Measurement of $BR(K^+ \rightarrow \mu^+\nu_\mu e^+e^-)$ with NA48/2 at CERN

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Abstract. The exclusive branching ratio of the radiative decay $K^\pm \rightarrow \mu^\pm\nu_\mu e^+e^-$ is measured using data collected by the NA48/2 experiment at CERN in 2003-2004. The measurement is performed in the phase-space region of the mass of the electron-positron pair, $m_{e^+e^-} \geq 140$ MeV/$c^2$. This is the most accurate measurement of the exclusive $BR(K^+ \rightarrow \mu^+\nu_\mu e^+e^-)$. The result, $BR(K^+ \rightarrow \mu^+\nu_\mu e^+e^- | m_{e^+e^-} \geq 140$ MeV/$c^2) = (7.81 \pm 0.23) \times 10^{-8}$ is compatible with the theoretical value, $8.51 \times 10^{-8}$, calculated using the Chiral Perturbation Theory framework at next-to-leading order.

1. Introduction

The branching ratios of the radiative four-lepton Kaon decays, $K^\pm \rightarrow l^\pm\nu_bl^+l^-$, where, $l, l' = e, \mu$ and $l = l'$ or $l \neq l'$, are precisely known in the Standard Model (SM) already at the tree level. However, their differential branching ratios in the high kinematic tail of the invariant mass of the $l^+l^-$ pair, $m_{l^+l^-} \geq 140$ MeV/$c^2$, are particularly interesting for two reasons: a) it is possible to have a precise and unambiguous calculation of the low-energy QCD effects within the framework of the Chiral Perturbation Theory (ChPT) at next-to-leading order; b) the suppression of the SM branching ratio in the high $m_{l^+l^-}$ region makes the decays sensitive...
to a new physics phenomenon, which can contribute in the production of the $l^+l'^-$ pair via some new particle with a non-zero mass [1, 2, 3].

In the following work the exclusive branching ratio, $BR(K^\pm \rightarrow \mu^\pm\nu_\mu e^+e^-)$ is measured in the kinematic region of $m_{e^+e^-} \geq 140 \text{ MeV}/c^2$. The corresponding theoretical value, $BR(K^\pm \rightarrow \mu^\pm\nu_\mu e^+e^- \mid m_{e^+e^-} \geq 140 \text{ MeV}/c^2) = 8.51 \times 10^{-8}$, is calculated [4] using the ChPT framework. The calculation includes next-to-leading order loop corrections, which lead to the Structure Dependent (SD) contribution (see Figure 1c) in the branching ratio. The Inner Bremsstrahlung (IB) contribution is mainly determined by the leading order amplitudes of the initial- and final-state radiation (see Figures 1a and 1b) and dominates if a calculation is done in the full phase space. Table 1 shows results of theoretical calculation [4] taking only IB or IB, SD and their interference (INT) contributions into account. Comparison is made for both the full and the interesting $m_{e^+e^-} \geq 140 \text{ MeV}/c^2$ phase space regions. The SD and INT contributions have no significant impact on the inclusive branching ratio. In contrast, their contribution is about 40% in the exclusive branching ratio in the $m_{e^+e^-} \geq 140 \text{ MeV}/c^2$ region.

Table 1: Comparison of branching ratios obtained with only IB and with all IB+SD+INT contributions included in the calculation of the ChPT form factors. Results for both the full phase space region and the high mass region of the $e^+e^-$ pair are presented.

|                  | IB          | IB+SD+INT  |
|------------------|-------------|------------|
| Full phase space | $2.49 \times 10^{-5}$ | $2.49 \times 10^{-5}$ |
| $m_{e^+e^-} \geq 140 \text{ MeV}/c^2$ | $4.98 \times 10^{-8}$ | $8.51 \times 10^{-8}$ |

The first measurement of $BR(K^\pm \rightarrow \mu^\pm\nu_\mu e^+e^- \mid m_{e^+e^-} \geq 140 \text{ MeV}/c^2) = (12.3 \pm 3.2) \times 10^{-8}$ was carried out at CERN in 1976 [5]. The mean value is higher than the theoretical one but the difference is not significant due to the large uncertainty of the measurement. A more precise measurement of the exclusive branching ratio but with a more severe cut on the phase space was performed later by the E865 experiment at BNL in 2002 [6]. The result, $BR(K^\pm \rightarrow \mu^\pm\nu_\mu e^+e^- \mid m_{e^+e^-} \geq 145 \text{ MeV}/c^2) = (7.06 \pm 0.16 \text{ (stat.)} \pm 0.26 \text{ (syst.)}) \times 10^{-8}$ is in agreement with the ChPT expectation.

