OPTIMIZING AN ALUMINUM EXTRUSION PROCESS

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ABSTRACT

Minimizing the amount of scrap generated in an aluminum extrusion process. An optimizing model is constructed in order to select the best cutting patterns of aluminum logs and billets of various sizes and shapes. The model applied to real data obtained from an existing extrusion factory in Kuwait. Results from using the suggested model provided substantial reductions in the amount of scrap generated. Using sound mathematical approaches contribute significantly in reducing waste and savings when compared to the existing non scientific techniques.

Keywords: Minimizing, Cutting Patterns, Aluminum Logs, Mathematical Approaches

1. INTRODUCTION

Aluminum is the third most abundant element in the Earth’s crust and it constitutes 7.3 percent by mass. The Aluminum industry contributes significantly to the global economy as well as too many individual economies. The industry employs over a million people worldwide. Aluminum smelting is a capital-intensive, technology-driven industry concentrated in a few relatively dominant companies. Aluminum consumption has enjoyed substantial average growth over the last few decades due to general economic growth and to its substitution of other materials. Kuwait is a member of the Gulf Cooperation Council (GCC); the GCC countries will boost their share of global aluminum output to 15-17% by the end of the decade. There are two types of aluminum industries; the first is the extrusion industry where profiles of different sizes, colors and shapes are produced, while the second is the fabrication industry where various products such as windows, fences and doors are designed from aluminum profiles.

In the aluminum extrusion industry, logs and billets are cut using various stock cutting patterns; the amount of scrap generated is dependent on the cutting method used. The Stock Cutting Problem (SCP) is discussed thoroughly in the literature. One of the first articles was presented by Gilmore and Gomory (1961) where integer programming was utilized for the cutting stock problem. The problem compromised a large number of variables which generally makes the computation infeasible. Gilmore and Gomory (1964) examined the cutting stock problems involving two or more dimensions. Haessler (1971) described a heuristic procedure for scheduling production-rolls of paper through a finishing operation to cut them down to finished roll sizes. The objective was to minimize the cost of trim-loss and that of the reprocessing. Covesdale and Wharton (1976) presented a heuristic procedure for a nonlinear cutting stock problem; the problem was solved using the pattern enumeration technique.

Sumichrast (1986) addressed a scheduling problem in the woven fiber glass industry as an example of the cutting stock problem with the objective of controlling the wasted production capacity rather than wasted material. A heuristic was developed for the purpose of scheduling the production process. Stadtler (1990) used the column generation method of Gilmore and Gomory (1964) for minimizing the amount of scrap generated from fabricated aluminum made for window frames. Krichagina et al. (1998) examined the cutting process of sheets in a paper plant. The main objective was to minimize the long-run average cost of paper waste. In this regard, a two step procedure consisting of linear programming and Brownian control was developed.
Liang et al. (2002) applied an evolutionary algorithm (EP) for cutting stock problems with and without contiguity. Results showed that the EP algorithm is more effective and superior when compared to the genetic algorithm used. Parada et al. (2003) proposed a meta-heuristic approach for solving a non-guillotine stock cutting problem. The approach was a combination of the principles of the constructive and evolutive methods. The results showed an error reduction of around 2%. Hifi (2004) proposed an algorithm for solving a two dimensional constrained cutting stock problem. In the algorithm and for depth search, hybrid approach combining hill climbing strategies and dynamic programming were employed. Cui (2005) developed an algorithm that utilizes the knapsack algorithm and an implicit enumeration technique. The algorithm was applied to real cutting stock data of the manufacture electric generators. Khalifa et al. (2006) built a one dimensional cutting stock problem using genetic algorithm. The objective was to reduce the amount of waste generated in constructing steel bars.

Saad et al. (2007) addressed the problem of scrap generated from cutting cylindrical logs produced by an aluminum extrusion company. In this regard, a multi-objective cutting stock problem was constructed. A solution procedure was developed considering the scrap generated as a fuzzy parameter. Chen (2008) presented a recursive heuristic algorithm for the constrained two-dimensional stock cutting problems. The algorithm was tested and the computational results produced good solutions in short computing time for problems of different scales. Alves et al. (2009) used several constrained and non-constrained integer programming using column generation. lower bounds for the different minimization patterns were derived. Using actual data, the outcome of these models showed improvement of the lower bounds. Hajeeh (2010) addressed the problem of waste generated in an aluminum fabrication industry. A heuristic was propped for optimizing the cutting of aluminum profiles. The heuristic produced less scrap when compared to the existing procedure used in the company. Macedo et al. (2010) proposed an integer linear programming model to solve the two-dimensional stock cutting problem with guillotine constraint. A computer software was used to examine the behavior of the models with data from a wood industry. The lower bound of the model was found to be superior to those of other methods.

