Five models for lepton mixing

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Abstract
We produce five flavour models for the lepton sector. All five models fit perfectly well—at the 1σ level—the existing data on the neutrino mass-squared differences and on the lepton mixing angles. The models are based on the type I seesaw mechanism, on a \(Z_2\) symmetry for each lepton flavour, and either on a (spontaneously broken) symmetry under the interchange of two lepton flavours or on a (spontaneously broken) \(CP\) symmetry incorporating that interchange—or on both symmetries simultaneously. Each model makes definite predictions both for the scale of the neutrino masses and for the phase \(\delta\) in lepton mixing; the fifth model also predicts a correlation between the lepton mixing angles \(\theta_{12}\) and \(\theta_{23}\).

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1 Introduction

The problem of explaining the observed features of lepton mixing became significantly more awkward with the recent definitive (more than 5\(\sigma\)) indications for a non-zero mixing angle \(\theta_{13}\) [1]. These indications have rendered obsolete the idea of a \(\mu-\tau\) interchange symmetry in the neutrino mass matrix [2]. Moreover, a not so well-established (2\(\sigma\)) indication that the mixing angle \(\theta_{23}\) is not maximal\(^1\) suggests that the idea of a \(CP\) symmetry incorporating the \(\mu-\tau\) interchange [6] should be discarded too.

At this juncture, one may either search for other ideas or try and somehow modify the approaches mentioned in the previous paragraph in order to make them compatible with the data. In this paper we take the latter path. Specifically, while in the models of refs. [2] [7] the neutrino Dirac mass matrix \(M_D\) was of the form diag \((a, b, b)\) and in the model of ref. [6] it was diag \((a, b, b^*)\), in this paper we present models where \(M_D = \text{diag} (a, b, c)\). We shall see that this still allows us to retain some predictive power. In order to enhance that power, we also apply in this paper an idea originally proposed in ref. [7], which allows us to obtain one vanishing off-diagonal matrix element in the inverse of the light-neutrino Majorana mass matrix \(M\) (in the basis where the charged-lepton mass matrix is diagonal).

In general, the matrix \(M\), which is symmetric, contains nine physical parameters: the six moduli of its matrix elements and three rephasing-invariant phases. Those nine parameters correspond to nine observables: three neutrino masses, three mixing angles, one Dirac phase (\(\delta\)) in the mixing, and two Majorana phases among the three neutrino masses. The first four models that we present in this paper have only five physical parameters and are thus comparable, in predictive power, to models or \textit{Ansätze} with two ‘texture zeros’ [8] [9]. Those models are able to predict \(\delta\) and the absolute scale of the neutrino masses.\(^2\) Moreover, our models provide excellent fits to the existing data on the three mixing angles and on the two neutrino mass-squared differences. Our fifth model has only four physical parameters and is able to predict one correlation between two mixing angles.

All the models that we propose in this paper utilize the type I seesaw mechanism with three right-handed neutrinos. Thus, the leptonic multiplets in our models are the following:

- Three right-handed charged-lepton gauge-\(SU(2)\) singlets \(\alpha_R\), where \(\alpha\)

\(^1\)In this paper we use exclusively the global fit of the neutrino-oscillation data by Fogli \textit{et al.} [3]. Other fits are contained in refs. [4] [5].

\(^2\)We omit predictions for the two Majorana phases, since they are, in general, experimentally irrelevant.
may take the values $e$, $\mu$, and $\tau$;

- three right-handed neutrino singlets $\nu_{\alpha R}$;
- three left-handed doublets $D_{\alpha L} = (\nu_{\alpha L}, \alpha_L)^T$.

Our models also have a very simple scalar sector:

- Two Higgs doublets $\phi_{1,2} = (\phi_{1,2}^+, \phi_{1,2}^0)^T$. Their conjugate doublets are $\tilde{\phi}_{1,2} = (\phi_{1,2}^0, -\phi_{1,2}^-)^T$;
- two real gauge singlets, $\chi_1$ and $\chi_2$.

The outline of this paper is as follows. In section 2 we introduce a matrix $A$, derived from the matrix $M$ but which does not suffer from the rephasing ambiguity of the latter; it turns out that most predictions of our models can be stated in terms of $A$-matrix elements. In section 3 we present our first model, which uses a $\mu$–$\tau$ interchange symmetry. In section 4 we present our second model, which uses a $CP$ symmetry incorporating the $\mu$–$\tau$ interchange. In section 5 we note that in both previous models one may use an $e$–$\tau$ interchange instead of the $\mu$–$\tau$ interchange and still obtain a perfect fit to the data; we moreover note that, by combining the $e$–$\tau$ interchange symmetry with the corresponding generalized $CP$ symmetry, one obtains a fifth model that is still compatible with the data. In section 6 we elaborate, by means of a fit to the phenomenological data and of scatter plots, the predictions of our models. We briefly summarize our achievements in section 7.

