Reconstructing the Cosmic Expansion History up to Redshift $z = 6.29$ with the Calibrated Gamma-Ray Bursts

Hao Wei

Department of Physics, Beijing Institute of Technology, Beijing 100081, China

Shuang Nan Zhang

Department of Physics and Tsinghua Center for Astrophysics, Tsinghua University, Beijing 100084, China

Key Laboratory of Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Physics Department, University of Alabama in Huntsville, Huntsville, AL 35899, USA

ABSTRACT

Recently, Gamma-Ray Bursts (GRBs) were proposed to be a complementary cosmological probe to type Ia supernovae (SNIa). GRBs have been advocated to be standard candles since several empirical GRB luminosity relations were proposed as distance indicators. However, there is a so-called circularity problem in the direct use of GRBs. Recently, a new idea to calibrate GRBs in a completely cosmology independent manner has been proposed, and the circularity problem can be solved. In the present work, following the method proposed by Liang et al., we calibrate 70 GRBs with the Amati relation using 307 SNIa. Then, following the method proposed by Shafieloo et al., we smoothly reconstruct the cosmic expansion history up to redshift $z = 6.29$ with the calibrated GRBs. We find some new features in the reconstructed results.

PACS numbers: 98.80.Es, 95.36.+x, 98.70.Rz, 98.80.-k

* email address: haowei@bit.edu.cn
I. INTRODUCTION

The current accelerated expansion of our universe \cite{1} has been one of the most active fields in modern cosmology since its discovery in 1998 from the observations of type Ia supernovae (SNIa) \cite{2}. Later, the observations of cosmic microwave background (CMB) anisotropy \cite{3} and large-scale structure \cite{4} confirmed this discovery. Although today there are already many observational methods, SNIa have been proved to be one of the most powerful tools to probe this mysterious phenomenon. In 1992, the famous Phillips relation was found \cite{5}, which claimed that for nearby SNIa there exists a clear correlation between their intrinsic brightness at maximum light and the duration of their light curve. Since then, some empirical technics \cite{6, 7, 8} have been developed to use the Phillips relation to make SNIa into standard candles. See e.g. \cite{9, 10} for brief historical reviews.

However, SNIa are plagued with extinction from the interstellar medium. Hence, the currently maximum redshift of SNIa is only about \( z \approx 1.755 \). As argued in \cite{10}, the observations at \( z > 1.7 \) are fairly important to distinguish cosmological models. On the other hand, the redshift of the last scattering surface of CMB is \( z \simeq 1090 \). So, the observations at intermediate redshift are important. Recently, Gamma-Ray Bursts (GRBs) were proposed to be a complementary probe to SNIa. So far, GRBs are the most intense explosions observed in our universe. Their high energy photons in the gamma-ray band are almost immune to dust extinction. Up to now, there are many GRBs observed at \( 0.1 < z \leq 8.1 \), whereas the maximum redshift of GRBs is expected to be 10 or even larger \cite{11}. Strictly speaking, GRBs are not standard candles, with radiated energies spanning several orders of magnitude. But the use of empirical luminosity correlations to standardize them as distance indicators has been proposed by several authors. In some sense, this is reminiscent of the case of the Phillips relation for SNIa. These empirical correlations include the Amati relation \cite{12}, those derived from it (for examples, the Ghirlanda relation \cite{13}, the Yonetoku relation \cite{14}, the Liang-Zhang relation \cite{15}, the Firmani relation \cite{16}), and others \cite{17, 18, 19, 20}. Therefore, the so-called GRB cosmology emerges recently. We refer to e.g. \cite{11, 21, 22, 23} for comprehensive reviews.

To our knowledge, in \cite{24} GRBs were used to constrain \( \Omega_m \) and dark energy for the first time. Similar works also include \cite{25} for examples. However, there is a so-called circularity problem in the direct use of GRBs \cite{21}. In this case, to calibrate the empirical GRB luminosity relations, one need to assume a particular cosmological model with some model parameters \textit{a priori}, mainly due to the lack of a set of low redshift GRBs at \( z < 0.1 \) which are cosmology independent. When one uses these “calibrated” GRBs (which are actually cosmological model dependent) to constrain cosmological models, the circularity problem occurs. To alleviate the circularity problem, some statistical methods have been proposed, such as the scatter method \cite{26}, the luminosity distance method \cite{26}, and the Bayesian method \cite{27}. These methods were used extensively in the literature \cite{15, 28, 29}. However, they still cannot solve the circularity problem completely. Another method trying to avoid the circularity problem was proposed in \cite{30}, in which the parameters of the empirical GRB luminosity relation and the cosmological model parameters were fitted to the observational data simultaneously. However, for \textit{any} given cosmological model, this method can always obtain some parameters for the cosmological model and the empirical GRB luminosity relation. In this sense, \textit{any} cosmological model is “viable” (except for a few obviously absurd models). So, it cannot be used to rule out any cosmological model. Therefore, it is also an unsatisfactory method to solve the circularity problem completely.

