Nonlinear Active Disturbance Rejection Control of VGT-EGR System in Diesel Engines

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Abstract: In this paper, a nonlinear active disturbance rejection control (NLADRC) strategy based on nonlinear extended state observer (NLESO) is proposed to solve the unmodeled dynamics, coupling and disturbance due to change of working point in the variable geometry turbine (VGT) and exhaust gas recirculation (EGR) system, so as to achieve accurate control of intake manifold pressure and mass air flow in a diesel engine. To achieve decoupling, the double-input double-output (DIDO) VGT-EGR system is decomposed into two single-input single-output (SISO) subsystems, and each subsystem has a separate nonlinear active disturbance rejection controller. At the same time, the convergence proof of the designed NLESO is also given theoretically. Finally, the NLADRC controller is compared with linear active disturbance rejection controller and proportional–integral–derivative (PID) controller. Through simulation, it is indicated that the proposed NLADRC controller has better transient response performance, resistance to external disturbance and robustness to the change of engine operating point.

Keywords: nonlinear active disturbance rejection controller; variable geometry turbine; exhaust gas recirculation; diesel engine

1. Introduction

As one of the main power sources of trucks and ships, diesel engines have the characteristics of strong power performance, good fuel economy and long service life. However, the increasing number of diesel engines makes energy shortage more serious. In addition, the exhaust emissions of diesel engines are mainly nitrogen oxide (NOx) and particulate matter (PM), which are one of the main causes of haze formation. With the increasingly stringent emission regulations and the continuous improvement of power and economy requirements, variable geometry turbine (VGT) and exhaust gas recirculation (EGR) technology has become widely used in diesel engines. Part of the exhaust gas generated in the cylinder is delivered to the intake manifold through the EGR valve to reduce NOx emissions. However, excessive exhaust gas entering the intake manifold can cause insufficient fuel combustion. The other part of the exhaust gas through the VGT valve drives the turbine, which drives the compressor to absorb more fresh air and improve the combustion efficiency. However, excessive exhaust gas emissions will reduce the exhaust gas back to the intake manifold, thus increasing NOx emissions. In addition, complex system dynamics phenomena such as sign inversion, nonminimum phase and overshoot also exist in VGT-EGR system [1–3]. Therefore, the precise control of VGT-EGR system in diesel engines is very important to reduce emissions. At the same time, cross coupling and complex system dynamics make it difficult to control VGT-EGR system accurately.
For the control of VGT-EGR system in diesel engines, many control strategies have been proposed by scholars. To achieve the required emission level and safe operation of the engine and turbocharger, Wahlström et al. [4] proposed a proportional–integral–derivative (PID) structure that controls the air/fuel ratio $\lambda$ and the intake manifold EGR fraction $x_{egr}$. The air/fuel ratio $\lambda$ is controlled by the EGR valve and the EGR fraction $x_{egr}$ is controlled by the VGT position. In addition, Wahlström and Eriksson [5] proposed a control structure composed of PID controller and nonlinear compensator. Compared with the control structure without nonlinear compensator, this control structure deals with nonlinear effects and reduces EGR error. Beside PID control method, other commonly used control methods are also applied in the control of VGT-EGR system in diesel engines, such as sliding mode control [6,7], model predictive control (MPC) [8,9] and control Lyapunov function (CLF) [10,11]. From the perspective of system theory, Shen, Wu and Zhang applied the optimal control strategy based on multi-valued logic to the control of internal combustion (IC) engines [12,13] and hybrid electric vehicles (HEVs) [14]. In addition, for some related works on optimization, and advanced control strategies for IC engines, please refer to [15–19].

However, the VGT-EGR system is a complex and uncertain system, which contains the unmodeled dynamics and the changes of different working points of the engine also have coupling effects on the system. Therefore, the control strategy of VGT-EGR system needs to be independent of the accurate model and has strong robustness to disturbance. Active disturbance rejection control (ADRC) is a method proposed by researcher Han Jingqing [20], which inherits the advantages of PID and is less dependent on the accurate model. The coupling and external disturbance of the system are regarded as the total disturbance and extended into a new state, which is observed by the extended state observer. Then, the system is transformed into an integral series control system by compensating the disturbance into the control input. In addition, to solve the contradiction between speediness and overshoot, a tracking differentiator is designed. Finally, a more efficient nonlinear state error feedback control law is used to replace the traditional PID control law. Therefore, ADRC plays an increasingly important role in the control of nonlinear system and multi-input multi-output (MIMO) system such as quadrotor [21,22], aero engine [23], and diesel engines [24–26]. In addition, Wang et al. [27] introduced the ADRC method into the control of the differential drive assist steering system of electric vehicles, and selected the standard working conditions for simulation and experimental verification. The results showed that compared with the PID controller, the ADRC controller can not only reduce the steering effort of the driver obviously, but also have better control performance in tracking accuracy and smooth road feeling of the driver. Shi et al. [28] proposed a hybrid ADRC control for the control of superheated steam temperature in coal-fired power plants. Numerical simulation results showed that the hybrid ADRC can improve the performance of tracking and disturbance rejection under the condition of good robustness. At the same time, experiments were carried out on a 150 MW power plant simulator. The experimental results showed that the hybrid ADRC can improve the control performance of superheated steam temperature and its structure is simpler than cascade control. Zhou et al. [29] proposed the ADRC technology as a speed loop controller for permanent magnet synchronous motor. The ADRC is used to measure and compensate unknown disturbances such as rotational inertia and stator resistance, so that the system has strong against the system parameter change and external disturbance, and the decoupling control of permanent magnet synchronous motor is realized. Simulation and experimental results show that the controller has strong robustness, stability and accurate dynamic tracking performance.

However, the ADRC was originally proposed by Han in a nonlinear form, but nonlinear active disturbance rejection control (NLADRC) requires too many parameters to be adjusted. Gao proposed a bandwidth-based parameter tuning method for the first time, which simplified the ADRC from the original nonlinear form to the linear form, which greatly reduced the parameters of the controller and greatly promoted the application of ADRC. However, linear active disturbance rejection control (LADRC) cannot meet the requirements of the system which needs high control precision and fast response speed. In the decoupled control of diesel engines, Song and Xie et al. [24,25] decoupled the
double-input double-output (DIDO) system into two independent single-input single-output (SISO) systems, each of which can be designed as an ideal monolithic object for control. The nonlinear, uncertain and time-varying process dynamics and the external disturbance are uniformly treated as a total disturbance, which is estimated and compensated by extended state observer (ESO) and eliminated in the control input, thus achieving the purpose of controlling the intake manifold pressure and air quality flow rate of diesel engines by LADRC controller. Based on the above work, our goal is to design a control strategy for diesel engines with satisfactory transient response performance, strong resistance to external disturbances and robust performance for engine operating conditions.

