Charge quantization in the largest leptoquark-bilepton chiral electroweak scheme

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Abstract

The uniqueness of the hypercharge assignments in the three fermion families leptoquark-bilepton SU(3)$_C$×SU(4)$_L$×U(1)$_N$ model is established. Although the gauge group contains an explicit U(1) factor, freedom from triangle anomalies combined with the requirement of nonvanishing charged fermion masses uniquely fix the electric charges of all fermions independently of the neutrinos being massless or not. The electric-charge quantization, family replication, and the existence of three colors are interwoven.

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So far, the standard model of electroweak and strong interactions \[1\] has been quite successful in its compatibility with almost all available experimental data \[2,3\]. It nevertheless leaves some fundamental theoretical questions unexplained. In the standard model, each family of fermions is anomaly-free and this is true as well for grand unified models, supersymmetric extensions, except the supersymmetric preon model \[4\], technicolor, superstring theories, and most compositeness scenarios where the number of families remains completely unrestricted on theoretical grounds. The chiral anomaly is cancelled between quarks and leptons in each family and the indetermination about the inter-relation between families constitute the so-called flavor question. At present, we know of three families, but the standard model does not explain why this number has to be three, even so the number of neutrino flavors within the electroweak standard model is \(N_\nu = 3.00 \pm 0.09\) and the experimental determination of this number is model dependent \[5\]. Some very fundamental aspects of the standard model such as the flavor question might be understood by embedding the three-family version in a Yang-Mills theory with the gauge semisimple group

\[ G_0 \equiv \text{SU}(3)_C \otimes G_W \otimes \text{U}(1)_{L+R} \]

just enlarging the SU(2)\(_L\) weak isospin group to \(G_W = \text{SU}(3)_L\) (331 model \[6–8\]) which is the minimal gauge group that at the leptonic level admits charged fermions and their antiparticles as members of the same multiplet \[9\]. The key predictions of the \(G_0\) alternative models are leptoquark fermions with electric charges \(\pm 5/3\) and \(\mp 4/3\) and bilepton gauge bosons \[10\] with lepton number \(L = \pm 2\). The leptoquark fermions are color-triplet particles which possess barion and lepton numbers. Another interesting feature of these leptoquark-bilepton chiral models is that the weak mixing angle of the standard model has an upper limit. Therefore, it is possible to compute an upper bound to the mass scale of the \(G_W = \text{SU}(3)_L\) breaking of about 1.7 TeV \[11\].

Considering the lightest particles of the model as the sector in which a symmetry is manifested, the lepton sector could be the part of the model determining new approximate symmetries. In fact, if right-handed neutrinos are introduced, there arises a more interesting
possibility of having $\nu_l$, $l = e, \mu, \tau$, and the charge conjugate fields $\nu^c_l$, $l^c$ in the same multiplet for each family flavor. Model building in that direction, if each family of fermions is treated separately, culminates with the highest symmetry $G_W = SU(4)_L$ to be considered in the electroweak sector (341 model \[12–15\]). In the 331 and 341 leptoquark-bilepton models the number of families must be divisible by the number of color degrees of freedom in order to cancel anomalies. This novel method of anomaly cancellation requires that at least one family transforms differently from the others, thus breaking generation universality. To accommodate the replication of three fermion families, the number of families, number of colors, and fractional electric charge values become related \[7,16,17\]. Having established that connection the flavor question is solved with a relation between the strong and electroweak parts of the model which does not exist in the context of the standard model. In the minimal standard model there is a remarkable failure concerning the connection among family replication and the electric charge quantization \[18\]. In fact, the charge quantization is realized only within each family \[19,20,23\]. Nevertheless, taking the three families together the effect of dequantization occurs \[19,21,23,24\]. The possibility of charge quantization with three families in the minimal 331 model was shown recently \[25\].

