Vilkovisky-DeWitt Effective Potential and the Higgs-Mass Bound

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We compute the Vilkovisky-DeWitt effective potential of a simplified version of the Standard Electroweak Model, where all charged boson fields as well as the bottom-quark field are neglected. The effective potential obtained in this formalism is gauge-independent. We derive from the effective potential the mass bound of the Higgs boson. The result is compared to its counterpart obtained from the ordinary effective potential.

1 Introduction

The gauge dependence of the effective potential was first pointed out by Jackiw in early seventies. This finding raised concerns on the physical significance of the effective potential. In a later work by Dolan and Jackiw, the effective potential of scalar QED was calculated in a set of $R_ξ$ gauges. It was concluded that only the limiting unitary gauge gives sensible result on spontaneous symmetry-breaking. This difficulty was partially resolved by the work of Nielsen. In his paper, Nielsen derived a simple identity characterizing the mean-field and the gauge-fixing-parameter dependences of the effective potential, namely,

$$\xi \frac{\partial}{\partial \xi} + C(\phi, \xi) \frac{\partial}{\partial \phi} V(\phi, \xi) = 0, \quad (1)$$

where $\xi$ is the parameter appearing in the gauge-fixing term $L_{gf} = -\frac{1}{2\xi} (\partial_\mu A^\mu)^2$. The above identity implies that the local extrema of $V$ for different $\xi$ are located along the same characteristic curve on $(\phi, \xi)$ plane, which satisfies $d\xi = \frac{d\phi}{C(\phi, \xi)}$. Hence covariant gauges with different $\xi$ are equally good for computing $V$. On the other hand, a choice of the multi-parameter gauge $L_{gf} = -\frac{1}{2}(\partial_\mu A^\mu + \sigma \phi_1 + r \phi_2)^2$ would break the homogeneity of Eq. (1). Hence effective potential calculated in this gauge has no physical significance.

Recently it was pointed out that the Higgs mass bound as derived from the effective potential is gauge-dependent. The gauge dependence enters in the calculation of one-loop effective potential, a quantity that is crucial for the determination of the Higgs mass bound. Boyanovsky, Loinaz and Willey has proposed a resolution of first-class constraints in the theory and a projection to the physical states. Such a procedure necessarily breaks the manifest Lorentz invariance of the theory. Consequently we expect it is highly non-trivial to apply this formalism to the Standard Model (SM).

In our work, we introduce the formalism of Vilkovisky and DeWitt for constructing an gauge-independent effective potential, and therefore obtaining a gauge-independent lower bound for the Higgs mass. We present the idea with a toy model which corresponds to neglect all charged boson fields in the SM. The generalization to the full SM is straightforward. In fact, the applicability of Vilkovisky-DeWitt formulation to non-abelian gauge theories has been extensively demonstrated in literature.

The outline of this presentation is as follows. In Section 2, we briefly review the formalism of Vilkovisky and DeWitt using scalar QED as an example. We shall illustrate that the effective action of Vilkovisky and DeWitt is equivalent to the ordinary effective action constructed in the Landau-DeWitt gauge. In Section 3, we calculate the effective potential of the simplified standard model, and the relevant renormalization constants of the theory are computed using the Landau-DeWitt gauge. The effective potential is then extended to large vacuum expectation value of the scalar field by means of renormalization group analyses. In Section 4, the mass bound of the Higgs boson is derived and compared to that given by ordinary effective action. Section 5 is the conclusion.

2 Vilkovisky-DeWitt Effective Action of Scalar QED

The formulation of Vilkovisky-DeWitt effective action is motivated by the parametrization dependence of the ordinary effective action, which can be written generically as

$$\exp \frac{i}{\hbar} \Gamma[\Phi] = \exp \frac{i}{\hbar} (W[j] + \Phi^i \frac{\delta \Gamma}{\delta \phi^i})$$

$$= \int [D\phi] \exp \frac{i}{\hbar} (S[\phi] + (\Phi^i - \phi^i) \cdot \frac{\delta \Gamma}{\delta \phi^i}). \quad (2)$$