Kasimbeyli et al. (2011) proposed a linear integer programming with two conflicting objectives for a one-dimensional cutting problem. A special heuristic algorithm was used to find the optimum cutting pattern. Berberler et al. (2011) developed a dynamic programming algorithm to address the one-dimensional stock cutting problem. The results obtained from this algorithm was compared to others and results showed its efficiency and superiority. Cui and Huang (2012) proposed a heuristic to address constrained T-shaped patterns with the objective of maximizing the pattern value and meeting demand. The computation of 58 benchmark instances showed that the algorithm is superior to the two-stage patterns approaches. De Valle et al. (2012) developed an algorithm based on non-fit polygen to examine the two dimensional cutting/packing problem. The algorithm also solved problems with items of irregular shapes. Mobasher and Ekici (2013) developed a mixed integer linear and used the column generation method to the study a cutting stock problem with set up cost. The main objective was to find a cutting pattern at minimum production cost.

In the current research work, the extrusion process in a specific industry in Kuwait is thoroughly studied with objective of finding ways for reducing the large amount of scrap generated. The article organized as follows: it start by describing the aluminum profile production process, the amount of scrap generated in the chosen industry from using the existing cutting techniques. Next, the structure of the developed optimization models is provided. For illustration, an example is presented to compare the amount of scrap generated using the existing conventional cutting patterns and the size of scrap generate from the proposed optimization model. Results and discussion section comes next, the article ends with concluding remarks.

2. MATERIALS AND METHODS

2.1. Aluminum Profile Production

Aluminum profile production (extrusion) process passes through several stages starting with castings where logs are produced; the logs are next cut into standard billets and are put into extrusion machine to manufacture profiles of different shapes and sizes. The extruded aluminum profiles are placed in the aging furnace in order to increase their durability and strength. Next, the profiles are polished thoroughly and depending on request are either sent for painting, or anodizing before shipping to the customer. In Fig. 1, the detailed process is presented for a specific extrusion company in Kuwait.
The type of billets and logs used in the extrusion process in the company along with their lengths and weight is shown in Table 1. The monthly weight and percentage of scrap generated during a specific year in the aluminum extrusion by the same company is as given in Table 2.

2.2. Optimization Models

An efficient model for minimizing scrap in the profile production process is based on developing cutting patterns, where each cutting pattern uses several billets and the total weight cut is less than the total weight of the log used. All possible patterns are investigated and the weight of the scrap generated is calculated. Two models are presented in this research work, details are given below.

Table 1. Lengths and weights of standard billets and logs used by the aluminum extrusion company

| Billet Type | Length (m) | Weight (kg) | Log Type | Length (m) | Weight (kg) |
|-------------|------------|-------------|----------|------------|-------------|
| 1           | 0.48       | 32.16       | 1        | 2.59       | 173.4       |
| 2           | 0.52       | 34.25       | 2        | 2.64       | 176.8       |
| 3           | 0.56       | 37.55       | 3        | 2.69       | 180.2       |
| 4           | 0.58       | 38.78       | 4        | 2.74       | 183.6       |
| 5           | 0.61       | 40.55       | 5        | 2.79       | 187.0       |
| 6           | 0.65       | 43.55       | 6        | 2.84       | 190.4       |

Table 2. Weight and percentage of monthly total production scrapped in the extrusion stage

| Month       | Total production (kg) | Scrap weight (kg) | Percentage scrap (%) |
|-------------|-----------------------|-------------------|----------------------|
| January     | 249255                | 64722             | 26                   |
| February    | 280077                | 75978             | 27                   |
| March       | 295545                | 77954             | 27                   |
| April       | 343386                | 91102             | 27                   |
| May         | 245831                | 65408             | 27                   |
| June        | 321935                | 68588             | 21                   |
| July        | 268012                | 61195             | 23                   |
| August      | 219373                | 52435             | 24                   |
| September   | 317112                | 78005             | 25                   |
| October     | 278610                | 63709             | 23                   |
| November    | 347567                | 83844             | 24                   |
| December    | 385956                | 86888             | 23                   |
| Mean        | 296055                | 72636             | 25                   |

Standard deviation 48643 11328

Model I

Two models are presented in this research work. The first is shown in (1) which is based on different billets:

