Implications of a very light pseudoscalar boson on lepton flavor violation

Adriana Cordero-Cid, G. Tavares-Velasco, and J. J. Toscano
Facultad de Ciencias Físico Matemáticas, Benemérita Universidad Autónoma de Puebla, Apartado Postal 1152, Puebla, Pue., México
(Dated: December 4, 2018)

A long-lived very light pseudoscalar boson would favor lepton flavor violating transitions of charged leptons. Its implications on the \( l_i \to l_j \gamma \gamma \), and \( l_i \to l_j e^+ e^- \) transitions are investigated. Assuming that \( 2m_e < m_\phi < m_\mu \), it is found that the inequality \( B(l_i \to l_j \gamma \gamma) < B(l_i \to l_j e^+ e^-) \) is hold. The experimental constraints on the decays \( l_i \to l_j \gamma \), \( l_i \to l_j l_j \), and \( l_i \to l_j \gamma \gamma \) are used to bound the \( \phi l_i l_j \) couplings.

PACS numbers: 12.60.Fr,14.80.Bn,11.30.Fs

In the standard model (SM) the couplings of the Higgs boson to the remaining massive particles are thoroughly determined, which should be considered an outstanding feature of the model. This allowed the CERN large electron positron (LEP) collider to conclude its operative stage with a significantly strong lower bound on the Higgs boson mass of around 115 GeV [1]. However, as long as additional scalars are included in the theory, the Higgs boson masses become more difficult to bound due to the proliferation of free parameters. The bonus is the appearance of interesting new physics effects such as lepton flavor violating (LFV) and flavor changing neutral currents (FCNC) transitions, which can be mediated by the new Higgs bosons at the tree level. In particular, there are well motivated theoretical arguments that favor the existence of a very light scalar or pseudoscalar particle, which would have remained undetected so far because it would interact very weakly with ordinary matter. Since the Higgs boson masses are typically of the same order of the Fermi scale, extended scalar sectors do not lead automatically to the presence of light scalars, unless an unnatural fine tuning is implemented between all the parameters of the Higgs potential. However, an exception occurs if the theory possesses an approximate global symmetry. If such a symmetry is spontaneously broken, a massless Goldstone boson arises, but if it is only approximate, a massive state arises, which is naturally light. This symmetry is only approximate because a small explicit symmetry breaking can be introduced in the classical Lagrangian or generated through quantum effects such as anomalies. The most common example is the axion [2], which is associated with the spontaneously broken Peccei-Quinn symmetry [3]. Other known examples are familons [4] and Majorons [5], which are associated with spontaneously broken family and lepton number symmetries, respectively. One example of a global symmetry broken explicitly through a small parameter in the Higgs potential is the two-Higgs doublet model (THDM) [6], where the pseudoscalar boson \( A \) acquires a mass proportional to the small parameter: \( m_A^2 = -\lambda_5 v^2 \), with \( v = 246 \text{ GeV} \) the Fermi scale [6]. In this case, the Higgs potential possesses an exact global \( U(1) \times U(1) \) symmetry in the limit of vanishing \( \lambda_5 \). As pointed out in Refs. [6, 7], this particle has a very interesting phenomenology if its mass is below 200 MeV.

We are interested in studying some phenomenological implications of LFV transitions mediated by a very light pseudoscalar boson \( \phi \), with \( 2m_e < m_\phi < m_\mu \). We will focus on the three-body transitions \( l_i \to l_j \gamma \gamma \) and \( l_i \to l_j e^+ e^- \), with \( l_i = \mu, \tau \) and \( l_j = e, \mu \). Although Higgs-mediated LFV effects have long attracted considerable attention [7, 8], the current evidences of nonzero mass for the neutrinos [9, 10] have renewed the interest in this issue, and particularly in the potential role that spin zero particles may play [11, 12]. In our case, the key ingredient is the \( \phi \) mass range, which imposes severe kinematical restrictions on the possible \( \phi \) channel decays. In fact, in this mass range, the only kinematically-allowed tree-level decay mode is \( \phi \to e^+ e^- \), though the one-loop induced mode \( \phi \to \gamma \gamma \) can also be competitive since a highly suppressed \( \phi e^+ e^- \) vertex is expected in general. Notice that, for such a light \( \phi \), the LFV decays \( \phi \to l_i l_j \) are not kinematically allowed. Although the relative significance of each channel would be model-dependent generally, it is also clear that the total decay width \( \Gamma_\phi \) would be very small. This means that such a \( \phi \) scalar boson is a long-lived particle. Our main motivation is thus that a particle with such peculiarities can naturally favor LFV transitions, not only because its mass is much smaller than any heavy lepton, but also because it is a very long-lived particle. A very long-lived \( \phi \) has the property that \( \Gamma_\phi \ll m_\phi \), so the narrow-width approximation can be used when calculating the three-body decays \( l_i \to l_j \gamma \gamma \) and \( l_i \to l_j e^+ e^- \). These facts, together with the fact that \( \phi \to e^+ e^- \) and \( \phi \to \gamma \gamma \) are the only decay modes of \( \phi \), will allow us to establish an interesting relation between the \( l_i \to l_j \gamma \gamma \) and \( l_i \to l_j e^+ e^- \) transitions.

