Geometrical Construction of Type I

Superstring Vacua

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Abstract

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Abstract

The parameter-space orbifold construction of open and unoriented toroidal and (target-space) orbifold compactifications is briefly reviewed, with emphasis on the underlying geometrical framework. A class of chiral four-dimensional type-I vacua with three generations is also discussed.

Superstring theories\(^1\) are defined perturbatively as Conformal Field Theories with a field content that saturates the conformal anomaly. Superstring amplitudes are correlation functions of suitable combinations of chiral vertex operators on arbitrary Riemann surfaces\(^2\). Limiting the perturbative expansion to only closed oriented surfaces correspond to defining closed oriented (type II or heterotic) superstring models. Whenever the bulk algebra exhibits left-right symmetry, it is possible to extend the expansion to non-orientable and bordered Riemann surfaces, thus describing also unoriented closed and open (type I) superstrings. The consistency conditions can be fully encoded in a set of sewing constraints connecting the ‘basic building blocks’, namely coefficients of suitable correlation functions on low genus surfaces, both for closed oriented models\(^3\) and for unoriented closed and open models\(^4\)\(^5\)\(^6\)\(^7\). Equivalently, the interest can be turned to the analysis of perturbative spectra, encoded in the one-loop partition function. This approach deserves several observations: first, closed superstrings are characterized by modular invariant partition functions\(^8\), a property no longer true for amplitudes on surfaces endowed with holes and crosscaps. Second, it has long been known that models of only open superstrings are non-unitary, due to the presence of closed superstring states in intermediate channels of open diagrams\(^9\). Third, multiplicities associated to boundary fields (open vertex operators) are promoted to gauge (Chan Paton) factors of some classical Cartan groups\(^10\). These are (partly) fixed by tadpole conditions, that enforce the inclusion of open and unoriented contributions\(^11\). As a result, type I vacua are

\(^1\)The absence of tadpole conditions allows in fact type I models with only closed unoriented superstrings \(^2\)\(^3\)\(^4\)\(^5\)\(^6\)\(^7\)\(^8\)\(^9\)\(^10\)\(^11\).
parameter space orbifolds \[1\] of corresponding ‘parent’ type IIB vacua. Four contributions enter their one-loop partition function. The first one is the torus amplitude, that encodes the spectrum of the closed oriented ‘parent’ model. In order to construct a class of ‘open descendants’, one projects the closed spectrum into an unoriented one adding to the (halved) torus the Klein bottle amplitude. Then, the two open contributions, annulus and Möbius strip amplitudes, complete the description of the open unoriented spectrum. The construction resembles closely what happens in \( Z_2 \) orbifolds \[12\], where the closed spectrum is projected in a \( Z_2 \)-invariant way and ‘twisted’ sectors corresponding to strings closed only on the orbifold are added and projected. The \( Z_2 \) group is enforced by the ‘twist operator’ \( \Omega \) (now commonly referred to as world-sheet parity operator) that interchanges left and right sectors, while open superstrings, closed only on the double cover, play the role of ‘twisted sectors’. Thus, the orbifold should be thought of in parameter space rather than in target space, and the natural action of ‘twist’ requires that the closed ‘parent’ model be left-right symmetric. It is worth noticing that all open descendants of non left-right symmetric ‘parent’ models discussed in the literature are related by T-duality \[13\] to left-right symmetric ones.

Type I vacua were considered, in the past, less appealing than heterotic ones, essentially for phenomenological reasons. In the last two years, however, much progress has been made in understanding the non-perturbative structure of superstrings. All superstring vacua emerge as asymptotic expansions around specific points of moduli space of an unknown fundamental (M \[14\] or F \[15\]) theory. Moreover, a number of duality symmetries \[16\] connecting the various perturbative vacua have been found or conjectured that make plausible such a unified scenario, and that correspond to different compactifications of M or F theory. For instance, there is a strong-weak coupling duality between the type I \( SO(32) \) superstring and the \( SO(32) \) heterotic string in \( d = 10 \) \[17\], and a T-duality connecting the latter to the \( E_8 \times E_8 \) heterotic string in \( d = 9 \) \[18\]. All three can be thought of as ‘derived’ from the compactification of eleven dimensional M theory on \( S^1/Z_2 \), that gives rise to the \( E_8 \times E_8 \) heterotic string \[19\].

