Mathematical simulation of a pressure field exemplified by dual porosity reservoir

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Abstract. The authors analyze the process of fluid transfer in a formation using the example of a porous fractured reservoir rock. A reservoir of this type has a natural-fracture system and is described by a dual porosity model. The most detailed description of a filtration process is provided by Warren-Root equations which analyze fluid redistribution between the matrix and the natural fracturing pattern. In this paper we develop a numerical model of fluid flow processes for a fractured-porous reservoir. The finite difference method is applied to solve partial differential equations. The problem is approximated by an implicit difference scheme. The system of linear algebraic equations at each time layer is solved by the matrix sweep method.

1. Introduction

Well test is used to study the productive layers of production and injection wells during their test, development, and operation in order to obtain information about their productivity, injection capacity, and filtration parameters, to determine the boundaries of the formation and features of drainage zones, type of reservoir bed, and formation anisotropy by permeability. The objective of analyzing and interpreting the well test data is to determine the system parameters based on the known input and output signals. The input signal refers to the change in the well operation mode, and the output signal refers to the response of the "well - formation" system in the form of the bottomhole pressure change. This problem is called the inverse problem of hydrodynamics. The main types of traditional pressure transient analysis are the pressure buildup curve for production wells and the pressure drawdown curve for injection wells. Essentially, these methods analyze the data of the bottomhole pressure curves with a change in flow rate or injection capacity during the well shutdown period.

Hydrodynamic methods for determining the parameters of the formation and the formational pressure for dual porosity reservoirs differ significantly from the standard methods due to high heterogeneity. Fractured reservoirs are characterized by an intensive exchange of fluid between cracks and porous blocks [¹], which makes particular adjustments to the well-known traditional methods of determining filtration parameters.
Currently, much attention is paid to the choice of technologies and the improvement of the development of fractured-porous reservoirs based on mathematical modeling [2,3]. Therefore, the study of the processes occurring in fractured-porous reservoirs is a relevant objective.

As a rule, the properties of the productive formation of such reservoirs vary considerably from well to well [4]. The processes of mass transfer in fractured-porous reservoirs shall be studied to select an effective technology for extracting oil from these formations. The exchange of fluids between the blocks (hereinafter referred to as the matrix) and the natural fracturing pattern shall be taken into account in the process of filtering in the reservoirs of this type. The Warren Root model is used in this work to study the system [5]. The model considers two systems with different values of geometric dimensions and permeability properties. The Warren Root model assumes that the porous reservoir is represented by similar rectangular parallelepipeds with high porosity and low permeability. The matrix is divided by natural fracturing pattern with high permeability and low porosity. It is also assumed that fluid moves to the well through the fracture network, and the matrix feeds the entire natural fracturing pattern continuously. The redistribution of fluid between the matrix and fractures depends on the shape and size of the matrix blocks; the smaller the blocks, the easier the fluid flows between them [6]. The piezoelectricity equations are used to describe the liquid filtration mechanism in the “fracture network-matrix” system (6–7) [7].

2. Research objective

\[
\frac{\partial P_f}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{k_f}{\mu} \frac{\partial P_f}{\partial r} \right) - S \frac{k_m}{\mu} (P_f - P_m) = 0
\]

(1)

\[
\frac{\partial P_m}{\partial t} + S \frac{k_m}{\mu} (P_f - P_m) = 0
\]

(2)

\[
\left. P_f \right|_{r=0} = P_0 - \Delta P, \quad P_f \bigg|_{r=\infty} = P_0,
\]

(3)

\[
S = \frac{4 \cdot n (n + 2)}{3 \cdot a \cdot b \cdot c}
\]

(4)

\[
L = \frac{3 \cdot a \cdot b \cdot c}{a \cdot b + b \cdot c + c \cdot a}
\]