The contributions made in the paper can be summarized as follows.

1. Based on the work of Song and Xie [24,25] on the LADRC controller in diesel engines, we designed the NLADRC controller for VGT-EGR system in order to improve the control accuracy.
2. Based on the theoretical result of Guo [30] on ADRC, we give the convergence proof of the designed NLSEO, which theoretically ensures the rationality of the designed controller.
3. In addition, the NLADRC controller designed by us is compared with PID controller [24] and LADRC controller. First, the NLADRC controller we designed can quickly track the given signal without overshoot. Secondly, the control effect of the controller is not affected by the external square wave and sinusoidal control disturbance. Finally, the controller can track the given signal well after changing the diesel engine speed and fuel injection without resetting the parameters of the controller.

The rest of this paper is organized as follows: The next section describes the dynamics of diesel engines. Combined with the dynamics of Section 2, the nonlinear extended state observer is designed for intake manifold pressure and air mass flow in Section 3. Section 4 gives the convergence proof of the designed NLSEO. Section 5 presents the design of NLADRC controller and the comparative simulation results of NLADRC controller, LADRC controller and PID controller. Finally, the conclusion is given in the last section.

2. System Dynamics

Figure 1 shows the diesel engine model, which is mainly composed of seven parts: cylinder, compressor, turbine, intake manifold, exhaust manifold, VGT valve and EGR valve.
According to the law of ideal gas and the law of conservation of mass, the dynamics of manifold pressure can be modeled \([1]\) as

\[
\dot{p}_{im} = \frac{RT_{im}}{V_{im}} (W_c + W_{egr} - W_e),
\]

\[
\dot{p}_{em} = \frac{RT_{em}}{V_{em}} (W_c + W_f - W_{egr} - W_t),
\]

where \(p_{im}, T_{im}\) and \(V_{im}\) respectively represent the intake manifold pressure, temperature and volume. In addition, \(p_{em}, T_{em}\) and \(V_{em}\) respectively represent the exhaust manifold pressure, temperature and volume. \(W_c, W_e, W_f, W_{egr}\) and \(W_t\) represent the mass air flow, the total mass flow from the intake manifold into the cylinder, the fuel mass flow into the cylinders, the mass flow through the EGR valve, and the turbine mass flow, respectively.

The fuel mass flow \(W_f\) into the cylinder can be expressed \([1]\) as

\[
W_f = \frac{10^{-6}}{120} u_f n_e n_{cyl},
\]

where \(u_f\) represents the injected fuel, \(n_e\) represents the engine speed and \(n_{cyl}\) is the number of cylinders.

The mass flow through the EGR valve \(W_{egr}\) and the turbine mass flow \(W_t\) are expressed \([25]\) as

\[
W_{egr} = A_{egr}(u_{egr}) \frac{\sqrt{2 p_{im}(p_{em} - p_{im})}}{\sqrt{RT_{em}}},
\]

\[
W_t = A_{vgt}(u_{vgt}) \frac{\sqrt{2 p_{amb}(p_{em} - p_{amb})}}{\sqrt{RT_{em}}},
\]

where \(u_{egr}\) and \(u_{vgt}\) are input signals for diesel engines and represent the open ratio of VGT and EGR valve, respectively. It is 0% when the valves are closed and 100% when the valves are fully open. \(A_{egr}(u_{egr})\) is a polynomial function of \(u_{egr}\), so is \(A_{vgt}(u_{vgt})\).

The compressor power \(P_c\) and turbine power \(P_t\) can be calculated \([25]\) as

\[
P_c = \frac{W_c c_p T_{amb}}{\eta_c} \left( \left( \frac{p_{im}}{p_{amb}} \right)^\mu - 1 \right),
\]

\[
P_t = W_t c_p T_{em} \eta_t \left( 1 - \left( \frac{p_{amb}}{p_{em}} \right)^\mu \right),
\]

where \(T_{amb}\) and \(p_{amb}\) represent ambient temperature and pressure. \(\eta_m\), \(\eta_c\) and \(\eta_t\) represent the mechanical efficiency in the turbocharger, the isentropic efficiency of compressor and the isentropic efficiency of turbine, respectively. \(c_p\) is the constant pressure specific heat capacity, and \(\tau_t\) is time constant. The power \(\mu\) can be expressed as \(\mu = 1 - \frac{1}{\gamma}\) by the specific heat capacity ratio \(\gamma\).

The relationship between compressor power \(P_c\) and turbine power \(P_t\) can be expressed as

\[
P_c = \frac{1}{\tau_t} (-P_c + \eta_m P_t).
\]

The dynamic process of the exhaust process is very fast, so it is considered a stable process, then \(\dot{p}_{em} = 0\), Equation (2) can be simplified \([25]\) as

\[
W_e + W_f - W_{egr} - W_t = 0.
\]

Define \(g(u_{vgt}) = A_{vgt}(u_{vgt})\) and \(f(u_{egr}) = A_{egr}(u_{egr})\), and their Taylor series at VGT valve opening \(u_{vgt} = u_0\) and EGR valve opening \(u_{egr} = u_1\) are
\[
\begin{align*}
\dot{g}(u_{vgt}) &= g(u_0) + g'(u_0)(u_{vgt} - u_0) + \ldots + \frac{g^n(u_0)}{n!}(u_{vgt} - u_0)^n + R_n(u_{vgt}), \\
\dot{f}(u_{egr}) &= f(u_1) + f'(u_1)(u_{egr} - u_1) + \ldots + \frac{f^n(u_1)}{n!}(u_{egr} - u_1)^n + R_n(u_{egr}),
\end{align*}
\]

where \(R_n(u_{vgt}) = o\left((u_{vgt} - u_0)^n\right)\), \(R_n(u_{egr}) = o\left((u_{egr} - u_1)^n\right)\), and they represent polynomials of higher order than \(n\).