2. The NA48/2 Experiment

The main goal of the fixed-target experiment NA48/2 was to measure the direct CP violation in the $K^\pm \rightarrow \pi^\pm\pi^+\pi^-$ and $K^\pm \rightarrow \pi^\pm\pi^0\pi^0$ decays [7]. The experiment was operating on the primary beam of 400 GeV/c momentum protons supplied by the SPS accelerator facility at CERN. A secondary beam of simultaneous and collinear $K^+$ and $K^-$ with the momentum $(60 \pm 3.7) \text{ GeV}/c$ was extracted after the Beryllium target. Kaon decays in vacuum inside the 114 m long fiducial volume were used for measurements. The magnetic spectrometer was placed downstream after the decay volume to measure charged tracks and was operating in Helium at the atmospheric
pressure. It consisted of four drift chambers (DCH) and a dipole magnet installed in between the 2nd and 3rd DCHs. The dipole magnet was providing 120 MeV/c horizontal momentum kick. The momentum resolution of the spectrometer was \( \sigma_p/p = (1.02 \pm 0.044 \cdot p)/\% \), where, \( p \) is given in GeV/c. The magnetic spectrometer was followed downstream by the charged track hodoscope (HOD) to trigger events with charged tracks and measure their time. Its resolution was about 200 ps. The HOD was followed by the 27\( X_0 \) deep, almost homogeneous electromagnetic calorimeter (LKr) filled with liquid Krypton. The LKr had excellent energy and spatial resolution. As an example, \( \sigma_E/E = 1.4\% \) and \( \sigma_x = \sigma_y = 1.5 \text{ mm} \) at \( E = 10 \text{ GeV} \). The LKr was followed by the hadronic calorimeter and the muon veto system (MUV). The MUV consisted of three orthogonal planes of scintillator strips to measure time and coordinates of tracks. Each MUV plane was preceded with a 80 cm thick iron wall for absorbing hadrons. An additional 40 cm thick iron wall was placed after the last MUV plane to stop hadrons scattered from the material at the rear of the beamline. A complete description of the NA48/2 beamline and the detector can be found in [8].

3. Measurement of \( BR(K^\pm \rightarrow \mu^\pm \nu_\mu e^+ e^- | m_{e^+ e^-} \geq 140 \text{ MeV}/c^2) \)

The signal \( K^\pm \rightarrow \mu^\pm \nu_\mu e^+ e^- \) decay (Figure 2a) has a similar three-track event topology as the \( K^\pm \rightarrow \pi^\pm \pi^+ \pi^- \) decay. Therefore, its measurement is based on the NA48/2 main physics trigger for selection of the three-track events. In total, order of \( 10^{11} K^\pm \) decays were recorded during the whole operation of the experiment using the main trigger.

3.1. Signal event selection

Events with the reconstructed good-quality vertices of three tracks within the 98 \( m \) long fiducial volume are selected. The total charge of the three tracks is required to be \( \pm 1 \) in order to avoid events with the fake three-track vertices. All tracks are required to be within the geometrical acceptance of the DHCs, HOD, LKr and MUV detectors. Two out of the three tracks have to be identified as electrons using the LKr calorimeter. The third track is identified as muon if it behaves as a minimum ionising particle (MIP) in the LKr and is detected by the MUV. Further selection criteria are applied to reject background events. The requirement \( m_{e^+ e^-} \geq 140 \text{ MeV}/c^2 \) helps to completely get rid of the background from the \( K^\pm \rightarrow \pi^\pm \pi_0(\pi_0 \rightarrow e^+ e^- \gamma) \) and \( K^\pm \rightarrow \mu^\pm \nu_\mu \pi_0(\pi_0 \rightarrow e^+ e^- \gamma) \) decays. A cut on the invariant mass of the muon-neutrino pair, \( M_{\mu \nu} > m_{\pi^\pm} + 3 \times \sigma(M_{\mu \nu}) = 170 \text{ MeV}/c^2 \), is applied to suppress the \( K^\pm \rightarrow e^+ e^- \pi^\pm(\pi^\pm \rightarrow \mu^\pm \nu_\mu) \) decay background from 10\% down to a negligible level. Here, \( M_{\mu \nu} = \sqrt{(p_{K^\pm} - p_{e^+ e^-})^2} \) and the resolution \( \sigma(M_{\mu \nu}) = 10 \text{ MeV}/c^2 \) in the peak region of \( m_{\pi^\pm} \) is found from Monte-Carlo simulation. The requirement \( M_{\mu \nu} > 170 \text{ MeV}/c^2 \) decreases the signal acceptance by 11\%.

