The Gap at $\nu = 5/2$ and the Role of Disorder in Fractional Quantum Hall States

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(Dated: 10th November 2018)

Theoretical results for the gaps of fractional quantum Hall states are substantially larger than experimental values determined from the activated behaviour of charge transport. The disparity in the case of the enigmatic $\nu = 5/2$ state is worrying as it amounts to a factor 20 to 30. We argue that disorder effects are responsible for this disparity and show how intrinsic gaps can be extracted from the measured transport gaps of particle-hole symmetric states within the same Landau level. We present new theoretical results for gaps at $\nu = 5/2$ and $7/2$, as well as at $\nu = 1/3$, $2/5$, $3/7$ and $4/9$, based on exact diagonalizations, taking account of the finite thickness of the two-dimensional electron layer and Landau level mixing effects. We find these to be consistent with the intrinsic gaps inferred from measured transport gaps. While earlier analyses (Du et al, Phys. Rev. Lett. 70, 2944 (1993)) assumed constant broadening for each sample, our results for the disorder broadening depend on the filling fraction, and appear to scale with the charge of the elementary excitations of the corresponding fractional state. This result is consistent with quasiparticle mediated dissipative transport.

The gaps obtained from analyzing the activated temperature-dependence of the longitudinal conductance near the center of fractional quantized Hall (FQH) plateaus in GaAs heterostructures [1-3] disagree with the values obtained from direct diagonalizations of finite systems [3]. This is the case even taking account of the softening of the Coulomb interaction between electrons resulting from the non-zero thickness of the 2D electron layer and of Landau level mixing effects. The discrepancies can be around a factor of two in the highest mobility samples for FQH states in the lowest Landau level (LLL). For FQH states in the second Landau level the discrepancies are even larger, as much as a factor of 20 at filling fraction $\nu = 5/2$ [3,4,5,6] and a factor of 30 at $\nu = 7/2$ [3]. Such large discrepancies make one wonder whether the $\nu = 5/2$ state has been correctly identified.

Here, we argue that disorder effects are responsible for these discrepancies. We show how the intrinsic gap of FQH states, that are strongly affected by disorder, can be estimated directly from measurements of the transport gaps using a simple model. These estimates are consistent with results we obtain from exact diagonalizations of finite systems provided we take account both of the non-zero width of the electron wave-function in the direction perpendicular to the two-dimensional electron layer and of Landau level mixing (LLM). Our analysis also provides estimates for the reduction of the measured activation gaps relative to the disorder-free intrinsic gaps—the so-called ‘disorder broadening’. We find that the disorder-induced gap reduction depends on the FQH state studied and is roughly proportional to the fractional charge of the corresponding elementary excitations. This is not in agreement with previous analyses of FQH gaps near $\nu = 1/2$ which assumed a filling factor independent disorder broadening of electronic states [3]. These gave intrinsic gaps which scaled as $|B_0 - B_{1/2}|$, where $B_0$ is the magnetic field at which the filling is precisely $\nu$, and implied an effective mass of the composite fermions which is independent of the filling fraction, contrary to theoretical predictions [7]. Our results show that, using a combination of the scaling analysis (described here) and comparisons with the results of exact diagonalizations, it will be possible to extract from measurements of activation energies reliable estimates both for the intrinsic FQH gap and for the disorder-induced gap reduction.

This work has been motivated by the observation of a transport gap for a FQH state at $\nu = 7/2$ by Eisenstein et al. [3], and by their report of transport gaps for the $\nu = 7/2$ and $5/2$ FQH states of $\Delta^a(7/2) = 0.07K$ and $\Delta^a(5/2) = 0.31K$. The latter is almost a factor of three larger than the earlier value $\Delta^a(5/2) = 0.11K$ (first reported in reference [4] and confirmed in [5]). The smaller gaps at $\nu = 5/2$ were obtained for samples with electron density $n_S = 2.3 \times 10^{11}/cm^2$, whereas the most recent results are for $n_S = 3 \times 10^{11}/cm^2$ [3]. The factor of three difference between the new and old results for $\Delta^a(5/2)$ cannot result from Coulomb interaction effects alone, as these scale with $\sqrt{n_S}$.

