Spotlighting the sensitivities of T2HK, T2HKK and DUNE

Kaustav Chakraborty,1,2,∗ K. N. Deepthi,1,† and Srubabati Goswami1,‡

1Theoretical Physics Division, Physical Research Laboratory, Ahmedabad - 380009, India
2Discipline of Physics, Indian Institute of Technology, Gandhinagar - 382355, India

Neutrino oscillation physics has entered the precision era and the potential forthcoming experiments Hyper-Kamiokande and Deep Under-ground Neutrino Experiment (DUNE) are expected to lead this endeavor. In this paper we perform a comprehensive study of the octant, mass hierarchy and CP discovery sensitivities of DUNE, T2HK & T2HKK in their individual capacity and investigate the synergies of the aforementioned experiments with the on going T2K and NOνA experiments. We present a comparative account of the probabilities at the three baselines and explore in detail the physics issues which can cause the discrepancies in the sensitivities among the different experiments. We also find out the optimal exposure required by these experiments for achieving 5σ hierarchy and octant sensitivity and to discover CP violation at 3σ for 60% values of δCP. In addition we vary the neutrino-antineutrino runtime ratios for T2HK & T2HKK and check if the sensitivities are affected significantly due to this.

I. INTRODUCTION

Neutrino oscillation physics is an emerging field of research that has been posing many challenges over the past few decades. Nevertheless, with the help of many phenomenal experiments much progress has been made in precisely determining the oscillation parameters θ12, θ13, |Δm23| and Δm21. This leaves determination of neutrino mass hierarchy i.e. the sign of Δm23, CP phase δCP and the octant of θ23 as the primary objectives of the on-going and the upcoming potential neutrino oscillation experiments. In a three flavour framework there can be two possible ordering of the mass eigenstates mi. If m1 < m2 < m3 then we get the normal hierarchy (NH) and if m3 < m2 ≈ m1 then it is called the inverted hierarchy (IH). Octant of θ23 refers to whether θ23 < 45◦ i.e. it lies in the lower octant (LO) or if θ23 > 45◦ i.e it is located in the higher octant (HO). For δCP the values 0 and ±180◦ correspond to CP conservation and ±90◦ corresponds to maximal CP violation.

In this regard the currently running NOνA and T2K experiments have recently published some interesting results. T2K data shows a preference for δCP = −90◦, maximal θ23 and a mild indication for normal mass hierarchy [1]. Whereas, the combined νμ → νe appearance and disappearance channel data of NOνA [2] has recently suggested that for all values of δCP inverted mass hierarchy along with θ23 < 45◦ is disfavored at 93% C.L. and there are two degenerate best-fit points (1) sin 2θ23 = 0.404, δCP = 266.4◦ (2) sin 2θ23 = 0.623, δCP = 133.2◦ when neutrino masses obey normal hierarchy. Global analysis of oscillation data indicates δCP = −90◦ [3–5]. There is a preference towards HO and a weak hint for NH. The difficulties in accurate determination of these parameters faced by the current experiments are the parameter degeneracies allowing for multiple solutions. This can be surpassed by the future experimental proposals like T2HK [6] , T2HKK [7] and DUNE [8] thereby improving the hierarchy, octant and δCP sensitivity. Among these DUNE is a proposed Fermilab based experiment with a Liquid Argon Time Projection Chamber detector placed at the Sanford Undergrounds Research Facility (SURF) which is 1300 km downstream to the initial neutrino beam. T2HK is an experiment based in Japan which plans to send neutrino beam from Tokai to Kamioka through a baseline of 295 km to two Hyper-Kamiokande detectors. Recently an alternative proposal in which one of the detec-

∗Email Address: kaustav@prl.res.in
†Email Address: deepthi@prl.res.in
‡Email Address: sruba@prl.res.in
tors in Japan will be shifted to Korea has been mooted. This is termed as T2HKK [7]. There have been several studies on the capabilities of DUNE [9–14] and T2HK [6, 15, 16] for determination of the three major unknowns mentioned above. The T2HKK proposal studied the hierarchy and $\delta_{CP}$ sensitivity of the set up with respect to three off-axis angles $1.5^\circ, 2^\circ, 2.5^\circ$ [7]. With the inception of this proposal, the physics possibilities of T2HKK regarding determination of hierarchy, octant and $\delta_{CP}$ has been studied in ref. [17–20]. In ref. [17], the mass hierarchy and CP sensitivity of DUNE, T2HK, DUNE+T2HK were studied in detail. The also elaborated on the optimization of alternative designs for DUNE and the T2HKK proposal. In [18] a hybrid setup in which the antineutrino run of T2HK is substituted by antineutrinos coming from muon decay at rest ($\mu$-DAR) has been studied for determination of hierarchy, octant and $\delta_{CP}$. The author of ref. [20] has studied the sensitivity of T2HK, T2HKK and DUNE to determine mass hierarchy and to measure the CP phase $\delta_{CP}$ and also made a comparative analysis of DUNE+T2HK and DUNE+T2HKK. In ref. [19] the role of systematic uncertainties in the determination of hierarchy, octant and $\delta_{CP}$ in the three set ups were studied. However octant sensitivity of T2HKK and a detailed comparative study of the three experiments have not been presented in any of these references.

In this work, one of our major aims is to study the octant sensitivity of the T2HKK setup. It is well known that one of the challenges for precise determination of $\delta_{CP}$ is the octant-$\delta_{CP}$ degeneracy. Therefore the CP discovery potential of a set up is intimately connected with its octant resolution capability if such degeneracies are present. Thus it is important to investigate to what extent an experiment can resolve these degeneracies. With this aim we do a comprehensive analysis of the octant discovery potential of T2HKK and compare it with the same for the other two setups. Further, we give a detailed account of the underlying physics reasons which are causing differences in the octant sensitivity of these three experiments. Additionally we also do a comparative analysis of the hierarchy and CP discovery potential of the three set ups for the sake of completeness. Thus our work provides a ready reference for the comparison amongst the hierarchy, CP discovery and octant sensitivities and the underlying physics issues of these facilities. In view of the fact that by the time these experiments are operative, the current experiments T2K and NO$\nu$A will already have their results we also show to what extent the inclusion of this information can improve the sensitivities. In addition, we also estimate the optimal exposures/run times of these setups to achieve $5\sigma$ sensitivity in octant and hierarchy determination and $3\sigma$ sensitivity to CP discovery potential for 60% values of the CP phase $\delta_{CP}$. Furthermore we consider two different neutrino and antineutrino runtime ratios (3:1 and 1:1) in T2HK and T2HKK apart from the proposed 1:3 ratio and study to what extent the sensitivities are affected.

The outline of the paper is as follows. In section II we present the relevant probabilities and discuss the degeneracies that are faced by the different experiments in terms of probabilities. The following section contains the results on octant, hierarchy and CP discovery potentials of the three experiments with and without inclusion of T2K and NO$\nu$A results. A concise discussion regarding the experimental setups have also been included in this section. In section V we obtain the optimal exposures required by various setups to achieve $5\sigma$ octant and hierarchy sensitivity. We also present the exposure needed to achieve $3\sigma$ sensitivity to discovery of CP violation for 60% values of $\delta_{CP}$. In the next section we study the effect of varying the proportion of neutrino and antineutrino run times in the T2HK and T2HKK experiment to explore if this can cause any discernible effect as opposed to the proposed runtime ratio. We make our concluding remarks in the final section.

II. PROBABILITY DISCUSSIONS

Neutrino oscillation experiments measure the event rates which in turn depends on the probabilities. The probability relevant for determination of the three unknowns in the long baseline experiments is the appearance probability $P_{\mu e}$. An approximate analytic expression for this can be obtained
Oscillation parameters | True value | Test value
--- | --- | ---
\(\sin^2 2\theta_{13}\) | 0.085 | 0.07 – 0.1
\(\sin^2 \theta_{12}\) | 0.304 | –
\(\theta_{23}\) | \(42^\circ\) (LO), \(48^\circ\) (HO) | \(39^\circ\) – \(51^\circ\)
\(\Delta m^2_{21}\) | \(7.40 \times 10^{-5}\) eV\(^2\) | –
\(\Delta m^2_{31}\) | \(2.50 \times 10^{-3}\) eV\(^2\) | \((2.35 – 2.65) \times 10^{-3}\) eV\(^2\)
\(\delta_{CP}\) | \(-180^\circ\) to \(+180^\circ\) | \(-180^\circ\) to \(+180^\circ\)

TABLE I: True values and marginalization ranges of neutrino oscillation parameters used in our numerical analysis.

