Vectorial eigenvector method for simulating the polarization dependent resonator

Youyou Hu, Xiaofeng He, Sisi Liu, Xiahui Tang, and Bo Li*

Wuhan National Laboratory for Optoelectronics, School of Optical and Electronic Information, Huazhong University of Science and Technology, Wuhan 430074, China

*libohust@gmail.com

Abstract: Derived from the eigenvector method (EM), a vectorial eigenvector method (VEM) for the mode calculation of polarization dependent resonator is presented. New transfer matrices of laser resonant cavities with polarization-selective devices are built by using Jones matrix representation. The eigenvectors computed by solving the matrix eigenequation manifest the mode characteristics of resonators, including cylindrical vector (CV) modes. Comparing with EM, the VEM inherits EM's superiorities and extends scalar models to vector models. Then, an example utilizing VEM to simulate axicon mirror resonator was given, which proved the VEM feasible and efficacious. Finally, upon a fast axial flow (FAT) CO\textsubscript{2} laser with an axicon mirror as rear mirror, we experimentally corroborated the validity of the VEM and a working-stable 1.5 kW radially polarized beam was obtained.

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1. Introduction

In the past ten years, cylindrical vector (CV) beams, such as azimuthally polarized beams and radially polarized beams, have attracted great interests of many research groups. Heretofore, many methods of generating CV beams by a twisted nematic liquid crystal [1], inserting phase elements in the laser resonator [2], space-variant dielectric subwavelength dielectric gratings [3], a conical Brewster prism [4], diffraction gratings with polarization selectivity [5–7], Birefringence-induced bifocusing [8] and metasurfaces [9] have been reported. Polarization selective diffraction gratings and axicon mirrors are commonly utilized as rear mirrors to generate high power CV beams [5–7,10] because of their high thermal stability and high polarization selectivity. Due to the special polarization distribution of cylindrical symmetry, CV beams have been extensively explored for the applications of electron acceleration [11], optical trapping [12], optical microscopy [13], optical storage [14], laser processing [15,16], plasma excitation [17] and so on. For example, high power radially polarized laser is of great significance to improve the speed and quality of laser cutting and welding in industrial applications.

Meanwhile, the modes and the energy distribution of these polarization sensitive resonators are very significant for the generation and application of CV beams. Although the Fox-Li iteration method has been used to analyze vectorial optical field in the laser resonator, it requires hundreds times of iteration and it’s pretty hard to distinguish the distribution of each mode. Besides, the eigenvector method (EM), based on finite element method (FEM), is also given to simulate laser resonators, but it's only applicable to scalar optical field [18,19].

In this paper, upon the scalar eigenvector method (SEM), we present a vectorial eigenvector method (VEM), which can be used to compute the CV modes of resonators. The VEM not only inherits the advantages of SEM that deriving lots of modes distributions at a time, but also applies to polarization dependent resonators, such as laser resonant cavities with diffraction gratings, axicon mirrors, wave plates, linear polarizers and so on. In addition, the examples utilizing VEM to simulate axicon mirror resonators and study the polarization mode selection of radial and azimuthal polarization were given to prove the correctness of the VEM. Ultimately, we demonstrated an experiment on a test platform of fast axial flow (FAT)
CO₂ laser. We obtained a high quality 1.5 kW radially polarized beam, and the spot mode of the beam is consistent with the calculated results by the VEM.

2. Principle of the vectorial eigenvector method

Owing to the self-contradictions of paraxial wave optics theories, nonparaxial vector beam theories like the perturbation series method, angular spectrum representation, vectorial Rayleigh-Sommerfeld diffraction integral have been used to solve the exact solution of electromagnetic field [20–23]. Among them, the vectorial Rayleigh-Sommerfeld diffraction integral has been widely applied in the transmission of azimuthally and radially polarized beams [24,25]. In paraxial approximation, the vectorial Rayleigh-Sommerfeld diffraction integral can be expressed as:

\[
E_x(x, y, z) = -i \frac{\exp(ikz)}{\lambda z} \int_{-\infty}^{\infty} E_x(x_0, y_0, 0) \exp\left(\frac{ik}{2z}[(x-x_0)^2+(y-y_0)^2]\right) dx_0 dy_0, \tag{1}
\]

\[
E_y(x, y, z) = -i \frac{\exp(ikz)}{\lambda z} \int_{-\infty}^{\infty} E_y(x_0, y_0, 0) \exp\left(\frac{ik}{2z}[(x-x_0)^2+(y-y_0)^2]\right) dx_0 dy_0, \tag{2}
\]

\[
E_z(x, y, z) = \frac{i}{\lambda} \frac{\exp(ikz)}{z^2} \int_{-\infty}^{\infty} \left[(x-x_0)E_x(x_0, y_0, 0)+(y-y_0)E_y(x_0, y_0, 0)\right]
\times \exp\left(\frac{ik}{2z}[(x-x_0)^2+(y-y_0)^2]\right) dx_0 dy_0. \tag{3}
\]