To overcome the circularity problem completely, one should calibrate GRBs in a cosmology independent manner. Due to the lack of a set of low redshift GRBs at \( z < 0.1 \) which are cosmology independent, it was proposed to calibrate the empirical GRB luminosity relation using a sufficient number of GRBs within a small redshift bin centered around any redshift \cite{31} (see also e.g. \cite{21}). However, this method might be unrealistic, since the current sample of observed GRBs is not large enough. Recently, a new idea to calibrate GRBs in a completely cosmology independent manner has been proposed in \cite{32, 53} independently. Similar to the case of calibrating SNIa as secondary standard candles by using Cepheid variables which are primary standard candles, we can also calibrate GRBs as standard candles with a large amount of SNIa. This idea of the distance ladder was also briefly mentioned in \cite{54}. However, in fact, \cite{54} used the \( \Lambda \)CDM model rather than SNIa data themselves to calibrate GRBs and hence the calibration is still cosmology dependent actually. In \cite{52}, the GRB luminosity relation was calibrated with the empirical formula of the luminosity distance of SNIa, while the physical meaning of this empirical formula is still unclear and its reliability should be tested carefully. Instead, in \cite{53} the GRB luminosity relation was calibrated with the interpolated distance moduli from the Hubble diagram of SNIa themselves. We refer
to the original papers [32, 33] for details. Using these cosmology independently calibrated GRBs, we can constrain the parameters of cosmological models without circularity problem. In fact, the constraints on ΛCDM model and $w_0 - w_a$ parameterized dark energy model have been considered in [32, 33, 35].

In the present work, we will calibrate GRBs to reconstruct the cosmic expansion history up to redshift $z = 6.29$. The calibration method of GRB relation adopted here is the one proposed in [33]. However, we will use the datasets of SNIa and GRBs which are larger than the ones used in [33]. So, more calibrated high redshift GRBs can be available with smaller uncertainties. In Sec. II we use 38 GRBs at $z < 1.4$ to calibrate the Amati relation while the distance moduli of these 38 low redshift GRBs are interpolated from 307 SNIa data [36, 37]. Then, the distance moduli of 32 GRBs whose $z > 1.4$ can be derived from the calibrated Amati relation. In Sec. III following the method proposed in [38, 39], we smooth 307 SNIa data and 32 calibrated GRBs data whose $z > 1.4$ to reconstruct the cosmic expansion history up to $z = 6.29$. Conclusion and discussions are briefly given in Sec. IV.

II. CALIBRATING GRBS WITH AMATI RELATION

In [33], the authors used 192 SNIa compiled by Davis et al. [40] to interpolate the distance moduli of low redshift GRBs. Recently, the Supernova Cosmology Project (SCP) collaboration released their latest 307 SNIa dataset [36, 37]. This so-called Union compilation is the currently largest SNIa dataset. Obviously, a larger SNIa dataset could bring a better interpolation. On the other hand, in [33] the 69 GRBs compiled by Schaefer [10] have been used. As in [10], the derived distance moduli of high redshift GRBs in [33] are the weighted average of all available distance moduli from five GRB luminosity relations. For each GRB luminosity relation, only about a dozen GRBs at $z < 1.4$ can be used to calibrate the corresponding relation [33]. Instead, in the present work, we calibrate GRBs only with the Amati relation, so that we can use a larger GRBs dataset for single GRB luminosity relation. Here, we adopt the 70 GRBs compiled by Amati et al. [41]. Notice that in [42] there is another 76 GRBs compilation which heavily overlaps the one of [41]. However, since the fluence data given in [42] are not in the form of bolometric fluence, they are not convenient for our computing. As in [33], we choose $z = 1.4$ to be the divide line to separate GRBs.
into two groups, since in the 307 SNIa dataset of [36] there is only one SNIa (2003ak) whose $z > 1.4$. In the 70 GRBs compiled by Amati et al. [41], there are 38 GRBs at $z < 1.4$ and 32 GRBs at $z > 1.4$. The maximum redshift of these 70 GRBs is $z = 6.29$ for GRB 050904.

Several years ago, Amati et al. found the $E_{p,i} - E_{iso}$ correlation in GRBs as $E_{p,i} = K \times E_{iso}^m$, by using 12 GRBs with known redshifts [12], where $E_{p,i} = E_{p,obs} \times (1 + z)$ is the cosmological rest-frame spectral peak energy; the isotropic-equivalent radiated energy is given by

$$E_{iso} = 4\pi d_L^2 S_{bolo}(1 + z)^{-1},$$

in which $S_{bolo}$ is the bolometric fluence of gamma rays in the GRB at redshift $z$, and $d_L$ is the luminosity distance of the GRB. Later, Amati et al. have updated it in [43] and [41]. Up to now, some theoretical interpretations have been proposed for the Amati relation [21]. It might be geometrical effects due to the jet viewing angle with respect to a ring-shaped emission region [44], or with respect to a multiple sub-jet model structure [45]. An alternative explanation of the Amati relation is related to the dissipative mechanism responsible for the prompt emission [46]. For convenience, similar to [10], we can rewrite the Amati relation as

$$\log \frac{E_{iso}}{\text{erg}} = \lambda + b \log \frac{E_{p,i}}{300 \text{ keV}},$$

where log indicates the logarithm to base 10, whereas $\lambda$ and $b$ are constants to be determined. In the literature, the Amati relation was calibrated with the $E_{iso}$ computed by assuming a $\Lambda$CDM cosmology with particular model parameters. As mentioned above, this is cosmology dependent and the circularity problem follows. Here, we instead use the method proposed in [33] to calibrate the Amati relation in a cosmology independent manner.

