Meson Strings and Flavor Branes

Masako BANDO\textsuperscript{1}, Akio SUGAMOTO\textsuperscript{2,3} and Sachiko TERUNUMA\textsuperscript{3}

\textsuperscript{1}Physics Division, Aichi University, Aichi 470-0296, Japan
\textsuperscript{2}Department of Physics, Ochanomizu University, Tokyo 112-8610, Japan
\textsuperscript{3}Graduate School of Humanities and Sciences, Ochanomizu University, Tokyo 112-8610, Japan

Abstract

We investigate the shape of meson strings in the five-dimensional curved space and the potential between the quark and anti-quark in a QCD-like string model based on D6 flavor branes in the presence of D4 color branes wrapping one of the compactified dimension on an $S^1$. The flavor branes on which both ends of a meson string lie are assumed to be separated in this five dimensional space, depending on the values of the constituent quark masses. It is shown in this picture that a meson string with different flavors at two ends changes shape at a critical distance. There is, however, no critical distance for a meson with the same flavor. At this critical distance, the potential between a quark and anti-quark with different flavors gives a point of reflection and changes shape near this point. Accordingly, the attractive force between a quark and an anti-quark seems to become stronger when the distance between the flavor branes connecting meson strings becomes larger. This indicates that quark systems with different flavors can form high-density states.
1 Introduction

The origin of families is one of the most important subjects of particle physics. It is well known that quarks and leptons appear as three repetitive families, each of which forms a multiplet of the strong, weak and electromagnetic gauge symmetries. The coupling constants of these gauge interactions are uniquely determined by the symmetry principle and are independent of the flavors. By contrast, flavor interactions determined by the Yukawa coupling to Higgs fields are mysterious; their coupling strength exhibits hierarchical structure depending on the flavor of the matter fields. Many proposals for the origin of flavor have been made: It may originate from some family quantum number, Abelian or non-Abelian, by imposing a horizontal symmetry. However, such hierarchical structure could not be derived from such symmetries. One promising idea is to extend our 4-dimensional space to a higher-dimensional space and to attribute the origin of flavor to the structure of the extra dimensions. If we can gain some information from such an idea with higher-dimensional space-time, we may be able to take an important step toward the understanding of flavor.

In the previous papers [1, 2], we studied QCD strings based on the picture that a quark is identified as a colored flavored string, one end of which has a color (color end) and the other end of which has a flavor (flavor end). Therefore, the string of a free quark emerges from a point on a color brane and terminates at a point on a flavor brane. Furthermore, apart from the stack of color branes, the flavor branes are considered to be separated flavor by flavor in the direction of the extra dimension. Hereafter, we denote the coordinate of this extra dimension by $u$. A meson consists of two strings from a quark and an anti-quark, but they are joined to form a single meson string, because color ends of two strings can be annihilated, and only flavor ends appear on individual flavor branes. Similarly, a baryon consists of three strings, with the flavor ends of the three strings located on individual flavor branes, and the color ends of the three strings, if the colors are red, green and blue, can annihilate to form a junction. In this way, we studied the exotic hadron of pentaquarks in the previous papers [1, 2]. In such a picture, the hierarchical structure of the masses of quarks can be naturally explained by taking proper allocations of flavor branes in the curved space. In addition, the existence of a horizon-like singularity coming from the stack of $N_c$ color branes can account for the non-perturbative linear potential between a quark and an anti-quark.

In this paper, we study the shape of meson strings in the five-dimensional
curved space and calculate the potential between the quark and anti-quark in
detail. Leaving the details to Ref. [1], we briefly explain the picture of flavor
branes existing in the space extended to extra dimensions. Here we adopt a calisitic
QCD-like model in the string theory developed by Witten and others [3]. In
this model, all the supersymmetries are broken by the compactification of one
of the extra dimensions on an $S^1$ which is wrapped by D4 color branes. In the
model, the gauge theory is represented by open strings, while the gravity theory is
represented by closed strings. Because the two theories are dual, we can replace the
non-perturbative QCD by the corresponding classical gravity theory. The original
space of the gravity theory is flat. However, once heavy sheets of $N_c$ color ($D_4$)
branes are placed perpendicular to the extra dimension $u$, it is deformed in this
extra dimension. Because each endpoint of a meson string lies on a flavor brane,
meson strings possesses different shapes, according to the variety of the flavors of
the quarks on two ends.

Let us simplify the ten-dimensional space-time of string theory to a 5-dimensional
space-time, including only the extra coordinate $u$ in addition to the usual Minkowski
space, $(t, z, x_\perp)$. The world volumes of the stack of $N_c$ color ($D_4$)
branes and the separately allocated flavor branes extend along the Minkowski space but are perpen-
dicular to the $u$ direction. This curved space can be expressed as

$$ds^2 = f(u)(-dt^2 + dz^2 + dx_\perp^2) + g(u)du^2,$$

(1)

with the functions $f(u)$ and $g(u)$. In a realistic QCD-like model, they are [3]

$$f(u) = (u/R')^{3/2}, \quad g(u) = (f(u)h(u))^{-1},$$
$$h(u) = 1 - (U_{KK}/u)^3,$$

(2)

with $R'$ and $U_{KK}$ given by

$$R'^3 = 2\pi\alpha_cN_c\alpha'/M_{KK}, \quad \text{and} \quad U_{KK} = \frac{8\pi}{9}\alpha_cN_c\alpha'M_{KK},$$

(3)

where $(2\pi\alpha')^{-1}$ is the string tension of the starting Lagrangian, being $O(M_{\text{Planck}}^2)$,
and $\alpha_c$ is the QCD (with $N_c$ colors) coupling. The warp factor $f(u)$ is a monoton-
ically increasing function of $u$. * In the following, we write $\alpha'$ (of the order of the Planck length squared) dependence explicitly. This was omitted in Ref. [1].

