Measuring of $|V_{ub}|$ in the forthcoming decade

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abstract

I first introduce the importance of measuring $V_{ub}$ precisely. Then, from a theoretician’s point of view, I review (a) past history, (b) present trials, and (c) possible future alternatives on measuring $|V_{ub}|$ and/or $|V_{ub}/V_{cb}|$. As of my main topic, I introduce a model-independent method, which predicts $\Gamma(B \to X_u l\nu)/\Gamma(B \to X_c l\nu) \equiv (\gamma_u/\gamma_c) \times |V_{ub}/V_{cb}|^2 \simeq (1.83 \pm 0.28) \times |V_{ub}/V_{cb}|^2$ and $|V_{ub}/V_{cb}| \equiv (\gamma_c/\gamma_u)^{1/2} \times [B(B \to X_u l\nu)/B(B \to X_c l\nu)]^{1/2} \simeq (0.74 \pm 0.06) \times [B(B \to X_u l\nu)/B(B \to X_c l\nu)]^{1/2}$, based on the heavy quark effective theory. I also explore the possible experimental options to separate $B \to X_u l\nu$ from the dominant $B \to X_c l\nu$: the measurement of inclusive hadronic invariant mass distributions, and the ‘$D - \pi$’ (and ‘$K - \pi$’) separation conditions. I also clarify the relevant experimental backgrounds.

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Measuring of $|V_{ub}|$ in the forthcoming decade

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I first introduce the importance of measuring $V_{ub}$ precisely. Then, from a theoretician’s point of view, I review (a) past history, (b) present trials, and (c) possible future alternatives on measuring $|V_{ub}|$ and/or $|V_{ub}/V_{cb}|$. As of my main topic, I introduce a model-independent method, which predicts $\Gamma(B \rightarrow X_u \nu)/\Gamma(B \rightarrow X_d \nu) \equiv (\gamma_u/\gamma_c) \times |V_{ub}/V_{cb}|^2 \approx (1.83 \pm 0.28) \times |V_{ub}/V_{cb}|^2$ and $|V_{ub}/V_{cb}| \equiv (\gamma_c/\gamma_u)^{1/2} \times [\sigma(B \rightarrow X_u \nu)/\sigma(B \rightarrow X_d \nu)]^{1/2} \approx (0.74 \pm 0.06) \times [\sigma(B \rightarrow X_u \nu)/\sigma(B \rightarrow X_d \nu)]^{1/2}$, based on the heavy quark effective theory. I also explore the possible experimental options to separate $B \rightarrow X_u \nu$ from the dominant $B \rightarrow X_d \nu$: the measurement of inclusive hadronic invariant mass distributions, and the ‘$D - \pi$’ (and ‘$K - \pi$’) separation conditions. I also clarify the relevant experimental backgrounds.

1. INTRODUCTION

A precise determination of Cabibbo Kobayashi Maskawa (CKM) matrix elements [1] is the most important goal of the forthcoming $B$-factories [2], CLEO-III, KEK-B, SLAC-B, HERA-B, LHC-B. Their precise values are urgently needed for analyzing CP-violation and for testing the Standard Model (SM) through the unitarity relations among them [3]. Furthermore, the accurate knowledge of these matrix elements can be useful in relating them to the fermion masses and also in the searches for hints of new physics beyond the SM [4]. Even precision measurements on Top-physics will be affected, because the value of $V_{td}$ is related to $V_{ub}$ through the unitary relation.

The CKM matrix element $V_{ub}$ is important to the SM description of CP-violation. If it were zero, there would be no CP-violation from the CKM matrix (i.e. in the SM), and we have to seek for other sources of CP violation in $K_L \rightarrow \pi \pi$. Observations of semileptonic $b \rightarrow u$ transitions by the CLEO [5] and ARGUS [6] imply that $V_{ub}$ is indeed nonzero, and it is important to extract the modulus $|V_{ub}|$ from semileptonic decays of $B$ mesons as accurately as possible.

