Wiener Index of Some Operations on Fuzzy Graphs

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This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

In this paper, the concept of the Wiener index in some operations on fuzzy graphs was introduced and investigated. The bound of $W(G)$ of some operations on fuzzy graphs are obtained like union, join, Cartesian product, composition, complete fuzzy graph, and bipartite complete fuzzy graph.

Keywords: Fuzzy graph; wiener index.

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1 Introduction

Fuzzy graph is one of the application tool in the field of mathematics, which allow the users to describe the relationship between any notions easily, because the nature of fuzziness is favorable for...
the environment [1] - [3]. Harold Wiener suggested the first definition of the Wiener index $W(G)$ as "The sum of the distance between all the pairs vertices of a graph $G$" in (1947) [4, 5]. Fuzzy graphs are beneficial to give a more accurate and flexible system as compared to the classical models. A topological index is a numerical quantity for the structural graph of a molecule. Generally, the topological indices are familiar in chemistry [6, 7]. Still, but a graph structure is of the mathematical background, and Harold, who developed it in his theory and constructed a huge branch of chemical graph theory. Wiener usage indices to find the properties of the type of allanes known as Paraffin. Moreover, it isn’t only an application in chemistry, but it can be applied in all areas, including computer science, networking human traffic, and internet routing [8, 9, 10]. Perhaps the fastest growing area within a graph and fuzzy graph is the study of Wiener index, the reason being its many and varied applications in such fields, which motivated us to find the Wiener index due to the work of operation on fuzzy graph and the possibility of using it. For instance, Yeh and Gutman in [11] computed the Wiener index in the case of graphs obtained using certain binary operations (such as product, join, and composition).

2 Preliminaries

In this section, we review some basic definitions related to fuzzy graphs, Wiener index in fuzzy graphs, and some operations in fuzzy graphs.

A fuzzy graph $G = (\mu, \rho)$, where $\mu$ is a fuzzy subset of $V$ and $\rho$ is a fuzzy relation on $\mu$ such that $\rho(u, v) \leq \mu(u) \land \mu(v); \forall u, v \in V$.

We assume that $V$ is finite and nonempty, $\rho$ is reflexive and symmetric. For all $u, v \in V$ The order $p$ and size $q$ of a fuzzy graph $G$ are defined as $p = \sum_{v \in V(G)} \mu(v)$ and $q = \sum_{u, v \in V(G)} \rho(u, v)$.

A fuzzy graph, $G = (\mu, \rho)$, is said to be a complete fuzzy graph if $\rho(u, v) = \min\{\mu(u), \mu(v)\}$, for all $u, v \in V$ and is denoted by $K_p$. A fuzzy graph $G = (\mu, \rho)$ is said to be bipartite if the vertex set $V(G)$ can be partitioned into two nonempty subsets $V_1$ and $V_2$ such that

(i) $\rho(u, v) = 0$ if and only if $u \in V_1$ or $v \in V_2$;
(ii) and $\rho(u, v) \leq \min\{\mu(u), \mu(v)\}$ if $u \in V_1$ and $v \in V_2$. A bipartite fuzzy graph $G = (\mu, \rho)$ is said to be a complete bipartite fuzzy graph if $\rho(u, v) = \min\{\mu(u), \mu(v)\}$ for all $u \in V_1$ and $v \in V_2$.

A path $p$ of length $n$ is a sequence of distinct vertices $u_0, u_1, ..., u_n$ such that $\mu(u_i, u_{i+1}) > 0$, $i = 1, 2, 3, ..., n$ and the degree of membership of the weakest edge in the Path is defined as its strength. If $u_0 = u_0$ and $\geq 3$, then $P$ is called a cycle and a cycle $P$ is called a fuzzy cycle if it contains more than one weakest edge. The strength of connectedness between two vertices $u$ and $v$ is defined as the maximum of the strengths of all paths between $u$ and $v$ and is denoted by $CONNG(u, v)$.

