Generalized intuitionistic fuzzy soft matrices and their application

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Abstract. The purpose of this article is to define the generalized intuitionistic fuzzy soft matrices and to study their operations. Then we obtain some relevant properties of generalized intuitionistic fuzzy soft matrices. To apply the generalized intuitionistic fuzzy soft matrices for decision making problems solving, we define the Rate matrix, the Resultant matrix and the Goal matrix. Then by developing an algorithm, finally the decision is obtained based on the maximum score in the Goal matrix. This study is a generalization the concept of intuitionistic fuzzy soft matrices.

1. Introduction
Most of the traditional tools for formal modeling, reasoning, and computation are crisp sets and deterministic. However, in the real life, there are many complex problems in engineering, economic, environmental, social sciences, medicine etc. Those problems involve data that are not always firm, precise and deterministic, because of different types of problems are uncertain. Such uncertainty problems can be addressed with the help of theories, such as the probability theory, the fuzzy sets theory, intuitionistic fuzzy sets theory, etc. One of the fuzzy set theories is on fuzzy algebra. Nazra [1] has studied on ideals and their fuzzifications in implicative semigroups as a theoretical mathematics problems related to the concept of the fuzzy sets.

Molodtsov [2] had initiated a new concept called "Soft Set Theory" which is a new approach to deal with things that contains uncertainty or ambiguity. The soft sets theory has rich potential to be applied in resolving problems in the fields of economics, social sciences, medicine, etc. Researchers in the field of the soft sets theory had high attention in recent years. Maji et al. [3] are who first introduced the soft set theory in decision-making problems. Then Maji et al. [4] introduced the concept of fuzzy soft sets which are a combination of soft sets and fuzzy sets. As a combination between the concept of hesitant fuzzy sets, intuitionistic fuzzy sets and soft sets, Nazra et al. [5,6] have studied on hesitant intuitionistic fuzzy soft sets and its generalization.

Furthermore, Cagman and Enginoğlu [7] defined fuzzy soft matrices which are representations of fuzzy soft sets and constructed a fuzzy soft max-min decision making method. Likewise, Borah et al. [8] developed a fuzzy soft matrix theory and its application. These theories continue to grow, where Deli and Cagman [9] introduced the intuitionistic fuzzy parameterized soft sets theory. They also have applied the theory for the problems that contain uncertainty.
Researchers have attempted to generalize the previous theories. Babitha and John [10] and also Dinda et al. [11] introduced a concept on generalized intuitionistic fuzzy soft sets and applied it to solve various decision making problems. Rajarajeswari and Dhanalakshmi [12] explained the concept of intuitionistic fuzzy soft matrices with some mathematical operations. Researches on the intuitionistic fuzzy soft matrices were further developed by Mondal and Roy [13] and Basu et al. [14]. They have introduced the definition of an intuitionistic fuzzy soft matrix, constructed some mathematical operations and defined a weighting operator that is useful for solving the decision making problems. In 2014 there was a study conducted by Saikia et al. [15] on generalized fuzzy soft matrices as a development of fuzzy soft matrices.

Refer to the method developed by Saikia et al. [15] in formulating generalized fuzzy soft matrices, and the method used by Babitha and John [10] and Dinda et al. [11] to study generalized intuitionistic fuzzy soft sets, it is interesting to study on generalized intuitionistic fuzzy soft matrices. In this paper we give some properties of generalized intuitionistic fuzzy soft matrices and their application in a decision making problem.

2. Preliminaries

First of all, we review the concept on fuzzy soft sets, fuzzy soft matrices and generalized intuitionistic fuzzy soft sets.

Given a set \( H \). A set \( X = \{(h, \mu(h))| h \in H\} \) where \( \mu \) is a mapping given by \( \mu: H \rightarrow [0,1] \) is called a fuzzy set over the set \( H \). Here \( \mu \) and \( \mu(h) \) are called the membership function of \( X \) and the membership value of \( h \) respectively. We sometimes call \( \mu \) as a fuzzy set over the set \( H \).

