Comparison of Different Empirical Models for Analysing the Thermal Conductivities of Various Materials for Use in Nanofluid Preparation

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Abstract—For over the past ten years, various empirical models have been studied for analysing the thermal conductivity of different materials. In this analysis, different empirical correlations and models of thermal conductivity have been compared. The thermal conductivities of four types of oxide materials (SiO2, TiO2, CuO, Al2O3) and MWCNTs with volume fractions from 0.5 to 5% in a temperature range of 273–373 K and various nanoparticle shapes were compared. The results illustrated that the thermal conductivity of nanofluids based upon various nanoparticles increased with increasing volume fraction. In comparison, the effective thermal conductivity of nanofluids based on MWCNTs was enhanced much more than that of other types of nanofluids. Furthermore, Maxwell’s model was considered the basis for predicting the effective thermal conductivity of nanofluids. According to the basic models, a new correlation was proposed for predicting the effective thermal conductivity as a function of temperature and nanoparticle volume concentration. The nanoparticle shape has a great impact on the thermal conductivity of nanofluids. Regarding the precise heat transfer enhancement, the analysed nanofluids are suggested to be heat carriers in thermal systems, particularly solar thermal collectors. Typically, the use of nanofluids in solar thermal collectors improves the thermal efficiency of collectors in terms of the precise heat transfer of nanofluids.

Keywords: nanofluid, thermal conductivity, empirical models, correlations, shape factor, MWCNT

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INTRODUCTION

With increasing population, energy demands tend to be maximized on consumption, particularly for electricity and thermal energy. Due to the COVID-19 pandemic, primary energy consumption and carbon emissions decreased by 4.5 and 6%, respectively, in 2020, but wind, solar and hydroelectricity all grew despite the fall in overall energy demand [1]. Among the other types, solar energy is the most preferred for use as a low-cost and eco-friendly energy source. There are different ways to convert solar energy into another type; by means of solar collectors, solar energy can be converted into thermal energy. Despite the advances of using solar collectors as a solar energy utilizer, there are disadvantages of low heat transfer on heat carrying capacity. To boost the performance of solar thermal applications, as first proposed by Choi, a new type of heat transfer that homogenizes a mixture of nanoparticles at a low volume fraction into base fluids, such as water, ethylene glycol, and oil, might noticeably increase thermal performance [2, 3]. It is important that the thermal conductivity of nanofluids also plays a great role when used as a heat carrier. Akhatov et al. [4] experimentally analysed the enhancement of the thermal conductivity of nanofluids based on SiO2 while using it as a heat carrier in a “tube in tube” heat exchanger. To investigate the thermal conductivity of nanofluids, several empirical models have been proposed by several investigators, such as Maxwell [5], Hamilton and Crosser [6], Wasp [7], Xue [8] and Sokhansefat [9]. Their empirical models and equations expressed the impact of nanoparticle concentration on the thermal conductivity of the suspension. This led to the development of new theoretical models and correlations that might evaluate the thermal conductivity of nanofluids, which has shown the special effects of solid/liquid boundaries and the micromixing convection influenced by nanoparticle Brownian motion, as expressed in the models of Choi and Yu [10], Kumar et al. [11], Koo and Kleinstreuer [12], and Jang and Choi [13–15]. Nevertheless, they differ from each other, which means that there are some limitations in their applications. Moreover, there are some comparisons for empirical models that can predict the effective thermal conductivity of nanofluids, as provided by Sidik [16], who obtained curves that represent the influence of the volume fraction of nanoparticles on thermal conductivity. Despite these advantages, there are some drawbacks that can impact the enhancement of the...
heat transfer capabilities of nanofluids, such as agglomeration, coagulation and sedimentation. The process of sedimentation in nanofluids based upon Al2O3 and SiO2 nanoparticles has been studied by Akhatov et al. [17], where the sedimentation ratio varied in the interval [0.005; 0.14]. Furthermore, the sedimentation process depends on both the size and concentration of nanoparticles.

The objective of this work is to compare and introduce several empirical models to analyse the thermal conductivities of various types of oxide materials, such as SiO2, TiO2, CuO, Al2O3, and MWCNTs for application in solar thermal applications, particularly various types of solar collectors.

