Computation of the structure of magnetized strange quark star

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Abstract

In this work, we have calculated some properties of the spin polarized strange quark matter (SQM) in a strong magnetic field at zero temperature using the MIT bag model. We have shown that the equation of state of spin polarized SQM is stiffer than that of the unpolarized case. We have also computed the structure properties of the spin polarized strange quark star (SQS) and have found that the presence of magnetic field leads to a more stable SQS compared to the unpolarized SQS.

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I. INTRODUCTION

Strange quark stars (SQS) are those which are built mainly from self bound strange quark matter (SQM). The surface density of SQS is equal to the density of SQM at zero pressure ($\sim 10^{15}$ $g/cm^3$), which is fourteen orders of magnitude greater than the surface density of a normal neutron star. The central density of these stars is about five times greater than their surface density [1–4]. The existence of SQS which are made of SQM was first proposed by Itoh [5] even before the full developments of QCD. Later Bodmer [6] discussed the fate of an astronomical object collapsing to such a state of matter. In 1970s, after the formulation of QCD, the perturbative calculations of the equation of state of the SQM was developed, but the region of validity of these calculations was restricted to very high densities [7]. The existence of SQS was also discussed by Witten [8] who conjectured that a first order QCD phase transition in the early universe could concentrate most of the quark excess in dense quark nuggets. He suggested that the true state of matter was SQM. Witten proposal was that the SQM composed of light quarks is more stable than nuclei, therefore SQM can be considered as the ground state of matter. SQS would be the bulk SQM phase consisting of almost equal numbers of up, down, and strange quarks plus a small number of electrons to ensure the charge neutrality. A typical electron fraction is less than $10^{-3}$ and it decreases from the surface to the center of SQS [1–4]. SQM would have a lower charge to baryon ratio compared to the nuclear matter and can show itself in the form of SQS [8–11].

The collapse of a massive star may lead to the formation of a SQS. A SQS may be also formed from a neutron star and is denser than the neutron star [12]. If sufficient additional matter is added to a SQS, it will collapse into a black hole. Neutron stars with masses of $1.5 - 1.8 M_\odot$ with rapid spins are theoretically the best candidates for conversion to the SQS. An extrapolation based on this indicates that up to two quark-novae occur in the observable universe each day. Besides, recent Chandra observations indicate that objects RX J185635-3754 and 3C58 may be bare SQS [13].

It is known that the compact objects such as the neutron stars, pulsars, magnetars, and strange quark stars are under the influence of the strong magnetic field, which typically is about $10^{15} - 10^{19}$ $G$ [1–4, 14, 15]. Therefore, in astrophysics, it is of special interest to study the effect of strong magnetic field on SQM properties which can be found in the core of neutron stars and also in the SQS. We note that in the presence of magnetic field,
the conversion of neutron stars to bare quark stars can not take place unless the value of magnetic field exceeds $10^{20} \, G$ \[16\].

Recently, we have calculated the structure of unpolarized SQS at zero temperature \[17\] and finite temperature \[18\]. In this article, we focus on SQS which is purely composed of the spin polarized SQM, and investigate the effects of strong magnetic field on different properties of such an star. In section 2, we study the spin polarized SQM in the absence and presence of the strong magnetic field. In section 3, by numerically solving the Tolman-Oppenhaimer-Volkoff equation, we obtain the structure properties of the spin polarized SQS. Moreover, we discuss the stability of spin polarized SQS.

II. ENERGY CALCULATION FOR THE SPIN POLARIZED SQM

We consider the spin polarized SQM composed of $u$, $d$, and $s$ quarks with spin up ($+$) and down ($-$). We denote the number density of quark $i$ with spin up by $\rho_i^{(+)}$, and spin down by $\rho_i^{(-)}$. We introduce the polarization parameter $\xi_i$ by

$$\xi_i = \frac{\rho_i^{(+)} - \rho_i^{(-)}}{\rho_i}$$

where $0 \leq \xi_i \leq 1$ and $\rho_i = \rho_i^{(+)} + \rho_i^{(-)}$. Under the conditions of beta-equilibrium and charge neutrality, we get the following relation for the number density,

$$\rho = \rho_u = \rho_d = \rho_s,$$

where $\rho$ is the total baryonic density of the system.

