Pion-pion, pion-kaon, and kaon-kaon interactions in the one-meson-exchange model

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The SU(3)-symmetric one-meson-exchange mechanisms are used in order to describe the ππ, πK, and KK interactions at low energies (\(\sqrt{s} < 1.5\) GeV). In the model, both s- and t-channel exchange diagrams are considered in ππ and πK scatterings, in which the coupling constants and cut-off masses in form factors are determined to reproduce experimental phase shifts with \(J < 3\). The form factors are examined both in monopole and Gaussian types. At the same time as giving quantitative predictions for KK interaction with \(J = 0 \sim 3\), the investigation of the \(\eta\) channel effects, the \(\eta\eta\) channel in ππ scattering, and the \(\eta K\) channel in πK scattering are also interesting discussion points in this paper. The scattering lengths of all states in ππ, πK, and KK interactions and the pole positions of the S-matrix are shown.

Subject Index D32

1. Introduction

The strong forces acting between mesons at low energies (\(\sqrt{s} < 1.5\) GeV), especially the pion–pion (ππ) and pion–kaon (πK) forces, have been the subject of investigation for some decades, experimentally as well as theoretically. In many theoretical models, such as the SU(3) symmetric meson-exchange models [1] and the SU(3) and SU(2) chiral perturbation models [2,3], hadron–hadron interactions have been treated for two decades. Oller and Oset [2] used a non-perturbative coupled-channel approach to deal with the meson–meson interaction in the strangeness \(S = 0\) sector at energies below \(\sqrt{s} = 1.2\) GeV and found the poles for the \(f_0(980)\) and \(a_0(980)\) resonances. In 1999, a non-perturbative method, which combined constraints from chiral symmetry breaking and coupled-channel unitarity to describe the meson–meson interaction up to about 1.2 GeV in perturbation theory, was proposed by Oller, Oset, and Pelaez [3].

The purpose of our work is to construct a unified hadron–hadron potential model that is appropriate for all baryon–baryon (BB) [4], meson–baryon (MB) [5], and meson–meson (MM) interactions. This work is the first step towards extending our model to include meson–meson interactions. In this step, we do not explicitly try to obtain a model consistent with the BB and MB potential model.

In this paper, we not only construct the ππ and πK interactions but also give a prediction of the KK (\(\bar{K}K\)) interaction based on the SU(3)-symmetric one-meson-exchange mechanisms. Concretely, by using common parameters (coupling constants and form factors), we determine the coupled-channel ππ–KK–ηη \(\eta K\) potentials in order to reproduce low-energy (\(\sqrt{s} < 1.5\) GeV) ππ and πK scatterings and quantitatively predict the KK interaction.
In Sect. 2, we give the formalism of the model. The vector, scalar, and tensor meson-exchange potentials are determined based on the SU(3) symmetric interaction Lagrangians. The method, which is used to calculate the potentials, is described in detail in Sect. 2.2. The potentials are given in the momentum space. The coupled-channel Lippmann–Schwinger (LS) equations are solved with relativistic kinematics in order to obtain the T-matrix. The S-matrix is then acquired from the on-shell T-matrix. Solutions of the LS equations are obtained by a kind of numerical matrix inversion method that was developed by Haftel and Tabakin [6]. In s-channel exchange diagrams, we introduce bare masses for exchanged mesons and perform renormalization calculations to determine the poles of the S-matrix that correspond to physical resonance poles. In order to discuss the η channel effects, we consider the potential model without η channels. We find that the η channel effects are not small, but these effects can be fairly well compensated by the readjustment of coupling constants and form factors mainly in s-channel exchange diagrams.

In Sect. 3, numerical results are compared with experimental analysis and quantitative predictions for the KK interaction are given. In Sect. 4, we mainly discuss the effects of η channels in ππ–K K–ηη scattering. The last section is devoted to summarizing this paper and giving future prospects.

2. Formalism of the model

In meson–meson potentials, we consider two kinds of one-meson-exchange diagrams, i.e., the t- and s-channel exchange diagrams that are shown in Fig. 1. We do not need to treat the u-channel exchange, because it is already taken into account as the t-channel exchange. For example, the K∗ exchange in the π K interaction, which provides an exchange force, is regarded as a t-channel exchange (t = (p′ K–pπ)²).

We use ρ, K∗, ω, φ, f2, ε, and κ as exchanged mesons. We emphasize that both the t- and s-channel ρ(K∗) exchanges have to be taken into account for the ππ(π K) channel to obtain a quantitative description of the experimental phase shifts. The scalar–isoscalar ε(1400), the isoscalar tensor (spin-two) meson f2(1270), and the strange scalar meson κ(1430) in s-channel exchange are necessary but their t-channel contributions are neglected because their large mass bring too short range an interaction. Both the t- and s-channels are used to construct the π K interaction with ρ, K∗, and κ exchanged mesons. The KK interaction acts only on t-channel exchange diagrams with ρ, ω, and φ meson exchange. When extending the model for investigating the effect of ηη and ηπ channels in ππ scattering and ηK in π K scattering, ε and K∗ contribute to the coupling.
2.1. Interaction Lagrangians

For vertices in meson-exchange diagrams, we assume the following types of interaction Lagrangians $\mathcal{L}_{ppm}$ ($m = s, v, t$):

\[
\begin{align*}
\mathcal{L}_{pps} &= \frac{f_{pps}}{m_\pi} \partial^\mu \phi_p(x) \partial_\mu \phi_p(x) \phi_s(x), \\
\mathcal{L}_{ppv} &= g_{ppv} \left( \partial^\mu \phi_p(x) \phi_p(x) - \phi_p(x) \partial^\mu \phi_p(x) \right) \phi_v^\mu, \\
\mathcal{L}_{ppt} &= g_{ppt} \frac{2}{m_\pi} \partial^\mu \phi_p(x) \partial_\nu \phi_p(x) \phi_t^{\mu\nu}.
\end{align*}
\]

In the above equations, $\phi_s, \phi_p, \phi_v^\mu$, and $\phi_t^{\mu\nu}$ are the field operators for scalar, pseudoscalar, vector, and tensor mesons, respectively. By treating the interaction Hamiltonian $H_I = -\int L \, d^3x$ in time-ordered perturbation theory, we construct the potentials from scalar-, vector-, and tensor-exchange diagrams.