---

*For reference, we also give the metric of the proto-type model of Mardacena [4], possessing $N = 4$ supersymmetries,

$$f(u) = g(u)^{-1} = (u/R)^2,$$

(4)
In such a deformed space including the one extra dimension $u$, there exist “flavor branes” on which the endpoints of the hadron strings are placed, while quarks are expressed as open strings having one endpoint on the flavor brane and the other endpoint on the “color brane”. The shape of a hadron string chooses its most economical path in this curved space. It is analogous to a catenary under the influence of gravity on the earth. In our case, gravity is along the $u$-direction, and its nature is different from that of gravity on the earth.

Here we concentrate our attention on a meson string with different flavors at either end and calculate their shapes and the potentials between the quark and anti-quark, especially those with different flavors.

2 QCD strings in curved space

First, we give a brief survey of the formulae needed in this paper\[1\]. The string action in the background curved space is expressed as

$$S = \frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-(\dot{X}^M \dot{X}_M)(X'^N X'_N) + (\dot{X}^M X'_M)^2},$$

(6)

where the dot and the prime represent derivatives with respect to $\tau$ and $\sigma$, respectively, and $X^M(\tau, \sigma)$ describes the configuration of the worldsheet of a string with the two parameters $\tau$ and $\sigma$. Here, the outside space (i.e. the target space) of $X^M$ is curved, and the contraction $\dot{X}^M \dot{X}_M$, etc., represents that with the metric $G_{MN}(x)\dot{X}^M \dot{X}^N$, etc.

If we fix the parameterization of the worldsheet, choosing $\sigma = z$ and $\tau = t$, and take the static limit, $\dot{X}^M = 0 (M \neq 0)$, the string action (6) in the time interval $\Delta t$ reads

$$S = \frac{\Delta t}{2\pi\alpha'} \int dz L,$$

(7)

with the “Lagrangian”

$$L = \sqrt{f(u)^2(1 + (x'_\perp)^2))} + f(u)g(u)(u')^2.$$

(8)

By regarding $z$ as “time”, the following equation of motion is derived:

$$u' = \frac{1}{(-H)} \sqrt{\frac{f}{g}} (f^2 - (-H)^2 - (p_\perp)^2),$$

(9)

with the radius $R$ of $AdS_5$ space given by

$$R^4 = 8\pi\alpha' N_c\alpha'^2,$$

(5)

In this case $f(u)$ is also an increasing function of $u$. 


with the three conserved quantities

\[ (-H) = \frac{f(u)^2}{L}, \quad \text{and} \quad (p_\perp) = x'_\perp(-H). \] (10)

The shape of the string \( u(z) \) is determined by Eq. (9), once we fix the boundary conditions, and the coordinates of the endpoints of the string are denoted by \((U_1, Z_1)\) and \((U_2, Z_2)\).

The “action” during the change of \( u \) from \( U_1 \) to \( U_2 \) is given by

\[ \frac{1}{2\pi \alpha'} \int_{U_1}^{U_2} dz L = \frac{1}{2\pi \alpha'} \int_{U_1}^{U_2} du \frac{1}{u'} \frac{f(u)^2}{(-H)}, \] (11)

and thus the energy stored inside a string is

\[ E = \frac{1}{2\pi \alpha'} \int_{U_1}^{U_2} du \sqrt{ \frac{f(u)^3 g(u)}{f(u)^2 - (-H)^2 - (p_\perp)^2} }. \] (12)

We can also estimate the change of \( z \) from \( U_1 \) to \( U_2 \) as

\[ r = Z_2 - Z_1 = \int_{U_1}^{U_2} \frac{du}{u'} = (-H) \int_{U_1}^{U_2} du \sqrt{ \frac{g(u)}{f(u)^2 - (-H)^2 - (p_\perp)^2} } , \] (13)

with the string shape determined by Eq. (9). Note that this general formulation is also applicable to the more complicated web-like exotic hadrons and string systems.

### 3 Meson string

Now let us consider a meson string representing a bound state of \((q_1 \bar{q}_2)\). This is a string connecting the flavor brane at \((u = U_1, \ z = Z_1)\) of quark \( q_1 \) and the flavor brane at \((u = U_2, \ z = Z_2)\) of anti-quark \( \bar{q}_2 \). We take \( z \) to be the direction in which the string is stretched. In our picture, quarks are not point particles but also strings. However, it is acceptable to regard point particles \( q_1 \) and \( \bar{q}_2 \) as being placed on the flavor branes at \( u = U_1 \) and \( u = U_2 \), respectively, with the gluonic QCD string connecting them. Then, our meson string can be understood as the entire system consisting of the quark, anti-quark and the gluonic QCD string connecting them. Then, the distance \( r = Z_2 - Z_1 \) can be interpreted as the distance between the point quark \( q_1 \) and the anti-quark \( \bar{q}_2 \) in the ordinary Minkowski space. We can choose \( x_\perp = 0 \) for mesons. Then, the string connecting the quark \( q_1 \) and the anti-quark \( \bar{q}_2 \) appears on the \((z, u)\)-plane. Then, the equation of motion takes the simple form

\[ u' \equiv \frac{du}{dz} = \frac{1}{f_0^2} \sqrt{ \frac{f(u)}{g(u)} (f^2(u) - f_0^2) } , \] (14)
with
\[ E = \frac{1}{2\pi\alpha'} \int_{U_1}^{U_2} \left[ \frac{\int_{U_0}^{U_1} \, du' \, \sqrt{f(u)^3 g(u)}}{\int_{U_0}^{U_1} \, du' \, \sqrt{f(u)^2 - f_0^2}} \right]. \] (15)

