Covalent hadronic molecules induced by shared light quarks

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Abstract
After examining Feynman diagrams corresponding to the 

\[ D^{(*)}\Sigma^{(*)}_c, D^{(*)}\Lambda_c, D^{(*)}K^*, \]

and 

\[ D^{(*)}\bar{D}^{(*)}\] hadronic molecular states, we propose a possible binding mechanism induced by shared light quarks. This mechanism is similar to the covalent bond in chemical molecules induced by shared electrons. We use the method of QCD sum rules to calculate its corresponding light-quark-exchange diagrams, and the obtained results indicate a model-independent hypothesis: the light-quark-exchange interaction is attractive when the shared light quarks are totally antisymmetric so they obey the Pauli principle. We build a toy model with four parameters to formulize this picture and estimate binding energies of some possibly-existing covalent hadronic molecules. A unique feature of this picture is that the binding energies of the 

\[ (I)J^P = (0^{+}) DB^*/D\bar{B}^* \]

hadronic molecules are much larger than those of the 

\[ (I)J^P = (0^{+}) D\bar{D}^*/D\bar{D}^* \]

ones, while the 

\[ (I)J^P = (1/2^{+}) D\Sigma^*_c/D\Sigma^*_{c\bar{c}}/B\Sigma_c/B\Sigma_{c\bar{c}} \]

hadronic molecules have similar binding energies.

Keywords: hadronic molecule, covalent bond, QCD sum rules

1. Introduction
Since the discovery of the \( X(3872) \) by Belle in 2003 [1], lots of charmonium-like \( XYZ \) states were discovered in the past two decades [2]. Some of these structures may contain four quarks and are good candidates for hidden-charm tetraquark states. In recent years, the LHCb Collaboration continually observed six \( P_c/P_{cJ} \) states [3–6], which contain five quarks and are good candidates for hidden-charm pentaquark states. Although there is still a long way to fully understand how the strong interaction binds these quarks and antiquarks together, the above exotic structures have become one of the most intriguing research topics in hadron physics. Their theoretical and experimental studies are significantly improving our understanding of the non-perturbative behaviors of the strong interaction at the low energy region. We refer to the reviews [7–22] and references therein for detailed discussions.

Some of the \( XYZ \) and \( P_c/P_{cJ} \) states can be interpreted as hadronic molecular states, which consist of two conventional hadrons [23–32]. For example, the \( P_c \) states were proposed to be the 

\[ D^{(*)}\Sigma^{(*)}_c \]

hadronic molecular states in [33–39] bound by the one-meson-exchange interaction \( \Pi_M \), as depicted in figure 1(a). Besides, we know from QCD that there can be the double-gluon-exchange interaction \( \Pi_G \) between 

\[ D^{(*)} \]

and 

\[ \Sigma^{(*)}_c \]

, as depicted in figure 1(b).

In this paper we propose another possible interaction between 

\[ D^{(*)} \]

and 

\[ \Sigma^{(*)}_c \]

induced by the light-quark-exchange term \( \Pi_{Q} \), as depicted in figure 1(c). This term indicates that 

\[ D^{(*)} \]

and 

\[ \Sigma^{(*)}_c \]

are exchanging and so sharing two light up/down quarks, as depicted in figure 2. It can induce an interaction between 

\[ D^{(*)} \]

and 

\[ \Sigma^{(*)}_c \]

, either attractive or repulsive. Note that the two interactions, \( \Pi_M \) at the hadron level and \( \Pi_{Q} \) at the quark-gluon level, can overlap with each other. The quark-exchange effect has been studied in [40] by Hoodbhoy and Jaffe to explain the European Muon Collaboration (EMC) effect in three-nucleon systems, and later used in [41, 42] to study some other nuclei. We also refer to [43] for some relevant discussions.

In this paper we shall systematically examine the Feynman diagrams corresponding to the 

\[ D^{(*)}\Sigma^{(*)}_c, D^{(*)}\Lambda_c, D^{(*)}K^*, \]

and 

\[ D^{(*)}\bar{D}^{(*)}\] hadronic molecular states. We shall apply the method of QCD sum rules to investigate the light-quark-exchange term \( \Pi_{Q} \), and study its contributions to these states. Based on the obtained results, we shall study the binding mechanism induced by shared light quarks. This mechanism is somewhat similar to the covalent bond in
chemical molecules induced by shared electrons, so we call such hadronic molecules ‘covalent hadronic molecules’.

Based on the obtained results, we shall further propose a model-independent hypothesis: the light-quark-exchange interaction is attractive when the shared light quarks are totally antisymmetric so that they obey the Pauli principle. We shall apply this hypothesis to predict some possibly-existing covalent hadronic molecules. We shall also build a toy model to formulize this picture and estimate their binding energies. Our model has four parameters, which are fixed by considering the $P_c/P_{c*}$ and the recently observed $T_{cc}^{0}$ [44, 45] as possible covalent hadronic molecules.

Take the $X(3872)$ as another example. We shall find that the light-quark-exchange term $\Pi_{Q}$ does not contribute to the $D^{(*)}D^{(*)}$ molecules, suggesting the $D^{(*)}D^{(*)}$ covalent hadronic molecules not to exist. However, there can still be the $D^{(*)}\bar{D}^{(*)}$ hadronic molecules induced by some other binding mechanisms, such as the one-meson-exchange interaction [23–32]. Especially, the $X(3872)$ can be interpreted as such a $D\bar{D}$ hadronic molecule, while it was suggested to be a compact tetraquark state in [46–50], a conventional $cc$ state in [51, 52], and the mixture of a $cc$ state with the $DD$ component in [53, 54].

This paper is organized as follows. In section 2 we systematically investigate correlation functions of the $D^{-}\Sigma_{c}^{++}$, $D^{0}\Sigma_{c}^{0}$, $I=1/2\ D_{c}$, and $I=3/2\ D_{c}$ hadronic molecules. In section 3 we apply the method of QCD sum rules to investigate the light-quark-exchange term $\Pi_{Q}$, and study its contributions to these molecules. In section 4 we follow the same procedures and study the $D^{0}\Sigma_{c}^{0}/D_{c}/\bar{D}^{0}\Sigma_{c}^{0}$, $D^{0}\Lambda_{c}$, $D^{(*)}\bar{D}^{(*)}$ hadronic molecules. Based on the obtained QCD sum rule results, we propose the above hypothesis in section 5, and predict more possibly-existing covalent hadronic molecules. In section 6 we build a toy model to formulize the covalent hadronic molecule picture, and the obtained results are summarized and discussed in section 7.

2. Correlation functions of $D^{-}\Sigma_{c}^{++}/D^{0}\Sigma_{c}^{0}/D\Sigma_{c}$ molecules

In this section we investigate correlation functions of the $D^{-}\Sigma_{c}^{++}$, $D^{0}\Sigma_{c}^{0}$, $I=1/2\ D_{c}$, and $I=3/2\ D_{c}$ hadronic molecular states.

2.1. $D^{-}\Sigma_{c}^{++}$ correlation function

In this subsection we investigate the correlation function of the $D^{-}\Sigma_{c}^{++}$ molecule. Firstly, we investigate the correlation functions of the $D^{-}$ meson and the $\Sigma_{c}^{++}$ baryon. Their corresponding interpolating currents are

$$J_{D}^{i}(x) = \bar{c}_{a}(x)\gamma_{5}d_{a}(x),$$

$$J_{\Sigma_{c}^{++}}^{i}(x) = \frac{1}{\sqrt{2}}\epsilon_{abc}\bar{u}_{c}(x)\gamma^{\mu}u_{b}(x)\gamma_{\mu}\gamma_{5}c_{a}(x),$$

where $a \cdots c$ are color indices; $C = i\gamma_{5}\gamma_{0}$ is the charge-conjugation operator; the coefficient $1/\sqrt{2}$ is an isospin factor.

We write down two-point correlation functions of the $D^{-}\Sigma_{c}^{++}$ baryon in the coordinate space:

$$\Pi_{D}^{i}(x) = (0)\langle J_{D}^{i}(x)J_{D}^{i*(-1)(0)}\rangle_{0}$$

$$= -\text{Tr}[iS_{q}^{\text{aff}}(x)\gamma_{5}S_{q}^{\text{aff}}(-x)\gamma_{5}],$$

$$\Pi_{\Sigma_{c}^{++}}^{i,i'}(x) = (0)\langle J_{\Sigma_{c}^{++}}^{i}(x)J_{\Sigma_{c}^{++}}^{i'}(0)\rangle_{0}$$

$$= \epsilon_{abc}e_{a'b'c'}\text{Tr}[iS_{q}^{\text{aff}}(x)\gamma_{b'}\gamma_{a'}C(iS_{q}^{\text{aff}}(x)\gamma_{5}C\gamma_{5})]$$

$$\times \gamma_{\mu}\gamma_{5}S_{q}^{\text{aff}}(x)\gamma_{\mu'}\gamma_{5}. \quad (4)$$

Their corresponding Feynman diagrams are depicted in figures 3(a) and (b), respectively. In the present study we do not differentiate propagators of the light up and down quarks,
and use $iS_{qq}^{ab}(x) = iS_{qq}^{ab}(0) = iS_{down}^{ab}(0)$ to denote both of them in the coordinate space; besides, we use $iS_{q}^{ab}(x)$ to denote the propagator of the heavy charm quark in the coordinate space:

$\begin{align}
\text{(a) $\Pi^{D^*}$} & & \text{(b) $\Pi^\Sigma^*$} \\
\text{(c) $\Pi_0^{D^*\Sigma^*_+}$} & & \text{(d) $\Pi_G^{D^*\Sigma^*_+}$}
\end{align}$

$\text{Figure 3.}$ Feynman diagrams corresponding to: (a) $\Pi^{D^*}(x)$, (b) $\Pi^\Sigma^*(x)$, (c) $\Pi_0^{D^*\Sigma^*_+}(x)$, and (d) $\Pi_G^{D^*\Sigma^*_+}(x)$. $\Pi^D(x)$ and $\Pi^\Sigma^*(x)$ are correlation functions of the $D^-$ meson and the $\Sigma^*$ baryon, respectively. The correlation function of the $D^*\Sigma^*_+$ molecule satisfies $\Pi^{D^*\Sigma^*_+}(x) = \Pi_0^{D^*\Sigma^*_+}(x) + \Pi_G^{D^*\Sigma^*_+}(x)$ with $\Pi_0^{D^*\Sigma^*_+}(x) = \Pi^D(x) \times \Pi^\Sigma^*(x)$.

Then we put the $D^-$ meson and the $\Sigma^*$ baryon at the same location, and construct a composite current corresponding to the $D^*\Sigma^*_+$ molecule,

$\begin{align}
\text{(a) $\Pi^{D^*}$} & & \text{(b) $\Pi^\Sigma^*$} \\
\text{(c) $\Pi_0^{D^*\Sigma^*_+}$} & & \text{(d) $\Pi_G^{D^*\Sigma^*_+}$}
\end{align}$

and use $iS_{qq}^{ab}(x) = iS_{qq}^{ab}(0) = iS_{down}^{ab}(x)$ to denote both of them in the coordinate space; besides, we use $iS_{q}^{ab}(x)$ to denote the propagator of the heavy charm quark in the coordinate space:

$\begin{align}
\text{(5):} & & \quad \Pi^{D^*}(x) \\
\text{(6):} & & \quad \Pi^\Sigma^*(x) \\
\text{(5):} & & \quad \Pi_0^{D^*\Sigma^*_+}(x) \\
\text{(6):} & & \quad \Pi_G^{D^*\Sigma^*_+}(x)
\end{align}$

$\text{Figure 3.}$ Feynman diagrams corresponding to: (a) $\Pi^{D^*}(x)$, (b) $\Pi^\Sigma^*(x)$, (c) $\Pi_0^{D^*\Sigma^*_+}(x)$, and (d) $\Pi_G^{D^*\Sigma^*_+}(x)$. $\Pi^D(x)$ and $\Pi^\Sigma^*(x)$ are correlation functions of the $D^-$ meson and the $\Sigma^*$ baryon, respectively. The correlation function of the $D^*\Sigma^*_+$ molecule satisfies $\Pi^{D^*\Sigma^*_+}(x) = \Pi_0^{D^*\Sigma^*_+}(x) + \Pi_G^{D^*\Sigma^*_+}(x)$ with $\Pi_0^{D^*\Sigma^*_+}(x) = \Pi^D(x) \times \Pi^\Sigma^*(x)$.

Then we put the $D^-$ meson and the $\Sigma^*$ baryon at the same location, and construct a composite current corresponding to the $D^*\Sigma^*_+$ molecule,

$\begin{align}
\text{(5):} & & \quad \Pi^{D^*}(x) \\
\text{(6):} & & \quad \Pi^\Sigma^*(x) \\
\text{(5):} & & \quad \Pi_0^{D^*\Sigma^*_+}(x) \\
\text{(6):} & & \quad \Pi_G^{D^*\Sigma^*_+}(x)
\end{align}$

$\text{Figure 3.}$ Feynman diagrams corresponding to: (a) $\Pi^{D^*}(x)$, (b) $\Pi^\Sigma^*(x)$, (c) $\Pi_0^{D^*\Sigma^*_+}(x)$, and (d) $\Pi_G^{D^*\Sigma^*_+}(x)$. $\Pi^D(x)$ and $\Pi^\Sigma^*(x)$ are correlation functions of the $D^-$ meson and the $\Sigma^*$ baryon, respectively. The correlation function of the $D^*\Sigma^*_+$ molecule satisfies $\Pi^{D^*\Sigma^*_+}(x) = \Pi_0^{D^*\Sigma^*_+}(x) + \Pi_G^{D^*\Sigma^*_+}(x)$ with $\Pi_0^{D^*\Sigma^*_+}(x) = \Pi^D(x) \times \Pi^\Sigma^*(x)$.