Now, we calculate the energy density of spin polarized SQM. To calculate the total energy of spin polarized SQM, we use MIT bag model in which the total energy is the sum of kinetic energy of quarks plus a bag constant ($B_{bag}$) \[19\]. The bag constant $B_{bag}$ can be interpreted as the difference between the energy densities of the noninteracting quarks and the interacting ones. Dynamically it acts as a pressure that keeps the quark gas in constant density and potential. In MIT bag models, different values are considered for the bag constant such as $55$ and $90 \, MeV/fm^3$. We calculate the energy density of SQM in the absence and presence of the magnetic field in the following two separate sections.
A. Energy density of spin polarized SQM in the absence of magnetic field

The total energy of the spin polarized SQM in the absence of magnetic field ($B = 0$) is given by

$$\varepsilon_{\text{tot}}^{(B=0)} = \varepsilon_u + \varepsilon_d + \varepsilon_s + B_{\text{bag}}, \quad (3)$$

where $\varepsilon_i$ is the kinetic energy per volume of quark $i$,

$$\varepsilon_i = \sum_{p=\pm} \sum_{k(p)} \sqrt{m_i^2 c^4 + \hbar^2 k(p)^2 c^2}. \quad (4)$$

We ignore the masses of quarks $u$ and $d$, while we consider $m_s = 150$ MeV for quark $s$. After doing some algebra, supposing that $\xi_s = \xi_u = \xi_d = \xi$, we get the following relation for the total energy of the spin polarized SQM,

$$\varepsilon_{\text{tot}}^{(B=0)} = \frac{3}{16\pi^2 \hbar^4} \sum_{p=\pm} \left[ \frac{\hbar}{c^2} k_F^{(p)} E_F^{(p)} \left( 2\hbar^2 k_F^{(p)} c^2 + m_s^2 c^4 \right) - m_s^4 c^5 \ln \left( \frac{h k_F^{(p)} + E_F^{(p)}}{m_s c} \right) \right]$$

$$+ \frac{3 \hbar c \pi^{2/3}}{4} \rho^{4/3} \left[ (1 + \xi)^{4/3} + (1 - \xi)^{4/3} \right] + B_{\text{bag}}, \quad (5)$$

where

$$k_F^{\pm} = \left( \pi^2 \rho \right)^{1/3} (1 \pm \xi)^{1/3}, \quad (6)$$

and

$$E_F^{\pm} = \left( \hbar^2 k_F^{(\pm)} c^2 + m_s^2 c^4 \right)^{1/2}. \quad (7)$$

In Fig. 1, we have plotted the total energy density of spin polarized SQM as a function of the density for different values of the polarization ($\xi$) in the absence of magnetic field. Fig. 1 shows that the energy is an increasing function of the density, however the increasing rate of energy versus density increases by increasing polarization. For each density, we see that the energy of spin polarized SQM increases by increasing polarization, specially at high densities.

For the spin polarized SQM, we can also calculate the equation of state (EoS) using the following relation,

$$P(\rho) = \rho \frac{\partial \varepsilon_{\text{tot}}}{\partial \rho} - \varepsilon_{\text{tot}}, \quad (8)$$

where $P$ is the pressure and $\varepsilon_{\text{tot}}$ is the energy density which in the absence of magnetic field, is obtained from Eq. (3). In Fig. 2, we have shown the pressure of spin polarized SQM.
as a function of the density for various values of the polarization parameter in the absence
of magnetic field. We see that for a given density, the pressure increases by increasing
polarization. This shows that the EoS of spin polarized SQM is stiffer than that of the
unpolarized case. From Fig. 2 it can be seen that by increasing polarization, the density
 corresponding to zero pressure takes lower values.

B. Energy density of spin polarized SQM in the presence of magnetic field

In this section, we consider the spin polarized SQM which is under influence of a strong
magnetic field ($B$). For this system, the contribution of magnetic energy is $E_M = -\mathbf{M} \cdot \mathbf{B}$. If we consider the magnetic field along $z$ direction, the contribution of magnetic energy of
the spin polarized SQM is given by

$$E_M = - \sum_{i=u,d,s} M_z^{(i)} B,$$

where $M_z^{(i)}$ is the magnetization of system corresponding to particle $i$ which is given by

$$M_z^{(i)} = N_i \mu \xi_i.$$  \hspace{1cm} (10)

