Quark polarimetry with a recursive fragmentation model including the spin degree of freedom

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Abstract. Several estimators of the initial transverse polarization of a quark fragmenting into a jet are proposed and compared using Monte Carlo simulations. These are based on the simplified version of a recursive model [A. Kerbizi et al, Phys. Rev. D97, 074010 (2018)] which includes the quark spin degree of freedom.

1. Introduction

The precise transverse profile of a jet produced by the fragmentation of a quark depends on the polarization vector \(S = (S_T, S_L)\). In particular, the Collins effect [1] results in a single-hadron distribution of the form

\[
dN_{q_a \to h + X} = (dz_h / z_h) d^2 p_T D_0(\Delta z_h, |p_T|) [1 + |S_T| A_h(z_h, |p_T|) \sin \phi_p S] ;
\]

\(S_T = (S_x, S_y)\) is the transversity vector; \(S_L = S_z\) is twice the helicity; \(D_0\) is the unpolarized fragmentation function, \(A_h\) is the Collins analyzing power; \(\phi_p S = \phi_p - \phi_S\) where \(\phi_V\) is the azimuth of a vector \(V\); \(p\) and \(k_{in}\) are the 4-momenta of the observed hadron \(h\) and the quark which initiates the jet; \(z_h = p^+/k_{in}^+\) with \(V^+ = V^0 \pm V^z\). The \(z\)-axis is chosen along \(k_{in}\).

For a two-particle distribution, one may separate the global variables \(z_{dh} = z_{h1} + z_{h2}\) and \(p_T = p_{T1} + p_{T2}\) of the di-hadron \(dh = \{h_1, h_2\}\), from the relative variables \(\hat{z}_1 = z_{h1}/z_{dh}\) and \(R_T = (1 - \hat{z}_1)p_{T1} - \hat{z}_1 p_{T2}\). Then

\[
dN_{q_a \to h_1 + h_2 + X} = d^2 p_T d^2 R_T \frac{dz_{h1} dz_{h2}}{z_{h1} z_{h2}} D_0(\mathcal{X})
\times \left\{ 1 + |S_T| [A_{dh P}(\mathcal{X}) \sin \phi_P S + A_{dh R}(\mathcal{X}) \sin \phi_R S] + S_L A_{dh L}(\mathcal{X}) \sin \phi_R P \right\}
\]

where \(\mathcal{X}\) is the argument list \(\{z_{dh}, \hat{z}_1, R_T, |\phi_R P|\}\), \(A_{dh P}\) and \(A_{dh R}\) are the global and relative Collins asymmetries of the pair. \(A_{dh L}\) is a measure of jet handedness [2, 3] associated to the quark helicity. Integrating over \(p_T\) yields

\[
dN_{q_a \to h_1 + h_2 + X} = d^2 R_T \frac{dz_{h1} dz_{h2}}{z_{h1} z_{h2}} D_0(\hat{z}_{dh}, \hat{z}_1, R_T) \left\{ 1 + S_T A_{dh R}(z_{dh}, \hat{z}_1, R_T) \sin \phi_R S \right\}.
\]
$A_{\text{dhR}}$ is called the di-hadron analyzing power, $A_{\text{dhR}} \times \bar{D}_0$ the interference fragmentation function. The above asymmetries can serve as quark polarimeters in the measurement of the transversity distribution in the nucleon, $h_1(x)$, or for testing new physics. If $k_{in}$ is not precisely defined, due to experimental errors or to gluon emission, $A_h$, $A_{dhp}$ and $A_{dhl}$ are blurred, but not $A_{\text{dhR}}$ or $A_{\text{dhR}}$.

The present quark fragmentation models used in simulation codes like PYTHIA are blind to the quark spin degree of freedom, therefore these codes cannot generate Collins- or jet handedness asymmetries. However this situation is changing with the development of recursive models for the fragmentation of polarized quarks. One of them $[4, 5, 6]$ is based on the string fragmentation asymmetries. However this situation is changing with the development of recursive models for the quark spin degree of freedom, therefore these codes cannot generate Collins- or jet handedness asymmetries.

In Section 2. Examples of estimators of transversity are given in Section 2. In Section 3 the string+3P0 model is briefly reviewed, in a simplified version of the string+3P0 model. Examples of estimators for the transversity are given and discussed in Section 4.

