Turbulence prevents core particle depletion in stellarators

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In reactor-relevant plasmas, neoclassical transport drives an outward particle flux in the core of large stellarators and predicts strongly hollow density profiles. However, this theoretical prediction is contradicted by experiments. In particular, in Wendelstein 7-X, the first large optimized stellarator, flat or weakly peaked density profiles are generally measured, indicating that neoclassical theory is not sufficient and that an inward contribution to the particle flux is missing in the core. In this Letter, it is shown that the turbulent contribution to the particle flux can explain the difference between experimental measurements and neoclassical predictions. The results of this Letter also prove that theoretical and numerical tools are approaching the level of maturity needed for the prediction of equilibrium density profiles in stellarator plasmas, which is a fundamental requirement for the design of operation scenarios of present devices and future reactors.

INTRODUCTION

The stellarator is the main alternative to the tokamak in the quest for achieving net energy from controlled nuclear fusion, due to its intrinsic steady-state operation and absence of current-driven instabilities. Nonetheless, neoclassical transport—associated with the inhomogeneity of the magnetic field and with particle collisions—has traditionally handicapped the performance of stellarators. For this reason, its minimization is one of the main targets in stellarator optimization. Wendelstein 7-X (W7-X) has demonstrated that large stellarators with optimized neoclassical transport can be designed [1] and built with high accuracy [2] and that this has been fundamental for achieving triple product record values [3].

However, even in optimized stellarators, neoclassical transport may play a major role in the plasma core [4]. In this region, high temperatures are achieved, causing strong neoclassical particle and energy fluxes due to their unfavourable scaling with the temperature in low collisionality plasma regimes. For example, in the $1/\nu$ regime, the neoclassical particle and energy fluxes scale, respectively, as $T_s^{7/2}$ and $T_s^{9/2}$, with $T_s$ the temperature of the species $s$ (see e.g. [3]). In what follows, we only deal with plasmas consisting of singly charged ions and electrons, so that $s$ can take the values $s = i$ and $e$, respectively.

Stellarator plasmas with core particle transport dominated by neoclassical processes have been predicted to present problems of core particle depletion [5, 6]. If the core particle source is negligible, as is the case in large stellarators with peripheral fueling, the steady-state particle balance equation for each of the species reads $\Gamma_s = \Gamma_{s}^{\text{neo}} + \Gamma_{s}^{\text{turb}} = 0$. Here, $\Gamma_s$ is the flux-surface-averaged radial particle flux, and it is assumed to be the sum of a neoclassical $\Gamma_{s}^{\text{neo}}$ and a turbulent contribution

![Figure 1: Neoclassical radial particle flux as a function of $a/L_T$ and $a/L_{ns}$ for the standard W7-X configuration.](image-url)

Here, $r/a = 0.25$, $n_e = n_i = 6.3 \times 10^{19} \text{ m}^{-3}$, $T_e = 2.2 \text{ keV}$, $T_i = 1.1 \text{ keV}$ and $a/L_{Ti} = 0.81$. The circle corresponds to the experimental discharge analyzed in the final part of this Letter. The dashed line shows the combination of $a/L_{Ts}$ and $a/L_{ns}$ that gives $\Gamma_s^{\text{neo}} = 0$ in Eq. (2).
Contour $\Gamma$. Assuming $\Gamma_{s,\text{turb}} = 0$, the above steady-state conditions reduce to $\Gamma_{s,\text{neo}} = \Gamma_{s,\text{turb}} = 0$. This relation, which additionally implies that no net radial current exists in the plasma, i.e., that the particle fluxes are ambipolar, imposes a constraint on the plasma profiles. The neoclassical particle flux can be written as

$$\frac{\Gamma_{s,\text{neo}}}{n_s} = - L_{11}^s \left( \frac{1}{n_s} \frac{d n_s}{d r} - \frac{Z_s e E_r}{T_s} + \delta_{12}^s \frac{1}{T_s} \frac{dT_s}{d r} \right),$$

