Making decisions in national energy markets with bifurcation analysis

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Abstract. A mathematical model for international integration of national energy markets was proposed by references. Lowering the dimension conveniently to 3 dimensions using system dynamics methodology, the dynamical system obtained could present a commutation region proposing that the system is a piecewise smooth system PWS with local bifurcations: Saddle-node and transcritical bifurcations. The bifurcation diagram is interpreted as a map of all possible system prospective scenarios, in this sense, the making decisions over systems have leverage points in its bifurcation parameters. In the national energy markets case, the bifurcations parameters depend on the generation lifetime and the investments rate, then two trend scenarios were defined: the absolute disappearance of the national supply of electricity or convergency at an equilibrium value of the national supply with which it is possible to serve the market. It is concluded that any investment and small useful life of the generation plants is not enough for attending the national electricity demand in the medium and long term.

1. Introduction

One of the main issues of economic activity in any country is their energy market. In this way, models have been developed for making decisions for the energy management, linking their market variables. For example, authors have used statistical techniques [1–5], neural networks [6–9], fuzzy systems [10] and recurrent neural networks [11] for establishing the electricity price. Other techniques have been used for modeling energy markets: system dynamics [12–17] optimization models, equilibrium models and simulation models, as is shown in [18]. Dynamical systems approach for energy markets was presented in [19] and [20].

This paper has analyzed the system of equations obtained for the modeling of an energy market presented by [20] restricted to three dimensions, which means not considering the energy demand as a state variable. The main purpose of this article is showing the results of bifurcations found in the mathematical model of national energy markets proposed by [20], to conclude on its prospective possibilities in the practice.

2. Modeling process

The modeling process developed in this paper for obtaining the equations systems was the proposed in the system dynamics methodology [21]. The causal diagram was taken from [20],
but the levels and flows diagram, which is interpreted from the causal diagram, was modified
with respect to the four dimensional system proposed by [20], as is shown in the Figure 1,
for obtaining a three dimensional system with building capacity, installed capacity and delayed
price as state variables. Following the levels and flows diagram were obtained the mathematical
model, as shown below.

\[
\frac{dx}{dt} = Inv - CC
\]  

(1)

The CC is the building capacity \( x \) over a construction time \( k_1 \) given in years, see Equation (2),
and the investments are a function that depends of the expected return \( ER \) given in MW/year,
the installed capacity \( y \) and an investment rate \( a \) given in year\(^{-1} \), see Equation (3).

\[
CC = \frac{x}{k_1}
\]  

(2)

\[
Inv = \max\{0, a \cdot ER \cdot y\}
\]  

(3)

The investment equation was defined in such a way that if the expected return is negative,
the investor has not interested for investing, while if the expected return is positive, the investor
will be motivated with the opportunity and will do investments in proportion with the installed
capacity and the expected return. The expected return equation is obtained as the percentual
difference between the electricity generation price \( P \) and it cost \( c \), making dimensionless the
expected return, see the Equation (4). It keeps the idea that if \( P > c \) \( (P < c) \), the expected
return is positive (negative).

\[
ER = \frac{P - c}{P}
\]  

(4)
The electricity generation price $P$ was defined so that if the reserve margin $RM$, taken as a dimensionless magnitude, is very high, then the price decreases, but if the reserve margin is very low, then it is sent a signal of expansion of electricity generation, dramatically increasing prices. The selected function is presented in Equation (5).

$$P = p \left( \frac{RM}{q} \right)^{-\beta}$$

(5)

If $RM = q$ in Equation (5), then $P(q) = p$, where $p$ and $q$ are a reference price and a reference reserve margin, respectively. The reference price $p$ is obtained from market values, while reference margin reserve $q$ is obtained from the Equation (6) when the demand is the reference demand, as will be shown below. The $\beta$ parameter corresponds to the elasticity of the price concerning the margin of capacity. The reserve margin $RM$ represents the relationship between supply and demand electricity and has different mathematical functions for its representation. In this paper, the RM was defined percentual, so that the units are corrected and the RM is a dimensionless quantity, see Equation (6). Note that negative values of the RM represents electricity deficit (-1< RM <0), while positive values of the RM represents the surplus of electricity (0< RM).

$$RM = \frac{y - D}{D}$$

(6)

