Conformal superfields and BPS states in $AdS_{4/7}$ geometries

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Abstract

We carry out a general analysis of the representations of the superconformal algebras $OSp(8/4,\mathbb{R})$ and $OSp(8^*/2N)$ in terms of harmonic superspace. We present a construction of their highest-weight UIR’s by multiplication of the different types of massless conformal superfields (“supersingletons”). Particular attention is paid to the so-called “short multiplets”. Representations undergoing shortening have “protected dimension” and may correspond to BPS states in the dual supergravity theory in anti-de Sitter space.

These results are relevant for the classification of multitrace operators in boundary conformally invariant theories as well as for the classification of AdS black holes preserving different fractions of supersymmetry.

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1 Introduction

Superconformal algebras and their representations play a crucial rôle in the AdS/CFT correspondence because of their dual rôle of describing the gauge symmetries of the AdS bulk supergravity theory and the global symmetries of the boundary conformal field theory [1, 2, 3].

A special class of configurations which are particularly relevant are the so-called BPS states, i.e. dynamical objects corresponding to representations which undergo “shortening”.

These representations can only occur when the conformal dimension of a (super)primary operator is “quantized” in terms of the R symmetry quantum numbers and they are at the basis of the so-called “non-renormalization” theorems of supersymmetric quantum theories [4].

There exist different methods of constructing the UIR’s of superconformal algebras. One is the so-called oscillator construction of the Hilbert space in which a given UIR acts [3]. Another one, more appropriate to describe field theories, is the realization of such representations on superfields defined in superspaces [6, 7]. The latter are “supermanifolds” which can be regarded as the quotient of the conformal supergroup by some of its subgroups.

In the case of ordinary superspace the subgroup in question is the supergroup obtained by exponentiating a non-semisimple superalgebra which is the semidirect product of a super-Poincaré graded Lie algebra with dilatation (SO(1,1)) and the R symmetry algebra. This is the superspace appropriate for non-BPS states. Such states correspond to bulk massive states which can have “continuous spectrum” of the AdS mass (or, equivalently, of the conformal dimension of the primary fields).

BPS states are naturally associated to superspaces with lower number of “odd” coordinates and, in most cases, with some internal coordinates of a coset space G/H. Here G is the R symmetry group of the superconformal algebra, i.e. the subalgebra of the even part which commutes with the conformal algebra of space-time and H is some subgroup of G having the same rank as G.

Such superspaces are called “harmonic” [8] and they are characterized by having a subset of the initial odd coordinates θ. The complementary number of odd variables determines the fraction of supersymmetry preserved by the BPS state. If a BPS state preserves K supersymmetries then the θ’s of the associated harmonic superspace will transform under some UIR of HK.

For 1/2 BPS states, i.e. states with maximal supersymmetry, the superspace involves the minimal number of odd coordinates (half of the original one) and HK is then a maximal subgroup of G. On the other hand, for states with the minimal fraction of supersymmetry HK reduces to the “maximal torus” whose Lie algebra is the Cartan subalgebra of G.

It is the aim of the present paper to give a comprehensive treatment of BPS states related to “short representations” of superconformal algebras for the cases which are most relevant in the context of the AdS/CFT correspondence, i.e. the d = 3 (N = 8) and d = 6 (for arbitrary N). The underlying conformal field theories correspond to world-volume theories of Nc copies of M2 and M5 branes in the large Nc limit [9]-[15] which are “dual” to AdS supergravities describing the horizon geometry of the branes [16].

The present contribution summarizes the results which have already appeared elsewhere [15, 17, 18, 19]. We first carry out an abstract analysis of the conditions for Grassmann (G-)analyticity [20] (the generalization of the familiar concept of chirality [7]) in a superconformal context. We find the constraints on the conformal dimension and R symmetry quantum numbers of a superfield following from the requirement that it do not depend on one or more Grassmann vari-
ables. Introducing G-analyticity in a traditional superspace cannot be done without breaking the R symmetry. The latter can be restored by extending the superspace by harmonic variables \([21, 8, 22-26]\) parametrizing the coset \(G/H_K\). We also consider the massless UIR’s (“supersingleton” multiplets) \([27, 28]\), first as constrained superfields in ordinary superspace \([29, 30, 31]\) and then, for a part of them, as G-analytic harmonic superfields \([8, 26, 32, 33, 31]\). Next we use supersingleton multiplication to construct UIR’s of \(\text{OSp}(8^*/2N)\) and \(\text{OSp}(8/4, \mathbb{R})\). We show that in this way one can reproduce the complete classification of UIR’s of ref. \([34]\). We also discuss different kinds of shortening which certain superfields (not of the BPS type) may undergo.

We conclude the paper by listing the various BPS states in the physically relevant cases of \(M_2\) and \(M_5\) branes horizon geometry where only one type of supersingletons appears.

Massive towers corresponding to 1/2 BPS states are the K-K modes coming from compactification of M-theory on \(\text{AdS}_7/4 \times S^4/7\) \([35, 9]\). Short representations of superconformal algebras also play a special role in determining \(N\)-point functions from OPE \([36, 37]\).

Another area of interest is the classification of AdS black holes \([38-40]\), according to the fraction of supersymmetry preserved by the black hole background.

In a parallel analysis with black holes in asymptotically flat background \([41]\), the AdS/CFT correspondence predicts that such BPS states should be dual to superconformal states undergoing “shortening” of the type discussed here.

2 The six-dimensional case

In this section we describe highest-weight UIR’s of the superconformal algebras \(\text{OSp}(8^*/2N)\) in six dimensions. Although the physical applications refer to \(N = 1\) and \(N = 2\), it is worthwhile to carry out the analysis for general \(N\), along the same lines as in the four-dimensional case \([12, 43]\). We first examine the consequences of G-analyticity and conformal supersymmetry and find out the relation to BPS states. Then we will construct UIR’s of \(\text{OSp}(8^*/2N)\) by multiplying supersingletons. The results exactly match the general classification of UIR’s of \(\text{OSp}(8^*/2N)\) of Ref. \([34]\).

