Branching ratios and CP asymmetries in charmless nonleptonic $B$ decays to radially excited mesons

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Nonleptonic two body $B$ decays including radially excited $\pi(1300)$ or $\rho(1450)$ mesons in the final state are studied using the framework of generalized naive factorization approach. Branching ratios and CP asymmetries of $B \to P\pi(1300)$, $B \to V\pi(1300)$, $B \to P\rho(1450)$ and $B \to V\rho(1450)$ decays are calculated, where $P$ and $V$ stand for pseudoscalar and vector charmless mesons. Form factors for $B \to \pi(1300)$ and $B \to \rho(1450)$ transitions are estimated in the improved version of the Isgur-Scora-Grinstein-Wise quark model. In some processes, CP asymmetries of more than 10% and branching ratios of $10^{-5}$ order are found, which could be reached in experiments.

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I. INTRODUCTION

Most of the research on nonleptonic two body $B$ decays have concentrated in processes where the final mesons are ground states or angular orbital excitations [1]. Radial excitations can be produced in $B$ decays. Decays involving radially excited mesons could be an alternative to those more traditionally studied, given additional and complementary information.

The physics involved in nonleptonic two body $B$ decays allows to study the interplay of QCD and electroweak interactions, to search for CP violation, and over constrain Cabibbo-Kobayashi-Maskawa (CKM) parameters in a precision test of the standard model [1, 2].

In the quark model, mesons are $qq'$ bound states of quark $q$ and antiquark $\bar{q}'$. The $qq'$ state has an orbital angular momentum $l$ and spin $J$. Pseudoscalar and vector mesons have orbital angular momentum $l = 0$. The angular orbital excitations: scalar, axial vector and tensor mesons have $l = 1$. Mesons can be classified in spectroscopy notation by $n^{2s+1}l_J$, where $s = 0$ or 1 for parallel or antiparallel quarks $q$ and $\bar{q}'$, respectively. Radial excitations are denoted by the principal quantum number $n$.

The radial excitations $\pi(1300)$ and $\rho(1450)$, with principal quantum number $n = 2$, $u$ and $d$ quark content, can be produced in nonleptonic two body $B$ decays. In spectroscopy notation $\pi(1300)$ is denoted by $2^1S_0$ and $\rho(1450)$ by $2^3S_1$. To simplify, we denote $\pi(1300)$ by $\pi'$ and $\rho(1450)$ by $\rho'$.

In Ref. [2], the authors interested in factorization breaking effects in $B$ decays, consider $B$ decays to final states with small decay constants, such as $B^0 \to D^+\pi^-$ and $\bar{B}^0 \to D^+\pi^-$ decays.

Production of charmless radially excited vector mesons in nonleptonic two body $B$ decays is considered in Ref. [4]. The authors make a prediction for the ratio $Br(B \to \rho'\pi)/Br(B \to \rho\pi)$. This ratio is given in terms of the form factor $A_0$, which is calculated in a constituent quark model [2]. We compare our calculations with their result and experimental data available.

In this paper, we present a study on the exclusive modes $B \to P\pi'$, $B \to V\pi'$, $B \to P\rho'$ and $B \to V\rho'$, where $P$ and $V$ are the pseudoscalar and vector mesons, $\pi$, $\eta$, $\eta'$ and $K$ and $\rho$, $\omega$, $K^*$ and $\phi$, respectively. We compute branching ratios of these processes using the effective weak Hamiltonian, with tree and penguin contributions. Matrix elements are calculated in the generalized naive factorization approach [2, 4]. The form factors for $B \to P$ and $B \to V$ transitions are calculated in the Bauer-Stech-Wirbel (WSB) model [7] and Light-Cone-Sum-Rule (LCSR) approach [8]. Form factors for $B \to \pi'$ and $B \to \rho'$ transitions are calculated in the improved version of the Isgur-Scora-Grinstein-Wise (ISGW) quark model, called ISGW2 model [4, 10].

We also calculate CP violating asymmetries in the framework of generalized naive factorization approach [11]. CP asymmetries allow to determine interior angles of the unitary triangle and test the unitarity of the CKM matrix. Specifically, in this work, we calculate direct CP violation for charged $B^\pm$ decays and CP asymmetries for neutral $B^0(\bar{B}^0)$ decays. For some channels asymmetries of order 10% are found.

The decay constants $f_{\pi'}$ and $f_{\rho'}$ are not well determined input parameters. The range of values for the $f_{\pi'}$ decay constant obtained from different methods and its impact in channels $B \to P\pi'$ and $B \to V\pi'$ are discussed in this paper.

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work. Some branching ratios are sensitive to the decay constants \( f_{q^0} \) and \( f_{q^+} \). This fact will allow to determine decay constants by experiment in cases where branching ratios are measured.

This paper is organized as follows. In Sec. II, we present the framework used to calculate branching ratios of nonleptonic two body \( B \) decays, effective Hamiltonian and the generalized naive factorization approach. Input parameters, mixing schemes, decay constants, and form factors are discussed in Sec. III. In Sec. IV, we discuss the amplitudes involving radially excited mesons and calculate numerical results for branching ratios. CP violating asymmetries for charged and neutral channels are presented in Sec. V. Our conclusions are given in Sec. VI. In the appendices, we give the amplitudes for \( B \to P\pi', V\pi', P\rho' \) and \( V\rho' \) processes.

II. FRAMEWORK

A. Effective Hamiltonian

The framework to study \( B \) decays is the effective weak Hamiltonian \cite{12}. For \( \Delta B = 1 \) transitions, it is written as

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ \sum_{j=u,c} V_{ij}^* V_{jk} (C_1(\mu)O_1^j(\mu) + C_2(\mu)O_2^j(\mu)) - V_{ib}^* V_{ic} \left( \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \right) \right] + \text{H.c.,}
\]

where \( G_F \) is the Fermi constant, \( C_i(\mu) \) are Wilson coefficients at the renormalization scale \( \mu \), \( O_i(\mu) \) are local operators and \( V_{ij} \) are the respective CKM matrix elements involved in the transitions. The local operators for \( b \to q \) transitions are

\[
\begin{align*}
O_1^j & = \bar{q}_\alpha \gamma^\mu L u_\alpha \cdot \bar{u}_\beta \gamma_\mu L b_\beta \\
O_2^j & = \bar{q}_\alpha \gamma^\mu L u_\beta \cdot \bar{u}_\beta \gamma_\mu L b_\alpha \\
O_{3(5)} & = \bar{q}_\alpha \gamma^\mu L b_\alpha \cdot \sum_q q'_\beta \gamma_\mu L(R) q'_\beta \\
O_{4(6)} & = \bar{q}_\alpha \gamma^\mu L b_\beta \cdot \sum_q q'_\beta \gamma_\mu L(R) q'_\alpha \\
O_{7(9)} & = \frac{3}{2} \bar{q}_\alpha \gamma^\mu L b_\beta \cdot \sum_{q'} e_{q'} q'_\beta \gamma_\mu R(L) q'_\beta \\
O_{8(10)} & = \frac{3}{2} \bar{q}_\alpha \gamma^\mu L b_\beta \cdot \sum_{q'} e_{q'} q'_\beta \gamma_\mu R(L) q'_\alpha,
\end{align*}
\]

where \( q = d \) or \( s \), \( O_1^j \) and \( O_2^j \) are the current-current operators \((j = u, c)\), \( O_3 - O_{10} \) the QCD penguins operators and \( O_3 - O_{10} \) the electroweak penguins operators. The indexes \( \alpha \) and \( \beta \) mean \( SU(3) \) color degrees, \( L \) and \( R \) are the left and right projector operators, respectively. The sum extends over active quarks \( u, d, s \) and \( c \) at the scale of \( B \) meson \( \mu = O(m_b) \).

In order to calculate the branching ratios and CP asymmetries in this work, we use the next to leading order Wilson coefficients for \( \Delta B = 1 \) transitions obtained in the naive dimensional regularization scheme (NDR) at the energy scale \( \mu = m_b \), \( \Lambda_{\text{QCD}}^{(5)} = 225 \text{ MeV} \) and quark top mass \( m_t = 170 \text{ GeV} \). These coefficients are taken from Ref. \cite{12}, see Table XXII. Those values are \( c_1 = 1.082, c_2 = -0.185, c_3 = 0.014, c_4 = -0.035, c_5 = 0.009, c_6 = -0.041, c_7/\alpha = -0.002, c_8/\alpha = 0.054, c_9/\alpha = -1.292 \) and \( c_{10}/\alpha = 0.263 \), where \( \alpha = 1/137 \) is the fine structure constant.

B. Generalized naive factorization approach

The decay amplitude of a nonleptonic two body \( B \) decay can be calculated using the effective weak Hamiltonian by

\[
\mathcal{M}(B \to M_1 M_2) = \langle M_1 M_2 | H_{\text{eff}} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} C_i(\mu) | O_i(\mu) \rangle,
\]

(3)
where the hadronic matrix elements $\langle O_i(\mu) \rangle$ are defined by $\langle M_1 M_2 | O_i(\mu) | B \rangle$ and $M_i$ are final state mesons. In the naive factorization hypothesis, hadronic matrix elements $\langle O_i(\mu) \rangle$ are evaluated by the product of decay constants and form factors. These matrix elements are energy $\mu$ scale and renormalization scheme independent, consequently there is no term to cancel the energy $\mu$ dependency in the Wilson coefficients, and the amplitudes for nonleptonic two body $B$ decays are scale and renormalization scheme dependent.

