1. Introduction

Heterostructures consisting of ferromagnetic and superconducting materials have received much attention from not only the fundamental physics in condensed matter but also applications such as spintronics devices [1–9]. As is well known in such structures, the Andreev reflection (AR) is an essential physics [10]. The AR is the scattering process occurs at the nonsuperconducting metal/superconductor (NSCM/SC) interface for which an incoming electron (hole) from NSCM to SC is retroreflected as a hole (electron) with opposite spin. In other words, the AR reflects electron-hole coherence effect of superconductor. Therefore, the AR provides us some information about superconducting states. Indeed, in the junction consisting of unconventional superconductors such as d-wave [11–16] and chiral p-wave [17, 18] pairing states, a zero-bias conductance peak (ZBCP) due to the Andreev bound state (ABS) formed at the interface of the junctions have been observed [19–22]. On the other hand, the AR in the ferromagnet/superconductor (FM/SC) junction shows mutually exclusive a feature of FM and SC. About the Stoner ferromagnet (Stoner-FM)/conventional SC junctions, de Jong and Beenaker clarified theoretically that the AR is affected by the exchange interaction in the FM and showed a possibility of the measurement of the polarization of FM using the FM/SC point contact AR spectroscopy [23]. After the proposal by de Jong and Beenaker, the polarization of FM was obtained from the conductance measurements in the FM/SC point contacts by Soulen et al., [24] and Upadhyay et al. [25]. The de Jong and Beenaker theory was also extended to the Stoner-FM/unconventional SC junctions [26–29] by extending the BTK theory [30] within the quasiclassical Andreev approximation. In the Stoner-FM/insulator (I)/d-wave SC junctions, the suppression of ZBCP due to the breaking of the retroreflectivity of AR called virtual AR (VAR) [26, 27] has been confirmed experimentally after the theoretical prediction [31, 32]. As an interesting magnetic effect other than FM, there are problems of spin-active interface [33–36]. The problem of the spin-active interface can be thought separately into a simple spin-filtering (SF) effect and a more complicated spin-mixing effect. Here, simple SF effect means that the strength of interface potential is only different between ↑-
and [\downarrow]-spin particles without the rotation of exchange field in insulator and spin flip scattering at the interface. For the spin-mixing effect, a spin-dependent ABS was clarified firstly by Fogelström [36]. Moreover, the spin-mixing effect leading to the spin singlet-triplet mixing revealed by Eschrig and co-workers [4, 37, 38] is currently one of the most important themes in this field [39–47]. For a simple SF effect, it has been investigated theoretically that the ferromagnetic insulator(FI) switches the Josephson junction from 0- to \pi-junction [48]. In Normal metal(NM) or Stoner-FM/FI/d-wave SC junctions, the splitting of ZBCP owing the FI been shown [26]. Furthermore, it has been also clarified that the effective cooling power can be obtained by the AR suppression due to FI in NM/FI/s-wave SC junctions compared to that in NM/FI/s-wave SC junctions [49]. As shown in these results, even a simple SF effect would expect an interesting phenomenon. Especially, more recently, for NM/FI/s-wave, d-wave SC junctions, we find the detailed mechanism of the influence of FI on AR which results the double conductance peaks at gag-edge for s-wave SC and the ZBCP-splitting for d-wave SC [50].

On the other hand, as a model of ferromagnet different from Stoner-FM, there is a ferromagnet by a spin-dependent bandwidth asymmetry or, equivalently, by a difference of effective mass between [\uparrow]- and [\downarrow]-spin particles [51–55]. In the spin bandwidth asymmetry ferromagnet(SBA-FM), the magnetization is given by the ratio of effective mass between [\uparrow]- and [\downarrow]-spin particles. For SBA-FM/1/ d-wave [56], p-wave [57] SC junctions, the ZBPC is suppressed due to the VAR increasing the magnetization, as in Stoner-FM case. Furthermore, as an interesting influence of the SBA-FM on charge transport, the conductance below the superconducting energy gap for minority spin larger than that for majority spin in SBA-FM/1/ s-wave SC junction and the asymmetric ZBPC splitting in SBA-FM/1/ d_{x^2-y^2}-wave SC junction are theoretically calculated [56]. These results mean that the SBA-FM produces the SF effect.

However, the mechanism of SF effect due to SBA-FM has not been elucidated at all. Thus, it is one of the important problems to clarify the mechanism and the difference of the SF effect between FI and SBA-FM. In addition to this, also related to superconducting Sr2RuO4 [58–61], it is an interesting problem to investigate the SF effect in the NM, Stoner- and SBA-FM/FI/chiral p-wave SC junctions which has not been studied sufficiently yet. In this paper, we will clarify a mechanism of the SF effect which differs from cause by the singlet-triplet spin-mixing, spin-flip scattering and spin–orbit coupling at the interface of juctions. It is shown clearly that the imaginary part of superconducting coherence function induced due to the exchange potential of FI and the wavenumber difference between [\uparrow]- and [\downarrow]-spin particles in FI is an essential for the SF effect. As an application, we will also study the SF effect on the Stoner- and SBA-FM/FI/chiral p-wave SC junctions. For the both Stoner and SBA-FMs, we can find the ZBPC shift similar to that in NM/FI/SC structures formed on the surface of three-dimensional topological insulator [62]. It has been shown that the conductance in each case for the Stoner-FM and SBA-FM shifts to opposite energy direction by the SF effect.

This paper is organized as follows. In section 2, the SF factor is presented together with the reflection coefficients and the tunneling conductance. In section 3, at first, the bias voltage dependences of the coherence function for the chiral p-wave superconductor induced by the SF effect are shown for the several incident angles. Then, we present the results for the NM/FI/chiral p-wave SC junction and FI/I(FI)/chiral p-wave SC junctions. In section 4, we summarize our results.

2. Model and formulation

In this section, we present the SF factor with reconstructed reflection coefficients and tunneling conductance based on extended BTK theory formulated in previous paper [57]. The FI/FM/SC heterostructure considered in here is a two-dimensional ballistic junction as shown in figure 1, where semi-infinite FM and SC correspond to region \(x < 0\) and \(x > 0\), respectively. A flat interface in the y-direction is located at \(x = 0\). Assuming a non-magnetic \(V_0\) and magnetic \(V_\infty\) potentials at \(x = 0\), the simple spin–filtering FI barrier for \(\sigma(= \uparrow \text{ or } \downarrow)\)-spin can be described by \(\nu_\sigma = (V_0 - \rho V_\infty) \delta(x)\) with \(\rho = 1(-1)\) for \(\sigma = \uparrow (\downarrow)\)-spin, where \(\delta(x)\) is the Dirac delta function. The single particle Hamiltonian for \(\sigma\)-spin in the FM is given by

\[
H_\sigma^F(r) = -\hbar^2 \nabla^2 / 2m_\sigma - \rho U_\infty - E_{FM}
\]

where \(m_\sigma\) is the effective mass for \(\sigma\)-band particles, \(U_\infty\) is the exchange potential, and \(E_{FM}\) is the Fermi energy. In this model, the \(H_\sigma^F(r)\) describes the pure Stoner-FM for \(m_\uparrow = m_\downarrow\) and the pure SBA-FM for \(m_\uparrow \neq m_\downarrow\) with \(U_\infty = 0\), respectively. By setting \(U_\infty = 0\) and converting \(m_\sigma\) and \(E_{FM}\) to those in the SC: \(m_\sigma \rightarrow m_\sigma\) and \(E_{FM} \rightarrow E_{SS}\), the single particle Hamiltonian in the SC is given by

\[
H_\sigma^S(r) = H_\sigma^B(r) = -\hbar^2 \nabla^2 / 2m_\sigma - E_{SS}
\]

