Soft superweak CP violation in a 331 model

J. C. Montero *, V. Pleitez † and O. Ravinez ‡

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona 145
01405-900–São Paulo, SP
Brazil
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Abstract

We show that it is possible to implement soft superweak CP violation in the context of a 331 model with only three triplets. All CP violation effects come from the exchange of singly and doubly charged scalars. We consider the implication of this mechanism in the quark and lepton sectors. In particular it is shown that in this model, as in most of those which incorporate the scalar mediated CP violation, it is possible to have large electric dipole moments for the muon and the tau lepton while keeping small those of the electron and the neutron. The CKM mixing matrix is real up to the two loop level.

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*E-mail:montero@ift.unesp.br
†E-mail:vicente@ift.unesp.br
‡Present address: Universidad Nacional de Ingeniería (UNI), Facultad de Ciencias Av. Tupac Amaru S/N apartado 31139, Lima-Peru. E-mail:pereyra@fc-uni.edu.pe
I. INTRODUCTION

The origin and the smallness of $CP$ violation are still open questions. In the context of the electroweak standard model\cite{1} the only source of $CP$ violation, in the quark sector, is the surviving phase in the charged current coupled to the vector boson $W^\pm$\cite{2}. This is called explicit or hard $CP$ violation. On the other hand there is no $CP$ violation in the lepton sector at lower orders since neutrinos are massless.

Although this is an interesting feature of the model it leaves open the question of why $CP$ is so feebly violated. It is well know that $CP$ is softly broken if it occurs through a dimensional coupling in the bare lagrangian and/or spontaneously. The possibility that $CP$ nonconservation arise exclusively through Higgs boson exchanges is a rather old one. It can be implemented in a spontaneous way as it was proposed by Lee\cite{3} or, explicitly in the parameters of the scalar potential, as proposed by Weinberg\cite{4,5}. Of course, both possibilities can be mixed. Since the works of Lee and Weinberg it has been known that in renormalizable gauge theories the violation of $CP$ has the right strength if it occurs through the exchange of a Higgs boson of mass $M_H$, i.e., it is proportional to $G_F m_f^2 / M_H^2$, where $m_f$ is the fermion mass. Since then, there have been many realizations of that mechanism in extensions of the electroweak standard model. Even if we insist that the $CP$ violation arises solely through the Higgs exchange we have several possibilities in the standard electroweak model with several doublets\cite{6}.

Some years ago it was proposed a model based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ gauge symmetry with exotic charged leptons\cite{7}. In this model since the scalar multiplets transform in a different way one from another the scalar potential is constrained in such a way that even with three triplets there is not spontaneous $CP$ violations\cite{8}. Here we will show that it is possible to have soft $CP$ violation because the coupling constant of the trilinear term in the scalar potential is complex and the vacuum expectation value (VEV) of the three triplets are also complex. Hence, $CP$ is a symmetry in the full bare lagrangian but in the trilinear term in the potential, in particular all Yukawa couplings are real at tree level. We will show that the model is a realization of the pure superweak $CP$ violation\cite{9} in the sense that all flavor changing phenomena other than $CP$ violation are accurately described by the real Cabbibo-Kobayashi-Maskawa (CKM) matrix since $CP$ violation is restricted to one operator of dimension three\cite{10}.

If the condition that except in the trilinear term $CP$ is a symmetry of the lagrangian is assumed, we have verified that it is possible to choose the physical phases in such a way that the only places of $CP$ violation in the lagrangian density are those related with the singly and doubly charged scalars which are present in the model. One interesting feature of the model is that the electric dipole moment of the muon and tau lepton can be of the order of magnitude of the respective present experimental bounds. In the lepton tau case it means that this quantity can be studied in a tau-charm factory.

This work is organized as follows. In Sec.\,11 we review the Higgs sector of the model. We consider in particular the minimization of the scalar potential and we also give the definitions of the Goldstone and the physical scalar fields of the several charged sectors. In Sec.\,111 we study the Yukawa interactions showing how the VEV’s phases can be absorbed in the fermion field in some places of the lagrangian density and that they only survive in the sector involving both simply and doubly charged scalar fields. In Sec.\,1IV we show that
there is not $CP$ violation in the vector boson-fermion interactions. The phenomenology of the model is considered in Sec. [V] while our conclusions are in the last section.

### II. A MODEL WITH THREE SCALAR TRIPLETs

As we said before, here we will consider a model with $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ symmetry with both exotic quarks and charged leptons [7]. In this model in order to give mass to all fermions it is necessary to introduce three scalar triplets. They transform as

$$
\chi = \begin{pmatrix} \chi^- \\ \chi^- \\ \chi^0 \end{pmatrix} \sim (3,-1), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (3,1), \quad \eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta^+ \end{pmatrix} \sim (3,0).
$$

The more general scalar potential invariant under the gauge symmetry is

$$
V(\chi, \eta, \rho) = \mu_1^2 \chi^\dagger \chi + \mu_2^2 \eta^\dagger \eta + \mu_3^2 \rho^\dagger \rho + (\alpha \epsilon_{ijk} \chi_i \rho_j \eta_k + H.c.)
$$

where

$$
(a \equiv \epsilon_{ijk} \chi_i \rho_j \eta_k).
$$

All terms of this potential but the $a_{10}$ term conserve the total lepton number $L$ (or $L + B$, where $B$ is the baryonic number). The minimum of the potential must be studied after the shifting of the neutral components of the three scalar multiplets in Eq. (1). Hence, we redefine the neutral components as follows:

$$
\eta^0 \rightarrow \frac{v_\eta}{\sqrt{2}} \left(1 + \frac{X_\eta^0 + iL_\eta^0}{|v_\eta|}\right), \quad \rho^0 \rightarrow \frac{v_\rho}{\sqrt{2}} \left(1 + \frac{X_\rho^0 + iL_\rho^0}{|v_\rho|}\right), \quad \chi^0 \rightarrow \frac{v_\chi}{\sqrt{2}} \left(1 + \frac{X_\chi^0 + iL_\chi^0}{|v_\chi|}\right).
$$

where $v_a = |v_a| e^{i\theta_a}$ with $a = \eta, \rho, \chi$.

The condition that the first derivative of the potential in Eq. (2) is zero (i.e., no linear terms in all neutral fields must survive) implies the following constraint equations:

$$
\frac{a_4}{2} |v_\eta|^2 |v_\chi| + \frac{a_5}{2} |v_\rho|^2 |v_\chi| + \mu_1^2 |v_\chi|^3 + \frac{1}{\sqrt{2}|v_\chi|} \text{Re} (\alpha v_\chi v_\rho v_\eta) = 0, \quad (4a)
$$

$$
\frac{a_2}{2} |v_\eta|^3 + \frac{a_4}{2} |v_\chi|^2 |v_\eta| + \frac{a_6}{2} |v_\rho|^2 |v_\eta| + \mu_2^2 |v_\eta|^3 + \frac{1}{\sqrt{2}|v_\eta|} \text{Re} (\alpha v_\chi v_\rho v_\eta) = 0, \quad (4b)
$$

$$
\frac{a_6}{2} |v_\eta|^2 |v_\rho| + \frac{a_5}{2} |v_\rho|^2 |v_\chi| + \mu_3^2 |v_\rho|^3 + \frac{1}{\sqrt{2}|v_\rho|} \text{Re} (\alpha v_\chi v_\rho v_\eta) = 0, \quad (4c)
$$

and, finally

$$
\text{Im} (\alpha v_\chi v_\rho v_\eta) = 0. \quad (4d)
$$
Before considering how many physical phases will survive in the potential (this must be done by taking at the same time the Yukawa interactions) we will study all scalar mass eigenstates in the model. Recall that we have to verify where the VEV’s phases appear in the several sectors of the lagrangian that is, in vertices and in mixing matrices. Hence, we will firstly write down explicitly the mass and mixing matrices of each charged sector of the model. Two of the phases in Eqs. (3) can be transformed away with a SU(3) transformation; whenever we do that we will mention it explicitly and we will choose \( \theta_\eta = \theta_\rho = 0 \).

