Formation and Evolution of Binary Neutron Stars

Simon F. Portegies Zwart and Lev R. Yungelson

Abstract. The formation and evolution of binaries which contain two neutron stars or a neutron star with a black hole are discussed in detail. The evolution of the distributions in orbital period and eccentricity for neutron star binaries are studied as a function of time. In the model which fits the observations of high mass binary pulsars best the deposition of orbital energy into common envelopes has to be very efficient and a kick velocity distribution has to contain a significant contribution of low velocity kicks. The estimated age of the population has to be between several 100 Myr and 1 Gyr. The birthrate of binary neutron stars is \( \sim 3.4 \times 10^{-5} \, \text{yr}^{-1} \) (assuming 100\% binarity) and their merger rate is \( \sim 2 \times 10^{-5} \, \text{yr}^{-1} \). The merger rate of neutron star binaries is consistent with the estimated rate of \( \gamma \)-ray bursts, if the latter are beamed into an opening angle of a few degrees. We argue that PSR B2303+46 is possibly formed in a scenario in which the common envelope is avoided while for the other three known high-mass binary pulsars a common envelope is required to explain their orbital period.

Key words: binaries: close – stars: neutrons – pulsars: general – Galaxy: stellar content

1. Introduction

Peculiarly enough, the first binary pulsar to be discovered, PSR B1913+16 (Hulse & Taylor 1975), turned out to contain two neutron stars one of which is visible as a recycled pulsar and the other is not visible. The majority of about thirty recycled pulsars discovered so far in the Galactic disc are found to have white dwarf companions. Four of these systems may have a neutron star companion instead, as is suggested by their mass.

An adequate model of the population of Galactic neutron star binaries is highly desirable, as their mergers are currently considered as the most viable sources of detectable bursts of gravitational wave radiation (Clark & Eardley 1977) and possibly \( \gamma \)-rays (Paczyński 1986). Estimates of the merger rates of the Galactic binaries which contain two neutron stars based on observations are strongly biased by the uncertainties in the pulsar lifetime, beaming factor and the fraction of unobserved low-luminosity pulsars (Curran & Lorimer 1995; van den Heuvel & Lorimer 1996). Therefore, it is useful to investigate the Galactic population of neutron star binaries by simulating their formation and the subsequent orbital decay due to gravitational wave radiation and to fit the orbital parameters to observations. The population of binary neutron stars is a part of the more general family of descendants of massive binaries. Hence, an adequate model has to comply with observations also in the stages preceding the formation of neutron stars (see Meurs & van den Heuvel 1989 for an example of this approach).

After the pioneering studies of Clark et al. 1979, Kornilov & Lipunov (1984a,b) and Dewey & Cordes (1987) the formation of single and binary neutron stars and the merger rate of the latter are modelled by, among others Tutukov & Yungelson (1993a,b), Portegies Zwart & Spreeuw (1996), Lipunov et al. (1996, 1997), Bagot (1996).

Since the paper by Flannery & van den Heuvel (1975), a lot of effort was spent on reproduction of the parameters of PSR B1913+16. But surprisingly, the rest of the family didn’t attract much attention. The primary aim of most studies was to constrain the orbital parameters of immediate progenitors of high-mass binary pulsars (HMBP, hereafter) and kicks imparted to neutron stars (see e.g. Wijers et al. 1992, Yamaoka et al. 1993; Fryer & Kalogera 1997; Bagot 1997; Lipunov et al. 1997). No attempts were made to restrict the models by simultaneous confrontation with all observed systems and to understand how typical they are in the total inferred population of double neutron stars. Though only four HMBP are definitely known1 a

1 We omit the globular cluster object B2127+11C which could be formed by a dynamical encounter rather than by binary evolution (Phinney & Sigurdsson 1991) and B1820-11 which may be young or have a non-neutron star as companion (Phinney & Verbunt 1991).
better understanding of their formation is helpful, as it may provide additional constraints on the probable rate of events accompanying their merger.

In Sect. 2 we discuss the formation scenarios for HMBP. (We use the terms “neutron star” as a generic term for the remnant of a high mass star and “pulsar” for a neutron star which is observable in radio.) Particular attention is given to the revision of the “standard” scenario suggested by Brown (1995). Section 3 gives the results of our population synthesis computations and the inferred orbital-period–eccentricity diagram for binary neutron stars. This probability diagram provides a unique opportunity to compare the combined results of evolutionary computations and population synthesis with observations of HMBP. A requirement for pulsars to be located in the regions of this diagram with high likelihood also allows to constrain the progenitor population and its evolution. We derive the estimate of the merger rate of the Galactic neutron stars from the model which fits best to the observed population. Discussion follows in Sect. 4 and conclusions are summarized in Sect. 5.

2. Formation of binary neutron stars

2.1. Scenarios of formation

Neutron stars are believed to be born in the supernova explosions triggered by the iron-core collapse in massive stars. The range of progenitor masses of neutron stars in isolation assumed in the present paper is 8 to 40 $M_\odot$. However, the lower limit may be actually even higher than 10 $M_\odot$; Ritossa et al. (1996) demonstrated that a 10 $M_\odot$ star ends its evolution as a 1.26 $M_\odot$ oxygen-neon white dwarf. This would require a reduction of the model occurrence rate of supernovae from single stars by $\sim 30\%$. If a star is stripped from its hydrogen envelope due to Roche-lobe overflow in a close binary the lower mass limit for the formation of a neutron star increases to about 12 $M_\odot$. The necessary condition for the formation of a neutron star is the requirement for the helium star remnant of a binary component to have a mass in excess of about 2.2 to 2.5 $M_\odot$ (Tutukov & Yungelson 1973; Habets 1985), which enables the formation of a carbon-oxygen core of a Chandrasekhar mass. The upper limit is less well known. The main factors contributing to this are uncertain stellar mass-loss rates, poor understanding of mixing processes in stellar interiors, the processes during the supernova itself and the uncertainty concerning the equation of state of the compact object which results from the supernova (see e.g. Brown et al. 1996). The “standard” assumption is that neutron stars are formed in binaries by stars initially less massive than 40 $M_\odot$ (van den Heuvel & Habets 1984). However, this limit may be as high as $\sim 60 M_\odot$ if helium stars have very strong stellar winds (Woosley et al. 1995). On the other hand, there are indications from theory as well as from observations that this limit may be as low as 20-25 $M_\odot$ (see Portegies Zwart et al. 1997b and Timmes et al. 1996) The wide spread in the mass of observed candidate black holes in binaries suggests that there may be factors other than the progenitor mass alone which determines the fate of a star (Ergma & van den Heuvel 1997).

The chain of events resulting in the formation of a bound pair of neutron stars is known now for more than 20 years, starting from studies by van den Heuvel & Heise (1972), Tutukov & Yungelson (1973), De Loore et al (1975), Flannery & van den Heuvel (1975) and van den Heuvel (1976). Let us introduce the notation $ms$, $gs$, $he$, $co$, $ns$ and $bh$, for a main-sequence star, giant, stripped helium core or helium star, the stripped carbon-oxygen core of a helium star, neutron star and a black hole, respectively (see Portegies Zwart & Verbunt 1996 for an extended description). A bullet above a stellar type indicates that this star is to experience a supernova explosion in the next evolutionary step. A subscript $e$ means that the orbit of the system is eccentric. A bound pair is indicated by symbols enclosed by round brackets: two main-sequence stars in a circular orbit are denoted as ($ms, ms$) and an eccentric binary neutron star as ($ns, ns_e$). A square bracket by the symbol indicates that this star fills its Roche lobe and stably transfers mass onto its companion. If the binary experiences a phase of unstable mass transfer the symbols are placed within braces. Using this notation the “standard” scenario for the formation of an ($ns, ns_e$) is:

$$I. \ (ms, ms)_e, [gs, ms), (he, ms), [he, ms), (co, ms), (ns, ms)_e, $$
$$\{ ns, gs \}, \{ ns, he \}, \{ ns, he \}, \{ ns, co \}, (ns, ns)_e.$$ 

Actually, all other scenarios for the formation of HMBP differ from scenario I in the number of Roche-lobe overflow and common envelope events experienced by the binary. The stages [he, ms] and {ns, he} typically are absent for helium stars more massive than $\sim 4 M_\odot$, which do not expand much beyond $\sim 10 R_\odot$ after core helium exhaustion (Paczynski 1971; Habets 1985; Woosley et al. 1995). The strong stellar wind of helium stars may also prevent the occurrence of both phases of mass transfer from the helium star.

A peculiar case are binaries with a primary star initially more massive than $\sim 40 M_\odot$ and a secondary less massive than 25 to 40 $M_\odot$. They may avoid the first Roche-lobe overflow at all due to their strong stellar wind which may 1) prevent them from expanding to a radius exceeding several hundred solar radii and blow away most of their envelopes during the Wolf-Rayet phase, and 2) increase the orbital separation via loss of angular momentum. However, the secondary star may overflow its Roche lobe when it evolves onto the giant branch. The orbital period subsequently reduces as a result of the spiral-in and the second supernova occurs in a short-period binary. Note that a kick can also decrease the orbital separation upon the formation of the first neutron star.
Scenario I involves stages when the neutron star is immersed into a common envelope (the \{ns, gs\} stages and, on some occasions, also \{ns, he\}). Recently certain doubts were expressed concerning this phase based on the work of Zel’dovich et al. (1972; see also Chevalier 1993; Brown 1995; Fryer et al. 1996) which have shown that neutrino cooling allows hypercritical accretion onto neutron stars, which is \(O(10^5)\) higher than the canonical photon Eddington limit of \(\sim 10^{-8} \, M_{\odot} \, \text{yr}^{-1}\). Common envelopes in massive binary systems may provide suitable conditions for hypercritical accretion; the net outcome may be the collapse of the neutron star into a black hole. Whether or not the binary survives the common envelope and a \((bh, ns)_e\) binary forms or the spiral-in continues until both stars merge into a single object is not clear. The radical conclusion (Brown 1995; Wettig & Brown 1996; Fryer & Kalogera 1997) is that the binaries in which neutron star enters a common envelope are not able to produce HMBP.

