Frontal Plane Modelling of Human Dynamics during Standing in Narrow-Stance

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Abstract. Standing ride type vehicles like electric skateboards have been developed in recent years. Although these vehicles have advantages as being compact and low cost due to their simple structure, it is necessary to improve the riding quality. Therefore, the system aiding riders to keep their balance on a skateboard by feedback control or feedforward control has been required. To achieve it, a human balance model should be built as simple as possible. In this study, we focus on the human balance modelling during standing when the support surface moves largely. We restricted the model on frontal plane and narrow stance because the restrictions allow us to assume single-degree-of-freedom model. The balance control system is generally assumed as a delayed feedback control system. The model was identified through impulse response test and frequency response test. As a result, we found the phase between acceleration of the skateboard and posture angle become opposite phase in low frequency range.

1. Introduction
This study focuses on electric skateboard which is one of the electric vehicles having been developed in recent years. While the vehicle is compact and low cost, it is difficult to stand on accelerating skateboard because commercial skateboards are used to be accelerated by a controller without considering driver’s dynamics. Therefore, we have been developed an electric skateboard to aid rider’s balance control through acceleration control of the skateboard. To achieve it, a model describing human body dynamics during standing on frontal plane is needed. The model is desired to be simple for applying the feedback control or the feedforward control.

The purpose of this study is to build a simple model expressing human balance dynamics on frontal plane. The model must be considered sensory feedback mechanism composed of vestibular, vision, and somatic sensation, however, the detail (filtering mechanism and sensing weight) was not established in past studies. There are a few studies of identification for human body dynamics on frontal plane when its support surface moves. Experiment using an electric skateboard allows us to give the large motion to a subject and to identify human body dynamics in low frequency range.

General modeling of human balance control has not been established because inner structures of the sensing system and the feedback system are obscure. Stepan and Kollar modeled vestibular sensing system and balance control based on the single degree-of-freedom system like an inverted pendulum [1], however, the validation was not evaluated. Some studies tried to build a model by adding external force and measuring human body dynamics during standing. Bingham et al. have
reported that balance control of the frontal-plane model was assumed as proportional-derivative delayed feedback control system with small delay and the parameters were obtained from results of impulse response test [2]. Goodworth analyzed dynamics of subjects standing on support surface rotating like a seesaw and described the balance control mechanism based on two degrees-of-freedom system by a complex feedback system in narrow stance [3] and wide stance [4], respectively. Additionally, they denoted that the system has nonlinear characteristic whose gain varies depending on amplitude of the support surface [5]. Some researchers expect that the nonlinear characteristic is generated by sensory reweighting [6] which changes feedback gains according to amplitude of external force.

From past studies, we found it is difficult to build a precise human balance model by transient response analysis from impulse response test because the response scatters over a wide range. Tokuno reported that the electromyogram data measured from impulse force experiment varies largely according to subject’s initial posture [7]. On the other hand, it is also difficult to build a stable model from frequency response analysis due to its nonlinear characteristic. To solve the problem, we simplified the model by restrictions of stance width and frequency range, and identified the model by utilizing impulse response test and frequency response test in combination.

In this study, we focused the investigation on a particular aspect of the subject. As we assumed the stand width of the human is fixed to narrow stance, it allows us to regard the mechanism of the model as a low dimensional model like a Bingham model in this report. And the frequency range of external force is restricted in low frequency range. Under the condition, we estimated two transfer function models of posture angle output and moment output. To estimate the models with stable, we estimated characteristic roots from impulse response analysis. And amplitude and phase of the response are evaluated from frequency response analysis. As a result, we derived two transfer function models which are appropriate for applying the feedback control and the feedforward control.

2. Assumption of human body system

This study aims to build a transfer function model of a human standing on an electric skateboard. From experimental data in terms of inputs and outputs of the system, the model is identified. Then, order of the transfer function model and degree-of-freedom of the subject are important. Although a mechanism of human is exactly composed of much degrees-of-freedom, we defined a simple model from the viewpoint of building a control subject.

