A New Beta Chaotic Map with DNA Encoding for Color Image Encryption

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Abstract
Images hold important information, especially in military and commercial surveillance as well as in industrial inspection and communication. Therefore, the protection of the image from abuse, unauthorized access, and damage became a significant demand. This paper introduces a new Beta chaotic map for encrypting and confusing the color image with Deoxyribonucleic Acid (DNA) sequence. First, the DNA addition operation is used for diffusing each component of the plain image. Then, a new Beta chaotic map is used for shuffling the DNA color image. In addition, two chaotic maps, namely the proposed new Beta and Sine chaotic maps, are used for key generation. Finally, the DNA XOR is applied concerning the produced key and shuffled DNA image to yield the cipher image. The experimental results prove that the proposed method surpassed the other methods in terms of Mean Square Error (MSE), Peak Signal-To-Noise Ratio (PSNR), entropy, and correlation coefficient.

Keywords: Beta chaotic map, DNA encoding, Image Encryption, Sine map, Confusion, Diffusion.

1. Introduction
The evolution of new technologies in computer sciences led to the widespread growth of information, such as those of images, sounds, and videos stored in digital forms on hard disks or distributed over the internet. Furthermore, abuse and illegal access to private information have come to be a severe...
challenge in the digital world. One of the effective methods for solving these concerns is encryption [1]. The conventional cryptographic techniques are based on a number of theoretic or algebraic concepts and hence they are inappropriate for multimedia data encryption because of their enormous sizes, high redundancy of pixels, interactive processes, complexity, and inadequacy for handling various data formats and demand of real-time responses. Recent researches suggest that the chaos-based image encryption schemes are highly efficient concerning speed and security for traditional cryptographic schemes [2].

Further, the properties of chaotic maps, such as being sensitive to the initial situation and system parameter, ergodicity, property of pseudorandom, and non-periodic and topological mixing, meet the cryptographic requirements. The idea of chaotic image encryption indicates the capability of chaotic maps of producing a pseudorandom number of sequences based on initial conditions and control parameters. Little variation in them provides an entirely different set of random numbers [2]. In addition, due to the recent advances in DNA technologies, DNA computing has entered the domain of cryptography, mainly in the field of image encryption [3].

This paper proposes an encryption method for a color image by combining DNA encoding and a new chaotic map that controls the encryption process, namely generating the key and shuffling the pixel position.

The paper is structured as follows: in section 2, related work is presented. Section 3 explains chaotic maps and DNA operations. In section 4, the proposed RGB image encryption and decryption are described in detail. Section 5 clarifies the evaluation of the results concerning the security analysis of the image. Finally, conclusions are given in section 6.