### A. Doubly charged scalars

In this sector we have the following mass matrix in the \((\rho^{++}, \chi^{++})^T\) basis

\[
\begin{pmatrix}
-\frac{a_8}{2} |v_\chi|^2 + \frac{A}{\sqrt{2}|v_\rho|^2} & \frac{\alpha v_\rho v_\chi}{\sqrt{2}} - \frac{a_8}{2} v_\rho v_\chi \\
-\frac{a_8}{2} v_\rho^* v_\eta & -\frac{a_8}{2} |v_\rho|^2 + \frac{A}{\sqrt{2}|v_\chi|^2}
\end{pmatrix}
\]

(5)

where we have defined \( A \equiv \text{Re}(\alpha v_\chi v_\rho v_\eta) \).

As expected, we have a doubly charged Goldstone boson \( G^{++} \) and a physical doubly charged scalar \( Y^{++} \)

\[
\begin{pmatrix}
\rho^{++} \\
\chi^{++}
\end{pmatrix} = \frac{1}{(|v_\rho|^2 + |v_\chi|^2)^{1/2}} \begin{pmatrix}
|v_\rho| & -|v_\chi| e^{-i\theta_\chi} \\
|v_\chi| e^{i\theta_\chi} & |v_\rho|
\end{pmatrix} \begin{pmatrix}
G^{++} \\
Y^{++}
\end{pmatrix},
\]

(6)

with the mass square of the \( Y^{++} \) field given by

\[
M_{Y^{++}}^2 = \frac{A}{\sqrt{2}} \left( \frac{1}{|v_\chi|^2} + \frac{1}{|v_\rho|^2} \right) - \frac{a_8}{2} \left( |v_\chi|^2 + |v_\rho|^2 \right)
\]

(7)

Notice that this mass is proportional to \( |v_\chi|^2 \) which is the VEV that is in control of the \( SU(3)_L \) symmetry and so it is the largest mass scale of the model. It means that the doubly charged scalar must be a heavy scalar.

### B. Singly charged scalars

Next, for the simply charged scalar fields we have, in the \((\eta_1^+, \rho^+, \eta_2^+, \chi^+)^T\) basis,

\[
\begin{pmatrix}
-\frac{a_9}{2} |v_\rho|^2 + \frac{A}{\sqrt{2}|v_\eta|^2} & -\frac{a_9}{2} v_\rho^* v_\eta + \frac{\alpha_9}{\sqrt{2}} & -\frac{\alpha_9}{2} v_\rho^* v_\chi & -\frac{a_9}{2} v_\rho v_\eta \\
-\frac{a_9}{2} v_\eta^* v_\rho + \frac{\alpha_9}{\sqrt{2}} & -\frac{a_9}{2} |v_\eta|^2 + \frac{A}{\sqrt{2}|v_\rho|^2} & -\frac{\alpha_9}{2} v_\eta^* v_\chi & -\frac{a_9}{2} v_\eta v_\rho \\
-\frac{a_{10}}{2} |v_\chi|^2 + \frac{A}{\sqrt{2}|v_\eta|^2} & -\frac{a_{10}}{2} v_\chi^* v_\eta & -\frac{a_{10}}{2} v_\chi^* v_\rho & -\frac{a_{10}}{2} |v_\chi|^2 + \frac{A}{\sqrt{2}|v_\eta|^2} \\
-\frac{a_{10}}{2} v_\eta^* v_\chi + \frac{\alpha_{10}}{\sqrt{2}} & -\frac{a_{10}}{2} v_\eta v_\chi & -\frac{a_{10}}{2} v_\eta v_\rho & -\frac{a_{10}}{2} |v_\eta|^2 + \frac{A}{\sqrt{2}|v_\chi|^2}
\end{pmatrix}
\]

(8)

Notice that if \( a_{10} = 0 \), \( \eta_1^+ \) and \( \rho^+ \) decouple from \( \eta_2^+ \) and \( \chi^+ \). Hence, the mass matrix in Eq. (8) is reduced to two \( 2 \times 2 \) mass matrices and we have two Goldstone bosons \( G_1^+ \) and \( G_2^+ \); and two massive fields \( Y_1^+ \) and \( Y_2^+ \).
\[
\begin{pmatrix}
\eta^+_i \\
\rho^+_i
\end{pmatrix} = \frac{1}{(|v_\eta|^2 + |v_\rho|^2)^{\frac{1}{2}}} \begin{pmatrix}
-|v_\eta| & |v_\rho| \\
|v_\rho| & -|v_\eta|
\end{pmatrix} \begin{pmatrix}
G^+_i \\
Y^+_i
\end{pmatrix},
\]
with
\[
m^2_{\eta^+_i} = \frac{A}{\sqrt{2}} \left( \frac{1}{|v_\eta|^2} + \frac{1}{|v_\rho|^2} \right) - \frac{a_9}{2} \left( |v_\rho|^2 + |v_\eta|^2 \right),
\]
and
\[
m^2_{\rho^+_i} = \frac{A}{\sqrt{2}} \left( \frac{1}{|v_\eta|^2} + \frac{1}{|v_\chi|^2} \right) - \frac{a_7}{2} \left( |v_\chi|^2 + |v_\eta|^2 \right).
\]

There are 5 phases in the matrix in Eq. (8), two of them can be transformed away with a \(SU(3)\) transformation (here when we do that we will chose \(\theta_\eta = \theta_\rho = 0\)). The other three phases can be absorbed by redefining three scalar fields. However these phases will appear in the Yukawa interactions or in the scalar potential in terms like \(a_{10}\chi^+\eta^0\rho^0\eta^0\). Hence, there is \(CP\) violation in the propagators of the singly charged scalars if \(a_{10} \neq 0\). Since we want \(CP\) softly broken we will consider here \(a_{10} = 0\) with the singly charged scalar given by the expressions above.

\section*{C. Neutral scalars}

Finally, in the neutral sector we have \(CP\) even fields, denoted here by \(H^0_i\), and \(CP\) odd fields, \((G^0_{1,2}, h^0) \equiv h^0_i\).

In the \(CP\) odd sector we have
\[
\begin{pmatrix}
\frac{A}{\sqrt{2}|v_\eta|^2} & \frac{A}{\sqrt{2}|v_\rho||v_\eta|} & \frac{A}{\sqrt{2}|v_\chi||v_\rho|} \\
\frac{\sqrt{2}|v_\rho|^2}{|v_\eta|^2} & -\frac{\sqrt{2}|v_\eta||v_\rho|}{|v_\chi|^2} & -\frac{\sqrt{2}|v_\rho||v_\chi|}{|v_\eta|^2} \\
\frac{\sqrt{2}|v_\eta|^2}{|v_\rho|^2} & -\frac{\sqrt{2}|v_\rho||v_\eta|}{|v_\chi|^2} & -\frac{\sqrt{2}|v_\eta||v_\chi|}{|v_\rho|^2}
\end{pmatrix}
\]
in the \((I_\eta, I_\rho, I_\chi)^T\) basis. There are in fact two neutral Goldstone bosons \(G^0_1\) and \(G^0_2\) (as it must be since we have two massive neutral vector bosons \(Z\) and \(Z\)') and a physical \(CP\) odd scalar field \(h^0\). Explicitly we have
\[
\begin{pmatrix}
I^0_\eta \\
I^0_\rho \\
I^0_\chi
\end{pmatrix} = \begin{pmatrix}
-\frac{|v_\eta|}{N_1} & -\frac{|v_\rho|^2|v_\eta|}{N_2} & \frac{|v_\rho||v_\chi|}{N_3} \\
\frac{|v_\rho|^2|v_\eta|}{N_1} & -\frac{|v_\eta||v_\rho|}{N_2} & \frac{|v_\eta||v_\chi|}{N_3} \\
0 & \frac{|v_\chi||v_\rho|}{N_2} & \frac{|v_\chi||v_\eta|}{N_3}
\end{pmatrix} \begin{pmatrix}
G^0_1 \\
G^0_2 \\
h^0
\end{pmatrix},
\]
where
\[
N_1 = (|v_\eta|^2 + |v_\rho|^2)^{\frac{1}{2}}, \quad N_2 = (|v_\eta|^2|v_\rho|^4 + |v_\eta|^4|v_\rho|^2 + |v_\rho|^4|v_\chi|^2)^{\frac{1}{2}},
\]
\[
N_3 = (|v_\eta|^4|v_\rho|^2 + |v_\eta|^2|v_\chi|^4 + |v_\rho|^4|v_\chi|^2)^{\frac{1}{2}},
\]
\[
N_4 = (|v_\eta|^4 + |v_\rho|^4 + |v_\chi|^4)^{\frac{1}{2}}.
\]
and
\[ N_3 = (|v_\chi|^2|v_\rho|^2 + |v_\chi|^2|v_\eta|^2 + |v_\rho|^2|v_\eta|^2)^{\frac{1}{2}}, \]  
(16)

and with the mass of the \( h^0 \) field given by
\[ M^2_{h^0} = A \sqrt{2} \left( \frac{1}{|v_\eta|^2} + \frac{1}{|v_\chi|^2} + \frac{1}{|v_\rho|^2} \right). \]  
(17)

