Reconstructing QCD ghost $f(R, T)$ models

M. Zubair and G. Abbas

1 Department of Mathematics, COMSATS Institute of Information Technology, Lahore-54000, Pakistan.
2 Department of Mathematics, COMSATS Institute of Information Technology, Sahiwal, Pakistan.

Abstract

We reconstruct $f(R, T)$ theory (where $R$ is the scalar curvature and $T$ is the trace of energy-momentum tensor) in the framework of QCD ghost dark energy models. In this study, we concentrate on particular models of $f(R, T)$ gravity which permits the standard continuity equation in this theory. It is found that reconstructed function can represent phantom and quintessence regimes of the universe in the background of flat FRW universe. In addition, we explore the stability of ghost $f(R, T)$ models.

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1 Introduction

Contemporary observational results form Supernovae type Ia (SNeIa) (Perlmutter et al. 1999) revealed the expanding behavior of the universe. This fact has further been affirmed by the observations of anisotropies in cosmic...
microwave background (CMB) (Spergel et al. 2004)), large scale structure (Hawkins, E. et al. 2003), baryon acoustic oscillations (Eisentein, D.J. et al. 2005) and weak lensing (Jain and Taylor 2003). The most promising feature of the universe is the dominance of exotic energy component with large negative pressure, known as dark energy (DE). A number of alternative models have been proposed in the framework of general relativity (GR) to explain the role of DE in the present cosmic acceleration (Bamba et al. 2012).

Recently, a new dynamical DE model is proposed on the basis of Veneziano ghost chromodynamics (QCD) called as ghost DE (GDE) (Urban and Zhitnitsky 2009; Ohta 2011). The existence of Veneziano ghost is necessary for the resolution of U(1) problem in low energy effective theory (Witten 1979; Veneziano 1979). Although in the usual Minkowski spacetime QFT, Veneziano ghost is unphysical and makes no contribution to the vacuum energy density but it exhibits significant physical effects in dynamical spacetime. In a curved spacetime this ghost provides the vacuum energy density proportional to $\Lambda_{QCD}^3 H$ (Zhitnitsky 2010), where $H$ is the Hubble parameter and $\Lambda_{QCD} \sim 100\text{Mev}$ is the QCD mass scale. For $H \sim 10^{-33}\text{eV}$, $\Lambda_{QCD}^3 H$ gives the numerical value in accordance with the observed energy density of DE. Therefore, GDE helps to get rid of fine tuning and coincidence problems (Urban and Zhitnitsky 2009; Ohta 2011). Cai et al. (2011) fitted this model and developed the constraints on model parameters using the recent observational data SNeIa, CMB, BAO, BNN and the Hubble parameter data.

GDE has attained significant attention and various aspects have been discussed such as interacting GDE models (Sheykhi and Movahed 2012), thermodynamics (Feng et al. 2012), correspondence with scalar field models (Karami and Fahimi 2013), $f(R)$ gravity (Jawad 2014; Saaidi et al. 2012), $f(T)$ gravity (Karami et al. 2013a) and Brans-Dicke theory (Ebrahimi and Sheykhi 2011). In (Cai et al. 2012), authors presented a generalized GDE of the form $\alpha H + \beta H^2$ and discussed its dynamical evolution. The energy density of the QCD GDE can be related to the radius of trapping horizon as (Garcia-Salcedo et al. 2013) $\rho_{GDE} = \frac{\alpha(1-\epsilon)}{r_T} = \alpha(1-\epsilon)\sqrt{H^2 + \frac{\kappa}{a^2}}$, here $\epsilon = \frac{\dot{r}_T}{2Hr_T}$ introduces a new time dependent component which is previously ignored. Garcia-Salcedo et al. (2013) presented the phase space analysis for GDE and discussed some issues related to the stability of this model. Jawad (2014) reconstructed the modified Horava-Lifshitz $F(R)$ gravity for this QCD GDE model and discussed the physical parameters.

The modification of Einstein-Hilbert action is another approach to un-
ravel the mysterious nature of DE and various candidates have been proposed namely, \( f(R) \) gravity (Capozziello and Faraoni 2011), \( f(T) \) (Ferraro and Fiorini 2007; Zubair 2015), where \( T \) is the torsion scalar, GaussBonnet gravity (De Felice and Tsujikawa 2010), \( f(R, T) \) gravity (Alvarenga et al. 2013a; Harko et al. 2011; Sharif and Zubair 2012, Shabani and Farhoudi 2013; Noureen and Zubair 2015, Noureen et al. 2015), where \( T \) is the trace of the energy-momentum tensor and \( f(R, T, R_{\mu\nu}T^{\mu\nu}) \) gravity (Nojiri and Odintsov 2010; 2011a; Haghani et al. 2013; Sharif and Zubair 2013a, 2013b). Cosmological reconstruction in modified theories is one of the significant aspects in cosmology. The reconstruction schemes in modified theories have been carried out under different scenarios (Elizalde et al. 2004; Capozziello et al. 2006; Carloni et al. 2012) to find out realistic cosmology which can explain the transition of matter dominated epoch to DE phase. In (Nojiri and Odintsov 2006) the general formulation of modified \( f(R) \) gravity is presented which can be reconstructed for FRW universe model. Several versions of modified gravity are formulated compatible with solar system tests which includes matter dominated phase, transition from deceleration to acceleration, accelerating epoch and \( \Lambda \)CDM cosmology consistent with recent observations. Nojiri and Odintsov (2007, 2011b) discussed the reconstruction scheme in different modified theories including scalar-tensor theory, \( f(R) \), \( f(G) \) and string-inspired, scalar-Gauss-Bonnet gravity.

