On 10D SYM Superamplitudes

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Abstract—Recently the spinor helicity and (two types of) superamplitude formalisms for 11D supergravity and 10D supersymmetric Yang–Mills theories were proposed in [1–3]. In this contribution we describe briefly the basic properties of these superamplitudes for the simpler case of 10D SYM.

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1. INTRODUCTION

The impressive recent progress in calculation of loop amplitudes in maximally supersymmetric gauge theory and supergravity, SYM and SUGRA, has been reached in the frame of on–shell amplitude calculus (see [4–7] and refs. therein) using intensively the so-called spinor helicity formalism and on–shell superfield approach.

These are based on the description of massless particle momentum and polarizations by single complex Weyl spinor (called helicity spinor, and its complex conjugate. The (light-like) momentum of $i$th particle is given by

$$p_{\mu(i)} := \sigma^{\mu\alpha}_{\alpha\beta} \lambda_{\beta(i)} \equiv \lambda_{\beta(i)} \sigma_{\mu} \bar{\lambda}_{\alpha(i)},$$

where $\sigma^{\mu\alpha}_{\alpha\beta}$ are relativistic Pauli matrices. The polarization vectors of spin 1 particles ("gluons") of negative and positive helicity are

$$e^{(+)\alpha}_{\alpha(i)} := \lambda_{\alpha(i)} / |\lambda_{\alpha(i)}|, \quad e^{(-)\alpha}_{\alpha(i)} = \mu_{\alpha} \lambda_{\alpha(i)} / |\mu \lambda_{\alpha(i)}|,$$

where $\mu_{\alpha} = (\mu_{\alpha})^\ast$ is a (constant) reference spinor, and

$$\langle \mu \lambda_{\alpha(i)} \rangle := e^{\alpha}_{\alpha(i)} \mu_{\alpha} = 2 \lambda_{\alpha(i)} / |\lambda_{\alpha(i)}|, \quad \langle \mu \lambda_{\alpha(i)} \rangle = \mu_{\alpha} \lambda_{\alpha(i)} / |\mu \lambda_{\alpha(i)}|.$$

The $n$-point scattering amplitudes, which we define with explicitly extracted momentum conservation delta function,

$$\delta^n \left( \sum_{i=1}^{n} p_i \right) \mathcal{A}(p_1, e_1; \ldots; p_n, e_n)$$

are independent of the choice of $\mu$ in (2), and obey the helicity constraints,

$$\hat{h}_{(i)} \mathcal{A}(l_1, \ldots, n) = h_{(i)} \mathcal{A}(l_1, \ldots, n).$$

Here $h_{(i)}$ is the helicity of the state, $h_{(i)} = \pm 1$ in the case of gluons, and

$$\hat{h}_{(i)} := \frac{1}{2} \lambda_{\alpha(i)} \frac{\partial}{\partial \lambda_{\alpha(i)}} - \frac{1}{2} \bar{\lambda}_{\alpha(i)} \frac{\partial}{\partial \bar{\lambda}_{\alpha(i)}}$$

is the helicity operator. Eq. (5) implies

$$\mathcal{A}(\ldots, e^{b\alpha}_{\alpha(i)} \lambda_{\alpha(i)}, e^{-b\alpha}_{\alpha(i)} \bar{\lambda}_{\alpha(i)}, \ldots) = e^{2bh_{(i)}} \mathcal{A}(\ldots, \lambda_{\alpha(i)}, \bar{\lambda}_{\alpha(i)}, \ldots).$$