**METHODOLOGY**

The thermophysical properties of suspensions have been studied using several mathematical equations and correlations. The thermal properties of the SiO2, TiO2, CuO, Al2O3, and MWCNT nanoparticles and base fluid (distilled water) are illustrated in Table 1.

While predicting thermal properties, four types of nanoparticles (SiO2, TiO2, CuO, Al2O3) and MWCNTs with volume fractions of 1–5% and nanoparticle diameters of 16–100 nm have been investigated. The preparation temperature ranged from 293 to 323 K, and the sonication time was shifted from 6 to 24 min. Furthermore, surfactant additives were not observed while preparing nanofluids based on selected materials.

**EMPIRICAL MODELS**

**Thermal Conductivity Models**

Generally, investigating the thermal conductivity of nanofluids requires an empirical Maxwell model, which provides the relation between the volume fraction of nanoparticles and the thermal conductivity of base fluids [5]. The mathematical expression of the model is given below:

\[
k_{nf} = k_{bf} \frac{k_{np} + 2k_{bf} + 2\varphi(k_{np} - k_{bf})}{k_{np} + 2k_{bf} - 2\varphi(k_{np} - k_{bf})},
\]

where \(\varphi\) is the concentration of nanoparticles, based on metal oxides, and \(k_{np}\) and \(k_{bf}\) indicate the thermal conductivity of nanoparticles and base fluids, respectively. According to the Maxwell hypothesis, the thermal conductivity of nanofluids depends upon the base fluids and volume fraction of nanoparticles.

However, the thermal conductivity of nanofluids also depends on the shape of the particles. To this end, Hamilton and Crosser [6] determined the relationship between the shape factors of nanoparticles and thermal conductivity. They expanded Maxwell’s model, which includes the shape factors of nanoparticles. The relationship might be seen by the expression of (2).

\[
k_{nf} = k_{bf} \frac{k_{np} + (n-1)k_{bf} - \varphi(n-1)(k_{bf} - k_{np})}{k_{np} + (n-1)k_{bf} + \varphi(k_{bf} - k_{np})},
\]

where \(\varphi\) is the concentration of nanoparticles, \(k_{np}\) and \(k_{bf}\) indicate the thermal conductivity of nanoparticles and base fluids, respectively, and \(n\) presents the shape factor, which can be found by the following relation;

\[
n = 3/\Psi,
\]

where \(\Psi\) shows the sphericity. The sphericity of the particle changes in the interval [0.5; 1.0] for cylindrical and spherical shapes.

Moreover, to investigate the thermal conductivity of nanofluids, one more empirical model was introduced by Wasp [7] that illustrates the ratio of thermal conductivity and volume concentration of nanoparticles:

\[
k_{nf} = k_{bf} \frac{k_{np} + 2k_{bf} - 2\varphi(k_{bf} - k_{np})}{k_{np} + 2k_{bf} + \varphi(k_{bf} - k_{np})}.
\]

Nevertheless, the presented models are not appropriate for investigating the thermal conductivity of nanofluids based on CNTs (carbon nanotubes). To explore the thermal conductivity of CNTs, Xue [8]...
introduced an empirical model, which is presented as follows:

\[
k_{nf} = k_{bf} \left( 1 - \varphi + 2\varphi \frac{k_{np}}{k_{np} - k_{bf}} \ln \left( \frac{k_{np} + k_{bf}}{2k_{bf}} \right) \right). \tag{5}\]

Furthermore, to obtain the effective thermal conductivity of the nanofluids, including the effect of the interfacial nanolayer, a new empirical model was introduced by Sokhansefat et al. [9], which introduces the relation between the only thermal conductivity of nanofluids and the volume concentration of nanoparticles:

\[
k_{nf} = k_{bf} \frac{k_{np} + 2k_{bf} - 2\varphi(k_{bf} - k_{np})(1 + \gamma)^3}{k_{np} + 2k_{bf} + \varphi(k_{bf} - k_{np})(1 + \gamma)^3}, \tag{6}\]

where, \( \gamma \) illustrates the ratio of nanolayer thickness to nanoparticle diameter, and according to Liang et al. [22], it is considered to be 0.1.

**IMPLEMENTATION OF MODELS**

In all analysed models, the effective thermal conductivity of nanofluids is given as a function of concentration.