In this article we shall review some aspects of the parameter space orbifold construction. Our aim is introducing the relevant ideas to describe a class of chiral four dimensional type I vacua recently presented in ref. \[20\]. Let us begin with the simplest irrational case, namely the class of open descendants of the toroidal compactification on a one-dimensional circle of radius \( R \) \[21\]. The (halved) closed partition function for the internal part is given
by

\[ T = \frac{1}{2} \sum q^{p_L^2/2} q^{p_R^2/2} \eta(\tau) \eta(\bar{\tau}) , \]

where \( p_{L,R} = (\sqrt{\alpha'/2}) (m/R \pm nR/\alpha') \). The action of \( \Omega \) on the Hilbert space of states consists of interchanging holomorphic and antiholomorphic parts. The ‘crosscap constraint’ of refs. \[4, 6\] allows some freedom in the choice of \( \Omega \) eigenvalues. We limit ourselves to the simplest ‘standard’ Klein bottle projection with all sectors symmetrized, while bearing in mind that in general there exist several choices for the non-oriented projection. The Klein bottle contribution

\[ \mathcal{K} = \frac{1}{2} \sum \frac{e^{-\pi \alpha' (n \tau)^2}}{\eta(2i\tau)} \]

completes the spectrum of the unoriented closed sector. While the torus amplitude is modular invariant, an \( S \)-transformation turns the Klein partition function into the closed crosscap-to-crosscap amplitude

\[ \tilde{\mathcal{K}} = \frac{v}{2} \sum \frac{q^{(2nR)^2/4\alpha'}}{\eta(q)} \]

where \( v \) denotes the (dimensionless) volume \( R/\sqrt{\alpha'} \). Notice that only the closed sectors labelled by even winding modes can flow through the crosscap, as demanded by the freely acting involution. The presence in this channel of massless characters may signal some inconsistency due to ‘tadpoles’ of ‘unphysical’ fields \[22\], the cancellation of which demands the inclusion of open superstrings. For irrational values of \( R \), the closed spectrum is a superposition of an infinite number of sectors labelled by the winding mode and coupled with their conjugates in the amplitude \( (1) \). They can thus be consistently reflected in front of a boundary or, which is the same, can flow in the transverse annulus amplitude with (in general) individual reflection coefficients. This can be achieved by including continuous ‘Wilson Lines’ on the boundaries that shift the momentum according to the (constant) values of background internal gauge fields. To illustrate this mechanism, it is useful to think of eq. \( (1) \) as the lattice sum of the Type IIB superstring compactified on a circle. We are thus omitting (as always in this paper) all factors inert with respect to the modular properties. In order to give rise to marginal deformations, the Wilson lines must be chosen in a Cartan subalgebra of \( SO(32) \), the unbroken gauge group of the ten-dimensional type I superstring \[23\]. A standard parametrization of the Wilson line is, for each boundary,

\[ U = \exp \bigoplus_{r=1,\ldots,16} i \alpha^r \sigma_2 , \]