2. Related work
A considerable amount of literature has been published on gray and color image encryption. Xiaoling Huang, in 2012, suggested an image encryption algorithm that generates a key by utilizing Chebyshev chaotic map. The performance of the algorithm showed that the keyspace was large and a slight change in initial settings influenced the performance [4]. Wu et al., in 2012, proposed an image encryption method using two-dimensional logistic chaotic maps and permutation-substitution network structure. The results showed that the proposed method could produce a random cipher image and withstand several attacks including the statistical and the differential attacks [5]. Zhang et al., in 2014, proposed an enhanced algorithm for encrypting an image via DNA encoding, 2D logistic and wavelet chaotic map. The chaotic system was used to disturb the locations and the value of the pixel. Then, DNA encoding was carried out. The experimental results clarified that the algorithm increases image security by providing a large keyspace, along with its capability to counter attack statistical and exhaustive attacks [6]. Zhou et al., in 2015, suggested an image encryption algorithm via skew tent map and line map for shuffling. The proposed algorithm was implemented in parallel to obtain a high performance regarding speed. The results illustrated the robustness of the proposed algorithm against a chosen plaintext attack [7]. Kumar et al., in 2015, suggested a new procedure that used a multiple of chaotic maps for diffusion an image. The results illustrated that the suggested algorithm was appropriate for encrypting an image with high security [8]. Niyat et al., in 2015, used DNA sequence operation, a hyper-chaotic system, namely the 1D logistic map, and sine map for color image encryption, as well as the Chen hyper chaotic system for shuffling. The proposed algorithm results indicated that it could counter statistical analysis and exhaustive attacks [9]. Jain and Rajpal, in 2016, proposed an image encryption algorithm using DNA and 1D and 2D chaotic logistic maps. A 1D chaotic map was used for producing a mask matrix. Then, DNA addition operation and 2D chaotic map were applied to permute the image. The results clarified that the algorithm is robust against known plaintext attack, statistical attacks, and differential attacks [10]. Liu and Miao, in 2016, proposed an image encryption algorithm via logistic chaotic map using a varying parameter that was used for shuffling an image. After that, a dynamical algorithm was employed for encrypting the image. The results clarified that the algorithm provides high security and is competitive with several image encryption algorithms [11]. Rostami et al., in 2017, proposed a parallel image encryption algorithm by chaotic windows based on the 1D logistic map. The image was divided into 16 × 16 blocks. Then, the XOR operation between a chaotic window and these blocks was performed to produce an encrypted image. The results showed that the algorithm was able to withstand the statistical attacks, brute force
attack, differential attack, chosen-plaintext attack, and chosen-ciphertext attack [12]. Zahmoul et al., in 2017, suggested a Beta chaotic map to generate chaotic sequences that were intended for the encryption. Different pseudo-random sequences were generated to shuffle the image pixels position and unclear the relationship between the encrypted and the original image. The proposed algorithm was qualified for thwarting many attack [13]. Niyat et al., in 2017, suggested color image encryption algorithm using hyper-chaotic system and cellular automata. Security analysis showed that the proposed method has a considerable keyspace and is able to withstand against noise and attacks. Adjacent pixels correlation in the encrypted image was decreased, and the quantity of entropy was equal to 7.9991 [1]. Chai et al., in 2017, suggested a chaos-based image encryption algorithm with DNA operations. First, a new wave-based permutation was applied to the plain image after converting it to DNA. Then, a 2D Logistic chaotic was used for the column circular permutation and row circular permutation of the DNA matrix. The security analysis proved that the algorithm provided a larger secure keyspace and high sensitivity to the secret. Experimental results showed that the algorithm is capable of resisting differential, noise entropy, known-plaintext, occlusion, and chosen-plaintext attacks [14]. Wu et al., in 2018, presented a new lossless and robust color image encryption algorithm employing DNA operation and one-way coupled-map lattices. Firstly, the DNA encoding rules were applied to the three components of the plain image to get three DNA matrices. After that, XOR operation was applied to the DNA matrices for two times. Finally, the cipher image was obtained by a diffusion process. The results proved the robustness of the proposed algorithm against histogram equalization, noise adding, JPEG compression, contrast adjustment and cropping [15]. Elamrawy et al., in 2018, suggested an image encryption algorithm employing DNA encoding followed by a 2D logistic chaotic map. The plain image was encoded to DNA coding and the 2D logistic map was used to permute the DNA coded image. The encrypted image was diffused by the 2D logistic map. The results demonstrated that the algorithm provided high entropy and a small correlation coefficient [16]. Girdhar and Kumar, in 2018, introduced a robust color image encryption by combining Lorenz-Rossler chaotic map and DNA encoding. Lorenz and Rossler chaotic map was employed to produce the arbitrary sequence, whereas the rules of DNA were used to encode the plain image. Moreover, cross-channel operation was applied to the plain image. The results revealed that the proposed algorithm provided superior correlation coefficient than the existing algorithms; the key sensitivity was high and the keyspace was large to resist exhaustive attacks [17].

3. DNA sequence operations and chaotic maps
This section presents the basic background related to DNA sequence operations and chaotic maps that are utilized in the proposed image encryption.

3.1 DNA rules and operations
DNA is a form of natural macromolecule which consists of nucleotides that hold a separate base. Four essential nucleic acids are employed to make the DNA sequence. These are Adenine (A), Cytosine(C), Guanine (G) and Thymine (T). A is constantly associates with T, while C regularly associates with G. The total numbers of potential combinations are twenty-four, while just eight characterize the complementary rule, as shown in Table-1. DNA encoding includes describing each nucleotide with a binary number that follows the complementary rule [2].

| DNA Sequence | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------|---|---|---|---|---|---|---|---|
| 00           | A | A | C | G | C | G | T | T |
| 01           | C | G | A | A | T | T | C | G |
| 02           | G | C | T | T | A | A | G | C |
| 11           | T | T | G | C | G | C | A | A |

