Model predictive control in optimizing stock portfolio based on stock prediction data using Holt-Winter’s exponential smoothing

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Abstract. This study aims to solve an optimization problem on stock portfolio. There are two main subtopics of this research, namely stock price prediction using Holt Winter's Exponential Smoothing method and stock portfolio optimization using Model Predictive Control (MPC) method. The steps that have been taken are: collecting and analysing stock price data, determination of smoothing parameters and stock price prediction, calculation of stock price prediction returns, determining the stock portfolio model and system constraints, converting the objective function into a quadratic programming form, and initialization of optimization parameters and program simulation. Based on the simulation results, all control variables are within the predetermined constraints. The application of MPC in optimizing all capital in a portfolio based on stock price predictions can provide satisfactory results. This is represented by the decisions given by the MPC which resulted in the investor's total capital increasing closer to the expected target.

1. Introduction

Investment is a commitment to sacrifice present consumption in order to increase future consumption [1]. One of the investment instruments that offer high return is stocks. In stock portfolio management, an investor diversifies or distributes capital in several stock assets as to produce an optimal return value and total capital moves closer to the target expected by investors. The problem of optimizing stock portfolio is basically a dynamic problem involving the stochastic dynamics of the ever-expanding stock price. Therefore, the stock portfolio will be more optimal when investors know the prediction of future stock prices. One method for predicting stock prices is Holt Winter's Exponential Smoothing method. This method can analyse data in a univariate manner (single variable) which contains seasonal patterns and trends. There are two types of Holt Winter's Exponential Smoothing model, namely the additive model and the multiplicative model. In general, the Holt Winter Multiplicative model is more often used, because this model is suitable in calculating periodic series predictions, where the amplitude or height of the seasonal pattern is proportional to the average level or level of the data series [2]. Research related to the application of the Holt Winter's Exponential Smoothing method has been carried out by Safitri et al [3] and Fitria et al [4].

In addition to stock price prediction, portfolio optimization also involves optimizing the system while still pay attention to constraints. The control method used is the Model Predictive Control (MPC). MPC is included in the predictive control based on the process model. Some of the advantages of MPC can handle some constraints on the system in designing controllers, handling multivariable
systems, and having the capability of feed forward controllers to compensate for measured disturbances. Research on the application of MPC has been carried out in various scientific fields. Initially, MPC was often applied in the field of ship control, one of the research in this field was conducted by Purnawan et al [5]. While in the financial control is carried out by Wang et al [6] entitled an experiment of control-theoretical model in dynamic portfolio management [7]. Another research with similar field were conducted by Syaifudin et al [8] and Fitria et al [9].

Modelling and optimization of stock portfolios has also been carried out by Subchan et al using Nadir Compromise Programming [10].

Based on the previous research studies, this paper proposes the application of MPC to optimize stock portfolios based on stock price prediction data using the Holt Winter’s Exponential Smoothing method, in order to obtain an optimal stock portfolio.

2. Mathematical modelling on stock price forecasting

The Holt Winter's Exponential Smoothing method is based on three smoothing elements, namely the stationary data element (mean), the trend element, and the seasonal element for each period, and provides three weightings in the prediction, namely $\alpha$, $\beta$ and $\gamma$. The magnitude of the coefficient $\alpha$, $\beta$ and $\gamma$ within the range of 0 to 1 is determined subjectively by minimizing the error value of the estimate [11]. The following shows a mathematical model of stock price prediction using the Holt Winter Multiplicative method [12].

$$L_{p+t} = \alpha \left( \frac{X_{p+t}}{I_t} \right) + (1 - \alpha) \left( L_{p+t-1} + T_{p+t-1} \right)$$  \hspace{1cm} (1)

$$T_{p+t} = \beta \left( L_{p+t} - L_{p+t-1} \right) + (1 - \beta) T_{p+t-1}$$  \hspace{1cm} (2)

$$I_{p+t} = \gamma \left( X_{p+t} - L_{p+t} \right) + (1 - \gamma) I_t$$  \hspace{1cm} (3)

where $= 1, 2, \ldots, (n - p)$ , with:

$L_{p+t}$ : Exponential smoothing at $(p + t)$ time

$T_{p+t}$ : Smoothing the trend (trend) element over $(p + t)$ time

$I_{p+t}$ : Seasonal element smoothing at $(p + t)$ time

$\alpha$ : Weight constant for exponential smoothing

$\beta$ : Weight constant for trend elements

$\gamma$ : Weight constant for seasonal elements

$X$ : Actual data

$p$ : The length of the seasonal element

$n$ : Amount of actual data

The stock price forecast in period $m$ is formulated as [12]:

$$F_{n+m} = (L_n + mT_n)I_{n-p+m}$$  \hspace{1cm} (4)

After the data forecasting results are obtained, the model error value is then measured. The value of the model error in this study is measured using the Mean Absolute Percentage Error (MAPE). MAPE measures the average percentage of the error between the actual data and the forecasted data. The MAPE value is obtained from Equation (5) [13].

$$MAPE = \frac{1}{n} \sum_{t=1}^{n} \left| \frac{X_t - F_t}{X_t} \right| \times 100\%$$  \hspace{1cm} (5)

with,

$X_t$ : Actual data value at $t$ time
\( F_t \) : The value of the predicted data at \( t \) time
\( n \) : Amount of actual data

### Table 1. MAPE value range classification

| MAPE Range | Meaning                      |
|------------|------------------------------|
| < 10%      | Very Good Forecasting Model Ability |
| 10 – 20%   | Good Forecasting Model Ability |
| 20 – 50%   | Decent Forecasting Model Ability |
| > 50%      | Bad Forecasting Model Ability  |

The ability to predict data is said to be good when the MAPE value is getting smaller. Table (1) shows the classification range for the MAPE value data [14].

### 3. Mathematical modelling on stock portfolio management

In this study, the portfolio formed consists of three risk assets (stocks), one risk-free asset (bank), and one capital loan asset. The share assets consist of shares of PT. X Tbk., PT. Y Tbk., And PT. Z Tbk. Figure (1) below illustrates a portfolio scheme in stock investing which describes the process of distributing capital on each asset in the portfolio.

At the beginning, an investor has a certain amount of capital that can be used to invest in the \( i \)-th stock asset, where \( i = 1,2,3 \). In addition, investors’ wealth also comes from risk-free assets, namely banks. Apart from assets in banks, investors also need additional capital to purchase stocks in a portfolio. This additional capital is in the form of investor loan assets of \( v \). If in the future the investor's capital is sufficient, the investor can make payments on the capital loan. Based on the scheme in Figure (1), a mathematical model in portfolio management can be formed for stock assets, as shown in Equation (6) [15].

\[
x_i(k + 1) = [1 + R_i(k)] [x_i(k) + p_i(k) - q_i(k)]
\]  

(6)

with,
- \( R_i(k) \) : Return of the \( i \) assets, with \( i = 1,2,3 \).
- \( x_i(k) \) : The amount of capital the investor invests in \( i \)-th risk assets
- \( p_i(k) \) : Transfer amount from risk-free assets to \( i \)-th risk assets
- \( q_i(k) \) : Transfer amount from risk assets to \( i \)-th risk-free assets

where \( p_i(k) \geq 0 \) and \( q_i(k) \geq 0 \), so that the equation can be fulfilled. Each sale and purchase transaction of shares is subject to a transaction fee that must be paid when the transaction takes place with a proportion of \( \alpha \) for the cost of buying shares and \( \beta \) for the cost of selling shares. The equation below shows the change from risk-free asset (bank) [15].

\[
x_n+1(k + 1) = [1 + r_1(k)] [x_4(k) + v(k) - (1+\alpha)\sum_{i=1}^{3}p_i(k) + (1-\beta)\sum_{i=1}^{3}q_i(k)]
\]  

(7)
where,
\( r_1(k) \): The interest rate on risk-free assets
\( v(k) \): The transfer amount between free assets and capital loan assets

If \( v(k) > 0 \), this indicates that the investor is borrowing capital. However, if \( v(k) < 0 \), it means that investors make payments on the loan credit. Changes from investor capital loans can be written in the following equation:

\[
x_{n+2}(k + 1) = [1 + r_2(k)][x_5(k) + v(k)]
\]  

(8)

Where \( r_2(k) \) denotes the interest rate on capital loans. Based on the description above, the total amount of investor capital in a portfolio is the accumulation of wealth or capital owned by investors in risky assets (stocks) and risk-free assets (banks), then there is a reduction in the amount of investor capital loans. This can be expressed as:

\[
y(k) = \sum_{i=1}^{n} x_i(k) - x_5(k)
\]  

(9)

