Better Late than Never; Scaling Computation in Blockchains by Delaying Execution

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ABSTRACT
Proof-of-Work (PoW) based blockchains typically allocate only a tiny fraction (e.g., less than 1% for Ethereum) of the average interarrival time ($\tau$) between blocks for validating transactions. A trivial increase in validation time (\(\tau\)) introduces the popularly known Verifier’s Dilemma, and as we demonstrate, causes more forking and increases unfairness. Large $\tau$ also reduces the tolerance for safety against a Byzantine adversary. Solutions that offload validation to a set of non-chain nodes (a.k.a. off-chain approaches) suffer from trust and performance issues that are non-trivial to resolve.

In this paper, we present Tuxedo, the first on-chain protocol to theoretically scale $\tau/\Delta \approx 1$ in PoW blockchains. The key innovation in Tuxedo is to separate the consensus on the ordering of transactions from their execution. We achieve this by allowing miners to delay validation of transactions in a block by up to $\xi$ blocks, where $\xi$ is a system parameter. We perform security analysis of Tuxedo considering all possible adversarial strategies in a synchronous network with maximum end-to-end delay $\Delta$ and demonstrate that Tuxedo achieves security equivalent to known results for longest chain PoW Nakamoto consensus. Additionally, we also suggest a principled approach for practical choices of parameter $\xi$ as per the application requirement. Our prototype implementation of Tuxedo atop Ethereum demonstrates that it can scale $\tau$ without suffering the harmful effects of naïve scaling in existing blockchains.

1 INTRODUCTION
One major problem of PoW blockchains such as Bitcoin and Ethereum is that they have useful compute power of orders of magnitude less than a typical desktop. For example, the gas limit of each Ethereum block corresponds to a block processing time ($\tau$) of approximately 150 milliseconds.\(^1\) Thus only 1% of the block interarrival time ($\Delta$) of 15 seconds in Ethereum is used for executing transactions. This prevents permissionless blockchains from accepting blocks that contain computationally-heavy transactions. Such computationally-heavy transactions are desirable for applications such as cryptographic trusted setup and privacy-preserving computation.

Problems due to large $\tau$. To see why one cannot arbitrarily increase $\tau$, we must understand the actions taken by a miner on receiving a new block. The miner first validates the block by executing all its transactions. It then forms its own block whose transactions it executes, and finally starts Proof-of-Work (PoW) to mine a new block.

Some permissionless systems, such as Ethereum, require each block to store a cryptographic digest of the latest blockchain state, which must be verified by miners while validating a block. This digest has many uses. First, it assists users with low computation resources (a.k.a., light clients) to efficiently validate, or prove to other light clients, a portion of the latest state. Second, it helps new nodes to quickly bootstrap and join the system. Third, it makes the system more compatible with stateless cryptocurrencies [8, 27].

As a result, higher validation time $\tau$ “eats into” PoW time. This gives significant advantages to miners with higher block processing power than others, and also opens up the system to various attacks. As we demonstrate in §7 and Appendix B, with a larger $\tau$, an adversary $\mathcal{A}$ who skips validation of received blocks and/or its own created blocks can mine more than its fair share of blocks relative to its mining power on the main chain. For example, our experiments show that when $\tau/\Delta = 0.2$, an adversary $\mathcal{A}$ controlling 33% of the mining power can mine as much as 68% of blocks on the main chain. A large $\tau$ also leads to the well-known Verifier’s Dilemma [18] where a rational miner has to make the hard choice between validating blocks or not. The first choice reduces its chances of mining the next block, and the second increasing its mining chances but comes with the risk of accepting invalid blocks. Moreover, increasing $\tau$ leads to a higher backlog of blocks to be processed at miners, which delays block forwarding. This leads to more forks and wasted mining power and lowers the adversarial tolerance of the system [21, 22]. For these reasons, PoW blockchains currently keep block validation time small relative to the block interarrival time, i.e. $\tau/\Delta \ll 1$.

Previous Work. Existing works that enable blocks with heavy computation do so through off-chain solutions [5, 7, 9, 11, 13, 26, 30]. Rather than having all miners execute transactions, which we term on-chain computation, these methods delegate computation to a subset of miners or groups of volunteer nodes. These solutions make additional security assumptions, beyond those required for PoW consensus, so that miners can validate the results that voluntary nodes submit. Also, most off-chain solutions make restrictive assumptions about the interaction between contracts, e.g., one smart contract does not internally invoke functions of other smart contracts. Such interactions are desirable and often occur in practice (see Appendix D). An on-chain solution if designed carefully can

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\(^1\)Measured using a virtual machine with 16 cores, 120GB memory, 6.4TB NVMe SSD, and 8.2 Gbps network bandwidth.
be made to automatically inherit the existing functionality of interaction between smart contracts and also the security guarantees of the underlying blockchain.

Our Approach. In this paper we propose Tuxedo, the first on-chain solution that can theoretically scale $\tau/I$ close to 1 while circumventing the problems discussed above. As a result, Tuxedo can increase the useful computing power of PoW blockchains significantly to allow transactions with non-trivial execution time.

The core idea behind Tuxedo is to separate the consensus on the ordering of transactions in the blockchain from their validation. We achieve this by allowing miners to delay validation of transactions in a block at height $i$ ($B_i$) until the arrival of the block at height $i + \xi$ ($B_{i+\xi}$) where $\xi$ is a system parameter. Essentially, $B_{i+\xi}$ contains the cryptographic digest of the state corresponding to the execution of all transactions up to an including those in $B_i$. Hence we refer to this approach as Delayed Execution of Transactions (DET). This way, the validation of transactions in a block can be done in parallel with the PoW, thereby side-stepping the competition between validation and PoW.

While the idea of DET may seem simple, securely adopting it in PoW systems turns out to be non-trivial. The major challenge arises due to the variability in the deadline (that is the arrival of $B_{i+\xi}$) for validating transactions of $B_i$. As block generation is a random process (often modeled as Poisson), there is the possibility (albeit rare) that an honest miner may fail to execute transactions in $B_i$ before receiving $B_{i+\xi}$. In that case, the honest miner will not be able to validate $B_{i+\xi}$ immediately on its arrival. As a remedy, in Tuxedo an honest miner always extends its longest validated chain.

It is also possible that a miner does not have the digest ready for the next block it wants to mine on the longest validated chain. For example, suppose $B_{i+\xi}$ is the last block in the longest validated chain and that the miner has executed all transactions in $B_i$. However, it has not yet validated $B_{i+1}$ and hence it does not have the digest ready to put into $B_{i+\xi+1}$ which it wants to mine. In such situations, Tuxedo allows miners to put a special default state in place of the miner has not yet validated $B_{i+\xi}$ immediately on its arrival. As a remedy, in Tuxedo an honest miner always extends its longest validated chain.

Using standard techniques from Queueing theory, we prove (§6.2) that these changes ensure that Tuxedo achieves Chain growth, Chain quality and Safety similar to known results for Longest chain PoW [14, 21, 22]. Also, we prove in §6.3 that the fraction of blocks with empty state mined by the honest miner can be reduced arbitrarily by setting $\xi$ appropriately. In summary, we make the following contributions:

- We illustrate through analysis and experiments that a naive increase of $r$ in legacy blockchains gives unfair advantages to miners with faster processing power. An adversary $\mathcal{A}$ can further exacerbate the unfairness by skipping validation of received blocks and creating blocks that it can process quickly.
- We design Tuxedo, a secure on-chain approach that can theoretically scale $\tau/I$ to 1 in PoW based permissionless blockchains.
- We theoretically prove security guarantees of Tuxedo under a synchronous network with end-to-end network delay $\Delta$ and fixed processing time $\tau$. Our analysis considers all possible strategies by a Byzantine adversary controlling up to $f_{\text{max}} < 0.5$ fraction of the mining power. We also present an approach to choose $\xi$ in order to achieve any desired fraction of honest blocks with non-empty state.
- We implement Tuxedo on top of the Ethereum Geth client and evaluate it in an Oracle cloud with 50 virtual machines emulating the top 50 Ethereum miners. Our evaluation demonstrates that Tuxedo does not suffer from certain fairness problems unlike Ethereum does, for a high value of $\tau/I$.

Paper Organization. In §2 we present our system model and assumptions. This is followed by a brief background of block validation process of legacy blockchains and attacks on them with high $r$ in §3. In §4 we introduce the concept of Delaying Execution of Transactions and describe how Tuxedo employs it to achieve high validation time. We next describe our implementation methodology in §5. We then present our theoretical analysis demonstrating security of Tuxedo in §6. §7 describes our prototype implementation of Tuxedo experimental setup and observations from experimental results. We describe the related work in §8. We discuss few FAQs in §9 and conclude the work in §10.

2 SYSTEM MODEL

We consider a permissionless system consisting of a set of miners. These miners form a connected network and run a blockchain protocol with Proof-of-Work (PoW) as the underlying consensus. All honest miners mine blocks on top of the longest validated chain known to them (see §4.3). Block generation in Tuxedo is assumed to follow a Poisson process with the rate $\lambda$ where $\lambda$ depends on the mining power of the network and difficulty of the PoW puzzle. Each miner $n_z$ controls $p_z$ fraction of the mining power. Hence, any arbitrary miner $n_z$ will generate blocks at a rate $\lambda_z = p_z \lambda$. Tuxedo allows execution of Turing Complete programs called Smart Contracts. A smart contract can be created by sending a transaction to deploy it on the blockchain. Once a contract appears in the blockchain, its exposed functionality can be invoked by other miners through transactions.

Smart contracts in Tuxedo have unique IDs, and they maintain state, where state corresponds to the unique set of key-value pairs stored at each miner and is controlled by the program logic of the smart contract. For any arbitrary smart contract $c_z$, we use $\sigma_z$ to denote the state of smart contract. In addition to contracts, Tuxedo maintains Accounts, which maintains tokens. Each account also has a globally unique ID which is the public key of a public-private key pair generated from a secure asymmetric signature scheme. Additionally, Tuxedo has Clients which own accounts, can generate transactions to create smart contracts, invoke their functions and transfer tokens from one account to another.

