Impact of Device Orientation on Error Performance of LiFi Systems

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Abstract—Most studies on optical wireless communications (OWCs) have neglected the effect of random orientation in their performance analysis due to the lack of a proper model for the random orientation. Our recent empirical-based research illustrates that the random orientation follows a Laplace distribution for a static user equipment (UE). In this paper, we analyze the device orientation and assess its importance on system performance. The probability of establishing a line-of-sight link is investigated and the probability density function (PDF) of signal-to-noise ratio (SNR) for a randomly-oriented device is derived. By means of the PDF of SNR, the bit-error ratio (BER) of DC biased optical orthogonal frequency division multiplexing (DCO-OFDM) in additive white Gaussian noise (AWGN) channels is evaluated. A closed form approximation for the BER of UE with random orientation is presented which shows a good match with Monte-Carlo simulation results.

Index Terms—Random orientation, DCO-OFDM, bit-error ratio (BER), light-fidelity (LiFi), visible light communication (VLC).

I. INTRODUCTION

Statistical data traffic confirms that smartphones will generate more than 86% percent of the total mobile data traffic by 2021 [1]. Light-Fidelity (LiFi) as part of the future fifth generation can cope with this immense volume of data traffic [2]. LiFi is a bidirectional networked system that utilizes visible light spectrum in the downlink and infrared spectrum in the uplink [3]. LiFi offers remarkable advantages such as utilizing a very large and unregulated bandwidth, energy efficiency and enhanced security. These benefits have put LiFi in the scope of recent and future research [4]. The majority of studies on optical wireless communications assume that the device always faces vertically upwards. Although this may be for the purpose of analysis simplification or due to lack of a proper model for device orientation, in a real life scenario users hold their device in a way that feels most comfortable. Device orientation can affect the users' throughput remarkably and it should be analyzed carefully. Only a few number of studies have considered the impact of random orientation in their analysis [5]–[13]. Device orientation can be measured by the gyroscope and accelerometer implemented in every smartphone [14]. Then, this information can be feedback to the access point (AP) by the limited-feedback schemes to enhance the system throughput [3], [15], [16].

The effect of random orientation on users’ throughput has been assessed in [5]. In order to tackle the problem of load balancing, the authors proposed a novel AP selection algorithm that considers the random orientation of user equipments (UEs). The downlink handover problem due to the random rotation of UE in LiFi networks is characterized in [6]. The handover probability and handover rate for static and mobile users are determined. The handover probability in hybrid LiFi/RF-based networks with randomly-oriented UEs is analyzed in [7]. The effect of tilting the UE on the BER performance by tilting the UE plane properly. A similar approach is used in [10] by finding the optimal tilting angle to improve both the signal-to-noise ratio (SNR) and spectral efficiency of M-QAM orthogonal frequency division multiplexing (OFDM) for indoor visible light communication (VLC) systems. Impacts of both UE's orientation and position on link performance of VLC are studied in [11]. The outage probability is derived and the significance of UE orientation on inter-symbol interference is shown. The optimum polar and azimuth angles for single user multiple-input multiple-output (MIMO) OFDM is calculated in [12]. A receiver with four photodetectors (PD) is considered and the optimal angles for each PD are computed. In [13], the impact of the random orientation on the line-of-sight (LOS) channel gain for a randomly oriented UE is studied. The statistical distribution of the channel gain is presented for a single light-emitting diode (LED) and extended to a scenario with double LEDs. All mentioned studies assume a predefined model for the random orientation of the receiver. However, little or no evidence is presented to justify the assumed models. For the first time, experimental measurements are carried out to model the polar and azimuth angles in [17]–[19]. It is shown that the polar angle can be modeled by either the Laplace distribution for static users or the Gaussian distribution for mobile users while the azimuth angle follows a uniform distribution. All these studies emphasize the significance of incorporating the random orientation into the analysis.

We characterize the device random orientation and investigate its effect on the users' performance metrics such as SNR and BER in optical wireless systems. We also derive the probability density function (PDF) of SNR for randomly-oriented device. Based on the derived PDF of SNR, the BER performance of a DC biased optical OFDM (DCO-OFDM) is evaluated as a use case. A closed form approximation for BER is proposed. The impact of device orientation on BER with some interesting observations are investigated. In this study, we only consider the LOS channel gain, and the impact of higher reflections on BER performance
has been investigated in our recent study \[20\].

Notations: |·| expresses the absolute value of a variable; \(\tan^{-1}(y/x)\) is the four-quadrant inverse tangent. Further, \([·]^T\) stands for transpose operator. We note that throughout this paper, unless otherwise mentioned, angles are expressed in degrees.

II. SYSTEM MODEL

A. LOS Channel Gain

An open indoor office without reflective objects for optical wireless downlink transmission is considered in this study. The geometric configuration of the downlink transmission is illustrated in Fig.1. It is assumed that an LED transmitter (or AP) is a point source that follows the Lambertian radiation pattern. Furthermore, the LED is supposed to operate within the linear dynamic range of the current-power characteristic pattern. The direct current (DC) gain of the LOS optical wireless downlink transmission is considered in this study.

Notations:

- \(|·|\): absolute value
- \([·]^T\): transpose operator
- \(\alpha\), \(\beta\), \(\gamma\): elemental intrinsic rotation angles yaw, pitch and roll denoted (extrinsic rotation). Current smartphones are able to report the rotations about the axes of the reference coordinate system of the rotating local coordinate system (intrinsic rotation) or three separate angles showing the rotation about each axes (extrinsic rotation).

The geometric configuration of the downlink transmission is illustrated in Fig. 1. It is assumed that an LED transmitter (or AP) is a point source that follows the Lambertian radiation pattern. Furthermore, the LED is supposed to operate within the linear dynamic range of the current-power characteristic curve to avoid the nonlinear distortion effect. The LED is fixed and oriented vertically downward.

