Radiative corrections to the decay $H \to hh$ in
the Minimal Supersymmetric Standard Model

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Abstract

We set up a suitable renormalization programme for the one-loop computation of the decay rate $\Gamma(H \to hh)$ in the Minimal Supersymmetric extension of the Standard Model. We then perform an explicit diagrammatic calculation, including the full contributions from top, bottom, stop and sbottom loops. We find that, for $\tan \beta$ close to 1 and $m_H \gtrsim 2m_t$, the results can significantly differ from those previously obtained in the effective potential approach. However, the latter method remains a good approximation in the region of parameter space which is most relevant for $H$ searches at large hadron colliders.
Radiative corrections to the parameters of the Higgs boson sector in the Minimal Supersymmetric extension of the Standard Model (MSSM) have recently received much attention.

After the discovery that top and stop loops can cause large corrections to the masses of the neutral CP-even Higgs bosons, radiative corrections to Higgs boson masses have been computed by a variety of methods: the renormalization group approach [3], the effective potential approach [4–6] and the diagrammatic approach [7–9].

The renormalization group approach assumes that there are two (or more) widely separated mass scales, for example

\[
M_{\text{SUSY}} \sim m_{\tilde{t}_1} \sim m_{\tilde{t}_2} \sim \ldots \sim m_H \sim m_{H^\pm} \sim m_A \gg m_Z \sim m_h \sim m_t ,
\]

and considers the effective theory for the degrees of freedom lighter than \(M_{\text{SUSY}}\). It then solves (non-supersymmetric) renormalization group equations to obtain running parameters down to the scale \(Q = m_Z\), imposing the tree-level relations of the MSSM as boundary conditions at the scale \(Q = M_{\text{SUSY}}\). This approach has the advantage of resumming the leading corrections, proportional to \(\log(M_{\text{SUSY}}/m_Z)\), so that even the case of \(M_{\text{SUSY}}\) orders of magnitude larger than \(m_Z\) can be dealt with in perturbation theory. On the other hand, if supersymmetry is to solve the naturalness problem of the Standard Model, one expects the various mass parameters of the MSSM to be scattered around the electroweak scale, \(G_F^{-1/2} \approx 250\ \text{GeV}\), so that assumption (1) breaks down.

The effective potential approach consists in identifying the Higgs boson masses and self-couplings with the corresponding derivatives of the one-loop effective potential, evaluated at the minimum. By definition, this approach evaluates all Higgs self-energies and vertices at vanishing external momentum. In the case of radiative corrections to the Higgs boson masses, this was shown to be a rather accurate approximation [8,9]. Actually, when the external momentum (i.e. the Higgs mass) approaches or exceeds the threshold of the internal particles, the full correction can be rather different from the zero-momentum one. However, in that case corrections themselves are small, either in the absolute sense or relatively to the (increased) tree-level mass. Other possible drawbacks of the effective potential approach are the gauge- and scale-dependence of the associated quantities. These are not serious problems in the computation of the mass corrections: the dominant ones come from quark and squark loops, which do not introduce spurious dependences on the gauge parameter into the results; also, wave-function renormalization effects, responsible for the scale dependence, are generally small with respect to the overall mass corrections.

The diagrammatic approach consists in performing the complete one-loop renormalization programme, specifying unambiguously the input parameters and the relations between renormalized parameters and physical quantities. This approach gives the most precise computational tool in the case of supersymmetric particle masses spread around the electroweak scale, and results which are formally gauge- and scale-independent. Since corrections can be numerically large, however, one has to pay attention and improve conveniently the naïve one-loop calculations when necessary. An example is the determination of the neutral CP-even masses, as discussed in detail in ref. [9].

\(^2\) Previous studies \([\text{[2]}]\) either neglected the case of a heavy top quark, or concentrated on the violations of the neutral Higgs mass sum rule, without computing corrections to the individual Higgs masses.

\(^3\) We recall that, although the effective potential is scale-independent, scale dependence enters its derivatives through the renormalized fields. More generally, the issue involved here is the dependence on the renormalization scheme, scale dependence being interpretable as a particular kind of scheme dependence.
Whilst radiative corrections to Higgs boson masses are by now well under control, the study of radiative corrections to Higgs boson couplings is still at a less refined stage. In most phenomenological [4,5,10–12] and experimental [13] studies, radiative corrections to the Higgs couplings to vector bosons and fermions have been taken into account only approximately, by improving the tree-level formulae with one-loop corrected values of the $h$–$H$ mixing angle, $\alpha$, and with running fermion masses, evaluated at the typical scale $Q$ of the process under consideration. Residual corrections are expected to be numerically small, with the possible exception of important threshold effects [14,15].