In this study, one interesting way is to consider the known cosmic evolution and use the field equations to find particular form of Lagrangian that can reproduce the given evolution background. The cosmological reconstruction has been investigated in the framework of \( f(R, T) \) gravity realizing the \( \Lambda \)CDM, phantom, non-phantom eras and unification of matter dominated and accelerated phases (Houndjo 2012; Sharif and Zubair 2013c, 2014a). In this paper, we are interested to develop an equivalence between \( f(R, T) \) gravity and GDE without utilizing any additional DE component. We reconstruct the \( f(R, T) \) model and discussed its evolution and stability. The paper is arranged as follows. In next section we present the formulation of field equations in \( f(R, T) \) gravity. In sections (2.1) and (2.2) we reconstruct the ghost \( f(R, T) \) model and discuss its stability for homogeneous matter perturbation. Section 3 concludes our results.
2 Reconstructing $f(R, T)$ Gravity

The $f(R, T)$ gravity is an appealing modification to the Einstein-Hilbert action by setting an arbitrary function of scalar curvature $R$ and trace of the energy-momentum tensor $T$. The action for this theory is defined as (Harko et al. 2011)

$$I = \int dx^4 \sqrt{-g} \left[ \frac{1}{16\pi G} f(R, T) + \mathcal{L}(M) \right],$$

where $\mathcal{L}(M)$ denotes the matter Lagrangian.

The energy-momentum tensor of matter component is determined as

$$T^{(M)}_{\alpha\beta} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}(M))}{\delta g^{\alpha\beta}}.$$ (2)

The corresponding field equations are found through the variation of (1) with respect to the metric tensor

$$\kappa^2 T_{\alpha\beta} - f_T(R, T) T_{\alpha\beta} - f_T(R, T) \Theta_{\alpha\beta} - R_{\alpha\beta} f_R(R, T) + \frac{1}{2} g_{\alpha\beta} f(R, T)$$

$$- (g_{\alpha\beta} \Box - \nabla_\alpha \nabla_\beta) f_R(R, T) = 0,$$ (3)

where $f_R = \partial f / \partial R$, $f_T = \partial f / \partial T$, $\Box = \nabla_\alpha \nabla^\alpha$; $\nabla_\alpha$ is the covariant derivative linked with the Levi-Civita connection symbol and $\Theta_{\alpha\beta}$ is defined by

$$\Theta_{\alpha\beta} = g^{\mu\nu} \delta T^{(M)}_{\mu\nu} \delta g^{\alpha\beta} = -2 T^{(M)}_{\alpha\beta} + g_{\alpha\beta} \mathcal{L}_M - 2 g^{\mu\nu} \frac{\partial \mathcal{L}_M}{\partial g^{\alpha\beta}} \frac{\partial}{\partial g^{\mu\nu}}.$$ (4)

The modified field equation for the choice of perfect fluid are given by

$$\kappa^2 T_{\alpha\beta} + f_T(R, T) T_{\alpha\beta} + p f_T(R, T) g_{\alpha\beta} - R_{\alpha\beta} f_R(R, T) + \frac{1}{2} g_{\alpha\beta} f(R, T)$$

$$- (g_{\alpha\beta} \Box - \nabla_\alpha \nabla_\beta) f_R(R, T) = 0.$$ (5)

In $f(R, T)$ gravity, the divergence of the energy-momentum tensor is not covariantly conserved and is given by (Harko et al. 2011)

$$\nabla^\alpha T_{\alpha\beta} = \frac{f_T}{\kappa^2 - f_T} \left[ (T_{\alpha\beta} + \Theta_{\alpha\beta}) \nabla^\alpha \ln f_T + \nabla^\alpha \Theta_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} \nabla^\alpha T \right].$$ (6)

In this study, we consider the flat FRW geometry described by the metric

$$ds^2 = dt^2 - a^2(t) dx^2,$$
where $a(t)$ is the scale factor and $dx^2$ comprises the spatial part of the metric. For the FRW metric with perfect fluid as matter content, the divergence of the energy-momentum tensor takes the form (Harko et al. 2011)

$$\dot{\rho} + 3H(\rho + p) = \frac{-1}{\kappa^2 + f_T} \left[ (\rho + p)\dot{T}f_{TT} + \dot{\rho}f_T + \frac{1}{2}\dot{T}f_T \right]. \quad (7)$$

The above equation shows that energy momentum tensor is not covariantly conserved due to the matter geometry coupling in this theory. To obtain the standard continuity equation, we need to have an additional constraint by taking the right side of the above equation equal to zero. In this situation the additional constraint is

$$(1 + \omega)Tf_{TT} + \frac{1}{2}(1 - \omega)f_T = 0. \quad (8)$$

In next section, we consider particular $f(R, T)$ models and develop their correspondence with the QCD GDE proposals.

