Testing Scenario Library Generation for Connected and Automated Vehicles, Part II: Case Studies

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Abstract—Testing and evaluation is a critical step in the development and deployment of connected and automated vehicles (CAVs), and yet there is no systematic framework to generate testing scenario library. In Part I of the paper, a general framework is proposed to solve the testing scenario library generation (TSLG) problem with four associated research questions. The methodologies of solving each research question have been proposed and analyzed theoretically. In Part II of the paper, three case studies are designed and implemented to demonstrate the proposed methodologies. First, a cut-in case is designed for safety evaluation and to provide answers to three particular questions in the framework, i.e., auxiliary objective function design, naturalistic driving data (NDD) analysis, and surrogate model (SM) construction. Second, a highway exit case is designed for functionality evaluation. Third, a car-following case is designed to show the ability of the proposed methods in handling high-dimensional scenarios. To address the challenges brought by higher dimensions, the proposed methods are enhanced by reinforcement learning (RL) techniques. Typical CAV models are chosen and evaluated by simulations. Results show that the proposed methods can accelerate the CAV evaluation process by \(255\) to \(3.75 \times 10^5\) times compared with the public road test method, with same accuracy of indices.

Index Terms—Testing Scenario Library, Connected and Automated Vehicles, Testing and Evaluation, Safety, Functionality, Reinforcement Learning

I. INTRODUCTION

TESTING and evaluation is a critical step in the development and deployment of connected and automated vehicles (CAVs), which is usually conducted via three steps: simulation test, closed facility test, and public road test. Closed facility test, which can test real CAV in a controlled environment, has potential to greatly improve the testing efficiency. The key of the improvement is generation of testing scenario libraries. A testing scenario library is defined as a set of critical scenarios, multi-start optimization and seed-fill based method is applied, where an auxiliary objective function is designed to provide searching directions. The proposed methods are justified by theoretical analysis.

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In the past few years, increasing research efforts have been proposed to solve the testing scenario library generation (TSLG) problem \([1][2][3][4][5] ([1] for more details). However, all methods have limitations in either scenario types that can be handled (e.g., low-dimensional scenarios only), CAV models that can be applied (e.g., a specific CAV only), or performance metrics that can be evaluated (e.g., safety evaluation only).

A. Overview of Part I

To overcome these limitations, in Part I of this study \([7]\), we propose an innovative framework and methods to provide accurate and efficient solutions to the TSLG problem for different scenario types, CAV models, and performance metrics. Four research questions are identified and resolved, i.e., scenario description, metric design, library generation, and CAV evaluation. The major idea is to define the criticality of scenarios and search critical scenarios to construct the library. To this end, a new definition of criticality is rigorously proposed as a combination of maneuver challenge and exposure frequency. The new definition is fundamentally different from most existing studies, which usually overvalue the challenge yet infrequent scenarios, e.g., worst-case scenario evaluation \([6]\) and accelerated evaluation \([5]\). To efficiently search for critical scenarios, multi-start optimization and seed-fill based method is applied, where an auxiliary objective function is designed to provide searching directions. The proposed methods are justified by theoretical analysis.

B. Motivation

Part I of this paper lays out the methodological foundation and proves the statistical accuracy and efficiency theoretically. To implement the proposed methodologies, however, there exist a few remaining issues as follows:

First, although the proposed framework is generic, some sub-problems vary case by case, e.g., auxiliary objective function design, naturalistic driving data (NDD) analysis, and surrogate model (SM) construction. Implementing different case studies provide guidelines of the overall framework.

Second, the efficiency of the proposed methods needs further validation by case studies. The accuracy of the methods...
has been proved (see Theorem 1 in Part I), while the efficiency is proved under certain conditions, which cannot be guaranteed rigorously in real applications (see Theorem 2 in Part I). Case studies are desirable for further validation of the efficiency.

Third, it is significant to show the ability of the proposed methods in handling different performance metrics. Most existing studies only focus on safety evaluation, which is essential but insufficient for a deployable CAV. Besides safety, functionality is another critical performance metric, which shows the CAV’s ability to complete driving tasks in a timely manner. Designing and implementing testing scenarios for functionality evaluation are necessary.

Fourth, applying the proposed methods directly to high-dimensional cases can be problematic. Considering the complexity of driving environment, most testing scenarios are naturally high-dimensional. However, most existing studies suffer from the “curse of dimensionality”. For example, the PEGASUS project \cite{3} applied an exhaustive searching method to find all scenarios, which is impossible for high-dimensional scenarios. The accelerated evaluation (AE) method was proposed based on calibrating an importance function by testing CA V \cite{5}. The car-following case \cite{5}, where the number of required tests grows exponentially with scenario dimensions when calibrating the importance function. In the car-following case \cite{5}, the AE method degraded to a white-box method with the assumption of knowing the exact CAV models, which is intractable for practical applications. Although the proposed methods in Part I are efficient, it still suffers from the curse of dimensionality to a certain extent. Therefore, enhancing the proposed methods in high-dimensional cases is required.

C. Contributions

To address the abovementioned issues, three categories of scenarios are designed, as shown in Fig. 1. (1) Cut-in case: a background vehicle (BV) makes a lane change in front of the testing CAV. (2) Highway exit case: the testing CAV needs to make a lane change to the right and exits the highway within a certain distance. (3) Car-following case: the testing CAV follows a BV for certain time. The purposes of designing such cases are explained below.

The cut-in case illustrates each step of the scenario library generation and evaluation framework regarding safety. A few specific questions are elaborated, i.e., auxiliary objective function design, NDD analysis, and SM construction. Moreover, because the cut-in case is low dimensional (i.e., two dimensions), it is convenient to visualize the results by figures and help readers better understand the proposed methods.

The highway exit case focuses on the functionality evaluation. Compared with safety evaluation, the major difference lies in the design of auxiliary objective function for the library generation, i.e., how to quantify the maneuver challenge regarding functionality. To this end, several new concepts are proposed, i.e., task, task solution, task solution difficulty, and task difficulty. The specific auxiliary objective function is designed for the highway exit case based on the concepts.

The car-following case is designed to show the ability of the proposed methods in solving the TSLG problem of high-dimensional scenarios. To this end, the proposed methods in Part I are enhanced by reinforcement learning (RL) techniques. Specifically, decision variables of scenarios are represented by Markov Decision-making Process (MDP) considering independence properties of the variables. The definition of scenario criticality is gracefully inherited in the new definition of state-action values, i.e., $Q$ values. To guarantee the convergence of $Q$ values, the temporal-difference (TD) RL theory is applied, and an iteration equation is designed. The RL-enhanced method shows the powerful ability of the framework proposed in Part I in handling high-dimensional scenarios.