This reduces the wealth of scenarios for the formation of HMBP to the following one:

II. \((ms, ms)_e, [gs, ms], \{he, ms\}, \{he, gs\}, \{he, he\}, (he, ns)_e, (he, ns)_e, (he, ns, ns)_e\).

In scenario II both binary components must initially have almost equal mass (within \(\sim 4\%\)) and a primary with a relatively shallow convective envelope. This allows almost conservative mass exchange in the \(\{gs, ms\}\) phase. Due to the large amount of mass which is accumulated by the secondary star its evolution raps-up considerably and it can overfill its Roche lobe before the primary star explodes as a type Ib supernova. The first-born \(ns\) is subsequently recycled in the stellar wind of its companion helium star. The stellar wind prevents the helium common envelope phase. As the outcome of hypercritical accretion is yet not clear, we consider below two options: the formation of a single black hole or the formation of a \((bh, ns)\) binary for the case where both components do not merge within the common envelope formalism.

Finally, there exist “wide” binaries which avoid mass exchange altogether. Both components may evolve independently, if the initial orbital period is \(\gtrsim 10^4\) days or if the initial mass of the components exceeds \(30–40 \, M_{\odot}\) and stellar wind drives the starts apart and prevents Roche-lobe overflow. The initial eccentricity is not affected by tidal effects and it is conserved until the first supernova event occurs. The eccentricity increases the survival probability for the binary when the primary and the secondary subsequently explode in supernovae. It is likely that kicks prevent formation of \((ns, ns)_e\) pairs for (most of the) wide binaries. If one or two are formed anyway, both pulsars will be young and the binary will be visible as two single pulsars which happen to be spatially close for about 10 Myr. Detection of binarity of pulsars is possible for orbital periods below \(\sim 20000\) yr (Lamb & Lamb 1976).

Binarity might be confirmed for very wide binaries if the proper motion of two stars are the same within about a km s\(^{-1}\) (Latham et al. 1984). Detection of a young pulsar in a wide binary may be an experimentum crucis for the hypothesis claiming that single stars and components of wide binaries do not produce radio pulsars because of the slow spin of the pre-supernova stars (Tutukov et al. 1984; Iben & Tutukov 1996). It will argue even against the occurrence of kicks within a few tens of km s\(^{-1}\) (Portegies Zwart et al. 1997a).

The neutron star in the close binary which was not disrupted by the first supernova cannot be detected as a radio pulsar due to free-free absorption of radio emission in the wind of its companion. Within 10 Myr to 20 Myr, before the second supernova occurs, the pulsar passes away. This first-born “dead” pulsar may be posthumously recycled in the stellar wind of its companion and/or during the common envelope phase (Bisnovatyi-Kogan & Komberg 1975; Smarr & Blandford 1976; Bhattacharya & van den Heuvel 1991). The interaction in a binary system is hypothesized to result in a sharp decrease of the magnetic field strength down to \(\sim 10^9\) G (see Bhattacharya 1996 for a list of suggested hypotheses and their classification). A weak field results in a prolonged lifetime for the recycled pulsar, about two orders of magnitude longer than for a “regular” pulsar born with a high magnetic field and with the same pulse period (see e.g. Fig. 1 in van den Heuvel & Lorimer 1996).

2.2. Common envelopes

The crucial stage in the formation of a HMBP in scenario I is the common envelope stage in which the envelope of the donor star is expelled from the binary system through the interaction with its companion. The net result of the common envelope phase is a dramatic reduction in the orbital separation. Estimates of this reduction may be obtained from a comparison of the binding energy of the common envelope \(\Delta E_b\) with the change of the orbital energy of the binary \(\Delta E_{\text{orb}}\) (see e.g. Iben & Livio 1993; Livio 1996 for discussion). The measure of this ratio is the common envelope parameter \(\alpha_{\text{ce}} = \Delta E_b/\Delta E_{\text{orb}}\). Tutukov & Yungelson (1979) suggested the following equation for the variation of the separation of binary components in the common envelope (the donor mass is \(M\) and its companion has mass \(m\)):

\[
\frac{(M_i + m)(M_i - M_f)}{2a_i} = \alpha_{\text{ce}} M_i m \left(\frac{1}{2a_i} - \frac{1}{2a_f}\right),
\]

where subscripts \(i\) and \(f\) refer to the initial and final states, respectively. This equation is based on the assumption that the spiral-in starts when the envelope surrounding the two cores has dimension of \(\sim 2a_i\) and the envelope is ejected from the gravitational potential of both components. Recent 3-D smoothed-particle hydrodynamics calculations for the initial stage of formation of common en-
velopes (Rasio & Livio 1996) confirm such an assumption, at least qualitatively, by showing that very little mass is expelled during this stage and that the envelope expands considerably.

On the other hand, Webbink (1984) suggested a more straightforward equation for $a_t/a_i$ variation:

$$M_i(M_t - M_i)\frac{a_t}{a_i} = \eta_{ce} m \left( \frac{M_t}{2a_t} - \frac{M_i}{2a_i} \right),$$

where $\lambda$ is a structural constant which depends on the density profile in the star [$\lambda = 3/(5 - n)$ for polytropical models with index $n$; the usual assumption is $\lambda = 0.5$], $\eta_{ce}$ is the common envelope parameter and $r_L$ is the fractional radius of the Roche-lobe of the donor. Actually we handle the product $\eta_{ce}\lambda$ as a single parameter.

For completely non-conservative common envelopes, both equations result in equal $a_t/a_i$ for $a_{ce} \approx 4\eta_{ce}\lambda$. Confrontation of population-synthesis calculations with observations does not provide very rigorous constraints on the common envelope parameter. Nevertheless, population synthesis of galactic symbiotic stars (Yungelson et al. 1995), Be/X-ray binaries (Portegies Zwart 1995), super-soft X-ray sources (Yungelson et al. 1996), close binary white dwarfs (Iben et al. 1997) and low-mass millisecond pulsars (Tauris 1996; Tauris & Bailes 1996) confirms that using a high value of the common-envelope parameter. Nevertheless, population-synthesis calculations with observations does not provide very rigorous constraints on the common envelope parameter. The model $\eta_{ce} = 2$ for PSR J1415-0750, a low mass binary pulsar, while for $\eta_{ce} = 0.5$ the spin-up of the neutron star in this particular case is problematic. Increasing $\eta_{ce}\lambda$ to values larger than unity decreases the discrepancy between the predicted and observed ratios of the birthrates for neutron star and black hole low-mass X-ray binaries (see Fig. 4 in Portegies Zwart et al. 1997b). On the basis of 3-D smoothed-particle hydrodynamics calculations Rasio & Livio (1996) claim that the deposition of energy into the common envelope must be highly efficient. Taken at face value, for Eq. 2 values of $\eta_{ce} > 1$ mean in that additional energy for the expulsion of the common envelope must come from sources other than the orbital energy. However, Eqs. 1 and 2 are by no means anything more than indicative order of magnitude estimates. We argue below that reproduction of the distribution function for: the orbital period, the eccentricity and the relative birthrate of observed HMBP are reconcilable with high values for the common envelope parameter.

2.3. The model

For the present study we used the population synthesis code SeBa described by Portegies Zwart & Verbunt (1996). The basic difference with the Tutukov & Yungelson (1993a,b) code is that they use Eq. 1 instead of Eq. 2 which is used by Portegies Zwart & Verbunt for the treatment of binaries which evolve non-conservatively in the semi-detached and common-envelope phases. A detailed comparison of evolutionary sequences for models with similar initial parameters (Portegies Zwart & Verbunt 1996) shows that the sets of assumptions in the two codes result in reasonably similar configurations prior to the second supernova explosion in the system (although the stages in between can be quite different). Moreover, the enhancement of the amount of angular momentum loss during non-conservative mass transfer in the present model [$\eta_J = 6$ instead of $\eta_J = 3$, see Eq. (38) in Portegies Zwart & Verbunt (1996)], makes the results more comparable\(^2\). Comparison of synthesis studies with the observed population of Be/X-ray binaries supports this choice for loss of angular momentum (Portegies Zwart 1995). For the present study the mass-loss law in the stellar winds from the helium stars was accepted after Langer (1989): $M = 5 \times 10^{-8} M^{2.5}_n \, \text{M}_\odot \, \text{yr}^{-1}$, which is about twice the value used by Portegies Zwart & Verbunt (1996).

After the initial conditions for each binary are selected it is evolved until the binary merges, is disrupted or a pair of neutron stars is formed. For the latter, further decay of the orbit is computed from linearized general relativity (Peters 1964). At each moment in time, the distributions of the orbital elements are computed assuming a continuous formation of new systems and taking into account the decay of the orbital elements of the present ($ns$, $ns$)$_c$ binary population.