The direct current (DC) gain of the LOS optical wireless channel between the AP and the UE is given by \[21\]:

\[
H = \frac{(m+1)A_{PD}}{2\pi d^2} g_t \cos^m \phi \cos \psi \mathbb{1}(\psi \leq \Psi_c),
\]

where \(\mathbb{1}(x \leq x_0) = 1\) for \(0 \leq x \leq x_0\) and 0 otherwise; \(A_{PD}\) is the PD physical area; the Euclidean distance between the AP and the UE is denoted by \(d\) with \((x_u, y_u, z_u)\) as the position of the AP and UE in the Cartesian coordinate system, respectively; the Lambertian order is \(m = -1/\log_2(\cos \Psi_{1/2})\) where \(\Psi_{1/2}\) is the transmitter seminangle at half power. The incidence angle with respect to the normal vector to the UE surface, \(n_u\), and the radiance angle with respect to the normal vector to the AP surface, \(n_{ks}\), are denoted by \(\phi\) and \(\psi\), respectively. These two angles can be obtained by using the analytical geometry rules as \(\cos \phi = \mathbf{d} \cdot \mathbf{n}_{ks}/d\) and \(\cos \psi = -\mathbf{d} \cdot \mathbf{n}_u/d\) where \(\mathbf{d}\) is the distance vector from the AP to the UE and \(·\) is the inner product operator. The gain of the optical concentrator is given as \(g_t = c^2/\sin^2 \Psi_c\) with \(c\) being the refractive index and \(\Psi_c\) is the UE field of view (FOV). After some simplifications, \[1\] can be written as:

\[
H = \frac{H_0 \cos \psi}{d^{m+2}} \mathbb{1}(\psi \leq \Psi_c),
\]

where \(H_0 = (m+1)A_{PD}g_t h^m\); and \(h = |z_u - z_a|\) is the vertical distance between the UE and the AP as shown in Fig. 1.

B. Rotation in the Space

A convenient way of describing the orientation is to use three separate angles showing the rotation about each axes of the rotating local coordinate system (intrinsic rotation) or the rotation about the axes of the reference coordinate system (extrinsic rotation). Current smartphones are able to report the elemental intrinsic rotation angles yaw, pitch and roll denoted as \(\alpha\), \(\beta\) and \(\gamma\), respectively \[22\]. Here, \(\alpha\) represents rotation about the \(z\)-axis, which takes a value in range of \([0, 360]\); \(\beta\) denotes the rotation angle about the \(x\)-axis, that is, tipping the device toward or away from the user, which takes value between \([-180^\circ, 180^\circ]\); and \(\gamma\) is the rotation angle about the \(y\)-axis, that is, tilting the device right or left, which is chosen from the range \([-90, 90]\). The elemental Euler angles are depicted in Fig. 2.

Now we derive the concatenated rotation matrix with respect to the reference coordinate system. The normal vector after rotation can be obtained as:

\[
n_u' = R n_u,
\]

where \(n_u = [n_1, n_2, n_3]^T\) is the original normal vector and \(n_u' = [n'_1, n'_2, n'_3]^T\) is the rotated normal vector via the rotation matrix \(R\). The rotation matrix can be decomposed as \(R = R_\alpha R_\beta R_\gamma\), where \(R_\alpha\), \(R_\beta\) and \(R_\gamma\) are the rotation matrices about the \(z\), \(x\) and \(y\) axes, respectively. Assume that the body frame and the reference frame are initially aligned so that \(n_u = [0, 0, 1]^T\); then, the rotated normal vector, \(n'_u\), via the rotation matrices \(R_\alpha\), \(R_\beta\) and \(R_\gamma\) is given in \[4\] shown at top this page.

The rotated normal vector can be represented in the spherical coordinate system using the azimuth, \(\omega\), and polar, \(\theta\) angles. That is, \(n'_u = [\sin \theta \cos \omega, \sin \theta \sin \omega, \cos \theta]^T\). As shown in Fig. 4 \(\theta\) is the angle between the positive direction
The channel gain for different locations of the UE with a fixed $\Omega = 45^\circ$ and $\Psi_c = 90^\circ$.

![Fig. 4: The effect of changing $\theta$ on $\cos \psi$ for different locations of the UE with fixed $\Omega = 45^\circ$ and $\Psi_c = 90^\circ$.](image)

### Table I: Simulation Parameters

| Parameter                        | Symbol | Value  |
|----------------------------------|--------|--------|
| AP location                      | $(x_a, y_a, z_a)$ | $(0, 0, 2)$ |
| LED half-intensity angle         | $\mu_0$ | $60^\circ$ |
| PD responsivity                 | $R_{PD}$ | 1 A/W |
| Physical area of a PD            | $A_{PD}$ | 1 cm² |
| Refractive index                 | $s$ | 1 |
| Downlink bandwidth               | $B$ | 10 MHz |
| Number of subcarriers            | $K$ | 1024 |
| Noise power spectral density     | $N_0$ | $10^{-21}$ W/Hz |
| Vertical distance of UE and AP   | $h$ | 2 m |