The subtraction and addition of DNA are achieved over modulo 4. Table-2 reports the addition and subtraction operations of DNA when rule one is utilized [3]. Table-3 shows the DNA-XOR operation [3].
3.2 Chaotic maps
Sine chaotic map and Beta chaotic map are utilized in this paper. Sine map is described by Equation 1 [9]:

\[ y_{n+1} = r \sin(\pi y_n) \]  

Where \( r \in (0, 4) \), \( y_n \in (0, 1) \), \( n = 0, 1, 2, \ldots \)

Equation 2 defines the beta chaotic map which is used in numerous applications, such as image compression and image object detection [15]

\[ \text{Beta}(x, x_1, x_2, x_c, p, q) = \begin{cases} (x - x_1)^p (x_2 - x_c)^q & \text{if } x \in [x_1, x_2] \\ 0 & \text{otherwise} \end{cases} \]  

Where

\[ x_c = \frac{(px_2 + qx_1)}{(p+q)} \]  
\[ p = b_1 + c_1 \times a \]  
\[ q = b_2 + c_2 \times a \]

4. The proposed image encryption
In this section, the components of the proposed image encryption method that integrates DNA sequence and chaotic maps are illustrated.

4.1 Key sequence generation
An essential process for the proposed encryption method is the key sequence generation via chaotic maps. A chaotic map is used for generating the key sequence because it is sensitive to initial state and control parameters. The initial values of the chaotic maps are generated by using a secret key of 128-bit \( K = \{k_1, k_2, \ldots k_{32}\} \), where \( k_i \) represents a 4-bit hexadecimal digit.

The Beta map with some modification is suggested to formulate a new Beta chaotic map, as shown in Equation 6. The suggested equations in 7, 8 and 9 are used to initialize the parameters of new Beta chaotic maps. Each equation is divided by \( 2^{23} \) for normalization purposes.

\[ \forall n, n \in \{0, 1, 2, \ldots \}, \text{newBeta}(x_n, x_s, x_e, x_c, p, q) = k \times \left((x_n - x_1)^p (x_2 - x_c)^q \right) \mod 1 \]  

\[ x_0 = \frac{k_1 k_2, k_{10}}{2^{23}} \mod 1 \]  
\[ x_s = \frac{k_11 k_{12}, k_{20}}{2^{23}} \mod 1 \]  
\[ x_e = \frac{k_21 k_{22}, k_{32}}{2^{23}} \mod 1 \]

In addition, the initial values \( (y_0 \text{ and } r) \) for sine chaotic map are obtained from the suggested equations in 10 and 11.

\[ y_0 = (x_0 + x_s + x_e) \mod 1 \]  
\[ r = 3.9 + \frac{(x_s + x_e)}{10} \mod 1 \]
The mixture of Sine chaotic map in Equation 1 and new Beta chaotic map in Equation 6 are proposed to generate a sequence, $z_n$, as formulated in Equation 12:

$$\forall n, n \in \{0, 1, 2, \ldots\}, z_n = (x_n + y_n) \mod 1$$

Three chaotic sequences, $S_x = \{x_1, x_2, \ldots, x_{M \times N}\}$, $S_y = \{y_1, y_2, \ldots, y_{M \times N}\}$ and $S_z = \{z_1, z_2, \ldots, z_{M \times N}\}$ are generated using Equations 13, 14 and 15, respectively.

$$\forall i, 1 \leq i \leq M \times N$$

$$x_i = [(x_i \times 10^{14}) \mod 256]$$

$$y_i = [(y_i \times 10^{14}) \mod 256]$$

$$z_i = [(z_i \times 10^{14}) \mod 256]$$

$M$ is the length of the image.

After that, Rule 1 in Table-1 is used to transform the three sequences $S_x$, $S_y$ and $S_z$ to DNA sequence. Then, three DNA encoded matrices, $S_{xe}$, $S_{ye}$ and $S_{ze}$, are obtained by performing DNA XOR operation, as follows:

$$S_{xe} = S_{xe} \oplus S_{yc}$$

$$S_{ye} = S_{yc} \oplus S_{zc}$$

$$S_{ze} = S_{zc} \oplus S_{ye}$$

(16)

### 4.2 Image encryption via DNA sequence and chaotic map

Confusion and diffusion are two important properties that should be satisfied by the proposed method. These properties are achieved by changing each pixel value of the plain image $I$ by DNA operation and shuffling image pixels using a new Beta chaotic map.

