Discrete Stochastic Approximation for Cross-layer Transmission Control in Wireless Communications

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Abstract

This paper proposes the use of discrete stochastic approximation (DSA) algorithm for a cross-layer adaptive modulation problem in wireless communications. In this system, the throughput in the physical (PHY) layer and quality of service (QoS) incurred by the queueing effects in the data link layer are required to be optimized simultaneously and in the long run. By assuming Markov decision process (MDP) modeling, we prove that the optimal transmission policy is characterized by a queue threshold vector and can be determined by a multivariate discrete convex optimization problem if the dynamic programming (DP) is submodular. We then propose to use DSA, a subgradient-based stochastic approximation (SA) algorithm with proven convergence rate, for approximating the optimal value of the queue threshold vector. By an application in a cross-layer transmission control problem in a network-coded two way relay channel (NC-TWRC), we compare the performance of DSA with that of simultaneous perturbation stochastic approximation (SPSA), the commonly used SA algorithm for threshold policy optimization problems. The results show that DSA converges faster than SPSA resulting in lower and controllable computational cost.

Index Terms

discrete stochastic approximation, dynamic programming, $L^3$-convexity, Markov decision process, optimal monotonic/threshold policy, submodularity.

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Fig. 1. Cross-layer transmission control system. A scheduler adapts the transmission rate according to the optimization concerns in both layers, e.g., packet/symbol delay and queue overflow in data link layer and transmission error and spectral efficiency in physical (PHY) layer, in the long run.

## I. INTRODUCTION

Consider the transmission control system in Fig. 1. It is assumed that messages encapsulated in equal length units, e.g., packets or symbols, from a higher layer (say, application layer) arrive at data link layer randomly. The packets/symbols are temporarily stored in a first-in-first-out (FIFO) queue before the transmission in the physical (PHY) layer. A scheduler controls the transmission rate, the number of packets per transmission or bits per symbol, based on the instantaneous gain of the wireless fading channel, queue occupancy and their expectations in the future. The objective is to find a policy or decision rule that optimizes packet/symbol delay and/or queue overflow in the data link layer and the transmission error rate and/or spectral efficiency in the PHY layer simultaneously and in the long run. The system in Fig. 1 is called cross-layer adaptive modulation in [1], [2] because it not only incorporates the idea of adaptive modulation in the PHY layer [3], [4], i.e., adjusting the modulation scheme in accordance with the time varying feature of wireless channel to enhance the throughput, but also takes into account the quality of service (QoS) incurred by the queueing effects in the data link layer.

To increase spectral efficiency, most communication networks adopt a multiple channel access scheme, and the system in Fig. 1 is usually extended to a multi-user cross multimedia access control (MAC) and PHY layer adaptive modulation system in Fig. 2. The transmission scheme in Fig. 2 can be categorized into centralized and decentralized control. For example, the time-division multiple access (TDMA) network in [5], multiple-input and multiple-output (MIMO) systems in [6], [7] and network-coded two way relay channels (NC-TWRCs) in [8]–[10] adopted centralized scheduling where the decisions of schedulers 1 to M in Fig. 2 were jointly made by a central controller based on the information of queue occupancies and channel gains in...
the entire system, and the dynamic channel access problems in [11], [12] adopted decentralized scheduling where it was assumed that cognitive radio $i$, i.e., scheduler $i$ in Fig. 2, adapted the transmission rate based on local information, e.g., the occupancy of queue $i$ and/or the observed channel gains. By assuming i.i.d. message arrival processes and finite-state Markov chain (FSMC) [13] modeled channels, the majority of these studies formulated the system by a Markov decision process (MDP) and considered dynamic programming (DP) algorithms for optimal policy searching.

However, the MDP model for the optimal control problem in Fig. 2 usually has either high dimension or large scale. For example, in the MIMO system in [6], [7] the MDP model had large state space containing the composite states of MIMO channels, and in the dynamic channel access problem in [11], [14] the state variable was a $2M$-tuple ($M$ queue states and $M$ channel states). In these systems, the curse of dimensionality\footnote{The complexity grows exponentially with the cardinality of the system parameters [15].} of DP becomes more evident. The high complexity may severely overload the processor and make the optimization problem intractable. To relieve the computation load, most related studies, e.g., [7], [10], [12], [16], [17], focus on proving the monotonicity of the optimal policy in queue occupancy/state by submodularity, a property of DP that commonly occurs in the departure control of queueing systems. It is because that in this case the optimal policy is a switching curve or plane that is fully characterized by a

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Fig. 2. General diagram of cross-layer transmission control in multiple access channels where $M$ users transmit simultaneously. The objective is to optimize the long-term cross MAC-PHY layer losses for all users.
set of optimal queue thresholds, and the optimization problem has much lower complexity than DP.

But, a natural question that follows is how to effectively determine these thresholds. Among those works that addressed this question, e.g., [12], [18], the most prevailing solution was simultaneous perturbation stochastic approximation (SPSA). SPSA [19] is a gradient-based stochastic approximation (SA) algorithm for multivariate optimization problems, where in each iteration the gradient is approximated by noisy measurements of the objective function at random perturbations of the current estimation. Without knowing the shape of the objective function, SPSA was proved to converge almost surely to a local minimum in [20] and achieve global convergence in probability in [21]. But, there are two shortcomings when it is applied to the queue threshold optimization problem in Fig. 1 and 2. One is that SPSA is originally designed for continuous, instead of discrete, optimization. There are some studies considering modified SPSA for discrete problems, e.g., a projection function was used to round each continuous estimation to the closest integer points in [18]. But, the assumptions of the convergence of the modified SPSA are usually hard to verify [22]. The other shortcoming, as pointed out in [23], is that the rate of convergence (ROC) of SPSA cannot be certainly determined. This could be a problem in numerical analysis where the error of the approximation process is an important index for evaluating the efficiency of an algorithm. It is also difficult to determine the expected computation budget\(^2\) without ROC because we do not know how close an estimation is to the optimizer.

