Deorbit Options for a Low Lift-to-Drag Ratio Space Emergency Rescue Vehicle*

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Space rescue is of crucial importance when sudden accidents occur during manned orbital flights. Various rescue plans have been proposed for returning crews to the Earth. However, some plans require several days to get the crew to a specified location, and some can only direct the crew to a vast sea area within a short period of time. This paper presents a rescue scheme using a low lift-to-drag ratio vehicle that can send the spacecraft crew to a designated landing point within 48 hours. The scheme presented combines two measures, namely a return trajectory maneuver and multiple-revolution orbital phasing, to fulfill the requirements of urgent deorbiting. The return trajectory maneuver extends the deorbit window, avoiding possible losses in system reliability induced by delays in space, while the orbital phasing steers the space emergency rescue vehicle to a selected deorbit point at a given time. A numerical simulation is done to verify the rescue scheme proposed. The results show that the scheme requires little re-entry maneuverability, few fuel consumption, and only one on-ground rescue site, which significantly simplifies the rescue system and reduces rescue cost.

Key Words: Space Rescue, Deorbit Window, Orbital Phasing, Orbital Maneuver

Nomenclature

\( \mu \): Earth gravitational constant, 398,600 km\(^3\)s\(^{-2}\)
\( \alpha \): Earth mean equator radius, 6,378.140 km
\( R_e \): Earth mean radius, 6,371 km
\( g_0 \): Earth gravity coefficient, 9.8 m/s\(^2\)
\( \lambda_4 \): longitude of point A, rad
\( \lambda_N \): longitude of ascending node, rad
\( a \): semi-major axis, km
\( i \): inclination, rad
\( \omega \): argument of perigee, rad
\( \Omega \): argument of latitude, rad
\( p \): orbital parameter
\( \delta_R \): geocentric angle of downrange, rad
\( \Delta R \): maneuverable return downrange, km
\( \Delta t \): time interval between present and deorbit, s
\( \Delta t_P \): time interval of phasing, s
\( \alpha, \beta \): Lagrange’s parameter
\( s \): semi-perimeter of basic triangle
\( \nu_0 \): characteristic velocity, m/s
\( J_2 \): 1.08263 \times 10^{-3}
\( \omega_0 \): Earth rotation angular, 7.292115 \times 10^{-5} \text{ rad/s}^{-1}
\( \varphi_A \): latitude of point A, rad
\( \Lambda_0 \): azimuth at deorbit point D, rad
\( e \): eccentricity
\( \Omega_2 \): right ascension of ascending node, rad
\( f \): true anomaly, rad
\( n \): mean orbital angular velocity, rad/s
\( r \): geocentric distance
\( \delta_H \): geocentric angle of crossrange, rad
\( \Delta H \): maneuverable return crossrange, km
\( N \): number of revolutions

\( \Delta t_R \): interval of return, s
\( \Delta t_{\text{min}} \): transfer time of minimum-energy ellipse, s
\( c \): length of chord
\( \eta \): iterating variable

SERV: space emergency rescue vehicle
L/D: lift-to-drag ration
CHV: characteristic velocity

1. Introduction

Space rescue has been a topic since the start of manned orbital flights, especially after the construction of space stations.\(^1\)\(^-\)\(^5\) When a sudden accident occurs, such as a fire disaster, collision, or crew illness that cannot be cured in-orbit, it may become unacceptable for the crew to remain in the space station. An urgent means of returning the crew to the Earth should then be available. In 1997, the Mir caught fire and collided with a cargo spaceship, which highlights the necessity for space rescue capability, particularly as launching a rescue vehicle to the space station responsively is not presently practical. The Mir and International Space Station (ISS) had chosen to dock a spaceship in case of an accident. A variety of space emergency rescue vehicles (SERV) have been proposed in the past,\(^2\)\(^-\)\(^3\) among which the most well-known is the lifting Crew Return Vehicle (CRV).\(^4\)\(^-\)\(^6\)

When discussing the concept of space rescue, some fundamental problems need to be resolved, including the performance of the SERV, the location and number of landing points, and the manner of deorbiting. The lift-to-drag ratio (L/D) is the most crucial parameter for a SERV. It determines the re-entry style, thus influencing the maneuverability, manner of landing, thermal protection, aerodynamic loads, and deorbit windows. A high L/D leads to better re-entry performance, but it also brings intractable thermal protection problems and a high cost.\(^7\) In practice, the latitudes of the landing points must be lower than the orbital inclina-
tion of the space station. More landing points mean more opportunities to return to Earth, but this also means more life-saving personnel and equipment for sufficient coverage. Concerns about deorbiting manner include the number of braking impulses, guidance approach, and deorbit window. Increasing impulses could satisfy more mission constraints and relax the fuel requirements. However, orbital maneuvers concurrently lower the reliability of the system. Finally, an important and unavoidable issue is the cost, which usually determines the eventual solution to the rescue system.