(5)

where \( P \) is the pressure (MPa), \( \phi_f \) is the porosity of fracture network (unit fraction), \( \phi_m \) is the matrix porosity (unit fraction), \( c_f \) is the compressibility of fracture network (1/MPa), \( c_m \) is the compressibility of matrix (1/MPa), \( k_f \) is the permeability of fracture network (m²), \( k_m \) is the permeability of matrix (m²), \( \mu \) is the oil viscosity (Pa·s), \( P_f \) is the formational pressure in fracture network (MPa), \( P_m \) is the formational pressure in matrix (MPa), \( h \) is the effective formation thickness, \( q \) is the fluid rate (m³/day), \( \pi \approx 3.14 \), \( S \) is the fractured rocks coefficient (1/m²), \( n \) is the number of mutually perpendicular sets of joints, \( L \) is the size of blocks (m), \( a \) is the matrix block side length (m), \( b \) is the matrix block side width (m), \( c \) is the matrix block side height (m).

The resulting model (1-5) characterizes the redistribution of pressure in the fracture and the fracture network [8, 9, 10]. There is no general analytical solution of the system of differential equations (1-2). It has a solution only for a particular case, which corresponds to the time [9]:

\[
t_D > 100 \omega \text{ at } \lambda \ll 1 \text{ and } t_D > 100 \lambda - \frac{1}{\lambda} \text{ for } \omega \ll 1
\]

(6)

Let us consider next the numerical solution of the problem (1-2).

The numerical solution of the problem (1-2) considering the boundary conditions (3) was obtained and analyzed in the work [6]. The finite difference method is used for the numerical solution of the linear system (1-2). Uniform grid step is introduced both for the temporal and spatial coordinates, Explicit (7-8) and implicit (10-11) difference schemes were used for the approximation of differential equations [11-16].

\[
\frac{P_{mi}^{j+1} - P_{mi}^j}{\tau} = S \frac{k_m}{\mu} \left( P_{mi}^j - P_f^j \right)
\]

(7)
Stability has been proved for both cases. In the case of using an explicit scheme, the time step shall be limited by the stability condition, or the Courant condition (9).

\[
\tau \leq \frac{h^2 \cdot \varphi_f \cdot c_{ef}}{2 \cdot k_f}
\]  

(9)

Such limitation leads to an increase in the amount of calculations. As a consequence, the use of an explicit difference scheme is inappropriate in this case. Therefore, an implicit difference scheme (7-8) was chosen for further research [17-22].

The resulting scheme is reduced to a system of linear algebraic equations with a block tridiagonal matrix. At each time layer the system is solved by the matrix sweep method [23-25]. The elements of the tridiagonal matrix are matrices with the dimension of 2×2. The resulting matrices were presented in our previous work [6].

3. Results of numerical simulation

Let us consider an example. Initially a production vertical well was turned into production with a flow rate of 250 m³/day. The well exploits one formation with the thickness of 20 meters. After operating for 100 days, the well is shut down at the bottom for hydrodynamic research by the pressure recovery curve method. At the time of shutdown for the research, at q = 0 m³/day, the formation pressure begins to recover. It is assumed that a constant pressure is maintained at the boundary of the formation, and there is no influence from neighboring operating wells. The following initial parameters (see Table 1) are defined for the formation, the well and the fracture network.

The results of numerical simulation are provided in figure 1. The calculation of the dynamics of pressure change with the time is provided for the initial data (Table 1) during the well operation and during the hydrodynamic research by the pressure recovery curve method [26, 27, 28]. The grid step is uniform.

Figure 1. Dynamics of pressure change during the well operation and shut-in.

Figure 2 shows the calculation of the pressure recovery curve for a 100-hour research for various steps along the spatial coordinate. We observe that the smaller the step is, the more accurate is the
result. The solutions on different steps are compared with the analytical solution. It was found that the pressure recovery curve obtained for a step of 0.001 agrees well with the analytical solution curve.