The first level of DNA confusion that is satisfied by DNA addition operation is clarified as follows:

**Step1:** the plain image $I$ is decomposed into three components $R(M, N)$, $G(M, N)$ and $B(M, N)$.

**Step2:** contents of $R, G$ and $B$ matrices that hold numbers in $[0, 255]$ are transformed to binary.

**Step3:** rule 1 in Table-1 is applied for yielding 3 DNA coding matrices $R_e, G_e$ and $B_e$.

**Step4:** DNA addition operation is applied for producing 3 DNA encoding matrices $R_e, G_e$ and $B_e$, as follows:

$$R_e = R_e + G_e$$

$$G_e = G_e + B_e$$

$$B_e = B_e + G_e$$

(17)

After performing the first level of confusion, the DNA matrices $R_e, G_e$ and $B_e$ are shuffled (diffusion level) to exclude the correlation among the neighboring by using a new Beta chaotic map. In the new Beta chaotic map, a sequence $x$ of a size $4MN$ is generated, where $M$ is image width and $N$ is image height. Then, the inverse of the decimal part is conserved and the integer part is eliminated for shuffling each value of $R_e, G_e$ and $B_e$ matrices. Next, the sequence $x$ is arranged in an ascending order to confirm that all indexes are generated.

$$[x_{new}, ind] = sort(x)$$

(18)

Where $x_{new}$ is the sorted sequence and $ind$ is the index of the element.

Finally, $R_e, G_e$ and $B_e$ are shuffled as follows

$$\forall i, 1 \leq i \leq M \text{ and } \forall j, 1 \leq j \leq 4N \text{ and}$$

$$R_s(i, j) = R_e(ind(i), x_{new}(j))$$

$$G_s(i, j) = G_e(x_{new}(i), ind(j))$$

$$B_s(i, j) = B_e(ind(i), ind(j))$$

(19)

(20)

(21)

The operation of DNA XOR between the produced sequences $S_{xe}, S_{ye}$ and $S_{ze}$ and $R_s, G_s, B_s$ is applied to produce an encrypted image $C$, as follows

$$C_R = R_s \oplus S_{xe}$$

$$C_G = G_s \oplus S_{ye}$$

$$C_B = B_s \oplus S_{ze}$$

(22)
The decryption process for an image is like the encryption process; however, the stages are handled in the reverse order.

5. Experimental results
The Signal and Image Processing Institute (SIPI) data set, maintained by the University of South California (USC), is used as an evaluation data to evaluate the performance of the proposed method [18]. The results of encryption and decryption of Lena (512 × 512), Baboon (256 × 256), and Peppers (256 × 256) are depicted in Figure-1. The figure clarifies that any information cannot be retrieved from the cipher images. In other words, the proposed method provides good encryption. All the results are obtained by setting the new Beta parameters as follows

\[ b_1 = 3, \quad b_2 = 5, \quad c_1 = 1, \quad c_2 = -1, \quad a = 0.7 \quad \text{and} \quad k = 0.99 \]

![Plain Image](image1) ![Encrypted Image](image2) ![Decrypted Image](image3)

**Figure 1-** Encryption and decryption of images by the proposed method.

5.1 Evaluation concerning MSE and PSNR
Mean Square Error (MSE) and Peak Signal-To-Noise Ratio (PSNR) metrics are used to assess the performance of the proposed encryption method. MSE measures the distinction between plain image and cipher image, as in Equation 23 [15].

\[
MSE = \frac{1}{MN} \sum_{i=0}^{M} \sum_{j=0}^{N} (I(i,j) - C(i,j))^2
\]  
(23)

Where

\( I(i,j) \) is plain image pixel value, \( C(i,j) \) is cipher image pixel value, \( M \) and \( N \) are the dimensions of the image.

The mathematical demonstration of PSNR is as in Equation 24 [3].

\[
PSNR = 20 \log_{10} \left( \frac{255}{\sqrt{MSE}} \right)
\]  
(24)

The MSE and PSNR values of cipher images and their corresponding decrypted images are reported in Tables- (4 and 5). The results reveal that the cipher image and the plain image are different, as explained by the large value of MSE and small value of PSNR.
Table 4-MSE value of the proposed method

| Image  | MSE Component of Image |
|--------|------------------------|
| Lena   | R | 10642.4271 | 9067.8538 | 7109.7853 | Average 8940.0221 |
| Baboon | G | 8614.0818 | 7863.5261 | 9578.9543 | 8685.5207 |
| Peppers| B | 8020.5557 | 11224.0195 | 11136.6674 | 10127.0809 |

