Analysis of the cornering stiffness uncertainty impact on the steering sensitivity of a two-axle automobile

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Abstract. In order to provide a reliable estimate of the steering sensitivity at the early design stage it is necessary to know the impact of the uncertainty of the main parameters affecting this characteristic. These parameters include the tire cornering characteristics featuring a large degree of uncertainty. The objective of the research was to find the impact of the tire cornering characteristics variation on the steering sensitivity. The analysis was performed in a multi-body simulation program with the use of a spatial non-linear model of the automobile motion. The model takes into account the detailed kinematics of the suspension and steering linkages. Interaction of the tires with the road is described by an original semi-empirical model providing independent calibration of the tire longitudinal and lateral force characteristics in the contact patch. The research has demonstrated the degree of importance of considering the tire cornering stiffness variation during the steering sensitivity analysis.

Introduction

Steering sensitivity is an important characteristic of the automobile handling. Reliability of this characteristic estimation at early design stages depends on the reliability of the analysis input data. Steering sensitivity depends mostly on the tire cornering stiffness, automobile moment of inertia about its vertical axis, and longitudinal coordinate of the vehicle center of mass [1]. The tire cornering characteristics feature a high degree of uncertainty. As it has been shown in [2], the variation of the cornering stiffness obtained for the same tire on different test rigs can reach up to 46%. In order to estimate reliability of the steering sensitivity analysis results it is necessary to estimate the impact of the tire cornering stiffness variation on this characteristic.

At present, estimation of the performance of the vehicle at early design stages is often performed by its dynamics simulation with the use of spatial non-linear multi-body models with suspension kinematics [3–9]. Models of this type provide good accuracy of the automobile handling characteristics estimation [1]. For complex multi-body mechanical systems, to which category automobiles with independent suspensions can be referred as well, there exists a problem not only to solve the equations of their motion but also to derive these equations. A solution to this problem is to use multi-body dynamics simulation (MBS) software [10–12]. In the MBS software, the user describes the mechanical system as a set of rigid bodies, joints, and force interactions from the library of standard elements, and the software generates the equations of motion automatically and provides built-in means for their numerical solution. MBS programs usually include built-in tire – road interaction models [13–17]. However, these tire models not always satisfy the analysts since their force calculation algorithms are either complicated and not fully disclosed in the manuals or need large amount of experimental data which are not available. As a result, an analyst doesn’t fully trust the
built-in model of the tire – road interaction. In this case, the development of relatively simple custom models of the tire – road interaction for specific problems by means of the MBS program or by means of an external programming language is an acceptable alternative [18].

The objective of this research is to analyse the impact of the tire cornering characteristics variation on the automobile steering sensitivity. The analysis will be performed in the Universal Mechanism MBS program with the use of a spatial non-linear multi-body model.

1. Automobile model
The research was done for a two-axle M1 category passenger vehicle with gross mass of 1262 kg, rear drive axle, and independent McPherson strut suspensions of all wheels. The automobile multi-body model consists of the subsystems shown in figure 1.

The sprung part model consists of the mass-inertial model of the body with spoilers and moldings and mass-inertial model of the frame with the main unsprung elements of the vehicle.

Suspension linkages are modeled as a set of rigid bodies connected by means of ideal joints and force elements. The masses and moments of inertia of the suspension bodies are calculated by their geometric models. The force elements are modeled as a pair of parallel spring and damper.

The steering system model couples the steering wheel rotation with the translation of the steering rack.
The vehicle model also includes the submodel of the powerplant with constant gear ratio transmission distributing the engine torque generated according to a control signal equally between the rear drive wheels, which corresponds to the transmission with all open differentials.

2. Tire – road interaction model

The forces and moments acting on the wheel from the road are calculated by a custom tire model developed in the Universal Mechanism MBS program. The tire – road interaction model is shown in figure 2. The model uses the following coordinate systems (see figure 2a):

- wheel coordinate system (WCS) \( OXYZ \) — a movable coordinate system with the origin at the wheel center, \( Z \) axis is perpendicular to the road plane, \( X \) axis is perpendicular to the wheel rotation axis;
- road fixed coordinate system (FCS) \( O, X_r, Y_r, Z_r \) — an orthogonal coordinate system fixed on the road.