In matter of constant density by expanding in terms of two small parameters \(\alpha\) and \(\sin \theta_{13}\) as \([21]\),

\[
P_{\mu e} = 4s^2_{13}s^2_{23} \sin^2(\hat{A} - 1)\Delta \frac{\sin(\hat{A} - 1)\Delta}{(A - 1)^2}
\]

\[
+ 2\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta + \delta_{CP}) \frac{\sin \hat{A} \Delta \sin(\hat{A} - 1)\Delta}{\hat{A} - 1}
\]

\[
+ \alpha^2 \sin^2 \theta_{12} s^2_{23} \frac{\sin^2 \hat{A} \Delta}{\hat{A}^2}
\]

where, \(\alpha = \Delta m^2_{21}/\Delta m^2_{31}\), \(\Delta = \Delta m^2_{31}L/4E\) and \(\hat{A} = A/\Delta m^2_{31}\). Here \(A = 2\sqrt{2}G_FN_eE\) is the matter potential, \(N_e\) is the electron number density inside the Earth, \(E\) is the neutrino energy and \(L\) is the baseline length. For antineutrinos the matter potential changes sign which implies \(\hat{A} \rightarrow -\hat{A}\) and \(\delta_{CP} \rightarrow -\delta_{CP}\). For IH, \(\Delta \rightarrow -\Delta\).

Figure 1 shows the behaviour of the probabilities as a function of energy for the baselines 295 km, 1100 km and 1300 km. These plots are done for a fixed value of \(\delta_{CP} = -90^\circ\). The bands correspond to variation over the octant of \(\theta_{23}\) in the range \(39^\circ\) – \(42^\circ\) for LO and \(48^\circ\) – \(51^\circ\) for HO. The figure shows that for NH the bands are wider for T2HK and DUNE baseline as compared to the T2HKK baseline. This indicates that the variation of the probability over octant is more for T2HK and DUNE than in T2HKK. For IH, the bands at the energies where the respective flux peaks for T2HK are wider but the bands for T2HKK and DUNE are narrower.

In order to elucidate this further we plot \(P_{\mu e}\) as a function of \(\theta_{23}\) for different baselines of T2K & T2HK (295 km, 0.6 GeV), T2HKK (1100 km, 0.6 GeV ), DUNE (1300 km, 2 GeV) in fig. (2), by fixing the corresponding energies at the values where the respective flux peaks. It is to be noted that 0.6 GeV corresponds to the first oscillation maxima for T2HKK baseline while it is close to the second oscillation maxima for the T2HKK baseline. The left (right) plot corresponds to \(P_{\mu e}\) vs \(\theta_{23}\) assuming normal (inverted) hierarchy. It can be seen that the lines corresponding to the baselines of 295 km (blue-solid), 1300 km (brown-dash-dotted) have positive slopes both for NH and IH. However, the curve for 1100 km (black-dashed) is much flatter with a small positive slope for NH and negative for IH.

With a view to understand this behaviour of the probability with \(\theta_{23}\) at different baselines we write the probability expression eqn. 2 in the following form \([22]\)

\[
P_{\mu e} = (\beta_1 - \beta_3) \sin^2 \theta_{23} + \beta_2 \sin 2\theta_{23} \cos(\Delta + \delta_{CP}) + \beta_3
\]
FIG. 1: Appearance probabilities $P_{\mu e}$ vs Energy for 295 km, 1100 km and 1300 km. The left panel is for NH and right panel for IH. The bands are due to variation over $\theta_{23}$. Here $\delta_{CP}$ and $\theta_{13}$ are in degrees.

where,

$$\beta_1 = \sin^2 2\theta_{13} \sin^2 \Delta(1 - \hat{A}) (1 - \hat{A})^2,$$

$$\beta_2 = \alpha \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \frac{\sin \Delta \hat{A} \sin \Delta(1 - \hat{A})}{A},$$

$$\beta_3 = \alpha^2 \sin^2 2\theta_{12} \cos^2 \theta_{13} \frac{\sin^2 \Delta \hat{A}}{A^2}.$$  

(3)

The $\theta_{23}$ dependence is seen to come from the first term of eq.(2), varying linearly with $\theta_{23}$ with a
slope given by \((\beta_1 - \beta_3)\). Note that over the range of \(\theta_{23}\) spanning 39° − 51°, \(\sin 2\theta_{23}\) stays close to 1 and so the second term of eq. (2) does not affect the behaviour of \(P_{\mu e}\) vs \(\theta_{23}\). The \(\beta_i\)s for the three different baselines are tabulated in table II for both the hierarchies. The first column corresponds to the L/E ratios of different setups. For the baselines 295 km and 1100 km the peak energies being the same the main difference between the \(\beta_i\)s enumerated in table II is due to the L/E ratio which is much higher for the 1100 km baseline. Between the 1300 km baseline and 295 km baseline the L/E ratio is approximately the same order and the \(\beta_i\)s are different due to the different energies. The higher energy implies a higher value for the matter potential \(\hat{A}\) for the 1300 km baseline entailing \((1 - \hat{A})\) to be smaller and hence \(\beta_1\) larger. This can be seen from the second column of table II. Whereas for 1100 km and 1300 km baselines the energies as well as the different L/E ratios attribute to the difference in the \(\beta_1\) factors. From the third column of II it can be seen that the \(\beta_3\) values of 1300 km and 295 km are smaller than \(\beta_1\) and \(\beta_2\). Thus the probabilities of these two experiments are expected to rise as \(\sim \sin^2 \theta_{23}\) with slope given by \(\approx \beta_1\). This can be seen from the brown (dash-dotted) and the blue (solid) lines respectively. Comparing these two cases for NH it can be seen that since \(\beta_1\) is higher for the 1300 km baseline the variation with \(\theta_{23}\) is more than that of 295 km. For IH, on the other hand, \(\beta_1\) is much smaller for the 1300 km baseline (for the neutrinos) when compared to 295 km baseline due to the suppression caused by matter effect. Hence the variation of \(P_{\mu e}\) with \(\theta_{23}\) is much less for 1300 km when compared to 295 km as can be seen from right panel of fig. 2. For the 1100 km baseline and NH \(\beta_3\) term is comparable to \(\beta_1\) term which makes the \(\theta_{23}\) variation much flatter. For IH, \(\beta_1\) is less than \(\beta_3\) and the probability decreases with \(\theta_{23}\). These patterns can be seen from the black (dashed) lines in the fig. 2.

| L/E (km/GeV) | \(\beta_1\) NH | \(\beta_1\) IH | \(\beta_2\) NH | \(\beta_2\) IH | \(\beta_3\) NH | \(\beta_3\) IH |
|-------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1300 km     | 0.122          | 0.028          | 0.018          | -0.009         | 0.003          | 0.003          |
| 295 km      | 0.094          | 0.077          | 0.013          | -0.011         | 0.002          | 0.002          |
| 1100 km     | 0.045          | 0.002          | -0.032         | 0.007          | 0.023          | 0.023          |

TABLE II: \(\beta_1\), \(\beta_2\) & \(\beta_3\) values for DUNE, 295km & 1100km baselines for T2HK & T2HKK.