The improved naive factorization approach is formulated to solve the problem of energy scale dependency by including some perturbative QCD contributions in Wilson coefficients. This is considered in order to isolate the Wilson coefficients $c_i^{\text{ eff}}$, which are scale $\mu$ independent. Schematically

$$\sum_i C_i(\mu) \langle O_i(\mu) \rangle = \sum_i C_i(\mu) g_i(\mu) \langle O_i \rangle_{\text{tree}} = \sum_i c_i^{\text{ eff}} \langle O_i \rangle_{\text{tree}},$$

where $g_i(\mu)$ are perturbative QCD corrections to Wilson coefficients and $\langle O_i \rangle_{\text{tree}}$ are tree level hadronic matrix elements. Explicit expressions for the effective Wilson coefficients $c_i^{\text{ eff}}$ are given in Refs. These coefficients are recalculated with the current CKM parameters. Effective Wilson coefficients, for transitions $b \to d$ and $b \to s$, are shown in Table I. They are evaluated at the factorisable scale $\mu = m_b$, with an averaged momentum transfer of $k^2 = m_b^2/2$, and using the central values for CKM parameters from Ref.

| $c_i^{\text{ eff}}$ | $b \to d$ | $b \to s$ |
|---------------------|-----------|-----------|
| $c_1^{\text{ eff}}$ | 1.1680    | 1.1680    |
| $c_2^{\text{ eff}}$ | -0.3652   | -0.3652   |
| $c_3^{\text{ eff}}$ | 0.0231 + i 0.0038 | 0.0233 + i 0.0043 |
| $c_4^{\text{ eff}}$ | -0.0477 - i 0.0113 | -0.0482 - i 0.0129 |
| $c_5^{\text{ eff}}$ | 0.0139 + i 0.0038 | 0.0140 + i 0.0043 |
| $c_6^{\text{ eff}}$ | -0.0499 - i 0.0113 | -0.0503 - i 0.0129 |
| $c_7^{\text{ eff}}/\alpha$ | -0.0303 - i 0.0326 | -0.0311 - i 0.0356 |
| $c_8^{\text{ eff}}/\alpha$ | 0.0551 | 0.0551 |
| $c_9^{\text{ eff}}/\alpha$ | -1.4268 - i 0.0326 | -1.4276 - i 0.0356 |
| $c_{10}^{\text{ eff}}/\alpha$ | 0.4804 | 0.4804 |

In the factorisable decay amplitude, the effective Wilson coefficients appear as linear combinations. Thus, to simplify decay amplitudes, $a_i$ coefficients are introduced

$$a_i \equiv c_i^{\text{ eff}} + \frac{1}{N_c} c_{i+1}^{\text{ eff}} \ (i = \text{odd}),$$

$$a_i \equiv c_i^{\text{ eff}} + \frac{1}{N_c} c_{i-1}^{\text{ eff}} \ (i = \text{even}),$$

where index $i$ runs over 1, ..., 10 and $N_c = 3$ is the color number of QCD. Effective coefficients $a_i$ for $b \to d$ and $b \to s$ transitions are shown in Table II.

## III. INPUT PARAMETERS AND FORM FACTORS

### A. Input parameters

The CKM matrix is parametrized in terms of Wolfenstein parameters $\lambda$, $A$, $\bar{\rho}$ and $\bar{\eta}$,

$$
\begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A\lambda^3 (\rho - i\eta) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 & A\lambda^2 \\
A\lambda^3 (1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1
\end{pmatrix},
$$

(6)
Ideal mixing is considered for ω rules in Ref. [21], the authors estimate the decay constant \( f = 0 \) and \( f \) constant are used. In order to calculate the chiral factor, which multiply the matrix elements of penguin terms, \( \eta \) and \( B \) are evaluated, the correct chiral behavior must be ensured. Thus, these matrix elements are multiplied by the factor \( \frac{1}{\langle 135 \rangle} \) and the values \( m_{\eta} = 4 \) MeV and \( m_{B} = 2257 \) MeV are used in calculations, see Ref. [16].

Since the WSB model [7] and LCSR approach [8] are used to calculate for branching ratios, form factors for \( B \rightarrow P \) and \( B \rightarrow V \) transitions are calculated using the ISGW2 quark model [10].

### B. Form factors

The WSB model [7] and LCSR approach [8] are used to calculate form factors for \( B \rightarrow P \) and \( B \rightarrow V \) transitions. Since the WSB model and LCSR approach provide form factors only for the above transitions, form factors for \( B \rightarrow \pi' \) and \( B \rightarrow \rho' \) transitions are calculated using the ISGW2 quark model [10].

| \( a_i \) | \( b \to d \) | \( b \to s \) |
|---|---|---|
| 1 | 0.046 | 0.046 |
| 2 | 0.024 | 0.024 |
| 3 | 72 | 72 |
| 4 | -400 - i 114 | -404 - i 114 |
| 5 | -28 | -28 |
| 6 | -453 - i 114 | -457 - i 114 |
| 7 | -0.87 - i 2.60 | -0.93 - i 2.60 |
| 8 | 3.26 - i 0.87 | 3.26 - i 0.87 |
| 9 | -92.5 - i 2.60 | -92.5 - i 2.60 |
| 10 | 0.35 - i 0.87 | 0.33 - i 0.87 |

### TABLE II. Effective coefficients for \( b \to d \) and \( b \to s \) transitions (in units of \( 10^{-4} \) for \( a_3, ..., a_{10} \)).
The transitions $B \to P$ and $B \to V$ can be written in terms of form factors by the following expressions

$$
\langle P(pp)|V_\mu|B(pb)\rangle \equiv \left[ (p_B + pp)_\mu - \frac{m_B^2 - m_P^2}{q^2} q_\mu \right] F_1(q^2) + \left[ \frac{m_B^2 - m_P^2}{q^2} \right] q_\mu F_0(q^2)
$$

and

$$
\langle V(p_V, \epsilon)|(V_\mu - A_\mu)|B(pb)\rangle \equiv -\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\alpha\beta}p_B^\mu p_V^\nu \frac{2V(q^2)}{(m_B + m_V)} - i \left[ (\epsilon_\mu - \epsilon^* \cdot q \frac{q_\mu}{q^2}) (m_B + m_V)A_1(q^2) \right.

- \left. \left( (p_B + p_V)_\mu - \frac{(m_B^2 - m_V^2)}{q^2} q_\mu \right) \frac{2m_V(\epsilon^* \cdot q)}{q^2} q_\mu A_0(q^2) \right]
$$

where $q = (p_B - pp)$ or $q = (p_B - p_V)$ and $\epsilon$ is the polarization of the vector meson $V$. The following restrictions are imposed over form factors in order to cancel poles at $q^2 = 0$

$$
F_1(0) = F_0(0),
2m_V A_0(0) = (m_B + m_V)A_1(0) - (m_B - m_V)A_2(0).
$$

In the case of the WSB model a single pole dominance model is used for the $q^2$ momentum squared dependency

$$
f(q^2) = \frac{f(0)}{(1 - q^2/m_0^2)},
$$

where $m_0^2$ is the pole mass given by the vector meson and $f(0)$ is the form factor at zero momentum transfer.

The WSB model is a relativistic constituent quark model where the meson-meson matrix elements are evaluated from the average integral corresponding to meson functions, which are solutions of a relativistic harmonic oscillator potential [7].

The LCSR approach uses the method of QCD sum rules on the light cone [8]. The second set of parameters are used in the calculations, see [8]. A fit parametrization is utilized for the $q^2$ dependency of the form factors.

In Table III, values for form factors involved in transitions $B \to P$ and $B \to V$ are shown, evaluated at zero momentum transfer, in the WSB quark model [7] and LCSR approach [8]. Form factor in $B \to \pi$ transition, evaluated in the LCSR approach, is small compared to the WSB model. This result is in accordance with current experimental data [12]. The pseudoscalar meson $\eta'$ is too heavy to be treated in the LCSR approach, its value is not reported. In general, the form factors calculated in LCSR approach are smaller than those calculated in the WSB model, for this reason the branching ratios are also smaller.

| Transition | $F_1 = F_0$ | $V$ | $A_1$ | $A_2$ | $A_0$ |
|------------|--------------|-----|------|------|------|
| $B \to \pi$ | 0.333 [0.258] |     |      |      |      |
| $B \to K$  | 0.379 [0.331] |     |      |      |      |
| $B \to \eta$ | 0.168 [0.275] |     |      |      |      |
| $B \to \eta'$ | 0.114 [-] |     |      |      |      |
| $B \to \rho$ | 0.329 [0.323] | 0.283 [0.242] | 0.283 [0.221] | 0.281 [0.303] |
| $B \to \omega$ | 0.232 [0.311] | 0.199 [0.233] | 0.199 [0.181] | 0.198 [0.363] |
| $B \to K^*$ | 0.369 [0.293] | 0.328 [0.219] | 0.331 [0.198] | 0.321 [0.281] |

The improved version of the ISGW model [9], the so called ISGW2 model [10], a non relativistic quark model is used in this work. Even tough, in the ISGW model is possible to calculate transitions to a radially excited pseudoscalar and vector mesons, the ISGW2 model is better because the constrains imposed by heavy quark symmetry, hyperfine distortions of wave functions, and form factor more realistic at high recoil momentum transfer. These additional features incorporated in the ISGW2 model allow us to make more reliable estimations.

In the ISGW2 model, $B \to P'$ transition is written as
\begin{equation}
\langle P(p_P)|V_\mu|B(p_B)\rangle = f^\prime_+ (q^2)(p_B + p_P)_\mu + f^\prime_- (q^2)(p_B - p_P)_\mu
\end{equation}

and matrix elements of vector and axial vector currents for $B \to V'$ transition are written as

\begin{equation}
\langle V(p_V, \epsilon)|V_\mu|B(p_B)\rangle = ig'(q^2)\epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu\nu}(p_B + p_V)^\alpha (p_B - p_V)^\beta
\end{equation}

and

\begin{equation}
\langle V(p_V, \epsilon)|A_\mu|B(p_B)\rangle = f'_-(q^2)\epsilon'_\mu + a'_+(q^2)(\epsilon^\ast \cdot p_B)(p_B + p_V)_\mu + a'_-(q^2)(\epsilon^\ast \cdot p_B)(p_B - p_V)_\mu,
\end{equation}

where $q = (p_B - p_P)$ or $(p_B - p_V)$ in the respective case.

