Cooperative Wi-Fi Deployment: A One-to-Many Bargaining Framework

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Background

- Nearly 10-fold increase between 2014 and 2019
Background

- Need more Wi-Fi to offload cellular traffic
  - More than half of total traffic will be offloaded (54% by 2019)
  - The Wi-Fi deployment rate is increasing (10.5 million by 2018)
Cooperative Wi-Fi Deployment

- **MNOs** cooperate with **VOs** to deploy public Wi-Fi
  - **Venue owner**: owner of public places

![Shopping malls](image1)

Shopping malls

![Cafes](image2)

Cafes

![Stadiums](image3)

Stadiums

![Hotels](image4)

Hotels
Cooperative Wi-Fi Deployment

- MNOs cooperate with VOs to deploy public Wi-Fi
  - Example: AT&T (MNO) and Starbucks (VO)

AT&T provides Wi-Fi for Starbucks from 2008 to 2014
Problem Description

- **Economic interactions** between a monopoly MNO and multiple VOs:
  - Q1: *Which VOs should the MNO cooperate with?*
  - Q2: *How much should the MNO pay to these VOs?*
  - Q3: *What negotiation sequence can maximize the MNO’s payoff?*

![Diagram showing MNO and VOs with questions](image)
Model

- Basic Settings:
  - A set $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$ of VOs, describe VO $n$ by $(X_n, R_n, C_n)$;
  - $X_n \geq 0$: expected traffic offloaded by the Wi-Fi at venue $n$;
  - $R_n \geq 0$: extra revenue Wi-Fi creates for VO $n$;
  - $C_n \geq 0$: cost for the MNO to deploy Wi-Fi at venue $n$.

- Negotiation Results
  - $b_n \in \{0, 1\}$: whether the MNO cooperates with VO $n$;
  - $p_n \in \mathbb{R}$: the MNO’s payment to VO $n$ (could be negative)
  - For all $n \in \mathcal{N}$, define
    \[
    b_n \triangleq (b_1, b_2, \ldots, b_n), \quad p_n \triangleq (p_1, p_2, \ldots, p_n).
    \]
Model

- **Notations:**
  - Negotiation variables \((b_n, p_n)\), VO attributes \((X_n, R_n, C_n)\)

- **MNO’s payoff:**
  \[
  U(b_N, p_N) = f \left( \sum_{n=1}^{N} b_n X_n \right) - \sum_{n=1}^{N} b_n C_n - \sum_{n=1}^{N} p_n
  \]

  \(- f (\cdot)\) is increasing and strictly concave with \(f (0) = 0\)

- **VO \(n\)’s payoff:**
  \[
  V_n(b_n, p_n) = b_n R_n + p_n
  \]
Model

- Social welfare:

\[
\Psi (b_N) = U (b_N, p_N) + \sum_{n=1}^{N} V_n (b_n, p_n)
\]

\[
= f \left( \sum_{n=1}^{N} b_n X_n \right) + \sum_{n=1}^{N} b_n (R_n - C_n)
\]

\[
= f \left( \sum_{n=1}^{N} b_n X_n \right) + \sum_{n=1}^{N} b_n Q_n
\]

- Social welfare only depends on \(b_N\)
- \(Q_n \triangleq R_n - C_n\) captures the factors excluding data offloading. In later analysis, describe VO \(n\) by \((X_n, Q_n)\), instead of \((X_n, R_n, C_n)\)
One-to-One Nash Bargaining

- Assume $|\mathcal{N}| = 1$, the problem degenerates to one-to-one bargaining.
- NBS (Nash Bargaining Solution) solves:

$$\max \ (u(b_1, p_1) - u(0, 0)) \cdot (v_1(b_1, p_1) - v_1(0, 0))$$

s.t. \quad u(b_1, p_1) - u(0, 0) \geq 0, \quad v_1(b_1, p_1) - v_1(0, 0) \geq 0,$$

var. \quad b_1 \in \{0, 1\}, \quad p_1 \in \mathbb{R}

- Disagreement points: $u(0, 0) = v_1(0, 0) = 0$
- NBS maximizes the product of the players’ payoff gains upon their disagreement points. With a higher disagreement point, the MNO (or VO) can obtain a larger payoff under the NBS.
To simplify description, show the NBS in the form of \((b_1^*, \pi_1^*)\), instead of \((b_1^*, p_1^*)\): \((\pi_1: \text{VO 1's payoff}; \ p_1: \text{payment from MNO to VO 1})\)

\[
(b_1^*, \pi_1^*) = \begin{cases} 
(1, \frac{1}{2}\Psi(1)) & \text{if } \Psi(1) \geq \Psi(0) = 0, \\
(0, 0) & \text{otherwise,}
\end{cases}
\]

