Prepeak energy evolution properties of sandstone subject to diametrical splitting with different loading rates

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Abstract. To understand the energy evolution properties during rock splitting under different loading rates, four groups of Brazilian test with a loading-unloading cycle corresponding to four orders of magnitude of controlled loading rate ($10^2$, $10^3$, $10^4$, $10^5$ kN/min) were conducted on red sandstone using the MTS 322 testing system. The energy characteristics were interpreted by mathematically integrating the force-displacement responses of the tested rock specimens. It was found that at each loading rate, the input energy was a linear function as the elastic energy and dissipated energy, while these three types of energy nonlinearly grew with the unloading level. Importantly, the loading rate was found to exert an insignificant influence on the evolution of the linear energy properties (referring to the linear functions between the three types of energy); the energy storage coefficient (ESC) and/or energy dissipation coefficient (EDC) remained constant under various loading rates, exhibiting an approximate independency on loading rate. Based on this, a unified formula for describing the rate-independence of linear energy properties was presented in the tested loading rate range. It is also noted that rock mechanical parameters such as the peak force and peak displacement and the ESC and EDC are closely comparable versus the increasing loading rate, and hence the energy parameters are likely to be used to characterize the rock mechanical ones. Finally, the relevancy between energy allocation and rock failure patterns was discussed.

1. Introduction
Deformation and destruction of rock material is essentially a comprehensive outcome induced by input, storage, dissipation and release of the deformation energy during external load disturbances [1-3], and thus its mechanism can be explained from the viewpoint of energy [4]. A substantial effort has been dedicated to study rock failure behaviours by means of energy interpretation in the community of laboratory rock mechanics [4-7]. Moreover, it is a known fact that loading rate is one of the key factors that can influence the rock failure behaviours [8-10], and in many engineering cases, rock suffers from external influences with a specific loading rate. Therefore, the investigation of rock destruction property normally involves the determination or variation of loading rate [11-14]. In addition to the influence of the material itself, the energy characteristics of rocks are also closely related to external conditions, because different external conditions will lead to different mechanical responses. Many investigators have studied the influence of loading rate on the indirect tensile strength [15-17] and some investigated the energy evolution characteristics of rock in compression with different loading rates [18-20]. However, few research works involving rock splitting tensile failure by means of energy interpretation subject to various loading rates are available to date [20].
Considering our previous studies in which the linear property of energy storage and dissipation of rock was revealed experimentally, whereas the influencing mechanism of loading rate on linear energy properties (the linear relations between the input energy, elastic deformation energy, and dissipated energy) has not been investigated [21], herein we further explore the impact of loading rate on the linear energy relations during rock splitting. Four orders of magnitude of loading rate were imposed on the rock samples in the Brazilian tests. The energy allocation performance in the process of rock splitting was interpreted by integrating the force-displacement responses and implication of the test results was discussed. A unified linear energy relations were deduced in consideration of the change of loading rate and the relevancy between energy allocation performance and failure pattern of rock samples was discussed. These findings further evidence that the linear energy relations are the natural properties of rock materials.

2. Test plan and material

2.1. Test device and procedures

Brazilian tests were carried out with four orders of magnitude loading rate ($v=10^{-1}, 10^0, 10^1, 10^2$ kN/min) using the MTS 312 testing system in Central South University. The testing system consists of a software control component on an individual computer and a testing machine. As shown in Figure 1, an artificial mould and two rigid rollers were adopted to realize diametrical load during the tests.

In the single-cyclic loading-unloading Brazilian test, the unloading level $i$ is computed as the ratio of a force value at the unloading point to the peak force [21], which varies in the range of 0–1. The testing procedure was described as follows: one sample firstly was continuously loaded until its final failure to determine its peak force $P$ under a given loading rate (e.g. $10^{-1}$ kN/min); based on the $P$ obtained, several unloading points or unloading levels were presumed to conduct the single cyclic loading-unloading test in which $i$ is nonzero. The complete force-displacement curve acquired in the single-cyclic loading-unloading test includes four parts: the initial loading curve, unloading curve, reloading curve, and post-peak curve, as displayed in Figure 2a. More exhaustive testing procedure to the single-cyclic loading-unloading test can be found in [22].  

![Figure 1 Testing system and diametrical loading mould used in the Brazilian test](image)

2.2. Energy determination principle

From the perspective of thermodynamic theory, assuming that all the work done by the outside environment is transformed into the energy inside the rock, the following relationship between input energy, elastic deformation energy, and dissipated energy exists [23]:

$$E_w = E_{el} + E_d$$

(1)

where $E_w$, $E_{el}$, and $E_d$ refer to the input energy, elastic deformation energy and dissipated energy, respectively.