Having accomplished some rendezvous and docking tests, China will construct its own space station and face the problem of space rescue. The Shenzhou spaceship, or a similar spacecraft with improved L/D, will play the role of the SERV. The ideal L/D of the spacecraft is 0.3–0.5. A propulsion module will be attached to the spacecraft to supply power for in-orbit maneuvers and deorbit braking. The SERV will separate from the propulsion module after braking, and return to the landing point utilizing lift control. The SERV can accommodate three astronauts and provide life support for 48 hours, meaning the SERV must catch a deorbit window within two days. In addition, no overseas rescue range is available for China. Considering the geographical location, adjacent environment, and rescue implementation, a location near the coast of Tsingtao in the Yellow Sea has been suggested as a unique landing point. As a result, in order to return to the landing site within 48 hours, the deorbit window design becomes a crucial problem that must be carefully explored.

The deorbit window is defined as the interval within which the rocket motor can ignite to accomplish the return mission. If the landing point and return trajectory are unchangeable, the ground track of the braking point is fixed on the Earth. The rocket motor can ignite only when its ground track passes the orbital plane while the SERV is in the proper phase. Thus, the deorbit window is very narrow. Two measures can be used to extend the deorbit window, namely the return trajectory maneuver and orbital phasing. A vehicle with return maneuverability is beneficial for increasing the deorbit opportunities. For instance, the deorbit window of the Space Shuttle is wider than that of a low-L/D spacecraft.11) The orbital phasing can direct the SERV into the desired phase at a given time. Generally, orbital phasing can be classified into two categories: Hohmann-style phasing and Lambert-style phasing. The two terminal ends of a Hohmann phasing orbit must be identical, while that of a Lambert phasing orbit can be selected arbitrarily. Lambert phasing appears to be more flexible, while Hohmann phasing is a special case of Lambert phasing. A Lambert maneuver with multiple revolutions is called the N-revolution Lambert maneuver, meaning that the spacecraft flies N revolutions in the transition orbit before transferring into the new orbit. In recent years, attention has been focused on the multiple-revolution transfer because it has the potential to consume less fuel than a zero-revolution transfer. The difficulty in designing a multiple-revolution transfer exists in the optimization of the number of revolutions, which can be expressed as a mixed integer programming problem.12,13)

This paper explores a methodology that combines the return trajectory maneuver and multiple-revolution Lambert phasing, which can catch a deorbit window within 48 hours for the SERV to return to a designated landing point. Design procedures and numerical simulations are presented. In the following discussion, it is assumed that the SERV and propulsion module have departed from the space station and are now coasting in orbit.

2. Return Trajectory Maneuver

This section studies the relationship between the deorbit window and return range based on return movement characteristics and spherical geometry equations. Only the scenario with the orbital inclination of the SERV less than π/2 is considered. Results can be generalized with little effort to situations with inclinations of larger than π/2.

2.1. Fixed return range

Firstly, the case of a fixed return range, which is the nominal situation for a low-L/D return vehicle, is investigated.

The spherical geometry is shown in Fig. 1. Point F denotes the landing point. Since point F rotates with the Earth, the SERV should aim at point F during braking in the inertial framework. The geographic coordinates of the two points are related by:

\[ \lambda_{F1} = \lambda_F + \omega_e \Delta t, \quad \phi_{F1} = \phi_F \]  

(1)

Point D represents the spherical projection of the deorbit point and point N represents the projection of the orbital ascending node. The longitude of the ascending node \( \lambda_{NI} \) is introduced to describe the location of the point, N. Considering the situation where the landing point is a sub-satellite point of the ascending arc (i.e., called the ascending arc case in the following analysis), the argument of latitude for the deorbit target point, \( F_1 \), can be derived according to the sine theorem in spherical triangle \( \triangle NN_F F_1 \) as:

Fig. 1. Spherical geometry of a fixed return range.
\[ u_{F1} = \sin^{-1}\left(\frac{\sin \varphi_{F1}}{\sin i}\right) \]  
\[ (2a) \]

Hence, the argument of latitude of the deorbit point, \( D \), is:

\[ u_D = u_{F1} - \delta_R \]
\[ (3) \]

In the spherical triangle \( NND_D \), the geocentric latitude of point \( D \) is derived by the sine theorem as:

\[ \varphi_D = \sin^{-1}(\sin u_D \sin i) \]
\[ (4) \]

To obtain the longitude of point \( D \), the cosine equation of the arc in the spherical triangle \( PDF_I \) is adopted:

\[ \cos \delta_R = \cos\left(\frac{\pi}{2} - \varphi_D\right) \cos\left(\frac{\pi}{2} - \varphi_{F1}\right) + \sin\left(\frac{\pi}{2} - \varphi_D\right) \sin\left(\frac{\pi}{2} - \varphi_{F1}\right) \cos \Delta \lambda_D \]

and hence:

\[ \lambda_D = \lambda_{F1} - \cos^{-1}\left(\frac{\cos \delta_R - \sin \varphi_D \sin \varphi_{F1}}{\cos \varphi_D \cos \varphi_{F1}}\right) \]
\[ (5) \]

