Particle Physics Inflation Model Constrained from Astrophysics Observations

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Abstract

The early Universe inflation [1] is well known as a promising theory to explain the origin of large scale structure of the Universe, a causal theory for the origin of primordial density fluctuations which may explain the observed density inhomogeneities and cosmic microwave fluctuations in the very early Universe, and to solve the early universe pressing problems for the standard hot big bang theory [2]. For a resonable inflation model, the potential during inflation must be very flat in, at least, the direction of the inflaton. To construct a resonable inflation model, or the inflaton potential, all the known related astrophysics observations should be included. For a general tree-level hybrid inflation potential, which is not discussed fully so far for the quartic term, the parameters in it are shown how to be constrained via the astrophysics data observed and to be obtained to the expected accuracy by the soon lauched MAP and PLANCK satellite missions [14], as well as the consistent cosmology requirements. We find the effective inflaton mass parameter is in the TeV range, and the quartic term’s self-coupling constant tiny, needs fine-tuning.

Key Words: Inflation model building, Supersymmetry Particle Physics Model, COBE normalization, Power spectral index.

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I. INTRODUCTION

For the past nearly twenty years, two major theories, inflation and topological defects to the early universe pressing problems [1] seem to have stood the test of time and one current goal is to determine which if any best fits the increasingly accumulated astrophysics data, especially the COBE’s cosmic microwave background anisotropy detections as well as the soon launched more accuracy MAP and PLANCK observations [14], and more reasonably interpret the origin of the Universe large scale structures. They usually are regarded as mutually exclusive theories in that defects formed before a period of inflation would rapidly be diluted to such a degree during the inflationary era as to make them of little interest to cosmology. But in some inflation models inspired from particle physics considerations the formation of topological defects can be naturally obtained at the end of the inflationary period [8]. On the other side, as is widely supposed, the initial conditions for the successful hot big bang are set by inflation, and then an adiabatic, Gaussian and more or less scale invariant density perturbation spectrum at horizon entry is predicted [9]. Such a perturbation is generated by the vacuum fluctuation during inflation so the dazzling prospect of a window on the fundamental interactions on scales approaching the Planck energy appears. The studying of inflation paradigm will help us to understand basic physics laws to the possibly highest scale in the Nature.

The inflationary Universe scenario [10] has the universe undergoing a period of accelerated expansion, the effect originally being to dilute monopoles (and any other defect formed before this period) outside of the observable universe, thereby dramatically reducing their density to below the observable limits. In a homogeneous, isotropic Universe with a flat Friedmann-Robertson-Walker (FRW) metric described by a scale factor $a(t)$, the acceleration is given via

$$\ddot{a} = -a(4\pi G/3)(\rho + 3p)$$

where $\rho$ is the energy density and $p$ the pressure. Usually the energy density which drives inflation is identified with a scalar potential energy density that is positive, and flat enough to result in an effective equation of state

$$\rho \approx -p \approx V(\phi)$$

satisfying the acceleration condition $\ddot{a} > 0$.

The scalar potential is associated with a scalar field known as the so-called inflaton. During the inflationary period, the inflaton potential is fairly flat in the direction the field evolves, dominating the energy density of the universe for a finite period of time. Over the period it evolves slowly towards a minimum of the potential either through a classical roll over or through a quantum mechanics tunnelling transition. Inflation then ends when the inflaton starts to execute decaying oscillations around its own vacuum value, and the hot Big Bang (reheating) ensues when the vacuum value has been achieved and the decay products have thermalised. Over the past decades there have been lots of inflation models constructed and there shall be certainly more with the coming of MAP and PLANCK satellites missions. With our knowledge so far we understand that any reasonable inflation model should satisfy at least that COBE normalization, cosmology observations constraint to the spectral index
and adequate e-folding inflation for consistence requirements \[10\]. In this line, the running-
mass models of inflation without quartic term is studied \[3\] and we will discuss a general
Tree-level inflation model, especially the constraint to the quartic term self-coupling constant 
\[15\].

