Preheating in the nonminimal derivative coupling to gravity

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Abstract

We revisit the reheating mechanism after the end of inflation in the non-minimal derivative coupling (NDC) to gravity with quadratic potential. This is because the inflaton of the NDC should describe the slow-roll inflation as well as the preheating stage after the end of inflation. We point out that the non-periodic inflaton solution implies the absence of parametric resonance, compared to the periodic oscillating inflaton for the canonical coupling (CC) to gravity. Furthermore, it is demonstrated that narrow and broad parametric resonances do not appear after the end of inflation in the NDC model by solving the differential equation numerically for the quantum field, which differs from the case of the the CC model obtained by solving Mathieu equation.

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1 Introduction

A period of accelerated expansion during the very early stage of the universe called inflation is able to account for several otherwise difficult to explain features of the observed universe. A simplest inflation model is based on a single slowly-rolling scalar field canonically coupled (CC) to gravity [1]. In the standard picture of the early universe, the universe passes through the period of reheating after the end of inflation.

The nonminimal derivative coupling (NDC) [2, 3] was firstly notified by coupling the inflaton kinetic term to the Einstein tensor such that the friction is gravitationally enhanced [4]. Later, this coupling has been considered as an alternative mechanism to increase friction of an inflaton rolling down its own potential. Actually, it makes a non-flat potential adequate for inflation without introducing ghost state [5, 6]. This implies that during inflation, the NDC increases friction, and flattens the potential effectively.

It is meaningful to note that there was a difference between CC and NDC even for taking the same quadratic potential [7]. The difference appears clearly in the reheating process after the end of inflation. Reheating is being considered as an important part of inflationary universe because it describes the production of Standard Model particles after the inflation [8]. At this stage, the classical periodic oscillating inflaton $\phi$ in the CC decays into massive bosons is due to parametric resonance [9, 10, 11]. In order to explain this phenomenon by introducing an interacting Lagrangian of $L_{\text{int}} = -\frac{1}{2} g^2 \phi^2 \chi^2$, the equation for quantum field $\chi$ can be reduced to the Mathieu equation [9], which is the well-known differential equation with periodic mass term when neglecting the expansion of the universe. This equation describes a harmonic oscillator with variable frequency (parametric oscillator). In particular, if the coupling $g$ is large enough, the periodic modulation of the field mass leads to strong instability via parametric resonance.

On the other hand, the inflaton in the NDC oscillates with time-dependent frequency which is surely a non-periodic function. The average solution $\phi$ in Eq. (26) mimics the non-periodic nature of the inflation observed in Ref. [7] numerically. Therefore, the equation of quantum field $\chi$ does not take a form of the Mathieu equation, and its solution could not be obtained analytically. However, the author in Ref. [12] has recently claimed that the parametric resonance instability is absent, by solving this equation with quadratic potential approximately.

In this work, we wish to revisit this important issue because the inflaton of the NDC
should describe the slow-roll inflation as well as the preheating stage after the end of inflation.

We will argue that there is no (narrow, broad) parametric resonance after the end of inflation in the NDC model because the field mass term is not a periodic function. We will also numerically confirm it by solving the NDC-equation for $\chi_k$ and by comparing those obtained from the Mathieu equation.

2 NDC with quadratic potential

Let us consider an inflation model including the NDC of a single scalar field $\phi$ with the quadratic potential [7, 13, 14]

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[ M_P^2 R + \frac{1}{M^2} G_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - 2V(\phi) \right], \quad V = \frac{m^2 \phi^2}{2},$$

