The Holographic Description of Baryon in Non-Critical String Theory

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Abstract

We consider a holographic model constructed from an intersecting brane configuration \( D4/D4/D4 \) in noncritical string theory. We study the baryon in the confined phase of this supergravity by considering the source term for the baryon. Also, the thermodynamics functions are studied. Moreover, we obtain the binding energy of the baryon in this holographic QCD-like model.

Keywords: AdS/CFT correspondence; Noncritical string theory; Baryon.
1 Introduction

The AdS/CFT correspondence is a useful duality between string theory in $d + 1$ dimensional space-time and conformal field theory in $d$ dimension [1-4]. It also can be expanded to the general cases of string-gauge dualities like non conformal and non supersymmetric theories. So, some more realistic effective theories can be constructed from the string theory. In the low energy physics, in case of effective boundary theories, it can be investigated by their classical supergravity duals. Recently, some holographic models are introduced via the gauge-gravity correspondence that studied some features of the low-energy QCD [7-18]. These models, called holographic QCD models, are constructed from the intersecting brane configuration, like the Sakai-Sugimoto model[10,11] that is a $D4/D8/D8$. In the holographic model arising from the critical string theory, the color brane backgrounds are ten-dimensional, so the dual gauge theories are supersymmetric. In order to break the supersymmetry, some parts of such backgrounds need to be compactified on some manifolds. This causes to produce some Kluza-Klein modes. The mass scale of these modes are at the same order as the masses of the hadronic modes. These modes are coupled to the hadronic modes. There is no mechanism to disentangle these unwanted modes from the hadronic modes yet. One can consider the color brane configuration in the noncritical string theory to overcome this problem. The result is a gravitational backgrounds located at the low dimensions. In these backgrounds the string coupling constants are proportional to $\frac{1}{N_c}$, so the large $N_c$ limit corresponds to the small string coupling constant. However, contrary to the critical holographic models, in large $N_c$ limit the 't Hooft coupling is of order one instead of infinity and the scalar curvature of the gravitational background is also of order one[19,21]. So the noncritical gauge-gravity correspondence is not very reliable. But studies show that the results of these models for the some low energy QCD properties like the meson mass spectrum, wilson loop and the mass spectrum of glueballs [22-24] are comparable with the lattice computations. Therefore,
the noncritical holographic models seem still useful to study the QCD stuff.
One of the noncritical holographic models is composed of $D4$ and anti $D4$ brane in six-dimensional noncritical string theory\cite{20,23}. The low energy effective theory on the intersecting brane configuration is a four-dimensional QCD-like effective theory with the global chiral symmetry $U(N_f)_L \times U(N_f)_R$. In this brane configuration, the six-dimensional gravity background is the near horizon geometry of the color $D4$ brane. Here, there is no compact sphere compare with the critical gravity backgrounds. This model is based on the compactified $AdS_6$ space-time with a constant dilaton. So the model dose not suffer from the large string coupling as the SS model. The meson spectrum \cite{22} and the structure of thermal phase \cite{25} are studied in this model. Some properties, like the dependence of the meson masses on the stringy mass of the quarks and the excitation number are different from the critical holographic models such as SS model.

Baryon was analyzed in the critical holographic models like the SS model\cite{26-43}. In the SS model, a $D4$ brane wrapped on a $S^4$ is the baryon vertex. $N_c$ fundamental strings attach the vertex to the flavor $D8$ brane. It was shown that this baryon corresponds to an instanton of the five-dimensional effective $U(N_f)$ gauge theory\cite{43}. The physical properties of this baryon like the mass, size, mass splitting, the mean radii, magnetic moments and various couplings were analyzed in the SS model\cite{31-42}. The obtained results show a better agreement with the experimental data compared to the Skyrme model results\cite{44}. But there are some problems. For example, the size of the baryon is proportional to $\lambda^{-1/2}$. In the large 't Hooft coupling (large $\lambda$) the size of the baryon becomes zero. So the stringy corrections have to be taken into account. Another problem is that the scale of the system associated with the baryonic structure is roughly half the one needed to fit to the mesonic data \cite{42}. So, all above information give us motivation to analyze the baryon in a noncritical holographic model with the $AdS_6$ background. In this background there is no compact $S^4$ sphere, so we consider an unwrapped $D0$ brane as a baryon vertex. Similar to the SS model, in this case also it is necessary that $N_c$ fundamental strings
attach the vertex to the probe flavor brane. Also the baryon vertex is attached to the color probe brane[43]. In this paper, using this description of the baryon we calculate the baryon energy. Also we study the thermodynamics functions and the binding energy of baryon. We compare our results with the results of the critical holographic model such as SS model, and show that the behavior of the thermodynamics function respect to the baryon density is similar to the SS model [33]. Also, we find a similar behavior for the baryon binding energy, unless the value of the binding energy is larger as compare with the SS model [14].

