Among dark-matter candidates are the WIMPs (Weekly Interacting Massive Particles). Low-threshold detectors could directly detect dark-matter by measuring the energy deposited by the particles. In this work we examine the cross section for the elastic scattering of WIMPs on nucleons, in the spin-dependent and spin-independent channels. WIMPs are taken as neutralinos in the context of the minimal super-symmetric extension of the standard model (MSSM). The dependence of the results with the adopted MSSM parameters is discussed.

1. Introduction

The Universe is mostly composed by dark matter (DM), which does not shine but interacts gravitationally with baryonic matter. The first evidence about the existence of such kind of matter were proposed by Zwicky in the 30s and in the 70s by Rubin et al. Measurements of the Cosmic Microwave Background (CMB) and of the gravitational-lensing effects have produced new evidences about the existence of dark matter.

Dark matter may be produced by thermal processes, such as the collision of plasmas, and non-thermal processes, such as particle decays. The mass of the dark matter particles is unknown. It can be as large as $10^{13}$ GeV. There exist several candidates to dark matter, e.g. WIMPZILLA, MACHO, axions, sterile neutrinos, WIMPs.
The WIMPZILLA is the heaviest candidate of all and it could be produced through non-thermal processes after the inflation epoch. These particles could decay in ultra-energetic cosmic rays. The MACHOs (massive astrophysical halo objects) are compact objects of baryonic matter. Some examples are brown-dwarf (mass of the order of $0.08 M_\odot$), Jupiter’s kind objects (mass of the order of $0.001 M_\odot$), white-dwarf, planets and primordial black holes. These objects can be detected through the gravitational micro-lensing effect. MACHOs may represent 20% of the dark matter in the galaxy. Models that considered MACHOs as dominant components of dark matter are ruled out. The baryonic component of dark matter can not explain the results of Planck, therefore there should be non-baryonic dark matter. The axions are bosons introduced to solve the CP-problem with masses of the order of $10^{-9}$ eV. The interaction between axions and photons through the Primakoff effect gives photons that could be detected. The sterile neutrino were proposed as warm dark matter candidate. The mass of these particles should be lower than 1 keV. There are several experimental efforts to set constrains on the mass of this neutrino and on the active-sterile mixing angle.

Weakly interacting massive particles (WIMPs) are the most probably dark-matter candidates. These particles interacts through weak interactions with matter, their mass could be in the range 1 GeV - 10 TeV, however, this mass-range can be reduced to 3 GeV - 20 GeV. Some of these candidates are the heavy photon (little Higgs theories), the Kaluza-Klein photon (multidimensional theories) and the neutralino (super-symmetric theories). Since the evidence suggests that dark matter is massive, stable, cold, and electrically neutral, neutralinos are preferred other than more exotic candidates. Recent measurements of the excess in the electron recoil from XENON1T experiment may be taken as indication about the existence of two-components-exothermic dark matter. The theoretical description implies the use of effective Lagrangians where the interactions of dark matter particles with neutral massive bosons is explicitly taken into account, as done in. There exist two different methods to detect WIMPs: direct and indirect detections. The former aims at measuring the energy deposited by a WIMP when it interacts with the detector, and the latter looks for annihilation or decay products of WIMPs such as gamma rays, neutrinos, and cosmic rays. In this work, we shall focus on the calculation of cross sections relevant for direct detection experiments.

Because of the extremely low signal-to-noise rates, direct-detection experiments need to be performed in low-background conditions, therefore, they are located in underground laboratories. A modulation of the signal by the movement of the Earth in its orbit, phenomenon called annual modulation, is also expected to be measured, as well as the diurnal modulation which is produced by the Earth movement around its axis.

The currently running detectors are located in the northern hemisphere, however there exist projects to settled detectors in the southern hemisphere, like in...
the planned ANDES laboratory in Agua Negra (Argentina)\cite{10} and the SABRE experiment (Australia, not yet operational)\cite{34}. The experimental data, when available, will allow for the comparison between northern and southern located detectors, hopefully.

In this work we have considered the lightest neutralino as a possible candidate for dark matter. We have computed the scattering cross section between the neutralino and protons (neutrons) as a function of different parameters of the MSSM (minimal super-symmetric extension of the standard model). Then, we have compared the theoretical results with the available experimental data.

The paper is organized as follows. In section 2 we provide a brief description of neutralinos’s properties and the formalism needed to compute the neutralino-nucleon cross section. In section 3 we show and discussed the results of the calculations. The conclusions are drawn in section 4.

2. Formalism

In the framework of the MSSM models, each elementary particle has a super-partner with a spin that differs by a half-integer. The mass-matrix of the sector of neutral fermions can be written, in terms of the bino, wino and higgsino mass parameters ($M$, $M'$ and $\mu$ respectively) as \cite{41,42,43,44,45,46}

$$Y = \begin{pmatrix}
M' & 0 & -M_{Zc\beta} s_W & M_{Zs\beta} c_W \\
0 & M & M_{Zc\beta} c_W & -M_{Zs\beta} c_W \\
-M_{Zc\beta} s_W & M_{Zc\beta} c_W & 0 & -\mu \\
M_{Zs\beta} s_W & -M_{Zs\beta} c_W & -\mu & 0
\end{pmatrix},$$

where $c_\beta$ ($s_\beta$) stands for $\cos \beta$ ($\sin \beta$) with $\tan \beta = v_1/v_2$ \cite{45,46,47,48}, $M_Z$ is the mass of the $Z$-boson, $\theta_W$ is the Weinberg angle and $c_{W'}$ ($s_{W'}$) is $\cos \theta_W$ ($\sin \theta_W$). In the Grand-Unified-Theory (GUT), the parameters $M$ and $M'$ are related by $M' = \frac{3}{5} M \tan^2 \theta_W$.