In the \( CP \) even sector we have the mass matrix
\[
\begin{pmatrix}
-2a_2 |v_\eta|^2 + \frac{A}{\sqrt{2}|v_\eta|^2} & -a_6 |v_\eta||v_\rho| - \frac{A}{\sqrt{2}|v_\eta||v_\rho|} & -a_4 |v_\eta||v_\chi| - \frac{A}{\sqrt{2}|v_\eta||v_\chi|} \\
-a_6 |v_\eta||v_\rho| - \frac{A}{\sqrt{2}|v_\eta||v_\rho|} & -2a_3 |v_\rho|^2 + \frac{A}{\sqrt{2}|v_\rho|^2} & -a_5 |v_\rho||v_\chi| - \frac{A}{\sqrt{2}|v_\rho||v_\chi|} \\
-a_4 |v_\eta||v_\chi| - \frac{A}{\sqrt{2}|v_\eta||v_\chi|} & -a_5 |v_\rho||v_\chi| - \frac{A}{\sqrt{2}|v_\rho||v_\chi|} & -2a_1 |v_\chi|^2 + \frac{A}{\sqrt{2}|v_\chi|^2}
\end{pmatrix},
\]  
(18)
in the \( (X_0^\eta, X_0^\rho, X_0^\chi)^T \) basis. All those scalars are physical but we will not write the respective mass eigenstates \([12]\). It is enough to stress that \( X_a^i = O^H_{ai} H^0_i \) where \( a = \eta, \rho, \chi; \ i = 1, 2, 3 \) and \( O^H \) is an orthogonal \( 3 \times 3 \) matrix.

### III. YUKAWA INTERACTIONS

Here we will consider the most general Yukawa interaction with real coefficients which is invariant under the gauge symmetry. However, there are flavor changing neutral currents through the neutral scalars and since these fields get nonzero VEVs as in Eq. (3) it is not straightforward to see which phases can be absorbed by redefining the fermion fields or which of them, and where, survive in the lagrangian density.

In this section we will show that it is possible to choose that all Yukawa interactions which have a counterpart in the standard model are \( CP \) conserving. Hence, all \( CP \) violation effects arise from the singly and/or doubly charged scalar-fermion interactions. The vector gauge interactions with the fermions are also \( CP \) conserving including the singly and doubly charged bileptons as we will show in Sec. [IV].

#### A. Quark-scalar interactions

First, let us consider the quark-scalar interactions. The quark multiplets are the following
\[
Q'_{1L} = \begin{pmatrix} u_1' \\ d_1' \\ J_1' \end{pmatrix}_L \sim (3, \frac{2}{3}), \quad Q'_{mL} = \begin{pmatrix} d_m' \\ u_m' \\ J_m' \end{pmatrix}_L \sim (3^*, -\frac{1}{3}), \quad m = 2, 3;
\]  
(19a)

and the respective right-handed components in singlets
\[
U'_{aR} \sim (3, 1, 2/3), \quad D'_{aR} \sim (3, 1, -1/3), \quad J'_{1R} \sim (3, 1, 5/3); \quad J'_{mR} \sim (3, 1, -4/3),
\]  
(19b)

where \( a = 1, 2, 3 \).
With the quark multiplets in Eqs. (19) and the scalar ones in Eq. (1) we have the Yukawa terms
\[- \mathcal{L}_Y = \overline{Q}_1 L \sum_\alpha (G_{1a} U^\prime_{1R} \eta + \tilde{G}_{1a} D^\prime_{1R} \rho) + \sum_i \overline{Q}_i L \sum_\alpha (F_{ia} U^\prime_{1R} \rho^*)
+ \tilde{F}_{ia} D^\prime_{1R} \eta^*) + \lambda_1 \overline{Q}_1 J_1 R \chi + \sum_{i,m} \lambda_{im} \overline{Q}_i L \chi^* + H.c., (20)\]

here \(\alpha = 1, 2, 3\); \(i, m = 2, 3\) and we will assume that all coupling constants in Eq. (20) are real. In the following subsections we will analyse, case by case, all the charged sectors.

1. u-like sector–neutral scalar interactions

After the spontaneous symmetry breaking the VEVs of the neutral scalars are arbitrary complex numbers, as discussed in Secs. I. Using the shifted neutral fields, given in Eq. (3), from Eq. (20) we obtain the interactions of the quarks with the neutral scalars.

In the u-like sector we have the interactions
\[- \mathcal{L}^u_Y = \overline{u} L \sum_\alpha G_{1a} U^\prime_{1R} \eta^0 + \sum_m \overline{u} m_L \sum_\alpha F_{ia} U^\prime_{1R} \rho^0 + H.c.. (21)\]

In this section we will not use the freedom of choosing two phases equal to zero because we want to see how many phases can be absorbed in the fermion fields. Redefining the phases of the following fields \[11\]
\[
U''_{1R} \equiv e^{i\theta_\eta} U_{1R}, \quad u''_m \equiv e^{-i(\theta_\rho + \theta_\eta)} u'_m,
\]
the mass matrix of the u-like sector become real and can be diagonalized by real orthogonal matrices \(O_{L,R}\) (recall that \(\alpha = 1, 2, 3\) and \(m = 2, 3\):
\[
(O_u^L)^T \Gamma^u O_R^u = M^u = \text{diag} (m_u, m_c, m_t) \quad (23)
\]

with
\[
\Gamma^u = \frac{1}{\sqrt{2}} \begin{pmatrix}
|v_\eta| G_{11} & |v_\eta| G_{12} & |v_\eta| G_{13} \\
|v_\rho| F_{21} & |v_\rho| F_{22} & |v_\rho| F_{23} \\
|v_\rho| F_{31} & |v_\rho| F_{32} & |v_\rho| F_{33}
\end{pmatrix}
\]

Symmetry eigenstates (singly and doubly primed fields) are related to the mass eigenstates fields \(u, c, t\) as follows:
\[
\begin{pmatrix}
\overline{u}'_{1L} \\
\overline{u}'_{2L} \\
\overline{u}'_{3L}
\end{pmatrix} = O_u^L \begin{pmatrix}
u \\
c \\
t
\end{pmatrix}, \quad \begin{pmatrix}
U''_{1R} \\
U''_{2R} \\
U''_{3R}
\end{pmatrix} = O_R^u \begin{pmatrix}
u \\
c \\
t
\end{pmatrix} (25)
\]

The Yukawa interaction in Eq. (21) can be written as
\[
- \mathcal{L}^u_Y = \overline{u}_L M^u U_R + \overline{u}_L M^u U_R \frac{X_0^0 - i I_0^0}{|v_\rho|} \\
+ \overline{u}_L (O_u^L)^T \Delta O_u^L M^u U_R \left( \frac{X_0^0 + i I_0^0}{|v_\eta|} - \frac{X_0^0 - i I_0^0}{|v_\rho|} \right) + H.c., (26)
\]
with
\[ \Delta \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (27) \]

Since there is no mixing among \( X \)'s and \( I \)'s we have no \( CP \) violation in this sector.

