Role of short-range and tensor correlations in nuclei

A Rios¹, W H Dickhoff² and A Polls³

¹Department of Physics, Faculty of Engineering and Physical Sciences, University of Surrey, Guildford, Surrey GU2 7XH, United Kingdom
²Department of Physics, Washington University, St. Louis, Missouri 63130, USA
³Departament d’Estructura i Constituents de la Matèria and Institut de Ciències del Cosmos, Universitat de Barcelona, Avda. Diagonal 647, E-08028 Barcelona, Spain

E-mail: a.rios@surrey.ac.uk

Abstract. The role of short-range and tensor correlations in nuclei can be investigated through a careful comparison with microscopic nuclear matter calculations. We focus on the momentum distribution, a one-body operator which is particularly sensitive to correlations. We identify a depletion of the population of hole states of around 15% in symmetric matter and a significant isospin dependence due to the tensor force. The increased role of short-range and tensor correlations for the minority species makes the case for further experimental scrutiny of nuclei with large neutron excess.

The nucleon-nucleon (NN) interaction is extremely repulsive at short distances [1]. As a consequence, the probability of finding two nucleons close to each other is substantially suppressed. In turn, such a decrease in the configuration space components implies the population of high-momentum components in the many-body wave function. These significant modifications cannot be described within an independent particle model and are generically referred to as correlations [2]. An additional mechanism that provides high-momentum components is associated with the NN tensor force, which is required, among other things, to obtain the correct quadrupole moment of the deuteron [1]. Particle number conservation requires the high-momentum components to be accompanied by a corresponding depletion of the population of states within the nuclear Fermi sea [3]. A characteristic feature of this depletion in nuclear matter is its essentially momentum-independent character [4]. For realistic NN interactions, different many-body techniques consistently predict a depletion of the nuclear Fermi sea of a little over 15%, of which around a third is caused by tensor correlations. An \((e,e’p)\) experiment on \(^{208}\)Pb at NIKHEF in a large domain of missing energy and a momentum range corresponding to the mean-field Fermi sea confirmed that a global depletion between 15 and 20% of proton orbits below the Fermi energy explains all the measured coincidence cross sections [5]. The differences with respect to nuclear matter calculations can be largely attributed to long-range correlations [6].

A particularly well-suited technique to study the depletion of nuclear systems is the SCGF method [2]. Within this approach, short-range (SRC) and tensor correlations (TC) are accounted for by a fully self-consistent treatment of ladder diagrams for the interaction between particles that propagate with respect to a correlated (by short-range and tensor effects) ground state [7]. The intermediate particle-particle and hole-hole states are fully dressed, with off-shell effects consistently taken into account. A new generation of SCGF numerical calculations have recently become available at finite temperature [7, 8]. Thermal effects are expected to smooth out the
Figure 1. Isospin asymmetry dependence of the Free Fermi gas (left panel) and the correlated momentum distribution from Av18 (right panel). The density ($\rho = 0.16$ fm$^{-3}$) and temperature ($T = 5$ MeV) are fixed and the results for different asymmetries correspond to different line-styles.

momentum distribution near the Fermi momentum, but they hardly affect the small and very high-momentum content of the ground state [3].

Interaction-induced correlations have a distinctive signature in the momentum distribution, removing strength at momenta below the Fermi surface and shifting it to high momenta. The momentum distribution for symmetric and isospin asymmetric matter, obtained with the SCGF method, are shown in the right panel of Fig. 1 for a fixed density, $\rho = 0.16$ fm$^{-3}$, and temperature, $T = 5$ MeV. For comparison, we show equivalent free Fermi gas (FFG) momentum distributions in the left panel. At such low temperature, the Fermi-Dirac distribution of the symmetric matter FFG (solid line) deviates very little from a step function, $\Theta(k_F - k)$. This indicates that thermal effects are small in the deep interior of the Fermi sea under these conditions. In contrast, for the correlated case (right panel), $n(k)$ for symmetric matter presents a sizable depletion and it is rather flat below $k_F$. This depletion is mainly caused by dynamical SRC and TC. As a consequence, the contribution to the density sum-rule of the states below $k_F$ is reduced to 75%, i.e. 10% of the strength is shifted to higher momenta due to NN correlations. The high-momentum tail in $n(k)$ for symmetric matter provides 52% of the total kinetic energy per particle, to be compared to the much smaller 25% in the FFG.

For a fixed total density, one can switch from symmetric to neutron matter by modifying the relative concentration of neutrons and protons, defined by the asymmetry parameter, $\alpha = \frac{n - p}{n + p}$. The SCGF method within the ladder approximation can be generalized to the case of partially isospin-polarized matter [9, 10]. Since the approach accounts for both the SRC and TC associated with the underlying NN interaction, it can be used to generate quantitative predictions for the importance of different types of correlations in isospin asymmetric systems. The study of the asymmetry dependence of SRC and TC is motivated to a large extent by the future study of rare isotopes with large neutron excess [11].