As the first step, we derive the distance moduli for the 38 low redshift ($z < 1.4$) GRBs of [41] by using cubic interpolation from the 307 SNIa compiled in [36]. We present the interpolated distance moduli of these 38 GRBs in the left panel of Fig. 1. The corresponding error bars are also plotted. As in [33], when the cubic interpolation is used, the error of the distance modulus $\mu$ for the GRB at redshift $z$ can be calculated by

$$\sigma_\mu = \left( \sum_{i=1}^{4} A_i^2 \sigma_{\mu,i}^2 \right)^{1/2},$$

where $A_i \equiv \prod_{j \neq i} (z_j - z) / \prod_{j \neq i} (z_j - z_i)$,

in which $j$ runs from 1 to 4 but $j \neq i$; on the other hand, $\epsilon_{\mu,i}$ are the errors of the nearby SNIa whose redshifts are $z_i$. Then, by using the well-known

$$\mu = 5 \log d_L + 25,$$

one can convert distance modulus $\mu$ into luminosity distance $d_L$ (in units of Mpc). From Eq. (1) with the corresponding $S_{bolo}$ given in Table 1 of [41], we can derive the $E_{iso}$ for these 38 GRBs at $z < 1.4$. We present them in the right panel of Fig. 1 whereas the $E_{p,i}$ for these 38 GRBs at $z < 1.4$ are read from Table 1 of [41]. Also, we present the errors for these 38 GRBs at $z < 1.4$, by simply using the error propagation. From Fig. 1, one can see that the intrinsic scatter is dominating over the measurement errors. Therefore, as in [10, 33], the bisector of the two ordinary least squares [47] will be used. Following the procedure of the bisector of the two ordinary least squares described in [47], we find the best fit to be

$$b = 1.725 \quad \text{and} \quad \lambda = 52.7322,$$

with 1σ uncertainties

$$\sigma_b = 0.010 \quad \text{and} \quad \sigma_\lambda = 0.0065.$$

The best-fit calibration line Eq. (2) with $b$ and $\lambda$ in Eq. (4) is also plotted in the right panel of Fig. 1.

Next, we extend the calibrated Amati relation to high redshift, namely $z > 1.4$. Since the $E_{p,i}$ for the 32 GRBs at $z > 1.4$ are given in Table 1 of [41], we can derive the $E_{iso}$ from the calibrated Amati relation Eq. (2) with $b$ and $\lambda$ in Eq. (4). Then, we derive the distance moduli $\mu$ for these 32 GRBs at $z > 1.4$ using

$$\sigma_\mu = \left( \sum_{i=1}^{4} A_i^2 \sigma_{\mu,i}^2 \right)^{1/2},$$

where $A_i \equiv \prod_{j \neq i} (z_j - z) / \prod_{j \neq i} (z_j - z_i)$,
Eqs. (1) and (3) while their $S_{\text{bolo}}$ can be read from Table 1 of [41]. On the other hand, the propagated uncertainties are given by [10]

$$\sigma_\mu = \left[ \left( \frac{5}{2} \sigma_{\log E_{\text{iso}}} \right)^2 + \left( \frac{5}{2 \ln 10} \sigma_{S_{\text{bolo}}} \right)^2 \right]^{1/2},$$

where

$$\sigma^2_{\log E_{\text{iso}}} = \sigma^2_b \log \left( \frac{E_{p,i}}{300 \text{ keV}} \right)^2 + \left( b \frac{\sigma_{E_{p,i}}}{\ln 10} \right)^2 + \sigma^2_{E_{\text{iso,sys}}},$$

in which $\sigma_{E_{\text{iso,sys}}}$ is the systematic error and it accounts the extra scatter of the luminosity relation. As in [10], by requiring the $\chi^2/dof$ of the 38 points at $z < 1.4$ in the $\log E_{p,i}/(300 \text{ keV}) - \log E_{\text{iso}}/\text{erg}$ plane about the best-fit calibration line to be unity, we find that

$$\sigma^2_{E_{\text{iso,sys}}} = 0.184.$$

We present the derived distance moduli $\mu$ with 1σ uncertainties for these 32 GRBs at $z > 1.4$ in Table I and Fig. 2. It is worth noting that they are obtained in a completely cosmology independent manner.

![Fig. 2: The Hubble diagram of 307 SNIa (black diamonds) and 32 high redshift GRBs (red stars) whose distance moduli are derived by using the calibrated Amati relation. The dashed line indicates $z = 1.4$.](image)

**III. SMOOTHLY RECONSTRUCTING THE COSMIC EXPANSION HISTORY WITH THE CALIBRATED GRBS**

Following the well-known procedure which is frequently used in the analysis of large-scale structure [48], Shafieloo et al. proposed a new method in [38] to smooth noisy data directly using a Gaussian smoothing
function, rather than the top hat smoothing function. The iterative method calculates the $\ln d_L(z)$ as

$$\ln d_L(z)_n^* = \ln d_L(z)_{n-1}^* + N(z) \sum_i \left[ \ln f_{\text{obs}}^{i+1}(z_i) - \ln f_{\text{obs}}^{i-1}(z_i) \right] \exp \left[ -\frac{\ln^2 \left( \frac{1+z}{1+z_i} \right)}{2\Delta^2} \right],$$  \hspace{1cm} (9)

where $\Delta$ is a constant to be given priorly, and the normalization parameter $N(z)$ is given by

$$N(z)^{-1} = \sum_i \exp \left[ -\frac{\ln^2 \left( \frac{1+z}{1+z_i} \right)}{2\Delta^2} \right].$$  \hspace{1cm} (10)

In this section, following the method proposed in [38, 39], we smooth the 307 SNIa data and the 32 calibrated GRBs data at $z > 1.4$ to reconstruct the cosmic expansion history up to $z = 6.29$.