2. \( d \)-like sector–neutral scalar interactions

Similarly, in the \( d \)-like sector, the interaction with the neutral Higgs scalars are
\[- \mathcal{L}_Y^d = \overline{d}_L \sum_{\alpha} \tilde{G}_{1\alpha} D_{\alpha R} \rho^0 + \sum_m \overline{d}_m L \sum_{\alpha} \tilde{F}_{m\alpha} D_{\alpha R}^\dagger \eta^0 + H.c.. \quad (28)\]

Making the following phase redefinition
\[ D_{\alpha R}'' \equiv e^{i\theta_{\rho}} D_{\alpha R}', \quad d_{mL}'' \equiv e^{-i(\theta_{\rho} + \theta_{\eta})} d_{mL}', \quad (29) \]
we obtain a real mass matrix which can be diagonalized with an orthogonal transformation
\[ \left( \mathcal{O}_L^d \right)^T \Gamma^d \mathcal{O}_R^d = M^d = \text{diag} (m_d, m_s, m_b) \quad (30) \]
with
\[ \Gamma^d \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} |v_{\rho}| \tilde{G}_{11} & |v_{\rho}| \tilde{G}_{12} & |v_{\rho}| \tilde{G}_{13} \\ |v_{\eta}| \tilde{F}_{21} & |v_{\eta}| \tilde{F}_{22} & |v_{\eta}| \tilde{F}_{23} \\ |v_{\eta}| \tilde{F}_{31} & |v_{\eta}| \tilde{F}_{32} & |v_{\eta}| \tilde{F}_{33} \end{pmatrix} \quad (31) \]

The symmetry eigenstates (singly and doubly primed fields) are related to the mass eigenstates (unprimed fields) as follows:
\[ \begin{pmatrix} d_{1L}' \\ d_{2L}' \\ d_{3L}' \end{pmatrix} = \mathcal{O}_L^d \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L, \quad \begin{pmatrix} D_{1R}'' \\ D_{2R}'' \\ D_{3R}'' \end{pmatrix} = \mathcal{O}_R^d \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R, \quad (32) \]

and the interactions in Eq. \( (28) \) become
\[- \mathcal{L}_Y^d = \overline{D}_L M^d D_R + \overline{D}_L M^d D_R \frac{X_0^0 - iI_0^0}{|v_\eta|} \]
\[ + \overline{D}_L \left( \mathcal{O}_L^d \right)^T \Delta \mathcal{O}_L^d M^d D_R \left( \frac{X_0^0 + iI_0^0}{|v_\rho|} - \frac{X_0^0 - iI_0^0}{|v_\eta|} \right) + H.c.. \quad (33) \]

Again, we see that if \( X \)'s and \( I \)'s do not mix among them we have not \( CP \) violation through the exchange of neutral fields. This is the case of the model with only three triplets in Sec. 1. On the other hand, we have this type of \( CP \) violation in the model with three triplets and one sextet [8].
3. Exotic quarks–neutral scalar interactions

In the sector involving exotic quarks we have

\[ - \mathcal{L}_Y = \lambda J^I_1 J^I_{1R} \chi^0 + \sum_{i,m} \lambda_{im} J^I_i J^I_{mR} \chi^{0*} + H.c. \]  

(34)

Making the redefinition of the right-handed components of the exotic quarks

\[ J''_{1R} \equiv e^{i\theta} J'_{1R}, \quad J''_{mR} \equiv e^{-i\theta} J'_{mR}, \]  

(35)

we have

\[ - \mathcal{L}_Y = \lambda J^I_1 J''_{1R} \frac{|v_\chi|}{\sqrt{2}} \left( 1 + \frac{X^0_\chi + iI^0_\chi}{|v_\chi|} \right) + \sum_{i,m} \lambda_{im} J^I_i J''_{mR} \frac{|v_\chi|}{\sqrt{2}} \left( 1 + \frac{X^0_\chi - iI^0_\chi}{|v_\chi|} \right) + H.c.. \]  

(36)

Notice that \( J'_1 \) (or \( J''_1 \)) does not mix with any other quarks but \( J'_2, 3 \) (or \( J''_2, 3 \)) mix between themselves since they have the same charge. Here we use, when necessary,

\[ \begin{align*}
J & \equiv J_1', \\
(J'_{L,R})_m & = (\mathcal{O}_L^J)_{mi}(j_{L,R}),
\end{align*} \]  

(37)

where \( J \) and \( j_n \) with \( n = 1, 2 \) denote mass eigenstates, i.e., the mass eigenstates in the exotic quark sector will be denoted when necessary \( J \) for the charge 5/3 quark and \( j_1, 2 \) for the two charge \(-4/3\) quarks. Hence Eq. (36) can be written in terms of mass eigenstates exotic fermions

\[ - \mathcal{L}_Y^I = m_J J^I_J J^I_J \left( 1 + \frac{X^0_\chi + iI^0_\chi}{|v_\chi|} \right) + \tilde{J} M^I_J \tilde{J}_R \left( 1 + \frac{X^0_\chi - iI^0_\chi}{|v_\chi|} \right) + H.c., \]  

(38)

where \( M^I = \text{diag}(m_{j_1}, m_{j_2}) \).

4. Singly charged scalar–quark interactions

The interaction lagrangian is

\[ - \mathcal{L}_{u-d}^{u-d} = \sum_{\alpha} \left( \overline{t}_{1L} G_{1a} U'_{\alpha R} \eta_1^- + \overline{u}_{1L} \tilde{G}_{1a} D'_{\alpha R} \rho^+ \right) + \sum_{i,\alpha} \left( \overline{t}_{iL} F_{i\alpha} U'_{\alpha R} \rho^- + \overline{u}_{iL} \tilde{F}_{i\alpha} D'_{\alpha R} \eta_1^+ \right) + H.c. \]  

(39)

Using the phase redefinition of Eqs. (22), (29) and (35) in Eq. (39) we have

\[ - \mathcal{L}_{u-d}^{u-d} = \frac{\sqrt{2}}{|v_\rho|} \overline{D}_L (\mathcal{O}_L^d)^T K_1 \mathcal{O}_L^u M^u U_R \rho^- + \overline{U}_L (\mathcal{O}_L^u)^T K_2 \mathcal{O}_L^d M^d D_R \frac{\sqrt{2}}{|v_\eta|} \eta_1^+ \]  

\[ + \overline{D}_L (\mathcal{O}_L^d)^T K_1 \Delta \mathcal{O}_L^u M^u U_R \left[ \frac{\sqrt{2}}{|v_\eta|} \eta_1^- - \frac{\sqrt{2}}{|v_\rho|} \rho^+ \right] \]  

\[ + \overline{U}_L (\mathcal{O}_L^u)^T K_2 \Delta \mathcal{O}_L^d M^d D_R \left[ \frac{\sqrt{2}}{|v_\rho|} \rho^- - \frac{\sqrt{2}}{|v_\eta|} \eta_1^+ \right] + H.c., \]  

(40)
being

\[ K_1 = \text{diag}(e^{-i\theta_\eta}, e^{i\theta_\rho}, e^{i\theta_\rho}), \quad \text{and} \quad K_2 = \text{diag}(e^{-i\theta_\rho}, e^{i\theta_\eta}, e^{i\theta_\eta}). \]  

(41)

Notice that if we consider the more general potential, i.e., with the \( a_{10} \) term, we have, according to the Eq. (38) a general mixing among all singly charged scalars. There is \( CP \) violation through the exchange of a singly charged scalar in this case. However, when we consider the case when \( a_{10} = 0 \) the mixing in that sector is given by Eqs. (39) and (41) and there is no \( CP \) violation in the interaction of Eq. (40) if we also chose that \( \theta_\eta = \theta_\rho = 0 \) by making a \( SU(3) \) transformation.

Finally, we have the interaction involving the exotic quarks,

\[
- \mathcal{L}_{Y}^{J^+} = \sum_{\alpha} \bar{J}_{L} G_{1\alpha} \mathcal{O}_{\alpha \beta}^{u} U_{\beta \gamma} \eta_2 e^{-i\theta_\eta} + \frac{m_{J} \sqrt{2}}{|v_\chi|} \bar{U}_{L} (\mathcal{O}_{L}^u)^T \Delta J_R e^{-i\theta_\chi} \]
\[
+ \frac{\sqrt{2}}{|v_\chi|} \bar{D}_{L} (\mathcal{O}_{L}^d)^T \bar{\Sigma} (\mathcal{O}_{R}^{J})^T M_{J} j_{R} e^{-i(\theta_\rho + \theta_\eta - \theta_\chi)} \chi^+ + H.c.,
\]  

(42)

where we have already used Eqs. (22) and (29) and defined

\[
\bar{\Sigma} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\]  

(43)

The phases of the fields \( u_{1L} \) and \( d_{1L} \) (and those of the \( J_{1L} \) and \( J_{mL} \) too) are, in principle, still free. However, we can not change the phases of these fields unless we change the phases in the Eqs. (21) and (28) (or also in Eqs. (41)). Notwithstanding, the phases in those equations have already being absorbed after the redefinition given in Eqs. (22) and (29), respectively. We can not absorb phases any more so we have \( CP \) violation through the exchange of the singly charged scalars which are coupled to the exotic and known quarks. Hence, we have shown that the only phase redefinition in the quark fields are those in Eqs. (22), (29) and (35). It means that the observable \( CP \) violating phase is \( \text{Im} v_\chi \) (or \(- \text{Im} \alpha\)). This will be also the case in the interactions with the doubly charged scalars.