Overall, the three cases help validate the accuracy and efficiency of the proposed methods for different scenario types (e.g., low- and high-dimensional scenarios) and performance metrics (e.g., safety and functionality). Typical CAV models are tested and evaluated in the case studies. Compared with simulation results of public road test, the proposed method can accelerate the evaluation process by 255 to $3.75 \times 10^5$ times.

D. Structure

The rest of the paper is organized as follows. For the convenience of readers, Section II briefly revisits the proposed methods in Part I. Section III studies the cut-in case for safety evaluation. Section IV studies the highway exit case for functionality evaluation. The RL-enhanced method is developed for high-dimensional car-following case in Section V. Finally, Section VI concludes the paper.

II. REVISIT THE PROPOSED METHODS IN PART I

In this section, we briefly revisit the proposed methods in Part I \cite{7} including library generation and CAV evaluation. Notations of related variables are listed in Table I.

A. Library Generation

The basic idea of library generation is to define the criticality of scenarios and search critical scenarios to construct the library. If pre-determined parameters of scenarios in the operational design domain (ODD) are denoted as $\theta$, and the vector of decision variables is denoted as $x$, then the criticality of a scenario is defined as

$$V(x|\theta) \overset{\text{def}}{=} P(S|x, \theta)P(x|\theta), \quad (1)$$
TABLE I
NOTATIONS OF VARIABLES IN THIS PAPER.

| Variable | Notation |
|----------|----------|
| $\theta$ | Pre-determined parameters of scenarios in the operational design domain. |
| $x$ | Decision variables of testing scenarios. |
| $A$ | Event of interest (e.g., accident) with a CAV model. |
| $\hat{S}$ | Event of interest (e.g., accident) with a surrogate model. |
| $\chi$ | Feasible set of the decision variables. |
| $\Phi$ | Set of decision variable vector of critical testing scenarios. |
| $V(x|\theta)$ | Criticality value of a scenario determined by $x$ and $\theta$. |
| $N(\chi), N(\Phi)$ | Total number of scenarios of the set $\chi, \Phi$. |
| $P(x_i|\theta)$ | Probability of sampling the scenario $x_i$ in the generated library with pre-determined parameters $\theta$. |
| $\hat{P}(A|\theta)$ | Estimated probability of the event $A$ with pre-determined parameters $\theta$. |
| $n$ | Total number of sampled testing scenarios. |

where $S$ denotes the event of interest (e.g., accident) with a surrogate model (SM) of CAVs. Since too many local optimal solutions exist in the criticality function, directly searching critical scenarios is inefficient. To solve this issue, the multi-start optimization and seed-fill based searching method is applied, where an auxiliary objective function is designed to provide searching directions. The SM and the auxiliary objective function will be discussed case by case.

B. CAV Evaluation

After the generation of library, testing scenarios are sampled from the library with $\varepsilon$-greedy policy, and the index is estimated based on the testing results. A minimal number of tests is required for attaching certain estimation precision.

The sampling distribution with $\varepsilon$-greedy policy is derived as

$$
\widehat{P}(x_i|\theta) = \left\{ \begin{array}{ll}
(1 - \varepsilon)V(x_i|\theta) / W, & x_i \in \Phi \\
\varepsilon / (N(\chi) - N(\Phi)), & x_i \notin \Phi
\end{array} \right.
$$

where $N(\chi)$ denotes the total number of feasible scenarios, $x \in \Phi$ denotes the set of critical scenarios, the selection of $\varepsilon$ is theoretically analyzed (see Theorem 3 in Part I), and $W$ is a normalization factor as

$$
W = \sum_{x_i \in \Phi} V(x_i|\theta).
$$

After testing the CAV in sampled scenarios, an index of the targeted performance metric (e.g., accident rate of safety performance) can be estimated as

$$
\hat{P}(A|\theta) = \frac{1}{n} \sum_{i=1}^{n} P(x_i|\theta) / \hat{P}(x_i|\theta) \hat{P}(A|x_i, \theta),
$$

where $n$ denotes the total number of the sampled testing scenarios, $A$ denotes the event of interest with the CAV model, and $\hat{P}(A|x_i, \theta)$ is estimated by the testing results.

The minimal number of tests is shown as follow. The testing process stops if the relative half-width of the estimation is less than $\beta$ [4][8][9] as

$$
\frac{2\alpha}{\hat{P}} Var(\hat{\mu}) \leq \beta,
$$

where $z_\alpha$ is a constant with the confidence level at $100(1 - \alpha)\%$, $\hat{P}(A|\theta)$ is the estimation of the index, and $Var(\hat{\mu})$ is the estimation variance, which decreases with increasing number of tests.

III. CUT-IN CASE STUDY

In this section, the testing scenario library for cut-in case regarding safety evaluation is generated and validated.

A. Problem Formulation

Similar to most existing studies [3][4], the decision variable vector of the cut-in case is simplified as two dimensions, i.e.,

$$
x = [R, \dot{R}]^T,
$$

where $R$ and $\dot{R}$ denote the range and range rate at the cut-in time respectively. For this simplification, the BV is assumed to keep constant velocity after the cut-in behavior, and parameters of road environments are pre-determined. All these pre-determined parameters are denoted as $\theta$.

The accident rate is utilized to measure the safety performance of CAVs in the cut-in case. The road test method is simulated to estimate the accident rate as a baseline. Specifically, if a testing CAV drives on public roads, experiences $n$ specified cut-in scenarios, and has $m$ accident events, the accident rate of event $A$ can be estimated by

$$
P(A|\theta) \approx \frac{m}{n}.
$$

The public road test is simulated based on naturalistic driving data (NDD), so the method is denoted as NDD evaluation method in this paper.

B. Library Generation

To implement the library generation method, three questions need to be answered specifically, i.e., auxiliary objective function design, NDD analysis, and SM construction.

1) Auxiliary Objective Function Design: To provide searching directions for critical scenarios, an auxiliary objective function is designed as the combination of maneuver challenge and exposure frequency.

First, the maneuver challenge is estimated by minimal normalized positive enhanced time-to-collision (mnpETTC). As discussed in [10][11], ETTC is one of most widely used indices of safety evaluation for varying velocity scenarios, defined as

$$
ETTC(t) = -\dot{R}(t) - \sqrt{\ddot{R}^2(t) - 2u_r(t)R(t)} / u_r(t),
$$

where $R(t)$ and $\dot{R}(t)$ are the range and range rate at time $t$ respectively, and $u_r(t)$ is the relative acceleration. Values of ETTC for different scenarios can be obtained by simulations. To make the index comparable with exposure frequency, a normalization factor is applied, denoted as $U_f$, which is calibrated by NDD analysis. The negative values, which denote safe situations, are set one. Then the ETTC is modified to denote the most dangerous situation of a scenario, i.e., the
minimal normalized positive ETTC (mnpETTC), which can be calculated as
\[ mnpETTC(t) = \min_i npETTC(t), \] (9)
where
\[ npETTC(t) = \begin{cases} \frac{ETTC(t)}{U_i}, & ETTC(t) \geq 0 \\ 1, & ETTC(t) < 0 \end{cases} \] (10)

Second, the exposure frequency of a scenario is estimated by the distance between the scenario and a common set (i.e., scenarios with high exposure frequency). The common set is determined by NDD analysis, and the distance is defined as
\[ d(x, \Omega) = \min_{y \in \Omega} \left[ \frac{1}{m} \sum_{i=1}^{m} \frac{(x_i - y_i)^2}{U_{F,i}} \right], \] (11)
where \( \Omega \) denotes the common set, \( m \) is the dimension of the decision variable vector, and \( U_{F,i} \) is the normalization factor for the \( i \)-th dimension, which is calibrated by NDD analysis.