2 The matrix $A$

Let

$$M = \begin{pmatrix} a & f & e \\ f & b & d \\ e & d & c \end{pmatrix}$$

be a symmetric $3 \times 3$ matrix. We assume $M$ to be invertible. Then,

$$M^{-1} = \frac{1}{\det M} \begin{pmatrix} bc - d^2 & de - cf & df - be \\ de - cf & ac - e^2 & ef - ad \\ df - be & ef - ad & ab - f^2 \end{pmatrix}.$$ 

(2)

We define the matrix $A$, which is a symmetric $3 \times 3$ matrix too, through

$$A_{\alpha\beta} = M_{\alpha\beta} (M^{-1})_{\alpha\beta}.$$ 

(3)
where we do not imply summation over $\alpha$ and $\beta$.

Then,

$$A = \frac{1}{\det M} \begin{pmatrix} abc & ad^2 & df - cf^2 & df - be^2 \\ df - cf^2 & abc & be^2 & df - ad^2 \\ df - be^2 & df - ad^2 & abc & cf^2 \\ df - be^2 & df - ad^2 & abc & cf^2 \end{pmatrix}. \tag{4}$$

An advantage of the matrix $A$ over the matrix $M$ is that, when $M$ gets rephased through

$$M_{\alpha\beta} \rightarrow e^{i(\psi_{\alpha}+\psi_{\beta})} M_{\alpha\beta}, \tag{5}$$

the matrix $A$ remains invariant.

Since

$$\det M = abc + 2def - ad^2 - be^2 - cf^2, \tag{6}$$

the matrix $A$ in eq. (4) satisfies

$$\sum_{\alpha=1}^{3} A_{\alpha\beta} = 1 \tag{7}$$

for any value of $\beta$. It follows from eq. (7) that only three elements of $A$ are linearly independent. Therefore $A$ has only six parameters: three moduli and three phases.

Equation (7) has several consequences. Let $\alpha \neq \beta \neq \gamma \neq \alpha$. Then,

$$A_{\gamma\alpha} = A_{\gamma\beta} \Leftrightarrow A_{\alpha\alpha} = A_{\beta\beta}, \tag{8}$$

which holds because $A_{\alpha\alpha} = 1 - A_{\beta\alpha} - A_{\gamma\alpha}$, $A_{\beta\beta} = 1 - A_{\alpha\beta} - A_{\gamma\beta}$, and $A$ is symmetric. Another consequence is

$$A_{\gamma\alpha} = A_{\gamma\beta}^* \Rightarrow A_{\gamma\gamma} = A_{\gamma\gamma}^*, \tag{9}$$

which holds because $A_{\gamma\gamma} = 1 - A_{\alpha\gamma} - A_{\beta\gamma}$. Still another consequence of eq. (7) is

$$A_{\gamma\alpha} = A_{\gamma\beta}^* \land A_{\alpha\alpha} = A_{\beta\beta}^* \Rightarrow A_{\alpha\beta} = A_{\alpha\beta}^*. \tag{10}$$

Finally, one further consequence of eq. (7) is

$$A_{\gamma\alpha} = A_{\gamma\beta}^* \land A_{\alpha\beta} = A_{\alpha\beta}^* \Rightarrow A_{\alpha\alpha} = A_{\beta\beta}^*. \tag{11}$$

3In this section, Greek-letter indices have range $\{1, 2, 3\}$. In the other sections of this paper, $M$ is interpreted as the neutrino mass matrix and those indices indicate the lepton flavours $e$, $\mu$ and $\tau$.

4The original matrix $M$, after allowing for the rephasing, has nine parameters: six moduli and three phases. So, $A$ contains less information than $M$. 

4
3 Model 1

Our first model has four $Z_2$ symmetries. The first three of them are

- $Z_2^{(e)}$: $D_{eL}$, $e_R$, $\nu_{eR}$, $\chi_1$, $\chi_2$ change sign;
- $Z_2^{(\mu)}$: $D_{\mu L}$, $\mu_R$, $\nu_{\mu R}$, $\chi_1$ change sign;
- $Z_2^{(\tau)}$: $D_{\tau L}$, $\tau_R$, $\nu_{\tau R}$, $\chi_2$ change sign.

