Research on Vibration Suppression for AGARD Wing Based on State Space Model

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Abstract. Based on the Observer method, the unsteady aerodynamic reduced order model is established. The aerodynamic, structural and control parts are in the form of state space equations. The state feedback control law is designed by linear quadratic regulator (LQR) method, and the active flutter suppression of aeroelastic system is realized. The three-dimensional AGARD elastic wing is selected as an example. The simulation results show that the active control law designed by the model reduction technique can effectively stabilize the response of the system, and the critical speed of closed-loop flutter increases greatly.

1. Introduction
The design of modern aeroelastic active control system requires a low-order state-space model. The computational fluid dynamics (CFD) method has considerable advantages in the simulation of nonlinear problems, but the engineering application is limited to a certain extent. It is not suitable for active control law design and conventional analysis. Therefore, it is necessary to apply the unsteady aerodynamic reduced order model (ROM) with high computational efficiency to the aeroelastic control system. The aerodynamic ROM reflects the important characteristics of the aerodynamic system, and its accuracy is equivalent to that of the CFD solver. It can be integrated into the conventional analysis of aeroelastic system control. Unsteady aerodynamic ROM can replace CFD solver, which is used for time domain simulation and integrated design of aeroelastic system. The flutter critical velocity of many aircraft in transonic velocity region will appear a unique "pit" phenomenon, which directly restricts the flight envelope of the aircraft. Active flutter suppression is to transform unstable mode branches into stable ones by closed-loop control, and then improve flutter characteristics of aircraft and enlarge flight envelope. Usually, the aerodynamic control surface is used as the actuator which is deflected to generate additional unsteady aerodynamic force, and the energy of the aeroelastic system is balanced in the form of artificial damping. Reference [1] establish aerodynamic ROM by system identification method. In Matlab Simulink active flutter suppression of BACT wing with plunge/pitching degrees of freedom is carried out. Reference [2] used the ARMA method to establish the active suppression model of the aero-servo-elastic(ASE) system by coupling structural state equations, transonic unsteady aerodynamic ROM and the aerodynamic model of the control surface. The suboptimal control method based on output feedback is used to design the control law. Reference[3] used Volterra series ROM to carry out ASE synthesis design for two-dimensional BACT wing. Compared with CFD/CSD direct simulation, the computational efficiency of the reduced-order model was improved by 2 ~ 3 orders of magnitude. Reference [4] used POD method and equilibrium cut-off method to obtain aerodynamic ROM and reduced-order aeroelastic system,
and designed active control law based on LQR method to achieve flutter suppression for wing segments.

The low order model suitable for active control system design has certain requirements for the number of degrees of freedom, the cost of model generation, robustness and precision. In this paper, the Observer method [5] is used to establish the aerodynamic reduced order model. The physical smooth input is selected to train the CFD model so that it has a high computational efficiency in the time domain and the construction cost of the model is relatively small. The reduced order aeroelastic control system is constructed in the form of state space equations for aerodynamic, structural and control parts. The standard elastic model AGARD wing is taken as the object, and LQR method is used to design the active flutter suppression through the state space model.

2. Construction of Aerodynamic Rom

2.1. Overview of Method

System impulse response sampling is also called Markov parameter. Extracting Markov parameters from input / output data is an important step in system identification. These parameters can be used to develop efficient state space models of aeroelastic systems. The Observer method has been successfully used to identify modal parameters of structural systems. The Markov parameters of the small damping system decay slowly. First, the Markov parameters of the corresponding observer are obtained, and then the Markov parameters of the system itself are recovered [6]. The system is indirectly identified by asymptotically stable observer model and pole assignment method [7]. Ref. [8] discussed the pole assignment method of the observer, whose value can be real number, complex number or zero (corresponding to the Deadbeat observer).

In this paper, the Observer method is extended to the fluid system as in [5]. Discrete linear state space models are represented as

\[ x(k+1) = Ax(k) + Bu(k) \]
\[ y(k) = Cx(k) + Du(k) \] (1)

When the above system is at zero initial condition \( x(0) = 0 \), its input/output relation is [8]

\[ y(k) = \sum_{\tau=-\infty}^{k-1} Y_{k-\tau} u(\tau) + Du(k) . \] (2)

\( Y_{k-\tau} \) is denoted by the product \( CA^{k-\tau-1}B \), Equation (1) is written in matrix form about the time series \( k \) [6]

\[ y = Y U \]
\[ Y = [D \ CB \ CAB \cdots \ CA^{l-2}B] \] (3)