A strong path $P$ from $u$ to $v$ is a $u - v$ geodesic if there is no shorter strong path from $u$ to $v$, and the length of $u - v$ geodesic is the geodesic distance from $u$ to $v$ denoted by $d_*(u, v)$.

The distance $d_*(u, v)$ between two vertices $u, v \in V(G)$ is the minimum number of edges on $G$'s path between $u$ and $v$.

Let $G$ be a connected fuzzy graph. For any path $P: u_0 - u_1 - u_2 - u_3 - ... - u_n$, the length of $P$ is denoted by $L(P)$, if $n = 0$, define $L(P) = 0$, and for $n \geq 1$, $L(P)$ is defined as the sum of the weights of the edges in $P$,

$L(P) = \sum_{i=1}^{m} \rho(u_i, u_{i+1})$

For any two vertices $u$ and $v$ in $G$, let $P = \{P_i : P_i$ is a $u - v$ path, $i = 1, 2, 3, ..., \}$. 

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The sum distance between $u$ and $v$ is defined as $d_s(u,v) = \min\{L(P_i) : P_i \in P ; i = 1,2,...\}$. Let $G$ be a connected fuzzy graph, the Wiener index $W(G)$ of $G$ is defined by

$$W(G) = \sum_{u,v \in V(G)} d_s(u,v).$$

Let $G = (\mu, \rho)$ be a fuzzy graph on $V$; let $u, v \in V$ we say that $u$ dominates $v$ in $G$ if there exists a strong edge between them.

A set $D$ of vertices of $G$ is strong dominating set of $G$ if every vertices of $V(G) - D$ is a strong neighbor of some vertex in $D$. The weight of a strong dominating set $D$ is defined as $w(G) = \sum_{u \in D} \rho(u,v)$, where $\rho(u,v)$ is the minimum of the membership values (weights) of the strong edges incident on $u$. The strong domination number of a fuzzy graph $G$ defined as the minimum weight of strong dominating sets of $G$ and it is denoted by $\gamma_s(G)$ or simply $\gamma_s$. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two fuzzy graphs, on $V_1, V_2$ respectively with $V_1 \cap V_2 = \phi$. The union of $G_1$ and $G_2$ and denoted by $G_1 \cup G_2$ is the fuzzy graph on $V_1 \cup V_2$, and defined $G = G_1 \cup G_2 = \{x \cup \mu_1, \mu_2, \rho_1, \rho_2\}$. Where $\mu = \{\mu_1, \mu_2\}$ and $\rho = \{\rho_1, \rho_2\}$. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two fuzzy graphs, on $V_1, V_2$ respectively with $V_1 \cap V_2 = \phi$. Then we denoted the join of two fuzzy graphs, $G_1$ and $G_2$, defined by $G_1 + G_2 = (V_1 + V_2, E_1 + E_2 + |V_1||V_2|)$ and defined as follows:

$$\{(x \cup \mu_1, \mu_2, \rho_1, \rho_2) \mid x \in V_1, \text{ or } \mu_2(x) \text{ if } x \in V_2\}.$$

Let $E'$ is the set of all edges joining the vertices of $V_1$ and $V_2$. The Cartesian product of $G_1$ and $G_2$ and denoted $G_1 \times G_2$ is the fuzzy graph on $V_1 \times V_2$ and defined by $G = G_1 \times G_2 = \{(x \times y) \mid (x, y) \in V_1 \times V_2\}$ such that

$$\{(x \times y) \mid (x \times y) = \min\{\mu_1(x), \mu_2(y)\} \forall (x, y) \in V_1 \times V_2\}.$$

$\{(x \times y) \mid (x \times y) = \min\{\rho_1(x, y), \rho_2(y, y)\} \forall (x, y) \in V_2\}.$

