Wave Functions and Leptonic Decays of Bottom Mesons In the Relativistic Potential Model

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(Dated: March 12, 2019)

Abstract

We study the wave functions and purely leptonic decays of $b$-flavored mesons (pseudoscalars, vector mesons, and higher excited states that are well established in experiment) in the relativistic potential model based on our previous works. The wave functions are obtained by solving the wave equation including the spin-spin and spin-orbit corrections in the effective potential. The decay constants of $B$, $B^*$ and some of their excited states that have been found in experiment are calculated with these wave functions. Then the branching fractions of the purely leptonic decay modes of these bottom mesons are studied. Our results are in well agreement with experimental data for decay modes that have been measured in experiments. We also provide predictions for some yet unmeasured channels, which are useful for experimental test in the future.

PACS numbers: 12.39.Pn, 13.20.He, 14.40.Nd

Keywords: leptonic decay

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I. INTRODUCTION

Recently, the ATLAS collaboration updated branching ratios of \( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) = (2.8^{+0.8}_{-0.7}) \times 10^{-9} \) and \( \mathcal{B}(B_d \rightarrow \mu^+\mu^-) < 2.1 \times 10^{-10} \) with larger collision data samples \(^1\) at 95% confidence level. The former decay channel imposes severe constraints on theoretical study, especially for physics beyond the Standard Model (BSM) \(^2, 3\). In general, the purely leptonic decays with final lepton-neutrino pair or lepton-lepton pair are considered as rare decays, which have relatively simpler physics than hadronic decays. Their decay rates are connected straightforwardly with the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements \(^4, 5\) and the bound-state properties of the bottom meson.

Compared with our previous work \(^6\) where only decay constants of pseudoscalar \(B\) and \(B_s\) mesons are considered, here we extend our earlier work by including the decay constants and pure leptonic decays of vector and higher excited states of bottom mesons. We upgrade the scenario for treating the energy-momentum conservation for the quark-antiquark inside the bottom mesons. The theoretical results for the leptonic decays are compared with experimental data. For the measured decay modes, our prediction are well consistent with experiment. For the yet unmeasured decay modes, our prediction could be useful for experimental test in the future.

The paper is organized as followings. In Section II, we briefly present the theoretical framework for relativistic potential model, give the solved wave function for the bottom mesons. The formulas to calculate the decay constants and branching ratios are also given here. Section III is devoted to numerical results and discussions. Finally, Section IV is for the conclusion and summary.

II. THEORETICAL FRAMEWORK

A. Relativistic potential model and bound-state wave functions

The heavy-light quark-antiquark bound-state systems have been extensively studied with the relativistic potential model in our previous works \(^6–9\). The bound state wave functions of mesons can be obtained by solving a Schrödinger type equation

\[
(H_0 + H')\Psi(\vec{r}) = E\Psi(\vec{r}),
\]

(1)
where $H_0 + H'$ is the effective Hamiltonian, which can be found in Ref. [9] and $E$ is meson’s energy. The term $H_0$ reads,

$$H_0 = \sqrt{p_1^2 + m_1^2} + \sqrt{p_2^2 + m_2^2} + V(r), \quad (2)$$

with

$$V(r) = -\frac{4}{3} \frac{\alpha_s(r)}{r} + br + c, \quad (3)$$

where $V(r)$ is the effective potential for the strong-interaction between the quark and antiquark [10–12]. The first term $-\frac{4}{3} \frac{\alpha_s(r)}{r}$ in $V(r)$ originates from the one-gluon-exchange diagram for the short distance contributions, and $br$ is for confinement effects in long distance, while $c$ is a phenomenological parameter for this heavy-light quark-antiquark system. The other term $H'$ contains spin-spin hyper-fine interactions and spin-orbit interactions, which are not given explicitly here (see Ref. [9]).

