Optimization of anti-bending deformation of aerogel sandwich thermal protection structure

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Abstract. The failure of the thermal protection structure is very important to the safety of the aircraft structure. In order to accurately predict the flexural behavior of the aerogel sandwich thermal protection structure, first of all, for the sandwich structure in the form of statically indeterminate beam with variable cross-section, the beam theory considering the transverse shear deformation of the core layer was deduced. Secondly, by comparing the deflections, the forms of deflection curve and the maximum strains of the upper panel, the accuracy of the theoretical prediction was analyzed. Then, according to the requirements of heat protection and integrity of the thermal protection structure, the optimization analyses of the parameters of the core layer, such as the thickness, the density and the elastic modulus, were carried out. The results showed that the beam theory considering the transverse shear deformation of the core layer had better prediction accuracy. The maximum deflection error was the smallest, which was -17%, and the form of the deflection curve was the most reasonable. Within the range of the physical parameters of the core layer studied, the theoretical basis and guiding direction were given for the regulation of the performance of the aerogel core layer.

1. Introduction

As the outermost structure of the aircraft, the thermal protection structure is used to protect the internal structure and payload of the spacecraft\textsuperscript{[1]}. Its heat resistance and bearing capacity directly affect the safety of the entire system. Since the manufacture of aerogel by Kistler\textsuperscript{[2]}, its excellent thermal insulation properties\textsuperscript{[3]} have been attracting attention. With the improvement of the mechanical properties of aerogel\textsuperscript{[4]}, the sandwich structure composed of aerogel core layer and high temperature resistant mullite fiber panel has become a promising aerospace thermal protection structure.

During the service of the thermal protection structures, they will be subjected to loads such as aerodynamic force, aerodynamic heat, aerodynamic noise, and propulsion pulse force\textsuperscript{[5]}. One of the main mechanical behaviors of thermal protection structures under multi-load coupling is flexural deformation. Therefore, evaluating the integrity of such sandwich structures under flexural deformation is a critical task. The classical thin plate theory does not consider the transverse shear deformation of the structures. When the thickness cannot be ignored, the transverse shear deformation needs to be considered. The bending linear theories of sandwich panels commonly used in engineering are mainly Reissner theory\textsuperscript{[6]}, Hoff theory and Pycakob theory. Sawhney Himanshu\textsuperscript{[7]} reduced the three main unknown functions in Mindlin first-order shear deformation theory to two, and proposed a new simple and effective first-
order shear deformation theory. The ideas of classical thin plate theory and Reissner plate theory are echoed by Euler-Bernoulli beam theory and Timoshenko beam theory when the bending of plates is studied in a two-dimensional plane.

This paper established a theory considering the transverse shear deformation of the core layer and analyzed its advantages. The parameters of the aerogel core layer were optimized and analyzed. It could provide theoretical basis and direction for guiding the optimal design of thermal protection structures.

2. Beam theory

The laminated structure is designed as the model shown in Fig.1(a). The parts of the model from top to bottom are the upper panel, the aerogel core layer, the lower panel, the glue layer and the metal plate. As shown in Fig.1(b), both ends of the structure are fixed ends, and a \( z \)-direction concentration line distribution load \( F \) is applied along the width direction in the midspan.

Compared with the panels and the metal plate, the shear modulus of the aerogel core layer is much smaller, and the thickness is much larger. The transverse shear deformation cannot be ignored. It is calculated that the neutral layer is located inside the metal plate using the material parameters in Table 1. Therefore, the subsequent analyses assume that the neutral layer is inside the metal plate.

The cross-section coordinate system is established as shown in Fig.1(a), and the \( z \)-coordinate origin is located on the neutral layer of the cross-section. Let the thicknesses of the upper panel, the core layer, the lower panel, the glue layer and the metal plate be respectively \( t_u, t_c, t_d, t_g, t_m \), the elastic moduli be respectively \( E_u, E_c, E_d, E_g, E_m \), the shear modulus of the core layer be \( G_{xz} \), and the cross-sectional width be \( b \).