2.1 The conformal superalgebra \(\text{OSp}(8^*/2N)\) and Grassmann analyticity

The part of the conformal superalgebra \(\text{OSp}(8^*/2N)\) relevant to our discussion is given below:

\[
\{Q^i_{\alpha}, Q^j_{\beta}\} = 2\Omega^{ij} c_{\alpha\beta} P_\mu , \\
\{S^i_{\alpha}, S^j_{\beta}\} = 2\Omega^{ij} \gamma_{\mu}^{\alpha\beta} K_\mu , \\
\{Q^i_{\alpha}, S^j_{\beta}\} = i\Omega^{ij} (\gamma^{\mu\nu})_{\alpha\beta} M_{\mu\nu} + 2\delta_{\alpha}^{\beta} (4T^{ij} - i\Omega^{ij} D) , \\
[D, Q^i_{\alpha}] = \frac{i}{2} Q^i_{\alpha} , \quad [D, S^i_{\alpha}] = -\frac{i}{2} S^i_{\alpha} , \\
[T^{ij}, Q^k_{\alpha}] = -\frac{1}{2}(\Omega^{kij} Q^i_{\alpha} + \Omega^{ij} Q^k_{\alpha}) , \\
[T^{ij}, T^{kl}] = \frac{1}{2}(\Omega^{ijkl} + \Omega^{ij} T^{kl} + \Omega^{jk} T^{il} + \Omega^{jl} T^{ki}) .
\]

Here \(Q^i_{\alpha}\) are the generators of Poincaré supersymmetry carrying a right-handed chiral spinor index \(\alpha = 1, 2, 3, 4\) of the Lorentz group \(\text{SU}^*(4) \sim \text{SO}(5, 1)\) (generators \(M_{\mu\nu}\) and an index \(i = 2\))
1, 2, ..., 2N of the fundamental representation of the R symmetry group USp(2N) (generators $T^{ij} = T^{ji}$); $S^{ij}_j$ are the generators of conformal supersymmetry carrying a left-handed chiral spinor index; $D$ is the generator of dilations, $P_\mu$ of translations and $K_\mu$ of conformal boosts.

It is convenient to make the non-standard choice of the symplectic matrix $\Omega^{ij} = -\Omega^{ji}$ with non-vanishing entries $\Omega^1_{2N} = \Omega^{2N-1} = \ldots = \Omega^N_{N+1} = 1$. The chiral spinors satisfy a pseudo-reality condition of the type $Q^i_\alpha = \Omega^{ij}Q^j_\beta c_{\beta\alpha}$ where $c$ is a 4 × 4 unitary “charge conjugation” matrix. Note that the generators $M, P, K, D$ form the Lie algebra of $SO(8^*) \sim SO(2, 6)$ and the generators $Q, S$ form an $SO(8^*)$ chiral spinor.

The standard realization of this superalgebra makes use of a superspace with even coordinates $x^\mu$ and left-handed spinor odd ones $\theta^a_i$. Unlike the four-dimensional case, here chirality is not an option but is already built in. The only way to obtain smaller superspaces is through Grassmann analyticity. We begin by imposing a single condition of G-analyticity:

$$q^1_{\alpha} \Phi(x, \theta) = 0 \quad (2.7)$$

(here and in what follows the lower-case notation refers to the matrix part of the generators). This condition amounts to removing the odd variable $\theta^{2N}$, i.e. to a superspace with coordinates $x^\mu, \theta^a_1, 2, \ldots, 2N-1$. From the definition of a superconformal primary field we have

$$s^i_\beta \Phi = 0, \quad (2.8)$$

which, together with (2.7) and the algebra (2.1)-(2.6) yields the consistency conditions

$$m_{\mu\nu} = 0, \quad (2.9)$$
$$t^{11} = t^{12} = \ldots = t^{12N-1} = 0, \quad (2.10)$$
$$4t^{12N} + \ell = 0 \quad (2.11)$$

(in (2.11) $\ell$ denotes the conformal dimension, i.e. the eigenvalue of $-iD$). Eq. (2.9) implies that the superfield $\Phi$ must be a Lorentz scalar. In order to interpret eqs. (2.10), (2.11), we need the Cartan decomposition of the algebra of USp(2N) into:

(i) raising operators (corresponding to the positive roots):

$$T^k_\mu = 2N-l, \quad k = 1, \ldots, N, \quad l = k, \ldots, 2N-k \quad (simple \ if \ l = k); \quad (2.12)$$

(ii) $[U(1)]^N$ charges:

$$H_k = -2T^k_1, \quad k = 1, \ldots, N; \quad (2.13)$$

(iii) the rest are lowering operators (corresponding to the negative roots). The Dynkin labels $a_k$ of a USp(2N) irrep are defined as follows:

$$a_k = H_k - H_{k+1}, \quad k = 1, \ldots, N-1, \quad a_N = H_N, \quad (2.14)$$

so that, for instance, the projection $Q^1$ of the supersymmetry generator is the HWS of the fundamental irrep $(1, 0, \ldots, 0)$.

Now it becomes clear that (2.10) is part of the USp(2N) irreducibility conditions whereas (2.11) relates the conformal dimension to the sum of the Dynkin labels:

$$\ell = 2 \sum_{k=1}^{N} a_k. \quad (2.15)$$
Let us denote the highest-weight UIR’s of the OSp(8*/2N) algebra by
\[ \mathcal{D}(\ell; J_1, J_2, J_3; a_1, \ldots, a_N) \]
where \( \ell \) is the conformal dimension, \( J_1, J_2, J_3 \) are the SU*(4) Dynkin labels and \( a_k \) are the USp(2N) Dynkin labels of the first component. Then the G-analytic superfields defined above are of the type
\[ \Phi(\theta^{1,2,\ldots,2N-1}) \Leftrightarrow \mathcal{D}(2\sum_{k=1}^{N} a_k; 0, 0, 0; a_1, \ldots, a_N) . \]  
(2.16)

The next step is to impose a second condition of G-analyticity with the generator \( q^2_{\alpha} \) which removes one more odd variable and leads to a superspace with coordinates \( x^\mu, \theta^{1,2,\ldots,2N-2} \). This implies the new constraints
\[ 4t^{2N-1} + \ell = 0 \Rightarrow a_1 = 0, \quad t^{2N} = 0 . \]  
(2.17)

Note that the vanishing of the lowering operator \( t^{2N} \) means that the subalgebra SU(2) \( \subset \) USp(2N) formed by \( t^{2N-1}, t^{2N} \) and \( t^{2N} - t^{2N-1} \) acts trivially on the particular USp(2N) irreps. This is equivalent to setting \( a_1 = 0 \), as in (2.17). Thus, the new G-analytic superfields are of the type
\[ \Phi(\theta^{1,2,\ldots,2N-2}) \Leftrightarrow \mathcal{D}(2\sum_{k=2}^{N} a_k; 0, 0, 0; a_2, \ldots, a_N) . \]  
(2.18)

From (2.1) it is clear that we can go on in the same manner until we remove half of the \( \theta \)'s, namely \( \theta^{N+1}, \ldots, \theta^{2N} \). Each time we have to set a new Dynkin label to zero. We can summarize by saying that the superconformal algebra OSp(8*/2N) admits the following short UIR’s corresponding to BPS states:
\[ \frac{p}{2N} \text{ BPS : } \mathcal{D}(2\sum_{k=p}^{N} a_k; 0, 0, 0; \ldots, 0, a_p, \ldots, a_N) , \quad p = 1, \ldots, N . \]  
(2.19)

### 2.2 Supersingletons

There exist three types of massless multiplets in six dimensions corresponding to ultrashort UIR’s (supersingletons) of OSp(8*/2N) (see, e.g., [44] for the case \( N = 2 \)). All of them can be formulated in terms of constrained superfields as follows.