Finally, the auxiliary objective function for safety evaluation in the cut-in case is formulated as
\[ \min_x J(x) = \min_x \left( mnpETTC(x) + w \times d(x, \Omega) \right), \] (12)
where \( w \in [0, 1] \) is a balance weight. Note the goal of the auxiliary objective function is to provide searching directions, so certain roughness (e.g., caused by \( w \)) is reasonable and acceptable.

2) NDD Analysis: NDD is analyzed to provide exposure frequency measurement, determine parameters of the auxiliary objective function, and calibrate the SM.

The NDD from the Safety Pilot Model Deployment (SPMD) program at University of Michigan [12] is utilized for the cut-in case. The SPMD database is one of the largest databases in the world that recorded naturalistic driving behaviors over 34.9 million miles from 2,842 equipped vehicles in Ann Arbor, Michigan. In the database, there are 98 sedans equipped with the data acquisition system and MobileEye, which enables measuring and recording the position and speed data between the host vehicle and preceding vehicles at an frequency of 10 Hz. The following query criteria are designed to extract all cut-in events from the database [4][13]: (a) the vehicles’ speeds at the cut-in time belong to \((2 m/s, 40 m/s)\); (b) the range at the cut-in time belongs to \((0.1 m, 90 m)\). All these criteria are consistent with the pre-determined parameters \( \theta \). As a result, 414,770 qualified cut-in events are successfully obtained. Fig. 2 shows the location distribution of the events. The exposure frequency distribution (i.e., \( P(x|\theta) \)) is shown in Fig. 3 where brighter color denotes higher exposure frequency, i.e., the common set. The range and range rate are discretised by \(2m\) and \(0.4m/s\) respectively. The NDD evaluation method is equivalently sampling testing scenarios from this probability distribution.

The parameters of the auxiliary objective function are determined. First, the common set is determined by finding a minimal rectangle or hyper-rectangle of scenarios with high probabilities (i.e., \( P(x|\theta) > 10^{-3} \)). As shown in Fig. 3 the dashed red rectangle denotes the boundaries of the common set in the cut-in case (i.e., \([6, 88]\) for range and \([-2.4, 1.2]\) for range rate). For more complex scenarios, the common set can be further simplified as the most frequent scenario, e.g., \( R = 14, \tilde{R} = 0 \). Second, the normalization factors are determined by the maximal distance between scenarios and the common set. For example, the maximal range rate between scenarios and the common set is smaller than 18, so the normalization factor of the range rate is set to 18. The values of the parameters for the cut-in case are listed in Table II.

3) Surrogate Model Construction: SM construction is a very important step in the library generation process. It denotes

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| \( m \)   | 2     | \( U_F \) | 100   |
| \( U_{F,1} \) | 18    | \( U_{F,2} \) | 20    |
| \( w \)   | 1.0   | \( v_{max} \) | 40 m/s |
| \( v_{min} \) | 2 m/s | \( a_{min} \) | -4 m²/s |
| \( a_{max} \) | 2 m²/s | \( \beta \) | 0.3   |
| \( \alpha \) | 0.95  | \( d_{acc} \) | 1 m   |
| \( \alpha_{IDM} \) | 2     | \( \beta_{IDM} \) | 18    |
| \( c_{IDM} \) | 4     | \( s_0 \) | 2     |
| \( L_{IDM} \) | 4     | \( T \) | 1     |
| \( b_{IDM} \) | 3     | -        | -     |
what we know about the common features of CAVs. Different types of CAVs may have common features as well as unique features brought by their own manufacturers. The “common features” capture fundamental functions of a well-developed vehicle behavior, e.g., keep safe distances and interact with surrounding vehicles safely. It is similar to human driving vehicles, where different drivers have different driving habits, but common features exist among all drivers. The benefit of capturing common features in the SM is that it can be applied to all types of CAVs. A well-generated library should be designed towards common features and includes more critical scenarios for most CAVs.

An ideal SM should be calibrated from actual CAV driving data similar to human driving model calibration [14]. At the current stage, however, there is very little open CAV data available for public research. Therefore, we propose to calibrate the SM based on the human driving data, i.e., NDD. It is a reasonable starting point because of the following reasons. First, the common features of human drivers are the natural baselines for CAV evaluation. Critical scenarios for human drivers are the most straightforward testing scenarios for CAVs. Second, CAV is essentially an application of “artificial intelligence”, the purpose of which is to mimic and outperform “human intelligence” [1]. Many CAV algorithms are obtained by imitating human driving behaviors, e.g., end-to-end learning method [15][16]. Third, a “human-like” CAV can improve safety in a mixed traffic condition, where CAV and human-driven vehicles coexist on the roadway. A similar concept of “roadmanship” was recently proposed for CAV evaluation [17].

In this case study, the commonly used intelligent driving model (IDM) is calibrated by the NDD [18] and selected as the SM for the car-following behaviors of CAVs after the cut-in event as

\[
u(k + 1) = \alpha_{\text{IDM}} \left( 1 - \left( \frac{v(k)}{\beta_{\text{IDM}}} \right)^{c_{\text{IDM}}} - \left( \frac{s_{\text{IDM}}(v(k), \dot{R}(k))}{\dot{R}(k) - L_{\text{IDM}}} \right)^2 \right)\]

where \( k \) denotes the discrete time step, \( u \) denotes the acceleration, \( \alpha_{\text{IDM}}, \beta_{\text{IDM}}, c_{\text{IDM}}, L_{\text{IDM}} \) are constant parameters, and

\[s_{\text{IDM}}(v(k), \dot{R}(k)) = s_0 + v(k)T + \frac{v(k)\dot{R}(k)}{2\sqrt{\alpha_{\text{IDM}}b_{\text{IDM}}}}, \quad (14)\]

where \( s_0, b_{\text{IDM}}, T \) are constant parameters. Similar to [19], the constraints of acceleration and velocity are added to make the model more practical (i.e., model accident-prone behaviors) as

\[v_{\text{min}} \leq v \leq v_{\text{max}}, a_{\text{min}} \leq u \leq a_{\text{max}}. \quad (15)\]

The calibrated values are listed in Table II Fig. 4 shows the safety performance of the selected SM, where the SM has accidents in scenarios of the yellow region.

