MODELLING AND FORECASTING OF WEB TRAFFIC USING HOLT'S LINEAR, BATS AND TBATS MODELS

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Abstract: In the recent era, internet consumption has increased. Due to this heavy and regular use of internet web traffic is increased. Sometimes due to high web traffic server may also affect. In this study, cumulative data on the number of visitors to Wikipedia, Facebook, Energy, Android, and Apple, are analyzed in detail. Some descriptive statistics of visitor’s pages in Wikipedia are given such as mean, minimum, maximum, standard deviation, skewness

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and kurtosis. The present study used different time series models like Holt’s Linear Trend, BATS and TBATS for different web pages. From the results, it was found that the Android page as well as apple page in Wikipedia holt’s Linear Trend model performance is better compared to other models. This kind of projection is helpful for web traffic to solve the server breakdown problem for larger users of the server. These Wikipedia pages have been chosen to Forecast the number of visitors to these pages through time series models BATS, TBATS, and Holt's Linear Trend Model in order to face future problems to mitigate over loading that may occur with the increase in the number of visitors to these pages and also to experiment the suitability of these models of Time series to Forecast the number of visitors and that to achieve the highest level of accuracy. MAPE for accuracy was used to compare model performance.

**Keywords:** Holt’s linear trend; forecasting; TBATS; BATS; time series; web traffic; Wikipedia; webpages.

**2010 AMS Subject Classification:** 37M10.

### 1. INTRODUCTION

Nowadays the web traffic consumption has been increased significantly and it became necessary to be forecasted in order to design the web server to balance and distribute the request loads [1]-[7]. Forecasting the load can allow building load balancing techniques to schedule client requests to meet the demands and to be dynamically scaled up and down for the server based on the forecasted visits[21]. Forecast of the upcoming traffic on multiple pages on a website is very challenging specifically if we are trying to forecast the traffic for Wikipedia pages [2] as it’s becoming the primal source of knowledge for billions of users [3], who are constantly contributing and expanding the content. Wikipedia consented users to access the traffic by querying each article over a given window [7], but we used in this research the available dataset created by the Kaggle competition [6]. [22-23] compared times series models and used for forecasting purposes. In this research, we employed the time series forecasting techniques to estimate future request loads based on historical visits[4], traditional time series such as Autoregressive integrated moving average “ARIMA”[5], Seasonal ARIMA “SARIMA”[4] have been used extensively for forecasting and based on a recent research Abotaleb [9] found that
Holt’s Linear Trend is giving accurate results in predicting the daily Infection cases for COVID-19 research in (Italy, China and USA) reaching to a conclusion that Holt’s model is giving more accurate results than ARIMA models. So, we used Holt’s Linear Model, and Trigonometric seasonality, Box-Cox transformation, ARMA errors, Trend and Seasonal components “TBATS” in which time series seasonality is not forced to be periodic, however, it’s allowed to be dynamic, in which results proved to be more accurate.

2. MATERIAL AND METHODS
This study was performed on data obtained from a database by the Kaggle competition from 01-07-2015 to 31-12-2016 [10]. We used cumulative data on the number of visitors to Wikipedia and four pages, namely Facebook, Energy, Android, and Apple, were selected. In this paper, we used time series models because these model a combination of Fourier with an exponential smoothing state-space model and a Box-Cox transformation, Both the BATS and TBATS models dealing with seasonality.

2.1. BATS and TBATS Models
TBATS is an improvement modification of BATS that allows multiple seasonal incorrect cycles. TBATS has the following equation [11].

Equation (1) is a Box-Cox transformation

\[
Y_t^{(\omega)} = \begin{cases} 
\frac{y_t^{(\omega)} - 1}{\omega} & \omega \neq 0 \\
\log y_t & \omega = 0
\end{cases}
\] (1)

Equation (2) represents the seasonal M pattern

\[
Y_t^{(\omega)} = l_{t-1} + \varphi b_{t-1} + \sum_{i=1}^{T} s_{t-m_i}^{(i)} + d_t
\] (2)

Equations (3), (4) and (5) are global trends and local trends

\[
l_t = l_{t-1} + \varphi b_{t-1} + \alpha d_t
\] (3)

\[
b_t = \varphi b_{t-1} + \beta d_t
\] (4)

\[
s_t^{(i)} = s_{t-m_i}^{(i)} + \gamma_t d_t
\] (5)
equation (6) is the error modeled by ARMA

\[ d_t = \sum_{i=1}^{p} \varphi_i d_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t. \]  

(6)

where \( m_1, \ldots, m_T \) denote that seasonal period, \( l_t \) and \( b_t \) denote that the level and trend of components of the time series at time \( t \), \( s_t^{(i)} \) denote that seasonal component at time \( t \), \( d_t \) represents to ARMA(p, q) component and \( \varepsilon_t \) is white noise process.