These three symmetries are broken spontaneously at the high (seesaw) scale when $\chi_{1,2}$ acquire vacuum expectation values (VEVs)

$$\langle 0 | \chi_1 | 0 \rangle \equiv U \cos \vartheta, \quad \langle 0 | \chi_2 | 0 \rangle \equiv U \sin \vartheta.$$  \hspace{1cm} (12)

The fourth $Z_2$ symmetry of model 1 is

$$Z_2^{(int)}: D_{\mu L} \leftrightarrow D_{\tau L}, \quad \mu_R \leftrightarrow \tau_R, \quad \nu_{\mu R} \leftrightarrow \nu_{\tau R}, \quad \chi_1 \leftrightarrow \chi_2, \quad \phi_2 \rightarrow -\phi_2.$$ \hspace{1cm} (13)

This symmetry is broken spontaneously at the low (Fermi or lower\footnote{Our Higgs doublets $\phi_{1,2}$ are not necessarily the ones which couple to the quarks. Their VEVs may be much lower (and their masses much higher) than the Fermi scale, suppressed for instance through a type II seesaw mechanism\cite{10}. Alternatively, our model 1 may be viewed as an ordinary two-Higgs-doublet model furnished with a $Z_2$ symmetry under which $\phi_2 \rightarrow -\phi_2$; that symmetry may apply to the quark sector in a variety of ways\cite{11}, for instance by inverting the signs of the right-handed down-type quarks. It has been shown\cite{12} that these models are quite capable of describing the phenomenology and experimental constraints that arise from the recent observations of a Higgs-like particle\cite{13}.}) scale when $\phi_2^0$ acquires a VEV.

The Lagrangian of Majorana mass terms is then of the form

$$L_{\text{Maj}} = -\frac{1}{2} \left[ m_{\bar{\nu}_{eR}} C \bar{\nu}_{eR}^T + m' \left( \bar{\nu}_{\mu R} C \bar{\nu}_{\mu R}^T + \bar{\nu}_{\tau R} C \bar{\nu}_{\tau R}^T \right) \right] + \text{H.c.}$$ \hspace{1cm} (14)

The Yukawa Lagrangian is

$$L_{\text{Yuk}} = -y_1 \bar{D}_{eL} e R \phi_1$$
$$-y_2 \left( D_{\mu L} \mu R + \bar{D}_{\tau L} \tau R \right) \phi_1 - y_3 \left( \bar{D}_{\mu L} \mu R - \bar{D}_{\tau L} \tau R \right) \phi_2$$
$$-y_4 \bar{D}_{eL} \nu_{eR} \tilde{\phi}_1$$
$$-y_5 \left( \bar{D}_{\mu L} \nu_{\mu R} + \bar{D}_{\tau L} \nu_{\tau R} \right) \tilde{\phi}_1 - y_6 \left( \bar{D}_{\mu L} \nu_{\mu R} - \bar{D}_{\tau L} \nu_{\tau R} \right) \tilde{\phi}_2$$
$$-y_7 \left( \chi_1 \bar{\nu}_{\mu R} + \chi_2 \bar{\nu}_{\tau R} \right) C \bar{\nu}_{eR}^T + \text{H.c.},$$ \hspace{1cm} (15)

where $C$ is the charge-conjugation matrix in Dirac space. So the charged-lepton mass matrix is diagonal,

$$M_\ell = \text{diag} \left( y_1 v_1, y_2 v_1 + y_3 v_2, y_2 v_1 - y_3 v_2 \right).$$ \hspace{1cm} (16)
where \( v_k = \langle 0 | \phi_k^0 | 0 \rangle \) for \( k = 1, 2 \). The neutrino Dirac mass matrix is diagonal too:

\[
M_D = \text{diag} (a, b, c),
\]

where \( a = y_4^* v_1 \), \( b = y_5^* v_1 + y_6^* v_2 \), and \( c = y_5^* v_1 - y_6^* v_2 \). The Majorana mass matrix of the right-handed neutrinos is given by

\[
M_R = \begin{pmatrix}
m & y_7 U \cos \vartheta & y_7 U \sin \vartheta \\
y_7 U \cos \vartheta & m' & 0 \\
y_7 U \sin \vartheta & 0 & m'
\end{pmatrix}.
\]

The expression for the effective light-neutrino Majorana mass matrix is

\[
M = -M_D^T M_R^{-1} M_D. \tag{19}
\]

Since \( M_D \) is diagonal and \( (M_R)_{\mu \tau} = 0 \), it immediately follows that

\[
(M^{-1})_{\mu \tau} = 0. \tag{20}
\]

Taking into account the imposed symmetries, one finds the scalar potential

\[
V = \sum_{k=1}^{2} \left[ \mu_k \phi_k^+ \phi_k + \lambda_k \left( \phi_k^+ \phi_k \right)^2 \right] + \lambda_3 \phi_1^+ \phi_1 \phi_2^+ \phi_2 + \lambda_4 \phi_1^+ \phi_2 \phi_2^+ \phi_1 + \left[ \lambda_5 \left( \phi_1^+ \phi_2 \right)^2 + \text{H.c.} \right] + \left( \mu_\chi + \sum_{k=1}^{2} \bar{\lambda}_k \phi_k^+ \phi_k \right) \left( \chi_1^2 + \chi_2^2 \right) + \lambda_\chi \left( \chi_1^2 + \chi_2^2 \right)^2 + \lambda_\chi' \chi_1^2 \chi_2^2 + \left( \lambda \phi_1^+ \phi_2 + \text{H.c.} \right) \left( \chi_1^2 - \chi_2^2 \right). \tag{21}
\]

