Bubbles from Nothing

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Abstract

Within the framework of flux compactifications, we construct an instanton describing the quantum creation of an open universe from nothing. The solution has many features in common with the smooth 6d bubble of nothing solutions discussed recently, where the spacetime is described by a 4d compactification of a 6d Einstein-Maxwell theory on $S^2$ stabilized by flux. The four-dimensional description of this instanton reduces to that of Hawking and Turok. The choice of parameters uniquely determines all future evolution, which we additionally find to be stable against bubble of nothing instabilities.
I. INTRODUCTION

Quantum gravitational effects must be important if the early universe were ever at temperatures of order the Planck scale. This would be the case if slow-roll inflation were preceded by a Big Bang, where the initial singularity marks where the semi-classical theory has completely broken down. Calculations near this regime remain prohibitively difficult. Nevertheless, it is possible that quantum gravitational effects can be relevant at far lower energy scales than $T_{Pl}$. This possibility is assumed in the field of quantum cosmology, where the semi-classical universe emerges from non-perturbative effects in quantum gravity, but at a scale where solutions can be trusted. This is called “creation of the universe from nothing” \[1–5\], although it should be thought of as the emergence of a semi-classical universe from a regime not describable as a classical spacetime. In principle, the choice of parameters is the only freedom, and initial conditions are determined solely by the instantons and their Euclidean action.

The quantum creation of the universe from nothing has, over the years, attracted a lot of attention. Most of the models discussed in the literature studied the formation of a closed universe, since it was difficult to see how a spatially infinite flat or open universe could emerge from a compact instanton.\(^1\) On the other hand, the instanton describing the nucleation of a bubble universe from a false vacuum provides an example of how to go around this objection. The symmetry of the decay process is such that the interior of the bubble can be shown to be a spatially infinite open universe \[8, 9\] whose Big Bang surface coincides with the lightcone emanating from the nucleation center of the bubble. Using a somewhat similar strategy, one could imagine an instanton that describes the formation of an open universe from nothing, in other words, a Euclidean solution similar to the bubble nucleation instanton, but without the false vacuum region. Solutions of this type were first described in \[10\]. One characteristic property of such instantons is that they are singular. It is precisely the existence of this singularity which eliminates the possibility of including the false vacuum region in the Lorentzian continuation of these instantons. On the other hand, the singularities are mild enough that their Euclidean action is finite, suggesting that one should consider these solutions as meaningful contributions to the path integral.

\(^1\) One can however create a flat or open universe by a compact instanton of non-trivial topology, see for example \[6, 7\].
Nevertheless, the validity of these types of instantons was called into question by several authors [11–13]. In particular, it was realized in [13] that the admittance of such singular instantons would imply the possibility for Minkowski space to decay by an analogous process.

The conceptual difficulty introduced by the Hawking-Turok (HT) singularities motivated people to look for alternative instantons describing creation of an open universe from nothing. One suggestion put forward in [14] was to consider the singularity as resulting from the dimensional reduction of a smooth higher-dimensional solution. This was described in [14] using a 5d solution whose $S^1$ extra dimension smoothly degenerates in a particular region of the four-dimensional space. Looking at this solution from a purely four-dimensional point of view, one recovers the same type of singular structure introduced in [10]. The higher-dimensional geometry is perfectly smooth, just like a bubble of nothing [15]. Adding extra dimensions to the instanton solutions forces one to incorporate new fields capable of stabilizing the new moduli. In the following we will show that adding these new fields has important consequences, not only in the Lorentzian continuation of the solution but for the Euclidean instanton itself.

An alternative proposal to effectively eliminate the singularities from Hawking-Turok instantons was presented in [16, 17]. These solutions are supplemented by the presence of a membrane that allows one to cut off the instantons before reaching the singularity.

The instanton solutions we introduce here exhibit characteristics of both of these resolutions in a natural way. In our scenario, the four-dimensional spacetime is a 6d flux compactification with all moduli sufficiently stabilized, and which is manifestly smooth. The four-dimensional causal structure is shown in Fig. (1).

We will consider the creation of an open 4d universe from a six dimensional theory. The existence of extra dimensions allows us to obtain a compact, smooth solution of the higher-dimensional Euclidean equations of motion which would be singular from a purely 4d point of view, similar to what was found in [14]. The analytic continuation of this solution into the Lorentzian regime leads to a Kaluza-Klein cosmological scenario. We are therefore led to include additional gauge fields in our model whose fluxes stabilize the extra dimensions and yield a viable cosmology. However, adding this flux over the compactified space creates an obstacle to the smooth degeneration of the extra dimensions. We solve this problem just as in the bubble of nothing solutions recently found in flux compactifications [18, 19], i.e., by introducing new degrees of freedom into the theory that regularize the singularities with
FIG. 1: The causal structure of a 4d-Minkowski bubble from nothing. Only the shaded region exists. From this four-dimensional point of view, the domain wall (thick line) is a singular boundary. Roughly speaking, the bottom half of this diagram is virtual, and the nucleation occurs at the narrowest point.

charged solitonic branes at the degeneration loci. One can think of these branes from a lower-dimensional perspective in a dualized theory [20] effectively recovering the idea envisioned in [16, 17]. The smooth 6d instanton and its Lorentzian continuation is illustrated in Fig. (2).