The lightest neutralino state may be written as a linear combination of binos ($\tilde{B}$), winos ($\tilde{W}_3$) and higgsinos ($\tilde{H}_1^0, \tilde{H}_2^0$)

$$\chi^0_1 = Z_{11} \tilde{B} + Z_{12} \tilde{W}_3 + Z_{13} \tilde{H}_1^0 + Z_{14} \tilde{H}_2^0,$$

where $Z_{11}$, $Z_{12}$, $Z_{13}$, $Z_{14}$ are the components of the eigenvector related to the lowest positive eigenvalue of the mass matrix (see Appendix \[A\] for details).

The effective Lagrangian density describing the neutralino-quark elastic-scattering in the MSSM is given by \cite{49}

$$\mathcal{L}_{\text{eff}} = \frac{g^2}{2 M_W} \sum_q \left( \bar{\chi} \gamma^\mu \gamma_5 \tilde{\psi}_q \gamma_\mu A_q \gamma_5 \psi_q 
+ \bar{\chi} \chi S_q \frac{m_q}{M_W} \bar{\psi}_q \psi_q \right).$$

(2)
In Equation (2) \( \psi_q \) represents the quark field, \( \chi \) is the dark matter field, \( g \) is the SU(2) coupling constant and \( M_W \) stands for the mass of the W boson. The axial-vector \((\gamma_{\mu}\gamma_5)\) and scalar terms of it are shown in Fig. 1. The coupling constants, \( A_q \) and \( S_q \) can be written as \[ A_q = -\frac{M_W^2}{M_q^2} \left((T_{3q}Z_{12} - \tan \theta_W(T_{3q} - e_q)Z_{11})^2 \right. \]
\[ + \frac{\tan^2 \theta_W e_q^2 Z_{11}^2}{2M_W^2} \left. + \frac{m_{\tilde{q}}^2 \tilde{q}^2}{M_W^2} \right] \]
\[ + \frac{T_{3q}(Z_{13}^2 - Z_{14}^2)}{2}, \]
\[ S_q = \frac{Z_{12} - \tan \theta_W Z_{11}}{2} \]
\[ \times \left( \frac{M_W^2}{M_{H^0}^2} g_{H^0}^2 k_q^{(2)} + \frac{M_W^2}{M_{H^0}^2} g_{H^0} k_q^{(1)} + \frac{M_W^2 e_{d_q}}{M_q^2} \right), \]
where \( M_{\tilde{q}} \) and \( M_{H^0} \) are the squark and higgsino masses respectively. \( T_{3q}, e_q \) and \( m_q \) are the quark weak isospin, charge and mass respectively, \( \epsilon \) is the sign of the lightest-neutralino mass eigenvalue. The corresponding values of \( d_q \) and \( k_q^{(i)} \) for the up-type and down-type quark are given in Table 1. The constants for the higgsinos are
\[ g_{H^0}^1 = -Z_{13} \cos \alpha + Z_{14} \sin \alpha, \]
\[ g_{H^0}^2 = Z_{13} \sin \alpha + Z_{14} \cos \alpha, \]
\[ \alpha = 0.5 \arctan \left( \frac{m_{ps}^2 + M_Z^2}{m_{ps}^2 - M_Z^2} \tan (2\beta) \right). \]
In the previous equations \( m_{ps} \) is the mass of the Higgs pseudo-scalar.

| quark | \( d_q \) | \( k_q^{(1)} \) | \( k_q^{(2)} \) | \( k_q^{(3)} \) |
|-------|-----------|-----------|-----------|-----------|
| up    | \( \frac{Z_{13}}{\sin \beta} \) | \( \frac{\sin \alpha}{\sin \beta} \) | \( \cos \alpha \) | \( \cos \beta \) |
| down  | \( \frac{-Z_{14}}{\cos \beta} \) | \( \frac{\cos \alpha}{\cos \beta} \) | \( \sin \alpha \) | \( \sin \beta \) |

Table 1. Values of \( d_q \) and \( k_q^{(i)} \) for the up-type and down-type quark adopted for the calculations.

