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Abstract. The paper discusses four contact models used in the dynamic analysis of rotor systems. A mathematical imperfection of two models by a real example is shown. It consists in a significant overstatement of the calculated whirl-whoop speed in case of the rotor slipping on the stator.

1. Introduction
When contact interaction appears in a mechanical system, an assumption on the absolute rigidity of body(ies) is not always acceptable, moreover, a simulation of the elastic behavior of system is not always true. In particular, for the “rigid” rotors (in quotation marks, not because the terminology of balancing theory is used, but because it is wrong at the contact in general), four basic contact models of a rotor with a stator can be proposed including with hydrodynamic bearings, but as yet for simplicity without lubrication, as figure 1 illustrates.

Figure 1. Normal contact force N between rigid rotor and rigid stator (1), between rigid rotor and fixed compliant stator (2), between flexible rotor and rigid stator (3), between rigid rotor and elastically restrained rigid stator (4).

2. Mathematical models for rotors contacting with stator
In an absolutely rigid case (figure 1.1), the normal pressure force is self-determined by the centrifugal force of the rotor, counteracting it:
where $m$ is rotor mass, $\Omega$ is angular velocity of rotor whirling, $\delta$ is radial clearance in the contact place.

In the generally accepted second model \[1\] (figure 1.2), the normal pressure is determined by the so-called contact stiffness $k_{c}$ and this force balances the initial and additional (proportional to the radial displacement $\rho_{c}$) rotor inertia forces:

$$N = k_{c}\rho_{c}$$

$$N = m\Omega^{2}\delta + m\Omega^{2}\rho_{c}.$$  

Hence, the equilibrium equation is

$$\left(k_{c} - m\Omega^{2}\right)\rho_{c} = m\Omega^{2}\delta.$$

In the third representation recommended by the author \[2\] (figure 1.3), the force $N$ is determined by the equivalent transverse stiffness of rotor $\tilde{k}$ and the force $N$ curbs inertial forces, including from contact displacement of rotor $\tilde{\rho}$:

$$N = \tilde{k}\tilde{\rho}$$

$$N = m\Omega^{2}\delta + m\Omega^{2}\tilde{\rho}.$$  

So, the equilibrium equation is

$$\left(\tilde{k} - m\Omega^{2}\right)\tilde{\rho} = m\Omega^{2}\delta.$$  

Finally, in the model No.4, the first proposed by Erich \[3\] (figure 1.4), the force $N$ is expressed by stiffness of the stator fixing $k_{s}$ and $N$ prevents centrifugal displacements of the rotor and stator masses:

$$N = k_{s}\rho_{s}$$

$$N = m\Omega^{2}\delta + m\Omega^{2}\rho_{s} + m\Omega^{2}\rho_{s},$$  

i.e. the equilibrium equation is

$$\left(k_{s} - (m + m_{s})\Omega^{2}\right)\rho_{s} = m\Omega^{2}\delta.$$  

**Figure 2.** Forces of normal pressure $N$ and contact friction $T$ and their resulting $Q$ (dynamic schemes correspond to figure 1).

In real dissipative rotor systems, for simplicity with equivalent coefficient of viscous damping $c$, the force of normal pressure $N$ and the tangentially directed friction force are inevitable at contact. It is close to the Coulomb formulation $T = \mu N$. Their resulting $Q$ is deviated from $N$ by an angle equal to the angle of friction, i.e. which satisfies the identity $\mu = \tan \gamma$. The corresponding basic contact models and
equilibrium equations for “rigid” rotors with incomplete mechanical impedance \((-m\Omega^2 + ic\Omega)\) are shown in figure 2 and table 1.

**Table 1. Mathematical models for rotors with contact friction on stator**

| No. | Contact forces | Equations of the forces balance in complex and expanded forms as well as equations determining amplitude and speed of the rotor whirl |
|-----|----------------|----------------------------------------------------------------------------------------------------------------------------------|
| 1   | \(N, \quad T=\mu N\) | \((-m\Omega^2 + ic\Omega)(\delta + N + iT = 0, \quad N = m\Omega^2 \delta, \quad \Omega = \frac{e}{\mu m}\) |
| 2   | \(N=(k_i + i\eta)\rho_i, \quad T=\mu(k_i + i\eta)\rho_i\) | \((-m\Omega^2 + ic\Omega)(\delta + \rho_i + N + iT = 0, \quad \left\{\begin{array}{l}
(k_i - m\Omega^2 - \mu\eta)\rho_i = m\Omega^2 \delta, \\
(c\Omega + \mu k_i + \eta)\rho_i = -c\Omega \delta,
\end{array}\right\} \rho_i = \frac{m\Omega^2}{k_i - m\Omega^2 - \mu\eta} \delta, \quad \Omega = \frac{e}{\mu m k_i + \eta} = \frac{e}{\mu m}\) |
| 3   | \(N=\vec{\rho} \cos \gamma, \quad T=\vec{\rho} \sin \gamma\) | \(Q=\vec{\rho}\) |
| 4   | \(N=k_i \rho, \cos \gamma \quad T=\vec{k} \rho, \sin \gamma\) | \(Q=k_i \rho, \) |