This paper considers the MDP modeled transmission control problem in Fig. 2 where the schedulers are making decisions as to whether or not to transmit. The purpose is to propose the use of a discrete stochastic approximation (DSA) algorithm for searching the optimal queue threshold policy. We start the work with a study on the single-user cross-layer transmission control system in Fig. 1 the fundamental building block of Fig. 2. We model the system in Fig. 1 by an MDP and formulate the optimal decision making process by a multivariate discrete convex optimization problem. We then extend the MDP model and discrete convex optimization problem to the multi-user system in Fig. 2 and use a DSA algorithm to approximate the optimal

\(^2\)The expected computation budget for an approximation algorithm is the minimum number of iterations that is required to produce an estimation with the accuracy above a given threshold. In some application scenarios where the computing resources are limited, the computation budget needs to be determined or predicted in advance.
transmission policy. The main results are as follows:

- We prove that the optimal policy for the transmission control problem in Fig. 1 is characterized by a queue threshold vector if the DP function is submodular. We prove that this optimal queue threshold vector can be searched by a multivariate optimization problem where the objective function is $L^\frac{1}{2}$-convex, a type of discrete convexity [24]. We show that these results are also valid in Fig. 2, the adaptive transmission control in multiple access channels.

- We propose the use of DSA in [22] for searching the optimal queue threshold vector. We show that DSA converges faster than SPSA by an application to a transmission control problem in a network-coded two-way relay channel (NC-TWRC), an example of Fig. 2.

The rest of the paper is organized as follows. In Section II we describe the MDP formulation of Fig. 1 clarify the assumptions of the system, state the optimization objective and describe DP algorithm. In Section III we formulate a discrete convex optimization problem based on the submodularity of DP. In Section IV we extend the MDP to model the multiple channel access system in Fig. 2 propose to use a DSA algorithm for searching the optimal policy and compare its convergence performance with SPSA by running experiments on an NC-TWRC system in [8].

II. MDP FORMULATION AND DYNAMIC PROGRAMMING

Consider the cross-layer transmission control system with wireless multipath fading channel in Fig. 1. Let time be divided into small intervals, called decision epochs and denoted by $t$. We assume the following.

Assumption 2.1: The decision making process is infinitely long, i.e., $t \in \{0, 1, \ldots, \infty\}$.

Assumption 2.2: Let $\{f(t)\}$ be i.i.d. random message arrival process, where $f(t)$ denotes the number of packets/symbols that arrive in the FIFO queue at $t$. The scheduler knows the statistics of $\{f(t)\}$.

Assumption 2.3: $\{\gamma(t)\}$, the variation process of instantaneous SNR of the fading channel, is i.i.d. and independent of $\{f(t)\}$. Let the full variation range of $\gamma(t)$ be partitioned into $K$ non-overlapping regions $\{[\Gamma_1, \Gamma_2), [\Gamma_2, \Gamma_3), \ldots, [\Gamma_K, \Gamma_{K+1})\}$, where $\Gamma_{K+1} = \infty$. Region $[\Gamma_k, \Gamma_{k+1})$ is called channel state $k$. Denote $h(t)$ as the channel state variable at decision epoch $t$. We say that $h(t) = k$ if $\gamma(t) \in [\Gamma_k, \Gamma_{k+1})$. Let the channel be modeled by an FSMC [13], where
$P_{h(t)h(t+1)} = Pr(h(t+1)|h(t))$, the channel transition probability, is determined by channel parameters and statistics. The scheduler obtains the statistics of \{$h(t)$\}. For all $t$, the scheduler knows the value of $h(t)$ before making the decision.

**Assumption 2.4:** Denote $a(t) \in A = \{0 \text{ – transmit}, 1 \text{ – not transmit}\}$ the action taken by the scheduler at $t$. Let $\delta$ be a nonnegative integer constant. Whenever $a(t) = 1$, $\delta$ packets/symbols are sent.

Let the system in Fig. 1 be modeled by a discounted infinite-horizon MDP. The system state at $t$ is $x(t) = (b(t), h(t)) \in \mathcal{X} = \mathcal{B} \times \mathcal{H}$, where $b(t) \in \mathcal{B} = \{0, 1, \ldots\}$ is the queue occupancy-state, $h(t) \in \mathcal{H} = \{1, 2, \ldots, K\}$ is the channel state and $\times$ denotes the Cartesian product. The state transition probability $P_{x(t)x(t+1)} = Pr(x(t+1)|x(t), a(t))$ is given by

$$P_{x(t)x(t+1)} = P_{b(t)b(t+1)} P_{h(t)h(t+1)}, \quad (1)$$

where $P_{b(t)b(t+1)}$ is the queue state transition probability. In a queue departure control system, the state variation is usually governed by the Lindley equation \cite{25}

$$b := \max\{b - a\delta, 0\} + f. \quad (2)$$

Therefore, the queue state transition probability can be determined by the statistics of \{$f(t)$\} as

$$P_{b(t)b(t+1)} = Pr(f(t) = b(t+1) - \max\{b(t) - a(t)\delta, 0\}). \quad (3)$$

The immediate cost $c: \mathcal{X} \times A \mapsto \mathbb{R}_+$ is given by

$$c(x(t), a(t)) = c_q(b(t), a(t)) + c_{tr}(h(t), a(t)), \quad (4)$$

where $c_q$ is associated with the loss in data link layer, e.g., packet/symbol delay, queue overflow\cite{1}, and $c_{tr}$ is associated with the loss in PHY layer, e.g., packet/symbol error rate, power consumption. The immediate cost function in the form of (4) is commonly seen in related works, e.g., \cite{1, 7, 12, 17}, etc.

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3The queue length, the maximum number of packets/symbols that can be stored in queue, could be finite, e.g., \cite{7, 8, 12}, or infinite, e.g., \cite{10}. We assume the general case in this paper, i.e., the queue length is infinite. $\mathcal{B} = \{0, 1, \ldots\}$ is a countable set. The case when queue length is finite, e.g., the NC-TWRC system in Section IV, can be considered as a special case where the results derived in this paper are also applicable.

