CASIMIR AND SHORT-RANGE GRAVITY TESTS

A. LAMBRECHT and S. REYNAUD
Laboratoire Kastler Brossel, ENS, UPMC, CNRS, Jussieu, 75252 Paris, France
serge.reynaud@upmc.fr , www.lkb.ens.fr

Comparison with theory of Casimir force measurements are used to test the gravity force law at ranges from 0.1 to 10 micrometers. The interest of such tests depends crucially on the theoretical evaluation of the Casimir force in realistic experimental configurations. We present the scattering approach which is nowadays the best tool for such an evaluation. We then describe the current status of the comparisons between theory and experiments.

1 Introduction

The Casimir effect is an observable effect of quantum vacuum fluctuations which deserves careful attention as a crucial prediction of quantum field theory [1,2,3,4,5,6].

Casimir physics also plays an important role in the tests of gravity at sub-millimeter ranges [7,8]. Strong constraints have been obtained in short range Cavendish-like experiments [9,10]. For scales of the order of the micrometer, similar tests are performed by comparing with theory the results of Casimir force measurements [11,12]. At even shorter scales, the same can be done with atomic [13,14,15] or nuclear [16,17] force measurements. A recent overview of these short-range tests can be found in [18].

In the following, we focus our attention on Casimir tests of the gravity force law. They are performed at distances from 0.1 to 10 micrometers for which the Casimir force dominates the standard gravity force. It follows that the hypothetical new force would be seen as a difference between the experimental result \( F_{\text{exp}} \) and theoretical prediction \( F_{\text{th}} \). This implies that these two quantities have to be assessed independently from each other. This situation should clearly forbid anyone to use theory-experiment comparison to prove (or disprove) a specific experimental result or theoretical model.

2 The problem of vacuum energy

Before entering this discussion, we want to emphasize that the Casimir effect has a fascinating interface with the puzzles of gravitational physics through the problem of vacuum energy [19,20,21].

Nernst was the first physicist to notice as soon as in 1916 that zero-point fluctuations of the electromagnetic field constituted a challenge for gravitation theory [22,23]. The very existence of these fluctuations dismisses the classical idea of an empty space. When the vacuum energy density is calculated by adding the zero-point energies over all field modes, an infinite value is obtained. When a high frequency cutoff is introduced, the sum is finite but still much larger than the mean energy observed through gravitational phenomena [24,25].
This problem has led famous physicists to deny the reality of vacuum fluctuations. In particular, Pauli crudely stated in his textbook on Wave Mechanics:

At this point it should be noted that it is more consistent here, in contrast to the material oscillator, not to introduce a zero-point energy of \( \frac{1}{2} \hbar \omega \) per degree of freedom. For, on the one hand, the latter would give rise to an infinitely large energy per unit volume due to the infinite number of degrees of freedom, on the other hand, it would be principally unobservable since nor can it be emitted, absorbed or scattered and hence, cannot be contained within walls and, as is evident from experience, neither does it produce any gravitational field.

A part of these statements is simply unescapable: the mean value of vacuum energy does not contribute to gravitation as an ordinary energy. This is just a matter of evidence since the universe would look very differently otherwise. But it is certainly no longer possible to uphold today that vacuum fluctuations have no observable effects. Certainly, vacuum fluctuations can be emitted, absorbed, scattered... as shown by their numerous effects in atomic and subatomic physics. And the Casimir effect is nothing but the physical effect produced by vacuum fluctuations when they are contained within walls.

3 The Casimir force in the ideal and real cases

Casimir calculated the force between a pair of perfectly smooth, flat and parallel plates in the limit of zero temperature and perfect reflection. He found universal expressions for the force \( F_{\text{Cas}} \) and energy \( E_{\text{Cas}} \)

\[
F_{\text{Cas}} = -\frac{dE_{\text{Cas}}}{dL}, \quad E_{\text{Cas}} = -\frac{\hbar c A^2}{720 L^3}
\]  

with \( L \) the distance, \( A \) the area, \( c \) the speed of light and \( \hbar \) the Planck constant. This universality is explained by the saturation of the optical response of perfect mirrors which reflect 100% (no less, no more) of the incoming fields. In particular the expressions \( F_{\text{Cas}} \) and \( E_{\text{Cas}} \) do not depend on the atomic structure constants. Of course, this idealization is no longer tenable for the real mirrors used in the experiments.