This paper is arranged as following. In next section we give a general comments on the
properties of inflaton potential, which must satisfy the COBE normalization condition be-
sides the flatness conditions. In section three we examine detailly a tree-level hybrid inflation
potential model, which can be regarded as a generalization of previously fully discussed some
inflation models \[20\]. We use the slow-roll approximation to derive an analytic expression
for the e-folds number N between a given epoch and the end of slow-roll inflation, and derive
the spectral index of the spectrum of the curvature perturbation of this model. Confronted
them with the COBE measurement of the spectrum on large scales ( the normalization),
the required e-folds number N and the observational constraint on the spectrum index over
the whole range of cosmological scales, we give the coupling constant of the inflaton model
an allowed region, to specify its parameter space by reducing its two free parameters to one .
Finally we give a discussion and conclusion.

II. GENERAL CONSIDERATIONS ON INFLATION MODEL

Cosmological inflation has been regarded as the most elegant solution to the horizon and
flatness problems of the standard Big Bang universe \[10\]. If considering the later stage ther-
amal inflation \[16\] it also beautifully solve the moduli problems \[12\]. Even though it explains
successfully why the current Universe appears so homogenous and flat in a very natural man-
ner, it has been difficult to construct a model of inflation without a small parameter(the
fine-tunning problem). The key point is to have a resoneable scalar field potential either from
a more underlying gravity theory like effective superstring thery or from a more fundmental
particle physics theory such as supergravity or the so-called M-theory \[4\]. In fact to any
case, one needs at least a scalar field component(inflaton) that rolls down the potential very
slowly with enough e-folds number to successfully generate a viable inflationary scenario.
This requires the potential to be almost flat in the direction of the inflaton. There are lots
of inflation models constructed so far. If gravitation wave contribution is negligible, at least
for the present situation, the curvature perturbation spectrum index is the most powerful
discriminator to inflation models. In this section we discuss the general properties of an
inflaton potential with the astrophysics considerations. In the effectve slow-roll inflation
scheme as physics requirments, the general inflaton potential \(V(\phi)\) must satisfy the flatness conditions

\[
\epsilon \ll 1 \quad (3)
\]

and

\[
|\eta| \ll 1 \quad (4)
\]

where the notations \[10\]

\[
\epsilon \equiv \frac{1}{2} M_{Pl}^2 (V'/V)^2 \quad (5)
\]

\[
\eta \equiv M_{Pl}^2 V''/V \quad (6)
\]

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the prime indicates differential with respect to $\phi$ and $M_{Pl} = (8\pi G)^{-1/2} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass (scale). When these are satisfied, the time dependence of the inflaton $\phi$ is generally given by the slow-roll expression

$$3H\dot{\phi} = -V',$$

where the quantity

$$H \simeq \sqrt{\frac{1}{3}M_{Pl}^{-2}V}$$

is the Hubble parameter during inflation. On a given scale, the spectrum of the primordial curvature perturbation, thought to be the origin of structure in the Universe, is given by

$$\delta_H^2(k) = \frac{1}{150\pi^2M_{Pl}^4} \frac{V}{\epsilon}$$

The right hand side is evaluated when the relevant scale $k$ leaves the horizon. On large scales, the COBE observation of the Cosmic Microwave Background (CMB) anisotropy corresponds to

$$V^{1/4}/\epsilon^{1/4} = 0.027M_{Pl} = 6.7 \times 10^{16}$$

The spectral index of the primordial curvature perturbation is given by

$$n - 1 = 2\eta - 6\epsilon$$

A perfectly scale-independent spectrum would correspond to $n = 1$, and observation already demands

$$|n - 1| < 0.2$$

Thus $\epsilon$ and $\eta$ have to be $\lesssim 0.1$ (barring a cancellation) and this constraint will get much tighter if future observations, such as the near future MAP and PLANCK satellite experiment missions, move $n$ closer to 1. Many models of inflation predict that this will be the case, some giving a value of $n$ completely indistinguishable from 1.