Where \(E_x(x_0, y_0, 0)\) and \(E_y(x_0, y_0, 0)\) are separately the electric field component of X and Y direction on source field, \(E_x(x, y, z)\), \(E_y(x, y, z)\) and \(E_z(x, y, z)\) are the electric field component of X, Y and Z direction on far field, \(\lambda\) is the wavelength and \(k\) is the wave vector.

In the case of approximative axis, regardless of the diffraction in the free space or through the small hole, \(E_z(x, y, z)\) is almost negligible [21]. Thus, it’s reasonable to think only of the electric field component of \(E_x(x, y, z)\) and \(E_y(x, y, z)\), and the vectorial Rayleigh-Sommerfeld diffraction integral degenerates into Fresnel diffraction integral of \(E_x\) and \(E_y\) components.

For polarization sensitive resonator, the vector characteristics of optical field can never be neglected. As shown in Fig. 1, the action of polarized devices is equivalent to a thin film attached to rear mirror.

![Fig. 1. Laser Resonator.](image)

According to Jones vector theory, a beam of arbitrarily polarized laser can be decomposed into a set of orthogonally polarized lights, such as a pair of orthogonally linearly polarized lights or a pair of left-handed and right-handed circularly polarized lights. So, \(U_1(x, y)\) and \(U_2(x, y)\), the fields distributions on mirror 1 and 2, can be discretized into:
\[ U_i(x,y) = \begin{bmatrix} U_{i1}(x,y) \\ U_{i2}(x,y) \end{bmatrix} = \begin{bmatrix} u_{i1}[1] & u_{i1}[2] & \ldots & u_{i1}[s] & u_{i1}[1] & u_{i1}[2] & \ldots & u_{i1}[s] \end{bmatrix}^T \] \tag{4}

\[ U_2(x,y) = \begin{bmatrix} U_{2x}(x,y) \\ U_{2y}(x,y) \end{bmatrix} = \begin{bmatrix} u_{2x}[1] & u_{2x}[2] & \ldots & u_{2x}[g] & u_{2x}[1] & u_{2x}[2] & \ldots & u_{2x}[g] \end{bmatrix}^T \] \tag{5}

Where \( T \) is a transpose operator, \( s \) is the mesh number of mirror 1, \( u_{1x}[m] \) and \( u_{1y}[m] \) (\( m = 1, 2, \ldots, s \)) are the sampling field of the \( m \)th elements on mirror 1, \( g \) is the mesh number of mirror 2, and \( u_{2x}[n] \) and \( u_{2y}[n] \) (\( n = 1, 2, \ldots, g \)) are the sampling field of the \( n \)th elements on mirror 2.

Then, the transmission matrices \( A_{12} \) from mirror 1 to 2 and \( A_{21} \) from mirror 2 to 1 are written as

\[
A_{12} = \begin{bmatrix}
A_{12x} & 0 \\
0 & A_{12y}
\end{bmatrix} = \begin{bmatrix}
A_{12x}[1,1] & A_{12x}[2,1] & \ldots & A_{12x}[s,1] & 0 & 0 & \ldots & 0 \\
A_{12x}[1,2] & A_{12x}[2,2] & \ldots & A_{12x}[s,2] & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \ldots & 0 & A_{12x}[1,1] & A_{12x}[2,1] & \ldots & A_{12x}[s,1] \\
0 & 0 & \ldots & 0 & A_{12x}[1,2] & A_{12x}[2,2] & \ldots & A_{12x}[s,2] \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \ldots & 0 & A_{12x}[1,s] & A_{12x}[2,s] & \ldots & A_{12x}[s,g]
\end{bmatrix}
\]

\[
A_{21} = \begin{bmatrix}
A_{21x} & 0 \\
0 & A_{21y}
\end{bmatrix} = \begin{bmatrix}
A_{21x}[1,1] & A_{21x}[2,1] & \ldots & A_{21x}[g,1] & 0 & 0 & \ldots & 0 \\
A_{21x}[1,2] & A_{21x}[2,2] & \ldots & A_{21x}[g,2] & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \ldots & 0 & A_{21x}[1,1] & A_{21x}[2,1] & \ldots & A_{21x}[g,1] \\
0 & 0 & \ldots & 0 & A_{21x}[1,2] & A_{21x}[2,2] & \ldots & A_{21x}[g,2] \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \ldots & 0 & A_{21x}[1,s] & A_{21x}[2,s] & \ldots & A_{21x}[g,s]
\end{bmatrix}
\]