where
where $\alpha^r$ are proportional to the constant values $A^r$ of the gauge field in the Cartan subalgebra. A contribution $Tr(U)^2$ (due to the presence of two boundaries) must be included in the path integral for the annulus partition function and a contribution $Tr(U^2)$ (due to the presence of a single boundary of double length) must be included for the Möbius strip. For instance, choosing $\alpha^r = 0$ for $r = 1, \ldots, N$, and $\alpha^r = \alpha$ for $r = N + 1, \ldots, 16$, the annulus partition function reads
\[
\mathcal{A} = \left[ \left( \frac{(2N)^2}{2} + M\bar{M} \right) \sum_m e^{-\pi \alpha' \frac{m^2}{R^2}} + (2N)M \sum_m e^{-\pi \alpha' \frac{(m+2\alpha)^2}{R^2}} + \frac{M^2}{2} \sum_m e^{-\pi \alpha' \frac{(m+2\alpha)^2}{R^2}} + M \sum_m e^{-\pi \alpha' \frac{(m-2\alpha)^2}{R^2}} \right] / \eta \left( \frac{i\tau}{2} \right),
\]
where $a = QRA$, with $Q$ the charge and $A$ the internal gauge field. The Möbius strip partition function completes the projection in the non-oriented sector
\[
\mathcal{M} = - \left[ \frac{2N}{2} \sum_m e^{-\pi \alpha' \frac{m^2}{R^2}} + \frac{M}{2} \sum_m e^{-\pi \alpha' \frac{(m+2\alpha)^2}{R^2}} + \frac{\bar{M}}{2} \sum_m e^{-\pi \alpha' \frac{(m-2\alpha)^2}{R^2}} \right] / \hat{\eta} \left( \frac{i\tau}{2} + \frac{1}{2} \right),
\]
'hattred' terms being, as usual, the real basis for $\mathcal{M}$ \cite{24}. From the boundary-to-boundary amplitude
\[
\tilde{A} = \frac{v}{2} \sum_n \left[ 2N + Me^{2\pi i n} + \bar{M}e^{-2\pi i n} \right]^2 \frac{q^{4\alpha}}{\eta(q)}
\]
\[\eta(q)
\]
\[\text{it is easy to read the tadpole condition for massless R-R sector. Taking into account the powers of two from the modular measure and the cooperative action of $\tilde{A}$ with $\tilde{K}$ and $\tilde{M}$, tadpole cancellation fixes the overall sign in eq. (5) and results in}
\[2N + M + \bar{M} = 32.
\]
Wilson lines thus drive the breaking of the Chan-Paton symmetry by moving part of the charge to massive sectors. In this simple example, the unbroken gauge group is generically $SO(2N) \otimes U(16 - N)$. Extra massless modes appear, however, for specific values of the background gauge field, giving rise to a group enhancement phenomenon. For instance, in eqs. (5) a half-integer value of $a$ leads to $SO(2N) \otimes SO(32 - 2N)$, while an integer value of $a$ restores the unbroken $SO(32)$. It should be appreciated that the whole construction is manifestly compatible with planar duality and factorization of amplitudes, as can be tested analyzing the proper sewing constraints. In the rational case, the closed spectrum
can be organized in terms of a finite number of characters of an extended algebra. There is a one to one correspondence between the number of chiral sectors and the number of Chan-Paton charges, that collapse to a corresponding finite number if the boundaries are to preserve the symmetry. This is really a subtle issue, because in some non-diagonal cases the algebra of open states is actually an extended algebra even if the bulk algebra is not [5].

Open descendants can be built for more general toroidal compactifications than those on (products of) circles [21]. A subtlety emerges due to the presence of ‘torsion’ in the lattice defining the torus, encoded in the antisymmetric NS-NS tensor B. In fact, the presence of B makes in general the theory not left-right symmetric, a property recovered only for ‘quantized’ values of the (constant) B field, that ceases anyway to be a modulus. Although projected out from the closed non-oriented spectrum, the B field plays an important role in the open sector, because its presence reduces the size of the Chan-Paton group by a factor $2^{r/2}$, with $r$ the rank of B. This observation will be crucial in the construction of chiral four dimensional models.

The geometrical nature of consistency conditions is more evident if we extend the parameter space orbifold construction to (target-space) irrational orbifolds. While generalizations are straightforward, we shall confine our attention to the one-dimensional $Z_2$ orbifold of the circle of radius $R$, following ref. [25]. Several new ingredients enter the game. First of all, in order to understand the contributions in the open and unoriented sectors, it is necessary to properly define the combined action of the (target-space) orbifold group and the ‘twist’. This can be achieved in the following geometrical setting: Klein bottle, annulus and Möbius strip are $Z_2$-orbifolds of the double cover torus with respect to the action of an anticonformal involution, the geometrical counterpart of $\Omega$. Only the orbifold sectors compatible with the involution can ‘descend’ on the corresponding surface. They amount precisely to the sections of the $Z_2$ line bundle on that surface. This is exactly what other authors recently called ‘gauging the orientifold group’ [26]. Let us see explicitly the $Z_2$ case. Defining the characters

$$\xi_{++} = \frac{1}{2} \left( \frac{1}{\eta} + \Theta_{34} \right), \quad \xi_{--} = \frac{1}{2\sqrt{2}} \left( \Theta_{23} + \Theta_{24} \right),$$