Here, we fix the constant value \((-H)\) at the point \(u = U_0\); that is, we have \((-H) = f(U_0) \equiv f_0\). This means that \(u = U_0\) is a stationary point of \(u\) giving \(u' = 0\). We can always find \(Z_0\) for which \(u(Z_0) = U_0\) and \(u'(Z_0) = 0\) hold, and therefore we can take \(Z_0 = 0\) hereafter without loss of generality.

There are two cases here. The first is that in which the stationary point, \((z, u) = (0, U_0)\), giving \(u' = 0\), exists on the string stretching between their ends. The second is that in which the stationary point \((0, U_0)\) is not on the string but is outside the string. In the former case, if \(Z_1 \leq 0\), then \(Z_2 \geq 0\), or vice versa. In the latter case, both \(Z_1\) and \(Z_2\) are positive or negative. In order to find a string profile, it is useful to note that the solution of Eq. (9) is symmetric under the exchange \(z \rightarrow -z\), so that if we find a solution \((z, u = u(z))\), then \((-z, u = u(z))\) is also a solution. Therefore, we have only to solve the equation for \(z \leq 0\). In Eq. (14), we have \(u' \geq 0\), and thus if we start to solve the solution from the point \((z = 0, U_0)\), \(u(z)\) increases monotonically in the positive \(z\) direction \((0 \leq z \leq Z_2)\).

We have to prepare two such solutions with \(0 \leq z \leq Z_1\) and \(0 \leq z \leq Z_2\). In the case that there is a stationary point on a meson string, we obtain a solution of the string profile by connecting at \(z = 0\) the above two solutions, the solution with \(0 \leq z \leq Z_2\), and the reversed solution of \(0 \leq z \leq Z_1\) with respect to the \(u\)-axis (by exchanging \(z\) and \(-z\)). In the other case, in which the stationary point lies outside the meson string, the profile of the meson string is the part \(U_1 \leq u \leq U_2\) of the solution of \(0 \leq z \leq Z_2\), where we have assumed \(U_1 \leq U_2\). The former string is a combination of the string from \((-|Z_1|, -|U_1|)\) to \((0, U_0)\) and that from \((0, U_0)\) to \((Z_2, U_2)\). On the other hand, in the latter case, the string can be expressed as that connecting the points \((0, U_0)\) and \((Z_2, U_2)\) by substructing the string from \((0, U_0)\) to \((Z_1, U_1)\). Here, the minimum value \(U_0\) is determined as a function of their separation parameter, \(r = Z_2 - Z_1\), via Eq. (13).

Now, from Eq. (13), the separation parameter reads
\[ r = Z_2 - Z_1 = f_0 \left[ \int_{U_0}^{U_1} \, du \, \sqrt{\frac{g(u)}{f(u)(f(u)^2 - f_0^2)}} \right], \] (16)
and the energy can be written from (12) as
\[ E = \frac{1}{2\pi\alpha'} \left[ \int_{U_0}^{U_2} \, du \, \sqrt{\frac{f(u)^3 g(u)}{f(u)^2 - f_0^2}} \right]. \] (17)
Figure 1: Typical profile of the shape of a string connecting $q_1$ and $\bar{q}_2$ in the QCD-like model.

In the above expressions, we take the signature $+$ in the first case, $Z_1 \leq 0 \leq Z_2$, and $-$ in the second case, $0 \leq Z_1 \leq Z_2$.

The string shape is determined in such a manner as to minimize this energy for a given separation $r$ between a quark and an anti-quark. A typical shape of a string for the case of a relatively large separation $r$ is depicted in Fig. 1, where the endpoints of the string terminate on the first brane and the second brane at $(Z_1, U_1)$ and $(Z_2, U_2)$, respectively.

In the large separation limit ($r \to \infty$), therefore, the string departing from the quark at $u = U_1$ quickly goes down vertically to the lowest possible value, $u = U_0 \approx U_{KK}$, and then moves horizontally from $Z_1$ to $Z_2$, approximately maintaining the relation $u \approx U_{KK}$. Then it goes up to the anti-quark point at $u = U_2$. This means that the basic shape of the meson string at a large separation is rectangular.

