The scalar sector in the Myers-Pospelov model

C. M. Reyes, L. Urrutia and J. D. Vergara

Abstract. We construct a perturbative expansion of the scalar sector in the Myers-Pospelov model, up to second order in the Lorentz violating parameter and taking into account its higher-order time derivative character. This expansion allows us to construct an hermitian positive-definite Hamiltonian which provides a correct basis for quantization. Demanding that the modified normal frequencies remain real requires the introduction of an upper bound in the magnitude $|k|$ of the momentum, which is a manifestation of the effective character of the model. The free scalar propagator, including the corresponding modified dispersion relations, is also calculated to the given order, thus providing the starting point to consider radiative corrections when interactions are introduced.

Keywords: Effective field theories
PACS: 11.10.-z, 11.30.Cp, 12.20.-m

1. INTRODUCTION

The basic aim of this and previous work [1] is to define a consistent quantization of the Myers-Pospelov (MP) model considered as an effective field theory which provides perturbative corrections to standard QED. This model incorporates Lorentz invariance violating (LIV) corrections to QED, which are codified by dimension five operators which are C, T conserving but P violating. Our main concern with the construction, which we expect to play a major role in defining the appropriate way of making sense of the effective model, is that we recover the Lorentz invariant results of standard QED when such dimension five operators are turned off. In other words, we are looking for a smooth interpolation procedure between a Lorentz violating description and a Lorentz invariant one for QED. This point of view has been successfully carried on in Ref. [2], where LIV is codified by a dimensionless parameter modifying the integration measure in momentum space appearing in the calculation of one-loop processes. In this case LIV arises only from radiative corrections associated to standard particle Lagrangians; that is to say the zeroth order approximation is just the Standard Model of particles. The fact that LIV should be treated as a perturbation over the standard dynamics is supported by the very stringent limits which terrestrial experiments together with astrophysical observations set upon the parameters which label such violation. This requirement poses additional interesting challenges to this problem because we need to deal with higher-order time derivatives (HOTD) theories. The perturbative treatment of these, over the normal lower-order time derivative cases, is a non-trivial task, but fortunately it is well described in the literature [3, 4, 5, 6, 7]. As an additional motivation to deal with quantum corrections in this model we mention the fine tuning problems recently reported [8].
In this work we will concentrate in the scalar sector of the model, that can be used as the charged matter realization of scalar electrodynamics, which will be considered in a future work. Preliminary descriptions of the photonic and fermionic sector were already given in Ref. [1]. We will set up the correct zeroth order quantization, identifying the new propagating normal modes described by their modified dispersion relations (MDR) up to second order in the LIV parameter. We will also present the corresponding propagator, with the final aim to calculate radiative corrections which are quadratical in the LIV parameters.

2. PERTURBATIVE APPROACH TO HIGHER ORDER TIME DERIVATIVE THEORIES

The general method for dealing with the canonical description of HOTD theories was given a long time ago in Ref. [9]. In order to highlight some general features of these theories we briefly review their basic properties in a non-degenerate mechanical setting \( \frac{\partial^2 L}{\partial q^{(k)}(t) \partial q^{(k)}(t)} \neq 0 \), where the fields depend only upon the time coordinate and

\[
q^{(k)}(t) = \frac{d^k q(t)}{dt^k}.
\]

The generalization incorporating additional coordinates \( q(t) \rightarrow q_I(t), I = 1, \ldots, N \) is direct and the introduction of space dependent fields goes along similar lines as in the standard transition from mechanics to field theory.

If the highest time derivative in the Lagrangian is of order \( k \), \( L = L(q(t), \ldots, q^{(k)}(t)) \), the corresponding phase space will be of dimension \( 2k \), been characterized by \( k \) coordinates: \( Q_0 = q(t), Q_1 = q^{(1)}(t), \ldots, Q_{k-1} = q^{(k-1)}(t) \) together with \( k \) momenta

\[
P_i(t) = \frac{\partial L}{\partial q^{(i+1)}} + \sum_{j=1}^{k-i-1} \left( -\frac{d}{dt} \right)^j \frac{\partial L}{\partial q^{(j+1)}}, \quad i = 0, \ldots, (k-1).
\]

