To Phenomenological Theory of Superconductivity. 
Superconductivity - not fading electrical current in dissipative medium

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Abstract

The basic stages of development of the theory of superconductivity are traced. Despite of remarkable successes of theory, the physical explanation of the phenomenon of superconductivity - of the not fading electrical current in dissipative medium - is not exists until now.

In the present paper on the basis of works (Klimontovich, 1990abc, 1999) the physical explanations of this phenomenon is considered. Will be show that the existence of not fading electrical current becomes possible due to occurrence of flicker noise and appropriate residual temporary correlations.

I. INTRODUCTION

In 1911 Dutch physicists H.Kamerlingh-Onnes has found out at temperature 4.12 the superconductivity of mercury - the electrical resistance of mercury suddenly disappeared and remained equal the zero at the further downturn of temperature. A bit later superconductivity was found out in others metals and also in some alloys.

Long years was supposed, that the superconductivity is the phenomenon of low temperature physics. The high-temperature superconductivity was found out for the first time only in 1986 by Bednorz and Muller.

In 1933 by Meissner and Ochxinfeld have found out that the weak magnetic field is pushed out from a massive superconductor - "Meissner effect". The value of a superconducting current in a sample, which is a part of a consecutive circuit, by a source of a current in the circuit is determined. The superconductivity disappears, when the value of a current reaches some critical value.

The first phenomenological theory of superconductivity was proposed in brothers London work in 1935 (see. London, 1954). They have admitted the existence of the superconductivity. On this basis they have shown, that the Meissner effect is inevitable consequence of the superconductivity phenomenon. A question on validity of the opposite statement remained open.

In 1950 on the basis of London equations the phenomenon of quantization of magnetic flow was predicted. It was revealed almost simultaneously by two groups of experimenters in 1961. They showed that the quant of magnetic flow is defined
by the particles with the double of electron charge. This result has confirmed the
Cooper effect. The last served as the basis of the Bardeen - Cooper - Schriffer (BCS)
microscopic theory of superconductivity (1957).

In 1950 the Ginsburg - Landau (GL) the stationary equation for some effective
wave function have offered. The coefficients of GL equation by measured values of
the critical magnetic field and the by the London penetration depth are defined.

The appreciable contribution to the theory of superconductivity was made By
Bogolubov (1958). In the Gor’kov paper (1959) the connection of GL and BCS
theories was established.

After opening of high-temperature superconductivity new mechanisms of this
phenomenon were discussed. However the enough convincing microscopic theory of
the high-temperature theory in the present time yet does not exists (see Abrikosov,
1987; Plakida, 1996). In such situation the phenomenological description of super-
conductivity plays the important role.

The of London and GL equations are not dissipative and correspond to approx-
imation of continuous medium. As all equations in approximation of continuous
medium are generally dissipative, that the following basic question arises: How in
the dissipative system exists not fading electrical current?

The possible answer on this question was discussed earlier (Klimontovich,
1990abc, 1995). It was shown that the existence of not fading electric current be-
comes possible due to occurrence of flicker noise and appropriate residual temporary
correlations (Kogan, 1985; Klimontovich, 1982, 1983). In present paper this works
serve by the basis of more general description of the superconductivity.

The possibility of connection between the two seemingly antagonistic phenom-
ena: flicker noise and superconductivity has the following explanation.

First, both, flicker noise and superconductivity are spatial coherent phenomena.
In the domain of flicker noise the distribution on wave numbers has a very sharp
maximum near to its zero value. Moreover, the dispersion is proportional to fre-
quency, and therefore tends to zero together with $\omega$. Thus has a place original Bose
condensation

II. THE GL AND LONDON EQUATIONS

Let’s designate number of Cooper pairs through $n^\ast(T)$ and shall enter the effec-
tive wave function of pairs of supercondutive electrons $\psi(R, t)$.

The Debye radius is much smaller of characteristic scale of Cooper pairs, together
with size of a point of continuous medium. On this reason the Coulomb interaction
not play an essential role in processes of superconductivity. The square of the module
of function $\psi(R, t)$ defines average density of number of electron pairs

$$|\psi(R, t)|^2 \equiv n^\ast(T) = \frac{n_S(T)}{2}. \quad (1)$$

In the GL work the stationary equation for the effective wave function is considered
only. The one from opportunities of the temporary description - the use for of
effective wave function, appropriate, the nonlinear Schrödinger equation (Hartree equation) for particles with the double electron charge \( e^* = 2e \). Just so acts, for example, Feinman, (1967).

\[
Ih \frac{\partial \psi(R,t)}{\partial t} = \frac{1}{2m^*} \left| \left( -ih \frac{\partial}{\partial R} - \frac{e^*}{m^*} A \right) \right|^2 \psi(R,t) + \varphi(R,t)\psi, \quad \text{div} A = 0. \tag{2}
\]

The GL potential \( \varphi_{GL}(R,t) \) will be defined below.