Substituting Equations (4), (8) and (9) into Equation (1), the dynamics of intake manifold pressure \(p_{im}\) becomes

\[
\dot{p}_{im} = \frac{RT_{im}}{V_{im}}(W_c + W_f) + K_0K_1 + K_0g'(u_0)u_{vgt},
\]

where

\[
\begin{align*}
K_0 &= \frac{-\sqrt{2RT_{im}}}{\sqrt{V_{em}V_{im}}}\sqrt{p_{amb}(p_{em} - p_{amb})}, \\
K_1 &= g(u_0) - g'(u_0)u_0 + \frac{g^n(u_0)}{n!}(u_{vgt} - u_0)^n + R_n(u_{vgt}).
\end{align*}
\]

Differentiate Equation (6) and convert the form, the representation of \(W_c\) can be expressed as

\[
\dot{W}_c = \frac{\dot{p}_c - \frac{W_c\tau_{em}}{\tau_{amb}}(\frac{p_{im}}{p_{amb}})^{\mu-1}\dot{p}_{im}}{n\tau_{amb}}.
\]

Substituting Equations (3), (5)–(8) and (10) into Equation (12), we obtain the dynamics of mass air flow \(W_c\) as

\[
\dot{W}_c = K_2\left(W_c + W_f\right) - K_4W_c - K_2K_3K_5 - K_2K_3f'(u_1)u_{egr},
\]

where

\[
\begin{align*}
\Pi_1 &= \frac{p_{im}}{p_{amb}}, \Pi_2 = \frac{p_{amb}}{p_{em}}K_2 = \frac{\eta_m\eta_c\eta_{\Pi}(1 - \Pi_2^\mu)}{\tau_{amb}\tau_1(\Pi_1^{\mu} - 1)}, \\
K_3 &= \sqrt{2\Pi_{im}(p_{em} - p_{im})} K_4 = \frac{1}{\tau_1 + \mu\Pi_1^{\mu-1}p_{im}} \\
K_5 &= f(u_1) - f'(u_1)u_1 + \ldots + \frac{f^n(u_1)}{n!}(u_{egr} - u_1)^n + R_n(u_{egr}).
\end{align*}
\]

3. NLES0 Design

In this part, we will design the NLES0 for VGT-EGR system according to the dynamics in Section 2. The decoupling control scheme is shown in Figure 2. As shown in the figure, the DIDO VGT-EGR system with \(u_{vgt}\) and \(u_{egr}\) as inputs and intake manifold pressure \(p_{im}\) and mass air flow \(W_c\) as outputs is decomposed into two subsystems. One of the SISO systems takes \(u_{vgt}\) as input and intake manifold pressure \(p_{im}\) as output, while the other takes \(u_{egr}\) as input and mass air flow \(W_c\) as output.

Figure 3 presents the block diagram of NLADRC controller designed for \(p_{im}\) loop of VGT-EGR system in diesel engines. NLADRC controller mainly consists of three parts: tracking differentiator (TD), nonlinear states error feedback control laws (NLSEF) and NLES0. The desired output signal \(p_{im-d}\) is arranged by TD, and then the transient process signal \(p_{im-v1}\) is obtained. \(p_{im-v1}\) subtracts the state signal \(z_1\) observed by NLES0, and its deviation is used as the input of NLSEF to produce control action \(u_0\). Then \(u_0\) subtracts the total disturbance \(z_2\) estimated by NLES0 and divides it by the compensation coefficient \(b\) to obtain the control output \(u_{nladrc}\) of NLADRC controller. Considering that the output signal of the controller may be affected by the external disturbance, the output \(u_{nladrc}\) of the controller plus the external disturbance \(d\) is the input signal \(u_{vgt}\) of the diesel engine.
Considering that the system may be affected by external interference, we add external interference $w$ to the system (11). In addition, the system (11) is converted to the total disturbance and input form as follows

$$\dot{p}_{im} = \frac{RT_{im}}{V_{im}} \left( W_c + W_f \right) + K_0 K_1 + K_0 g'(u_0) u_{vgt} + w$$

$$= \frac{RT_{im}}{V_{im}} \left( W_c + W_f \right) + K_0 K_1 + \left( K_0 g'(u_0) - b_{vgt} \right) u_{vgt} + b_{vgt} u_{vgt} + w$$

$$= f_{pim} + w + b_{vgt} u_{vgt} = F_{pim} + b_{vgt} u_{vgt}, \quad (14)$$

where $F_{pim}$ represents the total disturbance of $p_{im}$ loop. The system function $f_{pim}$ contains coupling and disturbance caused by the change of working point. The coefficient $b_{vgt}$ is an approximate estimate of $K_0 g'(u_0)$, and the estimated deviation $K_0 g'(u_0) - b_{vgt}$ between $b_{vgt}$ and $K_0 g'(u_0)$ is also included in $f_{pim}$, which is timely estimated and compensated. For the diesel engine physical system, we can notice that there are some positive constants $c_0, c_1, c_2$ and the positive integer $k$ that satisfy $|b_{vgt} u_{vgt}| + |f_{pim}| + |w| + |\frac{\partial f_{pim}}{\partial p_{im}}| + |\frac{\partial f_{pim}}{\partial t}| \leq c_0 + c_1 |p_{im}|^k$ and $|w| + |p_{im}| \leq c_2$.