The braking azimuth, \( \lambda_0 \), is defined as the included angle between the return longitudinal plane and local meridian plane (0 < \( \lambda_0 < \pi \), if \( i < \pi/2 \)). In spherical triangle \( PDF_I \), \( \lambda_0 \) can be expressed as:

\[ \lambda_0 = \cos^{-1}\left(\frac{\sin \varphi_{F1} - \sin \varphi_D \cos \delta_R}{\cos \varphi_D \sin \delta_R}\right) \]
\[ (6) \]

In the spherical triangle \( NND_D \), the relationship between the longitude of the ascending node, \( N \), in the deorbiting moment and the longitude of the ground track of the braking point, \( D \), can be described by:

\[ \cos \Delta \lambda_N = \frac{\cos u_D}{\cos \varphi_D} \]

Taking into account that point \( D \) can be in either the northern or southern hemisphere, it can be obtained

\[ \Delta \lambda_N = \text{sgn}(\varphi_D) \cdot \cos^{-1}\left(\frac{\cos u_D}{\cos \varphi_D}\right), \quad \lambda_{Nt} = \lambda_D - \Delta \lambda_N \]
\[ (7) \]

Accordingly, given the initial longitude \( \lambda_{N0} \) of the ascending node and the initial argument of latitude \( u_0 \) of the SERV, the condition that the sub-satellite point of the SERV is at point \( D \) after time interval \( \Delta t \) can be given by:

\[
\begin{align*}
\lambda_{N0} + \Delta \lambda_N - \omega_{oe} \Delta t &= \lambda_D - 2k_D \pi \\
u_0 + n \Delta t &= u_D + 2k_R \pi
\end{align*}
\]
\[ (8) \]

where \( k_D \) and \( k_R \) are the Earth rotation and SERV revolution numbers, respectively, during the interval, \( \Delta t \). They are both nonnegative integers. When \( k_D \) equals 1, the SERV will return within one day, while when \( k_R \) equals 1, the SERV will return within the current revolution. \( \omega_{oe} \) represents the angular velocity of the Earth with respect to the orbital plane when the secondary zonal harmonic term, \( J_2 \), of the Earth is taken into account. \( \omega_{oe} \) can be calculated by:

\[ \omega_{oe} = \omega_e - \frac{3}{2} J_2 \sqrt{\frac{\mu}{a_e^3}} \left(\frac{a_e}{a}\right)^{3.5} \cos i \left(1 - e^2\right) \]
\[ (9) \]

Let \( \Delta \lambda_t = \lambda_{N0} + \Delta \lambda_N - \lambda_D \) and \( \Delta u_t = u_D - u_0 \). Eq. (8) is rewritten as:

\[ \frac{\Delta \lambda_t + 2k_D \pi}{\omega_{oe}} = \frac{\Delta u_t + 2k_R \pi}{n} \]
\[ (10) \]

Solving \( k_R \) from the above equation, \( k_R \) can be obtained as

\[ k_R = n \Delta \lambda_t - \omega_{oe} \Delta u_t + \frac{k_D \pi}{2\pi \omega_{oe}} \]
\[ (11) \]

Note that \( k_D \) and \( k_R \) are nonnegative integers. From Eq. (11), when the initial condition is unsuitable, a long idle time has to be spent waiting for the deorbit window.

For the situation where the landing point is a sub-satellite point of the descending arc (i.e., called the descending arc case in the following analysis), all of the equations above are applicable except Eq. (2a), which should be replaced by:

\[ u_{F1} = \pi - \sin^{-1}\left(\frac{\sin \varphi_{F1}}{\sin i}\right) \]
\[ (2b) \]

### 2.2. Maneuverable return range

Generally, a re-entry vehicle still has some longitudinal and lateral maneuverability besides the ability to overcome uncertainties and errors during the return course. When the SERV’s return range (i.e., specifically re-entry range) is adjustable, the deorbit point, \( D \), will be extended to a deorbit region, which is illustrated by the shadow around point \( F_1 \) in Fig. 2. The supposition that the return range is maneuverably re-entry range (i.e., called the return range for a maneuverable return range) is adopted:

\[ \omega_{oe} = \omega_e - \frac{3}{2} J_2 \sqrt{\frac{\mu}{a_e^3}} \left(\frac{a_e}{a}\right)^{3.5} \cos i \left(1 - e^2\right) \]

Let \( \Delta \lambda_t = \lambda_{N0} + \Delta \lambda_N - \lambda_D \) and \( \Delta u_t = u_D - u_0 \). Eq. (8) is rewritten as:

\[ \frac{\Delta \lambda_t + 2k_D \pi}{\omega_{oe}} = \frac{\Delta u_t + 2k_R \pi}{n} \]

Solving \( k_R \) from the above equation, \( k_R \) can be obtained as

\[ k_R = n \Delta \lambda_t - \omega_{oe} \Delta u_t + \frac{k_D \pi}{2\pi \omega_{oe}} \]

Note that \( k_D \) and \( k_R \) are nonnegative integers. From Eq. (11), when the initial condition is unsuitable, a long idle time has to be spent waiting for the deorbit window.