Form factors in ISGW2 model are related to form factors in WSB model by the following relations

\begin{align*}
F_1(q^2) &= f'_+(q^2), \\
V(q^2) &= (m_B + m_V) g'(q^2), \\
A_1(q^2) &= (m_B + m_V)^{-1} f'(q^2), \\
A_2(q^2) &= -(m_B + m_V) a'_+(q^2), \\
A_0(q^2) &= \frac{1}{2m_V} [f'(q^2) + (m_B^2 - m_V^2) a'_+(q^2), + q^2 a'_-(q^2)].
\end{align*}

The form factors for $B \to \pi'$ and $B \to \rho'$ transitions, calculated at momentum transfer $q^2 = m_\pi^2$, are presented in Table IV. To estimate branching ratios, it is necessary to calculate form factors at different momentum transfers, namely at $q^2 = m_K^2, m_{\eta}^2, m_{\eta}'^2, m_{\eta}^{'2}, m_{K}^{'2}$, and $m_{\pi}^{'2}$.

| Transition | $F_1/F_0$ | $V$ | $A_1$ | $A_2$ | $A_0$ |
|-----------|-----------|----|------|------|------|
| $B \to \pi'$ | 0.25 | | | | |
| $B \to \rho'$ | 0.456 | 0.118 | -0.118 | 0.397 |

Mixing $\eta - \eta'$ effect is not included in the WSB model prediction. $SU(3)$ symmetry is used to consider mixing in $B \to \eta$ and $B \to \eta'$ transitions, which imply the relations $F^{B\pi}(0) = \sqrt{3} F^{B\eta}(0) = \sqrt{6} F^{B\eta'}(0)$ and

\begin{align*}
F^{B\eta} &= F^{B\eta'} \cos \theta - F^{B\eta} \sin \theta, \\
F^{B\eta'} &= F^{B\eta'} \sin \theta + F^{B\eta} \cos \theta,
\end{align*}

where $\theta = 15.4^\circ$ is the mixing angle \cite{23}. Using $F^{B\pi}(0) = 0.333$ from the WSB model, form factors $F^{B\eta}(0) = 0.181$ and $F^{B\eta'}(0) = 0.148$ are obtained. In the LCSR approach, using the form factor $F^{B\pi}(t)$ and $SU(3)$ symmetry, the form factors $F^{B\eta}(t)$ and $F^{B\eta'}(t)$ are estimated.

The $\rho^0 - \omega$ mixing is introduced in hadronic matrix element $B \to \rho^0$. Nevertheless, the effect in $B \to \omega$ transitions is negligible and it is not included in branching ratios calculations. In the limit of isospin symmetry, physical states $\rho^0$ and $\omega$ are expressed in terms of isospin eigenstates $\rho^I$ and $\omega^I$ by a rotation matrix

\begin{align*}
|\rho^0\rangle &= |\rho^I\rangle + \epsilon |\omega^I\rangle \\
|\omega\rangle &= |\omega^I\rangle - \epsilon' |\rho^I\rangle,
\end{align*}

where numerical values for mixing parameters are $(1 + \epsilon) = (0.092 + 0.016i)$ and $(1 - \epsilon') = (1.011 + 0.030i)$. Including isospin effects, hadronic matrix element for $B \to \rho^0$ transition is modified by the factor $(1 + \epsilon)$, see Ref. \cite{25}.
IV. AMPLITUDES AND BRANCHING RATIOS

The amplitudes for processes studied in this work are explicitly written in the Appendices. These amplitudes are given in terms of decay constants and form factors, and contain all the contributions of the effective weak Hamiltonian.

Appendix A contains the amplitudes for \( B \to P \pi' \) decays, where \( P \) is the pseudoscalar meson \( \pi, \eta, \eta' \) or \( K \). The amplitude for the process \( B^0 \to \pi^- \pi^0 \) is written, but it can be obtained directly from \( B^0 \to \pi^+ \pi'^+ \), interchanging \( \pi \) by \( \pi' \). Similarly, the amplitude for \( B^- \to \pi^- \pi^0 \) can be obtained from \( B^- \to \pi^+ \pi'^+ \).

To compare \( B \to P \pi' \) with \( B \to P \pi \) modes, besides obvious differences in decay constants and form factors, a point to remark is the following one. In the penguin sector the more important contributions come from terms \( a_6 \) and \( a_8 \). Particularly, the \( a_6 \) and \( a_8 \) coefficients are enhanced by a chiral factor, which is proportional to the squared mass of pseudoscalar \( P \) or pseudoscalar radial excitation \( \pi' \). In the case of radial excitation \( \pi' \), this contribution can be two orders of magnitude bigger than contribution of the pseudoscalar meson \( \pi \). This enhancement effect is shown in the branching ratios of channels \( B \to \pi \pi' \), \( B \to \eta \pi' \) and \( B \to \eta' \pi' \).

In Appendix B, the amplitudes for \( B \to V \pi' \) processes are shown, where \( V \) is the vector meson \( \rho, \omega, K^* \) or \( \phi \). In these modes, the increased factor in the \( \alpha \) orders of magnitude bigger than contribution of the pseudoscalar meson \( \pi \). The branching ratios listed in Tables V and VI are CP averaged conjugate modes. Charged and neutral channels are shown, where \( \nu \) is the vector meson \( \rho, \omega, K^* \) or \( \phi \). From decay amplitude and input parameters, branching ratios are straightforward calculated. The decay rate for processes \( B \to P \pi' \) is given by

\[
\Gamma(B \to P \pi') = \frac{\lambda^{1/2}(m_B^2, m_P^2, m_{\pi'}^2)}{16\pi m_B^4} |M(B \to P \pi')|^2,
\]

where \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz) \). The decay rate for processes \( B \to V \pi' \) and \( B \to P \rho' \) are calculated using Eq. (18). In this case, the squared amplitude is proportional to \( |\epsilon_V \cdot p_{\pi'}|^2 \) and \( |\epsilon_{\rho'} \cdot p_P|^2 \), respectively. In processes \( B \to V \rho' \), the squared amplitude is involved due to interfering terms proportional to \( X_{B\rho',V} \) and \( X_{B\rho',V'} \) contributions.

The branching ratios listed in Tables V and VI, are CP averaged conjugate modes. Charged and neutral channels are calculated by

\[
\frac{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)}{2}, \quad \frac{\Gamma(B^0 \to f) + \Gamma(B^0 \to \bar{f})}{2},
\]

where \( f \) is the two body meson final state, i.e. \( f = P \pi', V \pi', P \rho' \) or \( V \rho' \).

Form factors in transitions \( B \to P \) and \( B \to V \) are calculated using two representative methods, the WSB quark model \( 7 \) and LCSR approach \( 8 \). The branching ratios calculated in LCSR approach are listed between squared brackets in Tables V and VI. In transitions \( B \to \pi' \) and \( B \to \rho' \) the improved version of the ISGW non relativistic quark model \( 10 \) is used.

In Table V, branching ratios for processes \( B \to P \pi' \) and \( B \to V \pi' \) are shown, where \( P \) is the pseudoscalar meson \( \pi, \eta, \eta' \) or \( K \) and \( V \) is the vector meson \( \rho, \omega, K^* \) or \( \phi \). Branching ratios are calculated using two different values in decay constant \( f_{\pi'} = 26 \) MeV (first and second column) and \( f_{\pi'} = 0 \) MeV (third column).

In Table V, channels with \( K, K^* \) or \( \phi \) in the final state, have equal branching ratio calculated in the WSB model or in the LCSR approach. The branching ratio in processes \( B \to \pi^+ \pi'^+ \) and \( B \to \rho^- \pi'^+ \) have independent value of the decay constant \( f_{\pi'} \), since decay amplitude has only one contribution proportional to \( f_{\pi} \) or \( f_{\rho} \), respectively.
Table V. Branching ratios (in units of $10^{-6}$) averaged over CP conjugate modes for $B \to P\pi'$ and $B \to V\pi'$ decays, using the WSB [7] model and LCSR approach [8], decay constants $f_{\rho'} = 26$ MeV and $f_{\rho'} = 0.0$ MeV.

| Mode                  | $B$     | Mode                  | $B$   |
|-----------------------|---------|-----------------------|-------|
| $B^0 \to \pi^+\pi^-$  | 5.8 [5.8] | $B^0 \to \rho^-\pi^+$ | 14.6 [14.6] |
| $B^0 \to \pi^-\pi^+$  | 4.14 [26.7] | $B^0 \to \rho^+\pi^-$ | 23.2 [30.2] |
| $B^0 \to \pi^0\pi^0$  | 6.7 [4.5] 0.03 | $B^0 \to \rho^0\pi^0$ | 2.8 [3.8] 0.02 |
| $B^- \to \pi^-\pi^0$  | 6.0 [6.0] 3.0 | $B^- \to \rho^-\pi^-$ | 13.8 [17.7] 0.04 |
| $B^- \to \pi^-\pi^+$  | 0.47 [0.29] 0.005 | $B^- \to \rho^-\pi^0$ | 14.1 [16.1] 7.8 |
| $B^0 \to \eta\pi^0$   | 4.2 [2.9] 0.05 | $B^0 \to \omega\pi^0$ | 7.1 [14.8] 0.05 |
| $B^- \to \eta\pi^-$   | 15.0 [10.1] 0.1 | $B^- \to \omega\pi^-$ | 7.6 [24.1] 0.1 |
| $B^0 \to \eta'\pi^0$  | 2.3 [1.6] 0.007 | $B^0 \to K^{*-\pi^-}$ | 3.0 [3.0] 3.0 |
| $B^- \to \eta'\pi^-$  | 8.6 [5.7] 0.02 | $B^0 \to K^{*0}\pi^0$ | 0.75 [75.7] 0.9 |
| $B^0 \to K^-\pi^+$    | 6.5 [6.5] 6.5 | $B^- \to K^{*-\pi^-}$ | 1.8 [1] 1.6 |
| $B^0 \to K^0\pi^+$    | 4.0 [4.0] 3.7 | $B^- \to K^{*0}\pi^-$ | 3.6 [3.6] 3.6 |
| $B^- \to K^-\pi^0$    | 3.8 [3.8] 3.5 | $B^0 \to \phi\pi^0$ | 0.004 [0.004] 0.004 |
| $B^- \to K^0\pi^-$    | 7.9 [7.9] 7.9 | $B^- \to \phi\pi^-$ | 0.008 [0.008] 0.008 |

