A note on half-supersymmetric bound states in M-theory and type IIA

Henric Larsson
Institute for Theoretical Physics
Göteborg University and Chalmers University of Technology
SE-412 96 Göteborg, Sweden
E-mail: solo@fy.chalmers.se

Abstract
By using $O(7, 7)$ transformations, to deform D6–branes, we obtain half-supersymmetric bound state solutions of type IIA supergravity, containing D6, D4, D2, D0, F1-branes and waves. We lift the solutions to M-theory which gives half-supersymmetric M-theory bound states, e.g. KK6–M5–M5–M5–M2–M2–M2–MW. We also take near horizon limits for the type IIA solutions, which gives supergravity duals of 7–dimensional non-commutative open string theory (with space-time and space-space non–commutativity), non-commutative Yang-Mills theory (with space-space and light-like non–commutativity) and an open D4–brane theory.

1 Introduction

Many different dual supergravity solutions corresponding to bound states have been constructed \cite{1, 2, 3}, using different solution generating methods \cite{1, 2, 3, 4, 5}. Most of these bound states have D3, D4, D5, NS5 or M5–branes as the highest rank brane. These bound states are of interest since they are useful in the investigation of non–commutative Yang-Mills theory (NCYM) \cite{6, 7, 5}, non–commutative open string theory (NCOS) \cite{8, 9, 12, 13, 14} and wound string theory \cite{15}. More specifically, they are used to construct supergravity duals of non–commutative open brane (or gauge) theories, by taking appropriate near horizon limits. E.g., for a bound state which has been obtained by an NS-NS $B_{01}$ deformation of a D$p$–brane, one has to take an ‘electric’ near horizon limit, while for a bound state with $B_{12}$ one has to take a ‘magnetic’ near horizon limit. In the first case one obtains the supergravity dual of $(p+1)$-dimensional space-time non–commutative open
string theory, which can be seen by investigating the properties of the open string quantities (see e.g. [5])

\[
\frac{G_{\mu\nu}}{\alpha'} = \frac{1}{\alpha'_{\text{eff}}} H^{-1/2} \eta_{\mu\nu} , \quad G_{OS}^2 = g H^{3-p} , \quad \Theta^{01} = \alpha'_{\text{eff}} ,
\]

where \( G_{\mu\nu} \) is the open string metric, \( G_{OS}^2 \) is the open string coupling constant and \( \Theta^{01} \) is the non–commutativity parameter. Note that the open string metric and coupling constant approach constant values in the decoupling limit \( \frac{R}{r} \to \infty \), since \( H = 1 + \left( \frac{R}{r} \right)^{7-p} \to 1 \). This implies that the tension of open strings, in the decoupling limit, is \( T = 1/\alpha'_{\text{eff}} \), which means that massive open string modes are kept in the spectrum. In the second case one obtains the supergravity dual of non–commutative super-Yang-Mills, with \( \Theta^{12} \neq 0 \) and not an open string theory, since the massive open strings decouple. For a more detailed introduction to supergravity duals and the definition of ‘electric’ and ‘magnetic’ near horizon limits, see Section 3.2 below and [5, 14].

In this note, the main interest is to construct supergravity solutions, corresponding to half-supersymmetric bound states, for which the D6–brane (IIA) or the KK6–brane (M-theory) is the brane of highest dimensionality\(^1\). All these bound states are half-supersymmetric since we start with a half-supersymmetric D6–brane and use \( O(7,7) \) transformations to obtain the bound states (more specifically we only use T-duality and gauge transformations, see Section 2 and [3, 4] for details). We also construct a half-supersymmetric type IIB supergravity solution, corresponding to a D7–D5–D5–D5–D3–D3–D3–D1–F1 bound state (see Appendix A). This bound state is included since it can not be obtained from the D6–brane bound states, by using T-duality.

The next step, towards constructing all possible supergravity solutions, corresponding to M-theory and IIA/B bound states, would be to construct bound states where the D8–brane is the brane of highest dimensionality. However, since the D8–brane only exists in massive type IIA supergravity, one would have to adjust the deformation procedure that is used in this note (see Section 2), in order to be able to deform the D8–brane. Bound states containing D8–branes will not be discussed further in this note (except in Section 5 Discussion).

As an application, we take near horizon limits of the type IIA bound states and obtain supergravity duals of NCOS (with space-time and space-space non–commutativity), NCYM (with space-space and light-like non–commutativity) and an open D4–brane theory.

\(^1\)For D6–brane bound states with lower supersymmetry, see e.g. [16].
This note is organized as follows: In Section 2 we give an introduction to the most important features of the solution generating technique that we will use in Section 3 and 4. In the next Section we construct various bound states containing D6–branes and lower rank D–branes as well as F-strings and waves. We also take various near horizon limits, which give supergravity duals of different non–commutative open brane (or gauge) theories. In Section 4 we lift the type IIA bound state solutions to 11 dimensions, which gives half-supersymmetric (since uplifting to 11 dimensions preserves supersymmetry) bound states containing KK6, M5, M2–branes and M-waves. We conclude with section 5 which is discussion.

