Interacting polytropic gas model of phantom dark energy in non-flat universe

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Abstract

By introducing the polytropic gas model of interacting dark energy, we obtain the equation of state for the polytropic gas energy density in a non-flat universe. We show that for even polytropic index by choosing $K > B a^\frac{n}{3}$, one can obtain $\omega^e_{\Lambda} < -1$, which corresponds to a universe dominated by phantom dark energy.
1 Introduction

Type Ia supernovae observational data deduced by two research groups, the High-$z$ Supernovae Search Team [1] and the Supernovae Cosmology Project [2], show that the universe appears to be accelerating at present. Observational evidences also clear that the universe is nearly spatially flat and consists of about 70% dark energy (DE), 30% dust matter (cold dark matter plus baryons), and negligible radiation [3]. However the nature of DE is still unknown, people have proposed some candidates to describe it. The cosmological constant, $\Lambda$, is the most obvious theoretical candidate of DE which has the equation of state $\omega = -1$. Astronomical observations indicate that the cosmological constant is many orders of magnitude smaller than estimated in modern theories of elementary particles [4]. Also the "fine-tuning" and the "cosmic coincidence" problems are the two well known difficulties of the cosmological constant problems [5].

There are different alternative theories for the dynamical DE scenario which have been proposed by people to interpret the accelerating universe. i) The scalar-field models of DE including quintessence [6, 7], phantom (ghost) field [8, 9, 10], K-essence [11, 12, 13] based on earlier work of K-inflation [14, 15], tachyon field [16, 17, 18], dilatonic ghost condensate [19, 20, 21], quintom [22, 23, 24], and so forth. ii) The interacting DE models including Chaplygin gas [25, 26], holographic DE models [27, 28, 29, 30], and braneworld models [31, 32], etc. The interaction between DE and dark matter has been discussed in ample details by [33, 34, 35, 36]. The recent evidence provided by the Abell Cluster A586 support the interaction between dark energy and dark matter [37]. However, there are no strong observational bounds on the strength of this interaction [38].

Here we study the interaction between the DE and the cold dark matter (CDM). To do this we use the method introduced by [39]. Using the polytropic gas model of DE, we obtain the equation of state for an interacting polytropic gas energy density in a non-flat universe. We show that the interacting polytropic gas DE in a non-flat universe behaves as a phantom type DE, i.e. the equation of state of DE crosses the cosmological constant boundary $\omega = -1$ during the evolution.

2 Interacting polytropic gas

The polytropic gas equation of state (EoS) is given by

$$p_\Lambda = K \rho_\Lambda^{1 + \frac{1}{n}}, \quad (1)$$

where $K$ is a positive constant and $n$ is the polytropic index [40]. The different phenomenological models with EoS as $p = -\rho + f(\rho)$ and specifically containing a polytropic part $f(\rho) = A\rho^\alpha$ with constant $A$ and $\alpha$ have been already studied in the three approaches including EoS, scalar-tensor theory and F(R) gravity descriptions [23, 41, 42]. The polytropic EoS plays a very important role in astrophysics. It is still very useful as simple example which is nevertheless not too dissimilar from realistic models. More importantly, there are cases where a polytropic EoS is a good approximation to reality. An example is a gas where the pressure is dominated by degenerate electrons in white dwarfs or degenerate neutrons in neutron stars. Another example is the case where pressure and density are related adiabatically in main sequence stars. These motivate us to consider only the polytropic part of $p = -\rho + f(\rho)$ for the EoS fluid description of DE in cosmology.
Using Eq. (1), the continuity equation can be integrated to give
\[
\rho_\Lambda = \left( \frac{1}{Ba^2 - K} \right)^n,
\] (2)
where \( B \) is a positive integration constant and \( a \) is the cosmic scale factor. We consider a universe containing an interacting polytropic gas energy density \( \rho_\Lambda \) and the CDM, with \( \omega_m = 0 \). The energy equations for DE and CDM are
\[
\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -Q, \tag{3}
\]
\[
\dot{\rho}_m + 3H\rho_m = Q, \tag{4}
\]
where following [39], \( Q = \Gamma \rho_\Lambda \) is an interaction term and \( \Gamma = 3b^2H(1+\Omega_k) \) is the decay rate of the polytropic gas component into CDM with a coupling constant \( b^2 \). Also \( \Omega_\Lambda = \frac{\rho_\Lambda}{3M_p^2H^2} \) and \( \Omega_k = \frac{k}{a^2H^2} \). The choice of the interaction between both components was to get a scaling solution to the coincidence problem such that the universe approaches a stationary stage in which the ratio of DE and DM becomes a constant [43].