The spatial dependence of the pair potential is taken as \(\Delta(r) = \Delta(\Theta(x))\) for simplicity, where \(\Delta\) is the 2 × 2 matrix in spin-space and \(\Theta(x)\) is the Heaviside step function. Here, we consider only the \(S_z = 0\) triplet pairing state, where the elements of pair potential are given by \(\Delta_{0-} = \Delta_{0+} = 0\), and \(\Delta_{1-} = \Delta_{1+}\) [63]. Thus, the effective Bogoliubov–de Gennes(BdG) equation of our system can be reduced to decoupled equation for the wave fuctions \((u_\sigma^l(r), v_\sigma^l(r))^T\) in \(l = \text{FI or SC side and is given by}

\[
\begin{align*}
\frac{d}{dr} \begin{pmatrix}
\begin{pmatrix}
u_\sigma u_\sigma^l(r) \\
\nu_\sigma v_\sigma^l(r)
\end{pmatrix}
\end{pmatrix} & = H_\sigma^l(r) \begin{pmatrix}
\begin{pmatrix}
u_\sigma u_\sigma^l(r) \\
\nu_\sigma v_\sigma^l(r)
\end{pmatrix}
\end{pmatrix} \\
& = -\hbar^2 \nabla^2 \begin{pmatrix}
u_\sigma u_\sigma^l(r) \\
\nu_\sigma v_\sigma^l(r)
\end{pmatrix} / 2m_\sigma - \rho U_\infty - E_{FM} \begin{pmatrix}
u_\sigma u_\sigma^l(r) \\
\nu_\sigma v_\sigma^l(r)
\end{pmatrix}
\end{align*}
\]
\[
\begin{pmatrix}
H_0^s(r) & \Delta^s(r) \\
\Delta^s(r) & -H_0^s(r)
\end{pmatrix}
\begin{pmatrix}
u^s_1(r) \\
v'^s_1(r)
\end{pmatrix} = \varepsilon \begin{pmatrix}
u^s_1(r) \\
v'^s_1(r)
\end{pmatrix},
\]

(2.1)

where \( \varepsilon \) is the energy of the quasiparticle and \( \sigma \) is the inversion of \( \sigma \)-spin. The BdG equation can be solved under the quasiclassical Andreev approximation where the wave vector dependence of \( \Delta \) is replaced by the angle \( \theta_s \) between the direction of the trajectory of quasiparticles in the SC and the interface normal [11, 12]. Then, a chiral \( p \)-wave pair potential can be taken as \( \Delta_+ = \Delta_0 e^{i\theta_s}, \Delta_- = \Delta_0 e^{i(\pi - \theta_s)} \) with the magnitude of pair potential \( \Delta_0 \) at zero temperature \( T = 0 \) [63]. In the quasiclassical Andreev approximation, the magnitude of wave vectors \( k_x \) in the FM is given by \( k_\sigma = \sqrt{2m_\sigma/k^2}(E_{FM} + \rho U_{2\sigma}) \). In the SC, the magnitude of wave vectors of the electronlike quasiparticles (ELQ) and the holelike quasiparticles (HLQ) can be denoted by \( k_{ELQ(HLQ)} = k_s = \sqrt{2m_s E_{FS}/\hbar^2} \). Since the translational symmetry holds along the flat interface, the \( y \)-component of all wave vectors is conserved

\[
k_x \sin \theta_x = k_y \sin \theta_y = k_z \sin \theta_z,
\]

(2.2)

where \( \theta_{x(y)} \) is an angle of \( k_{x(y)} \) with respect to the interface normal in the FM. In addition, the translational invariance along the flat interface of the Hamiltonian reduces the BdG equation to the effective one-dimensional equation. Below, for \( E_{FM} = E_{FS} = E_0 \), we assume \( k_1 < k_i < k_f \) and \( m_1 < m_i < m_f \) with \( m_1/m_i = m_i/m_f \). Under this assumption, the magnitude of magnetism is parameterized by \( \gamma = m_1/m_i \geq 1 \) for the pure SBA-FM and \( \chi = U_{2\sigma}/E_{FS}(0 \leq \chi \leq 1) \) for the pure Stoner-FM. Then, the magnetization \( M \) in the FM can be calculated by \( M = P_1 - P_i \) where \( P_1 = \gamma(1 + \chi)/(\gamma(1 + \chi) + 1 - \gamma) \) and \( P_i = (1 - \chi)/(\gamma(1 + \chi) + 1 - \gamma) \). In this situation, for example, there are four scattering processes for the injection of \( \uparrow \)-spin electrons from the FM side at an angle \( \theta_i \) to the interface as shown in figure 1: normal reflection (NR) with angle \( \theta_i \), AR as holes with angle \( \theta_i \) ELQ and HLQ transmitted with angle \( \theta_i \) to the SC side. Considering \( k_1 < k_i < k_f \) under the conservation of \( y \)-component wave vectors equation (2.2), the scattering processes for the injection of \( \downarrow \)-spin electrons are described by converting \( \theta_i \) to \( \theta_i \). Thus, using the internal phase factor of pair potential \( e^{i\theta_s} = e^{i\Delta_0/(\Delta_0^2)} \), the solutions of the BdG equation for the \( \uparrow \)-spin and \( \downarrow \)-spin electrons injected from the FM into the SC are described as

\[
\begin{align*}
\begin{pmatrix}
\nu^F_{1(1)} \\
\nu^F_{1(2)}
\end{pmatrix} = & \begin{pmatrix}
1 \\
0
\end{pmatrix} e^{ik_{1(1)}x \cos \theta_i} + a_{1(1)} \begin{pmatrix}
0 \1
\end{pmatrix} e^{ik_{1(1)}x \cos \theta_i} + b_{1(1)} \begin{pmatrix}
1 \0
\end{pmatrix} e^{-ik_{1(1)}x \cos \theta_i} \\
\nu^{F*}_{1(1)} = & \begin{pmatrix}
\nu^F_{1(1)} & \nu^F_{1(2)}
\end{pmatrix} \begin{pmatrix}
u^F_{1(1)} & \nu^F_{1(2)}
\end{pmatrix}^{-1}
\end{align*}
\]