The parametrization dependence of the ordinary effective action arises because the difference $\eta^i \equiv (\Phi^i - \phi^i)$ is
not a vector in the field configuration space, hence the product \( \eta^i \cdot \frac{\delta}{\delta \phi_i} \) is not a scalar under reparametrization. The remedy to this problem is to replace \(-\eta^i\) with a two-point function \( \sigma^i(\Phi, \phi) \) which, at the point \( \Phi \), is tangent to the geodesic connecting \( \Phi \) and \( \phi \). The precise form of \( \sigma^i(\Phi, \phi) \) depends on the connection \( \Gamma_{jk}^i \) of the configuration space. It is easy to show that

\[
\sigma^i(\Phi, \phi) = -\eta^i - \frac{1}{2} \Gamma_{jk}^i \eta^j \eta^k + O(\eta^3) \tag{3}
\]

For scalar QED described by the Lagrangian:

\[
L = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + (D_\mu \phi)^i (D^\mu \phi) - \lambda (\phi^i \phi - \mu^2)^2, \tag{4}
\]

with \( D_\mu = \partial_\mu + ieA_\mu \) and \( \phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \), we have

\[
\Gamma_{jk}^i = \{ jk \} + T_{jk}^i, \tag{5}
\]

where \( \{ jk \} \) is the Christoffel symbol of the field configuration space which has the following metric:

\[
G_{\phi_\alpha(x)\phi_\beta(y)} = \delta_{\alpha\beta} \delta^i(x-y), \quad G_{A_\mu(x)A_\nu(y)} = -g^{\mu\nu} \delta^i(x-y), \quad G_{A_\mu(x)\phi_\alpha(y)} = 0. \tag{6}
\]

We note that the metric of the field configuration space is determined by the quadratic part of the Lagrangian according to the prescription of Vilkovisky. In the above flat-metric, we have \( \{ jk \} = 0 \). However, the Christoffel symbols would be non-vanishing in the parametrization with polar variables \( \rho \) and \( \chi \) such that \( \phi_1 = \rho \cos \chi \) and \( \phi_2 = \rho \sin \chi \). \( T_{jk}^i \) is a quantity pertinent to generators \( g^i_\alpha \) of the gauge transformation. Explicitly, we have

\[
T_{jk}^i = -B_{ij} D_k g^a_\alpha + \frac{1}{2} g^a_\alpha D_\rho K^a_j B^a_\beta + j \leftrightarrow k, \tag{7}
\]

where \( B^a_\beta = N^{\alpha \beta} g_{\beta \delta} \) with \( N^{\alpha \beta} \) being the inverse of \( N_{\alpha \beta} \equiv g^a_{\alpha \beta} G_{kl} \). In scalar QED, the generators \( g^i_\alpha \) are given by

\[
g^a_\alpha(x) = -eab \phi_b(x) \delta^4(x-y), \quad g^a_\mu(x) = -\partial_\mu \delta^4(x-y). \tag{8}
\]

with \( e^{12} = 1 \). The one-loop effective action of scalar QED can be calculated from Eq. (4) with the quantum fluctuations \( \eta^i \) replaced by \( \sigma^i(\Phi, \phi) \). The result is written

\[
\Gamma[\Phi] = S[\Phi] - i\hbar \frac{1}{2} \ln \det G + i\hbar \frac{1}{2} \ln \det \tilde{D}_{ij}^{-1}, \tag{9}
\]

where \( S[\Phi] \) is the tree-level effective action; \( \ln \det G \) arises from the function space measure \( [D\phi] \equiv \prod_\omega d\phi(x)\sqrt{\det G} \), and \( \tilde{D}_{ij}^{-1} \) is the modified inverse-propagator:

\[
\tilde{D}_{ij}^{-1} = \frac{\delta^2 S}{\delta \Phi_i \delta \Phi_j} - \Gamma_{jk}^i \frac{\delta S}{\delta \Phi_k}. \tag{10}
\]

To study the symmetry-breaking behaviour of the theory, we focus on the effective potential which can be obtained from \( \Gamma[\Phi] \) by setting the classical fields \( \Phi \)’s to constants.