Let’s wrote the appropriate equation of a continuity

\[
\frac{\partial |\psi(R,t)|^2}{\partial t} + \frac{\partial j_S(R,t)}{\partial R} = 0. \tag{3}
\]

The current created by electron pairs is defined by expression:

\[
j_S(R,t) = -\frac{ie^*}{2m^*} (\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{e^*}{2m^*c} |\psi|^2 A(R). \tag{4}
\]

Let’s present complex function \( \psi(R,t) \) as

\[
\psi(R,t) = |\psi(R,t)| \exp (i\theta(R,t)) = a(R,t) \exp (i\theta(R,t)). \tag{5}
\]

Then the electric current created by the superconductive pairs is defined by expression:

\[
j_S(R,t) \equiv e^* |\psi(R,t)|^2 u_S(R,t) = \frac{e^*}{m^*} \left( \frac{\partial \theta}{\partial R} - \frac{e^*}{hc} A \right) |\psi(R,t)|^2. \tag{6}
\]

From here the expression for the mean velocity of superconductive electron follow:

\[
u_S(R,t) = \frac{\hbar}{m^*} \frac{\partial \theta}{\partial R} - \frac{e^*}{m^*c} A, \quad \text{div} A = 0. \tag{7}
\]

It consists from the sum of the potential and the vortical components. The current \( j_S(R,t) \) in a normal state is equal to zero.

In massive metals the electron density \( |\psi(R,t)|^2 = \text{const.} \) and the Maxwell equations look like

\[
\text{rot} B = \frac{4\pi}{c} j_S, \quad E = 0. \quad B = \text{rot} A, \quad \text{div} j_S = 0. \tag{8}
\]

Under condition of a constancy of a phase \( \theta(R) = \text{const.} \), that has a place for continuous (without holes) superconductor, the potential component of the velocity \( u_S(R,t) \) is equal to zero and the current is connected to the vector potential by the equation of London

\[
j_S(R) = - |\psi(R,t)|^2 \frac{e^2}{m^*c} A \equiv - |\psi(R,t)|^2 \frac{2e^2}{mc} A. \tag{9}
\]

From it in a combination to the Maxwell equations the closed equation for the magnetic field follows:
\[
\frac{\partial^2 B}{\partial R^2} + \frac{1}{\delta_L^2} B = 0. \tag{10}
\]

Designation for the depths of penetration - the London parameter \(\delta_L\) here is used

\[
\delta_L^2 = \frac{m^* e^2}{4\pi e^2 |\psi(T)|^2} \equiv \frac{mc^2}{4\pi e^2 n_S}, \tag{11}
\]

Thus the theory of London, in which the fact of the existence, of not fading electrical current is accepted, gives the explanation of Meissner effect. The expression, received by such way, for penetration depth \(\delta_L\), corresponds to experimental data. Typical size \(\delta_L\) of the order \(10^{-5} \text{ cm}\).

So, the theory of London is based on the assumption of the existence of a not fading electrical current, and gives the explanation of Meissner effect. The equations describing the screening of magnetic field and the current are classical ones - does not contain the Planck constant.

At an establishment of the equation of London the equation of continuity was used only. Let’s show, that the equation of London satisfies to complete system of the reversible hydrodynamical equations, which turns out on the basis of the Schroedinger equation.

Being based on the Schroedinger equation for the effective wave function it is possible to receive and equation for average velocity of superconducting electrons:

\[
\frac{\partial u_S}{\partial t} + \left( u_S \frac{\partial}{\partial R} \right) u_S = \frac{e^*}{m^*} \left( E + \frac{1}{c} [u_S B] \right) + \frac{1}{m_S} \frac{\partial U_{\text{quant}}}{\partial R}. \tag{12}
\]

Designation is entered here

\[
U_{\text{quant}} = \frac{\hbar^2}{2\sqrt{\rho_S}} \frac{\partial^2 \sqrt{\rho_S}}{\partial R^2}, \tag{13}
\]

for "the quantum potential energy", through which enters a quantum source”.

In the theory of London a magnetic field and, as a consequence, and hydrodynamical velocity are small. In linear approximation the equation of motion has form:

\[
\frac{\partial u_S}{\partial t} = -\frac{e^*}{m^* n} \frac{\partial A}{\partial t}. \tag{14}
\]

This equation is consequence of the Schroedinger equation, therefore does not contain a dissipation. In view of a constancy of the density of electron pairs can it be rewrite as the London equation

\[
u_S(R) = -\frac{e^*}{m^* c} A(R). \tag{15}\]

Thus the reversible equation (14) is carried out just by virtue of the London equation.

The London theory does not explain the phenomenon of existence of superconductive current, but essentially bases on fact of its existence. The condition is used also: \(|\psi(R, t)|^2 = \text{const}\) is used also. Thus the question on dependence of number of superconducting electrons on temperature there is without the answer.
III. COMPARISON WITH THE GL THEORY

The following essential step in development of the phenomenological theory of superconductivity was made in work GL. However, the nature of phase transition in superconductors was not concretized. It was found out only at creation if the BCS theory.