As shown in Fig.1(c), the dashed line \( \omega_2(x) \) represents the position of the neutral layer of the sandwich part beam, and the dotted line \( \omega_1(x) \) represents the neutral layer position of the metal plate without the sandwich part. However, due to the change of the position of the neutral layer at the variable section, the \( x \)-coordinates of the deflection curves \( \omega_1(x) \) and \( \omega_2(x) \) of the overall structure at both ends of the abrupt section are discontinuous with a difference of \( \Delta x \). In order to maintain continuity and facilitate the solution, the deflection curve \( \omega_1(x) \) is solved for the structure containing the sandwich part.

When the neutral layer of the metal plate is taken as the origin of the \( z \)-axis, taking the upper panel as an example, the \( x \)-direction displacement can be expressed as:

\[
u = -(z_3 - \Delta z) \frac{du}{dx} \left( z_4 - z_3 \right) \phi - \left( z - z_4 \right) \frac{du}{dx}, \quad z_4 \leq z \leq z_5
\]

(1)

Where \( \omega \) is the deflection curve of the neutral layer, and \( \phi \) is the corner of the cross-section of the core layer. When only the transverse shear deformation of the core layer is considered, the shear strain is:

\[
\gamma_{xz} = \frac{du}{dz} + \frac{du}{dx} - \phi = \left( \frac{du}{dx} - \phi \right) \quad \text{(core layer)}
\]

\[
0 \quad \text{(panels, glue core and metal plate)}
\]

(2)

Where \( u \) is the \( x \)-direction displacement of any point on the beam section, which can be obtained by referring to Eqs.(1).

The strain energy of the entire structure can be expressed as:

\[
U_1 = \frac{1}{2} \iint \left( E_x \epsilon_x^2 + G_{xz} \gamma_{xz}^2 \right) \, dx \, dy \, dz = \frac{1}{2} \iint \left( E_x \left( \frac{du}{dx} \right)^2 + G_{xz} \gamma_{xz}^2 \right) \, dx \, dy \, dz
\]

(3)
Let \( q = q(x) \) be the distributed load on the beam. \( M(0) \) and \( M(L) \) are the moments at both ends of the beam. \( Q(0) \) and \( Q(L) \) are the shear forces at both ends of the beam. Then when the beam is bent, the potential energy of the external force is:

\[
U_2 = \int_0^L q \phi(x) \, dx + (M(0) - Q(0))_0 \tag{4}
\]

Then the total potential energy of the system is:

\[
U = U_1 + U_2 \tag{5}
\]

Expand Eqs.(5) using the above equations. Make a first variation, then set it to zero. After calculating and arranging, we can get:

\[
\begin{align*}
\int_0^L \left( \frac{d}{dx} \left( 2B \frac{d^2 \phi}{dx^2} + D \phi - 2D \frac{d \phi}{dx} + q \right) \right) \, dx + \int_0^L \left( \frac{d}{dx} \left( 2B \frac{d^2 \phi}{dx^2} + D \phi - 2D \frac{d \phi}{dx} + q \right) \right) \, dx + \int_0^L \left( \frac{d}{dx} \left( 2B \frac{d^2 \phi}{dx^2} + D \phi - 2D \frac{d \phi}{dx} + q \right) \right) \, dx + \int_0^L \left( \frac{d}{dx} \left( 2B \frac{d^2 \phi}{dx^2} + D \phi - 2D \frac{d \phi}{dx} + q \right) \right) \, dx = 0
\end{align*}
\]

Where \( A, B, C, D, E, F \) are constant coefficients. For brevity, omit the expression.

Since \( \delta \omega \) and \( \delta \phi \) are completely arbitrary, the conditions for the above equations to hold are:

\[
\begin{align*}
E \frac{d^4 \phi}{dx^4} = 0 & \quad (0 \leq x \leq d), & F \frac{d^4 \phi}{dx^4} = 0 & \quad (d < x < (d + L)), & M \frac{d^4 \phi}{dx^4} = 0 & \quad (d + L) \leq x \leq L \tag{7}
\end{align*}
\]

\[
\begin{align*}
\frac{d}{dx} \left( 2B \frac{d^2 \phi}{dx^2} + D \phi - 2D \frac{d \phi}{dx} + q \right) \, dx + \frac{d}{dx} \left( 2B \frac{d^2 \phi}{dx^2} + D \phi - 2D \frac{d \phi}{dx} + q \right) \, dx + \frac{d}{dx} \left( 2B \frac{d^2 \phi}{dx^2} + D \phi - 2D \frac{d \phi}{dx} + q \right) \, dx = 0
\end{align*}
\]

3. Precision analysis

According to the research on specific objects, the size of the metal plate is set to 450 mm \( \times \) 100 mm, and the size of the sandwich structure is 300 mm \( \times \) 100 mm. As shown in Fig.1(a), the thickness of each layer from top to bottom is: 2 mm, 26.5 mm, 1 mm, 0.5 mm and 3 mm. The action range of the load \( F \) is set as the width of 10 mm in the middle of the span, and the load concentration is \( q = 16 \) N/mm.