5. Doubly charged scalar–quark interactions

The doubly charged Higgs scalars couple the known quarks with the exotic ones and by using Eqs. (22) and (29) we have

\[
- \mathcal{L}_{Y}^{J^-} = \sum_{\alpha} \bar{J}_{L} G_{1\alpha} \mathcal{O}_{\alpha \beta}^{D} D_{\beta \gamma} e^{-i\theta_\rho} \rho^{++} + \sum_{i,l,\alpha,\beta} \bar{J}_{Li} (\mathcal{O}_{L}^D)_{li} F_{\alpha \beta} \mathcal{O}_{\alpha \beta}^{u} U_{\beta \gamma} e^{-i\theta_\eta} \rho^{--} + H.c.,
\]  

(44a)

and

\[
- \mathcal{L}_{Y}^{J^-} = \frac{m_{J} \sqrt{2}}{|v_\chi|} \bar{D}_{L} (\mathcal{O}_{L}^D)^T \Delta J_R e^{-i\theta_\chi} \chi^{--} + \frac{\sqrt{2}}{|v_\chi|} \bar{U}_{L} \bar{\Sigma} (\mathcal{O}_{L}^{D})^T \mathcal{O}_{R}^{J} M_{J} j_{R} e^{i(\theta_\rho - \theta_\eta - \theta_\chi)} \chi^{++} + H.c.,
\]  

(44b)
From the same argument of the preceding subsections we can see that there is $CP$ violation in the doubly charged Higgs exchange too. The fields $\rho^{++}$ and $\chi^{++}$ are given in terms of the respective mass eigenstates in Eq. (6). Fermion fields are all the mass eigenstates.

B. Lepton–scalar interactions

Now, let us consider the leptonic sector. The leptons are assigned to the following representations:

$$\Psi_{aL} = \begin{pmatrix} \nu_{aL} \\ l_{aL}^- \\ E_{aL}^+ \end{pmatrix} \sim (3, 0); \quad l_{aR}^- \sim (1, -1), \quad E_{aR}^+ \sim (1, +1), \quad a = e, \mu, \tau. \quad (45)$$

The Yukawa interactions in this sector are

$$- \mathcal{L}_Y = C_{ab} \psi_{aL} l_{bR} \rho + C_{ab} \psi_{aL} E_{bR}^+ \chi + H.c.. \quad (46)$$

Next, we will consider each type of interactions as we did in the quark sectors.

1. Lepton–neutral scalar interactions

In this sector the relation between the symmetry eigenstate fields ($E_i^a, l_i^aL; \ a = e, \mu, \tau$) and the mass eigenstate fields ($l_i = e, \mu, \tau; \ E_i = E_1, E_2, E_3$) is obtained through orthogonal matrices (note that in the leptonic sector $i = 1, 2, 3$)

$$l_{aL}^i = O_{Lai} l_{L}, \quad l_{aR}^i = O_{Rai} l_{R}, \quad E_{aL}^i = O_{Lai}^E E_{L}, \quad E_{aR}^i = O_{Rai}^E E_{R}, \quad (47)$$

The respective mass matrices are defined as follows:

$$M^e = \frac{|v_\rho|}{\sqrt{2}} (O_L^e)^T G^e O_R^e, \quad M^E = \frac{|v_\chi|}{\sqrt{2}} (O_L^E)^T G^E O_R^E, \quad (48)$$

with $M^e = \text{diag}(m_e, m_\mu, m_\tau)$ and $M^E = \text{diag}(m_{E_1}, m_{E_2}, m_{E_3})$. Notice that the exotic lepton masses are proportional to the larger VEV, $v_\chi$. We see that there is no $CP$ violation in this sector too.

In terms of the physical lepton fields we have

$$- \mathcal{L}_Y = \overline{l}_{eL} M^e l_{eR} + \overline{E}_{LM^E E_R} + \overline{l}_{eL} M^e l_{eR} \frac{X_0 + i \theta^0}{|v_\rho|} + \overline{E}_{LM^E E_R} \frac{X_0 + i \theta^0}{|v_\chi|} + H.c., \quad (49)$$

and we have redefined the right-handed lepton components

$$l_{iR}'' = e^{i\theta_l} l_{iR}, \quad E_{iR}'' = e^{i\theta_\chi} E_{iR}, \quad (50)$$

but however we have omitted the double primed in Eq. (49).
Lepton–singly charged scalar interactions

The interactions involving singly charged scalars are

\[ -L_{\nu - l, \psi} Y = \sqrt{2} \frac{\overline{v}_L M^e l_R \rho^+ + \sqrt{2} \overline{v}_L K M^E E_R e^{-i(\theta_x - \theta_\rho)} \chi^- + H.c.}{|v_\rho|} \]

(51)

where we have defined

\[ \overline{v}_L = \nu_L O e L e - i \theta_\rho, \]

(52)

and \( K \equiv (O_L)^T O_L^E \). We see that even if we choose \( \theta_\rho = 0 \) the phase \( \theta_\chi \) survives and we have \( CP \) violation by the singly charged scalar exchanging.

Lepton–doubly charged scalar interactions

The interactions with the doubly charged scalars in the lepton sector are given by

\[ -L_{l - E} Y = \sqrt{2} \frac{\overline{E}_L K^T M^e l_R \rho^+ + \sqrt{2} \overline{E}_L K M^E E_R e^{-i(\theta_x - \theta_\rho)} \chi^- + H.c.}{|v_\rho|} \]

(53)

As in the previous case we have \( CP \) violation in the doubly charged scalar sector even if we choose \( \theta_\rho = 0 \).

IV. GAUGE INTERACTIONS

Next, we will verify in what conditions all phase redefinitions that have been done in the previous section do not appear in the vector boson-fermion interactions.

A. Quark–vector boson interactions

With the redefinition of the phases in Eqs. (22) and (29) the mixing matrix in the charged currents coupled to the \( W^\pm \) is real

\[ L_{Y W^\pm} = -\frac{g}{\sqrt{2}} \overline{U}_L \gamma^u V_{CKM} D_L W^+_\mu + H.c., \]

(54)

with the CKM matrix defined as \( V_{CKM} = (O_L^q)^T O_L^d \). So we have no \( CP \) violations in this sector. Similarly, we have in the charged currents coupled to the vector bileptons \( V^+ \) and \( U^- \),

\[ L_{Y V^+ - q} = -\frac{g}{\sqrt{2}} \left( J^\mu_1 L \gamma^\mu u_1 L - \sum_{i,m} \overline{d}_i L \gamma^\mu J^\mu_{mL} \right) e^{-i(\theta_\rho + \theta_\eta)} V^+_\mu + H.c., \]

(55)

and

\[ L_{Y U^- - q} = -\frac{g}{\sqrt{2}} \left( J^\mu_1 L \gamma^\mu d_1 L - \sum_{i,m} \overline{u}_i L \gamma^\mu J^\mu_{mL} \right) e^{-i(\theta_\rho + \theta_\eta)} U^-_\mu + H.c. \]

(56)

However, we always can choose \( v_\eta = v_\rho = 0 \) by using a \( SU(3) \) transformation. Hence, we will not have \( CP \) violation in the bilepton sector.
B. Lepton–vector boson interactions

The charged current interactions with the vector bosons in the leptonic sector are

\[ \mathcal{L}^{CC}_l = -\frac{g}{\sqrt{2}} \sum_a \left( \bar{\nu}_a L \gamma^\mu l a L W^\mu + \bar{\psi}_a L \gamma^\mu \nu a L V^\mu + \bar{\psi}_a L \gamma^\mu l a L U^{\mu+} + H.c. \right), \]

where all fields are still symmetry eigenstates (but we have omitted the prime). Then, the charged current interactions in terms of the physical basis, using Eq. (47), is given by

\[ \mathcal{L}^{CC}_l = -\frac{g}{\sqrt{2}} \left( \bar{\nu}_L \gamma^\mu l L W^\mu + \bar{\psi}_L \gamma^\mu K^T V^\mu + e^{i\theta_\rho} - \bar{\psi}_L \gamma^\mu K l L U^{\mu+} + H.c. \right), \]

where \( K \equiv (O^e_L)^T O^\psi_L \) and we have used the redefinition of the neutrino fields in Eq. (52). Although a phase appears we can always choose it as being zero by an \( SU(3) \) transformation.