(i) The first type is described by a superfield \( W^{(i_1,\ldots,i_n)}(x, \theta) \), \( 1 \leq n \leq N \), which is antisymmetric and traceless in the external USp(2N) indices (for even \( n \) one can impose a reality condition). It satisfies the constraint (see [29] and [15])
\[ D^{(k}_{\alpha} W^{(i_1,\ldots,i_n)}(x, \theta) = 0 \Rightarrow \mathcal{D}(2; 0, 0, 0; \ldots, 0, a_n = 1, 0, \ldots, 0) \]  
(2.20)

where the spinor covariant derivatives obey the supersymmetry algebra
\[ \{ D^i_{\alpha}, D^j_{\beta} \} = -2i\Omega^{ij}_{\gamma\alpha\beta} \partial^\gamma . \]  
(2.21)

The components of this superfield are massless fields. In the case \( N = n = 1 \) this is the on-shell \((1,0)\) hypermultiplet and for \( N = n = 2 \) it is the on-shell \((2,0)\) tensor multiplet [29, 30].
(ii) The second type is described by a (real) superfield without external indices, \( w(x, \theta) \). The corresponding constraint is second-order in the spinor derivatives:

\[
D^{i [\alpha} D^{j]} w = 0 \quad \Rightarrow \quad \mathcal{D}(2; 0, 0; 0, 0, \ldots, 0).
\]  

(2.22)

(iii) Finally, there exists an infinite series of multiplets described by superfields with \( n \) totally symmetrized external Lorentz spinor indices, \( w_{(\alpha_1 \ldots \alpha_n)}(x, \theta) \) (they can be made real in the case of even \( n \)). Now the constraint takes the form

\[
D^I w_{(\alpha_1 \ldots \alpha_n)} = 0 \quad \Rightarrow \quad \mathcal{D}(2 + n/2; n, 0, 0, 0, \ldots, 0).
\]

(2.23)

As shown in ref. [18], the six-dimensional massless conformal fields only carry reps \((J_1, 0)\) of the little group SU(2) \(\times\) SU(2) of a light-like particle momentum. This result is related to the analysis of conformal fields in \(d\) dimensions [46, 47]. This fact implies that massless superconformal multiplets are classified by a single SU(2) and USp(2\(N\)) R-symmetry and are therefore identical to massless super-Poincaré multiplets in five dimensions. Some physical implication of the above circumstance have recently been discussed in ref. [48] where it was suggested that certain strongly coupled \(d = 5\) theories effectively become six-dimensional.

2.3 Harmonic superspace

The massless multiplets (i), (ii) admit an alternative formulation in harmonic superspace (see [32, 33, 31] for \(N = 1, 2\)). The advantage of this formulation is that the constraints (2.20) become conditions for G-analyticity. We introduce harmonic variables describing the coset USp(2\(N\))/[U(1)]\(^N\):

\[
u \in \text{USp}(2N) : \quad u^I_i u^J_j = \delta^I_J, \quad u^I_i \Omega^{IJ} u^J_j = \Omega^I_J, \quad u^I_i = (u^I_i)^*.
\]

(2.24)

Here the indices \(i, j\) belong to the fundamental representation of USp(2\(N\)) and \(I, J\) are labels corresponding to the [U(1)]\(^N\) projections. The harmonic derivatives

\[
D^{IJ} = \Omega^{K(I} u^J_K} \frac{\partial}{\partial u^K_i}
\]

(2.25)

form the algebra of USp(2\(N\))\(_R\) (see (2.6)) realized on the indices \(I, J\).

Let us now project the defining constraint (2.20) with the harmonics \(u^K_i u^1_{i_1} \ldots u^n_{i_n}\), \(K = 1, \ldots, n\):

\[
D^K_\alpha W^{12 \ldots n} = D^K_\alpha W^{12 \ldots n} = \ldots = D^K_\alpha W^{12 \ldots n} = 0
\]

(2.26)

where \(D^K_\alpha = D^K_\alpha u^K_i\) and \(W^{12 \ldots n} = W^{(i_1 \ldots i_n)} u^1_{i_1} \ldots u^n_{i_n}\). Indeed, the constraint (2.20) now takes the form of a G-analyticity condition. In the appropriate basis in superspace the solution to (2.26) is a short superfield depending on part of the odd coordinates:

\[
W^{12 \ldots n}(x_A, \theta^1, \theta^2, \ldots, \theta^{2N-n}, u).
\]

(2.27)

In addition to (2.26), the projected superfield \(W^{12 \ldots n}\) automatically satisfies the USp(2\(N\)) harmonic irreducibility conditions

\[
D^K 2N-K W^{12} = 0, \quad K = 1, \ldots, N
\]

(2.28)
(only the simple roots of USp(2N) are shown). The equivalence between the two forms of 
The constraint follows from the obvious properties of the harmonic products $u^K_i u^K_i = 0$ and 
$\Omega^{ij} u^L_i u^L_j = 0$ for $1 \leq K < L \leq n$. The harmonic constraints (2.28) make the superfield 
ultrashort. Finally, in case (ii), projecting the constraint (2.22) with $u^I_i u^I_j$ where $I = 1, \ldots, N$ (no summation), we obtain the condition

$$D^I_I D^J_J w = 0.$$  \hspace{1cm} (2.29)

It implies that the superfield $w$ is linear in each projection $\theta^{\alpha I}$. 

