DARK MATTER SCALING RELATIONS

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ABSTRACT

We investigate the structure of dark matter halos by means of the kinematics of a very large sample of spiral galaxies of all luminosities.

The observed rotation curves show an universal profile which is the sum of an exponential thin disk term and a spherical halo term with a flat density core. We find that the Burkert profile proposed to describe the dark matter halo density distribution of dwarf galaxies also provides an excellent mass model for the dark halos around disk systems up to 100 times more massive. Moreover, we find that spiral dark matter core densities \( \rho_0 \) and core radii \( r_0 \) lie in the same scaling relation \( \rho_0 = 4.5 \times 10^{-2} (r_0/kpc)^{-2/3} M_\odot pc^{-3} \) of dwarf galaxies with core radii up to 10 times smaller.

At the highest masses \( \rho_0 \) decreases with \( r_0 \) faster than the \( -\frac{4}{3} \) power law implying a lack of objects with disk masses \( > 10^{11} M_\odot \) and central densities \( > 1.5 \times 10^{-2} (r_0/kpc)^{-3} M_\odot pc^{-3} \) that can be explained by the existence of a maximum mass of \( \approx 2 \times 10^{12} M_\odot \) for an halo hosting a spiral galaxy.

Subject headings: Galaxies: spiral, kinematics and dynamics Cosmology: dark matter

1. INTRODUCTION

It is now well established that spiral galaxies have universal rotation curves (URC) that can be characterized by one single free parameter, the luminosity (e.g. Rubin et al. 1980; Persic & Salucci, 1991, Persic, Salucci & Stel 1996 (PSS)). For instance, low-luminosity spirals show ever-increasing rotation curves (RC) out to the optical radius \(^1\), while, in the same region, the RC of high-luminosity spirals are flat or even decreasing.

It has been demonstrated by Persic & Salucci (1988, 1990) and Broeils (1992) that, as the galaxy luminosity decreases, the light is progressively unable to trace the radial distribution of the gravitating matter (see also Salucci, 1997). This discrepancy is, in general, interpreted as the signature of an invisible mass component (Rubin et al. 1980; Bosma 1981). As pointed out by PSS the universality of the rotation curves, in combination with the invariant distribution of the luminous matter, implies an universal dark matter distribution with luminosity-dependent scaling properties.

On the theoretical side, recent high-resolution cosmological N-body simulations have shown that cold dark matter halos achieve a specific equilibrium density profile (Navarro, Frenk & White 1996, NFW; Cole & Lacey 1997; Fukushige & Makino 1997; Moore et al. 1998; Kravtsov et al. 1998). This can be characterized by one free parameter, e.g. \( M_\text{200} \), the dark mass enclosed within the radius inside which the average over-density is 200 times the critical density of the Universe. In the innermost regions the dark matter profiles show some scatter around an average profile which is characterized by a power-law cusp \( \rho \sim r^{-\gamma} \), with \( \gamma = 1 - 1.5 \) (NFW, Moore et al. 1998, Bullock et al., 1999).

Until recently, due to both the limited number of suitable rotation curves and a poor knowledge of the exact amount of luminous matter present in the innermost regions of spirals, it has been difficult to investigate the internal structure of dark matter halos. The situation is more favorable for (low surface brightness) dwarf galaxies which are strongly dark matter dominated even at small radii. The kinematics of these systems shows an unsuitability of the dark halo density profiles, but, it results in disagreement with that predicted by CDM, in particular because of the existence of dark halo density cores. (Moore 1994; Burkert 1995). The origin of these features is not yet understood (see e.g. Navarro, Eke & Frenk 1996; Burkert & Silk 1997, Gelato & Sommer-Larsen 1999), but it is likely that it involves more physics than a simple hierarchical assembly of cold structures. To cope with this observational evidence, Burkert (1995) proposed an empirical profile that successfully fitted the halo rotation curves of four dark matter dominated dwarf galaxies.

