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ERHARD SCHOLZ

ABSTRACT. The hypothesis that gravitational self-binding energy may be the source for the vacuum energy term of cosmology is studied in a Newtonian Ansatz. For spherical spaces the attractive force of gravitation and the negative pressure of the vacuum energy term form a self stabilizing system under very reasonable restrictions for the parameters, among them a characteristic coefficient \( \beta \) of self energy. In the Weyl geometric approach to cosmological redshift, Einstein-Weyl universes with observational restrictions of the curvature parameters are dynamically stable, if \( \beta \) is about 40% smaller than in the exact Newton Ansatz or if the space geometry is elliptical.

1. Introduction

The physical nature of the cosmological vacuum is a terrain wide open for questions. They have become increasingly pressing, since observational data are indicating an important role for the vacuum contribution to the total balance of the Einstein equation in realistic cosmological models.

Cosmic vacuum is characterized by a thermodynamical neutrality in the following sense: The expansion work of a small vacuum volume during space expansion has to compensate the increase of the energy content while enlarging the volume. The characteristic relation \( p = -\rho_{\text{vac}} \) (setting \( c = 1 \)) between vacuum pressure \( p \) and the vacuum energy density \( \rho_{\text{vac}} \) follows from this property. The energy momentum tensor of the vacuum \( T_{\text{vac}} \) (denoting tensors of the type \( T = (T_{ij}) \) by their coordinate free expression \( T \) ) therefore satisfies the relation \( T_{\text{vac}} = \rho_{\text{vac}} g \) with \( g = (g_{ij}) \) the Lorentz metric of spacetime; in coordinate expressions

\[
T_{ij}^{\text{vac}} = \rho_{\text{vac}} g_{ij} .
\]

\( T_{ij}^{\text{vac}} \) has the form of an energy momentum tensor derived from a cosmological constant term in the Lagrange action, if \( \rho_{\text{vac}} \) is supposed constant.

Recently H.-J. Fahr e.a. have proposed considering gravitational self-binding energy of cosmic matter distributions, and its density \( \rho_{\text{grav}} \), as a possible source of vacuum energy (Fahr/Overduin 2001, Fahr/Heyl 2007)

\[
\rho_{\text{vac}} = \rho_{\text{grav}} .
\]

In this case vacuum energy density can no longer be considered constant, like in its characterization by a classical cosmological constant (\( \Lambda \)-) term. It
rather will be dependent on the matter-energy content of the universe. A similar idea has been proposed earlier by (Fischer 1993).

This is a very welcome modification of the received approach. Fahr and Heyl give a completely convincing argument, why this should be so:
“A constant vacuum energy doing an action on space by accelerating its expansion, without itself being acted upon, does not seem to be a concept conciliant with basic physical principles” (Fahr/Heyl 2007, 2).

Present cosmological models often assume a cosmological constant approach and are subject to the verdict of this simple basic criticism.

It is an open problem how to characterize gravitational self-binding energy in cosmological models. Fahr and Heyl use an approach with a Poisson equation for the cosmic potential with respect to radial coordinates (ibid., equ. (16)). They consider an exchange between vacuum energy and mass energy in both directions, mass creation from vacuum energy in the one direction and vacuum energy induced from self energy of gravitating masses in the other. From this they derive a new cosmological model which they call the economic universe.

As matter density on the cosmological level is extremely small, it is not per se nonsensical to investigate Newtonian approximations for potential and self-binding energy of a homogeneous mass distribution. Although the potential of a homogenous mass distribution in euclidean space is infinite, it may be finite in closed spaces like the 3-sphere $S^3$ or spherical spaces finitely covered by it. Geometrically most interesting is the positively curved space of non-euclidean geometry, arising from $S^3$ by antipodal identification. In classical geometrical language it is the elliptical space $E^3$ or, in more recent terminology, the round projective 3-space with the metric inherited from the classically metricized 3-sphere. It thus seems worthwhile to study Robertson-Walker cosmologies with closed spacialike fibres and their dynamics under the assumption of a variable vacuum term given by the gravitational self-binding energy in Newtonian approximation.