Using the method described in [7] and developed in [8, 9], the wave equation in Eq. (1) can be solved numerically. The wave functions in momentum space can be written as,

$$\Psi_{nlm}(\hat{k}) = \varphi_{nl}(k)Y_{lm}(\hat{k}), \quad (4)$$

where the subscripts $nlm$ stands for $n$-th radial wave function ($n = 1$ is the lowest), $l$ orbital angular momentum quantum number ($l = 0, 1, 2, \ldots$), and $m$ the magnetic quantum number corresponding to $l$. $\varphi_{nl}(k)$ is the radial wave functions and $Y_{lm}(\hat{k})$ is the spherical harmonics. The normalization condition for the wave function is

$$\int dk^3 |\Psi_{nlm}(\hat{k})|^2 = 1. \quad (5)$$

The details for solving the wave equation can be found in our previous works in Refs. [7–9], which will not be given here for briefness. By solving the wave equation numerically, the wave function can be obtained. In practice, it is convenient to give an analytical form for radial wave functions by fitting the numerical solution. We find the wave function can be fitted with the following exponential form,

$$\varphi_{nl}(\hat{k}) = a_1 e^{a_2 |\hat{k}|^2 + a_3 |\hat{k}| + a_4}. \quad (6)$$

Next, we give the obtained results for the parameters $a_1 \sim a_4$ for each quantum states.
FIG. 1. Radial wave functions of pseudoscalar mesons, normalized by multiplying a constant $Y_{00} = 1/\sqrt{4\pi}$.

TABLE I. Parameters of fitting functions for pseudoscalar $B$ mesons.

| Mesons     | $a_1$ (GeV$^{-3/2}$) | $a_2$ (GeV$^{-2}$) | $a_3$ (GeV$^{-1}$) | $a_4$ |
|------------|----------------------|-------------------|-------------------|-------|
| $B^\pm/B_d$ | $1.66_{-0.11}^{+0.07}$ | $-1.07_{-0.16}^{+0.12}$ | $-0.98_{-0.12}^{+0.17}$ | $-0.13_{-0.04}^{+0.02}$ |
| $B_s$      | $1.97_{-0.09}^{+0.05}$ | $-1.09_{-0.18}^{+0.12}$ | $-0.69_{-0.10}^{+0.07}$ | $-0.47_{-0.07}^{+0.03}$ |
| $B_c$      | $1.04_{-0.03}^{+0.02}$ | $-0.51_{-0.08}^{+0.07}$ | $-0.44_{-0.07}^{+0.04}$ | $-0.43_{-0.04}^{+0.02}$ |

(1) For pseudoscalars $J^P = 0^-$, we ignore the difference between the light quark masses $m_u$ and $m_d$, therefore the radial wave functions of $B^\pm$ and $B_d$ shall be the same. The wave functions of $B^\pm/B_d$, $B_s$, and $B_c^\pm$ are depicted in FIG. 1. It is noted that the differences between $B^\pm/B_d$ and $B_s$ is relatively smaller than that between $B^\pm/B_d$ and $B_c^\pm$, since the mass of $c$ quark is greatly larger than that of the light-quarks. The results for the parameters $a_1, a_2, a_3, a_4$ we obtained are listed in Table I.

(2) For vector mesons $J^P = 1^-$, in our model [8, 9], they are mixing-states of S-wave ($l = 0$) eigenstate $|^3S_1\rangle$ and D-wave ($l = 2$) eigenstate $|^3D_1\rangle$. The S-wave and D-wave wave functions should be given separately. We use $\Psi^S_V(\vec{k})$ to denote the S-wave radial wave function, and $\Psi^D_V(\vec{k})$ as the D-wave wave function.