The cosmological evolution of flux compactifications has been extensively studied [21], including their global spacetime structure [22], but observational implications have only recently begun to be uncovered [20, 23–30]. These studies portray a complicated transdimensional multiverse whose *pocket bubble universes* have different numbers of large dimensions in an eternally inflating background.

Importantly, this intricate multiverse does not admit sensible solutions eternal to the past [31], and it is therefore still necessary to seek a theory of initial conditions. (For related work see [32].)
II. BUBBLE FROM NOTHING UNIVERSE

A. Compactification solutions

Following the arguments given above, we will study the quantum creation of a 4d open universe in the framework of a higher-dimensional theory. A simple scenario which contains enough richness to build models is a 6d Einstein theory compactified on $S^2$. To stabilize the $S^2$, we need a two-form field strength, so we include a Maxwell field. To have smooth magnetically charged branes in our theory, we embed this $U(1)$ in $SU(2)$ broken by an adjoint Higgs. We must ensure that the extra dimensions are sufficiently stabilized to remain compactified through subsequent cosmological evolution.

Here we briefly review the different vacuum solutions found in our model, described by the 6d action

$$S = \int d^6 x \sqrt{-g} \left( \frac{1}{2\kappa^2} R - \frac{1}{4} F_{MN}^a F^{a MN} - \frac{1}{2} D_M \Phi^a D^M \Phi^a - V(\Phi) - \Lambda \right),$$

with $M, N = 0, \ldots, 5$ and $\kappa^2 = M_P^{-4}$, where $M_P$ is the 6d reduced Planck mass, and $\Lambda$ denotes the six-dimensional cosmological constant. The remaining terms in the Lagrangian are given
by

\[ V(\Phi) = \frac{\lambda}{4} \left( \Phi^a \Phi^a - \eta^2 \right)^2, \]

\[ F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a + \epsilon^{abc} A_N^b A_M^c, \]

\[ D_M \Phi^a = \partial_M \Phi^a + \epsilon^{abc} A_M^b \Phi^c. \]

It was shown long ago \cite{33} that this theory admits 4d vacua by turning on a magnetic flux along the internal 2-sphere compactification manifold. One can easily generalize this type of solution to arbitrary flux number \( n \) using the monopole-like ansatz

\[ ds^2 = g_{MN} dx^M dx^N = g_{\mu\nu} dx^\mu dx^\nu + C^2 d\Omega_2^2, \]

\[ \Phi^a = \eta p_c (\sin \theta \cos n\varphi, \sin \theta \sin n\varphi, \cos \theta)^a, \]

\[ A_{\mu}^a = A_{\nu}^a = 0, \]

\[ A_{\theta}^a = \frac{1 - w_c}{e} (\sin n\varphi, - \cos n\varphi, 0)^a, \]

\[ A_{\varphi}^a = \frac{n (1 - w_c)}{e} \sin \theta (\cos \theta \cos n\varphi, \cos \theta \sin n\varphi, - \sin \theta)^a, \]

with \( n \in \mathbb{Z} \). For \( n > 1 \), the equations of motion constrain the possible values of the constants in the ansatz Eqs. \cite{6,9} to be \( p_c = 1 \) and \( w_c = 0 \), and the equations of motion reduce to

\[ 3H^2 + \frac{1}{C^2} = \kappa^2 \left( \frac{n^2}{2e^2 C^4} + \Lambda \right), \]

\[ 6H^2 = \kappa^2 \left( \Lambda - \frac{n^2}{2e^2 C^4} \right), \]

where \( H \) denotes the effective 4d Hubble parameter, and \( C \) describes the size of the compactification manifold. The generic solution of these equations is given by \cite{19}

\[ C^2 = \frac{1}{\kappa^2 \Lambda} \left( 1 - \sqrt{1 - \frac{3\kappa^4 \Lambda n^2}{2e^2}} \right), \]

\[ H^2 = \frac{2\kappa^2 \Lambda}{9} \left[ 1 - \frac{e^2}{3 \Lambda \kappa^4 n^2} \left( 1 + \sqrt{1 - \frac{3\kappa^4 \Lambda n^2}{2e^2}} \right) \right]. \]