Once the main motivations of this work were discussed, we proceed to calculate the decay rates for the above mentioned channels. It is well known that a Yukawa sector associated with an extended Higgs sector predicts somewhat general couplings of the Higgs bosons to the fermions. One of the most interesting features of such a scalar sector is the presence of nondiagonal interactions both in the lepton sector and the quark sector. In the context of a general renormalizable theory, we assume that the couplings of the pseudoscalar \( \phi \) to the leptons are naturally suppressed by...
Note that apart from the Feynman diagram shown in Fig. 1(ii), the decay \( \phi \to e\gamma \) where \( \Gamma(\phi \to e\gamma) \) is a very good approximation indeed because \( \Gamma(\phi \to e\gamma) \) is thirteen orders of magnitude smaller than \( m_\phi \). As will be seen below, this is the case for transitions involving the two first lepton families since the current experimental constraints on \( \mu \to e\gamma \) indicate that \( |\mu_e| \ll 1 \). Below we will consider \( \lambda_{ij} \) as free parameters, including the nondiagonal ones, since it is reasonable to expect that \( |\lambda_{ii}| \leq 1 \) in a general context.

We now proceed to analyze the \( l_i \to l_j \gamma \gamma \) and \( l_i \to l_j e^+ e^- \) decays. From the Feynman diagrams depicted in Fig. 4 the branching fractions can be written, in the narrow width approximation, as

\[
B(l_i \to l_j \gamma \gamma) = B(l_i \to l_j \phi) B(\phi \to \gamma \gamma),
\]

\[
B(l_i \to l_j e^+ e^-) = B(l_i \to l_j \phi) B(\phi \to e^+ e^-).
\]

Note that apart from the Feynman diagram shown in Fig. 4(ii), the decay \( l_i \to l_j \gamma \gamma \) can also proceed via those reducible graphs in which one of the photons is emitted from an external lepton, or also via box diagrams. However, general considerations suggest that this class of contributions is marginal [12]. From the above expressions, it follows that

\[
B(l_i \to l_j \gamma \gamma) = \frac{\Gamma(\phi \to \gamma \gamma)}{\Gamma(\phi \to e^+ e^-)} B(l_i \to l_j e^+ e^-),
\]

where

\[
\Gamma(\phi \to e^+ e^-) = \frac{\alpha \lambda_{ee}^2 m_\phi}{2 s_{2W}^2} \left( \frac{m_e}{m_Z} \right)^2 \left( 1 - \frac{4 m_e^2}{m_\phi^2} \right)^{3/2},
\]

with \( s_{2W} = 2 \sin \theta_W \cos \theta_W \), being \( \theta_W \) the weak angle. On the other hand, the decay width for the two photon mode is given by [13]

\[
\Gamma(\phi \to \gamma \gamma) = \frac{\alpha^3 m_\phi}{16 \pi^2 s_{2W}^2} \left( \frac{m_\phi}{m_Z} \right)^2 |F|^2,
\]

with

\[
F = \sum_{f=1,2} N_C^f Q_f^2 \lambda_{ff} x f(x),
\]

and

\[
f(x) = \begin{cases} \left( \arcsin \frac{x}{\sqrt{x}} \right)^2 & x \geq 1, \\ \left( \arccosh \frac{1}{\sqrt{x}} - \frac{x}{2} \right)^2 & x < 1, \end{cases}
\]