![Figure 2. Tire – road interaction model: forces, moments and coordinate systems of the model – b) – forces and velocities in the longitudinal and lateral planes](image)

The vertical reaction is calculated by the visco-elastic model:

\[
R_Z = P_{z-st} \left( \frac{h_z}{h_{z-st}} \right)^{1.5} - b_z \cdot V_Z, \tag{1}
\]

where \( P_{z-st} \) is the wheel static load, \( h_{z-st} \) is the tire static deflection; \( b_z \) is vertical visco-elastic resistance force coefficient; \( h_z \) is the tire deflection:

\[
h_z = \max\left(0, r_0 \cdot \cos(\gamma) - \eta\right), \tag{2}
\]

where \( r_0 \) is the unloaded tire radius; \( \eta \) is the loaded tire radius.

The reaction in the road plane:

\[
R = \mu_{sxy}(S) \cdot R_Z, \tag{3}
\]

here \( S \) is the tire slip ratio; \( \mu_{sxy}(S) \) is the tire – road friction coefficient.
where \( \mu_{sx}, \mu_{sy} \) are the tire – road friction coefficients in longitudinal and lateral directions.

\[
\mu_{sx} = \mu_{sx\text{max}} \left( 1 - e^{-\frac{S_x}{S_{0x}}} \right),
\]

\[
\mu_{sy} = \mu_{sy\text{max}} \left( 1 - e^{-\frac{S_y}{S_{0y}}} \right),
\]

where \( S_x, S_y \) are the tire longitudinal and lateral slip ratios; \( S_{0x}, S_{1x}, S_{0y}, S_{1y} \) are parameters defining the shape of the \( \mu_{sx} \) and \( \mu_{sy} \) characteristics; \( \mu_{sx\text{max}}, \mu_{sy\text{max}} \) are the friction ellipse parameters.

The slip ratios \( S_x, S_y \) are calculated by the following formulae:

\[
S_x = \frac{V_{sx}}{\omega_y r_{e0}},
\]

\[
S_y = \frac{V_{sy}}{\omega_x r_{e0}},
\]

where \( r_{e0} \) is the effective radius of a free rolling tire:

\[
r_{e0} = \frac{3 \cdot \eta}{1 + \frac{2 \cdot \eta}{n_0}},
\]

\( V_{sx}, V_{sy} \) are longitudinal and lateral slip velocities:

\[
V_{cxX} = V_X - \omega_y \cdot r_{e0},
\]

\[
V_{csY} = V_Y - \omega_x \cdot r_{e0},
\]

The tire – road friction force \( \vec{R} \) is directed opposite to the slip velocity \( \vec{V}_s \) defined as:

\[
V_s = \sqrt{V_{sx}^2 + V_{sy}^2};
\]

The angle \( \alpha \) between the slide velocity vector and the wheel WCS \( X \) axis can be found from the following equations:

\[
\sin \alpha = \frac{V_{sy}}{V_s}.
\]
\[ \cos \alpha = \frac{V_{sX}}{V_s} , \] (14)

The projections of the tire – road friction force in the road plane are calculated in the following way:

\[ R_X = -R \cdot \cos \alpha ; \] (15)
\[ R_Y = -R \cdot \sin \alpha . \] (16)

The moments acting on the wheel are defined as follows:

\[ M_X = R_Y \cdot \eta - R_Z \cdot \eta \cdot \frac{\sin(\gamma)}{\cos(\gamma)} ; \] (17)
\[ M_Y = -R_X \cdot \eta + M_f ; \] (18)
\[ M_Z = 0 , \] (19)

where \( M_f \) is the tire rolling resistance moment:

\[ M_f = -R_Z \cdot r_{e0} \cdot f \cdot \text{sign}(\omega_y) , \] (20)

where \( f \) is the tire rolling resistance coefficient:

\[ f = f_0 + k_f \cdot (V_X^2) , \] (21)

where \( f_0 \) is the tire rolling resistance coefficient at a low speed (about 5 km/h); \( k_f \) is the tire rolling resistance growth factor describing increase in the rolling resistance with the growth of the forward velocity.