In the case of the Higgs boson self-couplings, which control decays like $H \to hh$, $H \to AA$ and $h \to AA$ when the latter are kinematically allowed, it is known [4] that radiative corrections can be numerically large. Radiative corrections to cubic Higgs boson self-couplings have been computed, at different levels of approximation, both in the effective potential approach [5,16,11] and in the renormalization group approach [17]. Given the fact that, in addition to the masses of the virtual particles in the one-loop diagrams, two different mass scales are involved in the decays $H \to hh$, $H \to AA$ and $h \to AA$, the mass of the decaying particle and the mass of the decay products, one might suspect that momentum-dependent effects, which are neglected in the renormalization group and in the effective potential approaches, could play a role.

The purpose of the present work is to perform the diagrammatic calculation of $\Gamma(H \to hh)$ at the one-loop level, and to compare the results with those obtained in other approaches. The main motivations for choosing this particular decay are the relative simplicity of the calculation and the fact that, even after the inclusion of the leading radiative corrections, $H \to hh$ is the dominant $H$ decay mode over a large region of parameter space. A detailed discussion of the MSSM Higgs branching ratios at the one-loop level would require extending the present calculation to other decay modes, as currently under study.

This paper is organized as follows. We begin by setting up a convenient one-loop renormalization programme for the decay rate $\Gamma(H \to hh)$ in the MSSM. We then perform a complete computation of the contributions due to top, stop, bottom and sbottom loops. Finally, we compare our results with those obtained at the tree level and in the effective potential approach, and we discuss their phenomenological implications for the detection of $H$ at future colliders.

The notation of the present paper will closely follow that of [8,9], unless otherwise stated. In [9], radiative corrections to the neutral Higgs boson masses were computed in the $\overline{\text{DR}}$ scheme [18], using the physical mass $m_A$ and $\beta \equiv \beta^{\overline{\text{DR}}}(Q = m_Z)$ as input parameters, and explicit formulae for the physical masses $m_h$ and $m_H$ were given. Here we shall adopt the same renormalization scheme for the computation of $\Gamma(H \to hh)$ at the one-loop level. We define the $H$ and $h$ fields in the CP-even neutral Higgs sector by $H = \cos \alpha S_1 + \sin \alpha S_2$ and $h = -\sin \alpha S_1 + \cos \alpha S_2$. As explained in [4], for a satisfactory convergence of the perturbative expansion it is important to define the mixing angle $\alpha$ in terms of a mass matrix which includes the leading, momentum-independent one-loop self-energy corrections.

The decay rate for the process under consideration reads

$$\Gamma(H \to hh) = \frac{|A(H \to hh)|^2}{32\pi m_H} \sqrt{1 - \frac{4m_h^2}{m_H^2}},$$

(2)

where the one-loop-corrected amplitude $A$ is given by

$$A = \sum_{i,j,k} (Z_{H}^{1/2})_{H_i} (Z_{h}^{1/2})_{h_j} (Z_{h}^{1/2})_{h_k} \lambda_{ijk} + \Lambda_{Hhh}.$$  

(3)
In eq. (3), $i, j, k = h, H$ and $\lambda_{ijk}$ are the tree-level cubic Higgs couplings in the CP-even sector:

$$\lambda_{HHH} = -\frac{g m_Z}{2 \cos \theta_W} 3 \cos(\beta + \alpha) \cos(2\alpha),$$

$$\lambda_{Hhh} = -\frac{g m_Z}{2 \cos \theta_W} \left[-2 \cos(\beta + \alpha) \sin(2\alpha) - \sin(\beta + \alpha) \cos(2\alpha)\right],$$

$$\lambda_{hhh} = -\frac{g m_Z}{2 \cos \theta_W} \left[2 \sin(\beta + \alpha) \sin(2\alpha) - \cos(\beta + \alpha) \cos(2\alpha)\right],$$

$$\lambda_{hhh} = -\frac{g m_Z}{2 \cos \theta_W} 3 \sin(\beta + \alpha) \cos(2\alpha),$$

where $g$, $m_Z \equiv \sqrt{(g^2 + g'^2)(v_1^2 + v_2^2)/2}$ and $\cos \theta_W \equiv g/\sqrt{g^2 + g'^2}$ are ($\overline{DR}$) renormalized parameters. The last term $\Lambda_{HHh}$ is the ($\overline{DR}$) renormalized one-loop proper vertex, evaluated with on-shell external momenta.