2 \textit{AdS}_6 \textbf{Backgrounds}

In analogy with the critical models, in this noncritical model, the gravity background is generated by near-extremal \( D4 \) branes wrapped over a circle with the anti-periodic boundary conditions. Two stacks of flavor branes, that is one of \( D4 \) branes and the other one of anti-\( D4 \) branes are added to this geometry which called flavor probe branes. The color branes extend along the directions \( t, x_1, x_2, x_3, \tau \) while the probe flavor branes fill the whole Minkowski space and stretch along the radius \( U \) up to infinity. The strings attached color \( D4 \)-brane to a flavor brane transform as quarks, while strings hanging between a color \( D4 \) and a flavor \( \overline{D4} \) transform as anti-quarks. The chiral symmetry breaking is achieved by a reconnection of the brane-anti-brane pairs. Under the quenched approximation (\( N_c \gg N_f \)), the backreactions of flavor branes on the color branes can be neglected. Just like the SS model, the \( \tau \) coordinate is wrapped on a circle and the anti-periodic condition is considered for the fermions on the thermal circle. The final low energy effective theory on the background is a four-dimensional QCD-like effective theory with the global chiral symmetry \( U(N_f)_L \times U(N_f)_R \).

The near horizon gravity background at low energy is [20,23]

\[
 ds^2 = \left( \frac{U}{R} \right)^2 (-dt^2 + dx_i dx_i + f(U) d\tau^2) + \left( \frac{R}{U} \right)^2 \frac{dU^2}{f(U)} 
\]  

(1)
The background geometry consists of a constant dilaton and a RR six-form field strength as follows,

$$F_{(6)} = Q_c \left(\frac{U}{R}\right)^4 dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge du \wedge d\tau$$

$$e^\phi = \frac{2\sqrt{2}}{\sqrt{3}Q_c},$$

and

$$f(U) = 1 - \left(\frac{U_{KK}}{U}\right)^5, \quad R^2 = \frac{15}{2},$$

where $Q_c$ is proportional to the number of color branes $N_c$.

To avoid the singularity, the coordinate $\tau$ satisfies the following periodic condition,

$$\tau \sim \tau + \delta \tau, \quad \delta \tau = \frac{4\pi R^2}{5U_{KK}}.$$

The Kluza-Klein mass scale of this compact dimension is

$$M_{KK} = \frac{2\pi}{\delta \tau} = \frac{5}{2} \frac{U_{KK}}{R^2}.$$

and dual gauge field theory in this background is non supersymmetric.

### 3 Baryon in the Noncritical Holographic Model

In this section, we consider the baryon in the noncritical holographic model with the $AdS_6$ background. In the critical holographic model like the SS model, $D4$ brane wrapping the compact $S^4$ is introduced as a baryon vertex which has $N_c$ units of electric charge. It is shown that a $D4$ brane wrapping $S^4$ looks like an object with electric charge with respect to the gauge field on $D8$ and it is possible to say that $D4$ brane spread inside $D8$ brane as an instanton.

Here we consider noncritical holographic model which has no compact sphere. So one can introduce an unwrapped $D0$ brane as a baryon vertex. There is a Chern-Simons term
on the $D0$ brane world-volume. Also in this case one needs to attach $N_c$ strings to the baryon vertex. The other end the corresponding strings must be attached to the probe flavor $D4$ branes. Also the baryon vertex will be attached to the probe branes [43]. Now, we turn on only the zero component of the gauge field on the world-volume of the flavor $D4$ brane and assume that this component depends only on the compact coordinate $\tau$. Therefore, the abelian effective action on the $D4$ brane is written by,

$$S_{D4} = -N_f T_4 e^{-\phi} \int d^5 x \sqrt{-det(g_{MN} + 2\pi \alpha' F_{MN})} + \mu \int C_5,$$  

where $T_4 = (2\pi)^{-4}(l_s)^{-5}$ is the tension of $D4$ brane, the $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$ $(M, N = 0, 1, \ldots, 5)$ is the field strength tensor and the $A_M$ is the $U(N_f)$ gauge field on the $D4$ brane. The second term in the above action, is the Chern-Simons action which has to be zero[25]. So, we can neglect this term.

