The Josephson Effect provides a primary signature of single spin superconductivity (SSS), the as yet unobserved superconducting state which was proposed recently as a low temperature phase of half-metallic antiferromagnets. These materials are insulating in the spin-down channel but are metallic in the spin-up channel. The SSS state is characterized by a unique form of p-wave pairing within a single spin channel. We develop the theory of a rich variety of Josephson effects that arise due to the form of the SSS order parameter. Tunneling is allowed at a SSS-SSS' junction but of course depends on the relative orientation of their order parameters. No current flows between an SSS and an s-wave BCS system due to their orthogonal symmetries, which potentially can be used to distinguish SSS from other superconducting states. Single spin superconductors also offer a means to probe other materials, where tunneling is a litmus test for any form of “triplet” order parameter.

Recently one of the authors has used local spin density functional calculations to identify a few good candidates for half-metallic antiferromagnets (HM AFMs). Half-metallic (HM) materials have the property that charge transport is 100% spin polarized: one spin channel is metallic (chosen as 'up' by convention), while the down channel is insulating. HM antiferromagnets are distinguished from the HM ferromagnets by having no macroscopic magnetization, yet charge transport is still 100% polarized. A pairing instability in a HM AFM leads to superconductivity in only the metallic channel, a condensation that has been called single spin superconductivity (SSS). We note briefly the special properties of these materials, and then discuss effects that can be observed in tunneling.

Several HM ferromagnetic materials are strongly indicated from theoretical studies, such as CrO$_2$, the Heusler alloys UNiSn and NiMnSb and probably the colossal magnetoresistance manganates. The properties of some of these materials have supported the HM behavior. Since HM materials have the remarkable property that (neglecting spin-orbit coupling) the spin moment is quantized, a necessary condition for HM antiferromagnetism is that magnetic ions within the unit cell have antialigned moments which cancel. This criterion can be applied to limit the thousands of possible magnetic compounds within even a given crystal structure, to a small class that can be studied individually. Ref. identifies, within the double perovskite structural class La$_2$M'M''O$_6$, the two candidates La$_2$VCuO$_6$ and La$_2$MnVO$_6$, from a search of six pairs of transition metal ions giving a double perovskite structure. The MnV material is more likely to be an exotic superconductor (See Ref. ). A third compound, La$_2$MnCoO$_6$, also has a HM AFM phase but it is clearly unstable to ferromagnetism. The density of states for the HM AFM phase of La$_2$VCuO$_6$ is shown in Fig. 1.

The proposed SSS phase is a new kind of superconductor, with unique properties arising due to the feature that only spin up electrons participate in pairing. A Cooper pair has total spin $S = 1$, something familiar from superfluid $^3$He and some theories of p-wave pairing in heavy fermion superconductors. Indeed, the systems have many common features, but the fact that only one spin component is present in SSS leads to important distinctions, specifically, $S_z = 1$ and the supercurrent is 100% polarized. In effect, the spin quantum variable is irrelevant to the bulk superconducting ground state. Fermi statistics require that the orbital pair wave function is odd under exchange, characteristic of the triplet form of pairing.

In Ref. we compared and contrasted the SSS phase with the conventional BCS theory of pairing, and enumerated the symmetries of all allowed order parameters for cubic, tetragonal, and hexagonal crystal lattices. The
phase diagram for the cubic lattice is shown in Fig. 2.

![Phase Diagram](image)

**FIG. 2.** The Ginzburg-Landau phase diagram for SSS with \( \Gamma_{4,5} \) (\( T_{1u} \) or \( T_{2u} \)) cubic symmetry has five phases near \( T_c \). The cubic group is only partly broken in each of these phases. The details are given in Ref. [3]. The order parameter is \( \Delta \).

Several experimental signatures for HM AFMs (i.e. the normal phase) have been proposed: non-Korringa behavior in NMR, resistivity and magnetic susceptibility, or more exotically through spin-polarized angle resolved positron annihilation experiments. Further tests are possible in the superconducting phase. The unconventional pairing of SSS would be reflected in the existence of multiple superconducting phases (See Fig. 2). Also, thermodynamic quantities which have an exponential dependence on the temperature in SSS due to the nodes of the gap function \([3]\). In principle, the exponent may be used to distinguish point nodes from line nodes, and hence gain symmetry information, but such measurements on unconventional superconductors are very demanding of sample quality.