In the 341 model, the electric charge operator is embedded in the neutral generators of the $SU(4) \otimes U(1)$ group

$$Q = \Lambda_3 + \xi \Lambda_8 + \zeta \Lambda_{15} + \epsilon N$$

with the embedding parameters $\xi = -1/\sqrt{3}$, $\zeta = -2\sqrt{6}/3$ and $N$ is the new $U(1)$ charge. The neutral generators of $SU(4)_L$ are

$$\Lambda_3 = \frac{\lambda_3}{2} = \frac{1}{2} \text{diag}(1, -1, 0, 0),$$

$$\Lambda_8 = \frac{\lambda_8}{2} = \frac{1}{2\sqrt{3}} \text{diag}(1, 1, -2, 0),$$

$$\Lambda_{15} = \frac{\lambda_{15}}{2} = \frac{1}{2\sqrt{6}} \text{diag}(1, 1, 1, -3).$$

The model treats the color singlet leptons democratically in each of the three families
\[
f_{iL} = \begin{pmatrix}
\nu_i \\
\ell_i \\
\nu_i^c \\
\ell_i^c
\end{pmatrix}_L \sim (1, 4, N_i) \tag{2}
\]

where \(i = 1, 2, 3\) is a flavor index and \(\nu_i^c, \ell_i^c\) denote charge conjugated fields. There are no leptonic flavor singlets because right-handed charged leptons are not independent degrees of freedom and can be obtained through charge conjugation of the fields contained in the multiplet \(f_{iL}\). The quarks have the attributions

\[
Q_{1L} = \begin{pmatrix}
u_1 \\
d_1 \\
u_1^c \\
J
\end{pmatrix}_L \sim (3, 4, N_{Q_{1L}}) \tag{3}
\]

\[
Q_{\alpha L} = \begin{pmatrix}
j_\alpha \\
d_\alpha' \\
u_\alpha \\
d_\alpha \\
J_R \sim (3, 4^*, N_{Q_{\alpha L}})
\end{pmatrix}_L \sim (3, 4^*, N_{Q_{\alpha L}}) \tag{4}
\]

and the associated right-handed projections transform as singlets under SU(4)

\[
u_{1R} \sim (3, 1, N_{u_{1R}}) \tag{5a}
\]

\[
d_{1R} \sim (3, 1, N_{d_{1R}}) \tag{5b}
\]

\[
u_{1R}' \sim (3, 1, N_{u_{1R}}') \tag{5c}
\]

\[
J_R \sim (3, 1, N_{J_R}) \tag{5d}
\]

\[
j_{\alpha R} \sim (3, 1, N_{j_{\alpha R}}) \tag{5e}
\]

\[
d_{\alpha R}' \sim (3, 1, N_{d_{\alpha R}}') \tag{5f}
\]

\[
u_{\alpha R} \sim (3, 1, N_{u_{\alpha R}}) \tag{5g}
\]

\[
d_{\alpha R} \sim (3, 1, N_{d_{\alpha R}}) \tag{5h}
\]
where $\alpha = 2, 3$. The 341 original symmetry is broken and quark masses are generated by the following Higgs multiplets

$$
\begin{align*}
\eta &\sim (1, 4, N_\eta), \\
\rho &\sim (1, 4, N_\rho), \\
\chi &\sim (1, 4, N_\chi).
\end{align*}
$$

The lepton mass term transforms as $(f_L)\overline{c}f_L \sim (1, 6A \oplus 10_S)$. In order to obtain massive charged leptons it is necessary to introduce the multiplet

$$
H^* = \begin{pmatrix}
H_1^0 & H_1^+ & H_2^0 & H_2^- \\
H_1^+ & H_1^{++} & H_3^+ & H_3^0 \\
H_2^0 & H_3^+ & H_4^0 & H_4^- \\
H_2^- & H_3^0 & H_4^- & H_2^{--}
\end{pmatrix} \sim (1, 10_S^*, N_{H^*})
$$

