ISAR imaging of space objects using encoded apertures

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ABSTRACT
A major threat to satellites is space debris, which are observed in orbit for a short time due to their low mass and high rotational speed. This major limitation causes them not to be fully illuminated in one snapshot, resulting in their incomplete image reconstruction and identification. In this paper, we propose a method to decrease the number of snapshots in a given observation time and use a limited number of spot beams per snapshot, which we call the encoded aperture. To recover the space debris images, an inverse problem is defined based on compressive sensing methods. Also, we show that for satellite imaging the Total Variation (TV) norm is more appropriate. We develop a procedure to recover space debris and satellites using $L_1$ and TV norms. Using simulation results, the proposed method is compared with the well-known Sparse Bayesian Learning (SBL) and $S_0$ methods in terms of the number of snapshots, minimum Mean Square Error (MSE), Signal-to-Noise Ratio (SNR), and running time. It is shown that the proposed method can successfully recover the images of space objects using a fewer number of snapshots.

1. Introduction

Inverse Synthetic Aperture Radar (ISAR) is an imaging radar for identifying, recognizing, and distinguishing moving objects from each other (Chen and Martorella 2014). This type of radar can extract a Two-dimensional (2D) image of space objects in both range and azimuth (or cross-range) directions. In addition, it can work in almost all-weather conditions, making it an appropriate choice for imaging of rotating objects (Ozdemir 2012) such as space debris. These objects may lie in different orbits due to the remaining parts of disabled satellites, spacecraft, meteors, etc. The dimensions of space debris are very diverse and due to their low mass, they can move at high speeds. As a result, an important challenge for spacecraft or satellite launching is their impact on orbital debris, which could create expensive economic and research costs (Anger et al. 2019). Accordingly, space debris imaging has received increasing attention in space research for which several optical or radar-based methods have been reported (Sciré, Piergentili, and Santoni 2017; Zhu, Zhu, and Liao 2015). In this regard, ISAR is a good choice for creating spatial images of space debris, which may also be used for imaging and monitoring active satellites. To make acceptable images
of space debris, two major issues are important to note. First, these objects have a high rotational speed due to their low mass, which leads to generating high-bandwidth Doppler frequencies. Secondly, the low observation time of space objects in orbit is a critical limitation in ISAR imaging and thus an important motivation for reducing the processing time (Zhu, Zhu, and Liao 2015). In order to increase the resolution of ISAR images, given the sparse nature of space debris images, Compressive Sensing (CS) methods are reasonable techniques to use. In (Hu et al. 2014), a method based on randomly stepped frequency radar is presented by benefiting from sparse recovery and CS, which improves the range and azimuth resolutions. Also, a method is addressed by (Kang et al. 2021) based on orthogonal coding signals with different delays and a modified Smoothed L0-norm (SL0) algorithm. By combining the motion characteristics of space debris, a method is presented by (Zhu, Zhu, and Liao 2015) for extracting high-resolution images using CS techniques. The method of Multiple Azimuth Beams addressed by (Chen et al. 2018) allows ISAR imaging of a large area using simple processing, electronic scan, and digital beamforming. Also, (Wang, Xu, and Jin 2018) has recovered the scatterers using consecutive observations of space debris based on CS. Moreover, radar imaging for spinning space debris with a dimension smaller than the radar range resolution is developed by (Wang et al. 2010) and (Baskakov et al. 2019). An ISAR imaging method based on sparse aperture and micro-Doppler removal using low-rank and sparsity properties is proposed by (Zhang, Liu, and Li 2021). Also, in (Zhang et al. 2021), an imaging method is developed based on the convolutional reweighted L1-norm (L1) minimization problem to recover ISAR images from sparse apertures. We focus on the problem of rapid rotation of space debris, which requires having enough snapshots to fully recover the image. On the other hand, due to the low observation time of space debris, access to a sufficient number of snapshots in a limited time is problematic. To solve this problem and improve the quality of the retrieved images, we use several snapshots and evaluate the performance of the L1 and TV norms to more successfully recover ISAR images of satellites and space debris.