Table VI. Branching ratios (in units of $10^{-6}$) averaged over CP conjugate modes for $B \to P\rho'$ and $B \to V\rho'$ decays, using the WSB [7] model and LCSR approach [8], and decay constant $f_{\rho'} = 128$ MeV.

| Mode                  | $B$     | Mode                  | $B$  |
|-----------------------|---------|-----------------------|------|
| $B^0 \to \pi^+\rho^-$ | 10.4 [6.8] | $B^0 \to \rho^-\rho^+$ | 38.7 [38.7] |
| $B^0 \to \pi^-\rho^+$ | 13.2 [13.2] | $B^0 \to \rho^+\rho^-$ | 29.8 [22.5] |
| $B^0 \to \pi^0\rho^0$ | 0.02 [0.01] | $B^0 \to \rho^0\rho^0$ | 0.15 [0.13] |
| $B^- \to \pi^-\rho^-$ | 6.0 [4.0] | $B^- \to \rho^-\rho^-$ | 7.9 [6.4] |
| $B^- \to \pi^-\rho^+$ | 7.4 [7.4] | $B^- \to \rho^-\rho^+$ | 21.1 [21.1] |
| $B^0 \to \eta\rho^0$  | 0.008 [0.006] | $B^0 \to \omega\rho^0$ | 0.04 [0.03] |
| $B^- \to \eta\rho^+$  | 3.4 [2.3] | $B^- \to \omega\rho^+$ | 4.1 [6.0] |
| $B^0 \to \eta'\rho^0$ | 0.044 [0.039] | $B^0 \to K^{*-\rho^+}$ | 8.0 [8.0] |
| $B^- \to \eta'\rho^-$ | 2.1 [1.4] | $B^0 \to K^{*0}\rho^0$ | 6.6 [7.0] |
| $B^0 \to K^-\rho^+$   | 1.1 [1.1] | $B^- \to K^{*-\rho^0}$ | 6.2 [6.6] |
| $B^0 \to K^0\rho^0$   | 0.6 [0.1] | $B^- \to K^{*-\rho^0}$ | 9.5 [9.5] |
| $B^- \to K^-\rho^0$   | 0.7 [0.6] | $B^0 \to \phi\rho^0$ | 0.01 [0.01] |
| $B^- \to K^0\rho^-\rho^-$ | 0.013 [0.013] | $B^- \to \phi\rho^-$ | 0.02 [0.02] |

Using the value $f_{\rho'} = 26$ MeV in calculations, channels $B^0 \to \pi^-\pi^+$, $B^- \to \eta\pi^-$, $B^0 \to \rho^-\rho^+$, $\bar{B}^0 \to \rho^+\rho^-$ and $\bar{B}^0 \to K^-\rho^+$, have branching ratios of the order of $10^{-5}$. The channels with branching ratios of order below $10^{-6}$ are $B^- \to \pi^0\pi^-$, $B^0 \to K^{*0}\pi^0$, $B^0 \to \phi\pi^0$ and $B^- \to \phi\pi^-$. Numerical values for branching ratios of processes $B \to P\rho'$ and $B \to V\rho'$ are listed in Table VI. The branching ratios are calculated using the decay constant $f_{\rho'} = 128$ MeV. Branching ratio prediction for the decays $B^0 \to \pi^-\rho^+$, $B^0 \to \rho^-\rho^+$, $B^0 \to \rho^+\rho^-$, $B^0 \to K^-\rho^+$ are of order $10^{-5}$. The branching ratio of the channels $B^0 \to \pi^0\rho^0$, $B^0 \to \eta\rho^0$, $B^0 \to K^0\rho^0$, $B^- \to K^-\rho^0$ and $B^- \to K^0\rho^-$ are suppressed, of order below $10^{-6}$. The channels that include two vector mesons in final states $\bar{B}^0 \to \rho^0\rho^0$, $B^0 \to \omega\rho^0$, $B^0 \to \phi\rho^0$ and $B^- \to \phi\rho^-$, have also branching ratios of order below $10^{-6}$. The modes $B^0 \to \pi^+\rho^-$, $B^- \to \pi^0\rho^-$, $B^- \to \eta\rho^-$, $B^0 \to \rho^+\rho^-$ have different branching ratios when form factors are calculated using the WSB model or the LCSR approach.

The authors in Ref. [4] can calculate the ratios

$$R_{\rho^+} = \frac{Br(B^0 \to \pi^-\rho^+)}{Br(B^0 \to \pi^-\rho^0)}, \quad R_{\rho^0} = \frac{Br(B^- \to \pi^-\rho^0)}{Br(B^- \to \pi^-\rho^-)},$$

and obtain approximately $R_{\rho^+} \approx R_{\rho^0} \approx 2$. Using the world averaged experimental data from Ref. [13], $Br(B^0 \to \pi^-\rho^+) = 10.9 \pm 1.4 \times 10^{-6}$ and $Br(B^- \to \pi^-\rho^0) = 8.7 \pm 1.1 \times 10^{-6}$, it is possible to predict the branching ratios.
For the charged modes $B^± \to \rho^±$ and $B^± \to \omega \pi^0$, the form factors in the transitions $B^\pm \to \rho^0$ have direct CP violating asymmetries of order 10%. In the modes $B^0 \to K^\pm \pi^0$, $B^\pm \to \rho^\pm \pi^0$, $B^\pm \to K^\pm \pi^0$, and $B^\pm \to \phi \pi^\pm$, the CP violating asymmetry is equal to zero. Direct CP violating asymmetry in channels $B^\pm \to \pi^\pm \pi^0$ and $B^\pm \to \omega \pi^\pm$ depend on the use of WSB model or the LCSR approach.
In the channels $B^\pm \to \eta \pi^\pm$, $B^\pm \to K^\pm \pi^0$, and $B^\pm \to K^{*\pm} \pi^0$, direct CP violation asymmetries are not sensible to the value of decay constant $f_{\pi^\pm}$. When the value $f_{\pi^\pm} = 0$ MeV, direct CP violation asymmetry in channel $B^\pm \to \pi^\pm \pi^0$ is equal to zero. In the same case, modes $B^\pm \to \pi^0 \pi^\pm$, $B^\pm \to \eta \pi^\pm$, $B^\pm \to \rho^0 \pi^\pm$ and $B^\pm \to \omega \pi^\pm$ have an increase in the direct CP violation asymmetry. On the contrary, channel $B^\pm \to \rho^0 \pi^0$ has a decrease.

In Table VIII, direct CP violating asymmetries for the channels $B^\pm \to P^{\rho'}$, $B^\pm \to V^{\rho'}$ are shown, using the WSB model and LCSR approach, and the decay constant $f_{\rho'} = 128$ MeV.

Direct CP violating asymmetry corresponding to the channels $B^\pm \to K^\pm \rho^0$, $B^\pm \to \omega \rho^0$ and $B^\pm \to K^{*\pm} \rho^0$ are bigger than 10%, which make them good candidates to be observed experimentally. In channels $B^\pm \to \bar{K}^0 \rho^\pm$, $B^\pm \to K^{*0} \rho^\pm$ and $B^\pm \to \phi \rho^\pm$, the decay amplitude has only one contribution in consequence the direct CP violating asymmetry is automatically equal to zero. The rest of channels have direct CP violating asymmetries of less than 10% order.

The channels $B^\pm \to \pi^0 \rho^\pm$, $B^\pm \to \pi^\pm \rho^0$ and $B^\pm \to \omega \rho^0$ have direct CP violating asymmetries which depend on the use of the WSB model or the LCSR approach in evaluating the form factors in transitions $B \to \pi$ and $B \to \omega$.

### Table VII. Direct CP violating asymmetries in percent for $B^\pm \to P^{\rho'}$ and $B^\pm \to V^{\rho'}$ decays, using the WSB [7] model and LCSR approach [8], decay constants $f_{\rho'} = 26$ MeV and $f_{\rho'} = 0.0$ MeV.

| Final state | $A_{CP}$ | Final state | $A_{CP}$ |
|-------------|----------|-------------|----------|
| $\pi^0 \pi^0$ | -1.4 [-1.1] 0.0 | $\rho^0 \pi^0$ | -5.6 [-5.5] -20.6 |
| $\pi^0 \pi^\pm$ | -0.4 [-0.5] 1.6 | $\rho^\pm \pi^0$ | -20.7 [-21.0] 4.8 |
| $\eta \pi^\pm$ | 4.8 [4.8] -4.6 | $\omega \pi^0$ | -6.3 [-5.8] 13.4 |
| $\eta' \pi^\pm$ | 5.3 [5.4] 20.3 | $K^{*\pm} \pi^0$ | -25.4 [-24.9] -27.7 |
| $K^\pm \pi^0$ | -12.5 [-12.6] -13.6 | $\bar{K}^{*0} \pi^\pm$ | 0.0 [0.0] 0.0 |
| $\bar{K}^0 \pi^\pm$ | 0.0 [0.0] 0.0 | $\phi \pi^\pm$ | 0.0 [0.0] 0.0 |

### Table VIII. Direct CP violating asymmetries in percent for $B^\pm \to P^{\rho'}$ and $B^\pm \to V^{\rho'}$ decays, using the WSB [7] model and LCSR approach [8], and decay constant $f_{\rho'} = 128$ MeV.

