Factorization and Resummation in Soft-Collinear Effective Theory

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Abstract

We review soft-collinear effective theory (SCET), and apply it to discuss quark electromagnetic form factor, then present the resumed transverse momentum distribution of Higgs-boson production via gluon fusion under this framework, where we derive a relatively full differential formula in transverse momentum $Q_T$ space like one which have been obtained by Dokshitzer-D’Yanov-Troyan (DDT) in perturbative Quantum Chromodynamics (pQCD). Furthermore, our above result can be generalized to even higher order. Comparing our formula with the integral formula of Collins-Soper-Sterman (CSS) in impact parameter $b$ space, we establish the relationship between the anomalous dimension of operator together with matching coefficients in SCET and the well-known coefficients A, B and C in pQCD, which also provides a relative natural and convenient method to treat the similar questions as ones of CSS, such as the matching in nonperturbative region. Finally, the joint resummation method in SCET is briefly discussed.

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I. INTRODUCTION

Quantum Chromodynamics (QCD) is a highly nonlinear gauge field theory. Comparing with Quantum Electrodynamics (QED), QCD has very complex mass singularity due to the self-interactions of gluons and the failure of perturbation at low energy region. However, QCD is a field theory with asymptotic freedom, which means perturbative approach is trustful within high energy region. Thus factorization, which separates long distance or nonperturbative and short distance or perturbative contributions of a process, i.e. subtracting infrared (IR) divergences from observable, particularly collinear divergences, is necessary to apply QCD to high energy hard processes. Moreover, when in a process there are two separating scales, for example, the distribution in the edge of phase space, double logarithmic (DL) terms would appear that may spoil perturbative expansion. Thus, we need resummation technique to control such behavior.

There exist two methods to satisfy above requirements. The first one is perturbative QCD (pQCD) approach based on the analysis of Feynman diagrams: Collins-Soper-Sterman (CSS) formulism [10, 11]; the other one is effective field theory (EFT) approach based on Lagrangian and operators [2, 3]. The spirit of factorization in pQCD approach can be briefly summarized as follows: first find leading order (LO) IR divergence of Feynman diagrams through Landau equation; then separate these IR divergences into soft and collinear parts by utilizing gauge invariance and unitarian relations, thus while soft divergences are cancelled, the left collinear divergences only remain in initial states; at last absorb them into jet functions. As for resummation, because conventional renormalization group equation (RGE) can not resum the terms such as DL between two scales, pQCD utilizes a new type of equations which is derived by the differentiation of jet-function from the factorized cross section with respect to the axial parameter in axial gauge, in order to disentangle soft and collinear contributions which are origins of DL terms, and so DL terms can be resummed by solving this equation.

However, the proof of factorization in pQCD is tedious, and resummation in pQCD is inconvenient as well. Recently, soft-collinear effective theory (SCET) has made great simplifications on the proof of factorization in B meson decays [1, 2] and high energy hard scattering processes [3, 5], including resummation of large logs in certain regions of phase space, for example, $e^+e^-$ annihilation into two jets of thrust $T \rightarrow 1$ [5, 6], the deep inelastic
scattering (DIS) in the threshold region $x \to 1$ and Drell-Yan (DY) process in the case of $z \to 1$. The reason is that SCET can be viewed as an operator realization of the pQCD analysis when the modes participating the interactions of interest are soft and collinear, just like chiral dynamics vs QCD at low energy region. EFT provides a simple and systematic method for factorization of hard, collinear and usoft or soft degrees of freedom at operator level, especially usoft modes can be decoupled from collinear modes by making a field redefinition, and large logarithms such as $\log(Q^2/\Lambda^2)$, where $Q, \Lambda$ are two typical scales that characterize the processes, can be resummed naturally through RGE running.

In the following, after a brief introduction to SCET, we will discuss quark form factor and resummed $Q_T$ distributions of Higgs-boson production via gluon fusion in small $Q_T$ region within the framework of SCET, which was studied using CSS formulism in pQCD before.

II. REVIEW OF SCET

SCET is appropriate for the kinematic regions of collinear and usoft(soft) modes with momenta scaling: $p_c = (p^+, p^-, p_\perp) = (n \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$ and $p_{us} \sim Q(\lambda^2, \lambda^2, \lambda^2)$ or $p_s \sim Q(\lambda, \lambda, \lambda)$, where the light-like vectors $n, \bar{n}$ satisfy $n \cdot \bar{n} = 2$ and the perpendicular components of any four vector $V$ are defined by $V^\mu_\perp = V^\mu - (n \cdot V)\bar{n}^\mu/2 - (\bar{n} \cdot V)n^\mu/2$ and in the rest of the report the common scale $Q$ is often omitted. As any other EFT, SCET should reproduce the infrared behavior of the full theory, which is ensured by using the method of regions for Feynman integrals, i.e., by expanding the integrand in different momentum regions which contribute to the integrals. This fact provides us a direct matching calculation to determine the Wilson coefficients and anomalous dimensions of the operators in SCET. Because the matching procedure is independent of regularization, we can calculate on-shell scaleless matrix elements and use dimensional regularization to regulate UV and IR divergences, and in this case the loop integrals in SCET are zero, and results in $\text{IR}_{QCD} = \text{IR}_{\text{SCET}} = -\text{UV}_{\text{SCET}}$.