\( Y \) includes all the Markov parameters that need to be solved. For small damping systems, the scale of \( U \) may be too large to obtain the pseudo inverse \( U^+ \) by numerical calculation. The observer method solves this problem by introducing an observer into the input / output relation matrix, whose eigenvalue can be arbitrarily specified. Add and subtract \( Gy(k) \) at the right side of (1), then the observer model is constructed as [6]

\[ x(k+1) = \bar{A}x(k) + \bar{B}v(k) \]
\[ y(k) = Cx(k) + Du(k) \] (4)

\( \bar{A} = A + GC \), \( \bar{B} = [B + GD, -G] \)

\( G \) is an arbitrary matrix of \( n \times q \). If the original system is observable, the purpose of \( G \) is to configure the eigenvalue of matrix \( \bar{A} \) to be stabilized. At this point, the corresponding Markov parameter is called observer Markov parameter, and the input / output relationship is [6]
where

\[ y(k) = \sum_{r=0}^{k-1} \bar{Y}_{k-r-1}(\tau) + Du(k) \]  

(5)

Where

\[ \bar{Y}_{k-r-1} = C\bar{\lambda}^{k-r-1}\bar{B} \]  

(6)

The Markov parameters of the original system are restored based on the Deadbeat observer, and the unsteady aerodynamic state space model is constructed by using the Markov parameters combined with the eigenvalue realization algorithm.

2.2. Validation of Method

In order to verify the aerodynamic modeling method based on Observer technology, a AGARD CT6 example [9] is selected, which is often cited in many references. The airfoil is subjected to forced torsional motion around 1/4 chord length point. The CFD numerical simulation is carried out by using the N-S equations along with the dynamic grid technique, with \( M_\infty = 0.796 \) is simulated. The variation of lift coefficient with attack of attack is shown in figure 1.

![Figure 1. Comparison of lift coefficient between ROM and experiment.](image)

The reduced-order model shows the nonlinear effect of aerodynamic force reasonably. The predicted response is consistent with the wind tunnel experimental data [9], and retains the mainstream nonlinear characteristics. It only takes a few minutes to predict the response of the unsteady aerodynamics by means of state space model.

3. Design of Aeroelastic Active Control Law

3.1. Aeroelastic Control System Model

Coupling the aerodynamic state equations with structural state equations, aeroelastic system is written as

\[
\begin{align*}
\mathbf{x}_{\text{open}}(k + 1) &= \mathbf{A}_{\text{open}} \mathbf{x}_{\text{open}}(k) + \mathbf{B}_{\text{open}} \mathbf{F}(k) \\
\mathbf{y}_{\text{open}}(k) &= \mathbf{C}_{\text{open}} \mathbf{x}_{\text{open}}(k)
\end{align*}
\]  

(7)

The feedback control is realized by the active deflection of the control surface to generate additional aerodynamic forces. In this paper, it is assumed that the elastic deformation of the control surface relative to the main wing surface is negligible and the rigid body rotation of the control surface is only considered. In addition, the control surface is relatively smaller than the main wing surface, thus the interaction of aerodynamic interaction is not obvious. As a result, the unsteady aerodynamic force can be decomposed into both the main wing and the control surface[10]. An aerodynamic state equation corresponding to the mode of the control surface is established by means of the aerodynamic ROM technique.
\[ x_c(k+1) = A_c x_c(k) + B_c \delta(k) \]
\[ f_c(k) = C_c x_c(k) + D_c \delta(k) \]  \hspace{1cm} (8)

Where \( \delta \) is the deflection angle of the control surface, \( f_c \) is the additional aerodynamic coefficient caused by the deflection of the control surface.

Set \( x_{ace} = \begin{bmatrix} x_{ace}^\text{open} \end{bmatrix}^T \), \( F_c = df_c \). The open loop state equation of aeroelastic control system is obtained by joining equation (7) with equation (8), get

\[ x_{ace}(k+1) = A_{ace} x_{ace}(k) + B_{ace} \delta(k) \]
\[ y_{ace}(k) = C_{ace} x_{ace}(k) \]  \hspace{1cm} (9)

Where

\[
A_{ace} = \begin{bmatrix} A_{\text{open}} & qB_{\text{open}}C_c \\ 0 & A_c \end{bmatrix}, \quad B_{ace} = \begin{bmatrix} qB_{\text{open}}D_c \\ B_c \end{bmatrix}, \quad C_{ace} = \begin{bmatrix} C_{\text{open}} & 0 \end{bmatrix}.
\]

3.2. Vibration suppression design

The AGARD 445.6 wing is a classical aeroelastic test example [11]. In this paper, the geometric shape of the model is selected and the structural data of the model are used. The trailing edge control surface is installed at the outboard wing, and the structure layout is shown in figure 2. The span of the control surface is 30% of the wing span and the chord length of the control surface is 25% of the wing chord.

