Prescriptions for the scaling variable of the nucleon structure function in nuclei

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Abstract

We tested several choices of the in-medium value of the Bjorken scaling variable assuming the nucleon structure function in nucleus to be the same as that of free nucleon. The results unambiguously show that it is different.

1 Introduction

As well known, the deep inelastic scattering (hereafter DIS) of leptons on nucleons begins by the formation of parton with the size (Compton wave length in the rest frame) \((mx)^{-1} = \frac{0.21}{x} \text{ fm}\), \(x = Q^2/(2mq_0)\), \(x\) and \(q_0\) are the Bjorken scaling variable and the energy of virtual photon in the rest frame of nucleus. Accordingly three interaction regions are inherent for the DIS on nuclei:

I. Correlation region, \(0 < x < 0.2\). In this region the size of parton exceeds the distance between nucleons \(r_0 = 1.2 \text{ fm}\), and therefore two or even several nucleons take part in the process. For this reason the correlations between nucleons, both short-range and long-range ones, are of importance.

II. One-nucleon region, \(0.2 < x < 0.8\). In this region \((mx)^{-1} < r_0\), and therefore the virtual \(\gamma\)-quantum is absorbed by one nucleon only.

III. Competition region, \(0.8 < x \sim 1\). In this region for a not very large \(Q^2\) the competition occurs between DIS, elastic lepton–nucleon scattering and the possible formation of heavier baryons through the reaction \(\ell N \rightarrow \ell' B\), \(B\) being \(\Delta_{33}, N^*\) etc.

In the one-nucleon region we are dealing with the in-medium nucleon structure function; \(F_{2m}(x, Q^2, p, \varepsilon)\) depending upon the momentum \(p\) and binding energy \(\varepsilon\) of nucleon in nucleus in addition to \(x\) and \(Q^2\) must be averaged over the energy-momentum distribution \(S(p, \varepsilon)\) of nucleon, \(i.e.

\[
\frac{F_{2A}(x, Q^2)}{x_A} = \int d^3pd\varepsilon S(p, \varepsilon) \frac{F_{2m}(x, Q^2, p, \varepsilon)}{x_N},
\]

(1)
where $x_A = \frac{4mx}{M_A}$ is the scaling variable of target nucleus, $M_A$ is its mass and $x_N$ is the in-medium scaling variable of nucleon. The immediate question is as follows: is the in-medium structure function $F_{2m}$ the same as that of free nucleon? Note that in Eq. (1) we used the in-medium scaling variable $x_N$ which is different from $x$. The usual hope was that choosing an appropriate definition of $x_N$ one may absorb the in-medium dependence of the function $F_{2m}$ and describe the data using the free-nucleon structure function, i.e. putting $F_{2m}(x, Q^2, p, \varepsilon) = F_2(x_N, Q^2)$. The analysis of the available data showed that this is not the case [1]. But as discussed in [2] all the previous calculations are based on seemingly evident but erroneous assumption that the quantity $S(p, \varepsilon)$ is the ground-state spectral function of the target nucleus. Actually it is the spectral function of the doorway states for one-nucleon transfer reactions. Indeed, the nucleon hole (which is just the relevant doorway state) is formed in the ground state of target nucleus when the struck nucleon is destroyed by DIS. This state is not the eigenstate of nuclear Hamiltonian thus being fragmented over the actual states of residual nucleus because of the correlations between nucleons. The observed spreading width of the hole states is 20 MeV [3] the fragmentation time thus being $3 \cdot 10^{-23}$ sec. But the interaction times of DIS is $2q/Q^2 = (mx)^{-1} = \frac{0.2}{x} \cdot 10^{-24}$ sec thus being less than $3 \cdot 10^{-24}$ sec for $x > 0.3$. So the DIS interaction time in the one-nucleon region is an order of magnitude less than that of the fragmentation and therefore the correlation processes do not have time to come into play. As a consequence the quantity $S(p, \varepsilon)$ entering (1) is the spectral function of the doorway states. As discussed in [4] it can be unambiguously calculated in a model-independent way in contrast to the ground-state spectral function. So the theory of doorway states provides a natural way for testing the models of nucleon structure functions in nuclei.