In constructing SCET, one should first identify all possible modes in initial and final states, then with them construct all possible propagator and vertex. In actual application, there always involves two kinds of SCET:

- **SCET$_{II}$**: soft and collinear modes for exclusive or semi-inclusive processes, such as
$B \rightarrow \pi \nu e, \pi \pi, D \pi$ with momentum scaling as

$$B, D \sim \mathcal{O}(\lambda, \lambda, \lambda), \quad \pi \sim \mathcal{O}(\lambda^2, 1, \lambda),$$

where $\lambda = \frac{\Lambda}{Q}$.

- **SCET$_I$:** usoft and collinear modes for inclusive processes, say $B \rightarrow X^* \gamma$ at the end point region and $e^- p \rightarrow e^- X$ at the threshold region with momentum scaling as

$$B \sim \mathcal{O}(\lambda^2, \lambda^2, \lambda^2), \quad X^* \sim \mathcal{O}(\lambda^2, 1, \lambda),$$

where $\lambda = \sqrt{\frac{\Lambda}{Q}}$. We notice that SCET$_{II}$ can be also viewed as an EFT of SCET$_I$, i.e., SCET$_I \rightarrow$ SCET$_{II}$. Hence we first discuss SCET$_I$, at LO of $\lambda$, the collinear parts of the lagrangian in SCET$_I$ are

$$L_c = L_{cq} + L_{cg},$$

where

$$L_{cg} = \frac{1}{2g^2} tr\{[iD^\mu + gA^\mu_{n,q}, iD^\nu + gA^\nu_{n,q}]\}^2$$

$$+ 2 tr\{\bar{c}_{n,p} [iD_\mu, [iD^\mu + A^\mu_{n,q}, c_{n,p}]]\} + \frac{1}{\alpha} tr\{[iD_\mu, A^\mu_{n,q}]\}^2 \sim \mathcal{O}(\lambda^4),$$

here the last is the gauge fixing term with parameter $\alpha$ and $c_{n,p}$ denotes collinear ghost field, and

$$L_{cq} = \bar{\xi}_{n,p'} [in \cdot D + iD^c - \frac{1}{i\bar{n} \cdot Dc} \frac{\bar{n} \cdot D_{\perp}}{2} \xi_{n,p} \sim \mathcal{O}(\lambda^4),$$

with

$$in \cdot D = in \cdot D_{us} + in \cdot A_n, \quad iD_{us} = i\partial + gA_{us},$$

$$i\bar{n} \cdot D^c = \bar{\mathcal{P}} + g\bar{n} \cdot A_n, \quad i\bar{D}^c_{\perp} = \mathcal{P}_{\perp} + gA^\perp_{n}.$$  

The power counting in SCET is straightforward once the scaling of fields is obtained from the scaling of their corresponding propagators. The collinear fermion field is a two component spinor in the $n$ direction, after its small components integrated out by solving the equation of motion, and can be expanded as

$$\xi_n(x) = \sum_{\bar{p}} e^{-i\bar{p} \cdot x} \xi_{n,p}(x),$$  

where $p = \bar{p} + \mathcal{O}(\lambda^2)$ and $n \cdot \bar{p} = 0$, $\bar{p}$ is called label momentum. The order $\lambda^2$ components are resided in the space-time dependence of the fields $\xi_{n,p}(x)$. The label operators $\mathcal{P}$ and $\mathcal{P}_{\perp}$ pick out $\mathcal{O}(1)$ and $\mathcal{O}(\lambda)$ label momentum of collinear fields respectively, which is introduced
to facilitate power counting in SCET and conservation of momenta [3]. The label operator \( \mathcal{P}^\mu = \mathcal{P}_{\perp}^\mu + \mathcal{P}_{\parallel}^\mu \) is defined by

\[
\mathcal{P}^\mu(\phi_1^\dagger \cdots \phi_p \cdots) = (\tilde{p}^\mu + \cdots - \tilde{q}^\mu - \cdots) \times (\phi_1^\dagger \cdots \phi_p \cdots)
\]

for collinear fields and

\[
iD^\mu = \mathcal{P}_{\parallel}^\mu + (in \cdot \partial + gn \cdot A_{us}) \mathcal{P}_{\parallel}^\mu/2.
\]

The lagrangian of usoft modes is the same as that of QCD.

There are three types of symmetries in SCET. The first one is of spin structures, as the collinear quark fields are two-component spinors, the independent Dirac matrixes are \( \{ \bar{n}, \bar{n}\gamma_5, \bar{n}\gamma_\perp \} \) or \( \{ 1, \gamma_5, \gamma_\perp \} \). The second is re-parameters (RP) transformations invariance under the infinitesimal transformations:

- \( n \to n + \triangle_\perp, \quad \bar{n} \to \bar{n} \);
- \( n \to n, \quad \bar{n} \to \bar{n} + \epsilon_\perp \);
- \( n \to e^{\alpha} n, \quad \bar{n} \to e^{-\alpha} \bar{n} \).