We see from these solutions that the landscape of 4d vacua includes compactifications of the form \( AdS_4 \times S^2, \mathbb{R}^{1,3} \times S^2 \), and \( dS_4 \times S^2 \). This is the same landscape of vacua of Einstein-Maxwell theory in 6d \cite{20,34}. The reason for this can be traced back to the conditions \( p = 1 \) and \( w = 0 \), which effectively reduce the theory to the abelian Einstein-Maxwell model. Instanton solutions describing transitions between these vacua have been recently
discussed in several papers [20, 23, 24, 35, 36]. Furthermore, in [18, 19], we identified a new
instability for such vacua, the decay via a charged bubble of nothing [15]. In this paper we
will study, in some sense, the opposite of the bubble of nothing instability, a process that
creates an open universe from nothing.

B. The instanton solution

In order to find the gravitational instanton that interpolates between nothing and one of
the compactified vacuum solutions discussed above, we consider the Euclidean metric ansatz

\[ ds^2 = B^2(r) \left( d\psi^2 + \sin^2 \psi d\Omega_2^2 \right) + dr^2 + C^2(r) \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right). \]  

(14)

We will show below that the Lorentzian continuation of this spacetime will indeed have a
region that describes an open FRW universe with an extra-dimensional \( S^2 \). Furthermore,
the ansatz for the matter fields is the \( r \)-dependent generalization of the \( n = 1 \) flux compact-
ification solution of Eqs. (6-9) into the Euclidean region, namely,

\[ \Phi^a = \eta \, p(r) \left( \sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta \right)^a, \]  

(15)

\[ A^a_\mu = A^a_r = 0, \]  

(16)

\[ A^a_\theta = \frac{1 - w(r)}{e} \left( \sin \varphi, - \cos \varphi, 0 \right)^a, \]  

(17)

\[ A^a_\varphi = \frac{1 - w(r)}{e} \sin \theta \left( \cos \theta \cos \varphi, \cos \theta \sin \varphi, - \sin \theta \right)^a. \]  

(18)

There are different boundary conditions that one can impose on the functions present in
this ansatz that would lead to a compact gravitational instanton. Some of these solutions
were already identified in the literature, and their interpretations are quite distinct from the
one we present in this paper. Let us briefly mention some of the known cases.

One could consider an instanton with boundary conditions

\[ B(0) = 0, \quad B(r_h) = 0, \quad C(0) = C_0, \quad C(r_h) = C_h. \]  

(19)

Solutions of this type were considered in the purely Einstein-Maxwell model where they were
interpreted as the creation of an inflating magnetically charged black brane in \( dS_6 \) [24], or
its inverse process, the decompactification transition. The limiting case where \( C_0 = C_h \) is
the higher-dimensional analogue of the Nariai solution.
In this paper we will consider the boundary conditions

$$B(0) = B_0, \quad B(r_h) = 0, \quad C(0) = 0, \quad C(r_h) = C_h.$$  \tag{20}$$

Similar conditions were found in our previous work describing the decay of $dS_4 \times S^2$ flux vacua via a bubble of nothing. We will now show that the same boundary conditions can also be used to describe the creation of an open universe from nothing.

In the rest of the paper we will take for simplicity the case $n = 1$ and set

$$\Lambda = \frac{e^2}{2\kappa^4},$$

in other words, we will focus here on finding the instanton asymptotic to a Minkowski ($\mathbb{R}^{1,3} \times S^2$) compactification, or more precisely the open FRW compactification with vanishing effective 4d vacuum energy.

As shown in [19], the presence of magnetic flux threading the extra dimensions can only be made compatible with the boundary conditions imposed in Eq. (20) if we incorporate a solitonic magnetically charged brane at $r = 0$. One can show that the most general smooth solution describing this core is given by the Taylor expansion

$$p(r) = p_1 r + \cdots,$$

$$w(r) = 1 + w_2 r^2 + \cdots,$$

$$B(r) = B_0 + B_2 r^2 + \cdots,$$

$$C(r) = r + C_3 r^3 + \cdots.$$  \tag{21}$$

Note that the local geometry around this point is similar to the charged generalization of the bubble of nothing solutions discussed in [19]. Comparing this solution to the Hawking-Turok instanton, we see that it is this codimension three soliton which permits a regular geometry, avoiding the singularity present in the 4d description.