(2.3)
with

\[
    u_{\pm} = \frac{1}{\sqrt{2}} \left( 1 + \sqrt{\frac{\varepsilon^2 - |\Delta_\pm|^2}{\varepsilon}} \right), \quad v_{\pm} = \frac{1}{\sqrt{2}} \left( 1 - \sqrt{\frac{\varepsilon^2 - |\Delta_\pm|^2}{\varepsilon}} \right),
\]

where the probability coefficients \( a_{1(\uparrow)} \), \( b_{1(\uparrow)} \), \( c_{1(\downarrow)} \), and \( d_{1(\downarrow)} \) are for AR, NR, transmission ELQ and HLQ. From the boundary conditions at the FI (\( \Gamma(x = 0) \))

\[
    u_{\uparrow\uparrow,\uparrow}(x = 0) = u_{\uparrow\uparrow,\downarrow}(x = +0), \quad v_{\uparrow\uparrow,\uparrow}(x = 0) = v_{\uparrow\uparrow,\downarrow}(x = +0),
\]

\[
    \frac{\hbar^2}{2m_S} \frac{d^2 v_{\uparrow\downarrow,\downarrow}}{dx^2} \bigg|_{x = +0} = \frac{\hbar^2}{2m_{FI}} \frac{d^2 v_{\uparrow\downarrow,\downarrow}}{dx^2} \bigg|_{x = -0} = V_{1(\downarrow)} u_{\uparrow\uparrow,\downarrow}(x = 0),
\]

\[
    \frac{\hbar^2}{2m_S} \frac{d^2 v_{\uparrow\uparrow,\downarrow}}{dx^2} \bigg|_{x = +0} = \frac{\hbar^2}{2m_{FI}} \frac{d^2 v_{\uparrow\uparrow,\downarrow}}{dx^2} \bigg|_{x = -0} = V_{1(\downarrow)} v_{\uparrow\uparrow,\downarrow}(x = 0),
\]

(2.4)

the AR \( |a_{1(\uparrow)}|^2 \) and NR \( |b_{1(\uparrow)}|^2 \) reflection probabilities for the injection of electron with \( \uparrow(\downarrow) \)-spin can be obtained as

\[
    |a_{1(\uparrow)}|^2 = \frac{G_{N1(\downarrow)} G_{N1(\uparrow)} |\Gamma_\uparrow|^2}{1 + |\Gamma_\uparrow \Gamma_\downarrow|^2 (1 - G_{N1(\downarrow)})(1 - G_{N1(\uparrow)}) + D + P_{1(\downarrow)}},
\]

(2.5)

\[
    |b_{1(\uparrow)}|^2 = \frac{1 - G_{N1(\downarrow)} + |\Gamma_\uparrow \Gamma_\downarrow|^2 (1 - G_{N1(\downarrow)})(1 - G_{N1(\uparrow)}) + D + P_{1(\downarrow)}}{1 + |\Gamma_\uparrow \Gamma_\downarrow|^2 (1 - G_{N1(\downarrow)})(1 - G_{N1(\uparrow)}) + D + P_{1(\downarrow)}},
\]

(2.6)

with the conductance \( G_{N1(\downarrow)} \) for \( \uparrow(\downarrow) \)-spin when the SC is in the normal state

\[
    G_{N1(\downarrow)} = \frac{4\pi n_{S(\downarrow)} \cos \theta_S}{(\cos \theta_S + \kappa_{1(\downarrow)})^2 + Z_{1(\downarrow)}},
\]

(2.7)

where \( \kappa_{1(\downarrow)} = \gamma^{-1/2}(1/2)k_{1(\downarrow)}/k \) is the normalized wavenumber

\[
    \kappa_{1(\downarrow)} = \sqrt{\gamma^{-1/2} \cos^2 \theta_S + \gamma^{-1/2}(1 - \gamma^{-1/2}) + (\gamma^{-1/2} \chi)},
\]

(2.8)

and \( Z_{1(\downarrow)} \) is the normalized FI barrier parameter

\[
    Z_{1(\downarrow)} = Z_0 = ( + )Z_{ex}, \quad Z_0 = \frac{2m_S V_0}{\hbar^2 k_S}, \quad Z_{ex} = \frac{2m_S V_{ex}}{\hbar^2 k_S}.
\]

The coherence function \( \Gamma_\pm \) is given by

\[
    \Gamma_\pm = \Gamma_\pm e^{\pm \phi_\pm}, \quad \phi_\pm = \frac{|\Delta_\pm|}{\varepsilon + \sqrt{\varepsilon^2 - |\Delta_\pm|^2}}.
\]

(2.9)

Using the real part of the coherence function product \( \text{Re}[\Gamma_+, \Gamma_-] = (\Gamma_+ \Gamma_- + \Gamma_- \Gamma_+ )/2 \), the term of denominator \( D \) is given by

\[
    D_1(\downarrow) = 2 \text{Re}[\Gamma_+ \Gamma_- D_{1(\downarrow)}],
\]

with

\[
    D_{1(\downarrow)} = \frac{\kappa_{1(\downarrow)}^2 (\cos^2 \theta_S - Z_{\downarrow(\downarrow)}^2) + \kappa_{1(\downarrow)}^2 (\cos^2 \theta_S - Z_{\downarrow(\downarrow)}^2)}{((\cos \theta_S + \kappa_{1(\downarrow)})^2 + Z_{1(\downarrow)}^2)((\cos \theta_S + \kappa_{1(\downarrow)})^2 + Z_{1(\downarrow)}^2)}.
\]

(2.10)

Similarly, the imaginary part using \( \text{Im}[\Gamma_+, \Gamma_-] = (\Gamma_+ \Gamma_- - \Gamma_- \Gamma_+ )/2 \), the term \( P_{1(\downarrow)} \) is defined as

\[
    P_{1(\downarrow)} = 2i \text{Im}[\Gamma_+ \Gamma_- S_{1(\downarrow)}].
\]

(2.11)

with

\[
    S_{1(\downarrow)} = \frac{2 \cos \theta_S (Z_{1(\downarrow)}(\cos^2 \theta_S - \kappa_{1(\downarrow)}^2) - Z_{\downarrow(\downarrow)}^2) - Z_{1(\downarrow)}(\cos^2 \theta_S - \kappa_{1(\downarrow)}^2) - Z_{\downarrow(\downarrow)}^2)}{((\cos \theta_S + \kappa_{1(\downarrow)})^2 + Z_{1(\downarrow)}^2)((\cos \theta_S + \kappa_{1(\downarrow)})^2 + Z_{1(\downarrow)}^2)}.
\]

(2.12)

Here, we consider the spin dependence of terms \( D \) and \( P_{1(\downarrow)} \). The product of coherence function \( \Gamma_+ \Gamma_- \) of \( D \) and \( P_{1(\downarrow)} \) does not contain the effect of FM and FI because the magnetic proximity effect is not considered in our model. It can be seen obviously that the effect of FM and FI appears in the \( D_{1(\downarrow)} \) and \( S_{1(\downarrow)} \) defined above via \( \kappa_{1(\downarrow)} \) and \( Z_{1(\downarrow)} \). Since the \( D_{1(\downarrow)} \) is symmetric with respect to the replacement of spin \( \uparrow \leftrightarrow \downarrow \), the term \( D \) is equal to \( \uparrow \)-spin and \( \downarrow \)-spin. In contrast to the \( D_{1(\downarrow)} \), the magnitude of \( S_{1(\downarrow)} \) is asymmetric on swapping of spin \( \uparrow \leftrightarrow \downarrow \). As the result, the \( P_{1(\downarrow)} \) depends on the direction of spin.
To clarify an origin of the SF effect, we focus on the terms $G_{N(t)} \mathcal{D}$ and $\mathcal{P}_j(i)$ in ARs $|a^{th}_{\sigma_{1}}|^2$ and $|b^{th}_{\sigma_{2}}|^2$ probabilities in equation (2.5) and equation (2.6). Because, as seen in above obtained formula, the inclusion of the FM and FI on $|a^{th}_{\sigma_{1}}|^2$ and $|b^{th}_{\sigma_{2}}|^2$ is included in these terms. In these terms, although the $G_{N(t)} \mathcal{D}$ naturally includes the SF effect, it already has meaning as a conductance in FM/FI (or 1)/NM junctions. On the other hand, the $\mathcal{D}$ and $\mathcal{P}_j(i)$ including $\Gamma_{\uparrow}, \Gamma_{\downarrow}$ appear only in FM/FI (or 1)/SC junctions. However, since $\mathcal{D}$ is the symmetric for $\uparrow \downarrow \downarrow \uparrow$, it does not contribute the SF effect. In contrast, the $\mathcal{P}_j(i)$ changes by $\uparrow \downarrow \downarrow \uparrow$ via $S_{1}(t)$ and will cause the SF effect in addition to $G_{N(t)} \mathcal{D}$. In fact, this will be easily seen in $|a^{th}_{\sigma_{1}}|^2$. In the exchange of spin $\uparrow \downarrow \downarrow \uparrow$ in $|a^{th}_{\sigma_{1}}|^2$, only $\mathcal{P}_j(i)$ in the denominator changes. Therefore, we choose the $\mathcal{P}_j(i)$ as the SF factor.