In the sequel we avoid writing the explicit time dependence in the fields. The equation of motion for \( q \) will be of order \( 2k \) requiring the fixing of \( 2k \) initial conditions, which is consistent with the existence of \( 2k \) degrees of freedom in phase space. The Hamiltonian is

\[
H(Q_0, \ldots, Q_{k-1}; P_0, \ldots, P_{k-1}) = \sum_{i=0}^{k-1} P_i Q_i - L(Q_0, \ldots, Q_{k-1}, q^{(k)}(P_{k-1}, Q_0, \ldots, Q_{k-1}))
\]

where the non-degeneracy assumption implies that we can solve \( q^{(k)} \) as a function of \( Q_0, \ldots, Q_{k-1}, P_{k-1} \). The above expression is linear in the momenta \( P_i, i = 0, \ldots, k-2 \), thus making the Hamiltonian \( H \) unbounded from below, independently of the interaction terms included in the Lagrangian.

Since we are interested in dealing with HOTD corrections in the action as perturbations upon standard theories, we must rely on a perturbation procedure which (i) retains
the original number of degrees of freedom and (ii) produces a free Hamiltonian which is bounded from below as an adequate starting point for quantization. Such a method has been already developed in Refs. [5], [6] and we present here a brief summary of it.

To point out some of its basic features let us consider the simplest framework of a Lagrangian depending upon accelerations \( L(q, \dot{q}, \ddot{q}) \), where the HOTD contribution arising from \( \frac{1}{2} g \dddot{q}^2 \) is only present as a perturbation characterized by a small parameter \( g \)

\[
L(q, \dot{q}, \ddot{q}) = L_0(q, \dot{q}) + \frac{1}{2} g \dddot{q}^2.
\]  

(4)

The standard procedure of extremizing the action leads to

\[
\delta S = \delta \int_{t_1}^{t_2} dt \left[ L(q, \dot{q}, \ddot{q}) \right] = \left[ P_0 \delta q + P_1 \delta \dot{q} \right]_{t_1}^{t_2} + \int_{t_1}^{t_2} dt E\left( q, \dot{q}, \ddot{q}, q^{(4)} \right) \delta q(t),
\]

(5)

where

\[
E(q, \dot{q}, \ddot{q}, q^{(4)}) = 0,
\]

(6)

is the fourth order equation of motion. From Eq. (5) the momenta can be directly read off as

\[
P_0 = \frac{\partial L_0}{\partial \dot{q}} - gq^{(3)}, \quad P_1 = g\ddot{q}.
\]

(7)

From the simple form assumed for the HTOD term, which is appropriate for the non-degenerate situation, we can solve for \( \dddot{q} \) in terms of \( P_1 \) and \( q^{(3)} \) in terms of \( P_0 \). Nevertheless, notice that both substitutions carry the non-analytical factor \( 1/g \). This is precisely what makes non-trivial a perturbative expansion around \( g = 0 \).

The Hamiltonian \( H \) and symplectic form \( \Omega \) are defined according to the Ostrogradski method as

\[
H(q, \dot{q}, P_0, P_1) = P_0 \dot{q} + P_1 \ddot{q} - L, \quad \Omega = dP_0 \wedge dq + dP_1 \wedge d\dot{q}.
\]

(8)

The dangerous contributions to the Hamiltonian arise from the non-analytic term \( P_1^2 / 2g \) together with the unbounded piece \( P_0 \ddot{q} \).

Let us summarize now the general perturbation scheme for the non-degenerate case developed in Ref. [6], which applies to systems of the general form

\[
L = L_0(q, \dot{q}) + g L_1(q, \dot{q}, \ldots, q^{(n)}).
\]

(9)

In order to obtain the appropriate Hamiltonian to order \( g^k \), one starts by solving the equations of motion to order \( g^{(k-1)} \) under the requirement of expressing all higher order derivatives \( q^{(n)}, n \geq 2 \) only in terms of \( q \) and \( \dot{q} \), which provides the first step to define the effective variables of the problem. This will allow us to rewrite

\[
H = H(q, \dot{q}), \quad \Omega = \omega(q, \dot{q}) d\dot{q} \wedge dq,
\]