In Fig. 1, the momentum distributions for different asymmetries, from symmetric matter ($\alpha = 0$) to extremely neutron-rich matter ($\alpha = 0.8$), correspond to different line styles. As asymmetry increases, the system becomes more neutron-rich and, consequently, the Fermi momentum of neutrons (protons) increases (decreases). This trivial effect can be observed in both the left and right panel. For the FFG, the neutron momentum distribution does not change substantially with asymmetry. For protons, however, $n(k)$ is substantially affected by asymmetry and it is modified at all momenta as the system becomes more neutron-rich. This is a direct consequence of the fact that protons become classical as their density decreases. Consequently, this effect should be ascribed to thermal correlations rather than dynamical correlations [3].
For the correlated case, the most abundant component (neutrons for positive $\alpha$) gets less depleted when the asymmetry increases, i.e. neutrons become “less correlated”. This is in contrast to the FFG results, which, for neutrons, exhibit no change inside the Fermi sea. This behavior can be explained in simple terms as follows. Although the total number of pairs is the same when increasing the asymmetry at constant density, some of the proton-neutron pairs are replaced by neutron-neutron pairs. The latter correlations are weaker than the proton-neutron ones, due to the absence of tensor effects, and therefore neutrons become less correlated at large $\alpha$’s. Conversely, the momentum distributions of the less abundant species (protons) become more depleted with asymmetry.

The information about the isospin dependence of the depletion is summarized in the left panel of Fig. 2, where $n_{\nu}(0)$ is plotted as a function of the asymmetry. The FFG results (orange lines) are compared to those obtained with a wide range of realistic NN interactions. The FFG provides a measure of thermal effects in the depletion. For the non-interacting case, $n(0)$ for neutrons does not change when the asymmetry increases, indicating that they are totally degenerate. The corresponding occupation for protons (the less abundant component) is close to 1 only at small asymmetries. As a matter of fact, in the limit $\alpha \rightarrow 1$, protons become an impurity gas in a Fermi sea of neutrons, thus behaving as a classical gas with $n_p(0) \rightarrow 0$. The steepness of the change in $n_p(0)$ would be smaller for higher densities, i.e. more degenerate systems.

For the correlated depletion, the occupation of the zero momentum state is an increasing (decreasing) function of the asymmetry for the more (less) abundant component. The behavior is similar for all NN interactions, with a changing offset due to the different predictions for the depletion of symmetric matter. Asymmetries of stable nuclei belong to the lower range of asymmetries, i.e. $\alpha \sim 0.2$ for $^{208}$Pb. The SCGF predictions should be valid in this range, where thermal effects are unimportant. Unfortunately, the difference between the occupation of protons at this asymmetry and the symmetric case is only $\sim 2\%$, too small to allow experimental verification. Nuclei at larger asymmetries produced at future rare isotope facilities may provide a better testing ground. Moreover, integrated effects over the whole neutron and proton density profiles might also enhance the effect of asymmetry. Let us note that, for finite nuclei, one should also take into account surface properties and their isospin dependence [12].

A surprising feature arises when comparing the different NN interaction results in the left panel of Fig. 2. In spite of the differences observed in both symmetric and pure neutron matter for the different potentials, the asymmetry dependence of $n_{\nu}(0)$ is relatively similar.
This is intriguing because these forces have a rather different short-range behavior and tensor structure. Given the almost linear dependence of \( n_\nu(0) \) with asymmetry, a better insight into these differences can be gained by plotting the difference \( n_\nu(0) - n_p(0) \). In the right panel of Fig. 2, we present this “iso-depletion” for different interactions as a function of the asymmetry at \( T = 5 \) MeV and \( \rho = 0.16 \) fm\(^{-3}\). The iso-depletion is the same for a wide variety of modern NN potentials, independently of their short-range or operatorial structure. This suggests that the isospin dependence of \( n(0) \) is fixed by the phase-shifts, most probably via their isospin dependence.

The right panel of Fig. 2 shows two exceptions to this common behaviour. On the one hand, the iso-depletion of the FFG falls below the other results. This is expected, because thermal effects are comparatively smaller than dynamical correlations for this particular density and temperature. On the other hand, immediately below the majority of the interactions, one finds the results corresponding to the Av4’ potential. This interaction is an oversimplified version of Av18, with an unrealistically simple operatorial structure that only has a spin-isospin part and no tensor components [13]. The fact that the Av4’ results lie significantly below the others demonstrates that correlations in asymmetric matter are largely driven by tensor effects, i.e. the tensor force increases substantially the difference between neutron and proton momentum distributions with isospin asymmetry. Moreover, the comparison with Av6’ and Av8’ suggests that, once the tensor components are included in a force, the iso-depletion becomes universal. Let us stress the fact that such an agreement for different potentials is surprising in view of their different momentum-space and operatorial structures.

We close this discussion by noting that the employed interactions appear to slightly underestimate the depletion of the experimentally determined depletion of the deep proton mean-field orbits in \(^{208}\)Pb [5]. While long-range correlations have an additional influence on the depletion of the Fermi sea, it is expected that they are more important near the Fermi energy. For a proper comparison with finite nuclei, however, these long-range volume effects are presumably irrelevant, since they must anyway be replaced by the surface dominated physics of low-lying nuclear states. Nevertheless, while there remains a few % uncertainty as to the exact amount that is experimentally required, it is also clear that different interactions, including modern ones, lead to very similar predictions for the depletion of the nuclear Fermi sea within the SCGF method.

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