The GL theory not give also the answer on the basic question: Why in superconductor, which, as any continuous medium is a dissipative system, is possible not fading electrical current?

The stationary GL equation has a form:

\[- \frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial R^2} + \left( \alpha \frac{T - T_C}{T_C} + b |\psi|^2 \right) \psi = 0. \]  

(16)

It presents the "synthesis" of the quantum mechanics and the Landau theory of a second order phase transition. The BCS theory allows to connect coefficients \( \alpha, b \) of the GL theory with parameters of a normal state of superconductor.

There are two opportunities of temporary generalization of this equation. Above was is used the quantumechanic generalization. It has resulted to the nonlinear Schroedinger equation (2). It corresponds to the Hartree equation with the GL potential

\[ \varphi_{G-L} = \alpha \frac{T - T_C}{T_C} + b |\psi|^2. \]  

(17)

Above at the description of temporary evolution the preference to quantum was given up. In result came to the nonlinear Schroedinger equation (the Hartree equation):

\[ i\hbar \frac{\partial \psi}{\partial t} = - \frac{1}{2m^*} \left( - \hbar \frac{\partial}{\partial R} - \frac{2e}{c} A \right)^2 \psi + \varphi_{G-L} \psi. \]  

(18)

The members responsible for phase transition, are expressed through appropriate potential - potential "GL":

\[ \varphi_{GL} = \left( \alpha \frac{T - T_C}{T_C} + b |\psi|^2 \right). \]  

(19)

The presence of additional potential \( \varphi_{GL} \) does not change a form of the continuity equation. The second hydrodynamical equation accepts the form:

\[ \frac{\partial u_S}{\partial t} + \left( u_S \frac{\partial}{\partial R} \right) u_S = \frac{e^*}{m^*} \left( E + \frac{1}{c} [u_S B] \right) - \frac{1}{m^*} \frac{\partial (U_{quant} + \varphi_{G-L})}{\partial R}. \]  

(20)

Through the additional force the dependence of number of superconductive electron pairs from temperatures enters. This dependence in the London theory is not taken
into account. Under former conditions (constancy of a phase and density of superconducting electrons) the London equation satisfies in linear approximation to the last equation.

The Hartree equation with potential GL is reversible and, gives only dynamic description of a superconductor. As, however, the number the superconducting electrons pairs depends on temperature, that more adequate is not "dynamic", but "chemical" way of temporary generalization of the stationary GL equation. This question repeatedly was discussed in the literature (see, for example, Elesin and Kopaev, 1981; Oraevskii, 1993). Let’s consider the appropriate generalization of the G-L equation for the description of temporary processes in superconductors.

IV. RELAXATION GL AND KINETIC EQUATIONS

Let’s give back now the preference to dissipative processes, which take place at phase transitions. Thus instead of the GL equation we come to the relaxation GL equation (RGLE).

For system of superconducting pairs the characteristic length - the coherence length at zero temperature and the coefficient of spatial diffusion are defined by expressions:

$$\xi_0^2 = \frac{\hbar^2}{2m^*\alpha} \approx \frac{p_F^2}{m_T}\lambda_B^2; \quad D = \frac{\hbar}{2m^*}. \quad (21)$$

Expressions for time diffusion time and, appropriate, from here follow of the friction coefficient from here follow

$$\tau_D = \frac{1}{\gamma} = \frac{\xi_0^2}{D} = \frac{\hbar}{\alpha}; \quad \gamma = \frac{\alpha}{\hbar}. \quad (22)$$

The RGLE at ($A = 0$) has form:

$$\frac{\partial \psi(R,t)}{\partial t} = -\frac{1}{2\gamma} \left( \frac{T - T_C}{T_C} + \frac{\left| \psi \right|^2(R,t)}{n} \right) \psi + D \frac{\partial^2 \psi(R,t)}{\partial R^2}. \quad (23)$$

It serves an example of the reaction diffusion equation. In it interactions with normal electrons and phonons, dissociation and formation of Cooper pairs it are taken into account only through coefficients.

The RGLE follows from the Schroedinger equation by formal replacement of the time $t$ by imaginary time $it$. Let’s consider other way of the description of relaxation processes, which is, indubitably, more consecutive.

Let’s begin from a case, when the distribution of values of phase is not essential. The appropriate Fokker-Planck equation for the local distribution function $f(n^*, R, t)$ of values of $|\psi|^2 = n^*$ has the following form:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial n^*/n} \left[ D_{n^*} \frac{n^*}{n} \frac{\partial f}{\partial n^*/n} \right] + \frac{\partial}{\partial n^*/n} \left[ \gamma \left( \frac{T - T_C}{T_C} + \frac{n^*}{n} \right) \frac{n^*}{n} f \right] + D \frac{\partial^2 f}{\partial R^2}. \quad (24)$$
The diffusion coefficient $D_{n^*}$ is defined by the expression:

$$D_{n^*} = \frac{1}{N_{ph}} \frac{k_B T}{\hbar}.$$  \hspace{1cm} (25)

This results to appropriate the smoothed Boltzmann distribution.

