The continuum and interference effect in $e^+e^- \rightarrow D^0\bar{D}^0$, $D^+D^-$ processes

Mao-Zhi Yang

CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, China
Institute of High Energy Physics, P.O.Box 918(4), Beijing 100049, China
(Dated: April 18, 2018)

In the $e^+e^-$ annihilation processes $e^+e^- \rightarrow D^0\bar{D}^0$, $D^+D^-$ just above the threshold of $D\bar{D}$, there are not only the resonance contribution $e^+e^- \rightarrow \psi(3770) \rightarrow D^0\bar{D}^0$, $D^+D^-$, but also the continuum contribution through virtual photon directly $e^+e^- \rightarrow \gamma^* \rightarrow D^0\bar{D}^0$, $D^+D^-$. The amplitudes through virtual photon directly and through resonance can interfere seriously. We consider the continuum and interference effect in the $D\bar{D}$ production process in $e^+e^-$ annihilation. We find that the effect is significant near and above the threshold of the $D\bar{D}$ mesons.

PACS numbers: 13.25.Gv, 13.66.Bc, 14.40.Gx

The $e^+e^-$ annihilation processes $e^+e^- \rightarrow D^0\bar{D}^0$ and $D^+D^-$ just above the threshold of $D\bar{D}$ have been measured by CLEO collaboration and also by several other collaborations. The total cross section for $e^+e^- \rightarrow D^0\bar{D}^0$ measured by experiments is $\approx 5 \pm 2$ pb, which is consistent with zero [1]. This result indicates that the branching ratio of $\psi(3770) \rightarrow D\bar{D}$ is small. In addition, except $J/\psi \pi^+\pi^-$ final state and some radiative transition processes, no statistically significant signals of non-$D\bar{D}$ decays of $\psi(3770)$ have been detected [2-3]. From the detected non-$D\bar{D}$ decay modes of $\psi(3770)$ by experiment [2], one can estimate that the summed branching fractions of $\psi(3770)$ to non-$D\bar{D}$ final states is at most 2 or 3%. Therefore about 98 or 97% of $\psi(3770)$ resonances decay to $D\bar{D}$ final states.

It is generally assumed that the process $e^+e^- \rightarrow D\bar{D}$ just above the threshold of the $D\bar{D}$ is almost completely contributed by $\psi(3770)$, while the continuum contribution of virtual photon is not important.

BESII and CLEO-c have measured the cross sections of $e^+e^- \rightarrow D^0\bar{D}^0$, and $D^+D^-$ at the center-of-mass energy $\sqrt{s} = 3.773$ GeV [4, 5]. Based on these data, we show in this paper, if one accepts that the branching ratio of $\psi(3770) \rightarrow D\bar{D}$ is as large as 97 or 98%, to understand the data of the cross section of $e^+e^- \rightarrow D\bar{D}$, one has to consider both the continuum contribution of $e^+e^- \rightarrow \gamma^* \rightarrow D\bar{D}$ and the interference effect between the amplitudes of $e^+e^- \rightarrow \psi(3770) \rightarrow D\bar{D}$ and $e^+e^- \rightarrow \gamma^* \rightarrow D\bar{D}$.

As shown in Fig. 1, the process of $e^+e^-$ annihilating into $D\bar{D}$ can occur either through the intermediate resonance $\psi(3770)$ or directly through the virtual photon. To calculate these diagrams, one needs to know the coupling of $\psi(3770)$ with $D\bar{D}$, the coupling of $\psi(3770)$ with the virtual photon, and the form factor $F_{D\bar{D}}$ describing the direct interaction of $D\bar{D}$ with the electromagnetic current. The coupling $g_{\psi D\bar{D}}$ can be defined by

$$
\langle D(p_1)\bar{D}(p_2)|\psi(p)\rangle = -ig_{\psi D\bar{D}}\epsilon^{(\lambda)} \cdot (p_1 - p_2)(2\pi)^4\delta^4(p - p_1 - p_2),
$$

where $\psi$ denotes $\psi(3770)$, $\epsilon^{(\lambda)}$ is the polarization vector of $\psi(3770)$, and $\lambda$ stands for the polarization state. The coupling of $\psi$ with the electromagnetic current can be described by the following equation