III. ORIENTATION ANALYSIS

Before analyzing user’s performance metrics such as average SNR and BER, let us define the critical elevation (CE), $\theta_{ce}$, which defines the elevation angle at the boundary of the field of view of the receiver. As shown in Fig. 3, the CE angle for a given position of UE, $(x_u, y_u)$, and user’s direction, $\Omega$, is the elevation angle for which $\psi = \Psi_c$. Thus, $\theta \geq \theta_{ce}$ results in $\psi \geq \Psi_c$, and the channel gain would be zero based on (1). This angle which is defined by both the UE position and its direction, $\Omega$ which is given as follows:

$$\theta_{ce} = \cos^{-1}\left(\frac{\cos \Psi_c}{\sqrt{l_1^2 + l_2^2}}\right) + \tan^{-1}\left(\frac{\lambda_1}{\lambda_2}\right),$$  \hspace{1cm} (6)

where the coefficients $\lambda_1$ and $\lambda_2$ are given as:

$$\lambda_1 = \frac{r}{d} \cos \left(\Omega - \tan^{-1}\left(\frac{y_u - y_a}{x_u - x_a}\right)\right),$$  \hspace{1cm} (7)

$$\lambda_2 = \frac{h}{d},$$

where $r = \sqrt{(x_u - x_a)^2 + (y_u - y_a)^2}$ is the horizontal distance between the AP and the UE. Proof of (6) is provided in Appendix A. As can be seen from (7), the parameter $\lambda_1$ contains the direction angle, $\Omega$. The physical concept of positive $\lambda_1$ is that the UE is facing to the AP while if it is not facing to the AP, $\lambda_1$ is negative. On the other hand, since always $z_u < z_a$, we have $\lambda_2 > 0$. It should be mentioned that the acceptable range for $\theta_{ce}$ is $[0, 90]$ as the polar angle, $\theta$, given in (4) takes values between $0^\circ$ and $90^\circ$. Note that for a given location of UE, the minimum CE angle, $\theta_{th}$, is obtained for $\Omega = \pi + \tan^{-1}\left(\frac{y_u - y_a}{x_u - x_a}\right) = \Omega_{th}$ which is given as:

$$\theta_{th} = \Psi_c + \sin^{-1}\left(\frac{h}{d}\right) - \frac{\pi}{2}. \hspace{1cm} (8)$$

The effect of changing the elevation angle, $\theta$, on the LOS channel gain for different locations of the UE with a fixed direction angle, $\Omega = 45^\circ$ and $\Psi_c = 90^\circ$, is shown in Fig. 4. Here, $\Psi_c = 90^\circ$ and other parameters are presented in Table I. It can be seen that for the UE’s locations of $L_4 = (-3, -3)$ and $L_5 = (-4, -1)$ by increasing the elevation angle, the LOS channel gain decreases. After $\theta_{ce} = 25.24^\circ$ and $\theta_{ce} = 29.5^\circ$ for $L_4$ and $L_5$, respectively, the AP is out of the UE’s FOV and hence the LOS channel gains are zero. However, for this specific $\Omega = 45^\circ$ if the UE is located at positions like $L_1 = (3, 3)$ or $L_2 = (4, 1)$, the LOS channel gain does not become zero if the elevation angle changes between $0^\circ$ and $90^\circ$. 

\[ n'_u = R_\alpha R_\beta R_\gamma \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \alpha \sin \gamma & -\sin \alpha \sin \gamma & \cos \alpha \sin \gamma + \cos \alpha \sin \gamma \\ \cos \alpha \sin \gamma & -\cos \alpha \cos \gamma \sin \beta & \cos \alpha \cos \gamma \sin \beta \end{bmatrix} \]

\[ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \]
It is noted that under the condition of $\theta < \theta_{th}$, the AP is always within the UE’s field of view for any direction of $\Omega$. For a given UE’s location, we are also interested in the range of $\Omega$ for which the LOS channel is active. Let’s denote this range as $\mathcal{R}_{\Omega, \theta}$. This range can be determined according to the following Proposition.

**Proposition.** For a given UE’s location, the range of $\Omega$ for which the LOS channel gain is non-zero is $[0, 2\pi]$ if $\theta$ is smaller than or equal to a threshold angle $\theta_{th} = \Psi_c + \sin^{-1}\left(\frac{h}{d} - \frac{r}{2}\right)$. Otherwise it is given as follows:

$$\mathcal{R}_{\Omega, \theta} = \left\{ \begin{array}{ll} [0, \Omega_{r1}] \cup [\Omega_{r2}, 2\pi], & \text{if } \Lambda'(\Omega_{r1}) < 0 \\ (\Omega_{r1}, \Omega_{r2}), & \text{if } \Lambda'(\Omega_{r1}) \geq 0 \end{array} \right.,$$

where $\Lambda'(\Omega) = -\kappa_1 \sin\left(\Omega - \tan^{-1}\left(\frac{y_0 - y_a}{x_u - x_a}\right)\right) + \kappa_2$ with:

$$\kappa_1 = \frac{r}{d} \sin \theta, \quad \kappa_2 = \frac{h}{d} \cos \theta.$$  

(9)

Also $\Omega_{r1} = \min\{\Omega_1, \Omega_2\}$ and $\Omega_{r2} = \max\{\Omega_1, \Omega_2\}$, where:

$$\begin{align*}
\Omega_1 &= \cos^{-1}\left(\frac{\cos \Psi_c - \kappa_2}{\kappa_1}\right) + \tan^{-1}\left(\frac{y_0 - y_a}{x_u - x_a}\right), \\
\Omega_2 &= -\cos^{-1}\left(\frac{\cos \Psi_c - \kappa_2}{\kappa_1}\right) + \tan^{-1}\left(\frac{y_0 - y_a}{x_u - x_a}\right).
\end{align*}$$

(11)

**Proof:** See Appendix B.

The LOS channel gain versus $\Omega$ for locations of $L_1$ and $L_5$ (see the inset of Fig. 3) with $\theta = \theta_{th}$ (dash line) and $\theta = 41^\circ \geq \theta_{th}$ (solid line) are shown in Fig. 5. Note that for $L_1$ and $L_5$, we have $\theta_{th} = 25.24^\circ$ and $\theta_{th} = 25.88^\circ$, respectively. As can be seen, if $\theta < \theta_{th}$, then, for any $\Omega$, the LOS channel gain is always non-zero (dash lines). Based on the proposition, the range of $\Omega$ for which the LOS channel gain is non-zero with $\theta = 41^\circ > \theta_{th}$ is $[0, 167.8] \cup [282.2, 360]$ for $L_1$ and $[70.1, 318]$ for $L_2$.