In calculating the conductance for a situation $k_{\downarrow} < k_{\uparrow} < k_{\downarrow}$ set here, we consider the critical angle $\theta_c$ for $\uparrow\downarrow\downarrow\uparrow$-spin particles injection. The $\theta_c$ is originated from the translational symmetry along the interface for the particle with $\downarrow\downarrow\downarrow\uparrow$-spin and can be expressed by $\theta_c = -\sqrt[3]{1/\gamma \gamma^{-1/2}} (1 + 1 + 1/2)$. For the $\uparrow\downarrow\downarrow\uparrow$-spin injection $\theta_\uparrow > \theta_c$, the $x$-component of the wave vector of AR with $\downarrow\downarrow\downarrow\uparrow$-spin becomes purely imaginary. That is, the evanescent mode exists in the AR process for $\uparrow\downarrow\downarrow\uparrow$-spin injection, which is called VAR [26]. With taking care of the critical angle $\theta_c$ and the probability conservation of quasiparticle flow [57], the bias voltage $V$ dependence of angle resolved conductance $G_{S(t)}(i)$ for $\uparrow\downarrow\downarrow\uparrow$-spin of the system at $T = 0$ where $\varepsilon = eV$ can be calculated by the extended BTK formula $G_{S(t)}(i) = 1 + |g_{1}^{th}(i)|^2 - |b_{1}^{th}(i)|^2$ as

$$
G_{S(t)}(i) = \frac{1 - |\Gamma_{\uparrow},, \Gamma_{\downarrow},, |(1 - G_{N(t)}) + |G_{S(t)}(i)|^2}{1 + |\Gamma_{\uparrow},, \Gamma_{\downarrow},, |(1 - G_{N(t)})} \Theta(|\theta_{\uparrow} - \theta_{\downarrow},,| - \gamma), \quad G_{S(t)}(i) = 0 \quad \text{for} \quad |\theta_{\downarrow} - \theta_{\uparrow},,| > \gamma \quad (2.13)
$$

where $\mathcal{P}_j(i)$ is the group velocity ratio for $\sigma$ and $\bar{\sigma}$-spins in FM and is given by

$$
g_{1}^{th}(i) = \frac{-\gamma^{1/2} \gamma^{1/2} - \gamma^{1/2} \gamma^{1/2}}{\gamma^{1/2} \gamma^{1/2} - \gamma^{1/2} \gamma^{1/2}} \Theta(|\theta_{\uparrow} - \theta_{\downarrow},,| - \gamma), \quad \text{for} \quad |\theta_{\downarrow} - \theta_{\uparrow},,| > \gamma \quad (2.14)
$$

The group velocity ratio $g_{1}^{th}(i)$, not only guarantees the probability conservation but also works as a SF factor. Here we note that the evanescent mode of AR for $\theta_{\uparrow} > \theta_c$ does not contribute to the conductance in our extended BTK formula, since the corresponding $g_{1}^{th}$ on $1 + g_{1}^{th}|a^{th}_{\sigma_{1}}|^2 - |b_{1}^{th}|^2$ is purely imaginary. The angle averaged conductance $C_{S(t),[\sigma]}$ is defined as $C_{S(t),[\sigma]} = \int_{-\pi/2}^{\pi/2} d\theta \cos \theta_{\uparrow} B_{\sigma} G_{S(t),[\sigma]}$ and the total conductance is given by $C_{S(t)} = C_{S(t),[\uparrow]} + C_{S(t),[\downarrow]}$.

3. Results

3.1. Imaginary part of coherence function $\Gamma_{\uparrow}, \Gamma_{\downarrow}$ for a chiral $p$-wave pair potential

In $p$-wave pair potential chosen here, i.e., $\Delta \psi \psi_{\sigma_{2}} \psi_{\sigma_{1}}$, $\text{Im}[\Gamma_{\uparrow}, \Gamma_{\downarrow}]$ is given by

$$
\text{Im}[\Gamma_{\uparrow}, \Gamma_{\downarrow}] = \text{Im}[\Gamma_{\uparrow}\Gamma_{\downarrow}] \sin \theta_{\downarrow} - \cos \theta_{\downarrow} - 2i \text{Re}[\Gamma_{\uparrow}\Gamma_{\downarrow}] \sin \theta_{\downarrow} \cos \theta_{\downarrow}, \quad (3.1)
$$

where $\text{Im}[\Gamma_{\uparrow}, \Gamma_{\downarrow}] = \Gamma_{\downarrow} + \Gamma_{\downarrow} - \Gamma_{\downarrow} \Gamma_{\downarrow}$ and $\text{Re}[\Gamma_{\uparrow}, \Gamma_{\downarrow}] = \Gamma_{\downarrow} + \Gamma_{\downarrow} + \Gamma_{\downarrow} \Gamma_{\downarrow}$. Here we note that $\Gamma_{\downarrow} + \Gamma_{\downarrow} + \Gamma_{\downarrow} \Gamma_{\downarrow} = 0$ at $v = 0$ since $\Gamma_{\downarrow} = -i$. In figure 2, we show the normalized Im$[\Gamma_{\uparrow}, \Gamma_{\downarrow}]$ for several incident angles $\theta_{\downarrow}$ at $\theta_{\downarrow} = 0$ where the ZES exists, $\text{Im}[\Gamma_{\uparrow}, \Gamma_{\downarrow}] / 2 = -i \Gamma_{\downarrow} \Gamma_{\uparrow} - \Gamma_{\downarrow} \Gamma_{\uparrow} \Gamma_{\downarrow} / 2$ shows a sinusoidal curve in the region $v^2 \leq |\Delta_{0}|$ (figure 2(a)). For finite angles except $\theta_{\downarrow} = \pm \pi/2$, it is found that $\text{Im}[\Gamma_{\uparrow}, \Gamma_{\downarrow}] / 2 = -i \sin \theta_{\downarrow} \cos \theta_{\downarrow} + \theta_{\downarrow} v$ and $\text{Im}[\Gamma_{\uparrow}, \Gamma_{\downarrow}] / 2 = i \theta_{\downarrow} + \theta_{\downarrow} \theta_{\downarrow} \sin \theta_{\downarrow}$ for $|v| \geq \Delta_{0}$ (see figures 2(b)-(c)). As $\theta_{\downarrow}$ approaches $\pm \pi/2$, $(\Gamma_{\downarrow} \Gamma_{\downarrow} - \Gamma_{\downarrow} \Gamma_{\downarrow} \Gamma_{\downarrow}) \sin \theta_{\downarrow}$ in first terms becomes dominant since the cos $\theta_{\downarrow}$ becomes. Thus, at $\theta_{\downarrow} = \pm \pi/2$, the $\text{Im}[\Gamma_{\uparrow}, \Gamma_{\downarrow}]$ reverses and shows the same behavior as s-wave pair potential (see figure 2(f)). However, since the angle $\theta_{\downarrow} = \pm \pi/2$ corresponds to the incident parallel to the interface, $\text{Im}[\Gamma_{\uparrow}, \Gamma_{\downarrow}] / 2$ does not appear. The SF effect for each incident angles is determined by $\mathcal{P}_j$. Although the $\mathcal{P}_j$ cannot actually integrate independently in the denominator of AR and NR probabilities, for easy understanding, we introduce the angle averaged SF factor $\mathcal{F}_j = \int_{-\pi/2}^{\pi/2} d\theta \cos \theta_{\uparrow} B_{\sigma} / 2$ in the following.