However, it is still debatable how the thermal conductivity of nanofluids is theoretically related to the temperature because the thermal conductivity of nanoparticles can also change with increasing temperature as the thermal conductivity of base fluids changes. In reference [23], Mehrpooya gave the correlation in Eq. (7) as a function of temperature for the effective thermal conductivity of nanoparticles based on \( \text{Al}_2\text{O}_3 \) nanoparticles from an experimental study.

\[
k_{p}(T) = 5.5 + 34.5e^{\exp(-0.0033(T - 273))}. \tag{7}\]

According to the correlation, the effective thermal conductivity of nanofluids based on \( \text{Al}_2\text{O}_3 \) nanoparticles can be predicted as the expanding temperature.

Moreover, Xuan et al. [24] proposed a new correlation for the effective thermal conductivity of nanofluids as a function of temperature considering the Brownian motion of nanoparticles as follows:

\[
k_{nf} = k_{bf} \left( k_{np} + 2k_{bf} - 2\varphi(k_{bf} - k_{np}) \right) \left( k_{np} + 2k_{bf} + \varphi(k_{bf} - k_{np}) \right) \left( 1 + \gamma \right)^3 + \frac{\rho_{np}C_{p}}{2} \frac{KT}{\sqrt{3\pi\mu_{bf}r_{c}}}, \tag{8}\]

where, \( r_{c} \) illustrates the mean radius of gyration of the cluster, when cluster consists from one particle, then \( r_{c} = r_{np} \). However, in this correlation, the thermal conductivity of nanoparticles is considered constant at a specific temperature.

Furthermore, it is important to know how the thermal conductivity of base fluids changes with increasing temperature. To explore the thermal conductivity of the base fluid, mathematical analyses were carried out on data obtained from experiments. To predict the thermal conductivity of base fluids, we have carried out numerical analyses as a linear interpolation, which can illustrate the new correlation for finding the thermal conductivity of base fluids at given temperatures. The correlation is given as follows:

\[
k_{bf}(T) = k_{bf}(T_i) + \frac{k_{bf}(T_{i+1}) - k_{bf}(T_i)}{T_{i+1} - T_i}(T - T_i), \tag{9}\]

where \( k_{bf} \) - thermal conductivity of base fluid, \( T_i \) and \( T_{i+1} \) temperatures on \( i \) and \( i + 1 \) steps. For analysis the data, are given in Table 2.

Moreover, there is another way to predict the thermal conductivity of nanofluids by using given data. Regarding regression analyses, it can be possible to use a polynomial equation to find the thermal conductivity of base fluids. The new correlation is expressed as follows:

\[
k_{bf}(T) = A + BT + CT^2 + DT^3, \tag{10}\]

**Table 2.** Thermophysical properties of pure water [25]

| \( T, \text{K} \) | \( \rho, \text{kg/m}^3 \) | \( k_{u}, \text{w/m K} \) | \( c_{p}, \text{kJ/kg K} \) | \( \mu_{bf}, \text{Pa s} \) |
|---|---|---|---|---|
| 273 | 999.3 | 0.558 | 4226 | 0.001794 |
| 293 | 998.2 | 0.597 | 4182 | 0.000993 |
| 313 | 992.2 | 0.633 | 4175 | 0.000658 |
| 333 | 983.2 | 0.658 | 4181 | 0.000472 |
| 353 | 971.8 | 0.673 | 4194 | 0.000352 |
| 373 | 958.4 | 0.682 | 4211 | 0.000278 |
where $A$, $B$, $C$ and $D$ are coefficients of regression. If we use the data from Table 2, the values of coefficients could be approximately $A = -1.0825$; $B = 0.01064$; $C = -0.00002004$; and $D = 1.13296E-08$.

Regarding Eq. (7) and Eqs. (9–10), the abovementioned models can be illustrated as a function of temperature and concentration as follows:

$$k_{nf} = f(\varphi, T).$$

$$k_{nf} = \left[\frac{\left(\varphi T + 2(1 - \varphi) T^2 + 1.13E^{-08} T^3\right) + 2(1 - \varphi)}{\left(\varphi k_{np} + 2(1 - \varphi) k_{np} T + 1.13E^{-08} T^3\right) - 2(1 - \varphi)}\right]$$

**RESULTS AND DISCUSSION**

**Influence of Volume Concentration on Thermal Conductivity**

Predicting the thermal conductivity of nanofluids based on the various nanoparticles used in the abovementioned empirical models illustrates the relation between the thermal conductivity of nanofluids in the presence of SiO$_2$, TiO$_2$, CuO, and Al$_2$O$_3$ nanoparticles and the concentration of nanoparticles in the base fluid.