Based on Equation (6) to Equation (9), a discrete-time state-space equation can be formed:

\[
x(k + 1 | k) = Ax(k | k) + Bu(k | k)
\]  

(10)

\[
y(k | k) = Cx(k | k)
\]  

(11)

with,

\[
x(k + 1 | k) = \begin{bmatrix} x_1(k + 1) \\ \vdots \\ x_n(k + 1) \\ x_{n+1}(k + 1) \\ x_{n+2}(k + 1) \end{bmatrix}, \quad u(k | k) = \begin{bmatrix} p_1(k) \\ \vdots \\ q_1(k) \\ v(k) \end{bmatrix}
\]

\[
A = \begin{bmatrix} 1 + R_1(k) & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 + R_1(k) & 0 & 0 \\ 0 & \cdots & 0 & 1 + r_1(k) & 0 \\ 0 & \cdots & 0 & 0 & 1 + r_2(k) \end{bmatrix}
\]

\[
B = \begin{bmatrix} 1 + r_1(k) & \cdots & -(1 + R_1(k)) & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \\ (1 + r_1(k))(-1 - \omega) & \cdots & (1 + r_1(k))(-1 - \beta) & \cdots & (1 + r_1(k)) \end{bmatrix}
\]

\[
C = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ -1 \end{bmatrix}
\]

Some of the constraints in the stock portfolio optimization problem are defined as follows:

\[
0 \leq p_i(k) \leq p_{i_{max}}, \quad i = 1, 2, 3
\]  

(12)

\[
0 \leq q_i(k) \leq q_{i_{max}}, \quad i = 1, 2, 3
\]  

(13)

\[
-v_{max} \leq v(k) \leq v_{max}
\]  

(14)

\[
0 \leq x_5(k) + v(k) \leq d_0
\]  

(15)
\[ x_4(k) + v(k) - (1+\alpha) \sum_{i=1}^{n} p_i(k) + (1 - \beta) \sum_{i=1}^{n} q_i(k) \geq 0 \]  

(16)

\[ x_i(k) + p_i(k) - q_i(k) \geq 0 \quad i = 1, 2, 3 \]  

(17)

Based on the constraint, equations (12) to (17) can be rewritten into the inequality form as follows:

\[ P_1 u(k) \leq h_1 \]  

(18)

\[ b_1 \leq S_1 u(k) \leq b_2 \]  

(19)

with,

\[ u(k) = [p_1(k) \ldots p_n(k) q_1(k) \ldots q_n(k) v]^T \]

\[ P_1 = \begin{bmatrix} 1 + \alpha & \ldots & -(1 - \beta) & \ldots & -1 \\ 0 & \ldots & 0 & \ldots & -1 \\ 0 & \ldots & 0 & \ldots & 1 \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \ldots & 0 & \ldots & 0 \end{bmatrix}, \quad S_1 = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \end{bmatrix}_{7 \times 7} \]

\[ h_1 = [x_{n+1}(k) \ldots x_{n+2}(k) \ldots x_{n}(k)]^T \]

\[ b_1 = [0 \ldots 0 \ldots -v_{\text{max}}]^T \]

\[ b_2 = [p_{1\text{max}} \ldots p_{n\text{max}} q_{1\text{max}} \ldots q_{n\text{max}} v_{\text{max}}]^T \]

4. Application of model predictive control in stock portfolio management

An investor certainly hopes to get maximum returns that are close to the desired target. To achieve this condition, the difference or error between the target and the output of achieving all capital must be minimized. Based on this, an objective function is given as follows:

\[ J = \sum_{j=0}^{N_p} e^T(k+j)Qe(k+j) + u^T(k+j-1)Ru(k+j-1) \]  

(20)

\[ e(k+j) = y(k+j) - r(k+j) \]  

states the error value between the target (reference trajectory) and the output at the time of \((k+j)\) step. \(N_p\) indicates the extent to which the prediction was made (prediction horizon). Matrix \(Q\) and \(R\) are semi-definite positive error weight and control weights, respectively. The optimization method used in linear MPC has a quadratic programming form, so that the objective function can be restated as:

\[ J(u(k)) = \hat{u}^T(k)H\hat{u}(k) + 2f^T\hat{u}(k) \]  

(21)

with,

\[ H = (\bar{B}^T\bar{Q}\bar{B} + \bar{R}) \]  

(22)

\[ f = \bar{B}^T\bar{Q}(\bar{A}x - r) \]  