Let the state variable $x_1(t) = p_{im}(t)$, $x_2(t) = F_{pim}(t) = f_{pim}(t, x_1(t)) + w(t)$. Then system (14) is expressed in the form of extended state space

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_{vgt} \\ 0 \end{bmatrix} u_{vgt}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} F_{pim}(t). \quad (15)$$
According to system (15), the NLESO for the intake manifold pressure $p_{im}$ is designed as

$$\begin{align*}
\dot{z}_1 (t) &= z_2 (t) + b_{egr} u_{egr} (t) + g_1 \left( \frac{z_1 (t) - z_1 (t)}{\varepsilon} \right), \\
\dot{z}_2 (t) &= \frac{1}{\epsilon} g_2 \left( \frac{z_1 (t) - z_2 (t)}{\varepsilon} \right),
\end{align*}$$

where $z_1 (t)$ and $z_2 (t)$ are estimates of the intake manifold pressure and the total disturbance, respectively. $\epsilon$ is constant gain. $g_1 \left( \frac{z_1 (t) - z_1 (t)}{\varepsilon} \right)$ and $g_2 \left( \frac{z_1 (t) - z_2 (t)}{\varepsilon} \right)$ are functions of the state deviation expressed as $g_1 \left( \frac{z_1 (t) - z_1 (t)}{\varepsilon} \right) = a_1 \left( \frac{z_1 (t) - z_1 (t)}{\varepsilon} \right) + fal \left( \frac{z_1 (t) - z_1 (t)}{\varepsilon}, 0.5, 1 \right)$, and $g_2 \left( \frac{z_1 (t) - z_2 (t)}{\varepsilon} \right) = a_2 \left( \frac{z_1 (t) - z_2 (t)}{\varepsilon} \right)$. Parameters $a_1$ and $a_2$ are positive, and nonlinear function $fal(e, \alpha, \delta)$ is specifically expressed as

$$fal(e, \alpha, \delta) = \begin{cases} 
|e|^\alpha \text{sign}(e), |e| > \delta, \\
\frac{\varepsilon}{\alpha}, |e| \leq \delta,
\end{cases}$$

where $\alpha$ and $\delta$ are the parameters to be adjusted, and $\delta$ represents the width of the linear region.

Similar to the simplification process of Equation (11), Equation (13) can be simplified in the form of input $u_{egr}$ and disturbance similar to (14)

$$\dot{W}_c = f_{\dot{W}_c} + w + b_{egr} u_{egr},$$

according to the similar design method of $p_{im}$ loop, NLESO can be designed for mass air flow $W_c$.

4. Convergence Analysis of NLESO for the Intake Manifold Pressure

In the above section, we designed NLESO using the intake manifold pressure $p_{im}$ loop as an example. In this section, we will provide the convergence analysis of the proposed NLESO.

**Theorem 1.** The proposed NLESO given in (16) for the intake manifold pressure loop dynamics (15) have the following two properties:

1. For every positive constant $\alpha$, $\lim_{\varepsilon \to 0} |x_i (t) - z_i (t)| = 0$ uniformly in $t \in [a, \infty)$.

2. $\lim_{t \to \infty} |x_i (t) - z_i (t)| \leq O \left( \varepsilon^{\alpha - i} \right), i = 1, 2$

where $\lim_{t \to \infty} |x_i (t) - z_i (t)| = \limsup |x_i (t) - z_i (t)|$, it means that the upper limit is the upper bound of the limit of the series of convergents. The state variables $x_1 (t)$ and $x_2 (t)$ respectively represent the intake manifold pressure $p_{im}$ and the total disturbance $F_{im}$. In addition, the states of observer $z_1 (t)$ and $z_2 (t)$ are approximations of $x_1 (t)$ and $x_2 (t)$, respectively.

**Proof of Theorem 1.** Set

$$e_i (t) = x_i (t) - z_i (t), \eta_i (t) = \frac{e_i (t)}{\varepsilon^{\alpha - i}}, i = 1, 2.$$  \hspace{1cm} (18)

Then the derivative of $\eta_1 (t)$ can be calculated as

$$\eta_1 (t) = \dot{e}_1 (t) = \dot{x}_1 (et) - \dot{z}_1 (et)$$

$$= x_2 (et) + b_{egr} u_{egr} (et) - z_2 (et) - b_{egr} u_{egr} (et) - g_1 \left( \frac{x_1 (et) - z_1 (et)}{\varepsilon} \right)$$

$$= x_2 (et) - z_2 (et) - g_1 (\eta_1 (t)) = \eta_2 (t) - g_1 (\eta_1 (t)).$$
Similarly, the derivative of \( \eta_2 (t) \) can be expressed as

\[
\eta_2 (t) = e \dot{x}_2 (et) = e \left( x_2 (et) - z_2 (et) \right) = e \left( \dot{F}_{p,m} (et) - \frac{1}{e} g_2 \left( \frac{x_1 (et) - z_1 (et)}{e} \right) \right) = -g_2 (\eta_1 (t)) + e \dot{F}_{p,m} (et).
\]

We construct positive definite function as

\[
V (\eta (t)) = \eta (t)^T P \eta (t) + \int_0^{\eta (t)} f a l (s, 0.5, 1) \, ds,
\]

where

\[
P = \begin{bmatrix}
\frac{a_1^2 b_2 + a_3 b_1}{a_1 a_2} & \frac{-b_2}{a_2} \\
\frac{a_1^2 b_2 + a_3 b_1}{a_1 a_2} & \frac{b_2}{a_2} \\
\end{bmatrix}, \quad \begin{cases}
b_2 > 1 \\
2b_1 - b_2 > 1 \\
b_1 \neq b_2
\end{cases}
\]

The derivative of \( V (\eta (t)) \) with respect to \( t \) along the \( \eta (t) \) can be calculated as

\[
\frac{d}{dt} V (\eta (t)) = \frac{\partial V}{\partial \eta_1 (t)} \dot{\eta}_1 (t) + \frac{\partial V}{\partial \eta_2 (t)} \dot{\eta}_2 (t) = \frac{\partial V}{\partial \eta_1 (t)} (\eta_2 (t) - g_1 (\eta_1 (t))) - \frac{\partial V}{\partial \eta_2 (t)} g_2 (\eta_1 (t)) + \frac{\partial V}{\partial \eta_2 (t)} e \dot{F}_{p,m} (et). 
\]

Next, we will calculate \( \dot{F}_{p,m} (et), \frac{\partial V}{\partial \eta_1 (t)} (\eta_2 (t) - g_1 (\eta_1 (t))) - \frac{\partial V}{\partial \eta_2 (t)} g_2 (\eta_1 (t)) \), and \( \frac{\partial V}{\partial \eta_2 (t)} \) respectively. First, we have

\[
\dot{F}_{p,m} (et) = \frac{d}{ds} f (s, x_1 (s)) \bigg|_{s=et} + \tilde{\psi} (et) = e \frac{\partial}{\partial t} f (et, x_1 (et)) + \frac{\partial}{\partial x_1} f (et, x_1 (et)) (x_2 (et) + b_{eg} u_{egt} (t)) + \tilde{\psi} (et),
\]

there exists a constant \( M > 0 \) that satisfies \( M \geq |\dot{F}_{p,m} (et)| \).