We see from Eq. (58) that for massless neutrinos we have no mixing in the charged current coupled to \( W^\mu + \) but we still have mixing in the charged currents coupled to \( V^\mu + \) and \( U^{\mu+} \). That is, if neutrinos are massless we can always choose \( \bar{\nu}'_L \equiv \bar{\nu}_L O^e_L \). However, the charged currents coupled to \( V^\mu + \) and \( U^{\mu+} \) are not diagonal in flavor space since the mixing matrix \( K \) survives. Thus, there is not \( CP \) violation in this sector if we use the freedom to choose \( \theta_\rho = 0 \).

The pure gauge boson interactions also conserve \( CP \) if the \( SU(3)_L \) gauge and \( U(1)_N \) vector bosons \( W^a \) with \( a = 1, \ldots, 8 \), and \( B \), respectively transform as

\[ (W^1_\mu, W^2_\mu, W^3_\mu, W^4_\mu, W^5_\mu, W^6_\mu, W^7_\mu, W^8_\mu, B_\mu) \xrightarrow{CP} -(W^{1\mu}, -W^{2\mu}, W^{3\mu}, -W^{4\mu}, W^{5\mu}, -W^{6\mu}, W^{7\mu}, W^{8\mu}, B^\mu). \]

It means that the physical fields transform as

\[ (W^+_\mu, V^+_\mu, U^{++}_\mu, A_\mu, Z_\mu, Z'_\mu) \xrightarrow{CP} -(W^{-\mu}, -V^{-\mu}, -U^{-\mu}, A^\mu, Z^\mu, Z'^\mu), \]

We have shown that if all term, except the trilinear term in the scalar potential conserve \( CP \) this symmetry will be broken when the neutral scalars gain a complex VEV.

V. PHENOMENOLOGICAL CONSEQUENCES

As it is well known, the violation of the \( CP \) symmetry was discovered in 1964 in the \( K^0 - \bar{K}^0 \) system \[13\]. Up to now, it is only in this particular system in which \( CP \) violating effects has been seen. If the source of the \( CP \) violation is the weak interactions we expect also to see its effects in \( B \) decays. However, only a general discussion is presented here concerning the mesons case \[14\]. On the other hand a detailed study of the electric dipole moments of the neutron and charged leptons is shown.
A. Quark sector

1. CP violation in the neutral meson systems

In Fig. 1 we show the tree level contributions to the mass difference $\Delta M_K = 2 \text{Re} M_{12}$ (where $M_{12} = \langle K^0 | H_{eff} | \bar{K}^0 \rangle$). These diagrams exist because of the flavor changing neutral currents in Eq. (33). The $H^0$'s contributions to $\Delta M_K$ have been considered in Ref. [15].

For $m_H \sim 150 \text{ GeV}$ the constraint coming from the experimental value of $\Delta M_K$ implies $(O_{dL})_{11} (O_{dL})_{12} < 0$. There are also tree level contributions to $\Delta M_K$ coming from the $Z'$ exchange which were considered in Ref. [16]. Similarly with the mass difference of $B^0_d - \bar{B}^0_d$, $B^0_s - \bar{B}^0_s$ and $D^0 - \bar{D}^0$ systems. However, in the present model, the $CP$ violating parameters like $\epsilon_K$ have only contributions coming from box diagrams involving the one or two doubly charged and one singly charged scalars as can be seen from Fig. 2. The direct $CP$ violation parameter $\epsilon'_K$ has contributions at the 1-loop level too, as is shown in Fig. 3. However, there is no penguin contributions as we will show later.

Although we will not make here a detailed calculation of $\epsilon_K$ and $\epsilon'_K$ and the respective parameters in the $B^0_d - \bar{B}^0_d$ and $B^0_s - \bar{B}^0_s$ systems we can notice that in principle the model can give values for these $CP$ violating parameters which are in accord with data since they depend on different mixing matrices $O_{L,R}^{u,d}$. From Eqs. (11a) and (11b) we see that $\epsilon_K$ comes from diagrams like those shown in Fig. 2(a) and 2(b). (There are two other diagrams as the ones in Figs. 2 but with the lines of $Y$ and $J,j_{1,2}$ interchanged.) There are diagrams similar to those in Figs. 2 but with one of the scalar bileptons being changed by a vector bilepton and with no mass insertion. This contributions are less suppressed by the mixing angles but for the vector bilepton mass.

Similar diagrams do exist for the other $B$ mesons. The coefficient of the amplitude produced by diagrams like that in Fig. 2 are proportional to the several mixing matrix elements and Yukawa couplings. For instance, for neutral $K$ system, from Fig. 2(a), up to a $\sin(4\theta_\alpha)$ factor the amplitude is proportional to

$$G_{1\alpha} (O_{R}^{d})_{\alpha 1} (O_{L}^{d})_{21} G_{1\beta} (O_{R}^{d})_{\beta 1} (O_{L}^{d})_{11},$$

while for $B$ mesons, similar diagrams to that in Fig. 2(a) imply that the amplitudes are proportional to

$$G_{1\alpha} (O_{R}^{d})_{\alpha 1} (O_{L}^{d})_{31} G_{1\beta} (O_{R}^{d})_{\beta 1} (O_{L}^{d})_{11},$$

for the neutral $B_d$ system and,

$$G_{1\alpha} (O_{R}^{d})_{\alpha 2} (O_{L}^{d})_{31} G_{1\beta} (O_{R}^{d})_{\beta 3} (O_{L}^{d})_{21},$$

for the neutral $B_s$ system. We see from Eqs. (11) that the orthogonality condition implies that if we have chosen two of the $\epsilon_{K,B_d,B_s}$ parameters the third one is fixed. A similar analyse follows from Fig. 2(b) and the equivalent diagrams for the $B$ systems.

In the $D$ mesons case the mixing matrix is different since in these models the left-handed mixing matrix $O_L^u$ survives in different places of the lagrangian so it is not natural to set them equal to zero (see below). Hence, we see that all $CP$ phenomenology in the meson systems can be accommodated, in principle, in the present model.
Concerning the direct \( CP \) violating parameters \( \epsilon' \) its contribution come from diagrams like the one in Fig. 3. The vertices are given in Eqs. (42) and (44b). There that there are a GIM-like cancellation between the contributions of \( j_1 \) and \( j_2 \). It means that the suppression of \( \epsilon' \) does not give a strong constraint on the masses of \( j_1, j_2 \). There are similar diagrams with the \( Y^- \) substituted by a \( V^- \) vector bilepton and mass insertions in the exotic quarks lines. As in the case of the \( \epsilon \) parameters, once we have chosen the appropriate value of the mixing matrix elements (and Yukawa couplings) for explaining the observed value in the \( K^0 - \bar{K}^0 \) system, the \( \epsilon \) related to the other \( B \) or \( B_s \) systems is at least of the same order of magnitude and the third one rather small. This is as expected since this sort of model has a superweak character. More details will be given elsewhere [14].

2. Electric dipole moment

It is well known that the discovery of a non-zero electric dipole moment (EDM) for the neutron (or another elementary non-degenerate system) would be a direct evidence for both \( CP \) and \( T \) violation. The current experimental upper bound is [17]

\[
|d_n| < 1.1 \times 10^{-25} \text{ e cm.}
\]  

(62)

In the standard model the neutron electric dipole moment (EDM) arises at the three-loop level and for this reason is very small, \( \sim 10^{-34} \text{ e cm} \) [18].

