Holographic Lattice in Einstein-Maxwell-Dilaton Gravity

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Abstract

We construct an ionic lattice background in the framework of Einstein-Maxwell-dilaton theory in four dimensional space time. The optical conductivity of the dual field theory on the boundary is investigated. Due to the lattice effects, we find the imaginary part of the conductivity is manifestly suppressed in the zero frequency limit, while the DC conductivity approaches a finite value such that the previous delta function reflecting the translation symmetry is absent. Such a behavior can be exactly fit by the Drude law at low frequency. Moreover, we find that the modulus of the optical conductivity exhibits a power-law behavior at intermediate frequency regime. Our results provides further support for the universality of such power-law behavior recently disclosed in Einstein-Maxwell theory by Horowitz, Santos and Tong.

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I. INTRODUCTION

In the past few years, there has been much progress in understanding strongly coupled condensed matter systems using the gauge/gravity duality (for recent reviews, see for instance [1–5]). Many interesting phenomena including the superconductivity, superfluidity and non-Fermi liquid behavior have been addressed in the holographic framework. Nevertheless, most of these works only consider homogeneous gravitational backgrounds, which possess spatial translation invariance, and so the momentum is conserved in these directions. Due to the conservation of the momentum, many crucial characteristics of practical condensed matter materials at zero frequency are far from reached along this route. One known phenomenon is that a delta function is always present at zero frequency for the real part of the optical conductivity in the dual field theory, reflecting an infinite DC conductivity which obviously is a undesired result for the normal phase of real materials.

Therefore, in order to build more realistic condensed matter system in holographic approach, the spatial translational symmetry in the bulk must be broken such that the momentum in these directions becomes dissipative. One idea to implement this is introducing the effects of lattice or impurities in the gravitational background [6–21]. From the viewpoint of AdS/CFT correspondence, such a symmetry breaking in the bulk geometry can lead to significant changes for some observables in dual field theory on the boundary, for instance, the optical conductivity and the Fermi surface structure. Especially, in recent work [6, 7], a holographic lattice model has been constructed in the Einstein-Maxwell theory by introducing a periodic scalar field to simulate the lattice, or directly imposing a periodic condition for the chemical potential on the boundary, which is also called as ionic lattice. Several surprising results have been disclosed by fully solving the coupled perturbation equations numerically. Firstly, the delta function at zero frequency spreads out and the conductivity can be well described by the standard Drude formula at low frequency. Secondly, in an intermediate frequency regime, a power law behavior is found as

\[ |\sigma(\omega)| = \frac{B}{\omega^\gamma} + C, \]

with \( \gamma \approx 2/3 \) in four dimensional spacetime, which is robust in the sense that it is inde-

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1 An alternative approach is considering holographic massive gravity in which the conservation of momentum is broken by a graviton mass term [22, 24].
pended of the parameters of the model, such as the lattice spacing, lattice amplitude and temperature. Remarkably, this power law behavior is in good agreement with those data measured in experiments on the cuprate\(^2\) [25, 26]. Another surprising result is that in the holographic superconducting phase [8], a delta function at zero frequency in the real part of the optical conductivity does not disappear due to the presence of the lattice, reflecting that a genuine superconducting phase is obtained indeed. In particular, the power law behavior with \(|\sigma(\omega)| \propto \omega^{-2/3}\) maintains in the mid-infrared regime of frequency. In addition, the power law behavior \(^1\) has also been observed in holographic massive gravity albeit the exponent \(\gamma\) depends on the mass of the graviton[22, 23].

Until now, such power law behavior of \(^1\) has not been well understood from the dual gravitational side yet. In this paper we intend to testify above observations in a different framework, namely the Einstein-Maxwell-dilaton theory. In this setup Gubser and Rocha proposed a charged black brane solution which contains some appealing features for the study of AdS/CMT correspondence[27–29]. First of all, such black branes have vanishing entropy in the extremal case with zero temperature, which is very desirable from the side of condensed matter materials. Secondly, a linear dependence of the entropy and heat capacity on temperature at low temperature can be deduced [27, 30], which seemingly indicate that the dual field theory is Fermi-like[31–34] \(^3\). The purpose of our this paper is constructing a lattice background based on this black brane solution and explore whether such lattice effects will lead to the same universal behavior of the optical conductivity as those in RN-AdS black brane [6–8].