In the self-consistent approximation on the first moment (when $f(n^*, R, t) = \delta(n^* - n^*(R, t))$) we have the following equation:

$$\frac{\partial n^*(R, t)/n}{\partial t} = \left[ D_{n^*} - \gamma \left( \frac{T - T_C}{T_C} + \frac{n^*(R, t)}{n} \right) \frac{n^*(R, t)}{n} \right] + D \frac{\partial^2 n^*(R, t)/n}{\partial R^2}. \hspace{1cm} (26)$$

In it two important factors are not yet taken into account: evolution of a phase and action electrical and magnetic fields. Without its the explanation neither superconductivity, nor Meissner effect it is impossible. The corresponding more general kinetic equation will be carried out below.

In the stationary and spatial - homogeneous state is received the algebraic equation for function $n^*(T)$:

$$\left( \frac{n^*(R, t)}{n} \right)^2 + \frac{T - T_C}{T_C} \frac{n^*(R, t)}{n} = \frac{D_{n^*}}{\gamma} = \frac{1}{N_{ph}} \frac{k_B T}{\hbar \gamma} = \frac{1}{N_{ph}} \frac{k_B T}{\alpha}. \hspace{1cm} (27)$$

This equation defines a finite solution at all values of temperature. For temperatures is significant larger critical one, the solution has following form:

$$n^*(R, t) = \frac{1}{N_{ph}} \frac{k_B T}{\alpha} \frac{T_C}{T - T_C} n; \hspace{1cm} (28)$$

In the critical point

$$n^*(R, t) = \left[ \frac{1}{N_{ph}} \frac{k_B T}{\alpha} n \right]$$

and, at last, for temperatures, below critical, the density of numbers of superconducting electrons is defined by expression:

$$n^*(R, t) = \frac{T_C}{T_C} n,$$  \hspace{1cm} (29)

which coincides with result of the Landau theory.

V. WHETHER EXISTS NOW THEORY OF SUPERCONDUCTIVITY?

Two variants of temporary generalization of the stationary GL were considered. In the first case temporary evolution is described on the basis of the reversible Hartree equation with potential, which is determined by distribution of density of superconducting electrons. As well as in the London theory, still have open a
question on a nature of superconductive electric current in a dissipative medium. In the second case were used the appropriate relaxation equation.

In both cases there is open the question on a physical nature not fading electrical current. Thus despite of doubtless successes, rather complete theory the superconductivity now does not exist yet.

In such situation the aspiration is natural to construct the evolutionary equations for the description of superconductivity with the simultaneous account as dynamic, so and the dissipative contributions.

Will be shown, that the existence of the superconducting current becomes possible due to occurrence of flicker noise and, appropriate, residual temporary correlations.

VI. DYNAMIC REACTION DIFFUSION EQUATION IN THE THEORY SUPERCONDUCTIVITY

Was shown that hydrodynamical equations at rather small velocity does not contain the Planck constant. Let’s remind also, that classical expression for the London length invariant concerning replacement:

\[ e^*, m^*, n^* \rightarrow e, m, n \]  

and in the theory of London there is no dependence on a kind of statistics: Fermi or Bose. The role of the electron pairs is reduced to the following.

The first, due to them in the nonideal fermi-gas there is a phase transition - there is the equilibrium state with lower energy, than appropriate state of free electrons.

The secondly, the superconductive state represents continuous medium, in which the size of a point is defined by the quantum parameter - the de Broglie wave length \( \lambda_B = \hbar/p_{T_C} \). This length considerably exceeds the Debye radius. Thanks to this, the superconductor represents unusual continuous quantum medium. In it "physical Knudsen number" - the basic small parameter of continuous medium, is defined by expression:

\[ (K_{ph})_n = \frac{\lambda_B}{L} = \frac{\hbar}{p_{T_C}L} \]  

and, hence, is the quantum characteristic. For a superconductor role of parameter \( L \) play the size of the Cooper pair \( \xi_0 = \hbar/\sqrt{2m^*}\alpha \) and the London length \( \lambda_B \). For superconductors of the London type the greatest is dimensionless small parameter (Klimontovich, 1995):

\[ (K_{ph})_n = \frac{\lambda_B}{\xi_0} \approx \frac{p_{T_C}}{p_F} \ll 1. \]  

Thus, the phase transition in the superconducting state, caused by existence of the Cooper pairs, it is possible to treat as transition to quantum continuous medium. By the basic dimensionless small parameter which serves the quantum physical Knudsen number.
Be taking into account that the dynamic in the London theory is classical, the dissipative kinetic equation for the local distribution function \( f(n^*, R, v, t) \) of values of density of electron pairs it is possible to write in form:

\[
\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial R} + \frac{e^*}{m^*} \left( E(R, t) + \frac{1}{c} [u_S B(R, t)] \right) \frac{\partial f}{\partial v} = I(v) + I(R) + I(n^*) ; \quad \text{div} E = 0
\] (34)

The norm of the distribution function \( f(n^*, R, v, t) \) will be carried out on the average of particles, which depends on temperature.

