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Unified equation of state for the outer and inner crusts of magnetars

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Abstract. Magnetars form a subclass of neutron stars characterized by magnetic fields of order $10^{14} - 10^{15}$ G at their surface. According to numerical simulations, the magnetic fields in their interior could potentially be even stronger. Such magnetic fields are so extreme that the internal constitution of neutron stars may be altered. The effects of Landau-Rabi quantisation of electron motion on the equation of state and on the equilibrium composition of the crust of a neutron star are investigated for a wide range of magnetic field strengths. Both the outer and inner parts of the crust are treated in a unified and consistent way within the nuclear-energy density functional theory.

1. Introduction

Whereas most pulsars are endowed with magnetic fields of order $10^{12}$ G, some neutron stars may be formed with extremely high magnetic fields of order $10^{14} - 10^{15}$ G, as first proposed by Thompson and Duncan [1]. According to numerical simulations [2–6], the internal magnetic fields can reach $\sim 10^{18}$ G. The existence of such highly magnetised neutron stars so-called magnetars has been confirmed by various astrophysical observations (see, e.g., Ref. [7] for a review).

We have recently shown that the equilibrium properties of the outer and inner crusts of a neutron star could be altered if the magnetic field is strong enough due to Landau-Rabi quantisation of electron motion [8–11]. In this paper, we present new results for intermediate magnetic field strengths.

2. Equilibrium properties of magnetar crusts

2.1. Outer crust

We determine the equilibrium properties of the outer crust of a cold nonaccreted neutron star by minimising the Gibbs free energy per nucleon at each pressure $P$, as detailed in Ref. [8]. In this model, atoms are supposed to be fully ionised and arranged in a perfect body-centred cubic lattice. We consider only pure layers composed of a single nuclear species with charge number $Z$ and mass number $A$. Electrons are highly degenerate and are only weakly perturbed by the...
ions. For the typical magnetic fields $B$ associated with magnetars, electrons are relativistic since $B_\ast \equiv B/B_{\text{rel}} > 1$, where

$$B_{\text{rel}} = \frac{m_e^2 c^3}{e \hbar} \simeq 4.41 \times 10^{13} \text{ G}$$

($m_e$ is the electron mass, $c$ the speed of light, $e$ the elementary electric charge, and $\hbar$ is the Planck-Dirac constant). The electron motion perpendicular to the magnetic field is quantised into Landau-Rabi orbitals, first calculated by Rabi [12].

**Table 1.** Composition of the outer crust of a cold nonaccreted magnetar. Results were obtained numerically for a magnetic field strength $B_\ast = 500$. $Z$ and $A$ are the charge and mass number of the equilibrium nucleus, $\bar{n}_{\text{min}}$ ($\bar{n}_{\text{max}}$) is the minimum (maximum) mean baryon number density at which that nucleus is present and $P_{1 \rightarrow 2}$ is the transition pressure between two adjacent layers. Properties in the upper part of the table are fully determined by experimental atomic masses.