Usually, $\phi$ is supposed to be charged under at least a $Z_2$ symmetry, that is there is no change for the system under

$$\phi \rightarrow -\phi$$

which is unbroken during inflation. Then $V' = 0$ at the origin, and inflation typically takes place near the origin. As a result $\epsilon$ negligible compared with $\eta$, and

$$n - 1 = 2\eta \equiv 2M_{Pl}^2V''/V$$

We assume that this is the case as in most inflation models, in what follows. If it is not, the inflation model-building, as in the slow roll approximation, is even more tricky. Another point should be clear that the possibly later stage thermal inflation only lasts a few e-foldings and at a lower energy scale around the electroweak symmetry breaking scale, which affects
less the extremely successful Hot Big Bang Nucleo-synthesis which is at the MeV scale. It
takes place while a lighter scalar field (with mass around 100GeV) with nonzero vacuum
expectation value, is trapped by thermal effects in the false vacuum at the lighter scalar field
value as zero. More components inflation model only make physics picture complex. In this
paper we consider two scalar fields one of which is the Higgs-like scalar field to make sure the
graceful exit after the inflation end, and contribute to the vacuum energy expectation value
when taking a critical value; another explicit one is the inflaton which drives the cosmic
inflation (exponential) expansion.

III. A TREE-LEVEL HYBRID INFLATION MODEL

In this section we will present the allowed parameter regions for a particular vaccum-
dominated potential that are not given out before. A similar form potential with only
different signs appears in a Supersymmetry particle physics model \[17\]. The focused vacuum
dominated potential we consider has the usual form with dimension four for the sake of
renormalizability in mind

\[
V = b(M^2 - h^2)^2/2 + m^2\phi^2/2 + \lambda\phi^4/4, \tag{15}
\]

where \(b\) is a coupling constant, \(m\) is the \(h\) expectation-value dependent mass parameter
for the inflaton and \(h\) is the Higgs-like scalar field for the graceful exit in the new inflation
scenario \[20\]. If the expectation value of \(h\) equals \(M\) during evolution the model turns out to
be the general chaotic inflationary potential \[3\]; if not then the usual hybrid inflation model
with the resulting \(V_0\) as the dominated vacuum \[16\].

\[
V = V_0 + m^2\phi^2/2 + \lambda\phi^4/4. \tag{16}
\]

with all parameters positive in it, which can be regarded as a generalization of previously
fully discussed inflation models \[20\]. Due to symmetry considerations we discard the cubic
term. Higher order inflaton terms may appear in some Susy particle physics effective models
\[10,18,19\]. There are two particular limits of vacuum energy inflation, according to whether
the energy density is dominated by the vacuum energy density or by the inflaton energy
density. We assume the former in our case as preference in the slow-roll approximation \[3\].
With the COBE normalization \(V^{1/4}/\epsilon^{1/4} = 0.27M_{Pl}\) and \(\epsilon \equiv \frac{1}{2}M_{Pl}^2(V'/V)^2\), in our case the
false vacuum energy density

\[
V_0 \approx 0.34 M_{Pl}^2(\phi_1^2 + \lambda\phi_1^3)^{2/3}/2^{1/3}. \tag{17}
\]

where \(\phi_1\) is the inflaton value when COBE scale leaves the horizon. To reduce free parameters
number we define

\[
y_1 = m^2/\phi_1^2. \tag{18}
\]

By cosmology observations to the power spectral index constraint

\[
|n - 1|/2 = \eta = M_{Pl}^2V''/V < 0.1 \tag{19}
\]
and if we take the nowadays observation value upper limit 0.1 as a potentially changing parameter \( x \), we have

\[
\eta = \frac{M_{Pl}^2 (m^2 + 3\lambda \phi^2)}{V_0} < x. \tag{20}
\]

Taking the potential form into equation (19) and using our definition for \( y_1 \) we can define a function of \( y_1 \) as

\[
f(y_1) = \frac{y_1 + 3\lambda}{(y_1 + \lambda)^{2/3}} < x \frac{0.3^4}{2^{1/3}}. \tag{21}
\]

