Crack detection of beam-type structures following the Bayesian system identification framework

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Abstract. This paper puts forward a method for the detection of crack locations and extents on a structural member utilizing measured dynamic responses following the Bayesian probabilistic framework. In the proposed crack detection method a beam with different number of cracks is modelled using different classes of models. The Bayesian model class selection method is then applied to select the “most plausible” class of models in order to identify the number of cracks on the structural member. The objective of the proposed method is not to pinpoint the crack locations and extents but to calculate the posterior (updated) probability density function (PDF) of crack parameters (i.e., crack locations and extents). The method explicitly handles the uncertainties introduced by measurement noise and modelling error. This paper presents not only the theoretical development of the proposed method but also the numerical and experimental verifications. In the numerical case studies, noisy data generated by a Bernoulli-Euler beam with semi-rigid connections is used to demonstrate the procedures of the proposed method. The method is finally verified by measured dynamic responses of a cantilever beam utilizing laser Doppler vibrometer.

1. Introduction
Changes in dynamic characteristics have frequently been employed as a means of structural damage detection. A comprehensive review of recent developments can be found in reference [14]. Due to advances in sensor technologies, inspection devices such as laser Doppler vibrometers and shearographs have been developed to enable the measurement of accurate dynamic structural responses. Many researchers have studied the use of dynamic measurement in crack detection on structural members. For detecting the existence of cracks and the corresponding locations, a non-model based approach, which relies on the measured responses of the undamaged (healthy) and possibly damaged structural member, is commonly used (e.g. [10]). However, a model-based approach, which involves modelling the structural members, has to be adopted if the crack extent (depth) is to be quantified.

To study the feasibility of the model-based approach, the majority of methods were focusing on single crack situations [11][8]. To extend the approach to a multi-crack situation, Ostachowicz and Krawczuk [12] studied the forward problem of a beam structure with two cracks. They expressed the changes in modal parameters as a function of crack locations and extents. Law and Lu [9] proposed to use measured time-domain responses in detecting multi-cracks on a beam structure through optimization algorithms. All of the abovementioned crack detection methods are only applicable in

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single-crack situations or when the number of cracks is known in advance, which is normally not possible in real situations.

The proposed crack detection methodology addresses this difficulty by dividing the process into two stages. The number of cracks is identified in the first stage, and the uncertainty of crack characteristics are calculated in the second stage.

The organization of this paper is as follows. In Section 2, the proposed methodology is presented together with the related background theories, such as the modelling of the cracked beam and the Bayesian model class section. Sections 3 and 4 report the results of numerical examples and experiment verification. Conclusions are drawn at the end of this paper.

2. Methodology and background theories

2.1. Modelling of beam with multiple cracks

The modelling of a beam with \( N_C \) cracks is briefly reviewed in this section. The beam is divided into \( N_C + 1 \) segments, each with length \( l_i \), for \( i = 1, \ldots, N_C + 1 \), where \( \sum_{i=1}^{N_C+1} l_i = L \). Each segment is modelled as an Euler-Bernoulli beam with the equation of motion for vibration under an arbitrary force \( P(t) \) as:

\[
EI \frac{\partial^4 y(x,t)}{\partial x^4} + m \frac{\partial^2 y(x,t)}{\partial t^2} = 0
\]  

where \( EI \) is the flexural rigidity, \( m \) is the mass per unit length, and \( y \) is the transverse deflection of the beam, which is a function of the position \( x \) along the beam and time \( t \). By using separation of variables \( y(x,t) = \phi(x)z(t) \), the displacement \( y(x,t) \) is described as the product of the modal function \( z(t) \) and the mode shape function \( \phi(x) \). The general solution of the mode shape functions \( \phi_i(x) \), \( i = 1,\ldots,N_C + 1 \), for the \( i \)-th segment can be expressed as:

\[
\phi_i(x) = C_i \sinh(\beta_i x) + D_i \cosh(\beta_i x) + E_i \sin(\beta_i x) + F_i \cos(\beta_i x)
\]

where \( \beta_i^2 = \omega^2 m / EI \); \( \omega \) is the angular natural frequency of the system in radians per second, and \( C_i, D_i, E_i, \) and \( F_i \) are unknown coefficients to be calculated from the boundary and continuity conditions.