It can be inferred from the Proposition that for a given UE’s location and $\theta$, the probability that the LOS path is not within the UE’s FOV (due to variation of $\Omega$) is $\Pr\{H = 0\} = 1 - \Pr\{\Omega \in \mathcal{R}_{\Omega, \theta}\}$. Fig. 6 shows the $\Pr\{H = 0\}$ versus the horizontal distance between the UE and the AP, $r$, for different UEs’ FOVs. The results are shown for $\theta = 41^\circ$. As can be observed, $\Pr\{H = 0\} = 1$, for UEs with a narrow FOV (i.e., $\Psi_c = 30^\circ$ and $40^\circ$) when they are located in the vicinity below the AP. As the horizontal distance, $r$, increases, $\Pr\{H = 0\}$ first decreases and then it increases as it goes away from the AP. For wide FOVs (i.e., $\Psi_c = 60^\circ, 80^\circ$ and $90^\circ$), $\Pr\{H = 0\}$ is zero when the UE is in the vicinity below the AP, and then it starts to increase at a certain $r$. This can be derived based on (8) for $\Psi_c \geq \theta$ that is $r \geq h \tan(\Psi_c - \theta)$. Note that the high value of losing the LOS link particularly for narrower FOVs is due to the fact that a single AP is considered and the effect of reflection is ignored. A study of such effects has been presented in our recent work [20].

Let $\mathcal{R}_\Omega$ denote the range for which the LOS channel gain is always non-zero regardless of $\theta$, i.e., $\forall \theta \in [0, 90]$. The range, $\mathcal{R}_\Omega$, can be determined according to the following Corollary.

**Corollary.** For a given UE’s location, the range of $\Omega$ for which the LOS channel gain is non-zero for all $\theta \in [0, 90]$ can be obtained as:

$$\mathcal{R}_\Omega = \mathcal{R}_{\Omega, \theta} \mid_{\theta = 90^\circ}.$$  

(12)

Proof of this corollary is similar to the proof of proposition 1. Noting that the worst elevation angle that leads to the smallest range of $\Omega$ is $\theta = 90^\circ$. The physical concept of $\mathcal{R}_\Omega$ is that when the UE faces the AP, we have $\Omega \in \mathcal{R}_\Omega$. Otherwise, if the UE faces the opposite direction of the AP, $\Omega \notin \mathcal{R}_\Omega$. In fact, $\mathcal{R}_\Omega$ provides a stable range for which the user can change the elevation angle between 0 and 90 without experiencing the AP out of its FOV. We note that the range given in (12) is valid if $\Psi_c \geq \cos^{-1}\left(\frac{h}{d}\right)$ (this condition can be readily seen by substituting $\theta = 90^\circ$ in (10) and then replacing the results in (11)).

IV. Bit-Error Ratio Performance

In this section, we evaluate the BER performance of DCO-OFDM in LiFi networks. We initially derive the SNR statistics on each subcarrier, then based on the derived PDF of SNR, the BER performance is assessed.
A. SNR Statistics

The received electrical SNR on k-th subcarrier of a LiFi system can be acquired as:

\[ S = \frac{R_{PD}^2 H^2 P_{\text{opt}}^2}{(K - 2) \eta^2 \sigma_k^2}, \]

(13)

where the PD responsivity is denoted by \( R_{PD} \); \( H \) is the LOS channel gain given in (1); \( P_{\text{opt}} \) is the transmitted optical power; \( K \) is the total number of subcarriers with \( K/2 - 1 \) subcarriers bearing information. Furthermore, \( \eta \) is the conversion factor [23]. The condition \( \eta = 3 \) can guarantee that less than 1% of the signal is clipped so that the clipping noise is negligible [3], [24]. In (13), \( \sigma_k^2 = N_0 B/K \) is the noise power on k-th subcarrier where \( N_0 \) stands for the noise spectral density and \( B \) represents the modulation bandwidth. Based on the experimental measurement of the device orientation, it is shown in [17] that the LOS channel gain, \( H \), follows a clipped Laplace distribution as:

\[ f_H(h) = \frac{\exp\left(-\frac{|h - \mu_H|}{b_H}\right)}{b_H \left(2 - \exp\left(-\frac{h_{\text{min}} - \mu_H}{b_H}\right)\right)} + c_H \delta(h), \]

(14)

where \( c_H = F_{\cos \varphi} (\cos \Psi_c) \), which is given as:

\[ c_H = F_{\cos \varphi} (\cos \Psi_c) \approx \begin{cases} 1 - \frac{1}{2} \exp \left(-\frac{\theta_{\text{ce}} - \mu_0}{b_0}\right), & \theta_{\text{ce}} < \mu_0 \\ \frac{1}{2} \exp \left(-\frac{\theta_{\text{ce}} - \mu_0}{b_0}\right), & \theta_{\text{ce}} \geq \mu_0 \end{cases} \]

(15)

where \( b_0 = \sqrt{\sigma_{\varphi}^2/2} \). The parameters \( \mu_0 \) and \( \sigma_\varphi \) are the mean and standard deviation of the elevation angle, which are obtained based on the experimental measurements. For static users, they are reported as \( \mu_0 = 41^\circ \) and \( \sigma_\varphi = 7.68^\circ \). Proof of [15] is provided in Appendix C. Furthermore, for the detailed proof of (14), we refer to Eq. (56) and (57) of [17]. The mean and scale factor of channel gain, \( \mu_H \) and \( b_H \) respectively, are:

\[ \mu_H = \frac{H_0}{d^{m+2}} (\lambda_1 \sin \mu_\theta + \lambda_2 \cos \mu_\theta), \]