### 3 Ghost $f(R, T)$ Models

Herein, we consider the $f(R, T)$ functions of the form

- $f(R, T) = R + 2f(T)$,
- $f(R, T) = f_1(R) + f_2(T)$.

#### 3.1 $f(R, T) = R + 2f(T)$

In the first place, we propose a particular case with $f(R, T) = R + 2f(T)$. This model corresponds to gravitational Lagrangian with time dependent cosmological constant being function of trace of the energy-momentum tensor (Poplawski 2006). Here, $f(T)$ is a correction to the usual Einstein Hilbert action. Such model appears to be interesting and has been widely studied in literature (Houndjo 2012; Sharif and Zubair 2013c, 2014a). The corresponding field equations are

$$T_{\alpha\beta} + 2f_T(T)T_{\alpha\beta} + 2pf_T(T) - R_{\alpha\beta} + \frac{1}{2}g_{\alpha\beta}(R + 2f(T)) = 0. \quad (9)$$
The 00 and 11 component of filed equation (9) can be represented as
\[ 3H^2 = \rho_M + \rho_\theta, \quad -(2\dot{H} + 3H^2) = p_M + p_\theta, \]  \tag{10}

where dot represents differentiation with respect to time, \( H = \dot{a}/a \) is the Hubble parameter and energy density \( \rho_\theta \) and pressure \( p_\theta \) of dark energy components are obtained as
\[ \rho_\theta = [2(\rho_M + p_M)f_T(T) + f(T)], \quad p_{de} = -f(T). \]  \tag{11}

The corresponding EoS parameter is
\[ \omega_\theta = \frac{-f(T)}{2(\rho_M + p_M)f_T(T) + f(T)}. \]  \tag{12}

- Garcia-Salcedo GDE Model

Our aim is to reconstruct the \( f(R, T) \) gravity according to QCD GDE model. The QCD GDE energy density related to the dynamics of trapping horizon is given by (Garcia-Salcedo et al. 2013)
\[ \rho_{GDE} = \frac{\alpha(1 - \epsilon)}{\tilde{r}_T} = \alpha(1 - \epsilon)\sqrt{H^2 + \frac{\kappa}{a^2}}, \quad \epsilon = \frac{\tilde{\dot{r}}_T}{2H\tilde{r}_T}. \]  \tag{13}

For the flat case with spatial curvature \( \kappa = 0 \), above relation becomes
\[ \rho_{GDE} = \alpha \left(1 + \frac{\dot{H}}{2H^2}\right)H. \]  \tag{14}

Using the energy conservation equation \( \dot{\rho}_\theta + 3H\rho_\theta(1 + \omega_\theta) = 0 \), the EoS parameter is set of the form
\[ 1 + \omega_{GDE} = -\frac{\dot{\rho}_{GDE}}{3H\rho_{GDE}} = \frac{1}{3} \left(\frac{\dot{\epsilon}}{H(1 - \epsilon)} + 2\epsilon\right). \]  \tag{15}

Equating the EoS parameters of dark energy components \( \omega_\theta \) and \( \omega_{GDE} \) and hence using the constraint (8), it leads to
\[ 4T^2f_{TT} + \left[\left(1 - \frac{1}{3} \left(\frac{\dot{\epsilon}}{H(1 - \epsilon)} + 2\epsilon\right)\right)^{-1} - 1\right] f = 0. \]  \tag{16}
Figure 1: Evolution of $f(T)$ versus $T$ with $H_0 = 67.3$ and $\Omega_{M0} = 0.315$.

It is evident that all the parameters in above equation are not defined in terms of $T$ which makes it difficult to find the analytic solution for $f(T)$. We are interested to determine $f(T)$ function coming from the QCD GDE model. In fact one can reconstruct the actions in modified theories for known cosmic history in terms of Hubble parameter. Here, we consider the Hubble parameter presented in (Nojiri and Odintsov 2008)

$$H(t) = m(t_p - t)^{-\eta},$$

where $m$ and $\eta$ are positive constants and $t < t_p$, $t_p$ is the probable time when finite-time future singularity may appear. In (Nojiri and Odintsov 2008), Nojiri et al. presented the classification of finite future singularities. The $H(t)$ defined in (17) specifies these singularities as (Nojiri and Odintsov 2005, 2008; Bamba et al. 2010, 2012): type I ("Big rip singularity") correspond to $\eta \geq 1$, type II to the $-1 < \eta < 0$, type III to the $0 < \eta < 1$ and type IV singularity can appear for $\eta < -1$, but $\eta$ is not an integer. Nojiri and Oditsov (2008) discussed the future evolution of quintessence/phantom-dominated epoch in modified $f(R)$ gravity which unifies the early-time inflation with late-time acceleration inconsistent with observational tests. They discussed the models where these singularities may occur. The occurrence of finite time future singularities has also been studied in $f(T)$, modified Gauss-Bonnet and $f(R,G)$ gravities (Bamba et al. 2010, 2012). We are interested to discuss the specific case with $\eta = 1$ and $a(t) = a_0(t_p - t)^{-m}$, $a_0 > 0$. This model represents the phantom phase of cosmos which may result in type I
singularity. For this choice of scale factor, the solution of Eq. (16) is given by