Where \( A_{12x}[m,n], A_{12y}[m,n], A_{21x}[n,m] \) and \( A_{21y}[n,m] \) (\( n = 1, 2, \ldots, g \) and \( m = 1, 2, \ldots, s \)) are derived from Eq. (1) and Eq. (2) [18,19].

Moreover, for the polarization selective devices, diffraction gratings or axicon mirrors, azimuthal and radial polarization have different reflection coefficients and phase shifts. So, additional matrices of polarization selection should be established. When utilized as rear mirrors of resonators, the grating mirrors or axicons have influences on amplitudes and phases of optical field. The transformation of beam at the gratings or axicons changes the polarization as follows:

\[ P_i = \text{Diag}(R_{i1}\exp(iP_{i1}), \ldots, R_{i1}\exp(iP_{im}), R_{i2}\exp(iP_{in}), \ldots, R_{i2}\exp(iP_{in})). \] \tag{8}
Where \( \text{Diag} \) is an operator to create a diagonal matrix, \( R_{\text{te}} \) and \( P_{\text{te}} \) are the reflection coefficient and phase shift of azimuthal polarization, and \( R_{\text{tm}} \) and \( P_{\text{tm}} \) are the the reflection coefficient and phase shift of radial polarization.

In order to realize the transformation of coordinate system between Cartesian coordinate system and cylindrical coordinate system, transformation matrices of coordinate are written as

\[
T_1 = \begin{bmatrix}
\cos(\phi_1) & 0 & \ldots & 0 & \sin(\phi_1) & 0 & \ldots & 0 \\
0 & \cos(\phi_2) & \ldots & 0 & 0 & \sin(\phi_2) & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & \ldots & \cos(\phi_\theta) & 0 & 0 & \ldots & \sin(\phi_\theta) \\
-\sin(\phi_1) & 0 & \ldots & 0 & \cos(\phi_1) & 0 & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & -\sin(\phi_2) & \ldots & 0 & 0 & \cos(\phi_2) & \ldots & 0 \\
\end{bmatrix}, \quad (9)
\]

\[
T_2 = T_1^T. \quad (10)
\]

Where \( T_1 \) transforms field in Cartesian coordinate system to field in cylindrical coordinate system, and \( T_2 \) makes inverse operation.

Waveplate is another optical device that alters the polarization state of a light wave travelling through it. Two cascaded \( \lambda/2 \) plates have been demonstrated for polarization conversion between azimuthal and radial polarization [26].

For \( \lambda/2 \) plate, if the angle between the fast axes and X-axis is \( \theta \), of which the Jones matrix can be shown to be

\[
G = \begin{bmatrix}
\cos(2\theta) & -\sin(2\theta) \\
\sin(2\theta) & \cos(2\theta)
\end{bmatrix}. \quad (11)
\]

Assuming that \( \theta_1 \) and \( \theta_2 \), the angles between the fast axes of \( \lambda/2 \) plate 1 and 2 and X-axis, are 0 and \( \pi/4 \), then the Jones matrix of two cascaded \( \lambda/2 \) plates is

\[
G_1 = \begin{bmatrix}
\cos(2\theta_2) & -\sin(2\theta_2) \\
\sin(2\theta_2) & \cos(2\theta_2)
\end{bmatrix} \begin{bmatrix}
\cos(2\theta_1) & -\sin(2\theta_1) \\
\sin(2\theta_1) & \cos(2\theta_1)
\end{bmatrix} = \begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix}. \quad (12)
\]

In the field of high power CO2 lasers, \( \lambda/2 \) plate is usually replaced by two cascaded reflective \( \lambda/4 \) plates. And the matrix of the two cascaded \( \lambda/2 \) plates for VEM as follows:

\[
P_2 = \begin{bmatrix}
0 & 0 & \ldots & 0 & -1 & 0 & \ldots & 0 \\
0 & 0 & \ldots & 0 & 0 & -1 & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & \ldots & 0 & 0 & 0 & \ldots & -1 \\
1 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 \\
0 & 1 & \ldots & 0 & 0 & 0 & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & \ldots & 1 & 0 & 0 & \ldots & 0
\end{bmatrix}. \quad (13)
\]