2. Signal model

According to (Zhu, Zhu, and Liao 2015), the baseband echo or the received signal after de-chirping and translational motion compensation consists of $K$ scatterers as

$$S_r(t_m) = \sum_{k=1}^{K-1} \sigma_k \exp\left[j \frac{4\pi r_k \cos(\omega t_m + \phi_k)}{\lambda}\right],$$

where $\lambda$ is the radar signal wavelength, $\sigma_k$ is the reflection coefficient of the $k$-th scatterer, $r_k$ and $\phi_k$ show the respective polar coordinates, $\omega$ is the angular velocity, and $t_m$ denotes the slow time, i.e., the time of the received signal in the azimuth direction. Then, the instantaneous phase of the $k$-th scatterer is given by

$$\theta(t_m) = \frac{4\pi r_k \cos(\omega t_m + \phi_k)}{\lambda},$$

where the instantaneous phase derivative is defined as
Next, given that \( \theta_d(t_m) = 2\pi f_d \), we get

\[
f_d = \frac{d\theta(t_m)}{2\pi dt_m} = \frac{-2r_k \omega \sin(\omega t_m + \phi_k)}{\lambda},
\]

where \( f_d \) is the Doppler Frequency. The Doppler bandwidth of the received signal is defined as ‘(Zhu, Zhu, and Liao 2015)’

\[
\Delta f_d = \frac{4r_k \omega}{\lambda},
\]

in which large angular velocities produce large Doppler bandwidths ‘(Zhu, Zhu, and Liao 2015)’.

3. Proposed ISAR imaging method

Due to the fast movement of space debris, their imaging in a short observation time is difficult. Here, we present a method for imaging satellite and space debris as follows:

1. Instead of a single snapshot for imaging, we use multiple snapshots.
2. To reduce the amount of data, computations, and processing time in each snapshot, we use randomly selected spot beams in each snapshot, which we call encoded aperture imaging.
3. We use the Bernoulli distribution to determine active/inactive spot beams in each encoded aperture.
4. We use \( L_1 \) and TV norms for image recovery using a limited number of snapshots and spot beams.

3.1. Encoded aperture

To capture images of low-mass, high-speed, and space-spinning debris, a single snapshot may not produce a high-quality image. This difficulty can be effectively compensated by using more snapshots, each of which with \( N \) spot beams radiating simultaneously. However, this results in the need for more processing time for image recovery, which may be undesirable due to the short time of observing space debris. On the other hand, the time interval to receive echoes from radar targets is determined by the Pulse Repetition Interval (PRI), which accordingly limits the image processing time. Also, if we increase the PRI, the ambiguity in the Doppler measurement increases, which leads to losing the image quality ‘(Zhu, Zhu, and Liao 2015)’. To overcome this difficulty, in the proposed algorithm, we generate a few randomly produced spot beams in each snapshot using the Bernoulli distribution ‘(Baraniuk et al. 2008)’, (namely an encoded aperture) to produce an incomplete image of an object. Then, by using a few encoded apertures in an inverse problem and some recovery methods, a complete and improved image is retrieved. To illustrate, Figure 1(a) shows an encoded aperture consisting of three spot beams, and Figure 1(b) illustrates \( M \) encoded apertures with five spot beams at each
The radar Half Power Beam Widths for each spot beam are defined in elevation and azimuth directions by $\theta$ and $\varphi$, respectively. (Plouchart et al. 2019) show that the practical realization of such narrow spot beams is possible even up to 10,000 beams per second.