At a given unloading level, the accumulated energy input into rock sample can be deduced by integrating the loading curve (see Figure 2b). Likewise, the elastic deformation energy is
obtained by the integral area of unloading curve. The specific calculation formulas of these two energy components (input energy and elastic deformation energy) are as follows:

\[
E_{ai} = \int_{u_1}^{u_2} f_i(u) \, du
\]

\[
E_{ei} = \int_{u_1}^{u_2} f_e(u) \, du
\]

(2)

where \(f_i(u)\) and \(f_e(u)\) are two functions describing the initial loading curve and unloading curve on a force-displacement relation, respectively. Upon these two functions, the dissipated energy is deduced by Eq. 1.

![Figure 2 Illustrations of force-displacement relation and energy calculation principle](image)

2.3. Testing material

The red sandstone tested in this work was taken from eastern China’s Shandong province, Linyi city, which is mainly composed of quartz (Qtz), plagioclase (Pl), and calcite (Cal) [24]. During preparation, rock cores with 50 mm diameter were firstly drilled from a raw rock block and subsequently cut and polished into 50 mm long cylinders. The mean wave speed and mean density of the rock samples prepared are 2479 kg/m³ and 3169 m/s, respectively. The texture is uniform, and no macroscopic cracks appear on the surface of these samples. The prepared samples are artificially divided into four groups for the use of correlative testing group corresponding to the four loading rates.

3. Results and discussion

3.1. Test curve

Figure 3 shows the representative force–displacement curves of the rock samples tested under different loading rates, wherein Figure 3a shows the test curves resulted from monotonically increasing load (in this testing case, \(i=0\)) and Figure 3b-f depict those induced from single-cyclic load. It can be observed that, when the rock samples were subjected to monotonically increasing load, the force-displacement curve under different loading rates exhibit consistency in their changing paces. That is, during the pre-peak loading, the curve is concave up first and then experiences a straight segment and finally it goes through a very short period of concave down before reaching the peak force.
3.2. Nonlinear energy characteristics

Figure 4 plots the changes in the input energy, elastic deformation energy and dissipated energy with an increase in unloading level. It is seen from Figure 4 that under different loading rates, the input energy, elastic deformation energy and dissipated energy all exhibit a quadratic functional growth with the increase of the unloading level. Under various loading rates and unloading levels, the degree of dispersion of the input energy, elastic deformation energy and dissipated energy data points is different. The dissipated energy shows the largest data scatter, followed by the input energy, and the elastic deformation energy has the lightest data scatter (data scatter can also be characterized by the coefficient of determination, $R^2$). This observation indicates that the effect of loading rate on elastic energy is the smallest, among these three energy components. However, although the loading rate has a certain impact on the scatter degree of energy data, the loading rate effect is negligible. In other words, the energy evolution can be expressed by a unified function as the unloading level in the tested range of loading rate.

3.3. Linear energy properties

Figure 5 demonstrates the relations of elastic deformation energy and dissipated energy against the input energy under distinct loading rates. The positive linear energy relations were observed under each loading rate. More specifically, with an increase of the input energy, the elastic deformation energy and dissipated energy increased as linear functions, and the increasing rate of elastic deformation energy is obviously faster than that of dissipated energy. In reality, the proportions of elastic deformation energy and dissipated energy in the splitting process of the red sandstone can be accurately characterized by the slope terms of the two types of linear energy relations above, corresponding the energy storage coefficient (ESC) and energy dissipation coefficient (EDC), respectively; the ESC and EDC characterizes the relative amount of the elastic deformation energy and dissipation energy accounting for the total strain energy during pre-peak splitting of the red sandstone, respectively [21]. The ESC and EDC of the linear energy relations under loading rates of $10^1$, $10^0$, $10^1$ and $10^2$ kN/min are 0.6488 and 0.3562, 0.6227 and 0.3773, 0.6383 and 0.3617, 0.6445 and 0.3557, respectively.
As presented in Figure 6, it appears that the loading rate leads to an inappreciable impact on the linear energy relations. Therefore, the increases in elastic deformation energy or dissipated energy as a linear function of input energy deduced from the testing rate range was fitted by a unified formula. The corresponding functional formulas of the linear energy relations are

\[
\begin{align*}
E_{ei} &= aE_{i} + b \\
E_{di} &= (a - 1)E_{i} - b
\end{align*}
\]

where \(a\) and \(b\) are two fitting constants; considering test results under the four loading rates, the values of \(a\) and \(b\) in the above formulas are determined as 0.6437 and 0.0109, respectively. These linear energy correlations indicate that the input energy is allocated into elastic deformation energy and dissipation energy as percentage in the testing range of loading rate.