(16)

\[ b_H = \frac{H_0}{d^{m+2}} |\lambda_1 | \cos \mu_\theta - 2 \sin \mu_\theta|, \]

(17)

where \( H_0 \) is given below (2). The factors, \( \lambda_1 \) and \( \lambda_2 \), are given in [4]. The support range of \( f_H(h) \) is \( h_{\text{min}} \leq h \leq h_{\text{max}} \) where \( h_{\text{min}} \) and \( h_{\text{max}} \) are given as:

\[ h_{\text{min}} = \begin{cases} \frac{H_0}{d^{m+2}} \cos \Psi_c, & \cos \varphi < \cos \Psi_c \\ \frac{H_0}{d^{m+2}} \min \{\lambda_1, \lambda_2\}, & \text{o.w.} \end{cases}, \]

(18)

\[ h_{\text{max}} = \begin{cases} \frac{H_0}{d^{m+2}} \lambda_2, & \text{if } \lambda_1 < 0 \\ \frac{H_0}{d^{m+2}} \sqrt{\lambda_1^2 + \lambda_2^2}, & \text{if } \lambda_1 \geq 0 \end{cases}. \]

(19)

The cumulative distribution function (CDF) of LOS channel gain can be also obtained by calculating the integral of (14), which is given as:

\[ F_H(h) = c_H + \begin{cases} \exp\left(\frac{h - \mu_H}{b_H}\right) - \exp\left(\frac{h_{\text{min}} - \mu_H}{b_H}\right), & h_{\text{min}} \leq h \leq \mu_H \\ 2 - \exp\left(\frac{h_{\text{min}} - \mu_H}{b_H}\right) - \exp\left(-\frac{h - \mu_H}{b_H}\right), & \mu_H \leq h \leq h_{\text{max}} \end{cases}. \]

(20)

The relationship between channel gain and received SNR of DCO-OFDM is given in (13). Using the fundamental theorem of determining the distribution of a random variable [25], the PDF of SNR can be obtained as follows:

\[ f_S(s) = \frac{f_H\left(\sqrt{s/S_0}\right)}{2S_0 \sqrt{s/S_0}} = \frac{\exp\left(-\sqrt{s/S_0} \mu_H |\lambda_1 | \cos \mu_\theta - 2 \sin \mu_\theta| - \sqrt{s/S_0} h_{\text{min}} \right)}{2b_H \sqrt{S_0} \theta_{\text{ce}}}, \]

(21)

where \( S_0 = \frac{R_{PD}^2 P_{\text{opt}}^2}{(K/2)\eta^2 \sigma_k^2} \) and with the support range of \( s \in (s_{\text{min}}, s_{\text{max}}) \), where \( s_{\text{min}} = S_0 h_{\text{min}}^2 \) and \( s_{\text{max}} = S_0 h_{\text{max}}^2 \), with \( h_{\text{min}} \) and \( h_{\text{max}} \) given in (18) and (19), respectively.

By calculating the integral, \( F_S(s) = \int_{s_{\text{min}}}^{s} f_S(s) \, ds \), the CDF of SNR on k-th subcarrier can be obtained. The CDF of SNR can be also acquired by substituting \( h = \sqrt{s/S_0} \) in (20), i.e., \( F_S(s) = F_H\left(\sqrt{s/S_0}\right) \).

Fig. 2 shows the PDF and CDF of the received SNR obtained from analytical results compared with the Monte-Carlo simulation results. The UE is located at position \( L_3 \) and the transmitted optical power is 1 watt. The results are provided for two directions: \( \Omega = 45^\circ \) and \( \Omega = 225^\circ \). Other simulation parameters are given in Table 1. As it can be seen, the analytical models for both PDF and CDF of the received SNR match the simulation results. The factor \( c_H \) for \( \Omega = 45^\circ \) is 0. This factor for \( \Omega = 225^\circ \) is 0.975 for simulation results and 0.979 for analytical model. These results confirm the accuracy of the analytical model.

B. BER Performance

In this subsection, we aim to evaluate the effect of UE orientation on the BER performance of a LiFi-enabled device as one use case. BER is one of the common metrics to evaluate the point-to-point communication performance. Assuming the M-QAM DCO-OFDM modulation, the average BER per subcarrier of the communication link can be obtained as [26]:

\[ P_e = \int_{s_{\text{min}}}^{s_{\text{max}}} P_e(s) f_S(s) \, ds, \]

(22)
where \( P_e \) determines the BER of \( M \)-QAM DCO-OFDM in additive white Gaussian noise (AWGN) channels, which can be obtained approximately as [27]:

\[
P_e(s) \approx \frac{4}{\log_2 M} \left( 1 - \frac{1}{\sqrt{M}} \right) \left( \sqrt{\frac{3s}{M - 1}} \right), \tag{23}
\]

where \( Q(\cdot) \) is the Q-function. Substituting (21) and (23) into (22) and calculating the integral from \( s_{\text{min}} \) to \( s_{\text{max}} \), we get the average BER of the \( M \)-QAM DCO-OFDM in AWGN channels with randomly-oriented UEs. After calculating the integral and some simplifications, the approximated average BER is given as:

\[
\tilde{P}_e \approx \begin{cases} 
-\Delta_0 + \frac{1}{2} c_M P_e(S_0\mu_H^2) + \frac{1}{2} c_M M, & \mu_H \leq h_{\text{min}}, \\
\frac{1}{2} c_M M, & h_{\text{min}} < \mu_H \leq h_{\text{max}}.
\end{cases} \tag{24}
\]

where

\[
\Delta_0 = \frac{2}{\log_2 M} \left( 1 - \frac{1}{\sqrt{M}} \right) \left( \exp \left( \frac{\mu_H - h_{\text{min}}}{h_0} \right) \right),
\]

\[
c_M = \frac{4}{\log_2 M} \left( 1 - \frac{1}{\sqrt{M}} \right).
\]

The proof is provided in Appendix D.