In order to understand this almost rectangular shape of the meson string, the following expression of the energy is useful:

$$E = \frac{1}{2\pi\alpha'} \int_{U_1}^{U_2} \sqrt{f(u)(f(u)dz^2 + g(u)du^2)}. \tag{18}$$

The approximately rectangular shape consists of two parts, the vertical and hori-
zontal parts. Using the above expression of energy, we find

$$E_{\text{basic rectangular}} \sim E_{\text{vertical}} + E_{\text{horizontal}},$$  \hspace{1cm} (19)

where

$$E_{\text{vertical}} = \frac{1}{2\pi \alpha'} \left[ \int_{U_{KK}}^{U_2} \sqrt{f(u)g(u)} du \right],$$  \hspace{1cm} (20)

$$E_{\text{horizontal}} = \frac{1}{2\pi \alpha'} \int_{Z_1}^{Z_2} f(u) dz.$$  \hspace{1cm} (21)

Because $f(u)$ is an increasing function of $u$, $f(u)$ is smallest at $u = U_{KK}$. Here, the region of $u$ is restricted by $u \geq U_{KK}$, due to the existence of the horizon-like singularity at $u = U_{KK}$. Therefore, the horizontal part on which $u$ is as small as possible near $u = U_{KK}$ is energetically favored.

Let us define the quark mass (the constituent quark mass) $m_i$ as the vertical part of the string energy in the limit of large separation, namely the energy of the string stretching along the vertical line from the quark at $U_i$ on the $i$-th flavor brane to the horizon-like brane at $u = U_{KK}$:

$$m_i \equiv \frac{1}{2\pi \alpha'} \int_{U_{KK}}^{U_i} \sqrt{f(u)g(u)} du.$$  \hspace{1cm} (22)

Then, the remaining energy obtained after subtracting the quark masses can be understood as the potential energy $V(r)$ between the quark and anti-quark,

$$V(r) = E_{\text{tot}}(r) - m_{q_1} - m_{q_2}.$$  \hspace{1cm} (23)

In the limit of large separation, the basic rectangular shape appears, and its horizontal part gives a typical potential energy,

$$V(r) \approx \frac{1}{2\pi \alpha'} \int_{Z_1}^{Z_2} f(u) du \approx \frac{1}{2\pi \alpha'} f(U_{KK}) r, \hspace{0.5cm} r = Z_2 - Z_1.$$  \hspace{1cm} (24)

Therefore, in the limit of large separation, the QCD-like string model yields a linear potential proportional to $r$ with the coefficient

$$k = \frac{f(U_{KK})}{2\pi \alpha'}. \hspace{1cm} (25)$$

\footnote{Note that in Maldacena’s prototype model [4], there is no horizon-like singularity, namely $f(U_{\text{MIN}}) = f(0) = 0$. Therefore, in this case, the linear potential does not appear, and the potential becomes Coulomb-like, which reflects the conformal symmetry at large distance.}
4 Parameters of the QCD meson string

Now let us determine the parameters appearing in the QCD-like string model from the information connecting the QCD strings and hadron phenomena.

We denote the tension of QCD strings as $k$, which is expressed in terms of the Regge slope, $\alpha_{\text{QCD}}$, \(^\dagger\) as

$$k = \frac{1}{2\pi \alpha'_{\text{QCD}}}.$$  

Let us derive this relation. First, if we rotate the QCD string relativistically and classically estimate its energy (or its mass $M$) and its angular momentum $J$, then we have the relation coming from the so-called Regge trajectory,

$$J = \alpha'_{\text{QCD}} M^2 = \frac{1}{2\pi k} M^2.$$  

Next, utilizing the experimental data for the $J$ dependence of $M^2$ (Regge slope) \[^5\], we fix $\alpha'_{\text{QCD}} = 0.9(\text{GeV})^{-2}$. we then obtain the coefficient of the linear potential as

$$E = kr, \quad k = 0.88 \text{GeV/fermi} = 0.176(\text{GeV})^2,$$  

which is identified with

$$k = \frac{1}{2\pi \alpha'} f(U_{KK})$$  

in our model. Then, using the expression for $f(u)$ in Eq. (2), we have

$$k = \frac{1}{2\pi \alpha'} f(U_{KK}) = \frac{1}{2\pi \alpha'} \left(\frac{U_{KK}}{R'}\right)^{3/2},$$  

where $R'$ and $U_{KK}$ are given in terms of $M_{KK}$ as

$$R'^3 = \frac{2\pi \alpha_c N_c \alpha'}{M_{KK}}, \quad \text{and} \quad U_{KK} = \frac{8\pi}{9} \alpha_c N_c \alpha' M_{KK}.$$  

Therefore, we have the following expression of $k$ in terms of $M_{KK}$:

$$k = \frac{8}{27} M_{KK}^2 (\alpha_c N_c).$$  

We should remark here that in the above equation, the parameter $\alpha'$, the string tension of the order of the Planck scale in the original Lagrangian, disappears, and

\(^\dagger\)The parameter $\alpha'_{\text{QCD}}$ in the meson potential is the Regge slope of QCD strings, and it is different from the tension $\alpha'$ appearing in our original action. The latter is of the order of the Planck length squared.
hence $k$ and $M_{KK}$ are the parameters of QCD scale. By taking $(\alpha_c N_c) \sim 1$, we obtain

$$M_{KK} \approx \sqrt{\frac{27}{8}} \times 0.176 \text{ GeV} \approx 0.77 \text{ GeV}. \quad (33)$$