In the sequel we study the densities of gravitational self-binding energy of homogeneous mass distributions for the two most simple spherical spaces, $S^3$ and $E^3$ (section 2). It will be shown that it differs only by a typical factor $\beta$ in the expression

$$\rho_{\text{vac}} = \beta G_N \rho_{\text{tot}}^2 f^2,$$

where $G_N$ denotes the Newton constant and $f$ the scaling function of the spherical space. Interestingly the total energy density $\rho_{\text{tot}}$ and vacuum energy show different scaling behaviour

$$\rho_{\text{tot}} \sim f^{-3}, \quad \rho_{\text{vac}} \sim f^{-4}.$$

Because of the different fall off of the attractive term $\rho_{\text{tot}}$ and the expansive term $p = -\rho_{\text{vac}}$ solutions close to the static Einstein universe are dynamically stable. The simplified Raychaudhury equation has a Ljapunov stable neighbourhood of the Einstein universe, while outside certain bounds an unlimited expansion occurs (section 3).

Of course, non-expanding cosmological models may acquire physical meaning only if cosmological redshift has a field theoretic origin rather than “space expansion”, which is just another view of a time dependent spatial metric.
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Weyl geometry allows an intriguing geometric characterization of such an assumption. Moreover, Weyl geometric versions of Einstein universes have good empirical properties. Their parameter determined by supernovae data corresponds to a value $\beta \approx 2$ for the characteristic coefficient of the gravitational energy, mentioned above. This value is consistent with a Lyapunov stable regime of the $E^3$, while it leads to instability in an exact Newtonian Ansatz for the gravitational binding energy in the sphere $S^3$ itself. Empirical tests of the $E^3$ hypothesis, or for other spherical spaces, can be designed by studying symmetry constellations of quasars close to the redshift of the ‘cosmic equator’ in the Einstein-Weyl universe (section 4).

We draw the conclusion that already a Newton approximated gravitational self-binding approach to the vacuum term leads to a vindication of slightly generalized Einstein universes as dynamically stable and empirically interesting models (section 5).

2. Gravitational self-binding energy in spherical spaces

For a heuristic motivation let us consider discrete masses $m_i$ of the same amount $m$ distributed in euclidean space on the nodes of a bounded, cubically symmetric lattice with edge $a = a_0$ ($i \leq N$). Every two mass elements attract each other by Newton’s law. Increasing the lattice parameter to $a = a_0 + da$ requires a finite work $dE$. The (absolute value) of the self-binding energy is given by expansion to infinite lattice edge lengths $a \to \infty$, summarily written:

$$E(a_0) = \int_{a_0}^{\infty} dE$$

This allows to determine self-binding energy of the lattice. For an (unbounded) lattice extending over the whole space it is infinite.

This is different in a spherical space. In the sequel we first consider the 3-sphere itself and calculate the self-binding energy of homogeneously distributed discrete masses of averaged gross mass density $\rho$ (respectively, for $c = 1$, energy density) in a continuity approximation according to Newton’s law. Actually we assume large “empty” bits of space between discretely distributed masses, carrying the corresponding gravitational field. Like in nuclear physics, where binding energy is emitted to the (electromagnetic) interaction field, the self-binding energy set free by the gravitationally interacting masses is assumed to be distributed homogeneously in the vacuous space parts between the masses. Its density will be denoted by $\rho_{grav}$. The gross mass (energy) $\rho$ will be reduced by self-binding effects to the net mass (energy) density $\rho_m$,

$$\rho_m = \rho - \rho_{grav} \ .$$

Of course the total energy density $\rho_{tot} = \rho_m + \rho_{grav}$ remains always the same,

$$\rho_{tot} = \rho \ ;$$

and is only differently distributed between net mass energy density $\rho_m$ and gravitational energy density $\rho_{grav}$. 

On a sphere $S^3$ (later a spherical space) of radius $R = R_0 f$

with scaling factor $f$ we consider a mass element $m$. The differential (gross) mass $dm'$ on an infinitesimal strip of width $da$ around the set of all points making central angle $\alpha \leq \pi$ with respect to $m$ (a 2-sphere) is given by

$$dm' = \rho V_{S^2}(\alpha) da = \rho V_{S^2}(\alpha) R d\alpha,$$

where $V_{S^2}(\alpha) = 4\pi R^2 \sin \alpha$ is the 2-volume (area) of the 2-sphere corresponding to $\alpha$. We assume gravitational forces acting along the geodesic connections between each (well localized) mass element $dm''$ of $dm'$ with $m$ according to Newton’s law. The actions along the two complementary geodesic arcs $a = R\alpha$ and $a' = R(2\pi - \alpha)$, connecting $dm''$ and $m$, lead to inversely oriented forces on $m$. They are to be subtracted, if we consider both arcs:\[4\]