Then we put the $D^-$ meson and the $\Sigma^*$ baryon at the same location, and construct a composite current corresponding to the $D^*\Sigma^*_+$ molecule,

$\begin{align}
\text{(5):} & & \quad \Pi^{D^*}(x) \\
\text{(6):} & & \quad \Pi^\Sigma^*(x) \\
\text{(5):} & & \quad \Pi_0^{D^*\Sigma^*_+}(x) \\
\text{(6):} & & \quad \Pi_G^{D^*\Sigma^*_+}(x)
\end{align}$

$\text{Figure 3.}$ Feynman diagrams corresponding to: (a) $\Pi^{D^*}(x)$, (b) $\Pi^\Sigma^*(x)$, (c) $\Pi_0^{D^*\Sigma^*_+}(x)$, and (d) $\Pi_G^{D^*\Sigma^*_+}(x)$. $\Pi^D(x)$ and $\Pi^\Sigma^*(x)$ are correlation functions of the $D^-$ meson and the $\Sigma^*$ baryon, respectively. The correlation function of the $D^*\Sigma^*_+$ molecule satisfies $\Pi^{D^*\Sigma^*_+}(x) = \Pi_0^{D^*\Sigma^*_+}(x) + \Pi_G^{D^*\Sigma^*_+}(x)$ with $\Pi_0^{D^*\Sigma^*_+}(x) = \Pi^D(x) \times \Pi^\Sigma^*(x)$.

Then we put the $D^-$ meson and the $\Sigma^*$ baryon at the same location, and construct a composite current corresponding to the $D^*\Sigma^*_+$ molecule,
Its correlation function in the coordinate space is

$$\Pi^{D^{++}}(x) = \{(0)\langle T [D^{++}(x) J^{D^{++}}(0)\rangle(0)\}\}$$

$$= -\text{Tr}[\mathbf{i}S_q^{d\bar{d}}(x)\gamma_5 S_{q\bar{c}}^{d\bar{c}}(-x)\gamma_5]$$

$$\times e^{abc} e^{d\bar{b}\bar{c}} \text{Tr}[\mathbf{i}S_q^{d\bar{d}}(x)\gamma_\mu\gamma_5 \mathbf{C}(\mathbf{i}S_q^{d\bar{d}}(x))\mathbf{\overline{C}}\gamma_\nu]$$

$$\times \gamma_\mu\gamma_5 S_{q\bar{c}}^{d\bar{c}}(-x)\gamma_\mu\gamma_5$$

$$= \Pi_0^{D^{++}}(x) + \Pi_G^{D^{++}}(x),$$

(8)

where

$$\Pi_0^{D^{++}}(x) = \Pi_D^{D}(x) \times \Pi_0^{D^{++}}(x),$$

(9)

is the leading term contributed by non-correlated $D^-$ and $\Sigma^+_c$, and $\Pi_G^{D^{++}}(x)$ describes the double-gluon-exchange interaction between them. Their corresponding Feynman diagrams are depicted in figures 3(c) and (d), respectively.

The double-gluon-exchange term $\Pi_G^{D^{++}}(x)$ is at the $\mathcal{O}(\alpha_s^2)$ order, and it is expected to be suppressed according to the OZI rule. In the present study we shall not pay much attention to it, while we shall investigate another more important term from the next subsection.

2.2. $D_\Sigma^c$ correlation function

In this subsection we investigate the correlation function of the $D^0\Sigma^+_{c}$ molecule. The interpolating currents corresponding to the $D^0\Sigma^+_{c}$ meson, the $\Sigma^+_c$ baryon, and the $D^0\Sigma^+_{c}$ molecule are

$$J^{D^0}(x) = \bar{c}\gamma_\mu u(x)\gamma_5 u_d(x),$$

(10)

$$J^{\Sigma^+}(x) = e^{abc} u^T(x)\gamma_\mu d^a(p)\gamma_\nu\gamma_5 c_c(x),$$

(11)

$$J^{D^0\Sigma^+}(x) = J^{D^0}(x) \times J^{\Sigma^+}(x)$$

$$\times [\bar{c}\gamma_\mu u_d(x)\gamma_\nu\gamma_5 c_c(x)].$$

(12)

Their correlation functions in the coordinate space are:

$$\Pi^{D^0}(x) = -\text{Tr}[\mathbf{i}S_q^{d\bar{d}}(x)\gamma_5 S_{q\bar{c}}^{d\bar{c}}(-x)\gamma_5],$$

(13)

$$\Pi^{\Sigma^+}(x) = e^{abc} e^{d\bar{b}\bar{c}} \text{Tr}[\mathbf{i}S_q^{d\bar{d}}(x)\gamma_\mu\gamma_5 \mathbf{C}(\mathbf{i}S_q^{d\bar{d}}(x))\mathbf{\overline{C}}\gamma_\nu]$$

$$\times \gamma_\nu\gamma_5 S_{q\bar{c}}^{d\bar{c}}(-x)\gamma_\mu\gamma_5,$$

(14)

$$\Pi^{D^0\Sigma^+}(x) = -\text{Tr}[\mathbf{i}S_q^{d\bar{d}}(x)\gamma_5 S_{q\bar{c}}^{d\bar{c}}(-x)\gamma_5]$$

$$\times e^{abc} e^{d\bar{b}\bar{c}} \text{Tr}[\mathbf{i}S_q^{d\bar{d}}(x)\gamma_\mu\gamma_5 \mathbf{C}(\mathbf{i}S_q^{d\bar{d}}(x))\mathbf{\overline{C}}\gamma_\nu]$$

$$\times \gamma_\nu\gamma_5 S_{q\bar{c}}^{d\bar{c}}(-x)\gamma_\mu\gamma_5$$

$$+ \text{Tr}[\mathbf{i}S_q^{d\bar{d}}(x)\gamma_\mu\gamma_5 \mathbf{C}(\mathbf{i}S_q^{d\bar{d}}(x))\mathbf{\overline{C}}\gamma_\nu]$$

$$\times \gamma_\nu\gamma_5 S_{q\bar{c}}^{d\bar{c}}(-x)\gamma_\mu\gamma_5$$

$$= \Pi_0^{D^0\Sigma^+}(x) + \Pi_G^{D^0\Sigma^+}(x),$$

(15)

In the above expressions, $\Pi^{D^0}(x)$ and $\Pi^{\Sigma^+}(x)$ are correlation functions of $D^0$ and $\Sigma^+_c$, respectively; $\Pi_0^{D^0\Sigma^+}(x) = \Pi^{D^0}(x) \times \Pi_0^{\Sigma^+}(x)$ is the leading term contributed by non-correlated $D^0$ and $\Sigma^+_c$, and $\Pi_G^{D^0\Sigma^+}(x)$ describes the double-gluon-exchange interaction between them. Their correspond-
Figure 4. Feynman diagrams corresponding to: (a) $\Pi_{\bar{D}^0}$, (b) $\Pi^{\Sigma^+}$, (c) $\Pi^0_{\bar{D}^0\Sigma^+}$, (d) $\Pi_{\bar{D}^0\Sigma^+}$, and (e) $\Pi_{Q\bar{D}^0\Sigma^+}$. $\Pi_{\bar{D}^0\Sigma^+}$ and $\Pi^{\Sigma^+}$ are correlation functions of the $\bar{D}^0$ meson and the $\Sigma^+$ baryon, respectively. The correlation function of the $\bar{D}^0\Sigma^+$ molecule satisfies $\Pi^{D^0\Sigma^+}(x) = \Pi^0_{D^0\Sigma^+}(x) + \Pi_{\bar{D}^0\Sigma^+}(x)$ with $\Pi^0_{D^0\Sigma^+}(x) = \Pi^{D^0}(x) \times \Pi^{\Sigma^+}(x)$. 

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\[ \langle 0 | \Pi \left[ \frac{1}{3} J^{0 \Sigma^+ c}(x) \right] \frac{1}{3} J^{0 \Sigma^+ c}(0) \rangle \rightarrow -\frac{1}{3} \Pi_{Q}^{0 \Sigma^+ c}(x), \]
\[ \langle 0 | \Pi \left[ \frac{2}{3} J^{0 \Sigma^- c}(x) \right] \frac{2}{3} J^{0 \Sigma^- c}(0) \rangle \rightarrow 0, \]
\[ -\langle 0 | \Pi \left[ \frac{2}{3} J^{0 \Sigma^+ c}(x) \right] \frac{2}{3} J^{0 \Sigma^- c}(0) \rangle \rightarrow \frac{2}{3} \Pi_{Q}^{0 \Sigma^+ c}(x), \]
\[ -\langle 0 | \Pi \left[ \frac{2}{3} J^{0 \Sigma^- c}(x) \right] \frac{2}{3} J^{0 \Sigma^+ c}(0) \rangle \rightarrow \frac{2}{3} \Pi_{Q}^{0 \Sigma^- c}(x). \]

(26)

2.4. \( I = 3/2 \, \overline{D} \Sigma^c_e \) correlation function

For completeness, we investigate the correlation function of the \( I = 3/2 \, \overline{D} \Sigma^c_e \) molecule in this subsection. Its corresponding interpolating current is

\[ J_{I=3/2}^{0 \Sigma^c_e}(x) = \frac{2}{3} J^{0 \Sigma^c_e}(x) + \frac{1}{3} J^{0 \Sigma^- c}(x), \]

with the correlation function to be

\[ \Pi_{I=3/2}^{0 \Sigma^c_e}(x) = \Pi_{0}^{0 \Sigma^c_e}(x) + \Pi_{G}^{0 \Sigma^c_e}(x) - 2 \Pi_{Q}^{0 \Sigma^c_e}(x). \]

In the above expression \( \Pi_{0}^{0 \Sigma^c_e}(x), \Pi_{G}^{0 \Sigma^c_e}(x), \) and \( \Pi_{Q}^{0 \Sigma^c_e}(x) \) are taken from the \( I = 1/2 \, \overline{D} \Sigma^c_e \) molecule.

Compared to equation (22), if the term \( \Pi_{Q}(x) \) induces an attractive interaction to the \( I = 1/2 \, \overline{D} \Sigma^c_e \) molecule, it would induce a repulsive interaction to the \( I = 3/2 \, \overline{D} \Sigma^c_e \) molecule; and vice versa.

3. QCD sum rule studies of \( D^- \Sigma^+_c \rightarrow \frac{1}{3} J^{0 \Sigma^c_e} \rightarrow \overline{D} \Sigma^c_e \) molecules

In this section we apply the method of QCD sum rules [55, 56] to investigate the light-quark-exchange term \( \Pi_{Q}(x) \), and study its contributions to the \( D^- \Sigma^+_c, \overline{D} \Sigma^+_c, I = 1/2 \, \overline{D} \Sigma^c_e, \) and \( I = 3/2 \, \overline{D} \Sigma^c_e \) hadronic molecular states.

In QCD sum rule analyses we consider the two-point correlation function in the momentum space:

\[ \Pi(q^2) = i \int d^4 x e^{i \beta x} \langle 0 | \Pi \left[ J(x) J(0) \right] | 0 \rangle, \]

(29)

where \( J(x) \) is an interpolating current. We generally assume it to be a composite current, coupling to the molecular state \( X \equiv |YZ\rangle \) through

\[ \langle Y \rangle |X\rangle = f_X u_X. \]

(30)

We write \( \Pi(q^2) \) in the form of dispersion relation as

\[ \Pi(q^2) = \int_{s_c}^{\infty} \frac{\rho(s)}{s - q^2 - i\epsilon} ds, \]

(31)

where \( s_c = 4m^2 \) is the physical threshold and \( \rho(s) \equiv \text{Im} \Pi(s)/\pi \) is the spectral density.

At the quark-gluon level we calculate \( \Pi(q^2) \) using the method of operator product expansion (OPE) up to certain order. According to equations (8), (15), (22), and (28), we further separate it into

\[ \Pi(q^2) \approx \Pi_{0}(q^2) + \Pi_{Q}(q^2), \]

(32)

and define their imaginary parts to be \( \rho_{0}(s) \) and \( \rho_{Q}(s) \). Here we have omitted the other term \( \Pi_{G}(q^2) \) since the light-quark-exchange term \( \Pi_{Q}(q^2) \) is much larger.

At the hadron level we evaluate the spectral density by inserting intermediate hadron states \( \sum |n\rangle \langle n| \) into

\[ \rho(s) = \sum_n \delta(s - M^2_n) \langle 0 | J^n | 0 \rangle = f_X^2 \delta(s - M^2_X) + \text{continuum}, \]

(33)

where we have adopted a parametrization of one pole dominance for the ground state \( X \) with a continuum contribution.

Given \( X \equiv |YZ\rangle \) to be a molecular state, its mass \( M_X \) can be expanded as

\[ M_X = M_F + M_L + \Delta M \equiv M_0 + \Delta M. \]

(34)

Then we insert equation (33) into (31), and expand it as

\[ \Pi(q^2) = \frac{f_X^2}{M_X^2 - q^2} + \cdots \]

\[ \approx \frac{f_X^2}{M_0^2 - q^2} - \frac{2M_0f_X^2}{(M_0^2 - q^2)^2} \Delta M + \cdots. \]

(35)

The former term is contributed by the non-correlated \( Y \) and \( Z \), and the latter term is contributed by their interactions. Compared to equation (32), we further obtain:

\[ \Pi_{0}(q^2) = \frac{f_X^2}{M_0^2 - q^2} + \cdots, \]

(36)

\[ \Pi_{Q}(q^2) = - \frac{2M_0f_X^2}{(M_0^2 - q^2)^2} \Delta M + \cdots. \]

(37)

We perform the Borel transformation to the above correlation functions at both hadron and quark-gluon levels. After assuming contributions from the continuum to be approximated by the OPE spectral densities \( \rho_{0}(s) \) and \( \rho_{Q}(s) \) above a threshold value \( s_0 \), we arrive at two sum rule equations:

\[ f_X^2 e^{-M_0^2/M^2} = \rho_{0}(M_0^2, s_0) = \int_{s_c}^{s_0} e^{-s/M^2} \rho_{0}(s) ds, \]

(38)

\[ - \frac{2M_0f_X^2}{M_0^2} \Delta M e^{-M_0^2/M^2} = \Pi_{Q}(M_0^2, s_0) = \int_{s_c}^{s_0} e^{-s/M^2} \rho_{Q}(s) ds. \]

(39)

There are two free parameters in equations (38) and (39): the Borel mass \( M_0 \) and the threshold value \( s_0 \). Differentiating equation (38) with respect to \( 1/M^2_0 \), we obtain
Given $M_0 = M_Y + M_Z$, this equation can be used to relate $M_B$ and $s_0$, so that there is only one free parameter left.