For convenience, we express the parameters in the integration appearing in the above in terms of non-dimensional variables. The quark mass given in Eq. (22) can be rewritten as

$$m_i \equiv \frac{U_{KK}}{2\pi\alpha'} \int_{v_0}^{v_i} \sqrt{f(u)g(v)} dv = \frac{4M_{KK}}{9} \int_{v_0}^{v_i} \sqrt{\frac{1}{1 - v^{-3}}} dv, \quad (34)$$

with the non-dimensional parameter $v = \frac{u}{U_{KK}}$. The expression for the energy of a string given in Eq. (17) can be written as

$$E = \frac{1}{2\pi\alpha'} \left[ \int_{U_0}^{U_2} \pm \int_{U_0}^{U_1} \right] du \sqrt{\frac{f(u)^3g(u)}{f(u)^2 - f_0^2}} \quad (35)$$

$$= \frac{4M_{KK}}{9} \left[ \int_{v_0}^{v_2} \pm \int_{v_0}^{v_1} \right] v^3 \sqrt{\frac{1}{(v^3 - v_0^3)(v^3 - 1)}} dv, \quad (36)$$

with the length $r$ given by

$$r = f_0 \left[ \int_{U_0}^{U_2} \pm \int_{U_0}^{U_1} \right] du \sqrt{\frac{g(u)}{f(u) (f(u)^2 - f_0^2)}} \quad (37)$$

$$= \frac{3}{2M_{KK}} \left[ \int_{v_0}^{v_2} \pm \int_{v_0}^{v_1} \right] \sqrt{\frac{v_0^3}{(v^3 - 1)(v^3 - v_0^3)}} dv, \quad (38)$$

where $v_0 = U_0/U_{KK}$.

The value $U_i = U_{KK}v_i$, determining the position of each flavor brane can be obtained from Eq. (22), once we fix the experimental values of the constituent quark masses $m_i$. § In Table 1 we list the values of the constituent quark masses with the corresponding values of $U_i$ evaluated from Eq. (22).

Note that the parameters $U_i$ are all expressed in units of $U_{KK} = \frac{8\pi}{9}\alpha' M_{KK}$, with the Planck scale parameter $\alpha'$ included. Therefore, the separation of the flavor branes in the extra direction is expressed in the extremely small units of $O(\text{GeV}/M_{\text{Planck}}^2) = O(10^{-38}\text{GeV}^{-1})$. However, this is the only parameter which explicitly includes the Planck scale parameter, $\alpha'$, and all the other physical quantities, such as the string energy $E$ and the separation length $r$ in the Minkowski space, are of the QCD scale ($\sim O(\text{GeV})$).

§The quark mass given by Eq. (34) is the constituent quark mass rather than the current quark mass, since the non-perturbative QCD effects in this mass are taken into account by the deformed space-time.
Table 1: The values $U_i$ (in units of $U_{KK}$) which determine the positions of the flavor branes.

| quark  | $m_u$ | $m_s$ | $m_c$ | $m_b$ | $m_t$ |
|--------|-------|-------|-------|-------|-------|
| mass(GeV) | 0.363 | 0.546 | 1.5   | 4.5   | 176   |
| $9m_i/4m_{KK}$ | 1.061 | 1.595 | 4.383 | 13.15 | 514.3 |
| $U_i$ (in the $U_{KK}$ unit) | 1.599 | 2.086 | 4.826 | 13.58 | 514.8 |

5 Numerical study of the meson string profile

Having fixed the parameters in our QCD model, we next study how the shape of meson string is deformed when the separation distance, $r$, decreases.

Let us consider a meson consisting of a light quark and a heavier anti-quark, which are located on the lower and higher branes, respectively. Here “higher and lower” means those of “larger and smaller” values of $u$. When the distance $r$ between a quark and an anti-quark becomes sufficiently small, the energetically favored shape is such that the string starting from the lighter quark goes up directly to the heavier quark at the higher position, which differs from the standard shape, which goes down to a stationary point and then up to the heavier quark. In this way, we can draw a rough sketch for the manner in which the shape of meson strings change depending on the separation distance $r$.

Now we report the results of a numerical calculation of the shape of meson strings, starting from $q_i$ of the $i$-th brane and stretching to $q_j$ on the $j$-th brane, by solving Eq. (14). First, in Fig. 2 we display the shape of the $(u\bar{d})$ string in the $(\bar{z}, \bar{u})$ plane, where

\[
\bar{z} = z M_{KK},
\]

\[
\bar{U} = u/U_{KK}.
\]

It is seen that the shape changes with the the separation distance $r$, as expected. If $r$ is very large, we see that the string almost reaches to the minimal point, $U_{KK}$. This is a typical shape of a meson string. By contrast, in the small separation limit, it shrinks to zero length with a total energy tending to zero. \footnote{We will come back to this short distance behavior in a subsequent section.}

Next, in the case of the $(u\bar{s})$ string, the shape is quite different from that of the $(u\bar{d})$ string. The calculated result for the string profile in the $(z, u)$ plane is shown in Fig. 3. We see that at large separations, the string has almost the same shape as the $(u\bar{d})$ string. However, if $r$ becomes smaller than the critical length $r_c(u\bar{s}) = 0.9217$ GeV$^{-1}$,
Figure 2: Profiles of a $u \bar{d}$ string, where $\bar{z} = zM_{KK}$ and $\bar{U} = u/U_{KK}$ in the QCD model.

Figure 3: Profiles of a $u \bar{s}$ string, where $\bar{z} = zM_{KK}$ and $\bar{U} = u/U_{KK}$ in the QCD-like model.

