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New pulse oximetry detection based on the light absorbance ratio as determined from amplitude modulation indexes in the time and frequency domains

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A B S T R A C T

The Pandemic COVID-19 situation, a pulse Oximetry is significant to detect a varying blood oxygen saturation of a patient who needed the device to operate with continuous, rapid, high accuracy, and immune of moving artifacts. In this article, three main schemes for low-complexity pulse oximetry detection are proposed. In the first scheme, the light absorbance ratio (R) is obtained by separating the red and infrared photoplethysmography (PPG) amplitude modulation (AM) signals from the frequency-division multiplexing (FDM) signal with two different bandpass filters (BPFs), determining the ratio of modulation index of red and infrared PPG AM signals. In the second scheme, the output PPG AM signals for the red and infrared light wavelengths from the BPFs are transformed into the frequency domain such that the AC components of both PPG AM signals are the magnitudes of the highest peaks in their respective sidebands, while the DC components are the magnitude of their carrier frequencies; then, the AC/DC ratio of the red PPG AM signal is divided by the AC/DC ratio of the infrared PPG AM signal is R. In the last scheme, the FDM signal is transformed into the frequency domain without being passed through any BPF, and R is obtained in the same way as in the same second scheme. Experimental results obtained by using the first scheme have an average error of about 0.7138%, for the second and the last scheme have an average error of about 1%, and all the methods agree with the corresponding mathematical model.

1. Introduction

Arterial blood oxygen saturation (SaO2) is an important vital sign of the human body. It is the proportion of the measured oxyhaemoglobin (HbO2) concentration relative to the total haemoglobin concentration in the blood. Currently, SaO2 can be assessed by using a technique named pulse oximetry. Pulse oximetry yields an empirical measurement of arterial blood oxygen saturation (SaO2) called pulse oxygen saturation (SpO2). Unlike SaO2, SpO2 is based on only two functional haemoglobins, namely, HbO2 and deoxyhaemoglobin (Hb). These haemoglobins can reversibly bind with oxygen molecules. Generally, the types of haemoglobin present in a healthy person are functional haemoglobins, so SpO2 can be considered approximately equal to SaO2 if SaO2 is greater than 70% [1–4]. The pulse oximetry technique does not require a blood sample from the patient, causes no pain and is easy to perform. Traditional pulse oximetry relies on a pulse oximetry probe with dual light sources and a single light detector attached to the fingertip or earlobe [5]. This technique is based on the principle of the Beer-Lambert law [6–7]. Therefore, before any device for measuring SpO2 using an optical process could be invented, various chemical processes were first proposed, such as Van Slyke’s method, the mixing syringe method, and methods using the principle of oxidation, for example, the Clark electrode and the galvanic electrode [8–11]. However, chemical processes require chemical expertise, and any such process requires a blood sample from the patient, which must first be sampled from the patient’s body.

In addition to chemical processes, there are also various optical processes available for measurement purposes. One of these optical processes is spectrophotometry, which is the basis of the technique known as oximetry. Oximetry refers to a method of measuring SaO2 using optical techniques, and pulse oximetry is one important oximetry technique. Almost all optical techniques are based on the Beer-Lambert law, especially spectrophotometry, which uses the isosbestic point wavelength to measure SaO2. At the isosbestic point, the extinction coefficients of both HbO2 and Hb are identical [12]. Moreover, the CO-
oximeter method has been proposed based on a principle similar to that of spectrophotometry. The difference is that a CO-oximeter can also measure the concentrations of other kinds of haemoglobins, such as methaemoglobin and carboxyhaemoglobin. To measure the concentrations of various kinds of haemoglobins, a CO-oximeter needs to use four light sources of different wavelengths. However, with these two oximeters, a patient’s blood sample is still needed to estimate \( \text{SpO}_2 \).

In the 1930s, non-invasive oximeters were invented, of which Kramer’s oximeter was the first example. Kramer’s oximeter used a single red light source to measure \( \text{SaO}_2 \); because the absorbance of blood depends on \( \text{SaO}_2 \), a single light source can be used for this measurement. At around the same time, Matthes’ oximeter was invented. Matthes’ oximeter emitted both red and green light, which passed through the earlobe to a photocell. Initially, optical techniques for measuring \( \text{SaO}_2 \) had no special name; this was true until Glen Milliken established the word “oximetry” [13]. Milliken’s oximeter was similar to Matthes’ oximeter, with the difference that the two light sources produced red and infrared light. In 1971, Takuo Aoyaki invented an oximeter using a red light source with a 630 nm wavelength and an infrared light source with a 900 nm wavelength. These two wavelengths have now become the basis of light source with a 630 nm wavelength and an infrared light source with modern pulse oximeters [14].