2 Deformation of Dp–branes

The solution generating technique that we will use was derived in \[4\] (see also \[5\]) following earlier work in \[3\]. For an NS deformation of a general D\(p\)–brane one first T-dualizes in the directions where one wants to turn on NS fluxes, and then one shifts \(B_2\) with a constant in these directions. After this one T-dualizes back again. In a more precise language, the deformation with parameter \(\theta^{\mu\nu}\) is generated by the following \(O(p + 1, p + 1)\) T-duality group element

\[
\Lambda = \Lambda_0 \ldots \Lambda_p \Lambda_\theta \Lambda_p \ldots \Lambda_0 = J \Lambda_\theta J = \Lambda_{\theta} = \begin{pmatrix} 1 & 0 \\ \theta & 1 \end{pmatrix},
\]  

where \(\theta^{\mu\nu}\) is dimensionless and carries indices upstairs since it starts life on the T-dual world volume \(\mathbb{E}\).

Starting with a D\(p\)–brane solution\[3\]

\[
ds^2 = g_{\mu\nu} dx^\mu dx^\nu + g_{ij} dy^i dy^j, \quad e^{2\phi},
\]

\[
C = \omega dx^0 \wedge \cdots \wedge dx^p + \gamma_{7-p},
\]

where \(x^\mu, \mu = 0, \ldots, p\), are coordinates in the brane directions while \(x^i, i = p + 1, \ldots, 9\), are coordinates in the transverse directions. \(\gamma_{7-p}\) is a transverse form, i.e., \(i_\mu \gamma_{7-p} = 0\), where \(i_\mu\) denotes the inner derivative in the \(\mu\) direction.

\[2\]See \[1\] for conventions and definitions of the various elements of \(O(p + 1, p + 1)\) appearing in the following discussion. Note that in this note \(\theta^{\mu\nu}\) is dimensionless while in \[2\] \(\frac{\theta^{\mu\nu}}{\omega}\) were dimensionless.

\[3\]We are using multi-form notation such that \(C\) is a sum of forms while \(B\) (see below) has fixed rank 2. Note also that in this example the exact form of e.g. \(g_{\mu\nu}\) and \(e^{2\phi}\) is not important.
Now following the above deformation procedure gives the following deformed configuration:

\[
\begin{array}{l}
\tilde{g}_{\mu\nu} = g_{\mu\rho}\left[1 - (\theta g)^2\right]^{\frac{1}{2}} \nu, \quad \tilde{g}_{ij} = g_{ij}, \\
\tilde{B}_{\mu\nu} = -g_{\mu\rho}\theta g^\rho g_{\sigma\lambda}\left[1 - (\theta g)^2\right]^{\frac{1}{2}} \lambda, \\
e^{2\tilde{\phi}} = e^{2\phi}\left(\frac{\det \tilde{g}}{\det g}\right)^{\frac{1}{2}}, \\
\tilde{C} = e^{-\frac{i}{2}B_{\mu\nu}dx^\mu \wedge dx^\nu} (\omega e^{\frac{1}{2}g^{\mu\nu} \epsilon_{\mu\nu} dx^0 \wedge \cdots \wedge dx^p + \gamma_{7-p}).
\end{array}
\]

There are two types of deformations that are possible: \(\theta^0\) and \(\theta^{ij}\), where \(i, j = 1, 2, \ldots, p\). The first one is called ‘electric’ since we mix the time direction with a spatial direction, while the second is called ‘magnetic’ since the time direction is not included.

Electric deformations are used if one would like to include F1–strings in the bound state, while magnetic deformations are used to include Dq–branes, where \(q = p - 2, p - 4, \ldots, p - 2n\), and \(n\) is the number of magnetic deformations (i.e., one turn on \(\theta^{12}, \theta^{34}, \ldots, \theta^{2n-1,2n}\)).

To include waves, one has to mix an electric and a magnetic deformation with equal ‘strength’, i.e., turn on e.g. \(\theta^0\) and \(\theta^{12}\) where \(\theta^0 = \pm \theta^{12}\).

### 3 Type IIA bound states and non–commutative theories

#### 3.1 Type IIA bound states

In this section, we will apply the above deformation procedure to the D6–brane. The undeformed, half-supersymmetric, D6–brane configuration is given by

\[
\begin{align}
\text{ds}^2 &= H^{-1/2}(-dx_0)^2 + \cdots + (dx_6)^2 + H^{1/2}(dr^2 + r^2 d\Omega_2^2), \\
C &= \frac{1}{gH}dx^0 \wedge \cdots dx^6 + \frac{1}{g}R \epsilon_1, \\
e^{2\phi} &= g^2 H^{-3/2}, \quad H = 1 + \frac{R}{r}, \quad R = gN \sqrt{\alpha'},
\end{align}
\]

where \(d\epsilon_1\) is the volume form of the 2-sphere and \(g\) is the closed string coupling constant. We will now apply an \(O(7, 7)\) transformation on a D6–brane (see
above and \[4\] for more details) in order to obtain bound states containing D6-branes and lower dimensional D–branes and possibly F-strings and waves. Deforming the D6-brane by turning on $\theta^{01}$, $\theta^{12}$, $\theta^{34}$ and $\theta^{56}$, gives the following deformed configuration:\footnote{For $\theta^{01} = 0$ the metric, NS-NS 2-form, dilaton and the RR 1- and 3-forms have been obtained in \[4\], using a different solution generating technique.}