The material parameters of each component of the structure are shown in Table 1.

| Structural components | Upper panel | Aerogel core layer | Lower panel | Glue layer | Metal plate |
|-----------------------|-------------|-------------------|-------------|------------|-------------|
| Elastic Modulus \( E/(\text{MPa}) \) | 4700 | 60 | 4700 | 1 | 118000 |
| Shear modulus \( G/(\text{MPa}) \) | 2020 | 1.5 | 2020 | 0.3571 | 42446 |

Fig.2 Deflection curves of the neutral layer of the metal plate
Establish a finite element model according to the above specific dimensions. All units are modeled with C3D8R three-dimensional solid units, with a total of 374,400 solid units. Both ends of the metal plate are constrained with fixed restraints. Loads are applied to element nodes at midspan on the back of the metal plate using displacement constraints. As shown in Fig.2, in the theoretical results without considering the transverse shear deformation, considering the transverse shear deformation of the core layer and the glue layer, and considering the transverse shear deformation of the structure, the maximum deflection values are -0.258 mm, -1.366 mm and -2.502 mm, respectively. Compared with the simulation result of -1.686 mm, the errors are -85%, -17% and 48%, respectively. It can be seen that the result considering the transverse shear deformation of the core layer and the glue layer are closer to the simulation result. On the whole, the theory considering the transverse shear deformation of core layer and glue layer is more advantageous.

4. Parameter optimization
The parametric analysis is carried out using the theory considering the transverse shear deformation of the core layer. To protect the internal structure and payload of the aircraft, the thermal protection system needs to have a certain thermal insulation and load-bearing capacity. To maximize the performance of the core layer as required under limited conditions, it is necessary to carry out parameter optimization analysis.

(1) The thickness direction of the core layer needs to have a certain thermal insulation capacity. The thermal insulation capability can be represented by the thermal resistance $R$:

$$R = \frac{t}{\lambda}$$  \hspace{1cm} (12)

where $t$ is the thickness of the material and $\lambda$ is the thermal conductivity.

(2) The weight of the core layer must be controlled within a certain range.

(3) Under the same load, the smaller the compressive strain of the upper panel, the better. The higher the flexural stiffness of the core layer, the better.

The relationship of the elastic modulus, density and thermal conductivity of aerogel can be obtained from the literature\(^{[10]}\). It is assumed that the elastic modulus of the aerogel is a function of the density. According to the mechanical test results of the aerogel composites under different fiber additions in the literature, the nonlinear relationship between the elastic modulus $E$ and the density $\rho$ is fitted:

$$E = 0.003543\rho^2 - 0.2393\rho - 18.02$$  \hspace{1cm} (13)

A graph of $\rho$-$\lambda$ is also provided in the literature, so it can be assumed that there is a function $g$ such that $\lambda = g(\rho)$. Combined with Eqs.(12), when the thermal resistance $R$ is fixed as a constant, we can get:

$$t = \varphi(\rho)$$  \hspace{1cm} (14)

From Eqs.(13) and Eqs.(14), it can be known that the thickness and density can be changed by taking the density as the independent variable. The optimization is carried out within the variation range of each physical quantity shown in Table 2. The value of the shear modulus is unknown, and the ratio of the elastic modulus to the shear modulus is temporarily set as $G/E=1/40$. And the density of the upper panel is 1800 kg/m\(^3\).

| Table 2 Variation range of relevant parameters of the core layer |
|---------------------------------------------------------------|
| **Physical quantities** | Density $\rho$ (kg/m\(^3\)) | Thermal conductivity $\lambda$ (W/(mK)) | Core thickness $t_c$ (mm) | Thermal resistance $R$ (°C/W) | Elastic Modulus $E_c$ (MPa) |
|------------------------|-----------------------------|-------------------------------|---------------------|-------------------|---------------------|
| Variation range        | 146-187                     | 0.02682-0.02838               | 13.00-13.76         | 484.7             | 22.4-62.02          |