The zero-age parameters of the simulated binary population are selected from independent distribution functions. The mass of the primary $M$ is chosen from a mass function of the form:

$$dN \propto M^{-2.5} dM.$$  \hspace{1cm} (3)

The mass of the secondary star $m$ is selected from a distribution function for the mass ratio $q_s = m/M$ :

$$\Psi(q_s) \propto (1 + q_s)^5.$$  \hspace{1cm} (4)

We explored the dependence of the model upon $\phi$ in the range from $-4$ to $+5$ and for a “flat” distribution $\Psi(q_s) = 1$. For the semi-major axis distribution we explored two options which are consistent with observational data: flat in $\log a$ between 0.1 $R_\odot$ and $10^6$ $R_\odot$ (Abt 1983) and a Gaussian with a mean of $10^{13.2} R_\odot$ and a dispersion of $10^{2.3} R_\odot$ (Duquennoy & Mayor 1991). The eccentricity for a binary was chosen between 0 and 1 from the thermal distribution: $E(e) = 2e$.

Observations of the space velocities of single radio pulsars suggest that neutron stars may receive a kick at birth (Gunn & Ostriker 1970). A clear summing up of arguments for asymmetric kicks is given by van den Heuvel\(^3\).

\(^2\) For binaries with a mass ratio below $\sim 0.4$ assumption of $\eta_J = 6$ is similar to mass loss through the second Lagrangian point, for larger mass ratios less angular momentum is lost.
& Paradijs (1997)\textsuperscript{3}. Following Hansen & Phinney (1996) and Hartman (1997) we used the distribution for isotropic kick velocities (Paczyński 1990)

\[ P(u)du = \frac{4}{\pi} \cdot \frac{du}{(1+u^2)^2}, \]

with \( u = v/\sigma \) and \( \sigma = 600 \text{ km s}^{-1} \). Implementation of this distribution of kicks by Hansen & Phinney and Hartman provides a reasonable model for the population of single pulsars close to Sun. As an alternatives we also made computations without kicks and with a Maxwellian kick-velocity distribution.

All stars are supposed to be born in binaries. The total birthrate is normalized to the present astration rate of \( 4 \times 10^3 M_\odot \text{ yr}^{-1} \), together with \( M_{\text{min}} = 0.1 M_\odot \) and Eq. 3, which is consistent with observational estimates for the Galactic disc (see van den Hoek 1997; van den Hoek & de Jong 1997). The inaccuracy in the number of binaries formed with \( M_1 \geq 8 M_\odot \) due to uncertainties in the astration rate and the shape of mass-function below \( \sim 0.3 M_\odot \) may be O(2). The assumed age of the Galactic disc is \( T = 10 \text{ Gyr} \) (Meynet et al. 1993; Carraro & Chiosi 1994). The star formation rate is assumed to be constant throughout the evolution of the Galaxy. For all models a total of \( 5 	imes 10^5 \) binaries with a primary mass between \( 8 M_\odot \) and \( 100 M_\odot \) are initialized.

3. Results

Our main purpose is to model the present Galactic population of (ns, ns)\textsubscript{e} binaries with the aim to reproduce their birthrate, period-eccentricity distribution and to obtain an estimate for their merger rate and extrapolate it to the local Universe.

3.1. Birth rates of neutron stars and related binaries

In each of the eight models, for which we present the results below, different assumptions are made concerning the kick velocity, the common envelope parameter, the mass ratio and semi-major axis distributions. Basic peculiarities of the models are summarized in Table 1. As a “standard” set of assumptions we consider the one with a flat initial distributions in \( q_o \) and \( a_0 \), common envelope parameter \( \eta_{CE} \lambda = 2 \), and the kick distribution given by Eq. 5.

In model A no kicks are imparted to neutron stars or black holes.

Model B is identical to model A, but now neutron stars do receive a velocity kick upon birth chosen randomly from the distribution given by Eq. 5. This model is “standard”. Black holes also receive a kick, but scaled by mass: \( v_{\text{bh}} = v_{\text{ns}} m_{\text{ns}}/m_{\text{bh}} \). No mass is ejected in the collapses to black holes, contrary to neutron stars. Small, but not vanishingly, kicks for black holes may be consistent with the velocity dispersion of \( \sim 40 \text{ km s}^{-1} \) of X-ray transients with black-hole candidates (White & van Paradijs 1996) which is higher than velocity dispersion of O-stars (\( \sim 10 - 20 \text{ km s}^{-1} \), Wielen 1992).

Model C is the only model in which the kick velocity is selected from a Maxwellian distribution with a 3-dimensional velocity dispersion of \( \sigma = 450 \text{ km s}^{-1} \).

Model D uses a smaller value for the common envelope parameter of \( \eta_{CE} \lambda = 0.5 \).

The initial semi-major axes of orbits in model E are selected from a Gaussian (see sect. 2.3).

In the models F and G the mass-ratio distribution is not flat: \( \phi = -4 \) in model E and +5 in model G (see Eq. 4).

Finally, in model H which used the standard assumptions of model B, the accretion rate onto a ns in a common envelope is \( 10^8 \times M_{\text{Edd}} \). All neutron stars which are able to increase their mass to \( \geq 2 M_\odot \) are to become black holes. No kicks are applied in these cases.

Table 1 gives an overview of a selection of the computed models. The sum of the entries in the columns (2) to (5), and (8) gives for each model the annual rate of explosive events resulting in the birth of a young neutron star or a black hole. The estimated rate of formation of single neutron stars is given by the sum of the entries in columns (2), (6) and (7). In all models these rates are close to \( 1.5 \times 10^{-2} \text{ yr}^{-1} \) and are almost independent of the assumed semi-major axes and mass-ratio distributions of the binaries and the rate of binary. This can be understood as a consequence of the relatively small number of high-mass secondaries. A smaller binary fraction together with the same astration rate results in an increase of the number of single stars and therefore a higher birthrate for single pulsars. If all stars are born single the pulsar formation rate becomes \( 1.9 \times 10^{-2} \text{ yr}^{-1} \). The model birthrate of pulsars may be compared to the inferred Galactic occurrence rate of type II and Ib/c supernovae: \( (12 \pm 6) \times 10^{-3} \text{ yr}^{-1} \) for SN II and \( (2 \pm 1) \times 10^{-3} \text{ yr}^{-1} \) for SN Ib/c (Cappellaro et al. 1997)\textsuperscript{4}. Based on the modeling of population of pulsars close to Sun, Hartman et al. (1997a) derive a pulsar formation rate of \( (9 \pm 3) \times 10^{-3} \text{ yr}^{-1} \), where a beaming factor of 0.3 was assumed.

The relative birthrate of single and binary neutron stars is most strongly affected by the assumptions concerning kicks (compare model A with the models B to H, see Table 1). The formation rate of single non-recycled neutron stars, column (2) in Table 1, increases considerably when kicks are applied. This is not surprising because the majority of the wide non-interacting binaries and a considerable fraction of the binaries with shorter orbital period that experience a rather stable phase of mass transfer, which causes the orbital separation to increase, are disso-

\textsuperscript{4} For the morphological type Sb, blue luminosity of \( 2 \times 10^{40} \text{L}_\odot \) and \( H_o = 75 \text{ km s}^{-1} \text{ Mpc}^{-1} \).
The formation rate of bound neutron stars, \((ns, \star)\), column (3), drops by an order of magnitude when kicks are present. This entry in Table 1 gives a rough estimate of the formation rate of X-ray binaries. Since the majority of them are high-mass binaries, their total number in the Galaxy can be estimated. Actually, these systems generally start as a Be/X-ray binary to evolve into an ordinary high-mass X-ray binary at a later instant. The X-ray lifetime of a Be/X-ray binary is between 1 and 10 Myr (the remaining main-sequence lifetime of the B-star companion of the neutron star). The estimated galactic population is consequently somewhere between \(10^3\) and \(10^4\). The lifetime of high-mass X-ray binaries is of the order of \(10^3\) yr to \(10^6\) yr, depending on the evolutionary status of the donor (e.g. Savonije 1979; Massevich et al. 1979; Hellings & DeLoore 1986). This results in a total of several tens of high-mass X-ray binaries in the Galaxy. These estimates of numbers of Be- and high-mass X-ray sources are consistent with observational estimates of \(> 2000\) and \(55 \pm 27\), respectively (Meurs & van den Heuvel 1989, and 50 to 80 by Iben et al. 1995). Models without a kick seem to overestimate the numbers of X-ray binaries by an order of magnitude.

Binaries which contain a black hole and a stellar companion are not as common as binaries which contain a neutron star. If their lifetime is comparable to the lifetime of high-mass X-ray binaries which contain a neutron star, their number has to be \(~ 1/10\) of the former, in agreement with the observations.

Derivation of the expected birthrate and the number of low-mass X-ray binaries from our calculations would require a more rigorous consideration of very severe constraints on the orbital periods and masses of donors in prospective candidates (Kalogera & Webbink 1996) and is beyond the scope of the present paper.

