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Electrical Method To Detect Objects In Sea Water
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Abstract. Analytical expressions, which help to calculate the electromagnetic field of the grounded wire with acceptable accuracy, were obtained based on a rigorous solution of the problem. It is shown that the field of a small electrode antenna is mainly determined by the field of free charges on the metallic electrodes. It is indicated that the electric field on big distances from the electrodes is represented as a superposition of the fields of point-sources. The calculation of the diffraction field of a conducting sphere, which is situated in the field of the grounded wire, was provided as an example.

1. Introduction

Electrode antennas are used in electromagnetic systems of sea area defense. These systems are divided into passive, active and combined systems. Active systems consist of transmitting and receiving antennas. Transmitting antennas create primordial electromagnetic field (EMF), which targets secondary electromagnetic sources in moving objects, and then their fields are registered by magnetic and/or electric receiving antennas [1]. Active electrical method is considered promising for shallow water regions [2]. The principle of this method is contained in creation of the ground current field of the transmitting electrode antenna in the controlled volume of water, and then registration of the field’s distortions caused by conducting or non-conducting objects. At its simplest, the ground current field in conducting medium can be generated by a system of two metallic electrodes, which are connected with a wire. Such system is called grounded wire or electrode antenna (EA). Analytical expressions to calculate electromagnetic field of a grounded wire were obtained only for direct current [3].

The goals of this article are: 1) derivation of general analytical expressions to calculate the electromagnetic field of the grounded wire with an acceptable accuracy, based on a rigorous solution of the problem of electrodynamics; 2) calculation of the secondary electromagnetic field of the object in the form of metallic sphere in the field of the electrode antenna.

2. Field of a grounded cable

The electrical field of the grounded wire (EA) is determined by free charges on the metallic electrodes [2].

The field of free charges can be presented by its potentials. There is an accurate solution for metallic water electrodes in the shape of circular thin disks in direct current. It is obtained as a limiting transition of the potential of the charged conducting ellipsoid.

The potential of two thin metallic disks with antiphased supply and with the diameter , the distance between which is (Fig.1), in random point of the space is following [5]:

$$U = \frac{2l U_0}{\pi \arctg \frac{b}{\Theta_1^{1/2}} - \arctg \frac{b}{\Theta_2^{1/2}}}$$

(1)
where $U_0$ – disk’s potential; \( \eta_{2(2)} = \sqrt{\rho^2 + (l \mp z)^2} \) – distances from the centers of the first and second disks to the viewpoint \( p \); \( \Theta_{1(2)} = 2/I \left\{ \left( \eta_{1(2)}^2 - b^2 \right)^2 + \left( \eta_{1(2)}^2 - b^2 \right)^2 + 4b^2 (l \mp z)^2 \right\}^{1/2} \) – elliptic coordinate of the viewpoint from the centers of the first and second water electrode.

Projection of the electric field intensity in cylindrical coordinates $E_z$ and $E_\rho$ can be determined using known equation $\vec{E} = -\nabla U$. It is obtained from (1):

$$
E_z = -\frac{dU}{dz} = -\frac{2U_0 b}{\pi} \left[ f(\Theta_1) \frac{(l-z)}{2} \left( 1 + \frac{\eta_1^2 + 3b^2}{\xi_1} \right) \right],
$$

(2)

$$
E_\rho = -\frac{dU}{d\rho} = \frac{2U_0 b \rho}{2\pi} \left[ f(\Theta_1) \frac{1}{2} \left( 1 + \frac{\eta_1^2 - b^2}{\xi_1} \right) \right] - f(\Theta_2) \frac{1}{2} \left( 1 + \frac{\eta_2^2 - b^2}{\xi_2} \right).
$$

(3)

Figure 1. The electrical field of the grounded wire

Where $\xi_{1(2)} = \left( \eta_{1(2)}^2 - b^2 \right)^2 + 4b^2 (l \mp z)^2 \right\}^{1/2}$, here symbol – corresponds to the index 1, and symbol + – to the index 2.
Equations (2) and (3) for the component of the electric field intensity are complicated to analyze. To simplify them it is needed to set that \( \eta_1 >> b \). In this case
\[
E_z = -\frac{2U_0b}{\pi} \left( \frac{l-z}{r_1^3} + \frac{l+z}{r_2^3} \right), \quad E_\rho = \frac{2U_0b\rho}{\pi} \left( \frac{1}{r_1^3} - \frac{1}{r_2^3} \right).
\]