We assume that \( U \) is at the high scale, while \( v = (|v_1|^2 + |v_2|^2)^{1/2} \) lies at the low scale\(^6\). The vacuum potential for \( U \) and \( \vartheta \) is

\[
V_0 = \left( \mu_\chi + \sum_{k=1}^{2} \bar{\lambda}_k |v_k|^2 \right) U^2 + \lambda_\chi U^4 + \frac{\lambda_\chi'}{4} U^4 \sin^2 2\vartheta + 2 \Re (\lambda v_1^* v_2) U^2 \cos 2\vartheta. \tag{22}
\]

\(^6\)In section \( ^2 \) \( M \) was a generic \( 3 \times 3 \) symmetric, non-singular matrix. From now on, \( M \) will specifically denote the light-neutrino Majorana mass matrix in the basis where the charged-lepton mass matrix is diagonal.

\(^7\)This of course requires some fine-tuning in \( V \). Namely, \( \bar{\lambda}_{1,2} \) must be very small, of order \( v^2/U^2 \), in order that \( \mu_1 \) and \( \mu_2 \) do not receive corrections of seesaw magnitude when \( \chi_{1,2} \) acquire VEVs. This is an ordinary ‘hierarchy problem’, to which we cannot offer a solution.
Unless $\lambda$ is very large, of order $U^2/v^2$, the last term in the right-hand side of eq. (22) is much smaller than the term just before it and may be neglected. Then $\vartheta$ will be either $n\pi/2$ or $(2n + 1)\pi/4$ (with integer $n$) depending on whether $\lambda'_\chi$ is, respectively, positive or negative. We assume that $\lambda'_\chi$ is negative and that $\vartheta = \pi/4$, i.e. that $\langle 0 | \chi_1 | 0 \rangle = \langle 0 | \chi_2 | 0 \rangle$. This equality holds to $O(v^2/U^2)$ when $\lambda$ and $\lambda'_\chi$ are of the same order.

Thus,

$$M^{-1} = -\text{diag} \left( \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \right) \begin{pmatrix} m & m'' & m'' \\ m'' & m' & 0 \\ m'' & 0 & m' \end{pmatrix} \text{diag} \left( \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \right),$$

where $m'' = y_t U / \sqrt{2}$. Hence,

$$M = -\frac{1}{m' \left( mm' - 2m'^2 \right)} \text{diag} (a, b, c) 
\times \begin{pmatrix} m'^2 & -m'm'' & -m'm'' \\ -m'm'' & mm' - m''^2 & m'^2 \\ -m'm'' & m'^2 & mm' - m''^2 \end{pmatrix} \text{diag} (a, b, c).$$

It follows from eqs. (23) and (24) that

$$A_{\mu\mu} = A_{e\tau}.$$

We conclude that model 1 leads to two complex conditions on $M$, eqs. (20) and (25). Therefore, the matrix $M$ in this model contains five physical parameters, as opposed to nine physical parameters in the general case.

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8This is the same approximation as in eq. (19), which neglects terms of order $v^2/U^2$ too.

9$A_{\mu\mu} = A_{e\tau}$ also follows from eqs. (23) and (24) but is equivalent to eq. (25), cf. eq. (8).
4 Model 2

Our second model also has the symmetries $\mathbb{Z}_2^{(e)}$, $\mathbb{Z}_2^{(\mu)}$, and $\mathbb{Z}_2^{(\tau)}$. However, now $\mathbb{Z}_2^{(\text{int})}$ is substituted by the $CP$ symmetry

\[
\begin{align*}
D_{eL}(x) &\to \gamma_0 C D_{eL}^T(x) \\
D_{\mu L}(x) &\to \gamma_0 C D_{\mu L}^T(x) \\
D_{\tau L}(x) &\to \gamma_0 C D_{\tau L}^T(x) \\
\nu_{e R}(x) &\to \gamma_0 C \bar{\nu}_{e R}^T(x) \\
\nu_{\mu R}(x) &\to \gamma_0 C \bar{\nu}_{\mu R}^T(x) \\
\nu_{\tau R}(x) &\to \gamma_0 C \bar{\nu}_{\tau R}^T(x) \\
\mu_R(x) &\to \gamma_0 C \bar{\mu}_R^T(x) \\
\tau_R(x) &\to \gamma_0 C \bar{\tau}_R^T(x)
\end{align*}
\]

where $x = (t, \bar{r})$ and $\bar{x} = (t, -\bar{r})$. Notice that, since $\chi_1$ and $\chi_2$ are gauge-singlet fields, they have no electroweak interactions, hence $CP$ may be arbitrarily defined for them; we have opted for $\chi_1^{CP} \chi_2$. Also notice that the Higgs doublets $\phi_1$ and $\phi_2$ transform with opposite signs under $CP$.