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two fuzzy graphs on $V_1, V_2$ respectively with $V_1 \cap V_2 = \phi$. The composition of $G_1$ and $G_2$ and denoted by $G_1 \circ G_2$ is the fuzzy graph on $V_1 \cup V_2$ and defined as $G = G_1 \circ G_2 = (G_1[G_2]) = (V_1 \circ V_2, E_1 \circ E_2)$ such that

$$\{(x \circ y) \mid (x \circ y) = \min\{\mu_1(x), \mu_2(y)\} \forall (x, y) \in V_1 \times V_2\}.$$

$\{(x \circ y) \mid (x \circ y) = \min\{\rho_1(x, y), \rho_2(y, y)\} \forall (x, y) \in V_2\}.$

Let $G_1$ and $G_2$ be two disjoint fuzzy graphs and $W_1, W_2$ be the Wiener index of $G_1$ and $G_2$, respectively. Then,

$$W(G_1) = \sum_{i=1}^{n} \mu(v)\mu(u)d_s(u,v).$$
\[ W(G_2) = \sum_{i=1}^{n} \mu(x)\mu(y)ds(x,y). \]

see [12].

Since \( G_1 \cup G_2 = (V_1 + V_2, E_1 + E_2) = (V_1, E_1) \cup (V_2, E_2) \) and since \( G_1 \cap G_2 = \phi \). Then,

\[ W(G_1 \cup G_2) = W(G_1) + W(G_2). \]

**Example 3.2.** Let \( G_1 = (\mu_1, \rho_1) \) and \( G_2 = (\mu_2, \rho_2) \) be two fuzzy graphs shown in Figures (3.1a) and (3.1b), respectively.

![Fuzzy Graphs](image)

By Theorem (3.2), in [13]. The Wiener index of \( G_1 \) and \( G_2 \) shown in Figures (3.1a) and (3.1b), as following:

\[ W(G_1) \leq 2mq - (m + 1)\gamma_s = 1.4 \]

and

\[ W(G_1) \leq 2mq - (m + 1)\gamma_s = 3.8. \]

The fuzzy graph \( G = G_1 \cup G_2 \) is given in Figure 3.1c, as following:

![Fuzzy Graph](image)

From Figure (3.1c), we have

\[ W(G_1 \cup G_2) = W(G_1) + W(G_2) = 1.4 + 3.8 = 5.2. \]

In the following Theorem, we give the Wiener index of the Join fuzzy graph \( G_1 \) and \( G_2 \), i.e \( W(G_1 + G_2) \).

**Theorem 3.3.** For any two disjoint fuzzy graphs \( G_1 \) and \( G_2 \), \( W_1, W_2 \) are the Wiener index of \( G_1 \) and \( G_2 \) and \( n_1 \geq 3, n_2 \geq 4 \) be the vertex number of a vertex in \( G_1 \) and \( G_2 \) respectively. Then

\[ W(G_1 + G_2) \leq p_2 W_1 + p_1 W_2 + p_1 p_2. \]

Where, \( p_1 \) and \( p_2 \) are the order of \( G_1 \) and \( G_2 \) respectively.

**Proof.** By the definition of Wiener index of a fuzzy graph, we have,

\[ W(G) = \sum_{i=1}^{n} \mu(v)\mu(u)ds(u,v); \ \forall \ u, v \in \mu^+. \]
Then,

\[ W(G_1 + G_2) = \sum_{i=1}^{n} \mu(v) \mu(u) d_s(u, v), \quad \forall \ u, v \in V_1 \cup V_2. \]

Since

\[ d_s(u, v) \leq d_s(u, x) + d_s(x, y) + d_s(y, v), \]

Then,

\[ \sum_{u,v \in V_1 \cup V_2} d_s(u, v) \leq \sum_{u,v \in V_1 \cup V_2} d_s(u, x) + \sum_{x,y \in V_1 \cup V_2} d_s(x, y) + \sum_{y,v \in V_1 \cup V_2} d_s(y, v). \]