4Queue overflow is considered if the queue length is finite, i.e., $\mathcal{B} = \{0, 1, \ldots, L\}$ where $L < \infty$ denotes the queue length.
Let $\beta \in [0, 1)$ be the discounted factor and $\theta : \mathcal{X} \mapsto A$ be a stationary deterministic policy. Denote the expected total discounted cost under policy $\theta$ as

$$V_\theta(x) = \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t c(x(t), \theta(x(t))) \right] \bigg| x(0) = x. \quad (5)$$

The objective is

$$\min_{\theta} V_\theta(x), \forall x \in \mathcal{X}. \quad (6)$$

Based on Assumption 2.2 and 2.3, the MDP is stationary. Therefore, $V_\theta(x)$ is in the form of Bellman equation [27]

$$V_\theta(x) = c(x, a) + \sum_{x'} P_{xx'}^\theta V_\theta(x'), \quad (7)$$

and (6) can be solved by DP [15]

$$V(x) := \min_a \left\{ c(x, a) + \beta \sum_{x'} P_{xx'}^a V(x') \right\}. \quad (8)$$

To assist the analysis in Section III we define an auxiliary function $Q$ as the minimand in (8), i.e.,

$$Q(x, a) := c(x, a) + \beta \sum_{x'} P_{xx'}^a V(x'). \quad (9)$$

Note, in (8) and (9), we drop the notation $t$ and use $y$ and $y'$ to denote variables at the current and next decision epochs, respectively, because the MDP under consideration is stationary.

### III. Discrete Convex Optimization

We show, in this section, that (6) can be converted to an $L^2$-convex optimization problem based on the submodularity of DP function $Q$. Before that, we clarify the definitions of submodularity and $L^2$-convexity as follows.

**Definition 3.1 (submodularity [24], [28]):** Let $e_i \in \mathbb{Z}^D$ be a $D$-tuple with all zero entries except the $i$th entry being one. $f : \mathbb{Z}^D \mapsto \mathbb{R}_+$ is submodular if $f(x + e_i) + f(x + e_j) \geq f(x) + f(x + e_i + e_j)$ for all $x \in \mathbb{Z}^D$ and $1 \leq i, j \leq D$.

**Definition 3.2 ($L^2$-convexity [24]):** $f : \mathbb{Z}^D \mapsto \mathbb{R}_+$ is $L^2$-convex if $\psi_f(x, \zeta) = f(x - \zeta 1)$ is submodular in $(x, \zeta)$, where $1 = (1, 1, ..., 1) \in \mathbb{Z}^D$ and $\zeta \in \mathbb{Z}$.

$^5$Since the state space $\mathcal{X}$ is countable and the action space $A$ is finite, according to [26], there exists an optimal policy that is stationary and deterministic. Therefore, we only consider stationary and deterministic policies $\theta$ in this paper.
Submodular DP is commonly seen in MDP modeled queue departure control problems, e.g., [7], [12], [17], that gives rise to a monotonic optimal policy. We show in the following theorem that the optimal transmission control policy in Fig. 1 is a set of optimal queue thresholds if (9) is submodular.

**Theorem 3.3:** If auxiliary function $Q$ is submodular in $(b, a)$ for all fixed $h$, then the optimal policy $\theta^*$, the solution of (6), is nondecreasing in $b$, i.e., $\theta^*$ is in the form of

$$
\theta^*(b, h) = I\{b \geq \phi^*_h\} = \begin{cases} 
1 & b \geq \phi^*_h \\
0 & b < \phi^*_h 
\end{cases}
$$

where $I\{\cdot\}$ is the indicator function which returns 1 if the expression in $\{\cdot\}$ is true and 0 otherwise and $\phi^*_h$ is the optimal queue threshold associated with channel state $h$.

**Proof:** Theorem 3.3 is the direct result of Theorem 4.10 and Proposition 4.11 in [17].

We can see from the following corollary that the monotonicity of $\theta^*$ in $b$ exists for DP function $Q$ that is $L^\natural$-convex in $(b, a)$, e.g., [17].

**Corollary 3.4:** Theorem 3.3 holds if $Q$ is $L^\natural$-convex in $(b, a)$ for all fixed $h$.

**Proof:** According to [29], an $L^\natural$-convex function is also submodular. Therefore, Theorem 3.3 is valid when $Q$ is $L^\natural$-convex in $(b, a)$.

**Remark 3.5:** The general approach for proving the submodularity/$L^\natural$-convexity of $Q$ is to show the submodularity/$L^\natural$-convexity of the immediate cost function $c$ and the stochastic monotonicity of $\sum_{x'} P_{xx'}^a$ for submodular/$L^\natural$-convex functions, e.g., [7], [12]. The submodularity of $Q$ in $(b, a)$ is a commonly seen property, e.g., [7], [12], [17]. In most cross-layer adaptive modulation systems, $c_q$ is in the form of $c_q(b, a) = \lambda h(b - a\delta)$, where $\lambda \geq 0$ is a weight factor and $h(y)$ is a nondecreasing convex function. For example, in [10], $c_q(b, a) = \lambda \max\{b - a\delta, 0\}$ accounts for the symbol delay cost. According to Lemma 4.7(c) in [17], $c_q$ is $L^\natural$-convex and submodular in $(b, a)$. Therefore, $c$ is submodular in $(b, a)$. Since submodularity is preserved by addition,

6 By Little’s law [30] the average customer delay is proportional to the average number of customers held in the queue in the long run for a given customer arrival rate. Here, $\max\{b - a\delta, 0\}$ is the number of packets/symbols remaining in the queue after decision $a$. Therefore, $c_q(b, a) = \lambda \max\{b - a\delta, 0\}$ accounts for the packet/symbol delay cost.