The requirement of \( \triangle_\perp \sim \lambda, \epsilon_\perp \sim \lambda^0, \alpha \sim \lambda^0 \) is to keep scaling behavior of the elements in \( SCET_1 \) under these transformations. The third symmetry is invariance of gauge transformations:

- **Collinear transformation** \( U_c, \partial^\mu U_c \sim O(\lambda^2, 1, \lambda) \) and \( \partial^\mu U_{c,q} \sim O(\lambda^2, \lambda^2, \lambda^2) \)

\[
\xi_n \to U_c \xi_n, \quad W_n \to U_c W_n, \quad A_n \to U_c A_n U_c^\dagger + \frac{i}{g} U_c [iD, U_c^\dagger],
\]

while usoft operators do not transform.

- **Usoft transformation** \( U_{us}, \partial^\mu U_{us} \sim O(\lambda^2, \lambda^2, \lambda^2) \)

\[
\xi_n \to U_{us} \xi_n, \quad W_n \to U_{us} W_n U_{us}^\dagger, \quad A_n \to U_{us} A_n U_{us}^\dagger,
\]

while usoft operators transform as in QCD. Where

\[
W_n(x) = \sum_{\text{perms}} \exp(-g \bar{n} \cdot A_{n,q})
\]

(2)

denotes a Wilson line of collinear gluons along the path in the \( \bar{n} \) direction, which is required to ensure gauge invariance of current operator in SCET; \( Y_n(x) = P \exp(ig \int ds \bar{n} \cdot A_{us}(ns+x)) \)
is usoft Wilson line of usoft gluons in $n$ direction from $s = 0$ to $s = \infty$ for final state particles
and $P$ means path-ordered product, while for initial state particles, $Y_n$ is from $s = -\infty$ to $s = 0$.

The above properties result in no extra-renormalization theorem of $\mathcal{L}$ and connecting
Wilson coefficients of different operators [12]. Until now the lagrangian of SCET is com-
pletely constructed, next we also need to determine various effective operators in SCET
through matching to count into contributions from high energy region.

For example, we match heavy $\rightarrow$ light current $b \rightarrow s\gamma$ at end point region [1], in following
we take the convention $\phi_{n,p}(x) = \phi_n(x)$ for collinear fields and use $\overline{\text{MS}}$ scheme, i.e., $\mu^2 \rightarrow \mu^2 e^{\gamma_E}/4\pi$,

$$J^{QCD} = \bar{s}\Gamma_b \rightarrow J^{SCET} = [\bar{\xi}_n W_n] \Gamma h_v$$

at the scale $\mu = p^- \sim m_b$,

$$J^{QCD} = (\bar{\xi}_n W_n)_{p^-} \Gamma h_v C(p^-,\mu) = [\bar{\xi}_n W_n] C(\bar{P}^\dagger,\mu) \Gamma h_v,$$

where $C(m_b) = 1 + \mathcal{O}(\alpha_s)$, and

$$Z = 1 + \frac{\alpha_s C_F}{4\pi} \left[ \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln\left(\frac{\mu^2}{m_b^2}\right) + \frac{5}{2\epsilon} \right].$$

Here $C_F = \frac{N_c^2 - 1}{2N_c}$ for $SU(N_c)$ QCD with $N_c = 3$.

After above matching procedure, we should running the operator in $SCET_I$ from scale
$\mathcal{O}(Q)$ to $\mathcal{O}(\sqrt{Q\Lambda})$ by RGE, so as to control the large logarithmic behavior. RGE at next-
to-leading-logarithmic-order (NLLO) thus is given by

$$\mu \frac{d}{d\mu} C(\mu) = -\frac{\alpha_s(\mu) C_F}{2\pi} \left[ 2\ln\left(\frac{\mu}{m_b}\right) + \frac{5}{2} \right] C(\mu).$$

As to the meaning of this RGE, we can set $\alpha_s = \alpha, C_F = 1 \sim QED$ at LLO

$$C(\mu) = \exp\left[-\frac{\alpha}{2\pi} \ln^2\left(\frac{\mu}{\bar{p}}\right)\right].$$

So it is indicated that Sudakov’s DL can be resummed through conventional RGE, which is
different from RGE in full theory. Thus EFT provides a natural method for resummation.

Next, we turn to the factorization in terms of SCET. The factorization in SCET is
represented as decoupling of collinear and usoft modes in $\mathcal{L}$ and relevant operators, which
can be accomplished by the following decoupling transformations for initial state particles:

$$\xi_{n,p} = Y \xi_{n,p}^{(0)}, \quad A_{n,p} = YA_{n,p}^{(0)} Y^\dagger, \quad W_n = Y W_n^{(0)} Y^\dagger.$$
Therefore, \( \mathcal{L} \) in terms of \( \xi_{n,p}^{(0)} \) and \( A_{n,p}^{(0)} \) can be written as

\[
\mathcal{L} = \xi_{n,p}' \frac{\hat{f}}{2} [in \cdot \mathcal{D} + gn \cdot A_{n,q} + \cdots ]\xi_{n,p}^{(0)}
\]

or \( \mathcal{L}(\xi_{n,p}, A_{n,q}, n \cdot A_{us}) = \mathcal{L}(\xi_{n,p}^{(0)}, A_{n,q}^{(0)}, 0) \), which is already a decoupled form. And whether an operator has the decoupled form depends on the case.

If we want to go to \( SCET_{II} \), the following steps should be performed:

(i) Matching QCD onto \( SCET_I \) at a scale \( \mu^2 \sim Q^2 \) with \( p_c^2 \sim Q^2 \lambda^2 \).

