( \tau \) has a maximum value in the range of \( \alpha \); that is, \( \tau = 9.13 \) when \( \alpha = 0 \).

Considering the external disturbance \( \nu(t) = \sin(t) \) and setting \( x(0) = [0.4, 0.2]' \), we obtained the curves for \( x_1(t), x_2(t), \) and \( E[x'(t)Rx(t)] \) (Fig. 3) from which we find that \( E[x'(0)Rx(0)] = E[x_1^2(0) + x_2^2(0)] = 0.2 \leq c_1^2 = 1 \) and \( E[x'(t)Rx(t)] < c_2^2 = 16 \). Therefore, the closed-loop system (5) is mean-square finite-time bounded with respect to \( (1, 4, 1, 1) \).

B. DYNAMIC OUTPUT FEEDBACK FINITE-TIME H\(_\infty\) CONTROL

To find the minimum value of \( \gamma \), the relationship between \( \gamma \) and \( \alpha \) is determined using Algorithm 1 (see Fig. 4).
From Fig. 4, \( \gamma \) decreases first as \( \alpha \) increases, and then increases as \( \alpha \) increases. The optimal solution \( \gamma = 0.1908 \) is obtained when \( \alpha = 1.3 \).

Setting \( \alpha = 1.3 \), then, according to Theorem 2, we derive a set of solutions

\[
\tilde{Q}_{11} = \begin{bmatrix} 3.8636 & 0.2661 \\ 0.2661 & 4.9417 \end{bmatrix}, \quad \tilde{Q}_{22} = \begin{bmatrix} 3.8779 & 0 \\ 0 & 3.8779 \end{bmatrix},
\]

\[
\tilde{P}_{22} = \begin{bmatrix} 1.1444 & 0 \\ 0 & 1.1444 \end{bmatrix}, \quad A_f = \begin{bmatrix} -208.9 & 0 \\ 0 & -208.9 \end{bmatrix},
\]

\[
B_f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad C_f = \begin{bmatrix} 0 & 0 \end{bmatrix}, \quad \gamma = 0.1908.
\]

Note, for \( B_f = 0, C_f = 0 \), the above solutions do not satisfy the requirements for the coefficients of the dynamic-output feedback controller. Therefore, setting \( X > 0 \), the following solutions are obtained,

\[
\tilde{Q}_{11} = \begin{bmatrix} 3.9163 & 0.2632 \\ 0.2632 & 4.9930 \end{bmatrix}, \quad \tilde{Q}_{22} = \begin{bmatrix} 3.9376 & 0 \\ 0 & 3.9376 \end{bmatrix},
\]

\[
\tilde{P}_{22} = \begin{bmatrix} 1.1087 & 0 \\ 0 & 1.1087 \end{bmatrix}, \quad A_f = \begin{bmatrix} -1245.3 & -55.5 \\ -55.5 & -836.2 \end{bmatrix},
\]

\[
B_f = \begin{bmatrix} -124.6 & -203.1 \\ -203.1 & 1620.7 \end{bmatrix}, \quad C_f = \begin{bmatrix} 30.9107 & -56.7711 \end{bmatrix}.
\]

Setting \( \gamma = 0.2 \), the relationship between \( \tau \) and \( \alpha \) (Fig. 5) shows that \( \tau \) decreases with increasing \( \alpha \). Its maximum value is \( \tau = 2.3 \) when \( \alpha = 0 \).

Considering the external disturbance \( v(t) = \sin(t) \) and letting \( \eta(0) = [0.1, 0.1, 0.1, 0.1]' \), we obtain the curves for \( x_1(t), x_2(t), \hat{x}_1(t), \hat{x}_2(t), \) and \( E[\eta'(t)\hat{R}\eta(t)] \) (Fig. 6). Magnifying the plot in the interval \( t \in [0, 0.1] \), (Fig. 7), we see that \( E[\eta'(0)R\eta(0)] = 0.04 \leq c_1^2 = 1 \) and
\[ E[|y(t)|^2] < c_2^2 = 16. \] Therefore, the closed-loop system (43) is mean-square finite-time bounded with respect to (1, 4, 1, 1, 1).

VI. CONCLUSION
We investigated the finite-time H∞ control problem for the Itô-type stochastic nonlinear time-delay systems. Two kinds of controllers, state feedback and dynamic-output feedback finite-time H∞ controllers, were designed. Moreover, their corresponding algorithms used in solving the state feedback controller and dynamic-output feedback controller were presented. They also simultaneously optimize the H∞ performance index and determine the maximum time-delay \( \tau \).

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