Transactions in Tuxedo are ordered in a Transaction Ordered List (TOL) and are included in a block. We use $T_i$ to denote the $i^{th}$ TOL. The contracts generated by transactions in TOLs $\{T_1, T_2, \ldots, T_i\}$ are denoted as $C_i = \{c_z | z = 1, 2, \ldots, i\}$ and the corresponding state as $S_i = \{\sigma_z | z = 1, 2, \ldots, i\}$. Each miner locally maintains states and updates it by executing a given TOL. Formally, with initial state

\[\sigma_0 = \{\} \text{ and } S_0 = \{\}\]

Pass et al. [21] use the term ‘asynchronous network’ for a network with the same constraints.
S\_i-1, the execution of the TOL T_i is denoted by:

\[ S_i = \Pi(S\_i-1, T_i) \]

where \( \Pi \) denotes the deterministic state transition function that executes transactions in \( T_i \) in the order they appear.

Let \( B_i = (B_0, B_1, \cdots, B_i) \) be the blocks known to \( n_a \). In addition to state, \( n_a \) maintains a transaction pool \( T^{(a)} \), which contains the set of valid transactions created by clients that are yet to be included in a block till \( B_i \). Hereon, when clear from the context, we drop the superscript from \( T^{(a)} \) for ease of notation.

**Assumptions.** We assume the underlying network to be synchronous with end-to-end delay of at most \( \Delta \), i.e., all messages sent by an honest miner gets delivered to every other honest miner within time \( \Delta \) from its release. Also, we assume that all honest miners process blocks in any particular chain serially at the rate of \( 1/\tau \), where \( \tau \) is the maximum time needed to validate a block. Like Ethereum, this can be enforced by requiring each transaction to specify the maximum time needed for its execution and keeping a cap \( \tau \) on the total execution time a block. Note that block processing is different from mining a block; mining involves solving the PoW puzzle, whereas processing is about executing the transactions inside a block. Also, a block with high validation time neither implies a large block size nor that the block has a large number of transactions in it. A small block containing a few computationally-heavy transactions can require a large validation time.

We also assume that the block processing at an honest miner does not contend with PoW, and the honest miners can simultaneously process blocks in distinct forks of the blockchain tree. We envision that our system will be adopted by blockchains such as Bitcoin and Ethereum where block processing can be done using CPUs while PoW requires ASICs. Furthermore, the number of simultaneous forks in them are quite small [12].

We assume the presence of an adversary \( A \), who can control up to \( f_{\text{max}} < 1/2 \) fraction of total mining power of the network and generate blocks at a rate \( \beta = f_{\text{max}} \lambda \). Adversarial miners can be Byzantine and can deviate arbitrarily from the specified protocol. The remaining miners are honest, control the remaining \((1 - f_{\text{max}})\) the fraction of the mining power, generates blocks at a rate \( \alpha = (1 - f_{\text{max}}) \lambda \), and strictly follow the specified protocol. \( A \) can see every message sent by honest parties immediately and can inject its messages at any point in time. Also, \( A \) can delay messages sent by the honest parties by a maximum of \( \Delta \) time.

### 3 BLOCK VALIDATION IN LEGACY BLOCKCHAINS

In this section, we first give some background on the block validation process in Ethereum and later demonstrate why increasing \( \Delta \) leads to reduced fairness in terms of the fraction of blocks mined by an honest miner. This background assists us in identifying the core problem behind smaller block validation time in existing systems. In the later sections (§4) we will describe how TUXEDO securely addresses these issues.

Ethereum is designed to force miners to validate blocks that they receive. To understand why, we must note that Ethereum state is not explicitly stored on the blockchain, only its digest is. For a miner to create a potential block of its own which includes a mining reward, it must put a digest of the new state resulting from this block in the header. However, the state resulting from executing the transactions of the previous block acts as an essential starting point to obtaining the correct state to put in its own block. Hence the miner is forced to execute transactions in the previous block which it received.

Once a miner successfully creates its own block, it starts mining, i.e., solving PoW on this block. During this PoW two things can happen: first, the miner receives a conflicting block created by a different miner at the same height as its own potential block; and second, the miner successfully solves the PoW, broadcasts its own block with the valid PoW and proceeds to create the next block, extending its very own recent block. Let us refer to the time spent validating the received block as the Validation phase; the time spent in creation of next block as the Creation phase, and time spent in PoW as the Mining phase. As honest miners do not solve for PoW during the validation and creation phase, we are interested in the time these phases takes to complete. For this purpose, we will first scrutinize these phases more carefully.

If the longest chain known to an honest miner \( n_a \) has length \( i - 1 \), with \( B_{i-1} \) at the tail of the chain as shown in Figure 1, \( n_a \) tries to mine the next block \( B'_i \) at height \( i \). Let \( S_i \) be the state after executing TOLs till \( B_{i-1} \) and \( T_{i-1} = \{t_1, t_2, t_3, t_4\} \) be the latest transaction pool. For \( B'_i \), \( n_a \) first picks up a TOL \( T'_i \) (e.g., \( T'_i = \{t_1, t_2, t_3\} \)), executes its transactions in order of their occurrence and starts PoW on block \( B'_i \). Let \( S'_i = \Pi(S_i, T'_i) \) be the updated state. Note that till \( n_a \) successfully solves the PoW puzzle, the updated state is not committed and remains cached at \( n_a \). As described earlier, while running the PoW algorithm for block \( B'_i \), one of two things can happen: either \( n_a \) receives a valid block \( B_i \) at height \( i \) from the network or \( n_a \) successfully solves the PoW. We now describe these as Case I and II, respectively.

**Case I.** Let \( n_b \) be the miner of the block \( B_i \) (containing ordered list \( T_i \)) that \( n_a \) receives. Without any coordination between \( n_a \) and \( n_b \), it is likely that \( T_i \neq T'_i \). In that case, \( n_a \) first validates \( B_i \) executing all the transactions in \( T_i \). On successful validation, \( n_a \) accepts the block and proceeds to create the block \( B_{i+1} \) at height \( i+1 \) by picking a new TOL \( T_{i+1} \) from \( T_{i-1} \setminus T_i \). Case I of Figure 1 illustrates this.

**Case II.** Unlike Case I, \( n_a \) commits the state update due to execution of \( T'_i \) and proceeds to create the block \( B'_{i+1} \) at height \( i+1 \) after picking a new TOL \( T'_{i+1} \) from \( T_{i-1} \setminus T'_i \). In our example, \( T'_{i+1} = \{t_3, t_4\} \). \( n_a \) then executes the new TOL \( T'_{i+1} \) and starts PoW for \( B'_{i+1} \). Case II of Figure 1 illustrates this.
Another way to describe the block validation and creation mechanism is through the time intervals during which a miner validates the received block, creates the next block and performs PoW. Let $t_0$ be the time instant of the arrival of the block at height $i$ and let $\tau$ be the validation and creation time of a block. In case I, the miner validates the received block in the time interval $(t_0, t_0 + \tau)$, creates the next block $B'_{i+1}$ in time interval $(t_0 + \tau, t_0 + 2\tau)$ and only at time $t + 2\tau$ starts PoW for $B'_{i+1}$. However in case II, since $n_a$ himself is the creator of the block $B'_{i}$, it skips validation of $B'_i$ and spends the time $(t_0, t_0 + \tau)$ in creating block $B'_{i+1}$ and starts PoW for $B'_{i+1}$ at time $t_0 + \tau$. The top half of Figure 2 demonstrates the timings along with the computations a miner needs to perform for Case I in more detail.

![Figure 2: Actions taken by a miner on receiving a block $B_i$ at time $t_0$ to validate $B_i$ and create the next block at height $i + 1$ in Ethereum (top) and DET with $\zeta = 2$ (bottom). In DET, $\varepsilon$ denotes the time spent by the miner to validate the received block and create the next one. Since validation/creation in DET involves only a small (constant) number of operations, $\varepsilon \ll \tau$.](image)

### 3.1 Consequences of high $\tau$ in legacy PoW Blockchains

Ideally when all miners are honest and with no network delay, one would expect that the fraction of blocks mined by a miner should be proportional to its mining power. In this section, we demonstrate that is not the case, and show that when $\tau/\lambda$ is high, the fraction of block mined by honest miners heavily depends on their relative transaction processing speed in addition to their mining powers.

Observe from Case II that the creator of a block spends $\tau$ units of extra time (i.e., between $(t_0 + \tau, t_0 + 2\tau)$) for PoW while the remaining miners are busy creating the next block. This extra time $\tau$ increases its chances of mining the next block as well. This effect gets exacerbated if the miner controls a large mining power (say 30%), because the miner will naturally mine blocks frequently and each of these blocks gives it an advantage to mine the next one as well.

More concretely, let $\lambda_a$ and $\tau_a$ be the block mining rate and block processing time of miner $n_a$, respectively. Let $c = \tau_a/\tau$ where $0 \leq c \leq 1$, i.e, $c$ is the ratio of block processing time of $n_a$ and remaining miners. $c = 0$ implies that $n_a$ can process a block instantly independent of $\tau$. With these parameters, $n_a$ will spend only $2\tau c$ units of time in case I before starting PoW for the next block. Similarly, in case II, $n_a$ will spend only $\tau c$ units of time creating the next block before starting PoW. Building on this intuition, we theoretically compute the fraction of blocks $n_a$ will mine in the longest chain for any given choice of $\lambda_a$ and $c$ in Appendix B.

Figure 3 illustrates results from our theoretical analysis. For example, with $\tau/\lambda = 0.26$, an honest miner who controls 30% of the mining power and can validate or create blocks twice as fast as others, i.e. $c = 0.50$, will mine at least 46% of the blocks. Further, a miner who skips both validation and creation of blocks, i.e with effective $c = 0$ will mine at least 53% of the blocks with 33% of the mining power. We also measure the same using our experimental setup described in §7 with realistic network delays and observe that the network delay exacerbates the attack and allows $\mathcal{A}$ to mine 68% of the blocks in the main chain.

![Figure 3: Fraction of blocks mined by miner $n_a$ with 0.33 fraction of the total mining power, i.e $\lambda_a = \lambda/3$ for varying $c = \tau_a/\tau$.](image)
cryptographic digest in it. Likewise, miners receiving a block execute all its transactions to verify its digest. Since block validation, block creation, and PoW mining are sequential, a large creation or validation time eats into PoW mining time, opening up the system to unfairness and attacks.

In Tuxedo, instead of making miners execute transactions and reporting a cryptographic digest of the updated state immediately, we delay the reporting of the digest by $\zeta \geq 2$ blocks. In particular the resulting state after executing all transactions up to a block at height $i$, $B_i$ is included in a block at height $i + \zeta$. Intuitively, this allows miners to process received blocks and create new blocks in parallel to the PoW mining phase. Thus larger $\tau$ does not eat into PoW time.