Infrared PPG signals. Therefore, if the PPG signals are interfered, the estimated result for \( R \) will be in error. Almost all such interference arises in the time-division multiplexing (TDM) form; in contrast, under two sinusoidal control signals of different frequencies, the obtained signals are two amplitude modulated signals (AM) of different carrier frequencies, corresponding to frequency-division multiplexing (FDM) [32–33]. To calculate \( R \) from such a FDM signal, bandpass filters (BPFs) and demodulators have been used to recover the normal PPG signals and calculate \( R \) in either the time domain or the frequency domain as described above.

PPG AM signals have two advantages: the first one is the effects of MAs are decreased [32], and the second one, the modulation index of a PPG AM signal also corresponds to the ratio between the AC and DC components of a conventional PPG signal, which is identical to the light absorbance (A), so, the modulation index is also identical to the light absorbance. Therefore, the ratio between the modulation indexes of the red and infrared PPG AM signals is equal to \( R \). Moreover, the frequency components of the PPG signal are shifted from a low frequency band to a higher frequency band because of the AM effect. In the low frequency band, the ratio between the AC and DC components in the frequency domain is equivalent to the ratio between the magnitude of the cardiac-frequency signal and the magnitude of the zero-hertz signal [20]. Therefore, there is a possibility that the ratio between the AC and DC components in the frequency domain is equivalent to the ratio between the magnitude of the cardiac-frequency signal and the magnitude of the zero-hertz signal [20]. Accordingly, \( R \) can be directly calculated from the two PPG AM signals. To calculate \( R \) from the PPG AM signals (from either the modulation index ratio or the spectrum ratio), it is unnecessary to first demodulate or recover the conventional PPG signal. Thus, the spectrum of the FDM signal, which is obtained directly from the amplifier circuit, can be used to calculate \( R \) without any BPF or demodulator.

Before the results are presented, the theory behind pulse oximetry and AM should be introduced. In the next section, the fundamental background on \( \text{SpO}_2 \) measurement and AM is presented, and the proposed method is then described, as well as the experimental setup of this article.
2. Pulse oxygen saturation measurement

As mentioned before, measuring SpO2 is a technique for estimating SaO2 based on the concentrations of Hb and HbO2. SpO2 is defined as shown in Eq. (1).

\[
SpO2 = \frac{c_{HbO2}}{c_{HbO2} + c_{Hb}}
\]  

(1)

From Eq. (1), if the concentrations of Hb and HbO2 have known, SpO2 can be determined easily; however, if SpO2 is to be determined based on a non-invasive optical technique, both concentrations cannot be directly obtained. Therefore, SpO2 is indirectly determined as follows. The optical technique for determining SpO2 is based on the principle of the Beer-Lambert law, which describes the relationship between the emitted light \( I_0 \) and the transmitted light \( I \) when \( I_0 \) is partially absorbed by a homogeneous medium that has an extinction coefficient of \( \varepsilon(\lambda) \) at the wavelength \( \lambda \), a concentration of \( c \), and an optical path length of \( d \). This relationship is shown in Eq. (2).

\[
I = I_0 e^{-\varepsilon(\lambda)cd}
\]  

(2)

The absorbance (A) is defined in Eq. (3).

\[
A = -\ln \frac{I}{I_0} = \varepsilon(\lambda)cd
\]  

(3)

To use a pulse oximeter, as shown in Fig. 1, the probe must be attached to the fingertip; however, a fingertip is not a homogeneous medium. The absorbance of the fingertip (A_finger) can be separated into two components. The first component \( A_{DC} \) represents the DC absorbance, which is predominantly related to substances such as skin, tissue, and blood vessels, and the second component \( A_{AC} \) represents the AC absorbance, which is predominantly related to the blood. The extinction coefficient for the DC absorbance component at a wavelength of \( \lambda \) is \( \varepsilon_{DC}(\lambda) \), the corresponding concentration is \( c_{DC} \), and the corresponding optical path length is \( d_{DC} \). On the other hand, the extinction coefficient for the AC absorbance component at the same wavelength is \( \varepsilon_{AC}(\lambda) \) the corresponding concentration is \( c_{AC} \), and the corresponding optical path length is \( d_{AC} \). So, \( A_{finger} \) can be approximated as a sum of two components, \( A_{DC} \) and \( A_{AC} \).