\[ f(T) = c_1 T^{\alpha_1} + c_2 T^{\alpha_2}, \]

where \( \alpha_1 = \frac{1}{2} (1 - \sqrt{1 - b}) \), \( \alpha_2 = \frac{1}{2} (1 + \sqrt{1 - b}) \) with \( b = 1/(3m) \) and \( c_1 \), \( c_2 \) are constants to be determined. Now evaluating the Friedmann equation (10) at \( t = t_0 \) implies

\[ [1 + 2 f(T_0)] \Omega_{M0} + \frac{f(T_0)}{3H_0^2} = 1. \]

After some manipulations it follows, it follows that

\[ f(T_0) = \frac{3H_0^2 \Omega_{\phi_0}}{b + 1}. \]

Hence using Eq. (20) and the constraint (8), we find the constants \( c_1 \) and \( c_2 \) as

\[
\begin{align*}
    c_1 &= \left( \frac{3H_0^2 \Omega_{\phi_0}}{b + 1} \right) \left( \frac{\alpha_2(1 - 2\alpha_2)}{\alpha_1(2\alpha_1 - 1) - \alpha_2(2\alpha_2 - 1)} \right) T_0^{-\alpha_1}, \\
    c_2 &= \left( \frac{3H_0^2 \Omega_{\phi_0}}{b + 1} \right) \left( \frac{\alpha_1(2\alpha_1 - 1)}{\alpha_1(2\alpha_1 - 1) - \alpha_2(2\alpha_2 - 1)} \right) T_0^{-\alpha_2}.
\end{align*}
\]

We show the plot of \( f(T) \) against \( T \) for different values of \( m \) in Figure 1. It can be seen that \( f(T) \) increases depending on the values of \( T \), which is in

Figure 2: Evolution of \( f(T) \) versus \( z \) with \( H_0 = 67.3 \) and \( \Omega_{M0} = 0.315 \).
Figure 3: Evolution of EoS parameter for ghost $f(R, T)$ model with $H_0 = 67.3$ and $\Omega_{M0} = 0.315$.

according with the representation of the function in Eq.(18). We set the present day values of Hubble parameter and fractional energy densities from the recent Planck observations as $H_0 = 67.3$ and $\Omega_{M0} = 0.315$ (Ade et al. 2013). The evolution of $f$ is also presented in terms of redshift $z$ as shown in Figure 2. The behavior of equation of state parameter $\omega_T$ in $f(R, T)$ gravity is shown in Figure 3. $\omega_T$ helps to identify three significant eras of cosmic expansion such as quintessence with $\omega_T > -1$, in this model universe escapes from entering the de Sitter and big rip phases, phantom $\omega_T < -1$, violating the null energy condition may result in big rip phase and $\omega_T = -1$ results in de Sitter phase. In our case it results in $\omega_T > -1$ representing the quintessence era of the universe as shown in Figure 3.

3.1.1 Stability of Ghost $R + f(T)$ model

Here, we propose to explore the stability of ghost $f(R, T)$ model against linear homogeneous perturbations. We assume a general solution $H(t) = H_h(t)$ for the dynamical equations in FRW background of $f(R, T)$ gravity. In (Alvarenga et al. 2013b; Sharif and Zubair 2013d, 2014a), we have presented the perturbed equations in the cosmological background of FRW universe for both general as well as specific cases and discuss the stability of ΛCDM, dS and power law solutions. We have also found the certain constraints which have to be satisfied to ensure that power law solutions may be stable and match the bounds prescribed by the energy conditions (Alvarenga et al.
The matter energy density in terms of $H_h$ satisfies the relation
\[ \rho_h(t) = \rho_0 e^{-3 \int H_h(t) dt}, \tag{22} \]
where $\rho_0$ is an integration constant. We propose to analyze the model around the arbitrary solution $H_h(t)$, the Hubble parameter and energy density can be perturbed as
\[ H(t) = H_h(t)(1 + \delta(t)), \quad \rho(t) = \rho_h(1 + \delta_m(t)). \tag{23} \]
The function $f(T)$ can be expanded in powers of $T_h(= \rho_h)$ as
\[ f(T) = f^h + f^h_T(T - T_h) + \mathcal{O}^2, \tag{24} \]
where $\mathcal{O}^2$ term includes all the terms proportional to the squares or higher powers of $T$ and the symbol $h$ means the functions and their derivatives are evaluated corresponding to the solution $H(t) = H_h(t)$.