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2. OVERVIEW OF MEASURING $|V_{ub}|$

2.1. Past history

Historically, the charged lepton energy spectrum $(d\Gamma/dE_l)$ has been measured, and the $b \rightarrow u$ events are selected from the high end of the charged lepton energy spectrum. This method is applied to both inclusive and exclusive semileptonic $B$ decays. However, this cut on $E_l$ is not very effective, since only less than 10% of $b \rightarrow u$ events survive this cut at the $B$ meson rest frame. (In the future asymmetric $B$-factories with boosted $B$ mesons, much less than 10% of $b \rightarrow u$ events would survive the $E_l$ cut over the $b \rightarrow c$ threshold.) We also note that the dependences of the lepton energy spectrum on perturbative and non-perturbative QCD corrections [7,8] as well as on the unavoidable specific model parameters (e.g. the parameter $p_s$ of the ACCM model [9]) are strongest at the end-point region, which makes the model-independent determination of $|V_{ub}/V_{cb}|$ almost impossible from the inclusive distribution of $d\Gamma/dE_l$.

For exclusive $B \rightarrow X_u l\nu$ decays, the application of heavy quark effective theory (HQET) is very much limited, since $u$-quark is not heavy compared to $\Lambda_{QCD}$. And the theoretical predictions for the required hadronic matrix elements are largely different depending on which model we use, as can be seen in the following, as an
example for $B^0 \rightarrow \rho^+ l^- \bar{\nu}$,

$$\gamma_\rho = \frac{\Gamma_{\text{theory}}(B^0 \rightarrow \rho^+ l^- \bar{\nu})}{|V_{ub}|^2}$$

$$= 8.3 \times 10^{12}/\text{sec}, \quad (10)$$

$$= 32.9 \times 10^{12}/\text{sec}, \quad (11)$$

$$= 18.7 \times 10^{12}/\text{sec}. \quad (12)$$

See also Fig. 1 for the explicit model dependence on the value of $\gamma_\rho$. There are certainly many more available models than the listed above. And every one of them is based on a few reasonable assumptions. However, such assumptions of one model are in general exclusive to the assumptions of the other models, e.g. monopole dominance or multipole dominance. And the usual practice of selecting a few models and averaging the few chosen results is physically groundless. These model dependences are not like the statistical errors.

2.2. Present trials

Measurement of exclusive charmless semileptonic decays can put constraints on the models and therefore restrict the model dependence in principle, if the ratio of rates for $\pi l \nu$ and $\rho l \nu$ as well as the $q^2$ dependence of the form-factors are precisely measured. CLEO has recently succeeded in measuring the branching ratio $BR(B^0 \rightarrow \rho l \nu)$ [13].

A neutrino reconstruction technique is used: The neutrino energy and momentum are determined by evaluating the missing momentum and energy in the entire event:

$$E_{\text{miss}} = 2E_{\text{beam}} - \sum_i E_i,$$

$$\vec{p}_{\text{miss}} = \sum_i \vec{p}_i. \quad (2)$$

More criteria are imposed to guard against events with false large missing energies: First, the net charge is required to be zero. Secondly, events with two identified leptons (implying two neutrinos) are rejected. Leptons are required to have momenta greater than 1.5 GeV in the case of $\pi l \nu$ and greater than 2.0 GeV in the case of $\rho l \nu$. In addition, the candidate neutrino mass is calculated as

$$M^2_{\nu} = E^2_{\text{miss}} - \vec{p}^2_{\text{miss}}. \quad (3)$$

Candidate events containing a neutrino are kept if $M^2_{\nu}/2E_{\text{miss}} < 300$ MeV. Then the semileptonic $B$ decay candidates ($\pi^0$, $\pi^+$, $\rho^0$, $\omega^0$, $\rho^+ \ell \nu$) are reconstructed using the neutrino four-vector found from the missing energy measurement. The beam constrained invariant mass, $M_{\text{cand}}$ is defined as

$$M^2_{\text{cand}} = E^2_{\text{beam}} - (\vec{p}_\nu + \vec{p}_\ell + \vec{p}_{(\pi \text{ or } \rho)})^2 \quad (4)$$

and with the use of the neutrino four-vector is essentially the same as any other full $B$ reconstruction analysis done at the $\Upsilon(4S)$.