For all interaction Lagrangians, we assume the derivative type of coupling. This type of coupling seems to be favorable from the viewpoint of chiral symmetry and the low-energy theorem. By comparing with the interaction Lagrangian $\mathcal{L}_2$ and $\mathcal{L}_4$ in the chiral perturbation model [3], we find that one-vector-exchange potentials give a physical interpretation for the main part of the $\mathcal{L}_2$ contributions while the scalar- and tensor-exchange potentials, which provide short-range interaction, only pick up some essential contributions from copious $\mathcal{L}_4$ contributions.

2.2. One-meson exchange potentials

In our calculations, all kinematical variables are defined in the center of mass system of two interacting mesons and employ relativistic kinematics. In practice, using the notation of Fig. 1, we define:

\[
\begin{align*}
p_1^\mu &= (\omega_1, p_1), & p_1'^\mu &= (\omega_1', p_1'), & p_2'^\mu &= (\omega_2, p_2), & p_2'^\mu &= (\omega_2, p_2), \\
p_1 &= \sqrt{m_1^2 + p_1^2}, & p_1' &= \sqrt{m_1'^2 + p_1'^2}, \\
p_2 &= \sqrt{m_2^2 + (p_1)^2}, & p_2' &= \sqrt{m_2'^2 + (p_1')^2}, \\
p_\alpha &= \sqrt{m_\alpha^2 + (p_1 - p_1')^2} = \sqrt{m_\alpha^2 + p_1^2 + p_1'^2 - 2p_1 p_1' \cos \theta}.
\end{align*}
\]

The above equations are applied for $t$-channel exchange. For $s$-channel exchange, the equations are almost the same but:

\[
p_\alpha = 0, \quad \omega_\alpha = m_\alpha.
\]

For vector exchange, using the relations, we always find

\[
\sum_{\lambda_\alpha} \epsilon^\mu(p_\alpha, \lambda_\alpha) \epsilon^{*\nu}(p_\alpha, \lambda_\alpha) = -g^{\mu\nu} + \frac{p_\alpha^\mu p_\alpha^\nu}{m_\alpha^2},
\]

where $\epsilon^\mu(p_\alpha, \lambda_\alpha)$ is the polarization vector for a vector meson with momentum $p_\alpha$ and helicity $\lambda_\alpha$. By using the interaction Lagrangian $\mathcal{L}_{ppv}$ given in Eq. (1), we obtained the potential for $t$-channel exchange.
vector meson exchange given by:

\[
\langle 1'2|V_s^{(t)}(z)|12 \rangle = \frac{g_2 g_3}{(2\pi)^3} F_1^{(s,1)}(\omega_p^2) F_2^{(s,1)}(\omega_p^2) \frac{1}{8m_{\alpha}} \sqrt{\omega_1' \omega_2' \omega_1 \omega_2} (p_1' + p_1)_\mu (p_2' + p_2)_\nu \times \left( -g^{\mu \nu} + \frac{p_2^\mu p_2^\nu}{m_2^2} \right) \left( \frac{1}{z - \omega_1' - \omega_2 - \omega_2'} + \frac{1}{z - \omega_1' - \omega_1 - \omega_2 - m_\alpha} \right). \tag{5}
\]

The form factors \(F^{(t)}\) will be given in Sect. 2.4. We need an additional factor \(\sqrt{\frac{1}{2}}\) with \(k = 0\) when both the initial and final states consist of different particles: \(k = 1\) when one of the states consists of identical particles, \(k = 2\) when both are identical.

For the potential by s-channel vector meson exchange, we obtain:

\[
\langle 1'2'|V_v^{(s)}(z)|12 \rangle = \frac{g_2 g_3}{(2\pi)^3} F_1^{(s,1)}(\omega_p^2) F_2^{(s,1)}(\omega_p^2) \frac{1}{8m_{\alpha}} \sqrt{\omega_1' \omega_2' \omega_1 \omega_2} (p_1' - p_1)_\mu (p_2' - p_2)_\nu \times \left( -g^{\mu \nu} + \frac{p_2^\mu p_2^\nu}{m_2^2} \right) \left( \frac{1}{z - m_0^0} + \frac{1}{z - \omega_1' - \omega_2 - \omega_1 - \omega_2 - m_{\alpha}} \right), \tag{6}
\]

where \(p_2^\mu = (m_{\alpha}, 0)\), \(\omega_p = \omega_1 + \omega_2\), \(\omega_p' = \omega_1' + \omega_2'\), \(m_0^0\) is bare mass, and \(m_{\alpha}\) is physical mass.

The s-channel potential for gradient coupling is given as follows:

\[
\langle 1'2'|V_v^{(s)}(z)|12 \rangle = \frac{g_2 g_3}{(2\pi)^3} F_1^{(s,2)}(\omega_p^2) F_2^{(s,2)}(\omega_p^2) \frac{1}{8m_{\alpha}} \sqrt{\omega_1' \omega_2' \omega_1 \omega_2} \times \left[ \frac{1}{m_2^2} \left[ (\omega_1' - \omega_2')^2 - 4P_1'^2 \right] \left[ (\omega_1 - \omega_2)^2 - 4P_1^2 \right] \right] \times \left( \frac{1}{z - m_0^0} + \frac{1}{z - \omega_1' - \omega_2' - \omega_1 - \omega_2 - m_{\alpha}} \right). \tag{7}
\]

In order to calculate the potential by the s-channel tensor meson \(f_2\) exchange, we need the polarization tensor of the spin-two meson:

\[
\sum_{\lambda_{\alpha}} \epsilon^{\mu \nu} (p_\alpha, \lambda_\alpha) \epsilon^{\rho \sigma} (p_\alpha, \lambda_\alpha^*) = \frac{1}{2} \left( P^{\mu \rho} P^{\nu \sigma} + P^{\mu \sigma} P^{\rho \nu} \right) - \frac{1}{3} P^{\mu \nu} P^{\rho \sigma}, \tag{8}
\]

where \(P^{\mu \nu} = -g^{\mu \nu} + \frac{p_\alpha^\mu p_\alpha^\nu}{m_\alpha^2}\).

