The Low-latency Region of a Communication Link

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Abstract—This letter shows the existence of a proportional fair rate allocation in communication links carrying both time-sensitive (TS) and non-time-sensitive traffic (NTS). This optimal point results from maximizing a simple throughput-delay trade-off that considers a) the NTS traffic load, and b) the difference between the maximum and the current delay of the TS packets. To show how the presented trade-off could be used to allocate NTS traffic in a realistic scenario, we use Google Stadia traffic traces to generate the TS flow. Results from this use-case confirm that the throughput-delay trade-off also works when both the packet arrival process and packet service time distributions are general.

Index Terms—Low-latency, time-sensitive traffic, proportional fair rate allocation, M/G/1

I. INTRODUCTION

The success of time-sensitive (TS) Internet services such as cloud-gaming, virtual reality, and real-time video transmission, depends to a great extent on the network’s ability to guarantee low end-to-end delays. This situation is especially challenging when non-time-sensitive traffic also shares the same link resources. In that situation, providing low latency guarantees depends mainly on limiting the incoming traffic to the link by using admission control, congestion control and traffic shaping techniques [1].

While the previous situation is a well-known problem in network engineering, we have not found a general definition of which is the low latency region (LLR) of a communication link that could be used as a reference. There could be two reasons: a) the LLR depends on the specific load and delay requirements of the active TS flows, and b) the non-existence of a clear trade-off between the traffic load and delay of a link, as both increase or decrease at the same time.

In this letter, we define the LLR of a communication link as follows: a TS flow operates inside the low-latency region if the average time that a TS packet spends in the link is lower than the mean inter-TS packet arrival time. Note that this is equivalent to say, that a new arriving packet must leave the system, on average, before another packet of the same flow appears.

Then, we show that inside the LLR exists a proportional fair rate allocation. It is formulated as the maximum difference between the throughput gain and the latency loss when non-time-sensitive (NTS) traffic is added to a link. We will refer to that point as the proportional fair LLR (PFLLR) rate allocation, or, equivalently, to the proportional fair NTS rate allocation. We show that the PFLLR rate allocation can be achieved in practice by estimating the time between TS packet arrivals, the mean packet delay of the TS packets, and controlling the amount of NTS traffic that can be allocated to a link.

Finally, we study the PFLLR rate allocation when a link carries Google Stadia traffic, a cloud-gaming service that requires both high-throughput and low latency to perform satisfactorily. We aim to obtain how much NTS traffic can be allocated, and evaluate how much it disturbs the TS flow in terms of the extra added latency. However, in addition to that, and more importantly, we confirm that we can estimate the PFLLR rate allocation even if the traffic arrival process is not Poisson.

This paper was partially inspired by [2], where the authors play with the packet aggregation level in a Wi-Fi network to guarantee both high-throughput and low-latency. Different queue management strategies to control queuing delay with concurrent video and best-effort traffic are evaluated in [3]. Finally, a congestion control solution for cloud-gaming is presented in [4].

II. SYSTEM MODEL

We consider that a link consists of a buffer and a transmitter as shown in Fig. 1. The link capacity, i.e., the rate at which the transmitter works, is of $R$ bits/second. We assume the buffer is large enough to be considered of infinite size. We also assume all arriving flows have the same priority. Therefore, traffic differentiation is not applied, and all arriving packets are served following their order of arrival.

Fig. 1: Link model. The buffer and the transmitter correspond to the network interface.

Besides, to keep the analysis simple, we do the following considerations:

1) There are only one TS flow and one NTS flow. The packet arrival process for both flows follows a Poisson process, with mean rates $\lambda_s$ and $\lambda_b$ packets/second, respectively. The mean aggregate packet arrival rate is given by $\lambda = \lambda_s + \lambda_b$. We consider the NTS flow is elastic, and so its packet arrival rate can be adjusted to the desired value.

2) The two flows have the same service distribution. Service times are independent and identically distributed.

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1To define the LLR we have used the mean packet delay instead of another percentile as it results in simple, closed-form, and insightful expressions.

2Google Stadia: https://stadia.google.com/

3In Sec. 1,2 we study a case where packets arrive following a general distribution, and the TS and NTS flows have different service distributions.
and follow a general distribution with mean \( E[S] \) seconds, and coefficient of variation \( C_s \). The mean transmission rate of the link in packets/sec. is \( \mu = 1/ E[S] \).