As is well known, the luminosity distance

$$d_L = c (1+z) \int_0^z \frac{dz}{H(z)},$$  \hspace{1cm} (11)

where $H(z)$ is the Hubble parameter, and $c = 2.9979 \times 10^{10}$ cm/s is the speed of light. For convenience, we introduce the dimensionless luminosity distance

$$D_L \equiv (1+z) \int_0^z \frac{dz}{E(z)},$$  \hspace{1cm} (12)

where $E \equiv H/H_0$ and $H_0 = H(z=0) = 100 h$ km/s/Mpc is the Hubble constant. Thus, $d_L = cH_0^{-1}D_L$ and consequently

$$\mu = 5 \log d_L + 25 = 5 \left( \log \frac{cH_0^{-1}}{\text{Mpc}} + \log D_L \right) + 25$$

$$= 5 \log f + 42.3841 = \frac{5}{\ln 10} \ln f + 42.3841,$$  \hspace{1cm} (13)

where $f \equiv D_L/h$ and hence $\ln D_L = \ln f + \ln h$. On the other hand, $\ln d_L = \ln f + \ln 2997.9$. Similar to [39], we can rewrite Eq. (9) as

$$\ln f(z)_n^* = \ln f(z)_{n-1}^* + N(z) \sum_i \left[ \ln f_{\text{obs}}^{i+1}(z_i) - \ln f_{\text{obs}}^{i-1}(z_i) \right] \exp \left[ -\frac{\ln^2 \left( \frac{1+z}{1+z_i} \right)}{2\Delta^2} \right],$$  \hspace{1cm} (14)

where $\ln f_{\text{obs}} = (\mu_{\text{obs}} - 42.3841)(\ln 10)/5$ from Eq. (13). When $n = 1$,

$$\ln f(z)_1^* = \ln f(z)_0^* + N(z) \sum_i \left[ \ln f_{\text{obs}}^{i+1}(z_i) - \ln f_{\text{obs}}^{i-1}(z_i) \right] \exp \left[ -\frac{\ln^2 \left( \frac{1+z}{1+z_i} \right)}{2\Delta^2} \right]$$

$$= \ln D_L(z)_0^* + N(z) \sum_i \left[ \ln f_{\text{obs}}^{i+1}(z_i) - \ln D_L(z)_0^* \right] \exp \left[ -\frac{\ln^2 \left( \frac{1+z}{1+z_i} \right)}{2\Delta^2} \right],$$  \hspace{1cm} (15)

where $D_L(z)_0^*$ is the dimensionless luminosity distance of the initial guess background model. The $\chi^2$ at any iteration is calculated as

$$\chi^2_n = \sum_i \frac{[\mu(z_i)_n - \mu_{\text{obs}}^{i+1}(z_i)]^2}{\sigma_{\mu_{\text{obs}},i}^2}. $$  \hspace{1cm} (16)
The best fit is at the minimum $\chi^2_n$. Note that the summation is over the 307 SNIa and 32 calibrated GRBs at $z > 1.4$. From Eq. (11) and $d_L = cH_0^{-1}D_L = fc/100$, we can find the Hubble parameter as

$$H(z) = \left\{ \frac{d}{dz} \left[ \frac{d_L(z)}{c(1+z)} \right] \right\}^{-1} = \left\{ \frac{d}{dz} \left[ f(z) \right] \right\}^{-1}.$$

Then, the deceleration parameter $q(z)$ and the total equation-of-state parameter $w_{\text{tot}}$ can be given by

$$q(z) = (1+z) \frac{H'(z)}{H(z)} - 1,$$

$$w_{\text{tot}}(z) = -1 + \frac{2}{3} (1+z) \frac{H'(z)}{H(z)},$$

where a prime denotes a derivative with respect to $z$. We intensively refer to the original papers [38, 39] for technical details.