**Definition 1.** [4] Let \( U \) be a universal set and \( E \) be a paramater set. Suppose \( A \subseteq E \). A pair \((f_A, A)\) is called a fuzzy soft set over \( U \) if \( f_A \) is a mapping given by \( f_A: A \rightarrow U^I \) where \( U^I \) is the collection of all fuzzy sets over \( U \).

Let \((f_A, A)\) be a fuzzy soft set over \( U \). Given a mapping \( \mu_{f_A}: U \times A \rightarrow [0,1] \) where \( \mu_{f_A}(u, e) \in [0,1] \) is a membership value of \( u \in U \) for each \( e \in A \). If \( \mu_{f_A}(u, e) = \mu_{ij} \) then it can be defined a matrix

\[
[\mu_{ij}]_{m \times n} = \begin{bmatrix}
\mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\
\mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\mu_{m1} & \mu_{m2} & \cdots & \mu_{mn}
\end{bmatrix}
\]

which is called a fuzzy soft matrix \((f_A, A)\) over \( U \) [14]. The collection of all fuzzy soft matrices over \( U \) is denoted by \( FSM_{m \times n} \). Here \( m := |U| \) and \( n := |A| \).

**Definition 2.**

Let \([\mu_{ij}] \in FSM_{m \times n} \) and \([\gamma_{ij}] \in FSM_{n \times p} \). Then we define the product of \([\mu_{ij}] \) and \([\gamma_{ij}] \) as

\[
[\mu_{ij}] \otimes [\gamma_{ij}] = [c_{ij}]_{n \times p}
\]

where

\[
c_{ij} = \{\max\{\min\{\mu_{ik}, \gamma_{kj}\}\}\}
\]

for \( k = 1, \cdots, n \).

**Definition 3.** [15] Let \( U \) be a set. An intuitionistic fuzzy set \( X \) over the set \( U \) is defined as

\[
X = \{(z, \mu_X(z), \gamma_X(z))| z \in U\}
\]

where \( \mu_X: U \rightarrow [0,1] \) and \( \gamma_X: U \rightarrow [0,1] \) such that \( 0 \leq \mu_X(z) + \gamma_X(z) \leq 1 \) for all \( z \in U \).

Here \( \mu_X(z) \) is called the membership value of \( z \) and \( \gamma_X(z) \) is the non-membership value of \( z \).
**Definition 4.** [16] Given a universal set $U$ and a parameter set $E$. Suppose that $IF^U$ is a collection of all intuitionistic fuzzy sets over $U$. Let $A \subseteq E$. A pair $(\hat{F}_A, E)$ is called an intuitionistic fuzzy soft set over $U$ if $\hat{F}_A$ is a mapping given by $\hat{F}_A : E \rightarrow IF^U$ and $\hat{F}_A(e) = \emptyset$ for $e \notin A$.

**Definition 5.** [9] Let $A \subseteq E$. Given a mapping $E : A \rightarrow IF^U$ and a fuzzy set $\alpha$ over $A$. Then $E_{\alpha} : A \rightarrow IF^U \times [0,1]$ is a function defined by

$$E_{\alpha}(a) = (E(a), \alpha(a)) = \left\{ (z, \mu_{E(a)}(z), \gamma_{E(a)}(z)) \mid z \in U \right\}, \alpha(a)$$

where $\mu_{E(a)}(z)$ and $\gamma_{E(a)}(z)$ are the membership and the non-membership values of $z \in U$ respectively.

$E_{\alpha}$ is called a generalized intuitionistic fuzzy soft set over $(U, E)$.

3. Main Results

In this section firstly we introduce the definition of generalized intuitionistic fuzzy soft matrices. Then we define some operations so that we get some properties corresponding to such operations.