Then, the potential for the tensor meson is given by:

\[
\langle 1'2'|V_t^{(s)}(z)|12 \rangle = \frac{g_2 g_3}{(2\pi)^3} F_1^{(s,2)}(\omega_p^2) F_2^{(s,2)}(\omega_p^2) \frac{1}{8m_{\alpha}} \sqrt{\omega_1' \omega_2' \omega_1 \omega_2} \frac{2}{3m_\alpha} \left( P_1' P_1 \right)^2 P_2(\cos \theta) \times \left( \frac{1}{z - m_0^0} + \frac{1}{z - \omega_1' - \omega_2' - \omega_1 - \omega_2 - m_{\alpha}} \right), \tag{9}
\]

where \(P_2(\cos \theta)\) is the Legendre polynomial.
2.3. Flavor SU(3)-symmetric coupling constants

For meson–meson–meson vertices, we explain the SU(3)-symmetric coupling constants. For scalar, vector, and tensor mesons, the SU(3) coupling constants and the mixing angles \( f \) are phenomenological quantities that are determined by fitting to the experimental data.

\[
L_{ppp} = \frac{F_s}{m_\pi} \text{Tr}[\partial^{\mu} P \partial_{\mu} P] S, \\
L_{ppv} = G_v \text{Tr}[\partial^{\mu} P] (P - \partial^{\mu} P) V, \\
L_{ppt} = F_t \frac{2}{m_\pi} \text{Tr}[\partial^{\mu} P] (\partial^{\nu} P) T^{\mu\nu} V,
\]
respectively. \( P \) is the \( 3 \times 3 \) matrix representation of the pseudoscalar octet, \( P = \lambda^a P^a, a = 1, \ldots, 8, \) and \( \lambda^a \) are the \( 3 \times 3 \) generators of \( SU(3) \).

\[
P_8 = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \frac{m_\pi}{\sqrt{6}} & \frac{\pi^+}{\sqrt{2}} & K^+ \\
\pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{m_\pi}{\sqrt{6}} & K^0 \\
K^- & K^0 & -\frac{\sqrt{2}}{\sqrt{3}} \eta_8
\end{pmatrix}, \quad P_1 = \eta_1, \quad (11)
\]

\[
V_8 = \begin{pmatrix}
\frac{\rho^0}{\sqrt{2}} + \frac{\phi}{\sqrt{6}} & \rho^+ & K^{*+} \\
\rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\phi}{\sqrt{6}} & K^{*0} \\
K^{*-} & K^{*0} & -\frac{\sqrt{2}}{\sqrt{3}} \phi
\end{pmatrix}, \quad V_1 = \omega, \quad (12)
\]

\[
S_8 = \begin{pmatrix}
\frac{a_0}{\sqrt{2}} + \frac{f_0}{\sqrt{6}} & a^+ & \kappa^+ \\
a^- & -\frac{a_0}{\sqrt{2}} + \frac{f_0}{\sqrt{6}} & \kappa^0 \\
\kappa^- & \kappa^0 & -\frac{\sqrt{2}}{\sqrt{3}} f_0
\end{pmatrix}, \quad S_1 = \sigma. \quad (13)
\]

For the tensor meson \( f_2 \) and scalar meson \( \epsilon_1 \), we assume them to be flavor \( SU(3) \) singlet mesons (\( T_1 = f_2 \) and \( S_1 = \epsilon \)). In the above expressions, we assume symmetric (D-type) coupling for scalar and tensor mesons and anti-symmetric (F-type) for vector mesons. The singlet–octet mixing angles \( \theta_S, \theta_V, \) and \( \theta_P \) are introduced by:

\[
\sigma_{\text{phys}} = \sigma \cos \theta_S + f_0 \sin \theta_S, \quad f_{0,\text{phys}} = f_0 \cos \theta_S - \sigma \sin \theta_S, \\
\omega_{\text{phys}} = \omega \cos \theta_V + \phi \sin \theta_V, \quad \phi_{\text{phys}} = \phi \cos \theta_V - \omega \sin \theta_V, \\
\eta_{\text{phys}}^1 = \eta_1 \cos \theta_P + \eta_8 \sin \theta_P, \quad \eta_{\text{phys}}^8 = \eta_8 \cos \theta_P - \eta_1 \sin \theta_P. \quad (14)
\]

The values \( \theta_P, \theta_V, \theta_S \) are determined by the \( SU(6) \) quark models, \( \theta_P = 24^\circ \) by the Gell-Mann–Okubo mass formula, \( \theta_P = 35.3^\circ \) by the ideal mixing between \( \omega \) and \( \phi \). By using the \( SU(3) \) coupling constants and the mixing angles \( \theta_P, \theta_V \), we determined the coupling constants \( g_{abc} \), which correspond to \( g_1 \) and \( g_2 \) in Eqs. (5, 7, and 9).

2.4. Form factors

Form factors play a very important role in meson–meson scattering. In this paper, we attempt two types of form factors: monopole \([1]\) and Gaussian type \([4,5]\). Meson-exchange potentials contain form factors that express the effects of short-range physics beyond the one-meson-exchange mechanism, e.g., many-meson exchange or the intrinsic structure of hadrons. Therefore, we treat the form factors as phenomenological quantities that are determined by fitting to the experimental data.
For $t$-channel exchange, the form factor of monopole type can be written as follows:

$$F_m^{(t)}(q^2_a) = \frac{\Lambda^2 - m^2_a}{\Lambda^2 + q^2_a}, \quad (15)$$

where $m_a$ and $q_a$ are the mass and 3-momentum of the exchanged mesons in the $t$-channel, respectively.

For $s$-channel exchange, we take two kinds of monopole form factors:

$$F_m^{(s,1)}(\omega_p^2) = \frac{\Lambda^2 + m^2_a}{\Lambda^2 + \omega_p^2}, \quad \text{and} \quad F_m^{(s,2)}(\omega_p^2) = \frac{\Lambda^4 + m^4_a}{\Lambda^4 + \omega_p^2}, \quad (16)$$

where $\omega_p$ is the total energy of the incoming (outgoing) state. The fourth-order monopole type is necessary for the gradient coupling of the scalar meson as well as the tensor meson to obtain sufficient convergence in the high-momentum region.