A FQH state at $\nu = 7/2$ is expected on theoretical grounds. Its structure should be very similar to that of the $\nu = 5/2$ state, as these two states are related by particle-hole conjugation symmetry, which becomes exact in the limit when LLM can be neglected. In that limit, if the energy gaps are purely controlled by the Coulomb interaction,

$$E_c = e^2/\kappa \ell_0, \quad (1)$$

the intrinsic gaps $\Delta'(\nu)$ of pure (disorder-free) systems can be written as

$$\Delta'(\nu) = \delta(\nu) E_c. \quad (2)$$

The symbol $\ell_0$ in equation (1) stands for the magnetic length, defined in terms of the magnetic field $B$, by $\ell_0 = \sqrt{\hbar c/eB}$, and $\kappa$ is the dielectric constant of the semiconductor material. For physically equivalent FQH states at fillings $\nu$ and $\nu'$ (those related by particle-hole conjugation symmetry), the coefficients $\delta(\nu)$ and $\delta'(\nu')$ in (2) will be the same and the difference in the gap values $\Delta'(\nu)$ and $\Delta'(\nu')$ will reflect the difference in the Coulomb energy scale $E_c$ at the magnetic
fields $B_e$ and $B_{e'}$ at which the FQH states occur. This will happen for $v' = 2 - v$, and in the second Landau level, when $\delta(2 + v) = \delta(2 + (2 - v))$, implying $\delta(5/2) = \delta(7/2)$. If, in addition, spin-mixing effects can be neglected, FQH states at filling fraction $\nu$ can be mapped to states at $v' = 1 - v$. As an example, we expect that gaps of fractional states at $\nu = 1/3, 2/3, 4/3$ and 5/3 will all be described by the same coefficient $\delta(1/3)$ as long as the Zeeman energy is large enough to suppress spin reversal in all these states, and as long as LLM effects can be neglected.

On the basis of equation (3), we analyze the measured gaps of symmetry related states as a function of $E_c$ to extract an estimate of the dimensionless coefficient $\delta(\nu)$. In Figure 1, we show the gap results from [1] for the $\nu = 5/2$ and 7/2 states as a function of $E_c$. The slope of the straight line through the two gap values yields $\delta(5/2) \approx 0.014$. This is just $\sim 35\%$ smaller than the theoretical estimate for a spin-polarized paired state of the Moore-Read type [9]. $\delta(5/2) \approx 0.022$, which was computed without taking account of LLM effects [3]. The intercept of the straight line gives an estimate of the gap reduction due to disorder, $\Gamma(5/2) \approx 1.24K$, which is only slightly less than the estimate for the intrinsic gap itself. We emphasize that the estimate for $\Gamma$ for the two states is based solely on the assumption that the 7/2 and 5/2 states are particle-hole conjugates of each other and that the Coulomb interaction dominates the value of the intrinsic gap. Although, in practice, the state at $\nu = 7/2$ is likely to be more strongly affected by LLM than that at $\nu = 5/2$, the assumption of particle-hole symmetry between the two states should still be approximately valid. In the following, we show that the effects of LLM reduce the theoretical value to $\delta(5/2) \approx 0.016$, so that the discrepancy between theoretical and experimental estimates of the gap at 5/2 essentially disappears. This provides further support for the identification of the FQH state at $\nu = 5/2$ as a paired state [10, 11].

We have also reanalyzed older results for FQH states at filling $\nu = p/(2p + 1)$ and $(p + 1)/(2p + 1)$. We define the gap reduction $\Gamma(\nu)$, as the difference between the theoretical activation gaps, $\Delta^a(\nu)$, and the intrinsic gap $\Delta'(\nu) = \delta(\nu)E_c$:

$$\Delta'(\nu) = \delta(\nu)E_c - \Gamma(\nu).$$

As for the 5/2 system, we assume that the dominant contribution to the FQH gaps comes from the Coulomb interaction of the electrons, so that the symmetry related states in the set $S_\nu = \{\nu, (1 - \nu), 1 + \nu, 2 - \nu\}$ are all described by the same coefficient $\delta_\nu$. If in addition, we assume that the gap reduction $\Gamma(\nu')$ has the same value $\Gamma_\nu$ for all states $\nu'$ in $S_\nu$, the activation gap is approximated by

$$\Delta'(\nu') \approx \delta_\nu E_c - \Gamma_\nu \quad \forall \nu' \in S_\nu.$$

This allows us to use the measured transport gaps $\Delta'(\nu')$ at $\nu = \nu, (1 - \nu), \ldots$ to obtain estimates, $\delta_\nu$ and $\Gamma_\nu$, for the intrinsic gap and the gap reduction for each family $S_\nu$.

The disorder scattering in the GaAs heterostructures, for which studies of the activated transport have been reported, is thought to be due mainly to ionized donors separated from the electron gas by a spacer layer of width $d$, where $d \sim 800\AA$ [3]. Our assumption that $\Gamma_\nu$ is the same for all states $\nu'$ in $S_\nu$ is equivalent to assuming that, for a given sample, the effect of these ionized donors on the FQH states and the low-lying excitations will be the same for the FQH states at filling fractions in the set $S_\nu$. In the limit $l_v/d \ll 1$ the response of the system on the length scale $d$ should be close to that of point-like quasiparticles, so that $\Gamma_\nu$ may depend on properties of the low-lying excitations like their charge $q$, which will be common to symmetry related states in the set $S_\nu$ but different for inequivalent sets $S_\nu$. However effects related to the internal structure of the excitations, which would result in an additional dependence of $\Gamma_\nu$ on the ratio $l_v/d$, are assumed to be small. When we compare with our results from exact diagonalizations, this assumption appears reasonable for the filling fractions close to 1/2 (see below).