Minimize $Z = W_1 + W_2 + W_3$

Subject to Equation 1:

\begin{align*}
  v_1 \xi_{11} + v_2 \xi_{21} + v_3 \xi_{31} + v_4 \xi_{41} + v_5 \xi_{51} + v_6 \xi_{61} &= W_i \\
  \omega_1 \xi_{12} + \omega_2 \xi_{22} + \omega_3 \xi_{32} + \omega_4 \xi_{42} + \omega_5 \xi_{52} + \omega_6 \xi_{62} &= W_j \\
  \omega_1 \xi_{13} + \omega_2 \xi_{23} + \omega_3 \xi_{33} + \omega_4 \xi_{43} + \omega_5 \xi_{53} + \omega_6 \xi_{63} &= W_k \\

\end{align*}

(1)

\[ y, \xi_{11}, \xi_{21}, \xi_{31}, \xi_{41}, \xi_{51}, \xi_{61}, \xi_{12}, \xi_{22}, \xi_{32}, \xi_{42}, \xi_{52}, \xi_{62}, \xi_{13}, \xi_{23}, \xi_{33}, \xi_{43}, \xi_{53}, \xi_{63} \geq 0 \] and integers

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Where:

- \( Z \) = Total weight of the scarp in kg
- \( W_i \) = Weight of scrap (kg) generated per 1.5 inch log (0.0381 m) of each billet used to extrude the desired profile
- \( W_2 \) = Weight of scrap (kg) generated from log cutting
- \( W_3 \) = Weight of scrap (kg) generated from producing longer profiles than demanded
- \( \nu_j \) = Weight (kg) of producing type \( j \) billet, \( j = 1, \ldots, 6 \)
- \( \omega_j \) = Weight (kg) of log used to produce the \( j \)th billet, \( j = 1, \ldots, 6 \)
- \( \xi_j \) = Number of type \( j \) billet used
- \( y \) = Number of logs used
- \( W_L \) = Weight (kg) of each log
- \( W_F \) = Total weight (kg) of profiles demanded

**Model II**

Although the above problem provides a good solution to the problem, however it has one disadvantage in that, where more than one log is needed, the model considers all logs to be one long log. A superior and more efficient model is based on cutting patterns as given in (2). Equation (3) represents the non-negativity constraint.

Minimize \( Z = \sum_{j=1}^{J} S_j \rho_j \)

Subject to Equation 2:

\[
\sum_{j=1}^{6} Q_j \rho_j \geq Q_{ML} \quad (2)
\]

\( \rho_j \) non negative integers

Where:

- \( \rho_j \) = Number of cutting pattern \( j \), \( j = 1, \ldots, 6 \)
- \( S_j \) = Amount of scrap generated from cutting pattern \( j \), \( j = 1, \ldots, 6 \)
- \( Q_j \) = Weight of cutting pattern \( j \) (kg), \( j = 1, \ldots, 6 \)
- \( Q_{ML} \) = Total weight of profiles demanded

The different cutting billet patterns of 1.5 m log used in the mathematical model are given in Table 3 along with the total length of the different billet combinations (patterns) and the amount of scrap generated in meters.

**2.3. Example**

As an example, the proposed model II has been used on die number 158B/7 for a demand 441 kg. Detailed mathematical programming formulation is as follows noting that \( Z \) and \( Q \) represent the amount of scrap generated from using two logs:

Minimize \( (Z + Q) \):

Subject to:

\[
\begin{align*}
7.04X_1 + 5.04X_2 + 34.04X_3 + 33.04X_4 + 31.04X_5 + 28.04X_6 + 35.07X_7 + 32.04X_8 + 31.04X_9 + 29.04X_{10} + 26.04X_{11} + 29.04X_{12} + 28.04X_{13} + 26.04X_{14} + 23.04X_{15} + 27.04X_{16} + 25.04X_{17} + 22.04X_{18} + 23.04X_{19} + 20.04X_{20} + 17.04X_{21} - Z &= 0 \\
96X_1 + 98X_2 + 69X_3 + 604X_4 + 72X_5 + 72X_6 + 68X_7 + 71X_8 + 72X_9 + 74X_{10} + 77X_{11} + 74X_{12} + 75X_{13} + 77X_{14} + 80X_{15} + 76X_{16} + 784X_{17} + 81X_{18} + 80X_{19} + 83X_{20} + 86X_{21} - Q &= 0
\end{align*}
\]

END

GIN     X_1
GIN     X_2
GIN     X_3
GIN     X_4
GIN     X_5
GIN     X_6
GIN     X_7
GIN     X_8
GIN     X_9
GIN     X_{10}
GIN     X_{11}
GIN     X_{12}
GIN     X_{13}
GIN     X_{14}
GIN     X_{15}
GIN     X_{16}
GIN     X_{17}
GIN     X_{18}
GIN     X_{19}
GIN     X_{20}
GIN     X_{21}