Table 5-PNSR value of the proposed method

| Image  | PSNR (dB) Component of Image |
|--------|-----------------------------|
| Lena   | R | 7.8604 | 8.5558 | 9.6122 | Average 8.6761 |
| Baboon | G | 8.7787 | 9.1746 | 8.3176 | 8.7570 |
| Peppers| B | 9.0888 | 7.6293 | 7.6633 | 8.1271 |

5.2 Evaluation concerning keyspace

The keyspace size illustrates the strength of the proposed method to stand against the brute-force attack. The parameters $x_0, x_a, x_b, k$ of the new beta map and $r$ and $y_0$ of sine map are represented in a secret key. In the proposed method, the secret key consists of the sine map and newBeta parameters. Hence, the keyspace of the proposed method is $(10^{143} \approx 2^{140}) \times 2^{24} \times 2^{128} = 2^{292}$ that seems to be sufficient for countering the brute force attack.

5.3 Evaluation concerning statistical attack

The suggested method for encryption is assessed in terms of histogram analysis, entropy ($H$) and correlation coefficient for demonstrating the ability of the suggested method for determining the statistical attack.

$$H(m) = -\sum_{i=0}^{L} p(m_i) \log_2 p(m_i)$$ (25)

Where $L = 255$, $m_i$ is the $i^{th}$ pixel value of an image, and $p(m_i)$ is the $m_i$ probability.

Correlation coefficient (CC) is calculated as in Equation 26, which measures the relationship of adjacent pixels [1].

$$r_{xy} = \frac{\text{cov}(x,y)}{\sqrt{D(x)D(y)}}$$ (26)

$$E(x) = \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$D(x) = \frac{1}{N} \sum_{i=1}^{N} (x_i - E(x_i))^2$$

$$\text{cov}(x,y) = \frac{1}{N} \sum_{i=1}^{N} (x_i - E(x_i))(y_i - E(y_i))$$

Where $x$ and $y$ are two neighboring pixels values in the image, $D(x)$ is the variance, and $N$ is the number of the chosen neighboring pixels of the image.

The histograms of red, green and blue components of encrypted Lena image are depicted in Figure-2. The results demonstrate that the cipher image histogram does not provide any information and is very flat.

![Figure 2-Red, green and blue components histogram of cipher images](image-url)
The cipher image entropy of the proposed method is nearer to 8 and somewhat better than that shown by previous works [1, 8, 15 and 17], as reported in Table-6. This points out that the proposed method provides high randomness, while the probability of inferring any information is too slight, which indicates that the proposed method can resist the statistical attack.

Table 6-The proposed method entropy compared vs. that of previously reported works [1, 8, 15 and 17]

| Method            | Name of Image | Component of Image | R     | G     | B     | Average |
|-------------------|---------------|--------------------|-------|-------|-------|---------|
| Proposed Method   | Lena (512x512)|                    | 7.9993| 7.9992| 7.9993| 7.9993  |
|                   | Lena (256x256)|                    | 7.9972| 7.9971| 7.9972| 7.9972  |
|                   | Baboon        |                    | 7.9972| 7.9971| 7.9972| 7.9972  |
|                   | Peppers       |                    | 7.9973| 7.9975| 7.9976| 7.9975  |
| [1]               | Baboon        |                    | 7.9972| 7.9972| 7.9972| 7.9972  |
| [8]               | Baboon        |                    | 7.9973| 7.9968| 7.9976| 7.9972  |
| [15]              | Lena (256x256)|                    | 7.9895| 7.9894| 7.9894| 7.9894  |
|                   | Baboon        |                    | 7.9899| 7.9896| 7.9888| 7.9894  |
|                   | Peppers       |                    | 7.9898| 7.9886| 7.9894| 7.9893  |
| [17]              | Lena (256x256)|                    | 7.9974| 7.9969| 7.9979| 7.9974  |
|                   | Baboon        |                    | 7.9970| 7.9972| 7.9973| 7.9972  |
|                   | Peppers       |                    | 7.9974| 7.9973| 7.9969| 7.9972  |

The correlation coefficient results that are obtained by randomly selecting 1000 pairs of neighboring pixels in three directions from the original image and the corresponding cipher image are reported in Tables- (7-9). The results show that the proposed method is capable of fighting the statistical attacks better than the other previously reported works [1, 8, 9, 15 and 17].