| Final state | $A_{CP}$ | Final state | $A_{CP}$ |
|-------------|----------|-------------|----------|
| $\pi^0 \rho^\pm$ | -3.9 [-5.8] 0.38 [0.38] | $\rho^0 \rho^0$ | 0.39 [0.39] |
| $\pi^\pm \rho^0$ | 5.3 [6.0] 0.39 [0.39] | $\rho^\pm \rho^0$ | 17.5 [15.2] |
| $\eta \rho^\pm$ | 5.2 [5.3] | $\omega \rho^\pm$ | -20.2 [-19.4] |
| $\eta' \rho^\pm$ | 5.0 [5.0] | $K^{*\pm} \rho^0$ | 0.0 [0.0] |
| $K^\pm \rho^0$ | -21.1 [-21.0] | $\bar{K}^{*0} \rho^\pm$ | 0.0 [0.0] |
| $\bar{K}^0 \rho^\pm$ | 0.0 [0.0] | $\phi \rho^\pm$ | 0.0 [0.0] |

In neutral $B^0$ decays, because of the $B^0 - \bar{B}^0$ mixing, it is required to include time dependent measurements in CP violation asymmetries. The CP violation time dependent asymmetry is defined as

$$A_f(t) = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to \bar{f})}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to \bar{f})} = C_f \cos(\Delta m t) + S_f \sin(\Delta m t),$$

where $f$ is a two body final state. The coefficients $C_f$ and $S_f$ are defined by

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad S_f = \frac{-2Im(\lambda_f)}{1 + |\lambda_f|^2},$$

given in terms of the ratio $\lambda_f$, which is defined by

$$\lambda_f = \frac{V_{tb} V_{td}^* (f | H_{eff} | B^0)}{V_{tb} V_{td}^* (f | H_{eff} | B^{0*})}.\tag{27}$$
The coefficients $C_f$ and $S_f$ are functions of $\lambda_f$. The quantity $\lambda_f$ is independent of phase conventions and physically meaningful, in consequence the coefficients $C_f$ and $S_f$ are observables. CP violation in the interference of decays with and without mixing is encoded in the coefficient $S_f \neq 0$. CP violation in decays means $C_f \neq 0$.

If the final state $f$ is a CP eigenstate, i.e., $CP[f] = \pm |f|$, and decay amplitudes are dominated by only one weak phase term contribution, then $\langle f | H_{\text{eff}} | B^0 \rangle = \langle \bar{f} | H_{\text{eff}} | B^0 \rangle$, $C_f = 0$ and $S_f = \eta_f \sin(2\phi)$, where $\eta_f$ is the CP eigenvalue of $f$ and $2\phi$ is the difference in weak phase between the $B^0 \to f$ and $B^0 \to \bar{B}^0 \to \bar{f}$ decay path. A contribution of another term to the decay amplitude with a different weak phase make the value of $S_f$ depends on the strong phase. In this situation is also possible that $C_f \neq 0$.

Table IX. CP violating asymmetry parameters $C_f$ and $S_f$ in percent for neutral $B^0(\bar{B}^0) \to P\pi^0$ and $B^0(\bar{B}^0) \to V\pi^0$ decays, using the WSB [7] model and LCSR approach [8], decay constants $f_{\pi'} = 26$ MeV and $f_{\rho'} = 0$ MeV.

| Final state $C_f$ | $S_f$ | Final state $C_f$ | $S_f$ |
|------------------|-------|------------------|-------|
| $\pi^0\pi^0$    | -0.7  [0.7] -9.0 | -2.7 [3.1] -30.5 $\rho^0\pi^0$ | 1.2 [1.1] -20.6 | 5.8 [4.7] -58.0 |
| $\eta\pi^0$     | 0.4   [0.5] 4.6  | 1.6 [2.1] 21.1 $\omega\pi^0$ | 0.8 [0.5] 13.4 | 3.6 [2.4] 46.6 |
| $\eta'\pi^0$    | 0.5   [0.6] 20.3 | 1.9 [2.5] 40.8 $K^{*+}\pi^\pm$ | -27.7 [27.7] -27.7 -17.7 [-17.7] -17.7 |
| $K^\mp\pi^\pm$  | -13.6 [13.6] 30.1 | 30.1 | $K^{*0}\pi^0$ | 0.09 [0.12] 0.0 | 71.1 [71.1] 71.1 |
| $K^0\pi^0$      | -0.1  [-0.1] -0.1 | 70.7 [70.7] 70.7 | $\phi\pi^0$ | 0.0 [0.0] 0.0 | 0.0 [0.0] 0.0 |

Table X. CP violating asymmetries parameters $C_f$ and $S_f$ in percent for neutral $B^0(\bar{B}^0) \to P\rho'$ and $B^0(\bar{B}^0) \to V\rho'$ decays, using the WSB [7] model and LCSR approach [8], and decay constant $f_{\rho'} = 128$ MeV.

| Final state $C_f$ | $S_f$ | Final state $C_f$ | $S_f$ |
|------------------|-------|------------------|-------|
| $\pi^0\rho^0$    | -48.8 [-53.9] -83.7 [-83.3] $\rho^0\rho^0$ | -20.6 [-20.6] -58.0 [-58.0] |
| $\eta\rho^0$     | -40.1 [-41.0] -91.6 [-89.3] $\omega\rho^0$ | 20.0 [20.6] 46.6 [46.6] |
| $\eta'\rho^0$    | -5.1 [-4.3] -48.4 [-47.6] $K^{*+}\rho^\pm$ | -27.7 [27.7] -17.8 [-17.8] |
| $K^\mp\rho^\pm$  | -14.96 [-15.0] -16.0 [-16.0] $K^{*0}\rho^0$ | -0.3 [-0.3] 66.3 [66.3] |
| $K^0\rho^0$      | 0.8 [0.9] 65.7 [65.7] | $\phi\rho^0$ | 0.0 [0.0] 0.0 [0.0] |

CP violating asymmetry coefficients $C_f$ and $S_f$ for neutral $B^0(\bar{B}^0)$ decays with radial excited mesons $\pi'$ and $\rho'$ in final state are shown in Tables IX and X, respectively. For the processes $B^0(\bar{B}^0) \to \phi\pi'$ and $B^0(\bar{B}^0) \to \phi\rho'$ the coefficients $C_f$ and $S_f$ are equal to zero, since there is only one contribution to the decay amplitude in the respective channels.

The calculations results for the coefficients $C_f$ and $S_f$ are practically equal when the form factors are estimated using the WSB model or the LCSR approach. Nevertheless, for the modes $B \to P\pi'$ and $B \to V\pi'$ there are a dependency with respect to use the values for the decay constant $f_{\pi'} = 26$ MeV or $f_{\rho'} = 0$ MeV. The channels with a strange meson in the final state have the same value of the coefficients using the two different values in decay constant $f_{\pi'}$.

The channels with $C_f \approx 0$ and $S_f \neq 0$ are $B^0 \to K^0\pi^0$, $B^0 \to K^{*0}\pi^0$, $B^0 \to K^0\rho^0$ and $B^0 \to K^{*0}\rho^0$. In these channels, where there is only present CP violation in the interference of the decay and in the mixing, it is possible to relate the coefficient $S_f$ to fundamental parameters in the standard model, i.e., interior angles of the unitary triangle.

Table XI. CP violating asymmetry parameters $C_f$, $S_f$, $\bar{C}_f$ and $\bar{S}_f$ in percent for $B^0(\bar{B}^0) \to \pi\pi'$, $B^0(\bar{B}^0) \to \rho\pi'$, $B^0(\bar{B}^0) \to \pi\rho'$ and $B^0(\bar{B}^0) \to \rho\rho'$ decays, using the WSB [7] model and LCSR approach [8].

| Final state $C_f$ | $S_f$ | $\bar{C}_f$ | $\bar{S}_f$ |
|------------------|-------|-------------|-------------|
| $\pi^+\pi^-$, $\pi^+\pi^0$ | 78.3 [68.2] -72.3 [-60.0] | 61.5 [72.3] 66.5 [77.0] |
| $\rho^+\pi^-$, $\rho^+\pi^0$ | 22.7 [34.6] -22.7 [-34.7] -97.3 [-93.8] -89.8 [-86.5] |
| $\pi^+\rho^-$, $\pi^+\rho^0$ | 14.2 [34.0] -9.1 [-29.3] 4.2 [4.0] 3.9 [3.8] |
| $\rho^+\rho^-$, $\rho^+\rho^0$ | 15.4 [15.4] -5.8 [-5.8] 13.9 [13.9] 14.0 [14.0] |

CP violation in neutral $B^0(\bar{B}^0)$ mesons is involved in case that a final state $f$ and its CP conjugate transformation state $\bar{f}$ are both common final states of $B^0$ and $\bar{B}^0$ mesons. The final states $f$ and $\bar{f}$ are not CP eigenstates, i.e.
are listed in Appendices. Nonrelativistic ISGW quark model \cite{9}, called ISGW2 model \cite{10}. The factorized decay amplitudes for these decays $B^0(t) \rightarrow f$, $B^0(t) \rightarrow \bar{f}$, $\bar{B}^0(t) \rightarrow \bar{f}$, $\bar{B}^0(t) \rightarrow f$, and $B^0(t) \rightarrow f$ are studied in terms of four basic matrix elements

\[
g = \langle f | H_{\text{eff}} | B^0 \rangle, \quad h = \langle f | H_{\text{eff}} | \bar{B}^0 \rangle,
\]
\[
\bar{g} = \langle \bar{f} | H_{\text{eff}} | B^0 \rangle, \quad \bar{h} = \langle \bar{f} | H_{\text{eff}} | \bar{B}^0 \rangle.
\]  

(28)

The following two CP violating asymmetries are introduced

\[
\bar{A}_f(t) = \frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(\bar{B}^0(t) \rightarrow f)}{\Gamma(B^0(t) \rightarrow f) + \Gamma(\bar{B}^0(t) \rightarrow f)} = \bar{C}_f \cos(\Delta m t) + \bar{S}_f \sin(\Delta m t)
\]  

(29)

and

\[
\bar{A}_f(t) = \frac{\Gamma(B^0(t) \rightarrow \bar{f}) - \Gamma(\bar{B}^0(t) \rightarrow \bar{f})}{\Gamma(B^0(t) \rightarrow \bar{f}) + \Gamma(\bar{B}^0(t) \rightarrow \bar{f})} = \bar{C}_f \cos(\Delta m t) + \bar{S}_f \sin(\Delta m t),
\]  