The effect of imperfect reflection is large in most experiments, and a precise knowledge of its frequency dependence is essential for obtaining a reliable theoretical prediction. Meanwhile, experiments are performed at room temperature so that the effect of thermal fluctuations has to be added to that of vacuum. Then, precise experiments are performed between a plane and a sphere whereas calculations are often devoted to the geometry of two parallel planes. The estimation of the force in the plane-sphere geometry involves the so-called Proximity Force Approximation (PFA) which amounts to averaging over the distribution of local inter-plate distances the force calculated in the two-planes geometry. But the PFA can only be valid when the radius \( R \) is much larger than the separation \( L \) and even in this case its accuracy has to be assessed.

4 The calculation of the force in the scattering approach

The best tool available for addressing these questions is the scattering approach. This approach has been used for years for evaluating the Casimir force between non perfectly reflecting mirrors. It is today the best solution for calculating the force in arbitrary geometries.

The basic idea is that mirrors are described by their scattering amplitudes. When studying first the geometry of two plane and parallel mirrors aligned along the axis \( x \) and \( y \), these amplitudes are specular reflection and transmission amplitudes \( (r \) and \( t) \) which depend on frequency.
\( \omega \), the transverse vector \( k \equiv (k_x, k_y) \) and the polarization \( p = \text{TE}, \text{TM} \) (all these quantities being preserved by scattering). Two mirrors form a Fabry-Perot cavity described by a global \( S \)-matrix which can be evaluated from the elementary \( S \)-matrices associated with the two mirrors. Thermal equilibrium is here assumed for the whole system cavity + fields. Care has to be taken to account for the contribution of evanescent waves besides that of ordinary modes freely propagating outside and inside the cavity. The properties of the evanescent waves are described through an analytical continuation of those of ordinary ones, using the well defined analytic behavior of the scattering amplitudes. At the end of this derivation, this analytic properties are also used to perform a Wick rotation from real to imaginary frequencies.

The sum of the phaseshifts associated with all field modes leads to the expression of the Casimir free energy \( \mathcal{F} \)

\[
\mathcal{F} = \sum_k \sum_p k_B T \sum_m \ln d(i\xi_m, k, p), \quad d(i\xi, k, p) = 1 - r(i\xi, k, p)e^{-2\kappa L}
\]

(2)

\( \sum_k \equiv A \int \frac{d^2k}{e^2} \) is the sum over transverse wavevectors with \( A \) the area of the plates, \( \sum_p \) the sum over polarizations and \( \sum_m' \) the Matsubara sum (sum over positive integers \( m \) with \( m = 0 \) counted with a weight \( \frac{1}{2} \)); \( r \) is the product of the reflection amplitudes of the mirrors as seen by the intracavity field; \( \xi \) and \( \kappa \) are the counterparts of frequency \( \omega \) and longitudinal wavevector \( k_z \) after the Wick rotation.

This expression reproduces the Casimir ideal formula in the limits of perfect reflection \( r \to 1 \) and null temperature \( T \to 0 \). But it is valid and regular at thermal equilibrium at any temperature and for any optical model of mirrors obeying causality and high frequency transparency properties. It has been demonstrated with an increasing range of validity in 34, 35 and 36. The expression is valid not only for lossless mirrors but also for lossy ones. In the latter case, it accounts for the additional fluctuations accompanying losses inside the mirrors.

It can thus be used for calculating the Casimir force between arbitrary mirrors, as soon as the reflection amplitudes are specified. These amplitudes are commonly deduced from models of mirrors, the simplest of which is the well known Lifshitz model 38, 39 which corresponds to semi-infinite bulk mirrors characterized by a local dielectric response function \( \varepsilon(\omega) \) and reflection amplitudes deduced from the Fresnel law.