Dividing equation (39) by (38), we obtain

$$\frac{-2M_0}{M_B} \Delta M = \frac{\Pi_Q}{\Pi_0} = \int_{s_c}^{s_b} e^{-s/\bar{M}_0} \rho_Q(s) ds.$$  \hspace{1cm} (40)

This equation can be used to calculate $\Delta M$.

We shall use equation (41) to study contributions of the term $\Pi_Q(s)$ to the $D_2 \Sigma_1^{-+}$, $D_2 \Sigma_2^{-+}$, and $I = 3/2 D_2 \Sigma_1^{-+}$ hadronic molecular states, separately in the following subsections. Before doing this, we note that $\Delta M$ is actually not the binding energy, but relates to some potential $V(r)$ between $Y$ and $Z$ induced by exchanged/shared light quarks. We use the $I = 1/2 D_2 \Sigma_1^{-+}$ hadronic molecular state as an example to qualitatively discuss this. After transforming the local current $J^{\Sigma_1^{-+}}(x)$ defined in equation (21) into its non-local form:

$$J^{\Sigma_1^{-+}}(x, y) = \frac{1}{3} J^{\Sigma_1^{-+}}(x, y) - \frac{2}{3} J^{\Sigma_1^{-+}}(x, y) - \frac{2}{3} J^{\Sigma_1^{-+}}(y),$$  \hspace{1cm} (42)

we calculate its non-local correlation function in the coordinate space to be:

$$\Pi^{\Sigma_1^{-+}}(x, y, x', y') \equiv \langle 0 | J^{\Sigma_1^{-+}}(x, y) J^{\Sigma_1^{-+}}(x', y') | 0 \rangle = -\text{Tr} [iS_{\mu \nu}^{dd}(\Delta x) \gamma_\mu iS_{\nu}^{dd}(\Delta x) \gamma_\nu]$$

$$\times e^{abc} e^{d'b'c'} \text{Tr} [iS_{d'}^{bb'}(\Delta x) \gamma_\mu C(iS_{b'}^{ad'})(\Delta x)^2 C_\mu^\nu]$$

$$\times \gamma_\alpha \gamma_5 iS_{\alpha}^{c'}(\Delta x) \gamma_\mu \gamma_5$$

$$-\text{Tr} [iS_{\mu \nu}^{dd}((\Delta x - r) \gamma_\mu C(iS_{\nu}^{bb'})(\Delta x)^2 C_\mu^\nu) iS_{\mu \nu}^{dd}((\Delta x - r) \gamma_\mu^\nu]$$

$$\times \gamma_\alpha \gamma_5 iS_{\alpha}^{c'}(\Delta x) \gamma_\mu \gamma_5$$

$$= \Pi_0^{\Sigma_1^{-+}}(\Delta x) + \Pi_G^{\Sigma_1^{-+}}(\Delta x, r) + \Pi_Q^{\Sigma_1^{-+}}(\Delta x, r).$$  \hspace{1cm} (43)

In the above expression we have assumed that $y - y' = x - x' = \Delta x$ and $y = x = y' = x' = r$. The leading term $\Pi_0^{\Sigma_1^{-+}}(\Delta x)$ does not change with the parameter $r$, while the light-quark-exchange term $\Pi_Q^{\Sigma_1^{-+}}(\Delta x, r)$ decreases as $\frac{r}{r} \rightarrow \infty$. Accordingly, we arrive at:

- Because we are using local currents in QCD sum rule analyses,

$$V(|r|) = 0 = \Delta M.$$  \hspace{1cm} (44)

Because the term $\Pi_Q(s)$ is color-unconfined, its contribution decreases as $r$ increases:

$$V(|r| \rightarrow \infty) \rightarrow 0.$$  \hspace{1cm} (45)

We may build a model and use the light-quark-exchange potential $V(r)$ to derive the binding energy of $X$, but this will not be done in the present study. Other than this, we shall calculate $\Delta M$ and qualitatively study several hadronic molecules possibly bound by this potential, through which we shall propose a model-independent hypothesis for such molecules.

The binding mechanism induced by the light-quark-exchange potential $V(r)$ is somewhat similar to the covalent bond in chemical molecules induced by shared electrons, so we call such hadronic molecules 'covalent hadronic molecules'.

3.1. $I = 1/2 D_2 \Sigma_1^{-+}$ sum rules

In this subsection we apply QCD sum rules to study the $I = 1/2 D_2 \Sigma_1^{-+}$ molecular. We have calculated its correlation function at the leading order of $\alpha_s$ and up to the $D$ (dimension) = 10 terms, including the perturbative term, the quark condensate $\langle \bar{q} q \rangle$, the gluon condensate $\langle g G g \rangle$, the quark-gluon mixed condensate $\langle g \bar{q} g \rangle$, and their combinations $\langle \bar{q} \bar{q} \rangle^3$, $\langle \bar{q} \bar{q} \rangle \langle g \bar{q} g \rangle$, $\langle \bar{q} \bar{q} \rangle \langle g \bar{q} g \rangle^2$. The extracted spectral density

$$\rho^{\Sigma_1^{-+}} = \rho_0^{\Sigma_1^{-+}} + \rho_0^{\Sigma_1^{-+}},$$  \hspace{1cm} (46)

is given in appendix A.

To perform numerical analyses, we use the following values for various QCD sum rule parameters in the present study [2, 57–65]:

$$m_s = 961.4 \pm 0.5 \text{ MeV},$$

$$m_c = 1.275 \pm 0.025 \text{ GeV},$$

$$\langle \bar{q} q \rangle = (0.240 \pm 0.010)^3 \text{ GeV}^3,$$

$$\langle \bar{s} s \rangle = (0.8 \pm 0.1) \times \langle \bar{q} q \rangle,$$

$$\langle g g G G \rangle = 0.48 \pm 0.14 \text{ GeV}^4,$$

$$\langle g \bar{q} g \rangle = -\frac{M_0^2}{m_c^2} \times \langle \bar{q} q \rangle,$$

$$\langle g \bar{q} g \rangle^2 = -M_0^2 \times \langle \bar{s} s \rangle,$$

$$M_0^2 = 0.8 \pm 0.2 \text{ GeV}^2.$$  \hspace{1cm} (47)

where the running mass in the $\overline{MS}$ scheme is used for the charm quark.

There are two free parameters in equations (38) and (39): the Borel mass $M_B$ and the threshold value $s_0$. We use equation (40) to constrain them by setting [2]:

$$M_0^{\Sigma_1^{-+}} = \frac{1}{3} (M_{B^0} + M_{B^+}) + \frac{2}{3} (M_{D^+} + M_{D^{*+}}) = 4321.66 \text{ MeV}. $$  \hspace{1cm} (48)

The derived relation between $M_B$ and $s_0$ is depicted in figure 5, which will be used in the following calculations.
There are two criteria to constrain the Borel mass $M_B$. The first criterion is to ensure the convergence of OPE series, by requiring the $D = 10$ terms $m_q(q^4)$ and $(g_5 q^2 G q^2)$ to be less than 15%:

\[ \text{Convergence} \equiv \left| \frac{\Pi^{D=10}(\infty, M_B)}{\Pi(\infty, M_B)} \right| \leq 15\%. \quad (49) \]

This criterion determines the lower limit of $M_B$. As shown in figure 6 using the solid curve, we find it to be $(M_B^{\text{min}})^2 = 3.11 \text{ GeV}^2$.

The second criterion is to ensure the validity of one-pole parametrization, by requiring the pole contribution to be larger than 40%:

\[ \text{Pole- Contribution} \equiv \frac{\Pi(s_0, M_B)}{\Pi(\infty, M_B)} \geq 40\%. \quad (50) \]

This determines the upper limit of $M_B$. As shown in figure 6 using the dashed curve, we find it to be $(M_B^{\text{max}})^2 = 3.48 \text{ GeV}^2$.

Altogether we extract the working region of $M_B$ to be $3.11 \text{ GeV}^2 < M_B^2 < 3.48 \text{ GeV}^2$, where we use equation (41) to calculate the mass correction to be

\[ \Delta M_{J=1/2, J=1/2}^{D\Sigma^+} = -95 \text{ MeV}. \quad (51) \]

We show its variation in figure 7 with respect to the Borel mass $M_B$. It is shown in a broader region $3.0 \text{ GeV}^2 < M_B^2 < 4.0 \text{ GeV}^2$, and we find it quite stable inside the above Borel window.

The mass correction $\Delta M_{J=1/2, J=1/2}^{D\Sigma^+}$ given in equation (51) is negative, suggesting the light-quark-exchange potential $V_{J=1/2, J=1/2}(r)$ to be attractive, so there can be the $D\Sigma^+_c$ covalent molecule of $I=1/2$ and $J=1/2$.

### 3.2. $D^- \Sigma^+_c$ sum rules

As shown in equation (8), the light-quark-exchange term $\Pi^{D\Sigma^+}(x)$ does not contribute to the correlation function of the $D^- \Sigma^+_c$ molecule, so the $D^- \Sigma^+_c$ covalent molecule does not exist.

### 3.3. $D^0 \Sigma^+_c$ sum rules

We follow section 3.1 to study the $D^0 \Sigma^+_c$ molecule, and calculate its mass correction to be

\[ \Delta M_{J=1/2, J=1/2}^{D^0 \Sigma^+_c} = -\Delta M_{J=1/2, J=1/2}^{D^0 \Sigma^-} = 95 \text{ MeV}. \quad (52) \]

This result suggests $V_{J=1/2, J=1/2}(r)$ to be repulsive, so the $D^0 \Sigma^+_c$ covalent molecule does not exist.

### 3.4. $I = 3/2$ $D \Sigma_c$ sum rules

We follow section 3.1 to study the $I=3/2$ $D \Sigma_c$ molecule, and calculate its mass correction to be

\[ \Delta M_{I=3/2, J=1/2}^{D \Sigma^-} = -2\Delta M_{I=1/2, J=1/2}^{D \Sigma^-} = 190 \text{ MeV}. \quad (53) \]

This result suggests $V_{I=3/2, J=1/2}(r)$ to be repulsive, so the $D \Sigma^-_c$ covalent molecule of $I=3/2$ and $J=1/2$ does not exist.

### 4. More hadronic molecules

In this section we follow the procedures used in section 3.1 and study more possibly-existing covalent hadronic molecules. We shall investigate the $D^{(*)}\Sigma_c$, $D^{(*)}\Sigma^-_c$, $D^{(*)}\Sigma^*_c$, $D^{(*)}\Lambda_c$, $D^{(*)}K^*$, $D^{(*)}K^{*0}$, $D^{(*)}D^{(*)}$ molecules, separately in the following subsections.
4.1. $D^*\Sigma_c^*/D^*\Sigma_c$ molecules

In this subsection we investigate the light-quark-exchange term $\Pi_0(q)$ and study its contributions to the $D^*\Sigma_c^*/D^*\Sigma_c$ molecules.

The current corresponding to the $D^*\Sigma_c$ molecule is

$$J_{\alpha}^{D^*\Sigma_c}(x) = J_0^{D^*}(x) \times J_2^{\Sigma_c}(x)$$

$$= [e_\alpha q_2^T(x) C_\gamma^\nu q_2(x)] \gamma_\mu \gamma_5 \epsilon_\nu(x).$$

(54)

Its correlation function is

$$\Pi_{\alpha \beta}^{D^*\Sigma_c}(q^2) = i \int d^4xe^{iqx} \langle 0 | T[J_{\alpha}^{D^*\Sigma_c}(x) J_0^{D^*}(0)] | 0 \rangle$$

$$= g_{\alpha \beta}^{D^*\Sigma_c}(q^2) \Pi_{J=3/2}^{D^*\Sigma_c}(q^2) + g_{\alpha \beta}^{1/2}(q^2) \Pi_{J=1/2}^{D^*\Sigma_c}(q^2),$$

(55)

where $\Pi_{J=3/2}^{D^*\Sigma_c}(q^2)$ and $\Pi_{J=1/2}^{D^*\Sigma_c}(q^2)$ are contributed by the spin-3/2 and spin-1/2 components, respectively. $g_{\alpha \beta}^{D^*\Sigma_c}(q^2)$ and $g_{\alpha \beta}^{1/2}(q^2)$ are coefficients of the spin-3/2 and spin-1/2 propagators, respectively:

$$g_{\alpha \beta}^{D^*\Sigma_c}(q^2) = \left( g_{\alpha \beta} - \frac{q_\alpha q_\beta}{3M} - \frac{2q_\alpha q_\beta}{3M^2} \right) \times (q^2 + M),$$

$$g_{\alpha \beta}^{1/2}(q^2) = q_\alpha q_\beta \times (q^2 + M).$$

(56)

(57)

In the present study we have only calculated the terms proportional to the Lorentz coefficient $g_{\alpha \beta}$, so we can only extract the mass correction to the spin-3/2 $D^*\Sigma_c$ molecule. We find it negative for the $I = 1/2$ one:

$$\Delta M_{I=1/2, J=3/2}^{D^*\Sigma_c} = -89 \text{ MeV}.$$  

(58)

This result suggests $V_{I=1/2, J=3/2}(r)$ to be attractive, so there can be the $D^*\Sigma_c$ covariant molecule of $I = 1/2$ and $J = 3/2$.