The intensity of the same receiving EA, which is located on the distance \( \rho \) parallel to the transmitting one, in the point \( z = 0 \) is determined
\[
U_1 = \frac{1}{l} \int_{-l}^{l} E_z^{(3)} dz = -\frac{4U_0b}{\pi} \left( \frac{1}{\rho} - \frac{1}{\sqrt{\rho^2 + 4l^2}} \right). \tag{4}
\]

Intensity of the receiving EA from the wire would be the following after integration of (4):
\[
U_2 = \frac{\mu_0\alpha I}{2\pi N} e^{i H_0^{(2)}(k_3\rho)}. \tag{5}
\]

The relationship between current in the wire \( I \) and intensity \( U_0 \) is determined using dissipation resistance of the thin disk \([5]\)
\[
I = 8\sigma_3 b U_0. \tag{6}
\]

The following is obtained after dividing (5) by (4) considering (6)
\[
\frac{U_2}{U_1} = -\frac{\mu_0\alpha I e^i \rho H_0^{(2)}(k_3\rho)}{k_3 \left[1 - \left(\frac{2l}{\rho}\right)^2\right]^{1/2}}.
\]

It is necessary to calculate the relationship between potentials using numerical values of the parameters: \( 2l = 3 \text{ m}, \ b = 0.125 \text{ m}, \ f = 4 \text{ kHz}, \ \rho = 1 \text{ m}, \ \sigma_3 = 1 \text{ S/m}, \)
\[
\frac{U_2}{U_1} = 2.53 \cdot 10^{-3} e^{2.3i}.
\]

It can be seen from the given example that the input of the current of the wire into total electric field of the small EA is lower than one percent and can be neglected compared to the field of the ground currents from water electrodes.

In general terms, accurate solution for the potential of the electric field of free charges on two thin metallic disks with antiphased supply in the point \( \rho, z \) can be obtained using Fourier-Bessel integral transformation method \([2]\)
\[
U = \frac{2U_0}{\pi} \int_{-\infty}^{\infty} e^{-q(l-z) - e^{-q(z+l)}} J_0(\gamma p) \frac{\sin(\gamma b)}{\gamma q} q d\gamma \tag{7}
\]

The potential (7) on the distance, which is three times bigger than radius of the water electrodes \( b \) can be calculated with acceptable accuracy using the following equation
\[
U = \frac{2U_0b}{\pi} \left( e^{ik_3\eta_1} - e^{-ik_3\rho_2} \right). \tag{8}
\]

Therefore, the potential of the EA on the distance \( r >> b \) from its electrodes can be calculated as an algebraic sum of the potentials of two point-sources with the alternative voltage \( U_0 \) and the circular frequency \( \omega \).

3. Calculation of the secondary field of the metallic sphere

Primordial field will be distorted when the object, which conductivity differs from conductivity of the environment, enters the field of the transmitting EA. Estimation of the value of its distortion is presented using the spherical object as an example.
Geometry of the problem is shown in the Fig. 2. The center of the sphere is located on the axis $Z$, alongside which the transmitting EA is situated, to simplify the analysis of the secondary field of the sphere. The receiving EA is located at the right angle to the axis $Z$. The sphere with the radius $a$ is located in the point $Z = \zeta$. Evident equations can be drawn from the geometry of the Fig. 2.

\[
\eta_1 = \sqrt{r_0^2 + l^2 - 2lr_0 \cos \theta}; \quad r_2 = \sqrt{r_0^2 + l^2 + 2lr_0 \cos \theta}; \quad \eta_0 = \sqrt{r_0^2 + a^2 - 2a \zeta \cos \theta},
\]

where $r_0$ – the distance from the center of the EA to a random point on the surface of the sphere. Equation (8) can be expanded as spherical functions [4]:

\[
U = -\frac{2}{\pi} k b U_0 \sum_{n=0}^{\infty} (2n+1) j_n(k_3) h_n^{(2)}(k_3 \eta_0) \left[ p_n(\cos \theta) - p_n(-\cos \theta) \right], \quad (9)
\]

where $j_n(x)$ – spherical Bessel function of the 1st kind; $h_n^{(2)}(x)$ – spherical Bessel function of the 3rd kind; $p_n(x)$ – Legendre polynomials.