We assumed that track of the electric skateboard is constrained on a straight line and stance width are fixed to narrow stance. Mechanism of the model was defined as a rigid body rotating around midpoint of both ankle joints like a simple inverted pendulum shown in figure 1. The simplified model mainly describes lower body dynamics because dynamics of center of mass (COM) in whole body is in good agreement with that in lower body in frequency response analysis when the stance width is set to narrow [5].

In experimental identification, we added external force to a subject by acceleration of the skateboard and measured posture angle $\theta$ and moment $\tau$ shown in figure 1 by motion capture system and load sensors. As a result, we obtained two transfer function models: posture angle output model and moment output model. To consider how to identify the models, orders of the transfer function models were provisionally determined by assuming a simple balance control method and equation of motion of the model. When the posture angle is relatively small, the equation of motion of the model is given by

\[(J + ml^2)\ddot{\theta} - mgl\theta = ml\ddot{x} + \tau,\]  

(1)

where $m$ is mass of a rigid body and $J$ is moment of inertia around COM. COM and moment of inertia can be estimated from ratio of height and weight of a subject by reference to past studies [8]. Since $\tau$ in the right side of the equation is generated by balance control of human body and unknown, we assumed it as a typical function [2] which is a proportional and derivative feedback control system with small delay as follows:
where $k_p$ and $k_d$ are feedback gains and $t_d$ is delay time.

From equations (1) and (2), we can derive transfer functions and check the order. The transfer functions are obtained as follows:

$$G_d(s) = \frac{mlt_s + 2mla}{D(s)}, \quad G_r(s) = \frac{mlk_t s^2 - ml(k_s t_d + k_r)s + 2mlk_r}{D(s)}$$

$$D(s) = (J + ml^2)t_d s^3 + [2(J + ml^2) - k_d t_d]s^2 + (2k_d + k_s t_d - mgl t_d)s + 2(k_p - mgl)$$

For linear approximation, exponential functions arisen from time delay have been transformed to polynomial by first-order Padé approximation as follows:

$$\exp(-t_d s) = \frac{1 - \frac{t_d}{2} s}{1 + \frac{t_d}{2} s}$$

Consequently, we found that the denominator in the transfer functions is third order and the numerators are first order (posture angle output) and second order (moment). However, these orders depend on the assumption of equation (2). From the result, we defined the order of denominator as third order, and determined the order of numerator from result of frequency response test.

This study aims to express human body dynamics through two transfer functions in equation (3). Since the human response has variability in experimental measurement, we analyzed the dynamics based on frequency response analysis. However, the transfer function models obtained from frequency response analysis tend to have positive unstable roots due to its nonlinear characteristics. As a measure to counter this problem, we estimated negative stable roots from impulse response analysis. Through a combination of the impulse response test and the frequency response test, we identified transfer function models.

3. Identification

3.1. Apparatus

Figure 2 shows an electric skateboard modified for the experiment. We mounted a microcomputer (STM32F4 Discovery, STMicroelectronics), a motor driver (24V23, Pololu) and a rotary encoder to commercial electric skateboard. Motion of the skateboard was controlled by robust servo control.
A system based on sliding mode control with 1.2 kHz sampling. Performance of the follow-up control will be discussed further later on.

Dynamics of a subject was measured by a motion capture system and two load devices. We affixed twelve reflecting markers on a subject shown in figure 3. From these trajectories, we can estimate COM of the subject. COM was estimated from COM of seven segments (head, upper body, pelvis, thighs and lower legs) shown in figure 4 which are derived with reference to [8]. In this calculation, we merged upper limbs with upper body and except foot segments.

