The LHC can probe small $x$ PDFs; the treatment of the infrared region

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Abstract

First, we show how to reduce the sensitivity of the NLO predictions of the Drell-Yan production of low-mass, lepton-pairs, at high rapidity, to the choice of factorization scale. In this way, observations of this process at the LHC can make direct measurements of parton distribution functions in the low $x$ domain: $x \lesssim 10^{-4}$. Second, we find an inconsistency in the conventional NLO treatment of the infrared region. We illustrate the problem using the NLO coefficient function of Drell-Yan production.

1 LHC Drell-Yan production as a probe of low $x$

The very high energy of the LHC allows a probe of the parton distribution functions (PDFs) of the proton at extremely small $x$, a region not accessible at previous accelerators. To extract the PDFs we describe the experimentally observed cross sections as a convolution of the PDFs and the cross section for the hard partonic subprocess, which is of the form

$$
d\sigma/d^3p = \int dx_1 dx_2 \, \text{PDF}(x_1, \mu_F) |\mathcal{M}(p; \mu_F, \mu_R)|^2 \, \text{PDF}(x_2, \mu_F),
$$

where a sum over the various pairs of PDFs is implied.

Here we focus on Drell-Yan production of a low mass $\mu^+\mu^-$ pair. At LO the production of a $\mu^+\mu^-$ system of mass $M$ and rapidity $Y$ arises from the subprocess $\gamma^* \to q\bar{q}$ with $x_{1,2} =$
So for $M = 6$ GeV, $Y = 4$ (a domain which is accessible to the LHCb experiment) we probe $x_1 = 4.7 \times 10^{-2}$, $x_2 = 1.6 \times 10^{-5}$. The problem is that, in the low $x$ region, the PDFs strongly depend on the choice of the factorization scale $\mu_F$, see Fig. 1(a). It is made worse due to the dominance of the gluon PDF at small $x$, which means that the LO $q\bar{q} \rightarrow \gamma^*$ subprocess is overshadowed by the NLO subprocess $gg \rightarrow q\bar{q}^*$. However, it is this very dominance which will allow us to introduce a procedure which greatly suppresses the scale dependence of the predictions.

![Figure 1](image.png)

**Figure 1:** (a) Sensitivity of $M = 6$ GeV Drell-Yan $\mu^+\mu^-$ production, as a function of rapidity $Y$, to the choice of factorization scale: $\mu_F = M/2$, $M$, $2M$, at LO, NLO, NNLO. (b) The bold lines correspond to the choice $\mu_F = \mu_0 = 1.4M$ which minimizes $C_{\text{rem}}^\text{NLO}$, and show the stability with respect to the variations $\mu = M/2$, $M$, $2M$ in the scale of the PDFs convoluted with $C_{\text{rem}}^\text{NLO}$ – the $\mu$ dependence is indicated by the symbolic equation at the top of the diagram. The dashed lines show that the stability disappears for other choices of $\mu_0$. The small crosses are the NNLO result. In this figure, taken from [1], the renormalization scale is fixed at $\mu_R = M$.

The plan is to choose the value of $\mu_F$ which minimizes the higher order $\alpha_s$ NLO, NNLO... contributions. To sketch the idea, we start with the LO expression for the cross section: 

$$\sigma(\mu_F) = \text{PDF}(\mu_F) \otimes C^{\text{LO}} \otimes \text{PDF}(\mu_F)$$

in the collinear approach. The effect of varying the scale from $m$ to $\mu_F$, in both the left and right PDFs, can be expressed, to first order in $\alpha_s$, as

$$\sigma(\mu_F) = \text{PDF}(m) \otimes \left( C^{\text{LO}} + \frac{\alpha_s}{2\pi} \ln \left( \frac{\mu_F^2}{m^2} \right) (P_{\text{left}} C^{\text{LO}} + C^{\text{LO}} P_{\text{right}}) \right) \otimes \text{PDF}(m),$$

(2)

where the splitting functions $P_{\text{right}} = P_{qq} + P_{qg}$ and $P_{\text{left}} = P_{\bar{q}q} + P_{\bar{q}g}$ act on the right and left PDFs respectively. We may equally well have incoming $\bar{q}$’s in $P_{\text{right}}$ and incoming $q$’s in $P_{\text{left}}$.

At NLO we may write

$$\sigma(\mu_F) = \text{PDF}(\mu_F) \otimes (C^{\text{LO}} + \alpha_s C^{\text{NLO}}) \otimes \text{PDF}(\mu_F),$$

(3)
where the $2 \rightarrow 2$ subprocesses $q\bar{q} \rightarrow g\gamma^*$ and $gg \rightarrow q\gamma^*$ are now calculated with better, than LLA, accuracy. However, we must subtract from $C_{\text{NLO}}$, the part of the contribution already included, to LLA accuracy, in the $\alpha_s$ term in (2). The remaining contribution, $C_{\text{rem}}^{\text{NLO}}(\mu_F)$, now depends on the scale $\mu_F$, coming from the $\mu_F$ dependence of the LO LLA term that has been subtracted off. The trick is to choose an appropriate scale, $\mu_F = \mu_0 = 1.4M$. The stability of the prediction, using MSTW NLO PDFs [2], is shown in Fig. 1(b). For $Y \gtrsim 3$, pure DGLAP PDF extrapolations become unreliable due to the absence of absorptive, $\ln(1/x)$, modifications. Rather, LHC data will provide a direct measure of PDFs in this low $x$ domain.