However, it is often difficult to prove that a $\pi \pi$ system indeed is dominantly from resonant $\rho$. CLEO attempts to show $\rho$ dominance by plotting the $\pi^+ \pi^-$ and $\pi^+ \pi^o$ summed mass spectrum. They also show a test case of $\pi^o \pi^o \ell \nu$, which cannot be $\rho$, since $\rho^o$ cannot decay to $\pi^o \pi^o$. There is an enhancement in the $\pi^+ \pi^-$ plus $\pi^+ \pi^o$ sum, while the $\pi^o \pi^o$ shows a relatively flat spectrum that is explained by background. CLEO proceeds by assuming they are seeing purely resonant decays in the vector channel.

Experimental importance here are that CLEO-III can be a powerful $B$-factory with possibly more than $10^5$ fully reconstructed semileptonic decay events. However, they have to find the way to avoid the most difficult problem; the large model dependence for exclusive $b \rightarrow u$ decays, as shown in Eq. (1), and as explained in previous Section 2.1. Better options shown in following Sections should be seriously pursued by CLEO-III experiment.

2.3. Future alternatives

(a) The possibility of measuring $|V_{ub}|$ via non-leptonic decays of $B$ mesons to exclusive two meson final states [13] has been theoretically explored. To avoid the theoretical difficulties of non-spectator decay diagrams, only those final states must be chosen in which no quark and antiquark pair has the same flavor. Within the factorization approximation and after considering the final state interactions, exclusive two body decay modes of $B$ mesons would certainly be worth of full investigation.
(b) It has also been suggested that the measurements of hadronic invariant mass spectrum [16,17] as well as hadronic energy spectrum [18] in the inclusive $B \to X_{c(u)} \ell \bar{\nu}$ decays can be useful in extracting $|V_{ub}|$ with better theoretical understandings. Experimentally, the hadron energy spectrum in semileptonic $B$-decays can be measured schematically as follows [19]: Working at the $\Upsilon(4S)$ resonance, which decays into $BB$, one requires one of the $B$-mesons to decay semileptonically and the other one hadronically. In the case of a symmetric $B$-factory, like CLEO, the energy of the hadrons stemming from the semileptonically decaying $B$-meson can be obtained by measuring the total energy of all the hadrons in the final state and then subtracting $m_{\Upsilon(4S)}/2$. In case of asymmetric $B$ factories, the hadron energy spectrum is harder to measure. One way is to reconstruct in a first step the whole $\Upsilon(4S)$ decay in its rest frame and then perform the analysis just described for the symmetric case. After imposing a relatively high lower-cut at $E_{had} = 1$ GeV (in order to avoid the region of phase space where the range in the invariant hadronic mass is too narrow to invoke quark-hadron duality), a much larger fraction ($\sim 25\%$) of the $b \to u \ell \bar{\nu}$ events is captured in the remaining window $1$ GeV $\leq E_{had} \leq m_D$ than in the lepton spectrum endpoint analysis.

(c) The measurement of ratio $|V_{ub}/V_{ts}|$ from the differential decay widths of the processes $B \to \rho \ell \nu$ and $B \to K^{*+} \ell \nu$ by using $SU(3)$-flavor symmetry and the heavy quark symmetry has also been proposed [20]. Then the ratio $|V_{ub}/V_{ts}|^2$ is extracted as

$$\frac{|V_{ub}|^2}{|V_{ts}|^2} \propto \frac{q^{2B \to K^*}}{q^{2B \to \rho}} \left( \frac{d\Gamma(B \to \rho \ell \nu)}{dq^2} \right)^{q_{max}^{2B \to \rho}} \left( \frac{d\Gamma(B \to K^* \ell \nu)}{dq^2} \right)^{q_{max}^{2B \to K^*}}.$$  

(5)

In the limit $q^2 \to q_{max}^2$, the $q^2$ distributions vanish due to the phase space suppression. In fact, CLEO collaboration has rather accurately determined the value of $|V_{cb}| \cdot f(q_{max}^2)$ for the process $B \to D^{*+} \ell \nu$ [21] by extrapolating the $q^2$ distribution. In the similar manner, the right-hand side of Eq. (5) can be determined by experiments.