![Figure 2. AGARD wing with control surface.](image)

Firstly, the open-loop aeroelastic system is studied. The 70-order unsteady aerodynamic state equations of AGARD wing is constructed by using the Observer method, and the calculated open-loop flutter velocity is \( V^* = 0.4264 \) when \( M_x = 0.678 \). For \( V^* = 0.45 \), the generalized coordinate response of the AGARD wing is shown in Fig 3, and the system is at an unstable divergent state. At this speed, the LQR method is used to design the feedback control, and the system flow chart is shown in Fig. 4. The Observer method is used to establish the 45-order aerodynamic model of the control surface. Coupling aerodynamic equation, structural equation and control surface equation, an open-loop aeroelastic control model is established which is a controlled object for design of control law.

![Figure 3. Generalized coordinate response of each mode without control.](image)
In closed loop case, the modal response of wing structure are shown in figure 5. The response decays rapidly after initial disturbance, which indicates that the stability of the system is improved obviously under the control system. Change $V^*$ until the aerelastic control system reaches the critical state. The closed-loop critical response is shown in figure 6 and figure 7. The critical $V_f^* = 0.5043$ The flutter velocity is 20.8% higher than the uncontrolled wind tunnel test value $0.4174$[11].

3.3. Aeroelastic control system reduction by equilibrium truncation
The order of the three-dimensional aerelastic control model is relatively high, and it is still necessary to reduce the order of the model in the engineering application of the control law design. In this paper, the balanced truncation method [12] is adopted. The controllable and observable Gramian matrix of the system is defined as [12]
\[
W_c = \int_0^\infty e^{At}BB^T e^{A^T} dt
\]
\[
W_o = \int_0^\infty e^{At}C^T e^{A^T} dt
\]
(10)

It is generally considered that the actual system is controllable and stable, then \( W_c \) and \( W_o \) are nonsingular, symmetric positive semidefinite matrix. The realization of the internal balance of the system is to find the nonsingular transformation matrix \( T \) by which the original system \( \{A, B, C, D\} \) is transformed to \( \{A_0, B_0, C_0, D\} \), and the controllable and observable Gramian matrix is transformed into the same diagonal matrix

\[
W_{c0} = W_{o0} = \Sigma = \text{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2).
\]
(11)

Where \( \sigma_i \) is the Hankel singular value that reflects the comprehensive controllability and observability of the system state, \( \sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \). Truncate the state variable with lower controllability and observability, then the original system becomes \( \{A_m, B_m, C_m, D\} \).

The equilibrium truncation reduction of the open-loop aeroelastic model at Mach number 0.678 is presented in this paper. The full-order model is composed of 200-order aerodynamic state equation, 90-order control surface aerodynamic equation and 8-order structural state equation. The full-order model is reduced to 80-order, 50-order, 40-order and 30-order respectively. The open-loop flutter velocity calculated by full-order model is 0.4195 and the corresponding wind tunnel test value is 0.4174. The comparison of open-loop flutter response between full-order model and 30-order truncated model are given in Fig. 8. Design of flutter suppression control law based on truncated reduced order aeroelastic control model, and it would be applied to full-order model. The results are shown in Table 1. From the results of flutter suppression, the closed-loop flutter velocity of different order truncated model is close, but the increase ratio is obviously lower than that of direct design of full-order model.

| System order | Closed-loop flutter velocity | Higher than calculated open loop velocity |
|--------------|------------------------------|------------------------------------------|
| Full order 298 | 0.5236 | 24.8% |
| 80           | 0.4501 | 7.3%  |
| 50           | 0.4526 | 7.9%  |
| 40           | 0.4501 | 7.3%  |
| 30           | 0.4519 | 7.7%  |

**Figure 8.** Comparison of open-loop flutter responses.

**Table 1.** Closed-loop results
4. Conclusions
The Observer method focuses on the external input / output relationship of the system, extracts the Markov parameters of the system, and directly establishes the state space equation of the system. This form is beneficial to the comprehensive design of the aeroelastic control system. In this paper, the Observer method is introduced into identification of transonic unsteady aerodynamics, and the reduced-order aerodynamic model with higher accuracy and lower order is established, which is coupled with structural and control equations to construct the state-space model of low-order aeroelastic control system that is convenient for engineering application. It provides a theoretical basis for the engineering realization of transonic low-order flutter active suppression. The transonic AGARD wing is selected as an example. The simulation results show that the low-order aeroelastic control model is suitable for active flutter suppression in engineering field.

5. References
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