(23)

\[ u(k) = [u(k) \quad u(k+1) \quad \ldots \quad u(k+N_p+1)]^T \]
\[ \hat{B} = \begin{bmatrix} CB & 0_{1 \times 7} & 0_{1 \times 7} & \cdots & 0_{1 \times 7} \\ CAB & CB & 0_{1 \times 7} & \cdots & 0_{1 \times 7} \\ CA^2B & CAB & CB & \cdots & 0_{1 \times 7} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^{N_p-1} & CA^{N_p-2}B & CA^{N_p-3}B & \cdots & CB \end{bmatrix}_{N_p \times 7N_p} \]

\[ \hat{A} = [CA \; CA^2 \; CA^3 \; \cdots \; CA^{N_p}]^T \]

\[ \hat{Q} = \begin{bmatrix} Q & 0 & \cdots & 0 \\ 0 & Q & \cdots & 0 \\ 0 & \vdots & \ddots & \vdots \\ 0 & \vdots & \vdots & Q \end{bmatrix}_{N_p \times N_p} , \quad \hat{R} = \begin{bmatrix} R & 0 & \cdots & 0 \\ 0 & R & \cdots & 0 \\ 0 & \vdots & \ddots & \vdots \\ 0 & \vdots & \vdots & R \end{bmatrix}_{7N_p \times 7N_p} \]

\[ 0_{1 \times 7} = [0 \; 0 \; \cdots \; 0]_{1 \times 7} \]

The constraints in the portfolio optimization problem are defined as follows:

\[ P\hat{u}(k) \leq h \quad (24) \]

\[ B_1 \leq S\hat{u}(k) \leq B_2 \quad (25) \]

with,

\[ P = \begin{bmatrix} P_1 & 0 & \cdots & 0 \\ 0 & P_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_1 \end{bmatrix}_{6N_p \times 7N_p} \]

\[ S = \begin{bmatrix} S_1 & 0 & \cdots & 0 \\ 0 & S_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \vdots & \vdots & S_1 \end{bmatrix}_{7N_p \times 7N_p} \]

\[ h = [h_1 \; h_1 \; \cdots \; h_1]^T \]

\[ B_1 = [b_1 \; b_1 \; \cdots \; b_1]^T \]

\[ B_2 = [b_2 \; b_2 \; \cdots \; b_2]^T \]

The optimal solution of the stock portfolio optimization problem is

\[ \{ u^*(k), u^*(k+1), \ldots, u^*(k+N_p-1) \} \]

By using the receding horizon principle in MPC, the optimal control value inputted into the system is the initial vector of the optimal solution. Thus, the control value assigned to the state variable is

\[ u(k) = u^*(k) \]

\[ u(k) \] is the control vector at \( k \) time and \( u^*(k) \) is the optimum control value at the \( k \) time.

5. Simulation and result

The share price prediction is carried out at the closing price of PT. X., PT. Y., And PT. Z for 90 days [16] in the range 3th February 2020 to 5th June 2020. Based on the results of the simulation using the R Studio software, the optimum values of \( \alpha \), \( \beta \), and \( \gamma \) are presented in Table 2.
Table 2. Parameter value $\alpha$, $\beta$, and $\gamma$

| Stock     | $\alpha$  | $\beta$  | $\gamma$ |
|-----------|-----------|----------|----------|
| PT. X Tbk | 0.886108  | 0.00324419 | 1        |
| PT. Y Tbk | 0.8776647 | 0.005092039 | 0.6494706 |
| PT. Z Tbk | 0.9163035 | 0.002541073 | 0.6529117 |

The smoothing parameter values $\alpha$, $\beta$, and $\gamma$ are then used in the calculation of the prediction model in Equations (1) - (4). The simulation results of stock price predictions is presented in Figure (2)-(4).

Figure 2. Actual data and price prediction of PT. X Tbk.

Figure 3. Actual data and price prediction of PT. Y Tbk

Figure 4. Actual data and price prediction of PT. Z Tbk.

