The Catastrophe Criterion for Instability of Deep Roadway based on Malignant Expansion of Plastic Zone

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Abstract. In order to solve the problem of discriminant criterion for deep roadway instability, the core of surrounding rock failure in deep roadway and its two types of instability mechanisms were analysed. Based on the cusp catastrophe model and taking the plastic radius function as the potential function of the roadway system, the instability discriminant of surrounding rock was derived and the catastrophic criterion for instability of deep roadway under the influence of disturbance was established. Aiming at the engineering application of this criterion, the implementation method and detailed procedure of stability discrimination for the deep roadway were proposed. The numerical results show that the malignant expansion of plastic zone often occurs in surrounding rock causing large deformation or transient instability in the ultra-high deviatoric stress environment. Under the influence of external disturbance, the plastic radius experienced two stages including slow increase and nonlinear acceleration. The discriminant conclusion of the deep roadway instability according to the calculation results of the catastrophic eigenvalues is consistent with the extension of the plastic zone during the instability process of surrounding rocks, which verifies that the established instability criterion is feasible and applicable.

1. Introduction

With the gradual exhaustion of shallow resources around the world, the exploitation of deep coal resources has become more and more common[1,2]. Compared with shallow mining, deep roadways are prone to cause large deformation, coal and gas outburst. Therefore, the instability identification of deep surrounding rock is one of the important subjects in rock mechanics and mining industry[3,4].

There are a lot of research results on the instability criterion of surrounding rock, mainly including stability classification, deformation rate or critical displacement criterion, energy method and stress criterion based on strength theory. The catastrophe theory is widely used in the instability discrimination of rock masses engineering in recent years. Based on the catastrophe theory, Fu[5] established the intuitive criterion for stability of underground engineering, including the catastrophe criterions on yield area and viscous-plastic shear strain. Mu et al.[6,7] established the cusp catastrophe model for instability of surrounding rock and derived the mechanical determination condition for its
instability. Liu[8] analysed the ratio of the stiffness of the internal bearing area to that of the relaxation zone and obtained the criterion of tunnel collapse using the cusp catastrophe theory.

In deep roadways, nonlinear large deformation, coal and gas outburst and other disasters are still not accurately and scientifically predicted. Therefore, in this paper the cusp catastrophe model for instability of deep surrounding rock was established and the instability criterion dominated by plastic radius was obtained based on the catastrophe theory.

2. Catastrophic instability of deep roadway and malignant expansion of the plastic zone

2.1 Malignant expansion of the plastic zone around deep roadway

Assuming that the underground chamber is circular and that the surrounding rock is homogeneous. $P_1$ and $P_3$ are the initial vertical and horizontal stresses respectively. $D_\varsigma$ is the vertical disturbance coefficient and $D_{3\varsigma}$ is the horizontal one. According to the boundary equation[9], the evolution of plastic zone in the surrounding rock can be obtained as shown in figure 1, where $R_0=2$ m, $c=3$ MPa, $\varphi=25^\circ$, $P_1=P_3=20$ MPa.

Figure 1 shows that the shape of plastic zone changes from circular to elliptical, rectangular and butterfly with the increase of the difference between the vertical and horizontal pressure. It is worth noting that the plastic damage depth in the butterfly-shaped plastic zone is much larger than that in other shapes obviously, thus the stability of surrounding rock is relatively poor.

The butterfly-shaped plastic zone is highly sensitive to the change of the stress field. When the external disturbance is small, the plastic radius increases slowly. When it continues to increase and exceeds the critical value $D_{3\varsigma}$, as shown in figure 2, the change rate of the plastic radius ($r$) increases sharply, resulting in that the plastic range can’t converge and $r$ tends to infinity. This phenomenon is called as malignant expansion[9]. In this case, the roadway will be completely instability.

2.2 Mechanism of catastrophic instability for the deep roadway

The key scientific issue for the mechanism of deep roadway instability is how the plastic zone develops and expands. According to the expansion rate and time of the plastic zone, the instability of the deep roadway is divided into the following two types.