because the $6^*_A$ will leave one lepton massless and two others degenerate for three generations. Therefore a vacuum expectation value of the decuplet is needed to produce a realistic leptonic mass matrix. Moreover, in order to avoid mixing among primed and unprimed quarks we introduce another multiplet $\eta'$ transforming as $\eta$ but with different vacuum expectation value. Notice that the introduction of the (anti)decuplet $H^*$ is not essential for the symmetry breaking. In fact, the 341 gauge symmetry breaks to $\text{SU}(3)_C \times \text{U}(1)_Q$ if the vacuum structures

$$
\langle \eta \rangle = (v_\eta, 0, 0, 0), \quad \langle \rho \rangle = (0, v_\rho, 0, 0), \quad \langle \chi \rangle = (0, 0, 0, v_\chi), \quad \langle \eta' \rangle = (0, 0, v_{\eta'}, 0),
$$

are realized. At tree level the charged leptons get a mass but neutrinos remain massless if $\langle H_3^0 \rangle \neq 0$, $\langle H_{1,2,4}^0 \rangle = 0$.

The electric charge operator is defined as the linear combination

$$
Q = T_3 + \frac{Y}{2}
$$

which annihilates the vacuum. Consequently,

$$
N_\eta = N_{\eta'} = N_{H^*} = 0, \quad \varepsilon = \frac{1}{N_\rho}, \quad N_\chi = -N_\rho,
$$

so that the hypercharge is the mixture

$$
N_\eta = N_{\eta'} = N_{H^*} = 0, \quad \varepsilon = \frac{1}{N_\rho}, \quad N_\chi = -N_\rho,
$$
\[ Y = \frac{T_8 + T_{15} + \frac{N}{N_\rho}}{2} \]  

with the components

\[ T_8 = \xi \Lambda_8 = -\frac{1}{2\sqrt{3}} \lambda_8, \]

\[ T_{15} = \xi \Lambda_{15} = -\sqrt{\frac{6}{3}} \lambda_{15}, \]

and \( T_3 = \Lambda_3 = \lambda_3/2 \). The most general Yukawa interactions in terms of weak eigenstates are

\[- \mathcal{L}_Y = \frac{1}{2} G_{ij} (f_i L)^c H^* f_j L \]

\[ + F_{1k} Q_{1L} u_{kR} \eta + F_{\alpha k} Q_{\alpha L} u_{kR} \rho^* \]

\[ + F'_{1k} Q'_{1L} d_{kR} \eta^* + F'_{\alpha k} Q'_{\alpha L} d_{kR} \rho^* \]

\[ + h_1 Q_{1L} u_{R} \eta^* \]

\[ + h_\alpha \beta Q_{\alpha L} d_{R} \rho^* \]

\[ + \Gamma_1 Q_{1L} J_R \chi + \Gamma_{\alpha \beta} Q_{\alpha L} J_\beta \chi^* + \text{H.c.} \]  

(11)

where \( i, j = e, \mu, \tau; k = 1, 2, 3 \); and \( \alpha, \beta = 2, 3 \). These couplings automatically contain a Peccei-Quinn symmetry \[26\] which can also be extended to the Higgs potential, solving the strong \( CP \) problem \[27\]. The \( U(1)_{N} \) gauge invariance of the Yukawa leptonic term gives three classical constraints

\[ N_i = 0, \quad i = e, \mu, \tau \]  

(12)

while for the quark flavors the set of classical constraints is

\[ N_{Q_{1L}} - N_{u_{kR}} = N_\eta, \]  

(13a)

\[ N_{Q_{\alpha L}} - N_{d_{kR}} = N_{\eta^*}, \]  

(13b)

\[ N_{Q_{1L}} - N_{d_{kR}} = N_\rho, \]  

(13c)

\[ N_{Q_{\alpha L}} - N_{u_{kR}} = N_{\rho^*}, \]  