Entries in columns (6) to (8) of Table 1 refer to neutron stars which were born first in the binaries and were subsequently recycled or merged with their stellar companions. Evidently, the interaction with companions influences the radio properties of neutron stars. We still call them “recycled” pulsars, though decay of the magnetic field and/or the action of the propeller mechanism can prevent a certain fraction of them to show up in radio. The birthrate of single recycled pulsars is given in column (6) of Table 1. Observational properties of these objects are discussed in detail by Hartman et al. (1997b). Typically, single recycled pulsars contribute for less than 1% to the total population of single radio pulsars\(^5\). This is consistent with the presence of eight isolated objects among observed

5. The birthrates found by Hartman et al. are smaller by about a factor two because they assumed a binarity fraction of 50%.
millisecond pulsars in the Galactic disc. In the absence of kicks (model A) \( \sim 1/7 \) of all pulsars are old and possibly recycled; this contradicts the observations (see however Deshpande et al. 1995 for arguments which suggest a high fraction of recycled pulsars).

The presence of kicks significantly increases the proportion of systems in which the neutron star merges with its stellar companion. The main reason for this is that a kick can decrease the orbital period or even shoot the neutron star directly into its companion (see also Brandt & Podsiadlowski 1995). In the first case, the smaller orbital separation of the binary causes a merger to occur more easily after a phase of unstable mass transfer. If the merger does not lead to the formation of a black hole the neutron star may, after ejection of the common envelope, emerge as a single recycled pulsar. These systems may increase the fraction of recycled pulsars by a factor of two.

3.2. Neutron star–neutron star binaries

Comparison of column (3) with column (8) from Table 1 shows that less than \( \sim 5\% \) of the Be- or high-mass X-ray binaries survive the second supernova and produce a recycled binary pulsar, about \( 25\% \) is dissociated in the second supernova and about \( 10\% \) merge into a single object. The remaining \( \sim 60\% \) never experience the second supernova. A minor proportion of the latter systems may become LMXB and, later, millisecond pulsars.

We confirm the result from previous computations that the birthrate of \((n_s, n_s)\) binaries drops by more than an order of magnitude when kicks are implied. This reflects in the birthrates in Table 1. More binaries are dissociated when the kick velocity is taken from a Maxwellian distribution instead of the distribution of Eq. 5, because of the larger average magnitude of the kick.

Directly after the second supernova, a binary containing two neutron stars of which one is visible as a young radio pulsar and the other may be recycled. The later born pulsar dies after about 10 Myr. A recycled pulsar may live up to a few billion years. For most of this time span the binary may be detectable as a recycled pulsar with a “dead” companion. A few known systems are listed in Table 2. For PSR J1518+49 and B2303+46 only the total mass is known and they may have a massive white dwarf instead of a neutron star as a companion. White dwarf companions will be ruled out if optical observations will not discover objects of a visual magnitude of \( \sim 26 \) for PSR J1518+49 and between 23 to 25 for the other pulsar (Fryer & Kalogera 1997).

The long pulse period and strong magnetic field of PSR B2303+46 do not exclude that it is the second born pulsar which we observe. In this case, its companion would be at most \( \sim 2 \times 10^7 \) yr older (the evolutionary timescale of a \( M \sim 10M_\odot \) star). The companion is either a dead pulsar which did not resurrect as a recycled pulsar or it is beaming away from our planet. This discussion also applies to PSR B1820-11 for which even the total mass is not known. It might be the young pulsar with a main-sequence star, a white dwarf or an unseen pulsar as companion (see Phinney & Verbunt 1991 and Portegies Zwart & Yungelson (1997), for discussion).

The birthrate of radio pulsars in neutron star binaries must equal the birthrate of recycled pulsars in similar systems (Bailes 1996). The absence of “young” pulsars in pairs with old ones among \( \sim 650 \) objects restricts the relative birthrate of \((n_s, n_s)\) systems to \(1/650\). Bailes’ criterion does not restrict the absolute birthrate of \((n_s, n_s)\) binaries; it lacks absolute calibration. Except for model A, none of the models contradicts this statement. If it is actually the young pulsar we observe in PSR B2303+46 and B1820-11 the limit on the relative birthrate increases to \(2/650\).

3.3. Orbital period–eccentricity plane for neutron-star binaries

The only observational parameters of HMBPs to which one may compare the results of evolutionary computations and population synthesis are the orbital period \( P_{\text{orb}} \) and eccentricity \( e \). Other characteristics of the binary like the center-of-mass velocity are ill known from observations. For a comparison of the characteristics of the individual pulsar, like the strength of the magnetic field, the lifetime and the pulse period, no theory is available. In the \( P_{\text{orb}} - e \) diagram it is only the emission of gravitational waves that effects the further evolution of the orbital parameters.

Figure 1 gives the distribution function for the orbital parameters of \((n_s, n_s)\) binaries at birth for a model without a kick (A) and for model B with a kick velocity distribution according to Eq. 5. The population of \((n_s, n_s)\) binaries with periods beyond \(10^4\) days is not shown. Since in models without kick most wide binaries survive the supernovae explosions bound, \( \sim 70\% \) of all \((n_s, n_s)\) have orbital periods greater than \(10^4\) day. Nondetection of young pulsars in wide binaries then argues for kicks of at least of several tens of \( \text{km s}^{-1} \), see Portegies Zwart et al. 1997a.

In the models with a kick this population contributes only little (\( \lesssim 10\% \)), see also Portegies Zwart & Verbunt 1996). In both models the adopted common envelope parameter in Eq. 2 is \( \eta_{\text{ce}} \lambda = 2 \) (which is roughly equivalent to \( a_{\text{ce}} = 0.5 \) in Eq. 1). The reason for this choice is the inability for smaller \( \eta_{\text{ce}} \lambda \) to reproduce the position of the observed high-mass recycled pulsars in the \( P_{\text{orb}} - e \) plane. Particularly, the probability for the formation of binary pulsars with \( P_{\text{orb}} \lesssim 2 \) days and \( e \lesssim 0.3 \) and \( P_{\text{orb}} \sim 10 \) days and \( e \sim 0.6 - 0.7 \) appears to be very small. The reason is that low efficiency of the deposition of orbital energy into the common envelope results in the formation of very close binaries. For comparison we give the \( P_{\text{orb}} - e \) diagram for \( \eta_{\text{ce}} \lambda = 0.5 \) in Fig. 4.
Table 2. Observed population of massive neutron star binaries. Their names are followed by the pulse period, the period derivative and the orbital elements, age and magnetic field strength. References: 1 - Nice et al. (1996), 2 - Wolszczan (1990), 3 - Lyne & McKenna (1989), Bhattacharya & van den Heuvel 1991, 4 - Hulse & Taylor (1975), 5 - Deich & Kulkarni (1996), 6 - Stokes et al. (1985). Data which is not available in the cited papers were taken from Taylor et al. (1993).

| PSR       | ref | $P$ [ms] | $\log \dot{P}$ | $P_{\text{orb}}$ [days] | $e$ | age [Myr] | $\log B$ [G] | Comments |
|-----------|-----|---------|----------------|------------------------|----|-----------|-------------|----------|
| J1518+49  | 1   | 40.94   | -19.4          | 8.634                  | 0.249 | 16000    | 9.1         |          |
| B1534+12  | 2   | 37.90   | -17.6          | 0.420                  | 0.274 | 250      | 10.0        |          |
| B1820-11  | 3   | 279.83  | -14.86         | 357.762                | 0.795 | 3.3      | 11.8        |          |
| B1913+16  | 4   | 59.03   | -17.1          | 0.323                  | 0.617 | 110      | 10.4        | Non-recycled? |
| B2127+11C | 5   | 30.53   | -17.3          | 0.335                  | 0.681 | 100      | 10.1        | In globular cluster M15 |
| B2303+46  | 6   | 1066.37 | -15.24         | 12.340                 | 0.658 | 30       | 11.9        | Non-recycled? |

If no kicks are imparted the $P_{\text{orb}} - e$ diagram has clear-cut borders (see Fig. 1). The shortest orbital period is determined by the requirement to accommodate a helium star with a neutron star companion in the pre-explosion orbit. The supernova tends to widen the binary orbit and the minimum eccentricity is obtained from the fact that at the instant of explosion all pre-neutron stars still have helium envelopes of $\Delta M \gtrsim 0.5 M_\odot$. A minor population with low $e$ and $20 \lesssim P_{\text{orb}} \lesssim 30$ day descends from massive systems which had eccentric orbits before the second explosion. The major, short-period, population in Fig. 1 descends from systems which evolved via scenario I with all mass exchanges being unstable, while wider systems had at least one phase of stable mass exchange before the formation of the first neutron star. Presence of these two populations was also clearly seen in the figures in Tutukov & Yungelson (1993a) and Portegies Zwart & Spreeuw (1996) papers. The lower panel in Fig. 1 demonstrates that, apart from reducing the birthrate, kicks have the effect of “smearing” the period–eccentricity distribution of the neutron star pairs. The eccentricity distribution is more strongly affected by the presence of a kick than the period distribution. Striking in the lower panel in Fig. 1 is the existence of a population of binary neutron stars with orbital periods below $\sim 0.1$ day, in contrast to the computations without a kick. These short periods are the result of a kick in the “right” direction to reduce the orbital period upon the second supernova. Due to the emission of gravitational waves this population is extremely short-living. If kicks are absent, only the Hulse-Taylor pulsar is born in the region in the $P_{\text{orb}} - e$ diagram with high formation probability. The extremely small probability for the formation of zero-age pulsars with parameters similar to that of J1518+49 and B2303+46 is appealing.