(30)

where the coefficients of $\cos(\Delta m t)$ and $\sin(\Delta m t)$ are defined by

\[
\bar{C}_f = \frac{|g|^2 - |h|^2}{|g|^2 + |h|^2}, \quad \bar{S}_f = \frac{-2Im(V_{tb \bar{d}} V_{tb \bar{d}}^*)}{1 + |h/g|^2},
\]  

(31)

and

\[
\bar{C}_f = \frac{|\bar{h}|^2 - |\bar{g}|^2}{|\bar{h}|^2 + |\bar{g}|^2}, \quad \bar{S}_f = \frac{-2Im(V_{tb \bar{d}} V_{tb \bar{d}}^*)}{1 + |\bar{h}/\bar{g}|^2}.
\]  

(32)

The condition for CP violation is that width decays $\Gamma(B^0(t) \rightarrow f) \neq \Gamma(\bar{B}^0 \rightarrow f)$ and $\Gamma(B^0(t) \rightarrow \bar{f}) \neq \Gamma(\bar{B}^0 \rightarrow \bar{f})$, which means in terms of CP violating asymmetry coefficients $\bar{C}_f \neq -\bar{C}_f$ and (or) $\bar{S}_f \neq -\bar{S}_f$. Numerical values in percent for the CP violating asymmetry parameters $\bar{C}_f$, $\bar{S}_f$, $\bar{C}_f$ and $\bar{S}_f$ in the $B^0(\bar{B}^0) \rightarrow \pi \pi'$, $B^0(\bar{B}^0) \rightarrow \rho \pi'$, $B^0(\bar{B}^0) \rightarrow \rho \rho'$ and $B^0(\bar{B}^0) \rightarrow \rho \rho'$ decays, are listed in Table IX. The form factors for the transitions $B \rightarrow \pi$ and $B \rightarrow \rho$ are calculated using the WSB model and the LCSR approach.

The CP violating asymmetry parameters for the final states $\rho^+ \rho^-$, $\rho^- \rho^+$ have the same value if they are calculated using the WSB model or the LCSR approach. The parameters for the final states with a $\pi'$ meson are only calculated using the values in decay constant $f_{\pi'} = 26$ MeV. When the value $f_{\pi'} = 0$ MeV is used, results are not reported, since there are zero contributions, $g = 0$ and $cg = 0$, with the consequence that $\bar{C}_f = -\bar{C}_f = 100\%$, $\bar{S}_f = \infty$ and $\bar{S}_f = 0$.

There is not yet measurements for CP violation asymmetries in the channels studied in this work \cite{12}. One of the reasons for this work is to estimate this asymmetries and to motivate the experimental measurement of them.

VI. CONCLUSIONS

In the framework of generalized naive factorization we calculate branching ratios and CP violating asymmetries of exclusive nonleptonic two body $B$ decays including the radial excited $\pi(1300)$ or $\rho(1450)$ meson in the final state. Branching ratios and CP violating asymmetries for the exclusive channels $B \rightarrow P \pi'$, $B \rightarrow V \pi'$, $B \rightarrow P \rho'$ and $B \rightarrow V \rho'$ (where, $P$ and $V$ denote a pseudoscalar and vector meson, respectively) have been estimated using all the contributions coming from the effective weak Hamiltonian $H_{\text{eff}}$.

The form factors in $B \rightarrow P$ and $B \rightarrow V$ transitions are estimated using the WSB model \cite{5} and the LCSR approach \cite{8}. In order to obtain form factors in $B \rightarrow \pi'$ and $B \rightarrow \rho'$ transitions, we use the improved version of the nonrelativistic ISGW quark model \cite{9}, called ISGW2 model \cite{10}. The factorized decay amplitudes for these decays are listed in Appendices.
We have obtained branching ratios for 52 exclusive channels. Some of these decays can be reached in experiments. In fact, decays $B^0 \rightarrow \pi^-\pi^+$, $B^- \rightarrow \eta\pi^-$, $B^0 \rightarrow \rho^-\pi^+$, $B^0 \rightarrow \rho^+\pi^-$, $B^0 \rightarrow \pi^-\rho^+$, $B^0 \rightarrow \rho^-\rho^+$, $B^0 \rightarrow \rho^+\rho^-$, and $B^- \rightarrow \rho^-\rho^0$ have branching ratios of the order of $10^{-5}$.

We also studied the dependency of branching ratios in channels $B \rightarrow P\pi'$ and $B \rightarrow V\pi'$ with respect to the decay constant $f'$. The more sensible modes to the value in decay constant $f'$ are $B^0 \rightarrow \pi^-\pi^+$, $B^0 \rightarrow \pi^0\pi^0$, $B^- \rightarrow \eta\pi^-$, $B^- \rightarrow \eta'\pi^-$, $B^0 \rightarrow \rho^-\pi^-$, $B^- \rightarrow \rho^0\pi^-$, $B^0 \rightarrow \omega\pi^0$, and $B^- \rightarrow \omega\pi^-$. These channels could be the best scenario to determine the decay constant $f'$ in nonleptonic two body $B$ decays.

In general, we can explain the large branching ratios in decays $B^0 \rightarrow \pi^-\pi^+$, $B^0 \rightarrow \pi^0\pi^0$, $B^0 \rightarrow \pi^+\pi^-$, and $B^- \rightarrow \rho^-\rho^0$ by the effect of the enhancement of the chiral factor that multiply the penguin contributions $a_6$ and $a_8$ in the effective weak Hamiltonian $H_{eff}$.

Direct CP violating asymmetry in channels $B^\pm \rightarrow K^\pm\pi^0$, $B^\pm \rightarrow \rho^\pm\pi^0$, $B^\pm \rightarrow K^{*\pm}\pi^0$, $B^\pm \rightarrow \rho^0$, $B^\pm \rightarrow \omega\rho^\pm$, and $B^\pm \rightarrow K^{*\pm}\rho^0$ are more than $10\%$ order. In the modes $B^\pm \rightarrow \eta\pi^\pm$, $B^\pm \rightarrow \rho^0\pi^\pm$, and $B^\pm \rightarrow \omega\pi^\pm$, estimation of direct CP violating asymmetry using the value of the decay constant $f' = 0$ MeV, give an increase with respect to the calculations using the value $f_\pi = 26$ MeV. On the contrary the channel $B^\pm \rightarrow \rho^\pm\pi^0$ has a decrease in its estimation. When the value in the decay constant $f' = 0$ MeV is used, estimation of direct CP violating asymmetry in modes $B^\pm \rightarrow \eta\pi^\pm$, $B^\pm \rightarrow \rho^0\pi^\pm$, and $B^\pm \rightarrow \omega\pi^\pm$ are more than $10\%$ order.

In the neutral modes $B^0 \rightarrow K^0\pi^0$, $B^0 \rightarrow K^{*0}\pi^0$, $B^0 \rightarrow K^0\rho^0$ and $B^0 \rightarrow K^{*0}\rho^0$, we have estimated the CP violating asymmetry coefficients $C_f \approx 0$ and $S_f$ more than $60\%$.

Finally, we want to mention that our predictions for the channels $B^0 \rightarrow \pi^-\rho^+$ and $B^- \rightarrow \pi^-\rho^0$ are lower as ones obtained by Ref. [4], although our value in the same order of magnitude that the only experimental branching ratio measured $B^- \rightarrow \pi^-\rho^0$ [26, 27].

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Appendix A: Matrix elements for $B \rightarrow P\pi'$ decays

\[
\mathcal{M}(B^0 \rightarrow \pi^-\pi^+) = -i\frac{G_F}{\sqrt{2}}F_0^{B \rightarrow \pi}(m_{\pi}^2)(m_{B}^2 - m_{\pi}^2) \left\{V_{ub}V_{ud}^{*}a_1 - V_{tb}V_{td}^{*}\left[a_4 + a_{10} + 2(a_6 + a_8)\frac{m_{\pi}^2}{(m_b - m_u)(m_d + m_u)}\right]\right\}
\]

(A1)

\[
\mathcal{M}(B^0 \rightarrow \pi^0\pi^0) = i\frac{G_F}{2\sqrt{2}}F_0^{B \rightarrow \pi'}(m_{\pi}^2)(m_{B}^2 - m_{\pi}^2) \left\{V_{ub}V_{ud}^{*}a_2 - V_{tb}V_{td}^{*}\left[-a_4 + \frac{1}{2}a_{10} - \frac{3}{2}(a_7 - a_9) - (2a_6 - a_8)\frac{m_{\pi}^2}{(m_b - m_d)(m_d + m_d)}\right]\right\}
\]

(A2)

\[
\mathcal{M}(B^- \rightarrow \pi^-\pi^0) = -i\frac{G_F}{2\sqrt{2}}F_0^{B \rightarrow \pi}(m_{\pi}^2)(m_{B}^2 - m_{\pi}^2) \left\{V_{ub}V_{ud}^{*}a_1 - V_{tb}V_{td}^{*}\left[a_4 + a_{10} + a_7 - 2a_8\frac{m_{\pi}^2}{(m_b - m_u)(m_d + m_u)}\right]\right\}
\]