3.2. Spin-filtering factor of ferromagnetic insulator

In the NM/FI/SC junction, i.e., $\gamma = 1.0$, $\theta_{\downarrow} = 0$, $S_{1}(t)$ is reduced to a simple form given by

$$
S_{1} = \frac{4 \cos \theta_{\downarrow} (Z_{\uparrow} - Z_{\downarrow}) Z_{oo}}{(4 \cos \theta_{\downarrow} + Z_{oo}) (4 \cos \theta_{\uparrow} + Z_{oo}), \quad S_{1} = -S_{1}. \quad (3.2)
$$

We can see easily that the FI does not work as a spin-filter for $Z_{oo} = Z_{o}$. In this case, for the injection of particles with $\uparrow\downarrow\downarrow\uparrow$-spin transmitting perfectly through the $Z_{o}(=0)$-interface, the NR occurs through the Andreev reflection.
of the AR particles with †-spin once reflected normally at the $Z_0 (= 2Z_0)$-interface. This implies that probabilities of the AR and NR for the injection of particles with †-spin are equal to those of the particles with †-spin, respectively. As the result, for $Z_{xx} = Z_0$, the shift-like behavior of ABS owing to the SF factor is expected. Figure 3 shows the $Z_{xx}$ dependence of $F_2$ for $Z_{xx} = 5.0$. All curves are sinusoidal symmetrical with respect to $eV = 0$ reflecting $\text{Im}[\Gamma^\dagger \Gamma^\prime]$. With the increase of $Z_{xx}$ the peak of $F_2$ increases and decreases after reaching a maximum value. The $Z_{xx}$ giving a maximum peak is calculated by $Z_{xx} = Z_0/\sqrt{3}$. It should be emphasized to focus on the behavior near $eV = 0$. Because the position of the minimum value of the denominator of AR $|a^{\text{AR}}|^2$ in equation (2.5) and NR $|b^{\text{AR}}|^2$ in equation (2.6) shifts from $eV = 0$ depending on the negative $F_2$. In figure 4, it is found that the peak (bottom) positions of the angle averaged AR(NR) probability $\int_{-\pi/2}^{\pi/2} d\theta \cos \theta |a^{\text{AR}}|^2(\int_{-\pi/2}^{\pi/2} d\theta \cos \theta |b^{\text{AR}}|^2)$ for †- and †-spins are shifted in opposite directions since $F_1$ and $F_2$ are inverted to each other with respect to $eV = 0$. Resulting normalized conductance $C_{S}/C_N$ are presented with the spin resolved conductance $C_{S,\sigma}/C_N$ in figure 5. One can see that, with increasing of $Z_{xx}$, $C_{S,\uparrow}/C_N$ and $C_{S,\downarrow}/C_N$ are symmetrically shifted to negative and positive $eV$, respectively. However, since the magnitude of ZBCP contributed from only particles injected at $\theta_0 = 0$ is weak, any significant effects like the ZBCP-splitting in $d$-wave or the double peak structure at gap-edge $eV = |\Delta_0|$ in $s$-wave cases are not seen in the total conductance $C_{S}/C_N$. Now, it should be noted that the direction of peak shift of $C_{S,\sigma}/C_N$ is determined by the sign of numerator ($Z_{xx}^2 - Z_0^2)Z_{xx}$ of $S_{\sigma}$. For example, in the case of $-Z_{xx}(|Z_{xx}| < Z_0)$ model, it can be easily predicted to shift in the opposite direction to the $C_{S,\sigma}$ obtained here. Thus, it will be expected to obtain an information on the sign of $Z_{xx} (|Z_{xx}| < |Z_0|)$ from measurement of the spin resolved conductance $C_{S,\sigma}$ in NM/Fl/SC junctions. Furthermore, these peak shifts by the SF factor $P_\sigma$ could be distinguished from those of the interface spin-mixing, spin–orbit coupling and Zeeman effects.

3.3. Spin-filtering factor of ferromagnet

By setting $Z_{xx} = 0$, the $S_{\uparrow}$ for the FM can be obtained as

$$S_{\uparrow} = -\frac{2 \cos \theta_0 K_{\uparrow}^{\text{FM}} Z_0}{((\cos \theta_0 + \kappa_1)^2 + Z_0^2)((\cos \theta_0 + \kappa_1)^2 + Z_0^2)}, \quad S_{\downarrow} = -S_{\uparrow}$$

where $K_{\uparrow}^{\text{FM}} = (\kappa_1^2 - \kappa_1^2)$. From the numerator, it can be found that the FM induces a spin-filter owing to the wavenumber difference. It is also obvious that the SF effect do not occur in the metallic limit $Z_0 = 0$. Therefore, in addition to suppression of AR, the SF effect originated in polarization of FM should be expected in the FM/1/...
SC junctions as reported for the SBA-FM \[56\]. However, this SF effect seems to be overlooked for the Stoner-FM.

### 3.4. Stoner ferromagnet

In the Stoner-FM case (\(\gamma = 1.0\)), the magnitude of \(k_{\sigma}^{\text{fm}}\) is calculated by \(2\chi\). Since \(0 \leq \chi \leq 1\), \(S_\sigma\) in a fixed \(\chi = 0\) will be in the order \(\sim 1/Z_0^3\) for high barrier as tunneling limit. For this reason, it seems that the SF effect owing to Stoner-FM has not been noticed in the tunnel limit. It is also found in equation (3.3) that \(S_\sigma\) in a fixed \(Z_0 \neq 0\)
decreases with increasing $\chi$. Thus, as shown in figure 6, the resulting $\Phi$ decreases for each increase in $Z_0$ and $\chi$. It should be noticed that both results show inverted behavior to those of the FI case. Therefore, the peak shift will be opposite to those of the FI case. However, this difference is derived from the model of Stoner-FM and FI assumed in here. If the magnetizations of Stoner-FM and FI are set in mutually different directions, the expected peak shift will be in the same direction. Figure 7 shows the $Z_0$-dependence of the angle averaged $S_{\uparrow\uparrow}$ for $\chi = 0.5$ giving $M = 0.5$. In both AR and NR cases, probabilities for $\uparrow (\downarrow)$-spin are shifted in the positive (negative) $eV$ direction. However, as mentioned above, the SF effect weakens as $Z_0$ increases, and both $\int_{-\pi/2}^{\pi/2} d\theta \cos \theta |P_{\downarrow} \sigma |h^{\uparrow\downarrow}_b|^2$ and $\int_{-\pi/2}^{\pi/2} d\theta \cos \theta |P_{\uparrow\downarrow} \sigma |h^{\uparrow\uparrow}_b|^2$ become symmetrical with respect to $eV = 0$. The magnitude difference between the $\uparrow$-spin and $\downarrow$-spin is due to the polarization $P_{z}$. As shown in figure 8, $P_{z}$ makes $C_{\downarrow}/C_N$ and $C_{\uparrow\downarrow}/C_N$ asymmetric with respect to $eV = 0$ for the barrier $Z_0$ where the effect of $S_{\uparrow}$ works. Eventually, $C_{\uparrow}/C_N$ and $C_{\uparrow\downarrow}/C_N$, composed of AR and NR probabilities become symmetrical with increasing $Z_0$.