2 Treatment of the infrared region in perturbative QCD

Interestingly, a spin-off of the above study highlighted an inconsistency in the conventional treatment of the infrared region [3]. Again we use Drell-Yan as an example. For the main NLO subprocess we have

$$
\frac{d\hat{\sigma}(gq \rightarrow q\gamma^*)}{d|t|} = \frac{\alpha^2 \alpha_s z}{9M^2} \frac{1}{|t|} \left[ ((1-z)^2 + z^2) + z^2 \frac{t^2}{M^4} - 2z^2 \frac{t}{M^2} \right],
$$

(4)

where $z = M^2/\hat{s}$ and $\sqrt{\hat{s}}$ is the incoming $gq$ c.m. energy. (Strictly speaking, $z$ is the ratio of the light-cone momentum fraction carried by the ‘daughter’ quark to that carried by the ‘parent’ gluon, $z = x_+^q / x_+^g$.) In order to calculate the inclusive cross section $d\sigma/dM^2$, it seems that we have to integrate over $t$ starting from $t = 0$. If this were necessary, then we would face an infrared divergency.

However, we follow the procedure of the previous Section, which we call the physical approach. To avoid double-counting, we subtract the LO DGLAP-generated contribution. Then the remaining contribution of (4) is

$$
\frac{d\hat{\sigma}_{\text{rem}}^{\text{NLO}}(gq \rightarrow q\gamma^*)}{d|t|} = \frac{\alpha^2 \alpha_s z}{9M^2} \frac{1}{|t|} \left[ ((1-z)^2 + z^2) \Theta(|t| - \mu_F^2) + z^2 \frac{t^2}{M^4} - 2z^2 \frac{t}{M^2} \right],
$$

(5)

which has no singularity as $t \to 0$. The LO DGLAP evolution has accounted for all virtualities $|t| = k^2 < \mu_F^2$; with the contribution of $k^2 < Q_0^2$ hidden in the phenomenological input PDF at $Q^2 = Q_0^2$.

On the other hand, the conventional prescription evaluates the inclusive cross section, $d\sigma/dM^2$, by integrating (4) over the infrared divergency at $t = 0$ using $4 + 2\epsilon$ dimensional space to regularize the integral [4, 5, 6]. Then the contribution from very small $t$ produces a $1/\epsilon$ pole, which is absorbed into the incoming PDF. The conventional prescription is as follows. The same $gq \rightarrow q\gamma^*$ diagram, but now generated by LO DGLAP evolution, is considered; it gives an $1/\epsilon$ pole which cancels the corresponding $1/\epsilon$ pole in hard matrix element (coefficient
function). However, we are left with $\epsilon/\epsilon$ terms of infrared origin, which produce a non-zero result as $\epsilon \to 0$. Unlike the finite $\epsilon/\epsilon$ terms of ultraviolet origin, which can be treated as point-like counter-terms in the Lagrangian, the infrared $\epsilon/\epsilon$ contribution makes no physical sense in QCD theory, since the confinement eliminates any interaction at very large distances.

To be explicit, the *conventional* prescription gives

$$
\frac{M^2 d\hat{\sigma}^{\text{NLO}}_{\text{rem}}(gq \to q\gamma^*)}{dM^2} = \frac{\alpha^2 \alpha_s z}{9 M^2} \left\{ \left[ (1 - z)^2 + z^2 \right] \ln \left( \frac{1 - z}{z} \right) + \frac{1}{2} + 3z - \frac{7}{2} z^2 \right\},
$$

which is to be compared with the result,

$$
\frac{M^2 d\hat{\sigma}^{\text{NLO}}_{\text{rem}}(gq \to q\gamma^*)}{dM^2} = \frac{\alpha^2 \alpha_s z}{9 M^2} \left\{ \left[ (1 - z)^2 + z^2 \right] \ln \left( \frac{1 - z}{z} \right) + \frac{1}{2} + z - \frac{3}{2} z^2 \right\},
$$

obtained, from (5), using the *physical* approach. We have checked that the spurious $\epsilon/\epsilon$ contribution in the conventional approach is responsible for the difference,

$$
\frac{\alpha^2 \alpha_s z}{9 M^2} \left\{ \left[ (1 - z)^2 + z^2 \right] \ln(1 - z) + 2z(1 - z) \right\},
$$

between (6) and (7).

The message is that to get a reliable result we must use the *physical* approach and to subtract from (4) the $1/t$ singularity exactly, so obtaining the non-singular expression (5). Dimensional regularisation is not appropriate in the infrared region. As an example we have used the NLO coefficient function for Drell-Yan production. However, the result applies more generally. It is relevant to other processes, such as deep inelastic scattering [3]. It is relevant to the NLO splitting functions.

It should be noted that the *physical* approach cannot be considered as an alternative factorization scheme. That is, the difference between the conventional and the physical coefficient functions cannot be compensated by a re-definition of the parton distributions (PDFs). The corresponding (infrared) $\epsilon/\epsilon$ corrections in the splitting functions do not coincide with those that come from the re-definition of the PDFs.

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