A load device is composed of four load cells (LMA-A-500N, KYOWA) shown in figure 5. By the device, moment acted from human body is estimated by

\[ N = \frac{L}{2}(R_{x2} + R_{y1} - R_{y1} - R_{x1}) + \frac{W}{2}(R_{x1} + R_{x2} - R_{y1} - R_{y2}), \]  

(5)
where $R_{li}$ and $R_{ri}$ ($i = 1, 2$) are ground reaction force described in figure 6. We set to $W = 0.23$ m and $L = 0.070$ m and shear force was ignored in moment calculation. Measured data was logged by 120Hz sampling.

### 3.2. Impulse response test

We derived characteristic roots of transfer function model described in equation (3) through impulse response test. Six healthy males (Subject A-F) in their twenties participated in this experiment. All subjects gave their informed consent before being tested using protocol approved by the institutional review board at Tokushima University. They instructed to look ahead and keep their arms spontaneously to the sides. The test were repeated twenty times.

In the test, reference acceleration is defined as Gaussian pulse as follows:

$$a_{\text{ref}} = A \frac{1}{(2\pi)^{1/2} \sigma} \exp \left( -\frac{t^2}{2\sigma^2} \right).$$

We set $\sigma = 0.1$ and $A(2\pi)^{1/2} \sigma^2 = 1.0$ ms$^{-2}$ in the test. Since the beginning time of acceleration was determined at random, subjects cannot predict board’s motion. We measured posture angle and moment acted from support surface. Representative result of subject A is shown in figure 7. We analysed measured data in a duration from 0.3 to 2.5 sec in each test.

Assuming that the characteristic roots of the transfer functions described in equation (3) are combination of a real root and a couple of complex root, it is possible to fit the experimental data by functions as follows:

$$\theta(t) = p_4 + p_5 \exp(\rho_1 t) + p_6 \exp(\rho_2 t) \sin \rho_1 t + p_7 \exp(\rho_2 t) \cos \rho_1 t$$

$$\tau(t) = p_8 + p_9 \exp(\rho_1 t) + p_{10} \exp(\rho_2 t) \sin \rho_2 t + p_{11} \exp(\rho_2 t) \cos \rho_2 t$$

We obtained characteristic roots from $\rho_1$, $\rho_2$, and $\rho_3$ by least square method. Distribution of the roots are plotted in figure 8. Table 1 shows average of the roots for six subjects. Strictly speaking, distribution and average of the roots were varied according to amplitude of the impulse. We set the amplitude of the impulse input to the point of keeping subject’s balance easily in this study. As a result, we obtained three stable characteristic roots of the system.

### 3.3. Frequency response test

After the impulse response test, we implemented frequency response test to determine numerators of the transfer functions. In the test, motion and reaction force were measured for 34.13 sec after 5
Table 1. Characteristic root of six subjects estimated from impulse response tests.

| Subject | Characteristic roots       |
|---------|---------------------------|
| A       | -2.98 -1.07±2.91i         |
| B       | -2.67 -1.93±3.06i         |
| C       | -1.60 -1.65±3.56i         |
| D       | -1.92 -1.25±3.17i         |
| E       | -2.95 -1.57±3.06i         |
| F       | -2.81 -1.77±3.84i         |

Figure 7. Experimental result and fitted line in impulse response test for estimation of characteristic roots. (Subject A)

Figure 8. Distribution of characteristic roots estimated from impulse response tests doing 20 times.

seconds preliminaries. Subjects and cautions in the test were same as the impulse response test and the was repeated six times.

The reference acceleration of the test was given by

\[ a_{ref} = A \sum_{i=1}^{17} \cos(\omega t + \phi_i) , \]

where \( \omega \) is determined as an integral multiple of \( \omega_0 = 2 \pi / T \approx 0.184 \text{ rad} \cdot \text{s}^{-1} \); \( \omega = (2i+2) \omega_0 \) (i = 1, \( \cdots \), 17) and phase \( \phi_i \) is designed to minimize moving distance of the skateboard. The frequency range
Figure 9. Reference acceleration of the skateboard in frequency response test.