In the most general case, the optical response of the mirrors cannot be described by a local dielectric response function. The expression (2) of the free energy is still valid in this case with reflection amplitudes to be determined from microscopic models of mirrors. Attempts in this direction can be found for example in 40, 41, 42.

5 The case of metallic mirrors

The most precise experiments have been performed with metallic mirrors which are good reflectors only at frequencies smaller than their plasma frequency \( \omega_p \). Their optical response is described by a reduced dielectric function usually written at imaginary frequencies \( \omega = i\xi \) as

\[
\varepsilon[i\xi] = \varepsilon'[i\xi] + \frac{\sigma[i\xi]}{\xi}, \quad \sigma[i\xi] = \frac{\omega_p^2}{\xi + \gamma}
\]

(3)

The function \( \varepsilon'[i\xi] \) represents the contribution of interband transitions and is regular at the limit \( \xi \to 0 \). Meanwhile \( \sigma[i\xi] \) is the reduced conductivity (\( \sigma \) is measured as a frequency and the SI conductivity is \( \varepsilon_0 \sigma \)) which describes the contribution of the conduction electrons.
A simplified description corresponds to the lossless limit $\gamma \to 0$ often called the plasma model. As $\gamma$ is much smaller than $\omega_P$ for a metal such as Gold, this simple model captures the main effect of imperfect reflection. However it cannot be considered as an accurate description since a much better fit of tabulated optical data is obtained with a non-null value of $\gamma$. Furthermore, the Drude model $\gamma \neq 0$ meets the important property of ordinary metals which have a finite static conductivity

$$\sigma_0 = \frac{\omega_P^2}{\gamma}$$

This has to be contrasted to the lossless limit which corresponds to an infinite value for $\sigma_0$.

When taking into account the imperfect reflection of the metallic mirrors, one finds that the Casimir force is reduced with respect to the ideal Casimir expression at all distances for a null temperature. This reduction is conveniently represented as a factor

$$\eta_F = \frac{F}{F_{\text{Cas}}}$$

$$F = -\frac{\partial F}{\partial L}$$

where $F$ is the real force and $F_{\text{Cas}}$ the ideal expression. For the plasma model, there is only one length scale, the plasma wavelength $\lambda_P = \frac{2\pi c}{\omega_P}$ in the problem (136nm for Gold). The ideal Casimir formula is recovered ($\eta_F \to 1$) at large distances $L \gg \lambda_P$, as expected from the fact that metallic mirrors tend to be perfect reflectors at low frequencies $\omega \ll \omega_P$. At short distances in contrast, a significant reduction of the force is obtained ($\eta_F \ll 1$), which scales as $L/\lambda_P$, as a consequence of the fact that metallic mirrors are poor reflectors at high frequencies $\omega \gg \omega_P$. In other words, there is a change in the power law for the variation of the force with distance. This change can be understood as the result of the Coulomb interaction of surface plasmons living at the two matter-vacuum interfaces.

As experiments are performed at room temperature, the effect of thermal fluctuations has to be added to that of vacuum fields. Significant thermal corrections appear at distances $L$ larger than a critical distance determined by the thermal wavelength $\lambda_T$ (a few micrometers at room temperature). Boström and Sernelius were the first to remark that the small non-zero value of $\gamma$ had a significant effect on the force at non-null temperatures. In particular, there is a large difference at large distances between the expectations calculated for $\gamma = 0$ and $\gamma \neq 0$, their ratio reaching a factor 2 when $L \gg \lambda_T$. It is also worth emphasizing that the contribution of thermal fluctuations to the force is opposite to that of vacuum fluctuations for intermediate ranges $L \sim \lambda_T$.

This situation has led to a blossoming of contradictory papers (see references in). As we will see below, the contradiction is also deeply connected to the comparison between theory and experiments.

### 6 The non-specular scattering formula

We now present a more general scattering formula allowing one to calculate the Casimir force between stationary objects with arbitrary geometries. The main generalization with respect to the already discussed cases is that the scattering matrix $S$ is now a larger matrix accounting for non-specular reflection and mixing different wavevectors and polarizations while preserving frequency. Of course, the non-specular scattering formula is the generic one while specular reflection can only be an idealization.

