Mode excitation by an antenna in global gyrokinetic simulations

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Abstract. In order to get a better understanding of the linear and non-linear plasma interaction of microinstabilities and associated turbulence with different specific modes, an antenna is implemented in the global gyrokinetic code ORB5. It consists in applying an external perturbation to the plasma to excite various types of modes and study their coupling with the rest of the system. The contributions of the antenna and plasma perturbed fields are considered separately and, optionally, the plasma response can be linearized by neglecting the perturbed plasma field contribution in the particle orbits.

As a proof of principle, we apply stationary $E \times B$ flows and measure their impact on the linear growth rate of ion temperature gradient (ITG) -driven instability. We distinguish zero and finite shearing rate radial profiles. In the latter case, we show that sheared $E \times B$ flows can destabilize ITG modes.

Future applications include the study of time-dependent zonal structures coupling with electrostatic instabilities, the radial propagation of avalanche-like structures, or the excitation of Alfvén eigenmodes.

1. Introduction
In order to disentangle the coupling between a specific set of modes and other ones in fusion plasma, we want to be able to apply external perturbations to the system. For this purpose, we introduce an antenna in the global gyrokinetic particle-in-cell code ORB5 [1]. It can be an external perturbation of charge and current density or, alternatively, electrostatic and magnetic potentials. The amplitude of this perturbation is of the order of the self-consistent plasma fluctuations.

Thanks to the numerous physical features in ORB5 (including electromagnetic fluctuations [2, 3], arbitrary order quasi-neutrality field solver [4], drift-kinetic electrons [5], multi-species collisions [6], sources [7], ...), the antenna opens a broad field of studies. For instance, the question of time-dependent $E \times B$ flows effectiveness in saturating turbulence [8] can be addressed. The antenna can also be used to excite zonal structures, like geodesic acoustic modes (GAMs, [9–11]) and zero-frequency zonal flows (ZFZFs, [12, 13]) and study their coupling with microinstabilities.
Another application is to investigate on the origin of non-linear zonal structures (avalanche-like features) observed to propagate at a frequency close to the one of GAMs [14]. On the electromagnetic side, the investigation of the excitation of Alfvén eigenmodes, like toroidal Alfvén eigenmodes (TAEs, [15,16]), is of strong interest to improve energetic particle confinement [17].

In this paper, we benchmark the well-known case of stationary applied $E \times B$ flows in linear phase [18–22]. The formalism is more general as it allows for time-dependent electromagnetic antenna perturbation in non-linear regime.

The paper is organized as follows. Section 2 describes the model of ORB5’s antenna. Section 3 presents the results of the simulations, with the system setup in subsection 3.1, a convergence study in subsection 3.2, a scan of the mode number in subsection 3.3, and scans of antenna’s amplitude with zero shearing rate in subsection 3.4 and finite shearing rate in subsection 3.5.

2. Model

ORB5’s ordering between the geometry-related small parameter $\epsilon_B$ and the electromagnetic fluctuation related small parameter $\epsilon_\delta$ reads

$$O(\epsilon_B) = O(\epsilon_\delta^2),$$

where

$$\epsilon_B = \rho_{th} \left| \frac{\nabla B}{B} \right|,$$

and

$$\epsilon_\delta = k_\perp \rho_{th} \frac{e\phi}{T_i}.$$

$\rho_{th}$ is the ion thermal Larmor radius, $|\nabla B/B|^{-1}$ is the length scale of the background magnetic field variation, $k_\perp$ is the fluctuation perpendicular wave number, $e$ is the elementary charge, $\phi$ is the fluctuating electrostatic potential, and $T_i$ is the ion temperature.

In ORB5, the distribution function $f(X,v_\parallel,\mu,t)$, where $X$ is the guiding center position, $v_\parallel$ is the velocity component parallel to the background magnetic field, $\mu$ is the magnetic momentum, and $t$ is the time, is decomposed as a time-independent background $f_0$ and a fluctuating part $\delta f$:

$$f = f_0 + \epsilon_\delta \delta f.$$

$\delta f$ is discretized with numerical markers, while $f_0$ is an analytical function satisfying $\{f_0,H_0\} = 0$, where $\{\ldots\}$ denotes Poisson bracket defined in eq. (15) of [23], and $H_0$ is the extended unperturbed Hamiltonian of the system, given by

$$H_0 = \frac{p_z^2}{2m_s} + \mu B - w$$

where $p_z = m_s v_\parallel + \epsilon_\delta q_s A_\parallel$ is the canonical gyrocenter momentum, $m_s$ is the species mass, $q_s$ is the species charge, $A_\parallel$ is the parallel component of the fluctuating magnetic potential, $B$ is the norm of the background magnetic field, and $w$ is the canonically conjugated variable of $t$.