Note that if the UE is tilted optimally towards the AP, the BER is minimum. For any arbitrary location and direction of UE, the optimum tilt (OT) angle is defined as the angle that provides maximum channel gain [8], [9], [28]. This angle is \( \theta_{\text{ot}} = \tan^{-1} \left( \frac{\lambda_0}{\lambda} \right) \) and the average BER for this tilt angle is \( \tilde{P}_e \approx P_e(S_0\mu_H^2) \) (since \( c_M = 0 \)).

Fig. 8 illustrates the BER performance of 4-QAM DCO-OFDM for three scenarios: i) a vertically upward UE, ii) a UE with a fixed polar angle and without random orientation iii) a realistic scenario in which the polar angle follows a Laplace distribution that considers the random orientation, i.e., \( \theta \sim \mathcal{L}(\mu_0, b_0) \). Here, we assume \( \mu_0 = 41^\circ \) and \( b_0 = 5.43^\circ \) as reported in [17]. Other simulation parameters are given in Table I. The results are provided for the UE’s location of \( L_1 \).

For this location, \( \theta_{\text{ot}} \approx 65^\circ \). Some interesting observations can be seen from the results shown in this figure. For the case of \( \theta = 41^\circ \) and \( \Omega = 135^\circ \), the vertically upward UE outperforms the other two scenarios. Also the gap between the exact and approximate BER is small especially for low transmission power. By changing \( \Omega \) from \( \Omega = 135^\circ \) to \( \Omega = 45^\circ \), the BER of the second and third scenarios change significantly and now outperform the vertically upward UE. The reason for this is because, in this case, the PD faces the AP. One interesting observation is that after \( P_{\text{opt}} > 3 \) watt, the BER does not decrease and is saturated. This is due to the constant term in (24), i.e., \( \frac{1}{2} c_M \), will be dominant compared to the power-dependent term, i.e., \( P_e(S_0\mu_H^2) \). In other words, due to the random orientation, there are cases that LOS link is out of the UE’s FOV and data is lost. The BER performance of second and third scenarios can still be better if \( \theta = \theta_{\text{ot}} \approx 65^\circ \).
For $\theta = \theta_{\text{rot}}$, the maximum LOS channel gain is achieved and under this condition the BER is minimum. This fact underlines that the device orientation is not always destructive. Furthermore, with $\theta = \theta_{\text{rot}}$, the UE’s random orientation has the minimum effect on the BER. We note that for a given location and $\Omega$, the $P_e$ given in (24) is always bounded to the BER of $P_e(s)$ obtained for $\theta = \theta_{\text{rot}}$ as it provides the maximum LOS channel gain.

Fig. 9 illustrates the BER performance comparison of vertically upward and randomly-oriented UEs for different modulation orders. For these results, the UE is located at $L_1$ with $\Omega = 45^\circ$ and the elevation angle is set to the optimum tilt angle, $\theta = \theta_{\text{rot}} = 65^\circ$. Other parameters are the same as given in Table I. As can be seen, randomly-oriented UEs are highly affected by the increase of the modulation order compared to vertically upward UEs. As the ratio of BER for modulation orders of $M = 4$ and $M = 16$ at $P_{\text{opt}} = 3$ watt for randomly-oriented UEs is $5 \times 10^3$ while this ratio for vertically upward UEs is just 22. The other observation is that randomly-oriented UEs with $M = 16$ and with $\theta = \theta_{\text{rot}}$ have almost the same performance as the vertically upward UEs with $M = 4$.

V. CONCLUSIONS AND FUTURE WORKS

We analyzed the device orientation and assessed its importance on system performance. The PDF of SNR for randomly-oriented device is derived, and based on the derived PDF, the BER performance of DCO-OFDM in AWGN channel with randomly-oriented UEs is evaluated. An approximation for the average BER of randomly-oriented UEs is calculated that closely matches the exact one. The role of CE angle that guarantees having LOS link in the UE’s FOV is investigated. Further, the significant impact of being optimally tilted towards the AP on the BER performance is shown. We note that even though we considered DCO-OFDM, the methodology can be readily extended to other modulation schemes, which can be the focus of future studies. Furthermore, other performance metrics such as throughput and user’s quality of service can also be assessed. Also, the device orientation impact can be evaluated in a cellular network with consideration of diffuse link.

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APPENDIX

A. Proof of (6)

Recalling that $\cos \psi = -d \cdot n_u/d$, replacing for $d = [x_u - x_a, y_u - y_a, z_u - z_a]^T$ and $n_u = [\sin \theta \cos \omega, \sin \theta \sin \omega, \cos \theta]^T$ and also noting that $\omega = \Omega + \pi$, we have:

$$
\cos \psi = \frac{(x_u - x_a)\sin \theta \cos \Omega + (y_u - y_a)\sin \theta \sin \Omega - (z_u - z_a)\cos \theta}{\sqrt{(x_u - x_a)^2 + (y_u - y_a)^2 + (z_u - z_a)^2}}
$$

$$
= \frac{\sqrt{(x_u - x_a)^2 + (y_u - y_a)^2 + (z_u - z_a)^2}}{\sqrt{(x_u - x_a)^2 + (y_u - y_a)^2 + (z_u - z_a)^2}} \cos \theta
$$