The radial wave functions are shown in FIG. 2. The contribution of the D-wave part to the leptonic decay is rather small. Therefore, the fitting functions are provided only for S-wave part using the same analytic form in Eq. (6), the parameters for vector mesons are
TABLE II. Parameters of fitting functions for vector $B$ mesons, S-wave part.

| Mesons   | $a_1$ (GeV$^{-3/2}$) | $a_2$ (GeV$^{-2}$) | $a_3$ (GeV$^{-1}$) | $a_4$   |
|----------|----------------------|-------------------|-------------------|---------|
| $B^*$    | 1.62$^{+0.06}_{-0.08}$ | $-1.09^{+0.14}_{-0.11}$ | $-1.23^{+0.13}_{-0.09}$ | 0.03$^{+0.01}_{-0.00}$ |
| $B_s^*$  | 1.45$^{+0.04}_{-0.06}$ | $-1.17^{+0.15}_{-0.11}$ | $-0.84^{+0.09}_{-0.10}$ | $-0.03^{+0.01}_{-0.00}$ |

(3) For higher excited states $1^+$ and $2^+$, only few states have been found in experiments up to now and information about their decay channels is very limited. Therefore, we only show the radial wave functions for $J^P = 1^+$ mesons in FIG. 3 and for $J^P = 2^+$ mesons in FIG. 4 without giving an analytic form. In principle the numerical form for the wave functions can be used directly to calculate the meson decays.

B. Decay constants

Using the wave function of the quark-antiquark bound-state, the state of a bottom meson can be written as [7, 13]

$$ |M(P)\rangle = \frac{1}{\sqrt{3N_L}} \sum_i \int d^3k_q d^3k_Q \delta^{(3)}(\vec{P} - k_q - k_Q)\Psi_{nlm}(k_q)X_{Sm_s}(a_i^{+}_{k_Q,s_1}, b_i^{+}_{k_q,s_2})|0\rangle $$

where $X_{Sm_s}$ is the spin wave function, $a_i^{+}_{k_Q,s_1}, b_i^{+}_{k_q,s_2}$ are creation operators, and $S, m_s, s_1, s_2$ are the corresponding spin-related quantum numbers. The superscript $i$ is color index and
the normalization factor $N_L$ is obtained via the normalization condition of the meson state

$$\langle M(\vec{P})|M(\vec{P}')\rangle = (2\pi)^3 2E\delta^{(3)}(\vec{P} - \vec{P}').$$  \hspace{1cm} (8)

The anti-commuting relation of the quark and anti-quark annihilation and creation operators are

$$\{a_{\vec{k},s}, a^\dagger_{\vec{k}',s'}\} = \delta_{ss'}\delta^{(3)}(\vec{k} - \vec{k}'),$$  \hspace{1cm} (9)

$$\{b_{\vec{k},s}, b^\dagger_{\vec{k}',s'}\} = \delta_{ss'}\delta^{(3)}(\vec{k} - \vec{k}').$$  \hspace{1cm} (10)

The energy and momentum conservation between the meson and its constituent quark and antiquark should hold when considering the decays of the bottom mesons. We take $k_q = (E_q, \vec{k}_q)$, $k_Q = (E_Q, \vec{k}_Q)$, and $P = (m_P, \vec{0})$ as the four-momenta of the light quark,
the heavy quark and the meson in rest frame, respectively. Due to the energy and momentum conservation, one has

\[ E_q + E_Q = m_P, \]
\[ \vec{k}_q = -\vec{k}_Q = \vec{k}. \]  

To keep the four-momentum conservation, the heavy quark is taken off-shell, while the light quark is kept on-shell in the Altarelli-Cabibbo-Corbo-Maiani-Martinelli (ACCMM) scenario [14, 15]. Here we extend the ACCMM scenario by taking both the light and heavy quark off-shell. The off-shell of the quarks are a simple treatment for including the energy and momentum carried by the color field around the quarks. Both the masses of the light and heavy quarks are taken to be running masses

\[ m_q(k) = \sqrt{E_q^2 - |\vec{k}|^2}, \quad m_Q(k) = \sqrt{E_Q^2 - |\vec{k}|^2}. \]  