The sum of second and third "collision integrals" we shall present as the sum of two contributions. First describes spatial diffusion of distribution function. Second - diffusion in the space of values of number of electron pairs:

\[
I(R) + I(n^*) = D \frac{\partial^2 f}{\partial R^2} + \frac{\partial}{\partial n^*} \left[ D_{n^*} n \frac{\partial f}{\partial n^*} \right] + \frac{\partial}{\partial n^*} \left[ \gamma \left( \frac{T - T_C}{T_C} + \frac{n^*}{n} \right) n f \right].
\] (35)

The diffusion \( D_{n^*} \), \( D \) and friction \( \gamma \) coefficients are defined by the formulas:

\[
D_{n^*} = \frac{1}{N_{ph}} \frac{k_B T}{\hbar}; \quad D = \hbar/2m^*; \quad \gamma = \frac{\alpha}{\hbar}.
\] (36)

"The collision integral" \( I(v) \) defines redistribution of pairs of electrons on velocity. It has the same properties, as the Boltzmann collision integral in the kinetic theory of gases.

Let's proceed from the kinetic equation to the equations for local functions \( \langle n^* \rangle_{R,t}, u_S(R, t) \). In the self-consistent approximation on the first moments for the distribution function \( f(n^*, R, v, t) \) it is represented as:

\[
f(n^*, R, v, t) = \delta (v - u_S(R, t)) \delta \left( n^* - \langle n^* \rangle_{R,t} \right).
\] (37)

In result is received the equation for average local density number particles:

\[
\frac{\partial}{\partial t} \langle n^* \rangle_{R,t} + \frac{\partial}{\partial R} \langle n^* \rangle_{R,t} u_S(R, t) = D_{n^*} - \gamma \left( \frac{T - T_C}{T_C} + \frac{\langle n^* \rangle_{R,t}}{n} \right) \frac{\langle n^* \rangle_{R,t}}{n} + D \frac{\partial^2}{\partial R^2} \langle n^* \rangle_{R,t}.
\] (38)

It contains the members, taking into account birth and disappearance of electron pairs ("chemical reaction") and a self-diffusion contribution. To this reason the flow of electron pairs is defined not only the convective transfer \( \langle n^* \rangle_{R,t} u_S(R, t) \), but also by the spatial diffusion. The similar results take place at the description of nonequilibrium processes in gases and plasma (Klimontovich, 1995.1999).

In the London equations the electron pairs density \( \langle n^* \rangle_{R,t} = \text{const.} \). Now in the stationary state we have the equations:

\[
D_{n^*} - \gamma \left( \frac{T - T_C}{T_C} + \frac{\langle n^* \rangle_{R,t}}{n} \right) \frac{\langle n^* \rangle_{R,t}}{n} = 0; \quad \frac{\partial u_S(R, t)}{\partial R} = 0.
\] (39)

This equation defines \( \langle n^* \rangle \) at all values of temperature. From the second equation follows, that the superconducting current is vortical.
VII. DISSIPATIVE LONDON EQUATION

With the help of the kinetic equation we shall receive the equation for velocity:

\[
\frac{\partial u_S}{\partial t} + \left( u_S \frac{\partial}{\partial R} \right) u_S = \nu \frac{\partial^2 u_S}{\partial R^2} + \frac{e^*}{m^*} \left( E(R, t) + \frac{1}{c} [u_S B(R, t)] \right) ; \quad \text{div} \, E = 0 \tag{40}
\]

We assume, that coefficient of diffusion, of viscosity and of temperature conductivity are identical. This gives the basis for replacement \( D \rightarrow \nu \).

At weak currents and weak magnetic field it is possible to neglect the nonlinear terms and to write down the equation for hydrodynamic velocity of electron pairs the following equation:

\[
\frac{\partial u_S}{\partial t} = \nu \frac{\partial^2 u_S}{\partial R^2} + \frac{e^*}{m^*} E. \tag{41}
\]

In a combination with the Maxwell equations:

\[
\text{rot} \, B = \frac{4 \pi}{c} j_S, \quad \text{rot} \, E = -\frac{1}{c} \frac{\partial B}{\partial t}, \quad \text{div} \, E = 0 \tag{42}
\]

we have the closed system of the equations for density and hydrodynamical velocity of electron pairs, and also for electrical and magnetic fields.

Thus we have "the dissipative London equation" (41). How to explain presence of a superconducting current in a dissipative system?