**Table 1.** The following initial and boundary parameters.

| Parameters                                    | Value    | Unit of measurement |
|-----------------------------------------------|----------|---------------------|
| Oil viscosity, $\mu$                          | 0.7E-3   | Pa·s                |
| Initial fracture pressure, $P_{f0}$           | 25.0E6   | MPa                 |
| Initial pressure in the matrix, $P_{m0}$      | 25.0E6   | MPa                 |
| Permeability of fractures, $k_f$              | 100.0E-15| m²                  |
| Permeability of the matrix, $k_m$             | 1.0E-15  | m²                  |
| Compressibility of fractures, $c_{ef}$        | 3.5E-9   | 1/Pa                |
| Compressibility of the matrix, $c_{em}$       | 3.8E-10  | 1/Pa                |
| Compressibility of oil, $c_o$                 | 3.0E-9   | 1/Pa                |
| Compressibility of water, $c_w$               | 4.0E-9   | 1/Pa                |
| Porosity of the natural fractures system, $\phi_f$ | 0.01   |                     |
| Porosity of the matrix, $\phi_m$              | 0.15     |                     |
| Number of mutually perpendicular fracture groups, $n$ | 3       |                     |
| Block length, $a$                             | 20       | m                   |
| Width of the block, $b$                       | 20       | m                   |
| Block height, $c$                             | 1        | m                   |
| Reservoir thickness, $h$                      | 20       | m                   |
| Radius of the well, $r_w$                     | 0.1      | m                   |
| Well supply circuit, $R_e$                    | 100.0    | m                   |

**Figure 2.** Pressure recovery curves for various steps along the spatial coordinate. Comparison with the analytical solution.

Due to the fact that at the initial stage of the research the pressure has a sharp gradient, it becomes necessary to make the grid near the well finer. Thus, we consider a non-uniform grid along the spatial coordinate.
We will use a corner-point geometry to make the approximation of the difference scheme consistent. To do this, we first define the nodes calculated according to the formula [11]:

$$\frac{r_{i+1}}{r_i} = \left( \frac{r_i}{r_0} \right)^{\frac{1}{N-1}}$$  (12)

The arithmetic mean of the coordinates, the geometric mean and the logarithmic mean can be used to determine the boundary. When using the logarithmic mean, it is necessary to use certain physical concepts, which makes the task closer to reality. Therefore, the logarithmic mean is used below to determine the boundary of the blocks connecting two nodes:

$$r_{i+\frac{1}{2}} = \frac{r_i - r_{i-1}}{\ln(\frac{r_i}{r_{i-1}})}$$  (13)

When using an explicit difference scheme [12], the time step size $\tau$ should be additionally limited by the condition (9), which leads to a significant increase in the amount of calculations approximately by 3 orders of magnitude. Therefore, we will apply, as in the previous case, the implicit difference scheme. Thus, we obtain the following difference scheme:

$$\frac{\varphi_f C_f p_i^{i+1} - p_i^i}{\tau} = \frac{1}{\mu_i \Delta r_i} \left[ r_{i+1/2} \left( \frac{p_i^{i+1} - p_{i+1}^{i+1}}{\Delta r_{i+1/2}} \right) + r_{i-1/2} \left( \frac{p_i^{i+1} - p_{i-1}^{i+1}}{\Delta r_{i-1/2}} \right) \right] + \frac{k_m}{\mu_i} \left( p_{m_i}^{i+1} - p_f^i \right)$$  (14)

The use of a non-uniform grid will allow to break the bottomhole formation zone into small cells, which will allow to define the influence of the effects like the skin effect and the influence of the wellbore. The correct information on pressure at initial time is very important for the analysis of hydrodynamic research, since it is the first hours of research that allow us to assess the contamination of the bottomhole formation zone.

### 4. Conclusion

This paper analyzes the method of numerical solution for a mass transfer model in a fractured reservoir. The Warren-Root model is taken as a basis. Explicit and implicit difference schemes are analyzed. Based on the analysis, it was revealed that the use of the implicit difference scheme is the most efficient, since the explicit difference scheme requires to choose a grid that satisfies the stability condition. We shall also note that the use of an implicit difference scheme allows to choose a non-uniform grid. The numerical model was solved using the matrix sweep method. As a result of the calculation, pressure curves over time were constructed. An implicit difference scheme for a non-uniform grid was proposed.

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