In [2] we performed the EMC calculations assuming the nucleon structure function in the doorway state $\lambda$ to be the same as that of free nucleon however dependent upon the in-medium scaling variable in this state:

$$F_{2m}(x, Q^2, p, \varepsilon_2) = F_2(x_N, Q^2), \quad x_N = \frac{mx}{m + \varepsilon_\lambda - \beta p_3}, \quad \beta = \frac{|q|}{q_0} = \left(1 + \frac{4m^2x^2}{Q^2}\right)^{1/2},$$

(2)

where $\varepsilon_\lambda < 0$ is the nucleon binding energy in the state $\lambda$ and the axis 3 is chosen along the momentum of virtual photon. The results do not agree with all the available EMC data thus indicating that $F_{2m}$ is different from $F_2$. In the present work we are testing two different choices of the in-medium scaling variable, the first belonging to Molochkov [5] and the second to Pandharipande and coworkers [6].
2 Analysis

2.1 Molochkov’s definition of $x_N$

Using the Bethe–Salpeter technique Molochkov derived the following expression of the nucleon structure function in nucleus:

$$F_2(x, Q^2) = \frac{\int d^4 p \ F_2(x_N, Q^2) \ |V_{AN}(P_A, p)|^2}{(2\pi)^4 \ x_N \ (p^2 - m^2)^2 \ ((P_A - p)^2 - M_{A-1}^2)}, \quad x_N = \frac{mx}{p_0 - \beta p_3}. \quad (3)$$

The vertex $V_{AN}(P_A, p)$ describes the wave function of nucleon in nucleus, see Eq. (6). The meaning of other entering quantities is clear from Fig. 1, where Eq. (3) is graphically represented. In a more detailed form

$$F_2(x, Q^2) = \int \frac{id^3 p d p_0}{(2\pi)^4} \ |V_{AN}(P_A, p)|^2 \ \times \ \left(\frac{p_0 - e_p + i\delta}{(p_0 - (M_A + E_{A-1}) - i\delta)(p_0 + e_p - i\delta)^2}\right)^2.$$ \hspace{1cm} (3a)

The integrand has the first-order pole $p_0 = M_A - E_{A-1}$ and the second-order one $p_0 = -e_p = -(m^2 + p^2)^{1/2}$ (the latter is negligible because the function $F_2(x, Q^2)$ vanishes at negative $x$ values) in the upper half-plane of $p_0$ and the second-order pole $p_0 = e_p$ together with the first-order one $p_0 = M_A + E_{A-1}$ in the lower half-plane. For the doorway state $\lambda$ of heavy nucleus

$$E_{A-1} = \left((M_A - m - \varepsilon_{\lambda})^2 + p^2\right)^{1/2} \approx M_A - m - \varepsilon_{\lambda} \quad (4)$$

(in this case the recoil may be neglected) and

$$E_{A-1} = e_p \quad \text{(4a)}$$

for the case of deuteron.

Closing the integration contour over $p_0$ in the upper half-plane we get $|\delta = e_p - (M_A - E_{A-1})|$

$$F_2(x, Q^2) = \frac{Amx}{M_A} \int \frac{d^3 p}{(2\pi)^3} \ \frac{M_A - E_{A-1} - \beta p_3}{mx} \times \frac{Mx}{M_R - E_{A-1} - \beta p_3, Q^2} \ |V_{AN}(P_A, p)|^2 \ \times \ M_A \ f_{\lambda}(p) \frac{\delta_{\lambda}}{4\pi}, \quad \delta_{\lambda} = e_p - m - \varepsilon_{\lambda}, \quad \bar{M}_A = \sum_{\lambda} \nu_{\lambda}(m + \varepsilon_{\lambda}) \quad (5)$$

