About a possibility of measuring the central temperature of the Sun through the regeneration of the $^7$Be neutrinos in the Earth

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Abstract. The solar neutrino’s $^7$Be line ($E_\nu = 0.862$ MeV) has a width of order the temperature in the center of the Sun ($\sim 1$ keV). The regeneration of neutrinos from remote structures of the Earth is suppressed due to the averaging of the effect over the width of the $^7$Be line (oscillation dying effect). We discuss a possibility of measuring the width of the beryllium line at large liquid scintillator detector (LENA) by measuring the electron neutrino flux at different nadir angles of neutrino trajectories.

1. Introduction

The energy profile of $^7$Be neutrinos from the Sun was discussed in [1]. It was found that the $^7$Be neutrinos line is broadened and has asymmetric form (Fig. 1). The left wing of the beryllium neutrinos energy profile (from the peak $E \sim 862.27$ keV) is determined by the Doppler shift caused by thermal velocities of $^7$Be nuclei and the right wing is mainly determined by the Sun’s internal temperature (averaged over $^7$Be neutrinos production region). In [1] it was proposed to measure Sun’s central temperature by measuring the shift of the peak from the laboratory value ($E \sim 861.64$ keV). It was also mentioned the influence of width of the $^7$Be line on vacuum oscillations.

In the present work we propose a new way to determine the Sun’s central temperature. We take into account the broadening of the $^7$Be line due to the central temperature of the Sun and the regeneration process of neutrinos in the Earth.

Let us consider a system of two mixed neutrinos with values of oscillation parameters $\Delta m^2$ and $\sin^2 2\theta$. The electron neutrino produced in the center of the sun is adiabatically converted to a combination of the mass eigenstates $\nu_1$ and $\nu_2$ which is determined by the mixing angle, $\theta_m$, in the production point: $\nu_e \rightarrow \cos \theta_m \nu_1 + \sin \theta_m \nu_2$. The angle $\theta_m$ is given by: $\tan^2 2\theta_m = \tan^2 2\theta \left(1 - \frac{2V_0^2 E}{\Delta m^2 \cos 2\theta}\right)^{-2}$, where $E$ is the neutrino energy, $V_0$ is the matter potential for neutrinos in the production point. $V_0 = \sqrt{2} G_F N_e$, $G_F$ is the Fermi coupling constant and $N_e$ is the electron number density; $V_0 = V_e(N_e^0)$. The conversion effect should be averaged over the neutrino production region, and in what follows we will describe this averaging.
by the effective mixing angle \( \theta^0_m \). Due to loss of coherence, neutrinos arrive at the surface of the Earth as incoherent fluxes of \( \nu_1 \) and \( \nu_2 \) with relative admixtures given by \( \cos^2 \theta^0_m \) and \( \sin^2 \theta^0_m \) correspondingly.

Let us consider oscillations inside the Earth. The probability to find \( \nu_e \) in the detector can be written as:

\[
P = \cos^2 \theta^0_m P_{1e} + \sin^2 \theta^0_m P_{2e} = \cos \theta^0_m P_{1e} + \sin \theta^0_m, \tag{1}
\]

where \( P_{1e} \) and \( P_{2e} \) are the probabilities of \( \nu_1 \rightarrow \nu_e \) and \( \nu_2 \rightarrow \nu_e \) transitions in matter of the Earth correspondingly. We can write \( P \) as:

\[
P = P^0 + \Delta P, \quad P^0 = \frac{1}{2} (1 + \cos 2\theta^0_m \cos \theta), \quad \Delta P = \cos \theta^0_m (P_{1e} - P^0_{1e}). \tag{2}
\]

Here \( P^0 \) is the probability to find \( \nu_e \) during the day time and \( \Delta P \) is the change of the probability due to the Earth’s matter effect during the night time. For beryllium neutrinos the matter effect of the Earth is determined by a small parameter: \( \epsilon \equiv \frac{2\nu \cdot E}{\Delta m^2} \approx 3.6 \cdot 10^{-3} \left( \frac{2.76}{\text{cm}^2} \right) \left( \frac{5.10^{-5}eV^2}{\Delta m^2} \right) \left( \frac{V_{\text{atm}}}{0.5} \right) \) which characterizes deviations of the mixing angle and the oscillation length, \( L_e \approx 43 \left( \frac{5.10^{-5}eV^2}{\Delta m^2} \right) \text{km}, \) in medium from their vacuum values.

As it was shown in [2] the probability of \( \nu_1 \rightarrow \nu_e \) oscillations equals

\[
P_{\nu_1 \rightarrow \nu_e} = \cos^2 \theta - \frac{1}{2} \sin^2 2\theta \int_0^L dx \ V(x) \sin \phi^m_{x \rightarrow L}, \tag{3}
\]

where

\[
\phi^m_{x \rightarrow x_n} = \int_{x_n}^{x} dx \ \Delta_m(x), \quad \Delta_m(x) = \frac{\Delta m^2}{2E} \sqrt{(\cos 2\theta - \epsilon(x))^2 + \sin^2 2\theta}. \tag{4}
\]