Table 7-Vertical correlation coefficient of the proposed method vs. that of previously reported works [1, 8, 9, 15 and 17]

| Method          | Name of Image | Component of Image | R     | G     | B     | Average |
|-----------------|---------------|--------------------|-------|-------|-------|---------|
| Proposed Method | Lena (512x512)|                    | -0.0742| 0.0009| 0.0738| 0.0002  |
|                 | Baboon        |                    | 0.0278| -0.0363| 0.0090| 0.0002  |
|                 | Peppers       |                    | -0.0041| 0.0102| -0.0065| -0.0001 |
| [1]             | Baboon        |                    | -    | -    | 0.0022| 0.0018  |
| [8]             | Lena          |                    | -0.0099| 0.0126| 0.0063| 0.0030  |
| [9]             | Baboon        |                    | 0.0744| 0.0788| 0.0748| 0.0760  |
| [15]            | Lena (256x256)|                    | -0.0026| 0.0199| 0.0120| 0.0098  |
| [17]            | Lena (256x256)|                    | 0.0026| 0.0009| -0.0030| 0.0009  |
|                 | Baboon        |                    | -0.0007| 0.0039| 0.0061| 0.0050  |
|                 | Peppers       |                    | 0.0023| 0.0053| 0.0005| 0.0027  |

Table 8-Horizontal correlation coefficient of proposed method vs. that of previously reported works [1, 8, 9, 15 and 17]

| Method          | Name of Image | Component of Image | R     | G     | B     | Average |
|-----------------|---------------|--------------------|-------|-------|-------|---------|
| Proposed Method | Lena (512x512)|                    |       |       |       |         |
|                 | Baboon        |                    |       |       |       |         |
|                 | Peppers       |                    |       |       |       |         |
| [1]             | Baboon        |                    |       |       |       |         |
| [8]             | Lena          |                    |       |       |       |         |
| [9]             | Baboon        |                    |       |       |       |         |
| [15]            | Lena (256x256)|                    |       |       |       |         |
| [17]            | Lena (256x256)|                    |       |       |       |         |
Table 9—Diagonal correlation coefficient of proposed method vs. that of previously reported works [1, 8, 9, 15 and 17]

| Method          | Name of Image | Component of Image | Average |
|-----------------|---------------|--------------------|---------|
| Proposed Method | Lena (512x512) | -0.0063 -0.0152 0.0211 -0.0001 |
|                 | Baboon        | -0.0198 0.0276 -0.0068 0.0003 |
|                 | Peppers       | 0.0624 -0.0084 -0.0537 0.0001 |
| [1]             | Baboon        | -0.0007 -0.0017 0.0007 -0.0010 |
|                 | Peppers       | 0.0007 -0.0017 0.0007 -0.0010 |
| [8]             | Lena          | 0.0085 0.0127 -0.0155 0.0019 |
| [9]             | Baboon        | 0.0733 0.0687 0.0701 0.0707 |
| [15]            | Lena (256x256) | 0.0052 -0.0064 -0.0161 -0.0058 |
|                 | Baboon        | 0.0015 0.0015 0.0025 0.0018 |
|                 | Peppers       | 0.0004 -0.0008 0.0008 0.0006 |

5.4 Evaluation concerning the differential attack

Number of Pixels Change Rate (NPCR) and Unified Average Changing Intensity (UACI) that are calculated in Equation 27 and 28, respectively, are used to show the effect of changing one pixel in the plain image on the cipher image [1].

\[
UACI(C1, C2) = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} |C1(i,j) - C2(i,j)|}{255} \times 100 \tag{27}
\]

Where

\[M \text{ and } N \text{ are image width and height, respectively.}\]

and

\[C1 \text{ and } C2 \text{ are encrypted images of the plain image and the modified one.}\]

\[
NPCR(C1, C2) = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} D(i,j)}{M \times N} \times 100 \tag{28}
\]

Where

\[D(i,j) = \begin{cases} 
0, & \text{if } C1(i,j) = C2(i,j) \\
1, & \text{if } C1(i,j) \neq C2(i,j) 
\end{cases}\]