The induced metric on the $D4$ brane is written as,

$$ds^2 = \left( \frac{U}{R} \right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu) + \left[ \left( \frac{R}{U} \right)^2 f(U) \tau^2 + \left( \frac{R}{U} \right)^2 f(U)^{-1} \right] dU^2.$$  

Thus, the $D4$ brane action has the following form,

$$S_{D4} = -N_f T_4 e^{-\phi} \int d^4 x dU \left( \frac{U}{R} \right)^5 \sqrt{\tau^2 f(U) + \left( \frac{R}{U} \right)^4 f(U)^{-1} - (2\pi \alpha' \dot{A}_0)^2},$$  

where the $\dot{A}_0$ and $\dot{\tau}$ are derivatives respect to the $U$ coordinate. The equations of motion for $\tau$ and $A_0(U)$ are,

$$\frac{d}{dU} \left( \frac{\dot{\tau} f(U)}{\sqrt{\tau^2 f(U) + \left( \frac{R}{U} \right)^4 f(U)^{-1} - (2\pi \alpha' \dot{A}_0)^2}} \right),$$  

$$\frac{d}{dU} \left( \frac{\dot{U}/R A_0}{\sqrt{\tau^2 f(U) + \left( \frac{R}{U} \right)^4 f(U)^{-1} - (2\pi \alpha' \dot{A}_0)^2}} \right),$$

respectively.

For convenience, we assume that $D4$ and $\overline{D4}$ branes have the maximum separation at the boundary. In analogy with the SS model[45], it implies that in the confined phase the $D4$ and $\overline{D4}$ branes connect together at the $U = U_{KK}$. In this case, we have $\dot{\tau} = 0$. We use
this simplified condition for the $\tau$.

By introducing the following variable,

$$U = \left( U_{K K}^5 + U_{K K}^3 z^2 \right)^{1/5}, \quad (11)$$

and using dimensionless parameters,

$$Z = \frac{z}{U_{K K}} \quad K(Z) = 1 + Z^2. \quad (12)$$

the $D4$ brane action rewritten as,

$$S_{D4} = -B \int d^4x \int dZ K^{3/10} \sqrt{1 - \hat{B} K^{3/5} (\partial_Z A_0)^2} \quad (13)$$

where the constants $B$ and $\hat{B}$ are,

$$B = \frac{2 U_{K K}^4 N_f T_4 e^{-\phi}}{R^3}, \quad \hat{B} = \left( \frac{5 \pi \hat{\alpha}_s}{U_{K K}} \right)^2 \quad (14)$$

Now, in analogy with the SS model, we introduce the source term for the baryon. This term arises from the coupling between the gauge field $A_0$ on the flavor brane and the $N_c$ units of electric charge on the baryon vertex. We assume that the baryon is distributed homogenously in the $R^3$ space. Therefor, we consider the source term as follows,

$$S_{\text{source}} = N_c n_B \int d^4x \int dZ \delta(Z) A_0(Z). \quad (15)$$

We assume the baryon action as a sum of the DBI action for the $D4$ brane and the source term. Thus, the Lagrangian density for the baryon can be written as,

$$\mathcal{L}_{\text{Baryon}} = -B K^{3/10} \sqrt{1 - \hat{B} K^{3/5} (\partial_Z A_0)^2} + N_c n_B \delta(Z) A_0(Z) \quad (16)$$

So, the equation of motion for the gauge field becomes,

$$\frac{d}{dZ} \frac{\partial \mathcal{L}}{\partial (\partial_Z A_0)} = \frac{1}{2} n_q \delta(Z), \quad \frac{1}{2} n_q \delta(Z). \quad (17)$$

where $n_q = n_B N_c$ is the quark density. The conjugate momentum of the gauge field can be defined as follows,

$$D \equiv \frac{\partial \mathcal{L}}{\partial (\partial_Z A_0)}. \quad (18)$$
So, from the equation of motion we obtain,

\[ D = \frac{1}{2} n_q Sgn(Z), \]  

where \( Sgn(Z) \) is the sign function and is determined by the symmetry between \( D4 \) and \( \overline{D4} \). Integrating relation (18) yields the classical solution for the gauge field,

\[ A_0(Z; n_q) = A_0(0) + \int_0^Z dZ \frac{n_q/2}{\sqrt{(B \dot{B})^2 K^{9/5} + \dot{B} n_0^2/4 K^{3/5}}}. \]  

