Experimental measurement of efficiency and transport coherence of a cold atom
Brownian motor in optical lattices

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Brownian motors (BMs) are devices that can rectify noise into work or directed motion in the absence of external forces. They are of interest for the understanding of fundamental principles in statistical physics and thermodynamics, and several studies have shown that they play a crucial part in transport phenomena in nature; see, for example, [1–3]. Since BM’s utilize noise, they can work in regions where the inherent noise is large compared to other interactions. Applications of BM’s, therefore, reach into the nano-scales, where they make ideal tools for powering up nano-machines [4–6]. Recent reviews of the subject can be found in [7–10].

Of particular interest for any motor is the quantification of its efficiency, usually defined as the ratio of produced work to input energy. Due to the peculiar nature of the energy source of BMs, determination of efficiency is not straightforward. There have been several theoretical discussions on the efficiency of BM’s [11–15], and different performance characteristics have been discussed in [16]. We present here experimental measurements of two performance characteristics of a BM realized with ultracold atoms in double optical lattices [17]: the efficiency, that is, the fraction of input power driving the directed motion, and the transport coherence, or the Péclet number, that is, the comparison between the drift and the diffusion. Usually, the efficiency is defined in terms of the amount of work obtained from the motor against a load. As no load is present in our case, we instead follow the convention [11,12] of defining “useful energy” as the energy needed to drive the directed motion of the atoms against friction. It has also been argued that including the dissipation due to friction against the directed motion provides a better definition of efficiency even when a load is present [11].

For a BM to be able to function, it has to have (i) present an asymmetry [18] and (ii) be out of thermal equilibrium [19]. In most cases, the symmetry breaking arises either from a time-asymmetric periodic driving force with zero average (rocked ratchet), or by flashing an asymmetric potential (flashed ratchet). However, as shown in our system [17], rectification can be achieved by switching between two symmetric potentials.

The model for the BM used in our experiment was introduced in [20]. Briefly, particles with mass m move in two symmetric potentials, $U_1 = A_1 \cos(kx)$ and $U_2 = A_2 \cos(kx + \phi)$, phase shifted by $\phi$, and are randomly transferred between the two with unequal transfer rates $\Gamma_{1\rightarrow 2} \neq \Gamma_{2\rightarrow 1}$. In addition, the particles experience a friction force $-\alpha_2 \dot{x}$ and a diffusive force $\xi_i(t)$ in either lattice $i=1,2$. This gives the equations of motion

$$ m\ddot{x} = -\nabla_x U_i(x) - \alpha_i \dot{x} + \xi_i(t). \quad (1) $$

Here, $\xi_i(t)$ satisfies the relations $\langle \xi_i(t) \rangle = 0$ and $\langle \xi_i(t)\xi_i(t') \rangle = 2 D_i \delta(t-t')$. Thus the atom is both subject to work from the potential $U_i$ and to fluctuations and dissipation given by the diffusion coefficient $D_i$ and the friction coefficient $\alpha_i$.

For an atom moving in a single periodic potential, the long-time average of the work goes to zero, and the atoms reach a steady state with kinetic temperature $D_i/\alpha_i$. This changes when it is transferred between the potentials, changing instantaneously its potential energy. The total work on an atom is therefore equal to the changes in potential energy summed over all jumps between the potentials. For identical potentials ($A_1 = A_2$, $\phi = 0$), no energy is gained by an atom transferred between the potentials; see Fig. 1a). In this situation, both potentials satisfy the symmetry condition $U_i(-x) = U_i(x)$, which entails that $\langle \dot{x} \rangle = -\langle \dot{x} \rangle$, and hence no BM effect is possible [21]. Introducing a nonzero $\phi$ between the lattices, the system still possesses glide reflection [22], but because of the unequal transfer rates between the potentials, there is no symmetry condition requiring $\langle \dot{x} \rangle = 0$, and therefore there will in general be a rectification [18]. An exception is the point $\phi = \pi$, where again $U_i(-x) = U_i(x)$, leading to zero current, with the input energy gained from the transfer between potentials only appearing as a heating of the atoms.
The experiment has been described in detail in [17, 24, 25]. In short, we use laser cooling to trap and cool cesium atoms and transfer them into a double optical lattice [24, 25]. These are potentials realized from the interference pattern of laser beams due to a second-order interaction between the induced atomic dipole moment and the periodic light fields [26]. The two potentials correspond to two different hyperfine levels, $F = 3$ and $F = 4$, of the electronic ground state of cesium. Each atom will be transferred between the two potentials at random times through optical pumping, with rates for transfer, scattering, and cooling set by the parameters of the laser fields (intensity and detuning). To collect data, we use absorption imaging to measure the mean momentum $\vec{p}$ as well as the size of the atomic cloud. The imaging is done in the horizontal plane to avoid any effects of gravity [27]. To access the mean momentum spread (the kinetic temperature) $\langle \delta \vec{p}^2 \rangle$ in Eq. (4), we use a time-of-flight technique that enables fast and accurate measurements of the distribution of the momentum, $\delta \vec{p}^2$ [28].