This is just our result [2] for the in-medium value of the scaling variable. Comparing Eq. (5) with Eqs. (29), (30) and (31) of Ref. [2] we get

$$\frac{|V_{AN}(P_A, p)|^2}{2(2\pi)^3 E_{A-1} \delta_{\lambda}^2 (2e_p - \delta_{\lambda})^2} = M_A \ \frac{f_{\lambda}(p)}{4\pi}, \quad \delta_{\lambda} = e_p - m - \varepsilon_{\lambda}, \quad \bar{M}_A = \sum_{\lambda} \nu_{\lambda}(m + \varepsilon_{\lambda}) \quad (6)$$
for the doorway state $\lambda$ ($f_{\lambda}(p)$ is the nucleon momentum distribution in this state, see [2] for details) and

$$V_{DN}(P_{D,p}) = \frac{M_D f_D(p)}{2(2\pi)^3 e_p \delta_D^2 (2e_p - \delta_D)^2}, \quad \delta_D = 2e_p - M_D,$$

$$\bar{M}_D = 2(M_D - \bar{e}_p), \quad \bar{e} = \int \frac{d^3 p}{4\pi} e_p f_D(p)$$

(6a)

for deuteron.

Molochkov however closed the integration contour in the lower half-plane so

$$F_2^A(x, Q^2) = \frac{A_m}{M_A} \int \frac{d^3 p}{(2\pi)^3} \left( \frac{p_0 - \beta p_3}{m x} \right) F_2 \left( \frac{m x}{p_0 - \beta p_3}, Q^2 \right)$$

\times \frac{|V_{AN}(P_A, p)|^2}{(p_0 + e_p)^2(M_A - E_{A-1} - p_0)(M_A + E_{A-1} - p_0)} \bigg|_{p_0 = e_p}

+ \frac{A_m}{M_A} \int \frac{d^3 p}{(2\pi)^3} \frac{M_A + E_{A-1} - \beta p_3}{m x} F_2 \left( \frac{m x}{M_A + E_{A-1} - \beta p_3}, Q^2 \right)

\times \frac{|V_{AN}(P_A, p)|^2}{2E_{A-1}(M_A + E_{A-1} - e_p)^2(M_A + E_{A-1} + e_p)^2}.$$

(7)

Performing the calculations he disregarded the second term of the rhs and included only part of the derivative in the first term by putting

$$\left( \frac{p_0 - \beta p_3}{m x} \right) F_2 \left( \frac{m x}{p_0 - \beta p_3}, Q^2 \right) \frac{|V_{AN}(P_A, p)|^2}{(p_0 + e_p)^2(M_A - E_{A-1} - p_0)(M_A + E_{A-1} - p_0)} \bigg|_{p_0 = e_p}

= \frac{4e_p^2(M_A + E_{A-1} - e_p)}{|V_{AN}(P_A, p)|^2} \left( \frac{p_0 - \beta p_3}{m x(M_A - E_{A-1} - p_0)} F_2 \left( \frac{m x}{p_0 - \beta p_3}, Q^2 \right) \right) \bigg|_{p_0 = e_p}.$$

We instead neglected only the $p_0$ dependence of the vertex $V_{AN}(P, p)$ and included both terms of the rhs of Eq. (7) thus obtaining

$$F_2^A(x, Q^2) = \frac{A_m}{M_A} \int \frac{d^3 p}{(2\pi)^3} \left( 1 + \frac{\delta}{e_p - \frac{\delta}{M_A + E_{A-1} - e_p}} \right) \frac{|V_{AN}(P_A, p)|^2}{4e_p^2 \delta^2(M_A + E_{A-1} - e_p)^2}$$

\times \left( \frac{e_p - \Delta - \beta p_3}{m x} \right) F_2 \left( \frac{m x}{e_p - \beta p_3}, Q^2 \right) + \frac{\Delta}{e_p - \beta p_3} \hat{F}_2 \left( \frac{m x}{e_p - \beta p_3}, Q^2 \right)

+ \frac{A_m}{M_A} \int \frac{d^3 p}{(2\pi)^3} \frac{M_A + E_{A-1} - \beta p_3}{2E_{A-1}(M_A + E_{A-1} - e_p)^2(M_A + E_{A-1} + e_p)^2}$$