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2 One could also consider the case $B(0) = B_0, \quad B(r_h) = B_h, \quad C(0) = 0, \quad C(r_h) = 0$, whose Lorentzian continuation could be interpreted as a warped $dS_3 \times S^3$ flux compactification, where $S^3$ is the squashed sphere $dr^2 + C^2(r)(d\theta^2 + \sin^2(\theta)d\phi^2)$. We will not consider such metrics.
FIG. 3: The bubble from nothing is characterized by both its Region I solution, shown here, and its Region II solution shown below. The core of the bubble is at \( r = 0 \), and the horizon at \( r = r_h \) leads to the future-directed region II of Fig. (1). All quantities are expressed in 6d reduced Planck units (i.e., \( \kappa = 1 \)).

On the other side, near the horizon at \( r = r_h \), the solution takes the form

\[
\begin{align*}
p(r) & = p_h + p^h_2(r - r_h)^2 + \cdots, \\
w(r) & = w_h + w^h_2(r - r_h)^2 \cdots, \\
B(r) & = \left( r - r_h \right) - B^h_3(r - r_h)^3 + \cdots, \\
C(r) & = C_h - C^h_2(r - r_h)^2 + \cdots.
\end{align*}
\]

In [19] we described the numerical methods used to find explicit solutions which interpolate between these two regions of the spacetime. The Euclidean solution is identical in form to the Lorentzian region I solution, an example of which we show in Fig. (3) using parameters \( \eta = 2, \lambda = 3, \, e = 1, \, \Lambda = \frac{1}{2} \). The corresponding region II is shown in Fig. (4).
FIG. 4: The future directed solution of Region II. The scale factor $B(t)$ grows linearly due to curvature domination, while the other fields rapidly approach their vacuum values.

C. The Lorentzian continuation: Open Inflation.

We can now analytically continue the instanton solution given in Eq. (14) by taking the $S^3$ polar angle $\psi \rightarrow \frac{\pi}{2} + it$, so that we arrive at the Lorentzian metric

$$ds^2 = B^2(r) \left(-dt^2 + \cosh^2 t \, d\Omega_2^2\right) + dr^2 + C^2(r) \left(d\theta^2 + \sin^2 \theta \, d\varphi^2\right).$$

(26)

Thus the magnetic soliton at $r = 0$ is inflating, i.e., it is a co-dimension three de Sitter brane. A horizon exists at $r = r_h$, where $B(r)$ vanishes and the extra dimensions have finite size $C_h > 0$. This is quite similar to what happens in the spacetime geometry of a global string [37]. The difference is that in our case the region beyond the horizon is stably compactified by the magnetic flux along the $S^2$ extra dimensions. One can obtain a geometry similar to the one described here in the global string case, along the lines of [18].
We extend the geometry beyond the horizon to obtain

\[ ds^2 = -dt^2 + B^2(t)d\mathcal{H}_3^2 + C^2(t)d\Omega_2^2. \]  

(27)

This is the line element of an anisotropic six-dimensional universe. The interesting point is that one can choose the parameters of the theory such that the subsequent evolution is in the basin of attraction of a flux vacuum, i.e., a four-dimensional open universe.

Any viable model should ensure the existence of a period of slow-roll inflation within the bubble universe. A realistic treatment is beyond the scope of this paper, and it seems likely one will need to add new ingredients to our toy model to satisfy observational constraints.

Nevertheless we have found an interesting point in parameter space that demonstrates \( \approx 17 \) e-folds of 4d slow-roll inflation without adding additional fields (or tuning of initial conditions, which are fixed). This is shown in Figs. (5-6) below. By fine-tuning the parameters in the Lagrangian, the bubble solution propels the radion \( C(t) \) outward to the top of the effective compactification potential. This in turn means that the evolution in the interior of the bubble is quickly dominated by the 4d effective vacuum energy associated with this potential, and so the effective 4d scale factor soon begins to grow exponentially. One can identify three different epochs for the evolution of the scale factor \( B(t) \) in Fig.(6). Like all bubble interiors, this universe begins curvature dominated, and \( B(t) \) grows linearly. This is rapidly followed by a vacuum energy dominated epoch which corresponds to a period of slow-roll inflaton. Finally, the radion suddenly rolls toward its vacuum, initiating an epoch of matter domination due to the massive field oscillations about the potential minimum. In a realistic setup the inflaton \( C(t) \) must couple to the standard model and reheat.

To achieve the inflationary solution, we chose parameters

\[ \Lambda = \frac{1}{2}, \quad e = 1, \quad \eta = 2.4070426, \quad \lambda = 6.7275, \]  

(28)

in 6d reduced Planck units, where \( \kappa = 1 \).

