Anomaly Inflow at Singularities

Julie D. Blum

Department of Physics, University of California at San Diego
9500 Gilman Drive, La Jolla, CA 92093 USA

Abstract

Many noncompact Type I orbifolds satisfy tadpole constraints yet are anomalous. We present a generalization of the anomaly inflow mechanism for some of these cases in six and four dimensions.
1. Introduction

The purpose of this note is to argue that the anomaly inflow mechanism [1] in the context of gauge and gravitational defects [2][3] can be extended to the singular case where the scale size of these defects vanish. We will consider noncompact type I orientifolds which are consistent and cancel tadpole anomalies. In type II in six dimensions, the interchange of sources of curvature and sources of the field strength of the NS antisymmetric tensor (NS fivebranes) under T-duality makes it appropriate to also regard gravitational defects as fivebranes. In type IIA or M theory the theory on the usual fivebranes is anomalous, and a current flows onto the brane from the outside. This current could complicate the proposed relationship [4] between the large $N$ theory of the M theory branes and supergravity on $AdS_7 \times S^4$. Alternatively, one might be able to deduce some nonperturbative correlations from the anomalous coupling similar to the $AdS_5$ case [5].

In [6] it was proposed that in some cases nonperturbative effects allowed one to regard singular gravitational defects of the type I $SO(32)$ theory as NS fivebranes of the heterotic $E_8 \times E_8$ theory and vice versa. We would like to extend some of these ideas to threebranes. In type IIB a gravitational defect of codimension three cannot be considered as a threebrane because there is no anomalous coupling to the Euler characteristic. Since there are no gauge fields, there can be no gauge defects of the appropriate codimension. In type I, however, we can have gauge defects of codimension three in the presence of gravitational defects. The threebrane potential is generally projected out of the theory. Nevertheless, there is the possibility of wrapping fivebranes on two-cycles. There is, thus, the possibility of regarding this kind of singularity as a threebrane. We will first discuss anomaly inflow at singularities of codimension five. Then we will argue, perhaps, naively that anomaly inflow currents of gauge charge can also occur in four dimensions in the presence of the above mentioned singularity.

2. Six Dimensional Anomalies

We will discuss here the noncompact ALE singularities in type I theory [7][8][9][10]. It has been shown [8][9] that tadpole anomaly cancellation implies the cancellation of spacetime anomalies in the fivebrane gauge group of these models. We now wish to show how the anomalies of the ninebrane gauge group are exactly those of a current flowing out of the ALE space and onto the singularity. The net effect is that the full ten-dimensional theory is not anomalous.
In the particular cases that concern us here the twelve-form for the ten-dimensional anomaly factorizes in the form $X_{12} = X_4 X_8$ where the six-dimensional anomaly is derived by descent from $X_8$ and the field strength of the R-R antisymmetric tensor satisfies $dH_{RR} = X_4$ with $X_4 = tr R^2 - tr F^2$ (numerical constants are being ignored here). The integral of $X_4$ gives the bulk contribution of the Euler characteristic $\chi$ minus that of the instanton number $I$ which is integral since $H_{RR}$ is quantized. One significant question is whether the bulk contributions of $\chi$ and $I$ are necessarily equal if $\chi = I$. We want to see whether anomaly inflow can play a role in theories without physical fivebranes. These theories are the $\mathbb{Z}_2$ orbifolds with Wilson lines breaking $SO(32)$ to $SO(16) \times SO(16)$ and the $\mathbb{Z}_{2N+1}$ orbifolds with Wilson lines yielding $SO(16) \times U(8)$.

The $\mathbb{Z}_2$ orbifolds satisfy $I = \chi + F$ while the $\mathbb{Z}_{2N+1}$ orbifolds have the relation $I = \chi + F - \frac{1}{2N+1}$ where $F$ is the number of physical fivebranes on the Coulomb branch (which can be fractional). By comparison with a $\mathbb{Z}_3$ orbifold of K3, we deduce that there is a “standard embedding” for this case which is consistent with there being no spacetime anomaly. This case is the only one with equal bulk contributions of $\chi$ and $I$ that corresponds to a possible compact orbifold and the only one without anomalies. The bulk contribution to $\chi$ for a $\mathbb{Z}_N$ orbifold is $N - 1/N$ while the boundary contribution is $1/N$. For $N > 3$ the bulk contribution to $I$ should be $F + N - 8 - 1/N$, and the boundary contribution should be $8 + 1/N$ for $N$ even and 8 for $N$ odd to be consistent with anomaly cancellation. (There are no anomalies for $F = 8$.) It would be interesting to verify these numbers directly. That there is a net gravitational contribution to the $H_{RR}$ charge for $F = 0$ is consistent with the interpretation of the singularity as fivebranes. When the bulk contribution of $I$ is negative ($F + N < 9$), this contribution could be interpreted as gravitational. In this case one expects the corrections to the gauge theory argued for in [6] to be important, and anomaly inflow currents to be gravitational. Note also that the $H_{RR}$ charge changes sign at $F = 8$. We have not discussed the $\mathbb{Z}_2$ case which would require a negative boundary contribution.