Figure 2 shows the effect of time evolution on the population of neutron star binaries from the standard model B with a kick, giving the probability distribution of them at the age of 100 Myr, 1 Gyr and 10 Gyr. The limit of 100 Myr is suggested by the characteristic age of PSR B1913+16. Figure 2 may be compared with the lower panel of Fig. 1.
which gives the probability distribution of these binaries at birth (which is more appropriate for relatively young pulsars like PSR B2303+46). PSR B1913+16 fits extremely well into the model population for its age. The rest of the object fall into regions with probability by a factor of a few lower, but still relatively high. The observed system object fall into regions with probability by a factor of a
sars like PSR B2303+46). PSR B1913+16 fits extremely
birth (which is more appropriate for relatively young pul-
sars older than PRS B1913+16 are found in the area of the
neutron star binaries present in the Galaxy are dead! (van den Heuvel & Lorimer 1996), the majority of the
pulsars, which depends on their specific accretion history.

The probability distribution shifts to larger orbital period as the population becomes older, because short-period objects merge due to the influence of gravitational wave radiation while the long-period binaries survive for a longer time. As the age of the population increases pulsars older than PRS B1913+16 are found in the area of the $P_{\text{orb}} - e$ diagram with a higher probability; the for-
mer pulsar drifts to the lower probability area. The middle
panel of Fig. 2 fits best with the observed four systems suggesting that the average age of the observed popula-
tion is close to 1 Gyr. It is, however, hard to quantify this
statement. Particularly, there may be a differential selection effects related to the different lifetimes of recycled pulsars, which depends on their specific accretion history.

The bottom panel of Fig. 2 represents the total pop-
ulation of binary neutron stars currently present in the
Galaxy. The $\langle ns, ns \rangle$ binaries we observe today are a sub-
set of this population. Since typical merger time for most
$\langle ns, ns \rangle$ binaries is several hundred Myr (see Fig. 7 lower
panel) and radio lifetime of recycled pulsars is $\lesssim 1$ Myr (van den Heuvel & Lorimer 1996), the majority of the
neutron star binaries present in the Galaxy are dead!

Figure 3 gives the probability distribution for $\langle ns, ns \rangle$ pairs younger than 100 Myr in the $P_{\text{orb}} - e$ plane for the model where the kicks are selected from a Maxwellian velo-
city distribution. The gray shading in this Figure is nor-
malized to that in the upper panel of Fig. 2. This model has a smaller $\langle ns, ns \rangle$ birthrate than model B, this re-
flects in the absence of the darkest shades in Fig. 3. The presence of a non-negligible, yet not observed, popula-
tion of $\langle ns, ns \rangle$ binaries with orbital periods larger than $\sim 10$ days in model B (see Fig. 2) reflects the higher prob-
ability of low-velocity kicks in the distribution given by
Eq. 5 relative to the Maxwellian. Though the latter model suggests almost equal probability for the detection for PSR B1534+12, B1913+16 and B2303+46, it gives an extremely low birthrate for binaries with orbital parameters similar to PSR J1518+49. Even high age does not increase the model galactic population of pulsars with similar orbital parameters. This discrepancy does not go away when the velocity dispersion of the Maxwellian distribution is decreased by a factor of 2. (Hansen & Phinney 1997, sug-
gested that a Maxwellian kick-velocity distribution with
$\sigma = 190 \text{ km s}^{-1}$ fits satisfactory with the observed single pulsar velocities.)

Model E, with initial Gaussian distribution over log $a$ differs only slightly from the model with an initial flat distribution in log $a$. It gives a similar birthrate of $\langle ns, ns \rangle$ binaries and reproduces the $P_{\text{orb}} - e$ relation equally satisfactorily. This means that most of $\langle ns, ns \rangle$ stars descend from initial systems having orbital separations in the interval where the probabilities for formation for both distributions are comparable.

Our computations included several additional runs with different parameters which produced models which we consider less satisfactory than the standard model. Model F with initial $q$-distribution strongly raising to 0 ($\phi = -4$, in Eq. 4) gives birthrates of single neutron stars and $\langle ns, ns \rangle$ binaries similar to those in the standard model B, but does not reproduce the $P_{\text{orb}} - e$ distribution satisfactory. Particularly, it does not reproduce the pop-
ulation with $P_{\text{orb}} \sim 10$ day and $e \lesssim 0.7$ and pulsars with $P_{\text{orb}} \sim 0.4$ days and $e \approx 0.2 - 0.3$. This is an under-
standable result of the high proportion of mergers in the first
(hydrogen) common envelope event and of strong reduc-
tion of the orbital separation in the systems which avoid mergers. Model G, with $q_e$ strongly raising to 1 ($\phi = +5$),
does not basically differ from the other models in the total production rate of pulsars and it reproduces the $P_{\text{orb}} - e$ relation rather well. However, it is the least satisfactory model respective to the ratio of formation rates of binary and single pulsars (see discussion section).

Figure 4 gives the $P_{\text{orb}} - e$ plane for the population younger than 100 Myr for $\langle ns, ns \rangle$ systems of model D. In this model the efficiency for the deposition of orbital energy into the common envelope during the spiral-in is con-
siderably smaller than in the standard model: $\eta_{\text{ce}} \lambda = 0.5$. The birthrate of $\langle ns, ns \rangle$ binaries in this model is consid-
erably smaller than in the standard model, as a result of
greater proportion of mergers under low $\eta_{\text{ce}}$. However, all four HMBP's appear in low-probability areas of the dia-
gram. This was the basic reason for pushing $\eta_{\text{ce}}$ to higher values in our efforts to obtain satisfactory models for the
population of binary neutron stars.

Figure 5 gives the $P_{\text{orb}} - e$ distribution of the neutron star binaries which originate from model H for two age lim-
its. This distribution is hardly affected by age (compare with model B in Fig. 2). Since systems which experience a common envelope are absent in this model, the average orbital separation is relatively large. The distribution is static for systems with $P_{\text{orb}} \gtrsim 1$ day; it is hard to discrim-
nate between the 100 Myr and the 10 Gyr populations.

The population which originates from model H is a subset of model B, and Fig. 5 consequently represents the $P_{\text{orb}} - e$ distribution of this subset in Fig. 2. Only the young pulsar PSR B2303+46 is present in the area with the highest probability in Fig. 5. Therefore it is appeal-
ing to suggest that it originates from scenario II. Similarly, van den Heuvel et al. (1994) argue that the evolution of the progenitor of this system was dominated by mass loss in a stellar wind rather than by Roche-lobe overflow. A common envelope phase is required to form short period binary pulsars. The apparent “success” of Wettig & Brown (1996) and Fryer & Kalogera (1997) in reproducing PSR B1913+16 and B1534+12 is mainly due to their fine tuning of the parameters of the pre-supernova binary. Such systems are not the dominant contributors to short orbital period HMBP.

Figure 6 gives the cumulative velocity distribution for the center of mass of the merging (ns, ns) binaries which results from the model computations. For comparison, we also give the kick distribution (Eq. 5). As expected, the model without the kick produces low velocity binary neutron stars and models B and C produce the higher velocity ones. The low-velocity shoulder for the model without a kick (left dash-dotted line), shown for comparison, is the result of very wide – non-interacting – binaries which happen to stay bound due to the orbital eccentricity and low mass loss during the supernova. The short period (ns, ns) binaries from the model without a kick reach much higher velocities (right dash-dotted line), but by far not as high as in the models where a kick is imparted to the newly formed neutron star. The difference between the models with a Maxwellian kick and a Paczyński kick becomes quite noticeable in the tail of the distribution: model B produces more high velocity neutron star binaries. The velocity distribution for HMBP which are produced in model H is very similar to that of model B (solid line) but it is shifted to lower velocities by about 50 km/s, as a result of the absence of very close systems among their progenitors. The velocity distribution for the single pulsars is similar to the kick distribution, the contamination by recycled pulsars (which typically have smaller velocities, Hartman et al. 1997b) is negligible. The distribution for HMBP has a lower proportion of high-velocity objects compared to the single pulsars because the high velocity tail of the kick distribution prevents the formation of (ns, ns) binaries. There are not references to the spatial velocities of binary pulsars in the refereed literature and it is impossible to confront Fig. 6 with observations. We should note, however, that discovery of a HMBP with velocity in excess of ∼250 km s⁻¹ would argue for the presence of kicks.

4. Discussion

4.1. Population of HMBP and their merger times

Population synthesis computations for the formation and further evolution of binary neutron stars provide estimates for their birthrate and the expected orbital elements of the present day population. The most crucial parameter for modeling of (ns, ns) binaries is the common envelope parameter. This parameter is not strongly constrained by observations. Population synthesis and smoothed particles hydrodynamic computations suggest that energy deposited into common envelopes may be high compared to the orbital binding energy. Our preferred model B is characterized by such a high efficiency: ηceλ = 2. Higher values of ηceλ up to 4 still result in agreement with the observed population but the birthrate of neutron star binaries becomes uncomfortably high compared to the formation rate of single radio pulsars. For ηceλ ≤ 1 the birthrate of neutron star binaries decreases significantly but the period-eccentricity diagram fits badly with the four observed systems. However, it might well be that ηceλ depends on the evolutionary stage of the binary; for an inspiralling neutron star this parameter might have a very different value than for a main-sequence star or white dwarf. Neutron stars can be very efficient in distilling the common envelope by energy released by e.g. small explosions (Fryer et al. 1996) or semi-steady burning on its surface during the spiral-in process.