\[
\rho_0(r) = \frac{\rho_0 r_0^3}{(r + r_0)(r^2 + r_0^2)}
\]

where \( \rho_0 \) and \( r_0 \) are free parameters which represent the central dark matter density and the scale radius. This sample has been extended, more recently, to 17 dwarf irregular and low surface brightness galaxies (Kravtsov et al. 1998, see however van den Bosch et al. 1999) which all are found to confirm equation (1). Adopting spherical symmetry, the mass distribution of the Burkert halos is given by

\[
M_b(r) = 4 M_0 \left\{ \ln \left( 1 + \frac{r}{r_0} \right) - \frac{1}{2} \ln \left[ 1 + \left( \frac{r}{r_0} \right)^2 \right] \right\}
\]

\(^1\) \( R_{\text{opt}} = 3.2 R_d \), with \( R_d \) the exponential disk length-scale
with $M_0$, the dark mass within the core given by:

$$M_0 = 1.6\rho_0 r_0^3$$

(3)

The halo contribution to the circular velocity is then:

$$V_h^2(r) = GM_0(r)/r.$$  

(4)

Although the dark matter core parameters $r_0$, $\rho_0$ and $M_0$ are in principle independent, the observations reveal a clear correlation (Burkert 1995):

$$M_0 = 4.3 \times 10^7 \left(\frac{r_0}{kpc}\right)^{7/3} M_\odot$$

(5)

which indicates that dark halos represent a 1-parameter family which is completely specified, e.g. by the core mass.

2. DM Halo Profiles in Spirals

PSS (see also Rubin et al. 1982) have derived, from 15000 velocity measurements of 1000 rotation curves (RC), $V_{syn}(r/R_{opt}, L_I/L_*)$, the synthetic rotation velocities of spirals sorted by luminosity (Figure 1, with $L_I$ the I-band luminosity and $L_I/L_*$ = 1/(1+2.9)$/3). Remarkably, individual RC's have a very small variance with respect to the corresponding synthetic curves (PSS, Rhee 1996, Rhee & van Albada 1996, Roscoe 1999, Swaters, 1999): spirals sweep a very narrow locus in the RC-profile/amplitude/luminosity space. On the other hand, the galaxy kinematical properties do significantly change with luminosity (e.g. PSS), so it is natural to relate the mass distribution with this quantity. The whole set of synthetic RC's has been modeled with the Universal Rotation Curve (URC), $V_{URC}(r/R_{opt}, L_I/L_*)$ which includes: (a) an exponential thin disk term:

$$V_{d,URC}^2(x) = 1.28 \beta V_{opt}^2 x^2 (I_0 K_0 - I_1 K_1)|_{1.6x}$$

(6)

and (b) a spherical halo term:

$$V_{h,URC}^2(x) = V_{opt}^2 (1 - \beta) (1 + a^2) \frac{x^2}{(x^2 + a^2)}.$$  

(7)

with $x \equiv r/R_{opt}$, $\beta \equiv (V_{d,URC}(R_{opt})/V_{opt})^2$, $V_{opt} \equiv V(R_{opt})$ and $a$ the halo core radius in units of $R_{opt}$. At high luminosities the contribution from a bulge component has been also considered (Salucci and Persic, 1999b).

The analysis of a recently published large sample of RC's (Persic & Salucci, 1995) has provided a suitable framework to investigate the dark halo density distribution in spirals. The starting points of this study are: a) the mass in spirals is distributed according to the Inner Baryon Dominance (IBD) regime: there is a characteristic transition radius $R_{IBD} \simeq 2R_d\left(\frac{V_{opt}}{2200 km/s}\right)^{1.2}$ for which, at $r \leq R_{IBD}$, the luminous matter totally accounts for the mass distribution, while, for $r > R_{IBD}$, the DM rapidly becomes the dominant dynamical component (Salucci and Persic, 1999a,b; Salucci et al, 2000; Ratnam and Salucci, 2000; Borriello and Salucci, 2000). Then, although the dark halo might extend down to the galaxy center, it is only for $r > R_{IBD}$ that it gives a non-negligible contribution to the circular velocity. b) The dark matter is distributed in a different way with respect to any of the various baryonic components (PSS, Corbelli and Salucci, 2000), and c) The HI contribution to the circular velocity, for $r < R_{opt}$, is negligible (e.g. Rhee, 1996; Verheijen, 1997).

The main aim of this letter is to expand the above results to derive the luminosity-averaged density profiles of the dark halos and to relate them with the Burkert profiles. Section 2 presents the analysis of a homogeneous sample of about 1100 rotation curves in which the dark halo contribution to the circular velocity is first derived and then matched to the Burkert halo mass models. In section 3 we discuss the results. We take $H_0 = 75 km/s/Mpc$ and $\Omega_0 = 0.3$, however no result depends on these choices.