The smoothing parameters are given by \( \alpha, \beta, \gamma_i \) for \( i=1 \ldots T \) and \( \emptyset \) is the dampening parameter, which gives more control over trend extrapolation when the trend component is damped [12]. For seasonal data the following equations representing Trigonometric exponential smoothing models

\[ s_t^{(i)} = \sum_{j=1}^{k_i} a_{j,t}^{(i)} \cos(\lambda_j^{(i)} t) \]  

(7)

\[ a_{j,t}^{(i)} = a_{j,t-1}^{(i)} + k_1^{(i)} d_t \]  

(8)

\[ \beta_{j,t}^{(i)} = \beta_{j,t-1}^{(i)} + k_2^{(i)} d_t \]  

(9)

where \( k_1^{(i)} \) and \( k_2^{(i)} \) are the smoothing parameters.

\[ \lambda_j^{(i)} = 2\pi j/m_i. \]  

This is an extended, modified single source of error version of single seasonal multiple sources of error representation suggested by [13] and is equivalent to index seasonal approaches when \( k_i = m_i/2 \) for even values of \( m_i \) and when \( k_i = (m_i - 1)/2 \) for odd values of \( m_i \). But most seasonal terms will require much smaller values of \( k_i \), thus reducing the number of parameters to be estimated.

In the single seasonal multiple sources of error setting [14] an alternative, but equivalent formulation of representation (2) is preferred [15] which can be obtained hyper-parameterizing the single seasonal multiple sources of error version of (2) using

\[ a_{j,t}^{(i)} = s_{j,t}^{(i)} \cos(\lambda_j^{(i)} t) - s_{jt}^{*(i)} \sin(\lambda_j^{(i)} t) \]  

(10)

\[ \beta_{j,t}^{(i)} = s_{j,t}^{(i)} \sin(\lambda_j^{(i)} t) - s_{jt}^{*(i)} \cos(\lambda_j^{(i)} t) \]  

(11)
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\[ s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)} \]  \hspace{1cm} (12)

Where

\[ s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*\,(i)} \sin \lambda_j^{(i)} + \left[ k_1^{(i)} \cos(\lambda_j^{(i)} t) + k_2^{(i)} \sin(\lambda_j^{(i)} t) \right] d_t \]  \hspace{1cm} (13)

\[ s_{j,t}^{*\,(i)} = -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*\,(i)} \cos \lambda_j^{(i)} + \left[ k_2^{(i)} \cos(\lambda_j \, t) - k_1^{(i)} \sin(\lambda_j \, t) \right] d_t \]  \hspace{1cm} (14)

\[ s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)} \]  \hspace{1cm} (15)

Equations (16) and (17) are seasonal patterns modeled by the Fourier model.

\[ s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*\,(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t \]  \hspace{1cm} (16)

\[ s_{j,t}^{*\,(i)} = -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*\,(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t \]  \hspace{1cm} (17)

The notation TBATS\((p,q,\{m_1,k_1\},\{m_2,k_2\},...,\{m_T,k_T\})\) is used for these trigonometric models.

2.2. Holt’s Linear Trend Method

The exponentially weighted moving average is also the averages of smoothing random variability with the following properties: (1) that is very important that older data have a declining weight; (2) it is very simple to calculate; and (3) the most important for data set is that minimal data is needed. Holt, C.E. 1957 had given equation.

Forecast Equation

\[ \hat{y}_{t+h|t} = m_t + h z_t \]  \hspace{1cm} (18)

Level Equation

\[ m_t = \alpha y_t + (1 - \alpha)(m_{t-1} + z_{t-1}) \]  \hspace{1cm} (19)

Trend Equation

\[ b_t = \beta^*(m_t - m_{t-1}) + (1 - \beta^*)z_{t-1} \]  \hspace{1cm} (20)

where \( m_t \) represent an estimate of the level of series at time \( t \), \( b_t \) denotes an estimate of the trend (slope) of the series at time \( t \), \( \alpha \) is the smoothing parameter for level, \( 0 \leq \alpha \leq 1 \) and \( \beta^* \) is the smoothing parameter for the trend, \( 0 \leq \beta^* \leq 1 \) that’s with simple exponential smoothing.
2.2.1. Hyperparameters & Overfit
BATS differs from TBATS only in the way it models seasonal effects. In BATS we have a more traditional approach where each seasonality is modeled by that Equation $s_t^{(i)} = s_{t-m_i} + \gamma_l d_t$ that's mean that BATS can only model integer period lengths. Approach taken in BATS requires $m_i$ seed states for season i, if this season is long the model may become intractable.

TBATS Choose the Final Model Under the hood TBATS will consider various alternatives and fit quite a few models. It will consider models. 1) with Box-Cox transformation and without it. 2) with the trend and without it. 3) with Trend Damping and without it. 4) with ARMA(p,q) process used to model residuals non-seasonal model and without it various amounts of harmonics used to model seasonal effects. The final model will be chosen using the Akaike information criterion (AIC). In particular auto, ARMA is used to decide if residuals need modeling and what p and q values are suitable.

