Structure of the Particle-Hole Amplitudes in No-core Shell Model Wave Functions

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We study the structure of the no-core shell model wave functions for ⁶Li and ¹²C by investigating the ground state and first excited state electron scattering charge form factors. In both nuclei, large particle-hole (ph) amplitudes in the wave functions appear with the opposite sign to that needed to reproduce the shape of the (e, e′) form factors, the charge radii, and the B(E2) values for the lowest two states. The difference in sign appears to arise mainly from the monopole Δhω = 2 matrix elements of the kinetic and potential energy (T+V) that transform under the harmonic oscillator SU(3) symmetries as (λ, μ) = (2, 0). These are difficult to determine self-consistently, but they have a strong effect on the structure of the low-lying states and on the giant monopole and quadrupole resonances. The Lee-Suzuki transformation, used to account for the restricted nature of the space in terms of an effective interaction, introduces large higher-order Δhω contributions to the inelastic form factors. The latter ph excitations aggravate the disagreement between the experimental and predicted (e, e′) form factors with increasing model spaces, especially at high momentum transfers. For sufficiently large model spaces the situation begins to resolve itself for ⁶Li, but the convergence is slow. A prescription to constrain the ph excitations would likely accelerate convergence of the calculations.

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I. INTRODUCTION

The ab initio no-core shell model (NCSM) permits calculations of wave functions in very large model-space sizes for nuclei at the beginning of the p-shell. For ⁶Li calculations up to 16ℏω have been achieved [1]. Among the successes of the model is its predicted energy spectra of light nuclei [1, 2, 3]. Towards the end of the p-shell a 10ℏω basis calculation has been achieved for A=11 [4]. In mass 12 the model provides a reasonable description of the low-momentum component of the vector and axial currents involved in the electro-weak transitions to the ground state on ¹⁰N when a three-body interaction is included [5]. In all of these calculations, a Lee-Suzuki [6] transformation of the nucleon-nucleon interaction is used to account for the restricted nature of the space in terms of an effective interaction. Group theoretical analyses [7] of the no-core shell model wave functions have shown that the predicted eigenstates of ¹²C and ¹⁶O have very large overlaps with a small sub-space of the full model space, with the sub-space being defined by the most deformed symplectic basis states. The purpose of this paper is to examine the structure of the multi-ℏω terms in the wave functions for the low-lying states at the beginning and end of the p-shell in more detail. For this we compare NCSM predictions with measured elastic and inelastic (e, e′) charge form factors in ⁶Li and ¹²C.

The shape of the electron scattering form factors provides a direct probe of the magnitude and structure of the higher shell components in the wave functions. The charge form factors have the additional advantage that two-body meson-exchange currents do not contribute significantly to the form factors below about 2 fm⁻¹ [8]. In all (e, e′) calculations presented here we use a bare one-body operator; as discussed below, the introduction of an effective operator (to compensate for the truncated model space) does not significantly affect our conclusions.

Our first main finding is that the ph amplitudes that contribute significantly to electron scattering appear with the opposite sign to that needed to replicate the experimental form factors, elastic and inelastic, and the charge radii. The 2ℏω contributions to the inelastic form factor change sign, in agreement with experiment, for sufficiently large model spaces for ⁶Li; however, higher-order terms do not within the model spaces we examined. Second we show that the symplectic (λ, μ), Δhω = 2 ph amplitudes in the wave functions are sensitive functions of the oscillator parameter.

II. THE ELASTIC C₀ FORM FACTORS

The ground state C₀ form factor is the Fourier transform of the charge density, and contributions from two-body charge operators and/or relativistic corrections are negligible for momenta up to about 2 fm⁻¹ [8]. In a harmonic oscillator (HO) basis, the 0ℏω p-shell charge form factor is given by

\[ F_{0p-0p}(q^2) = \sqrt{3}(1 - 2/3y)\exp(-y) \]  

(1)

where \( y = (bg/2)^2 \) and \( b \) is the oscillator parameter.