Therefore,

\[ \sum_{u,v \in V_1 \cup V_2} \mu(u) \mu(v) d_s(u, v) \leq \sum_{u,v \in V_1 \cup V_2} \mu(u) \mu(v) d_s(u, x) + \sum_{x,y \in V_1 \cup V_2} \mu(u) \mu(v) d_s(x, y) + \sum_{y,v \in V_1 \cup V_2} \mu(u) \mu(v) d_s(y, v). \]

Clearly, \( \sum_{x,y \in V_1 \cup V_2} \mu(u) \mu(v) d_s(x, y) \leq p_2 W_1 + p_1 W_2 + \sum_{x,y \in V_1 \cup V_2} \mu(u) \mu(v) d_s(x, y). \)

Hence,

\[ W(G_1 + G_2) \leq p_2 W_1 + p_1 W_2 + p_1 p_2. \]

We can verify that \( \sum_{x,y \in V_1 \cup V_2} \mu(u) \mu(v) d_s(x, y) \leq p_1 p_2. \)

**Example 3.4.** Let \( G_1 = (\mu_1, p_1) \) and \( G_2 = (\mu_2, p_2) \) be two fuzzy graphs shown in Figures (3.2a) and (3.2b).

![Figure 3.2a](image1.png)

**Fig. 3.2a**

\[ (G_1) : \quad a(0.3), b(0.6), c(0.7) \]

![Figure 3.2b](image2.png)

**Fig. 3.2b**

\[ e(0.4), f(0.5), g(0.8) \]

From the Figures (3.2a) and (3.2b), we have by Theorem (3.2), in [13]: The \( W(G_1) \leq 2mq - (m + 1)\gamma_s(G_1) = 22 \) and since the Figure 3.2b is strong cycle. Then by Theorem (3.4), in [13].

\[ W(G_2) \leq \frac{m(m-1)-1}{2}; \quad m = |E^+| = 1.8 + 2 = 2, \quad p_1 = 1.6, p_2 = 2.3. \]

The join of \( G_1 \) and \( G_2 \) is given in the following Figure (3.2C).
From Figure 3.2c, we have every edge in $G_1$ and $G_2$ are $\delta$-edge in $G_1 + G_2$. Now, we will calculate the distance between vertices in $G_1 + G_2$.

\[
d(s(v_i, v_j))
\]

| $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ |
|-------|-------|-------|-------|-------|-------|-------|
| 0     | 0.8   | 0.7   | 0.4   | 0.6   | 0.5   | 0.6   |
| 0.7   | 0.4   | 0.6   | 0.5   | 0.6   | 2.9   |
| 0.3   | 0.3   | 0.3   | 0.3   | 1.2   |
| 0.6   | 0.6   | 0.6   | 1.8   |
| 0.6   | 0.6   | 1.2   |
| 0.6   | 0.6   | 0.6   |

Then, the Wiener index of $G_1 + G_2$ given as following:

\[
W(G_1 + G_2) = \sum_{i=1}^{n} d_s(u, v) = 3.6 + 2.9 + 1.2 + 1.8 + 1.2 + 0.6 = 11.3.
\]

By using the above Theorem, we have,

\[
W(G_1 + G_2) \leq p_2 W(G_1) + p_1 W(G_2) + p_1 p_2 = 2.3 \times 2.2 + 1.6 \times 2 + 1.6 \times 2.3 = 11.94.
\]

For any Fuzzy graph $G$,

\[
W(G) \leq W(G^*).
\]

In the following Theorem, we give the Wiener index of the product of two fuzzy graphs, $G_1$ and $G_2$. Respectively.

**Theorem 3.5.** Let $G_1$ and $G_2$ be two connected disjoint fuzzy graphs, $W_1, W_2$ be the Wiener index of $G_1$ and $G_2$, and $n_1 \geq 3, n_2 \geq 4$ be the number of vertices $G_1$ and $G_2$ respectively. Then,

\[
W(G_1 \times G_2) \leq n_1^2 W(G_1) + n_2^2 W(G_2).
\]
Proof. Let $G_1$ and $G_2$ be two disjoint fuzzy graphs and $W_1, W_2$ be the Wiener index of $G_1$ and $G_2$, respectively. Then, by Theorem 1, in [11]

$W(G_1 \times G_2) = [G_1^2]^2 W(G_1) + [G_2^2]^2 W(G_2)^2$.