A superamplitude of $\mathcal{N} = 4$ SYM depends, besides $n$ sets of complex bosonic spinors, on $n$ sets of complex fermionic variables $\eta_{\alpha i}$ carrying the index of fundamental representation 4 of the $SU(4)$ R-symmetry group

$$\mathcal{A}(l_1; \ldots; n) = \mathcal{A}(\lambda_{(i)}, \bar{\lambda}_{(i)}, \eta_{(i)}; \ldots; \lambda_{(n)}, \bar{\lambda}_{(n)}, \eta_{(n)}),$$

$$\eta_{(i), \eta_{(i)}} = -\eta_{(i), \eta_{(i)}}.$$
it obeys \( n \) super-helicity constraints,
\[
\hat{h}(\lambda_{i}), \delta(A(\lambda_{i}), \lambda_{A})) = \delta(A(\lambda_{i}), \lambda_{A})),
\]
\[
\hat{h}(i) = \frac{1}{2} \lambda_{a} \frac{\partial}{\partial \lambda_{a}} - \frac{1}{2} \lambda_{a} \frac{\partial}{\partial \lambda_{a}} + \frac{1}{2} \lambda_{A} \frac{\partial}{\partial \lambda_{A}}.
\]
\[A = 1, \ldots, 4.\]

The dependence of the superamplitude on \( \eta_{A} \) is holomorphic: it is independent of \( \eta_{A} \). Furthermore, according to (10), the degrees of homogeneity in \( \eta_{A} \) is related to the helicity \( \hat{h} \) in (6), so that the decomposition of superamplitude on \( \eta_{A} \) includes amplitudes of different helicities.

These superamplitudes can be regarded as multi-particle generalizations of the so-called on-shell superfields
\[
\Phi(\lambda, \lambda, \eta^{A}) = f^{(A)} + \eta_{A} \chi^{A},
\]
\[
\eta^{A} = \frac{1}{2} \epsilon_{AB} \eta_{B}, \quad \eta^{A} = \frac{1}{4} \eta_{A} \ldots \eta_{A} \epsilon^{A},
\]
which obey the super-helicity constraint
\[
2 \hat{h} = \lambda^{a} \frac{\partial}{\partial \lambda^{a}} - \lambda^{a} \frac{\partial}{\partial \lambda^{a}} + \eta_{A} \frac{\partial}{\partial \eta_{A}}.
\]

The component fields in (11) describe the on-shell degrees of freedom of the \( \mathcal{N} = 4 \) SYM multiplet: positive and negative helicity gluons (\( f^{(A)} \)), 6 scalars (\( s^{AB} = -s^{BA} \)) and four \( \pm 1/2 \) helicity fermions (\( \chi^{A} \) and \( \chi_{A} \)).

Higher \( n > 3 \) (super)amplitudes can be reconstructed from the lower, \( n' \) point (super)amplitudes with \( 3 \leq n' \leq (n-1) \), using the BCFW recurrent relations [8] and its superfield generalization [9]. To start such calculations one needs to know the basic MHV and anti-MHV (MHV) 3-point superamplitudes which in the case of \( \mathcal{N} = 4 \) SYM read
\[
\delta(\eta_{A}) (23) + \eta_{A} (31) + \eta_{A} (12)
\]
\[
= \delta(\eta_{A}) (23) + \eta_{A} (31) + \eta_{A} (12)
\]
(19) to \(SO(1,1) \times SO(8)\), (where the index \(i\) is introduced to stress that, in the scattering problem, each set of spinor frame variables is defined up to its "own" \(SO(1,1) \times SO(8)\) gauge transformations). This makes possible to identify \(v_{aq(i)}\) with homogeneous coordinates of \(\mathbb{S}^8\), (18), which, in the light of the relation with light-like momenta (19), can be recognized as celestial sphere of a ten-dimensional observer (of the \(i\)th 10D observer) [12, 13].

The name of spinor frame variables indicates that the above constraints (19) can be obtained from two statements: i) that the variables \(v_{aq}\) form a \(16 \times 8\) block of a \(Spin(1,9)\) valued matrix

\[
V_a^{(b)} = \left( v_{aq}, v_{aq} \right) \in Spin(1,9),
q = 1, \ldots, 8, \quad a = 1, \ldots, 8,
\]

which is called spinor moving frame matrix, and ii) that the light-like momentum \(k_{ai}\) of a massless 10D particle, \(k_i k^i = 0\), is related to certain vector from associated \(SO(1,9)\) valued matrix (moving frame matrix)

\[
u_{ai}^{(b)} = \left( \frac{1}{2} v_{ai}^{(a)} u_{ai}^{(b)} - \frac{1}{2} u_{ai}^{(a)} v_{ai}^{(b)} \right) \in SO(1, D - 1)
\]

by (for further use, we restore the subscript \(i = 1, \ldots, n\) here)

\[
k_{ai} = p_i^{\alpha} u_{ai}^{\alpha},
\]