As seen from Fig. 1, \( A_{AC} \) is essentially equivalent to the absorbance of the blood \( A_{Blood} \), which depends on the absorbance of Hb and HbO2, so

\[
A_{finger} = A_{DC} + A_{AC} = \varepsilon_{DC}(\lambda)c_{DC}d_{DC} + (\varepsilon_{HbO2}(\lambda)c_{HbO2} + \varepsilon_{Hb}(\lambda)c_{Hb})d_{blood}
\]  

(4)

From Eq. (4), the \( A_{DC} \) must be eliminated in order to obtain \( A_{Blood} \). This problem can be solved by determining the difference between the fingertip absorbances at different times. The optical path length \( d_{blood} \) changes over time because of the contraction of the heart, as characterized by the systolic and diastolic phases. In Eq. (4), \( A_{DC} \) is typically constant, while \( A_{AC} \) varies; therefore, the difference in absorbance between the systolic (\( A_{Systole} \)) and diastolic (\( A_{Diastole} \)) phases can be expressed as shown in Eq. (5).

\[
A_{Difference} = A_{Systole} - A_{Diastole} = (\varepsilon_{HbO2}(\lambda)c_{HbO2} + \varepsilon_{Hb}(\lambda)c_{Hb})\Delta d_{blood} = A_{Blood}
\]  

(5)

Eq. (5) expresses the absorbance of the blood \( A_{blood} \), and \( \Delta d_{blood} \) is the absolute optical path length in the blood. At this point, the concentrations of oxyhaemoglobin and deoxyhaemoglobin are still unknown, and there is also one additional unknown \( \Delta d_{blood} \). This problem can be solved by using two light sources with different wavelengths. The most common wavelengths for pulse oximetry probes are red and infrared. To eliminate the effect of \( \Delta d_{blood} \), the light absorbance ratio is calculated; because both light signals are travelling through the same finger, the optical path lengths for both wavelengths are equal. The light absorbance ratio (R) is defined as shown in Eq. (6).

\[
R = \frac{\varepsilon_{HbO2}(\lambda_{RED})c_{HbO2} + \varepsilon_{Hb}(\lambda_{RED})c_{Hb}}{\varepsilon_{HbO2}(\lambda_{IR})c_{HbO2} + \varepsilon_{Hb}(\lambda_{IR})c_{Hb}}
\]  

(6)

By rearranging Eq. (1) and Eq. (6), SpO2 can be written as a function of R, as shown in Eq. (7) [7].

\[
SpO2 = \frac{\varepsilon_{IR}(\lambda_{IR}) - \varepsilon_{IR}(\lambda_{RED})R}{\varepsilon_{IR}(\lambda_{RED}) - \varepsilon_{IR}(\lambda_{IR})R}
\]  

(7)

Eq. (7) shows that it is not necessary to know the concentrations of haemoglobins; instead, SpO2 can be determined based only on the R-value. Moreover, the extinction coefficients in Eq. (7) are constant at specific wavelengths, so the empirical equation for estimating SpO2 can be further written as follows [34].

\[
SpO2 = 110 - 25R
\]  

(8)
Theoretically, R is the ratio of $A_{\text{blood}}$ between the red and infrared light signals, but in practise, R can be obtained from the light intensity signal shown in Fig. 2. This signal is called the PPG signal, and R can be calculated from the PPG signal via the following three methods.

### 2.1. Neighbour minimum valley and maximum peak method (Peak and valley Method)

From the signal in Fig. 2, the light intensities $I_R$ and $I_I$ are expressed as shown in Eqs. (9) and (10), respectively.

$$I_R = I_0 e^{-(\omega \tau + \omega \tau)} dL$$  \hspace{1cm} (9)

$$I_I = I_0 e^{-(\omega \tau + \omega \tau)} dL$$  \hspace{1cm} (10)

Because $I_R$ is the light intensity due to absorption by the AC component, the corresponding absorbance is given by Eq. (11):

$$- \ln \frac{I_R}{I_0} = e_{AC}(\lambda) \epsilon_{AC} dL = (e_{HbO_2}(\lambda) e_{HbO_2} + e_{Hb}(\lambda) e_{Hb}) dL = A_{\text{blood}}$$  \hspace{1cm} (11)

Therefore, R as calculated with the peak and valley method is.

$$\frac{- \ln I_R}{- \ln I_I} = \frac{(e_{HbO_2}(\lambda) e_{HbO_2} + e_{Hb}(\lambda) e_{Hb})}{(e_{HbO_2}(\lambda) e_{HbO_2} + e_{Hb}(\lambda) e_{Hb})} = R$$  \hspace{1cm} (12)

Eq. (12) shows that in this method, $I_I$ is discarded, and only $I_R$ and $I_I$ of the PPG signals from the red and infrared wavelength light sources are used to compute R.