Figure 3 shows the probabilities as a function of \(\delta_{CP}\) for the baselines 295 km, 1100 km and 1300
FIG. 3: Appearance probabilities vs $\delta_{CP}$ (degrees) for 295 km, 1100 km and 1300 km. The left panel is for neutrinos while the right panel is for antineutrinos. Each panel contains plots for both NH and IH. The band represents variation over $\theta_{23}$. km for both neutrinos and antineutrinos. Each figure has four probability bands green, yellow, red and blue corresponding to NH-LO, NH-HO, IH-LO, IH-HO respectively. The bands are obtained due to the variation in $\theta_{23}$ in the range $35^\circ - 42^\circ$ in LO and $48^\circ - 51^\circ$ in HO. This range suffices since in any case the position of the minima is determined by the disappearance channel to be near $\approx 90^\circ - \theta_{23}$. These kind of plots are helpful in understanding the various degeneracies at a probability level for values of $\theta_{23}$ not very near to maximal mixing. An overlap between the bands for different values of $\delta_{CP}$ (as can be seen by drawing a horizontal line through the bands) would indicate a degenerate solution with a wrong $\delta_{CP}$ value, whereas, intersection between the bands would signify a degenerate solution even with the same value of $\delta_{CP}$ [23]. For instance from the left hand plot
in the first row one can see that NH-LO (green band) is degenerate with IH-HO (blue band) in the range \(-180^\circ < \delta_{CP} < 0\) (lower-half plane, LHP) for the 295 km baseline. This gives rise to the wrong hierarchy (WH) – wrong octant (WO) solutions. On the other hand NH-HO (yellow band) in the LHP and IH-LO (red band) in the range \(0 < \delta_{CP} < 180^\circ\) (upper-half plane, UHP) are devoid of any degeneracy for the neutrinos. For the antineutrinos, as can be seen from the right panel of the first row NH-LO (green band) in LHP and IH-HO (blue band) in UHP are non-degenerate. Thus the octant degeneracy is seen to be opposite for neutrinos and antineutrinos and hence combined neutrino and antineutrino run is expected to resolve wrong octant solutions. Again, considering \(\delta_{CP} \sim 90^\circ\) in UHP, NH-HO (yellow band) in UHP is degenerate with IH-HO (blue band) around \(\delta_{CP} \sim 0\). This corresponds to wrong hierarchy-right octant solution. This kind of degeneracy also exists for antineutrinos and hence inclusion of antineutrino run cannot resolve the wrong hierarchy-right octant solutions. Similar conclusions are true for the DUNE baseline as can be seen from the figures in the bottom row. However, because of enhanced matter effects DUNE has much better hierarchy sensitivity and therefore the difference between NH and IH bands are much more for this case and wrong hierarchy solutions are not seen in the probability plots. However, for e.g. the wrong octant solutions between NH-HO (yellow band) and \(\delta_{CP} \sim 90^\circ\) and NH-LO (green band) and \(\delta_{CP} \sim 0\) is visible from the neutrino probability presented in the left-panel of the bottom row for DUNE baseline. But for antineutrinos this degeneracy is not present.

For the T2HK baseline since the peak coincides with the second oscillation maxima and the L/E is different the degeneracy pattern is somewhat different. In this case, as can be seen from the first plot in the second row, for neutrinos the NH probability can have hierarchy degeneracy only in the range \(-30^\circ\) to \(90^\circ\) excepting the range \(10^\circ < \delta_{CP} < 60^\circ\) for LO (green band) for which there is no degeneracy. For IH, on the other hand there is degeneracy over the entire range of \(\delta_{CP}\). Similarly the plot in the right panel of the 2nd row exhibits that for NH there is hierarchy degeneracy over the whole range. While for IH the hierarchy degeneracy occurs in the range \(-180^\circ\) to \(-120^\circ\) and \(130^\circ\) to \(180^\circ\) barring the small range \(-170^\circ < \delta_{CP} < -150^\circ\) for LO. From these discussions it is clear that for true NH, neutrino run is better for hierarchy sensitivity whereas for true IH, antineutrinos are more useful.

For octant degeneracy if we check the NH plot in the left panel then (the green and yellow bands) we see that over most of the \(\delta_{CP}\) range a horizontal line drawn through these bands intersect the probabilities at \(\delta_{CP}\) values belonging to different half planes giving rise to right-hierarchy – wrong octant – wrong \(\delta_{CP}\) solutions. Over the range of \(\delta_{CP}\) for which hierarchy degeneracy occurs there can also be wrong-hierarchy – wrong octant solutions in the same half plane of \(\delta_{CP}\). For IH also, over almost full CP range octant degeneracy is seen from the probability curves. Similar conclusions can be drawn from the antineutrino probabilities. Note that for the Korean baseline and energy for neutrinos the IH-LO band (red) lies above the IH-HO (blue) band unlike the other two experiments. For IH \(\beta_3\) being greater than \(\beta_1\) the slope is negative and hence LO has a higher probability. Similarly for the antineutrinos NH-LO (green band) has a higher probability than NH-HO (yellow band). We can also see that the the curves for LO and HO are much closer for the 1100 baseline. In addition, the widths of the octant bands are very small which is reflective of the fact that the variation in the probability with octant is much less as we have seen earlier. Thus the octant sensitivity is expected to be less for this baseline and energy combination. From the equation 2, we can write the difference of the probabilities for two different \(\theta_{23}\) values as,

\[ P_{\mu e}(\theta_{23}) - P_{\mu e}(\theta'_{23}) \approx (\beta_1 - \beta_3)(\sin^2 \theta_{23} - \sin^2 \theta'_{23}) \]  

(4)

Note that the \(\beta_3\) term is not included since it \(\sin 2\theta_{23} \sim 1\) over the range in which \(\theta_{23}\) is varied. Because of this the difference is independent of \(\delta_{CP}\). This implies the width of the \(\theta_{23}\) band is expected to be the same for all values of \(\delta_{CP}\) which can be seen from the figure 3. Inserting the values of \(\beta_1\) and \(\beta_3\) from table II and putting \(\theta_{23}\) as \(39^\circ\) and \(\theta'_{23}\) as \(42^\circ\) we find that the width of the LO band as 0.006 (DUNE), 0.005 (T2HK) and 0.002 (T2HKK). This is agreement with what is observed in the plots.
III. EXPERIMENTAL AND SIMULATION DETAILS

In this section we elucidate briefly on the experimental set ups that have been used in our analysis.

T2HK and T2HKK

T2HK (Tokai-to-Hyper-Kamiokande) is a natural extension to the existing T2K experiment. It’s default plan was to have two Hyper-Kamiokande detectors (cylindrical water tanks) of 187 kt at 295 km baseline in Kamioka. T2HKK, an alternative to T2HK, is a newly proposed option which plans to have one of it’s detectors at 295 Km in Kamioka mine and another at 1100 km in Korea. The water-cherenkov detector in Korea could be placed at one of the three suggested off-axis (OA) angles $1.5^\circ$, $2^\circ$ or $2.5^\circ$. The optimization of these OA angles to give maximum sensitivity to various neutrino oscillation parameters has been explored in ref. [7]. This study indicates that the optimal configuration is to place the detector at $1.5^\circ$ OA angle. Therefore, we have considered the 187 kt Korean detector at an OA angle of $1.5^\circ$ in our simulations. The proposed runtime for both configurations is 1:3 in neutrino and antineutrino modes and the total exposure of $27 \times 10^{21}$ Protons on Target(POT) which is obtained by a beam energy of 1.3 MW and 10 years of experiment run. The detector specifications and the systematic uncertainties are consistent with the proposal given in ref. [7]. The simulations in this work are carried out after matching with the signal, background event spectra and the sensitivities presented in ref. [7].

DUNE

Deep Underground Neutrino Experiment (DUNE) is a promising upcoming long-baseline neutrino oscillation experiment supported by Long-Baseline Neutrino Facility (LBNF). LBNF and DUNE facilities together will constitute a high intensity neutrino beam of 0.5-8 GeV energy, a near detector at Fermilab site, a 40 kt liquid argon time-projection chamber (LArTPC) as far detector 1300 km away at Sanford Underground Research Facility (SURF), South Dakota. Simulation of the DUNE experiment is done by considering a beam power of 1.2 MW which results in a total exposure of $10 \times 10^{21}$ Protons on Target(POT) for a 10 years experimental run. The $\nu : \bar{\nu}$ runtime ratio for DUNE is considered as 1:1. The experimental specifications for our simulation of DUNE are taken from the ref. [8].

T2K & NO$\nu$A

NO$\nu$A [24] and T2K [24] are currently operative long-baseline experiments. The NO$\nu$A experiment has a baseline of 812 km. For this experiment muon-neutrinos are directed towards a far detector of fiducial volume 14 kt located at Ash River Minnesota from the NuMI beam facility at Fermilab. In our simulation we have considered the projected exposure for NO$\nu$A which consists of 3 year $\nu$ and 3 year $\bar{\nu}$ runs and total exposure of $7.3 \times 10^{20}$ POT. The T2K experiment uses the muon-neutrino beam at JPARC facility. The beam is projected towards a detector of fiducial volume 22.5 kt located at Kamioka with a baseline of 295 km. We have considered a runtime of $4\nu+4\bar{\nu}$ with a total exposure as $8 \times 10^{21}$ POT.

IV. HIERARCHY, OCTANT AND CP DISCOVERY POTENTIAL OF T2HK, T2HKK AND DUNE

In this section, we present a comprehensive analysis of the hierarchy, octant and CP discovery sensitivity of the experiments T2HK, T2HKK and DUNE with their proposed configurations. In addition we also present the sensitivities including the projected run times of T2K and NO$\nu$A . For
performing the numerical simulation we have used the package General Long Baseline Experiment Simulator (GLoBES) [25, 26]. Our presentation facilitates a comparison of the performances of the different experiments. We perform a $\chi^2$ test where,

$$\chi^2_{\text{stat}} = 2 \sum_i \left( N_i^{\text{test}} - N_i^{\text{true}} + N_i^{\text{true}} \ln \frac{N_i^{\text{true}}}{N_i^{\text{test}}} \right),$$  \hspace{1cm} (5)

where $(N)_i^{\text{true}}$ corresponds to the simulated data and $(N)_i^{\text{test}}$ is the number of events predicted by the theoretical model. The systematic errors were incorporated in terms of pull variables. We have marginalized over the test parameters. The true values of the parameters and the marginalization ranges of the other parameters are as given in Table I.