For the situation where the landing point is a sub-satellite point of the descending arc (i.e., called the descending arc case in the following analysis), all of the equations above are applicable except Eq. (2a), which should be replaced by:

\[ u_{F1} = \pi - \sin^{-1}\left(\frac{\sin \varphi_{F1}}{\sin i}\right) \]

![Fig. 2. Ground track of the ADR for a maneuverable return range.](image-url)
ranges are small quantities with respect to the radius of the Earth. Therefore, when determining the allowable deorbit region (ADR), one can assume that the vehicle flies reversely from point $F_1$ to point $D_0$. The ADR therefore becomes an area around point $D_0$ within the longitudinal distance $\Delta R$ and lateral distance $\Delta H$, as shown by the shadow around point $D_0$ in Fig. 2.

As shown in Fig. 3, point $D$ is arbitrarily selected within the ADR. With respect to nominal deorbit point $D_0$, the longitudinal geocentric angle of point $D$ is $\Delta \delta_R$, measured along the return direction. The lateral geocentric angle is $\Delta \delta_H$, measured clockwise. $\Delta \delta_R$ and $\Delta \delta_H$ are calculated from the maneuverable re-entry range $\Delta R$ and $\Delta H$ using the following equations:

$$\Delta \delta_R = \frac{\Delta R}{R_e}, \quad \Delta \delta_H = \frac{\Delta H}{R_e}$$

According to the cosine theorem of arc in the spherical triangle $D_0DK$, $D_0D$ can be calculated as:

$$\overline{D_0D} = \cos^{-1}(\cos \Delta \delta_R \cos \Delta \delta_H)$$

The azimuth of arc $D_0D$ with respect to the origin longitudinal plane using the sine theorem can be obtained by:

$$\alpha_0 = \text{sgn}(\Delta \delta_H) \sin^{-1}\left(\frac{\sin \Delta \delta_H}{\sin \overline{D_0D}}\right)$$

The sign function, $\text{sgn}(\cdot)$, is introduced to accommodate different directions of $\Delta \delta_H$. When $\Delta \delta_R$ is positive, the cosine theorem of arc is used again in the spherical triangle $PD_0D$:

$$\cos\left(\frac{\pi}{2} - \varphi_D\right) = \cos\left(\frac{\pi}{2} - \varphi_{D0}\right) \cos \overline{D_0D}$$

$$+ \sin\left(\frac{\pi}{2} - \varphi_{D0}\right) \sin \overline{D_0D} \cos(A_0 + \alpha_0)$$

Thereby, the latitude of point $D$ can be calculated by:

$$\varphi_D = \sin^{-1}\left[\sin \varphi_{D0} \cos \overline{D_0D} \right.$$

$$\left. + \cos \varphi_{D0} \sin \overline{D_0D} \cos(A_0 + \alpha_0)\right]$$

According to the sine theorem in the spherical triangle $PD_0D$:

$$\frac{\sin(\lambda_D - \lambda_{D0})}{\sin \overline{D_0D}} = \frac{\sin(\alpha_0 + \alpha_0)}{\sin(\pi/2 - \varphi_D)}$$

The longitude of point $D$ is:

$$\lambda_D = \lambda_{D0} + \sin^{-1}\left[\frac{\sin\varphi_{D0} \cos \overline{D_0D}}{\cos \varphi_D}\right]$$

With similar derivations, when $\Delta \delta_R$ is negative, the geographic coordinate of point $D$ can be expressed as:

$$\varphi_D = \sin^{-1}\left[\frac{\sin\varphi_{D0} \cos \overline{D_0D}}{\cos \varphi_D}\right]$$

$$- \cos \varphi_{D0} \sin \overline{D_0D} \cos(A_0 - \alpha_0)$$

$$\lambda_D = \lambda_{D0} - \sin^{-1}\left[\frac{\sin(A_0 - \alpha_0) \sin \overline{D_0D}}{\cos \varphi_D}\right]$$

Given $\Delta R$ and $\Delta H$, the ADR can be determined from Eqs. (16) and (17).

When the deorbit point varies in the region, the return time $\Delta t_R$ will change accordingly. Because the variation is a trivial portion of the total interval, it is ignored in the following analysis. If more accurate simulations are needed, a quadratic polynomial could be adopted in terms of $\Delta R$ and $\Delta H$ to fit $\Delta t_R$.

### 2.3. Allowable deorbit region analysis

A scenario is presented in this subsection to elaborate the calculation of the ADR. Initial orbital elements are listed in Table 1, which approximates the rendezvous and docking orbit of the Space Palace Test Module. An orbit nearby will probably be chosen as the running orbit of China’s first space station.

For convenience, the landing field is located near the east coast of China in Yellow Sea. The geographic coordinates of the landing field center are $35.0^\circ$N, $120.0^\circ$E. The downrange from the nominal deorbit point to the landing point is 13,000 km, and the total return time $\Delta t_R$ is 2,100 s. Preliminary simulations are conducted to obtain the maneuverability of the SERV. In the simulations, constraints such as heat flux peak and maximum load are considered, and the maneuverability varies when these constraints change. The heat flux peak is constrained to 1,200 kW/m², and the maximum load is constrained to $3.5g_0$ during the simulation. The resulting total maneuverable downrange is 1,400 km, while the cross-range is 220 km. Considering that uncertainties and errors mainly influence longitudinal motion during re-entry, the surplus longitudinal maneuverable range for extending deorbit windows is set to 200 km, while the lateral is set to 100 km.