### 2.2. Derivative method

In this technique, R is calculated by separating the DC and AC components. To decrease the potential for confusion among variables, the notation for the optical path length $d$ is changed to $L$ through approximation of the derivatives, Eq. (16) is obtained.

$$dI(t) = -e(\lambda) \frac{dL(t)}{dt} (I_0 e^{-\omega \tau L(t)}) = -e(\lambda) \frac{dI(t)}{dt} I(t)$$  \hspace{1cm} (13)

Dividing both sides of Eq. (13) by $I(t)$ yields the result shown in Eq. (14).

$$\frac{dI(t)}{I(t)} = -e(\lambda) \frac{dI(t)}{dt}$$  \hspace{1cm} (14)

By comparing Eq. (14) to Eq. (3), the term $\frac{dI(t)}{I(t)}$ can be seen to be the light absorbance, which is equivalent to $A_{\text{blood}}$; therefore, R can be written in derivative form as shown in Eq. (15).

$$\frac{dI_R(t)}{I_R} = \frac{(e_{HbO_2}(\lambda) e_{HbO_2} + e_{Hb}(\lambda) e_{Hb})}{(e_{HbO_2}(\lambda) e_{HbO_2} + e_{Hb}(\lambda) e_{Hb})} = R$$  \hspace{1cm} (15)

Through approximation of the derivatives, Eq. (16) is obtained.

$$\frac{dI_R(t)}{dt} \approx I(t) - I(t)$$  \hspace{1cm} (16)

From Fig. 2, all the light intensities can be substituted into Eq. (15) to obtain Eq. (17):

$$\frac{dI_R(t)}{I_R} = \frac{(e_{HbO_2}(\lambda) e_{HbO_2} + e_{Hb}(\lambda) e_{Hb})}{(e_{HbO_2}(\lambda) e_{HbO_2} + e_{Hb}(\lambda) e_{Hb})} = \frac{A_{\text{blood}}}{A_{\text{blood}}} = R$$  \hspace{1cm} (17)

### 2.3. Spectral analysis method

The magnitude spectrum of PPG signal is shown in Fig. 3. In the frequency domain, the AC component is the magnitude of the highest peak in the cardiac-frequency band ($f_{\text{car}}$), while the DC component is the magnitude of the zero-hertz signal. The calculation of R via the spectral analysis method relies on the direct substitution of the DC and AC components into Eq. (17).

From a communication perspective, because a pulse oximetry probe has dual light sources and a single light detector, if both light sources are active simultaneously, the light detector will be unable to separate the red PPG signal from the infrared PPG signal; therefore, TDM is used. Traditionally, the two light sources are controlled by square wave signals with two different phases, and the signals are obtained in PAM form; however, in this article, the two light sources are considered to be controlled by two sinusoidal waves of different frequencies based on Sakkarin’s method [32]. The signal pattern obtained at the light detector when the dual light sources are controlled with sinusoidal waves is formed of two AM signals. Therefore, in the next subsection, the fundamentals of the AM technique are described.

### 3. Amplitude modulation

In telecommunication, AM and frequency modulation (FM) are common modulation techniques used in radio broadcasting. The simplest modulation technique is to vary some parameter of the sinusoidal waveform, such as the amplitude, frequency, or phase, versus the message signal. In this article, the double-sideband suppressed-carrier AM (AM-DSBSC) approach with a large carrier is assumed. Before the use of the above techniques to calculate R for estimating $\text{SpO}_2$ is demonstrated, in this section, basic AM is briefly described.
First, considering the message signal shown in Fig. 4(a) and the carrier signal shown in Fig. 4(b), the corresponding AM signal is described by Eq. (18) and depicted in Fig. 4(c).

\[ AM(t) = A \cos(\omega_c t) + m(t) \cos(\omega_m t) = (A + m(t)) \cos(\omega_c t) \]  

(18)

The term \( m(t) \cos(\omega_m t) \) is the AM-DSBSC signal, and the term \( A \cos(\omega_c t) \) is the carrier signal when they are combined; thus, Eq. (18) represents an AM-DSBSC signal with a large carrier, which is a well-known type of AM signal. By substituting \( m(t) = A_m \cos(\omega_m t) \) into Eq. (18) and Eq. (19) is obtained.

\[ AM(t) = A \cos(\omega_c t) + \frac{A_m}{2} \cos((\omega_c - \omega_m) t) + \frac{A_m}{2} \cos((\omega_c + \omega_m) t) \]  

(19)

When Eq. (19) is transformed into the frequency domain, the terms \( \omega_c - \omega_m \) and \( \omega_c + \omega_m \) are the lower sideband (LSB) and the upper sideband (USB), respectively. The magnitude spectrum of the AM signal is shown in Fig. 4(d).