As shown in [38], the reconstructed results are not sensitive to the chosen value of $\Delta$ and the initial guess model. So, we choose $\Delta = 0.55$ and use the flat $\Lambda$CDM model with $\Omega_{m0} = 0.15$ as the initial guess model (notice that $\Omega_{m0} = 0.15$ is the best fit of [56] and [41], which also fit $\Lambda$CDM model to GRBs data). For the flat $\Lambda$CDM model, $E(z) = \left[ \Omega_{m0}(1+z)^3 + (1 - \Omega_{m0}) \right]^{1/2}$. In Fig. 3 we present the computed $\chi^2$ for the reconstructed results at each iteration using 307 SNIa and 32 calibrated high redshift GRBs at $z > 1.4$. It is easy to see that the $\chi^2$ goes to its minimum value very fast at just fifth iteration (similar to the case of the first reference in [38]). The corresponding $\chi^2_{\min} = 322.045$. In the top-left panel of Fig. 4 we present the three reconstructed $\mu(z)$ lines with the likelihood within $1\sigma$, whereas the best-fit result with $\chi^2_{\min}$ is indicated by a blue solid line. In fact, these three reconstructed $\mu(z)$ lines cannot be significantly distinguished. From Eqs. (17)–(19), the $H(z)$, $q(z)$ and $w_{\text{tot}}(z)$ according to these three $\mu(z)$ lines are also plotted in the other panels of Fig. 4. From Fig. 4 we read that $H_0 = 68.817$ km/s/Mpc, $q_0 = -0.587$ and $w_{\text{tot}}(z = 0) = -0.724$. Interestingly, while the results are familiar at relatively low redshifts which cover the redshift range of SNIa, the reconstructed $H(z)$, $q(z)$ and $w_{\text{tot}}(z)$ decrease at high redshifts which are in the redshift range of GRBs. The $w_{\text{tot}}$ crossed the phantom divide $-1$.
| GRB      | z    | \( S_{\text{iso}} \) (10\(^{-5}\) erg cm\(^{-2}\)) | \( E_{\text{p},i} \) (keV) | \( \mu \)  |
|---------|------|------------------------|------------------|--------|
| 050318  | 1.44 | 0.42 ± 0.03            | 115 ± 25         | 44.25 ± 1.58 |
| 010222  | 1.48 | 14.6 ± 1.5             | 766 ± 30         | 43.97 ± 1.53 |
| 060418  | 1.489| 2.3 ± 0.5              | 572 ± 143        | 45.43 ± 1.60 |
| 030328  | 1.52 | 6.4 ± 0.6              | 328 ± 55         | 43.29 ± 1.56 |
| 070125  | 1.547| 13.3 ± 1.3             | 934 ± 148        | 44.47 ± 1.56 |
| 040912  | 1.563| 0.21 ± 0.06            | 44 ± 33          | 43.26 ± 2.08 |
| 090123  | 1.6  | 35.8 ± 5.8             | 1724 ± 466       | 44.56 ± 1.61 |
| 090510  | 1.619| 2.6 ± 0.4              | 423 ± 42         | 44.79 ± 1.54 |
| 080319C | 1.95 | 1.5 ± 0.3              | 906 ± 272        | 46.94 ± 1.63 |
| 030226  | 1.98 | 1.3 ± 0.1              | 289 ± 66         | 44.97 ± 1.59 |
| 000926  | 2.07 | 2.6 ± 0.6              | 310 ± 20         | 44.38 ± 1.53 |
| 011211  | 2.14 | 0.5 ± 0.06             | 186 ± 24         | 45.24 ± 1.55 |
| 071020  | 2.145| 0.87 ± 0.4             | 1013 ± 160       | 47.81 ± 1.56 |
| 050922C | 2.198| 0.47 ± 0.16            | 415 ± 111        | 46.83 ± 1.61 |
| 060124  | 2.296| 3.4 ± 0.5              | 784 ± 285        | 45.90 ± 1.67 |
| 021004  | 2.3  | 0.27 ± 0.04            | 266 ± 117        | 46.63 ± 1.73 |
| 051109A | 2.346| 0.51 ± 0.05            | 539 ± 200        | 47.28 ± 1.68 |
| 060908  | 2.43 | 0.73 ± 0.07            | 514 ± 102        | 46.82 ± 1.57 |
| 050820  | 2.612| 6.4 ± 0.5              | 1325 ± 277       | 46.30 ± 1.58 |
| 030429  | 2.65 | 0.14 ± 0.02            | 128 ± 26         | 46.08 ± 1.57 |
| 050603  | 2.821| 3.5 ± 0.2              | 1333 ± 107       | 47.02 ± 1.53 |
| 051401  | 2.9  | 1.9 ± 0.4              | 467 ± 110        | 45.75 ± 1.59 |
| 020124  | 3.2  | 1.2 ± 0.1              | 448 ± 148        | 46.25 ± 1.65 |
| 060526  | 3.21 | 0.12 ± 0.06            | 105 ± 21         | 46.03 ± 1.57 |
| 030323  | 3.37 | 0.12 ± 0.04            | 270 ± 113        | 47.84 ± 1.72 |
| 071214  | 3.42 | 0.87 ± 0.11            | 685 ± 133        | 47.45 ± 1.57 |
| 060707  | 3.425| 0.23 ± 0.04            | 279 ± 28         | 47.21 ± 1.54 |
| 060115  | 3.53 | 0.25 ± 0.04            | 285 ± 34         | 47.19 ± 1.54 |
| 060206  | 4.048| 0.14 ± 0.03            | 394 ± 46         | 48.54 ± 1.54 |
| 000131  | 4.5  | 4.7 ± 0.8              | 987 ± 416        | 46.54 ± 1.72 |
| 060927  | 5.6  | 0.27 ± 0.04            | 475 ± 47         | 48.47 ± 1.54 |
| 050904  | 6.29 | 2.0 ± 0.2              | 3178 ± 1094      | 49.96 ± 1.66 |