3.2. Retrieval of ISAR images using CS

A space object image observed at each encoded aperture by the spot beams in both range and azimuth directions is represented by a matrix $\mathbf{X} \in \mathbb{C}^{n \times n}$, where $n$ shows the maximum number of spot beams in each direction. We vectorize $\mathbf{X}$ as $\mathbf{x} \in \mathbb{C}^{N}$, with $r$ non-zero entries and $N = n \times n$, indicating sparsity of the vector. Also, the number of encoded apertures is $M$, each one randomly generated based on the Bernoulli distribution, with 0 and 1 denoting the presence and absence of a spot beam, respectively. By vectorizing each encoded aperture as the row of a matrix, we can construct

$$
\Phi = \begin{bmatrix}
\Phi_{11} & \Phi_{12} & \cdots & \Phi_{1N} \\
\Phi_{21} & \Phi_{22} & \cdots & \Phi_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\Phi_{M1} & \Phi_{M2} & \cdots & \Phi_{MN}
\end{bmatrix} \in \mathbb{C}^{M \times N},
$$

where $\Phi_{ij}$, $1 \leq i \leq M$, $1 \leq j \leq N$, are defined as

$$
\Phi_{ij} = \begin{cases} 
0 & j-\text{th spot beam at the } i-\text{th encoded aperture is absent} \\
1 & j-\text{th spot beam at the } i-\text{th encoded aperture is present}
\end{cases}.
$$

This matrix indicates whether the spot beams are active in both range and azimuth directions or not. Note that each encoded aperture normally forms an incomplete image of a space object in each snapshot, and $M = N$ encoded apertures are required to complete the image. However, due to the observation time constraint, we use $M < N$ encoded apertures to retrieve the space debris images using the $L_1$ norm and satellite images using the TV norm. Applying the $L_1$ norm is justified due to the fact that the space debris are physically smaller and their images are more sparse compared to those of a satellite. According to (Rossi, Haimovich, and Eldar 2014), in order to satisfy the Restricted Isometry Property condition, $\Phi$ should have the following conditions:
(I) It should be a non-square matrix with \( M < N \),
(II) Its entries are drawn from the Bernoulli distribution ’(Baraniuk et al. 2008)’,
(III) Its rows are orthogonal to have the least mutual coherence and further increase
the probability of successful recovery ’(Candes and Wakin 2008)’.

By representing the signals received from active spot beams in each snapshot based on
Equation (1), we define the vector \( \mathbf{y} \in \mathbb{C}^M \) for \( M \) snapshots as

\[
\mathbf{y} = \Phi \mathbf{x},
\]

where \( \mathbf{y} = [y_i(t_m)] \) and \( y_i(t_m) = S_i(t_m) \) is the received signal in the \( i \)-th snapshot and \( i = 1, \ldots, M \). Then, considering an inverse problem for (8), the space debris images given by \( \mathbf{x} \) can be retrieved using the \( L_0 \) norm. However, this sparse problem is non-convex and thus NP-hard, which can be solved by replacing the \( L_1 \) norm as ’(Zhang et al. 2015)’

\[
\min \| \mathbf{x} \|_1, \quad \text{s.t.} \quad \mathbf{y} = \Phi \mathbf{x},
\]

where \( \| \cdot \|_1 \) shows the \( L_1 \) norm. Equation (9) is also known as the Basis Pursuit (BP)
problem ’(Candes, Romberg, and Tao 2006)’. It has been shown that smaller mutual
coherence in the columns of \( \Phi \) leads to a more successful recovery of the unknown vector \( \mathbf{x} \’(Rossi, Haimovich, and Eldar 2014)’
. On the other hand, to retrieve satellite images in orbit, we recommend the TV norm defined in isotropic form as the \( L_2 \) norm of the discrete gradient \( \nabla \mathbf{X} \) as ’(Jonathan et al. 2020)’

\[
\| \mathbf{X} \|_{\text{TV}} := \sum_{i,j} \| \nabla \mathbf{X} \|_2, \quad \nabla \mathbf{X} = [D_h(\mathbf{X}) D_v(\mathbf{X})],
\]

where \( D_h(\mathbf{X}) \) and \( D_v(\mathbf{X}) \) are the horizontal and vertical differences defined as

\[
D_h(\mathbf{X}) = \mathbf{X}(i + 1,j) - \mathbf{X}(i,j),
\]

\[
D_v(\mathbf{X}) = \mathbf{X}(i,j + 1) - \mathbf{X}(i,j).
\]