A large $\tau$, however, introduces new scenarios not encountered in PoW systems with negligible $\tau$, such as those studied hitherto [14, 21, 22]. In one scenario, an honest miner may not be able to validate the digests present in its longest known chain. This happens if the longest chain has $B_{i+\zeta}$ as its last block and the miner has not yet executed transactions in $B_i$, either because subsequent blocks were generated quickly by honest miners, or were generated privately by an adversary and released all at once. As a remedy, in Tuxedo, a miner mines not on the longest known chain, but on the longest chain it has so far validated. Intuitively, we must choose a large $\zeta$ to reduce such occurrences. However, a very large $\zeta$ is undesirable as it delays the reporting of the updated state. Hence, we must pick a suitable $\zeta$ to balance this trade-off.

In another scenario, a miner may be able to validate all blocks in a chain, and yet not have the state ready to put in the next block it wants to mine. For example, if the last block is $B_{i+\zeta}$ and the miner has executed transactions in $B_i$ but not $B_{i-1}$, then it can indeed verify the state in $B_{i+\zeta}$ but does not have the state to put in $B_{i+\zeta+1}$. Tuxedo remedies this situation by allowing miners to report an empty state (e.g. all zeros) in the block they create.

We show in our analysis §6, that with the above remedies, the mining of honest miners never stalls, independent of any adversarial strategy. We exploit this fact to lower bound the guarantees of Tuxedo with known guarantees of PoW based Nakamoto consensus [14, 21, 22].

### 4.2 Delayed Execution of Transactions

The basic idea behind Delayed Execution of Transactions (DET) is to decouple the inclusion of transactions in the blockchain from the reporting (and hence validation) of the state resulting from those transactions. In Tuxedo transactions are ordered in a block without being immediately validated, and the state resulting from them is reported $\zeta$ blocks later. More formally, a block $B_i$ at height $i$ contains TOL $T_i$ and state $S_{i-\zeta} = \Pi(S_{i-\zeta-1}, T_{i-\zeta})$. Hence miners have a window of $\zeta$ blocks to pre-compute the state required for validation and this pre-computation can be done in parallel with the PoW. Figure 2 illustrates this for $\zeta = 2$ where the resulting states are delayed by 2 blocks.

As explained in Case I in Section 3, in existing blockchain designs, up to $2\tau$ time can "eat into" the PoW time. Thus, in order to get $\tau/I \approx 1$, we need to delay the execution of transactions by at least two blocks. In the rest of this section, we first describe DET with $\zeta = 2$ and then explain why an even larger $\zeta$ is needed.

**DET with $\zeta = 2$.** Let $B_{i-1}$ with state $S_{i-3}$ and TOL $T_{i-1}$ (as $\zeta = 2$) be the latest valid block known to miner $n_0$ (see Figure 2). For now, assume that $n_0$ has already computed (i) $S'_{i-2} = \Pi(S_{i-1}, T_{i-2})$ and (ii) $S''_{i-1} = \Pi(S_{i-2}, T_{i-1})$ and cached them prior to the arrival of the block $B_i$. Here $S'_{i-2}$ and $S''_{i-1}$ are the states locally computed by $n_0$ for TOL $T_{i-2}$ and $T_{i-1}$ respectively before arrival of block $B_i$ (ref. Figure 2). Upon arrival of $B_i$, $n_0$ validates it by checking whether the reported $S_{i-2}$ matches $S'_{i-2}$ (step 1). If it does, then $n_0$ accepts $B_i$ and starts computing $\Pi(S''_{i-1}, T_i)$ (step 3). Simultaneously, $n_0$ picks a new TOL $T'_{i+1} \in T_{i-1} \setminus T_i$, creates the block $B'_{i+1}$ by fetching the precomputed state $S'_{i-1}$ from its cache (step 2), and starts PoW for block $B'_{i+1}$ (step 4). This way, upon arrival of a block, $n_0$ is able to start PoW for the next block immediately.

### 4.3 Handling variable block interarrival

If blocks arrive exactly $I$ time apart from each other, then $\zeta = 2$ will be sufficient to scale $\tau/I \approx 1$. However, in reality, block interarrival times are random and can even be manipulated by the adversary to some extent. In case a sequence of $\zeta$ blocks following $B_i$ with TOL $T_i$ arrive closely spaced to each other, it is possible that a miner will not be able to compute the state $S_i$ before receiving $B_{i+\zeta}$. Hence, the miner will not be able to immediately validate $B_{i+\zeta}$. Without any precautionary measure, in such a situation, miners will be forced to defer creation of the next block, and hence the PoW on it till it computes $S_i$. If a large fraction of honest miners temporarily stop mining, an adversary $\mathcal{A}$ with faster block processing power will effectively enjoy higher fraction of mining power and may even pull off the "51% attack" during these periods. We address this issue by making two critical observations: first, the probability of this event occurring decreases with increasing $\zeta$, and second, we can ask honest miners to mine on the longest validated chain during such scenarios. We elaborate on these below.

**Need for higher $\zeta$.** To see why increasing $\zeta$ reduces the probability of the above mentioned undesirable event, we model DET as a queuing system where the transaction processing unit of a miner is analogous to the queue’s server. Each arriving block is a task input to a queue and each block is processed in $\tau$ units of time. In the absence of an adversary and network delays, the block arrival follows a Poisson process with rate $\alpha$. Assuming the input rate is independent of the queue size, this is essentially an $M/D/1$ queue. The sequence of blocks that a miner is yet to process in a given chain represents the contents of the queue.

If an arriving block enters a queue of size less than or equal to $\zeta - 2$ then its own state as well as that of the subsequent block have been pre-computed. The probability of the queue exceeding $\zeta - 2$ is the probability of the miner missing the deadline for computing the state which that block must contain. This tail probability of the
queue decreases with increasing $\zeta$, thus making larger $\zeta$ is more desirable. However, there is a trade-off here, because a larger $\zeta$ implies that blocks update the global state later, which is undesirable from a user’s point of view. Hence $\zeta$ must be chosen to balance this tradeoff. We leave detailed queuing models that take into consideration input variation of blocks due to $\Delta$, the presence of $\mathcal{A}$, and the trade-off due to larger $\zeta$ to §6.

Remark. Due to forks, miners in TUXEDO maintain multiple queues, one for each forked chain (see Figure §5), and process them in parallel. Blocks which are common to multiple chain (e.g. blocks $\{B_y, \cdots, B_i\}$ in Fig. 5 need to be processed only once. All our analysis will be valid with multiple queues because, as assumed in 2, a miner processes them in parallel and the input to each queue is still upper bounded by the block generation rate of miners. We believe that the assumption of parallel processing of forks is a reasonable one because, in practice the number of simultaneous forks in them are quite small [12] and is limited by the block-generation capability of the adversary. Lastly, we envision that our system will be adopted by blockchains such as Bitcoin and Ethereum where block processing can be done using CPUs while mining requires ASICs. Thus PoW mining and block processing do not compete for the same resources.

Extending Longest Validated Chain. Although higher $\zeta$ lowers the probability of queue of an honest miner crossing $\zeta − 2$ in the absence of adversary, additional care needs to be taken to provably prevent a Byzantine adversary from sabotaging the protocol. Thus we modify the chain selection rule of TUXEDO from a standard longest chain selection procedure. Recall from 4.1, the following changes are very crucial in lower bounding the chain growth property of TUXEDO which in turn is the core component in the security analysis of any PoW based blockchain including TUXEDO. Refer to §6 for the detailed security analysis of TUXEDO.

Honest miners in TUXEDO extend the longest chain they can validate. If a miner does not have the state to put in the next block, it puts a protocol specified default state, such as a sequence of zeros in place of the required state. Unless otherwise stated, we refer to this default state as the empty state. We refer to such blocks as ES blocks (i.e. blocks with an empty state). Similarly, we refer to blocks with the non-empty state as non-ES blocks. ES blocks can contain transactions (see §5) and non-ES block at height $i$ report $S_{i-\zeta}$. Also, during the entire duration, honest miners continue to process all unprocessed blocks with correct PoW that appears in a chain longer than the current mining head. On successfully validating a block, a miner re-configures its mining head to pick the new longest known validated chain.

4.4 Fees collection in TUXEDO

Every transaction in TUXEDO specifies the maximum amount of computation resources needed for its execution. Based on this specification, the fee of every transaction in the $i^{th}$ block, $B_i$ is collected in the same block. These fees are paid using the native token of TUXEDO, token1 (similar to Ether in Ethereum). Once the transaction gets executed, any leftover fees i.e., fees of unused computational resources are refunded in $B_{i+\zeta}$ where the state after the execution of $B_i$’s transactions is reported.\textsuperscript{3} This ensures that only blocks that can pay sufficient amount of gas, a unit of payment in Ethereum, for their transaction fees enter the blockchain.

Note that additional care needs to set the minimum transaction fee paid by a transaction. In particular, we cannot levy a small fixed fee for every transaction as in Ethereum, as such a design can lead to under utilization in terms of actual amount of gas usage. In particular, malicious users may over-specify the amount of required computation resources and actually use only a tiny fraction of the specified resource. One way to discourage such behavior is to take fees equivalent to the minimum of $\alpha$ ($0 < \alpha \leq 1$) times the specified gas usage and the true amount of gas used by the transaction. One many also consider alternative fee mechanisms depending upon the specific use cases. We leave the detailed analysis of the fee mechanism as future research.

Similar to Ethereum, TUXEDO also allows its smart contracts to transfer and receive tokens. However, since the transactions of the $i^{th}$ block, $T_i$, are executed after fees are collected for $\zeta − 1$ future blocks, additional care needs to be taken to prevent fees of future blocks from altering the execution results of past transactions. Specifically, TUXEDO restricts its smart contracts from using the native token. But, at the same time, TUXEDO allows its contracts to create their own tokens reminiscent of ERC’20 tokens in Ethereum and use them during execution. These tokens could be contract-specific, shared by several contracts, or shared by all contracts and is up to the contract designer. In our implementation, every smart contract uses the single ERC’20 token which we refer to as the token2. We describe the details of our implementation in §5.

5 IMPLEMENTATION DETAILS

Accounts and Tokens. Our implementation of TUXEDO has two kinds of accounts: user accounts and contract accounts. Each user account maintains both token1, the native currency, and token2, the ERC’20 token to be used in contracts, whereas contract accounts only maintains token2. contract accounts maintain executables that can be invoked by transactions.