Let us consider one other case. For unbroken $SO(32)$ anomaly considerations lead us to expect that the bulk contribution to $I$ will be $F + 2N - 24 - 1/N$, and the boundary will be $24 - N + 1/N$ for even $N$; and the bulk will be $F + 2N - 24 - 2/N$ and the boundary $24 - N + 1/N$ for odd $N$ which implies that one can cancel the R-R charge in the bulk for $F = 24 - N$ (even $N$) and $F = 24 - N + 1/N$ (odd $N$). The above allows for the construction of a compact K3 for $I = 24$ as expected if the boundary is included in the compactification with added curvature. For the nonabelian orbifolds without Wilson lines,
anomalies also cancel for \( I = 24 \). We will not try to extend this analysis to other choices of Wilson lines but will note that determining the bulk contributions generally will impose restrictions on the possibilities for Wilson lines at compact K3 singularities. In general, there will not be a choice of \( F \) to cancel anomalies and, thus, no possible compactification. We also note that the \( H_{RR} \) charge can change integrally leaving \( I \) constant, and we expect such a change in the transition to the Higgs branch.

Now we can see how anomaly inflow works in these theories. To cancel the anomaly

\[
c_6 \int d^{10} x X_6^1(\Lambda) X_4
\]

we need the counterterm

\[
c_6 \int d^{10} x H_{RR} X_7
\]

where \( \delta X_7 = dX_6^1(\Lambda) \) with \( \Lambda \) a gauge or gravitational parameter and \( c_6 \) is a constant. The integral of \( H_{RR} \) over the boundary of the ALE space is nonzero and quantized showing that there is a current through this boundary with divergence compensating for the anomalous current at the origin given by \((2.1)\). Our crucial assumption was that the bulk can contribute to the \( H_{RR} \) charge beyond the effect of physical fivebranes.

3. Four Dimensional Anomalies

Let us try to extend this analysis to some noncompact orbifolds of codimension three. The discussion in this section will be fairly speculative as there do not seem to be as yet the mathematical results for these orbifolds similar to the ALE ones. Because the tadpole equations generally do not have solutions, we will confine our analysis to \( \mathbb{Z}_N \) orbifolds with \( N \) odd. These cases have also been discussed by [11]. There are solutions of the tadpole constraints for all odd \( N \), but only the cases \( N = 3 \) or \( N = 7 \) which correspond to possible toroidal compactifications give theories without nonabelian gauge anomalies in four dimensions. The gauge group is determined by embedding the orbifold action into Wilson lines that break the ninebrane gauge group \( SO(32) \) to a subgroup. There are no perturbative fivebranes.

Our speculation is that an anomaly inflow mechanism involving nonperturbative fivebranes wrapped on two-cycles is the necessary ingredient to make sense out of these theories. As in six dimensions, the \( H_{RR} \) charge need not vanish since the orbifolds are noncompact. The charge should, however, be quantized. Since these fivebranes are also instantons
of zero scale size \([12]\) in the ninebrane gauge group, there will necessarily be some gauge field strengths of the ninebrane group turned on at the location of the fivebrane. Since the holonomy of a smooth supersymmetric \(N = 1\) compactification to four dimensions must be \(SU(3)\), we might expect that these instantons are embedded in \(SU(3)\) subgroups with the \(SU(3)\) symmetry restored when the instanton has vanishing scale size, and the cycle also has vanishing size.

By wrapping the fivebranes on two-cycles, we obtain a net threebrane charge which normally does not exist in type I theory. Assuming that the theories with these extra fivebranes are anomalous, there are chiral zero modes in four dimensions that contribute to the anomaly. Our main assumption will be that the coupling of the four dimensional anomaly in ten dimensions is determined by the threebrane charge induced from the wrapped fivebranes and is equal to the bulk part of the Dirac index induced from the fivebrane instantons. These fivebranes are all bound to the gravitational defect since there are no free fivebranes in these theories.