The HI contribution to the circular velocity term of eq. (7) does not bias the mass model: it can account for maximum-disk, solid-body, no-halo, all-halo, CDM and core-less halo mass models. In practice, the values of the free parameters $a$ and $\beta$ that result from fitting the URC

$$V_{URC}^2(x) = V_{h,URC}^2(x,\beta,a) + V_{d,URC}^2(x,\beta)$$

(8)

Let us stress that the halo velocity term of eq. (7) does not bias the mass model: it can account for maximum-disk, solid-body, no-halo, all-halo, CDM and core-less halo mass models. In practice, the values of the free parameters $a$ and $\beta$ that result from fitting the URC

Fig. 1.— Synthetic rotation curves (filled circles with error bars) and URC (solid line) with its separate dark/luminous contributions (dotted line: disk; dashed line: halo.) See PSS for details.

Fig. 2.— $\alpha$ vs $\beta$ and $\beta$ vs $V_{opt}$.
to the synthetic curves $V_{syn}$ select the actual model. Adopting $\beta \approx 0.72 + 0.44 \log \left( \frac{L}{L_\odot} \right)$ and $\alpha \approx 1.5 \left( \frac{L}{L_\odot} \right)^{1/5}$ (see PSS) or, equivalently the corresponding

$$a = a(\beta) \quad \beta = \beta(\log V_{opt})$$

that we have plotted in Fig. (2), the URC mass models reproduce the synthetic curves $V_{syn}(r)$ within their r.m.s. (see Fig. (1)). More in detail, at any luminosity and radius, $|V_{URC} - V_{syn}| < 2\%$ and the 1σ fitting uncertainties on $a$ and $\beta$ are about 20% (PSS).

We then compare the dark halo velocities obtained with eq. (7) and (9), with the Burkert velocities $V_b(r)$ of eqs. (2)-(4). We leave $\rho_0$ and $r_0$ as free parameters, i.e. we do not impose the relationship of eq. (5). The results are shown in Fig (3): at any luminosity, out to the outermost radii ($\sim 6R_d$), $V_b(r)$ is indistinguishable from $V_b,URC(r)$. More specifically, by setting $V_b,URC(r) \equiv V_b(r)$, we are able to reproduce the synthetic rotation curves $V_{syn}(r)$ at the level of their r.m.s. For $r >> 6R_d$, i.e. beyond the region described by the URC, the two velocity profiles progressively differ.

![Fig. 3. — URC-halo rotation curves (filled circles with error bars) and the Burkert model (solid line). The bin magnitudes are also indicated.](image)

The values obtained for $r_0$ and $\rho_0$ for the URC agrees with the extrapolation at high masses of the scaling law $\rho \propto r_0^{-2/3}$ (Burkert, 1995) established for objects with core radii $r_0$ ten times smaller (see Fig 4). Let us notice that the core radii are very large: $r_0 >> R_d$ so that an ever-rising halo RC cannot be excluded by the data. Moreover, the disk-mass vs. central halo density relationship $\rho_0 \propto M_d^{-1/3}$, found in dwarf galaxies (Burkert, 1995), where the densest halos harbor the least massive disks, holds also for disk systems of stellar mass up to $10^{11} M_\odot$ (see Fig 4).

The above relationship show a curvature at the highest masses/lowest densities that can be related to the existence of an upper limit in the dark halo mass $M_{200}$ which is evident by the sudden decline of the baryonic mass function of disk galaxies at $M_{b}^{max} = 2 \times 10^{11} M_\odot$ (Salucci and Persic, 1999a), that implies a maximum halo mass of