**Definition 6.** Suppose that $E_{\alpha}$ is a generalized intuitionistic fuzzy soft set over $(U, E)$. Then given a subset of $U \times E$ namely $\hat{R}_A = \{(u, e) \mid e \in A \subseteq E, u \in U\}$ which is called the relation form of $(U, E)$. The relation $\hat{R}_A$ is characterized by a membership function $\mu_{E(e)} : U \times E \rightarrow [0,1]$, and a non-membership function $\gamma_{E(e)} : U \times E \rightarrow [0,1]$, where $\mu_{E(e)}(u, e)$ and $\gamma_{E(e)}(u, e)$ are the membership and the non-membership values of $u$ respectively, related to a parameter $e$, while $\alpha_A : A \rightarrow [0,1]$ is a degree of preference of such membership and non-membership values. Suppose that $U = \{u_1, u_2, ..., u_m\}$ and $E = \{e_1, e_2, ..., e_n\}$, then $\hat{R}_A$ can be presented as the shown in Table 1.

| Table 1. Degree of preference of membership and non-membership |
|-------------------|----------------|----------------|
| $u_1$             | $\mu_{A_{11}}, \gamma_{A_{11}}, \alpha_{A_{11}}$ | $\mu_{A_{12}}, \gamma_{A_{12}}, \alpha_{A_{12}}$ | ... | $\mu_{A_{1n}}, \gamma_{A_{1n}}, \alpha_{A_{1n}}$ |
| $u_2$             | $\mu_{A_{21}}, \gamma_{A_{21}}, \alpha_{A_{21}}$ | $\mu_{A_{22}}, \gamma_{A_{22}}, \alpha_{A_{22}}$ | ... | $\mu_{A_{2n}}, \gamma_{A_{2n}}, \alpha_{A_{2n}}$ |
| $\vdots$          | $\vdots$       | $\vdots$       | $\checkmark$ | $\vdots$ |
| $u_m$             | $\mu_{A_{m1}}, \gamma_{A_{m1}}, \alpha_{A_{m1}}$ | $\mu_{A_{m2}}, \gamma_{A_{m2}}, \alpha_{A_{m2}}$ | ... | $\mu_{A_{mn}}, \gamma_{A_{mn}}, \alpha_{A_{mn}}$ |

where $(\mu_{A_{ij}}, \gamma_{A_{ij}}, \alpha_{A_{ij}}) = (\mu_{E(e_j)}(u_i, e_j), \gamma_{E(e_j)}(u_i, e_j), \alpha_A(e_j))$. Sketchily it can be written $(\mu_{A_{ij}}, \gamma_{A_{ij}}, \alpha_{A_{ij}}) = \hat{a}_{ij}(u_i, e_j) = \hat{a}_{ij}$. Therefore it can be defined the following matrix

$$\hat{a}_{ij} = \hat{a}_{ij} \mid_{m \times n} = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} & \cdots & \hat{a}_{1n} \\ \hat{a}_{21} & \hat{a}_{22} & \cdots & \hat{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{a}_{m1} & \hat{a}_{m2} & \cdots & \hat{a}_{mn} \end{bmatrix}$$

which is called the generalized intuitionistic fuzzy soft matrix $m \times n$ corresponding to a generalized intuitionistic fuzzy soft set $E_{\alpha}$ over $(U, E)$. Here $\hat{a}_{ij}$ is defined as follows

$$\hat{a}_{ij}(u_i, e_j) = \begin{cases} (\mu_{E(e_j)}(u_i, e_j), \gamma_{E(e_j)}(u_i, e_j), \alpha_A(e_j)) & \text{if } e_j \in A \\ (0,1,0) & \text{if } e_j \notin A. \end{cases}$$

The collection of all generalized intuitionistic fuzzy soft matrices over $(U, E)$ is denoted by $GIFS_{m \times n}$.

