Hydraulic losses in the initial section of a flow parts of at aggregates of liquid rocket engines

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Abstract. The article deals with the need for accurate hydraulic losses determination along the length of the flow part in the process of designing units for supplying liquid rocket engines (LRE). The dependencies that are usually used, as a rule, are of a criterion-empirical nature and are not suitable for their application, taking into account the tendency of an increase in the rotating speed of turbo-pumping LRE supply units to 100,000 r.p.m. The analysis necessary to select the friction resistance laws for the flow part elements of the rocket engine supply units was carried out. A numerical integration equation system method to determine the variation in the characteristic thicknesses of the spatial boundary layer along the channel length in the initial section of the flow and hydraulic losses (taking into account the inertial component of the flow core velocity) was considered and proposed, depending on the current flow regimes in the flow path elements of the LRE supply units.

1. Introduction

To solve the problem of determining the energy characteristics in the flow parts of rocket engine units accurately [1], [2], [3], the approach is used to split the flow in the channel into the main non-viscous flow and the flow in a thin boundary layer (BL), where the viscosity of working fluid displays and friction stresses occur [4].

The flow in the vane channel and rotation cavities can occur both with the presence of a flow core and with the merging of boundary layers, when there is no flow core. When considering the problem of flow in the flow parts of LRE supply units, many authors use original equations to calculate and analyse losses for friction stresses for both a flat plate and a round pipe [5-9].

To solve the problem of hydraulic losses determining, it is necessary to calculate the friction stress [5], which is often, specified in the form of the friction law for a flat plate for the turbulent flow regime [1-3], [5-11].

\[ Tr = \frac{\tau_0}{\rho U^2} = C \left( \frac{U \delta^*}{\nu} \right)^{-0.25}, \]  

(1)
where \( C = 0.01256 \) [6], [11], or \( C = 0.0128 \) [4]; \( U \) – the external flow speed (speed of the “non-viscous” flow core); \( \delta^* = \frac{1}{U^2} \int_0^\infty u(U - u)dy \) – the pulse loss thickness BL, \( u \) – the flow rate in the BL.

2. Resistance law and hydraulic losses determination

The friction law is determined from the solution of the BL integral momentum equation of Karman, for the incompressible liquid planar flow [11], [4], [6]

\[
\frac{d\delta^*}{dx} = -(2 + H) \frac{\delta^*}{U} \frac{dU}{dx} + Tr
\]

(2)

where \( H = \frac{\delta^*}{\delta^*} \), BL specific relative thickness [12]; \( \delta^* = \frac{1}{U} \int_0^\infty (U - u)dy \) – BL displacement thickness.

The following assumptions are used to solve the integral momentum equation [4]:

1. There is a boundary layer fusion;
2. The flow is steady, with a constant speed, i.e. \( dU/dx = 0 \);
3. The flow corresponds to the velocity distribution in the BL “1/7-th-power law”:

\[
\frac{u}{U} = \left( \frac{y}{\delta} \right)^{1/m}.
\]

(3)

When the profile degree \( m = 7 \) meets the Reynolds criterion \( \operatorname{Re}_\delta = \frac{2\delta \bar{u}}{\nu} = 10^5 \), where \( \delta \) – the boundary layer thickness. The dependence between the thicknesses is set as [12]

\[
N = \frac{\delta}{\delta^*} = \frac{(m + 1)(m + 2)}{m}, \quad H = \frac{\delta^*}{\delta^*} = \frac{m + 2}{n};
\]

(4)

4. The equation for the friction stresses on the wall as well as for the pipe is used as the initial [4]

\[
\tau = \frac{\lambda}{8} \rho \bar{u}^2,
\]

(5)

where \( \lambda \) – the friction resistance coefficient, \( \bar{u} = \dot{V} / (\pi R^2) = \text{const} \) – the average flow rate over the cross section with the pipe radius \( R \), and the volumetric flow rate is determined by the equation (figure 1)

![Figure 1. The flow pattern with the boundary layer thickness \( \delta \) and the flow core.](image)
where the flow velocity in the boundary layer depends on the distance from the friction surface \(u=f(y)\) is determined according to the equation (3), and \(u=U\) where \(r=\delta\) for the pipe and for the plate \(u=U\) for \(r\geq\delta\).

When considering the flow in the pipe based on equation (3) and the boundary conditions: \(u=0\) for \(r=R\) or \(y=R-r=0\), \(u=U\) for \(r=0\) or \(y=R=\delta\), we obtain a connection between the average consumption rate and the flow core velocity

\[
\frac{\bar{u}}{U} = \left(\delta \left(2R - \frac{n}{(n+1)} - 2\delta - \frac{n}{(2n+1)}\right) + (R-\delta)^2\right) R^{-2};
\]

(7)

5. Blasius friction resistance law applies [4]

\[
\lambda = 0,3164 \text{Re}^{-0.25} m/\delta.
\]

(8)

It should be noted that the power law velocity profile in the boundary layer has its drawbacks, such as the derivative of the velocity on the wall is zero, which is not consistent with the physical picture of the flow. Therefore, sometimes a “gradient” velocity profile is applied in the BL [12]

\[
u \quad \text{U} = 1 - \left(1 - \frac{y}{\delta}\right)^m.
\]