In this expression, the parameter \( x \) as an observation input runs from nowadays 0.1 to the hopefully 0.01 by the near future planned MAP and PLANK satellite missions. Easily we find

\[
df/dy_1 = \frac{(y_1 - 3\lambda)(y_1 + \lambda)^{-5/3}}{3} \tag{22}
\]

For \( y_1 > 3\lambda \) the \( df/dy_1 > 0 \), while for \( y_1 < 3\lambda \) the \( df/dy_1 < 0 \), so that gives

\[
f_{\text{min}}(3\lambda) = 3(\lambda/2)^{-1/3} \tag{23}
\]

which corresponds to, with relation (21) into account

\[
\lambda_{\text{max}} < x^3 1.97 \times 10^{-8} \tag{24}
\]

With obviously

\[
y_1 + \lambda < y_1 + 3\lambda \tag{25}
\]

and the relation (21), directly we have

\[
y_1 + \lambda < x^3 0.027^4/2. \tag{26}
\]

which together give the allowed parameters regions for quartic self-coupling constant \( \lambda \) and the reduced mass parameter with various observed or to be obtained parameter \( x \) values as inputs. Take today’s upper limit \( x = 0.1 \) we find the quartic self-coupling constant \( O(10^{-11}) \), far too small. For the reduced mass parameter it’s normal since inflation starts at the inflaton field value around \( 0.1M_{Pl} \), which gives the inflaton’s effective mass about 1000GeV.

With the certain e-folds number constraint for overcoming horizon and flatness problems

\[
N = \left| \int_{\phi_1}^{\phi_c} \frac{V}{V'} d\phi \right| / M_{Pl}^2. \tag{27}
\]

where \( \phi_c \) is the inflaton at the end of inflation. Insert the potential form and the COBE normalization we get another relation for the reduced mass parameter defined as

\[
y_c = m^2/\phi_c^2 = (\lambda + y_1) \exp(Ny_1 10^3/3.6 \times (\lambda + y_1)^{-2/3}) - \lambda. \tag{28}
\]

which limits the allowed parameters with astrophysics required e-folding numbers \( N \) and the spectrum index \( x \) as today’s observations required values as inputs. We can see the limit
cases are consistent with the previous results by numerical calculations \[15\] with figures \[20\]. The self-coupling constant is very tiny and the fine-tuning problem appears. (We also can get directly from the above relation curves of the reduced parameter \( y_c \) with parameter \( \lambda \)). It is clear that the reduced parameter \( y_c \) is approximately linear to \( \lambda \) when the exponential value is around 1.

Taking the power spectral index constraint relation (19) and the relation (21) with the above equation into account we can get a constraint relation, approximately allowed regions to reduced parameters \( y_c \) and \( \lambda \) as

\[ y_c + \lambda < (y_1 + \lambda)exp(1000N/3.608 \times (y_1 + \lambda)^{1/3}) < 2.657 \times 10^{-7}x^3exp(1.6518Nx) , \]

By which the approximately allowed parameter regions for \( \lambda \) vs \( y_c \) with \( x \) from 0.1 to future possibly 0.01 is straightforward worked out.

If put the power spectral index expression and the e-folding expression together we find

\[ |n - 1| = N^{-1/2}(1 + 3\lambda/y_1)ln(1 + \frac{y_c - y_1}{\lambda + y_1}). \]

qualitively that roughly is

\[ |n - 1| \propto N^{-1}. \]

which implies these two constraints have intrinsic connection. When \( |n - 1| \) < 0.1 as today’s cosmological observations available, then \( N \gtrsim 10 \). If \( |n - 1| \) < 0.01 as the soon satellite missions by MAP and PLANCK on design hopefully to give, then the \( N \gtrsim 100 \). This kind of generic character as expressed by relation (31) appears in a class of dynamical supersymmetry breaking particle physics models \[3\].