Figure 5. Normalized total conductance $C_{\uparrow}/C_N$ (solid) together with $C_{\downarrow}/C_N$ (dotted) and $C_{\sigma}/C_N$ (dashed) in NM/FI/chiral $p$-wave SC junction corresponding to figures 3 and 4 where we have set $Z_0 = 5.0$, and $Z_w = 1.0, 2.0, 3.0, 5.0, 4.0$.

Figure 6. The bias voltage dependence of the angle averaged spin-filtering factor $\mathcal{F}_\uparrow$ (solid line) and $\mathcal{F}_\downarrow$ (dash line) in Stoner-FM/I/ chiral $p$-wave SC junctions. We have set (a) $Z_0 = 1.0, 2.0, 3.0, 4.0, 5.0$ for $\chi = 0.5$ and (b) $\chi = 0.1, 0.25, 0.5, 0.75, 1.0$ for $Z_0 = 2.0$. 

\[ Z_0 = 5.0 \]

\[ Z_w = 1.0, 2.0, 3.0, 5.0, 4.0 \]

\[ \text{solid: } C_{\uparrow}/C_N \]

\[ \text{dott: } C_{\downarrow}/C_N \]

\[ \text{dash: } C_{\sigma}/C_N \]

\[ eV/\Delta_0 \]

\[ \text{Normalized Conductance} \]

\[ \Phi \]

\[ \chi = 5.0 \]

\[ Z_0 = 1.0, 2.0, 3.0, 4.0, 5.0 \]

\[ \text{solid: } \mathcal{F}_\uparrow \]

\[ \text{dott: } \mathcal{F}_\downarrow \]

\[ eV/\Delta_0 \]
3.5. Spin bandwidth asymmetry ferromagnet

For the SBA-FM case ($\chi = 0.0$), the magnitude of $\mathcal{K}^{FM}_{\sigma}$ is calculated as $(\gamma^{-1/2} - \gamma^{1/2})(\cos^2{\theta} - 1)(\gamma^{-1/2} + \gamma) + 1$. As shown in figure 9, $\mathcal{F}_S(\mathcal{F}_I)$ shows the same tendency as that of the Stoner-FM case for the dependency $Z_0$ and $\gamma$. However, its behavior is opposite, i.e., the same tendency as the NM/FI/p-wave SC case where $\mathcal{F}_S(\mathcal{F}_I)$ has a minimum value at a negative (positive) $eV$. Because of this, in our model being $\mathcal{K}^{FM}_{\sigma}$ negative (positive) for $\sigma$-spin particle injection in Stoner-FM/FI/chiral p-wave SC junction for $\chi = 0.75$, Subscripts ‘e’ and ‘h’ are inverted with respect to positive and negative bias voltages. The solid (dash) lines shows the $\int_{\pi/2}^{\pi/2} d\theta \cos{\theta} P_S^{(1)} g^{FM}_{\sigma} (\sigma_T^{\sigma})^2$ and $\int_{-\pi/2}^{\pi/2} d\theta \cos{\theta} P_S^{(1)} b^{FM}_{\sigma} (\sigma_T^{\sigma})^2$. We have set $Z_0 = 1.0, 2.0, 5.0$.

Figure 8. Normalized total conductance $C_S/C_N$ (solid) together with $C_{S,1}/C_N$ (dotted) and $C_{S,1}/C_N$ (dashed) in Stoner–FM/FI/chiral p-wave SC junction corresponding to figure 6 where we have set $\chi = 0.75$, and $Z_0 = 1.0, 2.0, 5.0$.

3.5. Spin bandwidth asymmetry ferromagnet

For the SBA-FM case ($\chi = 0.0$), the magnitude of $\mathcal{K}^{FM}_{\sigma}$ is calculated as $(\gamma^{-1/2} - \gamma^{1/2})(\cos^2{\theta} - 1)(\gamma^{-1/2} + \gamma) + 1$. As shown in figure 9, $\mathcal{F}_S(\mathcal{F}_I)$ shows the same tendency as that of the Stoner-FM case for the dependency $Z_0$ and $\gamma$. However, its behavior is opposite, i.e., the same tendency as the NM/FI/p-wave SC case where $\mathcal{F}_S(\mathcal{F}_I)$ has a minimum value at a negative (positive) $eV$. Because of this, in our model being $\mathcal{K}^{FM}_{\sigma}$ negative (positive) for $\sigma$-spin particle injection in Stoner-FM/FI/chiral p-wave SC junction for $\chi = 0.75$, Subscripts ‘e’ and ‘h’ are inverted with respect to positive and negative bias voltages. The solid (dash) lines shows the $\int_{\pi/2}^{\pi/2} d\theta \cos{\theta} P_S^{(1)} g^{FM}_{\sigma} (\sigma_T^{\sigma})^2$ and $\int_{-\pi/2}^{\pi/2} d\theta \cos{\theta} P_S^{(1)} b^{FM}_{\sigma} (\sigma_T^{\sigma})^2$. We have set $Z_0 = 1.0, 2.0, 5.0$.

Figure 7. The bias voltage dependence of the angle averaged (a) Andreev $P_S g^{FM}_{\sigma h} (\sigma_T^{\sigma})^2$ and (b) normal $P_S b^{FM}_{\sigma h} (\sigma_T^{\sigma})^2$ reflection amplitudes for $\sigma$-spin particle injection in Stoner-FM/FI/chiral p-wave SC junction for $\chi = 0.75$. Subscripts ‘e’ and ‘h’ are inverted with respect to positive and negative bias voltages. The solid (dash) lines shows the $\int_{\pi/2}^{\pi/2} d\theta \cos{\theta} P_S^{(1)} g^{FM}_{\sigma h} (\sigma_T^{\sigma})^2$ and $\int_{-\pi/2}^{\pi/2} d\theta \cos{\theta} P_S^{(1)} b^{FM}_{\sigma h} (\sigma_T^{\sigma})^2$. We have set $Z_0 = 1.0, 2.0, 5.0$. 