The $D$ demand in MW was represented with the function presented in Equation (7), where $d$ is a reference demand value obtained in the market and the parameter $\epsilon$ corresponds to the elasticity of the demand concerning the delayed price. So, if the delayed price $z$ in $$/kWh$, it is very high, the demand $D$ decreases, while if the delayed price $z$ is low, the demand $D$ takes advantage of the low prices to increase its consumption.

$$D = d(DP)^{-\epsilon}$$

(7)

The time evolution of the installed capacity $y$ is defined as the difference between the generation $CC$ and the depreciation $Dep$, given in MW/year, see Equation (8).

$$\frac{d}{dt} y = CC - Dep$$

(8)

The generation $CC$ was defined in the Equation (2). The depreciation $Dep$ is defined as the quotient between the generation capacity installed $y$ and the average useful life of the generation plants $k_2$, given in years, see Equation (9).

$$Dep = \frac{y}{k_2}$$

(9)

The time evolution of the delayed price or consumer price $z$ is defined by the price change (PC), given in ($$/kWh)/year, see Equation (10).

$$\frac{d}{dt} z = PC$$

(10)

The price change PC is the existing change between the price of generation and the price that the consumer perceives, during the time in which the consumer can perceive these changes, which for the model we have denominated time price adjustment $k_3$, see Equation (11).

$$PC = \frac{P - z}{k_3}$$

(11)
Finally, if the replacements between the presented equations are properly carried out, the differential system that we will call the mathematical model of a national electricity market is obtained, see Equation (12), from which, we will develop the remaining work in this article.

\[
\dot{x} = \max \left\{ 0, a \left[ 1 - \frac{c}{p} \left( \frac{w_x e^{-1/q}}{q} \right)^\beta \right] \cdot y \right\} - \frac{x}{k_1}, \\
\dot{y} = \frac{x}{k_1} - \frac{y}{k_2}, \\
\dot{z} = \frac{1}{k_3} \left[ p \left( \frac{w_x e^{-1/q}}{q} \right)^{-\beta} - z \right],
\]

where \( 0 \leq a \leq 1 \) is the investment rate in new energy capacity, \( c \in \mathbb{R}^+ \cup \{0\} \) is the unit generation cost, \( k_1, k_2, k_3 \in \mathbb{R}^+ \) is the construction time, the average useful life of the generation plants, and the time of adjustment of the real price to the price assumed by the consumer, respectively. The parameter \( p \in \mathbb{R}^+ \cup \{0\} \) is a reference value of the unit price, \( q \in \mathbb{R} \) is a reference value of the reserve margin, \( d \in \mathbb{R}^+ \cup \{0\} \) is a reference value of demand, \( \epsilon \in \mathbb{Q}^+ \cup \{0\} \) is the elasticity of demand with respect to price and \( \beta \in \mathbb{Z}^+ \cup \{0\} \) is the price elasticity with respect to the reserve margin. Finally, a unit review was realized concluding that the system is consistent.

### 3. Results

Different cases were defined for studying the proposed mathematical model in Equation (12).

#### 3.1. Planar case analysis I: \( \beta = 0 \)

Equation (12) can be studied as a planar system taking the price elasticity with respect to the reserve margin \( \beta \) as zero. It happens because with \( \beta = 0 \), the system is desacoupled in the equation \( \dot{z} = \frac{1}{k_3} (p - z) \), whose solution for \( z(0) = p \) is \( z(t) = p \), and in the planar system that follows, Equation (13).

\[
\dot{x} = \max \left\{ 0, a \left[ 1 - \frac{c}{p} \left( \frac{w_x e^{-1/q}}{q} \right)^\beta \right] \cdot y \right\} - \frac{x}{k_1}, \\
\dot{y} = \frac{x}{k_1} - \frac{y}{k_2},
\]

The next proposition is obtained from the Equation (13).

- Proposition 1. If Equation (13) has maximum zero, the system has a stable equilibrium point at the origin, but if the maximum is not zero, which occur when the reference value of the unit price is greater than the unit generation cost: \( p > c \), the origin has a saddle-node bifurcation in \( k_2 = B_1 \), where \( B_1 = p/(a(p - c)) \), such that, If \( k_2 < B_1 \), the origin is stable, and If \( k_2 > B_1 \), the origin is a saddle node.