Since $0 < \rho(u,v) < 1$ and by above Remark. Then,

$W(G_1 \times G_2) \leq n_1^2 W(G_1) + n_2^2 W(G_2)$.

Example 3.6. Let $G_1 = (\mu_1, \rho_1)$ and $G_2 = (\mu_2, \rho_2)$ be two fuzzy graphs shown in Figures (3.3a) and (3.3b).

From the Figures (3.3a) and (3.3b), by Theorem (3.2), in [13]. We have

$W(G_1) \leq 1.4$ and $W(G_2) \leq 3.4$.

The following Figure 3.3c, gives $G_1 \times G_2$.

From Figure 3.3c we can calculate The Wiener index in the same way in Example 3.6. Therefore, $W(G_1 \times G_2) = 51.2$.

Now, By Above Theorem, $n_1 = 3, n_2 = 4$. Then,

$W(G_1 \times G_2) \leq (4^2)(1.4) + (3^2)(3.4) = 53$.

The Wiener index $W$ of the composition of two fuzzy graphs, $G_1$ and $G_2$, given in the following result.

Theorem 3.7. Let $G_1$ and $G_2$ be two connected disjoint fuzzy graphs, $W_1, W_2$ be the Wiener index of $G_1$ and $G_2$, and $n_1 \geq 3, n_2 \geq 4$ be the vertices number of $G_1$ and $G_2$ respectively. Then

$W(G_1 \circ G_2) \leq \frac{W(G_1 \times G_2)}{2}$.  

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Proof. Since $G_1 \odot G_2 = (G_1[G_2]) = (V_1 \odot V_2, E_1 \odot E_2)$. Then $V(G_1 \odot G_2) = V(G_1 \times G_2)$, and the $(G_1 \odot G_2)$ have two or more edges of $(G_1 \times G_2)$. Also, for any two vertices $x$ and $y$, in $(G_1 \times G_2)$. And $(G_1 \odot G_2)$. Then,
$$d_s(x,y)_{| G_1 \odot G_2} \leq \frac{d_s(x,y)_{| G_1 \times G_2}}{2},$$
and
$$\mu(x)\mu(y)d_s(x,y)_{| G_1 \odot G_2} \leq \mu(x)\mu(y)\frac{d_s(x,y)_{| G_1 \times G_2}}{2}.$$

Taken sum,
$$\sum_{i=1}^{n} \mu(x)\mu(y)d_s(x,y)_{| G_1 \odot G_2} \leq \sum_{i=1}^{n} \mu(x)\mu(y)\frac{d_s(x,y)_{| G_1 \times G_2}}{2},$$

Hence,
$$W(G_1 \odot G_2) \leq \frac{W(G_1 \times G_2)}{2}.$$  

Example 3.8. Let $G_1$ and $G_2$ be two fuzzy graphs shown in Figures (3.4a) and (3.4b), respectively.

From Figures (3.4a) and (3.4b), and by Theorem (3.2), in [13]. We have $W(G_1) \leq 1.4$ and $W(G_2) \leq 3.4$.

The following Figure (3.4c), gives $G_1 \odot G_2$. 

![Diagram](image)
From Figure 3.4c, we can calculate The Wiener index in the same way in Example 3.6. Therefore, 
\[ W(G_1 \times G_2) = 23.1. \]

Now, By Above Theorem,
\[ W(G_1 \circ G_2) \leq W(G_1 \times G_2) = \frac{53}{2} = 26.5. \]

The Wiener index of the complete fuzzy graph is given in the following result.

**Theorem 3.9.** Let \( G = K_{n} \) be a complete fuzzy graph, with \( n \) vertices. Then

\[ W(K_n) \leq (n - 1)\gamma_s(K_n) = (n - 1)^2 \mu(v). \]

Where \( v \) has the minimum membership value in \( K_n \).