Using Eqs.(23) and (24) in FRW equation, we obtain
\[ (T_h + 3T_h f^h_T + 2T_h^2 f^h_{TT}) \delta_m(t) = 6H_h^2 \delta(t). \tag{25} \]
For the conserved energy momentum tensor the second perturbation equation is
\[ \dot{\delta}_m(t) + 3H_h(t) \delta(t) = 0. \tag{26} \]
Combining Eqs.(25) and (26), we get the first order equation for $\delta_m$ as
\[ \dot{\delta}_m(t) + \frac{1}{2H_h} (T_h + 3T_h f^h_T + 2T_h^2 f^h_{TT}) \delta_m(t) = 0. \tag{27} \]
The evolution of $\delta_m$ and $\delta$ is determined by the relations
\[ \delta_m(t) = \gamma \exp \left\{ -\frac{1}{2} \int \gamma_T dt \right\}, \quad \delta(t) = \frac{\gamma \gamma_T}{6H_h} \exp \left\{ -\frac{1}{2} \int \gamma_T dt \right\}, \]
\[ \gamma_T = \frac{T_h}{H_h}(1 + 3f^h_T + 2T_h f^h_{TT}). \tag{28} \]
Now, we examine the stability of ghost $f(R, T)$ model (18). The corresponding relations of $\gamma_T$ and $-\frac{1}{2} \int \gamma_T dt$ are given as

$$\gamma_T = \frac{1}{m} \{ T_0 (t_p - t)^{3m+1} + \alpha (2\alpha + 1) c_1 T_0^\alpha (t_p - t)^{3\alpha m+1} + \beta (2\beta c_1 T_0^\beta (t_p - t)^{3\beta m+1} \}, \tag{29}$$

$$-\frac{1}{2} \int \gamma_T dt = \frac{1}{2m} \{ \frac{T_0}{3m+2} (t_p - t)^{3m+2} + \frac{\alpha (2\alpha + 1) c_1 T_0^\alpha (t_p - t)^{3\alpha m+2} \}} + \frac{\beta (2\beta + 1) c_1 T_0^\beta (t_p - t)^{3\beta m+2} \}. \tag{30}$$

It can be seen that the above expressions do not decay in future evolution which results in increase of instability of our model against the homogeneous perturbations. Thus the $f(R, T)$ ghost model is not stable in this scenario which is in accordance with the results in (Cai et al. 2011). We also test the evolution of squared speed of sound for stability analysis of ghost $f(R, T)$ model. We plot $\nu_s^2$ against $T$ as shown in Figure 4. It can be seen that squared speed of sound is negative showing the instability in the model.

- **Modified GDE Model**

It can be seen that vacuum energy from the Veneziano ghost field in QCD turns out to be $H + \mathcal{O}(H^2)$ (Zhitnitsky 2012). However, in previous works of QCD GDE model people have only considered the case of $\rho_{GDE} \propto H$. 

![Figure 4: Evolution of $\nu_s^2$ versus $T$ with $H_0 = 67.3$, $\Omega_{M0} = 0.315$ and $m = 2$.](image-url)
Recently, Cai et al. (2012) introduced a modified form of QCD ghost model by involving the term $H^2$ so that energy density of QCD ghost is given by

$$\rho_{GDE} = \alpha H + \beta H^2,$$

(31)

where $\alpha$ is the same constant as that defined in the ordinary GDE and $\beta$ is another constant with dimension [energy]$^2$. They fitted this model with observational data including SNeIa, BAO, BBN, the Hubble parameter data and found that the modified GDE model, without having the two fundamental cosmological puzzles, like the $\Lambda$CDM fit the astronomical data very well. Sadeghi et al. (2013) discussed the flat universe model with varying $G$ and $\Lambda$ in the presence of modified GDE model (31). In (Karami et al. 2013b), modified GDE scalar field models has been discussed in FRW universe with both interaction and viscosity. The EoS parameter for model (31) is given by

$$1 + \omega_{\text{GDE}} = \frac{-m(\alpha + 2\beta H)}{3(\alpha + \beta H)}.$$  

(32)

Comparing Eqs. (12) and (32), we find the $f(T)$ function of the form

$$f(T) = C_1 \exp\{(-(2m - 3)\beta + (m - 3)\gamma)\ln(-\alpha + \sqrt{\alpha^2 + 4T\gamma}) + (m - 3)$$

$$\times (2\beta + \gamma)\ln(\alpha + \sqrt{\alpha^2 + 4T\gamma}) + \ln(\alpha(\beta + \gamma) + \sqrt{\alpha^2 + 4T\gamma})$$

$$\times 3(\beta + \gamma)\}/\{(2m(2\beta + \gamma))\},$$

(33)

where $\gamma = 3 - \beta$ and $C_1$ is a constant to be determined. To find the constant $C_1$, we set the initial condition by evaluating the Friedmann equation (10) at $t = t_0$ which implies

$$f(T_0) = H_0^2\{2m\alpha\gamma + 2m\beta(\alpha + \sqrt{\alpha^2 + 4\gamma T_0})\}/\{2\alpha\gamma + 2(\alpha + \sqrt{\alpha^2 + 4\gamma T_0})\},$$

(34)