(ii) Decoupling the usoft -collinear interactions with the field redefinitions, \( \xi_n = Y_n^\dagger \xi_n^{(0)} \) and \( A_n = Y_n^\dagger A_{n}^{(0)} Y_n \) for final state particles, while for initial states particles the daggers are reversed.

(iii) Matching \( SCET_I \) onto \( SCET_{II} \) at a scale \( \mu^2 \sim Q^2 \eta^2 \) with \( p_c^2 \sim Q^2 \eta^2 \), where \( \eta \sim \lambda^2 \).

Considering heavy to light current, for example, it leads to

\[
J_{QCD}^{SCET} = \bar{\psi} \Gamma h \rightarrow J_{SCET_I}^{SCET} = \bar{\xi}_n W_n \Gamma h_v;
\]

\[
J_{SCET_I}^{SCET} = \bar{\xi}_n W_n \Gamma h_v = \xi_n^{(0)} W_n^{(0)} \Gamma Y^\dagger h_v;
\]

and

\[
J_{SCET_I}^{SCET} = \xi_n^{(0)} W_n^{(0)} \Gamma Y^\dagger h_v \rightarrow J_{SCET_{II}}^{SCET} = \bar{\xi}_n W_n \Gamma S^\dagger h_v.
\]

We thus can obtain general factorized convolutional structure of any quantity \( \sigma \) from matching:

\[
\sigma \sim H \otimes J \otimes S.
\]

The function \( H \) encodes the short distance physics, the jet function \( J \) describes the propagation of energetic particles in collimated jets, and the soft function \( S \) contains relative long-distance physics.

III. QUARK ELECTROMAGNETIC FORM FACTOR IN QCD [3, 9]

As a demonstration of the statements in last section, we consider Sudakov effect of quark electromagnetic form factor in QCD [14, 15], i.e., double logarithmic asymptotic of current \( j^\mu = \bar{\psi} \gamma^\mu \psi \) in the following kinematics:
(a) on-shell case

\[ Q^2 = -(p - k)^2 \gg \Lambda^2 \gg m^2, \quad p^2 = k^2 = m^2. \]

(b) off-shell case

\[ Q^2 = -(p - k)^2 \gg -p^2 = -k^2 = M^2 \gg m^2. \]

where \( p, k \) are momenta of an initial and a final quark with mass \( m \) near the light cone in Breit frame, and \( \Lambda^2 \) is a parameter in the IR cut-off. In the above two cases we set \( \lambda^2 \sim \Lambda/Q \) and \( \lambda \sim M/Q \), respectively.

First, we consider the on-shell case (a), according to the step (i), the current in SCET at leading order of \( \lambda \) is

\[ j^\mu = [\bar{\xi}_n W_n][\gamma^\mu C(P^\dagger, \bar{P}, \mu^2)[W_n^\dagger \xi_n]], \tag{3} \]

where the reparameterization invariance (RPI) implies Wilson coefficient

\[ C(P^\dagger, \bar{P}, \mu^2) = C(P^\dagger \cdot \bar{P}, \mu^2). \]

Because EFT Lagrangian only takes coupling vertexes with the fields involved interacting in a local way, there is no direct coupling of collinear particles moving in the two separate directions defined by \( n \) and \( \bar{n} \). However usoft modes can still mediate between them. The calculations at one-loop level give

\[ C(Q^2, Q^2) = 1 + \frac{\alpha_s C_F}{4\pi}(-8 + \frac{\pi^2}{6}) \tag{4} \]

and the UV renormalization factor for the current in SCET is

\[ Z_V = 1 + \frac{\alpha_s C_F}{4\pi}[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{2}{\epsilon} \log(\frac{Q^2}{\mu^2})]. \tag{5} \]

Thus the RGE of Wilson coefficient \( C \) is

\[ \frac{d \log C(Q^2, \mu^2)}{d \log(\mu)} = \gamma_1(\mu), \tag{6} \]

\[ \gamma_1(\mu) \equiv A_q(\alpha_s) \log(\frac{Q^2}{\mu^2}) + B_q(\alpha_s) \]

\[ = -\frac{\alpha_s C_F}{4\pi}[4 \log(\frac{\mu^2}{Q^2}) + 6], \tag{7} \]
In this paper we use the notions $A$, $B$ and $C$ in order to distinguish the well known coefficients $A$, $B$ and $C$ in pQCD, but later we will find they are connected with each other. So we have $A^{(1)} = C$ and $B^{(1)} = -3C/2$ if $A \equiv \sum_n (\alpha_s/\pi)^n A^{(n)}$, etc. With (6), we can resumme terms such as double logarithmic from scale $\sim O(1)$ down to scale $\sim O(\lambda)$, we abbreviate this matching step as a chain $QCD|_{Q^2} \rightarrow SCET_I|_{Q^2\lambda^2}$.

According to the step (ii), usoft and collinear modes are decoupled after field redefinitions. As a result,

$$\langle k| [\bar{\xi_n} W_n] \gamma^\mu [W_n^\dagger \xi_n]|p \rangle \rightarrow \langle k| [\bar{\xi_n} W_n]|0\rangle \gamma^\mu \langle 0|T[Y_n Y_n]|0\rangle \langle 0|[W_n^\dagger \xi_n]|p \rangle,$$

where $T$ denotes time-ordered product.