Potentially powerful tools to analyze the superconducting state, including establishing that SSS occurs, are tunneling and the related phenomena that occur at interfaces. Single spin superconductivity offers unique tunneling effects. Junctions may be formed between two identical SSS materials, between SSS material and another superconductor (such as an s-wave superconductor, an unconventional (p-wave or d-wave) superconductor or a different SSS) or between a SSS and a normal metal (such as an ordinary metal, a ferromagnet, a half-metal or a HM AFM).

Unlike the case of bulk SSS where spin is irrelevant, the spin is not only relevant but crucial to SSS junctions. Proper treatment of the spin is the new ingredient required to generalize existing theories of tunneling to the single spin case. In the bulk of a SSS the order parameter always breaks the lattice symmetry and often can be characterized by a vector \( \hat{d} \). In addition, magnetocrystalline anisotropy (although involving energies of only \( \sim 10^{-4} \) eV) determines the direction of the atomic moments, and hence the preferred direction of the spin of the superconducting carriers. Spin-orbit coupling also induces a non-zero orbital moment, but we neglect that complication in this paper. Its effect should be much less important in the double perovskite HM AFMs than in the heavy fermion unconventional superconductors.

For definiteness we consider a model of SSS junctions based on the tunneling Hamiltonian

\[
H_T = \sum_{\vec{k},s;\vec{p},s'} \left[ T_{\vec{k}\vec{p}} \delta_{ss'} C_{\vec{k}s}^\dagger C_{\vec{p}s'} + \text{h.c.} \right]
\]

where \( \vec{k} \) and \( s \) are the wave vector and spin indices on the left side of the junction and \( \vec{p} \) and \( s' \) are the corresponding quantities on the right side. We assume the basic spin-diagonal form \( \delta_{ss'} \) of the tunneling matrix element. The same coordinate systems are used on both sides of the junction in Eqn. (1), however, the orientation of the spin axes may differ on the two sides of the interface. Neighboring crystals, or neighboring domains in a monocrystal, may have non-aligned carrier spins. It is convenient to transform to a spin system that diagonalizes the local Hamiltonian (so that the gapped channel is purely spin down). The tunneling Hamiltonian becomes

\[
H_T = \sum_{\vec{k},s;\vec{p},t} \left[ T_{\vec{k}\vec{p}} \left( e^{i\vec{\sigma} \cdot \hat{d}} \right)_{s,t} C_{\vec{k}s}^\dagger C_{\vec{p}t} + \text{h.c.} \right]
\]

where \( t \) is the natural spin coordinate on the right. The spins \( s \) and \( t \) differ by a rotation by the angle \( \theta \) about the axis \( \hat{d} \). \( \vec{\sigma} \) denotes the Pauli spin matrices.

The down spin electrons do not participate in tunneling since the gap is large compared to the temperature and the junction voltage, \( E_g \gg kT, eV \). The spin up part of the tunneling Hamiltonian is simply

\[
H_T = \sum_{\vec{k},s} \left[ T_{\vec{k}\vec{p}} d_{ss'}^\dagger \left( e^{i\vec{\sigma} \cdot \hat{d}} \right)_{s,t} C_{\vec{k}s}^\dagger C_{\vec{p}t} + \text{h.c.} \right]
\]

where \( d_{ss'}^\dagger(\theta) \) is the Wigner d-function. Apart from the restriction to spin up electrons, this form of the tunneling Hamiltonian is very similar to the familiar one from BCS theory. The single-particle tunneling current may be computed using standard techniques.