The cross section of the neutralino quark scattering can be computed as
\[ \sigma = V \int \frac{d^3p'}{(2\pi)^3} V \int \frac{d^3l'}{(2\pi)^3 \sum_{s,t,s',t'} W(q_\chi \rightarrow q_\chi) J_\chi V^{-1}.} \tag{3} \]

The integration is performed in the space of outgoing momenta \( p', l' \) of the neutralino and quark, respectively, and the sum is computed over the incoming state.
spins $s$, $t$ and the outgoing state spins $s'$, $t'$. The transition rate per unit volume is

$$W(q \chi \rightarrow q \chi) = \frac{1}{VT} |S(q \chi \rightarrow q \chi)|^2 = \frac{1}{VT} \left| -i \int dt H_{\text{int}}(q \chi \rightarrow q \chi) \right|^2,$$

where $H_{\text{int}}$ is the Hamiltonian and the flux of incoming neutralinos is

$$J_{\chi} = \sqrt{(p_0 l_0^2 - m_\chi^2 m_q^2)}.$$

In the previous equation $m_\chi$ and $p$ are the mass and the incoming momentum of the neutralino, $m_q$ and $l$ are the mass and the incoming momentum of the quark, respectively.

The Eq. (3) is reduced to

$$\sigma = \sum_{s',t'} \frac{1}{8s} \frac{1}{\sqrt{\beta_- \beta_+(2\pi)^2}} \times \int \frac{d^3p'}{p'^3} \frac{d^3l'}{l'^3} \delta^4(p' - p + l' - l)|M|^2,$$

where we have defined $\beta_\pm = \left(1 - \frac{(m_\chi \pm m_q)^2}{s} \right)$ and $s = (p + l)^2$.

For each mediator the expression of the factor $|M|^2$ for the axial-vector ($A - V$)
and scalar ($S$) terms of the interactions is given by

\[ M^{A-V}_Z = C^{A-V}_Z \left[ \bar{\chi} \gamma^\mu \gamma_5 \chi \right] \left[ \bar{\psi}_q \gamma_\mu \gamma_5 \psi_q \right], \]

\[ M^{A-V}_{\tilde{q}} = C^{A-V}_{\tilde{q}} \left[ \bar{\chi} \gamma^\mu \gamma_5 \chi \right] \left[ \bar{\psi}_q \gamma_\mu \gamma_5 \psi_q \right], \]

\[ M^{S}_H = C^{S}_H \left[ \bar{\chi} \chi \right] \left[ \bar{\psi}_q \psi_q \right], \]

\[ M^{S}_{\tilde{q}} = C^{S}_{\tilde{q}} \left[ \bar{\chi} \chi \right] \left[ \bar{\psi}_q \psi_q \right], \]

with

\[ C^{A-V}_Z = \frac{g^2}{4 \cos^2 \theta_W} \left( k^2 - M_Z^2 \right) T_{3q}(Z_{13}^2 - Z_{14}^2), \]

\[ C^{A-V}_{\tilde{q}} = \frac{g^2}{2(s - M_{\tilde{q}}^2)} \left[ (T_{3q} Z_{12} - \tan \theta_W (T_{3q} - e_q) Z_{11})^2 \right. \]

\[ + \left. \tan^2 \theta_W e_q^2 Z_{11}^2 + \frac{2m_q^2 d_q^2}{4M_W^2} \right], \]

\[ C^{S}_H = \frac{g^2 m_q}{4M_W} (Z_{12} - \tan \theta_W Z_{11}) \]

\[ \times \left( \frac{g_{H_2} k_2}{(k^2 - M_{H_2}^2)} + \frac{g_{H_1} k_1}{(k^2 - M_{H_1}^2)} \right), \]

\[ C^{S}_{\tilde{q}} = \frac{g^2 m_q}{4M_W} (Z_{12} - \tan \theta_W Z_{11}) \frac{e d_q}{(s - M_{\tilde{q}})}. \]

In the previous definitions $k$ is the momentum of the mediator. In the following we are going to analyse the axial-vector and scalar channel channels separately.

- **Axial-vector channel**
  The matrix elements for the axial-vector interaction is written as

\[ M_{A-V} = \left( C^{A-V}_Z + C^{A-V}_{\tilde{q}} \right) \left[ \bar{\chi} \gamma^\mu \gamma_5 \chi \right] \left[ \bar{\psi}_q \gamma_\mu \gamma_5 \psi_q \right]. \]

After some algebra (see appendix B.1 for detail), we have obtained

\[ \sum_{s, s', t, t'} |M|_{A-V}^2 = 32 \left( C^{A-V}_Z + C^{A-V}_{\tilde{q}} \right)^2 \]

\[ \times \left[ (p \cdot l)(p' \cdot l') + (p \cdot l')(p' \cdot l) \right. \]

\[ \left. + m_\chi^2 (l \cdot l') + m_q^2 (p \cdot p') + 2 m_q^2 m_\chi^2 \right] \]
Using Eq. (4) the cross section reads

\[\sigma_{A-V} = \frac{32}{8s\sqrt{\beta(2\pi)^2}} \int \frac{d^3p'}{p'^0} \frac{d^3l'}{l'^0} \delta^4(p' - p + l' - l) \]

\[\times \left( \tilde{C}_Z^{A-V} + \tilde{C}_q^{A-V} \right)^2 \]

\[\times [(p \cdot l)(p' \cdot l') + (p \cdot l')(p' \cdot l)] \]

\[+ m_\chi^2(l \cdot l') + m_q^2(p \cdot p') \]

\[+ 2m_q^2m_\chi^2 \]