$$M_{200}^{max} \propto \Omega_0/\Omega_b \times M_d^{max}$$

where $\Omega_0$ and $\Omega_b \approx 0.03$ (e.g. Burles and Tytler, 1998) are the matter and baryonic densities of the Universe in units of critical density. From the definition of $M_{200}$, by means of eq. (2) and (3), we can write $M_{200}$ in terms of the “observable” quantity $M_0$: $M_{200} = \eta M_0$. For $(\Omega_0, \Omega_b) = (0.3, 3)$, $\eta \approx 12$; notice that there is a mild dependences of $\eta$ on $z$ and $\Omega_0$ which is irrelevant for the present study. Combining eq. (3) and (10) we obtain an upper limit for the central density, $\rho_0 < 1 \times 10^{-20} (r_0/kpc)^{-3} g/cm^3$, which implies a lack of objects with $\rho_0 > 4 \times 10^{-25} g/cm^3$ and $r_0 > 30 kpc$, as is evident in Figure 3. Turning the argument around, the deficit of objects with $M_d \sim M_d^{max}$ and $\rho_0 > 4 \times 10^{-25} g/cm^3$, suggests that, at this mass scale, the total-to-baryonic mass fraction may approach the cosmological value $\Omega/\Omega_b \approx 10$.

3. DISCUSSION

Out to two optical radii, the Burkert density profile reproduces, for the whole spiral luminosity sequence, the DM halos mass distribution. This density profile, though at very large radii coincides with the NFW profile, approaches a constant, finite density value at the center, in a way consistent with an isothermal distribution. This is in contradiction to cosmological models (e.g. Fujisawa and Makino 1997) which predict that the velocity dispersion $\sigma$ of the dark matter particles decreases towards the center to reach $\sigma \rightarrow 0$ for $r \rightarrow 0$. After the result of this study, the dark halo inner regions, therefore, cannot be considered as kinematically cold structures but rather as “warm” regions with size $r_0 \propto \rho_0^{1.5}$. The halo core sizes are very large: $r_0 \sim 4 - 7 R_d$. Then, the boundary of the core region is well beyond the region where the stars are located and, as in Corbelli and Salucci (2000), even at the outermost observed radius there is not the slightest evidence that dark halos converge to a $\rho \sim r^{-2}$ (or a steeper) regime.

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1. The virial halo mass is given by $M_{200} \equiv 200 \times 4\pi/3 \rho_0 R_{200}^3 \Omega_0 (1 + z)^3 g(z)$ with $z$ the formation redshift, $R_{200}$ the virial radius, for $g(z)$ see e.g. Bullock et al., (1999); the critical density is defined as $\rho_c \equiv 3/(8\pi G^2) H_0^2$. 

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We find that the dark halos around spirals are essentially
an one-parameter family. It is relevant that the order pa-
rameter (the central density or the core radius) correlates
with the luminous mass (see Fig 4). We do however not
know how it is related to the global structural properties
of the dark halo, like the virial radius or the virial mass.
In fact, the RC out to 6R_D is completely determined by
the core parameters, i.e. the central core density and the
core radius, both of which are not defined in the CDM
scenario.

The location of spiral galaxies in the parameter space
of virial mass, halo central density and baryonic mass is
determined by different processes that occur on different
scales and at different red-shifts. Yet, this 3D space
degenerates into a single curve (see Figure 4, remind that:
ρ_0 = \frac{M_{200}}{r_0^3} and remind that: M_D = G \cdot \beta V_{opt}^2 R_{opt})
which describes the dark-luminous coupling.

Let us discuss the limitations of the present results.
First, here we have considered the luminosity dependence
of the dark halo structure. Although this is probably the
most relevant one, other dependences (Hubble type and
surface brightness) should also be investigated. Moreover,
the existence of a (weak) cosmic variance in the halo struc-
tural properties cannot be excluded until we analyze indi-
vidual objects (Salucci, 2000, Borriello and Salucci, 2000).
Secondly, we have derived the profile of DM halos out to
about six disk-scale lengths, i.e. out to a distance much
smaller than the virial radius. To assess the global valid-
ity of the proposed mass model data at larger radii are
obviously required.

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![Figure 4](...). Top Disk mass (in solar units) vs central halo density \( \rho_0 \) (in \( g/cm^3 \)) for normal spirals (filled circles). The straight line is the extrapolation to high luminosities of the relation of dwarfs. Bottom Central density vs core radii (in kpc) for normal spirals (filled circles), compared with the extrapolation of the relationship of dwarfs (dotted line, the point with errorbar represents a typical object of Burkert, 1995). The solid line is the eyeball fit; \( \rho_0 = 5 \times 10^{-24} r_0^{-2/3} e^{-(r_0/20)^2} \) g/cm³. The effect of a limiting halo mass is also shown (dashed line).