**TABLE I:** The numerical data of 32 calibrated GRBs at \( z > 1.4 \). The first 4 columns are read from Table 1 of [41], whereas the last column is derived by using the calibrated Amati relation. See text for details.
FIG. 4: The top-left panel is the three reconstructed \( \mu(z) \) lines with the likelihood within 1\( \sigma \), whereas the best-fit result with \( \chi^2_{\text{min}} \) is indicated by a blue solid line. In fact, these three reconstructed \( \mu(z) \) lines cannot be significantly distinguished. We also plot the \( \mu^{\text{obs}} \) of 307 SNIa (black diamonds) and 32 calibrated GRBs (red stars) in the \( \mu(z) \) panel for comparison. From Eqs. (17)–(19), the \( H(z), q(z) \) and \( w_{\text{tot}}(z) \) according to these three \( \mu(z) \) lines are also plotted in the other panels. See text for details.

IV. CONCLUSION AND DISCUSSIONS

In this work, we used the cosmology independent method proposed in [33] to calibrate the GRBs and derived the distance moduli \( \mu \) for 32 high redshift GRBs at \( z > 1.4 \), whereas the numerical results are given in Table I. We have used the 307 SCP Union SNIa compilation [36] and the 70 GRBs compiled in [41]. Since these GRBs are calibrated in a completely cosmology independent manner, one can use the calibrated 32 GRBs at \( z > 1.4 \) to constrain cosmological models without circularity problem completely. On the other hand, following the method proposed by Shafieloo et al. [38], in this work we smoothly
reconstructed the cosmic expansion history up to \( z = 6.29 \) using 307 SNIa compiled in [36] and 32 calibrated GRBs at \( z > 1.4 \). It is worth noting that this method is also model independent.

Here are some comments on the technical details [74]. Firstly, SNIa as standard candles are affected by several potential problems, e.g., dust extinction, color evolution, reliability of the Phillips relation up to high \( z \), uncertainty on the progenitors, possible existence of different sub-classes, etc. Thus, calibrating GRB with SNIa propagates these uncertainties and sources of systematics also into the calibrated spectrum-energy correlations. In addition, even when using a cosmology independent method, in this way GRBs are no more an “independent” cosmological probe. In other words, by calibrating with SNIa one can avoid the circularity problem intrinsic in the use of GRB spectrum-energy correlations and increase the accuracy in the determination of spectral parameters. However, with respect to using spectrum-energy correlations alone, one might introduce more systematics and introduce a “circularity” with SNIa themselves.

As mentioned above, the reconstructed \( H(z) \), \( q(z) \) and \( w_{tot}(z) \) are interesting in some sense. To make these results robust, we have tried various values of \( \Delta \) and various initial guess models. The resulted \( H(z) \), \( q(z) \) and \( w_{tot}(z) \) are still similar to the previous ones. The features in the reconstructed \( H(z) \), \( q(z) \) and \( w_{tot}(z) \) at high redshift \( z > 2 \) remain. Some remarks are in order. Firstly, one might doubt the validity of using GRBs as standard candles. If GRBs are not real standard candles, the reconstructed results at \( z > 1.4 \) are unreliable of course. In fact, the debate is not settled in the GRB community today. Some authors argued that GRBs cannot be used as standard candles, see [49, 50, 51, 52, 53, 70, 71, 72, 73] for examples. On the other side, some authors advocated that GRBs can be used as standard candles to probe cosmology, see [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63] and references therein. Today, the situation is still unclear and many authors choose to wait and see. Secondly, if the GRBs can be used as standard candles, since the sample of available GRBs at high redshifts is still not large enough, some bias might exist in the reconstructed \( H(z) \), \( q(z) \) and \( w_{tot}(z) \) and lead to the interesting features. So, before more and better high redshift GRBs are available, we cannot say anything conclusively. Finally, on the other side, some hints were found for oscillating \( H(z) \), \( q(z) \) and \( w_{tot}(z) \) in the literature, see e.g. [64, 65, 66, 67, 68] and references therein. So, the reconstructed results obtained here might be meaningful in some sense and deserve further investigations.

Obviously, due to the large scatter and the lack of a large amount of well observed GRBs, there is a long way to use GRBs extensively and reliably to probe cosmology. The Fermi Gamma-ray Space Telescope [69] which was launched recently might change this situation. The GRB sample will be appreciably larger, and the new GRBs will be better studied in some sense. In the coming Fermi era, we hope that the GRB cosmology could have a bright future.

ACKNOWLEDGEMENTS

We thank the anonymous referee for quite useful comments and suggestions, which helped us to improve this work. We are grateful to Prof. Rong-Gen Cai, Prof. Xinmin Zhang and Prof. Zu-Hui Fan for helpful discussions. We also thank Minzi Feng, as well as Nan Liang, Pu-Xun Wu, Yuan Liu, Wei-Ke Xiao, Rong-Jia Yang, Jian Wang, Lin Lin, Bin Fan, and Bin Shao, Yu Tian, Zhao-Tan Jiang, Feng Wang, Jian Zou, Zhi Wang, Xiao-Ping Jia, for kind help and discussions. This work was supported by the Excellent Young Scholars Research Fund of Beijing Institute of Technology.