where \( x = 4 m_e^2 / m_\phi^2 \), \( N_C^f \) is the color index, and \( Q_f \) is the electric charge in units of the positron charge. For the sake of illustration, we have evaluated the decay widths \( \Gamma(\phi \to e^+ e^-) \) and \( \Gamma(\phi \to \gamma \gamma) \) for \( \lambda_{ff} = 1 \). They are shown as function of \( m_\phi \) in Fig. 2. We can observe that \( \Gamma_\phi \) is of order \( 10^{-14} \) GeV at most, which means that the narrow width approximation used in obtaining Eq. 4 is a very good approximation indeed because \( \Gamma_\phi \) is thirteen orders of magnitude smaller than \( m_\phi \). Also, we can see that, depending on the value of \( m_\phi \), \( \Gamma(\phi \to \gamma \gamma) \) is up to three or one order of magnitude smaller than \( \Gamma(\phi \to e^+ e^-) \). Although in some particular models \( \Gamma(\phi \to \gamma \gamma) \) may reach values near \( \Gamma(\phi \to e^+ e^-) \), it is reasonable to assume that \( \Gamma(\phi \to \gamma \gamma) < \Gamma(\phi \to e^+ e^-) \) as the \( \gamma \gamma \) mode is always a loop-generated effect in a renormalizable theory. This means that Eq. 4 can be written as an inequality

\[
B(l_i \to l_j \gamma \gamma) < B(l_i \to l_j e^+ e^-),
\]
which is in agreement with the fact that the decay $l_i \to l_j \gamma \gamma$ can only arise at one-loop or higher orders, whereas the $l_i \to l_j e^+ e^-$ reaction can be induced at the tree level. It is worth emphasizing that (9) is only true under the assumption that $\lambda_{ee} \simeq 1$. If the very light $\phi$ was leptophobic, it would only decay into a photon pair, in which case the inequality (9) would not hold.

The current experimental constraints on $B(l_i \to l_j e^+ e^-)$ [14] allow us to translate (9) into the following bounds on the decays $l_i \to l_j \gamma \gamma$:

$$B(\mu^- \to e^- \gamma \gamma) < 1.0 \times 10^{-12}, \quad B(\tau^- \to e^- \gamma \gamma) < 2.0 \times 10^{-7}, \quad B(\tau^- \to \mu^- \gamma \gamma) < 1.9 \times 10^{-7}. \quad (10)$$

Notice that the constraint on $B(\mu^- \to e^- \gamma \gamma)$ is almost two orders of magnitude more stringent than the experimental one [14]: $B_{\text{Exp.}}(\mu^- \to e^- \gamma \gamma) < 7.2 \times 10^{-11}$.

The size of the parameters $\lambda_{ij}$ can be estimated using the current experimental constraints on the decay $l_i \to l_j \gamma \gamma$. It proceeds via the loop diagrams shown in Fig. 3 which involve the pseudoscalar boson and the charged leptons. The branching ratio can be written as

$$B(l_i \to l_j \gamma) = \frac{\alpha^3}{16\pi^2 s_{W}^4} \left( \frac{m_i}{\Gamma_i} \right) \left( \frac{m_j}{m_Z} \right)^2 \left| \sum_k \left( \frac{m_k}{m_Z} \right) \lambda_{ik} \lambda_{kj} |A_k|^2 \right|^2, \quad (13)$$

where $\Gamma_i$ is the $l_i$ total width and

$$A_k = \frac{1}{2} + \left( \frac{m_i}{m_k} - 1 \right) m_i^2 C_0(m_i^2, 0, 0, m_k^2, m_\phi^2) + \frac{m_k(m_k - m_i)}{m_i^2} \left[ B_0(0, m_k^2, m_\phi^2) - B_0(m_i^2, m_k^2, m_\phi^2) \right]. \quad (14)$$
FIG. 3: Feynman diagrams contributing to the $l_i \rightarrow l_j \gamma$ decay.

with the Passarino-Veltman scalar functions given by

$$B_0(m_k^2, m_{\phi}^2) = B_0(m_i^2, m_{\phi}^2) = 1 + \frac{\xi}{m_i^2} \arccosh \left( \frac{m_k^2 - m_i^2 + m_{\phi}^2}{2 m_k m_{\phi}} \right) - \frac{1}{2} \left( \frac{m_k^2 - m_i^2 - m_{\phi}^2}{m_i^2} \right) \log \left( \frac{m_k^2}{m_i^2} \right),$$

(15)

$$C_0(m_i^2, 0, 0, m_{\phi}^2, m_k^2) = \frac{1}{m_i^2} \left[ \text{Li}_2 \left( 1 - \frac{m_{\phi}^2}{m_k^2} \right) - \text{Li}_2 \left( \frac{2 m_i}{\eta + \xi} \right) - \text{Li}_2 \left( \frac{2 m_i}{\eta - \xi} \right) \right],$$

(16)

with $\xi^2 = \left( m_k^2 + m_i^2 - m_{\phi}^2 \right)^2 - 4 m_k^2 m_i^2$ and $\eta = m_k^2 + m_i^2 - m_{\phi}^2$.