Finally, the $2 \times 2$ matrices $Z_H$ and $Z_h$ correspond to a finite wave function renormalization, which must be taken into account in the computation of a physical amplitude. Such corrective factors can be interpreted as the effect of inserting ‘bubbles’ on the external legs of the tree-diagrams. More precisely, the two matrices $Z_H$ and $Z_h$ are nothing else than the (matrix-) residues of the renormalized (matrix-) propagator $G_{ij}(p^2)$ of the neutral CP-even Higgses, at the poles $p^2 = m_H^2$ and $p^2 = m_h^2$, respectively. Actually, $Z_H$ and $Z_h$ are extracted from the real part of the propagator, so in the following we shall implicitly consider only real parts of propagators and self-energies. The renormalized inverse propagator $\Gamma(p^2)$ was computed, in the $(S_1, S_2)$ basis, in ref. [4], and the one-loop-corrected masses $m_H$ and $m_h$ were extracted from it. We now rewrite $\Gamma(p^2)$ in the $(H, h)$ basis in terms of $m_H$ and $m_h$:

$$\Gamma_{ij}(p^2) = \frac{p^2 - m_H^2 + \hat{\Pi}_{HH}(p^2) - \hat{\Pi}_{HH}(m_H^2)}{\Gamma_{HH}(p^2)} \frac{p^2 - m_h^2 + \hat{\Pi}_{hh}(p^2) - \hat{\Pi}_{hh}(m_h^2)}{\Gamma_{hh}(p^2)},$$

In eq. (8), the $\hat{\Pi}_{ij}(p^2)$ are one-loop $\overline{DR}$-renormalized self-energies, and

$$\Gamma_{hh}(p^2) = \Gamma_{HH}(p^2) = \frac{1}{2} \sin 2(\beta + \alpha) \hat{\Pi}_{ZZ}(m_Z^2) + \frac{1}{2} \sin 2(\beta - \alpha) \Delta \hat{\Pi}_{AA}(m_A^2) + \Delta \hat{\Pi}_{HH}(p^2),$$

where for a generic self-energy $\hat{\Pi}(p^2)$ we define $\Delta \hat{\Pi}(p^2) \equiv \hat{\Pi}(p^2) - \hat{\Pi}(0)$. Around the pole $p^2 = m_H^2$, we have $G(p^2) \sim Z_H/(p^2 - m_H^2)$, where

$$Z_H = \begin{pmatrix} 1 - \hat{\Pi}'_{HH}(m_H^2) & -\Gamma_{HH}(m_H^2)/(m_H^2 - m_h^2) \\ -\Gamma_{HH}(m_H^2)/(m_H^2 - m_h^2) & 0 \end{pmatrix}.$$  

Around the pole $p^2 = m_h^2$, we have $G(p^2) \sim Z_h/(p^2 - m_h^2)$, where

$$Z_h = \begin{pmatrix} 0 & -\Gamma_{hh}(m_h^2)/(m_h^2 - m_H^2) \\ -\Gamma_{hh}(m_h^2)/(m_h^2 - m_H^2) & 1 - \hat{\Pi}'_{hh}(m_h^2) \end{pmatrix}.$$  

In eqs. (10) and (11), $\hat{\Pi}'(p^2) \equiv d\hat{\Pi}(p^2)/dp^2$.

Expanding eq. (3), we finally obtain

$$\mathcal{A} = \left[1 - \frac{1}{2} \hat{\Pi}'_{HH}(m_H^2) - \hat{\Pi}'_{hh}(m_h^2)\right] \lambda_{HHh} + \frac{2 \Gamma_{HH}(m_H^2)\lambda_{HHh} - \Gamma_{HH}(m_H^2)\lambda_{hhh} + \lambda_{HHh}}{m_H^2 - m_h^2}.$$  

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3In the Feynman rules these couplings are multiplied by $i$. Also, we denote by $i\mathcal{A}$ the proper vertex obtained from the Feynman diagrams.
We now proceed to the evaluation of the general formula for $\mathcal{A}$, eq. (12), in the particular case in which only diagrams corresponding to top-stop-bottom-bottom loops are taken into account. This should give the dominant one-loop correction to $\Gamma(H \rightarrow hh)$ \[17\].