The relation of moving frame (22) and spinor moving frame (21) is given by

\[
V \sigma_b \sigma_i V = \left( u_{ai}^{(a)} \right) \sigma_b \sigma_i = \frac{1}{2} \sigma^b \sigma_i u_{ai}^{(a)},
\]

which can be easily recognized as conditions of Lorentz invariance of the generalized Pauli matrices written for a specific Lorentz rotation associated to the vector frame.

Eq. (22) implies the following properties of the frame vectors (or vector harmonics; these were called light-cone harmonic variables [14, 15])

\[
u_{ai}^{(a)} u_{ai}^{(a)} = 0, \quad u_{ai}^{(a)} u_{ai}^{(a)} = 0, \quad u_{ai}^{(a)} u_{ai}^{(a)} = 2,
\]

\[
u_{ai}^{(a)} u_{ai}^{(a)} = 0, \quad u_{ai}^{(a)} u_{ai}^{(a)} = 0, \quad u_{ai}^{(a)} u_{ai}^{(a)} = -\delta_{ij}.
\]

With an appropriate representation of sigma matrices, (24) implies

\[
u_{ai}^{(a)} \sigma_{aj}^{\alpha} = 2 v_{aq}^{(a)} v_{aq}^{(b)} \sigma_{aj}^{\alpha}, \quad u_{ai}^{(a)} \delta_{aq}^{\alpha} = v_{aq}^{(a)} v_{aq}^{(b)},
\]

\[
u_{aq}^{(a)} \sigma_{aq}^{\alpha} = 2 v_{aq}^{(a)} v_{aq}^{(b)} \sigma_{aq}^{\alpha}, \quad u_{aq}^{(a)} \delta_{aq}^{\alpha} = v_{aq}^{(a)} v_{aq}^{(b)},
\]

\[
u_{aq}^{(a)} \sigma_{aq}^{\alpha} = 2 v_{aq}^{(a)} v_{aq}^{(b)} \sigma_{aq}^{\alpha}, \quad u_{aq}^{(a)} \delta_{aq}^{\alpha} = v_{aq}^{(a)} v_{aq}^{(b)}.
\]

where \(\sigma_{aq}^{\alpha} = \gamma_{aq}^{\alpha} I_{pq}\) with \(I = 1, \ldots, 8\) are \(SO(8)\) Clebsch–Gordan coefficients, which obey \(\gamma_{aq}^{\alpha} + \gamma_{aq}^{\beta} = \delta_{aq}^{\alpha} I_{pq}\) and \(\gamma_{aq}^{\alpha} + \gamma_{aq}^{\beta} = \delta_{aq}^{\alpha} I_{pq}\).

The 10D spinor helicity variables \(\lambda_{aq(i)} = \sqrt{p_i^{\alpha} v_{aq(i)}^{\alpha}}\) were introduced in [17] and used their to construct a Clifford superfield approach to superamplitude. The understanding of the Lorentz harmonic nature of spinor helicity variables from [17] allowed us to construct the spinor helicity formalism for 11D supergravity [1], simplify it for 10D SYM [2] and propose two versions of superamplitude formalism for 11D SUGRA and 10D SYM [1–3] (both simpler than the 10D Clifford superamplitude approach of [17]).

Eqs. (25) and (26) follow from (27)–(29). The relations (19) follow from (27) and (23). What remains to comment is how the statement in (18) occurs.