Hence using (34), constant $C_1$ is found of the form

$$C_1 = H_0^2\{2m\alpha\gamma + 2m\beta(\alpha + \sqrt{\alpha^2 + 4\gamma T_0})\}/\{2\alpha\gamma + 2(\alpha + \sqrt{\alpha^2 + 4\gamma T_0})\}$$

$$\times \exp\{(-((2m - 3)\beta + (m - 3)\gamma)\ln(-\alpha + \sqrt{\alpha^2 + 4T\gamma}) + (m - 3)$$

$$\times (2\beta + \gamma)\ln(\alpha + \sqrt{\alpha^2 + 4T\gamma}) + \ln(\alpha(\beta + \gamma) + \sqrt{\alpha^2 + 4T\gamma})$$

$$\times 3(\beta + \gamma)\}/\{(2m(2\beta + \gamma))\}.$$  

In Figure 5(a), we plot the function $f(T)$ versus $T$ for different values of $m$. Figure 5(b) shows the evolution of EoS parameter $\omega_{f(T)}$, and it can be seen...
that $\omega_{f(T)}>0$. In this study we set the model parameters according to Cai et al. 2012 results as $\alpha = 2$, $\beta = 0.3$ and $\gamma = 3.342$. Hence this model is not viable according to recent observations as it represents the matter dominated epoch. However, for particular choice of $m$ it favors the quintessence era as shown in Figure 6.

### 3.2 $f(R, T) = f_1(R) + f_2(T)$

In this section, we consider the Lagrangian as sum of two independent functions of $R$ and $T$. Consequently, the corresponding field equations can be arranged of the form

$$\dot{R}^2 f_{1RRR} + (\ddot{R} - H\dot{R}) f_{1RR} + [\kappa^2 T + (1 + \omega_{de})\rho_{de}] f_{1R} + (\kappa^2 + f_2 T) T = 0, \quad (35)$$

which is the third order differential equation in $f_1$ and $f_2$ involving contribution both from scalar curvature and matter density. To make the Lagrangian $f(R, T) = f_1(R) + f_2(T)$ consistent with the standard continuity equation, we need to set an additional constraint so that the right side of the equation \ref{eq:continuity}. In such scenario, we have (Alvarenga et al. 2013a; Sharif and Zubair 2014b)

$$(1 + \omega)T f_{2TT} + \frac{1}{2}(1 - \omega)f_{2T} = 0,$$

which results in functional form of $f_2(T)$ as

$$f_2(T) = \gamma_1 T^{\frac{1+\omega}{2(1+\omega)}} + \gamma_2, \quad (36)$$
where $\gamma_i$’s are integration constants.

- Garcia-Salcedo GDE Model

Initially, we consider the GDE model proposed by Garcia-Salcedo et al. (2013). Using the GDE model (14), matter energy density $\rho_m$ and $\rho_{GDE}(1 + \omega_{GDE})$ are represented in terms of Ricci scalar $R$ as

$$\rho_{GDE}(1 + \omega_{GDE}) = -\frac{\alpha \sqrt{2m+1} \sqrt{-R}}{(6m)^{3/2}},$$

$$\rho_m = -\frac{m R}{2(2m+1)} \left(1 - \frac{\alpha(2m + 1) \sqrt{6m(2m + 1)}}{6m^2 \sqrt{-R}}\right)$$  \hspace{1cm} (37)

Substituting the above results (36) and (37) in Eq. (35), we obtain a 3rd order nonlinear differential equation in terms of $f_1$ as

$$R^3 f_{RRR} + \frac{3 - m}{2} R^2 f_{RR} + \left(\frac{3m^2 \sqrt{-R}}{4} - \frac{\alpha(3m - 1)(2m + 1)^{3/2}}{4 \sqrt{6m}}\right) f_R$$

$$+ \frac{1}{8} (6m^2 R - \alpha(2m + 1)^{3/2} \sqrt{-6m R}) - \frac{3\gamma_1 \sqrt{-(2m + 1) R}}{4 \sqrt{2}}$$

$$\times \sqrt{1 - \frac{\alpha(2m + 1) \sqrt{6m(2m + 1)}}{6m^2 \sqrt{-R}}} = 0.$$  \hspace{1cm} (38)
The analytic solution of above equation is not on cards, therefore we solve this equation numerically by setting the following initial conditions (Capozziello et al. 2005; Houndjo 2012; Sharif and Zubair 2014b)

\[ f_R \big|_{t=t_0} = 1, \quad f_{RR} \big|_{t=t_0} = 0, \]

\[ f(R_0) = R_0 + \delta, \quad \delta = 6H_0^2(1 - \Omega_{m0} - \frac{\sqrt{3\Omega_{m0}\gamma_1}}{2H_0}) - \frac{\gamma_2}{3H_0^2}. \]

In numerical solutions, we set the present day values of parameters as \( H_0 = 67.3, \Omega_{M0} = 0.315 \) and \( \Omega_{DE} = 0.685 \) (Ade et al. 2013). The variation of \( f \) with \( R \) is shown in Figure 7. In this plot, we set \( m = 2 \) while other values of \( m \) do not make major changes on behavior of curves as presented in Figure 7. The \((f - R)\) plot drawn in Figure 7 provides sufficient data set which can be used to find closed analytic mathematical expression for \( f \) in terms of \( R \). These expressions can be determined by using a polynomial function for \( f \) as \( f = \Sigma_0^n a_i R^i \). The approximate analytic function corresponding to Figure 7 may be expressed as

\[
f(R) = -4.112 \times 10^7 - 127.768 R - 0.502 R^2 - 0.001 R^3 - 1.759 \times 10^{-6} R^4 \\
- 1.432 \times 10^{-9} R^5 - 6.192 \times 10^{-13} R^6 - 1.102 \times 10^{-16} R^7. \quad (39)
\]