According to the given conditions, the geographic coordi-

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**Table 1. Initial orbital elements of the SERV.**

| $a_0$, km | $e_0$ | $i_0$, deg | $\Omega_0$, deg | $\omega_0$, deg |
|-----------|-------|------------|----------------|---------------|
| 6728.140  | 0.0   | 42.8       | 45.0           | 0.0           |

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can be described using the following linear equations:

Taking the ascending arc as an example, the constraints of the deorbit point are constrained by the ADR. For a floating instrumentation ship, the sub-satellite point of the deorbit place is constrained by the ADR. Hence, there is no need to deploy it at the speciﬁc time interval. Since an orbital planar maneuver does not save much waiting time and consumes a large amount of fuel, it is not adopted in this paper.

To design the phasing orbit, terminal point is assumed as optimization parameters.

Fig. 4. Allowable deorbit region.

Fig. 5. ADR and landing point on a world map.

The deorbiting opportunity for a fixed return range will be discussed in Section 4 together with other cases.

3. Multiple-Revolution Lambert Orbital Phasing

3.1. Basic models

For a low-L/D re-entry vehicle, the adjustable return range is very limited. It is impossible to return to Earth within 48 hours using the re-entry maneuver only. Hence, orbital phasing is a necessary and effective way to extend the deorbit windows.

The procedure for multiple-revolution Lambert orbital phasing is shown in Fig. 6. The SERV diverts from the initial orbit to the phasing orbit at point \( F_1 \) with velocity increment \( \Delta v_1 \), and coasts for \( N \) revolutions to reach point \( F_2 \). The SERV then executes another velocity increment, \( \Delta v_2 \), and returns to the initial orbit. Because the orbital periods of the two mentioned orbits are different, the spaceship flies through different geocentric angles in a ﬁxed time interval. Thus, the SERV is able to arrive at the required deorbit point at the speciﬁed time. Since an orbital planar maneuver does not save much waiting time and consumes a large amount of fuel, it is not adopted in this paper.

To design the phasing orbit, terminal point \( F_2 \) is ﬁxed according to the deorbit requirements. For example, point \( F_2 \) is required to be one revolution ahead of deorbit point \( D \). The argument of latitude of point \( F_1 \) and the geographic coordinates of braking point \( D \) are assumed as optimization param-
Fig. 6. Multiple-revolution Lambert phasing maneuver.

Values for the three variables are chosen to minimize fuel consumption and waiting time before braking. Noting that the deorbit can only be performed when point $D$ passes across the orbital plane, total time interval $\Delta t$ can be calculated from the present to the deorbit moment. The time interval, $\Delta t_{F1}$, that the SERV coasts from initial position $u_0$ to point $F_1$ is obtained by:

$$\Delta t_{F1} = \frac{u_{F1} - u_0}{n}$$  \hspace{1cm} (19a)$$

Similarly, the time interval, $\Delta t_{F2}$, that the SERV coasts from point $F_2$ to the deorbiting position $u_D$ is obtained by:

$$\Delta t_{F2} = \frac{u_D - u_{F2}}{n}$$  \hspace{1cm} (19b)$$

Hence, phasing time $\Delta t_p$ can be obtained as:

$$\Delta t_p = \Delta t - \Delta t_{F1} - \Delta t_{F2}$$  \hspace{1cm} (20)$$

Given the positions of points $F_1$ and $F_2$, and phasing time $\Delta t_p$, the semi-major axis, $a_P$, of the phasing orbit can be obtained by iterating Lagrange’s time equation:

$$\Delta t_p = \sqrt{\frac{s^3}{\mu(1+\cos \eta)}[(2N+1)\pi + (\eta + \sin \eta) \mp (\beta - \sin \beta)]}$$  \hspace{1cm} (21)$$

When the transfer angle is $\Delta \phi \leq \pi$, the sign “−” is adopted in the foregoing equation; otherwise the sign “+” is adopted. The variable $\eta$ is the iterating parameter defined by:

$$\eta = \begin{cases} \alpha - \pi, & \Delta t_p \leq \Delta t_m; \\ \pi - \alpha, & \Delta t_p > \Delta t_m \end{cases}$$  \hspace{1cm} (22)$$

where $\alpha$ and $\beta$ are Lagrange parameters that can be calculated from:

$$\sin \frac{\alpha}{2} = \sqrt{\frac{r_1 + r_2 + c}{4a}} = \sqrt{\frac{s}{2a}}$$

$$\sin \frac{\beta}{2} = \sqrt{\frac{r_1 + r_2 - c}{4a}} = \sqrt{\frac{s - c}{2a}}$$  \hspace{1cm} (23)$$

where $s$ is the semi-perimeter of the basic triangle $O_x F_1 F_2$ and $c$ is the length of the chord. The following equation is used for the iteration:

$$f(\eta) = \sqrt{\frac{s^3}{\mu(1+\cos \eta)}} \times [(2N+1)\pi + (\eta + \sin \eta) \mp (\beta - \sin \beta)] - \Delta t_p$$  \hspace{1cm} (24)$$