Transactions. Each transaction $tx$ of TUXEDO is a tuple containing \{type, to, from, gas, $\star$\} where type either takes the value 1 or 2, to (resp. from) is the receiver (resp. sender) address, gas specifies the

\textsuperscript{3}We leave the exact refund policy as a design choice as we primarily focus on the capability of refunding fees if needed. TUXEDO will work same for schemes that does not refund fees as well.
maximum amount of gas $tx$ can use and $*$ represents the auxiliary information required for execution of $tx$. Transactions with type $1$ are addressed to user accounts and transfers token $1$ from the from address to the to address. The amount of token $1$ transfer is present in the auxiliary information denoted using $*$. Transactions of type $2$ are addressed to contract accounts and $*$ contains the identity of the functions to be invoked and the required function call parameters. For every such transaction, miners in Tuxedo transfer an amount of token $1$, as a transaction fee from its sender account to a prespecified address denoted by Deposit. If in case $tx$ from does not have enough token $1$, a miner discards $tx$.

**Two States.** In our implementation of Tuxedo every miner maintains two different states: $P$ and $S$ where $P$ is used to store information related to the amount of token $1$ in each user accounts and $S$ is used to store the information regarding contract execution and amount of token $2$ in all the accounts. Since payment (including fees) and refund transactions only modify $P$, such a segregation enables faster validation and block creation.

**Block Validation.** Let $S_{i-\xi}$ and $P_{j}$ be the contract and payment state at the end of block $B_{j}$. Also, $R_{j-\xi}$ represents the refund processed after the execution of $T_{j-\xi}$. The next block known to an honest miner $n_{a}$ be $B_{i-1}$. Let $B_{i}$ be the next arriving block. Let assume $B_{i}$ is a non-ES block and $n_{a}$ has already computed and cached the following state before its arrival.

\[
P'_{i-1} = \Pi(P'_{i-2}, R_{i-\xi-1}, \Phi_{i-1})
\]
\[
R_{i-\xi-1}, S'_{i-\xi-1} = \Pi(S'_{i-\xi-2}, T_{i-\xi-1})
\]
\[
R_{i-\xi}, S'_{i-\xi} = \Pi(S'_{i-\xi-1}, T_{i-\xi-1}).
\]

Note that applying $\Pi$ on any TOL $T_{j}$ also outputs the ordered list of refund transactions corresponding to $T_{j}$ for $\xi = 2$, this is depicted in Figure 6. On receiving the block $B_{j}$ containing TOL $T_{j}$ and digests of state $P_{j}$ and $S_{i-\xi}$, $n_{a}$ validates $B_{i}$ as follows: (i) $n_{a}$ first computes $P'_{i} = \Pi(P'_{i-1}, R_{i-\xi}, \Phi_{i})$, (ii) checks whether $P'_{i}$ matches with $P_{j}$, and (iii) $n_{a}$ also checks $S'_{i-\xi}$ matches $S_{i-\xi}$.

Alternatively, if $n_{a}$ has not pre-computed $S'_{i-\xi}$ and $B_{i}$ is a non-ES block, $n_{a}$ continues to mine on the previous mining head till it computes $S'_{i-\xi}$ and re-starts validating $B_{i}$ as above. However, if $B_{i}$ is an ES-block and $n_{a}$ has already validated the latest non-ES ancestor of $B_{i}, n_{a}$ computes $P'_{i}$ as $\Pi(P'_{i-1}, \Phi_{i})$ to check whether $P'_{i}$ matches with $P_{j}$ and skips step (iii) of validation. On successful validation, $n_{a}$ accepts $B_{i}$ and proceeds to create the next block as described below. Procedure **Validate** in Algorithm 1 presents the pseudo code for validation of blocks in Tuxedo.

**Block Creation.** On successful validation of the received block, to create the next block $n_{a}$ picks a new TOL $T'_{i+1}$ from $\mathcal{T} \setminus T_{i}$, (ii) computes $P'_{i+1} = \Pi(P'_{i}, R_{i-\xi+1}, \Phi'_{i+1})$, (iii) fetches $S'_{i-\xi+1}$ from cache (if available), and (iii) creates the next potential block $B'_{i+1}$ containing $T'_{i+1}$ and digests of $P'_{i+1}$ and $S'_{i-\xi+1}$. Also, the first non-ES block after a sequence of ES-blocks applies all accumulated refunds since the last non-ES block. Alternatively, if $S'_{i-\xi}$ is not available in the cache, $n_{a}$ puts an empty string in place of $S'_{i-\xi}$. After creating $B'_{i+1}, n_{a}$ immediately starts PoW on $B'_{i+1}$. Procedure **Create** in Algorithm 1 presents the pseudo code for validation of blocks in Tuxedo.

**Execution of contract transactions** In Tuxedo contract transactions are executed in parallel to PoW as shown in Figure 6. Specifically, during PoW for $B'_{i+1}, n_{a}$ computes $R'_{i-\xi}, S'_{i-\xi} = \Pi(S'_{i-\xi-1}, T'_{i-\xi})$. Also $n_{a}$ adds $T_{j}$ in the task queue (ref. §4.3) and executes $T_{j}$ as soon as it executes all $T_{j}$ for $j < i$, that appear prior to $T_{i}$.

### 6 ANALYSIS

We analyze the security of Tuxedo in the presence of a Byzantine adversary under all possible adversarial strategies. Our analysis has the following outline. We start by modeling the blocks to be processed at an honest miner as a queueing system. Our queuing model captures the variability in the block inter-arrival time and the fact that honest miners might be processing old blocks as newer blocks continue to arrive. We then use our queuing analysis and the fact that honest miners are allowed to mine blocks with an empty state to illustrate that honest miners can expand blocks created by other honest miners even in a network with the worst possible latency. This implies that the chain growth in Tuxedo is greater than or equal to the chain growth of PoW based Nakamoto consensus. This immediately implies that the lower bounds on chain-growth, chain quality and consistency shown in [14, 21, 22] apply to Tuxedo as well. Hence, Tuxedo provides guarantees, equivalent to the known guarantees of existing PoW system.

#### 6.1 Block Processing as a Queuing System

The arrival of blocks in PoW blockchain can be modeled as a Poisson process with arrival rate $\lambda$, or equivalently $1/\lambda$ is the expected inter-arrival time between two consecutive blocks [19]. As all honest miners take $\tau$ units of time to process a block, i.e. the processing rate of the server is $1/\tau$. On arrival of every new block $B_{i}$ with TOL $T_{i}$ that extends a chain longer than the current mining head at a miner $n_{a}, n_{a}$ adds the block to its queue. $n_{a}$ processes (that is,
Algorithm 1 TUXEDO for $\zeta \geq 2$

1: $T : \{tx_j \mid j = 1, 2, \cdots \}$ \quad \triangleright \text{Transaction pool at the miner}$
2: $C : \{R_j, S_j = \Pi(S_{j-1}, T_j)\}$ \quad \triangleright \text{Contract Cache}$
3: $P : \{P'_j = \Pi(P_{j-1}, R_j, \zeta, \Phi_j)\}$ \quad \triangleright \text{Payment Cache}$
4: $Q : \{(T_j \mid j = 1, 2, \cdots )\}$ \quad \triangleright \text{TOL that are yet to be processed}$
5: PROCESSTOL() \quad \triangleright \text{Non-blocking call to process existing TOL}$
6: 
7: \textbf{while} true \textbf{do} \quad \triangleright \text{On arrival of new block}$
8: 
9: \textbf{procedure} RECONFIGURE($B_k$) \quad \triangleright \text{Reconfigure}$
10: 
11: \textbf{if} $\text{VALIDATE($B_k$)}$ \textbf{then}$
12: 
13: \text{stop current PoW}$
14: 
15: \text{start PoW on $R_{k+1}'$}$
16: 
17: \textbf{end if}$
18: 
19: \textbf{end procedure}$
20: 
21: \textbf{procedure} VALIDATE($B_k$) \quad \triangleright \text{Validate}$
22: 
23: \text{valid} \leftarrow \text{false}; S_{k-\epsilon}, P_k, T_k \leftarrow B_k$
24: 
25: \textbf{if} $S_{k-\epsilon}$ \text{is empty then}$
26: 
27: \quad P'_k \leftarrow \Pi(P'_k-1, R_k, \zeta, \Phi_k)$
28: 
29: \textbf{if} $P'_k = P_k$ \text{then}$
30: 
31: \quad \text{valid} \leftarrow \text{true}$
32: 
33: \textbf{else}$
34: 
35: \quad \text{if} S'_{k-\epsilon} \text{is not in cache then}$
36: 
37: \text{valid} \leftarrow \text{false}; add T_k \text{to Q}$
38: 
39: \textbf{else}$
40: 
41: \quad P'_k \leftarrow \Pi(P'_k-1, R_{k-\epsilon}, \zeta, \Phi_k)$
42: 
43: \textbf{if} $P'_k = P_k \text{and} S'_{k-\epsilon} = S_{k-\epsilon}$ \text{then}$
44: 
45: \quad \text{add} P_k \text{to P}; add T_k \text{to Q}; \text{T} \leftarrow \text{T \setminus T_k}$
46: 
47: \text{valid} \leftarrow \text{true}$
48: 
49: \textbf{end if}$
50: 
51: \textbf{end if}$
52: 
53: \textbf{end if}$
54: 
55: \textbf{return} valid$
56: 
57: \textbf{end procedure}$
58: 
59: \textbf{procedure} CREATE($B_k$) \quad \triangleright \text{Create}$
60: 
61: \text{T}'_{k+1} \leftarrow \text{subset of T}$
62: 
63: \textbf{if} $S'_{k-\epsilon}$ \text{is in cache then}$
64: 
65: \quad P'_k \leftarrow \Pi(P'_k-1, R_{k-\epsilon}, \zeta, \Phi'_{k+1})$
66: 
67: \textbf{return} $S'(k-\epsilon, P'_k, T'_{k+1})$
68: 
69: \textbf{else}$
70: 
71: \quad P'_k \leftarrow \Pi(P'_k, \Phi'_{k+1})$
72: 
73: \textbf{return} empty-string $P'_k, T'_{k+1}$
74: 
75: \textbf{end if}$
76: 
77: \textbf{end procedure}$
78: 
79: \textbf{procedure} PROCESSTOL() \quad \triangleright \text{Process TOL}$
80: 
81: \textbf{while} true \textbf{do}$
82: 
83: \textbf{if} Q \text{is non empty then}$
84: 
85: \quad B_j \leftarrow \text{next block in Q}$
86: 
87: \text{add} B_j \text{to C}$
88: 
89: \text{if} $j > \text{current validated chain length then}$
90: 
91: \text{Non-blocking RECONFIGURE($B_j$)}$
92: 
93: \textbf{end if}$
94: 
95: \textbf{end if}$
96: 
97: \textbf{end while}$
98: 
99: \textbf{end procedure}$

6.2 Reduction to Nakamoto PoW

In this section we illustrate that the chain-growth of TUXEDO (as defined in [14, 21, 22]) in a time interval $T$ is greater than or equal to the known chain growth of PoW based Nakamoto consensus [14, 21, 22]. As mentioned earlier, we later use this fact to prove security of TUXEDO against all possible Byzantine adversaries.