In distinction to \(v_{aq}\), the complementary harmonic variables \(v_{aq}^{+}\) are not physical and serve as a set of reference spinors (a counterpart of 4D \(\mu_\alpha, \pi_\alpha\)). This is reflected by \(K_i\) gauge symmetry of the Lorentz harmonic description of massless particles which acts on spinor frame as

\[
K_i: v_{aq}^{+} \mapsto v_{aq}^{+} + \frac{1}{2} K_i^{\alpha} v_{aq}^{-} v_{aq}^{\alpha}, \quad v_{aq}^{-} \mapsto v_{aq}^{-}.
\]

The gauge symmetry \(\prod_i SO(1,1) \otimes SO(8) \cong K_{S_i}\) make possible to identify the Lorentz harmonics variables \(v_{aq(i)}, v_{aq(i)}^{+}\) with generalized homogeneous coordinates of the coset isomorphicto the celestial sphere,

\[
\left( \left\{ v_{aq(i)}, v_{aq(i)}^{+} \right\} \right) = \left( Spin(1,9) \right) \left( SO(1,1) \otimes Spin(8) \right) \cong K_{S_i},
\]

To make the statement in (31) manifest, we can introduce an arbitrary reference spinor frame \((v_{aq}, v_{aq}^{-})\) and then fix the \(\prod_i^\otimes SO(1,1) \otimes SO(8) \cong K_{S_i}\) auxiliary gauge symmetries by representing the \(i\)th spinor frame as [2, 3]:

\[
v_{aq}^{-} = v_{aq}^{-} + \frac{1}{2} K_i^{\alpha} v_{aq}^{+} v_{aq}^{\alpha}, \quad v_{aq}^{+} = v_{aq}^{+}.
\]

Then eight variables \(K_i^{\alpha}\) carrying the physical degrees of freedom in \(v_{aq}\) can be identified with (stereographic) projective coordinates of \(\mathbb{S}^8\) sphere.

\[\text{This helicity spinor–Lorentz harmonic correspondence was also noticed in [18] in a context of five dimensional field theories.}\]
When calculating amplitudes with our spinor frame based spinor helicity formalism, it is often convenient and/or instructive to fix only the $\prod_i K_i$ symmetry and to write the variables of the $\alpha$th spinor frame as \cite{2,3}

$$v_{\alpha i} = e^{-\alpha_i} C_{i \alpha p} (v_{\alpha p} + \frac{1}{2} K_i \gamma_i v_{\alpha q} C_p^{\alpha q}),$$

$$v_{\alpha i}^+ = C_{i \alpha p} e^{\alpha_i} v_{\alpha p},$$

where $C_{i \alpha p}$ and $C_{i \alpha p}$ are $SO(8)$ valued matrices ‘parametrizing’ the $SO(8)$ group, $C_{i \alpha p} = C_{i \alpha p}^{\gamma} C_{i \gamma}$, $\gamma_i$ and $\alpha_i$ are parameters of $SO(1,1)$.

2.2. The internal harmonics $(w_{\alpha i}^A, \bar{w}_{\alpha i})$ (17) are defined by their relation with a single set of reference internal frame variables $(w_{\alpha i}^A, \bar{w}_{\alpha i})$:

$$\bar{w}_{\alpha i} = C_{i q p} w_{\alpha B} e^{\delta_{i q A} \bar{U}_{A B}},$$

$$w_{\alpha i}^A = C_{i q p} w_{\alpha B} e^{\eta_q \bar{U}_{A B}},$$

$$\bar{w}_{\alpha i} = C_{i q p} w_{\alpha B} e^{\eta_q \bar{U}_{A B}},$$

$$w_{\alpha i}^A = C_{i q p} w_{\alpha B} e^{\delta_{i q A} \bar{U}_{A B}}.$$  

They are needed to construct the complex fermionic coordinate $\eta_{\alpha i} = (\bar{w}_{\alpha i})^*$ with $SU(4)$, the arguments of the superamplitude (16), starting from a real fermionic coordinate $\theta_{\alpha i} = (\bar{w}_{\alpha i})^*$ with $SO(8)$, s-spinor index $q = 1,\ldots,8$,

$$\eta_{\alpha i} = \theta_{\alpha i}^* \bar{w}_{\alpha i}.$$  

To understand the origin of these two types of fermionic variables, one may turn to the quantization of massless superparticle. We refer to \cite{2} for further details.

