Analysis of Mine Ventilation Network by 1D FEM: Simulation of Fans and Natural Draught

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Abstract. The version of simulating fans in mine ventilation network analysis by using the combination of FEM with the laminar solution method is considered. Two ways of fan insertion are discussed: in the form of a separate active element and in the form of combined element (an active element inserted into a passive element). The latter approach allows changing a position of the fan in a design stage without renumbering of a grid. The method of chords is used to linearize in general nonlinear fan characteristic. It is shown that the dependence of air discharge on the upstream-downstream pressure difference is an ascending curve for the fan. This provides a convergence for the iterative procedure of solving the FEM nonlinear equations system. A method of calculation of convective heat transfer between the air flow and the rock wall based on Lyon's solution for a pipe with the constant density of boundary heat flow is described. As the density of heat flow and the air properties are slowly varying functions along a mine working (a branch of a network), then discretizing each branch into small segments permits to consider these parameters as constant along segments. Calculation of nonhomogeneous temperature field and treatment of gravity allow modelling the natural draught effect. The results of solution of the test problems validating used algorithms are given. Calculation results of flow mass rate in a mine network within the year are also represented which make it possible to estimate the influence of natural draught on venting capability.

1. Introduction

The main aspects of application of the 1D finite element method (FEM) in combination with the laminar solution method (LSM) for calculation of the underground ventilating networks were considered earlier in [1]. It has been shown that the FEM-LSM together with the Newton method [2, 3] and the method of gradient [4, 5] is one of the most effective methods of calculation of hydraulic and ventilating networks among methods now used (see reviews in [6-8]). The fact that the FEM – LSM has a guaranteed convergence [11] is also important. It follows from the analogy of the elastic and plastic deformation of a rod to the laminar and turbulent flow in a pipe [1, 9]. The guaranteed convergence of the elastic solutions method (analog of LSM) is proved [10, 11] for a rod system suffering tension-compression.

Ventilating networks of the underground structures differ from other hydraulic networks by their active heat exchange between airstream and the surrounding rocks [12, 13]. The non-homogeneous...
temperature field in a mine ventilating network and as consequence the different air density in vertical and inclined network branches in the conditions of gravitation is the main reason of the natural draught [14, 15, 16]. The latter can lead to the ventilating stream inversion and the local circulation generation. The natural draught can considerably affect on ventilation system especially at pitch mining [14, 16, 17]. The additional fans used for the local regulation of air distribution in a network exert influence on the main fan operation also.

It is common in the air distribution analysis to divide network branches into active and passive [6, 18]. The branches simulating the fan operation are considered as the active elements. The branches without fans including both distributed and local resistances are the passive elements. In this paper we propose using the universal combined element which includes a fan, the distributed and local resistances and also considers action of gravity. Note that the use of combined elements instead of individual active elements simplifies the work with the computer program at the stage of the ventilation network design since it allows changing the fan position or add/remove fans without renumbering the FEM grid.

2. The basic relation for the combined finite element
In case of the standard passive finite element (branch) the pressure drop in a branch $p_{ij} = p_i - p_j$ consists of pressure losses on the distributed and local flow resistances

$$p_{ij} = \Delta p_{\text{dis}} + \Delta p_{\text{loc}}.$$  

(1)

To take in account the gravitation we add a term $\Delta p_g$ in (1). The fan can be modelled as a separate active element [18]. We will use the combined finite element therefore we also add a term considering the running fan $\Delta p_{\text{fan}}$ in (1). As a result the pressure difference in the combined element will consist of the four above-mentioned terms:

$$p_{ij} = \Delta p_{\text{dis}} + \Delta p_{\text{loc}} + \Delta p_g + \Delta p_{\text{fan}}.$$  

(2)