There has also been a recent theoretical progress on the exclusive $b \to u$ semileptonic decay form factors using the HQET-based scaling laws to extrapolate the form factors from the semileptonic $D$ meson decays [22].

(d) It is urgently important that all the available methods have to be thoroughly explored to measure the most important CKM matrix element $V_{ub}$ as accurately as possible in the forthcoming $B$-factories. In future asymmetric $B$-factories (or in hadronic $B$-factories) with microvertex detector, the hadronic invariant mass spectrum (or ‘$D - \pi$', ‘$K - \pi$' separation conditions) offer alternative ways to select $b \to u$ transitions that are much more efficient than selecting the upper end region of the lepton energy spectrum, with much less theoretical uncertainties [17]. Then we can use the simple relation,

$$\frac{|V_{ub}|}{|V_{cb}|} = \left( \frac{\gamma_u}{\gamma_c} \right)^{1/2} \times \left( \frac{B(B \to X_u \ell \nu)}{B(B \to X_c \ell \nu)} \right)^{1/2}$$

(6)

where $\gamma_u$, $\gamma_c$, and $\gamma_u/\gamma_c$ can be calculated model-independently within the HQET. However, we note that the individual exclusive decay width, e.g. $\gamma_\rho$ or $\gamma_\pi$, cannot be predicted model-independently, as shown in Eq. (1) and in Fig. 1. We will give in more detail on this alternative method in the following Section.

3. MEASURING $|V_{ub}|$ MODEL INDEPENDENTLY

3.1. Theoretical proposal

Over the past few years, a great progress has been achieved in our understanding of inclusive semileptonic decays of heavy mesons [8], especially in the lepton energy spectrum. However, it turns out that the end-point region of the lepton energy spectrum cannot be described by $1/m_Q$ expansion. Rather, a partial resummation of $1/m_Q$ expansion is required [23], closely analogous to the leading twist contribution in deep inelastic scattering, which could bring about significant uncertainties and presumable model dependences.

Even with a theoretical breakdown near around the end-point region of lepton energy spectrum, accurate prediction of the total integrated
semileptonic decay rate can be obtained within the HQET including the first non-trivial non-perturbative corrections as well as radiative perturbative QCD correction. The related uncertainties in calculation of the integrated decay rate have been also analyzed. The total inclusive semileptonic decay rate for \( B \rightarrow X_q \nu \) is given as

\[
\Gamma(B \rightarrow X_q \nu) = \frac{G_F^2 m_b^5}{192 \pi^3} |V_{qb}|^2 \times \left( \frac{z_0(x_q) - 2 \alpha_s(m_b^2)}{3 \pi} g(x_q) \right) \left( 1 - \frac{\mu_G^2 - \mu_\pi^2}{2 m_b^2} \right)
- z_1(x_q) \frac{\mu_G^2}{m_b^2} + \mathcal{O}(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3) \right)
\]

where

\[
x_q \equiv m_q/m_b,
\]

\[
z_0(x) = 1 - 8 x^2 + 8 x^6 - x^8 - 24 x^4 \log x,
\]

\[
z_1(x) = (1 - x^2)^4,
\]

and \( g(x) = (\pi^2 - 31/4)(1 - x^2)^2 + 3/2 \) is the corresponding single gluon exchange perturbative QCD correction. The expectation value of energy due to the chromomagnetic hyperfine interaction, \( \mu_G \), can be related to the \( B^* - B \) mass difference

\[
\mu_G^2 = \frac{3}{4} (M_{B^*}^2 - M_B^2) \approx (0.35 \pm 0.005) \text{GeV}^2
\]

and the expectation value of kinetic energy of b-quark inside the \( B \) meson, \( \mu_\pi^2 \), is given from various arguments

