The Casimir force on a piston in the spacetime with extra compactified dimensions

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Abstract

A one-dimensional Casimir piston for massless scalar fields obeying Dirichlet boundary conditions in high-dimensional spacetimes within the frame of Kaluza-Klein theory is analyzed. We derive and calculate the exact expression for the Casimir force on the piston. We also compute the Casimir force in the limit that one outer plate is moved to the extremely distant place to show that the reduced force is associated with the properties of additional spatial dimensions. The more dimensionality the spacetime has, the stronger the extra-dimension influence is. The Casimir force for the piston in the model excluding one plate under the background with extra compactified dimensions always keeps attractive. Further we find that when the limit is taken the Casimir force between one plate and the piston will change to be the same form as the corresponding force for the standard system consisting of two parallel plates in the four-dimensional spacetimes if the ratio of the plate-piston distance and extra dimensions size is large enough.

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In 1948 a remarkable macroscopic quantum effect describing the attractive force between two conducting and neutral parallel plates was predicted by Casimir [1]. The Casimir effect appears due to the disturbance of the vacuum of the electromagnetic field induced by the presence of boundary. Twenty years later Boyer researched on the Casimir effect for a conducting spherical shell to find that this kind of Casimir force is repulsive [2]. This effect is more complicated than we thought. Afterwards more efforts have been paid for the problem and related topics. All results brought attention to the fact that whether the Casimir force is attractive or repulsive depends on the geometry of the configuration strongly [3]. However, there are several reasons to be suspicious of the analysis of the Casimir effect problems. Maybe their results are not perfect. For example, we always investigated a massless scalar field in a confined region such as parallel plates, rectangular box and so on to find the vacuum energy while we let the field satisfy the Dirichlet boundary conditions at the borders of the region [3-8]. Having regularized the vacuum energy, we obtain the Casimir energy. Certainly the Casimir force can be received by means of derivative of Casimir energy with respect to the distance between two edges. Here it should be pointed out that these former considerations on the topic have not involved the contribution to the vacuum energy from the area outside the confined region which depends on its dimensions while we discard the divergent terms related to the boundary also depending on the geometry and dimensions during the regularization process. In order to ignore the flaws mentioned above, a slightly different model called piston was put forward [9]. The system is a single rectangular box with dimensions $L \times b$ divided into two parts with dimensions $a \times b$ and $(L - a) \times b$ respectively by a piston which is an idealized plate that is free to move along a rectangular shaft. In ref. [9] the author calculated the Casimir force on a two-dimensional piston as a consequence of fluctuations of a scalar field obeying Dirichlet boundary conditions on all surfaces and found that the force on the piston is always attractive as $L$ goes to the infinity, regardless of the ratio of the two sides. Immediately the issue attracted more attention. The Casimir force acting on a conducting piston with arbitrary cross section always keeps attractive although the existence of the walls weaken the force [10]. The three-dimensional Casimir piston for massless scalar fields obeying Dirichlet boundary conditions was also explored, and it was found that the total Casimir force is negative no matter how long the lengths of sides are [11]. In addition in the case of various boundary conditions the Casimir force on a piston may be repulsive [12, 13]. In a word the Casimir piston is a new important model revealing its own distinct effects and can be used to explore the related topics. This model is also simpler to be constructed as a device from the experimental point of view.

The model of higher-dimensional spacetime is a powerful ingredient to be needed to unify the interactions in nature. More than 80 years ago Kaluza and Klein put forward the issue that our universe has more than four dimensions [14, 15]. The Kaluza-Klein theory introduced an extra compactified dimension to unify gravity and classical electrodynamics in our world. The theory has been generalized and developed greatly. Recently the quantum gravity such as string theory or brane-world scenario is developed to reconcile the quantum mechanics and gravity with the help
of introducing seven additional spatial dimensions. In Randall-Sundrum model the matter fields may be localized on a four-dimensional brane considered as our real universe, and only gravitons can propagate in the extra space transverse to the brane [16, 17]. In addition, although the order of the compactification scale of the additional dimensions has not been confirmed and are also of considerable interest recently, larger extra dimensions were invoked in order to provide a breakthrough of hierarchy problem in some approaches [18-20]. Research on higher-dimensional spacetime is valuable and become a focus in the physical community, therefore the theory needs to be explored deeply, extensively and in various directions.