Efficiency $\{E_\alpha\} = \hat{E}_\alpha^2$, where $\hat{E}_\alpha = \langle E_\alpha \rangle / \sqrt{\langle E_\alpha^2 \rangle} \in [-1, +1].$ (5)

$\hat{E}_\alpha$ is indeed the signal/noise ratio per event. For a given precision the required number of events is inversely proportional to $\hat{E}_\alpha^2$.

A good estimator $E_\alpha$ must be simple, universal (i.e., usable in all types of jet-producing reactions like $e^+e^-$ annihilation, deep inelastic scattering an hight-PT collisions) and have a large efficiency. Its choice can be guided by a Monte Carlo simulation based on a polarized fragmentation model. In this article we present an example of this method for quark transverse polarization, using the the string+3P0 model. Examples of estimators for the transversity are given in Section 2. In Section 3 the string+3P0 model is briefly reviewed, in a simplified version [12]. Simulated results of $\hat{E}$ for several estimators are given and discussed in Section 4.

2. Exemples estimators of transversity

A simple estimator of transversity, based on the Collins effect, is

$$E_T = (E_x, E_y) = \hat{z} \times p_T / |p_T| = (-\sin \phi_p, \cos \phi_p),$$ (6)

where $p_T$ is the momentum a detected particle of the jet, selected for instance for its charge and its order in rapidity. With two hadrons $h_1$ and $h_2$ one can use the relative Collins effect, replacing $p_T$ by $R_T$. An estimator more efficient than (6), inspired by Eq. (13) of [4], could be

$$E'_T = z_h \hat{z} \times p_T / (p_T^2 + \lambda(p_T^2)),$$ (7)
where $\lambda$ is a parameter of the order of unity. This estimator takes advantage of the increase of $A_h(z_h,|p_T|)$ with $z_h$ and its vanishing for $|p_T| \to 0$ or $\infty$. Theoretically, the most efficient estimator of $S_\alpha$ is the derivative $\partial / \partial S_\alpha$ of the spin asymmetry. Rewriting the square bracket in Eq.(1) as $[1 - A_h(z_h,|p_T|) S \cdot (\hat{z} \times p_T) / |p_T|]$, we deduce

$$E_T(\text{optimal}) = -A_h(z_h,|p_T|) \hat{z} \times p_T / |p_T|. \quad (8)$$

To have an efficiency close to the optimal one, it not necessary to know the function $A_h(z_h,|p_T|)$ precisely. $E_\alpha(\text{optimal}) - E_\alpha$ is indeed of the second order in $E_\alpha(\text{optimal}) - E_\alpha$.

One can improve the polarimetry by gathering informations from all detected particles $h_n$ of the jet. The multi-hadron estimator

$$E_T^n = \sum_n Q_n z_{hn} \hat{z} \times p_{nT} / (p_{nT}^2 + \lambda (p_{nT}^2)), \quad (9)$$

which generalizes (7), takes into account the fact that the sign of the Collins asymmetry is linked to the charge $Q$ of the hadron.

3. The recursive string $+ \bar{3}P_0$ model

This model is a generalization of the Lund model of string fragmentation [13], but starting with amplitudes instead of probabilities and including the quark spin degree of freedom, represented by Pauli spinors. It can also be formulated as a multiperipheral model with quark propagators and quark-hadron vertex functions being $2 \times 2$ matrices. In the ladder approximation, one obtains the Markov chain

$$q_1 \to h_1 + q_2, \quad q_2 \to h_2 + q_3, \quad \text{etc.,} \quad (10)$$

which can be generated by the Monte Carlo method. The splitting function $F$ of the elementary process $q \to h + q'$, defined by

$$dN(q \to h + q') = F_{q \to h+q'}(Z, p_T, k_T, S_q) d^2 p_T dZ / Z \quad (Z = p^+ / k^+), \quad (11)$$

depends on the polarization $S_q$ of $q$ and is of the form

$$F_{q \to h+q'}(Z, p_T, k_T, S_q) = \text{Tr} \left[ T_{q \to h+q'}(Z, p_T, k_T) \rho(q) T_{q \to h+q'}^\dagger(Z, p_T, k_T) \right]. \quad (12)$$