(9)

where the characteristic thickness ratio could be expressed as

\[
N = \frac{\delta}{\delta^*} = \frac{(m+1)(2m+2)}{n}, \quad H = \frac{\delta^*}{\delta^*} = \frac{(m+1)(2m+1)}{m(m+1)},
\]

(10)

and the dependence of the average consumption rate and the flow core velocity is established by the equation

\[
\frac{\bar{u}}{U} = 1 - \frac{\delta}{R^2} \left(\frac{2R-2\delta}{m+1} + \frac{2\delta}{m+2}\right).
\]

(11)

It should be noted that the flow in the channels and cavities of liquid rocket engine supply units has both turbulent and laminar characteristics; both with merged boundary layers and with the flow core presence. Most of the channels in LRE supply units have a complex flow shape; the channel length is comparable with the hydraulic diameter, which does not allow boundary layers to develop in the flow path before they merge. In addition, Blasius friction resistance law has a range of application of \(4*10^3 < \text{Re} < 10^5\), which limits the use of considered dependencies significantly. Since at the entrance to the channel the boundary layer thickness tends to zero and the laminar nature of the flow in the BL takes place [4], it is necessary to apply the friction resistance laminar law for the initial section

\[
\lambda = 64 \text{Re}^{-1} m/\delta.
\]

(12)

Further, when the flow moves along the channel, a turbulent boundary layer begins to develop, which requires the turbulent Konakov resistance law application [13]

\[
\lambda = (1,8 \text{Log} (\text{Re}_m) - 1,5)^{-2}
\]

(13)

or the universal Prandtl resistance law for smooth pipes, which has no restrictions on Re [4]

\[
\frac{1}{\sqrt{\lambda}} = 2,01 \text{Log}(\text{Re}_m \sqrt{\lambda} - 0,8).
\]

(14)
In turn, the velocity distribution profile in the boundary layer (3) also depends on the Re criterion [4], and this dependence for a turbulent flow can be represented as

$$m = 0.003 \Re^{0.49} + 6,$$

where $$\Re = \frac{Ux}{v}$$.

Another important factor is that in the flow core presence, which always takes place in the initial segment, according to (6) and (7) and $$\overline{U} = \text{const}$$ the derivative $$dU/dx \neq 0$$, which means the equation application (1) for the flow core presence and the pressure gradient for internal flows in closed channels reduces the area of correct application. Due to the change in the flow core velocity $$U$$, it is necessary to determine the velocity differential in the Karman integral relation (2), which in turn depends on the BL thickness $$\delta$$ (7) and (11). Therefore, it is necessary to solve the Karman relation numerically (2) in order to determine the transition point of the laminar flow into turbulent flow in BL accurately.

With a one-dimensional flow in a channel of arbitrary cross section, taking into account the inertial component of the flow core velocity, an equation for determining pressure losses is obtained

$$\frac{dp}{dx} = -\rho U \frac{dU}{dx} - \frac{\tau_0}{F} \frac{dS}{dx} - \frac{p}{F} \frac{dF}{dx},$$

which is necessary for the numerical solution of hydraulic losses determining at a constant average flow rate.

As a result, the system of equations for the numerical solution could be composed, which include:
1. the basic part of systems from equations (2), (16);
2. velocity distribution profiles law variations in BL: A. (4) and (7); B. (10) and (11);
3. friction law variations: C. (1), m=7; D. (5), (12), (8), m=7; E. (5), (12), (13), m=7; F. (5), (12), (14), m=7; G. (1), (15); H. (5), (12), (8), (15); I. (5), (12), (13), (15); J. (5), (12), (14), (15).

Therefore, a numerical solution of these equation systems makes it possible to determine the variation in BL thicknesses along the channel length in the initial section of the flow in a round pipe, which is shown in figure 2 and 3.

![Figure 2](image_url). The change in the boundary layer thickness along the length of the channel for various friction laws.
Figure 3. The change in BL the thickness along the channel length without and taking into account the dependence of the velocity profile exponent on the Re number for various friction laws.

In order to compare hydraulic losses depending on the flow regimes and the Re criterion, we use the well-known equation for the loss coefficient [13]

$$\xi = \frac{\Delta p}{L} \frac{w^2}{2 \rho \Delta x}.$$  \hspace{1cm} (17)

Then the losses in the initial section of the round pipe for various laws could be reflected in figure 4.

Figure 4. The change in BL the thickness along the channel length without and taking into account the dependence of the velocity profile exponent on the Re number for various friction laws.
3. Conclusion
As can be seen from the pictures above, different friction resistance laws under various conditions give noticeably different results of hydraulic losses. For example, if the presence of the laminar flow regime in the BL at the initial section is not taken into account, this can lead to significant hydraulic efficiency deviations of the unit from the predicted, both overstated and underestimated.

Finally, to determine hydraulic losses in LRE supply unichannels accurately, it is necessary to solve the system of equations of a viscous incompressible liquid and boundary layer motions in boundary conditions and the flow regime characteristic of liquid rocket engine supply unichannels numerically.

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