where $r \in [0, a]$ is a radial coordinate labeling magnetic surfaces, $a$ is the minor radius, $E_r$ is the radial electric field, $n_s$ is the density and $Z_s e$ is the electric charge ($Z_s = 1$ for ions and $Z_s = -1$ for electrons). Quasineutrality implies $n_i = n_e$, and its radial derivative implies $d n_i/d r = d n_e/d r$. Eq. (1) is valid at low collisionality and has been discussed in detail in [3] and references therein. The neoclassical transport coefficients $L_{11}^s$ and $\delta_{12}^s$ generally depend on the magnetic configuration and plasma parameters, including $E_r$. However, if the electrons and main ions are assumed to be, respectively, in the neoclassical $1/\nu$ and $\sqrt{\nu}$ collisionality regimes—the relevant regimes in the core of reactor-relevant stellarator plasmas—we can approximate $\delta_{12}^s = 7/2$ and $\delta_{12}^e = 5/4$, and, after imposing $\Gamma_{s,\text{neo}} = \Gamma_{s,\text{turb}} = 0$, arrive at the constraint on the plasma profiles [6]. Namely,

$$\frac{1}{n_e} \frac{d n_e}{d r} = - \frac{7/2}{T_e + T_i} \frac{d T_e}{d r} - \frac{5/4}{T_e + T_i} \frac{d T_i}{d r}.$$  

Reactor-relevant plasmas are expected to display peaked temperature profiles, $d T_i/d r > 0$, with $T_e \approx T_i$. Since $\delta_{12}^e > 0$, this automatically leads to a hollow density profile $d n_e/d r < 0$. Because $\delta_{12}^i > 1$ (i.e., because the so-called thermodiffusion coefficient $L_{11}^i \delta_{12}^s$ is relatively large), the hollowness can become large enough to make the pressure gradient positive.

The above analytical argument illustrates the generality of the mechanism of core depletion by neoclassical transport. Of course, more accurate relations between the stationary density and temperature gradients can be obtained by means of numerical simulations. Fig. 1 shows the neoclassical radial particle flux $\Gamma_{s,\text{neo}}$, obtained with the neoclassical code KNOSSOS [7], over a wide range of values for the density and electron temperature gradients. In Fig. 1 and in the remainder of the Letter, we define $a/L_{T_e} := -a \ln T_e/d r$ and $a/L_{n_e} := -a \ln n_e/d r$. peaked (hollow) density profiles have $a/L_{n_e} > 0$ ($a/L_{n_e} < 0$). For the rest of the parameters, typical values from W7-X hydrogen plasmas sustained by Electron Cyclotron Resonance Heating (ECRH) are employed (see the caption of Fig. 1 and reference [8]), and $E_r$ is set by ambipolarity of the neoclassical fluxes. Outward fluxes ($\Gamma_{s,\text{turb}} > 0$) are obtained in most of the represented area, driven by $a/L_{T_e}$, indicating the importance of thermodiffusion. Indeed, the contour $\Gamma_{s,\text{neo}} = 0$ (white line) predicts a slightly hollow stationary density profile even in the presence of a practically flat electron temperature profile. For the larger values of $a/L_{T_e}$, obtained in W7-X, a distinctive hollow density profile should be measured if particle transport were described by neoclassical theory only.

Despite this robust neoclassical prediction, flat or weakly peaked density profiles have been generally measured in ECRH plasmas of W7-X (see e.g. [9]). This disagreement between neoclassical theory and experiments may indicate that a significant inward contribution ($\Gamma_s < 0$) to the particle flux has not yet been identified. Under this hypothesis, the present Letter addresses for the first time the particle transport problem in W7-X from the perspective of microturbulence and by means of an unprecedented number of nonlinear gyrokinetic simulations in stellarator geometry, carried out with the code stella [10]. First, a parameter scan is performed, demonstrating that the turbulence resulting from the onset of the ion and the electron temperature gradients produces an inward pinch in broad regions of parameter space. Then, a specific W7-X discharge is studied, performing turbulence simulations throughout the plasma radius. The resulting profile of the turbulent particle flux is compared with the shortfall in the flux inferred from a careful particle balance analysis that includes estimates of the particle source and computations of the neoclassical particle flux. The sign of the missing contribution to the particle flux needed to sustain the experimental density profile agrees with that of the calculated turbulent particle flux, not only at the core but also at the edge of the plasma. In particular, the simulations consistently predict a sign change of the turbulent flux at intermediate radial positions, reflecting the fact that the neoclassical flux is too large at the core and too small at the edge of the plasma to explain the experimentally determined particle flux.