To get expressions for $\Delta p_{\text{dis}}$ and $\Delta p_g$ at first we will consider a laminar flow in a single element (a finite pipe, a finite length working). In this case we obtain from the Navier-Stokes equation and the continuity equation [19] the extension of the Hagen-Poiseuille equation [19] to a case of gravity force action:

$$p_{ij} = \frac{8\pi \nu l_e}{S_e^2} G_e - \rho_e gh_{ji},$$  

(3)

where $l_e$ and $S_e$ are the length and the cross-sectional area of a pipe; $\nu$, $\rho_e$ and $T_e$ are the kinematic viscosity, density and temperature of a fluid; $g$ is the acceleration of gravity; $h_{ji} = h_j - h_i$ is the difference of depths for nodes $j$ and $i$. We point out that the mass flow rate $G_e$ is used in the equation (3) as for the non-uniform density the continuity equation for a pipe of variable section ($G = \text{const}$) is valid namely for the mass flow rate. As a result we get from (3)

$$\Delta p_{\text{dis}} = R_{\text{dis}} G_e,$$  

(4)

$$\Delta p_g = -\rho_e gh_{ji},$$  

(5)

where $R_{\text{dis}} = \frac{8\pi \nu l_e}{S_e^2}$ is the branch resistance coefficient for a laminar flow.

In the FEM [20] when we write the basic relation the inverse dependence is used instead of the (4)

$$G_e = K_{\text{dis}} \Delta p_{\text{dis}},$$  

(6)
\[ K_{dis} = \frac{1}{R_{dis}} = \frac{S_e^2}{8\pi v_e l_e}, \quad (7) \]

where \( K_{dis} \) is the conductivity coefficient for a laminar flow.

In the case of a turbulent flow according to the LSM the formula (6) \[1\] has the same form but the coefficient of the laminar conductivity (7) is replaced by the “secant” coefficient of the turbulent conductivity

\[ K_{dis} = \frac{G_e}{\Delta p_{dis}} = \frac{G(\Delta p_{dis})}{\Delta p_{dis}} = \frac{\text{Re}(\Delta p^*)}{\Delta p^*} \frac{8S_e^2}{\pi v_e l_e}, \quad (8) \]

where \( \text{Re} = \frac{w_e d_e}{v_e} = \frac{4G_e}{\pi d_e \mu_e} \) is the Reynolds number \( (d_e \text{ is the hydraulic diameter}); \)

\[ \Delta p^* = \Delta p_{dis} \frac{16\rho_e v_e^2}{\mu_e^2 l_e}, \]

is the dimensionless pressure drop \[1\]. Then the branch resistance coefficient for a turbulent flow will have the following form

\[ R_{dis} = \frac{1}{K_e} = \frac{\Delta p^*}{\text{Re}(\Delta p^*)} \frac{\pi v_e l_e}{8S_e^2}. \quad (9) \]

The fan pressure difference \( \Delta p_{fan} \) is determined by the fan characteristic \( \Delta p(Q) \), where \( \Delta p = p_d - p_s \) is the pressure difference between the diffuser and the fan suction, \( Q \) is the fan volumetric flow rate. Its signature \[14, 15\] corresponding to the steady fan operation is shown in fig. 1 by the curve in the first quadrant.

![Fan characteristics](image-url)

**Figure 1.** Fan characteristics (a): initial, at its direct (b) and inverse (c) locations.

Let us consider two possible locations of the fan in a branch. For a direct fan (fig. 1b) the flow through it will have a positive value \( (Q_e = Q > 0) \) and the fan pressure difference will have a negative value \( (\Delta p_{fan} = p_s - p_d = -\Delta p < 0) \). This case corresponds to the position of the characteristic \( \Delta p_{fan}(Q_e) \) in the fourth quadrant in figure 1a. For an inverse fan (figure 1c) the flow through it will be negative \( (Q_e = -Q < 0) \) and the fan pressure difference will be positive \( (\Delta p_{fan} = p_d - p_s = \Delta p > 0) \). This case corresponds to the position of the characteristic \( \Delta p_{fan}(Q_e) \)
in the second quadrant in fig. 1a. We can see that for both cases the characteristics are ascending that provides convergence of the iterative procedure of the FEM-LSM [1].

According to the LSM the relation between the pressure difference and the mass flow rate in each element of a network must be linear. We will use the piecewise - linear approximation of the fan characteristic $\Delta p(Q)$ according to the method of chords. One of such rectilinear segments (chords) is shown in fig. 1. For this segment the fan characteristic $\Delta p_{\text{fan}}(Q_e)$ will have the following form

$$\Delta p_{\text{fan}} = R_{\text{fan}} G_e - p_{j0},$$

where $R_{\text{fan}} = \frac{\Delta p_1 - \Delta p_2}{\rho(Q_2 - Q_1)}$; $p_{j0} = \pm \frac{\Delta p_1 Q_2 - \Delta p_2 Q_1}{Q_2 - Q_1}$ ("+" corresponds to the direct fan and "-" to the inverse one).