$$\xi_{+-} = \frac{1}{2} \left( \frac{1}{\eta} - \Theta_{34} \right), \quad \xi_{-+} = \frac{1}{2\sqrt{2}} \left( \Theta_{23} - \Theta_{24} \right), \quad (9)$$

with $\Theta_{ij} = \theta_i^{1/2} \theta_j^{1/2} / \eta$ a combination of standard elliptic functions, the Klein bottle partition function is

$$\mathcal{K} = \frac{1}{4\eta} \left[ \sum_m e^{-\pi \alpha' (m^2)^2} + \sum_n e^{-\pi \alpha (nR)^2} \right] + \xi_{-+} + \xi_{--}. \quad (10)$$
Notice that the first two terms are, respectively, the standard symmetrizations of states with \( p_L = p_R \) and the projections of states with \( p_L = -p_R \), allowed only on the orbifold. The last two terms project the twisted (R-independent) sectors where the fixed point ambiguity between spin fields has been resolved in a diagonal fashion. It should be appreciated that, as expected from the experience with the rational case, in the tube terminating at two crosscaps only states with even windings or momenta (thus from ‘untwisted sector’) can flow, as encoded in the transverse Klein bottle amplitude:

\[
\tilde{\mathcal{K}} = \frac{1}{4\eta} \left[ v \sum_{n \neq 0} q^{\frac{1}{4}} (2nR)^2 + \frac{1}{v} \sum_{m \neq 0} q^{\frac{\alpha'}{4}} (\frac{2m}{\alpha'})^2 \right] + \frac{1}{4} (\sqrt{v} + \frac{1}{\sqrt{v}})^2 \xi_{++} + \frac{1}{4} (\sqrt{v} - \frac{1}{\sqrt{v}})^2 \xi_{+-} \quad . \tag{11}
\]

Notice also that the coefficients in (11) are, as expected, squares of reflection coefficients in front of the crosscap. The sections in the open sector are very interesting because the orbifold allows for open strings with generalized boundary conditions. In particular, three kinds of open strings are present, namely strings with (standard) Neumann conditions at both ends (NN), strings with Dirichlet conditions at both ends (DD) and strings with mixed conditions (ND). Recently open strings with Dirichlet conditions have received much attention because their ends live on dynamical hyperplanes (Dirichlet p-branes or D-branes [27] [28]) whose excitations are open string modes. D-branes play also a fundamental role in connection with non-perturbative aspects of superstring theories: they are in particular carriers of Ramond-Ramond charges [29]. There is another crucial ingredient in the construction: the orbifold group acts in a non-trivial fashion on the Chan-Paton charge space. Indeed, in the simplest cases, the corresponding charge assignment is driven by the fusion rules, but in general it is more involved [5]. If we parametrize the multiplicities entering the partition function for the ‘Neumann’ and ‘Dirichlet’ Chan-Paton groups as

\[
Tr(1_N) = N_+ + N_- \quad , \quad Tr(1_{D_i}) = D_{i+} + D_{i-} \quad , \\
Tr(R_N) = N_+ - N_- \quad , \quad Tr(R_{D_i}) = D_{i+} - D_{i-} \quad , \quad \tag{12}
\]

where \( i \) labels the two fixed points, the annulus reads

\[
\mathcal{A} = \frac{1}{4\eta} [(N_+ + N_-)^2 \sum_{m \neq 0} e^{-\pi \tau \alpha'} (\frac{m}{\alpha'})^2] + \sum_{i,j} (D_{i+} + D_{i-}) (D_{j+} + D_{j-}) \sum_{n \neq 0} e^{-\frac{\pi R^2}{\alpha} (n + \Delta_{ij})^2} \\
+ \left[ \left( \frac{N_+^2}{2} + \frac{N_-^2}{2} + \sum_{i} \left( \frac{D_{i+}^2}{2} + \frac{D_{i-}^2}{2} \right) \right) \xi_{++} + [N_+ N_- + \sum_{i} (D_{i+} D_{i-})] \xi_{+-} \right]
\]
\[ \sum_i (N_i D_{i+} + N_i D_{i-}) \xi_{+-} + \sum_i (N_i D_{i+} + N_i D_{i-}) \xi_{-+} \] 