Holt’s Linear Trend Method, as with simple exponential smoothing, the level equation here shows that $m_t$ is a weighted average of observation and the one-step-ahead training forecast for time $\hat{y}_t$ and the one-step-ahead training forecast for time t. here given by $m_{t-1} + z_{t-1}$ . The trend equation shows that $z_t$ is a weighted average of the estimated trend at time t based on $m_t - m_{t-1}$ and $z_{t-1}$ the previous estimate of the trend. The smoothing parameters $\alpha$ and $\beta^*$, and the initial values $m_0$ and $z_0$ are estimated by minimising the SSE for the one-step training errors.

2.2.2. Overfit problem
If the model is overfitting on training data when the model performs well on the training data but does not perform well on the evaluation data so when we comparing training data with testing data the accuracy of testing data is better than training data

3. RESULT AND DISCUSSION
From table 1, we find that: from 01-07-2015 to 31-12-2016, the visitors to Facebook page in Wikipedia have increased during the period from (65) to (28328). Average daily visitors to the Facebook page in Wikipedia are (13447.63). Kurtosis value is (1.6) indicates the data follows a platykurtic distribution which shows a tail that's thinner than a normal distribution which means the number of outliers will not be large. Followed by a positive value of skewness (0.12) which
is between -0.5 and 0.5, the distribution is approximately symmetric.

The Visitors of Energy page in Wikipedia have increased from (16) to (5014) during the same period, with average daily Visitors of Energy page in Wikipedia about (2284). Kurtosis value is (1.6) indicates the data follows a platykurtic distribution which shows a tail that's thinner than a normal distribution which means the number of outliers will not be large. With the positive value of skewness (0.16) which is between -0.5 and 0.5, the distribution is approximately symmetric.

The visitors of Android page in Wikipedia have increased from (8) to (22740) during the same period, with the average daily visitors of Android page in Wikipedia about (11021). Kurtosis value is (1.6) indicates the data follows a platykurtic distribution which shows a tail that's thinner than a normal distribution which means the number of outliers will not be large. With the positive value of skewness (0.01) which is between -0.5 and 0.5, the distribution is approximately symmetric. The Visitors of Apple page in Wikipedia have increased from (4) to (4906) during the same period, with average daily Visitors of Apple page in Wikipedia about (2416). Kurtosis value is (1.7) indicates the data follows a platykurtic distribution which shows a tail that's thinner than a normal distribution which means the number of outliers will not be large. With the positive value of skewness (0.04) which is between -0.5 and 0.5, the distribution is approximately symmetric.

The parameter for the level smoothing is denoted by Alpha, and the parameter for the trend smoothing is denoted by Beta, α and β are constrained to 0-1 with higher values giving faster learning and lower values providing slower learning. From Table 2 it becomes clear that the best values of the level and the trends are 0.999 for level for all the series (Facebook page in Wikipedia), (Energy page in Wikipedia), (Android page in Wikipedia), (Apple page in Wikipedia), meaning fast learning in the day-to-day visits for pages in Wikipedia, and 0.14 for trend for the series (Facebook page in Wikipedia), and 0.56 for trend for the series (Energy page in Wikipedia), and 0.10 for trend for the series (Android page in Wikipedia), and 0.96 for trend for the series (Apple page in Wikipedia), which means slow learning for the trend.

From Table 3, Facebook page in Wikipedia: BATS is the best-suited model (0.845, {0,0}, 1, -),
In this model, Box-Cox transformation = 0.845, the order of ARMA error is (0, 0), the damping parameter = 1 (essentially doing nothing). Energy page in Wikipedia: BATS is the best-suited model (0.718, {5,0}, 1, -), In this model, Box-Cox transformation = 0.718, the order of ARMA error is (5, 0), the damping parameter = 1 (essentially doing nothing).

Android page in Wikipedia: BATS is the best-suited model (0.854, {0,0}, 1, -), In this model, Box-Cox transformation = 0.854, the order of ARMA error is (0, 0), the damping parameter = 1 (essentially doing nothing).

Apple page in Wikipedia: BATS is the best-suited model (0.972, {1,0}, 1, -). In this model, Box-Cox transformation = 0.972, the order of ARMA error is (1, 0), the damping parameter = 1 (essentially doing nothing). By comparing table 2 with table 3, we find: The forecasting accuracy by the Holt's Linear Trend model is very high because the values of the (sigma, AIC) were lower than TBATS Model for all series.

Table 4 is given the model fitting of TBATS models in all web pages considered in the study. In Facebook page in Wikipedia: BATS is the best-suited model (1, {0,0}, 1, {<6,2>}), In this model, the damping parameter = 1 (essentially doing nothing).