[1] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006) [hep-th/0603057].
[2] A. G. Riess et al., Astron. J. 116, 1009 (1998) astro-ph/9805201;
S. Perlmutter et al., Astrophys. J. 517, 565 (1999) astro-ph/9812133.
[3] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 170, 377 (2007) astro-ph/0603449;
E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 180, 330 (2009) arXiv:0803.0547.
[4] M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D 69, 103501 (2004) astro-ph/0310723;
U. Seljak et al. [SDSS Collaboration], Phys. Rev. D 71, 103515 (2005) astro-ph/0407372.
M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D 74, 123507 (2006) astro-ph/0608632.
[5] M. Phillips, Astrophys. J. 413, L105 (1993).
[6] M. Hamuy et al., Astron. J. 109, 1669 (1995);
M. Hamuy et al., Astron. J. 112, 2391 (1996) astro-ph/9609059.
[7] S. Perlmutter et al., Astrophys. J. 440, L41 (1995) astro-ph/9505023;
A. Kim, A. Goobar and S. Perlmutter, Publ. Astron. Soc. Pac. 108, 190 (1996) astro-ph/9505024;
S. Perlmutter et al., Astrophys. J. 483, 565 (1997) astro-ph/9608192.
[8] A. G. Riess, W. H. Press and R. P. Kirshner, Astrophys. J. 473, 88 (1996) astro-ph/9604143.
[9] R. Miquel, J. Phys. A 40, 6743 (2007) astro-ph/0603459.
[10] B. E. Schaefer, Astrophys. J. 660, 16 (2007) astro-ph/0612285.
[11] V. Bromm and A. Loeb, Astrophys. J. 575, 111 (2002) astro-ph/0201400;
J. R. Lin, S. N. Zhang and T. P. Li, Astrophys. J. 605, 819 (2004) astro-ph/0311363.
[12] L. Amati et al., Astron. Astrophys. 390, 81 (2002) astro-ph/0208230.
[13] G. Ghirlanda, G. Ghisellini and D. Lazzati, Astrophys. J. 616, 331 (2004) astro-ph/0405602.
[14] D. Yan et al., Astrophys. J. 609, 935 (2004) astro-ph/0309217.
[15] E. W. Liang and B. Zhang, Astrophys. J. 633, 611 (2005) astro-ph/0504404.
[16] C. Firmani et al., Mon. Not. Roy. Astron. Soc. 370, 185 (2006) astro-ph/0605073.
[17] J. P. Norris, G. F. Marani and J. T. Bonnell, Astrophys. J. 534, 248 (2000) astro-ph/9903253.
[18] E. E. Fenimore and E. Ramirez-Ruiz, astro-ph/0004176;
D. E. Reichart et al., Astrophys. J. 552, 57 (2001) astro-ph/0004302.
[19] R. Sato, K. Ioka, K. Toma, T. Nakamura, J. Kataoka, N. Kawai and T. Takahashi, arXiv:0711.0903 [astro-ph].
[20] J. Hakkila et al., arXiv:0803.1655 [astro-ph];
Z. B. Zhang and G. Z. Xie, arXiv:0711.1411 [astro-ph].
[21] G. Ghirlanda, G. Ghisellini and C. Firmani, New J. Phys. 8, 123 (2006) astro-ph/0610248.
[22] B. Zhang, Chin. J. Astron. Astrophys. 7, 1 (2007) astro-ph/0701520;
B. Zhang, astro-ph/0611774.
[23] P. Meszaros, Rept. Prog. Phys. 69, 2259 (2006) astro-ph/0605208;
V. Bromm and A. Loeb, arXiv:0706.2415 [astro-ph];
S. E. Woosley and J. S. Bloom, Ann. Rev. Astron. Astrophys. 44 (2006) 507 astro-ph/0609142.
[24] Z. G. Dai, E. W. Liang and D. Xu, Astrophys. J. 612, L101 (2004) astro-ph/0407497.
[25] M. Su, Z. Fan and B. Liu, arXiv:0611155.
H. Li, M. Su, Z. Fan, Z. Dai and X. Zhang, Phys. Lett. B 658, 95 (2008) astro-ph/0612606;
F. Y. Wang, Z. G. Dai and Z. H. Zhu, Astrophys. J. 667, 1 (2007) arXiv:0706.0938;
S. Qi, F. Y. Wang and T. Lu, arXiv:0807.4594 [astro-ph].
[26] G. Ghirlanda, G. Ghisellini, D. Lazzati and C. Firmani, Astrophys. J. 613, L13 (2004) astro-ph/0408350.
[27] C. Firmani et al., Mon. Not. Roy. Astron. Soc. 360, L1 (2005) astro-ph/0501395.
[28] F. Y. Wang and Z. G. Dai, Mon. Not. Roy. Astron. Soc. 368, 371 (2006) astro-ph/0512270.
[29] C. Firmani et al., Rev. Mex. Astron. Astrofis. 43, 203 (2007) astro-ph/0605267;
C. Firmani et al., Mon. Not. Roy. Astron. Soc. 372, L28 (2006) astro-ph/0605430.
[30] H. Li, J. Q. Xia, J. Liu, G. B. Zhao, Z. H. Fan and X. M. Zhang, Astrophys. J. 680, 92 (2008) arXiv:0711.1792.
[31] E. W. Liang and B. Zhang, Mon. Not. Roy. Astron. Soc. 369, L37 (2006) astro-ph/0512177.
[32] Y. Kodama et al., Mon. Not. Roy. Astron. Soc. 391, L1 (2008) arXiv:0802.3428.
[33] N. Liang, W. K. Xiao, Y. Liu and S. N. Zhang, Astrophys. J. 685, 354 (2008) arXiv:0804.4262.
[34] K. Takahashi, M. Oguri, K. Kotake and H. Ohno, astro-ph/0305260.
[35] R. Tsutsui et al., arXiv:0807.2911 [astro-ph].
[36] M. Kowalski et al. [Supernova Cosmology Project Collaboration], arXiv:0804.4142 [astro-ph].
The numerical data of the full sample are available at http://supernova.lbl.gov/Union.
[37] D. Rubin et al. [Supernova Cosmology Project Collaboration], arXiv:0807.1108 [astro-ph].
[38] A. Shafieloo, Mon. Not. Roy. Astron. Soc. 380, 1573 (2007) astro-ph/0703034;
A. Shafieloo et al., Mon. Not. Roy. Astron. Soc. 366, 1081 (2006) [astro-ph/0505329].