$$p(dm'') = G_N \left(\frac{dm'' m}{a^2} - \frac{dm'' m}{a'^2}\right)$$

During an expansion of the spherical space to $R' = R + dR$ the work to be done is comparable to that of an expanding lattice. The distance of $m$ and $dm'$ increases by $da = \alpha dR$. The contributions of forces from elements $dm''$ to the work done on $m$ add up to

$$p(dm') = 4\pi G_N \rho m R (\sin \alpha)^2 \left(\frac{1}{\alpha^2} - \frac{1}{(2\pi - \alpha)^2}\right) da .$$

In elliptical space $E^3$ antipodal points of $S^3$ are to be identified. Then the range of $\alpha$ is restricted to $0 \leq \alpha \leq \frac{\pi}{2}$ and the complementary arc becomes $(\pi - \alpha)$. Accordingly the last term has to be changed to $\frac{1}{(\pi - \alpha)^2}$. Integrating over $\alpha$ leads to the work done on $m$ in $S^3$

$$dE_m = 4\pi G_N \rho m R 4\pi \left( \int_0^\pi \frac{\pi - \alpha}{\alpha (2\pi - \alpha)^2 \sin^2 \alpha} \, d\alpha \right) dR$$

\[1\] Of course, we might also consider only the contribution of the main (shortest) arc. Then the coefficients $\beta$ derived below become slightly larger.
and in $E^3$ to:

$$dE_m = 4\pi G_N \rho m R \pi \left( \int_{0}^{\pi} \frac{(\pi - 2\alpha)}{\alpha(\pi - \alpha)^2} \sin^2 \alpha \, d\alpha \right) dR$$

The total mass $M = \rho V_{S^3}(R)$, with volume of the 3-sphere

$$V_{S^3}(R) = 2\pi^2 R^3,$$

remains constant. Thus for $S^3$

$$dE_m = 8G_N M \frac{m}{R_0} \int_{0}^{\pi} \frac{\pi - \alpha}{\alpha(2\pi - \alpha)^2} \sin^2 \alpha \, d\alpha \right) R^{-2} dR,$$

and similarly for elliptical space. Integration over the whole expansion $R \to \infty$ gives the contribution $E_m$ of $m$ to the self-binding energy in a 3-sphere of radius $R_0$

$$E_m = 8G_N M \frac{m}{R_0} \int_{0}^{\pi} \frac{\pi - \alpha}{\alpha(2\pi - \alpha)^2} \sin^2 \alpha \, d\alpha .$$

We can now add up over all mass elements $m$ to the total mass $M$; but then the energy for each mass element is counted twice (distance gaining work for $dm''$ and $m$ is considered from both sides). Cancelling this double count gives the self-binding energy of the mass $M$ homogeneously distributed in an $S^3$ of radius $R$:

$$E = \frac{1}{2} 8G_N M \rho V_{S^3} \frac{\pi}{R} \int_{0}^{\pi} \frac{(\pi - \alpha)}{\alpha(2\pi - \alpha)^2} \sin^2 \alpha \, d\alpha$$

We thus arrive at the gravitational self-binding energy density in the Newtonian Ansatz for a round $S^3$

$$\rho_{grav} = \frac{E}{V_{S^3}} = 4G_N \rho^2 \frac{V_{S^3}}{R} \int_{0}^{\pi} \ldots \, d\alpha$$

(7)

$$= 8\pi^2 G_N \rho^2 R^2 \int_{0}^{\pi} \frac{\pi - \alpha}{\alpha(2\pi - \alpha)^2} \sin^2 \alpha \, d\alpha.$$

For elliptical space it is

$$\rho_{grav} = 2\pi^2 G_N \rho^2 R^2 \int_{0}^{\pi} \frac{(\pi - 2\alpha)}{\alpha(\pi - \alpha)^2} \sin^2 \alpha \, d\alpha .$$

(8)

The general form of the gravitational self energy density is

$$\rho_{grav} = \beta G_N \rho^2 R^2$$

with a dimensionless coefficient $\beta > 0$ which is characteristic for the specific model.