2.4 Series of UIR’s of OSp(8*/2N) and shortening

It is now clear that we can realize the BPS series of UIR’s (2.19) as products of the different 
G-analytic superfields (supersingletons) (2.26). BPS shortening is obtained by setting the first 
$p - 1$ USp(2N) Dynkin labels to zero:

$$\frac{p}{2N} \text{ BPS : } W^{(0, \ldots, 0, a_p, \ldots, a_N)} (\theta^1, \theta^2, \ldots, \theta^{2N-p}) = (W^1 \ldots p)^{a_p} \ldots (W^1 \ldots N)^{a_N}. \hspace{1cm} (2.30)$$

We remark that our harmonic coset USp(2N)/[U(1)]^N is effectively reduced to USp(2N)/U(p) x 
[U(1)]^{N-p} in the case of $p/2N$ BPS shortening. Such a smaller harmonic space was used in Ref. [31] to formulate the (2,0) tensor multiplet. 

A study of the most general UIR’s of OSp(8*/2N) (similar to the one of Ref. [39] for the case 
of SU(2,2/N)) is presented in Ref. [34]. We can construct these UIR’s by multiplying the three 
types of supersingletons above:

$$w_{\alpha_1 \ldots \alpha_{m_1}} w_{\beta_1 \ldots \beta_{m_2}} w_{\gamma_1 \ldots \gamma_{m_3}} w^k W^{[a_1 \ldots a_N]} \hspace{1cm} (2.31)$$

where $m_1 \geq m_2 \geq m_3$ and the spinor indices are arranged so that they form an SU*(4) UIR 
with Young tableau $(m_1, m_2, m_3)$ or Dynkin labels $J_1 = m_1 - m_2, J_2 = m_2 - m_3, J_3 = m_3$. Thus 
we obtain four distinct series:

A) \hspace{1cm} $\ell \geq 6 + \frac{1}{2} (J_1 + 2J_2 + 3J_3) + 2 \sum_{k=1}^{N} a_k$ ;

B) \hspace{1cm} $J_3 = 0$, \hspace{0.5cm} $\ell \geq 4 + \frac{1}{2} (J_1 + 2J_2) + 2 \sum_{k=1}^{N} a_k$ ;

C) \hspace{1cm} $J_2 = J_3 = 0$, \hspace{0.5cm} $\ell \geq 2 + \frac{1}{2} J_1 + 2 \sum_{k=1}^{N} a_k$ ;

D) \hspace{1cm} $J_1 = J_2 = J_3 = 0$, \hspace{0.5cm} $\ell = 2 \sum_{k=1}^{N} a_k$. \hspace{1cm} (2.32)

The superconformal bound is saturated when $k = 0$ in (2.31). Note that the values of the 
conformal dimension we can obtain are “quantized” since the factor $w^k$ has $\ell = 2k$ and $k$ must

\footnote{As a bonus, we also prove the unitarity of these series, since they are obtained by multiplying massless unitary multiplets.}
be a non-negative integer to ensure unitarity. With this restriction eq. (2.32) reproduces the results of Ref. [34]. However, we cannot comment on the existence of a "window" of dimensions $2 + \frac{1}{2}J_1 + 2 \sum_{k=1}^{N} a_k \leq \ell \leq 4 + \frac{1}{2}J_1 + 2 \sum_{k=1}^{N} a_k$ conjectured in [34].

In the generic case the multiplet (2.31) is "long", but for certain special values of the dimension some shortening can take place [34]. We can immediately identify all these short multiplets. First of all, case D corresponds to BPS shortening. In the other cases let us first set $a_i = 0$, i.e. no BPS multiplets appear in (2.31). Then saturating the bound in case A (i.e., setting $k = 0$) leads to the shortening condition (see (2.23)):

$$\varepsilon^{\delta\alpha\beta\gamma} D_5^i (w_{\alpha \ldots \alpha_m} w_{\beta \ldots \beta_m} w_{\gamma \ldots \gamma_m}) = 0 \rightarrow \ell = 6 + \frac{1}{2} (J_1 + 2J_2 + 3J_3) .$$ (2.33)

Next, in case B we have two possibilities: either we saturate the bound ($k = 0$) or we use just one factor $w$ ($k = 1$). Using (2.22) and (2.23), we find

$$\varepsilon^{\delta\gamma\alpha\beta} D_5^i (w_{\alpha \ldots \alpha_m} w_{\beta \ldots \beta_m}) = 0 \rightarrow \ell = 4 + \frac{1}{2} (J_1 + 2J_2) ;$$ (2.34)

$$\varepsilon^{\delta\gamma\alpha\beta} D_5^i (D_5^j (w w_{\alpha \ldots \alpha_m} w_{\beta \ldots \beta_m})) = 0 \rightarrow \ell = 6 + \frac{1}{2} (J_1 + 2J_2) .$$ (2.35)

Similarly, in case C with $J_1 \neq 0$ we have three options, namely setting $k = 0 \rightarrow \ell = 2 + \frac{1}{2} J_1$ (which corresponds to the supersingleton defining constraint (2.23)) or $k = 1, 2$ which gives:

$$\varepsilon^{\delta\gamma\alpha\beta} D_5^i (D_5^j (w w_{\alpha \ldots \alpha_m} w_{\beta \ldots \beta_m})) = 0 \rightarrow \ell = 4 + \frac{1}{2} J_1 ,$$ (2.36)

$$\varepsilon^{\delta\gamma\alpha\beta} D_5^i (D_5^j (D_5^k (w^2 w_{\alpha \ldots \alpha_m}))) = 0 \rightarrow \ell = 6 + \frac{1}{2} J_1 .$$ (2.37)

Finally, in case C with $J_1 = 0$ we can take the scalar supersingleton (2.22) itself, i.e. set $k = 1 \rightarrow \ell = 2$, or set $k = 2, 3$:

$$\varepsilon^{\delta\gamma\alpha\beta} D_5^i (D_5^j D_5^k (w^2)) = 0 \rightarrow \ell = 4,$$ (2.38)

$$\varepsilon^{\delta\gamma\alpha\beta} D_5^i (D_5^j D_5^k (w^3)) = 0 \rightarrow \ell = 6 .$$ (2.39)

Introducing USp(2N) quantum numbers into the above shortening conditions is achieved by multiplying the short multiplets by a BPS object. The new short multiplets satisfy the corresponding USp(2N) projections of eqs. (2.22), (2.23), (2.33)-(2.39). We call such objects “intermediate short”.

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2 In a recent paper [49] the UIR’s of the six-dimensional conformal algebra SO(2,6) have been classified. Note that the superconformal bound in case A (with all $a_i = 0$) is stronger than the purely conformal unitarity bounds found in [34].
3 The three-dimensional case

In this section we carry out the analysis of the $d = 3 N = 8$ superconformal algebra $\text{OSp}(8/4, \mathbb{R})$ in a way similar to the above (the generalization to $\text{OSp}(N/4, \mathbb{R})$ is straightforward). Some of the short representations of the $N = 2$ and $N = 3$ cases were discussed in Ref. [50].