**Definition 7.** Let $[\hat{a}_{mn}] \in GISFM_{m \times n}$. Then $[\hat{a}_{ij}]$ is called:

a) a zero matrix, denoted by $\hat{0}$ if $\mu_{A_{ij}} = 0$, $\gamma_{A_{ij}} = 1$, $\alpha_{A_{ij}} = 0$ for all $i, j$.

b) a universal matrix, denoted by $\hat{U}$ if $\mu_{A_{ij}} = 1$, $\gamma_{A_{ij}} = 0$, $\alpha_{A_{ij}} = 1$ for all $i, j$.  

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Definition 8. Suppose that $\bar{a}_{ij} = [(\mu_{Aij}, \gamma_{Aij}, \alpha_{Aij})]$. $\bar{b}_{ij} = [(\mu'_{Aij}, \gamma'_{Aij}, \alpha'_{Aij})] \in \text{GIFSM}_{m \times n}$. $\bar{a}_{ij}$ is called the generalized intuitionistic fuzzy soft submatrix of $\bar{b}_{ij}$ denoted by $\bar{a}_{ij} \subseteq \bar{b}_{ij}$ if $\mu_{Aij} \leq \mu'_{Aij} \cdot \gamma_{Aij} \geq \gamma'_{Aij}$ and $\alpha_{Aij} \leq \alpha'_{Aij}$ for all $i, j$.

Proposition 9. Let $\bar{a}_{ij}$, $\bar{b}_{ij}$, $\bar{c}_{ij}$ be any three generalized intuitionistic fuzzy soft matrices over $(U, E)$. Then the following properties hold.

1. $\bar{O} \subseteq \bar{a}_{ij}$.
2. $\bar{a}_{ij} \subseteq \bar{b}_{ij}$.
3. $\bar{a}_{ij} \subseteq \bar{c}_{ij}$.
4. If $\bar{a}_{ij} \subseteq \bar{b}_{ij}$ and $\bar{b}_{ij} \subseteq \bar{c}_{ij}$ then $\bar{a}_{ij} \subseteq \bar{c}_{ij}$.

Proof. The proof of the above properties follows from Definitions 7 and 8.

Definition 10. Suppose $\bar{a}_{mn} = [(\mu_{Amn}, \gamma_{Amn}, \alpha_{Amn})]$. $\bar{b}_{mn} = [(\mu'_{Amn}, \gamma'_{Amn}, \alpha'_{Amn})] \in \text{GIFSM}_{p \times q}$. Then the generalized intuitionistic fuzzy soft matrix $\bar{c}_{mn} = [(\mu''_{Amn}, \gamma''_{Amn}, \alpha''_{Amn})] \in \text{GIFSM}_{p \times q}$ is called

i. union of $\bar{a}_{mn}$ and $\bar{b}_{mn}$ denoted by $\bar{a}_{mn} \cup \bar{b}_{mn}$ if $\mu''_{Amn} = \max(\mu_{Amn}, \mu'_{Amn})$, $\gamma''_{Amn} = \min(\gamma_{Amn}, \gamma'_{Amn})$, and $\alpha''_{Amn} = \max(\alpha_{Amn}, \alpha'_{Amn})$ for all $m, n$. 

ii. intersection of $\bar{a}_{mn}$ and $\bar{b}_{mn}$ denoted by $\bar{a}_{mn} \cap \bar{b}_{mn}$ if $\mu''_{Amn} = \min(\mu_{Amn}, \mu'_{Amn})$, $\gamma''_{Amn} = \max(\gamma_{Amn}, \gamma'_{Amn})$, and $\alpha''_{Amn} = \min(\alpha_{Amn}, \alpha'_{Amn})$ for all $m, n$.

iii. complement of $\bar{a}_{mn}$ denoted by $\bar{c}_{mn} = [\bar{\alpha}_{mn}]^c$ if $\mu''_{Amn} = 1 - \mu_{Amn}$, $\gamma''_{Amn} = 1 - \gamma_{Amn}$, and $\alpha''_{Amn} = 1 - \alpha_{Amn}$ for all $m, n$.