The expressions for $\tilde{\Pi}_{HH}(m_H^2)$ and $\tilde{\Pi}_{hh}(m_h^2)$ can be trivially obtained from the expressions of $\Delta \tilde{\Pi}_{HH}$ and $\Delta \tilde{\Pi}_{hh}$ given in ref. \[19\]. To obtain an explicit expression for $\Gamma_{hh}(p^2)$, eq. (13), we need $\tilde{\Pi}_{ZZ}(m_Z^2)$, $\Delta \Pi_{AA}(m_A^2)$ and $\Delta \Pi_{hh}(p^2)$. The first two were given\footnote{The expression of $\tilde{\Pi}_{ZZ}(m_Z^2)$, which was correctly given in the preprint version, contains a misprint in the published version: the coefficient appearing in the third line should read $(s_t^2 c_{LL} - c_t^2 c_{LR})^2$ and not $(s_t^2 c_{LL} + c_t^2 c_{LR})^2$. There are other misprints in the published version, which were not present in the preprint version: in the conventions stated in footnote number 2, the part of a gauge-boson self-energy diagram proportional to $g_{\mu\nu}$ should read $-ig_{\mu\nu}p^\nu$; in the expression for $\Delta_{11}/m_Z^2$, $d_{12}$ should read $2d_{12}$; the remaining ones are obvious.} in ref. \[19\]. For the last one we find:

\[
\Delta \tilde{\Pi}_{hh}(p^2) = \frac{3g^2m^2}{16\pi^2m_W^2} \sin \alpha \cos \alpha \left[ \frac{p^2}{6} + 3m_t^2\Delta F(m_t, m_t, p) - 3G(m_t, m_t, p) \right] \\
- \frac{3g^2m_b^2}{16\pi^2m_W^2} \sin \alpha \cos \alpha \left[ \frac{p^2}{6} + 3m_b^2\Delta F(m_b, m_b, p) - 3G(m_b, m_b, p) \right] \\
- \frac{3g^2}{16\pi^2m_W^2} \sum_{q=t,b,c} \sum_{a,b=1,2} c_{H_{q\bar{a}\bar{b}} c_{H_{q\bar{a}\bar{b}}}} \Delta F(m_{\bar{q}a}, m_{\bar{q}b}, p) .
\]

In eq. (13), $p \equiv \sqrt{p^2}$ and $\Delta F(m_1, m_2, p) \equiv F(m_1, m_2, p) - F(m_1, m_2, 0)$, whilst the coefficients $c_{ij\bar{a}\bar{b}}$ correspond to the trilinear Higgs-squark-squark couplings and are summarized in the Appendix. The functions $F$ and $G$ were given in \[19\].

Finally, we need to compute $\Lambda_{HHh}$. The three basic topologies of the diagrams contributing to $\Lambda_{HHh}$ are depicted in fig. 1. Accordingly, the result for $\Lambda_{HHh}$ can be written as the sum of three contributions:

\[
\Lambda_{HHh} = \Lambda^{(I)}_{HHh} + \Lambda^{(II)}_{HHh} + \Lambda^{(III)}_{HHh} ,
\]

where

\[
\Lambda^{(I)}_{HHh} = \frac{3g^2m^4}{16\pi^2m_3^2} \sin \alpha \cos^2 \alpha \left[ F^c(m_t, m_t, m_H) + 2F^c(m_t, m_t, m_h) \\
+ \left( 4m_t^2 - \frac{1}{2}m_H^2 - m_h^2 \right) f(m_t, m_t, m_H, m_t, m_H, m_h) \right] \\
+ \frac{3g^2m_b^4}{16\pi^2m_3^2} \cos \alpha \sin^2 \alpha \left[ F^c(m_b, m_b, m_H) + 2F^c(m_b, m_b, m_h) \\
+ \left( 4m_b^2 - \frac{1}{2}m_H^2 - m_h^2 \right) f(m_b, m_b, m_H, m_b, m_H, m_h) \right] ,
\]

\[
\Lambda^{(II)}_{HHh} = \frac{2-3g^3}{16\pi^2m_3^2} \sum_{q=t,b,c} \sum_{a,b,c} c_{H_{q\bar{a}\bar{b}} c_{H_{q\bar{a}\bar{b}} c_{H_{q\bar{a}\bar{b}}}}} f(m_{\bar{q}a}, m_{\bar{q}b}, m_{\bar{q}c}, m_H, m_h, m_h) ,
\]

\[
\Lambda^{(III)}_{HHh} = -\frac{3g^3}{16\pi^2m_3^2} \sum_{q=t,b,c} \sum_{a,b} \left[ c_{H_{q\bar{a}\bar{b}} c_{H_{q\bar{a}\bar{b}} c_{H_{q\bar{a}\bar{b}}}}} F^c(m_{\bar{q}a}, m_{\bar{q}b}, m_H, m_h, m_h) \\
+ 2c_{H_{q\bar{a}\bar{b}}} F^c(m_{\bar{q}a}, m_{\bar{q}b}, m_H) \right] .
\]