As the first step, we will concentrate on the lattice effects on the optical conductivity in normal phase through this paper, but leave the superconducting phase for further investigation. We will firstly present a charged dilatonic black brane solution in Einstein-Maxwell-Dilaton theory in Section [II] and then investigate the optical conductivity without lattice for later comparison. In Section [III] we explicitly construct an ionic lattice background by imposing a periodic chemical potential on the boundary and solving the coupled

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\(^2\) We note that there is an off-set constant \(C\) in the power law behavior, which is different from that observed in experiments.

\(^3\) A large class of Einstein-Maxwell-dilaton theories with different Maxwell coupling and scalar potential has been constructed and they also possess the same appealing characteristic such as vanishing extremal entropy and linear specific heat at low temperature (see for instance [35, 38]). But, such solutions are usually not an analytic form.
partial differential equations numerically with the DeTurck method. Then we concentrate on the perturbations over this lattice background and compute the optical conductivity in a holographic approach. Our conclusions and discussions are given in Section IV.

II. CONDUCTIVITY IN THE CHARGED DILATONIC BLACK BRANE WITHOUT LATTICE

For comparison, in this section we will calculate the optical conductivity in the charged dilatonic black brane without lattice. For relevant discussion on the holographic conductivity in other backgrounds in Einstein-Maxwell-Dilaton theory, we can refer to, for instance, [36–40].

A. The charged dilatonic black brane

We begin with the following action, which includes gravity, a $U(1)$ gauge field, and a dilaton $\Phi$ with a Maxwell non-minimal coupling $f(\Phi) = e^\Phi$ and a scalar potential $V(\Phi) = \cosh \Phi$ [27]

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ R - \frac{1}{4} e^\Phi F^{\mu\nu} F_{\mu\nu} - \frac{3}{2} (\partial_\mu \Phi)^2 + \frac{6}{L^2} \cosh \Phi \right].$$

(2)

Applying the variational approach to the above action, one can obtain the equations of motion for the fields $g_{\mu\nu}$, $A_\mu$ and $\Phi$, respectively

$$R_{\mu\nu} = \frac{1}{2} e^\Phi (F_{\mu\sigma} F^{\nu}_\sigma - \frac{1}{4} F^2 g_{\mu\nu}) + \frac{3}{2} \partial_\mu \Phi \partial_\nu \Phi - \frac{3}{L^2} g_{\mu\nu} \cosh \Phi,$$

(3)

$$\nabla_\mu (e^\Phi F^{\mu\nu}) = 0,$$

(4)

$$\Box^2 \Phi = \frac{1}{12} e^\Phi F^2 - \frac{2}{L^2} \sinh \Phi,$$

(5)

where $\Box^2 \equiv \nabla^\mu \nabla_\mu = g^{\mu\nu} \nabla_\mu \nabla_\nu$ is the covariant d’Alembertian operator. An analytical charged solution to these equations has previously been given in [27], which reads as

$$ds^2 = \frac{L^2}{z^2} \left( -f(z) dt^2 + \frac{dz^2}{f(z)} + g(z)(dx^2 + dy^2) \right),$$

(6)

$$A_t(z) = L \sqrt{3Q(1-z)} \frac{\sqrt{1+Q}}{1+Qz},$$

(7)

$$\Phi(z) = \frac{1}{2} \ln (1+Qz).$$

(8)
where
\[
\begin{align*}
f(z) &= (1 - z) \frac{p(z)}{g(z)}, \\
g(z) &= (1 + Qz)^{3/2}, \\
p(z) &= 1 + (1 + 3Q) z + (1 + 3Q (1 + Q)) z^2.
\end{align*}
\tag{9}
\]