4) Library Generation: The optimization and seed-fill based method is applied to search for critical scenarios and construct the library. In this case, 50 points are uniformly sampled as the initial starting points. As discussed in Theorem 4 in Part I, the threshold of critical scenarios is determined as

\[P(x|S, \theta) > \frac{m}{N(X) - N(\Phi)} > \frac{m}{N(X)} = 2.9 \times 10^{-4}, \quad (16)\]

where \( m = 1 \), and \( N(X) = 47 \times 76 = 3,420 \). The discretization intervals of the range and range rate are \( 2m \) and \( 0.4m/s \), and the boundaries of the range and range rate are \( [0, 90] \) and \( [-20, 10] \) respectively.

Fig. 5 shows the obtained probability distribution after the library generation process. The color denotes the probability of a scenario, i.e., the normalized criticality. Compared with Fig. 3 where only exposure frequency is considered, the new distribution encodes more domain knowledge, i.e., maneuver challenge and exposure frequency of scenarios. A library is constructed by the critical scenarios. In this case, the generated library contains a total number of 184 scenarios, which is about 5.38% of all scenarios.

C. CAV Evaluation

In this step, a specific CAV is evaluated with the generated library. For field implementation, a real CAV should be tested.
The testing scenarios are sampled from the generated library in Fig. 3. For the proposed method, applied as the baseline, where testing scenarios are sampled from the NDD distribution in Fig. 3. The NDD evaluation method is adaptive cruise control and autonomous emergency braking as a proof of concept to validate the proposed method. Although a simulated CA V model cannot exactly reflect dynamics of a real CA V , it is used in this paper as a demonstration of functionality evaluation. For further studies, the high-dimensional problem can be resolved by the RL-enhanced method proposed in section V.

IV. HIGHWAY EXIT CASE STUDY

In this section, the testing scenario library of highway exit case for functionality evaluation is generated and validated.

A. Problem Formulation

As shown in Fig. 1 (b), the decision variable vector of the highway exit scenario should include initial states of the CA V , number of BVs, and trajectories of each BV , which is high-dimensional. To simplify the problem and focus on the functionality evaluation, the initial position and velocity of the CA V are pre-determined as \( p_0 \) and \( v_0 \), the number of BVs is pre-determined as two, and all BVs keep their initial velocity unless the distance is less than a threshold \( d_{cf} \), when the following BV will change its speed to be the same as the leading BV. As a result, the decision variable vector is formulated as

\[
x = [p_{0,1}, v_{0,1}, p_{0,2}, v_{0,2}]^T,
\]

where \( p_{0,i}, v_{0,i} \) denote the initial position and velocity of the \( i \)-th BV. The discrete interval of time and position is chosen as \( \Delta t \) and \( \Delta p \) respectively. The parameter values used in this study are summarized in Table III. Although the simplified problem cannot exactly reflect the actual highway exit scenarios, it can be used as a demonstration of functionality evaluation. For further studies, the high-dimensional problem can be resolved by the RL-enhanced method proposed in section V.

B. Library Generation

The library generation methods are the same as the cut-in case. To make the paper concise, only the auxiliary objective function design for the functionality evaluation will be elaborated.

1) Auxiliary Objective Function Design: Similar to the cut-in case, the auxiliary objective function is composed of exposure frequency and maneuver challenge. To evaluate the maneuver challenge for generic functionality, four new concepts are proposed, i.e., task, task solution, task solution difficulty, and task difficulty. The “task” is defined based on the functionality, e.g., exit from the highway. The “task solution” denotes a feasible CA V trajectory to complete the task, i.e., \( f \in \mathbb{F} \). The “task solution difficulty” denotes the difficulty in completing the task solution, i.e., \( W(f) \), where \( W(f) \) is negative and larger \( W(f) \) denotes higher difficulty. Finally, the “task difficulty” denotes the difficulty of the task, which can be evaluated by the summation of all task solution difficulties as

\[
M_f(x) = \sum_{f \in \mathbb{F}} W(f),
\]
This definition can represent both the difficulty in finding a feasible solution to the task and completing that solution. For the specified highway exit case, the maneuver challenge is evaluated based on the proposed concepts. The task is to make a lane change to the right before reaching the off-ramp location. The task solution is defined as a feasible lane-change point \( f = (t, p) \), where \( t \) is the lane-change time and \( p \) is the lane-change position. The feasible lane-change zone \( f \in \mathcal{F} \) is determined by maximal/minimal velocity \( (v_{\text{max}}, v_{\text{min}}) \), highway exit location \( (L) \), safe time-to-collision gaps \( (t_{\text{min}}) \), and reachability of the CAV. The reachability denotes whether the CAV can reach certain position at certain time considering the maximal/minimal acceleration \( (a_{\text{max}}, a_{\text{min}}) \) and maximal/minimal velocity. Fig. 7 illustrates an example of the feasible lane change zone for a specific scenario, i.e., \( x = [25, 34.5, -100, 40]^T \). The initial position of the CAV is set zero. The lane change boundary is determined by the maximal/minimal velocity and the off-ramp location, denoted as the red dashed line. The feasible lane change zone, i.e., \( \mathcal{F} \), consists of three isolated zones, separated by the trajectories of BVs. The gaps between \( \mathcal{F} \) and the lane change boundary come from the reachability of CAVs. The gaps between \( \mathcal{F} \) and the trajectories of BVs come from the safe time-to-collision gaps.

For simplicity, we assume all task solutions of this case have the same task solution difficulty. Then, the task difficulty can be estimated as

\[
M_f(x) = \sum_{f \in \mathcal{F}} W(f) = -S(\mathcal{F}),
\]

where \( S(\mathcal{F}) \) denotes the area of the feasible lane-changing zone. To make the index comparable with exposure frequency, a normalization factor is applied, denoted as \( U_S \), which can be obtained by the area enclosed by the lane change boundary. Finally, the auxiliary objective function of the highway exit case is designed as

\[
\min_{x} J(x) = \min_{x} \left( S(\mathcal{F})/U_S + w \times d(x, \Omega) \right),
\]

where \( w \) is the weight, and \( d(x, \Omega) \) can be obtained similarly as in the cut-in case (Eq. (1)). The common set \( \Omega \) in this case can be constructed by most frequent scenarios.