3. Step steer test model

According to Russian State Standard 31507-2012 [19], steering sensitivity analysis is a part of the step steer test. During this test the steering wheel of a vehicle moving with a constant velocity (80 km/h for M1 category) is rotated to a specified angle and held in that position till the beginning of the steady cornering. The angle of the steering wheel rotation is increased incrementally at each new test until the lateral acceleration reaches the tire – road adhesion limit value or the maximum value determined by the safety conditions.

The steering sensitivity analysis results in the steering sensitivity characteristic which is a dependency between the steering wheel rotation angle \( \varphi_{\text{steer}} \) and steady state lateral acceleration \( a_y \) of the vehicle.

During the simulation all the bodies of the vehicle model are given initial speed 80 km/h which is then kept constant by a PI speed controller model. The steering wheel is rotated according to the step transient function shown in figure 3.

![Figure 3. Steering wheel angle](image-url)
Figure 4 shows animation frames of the step steer simulation. The vector $a$ shows current direction of the vehicle acceleration.

![Figure 4. Animation frames of the step steer simulation](image)

4. Analysis of the cornering stiffness variation impact

The analysis consisted in the repeated simulations of the step steer within the range of attainable lateral accelerations for the tires with the basic lateral friction coefficient – slip ratio characteristics and for less and more laterally stiff tires. Lateral friction coefficient – slip ratio characteristics for the three tire variants are shown in figure 5. The parameters of the model of the tire with basic characteristics have been identified from the available experimental data for the Continental 205/55 R16 tire [20].

Lateral accelerations obtained at different tire characteristics are shown in figure 6.

Table 1 shows results of the analysis of the model sensitivity to the changes in the stiffness of the lateral friction coefficient – slip ratio characteristics.

The analysis results show that the change in lateral accelerations due to the change of the tire lateral stiffness increases with the increase in the steer angle. At the same time, the mean accelerations for the stiff tire are higher than the ones for the basic tire, and the accelerations for the soft tire are lower than the accelerations for the basic tire. In the analysed range of the steer angle from 20° to 40°, when the tire lateral friction coefficient – slip ratio characteristics stiffness changes by 30 %, the change in the lateral accelerations is not higher than 3.1 %. It means that the known uncertainty of the tire cornering stiffness [2] will not have a considerable impact on the vehicle steering sensitivity: its change will not exceed a few percent.
Figure 5. Lateral friction coefficient – slip ratio characteristics.

Figure 6. Time histories of the lateral accelerations at different steering angles and tire characteristics

Table 1. Impact of the tire lateral characteristics stiffness variation on the lateral acceleration

| Steer angle $\phi_{steer}$, ° | Tire characteristics | Stiffness of the tire characteristics, % | Mean stationary lateral acceleration $a_y$, m/s$^2$ | Change of $a_y$, % |
|-------------------------------|----------------------|------------------------------------------|-----------------------------------------------|-----------------|
| 20                            | Basic 100            | 3.40                                     | –                                             |
|                               | Stiff 70             | 3.44                                     | 1.2                                           |
|                               | Soft 120             | 3.38                                     | 0.6                                           |
| 30                            | Basic 100            | 4.95                                     | –                                             |
|                               | Stiff 70             | 5.04                                     | 1.8                                           |
|                               | Soft 120             | 4.89                                     | 1.2                                           |
| 40                            | Basic 100            | 6.22                                     | –                                             |
|                               | Stiff 70             | 6.41                                     | 3.1                                           |
|                               | Soft 120             | 6.09                                     | 2.1                                           |

5. Conclusion
The paper has demonstrated the procedure of the analysis of the tire cornering characteristics uncertainty impact on the passenger vehicle steering sensitivity in an MBS program. The analysis was performed with the use of a spatial non-linear multi-body model of the vehicle with an original semi-empirical tire – road interaction model providing independent calibration of the longitudinal and lateral characteristics of the tire. The analysis has shown that simultaneous change in cornering
stiffness of all the tires by 30% has little impact on the steering sensitivity. This result means that
during the steering sensitivity analysis the 30% accuracy of the tire cornering stiffness is quite
tolerable.

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