(1)

where $M_P$ is a reduced Planck mass, $\tilde{M}$ is a mass parameter and $G_{\mu\nu}$ is the Einstein tensor. We find the CC model when replacing the second term by $-g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi$. Here we do not consider a conventional combination of CC+NDC $[-(g_{\mu\nu} - G_{\mu\nu}/\tilde{M}^2) \partial^\mu \phi \partial^\nu \phi]$ in Ref. [15] because this combination won’t make the reheating analysis transparent. From the action [11], one can easily derive the Einstein and scalar (inflaton) equations

$$G_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu},$$

(2)

$$\frac{1}{M^2} G^{\mu\nu} \nabla_\mu \nabla_\nu \phi + m^2 \phi = 0,$$

(3)

where $T_{\mu\nu}$ takes a complicated form including fourth-order terms

$$T_{\mu\nu} = \frac{1}{M^2} \left[ \frac{1}{2} R \nabla_\mu \phi \nabla_\nu \phi - 2 \nabla_\rho \phi \nabla_\mu \nabla_\nu \phi R^\rho_\nu + \frac{1}{2} G_{\mu\nu} (\nabla \phi)^2 - R_{\mu\rho\nu\sigma} \nabla^\rho \phi \nabla^\sigma \phi \right.$$

$$- \nabla_\mu \nabla^\rho \phi \nabla_\nu \nabla_\rho \phi + (\nabla_\mu \nabla_\nu \phi) \nabla^2 \phi$$

$$\left. - g_{\mu\nu} \left( \frac{\tilde{M}^2 m^2}{2} \phi^2 - R^{\sigma\rho} \nabla_\rho \phi \nabla_\sigma \phi + \frac{1}{2} (\nabla^2 \phi)^2 - \frac{1}{2} (\nabla^\rho \nabla^\sigma \phi) \nabla_\rho \nabla_\sigma \phi \right) \right].$$

(4)

Considering a flat FRW spacetime by introducing cosmic time $t$ as

$$ds_{\text{FRW}}^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j,$$

(5)
the first Friedmann and scalar equations derived from Eqs. (2) and (3) are given by

\[ H^2 = \frac{1}{3M_P^2} \rho_\phi = \frac{1}{3M_P^2} \left[ \frac{9H^2}{2M^2} \dot{\phi}^2 + \frac{1}{2} m^2 \phi^2 \right], \]  
(6)

\[ \frac{3H^2}{M^2} \ddot{\phi} + 3H \left( \frac{3H^2}{M^2} + \frac{2H}{M^2} \right) \dot{\phi} + m^2 \phi = 0, \]  
(7)

respectively \[7\]. Here \( H = \dot{a}/a \) is the Hubble parameter and the overdot (\( \dot{} \)) denotes the differentiation with respect to time \( t \). It is evident from Eq. (6) that the energy density \( \rho_\phi \) for the NDC is positive (ghost-free).

First of all, we would like to mention that Qiu and Feng \[16\] have recently suggested \( a(t) \) and \( H(t) \) during the rapid oscillation of the universe after inflation. According to their approach, we wish to look for a possible solution \( \phi(t) \) with a quadratic potential after the end of inflation by considering

\[ a(t) \sim t^p, \quad H(t) = \frac{p}{t}, \quad \phi(t) = \Phi(t) \cos[\sqrt{\frac{6}{5}} m \tilde{M} t^2], \quad \Phi(t) = \Phi_0 t^{-\frac{1}{2} - \frac{5}{3} p}. \]  
(8)

Taking into account Eq. (8), the scalar equation (7) can be solved exactly for

\[ q = \pm \frac{m \tilde{M}}{\sqrt{6p}}, \quad p = 0, \quad \frac{1}{3}, \quad \frac{5}{3}. \]  
(9)

Here we do not consider the case of \( p = 0 \) because it is a trivial solution \( \phi = 0 \) to Eq. (7). Then, substituting Eq. (8) with the case of \( q = \frac{m \tilde{M}}{\sqrt{6p}} \) and \( p = \frac{1}{3} \) into the right-hand side of Eq. (7) leads to

\[ \rho_\phi^{p=5/3} = \frac{m^2}{4t^4} \left( 7 - 5 \cos[\sqrt{\frac{6}{5}} m \tilde{M} t^2] \right) + \frac{5\sqrt{6} m \sin[\sqrt{\frac{6}{5}} m \tilde{M} t^2]}{M t^6} + \frac{25(\cos[\sqrt{\frac{6}{5}} m \tilde{M} t^2] + 1)}{M^2 t^8}, \]  
(10)

which is not proportional to

\[ H^2 = \frac{25}{9} \frac{1}{t^2}. \]  
(11)