The boundary conditions for a beam with semi-rigid connections at both ends are:

\[
\begin{align*}
\phi_i(0) &= \phi_{i+1}(l_{i+1}) = 0 \\
K_L \frac{d\phi_i(0)}{dx} &= EI \frac{d^2\phi_i(0)}{dx^2} & \text{for } i = 1,\ldots,N_C \\
K_R \frac{d\phi_{i+1}(l_{i+1})}{dx} &= EI \frac{d^2\phi_{i+1}(l_{i+1})}{dx^2}
\end{align*}
\]  

For the cantilever beam, the boundary conditions at the fixed and free ends, respectively, are:
where \( K_L \) and \( K_R \) is the stiffness coefficient of the rotational spring at the left and right end of the semi-rigid connected beam. For the cantilever beam, it is assumed that there is no rotational spring at the right end.

At the general \( i \)-th crack of the beam, the following four continuity conditions must be satisfied:

\[
\phi_i(l) = \phi_{i+1}(0) \\
\frac{d\phi_i(l)}{dx} - \frac{d\phi_{i+1}(0)}{dx} = \Delta_i \frac{d^2\phi_{i+1}(0)}{dx^2} \\
\frac{d^2\phi_i(l)}{dx^2} = \frac{d^2\phi_{i+1}(0)}{dx^2} \\
\frac{d^3\phi_i(l)}{dx^3} = \frac{d^3\phi_{i+1}(0)}{dx^3}
\]

for \( i = 1, \ldots, N_C \) (5)

where \( \Delta_i \) is the non-dimensional flexibility parameter to characterize the extent of the \( i \)-th crack. The relationship between the crack extent \( \Delta_i \) and the crack depth ratio \( \delta_i = \gamma_i / h \) can be found in Ostachowicz and Krawczuk [12] as:

\[
\Delta_i = 6\pi \delta_i^2 \left( \frac{h}{L} \right) f(\delta_i)
\]

(6)

where \( h \) is the beam depth, \( \gamma_i \) is the depth of the \( i \)-th crack, and the function \( f(\delta_i) \) is given by:

\[
f(\delta_i) = 0.6384 - 1.035\delta_i + 3.7201\delta_i^2 - 5.1773\delta_i^3 + 7.553\delta_i^4 - 7.332\delta_i^5 + 2.4909\delta_i^6
\]

(7)

A characteristic equation of the semi-connected at both ends and cantilever cracked beam can be obtained by substituting the conditions in (4) and (5) into (2), respectively. An infinite number of natural frequencies \( \omega_k \) and mode shapes \( \phi_k(x) \) for \( k = 1, \ldots, \infty \) of the system can then be calculated.

In the under-damped vibration case, the modal function of \( k \)-th mode \( z_k(t) \) is in the following form:

\[
z_k(t) = e^{-\zeta_k\omega_k t} \left( A_k \sin \omega_{D,k} t + B_k \cos \omega_{D,k} t \right)
\]

(8)

where \( A_k \) and \( B_k \) depend on the initial conditions of the \( k \)-th system and \( \omega_{D,k} = \omega_k \sqrt{1-\zeta_k^2} \) and \( \zeta_k \) are the damped frequency and the critical damping ratio of the \( k \)-th mode. The overall response of the beam can be calculated by the method of modal superposition. In general, only a small number of lower modes contribute to the dynamic response of the system, and this number depends on many factors, such as the support conditions and the types of excitations.

In the numerical examples, the uncertain parameter vector for a beam with \( j \) cracks is:
The reference system (healthy status) is represented by the model class $M_0$ ($j = 0$), in which the vector of uncertain model parameters is $\mathbf{a}_0 = \{\tilde{K}_L, \tilde{K}_R, \zeta\}^T$, where the subscript of $\mathbf{a}$ represents the number of cracks. In experiment verification, it is:

$$\mathbf{a}_j = \left\{\tilde{K}_L, \zeta_1, \zeta_2, \zeta_3, \zeta_4, I_1, I_2, \ldots, I_j, \Delta_1, \Delta_2, \ldots, \Delta_j\right\}^T$$  \hspace{1cm} (9)

2.2. Identification of the number of crack, corresponding locations and extents

If the model-based approach is followed for crack detection and the number of cracks is not known, beams with different numbers of cracks have to be modelled by different classes of models, as shown in Figure 1. The model class $M_j$ is employed in modelling a beam with $j$ cracks, and the parameters $I_j$ and $\Delta_j$ are used to describe the location and extent of the $j$-th crack.