The running masses \( m_q(k) \) and \( m_Q(k) \) are restricted to be positive in this work. With Eqs. (12) and (13), one can obtain

\[ \sqrt{m_q^2(k) + |\vec{k}|^2} + \sqrt{m_Q^2(k) + |\vec{k}|^2} = m_P. \]  

It is not enough to determine the explicit dependence of the running masses \( m_{q,Q}(k) \) on the quark momentum \( k \) with the above equation. We assume the ratio of \( m_q(k)/m_Q(k) \) is a fixed parameter in this work, i.e., we define the ratio for each quark-antiquark pair as

\[ R_i \equiv \frac{m_q(k)}{m_Q(k)}, \quad i = (\bar{u}b) \sim (\bar{d}b), (\bar{s}b), (\bar{c}b). \]  

In our numerical treatment in the following, we find that the fixed ratio \( R_i \) can indeed accommodate the experimental data for the measured leptonic decays of the bottom mesons, and the value of the ratio \( R_i \) is approximately around the ratio of current masses of the light and heavy quarks \( m_q/m_b \).

In general, decay constants of pseudoscalar mesons \( (B^\pm, B_d, B_s, B_c^\pm) \) are defined as

\[ \langle 0 | \bar{q} \gamma_\mu \gamma^5 b | B(P) \rangle = i f_P P_\mu. \]  

Substituting Eq. (7) into the Eq. (16), we obtain the decay constant

\[ f_P = \frac{3}{(2\pi)^3 m_P} \int d^3 k \, \psi_{100}(\vec{k}) \left( \sqrt{1 + \frac{m_q(k)}{E_q}} \sqrt{1 + \frac{m_Q(k)}{E_Q}} - \sqrt{1 - \frac{m_q(k)}{E_q}} \sqrt{1 - \frac{m_Q(k)}{E_Q}} \right). \]
For vector mesons $B^{*\pm}$, $B^*$, $B_s^*$, there are two types of decay constants that are defined according to different currents.

$$
\langle 0 | \bar{q} \gamma_\mu b | B^*(P, \epsilon) \rangle = m_{V*} f_{V*} \epsilon_\mu, \quad
\langle 0 | \bar{q} \sigma_{\mu\nu} b | B^*(P, \epsilon) \rangle = -i f_{V*}^T (P_\mu \epsilon_\nu - \epsilon_\mu P_\nu),
$$

where $\epsilon_\mu$ is the polarization vector, $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$ is the Dirac tensor matrix. Similarly, the decay constants can be obtained as

$$
f_{V*} = \sqrt{\frac{3}{(2\pi)^3 m_{V*}}} \int d^3k |\Psi_{n_0 n_0}(k)\rangle \left(\sqrt{1 + \frac{m_q}{E_q}} \sqrt{1 + \frac{m_Q}{E_Q}} \right) \left(1 + \frac{1}{3} \sqrt{1 - \frac{m_q}{E_q}} \sqrt{1 - \frac{m_Q}{E_Q}} \right),
$$

$$
f_{V*}^T = \sqrt{\frac{3}{(2\pi)^3 m_{V*}}} \int d^3k |\Psi_{n_0 n_0}(k)\rangle \left(\sqrt{1 + \frac{m_q}{E_q}} \sqrt{1 + \frac{m_Q}{E_Q}} \right) \left(-\frac{1}{3} \sqrt{1 - \frac{m_q}{E_q}} \sqrt{1 - \frac{m_Q}{E_Q}} \right),
$$

The vector mesons are mixing states of $|n_1^3 S_1\rangle$ and $|n_2^3 D_1\rangle$ with $n_1, n_2 = 1, 2, 3$ (details can be found in Refs. [8, 9]). The wave function $\Psi_{n_0 n_0}(k)$ in Eqs. (19a) and (19b) is the sum of all the S-wave states. The D-wave states do not contribute to the decay constants.