III. OTHER SOLUTIONS

In the previous section we considered parameters resulting in Hawking-Turok type instantons, where an end-of-the-world brane expands from nothing with an open universe interior of vanishing 4d cosmological constant. By adjusting the ratio of the 6d cosmological constant \( \Lambda \) and the gauge coupling \( e \), this can result in \( AdS_4 \) (big-crunch) or \( dS_4 \) cosmologies,
FIG. 5: Region I of the inflationary solution. The parameters which yield this solution are $\Lambda = \frac{1}{2}$, $e = 1$, $\eta = 2.4070426$, $\lambda = 6.7275$.

as shown in Eq. (13). We will not consider higher $n$ compactifications, which furthermore do not admit $O(3)$ invariant solitonic sources [38].

Within the same vacuum energy sector, one can further vary parameters in the theory, namely the scalar self-coupling $\lambda$ and the symmetry breaking scale $\eta$. These change the thickness and tension of the solitonic brane without altering the vacuum energy. The full four-dimensional (excluding $n$) parameter space will be given in a more complete description elsewhere. For now, we content ourselves with the study of the one-dimensional subspace of solutions with different values of $\eta$, and all other parameters fixed. We will see that this is sufficient to probe a wide variety of effective 4d “domain wall” tensions. Note that our model does not admit stable compactifications for $\eta < 1$ in 6d reduced Planck units, so we will not comment further on this range.

A. Bubbles of Nothing. ($1 < \eta < \eta_c$)

Following [15, 19], one can find bubble of nothing solutions in our model. These differ by having an apparently negative tension 4d effective domain wall, since the bubble is gravitationally attractive. They are interpreted as instantons contributing to the decay rate
FIG. 6: The future directed inflationary solution of Region II. Notice $B(t)$ grows exponentially before transitioning to power-law growth. While the matter fields $p(t)$ and $w(t)$ rapidly approach their stable vacuum values, the radion $C(t)$ serves as the inflaton, and only falls back to its stable compactification value after $\approx 17$ e-folds.

of flux compactifications to nothing. For small values of $\eta > 1$, one finds a geometry very similar to the generalized version of the original bubble of nothing [15]. From a 6d point of view they describe a gravitationally attractive “throat” where the $S^2$ extra dimensions smoothly degenerate as one approaches the soliton core. Increasing $\eta$, a small repulsive region develops near the core of the object, but long range attraction remains. This is the type of solutions that we previously referred to as the “punted bubble of nothing.” Further increasing of $\eta$ continues to increase the $dS_3$ radius of the bubble surface until the critical value $\eta_c$, at which point the radius is infinite, i.e., the bubble is flat.\footnote{Incidentally this is similar to the kind of behavior that was suggested in [39].} Solutions similar to

\footnote{Incidentally this is similar to the kind of behavior that was suggested in [39].}
these can be found in a different context in [40][41].

**B. The Critical Bubble.** ($\eta = \eta_c$)

One can choose the parameters such that the (2+1)d brane worldvolume is a perfectly flat “domain wall” end to a $\mathbb{R}^{1,3}$ or $AdS_4$ flux compactification\(^4\). Solutions of the $AdS_4$ type resemble smooth higher-dimensional versions of Randall-Sundrum compactifications, albeit to a 3d universe in our case. (See also [40][41] for related solutions.) One example of a flat bubble is given by the parameters $\eta_c = 1.447928125$, $\lambda = 6.7275$, $e = 1$, $\Lambda = 1/2$.

**C. Bubbles from Nothing.** ($\eta_c < \eta < \eta_d$)

Further increasing $\eta$ results in another inflating magnetic brane solution, but with different boundary conditions, with a horizon at some distance away from the core. This is the type of solution we described above, which resembles the domain wall solutions of [44][46] and global cosmic string solutions of [37]. As we explained, looking at these solutions beyond the horizon reveals a stable compactification, and they are therefore interpreted as instantons describing the creation of an open 4d universe from nothing. Increasing $\eta$ changes the value of the matter fields at the horizon, effectively changing the details of the 4d universe in this region. We have seen an example of this in the main part of the text, where we discussed the effect of the value of $\eta$ on the amount of inflation.

**D. Decompactification.** ($\eta > \eta_d$)

By increasing the value of $\eta$ beyond $\eta_d$, the open universe fails to remain compactified. Instead, the radion $C(t)$ rolls to infinity, and the Region II geometry is actually 6d de Sitter space. This solution should be thought of as simply a smooth magnetically charged brane in $dS_6$. The inflationary solution in Figs. (5-6) falls just below this threshold, so the point $\Lambda = 1/2$, $e = 1$, $\eta = 2.4070426$, $\lambda = 6.7275$ is very near the boundary to decompactification.

\(^4\) Note that the radius of curvature of a domain wall can never exceed the radius of curvature of the bulk. Hence the flattest domain wall in $dS_4$ has positive curvature, and $AdS_3$ domain walls only exist in $AdS_4$. 