In general, terms, the secondary field of the sphere will be determined using known expansion as zonal harmonics [4]. It is necessary to set that the initial charge of the sphere equals zero, and, therefore, the total charge of the sphere in the external field is zero (external field only uncouples charges). Potential of the charges which were induced on the sphere in locations of the receiving electrodes can be calculated as follows:

\[
U_p = \sum_{n=1}^{\infty} h_n^{(2)}(k_3 r) \left\{ a_{n0} P_n(\cos \theta) + \sum_{m=1}^{N} (a_{nm} \cos m \varphi + b_{nm} \sin m \varphi) \ell_n^m(\cos \theta) \right\}, \quad (10)
\]

where $\ell_n^m(\cos \theta)$ – associated Legendre polynomials.

**FIGURE 2.** Calculation of the secondary field of the metallic sphere.
The potential on the surface of the sphere in the conducting medium equals zero, therefore

\[
U(k_3 \eta_0) = -U_{p}(k_3 a) \quad (11)
\]

Expansion coefficients of the secondary field of the sphere as spherical harmonics can be found by substituting (8) and (9) in (10), and then multiplying both parts of the equation (11) on \( P_n^{m}(\cos \theta) \sin \theta \) and integrating them over \( \varphi \) from 0 to \( 2\pi \) and over \( \theta \) from 0 to \( \pi \). It is necessary to consider property of orthogonality of Legendre polynomials [6]

\[
\int_{0}^{\pi} P_n^{m}(\cos \theta) P_m^{n}(\cos \theta) \sin \theta d\theta = 0.
\]

It can be seen from (9) that primordial field does not equal zero only for odd terms of the series, therefore, only odd coefficients \( a_{2n+1,0} \), which have the following, will stay in (10)

\[
a_{2n+1} = i(4n + 1) \frac{4U_{0}b_{k_3}^{2}}{\pi} \frac{h_{2n+1}^{(2)}(k_3 \eta_0)}{h_{2n+1}^{(2)}(k_3 a)}, \quad (12)
\]

Coefficients \( a_{nm} \) and \( b_{nm} \) equal zero due to the spherical symmetry of the secondary field over azimuth angle \( \varphi \).

The relationship for the first two coefficients of the expansion (10) as spherical harmonics \( a_{30} \) / \( a_{10} \) for the sphere with the volume of 0,1 m\(^3\) and radius of \( a = 0,288 \) m, \( \sigma_3 = 1 \) (Ohm-m)\(^{-1}\), \( f = 4 \cdot 10^3 \) Hz, \( \eta_0 = 2l = 3 \) m, should be determined. In addition, \( k_3 \eta_0 = 0,188 \). Taking into consideration obvious equations for the spherical functions \( h_1^{(2)}(x) \) and \( h_3^{(2)}(x) \) [6]:

\[
h_1^{(2)}(x) = -\frac{e^{-ix}}{x} + i \frac{e^{-ix}}{x^2}, \quad h_3^{(2)}(x) = -i \frac{e^{-ix}}{x} - 3i \frac{e^{-ix}}{x^2} + 7i \frac{e^{-ix}}{x^3} + 6i \frac{e^{-ix}}{x^4},
\]

the estimation of the relationship can be obtained \( \frac{a_{30}}{a_{10}} = 3,5 \frac{(a)}{\eta_0} \), \( \frac{(a)}{\eta_0} = 3,5 \frac{(a)}{\eta_0} = 0,022 \), therefore, the calculation of \( U_{p} \) can be limited to one term of the series (10).

The situation when the object is located randomly in relation to the transmitting-receiving system is considered (Fig. 3).