In the experiment, we adjust the potential depths by controlling the intensities in the lattice beams such that $A_1 = A_2$. To assess the quantity $\langle \delta \vec{p}^2 \rangle$, we study the system at $\phi = 0$, where the potentials are identical and there is no BM effect since the transfer between the potentials does not change the energy of the system ($P_{\text{in}} = 0$). From the energy balance (2), we then obtain, in agreement with the equipartition theorem,

$$\frac{E_{\text{kin}}}{\phi = 0} = \left. \frac{\delta \vec{p}^2}{2m} \right|_{\phi = 0} = \frac{N \vec{D}}{2 \overline{\alpha}}$$

The association of $\overline{\vec{D}} / \overline{\alpha}$ with the kinetic temperature at $\phi = 0$ assumes that the diffusion and friction constants are independent of the relative phase of the lattices. Following the standard model of Sisyphus cooling [29], our model (1) assumes that diffusion and friction are spatially homogeneous. It should be noted, though, that in a more accurate model these coefficients are dependent on the position $x$ of the atom in the lattice. The spatial distribution of atoms in either lattice will have some dependence on $\phi$, which will translate into a dependence on the spatially averaged friction and diffusion coefficients. This is ignored in our model, introducing a degree of approximation in Eq. (1) for efficiency.

Absorption images were taken for five different potential depths. In Fig. (2) we show typical raw data for atoms kept 150 ms in the lattices. A clear drift is seen in images 2 and 4, while images 1, 3, and 5 show no drift, as expected. Images such as Fig. (2) have been taken for $0 \leq \phi \leq 2\pi$ for potential depths between 40 and 200 $\mu K$, and the analyzed data can be seen in Fig. (3) in terms of the drift velocity. The induced drifts are expected to be symmetric around $\phi = \pi$. However, slightly larger drifts are observed for $\phi = 2\pi / 3$ than for $\phi = 4\pi / 3$, most likely due to experimental limitations in the alignment and the intensity balance of the lattice beams. Also shown in Fig. (3) is the kinetic temperature for the same parameters. We find that the baseline of the kinetic temperature increases with the potential depth, while the amplitude of its variation with $\phi$ is roughly unchanged. As discussed earlier, the $\phi$-dependent kinetic temperature is represented by the second term on the right-hand side of Eq. (2),

$$P_{\text{in}} = \frac{\overline{\alpha} \overline{p^2}}{m^2} + \frac{\overline{\alpha \delta p^2}}{m^2} - \frac{N \overline{D}}{m}.$$
of Eq. (2), while the baseline (or kinetic temperature at \( \phi = 0 \)) is represented by the third term. Hence their difference is the variation of the kinetic temperature with \( \phi \). Our data show that this variation is approximately the same for different potential depths. Hence, the greater efficiency for larger potential depths is mainly due to the increase in the drift momentum. Using the data from Fig. 3 in Eq. (1), we obtain the efficiency as a function of \( \phi \), where the maximum efficiency is close to 0.3%; see Fig. 4.

An alternative way to characterize the rectified motion is by the coherence of the transport, where the linear transport is compared to the diffusion. This can be quantified using the Péclet number \([16]\),

\[
\text{Pe} \equiv \frac{|\langle \delta x \rangle|}{\tilde{D}_{\text{eff}} t}, \tag{6}
\]

where \( l \) is a characteristic length of the system, in our case the lattice constant, and \( \tilde{D}_{\text{eff}} \) is the effective spatial diffusion given by

\[
\tilde{D}_{\text{eff}} \equiv \lim_{t \to +\infty} \frac{\langle x^2(t) \rangle - \langle x(t) \rangle^2}{2t}. \tag{7}
\]

For atoms in dissipative optical lattices, where thermal fluctuations play an important role, \( \tilde{D}_{\text{eff}} \) becomes the spatial diffusion constant \( \tilde{D} = \langle (\delta x(t) - \delta x(0))^2 \rangle/(2t) \), where \( \delta x(t) = x(t) - \langle x(t) \rangle \) \([90]\). This quantity can be calculated from the expansion of the atomic cloud in the optical lattices, where the size of the cloud is given by

\[
\sigma_t = \sqrt{\sigma_0^2 + 2 \tilde{D} t}, \tag{8}
\]

with \( \sigma_t \) the root-mean-square radius at time \( t \).

In order to quantify the performance in terms of the Péclet number, series of absorption images of the time evolution of the atomic cloud, such as shown in Fig. 5, have been taken. The phase is set to achieve maximum drift for \( \phi = 2\pi/3 \). In Fig. 6(a), a series of such images have been analyzed and the drift is plotted against the holding time in the lattice. In Fig. 6(b), the width of the sample is shown against the holding time, from which the diffusion constant \( \tilde{D}_{\text{eff}} \) can be extracted by fitting to Eq. (8).

Combining the result with the measured average velocities of the sample, according to Eq. (6), gives the Péclet number for different potential depths; see Fig. 7.

In conclusion, we have adopted existing theory and presented experimental results for two measures of the performance of a Brownian motor, namely the efficiency and the Péclet number, in a system of ultracold atoms in a double optical lattice. The results indicate trends that give higher efficiency and transport coherence for deeper potentials, and are in agreement with the values of the Péclet number that were predicted for similar systems \([91]\). Although our BM prototype differs from other BM’s, the fundamental principles are the same, and hence these kinds of characteristic measurements allow for interesting comparisons between BM systems from
FIG. 6: (Color online) (a) Position of the center of mass of the atomic sample. (b) Root-mean-square radius of the atomic sample as a function of holding time for four different potential depths. The center of mass moves linearly as expected [17], and indicates faster drifts for higher potential depths. The size of the cloud grows with time due to diffusion according to Eq. (8).

FIG. 7: (Color online) The measured Pélet number, Eq. (6), as a function of potential depth. The data indicates a greater coherence in the transport for higher potential depths.

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