We present the results for four true hierarchy-octant combinations – NH-LO, NH-HO, IH-LO & NH-HO. The true value of $\theta_{23}$ for LO is considered as $42^\circ$ while for HO it is taken as $48^\circ$ throughout our analysis unless otherwise mentioned. All the figures in this section show the performances of T2HK, T2HKK and DUNE and their corresponding sensitivities are plotted in red, blue and green colors respectively. The solid lines represent the individual sensitivities of these experiments and the dashed lines correspond to the combined significance of the concerned experiment along with T2K and NO$\nu$A data.

**Octant Sensitivity**

To resolve the octant ambiguity of $\theta_{23}$ is one of the important goals of current and up-coming LBL experiments. This gained more prominence in the light of the tension between the recent results of T2K and NO$\nu$A on the octant of $\theta_{23}$ where the former favours near maximal value of $\theta_{23}$ and the latter excludes maximal $\theta_{23}$ with $2.6\sigma$ significance.

However, the upcoming LBL experiments T2HK, T2HKK and DUNE can overcome the various degeneracies that are effecting the octant sensitivity. The figure 4 shows the ability of the various configurations to exclude the wrong octant of $\theta_{23}$ plotted as a function of true $\delta_{CP}$. Octant sensitivity is calculated by assuming a true octant in data and considering the opposite octant as test in the theory. Marginalization is done over $\theta_{13}$, $\theta_{23}$ (over the range of opposite octant), $|\Delta m^2_{31}|$, hierarchy and $\delta_{CP}$.

We find that the sensitivity is higher for a true lower octant for all the experiments. There are two reasons for this. Firstly for non-zero $\theta_{13}$ the $\chi^2$ vs $\theta_{23}$ curve is not symmetric about $45^\circ$. The two degenerate solutions in opposite octants are approximately related by $\theta_{23}^{LO} = 91.5^\circ - \theta_{23}^{HO}$ [27]. This implies that for a true $\theta_{23}$ of $42^\circ$ the degenerate minima will be at $49.5^\circ$ while for a true $\theta_{23} = 48^\circ$ it will be at $43.5^\circ$. Thus the numerator in eq. 5 is higher for a true LO. On the other hand the denominator for the case of LO will be lower because, in general, LO has lower probabilities compared to HO and hence for LO the number of events will be reduced. Thus, the octant sensitivity is higher for LO. For a true LO greater than $5\sigma$ sensitivity is obtained for all the experiments. For true HO, only T2HK experiment can attain $5\sigma$ octant sensitivity. In general, the best octant sensitivity is achieved by the proposed T2HK experiment for all the cases. The octant sensitivity for T2HKK experiment is less as compared to T2HK, since as is evident from the probability discussions the octant sensitivity gets reduced for the 1100 km baseline. Addition of the data of T2K and NO$\nu$A helps to increase the the significance by approximately 15% in all cases. The octant sensitivity of DUNE is seen to be slightly less as compared to T2HK and T2HKK in most of the parameter range. This can be attributed to the lesser detector volume of DUNE. The underlying physics issues behind octant sensitivity of DUNE has been discussed in detail in [14].

**Mass Hierarchy Sensitivity**

Mass hierarchy sensitivity is calculated (using eq. (5)) by assuming a true hierarchy in the data and testing it by fixing the opposite hierarchy in theory. Marginalization is done over $\theta_{13}$, $\theta_{23}$, $|\Delta m^2_{31}|$.
FIG. 4: Octant sensitivity in DUNE, T2HK and T2HKK with all hierarchy-octant configurations, first(second) row represent NH(IH) and first(second) columns represent LO(HO)

and $\delta_{CP}$. The figure 5 demonstrates the mass hierarchy sensitivity of the proposed experiments. It is seen that for T2HK the hierarchy sensitivity is much reduced in the UHP for NH and in the LHP for the IH. This is due to the presence of hierarchy degeneracies between NH-LO and IH-LO (green and red band) as can be seen from fig. 3. Similarly for IH, the unfavourable zone for hierarchy determination is the LHP. Note that the WH-WO solutions are removed because of combined neutrino antineutrino run. For T2HKK the hierarchy sensitivity is much higher. In this case the detectors are at two different baselines hence oscillation effects at both baselines will contribute. From the mass hierarchy plots fig. 11 we can conclude that there had been a significant increment in the overall mass hierarchy sensitivity over the previous case of both detectors being at 295 km baseline. In particular the degeneracy faced by the T2HK setup in the unfavourable region of $\delta_{CP}$ can be resolved by moving one detector to Korea. This is because for the Korean detector the behaviour of the probability near second oscillation maxima shows that the degeneracy for neutrinos (NH) and for antineutrinos (IH) occur only over a small range of $\delta_{CP}$ values as discussed in the earlier section.

For DUNE because of matter effect the hierarchy degeneracy is lifted. DUNE has very high hierarchy sensitivity and for HO the $\Delta \chi^2$ value is $> 100$. Inclusion of T2K and NO$\nu$A results can enhance the sensitivity by $\approx 10\%$ in all the three cases.

**CP Sensitivity**

Calculation of CP discovery sensitivity is performed by simulating the data for all true $\delta_{CP}$ values and comparing them with CP conserving values $\delta_{CP} = -180^\circ, 0^\circ$&$180^\circ$. Marginalization is done over $\theta_{13}, \theta_{23}, |\Delta m^2_{31}|$ and hierarchy. The CP discovery potential of the experiments under consideration
FIG. 5: Mass hierarchy sensitivity in DUNE, T2HK and T2HKK with all hierarchy-octant configurations, first(second) row represent NH(IH) and first(second) columns represent LO(HO) is shown in figure 6. The maximum CP discovery sensitivity can be achieved for $\delta_{CP} = \pm 90^\circ$. For T2HK the CP discovery potential is seen to be less in one of the half-planes of $\delta_{CP}$. This is due to the presence of wrong hierarchy solutions. T2HKK can overcome this because it has a better hierarchy sensitivity and hence the hierarchy-$\delta_{CP}$ degeneracy is resolved. In general T2HKK has a better CP sensitivity because at the second oscillation maxima the CP effect is larger and can compensate for the loss in flux due to a higher baseline [7]. In ref. [7] it was shown that the difference in the CP asymmetry between CP conserving and CP violating values is more for the 1100 km baseline. At the $\chi^2$ level this gets reflected in the tension between the neutrino and the antineutrino contribution to the $\chi^2$. For the T2HK experiment at 295 km the oscillation peak and the flux peak coincide and the probability for $\delta_{CP} = 0$ and $\pm 180^\circ$ are equidistant from the probability at $\pm 90^\circ$. However for the 1100 km baseline as can be seen from the probability plot for either NH or IH as the true hierarchy, for neutrinos $\delta_{CP} = -90^\circ$ is closer to $\pm 180^\circ$ while for antineutrinos $\delta_{CP} = 0$ is closer to $\delta_{CP} = -90^\circ$. This creates a tension between the neutrino and the antineutrino $\chi^2$ which gives a better sensitivity to T2HKK as one of its baselines is at 1100 km. For DUNE, the wrong hierarchy solutions get resolved and hence the sensitivities do not suffer a drop in one half plane of $\delta_{CP}$ as in T2HK. In this case also, the neutrino and antineutrino tension can enhance the overall $\chi^2$ for CP violation [14]. However, since the statistics of DUNE is lower compared to T2HKK and T2HK it’s CP discovery sensitivity is lower.
FIG. 6: CP sensitivity in DUNE, T2HK and T2HKK with all hierarchy-octant configurations, first(second) row represent NH(IH) and first(second) columns represent LO(HO)

V. OPTIMAL EXPOSURES

In the earlier section we have delineated the sensitivities of the different experiments and compared the performances using the proposed plans. These being very high statistics experiments in some cases very high sensitivity is seen to be achieved. In this section we present the optimum exposure that is needed by each experiment to attain $5\sigma$ significance for hierarchy and octant sensitivity. We also give the exposure required for achieving $3\sigma$ sensitivity for 60% values of $\delta_{CP}$. We furnish our results for two representative hierarchy-octant configurations NH-LO and IH-HO. For true LO and HO we have taken representative values of $\theta_{23}$ as $42^\circ$ and $48^\circ$.