$\rho(q) = (1 + S_q \cdot \sigma) / 2$ is the spin density matrix of quark $q$. The splitting matrix $T$ cannot be chosen arbitrarily because the model must be symmetric under the reversal of the multiperipheral quark line (the “left-right” symmetry of the Lund model). Its general form is given in Eqs. (35) and (50) of [7]. Once $h$, $q'$, $Z$ and $p_T$ have been drawn, one calculates the spin density matrix of $q'$ for the next iteration by

$$\rho(q') = \left[ T_{q \to h+q'}(Z, p_T, k_T) \rho(q) T_{q \to h+q'}^\dagger(Z, p_T, k_T) \right] / \text{Tr} [\text{idem}] \quad (13)$$

Here we use a simpler version [12] of the model than the one applied in [7]. It corresponds to the choice (c) for the function $\bar{g}(\epsilon_h^2)$ below Eq.(50) of [7]. Then the auxiliary matrix function $H_q(k_T)$ introduced in Eq.(46) of [7] is the unit matrix times a normalization factor. We restrict ourselves to the emission of pseudo-scalar mesons, where the vertex function is simply $\Gamma_h = \sigma_z$ (the $2 \times 2$ analogue of $\gamma_5$). Instead of Eq.(51) of [7] we obtain

$$F_{q', h, q}(Z, p_T, k_T, S_q) = |C_{q', h, q}|^2 N_q^{-1} \epsilon_h^2 \left[ (1 - Z) / \epsilon_h^2 \right] \exp\left(-b_t \epsilon_h^2 / Z - b_t k_T^2 \right) \times \text{Tr} \left[ (\mu + \sigma_z \sigma \cdot k_T) \sigma_z \rho(q) \sigma_z (\mu^* + \sigma \cdot k_T^\dagger \sigma_z) \right] \quad (14)$$
with \( \epsilon_h = [m_h^2 + p_h^2]^{1/2} \), \( k'_T = k_T - p_T \) and
\[
N_a(\epsilon_h^2) = \int_0^1 dZ \left( \frac{1 - Z}{\epsilon_h^2} \right) a \exp(-b_L \epsilon_h^2/Z). \tag{15}
\]
a and \( b_L \) correspond to the parameters \( a \) and \( b \) of the Lund model. \( b_T \) governs the width of quark transverse momentum. \( C_{q',h,q} \) is proportional to the internal \((q'q)\) wave function of the hadron \( h \) in flavor space. It acts upon the relative abundances of the hadron species. The second line of Eq.(14) makes the difference with the spin-blind Lund model. The factor \( \sigma \cdot k'_T \) is inspired by the \( ^3P_0 \) wave function \( \propto \sigma \cdot k \) of a \((q'q)\) state. \( \mu \) is a complex parameter having the dimension of a mass. The qualitative results of the classical string-\( ^3P_0 \) mechanism of \( q\bar{q} \) pair production are reproduced by taking \( \text{Im}(\mu) > 0 \) [4].

### 4. Numerical results for the estimators of transverse polarization

We took the same parameters \( a = 0.9 \), \( b_L = 0.5 \text{ GeV}^{-2} \), \( b_T = 5.17 \text{ GeV}^{-2} \) and \( \mu = (0.42 + i0.76) \text{ GeV} \) as used in [7] and the same subroutines for the generation of the quark flavors and meson species. In principle we should have re-adjusted the parameters for the present version of the model. We think, however, that it will not change our qualitative comparisons between estimators. We generated 20 hadrons per event, without taking into account the finite mass of the intial string, but made a cut \( z_h \geq z_{h,\min} \). We also introduced a Gaussian-distributed \textit{primordial transverse momentum} \( k_T^{\text{prim}} \) of r.m.s. \( K \). The \( p_T \)'s of all the mesons in a jet were shifted by \( z_h k_T^{\text{prim}} \). It can as well simulate the effect of gluon radiation or an experimental error in the definition of the jet axis. We have compared the following estimators of the transverse polarization \( S_y \):