For studies of dynamical behaviour of cosmological models we can consider the form (9) as a slightly generalized Newtonian approximation of gravitational self-binding energy. Taking the correctness of physical dimensions of formula (9) into account it seems reasonable to express possible modifications of Newton’s dynamics in this context by variations of $\beta$, the characteristic factor.

$^2\beta$ may be affected already by small modifications of the Newton law at scales well beyond the supercluster level.
For the round sphere we have found
\begin{equation}
\beta_S = 8\pi^2 \int_0^\pi \frac{(\pi - \alpha)}{\alpha(2\pi - \alpha)^2} \sin^2 \alpha \, d\alpha \approx 6.86, \tag{10}
\end{equation}
while for the elliptical case (the round projective 3-space)
\begin{equation}
\beta_E = 2\pi^2 \int_0^{\pi/2} \frac{(\pi - 2\alpha)}{\alpha(\pi - \alpha)} \sin^2 \alpha \, d\alpha \approx 3.65. \tag{11}
\end{equation}
We shall see in the next section that specific values for $\beta$ are decisive for evaluating the qualitative dynamics of the model.

3. Consequences for the dynamics of cosmological models

If we assume, like Fahr e.a., that the vacuum energy $\rho_{\text{vac}}$ of cosmology is constituted by gravitational self-binding energy, equ. (1), we no longer need to hypothesize an agency like “dynamical dark energy” which strongly acts on matter and physical space time, but is not acted upon. The cosmological constant then turns out as a heuristic device serving as a formal placeholder which can be used to explore whether it is necessary to extend simple matter models by a vacuum energy term. If it turns out to be necessary, as recent observational evidence indicates, it requires a more physical explanation. Gravitational self-binding energy may offer a route to the solution of the riddle; at least it seems able to shed new light on it.

Already a rough qualitative consideration shows an interesting fall off behaviour of gravitational vacuum energy in spherical spaces:
\begin{equation}
\rho_{\text{tot}} \sim R^{-3}, \quad \rho_{\text{vac}} \sim R^{-4}. \tag{12}
\end{equation}
If one starts close to an equilibrium point characteristic for the Einstein universe,
\begin{equation}
\rho_{\text{vac}} = -p = \frac{1}{3} \rho_{\text{tot}}, \quad \rho_{\text{tot}} = \rho, \tag{13}
\end{equation}
called the hyle condition for a cosmic fluid in the sequel, a small surplus of mass density will lead to a contraction of the spherical space. Then $\rho_{\text{vac}} \sim R^{-4}$ and with it the negative pressure of the vacuum term will increase faster than $\rho \sim R^{-3}$. This may, under certain restrictions for the parameters, bring the contraction to a halt and revert it. Similarly, but conversely, for a fall of mass density below the hyle point the initial expansion may come to a halt, because the negative pressure falls faster than the total energy density. So we have good reasons to expect, under certain parameter restrictions, a Lyapunov stable oscillating behaviour of spherical space models with gravitational self-binding energy about the Einstein universe (the hyle condition).

To investigate the case more closely, we have to look at the reduced Raychaudhury equation for the scaling function $f$ of a Robertson Walker solution for the Einstein equation
\begin{equation}
\frac{f''}{f} = -\frac{4\pi G_N}{3} (\rho + 3p), \tag{14}
\end{equation}
\footnote{Compare (Earman 2001).}
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cf. (Ellis 1999, A44), (O’Neill 1983, 346). With (11) we get a nonlinear ordinary differential equation of the form

\[ f'' = -a_1 f^{-2} + a_2 f^{-3}. \]

The hyle condition \( \rho + 3p = 0 \) may be normalized to

\[ f(0) = 1, \quad a_1 = a_2 = 1. \]

Obviously the initial condition \( f'(0) = 0 \) leads to a static solution \( f \equiv 1 \), corresponding to the Einstein universe.

For initial conditions with small \( f'(0) \), an oscillating solution is obtained; for larger initial expansion, the solution expands monotonically (figure 2).

\[ \text{Figure 2. Solutions of } f'' = -a_1 f^{-2} + a_2 f^{-3} \text{ for } a_1 = a_2 = 1; \]
\[ \text{initial conditions top: } f(0) = 1, f'(0) = 0 \text{ (dashed), } f'(0) = 0.1 \text{ (undashed); bottom: } f'(0) = 1. \]

For different coefficients, e.g. \( a_2 > a_1 \), the oscillatory solutions loose their up-down symmetry but may still be periodic (fig. 3). The inversion point for the initial conditions \( f(0) = 1, f'(0) = -0.4 \) (fig. 3 left) is no cusp, but a differentiable turning point from contraction to expansion (fig. 4).