Next, plugging Eq. (8) with the case of \( q = \frac{m \tilde{M}}{\sqrt{6p}} \) and \( p = \frac{1}{3} \) into the right-hand side of Eq. (6) leads to

\[ \rho_\phi^{p=1/3} = \frac{m^2}{4} \left( 7 - 5 \cos[\sqrt{6} m \tilde{M} t^2] \right), \]  
(12)

which is also not obviously proportional to \( H^2 \) in Eq. (11). Note that although Eq. (8) is not the solution to Eq. (9), but it is surely the solution to Eq. (7).
On the other hand, following the averaging method \cite{12} the other possible solution can be obtained by rewriting Eqs. (6) and (7) in terms of $\tau$ defined as $t = \int (3H/m\tilde{M})d\tau$:

\[ H^2 = \frac{m^2}{6M_p}(\phi'^2 + \phi^2), \]  
\[ \phi'' + \left(\frac{H'}{H} + \frac{9H^2}{mM}\right)\phi' + 3\phi = 0, \]  

where the prime(\:') denotes the differentiation with respect to $\tau$. Then, the parametric solution to Eq. (13) could be assumed to be

\[ \phi = \frac{\sqrt{6}HM_p}{m} \sin[\tau + f(\tau)], \]  
\[ \phi' = \frac{\sqrt{6}HM_p}{m} \cos[\tau + f(\tau)], \]

where $f(\tau)$ is an arbitrary function. Noting that the differentiation of Eq. (15) should be Eq. (16), one finds that

\[ Hf'\cos[\tau + f(\tau)] + H'\sin[\tau + f(\tau)] = 0. \]  

Then, the differentiation of $\phi'$ is given by

\[ \phi'' = -(1 + f')\phi + (H'/H)\phi'. \]  

Plugging Eqs. (15), (16) and (16) into Eq. (14) leads to

\[ \left(2H' + \frac{9H^3}{mM}\right)\cos[\tau + f(\tau)] + (2H - Hf')\sin[\tau + f(\tau)] = 0. \]  

Then, solving Eqs. (17) and (19) leads to

\[ f' = \frac{\sin[\tau + f(\tau)]}{1 + \cos^2[\tau + f(\tau)]}\left\{2\sin[\tau + f(\tau)] + \frac{9H^2}{mM}\cos[\tau + f(\tau)]\right\}, \]  
\[ H' = -\frac{H\cos[\tau + f(\tau)]}{1 + \cos^2[\tau + f(\tau)]}\left\{2\sin[\tau + f(\tau)] + \frac{9H^2}{mM}\cos[\tau + f(\tau)]\right\}. \]

However, it is almost impossible to solve Eqs. (20) and (21) because $f(\tau)$ is an argument of cos- and sin-functions. At this stage, we note that an approximate solution might be obtained when introducing the averaging method \cite{12}. In this case, one finds the averaged differential equations

\[ f'_a = 2(\sqrt{2} - 1), \quad H'_a = -\frac{9H^3_a}{mM}\left(1 - \frac{1}{\sqrt{2}}\right). \]
Here the subscript \((a)\) implies an average over \(\tau(f'_a = \int_0^\pi f'(\tau)d\tau/\pi)\) for the fast varying quantities \(f'\) and \(H\) by assuming that the slow varying quantities \(H\) and \(f\) are fixed. As a result, we easily obtain the desired solutions

\[
f_a(\tau) = 2(\sqrt{2} - 1)\tau, \quad H_a(\tau) = \frac{1}{\sqrt{\frac{18}{mM}(1 - \frac{1}{\sqrt{2}})}},
\]

(23)

Moreover, we may derive the relation between \(t\) and \(\tau\) as

\[
t = \frac{2\sqrt{\tau}}{\sqrt{2m\tilde{M}(1 - \frac{1}{\sqrt{2}})}}
\]

(24)

which explicitly shows the behavior of \(\tau \propto t^2\). This is origin of why \(\phi(t)\) has the non-periodic solution after the end of inflation [7].