Here for the convenience of the numerical analysis, we have made a coordinate transformation based on the solution presented in \cite{27}. Next we are concerned with the Hawking temperature \(T\) of this charged black brane. A simple algebra shows that in our current coordinate system, the Hawking temperature is
\[
T = \frac{3\sqrt{1 + Q}}{4\pi L}.
\tag{10}
\]

However, due to the conformal symmetry on the boundary, we stress that whatever the coordinates system is adopted, the quantity which is physically meaningful is the ratio of \(T/\mu\) in asymptotically charged AdS space time, namely the temperature measured with the chemical potential as the unit. This quantity is invariant under the rescaling of the time coordinate. Therefore, through this paper we will use the chemical potential on the boundary to set the unit of the system. In our case we find the Hawking temperature measured with the chemical potential reads as
\[
\frac{T}{\mu} = \frac{\sqrt{3}}{4\pi L\sqrt{Q}}.
\tag{11}
\]

Obviously, this temperature with the chemical potential as the unit is a monotonically decreasing function of \(Q\). As \(Q\) tends to zero, it approaches to infinity, which corresponds to a Schwarzschild-AdS black brane. On the other hand, as \(Q\) goes up to infinity, the ratio \(T/\mu\) runs to zero, corresponding to an extremal black hole. An appealing characteristic of this black brane solution is that its entropy on the horizon is vanishing in the extremal case, in contrast to the usual RN-AdS black holes which approach a AdS2 geometry near the horizon. As a result, this sort of black brane is expected to be a more practical arena to implement a holographic scenario for a condensed matter system at low temperature. For more discussion on the thermodynamics of this charged dilatonic black brane, please refer to \cite{27}. In this end of this subsection, we remark that our current coordinate system may

\footnote{Here after having fixed the radius of the horizon at \(z = 1\), we have only one parameter \(Q\), which as a matter of fact corresponds to \(q/r_h\) in \cite{27}, where \(q\) is the charge and \(r_h = (mL^2)^{1/3} - q\) is the position of the horizon with mass \(m\). In \cite{27} the extremal limit is given by \(q = (mL^2)^{1/3}\), which in our case corresponds to \(Q \to \infty\) since \(r_h \to 0\).}
not be appropriate for numerical analysis in zero temperature limit since in this case the parameter $Q$ goes to infinity. We need transform the metric into some form closely following the treatment as given in [8]. Nevertheless, since in this paper we are not concerned with its zero temperature behavior, we argue that this coordinate system is good enough for us to observe the lattice effects in a rather large regime of the temperature.

B. Conductivity

To compute the conductivity in a holographic approach, the simplest way is turning on the gauge field fluctuation $a_x$ and the metric fluctuation $h_{tx}$. Before proceeding, we take a Fourier transformation for the perturbed fields

$$h_{tx}(t, z) \sim e^{-i\omega t}h_{tx}(z), \quad a_x(t, z) \sim e^{-i\omega t}a_x(z). \quad (12)$$

Here, for simplicity, we stay at zero momentum $k = 0$. Perturbing the equations of motion in (3) and (4) at the linear level gives the following two independent equations

$$a''_x + \left(\frac{f'}{f} + \Phi'\right) a'_x + \frac{\omega^2}{f^2} a_x + \frac{z^2 A'_t}{f} h'_{tx} + \left(\frac{2z A'_t}{f} - \frac{z^2 A'_t g'}{fg}\right) h_{tx} = 0, \quad (13)$$

$$h'_{tx} + \left(\frac{2}{z} - \frac{g'}{g}\right) h_{tx} + e^\Phi \frac{A'_t}{L^2} a_x = 0. \quad (14)$$

Substituting the second equation into the first one, one can easily obtain a decoupled equation for the fluctuations of the gauge field $a_x$ which is

$$a''_x + \left(\frac{f'}{f} + \Phi'\right) a'_x + \left(\frac{\omega^2}{f^2} - \frac{z^2 e^\Phi A'^2_t}{L^2 f}\right) a_x = 0. \quad (15)$$