\[
0.1 \text{ GeV}^2 \leq \mu_\pi^2 \leq 0.7 \text{ GeV}^2,
\]

which shows much larger uncertainties compared to \( \mu_G^2 \). The value of \( |V_{cb}| \) has been estimated from the total decay rate \( \Gamma(B \rightarrow X_u \nu) \) of Eq. (7) by using the pole mass of \( m_b \) and a mass difference \( (m_b - m_c) \) based on the HQET. As can be easily seen from Eq. (7), the \( m_b^6 \) factor, which appears in the semileptonic decay rate, but not in the branching fraction, is the largest source of the uncertainty, resulting in about \( 5 \sim 20\% \) error in the prediction of \( |V_{cb}| \) via the semileptonic branching fraction and \( B \) meson life time. Historically, the ACCMM model was motivated to avoid this \( m_b^6 \) factor, and at the same time to naively incorporate the bound state effect of initial \( B \) meson.

We can do a similar exercise to predict the value of \( |V_{ub}| \) from the integrated total decay rate of \( \Gamma(B \rightarrow X_u \nu) \), to find out

\[
|V_{ub}|^2 = \frac{192 \pi^3}{G_F^2 m_b^2} \cdot \Gamma(B \rightarrow X_u \nu) \times \left( \frac{1 - 2 \alpha_s(m_b^2)}{3 \pi} \left( \pi^2 - \frac{25}{4} \right) \right)
- \left( 1 - \frac{\mu_G^2 - \mu_\pi^2}{2 m_b^2} \right) \left( \frac{\mu_G^2}{m_b^2} \right)^{-1}.
\]

We use the pole mass of b-quark \( m_b = (4.8 \pm 0.2) \) GeV from a QCD sum-rule analysis of the \( Y \) system. To be conservative, we use here a larger error bar (larger by a factor 8) than that of the original analysis. We estimate the largest possible error of \( m_b \) as \( \mathcal{O}(\Lambda_{QCD}) \). And \( x_u \equiv m_u/m_b \approx 0 \) and we take \( \alpha_s(m_b^2) = (0.24 \pm 0.02) \). [Extrapolating the known 5% error of \( \alpha_s(m_b^2) \), we estimate about 10% error for \( \alpha_s(m_b^2) \).]

We get numerically

\[
\gamma_u \equiv \frac{\Gamma_{theory}(B \rightarrow X_u \nu)}{|V_{ub}|^2} \simeq (7.1 \pm 1.5) \times 10^{13}/\text{sec},
\]

and

\[
|V_{ub}| \simeq (3.6 \pm 0.4) \times 10^{-3}
\times \left[ \frac{\mathcal{B}(B \rightarrow X_u \nu)}{1.4 \times 10^{-3}} \right]^{1/2} \left[ \frac{1.52 \text{ psec}}{\tau_b} \right]^{1/2}.
\]

As previously explained, the largest uncertainty comes from the factor \( m_b^6 \), which gives the most part of the theoretical errors shown in Eq. (11). We remark that the semileptonic branching fraction of \( b \rightarrow u \) decay, \( \mathcal{B}(B \rightarrow X_u \nu) \), has to be precisely measured to experimentally determine the value of \( |V_{ub}| \) from Eq. (11). We will discuss on the experimental possibilities in details in the
next Section. Once the inclusive branching fraction \( \mathcal{B}(B \rightarrow X_u \ell \nu) \) is precisely measured, we can extract the value of \( |V_{ub}| \) within the theoretical error (\( \sim 10\% \)) similar to those of \( |V_{cb}| \). (Compare the inclusive \( \gamma_u \) of Eq. (11) and the exclusive \( \gamma_p \) of Eq. (6) for the theoretical predictions of the semileptonic \( b \rightarrow u \) decay.)