In fig. 7 we plot two sets of mass hierarchy $\chi^2$ vs exposure in kt-yr for T2HKK and DUNE in the left and the right panels respectively. The blue (red) solid lines correspond to the sensitivity of true NH-LO (IH-HO) configuration of only the concerned experiment. The dashed lines represent the same when added with $3\nu + 3\bar{\nu}$ NO$\nu$A and $4\nu + 4\bar{\nu}$ T2K runs. We marginalized over true $\delta_{CP}$ in the range $[-180^\circ, 180^\circ]$.

From the left panel in fig. 7 we observe that considering only T2HKK the optimal exposure for $5\sigma$ hierarchy sensitivity is $\sim 1080$ kt-yr for NH-LO and 680 kt-yr for IH-HO. This corresponds to volume of one detector. Therefore the total exposure for two detectors is 2160 kt-yr for NH-LO and 1360 kt-yr for IH-HO. For the fiducial volume of 187 kt for a single detector in true NH-LO configuration, this corresponds to $\approx 6$ years run time $i.e.$ 1.5 years in neutrino and 4.5 years in antineutrino mode. The more optimistic case would be with true IH-HO where it requires only 3.6 years ($0.9\nu + 2.7\bar{\nu}$) of run time. After adding T2K and NO$\nu$A information this exposure for each detector is reduced to 840 kt-yr for true NH-LO and 430 kt-yr for true IH-HO which for the proposed detector volume correspond to 4.5 years and 2.3 years of run time respectively. Performing the similar analysis for DUNE we see
that, a minimum $5\sigma$ significance can be obtained for 140 kt-yr in true NH-LO. This corresponds to a run time of 3.5 years (split equally in neutrinos and antineutrinos) for 40 kt detector volume. Adding the information from T2K and NO$\nu$A reduces it to 70 kt-yr corresponding to just 1.75 years run time for DUNE. For the case where IH-HO is the true combination $5\sigma$ significance can be obtained with an exposure of 125 kt-yr, which reduces to 60 kt-yr once the data from T2K and NO$\nu$A is added. In the first line of table III we summarize the exposures for the different setups for $5\sigma$ hierarchy sensitivity for the favourable case of true IH-HO.

Similarly we also find out the optimum exposure for $5\sigma$ octant sensitivity. For this case we present our results for T2HK and DUNE. Since the octant sensitivity of T2HKK is poorer as compared to T2HK, as seen in the earlier section IV, we present the minimal optimal exposure required for T2HK to resolve the octant of $\theta_{23}$ with $5\sigma$ sensitivity assuming true NH-LO and IH-HO.

We observe that overall $5\sigma$ significance can be obtained by the T2HK experiment for an exposure of 2500 kt-yr for one detector which corresponds to approximately 13.4 years of run time in IH-HO as can be seen from the red solid line in fig. 8. Including the information from T2K and NO$\nu$A the exposure reduces to 2100 kt-yr corresponding to a run time of 11 years approximately. The second panel represents the octant sensitivity for DUNE. In this case, for true IH-HO (red curves) it can be seen that the exposure for $5\sigma$ octant sensitivity is 800 kt-yr which with the proposed volume of 40 kt corresponds to a runtime of 20 years. Adding T2K and NO$\nu$A information the optimal exposure is 680 kt-yr which corresponds to 17 years runtime. This is not surprising since as we have seen in the previous section that DUNE doesn’t reach $5\sigma$ octant sensitivity in 10 years for IH-HO with their proposed volume. However, from the blue lines of fig. 8, we can see that the true hierarchy–octant configuration NH-LO gives $5\sigma$ sensitivity for comparatively less exposure for both the experiments. We list the exposures required for this optimistic case in table III. The number of operative years needed for T2HK to attain $5\sigma$ octant sensitivity is 3.8 yr in its individual capacity and 2.7 yrs when the data from T2K and NO$\nu$A experiments is added. However, DUNE requires 10 years of data taking to resolve octant of $\theta_{23}$ which reduces to 7.5 years when the T2K and NO$\nu$A results are taken into account.

In figure 9 we display the fraction of $\delta_{CP}(\text{true})$ values for which the experiments can observe CP violation with minimum $3\sigma$ significance. We perform this study for both NH and IH.

For calculating the fraction of $\delta_{CP}$ values we compare the true $\delta_{CP}$ values against the CP conserving test $\delta_{CP}$ values of 0° and ±180°. The fraction of $\delta_{CP}$ for more than $3\sigma$ CP discovery significance is the ratio of true $\delta_{CP}$ values for which the CP significance is more than $3\sigma$ to the total number of $\delta_{CP}$ values. For true $\theta_{23}$ we consider three values 42°, 45° and 48° and choose the minimum $\chi^2$ amongst this set. We have also marginalized over $\theta_{13}$, $\theta_{23}$, $\Delta m^2_{31}$ and hierarchy in test. The fig. 9 contains two plots for each experiment T2HKK(left) and DUNE(right). Each plot has two lines, green(solid) line represents true-NH while magenta(dashed) represent true-IH. We see that 60% cov-
FIG. 8: Octant Sensitivity in T2HK and DUNE versus exposure for true NH-LO

| Sensitivity                  | T2HKK | T2HK | T2HKK+T2K+NOνA | T2HK+T2K+NOνA | DUNE | DUNE+T2K+NOνA |
|-----------------------------|-------|------|----------------|---------------|------|---------------|
| Hierarchy ($\chi^2 = 25$)   | 680   | –    | 430            | –             | 140  | 70            |
| Octant ($\chi^2 = 25$)      | –     | 700  | –              | 500           | 400  | 300           |
| CP (60% at $\chi^2 = 9$, NH)| 400   | –    | –              | –             | 500  | –             |
| CP (60% at $\chi^2 = 9$, IH)| 400   | –    | –              | –             | 400  | –             |

TABLE III: Minimum exposures required for hierarchy, octant and CP in units of kt-yr

erage in $\delta_{CP}$(true) values is obtained by an exposure of 400 kt-yr of T2HKK (each detector) both for NH and IH. However for DUNE the exposure is 500 kt-yr for NH and 400 kt-yr for IH. This corresponds to approximately 2 years of running of T2HKK and 12.5 years of running of DUNE with their proposed volumes.

FIG. 9: Fraction of $\delta_{CP}$ in T2HKK and DUNE versus exposure for $3\sigma$ CPV sensitivity
VI. NEUTRINO AND ANTINEUTRINO RUN OPTIMIZATION IN T2HK & T2HKK

The proposed total runtime for the T2HK and T2HKK experiment is 10 years, which will consist of 2.5 years $\nu$ run and 7.5 years $\bar{\nu}$ run. This $1\nu : 3\bar{\nu}$ runtime ratio was chosen to keep the number of neutrino events comparable to that of the antineutrino events. In this section, we explore the possibilities of acquiring better sensitivities by considering $3\nu : 1\bar{\nu}$ and $1\nu : 1\bar{\nu}$ as alternative runtime ratios.

![Mass hierarchy sensitivity plots](image)

**FIG. 10:** Mass hierarchy $\chi^2$ vs $\delta_{CP}$ plots for T2HK (true NH first row and true IH second row). The labels signify the $\nu+\bar{\nu}$ runs.