(A3)
\begin{align*}
\mathcal{M}(B^0 \to \eta^{(')}\pi^0) &= -\frac{G_F}{2} f_\pi F_0^{B\to\eta^{(')}} (m_{\eta^{(')}}^2)(m_B^2 - m_{\eta^{(')}}^2) \\
&\quad \begin{cases}
V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^* \left[-a_4 + \frac{1}{2}(a_1 + 2a_6 - a_8)ight] \frac{m_{\pi^0}^2}{(m_b - m_d)(m_d + m_u)} + 3\frac{1}{2}(a_9 - a_7)
\end{cases}
&+ i \frac{G_F}{2} f^u_{\eta^{(')}} F_0^{B\to\pi^{(')}} (m_{\eta^{(')}}^2)(m_B^2 - m_{\pi^0}^2) \begin{cases}
V_{ub}V_{ud}^* a_2 + V_{cb}V_{cd}^* \frac{m_{\pi^0}^2}{(m_b - m_d)(m_s + m_s)} \left(\frac{f^u_{\eta^{(')}}}{f_{\eta^{(')}}} - 1\right) r_{\eta^{(')}}
\end{cases}
\end{align*}

\begin{align*}
\mathcal{M}(B^- \to \eta^{(')}\pi^-) &= \frac{G_F}{\sqrt{2}} f_\pi F_0^{B\to\eta^{(')}} (m_{\pi^0}^2)(m_B^2 - m_{\pi^0}^2) \\
&\quad \begin{cases}
V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^* \left[a_4 + a_10 + 2(a_6 + a_8)\right] \frac{m_{\pi^0}^2}{(m_b - m_u)(m_d + m_u)}
\end{cases}
&+ i \frac{G_F}{\sqrt{2}} f^u_{\eta^{(')}} F_0^{B\to\pi^{(')}} (m_{\eta^{(')}}^2)(m_B^2 - m_{\pi^0}^2) \begin{cases}
V_{ub}V_{ud}^* a_2 + V_{cb}V_{cd}^* \frac{m_{\eta^{(')}}^2}{(m_b - m_d)(m_s + m_s)} \left(\frac{f^u_{\eta^{(')}}}{f_{\eta^{(')}}} - 1\right) r_{\eta^{(')}}
\end{cases}
\end{align*}

\begin{align*}
\mathcal{M}(B^0 \to K^- \pi^+) &= -\frac{G_F}{\sqrt{2}} f_K F_0^{B\to\pi^{(')}} (m_{K^-}^2)(m_B^2 - m_{\pi^0}^2) \\
&\quad \begin{cases}
V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^* \left[a_4 + a_10 + 2(a_6 + a_8)\right] \frac{m_{K^-}^2}{(m_b - m_u)(m_d + m_s)}
\end{cases}
\end{align*}

\begin{align*}
\mathcal{M}(B^0 \to \bar{K}^0 \pi^0) &= -\frac{G_F}{2} f_K F_0^{B\to\pi^{(')}} (m_{\bar{K}^0}^2)(m_B^2 - m_{\pi^0}^2) \\
&\quad \begin{cases}
V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^* \left[a_4 - \frac{1}{2}a_10 + 2(a_6 + a_8)\right] \frac{m_{\bar{K}^0}^2}{(m_b - m_d)(m_d + m_s)}
\end{cases}
&- i \frac{G_F}{2} f_\pi F_0^{B\to\bar{K}^-} (m_{\bar{K}^0}^2) (m_B^2 - m_{\pi^0}^2) \begin{cases}
V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^* \frac{3}{2}(a_9 - a_7)
\end{cases}
\end{align*}

\begin{align*}
\mathcal{M}(B^- \to K^- \pi^0) &= -\frac{G_F}{2} f_K F_0^{B\to\pi^{(')}} (m_{K^-}^2)(m_B^2 - m_{\pi^0}^2) \\
&\quad \begin{cases}
V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^* \left[a_4 + a_10 + 2(a_6 + a_8)\right] \frac{m_{K^-}^2}{(m_b - m_u)(m_d + m_s)}
\end{cases}
&- i \frac{G_F}{2} f_\pi F_0^{B\to\bar{K}^-} (m_{K^-}^2) (m_B^2 - m_{\pi^0}^2) \begin{cases}
V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^* \frac{3}{2}(a_9 - a_7)
\end{cases}
\end{align*}

\begin{align*}
\mathcal{M}(B^- \to \bar{K}^0 \pi^-) &= -\frac{G_F}{\sqrt{2}} f_K F_0^{B\to\pi^{(')}} (m_{\bar{K}^0}^2)(m_B^2 - m_{\pi^0}^2) \\
&\quad \begin{cases}
V_{ub}V_{ts}^* \left[a_4 - \frac{1}{2}a_10 + 2(a_6 + a_8)\right] \frac{m_{\bar{K}^0}^2}{(m_b - m_d)(m_d + m_s)}
\end{cases}
\end{align*}
Appendix B: Matrix elements for $B \to V\pi'$ decays

\begin{align*}
\mathcal{M}(\bar{B}^0 \to \rho^- \pi^{++}) &= \sqrt{2} G_F f_\rho f_{\rho \pi} (m_\rho^2) m_\rho (\epsilon \cdot p_{\pi'}) \{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* [a_4 + a_{10}] \} \\
\mathcal{M}(\bar{B}^0 \to \rho^+ \pi^-) &= \sqrt{2} G_F f_\pi A_0^{B \to \rho} (m_\pi^2) m_\rho (\epsilon \cdot p_{\pi'}) \left\{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* \left[ a_4 + a_{10} - 2(a_6 + a_8) \frac{m_{\rho'}^2}{(m_b + m_u)(m_u + m_d)} \right] \right\} \\
\mathcal{M}(\bar{B}^0 \to \rho^0 \pi^0) &= \frac{G_F}{\sqrt{2}} m_\rho (\epsilon \cdot p_{\pi'}) \left( f_{\rho} F_{1}^{B \to \pi'} (m_\rho^2) \left\{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* \left[ a_4 + a_{10} - \frac{3}{2}(a_7 + a_9) \right] \right\} \\
&\quad + f_{\pi} A_0^{B \to \rho} (m_\pi^2) \left\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \left[ a_4 - \frac{1}{2} a_{10} - (2a_6 - a_8) \frac{m_{\pi'}^2}{(m_b + m_d)(m_d + m_d) + \frac{3}{2}(a_7 - a_9)} \right] \right\} \right) \\
\mathcal{M}(B^- \to \rho^0 \pi^-) &= G_F m_\rho (\epsilon \cdot p_{\pi'}) \left( f_{\rho} F_{1}^{B \to \pi'} (m_\rho^2) \left\{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* \left[ a_4 + a_{10} - 2(a_6 + a_8) \frac{m_{\pi'}^2}{(m_b + m_u)(m_u + m_d)} \right] \right\} \\
&\quad + f_{\pi} A_0^{B \to \rho} (m_\pi^2) \left\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \left[ -a_4 + \frac{1}{2} a_{10} + \frac{3}{2}(a_7 - a_9) \right] \right\} \right) \\
\mathcal{M}(\bar{B}^0 \to \omega \pi^0) &= \frac{G_F}{\sqrt{2}} m_\omega (\epsilon \cdot p_{\pi'}) \left( f_{\omega} F_{1}^{B \to \pi'} (m_\omega^2) \left\{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* \left[ a_4 + 2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9 - a_{10}) \right] \right\} \\
&\quad + f_{\pi} A_0^{B \to \omega} (m_\pi^2) \left\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \left[ -a_4 + \frac{1}{2} a_{10} - (2a_6 - a_8) \frac{m_{\pi'}^2}{(m_b + m_u)(m_u + m_d) + \frac{3}{2}(a_7 - a_9)} \right] \right\} \right) \\
\mathcal{M}(B^- \to \omega \pi^-) &= G_F m_\omega (\epsilon \cdot p_{\pi'}) \left( f_{\omega} A_0^{B \to \omega} (m_\omega^2) \left\{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* \left[ a_4 + a_{10} - 2(a_6 + a_8) \frac{m_{\pi'}^2}{(m_b + m_u)(m_u + m_d)} \right] \right\} \\
&\quad + f_{\pi} F_{1}^{B \to \pi'} (m_\pi^2) \left\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \left[ a_4 + 2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9 - a_{10}) \right] \right\} \right) \\
\mathcal{M}(\bar{B}^0 \to K^- \pi^+) &= \sqrt{2} G_F f_K F_{1}^{B \to \pi'} (m_{K^0}^2) m_K (\epsilon \cdot p_{\pi'}) \{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* [a_4 + a_{10}] \} \\
\mathcal{M}(B^- \to K^0 \pi^0) &= G_F m_{K^0} (\epsilon \cdot p_{\pi'}) \left( f_{\pi} A_0^{B \to K} (m_\pi^2) \left\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \frac{3}{2}(a_9 - a_7) \right\} \\&\quad + f_{\rho} F_{1}^{B \to \pi'} (m_{K^0}^2) V_{tb} V_{ts}^* \left\{ a_4 - \frac{1}{2} a_{10} \right\} \right) \\
\mathcal{M}(B^- \to K^- \pi^0) &= G_F m_K (\epsilon \cdot p_{\pi'}) \left( f_{\pi} A_0^{B \to K} (m_\pi^2) \left\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \frac{3}{2}(a_9 - a_7) \right\} \\
&\quad + f_{\rho} F_{1}^{B \to \pi'} (m_{K}^2) \{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* [a_4 + a_{10}] \} \right) \\
\mathcal{M}(B^- \to K^- \pi^0) &= G_F m_K (\epsilon \cdot p_{\pi'}) \left( f_{\pi} A_0^{B \to K} (m_\pi^2) \left\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \frac{3}{2}(a_9 - a_7) \right\} \\
&\quad + f_{\rho} F_{1}^{B \to \pi'} (m_{K}^2) \{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* [a_4 + a_{10}] \} \right)
\end{align*}
\[ \mathcal{M}(B^+ \rightarrow K^{*0}\pi^-) = -\sqrt{2}G_F f_K \cdot F_1^{B^+\rightarrow \pi^0}(m_{K^*}^2) m_K \cdot (\epsilon \cdot p_{\pi^0}) V_{tb} V_{ts}^* \left\{ a_4 - \frac{1}{2} a_{10} \right\} \] (B11)

\[ \mathcal{M}(B^0 \rightarrow \phi\pi^0) = -G_F f_{\phi} F_1^{B^0\rightarrow \pi^0}(m_{\phi}^2) m_{\phi}(\epsilon \cdot p_{\pi^0}) V_{tb} V_{ts}^* \left\{ a_3 + a_5 - \frac{1}{2} (a_7 + a_9) \right\} \] (B12)

\[ \mathcal{M}(B^+ \rightarrow \phi\pi^-) = -\sqrt{2} \mathcal{M}(B^0 \rightarrow \phi\pi^0) \] (B13)