Only the models in which a kick is imparted to the newly born neutron star from the distribution proposed by Paczyński (1990) with parameters suggested by Hartman et al. (1997a) give a satisfactory birthrate as well as a distribution over P orb − e. Increasing the fraction of low-velocity kicks in Paczyński distribution will however, increase the quality of the fit of the P orb − e plane. This distribution seems at the moment preferable because the reproduction of transverse velocities of young pulsars with measured proper motions requires the presence of a significant number of objects with low velocities (Hansen & Phinney 1996, Hartman et al. 1997a). The actual kick velocity distribution might have an even larger contribution of low velocity kicks than Eq. 5 suggests.

Apart from kick velocity, all other parameters have a relatively small effect. Birth- and merger rates vary within a factor of a few, but not by orders of magnitude. Therefore it is hard to decide on population estimates of birthrates which model parameters are favoured.

The observations may be best fit with the computed probability distribution in the P orb − e plane if the age of the recycled neutron stars is of the order of 100 million to 1 billion year. Note that there might well be effects other than the age alone which have its influence on the detectability of neutron star binaries; the pulse period, the strength of the magnetic field and the luminosity. All these parameters give rise to a different lifetime of the pulsar and different observational characteristics which make the selection effects hard to quantify. It is appealing that the youngest system PSR B2303+46 fits the relatively high probability region of P orb − e diagram for the youngest pulsars (Fig. 1, lower panel), the oldest one PSR J1518+49 fits best if the age of the population is as old as a few billion years (close to the one in the lower panel of Fig. 2) and intermediate age pulsars PSR B1913+16 and PSR B1534+12 are best reproduced.
by a population of a few hundred Myr (Fig. 2). Since the best fit between the observations and the models is at an age of the population between 100 Myr and 1 Gyr, this suggests that the majority of the recycled neutron stars are born with low magnetic field.

Taking into consideration the characteristic age, orbital period and the eccentricity of PSR B2127+11C, one may suggest that it is formed by scenario I instead of an exchange interaction in the high-density core of the globular cluster M15 (see Phinney & Sigurdsson 1991). However, to be primordial, the pulsar should be at least 10 Gyr old (the age of the globular cluster). This pulsar has to merge with its companion within a few 100 Myr and it is highly unlikely to observe it in the last percent of its lifetime as a (ns, ns) binary.

The birthrate of binary pulsars relative to singles is \( \sim 1/430 \) in the standard model, while in the most extreme model II it is \( \sim 1/1775 \). These numbers have to be reduced for the binary fraction, which is smaller than 100%. According to van den Heuvel & Lorimer (1996) the lifetime of recycled pulsars is a factor \( \sim 3 \) larger than their characteristic age and therefore it is between \( \sim 100 \) Myr and \( \sim 1 \) Gyr. On the other hand population synthesis studies suggest that \( \sim 90\% \) of the recycled pulsars are missed in pulsar surveys (Curran & Lorimer 1995). The lifetime of young pulsars is \( \sim 10 \) Myr. Combining these numbers we arrive at the conclusion that the relative birthrate of single pulsars and binary pulsars also reflects their relative galactic number density. If beaming factors are similar for old and young pulsars, relative numbers of ns and (ns, ns) in our models do not contradict observations (see Bailes 1996). As the birthrate of single pulsars in our model is consistent with the rate of supernovae and the birthrate of single pulsars, we may conclude that our models give basically the actual Galactic birthrate of binary neutron stars.

A fraction of the neutron stars in binaries may never become observable as recycled pulsars. The large pulse periods of X-ray pulsars suggest that observed recycled pulsars are resurrected at later stages. It may be shown that in the common-envelope stage that follows the X-ray pulsar stage, if \( B \gtrsim 10^{11} \) G, accretion at the Eddington rate doesn’t spin neutron stars to \( P \lesssim 100 \) ms (Ergma & Yungelson 1977). Recycling of a pulsar to \( P_{\text{eq}} \sim 30 \) ms requires the accretion of \( \Delta M \approx 0.17 \left( P_{\text{eq}}/\text{ms} \right)^{-4/3} \approx 0.002 M_\odot \) (e.g. Bhattacharya 1996). The only evolutionary stage in which this can happen is during accretion from the wind of the accompanying helium star. The “propeller mechanism” (Illarionov & Sunyaev 1975), which prevents accretion onto the neutron star may cause the neutron star to remain dead. Ergma & Yungelson (1997) derive an estimate for the orbital period at which the propeller mechanism is effective:

\[
P_{\text{orb}} \gtrsim 0.5 M_{\text{ns}}^{3/2} v_{1000}^{-3} (M_{\text{ns}} + M_{\text{he}})^{-1/2} M_{\text{he}}^{15/8} \ \text{hr},
\]

where \( v_{1000} \) is the velocity of the helium-star wind in \( 1000 \) km s\(^{-1} \) and masses are in \( M_\odot \). The spin slow-down time in the (ns, he) systems is comparable to the helium burning time. For typical \( v_{1000} \sim 1 – 2 \), the critical period is of the order of only several hours for a low mass helium star, which is quite short. If the propeller mechanism is effective, accretion is possible for short period binaries with a low mass helium star or for wide systems with a high mass helium star. In the computations of Ergma & Yungelson it appeared that the majority of the neutron stars are not able to accrete from helium companion winds. The first born pulsar has plenty of time (10 to 20 Myr) to die before its companion becomes a neutron star. If the propeller mechanism is effective, the first born pulsar will never be resurrected from the graveyard. In this case the birthrate of recycled pulsars in binaries may be smaller than the birthrate of neutron star binaries. According to Lipunov et al. (1997) this fraction may be as small as \( \sim 1\% \), depending on the assumed initial magnetic moment of the neutron star and the decay timescale of its magnetic field. There may be one such a binary among the observed pulsars, and in fact it is possible that PSR B2303+46 contains a non-revived pulsar next to a young one (see Table 2).

An observational estimate for the birthrate of HMBP can be derived from the estimate of their number in the Galaxy and their lifetimes as pulsars (Kulkarni & Narayan 1988). Using scaling factors, correction factors for incompleteness of the sample and beaming factor from Curran & Lorimer (1995) and lifetimes from van den Heuvel & Lorimer (1996) for PSR B1534+12, B1913+16, and B2303+46 we arrive at a birthrate of \( 2.7 \times 10^{-5} \) yr\(^{-1} \). This is comfortably close to our standard model B (Table 1). This derived birthrate heavily relies on the young PSR B2303+46. Excluding it reduces the birthrate to \( 0.8 \times 10^{-5} \) yr\(^{-1} \), which is obviously the estimate obtained by van den Heuvel & Lorimer (1996). There is no data available on scaling factor for PSR J1518+49, but we may expect that it will enter the estimate of the birthrate with a very small weight because of its long characteristic age.

To summarize, we may argue that our standard model B, which reproduces best the orbital periods and eccentricities of the observed HMBP is not in conflict with observations also in respect to the birthrate of these systems. Therefore we may expect, that this model gives a realistic estimate of the merger rate of neutron star binaries \( \approx 2 \times 10^{-5} \) yr\(^{-1} \). This number is not inconsistent with the estimated rate of \( 0.8 \times 10^{-5} \) yr\(^{-1} \) from van den Heuvel & Lorimer (1996) based on two observed pulsars with merger times shorter than the Hubble time and the assumption

\(^6\) The results of Ergma & Yungelson cannot be directly applied here for numerical estimates, since their population synthesis was made under different assumptions. Qualitatively it is clear that for the present computations accretion would be prohibited in even larger proportion of all systems, since here we typically produce wider (ns, he) binaries.
of a steady state in which the birthrate equals the merger rate. Actually, from Table 1 and Figs. 1 and 7 it is immediately clear that there does not exist a steady state in which the birthrate equals the merger rate. About 10% to 20% of all \((ns, ns)\) binaries merge after the recycled pulsar has died within \(0.5 – 1\) Gyr (see Fig 7). These binaries hang around undetected in the Galaxy until they merge. A considerable fraction of these (20% to 30%) are parked in wide orbits in the Galactic halo or escape from its potential well (see Fig. 8 and 9). Another \(~10\%\) merge within \(~10\) Myr, before even the young pulsar dies (see Fig 7). These systems may also avoid detection due to their short lifetime (their number relative to the number of single pulsars is only \(\sim 1/4000\) in model B for 100% binarity). Figure 7 suggests a revision of the “observational” estimate upward by a factor \(\sim 1.5\) to \(1.2 \times 10^{-5}\). This correction factor may increase even further taking non-recycled, hence non-observable, close binary neutron stars into account. (Curiously enough the situation with merging neutron stars is similar to that of merging white dwarfs which are candidate progenitors for SNe Ia; most of the mergers will occur when both white dwarfs have cooled beyond detectability, Iben et al. 1997.)

On the other hand, about \(1/3\) of all \((ns, ns)\) binaries born annually in the Galaxy have merger times exceeding \(10\) Gyr and are accumulating in it. The total number of dead non-merging pairs present in the Galactic disc may be as large as \(10^5\).