Notice that \( g_1 (\eta_1 (t)) = a_1 \eta_1 (t) + f a l (\eta_1 (t), 0.5, 1) \) and \( g_2 (\eta_1 (t)) = a_2 \eta_1 (t) \). Then, we have the following calculation

\[
\frac{\partial V}{\partial \eta_1 (t)} (\eta_2 (t) - g_1 (\eta_1 (t))) - \frac{\partial V}{\partial \eta_2 (t)} g_2 (\eta_1 (t)) = \left( \frac{a_2^2 b_2 + a_3 b_1}{a_1 a_2} \eta_1 (t) - b_2 \eta_2 (t) + f a l (\eta_1 (t), 0.5, 1) \right) (\eta_2 (t) - a_1 \eta_1 (t) - f a l (\eta_1 (t), 0.5, 1))
\]

\[
- \left( \frac{a_1^2 b_2 + a_3 b_1}{2a_1 a_2} \eta_2 (t) - b_2 \eta_1 (t) \right) a_2 \eta_1 (t)
\]

\[
= -b_1 \eta_1 (t)^2 - b_2 \eta_2 (t)^2 + (\eta_2 (t) - a_1 \eta_1 (t)) f a l (\eta_1 (t), 0.5, 1)
\]

\[
- \left( \frac{a_2^2 b_2 + a_3 b_1}{a_1 a_2} \eta_1 (t) - b_2 \eta_2 (t) + f a l (\eta_1 (t), 0.5, 1) \right) f a l (\eta_1 (t), 0.5, 1)
\]

\[
= -b_1 \eta_1 (t)^2 - b_2 \eta_2 (t)^2 + (1 + b_2) \eta_2 (t) f a l (\eta_1 (t), 0.5, 1)
\]

\[
- f a l (\eta_1 (t), 0.5, 1)^2 - \left( a_1 + \frac{a_2^2 b_2 + a_3 b_1}{a_1 a_2} \right) \eta_1 (t) f a l (\eta_1 (t), 0.5, 1)
\]

\[
\leq -b_1 \eta_1 (t)^2 - b_2 \eta_2 (t)^2 + (1 + b_2) \eta_2 (t) f a l (\eta_1 (t), 0.5, 1)
\]

\[
\leq -b_1 \eta_1 (t)^2 - b_2 \eta_2 (t)^2 + \frac{1 + b_2}{2} (\eta_2 (t)^2 + f a l (\eta_1 (t), 0.5, 1)^2).
\]
Due to $|f_{al}(\eta_1(t), 0.5, 1)| \leq |\eta_1(t)|$, we can further calculate as

\[
\begin{align*}
\frac{\partial V}{\partial \eta_1(t)} (\eta_2(t) - g_1(\eta_1(t))) - \frac{\partial V}{\partial \eta_2(t)} g_2(\eta_1(t)) \\
\leq -b_1\eta_1(t)^2 - b_2\eta_2(t)^2 + \frac{1 + b_2}{2} \left( \eta_2(t)^2 + f_{al}(\eta_1(t), 0.5, 1)^2 \right) \\
\leq -b_1\eta_1(t)^2 - b_2\eta_2(t)^2 + \frac{1 + b_2}{2} \left( \eta_2(t)^2 + \eta_1(t)^2 \right) \\
= -\left( \frac{2b_1 - b_2 - 1}{2} \eta_1(t)^2 + \frac{b_2 - 1}{2} \eta_2(t)^2 \right) = \eta(t)^T P_1 \eta(t) \triangleq -W(\eta(t)),
\end{align*}
\]

where

\[
P_1 = \begin{bmatrix}
\frac{2b_1 - b_2 - 1}{2} & 0 \\
0 & \frac{b_2 - 1}{2}
\end{bmatrix}.
\]

Obviously, $V(\eta(t))$ satisfies

\[
V(\eta(t)) = \eta(t)^T P_1 \eta(t) + \int_0^{\eta_1(t)} f_{al}(s, 0.5, 1) \, ds \\
\geq \eta(t)^T P_1 \eta(t) \triangleq V_1(\eta(t)).
\]

We notice that $P$ is a symmetric positive definite matrix, which means that $P$ has two positive eigenvalues denoted by $\lambda_{1_{min}}$ and $\lambda_{2_{max}}$. In addition, set $\beta_1 = \lambda_{1_{min}}$, we have

\[
V(\eta(t)) \geq V_1(\eta(t)) \geq \beta_1 \|\eta(t)\|^2. \tag{24}
\]

Let us calculate $V(\eta(t))$ as

\[
V(\eta(t)) = \eta(t)^T P_1 \eta(t) + \int_0^{\eta_1(t)} f_{al}(s, 0.5, 1) \, ds \\
\leq \eta(t)^T P_1 \eta(t) + \int_0^{\eta_1(t)} sds \\
= \eta(t)^T P_1 \eta(t) + \frac{\eta_1(t)^2}{2} \\
= \eta(t)^T P_2 \eta(t) \triangleq V_2(\eta(t)),
\]

where

\[
P_2 = \begin{bmatrix}
\frac{a_1^2 b_2 + a_2 b_1}{2a_1^2 d_1^2} + \frac{1}{2} & \frac{-b_2}{2} \\
\frac{-b_2}{2} & \frac{a_1^2 b_2 + a_2 b_1 + b_1}{2a_1^2 d_1^2}
\end{bmatrix}.
\]

Similarly, we can get the eigenvalues of symmetric positive definite matrix $P_2$ are $\lambda_{3_{min}}$ and $\lambda_{4_{max}}$, respectively. In addition, set $\beta_2 = \lambda_{4_{max}}$, we have

\[
V(\eta(t)) \leq V_2(\eta(t)) \leq \beta_2 \|\eta(t)\|^2. \tag{25}
\]

And for the positive definite function $W(\eta(t))$, we also have the expression as

\[
\beta_3 \|\eta(t)\|^2 \leq W(\eta(t)) \leq \beta_4 \|\eta(t)\|^2, \tag{26}
\]

where $\beta_3$ is the minimum eigenvalue of symmetric positive definite matrix $P_1$, and $\beta_4$ is the maximum eigenvalue of symmetric positive definite matrix $P_1$. 