The Vilkovisky - DeWitt effective potential of scalar QED has been calculated in various gauges and different parametrizations of scalar fields. The results all agree with one another. In this work, we calculate the effective potential and other relevant quantities in Landau-DeWitt gauge \( \tilde{g}_{ij} \), which is characterized by the gauge-fixing term: \( L_{gf} = -\frac{1}{2\xi}(\partial_\mu B^\mu - ie\eta^i \phi_1 + i e\Phi_1 \eta^i)^2 \), with \( \xi \to 0 \). In \( L_{gf} \), \( B^\mu \equiv A^\mu - A^\mu_0 \), and \( \eta \equiv \phi - \phi_0 \) are quantum fluctuations while \( A^\mu_0 \) and \( \phi_0 \) are classical fields. The advantage of performing calculations in the Landau-DeWitt gauge is that \( T_{jk}^i \) vanish in this case. In other words, Vilkovisky-DeWitt formalism coincides with the conventional one in the Landau-DeWitt gauge.

For computing the effective potential, we choose \( A^0_0 = 0 \) and \( \Phi = \frac{\rho_0}{\sqrt{2}} \), i.e. the imaginary part of \( \Phi \) is set to zero. In this set of background fields, \( L_{gf} \) can be written as

\[
L_{gf} = -\frac{1}{2\xi} \left( \partial_\mu B^\mu \partial_\nu B^\nu - 2e \rho_0 \chi \partial_\mu B^\mu + e^2 \rho_0^2 \chi^2 \right), \tag{11}
\]

where \( \chi \) is the quantum field defined by \( \eta = \frac{\rho_0 \chi}{\sqrt{2} \xi} \). We note that \( B_0 - \chi \) mixing in \( L_{gf} \) is \( \xi \) dependent, and therefore would not cancel out the corresponding mixing term in the classical Lagrangian of Eq. (4). This induces the mixed-propagator such as \( <0|T(A_\mu(x)\chi(y))|0> \) or \( <0|T(\chi(x)A_\mu(y))|0> \). The Faddeev-Popov ghost Lagrangian is given by

\[
L_{FP} = \omega^\ast(-\partial^2 - e^2 \rho_0^2 \omega). \tag{12}
\]

With each part of the Lagrangian determined, we are ready to compute the effective potential. Since we choose a flat-metric, the one-loop effective potential is completely determined by the modified inverse propagators \( \tilde{D}_{ij}^{-1} \). From Eqs. (4), (11) and (12), we arrive at

\[
\tilde{D}_{B^\mu, B^\nu}^{-1} = (-k^2 + e^2 \rho_0^2) g^{\mu \nu} + (1 - \frac{1}{\xi}) k^\mu k^\nu, \tag{13}
\]

\[
\tilde{D}_{B_0, \chi}^{-1} = i e \rho_0(1 - \frac{1}{\xi}), \tag{14}
\]

\[
\tilde{D}_{\chi, \chi}^{-1} = (k^2 - m_\chi^2 - \frac{1}{\xi} e^2 \rho_0^2), \tag{15}
\]

\[
\tilde{D}_{\rho_0, \rho_0}^{-1} = (k^2 - m_\rho_0^2), \tag{16}
\]

\[
\tilde{D}_{\omega, \omega}^{-1} = (k^2 - e^2 \rho_0^2)^2, \tag{17}
\]
where we have set $\rho_{cl} = \rho_0$, which is a space-time independent constant, and defined $m_0^2 = \lambda (3\rho_0^2 - 2\mu^2)$, $m_H^2 = \lambda (3\rho_0^2 - 2\mu^2)$. Using the definition $\Gamma[\rho_0] = (2\pi)^4 \delta^4(0)V_{eff}(\rho_0)$ along with Eqs. (12) and (13), and taking the limit $\xi \to 0$, we obtain $V_{eff}(\rho_0) = V_{thee}(\rho_0) + V_{1-loop}(\rho_0)$ with

$$V_{1-loop}(\rho_0) = -\frac{i\hbar}{2} \int \frac{d^n k}{(2\pi)^n} \ln[(k^2 - e^2 \rho_0)^{n-3}] \times (k^2 - m_0^2)(k^2 - m_0^2)(k^2 - m_0^2),$$