We can solve the above equation with purely ingoing boundary conditions at the horizon ($z \to 1$). And then, the conductivity can be read off from the formula

$$\sigma(\omega) = -i \frac{a^{(1)}_x}{\omega a^{(0)}_x}, \quad (16)$$

where $a^{(0)}_x$ and $a^{(1)}_x$ are determined by the asymptotic behavior of the fluctuation $a_x$ at the boundary ($z \to 0$)

$$a_x = a^{(0)}_x + a^{(1)}_x z + \ldots. \quad (17)$$

The numerical results for the real and imaginary parts of the conductivity are shown in FIG.1. In the high-frequency limit, the real part of the conductivity becomes constant. It
FIG. 1: The real (left) and imaginary (right) parts of the conductivity for different $Q$ (blue for $Q = 0.01$, red for $Q = 0.1$, green for $Q = 0.25$ and black for $Q = 0.5$).

is a common characteristic for the holographic dual theory. However, the imaginary part of the conductivity goes to infinity at $\omega = 0$ and the standard Kramers-Kronig relation implies that there is a delta function at $\omega = 0$ in the real part of the conductivity, which is of course not a genuine superconductivity phase but a consequence of translational symmetry. At small frequencies, some interesting behaviors exhibit in the real part of the conductivity. From FIG.1 we can observe that it develops a minimum and the minimum increases with the decrease of $Q$. Finally, it almost becomes a constant when $Q \to 0$, which is just the case for the Schwarzschild-AdS black brane.

III. CONDUCTIVITY IN THE CHARGED DILATONIC BLACK BRANE WITH LATTICE

In this section, we will firstly construct an ionic lattice background in Einstein-Maxwell-Dilaton theory, and then demonstrate how the optical conductivity would be greatly changed in low frequency limit due to the presence of the lattice.

A. A Holographic Charged Dilaton Lattice

As the first step, through this paper we will introduce the lattice only in $x$ direction but leave the $y$ direction translation invariance. As a result, one may take the following ansatz
for the background metric, which is compatible with the symmetry we considered above

\[ ds^2 = \frac{L^2}{z^2} \left[ -(1 - z)p(z)H_1 dt^2 + \frac{H_2 dz^2}{(1 - z)p(z)} + S_1(dx + Fdz)^2 + S_2dy^2 \right], \quad (18) \]

with

\[ A_t = L \sqrt{3Q(1 - z)} \psi(x, z), \quad (19) \]

and

\[ \Phi = \frac{1}{2} \ln \left( 1 + Qz \phi(x, z) \right), \quad (20) \]

where \( H_{1,2}, S_{1,2}, F, \psi \) and \( \phi \) are seven functions of \( x \) and \( z \), and will be determined by solving Eqs. (3), (4) and (5). Note that if \( H_1 = 1/g(z), \ H_2 = S_1 = S_2 = g(z), \ F = 0, \ \psi = \sqrt{1 + Q/(1 + Qz)} \) and \( \phi = 1 \), it will recover to the charged dilatonic black brane solution without lattice described by (6-9).

Now, we introduce the lattice by imposing an inhomogeneous boundary condition for the chemical potential as

\[ \psi(x, 0) = \sqrt{1 + Q(1 + A_0 \cos(k_0 x))}, \quad (21) \]

which is often referred to as an ionic lattice.

The nonlinear PDEs (3-5), together with the ansatz (18-20), are solved using the Einstein-DeTurck method\[41\]. We choose the charged dilatonic black brane (6-9) as the reference metric. We firstly linearize the PDEs (3-5) with the use of Newton-Kantorovich method and then change the differential equations into algebraic equations employing the spectral collocation method. Such a boundary value problem will be approximated by Fourier discretization in \( x \) direction and Chebyshev polynomials in \( z \) direction. As an example, we show a solution in FIG. 2 with \( A_0 = 0.4, \ k_0 = 2 \) and \( Q = 0.1 \), which corresponds to a lattice with \( T/\mu = 0.457 \).