Figure 1 illustrates the comparison of various models for predicting the effective thermal conductivity of nanofluids in the presence of various nanoparticles. When using Maxwell’s model, the difference between the thermal conductivities of various nanofluids increased remarkably as the volume concentration of nanoparticles increased. Accordingly, the highest enhancement in the thermal conductivity of water was observed in the presence of Al$_2$O$_3$ nanoparticles. However, other nanofluids also illustrated valuable results, while SiO$_2$/water nanofluids implemented the lowest remarks with nonlinear curves. According to the H&C model, the thermal conductivities of nanofluids were much different from each other. As shown in the figure, the Al$_2$O$_3$/water nanofluid was still dominant, with an indicator of approximately 0.7 W/m K at high concentrations, while CuO/water and TiO$_2$/water nanofluids illustrated similar results, at 0.68 W/m K and 0.66 W/m K, respectively. Following the results of Wasp’s model, the effective thermal conductivities of
nanofluids based on CuO, TiO₂, and Al₂O₃ were almost the same, where SiO₂/water showed the lowest value, at 0.58 W/m K at high concentrations. Considering the influence of the interfacial layer, the effective thermal conductivities of nanofluids were varied in the interval of [0.64; 0.68] (W/m K), which were not much different from other models. In summary, the most preferable model can be Maxwell’s model, where Sokhansefat’s model clarifies the effective thermal conductivity of nanofluids considering the interaction of the interfacial layer between nanoparticles and base fluids.

Figure 2 expresses the comparison of empirical models with the comparison of thermal conductivity enhancements of water in the presence of SiO₂, TiO₂, CuO and Al₂O₃ nanoparticles. According to the figures, the highest enhancement in the thermal conductivity of water containing CuO nanoparticles can be seen in the H&C model, with a nearly 24% improvement at a 5% concentration of nanoparticles. In the presence of TiO₂ nanoparticles, the thermal conductivity enhancement of water is approximately 20% in almost all models, except Wasp’s model. However, the marks are much different in the presence of Al₂O₃, with a 25% enhancement in the H&C model and a nearly 22% enhancement in the Sokhansefat and Maxwell models, which is approximately 7% more than Wasp’s model. Moreover, the thermal conductivity enhancements of water in the presence of various nanoparticles can be close to each other at low concentrations, even when using different models. However, the enhancement of water with SiO₂ nanoparticles is nearly the same for all models, with a 5% improvement at high concentrations. To conclude, Maxwell’s and Sokhansefat’s models, when including the nanolayer effect, are preferred for predicting the enhancement of the thermal conductivity of water with the existence of different nanoparticles. Furthermore, the effective thermal conductivity of nanofluids or thermal conductivity enhancement of water in the presence of various nanoparticles can be remarkably different from each other with the expansion of the nanoparticle volume concentration in all models.

**Influence of Temperature on Thermal Conductivity**

Figure 3 shows the ratio of thermal conductivity of nanofluids based on Al₂O₃ nanoparticles (2% vol concentration) with respect to temperature considering
the thermal conductivity of nanoparticles as a function of temperature and as a constant.

As shown in Fig. 3 there is a temperature impact on the effective thermal conductivity of nanofluids based on Al₂O₃ nanoparticles when comparing the empirical models as a function of temperature and as function of concentration of nanoparticles. According to the figure, there is no difference between the primary model, which is a function of the concentration of nanoparticles, and the modified model, as a function of temperature and concentration, including Eq. (7). When Sokhansefat’s model was compared with the empirical Xuan model, which is a function of the mean radius of gyration of the cluster, the specific heat capacity, the particle density, the viscosity of the base fluid and the temperature, the difference was nearly 0.03 (W/m K). However, the difference between the H&C and Xuan models was approximately 0.05 (W/m K), where Wasp’s model illustrated the same result as Xuan’s model. Maxwell’s model results are almost the same as the results from Sokhansefat’s model. It can be concluded that the effective thermal conductivity of nanofluids based on Al₂O₃ nanoparticles, where the thermal conductivity of nanoparticles is a function of temperature, did not differ from the empirical models, where the thermal conductivity of nanoparticles is constant.