The form factors of Gaussian type for $t$- and $s$-channel exchange are defined respectively by:

$$F_g^{(t)}(q^2_a) = e^{-\frac{q^2_a}{\Lambda^2}}, \quad (17)$$

$$F_g^{(s)}(\omega_p^2) = e^{-\frac{\omega_p^2}{\Lambda^2}}, \quad (18)$$

We should note the difference between the two kinds of form factors. In $t$-channel exchange, we have $F_m^{(t)}(q^2_a) < 1$ and $F_m^{(s)}(q^2_a) \sim 1$ for the momentum transfer $p_a \gg \theta$, which corresponds to forward scattering. In such a situation, short-range physics does not work and the latter may be reasonable. On the other hand, in $s$-channel exchange, $F_m^{(s,1)}(\omega_p^2) = F_m^{(s,2)}(\omega_p^2) = 1$ and $F_g^{(s)}(\omega_p^2) < 1$ for on-resonance scattering ($s = m^2_a$). But, in this case, the “momentum transfer” $|p_1| = |p_2|$ is not small in general. The short-range properties of mesons may be responsible. Therefore, Gaussian form factors seem to be more reasonable than monopole ones.

### 2.5. Description of scattering

We start with the partial wave expansion of the quasi-potential $V_j^{ii}(p'p|z)$.

$$V_j^{ii}(p'p|z) = 2\pi \int_{-1}^{1} d\cos \theta P_j(\cos \theta)(p'|V_j^{ii}(z)|p) \quad (19)$$

where $\theta$ is the angle between $p$ and $p'$. The $T$-matrix can be obtained from the Lippmann–Schwinger (LS) equation below:

$$T_j^{ii}(p'p|z) = V_j^{ii}(p'p|z) + \sum_{i''=1}^{N_c} \int_{0}^{\infty} dp'' p''^2 V_j^{i'i''}(p'p''|z) \frac{1}{z - \omega_p^{(i'')}} T_j^{i''i}(p''p|z). \quad (20)$$

By solving the LS equation, we determine the $T$- and $S$-matrices. We will solve the integral equation. Firstly, we solve the equation given by ($P$ is ignored for complex $z$)

$$R_j^{ii}(p'p|z) = V_j^{ii}(p'p|z) + P \sum_{i''=1}^{N_c} \int_{0}^{\infty} dp'' p''^2 V_j^{i'i''}(p'p''|z) \frac{1}{z - \omega_p^{(i'')}} R_j^{i''i}(p''p|z). \quad (21)$$

We define $\rho^{\frac{1}{2}} T \rho^{\frac{1}{2}} = \tilde{\rho}$ and $\rho^{\frac{1}{2}} R \rho^{\frac{1}{2}} = \tilde{R}$. $\rho$ is the density matrix. Then, we find the $T$-matrix from

$$\tilde{\rho} = (1 - i\pi \tilde{R})^{-1} \tilde{R}. \quad (22)$$

6/16
Table 1. Coupling constants and cut-off parameters $\Lambda$ (MeV) in the model.

| Parameters | Monopole          | Gaussian          |
|------------|-------------------|-------------------|
| $g_{\pi\pi\rho}$ | 0.52247           | 0.47320           |
| $g_{\pi\pi f_2}$ | $0.29255 \times 10^{-1}$ | $0.45438 \times 10^{-1}$ |
| $g_{\pi\pi \epsilon}$ | $0.28000 \times 10^{-1}$ | $0.18000 \times 10^{-1}$ |
| $g_{\pi\pi \kappa}$ | $0.20948 \times 10^{-3}$ | $0.12693 \times 10^{-3}$ |
| $\Lambda_{\pi\pi\rho}$ | 2704.325          | 1589.421          |
| $\Lambda_{\pi\pi \pi K^*}$ | 2405.654          | 2996.391          |
| $\Lambda_{\pi\pi \pi K}$ | 4191.844          | 2674.091          |
| $\Lambda_{\pi\pi \pi \omega, \phi}$ | 4562.535          | 4609.568          |
| $\Lambda_{\pi\pi \pi, \pi}$ | 2753.454          | 5805.239          |
| $\Lambda_{\pi\pi \pi \rho, \pi}$ | 3186.415          | 1748.241          |
| $\Lambda_{\pi\pi \pi \epsilon, \pi}$ | 2278.50           | 2500.582          |
| $\Lambda_{\pi\pi \pi \kappa, \pi}$ | 3141.034          | 3435.894          |
| $\Lambda_{\pi\pi \pi \kappa, \pi}$ | 3279.053          | 4764.548          |

The $S$-matrix is obtained from the on-shell $T$-matrix by using the equation

$$S = (1 + i\pi \tilde{R})^{-1}(1 - 2\pi i \tilde{R}).$$

From the $S$-matrix we can calculate the scattering lengths and pole positions of the meson–meson interaction. The $S$-matrix element in coupled-channel scattering is parameterized by

$$S(z) = \begin{pmatrix}
\eta(z)e^{2i\delta_{11}(z)} & i\sqrt{1 - \eta^2(z)}e^{i\delta_{12}(z)} \\
i\sqrt{1 - \eta^2(z)}e^{i\delta_{12}(z)} & \eta(z)e^{2i\delta_{22}(z)}
\end{pmatrix},$$

where $\eta(z)$ is called inelasticity and $\delta_{ij}$ are phase shifts. In this paper, we only discuss these quantities for the lowest channels (the $\pi\pi$, $\pi K$, and $KK$ channels). The potentials from the $s$-channel exchange diagrams have a pole at $\sqrt{s} = m_0$ (bare mass of the exchanged meson). Such singular potentials should be treated by the renormalization procedure. Details of this procedure are given in Refs. [7] and [8].

3. Results

In this section, we show the quantitative results of phase shifts $\delta_I^J$, in which $I$ is the isospin and $J$ is the total angular momentum of the $\pi\pi$ or $\pi K$ system. In addition, we also propose quantitative predictions for the $KK$ interaction.