In Table 5, the best-fitted models based on, lowest values of ME, RMSE, MAE, MPE, MAPE and ACF1, TBATS model is better than Holt's linear trend model for all the series (Facebook page in Wikipedia, Energy page in Wikipedia, Android page in Wikipedia, Apple page in Wikipedia). In other words, the forecasting accuracy by the TBATS model is very high and outperform the forecasting accuracy of Holt's Linear Trend Model, because the values of the accuracy criteria (ME, RMSE, MAE, MPE, MAPE and ACF1) were lower than the values of the accuracy criteria of Holt's Linear Trend Model for all series (Facebook page in Wikipedia, Energy page in Wikipedia, Android page in Wikipedia, Apple page in Wikipedia).

In Table 6, MAPEs are given for Holt’s linear model for all series. From Table 6, it is seen that the 7-day error averages can be ranked as Facebook page in Wikipedia, Android page in Wikipedia, Energy page in Wikipedia, and Apple page in Wikipedia in ascending order. This clearly states that the best forecasting for the Holt’s linear model is Facebook page of Wikipedia. In Table 7, MAPEs are given for the BATS model for Facebook page in Wikipedia, Energy page
in Wikipedia, Android page in Wikipedia, Apple page in Wikipedia. From Table 7, it is easily observed that the 7-day error averages can be ranked as Facebook page in Wikipedia, Android page in Wikipedia, Energy page in Wikipedia, and Apple page in Wikipedia in ascending order. These results show that the best forecasting for BATS linear model is Wikipedia's Facebook page. In Table 8, MAPEs are also presented for the TBATS model for Facebook page in Wikipedia, Energy page in Wikipedia, Android page in Wikipedia, Apple page in Wikipedia. In Table 8, similar results were seen in Tables 6 and 7, and it was seen that all three methods gave consistent results.

4. CONCLUSION

In the projection model, every model is showing a different error rate. In the present investigation; MAPE measure, from training data the best models with least MAPE error are Holt’s linear trend and Bats Models. The best models for Android page in Wikipedia and Apple page in Wikipedia on training data is Holt’s linear trend but the best model for Facebook page in Wikipedia and Energy page in Wikipedia on training data is the BATs model. However, the percentage of error (MAPE) on testing data for last 7 days in the data from (25-12-2016 to 31-12-2016), we concluded that the best model can be forecasting Facebook page in Wikipedia Holt’s Linear trend and Bats models because Facebook page in Wikipedia has least MAPE = 0.019 %, the best model can forecast. Energy page in Wikipedia is Bats model with MAPE = 0.1290%, for Android page in Wikipedia the best model with MAPE =0.0460%, and Apple page in Wikipedia TBATS model with MAPE = 0.34 %. This kind of study helps to understand the web traffic or different web pages trend in the future to invest towards it.

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HOLT’S LINEAR TREND AND TBATS MODELS

- Holt’s Linear Trend Model were fitted using R software.
- BATS and TBATS Model were fitted using R software.
- Data was used in fitting the Models1. (From 01-07-2015 To 31-12-2016) and testing data for last 7 days in the data from (25-12-2016 to 31-12-2016)

Table 1: Descriptive statistics of Visitors pages in Wikipedia

| Page                     | Mean    | Minimum | Maximum | Standard Deviation | Skewness  | Kurtosis |
|--------------------------|---------|---------|---------|--------------------|-----------|----------|
| Facebook page in Wikipedia | 13447.63 | 65      | 28328   | 8855.564           | 0.1255453 | 1.605639 |
| Energy page in Wikipedia  | 2284.327 | 16      | 5101    | 1551.386           | 0.1699649 | 1.633895 |
| Android page in Wikipedia | 11021.37 | 8       | 22740   | 7016.673           | 0.01107273| 1.635909 |
| Apple page in Wikipedia  | 2416.776 | 4       | 4906    | 1438.541           | 0.04719163| 1.723823 |

Table 2: Holt's Linear Trend Models fitted for Visitors pages in Wikipedia

| Page                     | Box-Cox | Smoothing transformation | Initial states | AIC    |
|--------------------------|---------|--------------------------|----------------|--------|
|                          | Lambda  | Alpha | Beta  | L   | B    | Sigma |        |
| Facebook page in Wikipedia | 0.826   | 0.9999 | 0.1466 | 22.1848 | 15.5383 | 5.6804 | 5311.728 |
| Energy page in Wikipedia  | 0.6822  | 0.9999 | 0.566 | 6.8721 | 1.65 | 0.9233 | 3338.653 |
| Android page in Wikipedia | 0.78    | 0.9999 | 0.1062 | 0.4811 | 8.357 | 7.9947 | 5682.872 |
| Apple page in Wikipedia  | 0.9675  | 0.9999 | 0.0297 | 10.0102 | 8.236 | 3.9439 | 4915.496 |
### Table 3: BATS Model fitted for Visitors pages in Wikipedia