When additional shells are added to the model space the new contributions to the form factor fall into two
main classes. The first of these are the *in-shell* contributions (e.g. $1s0d - 1s0d, 1p0f - 1p0f$, etc.) determined by the occupation numbers for the higher shells, and the second are from *cross-shell* $ph$ excitations (e.g. $0s - 1s, 0p - 1p, 0s - 2s$, etc.). At low $q$ the form factor is determined by the charge radius

$$ F(q^2) = 1 - \frac{< r^2 >}{6} q^2 + O(q^4) \quad (2) $$

and the higher in-shell contributions can be shown always to add constructively to the charge radius. The cross-shell excitations (which for HO wave functions contribute to the charge radius only for $\Delta \hbar \omega = 2 \, ph$ excitations across two shells) can add constructively or destructively. In a $2h\omega$ calculation for p-shell nuclei there are two possible cross-shell contributions, namely,

$$ F_{0s-1s}(q^2) = \sqrt{2/3y \ \exp(-y)} \quad (3) $$

$$ F_{0p-1p}(q^2) = \sqrt{10/3y(1-2/5y) \ \exp(-y)} $$

Both of these transform under SU(3) as $(\lambda, \mu) = (2, 0)$ and represent the $2h\omega$ symplectic contributions to the form factors. When these and/or higher shell $\Delta \hbar \omega = 2 \, ph$ excitations appear with a sign so as to enhance the predicted charge radius, they pull in the charge form factor in momentum space.

### A. C0 form factor for $^6$Li

Both elastic and inelastic scattering from $^6$Li have been studied extensively [9, 10, 11, 12, 13, 14]. Our calculations use the CD Bonn nucleon-nucleon interaction [15]. The predicted ground state energy of $^6$Li is least sensitive to the choice of the oscillator parameter $b$ over increasing basis size for $b = 1.79$ fm ($\hbar \omega = 13$ MeV) [14], which is the value we use in the present $(e,e')$ calculations. We note that, because of the increased computational difficulties in calculating the transition density matrix elements needed for the $(e,e')$ form factors, the present calculations are restricted to a maximum model space of $14h\omega$, to be compared with the $16h\omega$ model spaces used to calculate the energy spectra for $^6$Li.

Figure 1 compares the experimental data for the elastic form factor for $^6$Li with the model predictions. For model spaces up to $10h\omega$, the predicted form factor moves out further in momentum space as the basis is increased. For the largest model space examined, $14h\omega$, the tendency begins to reverse. In coordinate space the predicted charge density (Figure 2) is enhanced in the interior, with little change to the tail as higher shells are added. These trends reflect the structure and sign of $ph$ excitations introduced as the model space increases. There are two issues with the structure of the predicted ground state $ph$ excitations. First, the $\Delta \hbar \omega = 2 \, ph$ excitation for all shells included add destructively to the ground state charge radius. These suggest that the sign of the important symplectic excitations in the wave functions may be problematic. In momentum-space these excitations pull the form factor out further at higher $q$. These effects are shown in Figures 3.
FIG. 3: (color online) The ground state C0 form factor for $^6\text{Li}$. The figure displays the effect of particle-hole excitations on the predicted form factor by arbitrarily setting the one-body density matrix elements to zero. Experimental data are taken from [12].

and 4, where the relevant $ph$ one-body density matrix elements have been arbitrarily set to zero for the purposes of displaying their effect on the shape of the predicted form factor.

In Table 1 we show the contributions to the charge radius from in-shell versus cross-shell excitations, and the destructive interference from the $ph$ excitations is larger than the constructive interference from the higher in-shell excitations. This suggests that the predicted sign of the $\Delta h\omega = 2$ $ph$ excitations that transform under SU(3) as $(\lambda, \mu) = (2, 0)$ inhibits the convergence of the calculations.