| $Z$ | $A$ | $\bar{n}_{\text{min}}$ [fm$^{-3}$] | $\bar{n}_{\text{max}}$ [fm$^{-3}$] | $P_{1 \rightarrow 2}$ [MeV fm$^{-3}$] |
|-----|-----|---------------------------------|---------------------------------|---------------------------------|
| 26  | 56  | $4.50 \times 10^{-7}$           | $1.36 \times 10^{-6}$           | $1.18 \times 10^{-7}$           |
| 28  | 62  | $1.40 \times 10^{-6}$           | $5.34 \times 10^{-6}$           | $2.95 \times 10^{-6}$           |
| 28  | 64  | $5.51 \times 10^{-6}$           | $7.78 \times 10^{-6}$           | $6.15 \times 10^{-6}$           |
| 36  | 86  | $8.17 \times 10^{-6}$           | $1.25 \times 10^{-5}$           | $1.49 \times 10^{-5}$           |
| 34  | 84  | $1.29 \times 10^{-5}$           | $1.88 \times 10^{-5}$           | $3.23 \times 10^{-5}$           |
| 32  | 82  | $1.95 \times 10^{-5}$           | $2.55 \times 10^{-5}$           | $5.55 \times 10^{-5}$           |
| 30  | 80  | $2.65 \times 10^{-5}$           | $3.29 \times 10^{-5}$           | $8.60 \times 10^{-5}$           |
| 28  | 78  | $3.43 \times 10^{-5}$           | $7.59 \times 10^{-5}$           | $1.40 \times 10^{-4}$           |
| 28  | 80  | $7.78 \times 10^{-5}$           | $8.68 \times 10^{-5}$           | $1.58 \times 10^{-4}$           |
| 42  | 124 | $9.06 \times 10^{-5}$           | $1.22 \times 10^{-4}$           | $2.53 \times 10^{-4}$           |
| 40  | 122 | $1.26 \times 10^{-4}$           | $1.43 \times 10^{-4}$           | $3.20 \times 10^{-4}$           |
| 39  | 121 | $1.45 \times 10^{-4}$           | $1.47 \times 10^{-4}$           | $3.29 \times 10^{-4}$           |
| 38  | 120 | $1.50 \times 10^{-4}$           | $2.07 \times 10^{-4}$           | $4.01 \times 10^{-4}$           |
| 38  | 122 | $2.10 \times 10^{-4}$           | $2.47 \times 10^{-4}$           | $5.07 \times 10^{-4}$           |
| 38  | 124 | $2.51 \times 10^{-4}$           | $2.61 \times 10^{-4}$           | $5.39 \times 10^{-4}$           |

To determine the equilibrium composition of the outer crust, we have made use of the experimental atomic mass data from the 2016 Atomic Mass Evaluation (AME) [13, 14], supplemented by more recent measurements of copper isotopes [15]. For the isotopes for which no experimental data is available, we used the theoretical mass table HFB-24 from the BRUSLIB database [16]. These masses were obtained from self-consistent deformed Hartree-Fock-Bogoliubov calculations with the nuclear energy-density functional BSk24 [17]. Results are collected in tables 1, 2 and 3 for different magnetic field strengths. The outermost layers are made of isotopes with experimentally measured masses. While the stratification of the crust changes with the magnetic field strength, the composition of the deepest layers remains remarkably stable (only the transition pressures and the densities of the different layers are changed).

2.2. Inner crust

Minimising the Gibbs free energy per nucleon at fixed pressure as in the outer crust, is numerically more involved in the inner crust since the pressure now depends on the density of free neutrons in addition to that of electrons. Instead, we have performed the minimisation of
the Helmholtz free energy at fixed average baryon number density $\bar{n}$. This latter procedure was shown to be numerically equivalent to the former, the density discontinuities being vanishing small beyond the neutron-drip point [18]. We have obtained the equilibrium properties of the inner crust using the computer code developed by the Brussels-Montreal collaboration [18–

Table 2. Same as Table 1 for $B_\ast = 1500$.

| $Z$ | $A$ | $\bar{n}_{\text{min}}$ [fm$^{-3}$] | $\bar{n}_{\text{max}}$ [fm$^{-3}$] | $P_{1\rightarrow2}$ [MeV fm$^{-3}$] |
|-----|-----|-------------------------------|-------------------------------|--------------------------------|
| 26  | 56  | $1.72\times10^{-6}$           | $3.86\times10^{-6}$           | $2.64\times10^{-7}$            |
| 28  | 62  | $4.00\times10^{-6}$           | $1.69\times10^{-5}$           | $9.71\times10^{-6}$            |
| 28  | 64  | $1.75\times10^{-5}$           | $1.84\times10^{-5}$           | $1.08\times10^{-5}$            |
| 38  | 88  | $1.88\times10^{-5}$           | $2.44\times10^{-5}$           | $1.89\times10^{-5}$            |
| 36  | 86  | $2.51\times10^{-5}$           | $4.02\times10^{-5}$           | $5.07\times10^{-5}$            |
| 34  | 84  | $4.16\times10^{-5}$           | $5.95\times10^{-5}$           | $1.06\times10^{-4}$            |
| 32  | 82  | $6.17\times10^{-5}$           | $7.96\times10^{-5}$           | $1.79\times10^{-4}$            |
| 30  | 80  | $8.28\times10^{-5}$           | $1.01\times10^{-4}$           | $2.70\times10^{-4}$            |
| 46  | 128 | $1.06\times10^{-4}$           | $1.26\times10^{-4}$           | $3.83\times10^{-4}$            |
| 44  | 126 | $1.30\times10^{-4}$           | $1.37\times10^{-4}$           | $4.27\times10^{-4}$            |
| 42  | 124 | $1.41\times10^{-4}$           | $1.62\times10^{-4}$           | $5.66\times10^{-4}$            |
| 40  | 122 | $1.68\times10^{-4}$           | $1.80\times10^{-4}$           | $6.51\times10^{-4}$            |
| 39  | 121 | $1.83\times10^{-4}$           | $1.84\times10^{-4}$           | $6.62\times10^{-4}$            |
| 38  | 120 | $1.87\times10^{-4}$           | $1.97\times10^{-4}$           | $7.33\times10^{-4}$            |
| 38  | 122 | $2.00\times10^{-4}$           | $2.13\times10^{-4}$           | $8.28\times10^{-4}$            |
| 38  | 124 | $2.16\times10^{-4}$           | $2.20\times10^{-4}$           | $8.55\times10^{-4}$            |