3.1 The conformal superalgebra $\text{OSp}(8/4, \mathbb{R})$ and G-analyticity

The part of the conformal superalgebra $\text{OSp}(8/4, \mathbb{R})$ relevant to our discussion is given below:

\[
\begin{align*}
\{Q_\alpha^i, Q_\beta^j\} &= 2\delta^{ij}\gamma_{\alpha\beta}P_\mu, \\
\{Q_\alpha^i, S_\beta^j\} &= \delta^{ij}M_{\alpha\beta} + 2\epsilon_{\alpha\beta}(T^{ij} + \delta^{ij}D), \\
[T^{ij}, Q_\alpha^k] &= i(\delta^{ki}Q_\alpha^j - \delta^{kj}Q_\alpha^i), \\
[T^{ij}, T^{kl}] &= i(\delta^{jk}T^{il} + \delta^{il}T^{jk} - \delta^{il}T^{jk}).
\end{align*}
\]

Here we find the generators $Q_\alpha^i$ of $N = 8$ Poincaré supersymmetry with a spinor index $\alpha = 1, 2$ of the $d = 3$ Lorentz group $\text{SL}(2, \mathbb{R}) \sim \text{SO}(1,2)$ (generators $M_{\alpha\beta} = M_{\beta\alpha}$) and a vector index $i = 1, \ldots, 8$ of the R symmetry group $\text{SO}(8)$ (generators $T^{ij} = -T^{ji}$); $S_\alpha^i$ of conformal supersymmetry; $P_\mu, \mu = 0, 1, 2$, of translations; $D$ of dilations.

The standard realization of this superalgebra makes use of a superspace with coordinates $x^\mu, \theta^\alpha \cdot i$. In order to study G-analyticity we need to decompose the generators $Q_\alpha^i$ under $[\text{U}(1)]^4 \subset \text{SO}(8)$. Besides the vector representation $8_v$ of $\text{SO}(8)$ we are also going to use the spinor ones, $8_s$ and $8_c$. Denoting the four $\text{U}(1)$ charges by $\pm, (\pm), [\pm]$ and $\{\pm\}$, we decompose the three $8$-dimensional representations as follows:

\[
\begin{align*}
8_v: \quad Q^i &\to Q^{\pm}, Q^{(\pm)}, Q^{[\pm]}, \quad (3.5) \\
8_s: \quad \phi^\alpha &\to \phi^{+(\pm)}, \phi^{-(\pm)}, \phi^{+(\{\pm\})}, \phi^{-(\{\pm\})} \quad (3.6) \\
8_c: \quad \sigma^\alpha &\to \sigma^{+(\pm)}, \sigma^{-(\pm)}, \sigma^{+(\{\pm\})}, \sigma^{-(\{\pm\})} \quad (3.7)
\end{align*}
\]

The definition of the charge operators $H_i, i = 1, 2, 3, 4$ can be read off from the corresponding projections of the relation (3.2):

\[
\begin{align*}
\{Q_\alpha^{[+\{\pm\}}, S_\beta^{[-\{\pm\]}\} &= \frac{1}{2}M_{\alpha\beta} + \epsilon_{\alpha\beta}(D - \frac{1}{2}H_1), \\
\{Q_\alpha^{[+\{\pm\}}, S_\beta^{[-\{\pm\]}\} &= \frac{1}{2}M_{\alpha\beta} + \epsilon_{\alpha\beta}(D - \frac{1}{2}H_2), \\
\{Q_\alpha^{[+\{\pm\}}, S_\beta^{[-\{\pm\]}\} &= \frac{1}{2}M_{\alpha\beta} + \epsilon_{\alpha\beta}(D - \frac{1}{2}H_3 - \frac{1}{2}H_4), \\
\{Q_\alpha^{[+\{\pm\}}, S_\beta^{[-\{\pm\]}\} &= -\frac{1}{2}M_{\alpha\beta} - \epsilon_{\alpha\beta}(D - \frac{1}{2}H_3 + \frac{1}{2}H_4). (3.8)
\end{align*}
\]

Let us denote a quasi primary superconformal field of the $\text{OSp}(8/4, \mathbb{R})$ algebra by the quantum numbers of its HWS, $D(\ell; J; a_1, a_2, a_3, a_4)$, where $\ell$ is the conformal dimension, $J$ is the Lorentz spin and $a_i$ are the Dynkin labels (see, e.g., [51]) of the $\text{SO}(8)$ R symmetry. In fact, in our

\footnote{Ascribing one of the three 8-dimensional representations of $\text{SO}(8)$, $8_v$, $8_s$, $8_c$ (related by triality) to the supersymmetry generators is purely conventional. Since in all the other $N$-extended $d = 3$ supersymmetries the odd generators belong to the vector representation, we prefer to put an $8_c$ index $i$ on the supercharges.}
scheme the natural labels are the four charges \( h_i \) (the eigenvalues of \( H_i \)) which are related to the Dynkin labels as follows: \( a_1 = \frac{1}{2}(h_1 - h_2), \ a_2 = \frac{1}{2}(h_2 - h_3 - h_4), \ a_3 = h_3, \ a_4 = h_4. \) A HWS \( |a_i\rangle \) of \( SO(8) \) is by definition annihilated by the positive simple roots of the \( SO(8) \) algebra:

\[
T^{[++]|a_i\rangle} = T^{[++]|a_i\rangle} = T^{[++]|a_i\rangle} = T^{[++]|a_i\rangle} = 0.
\]