where $m_0^2$ and $m_0^2$ are solutions of the quadratic equation

$$(k^2 - 2e^2 \rho_0^2 + m_0^2)k^2 + e^2 \rho_0^4 = 0.$$  

In the above equation, the gauge-boson’s degree of freedom has been continued to $n - 3$ in order to preserve the relevant Ward identities. Our expression of $V_{1-loop}(\rho_0)$ agrees with previous results obtained in the unitary gauge. One could also calculate the effective potential in the ghost-free covariant gauges with $L_{gf} = -\frac{1}{4\pi}(\partial_\mu B^\mu)^2$. The cancellation of gauge-parameter dependence in the effective potential has been demonstrated in the case of massless scalar-QED with $\mu^2 = \frac{\sqrt{13}}{2} \mu_0^2$. It can be easily extended to the massive case.

It is instructive to rewrite Eq. (14) as

$$V_{1-loop}(\rho_0) = \frac{\hbar}{2} \int \frac{d^n k}{(2\pi)^n} \left( (n - 3)\omega_B(\bar{k}) + \omega_H(\bar{k}) \right) + \omega_+(\bar{k}) + \omega_-(\bar{k}),$$

where $\omega_B(\bar{k}) = \sqrt{k^2 + 2e^2 \rho_0^2}$, $\omega_H(\bar{k}) = \sqrt{k^2 + m_0^2}$ and $\omega_+(\bar{k}) = \sqrt{k^2} + m_0^2$. One can see that $V_{1-loop}$ is a sum of the zero-point energies of four excitations with masses $m_B = e\rho_0$, $m_H$, $m_+$, and $m_-$. Since there are precisely four physical degrees of freedom in scalar QED, we see that Vilkovisky-DeWitt effective potential does exhibit a correct number of physical degrees of freedom.

### 3 Vilkovisky-DeWitt Effective Potential of the Simplified Standard Model

In this section, we compute the effective potential of the simplified standard model where charged boson fields and all fermion fields except the top quark field are discarded. The gauge interactions of this model are prescribed by the following covariant derivative:

$$D_\mu t_L = (\partial_\mu + ig_L Z_\mu - \frac{2}{3} ieA_\mu)t_L,$$

$$D_\mu t_R = (\partial_\mu + ig_R Z_\mu - \frac{2}{3} ieA_\mu)t_R,$$

$$D_\mu \phi = (\partial_\mu + ig_L - g_R Z_\mu)\phi,$$

where $g_L = (-g_1/2 + g_2/3)$, $g_R = g_2/3$ with $g_1 = g/\cos\theta_W$ and $g_2 = 2e\tan\theta_W$. Clearly this toy model exhibits a $U(1)_A \times U(1)_Z$ symmetry where each $U(1)$ symmetry is associated with a neutral gauge boson. The $U(1)_Z$-charges of $t_L$, $t_R$ and $\phi$ are related in such a way that the following Yukawa interactions are invariant under $U(1)_A \times U(1)_Z$:

$$L_Y = -y_0 t_L \phi t_R - y_0 \phi^* t_L.$$  

Since Vilkovisky-DeWitt effective action coincides with ordinary effective action calculated in the Landau-DeWitt gauge, we hence calculate the effective potential in this gauge which has

$$L_{gf} = -\frac{1}{2\alpha} (\partial_\mu Z^\mu + \frac{i}{\sqrt{2}} \eta^\dagger \Phi - \frac{ig_1}{2} \Phi^\dagger \eta)^2 - \frac{1}{\beta} (\partial_\mu A^\mu)^2,$$

with $\alpha, \beta \to 0$. We note that $A^\mu$ and $Z^\mu$ are quantum fluctuations associated with the photon and the Z boson. Their classical backgrounds can be set to zero for computing the effective potential. Following the method of the previous section, we obtain

$$V_D(\rho_0) = \frac{\hbar}{2} \int \frac{d^{n-1} k}{(2\pi)^{n-1}} \left( (n - 3)\omega_Z(\bar{k}) + \omega_H(\bar{k}) \right) + \omega_+(\bar{k}) + \omega_-(\bar{k}) - 4\omega_F(\bar{k}),$$