Figure 10. Comparison between reference position and measured position of the skateboard to check performance of servo control of the electric skateboard.

Figure 11. Frequency response diagrams of subject A. Circles mean average and error bars mean standard deviation in six times tests.

(approximately corresponds to 0.1-1.0 Hz. Wave of the reference acceleration is plotted in figure 9. Amplitude $A_0$ was adjusted to make maximum of $a_{ref}$ 0.8 ms$^2$.

Because follow-up control is not perfect in our device, we compensated error between acceleration and reference acceleration of the skateboard in frequency domain. The error was checked by comparing measured position of skateboard with reference position and the result was shown in figure 10. The corrected acceleration of the skateboard in frequency domain is obtained as follows:
In the section, we discuss balance control system based on the experimental result. Figure 11 shows average and standard deviation frequency response diagram obtained from six times experiments in terms of subject A after compensating the servo errors. The left side figure and the right side figure show the diagram of posture angle output system and moment output system, respectively. The results indicate that variation of gain and phase are relatively small in the test. We found the system has peak frequency around 4.0 rad·s⁻¹. In particular, we noticed that the phase of posture angle output becomes anti-phase in low-frequency band below 2.0 rad·s⁻¹. The phenomenon supports Goodworth’s report indicating that a subject standing on constant accelerating board inclines his/her posture to counter direction from inertial force [9]. We suppose the anti-phase phenomenon in low frequency band is generated by inclining of upper body.

To obtain the transfer function models, some transfer function models were fit to the result of frequency response test using least square method. Because we cannot express the result of low frequency range (<2.0 rad·s⁻¹) in one degree-of-freedom model, the model were fitted in the range over 2.0 rad·s⁻¹.

Denominator of the transfer function model had been derived through the impulse response test. Because order of numerator of the transfer functions are unknown, we assumed from 1st to 5th order polynomials as order of numerator and determined appropriate order based on relevance ratio. As a result, orders of the numerators were determined 2nd in posture angle output model and 4th in moment output model, respectively. The transfer function models of all subjects were shown in table 2. We expect the model was effectiveness between 2.0 rad·s⁻¹ and 7.0 rad·s⁻¹.

### 4. Discussion of identified model

From the experimental identification, we obtained transfer function models. They are well fitted when the order of numerators is defined as 2nd (posture angle output) and 4th (moment output). In this section, we discuss a balance control system based on the experimental result.

In table 2, \( G_A(s) \) is non-proper function and it is generally transformed into a proper function as follows:

\[
A(\omega) = A_{ref}(\omega) \frac{X(\omega)}{X_{ref}(\omega)},
\]

where \( A(\omega) \) and \( A_{ref}(\omega) \) are Fourier transformed data of acceleration and reference acceleration of the skateboard, and \( X(\omega) \) and \( X_{ref}(\omega) \) are that of position and reference position.