(10)

from where we can read the bracket \( \{q, \dot{q}\} \). Next we look for an invertible change of variables \( Q(q, \dot{q}), P(q, \dot{q}) \) such that

\[
1 + O(g^{k+1}) = \{Q, P\} = \left( \frac{\partial Q}{\partial q} \frac{\partial P}{\partial \dot{q}} - \frac{\partial Q}{\partial \dot{q}} \frac{\partial P}{\partial q} \right) \{q, \dot{q}\}.
\]

(11)
Finally, the Hamiltonian $\hat{H}(Q, P) = H(q(Q, P), \dot{q}(Q, P))$ together with the Poisson bracket $\{Q, P\} = 1$ define the physical effective system to the order considered. This Hamiltonian will be bounded from below provided that the initial one obtained from $L_0$ is. The effective Lagrangian is given by $\tilde{\mathcal{L}}(Q, \dot{Q}) = \dot{P} \dot{Q} - \tilde{H}$ and the corresponding Euler-Lagrange equations reproduce those of the original system to the order considered. Since this Lagrangian is first order in the time derivatives, the quantization is straightforward. A formal proof of self-consistency to all orders is provided in Ref. [6]. As clearly shown in one of the examples of such reference, the physical meaning of reducing the original phase space to that generated by $q$ and $\dot{q}$ in the perturbative formulation is to suppress the excitation of high energy modes in a way consistent with the exact evolution and only to allow further excitations of the low energy modes already present in the zeroth order system. The method has been generalized to field theory in Ref. [7].

3. THE COMPLEX SCALAR FIELD

Before applying the method of Ref. [6] to our problem we need to identify the reduced phase space of this sector of the model which arises due to the presence of constraints, as it is shown in the next subsection.

3.1. Reduced phase space dynamics

We start from

$$\mathcal{L}_{scalar}(\phi^*, \phi) = -\phi^* (\partial^2 + m^2) \phi + ig\phi^* (n^\mu \partial_\mu)^3 \phi,$$

which describes the charged scalar field extension of the original MP Lagrangian. In the rest frame $n^\mu = (1, 0)$ the equations of motion are

$$\ddot{\phi} = \nabla^2 \phi - m^2 \phi + ig\phi^{(3)},$$

together with its complex conjugate, which exhibit the HOTD character of the model. Inserting a plane wave in the above equation provides the MDR as solutions of a cubic equation. The condition for recovering the standard energy-momentum relation in the limit $g \to 0$ requires that this cubic equation has three real solutions, one of which will be non-analytical in $g$, while the remaining two will be recovered in the perturbative approach to be used in the sequel. The above requirement demands

$$g^2 E_k^2 < \frac{4}{27}, \quad E_k = \sqrt{|k|^2 + m^2},$$

which sets an upper limit to $|k|$ in a way analogous to the photonic and fermionic cases. For future comparison we write here the MDR obtained directly by solving the cubic equation to order $g^2$

$$\omega_{\pm}(k) = \pm \left[ E_k \pm \frac{1}{2}gE_k^2 + \frac{5}{8}g^2 E_k^3 \right].$$
The coordinate space variables of the model are

$$\phi^*, \phi, \dot{\phi}, \ddot{\phi},$$

and our notation is

$$\phi^{(n)} = \frac{\partial^n \phi}{\partial t^n} , \ \phi^{(1)} = \dot{\phi} , \ \phi^{(2)} = \ddot{\phi}. \quad (17)$$

The presence of third-order time derivatives in the equation of motion (13), instead of fourth order ones, precludes the existence of constraints in the Hamiltonian formalism. In order to deal with this construction we have found it convenient to perform one integration by parts in the dimension five operator appearing in Eq. (12) . This leads to

$$\mathcal{L} = \dot{\phi}^* \dot{\phi} + \phi^* (\nabla^2 - m^2) \phi - ig \dot{\phi} \ddot{\phi}^*. \quad (18)$$