(13)

The \( \Delta_{ij} \) vanish if both ends of the Dirichlet strings are at the same fixed point, while they are \( 1/2 \) if the ends are at different fixed points. It is simple to verify that the transverse amplitude is a linear superposition of an (infinite) number of characters and the coefficients are perfect squares of the reflection coefficients in front of the boundaries, as demanded by sewing constraints. Notice that only (NN) strings and (DD) strings with ends at the same fixed point can flow in the Möbius strip. The other types of open strings are in fact oriented and do not contribute to the Möbius partition function

\[
\mathcal{M} = \frac{-1}{4\hat{\eta}} \left[ (N_+ + N_-) \sum_{m \neq 0} e^{-\pi \tau \alpha'(mR)^2} + \sum_i (D_{i+} + D_{i-}) \sum_{n \neq 0} e^{-\pi \tau R^2 n^2} \right] - \left[ \frac{N_+}{2} + \frac{N_-}{2} + \sum_i \left( \frac{D_{i+}}{2} + \frac{D_{i-}}{2} \right) \right] \hat{\xi}_{++}.
\]

(14)

The setting just illustrated concerns real charges. The involution, however, allows in this case additional phases which turn the Chan-Paton charges into complex ones. Consistency of the vacuum channel is ensured by the equality of the multiplicities of charges and anticharges for each Chan-Paton factor. Coming back to the real case in eqs. (13) and (14), the transverse amplitudes give rise to two tadpole conditions. The first is actually a pair of conditions, due to the incommensurability of the volume and the inverse volume. These conditions typically fix the total dimensionality of the Chan-Paton charge space both for Neumann and Dirichlet sector in a way analogous to eq. (8). The second tadpole condition is actually present only when some twisted sector becomes massless, and it is a constraint linking Neumann and Dirichlet groups.

In order to use this construction in supersymmetric models, a simultaneous twist of some ‘space-time’ coordinates is needed in such a way to preserve the antiperiodicity of the supercurrent. Open descendants of left-right symmetric superstring theories in arbitrary dimensions can be constructed in this way. In ref. [24] a number of models in six dimensions have been analyzed starting from convenient rational points in the moduli space. They exhibit a rich structure of Chan-Paton gauge groups. Introducing suitable (discrete) open-string Wilson lines, it is possible to enhance the Chan-Paton group up to \( U(16) \otimes U(16) \). The distinctive feature of these type I models is the presence in the perturbative spectra of a variable number of tensor multiplets. They play a fundamental role in the generalized Green-Schwarz anomaly cancellation mechanism [30]. Recently, using parameter-space orbifolds of Gepner models, many other examples of type I vacua in six
dimensions have been constructed with variable numbers of tensor multiplets (including zero) and rich patterns of Chan-Paton symmetry breaking [31]. Several authors have also obtained six-dimensional type I models using essentially the orbifold construction previously discussed [26] [32] [33]. Many of these models coincide with those of ref. [24] if specialized to rational values of the moduli.

Finally let us describe the only available class of four dimensional type I chiral models, recently discovered by merging all the ideas above [20]. We start from the $Z$-orbifold reduction of the type IIB superstring. In particular, the toroidal lattice comprises three copies (of sizes $R_i$) of a two-dimensional hexagonal lattice, where $Z_3$ has a natural action. Moreover, we choose a vanishing NS-NS antisymmetric tensor in order to avoid small-sized Chan-Paton groups (recall that the quantized values of $B$ reduce their maximum size by a factor $2^{r/2}$). The closed spectrum must exhibit $N = 2$ supersymmetry, then reduced to $N = 1$ by the unoriented truncation. To this end, as anticipated, it is necessary to twist some internal world-sheet fermions as well. The (light-cone) $SO(8)$ characters must be decomposed with respect to $SO(2) \otimes SU(3) \otimes U(1)$. Introducing

$$
\Xi_{0,\epsilon}(q) = \left( \frac{A_0 \chi_0 + \omega^\epsilon A_+ \chi_+ + \bar{\omega}^\epsilon A_- \chi_-}{H_{0,\epsilon}^{1/3}} \right)(q)
$$
$$
\Xi_{+,-}(q) = \left( \frac{A_0 \chi_0 + \omega^\epsilon A_+ \chi_0 + \bar{\omega}^\epsilon A_- \chi_-}{H_{+,-}^{1/3}} \right)(q)
$$
$$
\Xi_{-,+}(q) = \left( \frac{A_0 \chi_0 + \omega^\epsilon A_+ \chi_0 + \bar{\omega}^\epsilon A_- \chi_-}{H_{-,+}^{1/3}} \right)(q)
$$