and after performing the integration it yields

\[\sigma_{A-V} = \frac{1}{\pi} \left( \frac{\tilde{C}_Z^{A-V}}{\lambda_q^2} + \frac{\tilde{C}_q^{A-V}}{s - M_Z^2} \right)^2 \]

\[\times \left\{ \frac{s}{6} \left[ 1 - \frac{m_\chi^2 + m_q^2}{s} + \frac{4m_q^2m_\chi^2}{s^2} \right. \right. \]

\[\left. - \left( \frac{m_\chi^2 - m_q^2}{s} \right)^2 \frac{m_\chi^2 + m_q^2}{s} \right. \]

\[+ \left( \frac{m_\chi^2 - m_q^2}{s} \right)^4 \right\} \]

\[+ \frac{s}{2} \left( 1 - \frac{m_\chi^2 + m_q^2}{s} \right)^2 \]

\[+ \frac{m_\chi^2}{2} \left( 1 - \frac{m_\chi^2 - m_q^2}{s} \right)^2 \]

\[+ \frac{m_q^2}{2} \left( 1 + \frac{m_\chi^2 - m_q^2}{s} \right)^2 + 4m_q^2m_\chi^2 \right\} \]

where \(p_0\) is the energy of the neutralino. We have defined

\[\lambda_q^2 = 2m_q^2 - M_Z^2 - \frac{s}{2} \left( 1 - \frac{m_\chi^2 - m_q^2}{s} \right)^2 \]

\[\tilde{C}_Z^{A-V} = C_Z^{A-V} (k^2 - M_Z^2), \]

\[\tilde{C}_q^{A-V} = C_q^{A-V} (s - M_q^2). \]

- Scalar channel

In this case, the matrix elements is written as

\[M_S = (C_H^S + C_q^S) \left[ \bar{\chi} \right] \left[ \psi_q \right] \]
leading to the result (see appendix B.2)
\[
\sum_{s,s',l,l'} |M|^2_S = 16 \left( C_H^2 + C_q^S \right)^2 \left[ (p \cdot p')(l \cdot l') + m_\chi^2 (l \cdot l') \right] + m_q^2 (p \cdot p') + m_q^2 m_\chi^2 \].
\]
With this result the contribution due to the scalar channel to the cross-section is written as
\[
\sigma_S = \frac{1}{2\pi} \left[ \frac{\tilde{C}_{Hq}^2}{\lambda_H^q} + \frac{\tilde{C}_{q}^S}{\lambda_q^S} \right. \\
\left. + \frac{\tilde{C}_{q}^S}{s - M_q^2} \right]^2 \\
\times \left\{ \frac{s}{6} \left[ 1 - \frac{m_\chi^2 + m_q^2}{s} + \frac{4 m_q^2 m_\chi^2}{s^2} \right. \\
- \left( \frac{m_\chi^2 - m_q^2}{s} \right)^2 \frac{m_\chi^2 + m_q^2}{s} \right. \\
+ \left( \frac{m_\chi^2 - m_q^2}{s} \right)^4 \\
+ \frac{m_\chi^2}{2} \left( 1 - \frac{m_\chi^2 - m_q^2}{s} \right)^2 \\
+ \frac{m_q^2}{2} \left( 1 + \frac{m_\chi^2 - m_q^2}{s} \right)^2 + \frac{2 m_q^2 m_\chi^2}{s} \right\},
\]
where
\[
\lambda_H^q = 2 m_q^2 - M_H^2 - \frac{s}{2} \left( 1 - \frac{m_\chi^2 - m_q^2}{s} \right)^2,
\]
\[
\tilde{C}_{Hq}^S = C_{Hq}^S (k^2 - M_H^2),
\]
\[
\tilde{C}_{q}^S = C_{q}^S (s - M_q^2).
\]
To obtain the cross section of the neutralino-nucleon scattering we express the nucleon fields in terms of the quark fields. For the axial-vector interaction the cross section is written as
\[
\sigma_{A-V}^N = \sum_{q=u,d,s} \sigma_{A-V} \left( \Delta_q^N \right)^2,
\]
where \(\sigma_{A-V}^N\) is the cross section of Eq. (5) and \(\Delta_q^N\) are experimental values that describe the contribution of a quark \(q\) to the spin of the nucleon \(N\) (either a proton \((p)\) or a neutron \((n))\):
\[
\Delta u^p = \Delta d^p = 0.77,
\]
\[
\Delta d^p = \Delta u^n = -0.40,
\]
\[
\Delta s^p = \Delta s^n = -0.12.
\]
The cross section for the scalar channel is given by

$$\sigma_S^N = \sum_{q=u,d,s} \sigma_S \left( \frac{f_{T_q}^N m_N}{m_q} \right)^2 + \sum_{q=e,b,t} \sigma_S \left( \frac{2 f_{T_G}^N m_N}{m_q} \right)^2$$

where $\sigma_S^N$ is the cross section of Eq. (6). The factors $f_{T_q}^N$ (where $N$ denotes either proton or neutron) are the fractions of the nucleon-mass accounted for by a particular quark-flavour, defined as

$$f_{T_p}^p = 0.023, \quad f_{T_d}^p = 0.034,$$
$$f_{T_s}^p = 0.140, \quad f_{T_u}^p = 0.019,$$
$$f_{T_d}^n = 0.041, \quad f_{T_s}^n = 0.140,$$

with $f_{T_G}^N = 1 - \sum_{q=u,d,s} f_{T_q}^N$.