By this definition, the TV norm incorporates the differences between the adjacent points
of an image. In this work, we have used the NESTA algorithm ’(Jonathan et al. 2020)’ to
solve the optimization problems defined in Equations (9) and (10). The flowchart of the
proposed method for ISAR imaging of space debris and satellites is shown in Figure 2. In
the case of low-resolution recovered images, the number of the encoded apertures is
increased, and some steps are repeated. By increasing the number of snapshots, we can
be sure that most of the target points are lit randomly, and by doing so, the resolution of
the target image increases. When no space debris are detected, a satellite can be
launched. Otherwise, the necessary measures should be taken from the Earth station.

3.3. Observation time of space objects

The required time for observation of space objects includes spot beams generating and
steering, data acquisition, and processing time in each encoded aperture. This time totally
takes less than 100 \( \mu \)s and 10,000 spot beams can be generated per second ’(Plouchart
et al. 2019)’. Thus, by considering \( M_t \) as the time of each snapshot, the total time for \( M \)
snapshots is \( M_t \), which should be less than the observation time for ISAR imaging, which
is typically $3 - 5$ seconds ’(Sciré, Piergentili, and Santoni 2017)’. We will show that this value is achievable by applying the proposed method.

4. Simulation results

The ISAR specifications shown in Table 1 are as close to the actual conditions as possible ’(Anger et al. 2019)’. We also assume that we have 40 spot beams in each direction of range and azimuth, for which the maximum possible number of spot beams is 1600 spot beams. However, in the proposed method, we only randomly select a smaller number of spot beams. On the other hand, to successfully recover ISAR images, we need at least a limited number of snapshots, for which we present the results for 100, 200, and 300 snapshots.

4.1. Effect of the number of snapshots

We evaluate the performance of the proposed method for 100, 200, and 300 snapshots at $\text{SNR} = 5 \text{ dB}$’(Kang et al. 2021)’ in three different scenarios including; 1) only satellite in orbit, 2) only space debris in orbit, and 3) both satellite and space debris in orbit. Also, to recover the image, we compare $L_1$, TV, $\text{SL}_0$ norms, and SBL ‘(Huiping et al. 2015)’. Figure 3 shows the results for the first scenario. As seen, as opposed to the other methods, the TV norm has retrieved acceptable images using a small number of snapshots. One can note that the SBL has also weakly recovered the image, but it does impose a heavy computational burden compared to the TV norm.
4.2. Retrieval of ISAR images using CS

In the second scenario, we consider only space debris in orbit, for which the results are shown in Figure 4. As seen, the $L_1$ norm has clearly recovered the space debris using only 100 snapshots that are lower than those of the other methods. This success can be justified by noting that images of space debris can

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Table 1. ISAR imaging specifications.

| Parameter                                 | Value                           |
|-------------------------------------------|---------------------------------|
| Radar type                                | Pulse radar                     |
| Frequency band                            | X Band                          |
| Center frequency                          | 10.2 GHz                        |
| Bandwidth                                 | 4.4 GHz                         |
| Transmitter signal                        | LFM                             |
| PRF                                       | 200 Hz                          |
| Pulse length                              | 50 μs                           |
| Encoded aperture dimensions (range $\times$ azimuth) | $40 \times 40$                |
| Number of possible spot beams, active or inactive, per snapshot | 1600                            |
| Observation time                          | 3–5 s                           |
| Spot beams generating and processing time | $100 < \mu$s                    |
| Number of encoded apertures               | $>100$                          |
| ($M$) and $<1600$                         |                                 |
| Satellite orbital height                  | $524 \times 544$ km             |
| Satellite speed                           | $7430$ ms$^{-1}$                |
| Rotation speed of space debris            | $1$ rad $s^{-1}$                |
| Random distribution for $\Phi$            | Bernoulli                       |