We now have

\[
\mathcal{L}_{\text{Maj}} = -\frac{1}{2} [m \bar{\nu}_{e R} C \bar{\nu}_{e R}^T + m' \bar{\nu}_{\mu R} C \bar{\nu}_{\mu R}^T + m'' \bar{\nu}_{\tau R} C \bar{\nu}_{\tau R}^T] + \text{H.c.},
\]

where $m$ is real, and

\[
\mathcal{L}_{\text{Yuk}} = -y_1 \bar{D}_{e L} e_R \phi_1 - (y_2 \bar{D}_{\mu L} \mu_R + y_2^* \bar{D}_{\tau L} \tau_R) \phi_1 - (y_3 \bar{D}_{\mu L} \mu_R - y_3^* \bar{D}_{\tau L} \tau_R) \phi_2 - y_4 \bar{D}_{e L} \nu_R \phi_1 - (y_5 \bar{D}_{\mu L} \nu_R + y_5^* \bar{D}_{\tau L} \nu_R) \phi_1 - (y_6 \bar{D}_{\mu L} \nu_R - y_6^* \bar{D}_{\tau L} \nu_R) \phi_2 - (y_7 \bar{\chi}_1 \nu_R + y_7^* \bar{\chi}_2 \nu_R) C \bar{\nu}_{e R}^T + \text{H.c.},
\]

where $y_1$ and $y_4$ are real. So the charged-lepton mass matrix is once again diagonal,

\[
M_e = \text{diag} (y_1 v_1, y_2 v_1 + y_3 v_2, y_2^* v_1 - y_3^* v_2),
\]

just as the neutrino Dirac mass matrix, which is as in eq. (17) with $a = y_1^* v_1$, $b = y_2^* v_1 + y_3^* v_2$, and $c = y_5 v_1 - y_6 v_2$. The Majorana mass matrix of the right-handed neutrinos is given by

\[
M_R = \begin{pmatrix}
m & y_7 U \cos \vartheta & y_7^* U \sin \vartheta \\
y_7 U \cos \vartheta & m' & 0 \\
y_7^* U \sin \vartheta & 0 & m''
\end{pmatrix}
\]
and eq. (20) still holds. The scalar potential is

\[ V = \sum_{k=1}^{2} \left[ \mu_k \phi_k^\dagger \phi_k + \lambda_k \left( \phi_k^\dagger \phi_k \right)^2 \right] + \lambda_3 \phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2 + \lambda_4 \phi_1^\dagger \phi_2 \phi_2^\dagger \phi_1 + \lambda_5 \left( \phi_1^\dagger \phi_2 \right)^2 + \text{H.c.} \]

\[ + \left( \mu_\chi + \sum_{k=1}^{2} \lambda_k \phi_k^\dagger \phi_k \right) \left( \chi_1^2 + \chi_2^2 \right) + \lambda_\chi \left( \phi_1^\dagger \phi_2 \chi_2^2 - \phi_2^\dagger \phi_1 \chi_1^2 \right) \text{ H.c.}, \]

where \( \lambda_5 \) is real but \( \lambda \) is in general complex. Once again, if \( \lambda_\chi \) is negative then \( \vartheta = \pi/4 \). Thus,

\[ M^{-1} = - \text{diag} \left( \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \right) \left( \begin{array}{ccc} m & m'' & m''' \\ m'' & m' & 0 \\ m''' & 0 & m'' \end{array} \right) \text{ diag} \left( \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \right), \]

where \( m'' = y_f U / \sqrt{2} \). Hence,

\[ M^{-1} = \frac{1}{m |m'|^2 - 2 \Re (m'' m^n)} \text{ diag} (a, b, c) \]

\[ \times \left( \begin{array}{ccc} |m'|^2 & -m'' m' & -m'' m''' \\ -m'' m' & mm''^* - m'' m'^2 & \left|m'' \right|^2 \\ -m'' m''' & \left|m'' \right|^2 & mm' - m''^2 \end{array} \right) \text{ diag} (a, b, c). \]

Therefore this model predicts\(^{[10]}\)

\[ A_{\mu \nu} = A_{e \tau}^*. \]

So this model has once again five parameters, since it predicts the two complex eqs. (20) and (34).