TABLE I:

| Point charge radius for $^6\text{Li}$ in units of fm |
|------------------------------------------------------|
| Model space                  | $0h\omega$ | $2h\omega$ | $4h\omega$ | $10h\omega$ | $14h\omega$ | Expt. |
| full model space             | 2.23       | 2.08       | 2.1        | 2.14        | 2.21        | 2.38 ± 0.1 |
| $ph$ contributions omitted   | 2.23       | 2.26       | 2.31       | 2.38        | 2.41        |

B. C0 form factor for $^{12}\text{C}$

The trends seen for the elastic scattering form factor for $^6\text{Li}$ are also seen in the case of $^{12}\text{C}$. Again the calculations use the CD Bonn interaction [15]. The $\Delta h\omega = 2$ $ph$ excitations add destructively to the charge radius and pull the elastic C0 form factor out in momentum space, Figure 5. We note that the charge radius for $^{12}\text{C}$ (Table II) is over-predicted, which in part reflects the chosen oscillator parameter, $b = 1.663$ fm ($\hbar\omega = 15\text{MeV}$), which minimizes the ground state energy. We will discuss the choice and effect of $b$ in section V. As in the case of $^6\text{Li}$,
The C2 form factor is determined by the transition charge density; there is no significant contribution from two-body meson exchange currents below \( q \approx 2 \) fm\(^{-1}\). We examine the longitudinal form factor for scattering to the 2.186 MeV \((3^+ T=0)\) state in \(^6\)Li and the 4.44 MeV \((2^+ T=0)\) state in \(^{12}\)C. Data for the \(^6\)Li C2 transition have been measured to \( q \approx 3.5 \) fm\(^{-1}\) [12, 13, 14]. Extensive data are also available for the 4.44 MeV \((2^+\) state in \(^{12}\)C [17, 18, 19, 20].

The most significant contributions to the C2 form factors for \(p\)-shell nuclei in a \((0+2)\hbar\omega\) calculation are transitions within the \(p\)-shell \((0p \to 0p)\) and \(ph\) excitations across two shells that correspond to the excitation of the GQR. The latter transform under SU(3) as \((\lambda, \mu)L = (2, 0)2\). For HO wave functions the \(0p \to 0p\) and GQR form factors are [21]\

\[
F_{p-p}(q^2) = -\sqrt{8/15} \frac{y}{q^2} \exp(-y) 
\]

\[
F_{GQR}(q^2) = \sqrt{24/15} \frac{y(1-1/3y)}{q^2} \exp(-y),
\]

and as before \( y = (bq/2)^2 \). If a small admixture of the GQR is added to the \(0\hbar\omega\) state so as to enhance the B(E2), the form factor is suppressed at high \( q \). For larger model spaces higher powers of \( y \) are introduced.

The shape of the predicted \( F_L \) is often displayed in terms of the C2 matrix element [21]. In general, the \( C\lambda \) matrix element is defined in terms of the form factor \( F_\lambda \) as,

\[
B(C\lambda) = f^{-2} \frac{Z^2}{4\pi} \frac{(2\lambda+1)!!}{q^4} \frac{y}{A} \left( \frac{2\lambda+1}{2\lambda} \right)^2 F_\lambda^2
\]

and

\[
C2(q) \equiv B(C2)^{1/2} = A + Bq + Cy^2 + ...
\]

where \( f = f_{SN} f_{c.m.} \exp(-y) \), \( f_{SN} \) is the single-nucleon charge form factor [22] and the center of mass correction is \( f_{c.m.} = \exp(y/A) \) [23]. For a HO basis, the number of terms appearing in the polynomial [20] is determined by the number of shells included in the calculation. For \( p\)-shell nuclei the experimental \( C2(q) \) matrix elements for low-lying states generally decreases with increasing \( q^2 \), i.e., the coefficient ratio \( A/B < 0 \), where \( A > 0 \).