**Proof.** Let \( D \) be a strong dominating set in \( K_n \). Then, \( \gamma_s(K_n) = |D| = \min\{\mu(v); v \in K_n\} \). Since \( \sum \rho(v, v) \). Wher \( \rho(u, v) \) is the strong edge adjacent to \( v \), with minimum membership value, such that \( v \in D \). Then,

\[ \gamma_s(K_n) \leq (n - 1)\mu(v); v \in D. \]

Since \( \gamma_s(K_n) \leq \frac{n}{2} \leq \frac{n}{2} \). Hence,

\[ W(K_n) \leq (n - 1)\gamma_s(K_n) = (n - 1)^2 \mu(v) v \in D. \]

\[ \square \]

**Example 3.10.** Let \( G = (\mu, \rho) = K_{n} \) be a complete fuzzy graph shown in Figure (3.5).

![Graph](image)

**Fig. 3.5**

From Figure (3.5), \( W(K_n) = 0.2 + 0.2 + 0.2 + 0.3 + 0.4 + 0.3 = 1.6. \) Now by above Theorem, we have, \( \gamma_s = 0.6\) and \( n - 1 = 3 \). Then

\[ W(G) \leq (n - 1)\gamma_s = 3 \times 0.6 = 1.8. \]

**Theorem 3.11.** Let \( G = K_{p_1, p_2} \) be a bipartite complete fuzzy graph. With \( n \) vertices. Then

\[ W(K_{p_1, p_2}) \leq \min\{np_1, np_2\}. \]

Where \( p_1 = |V_1| \) and \( p_2 = |V_2| \).
Proof. Let $x, y$ and $z$ be three vertices in $K_{p_1,p_2}$. Then there is a path $P = x, y, z$, and we have two cases.

Case (1): If $x, z \in V_1$ and $y \in V_2$, then the strength of the path gives as following: (i) If $\mu(x) + \mu(z) < \mu(y)$. Then $L(P) = \mu(x) + \mu(z)$.

(ii) If $\mu(x) + \mu(y) < \mu(z)$. Then $L(P) = \mu(x) + \mu(y)$.

(iii) If $\mu(y) < \mu(z) + \mu(x)$. Then $L(P) = \mu(y) + \mu(y) = 2\mu(y)$.

From (i), (ii) and (iii), we obtain

$$L(P) \leq \min\{\mu(x) + \mu(z), 2\mu(y)\}.$$  

Taken sum

$$\sum L(P) \leq \min\{\sum \mu(x) + \mu(z), \sum 2\mu(y)\}.$$  

Hence,

$$W(K_{p_1,p_2}) \leq \min\{np_1, np_2\}.$$  

Case (2): If $x, z \in V_2$ and $y \in V_1$. Similarly case (1). Hence,

$$W(K_{p_1,p_2}) \leq \min\{np_1, np_2\}.$$  

Example 3.12. Let $G = (\mu, \rho) = K_{\mu}$ be a complete fuzzy graph shown in Figure (3.6).

From Figure (3.6), $W(K_{V_1, V_2}) = 0.1 + 0.1 + 0.2 + 0.2 + 0.3 + 0.3 = 1.2$. By the above Theorem, we have, $n = 4$, $V_1 = \{a, b\}$, $V_2 = \{c, d\}$, $p_1 = 0.5$ and $p_2 = 0.6$. Then

$$W(K_{V_1, V_2}) \leq \{np_1, np_2\} = \min\{4 \ast (0.5), 4 \ast (0.6)\} = \min\{2, 2.4\} = 2.$$  

4 Conclusions

In this paper, the Wiener index of some operations on a fuzzy graphs was introduced and investigated, such as union, join, Cartesian product and composition. The Wiener index of the complete fuzzy graph and bipartite complete fuzzy graph was introduced and investigated. With suitable examples.

Competing Interests

Authors have declared that no competing interests exist.
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