In the above equation, $N_i$ and $\mu_i$ are the number and magnetic moment of particle $i$, re-
respectively. By some simplification, the contribution of magnetic energy density of the spin
polarized SQM, $\varepsilon_M = \frac{E_M}{V}$, can be obtained as follows,

$$\varepsilon_M = - \sum_{i=u,d,s} \rho_i \mu_i \xi_i B.$$  \hspace{1cm} (11)

Consequently, the total energy density of spin polarized SQM in the presence of magnetic
field can be written as

$$\varepsilon_{tot}^{(B)} = \varepsilon_{tot}^{(B=0)} + \varepsilon_M.$$  \hspace{1cm} (12)

In Fig. 3 we have shown the total energy density of the spin polarized SQM as a
function of the polarization parameter ($\xi$), for $B = 5 \times 10^{18} G$ at various densities. From
Fig. 3 we have seen that the energy curve shows a minimum for each relevant density.
This behavior indicates that for each density there is a metastable state. We have also
seen that as the density increases, this metastable state is shifted to lower values of the
polarization parameter. Therefore, we can conclude that the metastable state disappears.
at high densities. We have also found that at high densities, the system becomes nearly identical with the unpolarized case. These results agree with those of reference [20]. In Fig. 4, we have plotted the total energy density of the spin polarized SQM versus the number density in the presence of magnetic field. We have seen that the total energy increases by increasing the density. We have found that the energy density of the spin polarized SQM in the presence of magnetic field is nearly identical with that of the unpolarized case which has been clarified in panel (b) of Fig. 4. As we will see in the next paragraph, this is due to the fact that the polarization parameter in the presence of magnetic field is very small, especially at high densities.

In Fig. 5, we have presented the polarization parameter corresponding to the minimum point of energy density as a function of the number density at $B = 5 \times 10^{18} G$. We see that the polarization parameter decreases by increasing the number density. From Fig. 5, it can be seen that for $\rho < 0.2 \text{ fm}^{-3}$, the decreasing rate of polarization versus density is substantially higher than for $\rho > 0.2 \text{ fm}^{-3}$. In Fig. 6 we have shown the polarization parameter versus the magnetic field for different values of the number density. For each density, we can see that the polarization increases by increasing the magnetic field. This figure also shows that the increasing rate of polarization versus magnetic field increases by increasing density.

We have also calculated EoS of spin polarized SQM in the presence of the magnetic field, where the contribution of magnetic pressure ($\frac{B^2}{8\pi}$) should be added to Eq. (8) in which the total energy density is obtained from Eq. (12). In Fig. 7, we have plotted EoS of spin polarized SQM where the magnetic field is switched on. We have found that this EoS is nearly identical with that of the unpolarized case. This is due to the fact that polarization at minimum of energy is very low, especially at high densities.

In Fig. 8, we have plotted the energy per baryon ($E/A$) for the spin polarized SQM as a function of pressure at $B = 5 \times 10^{18} G$. Our results for the case of SQM in the absence of magnetic field ($B = 0$) are also given for comparison. We have seen that the zero point of pressure in the presence of magnetic field has a lower $E/A$ compared to the case of SQM in the absence of magnetic field ($B = 0$). This indicates that, in the presence of magnetic field, the spin polarized SQM is more stable than that in the absence of magnetic field.
III. STRUCTURE OF THE SPIN POLARIZED SQS

The gravitational mass \((M)\) and radius \((R)\) of compact stars are of special interests in astrophysics. In this section, we calculate the structure properties of spin polarized SQS and compare the results of this calculation with those of the unpolarized case. Using the EoS of spin polarized SQM, We can obtain \(M\) and \(R\) by numerically integrating the general relativistic equations of hydrostatic equilibrium, Tolman-Oppenheimer-Volkoff (TOV) equations, which are as follows \[21\],

\[
\frac{dm}{dr} = 4\pi r^2 \varepsilon(r),
\]

\[
\frac{dP}{dr} = -\frac{G m(r) \varepsilon(r)}{r^2} \left( 1 + \frac{P(r)}{\varepsilon(r)c^2} \right) \left( 1 + \frac{4\pi r^3 P(r)}{m(r)c^2} \right) \left( 1 - \frac{2Gm(r)}{c^2r} \right)^{-1},
\]

(13)

where \(\varepsilon(r)\) is the energy density, \(G\) is the gravitational constant, and

\[
m(r) = \int_0^r 4\pi r'^2 \varepsilon(r') dr'
\]

(14)

has the interpretation of the mass inside radius \(r\). By selecting a central energy density \(\varepsilon_c\), under the boundary conditions \(P(0) = P_c, m(0) = 0\), we integrate the TOV equation outwards to a radius \(r = R\), at which \(P\) vanishes. This yields the radius \(R\) and mass \(M = m(R)\) \[21\].