G-analyticity is obtained by requiring that one or more projections of \( Q^i_\alpha \) annihilate the state. These projections must form an anticommuting subset closed under the action of the raising operators of \( SO(8) \) (3.9). Then, using the algebra (3.8) we examine the consistency of the G-analyticity conditions with the definition of a superconformal primary \( S^i_\alpha|\ell; J; a_k\rangle = 0 \). Thus we find the following set of G-analytic superspaces corresponding to BPS states:

\[
\begin{align*}
0 \text{ BPS:} & \quad q^+ + \Phi = 0 \rightarrow \Phi(\theta^{++}, \theta^{(\pm)}, \theta^{\pm|\pm}) \\
& \quad D(a_1 + a_2 + \frac{1}{2}(a_3 + a_4); 0; a_1, a_2, a_3, a_4) \\
1 \text{ BPS:} & \quad q^+ + \Phi = q^+_\alpha + \Phi = 0 \rightarrow \Phi(\theta^{++}, \theta^{(++)}, \theta^{[\pm]|\pm}) \\
& \quad D(a_2 + \frac{1}{2}(a_3 + a_4); 0; 0, a_2, a_3, a_4) \\
2 \text{ BPS:} & \quad q^+ + \Phi = q^+_3 + \Phi = 0 \rightarrow \Phi(\theta^{++}, \theta^{(++)}, \theta^{[\pm]|\pm}) \\
& \quad D(\frac{1}{2}(a_3 + a_4); 0; 0, 0, a_3, a_4) \\
2 \text{ BPS (II):} & \quad q^+ + \Phi = q^+_3 + \Phi = 0 \rightarrow \Phi(\theta^{++}, \theta^{(++)}, \theta^{[\pm]|\pm}) \\
& \quad D(\frac{1}{2}a_4; 0; 0, 0, a_4)
\end{align*}
\]

We remark that the states 1/4, 3/8 and 1/2 are annihilated by some of the lowering operators of \( SO(8) \). This means that the \( SO(8) \) subalgebras \( SU(2), SU(3) \) and \( SU(4) \), respectively, act trivially. These properties are equivalent to the restrictions on the possible values of the \( SO(8) \) Dynkin labels in (3.10). Note the existence of two types of 1/2 BPS states due to the two possible subsets of projections of \( q^i \) closed under the raising operators of \( SO(8) \) (3.9). This fact can be equivalently explained by the two possible embeddings of \( SU(4) \) in \( SO(8) \).

### 3.2 Supersingletons and harmonic superspace

The supersingletons are the simplest \( OSp(8/4, \mathbb{R}) \) representations of the 1/2 type in (3.10) and correspond to \( D(1/2; 0, 0, 0, 1, 0) \) or \( D(1/2; 0, 0, 0, 0, 1) \). The existence of two distinct types of \( d = 3 N = 8 \) supersingletons has first been noted in Ref. [32]. Each of them is just a collection of eight Dirac supermultiplets [28] made out of “Di” and “Rac” singletons [27].

In order to realize the supersingletons in superspace we note that the HWS in the two supermultiplets above has spin 0 and the Dynkin labels of the \( 8_s \) or \( 8_c \) of \( SO(8) \), correspondingly. Thus, we take a scalar superfield \( \Phi_\alpha(x^\mu, \theta^{\mu}_\alpha) \) (or \( \Sigma_\alpha(x^\mu, \theta^{\mu}_\alpha) \)) with an external \( 8_s \) index \( a \) (or an \( 8_c \) index \( \dot{a} \)). These superfields are subject to the following on-shell constraints [4]:

\[
\begin{align*}
type I: & \quad D^i_\alpha \Phi_\alpha = \frac{1}{8} \gamma^i_{\alpha\beta} \tilde{\gamma}^j_\beta D^j_\alpha \Phi_\beta; \\
type II: & \quad D^i_\alpha \Sigma_\dot{a} = \frac{1}{8} \gamma^i_{\dot{a}\dot{b}} \tilde{\gamma}^j_\dot{b} D^j_\alpha \Sigma_\dot{c}
\end{align*}
\]

\(^4\)See also [31] for the description of a supersingleton related to ours by \( SO(8) \) triality. Superfield representations of other \( OSp(N/4) \) superalgebras were considered in [3, 24].
which reduce them to a massless $8_s$ ($8_c$) scalar and $8_c$ ($8_s$) spinor.

The harmonic superspace description of these supersingletos can be realized by taking the harmonic coset $\text{SO}(8)/[\text{SO}(2)]^4 \sim \text{Spin}(8)/[\text{U}(1)]^4$. Since $\text{SO}(8)$ has three inequivalent fundamental representations, $8_s, 8_c, 8_v$, following [37] we introduce three sets of harmonic variables, $u^A, w^\dot{A}, v^I$, where $A, \dot{A}$ and $I$ denote the decompositions of an $8_s, 8_c$ and $8_v$ index, correspondingly, into sets of four $\text{U}(1)$ charges (see (3.5)-(3.7)). Each of these $8 \times 8$ real matrices belongs to the corresponding representation of $\text{SO}(8)$. This implies that they are orthogonal matrices:

$$u^A u^B = \delta^{AB}, \quad w^\dot{A} w^\dot{B} = \delta^{\dot{A}\dot{B}}, \quad v^I v^J = \delta^{IJ}.$$  \hspace{1cm} (3.13)

These matrices supply three copies of the group space, and we only need one to parametrize the harmonic coset. The condition which identifies the three sets of harmonic variables is

$$u^A \delta^I = v^I (\gamma^I)_{\dot{a}a}, \hspace{1cm} (3.14)$$

Further, we introduce harmonic derivatives (the covariant derivatives on the coset $\text{Spin}(8)/[\text{U}(1)]^4$):

$$D^{IJ} = u^A \delta^{IJ} \frac{\partial}{\partial u^A} + w^\dot{A} \delta^{IJ} \frac{\partial}{\partial w^\dot{A}} + v^I \frac{\partial}{\partial v^I}.$$  \hspace{1cm} (3.15)

They respect the algebraic relations (3.13), (3.14) among the harmonic variables and form the algebra of $\text{SO}(8)$ realized on the indices $A, \dot{A}, I$.

We now use the harmonic variables for projecting the supersingleton defining constraints (3.11), (3.12). From (3.14) it follows that the projections $\Phi^{++}[+]$ and $\Sigma^{++}[+]$ satisfy the following G-analyticity constraints:

$$D^{++} \Phi^{++}[+] = D^{++} \Phi^{++}[+] = D^{[+]} \Phi^{++}[+] = 0, \hspace{1cm} (3.16)$$

$$D^{++} \Sigma^{++}[+] = D^{++} \Sigma^{++}[+] = D^{[+] \Sigma^{++}[+] = 0}.$$  \hspace{1cm} (3.17)

where $D^I = v^I D^I$, $\Phi^A = u^A \Phi^A$ and $\Sigma^\dot{A} = w^\dot{A} \Sigma^\dot{A}$. This is the superspace realization of the 1/2 BPS shortening conditions in (3.10). In the appropriate basis in superspace $\Phi^{++}[+]$ and $\Sigma^{++}[+]$ depend on different halves of the odd variables as well as on the harmonic variables:

**type I:** \hspace{1cm} $\Phi^{++}[+] (x_A, \theta^{++}, \theta^{++}, \theta^{[+] \Sigma^{++}[+]}, u, w)$, \hspace{1cm} (3.18)

**type II:** \hspace{1cm} $\Sigma^{++}[+] (x_A, \theta^{++}, \theta^{++}, \theta^{[+] \Sigma^{++}[+]}, u, w)$, \hspace{1cm} (3.19)

In addition to the G-analyticity constraints (3.16), (3.17), the on-shell superfields $\Phi^{++}[+]$, $\Sigma^{++}[+]$ are subject to the $\text{SO}(8)$ irreducibility harmonic conditions obtained from (2.5) by replacing the $\text{SO}(8)$ generators by the corresponding harmonic derivatives. The combination of the latter with eq. (3.13) is equivalent to the original constraint (3.11).