### C. Branching ratios of purely leptonic decays

We calculate the purely leptonic decays of pseudoscalar and vector $b$-flavored mesons in this section. The decay modes we consider in this work include $B_{u(c)}^{\pm} \rightarrow l^\pm \nu_l$, $B_{u(c)}^{*\pm} \rightarrow l^\pm \nu_l$, $B_{d(s)}^0 \rightarrow l^+ l^-$ and $B_{d(s)}^{*0} \rightarrow l^+ l^-$. For the leptonic decays of charged bottom mesons, the decay amplitudes are dominated by the tree-level diagrams. The branching ratios are calculated to be

$$
B(B_q^{\pm} \rightarrow l^\pm \nu_l) = \frac{G_F^2 m_l^2 M_{B_q}}{8\pi} (1 - \frac{m_l^2}{M_{B_q}^2})^2 f_{B_q}^2 |V_{qb}|^2 \tau_{B_q},
$$

$$
B(B_q^{*\pm} \rightarrow l^\pm \nu_l) = \frac{G_F^2 M_{B_q}^3}{12\pi} (1 - \frac{3 m_l^2}{2 M_{B_q}^2} + \frac{1}{2} \frac{m_l^6}{M_{B_q}^4}) f_{B_q}^2 |V_{qb}|^2 \tau_{B_q},
$$

where $G_F$ is the Fermi constant, $V_{qb}$ the CKM matrix element, $M_{B_q^{(*)}}$ and $m_l$ the masses of $B_q^{(*)}$ meson and lepton, respectively. $\tau_{B_q^{(*)}}$ is the life time of the bottom meson.

For the leptonic decays of the neutral bottom mesons, the decays are induced by penguin diagrams. The effective Hamiltonian describes such decays is [16–18],

$$
\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} \lambda_q \sum_{i=1}^{10} C_i(\mu) Q_i(\mu),
$$

(21)
where \( \lambda_q = V_{tb} V_{tq}^* \) and the operators are (we use \( b \to s \) as an example and \( b \to d \) is similar),

\[
\begin{align*}
Q_1 &= (\bar{s}_a c_\beta)_{V-A} (\bar{c}_\beta b_\alpha)_{V-A}, \\
Q_3 &= (\bar{s} b)_{V-A} \sum_q (\bar{q} q)_{V-A}, \\
Q_5 &= (\bar{s} b)_{V-A} \sum_q (\bar{q} q)_{V+ A}, \\
Q_7 &= \frac{\alpha_{em}}{2\pi} m_b \bar{s}\alpha \sigma^{\mu\nu} (1 + \gamma^5) b_{\alpha} F_{\mu\nu}, \\
Q_9 &= \frac{\alpha}{2\pi} (\bar{s} b)_{V-A} (\bar{l} l)_V,
\end{align*}
\]

\[
\begin{align*}
Q_2 &= (\bar{s} c)_{V-A} (\bar{c} b)_{V-A}, \\
Q_4 &= (\bar{s}_a b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A} \\
Q_6 &= (\bar{s}_a b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A} \\
Q_8 &= \frac{\alpha_s}{2\pi} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1 + \gamma^5) T^a_{\alpha\beta} b_\beta C^a_{\mu
u} \\
Q_{10} &= \frac{\alpha}{2\pi} (\bar{s} b)_{V-A} (\bar{l} l)_A
\end{align*}
\]

where \( \alpha = \frac{e^2}{4\pi} \) is the electromagnetic coupling constant. Except for the contribution of the operators \( Q_7 \) and \( Q_9 \), the operators \( Q_{1-6,8} \) also contribute to the decay process \( B^{*0}_{d(s)} \to l^+ l^- \) up to next-to-leading (NLO) order in \( \alpha_s \) expansion in QCD. The contributions from the operators \( Q_{1-6,8} \) can be absorbed by a redefinition of two effective Wilson coefficients \( C^e_{7,9} \to C^{eff}_{7,9} \). The explicit form of \( C^{eff}_{7,9} \) can be found in Refs. [18–20], we do not repeat it here.