### A. Analysis of Mass hierarchy sensitivity

Fig. (10) and fig. (11) represent the mass hierarchy sensitivities of T2HK, T2HKK. In each plot three different run time ratios of $2.5\nu + 7.5\bar{\nu}$, $7.5\nu + 2.5\bar{\nu}$, $5\nu + 5\bar{\nu}$ are shown by blue, magenta and green curves respectively. From all the plots of fig. (10) corresponding to the true configurations – NH-LO, NH-HO, IH-LO and IH-HO we find that for T2HK all the three runtime ratios give hierarchy sensitivity in the same ballpark. The combinations $7.5 + 2.5$ and $5 + 5$ fare slightly better because of enhanced statistics except for the UHP of true IH-HO where $7.5 + 2.5$ is slightly lower than the other two cases. This is because in the UHP for IH-HO the neutrino probability is impaired by WH-WO and RH-WO degeneracies unlike that in antineutrinos. Thus, the wrong octant solutions can be removed by considering more antineutrino runs as can be seen from the UHP of true IH-HO configuration in the fig. 10 where the proposed $2.5\nu + 7.5\bar{\nu}$ gives better sensitivity than $7.5\nu + 2.5\bar{\nu}$. However for T2HKK from fig. (11) we see that $7.5\nu + 2.5\bar{\nu}$ and $5\nu + 5\bar{\nu}$ give better sensitivity when compared to the proposed ratio of $2.5\nu + 7.5\bar{\nu}$ for some values of $\delta_{CP}$. If we compare the best two cases we
infer that for $7.5\nu + 2.5\bar{\nu}$ the significance is quite high with respect to the significance of $2.5\nu + 7.5\bar{\nu}$ both in upper and lower half plane, except the region with $-20^\circ < \delta_{CP} < 40^\circ$ where $2.5\nu + 7.5\bar{\nu}$ gives better hierarchy sensitivity. The greater hierarchy sensitivity in T2HKK can be understood from fig. 3 which depicts the oscillation probabilities. For 1100 km baseline in the region with $\delta_{CP} < -20^\circ$ and $\delta_{CP} > 40^\circ$ the neutrino appearance probabilities are not degenerate w.r.t. hierarchy, hence sensitivity in this region is governed by neutrino appearance, in the region $-20^\circ < \delta_{CP} < 40^\circ$ the sensitivity is governed by antineutrino appearance because in this region neutrino appearance probability is degenerate but antineutrino appearance is non-degenerate. Considering both baselines the IH sensitivity behaves differently because of the non-degenerate behaviour of the probabilities at 1100 km baseline. Similar to NH we obtain better sensitivities at the regions $\delta_{CP} < -20^\circ$ and $\delta_{CP} > 40^\circ$ for $7.5\nu + 2.5\bar{\nu}$ and $-20^\circ < \delta_{CP} < 40^\circ$ for $2.5\nu + 7.5\bar{\nu}$. We can conclude from this discussions that in the regions of maximum CP violation $5\nu + 5\bar{\nu}$ give somewhat better sensitivity for T2HKK experiment.

### B. Analysis of Octant sensitivity

Since the octant degeneracies involving neutrinos and antineutrinos behave in an opposite way there are interesting interplay of run time ratios for octant sensitivity This issue has been delved in detail in [14] for DUNE. However for DUNE the WH solutions are largely absent and therefore one had to deal with only right hierarchy-wrong octant solutions. But for T2HK hierarchy degeneracy is also present and so one encounters wrong octant solutions for both right and wrong hierarchy, which further complicates the issue.
FIG. 12: Octant sensitivity $\chi^2$ vs $\delta_{CP}$ plots for T2HK (true NH first row and true IH second row). The labels signify the $\nu+\bar{\nu}$ runs.

Fig. 12 and fig. 14 show the octant sensitivity of T2HK and T2HKK experiments for various true values of $\delta_{CP}$ with respect to different true hierarchy–octant combinations. In each plot we show the octant sensitivity corresponding to three different run time ratios of $2.5\nu + 7.5\bar{\nu}$, $5\nu + 5\bar{\nu}$ and $7.5\nu + 2.5\bar{\nu}$ shown by solid, dotted and dash-dotted curves respectively. The blue curves correspond to right hierarchy whereas magenta curves correspond to wrong hierarchy. Marginalization over hierarchy would chose the lower values of $\chi^2$ in each case.

In the top left panel of fig. 12 where true hierarchy–octant configuration is NH-LO ($\theta_{23}(true) = 42^\circ$) it can be seen that the higher sensitivity for all values of $\delta_{CP}$ is obtained from the proposed run-time ratio of $2.5\nu + 7.5\bar{\nu}$ for the right hierarchy in the LHP of $\delta_{CP}$ and over the whole range of $\delta_{CP}$ for the wrong hierarchy. If hierarchy is known to be NH then in the upper half plane 5+5 or 7.5+2.5 gives better results. This can be understood by relating to the corresponding probabilities shown in the top panel of fig. 3. In the LHP of the top right panel of fig. 3, we can see that the NH-LO(green) band for antineutrinos do not suffer from octant degeneracies for both right and wrong hierarchy (the yellow and blue bands). Thus more antineutrinos help to have a higher octant sensitivity in both the cases. In the UHP for wrong hierarchy (the magenta curves) the antineutrino run helps in removing the WH-WO solutions occurring with the same CP. However since neutrinos do not suffer from any degeneracy between NH-LO and NH-HO in the UHP, 7.5+2.5 or 5+5 gives better results.

For IH-LO, in the LHP neutrino has octant degeneracy for wrong as well as right hierarchy for wrong $\delta_{CP}$ as can be seen by comparing the red band with the yellow and the blue bands respectively in fig. 3. But antineutrino probabilities do not have any octant degeneracy. Thus the plan with more neutrinos is worse than that with equal or more antineutrinos. On the other hand in the UHP neutrino probability does not suffer from octant degeneracy as can be seen from the red band in the left panel.
of fig. 3 while the antineutrino probabilities have octant degeneracies. Thus for both right and wrong hierarchy the run with greater proportion of neutrinos is better. For wrong hierarchy 5+5 fares much better as in the LHP.

| $\theta_{23}(test)$ (total minima) | $\theta_{23}(test)$ (disappearance minima) | App($\nu$) | App($\bar{\nu}$) | Disapp($\nu$)+Dispp($\bar{\nu}$) | Total |
|-----------------------------------|-------------------------------------------|------------|-----------------|-----------------------------|-------|
| 10+0                             | 43.5                                      | 43.3       | 13.94           | 0                           | 0.6   | 14.54 |
| 7.5+2.5                          | 43.6                                      | 43.2       | 12.79           | 9.63                        | 1.83  | 24.25 |
| 5+5                              | 43.7                                      | 43.2       | 11.44           | 13.66                       | 3.08  | 28.17 |
| 2.5+7.5                          | 43.8                                      | 43.2       | 9.37            | 15.67                       | 3.99  | 29.03 |
| 0+10                             | 44                                        | 43.1       | 0               | 15.79                       | 5.17  | 20.96 |

TABLE IV: Contributions of $\chi^2$ from appearance and disappearance channels for true NH-HO and $\delta_{CP} = -90^\circ$.

For NH-HO in the LHP we see that the WH-WO solutions give a much higher $\chi^2$ and a run plan with more neutrinos perform better. This can be easily understood by comparing the yellow band and the red band from which it can be observed that there is no such degeneracy in the neutrino mode. However for the antineutrino probability WH-WO-R$\delta_{CP}$ degeneracies can be observed. The NH-LO band (green) corresponding to the RH-WO solution is closer to the NH-HO band and thus the $\chi^2$ for the RH case is lower. In this case also neutrino probabilities do not show any degeneracy. However, it is seen that even then the 2.5+7.5 and 5+5 give slightly better sensitivity even though antineutrinos have degeneracy for this case. In order to understand this in table IV we display the contribution of the different components to $\chi^2$. We ignore a constant prior term in this table. From the table we can see that for 10+0 i.e only neutrino run the contribution from the antineutrinos to the appearance channel $\chi^2$ is zero. As we decrease the neutrino component and increase the antineutrino component the appearance channel contribution from the neutrinos get reduced whereas the antineutrino contribution is enhanced. Since neutrinos do not have any degeneracy for NH-HO whereas antineutrinos possess degeneracies with wrong CP the minima for neutrino and antineutrino do not come in the same position. The overall minima is controlled by the neutrinos because of more statistics. But since this point is not the minima for the antineutrinos they give a large octant sensitive contribution. Thus the tension between neutrinos and antineutrinos help in raising the $\chi^2$ for the cases of mixed runs in spite of degeneracies in the antineutrino channel. There is another interesting feature which can be noticed in this table which is that the disappearance channels also contribute towards octant sensitivity. This is contrary to our expectations because the leading term in this channel goes as $1 - \sin^2 \theta_{23} \sin^2 \Delta m^2_{31} L/4E$ and does not have any octant sensitivity.

To understand this behaviour in fig. 13 we plot the disappearance and appearance $\chi^2$ for neutrinos and antineutrinos separately as a function of test $\theta_{23}$. True $\theta_{23}$ is taken as $48^\circ$. It is seen from the figure and the table that the the global minima does not come at the disappearance minima but the appearance $\chi^2$ being a very steeply rising quantity tends to shift the global minima towards higher values of $\theta_{23}$. This shift is more as the antineutrino component is increased since the antineutrino $\chi^2$ for appearance channel is steeper as compared to the neutrino $\chi^2$. Since the global minima is not the disappearance minima there is finite octant sensitive contribution from the disappearance channel as well.