7 By Definition 3.1, $c_{ax}$ is always submodular in $a$ because the inequality in Definition 3.1 is vacuously satisfied for $f: \mathbb{Z} \mapsto \mathbb{R}_+$. Since submodularity is preserved by addition [31], $c$ is submodular in $(b, a)$.}

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expectation and minimization \cite{17, 29, 31},
\[ Q(x, a) := \lambda \max\{b - a\delta, 0\} + c_{tr}(h, a) + \beta P_{hh'} E_f \left[ V(\max\{b - a\delta, 0\} + f, h') \right] \] (11)
is submodular in \((b, a)\). However, in this case, the \(L^2\)-convexity of \(c\) requires the convexity of \(c_{tr}\) in \(a\) in addition. The examples of \(Q\) functions that are \(L^2\)-convex in \((b, a)\) can be found in \cite{7, 17}.

It is obvious that the optimal policy \(\theta^*\) is fully characterized by queue threshold set \(\{\phi_h^* : \Pi_{b \geq \phi_h^*} = \theta^*(b, h), h \in \mathcal{H}\}\) if Theorem 3.3 holds. Define a threshold vector \(\phi = (\phi_1, \phi_2, ..., \phi_{|H|})\), where \(\phi_h = \min\{b : \theta(b, h) = 1\} \in \mathcal{B}\). We show in the following theorem that (6) can be solved by a discrete convex optimization problem.

**Theorem 3.6:** Let \(\Phi = \mathcal{B}^{|H|}\). If Theorem 3.3 holds, then (6) is equivalent to
\[ \min_{\phi \in \Phi} J(\phi), \] (12)
where the objective function
\[ J(\phi) = \sum_x E \left[ \sum_{t=0}^{\infty} \beta^t c(x^{(t)}, \Pi_{b^{(t)} \geq \phi_{h(t)}}) \left| x^{(0)} = x \right. \right] \] (13)
is \(L^2\)-convex.

**Proof:** See Appendix A. \(\blacksquare\)

Note, (12) is a discrete optimization problem, minimization over discrete variable, where we know the objective function is \(L^2\)-convex, a kind of discrete convexity \cite{24}. In Section IV we will utilize the \(L^2\)-convexity of \(J\) to propose the use of DSA algorithm, a gradient-based SA algorithm, for approximating the minimizer in problem (12).

**IV. STOCHASTIC APPROXIMATION**

Since the objective function in (12) is an expectation, it is usually solved by SA, a simulation-based line search algorithm. In the existing literature, SPSA \cite{19} is the most popular SA algorithm for queue threshold policy optimization, e.g., \cite{12, 18}. But, the problem in (12) differs from those in the related works in that the shape of the objective function is known \textit{a priori}—\(J\) is \(L^2\)-convex. And, as aforementioned, SPSA is not originally designed for discrete optimization problems. Therefore, in this section, instead of SPSA, we propose the use of a DSA algorithm for solving problem (12). We also show the convergence performance of DSA by an application on a transmission control problem in multiple access channels.
A. Discrete Stochastic Approximation

\( L^2 \)-convexity is a kind of convexity for functions defined on discrete sets \([24]\). For any \( L^2 \)-convex function \( f : \mathbb{Z}^D \rightarrow \mathbb{R}_+ \) the piecewise linear interpolated \((PLI)\) function \( \tilde{f} : \mathbb{R}^D \rightarrow \mathbb{R}_+ \) is nonsmooth convex \([24]\). According to \([22], [32]\), the minimum and minimizer of \( \tilde{f} \) agree with those of \( f \), i.e., \( \min_x f(x) = \min_{\tilde{x}} \tilde{f}(\tilde{x}) \) and \( \arg \min_x f(x) = \arg \min_{\tilde{x}} \tilde{f}(\tilde{x}) \). Therefore, (12) is equivalent to the continuous optimization problem

\[
\min_{\phi \in \Phi} \tilde{J}(\tilde{\phi}),
\]

where \( \tilde{\Phi} \subseteq \mathbb{R}^D \) is the smallest interval such that \( \Phi \subseteq \tilde{\Phi} \) and \( \tilde{J} : \tilde{\Phi} \rightarrow \mathbb{R}_+ \) is the PLI function of \( J \). Since \( \tilde{\Phi} = \tilde{B} = [0, \infty)^{|H|} \) is a convex set and \( \tilde{J} \) is a convex function, (14) is in fact a convex minimization problem that has the same solution as (12). Then, a gradient-based method, applied on (14) returns a global minimum of (12). Note, the majority of developments in convex optimization is not directly applicable to (12) because they deal with minimizations over continuous parameters only. The advantage of having \( L^2 \)-convexity objective function in (12) is that the problem can be solved by (14) where all the approximation algorithms, e.g., the line search algorithms, designed for continuous convex optimization are applicable.

Consider a gradient descent method for (14). The gradient of \( \tilde{J} \), as shown in \([22]\), can be obtained by the measurements of \( J \) on \( \Phi \) as follows.

Let \( D \) be equal to the dimension of \( \tilde{\phi} \) in (14). Denote \([y], [y] \) and \([y] \) the largest integer less than \( y \), the smallest integer greater than \( y \) and the integer closest (by Euclidean distance) to \( y \), respectively. Let \( p = [\tilde{\phi}] \), \( q = \tilde{\phi} - p \) and

\[
U_d = \begin{cases} 
\emptyset, & d = 0 \\
\{\sigma(1), \ldots, \sigma(d)\}, & d \neq 0
\end{cases},
\]

where \( \sigma \) is the permutation of \((1, \ldots, D)\) such that \( \sigma(d) \) is the index of the \( d \)th largest of \( q_1, \ldots, q_D \), the components in \( q \). Define \( Y(d) \) such that

\[
Y(0) = J(p), \\
Y(d) = J(p + \chi_{U_d}),
\]

where \( \chi_{U_d} \) is the indicator function of \( U_d \).