2) NDD Analysis: The NDD from the Integrated Vehicle-Based Safety System (IVBSS) project is used to provide exposure frequency information [20][21]. In the IVBSS project, 108 randomly sampled drivers from different ages used sixteen Honda Accords vehicles in an unsupervised manner for over a 40-day period. Query criteria are designed to extract car-following events from the database as: (1) vehicle was traveling on a highway; (2) vehicle was traveling at a speed of at least 20 m/s (≈45 mph); (3) cruise control function was not activated; (3) surface condition is dry; (4) light condition is day. The resulting dataset represents a total of \( 5 \times 10^4 \) car-following events and 1.47 × 10⁶ points of car-following trajectories. The exposure frequency of a scenario can be estimated as

\[
P(x|\theta) = P(p_{0.1}|\theta)P(v_{0.1}, R, v_{0.2}|\theta),
\]

where \( R = p_{0.1} - p_{0.2}, P(p_{0.1}|\theta) \) denotes the initial position probability of the leading vehicle, which can be estimated by uniform distribution, and \( P(v_{0.1}, R, v_{0.2}|\theta) \) is obtained from the distribution of car-following trajectories in the NDD.

3) SM Construction: The MOBIL (‘minimizing overall braking induces by lane changes’) model was proposed to derive human lane-changing rules for discretionary and mandatory lane changes [22]. It provides the utility measurement method for deciding which gap has a desirable lane change position as

\[
U_{LG} = \hat{u} - \hat{u} + p_{LG} (\hat{u}_{new} - u_{new} + \hat{u}_{old} - u_{old}),
\]

where \( \hat{u} \) denotes the new acceleration of the CAV after the lane change, \( p_{LG} \) is the politeness factor, and \( u_{new}, u_{old} \) denote the acceleration of the new follower and old follower respectively. As it is desirable to complete the lane change, the politeness factor is set close to zero, e.g., \( p_{LG} = 0.1 \). To predict the CAV’s trajectories before the lane-change, the Model Predictive Control (MPC) [23] is applied, and the trajectory with higher predictive utility of lane change, i.e., \( U_{LG} \), will be chosen as the solution to the task.

4) Library Generation: Similar to the cut-in case, a hundred points are uniformly sampled as the initial starting points for the optimization problem, and the threshold of critical scenarios is determined as

\[
P(x|S, \theta) > \frac{1}{N(x)} = 6.1 \times 10^{-7},
\]

\begin{table}[h]
\centering
\caption{The Parameter Values of the Cut-In Case.}
\begin{tabular}{|c|c|c|}
\hline
Parameter & Value & Parameter & Value \\
\hline
\( v_{\text{max}} \) & 40 m/s & \( v_{\text{min}} \) & 20 m/s \\
\hline
\( a_{\text{max}} \) & 2 m/s² & \( a_{\text{min}} \) & -4 m/s² \\
\hline
\( L \) & 200 m & \( w \) & 1 \\
\hline
\( p_0 \) & 0 m & \( v_0 \) & 30 m/s \\
\hline
\( t_{\text{min}} \) & 0.5 s & \( \Delta t \) & 0.1 s \\
\hline
\( t \) & [0, 10] & \( p \) & [0, 200] \\
\hline
\( \Delta p \) & 5 m & \( p_{0,i} \) & [-100, 200] \\
\hline
\( v_{0,i} \) & [20, 40] & \( \Delta v \) & 1 m/s \\
\hline
\( d_{t,f} \) & 2 m & \( U_S \) & 500 \\
\hline
\end{tabular}
\end{table}

![Fig. 7. Illustration of the task difficulty evaluation of the highway-exit case.](image-url)
similar to Eq. (16). The size of total scenarios is \( N(X) = n_p^2 \times n_v^2 = 1.64 \times 10^6 \), where \( n_p = 61 \) and \( n_v = 21 \) denote the number of the feasible value of variables \( p_{0,i} \) and \( v_{0,i} \), respectively. After applying the critical scenario searching method, the testing scenario library of the highway exit case is generated. The total number of critical scenarios in the library is 1,895, which is about 0.12% of all scenarios.

C. CAV Evaluation

A typical CAV lane-change model is evaluated in this case study, where the lane-change utility is evaluated by average travel time, average time gap density, and remaining travel time of different lanes (see details in [23]). Similarly, the NDD evaluation method is used as the benchmark. In the proposed method, testing scenarios are sampled from the generated highway exit library, and events of task failures (i.e., cannot exit from the highway) are recorded. Similar to the cut-in case, the \( \epsilon \)-greedy sampling policy is applied with \( \epsilon = 0.10 \). The task failure rate is estimated to measure the functionality performance of the CAV model in the highway exit case. After the estimated task failure rate converges to a certain estimation precision, the estimated task failure rate is obtained, and the evaluation process is completed, as shown in Eq. (5).

Fig. 8 shows the comparison of the two evaluation methods. The legends and axis are the same as the cut-in case. Similar with the previous case study, both methods can obtain unbiased estimation of the failure rate with the relative half-width \( (\beta = 0.2) \). Fig. 8(b) shows that the proposed method achieves this estimation precision after \( 2.6 \times 10^3 \) tests, while the NDD evaluation method takes \( 6.6 \times 10^5 \) tests. The proposed method is about 255 times faster than the NDD evaluation method. As shown in Theorem 2 in Part I of this paper, the efficiency of the proposed method is influenced by the “dissimilarity” between the SM and the specific CAV model. It is the main reason why the efficiency of the proposed method in the highway case is lower than that in the cut-in case.

V. CAR-FOLLOWING CASE STUDY

Car-following is the most common scenario on public roads. The decision variables include the initial condition and acceleration profile of the leading BV, which is high-dimensional. To the best of our knowledge, all existing evaluation methods have the “curse of dimensionality” problem so that they cannot be applied to evaluate car-following case. For instance, the exhaustive search method proposed in the PEGASUS project [3] becomes intractable for high-dimensional scenarios. The accelerated evaluation method [4] suffers from the computation complexity in calibrating importance functions, which grows exponentially with the dimension.

To handle the high-dimensionality problem, the proposed method is enhanced by reinforcement learning (RL) techniques. The definition of scenario criticality is gracefully inherited in the new definition of state-action values, i.e., \( Q \) values. The RL-enhanced method shows powerful ability in generating library for car-following scenarios, which has great potential in other high-dimensional scenarios.

A. Difficulty of High-dimensional Scenarios

Because of the temporal and spatial complexity on public roads, most scenarios are naturally high-dimensional. Take a typical car-following scenario as an example, where a CAV is following a leading BV. The decision variable vector consists of the initial condition and acceleration profile of the BV as

\[
x = \begin{bmatrix} v_0, R_0, \dot{R}_0, u_1, u_2, \ldots, u_m \end{bmatrix}^T,
\]

where \( v_0 \) denotes the initial velocity of the leading BV, \( R_0 \) and \( \dot{R}_0 \) denote the initial range and range rate between the BV and CAV respectively, and \( u_1, u_2, \ldots, u_m \) denote the acceleration sequences of the BV. If the BV is controlled every 0.1s, for a 30s car-following scenario, the dimension of the scenario is 303. Since the computation complexity grows exponentially with the dimension, the problem is called “curse of dimensionality”.

