Mathematical model of the distribution of the coolant flow rate in the group distribution header and its use to determine the flow rate in the RBMK reactor channel

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Abstract. The paper proposes an elementary mathematical model of the distribution of coolant flows through water communications in the group distributing header (GDH) of a power unit with an RBMK reactor. This model used to determine the flow rate in the channel with a "forbidden" flow meter. An equation is given based on the mass balance of the coolant in the allocated volume. The results of the solution are used to construct an algorithm for determining the flow rate of the coolant in the channel. The results of the application of the technique on the measurement data from the power unit are presented. It is shown that the relative discrepancy with the measured value of the flow rate at a constant position of the flow control valve is on the order of several percent.

1. Introduction

In the practice of operating power units with RBMK reactors, situations arise when the readings of the coolant flow measurement system devices through the process channel cannot be trusted [1] (either because of an increase in measurement error or because of a failure in operability) [2]. There are known approaches to determining the flow rate using a mathematical model of the thermohydraulic channel of the channel [3] and information on the activation of the coolant [4].

The first approach is based on the use of a mathematical model of the thermohydraulic tract in the multiple forced circulation loop (CMPC) (see figure 1). The main circulation pumps continuously circulate the coolant in each loop of the CMPC. The measured pressure drop between the drum-separator and the pressure header is represented by the equation:

\[ P_{ph} - P_{ds} = \Delta P(W,G) + kG^2 \]  \hspace{1cm} (1)

where \( W \) is the channel power;
\( G \) is the flow rate of the coolant through the channel;
\( \Delta P(W,G) \) is channel thermal-hydraulic characteristic;
\( kG^2 \) is pressure loss across flow control valve;
\( k \) is model tuning factor.

Assuming that the pressure drop between the drum-separator and the pressure header, the thermohydraulic characteristic of the channel and the channel power are known, it is possible to determine the coolant flow rate by solving equation (1). The technique of thermohydraulic calculation of assemblies of fuel elements of nuclear reactors of the RBMK type is described, for example, in [5].
The second approach is based on the use of information on the activation of the coolant by fast neutrons during the passage of the core. In this case, the activity is measured by standard means of the fuel-element cladding tightness control system in a certain energy range (see figure 1). To determine the flow rate, a more complex mathematical model is used that relates the flow rate, power and concentration of radioactive nitrogen. In this case, the mathematical model (system of differential equations) as one of the blocks includes the same thermohydraulic calculation. The coolant flow rate is determined by solving the equation:

$$N_e = kN_m(W, G, l)$$

(2)

where $N_e$ is experimental value;
$N_m$ is mathematical model;
$l$ is the length of the steam-water communication;
$G$ is flow rate;
$k$ is model tuning factor.

In both the first and second cases, the successful solution of the problem is possible only after tuning the models according to the archived data of the power unit. It is also common that an isolated process channel is considered. At the same time, the coolant is supplied to the technological channels from group distributing headers (42 pcs.), to each of which several pipes for technological channels are connected (see figure 2). It is proposed to create a corresponding mathematical model and its application to determine the flow rate through the technological channel.
2. Mathematical model of the flow distribution in the group distributing header

Suppose that the water from GDH is evenly distributed over the water communications (see figure 2). Let $G_0$ is the flow rate at the entrance to the GDH; $G(x)$ is the flow rate in the section in the $x$ coordinate. Let's select the volume in the GDH from $x$ to $x + dx$ and get the balance equation.

The change in the mass of water in the allocated volume per unit of time is: \[ \frac{\partial}{\partial t} (\rho S dx) \], where $\rho$ is the density of the coolant; $S$ is the flow area of the GDH.

This change is due to the leakage of the coolant along the GDH and into the water communications. There is a leak along the GDH is

\[ \Delta G = \frac{G_0}{L} dx, \text{ where } L \text{ is GDH length.} \]

Thus, the balance equation is:

\[ \frac{\partial}{\partial t} (\rho S dx) = -\frac{\partial G}{\partial x} dx - \frac{G_0}{L} dx \]  

For stationary flow:

\[ \frac{\partial G}{\partial x} = -\frac{G_0}{L} \]

(4)

G(0) = 0

(5)

The solution to problem (4) - (5) is:

\[ G(x) = -\frac{G_0}{L} x + G_0 \]

(6)

Note that solution (6) shows the presence of a stagnant zone at the end of the GDH. Indeed, at $x = L$, the coolant velocity is 0, since $G(L) = 0$.

Equation (4) can be refined to describe the uneven flow rate of the coolant by introducing the function $\alpha(x)$ as follows:

\[ \frac{\partial G}{\partial x} = -\alpha(x) \frac{G_0}{L}, \int_0^L \alpha(x) dx = 1 \]

(7)

where $\alpha(x)$ is a function characterizing the uneven distribution of flow rate of the channels due to the different degrees of opening of the flow control valves.

