Neutrinoless double beta decay and neutrino physics

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Abstract. The connection of neutrino physics with neutrinoless double beta decay is reviewed. After presenting the current status of the PMNS matrix and the theoretical background of neutrino mass and lepton mixing, we will summarize the various implications of neutrino physics for double beta decay. The influence of light sterile neutrinos and other exotic modifications of the three neutrino picture is also discussed.

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1 Introduction

This contribution to the focus issue on Double Beta Decay deals with the connection of neutrinoless double beta decay ($\nu\beta\beta$) [1],

$$(A, Z) \rightarrow (A, Z + 2) + 2e^{-},$$

(1)
to neutrino physics. Observation of neutrinoless double beta decay would show that lepton number is violated, a finding that would be as important as observation of baryon number violation, i.e. proton decay. The huge importance of the process is the reason for extensive experimental and theoretical activities, as summarized in various recent reviews [2–8]. Indeed, the present time is an exciting time for neutrinoless double beta decay. The strongest limit on the life-time stemmed from 2001 [9], and was improved very recently [10]. A sizable number of new experiments is already running, under construction, or in the planing phase. Different isotopes and experimental techniques will be used, see table [1]. In case more than one experiment sees a signal, it will be possible to increase the credibility of the claims, test the nuclear matrix element calculations, and maybe even test the underlying mechanism of $\nu\beta\beta$. Recent reviews on the experimental aspects of $\nu\beta\beta$ can be found in [2,3,5,6], the focus issue contribution discussing this is by Zuber.

The fact that this contribution concentrates on neutrino physics needs to be stressed, because one always has to keep in mind that two main possibilities for $\nu\beta\beta$ exist [4]:
Table 1. Categorization of running and planned experiments\(^3\) and isotopes under consideration. Low energy resolution corresponds to about 10% at the expected peak, high energy resolution is 1% or less.

| Name             | Isotope          | source = detector; calorimetric with high energy res. | low energy res. | event topology | source ≠ detector event topology |
|------------------|------------------|------------------------------------------------------|----------------|----------------|---------------------------------|
| AMoRE            | \(^{100}\)Mo     | ✓                                                    |                |                |                                 |
| CANDLES          | \(^{48}\)Ca      | –                                                    | ✓              |                |                                 |
| COBRA            | \(^{116}\)Cd (and \(^{130}\)Te) | –                                                    |                | ✓              |                                 |
| CUORE            | \(^{130}\)Te     | ✓                                                    |                |                |                                 |
| DCBA             | \(^{82}\)Se or \(^{150}\)Nd | –                                                    |                |                | ✓                               |
| EXO              | \(^{136}\)Xe     | –                                                    | ✓              |                |                                 |
| GERDA            | \(^{76}\)Ge      | ✓                                                    | –              |                |                                 |
| KamLAND-Zen      | \(^{136}\)Xe     | –                                                    | ✓              |                |                                 |
| LUCIFER          | \(^{82}\)Se or \(^{100}\)Mo or \(^{116}\)Cd | ✓                                                    | –              |                |                                 |
| MAJORANA         | \(^{76}\)Ge      | ✓                                                    | –              |                |                                 |
| MOON             | \(^{82}\)Se or \(^{100}\)Mo or \(^{150}\)Nd | –                                                    | ✓              |                |                                 |
| NEXT             | \(^{136}\)Xe     | –                                                    | –              | ✓              |                                 |
| SNO+             | \(^{150}\)Nd     | –                                                    | –              | ✓              |                                 |
| SuperNEMO        | \(^{82}\)Se or \(^{150}\)Nd | –                                                    | –              |                | ✓                               |
| XMASS            | \(^{136}\)Xe     | –                                                    | ✓              |                |                                 |

(i) **Standard Interpretation:**
neutrinoless double beta decay is mediated by light and massive Majorana neutrinos (the ones which oscillate) and all other mechanisms potentially leading to \(0\nu\beta\beta\) give negligible or no contribution;

(ii) **Non-Standard Interpretations:**
neutrinoless double beta decay is mediated by some other lepton number violating physics, and light massive Majorana neutrinos (the ones which oscillate) potentially leading to \(0\nu\beta\beta\) give negligible or no contribution.

Here we will focus only on the standard interpretation of light neutrino exchange. Massive neutrinos are firmly established, and the vast majority of models and theories predicts neutrinos to be Majorana particles\(^{[11]}\). Therefore, an interpretation of neutrinoless double beta decay experiments in terms of neutrino physics is presumably the best motivated one. However, one always has to consider the possibility that other physics is at the origin of \(0\nu\beta\beta\). A recent review on the various beyond the Standard Model sources for \(0\nu\beta\beta\) can be found in\(^{[4]}\). Alternative interpretations will be dealt with in the contribution by Deppisch, Hirsch and Päs, ways to distinguish the mechanisms will also be discussed by Fogli and Lisi. Note that, as formulated by the Schechter-Valle theorem\(^{[12]}\), any mechanism will lead to a neutrino Majorana mass term. However, this mass is generated by a 4-loop diagram, and is therefore negligible, see\(^{[13]}\) for an explicit calculation.

As it turns out, if the three active neutrinos of the Standard Model are massive Majorana particles, observation of \(0\nu\beta\beta\) can give information on the neutrino mass ordering, the Majorana CP phases, and the neutrino mass scale. For the latter observable, it should be noted that two other ways to measure neutrino mass exist, namely in “Kurie-plot” experiments\(^{[14]}\) such as KATRIN, or via cosmological
observations\cite{15}. Different assumptions go into these complementary methods, and an interesting discussion arises when signals in two or even all three types of measurements are established. There is even more to $0
u\beta\beta$: it contains “flavor information”, in the sense that a number of the many proposed flavor symmetry models for lepton mixing has testable predictions for neutrino mass observables, which can be used to test the models, or rule them out.

A popular modification of the standard neutrino picture is to add one or two light sterile neutrinos, in order to explain various anomalous signals, such as the LSND/MiniBooNE results, the reactor anomaly, or certain cosmological/astrophysical observations. See\cite{16} for a detailed description of the current situation. The presence of light sterile neutrinos will strongly impact the standard discussion of $0\nu\beta\beta$, and is a good example on how the physics interpretation of $0\nu\beta\beta$ can get completely changed in case the underlying assumptions are wrong.

Extracting precise physics results from $0\nu\beta\beta$ requires precise determination of the nuclear matrix elements, which currently is not the case. Nevertheless, recent years saw a significant improvement of the calculations, and an extensive experimental program was launched\cite{17} in order to support and test the calculations (for the standard mechanism) as much as possible. These experimental activities are discussed in contributions by Ejiri and Frekers, and by Freeman, Grabmayr and Schiffer. Nuclear physics aspects will be dealt with by Faessler, Simkovic and Rodin, by Suhonen and Civitarese, by Menendez, by Engel and by Vogel.