Table 3. Same as Table 1 for $B_\ast = 2500$.

| $Z$ | $A$ | $\bar{n}_{\text{min}}$ [fm$^{-3}$] | $\bar{n}_{\text{max}}$ [fm$^{-3}$] | $P_{1\rightarrow2}$ [MeV fm$^{-3}$] |
|-----|-----|-------------------------------|-------------------------------|--------------------------------|
| 26  | 56  | $3.21\times10^{-6}$           | $6.21\times10^{-6}$           | $3.56\times10^{-7}$            |
| 28  | 62  | $6.46\times10^{-6}$           | $2.68\times10^{-5}$           | $1.41\times10^{-5}$            |
| 38  | 88  | $2.83\times10^{-5}$           | $4.33\times10^{-5}$           | $3.54\times10^{-5}$            |
| 36  | 86  | $4.46\times10^{-5}$           | $7.00\times10^{-5}$           | $9.14\times10^{-5}$            |
| 34  | 84  | $7.23\times10^{-5}$           | $1.02\times10^{-4}$           | $1.87\times10^{-4}$            |
| 32  | 82  | $1.06\times10^{-4}$           | $1.29\times10^{-4}$           | $2.80\times10^{-4}$            |
| 50  | 132 | $1.34\times10^{-4}$           | $1.65\times10^{-4}$           | $4.30\times10^{-4}$            |
| 46  | 128 | $1.74\times10^{-4}$           | $2.16\times10^{-4}$           | $6.66\times10^{-4}$            |
| 44  | 126 | $2.22\times10^{-4}$           | $2.34\times10^{-4}$           | $7.41\times10^{-4}$            |
| 42  | 124 | $2.41\times10^{-4}$           | $2.76\times10^{-4}$           | $9.76\times10^{-4}$            |
| 40  | 122 | $2.85\times10^{-4}$           | $3.03\times10^{-4}$           | $1.11\times10^{-3}$            |
| 40  | 124 | $3.08\times10^{-4}$           | $3.13\times10^{-4}$           | $1.15\times10^{-3}$            |
| 38  | 120 | $3.19\times10^{-4}$           | $3.32\times10^{-4}$           | $1.24\times10^{-3}$            |
| 38  | 122 | $3.37\times10^{-4}$           | $3.58\times10^{-4}$           | $1.40\times10^{-3}$            |
| 38  | 124 | $3.64\times10^{-4}$           | $3.70\times10^{-4}$           | $1.45\times10^{-3}$            |
and modifying it to take into account the effects of Landau-Rabi quantisation of electron motion. This code is based on the fourth-order extended Thomas-Fermi method with proton shell corrections added perturbatively using the Strutinsky integral theorem. This approach is a computationally very fast approximation to the fully self-consistent Hartree-Fock plus Bardeen-Cooper-Schrieffer method. The Coulomb lattice is described using the Wigner-Seitz approximation. We further assume that electrons are uniformly distributed. Nuclear clusters are supposed to be spherical and their local densities are parametrized as