Figure 4 illustrates the comparison of models for obtaining the effective thermal conductivity of nanofluids based on various oxide nanoparticles and MWCNTs with respect to concentration. The highest thermal conductivity was observed in nanofluids based on MWCNTs. Next, the H&C model for nanofluids based on Al₂O₃ nanoparticles showed a good increase with increasing nanoparticle concentration, where Sokhansefat’s model suggested the lowest marks. The difference between the enhancement of thermal conductivity of water in the presence of MWCNTs and other oxide materials was much greater, at approximately 40%. In summary, the thermal conductivity of nanofluids based on MWCNTs is much greater than that of other materials due to the good thermal conductivity and cylindrical shape of the particles. Moreover, we find Maxwell’s model to be basic and simple, where the shape of the particles is spherical and the materials are oxides.

**Influence of Shape Factor**

As given in the H&C model, the thermal conductivity may not only change according to the tempera-
ture and volume fraction but may also shift in different nanoparticles shapes, such as spherical and cylindrical. As illustrated in Eq. (2), thermal conductivity is related to the shape factor Eq. (3) and is related to the sphericity of particles. Accordingly, Eq. (2) can be rewritten with the following equations given in Table 3.

It should be considered that the shape factor also influences the thermal conductivity as temperature and volume fraction. According to the results given in Fig. 5, for all pursued materials, the best enhancement was seen in the nanofluids based on platelet-shaped nanoparticles.

As shown in the figure, the highest enhancement of thermal conductivity of water in the presence of CuO nanoparticles was seen with the shape of platelets, while spherical nanoparticle-based water illustrated the lowest improvement, at 25 and 10%, respectively. In terms of TiO$_2$, the results were close to the results of CuO, with 20% enhancement for platelets and approximately 10% for spheres. However, the enhancement of thermal conductivity of water with the existence of Al$_2$O$_3$ nanoparticles was remarkable for platelet-shaped particles (30%) compared with other shapes and other materials. However, the marks

| Table 3. Various equations for different shapes |
|-----------------------------------------------|
| Equations | Shape factors [26] | Shape of particles [26] |
| $k_{nf} = k_{bf} \frac{k_{np} + 3.8k_{bf} - \varphi \times 3.8(k_{bf} - k_{np})}{k_{np} + 3.8k_{bf} + \varphi(k_{bf} - k_{np})}$ | $n = 4.8$ | Cylindrical |
| $k_{nf} = k_{bf} \frac{k_{np} + 2k_{bf} - \varphi \times 2(k_{bf} - k_{np})}{k_{np} + 2k_{bf} + \varphi(k_{bf} - k_{np})}$ | $n = 3$ | Spherical |
| $k_{nf} = k_{bf} \frac{k_{np} + 2.7k_{bf} - \varphi \times 2.7(k_{bf} - k_{np})}{k_{np} + 2.7k_{bf} + \varphi(k_{bf} - k_{np})}$ | $n = 3.7$ | Brick |
| $k_{nf} = k_{bf} \frac{k_{np} + 4.7k_{bf} - \varphi \times 4.7(k_{bf} - k_{np})}{k_{np} + 4.7k_{bf} + \varphi(k_{bf} - k_{np})}$ | $n = 5.7$ | Platelet |
were almost similar in the presence of SiO2 nanoparticles, with a nearly 5% improvement. To conclude, the enhancement of thermal conductivity of water in the presence of various oxide materials can be shifted within the expanding volume concentration of particles and increased with the changing shapes of nanoparticles.

CONCLUSIONS

In the current analysis, different empirical correlations and models for predicting the effective thermal conductivity of nanofluids are compared. The thermal conductivities of four types of oxide materials (SiO2, TiO2, CuO and Al2O3) and MWCNTs with volume fractions from 0.5 to 5.0%, temperatures of 273–373 K and various nanoparticle shapes are analysed.

The thermal conductivity of nanofluids based on the pursued materials increased with the increase of volume fraction in all correlations differently. Moreover, Maxwell’s model is basic for other suggested correlations. According to the models, the enhancement of the effective thermal conductivity of water in the presence of various nanoparticles varied from 10 to 25% with increasing nanoparticle volume concentration. When considering the interfacial layer impact on the effective thermal conductivity of nanofluids, the results can be similar for all models in the presence of nanoparticles with low thermal conductivity.