The solution to equation (7) with (5) is:

\[ G(x) = -\int_0^x \alpha(z) \frac{G_0}{L} dz + G_0 \]

(8)

3. The algorithm for determining the flow rate in a channel with a "forbidden" flow meter

Since the GDH is considered in a linear dimension, we introduce into consideration, along with the GDH length, and $\Delta x$ is the channel diameter. Then the flow through the channel is defined as:

\[ G(x + \Delta x) - G(x) = -\int_x^{x+\Delta x} \alpha(z) \frac{G_0}{L} dz + G_0 - \left( -\int_0^x \alpha(z) \frac{G_0}{L} dz + G_0 \right) \]

\[ = -\int_x^{x+\Delta x} \alpha(z) \frac{G_0}{L} dz \]
Assuming that \( \alpha(z) \) is a piecewise constant function that differs from zero only in the places where the water communications of the channels are cut in, we obtain the expression for the flow rate through the \( j \)-th channel. If in the section from \( x \) to \( x+\Delta x \) there is a channel with index \( j \), then the amount of coolant flowing through this channel is:

\[
G_j = \alpha_j \frac{G_0}{L} \Delta x
\]

(9)

Coefficient \( \alpha_j \) is found according to the archived data as a result of measuring the coolant flow rate in the \( j \)-th channel:

\[
\alpha_j = \frac{\bar{G}_j L}{\Delta x}
\]

(10)

where \( \bar{G}_j \) is average value of the flow rate in the \( j \)-th channel during the observation time; \( \bar{G} = G_0 \) is the total value of all average values of expenses for this GDH.

Suppose that at time \( t \) the value of the flow rate in the \( j \)-th channel is unknown, but the flow rates in all other channels of this GDH are known, then from expressions (9) and (10) we can obtain:

\[
G_j(t) = \bar{G}_j \frac{1}{\sum_{i \neq j} \bar{G}_i} \sum_{i \neq j} G_i(t)
\]

(11)

Equation (11) reduces to:

\[
G_j(t) = \bar{G}_j \frac{\sum_{i \neq j} G_i(t)}{\sum_{i \neq j} \bar{G}_i}
\]

(12)

Thus, the flow rate in the \( j \)-th channel at time \( t \) is the product of the average flow rate in this channel up to time \( t \) by the ratio of the total flow rates in the GDH at time \( t \), excluding this channel, to the sum of average flows until time \( t \), again excluding this channel.

Estimation of the dispersion of the flow rate (12):

\[
D[G_j] = \left( \frac{\bar{G}_j}{\sum_{i \neq j} \bar{G}_i} \right)^2 D[\sum_{i \neq j} G_i] + \left( \frac{\sum_{i \neq j} G_i}{\sum_{i \neq j} \bar{G}_i} \right)^2 \left[ \sum_{i \neq j} D[G_i] + 2 \sum_{i \neq j} K_{ij} \right]
\]

(13)

where \( K_{ij} \) is the correlation moment between the flow rates in the \( i \)-th and \( j \)-th channels.

Let us estimate expression (13) from above. Suppose that the variance of all measured costs is the same, and the average costs themselves are equal, we get:

\[
D[G_j] < \left( \frac{\bar{G}_j}{\sum_{i \neq j} \bar{G}_i} \right)^2 \left( \sum_{i \neq j} D[G_i] \right) = \left( \frac{\bar{G}_j}{\sum_{i \neq j} \bar{G}_i} \right)^2 (n-1)D[G]
\]

(14)

\[
D[G_j] < \frac{3D[G]}{(n-1)}
\]

(15)

For \( n \) about 30 (the number of channels in one GDH), the dispersion of the flow rate calculated using the mathematical model (12) is an order of magnitude less than the dispersion of the flow rate measurement.
4. Results
The validity of this approach was tested on the archived data of one of the RBMK units as follows. The readings of the flow meters at time \( t \) were fictitiously forbidden and compared with those found by relation (12). For example, when averaging expenses for a month and restoring on the 18th day of the next month, the relative discrepancy did not exceed 4% (see table 1).

| Channel number | 63-21 | 63-22 | 63-23 | 63-24 | 63-40 | 63-41 | 63-42 | 63-43 |
|----------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Measured flow rate | 22.31 | 23.19 | 20.84 | 30.03 | 37.48 | 35.81 | 37.39 | 36.79 |
| Average flow rate (the previous month) | 20.37 | 22.22 | 20.87 | 30.17 | 37.35 | 35.68 | 26.94 | 36.74 |
| Estimated flow rate | 20.59 | 23.53 | 21.15 | 30.59 | 36.26 | 34.64 | 25.84 | 35.72 |
| Relative discrepancy of readings | 7.7  | 1.48  | 1.48  | 1.84  | 3.23  | 3.27  | 27.84 | 0.25  |

It can be seen from the table that with a fixed position of the flow control valve, this approach gives satisfactory results (the declared basic error of SHTORM-32M flow meters is ±1.5% [6-7]). Thus, the fundamental possibility of using the proposed approach for solving the problems of predictive diagnostics of flow meters based on the analysis of the archive [8-10] of the operating parameters of the power unit is shown.

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