with $\omega_{i}(\bar{k}) = \sqrt{k^2 + m_i^2}$ where $m_Z^2 = \frac{g_i^2}{4} \rho_i^2$, $m_W^2 = m_Z^2 + \frac{1}{2} (m_G^2 \pm m_G \sqrt{m_G^2 + 4m_W^2})$ and $m_t^2 = \frac{g_t^2}{4} \rho_t^2$. Performing the integration in Eq. (14) and subtracting the infinities with $\overline{\text{MS}}$ prescription, we obtain

$$V_D(\rho_0) = \frac{1}{64\pi^2} \left( m_H^4 \ln \frac{m_H^2}{k^2} + m_L^4 \ln \frac{m_L^2}{k^2} \right. + m_H^4 \ln \frac{m_+^2}{k^2} + m_-^4 \ln \frac{m_-^2}{k^2} - 4m_+^4 \ln \frac{m_-^2}{k^2} - \left. \frac{1}{128\pi^2} (3m_H^4 + 5m_L^4 + 3m_G^4 + 12m_X^2 - 12m_Y^4) \right).$$

Although $V_D(\rho_0)$ is obtained in the Landau-DeWitt gauge, we should stress that any other gauge with non-vanishing $T^0_\mu$ should lead to the same result. For later comparisons, let us write down the ordinary effective potential in the ghost-free Landau gauge (equivalent to removing the scalar part of Eq. (18)):

$$V_L(\rho_0) = \frac{1}{64\pi^2} \left( m_H^4 \ln \frac{m_H^2}{k^2} + 3m_L^4 \ln \frac{m_L^2}{k^2} \right.$$

$$\left. + 3m_G^4 \ln \frac{m_G^2}{k^2} \right).$$

Performing the integrations in $V_L$ and subtracting the infinities give

$$V_L(\rho_0) = \frac{1}{64\pi^2} \left( m_H^4 \ln \frac{m_H^2}{k^2} + 3m_L^4 \ln \frac{m_L^2}{k^2} \right.$$
\[ + m_G^4 \ln \frac{m_G^2}{\kappa^2} - 4m_t^4 \ln \frac{m_t^2}{\kappa^2} \]
\[ - \frac{1}{128\pi^2} (3m_H^4 + 5m_Z^4 + 3m_G^4 - 12m_t^4). \] (22)

We remark that \( V_L \) differs from \( V_{V_D} \) except at the point of extremum where \( \rho_0^2 = 2\mu^2 \). At this point, one has \( m_H^2 = 0 \) and \( m_Z^2 = m_Z^2 \) which leads to \( V_{V_D}(\rho_0 = 2\mu^2) = V_L(\rho_0^2 = 2\mu^2) \). That \( V_{V_D} = V_L \) at the point of extremum is a consequence of Nielsen identities.

To derive the Higgs mass bound from \( V_{V_D}(\rho_0) \) or \( V_L(\rho_0) \), one encounters a breakdown of the perturbation theory at, for instance, \( \frac{\lambda_i^2}{\kappa^2} \ln \frac{\lambda_i^2}{\kappa^2} > 1 \) for a large \( \rho_0 \). To extend the validity of the effective potential for a large \( \rho_0 \), the effective potential has to be improved by the renormalization group (RG) analysis. Let us denote the effective potential with one-loop functions of \( \rho \) and \( \mu \), and \( \gamma \) and \( \mu \). As the computation of two-loop functions differs from \( \gamma \), the anomalous dimension of the scalar field, are in fact gauge-independent in the \( \overline{MS} \) subtraction scheme. For \( \gamma \) in the Landau gauge, we have
\[ \gamma(\rho) = \frac{1}{64\pi^2} (-3g_1^2 + 4g_2^2). \] (28)