3. Results

In this section we present the results of our calculations of the neutralino-nucleon cross-section as a function of the different parameters of the MSSM. We have extracted the value of the masses from Ref. [48] and taken the squark-mass as 1500 GeV, the Higgs pseudo-scalar mass as 500 GeV and $g_s = 0.645527$. As said before, the form factors $\Delta_{kn}$ and $f_{T_q}^n$ were extracted from Ref. [49]. For the SUSY parameter, we have used $\tan \beta = 34.4583$ and varied the parameter $\mu$ and $M$ in order to obtain different values for the neutralino mass (see Table 2). The parameter $M$ has a lower constraint once the value of $\mu$ is fixed, which came from the fact that the neutralino mass (see Appendix A) is positive defined. We have explored two set of values for the neutralino mass: i) $m_\chi$ greater than 1 GeV; and ii) $m_\chi$ smaller than 1 GeV.

3.1. Results for $m_\chi > 1$ GeV

- Axial-vector contribution to the cross section

In Fig. 2 we present the nucleon-neutralino cross section for the axial-vector contribution as a function of the neutralino energy. The values of the parameters used to obtain the curves shown in the Figure, are listed in Table 2. The cross section for protons and neutrons are practically the same, therefore we have displayed the results for protons. In all the cases, the contribution to the cross section, when the $Z_0$-boson is the mediator, is larger than the contribution of the squark. The dependence on the mass parameter of the higgsino ($\mu$) is noticeable. For larger values of $\mu$ the cross-section becomes smaller.

If the value of $\tan \beta$ is changed the cross-section does not change appreciable, since the increase of the squark contribution due to changes in this parameter (ie; for $\tan \beta = 10$) does not match the contribution due to the $Z_0$ boson as mediator. Concerning the dependence of the cross section with
the quark flavour, we have seen from our results that the larger contribution comes from the strange-quark sector. We have compared our results with the limits determined by the available data \cite{51,38,52,53} and found that, for the values of $\mu$ used in this work, the calculated values are in good agreement with those limits.

- Scalar contribution to the cross section
  The scalar contribution to the cross section is presented in Fig. 3 as a function of the neutralino energy. Once again, the calculated values for neutrons and protons are practically the same. The contributions for all mediators to this channel, are of the same order of magnitude. The dependence on the mass parameter of the higgsino is noticeable. As for the case of the axial-vector terms we have studied the dependence of the cross section with the value of $\tan \beta$. Then, for e.g; $\tan \beta = 10$, the resulting value of the scalar contribution to the cross section decreases by one order of magnitude. At quark level the interaction with the top-quark has the largest cross-section. The squark as a mediator produces the largest cross section for the up, down and strange quarks. If we compared our results with experimental data \cite{39,54} we found some constraints on the value of $\mu$. For a neutralino mass $m_\chi = 60$ GeV the higgsino parameter $\mu$ should be greater than $530$ GeV, and for $m_\chi = 100$ GeV it should be $\mu > 1370$ GeV. For negative values of $\mu$ we have the following limits: $\mu < -193$ GeV for $m_\chi = 60$ GeV, and $\mu < -372$ GeV for $m_\chi = 100$ GeV.

### 3.2. $m_\chi < 1$ GeV

- Axial-vector contribution to the cross section
  The results for the neutralino-nucleon cross section of the axial-vector channel, when the neutralino mass is of order of few MeV, are shown in Fig. 4. The parameters used in the calculations are given in Table 3.

### Table 2. Values of $m_\chi$, $\mu$ and $M$ used in the calculation of the cross section of Fig. 2 and 3 for $\tan \beta = 3$.

| $m_\chi$ [GeV] | $\mu$ [GeV] | $M$ [GeV] |
|---------------|-------------|----------|
| 5             | 100         | 73.57    |
|               | 500         | 19.82    |
|               | 1000        | 14.01    |
| 35            | 100         | 161.40   |
|               | 500         | 76.24    |
|               | 1000        | 72.45    |
| 80            | 100         | 511.24   |
|               | 500         | 166.11   |
|               | 1000        | 162.12   |

The parameters used in the calculations are given in Table 3.
Figure 2. Axial-vector contribution to the cross section as a function of the neutralino energy, $E_\chi$, for nucleon-neutralino scattering. Left column: $m_\chi = 5$ GeV; middle column: $m_\chi = 35$ GeV; right column: $m_\chi = 80$ GeV. Top row: $Z_0$-boson contribution; middle row: squark contribution; bottom row: total axial-vector contribution. Solid line: $\mu = 100$ GeV; dashed line: $\mu = 500$ GeV; dotted line: $\mu = 1000$ GeV. The values for $M$ are listed in Table 2.