Similarly, we use the current

$$J_\alpha^{D^*\Sigma_c}(x) = J_0^{D^*}(x) \times J_2^{\Sigma_c}(x)$$

$$= [\bar{c}_\alpha(x) \gamma_\mu q_2(x)] \times [e^{abc} q_a^T(x) C_\gamma^\nu q_b(x) P_{0\mu\nu}(x) \epsilon_c(x)],$$

(59)

(60)

(61)

(62)

Its correlation function is

$$\Pi_{\alpha \beta}^{D^*\Sigma_c}(q^2) = i \int d^4xe^{iqx} \langle 0 | T[J_{\alpha}^{D^*\Sigma_c}(x) J_0^{D^*}(0)] | 0 \rangle$$

$$= g_{\alpha \beta}^{D^*\Sigma_c}(q^2) \Pi_{J=3/2}^{D^*\Sigma_c}(q^2) + g_{\alpha \beta}^{1/2}(q^2) \Pi_{J=1/2}^{D^*\Sigma_c}(q^2),$$

(63)

where $\Pi_{J=3/2}^{D^*\Sigma_c}(q^2)$, $\Pi_{J=1/2}^{D^*\Sigma_c}(q^2)$, and $\Pi_{J=1/2}^{D^*\Sigma_c}(q^2)$ are contributed by the spin-5/2, spin-3/2, and spin-1/2 components, respectively. $g_{\alpha \beta}^{D^*\Sigma_c}(q^2)$ and $g_{\alpha \beta}^{1/2}(q^2)$ are coefficients of the spin-5/2, spin-3/2, and spin-1/2 propagators, respectively:

$$g_{\alpha \beta}^{D^*\Sigma_c}(q^2) = (q^2 + M) \times \left( g_{\alpha \beta}^{2} + \frac{g_{\alpha \beta}^{2} g_{\alpha \beta}^{2}}{2} - \frac{g_{\alpha \beta}^{2} g_{\alpha \beta}^{2}}{4} \right),$$

$$g_{\alpha \beta}^{1/2}(q^2) = (q^2 + M) \times g_{\alpha \beta}^{2}. $$

(64)

(65)

(66)

In the present study we have calculated the terms proportional to the three Lorentz coefficients $g_{\alpha \beta}^{2}$, $g_{\alpha \beta}^{3}$, and $g_{\alpha \beta}^{4}$, so we can extract mass corrections to all the three $I = 1/2$ $D^*\Sigma_c$ molecules of $J = 5/2$, $J = 3/2$, and $J = 1/2$:

$$\Delta M_{I=1/2, J=5/2}^{D^*\Sigma_c} = -107 \text{ MeV},$$

$$\Delta M_{I=1/2, J=3/2}^{D^*\Sigma_c} = -47 \text{ MeV},$$

$$\Delta M_{I=1/2, J=1/2}^{D^*\Sigma_c} = 3.5 \text{ MeV}. $$

(67)

(68)

(69)

These results suggest $V_{I=1/2, J=5/2}(r)$ to be attractive, so there can be the $D^*\Sigma_c$ molecule of $I = 1/2$ and $J = 5/2$.

For completeness, we show variations of $\Delta M_{I=1/2}^{D^*\Sigma_c}$ in figure 8 with respect to the Borel mass $M_b$. Besides, we study the $I = 3/2$ $D^*\Sigma_c/D^*\Sigma_c$ molecules and obtain

$$\Delta M_{I=3/2}^{D^*\Sigma_c} \approx -2 \Delta M_{I=1/2}^{D^*\Sigma_c}. $$

(70)

4.2. $D^*\Lambda_c$ molecules

In this subsection we investigate the light-quark-exchange term $\Pi_0(q)$ and study its contributions to the $D^*\Lambda_c$ molecules.

The currents corresponding to the $D\Lambda_c$ and $D^*\Lambda_c$ molecules are

$$J_{\alpha}^{D\Lambda_c}(x) = J_0^{D}(x) \times J_1^{\Lambda_c}(x)$$

$$= [\bar{c}_\alpha(x) \gamma_\mu q_2(x)] \times [e^{abc} q_a^T(x) C_\gamma^\nu q_b(x) P_{0\mu\nu}(x) \epsilon_c(x)],$$

(71)

$$J_{\alpha}^{D^*\Lambda_c}(x) = J_0^{D^*}(x) \times J_1^{\Lambda_c}(x)$$

$$= [\bar{c}_\alpha(x) \gamma_\mu q_2(x)] \times [e^{abc} q_a^T(x) C_\gamma^\nu q_b(x) P_{0\mu\nu}(x) \epsilon_c(x)].$$

(72)
We use them to perform QCD sum rule analyses, and calculate mass corrections to the \( D^{*} \Lambda_c \) molecule of \( J = 1/2 \) and the \( D^{*} \Lambda_c \) molecule of \( J = 3/2 \). We find both of them to be positive. However, we do not obtain the mass correction to the \( D^{*} \Lambda_c \) molecule of \( J = 1/2 \), just like the \( D^{*} \Sigma_c \) molecule of \( J = 1/2 \). These results suggest \( V_{D^{*} \Lambda_c}^{1/2,1/2} \) and \( V_{D^{*} \Lambda_c}^{1/2,3/2} \) to be both repulsive, so the \( J = 1/2 \) \( D \Lambda_c \) and \( J = 3/2 \) \( D^{*} \Lambda_c \) covariant molecules do not exist.

4.3. \( D^{(*)} K^* \) molecules

In this subsection we investigate the light-quark-exchange term \( \Pi_{Q}(x) \) and study its contributions to the \( D^{(*)} K^* \) molecules. We do not investigate the \( D^{(*)} K^* \) molecules in the present study, because their bare masses \( M_{D^{(*)}} + M_K \) cannot be easily reached within the present QCD sum rule approach, due to the nature of \( K \) mesons as Nambu–Goldstone bosons.

The current corresponding to the \( D \bar{K}^* \) molecule is

\[
J_{D^{K*}}^{D^*}(x) = J_{D^*}^{D^*}(x) \times J_{K^*}^{D^*}(x) = [\bar{q}_d(x)\gamma_\alpha c_s(x)] \times [\bar{q}_b(x)\gamma_\alpha s_b(x)].
\]

Its correlation function is

\[
\Pi_{D^{K*}}^{D^*}(q^2) = i \int d^4x e^{i\xi x} \langle 0|\{J_{D^{K*}}^{D^*}(x)\overline{J}_{D^{K*}}^{D^*}(0)\}|0\rangle = \left( g_{\alpha\beta} - \frac{g_{\alpha\beta} q^2}{q^2} \right) \Pi_{D^{K*}}^{D^*}(q^2) + \cdots.
\]

We use \( J_{D^{K*}}^{D^*}(x) \) to perform QCD sum rule analyses, and calculate its mass correction to be

\[
\Delta M_{D^{K*}}^{D^*} = -180 \text{ MeV}.
\]

This suggests \( V_{D^{K*}}^{D^*}(r) \) to be attractive, so there can be the \( D \bar{K}^* \) covariant molecule of \( I = 0 \) and \( J = 1 \).

The current corresponding to the \( D^* K^* \) molecule is

\[
J_{D^{K*}}^{D^*}(x) = J_{D^*}^{D^*}(x) \times J_{K^*}^{D^*}(x) = [\bar{q}_d(x)\gamma_\alpha c_s(x)] \times [\bar{q}_b(x)\gamma_\alpha s_b(x)].
\]

Its correlation function is

\[
\Pi_{D^{K*}}^{D^*}(q^2) = i \int d^4x e^{i\xi x} \langle 0|\{J_{D^{K*}}^{D^*}(x)\overline{J}_{D^{K*}}^{D^*}(0)\}|0\rangle = \left( g_{\alpha\beta} - \frac{g_{\alpha\beta} q^2}{q^2} \right) \Pi_{D^{K*}}^{D^*}(q^2) + \cdots,
\]

where \( \Pi_{D^{K*}}^{D^*}(q^2) \), \( \Pi_{D^{K*}}^{D^*}(q^2) \), and \( \Pi_{D^{K*}}^{D^*}(q^2) \) are contributed by the spin-2, spin-1, and spin-0 components, respectively.

\[
\Pi_{D^{K*}}^{D^*}(q^2) = \frac{g_{\alpha\beta} g_{\alpha\beta}}{2}, \quad \Pi_{D^{K*}}^{D^*}(q^2) = \frac{g_{\alpha\beta} g_{\alpha\beta}}{4}, \quad \Pi_{D^{K*}}^{D^*}(q^2) = \frac{g_{\alpha\beta} g_{\alpha\beta}}{2}.
\]

We use \( J_{D^{K*}}^{D^*}(x) \) to perform QCD sum rule analyses, and extract mass corrections to the three isoscalar \( D^* K^* \) molecules of \( J = 2, J = 1, \) and \( J = 0 \):

\[
\Delta M_{D^{K*}}^{D^*} = -119 \text{ MeV},
\]

\[
\Delta M_{D^{K*}}^{D^*} = -46 \text{ MeV},
\]

\[
\Delta M_{D^{K*}}^{D^*} = -4.5 \text{ MeV}.
\]

These results suggest \( V_{D^{K*}}^{D^*}(r) \) to be attractive, so there can be the \( D^* K^* \) covariant molecule of \( I = 0 \) and \( J = 2 \).

We show variations of \( \Delta M_{D^{K*}}^{D^*} \) in figure 9 with respect to the Borel mass \( M_B \). Especially, \( V_{D^{K*}}^{D^*}(r) \) depends significantly on the Borel mass \( M_B \) and so also on the threshold value \( s_\alpha \) for which we refer to section 5 for relevant model-independent discussions.

For completeness, we study the isovector \( D^{(*)} K^* \) molecules and obtained

\[
\Delta M_{D^{K*}}^{D^*}/D^{K*} \approx -\Delta M_{D^{K*}}^{D^*}/D^{K*}.
\]

4.4. \( D^{(*)} \bar{D}^{(*)} \) molecules

The light-quark-exchange term \( \Pi_{Q}(x) \) does not contribute to the correlation functions of the \( D^{(*)} \bar{D}^{(*)} \) molecules, suggesting the \( D^{(*)} \bar{D}^{(*)} \) covariant molecules not to exist. However, there can still be the \( D^{(*)} \bar{D}^{(*)} \) hadronic molecular states induced by some other binding mechanisms, such as the one-meson-exchange interaction [23–32].
5. Covalent hadronic molecule

In the previous section we applied QCD sum rules to study the binding mechanism induced by shared light quarks. This mechanism is somewhat similar to the covalent bond in chemical molecules induced by shared electrons, so we call such hadronic molecules ‘covalent hadronic molecules’. Recalling that the two shared electrons must spin in opposite directions (and so totally antisymmetric obeying the Pauli principle) in order to form a chemical covalent bond, our QCD sum rule results indicate a similar behavior: the light-quark-exchange interaction is attractive when the shared light quarks have the same color and so theavor structure is totally antisymmetric so that obey the Pauli principle (these quarks may spin in the same direction given their flavor structure capable of being antisymmetric).

In this section we qualitatively study the above hypothesis. Its logical chain is quite straightforward. We assume the two light quarks \( q_A \) inside \( Y \) and \( q_B \) inside \( Z \) are totally antisymmetric. Hence, \( q_A \) and \( q_B \) obey the Pauli principle, so that they can be exchanged and shared. By doing this, wave functions of \( Y \) and \( Z \) overlap with each other, so that they are attracted and there can be the covalent hadronic molecule \( X = |YZ⟩ \). This picture has been depicted in figure 2. We believe it is better and more important than our QCD sum rule results, given it is model-independent and more easily applicable.

We apply it to study several examples as follows. The two exchanged light quarks have the same color and so the symmetric color structure; besides, we assume their orbital structure to be \( S \)-wave and so also symmetric; consequently, we only need to investigate their spin and flavor structures.

a. \( D^{*+}q_1q_2-Σ^+_c|u_dΣ^+_c⟩ \) covalent molecules. Let us exchange \( u_2 \) inside \( D^{*+} \) and \( u_3 \) inside \( Σ^+_c \). \( u_2 \) and \( d_3 \) also need to spin in opposite directions in order to form another \( Λ^+_c \), so \( u_2 \) and \( u_3 \) spin in the same direction with the symmetric spin structure.

b. \( D^0[q_1q_2-Σ^+_c|u_dΣ^+_c⟩ \) covalent molecule. Let us exchange \( u_2 \) inside \( D^0 \) and \( u_3 \) inside \( Σ^+_c \). \( u_1 \) and \( u_2 \) need to spin in opposite directions in order to form another \( D^0 \), so \( u_2 \) and \( u_3 \) spin in the same direction with the symmetric spin structure. The flavor structure of \( u_2 \) and \( u_3 \) is also symmetric, so they are totally symmetric (\( S = \) symmetric and \( A = \) antisymmetric):

| color | flavor | spin | orbital | total |
|-------|--------|------|---------|-------|
| \( u_2 \leftrightarrow u_3 \) | S | S | S | S | S |

Accordingly, the \( D^0Σ^+_c \) covalent molecules do not exist. This is consistent with our QCD sum rule result obtained in section 4.2.

c. \( D[q_1q_2-Σ^+_c|q_dΣ^+_c⟩ \) covalent molecule (\( q = u/d \)). After including the isospin symmetry, the exchange can take place between up and down quarks. Let us exchange \( q_2 \) inside \( D \) and \( q_3 \) inside \( Σ_c \). As discussed above, they have the symmetric spin structure, so they can be totally antisymmetric as long as their flavor structure is antisymmetric:

| color | flavor | spin | orbital | total |
|-------|--------|------|---------|-------|
| \( q_2 \leftrightarrow q_3 \) | S | A | S | S | A |

Accordingly, there can be the \( I = 1/2 \) \( DΣ_c \) covalent molecule, but not the \( I = 3/2 \) one. This is consistent with our QCD sum rule result obtained in sections 3.1 and 3.4. Similarly, we derive that there can be the \( I = 1/2 \) \( DΣ^+_c \) and \( I = 0 \) \( DK^+ \) covalent molecules, consistent with sections 4.1 and 4.3.

d. \( J = 5/2 \) \( D^{*}[q_1q_2-Σ^+_c|q_dΣ^+_c⟩ \) covalent molecule. Let us exchange \( q_2 \) inside \( D^* \) and \( q_3 \) inside \( Σ^+_c \). In the \( J_c = +5/2 \) component, all the quarks/antiquark spin in the same direction, so \( q_2 \) and \( q_3 \) have the symmetric spin structure. They can be totally antisymmetric as long as their flavor structure is antisymmetric:

| color | flavor | spin | orbital | total |
|-------|--------|------|---------|-------|
| \( q_2 \leftrightarrow q_3 \) | S | A | S | S | A |

Accordingly, there can be the \( I = 1/2 \) \( D^*Σ^+_c \) covalent molecule of \( J = 5/2 \), but not the \( I = 3/2 \) one. This is consistent with our QCD sum rule result obtained in section 4.1. Similarly, we derive that there can be the \( D^*K^+ \) covalent molecule of \( I = 0 \) and \( J = 2 \), consistent with section 4.3.
e. $J = 0$ $D^*([\bar{c} ]q_2) - K^*[s]q_3$ covalent molecule. Let us exchange $q_3$ inside $D^*$ and $q_1$ inside $K^*$. We perform the spin decomposition (see appendix B):

$$|l_{cq} \otimes l_{dq}; J = 0\rangle = \frac{\sqrt{3}}{2} |0 s \otimes 0 q_s; J = 0\rangle - \frac{1}{2} |1 s \otimes 1 q_s; J = 0\rangle.$$  

(85)

Hence, there exists both $s_{\bar{q},q_3} = 0$ (75%) and $s_{q_1,q_3} = 1$ (25%) components. The former becomes attractive when $I = 1$, and the latter becomes attractive when $I = 0$.

| color | flavor | spin | orbital | total |
|-------|--------|------|---------|-------|
| $q_2 \leftrightarrow \bar{q}_4$ | S | S | A | S | A |
| $q_3 \leftrightarrow q_4$ | S | A | S | S | A |

It is well known that there are both the para-hydrogen and ortho-hydrogen, where the two protons spin in opposite directions and in the same direction, respectively. Similarly, there might be two $J = 0$ $D^*K^*$ covalent molecules, i.e. the two components $|0 s \otimes 0 q_s; J = 0\rangle$ of $I = 1$ and $|1 s \otimes 1 q_s; J = 0\rangle$ of $I = 0$. However, our QCD sum rule studies performed in section 4.3 cannot differentiate these two hyperfine structures, because there we have summed over the $D^*$ and $K^*$ polarizations.