The ratio of CKM matrix elements \( |V_{ub}/V_{cb}| \) can be determined in a model-independent way by taking the ratio of semileptonic decay widths \( \Gamma(B \rightarrow X_u \ell \nu)/\Gamma(B \rightarrow X_c \ell \nu) \). As can be seen from Eq. (5), this ratio is theoretically described by the phase space factor and the well-known perturbative QCD correction only,

\[
\frac{\Gamma(B \rightarrow X_u \ell \nu)}{\Gamma(B \rightarrow X_c \ell \nu)} \simeq \frac{|V_{ub}|^2}{|V_{cb}|^2} \left[ 1 - \frac{2\alpha_s}{3\pi} \left( \frac{\pi^2 - 25}{4} \right) \right] \times \left[ z_0(x_c) - \frac{2\alpha_s}{3\pi} g(x_c) \right]^{-1} \quad , \quad (12)
\]

where we ignored the term \( \mu_c^2/m_b^2 \), which gives about 1\% correction to the ratio. We strongly emphasize here that the sources of the main theoretical uncertainties, the most unruly factor \( m_b^2 \) and the still-problematic non-perturbative contributions, are all canceled out in this ratio. By taking \( \alpha_s(m_b^2) = (0.24 \pm 0.02) \), and by using the mass difference relation from the HQET [22], which gives \( x_c \equiv m_c/m_b \approx 0.25 - 0.30 \). This ratio \( x_c \) is calculable from the mass difference \( (m_b - m_c) \), which also includes the uncertain parameter \( \mu_c^2 \) of Eq. (6) as a small correction factor.

The ratio of the semileptonic decay widths is estimated as

\[
\frac{\Gamma(B \rightarrow X_u \ell \nu)}{\Gamma(B \rightarrow X_c \ell \nu)} \equiv \left( \frac{\gamma_u}{\gamma_c} \right) \times \frac{|V_{ub}|^2}{|V_{cb}|^2} \quad (13)
\]

\[
\simeq (1.83 \pm 0.28) \times \frac{|V_{ub}|^2}{|V_{cb}|^2} \quad ,
\]

and the ratio of CKM elements is

\[
\frac{|V_{ub}|}{|V_{cb}|} \equiv \left( \frac{\gamma_c}{\gamma_u} \right)^{1/2} \times \left[ \frac{\mathcal{B}(B \rightarrow X_u \ell \nu)}{\mathcal{B}(B \rightarrow X_c \ell \nu)} \right]^{1/2} \quad (14)
\]

\[
\simeq (0.74 \pm 0.06) \times \left[ \frac{\mathcal{B}(B \rightarrow X_u \ell \nu)}{\mathcal{B}(B \rightarrow X_c \ell \nu)} \right]^{1/2} .
\]

Once the ratio of semileptonic decay widths (or equivalently the ratio of branching fractions \( \mathcal{B}(B \rightarrow X_u \ell \nu)/\mathcal{B}(B \rightarrow X_c \ell \nu) \)) is measured in the forthcoming asymmetric \( B \)-factories, this should give a powerful model-independent determination of \( |V_{ub}/V_{cb}| \). There is absolutely no theoretical model dependence in these ratios, Eqs. (11,13,14). As explained earlier, for example, in the ACCMM model \( 5 \) the model dependence comes in via the introduction of the parameter \( p_f \) in place of the factor \( m_b^2 \), which is now canceled in these ratios. We also note that by using the integrated total decay widths, instead of the lepton energy spectrum, another possible model dependence related to the endpoint region of the spectrum need not be even introduced.

**3.2. Experimental possibility**

As explained in the previous Section, in order to measure \( |V_{ub}/V_{cb}| \) (and \( |V_{cb}| \)) model-independently by using the relations Eqs. (11,13,14), it is experimentally required to separate the \( b \rightarrow u \) semileptonic decays from the dominant \( b \rightarrow c \) semileptonic decays, and to precisely measure the branching fraction \( \mathcal{B}(B \rightarrow X_u \ell \nu) \) or the ratio \( \mathcal{B}(B \rightarrow X_u \ell \nu)/\mathcal{B}(B \rightarrow X_c \ell \nu) \). At presently existing symmetric \( B \)-experiments, ARGUS and CLEO, where \( B \) and \( B \) are produced almost at rest, this required separation is possible only in the very end-point region of the lepton energy spectrum, because both \( B \) and \( B \) decay into the whole \( 4\pi \) solid angle from the almost same decay point, and it is not possible to identify the parent \( B \) meson of each produced particle. Hence all the hadronic information is of no use. However, in the forthcoming asymmetric \( B \)-experiments with microvertex detectors, BABAR and BELLE \( 3 \), where the two beams have different energies and the produced \( Y(4S) \) is not at rest in the laboratory frame, the bottom decay vertices will be identifiable. The efficiency for the full reconstruction of each event could be relatively high limited only by the \( \pi^0 \)-reconstruction efficiency of about 60\% \( 3 \), and this \( b \rightarrow u \) separation would be experimentally viable.