5 Models 3, 4, and 5

In the models of the previous two sections we have made the following assignment for the lepton flavours: we have assumed that the lepton flavours being

\(^{[10]}\)It also predicts \( A_{\mu \tau} = A_{\tau \mu}^* \), \( A_{e e} = A_{ee}^* \), and \( A_{\mu e} = A_{e \mu}^* \), but these three relations can be derived from eqs. (20) and (34), as was pointed out at the end of section 2.
interchanged either by the symmetry $Z_2^{(\text{int})}$ of eq. (13) or by the $CP$ symmetry of eq. (26) are the $\mu$ and $\tau$ flavours. Two other choices are possible—the lepton flavours being interchanged might be either $e$ and $\mu$ or $e$ and $\tau$. We have tested those two other choices against the experimental data and have found that the second choice fits those data just as well as models 1 and 2 above. Thus, we define model 3 as predicting

\[
(M^{-1})_{e\tau} = 0,
\]
\[
A_{e\mu} = A_{\mu\tau},
\]

and model 4 as predicting

\[
(M^{-1})_{e\tau} = 0,
\]
\[
A_{e\mu} = A^*_{\mu\tau}.
\]

It is also possible to simultaneously impose invariance under $Z_2^{(\text{int})}$ of eq. (13) and the $CP$ symmetry of eq. (26). In that case,

- Equation (14) holds with $m$ and $m'$ real, cf. eq. (27).
- Equation (15) holds with $y_{1-7}$ real, cf. eq. (28).
- Equation (21) holds with $\lambda_5$ and $\lambda$ real, cf. eq. (31).

So, finally one ends up with eq. (23) with $m$, $m'$, and $m''$ real. The $CP$ symmetry may be broken spontaneously through VEVs $v_{1,2}$ with a relative complex phase and this makes $a$, $b$, and $c$ complex; but that has no relevance for our predictions, since $a$, $b$, $c$ drop out in the matrix $A$.

It turns out that the model delineated above is unable to correctly fit the present data. But the model in which the $e$ and $\mu$ flavours are interchanged relative to the above works fine; hence we define model 5 as predicting

\[
(M^{-1})_{e\tau} = 0,
\]
\[
A_{e\mu} = A_{\mu\tau},
\]
\[
A_{\mu\tau} = A^*_{\mu\tau}.
\]

6 Observable predictions

We have fitted models 1–5 of the previous sections to the phenomenological data of ref. [3]. It turns out that our models fit the data so perfectly that
we were able to use just the $1\sigma$ intervals given in that paper. Thus, we have required that

\[ m_2^2 - m_1^2 \in [7.32, 7.80] \times 10^{-5} \text{ eV}^2, \]  
\[ \sin^2 \theta_{12} \in [0.291, 0.325], \]  
\[ \frac{m_3^2 - m_1^2 + m_2^2}{2} \in [2.33, 2.49] \times 10^{-3} \text{ eV}^2, \]  
\[ \sin^2 \theta_{13} \in [0.0216, 0.0266], \]  
\[ \sin^2 \theta_{23} \in [0.365, 0.410], \]

in the case of a normal neutrino mass spectrum. If the neutrino mass spectrum is inverted, then the requirements (38a) and (38b) remain, but the requirements (38c)–(38e) are substituted by

\[ \frac{m_1^2 + m_2^2}{2} - m_3^2 \in [2.31, 2.49] \times 10^{-3} \text{ eV}^2, \]  
\[ \sin^2 \theta_{13} \in [0.0219, 0.0267], \]  
\[ \sin^2 \theta_{23} \in [0.370, 0.431], \]

respectively. In eqs. (38) and (39), $m_{1,2,3}$ are the neutrino masses and $\theta_{12,13,23}$ are the lepton mixing angles as defined in the standard parameterization in eq. (13.79) of ref. [15]. Notice that we have not used in our fits the constraints in ref. [3] for the Dirac phase $\delta$ of lepton mixing, which we chose instead to be an observable to be predicted by our models. For comparison, at the $1\sigma$ level the authors of ref. [3] found

\[ \delta \in [0.77\pi, 1.36\pi] \quad \Rightarrow \quad \cos \delta \in [-1, -0.43], \]  
\[ \delta \in [0.83\pi, 1.47\pi] \quad \Rightarrow \quad \cos \delta \in [-1, -0.09] \]

for a normal and for an inverted spectrum, respectively.