The relevant part of the anomaly twelve-form for our consideration of the four dimensional nonabelian gauge anomaly is

\[
X_{12} = \frac{i}{(2\pi)^5} \left( \frac{1}{720} tr F^6 - \frac{1}{24 \cdot 48} tr F^4 tr R^2 \right).
\] (3.1)

Our next assumption is that in constructing these orbifolds there is a clean division between the field strengths due to the induced threebranes in the transverse dimensions and the field strengths of the four dimensional gauge group which should vanish in the transverse dimensions. In that case the twelve-form factorizes as

\[
X_{12} = \frac{8i}{6(2\pi)^2} tr_{G} F^3 \times \frac{1}{(2\pi)^3 48} (tr_{G'} F^3 - \frac{1}{8} tr_{G'} F tr R^2)
\] (3.2)

where the first term is proportional to the anomaly polynomial for the four-dimensional nonabelian gauge anomaly, and the integral of the second term denoted \(X_6\) is the bulk part of the Dirac index in six dimensions. Here \(G\) is the four-dimensional gauge group and \(G'\) is the gauge group in the transverse dimensions with expectation values induced by the wrapped fivebranes.

With the above assumptions, we can see how anomaly inflow works in four dimensions. The anomaly derived from \(X_{12}\) is

\[
c_4 \int d^{10} x X_4 (\Lambda) X_6
\] (3.3)
so we need a counterterm

\[ c_4 \int d^{10}x G_5^G' \omega_5^G \quad (3.4) \]

where \( dG_5^{G'} = -X_6 \) with \( G_5^{G'} = dA_4 - \omega_5^{G'} \), \( \delta \omega_5^G = dX_4(\Lambda) \), and \( c_4 \) is a constant. Here, \( A_4 \) is the ten-dimensional dual of the four-form obtained by reducing the six-form that couples to the fivebrane on a two-cycle. We obtain a gauge invariant five-form field strength by allowing this \( A_4 \) to transform under \( G' \) gauge (or gravitational) transformations and adding a \( \omega_5^{G'} \) with \( d\omega_5^{G'} = X_6 \). The variation of the counterterm under four-dimensional gauge transformations induces a current transverse to the defect whose divergence cancels that of the current at the location of the “threebranes” (3.3).

Although we cannot analyze this issue here, it is plausible that in resolving the singularities the threebrane charge changes integrally such that there are phase transitions similar to [13][8] where the fivebrane charge changes. The “T-dual” of this mechanism may have relevance to the supergravity/gauge theory correspondence [4]. In fact, fourbranes obtained by wrapping sevenbranes on three-cycles similar to the constructions of [14] could participate in anomaly inflow at anomalous orbifolded orientifolds [15][16][11][17]. (One could also obtain potentially anomalous axion strings by wrapping sevenbranes on five-cycles.) In closing, we emphasize that we have made large assumptions in deriving these models of anomaly inflow that need to be studied in a more mathematical framework. However, these models give new life to a whole class of gauge theories that are truly string theories.

This work was supported in part by a UCSD contract.
References

[1] C. Callan and J. Harvey, Nucl. Phys. B250 (1985) 427.
[2] J. Blum and J. Harvey, Nucl. Phys. B416 (1994) 119.
[3] J. Blum and J. Harvey, unpublished.
[4] J. Maldacena, hep-th/9711200.
[5] T. Banks and M. Green, hep-th/9804170.
[6] J. Blum, hep-th/9712233.
[7] M. Douglas and G. Moore, hep-th/9603167.
[8] K. Intriligator, Nucl. Phys. B496 (1997) 177.
[9] J. Blum and K. Intriligator, Nucl. Phys. B506 (1997) 223.
[10] J. Blum and K. Intriligator, Nucl. Phys. B506 (1997) 199.
[11] Z. Kakushadze, hep-th/9803214; hep-th/9804184.
[12] E. Witten, Nucl. Phys. B460 (1996) 541.
[13] P. Aspinwall, Nucl. Phys. B496 (1997) 149.
[14] E. Witten, hep-th/9805112.
[15] S. Kachru and E. Silverstein, hep-th/9802183.
[16] O. Aharony, Y. Oz, and Z. Yin, hep-th/9803051.
[17] A. Fayyazuddin and M. Spalinski, hep-th/9805096.