3.2. Data driven estimation of background

Diagrams of the background decays that are not completely suppressed with the event selection cuts are shown in Figures 2b - 2e. They contribute in the signal selection due to particle misidentification or/and detection inefficiency. The \( K^\pm \rightarrow \pi^\pm \pi^0 \pi^0 \) decay \( (K_{3\pi0}) \), where \( \pi^\pm \) decays further into the \( \mu^\pm \nu_\mu \) pair (Figure 2b) and only two electrons out of four electrons and two photons are detected, mimics a signal event. The \( K^\pm \rightarrow \pi^\pm \pi^+ \pi^- \) decay \( (K_{3\pi}) \) can mimic the signal if one of the pions decays into \( \mu \nu_\mu \) and the remaining pions are both misidentified as electrons (Figure 2c). The misidentification probability is small but given the large branching ratio of the \( K_{3\pi} \) decay the double-misidentified pions as electrons are still non-negligible background. The \( K^\pm \rightarrow e^\pm \nu_\mu \pi^+ \pi^- \) decay \( (K_{e4}) \) can contribute in the signal selection in two different ways. In one scenario one of the pions is misidentified as an electron and another one as a muon (Figure 2d). In the second scenario one of the pions is misidentified as an electron while the second pion decays into a \( \mu^\pm \nu_\mu \) pair (Figure 2e). Data driven approach is used to
estimate contribution of the irreducible background decays, $K_{3\pi0}$, $K_{3\pi}$ and $K_{e4}$ into the final selection of events. The idea of the data-driven method lies in selecting events with same-sign pair of electrons while other selection cuts are the same as for the signal event selection. The muon charge has to have the opposite sign to the same-sign electron pair. The misidentification probability of pions as electrons and the detector and trigger inefficiencies and acceptance do not depend on the electric charge. Therefore, the expected number of background events of a given type in the signal region (opposite-sign ee-pair) is equal to the observed number of background events of the given type in the control region (same-sign ee-pair) multiplied by a corresponding combinatorial factor. The combinatorial factor for the $K_{3\pi0}$ background (Figure 2b) is 2, since there are twice more possibilities of having $e^+e^-$ pairs selected than the same sign pairs, $e^+e^+$ or $e^-e^-$. For other irreducible background decays (Figures 2c-2e), the corresponding combinatorial factors are equal to 1. Figure 3 shows distributions of squared missing mass, defined as $M_{\text{miss}}^2 = (P_{K^\pm} - P_{\mu^+e^-})^2$, before the last selection cut of events is applied. The dashed vertical lines in red represent the final selection cut. This cut helps to remove a large fraction of the estimated background (solid blue histogram), which tends to contribute in the higher $M_{\text{miss}}^2$ region due to misidentified pions as electrons. Figure 4 shows a distribution of a dimensionless observable, $z = (m_{e^+e^-}/m_{K^\pm})^2$, in data (black histogram) after the final selection. In total, 1663 events are selected while the total background contribution is estimated to be about 3%. The $z$-distribution is compared with two different Monte-Carlo simulated samples of the signal. The shape of the $z$-distribution in the signal sample simulated with only IB amplitudes (red histogram) does not match the distribution in data. In contrast, the distribution in the signal sample produced with all IB+SD+INT contributions included in the Monte-Carlo simulation (solid cyan histogram) correctly describes the measured spectrum. In both Figure 3 and Figure 4 the simulated signal samples are normalised to the number of selected events in data minus the estimated number of events of the total background.