\[
\begin{align*}
\tilde{g}^2 &= \frac{e^{\tilde{\phi}}}{g^2 h h_{34} h_{56}}, \\
\tilde{B} &= \frac{\theta^{01}}{H h} dx^0 \wedge dx^1 - \frac{\theta^{12}}{H h} dx^1 \wedge dx^2 - \frac{\theta^{34}}{H h_{34}} dx^3 \wedge dx^4 - \frac{\theta^{56}}{H h_{56}} dx^5 \wedge dx^6, \\
g\tilde{C}_7 &= \frac{1}{H h h_{34} h_{56}} dx^0 \ldots \wedge dx^6 + \frac{\theta^{34} \theta^{56}}{H^3 h h_{34} h_{56}} \left( \theta^{12} dx^1 \wedge dx^2 - \theta^{01} dx^0 \wedge dx^1 + \theta^{12} dx^0 \wedge \ldots \wedge dx^6 \right), \\
g\tilde{C}_5 &= \frac{\theta^{56}}{H h h_{34}} dx^0 \ldots \wedge dx^4 - \frac{\theta^{34}}{H h_{56}} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^5 \wedge dx^6 \\
&\quad - \frac{1}{H h_{34} h_{56}} \left( \theta^{01} dx^2 \wedge \ldots \wedge dx^6 + \theta^{12} dx^0 \wedge dx^3 \wedge dx^4 \wedge dx^5 \wedge dx^6 \right) \\
&\quad + \frac{1}{H^2} \left( \theta^{12} \theta^{34} \frac{1}{h_{34}} dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 + \frac{\theta^{12} \theta^{56}}{h_{56}} dx^1 \wedge dx^2 \wedge dx^5 \wedge dx^6 \right) \\
&\quad + \frac{\theta^{34} \theta^{56}}{h_{34} h_{56}} dx^3 \wedge dx^4 \wedge dx^5 \wedge dx^6 - \frac{\theta^{34} \theta^{01}}{h_{34} h} dx^0 \wedge dx^1 \wedge dx^3 \wedge dx^4 \\
&\quad - \frac{\theta^{56} \theta^{01}}{h_{56} h} dx^0 \wedge dx^1 \wedge dx^5 \wedge dx^6 \right) \wedge R\epsilon_1, \\
g\tilde{C}_3 &= \frac{\theta^{34} \theta^{56}}{H h} dx^0 \wedge dx^1 \wedge dx^2 + \frac{\theta^{12} \theta^{56}}{H h_{34}} dx^0 \wedge dx^3 \wedge dx^4 + \frac{\theta^{12} \theta^{34}}{H h_{56}} dx^0 \wedge dx^5 \wedge dx^6 \\
&\quad + \frac{\theta^{01} \theta^{34}}{H h_{56}} dx^2 \wedge dx^5 \wedge dx^6 + \frac{\theta^{01} \theta^{56}}{H h_{34}} dx^2 \wedge dx^3 \wedge dx^4 - \tilde{B} \wedge R\epsilon_1, \\
g\tilde{C}_1 &= R\epsilon_1 - \frac{\theta^{12} \theta^{34} \theta^{56}}{H} dx^0 - \frac{\theta^{01} \theta^{34} \theta^{56}}{H} dx^2, \\
\end{align*}
\]

\[
d\tilde{s}^2 = \frac{H^{-1/2}}{h} (\tilde{h}_{12} (dx^0)^2 + (dx^1)^2 + h_{01} (dx^2)^2) - 2 \frac{H^{-3/2}}{h} \theta^{01} \theta^{12} dx^0 dx^2 \\
+ H^{-1/2} \left( \frac{1}{h_{34}} ((dx^3)^2 + (dx^4)^2) + \frac{1}{h_{56}} ((dx^5)^2 + (dx^6)^2) \right) \\
+ H^{1/2} (dx^2 + r^2 d\Omega^2_2),
\]

where $H = \frac{1}{h h_{34} h_{56}}$ and $h_{ij}$ are the components of the metric on the D6-brane.
| Brane | $x^1$ | $x^2$ | $x^3$ | $x^4$ | $x^5$ | $x^6$ |
|-------|-------|-------|-------|-------|-------|-------|
| D6    | X     | X     | X     | X     | X     | X     |
| D4    | X     | X     | X     | X     | $-$   | $-$   |
| D4    | $-$   | $-$   | X     | X     | X     | X     |
| D4    | X     | X     | $-$   | $-$   | X     | $-$   |
| D4    | X     | X     | $-$   | $-$   | $-$   | X     |
| D2    | $-$   | $-$   | X     | X     | $-$   | $-$   |
| D2    | $-$   | $-$   | $-$   | $-$   | $-$   | X     |
| D0    | $-$   | $-$   | $-$   | $-$   | $-$   | $-$   |