Taking into account these considerations, the link is modelled as an M/G/1 queueing system \( [3] \). The mean sojourn time for a packet of flow \( s \), \( E[D_s(\lambda_s, \lambda_b)] \) and for a packet of flow \( b \), \( E[D_b(\lambda_s, \lambda_b)] \), is the same:

\[
E[D_s(\lambda_s, \lambda_b)] = E[D_b(\lambda_s, \lambda_b)] = E[S] + \frac{\lambda E[S^2]}{2(1 - a)} = \frac{1}{\mu} \left( 1 + \frac{a}{1 - a} \right) = \frac{\Gamma(\lambda, \theta)}{\mu - \lambda},
\]

where \( E[S^2] = E^2[S](1 + C_s^2) \) is the second moment of the service time, \( a = \lambda/\mu \) is the link utilization, and \( \theta = (1 + C_s^2)/2 \). Lastly, \( \Gamma(\lambda, \theta) = 1 - a + a\theta \) is the ratio between the delay of a M/G/1 and a M/M/1 queue. Note that

\[
a = a_s + a_b = \lambda_s/\mu + \lambda_b/\mu,
\]

where \( a_s \) and \( a_b \) are the link utilization by the TS and NTS flows, respectively.

### III. LOW-LATENCY REGION

Following our previous definition, in the absence of NTS traffic \( (\lambda_b = 0) \), a TS flow \( s \) is working in the low-latency region of a link if the following condition is satisfied:

\[
\frac{1}{\lambda_s} \geq E[D_s(\lambda_s, 0)] = \frac{\Gamma(\lambda_s, \theta)}{\mu - \lambda_s}.
\]

The highest value of \( \lambda_s \) that satisfies (2) is

\[
\lambda_s^+ = \frac{\mu}{1 + \sqrt{\theta}},
\]

and therefore, the LLR includes all \( \lambda_s \in [0, \lambda_s^+] \). Note that the value of \( \lambda_s^+ \) depends on the service time distribution of the TS traffic. For example, if the service time distribution is exponential, \( \lambda_s^+ \) is half of the link capacity.

Considering Little’s theorem, (2) can be rewritten as:

\[
\lambda_s E[D_s(\lambda_s, 0)] = E[N_s(\lambda_s, 0)] \leq 1,
\]

where \( E[N_s(\lambda_s, 0)] \) is the mean number of TS packets in the link. It is equivalent to say that on average, the NTS flow will be inside the LLR if the mean number of packets in the link is equal or less than 1, and so, on average, an incoming packet will receive service immediately, or after the on-going transmission finishes.

Fig. 2 (left-side) shows the values of \( \lambda_s^+ \) as the intersection between \( 1/\lambda_s \) and \( E[D_s(\lambda_s, 0)] \) for different service time distributions. As expected, higher \( C_s \) values reduce the low-latency region. The right side of Fig. 2 confirms that \( \mathbb{E}[N_s(\lambda_s, 0)] \leq 1 \) when working inside the LLR.

### IV. NTS RATE ALLOCATION

Let us consider that an NTS flow \( b \) is added to the link, and so it shares the link resources with the TS flow \( s \). We are then interested in finding how much NTS traffic can be admitted to the link while keeping the TS flow working inside the LLR.

#### A. Max NTS rate allocation

A solution to the previous problem can be found by solving

\[
\frac{1}{\lambda_s} \geq E[D_s(\lambda_s, \lambda_b)] = \frac{\Gamma(\lambda_s, \theta)}{\mu - \lambda_s}.
\]

Then, we can obtain \( \lambda_b^+ \) as the highest feasible NTS rate allocation, i.e.,

\[
\lambda_b^+ = \left( \frac{\mu - \lambda_s}{\Gamma(\lambda_s, \theta)} \right) - \lambda_s = \frac{1}{E[D_s(\lambda_s, 0)]} - \lambda_s,
\]

which explicitly depends on the link delay of the TS packets.