#### 3.2. Planar case analysis II: \( \beta \neq 0 \) and \( \epsilon = 0 \)

Another way for obtaining a planar system is taking the price elasticity concerning the reserve margin \( \beta \) different to zero, and consider the elasticity of demand concerning price \( \epsilon \) as zero, which define the system in Equation (14). Let \( \epsilon = 0 \) then:

- From Equation (7), the demand \( D \) is the reference demand \( d \): \( D = d(DP)^{-\epsilon} = d \).
- From Equation (6), the reserve margin RM is the reference reserve margin \( q \): RM\( = (y - d)/d = q \).
The last case considered is when $\epsilon$ are defined in the regions $(1)$.

This PWS system has an stable equilibria (virtual if $f$ visible of $\epsilon \cdot k_1$ $(p - z)$, whose solution for $z(0) = z(t) = p$, but with a different planar case:

$$
\dot{x} = \max \{0, ayG\} - \frac{x}{k_1}; \quad G = 1 - \frac{z}{p} \left( \frac{y}{d_q} - \frac{1}{q} \right)^{\beta} 
$$

$$
\dot{y} = \frac{x}{k_1} - \frac{y}{k_2}
$$

(14)

The planar case in Equation (14) satisfies the next proposition.

Proposition 2. Let’s consider $X = (x, y)^T$. When the maximum in the Equation (14) is zero, the system has a stable equilibrium point at the origin, but if the maximum in the Equation (14) is not zero, then the system is a Piecewise Smooth system PWS with commutation region $\Sigma := \{(x, y) \in \mathbb{R}^2 | y = M\}$, such that $M = d + dq(p/c)^{1/\beta}$, defined as Equation (15).

$$
\dot{X} = \begin{cases} 
 f_1(X) & X \in S_1 \\
 f_2(X) & X \in S_2
\end{cases}
$$

where $f_1$ and $f_2$ are the vector fields: $f_1(X) = (-x/k_1, x/k_1 - y/k_2)^T$ and $f_2(X) = (ayG - x/k_1, x/k_1 - y/k_2)^T$ defined in the regions $S_1 = \{(x, y) \in \mathbb{R}^2 | y > M\}$ and $S_2 = \{(x, y) \in \mathbb{R}^2 | y < M\}$. This PWS system has an stable equilibrium (virtual if $q < (p/c)^{1/\beta}$ from the field $f_1$ in the origin and two equilibria $Eq_1(0, 0)$ (visible if $q < (p/c)^{1/\beta}$) and $Eq_2(x_2, y_2)$ visible from field $f_2$, where $x_2 = (k_1/k_2)y_2$ and $y_2 = d + dq(p/c)(1 - (1/(a \cdot k_2)))^{1/\beta}$.

In case when the equilibria $Eq_1(0, 0)$ and $Eq_2(x_2, y_2)$ are visible, the system presents a transcritical bifurcation in $k_2 = B_2$, such that $Eq_1(Eq_2)$ is a stable node (unstable of saddle type) for $k_2 < B_2$ and an unstable of saddle type (stable) for $k_2 > B_2$, where $B_2 = a(1 - (p/c)^{1/\beta})$.

3.3. Case analysis III: $\epsilon \neq 0 \beta \neq 0$

The last case considered is when $\epsilon \neq 0 \beta \neq 0$.

Proposition 3. Let’s consider $X = (x, y, z)^T$. If the differential system in Equation (12) has maximum zero, the system has a stable equilibria point in $(0, 0, (1)^{1/\beta}pq^2)$. But if the maximum in the system Equation (12) is non zero, then the system is a Piecewise Smooth System with commutation region $\Sigma := \{(x, y, z) \in \mathbb{R}^3 | y = M/z^\epsilon\}$, $M = d + dq(p/c)^{1/\beta}$, defined as Equation (16).