Table 1: The D6–(D4)$^3$–(D2)$^3$–D0 bound state

with

$$ h_{i,i+1} = 1 + \theta^{i,i+1} H^{-1}, \quad i = 1, 3, 5, \quad h_{01} = 1 - (\theta^{01})^2 H^{-1}, $$

$$ h = 1 - (\theta^{01})^2 H^{-1} + (\theta^{12})^2 H^{-1}. $$

This solution gives rise to many different bound states depending on the values of the $\theta$ (deformation) parameters. If $\theta^{01} = \pm \theta^{12} \neq 0$ there is a wave included in the bound state. E.g., if all the other parameters are nonzero then we have a D6–(D4)$^3$–(D2)$^3$–D0–F1–W bound state. In this case the metric can be rewritten in the following way (with $\theta^{01} = -\theta^{12} = \theta$ and $h = 1$):

$$ ds^2 = H^{-1/2} \left[ (-dx^0 + dx^2)(dx^0 + dx^2) + (dx^1)^2 - H^{-1} \theta^2 (-dx^0 + dx^2)^2 \right. $$

$$ + \left. \frac{1}{h_{34}} ((dx^3)^2 + (dx^4)^2) + \frac{1}{h_{56}} ((dx^5)^2 + (dx^6)^2) + H (dr^2 + r^2 d\Omega_2^2) \right], \quad (7) $$

which implies that there is a wave in the 2nd direction. This can be seen by going to light-like coordinates $x^\pm = \frac{1}{\sqrt{2}}(x^2 \pm x^0)$.

Instead if $|\theta^{01}| > |\theta^{12}|$ one can perform a Lorentz transformation to a frame where $\theta^{01} \neq 0$ and $\theta^{12} = 0$. Then if all the other parameters are nonzero, we have a D6–(D4)$^2$–D2–F1 bound state. Finally, if $|\theta^{01}| < |\theta^{12}|$, one can perform a Lorentz transformation to a frame where $\theta^{01} = 0$ while $\theta^{12} \neq 0$, which gives a D6–(D4)$^3$–(D2)$^3$–D0 bound state provided also $\theta^{34}$ and $\theta^{56}$ are nonzero. In Table 1 we show in which directions the different branes are oriented (for the D6–(D4)$^3$–(D2)$^3$–D0 bound state).

---

5 Here (D4)$^3$ means that there are three D4–branes in the bound state.

6 We thank P. Sundell for pointing this out to us. See also [18, 19].
Using T-duality, all these bound states can be used to obtain bound states in type IIB with a D5–brane (see [2] for the complete \((p, q)\) 5–brane bound states) or a D7–brane as the highest rank brane. However, T-duality of the deformed D6–brane solution \((3)\) can not be used in order to obtain a D7–brane bound state, with a rank 8 \((\theta^{i,i+1} = 0, 2, 4, 6)\) deformation. This D7–(D5)3–(D3)3–D1–F1 bound state is therefore included in Appendix A.

### 3.2 Non–commutative theories

In the three different cases above it is possible to take near horizon limits, e.g., if one wants to obtain supergravity duals of non–commutative open brane (or gauge) theories. In the first case, with \(\theta^{01} = -\theta^{12} = \theta\) (which correspond to a light-like deformation, since \(B_{-1} \neq 0\) and \(B_{+1} = 0\)) and \(\theta^{i,i+1} \neq 0\) (\(i = 3, 5\)), one has to take a magnetic near horizon limit \([4]\), i.e., keeping the following quantities fixed:\footnote{This limit gives a finite supergravity lagrangian, since \(d \tilde{s}^2/\alpha', \tilde{B}_2/\alpha'\) and \(\tilde{C}_q/(\alpha')^{q/2}\) are kept fixed in the \(\alpha' \rightarrow 0\) limit.}

\[
x^\mu, \quad u = \frac{r}{\alpha'}, \quad \tilde{\theta} = \alpha' \theta, \quad \tilde{\theta}^{i,i+1} = \alpha' \theta^{i,i+1}, \quad g_{\text{YM}}^2 = g(\alpha')^{3/2},
\]

in the \(\alpha' \rightarrow 0\) limit. Taking this limit for the \(\theta^{01} = -\theta^{12}\) case, gives the supergravity dual of NCYM theory with light-like \((\Theta^{+1} = \sqrt{2} \tilde{\theta}, \Theta^{-1} = 0)\), as well as space-space non–commutativity \((\Theta^{i,i+1} = \tilde{\theta}^{i,i+1}, i = 3, 5)\)\footnote{For other NCYM theories with light-like non–commutativity see e.g. \([12, 19]\).}. This can be seen from the metric \((7)\) by going to light-like coordinates \(x^\pm = \frac{1}{\sqrt{2}} (x^2 \pm x^0)\).

The above limit can also be used in all other cases when \(\theta^{01} = 0\). Then one obtains supergravity duals of NCYM with space-space non–commutativity \((\Theta^{i,i+1} = \tilde{\theta}^{i,i+1}, i = 1, 3, 5)\), which was shown in \([7]\).

In the case when \(\theta^{01} = \theta^{12} = \theta^{34} = 0\) and \(\theta^{56} \neq 0\), one can also interpret the supergravity solution as being dual to an open D4–brane theory (since there is a critical RR 5–form \(T\) in the solution, which is responsible for the finite tension \(\frac{1}{g_{\text{YM}}^2} = \frac{1}{g_{\text{YM}}^2}\) of an open D4–brane), containing light open D4–branes analogous to the open Dq–brane theories in \([14, 14]\). Using the results in Section 4 in \([14]\) (defining the coupling in a similar fashion) gives that the open
D4–brane theory, with length-scale \( \ell_{D4} \) and coupling constant \( G_{D4}^2 \), and the 7–dimensional NCYM theory (with only \( \tilde{\theta}^{56} \neq 0 \)) are related as follows:

\[
g_{YM}^2 = G_{D4}^{4/5} \ell_{D4}^3, \quad \Theta^{56} = \tilde{\theta}^{56} = G_{D4}^{-4/5} \ell_{D4}^2.
\]

(9)

This implies that 7-dimensional NCYM (with only \( \tilde{\theta}^{56} \neq 0 \)) arises in the limit of small \( G_{D4}^2 \) and small length scale \( \ell_{D4} \) (keeping \( \tilde{\theta}^{56} = G_{D4}^{-4/5} \ell_{D4}^2 \) fixed), which is the weakly coupled low energy limit of the open D4–brane theory, where we expect to have an effective field theory description.