We can re-write (3) as \( \lambda_b^+ = \mu - \kappa^+ \lambda_s \), with \( \kappa^+ \) being the minimum link capacity required by the TS flow to work in the LLR. The value of \( \kappa^+ \) is given by \( \kappa^+ = 1 + \frac{\theta}{(1 - a)\mu} \). Let us define \( \beta \) to refer to the second term of \( \kappa^+ \), i.e., \( \beta = \frac{\theta}{(1 - a)\mu} \). Note that \( \beta \lambda_s \) is the amount of link capacity that has to remain unused.

For example, considering the service times are exponentially distributed, \( \kappa^+ = 2 \), and so \( \lambda_b^+ = \mu - 2\lambda_s \). Also, observe that, in general, \( \kappa^+ \lambda_s^+ = \mu \).

#### B. Proportional fair NTS rate allocation

Let \( \lambda_b^* \leq \lambda_b^+ \) be the proportional fair NTS rate allocation, i.e., the value at which the trade-off between the delay of TS packets and the NTS throughput is maximum. \( \lambda_b > \lambda_b^* \) values result in a higher delay increase for TS packets than the link throughput gain. Similarly, for \( \lambda_b < \lambda_b^* \) values, we observe the opposite result.

To find \( \lambda_b^* \), we formulate the throughput-delay trade-off as the difference between the link throughput gain, and the delay loss for the TS traffic,

\[
g(\lambda_s, \lambda_b) = G_T(\lambda_s, \lambda_b) - G_D(\lambda_s, \lambda_b),
\]

with respect to the case there is no NTS traffic.

The NTS throughput gain is

\[
G_T(\lambda_s, \lambda_b) = \frac{(\lambda_b + \lambda_s) - \lambda_s}{\lambda_s} = \frac{\lambda_b}{\lambda_s},
\]

and the TS delay loss is

\[
G_D(\lambda_s, \lambda_b) = \frac{\mathbb{E}[D_s(\lambda_s, \lambda_b)] - \mathbb{E}[D_s(\lambda_s, 0)]}{\mathbb{E}[D_s(\lambda_s, 0)]}
\]

\[
= \frac{1}{\mu} + \frac{1}{\mu} \frac{a}{(1 - a)\theta} - \frac{1}{\mu} \frac{a}{(1 - a)\theta} - \frac{1}{\mu} \frac{a}{(1 - a)\theta} = \frac{\beta(\lambda - \lambda_s)}{\mu - \lambda} = \frac{\beta \lambda_b}{\mu - \lambda_s - \lambda_b}.
\]
Then, (5) results in
\[ g(\lambda_a, \lambda_b) = \frac{\lambda_b}{\lambda_a} - \frac{\beta \lambda_b}{\mu - \lambda_a - \lambda_b} = \lambda_b \left( \frac{1}{\lambda_a} - \frac{\beta}{\mu} \right). \]  
(6)

Finally, we are interested in finding
\[ \lambda_b^* = \arg\max_{\lambda_b} g(\lambda_a, \lambda_b), \]
that is the proportional fair NTS rate allocation.

Since \( g(\lambda_a, \lambda_b) \) is concave, and has its maximum in the range \( \lambda_b \in [0, \lambda_a^*] \), we find \( \lambda_b^* \) by deriving (6) with respect to \( \lambda_b \). The first derivative of \( g(\lambda_a, \lambda_b) \) is
\[ \frac{dg(\lambda_a, \lambda_b)}{d\lambda_b} = \frac{1}{\lambda_a} - \frac{\beta (\mu - \lambda_a)}{(\mu - \lambda_a - \lambda_b)^2}, \]
and it is equal to zero for
\[ \lambda_b^* = \mu - \lambda_a - \sqrt{\beta \lambda_a (\mu - \lambda_a)}, \]
\[ = \mu - \lambda_a \left( 1 + \sqrt{\frac{\mu - \lambda_a}{\lambda_a}} \right) = \mu - \kappa_\lambda \lambda_a, \]  
(7)

with \( \kappa_\lambda = 1 + \sqrt{\frac{\mu - \lambda_a}{\lambda_a}} \). Similarly to the max NTS rate allocation, the \( \kappa_\lambda \lambda_a \) term is the link capacity required by the TS flow to work at the PFLLR rate allocation. Note that \( \kappa_\lambda \geq \kappa^* \) for \( \lambda_a \leq \lambda_a^* \).