It is useful to generally discuss how many light quarks at most are there in the lowest orbit ($q = u/d$):

1. In the $\bar{D}^*[\bar{c}]q_2K^*[s]q_3$ covalent molecule, the two exchanged light quarks $q_2$ and $q_4$ have the same color and so the symmetric color structure; besides, we assume their orbital structure to be S-wave and so also symmetric; consequently, there are two possible configurations satisfying the Pauli principle ($S =$ symmetric and $A =$ antisymmetric):

| color | flavor | spin | orbital | total |
|-------|--------|------|---------|-------|
| $q_2 \leftrightarrow q_4$ | S | A | S | S | A |
| $q_3 \leftrightarrow q_4$ | S | A | S | S | A |

Hence, two antiquarks can share (at most) two quarks, with the quantum numbers either $(I)J^P = (0)1^+$ or $(1)0^+$. 2. In the $\bar{D}^*[\bar{c}]q_2\Sigma^*(3940)[q_3q_4c_3]$ covalent molecule, there is three light up/down quarks. We assume the two exchanged quarks to be $q_2$ and $q_3$ with the same color.

There are also two possible configurations:

| color | flavor | spin | orbital | total |
|-------|--------|------|---------|-------|
| $q_2 \leftrightarrow q_3$ | S | A | S | S | A |
| $q_2 \leftrightarrow q_4$ | A | S | S | S | A |
| $q_3 \leftrightarrow q_4$ | A | S | S | S | A |

They satisfy the condition that any two of the three light quarks are totally antisymmetric so they obey the Pauli principle. Hence, one quark and one antiquark can share (at most) three quarks, with either $(I)J^P = (1)\frac{1}{2}1^+$ or $(3)\frac{1}{2}1^+$. However, the $D^*\Lambda_b$ system does not satisfy this condition.

3. In the $\Sigma^*(3940)[q_2q_4c_4] - \Sigma^*(3940)[q_3q_4c_3]$ covalent molecule, there is four light up/down quarks. We assume that $q_1$ and $q_4$ can be exchanged with the same color, and $q_2$ and $q_3$ also be exchanged with the same color. There can be four possible configurations:

| color | flavor | spin | orbital | total |
|-------|--------|------|---------|-------|
| $q_1 \leftrightarrow q_4$ | S | A | S | S | A |
| $q_2 \leftrightarrow q_4$ | S | A | S | S | A |
| $q_3 \leftrightarrow q_4$ | S | A | S | S | A |

They satisfy that any two of the four light quarks are totally antisymmetric so they obey the Pauli principle. Hence, two quarks can share (at most) four quarks, with $(I)J^P = (0)2^+/(1)1^+/(2)0^+$. However, neither $\Sigma^*(3940)\Lambda_b$ nor $\Lambda_b\Lambda_b$ satisfies this condition.

We apply the above model-independent hypothesis to qualitatively predict more covalent hadronic molecules:

- Induced by shared light up/down quarks, there can be the $I = 0 \bar{D}^*[\bar{c}]q_2 - B^*[s]q_3q_4$, $I = 0 \bar{D}^*[\bar{c}]q_2 - \Xi^*_b[s]q_3q_4$, $I = 0 \bar{D}^*[\bar{c}]q_2 - \Xi^*_b[s]q_3q_4$, and $I = 12 \bar{D}^*[\bar{c}]q_2 - \Xi^*_b[s]q_3q_4$ covalent molecules, etc. We roughly estimate their binding energies to be at the 10 MeV level, considering the $P_c/P_{cs}$ and $X(2900)$ [66, 67] as possible covalent hadronic molecules. We list them in table 1, which are still waiting to be carefully analysed.

- If the strange quark is also exchangeable, there can be the $\bar{D}^*[\bar{c}]q_2 - B^*[s]q_3q_4$, $\bar{D}^*[\bar{c}]q_2 - \Xi^*_b[s]q_3q_4$, etc. These states are induced by shared light up/down strange quarks, with the $SU(3)$ light flavor structure being antisymmetric.

- If the heavy-quark-exchange interaction is negligible, there might be the $I = 0 \bar{D}^*[\bar{c}]q_2 - D^*[s]q_3q_4$, $I = 0 \bar{D}^*[\bar{c}]q_2 - \Xi^*_b[s]q_3q_4$, and $I = 12 \bar{D}^*[\bar{c}]q_2 - \Xi^*_b[s]q_3q_4$ covalent molecules, etc. These states are still induced by shared light up/down quarks.
Table 1. Possibly-existing covalent hadronic molecules $|IJ\rangle^P$ induced by shared light up/down quarks, derived from the hypothesis that the light-quark-exchange interaction is attractive when the shared light quarks are totally antisymmetric so that they obey the Pauli principle. Here, $q$ and $s$ denote the light up/down and strange quarks, respectively; $Q$ and $Q'$ denote two different heavy quarks; the symbols $[AB] = AB - BA$ and $[AB] = AB + BA$ denote the antisymmetric and symmetric SU(3) light flavor structures, respectively. The states with $✓$ have been confirmed in QCD sum rule calculations of the present study, but the states with ? and ??? have not. This is because the latter contain relatively-polarized components, while one needs to sum over polarizations of these components within QCD sum rule method and so can not differentiate these hyperfine structures. Moreover, the states with ??? contain light up/down quarks with the symmetric flavor/isospin structure, whose masses are (probably) considerably larger than their partners with the antisymmetric flavor/isospin structure. The states without any identification are still waiting to be carefully analysed in our future QCD sum rule studies.

| $Qq$, $1^0_2$ | $\tilde{Q}q$, $1^0_2$ | $Q(|qq\rangle, 0^+_2)$ | $Q(|qq\rangle, 1^+_2)$ | $Q(|qq\rangle, 1^3_2)$ | $Q(|sq\rangle, 1^1_2)$ | $Q(|sq\rangle, 1^1_2)$ | $Q(|sq\rangle, 1^3_2)$ |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $(0)^0_2$      | $(0)^1_2$ (? ) | $(0)^0_2$      | $(0)^1_2$      | $(0)^3_2$      | $(0)^1_2$      | $(0)^1_2$      | $(0)^3_2$      |
| $(1)^0_2$      | $(1)^1_2$      | $(1)^0_2$      | $(1)^1_2$      | $(1)^3_2$      | $(1)^1_2$      | $(1)^1_2$      | $(1)^3_2$      |
| $(0)^2_2$      | $(0)^3_2$      | $(0)^2_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      |
| $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      |
| $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      |

| $Q(|qq\rangle, 0^+_2)$ | $Q(|qq\rangle, 1^+_2)$ | $Q(|qq\rangle, 1^3_2)$ | $Q(|sq\rangle, 1^1_2)$ | $Q(|sq\rangle, 1^1_2)$ | $Q(|sq\rangle, 1^3_2)$ |
|----------------|----------------|----------------|----------------|----------------|----------------|
| $(0)^1_2$      | $(0)^0_2$      | $(0)^0_2$      | $(0)^0_2$      | $(0)^0_2$      | $(0)^0_2$      |
| $(1)^0_2$      | $(2)^0_2$      | $(2)^0_2$      | $(2)^0_2$      | $(2)^0_2$      | $(2)^0_2$      |
| $(0)^2_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      |
| $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      |
| $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      |

| $Q(|qq\rangle, 1^3_2)$ | $Q(|qq\rangle, 1^1_2)$ | $Q(|qq\rangle, 1^1_2)$ | $Q(|qq\rangle, 1^3_2)$ | $Q(|sq\rangle, 1^1_2)$ | $Q(|sq\rangle, 1^1_2)$ | $Q(|sq\rangle, 1^3_2)$ |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $(0)^1_2$      | $(0)^0_2$      | $(0)^0_2$      | $(0)^0_2$      | $(0)^0_2$      | $(0)^0_2$      | $(0)^0_2$      |
| $(1)^0_2$      | $(2)^0_2$      | $(2)^0_2$      | $(2)^0_2$      | $(2)^0_2$      | $(2)^0_2$      | $(2)^0_2$      |
| $(0)^2_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      |
| $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      | $(0)^3_2$      |
| $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      | $(0)^1_2$      |
Especially, we propose to search for the $D^0 [\bar{c}d] – K^{*0} [\bar{s}d]$, $D^+ [\bar{c}d] – D^{*+} [\bar{s}d]$, and $D^{*+} [\bar{c}d] – B^{*0} [\bar{b}d]$ covalent molecules of $I = 1$ and $J = 0$. These states might exist, when the two light quarks spin in opposite directions and the two antiquarks also spin in opposite directions, just like the para-hydrogen.

6. A toy model to formulize covalent hadronic molecules

In the previous section we have qualitatively discussed the hypothesis: the light-quark-exchange interaction is attractive when the shared light quarks are totally antisymmetric so that they obey the Pauli principle. In this section we further build a toy model to quantitatively formulize it, and estimate binding energies of some possibly-existing covalent hadronic molecules.

We shall use the following formula to estimate binding energies of the $D^{(*)} \Sigma^{(*)}_{c}$, $D^{(*)} B^{(*)}$, and $\Sigma_{c}^{(*)} \Sigma_{b}^{(*)}$ hadronic molecules:

$$B = N_{A} A – N_{R} R – N_{e} – \kappa \langle \lambda_{c} \cdot \lambda_{c} \cdot s_{c} \cdot s_{c} \rangle.$$  \hspace{1cm} (86)

This formula will be explained in section 6.1 in details, and similar formulae are used for some other covalent hadronic molecules. There are altogether four parameters:

$$A \sim 30 \text{ MeV},$$

$$R \sim 17 \text{ MeV},$$

$$\epsilon \sim 6 \text{ MeV},$$

$$\kappa \sim 13 \text{ MeV},$$

which are estimated by considering the $P_{c}/P_{cc}$ and the recently observed $T_{cc}$ as possible covalent hadronic molecules.

6.1. Parameters

As discussed in the previous section, there exists an attractive interaction when exchanging up and down quarks with the configuration:

$$q \leftrightarrow q'.$$

These two light up/down quarks have the same color and their relative orbital structure is $S$-wave. Besides, they have the antisymmetric flavor structure and the symmetric spin structure, so with the quantum numbers $(I)J^{P} = (0)1^{-}$. We use the attractive bond energy $A$ to describe this attraction, which is estimated to be $A \sim 30 \text{ MeV}$ for each bond, with $N_{A}$ the number of such bonds. It is illustrated in figure 10 using the solid curve.

![Figure 10](https://example.com/figure10.png)

| color | flavor | spin | orbital | total |
|-------|--------|------|---------|-------|
| $q \leftrightarrow q'$ | S | A | S | S | A |

We shall use the following formula to estimate binding energies of the $D^{(*)} \Sigma^{(*)}_{c}$, $D^{(*)} B^{(*)}$, and $\Sigma_{c}^{(*)} \Sigma_{b}^{(*)}$ hadronic molecules, etc.

As discussed in the previous section, the two exchanged light up/down quarks can form another configuration of $(I)J^{P} = (1)0^{-}$:

$$q \leftrightarrow q'.$$

However, its induced interaction is weaker, so we do not take this configuration into account in the present study. It is illustrated in figure 10 using the dashed curve.

There exists an $(I)J^{P} = (0)0^{-}$ up–down quark pair inside the proton/neutron with the configuration:

| color | flavor | spin | orbital | total |
|-------|--------|------|---------|-------|
| $q_{1} \leftrightarrow q_{2}$ | A | A | A | S | A |

One can not exchange $q_{1}$ (nor $q_{2}$) with another light up/down quark $q_{3}$, at the same time keeping: (a) any two of the three light quarks are totally antisymmetric, and (b) the proton/neutron remains unchanged. As an example, we exchange $q_{1} \leftrightarrow q_{2}$ and keep (b), but then (a) is not satisfied:

| color | flavor | spin | orbital | total |
|-------|--------|------|---------|-------|
| $q_{1} \leftrightarrow q_{2}$ | A | A | A | S | A |
| $q_{2} \leftrightarrow q_{3}$ | A | A | A | S | A |
| $q_{1} \leftrightarrow q_{3}$ | S | S | S | S | S |

Therefore, the above up–down quark pair is in some sense ‘saturated’. This suggests that the $(I)J^{P} = (0)0^{-}$ up–down quark pairs inside protons/neutrons can not be exchanged, so they are capable of forming repulsive cores in the nucleus.

We use the repulsive bond energy $R$ to describe the repulsion between two repulsive cores, which is estimated to be $R \sim 17 \text{ MeV}$ for each bond, with $N_{R}$ the number of such bonds. It is illustrated in figure 10 using the dotted curve. Besides, we shall find in sections 6.3 and 6.4 that the two charm quarks can also form such repulsive cores in $D^{(*)} D^{(*)}$ and $\Sigma_{c}^{(*)} \Sigma_{c}^{(*)}$ hadronic molecules, etc.
The third parameter is the residual energy $\epsilon$, which is estimated to be $\epsilon \sim 6$ MeV for each component hadron, with $N$ the number of components. We use it to describe the part of kinetic energy that can not be absorbed into $A$ and $R$.