| Page                                     | Parameters          | prediction error |
|------------------------------------------|---------------------|------------------|
| Facebook page in Wikipedia               | Lambda 0.845331     | 0.845331         |
| BATS(0.845, {0,0}, 1, -)                 | Alpha 1.080399      | 1.080399         |
|                                          | Beta 0.1306299      | 0.1306299        |
|                                          | Damping 1           | 1                |
|                                          | AR -                | -                |
|                                          | Sigma 6.735756      | 6.735756         |
|                                          | AIC 7031.399        | 7031.399         |
| Energy page in Wikipedia                 | Lambda 0.717972     | 0.717972         |
| BATS(0.718, {5,0}, 1, -)                 | Alpha 1.981873      | 1.981873         |
|                                          | Beta 0.006224636    | 0.006224636      |
|                                          | Damping 1           | 1                |
|                                          | AR -                | -                |
|                                          | Sigma 0.219562      | 0.219562         |
|                                          | AIC 5815.88         | 5815.88          |
| Android page in Wikipedia                | Lambda 0.853521     | 0.853521         |
| BATS(0.854, {0,0}, 1, -)                 | Alpha 1.409863      | 1.409863         |
|                                          | Beta -0.01404994    | -0.01404994      |
|                                          | Damping 1           | 1                |
|                                          | AR -                | -                |
|                                          | Sigma 14.09601      | 14.09601         |
|                                          | AIC 7716.226        | 7716.226         |
| Apple page in Wikipedia                  | Lambda 0.971881     | 0.971881         |
| BATS(0.972, {1,0}, 1, -)                 | Alpha 0.3868964     | 0.3868964        |
|                                          | Beta -0.004016503   | -0.004016503     |
|                                          | Damping 0.999781    | 0.999781         |
|                                          | AR -                | -                |
|                                          | Sigma 0.705365      | 0.705365         |
|                                          | AIC 5181.524        | 5181.524         |

### Table 4: TBATS Model fitted for Visitors pages in Wikipedia

| Page                                     | Parameters          | Gamma-1 Values | Gamma-2 Values | prediction error |
|------------------------------------------|---------------------|----------------|----------------|------------------|
| Facebook page in Wikipedia               | Alpha 1.075378      | 1.075378       | -0.0005057618  | 27.94339         |
| TBATS(1, {0,0}, 1, {<6,2>})              | Beta 0.1329358      | 0.1329358      | -0.0005904406  | 0.121119         |
|                                          | Damping 4.842195e-05| 4.842195e-05  | -0.006112289   | 5.010726         |
|                                          | AR -                | -0.527266      | 5.010726       | 5189.507         |
|                                          | Sigma 7055.907      | 7055.907       | 5189.507       |                  |
|                                          | AIC 7055.907        | 7055.907       | 5189.507       |                  |
| Energy page in Wikipedia                 | Alpha 1.130439      | 1.130439       | -0.0005904406  | 10.30364         |
| TBATS(1, {0,0}, 1, {<6,2>})              | Beta 0.3640595      | 0.3640595      | -0.0006112289  | 10.30364         |
|                                          | Damping -0.0005904406| -0.0005904406| -0.006112289   | 10.30364         |
|                                          | AR -                | -0.092203      | -0.097295      | 10.30364         |
|                                          | Sigma 1.121119      | 1.121119       | 10.30364       | 5815.88          |
|                                          | AIC 5815.88         | 5815.88        | 10.30364       |                  |
| Android page in Wikipedia                | Alpha 1.478911      | 1.478911       | -0.0005904406  | 52.69904         |
| TBATS(1, {0,0}, 1, {<6,2>})              | Beta 0.01387797     | 0.01387797     | -0.000609636   | 52.69904         |
|                                          | Damping -0.002673922| -0.002673922  | 0.0006609636   | 52.69904         |
|                                          | AR -                | -0.092795      | -0.092795      | 52.69904         |
|                                          | Sigma 1.121119      | 1.121119       | 52.69904       | 5972.422         |
|                                          | AIC 5972.422        | 5972.422       | 5972.422       |                  |
| Apple page in Wikipedia                  | Alpha 0.971881      | 0.971881       | -0.004016503   | 4.047286         |
| TBATS(1, {1,0}, 1, -)                    | Beta 0.3868964      | 0.3868964      | -0.004016503   | 4.047286         |
|                                          | Damping 0.999781    | 0.999781       | 4.047286       | 5181.524         |
|                                          | AR -                | -0.092795      | -0.092795      | 4.047286         |
|                                          | Sigma 4.047286      | 4.047286       | 4.047286       | 5181.524         |
|                                          | AIC 5181.524        | 5181.524       | 4.047286       | 5181.524         |
Table 5: Fitted Holt’s Linear Trend, BATS and TBATS Model fitted for Vistorspages in

| Model           | ME     | RMSE     | MAE     | MPE     | MAPE    | MASE    | ACF1    |
|-----------------|--------|----------|---------|---------|---------|---------|---------|
| Facebookpage in Wikipedia | -0.382917 | 28.20683 | 15.0246 | -0.02479222 | 0.3233684 | 0.291907 | 0.06510676 |
| Energy page in Wikipedia | -0.05710684 | 10.44684 | 4.623831 | -0.01219886 | 0.539105 | 0.5077221 | -0.0184911 |
| Android page in Wikipedia | -0.3773315 | 58.70418 | 20.22983 | -0.1105207 | 0.7482277 | 0.4897302 | 0.3402883 |
| Apple page in Wikipedia | -0.1331319 | 5.00807  | 3.324313 | 0.1261135  | 0.6548812 | 0.3745121 | 0.02402595 |