According to the step (iii), note that the IR cut-off scale $\Lambda^2 >> p^2 = k^2 = m^2$, the transformed operator can directly match onto the operator in SCET$_{II}$ associated with re-scaling the external modes in SCET$_I$,

$$p_c = Q(0,1,0) \sim Q(\lambda^4,1,\lambda^2) \rightarrow p_c \sim Q(\eta^2,1,\eta),$$

$$p_{us} \sim Q(\lambda^2,\lambda^2,\lambda^2) \rightarrow p_s \sim Q(\eta,\eta,\eta).$$

In this case, there is no intermediated scale which separates SCET$_I$ and SCET$_{II}$, and then the Wilson coefficient for this step is one and the anomalous dimension is the same as SCET$_I$. Although the RGE of Wilson coefficient still is (6), it runs from the scale $\sim O(\lambda)$ to the scale $\sim O(\eta)$. We will abbreviate this step as $SCET_I|_{Q^2\lambda^2} \Rightarrow SCET_{II}|_{Q^2\lambda^4}$. Collecting all the results above, we obtain the well known Sudakov form factor $S(Q,\Lambda)$, leaving other coefficients omitted,

$$S(Q,\Lambda) = \exp(- \int^{Q}_\Lambda \gamma_1(\mu) d \log \mu).$$

(9)

For the off-shell case (b), it can be taken as a sub-diagram of the on-shell case, from kinematical considerations, of which the external legs are amputated. Thus, the step (i) is unchanged, but in the step (ii) $\langle 0|T[Y_n Y_n]|0\rangle$ changes into

$$\int_0^\infty \int_0^\infty ds dt e^{iQ\lambda^2(s+t)} \langle 0|T[Y_n(0,\bar{n}s)Y_n^\dagger(0,-nt)]|0\rangle,$$

(10)

where $1/(Q\lambda^2)$ is the effective contour length as shown in [15] and

$$Y_n(0,\bar{n}s) \equiv P \exp(i \int_0^s d\beta \bar{n} \cdot A_{us}(\bar{n},\beta)).$$
\[ Y_n(0, -nt) \equiv P \exp(-ig \int_0^t d\beta n \cdot A_{us}(-n\beta)). \]

And in the step (iii) the jets with off-shellness \(-p^2 = -k^2 = M^2 >> Q^2\lambda^4\) are integrated out in SCET_{II} and only Eq.\(10\) is left in the step (iii). Actually, the running behaviors of Eq.\(10\) and \(F_{IR}\) in \[14\] are identical. The above steps adopted for (b) can be abbreviated as \(QCD|_{Q^2} \rightarrow SCET_I|_{Q^2,\lambda^2} \rightarrow SCET_{II}|_{Q^2,\lambda^4}\). Finally, Sudakov factor in the off-shell case is

\[ S(Q, Q\lambda^2) = \exp(- \int_{M}^{Q} \gamma_1(\mu)d\log \mu + \int_{Q\lambda^2}^{M} \gamma_2(\mu)d\log \mu), \tag{11} \]

where \(\gamma_2(\mu) = 2\Gamma_0(g) - \log(\mu^2/(Q^2\lambda^4))\Gamma_{cusp}(g) = -\log(\mu^2/(Q^2\lambda^4))\alpha_s C_F/\pi + \mathcal{O}(\alpha_s^2)\) as shown in \[14\]. Thus, SCET automatically derives the results based on the analysis of separating momentum space into collinear, soft and IR regions.

IV. \(Q_T\) RESUMMATION OF HIGGS-BOSON PRODUCTION \[9\]

Now we focus on resummed parts of full transverse momentum distribution of Higgs-boson produced via gluon fusion, while the remaining terms corresponding to \(Y\) \[11\] and the procedures of incorporating non-perturbative region \(Q_T \sim \Lambda_{QCD}\) are omitted.

The dominant process for Higgs-bosons production at the Large Hadron Collider(LHC) in the Standard Model are gluon fusion through a heavy quark loop, mainly the top quark: \(p_1(P_1) + p_2(P_2) \rightarrow gg \rightarrow H(Q) + X\), with \(P_1 = (0, 2p, 0)\) and \(P_2 = (2p, 0, 0)\). It is convenient to start from the effective Lagrangian for Higgs-bosons and gluon couplings \[16\]:

\[ \mathcal{L}_{Hgg} = h(\alpha_s(Q))G^a_{\mu\nu}G^{a\mu\nu}H, \tag{12} \]

where

\[ h(\alpha_s) = \frac{\alpha_s}{12\pi}(\sqrt{2}G_F)^{1/2}[1 + \frac{11}{4\pi} \frac{\alpha_s}{\pi}]. \]

From (12), the operator for one Higgs-boson production is \(\mathcal{H} = G^a_{\mu\nu}G^{a\mu\nu}\).