(15)

where $\{A_0, A_+, A_-\}$ are supersymmetric characters of conformal weights $\{1/2, 1/6, 1/6\}$ respectively, $\{\chi_0, \chi_+, \chi_-\}$ are level-one SU(3) characters of conformal weights $\{0, 1/3, 1/3\}$ respectively, and

$$
H_{0,\epsilon}(q) = q^{1/2} \prod_{n=1}^{\infty} (1 - \omega^\epsilon q^n)(1 - \bar{\omega}^\epsilon q^n)
$$
$$
H_{+,-}(q) = H_{-,+}(q) = 3^{-1/4} q^{-1/3} \prod_{n=0}^{\infty} (1 - \omega^\epsilon q^{n+1/3})(1 - \bar{\omega}^\epsilon q^{n+1/3})
$$

(16)

with $\epsilon = 0, \pm 1$ and $\omega = e^{2\pi i/3}$, one can write the closed orbifold partition function in the form

$$
T = \frac{1}{3} \Xi_{0,0}(q) \Xi_{0,0}(\bar{q}) \sum q^{1/2} p_{L,a} G^{ab} p_{L,b} q^{1/2} p_{R,a} G^{ab} p_{R,b} + \frac{1}{3} \sum_{\epsilon = \pm 1} \Xi_{0,\epsilon}(q) \Xi_{0,\epsilon}(\bar{q})
$$
$$
+ \frac{1}{3} \sum_{\eta = \pm 1} \sum_{\epsilon = 0, \pm 1} \Xi_{\eta,\epsilon}(q) \Xi_{-\eta,-\epsilon}(\bar{q})
$$

(17)

where $G$ is the lattice metric. Notice that the Klein-bottle projection contains only the sublattice with $p_L = p_R$. No other possibilities are allowed by the $Z_3$ involution. By
defining for $\sigma = 0, \pm 1$ and $\eta = 0, \pm 1 \ (mod \ 3)$

$$\rho_{\sigma,\eta} = \sum_{\epsilon=0,\pm 1} \frac{\omega^{\eta \epsilon}}{H_{\sigma \epsilon}^3}$$  \hspace{0.5cm} (18)

we can introduce the following $R_i$-independent untwisted characters

$$\Lambda_{0,\eta} = A_0 \chi_0 \rho_{0,\eta} + A_+ \chi_+ \rho_{0,\eta-1} + A_- \chi_- \rho_{0,\eta+1} \ , \hspace{0.5cm} (19)$$

and the following (obviously $R_i$-independent) twisted characters

$$\Lambda_{+,\eta} = A_0 \chi_+ \rho_{+,\eta} + A_+ \chi_0 \rho_{+,\eta-1} + A_- \chi_- \rho_{+,\eta-1} \ ,$$

$$\Lambda_{-,\eta} = A_0 \chi_- \rho_{-,\eta} + A_- \chi_0 \rho_{-,\eta-1} + A_+ \chi_+ \rho_{-,\eta-1} \ . \hspace{0.5cm} (20)$$

In terms of (19) and (20), it is relatively easy to describe the perturbative spectrum of the open descendants. The Klein bottle is

$$K = \frac{1}{6} \Xi_{0,0} \sum' e^{-\pi \tau \alpha^* m_a G_{ab} m_b} + \frac{1}{2} \Lambda_{0,0}$$ \hspace{0.5cm} (21)

where the primed sum is over the non-zero modes. The form of $K$ makes evident how in the closed GSO projection each character is paired with its conjugate, but $\Lambda_{0,0}$ is the only self-conjugate one. We can thus anticipate that all characters flow in the tube as well as in the transverse Klein amplitude due to the presence of only $\Lambda_{0,0}$ in eq. (21). Compatibility with the mirror-like involution defining the annulus selects the sections corresponding to strings with Neumann boundary conditions at both ends. Only D-9-branes are thus present for this class of models and the annulus partition function takes the form

$$A = \frac{1}{6} (N + M + \bar{M})^2 \Xi_{0,0} \sum' e^{-\pi \tau \alpha^* m_a G_{ab} m_b} + \frac{1}{2} (N^2 + 2M\bar{M}) \Lambda_{0,0}$$

$$+ \frac{1}{2} (M^2 + 2NM) \Lambda_{0,+} + \frac{1}{2} (\bar{M}^2 + 2N\bar{M}) \Lambda_{0,-} \ . \hspace{0.5cm} (22)$$