The Casimir free energy can be written as a generalization of equation (2)

$$\mathcal{F} = k_B T \sum_m' \text{Tr} \ln \mathcal{D}(i\xi_m)$$

$$\mathcal{D} = 1 - R_1 \exp^{-\kappa L} R_2 \exp^{-\kappa L}$$
The symbol \( \text{Tr} \) refers to a trace over the modes at a given frequency. The matrix \( \mathcal{D} \) is the denominator containing all the resonance properties of the cavity formed by the two objects 1 and 2 here written for imaginary frequencies. It is expressed in terms of the matrices \( \mathcal{R}_1 \) and \( \mathcal{R}_2 \) which represent reflection on the two objects 1 and 2 and of propagation factors \( \exp^{-K L} \). Note that the matrices \( \mathcal{D}, \mathcal{R}_1 \) and \( \mathcal{R}_2 \), which were diagonal on the basis of plane waves when they described specular scattering, are no longer diagonal in the general case of non specular scattering. The propagation factors remain diagonal in this basis with their diagonal values written as in (2). Clearly the expression (6) does not depend on the choice of a specific basis. But it may be written in specific basis fitting the geometry under study.

The multiple scattering formalism has been used in the past years by different groups using different notations (see as examples 50, 51, 52) and numerous applications have been considered. In particular, the case of corrugated plates or gratings has been extensively studied 53, 54, 55, 56 and it has given rise to interesting comparisons with experiments 57, 58, 59. Note also that calculations have been devoted to the study of atoms in the vicinity of corrugated plates 60, 61, 62.

7 The plane-sphere geometry beyond PFA

Recently, it has also become possible to use the general scattering formula to obtain explicit evaluations of the Casimir force in the plane-sphere geometry. Such calculations have first been performed for perfectly reflecting mirrors 63. They have then been done for the more realistic case of metallic mirrors described by a plasma model dielectric function 64. Even more recently, calculations were made which treat simultaneously plane-sphere geometry and non zero temperature, with dissipation taken into account 65.

In these calculations, the reflection matrices are written in terms of Fresnel amplitudes for plane waves on the plane mirror and of Mie amplitudes for spherical waves on the spherical mirror. The scattering formula is then obtained by writing also transformation formulas from the plane waves basis to the spherical waves basis and conversely. The energy takes the form of an exact multipolar formula labeled by a multipolar index \( \ell \). When doing the numerics, the expansion is truncated at some maximum value \( \ell_{\text{max}} \), which degrades the accuracy of the resulting estimation for very large spheres \( x \equiv L/R < x_{\text{min}} \) with \( x_{\text{min}} \) proportional to \( \ell_{\text{max}}^{-1} \).

The results of these calculations may be compared to the experimental study of PFA in the plane-sphere geometry 66. In this experiment, the force gradient is measured for various radii of the sphere and the results are used to obtain a constraint \( |\beta_G| < 0.4 \) on the slope at origin \( \beta_G \) of the function \( \rho_G(x) \)

\[
\rho_G = \frac{G}{G_{\text{PFA}}} = 1 + \beta_G x + O(x^2) \quad , \quad x \equiv \frac{L}{R}
\]

(7)

The slope obtained by interpolating at low values of \( x \) our theoretical evaluation of \( \rho_G \) reveals a striking difference between the cases of perfect and plasma mirrors. The slope \( \beta_G^{\text{perf}} \) obtained for perfect mirrors is larger than that \( \beta_G^{\text{Gold}} \) obtained for gold mirrors by a factor larger than 2

\[
\beta_G^{\text{perf}} \sim -0.48 \quad , \quad \beta_G^{\text{Gold}} \sim -0.21
\]

(8)

As a result, \( \beta_G^{\text{Gold}} \) is compatible with the experimental bound whereas \( \beta_G^{\text{perf}} \) is not 64.