Fig. 3 shows the PFLLR rate allocation (\( \lambda_b^* \) values) for different \( \lambda_a \) and \( C_S \) values. For low values of \( \lambda_a \), almost full link capacity can be achieved. However, it rapidly reduces when \( \lambda_a \) increases. Also, increasing the variability of service times reduces the PFLLR rate allocation.

V. A PRACTICAL APPROXIMATION

In this section, we show that the following practical approximation to (6).
\[ f(\lambda_a, \lambda_b) = \lambda_b \left( \mathbb{E}[D_s(\lambda_a, \lambda_b^*)] - \mathbb{E}[D_s(\lambda_a, \lambda_b)] \right), \]  
(8)
can be used to accurately estimate the proportional fair NTS rate allocation. Note that (8) is simply the product between the NTS throughput (the gain), and the difference in delay between the maximum tolerable and the current delay of the TS packets (the loss). To obtain it we have simply approximated the term \( \beta/(\mu - \lambda) \) in (6) by \( \mathbb{E}[D_s(\lambda_a, \lambda_b)] \).

VI. USE-CASE: GOOGLE STADIA

In this section, we aim to illustrate the existence of the PFLLR rate allocation, and the applicability of (8), when the aggregate traffic arrival process is not Poisson, and the TS and NTS traffic flows are characterized by different traffic arrival processes and service time distributions. We also examine the cumulative distribution function (cdf) of the delay of the TS packets, \( D_s(\lambda_a, \lambda_b) \), for different values of \( \lambda_b \) to observe how it changes when the NTS traffic increases.

We use an event-based C++ simulator that reproduces the system model described in Section II. The TS traffic is generated using a set of traffic traces collected while playing with Google Stadia’s Tomb Raider (GS). The traces represent the downlink traffic (mostly video contents, from the server to the client, sent at a rate of 60 frames/second) for the three different video resolutions available in GS (720p, 1080p, and 2160p). The duration of each trace is 30 seconds. Their main characteristics are shown in Fig. 6 and in Tbl. 1. We can observe that GS packets arrive in batches of mean size \( \mathbb{E}[^x] \).

In those conditions, packet arrivals are not Poisson, even if the coefficient of variation of the inter-packet arrival time, \( C_v \), for 720p and 1080p is close to 1. NTS traffic arrives to the link following a Poisson process. NTS packet sizes are assumed to be exponentially distributed, with an average size of \( \mathbb{E}[L_b] = 10000 \) bits. The capacity of the link is set to \( R = 100 \) Mbps.

The traces can be found at [https://www.upf.edu/web/warg/wn-datasets](https://www.upf.edu/web/warg/wn-datasets)
Instead, for \( \lambda < 0 \) the negative effect of the NTS traffic on the TS delay is significant.

The proportional fair NTS rate allocation (circle). The proportional fair NTS rate allocation for the best-effort flow is 65, 50 and 30 Mbps, for 720p, 1080p, and 2160p video resolutions, respectively.

For each resolution, we also plot the mean packet delay for GS packets, \( \mathbb{E}[\tau] \), and the mean packet size, \( \mathbb{E}[\sigma] \). For example, Fig. 7 shows that when the NTS traffic approaches 50 Mbps, if Stadia is sending video at a resolution 1080p, two options are possible: a) block any extra NTS traffic arriving to the link, and keep the TS traffic working at a resolution of 1080p, or b) switch to a lower video resolution (720p) and allow up to 60 Mbps of NTS traffic.

**TABLE I: Characteristics of Tomb Raider Downlink traffic.**

| Resolution (unit) | Load (Mbps) | \( \mathbb{E}[\tau] \) (ms) | \( \mathbb{E}[\sigma] \) (Bytes) | \( \mathbb{E}[L_s] \) (Bytes) | \( \mathbb{E}[\sigma] \) (packets) |
|------------------|-------------|-----------------|-----------------|-----------------|-----------------|
| 720p (HD)        | 10.25       | 1.700           | 997.5           | 0.40            | 2.18            |
| 1080p (FHD)      | 27.47       | 1.417           | 1123.2          | 0.23            | 4.33            |
| 2160p (4K)       | 39.89       | 1.293           | 1144.2          | 0.19            | 5.74            |

This paper defines the low-latency region of a communication link. It also shows the existence of a proportional fair rate allocation inside the low-latency region when the link carries TS and NTS traffic.

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