### 3.3. Signal normalisation

The total number of Kaon decays in the 98 $m$ long decay volume is measured by using the $K^\pm \rightarrow \pi^\pm \pi^\pm \pi^-$ decay. Background-free selection of this decay events can be performed by selecting the three-track events (same as for the signal selection) and applying two-sided cut (as shown in Figure 5 with dashed vertical lines in red) on a distribution of the mass of the three tracks around the mass peak of $K^\pm$. Order of $10^9$ $K^\pm \rightarrow \pi^\pm \pi^\pm \pi^-$ decay events are selected. The acceptance of the $K^\pm \rightarrow \pi^\pm \pi^\pm \pi^-$ decay channel, (24.04 ± 0.01)%, is found from Monte-Carlo simulation. Using the number of selected $K^\pm \rightarrow \pi^\pm \pi^\pm \pi^-$ events and the acceptance and the known $BR(K^\pm \rightarrow \pi^\pm \pi^\pm \pi^-)$, the total number of the $K^\pm$ decays is calculated to be $(1.56 \pm 0.01) \times 10^{11}$. Similarity of the event topologies of the normalisation $K^\pm \rightarrow \pi^\pm \pi^\pm \pi^-$ and the signal decays leads to the first order cancelation of some common systematic effects such as detector and trigger inefficiencies, etc.

Figure 2: Diagrams of signal (a) and $K_{3\pi0}$ (b), $K_{3\pi}$ (c) and $K_{e4}$ (d, e) background decays.
### Results

The signal acceptance is found to depend on the z-region according to Monte-Carlo simulation and to vary from 12% to 15%. In order to properly account for the variation of the acceptance the signal decay branching ratio is measured in 15 bins of z. The result of the measurement is presented in Figure 6. The total exclusive branching ratio is the sum of the measured branching ratios over all bins of z and is equal to $BR(K^z \rightarrow \mu^\pm \nu_\mu e^\mp e^- | m_{e^+e^-} \geq 140 \text{ MeV}/c^2) = (7.81 \pm 0.21 \text{ (stat.)} \pm 0.08 \text{ (syst.)} \pm 0.06 \text{ (ext.)}) \times 10^{-8}$. The uncertainty of the measurement due to the uncertainty of $BR(K^\pm \rightarrow \pi^\pm \pi^\mp \pi^-)$ is presented separately. It is referred as an external uncertainty.
Table 2: List of the sources of the uncertainty of the measurement. The list is divided into three
groups of the statistical, systematical and external uncertainty sources.

| Uncertainty source                  | $\Delta BR/BR \times 10^{2}$ |
|-------------------------------------|-------------------------------|
| Data statistics                     | 2.54                          |
| Normalisation channel statistics    | 0.02                          |
| Radiation corrections               | 0.70                          |
| Background statistics               | 0.02                          |
| Background systematics              | 0.30                          |
| Trigger efficiency                  | 0.54                          |
| Muon identification efficiency      | 0.13                          |
| Electron identification efficiency  | 0.04                          |
| Signal channel acceptance           | 0.12                          |
| Normalisation channel acceptance    | 0.03                          |
| $BR(K^\pm \rightarrow \pi^+\pi^-\pi^-)$ | 0.72                          |
| Total                               | 2.88                          |

uncertainty. Table 2 presents a detailed list of all uncertainties of the measurement. The data statistical uncertainty is the main source of the total uncertainty. The main contributors in the systematical uncertainties are the precision of the knowledge of $BR(K^\pm \rightarrow \pi^+\pi^-\pi^-)$ and modelling of the radiative corrections. The uncertainty due to the latter source is estimated in a conservative way by switching on/off the radiative corrections in Monte-Carlo simulation of the signal. The uncertainties of the background estimation and the trigger efficiency also have significant impact on the precision. The other sources of uncertainty are negligible.

4. Conclusions
Model independent measurement of the exclusive branching ratio of the radiative decay $K^\pm \rightarrow \mu^\pm \nu e^+e^-$ has been performed in the region $m_{e^+e^-} \geq 140$ MeV/$c^2$. In total, $(1.56 \pm 0.01) \times 10^{11}$ events of $K^\pm$ decays collected by the NA48/2 experiment at CERN were used for the measurement. The result is in agreement with the ChPT prediction. The uncertainty of the measurement is dominated by the statistical component.

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