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Figure 3. Retrieved images for only satellite in orbit using $L_1$, TV, $SL_0$ norms, and SBL. Columns (a), (b), and (c) are for 100, 200, and 300 snapshots, respectively.
be considered reasonably sparse. Also, noting that the required time for each snapshot is 100 μs, the total time for 100 to 300 snapshots in comparison to the rotation speed of space debris, which is 1 rad s⁻¹ (Ru M. et al. 2022), is relatively small. This, as a result, does not affect the performance of the proposed method in image recovery. In the third scenario, there exists a satellite in the vicinity of space debris in orbit. The results shown in Figure 5 reveal that, unlike the other methods, the TV norm has reliably recovered the satellite using a smaller number of snapshots.

Figure 4. Retrieved images for only space debris in orbit using $L_1$, TV, $SL_0$ norms, and SBL. Columns (a), (b), and (c) are for 100, 200, and 300 snapshots, respectively.

Figure 5. Retrieved images for a satellite in the vicinity of space debris in orbit using $L_1$, TV, $SL_0$ norms, and SBL. Columns (a), (b), and (c) are for 100, 200, and 300 snapshots, respectively.
4.3. Performance of methods in presence of noise

In the next simulation in Figure 6, both satellite and space debris are in orbit, and image recovery is investigated in the presence of noise at SNR = −5, 5, and 15 dB. It is observed that the TV norm has acceptably recovered both objects at 5 dB while the others have failed, indicating the better performance of the TV norm in such scenarios.

4.4. Comparison of MSEs in image recovery

The Mean-Square Errors (MSEs) of different methods are compared for only satellite in orbit at SNR = −5, 0, 5, 10, and 15 dB. The results are the average of 100 independent trials of the experiment. As seen in Figure 7(a), the MSEs for both L₁ and TV norms are

![Figure 6. Retrieved images for a satellite in the vicinity of space debris using L₁, TV, SL₀ norms, and SBL. Columns (a), (b), and (c) are for −5, 5, and 15 dB SNR, respectively.](image)

![Figure 7. (a) MSEs of L₁, TV, SL₀ norms, and SBL for SNR = −5 to 15 dB. (b) Running time of L₁, TV, SL₀ norms and SBL for 100 to 300 snapshots.](image)
lower than those of the other methods at all SNRs. These results are in agreement with the performance of the corresponding algorithms in Figure 3.

4.5. Comparison of running times

The run times of the recovery methods are compared in Figure 7(b) for 100 to 300 snapshots at 5 dB SNR. As shown, the SL₀ is the fastest; then, L₁ and TV norms, and finally SBL is the slowest. However, it is imperative to note that although SL₀ is fast, its performance in image recovery is much lower for a small number of snapshots and at low SNRs, as similarly resulted in Figures 3–7a. Also, SBL is also slow due to the inclusion of prior information in the computations.

5. Conclusion

Given that space debris is a significant threat to satellites before launch or in orbit, we proposed an encoded aperture method for ISAR imaging of space objects. Due to the fast rotation of space debris and having a limited time for observation, we focused on reducing the number of snapshots as much as possible. To do so, we used a few spot beams in each snapshot randomly generated by the Bernoulli distribution. To recover the space object images, we applied L₁ and TV norms. The performance of these methods was evaluated for a different number of snapshots and various SNRs. Accordingly, the corresponding MSEs and running times were compared. The simulation results showed that the TV norm can image a satellite in the absence/presence of space debris in orbit using a fewer number of snapshots compared to L₁, SL₀, and SBL methods. Also, by comparing the MSEs, the TV norm performed better than the other methods at low SNRs. Moreover, both L₁ and TV norms were faster than the SBL in terms of running time. Of course, SL₀ achieved much faster speed than the others, but at the cost of generating greatly low-resolution images that are mostly unusable. On the other hand, L₁ outperformed the SBL for space debris in the absence of satellites due to the sparse identity of the respective images. These results led us to design an ISAR imaging procedure by using the L₁ norm for space debris with no satellite in orbit and the TV norm for satellites in the absence/presence of space debris in orbit.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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