Although the method proposed in Part I cannot be directly applied in high-dimensional scenarios, different from other studies, it can be enhanced easily by RL techniques.
The core concept of the RL-enhanced method is to formulate the TSLG problem as a Markov Decision Process (MDP) problem considering independence properties. The entire scenario space is represented as a series of decision trees (i.e., a forest). Every branch from the root node (i.e., initial state) to the leaf node (i.e., terminal state) specifies a testing scenario. The library generation problem turns into finding the critical branches for CAV evaluation.

The RL techniques are applied to solve the MDP problem. The definition of criticality is gracefully inherited by the new definition of state-action values, i.e., $Q$ values. To guarantee the convergence of $Q$ values, an iteration equation is derived by the temporal-difference (TD) RL theory. After the convergence, the critical tabular $Q$ values (e.g., $Q > 0$) are selected to construct the library. Fig. 9 is an illustration of the generated library. The nodes denote states, and the arrows denote actions, which are defined according to specific cases. The generated library consists of critical testing scenarios, which are denoted by branches of consecutive red nodes and arrows. The accuracy of the RL-enhanced method is proved theoretically.

1) Problem Formulation: After the formulation of MDP, the decision variable vector $x$ can be represented as a sequence of states and actions. The state is defined to represent the snapshot of testing scenarios at current time point. In the following case, the state contains three variables, i.e., speed of the leading BV ($v_{BV}$), range ($R$) between the leading BV and the testing CAV, and the range rate ($\dot{R}$). The leading BV’s acceleration ($u$) is defined as the action.

Let $s = (v_{BV}, R, \dot{R}) \in X$ denote the state, where $X$ is the feasible set of states, and $u$ denote the action. Then a testing scenario $x$ in Eq. (24) can be denoted as a branch of the decision tree (e.g., $s_1 \rightarrow s_2 \rightarrow s_3 \ldots$). It is assumed that the Markovian property holds considering the next action is only dependent on the current state, i.e., the acceleration of the BV is actually only dependent on the current speed of BV for free driving. Moreover, the uncertainty (e.g., observation noise and decision uncertainty) of CAVs is not considered, so the state transition is deterministic.

The exposure probability of a testing scenario can be denoted as

$$P(x|\theta) = P(s_1|\theta)P(u_1|s_1,\theta)\ldots P(u_m|s_m,\theta), \quad (25)$$

where $m$ denotes the total time steps for the case, $\theta$ denote the pre-determined parameters in the ODD. To simplify the notations, $\theta$ will be omitted from now on. Therefore, Eq. (25) is re-written as

$$P(x) = P(s_1)P(u_1|s_1)\ldots P(u_m|s_m). \quad (26)$$

The TSLG problem is essentially to find a new probability distribution of testing scenarios to replace the distribution from NDD.

2) RL-enhanced Library Generation Method: As shown in Eq. (1), the criticality of a scenario is defined as

$$V(x) = P(S|x)P(x) = P(S)P(x|S), \quad (27)$$

where $P(S)$ is a constant, which can be obtained by simulations. By determining $P(x|S)$, a new distribution of testing scenarios can be obtained by the normalized criticality function, i.e., a library is generated. Similar to the problem formulation in Eq. (26), $P(x|S)$ is denoted as

$$P(x|S) = P(s_1|S)P(u_1|s_1, S)\ldots P(u_m|s_m, S), \quad (28)$$

where

$$P(s_1|S) = \frac{P(S|s_1)P(s_1)}{\sum_{s_1 \in \mathcal{X}} P(S|s_1)P(s_1)}, \quad (29)$$

$$P(S|s_1) = \sum_{u_1 \in \mathcal{U}} P(S|u_1, s_1)P(u_1|s_1), \quad (30)$$

$$P(u_k|s_k, S) = \frac{P(S|u_k, s_k)P(u_k|s_k)}{\sum_{u_k \in \mathcal{U}} P(S|u_k, s_k)P(u_k|s_k)}, \quad (31)$$

where $k = 1, \ldots, m$, and $\mathcal{U}$ denotes the feasible acceleration set of the leading BV. Obviously, if $P(S|u_k, s_k)P(u_k|s_k)$ is obtained for all $s_k \in X$ and $u_k \in \mathcal{U}$, we can first obtain $P(u_k|s_k, S)$ and $P(S|s_k)$ by Eq. (30), then calculate $P(s_1|S)$ by Eq. (29), and finally obtain $V(x)$ by Eq. (27)–(28), i.e., a library is generated.

To this end, the value of state-action pairs is defined as

$$Q(s_k, u_k) = P(S|u_k, s_k)P(u_k|s_k). \quad (32)$$

When values of all state-action pairs are obtained, the testing scenario library is generated. The definition of $Q$ is consistent with the proposed definition of criticality. As shown in Eq. (32), the left term, i.e., $P(S|u_k, s_k)$, denotes “the probability of the event $S$ if the scenario is currently at the state $s_k$ and take the action $u_k$”, which measures the maneuver challenge. The right term, i.e., $P(u_k|s_k)$, denotes “the probability of taking action $u_k$ if the scenario is currently at the state $s_k$”, which denotes the exposure frequency.

The remaining problem is how to guarantee that $Q$ values will converge to the values defined in Eq. (32). To this end, the temporal-difference (TD) RL theory is applied. The TDRL method updates the value of a state-action pair based on the estimation of the next state value (i.e., the TD(0) method in [25]). One major idea of TD-RL method is to perform the
iterative update based on a sort of error, which measures the difference between the current estimated value of \( Q(s_k, u_k) \) and the new estimated value from the next state, i.e., the TD error. Let \( \delta_t \) denote the TD error at the time \( t \), then an iteration equation can be obtained as

\[
Q(s_t, u_t) \leftarrow Q(s_t, u_t) + \alpha \delta_t,
\]

where \( \alpha \) is the learning rate, e.g., 0.1.

Now, a theorem is proposed to calculate the TD error.

**Theorem 1.** If the state and action are \( s_t, u_t \) at the time \( t \), and the next state is \( s_{t+1} \), then the TD error is

\[
\delta_t = \left( \sum_{u_{t+1} \in U} Q(s_{t+1}, u_{t+1}) \right) P(u_t|s_t) - Q(s_t, u_t),
\]

where \( Q \) is defined as in Eq. \( 32 \).

**Proof.** The TD error is the difference between the estimated value of \( Q(s_k, u_k) \) and its estimation from the next state. Therefore, if the first term on the right in Eq. \( 34 \) is the estimation of \( Q(s_k, u_k) \) based on the next state, the theorem can be proved.