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We have an expression of \( \frac{\partial V}{\partial \eta_2 (t)} \) as
\[
\frac{\partial V}{\partial \eta_2 (t)} = \frac{a_1^2 b_2 + a_2 b_1 + b_1}{2 a_1 a_2} \eta_2 (t) - b_2 \eta_1 (t) = \frac{\partial V_1}{\partial \eta_2 (t)},
\]
so we can get
\[
\left| \frac{\partial V}{\partial \eta_2 (t)} \right| = \left| \frac{\partial V_1}{\partial \eta_2 (t)} \right| \leq \| \frac{\partial V_1}{\partial \eta (t)} \| \leq 2 \| P \| \| \eta (t) \|.\]

Because \( P \) is a symmetric positive definite matrix, we have
\[
2 \sqrt{\lambda_{\text{max}} (P^TP) \| \eta (t) \|} = 2 \lambda_{\text{max}} (P) \| \eta (t) \|,
\]
and set \( \beta = 2 \lambda_{\text{max}} (P) = 2 \lambda_{\text{max}} (P) \), we can know that \( \frac{\partial V}{\partial \eta_2 (t)} \) satisfies
\[
\left| \frac{\partial V}{\partial \eta_2 (t)} \right| \leq \beta \| \eta (t) \|. \tag{27}
\]

So, Equation (20) combined with Equations (21) and (23)–(27), we have
\[
\frac{d}{dt} V (\eta (t)) = \frac{\partial V}{\partial \eta_1 (t)} (\eta_2 (t) - g_1 (\eta_1 (t))) - \frac{\partial V}{\partial \eta_2 (t)} g_2 (\eta_1 (t)) + \frac{\partial V}{\partial \eta_2 (t)} F_{\text{int}} (t) \\
\leq -W (\eta (t)) + \epsilon M \beta \| \eta (t) \| \\
\leq -\frac{\beta_3}{\beta_2} V (\eta (t)) + \frac{\sqrt{\beta_1}}{\beta_1} \epsilon M \beta \sqrt{V (\eta (t))}. \tag{28}
\]

Because
\[
\frac{d}{dt} \sqrt{V (\eta (t))} = \frac{d}{dt} \sqrt{V (\eta (t))} \frac{d}{dt} V (\eta (t)) = \frac{1}{2 \sqrt{V (\eta (t))}} \frac{d}{dt} V (\eta (t)),
\]
from Equation (28) we can get
\[
\frac{d}{dt} \sqrt{V (\eta (t))} \leq -\frac{\beta_3}{2 \beta_2} \sqrt{V (\eta (t))} + \frac{\sqrt{\beta_1} \epsilon M \beta}{2 \beta_1}. \tag{29}
\]

Combining Equation (24) with Equation (29), we know that \( \| \eta (t) \| \) satisfies
\[
\| \eta (t) \| \leq \frac{\sqrt{V (\eta (t))}}{\beta_1} \leq \frac{\sqrt{\beta_1 V (\eta (0))}}{\beta_1} e^{-\frac{\beta_3}{\beta_2} t} + \frac{\epsilon M \beta}{2 \beta_1} \int_0^t e^{-\frac{\beta_3}{\beta_2} (t-\tau)} d\tau, \tag{30}
\]
which together with Equation (18) satisfies
\[
|e_i (t)| = e^{2^{-i}} \left| \eta_i \left( \frac{t}{\varepsilon} \right) \right| \leq e^{2^{-i}} \| \eta \left( \frac{t}{\varepsilon} \right) \| \\
\leq e^{2^{-i}} \left[ \frac{\sqrt{\beta_1 V (\eta (0))}}{\beta_1} e^{-\frac{\beta_3}{\beta_2} t} + \frac{\epsilon M \beta}{2 \beta_1} \int_0^t e^{-\frac{\beta_3}{\beta_2} (t-\tau)} d\tau \right] \tag{31}
\]
\[
\rightarrow 0, \quad i = 1, 2
\]
uniformly in \( t \in [a, \infty) \) as \( \varepsilon \rightarrow 0. \) \( \square \)
5. NLADRC Controller Design and Simulation Results

In this part, we will design the discrete NLADRC controller for VGT-EGR system and give the simulation results. Due to the discrete controller adopted in the subsequent simulation, we discretize (16), and then discrete NLESO for the intake manifold pressure $p_{im}$ is expressed as

$$\begin{align*}
z_1(k+1) &= z_1(k) + h \left( z_2(k) + b_{vgt} u_{vgt}(k) + g_1 \left( \frac{x_1(k)-z_1(k)}{\varepsilon} \right) \right), \\
z_2(k+1) &= z_2(k) + h \left( \frac{1}{2} g_2 \left( \frac{x_1(k)-z_1(k)}{\varepsilon} \right) \right),
\end{align*}$$

(32)

where $h$ is the sampling time.

The discrete form of TD is expressed [20] as

$$\begin{align*}
v_1(k+1) &= v_1(k) + hv_2(k), \\
v_2(k+1) &= v_2(k) + hf_{han}(v_1(k) - v(k), v_2(k), r, h),
\end{align*}$$

(33)

where $r$ is the parameter indicating the speed of the transition process, $v(k)$ is the desired intake manifold pressure, $v_1(k)$ is the transition process of the desired intake manifold pressure, $v_2(k)$ is the derivative of the transition process, and $f_{han}(x_1, x_2, r, h)$ is the synthesis function of the fastest control expressed [20] as

$$\begin{align*}
d &= rh, \\
d_0 &= hd, \\
y &= x_1 + hx_2, \\
a_0 &= \sqrt{d^2 + 8r |y|}, \\
a &= \begin{cases} x_2 + \left( \frac{a_0-d}{2} \right) \text{sign}(y), |y| > d_0, \\
x_2 + \frac{y}{r}, |y| \leq d_0, \end{cases} \\
f_{han} &= \begin{cases} -r \text{sign}(a), |a| > d, \\
-r_\frac{d}{a}, |a| \leq d. \end{cases}
\end{align*}$$

The NLSEF is designed as

$$\begin{align*}
e(k) &= v_1(k) - z_1(k), \\
u(k) &= k_p f_{pal}(e(k), 3/4, 0.01).
\end{align*}$$

(34)

where $k_p$ is the proportional gain.