The contribution of this work is to add external fields $\phi_{\text{ant}}$ and $A_{\parallel,\text{ant}}$ of the same order as $\phi$ and $A_\parallel$ in this formalism, i.e. $O(\epsilon_B) = O(\epsilon_{\text{ant}}^2)$ with $\epsilon_{\text{ant}} = k_\perp \rho_{th} e\phi_{\text{ant}}/T_i$.

One can introduce those fields in a Hamiltonian of first order in $\epsilon_{\text{ant}}$

$$H_{1,\text{ant}} = q_s \left( \phi_{\text{ant}} - A_{\parallel,\text{ant}} \frac{p_z}{m_s} \right).$$
similar to the Hamiltonian of first order in $\epsilon$ 

$$H_{1,\delta} = q_s \left\langle \phi - A \frac{p_z}{m_s} \right\rangle$$

(7)

where $\langle \cdot \rangle$ denotes the gyroaveraging operator.

Those Hamiltonians are used in Vlasov equation to evolve $\delta f$ in absence of sources and collisions:

$$\frac{d\delta f}{dt} = - \epsilon \{ f_0, H_{1,\delta} \} - \epsilon_{\text{ant}} \{ f_0, H_{1,\text{ant}} \}$$

(8)

$$= - \dot{X} \cdot \nabla f_0 - \dot{p}_z \frac{\partial f_0}{\partial p_z}$$

(9)

where the characteristics are given by

$$\dot{Z} = \{ Z, H_0 \} + \epsilon \{ Z, H_{1,\delta} \} + \epsilon_{\text{ant}} \{ Z, H_{1,\text{ant}} \} \quad \forall Z = X, p_z.$$  

(10)

The explicit form reads

$$\dot{X} = \frac{\hat{b}}{q_s B_0} \times \nabla \left( \mu B + \epsilon q_s \left\langle \phi - A \frac{p_z}{m_s} \right\rangle + \epsilon_{\text{ant}} q_s \left\langle \phi_{\text{ant}} - A \frac{p_z}{m_s} \right\rangle \right)$$

$$+ \frac{B^*}{B_0} \left( \frac{p_z}{m_s} - \epsilon q_s \frac{A_0}{m_s} \frac{A}{m_s} \right) - \epsilon_{\text{ant}} q_s \frac{A_{\text{ant}}}{m_s}$$

(11)

$$\dot{p}_z = - \frac{B^*}{B_0} \cdot \nabla \left( \mu B + \epsilon q_s \left\langle \phi - A \frac{p_z}{m_s} \right\rangle + \epsilon_{\text{ant}} q_s \left\langle \phi_{\text{ant}} - A \frac{p_z}{m_s} \right\rangle \right).$$

(12)

When evolving the characteristics, linear simulations are neglecting the term in $\epsilon_{\delta}$ in eq. (10) in front of the first one, but the term in $\epsilon_{\text{ant}}$ is kept in order not to kill the effect of the antenna on the evolution of the self-consistent fluctuation. In other words, we linearize the self-consistent plasma fluctuation but not the external perturbation.

3. Simulations

3.1. ITG setup

We consider a hydrogen plasma of major radius $R_0 = 1.19$ m, minor radius $a = 0.21$ m, and on-axis magnetic field $B_0 = 1$ T.

The pseudo-safety factor is $q(\rho) = 1.25 + 3\rho^2/a^2$, where $\rho$ is the geometric radial coordinate. The associated safety factor is

$$q(\psi) = \frac{\bar{q}(\rho)}{2\pi} \int_0^{2\pi} d\theta \frac{1}{1 + \frac{\rho}{R_0} \cos \theta}$$

(13)

We assume constant density gradient, and temperature profiles

$$T_i(s) = T_e(s) = T_0 e^{-\frac{2\pi}{T_e} \tanh \left( \frac{s - s_0}{T_e} \right)}$$

(14)

where $T_0 = 1$ keV, $\Delta_T = 0.31$, $L_T = 0.77$, and $s_0 = 0.617$. $s$ is the radial coordinate defined by $s := \sqrt{\psi/\psi_{\text{edge}}}$, where $\psi$ is the poloidal flux.