- If cooperation increases social welfare, the Wi-Fi will be deployed and they will equally share the generated revenue.
- Recall \(\Psi(1) = f(X_1) + Q_1\), hence the NBS depends on \(X_1\) and \(Q_1\). That’s to say, we only need to know \(Q_1\), instead of \((R_1, C_1)\).
Example 1: One-to-One Bargaining

\[ f(x) = x^{0.5} \]

NBS: \( b_{\text{red}} = 1, \pi_{\text{red}} = 1 \)

MNO payoff: \( U_0 = 1 \)

\( X_{\text{red}} = 16, \quad Q_{\text{red}} = -2 \)
Example 2: One-to-Many Sequential Bargaining

Example 2

\[ f(x)=x^{0.5} \]

\[ X_{\text{red}}=16, \quad Q_{\text{red}}=-2 \]
\[ X_{\text{white}}=9, \quad Q_{\text{white}}=-2.5 \]

- Bargain with VOs **sequentially**: VO red→VO white
- **Backward induction**:
  - **Step 2**: assuming the MNO reaches \((b_1, \pi_1)\) in Step 1, we study the one-to-one bargaining between the MNO and VO white;
  - **Step 1**: Based on VO white’s response in step 2, we study the one-to-one bargaining between the MNO and VO red.
Example 1

\[ f(x) = x^{0.5} \]

NBS: \( b_{\text{red}} = 1, \pi_{\text{red}} = 1 \)
MNO payoff: \( U_0 = 1 \)

\[ X_{\text{red}} = 16, Q_{\text{red}} = -2 \]

Example 2

\[ f(x) = x^{0.5} \]

NBS: \( b_{\text{red}} = 1, \pi_{\text{red}} = 0.875, b_{\text{white}} = 0, \pi_{\text{white}} = 0 \)
MNO payoff: \( U_0 = 1.125 \)

\[ X_{\text{red}} = 16, Q_{\text{red}} = -2 \]
\[ X_{\text{white}} = 9, Q_{\text{white}} = -2.5 \]

- The existence of VO white allows the MNO to extract more revenue from the cooperation with VO red (think it as a backup plan)
- Different steps of bargaining generate externalities to each other, this is due to the concavity of the offloading benefit \( f \)
One-to-Many Bargaining I: Sequential Bargaining with Exogenous Sequence
Sequential Bargaining with Exogenous Sequence

Figure: Illustration of Sequential Bargaining
Step N and Step N-1

**NBS for step N**

\[(b^*_N, \pi^*_N) = \begin{cases} 
(1, \frac{1}{2}(b_{N-1})) & \text{if } \Delta_N(b_{N-1}) \geq 0, \\
(0, 0) & \text{otherwise}, 
\end{cases}\]

Define \(\Delta_N(b_{N-1}) = \Psi((b_{N-1}, 1)) - \Psi((b_{N-1}, 0))\)

**NBS for step N-1**

\[(b^*_{N-1}, \pi^*_{N-1}) = \begin{cases} 
(1, \frac{1}{2}(b_{N-2})) & \text{if } \Delta_{N-1}(b_{N-2}) \geq 0, \\
(0, 0) & \text{otherwise}, 
\end{cases}\]

where we define

\[\Delta_{N-1}(b_{N-2}) = \Psi((b_{N-2}, 1, b^*_N((b_{N-2}, 1)))) - \pi^*_N((b_{N-2}, 1))
- \Psi((b_{N-2}, 0, b^*_N((b_{N-2}, 0)))) + \pi^*_N((b_{N-2}, 0)).\]
Step $k$

**NBS for step $k$**

$$(b_k^*, \pi_k^*) = \begin{cases} 
(1, \frac{1}{2}\Delta_k (b_{k-1})) & \text{if } \Delta_k (b_{k-1}) \geq 0, \\
(0, 0) & \text{otherwise},
\end{cases}$$

where we define

$$\Delta_k (b_{k-1}) = \Psi \left( (b_{k-1}, 1, b_{k+1}^* ((b_{k-1}, 1)), \ldots, b_N^* ((b_{k-1}, 1, \ldots)) \right) - \pi_{k+1}^* ((b_{k-1}, 1)) - \ldots - \pi_N^* ((b_{k-1}, 1, \ldots)) - \Psi \left( (b_{k-1}, 0, b_{k+1}^* ((b_{k-1}, 0)), \ldots, b_N^* ((b_{k-1}, 0, \ldots)) \right) + \pi_{k+1}^* ((b_{k-1}, 0)) + \ldots + \pi_N^* ((b_{k-1}, 0, \ldots)).$$

**Remark:** The MNO's payoff under a particular bargaining sequence is fixed, and can be computed by a recursive algorithm (omitted)
One-to-Many Bargaining II: Sequential Bargaining with Endogenous Sequence
Bargaining sequence affects cooperation outcomes, money transfer, social welfare, and the MNO’s payoff
Optimal Sequencing Problem

Question: Which bargaining sequence maximizes the MNO’s payoff?
- Cannot obtain the closed-form solution of the MNO’s payoff
- Checking all $|\mathcal{N}|!$ possibilities is time-consuming

Key idea: prove structural properties related to VOs’ types
Categorization of VOs and Structural Properties

**Definition 1: VO Type**

- **VO** $n \in \mathcal{N}$ belongs to:
  - **type 1**, if $Q_n \geq 0$;
  - **type 2**, if $Q_n < 0$ and $f(X_n) + Q_n \geq 0$;
  - **type 3**, if $Q_n < 0$ and $f(X_n) + Q_n < 0$.