The problem is how to identify the “optimal” model class using a set of measurements $D$. By following the concept of model updating, one may consider carrying out a minimization for each model class to minimize the discrepancy between the measured and modelled responses, and “pick up” the “optimal” model class as that which can give the best fit to the measurement. It must be pointed out that the selection of the “optimal” model class based on a given set of data is not trivial. It is clear that the model class of a beam with more cracks consists of more model parameters (see Figure 1), which will always provide a better fit to the measurement when compared to a model class with fewer parameters. Hence, the selection of model class based solely on the fitting between the modelled and the measured dynamic responses can be very misleading, as the most complex model class will always be selected.

In addressing this problem, the first stage of the proposed methodology relies on the Bayesian model class selection method [2] in selecting the “optimal” model class to identify the number of cracks on.
the beam. Due to space limitations here, the Bayesian model class selection method will only be briefly reviewed. Interested readers should consult reference [2]. The original goal of the method is to select the “optimal” class of models from a given list of $N_M$ model classes. The selection is based on the probability of the model class conditional on the set of measurements $D$ [2]:

$$P(M_j|D, U) = \frac{p(D|M_j, U)P(M_j|U)}{p(D|U)} \quad (11)$$

where $U$ expresses the user’s judgment about the initial plausibility of the classes of models, expressed as a prior probability $p(M_j|U)$ on the model class $M_j$, such that $\sum_{j=1}^{N_M} P(M_j|U) = 1$. Unless there is prior information about the number of cracks on the beam, the prior probability $P(M_j|U)$ is taken as $1/N_M$; $1/p(D|U)$ is treated as a normalizing constant. The most important term in (11) is $p(D|M_j, U)$, which is known as the “evidence” for the model class $M_j$ provided by the data $D$. The class of models to be used is obviously the one that maximizes the probability $P(M_j|D, U)$ and this is generally equivalent to the one that maximizes the evidence $p(D|M_j, U)$ with respect to $M_j$. In the application of the Bayesian model class selection method in the detection of the number of cracks, subjective judgment from engineers is not preferred. As a result, $U$ is dropped in $p(D|M_j, U)$ because it is assumed that $M_j$ alone specifies the PDF for the data. Hence, the evidence $p(D|M_j, U) = p(D|M_j)$ hereafter.

For a globally identifiable case [1][4], the evidence can be calculated based on an asymptotic approximation [13]:

$$p(D|M_j) \approx p(D|\hat{a}_j, M_j)(2\pi)^{N_j/2} p(\hat{a}_j | M_j) |H_j(\hat{a}_j)|^{-1/2} \quad (12)$$

where $\hat{a}_j$ denotes the optimal model in the model class $M_j$ (the set of optimal model parameters of $a_j$). $N_j$ is the number of uncertain model parameters in $\hat{a}_j$, and $H_j(\hat{a}_j)$ is the Hessian of the function $g(a_j) = -\ln [p(a_j|M_j)p(D|a_j, M_j)]$ evaluated at the optimal model $\hat{a}_j$.

The evidence $p(D|M_j)$, $j = 1, \ldots, N_M$, in (12) consists of two factors. The first factor $p(D|\hat{a}_j, M_j)$ is the likelihood factor. This will be larger for those model classes that give a better “fit” to the data $D$. This favours model classes with more parameters (model classes with higher complexity). The second factor $(2\pi)^{N_j/2} p(\hat{a}_j | M_j) |H_j(\hat{a}_j)|^{-1/2}$ is called the Ockham factor [3]. Beck and Yuen [2] showed that the value of the Ockham factor decreases as the number of uncertain parameters in the model class increases and therefore provides a mathematically rigorous and robust penalty against parameterization. The combination of these two factors enables the selection of the “simplest” model class that can provide a “good fit” to the measurement.