In addition to the amplitude and frequency of an AM signal, the modulation index (\( \mu \)) is also an important parameter. From Fig. 4(c), \( \mu \) can be obtained as shown in Eq. (20).

\[ \mu = \frac{A_m}{A} = \frac{A_{\text{max}} - A_{\text{min}}}{A_{\text{max}} + A_{\text{min}}} = \frac{|m(t)|_{\text{max}}}{A} \]  

(20)

In the case of many sources of messages wanting to simultaneously send information in the same channel, various different carrier frequencies are allocated to these message sources for modulation, after which the modulated signals are summed together and transmitted through the provided channel. This process is known as the FDM technique. At the receiver site, demodulation cannot be directly performed; instead, the FDM signal must be filtered with a BPF with a centre frequency corresponding to each carrier frequency. After that, the AM detector is used to recover information as mentioned above. Eq. (21) shows the FDM signal obtained by multiplexing two AM signals formed from two message signals (\( m_1(t) \) and \( m_2(t) \)) and carrier signals of two different frequencies (\( \omega_c \) and \( \omega_m \)).

\[ FDM(t) = A_c \cos(\omega_c t) + m_1(t) \cos(\omega_m t) + A_c \cos(\omega_c t) + m_2(t) \cos(\omega_m t) \]  

(21)

The magnitude spectrum of the FDM signal considered in this example is shown in Fig. 4(e).

4. New methods for calculating the light absorbance ratio

Previously, the principles of pulse oximetry and AM were reviewed. The pulse oximetry principle was explained, illustrating why the non-invasive measurement of SpO2 requires two light sources with two different wavelengths. Finally, the mathematical expression was presented to show that in order to measure SpO2, it is not necessary to directly measure the concentration of any haemoglobin type in blood. Only the light intensities need to be measured to calculate R in order to estimate SpO2.
Fig. 5. The analogue front end. (a) The oscillator circuits generate (a-1) a cosine signal with a frequency of 1070 Hz and (a-2) a cosine signal with a frequency of 1550 Hz. (b) LED driver circuits. (c) Datex-Ohmeda’s pulse oximetry probe with a common LED anode. (d) Trans-impedance amplifier circuit.

Generally, a pulse oximetry probe includes two light sources and a single light detector. The single light detector needs to detect both light signals separately. Traditionally, the two light sources are driven by two square wave signals with different phases. From a communication point of view, this corresponds to TDM; when the signal from the light detector is considered, these signals are two PAM signals. In Sakkarin’s research [32], the control signals were changed from square wave signals to cosine signals. With this method, two cosine signals of different frequencies are used to control the light sources, so the signal received at the light detector is a combination of two PPG AM signals in FDM form.

The carrier signals of the two PPG AM signals are the cosine signals used to drive the light sources, and the message signals are the PPG signals. The advantage of controlling the two light sources with two different-frequency cosine signals is that ambient light will not disturb the PPG signals because the frequency bands of the PPG signals are moved to a higher frequency. Sakkarin calculated R by using two BPFs to separate the two PPG AM signals and finally demodulating the PPG AM signals into PPG signals to be processed using the conventional method.

However, when an AM signal is considered in the time domain, its modulation index is equal to the ratio of the amplitude of the message signal to its DC offset. If the message signal is a PPG signal, the modulation index of a PPG AM signal corresponds to the AC ratio of the PPG signal. Meanwhile, T. L. Rusch [20] has explained the method for calculating R in the frequency domain. The PPG AM signals in the frequency domain correspond to T. L. Rusch’s method: the AC component is the magnitude of the highest peak in the USB (or LSB) of the PPG AM signal, and the DC component is the magnitude of the peak at its carrier frequency. Finally, the combined signal consisting of the red and infrared PPG AM signals are in FDM form can be used to calculate R in the same way as in the second scheme. With this method, the PPG AM signals do not need to be demodulated to calculate R, and the FDM signal does not need to be filtered by any BPF. Accordingly, this article has proposed two types of novel methods for calculating R: in the time domain and in the frequency domain.