Next we give the branching ratio of the pure leptonic decay of the neutral bottom mesons.

The branching ratio of \( B^{*0}_{d(s)} \to l^+ l^- \) is \([21]\):

\[
\tilde{\mathcal{B}}(B_q \to l^+ l^-) = \frac{G_F^2 M_{B_q} m^2_l \alpha^2}{4\pi^3 \Gamma^q_H} f^2_{B_q} |\lambda_q|^2 \sqrt{1 - \frac{4m^2_l}{M^2_{B_q}}} |C_{10}|^2.
\] (22)

The hat over \( \mathcal{B} \) indicates that it is the averaged time-integrated branching ratio that depends on the details of \( B^0_q - \bar{B}^0_q \) mixing \([22]\). \( \Gamma^q_H \) denotes the total decay width of the heavier mass-eigenstate. In Ref. [21], the authors define a different Wilson coefficient \( C_A \) and its relation to \( C_{10} \) in Eq. (21) is straightforward: \( |C_A| = \frac{\sin^2 \theta_W}{2} |C_{10}| \), where \( \theta_W \) is the weak-mixing angle (the Weinberg angle).

For the process of the vector meson decaying into charged lepton-anti-lepton pair, the branching ratio reads \([23]\),

\[
\tilde{\mathcal{B}}(B_q^* \to l^+ l^-) = \frac{1}{\Gamma_{B_q^*}} \frac{G_F^2 M_{B_q^*}^2 \alpha^2}{96\pi^3} f^2_{B_q^*} |\lambda_q|^2 \left( |C^{eff}_9| + 2 \frac{m_b f_{B_q^*}^T}{M_{B_q^*} f_{B_q^*}^T} C^{eff}_7 |C_{10}|^2 \right),
\] (23)

where the contributions of order \( \mathcal{O}(m^2_l/M^2_{B_q^*}) \) are neglected and \( m_c \ll m_b \) is considered.
TABLE III. Numerical inputs.

| Parameter | Value | Unit | Parameter | Value | Unit |
|-----------|-------|------|-----------|-------|------|
| $G_F$     | $1.16638 \times 10^{-5}$ | GeV$^{-2}$ | $|V_{ub}|$ | $3.94(36) \times 10^{-3}$ | – |
| $\alpha_s^{(5)}(M_z)$ | 0.1181(11) | – | $|V_{cb}|$ | $4.22(8) \times 10^{-2}$ | – |
| $\alpha^{(5)}(M_Z)$ | $1/127.955(10)$ | – | $|V_{td}|$ | $8.1(5) \times 10^{-3}$ | – |
| $M_Z$     | 91.1876(21) | GeV | $|V_{ts}|$ | $3.94(23) \times 10^{-2}$ | – |
| $M_t$     | 173.1(9) | GeV | $|V_{tb}|$ | 1.019(25) | – |
| $M_{B^\pm}$ | 5279.32(14) | MeV | $M_{B^*}$ | 5324.65(25) | MeV |
| $M_{B_s}$ | 5366.89(19) | MeV | $M_{B_s^*}$ | $5415.4^{+1.8}_{-1.5}$ | MeV |
| $M_{B_d}$ | 5279.63(15) | MeV | $M_{B_c}$ | 6274.9(8) | MeV |
| $\tau_{B^\pm}$ | 1.638(4) | ps | $\tau_{B_d}$ | 1.520(4) | ps |
| $2/(\Gamma_{B_sL}+\Gamma_{B_sH})$ | 1.509(4) | ps | $\Gamma_{B_sL}-\Gamma_{B_sH}$ | 0.088(6) | ps$^{-1}$ |

III. NUMERICAL CALCULATION

In this section, we calculate the decay constants in Eqs. (17), (19), and leptonic decay branching ratios in Eqs. (20), (22), (23) numerically, and compare them with experimental measurements.

