Failure prediction of AZ31B Mg sheet at room temperature considering material anisotropy and differential work hardening

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Abstract. Failure predictions of AZ31B Mg sheet at room temperature were conducted considering material anisotropy as well as differential work hardening for the range from uniaxial to balanced biaxial loading paths. The maximum shear stress criterion was used as a failure criterion, and both the associated and non-associated flow rules were employed along with the quadratic anisotropic functions to derive the analytic expressions for failure. Different levels of material anisotropy were considered in formulation, and procedures of determining the material coefficients considering the differential work hardening behavior were provided. Failure predictions using the analytic expressions were compared with the experimental results obtained by the hemispherical dome test. The results show that in the failure prediction of AZ31B Mg sheet, both the material anisotropy and the differential work hardening need to be addressed properly to make an accurate failure prediction.

1. Introduction
Magnesium is the lightest metal among the metals being commercially used in industries. Rolled sheet products of Mg alloy show a low formability at room temperature (RT) because of its limited slip system at RT [1]. Additionally, Mg sheets show strong anisotropy [2] as well as a differential work hardening behavior at RT [3,4]. It is quite natural to conjecture that this constitutive behavior of Mg sheet at RT might be closely related to its deformation limit and failure.

In this work, the failure behavior of Mg sheet made of AZ31B alloy, a typical alloy for the rolled sheet products, was investigated. For the failure prediction, analytical expressions were derived for the stress states ranging from uniaxial to balanced biaxial tension. For the onset condition of failure, the maximum shear stress criterion was employed. The condition for yielding under the plane stress condition was determined by the quadratic anisotropic yield criterion proposed by Hill [5]. To accommodate more anisotropy parameters in the analytical expressions of failure, the non-associated flow rule (NASF) [6] was applied to the formulation with a plastic potential based on the quadratic anisotropic function.

To identify the material coefficients contained in the equations of failure prediction, uniaxial tensile tests and bulge tests were conducted. Material coefficients taking into account the differential work hardening of AZ31B Mg sheet were determined. The failure strains of AZ31B Mg sheet under various loading paths were measured by the hemispherical dome test, and compared to the predictions made by the equation.
Note that this paper is a brief summary of the author’s work concerning the failure prediction of AZ31B Mg sheet published before. For more details, refer to the work of Ahn and Seo [7].

2. Formulation
All the formulations in this work are based on the condition that the material symmetry axes are coincident with the directions of principal stresses, and the transverse direction (TD) of the Mg sheet is the material reference direction as well as the major loading direction. Conditions of plane stress were assumed, and the direction 1 of the in-plane principal stress is parallel to the rolling direction (RD) of the Mg sheet.

2.1. Maximum shear stress criterion
Under the plane stress condition, the maximum shear stress is no longer present within 1-2 plane if the both principal stresses are positive. If the increasing loads are proportional, failure occurs, according to the maximum shear stress criterion, as the increasing principal stresses first reach one of the following conditions:

\[
\frac{\sigma_1}{2} = k_{RD}, \quad \frac{\sigma_2}{2} = k_{TD},
\]

where \(k_{RD}\) and \(k_{TD}\) are the critical shear stresses of failure along the RD and TD, respectively. If the direction 2 is the direction of major loading (i.e., \(\sigma_2 > \sigma_1\)) and \(k_{RD}\) is greater than \(k_{TD}\), then the condition \(\frac{\sigma_2}{2} = k_{TD}\) is the criterion for failure.

2.2. Yield criterion and plastic potential
By using the quadratic anisotropic yield criterion proposed by Hill [5], the effective stress \(\bar{\sigma}\) under the plane stress condition can be defined as:

\[
\bar{\sigma} = \frac{1}{\sqrt{X^2 + (1 + \frac{Y^2}{X^2} - \frac{2}{B^2})\sigma_1\sigma_2} + \sigma_1^2} = Y(\bar{\varepsilon}),
\]

where the coefficients \(X\) and \(Y\) are the yield stresses along the RD and TD, respectively, while the coefficient \(B\) is the balanced biaxial yield stress. \(Y(\bar{\varepsilon})\) denotes the yield stress along the TD as a function of the effective strain \(\bar{\varepsilon}\). From the same quadratic anisotropic yield criterion by Hill [5], the plastic potential \(\Phi\) was obtained by using two R-values (width to thickness strain ratios) in the form

\[
\Phi = \frac{1 + R_{00}}{R_0} \sigma_1^2 - 2\sigma_1\sigma_2 + \frac{1 + R_{90}}{R_{90}} \sigma_2^2,
\]

where \(R_{00}\) and \(R_{90}\) denote R-values measured along the RD and TD, respectively.

2.3. Non-associated flow rule
Under the framework of the NAFR, the plastic strain increment is obtained from the gradient of the plastic potential, i.e.,

\[
d\varepsilon_i^p = d\lambda \frac{\partial \Phi}{\partial \sigma_i},
\]
where \( d\lambda \) is the plastic multiplier to define the magnitude of the plastic strain increment. By substituting equation (3) into equation (4), \( d\lambda \) can be obtained in the form

\[
d\lambda = \frac{\sqrt{K_0\beta^2 - 2\beta + K_90}}{K_90 - \beta} \, d\varepsilon_2^p,
\]

where

\[
K_0 = \frac{1 + R_0}{R_0}, \quad K_90 = \frac{1 + R_90}{R_90} \quad \text{and} \quad \beta = \frac{\sigma_1}{\sigma_2}.
\]

Here, the stress ratio \( \beta \) has a range varying from 0 (i.e., uniaxial tension) to 1 (i.e., balanced biaxial tension). Considering the plastic work equivalence principle with equation (4), the following relationships are valid:

\[
dW^p = \sigma d\varepsilon^p \Rightarrow \sigma d\varepsilon^p = \sigma_0 d\varepsilon_0^p = \sigma_1 d\varepsilon_1^p = \sigma_2 d\varepsilon_2^p.
\]