Figure 4: Profiles of a $u \bar{b}$ string, where $\bar{z} = zM_{KK}$ and $\bar{U} = u/U_{KK}$ in the QCD-like model.
it directly connects the two branes without passing through the region \( u \leq U_1 \). This tendency is more prominent in the shape of the \( u\bar{b} \) string, as depicted in Fig. 4. In this case, the critical length is \( r_c(u\bar{b}) = 1.427 \text{ GeV}^{-1} \). In order to derive physical meaning from our results, we study the potential between a quark and an anti-quark connected by a string in the next section.

### 6 The potential between a quark and an anti-quark

The string potential at a finite distance \( r = Z_2 - Z_1 \) is defined as

\[
V(r) = \frac{1}{2\pi \alpha'} \int_{U_1}^{U_2} \sqrt{f(u) (f(u) du^2 + g(u) dz^2)} - m_1 - m_2. \tag{41}
\]

We have seen that the critical length \( r_c \) becomes larger for strings connecting the \( u \) quark of the first brane to the heavier anti-quark. Table 2 lists the critical lengths for various mesons. The existence of this critical length is a characteristic feature of a string connecting different flavors, and it is also very important to determine the quantitative features of the interaction between a quark and an anti-quark.

When we plot the potential \( V = V(r) \) between a \( u \)-quark and a \( \bar{q} \)-quark as a function of \( r \), we need to connect the short distant part, \( r \leq r_c \), and the long distant part, \( r \geq r_c \). Thus, we can study the behavior of the potential by considering three regions for the distance, \( r \):

1. Region 1: Long distance region ( \( r \gg r_c \))

   From Eq. (41), it is easily seen that the potential converges to a single straight line, \( V(r) \). This implies that, for the large separation case (region 1) the potential is universal, and it is almost independent of the flavors of the quark and antiquark;

   \[
   V(r) \to \frac{f(U_{KK})}{2\pi \alpha'} r + C. \tag{42}
   \]

| string | \( ud \) | \( u\bar{s} \) | \( u\bar{c} \) | \( u\bar{b} \) |
|--------|--------|--------|--------|--------|
| \( r_c \) (in GeV\(^{-1}\)) | 0      | 0.9217 | 1.352  | 1.427  |
| \( r_c M_{KK} \)      | 0      | 0.710  | 1.041  | 1.099  |
Here we have an almost \( r \) independent constant term \( C \), which comes from the difference between the real string energy and that calculated using an approximate in rectangular string,

\[
C \to r_c \ll r, \quad E(r) - V(\text{basic rectangular}),
\]

where \( C \) is almost independent of the quark and anti-quark flavors. This constant comes from the difference between the energy calculated from an exactly rectangular string around the point \((Z_2, U_{KK})\) and that of the real, smooth meson shape, which is almost independent of the flavors, because the difference exists only in the region near the point \((Z_2, U_{KK})\).

2. Region 2: Intermediate region ( \( r \sim r_c \))

The critical distance \( r_c \), which separates the short and long distant parts of a potential, can be characterized as an inflection point for the potential \( V(r) \), i.e.

\[
\left. \frac{d^2 V(r)}{dr^2} \right|_{r=r_c} = 0.
\]

The reason for this is as follows. From the above discussion, we understand that the derivative \( u'(z) \) at the position of the u-quark endpoint in the short distance case and that in the long distance case have opposite signs, since a string in the former case is rising up from the position of the u-quark endpoint to that of the anti-quark endpoint, but in the latter case it goes down from the u-quark endpoint. Therefore, \( u'(z) \) at the position of u-quark vanishes at the critical distance \( r_c \). From Eqs. (14) and (15), we have

\[
\frac{dE}{dr} = \frac{dE}{du} u' = \frac{1}{2\pi\alpha'} \frac{f^2}{f_0^2},
\]

from which we obtain

\[
\frac{d^2E}{dr^2} = \frac{1}{2\pi\alpha'} \frac{2f}{f_0^2} \frac{df}{du} u'.
\]

Then, we can easily understand that the sign of \( \frac{d^2E}{dr^2} \) is determined by the sign of \( u'(z) \). Therefore, \( r_c \) becomes an inflection point for the potential \( V(r) \), because \( \frac{d^2E}{dr^2} = 0 \) at \( r = r_c \), and is positive for \( r \leq r_c \) and negative for \( r \geq r_c \).

We have no such inflection point for mesons connecting branes of the same flavor. The appearance of the inflection point is a characteristic feature of mesons connecting branes of different flavors. Note that the critical length depends on the flavor of the meson; in the case of the \( u\bar{d} \) meson, \( r_c = 0 \),
Figure 5: The string potential as a function of $r = Z \text{ GeV}^{-1}$ for the strings $u\bar{d}$ (the upper curve), $u\bar{s}$ (the middle curve) and $u\bar{b}$ (the lower curve).

while $r_c = 0.9217 \text{ GeV}^{-1}$ for $u\bar{s}$, and it becomes larger for mesons with heavier flavors.