Since the higher-dimensional spacetime described by Kaluza-Klein theory is important and indispensable, it is crucial to discuss several models including the Casimir effect problem in this background. The precision of the measurement has been greatly improved practically [21-24], leading the Casimir effect to be remarkable observable and trustworthy consequence of the existence of quantum fluctuations and to become a powerful tool for the topics on the model of Universe with more than four dimensions. It must be emphasized that the attractive Casimir force between the parallel plates vanishes when the plate gap is very large and no repulsive force appears according to the experimental results. Some topics were examined in the context of Kaluza-Klein theory. As the first step of generalization to investigate the higher-dimensional spacetimes, we show analytically that the extra-dimension corrections to the Casimir effect for a rectangular cavity in the presence of a compactified universal extra dimension are very manifest [25]. The Casimir effect for parallel plates in the spacetime with extra compactified dimensions was also studied [26-29]. We prove rigorously that there must appear repulsive Casimir force between the parallel plates when the plates distance is sufficiently large in the spacetime with compactified additional dimensions, and the higher the dimensionality is, the greater the repulsive force is. It should be pointed out that the value of the repulsive Casimir force which is obtained theoretically is within the experimental reach. Therefore the results obtained in the context of Kaluza-Klein theory conflict with the experimental phenomena mentioned above [27-29], which means that the model of higher-dimensional spacetime with extra compactified spatial dimensions needs further research.

It is necessary and significant to study the force-on-the-piston problem in a higher-dimensional spacetime within the frame of Kaluza-Klein theory. We wonder how the influence from extra dimensions on the Casimir effect of the piston. This problem, to our knowledge, has not been examined. For simplicity and comparison to the conclusion of standard parallel-plates system, the model of one-dimensional piston is chosen. The main purpose of this paper is to study the Casimir effect for the system consisting of three parallel plates in the Universe with \( d \) compactified spatial dimensions. We obtain the expression of force by means of the differential of the total vacuum energy including the contribution outside the three-parallel-plate device with respect to the distance between two plates in the system. We regularize the force to obtain the Casimir force on the piston when one outer plate is moved to the remote place. We focus on the influence of dimensionality of the spacetime on the Casimir force between one plate and a piston and compare
our results with the models like one-dimensional piston or parallel plates in the four-dimensional spacetime. Our discussions and conclusions are emphasized in the end.

In a higher-dimensional spacetime, we start to consider the massless scalar fields obeying Dirichlet boundary conditions within a one-dimensional piston. As a piston, one plate is inserted into a system consisting of two parallel plates. The piston is parallel to the plates and divides the parallel-plate-system into two parts labeled by \( A \) and \( B \) respectively. In part \( A \) the distance between the left plate and the piston is \( a \), and the distance between the piston and the right plate in part \( B \), the remains of the separation of two plates, is certainly \( L - a \), which means that \( L \) denotes the whole plates gap. The total vacuum energy for the three-parallel-plate system described above can be written as the sum of three terms,

\[
E = E^A(a) + E^B(L - a) + E^{out}
\]

where \( E^A(a) \) and \( E^B(L - a) \) represent the energy of part \( A \) and \( B \) respectively, and the terms depend on their each size in parts. \( E^{out} \) describes the vacuum energy outside the system and is independent of characters inside the system. Having regularized the total energy density, we obtain the Casimir energy density,

\[
E_C = E^A_R(a) + E^B_R(L - a) + E^{out}_R
\]

where \( E^A_R(a) \), \( E^B_R(L - a) \) and \( E^{out}_R \) are finite parts of terms \( E^A(a) \), \( E^B(L - a) \) and \( E^{out} \) in Eq.(1) respectively. In particular, it is also pointed out that \( E^{out}_R \) is not a function of the position of the piston, the Casimir force on the piston is given by the derivative of the Casimir energy with respect to the plates distance \(-\frac{\partial E_C}{\partial a}\) and can be written as,

\[
F_C = -\frac{\partial}{\partial a}[E^A_R(a) + E^B_R(L - a)]
\]

which means that the contribution of vacuum energy from the exterior region does not affect the Casimir force on the piston.

Here we set out to consider the massless scalar field in the three-parallel-plate system in the spacetime with \( d \) extra compactified dimensions in the context of Kaluza-Klein theory. Along the additional dimensions the wave vectors of the field have the form \( k_i = \frac{n_i \pi}{R} \), \( i = 1, 2, \ldots, d \), respectively, \( n_i \) an integer. Now we choose that the extra dimensions possess the same size as \( R \). The fields satisfy the Dirichlet condition, leading the wave vector in the directions restricted by the plates to be \( k_n = \frac{n \pi}{D} \), \( n \) a positive integer and \( D \) the separation of the plates. Under these conditions, the zero-point fluctuations of the fields can give rise to observable Casimir forces among the plates.