Then, for the grating mirror resonator, we can get the total transfer matrix expressed as \( A = A_2 T_1^P P_1 T_1 A_{12} \). Similarly, for the copper axicons mirror resonator with four cascaded reflective \( \lambda/4 \) plates to achieve radial polarization mode selection, the total transfer matrix is written as \( A = A_2 P_2 T_1^P P_1 T_1 P_2 A_{12} \).
Under self-reproducing condition, the issue of solving the stable vector modes in a polarization dependent resonator is converted into solving the complex eigenvectors ($U$) corresponding to eigenvalues ($\gamma$) of the transfer matrix ($A$). So, each column vector in $U$ is an eigenmode distribution of the resonator. The eigenvalue $\gamma$ is concerned with the round-trip loss of the resonator $\delta = 1 - |\gamma|^2$, including the round-trip diffraction loss and the loss resulting from rear mirror.

3. Numerical simulations and analysis

The mode calculations of the equivalent circular semi-confocal resonators with axicon mirror as rear mirror are taken to demonstrate the feasibility and reasonability of VEM. The cylindrical coordinates are used in calculation and, partitions of resonator mirrors are made by space area discrete method instead of space angle discrete method, which can effectively improve the computational efficiency.

The calculated field distributions of three emblematical modes are listed in Table 1 with resonator parameters: diameter of the planar mirror 1 of $2a_1 = 20 \text{ mm}$, diameter of the circular spherical mirror 2 of $2a_2 = 20 \text{ mm}$, radius of the equivalent spherical mirror 2 of $R_2 = 10 \text{ m}$, azimuthally and radially polarized modes’ reflection coefficients of the rear mirror 2 of $R_{ue} = 0.99$, $P_{ue} = 0$ and $R_{tm} = 0.93$, $P_{tm} = 0$ respectively, equivalent resonator length of $L = 5\text{ m}$, wavelength of $\lambda = 10.6 \text{ um}$.

The degree of polarization (DP) of beams with axial polarization is given by [15]:

$$DP = \frac{(I_r - I_\phi)}{(I_r + I_\phi)}$$

Where $I_r$, $I_\phi$ are intensities of radial and azimuthal electric field components. $DP = 1$ and $DP = -1$ mean completely radial polarization and azimuthal polarization respectively.

According to the results of calculation, it has been authenticated that the center light intensities of CV modes are 0, which are consistent with the theoretical analysis that there are polarization singularities in the center of CV modes [10]. It’s confirmed that a group of orthogonally polarized TEM$_{01}$ modes can be combined into an azimuthally polarized beam or a radially polarized beam [27]. In addition, we have also obtained optical field distributions of higher order azimuthally polarized and radially polarized modes. We can find that both mode TE$_{01}^*$ and mode TEM$_{00}$ are the eigen modes of polarization sensitive resonator and TE$_{01}^*$ most possibly appears in the cavity due to its lowest loss.
Table 1. Optical field distribution corresponding to the eigenvalues with $Rte = 0.99$ and $Rtm = 0.93$

| No. | 1        | 2        | 3        |
|-----|----------|----------|----------|
| $|\gamma|\gamma|$ | 0.98372  | 0.97936  | 0.91417  |

**Distribution of $E_x$**

- No. 1
- No. 2
- No. 3

**Distribution of $E_Y$**

- No. 1
- No. 2
- No. 3

**Optical field distribution**

- No. 1
- No. 2
- No. 3

**Section map of X-Z plane**

- No. 1
- No. 2
- No. 3

| DP     | Mode  |
|--------|-------|
| $-1$ (Azimuthal polarization) | $TE_{01^+}$ |
| 0      | $TEM_{00}$ |
| 1 (Radial polarization)      | $TM_{02^+}$ |
Table 2. Optical field distribution corresponding to the Rte with Rtm = 1, Pte = 0 and Ptm = 0.

| No | Rte | 1 | 2 | 3 |
|----|-----|---|---|---|
| 0.99 | 1 | ![graphic](image1) | ![graphic](image2) | ![graphic](image3) |
| 0.95 | 2 | ![graphic](image4) | ![graphic](image5) | ![graphic](image6) |

Then, with the assistance of VEM, we study the characteristics of polarization dependent resonators. We set the parameters of polarization-selective device that Rtm = 1, Pte = 0 and Ptm = 0, and let Rte reduce from 1 to 0.95 by an interval of 0.005. As illustrated in Table 2, the calculated field distributions of the three largest eigenvalues of several cases are listed.