### PARAMETRIC DEPENDENCE OF THE TURBULENT PARTICLE FLUX

Let us write the radial turbulent particle flux as the sum of a diffusive and a convective contribution,

$$\frac{\Gamma_{s,\text{turb}}}{n_s} = -D \frac{1}{n_s} \frac{d n_s}{d r} + V,$$

with $D$ the diffusion coefficient (which, in general, depends on the density gradient) and $V$ the convection velocity. As $D$ is always positive, the diffusive term adds a contribution to the flux with opposite direction to that of the density gradient. In other words, the diffusive flux is inwardly or outwardly directed depending on whether the density gradient is hollow or peaked, respectively. On the other hand, $V$ can be both positive or negative, supporting the formation of hollow or peaked density profiles, respectively. More specifically, the sign of $V$ for a flat
FIG. 2: Turbulent radial particle flux in gyroBohm units as a function of the (a) normalized density gradient, (b) electron to ion temperature ratio, (c) normalized ion temperature gradient and (d) normalized electron temperature gradient. The point with $T_e/T_i = 1$, $a/L_T = a/L_T = 3$ and $a/L_n = 0$ (golden square) is shared by the four represented scans.

Starting with Fig. 2a, note that turbulence driven solely by either the electron or the ion temperature gradient (being $a/L_T = 3$ or $a/L_T = 3$) cannot lead to peaked density profiles, as convection is outwards ($V > 0$) for the flat density profile cases ($a/L_n = 0$). In contrast, when both temperature gradients coexist, turbulence gives rise to a significant inward convection ($V < 0$), being $\Gamma_{\text{turb}}/\Gamma_B \approx -0.6$ at a vanishing density gradient. Moreover, the normalized density gradient at which the particle flux is zero is $a/L_n \approx 0.7$. In other words, if all transport were due to turbulence (and for the fixed values of other parameters used in this plot), the equilibrium density profile would be clearly peaked.

From Fig. 2b, it can be observed that a larger electron to ion temperature ratio enhances the particle pinch. Furthermore, Fig. 2c shows that $a/L_T$ need not be large or comparable to $a/L_T$ in order to turn the particle flux inward, as well. Such exhaustive scans cover both reactor-relevant conditions, where ions and electrons are expected to be thermalized and have comparable values of density and temperature gradients, and conditions corresponding to the first W7-X campaigns, where the temperature and its gradient are noticeably larger for electrons than for ions in the core. For the sake of clarity, the reference point in parameter space is represented with a golden square.

The turbulent transport is modeled with the flux-tube δf gyrokinetic code stella [10], which has been extensively benchmarked for W7-X geometry [11] and applied to the study of turbulent impurity transport in this device [12, 13]. The gyrokinetic simulations are nonlinear, collisionless, electrostatic and account for kinetic ions and electrons. The calculations are performed at $r/a = 0.25$ for the standard W7-X configuration. Due to the low magnetic shear, we employed generalized twist-and-shift boundary conditions [14]. For further information on the resolution, boundary conditions, flux tube and other details of the gyrokinetic simulations presented in this Letter, we refer the reader to the Supplemental Material.

We carried out a parameter scan around the reference point in parameter space $\{T_e/T_i, a/L_T, a/L_T, a/L_n\} = \{1, 3, 3, 0\}$ in order to study the dependence of the turbulent particle flux on the following quantities: $a/L_n$, in Fig. 2a; the electron to ion temperature ratio $T_e/T_i$ in Fig. 2b; the normalized ion temperature gradient $a/L_T$ in Fig. 2c; and the normalized electron temperature gradient $a/L_T$ in Fig. 2d. Such exhaustive scans cover both reactor-relevant conditions, where ions and electrons are expected to be thermalized and have comparable values of density and temperature gradients, and conditions corresponding to the first W7-X campaigns, where the temperature and its gradient are noticeably larger for electrons than for ions in the core. For the sake of clarity, the reference point in parameter space is represented with a golden square.