Figure 7 indicates that the Galactic disc merger rate is determined mainly by the star formation rate during the last \(1–2\) Gyr. The average star formation rate over the history of the Milky Way is about 4 times higher than the current rate (e.g. van den Hoek 1997). The same is the case for other massive (\(\sim 10^{11}\) M\(_\odot\)) spiral and irregular galaxies (Gallagher et al. 1984; Sandage 1986). The star formation rate was much higher than present only in the first few Gyr of the existence of the Galaxy. Figure 7 shows that this early star burst contributes only little to the current merger rate of \((ns, ns)\) binaries. The same point has to be made with respect to elliptical galaxies. As most stars in them were formed in a short (\(\sim 1\) Gyr) burst, overwhelming majority of merger candidates already merged long ago. Figure 7 suggests that in a galaxy which was formed in a \(1\) Gyr burst, the present rate of \((ns, ns)\) mergers per unit mass is \(\sim (10 – 15\%)\) of that in the galaxy of the same mass but formed in \(10\) Gyr. In the extrapolation of the Galactic rate of merger to the local Universe the different contributions from spirals and ellipticals have to be considered (see Bagot 1996 for a parametric study of this issue). Scaling to the total number of galaxies (spirals as well as ellipticals) within \(100\) Mpc after Curran & Lorimer (1995) gives an upper limit for the number of mergers detectable by the advanced LIGO detector of \(1\) yr\(^{-1}\) for model B.

### 4.2. Merger rate in the Universe and \(\gamma\)-ray bursts

A rough estimate of the total number of merger events in the Universe may be obtained from scaling the Galactic merger rate by the ratio of blue-band luminosities of the Milky Way and the Universe: \(R \approx 10^{-7} h\) Mpc\(^{-3}\), where \(h\) is Hubble constant in \(100\) km s\(^{-1}\) (Phinney 1991). For \(H_\odot = 75\) km s\(^{-1}\) Mpc\(^{-1}\) this gives, extrapolating model B, \(\sim 4.6 \times 10^4\) events per year. This number may be compared to the number of \(\gamma\)-ray bursts, which are also suggested to originate from neutron star mergers (Paczynski 1986). The observational estimate of the rate of detectable \(\gamma\)-ray bursts in the Universe is several times \(10^{-6}\) per year per galaxy (Mao & Paczyński 1992; Piran 1996). The discrepancy with the observed rate (for model B) may be erased if \(\gamma\)-ray emission is beamed into an opening angle of \(\sim 10^\circ\) (Mao & Yi 1994; Piran 1996). The anisotropy of the energy release in \(\gamma\)-ray production is suggested by numerical merger models (Davies et al. 1994).

Virtually all \((ns, ns)\) binaries in the models with a kick escape from the potential well of a globular star cluster (see Fig. 6) and a considerable fraction is expected to escape from a dwarf galaxy. In the model without a kick, this fraction is only \(\sim 25\%\) (about \(10\%\) for dwarf galaxies). But in the latter model the majority of \((ns, ns)\) systems originate from wide non-interacting binaries which never produce a recycled pulsar and which will not merge within the age of the Universe. The \((ns, ns)\) pairs confined to parental stellar systems have typically large orbital periods and are therefore very fragile for three-body encounters. The result is that globular clusters are deficient of HMBP’s according to the models with a kick and also according to the models without a kick! In the presence of a kick the formation of \((ns, ns)\) binaries is not an efficient way to keep a neutron star bound to a globular cluster.

The maximum distance traveled by a \((ns, ns)\) binary before it merges may be computed using the distribution functions for the merger time and the center-of-mass velocity of the binaries. Figure 8 provides this distribution for the HMBPs. It is interesting to notice that in the models without a kick the typical distance traveled is considerably larger than in the models with a kick. Note also that the distributions for both models with a kick are indistinguishable. The binaries in the high-velocity tail of model B (see Fig. 6) merge on a short time scale (see Fig. 7) whereas the low-velocity binaries in the model with a Maxwellian kick live rather long.

The distance traveled perpendicular to the galactic plane in the potential of the Galaxy can be estimated from a relation between the \(z\)-component of the initial velocity \(v(z)\) of an object and the maximum galactic height \(z\) (Tutukov & Yungelson 1978):

\[
v(z) \approx [6z - 2000\ln(1 + 0.003z)]^{1/2},
\]
where $v$ is in km s$^{-1}$ and $z$ in pc. The maximum $z$ reached by a $(ns, ns)$ binary before coalescence was estimated using the velocity from the distribution given in Fig. 6 divided by $\sqrt{3}$ (assuming that the velocity is distributed isotropically). To correct for the pre-merger lifetimes which may be shorter than the travel time to the maximum $z$, the minimum of $z$ resulting from Eq. 7 and the maximum distance traveled (Fig. 8) was adopted.

Figure 9 shows that for the models without a kick the majority of the systems do not reach a high $z$ (left dash-dotted line), as expected from the enormous low-velocity tail in Fig. 6. The short period $(ns, ns)$ binaries from model A reach considerably higher Galactic heights (right dash-dotted line) but by far not as high as in the models with a kick. Note that the majority of the binaries in the low-$z$ shoulder (left dash-dotted line in Fig. 9) are not visible as systems with a recycled pulsar, since they did not experience a phase of mass transfer. The $(ns, ns)$ binaries from the models where a kick is imparted to the newly formed neutron star show a much more extended $z$ distribution. Actually, a considerable fraction of these systems escape from the potential of the Galaxy or are parked in the Galactic halo. Escape of some $(ns, ns)$ beyond the sensitivity limits of detectors may be a factor contributing to the underestimate of the galactic birthrate of HMBP. Since the life times of single radio pulsars are considerably shorter than that of recycled pulsars, of which the lifetime is comparable to the merger time, the galactic heights of the latter are expected to be considerably larger.

Even in the absence of kicks $(ns, ns)$ binaries, which escape mainly from dwarf galaxies, may form a common intergalactic field of these objects (Tutukov & Yungelson 1994). It is suggested that most of star formation in the Universe occurs at low redshift ($z \sim 1$, see e.g. Madau 1997, Connolly et al. 1997) and the main sites of star formation could be dwarf star-burst galaxies (e.g. Babul & Ferguson 1996). However, due to the large distances traveled by the $(ns, ns)$ before they merge, a significant fraction merge at a large distance form the host galaxy in which the binary was formed. If $\gamma$-ray bursts are related to $(ns, ns)$ mergers, a considerable fraction may occur at large distance from their host galaxy and they are not likely to be associated.

### 4.3. Black hole plus neutron star binaries

Our models suggest (Table 1) that the number of detected radio pulsars has to be at least 3 to 4 times larger than at present before one may hope to find a young pulsar accompanied by a black hole. The only exception is in model H in the version in which hypercritical accretion onto the neutron star does not always result in swallowing of the companion. In this model one may expect the presence of 2 or 3 $(bh, ns)$ systems among the known young pulsars (see Table 1). The number of $(bh, ns)$ binaries has then to be comparable to the number of HMBPs. Both these conclusions are in conflict with the observations. The merger rate of $(bh, ns)$ binaries is typically more than an order of magnitude smaller than that of $(ns, ns)$ binaries. An exception to this is model D with small $\eta_{\text{e}}$, which prevents the formation of $(ns, ns)$ systems by merging their progenitors in the common envelope phase.

For gravitational waves detectors the signal-to-noise ratio is (Bonazzola & Marck 1994)

$$S/N \propto \frac{(M_1 M_2)^{1/2}}{(M_1 + M_2)^{1/6}} D^{-1},$$

where $M_{1,2}$ are the masses of coalescing objects and $D$ is the distance to them. E.g., for the range of black hole masses 4 to 17 $M_\odot$ suggested by black-hole candidate binaries, one would expect that for $(bh, ns)$ mergers detection distance may be 1.5 to 2.5 larger than for $(ns, ns)$ mergers. Hence, the rates of detectable $(ns, ns)$ and $(bh, ns)$ mergers are expected to be comparable (about 1/yr). This has to be considered in the context of expected coalescence waveforms from the first detections (Thorne 1996). Also, the characteristics of $\gamma$-ray bursts expected from $(bh, ns)$ mergers may differ from the ones from $(ns, ns)$ mergers (Paczyński 1991).

Due to the large orbital period of $(bh, bh)$ binaries, their expected merger rate is negligible and this number is not given in Table 1. A rate of detectable $(bh, bh)$ mergers comparable to the rate of $(ns, ns)$ mergers was predicted by e.g. Tutukov & Yungelson (1993a) and Lipunov et al. (1997). According to our current understanding this is a result of underestimating the effect of high mass loss in the stellar winds of stars more massive than $\sim 40 M_\odot$.

### 5. Conclusions

We discussed in detail the formation and evolution of binaries which contain two neutron stars or black holes. The orbital parameters of present-day population in the Galaxy are studied. For a successful model we require that also the results of the preceding evolutionary stages (occurrence rates for supernovae and the formation of single pulsars, number of Be and high-mass X-ray binaries) comply with observations. As an additional restriction we require the observed high-mass binary pulsars to fit into the modeled $P_{\text{orb}} - c$ diagram for the appropriate age of the population. We summarize our conclusions:

1. Deposition of orbital energy into common envelopes has to be very efficient for in-spiraling neutron stars, and sources other than orbital energy have to be invoked.
2. A kick velocity distribution with a high velocity tail and the bulk in small velocities (as given by Paczyński 1990 with the parameters from Hansen & Phinney 1996) gives a model which is in agreement with the observations. The same velocity distribution
provides a satisfactory model for single pulsars (Hartman 1997). Increasing the fraction of low velocities may provide an even better fit to the observations.