## A. C2 Form Factor in \(^{12}\)C

Our calculations for \(^{12}\)C include model spaces up to \(6\hbar\omega\). We use the CD Bonn [15] and the AV8′ [24] nucleon-nucleon interactions, as well as the AV8′ plus the Tucson-Melbourne TM′(99) 3-body [27] interactions. The oscillator parameter was taken to be \( b = 1.663 \) fm \((\hbar\omega = 15\) MeV).

Figure 7 shows a comparison between the measured and predicted form factors for increasing sizes of the shell.
model space. At low momentum transfers the calculations under-predict the form factor. Above \( q \approx 1.5 \text{ fm}^{-1} \) the calculations over-predict the form factor, and this over-prediction becomes increasingly worse as the size of the model space is increased. As the model space is increased beyond \( 0\hbar \omega \), the \( 0p \rightarrow 1p \) excitations add destructively at low \( q \) (and thus destructively to the the B(E2) value) and constructively at high \( q \), moving the form factor out. The predicted form factor is enhanced slightly at small \( q \) relative to the \( 0\hbar \omega \) calculation, and we largely attribute this to the \( 0s \rightarrow 1d \) excitations, which appear with the correct sign. We note this differs from the elastic \( C_0 \) form factors where excitations from the \( 0s \) shell appear with the same sign as excitations from the \( 0p \) shell. It should be noted that the model spaces examined here are restricted to \( 6\hbar \omega \) and that much larger spaces may well show very different trends.

Figure 8 displays \( C_2(q) \), which we extracted from the measured form factor using \( b=1.7 \) fm. The experimental \( C_2(q) \) matrix element steadily drops with increasing \( q^2 \). Our multi-\( \hbar \omega \) calculations predict \( C_2(q) \) to have the opposite slope, in large part because of the sign of the \( (0p)^{-1}(1p) \) excitations in the \( 0^+ \) and \( 2^+ \) wave functions.

Figure 9 displays the corresponding transition charge density \( \rho(r) \) for the \( 0^+ \rightarrow 2^+ \) transition. The experimentally determined \( \rho(r) \) peaks at about 2 fm, while the \( \rho(r) \) predicted by the NCSM peaks at about 1.5 fm. As the model space is increased, the peak moves towards smaller \( r \).

**B. The \( C_2 \) form factor for \( ^6\text{Li} \)**

Figure 10 displays a comparison of the measured and predicted form factors for increasing basis size for \( b=1.79 \) fm. These calculations use the CD Bonn nucleon-nucleon interaction \[15\]. At low momentum transfers, \( q < 1.0 \text{ fm}^{-1} \), the calculations under-predict the form factor. At \( q > 1.5 \text{ fm}^{-1} \), the larger model spaces over-predict the magnitude of the form factor, and this over-prediction increases with the model space size. The general trends seen with the sign of higher shell contri-

![Figure 8: (color online) The C2 matrix element for the 4.44 MeV (2\(^+\) T=0) state in \(^{12}\text{C}\). The effect of using a three-body interaction (AV\(^8\)+TM) is small. Experimental data were taken from 17, 18, 19, 20.](image1)

![Figure 9: (color online) The transition charge density for the 4.44 MeV (2\(^+\) T=0) state in \(^{12}\text{C}\). As the model space is increased the transition density moves towards smaller \( r \).](image2)

![Figure 10: (Color online) The charge form factor for the 3\(^+\) 2.186 MeV state using \textit{ab initio} NCSM CD Bonn wave functions, where \( b = 1.79 \) fm. At this value of \( b \) (\( \hbar \omega = 13 \) MeV), the ground state energy converges fastest over basis size. Empirical data are taken from Stanford, Saskatoon, and Mainz 12, 13, 14.](image3)
FIG. 11: (Color online) The charge form factor is calculated with $b = 1.94$ fm ($h\omega = 11$ MeV) for comparison to figure 10. The predictions are suppressed at low $q$ and enhanced at high $q$ for all size model spaces. Experimental data are taken from [12, 13, 14].

