STABILITY OF SUPERNOVA Ia PROGENITORS AGAINST RADIAL OSCILLATIONS

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ABSTRACT

We analyze the possible existence of a pulsational instability excited by the $\epsilon$-mechanism during the last few centuries of evolution of a Chandrasekhar-mass white dwarf prior to its explosion as a Type Ia supernova. Our analysis is motivated by the temperature sensitivity of the nuclear energy generation rate ($\propto T^{23}$) in a white dwarf whose structural adiabatic index is near 4/3. Based on a linear stability analysis, we find that the fundamental mode and higher order radial modes are indeed unstable and that the fundamental mode has the shortest growth timescale. However, the growth timescale for such instability never becomes shorter than the evolutionary timescale. Therefore, even though the star is pulsationally unstable, we do not expect these radial modes to have time to grow and to affect the structure and explosion properties of Type Ia supernovae.

Subject headings: instabilities — stars: oscillations

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1. INTRODUCTION

The explosion of a Chandrasekhar-mass white dwarf as a Type Ia supernova is one of the most dramatic events that can befall a star and, at the same time, one of the most complex. The outcome depends on details of the ignition process (Woosley et al. 2004) and the uncertain physics of turbulent flame propagation (see Hillebrandt & Niemeyer 2000, Gamezo et al. 2003, and references therein). Both of these depend on the presupernova evolution of the star, which sets, e.g., the central density at the time of runaway and the distribution of ignition points. However, since pioneering studies in the 1970s by Paczynski (1972), Couch & Arnett (1973, 1974, 1975), and Iben (1978), comparatively little attention has been given to the evolution of the interior of the presupernova star. It is known that the star ignites carbon centuries before it finally explodes, commencing a long ramp-up that eventually concludes when convection can no longer carry away the excess energy generated by the degenerate nuclear runaway. It is during this phase that the Urca process may lead to a complex convective structure in the core and perhaps even oscillations related to the switching on and off of Urca neutrino losses (Iben 1978). In this paper we consider a comparatively simple question: “Is presupernova carbon burning inherently unstable, even in the absence of Urca losses?”

Carbon burning occurs at a rate that is extremely temperature sensitive, at least $T^{23}$ even at the end of the stable burning near $T \approx 7 \times 10^{8}$ K. The white dwarf is also supported chiefly by degenerate electrons that are relativistic, hence $\Gamma \approx 4/3$. In other contexts in stellar evolution, this combination often tends toward instability (e.g., Ledoux 1941; Maeder 1985; Baraffe et al. 2001). Large excursions in the radius are not strongly damped, and the nonlinearity of the reaction rate promotes instability. This is known as the $\epsilon$-mechanism (Unno et al. 1989).

Such an instability leads to oscillations around the hydrostatic equilibrium configuration and periodic phases of expansion and contraction on a dynamical timescale $\tau_{\text{dyn}} \approx (G\rho)^{-1/2}$. These types of oscillations are described as acoustic modes. They can be excited if an excitation mechanism operates in regions where the amplitude of eigenfunctions is large.

We have thus undertaken a linear stability analysis of the structure of Chandrasekhar-mass white dwarf models prior to explosion, based on a similar analysis performed for very massive stars (Baraffe et al. 2001). We pick up the white dwarf when it has just ignited carbon (nuclear energy generation substantially in excess of plasma neutrino losses) and is developing an extended convective core. The central temperature at this point is $\sim 3 \times 10^{8}$ K and it is $\sim 300$ yr until the star’s death. The evolutionary models and input physics used are described in § 2, and the results of the linear stability analysis are presented in § 3. A discussion follows in § 4.