Figure 10. The bias voltage dependence of the angle averaged (a) Andreev $P_S g^{FM}_{\sigma h} (\sigma_T^{\sigma})^2$ and (b) normal $P_S b^{FM}_{\sigma h} (\sigma_T^{\sigma})^2$ reflection amplitudes for $\sigma$-spin particle injection in Stoner-FM/FI/chiral p-wave SC junction for $\chi = 0.75$, Subscripts ‘e’ and ‘h’ are inverted with respect to positive and negative bias voltages. The solid (dash) lines shows the $\int_{\pi/2}^{\pi/2} d\theta \cos{\theta} P_S^{(1)} g^{FM}_{\sigma h} (\sigma_T^{\sigma})^2$ and $\int_{-\pi/2}^{\pi/2} d\theta \cos{\theta} P_S^{(1)} b^{FM}_{\sigma h} (\sigma_T^{\sigma})^2$. We have set $Z_0 = 1.0, 2.0, 5.0$. 

Figure 11. We show the angle averaged conductance $C_S$ and $C_{S,1}$ for $\gamma = 7.0 (M = 0.75)$. One can see...
that the conductance peaks shifts to $eV = 0$ with increasing $Z_0$. Since the critical angle of AR $\theta_c$ for $0.2 \lesssim M \lesssim 0.9$ is larger than that of Stoner-FM as demonstrated in our previous paper [57], the conductance peak becomes higher and shifts as compared with those of Stoner-FM case. On the other hand, as already mentioned in section I, the effects of the SBA-FM on the conductance has been calculated for $s$- and $d_{x^2-y^2}$-wave SC cases [56]. In [56], using the wavenumber ratio $k_{\bar{m}}^m = \sqrt{(\cos^2 \theta_2 + \gamma \theta_2^2/2 - 1)/(\cos^2 \theta_2 + \gamma \theta_2^2 - 1)}$ between $\sigma$ and $\bar{\sigma}$ instead of the group velocity ratio $g_{\bar{m}}^m$, the spin-dependent conductance for the injection of

Figure 9. The bias voltage dependence of the angle averaged spin-filtering factor $F_\uparrow$ (solid line) and $F_\downarrow$ (dashed line) in SBA-FM/1/ chiral $p$-wave SC junctions. We have set (a) $Z_0 = 1.0, 2.0, 3.0, 4.0, 5.0$ for $\gamma = 3.0$ and (b) $\gamma = 1.2, 1.67, 3.0, 7.0, 200$ for $Z_0 = 2.0$.

Figure 10. The bias voltage dependence of the angle averaged (a) Andreev $P_{g\bar{m}}^m |\Psi_{g\bar{m}}^m|^2$ and (b) normal $P_{b\bar{e}}{e'}^2$ reflection amplitudes for $\sigma$-spin particle injection in SBA-FM/1/ chiral $p$-wave SC junction for $\gamma = 7.0$. Subscripts ‘$e$’ and ‘$h$’ are inverted with respect to positive and negative bias voltages. The solid(dash) lines shows the $\int_{-\pi/2}^{\pi/2} d\theta \cos \theta \theta_{\bar{m}}^m |\Psi_{g\bar{m}}^m|^2$ and $\int_{-\pi/2}^{\pi/2} d\theta \cos \theta \theta_{\bar{m}}^m |\Psi_{b\bar{e}}{e'}^2|^2$. We have set $Z_0 = 1.0, 2.0, 5.0$. 

N. Yoshida
electron with $\sigma$ spin is calculated by $G_{\sigma} = 1 + k_{\sigma}^{\text{fm}}|a_1^{\sigma \phi_1}|^2 - |b_0^\phi|^2$. Figure 12 shows the angle averaged AR
\[
\int_{-\pi/2}^{\pi/2} d\theta \cos \theta \frac{P_1 k_{\sigma}^{\text{fm}}|a_1^{\sigma \phi_1}|^2}{\Delta_0} \text{cos} \gamma = \frac{1}{1 + \gamma}
\]
and the conductance $C_{\sigma} = \int_{-\pi/2}^{\pi/2} d\theta \cos \theta \frac{P_1 k_{\sigma}^{\text{fm}}|a_1^{\sigma \phi_1}|^2}{\Delta_0}$ normalized by $C_N$. As compared with $P_2 k_{\gamma}^{\text{fm}}|a_1^{\gamma \phi_1}|^2$, it is found that $P_2 k_{\gamma}^{\text{fm}}|a_1^{\gamma \phi_1}|^2$ reverses with respect to spin and shows inverted behavior for $eV = 0$ (figure 12(a)). The inverse of AR about spin can be easily seen from directly calculating $P_1 k_{\sigma}^{\text{fm}}|a_1^{\sigma \phi_1}|^2$. For example, using equation (2.5) and equation (2.8), it can be confirmed that $k_{\gamma}^{\text{fm}}|a_1^{\gamma \phi_1}|^2 = \frac{1}{1 + \gamma} k_{\sigma}^{\text{fm}}|a_1^{\sigma \phi_1}|^2$. In addition, the $\text{u}$- and $\text{l}$-spin polarizations for the SBA-FM are $P_1 = \gamma/(1 + \gamma)$ and $P_1 = 1/(1 + \gamma)$, respectively. Thus, since $P_{1}(1)\gamma^{-1} = P_{1}(1) k_{\sigma}^{\text{fm}}|a_1^{\sigma \phi_1}|^2$ converts to $P_{1}(1) k_{\gamma}^{\text{fm}}|a_1^{\gamma \phi_1}|^2$ with opposite spin-polarization in our model. Since the NR is equal to that in figure 10(b), in contrast to the asymmetric $C_{S,\sigma}/C_N$ and $C_{S}/C_N$ in figure 11, the resulting $C_{S_1}/C_N$ composed of mutually inverted $C_{S_1}/C_N$ and $C_{S_1}/C_N$ with respect to $eV = 0$ is symmetric. This difference between $C_{S,\sigma}/C_N$ and $C_{S}/C_N$ arise from satisfaction of the probability conservation. As shown above, the $k_{\gamma}^{\text{fm}}|a_1^{\gamma \phi_1}|^2$ does not satisfy the probability conservation of quasiparticle of

![Figure 11](image1.jpg)

**Figure 11.** Normalized total conductance $C_1/C_N$ (solid) together with $C_{S_1}/C_N$ (dotted) and $C_1/C_N$ (dashed) in SBA-FM/I/chiral $p$-wave SC junction corresponding to figure 9 where we have set $\gamma = 7.0$, and $Z_0 = 1.0, 2.0, 5.0$.

![Figure 12](image2.jpg)

**Figure 12.** The bias voltage dependence of the angle averaged (a) Andreev reflection amplitude $\int_{-\pi/2}^{\pi/2} d\theta \cos \theta P_1 k_{\sigma}^{\text{fm}}|a_1^{\sigma \phi_1}|^2$ and (b) the normalized spin-dependent conductance $C_{S,\sigma}/C_N$ and the normalized total conductance $C_{S}/C_N = C_{S_1}/C_N + C_{S_1}/C_N$ (black solid line) in SBA-FM/I/chiral $p$-wave SC junction for $\gamma = 7.0$ and $Z_0 = 2.0$. The upper right inset in (a) is the angle averaged Normal reflection. The red/blue solid line shows the injection of electron $|T(\downarrow\uparrow)|$-spin. The dashed line is the corresponding result for $\gamma = 7.0$ and $Z_0 = 2.0$ in figure 11.
flow. In fact, the quantity naturally derived from the probability conservation is \( g_{\sigma}^{\text{fin}} \langle a_{\sigma}^\dagger a_{\sigma} \rangle \). Although the conductance behavior between \( C_5/C_N \) and \( C_{5,s}/C_N \) is different, the mechanism of the SF effect owing to \( P_n \) will be common.