Figure 7: Honest blocks chosen in time interval $[s + \Delta, s + T - \Delta]$ where the chosen blocks are separated by at least a gap of $\Delta$.

![Figure 7: Honest blocks chosen in time interval $[s + \Delta, s + T - \Delta]$ where the chosen blocks are separated by at least a gap of $\Delta$.](image)

validates) these blocks in First In First Out (FIFO) order. As we have mentioned earlier, due to forks, there will be multiple queues at each miner (see Figure 5), but our analysis applies to any of them as arrival rate at each queue is dominated by the arrival rate in a single queue setting and a miner processes all queues in parallel.

Let $Q_a(t)$ denote the size of the queue of a miner $n_a$ at time $t$. If block $B_k$ enters the queue at time $t_k$, we use $Q_a(t_k)$ and $Q_a(t_k')$ to denote the size of queue immediately before and after time $t_k$ respectively. Note, $Q_a(t_k') = Q_a(t_k) + 1$.

Handling non-ES blocks. The ability miner $n_a$ to validate a received block $B_k$ and/or create a non-ES block on it is directly related to the number of blocks in the queue which $B_k$ enters. Notice that if $Q_a(t_k') > \zeta$ then the head of the queue contains TOL $T_i$ for $i \leq k - \zeta$ and the miner will not be able immediately validate $B_k$. Similarly, when $Q(t_n') = \zeta$, the miner will be able to validate $B_k$ but will not have the state to immediately mine a non-ES block on top of the received block.

**Lemma 6.1.** Let $B_k$ be the $m^{th}$ block starting from genesis block in the chain containing $B_k$. Call these blocks $b_0, b_1, b_2, \cdots, b_m$ with $b_0$ as the genesis block. Let $t'_m, t'_m, t'_{m-1}, \cdots, t'_1$ be the time when an honest miner hears the corresponding block for the first time. Then by $t_k + \Delta$ i.e. $t'_m + \Delta$, all honest miners would have processed the state required to validate $B_k$.

**Proof.** Let $Q_L$ and $Q_U$ be two hypothetical FIFO queues with constant service rate $1/\tau$ in which blocks $b_1, b_2, \cdots, b_m$ enter at $t'_1, t'_2, \cdots, t'_m$ and $t'_1 + \Delta, t'_2 + \Delta, \cdots, t'_m + \Delta$ respectively. Let $Q_L^{(k)}(t)$ be the position of the block $b_k$ (i.e. $b_{n_a}$) at the time of miner $n_a$ at time $t$, and $Q_{B_k}(t)$ be its position at the miner which created it.

Then the following two conditions hold:

$$Q_L^{(k)}(t) \leq Q_U^{(k)}(t), \forall t \geq t_k + \Delta$$

Hence,

$$Q_{B_k}(t + \Delta) \leq Q_U^{(k)}(t_k + \Delta) = Q_L^{(k)}(t_k) \leq Q_{B_k}(t)$$

Then the following two conditions hold:

$$Q_L^{(k)}(t_k) \leq Q_U^{(k)}(t_k), \forall t \geq t_k + \Delta$$

Hence,

$$Q_{B_k}(t_k + \Delta) \leq Q_U^{(k)}(t_k + \Delta) = Q_L^{(k)}(t_k) \leq Q_{B_k}(t)$$
Equation (4) implies that by time $t_k + \Delta$, the position of $B_k$ in the queue of all honest miners will be less than or equal to the position of $B_k$ in the queue at time $t_k$. Hence if $Q_{B_k}(t_k^*) \leq \zeta$, i.e. $B_k$ is a non-ES block, by time $t_k + \Delta$, the position $B_k$ will be lower than or equal to $\zeta$ at all honest miners and hence, all honest miners will be able to validate $B_k$ by time $t_k + \Delta$.

Lemma 6.1 implies that whenever an honest miner generates a block, $\Delta$ time after the block generation time, every honest miner will have the state required to validate the block generated by the honest miner. Hence, $\Delta$ time after generation of an honest block $B_k$, every honest miner extends a block which is at a height greater than or equal to the height of $B_k$. We use this argument in Lemma 6.2 to prove that whenever two honest miners generate blocks at time instants that are at least $\Delta$ apart, the height of the latter block is greater than the height of the former. As a result, it is easy to see that blocks $B_1, B_2, \ldots, B_N$ that we consider have a strictly increasing length.

**Lemma 6.2.** Let $(t(B_k))$ denote the length of the block starting from genesis block $b_0$ with $(t(b_0)) = 0$. Then $B_i$ for all $i \in [1,N]$ which were mined between $[s + \Delta, s + T - \Delta]$ as shown in Figure 7 have distinct length. Further, $(t(B_i)) > (t(B_j)) \forall i > j$.

**Proof.** Consider two consecutive blocks $B_k, B_{k+1}$ mined at time $t_k, t_{k+1}$ respectively (need not be part of the same chain). Let $n_k, n_{k+1}$ be the miners of $B_k, B_{k+1}$ respectively. Since $t_{k+1} > t_k + \Delta$, $n_{k+1}$ would have heard of $B_k$ prior mining $B_{k+1}$. Also, from Lemma 6.1 by time $t_k + \Delta$, $n_{k+1}$ will have the state to validate $B_k$. Thus from time $t_k + \Delta$ onwards, $n_{k+1}$ either will extend $B_k$ or any other validated block with same or greater length than $B_k$. This implies $(t(B_{k+1})) > (t(B_k))$. This is true for all pair of consecutive blocks and hence by transitivity of length comparison, we get $(t(B_i)) > (t(B_j)) \forall i > j$.

Next we will use the Lemma 6.1 and Lemma 6.2 to show that during an interval of size $T$, the height of the blockchain at every honest miner grows by at least $N$.

**Lemma 6.3.** (Chain Increase) Let $L(t)$ be the length of the longest validated chain at miner $n_j$ at time $t$. Let $L_{\min}(t), L_{\max}(t)$ be the minimum and maximum of chain lengths of all honest miners at any time $t$, i.e. $L_{\min}(t) = \min_j \{L_j(t)\}$ and $L_{\max}(t) = \max_j \{L_j(t)\}$, then in the scenario shown in Figure 7, chain length of all honest miners grows by at least $N$ blocks, i.e $L_{\min}(s + T) \geq L_{\max}(s) + N$.

**Proof.** From Lemma 6.2, $(t(B_{k+1})) > (t(B_k)) + 1, \forall i \in [N - 1]$. Let $L_{B_k}(t_k^*)$ be the length of the longest chain of miner of block $B_k$ at time $t_k^*$. Then from Lemma 6.2 we know,

$$L_{B_k}(t_{k+1}^*) \geq L_{B_k}(t_k^*) + 1 \tag{5}$$

From Lemma 6.1,

$$L_{\min}(s + \Delta) \geq L_{B_k}(t_k^*)$$

Also, $L_{\min}(s + \Delta) \geq L_{\max}(s)$ as all blocks generated before $s$ reaches every honest miner by time $s + \Delta$. Hence,

$$L_{\min}(s + T) \geq L_{\min}(s + N + \Delta) \geq L_{\min}(s + \Delta) + N \geq L_{\max}(s) + N$$

Notice that the scenario shown in Figure 7 is stochastically identical to the HYB experiment shown by Pass et al. in [21]. Hence the chain growth and chain quality of Tuxedo is identical to results presented in [21]. Also, it is easy to see that $B_1, B_2, \ldots, B_N$ includes all loners defined in [22] (or convergence opportunities defined in [21]) and hence the safety results from [22] holds for Tuxedo. For completeness we state the theorem here.

**Theorem 6.4.** (Safety (Thm. 8 in [22])) Let $B^*$ and $B^{**}$ be two distinct blocks at the same height. If $e^{-2\alpha\Delta t} > (1 + \delta)\beta$, then once an honest node adopts a chain that buries $B^*$ by $k$ blocks deep, no honest node will adopt a chain that buries $B^{**}$ by $k$ blocks, except for $e^{-2\alpha(\delta k)}$ probability.

**Proof.** Directly follows from Lemma 6.3 and proof of Theorem 8 of [22].

### 6.3 Choice of $\zeta$

As $Q(t) \geq \zeta$ at a honest miner implies that the miner will not be able to validate a received non-ES block, we compute an upper bound on $Pr\{Q(t) \geq \zeta\}$ under all possible adversarial strategies after making certain approximations. Recall that $Q(t) \geq \zeta$ do not violate security of Tuxedo and hence the guarantees provided in previous section still holds true.

A well-known result from queuing theory [20] is that in any queuing system with constant service rate $1/\tau$, the size of the queue at any time $t$ is given by:

$$Q(t) = \sup \{A(s) - s \over \tau\}, \tag{6}$$

where $A(s)$ is the number of arrivals during the interval $[t - s, t]$. In addition to the number of blocks generated during the time interval $[t - s, t]$, $A(s)$ may also include honest blocks from time interval $[t - s - \Delta, t - s]$ as these blocks might be delayed due to network. Furthermore, an adversary can deliberately withhold blocks mined prior to time $t - s$ and release them during $[t - s, t]$. However, as demonstrated in [21] that to withhold a block by longer than time $t_w$, adversary needs to generate a private chain longer than honest chain during that time.