The control effect after compensation by disturbance estimate $z_2(k)$ is as follows

$$u_{vgt} = \frac{(u(k) - z_2(k))}{b_{vgt}}.$$  

(35)

Similar to the controller design method of $p_{im}$ loop, we can also design NLADRC controller for $W_c$. Therefore, we do not need to introduce the controller design process of $W_c$ loop in detail.

Next, we will verify the control effect of the NLADRC controller for VGT-EGR system from the transient response performance, disturbance resistance performance and robustness on the diesel engine model created by Wahlström [1], and compare it with the LADRC controller and the PID controller. And the parameters adjustment of PID controller are described by Ziegler-Nichols method in [24]. In the simulation experiment, the diesel engine working point: engine speed $n_e$: 1900 r/min, fuel injection $u_δ$ step from 110 mg/cycle to 160mg/cycle. The parameters of LADRC controller and NLADRC controller are summarized in Tables 1–3.
Table 1. NLADRC controller parameters.

| Index | \( h \) | \( b \) | \( a_1 \) | \( a_2 \) | \( \varepsilon \) | \( kp \) |
|-------|-------|-------|-------|-------|-------|-------|
| VGT loop | 0.01 | -454 | 3 | 1 | 0.3 | 30 |
| EGR loop | 0.01 | -0.015 | 3 | 1 | 0.4 | 20 |

Table 2. LADRC controller parameters.

| Index | \( h \) | \( b \) | \( \beta_1 \) | \( \beta_2 \) | \( kp \) |
|-------|-------|-------|-------|-------|-------|
| VGT loop | 0.01 | -100000 | 130 | 4225 | 1.3 |
| EGR loop | 0.01 | -0.015 | 30 | 225 | 2.2 |

Table 3. Nonlinear function \( f_{al}(\epsilon, a, \delta) \) parameters.

| Index | \( \alpha_{vgt} \) | \( \delta_{vgt} \) | \( \alpha_{egr} \) | \( \delta_{egr} \) |
|-------|-------|-------|-------|-------|
| NLESO | 0.5 | 1 | 0.5 | 1 |
| NLSEF | 0.75 | 0.01 | 0.75 | 0.01 |

5.1. Transient Performance

Figure 4 and 5 present the tracking response curves of intake manifold pressure \( p_{im} \) and mass air flow \( W_c \) to square wave signal. In the figure, the black curve represents the expected intake manifold pressure, the red curve represents the response curve with NLADRC controller, the blue curve represents the response curve with LADRC controller, and the green curve represents the response curve with PID controller, and other figures in this paper are similar. Table 4 summarizes the setting time and overshoot of the response curves of NLADRC controller, LADRC controller and PID controller.

Table 4. Setting time and overshoot of three controllers.

| Index | NLADRC | LADRC | PID |
|-------|--------|-------|-----|
| \( T_s \) of \( p_{im} \) loop | 6 s | 12 s | 20 s |
| \( T_s \) of \( W_c \) loop | 6 s | 15 s | 20 s |
| \( \sigma \) of \( p_{im} \) loop | 0% | 1.5% | 3.3% |
| \( \sigma \) of \( W_c \) loop | 0% | 1.5% | 1.1% |

Figure 4. Response curve of intake manifold pressure \( p_{im} \).
Figure 5. Response curve of mass air flow $Wc$.

Compare the tracking effect of intake manifold pressure $p_{im}$ in Figure 4, or the tracking effect of mass air flow $W_c$ in Figure 5. Obviously, system with NLADRC controller tracks square wave signals better. In the figure, the NLADRC controller tracks the desired intake manifold pressure $p_{im,ref}$ or mass air flow $W_{c,ref}$ without overshoot, while the LADRC controller and PID controller have relatively large overshoot. The intake manifold pressure $p_{im}$ and mass air flow $W_c$ of diesel engines are the outputs of dynamic system, which have certain inertia and cannot change suddenly. However, the set values $p_{im,ref}$ and $W_{c,ref}$ are given outside the system and can be changed instantaneously. If we use their deviation to produce control effect directly, it will lead to the initial error is too large, and the controller will have a great control effect, which will have a great impact on the system and easy to produce overshoot. TD was mentioned earlier when we introduced the structure of the NLADRC controller designed for the $p_{im}$ loop. In the $p_{im}$ loop, TD converts transient intake manifold pressure $p_{im,d}$ into a slow-varying transition process signal $p_{im,v1}$. In this way, the slowly changing transition process signal $p_{im,v1}$ minus the output signal $p_{im}$ of diesel engines will not produce excessive deviation, and NLADRC controller will not produce excessive control effect, so that the intake manifold pressure $p_{im}$ of the diesel engine does not generate overshoot. In the previous NLESO design, we introduced the nonlinear function $f_{al}(e, \alpha, \delta)$, whose image is shown in Figure 6. When the parameter $\alpha$ of the nonlinear function is equal to 1, it becomes a linear form of the error. As can be seen from the figure, the linear form and the nonlinear form intersect at $(1, 1)$. When the error is greater than 1, the value of the nonlinear function is less than that of the linear form, so the control effect generated by the NLADRC controller is less than that of the linear form, so the system response with the NLADRC controller will not produce overshoot.