For increasing ratio \( \frac{a_2}{a_1} \) the periods of the oscillations increase. Above a certain bound \( \frac{a_2}{a_1} \geq \alpha \), the expansive power of the vacuum pressure prevails; the solution expands monotonically. Numerical investigations indicate a bound \( \alpha \approx 1.95 \) for \( f'(0) = 0 \) and lower for \( f'(0) \neq 0 \). Numerical explorations thus indicate a regime of Lyapunov stability about the static solution.

In this sense, the Einstein universe is theoretically vindicated from a dynamical point of view. Eddington’s famous charge of instability holds for the cosmic constant Ansatz of vacuum energy, but does not hold for the

\[ \text{figure} \]

\[ \text{In (Fischer 1993, equ. (5)) this equation has been derived by assuming without further ado that the negative pressure of the Einstein universe derives from gravitational self-energy, and the qualitative behaviour has been correctly sketched.} \]
gravitational self energy approach. Here the Einstein universe reappears as the static neutral mode of a Lyapunov stable regime of equation (15). It remains to be seen, whether this observation may be of empirical value.

4. Application to Einstein-Weyl models

Contrary to a widely shared opinion, cosmological redshift need not necessarily be the result of a “true” space expansion. We cannot exclude that it may be due to a vacuum loss of photon energy or a higher order gravitational effect. Mathematically the two physical hypotheses, space expansion and photon energy loss by other reasons, are interchangeable by using integrable Weyl geometry. The (Weylian) metric is given, in gauge, by a pair \((g, \varphi)\) consisting of a Lorentz metric \(g = (g_{ij})\) and a real differential 1-form \(\varphi = \sum \varphi_i dx^i\) (in short \(\varphi = (\varphi_i)\)). In our context, using a well known form of the Robertson-Walker metric, they acquire the form

\[
\sum_{ij} g_{ij} \quad (k = 0 \text{ or } \pm 1) \].

Gauge changes are obtained by rescaling the Riemannian component \(g\) of the gauge metric, \(\bar{g} = \Omega^2(t) g\), and concomitant gauge transformations of the differential form \(\bar{\varphi} = \varphi - d\log \Omega\). In the framework of Weyl geometry, the warp (scale) function of Robertson-Walker cosmologies may be “gauged away”; then the redshift characteristic of the classical warp function is expressed by the Weylian length (scale) connection \(\varphi\) only. If cosmological models in such a gauge (called Hubble gauge in (Scholz 2005)) turn out dynamically and empirically superior to the standard approach,
we have to take this as a strong indicator against the expanding space hypothesis for the Hubble effect (cosmological redshift) and in favour of the vacuum or field theoretic one.

A class of particularly simple models arises from regauging classical Robertson-Walker models with a linear warp function \( f(t) = Ht \) with constant \( H \geq 0 \). A distinguished gauge (Hubble gauge = Weyl gauge) leads to

\[
(17) \quad g : ds^2 = -dt^2 + R^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\Theta^2 + r^2 \sin^2 \Theta d\phi^2 \right), \quad \varphi = H dt.
\]

It indicates a spatial geometry of constant curvature \( \kappa = kR^{-2} \) and a time independent redshift characteristic with Hubble constant \( H \).

These cosmologies have been termed Weyl universes, and for \( k > 0 \) Einstein-Weyl universes. Two Weyl universes are isomorphic (in the sense of Weyl geometry) if their parameters (metrical modules)

\[
(18) \quad \zeta := H^{-2}k = H^{-2}R^{-2}
\]

coincide. Energy densities are

\[
\rho = \rho_m + \rho_{\text{vac}}, \quad \rho = \Omega \rho_{\text{crit}}, \quad \rho_m = \Omega_m \rho_{\text{crit}}, \quad \text{etc.}
\]

with critical density

\[
(19) \quad \rho_{\text{crit}} = \frac{3H^2}{8\pi G_N}.
\]