The effect of temperature is also correlated with the plane-sphere geometry. The first calculations accounting simultaneously for plane-sphere geometry, temperature and dissipation have been published very recently 65 and they show several striking features. The factor of 2 between the long distance forces in Drude and plasma models is reduced to a factor below 3/2 in the plane-sphere geometry. Then, PFA underestimates the Casimir force within the Drude model at short distances, while it overestimates it at all distances for the perfect reflector and plasma.
model. If the latter feature were conserved for the experimental parameter region $R/L (> 10^2)$, the actual values of the Casimir force calculated within plasma and Drude model could turn out to be closer than what PFA suggests. This would affect the discussion of the next section, which is still based on calculations using PFA.

8 Discussion of experiments

We end up this review by discussing the status of comparisons between Casimir experiments and theory. We emphasize that, after years of improvement in experiments and theory, we have to face a lasting discrepancies in their comparison.

On one side, the Purdue and Riverside experiments\footnote{67,68,69} appear to favor predictions obtained with $\gamma = 0$ rather than those corresponding to the expected $\gamma \neq 0$ (see Fig.1 in\footnote{68}). This result stands in contradiction to the fact that Gold has a finite conductivity. Note that these experiments are done at distances smaller than 0.75 $\mu$m where the thermal contribution is small, so that accuracy is a critical issue here.

On the other side, a new experiment at Yale\footnote{70} has been able to measure the force at larger distances (0.7$\mu$m-7$\mu$m) where the thermal contribution is larger and the difference between the predictions at $\gamma = 0$ and $\gamma \neq 0$ significant. The results favor the expected Drude model ($\gamma \neq 0$), but only after subtraction of a large contribution of the patch effect.

It is worth emphasizing that the results of the new experiment see a significant thermal contribution and fit the expected model. Of course, they have to be confirmed by further studies\footnote{71}. In particular, the electrostatic patch effect remains a source of concern in Casimir experiments\footnote{72,73}. It is not measured independently in any of the experiments discussed above. This means that the Casimir effect, which is now verified in several experiments, is however not tested at the 1% level, as has been sometimes claimed. This also entails that the tests of gravity at the micrometer range have still room available for improvement.

Acknowledgments

The authors thank A. Canaguier-Durand, A. Gérardin, R. Guérout, J. Lussange, R.O. Behunin, I. Cavero-Pelaez, D. Dalvit, C. Genet, G.L. Ingold, F. Intravaia, M.-T. Jaekel, P.A. Maia Neto, V.V. Nesvizhevsky for contributions to the work reviewed in this paper, and the ESF Research Networking Programme CASIMIR (www.casimirnetwork.com) for providing excellent opportunities for discussions on the Casimir effect and related topics.