3.1. Parameter values in the model

We use 18 free parameters to fit the meson–meson data. They are 4 coupling constants ($g$), 9 cut-off masses ($\Lambda$) and 5 bare masses ($m_0$). There are two separate sets of parameters, as required to fit the data for each kind of form factor. All the parameters determined so as to reproduce $\pi\pi$ and $\pi K$ scattering are given in Tables 1 and 2. All coupling constants in the vector meson-exchange process are interrelated to the $\pi\pi\rho$ coupling by $SU(3)$ symmetry. Others involving the scalar ($\epsilon, \kappa$) and tensor ($f_2$) mesons are independently adjusted to the data.

3.2. $\pi\pi$ scattering

In Fig. 2a, we show the isospin $I = 1$ $P$-wave $\pi\pi$ phase shift, $\delta_1^1$. In this partial wave, the $\rho$ meson exchange makes a very important contribution in both the $s$- and $t$-channel exchange mechanisms.
Table 2. Bare mass parameters in the model.

| Mesons | Monopole | Gaussian |
|--------|----------|----------|
| $\epsilon$ | 4137.5 | 3500.19 |
| $\kappa$ | 1493.636 | 1663.92 |
| $\rho$ | 1038.5 | 1617.117 |
| $K^*$ | 1096.109 | 1094.636 |
| $f_2$ | 1857.1383 | 1453.084 |

Fig. 2. Phase shift (a) $\delta_1^1$, (b) $\delta_2^2$. The dashed lines are results with conventional monopole form factors. The solid lines are those with Gaussian form factors. The experimental phase shift analyses are taken from Refs. [9–11].

The $s$-channel $\rho$ meson exchange almost dominates the strong attraction of phase shift $\delta_1^1$. The agreement of the theoretical results with the experimental data indicates that both the position and the width of the $\rho$ meson are described very well.

In fact, the resonance of $I = J = 1$ is at $m = 765$ MeV with a width of 146 MeV for the monopole form factor case (see Table 5). For the Gaussian form factor case, the resonance is at $m = 772$ MeV with a width of 138 MeV.

In Fig. 2b, we show the results of the $I = 0$, $J = 2$ phase shifts, $\delta_2^0$, which agree rather well with the experimental data. By including $s$-channel $f_2$ meson exchange in the potential, we get much more attractive phase shifts and a physical $f_2$ pole in $\pi \pi$ scattering. The $I = 0$, $J = 2$ channel is influenced by resonance in the $s$-channel—the $f_2(1270)$ meson. In fact, we have a resonance at $m = 1253$ MeV with a width of 164 MeV for the monopole form factor case (see Table 5). For the Gaussian case, the resonance is at $m = 1265$ MeV with a width of 168 MeV, nearer to the $f_2(1270)$ than the monopole case.

The $I = 2$, $J = 0$ and $I = J = 2$ states describe a simple one-channel of $\pi \pi$ scattering with only $\rho$ meson exchange. Phase shifts $\delta_2^0$ (see Fig. 3a) and $\delta_2^3$ (see Fig. 3b) are all repulsive. In fact, the repulsive nature of these phase shifts definitely comes only from $t$-channel $\rho$ meson exchange. To obtain the $I = 2$, $J = 0$ phase shifts, we use the coupling constant and the cut-off mass of the $\pi \pi \rho$ vertex in the $t$-channel. Our calculated $I = J = 2$ $\pi \pi$ phase shifts are fairly large in comparison with the experimental data.
The most interesting result of $\pi\pi$ interaction is the phase shift of the scalar–isoscalar channel, $\delta^0_0$ (see Fig. 4). In this partial wave, $K\bar{K}$ channels play an essential role in reproducing the experimental data. In fact, the resonance at around $\sqrt{s} = 980$ is interpreted as a quasi-bound state of the $K\bar{K}$ system, which is produced by strongly attractive $t$-channel vector meson ($\rho, \omega, \phi$) exchange contributions. In the $S$-wave with $I = 0$, the phase shift is very difficult to reconcile with the experimental data. The derivative coupling increases strongly with the pion momentum above 1 GeV and gives a sharp rise to a potential that difficulty changes the phase shifts below 1 GeV. When we adjust carefully the $\epsilon_1$ meson with the coupling constant, the cut-off mass, and the bare mass, good agreement between theory and experiment is obtained throughout the whole energy range. The bare mass and the coupling constant of the $\epsilon_1$ meson have been chosen to reproduce the phase shift in the high-energy region. As a result, both the strong direct $K\bar{K}$ interaction and the $\epsilon_1$ meson are necessary to obtain agreement with the experiment. We have a resonance at $m = 981$ MeV with a width of 16.8 MeV for the monopole form factor case (see Table 5). For the Gaussian case, the resonance shifts by tens of MeV. The elasticity $\eta^0_0$ which is shown in Fig. 5, is calculated with derivative coupling. The error
Fig. 5. $\eta_0$. Experimental phase shift analyses are taken from Refs. [9,11,16].

Fig. 6. Phase shift (a) $\delta_0^{1/2}$, (b) $\delta_1^{1/2}$. The dashed lines are results with conventional monopole form factors. The solid lines are those with Gaussian form factors. The experimental phase shift analyses are taken from Refs. [19–21] for $\delta_0^{1/2}$ and from Refs. [20] and [22] for $\delta_1^{1/2}$.

bars in the experimental data for $\eta_0$ are quite large. Within the error bars, the agreement of $\eta_0$ with the experimental data is good.

3.3. $\pi K$ scattering

Most of the parameters used in the $\pi K$ interaction are determined from the previous investigation of the $\pi\pi$ interaction and the $SU(3)$ symmetry relations. The $t$-channel exchange contributions are completely determined by the coupled-channel $\pi\pi-K\bar{K}$ interaction, whereas additional pole contributions in the $s$-channel with $\kappa$ meson exchange have to be adjusted separately.