In order to guarantee the convergence of our numerical solutions, we have demonstrated the decaying tendency of the charge discrepancy \( \Delta_N \) which is defined as \( \Delta_N = |1 - Q_N/Q_{N+1}| \), where \( Q_N \) is the charge on the boundary with \( N \) grid points. As shown in FIG. 3 an exponential decay implies our solutions are exponentially convergent with the increase of grid points. Moreover, we have also checked the behavior of the DeTurck vector field \( \xi^a \) which is defined in [6] and found that for our solution its magnitude is controlled below \( 10^{-10} \) indeed. In this sense our lattice configuration is just a ripple over the previous charged black brane.
FIG. 2: We show $H_2$, $F$, $\psi$ and $\phi$ for $k_0 = 2$, $A_0 = 0.4$, $Q = 0.1$ and $|T/\mu| = 0.457$.

B. Conductivity

To explore the lattice effects on the optical conductivity in a holographic approach, we need consider the perturbations over the lattice background with full backreactions. Therefore, we need in principle turn on all the gravitational fields as well as gauge and dilaton fields, and allow them to fluctuate with a harmonic frequency. Since we only take the lattice
FIG. 3: On the left side we show the charge density $\rho$ as a function of $x$ on the boundary, which can be read off by expanding $\psi = \mu + (\mu - \rho)z + \mathcal{O}(z^2)$. On the right we show the charge discrepancy $\Delta_N$ as a function of the number of grid points $N$, where the boundary charge is evaluated in a period as $Q = \int_0^{2\pi/k_0} dx \rho$. The vertical scale is logarithmic, and the data is well fit by an exponential decay: $\log(\Delta_N) = -6.56 - 0.197N$.

into account in $x$-direction, one economical way in numerical analysis is assuming that all perturbation quantities do not depend on the coordinate $y$. This ansatz reduces the perturbation variables to 11 unknown functions $\{h_{tt}, h_{tz}, h_{tx}, h_{zz}, h_{xx}, h_{yy}, b_t, b_z, b_x, \eta\}$ and all of them are just dependent of $x$ and $z$. For instance, we may have

$$\delta A_x = b_x(t, x, z) \sim e^{-i\omega t} b_x(x, z).$$

Plugging all the perturbations into the equations of motion and ignoring the higher order corrections, we obtain 11 linear partial differential equations. In addition, we adopt the harmonic gauge conditions and Lorentz gauge condition for gravity and Maxwell field, respectively. The last thing is to set the suitable boundary conditions. We find the most economical but self consistent way to do this at $z = 0$ is requiring the $x$-component of the Maxwell field to be

$$b_x(x, z) = 1 + j_x(x)z + \mathcal{O}(z^2),$$

while the other perturbations are vanishing at the zeroth order of $z$. Of course we point out that such a choice is not unique, for instance we are allowed to set the dilaton perturbation to be a non-zero constant on the boundary, the corresponding solution can be found and we
find such adjustment will not affect our following observations on the optical conductivity. Thus, for simplicity we set $\eta(x,0) = 0$. As far as the boundary conditions at $z = 1$ are concerned, we adopt the standard regularity conditions on horizon and consider only the ingoing modes. In addition, in numerical analysis the highly oscillating modes near the horizon can be filtered by changing the variables with a factor such as $(1 - z)^{-i\omega/4\pi T}$, and we refer to [6] for detailed discussions.

Finally, we solve all these linear equations using a standard pseudo-spectral collocation methods. In FIG. 4 we show a solution for $h_{tz}$ and $b_x$ with $A_0 = 0.4$, $k_0 = 2$, $T/\mu = 0.457$ and $\omega = 0.6$.

The optical conductivity in the direction of the lattice can be read off

$$\sigma(\omega, x) = \frac{j_x(x)}{i\omega}. \quad (24)$$

Note that the conductivity is a function of $x$. Since the boundary electric field $E_x = i\omega e^{-i\omega t}$ is homogeneous, the homogeneous part of the conductivity is the quantity we study below.

