Speed and Load Torque Estimation of Induction Motors based on an Adaptive Extended Kalman Filter

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Abstract—When we monitor running state of induction motor in field, the sensorless estimation of load torque and speed of induction motor has important significance, in this paper, a method to estimate load torque and speed of motor using adaptive extended kalman filter(AEKF) is presented, the covariance matrices of noises are estimated while the speed and load torque of induction motor are estimated using EKF in this method; this method solved the problem that the estimate results of EKF are affected greatly by the covariance matrices of noise, Simulation demonstrate that this method can get higher estimated accuracy.

Introduction

Induction motors are extensively used in industrial, agricultural and commercial domain etc., Speed and load torque are every important parameter when we monitor running state of induction motor, They can be measured directly using sensor, but direct measurement have many shortcoming (1)the installation of sensor is very difficult in many industrial fields, Sometimes the sensor cannot be install, (2)the installation of sensor will increase cost of system (3) the installation of sensor will reduce mechanical strength of system and robustness; the sensorless measuring technique was proposed to resolve those problem, and have became research focus recent year.

In recent year, many sensorless estimate methods were proposed, in these methods, the EKF get researchers’ special attention\textsuperscript{[1,2]} for its good dynamic performance and robustness, The EKF can estimate the state of system accurately even if the model of system is imprecise because the error of model is as noise in EKF, But the key problem is that the estimation results of using EKF is greatly affected by the covariance matrices of noise, the improper covariance matrices of noise will make the result of estimation divergence or have large estimate error. The mostly used method of get covariance matrices of noise is to try and regulate according estimate error repeatedly, obviously it is a tedious procedure, Also getting the optimal covariance noise matrices is difficult by the method. To solve this problem, the covariance matrices of noise are got by optimization using GA in \textsuperscript{[3,4]}, the covariance matrices of noise are got by optimization using SA in [5], but there is a common problem in getting covariance matrices of noise by optimization, the objective function used in optimization often is the mean square error of estimate value and actual value of estimated state, such as in [3], the speed is the estimated state, the objective function is the mean square error of estimated speed and actual speed, the optimization will be impossible if the speed can’t be measured.
In this paper, an estimate method of speed and load torque of induction motor using AEKF is presented. First, a Stochastic model including speed and load torque of induction motor is established through analyzing the real system. Next, the model was transformed to linear discrete equation by linearization and discretization in order to implement using EKF. Next, the covariance matrices of noise are estimate in EKF by introducing the principle of adaptive filter into EKF and the implementation steps of AEKF are given. Last, the effectiveness of AEKF are verified by simulation.

Model of Induction Motor

Deterministic model of induction motor. In the stator stationary frame, the mathematical model of induction motor can be expressed as follows:

$$\frac{d}{dt}i_{ra} = \frac{a_1}{\tau_2}i_{ra} + \frac{a_2}{\sigma\tau_2}v_{ra} - a_{11}i_{ra} + a_{12}v_{ra}\omega + \frac{1}{\sigma\tau_2}u_{ra}$$

(1)

$$\frac{d}{dt}i_{rb} = \frac{a_1}{\tau_2}i_{rb} - a_{11}i_{rb} + \frac{a_2}{\sigma\tau_2}v_{ra}\omega + \frac{1}{\sigma\tau_2}u_{rb}$$

(2)

$$\frac{d}{dt}v_{ra} = \frac{1}{\tau_2}i_{ra} - \frac{1}{\tau_2}v_{ra} - \frac{1}{\tau_2}v_{ra}\omega$$

(3)

$$\frac{d}{dt}v_{rb} = \frac{1}{\tau_2}i_{rb} + \frac{1}{\tau_2}v_{rb}\omega - \frac{1}{\tau_2}v_{rb}$$

(4)

The mechanical equation of induction motor can be expressed as follows:

$$\frac{d}{dt}\omega = -a_{12}v_{rb} + a_{11}v_{ra} - \frac{B}{J}\omega - \frac{p}{J}T_L$$