The parameters used in this work are collected in Table III, which are quoted from PDG [24].

(1) For pseudoscalar $B$ mesons ($J^P = 0^-$)

The $J^P = 0^-$ pseudoscalar meson is $|1^1S_0\rangle$ state. Besides the parameters in Table III, additional numerical input is the Wilson coefficient $C_{10}$ whose expression is known up to next-to-leading-order (NLO) in electro-weak (EW) corrections [25] and next-next-to-leading-order (NNLO) in QCD [26]. Its value is $|C_{10}(\mu_b = m_b \sim 5\text{GeV})| = 4.053 \pm 0.032$.

Our numerical results are listed in Table IV. The uncertainties of branching ratios mainly come from decay constants (whose relative error is 5% in our scenario) and CKM matrix elements (see the full review in paper [24]). For the two well-measured decay modes $B^+ \rightarrow \tau^+\nu_\tau$ and $B_s \rightarrow \mu^+\mu^-$, our results are in good agreement with experiment data. The other predicted branching ratios are all consistent with experimental data. They are well below the experimental upper limit. The branching ratios of $B^+ \rightarrow \mu^+\nu_\mu$ and $B_d \rightarrow \mu^+\mu^-$ are very...
TABLE IV. Decay constants, purely leptonic branching ratios of pseudoscalars.

| B-Meson | $R_i$ (Eq.(15)) | $f_B$ (MeV) | channel | this work | Exp. |
|---------|----------------|-------------|---------|----------|------|
| $(bg)$  | $1.0 \times 10^{-3}$ | 210(10) | $B^+ \rightarrow e^+\nu_e$ | $1.27(26) \times 10^{-11}$ | $< 9.8 \times 10^{-7}$ [27] |
|         |                |            | $B^+ \rightarrow \mu^+\nu_\mu$ | $5.4(1.1) \times 10^{-7}$ | $< 10.7 \times 10^{-7}$ [28] |
|         |                |            | $B^+ \rightarrow \tau^+\nu_\tau$ | $1.21(25) \times 10^{-4}$ | $1.25(28)(27) \times 10^{-4}$ [29] |
|         |                |            | $B_d \rightarrow e^+e^-$ | $3.00(49) \times 10^{-15}$ | $< 8.3 \times 10^{-8}$ [30] |
|         |                |            | $B_d \rightarrow \mu^+\mu^-$ | $1.28(21) \times 10^{-10}$ | $< 2.1 \times 10^{-10}$ [1] |
|         |                |            | $B_d \rightarrow \tau^+\tau^-$ | $2.68(44) \times 10^{-8}$ | $< 1.6 \times 10^{-3}$ [31] |
| $(bs)$  | $1.0 \times 10^{-2}$ | 229(11) | $B_s \rightarrow e^+e^-$ | $8.1(1.3) \times 10^{-14}$ | $< 2.8 \times 10^{-7}$ [30] |
|         |                |            | $B_s \rightarrow \mu^+\mu^-$ | $3.45(55) \times 10^{-9}$ | $2.8^{+0.8}_{-0.7} \times 10^{-9}$ [1] |
|         |                |            | $B_s \rightarrow \tau^+\tau^-$ | $7.3(1.2) \times 10^{-7}$ | $< 5.2 \times 10^{-3}$ [31] |
| $(bc)$  | 0.3            | 429(21) | $B_c^+ \rightarrow e^+\nu_e$ | $2.24(24) \times 10^{-9}$ | – |
|         |                |            | $B_c^+ \rightarrow \mu^+\nu_\mu$ | $9.6(1.0) \times 10^{-5}$ | – |
|         |                |            | $B_c^+ \rightarrow \tau^+\nu_\tau$ | $2.29(24) \times 10^{-2}$ | – |

near to the present experimental upper limit, which are very hopeful to be detected with the upgraded detectors at Belle II and/or LHCb [32] in the near future.