The canonically conjugated momenta corresponding to the coordinates (16) are

$$\Pi_{\phi^*} = \dot{\phi} - ig \ddot{\phi}, \ \Pi_\phi = \dot{\phi}^* + ig \ddot{\phi}^*, \quad (19)$$

$$\Pi_{\dot{\phi}} = -ig \phi^*, \ \Pi_{\ddot{\phi}} = 0, \quad (20)$$

respectively. The Eqs. (20) are primary constraints and the extended Hamiltonian density is

$$\mathcal{H}_E = \Pi_\phi \dot{\phi} - \frac{i}{g} \dot{\phi} \Pi_{\phi^*} - \phi^* (\nabla^2 - m^2) \phi + \frac{i}{g} \Pi_{\phi^*} \Pi_\phi + \alpha \Pi_{\phi^*} + \beta \left( \Pi_\phi + ig \phi^* \right). \quad (21)$$

The evolution of the primary constraints fixes the Lagrange multipliers $\alpha, \beta$, so that no additional constraints arise and the original ones are second class. The number of coordinate degrees of freedom per space point is then $\frac{1}{2} (2 \times 4 - 2) = 3$ which exceeds the usual two associated with the standard charged scalar field. We impose the constraints

$$\chi_1 = \Pi_{\dot{\phi}^*}, \ \chi_2 = \Pi_\dot{\phi} + ig \phi^*, \quad (22)$$

strongly by introducing the corresponding Dirac brackets and verify that the resulting brackets for the reduced space variables

$$\{ \phi, \Pi_\phi \}, \ \{ \phi^*, \Pi_{\phi^*} \}, \ \{ \dot{\phi}, \Pi_\phi \}, \quad (23)$$

remain unchanged with respect to the original Poisson brackets, thus preserving the original symplectic structure. The reduced phase space Hamiltonian density is then

$$\mathcal{H}_R = \Pi_\phi \dot{\phi} + \frac{i}{g} \Pi_\phi \left( \Pi_{\phi^*} - \dot{\phi} \right) - \phi^* (\nabla^2 - m^2) \phi, \quad (24)$$

which exhibits a non-positive definite character due to terms linear in the coordinates and momenta; as well as the non-analytic behavior in $1/g$ that prevents naive attempts of constructing perturbative schemes around $g = 0$. From Eq. (24) we obtain the equations
of motion
\[
\dot{\Pi}_\phi = (\nabla^2 - m^2)\phi^*, \quad \Pi_{\phi^*} = (\nabla^2 - m^2)\phi, \quad \Pi_\phi = -\left(\Pi_\phi - \frac{i}{g}\Pi_\phi^*\right),
\]
(25)

\[
\dot{\phi}^* = \frac{i}{g}\Pi_\phi, \quad \frac{\partial}{\partial t}\dot{\phi} = \ddot{\phi} = \frac{i}{g}\left(\Pi_{\phi^*} - \dot{\phi}\right).
\]
(26)

From them it is a direct matter to recover the Lagrangian equation (13) and, consistently in an independent way, the corresponding complex conjugate expression.

### 3.2. The perturbative expansion

#### 3.2.1. The symplectic form

Regarding the symplectic structure of the theory let us indicate our conventions for the field theory case. Denoting generically \(Z^A(t, x)\) the bosonic phase space fields (both coordinates and momenta), the Poisson brackets relations
\[
\{Z_A(t, x), Z_B(t', x')\} = W_{AB}(t; x, x'),
\]
(27)

are encoded in the corresponding symplectic two form \(\Omega(t)\) through the relation
\[
\Omega(t) = \frac{1}{2}\int d^3x d^3x' W_{AB}(t; x, x') dZ^A(t, x) \wedge dZ^B(t, x'),
\]
(28)

where \(W_{AB}(t; x, x') = -W_{BA}(t; x', x)\), which can be considered as an antisymmetric matrix in its discrete as well as continuous indices, is such that
\[
\int d^3x' W_{AB}(t; x, x') W^{BC}(t; x', x'') = \delta^C_A \delta^3(x - x'').
\]
(29)

Our next step in the perturbative construction is to use the equations of motion to second order in \(g\) in order to rewrite the exact symplectic form
\[
\Omega = \int d^3x \left(d\Pi_\phi \wedge d\phi + d\Pi_{\phi^*} \wedge d\phi^* + d\Pi_\phi \wedge d\phi^*\right),
\]
(30)

in terms only of the unperturbed variables \(\phi, \phi^*\) together with their first order time derivatives.