\(^8\)The piecewise linear interpolation is in fact a triangulation method defined in \([24]\).
Algorithm 1: DSA [22]

\textbf{input} : initial guess $\phi^{(0)}$ (a D-tuple), budget $N$, step size parameters $A$, $B$ and $\alpha$

\textbf{output}: $[\tilde{\phi}^{(N)}]$ \\

begin \\
\hspace{1em} for $n=0$ to $N-1$ do \\
\hspace{2em} $a^{(n)} = \frac{A}{(B+n)^\alpha}$; \\
\hspace{2em} calculate $g^{(n)}$ at $\tilde{\phi}^{(n)}$ according to (16) and (17) by using simulated objective function $\hat{J}$ in (18); \\
\hspace{2em} $\tilde{\phi}^{(n+1)} = \tilde{\phi}^{(n)} - a^{(n)} g^{(n)}$; \\
endfor \\
end

where $\chi_{U_d} \in \{0, 1\}^D$ is a characteristic vector whose $d$th entry is 1 when $d$ belongs to $U_d$ and 0 otherwise. Define $g = (g_1, \ldots, g_D)$ with the $d$th entry being

$$g_d = Y(\sigma(d)) - Y(\sigma(d) - 1).$$

(17)

According to Proposition 2 in [22], $g$ is the subgradient of $\tilde{J}$.

The DSA algorithm for (14) is shown in Algorithm 1, where the subgradient $g$ is calculated as in (16) and (17) by using $\hat{J}$, the simulated value of $J$. The method of obtaining $\hat{J}$ is to generate random process $\{x^{(t)}\}_{t=0}^{10}$ governed by the threshold policy $\phi$ for each starting state $x^{(0)} \in X$, then calculate $\hat{J}$ as

$$\hat{J}(\phi) = \frac{1}{N_r} \sum_{i=1}^{N_r} \sum_{t=0}^{T} \beta^t c(x^{(t)}, I\{b \geq \phi_h\}),$$

(18)

where $N_r$ is the number of the simulations and $T$ is the simulation length, the number of decision epochs in one simulation.

\footnote{Let the computation budget $N$ be given in total number of iterations, the step size parameters $A$, $B$ and $\alpha$ are chosen according to the guide in [33], i.e., $B < 0.1N$ and $A$ and $\alpha$ is chosen such that $A/(B+n)^\alpha ||g^{(1)}||$ achieves the desired change in the elements of $\phi$ in the early iterations.}

\footnote{In fact, $\{x^{(t)}\}$ is a Markov chain with the transition probability given by $Pr(x^{(t+1)}|x^{(t)}) = P^{\theta(x^{(t)})}_{x^{(t)}x^{(t+1)}}$, where $\theta(x) = I\{b \geq \phi_h\}$ and $x = (b, h)$.}

\footnote{$N_r$ is chosen by referring to [22], i.e., $N_r$ is set to a value that yields the confidence interval of 95\% for $\hat{J}(\phi^{(1)})$. The simulation length $T$ depends on $\beta$, i.e., the simulation stops until the increment of $\hat{J}$ over successive decision epochs is blow a small threshold.}
B. Application and Comparison

It is proved in [23] that DSA in Algorithm 1, when applied to problem (12), had the $n$th iteration converge at the rate of $1/n$, an ROC that is even better than continuous convex optimization problems. In this section, to show convergence performance, we use DSA algorithm to search the optimal queue threshold policy in a transmission control problem in NC-TWRC, an example of Fig. 2, and compare the results with an SPSA algorithm.

The multiple channel access transmission control problem in Fig. 2 is usually modeled by a high dimensional MDP, e.g., [7], [12]. In this model, the state and action are the combinations of those of $M$ users, e.g., $x = (b_1, h_1, b_2, h_2, \ldots, b_M, h_M)$ with $b_i$ and $h_i$ denoting the queue and channel states of user $i$, respectively, and $a = (a_1, a_2, \ldots, a_M)$ with $a_i$ denoting the queue departure control of user $i$. The state transition probability is

$$P_{xx'} = \prod_{i=1}^{M} P_{b_i|b'_i} P_{h_i|h'_i},$$

and the immediate cost is

$$c(x, a) = c_q(b_1, a_1, \ldots, b_M, a_M) + c_tr(h_1, a_1, \ldots, h_M, a_M), \quad (19)$$

where $c_q$ and $c_tr$ are losses incurred in MAC and PHY layers summed over $M$ users, respectively. Let $a^* = (a_1^*, a_2^*, \ldots, a_M^*) = \theta^*(x)$ where the optimal policy $\theta^*$ is the solution of (6). It follows that if the assumptions in Section II are satisfied and Theorem 3.3 and 3.6 in Section III hold for all $i \in \{1, 2, \ldots, M\}$ the optimal action $a_i^*$ is nondecreasing in $b_i$ and the optimal thresholds of queue $i$ can be found by solving (12).

Consider the NC-TWRC system [8] in Fig. 3. $\{f^{(t)}_1\}$ and $\{f^{(t)}_2\}$ are i.i.d. random symbol arrival processes associated with user 1 and 2, respectively. Both FIFO queues are assumed to be of finite length with $L_1$ and $L_2$ denoting the maximum number of symbols that can be stored in queue 1 and 2, respectively. For all $i \in \{1, 2\}$, $\{h^{(t)}_i\}$, the state variation process of channel $i$, is modeled by an $K$-state FSMC, $\{f^{(t)}_i\}$ and $\{h^{(t)}_i\}$ are assumed to be independent and the decision $a_i \in A_i = \{0, 1\}$ is made by the scheduler. Here, $a_i = 0$ denotes the action

$^{12}$Lim proved in [23] that SA applied to a piecewise linear (nonsmooth) function exhibited a convergence rate that was better than a smooth function. The reason, as explained in [23], is that the subgradient $g$ for a nonsmooth function is nonzero, instead of approaching zero for a smooth function, at the neighborhood of the optimizer $\phi^*$, i.e., the line search update process $\phi := \phi - \alpha g$ converges faster around $\phi^*$ for nonsmooth functions.