Finally, we obtain the average solutions for \(H\) and \(\phi\) as

\[
a_a \propto t^{\frac{2}{3(2 - \sqrt{2})}}, \quad H_a(t) = \frac{2}{3(2 - \sqrt{2})}t,
\]

(25)

\[
\phi_a(t) = \Phi_a(t) \sin \left[\tilde{m}^2 t^2\right] = \frac{\sqrt{6}M_P H_a(t)}{m} \sin \left[\tilde{m}^2 t^2\right],
\]

(26)

\[
\dot{\phi}_a(t) = \frac{\sqrt{6}M_P \tilde{M}}{3} \cos \left[\tilde{m}^2 t^2\right]
\]

(27)

with

\[
\tilde{m}^2 = \frac{m\tilde{M}}{2}(2 - \sqrt{2})(\sqrt{2} - \frac{1}{2})
\]

(28)

Importantly, we observe that \(\phi_a(t)\) oscillates with time-dependent frequency. As a result, it is not a periodic function of \(t\). It is worth to note that Eqs. (26) and (27) satisfy the first Friedmann equation (6) exactly, while they satisfy the scalar equation (7) approximately for large \(t\). Hence, we may regard Eqs. (25)-(27) as the best analytic solution which describes the reheating process after inflation in the NDC. We note that the approximate solution found in Ref. [12] takes the forms (25)-(27) obtained when replacing \(\sin[\tilde{m}^2 t^2]\) in Eq. (26) and \(\cos[\tilde{m}^2 t^2]\) in Eq. (27) by \(\cos[\tilde{m}^2 t^2]\) and \(-\sin[\tilde{m}^2 t^2]\), respectively.

In conclusion, the non-periodic nature of the inflation is regarded as a clear feature of reheating process in the NDC, when one compares it with the periodic inflation of \(\phi(t) \propto \sin(mt)/t\) in the CC [9][10][11]. The solution (26) mimics the non-periodic nature of the inflation observed in [7] numerically. Furthermore, Eq. (27) indicates that the velocity of inflaton \(\dot{\phi}\) oscillates without damping for the NDC [14], while it oscillates with damping for the CC \(\dot{\phi}(t) \propto \cos(mt)/t\) in [12]. We are ready for studying the parametric resonance because we are aware of an analytic form for the inflaton (26) in the reheating.
3 Parametric resonance

It was reported that the parametric resonance is absent for NDC when considering the decay of the scalar field $\phi$ into a quantum field $\chi$ [12], whereas the parametric resonance is present for CC. However, this statement is not clear. In this section, we wish to revisit this issue.

Now, let us consider the relevant quantum field Lagrangian is given by

$$L_\chi = -\frac{1}{2} \partial_{\mu} \chi \partial^{\mu} \chi - \frac{1}{2} m^2 \chi^2 - \frac{1}{2} g^2 \phi^2 \chi^2.$$  \hspace{1cm} (29)

The time evolution of the quantum fluctuation $\chi$ is governed by the classical equation of motion in the flat FRW universe (5) as

$$\ddot{\chi} + 3H \dot{\chi} - \frac{1}{a^2} \nabla^2 \chi + g^2 \phi^2 \chi = 0.$$ \hspace{1cm} (30)

With $\vec{x}$ and $\vec{k}$ representing the comoving position and momentum vectors, $\chi$ can be expressed as

$$\chi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[ a_k \chi_k(t) e^{-i\vec{k} \cdot \vec{x}} + a_k^\dagger \chi_k^*(t) e^{i\vec{k} \cdot \vec{x}} \right],$$ \hspace{1cm} (31)

where $a_k$ and $a_k^\dagger$ are annihilation and creation operators, respectively. We assume $m^2 = 0$ for simplicity. Plugging (31) into (30) leads to the equation for temporal part of the Fourier mode $\chi_k$ with $k = |\vec{k}|$

$$\ddot{\chi}_k + 3H \dot{\chi}_k + \left( \frac{k^2}{a^2} + g^2 \phi^2(t) \right) \chi_k = 0.$$ \hspace{1cm} (32)