We start with the $I = \frac{1}{2}$, $J = 0$ states of the $\pi K$ system. The phase shift $\delta_0^{1/2}$ is attractive. As shown in Fig. 6a, in the low-energy region ($\sqrt{s} < 1$ GeV), the calculated results are somewhat larger than the experimental ones. However, the higher-energy regions are in good agreement with the experimental data. The $\kappa$ resonance is also reproduced very well. The corresponding physical mass of the meson scalar is $m_\kappa = 1440$ MeV with the width $\Gamma_\kappa = 90$ MeV for the monopole form factor and $m_\kappa = 1428$ MeV with the width $\Gamma_\kappa = 86$ MeV for the Gaussian form factor.
Fig. 7. Phase shift (a) $\delta_{3/2}^0$, (b) $\delta_{1/2}^1$. The dashed lines show results with conventional monopole form factors. The solid lines show those with Gaussian form factors. The experimental phase shift analyses are taken from Refs. [20,23,24] for $\delta_{3/2}^0$ and from Refs. [20] and [23] for $\delta_{1/2}^1$.

The strong attractive phase shifts for the $I = \frac{1}{2}$ $P$-wave $\pi K$ interaction are shown in Fig. 6b. The agreement between the theoretical calculation and the experimental data is good in the low-energy region; however, it is a little large at higher energies ($E > 1.0$ GeV). The resonant state of the $P$-wave with isospin $I = \frac{1}{2}$, $J = 1$ is known as $K^*(892)$. In Table 5, we can see that the physical mass of vector meson exchange is $m_{K^*} = 885$ MeV with the width $\Gamma_{K^*} = 44$ MeV for the monopole form factor and $m_{K^*} = 895$ MeV, $\Gamma_{K^*} = 40$ MeV for the Gaussian form factor.

An excellent test for meson-exchange interaction is $\delta_{3/2}^0$ and $\delta_{1/2}^1$. The interactions in these partial waves are completely determined by the parameters in the $\pi \pi - K \bar{K}$ interaction. As we can see in Fig. 7, the agreement with the experimental data is rather good up to about 1.4 GeV. $\delta_{3/2}^0$ and $\delta_{1/2}^1$ are both repulsive in the $\pi K$ interaction.

3.4. Prediction of KK scattering

By extending our vector meson exchange potentials without any additional parameters, we construct the $KK$ interaction. The $KK$ interaction is simply constructed by only $t$-channel with $\rho$, $\omega$, and $\phi$ meson exchange. The properties of $KK$ interaction are described by phase shifts in $S$-, $P$-, $D$-, and $F$-waves.

By investigating the contributions from $\rho$, $\omega$, and $\phi$ meson exchanges, we can confirm the realizability of our predicted results. In Fig. 8a, the weakly attractive phase shift in the $I = 0$, $J = 1$ state is shown. This attractive $\delta_{1/2}^0$ is quite reasonable because the contributions to the $KK$ interaction are repulsive with $\omega$ and $\phi$ meson exchange, while with $\rho$ meson exchange the contribution is attractive. Otherwise, the $I = 1$, $L = 0$ $KK$ interaction is really strongly repulsive because of all the repulsive contributions from the $\rho$, $\omega$, and $\phi$ meson exchanges. Using both types of form factors, we obtain almost the same results of phase shifts, as shown in Figs. 8 and 9. Unfortunately, we have not yet found any experimental data with which to compare these results.

From the contributions of meson exchange in $KK$ interaction (see details in Table 3), we can see the important role of the $\rho$ meson in the meson-exchange model. At the isospin $I = 1$ state, the vector meson $\rho$ made both the $KK$ and $K \bar{K}$ interactions repulsive. The $\omega$ and $\phi$ meson exchanges make the $KK$ interaction repulsive and the $K \bar{K}$ interaction attractive. Therefore, we get a strongly repulsive
Fig. 8. Phase shift of $KK$ interaction (a) $\delta_{0}^{0}$, (b) $\delta_{0}^{3}$. The dashed lines are results with conventional monopole form factors. The solid lines are those with Gaussian form factors.

Fig. 9. Phase shift of $KK$ interaction (a) $\delta_{1}^{0}$, (b) $\delta_{1}^{2}$. The dashed lines are results with conventional monopole form factors. The solid lines are those with Gaussian form factors.

Table 3. The contributions of $KK$ and $K\bar{K}$.

| Isospin | $J = 0$ | $J = 1$ |
|---------|---------|---------|
| $KK$    | $I = 0$ | $\rho$  | $\omega$ | $\psi$ | $\rho$  | $\omega$ | $\psi$ |
|         |         | attractive | repulsive | repulsive | attractive | repulsive | repulsive |
| $I = 1$ |         | repulsive  | forbidden  | repulsive | forbidden  | repulsive | repulsive |
| $K\bar{K}$ | $I = 0$ | repulsive  | repulsive  | attractive | attractive | attractive | attractive |
| $I = 1$ |         | repulsive  | attractive | attractive | repulsive  | attractive | attractive |
4. Discussion

4.1. Scattering lengths

In this section, we show the result for the scattering lengths of all states in $\pi\pi$, $\pi K$, and $KK$ scatterings. In our treatment, the experimental data for the scattering lengths were not used in determining the parameter values given in Table 1. All the experimental values were only reserved for checking the interaction model at very low energies. In Table 4, we show the results of scattering lengths $a_I^J$ in comparison with the experimental data for all the channels of the $\pi\pi$, $\pi K$, and $KK$ interactions.

For the $\pi\pi$ scattering in the $I = 2$, $J = 0$, 2 states, because of the repulsive nature of the phase shifts, the scattering lengths have negative values. The scattering lengths $a_0^2$ and $a_2^2$ are completely generated by the $t$-channel interaction. The scattering lengths of the $I = 2$, $J = 0$, 2 states are a little smaller than those of the experimental data. The influence of the $\epsilon$ meson on the scattering length $a_0^0$ is weaker than other $t$-channel interactions. However, in both the monopole and Gaussian form factor cases, $a_0^0$ is in very good agreement with the experimental value. In contrast, the influence of the $\rho$ meson on $a_1^1$ is very strong: the calculated $a_1^1$ is nearly same as the experimental value in both the monopole and Gaussian types of form factors. In the case of $a_2^0$, the contributions of $t$-channel interactions and of the $f_2$ meson are strong. The calculated scattering length $a_2^0$ is smaller than that of the experimental data. On the whole, the theoretical results of scattering lengths are in good agreement with the experimental data, but $a_1^1$ in $\pi\pi$ scattering, $a_0^2$, and $a_1^2$ are a little large in comparison with the experimental data. For the $KK$ interaction, there are no experimental data, except for a very strongly repulsive $a_0^1$: the scattering lengths are considerably smaller than the $\pi\pi$ and $\pi K$ interactions.