$$
\langle 0|\bar{e}\gamma_{\mu}|\psi\rangle = f_\psi m_\psi \epsilon^{(\lambda)}_\mu,
$$

where $m_\psi$ is the mass of $\psi(3770)$, and $f_\psi$ its decay constant, which can be determined by the measured decay width of $\Gamma(\psi(3770) \rightarrow e^+e^-)$. In the following analysis, we use $f_\psi = 95.5$ MeV, which is extracted from the measured decay width of $\psi(3770) \rightarrow e^+e^- [2]$. From eq. (2), one can get the effective vertex of the resonance $\psi$, and the electromagnetic current: $iQ_{\mu}e^{(\lambda)}_\mu$. The propagator of $\psi$ resonance is taken as the Breit-Wigner form

$$
\frac{i}{p^2 - m_\psi^2 + im_\psi \Gamma_T},
$$

where $m_\psi$ and $\Gamma_T$ are the mass and width of $\psi(3770)$ respectively.
here $\Gamma_T$ is the total decay width of $\psi(3770)$. Then one can obtain the amplitude of Fig.1
\[
T_0 = ig_{\psi D\bar{D}}Q_{e_1}e^2f_\psi m_\psi(p_1-p_2)^\mu \times \frac{1}{s-m_\psi^2+im_\psi\Gamma_T} \frac{1}{s}(v(k_2))_\mu u(k_1),
\]
where $s = (k_1+k_2)^2$, $u(k_1)$ and $v(k_2)$ are the Dirac spinors for the electron and positron, respectively.

The coupling of the meson pair $DD$ directly with the electromagnetic current can be parameterized as the electromagnetic form factor $F_{DD}$
\[
\langle D(p_1)\bar{D}(p_2)\vert j_{\text{em}}^{\mu}\vert 0\rangle_{\text{dir}} = F_{DD}(q^2)(p_1-p_2)^\mu,
\]
where $j_{\text{em}}^{\mu} = Q_{e_1}e\gamma^\mu e$, is the electromagnetic current, $q = p_1-p_2$, and the subscript “dir” means the $DD$ pair couples directly with the electromagnetic current without the contribution of the resonance. With this definition of the electromagnetic form factor, the vertex of the virtual photon and $DD$ is $ieF_{DD}(q^2)(p_1-p_2)^\mu$. Then the amplitude of Fig.1 can be obtained
\[
T_0 = -ie^2F_{DD}(s)(p_1-p_2)^\mu \frac{1}{s}(v(k_2))_\mu u(k_1).
\]
Combining eqs.1 and 4, one can obtain the total amplitude of $e^+e^- \rightarrow D^0\bar{D}^0$ or $D^+D^-$
\[
T = ie^2v(k_2)_\mu u(k_1)(p_1-p_2)^\mu \frac{1}{s} \times [F_{DD}(s) + g_{\psi D\bar{D}}Q_{e_1}e^2\frac{m_\psi}{s-m_\psi^2+im_\psi\Gamma_T}e^{i\phi}],
\]
where an arbitrary phase factor $e^{i\phi}$ is added, which should be determined by experiment. With the above amplitude, one can obtain the cross section of $e^+e^- \rightarrow D^0\bar{D}^0$ and/or $D^+D^-$
\[
\sigma(e^+e^- \rightarrow D^0\bar{D}^0, D^+D^-) = \frac{\pi}{3}(s-4m_D^2)^{3/2} \frac{1}{s^{5/2}} \alpha^2
\]
\[
\times \frac{1}{F_{DD}(s) + g_{\psi D\bar{D}}Q_{e_1}e^2\frac{m_\psi}{s-m_\psi^2+im_\psi\Gamma_T}e^{i\phi}}^2,
\]
where $\alpha = 1/137$, is the fine structure constant.