Objective Function Value: 74.2.
Table 3. Scrap generated from different cutting patterns ($\gamma_j$) of 1.5 m Log (100.5 kg)

| Cutting patterns ($\gamma_j$) | Standard billets | Total length (m) | Total scrap (m) |
|-------------------------------|------------------|-----------------|----------------|
| $\gamma_1$                   | 3                | 1.44            | 0.174          |
| $\gamma_2$                   | 2 1              | 1.48            | 0.134          |
| $\gamma_3$                   | 1 1              | 1.04            | 0.536          |
| $\gamma_4$                   | 1 1              | 1.06            | 0.516          |
| $\gamma_5$                   | 1                | 1.09            | 0.486          |
| $\gamma_6$                   | 1 2              | 1.13            | 0.446          |
| $\gamma_7$                   | 1 1              | 1.04            | 0.536          |
| $\gamma_8$                   | 1 1              | 1.08            | 0.486          |
| $\gamma_9$                   | 1 1              | 1.10            | 0.476          |
| $\gamma_{10}$                | 1 1              | 1.13            | 0.446          |
| $\gamma_{11}$                | 1 1              | 1.17            | 0.406          |
| $\gamma_{12}$                | 2                | 1.12            | 0.456          |
| $\gamma_{13}$                | 1 1              | 1.14            | 0.436          |
| $\gamma_{14}$                | 1 1              | 1.17            | 0.406          |
| $\gamma_{15}$                | 1 1              | 1.21            | 0.366          |
| $\gamma_{16}$                | 2                | 1.16            | 0.416          |
| $\gamma_{17}$                | 1 1              | 1.19            | 0.346          |
| $\gamma_{18}$                | 1 1              | 1.23            | 0.356          |
| $\gamma_{19}$                | 1 2              | 1.22            | 0.316          |
| $\gamma_{20}$                | 1 1              | 1.26            | 0.134          |
| $\gamma_{21}$                | 2                | 1.30            | 0.276          |

Table 4. Comparison of extrusion stage scrap generated by AEC’s conventional method and the proposed mathematical model

| No | Die number   | Weight demanded (kg) | Conventional method | Optimization model |
|----|--------------|----------------------|---------------------|-------------------|
| 1  | 158 B/7      | 441.00               | 275.350             | 74.215            |
| 2  | 861 A/3      | 450.00               | 123.400             | 65.200            |
| 3  | 861 A/3      | 675.00               | 204.350             | 42.280            |
| 4  | 113 D/12     | 396.00               | 226.960             | 119.120           |
| 5  | 7613         | 153.00               | 63.720              | 52.400            |
| 6  | 2211         | 165.00               | 65.510              | 41.080            |
| 7  | 1096         | 259.00               | 185.430             | 49.920            |
| 8  | 1037 A/6     | 2574.00              | 311.360             | 208.080           |
| 9  | 863 A/3      | 888.00               | 76.830              | 64.320            |
| 10 | 126 A/B      | 244.80               | 328.120             | 205.900           |
| 11 | 463          | 130.05               | 196.400             | 146.560           |
|    | Total Scrap  | 2034.99              |                     | 1145.105          |

Total Scrap (kg.) = 100-total weight + 2.5( number of billets used)

3. RESULTS

Table 3 provides the amount of scrap generated by different cutting patterns. The amount of scarp generated is less than one meter. As shown pattern 3 produces the 0.536 meters, while patterns 2 and 20 produce around 0.134 meters which is the least. Table 4 presents the amount of scrap generated resulted for ten Dies along with the total weight demand. The amount of scrap generated using the conventional method used in the company is compared to that using the optimization model. The maximum scrap is generated by Die 1037 A/6.
with an amount of 208.08 kg while the least is produced by 2211 with around 41 kg.

4. DISCUSSION

When comparing the total amount of scrap generated for the different Dies as shown in Table 4, it is found that it is around 56% on average. The total scrap generated produced using the conventional procedure is around 2035 cm whereas it is around 1145 using the suggested optimization model.

5. CONCLUSION

The aluminum extrusion process produces a sizable amount of scrap; this mainly attributed to the techniques used and the lack of modern scientific experience of the staff. In order to reduce the large amount of scrap, a thorough evaluation of the exiting cutting method should be carried out. In addition to reduce scrap, an effective and optimal cutting method will contribute to efficient uses of time and other resources. For example the use of the mathematical developed, the reduction of the amount of scrap generated ranged from 21-82% and this constitutes large saving.

Efficient scientific approaches and tools should be used in the different processes within industries; optimization techniques are one of the strong tools that produce good results. It is recommended that the extrusion company should substitute the existing methods with more cost effective approaches which are based on sound scientific ones.

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