2.2.1 Gradual type

Under high deviatoric stress, the butterfly-shaped plastic zone around the deep roadway forms after excavation. The external disturbance leads to increase of the stress field. Then the butterfly-shaped plastic zone correspondingly expands. If the disturbance is a long-term process, such as periodic
vibration, the butterfly-shaped plastic zone will expand continuously. The slow deformation of the rock masses accumulates, which leads to nonlinear large deformation and failure of the roadway[9].

2.2.2 Catastrophic type
In coal mining, due to loading and unloading effects caused by blasting, roof collapse, fault activation, the regional stress field of the deep roadway becomes an ultra-high deviatoric stress field instantaneously, which leads to explosive growth of the butterfly-shaped plastic zone around the roadway or in front of the heading face, that is malignant expansion. At the same time, a large amount of elastic energy and gas energy stored in the plastic zone suddenly release into the roadway space, thus rock burst or coal and gas outburst occur[10].

3. The cusp catastrophe model
In 1972, Thom firstly proposed the catastrophe theory. At present, the cusp catastrophe model is the most widely used elementary catastrophe model in catastrophe theory[11].

The standard potential function of the cusp catastrophe model is:

$$V(x, u, v) = x^3 + ux^2 + vx$$

where $x$ is the state variable, $u$, $v$ are the control variables.

The equation of the equilibrium surface for the above potential function is:

$$V'(x) = 4x^3 + 2ux + v = 0$$

In $(x, u, v)$ space this equilibrium surface is a pleated surface composed of the upper, middle and lower leaves, as shown in figure 3. The coordinates of a point on the equilibrium surface represent the state of the system. The system is stable when the point is at the equilibrium position in the upper or lower leaves, while it is unstable when that in the middle leaf. The equation of singular point set corresponding to the equilibrium surface can be expressed as

$$V''(x) = 12x^2 + 2u = 0$$

![Figure 3. The cusp catastrophe model[5].](image)

According to equation (2) and equation (3), the equation of bifurcation set can be obtained as

$$8u^3 + 27v^2 = 0$$

The bifurcation set is the critical position of the catastrophic phenomenon in the system.

Figure 3 shows the relationship between the bifurcation set and the stability of the nonlinear system. It can be determined that whether the state of the system has a sudden change by the catastrophic eigenvalue $\Delta = 8u^3 + 27v^2$. 
4. Establishment of catastrophic criterion for deep roadway instability

It’s assumed that the circular roadway is homogeneous with the given initial stresses, and that the horizontal or vertical disturbance factor is $D_c$. It’s considered that there is a certain mapping relationship between the maximum plastic radius $r_\psi$ and the disturbance factor $c_D$. Then $r_\psi$ can be expressed as a continuous function of $D_c$, i.e. $r_\psi = f(D_c)$, which is also the potential function for the roadway surrounding rock system.

The function $r_\psi = f(D_c)$ is expanded at $D_c = 0$ according to the Taylor’s series, and the terms less than or equal to the 4th order are taken. It can be obtained:

$$\psi_r = a_0 + \sum_{i=1}^{4} a_i D_c^i$$

(6)

where $a_0 = f(0), a_1 = \left. \frac{\partial \psi_r}{\partial D_c} \right|_{D_c=0}$. We are setting $D_c = x - \xi$, then equation (5) can be rewritten as

$$\psi_r = a_4 x^4 + (a_3 - 4a_4 \xi)x^3 + (a_2 - 3a_3 \xi + 6a_4 \xi^2)x^2 + (a_1 - 2a_2 \xi + 3a_3 \xi^2 - 4a_4 \xi^3)x + (a_0 - a_1 \xi + a_2 \xi^2 - a_3 \xi^3 + a_4 \xi^4)$$

(7)

Taking $\xi = \frac{a_1}{4a_4}$, then equation (7) is changed to be

$$\psi_r = c_4 x^4 + c_3 x^3 + c_2 x^2 + c_1 x + c_0$$

(8)

where $c_4 = a_4, c_3 = a_3 - 3a_4 \xi + 6a_4 \xi^2, c_2 = a_2 - 2a_3 \xi + 3a_4 \xi^2 - 4a_4 \xi^3, c_0 = a_0 - a_1 \xi + a_2 \xi^2 - a_3 \xi^3 + a_4 \xi^4$