In Figure 2, we show the measured gaps taken from [1, 2, 3, 4] for three different very high-mobility samples at filling fractions $\nu = p/(2p + 1), \nu' = 1 - \nu$, and $\nu' = 2 - \nu$ for $p = 1$ (Figure 2a), $p = 2$ (Figure 2b) and $p = 3$ (Figure 2c) as function of $E_c$. Samples A and B of [3] have an electron density $n_S$ of $(1.12$ and 2.3)$\times 10^{11}/\text{cm}^2$ and mobilities $\mu = (6.8$ and 12)$\times 10^6\text{cm}^2/\text{Vs}$. The sample of [4] (which we will call sample C) has $n_S = 1.65 \times 10^{11}/\text{cm}^2$ and a mobility $\mu = 5 \times 10^6\text{cm}^2/\text{Vs}$. Gaps at $\nu = 5/3$ and 4/3 were reported for sample A in [5]. The dependence of the gap on total magnetic field in a tilted field experiment showed that at $\nu = 5/3$ the ground and low-lying excited states were spin-polarized. At $\nu = 4/3$ the ground state was not spin-polarized for tilt angles up to 65.1° while the excitations involved spin-reversals up to even larger tilt angles. We therefore assume that only the states at $\nu = 1/3, 2/3$ and 5/3 are related by symmetry and not the state at $\nu = 4/3$. We show the measured gaps in untitled field as a function of $E_c$ in Figure 3. We note
that the slopes of the three straight line fits through the gaps of samples A, B and C are quite similar yielding estimates of \( \delta_{1/3} = 0.064, 0.058 \) and 0.075, respectively. By contrast, the intercepts at \( E_c = 0 \), which yield estimates, \( \Gamma^{1/3}_{\nu} \), that vary by almost a factor 2, and reflect differences in sample quality. In Figures 2b and 2c we show the analysis of the states states at \( \nu = 2/5, 3/5 \) and \( \nu = 3/7, 4/7 \), respectively. We again note that the slope of the gaps as function of \( E_c \) are very similar for the two samples A and B. They yield estimates of \( \delta_{3/7} = 0.029 \) for both samples (Figure 2b) and \( \delta_{3/7} = 0.027 \) and 0.025 (Figure 2c) for samples A and B, respectively.

An alternative approach to determining \( \Gamma(\nu) \) is simply to attribute the difference between precise calculations of FQH gaps of disorder-free systems and measured gaps to the effects of disorder and use equation (3) as definition of \( \Gamma(\nu) \). Such calculations must of course include the effects of the finite thickness \( w \) of the two-dimensional electron system as well as LLM. We take account of LLM within the random phase approximation for the dielectric function \( \varepsilon(q, \omega) \)

\[
\varepsilon(q, \omega) = 1 - \mathcal{V}(q) \Pi(q, \omega),
\]

and represent the electron-electron interaction by

\[
U(r) = \int \frac{d^2q}{(2\pi)^2} \mathcal{V}(q) \frac{e^{i\vec{q}\cdot\vec{r}}}{\varepsilon(q,0)},
\]

where

\[
\mathcal{V}(q) = \frac{2\pi e^2}{\kappa q} e^{i\vec{q}\cdot\vec{w}} \text{erfc}(qw)
\]

is the interaction between electrons that are trapped at the interface in a Gaussian wave function of width \( w \). The polarization \( \Pi(q, \omega) \) in (5) is given by

\[
\Pi(q, \omega) = -\frac{m^*}{\pi \hbar^2} \sum_i \int_0^\infty \sum_{n=0}^{[i-1]} F(V(s) - n) \sum_{k=v+1}^\infty F(k - V(s)) \times \left( -1 \right)^{(k-n)} \frac{(k-n)}{(\omega/\omega_c)^2 + (k-n)^2} \left( n^k(x) \right) e^{-x}
\]

where \( x = (q\ell_0)^2/2 \) and \( \sum \) stands for the summation over spin \( s = \uparrow, \downarrow \). The symbol \([x]\) denotes the largest integer \( \leq x \). Equation (7) agrees with expression (A1) of Aleiner and Glazman [13], which describes the spin degenerate case, \( \nu(\uparrow) = \nu(\downarrow) = N \), with integer filling \( 2N \). The function \( F(z) \) is introduced to treat the case of fractional filling and measures the filling fraction of the Landau level \( n \), via \( F(z) = z \) for \( 0 < z < 1 \), \( F(z) \equiv 1 \) for \( z \geq 1 \) and \( F(z) \equiv 0 \) for \( z \leq 0 \). We have verified this method for incorporating finite width and LLM corrections at filling fraction \( \nu = 1/3 \) where we could check that our results are consistent with those by Yoshioka [12]. Expression (5) together with (6) and (7) lead to a modification of the electron interaction at short separation, which is controlled by the dimensionless parameter \( k = E_c/\hbar \omega_c \). Here \( \omega_c = eB/m^*c \) stands for the cyclotron frequency, with \( m^* \) the effective mass of the electrons. A full account of this method will be published elsewhere.