In the following section the question on connection of an opportunity of existence of a superconductive electric current and the presence of the flicker noise - ”of noise \( 1/f \). will be discussed. The assumption, that the flicker noise defines an opportunity existence coherent state - superconductivity, on first sight seems as paradoxical. However we shall see, that the flicker-noise presents a coherent state similar, in some measure, to condensation of bose-gas. Now, however, ”condensation" in space of wave numbers occurs in a dissipative medium.

VIII. FLICKER NOISE AND SUPERCONDUCTIVITY

The physical phenomenon "flicker noise" (" \( 1/\omega \)" noise) consists in abnormal behavior of a spectrum of fluctuation of any physical characteristics in the region of low frequencies \( \omega \). (Kogan, 1985; Klimontovich, 1982, 1983). The flicker noise is characterized as well is by abnormal large of time correlations \( \tau_{cor} \).

In spite of on efforts of many scientists until now absent the unified point of view on the nature of the flicker noise. Our point of view on this problem is presented in (Klimontovich, 1982, 1983, 1995).

The flicker noise exists in the region of frequencies is restricted from the side of large frequencies by the diffusion time \( \tau_D = L^2 / D \). Here \( D \) - the coefficient of spatial diffusion, and \( L \)– the minimal characteristic scale of sample.

For the minimal frequency \( \omega_{min} \) there are possible two definitions - ”subjective ”, dependent from the observation time \( \tau_{obs} \), and ”objective ”, is defined by the
parameters of all system. Experience shows that the dependence $1/\omega$ conserves in
during of the increasing of the observation time. This gives the reason to suppose
that the minimal frequency of the flicker noise spectrum is defined by the time life
$\tau_{ife}$. Thus, the region of the flicker noise is defined by inequalities:

$$\frac{1}{\tau_{ife}} \leq \frac{1}{\tau_{obs}} \ll \omega \ll \frac{1}{\tau_D} \frac{D}{L^2}. \quad (43)$$

In the flicker noise domain appears the new scale and corresponding volume:

$$L_\omega = \sqrt{\frac{D}{\omega}} \gg L, \quad V_\omega = L_\omega^3 \gg V. \quad (44)$$

At these conditions the dimension of a sample $d$ is not play the role and in the limit
of very large observation times it can count equal to zero, that corresponds to zero
volume $V$. Below under $L$ we shall use the minimal from characteristic scales.

Thus, we have the chain of inequalities for volumes

$$V = V_D \ll V_\omega \ll V_{obs} \leq V_{ife}. \quad (45)$$

According to stated representations the equilibrium flicker noise arises at diffusion
in limited volume. In the unlimited sample appropriate spectral density at diffusion
is defined by known expression:

$$(\delta n\delta n)_{\omega,k} = \frac{(yy)_{\omega,k}}{\omega^2 + (Dk^2)^2}, \quad (yy)_{\omega,k} = 2Dk^2(\delta n\delta n)_k. \quad (46)$$

Designation for the spectral density of appropriate the Langevin source here is en-
tered. For ideal gas $(\delta N\delta n)_k = n$. In a general case it is defined through the
isothermal compressibility of a sample.

For domain of the flicker noise are carried out the following inequalities $\sqrt{D/\omega} \gg
L, V_\omega \gg V$. First of them is possible to write down as $1/\omega \gg \tau_D = L^2/D$. The spectral density of the appropriate the Langevin source is defined by expression
(Klimontovich, 1990abc, 1995)

$$(yy)_{\omega,k} = 2Dk^2AV_\omega \langle \delta n_V\delta n_V \rangle \exp\left(-\frac{Dk^2}{2\omega}\right) \quad n_{eff} = AV_\omega \langle \delta n_V\delta n_V \rangle. \quad (47)$$

Here $\langle \delta n_V\delta n_V \rangle$ - correlator of fluctuations, averaged on volume $V$. For ideal gas
$\langle \delta n_V\delta n_V \rangle = n/V$.

Appropriate expression for intensity of a Langevin source has a form:

$$(yy)_{\omega,k} = 2Dk^2An\frac{V_\omega}{V} \exp\left(-\frac{Dk^2}{2\omega}\right). \quad (48)$$

Thus, for the domain of flicker of noise has place the replacement

$$n \rightarrow n_{eff} = An\frac{V_\omega}{V}. \quad (49)$$
By this the repeated diffusion is taken into account - each particle covers the diffusion volume $V_\omega$, which is a lot of greater volume of a sample $V$.

Thus each particle ”works” much times. It also is resulted that the effective density of particles in $V_\omega/V$ times more of real one. This is taken into account by replacement (49). The constant multiplier $A$ will be determined from a normalization condition.

In intensity of a Langevin source there is a strong dependence from frequencies and stronger dependence on wave number. Thus the dispersion on wave numbers is proportional to frequency $\omega$:

$$\langle (\delta k)^2 \rangle \sim \frac{1}{L_\omega^2} = \frac{\omega}{D},$$

therefore in a domain of flicker noise arises a very sharp distribution on to wave numbers - original Bose-condensation. It speaks about that in the field of the flicker noise arises the spatial coherence.