In the present model we can calculate the EDM of the \( d \) and \( u \) quarks at the 1-loop level. In principle the contributions are those shown in Figs. 4 and 5. However, we can see from the interactions in Eqs. (40), (42), (44a), and (44b) with the phase convention \( \theta_{\eta} = \theta_{\rho} = 0 \) and the coefficient \( a_{10} = 0 \), that neither the diagram in Fig. 4(a) nor that in 4(b) contribute to the EDM of the quark \( d \), thus we have \( d_d = 0 \), at this level of approximation. For the quark \( u \) we have not only the diagram in Fig. 5(a) but also the one in Fig. 5(b). We obtain (recall that \( \theta_{\chi} = -\theta_{\alpha} \))

\[
d_u = \frac{e m_u^2 m_J}{32 \pi^2 (|v_{\chi}|^2 + |v_{\eta}|^2) m_{Y^+}^2} \left( O_{L11}^u \right)^2 F(m_u, m_J) \sin(2\theta_{\alpha}),
\]

(63)

where

\[
F(m_u, m_J) = -\frac{m_{Y^+}^2}{2 m_u^2} \ln \frac{m_{Y^+}^2}{m_J^2} + \frac{m_{Y^+}^2}{2 m_u^2 \Delta_u} (m_{Y^+}^2 + m_u^2 - m_J^2) \ln \left[ \frac{m_J^2 + m_{Y^+}^2 - m_u^2 + \Delta_u}{m_J^2 + m_{Y^+}^2 - m_u^2 - \Delta_u} \right],
\]

(64)

and

\[
\Delta_u^2 = (m_{Y^+}^2 + m_J^2 - m_u^2)(m_{Y^+}^2 + m_J^2 - m_u^2).
\]

(65)

The measured parameter is the EDM of the neutron which in the quark model is given in terms of the constituent quarks’s EDM:

\[
d_n = \frac{4}{3} d_d - \frac{1}{3} d_u = -\frac{1}{3} d_u \\
\simeq -1.3 \times 10^{-22} \left( O_{L11}^u \right)^2 \sin(2\theta_{\alpha}) \text{ e cm,}
\]

(66)
where we have made the approximation $|v_\eta| \approx |v_\rho|$ in order to use the doubly charged vector bilepton mass $M_U$, given by $M_U^2 = (g^2/4)(|v_\chi|^2 + |v_\rho|^2)$, instead of $|v_\chi|^2 + |v_\rho|^2$ in Eq. (13); and $G_F/\sqrt{2} = g^2/8M_W^2$. We have used $M_U = 300$ GeV, $M_{Y^+} = 100$ GeV; $m_J = 50$ GeV and $m_u = 0.002$ GeV. However, the value of $d_u$ (or $d_u$) is not sensible to the masses of the exotic particles $m_{Y^+}$, $M_U$, and $m_J$, at least with $M_U$ lesser than 10 TeV. It means that only the mixing element $(O_{L11}^u)^2$, for any value of $\sin(2\theta_\alpha)$, have to be invoked in order to obtain an EDM of the neutron compatible with the data in Eq. (62). This is not in conflict with the CKM mixing matrix in the charged current coupled to the vector boson $W^\pm$ since the later is defined as $V_{\text{CKM}} = (O_L^u)^T O_L^d$. Since in Eq. (10) only the matrix element $O_{L11}^u$ related to the $u$-like quarks appears and, with the phase convention used here, the mixing matrices related to the $d$-like quarks do not enter at all in this sort of models and we cannot use $V_{\text{CKM}} = O_L^d$ as is usually done in the standard electroweak model.

In fact, the matrices $O_{L,d}^u$ will appear in the neutral currents coupled to the extra neutral vector boson $Z^0$. We have

$$
\mathcal{L}_{Z'} = -\frac{g}{2c_W} \left[ \bar{U}_L \gamma^\mu O_L^u Y_L^u(Z') O_L^u U_L + \bar{U}_R \gamma^\mu O_R^u Y_R^u(Z') O_R^u U_R 
+ \bar{D}_L \gamma^\mu O_L^d Y_L^d(Z') O_L^d D_L + \bar{D}_R \gamma^\mu O_R^d Y_R^d(Z') O_R^d D_R \right] Z'_\mu, \tag{67}
$$

with the matrices

$$
Y_L^u(Z') = Y_R^d(Z') = -\frac{1}{\sqrt{3h(s_W)}} \begin{pmatrix}
1 & 0 & 0 \\
0 & 2s_W - 1 & 0 \\
0 & 0 & 2s_W - 1
\end{pmatrix}, \tag{68}
$$

and

$$
Y_L^u(Z') = -\frac{4s_W}{\sqrt{3h(s_W)}} \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad Y_R^d(Z') = \frac{2s_W}{\sqrt{3h(s_W)}} \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}, \tag{69}
$$

where $h(s_W) = (1 - 4s_W^2)^{1/2}$. Notice that the right-handed neutral currents remain diagonal, but it is not so for the left-handed ones. Here the matrices $O_{L,d}^u$ and $O_{L,d}^d$ survive in a different form than they appear in the $V_{\text{CKM}}$ matrix. These matrices $O_{L,d}^u$, $O_{L,d}^d$ and $V_{\text{CKM}}$ have to be only determined from experiment since we cannot set none of them to be diagonal in a natural way.

### B. Lepton sector

Beside the neutron EDM, another ones which are experimentally well studied ( alas not definitively yet ) is the EDM of the electron and the muon. It is interesting that the EDM of the electron $(d_e)$ in the standard model is rather small, of the order of magnitude of $2 \times 10^{-38}$ e cm [13]. However, the value of $d_e$ could be even smaller, of the order of $10^{-41}$ e cm if there is a cancellation of the three-loop diagrams [20] even if QCD corrections are included [21]. On the other hand, the experimental upper limit is $\leq 4 \times 10^{-27}$ e cm [22]. Hence, a measurement of a nonzero large electron EDM ( and other elementary particles like muon, tau and neutron, see below ) would indicate new physics beyond the standard model.
Although we have $CP$ violation in the singly charged scalar sector, there is no contribution to the EDM of the leptons at the 1-loop level since neutrinos are massless. If neutrinos remain massless the contribution to electric dipole moment will arise at the three-loop level \[23\].

This is not the case for diagrams involving doubly charged scalars and the known leptons and the exotic ones (see Fig. 6). It is always possible to choose the doubly charged scalars those fields which carry this phase $i.e.$, the $CP$ violation in the leptonic sector occurs only through the exchange of both exotic leptons and of doubly charged scalar fields.

The Yukawa couplings of leptons with the doubly charged scalars are given in Eq. (53) where the scalar fields are still symmetry eigenstates. In fact, there is one Goldstone boson $G^{++}$ and a physical one $Y^{++}$; we denote its mass by $m_{Y^{++}}$. (In this model there is not lepton number violation in the interactions with the neutral scalars.) As we have shown in Secs. III B and IV B it is not possible to absorb all phases in the complete lepton lagrangian density. Since neutrinos are considered massless here, the only contributions to the leptonic EDM arise from the doubly charged scalars as shown in Fig. 6. From this we obtain

$$d_l = -\frac{e m_l}{32 \pi^2 m_{Y^{++}}^2} \sqrt{2} M_W^2 G_F O_l \sin(2 \theta_\alpha), \quad l = e, \mu, \tau;$$

where we have defined

$$O_l = \sum_j \mathcal{K}^2_{l,j} \frac{4 m_{E_j}^2}{M_U^2} [F_+(m_l, m_{E_j}) + F_-(m_l, m_{E_j})],$$

the matrix $\mathcal{K}$ has been introduced in Eqs. (52) and (53):

$$F_\pm(m_l, m_{E_j}) = \frac{m_{Y^{++}}^2}{2 m_l^2} \ln \frac{m_{Y^{++}}^2}{m_{E_j}^2} + \frac{m_{Y^{++}}^2}{2 m_l^2} \Delta_l \left( m_{Y^{++}}^2 \pm m_l^2 - m_{E_j}^2 \right) \ln \left[ \frac{m_{Y^{++}}^2 - m_{E_j}^2}{m_{Y^{++}}^2 - m_l^2} \right],$$

and

$$\Delta_l^2 = (m_{Y^{++}}^2 + m_{E_i}^2 - m_l^2)(m_{Y^{++}}^2 - m_{E_i}^2 - m_l^2).$$

For nondegenerate heavy leptons the mixing angles remain in Eq. (71). For instance, the contribution of $E_1$ to the electron EDM, using $m_{E_1} = 50$ GeV and $m_{Y^{++}} = 100$ GeV \[17\] is given by

$$d_e \approx 10^{-17} \left( \frac{M_W^2}{M_U^2} \right) \mathcal{K}^2_{e1} \sin(2 \theta_\alpha) \, e \, \text{cm.}$$