(5)

In steady-state, the state equation of load torque can be expressed as follows:

$$\dot{T}_L = 0$$

(6)

The deterministic model of induction motor can be expressed as follows:

$$\dot{x}(t) = f(x(t), u(t)) =
\begin{bmatrix}
\frac{a_1}{\tau_2}x_1 + \frac{a_2}{\sigma\tau_2}x_1 + \frac{1}{\sigma\tau_2}u_{ra} \\
\frac{a_1}{\tau_2}x_2 - \frac{a_2}{\tau_2}x_2 + \frac{1}{\sigma\tau_2}u_{rb} \\
\frac{L_m}{\tau_2}x_1 - \frac{1}{\tau_2}x_1 \\
\frac{L_m}{\tau_2}x_2 + \frac{1}{\tau_2}x_2 \\
-a_{12}x_1 + a_{12}x_2 - \frac{B}{J}\omega - \frac{p}{J}x_1 \\
0
\end{bmatrix}$$

(7)

$$y(t) = h(x(t), u(t)) =
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}$$

(8)

Where

$$x = \begin{bmatrix} i_{ra} & i_{rb} & v_{ra} & v_{rb} & \omega & T_L \end{bmatrix}^T$$

$$u(t) = \begin{bmatrix} u_{ra} & u_{rb} \end{bmatrix}$$

$$y = \begin{bmatrix} i_{ra} & i_{rb} \end{bmatrix}^T$$

Here, $v_{ra}$ and $v_{rb}$ are the stator stationary axis components of rotor flux, respectively, $u_{ra}$ and $u_{rb}$ are the stator stationary axis components of stator voltages, respectively, $i_{ra}$ and $i_{rb}$ are the stator stationary axis components of stator current, respectively, $R_1$ and $L_1$ are stator resistance and inductance, respectively, $R_2$ and $L_2$ are rotor resistance and inductance, respectively, $L_m$ is
mutual inductance, \( \omega \) is angular speed, \( J \) is the total inertia of the IM and load, \( B \) is mechanical friction coefficient, \( p \) is the number of pole pairs, \( T \) is load torque, \( \phi = -\left( R_1 / \sigma L + 1 - \sigma \right) \), \( a_2 = L_2 / \sigma L_2 \), \( a_3 = p^2 L_2 / J L_2 \), \( \sigma = 1 - T_p / L_2 \), \( \tau_2 = L_2 / R_2 \).

**Stochastic model of induction motor.** In actual system of induction motor, consider the process noise \( w(t) \), the measuring noise \( v(t) \) and the input noise \( \zeta(t) \), the stochastic model of induction motor can be expressed as follows:

\[
\begin{align*}
\dot{x}(t) &= f(x(t), u(t) + \zeta(t)) + w(t) \\
y(t) &= h(x(t), u(t) + \zeta(t)) + v(t)
\end{align*}
\]

where, the noise is subject to the following distribution:

\[
\begin{align*}
p(\omega) &\sim N(0, Q) \\
p(v) &\sim N(0, R) \\
p(\zeta) &\sim N(0, D)
\end{align*}
\]

**Algorithm of Aekf**

The principle of adaptive filter\(^{[1]}\) is the statistical properties of noise can be calculate according to the new measurement information, and let the filter be optimal, AEKF is introducing the principle of adaptive filter into the EKF, the statistical properties of noise can be calculate according to the new measurement information and the estimate results of EKF. So we should estimate the state of induction motor using EKF firstly.