The Newtonian Method is adopted for the iteration, with iterating equation and termination condition given by:

$$\eta_{n+1} = \eta_n - \frac{f(\eta_n)}{f'(\eta_n)}, \quad |\eta_{n+1} - \eta| < \varepsilon$$

where

$$f'(\eta_n) = \frac{3}{2} \left[ \frac{f(\eta_n) + \Delta t_p}{2} \right] \sin \eta \left[ 1 + \left( \frac{s - c}{s} \right)^2 \sin \eta \right] \sin \beta$$  \hspace{1cm} (25)$$

The expressions for the semi-major axis, $a_P$, and the orbital parameter, $p_P$, in terms of $\eta$ are:

$$a_P = \frac{s}{1 + \cos \eta}, \quad p_P = \frac{4a(s - r_1)(s - r_2)}{c^2} \cos \left( \frac{\lambda + \beta}{2} \right)$$  \hspace{1cm} (26)$$

The velocity increments $\Delta v_1$ and $\Delta v_2$ can be calculated using $a_P$ and $p_P$. During the iterating process, it should be noted that $-\pi < \eta < \pi$ and the perigee of the phasing orbit must not be lower than the upper boundary of the Earth’s dense atmosphere.

3.2. Design procedure and optimization

As some system checkout procedures and process programming are needed before deorbit, a time interval between terminal point $F_2$ and deorbit point $D$ is required. Point $F_2$ is supposed to be one revolution ahead of point $D$ in the design. The optimization parameter is $x = [\varphi_D, \lambda_D, u_{F1}]$, in which $\varphi_D$ and $\lambda_D$ should satisfy the constraints of Eq. (18). Generally, fuel consumption is lower if the phasing maneuver is performed earlier. Thus, $u_{F1} - u_0 \leq 2\pi$ is required, which means time interval $\Delta t_{F1}$ from the present point to point $F_1$ is less than one revolution. Furthermore, the revolution number, $N$, needs to be specified to determine the unique phasing orbit. Some integer programming methods can be adopted to optimize the number $N$, such as the branch-boundary or cutting-plane methods. Since parameter $x$ also needs to be optimized, it becomes a mixed integer programming problem, which is difficult to converge. Since there are few feasible solutions for number $N$, an enumeration method is used instead. The lowest altitude of the phasing orbits is restricted, and is denoted by $h_{\min}$. The lower boundary of the semi-major axes of phasing orbits then becomes:

$$a_{\min} = \frac{a_0 + h_{\min} + R_e}{2}$$  \hspace{1cm} (27)$$

The minimum orbital period, $T_{\min}$, is obtained accordingly. If fuel consumption is taken as the optimization index, the scope of number $N$ can be confined to:
\[ N_{\text{min}} \leq N \leq N_{\text{max}}, \]

\[
N_{\text{min}} = \max\left\{ \frac{\Delta t_p}{T_0} - 5, 0 \right\}, \quad N_{\text{max}} = \frac{\Delta t_p}{T_{\text{min}}} \quad (28)
\]

The optimization function, \( f_{\text{mincon}} \), embedded in Matlab is utilized to find the optimal value of \( x \). To avoid the semi-major axis of the resulting phasing orbit exceeding the boundary, a penalty function is introduced. Thus, the following index is used as the optimization object function:

\[
J(x, N) = \Delta v_{\text{ch1}} + \Delta v_{\text{ch2}} + \max\left\{ 0, \ h_{\text{min}} - h_p \right\} \quad (29)
\]

where \( h_p \) is the perigee altitude of the phasing orbit in meter units.

### 3.3. Simulation results

During the simulation, different initial longitude of the ascending node \( \lambda_{x_0} \) and argument of latitude \( u_0 \) are used to represent various relative directions between the SERV and landing point. The lower boundary of phasing orbit perigees is set to 150 km.

Let \( \lambda_{x_0} \) equal 0 and \( u_0 \) vary between 0 and 2\( \pi \). The results are shown in Fig. 7. Figure 7(a) gives the waiting time before braking. It can be found that the waiting time is divided into several stages when \( u_0 \) varies. Figure 7(b) depicts the variation of the semi-major axes of the phasing orbits with respect to the initial orbit. Comparing the two illustrations, it is suggested that every stage in Fig. 7(a) corresponds to a specific phasing type. For some stages, the phasing orbital semi-major axes are larger than that for the initial orbit and some stages correspond to smaller orbits. Some stages do not need phasing. The maximum waiting time before braking is 43.049 hr, and the minimum is 18.264 hr. The average is 29.786 hr, about 6 hr longer than a day. The difference between the semi-major axes of the phasing and initial orbits is less than 40 km, which is equivalent to 49 s in the orbit period.