In order to promote a further computation, let us ignore the expansion of the universe and assume a slow variation of $\Phi_a(t)$ compared to oscillation frequencies of the fields $\phi$ and $\chi$:

$$H_a \approx 0; \quad a_a \approx 1; \quad \Phi_a(t) \approx \text{const}. \hspace{1cm} (33)$$

If the coupling $g$ is large enough, one can ignore the friction term $3H \chi_k$. Then, Eq. (32) takes the form

$$\ddot{\chi}_k + \left( k^2 + g^2 \phi^2(t) \sin^2(\tilde{m}t^2) \right) \chi_k = 0.$$ \hspace{1cm} (34)

Let us define new variables like as

$$\xi = \sqrt{2} \tilde{m} t, \quad \omega_k^2 = \frac{k^2}{2\tilde{m}^2} + 2\tilde{\epsilon}, \quad \tilde{\epsilon} = \frac{g^2 \Phi_a^2}{8\tilde{m}^2}. \hspace{1cm} (35)$$
Then, Eq. (32) leads to the differential equation with the non-periodic mass term as
\[
\frac{d^2 \chi_k}{d\xi^2} + \left[ \omega_k^2 - 2\tilde{\epsilon} \cos(\xi^2) \right] \chi_k = 0,
\] (36)
which is called the NDC-differential equation.

On the other hand, the CC case with quadratic potential arrives at the Mathieu equation with \( z = mt \) [9, 11]
\[
\frac{d^2 \tilde{\chi}_k}{dz^2} + \left[ A_k - 2\tilde{q} \cos(2z) \right] \tilde{\chi}_k = 0
\] (37)
with
\[
A_k = \frac{k^2}{m^2} + 2\tilde{q}, \quad \tilde{q} = \frac{g^2 \Phi^2}{4m^2}.
\] (38)
Here \( \tilde{\chi}_k \) of the CC case is a quantum field corresponding to \( \chi_k \) of the NDC case. If the coupling \( g \) is large enough, periodic modulation of the field mass leads to strong instability via parametric resonance. According to Floquet’s theorem, a general solution to the Mathieu equation (37) takes the form
\[
\tilde{\chi}_k(z) = e^{i\mu z} P(z),
\] (39)
where \( P(z) \) is a periodic function with period \( \pi \). The Floquet exponent \( \mu(A_k, \tilde{q}) \) depends on parameters \( A_k \) and \( \tilde{q} \). In the case of positive \( \text{Re}[\mu] \), one has an exponential instability of the solution. The growth of the mode \( \tilde{\chi}_k \) corresponds to particle production, as in the case of particle production in the external gravitational field. The case of \( \tilde{q} < 1 \) leads to the narrow parametric resonance, while the case of \( \tilde{q} > 1 \) provides the broad parametric resonance.

At this stage, we should admit that one could not solve the non-periodic differential equation (36) with \( \epsilon = 2\tilde{\epsilon} \) directly. To find an approximate solution, one may expand \( \chi_k(\xi) \) as
\[
\chi_k = \chi_k^0 + \epsilon \chi_k^1 + \cdots
\] (40)
The author in Ref. [12] has argued that the expression (40) is valid if \( \chi_k^1 \) has no terms which grow without bound as \( \xi \to \infty \). Plugging (40) into (37) and keeping all terms up to second-order in \( \epsilon \), one finds two equations
\[
\frac{d^2 \chi_k^0}{dz^2} + \omega_k^0 \chi_k^0 = 0,
\] (41)
\[
\frac{d^2 \chi_k^1}{dz^2} + \omega_k^1 \chi_k^1 = \chi_k^0 \cos(\xi^2).
\] (42)
The homogeneous equation (41) of zeroth order yields a general solution

\[ \chi_k^0 = b_1 \sin(\omega_k \xi) + b_2 \cos(\omega_k \xi). \] (43)