Eqs. (37) and (38) implies that the complex fermionic coordinate is actually constructed from the real MW spinor and complex harmonics: $\eta_{\alpha i}^\dagger = \theta_{\alpha i}^* v_{\alpha i}^\dagger$, $v_{\alpha i}^\dagger := v_{\alpha i} \bar{w}_{\alpha i}$. This is a manifestation of a more general fact that the amplitude (16) can be considered as a function

$$A_n = A_n(\{p_i, v_{\alpha i}, \gamma_i, \eta_{\alpha i}\}).$$

of complex spinor frame variables $v_{\alpha i}^\dagger := v_{\alpha i} \bar{w}_{\alpha i}$ and $v_{\alpha i}^\dagger := v_{\alpha i} \bar{w}_{\alpha i}$ parametrizing the coset $Spin(1,9) / SU(4) \otimes U(1) \otimes SU(1,1) \otimes K_s$ (see \cite{3} for further details).

The analytic superamplitudes (16) ((39)) do not carry indices, are Lorentz invariant, invariant under $\prod_i [SO(1,1), \otimes SO(8), \otimes SU(4)] (\prod_i [SO(1,1), \otimes SO(8), \otimes SU(4)]$ and covariant under $\prod_i SO(2) = U(1)$, symmetry transformations.

2.3. Eqs. (37) and (38) reveal a non-manifest difference of the fermionic arguments of $D = 4$ and $D = 10$ superamplitudes. The former, $\eta_{\alpha i}$, carry the index of the same $SU(4)$ group, the R-symmetry group of $\mathcal{N} = 4$ 4D supersymmetry, while the latter, $\eta_{\alpha i}$, carry the indices of different $SU(4)$, gauge symmetry groups, the transformations of which are used as identification relations allowing us to treat the internal harmonics as coordinate of the coset (17). The matrices $\bar{U}_{A B} \in SU(4)$ in (34)–(36) are “bridges” between these $SU(4)$’s and the $SU(4)$ gauge symmetry used to define the reference internal frame matrix $\{w_{\alpha i}^A, \bar{w}_{\alpha i}\}$ and its counterpart with c-spinor indices $\{w_{\alpha i}^A, \bar{w}_{\alpha i}\}$. The $8 \times 4$ complex conjugate blocks of these matrices obey $w_{\alpha i}^A w_{\alpha q}^B = \delta_{i p}^B$ and

$$\bar{w}_{\alpha B}^A w_{\alpha q}^B = \delta_{i q}^A, \quad w_{\alpha i}^A w_{\alpha q}^B = 0, \quad \bar{w}_{\alpha q}^A \bar{w}_{\alpha B} = 0.$$  

Due to specific properties of $SO(8)$, these constraints actually imply that $\{w_{\alpha i}^A, \bar{w}_{\alpha i}\}$ and $\{w_{\alpha i}^A, \bar{w}_{\alpha i}\}$ are formed from the linear combinations of the columns of the $Spin(8)$ valued matrices $w_{\alpha q}^A$ and $w_{\alpha q}^B$ related to 8v-representation of the $SO(8)$ reference internal frame

$$U_i^{(J)} = \left( U_i^{\dagger J}, U_i^{\dagger T}, U_i^{\dagger (8)} \right)$$

$$= \left( U_i^{\dagger T} \frac{1}{2}(U_i + \bar{U}_i), \frac{1}{2}(U_i - \bar{U}_i) \right) \in SO(8)$$  

by $\gamma_{i q} U_i^{(J)} = w_{\alpha q}^{(R)} U_i^{(J)} w_{\alpha q}^{(q)}$. This includes the following factorization relations for two null-vectors of the internal 8v-frame, $U_i$ and $\bar{U}_i = (U_i)^*$,

$$U_{i q} := \gamma_{i q} U_i = 2 \bar{w}_{\alpha q}^A w_{\alpha q}^B,$$

$$\bar{U}_{i q} := \gamma_{i q} \bar{U}_i = 2 w_{\alpha q}^A \bar{w}_{\alpha q}.$$  

$U_i, U_i = 0$ and $\bar{U}_i \bar{U}_i = 0$ follow from Eqs. (42) and (40).
### 3. ANALYTIC SUPERAMPLITUDE FROM CONSTRAINED SUPERAMPLITUDE