Based on Figures (2)-(4), the multiplicative Holt-Winter method is considered very good in predicting data, because it is able to produce predictive error accuracy (MAPE) values of 1.6%, 1.29%, and respectively. 1.38%. Furthermore, Figure (5) shows the daily stock return values of the three companies.
In the initial stage, an investor only has a capital of 100.000.000 which is stored in a risk-free asset (bank). Investors will use this amount of capital to invest in the stock assets of PT. X Tbk., PT. Y Tbk., and PT. Z Tbk. The initial control value given is $p_i(0) = 0$ and $q_i(0) = 0$, for $i = 1, 2, 3$, and $v(0) = 0$, and given the initial state value $[x_1(0) \ x_2(0) \ x_3(0) \ x_4(0) \ x_5(0)] = [0 \ 0 \ 0 \ 10^8 \ 0]$. Variable $x_1$, $x_2$, and $x_3$ show the amount of capital invested in the three stock assets, while $x_4$ and $x_5$ respectively show the amount of investor capital in bank and loan accounts. The parameters used in the stock portfolio can be seen in Table 3.

| Parameter | Value |
|-----------|-------|
| $r_1$     | $3 \times 10^{-5}$ |
| $r_1$     | $3.1 \times 10^{-4}$ |
| $\alpha$ | $2 \times 10^{-3}$ |
| $\beta$  | $2 \times 10^{-3}$ |
| $d_0(k)$  | $5 \times 10^8$ |
| $x(0)$    | $[0 \ 0 \ 0 \ 10^8 \ 0]^T$ |
| $u(0)$    | 0 |
| $v(0)$    | 0 |
| $N_p$     | 10 |
| $r(k)$    | $3 \times 10^8$ |
| $p_{i\text{max}}$ | $1 \times 10^8$ |
| $q_{i\text{max}}$ | $1 \times 10^8$ |
| $v(k)_{\text{max}}$ | $1 \times 10^8$ |
| $v(k)_{\text{min}}$ | $-1 \times 10^8$ |

Portfolio optimization results are obtained based on the stock price prediction using Holt-Winter’s Exponential Smoothing. The simulation results for control variables in the system are shown in Figures (6)-(12). In Figures (6)-(8), it can be seen that the value of the control variables $p_i(k)$ and $q_i(k)$ for $i = 1, 2, 3$ which is the transfer of capital used to buy and sell shares of the three companies are within the range or the constraints that have been defined. The value of the control variables $p_i(k)$ and $q_i(k)$ are between 0 and the value of the maximum allowable limit, which is 100,000,000. During 90 days of observation, the activity of transferring assets between stocks and risk-free assets (banks) were very volatile. This is influenced by the movement of the stock price prediction.
Figure 6. Value $p_1$ and $q_1$ for PT. X Tbk stocks

Figure 7. Value $p_2$ and $q_2$ for PT. X Tbk stocks

Figure 8. Value $p_3$ and $q_3$ for PT. X Tbk stocks

The simulation results also apply to show the control value of variable $v(k)$ which is the transfer of capital between risk-free assets (banks) and loan account assets. Figure (9) represents the movement of the control variable $v(k)$.

Figure 9. Value of capital loan transfer in portfolio

Figure 10. Changes in total investor capital in stock assets

Figure (9) shows the transfer between the amount of capital in the bank and the amount of funds borrowed by investors. The values of $v(k)$ are in the range $-10^8$ and $10^8$. A positive value at $v(k)$
indicates that investors borrow the amount of capital that can be used to buy shares. Meanwhile, \( v(k) \) is negative, indicating that the investor is paying back the capital he borrowed previously. Next, a chart of changes in the capital owned by investors in each share, bank, and loan is presented (Figures 10 to 11).

Based on Figure (10), it can be seen that the value of daily stock returns affects the amount of capital invested in each stock every day. The MPC controller seeks to reduce the losses that will occur to investors when the share price of one company decreases. Based on Figure (11), it can be seen that changes in risk-free assets and capital loan assets are correlated with each other. The MPC controller acts as a decision maker regarding when is the right time to borrow capital and when is the right time to return the capital. When investors need additional capital to buy shares, the MPC controller decides to borrow a certain amount of capital.

![Figure 11. Changes in total investor capital in risk-free assets and capital loan assets](image)

![Figure 12. Overall change in total investor capital in the portfolio](image)

Based on Figure (12), it can be seen that the capital owned by investors in the portfolio has increased during 90 days of observation. In the initial condition, the amount of capital owned by investors is 100.000.000 and in the final condition, the amount of capital owned by investors moves closer to the expected reference trajectory, which is 300.000.000. On several days, it appears that investors' capital has decreased, this is because on that day the return of some stocks is negative, so that investors experience losses. However, this loss can be covered by the profits in the next several days, so that the overall amount of investor capital increases. The total final capital of investors in the portfolio on the 90 days of observation is 295.950.000.