The approximation to order \(g^2\) leads to
\[
\dot{\phi} = (-\hat{E}^2) \left[(1 + g^2\hat{E}^2)\phi + ig\dot{\phi}\right] + O(g^3),
\]
(31)

together with its complex conjugate, which we also substitute in the corresponding momentum expressions. Here we have introduced the notation
\[
\hat{E}^2 = (m^2 - \nabla^2_x).
\]
(32)
In this way the Hamiltonian density is

$$\mathcal{H} = \phi^* (1 - 2g^2 \hat{E}^2) \phi + ig \phi^* \hat{E}^2 \phi - ig \phi^* \hat{E}^2 \phi + \phi^* \hat{E}^2 \phi,$$

(33)

together with the symplectic form

$$\Omega = \int d^3x (-2igd\phi^* \hat{E}^2 \wedge d\phi + d\phi^* \wedge (1 - g^2 \hat{E}^2) d\phi)$$

$$+ \int d^3x (d\phi \wedge (1 - g^2 \hat{E}^2) d\phi^* - igd\phi^* \wedge d\phi).$$

(34)

From Eq. (34) we read

$$W_{AB}(x, x') = \begin{bmatrix}
0 & +2ig \hat{E}^2 & 0 & - (1 - g^2 \hat{E}^2) \\
-2ig \hat{E}^2 & 0 & -(1 - g^2 \hat{E}^2) & 0 \\
0 & (1 - g^2 \hat{E}^2) & 0 & ig \\
(1 - g^2 \hat{E}^2) & 0 & -ig & 0
\end{bmatrix} \delta^3(x - x').$$

(35)

The notation here is $A, B = 1, 2, 3, 4$, with $Z^1 = \phi$, $Z^2 = \phi^*$, $Z^3 = \hat{\phi}$ and $Z^4 = \hat{\phi}^*$. Inverting the matrix (35) to second order in $g$ we obtain

$$W^{AB}(x, x') = \begin{bmatrix}
0 & -ig & 0 & 1 + 3g^2 \hat{E}^2 \\
ig & 0 & 1 + 3g^2 \hat{E}^2 & 0 \\
0 & -(1 + 3g^2 \hat{E}^2) & 0 & 2ig \hat{E}^2 \\
-(1 + 3g^2 \hat{E}^2) & 0 & 2ig \hat{E}^2 & 0
\end{bmatrix} \delta^3(x - x'),$$

(36)

from where we read the non-zero brackets among the fields and their first order time derivatives.

$$\{ \phi(x,t), \phi^* (x', t) \} = -ig \delta^3(x - x'),$$

(37)

$$\{ \phi(x,t), \phi^* (x', t) \} = (1 + 3g^2 \hat{E}^2) \delta^3(x - x'),$$

(38)

$$\{ \phi^*(x,t), \phi (x', t) \} = (1 + 3g^2 \hat{E}^2) \delta^3(x - x'),$$

(39)

$$\{ \phi (x,t), \phi^* (x', t) \} = -2ig \hat{E}^2 \delta^3(x - x').$$

(40)

### 3.2.2. New canonical variables

The next step in the procedure is to introduce new canonical variables having the standard symplectic form in such a way that the final Hamiltonian density has the quadratic term in the momenta normalized to one. To this end we define, to order $g^2$, the following new coordinates $\tilde{\phi}, \tilde{\phi}^*$ and momenta $\Pi_{\tilde{\phi}}, \Pi_{\tilde{\phi}^*}$

$$\tilde{\phi} = \left( P\phi - \frac{ig}{2} Q\phi \right), \quad \Pi_{\tilde{\phi}} = (R\phi^* - ig \hat{E}^2 S\phi^*),$$

(41)
together with the corresponding complex conjugates: \( \tilde{\phi}^* = (\tilde{\phi})^*, \Pi_{\tilde{\phi}}^* = (\Pi_{\tilde{\phi}})^* \). The imposition of standard Poisson brackets among the tilde variables leads to the conditions

\[
P = Q, \quad R = S, \quad PR = 1 - \frac{3g^2}{2} E^2.
\]