4. The Higgs Mass Bound

The lower bound of the Higgs mass can be derived from the vacuum instability condition for the effective potential. To derive the mass bound, one begins with Eq. (24) which implies
\[ V_{tree}(t\rho_0, \mu_i, \lambda_i) = \frac{1}{4} \chi(t)(\lambda(t, \lambda_i)((\rho_0)^2 - 2\mu^2(t, \mu_i))^2, \] (29)
with \( \chi(t) = \exp \left( \int_0^{t} \frac{1}{1 + \gamma_0}(\rho_0) dx \right) \). Since \( \mu(t, \mu_i) \) decreases as \( t \) increases, we have \( V_{tree}(t\rho_0, \mu_i, \lambda_i) \approx \frac{1}{4} \chi(t)(\lambda(t, \lambda_i)(\rho_0)^2 \right) \) for a sufficiently large \( t \). Similarly, the one-loop effective potential \( V_{1-loop}(t\rho_0, \mu_i, \lambda_i, \gamma) \) is also proportional to \( V_{1-loop}(\rho_0, \mu_i, \gamma, \gamma, \lambda_i) \) with the same proportional constant \( \chi(t) \) as in \( V_{tree} \). Since we shall neglect all running effects in \( V_{1-loop} \), we have \( \hat{\gamma}(t, \gamma) = \gamma(t) \) and \( \mu(t, \mu_i) = \frac{\gamma}{\mu_0} \). In \( V_{1-loop} \), for a sufficiently large \( t \), we can again approximate \( V_{1-loop} \) by its quartic terms. In the Landau-DeWitt gauge with the choice \( \rho_0 = \kappa \), we have
\[ V_{V_D} \approx \left( \frac{\rho_0}{64\pi^2} \right)^4 \left[ 9\lambda_i^2 \ln(3\lambda_i) + \frac{g_1^4}{16} \ln \left( \frac{g_1^2}{4} \right) - y^1 \ln \left( \frac{y^2}{2} \right) \right] \]
\[ + A_+^2(g_{1i}, \lambda_i) \ln A_+(g_{1i}, \lambda_i) \]
\[ + A_-^2(g_{1i}, \lambda_i) \ln A_-(g_{1i}, \lambda_i) \]
\[ - \frac{3}{2} \left( 10\lambda_i^2 + \lambda_i g_{1i}^2 + \frac{5}{48} g_{1i}^4 - y^1 \right). \] (30)
where \( A_{\pm}(g_{1i}, \lambda_i) = g_{1i}^2/4 + \lambda_i/2 \left( 1 \pm \sqrt{1 + g_{1i}^2/\lambda_i} \right) \). Similarly, the effective potential in the Landau gauge reads:
\[ V_L \approx \left( \frac{\rho_0}{64\pi^2} \right)^4 \left[ 9\lambda_i^2 \ln(3\lambda_i) + \frac{g_1^4}{16} \ln \left( \frac{g_1^2}{4} \right) - y^1 \ln \left( \frac{y^2}{2} \right) \right] \]
\[ + \lambda_i^2 \ln(\lambda_i) - \frac{3}{2} \left( 10\lambda_i^2 + \lambda_i g_{1i}^2 + \frac{5}{48} g_{1i}^4 - y^1 \right). \] (31)
Combining the tree level and the one-loop effective potential, we arrive at

\[ V_{\text{eff}}(t\rho_0, \mu_i, \hat{g}_i, \kappa) \approx \frac{1}{4} \lambda(t, \lambda_i) \Delta \lambda(\hat{g}_i) (\rho_0)^4, \]

where \(\Delta \lambda\) denotes one-loop corrections given by Eqs. (30) or (31). Let \(t_{V1} = \rho_{V1}/\rho_0\). The condition for vacuum instability of the effective potential is then

\[ \lambda(t_{V1}, \lambda_i) + \Delta \lambda(\hat{g}_i) = 0. \]  

We note that couplings \(\hat{g}_i\) in \(\Delta \lambda\) is evaluated at \(\kappa = \rho_0\), which can be taken as the electroweak scale. Hence \(g_{V1} = g_{\cos \theta_W} = 0.67, g_{2\iota} = 2e \tan \theta_W = 0.31\), and \(y_i = 1\). The running coupling \(\lambda(t_{V1}, \lambda_i)\) also depends on \(y_1, g_2\) and \(y\) through \(\beta_\lambda\), and \(\gamma_\rho\) shown in Eq. (27).

