Study on UCAV Robust Maneuvering Decision in Automatic Air Combat Based on Target Accessible Domain

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ABSTRACT: The paper presents a deep study on the robust maneuvering decision of unmanned combat aerial vehicle (UCAV) in air combat. First, a method to describe the target accessible domain by angles is designed. Then the state prediction model of the target with variable weight is built based on the change of target tactical intention. Second, robust membership functions that reflect the air combat situation are denoted to building up the decision method, robust multi-objective optimization (RMO). Finally, grey wolf optimizer (GWO) is adopted to complete the optimization. The simulation result shows that, the model of target state prediction based on accessible domain can reflect the tactical intention and the state prediction can obtain better situation advantage than that without prediction. Moreover, the RMO can get the promising result and the GWO can satisfy the need of maneuvering decision.

CCS Concepts
• Computing methodologies → Modeling and simulation

1. INTRODUCTION
UCAV plays an increasingly important role in modern war. With the development of technology and the continuous improvement of military demands, autonomous air combat is the inevitable development trend of UCAV battlefield application, and autonomous maneuver decision is the key to realizing UCAV’s autonomous air combat. As the frontier of military application, UCAV autonomous air combat maneuver decision-making has been rarely studied in recent years, and most of the relevant literatures remain on the research of manned vehicle. Seen from the existing public results, the maneuver decision method can be divided into two categories: traditional decision method and optimal decision method.

Traditional methods, such as expert system[1], differential game[2], influence diagram[3], the Bayesian network[4] and relevant improvement methods[5][7], and so on, exist insufficient generalization ability of expert knowledge, the poor efficiency in uncertainty decision-making, and situation model is too simple, besides these methods obtain the decisions with the help of the action...
library which still didn’t overcome the shortcomings of itself[8]. The optimal decision methods transform the problems into the optimal solution of differential inequality to solve the objective function[9-10]. These methods can make decisions with battlefield 3d models, and generate 3d dynamic decision track of actual, which is convenient for maneuvering tracking. However, the optimization process of these methods causes much time consumption, bad real-time, and hysteresis for the current situation. Also, its maneuvering decision results for uncertain environments are less robust.

For the above, this paper proposes a UCAV robust maneuver decision method based on the state prediction of the target reachable domain. Aiming at the lag of the current situation decision, in order to make UCAV better for attack occupation, a method to judge the tactical intention of the target was designed based on the target reachable domain, and then the target prediction state was given to provide information for the decision. For the influence of uncertain factors in air combat, the membership function of robust situation in air combat is designed. In order to make the decision result robust, a robust multi-objective optimization function based on statistical principle is designed. And for less time consuming of the optimization method, an intelligent algorithm GWO is employed to solve the problem of maneuvering decision.

2. VARIABLE WEIGHT STATE PREDICTION METHOD BASED ON TARGET REACHABLE DOMAIN

2.1 Target Reachable Domain Design

Target reachable domain refers to the collection of all states that can be achieved in a time domain by traversing all the controlled quantities within the control constraints in the current state of the target. The target state prediction is to find the most likely target state to arrive at the next moment in the reachable domain, so that the UCAV can make maneuver decisions in advance. However, it is unrealistic to traverse all the controlled quantities of the target to calculate the reachable domain. Therefore, this paper establishes the basic manipulation action library to describe the forward reachable domain through Angle.

The basic manipulation action library has been widely used in maneuver decision-making, and NASA scholars proposed seven typical action libraries to simulate aircraft maneuver[11]. In order to better describe the reachable domain of the target and reduce the operation time, this paper selects five actions of the upper, lower, right and left directions of the ultimate overload flight and rectilinear flight as the predicted maneuver actions of the target, as shown in Figure 1.

![Figure 1. Five kinds of maneuvers simulation.](image)

In very short decision time-domain, the target reachable domain is approximately symmetric, and the target motion is projected on two vertical planes, as shown in Figure 2. In the X-Z plane, the connection between the final position and the initial position of the target maneuver and its initial velocity direction constitute an Angle, denoted as $\omega$. When the target adopts the limit state maneuver, the maximum value of $\omega$ is taken, denoted as $\omega_{max}$. When the target maintains the rectilinear flight in the current state, $\omega = 0$. Therefore, the Angle range of the target reachable domain is $[0, \omega_{max}]$. The azimuth angle can be obtained by projecting the final state of the target in the Y-Z plane, denoted as $\delta \in [0,360^\circ]$.
Then the state in the target reachable domain can be represented by set
\[ \zeta = \{ (\omega_t, \delta) \mid \omega_t \in [0, \omega_{\text{max}}], \delta \in [0, 360^\circ] \}. \]

Figure 2. Target motion decomposition schematic diagram.