### 2.4. Equations of failure prediction

Integration of equation (7) with the substitution of equations (2), (3) and (5) leads to the relationship

\[
\bar{\varepsilon} = \frac{\Phi}{\sigma} \sqrt{G(\beta)}
\]

where

\[
G(\beta) = \frac{K_0\beta^2 - 2\beta + K_90}{\left[\frac{Y^2}{X^2} - (1 + \frac{Y^2}{X^2} - \frac{Y^2}{B^2})\beta + 1\right] (K_90 - \beta)}.
\]

To fit the material hardening of AZ31B Mg sheet, the Swift equation

\[
\bar{\sigma} = Y(\bar{\varepsilon}) = K(\bar{\varepsilon}_1^p + \bar{\varepsilon}_2^p)^n
\]

was utilized. The coefficients \( K \), \( \bar{\varepsilon}_0^p \) and \( n \) in equation (10) are determined by fitting the uniaxial stress-strain curves measured along the TD. Combining equation (10) with equations (1), (4) and (8) leads to the principal plastic strains at failure as

\[
(\varepsilon_2^p)_f = \frac{Y^2}{X^2} - (1 + \frac{Y^2}{X^2} - \frac{Y^2}{B^2})\beta + 1)^{\frac{1}{n}} \left( \frac{2k_{TD}}{K} \right)^{\frac{1}{n}} - \varepsilon_0^p
\]

\[
G(\beta)
\]

and

\[
(\varepsilon_1^p)_f = \frac{K_90}{K_90 - \beta} (\varepsilon_1^p)_f \quad \text{for} \quad 0 \leq \beta \leq 1,
\]

where \((\varepsilon_1^p)_f\) and \((\varepsilon_2^p)_f\) are the components of principal plastic strain at failure [7].
3. Material characterization

3.1. Uniaxial tensile test and bulge test

The material hardening and anisotropic mechanical properties of AZ31B Mg sheet were characterized by the uniaxial tensile test and the bulge test. The measured hardening curves were shown in figure 1 and their characteristic mechanical values were summarized in table 1. As can be seen from figure 1, the AZ31B Mg sheet shows a differential work hardening behavior.

| Loading condition     | $E$ (GPa) | $\nu$ | YS (MPa) | UTS (MPa) | $\varepsilon_{UTS}$ (%) | TE (%) | $R$ | Failure stress (MPa) |
|-----------------------|-----------|-------|----------|-----------|------------------------|--------|----|---------------------|
| Simple tension (RD)   | 45.0      | 0.35  | 212.4    | 280.2     | 13.9                   | 21.6   | 2.9 | -                   |
| Simple tension (TD)   | 45.0      | 0.35  | 182.8    | 278.5     | 14.7                   | 23.6   | 2.2 | -                   |
| Balanced biaxial      | 350.8     | -     | -        | -         | -                      | -      | -  | -                   |

Figure 1. Hardening curves of AZ31B Mg sheet measured along the rolling and transverse directions as well as under the balanced biaxial loading condition [7].

3.2. Hemispherical dome test

The failure strains of AZ31B Mg sheet were measured by the hemispherical dome test with specimens of various widths to take the effect of deformation path into account. The measured strains were classified into two types, ‘safe’ and ‘fail’ strains, depending on their positions relative to the fractured surface (figure 2). The boundary between these two strains is considered as a failure limit of AZ31B Mg sheet.

3.3. Identification of material coefficients

The failure prediction equation in equation (11) has a total of 9 material coefficients. The coefficients $K_0$, $P$, and $n$ of the Swift equation were determined by fitting the hardening curve as shown in figure 1. It is very straightforward to obtain $K_0$ and $P_0$ from the measured R-values listed in table 1. The critical shear stress $k_{TD}$ was obtained from the failure stress of the bulge test (whose magnitude is equal to 350.8MPa). Identification of the coefficients of yield function $X$, $Y$ and $B$ is not so straightforward. These coefficients were determined by considering the differential work hardening of AZ31B Mg sheet (for the detailed procedures, see Ahn and Seo [7]). The resulting material coefficients obtained are summarized in table 2.
4. Result and discussion
Equation (11) was evaluated with the material coefficients listed in Table 2 and the resulting curve was plotted in Figure 2. Comparison with the experimentally measured failure strains shows that the failure prediction made by equation (11) is close to the experimental failure limit, although they are not perfectly in agreement with each other.

To see the effect of material anisotropy in the failure prediction of AZ31B Mg sheet, equation (11) was evaluated under the isotropy condition, i.e., the conditions $X = Y = B$ and $K = K_{90} = 2$ were applied to equation (11). The failure prediction curve under the isotropy condition was plotted in Figure 2, and it shows a large deviation from the experimental failure limit. This result shows the importance of considering the material anisotropy in the failure prediction of AZ31B Mg sheet which possesses strong anisotropy.

To see the effect of differential work hardening in failure prediction, equation (11) was evaluated with the initial yield stresses (YS) listed in Table 1, instead of using the coefficients $X$, $Y$ and $B$ which are obtained by considering the behaviour of differential work hardening. The resulting curve shown in Figure 2 also shows a great deviation from the experimental failure limit, which implies the assumption of isotropic hardening is no more valid in the failure prediction of AZ31B Mg sheet.

The results in the above show that ignoring either material anisotropy or differential work hardening might lead to a great discrepancy in the result of failure prediction, in particular for the AZ31B Mg sheet.

![Figure 2. Comparison of failure prediction curves with the experimental data of failure strains measured by the hemispherical dome test [7].](image)

References
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