In what follows we will summarize general and current aspects of neutrino phenomenology and theory in section\cite{2} The main part of this contribution is section\cite{3} where we aim to discuss the various aspects of double beta decay and standard neutrino physics. Light sterile neutrinos are added to the standard picture in section\cite{4} before we conclude in section\cite{5}

2. Neutrino physics

2.1. Neutrino mass and lepton mixing: theoretical origin

The theory behind neutrino mass and lepton mixing has been reviewed for instance in\cite{18}. The only assumption that is necessary for what follows, is that neutrinos are Majorana particles. A pragmatic way to achieve this is to accept the presence of the unique dimension 5 operator, inversely proportional to the scale $\Lambda$, at which new physics sets in\cite{19}

\[
\mathcal{L}_{\text{eff}} = \frac{1}{2} \frac{h_{\alpha\beta}}{\Lambda} \tau^T \Phi \Phi^T L_\alpha L_\beta \overset{\text{EWSB}}{\longrightarrow} \frac{1}{2}(m_\nu)_{\alpha\beta} \nu_\alpha \nu_\beta.
\]  

(2)

Charge conjugated spinors are denoted by the superscript $^c$, $L_\alpha = (\nu_\alpha, \alpha)^T$ are the lepton doublets of flavor $\alpha = e, \mu, \tau$ and $\Phi$ is the Higgs doublet with vacuum expectation value $v = 174$ GeV. $m_\nu$ is the neutrino mass matrix, given as the Yukawa coupling matrix
$h_{\alpha\beta}$ times $v^2$ divided by the high energy scale $\Lambda$. As can be seen, after electroweak symmetry breaking a Majorana mass matrix $m_\nu$ of order $v^2/\Lambda$ is generated. With the typical mass scale of $m_\nu \simeq 0.05$ eV, it follows that $\Lambda \simeq 10^{15}$ GeV.

There are several ways to generate the Weinberg operator $L_{\text{eff}}$ in terms of fundamental particles. There are three tree-level possibilities, called type I, II and III seesaw, the numbering accidently corresponding to their popularity. For the type I seesaw [20–23] one introduces right-handed neutrinos, which are weak singlets. One can integrate out their Majorana mass matrix $M_R$, resulting in

$$m_\nu = -m_D^T M_R^{-1} m_D,$$

(3)

where $m_D$ is a Dirac mass matrix in the term $\overline{N_R} m_D \nu_L$, expected to be of the order of the other Standard Model masses. The Weinberg operator is here realized with $\Lambda \simeq M_R$. The type II (or triplet) seesaw [24–26] requires scalar Higgs triplets, and the type III seesaw [27] fermion triplets. Combinations of the three terms are of course possible, as well as more evolved seesaw variants, such as double, linear, or inverse seesaw. There are also radiative mechanisms which induce a Majorana neutrino mass matrix via loop diagrams, involving new particles [28–30]. Very often the heavy (seesaw) messengers can generate the baryon asymmetry by their decays in the early Universe, the so-called leptogenesis mechanism [31,32]. Hence, establishing the Majorana nature of neutrinos and the presence of CP violation in the lepton sector would strengthen our belief in this already very appealing mechanisms. Note however that a link between $0\nu\beta\beta$ and the baryon asymmetry is not guaranteed. The same is true for the connection between low energy CP violation and leptogenesis: leptogenesis is very well possible if the low energy CP phases are all zero [33,34].

Whatever the origin of the dimension 5 operator, neutrinos are Majorana particles: $\nu_i^c \equiv C \overline{\nu}_i^T = \nu_i$. Diagonalizing the mass matrix $m_\nu$ via

$$m_\nu = U^* m_\nu^{\text{diag}} U^\dagger,$$

(4)

where $m_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$, results for the charged current term in the appearance of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix $U$:

$$L_{\text{CC}} = -\frac{g}{\sqrt{2}} \overline{\ell}_\alpha \gamma^\mu U_{\alpha i} \nu_i W^\mu. $$

(5)

We have gone here without loss of generality in the basis in which the charged lepton mass matrix is real and diagonal. The neutrino mass states $\nu_{1,2,3}$ are superpositions of neutrino flavor states $\nu_{e,\mu,\tau}$:

$$\nu_\alpha = U^*_{\alpha i} \nu_i.$$ 

(6)

In the next subsection the observational status of the parameters in $m_\nu$ will be discussed.

### 2.2. Neutrino mass and lepton mixing: observational status

Since the mass matrix is complex and symmetric there are 9 physical parameters in $m_\nu$, usually parameterized as three masses, three angles and three phases. The PMNS
These phases are physical only if neutrinos are Majorana particles \cite{36–38}. We have included here a diagonal phase matrix in neutrino oscillation experiments. In eq. (7) we have included a diagonal phase matrix \( P \) in the 13-rotation. A "symmetrical parametrization" is also possible, where the phases of all three generations.

\[
U = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} P, \tag{7}
\]

where \( s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij} \) and \( \delta \) is the "Dirac phase" responsible for CP violation in neutrino oscillation experiments. In eq. (7) we have included a diagonal phase matrix \( P \), containing the two "Majorana phases" \( \alpha \) and \( \beta \):

\[
P = \text{diag}(1, e^{i\alpha}, e^{i(\beta + \delta)}). \tag{8}
\]

These phases are physical only if neutrinos are Majorana particles \cite{36–38}. We have included here \( \delta \) in \( P \), as a consequence the \( U_{ei} \) elements are independent of \( \delta \). The above parametrization is the product of a 23-, 13- and 12-rotation, then multiplied with \( P \), with a phase in the 13-rotation. A "symmetrical parametrization" is also possible, in which each individual rotation matrix contains a phase \cite{37}. The resulting PMNS matrix is \cite{39}

\[
U = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\phi_{12}} \\
    -s_{12}c_{23}e^{i\phi_{12}} - c_{12}s_{13}s_{23}e^{-i(\phi_{23} - \phi_{13})} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i(\phi_{12} + \phi_{23} - \phi_{13})} & s_{13}e^{-i\phi_{13}} \\
    s_{12}s_{23}e^{i(\phi_{12} + \phi_{23})} - c_{12}s_{13}c_{23}e^{i\phi_{13}} & -c_{12}s_{23}e^{i\phi_{23}} - s_{12}s_{13}c_{23}e^{-i(\phi_{12} - \phi_{13})} & c_{13}c_{23}
\end{pmatrix} \tag{9}
\]

and its differences to the standard parametrization will be discussed whenever appropriate.

Neutrino oscillation experiments can probe in principle 6 of the nine parameters in \( m_{\nu} \), all angles, one phase and two mass-squared differences (including their sign). \( \ddagger \) In the symmetrical parametrization \cite{39} the phase combination \( \phi_{13} - \phi_{12} - \phi_{23} \) governs CP violation in oscillations, and one immediately recognizes CP violation as a three-generation phenomenon, involving the phases of all three generations.
Fitting the general formula

\[
P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re} \left\{ U^*_{\alpha i} U_{\beta j} U_{\beta i} U_{\alpha j} \right\} \sin^2 \frac{\Delta m^2_{ij} L}{4E} + 2 \sum_{i>j} \text{Im} \left\{ U^*_{\alpha i} U_{\beta j} U_{\beta i} U_{\alpha j} \right\} \sin \Delta m^2_{ij} L, \]

(10)

with \( E \) the neutrino energy, \( L \) the baseline, and including matter effects whenever necessary, to the results of various neutrino oscillation experiments gives the allowed ranges in table 2. Regarding the mixing angle \( \theta_{13} \), or the mixing matrix element \( |U_{e3}| \), there has recently been spectacular progress. Following T2K, all three reactor experiments have shown evidence for non-zero and sizable \( |U_{e3}| \), namely Double Chooz, Daya Bay and RENO:

- **Double Chooz**: \( \sin^2 2\theta_{13} = 0.086 \pm 0.051 \neq 0 \) at 1.9\( \sigma \),
- **Daya Bay**: \( \sin^2 2\theta_{13} = 0.092 \pm 0.017 \neq 0 \) at 5.2\( \sigma \),
- **RENO**: \( \sin^2 2\theta_{13} = 0.113 \pm 0.023 \neq 0 \) at 4.9\( \sigma \).