Figure 7(c) indicates the situation for characteristic velocity (CHV), with a maximum of 41.098 m/s. From the figure, it can be seen that no phasing is required within the scopes [106°, 204°] and [−34°, 10°] due to the maneuverability of the SERV. The sum of the two scopes is 142°, which covers about 40% of all the cases. Two CHV peak values are present at about 48.5° and 283°. After analyzing the simulation data, we conclude that the peaks are precisely the boundaries between the ascending and descending arcs, as shown in Fig. 7(d). In Fig. 7(d), the CHV line of the descending arc oscillates with large amplitude at 220°. The phenomenon is induced by the constraint on the phasing orbital perigee. The semi-major axes of the phasing orbits oscillate around the initial axis due to the perigee height constraint at that point.

Let \( u_0 \) equal 0 and \( \lambda_{x_0} \) vary between 0 and 2\( \pi \). The results are shown in Fig. 8. Because \( \lambda_{x_0} \) directly determines the time when the ADR will pass the orbital plane, \( \lambda_{x_0} \) is more prominent than \( u_0 \) in influencing the waiting time, semi-major axis and CHV. The maximum for the waiting time before braking is 47.494 hr. The maximum CHV is 56.209 m/s. The maximum semi-major axis difference between the phasing and initial orbits is less than 50 km.

To acquire general knowledge, a sampling simulation is conducted. The procedure generates samples of \( \lambda_{x_0} \) and \( u_0 \) every 3° within \([0, 360°]\), which means a total combination number of \( 120 \times 120 = 14,400 \). The results for the samples are computed, and the distribution is illustrated in Fig. 9.
bars plotted in the figures represent percentages of samples within different intervals. Figure 9(a) shows that the portion of CHV samples less than 5 m/s is up to 49.18%, among which 43.25% do not need phasing. The percentage of samples with a CHV more than 40 m/s is 3.37%. The maximum semi-major axis difference between phasing and initial orbits is less than 60 km, as shown in Fig. 9(b), which indicates a difference of 73 s in the orbit period. Figure 9(c) indicates the distribution of the waiting time, which shows that many samples need to wait for more than 24 hr before deorbit. Simulation data shows that 14.812% of the samples are within 12 hr and 35.688% are within 24 hr.

4. Discussion and Comparison

It would be interesting and useful to analyze the variations in deorbit windows when the conditions vary. For example, the influence of return range maneuverability or results without re-entry maneuver. Six different conditions were studied, and 14,400 samples for each condition were computed. The six conditions are described as follows:

Cond. 1: Fixed return range; no longitudinal or lateral maneuverability. Return to the Earth within 48 hr using orbital phasing only. Minimum fuel consumption.

Cond. 2: Maneuverable return range; \(|\Delta R| \leq 100 \text{ km}, |\Delta H| \leq 50 \text{ km.} \) Return to the Earth within 48 hr. Minimum fuel consumption.

Cond. 3: Maneuverable return range; \(|\Delta R| \leq 200 \text{ km}, |\Delta H| \leq 100 \text{ km.} \) Return to the Earth within 48 hr. Minimum fuel consumption.

Cond. 4: Maneuverable return range; \(|\Delta R| \leq 300 \text{ km}, |\Delta H| \leq 150 \text{ km.} \) Return to the Earth within 48 hr. Minimum fuel consumption.

Cond. 5: Maneuverable return range; \(|\Delta R| \leq 200 \text{ km,} |\Delta H| \leq 100 \text{ km.} \) Propulsion module can supply a maximum velocity increment of 200 m/s. Return to the Earth as quickly as possible within the constraints on fuel.

Cond. 6: In the above discussion, terminal point \( F_2 \) of the
phasing orbit is supposed to be arbitrarily selected in the ADR. In reality, if disturbances during orbital phasing are taken into account, point \( F_2 \) could potentially coast beyond the deorbit region. As a result, there would be no opportunity for braking. Considering this circumstance, Cond. 6 requires that terminal point \( F_2 \) be the nominal deorbit point, \( D_0 \), besides the requirements of Cond. 3.

Cond. 7: A combination of Cond. 5 and Cond. 6.

In Table 2, \( \Delta t_{\text{mean}} \) means the average of time spent returning to the Earth for all of the samples. \( \Delta t_{\text{max}} \) stands for the maximum time spent returning to Earth among all of the samples. The fourth and fifth columns show the percentage that the number of cases in which the time spent on returning to Earth is less than the given time covers of all the samples. \( \bar{v}_{\text{ch, mean}} \) denotes the mean value of CHV. \( v_{\text{ch}} = 0 \) represents that CHV is zero (i.e., there is no need for phasing).

From the preceding four conditions, it is recognized that re-entry range maneuverability prominently influences the waiting time, CHV, and the need for phasing. If \( |\Delta R| \leq 100 \text{km} \) and \( |\Delta H| \leq 50 \text{km} \), about 1/4 of the samples do not need phasing. The portion is about 3/5 if \( |\Delta R| \leq 200 \text{km} \) and \( |\Delta H| \leq 150 \text{km} \). Adopting orbital phasing means an interruption to the SERV’s normal mode and will reduce the reliability of the system. Hence, re-entry maneuverability is of much value. However, the re-entry maneuver does not obviously shorten the maximum waiting time because it is mainly determined by the initial longitude of the ascending node. Deorbit cannot be performed until the deorbit region passes the orbital plane.