Contribution due to the squark as a mediator is the same for protons or neutrons. However, if the mediator is a $Z_0$ boson, the neutralino-proton and neutralino-neutron cross sections are different, the neutron-neutralino cross section is larger than the proton-neutralino one. In both cases (protons and neutrons) the cross section decreases its value when the neutralino mass...
increases. The most important contribution to the axial-vector channel is given by the $Z_0$ boson as mediator.

If the value of $\tan \beta$ is modified (e.g. $\tan \beta = 10$), the contribution due to the squark as a mediator increases its value, but it does not change the total axial-vector contribution. For negative values of $\mu$ the cross section

Figure 3. Scalar contribution to the cross section as a function of the energy of the neutralino. Left column: $m_\chi = 5$ GeV; middle column: $m_\chi = 35$ GeV; right column: $m_\chi = 80$ GeV. From top to bottom row: heaviest Higgs contribution; lightest Higgs contribution; squark contribution; total scalar contribution. Solid line: $\mu = 100$ GeV; dashed line: $\mu = 500$ GeV; dotted line: $\mu = 1000$ GeV. The values for $M$ are listed in Table 2.
Table 3. Used values of $m_\chi$, $\mu$ and $M$ in the computation of the cross section of Fig. 4 and 5 for $\tan \beta = 3$.

| $m_\chi$ [GeV] | $\mu$ [GeV] | $M$ [GeV] |
|----------------|-------------|----------|
| 0.005          | 100         | 61.36    |
| 0.1            | 100         | 61.59    |

Figure 4. Axial-vector contribution to the cross section as a function of the energy of the neutralino, for nucleon-light neutralino scattering. Top row: $Z_0$-boson contribution; middle row: squark contribution; bottom row: total axial-vector contribution. Solid line: $\mu = 100$ GeV, $M = 61.36$, and $m_\chi = 5$ MeV, dotted line: $\mu = 100$ GeV, $M = 61.59$, and $m_\chi = 100$ MeV.

is much smaller that the one obtained with $\mu > 0$. If we considered the cross section between the neutralino and quarks, the largest cross section corresponds to the strange-quark and the smallest to the up-quark.
Scalar contribution to the cross section

In the case of the scalar channel the cross section for protons and neutrons are quite similar. In Fig. 5 we show the neutralino-proton cross section for this case. The three computed cross section for each one of the mediators, that is both Higgs and the squark, are of the same order of magnitude. If both Higgs are the mediator, the cross section increases its value when the neutralino mass is large, however, for the squark as a mediator the cross section is practically constant.

If tan $\beta = 10$ the total scalar contribution decreases its value one order of magnitude with respect to the previously presented case. Once again, the contribution due to the lightest (heaviest) Higgs as mediators decreases (increases) its value one order of magnitude. For negative values of $\mu$ the cross section is much smaller that the one for $\mu > 0$. The interaction with the top-quark has the largest cross section while the smallest corresponds to the up-quark. The squark as a mediator produces the largest cross section for the up, down and strange quarks, while for the rest of the quarks the larger contribution to the cross section is given by the lightest Higgs.

4. Conclusions

In this work we have studied the neutralino-nucleon cross section as a function of the neutralino mass and energy. We have considered different combinations of mass parameters of SUSY and different ratios between the vacuum expectation value of the Higgs. We have also analysed the quark-neutralino cross section for the different interactions, that is the axial-vector and scalar interactions.

We have found that the cross section for neutrons and protons are almost the same if the mass of the neutralino is larger than 1 GeV. For smaller values of the neutralino mass the cross section, in the axial-vector channel, for neutrons is larger than the one for protons. The dominant mediator for the axial-vector channel is the $Z_0$ boson while for the scalar channel all the contributions are quite similar. If the value of $\mu$ increases the neutralino-nucleon cross section decreases.

In this work we have used $\tan \beta = 3$ and $\tan \beta = 10$ for the calculations of the cross section. We have found that the axial-vector contribution to the total cross section does not change its value when $\tan \beta$ changes, but the scalar contribution decreases its value for larger $\tan \beta$. We have also analysed the case with negative values of $\mu$ and found that the cross section is smaller than the one obtained with positive values of $\mu$. From the calculated values we could set constraints on the value of $\mu$ by the comparison with the available observational data.
Appendix A. Mass matrix diagonalization

The mass-matrix in the framework of the MSSM is \cite{11}

\[
Y = \begin{pmatrix}
M' & 0 & -M_Z \beta s_W & M_Z \beta s_W \\
0 & \mu & M_Z \beta c_W & -M_Z s_W c_W \\
-M_Z \beta s_W & M_Z \beta c_W & 0 & -\mu \\
M_Z s_W & -M_Z s_W c_W & -\mu & 0
\end{pmatrix},
\]

Figure 5. Scalar contribution to the nucleon-light neutralino scattering cross section as a function of the energy of the neutralino for protons. From top to bottom row: heaviest Higgs, lightest Higgs, squark, and total scalar contributions, respectively. Solid line: $\mu = 100$ GeV, $M = 61.36$, and $m_\chi = 5$ MeV, dotted line: $\mu = 100$ GeV, $M = 61.59$, and $m_\chi = 100$ MeV.
where \(c_\beta (s_\beta)\) stands for \(\cos \beta (\sin \beta)\), \(M_Z\) is the Z boson mass, \(\theta_W\) is the Weinberg angle and \(c_W (s_W)\) is \(\cos \theta_W (\sin \theta_W)\). The parameters \(M\) and \(M'\) are related by
\[
M' = \frac{5}{3} M \tan^2 \theta_W \text{ (GUT)}.
\]