The fourth parameter $\kappa$ relates to the spin splitting. We use the following term to describe the interaction between the $c$ and $\bar c$ quarks when investigating $D^{(*)} \Sigma_c^{(*)}$ hadronic molecules:

$$\mathcal{H}_{\text{spin}} = -\kappa \langle \lambda_c \cdot \lambda_{\bar c} s_c \cdot s_{\bar c} \rangle,$$  \hfill (91)

where $\kappa$ is estimated to be $\kappa \sim 13$ MeV; $s_c$ and $s_{\bar c}$ are spins of the $c$ and $\bar c$ quarks; $\lambda_c$ and $\lambda_{\bar c}$ are their color charges. We use a similar term to describe the interaction between the $c$ and $b$ quarks when investigating $D^{(*)} B^{(*)}$ and $\Sigma_c^{(*)} \Sigma_b^{(*)}$ hadronic molecules:

$$\mathcal{H}_{\text{spin}}^0 = -\kappa \langle \lambda_c \cdot \lambda_b s_c \cdot s_b \rangle.$$ \hfill (92)

However, we do not include such terms when investigating $D^{(*)} D^{(*)}$ and $\Sigma_c^{(*)} \Sigma_c^{(*)}$ hadronic molecules, since the interaction between two charm quarks has been (partly) taken into account in the repulsive bond energy $R$. Note that the spin splitting effect in hadronic molecules still needs to be updated with future experiments, since we do not well understand it at this moment. See section 6.6 for more discussions.

6.2. Nucleus

Taking the proton/neutron as the combination of an $(IJ)^P = (0)1^+$ up–down quark pair together with another up/down quark, we can estimate binding energies of the $^2\text{H}$, $^3\text{H}$, $^3\text{He}$, and $^4\text{He}$, as illustrated in figure 11. In the present study we do not investigate other nuclei consisting of more nucleons, because there are at most two up and two down quarks in the lowest orbit. See section 6.6 for relevant studies on the hypernucleus.

The $^2\text{H}$ contains two shared light up/down quarks with the configuration of $(IJ)^P = (0)1^+$:

| color | flavor | spin | orbital |
|-------|--------|------|---------|
| $q_1 \leftrightarrow q_2$ | S | A | S | A |

We estimate its binding energy to be

$$B_{^2\text{H},0,J=1} = A - R - 2\epsilon \sim 1 \text{ MeV}. \hfill (93)$$

The $^3\text{H}$ and $^3\text{He}$ both contain three shared light up/down quarks with the configuration of $(IJ)^P = (1/2)1/2^+$:

| color | flavor | spin | orbital |
|-------|--------|------|---------|
| $q_1 \leftrightarrow q_2$ | S | A | S | S |
| $q_1 \leftrightarrow q_3$ | S | A | S | S |
| $q_2 \leftrightarrow q_3$ | S | S | A | S |

We estimate their binding energies to be

$$B_{^3\text{H}/^3\text{He},1/2,J=1/2} = 2A - 2R - 3\epsilon \sim 8 \text{ MeV}. \hfill (94)$$

6.3. $D^{(*)} D^{(*)}$, $D^{(*)} B^{(*)}$, $B^{(*)} B^{(*)}$ molecules

The $(IJ)^P = (0)0^+$ $DD$ hadronic molecule does not exist due to the Bose–Einstein statistics. So do the $D^*D^*$ molecules of $(IJ)^P = (0)0^+$ and $(IJ)^P = (0)2^+$. Actually, we can construct their corresponding currents, and explicitly prove them to be zero.

Similar to section 2, we investigate the $(IJ)^P = (0)1^+$ $DD$ hadronic molecule through its corresponding current:

$$\sqrt{2} J_D^{\alpha D^\alpha} = J_D^{\alpha D^\alpha} - J_D^{\alpha D^\alpha} \approx J_D^{\alpha D^\alpha} \approx \bar d_\alpha \gamma_\alpha c_\alpha \bar q_\alpha \gamma_\alpha q_\alpha = \bar a_\alpha \gamma_\alpha c_\alpha \bar a_\alpha \gamma_\alpha q_\alpha.$$ \hfill (96)

Its correlation function

$$\Pi_{(IJ)^P}^{DD^\alpha}(\alpha) = \langle 0 | \prod J_D^{\alpha D^\alpha}(\alpha) J_D^{DD^\alpha}(0) | 0 \rangle,$$ \hfill (97)

can be separated into (omitting the subscripts $\alpha\beta$ for
Their corresponding Feynman diagrams (without condensates) are depicted in figure 12. We calculate them using QCD sum rules and find:

- The term \( \Pi_{DD^*}^{D^*} \) exchanging both light and heavy quarks simply vanishes, i.e. \( \Pi_{DD^*}^{D^*} = 0 \).
- The term \( \Pi_q^{D^*} \) exchanging light quarks is positive, so its induced interaction is attractive.
- The term \( \Pi_c^{D^*} \) exchanging heavy charm quarks is negative, so its induced interaction is repulsive.

\[
\Pi_{DD^*}^{D^*}(x) = \Pi_{DD^*}^D(x) + \Pi_{DD^*}^{D^*}(x) \\
\quad = \Pi_{DD^*}^D(x) + \Pi_{DD^*}^{D^*}(x) \\
\quad + \Pi_{DD^*}^c(x) + \Pi_{DD^*}^{D^*}(x) + \Pi_{DD^*}^{D^*}(x),
\]

where

\[
\Pi_{DD^*}^D(x) = \Pi_D(x) \times \Pi_{DD^*}^D(x),
\]

\[
\Pi_{DD^*}^c(x) = -\text{Tr}[iS_q^{a/}(-x)\gamma_5iS_{q'}^{a'/}(x)\gamma_5] \\
\quad \times \text{Tr}[iS_{q''}^{b/}(-x)\gamma_5iS_{q''}^{b'/}(x)\gamma_5],
\]

\[
\Pi_{DD^*}^{D^*}(x) = -\text{Tr}[iS_q^{a/}(-x)\gamma_5iS_{q'}^{a'/}(x)\gamma_5iS_{q''}^{b/}] \\
\quad \times \text{Tr}[iS_{q''}^{b'/}(-x)\gamma_5iS_{q''}^{b'/}(x)\gamma_5].
\]

Their corresponding Feynman diagrams (without condensates) are depicted in figure 12. We calculate them using QCD sum rules and find:

- The terms \( \Pi_{DD^*}^{D^*} \) and \( \Pi_{DD^*}^{D} \) are almost opposite, i.e. \( \Pi_{DD^*}^{D^*}(x) \approx -\Pi_{DD^*}^{D}(x) \).

Therefore, our QCD sum rule results suggest that the two charm quarks in \( D^{(*)D^*} \) can form repulsive cores. Accordingly, we can estimate binding energies of \( D^{(*)D^*} \) hadronic molecules, as illustrated in figure 13(a).

The \( \langle \Omega^P \rangle = (01) D^{(*)}D^{(*)} \) hadronic molecule contains two shared light antiquarks with the configuration of \( \langle \Omega^P \rangle = (01) \) :

| color | flavor | spin | orbital | total |
|-------|--------|------|---------|-------|
| S     | A      | S    | S       | A     |

We estimate its binding energy to be

\[
B_{\Omega^P = 0, J = 1}^{DD^*} = A - R - 2\epsilon \sim 1 \text{ MeV},
\]

suggesting it possible to interpret the recently observed \( T_{cc}^{*} \) [44, 45] as the \( \langle \Omega^P \rangle = (01) \) \( DD^* \) covalent hadronic molecule.

The \( \langle \Omega^P \rangle = (01) D^{(*)}D^{(*)} \) hadronic molecules have similar binding energies:

\[
B_{\Omega^P = 0, J = 1}^{DD^*} = A - R - 2\epsilon \sim 1 \text{ MeV}.
\]

However, in the above estimations we have not considered the spin splitting effect, and we have also not considered the long-range light-meson-exchange interaction. These uncertainties prevent us to well determine whether these hadronic molecules exist or not.

Different from \( D^{(*)}D^{(*)} \) and \( B^{(*)}B^{(*)} \) hadronic molecules, the charm and bottom quarks in \( D^{(*)}B^{(*)} \) hadronic molecules cannot be exchanged, so they are not capable of forming
covalent hadronic molecules are much larger than those of the
Especially, binding energies of the
splitting effect described by the parameter κ, and estimate binding energies of \(D^{(*)}B^{(*)}\) hadronic molecules to be

\[
\begin{align*}
B^ {D^6}_{J=0,J=0} &= A - 2\epsilon - 1.33\kappa \sim 1\text{ MeV}, \\
B^ {D^8}_{J=0,J=0} &= A + 2\epsilon + 0.44\kappa \sim 24\text{ MeV}, \\
B^ {D^6*}_{J=0,J=0} &= A - 2\epsilon - 1.33\kappa \sim 1\text{ MeV}, \\
B^ {D^8*}_{J=0,J=0} &= A + 2\epsilon + 4\kappa \sim 70\text{ MeV}, \\
B^ {D^6*}_{J=0,J=2} &= A - 2\epsilon - 1.33\kappa \sim 1\text{ MeV}.
\end{align*}
\]

Especially, binding energies of the \((IJ)^P = (0)1^+\) and \(D^6/DB^8\) covalent hadronic molecules are much larger than those of the \((IJ)^F = (0)0^+\) hadronic molecules.

In the above estimations we have assumed that the charm and bottom quarks form the symmetric color representation \(6_c\), so that

\[
\langle \lambda_c \cdot \lambda_b \rangle = \frac{16}{3}.
\]

Besides, we need to perform the spin decomposition from the \([s_{bc}, s_{cq}]\) basis to the \([s_{cb}, s_{cqc}]\) basis (see appendix B), and select the components satisfying \(s_{cq} = 1\) to evaluate \(\langle s_c \cdot s_b \rangle\). Take the \((IJ)^F = (0)1^+\) hadronic molecule as an example, we obtain

\[
\langle 0_{cq}, 1_{bq}; 1|s_c \cdot s_b|0_{cq}, 1_{bq}; 1 \rangle = \frac{4}{3} \times \frac{1}{4} (1_{cb}, 1_{cq}; 1|s_c \cdot s_b|0_{cq}, 1_{bq}; 1) + \frac{1}{2} (1_{cb}, 1_{cq}; 1|s_c \cdot s_b|1_{cb}, 1_{cq}; 1) = -0.0833.
\]

We further use \(D^{(*)}\) to compose multi-\(D^{(*)}\) hadronic molecules, such as the \(DDD^*\) and \(DDB^*\) of \((IJ)^F = (1/2)1^+\), and \(DDD^*\) and \(DDB^*\) of \((IJ)^P = (0)0^+/(0)2^+\) in our model.

Besides, our model supports the existence of the \(DBB^{(*)}\) hadronic molecules, etc. As illustrated in figures 13(d), (f), they have much larger binding energies:

\[
\begin{align*}
B^ {DBB^*}_{J=1/2,J=1} &= 2A - 2R - 3\epsilon \sim 8\text{ MeV}, \\
B^ {DBB^*}_{J=0,J=2} &= 4A - 4R - 4\epsilon \sim 28\text{ MeV}.
\end{align*}
\]

More examples can be found in table 2.

6.4. \(\Sigma^{(*)}\Sigma^{(*)}/\Sigma^{(*)}\Sigma^{(*)}/\Sigma^{(*)}\Sigma^{(*)}\) hadronic molecules

We follow section 6.3 and find that the two charm quarks in \(\Sigma^{(*)}\Sigma^{(*)}\) hadronic molecules also form repulsive cores. Accordingly, we can estimate binding energies of \(\Sigma^{(*)}\Sigma^{(*)}/\Sigma^{(*)}\Sigma^{(*)}\) hadronic molecules, as illustrated in figure 14(a).

The \((IJ)^P = (0)1^+\) hadronic molecule contains four shared light up/down quarks with the configuration of \((IJ)^F = (0)2^+\):

| color | flavor | spin | orbital | total |
|-------|--------|------|---------|------|
| \(q_1\) | \(q_2\) | \(q_1\) | \(q_2\) | \(q_1\) | \(q_2\) | \(q_1\) | \(q_2\) |
| \(S\) | \(S\) | \(S\) | \(S\) | \(A\) | \(A\) | \(A\) | \(A\) |
| \(A\) | \(A\) | \(S\) | \(S\) | \(A\) | \(A\) | \(A\) |
| \(A\) | \(A\) | \(S\) | \(S\) | \(A\) | \(A\) | \(A\) |
| \(A\) | \(A\) | \(S\) | \(S\) | \(A\) | \(A\) | \(A\) |

We estimate its binding energy to be

\[
B^ {\Sigma^{(*)}\Sigma^{(*)}}_{J=0,J=1} = 2A - 2R - 2\epsilon \sim 31\text{ MeV}.
\]

The \(I = 0\) \(\Sigma^{(*)}\Sigma^{(*)}/\Sigma^{(*)}\Sigma^{(*)}\) hadronic molecules have similar binding energies:

\[
B^ {\Sigma^{(*)}\Sigma^{(*)}}_{J=0,J=2} = 2A - 2R - 2\epsilon \sim 31\text{ MeV}.
\]
Table 2. Binding energies of some possibly-existing covalent hadronic molecules, estimated in our toy model through the simplified formula $B = N_p A + N_s S - N_p R - N_c$, with $A \sim 30$ MeV, $S \sim 20$ MeV, $R \sim 17$ MeV, and $\epsilon \sim 6$ MeV. We do not take into account the spin splitting effect described by the parameter $\kappa \sim 13$ MeV here.

| Molecules | Binding energies | Molecules | Binding energies |
|-----------|------------------|-----------|------------------|
| $^3 \text{H}, D^0 D^0 / B^+ B^-$ | 1 MeV | $^3 \text{He}, D^0 D^0 D^0 / B^+ B^- B^-$ | 18 MeV |
| $^3 \text{H}/ \text{He}, D^0 D^0 D^0 / B^+ B^- B^-$ | 8 MeV | $^3 \text{He}, D^0 D^0 D^0 / B^+ B^- B^-$ | 42 MeV |
| $^3 \Sigma_c^0, \Sigma_c^0 / \Sigma_b^0, \Sigma_b^0$ | 28 MeV | $^3 \Sigma_c^0, \Sigma_c^0 / \Sigma_b^0, \Sigma_b^0$ | 62 MeV |
| $^3 \Sigma_c^0, \Sigma_c^0 / \Sigma_b^0, \Sigma_b^0$ | 31 MeV | $^3 \Sigma_c^0, \Sigma_c^0 / \Sigma_b^0, \Sigma_b^0$ | 96 MeV |

So do their corresponding $\Sigma_c^{(q)} \Sigma_c^{(q)}$ hadronic molecules. However, the $(IJ)^P = (01)^+/ (02^+)$ in our model.