When the speed and load torque of induction motor are as the state of system model and parameter of coefficient matrix, the system became a nonlinear system, the model must be transformed to linear system, the linear system model of induction motor can be expressed as follow:

\[
\begin{align*}
\dot{x}(t) &= F(x(t))x(t) + B(u(t) + \zeta(t)) + w(t) \\
y(t) &= H \dot{x}(t) + v(t)
\end{align*}
\]

where

\[
F(x(t)) = \frac{\partial f}{\partial x}(x(t), u(t), 0) = \begin{bmatrix}
a_1 & \frac{\alpha_1}{\tau_2} & \frac{\alpha_2}{\tau_2} & \frac{\alpha_3}{\tau_2} & 0 & 0 \\
0 & a_2 & \frac{\alpha_3}{\tau_2} & 0 & 0 \\
0 & \frac{L_2}{\tau_2} & 0 & \frac{1}{\tau_2} & \frac{1}{\tau_2} & 0 \\
0 & \frac{L_2}{\tau_2} & \frac{1}{\tau_2} & 0 & 0 & 0 \\
-\frac{\alpha_3}{\tau_2} & \frac{\alpha_2}{\tau_2} & 0 & 0 & \frac{pB}{J} & \frac{p}{J} \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[B = \frac{\partial f}{\partial u}(x(t), u(t), 0) = \begin{bmatrix}
\sqrt{\sigma L_1} & 0 & 0 & 0 & 0 & 0 \\
0 & \sqrt{\sigma L_1} & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[H = \frac{\partial h}{\partial x}(x_{1:2}, u) = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

The computer implementation of EKF algorithm is basing discrete system model, so we discrete the linear system model of induction motor, the sample time is \( T_s \), obtain the following model:

\[
\begin{align*}
\dot{x}(k) &= F(x(k))x(k) + B(u(k) + \zeta(k)) + w(k) \\
y(k) &= H \dot{x}(k) + v(k)
\end{align*}
\]
In order to calculate statistical properties of noise according to adaptive filter principle, the noise of system can be sort two part, one part is noise of process and the other part is measuring noise, then obtain the following model:

\[
\begin{align*}
\delta x(k) &= \Phi_k \delta x(k) + M_k u(k) + w'(k) \\
y(k) &= H \delta x(k) + v(k)
\end{align*}
\]

Where

\[
w'(k) = M_k \zeta(k) + W_k w(k)
\]

For \( \zeta(k) \) and \( w(k) \) is not relevant, we assume \( w(k) \) is subject to the following distribution:

\[
p(\omega') \sim N(0, Q')
\]

So the Implementation process of AEKF algorithm is as follows:

1.) initialization \( x(0), P(0), Q', R \)
2.) begin sampling, \( k = 1 \)
3.) State prediction

\[
\begin{align*}
X_{k|k-1} &= f(X_{k-1|k-1}, u_{k-1}, 0) \\
P_{k|k-1} &= \Phi_k P_{k-1|k-1} \Phi_k^T + Q_{k|k-1}
\end{align*}
\]

4.) State update

\[
\begin{align*}
K_k &= P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_{k|k-1})^{-1} \\
\hat{X}_{k|k} &= X_{k|k-1} + K_k (y_k - H_k X_{k|k-1}) \\
P_{k|k} &= (I - K_k H_k) P_{k|k-1}
\end{align*}
\]

5.) Estimate the covariance matrices of noise

\[
\begin{align*}
\tilde{q}_{1|k-1} &= \frac{1}{k-1} [(k-2) \tilde{q}_{1|k-2} + \tilde{y}_{k-1}] \\
\tilde{R}_{1|k} &= \frac{1}{k-1} [(k-2) \tilde{R}_{1|k-2} + \tilde{y}_{k-1} \tilde{y}_{k-1}^T - H_k P_{k|k-2} H_k^T] \\
\tilde{y}_{k-1} &= y_{k-1} - H_k X_{k|k-1} \\
\tilde{q}_{2|k-1} &= \frac{1}{k-1} [(k-2) \tilde{q}_{2|k-2} + \tilde{x}_{k-1}] \\
\tilde{x}_{k-1} &= x_{k-1} - \Phi_k \tilde{y}_{k-1}
\end{align*}
\]

\[
\begin{align*}
\tilde{Q}_{1|k-1} &= \frac{1}{k-1} [(k-2) \tilde{Q}_{1|k-2} + K_{k-1} \Phi_k^T \tilde{Q}_{1|k-2} X_{k-1}+ P_{k-1} - \Phi_{k-1} P_{k-1} \Phi_{k-1}^T] \\
+ &K_{k-1} \tilde{y}_{k-1} \tilde{y}_{k-1}^T K_{k-1}^T + P_{k-1} - \Phi_{k-1} P_{k-1} \Phi_{k-1}^T
\end{align*}
\]