Approximating the growth rate of the honest miners as a Poisson process with rate $\gamma$ where $\gamma = \alpha/(1 + \Delta\alpha)$, we can approximate the race between honest chain and the adversarial chain for time $t_w$ as a Skellam Distribution [24] with $\mu_1 = \beta t_w$ and $\mu_2 = \gamma t_w$. Specifically, let $N(t_w), X_{\mathcal{A}}(t_w)$ be the random variables denoting the chain growth of honest miner and number of blocks generated by $\mathcal{A}$ during a time interval of size $t_w$ respectively. Then the success probability of $\mathcal{A}$ withholding a block for longer than $t_w$ is $Pr[X_{\mathcal{A}}(t_w) - n(t_w) > 0]$. Since, $X_{\mathcal{A}}(t_w)$ and $N(t_w)$ are independent Poisson random variable, $X_{\mathcal{A}}(t_w) - N(t_w)$ follows a Skellam distribution with mean $\mu_1$ and $\mu_2$ as mentioned above.

Using results from Skellam distribution, given a small threshold $\eta$, we pick a value of $t^*$ such that

$$Pr[X_{\mathcal{A}}(t^*) - Y(t^*) \leq \eta] \tag{7}$$

and assume that $\mathcal{A}$ is not allowed to withhold a block for more than $t^*$ units of time. Under this assumption, we next upper bound the probability that queue of an honest miner will exceed any given $\zeta$ under all possible adversarial strategies.
Theorem 6.5. For any given $\epsilon_0, \epsilon_1, t^*$, let $s_0 = \max \{\alpha, \frac{t^*}{\epsilon_1}\}$ and
\( \lambda = (1 + \epsilon_0)\alpha + (1 + \epsilon_1)\beta. \) Let $Q(t)$ be the size of an honest miner’s queue at time $t$. Then
\[
\Pr[Q(t) \geq \zeta] \leq \sum_{i=0}^{s_0} \pi_i + 1 - \sum_{i=0}^{s_1} \frac{\lambda_i^i e^{-\lambda s_0}}{i!},
\]
where $\pi_i$ is the stationary distribution of $M/D/1$ queue with arrival rate $\lambda_i$.

Proof. Let $X_A(b), X_{\bar{A}}(b)$ be the random variable denoting the number of blocks mined by honest miners and adversary in a given time interval of length $b$ respectively. As we assume that $\bar{A}$ withholds a block for at most $t^*$ time before the honest miner accepts them, blocks in $A(s)$ are either mined by the adversary during $(t - s - t^*, t)$ or mined by honest nodes during $(t - s - \Delta, t)$. For any $\epsilon_0 > 0, \epsilon_1 > 0$, let $s_0 = \max \{\alpha, \frac{t^*}{\epsilon_1}\}$. Then $s \geq s_0, s + \Delta < (1 + \epsilon_1)s$ and $s + t^* < (1 + \epsilon_1)s$. Hence,
\[
A(s) \leq X_{\bar{A}}(s + \Delta) + X_A(s + t^*) \geq X_{\bar{A}}(s) + X_A(s + t^*) \geq X_{\bar{A}}((1 + \epsilon_0)s) + X_A((1 + \epsilon_1)s), \forall s \geq s_0.
\]

Let $X(s)$ be a random variable denoting the number of blocks generated by a Poisson process within a time interval of size $s$ with arrival rate $\lambda = (1 + \epsilon_0)\alpha + (1 + \epsilon_1)\beta$. Since independent Poisson random variables are additive, we have the equality in distribution,
\[
X(s) = X_{\bar{A}}((1 + \epsilon_0)s) + X_A((1 + \epsilon_1)s).
\]
Hence using equation 6, we have
\[
\Pr[Q(t) > b] \leq \Pr\left[ \bigcup_{s > b} \left\{ A(s) - \frac{s}{\tau} > b \right\} \right].
\]
(From equation 6)
\[
= \Pr\left[ \bigcup_{s \geq s_0} \left\{ A(s) - \frac{s}{\tau} > b \right\} \right] + \Pr\left[ \bigcup_{s \geq s_0} \left\{ A(s) - \frac{s}{\tau} > b \right\} \right] \geq \Pr[A(s_0) > b] \geq \Pr[A(s_0) > b].
\]
(13)
\[
\leq \Pr\left[ \bigcup_{s \geq s_0} \left\{ X(s) - \frac{s}{\tau} > b \right\} \right] + \Pr[A(s_0) > b] \geq \Pr[A(s_0) > b].
\]
(14)

The first term of equation 14 is the standard M/D/1 tail queue probability with arrival rate $\lambda$, processing rate $1/\tau$ and hence its tail distribution probability decreases with increasing $\zeta$. For any given $b, t^*, \epsilon, \tau,$ and $\Delta$,
\[
\Pr[A(s_0) > b] \leq 1 - \sum_{i=0}^{s_1} \frac{\lambda^i e^{-\lambda s_0}}{i!}.
\]
(15)

Using our worst-case analysis, we suggest concrete values of $\zeta$ one should consider to bound the probability of an honest miner’s queue exceeding $\zeta$. As we expect attacks to be intermittent (if any), we also numerically compute these bounds for an honest execution of the protocol, i.e., in a network without any adversary. Figure 8 and 18 plots the result of Theorem 6.5 under some example parameters.

Concrete choice of $\zeta$. For any given $\lambda, f_{\max}, \Delta, \tau$, and $\epsilon$, we evaluate $\zeta$ such that $\Pr(Q(t) \geq \zeta) < 0.01$. For all our evaluation we have used $\eta = 0.001$ in equation 7. Figure 8 illustrates our results for different values of $\tau/\Delta$ and $\zeta$. For each $\tau/\Delta$, we pick $\zeta$ that minimizes $\zeta$. For example, with 25% adversary and allowable processing time equal to half of average interarrival time, i.e. $\tau/\Delta = 0.5$, we get $\zeta = 33$ for $\tau/\Delta = 10$.

Remark. It is important to note the event $Q(t) \geq \zeta$ do not let the Byzantine adversary to violate the consistency of the protocol. Instead, it only allows the adversary to delay the reporting of update state for a very short duration of time. This is because, when $Q(t) \geq \zeta$, honest miners do not extend blocks mined by the adversary, and since the block processing rate is considerably higher than the block generation rate of the adversary, the queue at the honest miners will soon have less than $\zeta$ blocks. Hence, very soon the honest miners will start creating blocks with non-empty state.

7 EVALUATION

To evaluate computation scalability of TUXEDO we built a prototype of on top of Ethereum Geth client version 1.9.3. Our implementation consists of all parts of TUXEDO including an on-demand adversarial behavior to skip validation of transactions. In many experiments we compare the performance of TUXEDO with that of Ethereum. To facilitate such comparisons, we implement an adversary who skips validation and/or creation of blocks in Ethereum.

7.1 Experimental Setup

Our experimental setup consists of 50 virtual machines (VMs) running in Oracle Cloud. All expect one of the VMs are dual-core machines with 8GM of RAM running Ubuntu16.04. The remaining VM which we use as an adversarial node has 8 core CPU @2.19 GHz, 30GB of RAM. We deliberately assign one node a computational advantage over others to measure its effects on the fairness of Ethereum and TUXEDO. In our setup, all nodes have identical network bandwidth of 1GB/s download and 100MB/s upload speeds.

Node. Each VM in our experimental setup runs one TUXEDO node. The mining power of each node is set according to the distribution of top 50 Ethereum miners extracted from [2]. This corresponds to 99.98% of Ethereum’s total mining power. For each node in our setup, we simulate its block mining process by drawing the interarrival time between the blocks from an exponential distribution with parameter $\lambda/h$ where $h$ is the fraction of mining power controlled by the node. In Table 1 reports the percentage of mining power
controlled by top 14 miners that sum up to 97% of the total mining power.

|         | Honest | Adversarial |
|---------|--------|-------------|
| $\tau/\lambda$ | 0.011  | 0.122       |
|         | 0.205  | 0.186       |
|         | 0.740  | 0.379       |
|         | 0.011  | 0.122       |
|         | 0.205  | 0.186       |
|         | 0.740  | 0.379       |

Table 1: Percentage of mining power controlled by top 14 miners in descending order our experimental setup. For e.g. first and 14th miner controls 32.98% and 1.05% of the mining power respectively.

Network. To make a prototype of this Ethereum mining network, we collected data regarding the geographical location of the top 50 miners in Ethereum. Each node in our experiment emulates the geographical location of one such miner. We then form a randomly connected network of these nodes where the degree distribution follows a power law with exponent $-2.5$. Communication delays between every pair of nodes in the network are set accordingly to the ping delays observed between respective geographical locations [1]. We use Linux tc command to simulate the link delays.

Methodology. We test Tuxedo by deploying 50 contracts each implementing Quicksort, 2D matrix multiplication, and iteration with basic arithmetic operations. We then invoke functions from each contract with appropriate parameters to achieve the desired block processing time. Throughout our experiment, we ensure that each block contains ~165 transactions in total. As we simulate an adversary ($\mathcal{A}$) that skips validation of blocks and creates new ones with contracts whose execution results are already known to the $\mathcal{A}$, we deliberately restrict all of the above mentioned contracts to be stateless. Note that, as the primary metric of evaluating Tuxedo is processing time of a block, any choice of contracts will give us the same results as long as they achieve the desired block processing time.

7.2 Experiments and Results

We first evaluate the effect of increasing $\tau/\lambda$ in Ethereum with all miners being honest. We then repeat the experiment in the presence of an adversary $\mathcal{A}$, who skips validation of received blocks and creates blocks with transactions for which $\mathcal{A}$ already knows the execution results. We then perform the same set of experiments with Tuxedo and compare our results with Ethereum. Next for fixed $\tau/\lambda = 0.70$ we evaluate Tuxedo with increasing network delay.

In all experiments the first miner ($n_1$) controls ~33% of the network's mining power and can process ~1.67 times faster than other miners. In all experiments we keep block mining difficulty such that $1/\lambda = 15.0$. Hence $I$ is equal $2\tau + 1/\lambda$ for Ethereum experiments and $1/\lambda$ for Tuxedo experiments.

Fairness violations in Ethereum. Figure 9a illustrates the fraction of Ethereum blocks mined $n_1$ mines. Observe that with increasing $\tau/\lambda$, the fraction of blocks $n_1$ mines increases even when $n_1$ is honest corroborating our theoretical analysis (ref. Appendix B). When $n_1$ is adversarial, i.e. $n_1$ skips both validation and creation of blocks, $n_1$ mines significantly higher fraction of blocks. These fractions are higher than theoretically computed fraction in Figure 9a. Because unlike our simplistic assumption in theoretical analysis (ref.B), in the experiments $n_1$ mines for the entire duration of the experiment.