Note that choosing the \( H \) in the \( Q \)-term is motivated purely by mathematical simplicity. Because the continuity equations imply that the interaction term should be a function of a quantity with units of inverse of time (a first and natural choice can be the Hubble factor \( H \)) multiplied with the energy density. Therefore, the interaction term could be in any of the following forms: (i) \( Q \propto H\rho_\Lambda \) [44], (ii) \( Q \propto H\rho_m \) [45], or (iii) \( Q \propto H(\rho_\Lambda + \rho_m) \) [46]. The dynamics of interacting DE models with different \( Q \)-classes have been studied in ample details by [47]. The freedom of choosing the specific form of the interaction term \( Q \) stems from our incognizance of the origin and nature of DE as well as DM. Moreover, a microphysical model describing the interaction between the dark components of the universe is not available nowadays [48].

Following [39], if we define
\[
\omega_\Lambda^{\text{eff}} = \omega_\Lambda + \frac{\Gamma}{3H}, \quad \omega_m^{\text{eff}} = -\frac{\rho_\Lambda}{\rho_m} \frac{\Gamma}{3H}, \tag{5}
\]
then rewriting Eqs. (3) and (4) in their standard form reduces to
\[
\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda^{\text{eff}})\rho_\Lambda = 0, \tag{6}
\]
\[
\dot{\rho}_m + 3H(1 + \omega_m^{\text{eff}})\rho_m = 0. \tag{7}
\]
Following [39], we consider Friedmann-Robertson-Walker (FRW) metric for the non-flat universe as
\[
ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1-kr^2} + r^2d\Omega^2 \right), \tag{8}
\]
where \( k \) denotes the curvature of space \( k = 0, 1, -1 \) for a flat, closed open universe, respectively.

Taking the time derivative of Eq. (2) yields to
\[
\dot{\rho}_\Lambda = -3BH\dot{a}^n \rho_\Lambda^{\frac{2}{n}}. \tag{9}
\]
Substituting Eq. (9) in (6) gives

\[ \omega_{\Lambda}^{\text{eff}} = Ba^{\frac{3}{n}} \left(3M_{p}^{2}H^{2}\Omega_{\Lambda}\right)^{\frac{1}{n}} - 1, \]

\[ = - \frac{Ba^{\frac{3}{n}}}{K - Ba^{\frac{3}{n}}} - 1. \] (10)

We see that for \( K > Ba^{\frac{3}{n}} \), \( \omega_{\Lambda}^{\text{eff}} < -1 \), which corresponds to a universe dominated by phantom dark energy. Note that to have \( \rho_{\Lambda} > 0 \), from Eq. (2) the polytropic index should be even, \( n = (2, 4, 6, \cdots) \). Also note that for \( a \to a_{s} = (\frac{K}{B})^{\frac{1}{3}} \) then Eqs. (1)-(2) show that \( \rho_{\Lambda} \to \infty \) and \( p_{\Lambda} \to \infty \). Therefore the selected polytropic DE model shows a finite-time future singularity type III. The properties of different future singularities in the DE universe have been investigated in ample details by [23, 49].

Following [5], one can obtain a corresponding potential for the polytropic gas by treating it as an ordinary scalar field \( \phi(t) \). Using Eqs. (1), (2) together with \( \rho_{\phi} = \frac{1}{2} \dot{\phi}^{2} + V(\phi) \) and \( p_{\phi} = -\frac{1}{2} \dot{\phi}^{2} - V(\phi) \), we find

\[ \dot{\phi}^{2} = Ba^{\frac{3}{n}} \left(3M_{p}^{2}H^{2}\Omega_{\Lambda}\right)^{\frac{1}{n}} - 1, \]

\[ = - Ba^{\frac{3}{n}} \left(\frac{1}{K - Ba^{\frac{3}{n}}}\right)^{\frac{1}{n+1}}, \quad n = (2, 4, 6, \cdots), \] (11)

\[ V(\phi) = K - \frac{B}{2} a^{\frac{4}{n}} \left(\frac{1}{K - Ba^{\frac{3}{n}}}\right)^{\frac{1}{n+1}}, \quad n = (2, 4, 6, \cdots). \] (12)