To proceed to the numerical analysis of the cross section of $e^+e^- \rightarrow D\bar{D}$, one needs the knowledge of the coupling $g_{\psi D\bar{D}}$ and the form factor $F_{DD}(s)$. For the coupling $g_{\psi D\bar{D}}$, we can estimate it from the knowledge of the branching ratio of $\psi(3770) \rightarrow D^0\bar{D}^0$ and $\psi(3770) \rightarrow D^+D^-$. From eq.1 we can get the branching ratio
\[
Br(\psi(3770) \rightarrow D^0\bar{D}^0, D^+D^-) = \frac{g_{\psi D\bar{D}}^2 |\vec{p}_D|^3}{6\pi \frac{m_\psi^2}{s} \Gamma_T},
\]
where $\vec{p}_D$ is the three-momentum of the $D$ meson in $\psi(3770)$ decay. The isospin violation effect for $g_{\psi D\bar{D}}$ is not considered here, since it is only at one or two percentage level. We take the mass and the total width of $\psi(3770)$ as $m_\psi = 3771.1\text{MeV}$, $\Gamma_T = 23.0\text{MeV}$, and the total branching fraction of $\psi(3770)$ decays to $D^0\bar{D}^0$ and $D^+D^-$ to be $Br(\psi(3770) \rightarrow D\bar{D}) = 97\%$ in the numerical estimation, which is relevant to take $g_{\psi D\bar{D}} = 12.7$. The value of $g_{\psi D\bar{D}}$ and the results for the branching ratios of $\psi(3770) \rightarrow D\bar{D}$ are listed in Table I. The ratio of $\frac{Br(D^0\bar{D}^0)}{Br(D^+D^-)} = 1.47$, which is within the error of the experimental value: $2.43 \pm 1.50 \pm 0.43$.

Now we consider the cross section of $e^+e^- \rightarrow D^0\bar{D}^0$ and $D^+D^-$ with $g_{\psi D\bar{D}} = 12.7$. At first we calculate the cross section without the continuum contribution, i.e., taking the form factor $F_{DD}(s) = 0$ in eq. 5. The results are listed in Table II. Both CLEO-c and BESII have measured the cross sections of $e^+e^- \rightarrow D^0\bar{D}^0$ and $D^+D^-$ at $\sqrt{s} = 3.773\text{ GeV}$. Their results are in good agreement with each other. The observed cross sections are
\[
\begin{align*}
\sigma^{\text{obs}}_{D^0\bar{D}^0} &= (3.58 \pm 0.09 \pm 0.31) \text{ nb} \quad \text{CLEO-c} [5] \\
\sigma^{\text{obs}}_{D^+D^-} &= (2.56 \pm 0.08 \pm 0.26) \text{ nb} \quad \text{BESII} [4]
\end{align*}
\]
\[
\begin{align*}
\sigma^{\text{obs}}_{D^0\bar{D}^0} &= (3.60 \pm 0.07 \pm 0.07) \text{ nb} \\
\sigma^{\text{obs}}_{D^+D^-} &= (2.79 \pm 0.07 \pm 0.04) \text{ nb}
\end{align*}
\]

BESII has also given the cross sections with the initial state radiative corrections. This result can be compared directly with our tree-level calculation. The comparison can be found in Table II. It shows that, without the continuum contribution, both of the results of the cross sections of $e^+e^- \rightarrow D^0\bar{D}^0$ and $D^+D^-$ are seriously larger than the experimental data at $\sqrt{s} = 3.773\text{ GeV}$. To improve the theoretical prediction, one choice is to include the contribution of the direct virtual photon in the $e^+e^- \rightarrow D\bar{D}$ process.

\begin{center}
\begin{tabular}{|c|c|c|c|c|}
\hline
$\psi(3770)$ & $D^0\bar{D}^0$ & $D^+D^-$ & $D^0\bar{D}^0$ & $D^+D^-$ \\
\hline
$\sigma^{\text{cal}}_{D^0\bar{D}^0}$ & $6.5 \text{ nb}$ & $4.5 \text{ nb}$ & $3.60 \pm 0.07 \pm 0.07$ & $2.79 \pm 0.07 \pm 0.04$ \\
$\sigma^{\text{cal}}_{D^+D^-}$ & $4.5 \text{ nb}$ & $3.29 \pm 0.10 \pm 0.37$ & $2.56 \pm 0.08 \pm 0.26$ & $2.79 \pm 0.07 \pm 0.04$ \\
\hline
\end{tabular}
\end{center}

For considering the contribution of the intermediate virtual photon, we assume the electromagnetic form factor of $DD$ to be the following form
\[
F_{DD}(s) = \frac{m_\psi^2 F_0}{s},
\]
This behavior of $s$-dependence is similar to the form factors of light mesons [2, 3].