High fork rate in Ethereum. With high $\tau/\lambda$ in Ethereum, we observe that the fork rate of Ethereum increases. Let Mining Power Utilization (MPU) of a blockchain network be defined as the fraction of blocks mined by the miners that end up in the eventual longest chain. A blockchain with lower MPU implies that a lower fraction of mined blocks end up being in the blockchain and that many blocks are orphaned. Figure 10a illustrates the MPU of Ethereum network with increasing $\tau/\lambda$. Notice that despite having $\lambda = 1/15$ and $I = 2\tau + 1/\lambda$, the fork rate increases. This is because miners in Ethereum only forward a block when they have fully
validated its parent block. Thus with high $r/I$, miners in Ethereum more frequently encounter blocks whose parents are yet to be validated by the miners. As a result, with $r/I$, the effective propagation delay of blocks in the network increase which leads to higher forks and lowers the MPU of the network.

Figure 11b illustrates that the median propagation delay of Ethereum network. Median block propagation delay in the presence of an adversarial node is lower due to the fact that the adversarial node can forward blocks immediately as it processes all received blocks immediately. Also we observe that higher $r/I$ does not lower MPU of the adversarial node $n_1$ by much. Further, MPU of an adversarial node is much higher than its honest counterpart. Figure 11a illustrates the mining power utilization of top 2 miner in our experimental setup (ref. 1). This is because when $n_1$ is adversarial, it mines solo for longer duration. This allows $n_1$ to mine longer sequence of blocks more frequently while other miners were busy extending an older block. Hence, when network gets synchronized again, that is honest nodes have to backlog of blocks to process, the string of blocks mined by the $n_1$ enters the blockchain with high probability.

**Increasing $r/I$ in Tuxedo.** We demonstrate that higher $r/I$ does not affect miners in Tuxedo by repeating the above experiments in Tuxedo. Figure 9b illustrates that the fraction of blocks $n_1$ mines does not vary in Tuxedo despite high $r/I$ and an adversarial $n_1$. Also, unlike Ethereum high $r/I$ does not affect the mining power utilization of Tuxedo since all miners can immediately validate all received blocks. This is illustrated in Figure 10b. We use $\zeta$ based on our analysis in §6.3 for $t^* = 0$ and $Pr[Q(t) \geq \zeta] \leq 2^{-15}$.

Figure 12 demonstrates the queue observed by a arriving block in an honest miner for $r/I = 0.379$. Observe that presence of a skipping adversary does alter the queue size honest miners observe. Further, more than 99% of the blocks find a queue size of four or less and less than $< 0.1\%$ blocks finds a queue size of eight or higher. This implies that though $\zeta$ is chosen to be high, for majority of the blocks the users of Tuxedo will get the execution results of their transactions within four blocks.

**Increasing network delay.** For fixed $r/I = 0.74$ we evaluate Tuxedo with increasing delay. Specifically we increase link delay between each pair of connected node by a factor of $d = 1, 2, 4$. Figure 9c illustrates that higher delay does not affect the fraction of blocks mined by $n_1$ for both honest and adversarial $n_1$. We also observe that the MPU of Tuxedo does not decrease in Tuxedo despite higher network delay (see Figure 10c). This is because, the top five miners controlling approximately 75% of mining power in our experimental network are in close proximity with each other. Hence the increased delay does not affect the block propagation delay between them.

8 RELATED WORK

To best of our knowledge, our work is the first on-chain solution to increase the ratio between block validation time and average block inter-arrival time in a PoW based blockchain.

There have been attempts to enable the execution of computationally intensive smart contracts [7, 9, 11, 13, 26] through off-chain solutions where the execution of intensive transactions is delegated to a subset of miners or volunteer nodes. These solutions typically have high latency for off-chain computations and also make additional security assumptions beyond those required for PoW consensus. Also, off-chain solutions restrict certain interactions between contracts, e.g., one smart contract cannot internally invoke functions of other smart contracts. Such interactions are desirable and often occur in practice (see Appendix D). Arbitrum [13] requires nodes participating in the protocol to be rational and one participating node to be honest. Yoda [9] requires an unbiased source of randomness whereas current mechanisms of generating distributed randomness are highly expensive [25]. Ekedon [7] relies on SGX Enclaves and requires all enclaves to be trusted, an assumption that is made questionable by recent attacks [6, 28]. In Zokrates [11] participants are required to generate expensive non-interactive proofs for verification of off-chain computations.

A concurrent work ACE [30] provides an off-chain solution that securely scales the smart contract execution and enables interactive calls between smart-contracts by implementing a variant of the classic two-phase commit protocol. Thus, interactive contracts in ACE have a very long delay as they can be delayed by the slowest committee involved in the interaction. Also ACE requires committee members to broadcast the updated state to the entire network. This can lead to large bandwidth usage for contracts that update a large number of variables. Furthermore, in ACE more than half of the committee members for each contract must be honest for guaranteed safety and liveness of the contract execution, and it relies on a reputation based mechanism to realize such assumptions. Note that this is a much stronger assumption than assuming, as Tuxedo does, that half of the total mining power is honest.

Note that Tuxedo does not eliminate the benefits of off-chain based approaches. Instead, it further opens up the possibilities for
We discuss some questions about TUXEDO while mining requires ASICs. Furthermore, the number of simultaneous forks in them are quite small [12].

How TUXEDO differs from off-chain solutions that do not make additional security assumptions as that of the PoW blockchain like Rollups? Rollups is the second layer solution that requires ‘operators’ to stake a bond in the rollup contract. This is not proposed as the smart contract scaling solution but instead a high throughput solution that enhances the transaction throughput between the operators involved in the bond. It also fails in the case of interactive smart contract transactions.

Why there is need to modify the standard longest chain rule? Since validation times of blocks are large, a miner may be presented with a long chain which it cannot immediately validate. By mining on the longest validated chain, we ensure that a miner does not mine on any chain which contains a block with an invalid state. In addition, honest miners will not stop mining even if a long unvalidated chain is known to it. We use this fact in our security proofs against a general adversary.

10 CONCLUSION
We have presented TUXEDO which theoretically allows validation time of blocks in PoW based blockchains to be comparable to the average interarrival time, i.e. $\tau/I \approx 1$. Such a high validation time allows TUXEDO to scale execution of smart contracts on-chain. Hence, it makes blockchains accessible to applications with heavy computation. Another advantage of the on-chain approach is that all miners update state locally and hence obviate the need for transferring state updated due to transaction execution. Hence the bandwidth usage of TUXEDO is identical to existing system such as Ethereum.

We prove the security of TUXEDO in synchronous network with end-to-end delay of $\Delta$ in the presence of a Byzantine adversary considering all possible adversarial strategies. We also present a principled approach to pick $\zeta$ for any given choice of parameters. Although, state corresponding to a contract transactions gets reported $\zeta$ block later, our analysis and evaluation demonstrate that, most (> 99%) blocks in TUXEDO finds a queue size of less than five on its arrival. Hence in practice, miners will have execution results of transactions reasonably quickly. Furthermore, token transactions are executed immediately thus can be used in the low latency applications. Our experimental results demonstrate working of TUXEDO for $\tau/I = 0.70$ for an implementation over a standard Ethereum geth client.

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All honest miners. The probability of the event $\lambda$ is depicted in the top diagram of Figure 14. Similarly, the bottom diagram in the same Figure illustrates the miners that perform PoW at different intervals starting from different state. In this paper we only derive transition probabilities of the MC for $c > 1/2$ as one can easily derive for $c \leq 1/2$ using similar approach.

**Lemma B.1.** Let $X, Y$ be two independent random variable with exponential distribution with arrival rate $\lambda_x, \lambda_y$ respectively. Then the probability of the event $A = (X \leq \tau \land X \leq Y)$ is denoted using $N(\lambda_x, \lambda_y, \tau)$ and is equal to:

$$Pr[A] = N(\lambda_x, \lambda_y, \tau) = \frac{\lambda_x}{\lambda_x + \lambda_y} \left(1 - e^{-(\lambda_x + \lambda_y)\tau}\right)$$  \hspace{1cm} (16)

**Proof.** For any arbitrary time $t$ where $0 < t \leq \tau$, $Pr[t - dt \leq X \leq t] = f_X(t)dt = \lambda_x e^{-\lambda_xt}dt$ and $Pr[Y > t] = 1 - \lambda_y e^{-\lambda_yt}$. Thus to get the closed form expression for $Pr[A]$, we compute,

$$Pr[A] = \int_{0}^{\tau} f_X(t)Pr[Y > t]dt = \int_{0}^{\tau} \lambda_x e^{-\lambda_xt}e^{-\lambda_yt}dt$$  \hspace{1cm} (17)

Solving the above produces the desired result.

**Theorem B.2.** For a particular $c > 1/2$ and $N = \{\lambda_j\} = 1, 2, \ldots, |N|$, with every miner honestly following the protocol, the state transition probabilities are given as:

$$p_{1,1} = 1 - M_0(\lambda_1, 2\tau - c\tau) + p_1M_0(\lambda_1, 2\tau - c\tau)$$  \hspace{1cm} (18)

$$p_{u,1} = M_0(\lambda_u, 2\tau - c\tau) [N(\lambda_1, \lambda_u, 2\tau - c\tau) + M_0(\lambda_1 + \lambda_u, 2\tau - c\tau)p_1]$$  \hspace{1cm} (19)

$$p_{1,0} = p_2M_0(\lambda_1, 2\tau - c\tau)$$  \hspace{1cm} (20)

$$p_{u,0} = 1 - M_0(\lambda_1, 2\tau - c\tau) + M_0(\lambda_1, 2\tau - c\tau)\{N(\lambda_1, \lambda_u, 2\tau - c\tau) + M_0(\lambda_1 + \lambda_u, 2\tau - c\tau)p_0\}$$  \hspace{1cm} (21)

$$p_{u,u} = M_0(\lambda_1, 2\tau - c\tau)M_0(\lambda_1 + \lambda_u, 2\tau - c\tau)p_0$$  \hspace{1cm} (22)

with $N(\lambda_u, \lambda_u, t)$ is as given in Lemma B.1 and $M_0(\lambda_x, t)$ is the probability of $0$ arrival in a Poisson process in a time interval $t$ with arrival rate $\lambda_x$. Hence $M_0(\lambda_x, t) = e^{-\lambda_xt}.$

**Proof.** Transition of state 1 to 1 can happen either if $n_1$ mines the next block during $2\tau - c\tau$ interval or if $n_1$ mines the block after time $2\tau$. The former happens with a probability $1 - M_0(\lambda_1, 2\tau - c\tau)$. The latter happens with a probability $p_1$ conditioned on that the former event did not happen. Hence the probability of the latter is $p_1M_0(\lambda_1, 2\tau - c\tau)$. Also since these two events are mutually exclusive, $p_{1,1}$ is sum of the probability of the events. Similarly, starting with state 1, any other miner $n_u$ will mine the next block only if $n_1$ does not mine the block during an interval of length $2\tau - c\tau$ starting at $\tau$. Also since all miners will be mining after time $2\tau$ if no block was mined before that, the probability that the next winner would be $n_u$ is $p_u$. Hence the transition probability $p_{1,u}$ equal to $p_2M_0(\lambda_1, 2\tau - c\tau)$. Alternatively, starting from a state $u$ with $u \neq 1$, $n_u$ will mine the next block during time interval $(\tau, 2\tau]$ with probability $1 - M_0(\lambda_u, 2\tau - \tau)$. Otherwise $n_u$ can mine the block during $(2\tau, 2\tau]$. But as both $n_1$ and $n_u$ will be mining during $(2\tau, 2\tau]$, the probability of $n_u$ mining the block before $n_1$ is equal to $M_0(\lambda_u, \lambda_1, 2\tau - 2\tau)$. Lastly if $n_u$ mine the block in neither of these interval, $n_u$ will mine the next block with probability $p_0 = \lambda_u/\lambda$. Combining the above will give the transition probability $p_{u,u}$. The transition probability $p_{u,1}$ can be derived similarly.