3. Region 3: Short distance region ($r << r_c$)

In the limit $r \to 0$, the string shape becomes almost a straight line connecting the lighter quark $q_1$ to the heavier quark $q_2$, and there, the total energy of the meson becomes $m_2 - m_1$. Subtracting the rest masses, the potential becomes

$$V(r) \to (m_2 - m_1) - (m_1 + m_2) = -2m_1, \quad m_1 \leq m_2.$$  \hspace{1cm} (47)

It is noted that in our string theory, eventhough it is called QCD-like, we cannot precisely take into account the short distance effects. Consequently, perturbative QCD effects resulting from one-gluon exchange would contribute as additional effects. The dual gravity theory that we study here is effective in accounting for long distance non-perturbative effects of QCD, but it is ineffective in accounting for short distance perturbative effects of QCD. To implement the latter effects in the present framework, we have to estimate the higher-order effects. Here, we employ more convinient way to include such perturbative effects by taking account of the perturbative QCD results directly, as QCD is effective in the short distance region.
Figure 6: The final form of the potential for a quark-antiquark pair obtained by taking account of the perturbative QCD effects added to the potential of the string connecting $q_1$ and $\bar{q}_2$ in the QCD-like model.
The first order perturbative QCD potential is a sort of Coulomb potential, which is known to be expressed as

$$V_{\text{perturbative QCD}} = -\frac{4\alpha_{\text{QCD}}}{3r}, \quad \alpha_{\text{QCD}} = 0.33,$$  \hspace{1cm} (48)

which also is independent of the flavors of the quark and anti-quark.

Therefore, a potential between the quark and anti-quark that is applicable to the short distance region as well as the long distance region may consist of the sum of the classical potential of the QCD-like string theory $V(r)$, which we have studied in this paper, and the one-gluon exchange potential $V_{\text{perturbative QCD}}$ of QCD. We call their sum the “improved potential”:

$$V(r)_{\text{improved}} = V(r) + V_{\text{perturbative QCD}}.$$  \hspace{1cm} (49)

By including such perturbative effects of QCD, the short distance behavior of the potential $V(r)$ of Eq.(49) may be improved to some extent. Figure 6 displays the calculated result for the improved potentials with different flavors.}

Although the behavior of the potential converges to a common value, $-2m_1$, in the limit $r \to 0$ (region 3), and also it converges to a single linear potential line in the large separation limit (region 1), the shape of the potential does depend on the flavors around the critical length, $r_c$ (region 2). It is interesting that the potential between the $u$ and $\bar{q}_j$ at relatively small distances becomes more attractive for the case as of $m_i$ increases.

From this picture, we can conclude that the flavor dependence of the potential can be found for the improved potentials around the critical region. It is noted that the non-perturbative effects can be included in our string picture realizing a linear potential which can be directly derived from this QCD-like model.

7 Conclusion

In the picture presented here, the origin of generation is attributed to the flavor branes. The flavor branes are separately positioned perpendicularly to the direction of the extra dimension $u$, which causes quite different energies for the meson strings, depending on the flavors.

\footnote{If we take account of the spin dependent color-magnet interaction, they are, of course, flavor dependent. Such an interaction modifies the potential only in the very short range region ($\sim \delta(r)$), but we here consider the spin-independent potential by removing such spin-dependent effects.}
Based on the above picture, we have explored general formulations of the QCD-like string model and have obtained classical solutions of meson strings. In this QCD-like model, it is automatical that the quark-antiquark potential is universal and independent of their flavors. This is consistent with the experimental data of meson spectroscopy in the large separation limit. If the separation between a quark-antiquark pair decreases, however, their interaction becomes flavor dependent. As the most characteristic feature of our model, we have found that this attractive interaction becomes stronger as the distance between the branes of the quark and anti-quark increases depending on the difference of their flavors, especially in the middle region around critical separation, \( r \sim r_c \). If the endpoints of a string are on the same brane, there is no critical distance \( r_c \). We believe that our flavor dependent potential may be observed in flavored mesons whose endpoints lie on different branes. In order to check this result, we need information concerning the quark-antiquark potential in the case that their endpoints lie on different flavored branes. We hope that detailed analysis of hadron spectroscopy, especially for hadrons including different flavors, will clear whether or not our prediction is consistent with realistic QCD systems [6, 7, 8].

The general formulae which we have used in this paper are also applicable to more complicated shapes of hadron strings, because the differential equation Eq. (14) is also applicable to various types of hadron strings, baryon strings, and more complicated web-like exotic hadronic strings possessing junctions. One example of such hadrons is the pentaquarks that we have investigated in Ref. [1]. In this respect, the string picture may suggest the new concept that “color and flavor are located at the endpoints of a string, but the spin, on the other hand, may be distributed over a whole string”.

If our interpretation of flavor in terms of flavor branes has some importance, our picture of the origin of flavor may give new insight into hadron physics. Then multi-quark states including quarks of 2nd or 3rd generations may be packed in a more compact way; that is, if the distance between a quark and an anti-quark with different flavors becomes shorter than the critical length \( r_c \), their interaction may become stronger, in which case, the density of the multi-quark system would be higher. Then, a variety of high density nuclear matters arise when different

\[**\]Actually the observation of pentaquarks motivated us to formulate this picture of flavor branes. It is found that experiments on multi-quark states have not yet been confirmed with the exception of \( \Theta^+ \), which was first observed in Spring 8[9]. Even though, we expect that quark-systems may have much variety of exotic hadrons to yield multi-quark systems including pentaquarks in addition to ordinary mesons and baryons.
flavors are included. Such behavior may be realized as a kind of crystal connecting quarks by junctions and strings[11]. This may be an indication of the existence of hadron states including strangeness with high density and it might be related to the recent observation of so-called $\bar{K}$ nucleon clusters, which have high nucleon density at the center, specifically 4-9 times higher than normal nuclear density ($\rho = 0.17 \text{ fm}^{-1}$)[10]. Finally, our interpretation of flavors may not be irrelevant to the more speculative conjecture of the existence of a pasta phase of high density matter, appearing in the initial stage of supernova explosions[12].