(11)

Combining the reactor data with the other information rules out vanishing \( U_{e3} \) at more than 7\( \sigma \). As we will see, the precise value of \( \theta_{13} \) is not particularly important for 0\( \nu \beta \beta \), at least for the current and next generation of experiments. Of more importance for 0\( \nu \beta \beta \) is the sign of the atmospheric mass-squared difference \( \Delta m^2_{31} \) (see section 3.3), and the neutrino mass scale (section 3.2), both are currently unknown. An often overlooked issue for neutrinoless double beta decay is the value of \( \theta_{12} \), which is the best known mixing angle, but still induces a sizable uncertainty for life-time predictions in the inverted hierarchy, comparable to the nuclear physics uncertainty, as we will discuss in section 3.3.

The neutrino mass ordering is called normal (inverted) if the atmospheric mass-squared difference is larger (smaller) than zero:

- **normal**: \( m_2 = \sqrt{m_1^2 + \Delta m^2_{\odot}} \); \( m_3 = \sqrt{m_1^2 + \Delta m^2_A} \),
- **inverted**: \( m_2 = \sqrt{m_3^2 + \Delta m^2_{\odot} + \Delta m^2_A} \); \( m_1 = \sqrt{m_3^2 + \Delta m^2_A} \).

(12)

With unknown mass ordering and total mass scale there are three extreme cases, namely

- **normal hierarchy (NH)**: \( m_3 \simeq \sqrt{\Delta m^2_A} \gg m_2 \simeq \sqrt{\Delta m^2_{\odot}} \gg m_1 \),
- **inverted hierarchy (IH)**: \( m_2 \simeq m_1 \simeq \sqrt{\Delta m^2_A} \gg m_3 \),
- **quasi-degeneracy (QD)**: \( m_0^2 \equiv m_1^2 \simeq m_2^2 \simeq m_3^2 \gg \Delta m^2_A \).

(13)

The neutrino mass scale can be measured with three complementary methods. The most model-independent one is by examining electron spectra in \( \beta \)-decays, i.e. **Kurie-plot experiments**. The observable neutrino mass parameter is

\[
m_\beta \equiv \sqrt{\sum |U_{ei}|^2 m_i^2}. \]

(14)

\[\text{§ At the Neutrino 2012 conference in June 2012, Double Chooz have presented new data with 3.1}\sigma \text{ evidence for non-zero } U_{e3}, \text{ and also Daya Bay has increased its significance to more than 7}\sigma. \text{ Also the T2K results were updated.}\]
The current limit at 95% C.L. to this quantity from spectrometer approaches is 2.3 eV and 2.1 eV, obtained from the Mainz \cite{46} and Troitsk \cite{47,48} collaborations, respectively. The KATRIN experiment \cite{49,50} has a discovery potential of $m_\beta = 0.35$ eV with 5$\sigma$ significance and a design sensitivity of $m_\beta = 0.2$ eV (90% C.L.). It has been shown that neutrino mass determination is very robust with respect to new physics \cite{51}. Such kind of experiments have reached their ultimate size, and going lower in mass requires alternative approaches. In principle, MARE (using crystal bolometers) \cite{52} and Project 8 (measuring cyclotron radiation emitted by electrons) \cite{53} can reach limits of 0.1 eV.

Neutrino mass determination via \textbf{cosmological and astrophysical observations} sets constraints on the sum of masses
\begin{equation}
\Sigma = \sum m_i .
\end{equation}
Limits depend on the fitted data sets and the underlying cosmological model applied to the data, see \cite{15,54} for recent summaries. Therefore, it is difficult to set a robust and model-independent constraint, upper limits on the sum of masses range from 1.5 eV to less than 0.5 eV. It is fair to say that if indeed neutrino masses are such that $\Sigma \gtrsim 1$ eV, a rather unusual cosmological model would be needed.

The third approach to neutrino mass is \textbf{neutrinoless double beta decay}, which we will discuss in section 3.2. Obviously, in order to extract neutrino mass from $0\nu\beta\beta$ one needs to assume that neutrinos are Majorana particles, and are the leading contribution to the process. The expression on which the lifetime of $0\nu\beta\beta$ depends is
\begin{equation}
\langle m_{ee} \rangle = \left| \sum U_{ei}^2 m_i \right| ,
\end{equation}
and upper limits on $\langle m_{ee} \rangle$ can be translated in upper limits on $m_\beta$ and $\Sigma$.

Cosmology gives the strongest limits on neutrino mass, $0\nu\beta\beta$ and direct searches give comparable limits, see section 3.2. However, we stress again the conceptual differences of the different approaches to neutrino mass, to be further discussed in section 3.2.

3. \textbf{Neutrinoless double beta decay and neutrino physics}

Having set the stage, we can discuss the connection of neutrino physics and neutrinoless double beta decay.

3.1. \textbf{General aspects}

In general, the decay rate of $0\nu\beta\beta$ can be written as
\begin{equation}
\Gamma^{0\nu} = \sum_x G_x(Q, Z) |M_x(A, Z) \eta_x|^2 ,
\end{equation}
where the subscript $x$ denotes the underlying mechanism and $\eta_x$ are functions of the particle physics parameters responsible for the decay. The nuclear matrix elements $M_x(A, Z)$ depend on the mechanism and the nucleus. Finally, $G_x(Q, Z)$ are phase space factors (depending on the $Q$-value with $Q^5$ for most mechanisms including light
neutrino exchange) which can have dependence on the particle physics. Note that the possibility of destructive or constructive interference of different mechanisms is present. However, here we are only interested in the presence of one particle physics mechanism, the exchange of light massive Majorana neutrinos, the ones which are responsible for neutrino oscillations.

The Feynman diagram for $0\nu\beta\beta$ on the quark level in this interpretation is shown in figure 1. The amplitude of the process is for the $V-A$ interactions of the Standard Model proportional to

$$\sum G_F^2 U_{ei}^2 \frac{q + m_i}{q^2 - m_i^2} \gamma_{\pm} = \sum G_F^2 U_{ei}^2 \frac{m_i}{q^2 - m_i^2} \gamma_{\mu} \gamma_{\nu} \cong \sum G_F^2 U_{ei}^2 \frac{m_i}{q^2} \gamma_{\mu} \gamma_{\nu},$$

where $\gamma_{\pm} = \frac{1}{2}(1 \pm \gamma_5)$, $m_i$ is the neutrino mass, $q \simeq 100$ MeV is the typical neutrino momentum (corresponding to the typical nuclear distance of about 1 fm), and $U_{ei}$ an element of the first row of the PMNS matrix. The linear dependence on the neutrino mass is expected from the requirement of a spin-flip, as the neutrino can be thought of as being emitted as a right-handed state and absorbed as a left-handed state.