The strategy for solving Eq. (33) is to make an initial guess on \(\lambda_i\), which enters into \(\lambda(t)\) and \(\Delta \lambda,\) and repeatedly adjusting \(\lambda_i\) till \(\lambda(t)\) completely cancels \(\Delta \lambda\). For \(t_{V1} = 10^4\) (or \(\rho_0 = 10^4\) (GeV) which is the new-physics scale reachable by LHC, we find \(\lambda_i = 4.83 \times 10^{-2}\) for Landau-DeWitt gauge, and \(\lambda_i = 4.8 \times 10^{-2}\) for Landau gauge. For a higher instability scale such as the scale of grand unification, we have \(t_{V1} = 10^{13}\) or \(\rho_0 \approx 10^{15}\) GeV. In this case, we find \(\lambda_i = 3.13 \times 10^{-1}\) for both Landau-DeWitt and Landau gauges. The numerical agreement between \(\lambda_i\)'s of two gauges can be attributed to an identical \(\beta\) function for the running of \(\lambda(t)\), and a small difference in \(\Delta \lambda\) between two gauges. We recall from Eq. (23) that the evolution of \(\lambda\) in two gauges will be different if effects of next-to-leading logarithm are taken into account. In that case, the difference in \(\gamma_\rho\) between two gauges give rise to different evolutions for \(\lambda\). One may expect to see non-negligible differences in \(\lambda_i\) between two gauges for a large \(t_{V1}\).

The critical value \(\lambda_i = 4.83 \times 10^{-2}\) corresponds to a lower bound for the \(MS\) mass of the Higgs boson. Since \(m_H = 2\sqrt{\lambda_0}\), we have \((m_H)_{\text{MS}} \geq 77\) GeV. For \(\lambda_i = 3.13 \times 10^{-1}\), we have \((m_H)_{\text{MS}} \geq 196\) GeV. To obtain the lower bound for the physical mass of the Higgs boson, finite radiative corrections must be added to the above bounds. We will not pursue any further on these finite corrections since we are simply dealing with a toy model. However, we like to point out that this finite correction is gauge-independent as ensured by Nielsen identities.

5 Conclusion

We have computed the one-loop effective potential of an abelian \(U(1) \times U(1)\) model in the Landau-DeWitt gauge, which reproduces the result given by the gauge-independent Vilkovisky-DeWitt formalism. One-loop \(\beta\) and \(\gamma\) functions are also computed to facilitate the RG improvement of the effective potential. A gauge-independent lower bound for the Higgs self-coupling or equivalently the \(MS\) mass of the Higgs boson is derived.

We compare this bound to that obtained by the ordinary effective potential computed in Landau gauge. The numerical values of both bounds are almost identical due to the leading-logarithmic approximation we have taken. A complete next-to-leading approximation as well as an extension of this work to the full standard model will be reported in future publications.

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8. See, for example, B. S. DeWitt in Quantum Field Theory and Quantum Statistics: Essays in Honour of the 60th Birthday of E. S. Fradkin, eds. I. A. Batalin, C. J. Isham and G. A. Vilkovisky (Hilger, Bristol, 1987), p. 191.
9. A mere replacement of \(-\eta^i\) with \(\sigma^i(\Phi, \phi)\), as suggested in Ref. 1, is not satisfactory for calculations beyond one-loop, because \(\Gamma[\Phi]\) does not generate one-particle irreducible diagrams at the higher loops. A modified construction was given by DeWitt in Ref. 11. Since we will be only concerned with one-loop corrections, we shall adhere to the current construction in our subsequent discussions.
10. See, for example, G. Kunstatter, in Super Field Theories, eds. H. C. Lee, V. Elias, G. Kunstatter, R. B. Mann, and K. S. Viswanathan (Plenum, New York, 1987), p. 503.
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17. In the current gauge, \( \tilde{D}^{-1}_{ij} = D^{-1}_{ij} \) since \( \Gamma^i_{jk} \) vanishes. Furthermore, for ghost fields, it is the ghost propagator rather than its inverse that will appear in the effective action. This is due to the Grassmannian nature of ghost fields.
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