When \( \theta^{01} \neq 0 \) and \( \theta^{12} = 0 \) one has to take an electric near horizon limit, i.e., keeping the following quantities fixed \([5, 14]\):

\[
\tilde{x}^\mu = \sqrt{\alpha'_{\text{eff}}} x^\mu, \quad \tilde{r} = \sqrt{\alpha'_{\text{eff}}} r, \quad \theta^{01} = 1, \quad \theta^{i,i+1}, \quad g,
\]

(10)

in the \( \alpha' \to 0 \) limit. Note that this limit is very different from (8) since the limit is obtained by making the electric field critical (i.e., \( \theta^{01} = 1 \)). In the \( \theta^{01} \neq 0 \) and \( \theta^{12} = 0 \) case this limit gives the supergravity dual of 7–dimensional NCOS with the following open string quantities:

\[
\frac{G_{\mu\nu}}{\alpha'} = \frac{1}{\alpha'_{\text{eff}}} H^{-1/2} \eta_{\mu\nu}, \quad G_{OS}^2 = gH^{-3/4}, \\
\Theta^{01} = \alpha'_{\text{eff}}, \quad \Theta^{34} = \theta^{34} \alpha'_{\text{eff}}, \quad \Theta^{56} = \theta^{56} \alpha'_{\text{eff}},
\]

(11)

which means that we have both space-time and space-space non–commutativity.

However, since all the above theories are world volume theories of a deformed D6–brane, it is possible that some or all of them do not decouple from the bulk gravity. This is a possibility since the undeformed super-Yang-Mills theory on the D6–brane does not decouple \([24]\). In e.g. \([13]\) it was suggested that NCYM theory with light-like non–commutativity does not decouple, and in \([17]\) it was shown that NCYM with space-space non-commutativity is not a decoupled theory. For NCOS on the other hand it was argued in \([8, 21]\) that NCOS decouple for all D\(_p\)–branes with \( p < 7 \). Whether the open D4–brane theory decouples or not is unclear, but since it has the same supergravity dual as NCYM (with only \( \tilde{\theta}^{56} \neq 0 \)) it indicates that it is not a decoupled theory. A further investigation of this will be presented elsewhere.

\( ^{10} \)To be more precise, \( \theta^{01} = k \) where \( k \) is the constant in the harmonic function \( H \). In this note we always set \( k = 1 \).
4 M-theory bound states

Next, we lift the type IIA bound state solutions (6) to obtain half-supersymmetric M-theory bound state solutions\(^1\). Lifting the type IIA solution (6) to 11 dimensions on a circle with radius \(R_{11} = g\sqrt{\alpha'}\) gives\(^2\):

\[
d_{11}^2 = \left( h_{34}h_{56} \right)^{1/3} \left[ \frac{1}{h} (-h_{12}(dx^0)^2 + (dx^1)^2 + h_{01}(dx^2)^2) - 2 \frac{1}{Hh} \theta^{01}\theta^{12}dx^0dx^2 
+ \frac{1}{h_{34}}((dx^3)^2 + (dx^4)^2) + \frac{1}{h_{56}}((dx^5)^2 + (dx^6)^2) + H(dx^2 + r^2d\Omega_2^2) 
+ \frac{1}{Hh_{34}h_{56}}(dx^{10} - Re_1 + \frac{\theta^{12}\theta^{34}\theta^{56}}{H} dx^0 + \frac{\theta^{01}\theta^{34}\theta^{56}}{H} dx^2)^2 \right], \\
A_3 = \frac{\theta^{34}\theta^{56}}{Hh} dx^0 \wedge dx^1 \wedge dx^2 + \frac{\theta^{12}\theta^{56}}{Hh_{34}} dx^0 \wedge dx^3 \wedge dx^4 + \frac{\theta^{01}\theta^{34}}{Hh_{56}} dx^0 \wedge dx^5 \wedge dx^6 
+ \frac{\theta^{01}}{Hh} dx^2 \wedge dx^5 \wedge dx^6 + \frac{\theta^{34}}{Hh_{34}} dx^2 \wedge dx^3 \wedge dx^4 
+ \left( \frac{\theta^{01}}{Hh} dx^0 \wedge dx^1 - \frac{\theta^{12}}{Hh} dx^0 \wedge dx^2 - \frac{\theta^{34}}{Hh_{34}} dx^2 \wedge dx^3 \wedge dx^4 
- \frac{\theta^{56}}{Hh_{56}} dx^5 \wedge dx^6 \right) \wedge (dx^{10} - Re_1), \quad H = 1 + \frac{R}{r}. 
\]

Depending on the values of the \(\theta\) parameters, we obtain several different bound states, e.g., \(\theta^{01} = 0\), and \(\theta^{i,i+1} \neq 0\) \((i = 1, 3, 5)\) gives a half-supersymmetric KK6–(M5)\(^3\)–(M2)\(^3\)–MW bound state (i.e., uplifting of the D6–(D4)\(^3\)–(D2)\(^3\)–D0 bound state).