The decay width of $0\nu\beta\beta$ is therefore proportional to the square of the so-called effective mass

$$\langle m_{ee} \rangle = \left| \sum U_{ei}^2 m_i \right| = \left| m_{ee}^{(1)} e^{2i\alpha} + m_{ee}^{(2)} e^{2i\beta} + m_{ee}^{(3)} \right|, \quad (18)$$

which is visualized in figure 1 (right) as the sum of three complex vectors $m_{ee}^{(1,2,3)}$. The effective mass is a coherent sum, which implies the possibility of cancellations. Note that since we have included $\delta$ in $P$ (see (8)), the Dirac phase does not appear in $\langle m_{ee} \rangle$, which is the way it should be. In the symmetrical parametrization (9) the effective mass is given as

$$\langle m_{ee} \rangle = \left| c_{12} c_{13}^2 m_1 + s_{12}^2 c_{13} m_2 e^{2i\phi_{12}} + s_{13}^2 m_3 e^{2i\phi_{13}} \right|, \quad (19)$$

so only the two Majorana phases appear in $\langle m_{ee} \rangle$ [39].

Neutrinoless double beta decay is suppressed by the extremely small ratio of neutrino mass $m_i \lesssim 0.5$ eV and momentum transfer $|q| \simeq 10^8$ eV, and therefore only

\[\text{Figure 1. Left: quark level “lobster” diagram for neutrinoless double beta decay in case of light Majorana neutrino exchange. Right: geometrical visualization of the effective mass.}\]
Avogadro’s number provides a chance to create situations in which the Majorana nature of neutrinos is observable. Compare this for instance with the decay $K^- \rightarrow \pi^+ e^- e^-$, which has a similar momentum scale, and has an extremely tiny branching ratio of about

$$\text{BR}(K^- \rightarrow \pi^+ e^- e^-) \sim 10^{-33} \left(\frac{\langle m_{ee}\rangle}{\text{eV}}\right)^2,$$

(20)
to be compared with the experimental upper limit of $6.4 \times 10^{-10}$.

Turning to experimental aspects, it is important to note that the lifetime reach that can be obtained in an experiment depends on the background level:

$$\langle T_{1/2}^{0\nu}\rangle^{-1} \propto \begin{cases} a M \varepsilon t & \text{without background}, \\ a \varepsilon \sqrt{\frac{M t}{B \Delta E}} & \text{with background}. \end{cases}$$

(21)

Here $M$ is the mass, $t$ measurement time, $B$ background index with natural units of counts/(keV kg yr) and $\Delta E$ the energy resolution at the peak. Noting that the decay width depends quadratically on the particle physics parameter, it is clear that within background dominated experiments an improvement of the particle physics by a factor 2 implies a highly non-trivial combined improvement of 16 on background index, energy resolution, mass and measurement time.

There are similar (and more difficult to observe) processes called neutrino-less double beta $^+ \text{EC}$ decay $(A, Z) \rightarrow (A, Z-2) + 2 e^+ (0\nu\beta^+\beta^+)$, beta $^+\text{EC}$ decay electron capture $e^-_b + (A, Z) \rightarrow (A, Z-2) + e^+ (0\nu\beta^+\text{EC})$, or double electron capture $2 e^-_b + (A, Z) \rightarrow (A, Z-2)^* (0\nu\text{ECEC})$ of bound state electrons $e^-_b$ (discussed in this focus issue by Blaum, Eliseev and Novikov), which can also be searched for. Observation of one of those processes would also imply the non-conservation of lepton number. The rates depend on the particle physics parameters in the same way as $0\nu\beta\beta$ does, we can therefore focus on $0\nu\beta\beta$.

The effective mass depends on 7 out of the 9 physical parameters of low energy neutrino physics (only $\theta_{23}$ and the Dirac phase do not appear), hence contains an enormous amount of information. It is the only realistic observable in which the two Majorana phases appear. For the other five parameters there will be complementary information from oscillation experiments or other experiments probing neutrino mass. It is also noteworthy that $\langle m_{ee}\rangle$ is the $ee$ element of the neutrino mass matrix $m_\nu$, see eq. (14), which is a fundamental object in the low energy Lagrangian. In terms of the origin of neutrino mass, $\langle m_{ee}\rangle$ is $h_{ee} v^2/\Lambda$, see eq. (2).

In the standard and symmetrical parametrization of the PMNS matrix we have

$$|m_{ee}^{(1)}| = m_1 |U_{e1}|^2 = m_1 c_{12}^2 c_{13}^2,$$
$$|m_{ee}^{(2)}| = m_2 |U_{e2}|^2 = m_2 s_{12}^2 c_{13}^2,$$
$$|m_{ee}^{(3)}| = m_3 |U_{e3}|^2 = m_3 s_{13}^2.$$

(22)

The individual masses can, using eq. (12), be expressed in terms of the smallest mass and the mass-squared differences, whose currently allowed ranges, as well as those of the mixing angles, are given in table 2. A typical analysis of the effective mass would
Figure 2. Effective mass against the smallest neutrino mass for the $3\sigma$ ranges (top) and best-fit values (bottom) of the oscillation parameters. The green and red shaded areas are the general $3\sigma$ ranges, while the blue shaded areas can only be realized if the CP phases take non-trivial values. $(\pm, \pm)$ denote different CP conserving situations, corresponding to signs of $m_2$ and $m_3$, relative to positive $m_1$. Prospective future values of $\Sigma$ and $m_\beta$ are also given.

plot it against the smallest neutrino mass [56–66], while varying the Majorana phases and/or the oscillation parameters. This results in figure 2 for which the best-fit values and $3\sigma$ ranges of the oscillation parameters have been used. The blue shaded areas are interesting because they can only be covered if the CP phases are non-trivial (see section 3.4). It is also interesting to plot the effective mass against the other neutrino mass observables $\Sigma$ and $m_\beta$, which is shown in figure 3. Such plots are helpful if more than one of the complementary neutrino mass experiments finds a signal. The simple analytical expressions for $\langle m_{ee} \rangle$ in certain extreme cases are given in figure 4.
Figure 3. Effective mass against sum of masses $\Sigma$ and kinematic neutrino mass $m_\beta$ for the $3\sigma$ ranges of the oscillation parameters. The green and red shaded areas are the general $3\sigma$ ranges, while the blue shaded areas can only be realized if the CP phases take non-trivial values. Prospective future values of $\Sigma$ and $m_\beta$ are also given.

In order to translate the effective mass into lifetime, one needs to give the phase space factor $G(Q, Z)$ and the (range of) nuclear matrix elements. It is an ongoing discussion on which NMEs one should use, and what their uncertainty is. The contributions to the focus issue by Faessler, Simkovic and Rodin, by Suhonen and Civitarese, by Menendez, by Engel and by Vogel will discuss this at length. We will stick here for definiteness to the compilation from [45], displayed in figure 5 (see also recent compilations in [5, 76, 77]). The current limits on $0\nu\beta\beta$ and the required phase
Table 3. Life-time limits on neutrinoless double beta decay at 90% C.L. and limits on the effective mass extracted with the matrix element compilation from figure 5. Note that the range of nuclear matrix elements leads to a range for the upper limits on $\langle m_{ee}\rangle$. Phase space factors are taken from [67].