Taking into account Landau equation and IR power counting \[17\], the singular terms of \(Q_T\) distribution in the limit of \(Q_T \rightarrow 0\) originate from soft and collinear modes, which are emitted by partons from hadrons \(p_1, p_2\). Thus, only real gluon emission and the gluon-quark scattering diagrams contribute to the singular parts of \(Q_T\) distribution. In the case of gluon emission, the emitted gluon can be soft or collinear, while the quark can only be collinear.
SCET is appropriate to this semi-inclusive process with $\lambda^2 \sim Q_T/Q, Q \gg Q_T \gg \Lambda_{QCD}$ and $Q \sim m_H$. So the operator $H$ can matches at the lowest order of $\lambda$ onto

$$H = G_{\mu
u}^n C(P^\dagger \cdot \bar{P}, \mu^2)G_{\mu
u}^n = G_{\mu
u}^n C(Q^2, \mu^2)G_{\mu
u}^n.$$  \hspace{1cm} (13)

where

$$G_{\mu
u}^n = -(i/g)W_n^a[iD_\mu^n + gA_\mu^n, iD_\nu^n + gA_\nu^n]W_n.$$ And $n \leftrightarrow \bar{n}$ for $G_{\mu
u}^\bar{n}$. The calculations at one-loop level give

$$C(Q^2, Q^2) = 1 + \frac{\alpha_s}{4\pi}(A_g^H + \frac{C_A\pi^2}{2}) \hspace{1cm} (14)$$

and

$$Z_H = 1 + \frac{\alpha_s}{4\pi}\left[\frac{2C_A}{\epsilon^2} + \frac{2\beta_0}{\epsilon} - \frac{2C_A}{\epsilon} \log(\frac{Q^2}{\mu^2})\right]. \hspace{1cm} (15)$$

Here, $C_A = N_c, A_g^H = 11 + 2\pi^2, \beta_0 = (11C_A - 2n_f)/6$, and $n_f = 5$ is the number of active quark flavors. Thus, the RGE of $C(Q^2, \mu^2)$ is

$$\frac{d \log C(Q^2, \mu^2)}{d \log(\mu)} = \gamma_1(\mu), \hspace{1cm} (16)$$

where

$$\gamma_1(\mu) \equiv A_g(\alpha_s) \log(\frac{Q^2}{\mu^2}) + B_g(\alpha_s)$$

$$\gamma_1(\mu) = -\frac{\alpha_s}{\pi}[C_A \log(\frac{\mu^2}{Q^2}) + \beta_0] \hspace{1cm} (17)$$

with $A_g^{(1)} = C_A$ and $B_g^{(1)} = -\beta_0$. The evolution from the scale $\sim O(1)$ to the scale $\sim O(\lambda)$ gives

$$C(Q^2, Q^2\lambda^2) = C(Q^2, Q^2) \exp(- \int_{Q^2\lambda^2}^{Q^2} \frac{d\mu^2}{2\mu^2} \gamma_1(\mu)). \hspace{1cm} (18)$$

Similar to the case of on-shell Sudakov form factor in last section, Eq.(16) can be directly used to running $H$ from $Q$ to $Q\lambda^2$ with no loss of degrees of freedom. For example, the scaling of the gluon interacting with the incoming quark is $O(\eta^2, 1, \eta)$, which is constrained by kinematics. After performing field redefinitions, the relevant operator for Higgs-boson production at the scale $Q_T$, where large logarithmic has been resummed by the RGE evolution in SCET, is

$$H = C(Q^2, Q^2\eta^2)2\text{Tr}\{T[Y^\dagger_n Y_n G_{\mu\nu}^n Y_n^\dagger Y_n G_{\mu\nu}^n]\} \hspace{1cm} (19)$$

$$= C(Q^2, Q^2\eta^2)T[Y^{\dagger n} Y_n G_{\mu\nu}^n G_{\mu\nu}^n] \equiv C(Q^2, Q^2\eta^2)\tilde{H}.$$
where $\mathcal{Y}_{n(n)}$ is adjoint Wilson line from $-\infty$ to $0$ in $n(n)$ direction for incoming fields. Until now, we have completed the procedures corresponding to (i), (ii) and (iii) mentioned in previous section. Next, with the composite operator $\mathcal{H}$ at the renormalization scale $\mu \sim Q\lambda^2$, we can relate $\mathcal{H}$ to differential cross section at this scale:

$$\frac{1}{\sigma^{(0)}} \frac{d\sigma^{\text{resum}}}{dQ^2dydq_T^2} = \frac{d}{dQ_T^2} \int_0^{Q_T^2} dq_T^2 e^{-S_g(\mu,Q)} C^2(Q^2, Q^2) \frac{1}{\sigma^{(0)}} \frac{d\sigma^{\text{SCET}}(\mu)}{dQ^2dydq_T^2}, \quad (20)$$

where

$$\sigma^{(0)} = (\sqrt{2}G_F) \frac{\alpha_s^2(Q)m_H^2}{576S} \delta(Q^2 - m_H^2), \quad S = P_1^-P_2^+, \quad (21)$$

$$S_g(\mu, Q) = \int_{\mu^2}^{Q^2} \frac{d\mu^2}{\mu^2} [A_g(\alpha_s) \log(\frac{Q^2}{\mu^2}) + B_g(\alpha_s)] \quad (22)$$

and $d\sigma^{\text{SCET}}(\mu)/(dQ^2dydq_T^2)$ represents differential cross section calculated in $\text{SCET}_{II}$ with the composite operator $\hat{\mathcal{H}}$ at the renormalization scale $\mu \sim Q\lambda^2$.