**Observation:**

- **Type 1** VO’s cooperation with the MNO does not decrease the social welfare, *i.e.*, $\Psi (1, b_{-n}) \geq \Psi (0, b_{-n})$;
- **Type 2** VO’s cooperation with the MNO may or may not decrease the social welfare, which depends on other VOs’ attributes and positions;
- **Type 3** VO’s cooperation with the MNO decreases the social welfare, *i.e.*, $\Psi (1, b_{-n}) < \Psi (0, b_{-n})$. 
Categorization of VOs and Structural Properties

Theorem 1

There exits a group of optimal bargaining sequences satisfying the following two conditions:

(1) VO \( l_1, l_2, \ldots, l_{N_1} \) are of type 1;
(2) VO \( l_{N_1+N_2+1}, l_{N_1+N_2+2}, \ldots, l_N \) are of type 3.

For any optimal sequence that belongs to this group, if the MNO interchanges the bargaining positions of any two type 1 VOs (or two type 3 VOs), the MNO’s payoff will not change.

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Reduce Complexity from $|\mathcal{N}|!$ to $|\mathcal{N}_2|!$

- For example, there are 7 VOs, where $\{1, 2\}$, $\{3, 4, 5\}$, $\{6, 7\}$ are type 1, 2, 3, respectively.
  - By exhaust search, we need to check $7! = 5040$ possibilities;
  - By Theorem 1, we only need to check $3! = 6$ possibilities.
Special Case I: All Are Type 1

**Theorem 2**

- If all VOs are of type 1, the MNO’s payoff is independent of the bargaining sequence $\mathbf{l}$ and is given as:

$$ U_0 = \frac{1}{2^N} \sum_{b_N \in \mathcal{B}} \Psi (b_N), $$

where $\mathcal{B} \triangleq \{(b_1, b_2, \ldots, b_N) : b_n \in \{0, 1\}, \forall n \in \mathcal{N}\}$.

- **Remark:** Can write down the close-form solution of the MNO’s payoff.
Special Case II: All Are Sortable

**Definition 2**

A set $\mathcal{N}$ of VOs is said to be **sortable** if and only if for any $i, j \in \mathcal{N}$, we have $(X_i - X_j)(Q_i - Q_j) \geq 0$.

**Theorem 3**

If all VOs are **sortable**, we can construct a sequence $\mathbf{l}$ such that $X_{l_n} \geq X_{l_{n+1}}$, $Q_{l_n} \geq Q_{l_{n+1}}$, $\forall n \in \{1, 2, \ldots, N - 1\}$. Furthermore:

1. $\mathbf{l}$ is the **optimal** bargaining sequence;
2. Under $\mathbf{l}$, the MNO will and only will cooperate with VO $l_1, l_2, \ldots, l_k$, where $k \in \{0\} \cup \mathcal{N}$ is uniquely determined by two inequalities (omitted here).

**Remark:** Can quickly provide the **optimal** sequence without any searching.
Simulation 1: Advantage of Optimal Sequencing

- **Settings:** \( f(x) = x^{1/2}, |\mathcal{N}| = 5 \)
- Compared with the **worst** sequence, the optimal sequence improves the MNO’s payoff by 17% on average and by 46% at most;
- Compared with the **random** sequence, the optimal sequence improves the MNO’s payoff by 8% on average and by 15% at most.
Simulation 2: Influence of Offloading Benefit

- **Settings:** $f(X) = X^c$, $|\mathcal{N}| = 4$
- Optimal sequencing’s advantage is not obvious for small and large $c$
  - **Small $c$:** offloading benefit is small, hence most VOs are type 3, and the MNO does not cooperate with these VOs
  - **Large $c$:** function $f(\cdot)$’s concavity is small and the externalities among different steps of bargaining are weak
Conclusion

- Study cooperative public Wi-Fi deployment
- Consider one-to-many Nash bargaining
  - **Exogenous** bargaining sequence: analyze the MNO's payoff under a given bargaining sequence, with the consideration of externalities among VOs
  - **Endogenous** bargaining sequence: obtain the optimal bargaining sequence by leveraging the structural property
THANK YOU