| Isotope | $T_{0\nu}^{1/2}$ [yrs] | Experiment | $G$ [$10^{-14}$ yrs$^{-1}$] | $\langle m_{ee}\rangle_{\text{lim}}^\text{min}$ [eV] | $\langle m_{ee}\rangle_{\text{lim}}^\text{max}$ [eV] |
|---------|----------------|-------------|----------------|----------------|----------------|
| $^{48}$Ca | 5.8 $\times 10^{22}$ | CANDLES [68] | 6.35 | 3.55 | 9.91 |
| $^{76}$Ge | 1.9 $\times 10^{25}$ | HDM [9] | 0.623 | 0.21 | 0.53 |
| | 1.6 $\times 10^{25}$ | IGEX [69] | 0.25 | 0.63 | |
| $^{82}$Se | 3.2 $\times 10^{23}$ | NEMO-3 [70] | 2.70 | 0.85 | 2.08 |
| $^{96}$Zr | 9.2 $\times 10^{21}$ | NEMO-3 [71] | 5.63 | 3.97 | 14.30 |
| $^{100}$Mo | 1.0 $\times 10^{24}$ | NEMO-3 [70] | 4.36 | 0.31 | 0.79 |
| $^{116}$Cd | 1.7 $\times 10^{23}$ | SOLOTVINO [72] | 4.62 | 1.22 | 2.30 |
| $^{136}$Xe | 2.8 $\times 10^{24}$ | CUORICINO [73] | 4.09 | 0.27 | 0.57 |
| | 1.6 $\times 10^{25}$ | EXO-200 [10] | 4.31 | 0.15 | 0.36 |
| | 5.7 $\times 10^{24}$ | KamLAND-Zen [74] | 0.25 | 0.60 | |
| $^{150}$Nd | 1.8 $\times 10^{22}$ | NEMO-3 [75] | 19.2 | 2.35 | 5.08 |

Table 4. Details of the most advanced experiments. Given are life-time sensitivity and the expected limit on $\langle m_{ee}\rangle$, using the NME compilation from figure 5. Note that the range of nuclear matrix elements leads to a range for the expected sensitivity on $\langle m_{ee}\rangle$.

| Experiment | Isotope | Mass [kg] | Sensitivity $T_{0\nu}^{1/2}$ [yrs] | Status | Start of data-taking | Sensitivity $\langle m_{ee}\rangle$ [eV] |
|------------|---------|-----------|----------------|---------|----------------------|----------------|
| GERDA      | $^{76}$Ge | 18        | $3 \times 10^{25}$ | running | $\sim$ 2011 | 0.17-0.42 |
| | | 40        | $2 \times 10^{26}$ | construction | $\sim$ 2012 | 0.06-0.16 |
| | | 1000      | $6 \times 10^{27}$ | R&D | $\sim$ 2015 | 0.012-0.030 |
| CUORE      | $^{130}$Te | 200       | $6.5 \times 10^{26}\ast$ | construction | $\sim$ 2013 | 0.018-0.037 |
| | | | $2.1 \times 10^{26}\ast\ast$ | | | 0.03-0.066 |
| MAJORANA   | $^{76}$Ge | 30-60     | $(1 - 2) \times 10^{26}$ | construction | $\sim$ 2013 | 0.06-0.16 |
| | | 1000      | $6 \times 10^{27}$ | R&D | $\sim$ 2015 | 0.012-0.030 |
| EXO        | $^{136}$Xe | 200       | $6.4 \times 10^{25}$ | running | $\sim$ 2011 | 0.073-0.18 |
| | | 1000      | $8 \times 10^{26}$ | R&D | $\sim$ 2015 | 0.02-0.05 |
| SuperNEMO  | $^{82}$Se | 100-200   | $(1 - 2) \times 10^{26}$ | R&D | $\sim$ 2013-15 | 0.04-0.096 |
| KamLAND-Zen | $^{136}$Xe | 400       | $4 \times 10^{26}$ | running | $\sim$ 2011 | 0.03-0.07 |
| | | 1000      | $10^{27}$ | R&D | $\sim$ 2013-15 | 0.02-0.046 |
| SNO+       | $^{150}$Nd | 56        | $4.5 \times 10^{24}$ | construction | $\sim$ 2012 | 0.15-0.32 |
| | | 500       | $3 \times 10^{25}$ | R&D | $\sim$ 2015 | 0.06-0.12 |
Figure 4. The main properties of the effective mass as function of the smallest neutrino mass. Here $m_0$ denotes the common mass for quasi-degenerate neutrinos and $t_{12} = \tan \theta_{12}$. We indicate the relevant formulae and the three important regimes: hierarchical, cancellation and quasi-degeneracy. See [63].

Figure 5. Nuclear matrix element compilation for $0\nu\beta\beta$, different isotopes and calculational approaches.
space factors are given in table 3, together with the resulting limits on the effective mass. Note that the range of nuclear matrix elements leads to a range for the upper limits on $\langle m_{ee} \rangle$. The current limit on the effective mass is therefore

$$\langle m_{ee} \rangle \lesssim 0.4 \text{ eV}.$$  \hfill (23)

We stress here that the very recent result from EXO-200 \cite{10} has finally improved the long-standing best limit from the Heidelberg-Moscow experiment \cite{9}. Details of the most advanced experiments in what regards their expected limits on the effective mass can be found in table 4. It is worth to take a closer look at the interesting features of the effective mass shown in figure 4:

- in the normal hierarchy case, the effective mass is smallest, somewhere in the meV regime:

$$\langle m_{ee} \rangle^{\text{NH}} \sim \sqrt{\Delta m_{\odot}^2} \sin^2 \theta_{12} + \sqrt{\Delta m_A^2} \sin^2 \theta_{13} \simeq 0.004 \text{ eV}.$$  \hfill (24)

The meV scale of the effective mass should be the final goal of experiments. The half-lifes corresponding to meV effective masses are $10^{28}$ to $10^{29}$ yrs;

- for certain values of the parameters the effective mass can even vanish. This unfortunate situation will be discussed in section 3.3;

- for the inverted hierarchy the effective mass cannot vanish. In the limit of negligible $m_3 \left| U_{e3} \right|^2$ one has

$$\langle m_{ee} \rangle^{\text{IH}}_{\text{max}} \equiv \sqrt{\Delta m_A^2} c_{13}^2 \leq \langle m_{ee} \rangle^{\text{IH}} \leq \sqrt{\Delta m_A^2} c_{13}^2 \cos 2\theta_{12} \equiv \langle m_{ee} \rangle^{\text{inv}}_{\text{min}}.$$  \hfill (25)

Due to the non-maximal value of $\theta_{12}$ the minimal value of the effective mass is non-zero \cite{60}. Therefore, if limits below the minimal value

$$\langle m_{ee} \rangle^{\text{inv}}_{\text{min}} = \langle m_{ee} \rangle^{\text{IH}}_{\text{min}} = (1 - \left| U_{e3} \right|^2) \sqrt{\Delta m_A^2} \left( 1 - 2 \sin^2 \theta_{12} \right),$$  \hfill (26)

are reached by an experiment, the inverted mass ordering is ruled out if neutrinos are Majorana particles. If we knew by independent evidence that the mass ordering is inverted (by a long-baseline oscillation experiment or a galactic supernova observation) then we would rule out the Majorana nature of neutrinos. Of course, one has to assume here that no other lepton number violating mechanism interferes. The two scales of $\langle m_{ee} \rangle$ corresponding to the minimal and maximal value in case of the inverted hierarchy, given in eq. (25), should be the intermediate or long-term goal of experiments. The typical effective mass values of order 0.02 eV are one order of magnitude larger than those for the normal hierarchy and roughly one order of magnitude smaller than for quasi-degenerate neutrinos. They correspond to half-lifes of order $10^{26}$ to $10^{27}$ yrs;