Figure 14. Illustration of the hadronic molecules $\Sigma_c \Sigma_b^*$ and $\Sigma_c \Sigma_b^*$ of $(IJ)^P = (01)^+/ (02^+)$ in our model.

6.5. $D^0 \Sigma_c^0 / \bar{D}^0 \Sigma_b^* / B^+ \Sigma_c^0 / B^+ \Sigma_b^*$ molecules

The charm and anti-charm quarks in $\bar{D}^0 \Sigma_c^0$ hadronic molecules can not be exchanged, so they are not capable of forming repulsive cores. Accordingly, we include the spin splitting effect described by the parameter $\kappa$, and estimate their binding energies to

$$B_{I=1/2, J=1/2}^{0\Sigma_c^0} = A - 2 \epsilon - 0.67 \kappa \sim 9 \text{ MeV},$$

$$B_{I=1/2, J=3/2}^{0\Sigma_c^0} = A - 2 \epsilon + 0.33 \kappa \sim 22 \text{ MeV},$$

$$B_{I=1/2, J=1/2}^{0\Sigma_b^*} = A - 2 \epsilon - 0.67 \kappa \sim 9 \text{ MeV},$$

$$B_{I=1/2, J=3/2}^{0\Sigma_b^*} = A - 2 \epsilon + 0.33 \kappa \sim 22 \text{ MeV},$$

$$B_{I=1/2, J=1/2}^{0\Sigma_c^0} = A - 2 \epsilon - 0.67 \kappa \sim 9 \text{ MeV},$$

$$B_{I=1/2, J=3/2}^{0\Sigma_b^*} = A - 2 \epsilon + 0.33 \kappa \sim 22 \text{ MeV},$$

Similarly, we can estimate binding energies of $\bar{D}^0 \Sigma_c^0 / B^+ \Sigma_c^0 / B^+ \Sigma_b^*$ hadronic molecules, which are the same as their corresponding $D^0 \Sigma_c^0$ ones.

These hadronic molecules all contain three shared light up/down quarks with the configuration of $(IJ)^P = (1/2)^3/2^+$.

We further use two charmed mesons and one $\Sigma_c^{(q)}$ baryon to compose $D^0 \Sigma_c^0 / B^+ \Sigma_c^0 / B^+ \Sigma_b^*$ hadronic molecules. Especially, the $D(c \bar{q}_1) D(c \bar{q}_2) \Sigma_c^0 [q_3 q_4]$ illustrated in figure 15(b) contains four shared light up/down quarks with the configuration of $(IJ)^P = (02^+)$. 
We estimate its binding energy to be:

$$B_{1/2}^{DΣc} = 2A - 3\epsilon - 1.33\kappa \sim 25 \text{ MeV}.$$  \hfill (114)

More examples can be found in table 2.

6.6. Molecules with strangeness

In the previous subsections we only consider the up and down quarks as exchanged light quarks, and in this subsection we further take the light strange quark into account. We introduce another parameter $S$ to describe the attractive interaction induced by the shared strange and up/down quarks with the configuration of either $(I)J^P = (1/2)0^+$ or $(1/2)1^+$:

This parameter is estimated to be $S \sim 20 \text{ MeV}$ for each bond, with $N_c$ the number of such bonds. We still use the solid curve to illustrate it, but this solid curve is slightly thinner than that denoting the attractive bond $A$.

Taking the $Λ$ hyperon as the combination of an $(I)J^P = (0)0^+$ up–down quark pair together with a strange quark, we can estimate binding energies of some hypernuclei.

We illustrate these hypernuclei in figure 16. More possibly-existing covalent hadronic molecules with strangeness can be found in table 2.

6.7. Discussions on the spin splitting effect

In the present study we investigate the spin splitting effect through equations (91) and (92) with the same parameter $\kappa \sim 13 \text{ MeV}$. These two equations are used for $D^{(*)}Σ_c^{(*)}$ and $\Lambda^0R^{(*)}Σ_c^{(*)}/Σ_l^{(*)}Σ_l^{(*)}$ hadronic molecules, respectively; while we do not include such terms when investigating $D^{(*)}Λ^{(*)}/Σ_l^{(*)}Σ_l^{(*)}$ hadronic molecules, since the interaction between two charm quarks has been (partly) taken into account in the repulsive bond energy $R$.

Because we do not well understand the spin splitting effect in hadronic molecules at this moment, we still need to update it with future experiments, and there can be other approaches that better describe it. We take $D^{(*)}Σ_c^{(*)}$ hadronic molecules as an example, and discuss several possible improvements.

Firstly, it is possible to use two different $\kappa$’s for equations (91) and (92). It is also possible to use some formulae other than equations (91) and (92), such as the spin–spin interaction

$$\mathcal{H}_s^{''} = -\kappa' (S_{\beta(\cdot)} \cdot S_{\beta(\cdot)})$$  \hfill (116)

or the spin–orbit interaction, etc.

Secondly, in the present study we perform the spin decomposition from the $|s_{q_1}, s_{q_2}, s_{q_3}, \cdot; J, J_{\text{cav}} = 1\rangle$ basis to the $|s_{q_1}, s_{q_2}, s_{q_3}; J, J_{\text{cav}} = 1\rangle$ basis, and select the components satisfying $s_{q_1} = s_{q_2} = s_{q_3} = 3/2$ to evaluate $|s_{\cdot} \cdot s_{\cdot}\rangle$. This is because the three light up/down quarks of $(I)J^P = (1/2)3/2^+$ are assumed to
supply the attraction forming $D^{(*)}\Sigma^{(*)}$ hadronic molecules in our model, while the two heavy quarks c/c are assumed to be freely polarised. If this is not the case and c/c are also fully polarised, there might be two $D^*\Sigma^{(*)}$ hadronic molecules with either $s_Q = 0$ or $s_Q = 1$ (see appendix B):

$$|s_{Q1}, s_{Q2}; c, c; I_J, J_{s_{Q1} = 1} = |s_{Q1}, s_{Q2}; c, c; I_J, J_{s_{Q2} = 1} = 0, \frac{3}{2}; \frac{3}{2}; 0, \frac{3}{2}; \frac{3}{2}; 0, \frac{3}{2}; \frac{3}{2}; 1, \frac{3}{2}; \frac{3}{2}; 1$$ (117)

It can be seen from the recent LHCb experiment [6] that there might be two peaks near the $D^*\Sigma^+$ threshold, and we propose to further study them as well as the $P_c(4440)^-$ and $P_c(4457)^+$ in order to better understand the spin splitting effect of hadronic molecules in future experiments.

To end this section, we further simplify our toy model by neglection the spin splitting effect described by the parameter $\kappa \sim 13$ MeV, and estimate binding energies of some possibly-existing covalent hadronic molecules through the simplified formula,

$$B = N_Q A + N_S S - N_K R - N_c$$ (119)

which still have four parameters $A \sim 30$ MeV, $S \sim 20$ MeV, $R \sim 17$ MeV, and $\epsilon \sim 6$ MeV. The obtained results are summarized in table 2.

Very quickly, we arrive at the unique feature of our covalent hadronic molecule picture: binding energies of the $(IJ)^F = (01)^+ DB^*/D^*B$ hadronic molecules are much larger than those of the $(IJ)^F = (01)^+ DD^*/BB^*$ ones, while the $(IJ)^F = (1/2)1/2^+ D^*\Sigma_c/D^*\Sigma_b/B^*_\Sigma_c/B^*_\Sigma_b$ hadronic molecules have similar binding energies. This is due to the two identical heavy quarks in $DD^*/BB^*$ hadronic molecules forming repulsive cores, but the two different heavy quarks in $DB^*/D^*B$ and $D^*\Sigma_c/D^*\Sigma_b/B^*_\Sigma_c/B^*_\Sigma_b$ hadronic molecules do not.

7. Summary and discussions

In this paper we systematically examine Feynman diagrams corresponding to the $D^{(*)}\Sigma^{(*)}$, $D^{(*)}\Lambda_c$, $D^{(*)}\bar{K}$, and $D^{(*)}\bar{D}^{(*)}$ hadronic molecular states. Take the $D^*\Sigma_c$ molecule as an example, first we calculate correlation functions of $D$ and $\Sigma_c$ in the coordinate space to be $\Pi^{D}(x)$ and $\Pi^{\Sigma}(x)$, respectively. Then we calculate correlation functions of the $D^*\Sigma_c^+$, $D^*\Sigma_c^-$, $I = 1/2$ $D^*\Sigma_c$, and $I = 3/2$ $D^*\Sigma_c$ molecules, separated into:

$$\Pi^{D^*\Sigma_c^+}(x) = \Pi^{D^*\Sigma_c^+}_Q(x) + \Pi^{D^*\Sigma_c^+}_Q(x)$$

$$\Pi^{D^*\Sigma_c^-}(x) = \Pi^{D^*\Sigma_c^-}_Q(x) + \Pi^{D^*\Sigma_c^-}_Q(x) - \Pi^{D^*\Sigma_c^-}_Q(x)$$

$$\Pi^{D^*\Sigma_c^+}_{1/2}(x) = \Pi^{D^*\Sigma_c^+}_Q(x) + \Pi^{D^*\Sigma_c^+}_Q(x) + \Pi^{D^*\Sigma_c^+}_Q(x)$$

$$\Pi^{D^*\Sigma_c^-}_{1/2}(x) = \Pi^{D^*\Sigma_c^-}_Q(x) + \Pi^{D^*\Sigma_c^-}_Q(x) - \Pi^{D^*\Sigma_c^-}_Q(x)$$

where $\Pi^{D^*\Sigma_c^+}_Q(x) = \Pi^{D^*\Sigma_c^+}_Q(x) \times \Pi^{D^*\Sigma_c^+}_Q(x)$ is the leading term contributed by non-correlated $D$ and $\Sigma_c$; $\Pi^{D^*\Sigma_c^-}_Q(x)$ describes the double-gluon-exchange interaction between them, but we do not take it into account in the present study, because the other term $\Pi^{D^*\Sigma_c^-}_Q(x)$ is much larger.

The term $\Pi^{D^*\Sigma_c^-}_Q(x)$ describes the light-quark-exchange effect between $D$ and $\Sigma_c$, i.e. $D$ and $\Sigma_c$ are exchanging and so sharing light up/down quarks. We systematically study it using the method of QCD sum rules, and calculate the mass correction $\Delta M$ induced by this term. Note that the obtained results can be further applied to study the production and decay properties of these hadronic molecular states [68–70]. The parameter $\Delta M$ is actually not the binding energy, because we are using local currents in QCD sum rule analyses. We can relate it to some potential $V(r)$ between $D$ and $\Sigma_c$ induced by exchanged/shared light quarks, satisfying:

$$V(|r| = 0) = \Delta M, V(|r| \to \infty) \to 0$$

We systematically investigate the light-quark-exchange term $\Pi_Q$, and study its contributions to the $D^{(*)}\Sigma^{(*)}$, $D^{(*)}\Lambda_c$, $D^{(*)}\bar{K}$, and $D^{(*)}\bar{D}^{(*)}$ hadronic molecular states. We calculate their mass corrections, some of which are listed here:

$$\Delta M_{I = 1/2, J = 1/2}^{D^{(*)}\Sigma^{(*)}} = -95 \text{ MeV},$$
$$\Delta M_{I = 1/2, J = 3/2}^{D^{(*)}\Sigma^{(*)}} = -89 \text{ MeV},$$
$$\Delta M_{I = 1/2, J = 3/2}^{D^{(*)}\Sigma^{(*)}} = -86 \text{ MeV},$$
$$\Delta M_{I = 1/2, J = 5/2}^{D^{(*)}\Sigma^{(*)}} = -107 \text{ MeV},$$
$$\Delta M_{I = 0, J = 1}^{D^{(*)}\bar{K}} = -180 \text{ MeV},$$
$$\Delta M_{I = 0, J = 1}^{D^{(*)}\bar{D}^{(*)}} = -119 \text{ MeV}.$$
Its logical chain is quite straightforward. We assume the two light quarks \( q_A \) inside \( Y \) and \( q_B \) inside \( Z \) are totally antisymmetric. Hence, \( q_A \) and \( q_B \) obey the Pauli principle, so that they can be exchanged and shared. By doing this, wavefunctions of \( Y \) and \( Z \) overlap with each other, so that they are attracted and there can be a covalent hadronic molecule \( X = \{YZ\} \). This picture has been depicted in figure 2. We believe it is better and more important than our QCD sum rule results, given it is model-independent and more easily applicable.

We apply the above hypothesis to the reanalysis of the \( D^{(*)} \Sigma^{(*)} \), \( \bar{D}^{(*)} \Lambda_c \), \( D^{(*)} K^* \), and \( \bar{D}^{(*)} \bar{B}^{(*)} \) hadronic molecules, and the obtained results are generally consistent with our QCD sum rule results. However, there can be more hyperfine structures allowed/predicted by the hypothesis, similar to the case of para-hydrogen and ortho-hydrogen with the two protons spinning in opposite directions and in the same direction, respectively. These hyperfine structures can not be differentiated in our QCD sum rule studies, since there we need to sum over polarizations.