$$
\dot{X} = \begin{cases} 
 f_1(X) & X \in S_1 \\
 f_2(X) & X \in S_2
\end{cases}
$$

where the vector fields $f_1(X) = (-x/k_1, x/k_1 - y/k_2, (1/k_3)[pG_1 - z])^T$ with $G_1 = (y \cdot z^\epsilon/(d \cdot q) - 1/q)^{-\beta}$ and $f_2(X) = (ay[1 - (c/p)G_1] - x/k_1, x/k_1 - y/k_2, (1/k_3)[pG_1 - z])^T$ are defined in the regions $S_1 = \{(x, y, z) \in \mathbb{R}^3 | y > M/z^\epsilon\}$ and $S_2 = \{(x, y, z) \in \mathbb{R}^3 | y < M/z^\epsilon\}$. This PWS system has an stable equilibria (virtual if $q < (p/c)^{1/\beta}$) of $f_1$ in $Eq_0 = (0, 0, (1)^{1/\beta}pq^2)$ and two equilibria $Eq_1 = (0, 0, (1)^{1/\beta}pq^2)$ (visible if $q < (p/c)^{1/\beta}$) and $Eq_2 = (x_2, y_2, z_2)$ visible of $f_2$, where: $x_2 = (k_1/k_2)(1/c - 1/(a \cdot c \cdot k_2))^\epsilon \left( d + dq(p/c)(1 - (1/(a \cdot k_2)))^{1/\beta} \right)$, $y_2 = (1/c - 1/(a \cdot c \cdot k_2))^\epsilon \left( d + dq(p/c)(1 - (1/(a \cdot k_2)))^{1/\beta} \right)$, and $z_2 = c(a \cdot k_2/(a \cdot k_2 - 1))$. 

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When $E_{q1}$ y $E_{q2}$ of $f_2$ are visible, the system has a transcritical bifurcation in $k_2 = B_2$, so that $E_{q1}$ ($E_{q2}$) is a stable node (saddle node) for $k_2 < B_2$ and an unstable saddlenode (stable) for $k_2 > B_2$, where $B_2 = \frac{1}{a(1 - \frac{pq \beta}{p \beta p^{\beta}})}$.

Despite its possible to do an analytical proof, a continuation was realized with Auto07p for verifying the mentioned bifurcations, see Figure 2.

Figure 2. Transcritical bifurcation was obtained with Auto07p under the variation of useful life parameter $k_2$. In this figure the bifurcation with respect to the delayed price is shown.

4. Discussion
This section addresses the implications for electricity markets from the mathematical results obtained. For any of the cases studied in the previous section, it is notable that not making an investment or doing it insufficiently, makes the market supply tend to disappear over time. From the propositions, it is established as a condition that the investment is sufficient, that the useful life exceeds a bifurcation value called $B_i$, $i = 1, 2$. Aspects that would make a useful life could not exceed the bifurcation value could be the following: 1) The technology of the generation plants. It is related to the technology type, with the obsolescence time, with inadequate uses or with maintenance programs and 2) The disposition of the investor, represented in the parameter $a$. When the investments are very low or even null, the bifurcation value $B_i$ becomes very large, which means that the useful life of the plants must be very large also, for keeping the installed capacity of electricity generation.

Otherwise, the desirable scenario occurs when there exists enough energy supply for the energy demand, we mean when the supply can attend the peak demand. From the mathematical analysis in the previous section, it occurs when the investment motivation, the reference price, the cost and the reference reserve margin are arranged adequately to make the bifurcation parameter $k_2$ be greater than $B_i$, $i = 1, 2$.

5. Conclusions
A mathematical model was presented for the national electricity markets, built on the methodology of system dynamics. The model is a system of differential equations of the first order, non-linear, non-smooth, with three state variables: the capacity of electrical energy in
construction, the capacity of installed electric power and the price of electricity in the market. The system of differential equations was studied from three (3) cases, which were defined from the elasticities $\epsilon$ and $\beta$. For case I, saddle-node bifurcations was found, while for cases II and III we found a switching region that partitioned the phase space into three regions, one of which is the same switching region. A transcritical bifurcation is observed in cases II and III.

The mathematical analysis leads us to conclude on the importance of the useful life parameter of the power generation plants. If this time is lower to the bifurcation parameter, for whatever reasons, the offer tends to disappear with time, but if the useful lifetime exceeds the bifurcation value, the supply will tend to be placed at a non-zero equilibrium value, guaranteeing a positive offer to meet the demand.

The bifurcation value, for this application, is defined as an array of parameters that include the investor’s motivation (displayed in an investment rate), a reference price for electricity, the cost of electricity and a margin of system reference reserve. From the simulations, it is concluded that a low motivation of the investor, represented by very small investments, leads to the fact that the bifurcation value becomes very large, with which the supply of electric power will tend to disappear with time. About the other parameters in the array, no inferences were made, because they are not taken as indicators in the context of the electricity markets.

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