Table 2 Transfer function models of six subjects

| Subject | \( G_d(s) \) | \( G_t(s) \) |
|---------|-------------|-------------|
| A       | \(-0.06353s^2+0.4998s+1.191\) | \(-8.562s^3+39.14s^2-389.6s^2-308.3s-3632\) |
|         | \(s^3+5.117s^2+15.96s+28.57\) | \(s^3+5.117s^2+15.96s+28.67\) |
| B       | \(-0.03702s^2+0.6547s+2.029\) | \(-0.4313s^4+47.57s^3-167.0s^2-508.7s-2330\) |
|         | \(s^2+4.385s^3+13.24s^2+8.429\) | \(s^2+4.385s^3+13.24s^2+8.429\) |
| C       | \(-0.01876s^2+0.5073s+3.500\) | \(4.230s^4+34.99s^3-269.0s^2-1294s-5942\) |
|         | \(s^3+4.904s^2+20.72s+24.74\) | \(s^3+4.904s^2+20.72s+24.74\) |
| D       | \(0.01505s^2+0.5360s+2.537\) | \(-3.553s^4+38.08s^3-356.4s^2-613.8s-4689\) |
|         | \(s^2+4.416s^2+16.39s+22.23\) | \(s^2+4.416s^2+16.39s+22.23\) |
| E       | \(-0.1083s^2+1.485s-1.767\) | \(16.44s^4-46.46s^3+313.4s^2-2359s+6422\) |
|         | \(s^3+0.1865s^2+6.448s-46.51\) | \(s^3+0.1865s^2+6.448s-46.51\) |
| F       | \(0.02277s^2+0.2212s+4.207\) | \(10.10s^4+2.178s^3-26.34s^2-1762s-6086\) |
|         | \(s^3+6.354s^2+27.83s+50.17\) | \(s^3+6.354s^2+27.83s+50.17\) |
\[
\frac{T(s)}{A(s)} = \frac{c_4s^4 + c_3s^3 + c_2s^2 + c_1s + c_0}{s^3 + a_2s^2 + a_1s + a_0} = \frac{c'_2s^2 + c'_1s + c'_{0}}{s^3 + a_2s^2 + a_1s + a_0} + c'_3s + c'_4,
\]

where
\[
c'_0 = c_0 - (c_3 - a_2c_4)a_0
\]
\[
c'_1 = (c_1 - a_0c_4) - (c_3 - a_2c_4)a_1
\]
\[
c'_2 = (c_2 - a_0c_4) - (c_3 - a_2c_4)a_2
\]
\[
c'_3 = c_3 - a_2c_4.
\]

If we consider simply, feedback control method for balance control is redefined as the function which are added acceleration and jerk terms with respect to the skateboard to equation (2) as follows:
\[
\tau = -k_p\theta(t-t_d) - k_d\dot{\theta}(t-t_d) + \gamma_1\alpha(t-t_d) + \gamma_2\dot{\alpha}(t-t_d).
\]

We suppose that third and fourth terms of right hand side in equation (11) mean the counter moment generated from force sensing around both ankles. Although we assumed the delay time of acceleration feedback is same as posture angle feedback in this equation, it may be quite smaller than posture angle feedback.

From equations (1) and (11), we obtained a posture angle output transfer function model as follows:
\[
G_p(s) = \frac{\gamma_1t_d s^2 + (2\gamma_2 - \gamma_1t_d + mlt_d)s + 2(\gamma_1 + ml)}{(J + ml^2)t_ds^3 + (2(J + ml^2) - k_d j_d)s^2 + (2k_d + k_j j_d - mgl t_d)s + 2(k_p - mgl)}.
\]

Thus, when we assumed feedback of acceleration and jerk in balance control, the transfer function of posture angle output is expressed as equation (12). The result implies the existence of acceleration feedback terms in human balance control when the support surface moves.

5. Conclusion

We built simple transfer function models expressing balance control on frontal plane through impulse response test and frequency response test. For simplification, the mechanism was assumed as single degree-of-freedom and its frequency range is restricted from 2 rad∙s⁻¹ and 7 rad∙s⁻¹. By estimating characteristic roots of the system from impulse response test, stable transfer functions were surely obtained. While order of denominator of the transfer functions were defined as third, that of numerator were determined based on the relevance ratio. As a result, the order was determined as second order (posture angle output) and fourth (moment output). This implies that moment for balance control is generated by superposing acceleration and jerk feedback of skateboard on posture feedback of the human body. In frequency response test, acceleration of the skateboard and posture angle of the human body became an antiphase relation to each other when the frequency is below 2 rad∙s⁻¹. The most likely cause is inclination of upper body to cope with the inertial force in low frequency range.

In future work, we should verify the validation of the estimation transfer function models and range of effectiveness. To achieve it, we will apply state estimation, feedback control, and feedforward control to the models. In addition, we must consider two degrees of freedom mechanism because the antiphase phenomena arising in low frequency range is difficult to explain by single degree-of-freedom system.

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