Our results for the structure of spin polarized SQS in the absence and presence of the magnetic field are given separately in two following sections.

A. Structure of the spin polarized SQS in the absence of magnetic field

In Figs. 9 and 10, we have plotted the gravitational mass and radius of the spin polarized SQS in the absence of magnetic field versus the central energy density \((\varepsilon_c)\) for different values of the polarization parameter \((\xi)\). From these figures, we see that for each central density, the mass and radius of SQS decrease by increasing the polarization parameter. This is due to the fact that by increasing the polarization parameter, the pressure of spin polarized SQM increases, which leads to the stiffer equation of state for this system (Fig. 2). Figs. 9 and 10 show that for a given polarization parameter, the gravitational mass and radius of SQS increase by increasing the central density. From Fig. 9 it can be seen that the
the gravitational mass of SQS reaches a limiting value called the maximum mass. In Fig. 11 we have plotted our results for the gravitational mass of spin polarized SQS as a function of the radius (mass-radius relation) in the absence of magnetic field. For this system, we see that the gravitational mass increases by increasing the radius. It is seen that the rate of increasing mass versus radius increases by increasing the polarization. In Table 1 the maximum mass \( (M_{\text{max}}) \) and the corresponding radius \( (R) \) of spin polarized SQS have been given for different values of the polarization parameter \( (\xi) \) in the absence of magnetic field. We can see that both maximum mass and the corresponding radius decrease by increasing \( \xi \). This shows that increasing polarization leads to a more stable SQS.

B. Structure of the spin polarized SQS in the presence of magnetic field

In this section, we present our calculations for the structure of SQS in the presence of the magnetic field. It should be noted that the strong magnetic field changes the spherical symmetry of the system. However, for the magnetic fields less than \( 10^{19} \text{ G} \), this effect is negligible \cite{22, 23}, therefore, we can solve the TOV equations using a spherical metric, which leads to Eq. (13). Our results for the gravitational mass and radius of the spin polarized SQS in the presence of magnetic field versus the central energy density \( (\varepsilon_c) \) have been shown in Figs. 12 and 13 respectively. In these figures, our results for the unpolarized case of SQS \( (B = 0) \) are also given for comparison. Figs. 12 and 13 show that for all values of central density, the mass and radius of SQS decrease when the magnetic field is switched on. From Fig. 12, we see that as the central density increases, the gravitational mass of SQS increases and finally reaches a limiting value (maximum mass). In Table 2 we have given the maximum mass and the corresponding radius of SQS for two cases \( B = 0 \) (unpolarized SQS) and \( B = 5 \times 10^{18} \text{ G} \). It is shown that the presence of magnetic field leads to lower values for both maximum mass and the corresponding radius of SQS showing a more stable SQS compared to the unpolarized SQS.

IV. SUMMARY AND CONCLUSIONS

We have studied the spin polarized strange quark matter (SQM) for both cases in the absence and presence of magnetic field. We have calculated some of the bulk properties
of this system such as the energy, equation of state (EoS), and polarization. We have shown that the energy of spin polarized SQM in the absence of magnetic field increases by increasing polarization. Calculation of energy in the presence of magnetic field shows that for each density, there is a minimum point for the energy of SQM showing a metastable state. We have seen that the EoS of spin polarized SQM becomes stiffer as the polarization increases. We have also seen that the spin polarized SQM in the presence of magnetic field is more stable than the unpolarized SQM. The structure properties of spin polarized strange quark star (SQS) have been also calculated in the absence and presence of the magnetic field. We have seen that for each central density, the mass and radius of spin polarized SQS decrease by increasing polarization. We have also seen that both maximum mass and the corresponding radius of this system decrease by increasing polarization. We have indicated that in the presence of magnetic field, the maximum mass and the corresponding radius of the polarized SQS get lower values than those of unpolarized SQS. Therefore, we can conclude that the presence of magnetic field leads to a more stable SQS compared to the unpolarized SQS.

Our results for the maximum mass and radius of SQS (Tables 1 and 2) are consistent with those observed for the object SAX J1808.4-3658 [24]. We can conclude that this object is a good candidate for SQS.