2.2 Adaptive Variable Weight State Prediction Model

What maneuver the target takes is a matter of concern to UCAV when making maneuver decisions, because in air combat, any maneuver is based on a certain tactical intent. If the tactical intention of the target can be known in advance and its predicted state can be inferred, UCAV can use these information as the decision-making basis to make maneuver decisions and expand the battlefield situational advantage.

Figure 3. Target tactical intent and maneuverability

In this paper, the tactical intention of the target is divided into three categories: attack, escape and constant linear. When the target wants to carry out attacks, the speed will turn to UCAV, and the azimuth angle advantage of UCAV will be reduced. When the target escapes, the velocity direction deviates from UCAV to increase the distance as much as possible, which increases the orientation advantage of UCAV, as shown in the Figure 3. According to the target base manipulation action library and its influence on counterparty position situation, the three tactical intentions correspond to one maneuver action respectively, and the predicted state of the target is the weighted sum of the three final maneuver states that represent the tactical intention, as shown by

\[ X^{\text{pre}} = \mu_1 X^{\text{attack}} + \mu_2 X^{\text{escape}} + \mu_3 X^{\text{keep}}, \quad \sum_{i=1}^{3} \mu_i = 1 \quad (1) \]

According to the target basic manipulation action library given above, the reachable domain of four limit maneuver and linear flight can be divided into five parts, as shown in Figure 2. Among them, the rectilinear flight area is V, and the influence of the other four actions on the azimuth angle situation function is sorted and divided into four regions I~IV. Area I contains the area of action most advantageous to target attack and most unfavorable to UCAV, the target attack on the sub-favorable denoted as II, the action area that is most advantageous to target escape and least threatening to UCAV azimuth angle posture is IV, and the area of actions that are suboptimal for target escape should be III.

The current state of the target and the previous observation state are known. According to the state of the previous step, the forward reachable domain of the target can be obtained. And the intent weight can be adjusted according to the current state of the target in which area of I to V. At the same time, when the target makes escape or attack maneuver, it will fly in a straight line with a small probability at the next moment. In order to reduce the influence of this term on the predicted state, the weight adjustment expression is given by Eq. (2).
where \( \omega_0 \) is the given threshold and \( \Delta \mu \) is the change in weight units. The target is considered to move in a straight line when \( \omega_t \in [0, \omega_k] \), and \( \omega_t \in (\omega_k, \omega_{\text{max}}] \) means the target performs attack or escape maneuver.

In order to ensure the normalization of weight, the updated weight should be normalized and substituted into Eq. (1). It can be seen from this adaptive adjustment strategy that, when the target turns to UCAV attack, \( \mu_1 \) will increase and \( \mu_2 \) and \( \mu_3 \) decrease.

However, it is impractical to obtain the tactical intention by just one observation of the target state. It is necessary to constantly adjust the weight value through multiple observations to obtain more accurate target tactical intention, so as to obtain more accurate prediction of the target state and provide information support for UCAV maneuver decision-making.

3. ROBUST MANEUVER DECISION MODEL FOR UCAV AIR COMBAT

The air combat environment is full of many uncertain factors, and a lot of information is lost in the decision-making, which leads to the maneuvering decision-making of uncertainty, incomplete information and strong dynamics. Robust maneuver decision making is the application of robust decision-making method in the field of combat. Aiming at the uncertainty influence in the maneuver decision process, a robust strategy is needed to make it adaptive to most possible situations caused by the interaction of uncertain factors[12].

The real concern over the maneuver decision is how to complete a promising maneuver trajectory quickly. The widely used UCAV three degree of freedom model (3-DOF) can fits the need in this problem.

\[
\begin{align*}
\dot{x} &= V \cos \gamma \cos \psi \\
\dot{y} &= V \cos \gamma \sin \psi \\
\dot{z} &= V \sin \gamma \\
\dot{V} &= g(n_x - \sin \gamma) \\
\dot{\gamma} &= \frac{g}{V}(n_x \cos \phi - \cos \gamma) \\
\dot{\psi} &= \frac{gn_x \sin \phi}{V \cos \gamma}
\end{align*}
\]

where \( x, y, z \) are the UCAV location in inertial coordinate, \( \dot{x}, \dot{y}, \dot{z} \) are the velocity in each axis, respectively. \( \gamma, \psi, \phi \) are the Euler angle in trajectory coordinate. \( n_x, n_y \) are the overload, i.e. tangential overload and normal overload. \( g \) is the gravitational acceleration.