Just the above complex null vectors are used to construct the analytic superamplitude (16) from the basic constrained superamplitude of 10D SYM theory \( \mathcal{A}_{I_1 \cdots I_n} \left( (\rho^a_i, v_{aqi}^i, \tilde{\Theta}_{qi}^i) \right) \) [2]. This carries \( n \) 8v-indices of \( SO(8) \), “small groups” of the light-like momenta (23), depends on real spinor frame variables \( v_{aqi}^i \), densities \( \rho^a_i \) and real 8s-spinor fermionic variables \( \Theta_{qi}^i \), and obeys the set of superfield equations

\[
D_q^{(s)} \mathcal{A}_{I_1 \cdots I_n} = 2 \rho_q^{(s)} \gamma_q \mathcal{A}_{I_1 \cdots I_n} \tag{43}
\]

Here \( D_q^{(s)} = \frac{\partial}{\partial \Theta_{qi}^i} + 2 \rho_q^{(s)} \Theta_{qi}^i \) and the fermionic constrained superamplitude \( \mathcal{A}_{I_1 \cdots I_n} \) is defined by gamma trace part of the same equation (43) [2] for further details.

The analytic superamplitude (16) is expressed in terms of constrained superamplitude \( \mathcal{A}_{I_1 \cdots I_n} \) obeying (43) by [3] \( (U_{I_1} = U_j \cup \gamma_{I_1} \times e^{-2\delta}) \)

\[
\mathcal{A}_n (\rho^a_i, v_{aqi}^i; W_{a}, \eta_{I_1}, \ldots, U_{I_n}) = e^{-2\delta_{\rho}^{\rho\eta}} \mathcal{A} (\rho^a_i, v_{aqi}^i; \eta_{I_1}^{-1} \cdots U_{I_n}) \tag{44}
\]

4. We would like to conclude this contribution by presenting the gauge fixed version of the basic 3-point superamplitude of 10D SYM, the counterpart of the 4D MHV amplitude (15). In the gauge (33), (34) this reads

\[
\mathcal{A}_{3}^{D=10,SYM} = \frac{1}{2} \mathcal{K}^{-1} U_1 e^{-2\delta_{\beta}^{\beta+\beta}}
\]

\[
\times \delta^4 \left( \rho_1^{\rho} \eta_{I_1}^{-1} + \rho_2^{\rho} \eta_{I_2}^{-1} + \rho_3^{\rho} \eta_{I_3}^{-1} \right),
\]

where \( U_j \) is the null-vector from the reference internal frame (41), (42),

\[
\tilde{\rho}_i^{\rho} := \rho_i^{\rho} e^{-2\alpha}, \quad \tilde{\eta}_i := \eta_i e^{\alpha+\beta+\beta} \tag{46}
\]

with \( \alpha, \beta \) and \( \eta_{a}^{-1} \) defined in (33) and (34), and the complex null-vector \( \mathcal{K}^{-1} \) is defined by

\[
K^{-1}_{a} = K^{-1}_{\tilde{a}}, \quad K^{-1}_{\tilde{a}} = K^{-1}_{\tilde{a}} \quad \mathcal{K}^{-1}, \quad K^{-1}_{a} = K^{-1}_{a} - K^{-1}_{a}. \tag{47}
\]

First and second equalities in (47) follow from the momentum conservation conditions in 3-particle process. These requires \( \mathcal{K}^{-1} \) to be complex and nilpotent, \( \mathcal{K}^{-1} \mathcal{K}^{-1} = 0 \). The use of the on-shell amplitudes dependent on deformed complex light-like momenta to calculate higher \( n \) amplitudes of particles with real, physical light-like momenta is a characteristic property of the BCFW approach [8, 9]. The candidate BCFW recurrent relations for constrained superamplitudes of 10D SYM and 11D SUGRA can be found in [2] and [1]. The structure of the BCFW deformation of complex spinor frame variables relevant for the calculation of the analytic 10D and 11D superamplitudes was discussed in [3].

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