3. The best fit between the observations and the models is at an age of the population between several 100 Myr and 1 Gyr. This suggests that the majority of the recycled neutron stars are born with low magnetic fields.

4. In the model of the population of \((ns, ns)\) binaries which we consider satisfactory, their birthrate is \(\sim 3.4 \times 10^{-5}\) yr\(^{-1}\) (assuming 100% binarity). This is consistent with the birthrate derived from observed binary pulsars. The merger rate of \((ns, ns)\) binaries in this model is \(\sim 2 \times 10^{-5}\) yr\(^{-1}\). The merger rate of \((bh, ns)\) binaries is smaller by at least an order of magnitude, but they may contribute significantly to the rate of expected gravitational wave detections because of the higher total mass of the systems.

5. Binary neutron stars merge typically within 1 to 2 Gyr after formation. Hence, elliptical galaxies and the old stellar population in spiral and irregular galaxies may not contribute significantly to the rate of gravitational waves detections. When the detectors become sensitive to the events at cosmological distances this population may give an important contribution.

6. The rate of \((ns, ns)\) mergers is consistent with the estimated rate of \(\gamma\)-ray bursts if the latter are beamed into an opening angle of \(\sim 10^{\circ}\).

7. The scenario for the evolution of close binary stars which assumes hypercritical accretion and the subsequent collapse of the neutron star into a black hole in the common envelope is able to reproduce binary neutron stars with large orbital periods (\(\sim 10\) days), but fails to reproduce the short period systems.

8. In the scenario in which a neutron star accretes hypercritically and the binary avoids merging, the Galactic population of binaries which contain a black hole and a neutron star might well exceed the number of double neutron stars. The merger rate of these binaries is then approximately \(3 \times 10^{-5}\) yr\(^{-1}\).

9. The number of observed single radio pulsars has to increase to about 2000, before one can hope to find a young radio pulsar accompanied by a black hole.

10. A considerable fraction of the binary neutron stars escape from the tidal field of the Galaxy and travel up to \(\sim\) Mpc distances before they coalesce.

11. Short period neutron star binaries are, due to the high velocities received upon the supernova, virtually all ejected from the shallow potential well of globular star clusters. High-mass binary pulsars in globular clusters are therefore likely to be formed by a dynamical interaction.

Acknowledgements. We thank Dipankar Bhattacharya, Gerry Brown, Ene Ergma, Hanno Spreeuw, Alexander Tutukov, Ed van den Heuvel and Frank Verbunt for numerous stimulating discussions. Ed van den Heuvel is acknowledged for critically reading the manuscript. SPZ acknowledges Ramesh Narayan for the suggestion to use the entire \(P_{orb} - e\) plane for a parametric study of the \((ns, ns)\) population. LRY acknowledges support through the NWO Spinoza grant and the warm hospitality of the Astronomical Institute “Anton Pannekoek” and Meudon observatory. This work was supported by NWO Spinoza grant 08-0 to E. P. J. van den Heuvel and RFBR grant 960216351.

References

Abt, H. A., 1983, ARA&A 21, 343

Bagab, A., Ferguson, H. C., 1996, ApJ, 458, 100

Bagot, P., 1996, Ph. D. Thesis, University of Montpellier II.

Bagot, P., 1997, A&A, 322, 533

Bailes, M., 1996, in Compact stars in binaries, ed. J. van Paradijs, E. P. J. van den Heuvel, E. Kuulkers (Dordrecht: Kluwer), p. 213

Bhattacharya, D., 1996, in Compact stars in binaries, eds. J. van Paradijs, E. P. J. van den Heuvel, E. Kuulkers (Dordrecht: Kluwer), p. 243

Brandt, N., Podsiadlowski, P., 1995, MNRAS 247, 484

Brown, G., 1995, ApJ, 440, 270

Brown, G., Weingartner, J. C., Wijers, R. A. M. J. 1996, ApJ, 463, 297

Clark, J. P. A., Eardley, B., 1977, ApJ, 215, 311

Clark, J. P. A., van den Heuvel, E. P. J., Sutantio, W., 1979, A&A, 72, 120

Cappellaro, E., Turatto, M., Tsvetkov, D. Yu., et al. 1997, A&A, 322, 431

Carraro, G., Chiosi, G., 1994, A&A, 287, 761

Chevalier, R. 1993, ApJ, 411, L33

Connolly A.J. et al. 1997, ApJ 486, L11

Curran, S. J., Lorimer, D. R., 1995, MNRAS, 276, 347

Davies, M. B., Benz, W. Piran, T., Thielemann, F. K., 1994, ApJ, 431, 742

Deich, W. T. S., Kulkarni, S. R., 1996, in Compact stars in binaries, ed. J. van Paradijs, E. P. J. van den Heuvel, E. Kuulkers (Dordrecht: Kluwer), p. 279

De Loore, C.W.E., De Greve, J.P., van den Heuvel, E.P.J., De Cuyper, J.P., 1975, Mem. Soc. Astron. It., 45, 893

Deshpande, A. A., Ramachandran, R., Srinivasan, G., 1995, JIA&A, 16, 53

Dewey, R. J., Cordes, J. M., 1987, ApJ, 321, 780

Duchene, M. A., Mayor, M., 1991, A&A, 248, 485

Egma, E., van den Heuvel, E. P. J., 1997, A&A, submitted

Egma, E., Yungelson, L. R., 1997, A&A, in press

Flannery, B. P., van den Heuvel, E. P. J., 1975, A&A, 39, 61

Fryer, C. L., 1997, A&A, 459, 244

Fryer, C. L., Benz, W., Herant, M., 1996, ApJ, 460, 801

Gallagher, J., Hunter, D., Tutukov, A. V., 1984, ApJ, 284, 544

Gunn, J., Ostriker, J., 1970, ApJ, 160, 979

Habets, G., 1985, PhD Thesis, U. Amsterdam

Hansen, B. M. S., Phinney, E. S., 1996, BAAS, 189, 7402

Hansen, B. M. S., Phinney, E. S., 1997, astro-ph/9708071

Hartman, J. W., Bhattacharya, D., Wijers, R. A. M. J., Verbunt, F. 1997a, A&A, 322, 477

Hartman, J. W., 1997, A&A, 322, 127
Fig. 2. The probability distribution in model B for the present day orbital parameters of the Galactic disc neutron star binaries younger than 100 Myr (upper panel), 1 Gyr (middle panel) and 10 Gyr (lower panel). The gray scaling in this graph represents numbers in the Galaxy. The darkest shades in the upper, middle, and lower panels correspond to a total of Galactic 23, 110, and 1100 \((ns, ns)\) binaries with given combination of \(P_{\text{orb}}\) and \(e\). The “•” symbols in the upper panel represent the observed \((ns, ns)\) binaries, the “⋆” symbols mark the position of the a system which might originate from a dynamical encounter (the left “⋆”) and a system of which one companion is possibly not a neutron star (see Table 2).

Fig. 3. The probability distribution for the number of Galactic \((ns, ns)\) binaries younger than 100 Myr. The kicks are selected from a Maxwellian velocity distribution (model C in Table 1). The gray scaling is given in the upper left corner and is chosen to be the same as for the upper panel in Fig. 2.

Fig. 4. The probability distribution for model D (\(\eta_{\text{ke}}\lambda = 0.5\), and kick according to Eq. 5) at the age of the population younger than 100 Myr. The gray scaling, given in the upper left corner, is chosen independently of the other models and gives 8.9 \((ns, ns)\) binaries in the Galaxy for the darkest shade.
Fig. 5. The probability distribution for the Galactic number of \((ns, ns)\) binaries younger than 100 Myr (upper panel) and 10 Gyr (lower panel) in model H. The normalization gives 6.3 \((ns, ns)\) for the darkest shades in the upper panel and 522 for the lower panel.

Fig. 6. The cumulative distribution of the center-of-mass velocities for neutron star binaries which merge within 10 Gyr from models A (dash-dotted line to the right), B (solid line) and C (dashed line). For model A, for comparison, the line to the left represents the velocity distribution of all \((ns, ns)\) binaries and includes a rather large population of systems which do not merge within 10 Gyr. The dotted line represents the input kick-velocity distribution for model B (see Eq. 5).

Fig. 7. **Upper panel** - Normalized distribution of \((ns, ns)\) binaries over merger ages in model B. The limit of 10 Gyr is adopted as the age of the galactic disc. **Lower panel** - The cumulative distribution of the \((ns, ns)\) binaries over merger ages for the models A (dash dotted line), B (solid line) and C (dashed line). The fraction of mergers reached within \(10^{10}\) yrs indicates that a considerable fraction of systems does not merge within a Hubble time.

Fig. 8. The cumulative distribution of \((ns, ns)\) binaries over the distance traveled before coalescence occurs. The line styles represent the results from the same model computations as denoted in Fig. 7.
Fig. 9. The cumulative distribution of (ns, ns) binaries over the galactic height z reached before coalescence occurs. The galactocentric distance at which the merger occurs is computed using Eq. 7. The line styles are as in Fig. 6. The subset of (ns, ns) binaries from model A which merge within 10 Gyr (right dash-dotted line) reach higher z-values than those with orbital period so large that they do not merge within the age of the Universe (left dash-dotted line).