Substitution of expression (49) in the first formula (46) results in expression for spatially temporary spectral density in the region of flicker noise:

$$(\delta n\delta n)_{\omega,k} = \frac{2Dk^2}{\omega^2 + (Dk^2)^2}AV_\omega \langle \delta n_V\delta n_V \rangle \exp \left(-\frac{Dk^2}{2\omega}\right).$$

Here it is possible to execute integration on $k$ and to receive expression for appropriate temporary spectral density:

$$\langle (\delta n\delta n) \rangle = \frac{\pi}{\ln (\tau_{life}/\tau_D)} \langle \delta n_V\delta n_V \rangle \frac{1}{\omega}, \quad \frac{1}{\tau_{life}} \ll \frac{1}{\tau_{obs}} \ll \omega \ll \frac{1}{\tau_D}.$$ (52)

The constant multiplier $A$ is determined from a the normalization condition:

$$\int_{1/\tau_{life}}^{1/\tau_D} (\delta n\delta n)_{\omega} \frac{d\omega}{\pi} = \langle \delta n_V\delta n_V \rangle.$$ (53)

It is supposed, thus, that the basic contribution to the correlator $\langle \delta n_V\delta n_V \rangle$ is in the domain of the flicker noise.

**A. Residual temporary correlation**

The temporary correlation is connected to temporary spectral density by relation:

$$\langle \delta n\delta n \rangle = \int_{1/\tau_{life}}^{1/\tau_D} (\delta n\delta n)_{\omega} \frac{d\omega}{\pi}.$$ (54)

From here follows, that

$$\langle \delta n\delta n \rangle = \left(C - \frac{\ln (\tau/\tau_D)}{\ln (\tau_{life}/\tau_D)}\right) \langle \delta n_V\delta n_V \rangle \text{ at } \tau_D \ll \tau \ll \tau_{life},$$
Here are used the Euler constant.

Thus, in the domain of the flicker noise dependence from $\tau$ very much weak - logarithmic at the large value of argument. It gives the basis to speak about presence of residual temporary correlations.

Let’s define the appropriate time of correlation:

$$
\tau_{\text{cor}} = \int_{\tau_D}^{\tau_{\text{life}}} \frac{\langle \delta n \delta n \rangle_{\tau \tau} d\tau}{\langle \delta n \delta n \rangle_{\tau = \tau_D}}.
$$

For an estimation of correlation time is spent the integration at performance of the inequalities $\tau_D \ll \tau \ll \tau_{\text{life}}$. In result we find, that

$$
\tau_{\text{cor}} \sim \frac{\tau_{\text{life}}}{\ln \frac{\tau_{\text{life}}}{\tau_D}}.
$$

Thus, the time of correlation at the unlimited time life $\tau_{\text{life}}$ aspires to infinity.

Stated gives the basis for a conclusion, that in the region of the flicker noise has a place as spatial, and temporary coherence. It also opens an opportunity for an establishment connection of two coherent phenomena: the flicker noise and the superconductivity.

### B. Flicker noise and superconductivity

The flicker noise represents spatial - temporary coherent structure. It makes existence of connection between the flicker noise and the superconductivity less surprising.

In approximation of the first moments from the equation (12.12.10) was received the appropriate system of the hydrodynamical equations. It consists from two equations of a continuity with ” a chemical source ” (12.12.23), which at condition $\langle n_S \rangle_{R,t} = \text{const.}$ is equivalent to the equations (39). First of them allows to find dependence of this quantity $\langle N_S \rangle_{R,t}$ from temperature. Second shows, that the velocity field is vortical.

The equation for average velocity we shall rewrite as the equation for a whirlwind of electrical current and also we shall enter into it the appropriate Langevin source:

$$
\frac{\partial}{\partial t} \left[ \Omega + \frac{e^2 n_S}{m^* c} B \right] = \nu \frac{\partial^2 \Omega}{\partial R^2} + y_{\Omega}(R, t), \quad \Omega = \text{rot} j, \quad D = \nu.
$$

This equation should be complemented by the Maxwell equation:

$$
\text{rot} B = \frac{4\pi}{c} j.
$$
Thus for the whirlwind we have the equation of diffusion type with the Langevin source. For the spectral density of the Langevin source it is possible to use the expression similar to formula (47)

\[
(y_\Omega y_\Omega)_{\omega,k} = 2\nu k^2 AV \omega \exp\left(-\frac{\nu k^2}{2\omega}\right) \langle \delta \Omega V \delta \Omega V \rangle.
\] (60)

The normalization condition has the form:

\[
\int_{1/\tau_D}^{1/\tau_{life}} \frac{d\omega}{\pi} \int \frac{dk}{(2\pi)^3} (\delta \Omega \delta \Omega)_{\omega,k} = \langle \delta \Omega V \delta \Omega V \rangle.
\] (61)

Now we need the equation for the description of temporary evolution of a whirlwind of average current.