We have found that all our models are able to fit the data either in eqs. (38)—for a normal mass spectrum—or in eqs. (39)—for an inverted mass spectrum—or in both of them—perfectly. In models 1 to 4, the phase space for the five observables appears, inside the intervals quoted in those equations, uniformly filled in all our scatter plots, so that no prediction for any of those observables looks warranted. On the other hand, predictions for the overall neutrino mass scale and for the phase $\delta$ are possible. Another quantity that one may predict is the effective mass $m_{\beta\beta} \equiv |M_{ee}|$, which is relevant for neutrinoless double beta decay. In model 5, moreover, the angles $\theta_{12}$ and $\theta_{23}$ are correlated and significantly restricted relative to the intervals in eqs. (38) and (39), respectively.
Figure 1: For model 1, we present (a) $\cos \delta$ vs. $m_{\beta\beta}$ and (b) the sum of the three neutrino masses vs. $m_{\beta\beta}$. All points shown were found requiring a $1\sigma$ fit to the data of Fogli et al. [3]. Model 1 allows an inverted mass spectrum only.

Model 1 is compatible only with an inverted neutrino mass spectrum. The neutrino masses must be quite high, of order 0.2 eV or more for each neutrino, which risks violating cosmological bounds [16]. The phase $\delta$ is close to $\pi$ ($\cos \delta < -0.5$), which agrees with the (valid at the $1\sigma$ level) preference found in ref. [3]. The corresponding scatter plots are presented as figure 1.

For model 2 both a normal and an inverted neutrino mass spectrum are possible. However, if the spectrum is inverted then $\delta$ must be very close to $\pi/2$ or $3\pi/2$ and the three neutrinos must be almost degenerate with masses around 0.15 eV, which is disfavoured by some of the recent cosmological bounds [16]. For a normal spectrum neutrino masses may be lower and there is a close correlation between $\cos \delta$ and the scale of neutrino masses. Notice that for model 2 $\cos \delta$ is always positive, which disagrees with the findings of ref. [3] at the $1\sigma$ level; at the $2\sigma$ level, though, $\cos \delta$ is already unconstrained in ref. [3], so this should not be considered a severe handicap of our model. Plots for this case are given in figure 2. Models 3 and 4 are, in practice, extremely similar. Only an inverted neutrino mass spectrum is possible for them. The neutrino masses must moreover lie in a very narrow range: the lowest mass, $m_3$, must lie in between 0.008 eV and 0.012 eV. This is compatible with all current cosmological bounds. Moreover, those models force $\cos \delta < -0.4$, which is in agreement with the $1\sigma$ range obtained in ref. [3]—see eqs. (10). Plots for models 3 and 4 are presented in fig. 3.

In model 5 the neutrino mass matrix $M$ is effectively real, apart from the
unphysical phases of $a$, $b$, and $c$; neglecting those phases, $M$ may be diagonalised by a real orthogonal matrix $O$ as $O^TMO = \text{diag} (m_1, -m_2, m_3)$. The smallest neutrino mass is $m_3$, since only an inverted spectrum works in model 5. In that model the neutrino masses are exceedingly constrained: the sum of the light-neutrino masses must be $(0.110 \pm 0.003)$ eV and $m_{3\beta} = (0.020 \pm 0.001)$ eV. The matrix $O$ is characterized by Dirac phase $\delta = \pi$.

Besides these predictions, model 5 predicts a correlation between the mixing angles $\theta_{12}$ and $\theta_{23}$, which we depict in the scatter plots in fig. 4. One sees in fig. 4 (a) that model 5 is still viable, although rather marginally, at the 1$\sigma$ level.

7 Summary

In this paper we have constructed five flavour models for the lepton sector. They are two-Higgs-doublet models with a standard $Z_2$ symmetry which changes the sign of one of the doublets, or else with a $CP$ symmetry under which one of the doublets is odd. The models are not necessarily supersymmetric and use no soft breaking of symmetries, just spontaneous breaking.

\footnote{We have found that only this choice of Majorana phases works: the largest eigenvalue of $M$ in absolute value, \textit{viz.} $m_2$, must have sign opposite to the one of the other two eigenvalues.}
Figure 3: Plots for $\cos \delta$ vs. $m_{33}$ in model 3 (plot a) and in model 4 (plot b); and for the sum of the neutrino masses vs. $m_{33}$ in model 3 (plot c) and in model 4 (plot d). All points shown were found requiring a 1σ fit to the data of Fogli et al. [3]. Models 3 and 4 allow an inverted neutrino mass spectrum only.
Figure 4: The correlation between $\sin^2 \theta_{23}$ and $\sin^2 \theta_{12}$ in model 5. In plot (a) all the physical quantities were forced to lie inside their 1σ ranges of ref. [3] while in plot (b) the 3σ ranges were used instead. The cross in both plots marks the best-fit point of ref. [3]. Model 5 only allows an inverted mass spectrum, with the Majorana phases and $\delta = \pi$ fixed (see text).