The most important is a search of the whirl frequency equation in the case of rotor friction (on the stator) in order to validate a theoretical contact model. If we neglect an unbalance and imagine a rupture of the oil film in the bearings, i.e. if we discard the elastic and circulation forces, but we leave the viscous forces\(^4\), then \(\Omega\)-value is in accordance with rough (No.1) and standard (No.2) models:

\[\Omega = \frac{e}{\mu m k_i + \eta} = \frac{e}{\mu m}\]

\[^1\)Here is an opposition for the full mechanical impedance of the system or for the total (inertial, viscous and elastic) resistance force of the system per unit of its displacement.

\[^2\)In contact model No.2, the viscoelastic behavior of rubbing surfaces is assumed in accordance with the concept of complex elastic modulus \(E(1+i\eta)\), where \(\eta\) is the loss coefficient in the material(s). Otherwise (when \(\eta=0\)), at contact resonance \((\Omega^2 = \lambda^2 = k_i / m\), the amplitude \(\rho_i \rightarrow \infty\).

\[^3\)Mechanical impedance of the stator on the shock absorbers with equivalent coefficient \(c_i\) as a whole with the rotor is \([-m(m+\nu)\Omega^2 + ic_\nu\Omega]\).

\[^4\)Here is a separation on the elastic, viscous and circulating forces for the lubricant fluid. Unlike viscous forces, the elastic and circulation forces of zero order (conservative and non-conservative positional) and of first order
In a first approximation (a linear problem definition) of the lubrication theory, the coefficient of viscous damping $c = 12 \frac{m_{\text{oil}} vr}{\delta^3}$, where $m_{\text{oil}} = \rho \pi r^2 l$ is mass of lubricating fluid in the journal volume. These ratios are used in computational practice very long. Their application to rotor of real pump with the parameters listed in table 2 allows to obtain the following speed of rotor whirl for slipping against rotation, i.e. in case of the rolling with slipping along the stator (bearings):

$$\Omega = -24 \frac{m_{\text{oil}} vr}{m \mu \delta^3} \approx -276000, \text{ rad/s}$$

(1)

The result (1) is commensurate with speed of the pure rolling $\Omega_{\text{no slip}} = -\omega r / \delta$, where $\omega$ is rotation speed. For instance in case of the nominal rotation speed $\Omega_{\text{no slip}} \approx -457000 \text{ rad/s}$.

Table 2. Characteristics of real pump rotor

| Parameter                  | Symbol | Amount | Dimension |
|----------------------------|--------|--------|-----------|
| Mass                       | $m$    | 9.4    | kg        |
| Nominal speed              | $n$    | 10000  | min$^{-1}$|
| First critical frequency   | $f_0$  | 200    | Hz        |
| Number of bearings         | $z$    | 2      | –         |
| Radial clearance of bearings| $\delta$ | 0.035  | mm        |
| Radius of journals         | $r$    | 16     | mm        |
| Length of journals         | $l$    | 30     | mm        |
| Density of lubricant fluid | $\rho$ | 800    | Kg m$^{-3}$|
| Kinematic viscosity of lubricant fluid | $\nu$ | 3      | mm$^2$s   |
| Coefficient of slipping friction (in case of the rupture of oil film) | $\mu$ | 0.1    | –         |

The enormous value (1) is difficult to accept as fair, which means it is necessary to improve the mathematical model in some way. Obviously, this improvement should mainly relate to asystem compliance. Namely, it should be assumed that the rotor is not absolutely rigid and internal damping occurs at the continuous slipping on the stator (model No.3), or it is necessary to take into account the oscillations of the stator as a solid on elastic-damping supports (model No.4).

Let the system behavior corresponds to contact model No.3 and the motion equation is:

$$\Omega^3 - \left( \frac{\dot{k}}{m} - \frac{c^2}{m'} \right) \Omega - \frac{ck}{\mu m'} = 0$$

Among the roots of this equation as applied to the real pump rotor with stiffness $\dot{k} = m (2\pi f_0)^2$, only one will be with a real value, which is circular frequency of the rotor rolling with slipping on stator: $|\Omega| \approx 1146 \text{ rad/s}$. This value is already perceived as possible in practice. It is slightly lower than the first circular natural frequency of the pump rotor (1257 rad/s).

**Conclusion**

Rough and standard contact models (No.1 and No.2) should be applied to rotor systems with care. In order to solve the problem of rotor rolling on stator, the author's model (No.3) or Erich's model (No.4) is appropriate.
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References

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