For a given location of UE and a fixed angle of $\Omega$, by using the simple triangular rules, $\cos \psi$ can be represented as:

$$
\cos \psi = \lambda_1 \sin \theta + \lambda_2 \cos \theta = \sqrt{\lambda_1^2 + \lambda_2^2} \cos \left(\theta - \tan^{-1} \left(\frac{\lambda_1}{\lambda_2}\right)\right),
$$

(26)

where $\lambda_1$ and $\lambda_2$ are given as:

$$
\lambda_1 = \frac{r}{d} \cos \left(\Omega - \tan^{-1} \left(\frac{y_u - y_a}{x_u - x_a}\right)\right),
$$

$$
\lambda_2 = \frac{h}{d}.
$$

(27)

According to the definition of critical elevation angle, if $\theta = \theta_{\text{ce}}$, then, $\cos \psi = \cos \Psi_c$. Therefore, (26) results in:

$$
\theta_{\text{ce}} = \cos^{-1} \left(\frac{\cos \Psi_c}{\sqrt{\lambda_1^2 + \lambda_2^2}}\right) + \tan^{-1} \left(\frac{\lambda_1}{\lambda_2}\right).
$$

(28)

This completes the proof of the derivation of CE angle.

B. Proof of Proposition

For a given location of UE and a fixed elevation angle, one other representation of $\cos \psi$ given in (25) would be as a function of $\Omega$:

$$
\cos \psi = \kappa_1 \cos \left(\Omega - \tan^{-1} \left(\frac{y_u - y_a}{x_u - x_a}\right)\right) + \kappa_2 \Delta(\Omega),
$$

(29)

where the coefficients $\kappa_1$ and $\kappa_2$ are given as:

$$
\kappa_1 = \frac{r}{d} \sin \theta, \quad \kappa_2 = \frac{h}{d} \cos \theta.
$$

(30)

Note that since $\theta \in [0, 90]$, we have $\kappa_1 \geq 0$ and $\kappa_2 \geq 0$. As mentioned for $\theta = \theta_{\text{ce}}$, we have $\cos \psi = \cos \Psi_c$. Then, solving $\Delta(\Omega) - \cos \Psi_c = 0$ for $\Omega$, the roots are $\Omega_{\pm} = \min\{\Omega_1, \Omega_2\}$.
and $\Omega_2 = \max\{\Omega_1, \Omega_2\}$, where $\Omega_1$ and $\Omega_2$ are given as follows:

$$\Omega_1 = \cos^{-1}\left(\frac{\cos \Psi_e - \kappa_2}{\kappa_1}\right) + \tan^{-1}\left(\frac{y_a - y_b}{x_a - x_b}\right),$$

$$\Omega_2 = -\cos^{-1}\left(\frac{\cos \Psi_e - \kappa_2}{\kappa_1}\right) + \tan^{-1}\left(\frac{y_a - y_b}{x_a - x_b}\right).$$

(31)

For the special case of $\Psi_e = 90^\circ$, (31) is simplified as:

$$\Omega_1 = \cos^{-1}\left(-\frac{h \cot \theta}{r}\right) + \tan^{-1}\left(\frac{y_a - y_b}{x_a - x_b}\right),$$

$$\Omega_2 = -\cos^{-1}\left(-\frac{h \cot \theta}{r}\right) + \tan^{-1}\left(\frac{y_a - y_b}{x_a - x_b}\right).$$

(32)

Using the sinuous function properties if $\Lambda(\Omega) \leq 0$ for $\Omega \in [\Omega_1, \Omega_2]$, then the derivative of $\Lambda(\Omega)$ at $\Omega = \Omega_{\tau_1}$ is negative, i.e., $\frac{d\Lambda(\Omega)}{d\Omega}|_{\Omega=\Omega_{\tau_1}} < 0$. For simplicity of notation, let’s denote $\Lambda'(\Omega) = \frac{d\Lambda(\Omega)}{d\Omega}|_{\Omega=\Omega_{\tau_1}}$. Using (29), we have $\Lambda'(\Omega) = -\kappa_1 \sin\left(\Omega - \tan^{-1}\left(\frac{y_a - y_b}{x_a - x_b}\right)\right) + \kappa_2$. Therefore, the range of $\mathcal{R}_\Omega$ that guarantees $\Lambda'(\Omega) > 0$ would be $[0, \Omega_{\tau_1}] \cup [\Omega_{\tau_2}, 2\pi]$. Similarly, if $\Lambda(\Omega) \geq 0$ for $\Omega \in (\Omega_{\tau_1}, \Omega_{\tau_2})$, then the derivative of $\Lambda(\Omega)$ at $\Omega = \Omega_{\tau_1}$ is positive, i.e., $\frac{d\Lambda(\Omega)}{d\Omega}|_{\Omega=\Omega_{\tau_1}} > 0$. Consequently, in this case the range of $\mathcal{R}_\Omega$ that ensures $\Lambda'(\Omega) > 0$ would be $[\Omega_{\tau_1}, \Omega_{\tau_2}]$. This completes the proof of Proposition.