A typical effect due to the presence of lattice on the optical conductivity is demonstrated in FIG. 5. The dashed lines represent the conductivity without lattice while the solid lines represent the conductivity at the same temperature with lattice. Obviously, in high frequency regime the conductivity with lattice coincides with that without lattice. However, in low frequency regime the conductivity is greatly changed by the lattice. We observe that the imaginary part of the conductivity has been significantly suppressed, while the previous delta function at zero frequency for the real part spreads out, which is consistent with the Kramers-Kronig relation. Quantitatively we find such suppression can be well fit by the famous Drude law with two parameters

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}, \quad (25)$$

where $\tau$ is the scattering time and $K$ is an overall constant characterizing the DC conductivity. A blow up of the low frequency part is shown in FIG. 6 with $A_0 = 0.4$, $k_0 = 2$ and $Q = 0.5$. No matter how we change the temperature or lattice spacing, we find the Drude law can hold in all cases we have examined.

From Drude formula one easily finds that the peak of the imaginary part of the conductivity appears at $\omega\tau = 1$. When $\omega\tau > 1$, the frequency enters an intermediate regime, in which we find the modulus of the conductivity exhibits a power-law fall-off. In FIG. 7 we
FIG. 4: The real and imaginary parts of $h_{tz}$ and $b_z$ are shown for $A_0 = 0.4$, $k_0 = 2$, $|T/\mu| = 0.457$ and $\omega = 0.6$.

plot the modulus and argument of the conductivity in this regime and then fit the data with a power-law formula

$$|\sigma(\omega)| = \frac{B}{\omega^\gamma} + C,$$

which contains three free parameters $B, C$ and $\gamma$.Remarkably, when we fix the frequency
FIG. 5: The real and imaginary parts of the conductivity, both without the lattice (dashed line) and with the lattice (solid line and data points) for $A_0 = 0.4$, $k_0 = 2$ and $Q = 0.5$. Only the low frequency regime of the conductivity is changed by the lattice.

FIG. 6: A blow up of the low frequency part of the conductivity with $A_0 = 0.3$, $k_0 = 2$ and $Q = 0.5$. The data can be fit by the simple two parameter Drude form.

regime to be $1.3 \leq \omega \tau \leq 8$, we find the exponent $\gamma \simeq 2/3$ is a universal result, irrespective of what temperature, lattice amplitude or spacing we have taken. We collect our fitting data for the exponent in a table for various values of parameters, accompanying with a log plotting of the variation of the conductivity with the frequency in FIG. 8 in which it should be noticed that for each case an offset $C$ with different value has been subtracted from the vertical axis. This universality further supports the results originally presented in [7]. Our results indicate that such a universality may be extended to various lattice models in four
FIG. 7: The modulus $|\sigma|$ and argument $\arg \sigma$ of the conductivity with $A_0 = 0.4$, $k_0 = 2$ and $Q = 0.5$. The line of the left is a fit to the power-law in (26) for $1.3 \leq \omega \tau \leq 8$.

dimensional spacetime, implying the exponent might be singly related to the dimensionality of the space time.

When the lattice amplitude is large enough or the temperature is low enough, we will observe another interesting feature of the optical conductivity, namely the resonance, which has previously been observed in the context of holographic conductivity in [7]. Such a resonance can be attributed to the excitation of the quasinormal modes of the black hole with an integer multiples of the lattice wavenumber. In FIG. 9, we show a clear example with $A_0 = 0.6$, $k_0 = 1$ and $Q = 1$. In order to see the detailed structure of the resonances, in FIG. 10 we subtract the homogeneous background from the conductivity, and we find that the data can be fit by

$$
\sigma(\omega) = \frac{G_R(\omega)}{i\omega} = \frac{1}{i\omega} \frac{a + b(\omega - \omega_0)}{\omega - \omega_0},
$$

where $G_R(\omega)$ is the retarded Green function which has a pole at the complex frequency $\omega_0$ and $a, b$ are complex constants. In FIG. 10 our fitting fixes the quasinormal mode frequency to be $\omega_0/T = 2.2 - 0.21i$.

In the end of this section we intend to present some remarks on what we have observed in our lattice model and hopefully such observations are valuable for further investigation on linking the holographic lattice with the experiments in condensed matter physics.