As of the most straightforward separation method, the measurements of inclusive hadronic invariant mass \( m_{X} \) distributions in \( B \rightarrow X_{c,u} \ell \nu \)
can be very useful for the fully reconstructed semileptonic decay events. For $b \to c$ decays, one necessarily has $m_X \geq m_D = 1.86$ GeV. Therefore, if we impose a condition $m_X < m_D$, the resulting events come only from $b \to u$ decays, and about 90% of the $b \to u$ events would survive this cut. This is already in sharp contrast with the usual cut on charged lepton energy $E_l$.

In fact, one can relax the condition $m_X < m_D$, and extract almost the total $b \to u$ semileptonic decay rate \cite{1,2}, because the $m_X$ distribution in $b \to c$ decays is completely dominated by contributions of three resonances $D, D^*$ and $D^{**}$, which are essentially like $\delta$-functions,

$$\frac{d\Gamma}{dm_X} = \Gamma(B \to Rl\nu) \delta(m_X - m_R), \quad (15)$$

where the resonance $R = D, D^*$ or $D^{**}$. See Fig. 1. In other words, one is allowed to use the $b \to u$ events in the region even above $m_X \geq m_D$, first by excluding small regions in $m_X$ around $m_X = m_D, m_{D^*}, m_{D^{**}}$, and then by including the regions again numerically in the $m_X$ distribution of $b \to u$ decay from its values just around the resonances. There is still a non-resonant decay background at large invariant-mass region $m_X \geq m_D + m_\pi$ from $B \to (D + \pi)l\nu$ in using this inclusive $m_X$ distribution separation. However, with an additional $D - \pi$ separation condition, which we explain later, this many-pion producing $b \to u$ decay could be safely differentiated from the non-resonant $b \to c$ semileptonic decays. Instead of using the $D - \pi$ separation, if we impose a condition $m_X < m_D + m_\pi$, we would get about 95% of the total $b \to u$ events.

We note that there is possibly a question of bias. Some classes of final states (e.g., those with low multiplicity, few neutrals) may be more susceptible to a full and unambiguous reconstruction. Hence an analysis that requires this reconstruction may be biased. However, the use of topological information from microvertex detectors should tend to reduce the bias, since vertex resolvability depends largely on the proper time of the decay and its orientation relative to the initial momentum (that are independent of the decay mode). Also such a bias can be allowed for in the analyses, via a suitable Monte Carlo modeling. For more details on this inclusive hadronic invariant mass distribution $d\Gamma/dm_X$, please see Ref. \cite{18}.

Even without full reconstructions of all the final particles, one can separate $b \to u$ decays from $b \to c$ decays by using the particle decay properties \cite{13}. Since $D^{**} \to D^* + \pi$ and $D^* \to D + \pi$, the semileptonic $b \to c$ decays always produce at least one final state $D$ meson, compared to $b \to u$ decays which produce particles, $\pi$, $\rho$, ... that always decay to one or more $\pi$ mesons at the end. Therefore, the $b \to u$ decay separation could be achieved better with the accurate $D - \pi$ separation in particle detectors, and if combined with the hadronic invariant-mass distribution separation. Such $D - \pi$ separations would be possible via partial reconstructions of a whole event or by using special event characteristics, rather than by fully reconstructing a whole event. In order for this $D - \pi$ separation to be successfully implemented, one should have the separation with better than about 98% efficiency, because of $\Gamma(B \to X_u l\nu) \sim (0.02) \times \Gamma(B \to X_c l\nu)$. We can also think of $K - \pi$ separation as a natural extension, because the semileptonic $b \to c$ decays always produce at least one final state $K$ meson. These $D - \pi$ (and $K - \pi$) separation conditions are also applicable at the hadronic $B$-factories, HERA-B, LHC-B.