$^{13}$Usually, the packet arrival and channel state variation processes of $M$ users are assumed to be independent of each other. So, the state transition probability is the multiplication of those of $M$ users.
Fig. 3. Transmission control problem in NC-TWRC [8], an example of Fig. 2. The queues store messages in symbols from two users. \( \{f_1(t)\} \) and \( \{f_2(t)\} \) are i.i.d. symbol arrival processes of user 1 and 2, respectively. The state variations of channels are modeled by two FSMCs. A scheduler controls the outflows of two queues so that the symbol delay, queue overflow and transmission power and error rate are minimized in the long run.

of holding symbols in queue \( i \), while \( a_i = 1 \) denotes the action of transmitting one symbol from queue \( i \). This system allows network coding (XORing), i.e., if \( a_1 = a_2 = 1 \) the symbols are XORed and broadcast in order to save transmission power. We assume that the scheduler knows \( f_i(t) \) and \( h_i(t) \) before the decision making at \( t \) and obtains the statistics of \( \{f_i(t)\} \) and \( \{h_i(t)\} \) for all \( i \). Then, in this system, all assumptions in Section III are satisfied. The objective is to find a transmission control policy that minimizes the expected symbol delay, queue overflow, transmission power (saved by utilizing network coding) and error rate in the long run. It is clear that Fig. 3 is a special case of Fig. 2 where there are two users, i.e., \( M = 2 \), and the scheduling is centralized, i.e., the departures of both queues are controlled by one scheduler. Note, although we just consider the two user and centralized transmission control case, it should be clear that DSA in Algorithm 1 can be applied to any system in the form of Fig. 2 where the assumptions in Section III are satisfied and Theorems 3.3 and 3.6 in Section III hold for all users, e.g., the MIMO system in [7] and the dynamic channel access system in [12].

Fig. 3 is modeled by a stationary MDP in [8] with the system state being \( x = (b_1, b_2, h_1, h_2) \in \mathcal{X} \), where \( b_i \in \mathcal{B}_i = \{0, 1, \ldots, L_i\} \), \( h_i \in \mathcal{H}_i = \{1, 2, \ldots, K\} \) and \( \mathcal{X} = \mathcal{B}_1 \times \mathcal{B}_2 \times \mathcal{H}_1 \times \mathcal{H}_2 \), the action being \( a = (a_1, a_2) \in \mathcal{A} = \times_{i=1}^2 \mathcal{A}_i = \{0, 1\}^2 \), and the state transition probability being
\( P_{xx'} = \prod_{i=1}^{2} P_{b_i'b_i'} \). The immediate costs \( c_q \) and \( c_{tr} \) are defined as

\[
c_q(b_1, a_1, b_2, a_2) = \sum_{i=1}^{2} \left( \lambda c_d(b_i, a_i) + \xi c_o(b_i, a_i) \right), \tag{20}
\]

\[
c_{tr}(h_1, a_1, h_2, a_2) = \sum_{i=1}^{2} \eta c_e(h_i, a_i) + \tau c_c(a_1, a_2), \tag{21}
\]

where \( c_d, c_o, c_e \) and \( c_c \) account for the quantities of symbol delay, queue overflow, symbol error and transmission power consumption (where the transmission power saved by network coding is deducted), respectively, and \( \lambda, \xi, \eta \) and \( \tau \) are the associated unit costs.

Let the optimal actions be \((a_1^*, a_2^*) = \theta^*(x)\). According to Theorem 4.13 and Corollary 4.14 in [17], we assume uniformly distributed traffic rates for both users and choose the value of \( \lambda, \tau, \eta, \xi_o \) and \( \beta \) to make function \( Q L^t \)-convex in \((b_1, a_1, b_2, a_2)\). In this case, \( Q \) is also submodular in \((b_1, a_1, b_2, a_2)\) [29], i.e., Theorem 3.3 holds according to Corollary 3.4. Therefore, \( a_1^* \) is monotonic in \( b_1 \). So, we define a threshold vector \( \phi \) by stacking

\[
\phi_{b_2h_1h_2} = \min\{b_1 : \theta(x) = 1\} \tag{22}
\]

for all \((b_2, h_1, h_2) \in B_2 \times H_1 \times H_2\) and form the discrete convex optimization problem \( \min_{\phi} J(\phi) \) as (12) in Theorem 3.6. We then run two experiments on NC-TWRC in Fig. 3 with different system settings.

Note, (22) is the threshold on \( b_1 \), i.e., an SA algorithm solving \( \min_{\phi} J(\phi) \) with \( \phi \) given by (22) will return a policy that is monotonic in \( b_1 \) for sure. But, since \( Q \) is submodular in \((b_1, a_1, b_2, a_2)\), according to Theorem 3.3 the optimal policy \( \theta^* \) is in fact nondecreasing in \((b_1, b_2)\), i.e, the solution of \( \min_{\phi} J(\phi) \) is a \( \phi^* \) that results in a \( \theta^* \) that is monotonic in both \( b_1 \) and \( b_2 \). In this way, we can check the correctness of the output of an SA algorithm (The related results are shown in Figs. 5 and 7).

In the first experiment, we apply the DSA algorithm in a 7-queue state and 4-channel state TWRC, i.e., \( B_1 = B_2 = \{0, 1, \ldots, 6\} \) and \( H_1 = H_2 = \{1, 2, \ldots, 4\} \). To compare the performance, we also run an SPSA [18] as shown in Algorithm 2 on the same system. See TABLE I

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14 According to Theorem 4.13 and Corollary 4.14 in [17], we set \( \lambda = 0.05, \tau = 1, \eta = 1, \xi = 4 \) and \( \beta = 0.97 \) to make \( Q L^t \)-convex in \((b_1, b_2, a_1, a_2)\).