The inverse functions are given by

\[
\phi = \left( 1 + \frac{1}{2} g^2 \hat{E}^2 \right) \left( \frac{1}{P} \tilde{\phi} + \frac{1}{R} \frac{i g}{2} \Pi_{\tilde{\phi}} \right),
\]

\[
\dot{\phi} = \left( 1 + \frac{1}{2} g^2 \hat{E}^2 \right) \left( \frac{1}{R} \Pi_{\tilde{\phi}}^* - \frac{1}{P} i g \hat{E}^2 \tilde{\phi} \right).
\]

Substituting in the expression (33) for the Hamiltonian density we obtain, to order \( g^2 \),

\[
\mathcal{H} = \Pi_{\tilde{\phi}} \Pi_{\tilde{\phi}}^* + \frac{ig}{2} \left( \tilde{\phi}^* \hat{E}^2 \Pi_{\tilde{\phi}} - \Pi_{\tilde{\phi}} \hat{E}^2 \tilde{\phi} \right) + \tilde{\phi}^* \left( \hat{E}^2 + \frac{5g^2}{4} (\hat{E}^2)^2 \right) \tilde{\phi},
\]

where the momenta are given by

\[
\Pi_{\tilde{\phi}} = \frac{\partial \tilde{\phi}}{\partial t} + \frac{ig}{2} \hat{E}^2 \tilde{\phi}, \quad \Pi_{\tilde{\phi}}^* = \frac{\partial \tilde{\phi}^*}{\partial t} - \frac{ig}{2} \hat{E}^2 \tilde{\phi}^*.
\]

The Hamiltonian density can be conveniently written as

\[
\mathcal{H} = \frac{\partial \tilde{\phi}^*}{\partial t} \frac{\partial \tilde{\phi}}{\partial t} + \tilde{\phi}^* \left( \hat{E}^2 + g^2 (\hat{E}^2)^2 \right) \tilde{\phi}.
\]

3.2.3. The modified dispersion relations

Finally we construct the corresponding effective Lagrangian density

\[
L = \frac{\partial \tilde{\phi}^*}{\partial t} \frac{\partial \tilde{\phi}}{\partial t} + \frac{ig}{2} \left( \frac{\partial \tilde{\phi}^*}{\partial t} \hat{E}^2 \tilde{\phi} - \frac{\partial \tilde{\phi}}{\partial t} \hat{E}^2 \tilde{\phi}^* \right) - \tilde{\phi}^* \left( \hat{E}^2 + g^2 (\hat{E}^2)^2 \right) \tilde{\phi},
\]

leading to the equation of motion

\[
\frac{\partial^2 \tilde{\phi}(g)}{\partial t^2} + ig \hat{E}^2 \frac{\partial \tilde{\phi}(g)}{\partial t} + \left( \hat{E}^2 + g^2 (\hat{E}^2)^2 \right) \tilde{\phi}(g) = 0.
\]

Using a plane wave type solution one finds the MDR for positive frequencies to be

\[
\omega_1 (k) = E_k \left( 1 + \frac{1}{2} g E_k + \frac{5}{8} (g E_k)^2 \right), \quad \omega_2 (k) = E_k \left( 1 - \frac{1}{2} g E_k + \frac{5}{8} (g E_k)^2 \right).
\]

The energy dependence in the corresponding plane wave are \( e^{-i\omega_1 t} \) and \( e^{i\omega_2 t} \) respectively. Let us emphasize two points at this level: (i) the MDR (50) coincide with those derived from the exact equation in (15). (ii) the theory is charge conjugation invariant, which means that the field \( \phi(g) \) together with the charge conjugate field \( \phi^C(g) = \phi^*(g) \) are solutions of the equation of motion (49), as can be directly verified.
3.3. The quantization