2. EVOLUTIONARY MODELS PRIOR TO-explosion

2.1. Characteristic Evolutionary Timescales

Following ignition, a convective region develops in the white dwarf that eventually grows to encompass most of the star’s mass. Heat is transported outward, but both radiative losses and neutrino losses are negligible. Instead, the energy mostly goes into heating the convective portion of the star, and, to a lesser extent, expansion. The heat capacity is given by (Woosley et al. 2004)

$$c_p = \left( \frac{\partial e_{\text{ion}}}{\partial T} \right)_p + \left( \frac{\partial e_{\text{electrons}}}{\partial T} \right)_p$$

$$= \left( \frac{9.1 \times 10^{14}}{(8.6 \times 10^{13})T_a^{1/3}} \right) \text{ergs} \left(10^8 \text{ g K}^{-1}\right)^{-1},$$

(1)
where \( e \) is the specific internal energy, \( T_k = T/10^8 \) K, and \( \rho_0 = \rho/10^9 \) g cm\(^{-3}\). For the relevant temperature and density range, \( T_k \approx 2 - 7 \) and \( \rho_0 \approx 1 - 3 \), the ionic term dominates with a minor contribution from the electrons. To a factor of 2 accuracy, \( c_P \approx 10^{15} \) ergs g\(^{-1} \) (10\(^8 \) K\(^{-1}\)). The total thermal energy is

\[
H = \int_0^R \left[ \int_0^{T(r)} c_P(\rho(T), T)\rho(T)\,dT \right] 4\pi r^2\,dr.
\]

(2)

The correct definition for pure “thermal energy” should be expressed in terms of \( c_P \) rather than \( c_P \), since \( c_P \) also contains an expansion term \( P\,dV \). However, because of strong degeneracy in the interior of the white dwarf, \( c_P \approx c_P \), and equation (2) provides a good estimate of the thermal heat content. Moreover, for our purpose below, to estimate the gravitational heat content, this is the appropriate quantity to use. If the mass-averaged temperature is about half the central value and the convection zone, \( \approx 1 \) M\(_{\odot}\), then

\[
H \approx \left(7 \times 10^{48}\right) \frac{T_{k,c}}{7} \frac{M_{\text{conv}}}{M_{\odot}} \text{ ergs},
\]

(3)

where the index \( c \) stands for central quantities of the star. Actually, from computer models discussed in the next section, we know that the convective core grows from 0.2 to 1.15 M\(_{\odot}\) as the central temperature rises from \( 3 \times 10^8 \) to \( 7 \times 10^8 \) K (see Table 1). A better estimate of the heat in the convective region is

\[
H \approx 7 \times 10^{48} \left(\frac{T_{k,c}}{7}\right)^3 \text{ ergs}.
\]

(4)

This actually agrees to much better than a factor of 2 accuracy with the detailed computer results described in the next section.

The total nuclear power generated by the star is (Woosley et al. 2004)

\[
L \approx 7.0 \times 10^{44} \left(\frac{\rho_{0,c}}{2}\right)^{4.3} \left(\frac{T_{k,c}}{7}\right)^\nu \frac{X_{12}}{0.5} \text{ ergs s}^{-1},
\]

(5)

where \( \nu \approx 23 \) for the relevant range of temperature, and \( X_{12} \) is the carbon mass fraction. This expression is valid in the temperature range \( 4 \leq T_k \leq 7 \) K and becomes increasingly inaccurate at lower temperature, because of the temperature dependence of \( \nu \) and especially the complex dependence of the screening correction on temperature and density.

The nuclear timescale, which characterizes the increase of nuclear energy generation rate in the center, taking into account the reaction of the rest of the star, is

\[
\tau_L = \left(\frac{d\ln L}{dt}\right)^{-1} = \frac{3H}{\nu L} \approx \frac{3 \times 10^4}{\nu} \left(\frac{2}{\rho_{0,c}}\right)^{4.3} \left(\frac{7}{T_{k,c}}\right)^{(\nu-3)} \left(\frac{0.5}{X_{12}}\right)^2 \text{ s}.
\]

(6)

where it is assumed that \( L = dH/dt \) (see Table 1).

Since the central temperature or the total heat content of the star need only increase \( \approx 3\% \) for the power to double, the white dwarf evolution is governed by the rise of the power, \( L \). Thus, the nuclear timescale \( \tau_L \) and the time to explosion are the same, and both are considerably shorter, by about a factor of 5, than the thermal timescale \( \tau_H = ([d\ln H]/dt)^{-1} \). The star takes a few centuries to go from ignition at \( 3.0 \times 10^8 \) K to \( 7 \times 10^8 \) K but explodes a few minutes later. This is a considerably longer timescale than one would get by evaluating a local nuclear timescale \( \tau_{\text{nuc}} = (c_P T)/\nu (\nu \epsilon_{\text{nuc}}) \) just from the central energy generation rate and heat capacity, because the cooler middle regions of the white dwarf act as an appreciable heat reservoir for the energy generation. Until the central temperature exceeds \( 7 \times 10^8 \) K, convection keeps the two efficiently coupled (see the discussion in § 4).