6. Conclusion
From the analysis of the Holt Winter's Exponential Smoothing method and the MPC controller, the following conclusions are obtained:
1. The Holt Winter's Exponential Smoothing method, provide fairly good stock price prediction results. The results of the stock price prediction of PT. X Tbk., PT. Y Tbk., And PT. Z Tbk. for 90 days resulted in the accuracy of the MAPE prediction error of 1.6%, 1.29%, and 1.38%, respectively.
2. In stock portfolio optimization management, the MPC controller can be applied very well. MPC is able to act as a decision maker regarding the right time and amount for investors to sell or buy shares and borrow or return capital loans in the stock portfolio.
3. The application of MPC in optimizing all capital in a portfolio based on stock price predictions is able to provide satisfactory results. This can be seen through the decisions given by MPC which resulted in the total capital obtained by investors to approach the expected target.
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References
[1] Tandelilin E 2010 Portofolio dan Investasi: Teori dan Aplikasi. Edisi Pertama, Kanisius, Yogyakarta.
[2] Hines W W, Montgomery D C, and Borror D M G C M 2008 Probability and Statistics in Engineering, John Wiley & Sons.
[3] Safitri T, Dwidayati N, & Sugiman 2017 Perbandingan Peramalan Menggunakan Metode Exponential Smoothing Holt Winter dan Arima. Unnes Journal of Mathematics, Vol 6 No. 1.
[4] Fitria I, Alam M S K, Subchan S 2017 Perbandingan Metode ARIMA dan Exponential Smoothing pada Peramalan Harga Saham LQ45 Tiga Perusahaan dengan Nilai Earning Per Share (EPS) Tertinggi. Limits: Journal of Mathematics and Its Applications, Vol 14 Issue 2, Pages 113-125.
[5] Purnawan H, Asfihani T, Adzkiya D, and Subchan 2018 Disturbance compensating model predictive control for warship heading control in missile firing mission. Journal of Physics: Conference Series 1108-012035.
[6] Wang J, Xu C, and Inoue A 2007 An Experiment of Control-theoretical Model in Dynamic Portfolio Management. In Second International Conference on Innovative Computing, Information and Control (ICICIC 2007) (pp. 114-114), IEEE.
[7] Wang L 2009 Model Predictive Control System Design and Implementation using MATLAB. Springer. Melbourne.
[8] Syaifudin W H 2015 Penerapan Model Prediktif Kontrol (MPC) pada Optimasi Portofolio Saham. Prosiding Seminar Nasional Matematika dan Pendidikan Matematika Universitas Negeri Surabaya.
[9] Fitria I 2016 Investment Management Using Portfolio Optimization Stock Price Forecasting. Applied Mathematical Sciences, Vol 10 Issue 48.
[10] Subchan S, and Rahmawati E 2020 Pemodelan dan Optimasi Multi-Tujuan Portofolio Saham dengan Resiko Menggunakan Nadir Compromise Programming. Limits: Journal of Mathematics and Its Applications, 16 (2), 105-116.
[11] Armstrong J S 1937 Longe-Range Forecasting From Crystal Ball to Computer. Second Edition. University of Pennsylvania, United States of America.
[12] Chatfield C, and Yar M 1988 Holt-Winters Forecasting: Some Practical Issues. Journal of The Royal Statistical Society: Series D. Vol. 37, Pages 129-140.
[13] D C Montgomery, C L Jennings, and M Kulahci 2015 Introduction to Time Series Analysis and Forecasting. John Wiley and Sons, Inc, Canada.
[14] A H Hutasyuhu, W Anggraeni, and R Tyasnurita 2014 Pembuatan Aplikasi Pendukung Keputusan untuk Peramalan Persediaan Bahan Baku Produksi Plastik Blowing dan Inject Menggunakan Metode ARIMA di CV. Asia. Jurnal Teknik ITS, Vol. 3, No. 2, pp. A-169-A-174.
[15] Dombrovskiy V V, Dombrovskiy D V, and Lyashenko E A 2004 Investment Portfolio Optimization with Transaction Costs and Constraints Using Model Predictive Control, KORUS 2004 Proceedings The 8th Russian-Korean International Symposium, Vol. 3, Hal. 202-205.
[16] Anonim 2020 Stock Price Data PT. X Tbk, PT. Y Tbk., dan PT. Z Tbk, accessed through www.finance.yahoo.com on April, 20. 2020