BATS Model

| Model           | ME     | RMSE     | MAE     | MPE     | MAPE    | MASE    | ACF1    |
|-----------------|--------|----------|---------|---------|---------|---------|---------|
| Facebookpage in Wikipedia | -0.4404807 | 28.12292 | 14.98003 | -0.05470518 | 0.3211768 | 0.2910412 | 0.001672377 |
| Energy page in Wikipedia | 0.1745692 | 9.689009 | 4.375281 | 0.1208824 | 0.5060333 | 0.48043 | -0.04764241 |
| Android page in Wikipedia | 0.9798254 | 52.52552 | 18.28792 | -0.1709344 | 0.874007 | 0.4427197 | 0.07114531 |
| Apple page in Wikipedia | -0.02153136 | 4.976799 | 3.269713 | 0.6647574 | 1.135035 | 0.368361 | -0.06509243 |

TBATS Model

| Model           | ME     | RMSE     | MAE     | MPE     | MAPE    | MASE    | ACF1    |
|-----------------|--------|----------|---------|---------|---------|---------|---------|
| Facebookpage in Wikipedia | 0.04114569 | 27.94339 | 14.91394 | -0.009450621 | 0.382524 | 0.2897572 | -5.82108e-05 |
| Energy page in Wikipedia | 0.0346115 | 10.30364 | 4.601217 | 0.09734302 | 0.5599713 | 0.505239 | 0.02425513 |
| Android page in Wikipedia | 1.855597 | 52.69904 | 19.40097 | 0.4709047 | 1.305884 | 0.4696648 | 0.008057886 |
| Apple page in Wikipedia | 0.08187205 | 5.010726 | 3.299648 | 0.8127762 | 1.243233 | 0.3717333 | 0.002301955 |
### Table 6: MAPE Holt’s Linear Trend Model fitted for Visitors pages in Wikipedia

| Date       | Day    | Facebook page in Wikipedia | Actual | Forecasted | Error   |
|------------|--------|----------------------------|--------|------------|---------|
| 2016-12-25 | Sunday | 28011                       | 28011.96 | 0.003 %    |
| 2016-12-26 | Monday | 28059                       | 28061.94 | 0.01 %     |
| 2016-12-27 | Tuesday| 28111                       | 28111.94 | 0.003 %    |
| 2016-12-28 | Wednesday | 28161                   | 28161.95 | 0.003 %    |
| 2016-12-29 | Thursday| 28221                      | 28211.97 | 0.032 %    |
| 2016-12-30 | Friday  | 28269                       | 28262.01 | 0.025 %    |
| 2016-12-31 | Saturday| 28328                      | 28312.01 | 0.056 %    |
| **MAPE**   |        |                            |         | 0.019 %    |

| Date       | Day    | Energy page in Wikipedia  | Actual | Forecasted | Error   |
|------------|--------|----------------------------|--------|------------|---------|
| 2016-12-25 | Sunday | 4969                        | 4959.849 | 0.184 %    |
| 2016-12-26 | Monday | 4976                        | 4967.703 | 0.167 %    |
| 2016-12-27 | Tuesday| 4985                        | 4975.560 | 0.189 %    |
| 2016-12-28 | Wednesday | 4994                    | 4983.422 | 0.212 %    |
| 2016-12-29 | Thursday| 5002                        | 4991.287 | 0.214 %    |
| 2016-12-30 | Friday  | 5005                        | 4999.157 | 0.117 %    |
| 2016-12-31 | Saturday| 5014                        | 5007.030 | 0.139 %    |
| **MAPE**   |        |                            |         | 0.175 %    |

| Date       | Day    | Android page in Wikipedia | Actual | Forecasted | Error   |
|------------|--------|----------------------------|--------|------------|---------|
| 2016-12-25 | Sunday | 22439                       | 22444.35 | 0.024 %    |
| 2016-12-26 | Monday | 22479                       | 22491.73 | 0.057 %    |
| 2016-12-27 | Tuesday| 22519                       | 22539.13 | 0.089 %    |
| 2016-12-28 | Wednesday | 22585                   | 22586.55 | 0.007 %    |
| 2016-12-29 | Thursday| 22628                       | 22633.99 | 0.026 %    |
| 2016-12-30 | Friday  | 22666                       | 22681.45 | 0.068 %    |
| 2016-12-31 | Saturday| 22740                       | 22728.94 | 0.049 %    |
| **MAPE**   |        |                            |         | 0.046 %    |