### 2.2. Stellar Models

Computer models of accreting white dwarfs approaching criticality were constructed using the KEPLER stellar evolution code (Weaver et al. 1978). A composition of 30\% 12C and 70\% 16O by mass fraction was assumed, although the outcome will not depend appreciably on this assumption. The initial model consisted of a “warm” white dwarf with central temperature \( 10^8 \) K and a mass of \( 2.6 \times 10^{33} \) g (\( \approx 1.307 \) M\(_{\odot}\)). Test calculations that started from a cooler white dwarf resulted in a higher mass at ignition, but the results of the pulsation analysis were not affected by this modification. The white dwarf then accreted matter (also carbon and oxygen in the same proportion) at \( 10^{-7} \) M\(_{\odot}\) yr\(^{-1}\). The accretion was stopped when the runaway had started, but about 80 centuries before the final incineration, to allow the addition of the well-resolved surface layers. At this point, the star has reached a central density of \( \rho_{0,c} = 3.64 \) and a central temperature of \( T_{k,c} = 2.43 \). In the remaining time the star would have accreted less than \( 10^{-3} \) M\(_{\odot}\), which does not affect the structure of the star or the runaway (note that the runaway occurs many times that mass before reaching the Chandrasekhar mass).

The Lagrangian grid uses zone masses ranging from \( 10^{29} \) g in the center over some \( 10^{31} \) g in the middle of the star to a

### Table 1

| \( T_{k,c} \) | \( \rho_{0,c} \) | \( M_{\text{conv}} \) (M\(_{\odot}\)) | \( L \) (ergs s\(^{-1}\)) | \( dH/dt \) (ergs s\(^{-1}\)) | \( \tau_L \) (yr) | \( t_{\text{expl}} \) (yr) |
|---|---|---|---|---|---|---|
| 3.0 | 3.60 | 0.2 | \( 2.7 \times 10^{36} \) | \( 2.3 \times 10^{36} \) | 210 | 300 |
| 4.0 | 3.50 | 0.6 | \( 2.1 \times 10^{39} \) | \( 1.9 \times 10^{39} \) | 0.75 | 1.5 |
| 5.0 | 3.35 | 0.85 | \( 3.3 \times 10^{41} \) | \( 2.9 \times 10^{41} \) | 0.010 | 0.017 |
| 6.0 | 3.16 | 1.05 | \( 1.9 \times 10^{43} \) | \( 1.6 \times 10^{43} \) | \( 3.5 \times 10^{-4} \) | \( 4.6 \times 10^{-4} \) |

**Note.**—\( T_{k,c} \) is the central temperature in units of \( 10^8 \) K, \( \rho_{0,c} \) is the density in units of \( 10^9 \) g cm\(^{-3}\), \( M_{\text{conv}} \) is the convective core mass, \( L \) is the nuclear power defined in eq. (5), \( dH/dt \) is the time derivative of the total heat content \( H \) defined in eq. (4); \( \tau_L \) and \( t_{\text{expl}} \) are the nuclear and the explosion timescales, respectively, in years.
smooth gradient in zone size down to $10^{15} \text{ g}$ at the surface. Convection is treated using standard mixing length theory with a mixing length $l_{\text{mix}} = H_P$.

During the last $\sim 300$ yr prior to explosion, the nuclear energy generation rate, $\epsilon_{\text{nuc}}$, rises rapidly in the center of the white dwarf. This is illustrated in Figure 1, which displays the evolution of central temperature, density, and nuclear energy as a function of the time remaining before explosion $(t_{\text{expl}} = 0$ corresponds to the time of flame ignition or start of the explosion). While only a small fraction of the energy released goes into expanding the star, the expansion is significant because $\Gamma \approx 4/3$.