$KK$ interaction in the $I = 1$ state. The phase shift in the $I = 1$ D-wave is predicted to be a repulsive one. We also have the prediction of an attractive $F$-wave $KK$ interaction. All these results show that the relative strengths of the $\rho$, $\omega$, and $\phi$ meson-exchange contributions are very important.

Table 4. Units of $M^{(2L-1)}$, $L = J$ is the angular momentum. Experimental and theoretical scattering lengths for $\pi\pi$ and $\pi K$ scattering. The experimental data are taken from Ref. [25].

| $a_I^J$ | Experimental | Monopole | Gaussian |
|--------|--------------|----------|----------|
| $\pi\pi$ | $a_0^0$ | $0.223 \pm 0.009$ | 0.2242 | 0.2419 |
| | $a_2^0$ | $(1.833 \pm 0.036) \times 10^{-3}$ | $0.669 \times 10^{-3}$ | $0.880 \times 10^{-3}$ |
| | $a_1^1$ | $0.0381 \pm 0.0009$ | 0.0386 | 0.0328 |
| | $a_2^1$ | $-0.0444 \pm 0.0054$ | $-0.0178$ | $-0.0211$ |
| | $a_1^2$ | $(-0.246 \pm 0.025) \times 10^{-3}$ | $-0.175 \times 10^{-3}$ | $-0.198 \times 10^{-3}$ |
| $\pi K$ | $a_0^1$ | $0.224 \pm 0.022$ | 0.3812 | 0.3890 |
| | $a_0^2$ | $0.019 \pm 0.001$ | 0.0207 | 0.01755 |
| | $a_1^0$ | $-0.0448 \pm 0.0077$ | $-0.0571$ | $-0.0595$ |
| | $a_2^0$ | $(0.65 \pm 0.44) \times 10^{-3}$ | $-0.69 \times 10^{-3}$ | $-0.38 \times 10^{-3}$ |
| $KK$ | $a_0^0$ | — | 0.00131 | 0.00132 |
| | $a_0^1$ | — | $-0.0646$ | $-0.0639$ |
| | $a_0^2$ | — | $1.647 \times 10^{-6}$ | $1.673 \times 10^{-6}$ |
| | $a_0^3$ | — | $-6.904 \times 10^{-5}$ | $-6.621 \times 10^{-5}$ |
η states are almost the same; therefore, the ππ interaction and πK interaction, one has to search for the positions of poles in the physical amplitudes in the complex $E$ plane. In the $I = 0, L = 0$ states, we find a pole of the $f_0$ resonance at $(981.0 + i8.4)$ MeV with the monopole form factor, which means that a mass $M_{f_0} = 981.0$ MeV and a width $\Gamma_{f_0} = 16.8$. For the Gaussian form factor, the position of the pole shifts tens of MeV compared with the monopole form factor case, i.e., $(1018.5 - i11.4)$ MeV. We can see other resonances of other states in Table 5. For ππ and πK scatterings, there are five resonances that correspond to the physical mass of $f_0$, $f_2$, $\rho$, $\kappa$, and $K^*$. 

4.2. Poles and widths

In order to determine the physical mass and width of the $\rho$, $f_2$, $f_0$, $\kappa$, and $K^*$ resonances in the ππ, πK interactions, we investigate the $\eta\pi$ channel in ππ scattering and the $\eta K$ channel in πK scattering. When introducing the η channels in our model, we have to redetermine the parameters of the coupling constants of $I = 0$, $J = 0$, 2 for the ππ interaction and $I = \frac{1}{2}$, $J = 0$, 1 for the πK interaction as given in Table 1. In this section, we discuss the η channel effects in $I = 0$, $J = 0$, 2 for ππ scattering and $I = \frac{1}{2}$, $J = 0$, 1 for πK scattering. In $I = J = 0$ ππ scattering, we deal with the coupled-channel ππ−$K\bar{K}\eta\eta$ problem. Only $\epsilon_1$ and $K^*$ meson exchange contribute to $\eta\eta$ channel-coupling. $K^*$ ($t$-channel) exchange does not cause direct $\eta\pi$−$\eta\eta$ coupling. For this reason, the effect of the $\eta\eta$ channel on ππ scattering by $K^*$ exchange is very small. On the other hand, $s$-channel $\epsilon_1$ exchange provides the coupling between all three channels. This effect is not small, as shown in Fig. 10. However, it can be compensated by readjustment of the bare mass of $\epsilon_1$ and the coupling constant $g_{\pi\pi\epsilon_1}$.

When we add $t$-channel $K^*$ exchange in $\pi\pi-K\bar{K}\eta\eta(\pi\eta)$, the phase shifts of the isospin $I = 0(1)$ states are almost the same; therefore, the η channel effects on the $K^*$ exchange mechanism are small. The η channels with the $\epsilon_1$ exchange mechanism have a large effect on the $I = 0$, $J = 0$ phase shift. As shown in Fig. 10, the $\delta_0^0$ with η channel suddenly gets a large variation of shape in comparison with the experimental data. After fitting the coupling constants, cut-off parameters, and bare mass of the $\epsilon$ meson, we obtain the phase shifts in the low-energy region ($\sqrt{s} < 900$ MeV) to show a good agreement with the experimental data, while the phase shifts at $\sqrt{s} > 900$ MeV are small in comparison with experimental data.

In πK scattering with $I = \frac{1}{2}$, $J = 0$, the η channel with $\kappa$ meson exchange has a smaller effect than the $I = 0$, $J = 0$ state in ππ scattering. We make a small adjustment of the coupling-constant parameters in πK with the $\kappa$ meson exchange reactions but the phase shifts $\delta_0^1$ in Fig. 11a do not show any big variation. The $I = \frac{1}{2}$, $J = 1$ state is affected quite a lot when the η meson is taken into account.