Definition 11. Suppose $\bar{a}_{ij}$, $\bar{b}_{ij}$ $\in \text{GIFSM}_{m \times n}$. Then $\bar{a}_{ij}$ and $\bar{b}_{ij}$ is called disjoin if $\bar{a}_{ij} \cap \bar{b}_{ij} = \emptyset$.

Proposition 12. Suppose $\bar{a}_{mn}$, $\bar{b}_{mn}$, $\bar{c}_{mn} \in \text{GIFSM}_{p \times q}$. Then

1. $[(\bar{a}_{mn})^c]^c = \bar{a}_{mn}$.
2. $[(\bar{a}_{mn} \cup \bar{b}_{mn})]^c = \bar{a}_{mn}^c \cap \bar{b}_{mn}^c$.
3. $[(\bar{a}_{mn} \cap \bar{b}_{mn})]^c = \bar{a}_{mn}^c \cup \bar{b}_{mn}^c$.
4. $\bar{a}_{mn} \cap \bar{b}_{mn} = [\bar{b}_{mn}] \cap [\bar{a}_{mn}]$.
5. $\bar{a}_{mn} \cup \bar{b}_{mn} = [\bar{b}_{mn}] \cup [\bar{a}_{mn}]$.
6. $[(\bar{a}_{mn} \cap \bar{b}_{mn})] \cup [\bar{c}_{mn}] = [\bar{a}_{mn}] \cap ([b_{mn} \cap [\bar{c}_{mn}])$.
7. $[(\bar{a}_{mn} \cup \bar{b}_{mn})] \cup [\bar{c}_{mn}] = [\bar{a}_{mn}] \cup ([b_{mn} \cup [\bar{c}_{mn}])$.
8. $[\bar{a}_{mn}] \cap ([b_{mn} \cup [\bar{c}_{mn}]) = ([\bar{a}_{mn}] \cap [b_{mn}]) \cup ([\bar{a}_{mn}] \cap \bar{c}_{mn})$.
9. $[\bar{a}_{mn}] \cap ([b_{mn} \cup [\bar{c}_{mn}]) = ([\bar{a}_{mn}] \cap [b_{mn}]) \cup ([\bar{a}_{mn}] \cap \bar{c}_{mn})$.

Proof. The proof of the above properties follows from Definition 10.

4. An Application of generalized intuitionistic fuzzy soft matrices

In this section we implement generalized intuitionistic fuzzy soft matrices with its operations to solve a real life problem. Before that we give the following definitions.

Definition 13. Suppose that $\bar{a}_{ij} = [(\mu_{Aij}, \gamma_{Aij}, \alpha_{Aij})] \in \text{GIFSM}_{m \times n}$. Then the Rate matrix of a generalized intuitionistic fuzzy soft matrix $\bar{a}_{ij}$ is $R(\bar{a}_{ij}) = r_{ij}$, where $r_{ij} = (\mu_{Aij} - \gamma_{Aij}, \alpha_{Aij})$ for all $i, j$. 


Definition 14. Suppose that \( R = R([\hat{a}_{ij}]) \) is a Rate matrix of a generalized intuitionistic fuzzy soft matrix \([\hat{a}_{ij}]\). Then the Resultant matrix of \( R \) is \( R(R) = \{t_{ij}\} \) where \( t_{ij} = (\mu_{A_{ij}} - \gamma_{A_{ij}})\alpha_{A_{ij}} \) for all \( i, j \).

Definition 15. Suppose that \([\hat{a}_{ij}] = ([\mu_{A_{ij}}, \gamma_{A_{ij}}, \alpha_{A_{ij}}]) \), \([\hat{b}_{ij}] = ([\mu_{B_{ij}}, \gamma_{B_{ij}}, \alpha_{B_{ij}}]) \) \( \in \) \( \text{GIFSM}_{m \times n} \). Then the Goal matrix of \([\hat{a}_{ij}] \) and \([\hat{b}_{ij}] \) is \( G([\hat{a}_{ij}], [\hat{b}_{ij}]) = [g_{ij}] = T \left( R([\hat{a}_{ij}]) \right) - T \left( R([\hat{b}_{ij}]) \right) \) for all \( i, j \).