The expression for the pressure loss on local resistance can be found after a linearization procedure similar to the one that was used when deriving $\Delta p_{\text{dis}}$ (see (4, 6, 8, 9)):

$$\Delta p_{\text{loc}} = R_{\text{loc}} G_e,$$

where $R_{\text{loc}} = \frac{\xi |\Delta p_{\text{loc}}|}{\sqrt{2 \rho S}}$; $\xi$ is the local resistance coefficient.

Substituting formulas (4, 5, 10, 11) in (2), we will get

$$p_{ij} = (R_{\text{dis}} + R_{\text{loc}} + R_{\text{fan}}) G_e - (p_{j0} + p_e gh_{ij}),$$

from which we obtain the basic relation for the combined 1D element

$$G_e = K_e p_{ij} + G_0,$$

where $K_e = 1/R_e$; $R_e = R_{\text{dis}} + R_{\text{loc}} + R_{\text{fan}}$; $G_0 = K_e (p_{j0} + p_e gh_{ij}).$

Having in mind the expression $G_e = G_i = -G_j$ ($G_i$, $G_j$ are the nodal mass flow rates), we get the basic relation for the combined 1D element in a matrix form

$$[k]^{e} \begin{bmatrix} p_i \\ p_j \end{bmatrix} = \begin{bmatrix} G_i - G_0 \\ G_j + G_0 \end{bmatrix},$$

where $[k]^{e} = K_e \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the local conductivity matrix.

Executing summation of (14) over all elements, we get [1, 9] the system of the nonlinear algebraic equations relative to the unknown nodal network pressures. Solving this system by the method of simple iteration, we find the nodal pressures and, using (13), the mass flow rates in each branch of a network.

3. Finding of the flow temperature

We can model the natural draught, if we know how to find the flow temperature in each element (see (5)). For this purpose the problem of the convective heat exchange between a ventilating airstream and a rock must be solved. One way of solution is considered in [21]. It includes two solutions. The first is the solution of the problem of heat conductivity in the infinite rock massif from the airstream moving in a cylindrical cavity which is obtained by the Green functions method in combination with transformation Laplace method [12, 13]. The second is Lyon's solution [22] of the problem of finding the flow temperature in a pipe with the constant heat flow density through a wall. As the heat flow density and air properties change slowly along the mine working (a branch of a network), discretizing each branch into the small segments, we can consider these quantities as constant on these segments and apply Lyon's solution.
4. Testing problems
For testing the offered way of modelling fan in the FEM an example of calculation of the consecutive connection of five elements (fig. 2a) has been considered. The fifth element includes the fan VOD-30m2.

\[
\Delta p = 1252.50 + 67.426Q - 0.6361Q^2.
\]

Operation of the fan was modelled in two ways. According to the traditional way the network without fan was separately calculated and the fan operating point was found by crossing the fan and network characteristics in the point \(\Delta p = 155.773\) Pa, \(Q = 120.221\) m\(^3\)/s (fig. 2b). When calculating according to the offered way the fan was considered in the fifth combined element of a network. In this case the following parameters of the fan operating point were found: \(\Delta p = 155.783\) Pa, \(Q = 120.221\) m\(^3\)/s. It is evident that the results coincide with the accuracy lying within the accuracy of calculations.

For the analysis of influence of the natural draught the model mine ventilating network (fig. 3a) with the fan VRCD-4.5 fan was considered. The input average daily air temperature changed under the sinusoidal law with the amplitude of 17 °C respective to the average temperature of 0.7 °C. The authors’ software package "MineClimate" [21] allowing to determine microclimate parameters in a mine ventilating network was used to determine the change of the mass flow rate in the network within a year (zero day is on April 15th) (fig. 3). It is obvious that the flow rate also changes under the sinusoidal law respective to the average value of 317.6 kg/s with an amplitude of 10.7 kg/s (3.4% of average value).

5. Conclusions
The basic relation for the combined final element considering the distributed and local resistance, gravitation and the operating fan is obtained.

The way of representation of the fan characteristic providing convergence of the FEM-LSM is proved.

The results of solutions of the test problems validating the offered algorithms of simulation of fans and the natural draught are provided.
6. References

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Figure 3. The model mine ventilating network (a); the change of mass flow rate in network within a year (b).
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