(13d)

\[ N_{Q_{1L}} - N_{u'_{kR}} = N_{\eta'}, \]  

(13e)

\[ N_{Q_{\alpha L}} - N_{d'_{\alpha R}} = N_{\eta'^*}, \]  

(13f)
\[ N_{Q1L} - N_{J_R} = N_{\chi}, \]  
(13g)

\[ N_{Q\alpha L} - N_{J\alpha L} = N_{\chi^*}, \]  
(13h)

where \( N_\eta = -N_{\eta^*}, N_{\eta'} = -N_{\eta'^*}, N_\rho = -N_{\rho^*}, \) and \( N_\chi = -N_{\chi^*}. \) These conditions imply

\[ N_{Q1L} + N_{Q2L} = N_{u_R} + N_{s_R}, \]
\[ N_{Q1L} + N_{Q2L} = N_{d_R} + N_{c_R}, \]  
(14)

\[ N_{Q1L} + N_{Q2L} = N_{J2R} + N_{J_R}, \]

for the first and second families and

\[ N_{Q2L} - N_{Q3L} = N_{J2R} - N_{J3R}, \]
\[ N_{Q2L} - N_{Q3L} = N_{c_R} - N_{l_R}, \]  
(15)

\[ N_{Q2L} - N_{Q3L} = N_{s_R} - N_{b_R}, \]

which relates the second and third families. Therefore, we obtain the following two conditions,

\[ N_{Q2L} = N_{Q3L} \equiv N_{Q\alpha L}, \quad \alpha = 2, 3 \]  
(16)

and

\[ N_{J2R} = N_{J3R} \equiv N_{J\alpha R}. \]  
(17)

To consider the quantum constraints, let us set the following notation

\[ N_{u1R} = N_{u2R} = N_{u3R} \equiv N_{U_R}, \]
\[ N_{d1R} = N_{d2R} = N_{d3R} \equiv N_{D_R}. \]

for the up- and down-like standard flavors. It will be sufficient to look at the anomalies which contain \( U(1)_N \) factors,

\[ \text{Tr}[SU(3)_C]^2[U(1)_N]=0: \]
\[ 3(N_{Q1L} + 2N_{Q\alpha L}) - 3(N_{U_R} + N_{D_R}) - N_{J_R} - 2N_{J\alpha R} - N_{u_R'} - 2N_{d_R'} = 0, \]  
(18a)
where the last condition arises from a triangle graph with two external gravitons. Whatever the correct quantum gravity theory is, the mixed gauge-gravitational anomaly \[ \text{(18d)} \] must be canceled to ensure the general covariance of the theory. The other non-trivial anomaly is \([\text{SU}(4)_L]^3\) which also cancels between the families if the number of fermion families coincides with the number of SU(3)$_C$ color degrees of freedom \[ \text{(17)} \]. Through the leptonic classical conditions of Eq. \[ \text{(12)} \] we have
\[ \sum_i N_i = 0 \]
over the leptonic families. Therefore, the mixed gauge-gravitational constraint coincides with that of the \([\text{SU}(3)_C]^2[\text{U}(1)_N]\) anomaly involving the color gauge bosons. As such there are essentially only three independent quantum constraints.

The U(1)$_N$ gauge invariance of the quark Yukawa couplings gives the explicit classical constraints
\[ \begin{align*}
N_{UR} &= N_{Q_{\alpha L}} + N_{\rho}, \\
N_{DR} &= N_{Q_{\alpha L}}, \\
N_{JR} &= N_{Q_{\alpha L}} - N_{\rho}, \\
N_{\alpha_R} &= N_{Q_{1L}}, \\
N_{\alpha_R'} &= N_{Q_{\alpha L}},
\end{align*} \]
and also, by the quantum constraints, there are the additional conditions

\[ \begin{align*}
3(N_{Q_{1L}} + 2N_{Q_{\alpha L}}) + \sum_i N_i &= 0, \\
3(N_{Q_{1L}}^3 + 2N_{Q_{\alpha L}}^3) - 3(N_{UR}^3 + N_{DR}^3) - N_{JR}^3 - 2N_{\alpha_R}^3 - 2N_{\alpha_R'}^3 + \sum_i N_i^3 &= 0
\end{align*} \]
\( N_{Q_{1L}} = N_{Q_{aL}} + N_{\rho}, \) \hspace{1cm} (20a)