15 The step size parameters \( A, B, C, \alpha \) and \( \rho \) are chosen by referring to [33] in the same way as described in Algorithm 1.
TABLE I
STEP SIZE PARAMETERS USED IN EXPERIMENTS

| NC-TWRC       | A    | B    | C   | α | ρ |
|---------------|------|------|-----|---|---|
| DSA 7-queue, 4-channel | 0.075N | 0.095N | -  | 0.8 | -  |
| DSA 11-queue, 2-channel | 0.047N | 0.095N | -  | 0.8 | -  |
| SPSA 7-queue, 4-channel | 4 | 0.03N | 4  | 0.6 | 0.1 |
| SPSA 11-queue, 2-channel | 2.15 | 0.03N | 4  | 0.6 | 0.1 |

Algorithm 2: SPSA [18]

input: initial guess $\tilde{\phi}^{(0)}$ (a $D$-tuple), budget $N$, step size parameters $A$, $B$, $C$, $\alpha$ and $\rho$.

output: $\tilde{\phi}^{(N)}$.

begin
for $n=0$ to $N-1$ do

$a^{(n)} = \frac{A}{\sum_{d=1}^{D} a^{(n)}_{d}}$; \hspace{1cm} $c^{(n)} = \frac{C}{\sum_{d=1}^{D} c^{(n)}_{d}}$;

generate $\Delta = (\Delta_1, \ldots, \Delta_D)$, where $\Delta_1, \ldots, \Delta_D$ are independent r.v. with $\Delta_d \sim \text{Bernoulli}(\frac{1}{2})$;

$\tilde{\phi}^{+} = \tilde{\phi}^{(n)} + c^{(n)} \Delta$;

$\tilde{\phi}^{-} = \tilde{\phi}^{(n)} - c^{(n)} \Delta$;

Get projections $\hat{\phi}^{+}$ and $\hat{\phi}^{-}$ by the projection function

$$\hat{\phi} = \begin{cases} 
\lfloor \hat{\phi} \rfloor & \text{w/ prob. } \frac{\hat{\phi} - \lfloor \hat{\phi} \rfloor}{r} \\
\lceil \hat{\phi} \rceil & \text{w/ prob. } \frac{\hat{\phi} - \lceil \hat{\phi} \rceil}{r}
\end{cases},$$

(23)

where $r = \lceil \hat{\phi} \rceil - \lfloor \hat{\phi} \rfloor$;

obtain $g^{(n)} = (g^{(n)}_1, \ldots, g^{(n)}_D)$ where $g^{(n)}_d = \frac{j^+ - j^-}{2c^{(n)}_d \Delta_d}$ with $j^+$ and $j^-$ simulated by using $\hat{\phi}^{+}$ and $\hat{\phi}^{-}$, respectively;

$\tilde{\phi}^{(n+1)} = \tilde{\phi}^{(n)} - a^{(n)} g^{(n)}$;

endfor
end

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for the step size parameters of DSA and SPSA used in the experiment. Fig. 4 shows $J(\tilde{\phi}^{(n)})$, the value of the objective function at $\tilde{\phi}^{(n)}$, where $\tilde{\phi}^{(n)}$ is the rounded $n$th estimation of the minimizer, and $\|\tilde{\phi}^{(n)} - \phi^*\|$, the distance of $\tilde{\phi}^{(n)}$ to the minimizer. It can be seen that, in contrast to SPSA, DSA has $J(\tilde{\phi}^{(n)})$ converge within approximately the first 5 iterations and $\|\tilde{\phi}^{(n)} - \phi^*\|$ exponentially decaying. In Fig. 5 we show the outputs $\tilde{\phi}^{(N)}$ of both DSA and SPSA. As mentioned before, $a_1^+$ should be nondecreasing in $(b_1, b_2)$ due to the submodularity of $Q$ in $(b_1, a_1, b_2, a_2)$. It can be seen from Fig. 5 that the output policy of DSA has monotonic property in $b_2$ while that of SPSA does not, which means it is more likely for the policy searched by DSA to be optimal than the one searched by SPSA. In the second experiment, we run DSA and SPSA on an 11-queue state and 2-channel state NC-TWRC. As shown in Figs. 6 and 7, we get the same results as in the first experiment.

It should be noted that DSA is dimension dependent while SPSA is not. The gradient approximation in DSA requires $D + 1$ estimations of the objective function $J$, as compared to two estimations in SPSA. It means the work load at each iteration in DSA is heavier than that in SPSA if $D$ is large. In this paper, to overcome this problem, we use parallel simulations, i.e., simultaneously estimating $J$ for all $d$ in (16) when estimating subgradient $g$ in (17).

V. CONCLUSION AND FUTURE WORK

We proved that the submodularity of DP in an MDP modeled cross-layer transmission control system gave rise to an optimal queue threshold policy. We showed that this policy was characterized by a queue threshold vector which could be determined by a multivariate discrete convex minimization problem. We proposed to implement a DSA algorithm to approximate the minimizer, the optimal queue threshold policy, and showed by experiments that DSA converges faster than SPSA.

Based on the work in this paper, we point out several possible extensions of the research work. First of all, it is still of interest if the DSA algorithm can be applied when the scheduler decides not only whether or not to transmit but also the transmission rate, a more general case of the adaptive modulation problem. In addition, due to the dimension dependency, the trade-off

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16In TABLE II the step size values of DSA are chosen by referring to [22], [33], while the step size values of SPSA are chosen by referring to [33]. For both DSA and SPSA, $B < 0.1N$ and $A$ is tuned such that $A/(B + n)^{\alpha}\|g^{(1)}\| \approx 8.5$. 