Introducing new variables \( u/v \equiv \sqrt{2/\pi}(\xi \pm \omega_k) \), the inhomogeneous equation (42) of first order can be solved to give

\[
\chi_k^1 = c_1 \sin(\omega_k \xi) + c_2 \cos(\omega_k \xi) \\
+ \frac{1}{4\omega_k} \sqrt{\frac{\pi}{2}} \left\{ \cos(\omega_k \xi + \omega_k^2)[b_1 C(u) - b_2 S(u)] + \sin(\omega_k \xi + \omega_k^2)[b_1 S(u) + b_2 C(u)] \right\} \\
+ \cos(\omega_k \xi - \omega_k^2)[b_1 C(v) + b_2 S(v)] + \sin(\omega_k \xi - \omega_k^2)[-b_1 S(v) + b_2 C(v)] \\
+ \frac{2}{\pi} \left\{ \cos(\omega_k \xi) + \frac{1}{\sqrt{\pi} \omega_k^2} \left[ \cos(\omega_k \xi + \omega_k^2)[b_1 C(u) - b_2 S(u)] + \sin(\omega_k \xi + \omega_k^2)[b_1 S(u) + b_2 C(u)] \right] \right\}, \tag{44}
\]

where Fresnel-cosine integral \( C(u) \) and Fresnel-sine integral \( S(u) \) are defined by

\[
C(u) = \int_0^u \cos \left( \frac{\pi x^2}{2} \right) dx, \quad S(u) = \int_0^u \sin \left( \frac{\pi x^2}{2} \right) dx, \tag{45}
\]

respectively. These are surely non-periodic finite functions because \( \lim_{u \to \infty} C(u) = 1/2 \) and \( \lim_{u \to \infty} S(u) = 1/2 \). One may attempt to conclude that the parametric resonance is absent in Eq. (36) because \( \chi_k^1 \) (44) does not have any terms which grow without bound as \( \xi(u, v) \to \infty \). However, in deriving this approximate solution, Ghalee [12] has neglected \( \epsilon \) itself in Eq. (33) which plays an important role in determining its solution. Hence, we insist that the solution (44) is not the correct one.

In order to support it, one may rewrite the Mathieu equation (37) in terms of \( t \) as

\[
\frac{d^2 \tilde{\chi}_k}{dt^2} + [\omega^2 - \epsilon \cos(2mt)]\tilde{\chi}_k = 0, \quad \omega^2 = k^2 + \epsilon, \quad \epsilon = \frac{g^2 \Phi^2}{2}. \tag{46}
\]

Introducing

\[ \tilde{\chi}_k = \tilde{\chi}_k^0 + \epsilon \tilde{\chi}_k^1 + \cdots, \tag{47} \]

its equations are given by

\[
\frac{d^2 \tilde{\chi}_k^0}{dt^2} + \omega^2 \tilde{\chi}_k^0 = 0, \tag{48}
\]

\[
\frac{d^2 \tilde{\chi}_k^1}{dt^2} + \omega^2 \tilde{\chi}_k^1 = \tilde{\chi}_k^0 \cos(2mt). \tag{49}
\]
Figure 1: Top-left: oscillating mode $\chi_k$ for $\omega_k^2 = 1$ and $\tilde{\epsilon} = 0.5$ in the NDC, and top-right: growth of $\tilde{\chi}_k$ for $A_k = 1$ and $\tilde{q} = 0.5$ in the CC. Bottom-left: parametric plot for $(\chi_k, \chi'_k)$ in the NDC where the prime denotes the derivative with respect to $\xi$, and bottom-right: parametric plot for $(\tilde{\chi}_k, \tilde{\chi}'_k)$ in the CC where the prime denotes the derivative with respect to $z$.