\( N_J = N_{Q_{aL}} + 2N_{\rho}. \) \hspace{1cm} (20b)

Let us take the condition in Eq. (20a) and the \([\text{SU}(4)_{L}]^2[\text{U}(1)_N]\) anomaly constraint

\( N_{Q_{1L}} + 2N_{Q_{aL}} = 0 \) \hspace{1cm} (21)

which, in turn, may be related to give

\( N_{Q_{aL}} = -\frac{1}{3} N_{\rho} \) \hspace{1cm} (22)

which establish the \(\text{U}(1)_N\) quark charge relations in units of \(N_{\rho}\)

\( N_{U} = \frac{2}{3} N_{\rho}, \) \hspace{1cm} (23a)

\( N_{D} = -\frac{1}{3} N_{\rho}, \) \hspace{1cm} (23b)

\( N_{Q_{1L}} = \frac{2}{3} N_{\rho}, \) \hspace{1cm} (23c)

\( N_{Q_{aL}} = -\frac{1}{3} N_{\rho}, \) \hspace{1cm} (23d)

\( N_J = \frac{5}{3} N_{\rho}, \) \hspace{1cm} (23e)

\( N_{j_{aL}} = -\frac{4}{3} N_{\rho}, \) \hspace{1cm} (23f)

and

\( N_{u'_{R}} = N_{Q_{1L}} = \frac{2}{3} N_{\rho}, \) \hspace{1cm} (23g)

\( N_{d'_{aR}} = N_{Q_{aL}} = -\frac{1}{3} N_{\rho}. \) \hspace{1cm} (23h)

These \(N\) charges for all quark flavors, together with the lepton ones in Eq. (12), allow to find the electric charges of all fermions in the 341 model by using the general expression of Eq. (8) that is,

\( Q_{\nu_i} = 0, \)

\( Q_\ell = -1, \)
\[ Q_U = \frac{2}{3}, \]
\[ Q_D = -\frac{1}{3}, \]
\[ Q_{u_1} = \frac{2}{3}, \]
\[ Q_{d_\alpha} = -\frac{1}{3}, \]
\[ Q_J = \frac{5}{3}, \]
\[ Q_{j_\alpha} = -\frac{4}{3}. \]

Summarizing, following the approach of the electric charge quantization in models that contain an explicit U(1) factor in the semisimple gauge group, we have shown that the quantization of electric charge occurs in the largest leptoquark-bilepton chiral gauge extension when the three families are taken together even for massless neutrinos. In the minimal 341 model the neutrinos remain massless since there is a global symmetry which prevents them from getting a mass. This symmetry implies the conservation of the quantum number \( F = L + B \), where \( L = L_e + L_\mu + L_\tau \) is the total lepton number and \( B \) is the baryon number \[28\]. If we allow this symmetry to be explicitly broken, Majorana neutrinos arise in the model if \( \langle H_{1,4}^0 \rangle \neq 0 \) which conserves the structure of the leptonic sector in Eq. (11). Also, if we add right-handed neutrino singlets, Dirac mass terms \( \bar{f}_i L \eta \nu_i R \) arise in the Yukawa couplings, but the gauge invariance of these terms implies \( N_{\nu_i} = 0 \). Then the charge quantization is unavoidable and does not depends on the nature of the neutral fields. Moreover, family replication, charge quantization, the existence of three colors and absence of massless charged fermions are interconnected in the minimal 331 leptoquark-bilepton model and its largest extension.

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