The expressions for the functions $F^c$ and $f$, as well as for the coefficients $c_{ij\bar{a}\bar{b}}$ corresponding to the quartic Higgs-squark-squark couplings, are collected in the Appendix.
This concludes our analytical evaluation of the one-loop-corrected amplitude and decay width, eqs. (2) and (3). As a check equivalent to divergence cancellation, we have explicitly verified that the amplitude, and consequently the width, do not depend on the renormalization scale \(Q\), as expected for physical quantities. Actually, such \(Q\) independence holds up to higher-order terms, consistently with the one-loop accuracy of our computation. It is the result of a cancellation between the explicit \(Q\) dependence of self-energies and proper vertices, and the implicit \(Q\) dependence of the parameters contained in \(\lambda_{Hhh}\): in our case, consistency requires that we consider only the \(Q\) dependence of parameters that is due to top, stop, bottom and sbottom virtual effects. In the following numerical calculations, we set \(Q = m_Z\).

Consistency would also require that we specify the input parameters \(g, m_Z\) and \(\cos \theta_W\) [or, equivalently, \(\alpha \approx g^2 \sin^2 \theta_W/(4\pi), m_Z\) and \(\cos \theta_W\)] in the \(\overline{DR}\) scheme, and in a theory containing the stop and sbottom degrees of freedom besides the Standard Model particles. The \(\overline{DR}\) mass \(m_Z\) is related to the physical mass \(m_{Z,phys}\) by

\[
m^2_Z = m^2_{Z,phys} + \hat{\Pi}_{ZZ}(m^2_Z).
\]

Similarly, the \(\overline{DR}\) fine structure constant \(\alpha\) is related to \(\alpha^{-1}_{em} \approx 137\) by

\[
\frac{1}{\alpha} = \frac{1}{\alpha_{em}} - \frac{\Delta \alpha_{light}}{\alpha} + \frac{1}{2\pi} \sum_{i \in heavy} b_i \log \frac{m_i}{m_Z},
\]

where \(\Delta \alpha_{light}\) is the (large) contribution from charged leptons and the five observed quarks, and the remaining (small) contributions from the heavy particles in the model, denoted by the index \(i\), are proportional to the corresponding one-loop QED \(\beta\)-function coefficient \(b_i\). Finally, the \(\overline{DR}\) electroweak mixing angle is given by

\[
\cos^2 \theta_W = \frac{m^2_{W,phys}}{m^2_{Z,phys}} \left[ 1 + \frac{\hat{\Pi}_{WW}(m^2_W)}{m^2_W} - \frac{\hat{\Pi}_{ZZ}(m^2_Z)}{m^2_Z} \right].
\]

At the level of accuracy of our numerical examples, however, it is enough to work with the fixed input parameters \(m_Z = 91\) GeV, \(\sin^2 \theta_W = 0.23\) and \(\alpha^{-1} = 128\). This approximation is justified by the fact that there are other effects not accounted for in our results: one-loop corrections involving loops of gauge bosons, Higgs bosons, gauginos and higgsinos; also, order \(h^2_t\) or \(g^2_s\) (two-loop) corrections to the one-loop diagrams considered here.

Before moving to the numerical evaluation of our results, we would like to relate them to the results one obtains in the effective potential approach, which consists in approximating the amplitude \(A\) by

\[
A^{e.p.} \equiv - \left( \frac{\partial^3 V_{eff}}{\partial H \partial h \partial h} \right)_{min}.
\]

Since this amounts to computing the \(Hhh\) 3-point function at vanishing external momenta, we can easily obtain an explicit expression for \(A^{e.p.}\) by taking a special limit of our previous result:

\[
A^{e.p.} = \lambda_{Hhh} + \Lambda_{Hhh}|_{m_h=0,m_H=0}.
\]

As a check, we have computed \(A^{e.p.}\) from its definition \[21] and verified that eq. \[22\] gives an identical result.

To illustrate our results, we show in fig. 2 the one-loop-corrected width \(\Gamma(H \rightarrow hh)\), as a function of \(m_H\), corresponding to four representative parameter choices. For simplicity, in our
Numerical examples we take as soft supersymmetry-breaking parameters $\tilde{m}_Q = \tilde{m}_T = \tilde{m}_B \equiv m_{sq}$ and $A_t = A_b \equiv A$, in the conventions of refs. \cite{3,14}. For comparison, we also show the values of the width obtained by replacing the amplitude $A$ in eq. (3) with its ‘improved tree-level’ expression, $A^{\text{tree}} = \lambda_{Hhh}$, and with the effective-potential expression, $A^{\text{e.p.}}$ in eq. (22). The behaviour of the one-loop-corrected width $\Gamma(H \to hh)$ in the $(m_A, \tan \beta)$ plane is illustrated in fig. 3, which displays contours of constant width, for the representative parameter choice $m_t = 140$ GeV, $m_{sq} = 1$ TeV, $A = \mu = 0$.