| Date       | Day    | Apple page in Wikipedia   | Actual | Forecasted | Error   |
|------------|--------|----------------------------|--------|------------|---------|
| 2016-12-25 | Sunday | 4826                        | 4823.601 | 0.05 %     |
| 2016-12-26 | Monday | 4834                        | 4832.202 | 0.037 %    |
| 2016-12-27 | Tuesday| 4858                        | 4840.803 | 0.354 %    |
| 2016-12-28 | Wednesday | 4868                    | 4849.405 | 0.382 %    |
| 2016-12-29 | Thursday| 4883                        | 4858.007 | 0.512 %    |
| 2016-12-30 | Friday  | 4895                        | 4866.610 | 0.58 %     |
| 2016-12-31 | Saturday| 4906                        | 4875.214 | 0.628 %    |
| **MAPE**   |        |                            |         | 0.363 %    |
### Table 7: MAPE BATS Model fitted for Visitors pages in Wikipedia

| Date       | Day    | Facebook page in Wikipedia | Actual | Forecasted | Error  |
|------------|--------|----------------------------|--------|------------|--------|
| 2016-12-25 | Sunday | 28011                      | 28011.88 | 0.003 %    |
| 2016-12-26 | Monday | 28059                      | 28061.98 | 0.011 %    |
| 2016-12-27 | Tuesday| 28111                      | 28112.09 | 0.004 %    |
| 2016-12-28 | Wednesday | 28161                  | 28162.21 | 0.004 %    |
| 2016-12-29 | Thursday| 28221                      | 28212.35 | 0.031 %    |
| 2016-12-30 | Friday  | 28269                      | 28262.50 | 0.023 %    |
| 2016-12-31 | Saturday| 28328                      | 28312.67 | 0.054 %    |
| **MAPE**   |        |                            |         |            | 0.019 % |

| Date       | Day    | Energy page in Wikipedia | Actual | Forecasted | Error  |
|------------|--------|----------------------------|--------|------------|--------|
| 2016-12-25 | Sunday | 4969                       | 4961.055 | 0.16 %    |
| 2016-12-26 | Monday | 4976                       | 4971.897 | 0.082 %    |
| 2016-12-27 | Tuesday| 4985                       | 4982.497 | 0.05 %     |
| 2016-12-28 | Wednesday | 4994                  | 4994.245 | 0.005 %    |
| 2016-12-29 | Thursday| 5002                       | 5005.343 | 0.067 %    |
| 2016-12-30 | Friday  | 5005                       | 5017.327 | 0.246 %    |
| 2016-12-31 | Saturday| 5014                       | 5028.527 | 0.29 %     |
| **MAPE**   |        |                            |         |            | 0.129 % |

| Date       | Day    | Android page in Wikipedia | Actual | Forecasted | Error  |
|------------|--------|----------------------------|--------|------------|--------|
| 2016-12-25 | Sunday | 22439                      | 22442.02 | 0.013 %    |
| 2016-12-26 | Monday | 22479                      | 22483.89 | 0.022 %    |
| 2016-12-27 | Tuesday| 22519                      | 22525.77 | 0.03 %     |
| 2016-12-28 | Wednesday | 22585                 | 22567.66 | 0.077 %    |
| 2016-12-29 | Thursday| 22628                      | 22609.56 | 0.081 %    |
| 2016-12-30 | Friday  | 22666                      | 22651.47 | 0.064 %    |
| 2016-12-31 | Saturday| 22740                      | 22693.40 | 0.205 %    |
| **MAPE**   |        |                            |         |            | 0.07 %  |

| Date       | Day    | Apple page in Wikipedia  | Actual | Forecasted | Error  |
|------------|--------|----------------------------|--------|------------|--------|
| 2016-12-25 | Sunday | 4826                       | 4823.733 | 0.047 %    |
| 2016-12-26 | Monday | 4834                       | 4832.427 | 0.033 %    |
| 2016-12-27 | Tuesday| 4858                       | 4841.091 | 0.348 %    |
| 2016-12-28 | Wednesday | 4868                 | 4849.735 | 0.375 %    |
| 2016-12-29 | Thursday| 4883                       | 4858.364 | 0.505 %    |
| 2016-12-30 | Friday  | 4895                       | 4866.981 | 0.572 %    |
| 2016-12-31 | Saturday| 4906                       | 4875.591 | 0.62 %     |
| **MAPE**   |        |                            |         |            | 0.357 % |
Table 8: MAPE TBATS Model fitted for Visitors pages in Wikipedia

| Date     | Day    | Facebook page in Wikipedia | Actual | Forecasted | Error  |
|----------|--------|-----------------------------|--------|------------|--------|
| 2016-12-25 | Sunday | 28011                        | 28009.45 | 0.006 %    |
| 2016-12-26 | Monday | 28059                        | 28063.59 | 0.016 %    |
| 2016-12-27 | Tuesday | 28111                      | 28117.03 | 0.021 %    |
| 2016-12-28 | Wednesday | 28161                   | 28167.62 | 0.023 %    |
| 2016-12-29 | Thursday | 28221                    | 28217.72 | 0.012 %    |
| 2016-12-30 | Friday  | 28269                      | 28263.83 | 0.018 %    |
| 2016-12-31 | Saturday | 28328                    | 28311.13 | 0.06 %     |