A comparison between the proposed method in this study and the methods presented previously [1, 8, 15 and 17] based on NPCR and UACI is reported in Tables- (10 and 11). Table-10 shows that the NPCR of our proposed methods is close to 100% and UACI is more close to 33.58 than the values of the previously reported methods. This indicates that the sensitivity of the suggested method for the alteration of the plain image is high. In addition, the competence of resisting the differential attack and plaintext attack of the suggested method is more amenable than that reported by the previously reported methods.
Table 10-NPCR of the proposed method vs. that of previously reported works [1, 8, 15 and 17]

| Method          | Name of Image       | Component of Image | Average NPCR |
|-----------------|---------------------|--------------------|--------------|
| Proposed Method | Lena (512x512)      | R 99.6010          |              |
|                 | Lena (256x256)      | G 99.5998          | 99.6178      | 99.6062 |
|                 | Baboon              | B 99.6231          |              |
|                 | Peppers             |                   | 99.6674      | 99.6424 |
| [1]             | Baboon              | R 99.6536          |              |
|                 | Peppers             | G 99.6078          | 99.6520      | 99.6378 |
| [8]             | Lena (256x256)      | B 99.6277          |              |
|                 | Baboon              |                   | 99.6368      | 99.6292 |
|                 | Peppers             | R 99.6170          |              |
|                 |                     | G 99.6323          | 99.6384      | 99.6292 |
| [15]            | Lena (256x256)      |                     | 99.6277      | 99.6075 |
|                 | Baboon              |                     | 99.6252      | 99.6004 |
|                 | Peppers             |                     | 99.6286      | 99.6073 |
| [17]            | Lena (256x256)      |                     | 99.6230      | 99.6270 |
|                 | Baboon              |                     | 99.5510      | 99.5903 |
|                 | Peppers             |                     | 99.5530      | 99.5910 |

Table 11-UACI of the proposed method vs. that of previously reported works [1, 8, 15 and 17]

| Method          | Name of Image       | Component of Image | Average UACI |
|-----------------|---------------------|--------------------|--------------|
| Proposed Method | Lena (512x512)      | R 33.4670          |              |
|                 | Lena (256x256)      | G 33.4795          | 33.4730      | 33.4731 |
|                 | Baboon              | B 33.4268          |              |
|                 | Peppers             |                   | 33.5998      | 33.5440 |
| [1]             | Baboon              | R 33.4753          |              |
|                 | Peppers             | G 33.5090          | 33.4176      | 33.4673 |
| [8]             | Lena (256x256)      | B 33.4570          |              |
|                 | Baboon              |                   | 33.4705      | 33.4566 |
|                 | Peppers             | R 33.4570          |              |
|                 |                     | G 33.3459          | 33.327       | 33.3186 |
| [15]            | Lena (256x256)      |                     | 33.4280      | 33.4342 |
|                 | Baboon              |                     | 33.4966      | 33.4379 |
|                 | Peppers             |                     | 33.4311      | 33.4935 |
| [17]            | Lena (256x256)      |                     | 33.4959      | 33.4302 |
|                 | Baboon              |                     | 33.4350      | 33.4342 |
|                 | Peppers             |                     | 33.4874      | 33.4372 |

5.5 Evaluation concerning the noise attack

High secure cryptosystems must be resistant to all kinds of noise during transmissions, such as salt and pepper and Gaussian noise. The salt and pepper noise with five different densities $d=\{0.05,0.10,0.15,0.20,0.50\}$ and Gaussian noise with zero mean and five different variances $\{0.00001,0.0001,0.001,0.01,0.1\}$ are added to Lena cipher image. Table-12 clarifies quantitatively the MSE, PSNR and differences percentage between the Lena image and decrypted image, under salt and pepper noise and Gaussian noise, with different levels of noise for the proposed method.

Table 12-MSE, PSNR and differences between plain and decrypted images under salt and pepper noise and Gaussian noise with different levels of noise for proposed method

| Noise Type     | Ratio | MSE       | PSNR     | Differences% |
|----------------|------|-----------|----------|--------------|
| Salt & pepper  | 5%   | 986.1228  | 18.1915  | 8.53         |
|                | 10%  | 1912.9160 | 15.3138  | 16.52        |
|                | 15%  | 2763.7078 | 13.7159  | 23.90        |
|                | 20%  | 3517.5386 | 12.6684  | 30.46        |
|                | 50%  | 6983.4256 | 9.6901   | 60.58        |
Figure-3 depicts the Lena encrypted image, under salt and pepper noise with different five densities, and the corresponding decrypted image. Figure-4 shows the Lena encrypted image, under Gaussian noise with different four variances, and the corresponding decrypted image. From these figures, it can be noted that the decrypted Lena image can be recolonized.
128 × 128, 256 × 256 and 256 × 512) are cropped from the Lena cipher image and the corresponding decrypted images. There is 50% of data lost but it is clear that the decrypted image is still recognizable and contains the most important visual information, which implies that the proposed method is robust against the cropping attacks.