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Fig. 4. Convergence performance comparison between DSA and SPSA on a 7-queue state and 4-channel state NC-TWRC in Fig. 3 with computation budget $N = 150$ iterations: $J(\tilde{\phi}^{(n)})$ (left), the value of the objective function at the rounded estimation $\tilde{\phi}^{(n)}$ in $n$th iteration; $\|\tilde{\phi}^{(n)} - \phi^*\|$ (right), the Euclidean distance from $\tilde{\phi}^{(n)}$ to the minimizer $\phi^*$. The optimal threshold policy $\phi^*$ is determined by $\theta^*$ through (22), where $\theta^*$ is found by value iteration, a DP algorithm.

Fig. 5. The optimal action $a^*_1$ determined by the output threshold policies $\tilde{\phi}^{(N)}$ of DSA (left) and SPSA (right) on a 7-queue state and 4-channel state NC-TWRC in Fig. 3 with computation budget $N = 150$ iterations.

between the complexity and accuracy of DSA needs to be further studied in high dimensional systems, e.g., adaptive modulation over MIMO channels or multi-user systems with the number of users more than 2. Finally, for reinforcement learning purpose, one should compare the real-time performance of DSA to that of SPSA, i.e., experiments should be run to show whether DSA is able to converge to the optimum faster than SPSA in accordance with the time-varying system and/or MDP parameters.

**APPENDIX A**

First of all, we derive a property of $L^2$-convexity in the following lemma.
Fig. 6. Convergence performance comparison between DSA and SPSA on a 11-queue state and 2-channel state NC-TWRC in Fig. 3 with computation budget \( N = 150 \) iterations: \( J([\tilde{\phi}^{(n)}]) \) (left), the value of the objective function at the rounded estimation \( [\tilde{\phi}^{(n)}] \) in \( n \)th iteration; \( \|\tilde{\phi}^{(n)} - \phi^*\| \) (right), the Euclidean distance from \( \tilde{\phi}^{(n)} \) to the minimizer \( \phi^* \). The optimal threshold policy \( \phi^* \) is determined by \( \theta^* \) through (22), where \( \theta^* \) is found by value iteration, a DP algorithm.

Fig. 7. The optimal action \( a^*_1 \) determined by the output threshold policies \( [\tilde{\phi}^{(N)}] \) of DSA (left) and SPSA (right) on a 11-queue state and 2-channel state NC-TWRC in Fig. 3 with computation budget \( N = 150 \) iterations.

**Lemma A.1:** Let \( x = (x_1, x_2, \ldots, x_n) \in \mathbb{Z}^n \) and \( g(x) = \sum_{i=1}^{n} f_i(x_i) \) where \( f_i : \mathbb{Z} \mapsto \mathbb{R}_+ \) is \( L^a \)-convex for all \( i \), then \( g \) is \( L^a \)-convex in \( x \).

**Proof:** Consider function \( \psi_g(x, \zeta) = g(x - \zeta 1) = \sum_{i=1}^{n} f_i(x_i - \zeta) \). Let \( e_i \in \mathbb{Z}^n \) be a \( n \)-tuple with all zero entries except the \( i \)th entry being one. \( \psi_g \) is submodular in \( (x, \zeta) \) because for all \( 1 \leq i, j \leq n \)

\[
\psi_g(x + e_i, \zeta) + \psi_g(x + e_j, \zeta) - \psi_g(x, \zeta) - \psi_g(x + e_i + e_j, \zeta) \\
= g(x + e_i - \zeta 1) + g(x + e_j - \zeta 1) - g(x - \zeta 1) - g(x + e_i + e_j - \zeta 1) = 0, \quad (24)
\]
\[ \psi_g(x + e_i, \zeta) + \psi_g(x, \zeta + 1) - \psi_g(x, \zeta) - \psi_g(x + e_i, \zeta + 1) \]
\[ = g(x + e_i - \zeta 1) + g(x - (\zeta + 1) 1) - g(x - \zeta 1) - g(x + e_i - (\zeta + 1) 1) \]
\[ = f_i(x_i - \zeta + 1) + f_i(x_i - \zeta - 1) - 2f_i(x_i - \zeta) \geq 0. \]  (25)

According to Definition 3.2, \( g \) is \( L^\natural \)-convex in \( x \).

Define \( V_b(h, \phi_h) = \sum_b Q(b, h, I_{\{b \geq \phi_h\}}) \).

**Lemma A.2:** If \( Q \) is submodular in \((b, a)\), then \( V_b(h, \phi_h) \) is \( L^\natural \)-convex in \( \phi_h \) for all \( h \).

**Proof:** Due to the submodularity of \( Q \) in \((b, a)\), we have
\[
V_b(h, \phi_h + 1) + V_b(h, \phi_h - 1) - 2V_b(h, \phi_h)
\]
\[
= Q(\phi_h, h, 0) - Q(\phi_h - 1, h, 0) + Q(\phi_h - 1, h, 1) - Q(\phi_h, h, 1) \geq 0. \]  (26)

So \( V_b \) is \( L^\natural \)-convex in \( \phi_h \).

Let \( \theta \) be the policy determined by \( \phi \) through (10), then by (5), (8) and (9), we have
\[
V_\theta(x) = E \left[ \sum_{t=0}^{\infty} \beta^t c(x(t), I_{\{b(t) \geq \phi_{h(t)}\}}) \middle| x(0) = x \right]
\]
\[
= Q(b, h, I_{\{b(t) \geq \phi_{h(t)}\}}), \]  (27)

i.e., \( J(\phi) = \sum_x V_\theta(x) \). So \( \min_\theta V_\theta(x), \forall x \) is equivalent to \( \min_\phi J(\phi) \). Since
\[
J(\phi) = \sum_h \sum_b Q(b, h, I_{\{b(t) \geq \phi_h\}})
\]
\[
= \sum_h V_b(h, \phi_h), \]  (28)

\( J \) is \( L^\natural \)-convex in \( \phi \) by Lemma A.1 and A.2.

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\(^{17}\)By Definition A.2 \( f : Z \mapsto \mathbb{R}_+ \) is \( L^\natural \)-convex if \( f(x + 1) + f(x - 1) - 2f(x) \geq 0. \)
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