For IH-HO (blue band) in fig. 3 there is WH-WO degeneracy with NH-LO (green band) at same
FIG. 13: Octant sensitivity $\chi^2$ vs test $\theta_{23}$ (degrees) plots for T2HK.

FIG. 14: Octant sensitivity $\chi^2$ vs $\delta_{CP}$ plots for T2HKK (true NH first row and true IH second row). The labels signify the $\nu+\bar{\nu}$ runs.

CP value in the LHP for neutrinos. Antineutrinos on the other hand have degeneracy with both IH-LO (red band) and NH-LO (green band). However, still the runs with $2.5 + 7.5$ give similar results with $5 + 5$ and $7.5 + 2.5$. This can again be attributed to the tensions between neutrinos and antineutrinos as described for NH-HO. For the UHP IH-HO has degeneracy (both WH-WO and RH-WO, green and red bands respectively) for neutrinos. On the other hand for antineutrinos there is no degeneracy. Therefore $7.5 + 2.5$ give better sensitivities for the WH solutions. Note that for the WH solutions for neutrinos the degeneracy is at right CP value and hence neutrino and antineutrino minima both
occur for right CP. On the other hand for the right hierarchy solutions the neutrino minima occurs in the LHP while the antineutrino minima occurs in the UHP. This the tensions between neutrino and antineutrino occur here also and all the three runtime proportions give similar results.

Fig. 14 shows the octant sensitivity $\chi^2$ vs true $\delta_{CP}$ of T2HKK with NH-LO, NH-HO, IH-LO, IH-HO as true hierarchy–octant configurations, where each plot shows right hierarchy (blue) and wrong hierarchy (magenta) curves for three different run time ratios $2.5\bar{\nu} + 7.5\nu$ (solid), $5\nu + 5\bar{\nu}$ (dotted) and $7.5\nu + 2.5\bar{\nu}$ (dot-dashed). Note that we will get the solutions with lower $\chi^2$ if we marginalize over the hierarchy.

Owing to its longer baseline of 1100 km we have shown in fig. 11 that T2HKK has high hierarchy sensitivity when compared to its counter-proposal T2HK. Thus the wrong hierarchy-octant solutions do not occur here. As a result it can be understood from fig. 14 that the wrong hierarchy solutions NH-IH (magenta) curves have comparatively higher $\chi^2$ in all the four cases and they will get removed once the hierarchy is marginalized.

In section II we have given a detailed account of how the octant sensitivity at 1100 km baseline is very low because of the degeneracies. However, since T2HKK is a hybrid setup with one detector placed at 295 km and another at 1100 km, the probabilities at both baselines govern the physics of this experiment. Thus, one can attribute the considerably large octant sensitivity arising in fig. 14 to be mainly coming from the 295 km baseline. Note that these can be only right hierarchy solutions as the wrong ones get removed at 1100 km.

For instance, in the top left panel of the figure where we assume NH-LO as the true configuration, the proposed run time of $2.5\nu + 7.5\bar{\nu}$ (solid) or $5\nu + 5\bar{\nu}$ (dotted) give a better solution than $7.5\nu + 2.5\bar{\nu}$ in the LHP. This behaviour is the same for the right hierarchy solutions, shown by blue curves when true combination is NH-LO as can be seen from the top left panel of fig. 12. Similarly in the UHP $7.5\nu + 2.5\bar{\nu}$ or $5\nu + 5\bar{\nu}$ is better run, as neutrinos corresponding to NH-NH do not suffer from any degeneracy as seen from fig. 3.

To sum it up, for all true hierarchy-octant combinations, the conclusions corresponding to NH-NH solutions (blue curves) of fig. 14 follow the physics at 295 km baseline i.e. T2HK and can be understood from the detailed description of the NH-NH solutions (blue curves) of fig. 12 presented before.

C. Analysis of CP discovery potential

Fig. 15 shows the CP violation sensitivity of T2HK for different true hierarchy-octant combinations of NH-LO, NH-HO, IH-LO, IH-HO. Each plot shows the sensitivity corresponding to the run time ratios of $2.5\nu + 7.5\bar{\nu}$ (blue-solid), $5\nu + 5\bar{\nu}$ (magenta-dotted) and $7.5\nu + 2.5\bar{\nu}$ (green-dash-dotted). The figures show that in all cases the CP discovery potential is much less in one of the half-planes. This can be attributed to the presence of wrong hierarchy solutions as can be seen in fig 11. However, we checked that the minima always occurs with the right octant. It is known that the role of antineutrinos for T2HK baseline and energy is to remove the wrong octant solutions [28]. Therefore, more neutrinos will help because of enhanced statistics. This can be seen from the plots in fig. 15 which show that the 1:1 or 3:1 ratios give slightly better results excepting true NH-LO. For this case, as can be observed from the left plot in the top panel, around the maximal $\delta_{CP}$ in the LHP, the sensitivity of $5\nu + 5\bar{\nu}$ gives almost the same result as the proposed $2.5\nu + 7.5\bar{\nu}$. However, the sensitivity of $7.5\nu + 2.5\bar{\nu}$ i.e. for more neutrino run, is lower. This is because as seen from fig. 3 NH-LO is degenerate with NH-HO at $\delta_{CP} = 0, \pm 180$ and IH-HO for right $\delta_{CP}$. But, in the case of antineutrinos NH-LO has degeneracy only with IH-LO at right $\delta_{CP}$. As a result more antineutrinos are helping the sensitivity.

Fig. 16 displays the CP discovery potential of T2HKK. One major difference between the sensitivity of T2HK (in fig. 15 and T2HKK is that the UHP of true NH and the LHP of the true IH no longer suffer from degeneracy coming from the wrong hierarchy solutions as these get removed at the 1100 km baseline. On the other hand the wrong octant solutions are addressed by the 295 km baseline as seen earlier. Overall CP discovery potential of $5\nu + 5\bar{\nu}$ is comparatively better than the proposed run
FIG. 15: CP discovery potential $\chi^2$ vs $\delta_{CP}$ plots for T2HK (true NH first row and true IH second row). The labels signify the $\nu+\bar{\nu}$ runs.

of 2.5$\nu + 7.5\bar{\nu}$ years in all the four cases.

VII. CONCLUSIONS

In this paper we have performed a comprehensive comparative analysis of the hierarchy, octant and CP discovery sensitivities of the proposed high statistics experiments T2HK, T2HKK and DUNE. We have also obtained the sensitivities by including the projected full runs of T2K and NO$\nu$A. We have discussed the underlying physics issues which could cause the differences in the sensitivities. In particular we present a detailed discussion on the octant sensitivities of the T2HKK experiment bringing out the salient features of the relevant probabilities near the second oscillation maxima.

Our study shows that with their proposed fiducial volume and run time, T2HKK and DUNE experiments can achieve very high hierarchy sensitivity. The T2HK experiment on the other hand cannot resolve hierarchy-$\delta_{CP}$ degeneracy in the range $-180^\circ < \delta_{CP} < 0$ for IH and $0 < \delta_{CP} < 180^\circ$ for NH. Hence in these unfavourable regions the hierarchy sensitivity can reach maximum 3$\sigma$. In the favourable half-plane of $\delta_{CP}$ more than 5$\sigma$ sensitivity is possible for T2HK stand-alone and more than 6$\sigma$ including T2K and NO$\nu$A information. Highest hierarchy sensitivity can be achieved by DUNE. On the other hand T2HK has the highest octant sensitivity among all the three experiments. T2HKK and DUNE have comparable octant sensitivity. The CP discovery potential on the other hand is best for T2HKK. T2HK gives comparable sensitivity to T2HKK for favourable values of $\delta_{CP}$. However for unfavourable $\delta_{CP}$ values due to wrong hierarchy-wrong CP solutions the CP discovery potential suffers. DUNE does not have hierarchy degeneracy. But it’s sensitivity to CP discovery potential is lower because of lower volume as compared to T2HK and T2HKK.
We also compute the optimal exposure for $5\sigma$ hierarchy and octant sensitivity of all the experiments both stand-alone and in conjunction with T2K and NO\(\nu\)A results. The sensitivity study is done for two representative configurations assuming true NH-LO ($\theta_{23} = 42^\circ$) and IH-HO ($\theta_{23} = 48^\circ$). For hierarchy sensitivity better result is obtained for IH-HO. We find that for this case DUNE can attain $5\sigma$ sensitivity in approximately 3 years (equal neutrino and antineutrino run) and with T2K and NO\(\nu\)A information it is just 1.5 years with a 40 kt volume. T2HKK with it’s proposed volume can achieve this in 3.6 and 2.3 years respectively. Optimum exposure required to obtain $5\sigma$ octant sensitivity is found to be less when we assumed true NH-LO. In this case T2HK can deliver the result in 3.8 years and with T2K and NO\(\nu\)A the same can be achieved within 2.7 years. On the other hand, T2HKK and DUNE would need more than 10 years of exposure to attain this goal. We also present the optimal volume for which $3\sigma$ CP violation discovery sensitivity can be reached for 60% fraction of $\delta_{CP}$ values for both NH and IH. This is found to be 400 kt-yr for T2HKK for both true NH and IH. For DUNE the optimum exposure is 500 kt-yr (400 kt-yr) for DUNE for true NH (IH). Since T2HKK has larger volume, it can therefore achieve the same goal in lesser time.