3.1 Robust Design of Air Combat Situation Function

There are many design methods for the air combat situation function[13,14]. Four typical parameters are often used to represent the air combat situation, \( \Theta = (A, R, V, H) \). \( A \) represents the orientation relation between UCAV and the enemy aircraft, \( R \) and to represent the distance between UCAV and enemy aircraft, \( V \) is the flight speed of UCAV, and \( H \) is flight altitude of UCAV. In order to enhance the robustness of decision results, the membership functions of the above four parameters are designed robustly.
3.1.1 Azimuth angle membership function

In real air combat, the stern attack is the most ideal situation for UCAV, in which the UCAV can lock on the target easily as well as ensure its own security. So, the azimuth angle membership function is defined as follows

\[
\tilde{R} = [x_u - x_t, y_u - y_t, h_u - h_t]
\]

\[
\vec{V}_e = \begin{bmatrix}
V_u \cos \gamma_u \cos \psi_u \\
V_u \cos \gamma_u \sin \psi_u \\
V_u \sin \gamma_u
\end{bmatrix}
\]

\[
\theta_a = \arccos \left( \frac{\tilde{R} \times \vec{V}_e}{||\tilde{R}|| \cdot ||\vec{V}_e||} \right)
\]

\[
\theta_t = \arccos \left( \frac{\tilde{R} \times \vec{V}_e}{||\tilde{R}|| \cdot ||\vec{V}_e||} \right)
\]

\[
\eta_a(\theta_a) = \begin{cases}
1, & \pi \geq \theta_a \geq \pi - \lambda^\text{missile}_{\text{max}} \\
1 - 0.2\left(\frac{\lambda^\text{missile}_{\text{max}} - \pi + \theta_a}{\lambda^\text{missile}_{\text{max}} - \pi} \right)^2, & \pi - \lambda^\text{missile}_{\text{max}} < \theta_a < \pi - \lambda^\text{missile}_{\text{max}} \\
0.8 - 0.3[1 - \left(\frac{\theta_a - \lambda^\text{missile}_{\text{max}}}{\pi - \lambda^\text{missile}_{\text{max}}} \right)], & \theta_a < \pi - \lambda^\text{missile}_{\text{max}}
\end{cases}
\]

\[
\eta_t(\theta_t) = \begin{cases}
1, & \theta_t \geq 2\pi / 3 \\
1 - 0.5\left(\frac{2\pi / 3 - \theta_t}{2\pi / 3} \right), & \theta_t < 2\pi / 3
\end{cases}
\]

\[
\eta_a = \eta_a(\theta_a) \eta_t(\theta_t)
\]

where, the subscript \(u\) represents UCAV and \(t\) represents the target. \(\theta_a\) and \(\theta_t\) respectively represent the angle between UCAV and target velocity vector and line of sight, as shown in Figure 4. \(\lambda^\text{missile}_{\text{max}}\) and \(\lambda^\text{radar}_{\text{max}}\) respectively represent the maximum off-axis angle of the missile and the maximum field of view angle of the radar.

![Figure 4 Diagram of azimuth angle.](image)

3.1.2 Distance membership function

When the UCAV wants to attack the target, UCAV should be located with a suitable distance constrained by the weapon launching conditions. Considering the limitations of the maximum and minimum range of attacking, the relative distance membership function \(\mu_R\) is designed as

\[
\mu_R = \begin{cases}
1, & |\tilde{R}| \leq R^\text{attack}_{\text{max}} \\
e^{-\frac{|\tilde{R}| - R^\text{attack}_{\text{max}}}{2\sigma^2}}, & |\tilde{R}| > R^\text{attack}_{\text{max}} \\
e^{-\frac{|\tilde{R}| - R^\text{attack}_{\text{min}}}{2\sigma^2}}, & |\tilde{R}| < R^\text{attack}_{\text{min}}
\end{cases}
\]

where \(R^\text{attack}_{\text{max}}\) and \(R^\text{attack}_{\text{min}}\) are the maximum range and the minimum range of attacking of the weapon, respectively. \(\sigma\) is the standard deviation. When the relative distance is in the range of weapon attacking distance, the value of \(\mu_R\) can always be real number 1 as. This character makes UCAV having the robustness of distance variety, which will contribute to a robust decision.
3.1.3 Velocity membership function