As a first comment, we observe that there is a region of the $(m_A, \tan \beta)$ plane in which the decay $H \to hh$ is kinematically forbidden. At tree level, this region corresponds to $|\cos 2\beta| \geq 2(m_A^2 + m_Z^2)/(5m_A m_Z)$, which implies $m_Z/2 \leq m_A \leq 2m_Z$ and $\tan \beta \geq 3$. As expected, this region is deformed by the inclusion of radiative corrections, in a way which depends on $m_t, m_{sq}$, ... It is delimited by the thick solid line in fig. 3, and its existence is also evident in figs. 2b and 2c. For a given $\tan \beta$ in the above range, the forbidden region for $H \to hh$ essentially corresponds to $m_H^{\text{min}} < m_H \lesssim 2m_h^{\text{max}}$, where $m_H^{\text{min}}$ ($m_h^{\text{max}}$) is the lowest (highest) possible value of $m_H$ ($m_h$). We also recall that the small region of $m_H \sim m_H^{\text{min}}$, corresponding to $m_A \lesssim 50$ GeV, is almost entirely ruled out by the present LEP data \cite{13}. From figs. 2a, 2d and 3 we can also see that there is an additional line in the $(m_A, \tan \beta)$ plane where $\Gamma(H \to hh)$ vanishes, due to the vanishing of the amplitude. For the parameter choice of fig. 3, this occurs for $\tan \beta \lesssim 2$ and $m_A \sim m_W$, corresponding to $m_H \sim m_H^{\text{min}}$.

The general behaviour of $\Gamma(H \to hh)$ in the $(m_A, \tan \beta)$ plane is well represented in fig. 3. For $m_A \lesssim 2m_Z$, and in the kinematically allowed region, the decay rate depends mildly on $\tan \beta$, and rapidly decreases with $m_A$ approaching the critical line near $m_W$. For $m_A \gtrsim 2m_Z$, the partial width has a milder dependence on $m_A$ and a stronger dependence on $\tan \beta$: the largest values are obtained for $\tan \beta \sim 2–3$ and $m_A \sim 200–350$ GeV. For very large values of $\tan \beta$ the width becomes negligibly small in comparison with the competing channels, in particular $H \to bb$.

As for the dependence of the corrections on $m_t, A$ and $\mu$, in general $\Gamma(H \to hh)$ rapidly increases with increasing $m_t, A$ and $\mu$. For example, for $m_A = 500$ GeV, $\tan \beta = 1.5$ and $m_{sq} = 750$ GeV, one obtains

\[
\begin{align*}
&m_t = 120 \text{ GeV}, A = \mu = 250 \text{ GeV}, \quad \Rightarrow \quad \Gamma = 0.04 \text{ GeV}; \\
&m_t = 120 \text{ GeV}, A = \mu = 1 \text{ TeV}, \quad \Rightarrow \quad \Gamma = 0.06 \text{ GeV}; \\
&m_t = 180 \text{ GeV}, A = \mu = 250 \text{ GeV}, \quad \Rightarrow \quad \Gamma = 0.18 \text{ GeV}; \\
&m_t = 180 \text{ GeV}, A = \mu = 1 \text{ TeV}, \quad \Rightarrow \quad \Gamma = 0.37 \text{ GeV}.
\end{align*}
\]

The comparison between the present one-loop calculations and previous approximations can be done by looking at fig. 2. One can see that the ‘improved tree-level’ result, obtained by using the tree-level formula with one-loop-corrected values $m_h, m_H$ and $\alpha$, can be off by as much as a factor of $\sim 4$. The pure tree-level calculation would be in general in much worse agreement. On the other hand, the effective potential result is typically much closer to the full one. In particular, the agreement between the two methods is good for $\tan \beta \gg 1$ or $m_H < 2m_t, 2m_{t2}$. As expected, radiative correction effects are maximal for $\tan \beta \sim 1$, corresponding to maximal top Yukawa coupling for a given top mass. Also, momentum-dependent effects begin to play a role only when the top or the stop thresholds are approached. The effect of the top threshold

\footnote{In the evaluation of $\Gamma(H \to hh)$, one also needs the masses $m_h$ and $m_H$ as functions of the input parameters: for those we use the one-loop-corrected expressions in all three cases. To be consistent with this prescription, even when evaluating $A^{\text{tree}}$ we use the one-loop-corrected expression of the mixing angle $\alpha$.}
is always smooth, but is nevertheless clearly visible in figs. 2a–2d. Besides their effect on the proper vertex, the stop thresholds give rise to singularities in $\Pi'_{HH}(m^2_H)$, corresponding to a breakdown of perturbation theory near threshold: an example is the cusp appearing in fig. 2d, whose details should therefore not be trusted.