The transmitting EA is located on the axis Y in this case, and the receiving EA – or the axis X. It can be conclude from the geometry that \( z = z - h \), where \( h \) is height of the center of the sphere from the bottom of the water reservoir, \( \eta_0 \) – the distance from the center of the transmitting-receiving system to the center of the sphere, \( \rho = \sqrt{x^2 + y^2} \), \( \cos \gamma = \sin \theta \sin \varphi \), \( \sin \theta = \frac{h}{\eta_0} \). It is necessary to set the direction of the sphere’s movement through the center of the transmitting-receiving system at the angle \( \varphi = \frac{\pi}{4} \), then: \( \cos \gamma = \frac{\rho}{\sqrt{2} \eta_0} \), \( \eta_1^{(2)} = \sqrt{\rho^2 + h^2} + 2l \eta_0 \cos \gamma = \sqrt{\rho^2 + h^2} + 2l \eta_0 \) – the distances from the center of the sphere to transmitting water electrodes, at the same time, the distances to receiving water electrodes \( \eta_1^{(2)} \) are equal to the distances to corresponding transmitting water electrodes – \( \eta_1^{(2)} = \eta_1^{(2)} \).
The relationship between angles $\theta$ and $\theta'$ can be seen from the geometry of the pic.3: $\theta = \pi - \theta'$, i.e. $\cos \theta = -\cos \theta' = -\frac{h}{l_0}$.

Difference signal at the input of the differential amplifier of the receiver $U_\Sigma$ from (10) considering the first term (12) would be the following

$$U_\Sigma = i \frac{12U_0bk_3}{\pi} j_1(k_3l) \frac{h^{(2)}_{1}(k_3\eta_0)}{h^{(2)}_{1}(k_3a)} \left[ h^{(2)}_{1}(k_3\eta'_1)R'_1 \left( \frac{\rho_1}{\rho_2} \right) - h^{(2)}_{1}(k_3\eta')R_1 \left( \frac{\rho_1}{\eta} \right) \right],$$

where $\rho_{l(2)} = \sqrt{\rho^2 + l^2 + \sqrt{2}\rho l}$, $\eta_{l(2)} = \sqrt{\rho^2 + h^2 + l^2 + \sqrt{2}\rho l}$.

**FIGURE 3.** Situation when the object is located randomly in relation to the transmitting-receiving system

There is the example of the calculation of the real parts of transit characteristics $U_\Sigma(\rho)$ when $f = 4 \cdot 10^3$ Hz, $\sigma_3 = 1$ (Ohm-m)$^{-1}$, $l = 1.5$ m, $b = 0.125$ m, transmitter power 100 W ($U_0 = 20$ V) for three values of depth $h = 3; 4; 5; 6$ m, which correspond with curves 1, 2, 3 on the Fig. 4.

Transit characteristics are symmetrical against the center of the transmitting-receiving system. Moreover, their maximums are located on the distance of $l$. Periodic nature of the transit characteristic can be clearly seen when $h = 3$ m, however, it degenerates on higher depths. These results correspond with experimental data obtained on Gulf of Finland. Root-mean-square noise amplitude equals to 77 $\mu$V based on the experiment, therefore, the signal from the object when $h = 6$ m, which had at its maximum $\pm 8$ mV, was found.
It is proved that the field of the small EA determines by the field of the ground current of the water electrodes based on the rigorous solution of the problem. Simple analytical expressions of this field, which allow carrying out calculations with an acceptable accuracy, are obtained. The calculation of the secondary (diffraction) field of the metallic sphere excited by the field of the transmitting EA was carried out as the example. Theoretical calculations correspond with experimental data obtained on Gulf of Finland. The article is prepared as a part of Federal Target Program “Academic and teaching personnel of innovative Russia” for 2009-2013.

References

[1] E.A. Ivleev, “Electromagnetic systems of sea area defense”, Special vehicles and connection. 1, 21-23 (2008).
[2] Y.I. Kuzmin, “Diffraction of the grounded wire on the conducting sphere in sea water”, Reports AN VSH, 2(17), 102-114 (2011).
[3] Y.I. Kuzmin, “Remote detection of the anomalies in the stream of the liquid using ground current method”, Izv. ETU, 426, 51-55 (1990).
[4] E.F. Zimin, E.S. Kochanov, Measurement of the parameters of electric and magnetic fields in conducting mediums, M: Energoatomizdat, (1985).
[5] Stratton A.J., Electromagnetic theory, M: OGIZ – Gostehizdat, (1948).
[6] V. Smite, Electrostatics and electrodynamics, M: Foreign Literature, (1954).
[7] G. Arkfen, Mathematical methods in physics, M: Atomizdat, (1970).