Equation (11) shows that for \( K > Ba^{\frac{3}{n}} \), \( \dot{\phi}^{2} < 0 \). Therefore by definition \( \phi = i\psi \), the Lagrangian of the scalar field \( \phi(t) \) can be rewritten as

\[ L = \frac{1}{2} \dot{\phi}^{2} - V(\phi) = -\frac{1}{2} \dot{\psi}^{2} - V(i\psi). \] (13)

The energy density and the pressure corresponding to the scalar field \( \psi \) are as follows, respectively:

\[ \rho_{\psi} = -\frac{1}{2} \dot{\psi}^{2} + V(i\psi), \]
\[ p_{\psi} = -\frac{1}{2} \dot{\psi}^{2} - V(i\psi). \] (14)

One can conclude that the scalar field \( \psi \) is a phantom field. Therefore, a phantom-like equation of state can be generated from an interacting polytropic gas DE model in a non-flat universe.

To obtain a corresponding potential for the polytropic gas we start from the first Friedmann equation given by

\[ H^{2} = \frac{1}{3M_{p}^{2}} (\rho_{m} + \rho_{\Lambda}) - \frac{k}{a^{2}}. \] (15)

Since \( p_{m} = 0 \) for the CDM, hence the continuity equation can be integrated to give

\[ \rho_{m} = \frac{\rho_{m_{0}}}{a^{3}}. \] (16)
Substituting Eqs. (2) and (16) in Eq. (15) yields to

$$H = \left\{ \frac{1}{3M_p^2} \left[ \frac{\rho_m}{a^3} + \left( K - Ba^n \right)^{-n} \right] - \frac{k}{a^2} \right\}^{1/2}.$$  \hspace{1cm} (17)

Using Eq. (17), definition $\phi = i\psi$ and $\frac{d\phi}{da} = \frac{\dot{\psi}}{aH}$, we can rewrite Eq. (11) in terms of the integral of $a$ as

$$\psi = -\frac{2nM_p}{\sqrt{3}} \int \left[ \rho_m B^n (K - u^2)^{1-n} u^{2n} - 3kM_p^2 B^{2n/3} (K - u^2)^{1-2n} u^{2n} + (K - u^2) \right]^{-1/2} du,$$ \hspace{1cm} (18)

where $u := \sqrt{K - Ba^n}$.

Now if we consider a flat universe containing only the polytropic gas, i.e. $\rho_m \rightarrow 0$ and $k \rightarrow 0$, then Eq. (18) can be easily integrated to give

$$\psi = -\frac{2nM_p}{\sqrt{3}} \sin^{-1} \left( \frac{u}{\sqrt{K}} \right),$$
$$\quad = -\frac{2nM_p}{\sqrt{3}} \sin^{-1} \left( \sqrt{1 - Ba^n / K} \right),$$ \hspace{1cm} (19)

or

$$a^{\frac{2}{n}} = \frac{K}{B} \cos^2 \left( \frac{\sqrt{3}}{2nM_p} \psi \right).$$ \hspace{1cm} (20)

Substituting this for Eq. (12), we obtain the following potential

$$V(i\psi) = \frac{1}{2K^n} \left[ 1 + \sin^2 \left( \frac{\sqrt{3}}{2nM_p} \psi \right) \right]^{n+1}, \quad n = (2, 4, 6, \cdots).$$ \hspace{1cm} (21)

Hence, a coupled filed with this potential is equivalent to the polytropic gas model.

### 3 Conclusions

Here we have introduced a polytropic gas model of DE as an alternative model to explain the accelerated expansion of the universe. We have considered a non-flat FRW universe filled with this fluid which is in interact with the CDM. We showed that the interacting polytropic gas DE with even polytropic index and $K > Ba^n$ can be described by a phantom field which has $\omega^\text{eff}_\Lambda < -1$. Note that in the present work regarding the selected polytropic EoS, there are two things that have been investigated in ample details in comparison with other works in the literature. i) The interaction between the polytropic DE and the CDM which can not only solve the coincidence problem but also cause the EoS of DE crosses the cosmological constant boundary $\omega = -1$ during the evolution. ii) The non-flatness of the polytropic universe. However some experimental data have implied that our universe is not a perfectly flat universe and it possess a small positive curvature ($\Omega_k \sim 0.015$) [50]. Although it is believed that our universe is flat, a contribution to the Friedmann equation from spatial curvature is still possible if the number of e-foldings is not very large [51].

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