- in the quasi-degenerate regime, both mass orderings are indistinguishable, the effective mass in this case reads

$$\langle m_{ee} \rangle^{\text{QD}} = m_0 \left| c_{12}^2 c_{13}^2 + s_{12}^2 \ e^{2i\alpha} + s_{13}^2 \ e^{2i\beta} \right|.$$  \hfill (27)

There is a much debated claim \cite{78} of observation, which corresponds to a value of $\langle m_{ee} \rangle$ of about 0.2 to 0.6 eV.
Table 5. Approximate analytical expressions for the neutrino mass observables for the extreme cases of the mass ordering. For $0\nu\beta\beta$ the typical (isotope-dependent) half-lifes are also given.

| $\Sigma$ | $m_\beta$ | $\langle m_{ee}\rangle$ |
|----------|-----------|-------------------|
| NH $\sqrt{\Delta m_A^2}$ $\sqrt{\sin^2 \theta_{12} \Delta m_0^2 + |U_{e3}|^2 \Delta m_A^2}$ $|\sin^2 \theta_{12} \sqrt{\Delta m_0^2 + |U_{e3}|^2 \Delta m_A^2} e^{2(\alpha-\beta)}|$ $\simeq 0.05$ eV $\simeq 0.01$ eV $\approx 0.004$ eV $\Rightarrow T_{1/2}^{0\nu} \gtrsim 10^{28-29}$ yrs |
| IH $2\sqrt{\Delta m_A^2}$ $\sqrt{\Delta m_0^2}$ $\sqrt{\Delta m_A^2 \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \alpha}}$ $\simeq 0.1$ eV $\simeq 0.05$ eV $\approx 0.02$ eV $\Rightarrow T_{1/2}^{0\nu} \gtrsim 10^{26-27}$ yrs |
| QD $3m_0$ $m_0$ $m_0 \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \alpha}$ $\simeq 0.1$ eV $\Rightarrow T_{1/2}^{0\nu} \gtrsim 10^{25-26}$ yrs |

This expression cannot vanish either. It corresponds, for $\langle m_{ee}\rangle^{QD} \simeq 0.1$ eV, to half-lifes in the ballpark of $10^{25}$ to $10^{26}$ yrs, hence the QD mass scheme is currently tested;

- it is evident from the approximate expressions of the effective mass and figure 4 that the recently found sizable value of $U_{e3}$ is currently of minor importance for $0\nu\beta\beta$. The difference between the maximal effective mass in the normal hierarchy and the minimal value in the inverted hierarchy shrinks a bit [63]. In addition, the area in which the effective mass vanishes becomes larger. These properties pose no problem for current and next-generation experiments.

The approximate expressions for the effective mass and the other neutrino mass observables are summarized in table 5. Table 6 attempts to illustrate the complementarity of neutrino mass observables. Prospective sensitivity values of $m_\beta = 0.2$ eV, $\langle m_{ee}\rangle = 0.02$ eV, and $\Sigma = 0.1$ eV are assumed and the interpretation of positive and/or negative results in all 3 approaches is given.

3.2. Neutrino mass

With a current limit $\langle m_{ee}\rangle^{\exp}_{\max}$ on the effective mass, which corresponds to the quasi-degenerate regime, one can obtain the following limit on the neutrino mass:

$$m_0 \leq \langle m_{ee}\rangle^{\exp}_{\max} \frac{1 + \tan^2 \theta_{12}}{1 - \tan^2 \theta_{12} - 2 |U_{e3}|^2} \equiv \langle m_{ee}\rangle^{\exp}_{\max} f(\theta_{12}, \theta_{13}).$$  \hspace{1cm} (28)

Note that $m_0$ is in the QD regime identical to $m_\beta$, i.e. to the quantity measured in Kurie-plot experiments. The function $f(\theta_{12}, \theta_{13})$ varies from 2.75 to 3.43 at 1$\sigma$ and from 2.28 to 4.66 at 3$\sigma$. The limit on the effective mass is about 0.4 eV (see [23]), and hence $m_0 \leq 1.4$ eV and 1.9 eV, respectively. Therefore, the current limit on $m_0$ from $0\nu\beta\beta$ is very similar to the one from the Mainz experiment. In the QD regime one further has the relation $m_0 = \Sigma/3$.

Perhaps more interesting is the determination of the neutrino mass scale in future experiments if information from complementary neutrino mass observables is combined.
Table 6. “Neutrino mass matrix” for the present decade. It is assumed that KATRIN will reach its sensitivity limit of \( m_\beta = 0.2 \text{ eV} \), that \( 0\nu\beta\beta \)-experiments can obtain values down to \( \langle m_{ee} \rangle = 0.02 \text{ eV} \), and that cosmology can probe the sum of masses down to \( \Sigma = 0.1 \text{ eV} \). With “yes” a positive measurement is denoted, while “no” refers to no observation at the limit. M denotes Majorana neutrino, D Dirac neutrino, N-SI non-standard interpretation of \( 0\nu\beta\beta \), N-SC non-standard cosmology.

| KATRIN  | \( 0\nu\beta\beta \) | Cosmology |
|---------|----------------------|-----------|
| yes     | yes                  | yes       |
| no      | no                   | no        |
| QD + M  | QD + D               | QD        |
| N-SI    | low IH or NH or D    | low IH or (QD + D) |
| N-SC    | NH                   | NH        |

For instance, Ref. [79] (see also [80]) has performed a statistical analysis of prospective data, see figure 6. Using realistic errors for the experimental quantities, it was found that the neutrino mass can be determined (at 3\( \sigma \)) with 15% to 25% uncertainty, depending on the NME uncertainty. The precision is largely determined by cosmology, the uncertainty of the oscillation parameters is of little importance.

3.3. Neutrino mass ordering

From figure 2 the interesting possibility of ruling out the inverted mass ordering becomes obvious. The minimal value of the effective mass, repeated here for convenience, is non-
Figure 7. Required half-life sensitivities to exclude and touch the inverted hierarchy for different values of $\theta_{12}$, using the compilation of NMEs from figure 5. The upper plots show the necessary half-lifes for $\sin^2 \theta_{12} = 0.27$ (upper left) and $\sin^2 \theta_{12} = 0.38$ (upper right). The lower left plot includes the current $3\sigma$ uncertainty for $\theta_{12}$. The lower right plot shows the necessary half-lifes in order to touch the inverted ordering, which is independent on $\theta_{12}$. The small horizontal lines show expected half-life sensitivities at 90% C.L. of running and planned $0\nu\beta\beta$-experiments. When two sensitivity expectations are given for one experiment they correspond to near and far time goals. See [45].

zero and given by

$$\langle m_{ee} \rangle_{\text{inv}}^{\text{min}} = (1 - |U_{e3}|^2) \sqrt{\Delta m^2_A} \left(1 - 2 \sin^2 \theta_{12}\right).$$

(29)

If a limit on the effective mass below this value is obtained, the inverted ordering is ruled out if neutrinos are Majorana particles. In case the mass ordering is known to be inverted then the Majorana nature of neutrinos would be ruled out.