The velocity membership function consists of three factors, the velocity $v_v$ of the UCAV, the velocity of the target $v_t$ and their relative distance $\eta_{\rho}$. The form of designed robustness is denoted as

$$\eta_v = \begin{cases} e^{-\frac{(v_v - v_t)^2}{2\sigma_t^2}}, & \eta_v < 1 \\ 1, & \eta_v \geq 1 \end{cases}$$

where $v_v$ is the best velocity. The more similar $v_v$ and $v_t$ are, the higher value of $\eta_v$ is. When the difference between $v_v$ and $v_t$ is in the scale $[\sigma_t, \sigma_t]$, the value of $\eta_v$ will get to 1 permanently. As shown in figure 4.

The concrete expression of $v_v$ is shown as

$$v_v = v_t + (v_{max} - v_t) \left\{ \begin{array}{ll} \frac{[R] - R_{max}}{|R|}, & |R| > R_{max} \\ \frac{[R] - R_{min}}{|R|}, & |R| < R_{min} \end{array} \right.$$  

where $v_{max}$ and $v_{min}$ are the limitation of the UCAV velocity. When the relative distance between the UCAV and the target is smaller than the weapon minimum attacking distance, the value of $v_v$ will get to 1. If the relative distance is bigger than maximum attacking distance, the value of $v_v$ will be bigger on the contrary.

3.1.4 Altitude membership function

The attack zone boundary of air-to-air missile is positively proportional to its altitude. In air combat, the altitude exits a proper boundary, not the bigger the better. Because the flight altitude not only affects the performance of UCAV itself, but also affects the performance of air-to-air missile. Therefore, there should be an advantage altitudes difference in air combat, and this membership function is given by

$$\eta_h = \begin{cases} e^{\frac{1.5h - h}{3h}}, & h > 1.5h_i \\ 1, & 0.9h_i \leq h \leq 1.5h_i \\ \frac{h_i}{0.9h_i}, & h < 0.9h_i \end{cases}$$

where $H_i$ and $h_i$ are respectively the flight altitude of UCAV and the target. Considering the actual energy mobility and air combat, the scope of advantage height should be $[0.9H_i, 1.5H_i]$, and in this condition, $\eta_h = 1$.

4. ROBUST MULTI-OBJECTIVE OPTIMIZATION OF UCAV MANEUVER DECISION-MAKING

4.1 Robust objective optimization function based on statistical principle

It can be seen the air combat is a multi-objective optimization problem that the four situation functions tend to the dominant area in air combat. Kwon Lee[15] proposed a weighted robust multi-objective optimization method (RMO) based on the mean and variance of statistics, as shown in the Eq.(16).

$$J_{\min} = \alpha \mu + (1 - \alpha) \sigma_{\mu} / \sigma $$  

where $\mu$ and $\sigma$ are respectively the mean and the variance of statistics.
Where \( \mu \) and \( \sigma \) respectively represent mean and standard deviation. However, this weighting method can only find Pareto optimal solution of the convex set. To address this shortcoming, S. Ghanmi[16] proposed another RMO method which is given by

\[
J_{\text{max}} = (f_1(x), f_2'(x), \ldots, f_n(x), f_n'(x))
\]

(14)

Where

\[
f_i'(x) = \frac{\mu_i}{\sigma_i}
\]

(15)

and \( f_i'(x) \) is called a robust function of the target function. This method is proved to be superior in dealing with uncertain problems.

Based on the above, and considering the time requirements of air combat maneuver decision timeliness to the algorithm optimization, the optimization objective function is selected as

\[
J_{\text{max}} = \frac{\mu_\eta}{\sigma_\eta}
\]

(16)

where \( \mu_\eta \) is the mean of the four state membership functions, and \( \sigma_\eta \) is the standard deviation of it.

4.2 GWO Algorithm

GWO[17] is a novel developed swarm intelligence algorithm which mimics the social hierarchy and hunting strategy of grey wolf pack and possesses fast convergence as well as low computation cost. Thus, GWO is utilized to solve this real time maneuver decision problem. We no longer provide the search mechanism of GWO in this study, and more details can be referred from its original work.