**Appendix C: Matrix elements for \( B \rightarrow P\rho' \) decays**

\[ \mathcal{M}(B^0 \rightarrow \eta^{(')}\rho^0) = G_F m_{\rho'}(\epsilon \cdot p_{\eta^{(')}}) \left( f_{\rho'} F_1^{B^0\rightarrow \eta^{(')}\rho}(m_{\rho'}^2) \right) \{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \left[ -a_4 + \frac{1}{2} a_{10} + \frac{3}{2} (a_7 + a_9) \right] \} \]

\[ + f_{\eta^{(')}}^{u} A_0^{B^0\rightarrow \rho'}(m_{\eta^{(')}}^2) \left\{ V_{ub} V_{ud}^* a_2 + V_{cb} V_{cd}^* \frac{f_{\eta^{(')}}^c}{f_{\eta^{(')}}^u} \right\} \]

\[ -V_{tb} V_{td}^* \left[ a_4 + 2(a_3 - a_5) + \frac{1}{2} (a_9 - a_7 - a_{10}) - (2a_6 - a_8) \frac{m_{\eta^{(')}}^2}{(m_b + m_d)(m_s + m_s)} \left( \frac{f_{\eta^{(')}}^u}{f_{\eta^{(')}}^c} - 1 \right) r_{\eta^{(')}} \right] \]

\[ + (a_3 - a_5 - a_7 + a_9) \frac{f_{\eta^{(')}}^c}{f_{\eta^{(')}}^u} + \left( a_3 - a_5 + \frac{1}{2} (a_9 - a_7) \right) \frac{f_{\eta^{(')}}^u}{f_{\eta^{(')}}^c} \} \] (C1)

\[ \mathcal{M}(B^+ \rightarrow \eta^{(')}\rho^+) = \sqrt{2} G_F m_{\rho'}(\epsilon \cdot p_{\eta^{(')}}) \left( f_{\rho'} F_1^{B^+\rightarrow \eta^{(')}\rho}(m_{\rho'}^2) \right) \{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* [a_4 + a_{10}] \}

\[ + f_{\eta^{(')}}^{u} A_0^{B^+\rightarrow \rho'}(m_{\eta^{(')}}^2) \left\{ V_{ub} V_{ud}^* a_2 + V_{cb} V_{cd}^* \frac{f_{\eta^{(')}}^c}{f_{\eta^{(')}}^u} \right\} \]

\[ -V_{tb} V_{td}^* \left[ a_4 + 2(a_3 - a_5) + \frac{1}{2} (a_9 - a_7 - a_{10}) - (2a_6 - a_8) \frac{m_{\eta^{(')}}^2}{(m_b + m_d)(m_s + m_s)} \left( \frac{f_{\eta^{(')}}^u}{f_{\eta^{(')}}^c} - 1 \right) r_{\eta^{(')}} \right] \]

\[ + (a_3 - a_5 - a_7 + a_9) \frac{f_{\eta^{(')}}^c}{f_{\eta^{(')}}^u} + \left( a_3 - a_5 + \frac{1}{2} (a_9 - a_7) \right) \frac{f_{\eta^{(')}}^u}{f_{\eta^{(')}}^c} \} \] (C2)

\[ \mathcal{M}(B^0 \rightarrow K^0\rho^+) = \sqrt{2} G_F f_K A_0^{B^0\rightarrow \rho'}(m_{K^*}^2) m_{\rho'}(\epsilon \cdot p_K) \]

\[ \left\{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* \left[ a_4 + a_{10} - 2(a_6 + a_8) \frac{m_{K^0}^2}{(m_b + m_u)(m_u + m_s)} \right] \right\} \] (C3)

\[ \mathcal{M}(B^0 \rightarrow \bar{K}^0\rho^0) = G_F m_{\rho'}(\epsilon \cdot p_K) \left( f_{K} A_0^{B^0\rightarrow \rho'}(m_{K^0}^2) V_{tb} V_{ts}^* \left[ a_4 - \frac{1}{2} a_{10} - (2a_6 - a_8) \frac{m_{K^0}^2}{(m_b + m_u)(m_u + m_s)} \right] \right) \]

\[ + f_{\rho'} F_1^{B^0\rightarrow \rho'}(m_{\rho'}^2) \left\{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* \left[ a_4 + a_{10} - 2(a_6 + a_8) \frac{m_{K^0}^2}{(m_b + m_u)(m_u + m_s)} \right] \right\} \] (C4)

\[ \mathcal{M}(B^0 \rightarrow \bar{K}^0\rho^0) = G_F m_{\rho'}(\epsilon \cdot p_K) \left( f_{K} A_0^{B^0\rightarrow \rho'}(m_{K^*}^2) V_{tb} V_{ts}^* \left[ a_4 + a_{10} - 2(a_6 + a_8) \frac{m_{K^*}^2}{(m_b + m_u)(m_u + m_s)} \right] \right) \]

\[ + f_{\rho'} F_1^{B^0\rightarrow \rho'}(m_{\rho'}^2) \left\{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* \left[ a_4 + a_{10} - 2(a_6 + a_8) \frac{m_{K^*}^2}{(m_b + m_u)(m_u + m_s)} \right] \right\} \] (C5)

\[ \mathcal{M}(B^0 \rightarrow \bar{K}^0\rho^-) = -\sqrt{2} G_F f_K A_0^{B^0\rightarrow \rho'}(m_{K^*}^2) m_{\rho'}(\epsilon \cdot p_K) \]

\[ V_{tb} V_{ts}^* \left[ a_4 - \frac{1}{2} a_{10} - (2a_6 - a_8) \frac{m_{K^*}^2}{(m_b + m_d)(m_d + m_s)} \right] \] (C6)
Appendix D: Matrix elements for $B \to V \rho'$ decays

\[ \mathcal{M}(\bar{B}^0 \to \rho^- \rho^+) = X(\bar{B}^0 \rho^+ \rho^-) \{V_{ub}V_{us}^*a_1 - V_{tb}V_{ts}^*[a_4 + a_{10}]\} \]  

(D1)

\[ \mathcal{M}(\bar{B}^0 \to \rho^0 \rho^0) = \left[ X(\bar{B}^0 \rho^0 \rho^0) + X(\bar{B}^0 \rho^+ \rho^-) \right] \left\{V_{ub}V_{ud}^*a_2 + V_{tb}V_{td}^*[a_4 - \frac{1}{2}a_{10} - \frac{3}{2}(a_7 + a_9)]\right\} \]  

(D2)

\[ \mathcal{M}(B^- \to \rho^- \rho^0) = X(B^- \rho^- \rho^0) \{V_{ub}V_{ud}^*a_2 + V_{tb}V_{td}^*[a_4 - \frac{1}{2}a_{10} + \frac{3}{2}(a_7 + a_9)]\} \]  

(D3)

\[ \mathcal{M}(\bar{B}^0 \to \omega \rho^0) = X(\bar{B}^0 \omega \rho^0) \left\{V_{ub}V_{ud}^*a_2 - V_{tb}V_{td}^*[a_4 + 2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9 - a_{10})]\right\} + X(\bar{B}^0 \omega \rho^0) \left\{-V_{tb}V_{ts}^*[a_4 + \frac{1}{2}a_{10} + \frac{3}{2}(a_7 + a_9)]\right\} \]  

(D4)

\[ \mathcal{M}(B^- \to \omega \rho'^-) = X(B^- \omega \rho'^-) \{V_{ub}V_{ud}^*a_2 - V_{tb}V_{td}^*[a_4 + 2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9 - a_{10})]\} + X(B^- \omega \rho'^-) \left\{V_{ub}V_{ud}^*a_2 - V_{tb}V_{td}^*[a_4 + 2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9 - a_{10})]\right\} \]  

(D5)

\[ \mathcal{M}(\bar{B}^0 \to K^{*-} \rho^+) = X(\bar{B}^0 \rho^+, K^{*-}) \{V_{ub}V_{us}^*a_1 - V_{tb}V_{ts}^*[a_4 + a_{10}]\} \]  

(D6)

\[ \mathcal{M}(\bar{B}^0 \to \bar{K}^{*0} \rho^0) = X(\bar{B}^0 \bar{K}^{*0} \rho^0) \left\{V_{ub}V_{us}^*a_2 - V_{tb}V_{ts}^*[a_4 + \frac{1}{2}a_{10}]\right\} + X(\bar{B}^0 \bar{K}^{*0} \rho^0) \left\{-V_{tb}V_{ts}^*[a_4 + \frac{1}{2}a_{10}]\right\} \]  

(D7)

\[ \mathcal{M}(B^- \to K^{*-} \rho^0) = X(B^- K^{*-} \rho^0) \{V_{ub}V_{us}^*a_2 - V_{tb}V_{ts}^*[a_4 + \frac{1}{2}a_{10}]\} + X(B^- K^{*-} \rho^0) \left\{V_{ub}V_{us}^*a_1 - V_{tb}V_{ts}^*[a_4 + a_{10}]\right\} \]  

(D8)

\[ \mathcal{M}(B^- \to \bar{K}^{*0} \rho'^-) = -X(B^- \rho'^- \bar{K}^{*0})V_{tb}V_{ts}^*[a_4 + \frac{1}{2}a_{10}] \]  

(D9)

\[ \mathcal{M}(\bar{B}^0 \to \phi \rho^0) = X(\bar{B}^0 \phi \rho^0) \left\{V_{tb}V_{td}^*[a_3 + a_5 - \frac{1}{2}(a_7 + a_9)]\right\} \]  

(D10)

\[ \mathcal{M}(B^- \to \phi \rho'^-) = -\sqrt{2} \mathcal{M}(\bar{B}^0 \to \phi \rho^0) \]  

(D11)

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