4.1. Calculation of R in the time domain

As mentioned above, R can be determined from Eq. (17). When considering the modulation index in Eq. (20), so, the term $\frac{\mu_{\text{RED}} - \mu_{\text{IR}}}{2}$ is the AC component of the PPG signal and the term $\frac{\mu_{\text{RED}} + \mu_{\text{IR}}}{2}$ is the DC component of the PPG signal, or the average of the PPG signal; thus, $\mu_{\text{PPG}}$ is the AC ratio. Therefore, R can be written in terms of $\mu_{\text{RED}}$ and $\mu_{\text{IR}}$ as shown in Eq. (22). The relations among the various signals are shown in Fig. 6 (c-2).

$$R = \frac{\mu_{\text{RED}}}{\mu_{\text{IR}}}$$ (22)

4.2. Calculation of R in the frequency domain

In the method proposed by T. L. Rusch [20], R is calculated from the conventional PPG signal in the frequency domain.

For the case in which two cosine signals are used to drive two light sources of different frequencies, resulting in two PPG AM signals at the light detector. Fig. 6(d-2) shows the PPG AM signals in the frequency domain. To calculate R from the AM magnitude spectra, the AC and DC components, as shown in Fig. 6(d-2) and 6(e-2), are substituted into Eq. (17).
5. System design

The system designed in this article uses two cosine signals of different frequencies to independently drive two light sources: a frequency of 1070 Hz is used to drive the red light source, and a frequency of 1550 Hz is used to drive the infrared light source. This system consists of two main components: the analogue front end and the signal processing unit. The analogue front end is shown in Fig. 5. It consists of two oscillator circuits (Fig. 5(a)), light-emitting diode (LED) driver circuits (Fig. 5(b)), a commercial common-anode pulse oximetry probe from Datex-Ohmeda (Fig. 5(c)) and a trans-impedance amplifier circuit (Fig. 5(d)). This analogue front end serves to produce the PPG signals in PPG AM form.

5.1. Cosine oscillator circuit

An XR2206 integrated circuit is used as the sinusoidal oscillator. The output frequency \( f \) can be calculated from Eq. (23):

\[
R = \frac{AC_{\text{RED}}}{DC_{\text{RED}}} = \frac{AC_{\text{IR}}}{DC_{\text{IR}}}
\]

Fig. 6. Signal processing flowchart. (a) Experimental Model 1 is Sakkarin’s method [32]. By using BPFs with two different centre frequencies and a corresponding demodulator part, the output signals are obtained as two PPG signals (Fig. 6(a-2)). (b) Experimental Model 2 is Rusche’s method [20]. By transforming the PPG signals from Model 1 into the frequency domain, the magnitude spectra of the two PPG signals are obtained. (c) In Experimental Model 3, by filtering the FDM signal with two BPFs, two PPG AM signals are obtained; then, the modulation indexes of these two PPG AM signals are calculated, where \( \mu_{\text{RED}} \) is the modulation index of the red PPG AM signal, while \( \mu_{\text{IR}} \) is the modulation index of the infrared PPG AM signal. (d) In Experimental Model 4, by filtering the FDM signal with two BPFs, two PPG AM signals are obtained; then, these two AM signals are transformed with the FFT. The AC component is the amplitude of the highest peak in the sideband, while the DC component is the amplitude at the carrier frequency. (e) In Experimental Model 5, the FDM signal is transformed directly to obtain its magnitude spectrum, which shows the two bands of the AM signals; then, the AC and DC components can be calculated using the same strategy as in Model 4.
For the system considered in this article, the two capacitors $c$ in Fig. 5 (a) are 0.01 μF, and the potentiometer $R_1$ is used to adjust the frequencies of the cosine signals, 1070 Hz and 1550 Hz.

5.2. Trans-impedance amplifier circuit

The operational amplifier used in the system presented in this article is an OP07 integrated circuit. As shown in Fig. 5 (d), the feedback resistance is 5 MΩ, so the amplifier’s gain is $5 \times 10^6$ volts per ampere. However, with a feedback capacitor, the amplifier circuit also rejects high frequencies with a cut-off frequency of 3,138 Hz, acting as a low-pass filter (LPF). The output signal from the trans-impedance amplifier circuit is the FDM signal formed by the two PPG AM signals.

5.3. Signal processing

For data acquisition, a NI PCI 4016 Basic Multifunction I/O Board is used to convert the analogue FDM signal into a digital FDM signal at a sampling rate of 16,000 for 10 s. The flow chart of all system operations is shown in Fig. 6. For the study reported in this article, all operations were run in MATLAB software. The FFT function in MATLAB was used to transform the time-domain signals into the frequency domain.

![Experimental setup](image1)

**Fig. 7.** Experimental setup.

![FDM Signal](image2)

**Fig. 8.** The FDM signal detected at the light detector. The signal consists of two PPG AM signals: red PPG AM and infrared PPG AM signals.
6. Experimental setup

In this article, the experimental setup is shown in Fig. 7. For Fig. 7, the dual light source in the oximeter probe were controlled by two oscillator circuits, and the light detector connected to the trans-impedance amplifier circuit to generates the FDM signal and transmitted to the computer (3.40 GHz CPU processor, 4 GB of RAM) by the NI PCI 4016 Basic Multifunction I/O Board and continued processing with MATLAB.