In addition, the setting time of NLADRC controller is about 6 s, while that of LADRC controller and PID controller is 2 times and 3 times of that of NLADRC controller, respectively.
5.2. Disturbance Rejection

In the previous section, we compared the effect of three controllers on tracking square wave signals. In this part, we will verify the resistance of the three controllers to control disturbances. The control disturbance is \( d \) in the NLADRC controller structure described in Section 3. In this part, we take the \( p_{\text{int}} \) loop as an example to compare the resistance of the three controllers to external control disturbances. At 40 s, we set the desired intake manifold pressure, and over time the diesel engine’s intake manifold pressure reaches the desired value. In addition, then at 60 s, we add a square wave control interference with an amplitude of 10 and a width of 10 s. Figure 7 shows the resistance of the three controllers to square wave disturbances. It is obvious that the response curve of the NLADRC controller has a slight amplitude change after adding the disturbance, and quickly returns to the expected value after 2 s. However, the amplitude variation of the LADRC controller and PID controller is much larger than that of the NLADRC controller, and the recovery time is relatively long. Next, we analyze why NLADRC controller is better than LADRC controller and PID controller in resisting square wave disturbance. First, let us look at the first figure in Figure 8, which shows the output signals \( u_c \) of the three controllers. The NLESO in NLADRC controller accurately estimates the amplitude and period of the square wave form disturbance and compensates to the control effect \( u_c \). We can also see from the image of \( u_c \) that at 60 s the compensated disturbance in the NLADRC controller is closer to the added control disturbance. The output signal \( u_c \) of the controller plus the external control disturbance \( d \) is the input signal \( u_{\text{vgl}} \) of the diesel engine, which is shown in the second figure of Figure 8. As can be seen from the figure, the \( u_{\text{vgl, nladrc}} \) is basically unchanged after disturbance is added. This is because the compensated disturbance in \( u_c \) and the external disturbance \( d \) cancel each other. Therefore, compared with LADRC controller and PID controller, NLADRC controller can resist external interference better.

In addition to square wave form disturbance, sinusoidal form control disturbances is also considered to be added. Sinusoidal interference with an amplitude of 5 and a period of 20 s was added at 60 s, and Figure 9 and Figure 10 present the resistance effect of the three controllers against sinusoidal disturbance. For sinusoidal control interference, we have the same conclusion as the square wave form, which will not be repeated here.
Figure 7. Response curve of intake manifold pressure $p_{im}$ after adding square wave form control disturbance.

Figure 8. Output signal $u_c$ of controller and input signal $u_{vgt}$ of VGT valve.

Figure 9. Response curve of intake manifold pressure $p_{im}$ after adding sinusoidal control disturbance.
5.3. Robustness

Because the diesel engine needs to change the working condition frequently, the controller designed must be able to have strong robust performance to the changing working condition. In this part, we mainly verify whether the NLADRC controller has strong robust performance to the change of working conditions from the two aspects of speed $n_e$ and fuel injection $u_\delta$. First, we verify that the controller is robust to changes in speed. Before 80 s, the speed $n_e$ of diesel engines is 1900 r/min. The engine speed is reduced to 1700 r/min and 1500 r/min at the time of 80 s and 140 s, respectively. Figures 11 and 12 respectively show the response curves of intake manifold pressure $p_{im}$ and mass air flow $W_c$ before and after changing the speed. In addition, the fuel injection $u_\delta$ of the engine is increased from 50 mg/cycle to 200 mg/cycle with steps of 50 mg/cycle, and the simulation images are obtained without adjusting the control parameters, as shown in Figures 13 and 14. In the $p_{im}$ loop and the $W_c$ loop, the NLADRC controller can track the given signal faster and better without recalibrating parameters after changing the engine speed and fuel injection. Therefore, the NLADRC controller has strong robustness to the change of diesel engine operating conditions.

Although NLADRC controller has good tracking performance, resistance to external interference and robust performance to the change of working conditions for VGT-EGR system in diesel engines,
too many parameters of NLADRC controller are too troublesome to adjust, which may limit the wide application of NLADRC controller.

![Graph](image1.png)

**Figure 12.** The response curve of mass air flow at different engine speeds.

![Graph](image2.png)

**Figure 13.** The response curve of intake manifold pressure under different fuel injection conditions.

![Graph](image3.png)

**Figure 14.** The response curve of mass air flow under different fuel injection conditions.
6. Conclusions

In this paper, considering that the VGT-EGR system of diesel engines is a complex nonlinear system with unknown dynamics, the operating conditions of the system often change in actual operation. Therefore, the control strategy of intake manifold pressure and mass air flow of diesel engines based on NLADRC control theory is proposed. To ensure the rationality of the designed controller, the convergence proof of the designed NLESO is also given. Finally, the performance of the designed NLADRC controller is verified from the following three aspects: the tracking effect of the set signal, the resistance to external disturbance and the robust performance to the change of fuel injection and speed. Compared with the LADRC controller and PID controller, the designed NLADRC controller is effective for the VGT-EGR system in diesel engines.

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Nomenclature

| Symbol | Description |
|--------|-------------|
| VGT    | variable geometry turbine |
| EGR    | exhaust gas recirculation |
| MIMO   | multi-input multi-output |
| DIDO   | double-input double-output |
| SISO   | single-input single-output |
| PID    | proportional–integral–derivative |
| MPC    | model predictive control |
| CLF    | control Lyapunov function |
| ADRC   | Active disturbance rejection control |
| ESO    | extended state observer |
| LADRC  | linear active disturbance rejection control |
| NLADRC | nonlinear active disturbance rejection control |
| TD     | tracking differentiator |
| NLESO  | nonlinear extended state observer |
| NLSEF  | nonlinear states error feedback control laws |
| $p_{im}$ | intake manifold pressure [Pa] |
| $p_{em}$ | exhaust manifold pressure [Pa] |
| $T_{im}$ | intake manifold temperature [K] |
| $T_{em}$ | exhaust manifold temperature [K] |
| $V_{im}$ | intake manifold volume [m$^3$] |
| $V_{em}$ | exhaust manifold volume [m$^3$] |
| $p_{amb}$ | ambient pressure [Pa] |
| $T_{amb}$ | ambient temperature [K] |
| $P_c$ | compressor power [W] |
| $P_t$ | turbine power [W] |
| $u_{vgt}$ | VGT valve opening ratio [%] |
| $u_{egr}$ | EGR valve opening ratio [%] |
| $W_c$ | mass air flow [kg/s] |
| $W_f$ | fuel mass flow [kg/s] |
| $W_{egr}$ | EGR gas mass flow [kg/s] |
| $W_{ex}$ | exhaust gas mass [kg/s] |
| $W_e$ | mass flow into the cylinder [kg/s] |
$R$  gas constant [J/kgK]

$\eta_m$  turbocharger mechanical efficiency

$c_p$  constant pressure specific heat capacity [J/kgK]

$\eta_c$  compressor isentropic efficiency

$\eta_t$  turbine isentropic efficiency

$A_{egr}$  effective cross-sectional areas of the EGR valve [m$^2$]

$A_{vgt}$  effective cross-sectional areas of the VGT valve [m$^2$]

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