In addition, the prediction of the branching ratio of $B_c^+ \rightarrow \tau^+\nu_\tau$ is two orders of magnitude larger than that of $B^+$. Therefore it could be a possible channel to be measured in experiments in future.

(2) For vector $B$ mesons $J^P = 1^-$

Since the total decay widths of vector $B$-meson are not well measured in experiments up to now, in this work, their values of theoretical estimations will be used. As stated in Refs. [33–35], vector $B$-mesons’ decays are dominated by the electromagnetic processes $B^* \rightarrow B\gamma$ and thus we make the assumption that the total decay width $\Gamma$ approximately equals $\Gamma(B^* \rightarrow B\gamma)$. The respective values are $\Gamma_{B_s^*} \sim 0.068(18) \text{ KeV}$, $\Gamma_{B_s^{**}} \sim 0.148(20) \text{ KeV}$, and $\Gamma_{B^{*+}} \sim 0.468_{-0.075}^{+0.073} \text{ KeV}$. The “running” mass of b-quark is $m_b(Ms) = 4.18 \text{ GeV}$. The values of effective Wilson coefficients are $C_9^{\text{eff}} = C_9^{\text{eff}}(m_b, m_{B^*}) \sim C_9^{\text{eff}}(m_b, m_{B_s^{**}})^{4.560 + 0.612}$ and $C_7^{\text{eff}} = C_7^{\text{eff}}(m_b, m_{B_s^{**}})^{0.384 - 0.111}$ [23].

The results for the branching ratios of vector $B$-meson are shown in Table V. The uncertainties mainly come from the uncertainties of the decay width of vector mesons and CKM
TABLE V. Decay constants, purely leptonic branching ratios of vectors with $l = e, \mu$.

| B-Meson | $R_i$ (Eq.(15)) | $f_{B_q}$ (MeV) | $f_{T_{B_q}}^r$ (MeV) | channel | this work |
|---------|------------------|-----------------|----------------------|---------|-----------|
| (bq)    | $1.0 \times 10^{-3}$ | 223(16) | 201(14) | $B^{*-} \rightarrow e^+\nu_e$ | $9.0(2.5) \times 10^{-10}$ |
|         |                   |               |                      | $B^{*-} \rightarrow \mu^+\nu_\mu$ | $9.0(2.5) \times 10^{-10}$ |
|         |                   |               |                      | $B^{*-} \rightarrow \tau^+\nu_\tau$ | $7.5(2.1) \times 10^{-10}$ |
|         |                   |               |                      | $B^{*}_d \rightarrow l^+l^-$ | $3.16(77) \times 10^{-13}$ |
| (bs)    | $1.0 \times 10^{-2}$ | 242(17) | 219(15) | $B^{*}_s \rightarrow l^+l^-$ | $2.02(64) \times 10^{-11}$ |

parameters. In general, the branching ratios of the leptonic decay of vector B-mesons are very small.

IV. SUMMARY

To summarize, we study the leptonic decays of b-flavored mesons in the relativistic potential model. The decay constants of the bottom mesons and branching ratios of the leptonic decay modes are calculated. The predictions for the branching ratios of $B^+ \rightarrow \tau^+\nu_\tau$ and $B_s \rightarrow \mu^+\mu^-$ are well consistent with the experimental measurements. The other predicted branching ratios are well below the experimental upper limit. For $B^+ \rightarrow \mu^+\nu_\mu$ and $B_d \rightarrow \mu^+\mu^-$ decays, the predictions of the branching ratios are very near to the present experimental upper limit, which are very hopeful to be detected with the upgraded detectors at Belle II and/or LHCb in the near future. For the leptonic decays of the vector B-mesons, the branching ratios are very small.

ACKNOWLEDGMENTS

This work is supported in part by the National Natural Science Foundation of China under Contract Nos. 11875168 and 11375088.

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