5. SIMULATION VERIFICATION AND ANALYSIS

The feasibility of the variable weight state prediction method is verified by three tactical maneuver methods of target rectilinear flight, attack and escape. RMO decision making method based on current situation is compared with RMO decision making method based on target variable weight state prediction proposed in this paper.

5.1 Initial Parameter Setting

The initial location of UCAV is \((0\, \text{m}, \, 0\, \text{m}, \, 5000\, \text{m})\). Velocity is \(120\, \text{m/s}\). Both value of the pitch angle and yaw angle are \(0^\circ\). The controlled quantity is \( n_r \in [-1.5, 1.5] \). The initial location of the target is \((-2500\, \text{m}, \,-1500\, \text{m}, \, 5000\, \text{m})\). Velocity is \(150\, \text{m/s}\). The value of pitch angle is \(0\, \text{deg}\) and yaw angle is \(45^\circ\). The speed constraint of UCAV is \([50\, \text{m/s}, 200\, \text{m/s}]\), and height constraint is \([0\, \text{m}, 11000\, \text{m}]\).

The Assuming that the target starts to fly in a straight line and then turns to UCAV for attack, and the decision step size is \(1\, \text{s}\). The population size of GWO is set to 30 with iterations of 30. The simulation platform is Matlab R2013a, the processor frequency is 3.00GHz, and the memory is 3.15GB.

5.2 Results analyze

Figure 5 shows the flight display of the decision results. Given the initial conditions, the two kinds of decision paths are smooth to complete the attack advantage occupation of UCAV against the target. From the perspective of simulation track, the performance of robust decision making method proposed with prediction is superior to that of the method without prediction. For it can predict the target flight state, thus, the UCAV has an advantage in decision making and the number of simulation steps is smaller.

Figure 6 shows the prediction points of target state in RMO decision making process. By observing and modifying the weights step by step, UCAV's deviation of target state prediction becomes smaller and smaller, which proves the rationality and effectiveness of the variable weight state prediction model proposed in this paper.
Figure 5. 3D flight path display of decision.

Figure 6. Decision track based on prediction state.

Figure 7 shows the change curve of the membership function value of air combat situation of the two decision methods. It can be seen that both decision results meet the set decision termination condition, while the membership function value obtained by RMO with prediction basically converges to the dominant region, with more stable change and stronger decision robustness.

The advantages and disadvantages of air combat situation are constrained by the minimum values of membership functions of four parameters. Figure 8 shows the minimum values of membership functions of two decision-making methods. It can be seen that the method based on target prediction state is superior to the original in decision steps and situation evaluation.

Figure 7. The changing curves of decision membership function values.
Figure 8. Minimum membership function value curve.

Figure 9 shows the changes of decision control quantities of the two decision methods, and all three control quantities meet the constraint conditions, which proves the rationality and effectiveness of the algorithm's solution results.

Figure 9. Decision control quantity.

According to the weight change given in Figure 10, the tactical intention of the target can be seen. Within 0~10s, the target flies in a straight line with increasing weight $\mu_1$ and decreasing weight $\mu_2$ and $\mu_3$. After 10s, the target turns to UCAV for attack, and the weight $\mu_1$ increases rapidly, $\mu_2$ decreases rapidly and $\mu_3$ continues to decrease. So the changes of weights are consistent with the changes of target tactical intention, which reflects the effectiveness of the prediction model and thus provides more accurate decision information for UCAV maneuver decision.

Figure 10. The changes of weights.

In order to reduce the effect of randomness of optimization algorithm on decision results, 20 repeated Monte Carlo experiments were carried out, and the results were statistically analyzed. As can be seen from Table 1, in terms of the number of decision steps, the RMO decision method based on the prediction of target variable weight state proposed in this paper is more promising. Although its single-step decision time is slightly longer, it still meets the demand of decision step limitation 1s and is real-time.

|                    | RMO without prediction | RMO with prediction |
|--------------------|------------------------|---------------------|
| Average decision steps | 19.75                  | 16.55               |
6. CONCLUSION
Aiming at the problem of the influence of uncertain factors and decision lag in UCAV autonomous air combat, a robust maneuver decision method based on the state prediction of target reachable domain is proposed. The target prediction state is related to the tactical intention of the target, and the target intention is modified by the target reachable domain, so as to obtain the target prediction state. The robust multi-objective optimization function is designed based on the principle of situation function and statistics. GWO is utilized to solve this real-time optimization problem. By comparing the experiment results, the target state prediction mechanism can help UCAV obtain better situation advantage, which is of positive significance to the development of autonomous air combat.

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