### Table 5. Pole positions (MeV) of $S$-matrices for $\pi\pi$ and $\pi K$ scattering.

| $I$  | $J$ | Resonance energy (MeV) |
|------|-----|------------------------|
| ππ   | 0   | 981.2 − i8.4           |
|      | 2   | 1253.4 − i82.0         |
| πK   | 0/2 | 765.0 − i73.0          |
|      | 0   | 1439.8 − i45.1         |
|      | 1/2 | 884.7 − i21.8          |

**4.3. The $\eta$ channel effects**

In order to understand the effects of the η channels in the ππ and πK interactions, we investigate the $\eta\eta$ channel in ππ scattering and the $\eta K$ channel in πK scattering. When introducing the η channels in our model, we have to redetermine the parameters of the coupling constants of $I = 0$, $J = 0$, 2 for the ππ interaction and $I = \frac{1}{2}$, $J = 0$, 1 for the πK interaction as given in Table 1. In this section, we discuss the η channel effects in $I = 0$, $J = 0$, 2 for ππ scattering and $I = \frac{1}{2}$, $J = 0$, 1 for πK scattering. In $I = J = 0$ ππ scattering, we deal with the coupled-channel ππ−$K\bar{K}\eta\eta$ problem. Only $\epsilon_1$ and $K^*$ meson exchange contribute to $\eta\eta$ channel-coupling. $K^*$ ($t$-channel) exchange does not cause direct $\eta\pi$−$\eta\eta$ coupling. For this reason, the effect of the $\eta\eta$ channel on ππ scattering by $K^*$ exchange is very small. On the other hand, $s$-channel $\epsilon_1$ exchange provides the coupling between all three channels. This effect is not small, as shown in Fig. 10. However, it can be compensated by readjustment of the bare mass of $\epsilon_1$ and the coupling constant $g_{\pi\pi\epsilon_1}$.

When we add $t$-channel $K^*$ exchange in $\pi\pi-K\bar{K}\eta\eta(\pi\eta)$, the phase shifts of the isospin $I = 0(1)$ states are almost the same; therefore, the η channel effects on the $K^*$ exchange mechanism are small. The η channels with the $\epsilon_1$ exchange mechanism have a large effect on the $I = 0$, $J = 0$ phase shift. As shown in Fig. 10, the $\delta_0^0$ with η channel suddenly gets a large variation of shape in comparison with the experimental data. After fitting the coupling constants, cut-off parameters, and bare mass of the $\epsilon$ meson, we obtain the phase shifts in the low-energy region ($\sqrt{s} < 900$ MeV) to show a good agreement with the experimental data, while the phase shifts at $\sqrt{s} > 900$ MeV are small in comparison with experimental data.

In πK scattering with $I = \frac{1}{2}$, $J = 0$, the η channel with $\kappa$ meson exchange has a smaller effect than the $I = 0$, $J = 0$ state in ππ scattering. We make a small adjustment of the coupling-constant parameters in πK with the $\kappa$ meson exchange reactions but the phase shifts $\delta_0^1$ in Fig. 11a do not show any big variation. The $I = \frac{1}{2}$, $J = 1$ state is affected quite a lot when the η meson is taken into account.
Fig. 10. Phase shift $\delta_0^0$. The solid lines are results with conventional monopole type form factors. The dashed lines are those with the Gaussian type form factor. The red lines represent the no-$\eta$-meson model, the green lines the model with the $\eta$ meson, and the blue lines the models with the $\eta$ meson in which the parameters are adjusted. We compare these results with those of phase shifts analyses [9,16–18].

Fig. 11. Phase shift in the model with $\eta$ meson (a) $\delta_1^{1/2}$, (b) $\delta_1^{1}$. The dashed lines are results with conventional monopole form factors. The solid lines are those with Gaussian form factors. The red lines represent the no-$$\eta$$-meson model, the green lines the model with the $\eta$ meson, and the blue lines the models with the $\eta$ meson in which the parameters are adjusted. The experimental phase shift analyses are taken from Refs. [19–21] for $\delta_1^{1/2}$ and from Refs. [20] and [22] for $\delta_1^{1}$.

As a result, the $\eta$ channel effects in $\pi\pi$ scattering are larger than those in $\pi K$ scattering and fitting $\delta_0^0$ is not easy to do. In the future, we will refine our model with $\eta$ channels for $\delta_0^0$, $\delta_1^{1/2}$, $\delta_1^{1}$ and propose some new results in phase shifts $\delta_0^0$ in $\pi\eta K\bar{K}$ scattering and $\delta_1^0$ in $K\bar{K}$ scattering.

5. Summary

For the purposes of describing the $\pi\pi$, $\pi K$, and $KK$ interactions in a unified way, we constructed a potential model that assumes the one-meson-exchange model based on the interaction Lagrangian
that satisfies the $SU(3)$ symmetry. In order to determine the parameters of the potential, which include form factors and coupling constants, we used the phase shift data of the $\pi\pi$ and $\pi K$ scatterings at low energies. We gave a prediction for the $KK$ interaction by extending the $\pi\pi$ and $\pi K$ scatterings. In the $KK$ interaction, $t$-channel $\rho$, $\omega$, and $\phi$ meson exchange plays an important role. In order to confirm resonance positions, we calculated the $S$-matrix in the complex energy plane. The scattering lengths of all states in $\pi\pi$, $\pi K$, and $KK$ scatterings were proposed.

Moreover, the $\pi\pi-K\bar{K}-\eta\eta$ and $\pi K-\eta K$ interactions have been discussed to investigate the role of the $\eta$ meson in the $\pi\pi$ and $\pi K$ scatterings. In the future, we will treat the complete meson–meson interaction ($\pi\pi-\pi\bar{K}-\pi\eta\bar{\eta}$ and $\pi K-\eta K$) with the meson-exchanged model. In the $\pi\eta-K\bar{K}$ interactions, we will examine the $I = 1, J = 0$ states to show the phase shift and look for the pole of the $a_0$ meson. In the $K\bar{K}$ interaction, we will propose the phase shift $\delta_1^0$ and search for the existence of the $\varphi, \omega$ poles.

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