4.1. Optimization analysis of aerogel density under the constraint of panel compressive strength
The maximum allowable compressive strain of the upper panel is given as 2930 $\mu$ε. Keep the thermal resistance of the structure unchanged, and set the maximum compressive strain of the upper panel to the allowable value. The variation law of the thickness of the core layer, the weight of the core layer and the ultimate load of the structure with the density of the core layer is studied. As shown in Fig.3, the abscissa adopts the dimensionless density. The ratio of the corresponding parameters of the core layer (subscript c) and the upper panel (subscript u) is used for dimensionless processing.
Fig.3  The diagrams with the maximum compressive strain of the upper panel at the allowable value.

It can be seen from Fig.3 that within the research range, as the density increases, the ultimate load gradually decreases, the weight of the core layer keeps increasing, and the thickness of the core layer first decreases and then increases. Based on the changing law, the parameters of the core layer can be determined as needed. For example, 1) It is assumed that the ultimate load bearing requirement has been determined and is below the horizontal dashed line in Fig.3(a). When the minimum structural thickness is sought under the requirement of weight, the core density corresponding to the vertical dotted line can be selected. If there are still requirements for the weight, you can find the design value that satisfies the conditions in Fig.3(b) and Fig.3(c). 2) It is assumed that the density of the core layer has been determined. In order to bear a larger load, within the allowable range, try to choose a larger thickness to meet the strength requirements.

4.2.  Optimization of elastic modulus and shear modulus of aerogel core layer

In the previous analysis, it was assumed that \( G/E = 1/40 \). The influence of the ratio of \( G/E \) on the structural bearing capacity and optimal design is further clarified below.

Fig.4 The ultimate load of different density of core layer varying with \( G/E \) ratios with the maximum compressive strain of the upper panel at the allowable value.

When the maximum allowable compressive strain of the upper panel is the design limit, the change of the structural bearing capacity with the \( G/E \) ratio is shown in Fig.4. Keeping the elastic modulus \( E \) unchanged, with the increase of the \( G/E \) ratio, the ultimate load of the structure decreases significantly at the beginning, and then tends to be flat.

As the shear modulus \( G \) increases, the cross-sectional angle decreases. The compressive strain of the upper panel cannot be released, so that the allowable value of compressive strain is reached at a small deflection, which is the main reason for the phenomenon shown in Fig.4. Besides, when the shear modulus of the core layer tends to infinity, if the compressive strain of the upper panel is used as a design constraint, it will lead to an over-safe design of the structure.

Furthermore, Fig.4 shows the effect of the dimensionless density on the ultimate load. As the density of the core layer increases, the elastic modulus \( E \) and the shear modulus \( G \) of the core layer will increase. When the thickness is constant, the ultimate load of the structure appears to decrease with the increase of the density of the core layer. Although the increase of the elastic modulus helps to improve the flexural stiffness of the structure, under the condition of constant \( G/E \) ratio, the shear modulus \( G \) can also be increased proportionally. Thus, the turning angle of the cross section is limited and the compressive strain of the upper panel increases. Its overall effect makes the ultimate load of the structure decrease with the increase of density of the core layer.
5. Conclusion
Through theoretical derivation, precision analysis and parameter optimization, the following conclusions were obtained:

1. The formula of the beam theory considering the transverse shear deformation of the core layer is derived. Compared with the simulation results, the maximum deflection error of the theory considering the transverse shear deformation of the core layer and the glue layer was the smallest, which was -17%. The deflection distribution was also the most reasonable. So the bending theory had the most advantage.

2. Based on the bending theory considering the transverse shear deformation of the core layer, keep the thermal resistance of the structure unchanged, and set the maximum compressive strain of the upper panel to the allowable value. Within the research range, the bending resistance of the structure was the strongest when the density ratio was the smallest, and at this time, it had a larger thickness and a lighter weight. By analyzing the influence of the $G/E$ ratio on the bearing capacity of the structure, different optimization results were given when the compressive strain of the upper panel was used as constraints. The theoretical basis and guiding direction were given for the regulation of the performance of the aerogel core layer.

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