(20)

The “baryon charge chemical potential of a quark”, \( \mu \) is defined the boundary value of the gauge field \([33,46]\),

\[ \mu(n_q) \equiv \lim_{|Z| \to \infty} A_0(Z; n_q). \]  

(21)

Moreover the chemical potential of the baryon is defined by,

\[ \mu_B = m_B + N_c \mu. \]  

(22)

where the \( m_B \) is the rest mass of the baryon. The variation of the \( A_0(Z) \) as a function of \( Z \) coordinate has been shown in Figure 1. Also, Figure 2 shows variation of chemical potential, \( \mu \) as a function of baryon densities. In all figures \( n_B \) is normalized to \( (n_B/n_0) \), with the nuclear matter density, \( n_0 = 0.17 fm^{-3} \simeq 1.3 \times 10^6 MeV^3 \). Also we use the \( N_c = 3, N_f = 2, M_{KK} = 1 GeV \) and \( \alpha = 1 \) in our calculations. Figure 1 indicates that the gauge field in the large \( Z \) becomes constant for each value of \( n_B/n_0 \).

Now, we try to eliminate the gauge field in the action. In order to do this, a Legendre transformation for the Lagrangian density must be applied as follows,

\[ \mathcal{L} \to -\mathcal{L}_{Baryon} + D \dot{A}_0. \]  

(23)

where \( D \) is the conjugate momentum of gauge field which is defined by equations (18), (19). It should be noted that the Legendre transformation at the classical field theory of bulk can be described as a Legendre transformation between the canonical and grand
canonical ensembles at the boundary thermodynamics\[46]. \\
$A_0(Z)$ is an auxiliary field with no time dependence, so we can eliminate it by the equation \((20)\). So, we obtain the energy \(U(n_q)\) as follows,

\[
U(n_q) = \int dx^3 \int_{-\infty}^{+\infty} dZ (-\mathcal{L}) = BV \int_{-\infty}^{+\infty} dZ K^{3/10} \sqrt{1 + \frac{n_q^2}{4B^2 B} K^{-6/5}},
\]

\((24)\)

where \(V\) represent the integral of the space part. It is known that the chemical potential is related to free Helmholtz energy by the Gibbs equation \(\mu = \frac{\partial F(n_q)}{\partial n_q}\). The free Helmholtz energy is the internal energy \(U(n_q)\) at the zero temperature. So, the chemical potential can be obtained using the Gibbs equation and the equation \((24)\) as follows,

\[
\mu = \int_{-\infty}^{+\infty} dZ \frac{n_q/4}{\sqrt{(B\dot{B})^2 K^{9/5} + B n_q^2/4 K^{9/5}}}. \quad \text{} \tag{25}
\]

which is expected from the equations \((20)\) and \((21)\). In fact, \(\mu\) is the work required to bring a charge from the UV to the IR region against electric field which is same as the work done to add a quark to the system.

The free Helmholtz energy is given by the Gibbs relation,

\[
\frac{F}{V} = B \int_{-\infty}^{+\infty} dZ K^{3/10} (1 + \frac{n_q^2}{4B^2 B} K^{-6/5} - 1) \quad \text{} \tag{26}
\]

For convenience, we normalize the potentials with the \(V\). We can obtain the grand canonical potential by evaluating the on-shell action,

\[
\frac{\Omega}{V} = B \int_{-\infty}^{+\infty} dZ K^{3/10} \left(1 + \frac{n_q^2}{4B^2 B} K^{-6/5} - 1\right) \quad \text{} \tag{27}
\]

Therefore, the pressure is obtained as,

\[
P = -\frac{\Omega}{V} = B \int_{-\infty}^{+\infty} dZ K^{3/10} \left(1 - \frac{1}{\sqrt{1 + \frac{n_q^2}{4B^2 B} K^{-6/5}}}\right) \quad \text{} \tag{28}
\]


and \( \tilde{\mu}_B \equiv \mu_B - m_B \) is,

\[
\tilde{\mu}_B = N_c \int_{-\infty}^{\infty} dZ \frac{n_q/4}{\sqrt{(B \dot{B})^2 K^{9/5} + \dot{B} n_q^2/4 K^{3/5}}}. \tag{29}
\]