In the case of \( d \) additional compactified dimensions we find the frequency of the vacuum fluctuation within a region confined by two parallel plates whose separation is \( D \) to be,

\[
\omega_{\{n_i\}n} = \sqrt{k^2 + \frac{n^2\pi^2}{D^2} + \sum_{i=1}^{d} \frac{n_i^2}{R^2}}
\]
$k^2 = k_1^2 + k_2^2$  \hfill (5)

$k_1$ and $k_2$ are the wave vectors in directions of the unbound space coordinates parallel to the plates surface. Now $\{n_i\}$ represents a short notation of $n_1, n_2, \ldots, n_d, n_i$ a nonnegative integer. According to Ref.[3, 4, 7, 30-34], therefore the total energy density of the fields in the interior of two-parallel-plate system reads,

$$E(D, R) = \int d^2k \sum_{n=1}^{\infty} \sum_{\{n_i\} = 0}^{\infty} \frac{1}{2} \omega_{\{n_i\}n}$$

$$= \frac{\pi}{2} \frac{\Gamma(-\frac{3}{2})}{\Gamma(-\frac{3}{2})} \sum_{l=0}^{d-1} \left( \frac{d}{l} \right) E_{d-l+1} \left( \frac{\pi^2}{D^2}, \frac{1}{R_2^2}, \frac{1}{R_2^2}, \ldots, \frac{1}{R_2^2}; -\frac{3}{2} \right) + \frac{\pi^4}{2D^3} \frac{\Gamma(-\frac{3}{2})\zeta(-3)}{\Gamma(-\frac{3}{2})} \hfill (6)$$

in terms of the Epstein zeta function $E_p(a_1, a_2, \ldots, a_p; s)$ defined as,

$$E_p(a_1, a_2, \ldots, a_p; s) = \sum_{\{n_j\}} (\sum_j a_j n_j^2)^{-s} \hfill (7)$$

Here $\{n_j\}$ stands for a short notation of $n_1, n_2, \ldots, n_p, n_j$ a positive integer. We can regularize Eq.(6) by means of the following formula,

$$\Gamma(-\frac{3}{2})E_{d-l+1} \left( \frac{\pi^2}{D^2}, \frac{1}{R_2^2}, \frac{1}{R_2^2}, \ldots, \frac{1}{R_2^2}; -\frac{3}{2} \right)$$

$$= -\frac{1}{2} \Gamma(-\frac{3}{2})E_{d-l}(1, 1, \ldots, 1; -\frac{3}{2}) \frac{1}{R^3} + \frac{1}{2\sqrt{\pi}} \Gamma(-2)E_{d-l}(1, 1, \ldots, 1; -2) \frac{D}{R^3}$$

$$+ \frac{1}{R^3} \sum_{k=0}^{\infty} \frac{16^{-k}}{k!} \left( \frac{D}{R} \right)^{-k-\frac{3}{2}} \prod_{j=1}^{k} \left[ 16 - (2j - 1)^2 \right]$$

$$\times \sum_{n_1, n_2, \ldots, n_{d-l+1} = 1}^{\infty} \frac{nk^{-\frac{3}{2}}(n_2^2 + n_3^2 + \cdots + n_{d-l+1}^2)^{\frac{k-3}{4}}}{n_1^{\frac{3}{2}}}$$

$$\times \exp \left[ -\frac{2D}{a} \left( n_2^2 + n_3^2 + \cdots + n_{d-l+1}^2 \right)^{\frac{1}{2}} \right] \hfill (8)$$

to obtain the Casimir energy density of a system consisting of two parallel plates in the spacetime with $d$ extra compactified spatial dimensions.

In the context of Kaluza-Klein theory we return to discuss the new system where a piston is also a plate localizing parallelly between two parallel plates mentioned above. Choosing the variable $D = a$ in Eq.(6) and (8), we have the vacuum energy density for part $A$ of the system containing one plate and the piston with distance $a$ as follow,

$$E^A(a, R) = \frac{\pi}{2} \frac{\Gamma(-\frac{3}{2})}{\Gamma(-\frac{3}{2})} \sum_{l=0}^{d-1} \left( \frac{d}{l} \right) E_{d-l+1} \left( \frac{\pi^2}{a^2}, \frac{1}{R_2^2}, \frac{1}{R_2^2}, \ldots, \frac{1}{R_2^2}; -\frac{3}{2} \right) + \frac{\pi^4}{2a^3} \frac{\Gamma(-\frac{3}{2})\zeta(-3)}{\Gamma(-\frac{3}{2})} \hfill (9)$$
Similarly the vacuum energy density for the remains of the system labeled $B$ with plates distance $L - a$ by replacing the variable $D$ with $L - a$ in Eq.(6) is,

$$E^A(L - a, R) = \frac{\pi}{2} \frac{\Gamma(-\frac{3}{2})}{\Gamma(-\frac{3}{2})} \sum_{l=0}^{d-1} \binom{d}{l} \frac{\pi^2}{L^2} \frac{1}{R^2} \frac{1}{R^2} \cdots \frac{1}{R^2} \frac{3}{2} + \frac{\pi^4}{2(L - a)^3} \frac{\Gamma(-\frac{3}{2}) \zeta(-3)}{\Gamma(-\frac{1}{2})}$$