C. Proof of (15)

Using (26), the CDF of $\cos \psi$ can be obtained as:

$$F_{\cos \psi}(\tau) = \Pr\{\cos \psi \leq \tau\} = \Pr\left\{\sqrt{\lambda_1^2 + \lambda_2^2}\cos \left(\theta - \tan^{-1}\left(\frac{\lambda_1}{\lambda_2}\right)\right) \leq \tau\right\}$$

$$= 1 - F_\theta\left(\cos^{-1}\left(\frac{\tau}{\sqrt{\lambda_1^2 + \lambda_2^2}}\right) + \tan^{-1}\left(\frac{\lambda_1}{\lambda_2}\right)\right).$$

(33)

where $F_\theta(\theta)$ is the CDF of the elevation angle, $\theta$. Under the assumption of Laplacian model for the elevation angle, $F_\theta(\theta)$ is given as [17]:

$$F_\theta(\theta) = \begin{cases} 
\frac{1}{2} \exp\left(\frac{\theta - \mu_\theta}{b_\theta}\right), & \theta < \mu_\theta \\
1 - \frac{1}{2} \exp\left(-\frac{\theta - \mu_\theta}{b_\theta}\right), & \theta \geq \mu_\theta 
\end{cases},$$

(34)

where $G(0) = \frac{1}{2} \exp\left(-\frac{\mu_\theta}{b_\theta}\right)$ and $G(\frac{\pi}{2}) = 1 - \frac{1}{2} \exp\left(-\frac{\pi - \mu_\theta}{b_\theta}\right)$. Note that with reported values for $\mu_\theta$ and $b_\theta$ from [17], we have $G\left(\frac{\pi}{2}\right) - G(0) \approx 1$. Therefore,

$$F_\theta(\theta) \approx \begin{cases} 
\frac{1}{2} \exp\left(\frac{\theta - \mu_\theta}{b_\theta}\right), & \theta < \mu_\theta \\
1 - \frac{1}{2} \exp\left(-\frac{\theta - \mu_\theta}{b_\theta}\right), & \theta \geq \mu_\theta 
\end{cases},$$

(35)

Finally, by recalling the definition of the CE angle given in (6), $F_{\cos \psi}(\cos \Psi_e)$ can be approximately obtained as:

$$F_{\cos \psi}(\cos \Psi_e) \approx \begin{cases} 
1 - \frac{1}{2} \exp\left(\frac{\theta_{ce} - \mu_\theta}{b_\theta}\right), & \theta_{ce} < \mu_\theta \\
\frac{1}{2} \exp\left(-\frac{\theta_{ce} - \mu_\theta}{b_\theta}\right), & \theta_{ce} \geq \mu_\theta
\end{cases}. $$

This completes the proof of (15).

D. Proof of (24)

Substituting (21) and (23) into (22), we have:

$$P_e = c_0 \int_{s_{\min}}^{s_{\max}} Q\left(\sqrt{\frac{3s}{M-1}} - \frac{1}{\sqrt{s}} \exp\left(-\frac{1}{\sqrt{s}} - \frac{\sqrt{3} \mu_\theta}{b_\theta}\right)\right) \frac{1}{s} ds$$

$$+ c_{H/M} \int_{s_{\min}}^{s_{\max}} Q\left(\sqrt{\frac{3s}{M-1}} \frac{6(s)}{s}\right) \frac{1}{s} ds$$

(36)

with $c_0$ and $c_{H/M}$ given as:

$$c_0 = \frac{c_{M}}{b_H \sqrt{\pi}} \left(2 - \exp\left(-\frac{\mu_\theta}{\sqrt{3} b_H}\right)\right),$$

$$c_{M} = \frac{4}{\log_2 M} \left(1 - \frac{1}{\sqrt{M}}\right).$$

(37)

Note that if $s_{\min} = 0$, the second integral in (36) is $c_{H/M} Q(0) = \frac{c_{M}}{b_H}$, and referring to the definition of $c_{H}$, it is zero for $s_{\min} > 0$. Thus, the second integral can be expressed as $\frac{c_{M}}{2 b_H}$ and we need to simplify the first integral. For simplicity of notation, let define $c_1 = \sqrt{\frac{3}{M-1}}$, $c_2 = \sqrt{3} \mu_\theta$, and $c_3 = \sqrt{3} b_H$. Furthermore, let $x = \sqrt{s}$, thus, the first integral in (36) can be rewritten as (38) given at the top of the next page. The right side of (38) is based on the behavior of PDF of SNR. It can be either single exponential (if $c_2 \geq \sqrt{3} s_{\min}$) or double exponential (if $\sqrt{3} s_{\min} < c_2 \leq \sqrt{3} s_{\max}$), for example, see results shown in Fig. 7. Noting that

$$\int Q(c_1 x) e^{c_2 x} dx =$$

$$c_3 e^{c_2 c_1 x} Q(c_1 x) + \frac{c_3}{2} e^{c_2 c_1 x} \left(1 - 2 Q\left(c_1 x - \frac{1}{2 c_1 c_3}\right)\right),$$

also for given values of $c_1$, $c_2$ and $c_3$, we have $Q(c_1 c_2) \approx Q(c_1 \sqrt{s_{\max}})$ and also since $\mu_H \gg \sqrt{3} b_H$, then, $e^{-\frac{3 \mu_H}{b_H}} \approx 0$. Hence, $P_e$ can be approximated by (39) presented at the top of the next page. By substituting for the values of $c_0$, $c_1$, $c_2$, $c_3$ and noting that $\sqrt{3} s_{\min} = \sqrt{3} b_H s_{\min}$ and $\sqrt{3} s_{\max} = \sqrt{3} b_H s_{\max}$ (39) can be rewritten as:

$$\hat{P}_e \approx \begin{cases} 
\frac{c_{H/M}}{2} e^{-\frac{2 \pi}{\sqrt{3} b_H}} + \frac{c_{M}}{2}, & \mu_H \leq s_{\min} \\
4 \left(1 - \frac{\sqrt{3} \mu_\theta}{\log_2 M}\right) Q\left(\sqrt{\frac{3 \mu_\theta}{\log_2 M}}\right) + \frac{c_{H/M}}{2}, & \mu_H \leq s_{\max}
\end{cases},$$

(40)

This completes the proof of (24).

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