The thermodynamics functions versus the \( n_B/n_0 \) have been shown in figure 3. Also, in the small \((n_B/n_0)\) limit, we can simplify the thermodynamics functions as follows,

\[
\mu = \frac{N_c n_0 n}{2 B \dot{B}} \int_{0}^{\infty} dZ K^{-9/10} (1 - \frac{(N_c n_0 n)^2}{8 B^2 \dot{B}} K^{-6/5} + ...) = \frac{0.919 N_c n_0 n}{B \dot{B}} - \frac{0.047 (N_c n_0 n)^3}{B^3 \dot{B}^2} \tag{30}
\]

\[
U(n_B) = \frac{(N_c n_0 n)^2}{4 B \dot{B}} \int_{0}^{\infty} dZ K^{-9/10} = \frac{1.84 (N_c n_0 n)^2}{4 B \dot{B}} \tag{31}
\]

\[
\Omega = -\frac{(N_c n_0 n)^2}{4 B \dot{B}} \int_{0}^{\infty} dZ K^{-9/10} = -\frac{1.84 (N_c n_0 n)^2}{4 B \dot{B}} \tag{32}
\]

\[
P = \frac{(N_c n_0 n)^2}{4 B \dot{B}} \int_{0}^{\infty} dZ K^{-9/10} = \frac{1.84 (N_c n_0 n)^2}{4 B \dot{B}} \tag{33}
\]

At low densities, internal energy, pressure and grand potential are quadratic in \((n_B/n_0)\). But the behavior of chemical potential is different. Generally, the thermodynamics functions behave same as the SS model in the small baryon densities. This behavior well explained in ref[33]. The small density limit can be interpreted that in bulk the \( A_0 \) configuration for fixed charge is obtained by minimizing the induced DBI action on \( D4-\overline{D4} \). So, there is only the flavor meson mediated interactions between the point-like baryons. At the large \( N_c \) limit \( D4 \) mediated correlated gravitons are heavy and decouple, so the point-like baryon vertex on the bulk can be considered as the skyrmions with the infinite size at the boundary. So, only the omega exchanges remain at the large \( N_c \). At the low baryon density, we have the repulsive skyrmion-omega-skyrmion interaction and the positive energy density[33].
4 Baryon Binding Energy

In this section we are going to obtain the baryon binding energy from noncritical holographic model. In previous section, we obtain the Lagrangian density of the baryon, so we write the energy of the baryon as follows,

$$E_{Baryon} = \int dx^3 \int_{-\infty}^{+\infty} dZ (\mathcal{L}_{Baryon})$$

$$= 2 BV \int_{0}^{+\infty} dZ K^{3/10} \sqrt{1 + \frac{n_q^2}{4B^2B}K^{-6/5}}, \quad (34)$$

In order to obtain the baryon binding energy, in first step, we must subtract the self-energy of the $N_c$ quarks from this energy. Then by minimizing the result, we obtain the binding energy of the baryon. The energy of the $N_c$ fundamental quarks in the noncritical background $AdS_6$ can be written as,

$$S_{quarks} = \frac{N_c}{2\pi\alpha'} \int dt dU f(U)^{-1/2}, \quad (35)$$

where we consider the condition $\tau = 0$.

Using the equations (11) and (12) in the above action, the energy of $N_c$ fundamental quarks is written as follows,

$$E_{quarks} = -C \int dZ Z K^{3/10}, \quad (36)$$

where

$$C = \frac{2}{5} \frac{N_c U_{KK}}{2\pi\alpha}. \quad (37)$$

So, we can calculate the final energy as,

$$E = E_{Baryon} - E_{quarks}$$

$$= 2 V \int_{0}^{Z_\Lambda} dZ K^{3/10} \sqrt{1 + \frac{n_q^2}{4B^2B}K^{-6/5}} - 2 C \int_{0}^{Z_\Lambda} dZ Z K^{3/10} \quad (38)$$

Minimizing this energy respect to the $Z_\Lambda$ results the baryon binding energy. The equation (38) is solved numerically by the $N_f = 2$, $N_c = 3$, $M_{KK} = 1GeV$ and $\dot{\alpha} = 1$ values and
the baryon energy as a function of the $Z_A$ parameter has been shown in the Figure 4. As it is clear from this figure, by increasing the $Z_A$, energy of baryon configuration decreases and has a stable equilibrium point at $Z_A = Z_{min}$ with the minimum energy of $E_I = -5.003 \text{GeV}$. At $Z_A = Z_c$ the binding energy is zero again, and for $Z_A > Z_c$ the baryon would be dissociated. So, we obtain an stable range for the baryon configuration.