With \(\theta^{01} = \pm\theta^{12}\) and \(\theta^{i,i+1} \neq 0\) \((i = 1, 3, 5)\) we instead obtain a half-supersymmetric KK6–(M5)\(^3\)–(M2)\(^4\)–(MW)\(^2\) bound state (which is uplifting to M-theory of the D6–(D4)\(^3\)–(D2)\(^3\)–D0–F1–W bound state). In this case the metric can be rewritten in the following way (with \(\theta^{01} = -\theta^{12} = \theta\) and \(h = 1\)):

\[
d_{11}^2 = \left( h_{34}h_{56} \right)^{1/3} \left[ (-dx^0 + dx^2)(dx^0 + dx^2) + (dx^1)^2 - H^{-1}\theta^2(-dx^0 + dx^2)^2 
+ \frac{1}{h_{34}}((dx^3)^2 + (dx^4)^2) + \frac{1}{h_{56}}((dx^5)^2 + (dx^6)^2) + H(dx^2 + r^2d\Omega_2^2) 
+ \frac{1}{Hh_{34}h_{56}}(dx^{10} - Re_1 + \frac{\theta^{34}\theta^{56}}{H}(-dx^0 + dx^2)^2 \right], \quad (13)
\]

\(^1\)Note that uplift to 11 dimensions does not reduce supersymmetry.
\(^2\) We use the conventions used in \cite{14}. Note also that we do not include the auxiliary 6-form.
where the waves are in the 2nd and 10th directions.
Finally, if we have $\theta^{01} \neq 0$, $\theta^{12} = 0$ and $\theta^{i,i+1} \neq 0$, $i = 3, 5$, we obtain a $\text{KK6}-(\text{M5})^2-(\text{M2})^2$ bound state (i.e., uplifting of $\text{D6}-(\text{D4})^2-\text{D2}-\text{F1}$).

5 Discussion

In this note we have obtained type IIA bound states, containing D6, D4, D2, D0, F1–branes and waves, using $O(7, 7)$ transformations on D6–branes. Lifting these bound states to M-theory gives bound states containing KK6, M5, M2–branes and M–waves. In an Appendix we have also included a type IIB D7–(D5)$^3$–(D3)$^3$–D1–F1 bound state. For the type IIA bound states we take electric (10) and magnetic (8) near horizon limits, which give us supergravity duals of NCOS (with $\Theta^{01} = \alpha'_{\text{eff}}$, $\Theta^{34} = \theta^{34}\alpha'_{\text{eff}}$ and $\Theta^{56} = \theta^{56}\alpha'_{\text{eff}}$) and NCYM (with light-like and space-space non-commutativity). In the case of NCYM with only $\theta^{56} \neq 0$ we argue that the supergravity dual can also be interpreted as supergravity dual of a theory containing light open D4–branes, where the relation between the open D4–brane theory and NCYM is given in (9). This suggest that NCYM (with only $\theta^{56} \neq 0$) is the low energy limit of the open D4–brane theory, i.e., the open D4–brane theory is the UV completion of 7-dimensional NCYM.

In [17] it was shown that NCYM on the D6–brane does not decouple. For NCOS on the other hand it was argued in [8, 21] that NCOS decouple for all D$p$–branes with $p < 7$. The possible decoupling of the open D4–brane theory has to be investigated further. However, the fact that it has the same supergravity dual as NCYM (with only $\theta^{56} \neq 0$) might indicate that it is not a decoupled theory. Next, to study the strong coupling of NCOS (with only $\Theta^{01} \neq 0$), one has to lift the D6-F1 bound state to M-theory, which gives the KK6-M2 bound state, and take the appropriate near horizon limit. What complicates this set up is that the KK6–brane is in the $x^1$–$x^6$ directions while the M2–brane has one direction in the periodic $x^{10}$ direction, i.e., transverse to the KK6–brane. For large $\tilde{r}$ the metric approaches a smeared membrane, which in the case of the near horizon region of the M5-M2 bound state indicates the existence of an open membrane theory. In this case, however, one cannot adopt the same interpretation since one of the membrane directions is transverse to the KK6–brane. It remains to be seen what the strong coupling limit of 7-dimensional NCOS is, but we do not expect it to be an open membrane theory.

\[\text{The easiest way to obtain this supergravity dual is to lift the D6-F1 bound state after one has taken an electric near horizon limit.}\]
As the next step, it would be interesting to find out if NCOS and/or the open D4-brane theories really are decoupled or not, and also what their strong coupling limits are. One can also further investigate the M-theory bound states KK6–(M5)3–(M2)3–MW and KK6–(M5)3–(M2)4–(MW)2, in order to find dual theories. However, these theories are probably not decoupled, since they are strong coupling limits of NCYM with space-space non–commutativity and NCYM with space-space as well as light-like non–commutativity, respectively.

Deformations of a type IIA D8–brane should also be possible, but since the D8–brane only exists in massive type IIA, one would have to adjust the deformation procedure in order to include these deformations. This has not yet been done, but we expect that it is possible to deform the D8–brane with e.g. a magnetic rank 8 deformation \( \theta^{i,i+1} \), \( i = 1, 3, 5, 7 \), which should give a half-supersymmetric D8–(D6)4–(D4)6–(D2)4–D2 bound state.