One can translate the effective mass necessary to rule out (or touch) the inverted hierarchy into half-lifes, see table 4 and figure 7. We note that currently running experiments will not fully probe the inverted hierarchy regime. The crucial dependence on $\theta_{12}$ has recently been discussed in [45]. The current $3\sigma$ range of $\theta_{12}$ corresponds to
an uncertainty of a factor \( \simeq 1.77 \) in the minimal value of the effective mass, which is of the same order as the current uncertainty in the NMEs. The factor 1.77 due to \( \theta_{12} \) corresponds to a factor of \( 1.77^{2} \simeq 3.13 \) in half-life. In experiments with background, see (21), this means a rather non-trivial combined factor of \( 1.77^{4} \simeq 10 \) in the product of measuring time, energy resolution, background index and detector mass. Therefore, a precision determination of the solar neutrino mixing angle would be very desirable to evaluate the requirements and physics potential of upcoming \( 0\nu\beta\beta \)-experiments in order to test the inverted ordering [45].

### 3.4. Majorana phases

Determining a Majorana CP phase from neutrinoless double beta decay is probably the most difficult physics goal related to \( 0\nu\beta\beta \) [81,87]. Note that there are two Majorana phases and only one observable, \( \langle m_{ee} \rangle \), and thus only one (or a combination) of the phases can be extracted in principle. In addition, complementary information on the neutrino mass scale has to be put in for such a measurement, and only for the inverted ordering or the quasi-degenerate scheme a phase determination is conceivable at all.

A somewhat pessimistic point of view was presented in [88], whereas [89] found the requirements not too unrealistic. Recalling figure 3 it is clear that in experiments one should find results lying in the areas indicated with “CPV”, which are however smeared by experimental and theoretical uncertainties. Neglecting \( \theta_{13} \), the effective mass for the IH or QD cases is proportional to

\[
\langle m_{ee} \rangle \propto \left| \cos^{2} \theta_{12} + e^{2i\alpha} \sin^{2} \theta_{12} \right| = \sqrt{1 - \sin^{2} 2\theta_{12} \sin^{2} \alpha}.
\]

(30)

Therefore, the larger \( \theta_{12} \) is, the more promising it is to extract \( \alpha \) from measurements. Recall that ruling out the inverted mass ordering is easier if \( \theta_{12} \) is small. A detailed statistical analysis has been performed in [80], see figure 8. One can see that, as expected, for larger values of \( \theta_{12} \) the areas in parameter space become larger.

### 3.5. Vanishing effective mass

Unfortunately, the normal mass hierarchy can allow for complete cancellation of the effective mass. In terms of figure 4 this “cancellation regime” means that a triangle can be formed. Neglecting \( m_{1} \), one needs

\[
\frac{m_{2}}{m_{3}} = \frac{\tan^{2} \theta_{13}}{\sin^{2} \theta_{12}} \simeq 3 \tan^{2} \theta_{13} \simeq 0.08.
\]

(31)

The Majorana phases need to be such that the two surviving terms have opposite sign. For the general case one finds [90]

\[
\cos 2\alpha = \frac{m_{3}^{2} s_{13}^{4} - c_{13}^{4} (m_{1}^{2} c_{12}^{4} + m_{2}^{2} s_{12}^{4})}{2 m_{1} m_{2} s_{12}^{2} c_{12}^{4} c_{13}^{4}}, \quad \cos 2\beta = -\frac{m_{2}^{2} s_{13}^{4} + c_{13}^{4} (m_{2}^{2} s_{12}^{4} - m_{1}^{2} c_{12}^{4})}{2 m_{2} m_{3} s_{12}^{2} s_{13}^{2} c_{13}^{2}}.
\]

(32)
It may seem unnatural that the 7 parameters on which $\langle m_{ee}\rangle$ depends conspire in such a way that the effective mass vanishes. However, recall that the effective mass is the $ee$ element of the Majorana neutrino mass matrix. This matrix is generated by the underlying theory of mass generation, and texture zeros occur frequently in such flavor models (cf. section 3.6), see [91] for a general analysis and [92] for symmetries leading to $\langle m_{ee}\rangle = 0$.

Experimentally, the effective mass can be considered zero if it is below $10^{-4}$ eV, no future experiment that currently is envisaged can reach such low values. If one of the complementary neutrino mass measurements finds a signal, then this means that neutrinos are QD or IH, and therefore the effective mass cannot vanish.

3.6. Distinguishing flavor models

There is an industry of model building in order to explain the peculiar mixing structure of leptons that is so different to quark mixing [93–95]. Many models lead to the same

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Figure 8. Constraints on the Majorana phase $2\alpha$ (here called $\alpha_{21}$) at 95% C.L. from an observed $\langle m_{ee}\rangle_{\text{exp}} = 0.3$ eV and prospective data on $\Sigma$, as a function of the NME uncertainty factor. Shown are the regions in which the data are consistent with a CP conserving value (hatched), observed $\Sigma$ is inconsistent with $\langle m_{ee}\rangle_{\text{exp}}$ (light-shaded), and Majorana CP violation is established (red/dark-shaded). Taken from [80].

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See [4] for a list of corrections which can lead to non-zero $\langle m_{ee}\rangle$. Note that the renormalization group running in the effective theory is multiplicative and cannot generate a non-zero value. Typically, if non-zero, the effective mass slightly increases from low to high scale.
neutrino mixing scheme, in particular tri-bimaximal mixing (TBM):

\[
U = \begin{pmatrix}
\frac{\sqrt{2}}{3} & \frac{\sqrt{3}}{3} & 0 \\
-\frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} & 0 \\
\frac{1}{6} & -\frac{1}{6} & \frac{1}{\sqrt{2}}
\end{pmatrix} \Rightarrow m_\nu = \begin{pmatrix}
A \\
B \\
B
\end{pmatrix} \cdot \begin{pmatrix}
\frac{1}{2}(A + B + D) \\
\frac{1}{2}(A - B - D) \\
\frac{1}{2}(A + B + D)
\end{pmatrix}.
\]

The prediction \(U_{e3} = 0\) can be corrected to non-zero values by a variety of mechanisms, and there are also many alternatives to TBM \[96\]. It turns out that neutrino mass observables can help in disentangling the vast amount of flavor symmetry models. One example is that the flavor symmetry leads to correlations of the mass matrix elements, which imply correlations of observables. For instance, the effective mass could be correlated with the atmospheric neutrino parameter \(\sin^2 \theta_{23}\) \[97\]. Recall that in general \(\theta_{23}\) has no influence on \(\langle m_{ee}\rangle\).

Another point are neutrino mass “sum-rules”: the most general neutrino mass matrix giving rise to TBM is given above. As such, the (complex) eigenvalues \(A - B\), \(A + 2B\) and \(D\) are independent of the mixing angles: no matter what \(A, B, D\) are, the PMNS mixing is given as above. However, very often the structure of the mass matrix is simpler, and “sum-rules” between the neutrino masses arise. Examples are \(2/m_2 + 1/m_3 = 1/m_1\) or \(1/m_2 + 1/m_3 = 1/m_1\) (here the masses are understood to be complex, i.e. including their Majorana phases). As a consequence of the sum-rules, not all masses and phases are allowed, and detailed studies of the predictions can be found.
in [98,100]. See figure 9 for an explicit example where relations between the neutrino masses forbid certain areas in parameter space.

3.7. Exotic modifications of the three neutrino picture

There are a number of very exotic modifications to the standard picture discussed so far, which we will shortly discuss in this section.

