Studies of Turbulent Angular Momentum Transport in Taylor-Couette Flow via 2D LDV

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Abstract. The turbulent (i.e., fluctuation-driven) transport of angular momentum in Taylor-Couette flow may be observed locally and directly with second order velocity fluctuation correlations obtained by dual beam (2D) Laser Doppler Velocimetry (LDV). This method is complementary to torque measurements, and initial results are commensurate with earlier torque studies performed in turbulent regimes. Other results utilizing this technique with centrifugally-stable high Reynolds number \((Re)\) flows indicate that the fluid is quiescent and devoid of significant angular momentum transport, even up to \(Re \sim 10^6\). This latter result is thought to bear upon the hydrodynamic properties of cool astrophysical accretion disks.

1. Introduction

Here we report on the turbulent (i.e., fluctuation-driven) transport of angular momentum for both centrifugally-stable and centrifugally-unstable laboratory flows at high Reynolds number \((Re \sim 10^4\) to \(10^6\)). The transport is observed directly and locally as second order velocity fluctuation correlations obtained by dual beam (2D) Laser Doppler Velocimetry (LDV). The centrifugally-unstable flows studied here have theoretical applications related to the scaling behaviour of Rayleigh-Bénard convection \([4,5]\), and also find practical application in various turbo-machinery. On the other hand, the centrifugally-stable flows we have studied, in an anticyclonic regime at high Reynolds number \((\sim 10^6)\), have application as a laboratory analogue to astrophysical accretion disks. The turbulent transport of angular momentum in disks by purely hydrodynamic mechanisms (i.e. without magnetism) is a longstanding issue in astrophysics, and one which laboratory Taylor-Couette results may be relevant to [e.g. 6,8,10].
2. Description of Apparatus and Technique

A schematic of our experimental apparatus is given in Figure 1a. Note that we have modified the traditional Taylor-Couette design to include two independent intermediary end-rings between the inner and outer cylinders, allowing for differential rotation at the vertical boundaries. These rings allow for the reduction of friction-driven secondary flows (Ekman circulation). We are thus able to create a Couette-like velocity profile, i.e. the ideal profile for diminishing gap width (or frictionless end-caps), but in a wide-gap apparatus. For more information on these foundational results, along with more detailed information on this device and its operation, see [1].

The data we present here was obtained via LDV, a well-known technique yielding time-series of velocity data $V_i(t)$ from which various statistics may be estimated. To measure correlated fluctuations, that is $\langle v'_i v'_j \rangle$ where $v'_i = V_i - \langle V_i \rangle$, two synchronized orthogonal beam pairs acquired data under a condition of simultaneity [3]. We measured both radial ($r$) and azimuthal ($\theta$, i.e. circumferential) components of velocity from underneath the apparatus, as shown in Figure 1b.

![Figure 1. (a): Apparatus detail. Inner cylinder radius is 7.1 cm, outer cylinder 20.3 cm. Cylinder heights are about 28 cm, giving an aspect ratio $\Gamma$ of $\sim 2$ and a radius ratio $\eta \sim 0.35$. This figure has previously appeared in [6]. (b): Arrangement for 2D LDV data acquisition. Note that the beam pair measuring azimuthal velocity is orthogonal to the plane of the paper and so appear as one in the figure.](image-url)
Angular momentum transport in Taylor-Couette flows is typically observed as a torque upon the inner and/or outer cylinder. The 2D LDV method is complementary to the torque method, and also has the advantage of being a local and direct measurement. It also has an advantage over in-situ probes such as hotwires by being unperturbative to the fluid. Note that there are two practical requirements for this technique to be useful: to be able to observe the fluctuation correlation pair $\langle v'_i v'_r \rangle$ needed for angular momentum transport, one needs transparent end-cap(s) and a relatively wide annular gap width.

3. Initial Results

To compare with earlier studies featuring torque measurements, the measured velocity correlation is here given as a dimensionless torque per unit length, $G$, expressed as

$$ G = \frac{2\pi r^2 \langle v'_i v'_r \rangle}{\nu^2} $$

where $\nu$ is the kinematic viscosity. In torque measurements, $G$ is typically given as

$$ G = \frac{\tau}{\rho \nu^2} $$

where $\tau$ is the measured torque per unit length. Power-law scaling relations are sought of the form $G \propto Re^\alpha$, but for turbulent flow it is now recognized that no strict scaling exists, and that $\alpha$ is actually a mildly increasing function of $Re$ [7,9], a behaviour that has recently been addressed with a promising new theory [5].

We have recently pursued an initial study on the scaling of angular momentum transport with $Re$ for centrifugally-unstable turbulent flow using 2D LDV. For this study the turbulence was driven by the apparatus in a 'split' configuration; that is, with the inner cylinder (and ring) at some finite speed and with the outer cylinder (and ring) held stationary. A fit to the data for $Re$ between $2 \times 10^4$ and $2 \times 10^5$ indicates a scaling exponent $\alpha$ of about 1.7, as seen in Figure 2. This exponent is in general agreement with previous work utilizing torque at these Reynolds numbers [7,9]. The absolute magnitude of the angular momentum transport ($G \sim 10^8$ for $Re \sim 10^5$) is also in agreement with this previous work, despite the different proportions of our apparatus. Preliminary data for a wider range of $Re$ (down to $10^3$) indicates that the scaling exponent increases slightly with increasing $Re$, also consistent with earlier results. This ongoing work will be detailed in a future publication [2]. For now, we may say that 2D LDV appears to be a viable and complementary alternative to torque measurements for these flows.
We also applied the 2D LDV technique to centrifugally-stable high \(Re\) flows, where centrifugal stability was ensured by meeting the Rayleigh stability criterion (globally, that is \(\left[\Omega r^2\right]_{\text{inner}} < \left[\Omega r^2\right]_{\text{outer}}\)), while keeping Ekman circulation significantly reduced with appropriate intermediate ring speeds, as in [1]. Even at high \(Re\) (~ \(10^6\)), these flows were observed to have relative velocity fluctuation levels of only 1-2\%, and a normalized correlated fluctuation level \(\beta = \frac{\left\langle u'v'\right\rangle}{\left\langle u^2\right\rangle}\) on the order of \(10^{-7}\) to \(10^{-6}\). Experimental details of this investigation may be found in [6]. Such extremely low values, along with separate visual observations with Kalliroscope\textsuperscript{TM}, indicate a nearly quiescent flow: one that in fact was indistinguishable from laminar flow. The lack of turbulence and significant turbulent angular momentum transport in this anticyclonic regime is thought to bear upon the nonlinear hydrodynamic stability of cool astrophysical disks, as we recently addressed in [6]. Though there is ongoing debate on the exact instability mechanism(s) responsible for accretion disk turbulence (e.g. [8, 10]), the dominant mechanism is generally thought to be the magnetorotational instability (MRI). A new experimental MRI investigation, utilizing a magnetized liquid gallium alloy, is the topic of future research in the current apparatus.

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