Finally, we study if the 1:3 neutrino-antineutrino run as proposed by T2HK and T2HKK is the best option or the alternative ratios of 3:1/1:1 give better sensitivities. We find that for hierarchy sensitivity of T2HK this does not make much difference. But for T2HKK for certain range of $\delta_{CP}$ values 3:1 ratio for neutrinos and antineutrinos give better hierarchy sensitivity. Additionally, we observed that the $\nu\bar{\nu}$ run time ratio of 1:1 results almost as good (or sometimes better) sensitivity as the proposed 3:1 ratio. For octant sensitivity there is an interesting interplay between the $\nu\bar{\nu}$ run time ratios since the octant degeneracy is different for neutrinos and antineutrinos. However, $(\nu + \bar{\nu})$ combination can raise the $\chi^2$ since their corresponding minimas do not occur at the same point. Apart
from this the tension between the appearance and the disappearance channel shifts the minima from the disappearance minima thus leading to an octant sensitive contribution from this channel. For T2HK experiment when true hierarchy is IH and test hierarchy is marginalized we find that the octant sensitivity for all three run time ratios is similar. But when we assume true NH-LO the sensitivity of 3:1 is poorer than 1:3 and 1:1 which are almost similar. But for NH-HO in the UHP 1:1 and 3:1 ratios perform better. For T2HKK, overall there are parameter spaces near $\delta_{CP} = +90^\circ$, where 3:1 ratio give better results. Otherwise all the three scenarios are in the same ballpark. The CP discovery potential for T2HK is almost the same for the three run time ratios. For T2HKK the 3:1 and 1:1 ratios give slightly better results.

To conclude, all the three proposed next generation experiments DUNE, T2HK and T2HKK hold the promise to measure the unknown neutrino oscillation parameters with a high sensitivity.

VIII. ACKNOWLEDGEMENT

Authors would like to thank Monojit Ghosh for many helpful discussions.

[1] K. Abe et al. Combined Analysis of Neutrino and Antineutrino Oscillations at T2K. *Phys. Rev. Lett.*, 118(15):151801, 2017.
[2] P. Adamson et al. Constraints on Oscillation Parameters from $\nu_e$ Appearance and $\nu_\mu$ Disappearance in NOvA. *Phys. Rev. Lett.*, 118(23):231801, 2017.
[3] F. Capozzi, E. Lisi, A. Marrone, D. Montanino, and A. Palazzo. Neutrino masses and mixings: Status of known and unknown 3$\nu$ parameters. *Nucl. Phys.*, B908:218–234, 2016.
[4] Ivan Esteban, M. C. Gonzalez-Garcia, Michele Maltoni, Ivan Martinez-Soler, and Thomas Schwetz. Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity. *JHEP*, 01:087, 2017.
[5] P. F. de Salas, D. V. Forero, C. A. Ternes, M. Tortola, and J. W. F. Valle. Status of neutrino oscillations 2017. 2017.
[6] K. Abe et al. A Long Baseline Neutrino Oscillation Experiment Using J-PARC Neutrino Beam and Hyper-Kamiokande. 2014.
[7] K. Abe et al. Physics Potentials with the Second Hyper-Kamiokande Detector in Korea. 2016.
[8] R. Acciarri et al. Long-Baseline Neutrino Facility (LBNF) and Deep Underground Neutrino Experiment (DUNE). 2015.
[9] Sanjib Kumar Agarwalla, Suprabh Prakash, and S. Uma Sankar. Exploring the three flavor effects with future superbeams using liquid argon detectors. *JHEP*, 03:087, 2014.
[10] Monojit Ghosh, Srubabati Goswami, and Sushant K. Raut. Maximizing the DUNE early physics output with current experiments. *Eur. Phys. J.*, C76(3):114, 2016.
[11] Kalpana Bora, Debajyoti Dutta, and Pomita Ghoshal. Determining the octant of $\theta_{23}$ at LBNE in conjunction with reactor experiments. *Mod. Phys. Lett.*, A30(14):1550066, 2015.
[12] Vernon Burger, Atri Bhattacharya, Animesh Chatterjee, Raj Gandhi, Danny Marfatia, and Mehedi Masud. Configuring the Long-Baseline Neutrino Experiment. *Phys. Rev.*, D89(1):011302, 2014.
[13] K. N. Deepthi, C. Soumya, and R. Mohanta. Revisiting the sensitivity studies for leptonic CP-violation and mass hierarchy with T2K, NOuA and LBNE experiments. *New J. Phys.*, 17(2):023035, 2015.
[14] Newton Nath, Monojit Ghosh, and Srubabati Goswami. The physics of antineutrinos in DUNE and determination of octant and $\delta_{CP}$. *Nucl. Phys.*, B913:381–404, 2016.
[15] C. Soumya, K. N. Deepthi, and R. Mohanta. A comprehensive study of the discovery potential of NOvA, T2K and T2HK experiments. *Adv. High Energy Phys.*, 2016:9139402, 2016.
[16] Pilar Coloma, Patrick Huber, Joachim Kopp, and Walter Winter. Systematic uncertainties in long-baseline neutrino oscillations for large $\theta_{13}$. *Phys. Rev.*, D87(3):033004, 2013.
[17] Peter Ballett, Stephen F. King, Silvia Pascoli, Nick W. Prouse, and TseChun Wang. Sensitivities and synergies of DUNE and T2HK. *Phys. Rev.*, D96(3):033003, 2017.

[18] Sanjib Kumar Agarwalla, Monojit Ghosh, and Sushant K. Raut. A hybrid setup for fundamental unknowns in neutrino oscillations using T2HK (ν) and μ-DAR (ν̄). *JHEP*, 05:115, 2017.

[19] Monojit Ghosh and Osamu Yasuda. Effect of systematics in the T2HK, T2HKK, and DUNE experiments. *Phys. Rev.*, D96(1):013001, 2017.

[20] Sushant K. Raut. Matter effects at the T2HK and T2HKK experiments. *Phys. Rev.*, D96(7):075029, 2017.

[21] Evgeny K. Akhmedov, Robert Johansson, Manfred Lindner, Tommy Ohlsson, and Thomas Schwetz. Series expansions for three flavor neutrino oscillation probabilities in matter. *JHEP*, 04:078, 2004.

[22] Sanjib Kumar Agarwalla, Suprabh Prakash, and S. Uma Sankar. Resolving the octant of theta23 with T2K and NOνA. *JHEP*, 07:131, 2013.

[23] Monojit Ghosh, Pomita Ghoshal, Srubabati Goswami, Newton Nath, and Sushant K. Raut. New look at the degeneracies in the neutrino oscillation parameters, and their resolution by T2K, NOνA and ICAL. *Phys. Rev.*, D93(1):013013, 2016.

[24] Y. Itow et al. The JHF-Kamioka neutrino project. In *Neutrino oscillations and their origin. Proceedings, 3rd International Workshop, NOON 2001, Kashiwa, Tokyo, Japan, December 508, 2001*, pages 239–248, 2001.

[25] Patrick Huber, M. Lindner, and W. Winter. Simulation of long-baseline neutrino oscillation experiments with GLoBES. *Comput. Phys. Commun.*, 167:195, 2005.

[26] Patrick Huber, Joachim Kopp, Manfred Lindner, Mark Rolinec, and Walter Winter. New features in the simulation of neutrino oscillation experiments with GLoBES 3.0. *Comput. Phys. Commun.*, 177:432–438, 2007.

[27] Sushant K. Raut. Effect of non-zero theta(13) on the measurement of theta(23). *Mod.Phys.Lett.*, A28:1350093, 2013.

[28] Monojit Ghosh. Reason for T2K to run in dominant neutrino mode for detecting CP violation. *Phys. Rev.*, D93(7):073003, 2016.