Note that $\Phi^{++}[+]$, $\Sigma^{++}[+]$ are automatically annihilated by some of the lowering operators of $\text{SO}(8)$. This means that the supersingleton harmonic superfields effectively live on the smaller harmonic coset $\text{Spin}(8)/\text{U}(4)$.

---

\(^5\)A formulation of the above multiplet in harmonic superspace has been proposed in Ref. [11] (see also [35] and [34] for a general discussion of three-dimensional harmonic superspaces). The harmonic coset used in [31] is $\text{Spin}(8)/\text{U}(4)$. Although the supersingleton itself does indeed live in this smaller coset, its residual symmetry $\text{U}(4)$ would not allow us to multiply different realizations of the supersingleton. For this reason we prefer from the very beginning to use the coset $\text{Spin}(8)/[\text{U}(1)]^4$ with a minimal residual symmetry.
3.3 OSp(8/4, R) supersingleton composites

One way to obtain short multiplets of OSp(8/4, R) is to multiply different analytic superfields describing the type I supersingleton. The point is that above we chose a particular projection of, e.g., the defining constraint \( \frac{3}{11} \) which lead to the analytic superfield \( \Phi^{+(+)[+]} \). In fact, we could have done this in a variety of ways, each time obtaining superfields depending on different halves of the total number of odd variables. Thus, we can have four distinct but equivalent analytic descriptions of the type I supersingleton:

\[
\Phi^{+(+)[+]}(\theta^{++}, \theta^{(++)}, \theta^{[+] [+]}), \\
\Phi^{+(+) [-]}(\theta^{++}, \theta^{(++)}, \theta^{[-] [+]}), \\
\Phi^{+(--) (+)}(\theta^{++}, \theta^{(--)}, \theta^{[+] [+]}), \\
\Phi^{+(--) (--)}(\theta^{++}, \theta^{(--)}, \theta^{[-] [+]}).
\]

Then we can multiply them in the following way:

\[
(\Phi^{+(+)[+]})^{p+q+r+s}(\Phi^{+(+) [-]})^{q+r+s}(\Phi^{+(--) (++)})^{r+s}(\Phi^{+(--) (--)})^{s}
\]

thus obtaining three BPS series of OSp(8/4, R) UIR’s:

\[
\begin{align*}
\frac{1}{8} \text{ BPS:} & \quad D(a_1 + a_2 + \frac{1}{2}(a_3 + a_4), 0; a_1, a_2, a_3, a_4), \quad a_1 - a_4 = 2s \geq 0 ; \\
\frac{1}{4} \text{ BPS:} & \quad D(a_2 + \frac{1}{2}a_3, 0; 0, a_2, a_3, 0) ; \\
\frac{1}{2} \text{ BPS:} & \quad D(\frac{1}{2}a_3, 0; 0, 0, a_3, 0)
\end{align*}
\]

where \( a_1 = r + 2s \), \( a_2 = q \), \( a_3 = p \), \( a_4 = r \).

We see that using only one type of supersingletons cannot reproduce the classification \( \frac{3}{10} \), in particular, the 3/8 series. The latter can be obtained by mixing the two types of supersingletons:

\[
[\Phi^{+(+)[+]}(\theta^{++}, \theta^{(++)}, \theta^{[+] [+]})]^{a_1}[\Sigma^{+(+)[+]}(\theta^{++}, \theta^{(++)}, \theta^{[+] [+]})]^{a_4}
\]

(or the same with \( \Phi \) and \( \Sigma \) exchanged). Counting the charges and the dimension, we find exact matching with the 3/8 series in \( \frac{3}{10} \). Further, mixing two realizations of type I and one of type II supersingletons, we can construct the 1/4 series in \( \frac{3}{10} \):

\[
[\Phi^{+(+)[+]}]^{a_2+a_3}[\Phi^{+(+) [-]}]^{a_2}[\Sigma^{+(+)[+]}]^{a_4}.
\]

Finally, the full 1/8 series in \( \frac{3}{10} \) (i.e., without the restriction \( a_1 - a_4 = 2s \geq 0 \) in \( \frac{3}{22} \)) can be obtained in a variety of ways.

In this section we have analyzed all short highest-weight UIR’s of the OSp(8/4, R) superalgebra whose HWS’s are annihilated by part of the super-Poincaré odd generators. The number of distinct possibilities have been shown to correspond to different BPS conditions on the HWS. When the algebra is interpreted on the AdS\(_4\) bulk, for which the 3d superconformal field theory corresponds to the boundary M-2 brane dynamics, these states appear as BPS massive excitations, such as K-K states or AdS black holes, of M-theory on AdS\(_{4} \times S^7\). Since in M-theory there is only one type of supersingleton related to the M-2 brane transverse coordinates \( \frac{58}{58} \), according to our analysis massive states cannot be 3/8 BPS saturated, exactly as it happens in M-theory on \( M^4 \times T^7 \). Indeed, the missing solution was also noticed in Ref. \( \frac{59}{59} \) by studying AdS\(_4\) black holes in gauged \( N = 8 \) supergravity. Curiously, in the ungauged theory, which is in some sense the flat limit of the former, the 3/8 BPS states are forbidden \( \frac{41}{41} \) by the underlying \( E_{7(7)} \) symmetry of \( N = 8 \) supergravity \( \frac{60}{60} \).
3.4 Series of UIR’s of OSp(8/4, ℝ)

In the even-dimensional case \( d = 6 \) we had supersingleton superfields carrying either \( R \) symmetry indices or Lorentz indices or just conformal dimension. Multiplying them we were able to reproduce the corresponding general series of UIR’s. In the odd-dimensional case \( d = 3 \) we only have two supersingletons carrying SO(8) spinor indices. Multiplying them we could construct all the short objects of BPS type. Yet, for reproducing the most general UIR’s (see [34]), we need short objects with spin but without SO(8) indices. These arise in the form of conserved currents. The simplest one is a Lorentz scalar and an SO(8) singlet \( w \) of dimension \( \ell = 1 \).