\times \frac{|V_{AN}(P_A, p)|^2}{(2\pi)^3} \frac{M_A + E_{A-1} - \beta p_3}{m x} F_2 \left( \frac{m x}{M_A + E_{A-1} - \beta p_3}, Q^2 \right),$$

(8)

where

$$\Delta = \left( 1 + \frac{\delta}{e_p - \frac{\delta}{M_A + E_{A-1} - e_p}} \right)^{-1} \frac{\delta}{e_p} \frac{\delta}{M_A + E_{A-1} - e_p}, \quad \left( \frac{\delta}{e_p} \right) = \frac{\partial}{\partial p_0} \left( \frac{\delta}{e_p} \right), \quad \hat{F}_2(x, Q^2) = \frac{\partial F_2(x, Q^2)}{\partial x}.$$
First consider the doorway state $\lambda$ in heavy nucleus. As follows from Eqs. (5) and (6)

\[
\frac{(1 + \frac{4\delta}{e_p} - \frac{2\delta}{2E_{A-1}})}{4(2\pi)^3 e_p^2 (2E_{A-1} - \delta)^2} |V_{AN}(P, p)|^2 = \frac{M_A f_\lambda(p)}{M_A} \left( e_p + \delta - e_p \frac{2\delta}{2E_{A-1} - \delta} \right)(2e_p - \delta)^2 E_{A-1}
\]

\[
\sim \frac{M_A f_\lambda(p)}{4\pi} \frac{4e_p (e_p + \delta_\lambda)(e_p - \delta + \frac{\delta^2}{4e_p^2})}{4e_p^3 E_{A-1}} \approx \frac{M_A f_\lambda(p)}{4\pi} \left( 1 - \frac{\delta^2}{e_p^2} \right) \approx \frac{M_A f_\lambda(p)}{4\pi}
\]

(10)

because the small quantities $\frac{\delta}{2E_{A-1}} \ll 1$ and $\frac{\delta^2}{e_p^2} \ll 1$ can be neglected in (10). In the same way

\[
\frac{|V_{AN}(P_A, p)|^2}{2(2\pi)^3 E_{A-1}((M_A + E_{A-1})^2 - e_p^2)} = \frac{M_A f_\lambda(p)}{4\pi} \frac{\delta_\lambda (2e_p - \delta_\lambda)}{(M_A + E_{A-1})^2 - e_p^2},
\]

(11)

so

\[
F_{2A,\lambda}(x, Q^2) = \frac{Amx}{M_A} \int \frac{d^3p}{4\pi} f_\lambda(p) \left\{ \left[ e_p - \Delta_\lambda - \beta p_3 \hat{F}_2 \left( \frac{mx}{e_p - \beta p_3}, Q^2 \right) \right] \right.
\]

\[
+ \frac{\Delta_\lambda}{e_p - \beta p_3} \hat{F}_2 \left( \frac{mx}{e_p - \beta p_3}, Q^2 \right) \right. \left[ \frac{\delta_\lambda (2e_p - \delta_\lambda)}{(M_A + E_{A-1})^2 - e_p^2} \right]^2 \frac{M_A + E_{A-1} - \beta p_3}{mx} \hat{F}_2 \left( \frac{mx}{M_A + E_{A-1} - \beta p_3}, Q^2 \right) \right\}.
\]

(12)

In the case of deuteron

\[
\frac{(1 + \frac{4\delta}{e_p} - \frac{2\delta}{2E_{D-1}})}{4(2\pi)^3 e_p^2 \delta_D M_D} |V_{DN}(P_D, p)|^2 = \frac{M_D f_D(p)}{M_D} \left( 2e_p - \delta_D \right) \left( 1 + \frac{\delta_D}{2e_p} + \frac{\delta_D}{2e_p - \delta_D} - \frac{\delta_D}{M_D} \right)
\]

\[
= \frac{M_D f_D(p)}{4\pi} \frac{(2e_p - \delta_D)(2e_p + \delta_D - \frac{\delta_D^2}{M_D})}{4e_p^2} \approx \frac{M_D f_D(p)}{M_D} \frac{1 - \frac{\delta_D^2}{4e_p^2}}{4\pi} \approx \frac{M_D f_D(p)}{M_D}
\]