We also apply the above hypothesis to predict more possibly-existing covalent hadronic molecules, as summarized in table 1. We build a toy model to formalize this picture and estimate their binding energies. Our model has four parameters, which are fixed by considering the \( P_\parallel / P_{el} \) and the recently observed \( T_{el}^{\uparrow \downarrow} \) as possible covalent hadronic molecules. Some simplified results neglecting the spin splitting effect are summarized in table 2. Note that these results are obtained from the light-quark-exchange interaction only, and there can be some other interactions among hadrons, e.g. the light-quark-exchange term \( \Pi_{el}(x) \) does not contribute to the \( D^{(*)} \bar{B}^{(*)} \) molecules, suggesting that the \( D^{(*)} \bar{B}^{(*)} \) covalent hadronic molecules do not exist, but there can still be the \( D^{(*)} \bar{B}^{(*)} \) molecules possibly induced by the one-meson-exchange interaction. These interactions can also contribute to the binding energies given in table 2, which is one source of their theoretical uncertainties.

A unique feature of our covalent hadronic molecule picture is that binding energies of the \( (\bar{J}P)=(0)^-DB^*/DB^* \) hadronic molecules are much larger than those of the \( (\bar{J}P)=(0)^-DD^*/BB^* \) ones, while the \( (\bar{J}P)=(1/2)^1/2 \) \( DSigma_c/DSigma_b/BSigma_c/BSigma_b \) hadronic molecules have similar binding energies. This is due to that the two identical heavy quarks in \( DD^*/BB^* \) hadronic molecules form repulsive cores, but the two different heavy quarks in \( DB^*/DB^* \) and \( DSigma_c/DSigma_b/BSigma_c/BSigma_b \) hadronic molecules do not.

To end this paper, we note again that the one-meson-exchange interaction \( \Pi_{el} \) at the hadron level and the light-quark-exchange interaction \( \Pi_{el} \) at the quark-gluon level can overlap with each other. Hence, we attempt to understand the nuclear force based on the picture of covalent hadronic molecules very roughly: (a) the \( (\bar{J}P)=(0)^- \) up–down quark pairs inside protons/neutrons are in some sense ‘saturated’, so they can not be exchanged and form repulsive cores in the nucleus; (b) the other up/down quarks inside protons/neutrons can be freely exchanged/shared/moving, inducing some interactions among nucleons; (c) in the multi-nucleon nucleus there can be many up/down quarks being shared, so its binding mechanism transfers into the ‘metallic’ hadronic bond. Based on these understandings, we estimate binding energies of some nuclei using our toy model, as summarized in table 2. Finally, we propose another possibly-existing binding mechanism similar to the ‘ionic’ bond, but it might only be observable in the quark-gluon plasma.

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**Appendix A. Spectral densities**

In this appendix we list spectral densities extracted from the currents \( J^{D_{1}^{(*)} \Sigma_{1}^{(*)}} \), \( J^{D_{1}^{(*)} \Sigma_{1}^{(*)}} \), \( J^{D_{1}^{(*)} \Sigma_{1}^{(*)}} \), \( J^{D_{1}^{(*)} \Sigma_{1}^{(*)}} \), and \( J^{D_{1}^{(*)} \Sigma_{1}^{(*)}} \). These currents are defined in equations (7), (12), (21), and (27), respectively. In the following expressions, \( \mathcal{F}(s) = [(\alpha + \beta) m_y^2 - \alpha \beta] \), \( \mathcal{H}(s) = [m_z^2 - (1 - \alpha) s] \), and the integration limits are \( \alpha_{\min} = 1 - \sqrt{1 - 4 m_z^2 / s} \), \( \alpha_{\max} = 1 + \sqrt{1 - 4 m_z^2 / s} / 2 \), \( \beta_{\min} = \alpha_{\min} m_y^2 / \alpha_{\max} m_z^2 \), and \( \beta_{\max} = 1 - \alpha \).

The spectral density \( \rho_{el}(s) \) extracted from the current \( J^{D_{1}^{(*)} \Sigma_{1}^{(*)}} \) of \( I = 1/2 \) can be separated into

\[
\rho_{el}(s) = \rho_{el}^{(el)}(s) + \rho_{el}^{(Q)}(s) \tag{A1}
\]

The leading term \( \rho_{el}^{(el)}(s) \) is

\[
\rho_{el}^{(el)}(s) = m_c \left( \rho_{el}^{p_{\pi}}(s) + \rho_{el}^{\bar{q}q}(s) + \rho_{el}^{GG}(s) + \rho_{el}^{G\bar{G}}(s) + \rho_{el}^{G\bar{G}}(s) + \rho_{el}^{G\bar{G}}(s) + \rho_{el}^{G\bar{G}}(s) + \rho_{el}^{G\bar{G}}(s) + \rho_{el}^{G\bar{G}}(s) \right), \tag{A2}
\]

where

\[
\rho_{el}^{p_{\pi}}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} \int_{\beta_{\min}}^{\beta_{\max}} \frac{\mathcal{F}(s)^3}{1024 \pi^2 \alpha^3 \beta^3}, \tag{A3}
\]

\[
\rho_{el}^{\bar{q}q}(s) = m_c \left( \bar{q}q \right) \int_{\alpha_{\min}}^{\alpha_{\max}} \int_{\beta_{\min}}^{\beta_{\max}} \frac{\mathcal{F}(s)^3}{1024 \pi^2 \alpha^3 \beta^3}, \tag{A4}
\]

\[
\rho_{el}^{GG}(s) = \left( \alpha^2 \pi \right) \int_{\alpha_{\min}}^{\alpha_{\max}} \int_{\beta_{\min}}^{\beta_{\max}} \frac{\mathcal{F}(s)^3}{98304 \pi^2 \alpha^3 \beta^3}, \tag{A5}
\]

\[
\rho_{el}^{G\bar{G}}(s) = \left( (\alpha + \beta - 1) \pi \right) \int_{\alpha_{\min}}^{\alpha_{\max}} \int_{\beta_{\min}}^{\beta_{\max}} \frac{\mathcal{F}(s)^3}{98304 \pi^2 \alpha^3 \beta^3}. \tag{A6}
\]
\[ \rho_1^{(QG)}(s) = m_c \langle \bar{q} G q \rangle \int_{b_{\text{min}}}^{b_{\text{max}}} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} d\beta \ \mathcal{F}(s)^2 \times \left\{ \mathcal{F}(s)^2 \times \frac{(1 - \alpha - \beta)(1 + 2\beta - 1)}{2048 \pi^5 \alpha^2 \beta^3} \right\}. \]

\[ \rho_1^{(qG)}(s) = m_c \langle \bar{q} G q \rangle \int_{b_{\text{min}}}^{b_{\text{max}}} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} d\beta \ \mathcal{F}(s)^2 \times \left\{ \mathcal{F}(s)^2 \times \frac{-1}{64 \pi^4 \alpha^2 \beta^2} \right\}. \]

\[ \rho_1^{(\bar{q}G)}(s) = \langle \bar{q} G q \rangle \int_{b_{\text{min}}}^{b_{\text{max}}} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} d\beta \ \mathcal{F}(s)^2 \times \left\{ \mathcal{F}(s)^2 \times \frac{-1}{128 \pi^7 \alpha^2} \right\}. \]

\[ \rho_1^{(qG)}(s) = m_c \langle \bar{q} G q \rangle \int_{b_{\text{min}}}^{b_{\text{max}}} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} d\beta \ \mathcal{F}(s)^2 \times \left\{ \mathcal{F}(s)^2 \times \frac{-1}{512 \pi^7 \alpha^2} \right\}. \]

\[ \rho_1^{(qG)}(s) = m_c \langle \bar{q} G q \rangle \int_{b_{\text{min}}}^{b_{\text{max}}} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} d\beta \ \mathcal{F}(s)^2 \times \left\{ \mathcal{F}(s)^2 \times \frac{1}{4249152 \alpha^3 \beta^2} \right\}. \]

\[ \rho_1^{(qG)}(s) = m_c \langle \bar{q} G q \rangle \int_{b_{\text{min}}}^{b_{\text{max}}} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} d\beta \ \mathcal{F}(s)^2 \times \left\{ \mathcal{F}(s)^2 \times \frac{1}{32768 \pi^3 \alpha^3} \right\}. \]

\[ \rho_0^{QG}(s) = m_c \langle \bar{q} G q \rangle \int_{b_{\text{min}}}^{b_{\text{max}}} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} d\beta \ \mathcal{F}(s)^2 \times \left\{ \mathcal{F}(s)^2 \times \frac{-1}{128 \pi^7 \alpha^2} \right\}. \]

\[ \rho_0^{QG}(s) = m_c \langle \bar{q} G q \rangle \int_{b_{\text{min}}}^{b_{\text{max}}} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} d\beta \ \mathcal{F}(s)^2 \times \left\{ \mathcal{F}(s)^2 \times \frac{-1}{512 \pi^7 \alpha^2} \right\}. \]

The light-quark-exchange term \( \rho_0^{QG}(s) \) is
\[
\begin{align*}
\rho_4^{q\bar{q}^2}(s) &= \langle \bar{q}q \rangle^2 \int_{\alpha_{\max}}^{\alpha_{\min}} \alpha \int_{\beta_{\min}}^{\beta_{\max}} \beta \left( \mathcal{F}(s) - \frac{1}{2} \right) \frac{1}{384 \pi^4 \alpha \beta} \, d\alpha \, d\beta \\
\rho_4^{q\bar{q}\langle G^f \rangle}(s) &= \langle \bar{q}q \rangle \langle \bar{q}q \rho \rangle \int_{\alpha_{\max}}^{\alpha_{\min}} \alpha \int_{\beta_{\min}}^{\beta_{\max}} \beta \left( \mathcal{F}(s) - \frac{1}{2} \right) \frac{1}{384 \pi^4 \alpha \beta} \, d\alpha \, d\beta \\
+ \mathcal{H}(s) &\times \frac{7}{3072 \pi^4} \end{align*}
\]

The spectral densities \( \rho^{D_{\Sigma}^{1/2}}(s) \), \( \rho^{D_{\Sigma}^{1/2}}(s) \), and \( \rho^{D_{\Sigma}^{1/2}}(s) \) extracted from the currents \( J^{D_{\Sigma}^{1/2}} \), \( J^{D_{\Sigma}^{1/2}} \), and \( J^{D_{\Sigma}^{1/2}} \) are related to \( \rho^{D_{\Sigma}^{1/2}}(s) \) through

\[
\rho^{D_{\Sigma}^{1/2}}(s) = \rho_0^{D_{\Sigma}^{1/2}}(s), \quad (A4)
\]

\[
\rho^{D_{\Sigma}^{1/2}}(s) = \rho_0^{D_{\Sigma}^{1/2}}(s) - \rho_0^{D_{\Sigma}^{1/2}}(s), \quad (A5)
\]

\[
\rho_{I=3/2}^{D_{\Sigma}^{1/2}}(s) = \rho_0^{D_{\Sigma}^{1/2}}(s) - 2 \rho_0^{D_{\Sigma}^{1/2}}(s), \quad (A6)
\]

### Appendix B. Spin decompositions

In this appendix we list some spin decompositions used in the present study. The transitions from the \( |s_{\bar{q}q}, s_{\bar{q}q}; J \rangle \) basis to the \( |s_{\bar{q}q}, s_{\bar{q}q}; J \rangle \) basis are:

\[
|s_{\bar{q}q}, s_{\bar{q}q}; J \rangle \rightarrow |s_{\bar{q}q}, s_{\bar{q}q}; J \rangle,
\]

\[
|0, 0; 0 \rangle = \frac{1}{2} \left| 0, 0; 0 \rightangle + \frac{1}{2} \left| 1, 1; 0 \rightangle,
\]

\[
|0, 1; 1 \rangle = \frac{1}{2} \left| 0, 1; 1 \rightangle - \frac{1}{2} \left| 1, 1; 1 \rightangle + \frac{1}{\sqrt{2}} \left| 1, 1; 1 \rightangle,
\]

\[
|1, 0; 0 \rangle = -\frac{1}{2} \left| 1, 0; 0 \rightangle + \frac{1}{2} \left| 1, 1; 0 \rightangle + \frac{1}{\sqrt{2}} \left| 1, 1; 1 \rightangle,
\]

\[
|1, 1; 1 \rangle = \frac{1}{2} \left| 1, 1; 1 \rightangle + \frac{1}{\sqrt{2}} \left| 1, 1; 1 \rightangle,
\]

\[
|1, 1; 2 \rangle = \left| 1, 1; 2 \rightangle.
\]

(B1)

The transitions from the \( |s_{\bar{q}q}, s_{\bar{q}q}; J \rangle \) to in the \( |s_{\bar{q}q}, s_{\bar{q}q}; J \rangle \) basis are:

\[
|s_{\bar{q}q}, s_{\bar{q}q}; J \rangle \rightarrow |s_{\bar{q}q}, s_{\bar{q}q}; J \rangle,
\]

\[
|0, 1; 1 \rangle = \frac{1}{2} \left| 0, 1; 2 \rightangle + \frac{1}{2} \left| 1, 1; 2 \rightangle - \frac{1}{\sqrt{2}} \left| 1, 1; 2 \rightangle,
\]

\[
|1, 1; 1 \rangle = -\frac{1}{2} \left| 1, 1; 2 \rightangle - \frac{1}{2} \left| 1, 1; 2 \rightangle + \frac{1}{\sqrt{2}} \left| 1, 1; 2 \rightangle.
\]

(B2)

The transitions from the \( |s_{\bar{q}q}, s_{\bar{q}q}; J \rangle \) to in the \( |s_{\bar{q}q}, s_{\bar{q}q}; J \rangle \) basis are:

\[
|s_{\bar{q}q}, s_{\bar{q}q}; J \rangle \rightarrow |s_{\bar{q}q}, s_{\bar{q}q}; J \rangle,
\]

\[
|0, 1; 1 \rangle = \frac{1}{2} \left| 0, 1; 2 \rightangle + \frac{1}{2} \left| 1, 1; 2 \rightangle - \frac{1}{\sqrt{2}} \left| 1, 1; 2 \rightangle,
\]

\[
|1, 1; 1 \rangle = -\frac{1}{2} \left| 1, 1; 2 \rightangle - \frac{1}{2} \left| 1, 1; 2 \rightangle + \frac{1}{\sqrt{2}} \left| 1, 1; 2 \rightangle.
\]
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