IV. DISCUSSION AND CONCLUSION

Models of inflation driven by a false vacuum formed by the Higgs-like scalar field are mainly different from true vacuum cases in their no zero false vacuum energy density, which are simple but also can reflect the astrophysics observations. We discuss a regular tree-level hybrid inflation model, whose special case is the general chaotic inflation model (not a toy model \[7\]) here to show how to constraint its parameters when confronting data and cosmology consistence requirements, and give several new parameter relations and the allowed regions, which can be used as a prototype model for the two planned Microwave Anisotropy Probe (MAP) and PLANCK satellite missions tests. The results we have obtained based on a reasonable assumption that the spectral index constraint we concentrate on is naturally satisfying the flatness conditions. Otherwise the slow roll approximation is not applicable.

The origin of this tree-level hybrid inflation model or its more complicated extentions may arise from some kind of supersymmetry particle physics or supergravity models which is generally cosidered as the appropriate framework for a description of the fundamental interactions at higher energy scale, and in particular for the description of their scalar interaction potential in the D-term and the gauge coupling F-Term \[21\]. Here we only study the essence of them in order to get more viable inflation models from supersymmetry particle physics and supergravity as well as superstring (M-)theories. No matter what kind
of theoretical model to be built it must satisfy at least the above observations, especially
the spectral index constraint at x=0.01. The parameter space then in our case is very tiny
that asks us to build the inflation models from a more natural way to avoid the fine-tunning
problem, which is a challenge facing us. Within next few years with the rapidly increasing
in the variety and more accuracy of cosmological observation data, like the measurements
of temperature anisotropies in cosmic microwave background at the accuracy expected from
MAP and PLANCK soon, it is possible for us to discriminate among inflation models.

As a flood of high-quality cosmological data is coming from experiments in outerspace,
on earth and underground, even in the sea or at the bottom of the sea we are really entering
the observation constrainning theory times. There are mainly six projects on the way. Here
we only discuss the most related one to our topic, inflation model building:

The CMB Map of the Universe. COBE mapped the CMB with an angular resolution of
around $10^0$; two new satellite missions, NASA’s MAP(launch in 2000) and ESA’s PLANCK
Surveyor (launch 2007), will map CMB with 100 times better resolution ($0.1^0$) and with a
detail map of our Universe at 300,000 years. From these maps of the Universe as it existed at
a simpler time, long before the first stars and galaxies, will come a gold mine of information:
a definitive measurement of matter fraction of the Universe today; a characterization of
the primeval lumpiness and possible detection of the relic gravity waves from inflation as
well as a determination of the Hubble constant to a precision of better than 0.05. Direct
measurements of the expansion rate using standard candles, gravitational time delay, SZ
imaging besides the precision CMB map will pin down the elusive Hubble constant once and
for all. It is the fundamental parameter that sets the size- in time (the puzzling Universe
age problem) and space- of the Universe. Its value is critical to testing the self consistency
of the early Universe models. After all, the precision maps of the CMB that will be made
are crucial to establishing the astroparticle physics inflation theory.

For the past two decades the hot big bang model as been referred to as the standard
cosmology- and for good reasons [13]. For just as long particle cosmologists have known
there are fundamental questions that are not answered by the standard cosmology and
point to a grander theory. The best candidate for that grander theory is the viable particle
physics inflation theory plusing dark components (dark energy, hot and cold dark matter)
as the Universe dominated contents. It holds that the Universe is flat, that slowly moving
elementary particles left over from the earliest moments provide the cosmic infrastructure
[22], and that the primeval density inhomogeneities that seed all the structure arose from
quantum fluctuations. There is now lots of prima facie evidence that supports the two
basic tenets of this paradigm. An avalanche of high quality cosmological observations will
soon make this case stronger or even will break it. Key questions remain to be answered;
foremost among them are: a viable inflation model to be built, elucidation of the dark-energy
component and the identification as well as detection of the cold dark matter particles. The
next, at least, two decades are exciting times in Particle Physics Cosmology with the planned
continuous astrophysics experiments going.

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