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Finally, in the limit of large $\eta$ one encounters the situation where the values of the matter fields necessarily do not change from those at the core of the defect, i.e., $w_h = 1, p_h = 0$. One can think of these solutions as describing a monopole brane whose core is larger than the Hubble distance of $dS_6$. This is nothing more than topological inflation \[47, 48\] in the 6d theory. A point in parameter space just within this regime is $\Lambda = \frac{1}{2}, e = 1, \eta = 2.57, \lambda = 6.7275$.

It is interesting to note that the instanton solutions presented in this case pick out an anisotropic slicing of $dS_6$ associated with the orientation of the defect. One could thus envision a four-dimensional version of this scenario in which the instanton singles out an anisotropic slicing of 4d de Sitter space. We will describe such a case in a subsequent paper.

**IV. CONCLUSIONS**

We have identified 6d instantons that describe the creation from nothing of an open 4d universe in the context of flux compactifications. The higher-dimensional nature eliminates the singularities found in the 4d instantons of this type \[10\]. The core of the brane allows for a smooth degeneration of the extra-dimensional space, acting at the same time as a source for the magnetic flux that is ultimately responsible for the stabilization, in the asymptotic region, of the modulus associated with the extra dimensions.

The region of the geometry near the brane resembles an inside-out version of the bubble of nothing solutions previously investigated \[19\]. In a sense this type of instanton causes the “opposite” process as the bubble of nothing decay. This reasoning might suggest that a universe created from a bubble from nothing could decay by a bubble of nothing. We found evidence here that this is not the case, since the parameters of the theory alone seem to determine the 4d effective tension of the domain wall leading to nothing. We have found a monotonic deformation of bubble of nothing solutions to bubble from nothing solutions as one increases the tension of the brane. The critical solution between these two classes is a flat brane, which in turn suggests that flux compactifications reach an island of stability against decay via bubbles of nothing in a similar manner that Coleman-De Luccia transitions are absent for sufficiently negative vacuum energy \[8\]. This also suggests that a supersymmetric version of our theory will possess critical bubble solutions, rendering it stable with respect to bubble of nothing decays, as expected on general grounds. Due to the impossibility of
Minkowski domain walls in de Sitter space, the Bubble of nothing ↔ Bubble from nothing transition is less sharp in de Sitter flux compactifications.

Most of the bubble from nothing solutions of the type described in this paper would not yield a realistic 4d universe, since they do not have enough slow-roll inflation within the bubble. Nevertheless, we have found a neighborhood in parameter space that leads to a significant number of e-foldings. While the parameters are fine-tuned, the initial conditions are entirely determined by regularity of the instanton. Such initial conditions would seem completely fine tuned without quantum cosmology, since we would have to position the inflaton at the top of its potential to tremendous accuracy by hand.

The only dynamical quantity which we chose by hand is the flux number \( n \), although this is in principle determined by the corresponding value of the Euclidean action. Interestingly, this opens up the possibility of having a discrete prediction for values of the curvature parameter \( \Omega_k \) today, alleviating the problems with the continuum of solutions found in the Hawking-Turok instantons. This is related to the fact that our instanton is not singular, so we do not have the extra degree of freedom found in the original HT instantons. Our instanton could also lead to distinctive signatures in cosmological perturbations that could be investigating along the lines of [49].

We should mention that even if the 4d universe obtained from our toy model were to have enough e-foldings of slow-roll inflation, one must additionally ensure that the spectrum of perturbations is consistent with observations possibly requiring the inclusion of further ingredients in the theory to flatten and lower the inflaton potential.