Lastly, state transition from a state $u$ to $o$ with $u \neq o \neq 1$ can happen if neither $n_1$ nor $n_u$ mines a block prior time $2\tau$. Hence the transition probability $p_{u,o}$ is equal to $M_0(\lambda_u, 2\tau - c\tau)M_0(\lambda_1 + \lambda_u, 2\tau - 2\tau)p_0$. □

Using the above state transition probabilities and mining power from Table 1 we numerically compute the stationary probabilities of the Markov chain with all miners being honest. Figure 3 illustrates our results for different $c$ with varying $\tau$.

**Higher $\tau$ in the presence of an Adversary.** Let node $n_1$ with arrival rate $\lambda_1$ be controlled by an adversary $\mathcal{A}$. We consider two different behaviors of the adversarial node $n_1$. First, $n_1$ validates the received blocks as per the protocol but instantly creates a block by putting transactions whose execution results are already known to $n_1$. In second $n_1$ skips validation of the received block as well and instantly starts PoW on top a new full block. The former attack is very practical as any miner can do that without any additional computational resources. The later damages fairness more severely but requires $n_1$ to produce final state due to transactions in the received block without executing them. An adversary can launch the later attack if it can download the modified state due to previous

![Figure 14: Miners who are active during each time interval from the instant when last block gets generated in a situation where all miners are honest.](image-url)
block from the creator of the previous block. Figure 15 illustrates which miners do PoW at different time intervals starting from the instant of successful PoW on the previous block. The diagram at the top is when n1 skips only creation and in the diagram at the bottom is when n1 skips both validation and creation. Here, we will only derive the transition probabilities for the latter, and the transition probabilities for the former can be derived similarly.

**Theorem B.3.** Given \( \tau \) and \( \Lambda = \{ \lambda_j | j = 1, 2, \ldots, |N| \} \), with \( \lambda_1 \) as arrival rate of the adversarial node n1, the transition probabilities for the Markov chain when n1 skips validation of received blocks and the creation of new ones are:

\[
\begin{align*}
p_{1,1} &= 1 - M_0(\lambda_1, 2\tau) + p_1 M_0(\lambda_1, 2\tau) \quad (23) \\
p_{1,0} &= p_0 M_0(\lambda_1, 2\tau) \quad (24) \\
p_{u,1} &= 1 - M_0(\lambda_1, \tau) + M_0(\lambda_1, \tau)[N(\lambda_1, \lambda_u, \tau) + M_0(\lambda_1 + \lambda_u, \tau)p_1] \quad (25) \\
p_{u,u} &= M_0(\lambda_1, \tau)[N(\lambda_u, \lambda_1, \tau) + M_0(\lambda_1 + \lambda_u, \tau)p_u] \quad (26) \\
p_{u,0} &= M_0(\lambda_1, \tau)M_0(\lambda_1 + \lambda_u, \tau)p_0 \quad (27)
\end{align*}
\]

**Proof.** When n1 mines the last block, all other nodes will start PoW for the next block only after 2\( \tau \) time units whereas n1 will do PoW for the entire 2\( \tau \) interval. Hence transition from state 1 to 1 will happen if either n1 mines the block in the first 2\( \tau \) time interval or n1 mine the after 2\( \tau \) time units. The former happens with a probability 1 - \( M_0(\lambda_1, 2\tau) \) and the later can happen with probability \( p_1 \) conditioned on the former not happening. Hence \( p_{1,1} = 1 - M_0(\lambda_1, 2\tau) + p_1 M_0(\lambda_1, 2\tau) \). Similarly, the transition from state 1 to another state can only happen if n1 did not mine during the first 2\( \tau \) time units. Also since all nodes will mining after 2\( \tau \) time units the transition probability \( p_{1,0} \) is equal to \( p_0 M_0(\lambda_1, 2\tau) \).

When a node \( n_u \) \( u \neq 1 \) mines the last block, n1 instantly starts PoW for the next block. Hence for the first \( \tau \) units of time only \( n_1 \) will be mining as even \( n_u \) will be busy creating the next block. During time (\( \tau, 2\tau \)) both n1 and \( n_u \) be mining and after 2\( \tau \) the rest of the miners will start PoW for the next block. Thus n1 can mine the next block either during the first \( \tau \) or in the interval (\( \tau, 2\tau \)) or after \( 2\tau \). The first can happen with a probability 1 - \( M_0(\lambda_1, 2\tau) \), the second with probability \( N(\lambda_1, \lambda_u, \tau) \) conditioned on the fact that the first did not occur, and lastly n1 will mine a block after time \( 2\tau \) with probability \( p_1 \) in case no block was mined prior to \( 2\tau \). Combining the above will give us the transition probability \( p_{u,1} \).

Similarly transition from state u to itself happens when \( n_u \) mines the next block either during time interval (\( \tau, 2\tau \)) or after time \( 2\tau \). The former happens with probability \( N(\lambda_u, \lambda_1, \tau) \) conditioned on the event that n1 did not mine the next block during first \( \tau \) time units and the later with probability \( p_u \) conditioned on neither \( n_u \) nor n1 mining a block before \( 2\tau \). Finally, transition to state to a state \( u, u \neq u \neq 1 \), will only happen with if neither \( n_u \) nor n1 mine the next block before \( 2\tau \). Hence the transition probability \( p_{u,0} \) is equal to \( M_0(\lambda_1, \tau)M_0(\lambda_1 + \lambda_u, \tau)p_0 \).

**Closed-form probabilities.** In a system with \( n \) miners, one needs to solve a system of \( n \) linear equations to get closed-form equations for the stationary probabilities. We do not solve these as a part of this paper. But we compute closed-form stationary distribution for a particular case is given below.

Let there be \( K \) nodes in the network with an equal mining power of each node. Among these \( K \) nodes an adversary \( A \) controls a f fraction of the node and all adversarial nodes skips both validation and creation of blocks.

Consider an honest node which has just mined a block and just finished creating a new block. We assume that \( K \) is large enough to ensure that the probability of this honest miner successfully mining in the next \( \tau \) units is approximately zero. In other words, we assume that the probability of all honest nodes together mining in this interval is zero and so only \( A \) mines in this interval. Under this assumption the Markov chain discussed so far can be reduced to a MC with only two states as depicted in Figure 17.

State 1 (resp. state 2) represents the last state was mined by a adversarial (resp. honest) node. The transition probabilities are given as:

\[
\begin{align*}
p_{0,0} &= p_{1,0} = 1 - M_0(f\lambda, 2\tau) + f M_0(f\lambda, 2\tau) \quad (28) \\
p_{0,1} &= p_{1,1} = (1 - f) M_0(f\lambda, 2\tau) \quad (29)
\end{align*}
\]

Let \( P_r \) be the transition probability matrix of this Markov chain and let \( \Psi = (\psi_1, \psi_2) \) be the stationary probabilities of the states 1, 2 respectively. Then we solve, \( \Psi P_r = \Psi \) to get the stationary
A general Markov chain for the skipping adversary can be reduced to the Markov chain above for large $n$ where each adversary controls a $f$ fraction of compute power and honest mining power is distributed among the remaining $n-1$ honest power such that the maximum amount of compute power a single miner controls extremely small.

probabilities and these values are:

$$
\psi_1 = 1 - M_0(\lambda_1, \tau) + p_1 M_0(\lambda_1, \tau)
$$

$$
\psi_2 = 1 - \psi_1
$$

C QUEUE OVERFLOW IN HONEST EXECUTION.

In this section we will evaluate the performance of Tuxedo when all parties are honest as we believe this will be the most likely case.

For any given $\zeta, \lambda, \Delta, \text{ and } \tau$ we compute the bounds using equation 8 by putting $\beta = 0$ and $t^* = 0$. This corresponds to a execution of Tuxedo in the absence of an adversary. Figure 18 illustrates the upper bound on the fraction of time queue at a honest miner will have more than $\zeta$ blocks for different values of $\zeta$. All plots are for $\tau/\Delta = 0.5$. Note that in the absence of $\mathcal{A}$, for $I/\Delta = 10$, with $\zeta$ as low as 20 queue, less than one in a billion honest blocks will hit a queue larger than $\zeta$.

D INTERACTION BETWEEN CONTRACTS

We measure the interaction between contracts for 50k blocks starting at a height of 6.5 Million. To measure this information, we sync an Ethereum Geth Client in archive mode. Such a node stores the debug trace of all transactions starting from the genesis block. We loop through debug traces of each transaction in every block in the given range and use EVM CALL, DELEGATECALL, and STATICCALL opcode to determine whether the transaction invoke a function which internally calls functions from other transactions. Figure 19 illustrates our findings. Specifically, we observe that in this corresponding range, in each block more than 50% of the transactions are addressed to a smart contract. Also, among all the transactions approximately 20% of the transactions internally invokes function calls to other contracts.

Figure 18: Upper bound on the probability of queue at a honest miner crossing the threshold in the absence of an adversary for $\tau/\Delta = 0.5$.