They (have to) satisfy the hyle condition (13), which means \( \Omega_{\text{vac}} = \frac{\Omega}{3} \) and \( \Omega_m = \frac{\Omega}{3} \Omega \). Moreover, in this model class the metrical parameter is determined by the total energy density:

\[
(20) \quad \zeta = \Omega - 1
\]

The class shows surprisingly good empirical properties. Supernovae data stand in excellent agreement with positively curved Weyl universes; recent data indicate\[\] \[
\zeta = 2.6 \pm 0.4.
\]

This fit hints to much higher values for mean mass energy density and for vacuum energy density, \( \Omega_m \approx 2.3 \) and \( \Omega_{\text{vac}} \approx 1.2 \), than accepted at the moment by the majority of cosmologists. Recent weighing of nearby galaxy groups by Ramallah e.a. indicates, however, the existence of much higher mass densities in galaxy groups than presently expected, with the consequence that \( \Omega_m \) might go up to \( \approx 3 \) (Lieu 2007, 10). The final word on mass densities in the universe seems not yet to be spoken. There is no reason to discard the hypothesis of Einstein-Weyl models in cosmology on the basis of presently preferred values for mass densities.

\[\]4It may be not by chance that Fahr e.a.’s economical universe condition leads exactly to this type with a linear expansion function. These authors continue to work in the classical (i.e., semi-Riemannian) Robertson-Walker framework.

\[\]5The fit has been improved with respect to (Scholz 2005) by the more recent data of (Riess e.a. 2007). With a mean square error \( \sigma \approx 0.21 \) for the deviation of the model magnitudes from the data points, the fit of the Einstein-Weyl model is now slightly better than that of the standard model with \( \sigma \approx 0.27 \). The mean standard data error for magnitudes is \( \sigma_{\text{dat}} \approx 0.24 \).
Vacuum energy density, on the other hand, behaves much more trustworthy in the new framework. The standard model of cosmology displays a surprising and even crazy looking shift between mass and vacuum energy densities during cosmological “evolution”, which allows relations $\Omega_{\text{vac}} \approx 0.75, \Omega_m \approx 0.25$ only for a cosmologically short transitory period (Carroll 2001). The Weyl geometric models are not affected by such anomalies. Here $\Omega_m$ and $\Omega_{\text{vac}}$ remain in a narrow interval corridor. They remain basically constant, with small fluctuations estimated below. It therefore seems highly interesting to check, whether the hypothesis of a gravitational self-binding energy as origin of vacuum energy is consistent with observational data.

The generalized Newtonian approximation for self-binding energy in spherical spaces (9) is consistent with the hyle condition if and only if

$$\rho_{\text{vac}} = \beta_0 G_N \rho^2 R^2 = \frac{\rho}{3}.$$ 

Using (19), (18) leads to

$$\beta_0 = \frac{8\pi \zeta}{9(\zeta + 1)}.$$ 

For $\zeta \approx 2.6$, determined by the supernovae data, we find

$$\beta_0 \approx 2.02.$$ 

In the exact Newton Ansatz for the sphere, (10), the gravitational self-energy of the spherical Einstein universe lies more than a factor 3 higher, while the elliptical space (the round projective space) comes down to a factor

$$\frac{a_2}{a_1} \approx \frac{3.65}{2.02} \approx 1.7$$

in the parameters of the simplified Raychaudhury equation (15). The latter constellation lies in the Lyapunov stable regime, the former does not (see fig. 3).

We conclude: In the framework of Newtonian gravitational self-binding energy as source of cosmic vacuum energy the elliptical Einstein-Weyl universe is dynamically consistent with the supernovae data. The spherical Einstein-Weyl universe is consistent in this sense only if the characteristic coefficient $\beta$ of a Newtonian approximation in equ. (9) is about 40 % lower than in the exact Newton Ansatz. Otherwise it leads to an (unboundedly) expanding solution.

The time unit $[t]$ in the calculation of fig. 3 is

$$[t] = H_0^{-1} \sqrt{\frac{2}{\pi(\zeta + 1)}}.$$ 

For $\zeta \approx 2.6$ that is $[t] \approx 0.4 H_0^{-1}$. Typical periodicities of solutions are at the order of magnitude of 10 Hubble times, and the oscillation factor at the order of magnitude 10 (fig. 3).