In further study, we use Eq.\( (39) \) to show the evolution of EoS parameter and null energy condition for \( f(R, T) = f_1(R) + f_2(T) \) ghost DE model. Figure
8(a) shows the variation of NEC versus power law parameter \(m\) and time \(t\). It can be seen that NEC is violated \(i.e., \rho_T + p_T < 0\) which necessitates \(\omega_T < -1\) (the phantom regime of the cosmos). To see this behavior of \(f(R, T)\) model (39), we plot the evolution of \(\omega_T < -1\) versus \(t\) and \(m\) as shown in Figure 8(b).

The energy density \(\rho_\theta\) and pressure \(p_\theta\) for the \(f(R, T)\) model, are defined as

\[
\rho_\theta = \frac{1}{f_{1R}} \left[(1 - f_{1R})\rho + \alpha_1 \sqrt{T} + \alpha_2 + \frac{1}{2}(f_1 - Rf_{1R}) - 3H\dot{R}f_{1RR} \right],
\]

\[
p_\theta = \frac{1}{f_{1R}} \left[\frac{1}{2}(Rf_{1R} - f_1) - \frac{\alpha_1}{2} \sqrt{T} + \alpha_2 + (\ddot{R} + 2H\dot{R})f_{1RR} + \dot{R}^2 f_{1RRR} \right].
\]

Using the above relations of \(\rho_\theta\) and \(p_\theta\), we define the squared sound speed of GDE \(v_s^2 = p_\theta/\rho_\theta\). In plot 9, we show the evolution of \(v_s^2\) for the \(f(R, T)\) model (39). It can be seen that \(v_s^2\) is less than zero.

• Modified GDE Model

Here, we reconstruct the \(f(R, T) = f_1(R) + f_2(T)\) function corresponding to modified GDE model (Cai et al. 2012). Using the model (31), we set
expressions for $\rho_m$ and $\rho_{GDE}(1 + \omega_{GDE})$ as given below

$$\rho_m = \frac{-mR}{2(2m+1)} \left( 1 - \frac{1}{3} \left( \beta + \frac{\alpha \sqrt{6(2m+1)}}{\sqrt{-mR}} \right) \right), \quad (40)$$

$$\rho_\theta(1 + \omega_\theta) = \frac{-1}{18(2m+1)} \left( \alpha \sqrt{-6m(2m+1)R - 2\beta R} \right). \quad (41)$$

Using Eqs. (40) and (41), we finally find the following results

$$R^3 f_{RRR} + \frac{3 - m}{2} R^2 f_{RR} - \left( \frac{3m^2 R}{2} \left( 1 - \frac{1}{3} \left( \beta + \frac{\alpha \sqrt{6(2m+1)}}{\sqrt{-mR}} \right) \right) \right)$$

$$+ \frac{m}{12} \left( \alpha \sqrt{-6m(2m+1)R - 2\beta R} \right) f_R + \frac{3m^2 R}{4} \left( 1 - \frac{1}{3} \left( \frac{\alpha \sqrt{6(2m+1)}}{\sqrt{-mR}} \right) \right)$$

$$+ \beta) - \frac{3m(2m+1)\alpha_1}{4} \sqrt{\frac{-mR}{2(2m+1) \left( 1 - \frac{1}{3} \left( \beta + \frac{\alpha \sqrt{6(2m+1)}}{\sqrt{-mR}} \right) \right)}} = 0, \quad (42)$$

which is a 3rd order nonlinear differential equation. Again we solve this equation numerically using the initial conditions as in previous case. We show the variation of $f$ with respect to $R$ in Figure 10 with the choice of parameters $H_0 = 67.3, \Omega_M = 0.315, \alpha = -10^{-6}, \gamma_1 = \gamma_2 = 1$ and $m = 2$. 

Figure 9: Evolution of $v_s^2$ versus $t$ with $H_0 = 67.3, \Omega_M = 0.315, \alpha = -10^{-6}, \gamma_1 = \gamma_2 = 1$ and $m = 2$. 

Using Eqs. (40) and (41), we finally find the following results
\[ f(R) = 8.139 \times 10^{32} + 4.079 \times 10^{33}R + 4.912 \times 10^{31}R^2 + 2.207 \times 10^{29}R^3 \\
+ 4.836 \times 10^{26}R^4 + 5.716 \times 10^{23}R^5 + 8.577 \times 10^{20}R^6 + 5.999 \times 10^{18}R^7 \tag{43} \]

Using Eq. (43), we discuss the variation of NEC and also EoS parameter for the obtained \( f(R, T) \) model. In Figure 11, we show evolution of NEC and EoS parameter versus \( m \) and \( t \). It can be seen that NEC is violated for \( m \geq 60 \) as shown in Figure 11(a) and EoS parameter favors the phantom regime for this choice of \( m \). The evolution of squared speed of sound is presented in Figure 12.