Assuming $M_U = 300$ GeV and the factor with the mixing angles $\mathcal{K}^2_{e1} \approx 10^{-8}$ we obtain $d_e \approx 10^{-27} \, e \, \text{cm}$ for any value of $\theta_\alpha$, which is compatible with the experimental upper limit of $10^{-27} \, e \, \text{cm} \ [22]$. For the muon we have an experimental EDM’s upper limits of $< 10^{-19} \, e \, \text{cm} \ [24]$. It means a constraint in $\mathcal{K}^2_{\mu2} \leq 1$. For the tau lepton a limit of $10^{-17} \, e \, \text{cm}$ is derived from $\Gamma(Z \rightarrow \tau^+ \tau^-) \ [25,26]$. In the present model it is at least as large as $10^{-19} \, e \, \text{cm}$. In particular, notice that from Eq. (70)
Using $F_+(m_l, m_{E_j}) = F_-(m_l, m_{E_j}) \equiv F(m_l, m_{E_j}) \approx -(m_X^2/2m_l^2)\ln(m_X^2/m_{E_j}^2)$ we obtain from Eq. (75)

\[
\frac{d_\mu}{d_e} = \frac{m_\mu \sum_j K_{\mu j}^2 m_{E_j}^2 F(m_\mu, m_{E_j})}{m_e \sum_j K_{e j}^2 m_{E_j}^2 F(m_e, m_{E_j})}.
\]

(75)

Notice that if

\[
\frac{\sum_j K_{\mu j}^2 m_{E_j}^2 \ln(m_X^2/m_{E_j}^2)}{\sum_j K_{e j}^2 m_{E_j}^2 \ln(m_X^2/m_{E_j}^2)} \gg \frac{m_\mu}{m_e}.
\]

(77)

is satisfied, we can have $d_\mu \gg d_e$. For instance, assuming that $E_1(E_2)$ dominates the EDM of the electron (muon) this condition implies that (neglecting the logarithmic in both the numerator and denominator)

\[
\left| \frac{K_{\mu 2} m_{E_2}}{K_{e 1} m_{E_1}} \right| \approx 10^4 \sqrt{\frac{m_\mu}{m_e}} \sim 10^5,
\]

(78)

this value of the ratio $|K_{\mu 2} m_{E_2}/K_{e 1} m_{E_1}|$ is easily obtained for the $K_{e 1}$ and $K_{\mu 2}$ given above if $m_{E_2} \approx 10m_{E_1}$. We see that in fact, the EDM of the muon can be larger than the EDM of the electron. A similar situation happens with the tau lepton if $E_3$ dominates here,

\[
\left| \frac{K_{\tau 3} m_{E_3}}{K_{e 1} m_{E_1}} \right| \approx 10^5 \sqrt{\frac{m_\tau}{m_e}} \sim 6 \times 10^6.
\]

(79)

If Eqs. (78) and (79) are satisfied, the muon and the lepton tau can have an EDM which is as large as their respective present experimental limit. For example, analyzing the process $e^+e^- \rightarrow \tau\tau\gamma$, the L3 collaboration has obtained the value $d_\tau = (0.0 \pm 1.5 \pm 1.3) \times 10^{-16}$ cm \[27\]. The conditions in Eqs. (78) and (79) are equivalent to the assumption that the mixing matrix $K \equiv (O_L^e)^T O_L^E$ is almost completely diagonal. If the tau lepton has in fact a large EDM it will induce anomalous couplings of the $Z$ boson to fermions which could be be seen in tau-charm factories \[28\].

In this model there is no rare decays such as $\mu \rightarrow 3e$ at tree level. However, the same loop diagrams that contribute for the EDM of the leptons imply transition magnetic and electric moments, like $\mu \rightarrow e\gamma$. However, it will constrain only the matrix elements $\left[(O_L^e)^T O_L^E\right]_{\mu 1}$. 

VI. CONCLUSIONS

The model of $CP$ violation of this work seems like an admixture of both spontaneous breaking as in Lee’s two doublets model \[3\] and the charged-Higgs-boson exchange of Weinberg’s three doublets one \[4\]. However, some differences are important to be pointed out. In a pure charged-scalar-exchange where the phases are in the coupling constants of the scalar potential it was shown that there is also $CP$ violation through the exchange of neutral Higgs
bosons\textsuperscript{[29]}. This is correct in models with $SU(2)_L \otimes U(1)_Y$ symmetry with several scalar doublets, that is, with all of them having the same quantum number. For this reason a more general mixing among the scalar fields of the same charge is possible. In the present model all triplets have different $U(1)_N$ charge so this constraint their interaction terms. Hence, there is not mixing among the real and imaginary part of the neutral scalar fields even with three scalar triplets as can be seen from Eqs. (13) and (18). If we had considered $a_{10} \neq 0$ in the scalar potential in Eq. (3) we would have $CP$ violation in the propagator of single charged scalar like in the Weinberg model but since the $a_{10}$ term in the scalar potential does not contribute to the mass matrix of the neutral Higgs there is not mixing among $CP$ even and $CP$ odd neutral scalars.

There are other interesting features of this model: i) Notice that since we have no $CP$ violation in the neutral scalar sector the Weinberg’s three-gluon operators involving neutral Higgs boson exchanges do not contribute to the EDM of the neutron at all\textsuperscript{[30]}. There is also no contribution to the EDM of the electron and neutron coming from the diagrams of Ref.\textsuperscript{[31]} which involve $CP$ violation in the propagator of the neutral Higgs bosons and it does not depend on the phase convention since in Eqs. (26) and (33) phases do not appear at all; ii) In this model radiative corrections up to 2-loop level will not induce phases in the CKM matrix. These type of diagrams are the same which would contribute to the penguin diagram, say of the $\epsilon'_{K}$ parameter, hence there is no penguin contributions to the $CP$ violating parameters at least at the 2-loop level. Hence, it means that $arg\det M$ vanish up to this loop order. Furthermore, if we assume that $CP$ is also conserved in the pure QCD part then $\theta = 0$ at the tree level and $\bar{\theta}$ will be finite and calculable. Of course, this is not such a natural solution to the $\theta$-vacua problem but at least it is in the same foot than the assumption that all Yukawa couplings are real at the tree level.

This model of $CP$ violation seems like a particular realization of the soft superweak model proposed recently by Georgi and Glashow\textsuperscript{[32]}. In that model the violation of $CP$ is due to the coupling of the left-handed doublet to the heavy sector in the mass eigenstate basis involve a complex matrix. In our model, the heavy sector corresponds to the exotic quarks $J,j_n$, the exotic leptons $E_j$ and the scalars $Y^+$ and $Y^{++}$; the only complex numbers are the phase $\theta_\alpha$ appearing in the trilinear term in the scalar potential and the complex VEVs (only $\theta_\chi$ after using the $SU(3)$ freedom to eliminate two phases).

With three triplets and one sextet which are needed in the model of Ref.\textsuperscript{[33]} it is possible to have truly spontaneous violation of the $CP$ symmetry\textsuperscript{[3]}. In this case, the minimization condition of the scalar potential implies more complicated constraint equations on the imaginary part of the neutral scalars so that two of the phases of the VEV survive in the lagrangian density. The phenomenology of this model has been studied in Ref.\textsuperscript{[15]}.

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Figure Captions

Fig. 1 Tree level contributions to $\Delta M_K$.

Fig. 2 Some of the box diagram contributions to $\varepsilon$ and $M_{12}$.

Fig. 3 The box diagram contribution to $\varepsilon'$.

Fig. 4 Possible diagram contributing to the EDM of the quarks $d$.

Fig. 5 Possible diagram contributing to the EDM of the quarks $u$.

Fig. 6 Diagrams contributing to the EDM of the charged leptons.
Figure 1
Figure 2
Figure 2
\begin{figure}[h]
\centering
\begin{tikzpicture}
\node (d) at (0,0) {$d_L$};
\node (j1R) at (2,0) {$j_{1R}(j_{2R})$};
\node (uL) at (4,0) {$u_L$};
\node (y-1) at (1,-1.5) {$Y^-$};
\node (y++) at (3,-1.5) {$Y^{++}$};
\node (sR) at (0,-3) {$\bar{s}_R$};
\node (j1L) at (2,-3) {$\bar{j}_{1L}(\bar{j}_{2L})$};
\node (uR) at (4,-3) {$\bar{u}_R$};
\draw[->] (d) -- (j1R);
\draw[->] (j1R) -- (uL);
\draw[->] (sR) -- (j1L);
\draw[->] (j1L) -- (uR);
\draw[dashed] (y-) -- (j1R);
\draw[dashed] (j1R) -- (y++);
\draw[dashed] (sR) -- (j1L);
\draw[dashed] (j1L) -- (uR);
\end{tikzpicture}
\caption{Figure 3}
\end{figure}
Figure 4
Figure 4
Figure 5
Figure 5
Figure 6
Figure 6