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[1] A. Vilenkin, “Creation Of Universes From Nothing,” Phys. Lett. B 117, 25 (1982).
[2] J. B. Hartle and S. W. Hawking, “Wave Function Of The Universe,” Phys. Rev. D 28, 2960 (1983).
[3] A. D. Linde, “Quantum Creation Of The Inflationary Universe,” Lett. Nuovo Cim. 39, 401 (1984).
[4] V. A. Rubakov, “Quantum Mechanics in the Tunneling Universe,” Phys. Lett. B148, 280-286 (1984).
[5] A. Vilenkin, “Quantum Creation of Universes,” Phys. Rev. D30, 509-511 (1984).
[6] Y. B. Zeldovich and A. A. Starobinsky, “Quantum creation of a universe in a nontrivial topology,” Sov. Astron. Lett. 10, 135 (1984).
[7] A. D. Linde, “Creation of a compact topologically nontrivial inflationary universe,” JCAP 0410, 004 (2004).
[8] S. R. Coleman and F. De Luccia, “Gravitational Effects On And Of Vacuum Decay,” Phys. Rev. D 21, 3305 (1980).
[9] J. R. Gott, “Creation Of Open Universes From De Sitter Space,” Nature 295, 304 (1982).
[10] S. W. Hawking and N. Turok, “Open inflation without false vacua,” Phys. Lett. B 425, 25 (1998); N. Turok and S. W. Hawking, “Open inflation, the four form and the cosmological constant,” Phys. Lett. B 432, 271 (1998).
[11] A. D. Linde, “Quantum creation of an open inflationary universe,” Phys. Rev. D58, 083514 (1998); (See also: S. W. Hawking, N. Turok, “Comment on 'quantum creation of an open universe', by Andrei Linde,” gr-qc/9802062).
[12] R. Bousso, A. D. Linde, “Quantum creation of a universe with omega does not = 1: Singular and nonsingular instantons,” Phys. Rev. D58, 083503 (1998).
[13] A. Vilenkin, “Singular instantons and creation of open universes,” Phys. Rev. D 57, 7069 (1998).
[14] J. Garriga, “Smooth *creation* of an open universe in five dimensions,” arXiv:hep-th/9804106.
[15] E. Witten, “Instability Of The Kaluza-Klein Vacuum,” Nucl. Phys. B 195, 481 (1982).
[16] J. Garriga, “Open inflation and the singular boundary,” Phys. Rev. D 61, 047301 (2000).
[17] R. Bousso and A. Chamblin, “Open inflation from non-singular instantons: Wrapping the universe with a membrane,” Phys. Rev. D 59, 063504 (1999).

[18] J. J. Blanco-Pillado, B. Shlaer, “Bubbles of Nothing in Flux Compactifications,” Phys. Rev. D82, 086015 (2010).

[19] J. J. Blanco-Pillado, H. S. Ramadhan, B. Shlaer, “Decay of flux vacua to nothing,” JCAP 1010, 029 (2010).

[20] J. J. Blanco-Pillado, D. Schwartz-Perlov and A. Vilenkin, “Quantum Tunneling in Flux Compactifications,” JCAP 0912, 006 (2009).

[21] A. Chodos, S. L. Detweiler, “Where Has the Fifth-Dimension Gone?,” Phys. Rev. D21, 2167 (1980); P. G. O. Freund, “Kaluza-Klein Cosmologies,” Nucl. Phys. B 209, 146 (1982); E. Alvarez, M. Belen Gavela, “Entropy from Extra Dimensions,” Phys. Rev. Lett. 51, 931 (1983); M. Gleiser, S. Rajpoot and J. G. Taylor, “Search For Higher Dimensional Cosmologies,” Phys. Rev. D 30, 756 (1984); S. Randjbar-Daemi, A. Salam, J. A. Strathdee, “On Kaluza-Klein Cosmology,” Phys. Lett. B135, 388-392 (1984); Y. S. Myung, B. H. Cho, Y. Kim et al., “Higher Dimensional Einstein-Maxwell Theory With Inflation,” Phys. Rev. D32, 1011-1013 (1985); K. i. Maeda, “Stability And Attractor In Kaluza-Klein Cosmology. 1,” Class. Quant. Grav. 3, 233 (1986);

[22] A. D. Linde, M. I. Zelnikov, “Inflationary Universe With Fluctuating Dimension,” Phys. Lett. B215, 59 (1988).

[23] S. M. Carroll, M. C. Johnson and L. Randall, “Dynamical compactification from de Sitter space,” JHEP 0911, 094 (2009).

[24] J. J. Blanco-Pillado, D. Schwartz-Perlov and A. Vilenkin, “Transdimensional Tunneling in the Multiverse,” JCAP 1005, 005 (2010).

[25] D. Schwartz-Perlov and A. Vilenkin, “Measures for a Transdimensional Multiverse,” JCAP 1006, 024 (2010).

[26] J. J. Blanco-Pillado, M. P. Salem, “Observable effects of anisotropic bubble nucleation,” JCAP 1007, 007 (2010).

[27] P. W. Graham, R. Harnik, S. Rajendran, “Observing the Dimensionality of Our Parent Vacuum,” Phys. Rev. D82, 063524 (2010).