Since they are seesaw models, they include an extra high scale at which we introduce two real scalar gauge singlets and three right-handed neutrinos. Models 1–4 have five physical parameters each, while model 5 only has four parameters. They all fit very well the phenomenological data for the lepton mixing angles and for the neutrino mass-squared differences. With the exception of model 2, all our models predict an inverted neutrino mass spectrum and a negative $\cos \delta$. In models 1 and 2 the neutrino mass scale tends to be quite high, in possible conflict with cosmological bounds, but in models 3, 4, and 5 the neutrinos are fairly light. Model 5 furthermore predicts a well-defined correlation between the mixing angles $\theta_{12}$ and $\theta_{23}$.

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References

[1] Y. Abe et al. (Double Chooz Coll.), Indication for the disappearance of reactor electron antineutrinos in the Double Chooz experiment, Phys. Rev. Lett. 108 (2012) 131801;
F.P. An et al. (Daya Bay Coll.), Observation of electron-antineutrino disappearance at Daya Bay, Phys. Rev. Lett. 108 (2012) 171803;
J.K. Ahn et al. (RENO Coll.), Observation of reactor electron antineutrino disappearance in the RENO experiment, Phys. Rev. Lett. 108 (2012) 191802.

[2] W. Grimus and L. Lavoura, Softly broken lepton numbers and maximal neutrino mixing, J. High Energy Phys. 0107 (2001) 045.

[3] G.L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, and A.M. Rotunno, Global analysis of neutrino masses, mixings and phases: Entering the era of leptonic CP violation searches, Phys. Rev. D 86 (2012) 013012.

[4] D.V. Forero, M. Tórtola, and J.W.F. Valle, Global status of neutrino oscillation parameters after Neutrino–2012, Phys. Rev. D 86 (2012) 073012.

[5] M.C. Gonzalez-Garcia, M. Maltoni, J. Salvado, and T. Schwetz, Global fit to three neutrino mixing: Critical look at present precision, J. High Energy Phys. 1212 (2012) 123.

[6] W. Grimus and L. Lavoura, A nonstandard CP transformation leading to maximal atmospheric neutrino mixing, Phys. Lett. B 579 (2004) 113.

[7] W. Grimus and L. Lavoura, A discrete symmetry group for maximal atmospheric neutrino mixing, Phys. Lett. B 572 (2003) 189.

[8] P.H. Frampton, S.L. Glashow, and D. Marfatia, Zeroes of the neutrino mass matrix, Phys. Lett. B 536 (2002) 79.

[9] L. Lavoura, Zeros of the inverted neutrino mass matrix, Phys. Lett. B 609 (2005) 317.
[10] W. Grimus, L. Lavoura, and B. Radovčić, *Type II seesaw mechanism for Higgs doublets and the scale of new physics*, Phys. Lett. B 674 (2009) 117.

[11] G.C. Branco, P.M. Ferreira, L. Lavoura, M.N. Rebelo, M. Sher, and J.P. Silva, *Theory and phenomenology of two-Higgs-doublet models*, Phys. Rept. 516 (2012) 1.

[12] P.M. Ferreira, R. Santos, M. Sher, and J.P. Silva, *Implications of the LHC two-photon signal for two-Higgs-doublet models*, Phys. Rev. D 85, (2012) 077703; S. Chang, S.K. Kang, J.-P. Lee, K.Y. Lee, S.C. Park, and J. Song, *Comprehensive study of two Higgs doublet model in light of the new boson with mass around 125 GeV*, [arXiv:1210.3439]; G. Bélanger, B. Dumont, U. Ellwanger, J.F. Gunion, and S. Kraml, *Higgs couplings at the end of 2012*, [arXiv:1212.5244]; C.-Y. Chen and S. Dawson, *Exploring two Higgs doublet models through Higgs production*, [arXiv:1301.0309].

[13] G. Aad et al. (ATLAS Coll.), *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, Phys. Lett. B 716 (2012) 1; S. Chatrchyan et al. (CMS Coll.), *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, Phys. Lett. B 716 (2012) 30.

[14] W. Grimus and L. Lavoura, *The seesaw mechanism at arbitrary order: Disentangling the small scale from the large scale*, J. High Energy Phys. 0011 (2000) 042.

[15] J. Beringer et al. (Particle Data Group), *Review of particle physics*, Phys. Rev. D 86 (2012) 010001.

[16] R. de Putter et al., *New neutrino mass bounds from SDSS-III data release 8: Photometric luminous galaxies*, Astrophys. J. 761 (2012) 12; Z. Hou et al., *Constraints on cosmology from the cosmic microwave background power spectrum of the 2500-square degree SPT-SZ survey*, [arXiv:1212.6267]; P.A.R. Ade et al. (Planck Coll.), *Planck 2013 results. XVI. Cosmological parameters*, [arXiv:1303.5076].