(13)

and

\[
\frac{|V_{DN}(P_D, p)|^2}{2(2\pi)^3 e_p M_D^2 (2e_p + M_D)^2} = \frac{M_D f_D(p)}{4\pi} \left( \frac{\delta_D}{2e_p + M_D} \right)^2,
\]

(14)

so

\[
F_{2D,N}(x, Q^2) = \frac{2mx}{M_D} \int \frac{d^3p}{4\pi} f_D(p) \left\{ \left[ e_p - \Delta_D - \beta p_3 \hat{F}_2 \left( \frac{mx}{e_p - \beta p_3}, Q^2 \right) \right] \right.
\]

\[
+ \frac{\Delta_D}{e_p - \beta p_3} \hat{F}_2 \left( \frac{mx}{e_p - \beta p_3}, Q^2 \right) \right. \left[ \frac{\delta_D}{2e_p + M_D} \right]^2 \frac{M_D + e_p - \beta p_3}{mx} \hat{F}_2 \left( \frac{mx}{M_D + e_p - \beta p_3}, Q^2 \right) \right\}.
\]

(15)
2.2 Prescription of Ref. [6]

This prescription is based on the following consideration: to obtain by DIS on bound nucleon with the momentum $p$ the same hadronic state as that on free nucleon the momentum transfer $q$ must be the same, but transferred energy $q_0$ must be larger to overcome the binding. As a result of the energy-momentum conservation

$$e_p + q' = M_A - E_{A-1} + q_0$$

we get

$$q'_0 = q_0 - e_p + (M_A - E_{A-1}) = q_0 - \delta,$$

$$Q^2 = Q^2 + q_0^2 - q'_0^2 \approx Q^2 + 2q_0\delta = \left(1 + \frac{\delta}{mx}\right)Q^2,$$

$$x_N = \frac{Q^2}{2pq'} = \left(1 + \frac{\delta}{mx}\right)\frac{Q^2}{2(e_pq'_0 - pq)} = \frac{mx + \delta}{e_p - \beta p_3},$$

so

$$F_{2A}(x, Q^2) = \int \frac{d^3p}{4\pi} f(p) \frac{m x}{e_p} (e_p - \beta p_3) F_2 \left( \frac{mx + \delta}{e_p - \beta p_3}; \left(\frac{\delta}{mx}\right)Q^2 \right).$$

It is worth mentioning that putting $\delta = 0$ we get the net effect of the Fermi-motion.

3 Results

We calculated the isoscalar part of the EMC ratio $^1$

$$R_A(x, Q^2) = \frac{F_{2A,p}(x, Q^2) + F_{2A,n}(x, Q^2)}{F_{2D,p}(x, Q^2) + F_{2D,n}(x, Q^2)},$$

where the denominator is the structure function of deuteron, and

$$F_{2A,p}(x, Q^2) = \frac{1}{Z} \sum_\lambda \nu_\lambda F_{2A,\lambda}(x, Q^2), \quad F_{2A,n}(x, Q^2) = \frac{1}{N} \sum_\lambda \nu_\lambda F_{2A,\lambda}(x, Q^2).$$

According to the Caushy’s theorem the results of the calculations using our prescription [2], formula (5), and the Molochkov’s one [5], formula (8), must coincide (the difference may be caused by the approximations /neglection of some small contributions/ used in one or another method). It is impossible to demonstrate this analytically because of the absence of analytical expressions for both the structure functions and the vertices (it is interesting