It proceeds along the standard lines of the complex scalar field, the only difference
being that the MDR are given by Eq. (50). In fact, we introduce the independent set of
creation-annihilation operators
\[
[a(k), a^\dag(k')] = \delta^3(k - k'), \quad [b(k), b^\dag(k')] = \delta^3(k - k'),
\]
(51)
with standard vacuum
\[
a(k)|0\rangle = 0 = b(k)|0\rangle.
\]
(52)
In terms of these operators, we expand the complex field $\tilde{\phi}$ as
\[
\tilde{\phi}(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3}} \frac{1}{\sqrt{2\Omega(|k|)}} \left( a(k)e^{-i(\omega_1t - k \cdot x)} + b^\dag(k)e^{i(\omega_2t - k \cdot x)} \right),
\]
(53)
together with its hermitian conjugate. The normalization factor $\Omega(|k|)$ is determined by
demanding the equal time commutation relation
\[
[\tilde{\phi}(x), \Pi\tilde{\phi}(x') = i\delta^3(x - x'),
\]
(54)
and it is given by
\[
\Omega(|k|) = E_k \left( 1 + \frac{5}{8}g^2E_k^2 \right).
\]
(55)
The remaining canonical commutation relations among the field operators can also be
recovered, in virtue of the relations (51) and (55). Starting from (47), the normal ordered
Hamiltonian turns out to be
\[
H = \int d^3k \left( \omega_1 a^\dag(k)a(k) + \omega_2 b^\dag(k)b(k) \right).
\]
(56)
This result can be considered as an additional consistency check of the choice (55)
to order $g^2$. The interpretation of $a^\dag(k), a(k)$ ($b^\dag(k), b(k)$) as creation and annihilation
operators for positively (negatively) charged particles also follows directly. In fact, the
normal ordered charge operator is given by
\[
Q = \int d^3k \left[ a^\dag(k)a(k) - b^\dag(k)b(k) \right].
\]
(57)
The Feynman propagator is
\[
i\Delta_F(x - y) = \langle 0|T(\tilde{\phi}(x)\tilde{\phi}^\dag(y))|0\rangle,
\]
(58)
which can be separated into retarded and advanced pieces
\[
i\Delta_F(x - y) = \theta(x_0 - y_0)\langle 0|\tilde{\phi}(x)\tilde{\phi}^\dag(y)|0\rangle + \theta(y_0 - x_0)\langle 0|\tilde{\phi}^\dag(y)\tilde{\phi}(x)|0\rangle.
\]
(59)
The standard calculation yields

$$\Delta_F(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik(x-y)}}{k^2 - k_0 gE_k^2 - E_k^2 (1 + g^2 E_k^2) + i\varepsilon},$$

where the denominator reproduces exactly the momentum space version of the effective equation of motion (60).

ACKNOWLEDGMENTS

LU thanks the organizers of the Third Mexican Meeting on Mathematical and Experimental Physics for their invitation. The work of CMR, JDV and LU has been partially supported by the projects DGAPA-UNAM # IN109017 and CONACYT # 55310. JDV also acknowledges partial support from the project CONACYT # 47211.

REFERENCES

1. C. M. Reyes, L. Urrutia and J. D. Vergara, Quantum corrections in the Myers-Pospelov model: a progress report, submitted for publication in the Proceedings of the Conference From Quantum to Emergent Gravity: Theory and Phenomenology, June 11-15, 2007, Trieste, Italy.
2. J. Alfaro, Quantum gravity and Lorentz invariance deformation in the standard model, Phys. Rev. Lett. 94, 221302 (2005); J. Alfaro, Quantum gravity induced Lorentz invariance violation in the Standard Model: Hadrons, Phys. Rev. D 72, 024027 (2005).
3. X. Jaen, J. Llosa and A. Molina, A Reduction of order two for infinite order lagrangians, Phys. Rev. D 34, 2302 (1986).
4. J. Z. Simon, Higher Derivative Lagrangians, Nonlocality, Problems And Solutions, Phys. Rev. D 41, 3720 (1990).
5. D. A. Eliezer and R. P. Woodard, The Problem of Nonlocality in String Theory, Nucl. Phys. B 325, 389 (1989).
6. T. C. Cheng, P. M. Ho and M. C. Yeh, Perturbative approach to higher derivative and nonlocal theories, Nucl. Phys. B 625, 151 (2002).
7. T. C. Cheng, P. M. Ho and M. C. Yeh, Perturbative approach to higher derivative theories with fermions, Phys. Rev. D 66, 085015 (2002).
8. P. M. Grignino and H. Vucetich, Quantum corrections to Lorentz invariance violating theories: Fine-tuning problem, Phys. Lett. B 651, 313 (2007).
9. M. Ostrogradski, Mémoire sur les équations différentielles relatives aux problèmes des isopérimètres, Mem. Acad. St.-Pétersbourg VI, 385 (1850).