In Table 2, we can find that when Rte = 1 and Rtm = 1, polarization dependent resonator degenerates into an ordinary spherical mirror resonator. The optical field distribution of each mode is in accordance with the results computed by SEM, which once again proves the correctness of VEM. For polarization dependent resonator, the modes depend on the difference of reflectivity between azimuthal and radial polarization. When (Rte - Rtm) > ΔRmin, azimuthally polarized mode TE01* most probably appears in the resonator due to its lowest loss. Similarly, when (Rtm - Rte) > ΔRmin, radial polarization mode TM01* would be selected by mode competition. ΔRmin is the minimum difference of reflectivity, approximately equal to 0.05, which is consistent with the experience value of 5% [28].

To sum up, the calculated results are the same as that obtained by the known analytical solution or numerical method (Fox-Li method), which proves that VEM is highly feasible and reasonable for mode simulations of polarization dependent resonators. We will use the VEM to guide our design of high power radially polarized CO2 laser.
4. Experiments

The experiments for generating radially polarized beam were carried out on a FAT CO₂ laser. As we all know, axicon mirror with polarization sensitive dielectric has been used for generation of radially polarized optical beam [29]. However, it's extremely difficult and expensive to coat a metal axicon mirror with polarization selective film. So, We chose the copper axicon mirrors with azimuthal polarization selection, which are directly processed by diamond turning. Moreover, two \( \lambda/2 \) phase shifters can be used to achieve the conversion of azimuthal polarization and radial polarization in the cavity. As sketched in Fig. 2, we have redesigned a 45-degree three-fold cavity structure. The axicon mirrors with azimuthal polarization selection and four \( \lambda/4 \) phase shifters forming two \( \lambda/2 \) phase shifters are used to obtain the output of radially polarized light. The length of the resonator is 5.6 m, and the radius of curvature of the 50% transmittance ZnSe output mirror is 15 m.

![Fig. 2. Resonator structure, 1-compound axicon mirror, 2~5-\( \lambda/4 \) phase retardation mirror, 6-output mirror, 7-gain section, where \( \theta = 45^\circ \)](image)

As illuminated in Fig. 3(a), we used the VEM to calculate the ideal output mode of the 45-degree three-fold resonator, which is a radially polarized annular Gauss mode \( \text{TM}_{01}^* \). Based on the above-mentioned system, the 1.5 kW laser radiation operating at 10.6 μm is yielded and the beam pattern is recorded by an acrylic plate, which is set about 2.2 m away from the output coupler. In accordance with the simulated result, a doughnut shaped burn pattern with null intensity in the beam center is shown in Fig. 3(b). A linear polarizer PAZ-35-WC-2 provided by II-VI INFRARED is employed to test the polarization of the doughnut mode at 1.5 kW. By rotating the polarizer axis (the black double-headed arrow) in horizontal, vertical, and 45° and 135° directions, respectively, the burn patterns demonstrated in Figs. 3(c)-3(f) confirm the laser beam to be radially polarized.

![Fig. 3. (a) Simulated output light field and Burned acrylic patterns for (b) the output without polarizer, the doughnut mode through a polarizer oriented (c) horizontally, (d) vertically, (e) at 45° direction, and (f) at 135° direction](image)

At the same time, we find some irregular stripes on the burn pattern. Two main reasons are considered to lead to such a problem. On the one hand, it is due to the deviation in the processing of the axicon mirrors. On the other hand, when operating in high power status, the bronze axicon mirrors absorb a large amount of heat and cause distortion.
5. Conclusions

In this paper, a vectorial eigenvector method is presented for the simulation of polarization dependent resonator. The VEM is based on finite element method and evolved from scalar eigenvector method. New transmission matrices of laser resonant cavities with polarization-selective devices are built by using Jones matrix representation. The VEM can derive a lot of mode distributions at a time, especially cylindrical vector modes. We take an axicon mirror resonator as an example to verify its effectiveness. Besides, with the assistance of VEM, we confirmed that the differences of reflectivity in 5% between radial polarization and azimuthal polarization can realize polarization mode discrimination, which contributes to our designs of high power industrial radially polarized lasers. Finally, upon a FAT CO$_2$ laser, we designed a new 45-degree three-fold cavity structure and the 1.5 kW radially polarized beam was obtained, which proved the correctness of the aforementioned method.

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