[28] J. Adamek, D. Campo, J. C. Niemeyer, “Anisotropic Kantowski-Sachs Universe from Gravitational Tunneling and its Observational Signatures,” Phys. Rev. D82, 086006 (2010).
[29] M. P. Salem, “A Signature of anisotropic bubble collisions,” Phys. Rev. D82, 063530 (2010).
[30] J. M. Arnold, B. Fornal, K. Ishiwata, “Finite Temperature Structure of the Compactified Standard Model,” arXiv:1103.0002 [hep-th]; J. M. Arnold, B. Fornal, M. B. Wise, “Standard Model Vacua for Two-dimensional Compactifications,” JHEP 1012, 083 (2010).
[31] A. Borde, A. H. Guth and A. Vilenkin, “Inflationary space-times are incomplete in past directions,” Phys. Rev. Lett. 90, 151301 (2003).
[32] H. Firouzjahi, S. Sarangi and S. H. H. Tye, “Spontaneous creation of inflationary universes and the cosmic landscape,” JHEP 0409, 060 (2004).
[33] E. Cremmer and J. Scherk, “Spontaneous Compactification Of Space In An Einstein Yang-Mills Higgs Model,” Nucl. Phys. B 108, 409 (1976).
[34] S. Randjbar-Daemi, A. Salam and J. A. Strathdee, “Spontaneous Compactification In Six-Dimensional Einstein-Maxwell Theory,” Nucl. Phys. B 214, 491 (1983).
[35] I. S. Yang, “Stretched extra dimensions and bubbles of nothing in a toy model landscape,” Phys. Rev. D 81, 125020 (2010).
[36] A. R. Brown, A. Dahlen, “Small Steps and Giant Leaps in the Landscape,” Phys. Rev. D82, 083519 (2010); A. R. Brown and A. Dahlen, “Bubbles of Nothing and the Fastest Decay in the Landscape,” arXiv:1010.5240 [hep-th].
[37] R. Gregory, “Non-singular global strings,” Phys. Rev. D 54, 4955 (1996); R. Gregory, C. Santos, “Space-time structure of the global vortex,” Class. Quant. Grav. 20, 21-36 (2003).
[38] E. J. Weinberg and A. H. Guth, “Nonexistence Of Spherically Symmetric Monopoles With Multiple Magnetic Charge,” Phys. Rev. D 14, 1660 (1976).
[39] G. Dvali, G. Gabadadze, M. Shifman, “Diluting cosmological constant in infinite volume extra dimensions,” Phys. Rev. D67, 044020 (2003).
[40] E. Roessl, M. Shaposhnikov, “Localizing gravity on a ’t Hooft-Polyakov monopole in seven-dimensions,” Phys. Rev. D66, 084008 (2002).
[41] I. Cho, A. Vilenkin, “Gravity of superheavy higher-dimensional global defects,” Phys. Rev. D68, 025013 (2003); I. Cho and A. Vilenkin, “Inflating Magnetically Charged Braneworlds,” Phys. Rev. D69, 045005 (2004).
[42] A. G. Cohen and D. B. Kaplan, “Solving the hierarchy problem with noncompact extra dimensions,” Phys. Lett. B 470, 52 (1999); R. Gregory, “Nonsingular global string compactifications,” Phys. Rev. Lett. 84, 2564 (2000); P. Berglund, T. Hubsch and D. Minic, “Exponential
hierarchy from space-time variable string vacua,” JHEP **0009**, 015 (2000); B. de Carlos and J. M. Moreno, “A Cigar - like universe,” JHEP **0311**, 040 (2003); B. de Carlos, J. M. Moreno, “Regular compactifications and Higgs model vortices,” Phys. Rev. **D70**, 084032 (2004).

[43] L. Randall, R. Sundrum, “An Alternative to compactification,” Phys. Rev. Lett. **83**, 4690-4693 (1999).

[44] A. Vilenkin, “Gravitational Field of Vacuum Domain Walls,” Phys. Lett. **B133**, 177-179 (1983).

[45] J. Ipser, P. Sikivie, “The Gravitationally Repulsive Domain Wall,” Phys. Rev. **D30**, 712 (1984).

[46] J. Garriga, M. Sasaki, “Brane world creation and black holes,” Phys. Rev. **D62**, 043523 (2000).

[47] A. Vilenkin, “Topological inflation,” Phys. Rev. Lett. **72**, 3137-3140 (1994).

[48] A. D. Linde, “Monopoles as big as a universe,” Phys. Lett. **B327**, 208-213 (1994).

[49] J. Garriga, X. Montes, M. Sasaki, T. Tanaka, “Spectrum of cosmological perturbations in the one bubble open universe,” Nucl. Phys. **B551**, 317-373 (1999); S. Gratton, N. Turok, “Cosmological perturbations from the no boundary Euclidean path integral,” Phys. Rev. **D60**, 123507 (1999); T. Hertog, N. Turok, “Gravity waves from instantons,” Phys. Rev. **D62**, 083514 (2000); S. Gratton, T. Hertog, N. Turok, “An Observational test of quantum cosmology,” Phys. Rev. **D62**, 063501 (2000).