One of the other astrophysical implications of our results is calculation of the surface redshift ($z_s$) of SQS. This parameter is of special interest in astrophysics and can be obtained from the mass and radius of the star using the following relation [4],

$$z_s = (1 - \frac{2GM}{Rc^2})^{-\frac{1}{2}} - 1.$$  \hspace{1cm} (15)

Our results corresponding to the maximum mass and radius of SQS lead to $z_s = 0.45 \ m/s^{-1}$ in the absence of magnetic field and $z_s = 0.44 \ m/s^{-1}$ for the magnetic field $B = 5 \times 10^{18} \ G$. This indicates that the presence of magnetic field leads to the (nearly) lower values for the surface redshift.

**Acknowledgements**

This work has been supported by Research Institute for Astronomy and Astrophysics of Maragha. We wish to thank Shiraz University and Tafresh University Research Councils.
One of us (A. R. Peivand) also wishes to thank M. Mirza.
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TABLE I: Maximum gravitational mass ($M_{\text{max}}$) and the corresponding radius ($R$) of the spin polarized SQS for different values of the polarization parameter.

| Star                      | $M_{\text{max}}$ (M$_{\odot}$) | $R$ (km) |
|---------------------------|---------------------------------|----------|
| Unpolarized SQS ($\xi = 0$) | 1.35                            | 7.6      |
| Polarized SQS ($\xi = 0.33$) | 1.33                            | 7.5      |
| Polarized SQS ($\xi = 0.66$) | 1.27                            | 7.2      |
| Polarized SQS ($\xi = 1$)  | 1.17                            | 6.7      |
TABLE II: Maximum gravitational mass ($M_{\text{max}}$) and the corresponding radius ($R$) of SQS for $B = 0$ and $5 \times 10^{18} G$.

| Star                        | $M_{\text{max}}$ ($M_\odot$) | $R$ (km) |
|-----------------------------|-------------------------------|----------|
| Unpolarized SQS ($B = 0$)  | 1.35                          | 7.6      |
| Polarized SQS ($B = 5 \times 10^{18} G$) | 1.31                      | 7.4      |
FIG. 1: The total energy density of spin polarized SQM as a function of the density ($\rho$) at different values of the polarization parameter ($\xi$) in the absence of magnetic field.

FIG. 2: As Fig. 1 but for the equation of state of spin polarized SQM.
FIG. 3: The total energy density of polarized SQM as a function of the polarization parameter ($\xi$) for $B = 5 \times 10^{18}$ G at different densities ($\rho$).
FIG. 4: (a) The total energy density of spin polarized SQM versus the density ($\rho$) at $B = 5 \times 10^{18} G$. (b) Comparison between the total energy for two cases of $B = 5 \times 10^{18} G$ and $B = 0$. 
FIG. 5: The polarization parameter ($\xi$) corresponding to the minimum points of the energy density versus the density ($\rho$) at $B = 5 \times 10^{18} \, G$. 
FIG. 6: The polarization parameter ($\xi$) corresponding to the minimum points of the energy density versus the magnetic field ($B$) for different values of density ($\rho$).
FIG. 7: The pressure ($P$) versus density ($\rho$) for spin polarized SQM at $B = 5 \times 10^{18}$ G.

FIG. 8: The energy per baryon versus the pressure ($P$) for spin polarized SQM at $B = 0$ (full curve) and $B = 5 \times 10^{18}$ G (dashed curve).
FIG. 9: The gravitational mass of spin polarized SQS versus the central density ($\varepsilon_c$) for different values of the polarization parameter ($\xi$) in the absence of magnetic field.
FIG. 10: As Fig. 9 but for the radius of spin polarized SQS.

FIG. 11: The mass-radius relation for spin polarized SQS in the absence of magnetic field at different values of the polarization parameter ($\xi$).
FIG. 12: The gravitational mass versus the central density ($\varepsilon_c$) for the spin polarized SQS at $B = 0$ and $B = 5 \times 10^{18} \, G$. 

\[ \varepsilon \left(10^{14} \text{ g/cm}^3\right) \]

Mass (M$_{\text{Sun}}$)

$\varepsilon_1 (10^{14} \text{gr/cm}^3)$

$B = 0$

$B = 5 \times 10^{18} \, G$
FIG. 13: As Fig. 12 but for the radius of spin polarized SQS.