\[
I_S = e d_{1,1}^\dagger(\theta) \sum_{\vec{k},\vec{p}} |T_{\vec{k}\vec{p}}|^2 \text{Tr} \left[ A_R(\vec{k},\epsilon) A_L(\vec{p},\epsilon + eV) [n_{\Gamma^+}(\epsilon) - n_{\Gamma^+}(\epsilon + eV)] \right]
\]

where \( A_L \) and \( A_R \) are the spectral functions for the two sides of the interface. The spin-1 Wigner d-function is
Green’s function is given by (using standard notation)

\[ \Phi_{ret}(i\omega) = d_{1,1}^l(\theta) \sum_{k,\bar{p}} \Delta_{L,R} T_{k\bar{p}} \left( \frac{1}{E_k + E_{\bar{p}}} - \frac{1}{i\omega + E_k - E_{\bar{p}}} \right) \left( 1 - n_{F^+}(E_k) \right) \]

\[ + \left( n_{F^+}(E_k) - n_{F^+}(E_{\bar{p}}) \right) \left( \frac{1}{i\omega + E_k - E_{\bar{p}}} - \frac{1}{i\omega + E_{\bar{p}} - E_k} \right) \]

(5)

where \( \Delta_{L,R} \) is the gap function.

The expressions given above may be used to describe many different SSS junction effects. Due to space limitation we will focus on a few of particular interest. Consider the case of a SSS-SSS’ junction. Tunneling is allowed, but only to the extent that a carrier can propagate once its axis of spin quantization projects onto the axis of the neighboring HM material. For example, the DC critical current for tunneling between two SSS regions with \( d=(1,0,0) \) of \( \Gamma_4 \) \((T_{1u})\) cubic gap symmetry is given by

\[ I_j^{V=0} = \frac{\sigma_0 \pi \Delta}{4} \left( \cos(\theta) + 1 \right) \sin(\phi) \cos(\varphi) \tanh\left( \frac{\beta}{2} \Delta \right) \]

(6)

where \( \Delta \) is the RMS gap and \( \phi \) is the phase difference of the order parameter. The maximal normal conductance is \( \sigma_0 = 2\pi e^2 N_L N_R |T_0|^2 \) where \( N_L,R \) is the density of states and \( T_0 \) is the magnitude of the tunneling matrix element. Note that the angle \( \varphi \), the relative orientation of the order parameter on the two sides of the junction, can have a large effect on Josephson tunneling. Even if the superconducting state has the same symmetry on both sides, the tunneling goes to zero when \( \varphi = \pi/2 \). The critical AC Josephson current has the same angle dependence, and it oscillates at the classic frequency \( \nu_j = 2eV/h \). Also, the superconducting density of states is given by the usual expression

\[ \varrho(eV) = \left( \frac{dI}{dV} \right)_{SN} / \left( \frac{dI}{dV} \right)_{NN} \]

(7)

which is independent of the orientation.

In other cases the spin projection factor may cause the second order Josephson coupling to vanish entirely. The archetypical example is the coupling between an s-wave (BCS) superconductor and a half-metal, normal or superconducting. The states of different total spin are orthogonal. Physically, it is impossible for the down spin tunnel into the half-metal, unless enough energy is supplied to overcome either the gap in the s-wave superconductor or the gap in the down spin channel of the half-metal. As a result, the half metal (or SSS) should behave as if it were an insulator from the point of view of the s-wave superconductor. This effect has been discussed in the context of triplet-singlet junctions \[12\] \[13\].

Similarly, there should be no Josephson tunneling to any unconventional superconductor with pure spin singlet pairing. Since tunneling is allowed from a single spin to p-wave superconductor in either the Anderson-Brinkman-Morel or the Balian-Werthamer state, the SSS Josephson effect could, in principle, be used as a litmus test to distinguish p-wave from d-wave pairing. That is, the knowledge that half-metallic materials must form single spin pairs (and not d-wave pairs) makes them a kind of standard by which to probe other unconventional superconductors.

Finally, we note that the Josephson coupling between SSS and p-wave superconductors suggests the possibility of induced superconductivity. In particular, it has been suggested recently that \( \text{Sr}_2\text{RuO}_4 \) may be displaying triplet superconductivity \[12\] \[13\]. If true, its lattice is similar enough to those of the candidate HM AFMs that a good interface may be possible. Then it may be possible to use \( \text{Sr}_2\text{RuO}_4 \) to induce SSS.

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\[1\] E-mail: rudd@dave.nrl.navy.mil
\[2\] E-mail: pickett@physics.ucdavis.edu

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