Let’s return to the formula (60) and we shall rewrite it as the fluctuation dissipation relation (FDR):

\[
(y_\Omega y_\Omega)_{\omega,k} = 2\gamma(\omega, k) A \langle \delta \Omega V \delta \Omega V \rangle.
\] (62)

The designation for appropriate dissipative coefficient here is entered at presence both temporary, and spatial dispersion

\[
\gamma(\omega, k) = \nu k^2 V \omega \exp\left(-\frac{\nu k^2}{2\omega}\right).
\] (63)

By return the Fourier transformation it is possible to receive expression for appropriate the dissipative operator. We use its elementary representation in form "1/\tau_{rel}".

In this approximation the relaxation time about time of correlation, i.e.

\[
\tau_{rel} \sim \tau_{cor} \sim \tau_{life}/\ln \frac{\tau_{life}}{\tau_D} \gg \tau_{obs} \gg \tau_D.
\] (64)

In result for a whirlwind of an electrical current instead of (12.13.24) is received the following ”model” equation for average value of a whirlwind

\[
\frac{\partial}{\partial t} \left[\Omega + \frac{e^2 n S}{m^* c} B\right] = -\frac{1}{\tau_{rel}} \Omega, \quad \Omega = \text{rot} j.
\] (65)

It should be solved together with the Maxwell equation (59).

As the relaxation time is the order of the time life of installation, in zero approximation on dimensionless parameter

\[
\frac{\tau_{obs}}{\tau_{life}} \text{ at } \tau_{life} \gg \tau_{obs} \gg \tau_D
\] (66)

in the equation (66) it is possible to neglect by dissipation. We come, such to the equation
\[ \frac{\partial}{\partial t} \left[ \Omega + \frac{e^2 n_s}{m^* c} B \right] = 0. \]

At integration on time a constant of integration it is possible to accept for zero, that will be coordinated to the Meissner. In result we come to the London equation

\[ \text{rot } j = \frac{e^2 n_s}{mc} B. \]  \hspace{1cm} (67)

We consider it together with the Maxwell equation (59). It enables to receive the closed equation for a magnetic field

\[ \frac{\partial^2 B}{\partial R^2} - \frac{1}{\delta_L^2} B = 0, \]  \hspace{1cm} (68)

which, as we already know, describes the Meissner effect.

So, many times diffusion and, as a consequence, the flicker noise allows to understand an opportunity of simultaneous existence of the not fading electrical current and screening of a magnetic field. Here it is difficult to tell, which of these two phenomena is more fundamental, so they are bound among themselves. Both of them appear possible thanking spatially temporary coherent fluctuations of a whirlwind of a current or magnetic field.

Let’s return to inequalities (66). We shall estimate values of the observation times, at which becomes possible to observe ”not fading” superconducting electrical current.

The diffusion time \( \tau_D \) defines \( \omega_{\text{max}} \) for domain of the flicker noise: \( \omega_{\text{max}} \sim 1/\tau_D \). The minimal scale is the London length \( \delta_L \sim 10^{-5} \) cm. The diffusion coefficient can be estimated by one of two formulas: \( D \approx \hbar/2m^* \); \( v_F l \) (\( l \) - the effective length of free paths of electron pairs). From these formulas follows, that

\[ (\tau_{\text{obs}})_{\text{min}} \geq D = \delta_L^2 / D \sim 10^{-10}-10^{-11} \text{ sec}. \]  \hspace{1cm} (69)

Let’s address now to definition of concepts ”not fading” or ”superconductive ” current.

One of definition of these concepts is ”measuring”. It is connected with the observation time. Outside of the observation time it is impossible to guarantee the constancy of a current.

However, as in process of increase of the observation time it is impossible to find out attenuation of a current, it is natural to assume, that the constancy of a current has a place within the limits of greatest temporary interval \( \tau_{\text{life}} \), i.e. ” the time life ” of installations.

**IX. CONCLUSION**

So, the attempt is made to connect two, apparently, incompatible phenomena: the existence of the flicker noise and the superconductivity in a dissipative medium.
The flicker noise arises due to formation of the spatial coherent structure at the many times diffusion process. Due to this the dissipation, caused with viscous friction, is replaced on dissipation with characteristic time about time of life of installation. It also opens an opportunity for existence not fading (within the limits of time life of installation) current, and also of the screening of a magnetic field.

The connection of the flicker noise and superconductivity allows to spill additional light on the question on an opportunity of existence superconducting current in the dissipative medium. According to stated, the occurrence of the flicker-noise, and consequently also superconductivity, is promoted by the smallest scale of length.

In the theory, considered above, the role of such parameter play the London length. In this connection it is possible to expect, that preferable to occurrence of superconductivity are the substances having layered structure.

At last, the phase transition results to occurrence of quantum continuous medium, for which the basic small parameter is the quantum the physical Knudsen number.

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