![Graph](image)

**FIG. 2:** The cross section of $e^+e^- \rightarrow D^0\bar{D}^0$ at $\sqrt{s} = 3.773$ GeV, varying with the parameter $F_0$ and the relative phase $\phi$. The plots $a$, $b$, $c$, $d$ and $e$ are for $\phi = 0, \pi/12, \pi/6, \pi/4$ and $\pi/2$, respectively.

Now substitute eq. (10) into eq. (8), one can calculate the cross section depending on the center-of-mass energy $\sqrt{s}$, provided one knows the values of the parameter $F_0$ and the relative phase $\phi$ between the resonance and the direct virtual photon contribution in eq. (8). To estimate the allowed values for $F_0$ and $\phi$, we plot the cross section of $e^+e^- \rightarrow D^0\bar{D}^0$ in Fig. 2 by varying the values of $F_0$ and $\phi$. The two dashed horizontal lines stand for the experimental result with the upper and lower errors [4]. The curves between this two dashed lines are allowed by the experimental data. Similar plots can be obtained for the cross section of $e^+e^- \rightarrow D^+D^-$.

![Graph](image)

**FIG. 3:** The cross section of $e^+e^- \rightarrow D^0\bar{D}^0$ changing with the center-of-mass energy $\sqrt{s}$. The solid curve is for $(F_0, \phi) = (5.0, \pi/2)$, the dotted one for $(6.0, \pi/4)$, and the dashed one for $(8.0, \pi/6)$.

How important is the continuum contribution in the resonance region in the process of $e^+e^- \rightarrow D\bar{D}$? The plot for each contribution by taking $(F_0, \phi) = (5.0, \pi/2)$ is shown in Fig. 4 where the dotted curve is the single contribution of the resonance $\psi(3770)$, the dashed curve is the single contribution of the virtual photon, and the solid one is the total contribution. At $\sqrt{s} = 3.773$ GeV, the single contribution of the resonance $\psi(3770)$ is 6.5 nb, the continuum contribution is 0.13 nb, while the combined contribution is 4.8 nb. This shows that the continuum contribution can change the final cross section as large as 30%. The interference effect is destructive. In the resonance region, the peak for the curve of the cross section is slightly shifted, and seriously lowered by the continuum virtual photon contribution.

![Graph](image)

**FIG. 4:** The cross section of $e^+e^- \rightarrow D^0\bar{D}^0$ changing with the center-of-mass energy $\sqrt{s}$. The solid curve is the total contribution of the virtual photon and the resonance $\psi(3770)$, where $(F_0, \phi) = (5.0, \pi/2)$ is taken, while the dotted one is for the case without the continuum virtual photon contribution, and the dashed one is only the virtual photon contribution.

The $s$-dependence of the cross section of $e^+e^- \rightarrow D^0\bar{D}$ are shown in Fig. 3 by taking $(F_0, \phi) = (8.0, \pi/6)$, $(6.0, \pi/4)$ and $(5.0, \pi/2)$. Both of these sets of parameters can give the correct prediction at $\sqrt{s} = 3.773$ GeV. The two dashed horizontal lines are the experimental bound at $\sqrt{s} = 3.773$ GeV. The solid curve is for $(F_0, \phi) = (5.0, \pi/2)$, the dotted one for $(6.0, \pi/4)$, and the dashed one for $(8.0, \pi/6)$. The curves for the cross section changing with the center-of-mass energy $\sqrt{s}$ are very sensitive to the form-factor parameter $F_0$ and the relative phase $\phi$. With precisely measured cross section of $\sigma_{DD}(s)$ at enough different points of $\sqrt{s}$, these parameters can be possibly fitted well. This measurement can be performed in BESIII and CLEO-c.

![Graph](image)

**FIG. 5:** The cross section of $e^+e^- \rightarrow D^0\bar{D}^0$ changing with the center-of-mass energy $\sqrt{s}$. The solid curve is for $(F_0, \phi) = (5.0, \pi/2)$, the dotted one for $(6.0, \pi/4)$, and the dashed one for $(8.0, \pi/6)$.