$^1$The MRST 2002 NLO [7] parametrization of the parton distributions in a free nucleon and the Bonn-B wave function for the deuteron [8] were used.
to mention in this connection that neglecting the difference between $\Delta$ and $\delta$ in formula (8) we get

$$
\frac{e_p - \delta - \beta p_3}{m_x} F_2 \left( \frac{m x}{e_p - \beta p_3}, Q^2 \right) + \frac{\delta}{e_p - \beta p_3} \hat{F}_2 \left( \frac{m x}{e_p - \beta p_3}, Q^2 \right) = \frac{e_p - \delta - \beta p_3}{m_x} \left( F_2 \left( \frac{m x}{e_p - \beta p_3}, Q^2 \right) + \left[ \frac{m x}{e_p - \delta - \beta p_3} - \frac{m x}{e_p - \beta p_3} \right] \hat{F}_2 \left( \frac{m x}{e_p - \beta p_3}, Q^2 \right) \right)
$$

the quantities in the parenthesis thus being the first two terms of the Tailor’s expansion of the function $F_2 \left( \frac{m x}{e_p - \delta - \beta p_3}, Q^2 \right)$, see (5). But the numerical results for the ratios are found to be the same. The results for the deuteron structure functions within both the above prescriptions are shown in Table 1. The comparison clearly shows that the coincidence occurs for the deuteron structure functions too. It is important to mention that performing the Molochkov’s calculations we neglected the possible $p_0$ dependence of the nuclear vertices $V_{AN}(P_A, p)$. The coincidence of the calculations within the methods [2] and [5] indicates that such a dependence is insignificant.

The results of the calculations using the prescriptions [2, 5, 6] for the in-medium scaling variable $x_N$ and that for the Fermi-motion are shown in Fig. 2 together with the SLAC data [9]. As clearly seen from the figure none of the prescriptions for $x_N$ leads to agreement with experiment. The same result is obtained for the EMC data [10]–[14]. This enables us to state that the structure function of nucleon in nucleus is different from that of free nucleon.

The $Q^2$ dependence of the results is insignificant. This is illustrated in Fig. 3 where the EMC ratios for $^{56}$Fe are calculated at $Q^2 = 5 \text{ GeV}^2$ and $100 \text{ GeV}^2$. 
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Table 1: Comparison between the ratios $D/np = F_{2D}(x, Q^2) \left( F_{2n}(x, Q^2) + F_{2p}(x, Q^2) \right)^{-1}$ calculated within the prescriptions [2] and [5].

| $x$  | $Q^2$ | $D/np$ [2] | $D/np$ [5] |
|------|-------|------------|------------|
| .100 | 5.0   | .9989      | .9993      |
| .123 | 5.0   | .9984      | .9988      |
| .148 | 5.0   | .9977      | .9982      |
| .205 | 5.0   | .9960      | .9965      |
| .235 | 5.0   | .9948      | .9954      |
| .268 | 5.0   | .9935      | .9941      |
| .303 | 5.0   | .9920      | .9926      |
| .340 | 5.0   | .9905      | .9910      |
| .380 | 5.0   | .9888      | .9893      |
| .420 | 5.0   | .9875      | .9878      |
| .460 | 5.0   | .9864      | .9865      |
| .500 | 5.0   | .9862      | .9861      |
| .540 | 5.0   | .9871      | .9866      |
| .580 | 5.0   | .9900      | .9888      |
| .620 | 5.0   | .9961      | .9942      |
| .660 | 5.0   | 1.0075     | 1.0045     |
| .700 | 5.0   | 1.0290     | 1.0243     |
| .740 | 5.0   | 1.0656     | 1.0582     |
| .780 | 5.0   | 1.1316     | 1.1196     |
| .820 | 5.0   | 1.2855     | 1.2642     |
| .860 | 5.0   | 1.4979     | 1.4577     |
Figure 1: Graphical representation of Eq. (3).

Figure 2: The EMC ratios at $Q^2 = 5\,\text{GeV}^2$ (left) and $100\,\text{GeV}^2$ (right) for $^{56}\text{Fe}$; the data are from ref.[9] (asterisks; the results [2] and [6] are undistinguishable), [6] (filled squares) and those for the Fermi-motion (open squares).
Figure 3: The EMC ratios at $Q^2 = 5$ GeV$^2$ within the prescriptions [2, 5] (asterisks; the results [2] and [6] are indistinguishable), [6] (filled squares) and those for the Fermi-motion (open squares).