**Acknowledgments**

We thank M. Cederwall, U. Gran, M. Nielsen, B.E.W. Nilsson and P. Sundell for discussions and comments.

**A Deformation of the D7–brane**

In this appendix we will deform the D7–brane with a rank 8 deformation, by turning on \( \theta^{01}, \theta^{23}, \theta^{45} \) and \( \theta^{67} \). Note that a rank 8 deformation is not possible for the D6-brane. The resulting D7–brane bound state can therefore not be obtained from the deformed D6–brane, using T-duality. The undeformed D7–brane configuration is given by (see e.g. [22])

\[
\begin{align*}
\text{ds}^2 &= H^{-1/2}(-(dx_0)^2 + \cdots + (dx_7)^2) + H^{1/2}(dr^2 + r^2 d\Omega_4^2), \\
C_8 &= \frac{1}{gH} dx^0 \wedge \cdots dx^7, \tag{14} \\
e^{2\phi} &= g^2 H^{-2}, \quad H = k + \frac{g}{2\pi} \log \frac{r}{R},
\end{align*}
\]

\[\text{Note that we do not include the auxiliary RR } C_0 \text{ potential. For an introduction to 7–branes and their transformations properties under SL}(2, Z), \text{ see } [22].\]
where $k$ is an arbitrary constant. Deforming the D7–brane with $\theta^{01}, \theta^{23}, \theta^{45}$ and $\theta^{67}$ gives

$$ds^2 = H^{-1/2} \left( \frac{1}{h_{01}}(-dx_0^2 + dx_1^2) + \frac{1}{h_{23}}(dx_2^2 + dx_3^2) + \frac{1}{h_{45}}(dx_4^2 + dx_5^2) + \frac{1}{h_{67}}(dx_6^2 + dx_7^2) \right) + H^{1/2}(dr^2 + r^2d\Omega_4^2),$$

$$e^{2\phi} = \frac{g^2H^{-2}}{h_{01}h_{23}h_{45}h_{67}},$$

$$\tilde{B} = \frac{\theta^{01}}{Hh_{01}} dx^0 \wedge dx^1 - \frac{\theta^{23}}{Hh_{23}} dx^2 \wedge dx^3 - \frac{\theta^{45}}{Hh_{45}} dx^4 \wedge dx^5 - \frac{\theta^{67}}{Hh_{67}} dx^6 \wedge dx^7,$$

$$g\tilde{C}_8 = \frac{1}{Hh_{01}h_{23}h_{45}h_{67}} dx^0 \wedge \cdots \wedge dx^7,$$

$$g\tilde{C}_6 = \frac{\theta^{01}}{Hh_{23}h_{45}h_{67}} dx^2 \wedge \cdots \wedge dx^7 - \frac{\theta^{23}}{Hh_{01}h_{45}h_{67}} dx^0 \wedge dx^1 \wedge dx^4 \wedge \cdots \wedge dx^7 - \frac{\theta^{67}}{Hh_{01}h_{23}h_{45}} dx^0 \wedge \cdots \wedge dx^5,$$

$$g\tilde{C}_4 = \frac{\theta^{01}\theta^{23}}{Hh_{45}h_{67}} dx^4 \wedge dx^5 \wedge dx^6 \wedge dx^7 + \frac{\theta^{01}\theta^{45}}{Hh_{23}h_{67}} dx^2 \wedge dx^3 \wedge dx^6 \wedge dx^7 + \frac{\theta^{01}\theta^{67}}{Hh_{01}h_{45}} dx^0 \wedge dx^1 \wedge dx^3 \wedge dx^5 + \frac{\theta^{23}\theta^{45}}{Hh_{01}h_{23}} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3,$$

$$g\tilde{C}_2 = \frac{\theta^{01}\theta^{23}\theta^{45}}{Hh_{67}} dx^6 \wedge dx^7 - \frac{\theta^{01}\theta^{23}\theta^{67}}{Hh_{45}} dx^4 \wedge dx^5 - \frac{\theta^{01}\theta^{45}\theta^{67}}{Hh_{01}} dx^0 \wedge dx^1,$$

$$g\tilde{C}_0 = \frac{\theta^{01}\theta^{23}\theta^{45}\theta^{67}}{H} ,$$

with $h_{i,i+1} = 1 + (\theta^{i,i+1})^2H^{-1}$, $i = 2, 4, 6$, $h_{01} = 1 - (\theta^{01})^2H^{-1}$.}

For $\theta^{i,i+1} \neq 0$ ($i = 0, 2, 4, 6$) we obtain a D7–(D5)$^3$–(D3)$^3$–D1–F1 bound state. If $\theta^{01} = 0$ the F1–string will no longer be included. Instead, if one magnetic $\theta$ vanishes the D1–brane is removed, while two magnetic $\theta$'s equal to zero removes the D1 and D3–branes and finally, all three equal to zero removes the D1, D3 and D5–branes.
References