In the numerical analysis, we did not include the possible effect of the lower resonance $\psi(3686)$, which is just...
under the threshold of $D\bar{D}$, because we do not know any information about its coupling with $D\bar{D}$. When performing the experimental analysis, the contribution of this resonance can be added into the formula of eq. (5). With large amount of experimental data about the cross section of $e^+e^- \rightarrow D\bar{D}$ at different point of $\sqrt{s}$, it is possible to detect whether the effect of the resonance $\psi(3686)$ is important. In general, eq. (5) can be generalized as

$$\sigma_{D\bar{D}} = \frac{\pi}{3} \frac{(s - 4m_D^2)^{3/2}}{s^{5/2}} \alpha^2 \times |\mathcal{F}_{D\bar{D}}(s) + \sum_i \frac{g_{D\bar{D}}M_i}{s - m_i^2 + im_i\Gamma_i}e^{i\phi_i}|^2, \quad (11)$$

where the summation is over all the possible resonances which can couple to $D\bar{D}$. However, in practice, one need only consider the resonances which give large contribution.

It is still possible to understand the experimental data of the cross section of $e^+e^- \rightarrow D\bar{D}$ theoretically by decreasing the coupling of $\psi(3770)$ with $D\bar{D}$, $g_{\psi D\bar{D}}$. However, this will predict a drastically smaller branching ratio of $\psi(3770) \rightarrow D\bar{D}$, then give a larger branching ratio for the non-$D\bar{D}$ decays of $\psi(3770)$. From the analysis of Ref. [5] it is concluded that the branching ratio of the charmless non-$D\bar{D}$ decays of $\psi(3770)$ may be as large as 13% or 18%. Such a large branching fraction for non-$D\bar{D}$ decays of $\psi(3770)$ is not favored by the recent measurement of CLEO collaboration [1]. Without considering the direct virtual photon contribution, it would be a puzzle for understanding consistently the measured cross section of $e^+e^- \rightarrow D\bar{D}$ at the resonance region and the experimental data about the non-$D\bar{D}$ decays of $\psi(3770)$ [2, 3]. Now the data can be understood consistently by considering the continuum and interference effect caused by the direct virtual photon in the annihilation process of $e^+e^- \rightarrow D\bar{D}$.

In summary, we have considered the continuum and interference effect in the process of $e^+e^- \rightarrow D\bar{D}$ near and above the threshold of $D\bar{D}$. We find the effect of the direct virtual photon is crucial. By including the contribution of the direct virtual photon in $e^+e^- \rightarrow D\bar{D}$, the recent experimental data about the non-$D\bar{D}$ decays of $\psi(3770)$ and the measured cross section of $e^+e^- \rightarrow D\bar{D}$ can be understood consistently.

The author would like to thank Hai-Bo Li for stimulating discussions. This work is supported in part by the National Natural Science Foundation of China under contract Nos. 10205017, 10575108, and the Knowledge Innovation Project of CAS under contract U-530 (IHEP).

---

* Electronic address: yangmz@mail.ihep.ac.cn

[1] D. Besson et al. (CLEO Collaboration), Phys. Rev. Lett. 96, 092002 (2006).
[2] W.-M. Yao et al. (Particle Data Group), J. Phys. G. 33, 1 (2006).
[3] N.E. Adam et al. (CLEO Collaboration), Phys. Rev. Lett. 96, 082004 (2006); T.E. Coan et al. (CLEO Collaboration), Phys. Rev. Lett. 96, 182002 (2006).
[4] M. Ablikim et al. (BES Collaboration), Phys. Lett. B 603, 130 (2004).
[5] Q. He et al. (CLEO Collaboration), Phys. Rev. Lett. 96, 121801 (2005).
[6] R. Chistov et al. (Belle Collaboration), Phys. Rev. Lett. 93, 051803 (2004).
[7] V. Chernyak, hep-ph/9906387; A.V. Manohar, Phys. Lett. B 244, 101 (1990); J.-M. Gerard, G. Lopez Castro, Phys. Lett. B 425, 101 (1990); J.-M. Gerard, G. Lopez Castro, Phys. Lett. 244, 101 (1990); J.-M. Gerard, G. Lopez Castro, Phys. Lett. B 425, 101 (1990); J.-M. Gerard, G. Lopez Castro, Phys. Lett. B 425, 101 (1990);
[8] P. Wang, X.H. Mo, C.Z. Yuan, Phys. Lett. B 557, 192 (2003).
[9] P. Wang, C.Z. Yuan, X.H. Mo, Phys. Rev. D 70, 114014 (2004).