The contributions in the predicted C2 form factors are very similar to those seen for $^{12}$C. However, our ability to go to considerably higher shells in the case of $^6$Li allows us to explore these trends in more detail. The peak of the predicted form factor occurs at higher $q$ than experiment; the biggest shift of the predicted form factor away from the observed peak occurs between the $0h\omega$ and the $2h\omega$ model space, and a shift in the peak position to even higher $q$ continues until $8h\omega$. Above $8h\omega$ the peak position of the form factor begins to improve. Above the peak, at momenta $q > 1.3$ fm$^{-1}$, the additional contributions from higher shell continue to enhance the form factor. For low momentum transfers (below the peak of the form factor) we see a slow convergence to a magnitude lower than experiment. This low momentum trend is consistent with the trend of predicted B(E2) values, as summarized in Table III.

We also examined the form factor for a set of calculations with a different oscillator parameter, namely, $b = 1.94$ fm ($h\omega = 11$ MeV), Figure 11. The $b = 1.94$ fm form factors display similar qualitative behavior as the $b = 1.79$ fm calculations. The peak of the former occurs at higher $q$ than experiment and continues to shift outward until about $10h\omega$. For large model spaces the situation starts to improve. Above the peak of the form factor the higher shell contributions move the form factor further out in $q$ with increasing model space.

The enhancements for the form factors at large $q$ are determined by the sign and magnitude of the higher shell contributions in the wave functions. As in the case of $^{12}$C, this is most striking in the case of the $2h\omega$ configurations, where the $0p \rightarrow 1p0f$ $ph$ excitations add destructively at low $q$ and constructively at high $q$. The slow convergence of the B(E2) to a value smaller than experiment is due in large part to the fact that these $ph$ excitations add destructively to the matrix element.

A more detailed understanding of the convergence of the lower momentum terms in the form factor with increasing model space can be seen by examining the C2 matrix element. To obtain an experimental C2 matrix element, we chose an oscillator parameter $b = 1.70$ fm, which is close to the value necessary to give the measured rms charge radius. The C2 calculated matrix elements are displayed in Figures 12 and 13. The experimental C2 matrix element decreases with increasing momentum transfer in contrast to the predictions of the model.

We graph the ratio of the coefficients $B/A$ appearing in eq. 6 in Figure 14 as a function of basis size. For the smaller model spaces this ratio has the wrong sign, but as the model space increases the sign eventually changes in qualitative agreement with experiment. For $b = 1.79$ fm, the ratio changes sign between $4h\omega$ and $6h\omega$ model spaces; for the $b = 1.94$ fm, the sign switches between $8h\omega$ and $10h\omega$.

Except for the largest model space examined, we see an approximate linear relationship between basis size and
$y = (bq/2)^2$

$C_2$ matrix element (in $\text{fm}^{-2}$)

FIG. 13: (Color online) The $C_2$ matrix element with $b = 1.94$ fm. Experimental data are taken from [12, 13, 14] and a $C_2$ matrix element extracted using $b = 1.70$ fm.

$B/A$ and $C/A$, Figs. 14, 15. The $14\hbar\omega$ model space calculation suggests that the rate of convergence starts to increase faster than this linear relationship suggests.

FIG. 14: (Color online) We fitted the $C_2$ matrix element to the polynomial $A + By + Cy^2$. The ratio of $B/A$ is graphed along the $y$-axis. Since $A$ is always positive, the change in sign is exclusively in $B$. Experimental data was taken from [12, 13, 14] and a $B/A$ ratio was extracted using $b = 1.70$ fm.

$B/A$ and $C/A$, Figs. 14, 15. The $14\hbar\omega$ model space calculation suggests that the rate of convergence starts to increase faster than this linear relationship suggests.

FIG. 15: (Color online) The ratio $C/A$ (eq. (3)) from a fit to the $C_2$ matrix element in $^6\text{Li}$ as a function of the model space. Experimental data are taken from [12, 13, 14], and a $C/A$ ratio was extracted using $b = 1.70$ fm.