To proceed further, we use the following two facts: the soft and collinear modes are decoupled in lagrangian at LO of $\lambda$ in $\text{SCET}_{II}$; since the spins of hadron are summed over and a colored octet operator vanishes between the color singlet states, we have

$$\langle p_n | B_{n,\omega_1} B_{n,\omega_2} | p_n \rangle \propto \delta^{ab} g_{\perp}^{\mu\nu} \langle p_n | \Tr[B_{n,\omega_1} B_{n,\omega_2}] | p_n \rangle$$

with $g_{\perp}^{\mu\nu} = g^{\mu\nu} - (n^\mu n^\nu + \bar{n}^\mu \bar{n}^\nu)/2$ and $B_{n,\omega}^{\alpha} = \bar{n}^\nu G_{n,\mu \nu}^\alpha$. Thus, the SCET cross section can be written as a factorized form:

$$\int_0^{Q_T^2} dq_T^2 C^2(Q^2, Q^2) \frac{1}{\sigma^{(0)}} \frac{d\sigma^{\text{SCET}}(\mu)}{dQ^2dydq_T^2} = g_{p_1}(x_1, \mu) g_{p_2}(x_2, \mu), \quad (23)$$

where $x_1 = Qe^y/\sqrt{S}$, $x_2 = Qe^{-y}/\sqrt{S}$ for $Q_T^2 << Q^2$. Because of KLN theorem, the contributions from the soft modes are free of IR divergences. So only the collinear divergences are survived. Through matching the SCET cross section onto a product of two PDFs given by $B$, which are equivalent to the conventional PDFs $f_{a/p_1(2)}(\xi_{1(2)}, \mu)$ at leading order of $\lambda$, the remained IR divergences can be absorbed into these nonperturbative inputs $\lambda$, of which the evolutions are controlled by the DGLAP equations. This step of matching Eq.(23) onto

$$\sum_{a,b} f_{a/p_1(2)}(\xi_1, \mu) f_{b/p_2(2)}(\xi_2, \mu)$$

leads to

$$g_{p_1(2)}(x_{1(2)}, \mu) \equiv \sum_{a=\bar{g},q} (f_{a/p_1(2)} \otimes C_{ga})(x_{1(2)}, \mu)$$

$$= \sum_a \int_{x_{1(2)}}^1 \frac{d\xi}{\xi} f_{a/p_1(2)}(\xi, \mu) C_{ga}(\frac{x_{1(2)}}{\xi}, \mu), \quad (24)$$

Obviously, at the tree-level only $C_{gg}^{(0)}(z) = \delta(1 - z) \neq 0$. To extract $\mathcal{O}(\alpha_s)$ coefficients $C_{ga}^{(1)}$, we can take a short-cut. As demonstrated in [12], EFT is the language of method of regions,
and we can use the results of [13], which are obtained by the method of regions in pQCD framework, to simplify our calculations. The only difference between the pQCD and SCET, if we set the factorization scale \( \mu_F = \mu \) in both theories, is that the choices of renormalization scale in pQCD [13] is \( \mu = Q \), while in SCET is \( \mu \sim Q \eta \). Fortunately, this discrepancy is compensated by the expansion of Sudakov factor \( \exp(-S_g(\mu, Q)) \), and the results for \( \mathcal{O}(\alpha_s) \) coefficients \( C_{ab}^{(1)} \) are the same as the known formula [13] in pQCD approach, i.e.,

\[
C_{ga}^{(1)}(z) = -\frac{1}{2} P_{ga}^\epsilon(z) + \frac{1}{4} \delta_{ga} \delta(1-z) (C_A \frac{\pi^2}{3} + A_g^H),
\]

where the scale \( \mu \) has been set to \( Q_T \) in order to minimize the logarithmic in \( C_{ga}^{(1)}(z) \), and the approximation \( \alpha_s(Q) = \alpha_s(Q_T) \) is taken up to NNLLO in \( C_2^2(Q, Q^2) \). And \( P_{ga}^\epsilon(z) \) represent the \( O(\epsilon) \) terms in the DGLAP splitting kernels, which are given by

\[
P_{gg}^\epsilon(z) = 0.
\]

Combining Eq.(21)-(24), we obtain resummation formula for transverse momentum distributions of Higgs-bosons production in SCET:

\[
\frac{1}{\sigma^{(0)}} \frac{d\sigma^{\text{resum}}}{dQ^2 dy dQ_T^2} = \frac{d}{dQ_T^2} \sum_{ab} e^{-S_g(Q_T, Q)} (f_{a/p_1} \otimes C_{ga})(x_1, Q_T) (f_{b/p_2} \otimes C_{gb})(x_2, Q_T).
\]