Safety factor and temperature profiles are shown in figure 1.
Figure 1. Safety factor and temperature profiles.

Quantities will be normalized with respect to profiles evaluated at $s = s_0$, in particular the ion sound speed $c_s = \sqrt{T_0/m_i}$, and the ion sound radius $\rho_s = c_s/\Omega_i$, where $\Omega_i = q_iB_0/m_i$ is the gyrofrequency.

The value of $\rho^* := \rho_s/a$ is equal to $1/65$.

For this study, the approximations are as follows. Magnetic equilibrium has circular concentric flux surfaces. Electrons are considered as adiabatic. Only the electrostatic fluctuations are taken into account ($A_\parallel = A_\parallel^{\text{ant}} = 0$). Quasi-neutrality equation uses the long wavelength approximation ($k_\perp \rho_{th} \ll 1$). Finally, we use the linearized equations of motions as described in section 2.

The antenna will consist in a purely radial, stationary electrostatic potential $\phi_{\text{ant}}(s)$ with an associated shearing rate given by [24]

$$\omega_{E_{\text{ant}} \times B} = \frac{s}{q} \frac{d\psi}{ds} \frac{d^2 \phi_{\text{ant}}}{d\psi^2}. \quad (15)$$

Unless specified, the main numerical parameters are $N_p = 10^6$ markers, $N_s \times N_\chi \times N_\varphi = 64 \times 128 \times 64$ cells, cubic splines, a time step $\Delta t = 20 \Omega_i^{-1}$, and a toroidal wave number $n = 6$.

3.2. Convergence study

We use the power balance derived in [23]:

$$\frac{dE_{\text{pot}}}{dt} = - \frac{dE_{\text{kin}}}{dt} \quad (16)$$

with

$$E_{\text{kin}} = \sum_{s=1} f H_0 dV dW, \quad (17)$$

$$E_{\text{pot}} = \frac{1}{2} \sum_{s} \int f H_{1,\delta} dV dW \quad (18)$$

where $\int dV dW$ denotes the integral over phase space.

From the same reference, we will use the relation

$$\frac{dE_{\text{kin}}}{dt} = - \int f \nabla H_{1,\delta} \cdot \dot{\mathbf{X}} \bigg|_0 dV dW, \quad (19)$$
where $\dot{X}\big|_0 = \{X, H_0\}$, to compute the instantaneous growth rate of the instability:

$$\gamma_{\text{kin}} = -\frac{1}{2E_{\text{pot}}} \frac{dE_{\text{kin}}}{dt}. \quad (20)$$

In this particular case of purely static radial antenna, we do not need to add any term to the power balance because $\nabla H_{1,\text{ant}} \cdot \dot{X}\big|_0 = 0$.

The instantaneous growth rate can also be computed by finite difference in time on $\ln(E_{\text{pot}})$ to get

$$\gamma_{\text{pot}} = -\frac{1}{2} \frac{d\ln(E_{\text{pot}})}{dt}. \quad (21)$$

The convergence of the $L^1$ norm of $\gamma_{\text{pot}} - \gamma_{\text{kin}}$ with respect to marker resolution is shown in figure 2.

![Figure 2. Convergence of the error between $\gamma_{\text{pot}}$ and $\gamma_{\text{kin}}$ averaged in time with respect to number of markers $N_p$, with and without antenna.](image)

As expected, the error converges as $1/\sqrt{N_p}$, with a respective value of 5% and 3% with and without antenna for $10^6$ markers. This difference is due to the value of the average growth rate itself which is different in between those two cases. For $N_p \gtrsim 10^7$, the convergence slows down because other discretization errors start to dominate, such as grid resolution and time step.

From now on, $\gamma$ will be used for $\gamma_{\text{kin}}$.

### 3.3. Dispersion relation

The instantaneous growth rate for $n = 6$ is shown in figure 3, with and without antenna.

After the transient phase, $\gamma$ oscillates around $(3.44 \pm 0.15) \cdot 10^{-2} c_s/a$ without antenna and around $(1.78 \pm 0.86) \cdot 10^{-2} c_s/a$ with the antenna. This behavior of highly variable $\gamma$ with the antenna is characteristic from competition between two modes of close growth rate.

The average and standard deviation values of $\gamma$ as a function of $n$ are reported in figure 4.