Plugging the equation

\[
P(S|u_t, s_t) = P(S|s_{t+1}) = \sum_{u_{t+1} \in U} P(S|u_{t+1}, s_{t+1}) P(u_{t+1}|s_{t+1})
= \sum_{u_{t+1} \in U} Q(s_{t+1}, u_{t+1}),
\]

into Eq. \( 32 \), we obtain

\[
Q(s_t, u_t) = \left( \sum_{u_{t+1} \in U} Q(s_{t+1}, u_{t+1}) \right) P(u_t|s_t),
\]

which concludes the theorem.

Finally, the iterative update equation of Eq. \( 33 \) can be expressed as Eq. \( 37 \) (see next page).

### C. Car-following Case Study

In this section, the testing scenario library of the car-following case for safety evaluation is generated based on the RL-enhanced method. To search for critical scenarios, states are classified into three zones, i.e., collision zone, dangerous zone, and safe zone. The critical scenarios are searched in the collision zone and dangerous zone. The same SM is applied as in Eq. \( 13 \). To validate the accuracy and efficiency of the generated library, the CAV car-following model in \( 4 \), which combines the adaptive cruise control and autonomous emergency braking functions, is evaluated.

1) **Library Generation:** The NDD of the highway exit case is used in the car-following case, where both the car-following and free-driving events are extracted. The car-following events are utilized to calculate the exposure frequency of states, while the free-driving events are utilized to estimate the exposure frequency of actions. We discretise the scenarios by \( 1m, 1m/s, 1m/s, \) and \( 0.2m^2/s \) for range \( R \in (0, 115] \), range rate \( R \in [-10, 8] \), velocity \( v \in [20, 40] \), and acceleration \( a \in [-4, 2] \), respectively, and the leading BV is controlled every 1s. For a 30s car-following case, the size of the entire scenario space is \( N(X) = 115 \times 31 = 3575 \) \( \approx 1.2 \times 10^6 \). The size of the entire state space is \( N(X) = 21 \times 115 \times 45 = 10,885 \), and the size of the entire state-action space is \( N(X) N(U) = 45,885 \times 31 \approx 1.4 \times 10^7 \), both of which are much smaller than the entire scenario space \( N(X) \).

To improve the searching efficiency of critical scenarios, the state space is classified into three zones, i.e., collision zone, dangerous zone, and safe zone. The collision zone is defined by the states \( X_c = \{ s \in X | R \leq d_{acci} \} \), where \( d_{acci} \) is a distance threshold for an accident, e.g., 1m. The safe zone, i.e., \( X_s \), is defined by the states which cannot lead to an accident with even the most extreme actions of the leading BV, i.e., BV decelerates with maximal deceleration. The dangerous zone, i.e., \( X_d \), contains the states which have probabilities leading to an accident. Then values of \( P(S|u_k, s_k) \) for states in different zones can be obtained as

\[
P(S|u_k, s_k) = \begin{cases} 0, & s_k \in X_s \\ 1, & s_k \in X_c \end{cases}
\]

As shown in Fig. 10 (a) a non-trivial car-following testing scenario should start form a dangerous state (i.e., root state) and stop at a collision state or a safe state (i.e., terminal state). A critical scenario should belong to the collision zone or start from a dangerous state and stop at a collision state. By simulations of the SM, the dangerous zone is obtained, which consists about 5,000 states (10% of all the states).

What left to be determined are initialization of \( Q \) values, training policy and stop criteria. The initial \( Q \) values are obtained from the NDD. To improve the training efficiency, a uniform distribution is applied as the training policy, which guarantees that all state-action pairs can be visited for unlimited number of times if the training has not stopped \( 25 \). The absolute value of the TD error is defined as the stop criteria as \( |\delta_t| < \delta_0 \), where \( \delta_0 \) is a pre-determined threshold, e.g., \( 10^{-10} \).

The training is conducted with Matlab 2018, in a workstation equipped with Intel i7-7700 CPU and 16G RAM. It takes about 20 minutes to reach convergence. Fig. 11 (a) shows the convergence of the absolute TD error with learning iterations. The values of state-action pairs converge after about \( 3 \times 10^6 \) steps of iterations. Fig. 11 (b) shows an example of the probability distribution of the actions for a dangerous state, i.e., \( s = (38, 6, -2) \). The distribution from NDD is denoted as the blue line (i.e., \( P(u|s) \)), while the generated distribution by the RL-enhanced method is denoted as the red line (i.e., \( P(u|s, S) \)). It shows that the generated distribution behaves more aggressively than NDD with higher probabilities at extreme decelerations. The highest probability lies in \( u = -3.4 m/s^2 \), instead of \( u = -4 m/s^2 \), which is consistent with the proposed definition of criticality combined of both maneuver challenge and exposure frequency.

2) **CAV Evaluation:** After the above steps, the testing scenario library of the car-following case is generated. Testing scenarios can be sampled from the trained table. The initial state is generated by Eq. \( 29 \), and accelerations of the BV are generated by Eq. \( 31 \). Similar to the previous cases,
\[
Q(s_t, u_t) \leftarrow Q(s_t, u_t) + \alpha \left( \sum_{u_{t+1} \in U} Q(s_{t+1}, u_{t+1}) P(u_t | s_t) - Q(s_t, u_t) \right).
\] (37)

Fig. 10. Illustration of the state transitions for car-following scenarios. States are classified into three zones, i.e., collision zone, dangerous zone, and safe zone.

Fig. 11. The training results of the absolute TD error (a) and probability distribution of the actions at the state \(s = (38, 6, -2)\) (b).

Fig. 12. The safety evaluation results of the car-following case: (a) estimation results of the accident rate; (b) relative half-width of the estimation results.

greedy sampling policy is applied in the sampling process with \(\epsilon = 0.1\). As shown by the red line in Fig. 11(b), the probability of acceleration greater than -3 is zero, i.e., out of the library. By adopting the \(\epsilon\)-greedy policy, however, these acceleration values can be sampled with a small probability. Similarly, the initial state has a small probability to be sampled from the safe states as well. The same CAV car-following model used in the cut-in case study [4] is evaluated with the generated library. Safety is selected as the performance metric and accident rate is used to represent safety. The NDD evaluation method is used as the baseline.

Fig. 12 shows comparison of the two evaluation methods. The blue line denotes results of the NDD evaluation method, and the red line denotes results of the proposed method. As shown in Fig. 12 both methods can obtain accurate estimation of accident rate with the same estimation precision (\(\beta = 0.2\)). Fig. 12 (b) shows that the proposed method achieves this
estimation precision after 50 tests, while the NDD evaluation method takes $1.875 \times 10^7$ tests. The proposed method is about $3.75 \times 10^5$ times faster than the NDD evaluation method. As discussed in Theorem 2 of Part I, the results suggest that the modified IDM model in Eq. (13) is a good surrogate of the selected testing CAV model regarding safety evaluation.