[39] P. X. Wu and H. W. Yu, JCAP 0802, 019 (2008) [arXiv:0802.2017].

[40] T. M. Davis et al., Astrophys. J. 666, 716 (2007) [astro-ph/0701510].

[41] L. Amati et al., Mon. Not. Roy. Astron. Soc. 391, 577 (2008) [arXiv:0805.0377].

[42] G. Ghirlanda, L. Nava, G. Ghisellini, C. Firmani and J. I. Cabrera, arXiv:0804.1675 [astro-ph].

[43] L. Amati, Mon. Not. Roy. Astron. Soc. 372, 233 (2006) [astro-ph/0601553].

[44] D. Eichler and A. Levinson, Astrophys. J. 614, L13 (2004) [astro-ph/0405014];

A. Levinson and D. Eichler, Astrophys. J. 629, L13 (2005) [astro-ph/0504125].

[45] R. Yamazaki, K. Ioka and T. Nakamura, Astrophys. J. 606, L33 (2004) [astro-ph/0401044];

K. Toma, R. Yamazaki and T. Nakamura, Astrophys. J. 635, 481 (2005) [astro-ph/0504624].

[46] M. J. Rees and P. Meszaros, Astrophys. J. 628, 847 (2005) [astro-ph/0412702].

[47] T. Isobe, E. D. Feigelson, M. G. Akritas and G. J. Babu, Astrophys. J. 364, 104 (1990).

[48] P. Coles and F. Lucchin, The origin and evolution of large-scale structure, Wiley, New York (1995);

V. J. Martinez and E. Saar, Statistics of galaxy distribution, Chapman & Hall, London (2002).

[49] L. X. Li, Mon. Not. Roy. Astron. Soc. 379, L55 (2007) [arXiv:0704.3128];

L. X. Li, arXiv:0806.2770 [astro-ph].

[50] F. Fiore, D. Guetta, S. Piranomonte, V. D’Elia and L. A. Antonelli, arXiv:0704.2189 [astro-ph].

[51] L. Amati, Nuovo Cim. 121B, 1081 (2006) [astro-ph/0611189].

[52] B. E. Schaefer and A. C. Collazzi, Astrophys. J. 656, L53 (2007) [astro-ph/0701548].

[53] G. Ghirlanda, L. Nava, G. Ghisellini and C. Firmani, astro-ph/0702352.

[54] B. Gendre, A. Galli and M. Boer, AIP Conf. Proc. 1000 (2008) 72 [arXiv:0711.2222].

[55] F. Virgili, E. Liang and B. Zhang, arXiv:0801.4751 [astro-ph].

[56] L. Amati, Nuovo Cim. 121B, 1081 (2006) [astro-ph/0611189].

[57] E. V. Linder, Astropart. Phys. 25, 167 (2006) [astro-ph/0412010].

[58] H. Wei and S. N. Zhang, Phys. Rev. D 71, 063533 (2005) [hep-th/0407432].

[59] S. Capozziello and L. Izzo, arXiv:0806.1120 [astro-ph].

[60] H. J. Mosquera Cuesta et al., astro-ph/0609262.

[61] H. J. Mosquera Cuesta, H. Dumet M. and C. Furlanetto, JCAP 0807, 004 (2008) [arXiv:0708.1355];

H. J. Mosquera Cuesta et al., astro-ph/0610796.

[62] Y. Kaneko et al., AIP Conf. Proc. 836, 133 (2006) [arXiv:0605427].

[63] Y. Kaneko et al., astro-ph/0601188.

[64] N. R. Butler, D. Kocevski, J. S. Bloom, and J. L. Curtis, Astrophys. J. 671, 656 (2007) [arXiv:0706.1275].

[65] L. Nava, G. Ghirlanda, G. Ghisellini and C. Firmani, arXiv:0807.4931 [astro-ph].

[66] We thank the anonymous referee for pointing out these issues.