An approximate solution is given by

$$\tilde{\chi}_0^0(t) = b \sin(\omega t) + c \cos(\omega t),$$

$$\tilde{\chi}_1^1(t) = \tilde{c}_1 \cos(\omega t) + \tilde{c}_2 \sin(\omega t) + \frac{1}{4m(m^2 - \omega^2)} \left\{ \cos(\omega t)[b \omega \sin(2mt) + cm \sin^2(mt)] \right.$$  
$$+ \left. \sin(\omega t)[b \sin^2(mt) - cm \sin(2mt)] - m \cos^2(mt)[b \sin(\omega t) + c \cos(\omega t)] \right\}. \quad (51)$$

We stress to note that $\tilde{\chi}_1^1(t)$ is an oscillating function for any $t$, and it does not blow up unless $m = \omega$. This contradicts to the solution (39) to the Mathieu equation. Hence this approach to obtaining approximate solutions (50) and (51) could not be trusted.

4 Numerical analysis: no parametric resonance

In this section, let us numerically solve the NDC-equation (36), and also solve Mathieu equation (37) of the CC case in order to compare to the NDC one. We observe inequalities of $\omega_k^2 \geq 2\tilde{\epsilon}$ and $A_k \geq 2\tilde{q}$. For this purpose, we choose three proper cases:
As was shown in Fig. 1[(i) case], we find that the top-left corresponds to the homogeneous oscillating mode for the NDC, whereas the top-right is a growth mode which represents the narrow parametric resonance for the CC ($\tilde{q} < 1$). The bottom-left and -right confirm the oscillating and growing modes, respectively. The case (ii)[Fig. 2] does not show the
Figure 4: Sketch of the stability-instability chart of the Mathieu equation. Gray bands indicate regions of stability, while white bands denote region of instability. The line $A_0 = 2\tilde{q}$ shows the values of $A_k$ and $\tilde{q}$ for $k = 0$. We choose $A_k = 1, 20$ for comparison test.

difference between NDC and CC significantly because two belong to oscillating modes. We observe from Fig. 3 that the case (iii) indicates clearly that the top-left denotes an oscillating mode for the NDC, while the top-right is a rapidly growing mode which represents broad parametric resonance for the CC ($\tilde{q} > 1$). Moreover, in the case of CC, the resonance is much more efficient if $\tilde{q} \gg 1$. These observations for the CC could be confirmed from Fig. 4, which shows the stability-instability chart of the Mathieu equation. In addition, we have solved the NDC-equation (36) for different $\omega_k^2$ and $\tilde{\epsilon}$ numerically, and compared those obtained from different sets of $A_k$ and $\tilde{q}$. We have also arrived at the same result.

5 Discussions

If the NDC is a promising coupling for obtaining a successful inflation rather than the CC, the inflaton of the NDC should describe the slow-roll inflation as well as the preheating stage after the end of inflation. It turned out that this coupling has been considered as an alternative mechanism to increase friction of an inflaton rolling down its own potential [5, 6].
However, after the end of inflation, the inflaton in the NDC oscillates with time-dependent frequency which is surely a non-periodic function. The solution of the form \(26\) mimics the non-periodic nature of the inflation observed in [7] numerically, while Eq. \(27\) dictates that the velocity of inflaton \(\dot{\phi}\) oscillates without damping for the NDC [14].

In order to see whether the parametric resonance occurs or not in the NDC, we have introduced the Lagrangian \(29\) for the quantum field \(\chi\). The differential equation \(36\) of quantum mode \(\chi_k\) did not take a form of the Mathieu equation \(37\) with periodic mass term, and thus its solution could not be obtained analytically. First, we have argued that there is no (narrow, broad) parametric resonance after the end of inflation in the NDC model because the field mass term is not a periodic function. We have also numerically confirmed it by solving the NDC-equation \(36\) for \(\chi_k\) and by comparing those obtained from the Mathieu equation \(36\).

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