The most obvious modification is that neutrinos are Dirac particles, in which case there is no neutrinoless double beta decay and reading this review was all in vain. Though in fact Dirac neutrinos are the most straightforward SM extension that can explain massive neutrinos, the associated Yukawa couplings to generate masses less than eV are at least 6 orders of magnitude smaller than the electron Yukawa coupling. For muon and tau neutrinos the coupling is even smaller than the one of the associated charged lepton. Since the masses in quark doublets are very close to each other, one considers the extreme hierarchy in the lepton masses as unnatural and fine-tuned. This is one of the reasons why the Weinberg operator (2) and its explicit realizations in terms of seesaw mechanisms, which suppress neutrino mass naturally, are considered as realistic origin of neutrino mass.

Dirac neutrinos can be written as two maximally mixed Majorana neutrinos with common mass $m_i$ and opposite CP parity. The effective mass is then

$$\sum_i \sqrt{\frac{1}{2}} |U_{ei}|^2 (m_i + m_i e^{i\pi}) = 0.$$  (33)

A small splitting of the degeneracy of a state can be described with the mass matrix

$$m_i \begin{pmatrix} \epsilon & 1 \\ 1 & 0 \end{pmatrix} \rightarrow \tilde{U} = \sqrt{\frac{1}{2}} \begin{pmatrix} 1 + \frac{\epsilon}{4} & -1 + \frac{\epsilon}{4} \\ 1 - \frac{\epsilon}{4} & 1 + \frac{\epsilon}{4} \end{pmatrix}$$

and

$$m_i^\pm = m_i (\pm 1 + \frac{\epsilon}{2}),$$  (34)

with the indicated new eigenstates and mixing (sub-)matrix. These Pseudo-Dirac neutrinos lead to a very small contribution to the effective mass of about $\frac{1}{2} \delta m^2 / m_i$, with $\delta m^2 = (m_i^+)^2 - (m_i^-)^2$. See [101] for current constraints on $\delta m^2$ from oscillation experiments. If all three states are Pseudo-Dirac the effective mass is basically zero [102].

If one or two are Pseudo-Dirac interesting predictions for the effective mass arise. This can happen in “bimodal” or “schizophrenic” scenarios [103], in which at leading order one or two mass states are Dirac particles while the other one is Majorana. For instance, if $\nu_2$ is a Dirac particle then the effective mass in the inverted hierarchy is

$$\langle m_{ee} \rangle \simeq \sqrt{\Delta m^2_{\text{A}} c_{12}^2 c_{13}^2},$$

roughly a factor of two larger than the minimal value in the standard case [103], see (25). A generalization to all possibilities can be found in [104].

Another exotic property is CPT violation. Interesting consequences for $0\nu\beta\beta$ have been considered in [105], where a simple one family example was discussed. Strictly speaking, the neutrinos cannot fulfill the Majorana condition, but neutrinoless double beta decay still can take place.
The situation is slightly different for tachyonic neutrinos, whose equation of motion does not allow to define a charge conjugation, and $0\nu\beta\beta$ has been argued to be absent [106, 107].

4. Light sterile neutrinos

The majority of neutrino experiments is consistent with the 3-neutrino picture. The noteworthy exception is a number of measurements and observations in particle physics, astrophysics and cosmology that can be explained with the presence of light (eV or sub-eV) sterile neutrinos. See [16] for a review on the current situation. Typical values are $\Delta m^2_{\text{st}} \simeq 1\text{ eV}^2$, and mixing of order $U_{e4} \simeq 0.15$ [108]. Scenarios with two or more sterile neutrinos are mildly disfavored by cosmology, since they contribute with two eV-scale masses to the sum of masses and with additional degrees of freedom [109]. On the other hand, scenarios with two sterile neutrinos provide better fits to the anomalous experimental results. Moreover, while oscillation analysis require typically mass-squared differences close to eV, cosmology would work better with less than eV masses. A number of dedicated oscillation experiments will put the sterile neutrino hypothesis to the test [16], and definite answers might be present within this decade.

What is the effect for neutrinoless double beta decay? This has been discussed several times in the past [110–112]. For one sterile neutrino, the effective mass is now a sum of four terms, the additional term quantifies the sterile contribution:

$$\langle m_{ee}\rangle = |U_{e1}|^2 m_1 + |U_{e2}|^2 m_2 e^{2i\alpha} + |U_{e3}| m_3 e^{2i\beta} + |U_{e4}|^2 m_4 e^{2i\Phi_1} \rangle_{\text{act}} + |\langle m_{ee}\rangle|_{\text{st}}. \quad (35)$$

Here $|\langle m_{ee}\rangle|_{\text{act}}$ is the 3-neutrino contribution discussed so far, and $|\langle m_{ee}\rangle|_{\text{st}}$ the sterile one.

Note that in case sterile neutrinos are present, the symmetrical parametrization [9] has several advantages [39]. For instance, if there is one sterile neutrino there are six independent rotations and six CP phases (3 Dirac and 3 Majorana). If each rotation contains a phase, the very often awkward attribution of CP phases to the PMNS matrix is not necessary, and taken care of automatically.

Suppose the easiest (and least problematic in what regards cosmology) case that there is only one sterile neutrino, and that it is heavier than the 3 active ones. The complete list of possibilities including the case of two sterile neutrinos can be found in Refs. [113, 115]. These other cases are usually corresponding to quasi-degenerate mass schemes. Recall from above that $|\langle m_{ee}\rangle|_{\text{NH}}$ can vanish and that $|\langle m_{ee}\rangle|_{\text{IH}}$ cannot vanish, having a typical value of 0.02 eV. With the typical sterile neutrino parameters given above one has

$$|\langle m_{ee}\rangle|_{\text{st}} \simeq \sqrt{\Delta m^2_{\text{st}}} |U_{e4}|^2 \simeq 0.02 \text{ eV} \left\{ \begin{array}{ll} \gg |\langle m_{ee}\rangle|_{\text{NH}} \,, \\ \simeq |\langle m_{ee}\rangle|_{\text{IH}} \,. \end{array} \right. \quad (36)$$

$^+$ Obviously, such sterile neutrinos can also be tested also in the other neutrino mass approaches, i.e. KATRIN and cosmology.
Therefore, if the active neutrinos are normally ordered, the effective mass cannot vanish anymore, whereas it can vanish when they are inversely ordered [113]. Hence, the usual standard phenomenology has been completely turned around! Given that the addition of light sterile neutrinos is presumably the simplest modification of the standard picture, this example shows that when discussing the physics potential of 0νββ one should carefully list one’s assumptions. Figure 10 is a more detailed analysis of the situation [113], in which the smallest mass is plotted against the effective mass, where the sterile neutrino parameters are from the global fit results of [108].

5. Summary

In this contribution we have focussed on the presumably best motivated neutrino physics aspect of neutrinoless double beta decay, but there are many other scenarios which can lead to 0νββ, including left-right symmetry, supersymmetry, extra dimensions, etc. A similar discussion to the one presented here can be performed for all of them, involving tests in collider physics, lepton flavor violation and so on. If a signal in neutrinoless double beta decay is established in one or more experiment, an exciting physics program will start, aiming to pin down the underlying mechanism.

It is indeed an exciting time for neutrinoless double beta decay and neutrino physics. A number of experiments is testing new life-time regimes of neutrinoless double beta decay, impressively encompassing enormous experimental difficulties. Theoretical and experimental progress of the nuclear physics part is also rapidly increasing. Combined with the progress in neutrino oscillation physics, and its further improvement in the years to come, we are about to enter new and unexplored regimes. The future will show which part of the large physics potential can be realized, or if even new possibilities...
open up.

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