The model displays a very slow pulsation about the static neutral mode with a moderate amplitude. For any observational purpose the dynamical model is very well approximated by the corresponding static Einstein-Weyl universe. The redshift component arising from expansion has, in principle,
to be superimposed to the one due to the Hubble form $\varphi = H dt$; but observationally it is negligible. It is intriguing to see how gravitational self energy, already in its Newtonian approximation, is apt to stabilize the geometry of Einstein-Weyl universes.

With $\zeta \approx 2.6^{+0.3}_{-0.4}$ the first conjugate point of the Einstein-Weyl universe (both for $S^3$ and the $E^3$) has redshift close to $z \approx 6^{+1.3}_{-0.7}$. The “equator” of the sphere corresponds to a redshift $z \approx 1.65^{+0.25}_{-0.14}$. If cosmological geometry of the spacelike fibres is elliptic, we should see objects close to the “equator” twice, in opposite directions and with slightly different redshifts. A simple empirical test could rely on quasar observations close to $z \approx 1.6$, which are exceptionally bright or share exceptional spectral, radio, or X-ray characteristics. They should appear in characteristic pairs. An empirical test for symmetry constellations of quasars expected in more complicated spherical spaces could be designed similarly. This would be a continuation, in a new research context, of the search for indicators of a non-trivial topology of the “universe” in the large, which has been attempted in the standard approach exploiting the latter’s peculiar view of the cosmic microwave background (Cornish e.a. 1998, Weeks 1998).

5. Conclusion

This paper contributes to the studies of gravitational self-binding energy which seem to offer a theoretically fruitful, and perhaps even empirically promising, route towards attacking the cosmic vacuum riddle. At the least they open up new perspectives at a theoretical level. It seems remarkable that already a Newtonian approximation for gravitational binding energy leads to unexpected dynamical consequences for traditional cosmological models which have been discarded for a long time. In particular the Einstein universe turns out as a neutral, stationary core state of a Lyapunov stable regime of cosmological models with closed spacelike fibres. In this sense, the Einstein universe comes out vindicated against Eddington’s charge of instability. A similar idea has been indicated in (Fischer 1993). Eddington’s warning applies, of course, to a cosmic constant ($\Lambda$-) Ansatz for vacuum energy.

Our investigation shows how the replacement of the formal-heuristic device of a $\Lambda$-term in the Lagrangian of cosmological models by a more physical hypothesis for vacuum energy may be able to change the overall dynamical behaviour of Robertson-Walker cosmologies drastically. The contributions of mass and gravitational self-energy to the total energy momentum seems to behave, under certain not unrealistic restrictions, like a self-stabilizing fluid. This is the justification for the word “hyle” condition of the whole stabilized system (13). Expectations of this kind seem to have been around among the first generation of relativists. Tullio Levi-Civita talked about the possibility of “real fluids” with negative pressure (Levi-Civita 1926, 359, 7

7 hyle $\sim$ (Greek) original substance. The original substance of Thales, the oldest Ionian natural philosopher known by name, was a kind of primordial fluid (“water”). In (Scholz 2007) a much more formal isentropic fluid Ansatz has been studied. It also led to stabilizing conditions.
This formulation made sense only with reference to cosmological considerations. At the end of the book he discussed the Einstein universe; but there seem to be no further notes on this question. Already this result underpins the demand that a physical concept of the cosmic vacuum (in contrast to a purely formal-heuristic one) ought to take actions in both directions into account, from vacuum on spacetime and matter, and from matter on vacuum. This should cast doubts on a “dynamical dark energy” as an agency which violates this basic principle.

So far our conclusions refer to primarily methodological questions. But there is more to resume. Our investigation has shown that the observational data on supernovae magnitudes are consistent with a Weyl geometric approach in a slightly modified Newtonian approximation with characteristic coefficient $\beta$ about 60% of the exact Newton Ansatz, or smaller. An unmodified Newton Ansatz for gravitational self-binding energy is consistent with the empirical data (if and) only if the spatial geometry is of a more complex topology than the sphere. A closer look at quasar data may be able to decide whether typical symmetry constellations of quasars, or other exceptional astronomical objects, about the ‘cosmic equator’ close to $z \approx 1.65$ do occur empirically. Most easily testable will be the simple antipodal symmetry of the elliptic case, $\mathcal{E}^3$.

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