[1] J.M. Izquierdo, N.D. Lambert, G. Papadopoulos and P.K. Townsend, Nucl. Phys. B460 (1996) 560, [hep-th/9508177].
J.C. Breckenridge, G. Michaund and R.C. Myers, Phys. Rev. D55 (1997) 6438, [hep-th/9611174].
M.S. Costa and G. Papadopoulos, Nucl. Phys. B510 (1998) 217, [hep-th/9612204].
J.G. Russo and A.A. Tseytlin, Nucl. Phys. B490 (1997) 121, [hep-th/9611047].
N. Ohta and J-G. Zhou, Int. J. Mod. Phys. A13 (1998) 2013, [hep-th/9706153].
M. Cederwall, U. Gran, M. Holm and B.E.W. Nilsson, JHEP 9902 (1999) 003, [hep-th/9812144].
J.X. Lu and S. Roy, Nucl. Phys. B560 (1999) 181, [hep-th/9904129].
J.X. Lu and S. Roy, JHEP 0001 (2000) 034, [hep-th/9905014].
E. Bergshoeff, R. Cai, N. Ohta and P.K. Townsend, Phys. Lett. B495 (2000) 201, [hep-th/0009147].

[2] M. Cederwall, U. Gran, M. Nielsen, and B.E.W. Nilsson, JHEP 0001 (2000) 037, [hep-th/9912106].

[3] A. Hashimoto and N. Itzhaki, Phys. Lett. B465B (1999) 142, [hep-th/9907164].

[4] P. Sundell, [hep-th/0011283] to be published in Int. J. Mod. Phys. A.

[5] D. Berman, V.L. Campos, M. Cederwall, U. Gran, H. Larsson M. Nielsen, B.E.W. Nilsson and P. Sundell, JHEP 0105 (2001) 002, [hep-th/0011282].

[6] J.Maldacena and J.G. Russo, JHEP 9909 (1999) 025, [hep-th/9908134].
J.F.L. Barbon and E. Rabinovici, JHEP 9912 (1999) 017, [hep-th/9910019].
T. Harmark and N. Obers, JHEP 0003 (2000) 024, [hep-th/9911164].
R-G. Cai and N. Ohta, JHEP 0003 (2000) 009, [hep-th/0001213].

[7] M. Alishahiha, Y. Oz and M.M. Sheikh-Jabbari, JHEP 9911 (1999) 007, [hep-th/9909215].

[8] R. Gopakumar, J. Maldacena, S. Minwalla and A. Strominger, JHEP 0006 (2000) 036, [hep-th/0005048].

[9] N. Seiberg, L. Susskind and N. Toumbas, JHEP 0006 (2000) 021, [hep-th/0005040].
J.X. Lu, S. Roy and H. Singh, JHEP 0009 (2000) 020, [hep-th/0006193].
T. Harmark, JHEP 0007 (2000) 043, [hep-th/0006023].
J. G. Russo and M. M. Sheikh-Jabbari, JHEP 0007 (2000) 052, [hep-th/0006203].
R-G. Cai and N. Ohta, Phys. Rev. D104 (2000) 1073, [hep-th/0007106].
J.X. Lu, S. Roy and H. Singh, Nucl. Phys. B595 (2001) 298, [hep-th/0007168].
J.G. Russo and M. M. Sheikh-Jabbari, [hep-th/0009141].
U. Gran and M. Nielsen, [hep-th/0104165].
T. Harmark and N. Obers, JHEP 0103 (2001) 032, [hep-th/0011282].
J.L.F. Barbón and E. Rabinovici, [hep-th/0104169].
[10] R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger, JHEP 0008 (2000) 008, hep-th/0006062.
   E. Bergshoeff, D.S. Berman, J.P. van der Schaar and P. Sundell, Phys. Lett. B492 (2000) 193, hep-th/0006112.
   D.S. Berman and P. Sundell, JHEP 0010 (2000) 014, hep-th/0007052.
   Y. Michishita, JHEP 0009 (2000) 036, hep-th/0008247.

[11] T. Harmark, Nucl. Phys. B593 (2001) 76, hep-th/0007147.

[12] M. Alishahiha, Y. Oz and J.G. Russo, JHEP 0009 (2000) 002, hep-th/0007215.

[13] J.X. Lu, JHEP 0108 (2001) 049, hep-th/0102056.

[14] H. Larsson and P. Sundell, JHEP 0106 (2001) 008, hep-th/0103188.

[15] J. Gomis and H. Ooguri, hep-th/0009181.
   U.H. Danielsson, A. Guijosa and M. Kruczenski, JHEP 0010 (2000) 020, hep-th/0009182.
   U.H. Danielsson, A. Guijosa and M. Kruczenski, JHEP 0103 (2001) 041, hep-th/0012183.

[16] M. Sato, hep-th/0101226.

[17] M. Alishahiha, H. Ita and Y. Oz, JHEP 0006 (2000) 002, hep-th/0004011.

[18] G-H. Chen and Y-S. Wu, Phys. Rev. D63 (2001) 086003, hep-th/0006013.

[19] O. Aharony, J. Gomis and T. Mehen, JHEP 0009 (2000) 023, hep-th/0006230.
    R-G. Cai and N. Ohta, JHEP 0010 (2000) 036, hep-th/0008119.

[20] O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. 323 (2000) 183, hep-th/9905111.

[21] S.S. Gubser, S. Gukov, I.R. Klebanov, M. Rangamani and E. Witten, hep-th/0009140.

[22] P. Meessen and T. Ortin, Nucl. Phys. B541 (1999) 195, hep-th/9806120.