The lightest neutralino mass \((m_\chi)\) is the eigenvalue of the lowest mass-eigenstate of \(Y\). Following Ref. [41], a squared, complex and unitary matrix \((N)\), (it can be real assuming CP invariance), transform \(Y\) such that
\[
M_{\text{diag}} = N^\dagger Y N = \text{diag} \left( \epsilon_1 \tilde{M}_1^0, \epsilon_2 \tilde{M}_2^0, \epsilon_3 \tilde{M}_3^0, \epsilon_4 \tilde{M}_4^0 \right), \quad (A.1)
\]
where \(\epsilon_i = \pm 1\) and \(M_i^0 > 0\). Taking into account the \(i\)-th row, one can find the following system of equations
\[
\begin{pmatrix}
Y_{11} - \lambda_i & Y_{21} & Y_{31} & Y_{41} \\
Y_{12} & Y_{22} - \lambda_i & Y_{32} & Y_{42} \\
Y_{13} & Y_{23} & Y_{33} - \lambda_i & Y_{43} \\
Y_{14} & Y_{24} & Y_{34} & Y_{44} - \lambda_i
\end{pmatrix}
\begin{pmatrix}
Z_{11} \\
Z_{12} \\
Z_{13} \\
Z_{14}
\end{pmatrix} = 0.
\quad (A.2)
\]

We have called \(\lambda_i\) to the eigenvalue \(\epsilon_i \tilde{M}_i^0\). After some algebra, the characteristic polynomial can be written as
\[
0 = \lambda_i^4 - \zeta \lambda_i^3 + \xi \lambda_i^2 + \gamma \lambda_i + \delta,
\]
with
\[
\zeta = M + M', \\
\xi = MM' - \mu^2 - M_Z^2, \\
\gamma = (M + M') \mu^2 + (M' \cos^2 \theta_W + M \sin^2 \theta_W) M_Z^2 \\
- \mu M_Z^2 \sin 2\beta, \\
\delta = -MM' \mu^2 + (M' \cos^2 \theta_W + M \sin^2 \theta_W) \mu M_Z^2 \sin 2\beta.
\]
The solutions of the characteristic polynomial are

\[
\begin{align*}
(\lambda_i)_1 &= \frac{\xi}{4} - \sqrt{\frac{a}{4} - \frac{C_2}{6} - \frac{U}{12a}} \\
&\quad + \sqrt{-\frac{a}{4} - \frac{C_2}{3} + \frac{C_3}{\sqrt{4a - \frac{8C_2}{3} - \frac{U}{2a}}}} + \frac{U}{12a}, \\
(\lambda_i)_2 &= \frac{\xi}{4} - \sqrt{\frac{a}{4} - \frac{C_2}{6} - \frac{U}{12a}} \\
&\quad - \sqrt{-\frac{a}{4} - \frac{C_2}{3} + \frac{C_3}{\sqrt{4a - \frac{8C_2}{3} - \frac{U}{2a}}}} + \frac{U}{12a}, \\
(\lambda_i)_3 &= \frac{\xi}{4} + \sqrt{\frac{a}{4} - \frac{C_2}{6} - \frac{U}{12a}} \\
&\quad - \sqrt{-\frac{a}{4} - \frac{C_2}{3} + \frac{C_3}{\sqrt{4a - \frac{8C_2}{3} - \frac{U}{2a}}}} + \frac{U}{12a}, \\
(\lambda_i)_4 &= \frac{\xi}{4} + \sqrt{\frac{a}{4} - \frac{C_2}{6} - \frac{U}{12a}} \\
&\quad + \sqrt{-\frac{a}{4} - \frac{C_2}{3} + \frac{C_3}{\sqrt{4a - \frac{8C_2}{3} - \frac{U}{2a}}}} + \frac{U}{12a}.
\end{align*}
\]

We have defined, following Ref. 41:

\[
\begin{align*}
C_2 &= \xi - \frac{3}{8}\xi^2, \\
C_3 &= -\frac{1}{8}\xi^3 + \frac{1}{2}\xi^2 + \gamma, \\
C_4 &= \delta + \frac{1}{4}\xi\gamma + \frac{1}{16}\xi^2 - \frac{3}{256}\xi^4, \\
U &= -\frac{1}{3}C_2^2 - 4C_4, \\
S &= -C_3^2 - \frac{2}{27}C_2^3 + \frac{8}{3}C_2C_4, \\
D &= -4U^3 - 27S^2, \\
a &= \left(-\frac{S}{2} + \frac{1}{2}\sqrt{-\frac{D}{27}}\right)^{1/3}.
\end{align*}
\]