To allow a better understanding of the origin of our results, we describe in more detail the sources of numerically large corrections. Wave-function renormalization effects in eq. (12) are in general negligible, also thanks to the use of the one-loop-corrected mixing angle $\alpha$ in the definition of the $(H, h)$ basis\(^6\). Large corrections come only from the proper vertex $\Lambda_{HHh}$, with the imaginary part never very large but sometimes non-negligible, and diagrams of type (II) usually give small contributions, unless $H$ is close to a stop threshold.

We conclude with some comments on the phenomenological implications of our results. To consistently examine the effects of these corrections on the $H$ branching ratios, one should also compute the remaining partial widths at the one-loop level, which goes beyond the aim of the present paper. At present, only approximate computations exist, but some qualitative considerations are nevertheless possible. For example, one might wonder if the phenomenological analyses of $H$ signals at hadron colliders \cite{10,12}, which used the effective potential approximation in the computation of $\Gamma(H \rightarrow hh)$, are going to be significantly affected. The answer is negative: $H \rightarrow hh$ is an important decay mode only for small $\tan \beta$ and $m_H < 2m_t, 2m_{\tilde{t}}$, and in this region the effective potential approach is a rather accurate approximation. The residual small corrections are negligible compared with the uncertainties in the evaluation of the production cross-sections. However, one could think that, in the happy event that $H$ is discovered, the effects studied in the present paper might be important for a detailed study of its properties at a high-energy and high-luminosity $e^+e^-$ collider.

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\(^6\) For example, had we used the tree-level definition of $\alpha$, the $\Gamma_{HH}(p^2)$ appearing in eq. (12) would have an expression different from (11) and would contain large corrections, proportional to $m_t^4$: this would jeopardize the validity of the perturbative expansion underlying eq. (12).
Appendix

We give here the explicit expressions of the coefficients \(c_{i\bar{a}b}\) and \(c_{ij\bar{q}b}\):

\[
\begin{align*}
    c_{Hi\bar{t}1} &= - \left[ d_{11}^t m_Z^2 \cos(\alpha + \beta) + \frac{m_t^2 \sin \alpha}{\sin \beta} + 2 s_t c_t B_{tH} \right], \\
    c_{Hi\bar{t}2} &= - \left[ d_{22}^t m_Z^2 \cos(\alpha + \beta) + \frac{m_t^2 \sin \alpha}{\sin \beta} - 2 s_t c_t B_{tH} \right], \\
    c_{Hi\bar{t}2} &= - \left[ d_{12}^t m_Z^2 \cos(\alpha + \beta) + (s_t^2 - c_t^2) B_{tH} \right], \\
    c_{Hb\bar{b}1} &= - \left[ d_{11}^b m_Z^2 \cos(\alpha + \beta) + \frac{m_b^2 \cos \alpha}{\cos \beta} + 2 s_b c_b B_{bH} \right], \\
    c_{Hb\bar{b}2} &= - \left[ d_{22}^b m_Z^2 \cos(\alpha + \beta) + \frac{m_b^2 \cos \alpha}{\cos \beta} - 2 s_b c_b B_{bH} \right], \\
    c_{Hb\bar{b}2} &= - \left[ d_{12}^b m_Z^2 \cos(\alpha + \beta) + (s_b^2 - c_b^2) B_{bH} \right];
\end{align*}
\]

\[
\begin{align*}
    c_{Hh\bar{a}b} &= \frac{1}{4} \sin 2\alpha \left( 2d_{ab}^h m_Z^2 - \delta_{ab} \frac{m_h^2}{\sin^2 \beta} \right), \\
    c_{Hh\bar{b}a} &= \frac{1}{4} \sin 2\alpha \left( 2d_{ab}^h m_Z^2 + \delta_{ab} \frac{m_h^2}{\cos^2 \beta} \right), \\
    c_{hh\bar{a}b} &= \frac{1}{2} \left( d_{ab}^h m_Z^2 \cos 2\alpha - \delta_{ab} \frac{m_h^2 \cos^2 \alpha}{\sin^2 \beta} \right), \\
    c_{hh\bar{b}a} &= \frac{1}{2} \left( d_{ab}^h m_Z^2 \cos 2\alpha - \delta_{ab} \frac{m_h^2 \sin^2 \alpha}{\cos^2 \beta} \right).
\end{align*}
\]