6.) if \( k < n \), then \( k = k + 1 \), go to step(3)

7.) end
Simulation

The simulation has been done using Matlab/Simulink, the structure of the IM drive proposed in this paper is an open direct supply motor model, as illustrated in Fig.1. The load torque is constant of 20, the measured stator voltages and currents from motor are used to estimate speed and load torque of motor, the measured speed and load torque are used to compare with the estimated speed and load torque. The parameter of the 3-phase 4 pole squirrel cage IM is $R_i = 1.115$, $L_1 = 0.209674$, $R_s = 1.083$, $L_2 = 0.209674$, $L_m = 0.2037$, $p = 2$, $J = 0.02$, $B = 0.005752$.

![Simulation diagram of power supplied directly induction motor](image-url)

Figure 1. Simulation diagram of power supplied directly induction motor

The initial value of $x(0)$ and $P(0)$ in AEKF are selected as follows:

$$x(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$P(0) = \text{diag}[1 \ 1 \ 1 \ 1 \ 1]$$

The initial value of Q and R are selected as table 1, the first group Q and R are generated randomly, the second group Q and R are selected to same with the Q and R obtained by trial and error that let EKF acquire good estimate results.

| No. | Q         | R         |
|-----|-----------|-----------|
| 1   | diag(0.422, 0.916, 0.792, 0.959, 0.656, 0.036) | diag(0.849, 0.934) |
| 2   | diag(1e-5, 1e-5, 1e-5, 1e-5, 1e-2, 1e2) | diag(1e-5, 1e-5) |

The estimate results of EKF according to different Q and R are showed in Fig.2, (a) is the estimate results of speed, (b) is the estimate results of load torque, curve 1 is the estimate results of the first group Q and R, curve 2 is the estimate results of the second group Q and R, we can conclude that the estimate results of EKF is greatly affected by the selected Q and R, the improper Q and R will generate large estimate error.

The estimate results of AEKF according to different Q and R are showed in Fig.3, (a) is the estimate results of speed, (b) is the estimate results of load torque, curve 1 is the estimate results of the first group Q and R, curve 2 is the estimate results of the second group Q and R, we can conclude that the estimate results of AEKF is almost unaffected by the selected Q and R, and have high accuracy.
In order to analyze the estimate results of AEKF further, we also compare the accuracy of EKF and AEKF, showed in table 2, $E_{w1}$ is steady-state error of speed using EKF, $E_{TL1}$ is steady-state error of load torque using EKF, $E_{w2}$ is steady-state error of speed using AEKF, $E_{TL2}$ is steady-state error of load torque using AEKF, The column denoted by 1 is the estimate results of first group Q and R, The column denoted by 2 is the estimate results of second group Q and R.

Table 2 indicate that AEKF has higher steady state accuracy than EKF, and the steady state accuracy is less affected by the selected Q and R, The column denoted by 2 indicate further that the estimate results of AEKF is better than the best estimate results obtained by trial and error.

Conclusions

A speed and load torque of induction motor estimation method using AEKF are presented in this paper, the covariance matrices of noise are estimated while the state of induction motor system are estimated, thus the covariance matrices of noise change adaptively with the estimation process, In simulation, two group of Q and R are selected to use in EKF and AEKF, the estimate results of EKF and AEKF according to the two group Q and R are compared, the estimate results indicate that the proposed method overcome the shortcomings of EKF, can get high-precision estimate results according to different covariance matrices of noise.
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