We regularize Eq.(9) and Eq.(10) and then substitute the two regularized expressions into Eq.(3) to obtain the Casimir force per unit area on the piston,

$$F'_C = -\frac{\pi^4}{120} \frac{1}{a^4} + \frac{\pi^4}{120} \frac{1}{(L - a)^4} + \frac{\sqrt{\pi}}{4} \sum_{l=0}^{d-1} \left( \frac{d}{l} \right) \left\{ \frac{1}{a R^3} \sum_{k=0}^{\infty} \frac{16^{-k}}{k!} (k + \frac{3}{2})(\frac{a}{R})^{-\frac{k-\frac{1}{2}}{2}} \prod_{j=1}^{k} [16 - (2j - 1)^2] \right. \times \sum_{n_1, n_2, \ldots, n_{d-l+1} = 1}^{\infty} n_1^{-\frac{k-\frac{1}{2}}{4}} (n_2^2_n + n_3^2_n + \cdots + n_{d-l+1}^2) \times \exp \left\{ -\frac{2a}{R} n_1 (n_2^2 + n_3^2 + \cdots + n_{d-l+1}^2) \right\} \\ \left. + \frac{1}{(L - a) R^3} \sum_{k=0}^{\infty} \frac{16^{-k}}{k!} (k + \frac{3}{2})(\frac{L - a}{R})^{-\frac{k-\frac{1}{2}}{2}} \prod_{j=1}^{k} [16 - (2j - 1)^2] \right. \times \sum_{n_1, n_2, \ldots, n_{d-l+1} = 1}^{\infty} n_1^{-\frac{k-\frac{1}{2}}{4}} (n_2^2_n + n_3^2_n + \cdots + n_{d-l+1}^2) \times \exp \left\{ -\frac{2(L - a)}{R} n_1 (n_2^2 + n_3^2 + \cdots + n_{d-l+1}^2) \right\} \\ \left. + \frac{2}{R^4} \sum_{k=0}^{\infty} \frac{16^{-k}}{k!} (\frac{L - a}{R})^{-\frac{k-\frac{1}{2}}{2}} \prod_{j=1}^{k} [16 - (2j - 1)^2] \right. \times \sum_{n_1, n_2, \ldots, n_{d-l+1} = 1}^{\infty} n_1^{-\frac{k-\frac{1}{2}}{4}} (n_2^2_n + n_3^2_n + \cdots + n_{d-l+1}^2) \times \exp \left\{ -\frac{2(L - a)}{R} n_1 (n_2^2 + n_3^2 + \cdots + n_{d-l+1}^2) \right\} \right\}$$

which has corrections from extra dimensions. Further we take the limit $L \to \infty$ which means that the right plate in part $B$ is moved to a very distant place, then we obtain the following expression for the Casimir force per unit area on the piston,
where the correction function \( C_d(\mu) \) is defined as,