Finally we present the relative change of the electron neutrino flux as:

\[
\frac{\Delta P}{P^0} = -f(\Delta m^2, \theta) \frac{1}{2} \int_0^L dx \ V(x) \sin \phi^m_{x \rightarrow L}, \quad \text{where} \quad f(\Delta m^2, \theta) = \frac{2 \cos \theta^0_m \sin 2\theta}{1 + \cos \theta^0_m \cos 2\theta}. \tag{5}
\]

The dependence of \( f(\Delta m^2, \theta) \) on \( \tan^2 \theta \) for \( \Delta m^2 \) is shown in Fig. 2. According to Fig. 2 the function \( f(\Delta m^2, \theta) \) has a maximum \( f_{\text{max}} \approx 0.4 \) at about \( \tan^2 \theta = 0.3. \)

Let us comment on a possible effect of the third neutrino. We consider the scheme in which \( \nu_3 \) is separated from two others by the mass gap \( \Delta m^2_{\text{atm}} \sim 2 \cdot 10^{-3} \text{ eV}^2 \). (We assume that this

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**Figure 1.** The energy spectrum of the solar \( ^7 \text{Be} \) neutrinos. \( q_{\text{lab}} = 0.86184 \text{ MeV} \). This graph is taken from [1].

**Figure 2.** The dependence of \( f(\Delta m^2, \theta) \) on \( \tan^2 \theta \) for \( \Delta m^2 = 5 \cdot 10^{-5} \text{ eV}^2, 7 \cdot 10^{-5} \text{ eV}^2 \) and \( 10^{-4} \text{eV}^2 \) [3].
The relative change of the electron neutrino flux must be averaged over neutrino energy

\[ A_e = \int dE' g(E', E) \frac{\Delta P}{P_0}. \] (6)

Let us, for illustration, consider a Gaussian energy profile

\[ g(E, E') = \frac{1}{\sqrt{2\pi} \Gamma_{Be}} e^{-\frac{(E - E')^2}{2(\Gamma_{Be})^2}}, \]

where \( \Gamma_{Be} \approx 2 \text{ keV} \). We may write the averaged relative change of the electron neutrino flux as:

\[ A_e = -f(\Delta m^2, \theta) \frac{1}{2} \int_0^L dx \frac{V(x)}{F(L - x)} \sin \phi^m_{L \rightarrow x_f}, \] (7)

where \( F(L - x) = e^{-\frac{2(\Gamma_{Be})^2(l_\nu - x)^2}{E^2 l^2}} \).

According to eq. (7) the relative change of the probability becomes insensitive from matter density profiles over distances \( \sim 4200 \text{ km} \) from the detector.

The relative change of the electron neutrino flux \( (A_e = \frac{\Delta P}{P_0}) \) as function of the nadir angle of the neutrino trajectory is illustrated in Figs. 3 and 4. The solid lines in the plots are the averaged effect over the spectrum of the \(^7\text{Be} \) neutrinos and the dotted lines are without averaging.

The key observation is that the period of oscillation probability in the energy scale \( \Delta E_T \) is comparable with \( \Gamma_{Be} \):

\[ \Delta E_T \sim \Gamma_{Be} \] (8)

The period is defined by

\[ \frac{d\phi}{dE} \Delta E_T = 2\pi, \] (9)

where \( \phi = 2\pi L/l_\nu \) is the oscillation phase and \( L \) is the length of neutrino trajectory in the Earth. Explicitly we find from (9)

\[ \Delta E_T = E_{l_\nu} \frac{l_\nu}{L}. \] (10)

With decrease of \( L \) (increase of \( \omega \)) the period \( \Delta E_T \) increases and averaging becomes weaker as one can see in Figs. 3 and 4. The averaging becomes stronger with decrease of \( \omega \), that is, for deeper trajectories.

It is the feature (8) which gives the sensitivity to the width of line. Indeed, if \( \Delta E_T \ll \Gamma_{Be} \) one would see completely averaged oscillation effect. In the opposite case \( \Delta E_T \gg \Gamma_{Be} \) the averaging is negligible and sensitivity to the width is lost. So at the condition (8) we deal with partial averaging of oscillations in the Earth.

3. Conclusions

In the present work we propose to measure the central temperature of the Sun via measuring the change of the electron neutrino flux at the future large neutrino detectors like LENA [5]. Also at such detectors one may determine solar \( \Delta m^2 \) with precision less than one percent.

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Figure 3. The relative change of the electron neutrino flux \( A_e = \frac{\Delta P_e}{P_e} \) as function of the nadir angle of the neutrino trajectory. Solid line is the averaged effect over the spectrum of the \(^7\text{Be}\) neutrinos and dotted line without averaging. There are about 300 “oscillations”. \( \tan^2 \theta = 0.3 \) and \( \Delta m^2 = 5 \cdot 10^{-5}\text{eV}^2 \).

Figure 4. The relative change of the electron neutrino flux for mantle crossing trajectories \( \omega = 0.58 \ldots .65 \) and \( 1.3 \ldots .1.35 \). Solid line is the averaged effect over the spectrum of the \(^7\text{Be}\) neutrinos and dotted line without averaging. \( \tan^2 \theta = 0.3 \) and \( \Delta m^2 = 5 \cdot 10^{-5}\text{eV}^2 \).

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