References

1. P.W. Milonni, *The quantum vacuum* (Academic, 1994).
2. S.K. Lamoreaux, Resource Letter in *Am. J. Phys.* 67 850 (1999).
3. M. Bordag, U. Mohideen and V.M. Mostepanenko, *Phys. Rep.* 353 1 (2001).
4. K.A. Milton, *J. Phys. A* 20 4628 (2005).
5. R.G. Barrera and S. Reynaud eds., *Focus on Casimir Forces, New J. Phys.* 8 (2006).
6. D. Dalvit *et al* eds., *Casimir physics*, Lecture Notes in Physics 834 (Springer, 2011)
7. E. Fischbach and C. Talmadge, *The Search for Non Newtonian Gravity* (AIP Press/Springer Verlag, 1998).
8. E.G. Adelberger, B.R. Heckel and A.E. Nelson, *Ann. Rev. Nucl. Part. Sci.* 53 77 (2003).
9. D.J. Kapner, T.S. Cook, E.G. Adelberger *et al*, *Phys. Rev. Lett.* 98 021101 (2007).
10. E.G. Adelberger *et al*, in this book.
11. A. Lambrecht and S. Reynaud, *Poincaré Seminar on Vacuum Energy and Renormalization* 1 107 (2002) [arXiv:quant-ph/0302073] and references therein.
12. R. Onofrio, New J. Phys. 8 237 (2006).
13. P. Wolf, P. Lemonde, A. Lambrecht et al, Phys. Rev. A 75 063608 (2007).
14. S. Lepoutre et al, EPL 88 20002 (2009).
15. B. Pelle et al, in this book.
16. V.V. Nesvizhevsky et al, Phys. Rev. D 77 034020 (2008).
17. V.V. Nesvizhevsky et al, in this book.
18. I. Antoniadis, S. Baessler, M. Buchner, V.V. Fedorov, S. Hoedl, V.V. Nesvizhevsky, G. Pignol, K.V. Protasov, S. Reynaud and Yu. Sobolev, Compt. Rend. Acad. Sci. (2011).
19. C. Genet, A. Lambrecht and S. Reynaud, in On the Nature of Dark Energy eds. U. Brax, J. Martin, J.P. Uzan, 121 (Frontier Group, 2002) [arXiv:quant-ph/0210173].
20. M.-T. Jaekel and S. Reynaud, in Mass and Motion in General Relativity, eds L. Blanchet, A. Spallicci and B. Whiting 491 (2011) [arXiv:0812.3936] and references therein.
21. M.-T. Jaekel et al, in this book.
22. W. Nernst, Verh. Deutsch. Phys. Ges. 18 83 (1916).
23. P.F. Browne, Apeiron 2 72 (1995).
24. S. Weinberg, Rev. Mod. Phys. 61 1 (1989).
25. R.J. Adler, B. Casey and O.C. Jacob, Am. J. Phys. 63 620 (1995).
26. W. Pauli, Die Allgemeinen Prinzipien der Wellenmechanik, in Handbuch der Physik 24 1 (1933); translation reproduced from Enz C.P., in Enz C.P. andMehra J. eds, Physical Reality andMathematical Description (Reidel, 1974) p.124.
27. C. Cohen-Tannoudji, J. Dupont-Roc and G. Gryenberg, Atom-Photon Interactions (Wiley, 1992).
28. Itzykson and J.-B. Zuber, Quantum Field Theory (McGraw Hill, 1985).
29. H.B.G. Casimir, Proc. K. Ned. Akad. Wet. 51 793 (1948).
30. A. Lambrecht and S. Reynaud, Euro. Phys. J. D 8 309 (2000).
31. J. Mehra, Physica 37 145 (1967).
32. J. Schwinger, L.L. de Raad and K.A. Milton, Ann. Phys. 115 1 (1978).
33. B.V. Deriagin, I.I. Abrikosova and E.M. Lifshitz, Quart. Rev. 10 295 (1968).
34. M.T. Jaekel and S. Reynaud, J. Physique I 1 1395 (1991) [arXiv:quant-ph/0101067].
35. C. Genet, A. Lambrecht and S. Reynaud, Phys. Rev. A 67 043811 (2003).
36. A. Lambrecht, P.A. Maia Neto and S. Reynaud, New J. Phys. 8 243 (2006).
37. A. Lambrecht, A. Canaguier-Durand, R. Guérout, and S. Reynaud, in [arXiv:1006.2959].
38. E.M Lifshitz, Sov. Phys. JETP 2 73 (1956).
39. I.E. Dzyaloshinskii, E.M. Lifshitz and L.P. Pitaevskii, Sov. Phys. Uspekhi 4 153 (1961).
40. L.P. Pitaevskii, Phys. Rev. Lett. 101 163202 (2008); Phys. Rev. Lett. 102 189302 (2009); B. Geyer et al, Phys. Rev. Lett. 102 189301 (2009).