IV. INCLUSION OF A 3-BODY INTERACTION

The inclusion of a 3-body interaction leads to an improved predicted level spectrum in $^6\text{Li}$, particularly for the splitting between the ground state and the first $3^+$ state. In addition, the magnetic form factor for the $0^+ \rightarrow 1^+$ transition in $^{12}\text{C}$ is significantly improved when a 3-body interaction is included [6]. This is because the predicted form factor is very sensitive to the strength of the spin-orbit interaction. However, the present ($\Delta L = \lambda$, $\Delta S = 0$) $C\lambda$ charge form factors are largely insensitive to the strength of the spin-orbit interaction and consequently to the 3-body interaction. In Figure 16 we compare the predicted $4\hbar\omega$ and $6\hbar\omega$ predictions for the elastic $C_0$ form factor for $^6\text{Li}$. Figure 17 shows the equivalent calculations for the transition $C_2$ form factor to the $3^+$ state. In both cases the inclusion of the 3-body interaction has little effect on the predicted form factor although it does improve the shape at higher $q$ very slightly. Figure 7 shows the effect of the 3-body interaction for the inelastic $C_2$ form factor of $^{12}\text{C}$, which is also very small.

V. DEPENDENCE ON THE OSCILLATOR PARAMETER

The unexpected sign for some of the higher shell components in our NCSM calculations bears strong resemblance to a similar problem found in standard multi-$\hbar\omega$ HO shell model calculations. When HO standard shell model calculations are extended to include multi-$\hbar\omega$ configurations the lack of self-consistency (in the Hartree-Fock sense) causes some of the higher shell components in the wave functions to be unphysical [26, 27, 28, 29, 30].
The main problem arises because matrix elements of the kinetic energy ($T$) and the two-body interaction ($V$) across two shells ($\Delta \hbar \omega = 2$) are large and opposite in sign, and they cannot be calculated reliably. The dependence of the C2 matrix element on $b$ implies that the magnitude and even the sign of the $\langle ph | T + V | 0 \hbar \omega \rangle$ matrix elements depend on $b$. These off-diagonal matrix elements across two shells in turn affect the sign of the leading $ph$ excitations in the wave functions, as well as all of the similar $\Delta \hbar \omega = 2$ matrix elements up to the maximum shell included in the calculation.

### A. Dependence of $^6$Li Form factors on the Oscillator Parameter

We investigated the effect of the oscillator parameter on the predicted form factor within the $2\hbar \omega$ model space, using four different values of $b$ ranging from $b = 1.66 - 1.94$ fm ($\hbar \omega = 15 - 11$ MeV), as shown in Figure 18. As $b$ varies so does the predicted shape of the form factor: the width of the peak becomes narrower, and the peak itself shifts to lower momentum values with increasing $b$. The change in slope of the C2 matrix element with $b$ (Figure 18b) suggests that for sufficiently small $b$, the slope of the C2 matrix element will become negative, qualitatively agreeing with experiment. But such a small value of $b$ would likely result in very slow convergence of the calculations.

### B. Dependence of the $^{12}$C Form Factor on the Oscillator Parameter

In Figure 19 we display the dependence of the predicted C2 matrix element in $^{12}$C on the oscillator parameter. These calculations were restricted to a $(0 + 2)\hbar \omega$ model space. As the oscillator parameter is varied, the value of $\langle ph | T + V | 0 \hbar \omega \rangle$ changes considerably and eventually changes signs. For sufficiently small $b (< 1.33 fm)$ the slope of C2($q$) becomes negative, in qualitative agreement with experiment.