Suppose that we are given a set \( U \) of objects related to a set \( E \) of features. Then the set \( E \) has a view number of parameters as a set \( S \). Assume that there are two decision makers need to take a decision. They want to know for every object in set \( U \) which parameter in set \( S \) is very related to.

Suppose that a married couple, Mr. X and Mrs. X, want to buy \( n \) number of different dresses (denoted by the set \( U = (d_1, d_2, ..., d_n) \)) with a set \( E = (e_1, e_2, ..., e_m) \) of features related to a set \( S = (s_1, s_2, ..., s_p) \) of \( p \) stores. By applying the generalized intuitionistic fuzzy soft matrix theory to find the proper store for buying every dress \( d_i \), every decision maker can construct the generalized intuitionistic fuzzy soft matrices over \( (U,E) \) and \( (E,S) \). We denote the generalized intuitionistic fuzzy soft matrices \( [\hat{a}_{ij}]_{n \times m} = ([\mu_{A_{ij}}, \gamma_{A_{ij}}, \alpha_{A_{ij}}]) \) and \( [\hat{b}_{ij}]_{m \times p} = ([\mu_{B_{ij}}, \gamma_{B_{ij}}, \alpha_{B_{ij}}]) \) \( (l = 1 \text{ for husband and } l = 2 \text{ for wife}) \) corresponding to generalized intuitionistic fuzzy soft sets over \( (U,E) \) and \( (E,S) \) respectively.

To solve this particular problem the decision makers go through the following algorithm.

Algorithm
1) Define the generalized intuitionistic fuzzy soft matrices \([\hat{a}_{ij}]_{n \times m} \) and \([\hat{b}_{ij}]_{m \times p} \) for \( l = 1, 2 \).
2) Calculate \([\hat{c}_{ij}] = [\hat{a}_{ij}] \cap [\hat{a}_{ij}] \) and \([\hat{d}_{ij}] = [\hat{b}_{ij}] \cap [\hat{b}_{ij}] \).
3) Determine generalized intuitionistic fuzzy soft complement matrices \([\hat{e}_{ij}]^c \) and \([\hat{d}_{ij}]^c \).
4) Calculate the Rate matrices \( R_1 = [r1_{ij}]_{n \times m} = R([\hat{c}_{ij}]) \), \( R_2 = [r2_{ij}]_{m \times p} = R([\hat{d}_{ij}]) \), \( R_3 = [r3_{ij}]_{n \times m} = R([\hat{e}_{ij}]^c) \), and \( R_4 = [r4_{ij}]_{m \times p} = R([\hat{d}_{ij}]^c) \).
5) Calculate the Resultant matrices \( T(R_1), T(R_2), T(R_3) \) and \( T(R_4) \). It is clear that \( T(R_1) \) is a fuzzy soft matrix by the definition.
6) Compute \( [x_{ij}]_{n \times p} := T(R_1) \otimes T(R_2) \) and \( [y_{ij}]_{n \times p} := T(R_3) \otimes T(R_4) \).
7) Calculate the Goal matrix \( G = [x_{ij}] - [y_{ij}] = [g_{ij}] \).
8) Find the dress \( d_i \) for which \( max\{g_{ij} | j = 1, ..., p\} \).

Finally we conclude that for every \( i \) (i.e the dress \( d_i \)), if \( max\{g_{ij} | j = 1, ..., p\} = g_{ij_o} \), it means the store \( s_{j_o} \) is the right choice to buy the dress \( d_i \).

5. Conclusion
In this paper, the notion of the generalized intuitionistic fuzzy soft matrix theory is introduced. By defining several operations, we get some properties of generalized intuitionistic fuzzy soft matrices. Then we implement the concept of generalized intuitionistic fuzzy soft matrices to develop an algorithm for solving a decision-making problem.

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