\[
E^\pm = p_x/|p_T| = \cos \phi_p \text{ of the fastest positive (for } E^+ \text{) or negative (for } E^- \text{) hadron},
\]
\[
E^m = \sum_n Q_n p_{nx}/|p_{nT}|, \text{ where } n \text{ labels the } n^{\text{th}} \text{ detected hadron},
\]
\[
E'^\pm = z_h p_x/(p_T^2 + 0.5|\mu|^2 + 0.5 z_h^2 K^2) \text{ of the fastest positive or negative hadron},
\]
\[
E'^m = \sum_n Q_n z_h p_{nx}/(p_{nx}^2 + 0.5|\mu|^2 + 0.5 z_h^2 K^2),
\]
\[
E^R = R_x/|R_T|, \text{ where } R = (z_{h+} P_{h+} - z_{h-} P_{h-})/(z_{h+} + z_{h-}) \text{ for the fastest } h^+ \text{ and } h^-,
\]
\[
E'^R = z_{h+} - z_{h-} - R_x/(R_T^2 + 1.0).
\]

The unprimed \( E \)'s are of the type of (6). \( E'^\pm \) and \( E'^m \) are of the type of (7) with \( \lambda = 0.5 \), assuming \( (p_T^2) \sim |\mu|^2 + (z_h K)^2 \). \( E^m \) and \( E'^m \) are \textit{multi-hadron} estimators. \( E^R \) and \( E'^R \), based on the relative Collins effect or di-hadron asymmetry, are not sensitive to \( K \).

Table 1 shows the “root efficiencies” \( \bar{E} = \langle E \rangle /\sqrt{\langle E^2 \rangle} \), obtained in a simulation of \( 10^5 \) jets from \( u \) quarks fully polarized along \( +\hat{y} \). The r.m.s. of \( p_T \) (in GeV) is also indicated. Three values of \( K \) were considered. We retained only particles with \( z_h \geq z_{h,\min} = 0.1 \). This restriction makes some estimators unavailable in parts of the events (then they are assigned the value 0). The fraction of events where the estimator \( E \) is available is given in the last line. The mean charged multiplicities are \( \langle N^+ \rangle = 1.06 \) and \( \langle N^- \rangle = 0.58 \).

**Discussion of the results.**

- For \( K =1 \text{ GeV} \) the efficiencies of all estimators are significantly reduced, except for the di-hadron ones \( E^R \) and \( E'^R \) which are not affected by the primordial \( k_T \). Note that an increase of \( \bar{E} \) by a factor 1.5 reduces the required number of events by a factor 2.25 to get the same precision.
- Replacing an \( E \) by the corresponding \( E' \) increases the efficiency, particularly for \( E^R \).
- The \textit{multi-hadron} estimators \( E^m \) or \( E'^m \) have the largest efficiencies, for all values of \( K \).
Table 1. Root efficiencies $\hat{E}$ of several estimators (see text).

| $K$ (GeV) | rms($p_T$) | $\hat{E}^+$ | $\hat{E}'^+ | \hat{E}^- | \hat{E}'^- | \hat{E}^m | \hat{E}'^m | \hat{E}^R | \hat{E}'^R |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.733 | 0.171 | 0.188 | -0.107 | -0.132 | 0.181 | 0.215 | 0.118 | 0.157 |
| 0.5 | 0.751 | 0.156 | 0.166 | -0.104 | -0.126 | 0.172 | 0.197 | 0.118 | 0.157 |
| 1.0 | 0.801 | 0.128 | 0.132 | -0.096 | -0.113 | 0.154 | 0.168 | 0.118 | 0.157 |

available fraction: 0.838 0.838 0.516 0.516 1.0 1.0 0.457 0.457

5. Conclusion

Our study shows that quark polarimetry can be significantly improved by a judicious choice of the estimator. This choice can be first guided by Monte-Carlo simulations based on a theoretical model. The final optimization should be made using equation like (8) with experimental data. The choice may also be influenced by the experimental conditions like the resolution and the acceptance of the detectors, but then the estimator is no more “universal”.

In the simulation we assumed that the quark flavor is $u$. A $d$-quark would give approximately opposite asymmetries. When the flavor is not a priori known, the polarization estimator has to be coupled with a flavor estimator.

The choice of estimator will be still more critical for the polarimetry of quark helicity based on jet handedness. This effect is predicted by the existing theoretical models [4, 7, 11]. To avoid the blurring by the primordial transverse momentum, the handedness estimator should involve three hadrons [2, 3]. This makes the number of variables rather large and the estimator choice more difficult than for transversity.

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