Also we can describe the $Z_A = Z_{min}$ point as the size of the baryon. Note that we obtained a similar behavior for the binding energy versus $Z_A$ compared to Ref.[14] with the $D4/D8 - D\overline{8}$ configuration.

We show the values of baryon binding energy and the $Z_{min}$ for different values of $n$ in Table 1. This Table indicated that for small $n$ values the binding energy is independent of $n$ value. It means that in $(n_B/n_0) < 1$, we can simplify the integrands to obtain the approximated energy as,

$$\dot{E} = 2 V \int_0^{Z_A} dZ \frac{K^{3/10}}{K^{3/10}} \left( 1 + \frac{(N_c n_0 n)^2}{8 B^2 B} K^{-6/5} + ... \right) - 2 C \int_0^{Z_A} dZ Z K^{3/10}$$

which has analytical solution in terms of the following hypergeometric functions,

$$\dot{E} = \frac{0.4 V}{BB} Z_A \left[ 4.94 B^2 \dot{B} F([-0.3, 0.5], [1.5], -Z_A^2) + 0.62 n_0^2 F([0.5, 0.9], [1.5], -Z_A^2) \right] - 2 C Z_A F([-0.3, 0.5], [1.5], -Z_A^2),$$

Table 1: The values of the $Z_{min}$ and the $E_I$ for the various baryon chemical potential.

| $n$   | 10  | 7   | 5   | 1   | 10^{-3} |
|-------|-----|-----|-----|-----|---------|
| $Z_{min}$ | 20.1| 20.2| 20.2| 20.2| 20.2    |
| $E_I$  | -5.11| -5.12| -5.13| -5.14| -5.14   |
Figure 1: The gauge field $A_0(Z)$ vs $Z$.

Figure 2: The chemical potential vs baryon charge $(n_B/n_0)$.
Figure 3: (a) The internal energy, (b) Baryon chemical potential, (c) grand potential and (d) pressure vs baryon charge density $\frac{n_B}{n_0}$, where $n_B$ is the baryon density and $n_0$ is the nuclear matter density.
Figure 4: The behavior of baryon energy vs the cutoff parameter $Z_\Lambda$. The minimum of the energy shows the binding energy of the baryon and the $Z_\Lambda = Z_{\text{min}}$ can be regarded as the baryon size.

5 Conclusion

In this study, we considered the baryon in the noncritical holographic model constructed by intersection $D4/D4-\overline{D4}$ branes. This model have the $AdS_6$ background geometry. Here, we considered the baryon action as the sum of the $DBI$ action and a delta function source of the gauge field analogy to the SS model. We obtained the thermodynamics functions in terms of the model parameters and studied the behavior of these function respect to the baryon density. At low baryon densities, internal energy, grand potential and the pressure are quadratic in $(n_B/n_0)$. But the chemical potential has different behavior as indicated in equation (30). These behaviors are similar to the results obtained from the SS model[33].

Also, we obtained the baryon binding energy in this model. In order to do this, we subtracted the self-energy of the quarks from the total baryon energy. Then by minimizing the result respect to the cutoff parameter, we obtained the binding energy for the baryon.
We presented this energy in the Figure 4. According to this Figure, we can easily find that the baryon energy is zero at $Z_\Lambda = 0$ or $U_\Lambda = U_{kk}$ which is the lower bound for $U$ coordinate in the model where the radius of $S^1$ diminishes to zero and no stable baryon configuration exists. As $Z_\Lambda$ increases, the energy of baryon configuration gets smaller. At $Z_\Lambda = Z_{min}$ there is an stable equilibrium point which corresponds to the size of baryon in the model. For $Z_\Lambda > Z_{min}$ the energy increases and also at $Z_\Lambda = Z_c$ the energy vanishes again. It reveals the fact that for $Z_\Lambda > Z_c$ there is no stable baryon configuration and the baryon would be dissociated. This behavior of baryon energy is similar to the one we obtained in ref[14].

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