To compute the eigenvectors we divide each equation of Eq. (A.2) by $Z_{i1}$ (that is a
not null value) and solve the system to obtain

\[
\frac{Z_{i2}}{Z_{i1}} = \frac{\lambda_i - M'}{\tan \theta_W (M - \lambda_i)},
\]

\[
\frac{Z_{i3}}{Z_{i1}} = \frac{1}{2z} \left[ -4 (M - \lambda_i) (\lambda_i - M') \mu \right.
\]

\[
\left. + M_Z^2 (-M + 2\lambda_i - M' + (M - M') \cos (2\theta_W)) \sin (2\beta) \right],
\]

\[
\frac{Z_{i4}}{Z_{i1}} = \frac{1}{z} \left[ (\lambda_i - M') (2 (M - \lambda_i) \lambda_i + M_Z^2 (1 + \cos (2\beta))) \right.
\]

\[
\left. + 2 (M' - M) M_Z^2 \cos^2 \beta \sin^2 \theta_W \right],
\]

where

\[
z = 2 \sin \theta_W (M - \lambda_i) M_Z (\mu \cos \beta + \lambda_i \sin \beta).
\]

Finally, using the normalization condition, that is

\[
Z_{i1}^2 + Z_{i2}^2 + Z_{i3}^2 + Z_{i4}^2 = 1,
\]

we obtain the eigenvector. We found that the eigenvalue \((\lambda_i)_1\) is positive and has the lowest value, therefore it could be associated to the neutralino mass.

**Appendix B. Matrix elements**

**B.1. Axial-Vector case**

The matrix element for the axial-vector interaction is

\[
M^{A-V} = C^{A-V} \left[ \bar{\chi} \gamma^\mu \gamma^5 \chi \right] \left[ \bar{\psi} q \gamma^\mu \gamma^5 \psi q \right].
\] (B.1)

Therefore one can split the expression into two contributions, one from the quark sector and the other one from the neutralino sector. Since the quark current is

\[
J^q_\mu = \bar{\psi} q \gamma^\mu \gamma^5 \psi q,
\]

the quark contribution can be written as

\[
\sum_{l,t,t'} (J^q_\mu J_\alpha^\dagger) = \sum_{l,t'} \left[ \bar{\psi}_q (l', t') \gamma_\mu \gamma_5 \psi_q (l, t) \right]
\]

\[
\times \left[ \bar{\psi}_q (l', t') \gamma_\alpha \gamma_5 \psi_q (l, t) \right]^\dagger
\]

\[
= \text{tr} \left\{ \gamma_\mu \gamma_5 (l + m_q) \gamma_\alpha \gamma_5 (l' + m_q) \right\}
\]

\[
= 4 \left[ (l_\mu l'_\alpha - g_{\mu\alpha} (l \cdot l') + l_\alpha l'_\mu) - m_q^2 g_{\mu\alpha} \right].
\] (B.2)

For the neutralino sector, which current is

\[
J^\mu_\chi = \bar{\chi} \gamma^\mu \gamma^5 \chi,
\]

we obtain

\[
\sum_{s,s'} (J^\mu_\chi J^\alpha_\chi^\dagger) = \sum_{s,s'} \left[ \bar{\chi}(p', s') \gamma^\mu \gamma^5 \chi(p, s) \right]
\]

\[
\times \left[ \bar{\chi}(p', s') \gamma_\alpha \gamma^5 \chi(p, s) \right]^\dagger
\]

\[
= 4 \left[ (P^\mu P'^\alpha - g^{\mu\alpha} (p \cdot p') + p^\alpha p'^\mu) - m_\chi^2 g^{\mu\alpha} \right].
\] (B.3)
Finally the squared invariant matrix element is
\[
\sum_{s,s',t,t'} |M^{A-V}|^2 = 32 |C^{A-V}|^2 [(p \cdot l)(p' \cdot l') + (p \cdot l')(p' \cdot l)]
+ m^2_{\chi}(l \cdot l') + m^2_{q}(p \cdot p')
+ 2m^2_{q}m^2_{\chi} .
\]  
(B.4)

B.2. Scalar case

The matrix element for the scalar interaction is
\[
M^S = C^S[\bar{\chi}\chi][\bar{\psi}p\psi]
\]  
(B.5)

Once again, one can split the expression into two contributions, one from the quark sector and the other one from the neutralino sector. For the quark sector, we derive
\[
\sum_{t,t'} (J^q J^{q\dagger}) = \sum_{t,t'} \left[ \bar{\psi}_q(l',t')\psi_q(l,t) \right]\left[ \bar{\psi}_q(l',t')\psi_q(l,t) \right]^\dagger
= 4(l \cdot l' + m^2_{q})
\]  
(B.6)

The neutralino current is \( J_\chi = \bar{\chi}\chi \), therefore
\[
\sum_{s,s'} (J_\chi J^{\dagger}) = \sum_{s,s'} [\bar{\chi}(p',s')\chi(p,s)] [\bar{\chi}(p',s')\chi(p,s)]^\dagger
= 4(p \cdot p' + m^2_{\chi})
\]  
(B.7)

Finally the squared invariant matrix element is written
\[
\sum_{s,s',t,t'} |M^S|^2 = 16 |C^S|^2 [(p \cdot p')(l \cdot l') + m^2_{\chi}(l \cdot l')]
+ m^2_{q}(p \cdot p') + m^2_{q}m^2_{\chi} .
\]  
(B.8)

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