\[ n_q(r) = n_{B,q} + n_{\Lambda,q} \left\{ 1 + \exp \left[ \left( \frac{C_q - R_c}{r - R_c} \right)^2 - 1 \right] \exp \left( \frac{r - C_q}{a_q} \right) \right\}^{-1} \]

where \( q = p, n \) denotes protons or neutrons respectively and \( n_{B,q}, n_{\Lambda,q}, C_q, a_q, \) and \( R_c \) are geometrical parameters of the Wigner-Seitz cell. We only take into account the magnetic-field effects on the electron gas using the analytical approximations implemented in the routines developed by Potekhin and Chabrier [21].

**Table 4.** Composition of the inner crust of a cold nonaccreted magnetar for different magnetic-field strengths \( B_\star \). \( Z \) and \( N \) are respectively the mean numbers of protons and neutrons in the Wigner-Seitz cell, \( \bar{n} \) is the mean baryon number density of the considered layer.

| \( \bar{n} \) [fm\(^{-3}\)] | \( B_\star = 500 \) | \( B_\star = 1500 \) | \( B_\star = 2500 \) |
|-----------------|-----------------|-----------------|-----------------|
|                 | \( Z \) | \( N \) | \( Z \) | \( N \) | \( Z \) | \( N \) |
| 5.474\times10^{-4} | 40 | 195 | 40 | 211 | 41 | 147 |
| 9.864\times10^{-4} | 40 | 283 | 41 | 262 | 41 | 271 |
| 1.777\times10^{-3} | 41 | 426 | 40 | 374 | 40 | 460 |
| 3.203\times10^{-3} | 40 | 555 | 40 | 548 | 41 | 572 |
| 5.772\times10^{-3} | 40 | 704 | 40 | 669 | 40 | 624 |
| 1.040\times10^{-2} | 40 | 824 | 40 | 855 | 40 | 832 |
| 1.874\times10^{-2} | 40 | 934 | 40 | 965 | 40 | 904 |
| 3.378\times10^{-2} | 40 | 1061 | 40 | 1068 | 40 | 1044 |
| 6.087\times10^{-2} | 40 | 1185 | 40 | 1193 | 40 | 1189 |

Calculations were carried out using the same generalized Skyrme functional BSk24 [17] as in the outer crust. The influence of the magnetic field on the composition is found to be quite small for the range of magnetic-field strengths considered, as shown in table 4. As in the absence of magnetic fields [22], we find that most layers are still made of clusters with \( Z = 40 \) and become progressively more neutron rich with increasing density.

The results we obtained for the equation of state over the whole crust region are plotted in figure 1. The effects of the magnetic field lessen with increasing density as electrons fill more and more Landau-Rabi levels. At densities above \( \bar{n} \approx 0.01 \text{ fm}^{-3} \) the equation of state matches with that obtained in the absence of magnetic fields. We find that the equation of state remains almost unchanged in the inner crust region for magnetic field strengths below \( B_\star = 500 \). As can be seen in figure 1, the neutron-drip density delimiting the boundary between the outer and inner crusts does not vary monotonically with \( B_\star \) depending on the filling of Landau-Rabi levels (for more details, see [9, 10]).

### 3. Conclusion

We have determined the equation of state and the composition of the outer and inner crusts of magnetars for different magnetic-field strengths taking into account Landau-Rabi quantisation
Figure 1. Pressure $P$ in MeV fm$^{-3}$ as a function of the mean baryon number density $\bar{n}$ in fm$^{-3}$ in the outer (black lines) and inner (blue lines) crusts of cold nonaccreted magnetars for different magnetic field strengths $B_\star$. The inset is a close-up view of the neutron-drip transition, marked by the symbol $\bigcirc$.

of electron motion. Our calculations were carried out in a unified and consistent way within the nuclear-energy density functional theory. The shallowest regions of the crust are found to be the most affected by the magnetic field. At densities above $\approx 0.01$ fm$^{-3}$, the equation of state and the composition appear to be almost unaltered by the magnetic field.

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