| 32 × 32 | 64 × 64 | 128 × 128 | 256 × 256 | 256 × 512 |
|---------|---------|-----------|-----------|-----------|
| ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png) | ![Image](image4.png) | ![Image](image5.png) |

**Figure 5**-Cipher and decrypted images under cropping attack with different cropping window size (32 × 32, 64 × 64, 128 × 128, 256 × 256 and 256 × 512, respectively) for the proposed method.

5.7 Evaluation concerning the key sensitivity analysis

Good image encryption should be sensitive to a slight change in the secret key. Keys sensitivity can be observed in the proposed methods by modifying a single bit in the secret key and a minor alteration is performed for chaotic maps parameters. In this experiment, some of the secret keys are modified, where only one parameter is modified at each time.

Suppose the secret key \( K \) and the chaotic parameters \( y = 0.0180513858795166, r = 3.97053072452545 \) and \( x_2 = 0.590390682220459 \) are used to encrypt Lena image by the proposed method. Figure-6b qualitatively depicts the decryption of the encrypted Lena image when the secret key \( K \) is modified to \( K' \) while keeping the other keys, \( r, y \) and \( x_2 \) unchanged. Moreover, the parameter \( y \) is changed to \( y' = 0.0180513858795167 \) and the decryption process is depicted in Figure-6c. Also, the \( r \) parameter is changed to \( r' = 3.97053072452544 \) and the decryption of the encrypted image is shown in Figure-6d. Finally, the \( x_2 \) parameter is changed to \( x'_2 = 0.590390682220458 \) and the decryption process is shown in Figure-6e.

![Image](image6.png)

**Figure 6**-Key sensitivity results for the proposed method. a. Lena image. b. The decrypted image with \( K' \). c. The decrypted image with \( y' \). d. The decrypted image with \( r' \). e. The decrypted image with \( x'_2 \)
From Figure-6, it can be noted that, in all cases, the decryption process is not capable of recovering the original image when a slight change was performed to the secret key. This indicates that the proposed methods are extremely sensitive to the secret keys.

5.8 NIST test
The statistical tests suite provided by the National Institute of Standards and Technology (NIST) are used to evaluate the randomness of the proposed methods. The $p$-value for statistical tests is compared with the significance level, $\alpha$, that is set to 0.001. If the value of the test is greater than $\alpha$ then the method is passed the test. Otherwise, it fails to pass the test. Table-13 reports the $p$-value of ten statistical tests for Lena, Baboon and Peppers of the proposed method. The results reveal that the proposed method achieves a high security level by successfully pass all statistical tests of the NIST test.

| Test Name                  | Lena   | Baboon  | Peppers  | Pass |
|----------------------------|--------|---------|----------|------|
| Frequency                  | 0.911413| 0.253551| 0.602458 | ok   |
| Block Frequency            | 0.122325| 0.911413| 0.911413 | ok   |
| Cumulative Sums            | 0.949602| 0.949602| 0.148094 | ok   |
| Runs                       | 0.739918| 0.213309| 0.213309 | ok   |
| Longest Run                | 0.671779| 0.804337| 0.804337 | ok   |
| Rank                       | 0.253551| 0.082177| 0.213309 | ok   |
| FFT                        | 0.739918| 0.350485| 0.468595 | ok   |
| Non Overlapping Template   | 0.991468| 0.976060| 0.976060 | ok   |
| Serial                     | 0.468595| 0.468595| 0.602458 | ok   |
| Linear Complexity          | 0.534146| 0.671779| 0.911413 | ok   |

6. Conclusions
This paper suggests a new encryption method for color image. Two chaotic maps including sine map and the proposed new Beta chaotic map are used to produce the key. Moreover, the diffusion and confusion properties are fulfilled by utilizing DNA operations and the proposed new Beta chaotic map, respectively. From the results presented, we can confirm the sensitivity of the proposed method to a slight change in the secret key and that it can counter several attacks, namely brute force attack, statistical attack, differential attack, plaintext attack, noise attack and cropping attack.

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