For collecting data, the oximeter probe was attached to volunteer’s fingertip. The volunteers were told to sit in relax position, have a minimum movement during the measurement. The experiment was operated in a non-variant ambient light room. The sample signals were acquired from 9 healthy volunteers: 7 male, 2 female, average age is 25 years.

This article has proposed three new models for determining R (The 3rd model (Fig. 6(c)) is time-domain, and the 4th and 5th models (Fig. 6(d) and 6(e) are frequency domain) and compared them to two traditional methods (The 1st and 2nd models [32,20] (Fig. 6(a) and 6(b)). All the experimental models are shown in Fig. 6. The recorded FDM signal are processed simultaneously. To calculate SpO₂, Eq. (8) is used.

7. Results and discussion

7.1. Frequency-Division multiplexing signal from the light detector

The signal at the detector obtained by driving dual light sources with different cosine signals is shown in Fig. 8.
7.2. Conventional R calculation

To obtain R, the FDM signal is filtered with two BPFs and demodulators. The PPG signals for the red and infrared light wavelengths are obtained as shown in Fig. 9.

From these PPG signals, Eq. (17) is used to calculate R, and Eq. (8) is used to estimate SpO\textsubscript{2}; for Model 1, the R = 0.602 and SpO\textsubscript{2} = 94.92\% are achieved. In simultaneous time, the spectrums of the PPG signals in the frequency domain (Model 2), as shown in Fig. 10(a) and (b).

The magnitude of the zero-hertz signal component is the DC component of the PPG signal, and the magnitude of the highest peak at the cardiac frequency is the AC component of the PPG signal. The frequencies of the highest peak magnitudes in Fig. 10(a) and (b) are determined according to the heart rate frequency which is 1.2 Hz and corresponds to a heart rate of 78 beats per minute in Model 1. The R value calculated from the Model 2’s signals in Fig. 10(a) and (b) is R = 0.812, and SpO\textsubscript{2} = 89.40\%.

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**Fig. 11. PPG AM signals.** (a) red PPG AM signal with a modulation index of 0.0059 and (b) infrared PPG AM signal with a modulation index of 0.101.

**Fig. 12. Normalized magnitude spectrum of the AM PPG signals.** (a) red PPG AM signal: (a-1) magnitude of the carrier signal at 1070 Hz and (a-2) expanded to show the shifted cardiac frequency within the 1070 Hz band. (b) infrared AM PPG signal: (b-1) magnitude of the carrier signal at 1550 Hz and (b-2) expanded to show the shifted cardiac frequency within the 1550 Hz band.
7.3. New models for Calculating $R$

In the time-domain, the PPG AM signals from the BPFs are shown in Fig. 11.

By using Eq. (22) of Model 3 to calculate $R$ in the time domain, the $R = 0.582$ and SpO$_2 = 95.49\%$ are obtained.

While in the frequency-domain, the normalized magnitude spectrum results used to calculate $R$ in the frequency domain in Model 4 are shown in Fig. 12(a) and (b).

The magnitude of the carrier-frequency component is the DC
can be calculated and SpO\textsubscript{2} are all very similar; therefore, to achieve greater accuracy, a new mathematical model for evaluating SpO\textsubscript{2} is determined by using the linear frequency domain. Which this expression is calculated using the linear mathematical model for evaluating SpO\textsubscript{2}.

The R values calculated using Models 2, 4, and 5 require increased complexity because it would be necessary to use more harmonic frequencies of the fundamental cardiac frequency. In this case, the error in the frequency-domain group, only the fundamental cardiac frequency is considered in the AC component; achieving greater accuracy would incur more error because in the frequency domain, the part of the AC component that is used is the part at the fundamental cardiac frequency or the first peak in the sideband (the first peak in the sideband corresponds to the time-domain method. So, the frequency-domain group is demonstrated that eventually to estimate SpO\textsubscript{2} from Eq. (24) are more accurate than using Eq. (8).

The SpO\textsubscript{2} results obtained by using Eq. (24) and Eq. (8) which is estimated from R in the frequency-domain. It shows that estimated SpO\textsubscript{2} from Eq. (24) are more accurate than using Eq. (8). Thus, it is demonstrated that eventually to estimate SpO\textsubscript{2} from the FDM signal in the frequency-domain, the system does not need to be demodulated and including the use of the BPFs to separate the signal, but it can be obtained directly from the magnitude spectrum of the FDM signal, thereby reducing the overall complexity of the overall processing system.