\[
C_d(\mu) = \frac{\sqrt{\pi}}{4} \sum_{l=0}^{d-1} \binom{d}{l} \left\{ -\frac{1}{\mu} \sum_{k=0}^{\infty} \frac{16^{-k}}{k!} (k + \frac{3}{2}) \mu^{-k-\frac{3}{2}} \prod_{j=1}^{k} (16 - (2j - 1)^2) \right\} 
\times \sum_{n_1, n_2, \ldots, n_{d-l+1}=1}^{\infty} n_1^{-k-\frac{5}{2}} (n_2^2 + n_3^2 + \cdots + n_{d-l+1}^2)^{-\frac{2k-3}{4}} 
\times \exp[-2\mu n_1(n_2^2 + n_3^2 + \cdots + n_{d-l+1}^2)] 
- 2 \sum_{k=0}^{\infty} \frac{16^{-k}}{k!} \mu^{-k-\frac{3}{2}} \prod_{j=1}^{k} (16 - (2j - 1)^2) 
\times \sum_{n_1, n_2, \ldots, n_{d-l+1}=1}^{\infty} n_1^{-k-\frac{5}{2}} (n_2^2 + n_3^2 + \cdots + n_{d-l+1}^2)^{-\frac{2k-5}{4}} 
\times \exp[-2\mu n_1(n_2^2 + n_3^2 + \cdots + n_{d-l+1}^2)] \right\} \right\} (13)
\]
dimensionality of spacetime and the ratio of plates distance and the size of extra dimensions. The more dimensions the spacetime has, the greater the absolute value of the correction function is, which means that there will appear stronger influence on the Casimir force on the piston in the higher-dimensional spacetime. The expression \( C_d(\mu) \) is also a function of ratio denoted in Eq.(14). When the ratio increases, the absolute value of the functions decreases fast. When the plates separation is more than several times larger than the extra dimensions radius, the absolute value will approach to zero. The manifestation of extra dimensions influence on the Casimir force on the piston under the limiting \( L \to \infty \) appears only when the distance between one plate and the piston is about equal to the size of extra dimensions. As mentioned above, if the extra dimensions possess larger size, the extra-dimension corrections will appear clearly in practice, therefore this model of one-dimensional piston can become a window to examine the high-dimensional spacetime. It should also be pointed out that the values of all correction functions in the world with different dimensionality keep negative, so the total Casimir force on the piston also remains attractive. The experimental evidence shows that no repulsive force generates in the case of parallel plates. It has been proved theoretically and rigorously that the Casimir force between parallel plates become repulsive as the plates are sufficiently far away from each other in the higher-dimensional spacetime described by Kaluza-Klein theory and the conclusion conflicts with the experimental phenomena and is inevitable [28, 29]. In this work our arguments on the piston in the same environment drawn above under the limiting \( L \to \infty \) is consistent with the experimental phenomena. It is useful to consider the system with a piston further. After one plate has been moved to the remote place, the three-parallel-plate model can be thought as an ordinary system consisting of two parallel plates in which one plate is thought as a piston. Actually it is interesting that the two kinds of results from three-parallel-plate model with limiting \( L \to \infty \) and original two-parallel-plate system respectively are completely different. The theoretical finding in this work, the Casimir force per unit area on the piston, is consistent with the measurement at least qualitatively. The deviation produces apparently as the plate-piston gap is close to the extra-dimension size. Maybe the corrections are beyond the experimental reach because the order of the compactification scale of the additional spatial dimensions can be extremely tiny. The three-parallel-plate model, called one-dimensional piston, must replace the standard parallel plates system unless the higher-dimensional approach described by Kaluza-Klein theory is excluded.

In this work the model of three parallel plates in which the middle one is called piston is studied in the higher-dimensional spacetime described by Kaluza-Klein theory. The expression of Casimir force per unit area on the piston is obtained. When one outer plate is moved away, we also get the exact form of reduced Casimir force per unit area between one plate and the piston. In the limiting case we discover that the force is always attractive and depend on the properties of extra compactified dimensions. The more extra dimensions will produce stronger influence. When the separation of one plate and the piston is larger enough than the size of additional spatial dimensions and the limit \( L \to \infty \) is taken, the Casimir force per unit area between them will be the same as the
results for the standard system consisting of two parallel plates in the four-dimensional spacetime. The results of the standard system in four-dimensional spacetimes are favoured in practice. Further we can argue that the model of two parallel plates and one piston under the condition that one outer plate has been moved extremely far away should substitute the standard two-parallel-plate model to describe the Casimir effect for parallel plates from experiment if the universe has additional compactified dimensions. We also should not neglect that the expressions of Casimir force for the two models derived and calculated in the same high-dimensional spacetime and the frame of the same Kaluza-Klein theory are completely different, in which at least one is attractive and the other is repulsive. Clearly in the high-dimensional background, our results from plate-piston-plate model in the limiting case avoid the flaw of results from general two-parallel-plate system. We should make use of model of one-dimensional piston to explain the measurement of Casimir effect from two parallel plates not a standard two-parallel plate model. Of course the further consequences and related topics are under progress.

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Figure 1: The solid, dot, dashed and dot-dashed curves of the correction functions of ratio of plate-piston distance and extra-dimension radius $\mu = \frac{a}{R}$ in $(4 + d)$-dimensional spacetime for $d = 1, 2, 3, 4$ respectively.
Figure 2: The solid, dot and dashed curves of the correction functions of ratio of plate-piston distance and extra-dimension radius $\mu = \frac{a}{R}$ in $(4 + d)$-dimensional spacetime for $d = 5, 6, 7$ respectively.