FIG. 3: Spectra of the particle flux (top) and phase difference between the density ($\delta n_k$) and electrostatic potential ($\phi_k$) fluctuations (bottom), with $a/L_T = 0$ (left) and $a/L_T = 3$ (right). The dominant phase difference for each $k_y \rho_i$ is denoted by a dot. Positive values are highlighted in red and negative values in blue.
wards. The negative convection starts to manifest from values as modest as $a/L_T \sim 0.25$ when $a/L_T = 3$, and its magnitude increases roughly linearly with the size of $a/L_T$. Similarly, Fig. 2d shows that, although the case with only $a/L_T = 3$ drives positive flux, adding an electron temperature gradient as moderate as $a/L_{T_e} \gtrsim 1.0$, turns the particle flux negative.

In summary, the existence of an inward turbulent particle flux is very robust and is found over broad regions of the scanned parameter space, whenever the temperature gradient of ions and electrons are both finite and not too small. In [15, 16] it is found that this turbulent pinch holds for different stellarator devices, which aligns with other references that report inward turbulent particle fluxes in LHD [17, 18].

Finally, we would like to shed some light on why the electron temperature gradient is key for obtaining inward turbulent particle fluxes. Particle fluxes are driven by density and electrostatic potential fluctuations that are out of phase. For example, if the electrons were treated adiabatically, the density and potential would be in phase and no particle fluxes would be driven. The sign of the particle flux is thus correlated with the sign of the density and electrostatic potential fluctuations. Particle fluxes are driven by the sign of the density and electrostatic potential fluctuations that are out of phase. For example, if the electrons were treated adiabatically, the density and potential would be in phase and no particle fluxes would be driven. The sign of the particle flux is thus correlated with the sign of the density and electrostatic potential fluctuations.

The radial profile of the neutral density, $n_0(r)$, and the associated ionization source are estimated using a short-mean-free-path, one-dimensional neutral transport model given by

$$\frac{1}{r} \frac{d}{dr} r D_{CX} \left( \frac{dn_0}{dr} + \frac{1}{T_0} \frac{d}{dr} n_0 \right) = \nu_{ion} n_0$$ (4)

(see e.g. [20]), where $D_{CX} = T_0/m_0 \nu_{CX}$ is the charge exchange diffusivity coefficient, $T_0$ and $m_0$ are the temperature and mass and $\nu_{ion}$ and $\nu_{CX}$ are the temperature- and density-dependent ionization and charge exchange frequencies. Neutrals and plasma ions are assumed isothermal, $T_0(r) = T_i(r)$, due to frequent charge exchange reactions between them. Fig. 4b shows the neutral profile obtained with this model for a set of Wendelstein 7-X stationary plasma profiles (Fig. 4a).

![Figure 4](image)

**FIG. 4:** Plasma profiles (a) and estimated neutral densities (b) for discharge #20180920.017.

Equation (4) determines the neutral profile up to a multiplicative constant that can be determined from the global particle confinement time $\tau_p$. In [19], a confinement time of $\tau_p = 0.258$ s was estimated for a similar discharge using a single-reservoir plasma particle balance. The neutral transport model (4) has recently been compared with neutral density measurements in W7-X (see figure 14 in [8]), showing varying degrees of agreement for $\tau_p$ between 0.15 and 1.0 s. In Fig. 4b, $\tau_p$ is varied from 0.1 s to 1.0 s to account for uncertainties in this number.

Let us denote by $\Gamma^{\exp}$ the flux profiles calculated, for this range of values of $\tau_p$, by integrating the neutral ionization source $\nu_{ion} n_0$ over the plasma radius. The neutral density $n_0$ is obtained from (4) using the experimental profiles shown in Fig. 4a. The results for $\Gamma^{\exp}$ are displayed in Fig. 5a. There is a region in the core (the width of which depends on the assumed $\tau_p$) where the estimated central particle sources are too small to sustain the outward neoclassical radial flux $\Gamma^{\exp}$ calculated with $\text{DKES}$ [21]. One is led to conclude that an additional particle flux must exist which moves particles towards the magnetic axis in that region. Further outside, on the contrary, the additional flux must be outward directed and account for essentially all the particle flux, since the...
neoclassical contribution falls 1 to 3 orders of magnitude short with respect to $\Gamma_s^{\text{exp}}$.

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