Mode numbers $2 \leq n \leq 11$ are unstable. Without antenna, the most unstable one is $n = 6$. Standard deviation values indicate that mode competition occurs at large values of $n$, in particular with the antenna and when marginal stability is approached.

For this specific antenna profile, all modes are stabilized, but subsection 3.5 will show that it is not always the case.
3.4. Zero shearing rate profile

In this subsection, we use a constant profile of $\frac{d\phi_{\text{ant}}}{d\psi}$, so that according to eq. (15), $\omega_{E_{\text{ant}} \times B}(s) = 0$.

Figure 5 indicates that the mode is stabilized by any value of the $E \times B$ flow, and that the flow direction does not matter much. The ITG-driven instability gets killed for $|v_{E_{\text{ant}} \times B}| \gtrsim 4 \cdot 10^{-2} c_s$, which is in good agreement with [18].

Figure 6 shows that radially elongated structures are not tilted (the poloidal position where the mode structure is radially aligned does not depend on $v_{E_{\text{ant}} \times B}$). The stabilization is rather due to the shift of the position of maximum mode amplitude towards more favorable $\nabla B$ region.

**Figure 3.** Time traces of linear growth rate $\gamma$ of mode $n = 6$ with and without antenna.

**Figure 4.** Average growth rate as a function of the mode number. Shaded areas indicate standard deviation in time.
Figure 5. Growth rate of the mode $n = 6$ as a function of the zero shearing rate flow amplitude. Shaded area indicates standard deviation in time.

Figure 6. Poloidal plane cut of electrostatic potential for negative, null, and positive flow. Red lines indicate the poloidal position where the mode structure is radially aligned.

3.5. Non-zero shearing rate profile
In this subsection, we use a linear finite shearing rate profile with a null $E \times B$ drift at the peak gradient position:

$$\left\{ \begin{align*}
\omega_{E \times B}(s) &= \frac{\alpha}{\alpha_0} \omega_{E \times B}(s_0), \\
v_{E \times B}(s_0) &= 0.
\end{align*} \right. \quad (22)$$

Figure 7 states that the ITG-driven instability can be killed for values of $\omega_{E \times B}(s_0) \gtrsim 7 \cdot 10^{-2} c_s/a \approx 2\gamma(\phi_{ant} = 0)$, that is to say a shearing rate of the order of the growth rate without antenna, in agreement with [20]. Conversely, the instability can be further destabilized by values of shearing rate $-10^{-1} c_s/a < \omega_{E \times B}(s_0) < 0$. The explanation comes with figure 8, which shows how radially elongated structures get tilted by the sheared flow. The most unstable case is shown in subfigure 8b, in which mode structure is radially aligned at the least favorable $\nabla B$ region. In that case, antenna’s shearing rate is assumed to compensate diamagnetic one.
Figure 7. Growth rate of the mode $n = 6$ as a function of the shearing rate. Shaded area indicates standard deviation in time.

$$\omega_{E\text{ant}} \times B = \begin{cases} 
\text{a. } -0.13 \, c_s/a \\
\text{b. } -0.056 \, c_s/a \\
\text{c. } +0.037 \, c_s/a
\end{cases}$$

Figure 8. Poloidal plane cut of electrostatic potential for different values of the shearing rate. Red lines indicate the poloidal position where the mode structure is radially aligned.

4. Discussion
ORB5’s antenna has been validated in electrostatic, linear regime. Convergence of the power balance has been verified. In agreement with [18, 19], two different mechanisms of ITG (de)stabilization have been recovered. The first one is the poloidal shift of the mode towards more or less favorable $\nabla B$ regions with a zero shearing rate $E \times B$ profile. The second one, with a finite shearing rate $E \times B$ profile, is the tilt of the radially elongated structures which moves the poloidal position where the mode structure is radially aligned towards more or less favorable $\nabla B$ regions.

This achievement opens the door to novel studies. As a first step, one could apply non-stationary $E \times B$ sheared flows to study their effect on mode stabilization, and eventually saturation of associated turbulence. Another application could be to excite zonal structures, like GAMs or ZFZFs, at the plasma boundary and study their radial propagation. Alternatively, an ITG-like perturbation could be imposed to measure the non-linear zonal response of the plasma. Finally, an electromagnetic perturbation could be used to excite Alfvén eigenmodes and study their coupling with microinstabilities.
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