It can be realized as a bilinear of two supersingletons of the same type, e.g., \( w = \Phi_a \Phi_a \) or \( w = \Sigma_a \Sigma_a \). Using (3.11) or (3.12) one can show that it satisfies the constraint (a non-BPS shortness condition)

\[
D_i^a D^{j\alpha} w = \frac{1}{8} \delta^{ij} D^k D^{k\alpha} w.
\]

(3.25)

The other currents carry SL(2, ℝ) spinor indices, \( w_{\alpha_1...\alpha_2J} \), have dimension \( \ell = 1 + J \) and satisfy the constraint [61]

\[
D^{\alpha a} w_{\alpha_2\alpha_2...\alpha_2J} = 0.
\]

(3.26)

They can be constructed as bilinears of the two types of supersingletons (for half-integer spin) or of two copies of the same type (for integer spin). For example, the two lowest ones (\( J = 1/2 \) and \( J = 1 \)) are

\[
w_{\alpha a} = \gamma^j_{bb} \left( D^i_a \Phi_b \Sigma_i - \Phi_i \Sigma_i^j \right) ,
\]

(3.27)

\[
w_{\alpha\beta} = D^{i\alpha a} \Phi_a \left( \gamma^j_{ab} \phi_b \phi_j^{\alpha a} \right) + 32i (\Phi_a \partial_{\alpha\beta} \Phi'_a - \partial_{\alpha\beta} \Phi_a \Phi'_a).
\]

(3.28)

The generic “long” UIR of OSp(8/4, ℝ) can now be obtained as a product of all of the above short objects:

\[
w_{\alpha_1...\alpha_2J} w^k \text{BPS}[a_1, a_2, a_3, a_4].
\]

(3.29)

Here we have used the first factor to obtain the spin, the second one for the conformal dimension and the BPS factor for the SO(8) quantum numbers. The unitarity bound is given by

\[
\ell \geq 1 + J + a_1 + a_2 + \frac{1}{2} (a_3 + a_4)
\]

(3.30)

and is saturated if \( k = 0 \) in (3.29). The object (3.29) is short if: (i) \( J \neq 0 \) and \( k = 0 \) (then it satisfies the intersection of (3.26) with the BPS conditions); (ii) \( J = 0 \) and \( k = 1 \) (then it satisfies the intersection of (3.25) with the BPS conditions); (iii) \( J = 0 \) and \( k = 0 \) (then it is BPS short). These results exactly match the classification of Ref. [34].

4 Conclusions

Here we give a summary of the different types of BPS states which are realized as products of supersingletons described by G-analytic harmonic superfields. We shall restrict ourselves to the physically interesting cases of \( M_2 \) and \( M_5 \) branes horizon geometry where only one type of such supersingletons appears. This construction gives rise to a restricted class of the most general BPS states.
4.1 OSp\((8^*/4)\)

The BPS states are constructed in terms of the \((2,0)\) \(d=6\) tensor multiplet \(W_{ij}^{12}(\theta^{1,2})^{p+q}(\theta^{1,3})^{q}\).

\[ (W^{12}(\theta^{1,2})^{p+q}(W^{13}(\theta^{1,3}))^{q} ) . \quad (4.1) \]

| BPS | USp(4) | Dimension | Harmonic space |
|-----|--------|-----------|----------------|
| \(\frac{1}{2}\) | (0,p) | 2p | USp(4)/U(2) |
| \(\frac{1}{4}\) | (2q,p) | 2p+4q | USp(4)/[U(1)]\(^2\) |
| (2q,0) | 4q | USp(4)/U(2) |

4.2 OSp\((8/4,\mathbb{R})\)

The type I BPS states are constructed in terms of the \(N=8\) \(d=3\) matter multiplet \(\Phi_a\) carrying an external \(8_s\) \(SO(8)\) spinor index in four equivalent G-analytic realizations:

\[ [\Phi^+(+)(\theta^{++}(\theta^{++},[\pm]),[\pm]{}^{\pm})]^{p+q+r+s} \times \\
[\Phi^-(+)(\theta^{++}(\theta^{++},[-]),[\pm]{}^{\pm})]^{q+r+s} \times \\
[\Phi^-(+)(\theta^{++},(-)[-],[\pm]{}^{\pm})]^{r+s} \times \\
[\Phi^-(+)(\theta^{++},(-)[-],[\pm]{}^{\pm})]^{s} . \quad (4.2) \]

| BPS | SO(8) | Dimension | Harmonic space |
|-----|-------|-----------|----------------|
| \(\frac{1}{2}\) | (0,0,p,0) | \(\frac{1}{2}p\) | Spin(8)/U(4) |
| \(\frac{1}{4}\) | (0,q,p,0) | \(\frac{1}{2}(p + 2q)\) | Spin(8)/U(2)\(\times U(2)\) |
| \(\frac{1}{8}\) | (r+2q,q,p,r) | \(\frac{1}{2}(p + 2q + 3r + 4s)\) | Spin(8)/[U(1)]\(^4\) |

The type II BPS states are constructed in terms of the \(N=8\) \(d=3\) matter multiplet \(\Sigma_{\dot{a}}\) carrying an external \(8_c\) \(SO(8)\) spinor index in four equivalent G-analytic realizations:

\[ [\Sigma^+(+)(\theta^{++}(\theta^{++},[\pm]),[\pm]{}^{\pm})]^{p+q+r+s} \times \\
[\Sigma^+(+)(\theta^{++}(\theta^{++},[-]),[\pm]{}^{\pm})]^{q+r+s} \times \\
[\Sigma^+(+)(\theta^{++},(-)[-],[\pm]{}^{\pm})]^{r+s} \times \\
[\Sigma^+(+)(\theta^{++},(-)[-],[\pm]{}^{\pm})]^{s} . \quad (4.3) \]
| BPS   | SO(8)             | Dimension       | Harmonic space     |
|-------|-------------------|-----------------|-------------------|
| $\frac{1}{2}$ | (0,0,0,p)        | $\frac{1}{2}p$  | Spin(8)/U(4)      |
| $\frac{1}{4}$ | (0,q,0,p)       | $\frac{1}{2}(p + 2q)$ | Spin(8)/U(2) × U(2) |
| $\frac{1}{8}$ | (r+2s,q,r,p)    | $\frac{1}{2}(p + 2q + 3r + 4s)$ | Spin(8)/[U(1)]$^4$ |

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