From our results, it may be expected that the measurement of the position and height of the conductance peak for weak barrier \( Z_0 \) excluding tunneling and metallic limits give an information on the cause of the SF effect as well as the distinction between the pure Stoner- and pure SBA-FMs.

### 3.6. Stoner-Spin bandwidth asymmetry mixed ferromagnet

From the above results, it is expected that the peak position varies depending on the degree of mixing in Stoner and SBA composite ferromagnet. In figure 13, we show the \( (\chi, \gamma) \)-dependence of the angle averaged \( C_5/C_N \) and \( C_{5,s}/C_N \) for \( M = 0.9, Z_0 = 2.0 \). As the proportion of SBA-FM \( \gamma \) increases from the pure Stoner-FM \( \chi = 0.9, \gamma = 1.0 \), the \( C_{5,1}/C_N \) shows that the peak position moves from positive to negative \( eV \). Since the \( M \) is near the half-metal limit, \( C_{5,1}/C_N \) does not contribute much to the total conductance \( C_5/C_N \) (figure 13(b)). Hence, the peak position of angle averaged \( C_5/C_N \) (figure 13(c)) moves from positive to negative \( eV \) according to \( C_{5,1}/C_N \) (figure 13(a)). In such a mixed FM, it may be seen which mechanism of Stoner-\( \chi \) and SBA-\( \gamma \) is more dominant from the peak position except \( eV = 0 \).

### 3.7. Spin-filtering effects in the FM/FI/SC junction

Although the effect as the splitting of ZBCP is not seen in the NM/FI/chiral \( p \)-wave SC junction, a remarkable FI effect can be expected because the symmetrically shift between \( C_{5,1}/C_N \) and \( C_{5,1}/C_N \) is broken by the polarization \( P_n \) in the FM/FI/chiral \( p \)-wave SC junctions. It can also be predicted from \( \kappa_{\sigma} = \kappa_{\gamma} \) in FM that the effect of FI will not disappear even if \( Z_{ex} = Z_0 \). Indeed, for example, the \( S_{\uparrow} \) for \( \uparrow \)-spin incident electrons is given as

\[
S_{\uparrow} = \frac{-4 \cos \theta_5 Z_0 (\cos \theta_5 - \kappa_{\uparrow})}{(\cos \theta_5 + \kappa_{\uparrow})(\cos \theta_5 + \kappa_{\downarrow})^2 + 4Z_0^2}.
\]

Thus, the SF effect of FI for \( Z_{ex} = Z_0 \) will not disappear by synergy with FM but the increment of \( Z_{ex} \) reduces conductance through the suppression of AR. The \( Z_{ex} \) dependences of angle averaged \( C_5/C_N \) in the FM/FI/\( p \)-wave SC junctions are shown for fixed \( M = 0.9, Z_0 = 2.0 \) in figure 14. For pure Stoner-FM case (figure 14(a)), with increasing \( Z_{ex} \) the position of conductance peak moves to a negative bias voltage direction since the conductance peak shifts for \( C_{5,1}/C_N \) and \( C_{5,1}/C_N \) by FI are opposite to that due to Stoner-FM (see figure 5 and figure 8). However, as \( Z_{ex} \) approaches \( Z_0 \), \( C_{5,1}/C_N \) becomes dominant in the total conductance \( C_5/C_N \) and the
influence of Stoner-FM is reflected more strongly than that of FI. As a result, a conductance peak for \( Z_{\text{ex}} = Z_0 \) is seen at a positive voltage. On the other hand, in the case of SBA-FM (figure 14(b)), it can be seen that the peak position is shifted to a more negative bias voltage with increasing \( Z_{\text{ex}} \), since the shift direction by FM and FI is the same. Although these behaviors can be seen even in the case of the mixed FM given by (\( \chi \sim 0.64, \gamma \sim 4.1 \)) (figure 14(c)), the deviation of the peak to the negative bias voltages is reduced by the effect of Stoner-\( \chi \).

However, in a mixed FM, the peak shift by FI may provide information on the dominant magnetic factor of FM.

4. Summary

In this paper, we have presented a general SF factor induced in the FM/FI/SC junctions. In roughly speaking, the SF factor is represented by a simple product of the exchange potential in FI and FM and the imaginary part of \( \Gamma \). The SF factor shifts the conductance \( \chi \) is different from the shift of ABS by the spin-mixing scattering and Zeeman splitting. The SF factor appears commonly in the denominator of the AR and NR reflection probabilities and shows sinusoidal behavior as a function of energy by reflecting the \( \chi \) and \( \gamma \) of the magnetic effect. That is, the positions of the minimum value of the denominator of AR and NR probabilities for \( \Gamma \) and \( \Gamma \) are shifted in opposite \( eV \) direction depending on the sign of the SF factor. As the result, the \( C_{G\chi} \) of the AR and NR probabilities is shifted. This is a scenario of the SF factor in the FM/NM/SC junctions. Especially, for a simply FI without the spin-mixing effect, the pure Stoner- and SBA-FM cases, we can see easily how the SF effect occurs. For the FI case, the SF factor appears only when \( 0 < Z_{\text{ex}} < Z_0 \) and induces a symmetrical shift with respect to \( \Gamma \) and \( \Gamma \)-spins. However, the total conductance \( C_{G\chi} \) in the NM/FI/chiral \( p \)-wave SC junction does not show any ZBCP-splitting or ZBCP-shift like \( d \)-wave case except for the suppression of ZBCP since only the incident angle \( \theta_{\delta} = 0 \) contributes to the formation of ZES. In the pure Stoner- and SBA-FM cases, the SF factor does not appear in the metallic limit \( Z_0 = 0 \) and are weakened in the tunneling limit, e.g. \( Z_0 \geq 5.0 \). Hence, the SF effect in FM/1/chiral \( p \)-wave SC junctions have not been shown in many studies targeting the junctions in the metallic limit or the tunneling limit. For both the pure Stoner- and SBA-FM/1/chiral \( p \)-wave SC junctions, the ZBCP shifts in mutually opposite directions have been shown in an intermediate barrier potential. Furthermore, it has been shown that these ZBCP shifts may be tunable by FI. From these obtained results, it will be expected that the conductance in FM/FI/chiral \( p \)-wave junctions give us a useful information to the mechanism of the ferromagnet and a tips for developing spintronics devices by using the concept of simply SF factor. However, since the electronic band structures of the FM/FI and SC are not taken into consideration, it is difficult to mention control and detect of parameters \( \gamma, \chi \) and \( Z_{\text{ex}} \) for actual experiments in the present model. In addition, other realistic effects such as the singlet-triplet spin mixing scattering, the spin-orbit scattering, the superconducting (or inverse) proximity effect, the interface roughness, and also the

![Figure 14](image-url) - The \( Z_{\text{ex}} \)-dependence of the normalized total conductance \( C_{G\chi}/C_N \) in FM/FI/chiral \( p \)-wave SC junction for fixed \( M = 0.9 \) and \( Z_0 = 2.0 \). The ferromagnet is (a) pure Stoner-FM (\( \chi = 0.9 \)), (b) pure SBA-FM (\( \gamma = 19 \)), and (c) mixed-FM (\( \chi \sim 0.64, \gamma \sim 4.1 \)).
detailed effect of evanescent AR process near the interface are ignored. In a future work, inclusion of these effects should be necessary for comparison with actual experiments and applications.

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