After setting $\psi_r = cx^4, \mu = \frac{c_2}{c_4}, \nu = \frac{c_1}{c_4}, \omega = \frac{c_0}{c_4}$, equation (8) is transformed into the standard potential function of the cusp catastrophe model,

$$V(x) = x^4 + \mu x^2 + \nu x + \omega$$

(9)

Based on

$$V'(x) = \frac{dV}{dx} = 4x^3 + 2\mu x + \nu = 0$$

(10)

and

$$V''(x) = \frac{d^2V}{dx^2} = 12x^2 + 2\mu = 0$$

(11)

The bifurcation set equation for the cusp catastrophe model is obtained as

$$8\mu^3 + 27\nu^2 = 0$$

(12)

Therefore, $\Delta = 8\mu^3 + 27\nu^2$ can be used as the criterion to judge whether the catastrophic instability occurs in the surrounding rock system of the deep roadway. When $\Delta \leq 0$, the surrounding rock system is in an unstable state, and the catastrophe phenomenon will occur in the roadway.

In addition, the stability of deep roadway can be identified by the following procedure (figure 4). Firstly the plastic radius $\psi_r(k)$ of the surrounding rock corresponding to the external disturbance $D_c$ is collected, and the plastic radius of each state constitutes a time series, that is
where $\psi_r(k)$ is equal to the maximum plastic radius, thus

$$\psi_r(k) = \max \{r_1, r_2, r_3, r_4, ... r_i\}$$

where $r_i$ is the plastic radius of the $i$-th key position in the surrounding rock under different states.

Figure 4. The discrimination procedure for instability of the deep surrounding rock.

5. The numerical validation

5.1 The numerical model and parameters for calculation

A deep circular roadway is selected for numerical calculation. The buried depth of the roadway with a diameter of 2 m is 800 m, and the surrounding rock medium is considered to be homogeneous soft
rock. The initial horizontal and vertical ground stress are both 20 MPa. The vertical disturbance factor is \( D_c \). The regional geological conditions of the roadway are relatively simple.

A plane strain model \((20 \text{ m} \times 20 \text{ m} \times 0.1 \text{ m})\) for the roadway is established, as shown in figure 5. The front and back boundaries of the model are constrained in its normal direction, while the top, bottom, left and right boundaries are fixed. The Mohr-Coulomb criterion is used, and the parameters for calculation are shown in table 1. The distributive characteristics of the plastic zone around the deep roadway subjected to the initial stress and different disturbances were calculated.

![Figure 5. The numerical model.](image)

| Bulk modulus (GPa) | Shear modulus (GPa) | Density (kg/m³) | Cohesion (MPa) | Internal friction angle (°) | Tensile strength (MPa) |
|-------------------|---------------------|-----------------|---------------|-----------------------------|-----------------------|
| 1.80              | 1.02                | 2500            | 3.03          | 28.00                       | 1.90                  |

5.2 The instability discrimination for the deep surrounding rock.

The analysis for instability of surrounding rock of the deep roadway focuses on the investigation to the morphologic distribution and evolution process of the plastic zone. The change process of the plastic radius \( \psi_r(k) \) was recorded when the disturbance factor \( D_c \) ranging from 1 to 2.9, as shown in figure 6.

![Figure 6. The changing curve of the plastic radius under disturbance.](image)

After calculation based on the proposed procedure, each parameter in the cusp catastrophe model for plastic radius under different disturbances are shown in table 2.

Figure 6 shows that the plastic radius of surrounding rock in the deep roadway slowly expands as the disturbance factor \( D_c \) increases from 1.0 to 2.0. After the disturbance factor exceeds 2.0, the plastic radius presents accelerated increase. The acceleration of the plastic radius begins to slow down until \( D_c=2.7 \). At this point, the plastic radius reaches 11.983 m, which is 6.84 times of the initial state. It is
indicated that the plastic zone of deep surrounding rock is very sensitive to the external disturbances and develops rapidly. In addition, it can be inferred from the trend of change rate of the post-stage (Dc = 2.0 ~ 2.8) on the curve that the plastic zone has a malignant expansion phenomenon.