3. PULSATION ANALYSIS

The system of equations that are linearized are the basic hydrodynamic equations written under the assumption of spherical symmetry (see Unno et al. 1989, p. 89). Details of the method and pulsation code can be found in Baraffe et al. (2001). The unperturbed, equilibrium state is described by hydrostatic equilibrium. The nuclear energy term appearing in the energy conservation equation includes neutrino energy loss. An important uncertainty in the present analysis is due to the assumption that convection is frozen in, neglecting the perturbation of the convective flux. This simplification is based on the argument that the convective timescales in the interior of the white dwarf remain significantly larger than the period of pulsation of the excited modes (see below). A sequence of evolutionary models is analyzed, starting from a central temperature $T_{8,c} = 3$, $\sim 300$ yr before explosion, to $T_{8,c} = 7$, corresponding to $\sim 10$ minutes before explosion. All models analyzed show that the fundamental and higher-order modes are unstable. As in Baraffe et al. (2001), the assumed time dependence for the eigenfunctions is of the form $\exp (\omega t) \times \exp (\kappa r)$, where $\sigma = 2\pi / \Pi$ is the eigenfrequency, $\Pi$ is the corresponding period, and $K$ is the stability coefficient. The $e$-folding time characterizing the growth timescale of the pulsation amplitude is defined by $\tau_d = 1/K$. In all models analyzed, this fundamental growth timescale is, by at least 1 order of magnitude, shorter than $\tau_d$ of the higher-order modes. Our analysis below can thus be restricted to the fundamental mode, since it is the mode that grows the fastest.

### TABLE 2

| $T_{8,c}$ | $\rho_{8,c}$ | $R$ (Mm) | $t_{\text{expl}}$ (s) | $\tau_{\text{nuc}}$ (s) | $\tau_d$ (s) | $\Pi_0$ (s) |
|-----------|----------------|----------|-------------------------|-------------------------|-------------|-------------|
| 3         | 3.60            | 1.75     | $9.68 \times 10^9$      | $2.99 \times 10^8$      | $2.95 \times 10^4$ | 1.95        |
| 4         | 3.50            | 1.77     | $4.48 \times 10^8$      | $8.31 \times 10^7$      | $6.19 \times 10^4$ | 1.95        |
| 5         | 3.35            | 1.79     | $5.46 \times 10^8$      | $8.63 \times 10^7$      | $4.92 \times 10^4$ | 1.96        |
| 6         | 3.16            | 1.82     | $1.47 \times 10^4$      | $2.26 \times 10^3$      | $1.03 \times 10^4$ | 1.97        |
| 7         | 2.97            | 1.85     | $6.57 \times 10^2$      | $11.3$                   | $4.22 \times 10^3$ | 1.98        |

**Note.**—Models are characterized by the central temperature $T_{8,c}$, central density $\rho_{8,c}$, and white dwarf radius $R$. The value $t_{\text{expl}}$ is the remaining time before explosion, and $\tau_{\text{nuc}} = c_p T / (\rho_{\text{nuc}})$ is the local nuclear timescale defined in the center. The growth timescale (see text) and the period of the fundamental mode are denoted by $\tau_d$ and $\Pi_0$, respectively.
Table 2 summarizes the stability analysis results. The fundamental mode pulsation period $\Pi_0$ is $\sim 2$ s, close to the dynamical timescale. Note that the shortest convective timescale is $\sim 10^4$ s in the first model analyzed ($T_{\text{nuc}, c} = 3$) and $\sim 10^5$ s in the last models. For a model with $T_{\text{nuc}, c} = 5$ ($\sim 6$ days before explosion), Figure 2 displays the work $dW/dM$, which is proportional to the energy transferred to the pulsation, with $W$ the so-called work integral (Cox 1980, p. 114; Unno et al. 1989). Positive work indicates driving regions, whereas negative work corresponds to damping zones. The positive value of $dW/dM$ in the center is due to the perturbation of the nuclear energy generation rate and characterizes the $\epsilon$-mechanism. As expected for a gas with $\Gamma_1 \approx 4/3$, the displacement $|\delta r|/r$ is essentially constant through the whole structure. Damping processes are small in the rest of the white dwarf, and the total work integral $W$ is essentially determined by the driving zone in the center.