The above suggested models for predicting the effective thermal conductivity of nanofluids can be signified as a function of volume concentration. However, as given in Eqs. (7), (9), and (10), models can be modified as a function of temperature and volume concentration. Figure 3 shows the comparisons of primary, modified and Xuan’s models for all materials. As a result, it can be proposed that predicting the effective thermal conductivity of nanofluids can give the same results in two cases, which in the thermal conductivity of nanoparticles depend on temperature or a constant. It can be proven with the results in Fig. 3 that the effective thermal conductivity of nanofluids based on Al2O3 nanoparticles was the same in the primary and modified models.

Furthermore, it should be noted that the shape of particles does not remarkably influence the effective thermal conductivity of nanofluids at low concentrations and with nanoparticles such as SiO2 (1.2 W/m K, at 298 K). In addition, in the presence of platelet-shaped nanoparticles, nanofluids can illustrate the highest enhancement. Despite having the highest
enhancement in thermal conductivity, platelet-shaped nanoparticles can cluster or agglomerate, which can reduce the thermal conductivity of nanofluids. Due to the mentioned effects, cylindrical shaped nanofluids are preferred in terms of high thermal conductivity; for example, MWCNT-based nanofluids are expressed in Fig. 4.

In conclusion, Maxwell’s model is basic for all models; in other models, the impact of the interfacial layer or Brownian motion is considered. Moreover, taking into account Eqs. (7), (9), and (10), Maxwell’s model can be modified as a new correlation for predicting the effective thermal conductivity of nanofluids based on various nanoparticles.

The types of nanofluids analysed above may be preferable and effective for application as a heat transfer fluid in solar thermal systems, particularly for solar collectors as a heat transfer fluid.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

REFERENCES

1. Statistical Review of World Energy, 2021, 70th ed., pp. 1–72.
2. Eastman, J.A., Choi, U.S., Li, S., Soyez, G., Thompson, L.J., and DiMelfi, R.J., Novel thermal properties of nanostructured materials, Mater. Sci. Forum, 1999, vol. 312, pp. 629–634.
3. Choi, S.U.S., Enhancing thermal conductivity of fluids with nanoparticles, Developments and Applications of Non-Newtonian Flows, New York: American Society of Mechanical Engineers, 1995, vol. 66, pp. 99–105.
4. Akhatov, Zh.S., Mirzaev, S.Z., Wu., Zh., Telyaev, S.S., Zhuraev, E.T., and Zhuraev, T.I., Research on thermophysical properties of nanoliquids based on SiO2 nanoparticles for use as a heat-transfer medium in solar-thermal converters, Appl. Sol. Energy, 2018, vol. 54, no. 1, pp. 50–60.
5. Maxwell, J.C., A Treatise on Electricity and Magnetism, Oxford: Clarendon Press, 1881, vol. 1.
6. Hamilton, R. and Crosser, O., Thermal conductivity of heterogeneous two component systems, Ind. Eng. Chem. Fund., 1962, no. 1. pp. 187–191.
7. Wasp, E.J., Kenny, J.P., and Gandhi, R.L., Solid-Liquid Flow: Slurry Pipeline Transportation, Bulk Materials Handling, vol. 1, Switzerland: Trans Tech Publications, 1977.
8. Xue, Q., Model for the effective thermal conductivity of carbon nanotube composites, Nanotechnology, 2006, vol. 17, pp. 1655–1660.
9. Sokhansefat, T., Kasaeani, A., and Kowsary, F., Heat transfer enhancement in parabolic trough collector tube using Al2O3/synthetic oil nanofluid, Renewable Sustainable Energy Rev., 2014, vol. 33, pp. 636–644.
10. Yu, W. and Choi, S., The role of interfacial layers in the enhanced thermal conductivity of nanofluids: a renovated Maxwell model, J. Nanopart. Res., 2003, no. 5, pp. 167–171.
11. Kumar, D.H., Patel, H.E., Kumar, V.R.R., Sundararajan, T., Pradeep, T., and Das, S.K., Model for heat conduction in nanofluids, Phys. Rev. Lett., 2004, vol. 93, pp. 144301-1–144301-4.
12. Koo, J. and Kleinstreuer, C., Erratum: a new thermal conductivity model for nanofluids, J. Nanopart. Res., 2004, no. 6, pp. 577–588.
13. Jang, S.P. and Choi, S.U.S., Effects of various parameters on nanofluid thermal conductivity, J. Heat Transfer, 2007, vol. 129, pp. 617–623.
14. Jang, S.P. and Choi, S.U.S., Role of Brownian motion in the enhanced thermal conductivity of nanofluids, Appl. Phys. Lett., 2004, vol. 84, pp. 4316–4318.