### C. Dependence of the Giant Resonances on the Oscillator Parameter

Hoshino et al. [32] have pointed out that the problem with the sign of matrix elements of $(T + V)$ in multi-$\hbar \omega$ shell model calculations also manifests itself in the
predicted excitation of the GQR and GMR. The excitation energy of the GMR reflects the compressibility of the nucleus. Both are intrinsic properties of the nucleon-nucleon interaction; and, the excitation energy should not depend on the properties of the HO well. In Figure 20 we show the predicted E0 strength for two different values of the oscillator parameter for our (0+2)\(\hbar\omega\) model space. The large shift in the predicted excitation of the GMR from \(\approx 35\) MeV to \(\approx 60\) MeV occurs because of the change in the off-diagonal \(\langle ph | T + V | 0\hbar\omega\rangle\) matrix elements for the two values of the oscillator parameter. An analogous problem is seen with the E2 strength, Figure 21. We note that the sensitivity of the excitation energy of the giant resonances to the oscillator parameter would likely be considerably less for larger model spaces. But our (0+2)\(\hbar\omega\) model space calculations exhibit similar sensitivity to that seen by Hoshino et al. [32]. As in the case of the \((e, e')\) form factors, the problem can be directly traced to the \(\Delta\hbar\omega = 2\), \((\lambda, \mu) = (2, 0)\) \(ph\) excitations.

D. Effect of an effective two-body electron scattering operator

Since the model space sizes discussed here are finite, it is important to address the issue of the impact of effective \((e, e')\) operators and whether they can correct for the sign of the \(\Delta\hbar\omega = 2\) \(ph\) excitations in the wave functions. Stetcu et al. [34] investigated how a two-body effective contribution affects the E2 and C2 operators. In the \(0\hbar\omega\) space, they found that the two-body operator moves the form factor in the same direction as the larger \((0+2)\hbar\omega\) calculations move; that is, the two-body operator results in the same unphysical enhancement of the form factor at large \(q\).

VI. CONCLUSION

We have calculated the elastic C0 and the first excited state C2 charge form factors and the corresponding C2 matrix elements in \(^6\text{Li}\) and \(^{12}\text{C}\) within the NCSM using one-body bare operators. These calculations reveal two primary findings. First, the magnitude and sign of higher shell \(ph\) amplitudes in the wave functions do not behave as expected. Higher shell contributions add destructively at low \(q\) and constructively at high \(q\) to the form factors, contrary to experimental and theoretical expectations. The relative sign of the symplectic \((\lambda, \mu) = (2, 0)\), \(\Delta\hbar\omega = 2\) amplitudes cause them to add destructively to the charge radii. The large \(0s \to ns\) (and \(0p \to np\) for \(^{12}\text{C}\)) amplitudes, introduced by the Lee-Suzuki transfor-
nformation, also affect the shape of the form factors, further increasing the magnitude of the form factors at high momentum transfers. In the larger model spaces we explored for $^6\text{Li}$, the sign of the $2\hbar \omega$ contributions to the inelastic form factor changes, but convergence onto experiment is slow.

The second main finding is the strong dependence of the magnitude and sign of the off diagonal $\Delta \hbar \omega = 2$ matrix elements of $T + V$ on the oscillator parameter. As a result, the observables (the C2 form factor, GMR, GQR) also heavily depend on the choice of oscillator parameter. Furthermore, the $\phi h$ configurations in the low lying wave functions appear with an unexpected sign. These results indicate a lack of self-consistency in the NCSM similar to that found in the standard HO shell model. While there is no known solution, the effects may be minimized by including a Hartree-Fock condition in the calculations. Further, the prescription to handle the $\Delta \hbar \omega = 2$ and higher $\phi h$ excitations may lead the NCSM to have as much success in predicting momentum-based observables as in predicting energy spectra.

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Certainly this method bears further investigation. But in any case a correct treatment of the symplectic terms in the wave functions is crucial to obtaining a realistic description of electron scattering form factors within a HO shell model basis. Invoking eq. (7) and/or another prescription to handle the $\Delta \hbar \omega = 2$ and higher $\phi h$ excitations may lead the NCSM to have as much success in predicting momentum-based observables as in predicting energy spectra.

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