It can be seen from Table 2 that when Dc ≤ 2.7, ∆ > 0, thus the deep roadway is stable. When Dc = 2.8, ∆ = -42.890 < 0, which indicates that the instability and failure occur in the roadway, and that the state of the surrounding rock system changes suddenly from stability to instability.

### Table 2. The calculation results of the parameters in the cusp catastrophe model.

| Dc  | a4   | a3    | a2   | a1   | a0   | ξ    | c4   | c2   | c1   | c0   | μ    | ν    | Δ    |
|-----|------|-------|------|------|------|------|------|------|------|------|------|------|------|
| 2.0 | 7.669| -42.688| 87.257| -76.508| 26.025| -1.392| 7.669| -1.846| 1.014| 2.255| -0.241| 0.132| 0.361|
| 2.2 | 3.352| -17.170| 31.827| -24.116| 7.856 | -1.280| 3.352| -1.151| 1.086| 2.123| -0.343| 0.324| 2.510|
| 2.4 | -1.110| 10.792| -32.248| 39.426| -15.132| -2.431| -1.110| 7.109 | 10.203| 6.425 | -6.406| -9.194| 4.41×10^4 |
| 2.5 | 0.241| 1.926  | -11.050| 17.591| 15.802 | 1.999 | 0.241| 7.109 | 77.166| -9.783| -6.962| -9.194| 3.180 |
| 2.6 | 3.663| -21.186| 45.634| -42.126| 15.802 | -1.446| 3.663| -16.825| 77.166| 2.266 | -0.85 | -9.194| 4.667 |
| 2.7 | 2.628| -14.016| 27.645| -22.785| 8.296  | -1.334| 2.628| -0.312 | 1.258 | 2.144 | -0.149| 0.343 | -42.890 |
| 2.8 | -1.659| 16.427 | 50.577| 63.137 | 8.296  | -2.475| -1.659| 10.415| 14.049| 7.572 | -6.277| -8.468| -21.146 |
| 2.9 | -4.513| 37.239 | -105.384| 124.687| -25.674| -2.063| -4.513| 9.838 | 6.824 | 3.447 | -2.180| -1.512| -21.146 |

| Status | Stable | Stable | Stable | Stable | Stable | Stable | Instability | Instability |
|--------|--------|--------|--------|--------|--------|--------|-------------|-------------|

When Dc increases to 2.8, the instability of the surrounding rock occurs. Meanwhile, the plastic zone size also has a step extension (as shown in figure 7). The plastic limbs of the circular roadway sharply expanded along the bisector of the principal stresses. When Dc = 2.9, the plastic zone area further enlarges, but the plastic radius doesn’t significantly increase, which also aggravates the failure degree of the roadway surrounding rock.

![Figure 7. The morphology of plastic zone before and after instability of the surrounding rock.](image.png)

In summary, the evolution of the plastic zone and instability of the surrounding rock in numerical calculation are consistent with the discriminant result obtained from the instability criterion. Thus it is feasible and applicable to discriminate the stability of the deep roadway under the disturbance.
6. Conclusions
(1) The core for the catastrophic instability of the deep roadway is the expansion and evolution of the plastic zone. In ultra-high deviatoric stress field caused by external disturbance, the malignant expansion of the plastic zone is prone to bring in surrounding rock, leading to large deformation or transient instability. The mechanisms for gradual and catastrophic types are interpreted respectively.

(2) Based on the cusp catastrophe model and taking the plastic radius function as the potential function of the roadway system, the criterion for catastrophic instability of the deep roadway under the influence of disturbance was established. For practical engineering, the method and detailed procedure for stability discrimination of the deep surrounding rock were proposed.

(3) The development of the plastic zone around deep roadway is very sensitive to external disturbance, and the plastic radius experiences slow increase and non-linear acceleration. The results show that when $D_c = 2.8$, the catastrophic eigenvalue $\Delta < 0$. So the instability of the deep roadway can be determined. Through numerical calculation the feasibility and applicability of the instability criterion for the deep roadway were verified.

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