NOMENCLATURES

CuO copper oxide
TiO2 titanium dioxide
Al2O3 aluminium oxide
SiO2 silicon dioxide
MWCNT multiwalled carbon nanotube
T temperature, K
K Boltzmann constant, \(1.381 \times 10^{-23} \text{J/K}\)
n shape factor
Ψ sphericity
γ ratio of nanolayer thickness to nanoparticle diameter
β volumetric coefficient of thermal expansion, 1/K

Greek symbols

ρ density, kg/m³
φ nanoparticle volume fraction, %

Subscripts

cp specific heat, \(\text{kJ/kg K}\)
knf thermal conductivity of nanofluids, \(\text{W/m K}\)
kp particle thermal conductivity, \(\text{W/m K}\)
kbf base fluid thermal conductivity, \(\text{W/m K}\)
r c mean radius of cluster gyration, nm
r np particle radius, nm
15. Jang, S.P., Lee, J.H., Hwang, K.S., and Choi, S.U.S., Particle concentration and tube size dependence of viscosities of Al$_2$O$_3$-water nanofluids flowing through micro and mini tubes, *Appl. Phys. Lett.*, 2007, vol. 91, p. 243112.
16. Omer A. Alawi, Nor Azwadi Che Sidik, Hong Wei Xian, Tung Hao Kean, and S.N. Kazi, Thermal conductivity and viscosity models of metallic oxides nanofluids, *Int. J. Heat Mass Transfer*, 2018, vol. 116, pp. 1314–1325.
17. Akhatov, J.S., Juraev, E.T., Juraev T.I., and Avdievich, V.N., Study of sedimentation process in nanofluids with various concentrations of SiO$_2$ and Al$_2$O$_3$ nanoparticles, *Appl. Sol. Energy*, 2018, vol. 54, no. 6, pp. 428–432.
18. Vajjha, R.S. and Das, D.K., Experimental determination of thermal conductivity of three nanofluids and development of new correlations, *Int. J. Heat Mass Transfer*, 2009, vol. 52, pp. 4675–4682.
19. Vajjha, R.S., Das, D.K., and Kulkarni, D.P., Development of new correlations for convective heat transfer and friction factor in turbulent regime for nanofluids, *Int. J. Heat Mass Transfer*, 2010, vol. 53, pp. 4607–4618.
20. Aziz, A., Jamshed, W., and Aziz, T., Mathematical model for thermal and entropy analysis of thermal solar collectors by using Maxwell nanofluids with slip conditions, thermal radiation and variable thermal conductivity, *Open Phys.*, 2018, no. 18, pp. 123–136.
21. El-Mahallawi, I., Shash, A.Y., and Eid Amer, A., Nanoreinforced cast Al–Si alloys with Al$_2$O$_3$, TiO$_2$ and ZrO$_2$ nanoparticles, *Metals*, 2015, no. 5, pp. 802–821.
22. Liang, H., You, S., and Zhang, H., Comparison of three optical models and analysis of geometric parameters for parabolic trough solar collectors, *Energy*, 2016, vol. 96, pp. 37–47.
23. Marefat, M., Mehrpooya, M., and Shafii, M.B., Optical and thermal analysis of a parabolic trough solar collector for production of thermal energy in different climates in Iran with comparison between the conventional nanofluids, *J. Cleaner Prod.*, 2018, vol. 175, pp. 294–313.
24. Xuan, Y., Li, Q., and Hu, W., Aggregation structure and thermal conductivity of nanofluids, *Am. Inst. Chem. Eng. J.*, 2003, vol. 49, no. 4, pp. 1038–1043.
25. Kreith, F. and Black, W., *Basic Heat Transfer*, New York: Harper and Row, 1980.
26. Sheikholeslami, M., Shahe, A., Ramzan, M., and Li, Z., Investigation of Lorentz forces and radiation impacts on nanofluid treatment in a porous semi annulus via Darcy law, *J. Mol. Liq.*, 2018, vol. 272, pp. 8–14.