The conventions for the squark masses and mixing angles, and the symbols \(d_{ab}^q\), \(B_{qH}\), etc., were all defined in [6,7]. The symbol \(\delta_{ab}\) is the Kronecker delta. Notice that the \(c_{Hh\bar{a}b}\) coefficients disagree with those reported in ref. [19], as already observed in ref. [15].

The function \(F^c\), corresponding to the two-point scalar loop integral, is given by

\[
F^c(m_1, m_2, m_3) = F(m_1, m_2, m_3) - i\pi \theta(m_3 - m_1 - m_2) \sqrt{\left[ 1 - \left( \frac{m_1 + m_2}{m_3} \right)^2 \right] \left[ 1 - \left( \frac{m_1 - m_2}{m_3} \right)^2 \right]}. 
\]

The function \(f(m_a, m_b, m_c; p_1, p_2, p_3)\) corresponds to the three-point scalar loop integral

\[
\begin{align*}
f(m_a, m_b, m_c; p_1, p_2, p_3) &= \int_0^1 dx \int_0^x dy \left[ p_1^a x^2 + p_1^2 y^2 + (p_1^a - p_1^2 - p_1^2) x y \right. \\
&\quad \left. + (-p_2^a + m_b^2 - m_c^2) x + (p_2^a - p_2^2 + m_b^2 - m_c^2) y + m_c^2 - ie \right]^{-1}. 
\end{align*}
\]

8
It was studied and computed in ref. [20]. For an on-shell decay its explicit expression can be written in the form

\[ f(m_a, m_b, m_c; p_1, p_2, p_3) = \frac{1}{D} \left[ Sp(A_{1+}^1) + Sp(A_{1-}^1) - Sp(A_{1+}^-) - Sp(A_{1-}^-) + (1 \rightarrow 2) + (1 \rightarrow 3) \right] , \]

where \( Sp(x) \) is the Spence function and

\[ A_{1\pm}^\pm = \frac{\pm p_1^2 - m_a^2 + m_b^2 + B_1}{B_1 \pm C_1} , \]

\[ B_1 = \frac{1}{D} \left[ p_1^2(p_1^2 - p_2^2 - p_3^2 + 2m_c^2 - m_a^2 - m_b^2) + (p_3^2 - p_2^2)(m_a^2 - m_b^2) \right] , \]

\[ C_1 = \sqrt{p_1^4 - 2(m_a^2 + m_b^2)p_1^2 + (m_a^2 - m_b^2)^2 + i\epsilon} , \]

\[ D = \sqrt{p_1^4 + p_2^4 + p_3^4 - 2p_1^2p_2^2 - 2p_1^2p_3^2 - 2p_2^2p_3^2} . \]

The replacements \((1 \rightarrow 2)\) and \((1 \rightarrow 3)\) mean the following:

\[ 1 \rightarrow 2 \equiv (p_1, p_2, p_3, m_a, m_b, m_c) \rightarrow (p_2, p_3, m_b, m_c, m_a) \]

\[ 1 \rightarrow 3 \equiv (p_1, p_2, p_3, m_a, m_b, m_c) \rightarrow (p_3, p_1, p_2, m_c, m_a, m_b) \]

Analogously to the function \( F^c \), the function \( f \) develops an imaginary part above thresholds.
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Figure captions

Fig.1: The three basic topologies of the diagrams involving top, bottom, stop and sbottom exchanges and contributing to the one-loop $Hhh$ proper vertex ($q = t, b; a, b, c, = 1, 2$).

Fig.2: The decay width $\Gamma(H \rightarrow hh)$, as a function of $m_H$, corresponding to the four indicated parameter choices. Solid lines correspond to the full diagrammatic calculation, dashed lines to the effective potential approach, dash-dotted lines to the ‘improved tree-level’ result.

Fig.3: Contours in the $(m_A, \tan \beta)$ plane, corresponding to constant values of $\Gamma(H \rightarrow hh)$, for the representative parameter choice $m_t = 140$ GeV, $m_{sq} = 1$ TeV, $A = \mu = 0$. The thick solid lines correspond to $\Gamma(H \rightarrow hh) = 0$ and in particular delimit the region where the decay $H \rightarrow hh$ is kinematically disallowed.