Many problems are left to be solved. First, we ignored the effects of spin, and therefore, we cannot say definitely which meson, i.e., a pseudoscalar meson or a vector meson, corresponds to the string we have studied. Also the hyperfine interaction and parity should be taken into account in the string picture of hadrons. Recently, studies of low energy QCD based on the AdS/CFT correspondence have been carried out many researchers; a holographic dual of QCD with massless flavors has been intensively studied by T. Sakai and S. Sugimoto[13], who have elucidated the mechanism of chiral symmetry breakdown within this framework and also derived the Chern-Simons term, which accounts for baryon states as skyrmions. An interesting fact is that the model they studied is closely related to the hidden local symmetry approach[14]. Although the model beautifully expresses the mechanism of chiral symmetry breaking and the spin structure of the system, the flavor structure with non-vanishing quark masses is not well implemented. In this sense, our model may be a complimentary approach. We hope a full understanding of non-perturbative QCD and the insight of the origin of flavor can be gained in near future.

**Acknowledgements**

The authors are grateful to T. Kugo for his kind and patient help and Y. Imamura, S. Sugimoto and T. Sakai for useful discussions on string theories, especially on the AdS/CFT correspondence. Also, we express our thanks to the participants at the international symposium “Flavor Physics and its Origin”, held at Ochanomizu University in December, 2005, for valuable comments.

M. B. and A. S. are partially supported by Grants-in-Aid for Scientific Research (No.17540238) from the Ministry of Education, Culture, Sports, Science and Technology, Japan.
References

[1] M. Bando, T. Kugo, A. Sugamoto and Y. Terunuma, Prog. Theor. Phys.. 2 (2004), 231; [hep-ph/0410225]

[2] A. Sugamoto, talk given at 2nd international symposium on “New Developments of Integrated Sciences” held at Ochanomizu U. on March 16 (2004); [hep-ph/0404019]

[3] E. Witten, Adv. Theor. Math. Phys. 2 (1998), 505; [hep-th/9803131]; J.High Energy Phys. 07 (1998), 006; [hep-th/9805112]
D. J. Gross and H. Ooguri, Phys. Rev. D58 (1998), 106002; [hep-th/9805129]
M. Kurczenski et al., J.High Energy Phys. 05 (2004), 041; [hep-th/0311270]

[4] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998), 231; [hep-th/9711200]
J. Babington, J. Erdmenger, N. J. Evans, Z. Guralnik, and I. Kirsch Phys. Rev. D69 (2004), 066007; [hep-th/0306018]
M. Kurczenski et al., J.High Energy Phys. 07 (2003), 049; [hep-th/0304032]

[5] S. Eidelman et al., Phys. Lett. B 592, (2004), 1; Review of particle properties.

[6] C. Quigg and J.L. Rosner, Phys. Rev. D23 (1981), 2625.

[7] E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K.D. Lane and T.M. Yan, Phys. Rev. Letters 34 (1975), 369.

[8] H. Boschi-Filho, Nelson R.F. Braga and C.N. Ferreira, [hep-th/0512295]

[9] T. Nakano et al. (LEPS collaboration), Phys. Rev. Lett. 91 (2003), 012002.

[10] Y. Akaishi, A. Dote and T. Yamazaki, [nucl-th/0501040v1 17 Jan 2005.

[11] Y. Igarashi, M, Imachi, T. Matsuoka, K. Ninomiya, S. Otsuki, S. Sawada and F. Toyoda, Suppliment of the Prog. THeor. Physcs, No. 63, 1978, P149.

[12] G. Watanabe, T. Maruyama, K. Sato, K. Yasuoka and T. Ebisuzaki, Phys. Rev. Letters, 94, 031101(2005).

[13] T. Sakai and S. Sugimoto, [hep-th/0412141]

[14] M. Bando, T. Kugo and K. Yamawaki, Phys. Rep. 164 (1988), 217.
[15] Angel Paredes and Pere Talavera, Nucl.Phys.B713:438-464,2005 e-Print Archive: hep-th/0412260 also see M. Kruczenski, L A. P. Zayas, J. Sonnenschein and D. Vaman, JHEP 0506:046,2005, e-Print Archiv: hep-th/0410035 for the flavor dependence of Regge trajectories for mesons in the holographic dual of large-N(c) QCD (although some parts, especially the part deriving the linear potential, include mistakes).

Note added: After submitting this paper we received a letter informing us that the paper titled “Multiflavor excited mesons from the fifth dimension” [15] has already proposed a method to analyse meson strings very clearly, and it investigates meson strings in detail on the basis of Mardacena metric, i.e., in the SUSY case. The authors of that paper focus mainly on spinning mesons and intensively investigated Regge trajectories. Contrastly we investigated the quark-antiquark potential in the realistic case with SUSY breaking (the Witten metric) in detail. We have found that the shape of potential differs on their flavors only in the middle region and that the shape of the potentials is almost independent of their flavors in the large separation limit. We found that the existence of the critical length is especially important. It would be interesting to compare their results in the SUSY case with our realistic results and to understand how the flavor branes play essential roles in flavor physics from QCD-like string models in which the origin of generation comes from the existence of flavor branes.