There possibly is a source of background to this $D - \pi$ separation condition from the cascade decay of $b \to c \to sl\nu$. Recently ARGUS and CLEO \cite{14} have separated this cascade decay background from the signal events to extract the model-independent spectrum of $\frac{d\Gamma}{dE_l}(B \to X_c l\nu)$ for the whole region of electron energy, by taking care of lepton charge and $B - \bar{B}$ mixing systematically. In the future asymmetric $B$-factories with much higher statistics, this cascade decay may not be any serious background at all except for the case with very low energy electron production.

4. DISCUSSIONS & CONCLUSIONS

The precise value of $V_{ub}$ is urgently needed for understanding the origin of CP-violation, for testing the SM through the unitarity relations among
them, and also in the searches for hints of new physics beyond the SM. We propose that the ratio of CKM matrix elements $|V_{ub}/V_{cb}|$ can be determined in a model-independent way by taking the ratio of semileptonic decay widths $\Gamma(B \to X_u l\nu)/\Gamma(B \to X_c l\nu)$, which is theoretically described by the phase space factor and the well-known perturbative QCD correction only, and which predicts

$$\frac{\Gamma(B \to X_u l\nu)}{\Gamma(B \to X_c l\nu)} \equiv \left( \frac{\gamma_u}{\gamma_c} \times \frac{|V_{ub}|}{|V_{cb}|} \right)^2 \approx (1.83 \pm 0.28) \times \left( \frac{|V_{ub}|}{|V_{cb}|} \right)^2,$$

and

$$\frac{|V_{ub}|}{|V_{cb}|} \equiv \left( \frac{\gamma_c}{\gamma_u} \right)^{1/2} \times \left[ \frac{B(B \to X_u l\nu)}{B(B \to X_c l\nu)} \right]^{1/2} \approx (0.74 \pm 0.06) \times \left[ \frac{B(B \to X_u l\nu)}{B(B \to X_c l\nu)} \right]^{1/2},$$

based on the heavy quark effective theory. Once the ratio of semileptonic decay widths (or equivalently the ratio of branching fractions $B(B \to X_u l\nu)/B(B \to X_c l\nu)$) is measured, this ratio will give a powerful model-independent determination of $|V_{ub}/V_{cb}|$.

In the forthcoming asymmetric $B$-factories with microvertex detectors, the total separation of $b \to u$ semileptonic decays from the dominant $b \to c$ semileptonic decays to determine the ratio would be experimentally viable. We explore the possible experimental options: the measurement of inclusive hadronic invariant mass distributions, and the ‘$D - \pi$’ (and ‘$K - \pi$’) separation conditions. We also clarify the relevant experimental backgrounds. In view of the potential importance of $B(B \to X_u l\nu)/B(B \to X_c l\nu)$ as a new theoretically model-independent probe for measuring $|V_{ub}/V_{cb}|$, we would like to urge our experimental colleagues to make sure that this $b \to u$ separation can indeed be successfully achieved.

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Figure 1. The $m_X$ distributions in $B \to X_{c,u}\ell\nu$ with $|V_{ub}/V_{cb}| = 1$. The $b \to c$ transition is dominated by the $X_c = D, D^*, D^{**}$. On the other hand, the $b \to u$ transition is largely nonresonant. The cases with $X_u = \pi, \rho$ are shown explicitly. The inclusive $m_X$ distribution for $b \to u$ was obtained from the ACCMM model with hadronic mass constraint of $m_X > 2m_\pi$. Note that the individual exclusive decay width, e.g. $\gamma_{\rho}$ as in Eq (1), depends strongly on the models, even though the total inclusive decay rate is calculable model-independently within the HQET.