VI. CONCLUSIONS AND FUTURE STUDY

In Part I of this paper, the testing scenario library generation (TSLG) problem was proposed and analysed theoretically. Part II of the paper demonstrated the implementation process by three case studies, i.e., cut-in, highway exit, and car-following. The cases were designed to reflect the general framework as well as unique features including auxiliary objective function design for different performance metrics (i.e., safety and functionality), Naturalistic Driving Data (NDD) analysis, and surrogate model (SM) construction. More importantly, the proposed method in Part I was enhanced by a temporal-difference reinforcement learning (TD-RL) method to generate high-dimensional scenarios efficiently. Results show that the proposed method can effectively and efficiently generate the testing scenario library, which can accelerate the evaluation process by $255 \times 3.75 \times 10^5$ times compared with the NDD evaluation method, with the same accuracy.

To the best of our knowledge, this is the first study that identifies the entire TSLG problem and solves it systematically for both low and high dimensional scenarios, different performance metrics, and CAV models. It provides guidelines in generating testing scenario libraries for closed testing facilities to enable accurate and efficient CAV evaluation.

There are many interesting topics that can be further investigated. First, the surrogate models used in this paper are simple, which can be improved to represent the common features of CAVs more accurately, especially on the tail distribution or rare event zone. Second, the library is generated off-line based on the SM. It is inevitable that dissimilarities exist between the SM and testing CAV, which is the major source of testing variance. To further improve the evaluation efficiency, the testing process can be divided into several stages, and the testing results of the previous stage can be used to update the generated library and improve the testing of the next stage adaptively. These topics are left for future studies.

REFERENCES

[1] L. Li, Y.-L. Lin, N.-N. Zheng, F.-Y. Wang, Y. Liu, D. Cao, K. Wang, and W.-L. Huang, “Artificial intelligence test: a case study of intelligent vehicles,” Artificial Intelligence Review, vol. 50, no. 3, pp. 441–465, 2018.

[2] J. Zhou and L. del Re, “Reduced complexity safety testing foradas & adf,” IFAC, vol. 50, no. 1, pp. 5985–5990, 2017.

[3] H. Hunger, “Test specifications for highly automated driving functions: Highway pilot,” Tech. Rep., 2017. [Online]. Available: [https://www.pegasusprojekt.de]

[4] D. Zhao, H. Lam, H. Peng, S. Bao, D. J. LeBlanc, K. Nobukawa, and C. S. Pan, “Accelerated evaluation of automated vehicles safety in lane-change scenarios based on importance sampling techniques.” IEEE Transactions on Intelligent Transportation Systems, vol. 18, no. 3, pp. 595–607, 2017.

[5] D. Zhao, X. Huang, H. Peng, H. Lam, and D. J. LeBlanc, “Accelerated evaluation of automated vehicles in car-following maneuvers.” IEEE Transactions on Intelligent Transportation Systems, vol. 19, no. 3, pp. 733–744, 2018.

[6] D. Jung, D. Jung, C. Jeong, Y. Kou, and H. Peng, “Worst case scenarios generation and its application on driving,” SAE Technical Paper, Tech. Rep., 2007.

[7] S. Feng, Y. Feng, C. C. Yu, Y. Zhang, and X. Liu, “Testing scenario library generation for connected and automated vehicles, part I: Methodology.” IEEE Transactions on Intelligent Transportation Systems, under review.

[8] L. Wasserman, All of statistics: a concise course in statistical inference. Springer Science & Business Media, 2013.

[9] S. M. Ross, Introductory statistics. Academic Press, 2017.

[10] K. Vogel, “A comparison of headway and time to collision as safety indicators,” Accident analysis & prevention, vol. 35, no. 3, pp. 427–433, 2003.

[11] R. Chen, R. Sherony, and H. C. Gabler, “Comparison of time to collision and enhanced time to collision at brake application during normal driving,” SAE Technical Paper, Tech. Rep., 2016.

[12] D. Bezziina and J. Sayer, “Safety pilot model deployment: Test conductor team report,” Report No. DOT HS 812, vol. 812, p. 171, 2014.

[13] X. Gong, Y. Guo, Y. Feng, J. Sun, and D. Zhao, “Evaluation of the energy efficiency in a mixed traffic with automated vehicles and human controlled vehicles,” arXiv preprint arXiv:1806.00377, 2018.

[14] T. A. Ranney, “Models of driving behavior: a review of their evolution,” Accident Analysis & Prevention, vol. 26, no. 6, pp. 733–750, 1994.

[15] M. Bojarski, D. Del Testa, D. Dworakowski, B. Firner, B. Flepp, P. Goyal, L. D. Jackel, M. Monfort, U. Muller, J. Zhang et al., “End to end learning for self-driving cars,” arXiv preprint arXiv:1604.07316, 2016.

[16] J. Zhang and K. Cho, “Query-efficient imitation learning for end-to-end autonomous driving,” arXiv preprint arXiv:1605.06450, 2016.

[17] L. Fraade-Blanar, B. Marjory S., A. James M., and K. Nidhi, “Measuring automated vehicle safety: Forging a framework,” Santa Monica, CA: RAND Corporation, Tech. Rep., 2018.

[18] J. W. Ro, P. S. Roop, A. Malik, and P. Ranjitkar, “A formal approach for modeling and simulation of human car-following behavior,” IEEE Transactions on Intelligent Transportation Systems, vol. 19, no. 2, pp. 639–648, 2018.

[19] S. Hamdar and H. Mahmassani, “From existing accident-free car-following models to colliding vehicles: exploration and assessment,” Transportation Research Record: Journal of the Transportation Research Board, no. 2088, pp. 45–56, 2008.

[20] J. R. Sayer, S. E. Bogard, M. L. Buonarosa, D. J. LeBlanc, D. S. Funkhouser, S. Bao, A. D. Blankspoor, and C. B. Winkler, “Integrated vehicle-based safety systems light-vehicle field operational test key findings report,” 2011.

[21] F. Feng, S. Bao, J. R. Sayer, C. Flannagan, M. Manser, and R. Wunderlich, “Can vehicle longitudinal jerk be used to identify aggressive drivers? an examination using naturalistic driving data,” Accident Analysis & Prevention, vol. 104, pp. 125–136, 2017.

[22] A. Kesting, M. Treiber, and D. Helbing, “General lane-changing model for car-following models,” Transportation Research Record, vol. 1999, no. 1, pp. 86–94, 2007.

[23] J. B. Rawlings and D. Q. Mayne, Model predictive control: Theory and design. Nob Hill Pub. Madison, Wisconsin, 2009.

[24] J. Nilsson, J. Silvmin, M. Brannstrom, E. Coelingh, and J. Fredriksson, “If, when, and how to perform lane change maneuvers on highways,” IEEE Intelligent Transportation Systems Magazine, vol. 8, no. 4, pp. 68–78, 2016.

[25] R. S. Sutton and A. G. Barto, Reinforcement learning: An introduction. MIT press, 2018.
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