8. Conclusion

The results of the five models are shown in Figs. 9–13, and the calculated errors are shown in Table 3. The error results can be divided into two groups: a time-domain group and a frequency-domain group. The SpO\textsubscript{2} errors are larger in the frequency-domain group than in the time-domain group because of the treatment of the AC component. In the frequency-domain group, only the fundamental cardiac frequency is considered in the AC component; achieving greater accuracy would require increased complexity because it would be necessary to use more harmonic frequencies of the fundamental cardiac frequency. In this case, not only the amplitudes of the harmonics but also their phases would need to be determined. Therefore, a new mathematical model for estimating blood oxygen saturation in the frequency domain is proposed in Eq. (24). When Eq. (24) is used to calculate SpO\textsubscript{2} based on the R-value calculated in the frequency domain, the error drops to 1%. The results show that with the new methods for calculating R using the modulation indexes, corresponding to Models 3 and 4, demodulators are unnecessary, and eventually, with Model 5, there is also no need for BPFs. With no demodulators and BPFs in Model 5, the analogue front-end design is similar, also the signal processing. Finally, for Models 1, 2, 3, and 4, which used the BPFs, the wavelet base technique for separate PPG AM signals can be substitute to the BPFs.

Table 2
The results of SpO\textsubscript{2} calculated by using Eq. (24) for the frequency domain and using Eq. (8) for the time-domain.

| Volunteer | Model | 1     | 2     | 3     | 4     | 5     |
|-----------|-------|-------|-------|-------|-------|-------|
| R         | R     | 0.5790| 0.8569| 0.5546| 0.8567| 0.8588|
| SpO\textsubscript{2} | R     | 95.5257% | 95.3138% | 96.1359% | 95.3148% | 95.3027% |
|           | R     | 0.6050| 0.9095| 0.5785| 0.8427| 0.8411|
| SpO\textsubscript{2} | R     | 94.8750% | 95.2627% | 95.5367% | 95.3915% | 95.4003% |
|           | R     | 0.5289| 0.8471| 0.5266| 0.8530| 0.8624|
| SpO\textsubscript{2} | R     | 96.7769% | 95.3674% | 96.8359% | 95.3535% | 95.2841% |
|           | R     | 0.6325| 0.9298| 0.5782| 0.9155| 0.9018|
| SpO\textsubscript{2} | R     | 94.1884% | 94.9322% | 95.5438% | 94.9939% | 95.0689% |
|           | R     | 0.6152| 0.8893| 0.5973| 0.8987| 0.8872|
| SpO\textsubscript{2} | R     | 94.6192% | 95.1368% | 95.0670% | 95.0857% | 95.1484% |
|           | R     | 0.6104| 0.8827| 0.5709| 0.8783| 0.8721|
| SpO\textsubscript{2} | R     | 94.7410% | 95.1733% | 95.7280% | 95.1969% | 95.2308% |
|           | R     | 0.5382| 0.8454| 0.5321| 0.8826| 0.8659|
| SpO\textsubscript{2} | R     | 96.5457% | 95.3764% | 96.6978% | 95.1737% | 95.2646% |
|           | R     | 0.5956| 0.8712| 0.5714| 0.8849| 0.8766|
| SpO\textsubscript{2} | R     | 95.1094% | 95.2361% | 95.7152% | 95.1609% | 95.2064% |
|           | R     | 0.5950| 0.8226| 0.5809| 0.8510| 0.8362|
| SpO\textsubscript{2} | R     | 95.1261% | 95.5010% | 95.4764% | 95.3461% | 95.4270% |

Table 3
Average Errors and Variances Using Eqs. (8) and (24).

| Model | Equation (8) | Equation (24) |
|-------|--------------|---------------|
|       | Average SpO\textsubscript{2} error | Variance | Average SpO\textsubscript{2} error | Variance |
| 2     | 6.2927 | 39.5984 | 1.0353 | 1.0719 |
| 3     | 0.7138 | 0.5095 | 0.7138 | 0.5095 |
| 4     | 7.87 | 61.9369 | 1.0279 | 1.0566 |
| 5     | 7.7758 | 60.4630 | 1.0760 | 1.1578 |
CRediT authorship contribution statement

Ananta Sinchai: Methodology, Software, Writing – original draft.
Ananta Sinchai: Software, Investigation, Writing – review & editing.
Panwit Tuwanut: Investigation, Writing – review & editing. Paramote Wardkein: Supervision, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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