The open spectrum is properly projected by the Möbius-strip amplitude. A suitable basis of ‘hatted’ real characters can be defined, barring some subtleties, with the aid of the rational model at $R_i = \sqrt{3}$, where an $SU(3)^{\otimes 3}$ symmetry is present. In terms of these characters, the result is

$$M = -\frac{1}{6} Tr(1) \hat{\Xi}_{0,0} \sum' e^{-\pi \tau \alpha^* m_a G_{ab} m_b} - \frac{N}{2} \hat{\Lambda}_{0,0} - \frac{M}{2} \hat{\Lambda}_{0,+} - \frac{\bar{M}}{2} \hat{\Lambda}_{0,-} \ , \hspace{0.5cm} (23)$$

where the negative signs anticipate the tadpole cancellations. $K$, $A$ and $M$ lead to sensible transverse channel amplitudes, and the decoupling of unphysical states is ensured if

$$N + M + \bar{M} = 32$$

$$2N - M - \bar{M} = -8 \ , \hspace{0.5cm} (24)$$
respectively from the untwisted and twisted massless sectors. Because of the (numerical)
equality of $M$ and $\bar{M}$, eqs. (24) are solved by $N = 8$ and $M = 12$.

Let us analyze the resulting (closed and open) massless spectrum: the original type
IIB closed spectrum is truncated to an $N = 1$ supergravity multiplet, coupled to the
universal linear multiplet and to 9 chiral multiplets from the untwisted sector, as well
as to 27 chiral multiplets (one for each fixed point) from the twisted sector. The open
unoriented spectrum exhibits a Chan-Paton gauge group $SO(8) \otimes SU(12) \otimes U(1)$, with
three generations of chiral multiplets in the $(8, 12^*)_{-1}$ and in the $(1, 66)_2$. Absence of
anomalies is guaranteed by tadpole cancellations, apart from the $U(1)$ factor, which is
disposed of by a Higgs mechanism giving a large mass to the corresponding $U(1)$ gauge
boson. This is the analogue of what happens in the heterotic theory [34], and in fact the
relation between heterotic and type I dilaton in $d$ dimensions is [20]

$$
\phi_I^{(d)} = \frac{6 - d}{4} \phi_H^{(d)} - \frac{(d - 2)}{16} \log \det G_H^{(10 - d)}
$$

where $G_H^{(10 - d)}$ is the internal metric in the heterotic-string frame. The duality between
Type I and Heterotic models in $d = 10$ survives in $d = 4$, but becomes a weak-weak
coupling duality rather than a strong-weak one. In [20] a candidate heterotic dual to
this chiral type I model is described, with additional chiral multiplets in the $(8_c, 1)$ from
twisted sectors. A final remark is concerned with the Kähler manifold of the untwisted
scalars in the low-energy supergravity [35]. This must be a Kähler submanifold of the
corresponding quaternionic manifold of the parent type IIB orbifold. The unoriented
truncation, however, is highly non-trivial due to several mixtures between moduli to give
the ‘real slice’. The manifold can be uniquely selected to be $Sp(8, R)/(SU(4) \times U(1))$.
The $SU(4)$ factor is suggestive of the relation between these type I vacua and the putative
$(12-d)$ F-theory compactified on Calabi-Yau fourfolds.

To summarize, we have analyzed some instances of perturbative type I vacua stressing
the elegance of the construction and some ‘exotic’ features emerging from it. The interest
in the subject is increasing, in light of some recent results suggesting that D-branes and
associated open-string excitations play a role as fundamental degrees of freedom in the
‘complete’ non-perturbative formulation of superstring theories [28].

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