41. D.A.R. Dalvit and S.K. Lamoreaux, Phys. Rev. Lett. 101 163203 (2008); Phys. Rev. Lett. 102 189304 (2009); R.S. Decca et al, Phys. Rev. Lett. 102 189303 (2009).
42. V.B. Svetovoy, Phys. Rev. Lett. 101 163603 (2008); Phys. Rev. Lett. 102 219903 (E) (2009).
43. C. Genet, F. Intravaia, A. Lambrecht, and S. Reynaud, Ann. Found. L. de Broglie 29 311 (2004).
44. F. Intravaia and A. Lambrecht, Phys. Rev. Lett. 94 110404 (2005).
45. C. Genet, A. Lambrecht and S. Reynaud, Phys. Rev. A 62 012110 (2000) and references therein.
46. M. Boström and Bo E. Sernelius, Phys. Rev. Lett. 84 4757 (2000).
47. S. Reynaud, A. Lambrecht and C. Genet, in Quantum Field Theory Under the Influence of External Conditions, ed. K.A.Milton (Rinton Press, 2004) p.36 [arXiv:quant-ph/0312224].
48. I. Brevik, S.A. Ellingsen and K. Milton, New J. Phys. 8 236 (2006).
49. G.-L. Ingold, A. Lambrecht and S. Reynaud, Phys. Rev. E 80 041113 (2009).
50. T. Emig, *J. Stat. Mech.: Theory Exp.* P04007 (2008).
51. O. Kenneth and I. Klich, *Phys. Rev. B* **78** 014103 (2008).
52. K.A. Milton and J. Wagner, *J. Phys. A* **41** 155402 (2008).
53. R.B. Rodrigues, P.A. Maia Neto, A. Lambrecht and S. Reynaud, *Phys. Rev. Lett.* **96** 100402 (2006); *Phys. Rev. Lett.* **98** 068902 (2007).
54. R.B. Rodrigues, P.A. Maia Neto, A. Lambrecht and S. Reynaud, *Phys. Rev. A* **75** 062108 (2007).
55. R.B. Rodrigues, P.A. Maia Neto, A. Lambrecht and S. Reynaud, *EPL* **76** 822 (2006).
56. A. Lambrecht and V.N. Marachevsky, *Phys. Rev. Lett.* **101** 160403 (2008); A. Lambrecht, *Nature* **454** 836 (2008).
57. H.B. Chan, Y. Bao, J. Zou *et al., Phys. Rev. Lett.* **101** 030401 (2008).
58. H.C. Chiu, G.L. Klimchitskaya, V.N. Marachevsky *et al., Phys. Rev. B* **80** 121402 (2009).
59. Y. Bao, R. Gurout, J. Lussange *et al., Phys. Rev. Lett.* **105** 250402 (2010).
60. D.A.R. Dalvit, P.A. Maia Neto, A. Lambrecht and S. Reynaud, *Phys. Rev. Lett.* **100** 040405 (2008).
61. R. Messina, D.A.R. Dalvit, P.A. Maia Neto, A. Lambrecht and S. Reynaud, *Phys. Rev. A* **80** 022119 (2009).
62. A.M. Contreras-Reyes, R. Gurout, P.A. Maia Neto, D.A.R. Dalvit, A. Lambrecht and S. Reynaud, *Phys. Rev. A* **82** 052517 (2010).
63. P.A. Maia Neto, A. Lambrecht and S. Reynaud, *Phys. Rev. A* **78** 012115 (2008).
64. A. Canaguier-Durand, P.A. Maia Neto, I. Cavero-Pelaez, A. Lambrecht and S. Reynaud, *Phys. Rev. Lett.* **104** 230404 (2009).
65. A. Canaguier-Durand, P.A. Maia Neto, A. Lambrecht and S. Reynaud, *Phys. Rev. Lett.* **104** 040403 (2010).
66. D.E. Krause, R.S. Decca, D. Lopez and E. Fischbach, *Phys. Rev. Lett.* **98** 050403 (2007).
67. R.S. Decca, D. López, E. Fischbach *et al., Annals Phys.* **318** 37 (2005).
68. R.S. Decca, D. López, E. Fischbach *et al., Phys. Rev. D* **75** 077101 (2007).
69. G.L. Klimchitskaya, U. Mohideen and V.M. Mostepanenko, *Rev. Mod. Phys.* **81** 1827 (2009).
70. A.O. Sushkov, W.J. Kim, D.A.R. Dalvit and S.K. Lamoreaux, *Nature Physics* **7** 230 (2011).
71. K. Milton, *Nature Physics* **7** 190 (2011).
72. C.C. Speake and C. Trenkel, *Phys. Rev. Lett.* **90** 160403 (2003).
73. W.J. Kim, A.O. Sushkov, D.A.R. Dalvit and S.K. Lamoreaux, *Phys. Rev. A* **81** 022505 (2010).