4 Conclusions

Modified theories of gravity have appeared as convenient candidates to address issues of accelerated cosmic expansion and predict the destiny of the universe. Harko et al. (2011) generalized \( f(R) \) gravity by introducing an arbitrary function of the Ricci scalar \( R \) and the trace of the energy-momentum tensor \( T \). The dependence of \( T \) may be introduced by exotic imperfect fluids or quantum effects (conformal anomaly). \( f(R, T) \) is a general modified gravity formulated on the basis of curvature matter coupling and provides
Figure 11: (a) Evolution of NEC and (b) EoS parameter $\omega_{f(R,T)}$ for modified ghost $f(R,T)$ model (43) with $H_0 = 67.3$, $\Omega_{M0} = 0.315$, $\alpha = 2$, $\beta = 0.3$ and $\gamma_1 = \gamma_2 = 1$.

Figure 12: Evolution of $v_s^2$ for modified ghost $f(R,T)$ model (43) versus $t$ with $H_0 = 67.3$, $\Omega_{M0} = 0.315$, $\alpha = 2$, $\beta = 0.3$, $\gamma_1 = \gamma_2 = 1$ and $m = 2$. 
an alternative way to explain the current cosmic acceleration with no need of introducing either the existence of extra spatial dimension or an exotic component of dark energy.

$f(R, T)$ gravity has gained significant attention to handle the issue of accelerated cosmic expansion and various aspects have been explored. In this respect the search of most appropriate form of Lagrangian is still under consideration. $f(R, T)$ theory of gravity has been reconstructed under various scenarios including de Sitter, power law solutions, phantom, non-phantom eras, anisotropic universe model and class of HDE models (Houndjo 2012; Sharif and Zubair 2013c, 2014a). In this work, we reconstruct $f(R, T)$ gravity corresponding to Garcia-Salcedo and modified QCD ghost DE models. We consider the $f(R, T)$ models of the form $f(R, T) = R + 2f(T)$ and $f(R, T) = f_1(R) + f_2(T)$. A particular model of scale factor is considered representing the phantom phase of the cosmos which may result in type I singularity (Nojiri and Odintsov 2008). The major concern of theories involving non-minimal matter geometry coupling is the divergence of energy momentum tensor is not covariantly conserved. We have obtained the explicit form of functions $f(T)$ and $f(R)$ using the constraint of standard continuity equation. In the following we summarize our findings

- $f(R, T) = R + 2f(T)$

In the first place we consider a $f(R, T)$ model representing a correction to Einstein gravity in the form of time dependent cosmological constant. We have selected two generalized form of GDE models, one suggested by Garcia-Salcedo et al. (2013) and other model is suggested by Cai et al. (2012) named as MGDE. The $f(T)$ model corresponding to Garcia-Salcedo GDE is introduced in Eq.[18]. The evolution of $f(T)$ and EoS parameter is shown in Figures 1-3. Figure 3 shows that this model represents the quintessence era of the universe which is consistent with the WMAP9 observations $-1.71 < \omega < -0.34$ (Hinshaw et al. 2013). We examine the stability of ghost $f(R, T)$ model against linear homogeneous perturbations. We find that ghost $f(R, T)$ model is not stable in this scenario. We also test the squared speed of sound $\nu_s^2$ and plot its evolution in Figure 4. It shows the instability of the ghost $f(R, T)$ model. In Section 3.1, the reconstruction of $f(T)$ function is also developed corresponding to MGDE model. The evolution of $f(T)$ versus $T$ and $f(R, T)$ EoS parameter for model [33] is shown in Figure 5. We find that this model is not viable according to recent observations.
\[ f(R, T) = f_1(R) + f_2(T) \]

In this scheme \( f_2(T) \) is found using the constraint for conservation equation in \( f(R, T) \) gravity. Substituting this \( f_2(T) \) function and parameters for the particular GDE models in dynamical equations of \( f(R, T) \) gravity results in 3rd order nonlinear equations in terms of \( f_1 \). We solve such type of equations using numerical technique and find the approximate analytic function corresponding to plots of resulting functions. In case of Garcia-Salcedo \( f(R, T) \) model, we show the evolution of NEC and EoS parameter in Figure 8. We find that \( \omega \) favors the phantom regime of the universe and lies in the range \(-1.08 < \omega < -1.02\) which is consistent with ranges set by Planck and WMAP9 data (Ade et al. 2013; Hinshaw et al. 2013) in the following form

\[ \omega = -1.13^{+0.13}_{-0.14}, \quad \text{(Planck+WP+SNLS)} \]
\[ \omega = -1.084 \pm 0.0063, \quad \text{(WMAP+eCMB+BAO+H_0+SNe)} \]

In case of MGDE DE candidate, we set the model parameters following (Cai et al. 2012) and show the variation of NEC and EoS parameter in Figure 11. It is found that \( \omega \) lies in the range \(-6 \times 10^{-35} < 1 + \omega < -1\) representing the phantom era of the universe consistent with the recent observations (Ade et al. 2013; Hinshaw et al. 2013).

5 Conflict of Interest

The authors declare that they have no conflict of interest.

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