For the particular sample of [8] we have repeated the calculation of [3] using the interaction amended for LL mixing and taking account of the non-zero thickness of the wavefunction. We have computed the quasiparticle and quasihole energies as described in [3] and, by extrapolating to the thermodynamic limit, we have estimated the intrinsic gaps at \( \nu = 5/2 \) and \( \nu = 7/2 \). The gaps are \( \delta^h(5/2) = 0.016 \) and \( \delta^h(7/2) = 0.015 \), and are close to the estimate \( \delta^h_{5/2} = 0.14 \) obtained from the experimental values of \( \Delta^a \) at \( \nu = 5/2 \) and 7/2, using [13] as discussed above. We have also calculated the finite width and LLM corrections for all the other states reported in references [1, 2, 8]. These results are listed in Table 1. As can be seen, the estimates \( \delta^v_{\nu, \ell} \) of the gap coefficients, calculated on the basis of the simple Ansatz [4] from
| ν  | ν'  | Ref | $\hat{\Delta}''$ | $\delta^h(\nu)$ | $\delta^h(\nu')$ | $\Gamma(\nu)$ | $\Gamma(\nu')$ |
|----|-----|-----|----------------|----------------|----------------|---------------|---------------|
| 1/3 | 2/3 | [4] | 0.069(9) | 0.75 | 0.074 | 7.2K | 6.4K |
| 1/3 | 5/3 | [4] | 0.077(4) | 0.075 | 0.057 | 7.2K | 5.3K |
| 1/3 | 2/3 | [4]A | 0.063(2) | 0.077 | 0.073 | 6.5K | 5.3K |
| 1/3 | 5/3 | [4]A | 0.064(1) | 0.077 | 0.052 | 6.5K | 3.1K |
| 2/5 | 3/5 | [4]A | 0.029(3) | 0.036 | 0.034 | 3.0K | 2.6K |
| 3/7 | 4/7 | [4]A | 0.027(5) | 0.025 | 0.025 | 2.3K | 2.3K |
| 4/9 | 5/9 | [4]A | 0.013(6) | 0.019 | 0.019 | 2.2K | 2.1K |
| 1/3 | 2/3 | [4]B | 0.058(2) | 0.077 | 0.076 | 9.4K | 7.8K |
| 2/5 | 3/5 | [4]B | 0.029(3) | 0.036 | 0.036 | 4.3K | 3.9K |
| 3/7 | 4/7 | [4]B | 0.025(3) | 0.025 | 0.025 | 3.6K | 3.5K |
| 4/9 | 5/9 | [4]B | 0.007(5) | 0.020 | 0.020 | 3.3K | 3.0K |
| 5/2 | 7/2 | [6] | 0.014 | 0.016 | 0.015 | 1.5K | 1.4K |

Table I: The values for the intrinsic gap $\hat{\Delta}''$ (see [3]) estimated by fitting the measured activation gaps $\Delta^a$ to the Ansatz [3] for different samples, together with our theoretical values $\delta^h(\nu)$ for the gaps obtained from exact diagonalization studies (cf. [3]) and the corresponding gap reduction $\Gamma(\nu)$ (see text). The theoretical values $\delta^h(\nu)$ include finite width and Landau level mixing corrections. Numbers in parentheses denote the error of the last quoted digit of $\hat{\Delta}''$. These are calculated from the quoted experimental error [1] or from the discretization error when extracting numerical data from experimental plots [2, 8]. For the latter as well as for $\nu = 5/2$ and $7/2$, no experimental uncertainty is specified. If for the values of $\Delta^a$ of references [2, 3] similar uncertainties are assumed as specified in reference [1], errors for samples A and B are 4-5 times bigger than quoted.

Figure 3: $\Gamma(\nu)$ for various samples of different high electron mobility plotted as a function of the charge of the elementary excitations. The full symbols refer to sample A, open triangles and diamonds to sample B [4, 8], the data on the dash-dotted line represent $\nu = 5/2$ and $7/2$ [8]. The triangles refer to $\nu$, diamonds to $1 - \nu$.

The authors acknowledge useful discussions with B.I. Halperin.

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