![Figure 5 Linear energy properties of red sandstone under various loading rates](image)

![Figure 6 Unified linear energy properties of red sandstone subject to various loading rates](image)

3.4. Compatibility of energy and mechanical properties under different loading rates
As the magnitude of the loading rate increases, the changes in ESC, EDC, peak force and peak displacement are demonstrated in Figure 7. Apparently, similar to the peak force and peak displacement, the ESC and EDC almost keep unchanged with the increase of loading rate and show a consistent response to the loading rate. This practically suggests that the energy parameters and mechanical parameters have good consistency in characterizing the failure behaviours and that they respond similarly to the loading rate for the same rock type and that ESC and EDC are essential properties of rock materials.

Generally, the loading rate effect of tensile strength is commonly obtained in rock mechanics test [16]. In the present work, the loading rate effect is unnoticeable. The reason may be that the span covering the four loading rates is not wider enough to exhibit the rate effect of peak force or the dispersion among peak forces of rock samples masks the rate effect law.

Figure 7 Variation trend of energy parameters and mechanical parameters

3.5. Relevancy between patterns of energy allocation and rock failure

The degree of rock broken is closely related to energy dissipation during progressive rock failure, and the energy dissipated during loading may be potential in characterizing the damage or broken degree of rock materials [25]. Figure 8 illustrates the percentage of energy distribution and the failure modes of rock samples subject to four different loading rates. It can be seen that rock samples were separated into two big blocks, basically along the diametrical direction of the sample end surface. No friction scratch trace can be noted on the splitting surface and few fine-grained rock clasts were produced during formation of rock fragments, presenting the obvious tensile characteristics. Moreover, it is also observed that the failure mode and broken degree of the rock samples under different loading rates are almost the same, which indicates that the varied rate of diametrical loading does not affect the global macro rock broken state, i.e. the fracture area is basically equal under different loading rate. Noticeably, there seems a good correspondence between the energy allocation performance and sample broken state. That is, the percentage of dissipated energy or elastic deformation energy under changing loading rate nearly remains constant, indicating that amount of energy consumed for macro fracture surface is almost equal under different loading rates. Although there is a local compression failure region near the steel rollers, it does not change the global energy consumption percentage of rock samples despite of the loading rate. According to the linear energy correlations, the ESC and EDC practically reflect the proportions of elastic deformation energy and dissipated energy in the cumulative input energy prior to the peak force, respectively. Therefore, it is concluded that ESC and EDC can be used to characterize the degree of rock damage and destruction.

The loading rate involved in this work only changes from $10^{-1}$ to $10^{2}$ kN/min, and it is found that the energy evolution properties exhibit insignificant loading rate effect. In view of this, more work with wider range of loading rate is desired to further validate the influence of loading rate on the energy allocation properties.
Figure 8 Illustration of percentage distribution of elastic deformation energy and dissipated energy during Brazilian splitting

4. Conclusions
In this study, four groups of single-cyclic loading-unloading Brazilian tests were performed to explore the energy evolution properties of red sandstone subject to different loading rates. The major conclusions are summarized as follows:
1. Under different loading rates, the nonlinear energy characteristics and the linear energy correlations during tensile splitting of the red sandstone were experimentally arrested.
2. The loading rate exerts an ignorable effect on the energy storage coefficient and energy dissipation coefficient. In addition, energy storage coefficient or energy dissipation coefficient exhibits a similar unchanging trend to the peak load and peak displacement under the four loading rates. Based on this, a unified formula for the linear energy properties in the tested range of loading rate is presented. The rate-independency of energy storage coefficient and energy dissipation coefficient proves that they can be termed as two essential energy indicators and that the linear energy correlations are natural properties of rock materials.
3. The proportional energy allocation behaviour seems to have close relevancy with the broken state of rock specimens. Under different loading rates, the failure pattern of rock samples is nearly the same, which results in very a similar proportion of energy allocation during rock splitting.

Acknowledgements:
This work was supported by the National Natural Science Foundation of China (Grant No. 41877272).

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