4. DISCUSSION

Linear stability analysis alone does not provide information about the maximum pulsation amplitude that can be finally reached. For this purpose, full hydrodynamical calculations are needed. However, comparison between the growth timescale $\tau_d$ of an excited mode and the evolutionary timescale $t_{\text{expl}}$ can indicate whether the perturbation may have time to grow in order to reach large amplitudes. Before doing such comparisons, it is useful to consider the uncertainties affecting the evolutionary timescale.

As previously discussed in § 2.1, the relevant timescale characterizing the evolution to explosion is considerably longer than the local nuclear timescale because of the heat reservoir provided by convective coupling to the rest of the star. For an $\epsilon$-mechanism, the growth timescale is expected to be closely related to the local nuclear timescale, since $\tau_{\text{nuc}}$ characterizes the increase of local nuclear energy generation rate in the central region where the $\epsilon$-mechanism takes place (see Fig. 2). The discussion below will thus be based on $\tau_{\text{nuc}}$ and its comparison with the time to explosion. Qualitatively, convection can lengthen the evolutionary timescale compared to the local nuclear timescale, and thus $\tau_{\text{nuc}} \lesssim t_{\text{expl}} \approx \tau_d$.

For the sake of a quantitative comparison, we have performed test evolutionary sequences with different convection efficiencies in terms of the mixing length parameter $L_{\text{mix}}$. All test sequences start from the same model with central temperature $T_{\text{nuc}, c} = 3$. The nuclear timescale $\tau_{\text{nuc}}$ as a function of the remaining time before explosion is displayed in Figure 3 for different cases of convection efficiency: the standard case with $L_{\text{mix}} = H_P$ (solid line), $L_{\text{mix}} = 100H_P$ (dotted line), $L_{\text{mix}} = H_P/100$ (dotted line), and a case with no convection (dash-dotted line). The long-dashed line corresponds to the case $t_{\text{expl}} = \tau_{\text{nuc}}$. As expected, when convection is inhibited, the runaway timescale is essentially determined by the local nuclear timescale. On the other hand, the more efficient the convection, the longer the evolutionary timescale compared to $\tau_{\text{nuc}}$. Figure 3 indicates that even in the unrealistic case with $L_{\text{mix}} = 100H_P$, the evolutionary timescale is at most $\sim 70$ times greater than $\tau_{\text{nuc}}$. Therefore, although the treatment of convection is crucial and still very uncertain during these last phases of evolution, one can reasonably expect that such uncertainty will not lengthen the evolutionary timescale by more than $100\tau_{\text{nuc}}$.

Figure 4 displays the growth timescale $\tau_d$ of the fundamental mode and the local nuclear timescale $\tau_{\text{nuc}}$ as a function of the evolutionary timescale $t_{\text{expl}}$, for the standard case of convection efficiency (see also Table 2). As expected and illustrated in Figure 4, $\tau_d$ follows $\tau_{\text{nuc}}$, with $\tau_d/\tau_{\text{nuc}} \approx (5\times 10^2)-10^3$, the same relation of proportionality being found independently of convection efficiency. Since the tests on convection efficiency...
suggest that $t_{\text{expl}} < 100t_{\text{nuc}}$, the quantity $\tau_d$ thus remains systematically larger than the evolutionary timescale $t_{\text{expl}}$, as illustrated in Figure 4. For standard convection, $\tau_d$ is larger than $t_{\text{expl}}$ by a factor of $\sim 30$ at the beginning of the sequence ($T_{8,c} = 3$) down to a minimum factor of $\sim 6.5$ for $T_{8,c} = 7$. Similar behavior is found for the test sequences with varying convection efficiency, with the smallest ratio $\tau_d/t_{\text{expl}} \approx 5$ found for the case $L_{\text{mix}} = 100H_P$. Such results suggest that the present vibrational instability does not have time to grow to reach significant amplitudes.

5. CONCLUSION

Based on a linear stability analysis of Chandrasekhar-mass white dwarf models prior to explosion, we have found that the central conditions are favorable to the excitation of radial eigenmodes through the $\epsilon$-mechanism. Such pulsational instability could, in principle, result in phases of expansion and contraction with growing amplitude and thus affect the structure of the supernova progenitor. We also found, however, that the growth timescale for the pulsation amplitude remains systematically longer than the evolutionary timescale. Such instability does not have time to grow significantly before the explosion of the white dwarf.

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