The analysis of the loop amplitude shows that the dominant contribution comes from $l_k = l_i$. In such a case, one can take $A_k \approx -1/2$. Eq. (13) thus translates into the following bound when the experimental constraint is taken into account

$$|\lambda_{ii}|^2|\lambda_{ij}|^2 < \frac{64 \pi^4 s_{2W}^4}{k^3} \alpha^3 \left( \frac{\Gamma_i}{m_i} \right) \left( \frac{m_Z}{m_i} \right)^3 \left( \frac{m_Z}{m_j} \right) B_{\text{Exp.}}(l_i \rightarrow l_j \gamma).$$

(17)

Using the experimental constraints on the $l_i \rightarrow l_j \gamma$ decays [14] we obtain

$$|\lambda_{\mu e}| |\lambda_{\mu e}| < 1.79 \times 10^{-3},$$

(18)

$$|\lambda_{\tau \mu}| |\lambda_{\tau \tau}| < 1.95 \times 10^{-1},$$

(19)

$$|\lambda_{\tau e}| |\lambda_{\tau e}| < 3.15.$$}

(20)

It is worth examining the bounds that can be obtained from the three-body decays $l_i \rightarrow l_j e^- e^+$ and $l_i \rightarrow \gamma \gamma$. The branching ratio of the former decay can be written as

$$B(l_i \rightarrow l_j e^- e^+) = \frac{\alpha^2}{8 \pi s_{2W}^2} \left( \frac{m_j}{\Gamma_i} \right) \left( \frac{m_i}{m_Z} \right)^2 \left( \frac{m_e}{m_Z} \right)^2 |\lambda_{ij}|^2 |\lambda_{ee}|^2 \int_0^1 \frac{x(1-x)^2}{(x-y)^2 + yz} dx,$$

(21)

with $x = p^2/m_i^2$, $y = m_{\phi}^2/m_i^2$, and $z = \Gamma_{\phi}/m_i^2$, $p$ being the four-momentum of the exchanged pseudoscalar. For $m_{\phi} \approx m_{\mu}/2$, the experimental bounds [14] yield

$$|\lambda_{\mu e}| |\lambda_{ee}| < 3 \times 10^{-6},$$

(22)

$$|\lambda_{\tau \mu}| |\lambda_{ee}| < 4.5,$$

(23)

$$|\lambda_{\tau e}| |\lambda_{ee}| < 66.0.$$}

(24)
These upper bounds depend smoothly on the $\phi$ mass and do not deviate significantly from the above values in the interval $0.01 \text{ GeV} \leq m_{\phi} \leq 0.125 \text{ GeV}$, except for the bound on $|\lambda_{\mu e}|/|\lambda_{e\gamma}|$, which relaxes up to $10^{-5}$ when $m_{\phi} \approx 0.01 \text{ GeV}$.

As for the three-body decay $l_i \to l_j \gamma \gamma$, Ref. [14] only reports an experimental bound on $\mu \to e\gamma\gamma$. Assuming the existence of a very light pseudoscalar, the corresponding branching ratio can be computed via the narrow width approximation: $B(\mu \to e\gamma\gamma) \simeq B(\mu \to e\phi)B(\phi \to \gamma\gamma)$, where

$$B(\mu \to e\phi) \simeq \frac{\alpha|\lambda_{\mu e}|^2}{4\Gamma_{\mu}} \left(\frac{m_{\mu}}{m_{\phi}}\right) \left(\frac{m_{Z}}{m_{\phi}}\right)^2 \left(1 - \frac{y_{\mu}}{m_{\phi}}\right)^2, \quad (25)$$

with $y_\mu = m_{\phi}/m_{\mu}$. Numerical calculation, when combined with the experimental constraint on $B(\mu \to e\gamma\gamma)$, yields

$$|\lambda_{\mu e}| < 2.14 \times 10^{-8}, \quad (26)$$

for $m_{\phi} = m_{\mu}/2$. This bound is stronger than the one obtained from the two-body decay $\mu \to e\gamma$ as $\lambda_{e\gamma}$ is expected to be of order unity at most. This stems from the fact that the three-body decay $\mu \to e\gamma\gamma$ becomes significantly enhanced on resonance of the $\phi$ boson.