This form is different from the known CSS formula in \( \vec{b} \) space [10, 11, 19]:

\[
\frac{1}{\sigma^{(0)}} \frac{d\sigma^{\text{resum}}}{dQ^2 dy dQ_T^2} = \int_0^\infty \frac{db}{2\pi} b J_0(bQ_T) \sum_{ab} e^{-S_g(\vec{b}, Q)} (f_{a/p_1} \otimes C_{ga})(x_1, \frac{c}{\vec{b}}) (f_{b/p_2} \otimes C_{gb})(x_2, \frac{c}{\vec{b}}),
\]

where \( c = 2e^{-\gamma_E} \) and \( \gamma_E \) is Euler’s constant. As we know, the reason of using the \( \vec{b} \) space in pQCD is to preserve conservation of momenta, while in SCET the conservation of momenta is automatically satisfied because the matrix elements is calculated at scale \( Q_T \) with the matched effective operator. Previously, a similar formula has been derived by the authors of [20], which is incomplete, only at NLLO, and corresponds to our result of setting \( C_{ga} = \delta_{ga} \delta(1-z) \) in Eq.(26). Later, the authors in [21] generalized the DDT formula to NNLLO with the CSS formula, in which its coefficients \( \tilde{A}, \tilde{B} \) and \( C \) at \( \mathcal{O}(\alpha_s) \) are just our \( A^{(1)}(1), B^{(1)} \) and \( C^{(1)} \). We also expect that they would agree at higher orders, and thus we propose that at NNLLO \( A^{(2)} = \tilde{A}^{(2)} = A^{(2)} \) and \( B^{(2)} = \tilde{B}^{(2)} = B^{(2)} + 2(A^{(1)})^2 \zeta(3) \), if all the coefficients
are defined by $A = \sum_n (\frac{a_n}{n}) A^{(n)}$ etc. The first relation has been noticed by the author of \cite{7}. The connections between $A$, $B$ and $C$ in SCET and $A$, $B$ and $C$ in pQCD, therefore, are established through this way. The numerical results of the $Q_T$ resummation formula in SCET and that of traditional CSS formalism in pQCD are in agreement at NNLLO, for details we suggest to refer to \cite{21}.

We summarize the steps for $Q_T$ resummation with a chain:

$$QCD|_{Q^2} \rightarrow SCET_I|_{Q^2 \lambda^2} \Rightarrow SCET_{II}|_{Q^2 \eta^2 >> \mu_0^2} \rightarrow DGLAP|_{\mu_0^2},$$

where the last arrow indicates that the evolutions below $SCET_{II}$ are governed by the DGLAP equations.

V. DISCUSSION

- Using the above methods to the production of lepton pair via virtual photon, we can get the similar result as Eq.(26) corresponding to that of \cite{19}.
- From our above analysis, it can be seen that SCET provides a natural framework of $Q_T$ resummation by conventional RGE in EFT.
- Because of the differential form of Eq.(26) in $Q_T$ space, it provides a simple and natural approach to numerical calculation and covering the effects in non-perturbative region with $Q_T \sim \Lambda_{QCD}$ \cite{21}.
- The reformulation of joint resummation can also be made straightforwardly in SCET. In fact, the conclusions of threshold resummation for $d\sigma^{\text{resum}}/dQ^2$ in moments $\bar{N} = e^{\gamma_E} N$ space for Higgs-boson production processes can be shown by the chain \cite{8}: in the case of $z \rightarrow 1$ and $\lambda^2 = 1 - z \sim \frac{1}{\bar{N}}$,

$$QCD|_{Q^2} \rightarrow SCET_I|_{Q^2 \lambda^2} \Rightarrow SCET_{II}|_{Q^2 \eta^2 >> \mu_0^2} \rightarrow DGLAP|_{\mu_0^2},$$

We observe that the two chains of Higgs-boson production processes in SCET have identical structure. This suggests that we can do threshold and $Q_T$ resummation for $d\sigma^{\text{resum}}/dQ^2dQ_T^2$ simultaneously. The relevant scale $\lambda^2 = 1/\chi(\bar{N}, \bar{b} \equiv bQ e^{\gamma_E}/2)$ of \cite{22} is replaced by the $1/\chi(\bar{N}, Q/Q_T)$ in SCET, which is an interpolation between $\lambda^2 \sim \frac{1}{\bar{N}}$ and $\lambda^2 \sim Q_T/Q$. That is to say

$$\chi(\bar{N}, Q/Q_T) = Q/Q_T + \frac{\bar{N}}{1 + 1/(4\bar{N})Q/Q_T}$$
approaches to $\bar{N}$ for $Q/Q_T \ll \bar{N}$ and to $Q/Q_T$ for $Q/Q_T \gg \bar{N}$, respectively. The matching steps for joint resummation then can be written as

$$QCD\big|_{Q^2} \rightarrow SCET_I\big|_{Q^2/\chi} \Rightarrow SCET_{II}\big|_{Q^2/\chi^2} \gg \mu_0^2 \rightarrow DGLAP\big|_{\mu_0^2},$$

which leads to similar result as Eq.(26) corresponding to that of [22].

VI. CONCLUSION

We have shown $Q_T$ resummation of Higgs-boson production in the framework of SCET and given a simple relationship between anomalous dimension of operator in SCET and $O(\alpha_s)$ coefficients A and B in pQCD, and $O(\alpha_s)$ coefficient C is reproduced by matching the matrix elements of hadrons at the scale $Q_T$ onto a product of two PDFs in $SCET_{II}$, and their connections at higher order are obtained by comparing the formulas in the two frames. We also show that the reformulation of joint resummation can be performed in SCET straightforwardly. Finally, we want to point out that any processes, which are limited to the soft and collinear regions by kinematics, can be treated in SCET following the steps outlined above, and moreover, for the high energy hard scattering processes involving the colored final state which is neither soft in the sense of SCET-including the heavy quark effective theory (HQET) nor collinear, the method for $Q_T$ resummation based on SCET collapses.

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