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5 We thank J. Santos for drawing our attention to this phenomenon.
FIG. 8: The values of the exponent $\gamma$ with different $Q$, $A_0$ and $k_0$ for $1.3 \leq \omega \tau \leq 8$. The table and the figure on the top have $k_0/A_0 = 5$, whereas the table and the figure at the bottom have $k_0 = 3$.

FIG. 9: The resonances of the conductivity for a lattice with $A_0 = 0.6$, $k_0 = 1$ and $Q = 1$. 

First of all, the value of the exponent is universal in the sense that it is independent of the parameters in the model. However, it does depend on what the intermediate regime of the frequency we would pick out. In particular, we find it is quite sensitive to the initial point of the intermediate interval. For instance, if we select an interval $1.5 \leq \omega \tau \leq 8$ as the intermediate regime, we find the exponent has shifted to $\gamma \approx 0.71$, irrespective of the values of the parameters. Keep going further, for the interval $5 \leq \omega \tau \leq 16$ which perhaps is far from the Drude peak, we find the data of modulus can still be well fit with a power law formula, while for this regime the exponent increases up to 0.96 in our model (See FIG.11). Mathematically such a dependence should not be quite surprising since we have introduced an offset $C$ such that we have a lot of room to fit the data with a power law formula. Physically, our understanding is that such holographic scenarios contain a lot of room to fit data in experiments not only for cuprate but also for other materials, provided that we select an intermediate frequency regime appropriately.

Secondly, the involvement of offset $C$ is not desirable from the side of condensed matter experiments. We find in general such an offset can not be avoided in order to fit the power law of the conductivity. But in some subsets of the parameter space, we do find

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The fact that the exponent is close to one probably implies that our model contains a regime which could be viewed as a graceful exit from the Drude regime since from Eq.(25) we have $|\sigma(\omega)| \sim K/\omega$ as $\omega \tau \gg 1$. 

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FIG. 10: The resonances of the conductivity with the homogeneous background subtracted off. The data points are well fit by (27).
FIG. 11: The values of the exponent $\gamma$ with different $Q$ and $k_0$ when fitting the data in the regime $5 \leq \omega \tau \leq 16$, where we have fixed $k_0/A_0 = 5$.

that the data can be fit well even without the offset $C$.

• Thirdly, the resonance is an independent phenomenon from the power law behavior. Both of them could occur at different frequency regimes. While once one increases the lattice amplitude, the position of the resonance may enter the intermediate frequency regime such that the power law behavior presented in FIG.11 may be swamped.

IV. CONCLUSIONS AND DISCUSSIONS

In this paper, we have constructed a holographic lattice model in Einstein-Maxwell-Dilaton theory and investigated the lattice effects on the optical conductivity of the dual field theory on the boundary. Due to the breaking of the translational symmetry in the presence of the lattice, the delta function previously appeared in the real part of the conductivity is smeared out and the behavior of the conductivity at the low frequency regime can be exactly described by a simple Drude formula. We have also observed a power law behavior for the modulus of the conductivity at an intermediate frequency regime, as that firstly found in [6–8]. Moreover, a resonance phenomenon has been observed when the lattice amplitude is large or the temperature is low enough. Our results furthermore justify the effects of the violation of translation symmetry in the presence of the lattice and test the robustness of the power law behavior at the intermediate regime.
A lot of work can be further done in this direction. First of all, as we stated the zero temperature limit is out of touch in our current paper. Secondly, we can construct a holographic superconductor model with lattice in this theory and study the appearance of the energy gap in superconducting phase. More importantly, we expect that we can understand the simple Drude form in this model through an analytical treatment, in which we can approximately obtain an analytical solution of the conductivity at $\omega \to 0$ by matching the inner region and outer region as suggested in [9, 23, 32, 42]. However, it is difficult to have an analytical treatment at the intermediate frequency regime by employing the matching method directly since the expansion of the perturbations at intermediate frequency is probably problematic. Our work on above issues is under process and we expect to report our results elsewhere in future.

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