We now would like to comment on some realistic models suited for the class of effects that we just have analyzed. Specific examples of models which allow a very light pseudoscalar are THDMs. It was shown recently [8] that although the parameter space of this class of models has been tightly constrained by the most recent measurements on the muon anomalous magnetic moment and other low energy processes, the existence of a very light scalar with a mass at the MeV level can still be possible if some parameters of the model are fine-tuned. As has been pointed out in Ref. [15], other examples of theories with extended Higgs sectors that allow a very light pseudoscalar are the minimal composite Higgs model (MCHM) [16] and the next-to minimal supersymmetric standard model (NMSSM) [17]. In both of these models a light pseudoscalar is allowed because its mass is controlled by the explicit breaking of a spontaneously broken $U(1)$ symmetry. The pseudoscalar is an axion due to the fact that $(1)$ symmetry has a QCD anomaly. Current bounds on the mass of such a pseudoscalar are at the MeV level.

Lepton flavor violating transitions can be considerably suppressed if they are mediated by a very heavy particle and the involved couplings are negligible, but they can be favored if either of these conditions are sufficiently relaxed. For instance, in the Cheng-Sher ansatz is assumed that these effects can be mediated by a relatively light scalar, with a mass of the order of the Fermi scale, at the same time that weak LFV couplings are postulated. In this work, the Cheng-Sher ansatz spirit was retained, but the requirement of a relatively light scalar boson was maximally relaxed, admitting the possible existence of a very light pseudoscalar boson. An inequality between the branching ratios for the three-body decays $l_i \to l_j \gamma \gamma$ and $l_i \to l_j e^+e^-$ [Eq. (4)] arises as a new ingredient introduced by the long-lived nature of this particle. As far as the strength of the $\phi l_l l_l$ vertex is concerned, current experimental data show that, for leptons of adjacent families ($\phi \mu e$ and $\phi \tau \mu$), it is still more suppressed than expected for a Higgs boson with mass of the order of the Fermi scale.

Acknowledgments

We acknowledge support from Conacyt under grant U44515-F.

[1] A. Heister et al. Phys. Lett. B 526, 191 (2002).
[2] S. Weinberg, Phys. Rev. Lett. 40, 223 (1978); F. Wilczek, Phys. Rev. Lett. 40, 279 (1978).
[3] R. D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38, 1440 (1977); Phys. Rev. D 16, 1791 (1977).
[4] F. Wilczek, Phys. Rev. Lett. 49, 1549 (1982).
[5] Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, Phys. Lett. B 98, 265 (1981); G. B. Gelmini and M. Roncadelli, Phys. Lett. B 99, 411 (1981).
[6] D. L. Anderson, C. D. Carone, and M. Sher, Phys. Rev. D 67, 115013 (2003).
[7] F. Larios, G. Tavares-Velasco, and C. P. Yuan, Phys. Rev. D64, 055004 (2001); Phys. Rev. D 66, 075006 (2002).
[8] T. P. Cheng and M. Sher, Phys. Rev. D 35, 3484.
[9] M. Sher and Y. Yuan, Phys. Rev. D 44, 1461 (1991).
[10] Y. Fukuda et al. Phy. Rev. Lett. 77, 1683 (1996).
[11] S. Nie and M. Sher, Phys. Rev. D 58, 097701 (1998); J. L. Díaz-Cruz and J. J. Toscano, Phys. Rev. D62, 116005 (2000); M. Sher, Phys. Lett. B 487, 151 (2000); T. Han and D. Marfatia, Phys. Rev. Lett. 86, 1442 (2001); D. Black, T. Han, H-J. He, and M. Sher, Phys. Rev. D 66, 053002 (2002); A. Atre, V. Barger, and T. Han, Phys. Rev. D 71, 113014 (2005); E. Arganda, Ana M. Curiel, M. J. Herrero, and D. Temes, Phys. Rev. D 71, 035011 (2005); S. Kanemura, T. Ota, and K. Tsumura, arXiv: hep-ph/0505191; J. L. Díaz-Cruz, R. Noriega-Papaqui, and A. Rosado, Phys. Rev. D71, 015014 (2005).

[12] A. GemINTERN, S. Bar-Shalom, G. Eilam, and F. Krauss, Phys. Rev. D 67, 115012 (2003).

[13] L. Resnick, M. K. Sundaresan, and P. J. S. Watson, Phys. Rev. D 8, 172 (1973).

[14] S. Eidelman et al., Phys. Lett. B 592, 1 (2004).

[15] B. Dobrescu, G. Landsberg, and K. Matchev, Phys. Rev. D 63, 075003 (2001).

[16] B. A. Dobrescu, Phys. Rev. D 63, 015004 (2001).

[17] B. A. Dobrescu and K. T. Matchev, J. High Energy Phys. 09, 031 (2000).