A comparison between Cond. 3 and Cond. 5 indicates that taking full advantage of the 200 m/s velocity increment could bring a 12 hr reduction in the average waiting time. The percentage of samples that can return to the Earth in 12 hr increases by 14.903%, and for those returning in 24 hr increases by 50.486%. Of all the samples, 86% can return within one day. The maximum waiting time is reduced to 33.843 hr. For a 41 weighted combination of the SERV and propulsion module, 200 m/s velocity increments means 258 kg of fuel, which is about 6.45% of the total mass. Therefore, this measure is efficient in reducing the waiting time.

Comparing Cond. 3 and Cond. 6 indicates that the waiting time does not change much when the terminal point, \( F_2 \), of the phasing orbit is confined to the nominal deorbit point \( D_0 \) while the CHV will increase. The mean value of CHV is about twice as much as for Cond. 3. Because the measure avoids uncertainties in exceeding the boundaries of ADR, it is still worthy.

Consequently, the recommended plan is Cond. 7, namely a combination of Cond. 5 and Cond. 6. In this situation, 43.25% of the samples do not need phasing, and about 70% can return to the Earth within 24 hr.

### 5. Conclusion

This paper discusses a space emergency rescue scheme with a low lift-to-drag ratio vehicle to return a spacecraft crew to a designated landing point within 48 hr. Theoretical analysis and numerical simulations show that return range maneuverability prominently influences return opportunities, not only in extending deorbit windows, but also in avoiding reductions in system reliability induced by orbital phasing. Additionally, orbital phasing can steer the SERV to a selected deorbit point at a given time. The nominal deorbiting point, \( D_0 \), is found to be the best choice for the terminal point of orbital phasing, which can avoid exceeding the boundaries of the allowable deorbit region and does not obviously increase the waiting time before braking.

The scheme that combines return range maneuvers and multiple-revolution orbital phasing can satisfy the requirements of a rescue mission, and thus offers a feasible scheme for China’s space emergency rescue. However, the concept proposed here is only a preliminary study. Influences of the uncertainties and errors during the procedure were not specifically investigated. There are still many challenges in the process of reaching engineering implementation.

### References

1. Konecci, E. B.: Space Rescue, AIAA Paper 66-905, Dec. 1966.
2. Halstead, J. D., Widhalm, J. W., and Whitsett, C. E.: Design of an Interim Space Rescue Ferry Vehicle, J. Spacecraft, 25, 2 (1988), pp. 180–186. doi:10.2514/3.25968
3. Naftel, J. C., Powell, R. W., and Talay, T. A.: Performance Assessment of a Space Station Rescue and Personnel/Logistics Vehicle, J. Spacecraft, 27, 1 (1990), pp. 76–81. doi:10.2514/3.26109
4. Daniel, D. M., Michelle, A. G., Patrick, A. T., Yuan, W., Renjith, R. K., and Michael, L. H.: International Space Station (ISS) Accommodation of a Single U.S. Assured Crew Return Vehicle (ACRV), NASA/ TM-97-206272, Dec. 1997.
5. Philip, S., Lyndon, B. J., Glenn, C. H., Denis, S., Richard, G., and David, G.: Considerations for Medical Transport from the Space Station via an Assured Crew Return Vehicle (ACRV), NASA/TM-2001-210198, July 2001.
6. Ellen, M. G. and Kanan, B. S.: Crew Return Vehicle (CRV) Deorbit Opportunities, AIAA Paper 2003-5833, Aug. 2003.
7. MacConochie, I. O.: A Study of a Lifting Body as Space Station Crew
Exigency Return Vehicle (CERV), NASA/CR-2000-210548, Oct. 2000.

8) Barry, A. G.: Minimum Energy Deorbit, J. Spacecraft, 3, 7 (1966), pp. 1030–1033.

9) Baldwin, M., Lu, P., and Pan, B.: On Autonomous Optimal Deorbit Guidance, AIAA Paper 2009-5667, Aug. 2009.

10) Morgan, C. B. and Lu, P.: Optimal Deorbit Guidance, J. Guid. Control Dynam., 35, 1 (2012), pp. 93–103. doi:10.2514/1.53937

11) Cox, K. J.: Space Shuttle Guidance, Navigation and Control: Deorbit and Atmospheric Operations, NASA Reports N72-23912, Dec. 1971.

12) Zhang, G., Mortari, D., and Zhou, D.: Constrained Multiple-Revolution Lambert’s Problem, J. Guid. Control Dynam., 33, 6 (2010), pp. 1779–1786. doi:10.2514/1.49683

13) Arlulkar, P. V. and Naik, S. D.: Solution Based on Dynamical Approach for Multiple-Revolution Lambert Problem, J. Guid. Control Dynam., 34, 3 (2011), pp. 920–923. doi:10.2514/1.51723

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