| Date     | Day    | MAPE                                        |
|----------|--------|---------------------------------------------|
|          |        | 0.022 %                                    |

| Date     | Day    | Energy page in Wikipedia                  |
|----------|--------|-------------------------------------------|
| 2016-12-25 | Sunday | 4969                                      | 4961.410 | 0.153 %    |
| 2016-12-26 | Monday | 4976                                      | 4968.871 | 0.143 %    |
| 2016-12-27 | Tuesday | 4985                                   | 4975.989 | 0.181 %    |
| 2016-12-28 | Wednesday | 4994                  | 4983.894 | 0.202 %    |
| 2016-12-29 | Thursday | 5002                                  | 4990.741 | 0.225 %    |
| 2016-12-30 | Friday  | 5005                                     | 4998.485 | 0.13 %     |
| 2016-12-31 | Saturday | 5014                                | 5007.631 | 0.127 %    |

| Date     | Day    | MAPE                                        |
|----------|--------|---------------------------------------------|
|          |        | 0.166 %                                    |

| Date     | Day    | Android page in Wikipedia                 |
|----------|--------|-------------------------------------------|
| 2016-12-25 | Sunday | 22439                                    | 22442.64 | 0.016 %    |
| 2016-12-26 | Monday | 22479                                    | 22484.13 | 0.023 %    |
| 2016-12-27 | Tuesday | 22519                                   | 22518.31 | 0.003 %    |
| 2016-12-28 | Wednesday | 22585                                | 22558.48 | 0.117 %    |
| 2016-12-29 | Thursday | 22628                                  | 22607.34 | 0.091 %    |
| 2016-12-30 | Friday  | 22666                                   | 22647.34 | 0.082 %    |
| 2016-12-31 | Saturday | 22740                               | 22685.96 | 0.238 %    |

| Date     | Day    | MAPE                                        |
|----------|--------|---------------------------------------------|
|          |        | 0.081 %                                    |

| Date     | Day    | Apple page in Wikipedia                |
|----------|--------|-----------------------------------------|
| 2016-12-25 | Sunday | 4826                                    | 4823.971 | 0.042 %    |
| 2016-12-26 | Monday | 4834                                    | 4833.684 | 0.007 %    |
| 2016-12-27 | Tuesday | 4858                                   | 4842.021 | 0.329 %    |
| 2016-12-28 | Wednesday | 4868                              | 4850.823 | 0.353 %    |
| 2016-12-29 | Thursday | 4883                                  | 4859.679 | 0.478 %    |
| 2016-12-30 | Friday  | 4895                                   | 4867.321 | 0.565 %    |
| 2016-12-31 |         | 4906                                    | 4876.286 | 0.606 %    |

| Date     | Day    | MAPE                                        |
|----------|--------|---------------------------------------------|
|          |        | 0.34 %                                     |
Figure 1 Actual Data of Number visitors Facebook page in Wikipedia

Figure 2 Forecasts From Holt's Method For Facebook page in Wikipedia
Figure 3 Decomposition by BATS Model Facebook page in Wikipedia

Figure 4 Forecasts From BATS Model in Facebook page in Wikipedia
Figure 5 Decomposition by TBATS Model for Facebook page in Wikipedia

Figure 6 Forecasts from TBATS Model for Facebook page in Wikipedia
Figure 7 Actual data For Visitors Energy page in Wikipedia

Figure 8 Forecasts from Holt's Method for Energy page in Wikipedia
Figure 9 Decomposition by BATS Model for Energy page in Wikipedia

Figure 10 Forecasts from BATS Model for Energy page in Wikipedia
Figure 11 Decomposition by TBATS Model for Energy page in Wikipedia

Figure 12 Forecasts from TBATS Model for Energy page in Wikipedia
Figure 13 Actual data for Visitors for Android page in Wikipedia

Figure 14 Forecasts from Holt's Method for Android page in Wikipedia
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Figure 15 Decomposition by BATS Model for Android page in Wikipedia

Figure 16 Forecasts from BATS Model for Android page in Wikipedia
Figure 17 Decomposition by TBATS Model for Android page in Wikipedia

Figure 18 Forecasts from TBATS Model for Android page in Wikipedia
Figure 19 Actual data for Visitors Apple Page in Wikipedia

Figure 20 Forecasts from Holt's Method for Apple Page in Wikipedia
Figure 21 Decomposition by BATS Model For Apple Page in Wikipedia

Figure 22 Forecasts from BATS Model for Apple Page in Wikipedia
Figure 23 Decomposition by TBATS Model for Apple Page in Wikipedia

Figure 24 Forecasts From TBATS Model For Apple Page in Wikipedia
CONFLICT OF INTERESTS
The author(s) declare that there is no conflict of interests.

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