The proposed algorithm for identifying the number of cracks on the beam is summarized as follows:

1. Initialize the index $j = 0$, and calculate the evidence $p(D|M_j)$ for the beam without crack by (12).
2. Increase the index $j$ by 1 ($j = j + 1$), and calculate the evidence $p(D|M_j)$ for the beam with single crack.
3 Compare the evidence of \( p(D | M_{j-1}) \) with that of \( p(D | M_j) \). If \( p(D | M_{j-1}) > p(D | M_j) \), then \( M_{j-1} \) is the “best” class of models. Otherwise, increase the index \( j \) by 1 (\( j = j + 1 \)) and repeat this step.

By following this simple algorithm, the proposed methodology can identify the number of cracks, say \( N_C \), by calculating the evidence of the model classes \( M_0, M_1, ..., M_{N_C+1} \). The maximum number of cracks to be considered \( N_M \) is equal to \( N_C + 1 \).

It should be noted that once number of cracks is identified, the corresponding crack locations and extents are obtained at the same time. The Bayesian identification framework [1] is then employed to calculate the PDF of the identified crack locations and extents.

3. Numerical case study
A Bernoulli-Euler beam with length 0.4 m is employed and semi-rigid connections in the numerical case study presented here to verify the proposed crack detection methodology. Both depth \( h \) and width \( b \) are 0.01m. The Young’s modulus \( E \) and mass per unit length \( m \) are 200GPa and 0.79 kg/m, respectively. The beam is assumed to be classically damped with a critical damping ratio of 1% for all modes (\( \zeta = 0.01 \)). Measurement noise is considered by adding a 5% white noise to the calculated dynamic responses in all cases. Only 0.4 sec of data with 1000 Hz sampling frequency is employed in the crack detection process and, therefore, the number of measured data points is 401. Four sensors are evenly installed on the beam for measuring the vertical vibration at 0.08 m, 0.16 m, 0.24 m, and 0.32 m from the left end of the beam.

The results of two cases (Cases A and B) are presented in this paper. The true values of crack number, rotational stiffness and crack parameters of them are summarized in Table 1.

3.1. Cases A and B
This case considers two cracks existed in the beam. The proposed crack detection methodology starts by calculating the logarithm of the evidence in (12) for \( M_0, M_1, \) and \( M_2 \). Comparing the relative evidence of the classes of models as shown in Table 2, it is obvious that the class of model \( M_2 \) has the highest relative evidence. The algorithm continues to calculate the relative evidence of \( M_3 \), which is equal to \( 2.5 \times 10^{-3} \) and is smaller than that of \( M_2 \). Therefore, it can be concluded that there are only two cracks on the beam. The proposed methodology successfully identifies the true number of cracks \( N_C = 2 \) in this case.

The identified locations of the first and second cracks are 0.1412 m and 0.0349 m, respectively. These are close to the true values (0.14 m and 0.04 m). Their corresponding coefficients of variation (COV) are 2.87% and 7.22%. The identified extents of the first and second cracks are 0.0490 and 0.0283, respectively, which are again very close to the true values (0.05 and 0.03). The COV are 35.06% and 37.64%, respectively. The proposed methodology successfully identifies the crack parameters in this case. The identified normalized spring stiffness \( K_L \) and \( K_R \) are 0.2060 and 0.4269 with 55.47% and 14.96% COV, respectively. The identified damping ratio (\( \zeta \)) is 0.01 with 0.18% COV.

Case B presents a situation in which there are three cracks with different crack depths. The relative evidence of the classes of models with zero to four cracks are calculated and summarized in Table 2. The relative evidence increases from \( M_0 \) to \( M_3 \) (from \( 1.7 \times 10^{-2021} \) to 1) and decreases from \( M_3 \) to \( M_4 \) (from 1 to \( 3.4 \times 10^{-4} \)), demonstrating that the correct number of cracks is three. The identified crack locations of first, second and third crack are 0.1406, 0.0393 and 0.0803, respectively. Their corresponding COV are 5.78%, 1.33% and 1.66%. The identified crack extents of first, second and third crack are 0.0494, 0.0248 and 0.0401 with COV 17.95%, 89.22% and 13.72%, respectively. All of the identified crack locations and extents are close to the true value. The identified normalized
spring stiffness $\tilde{K}_L$ and $\tilde{K}_R$ are 0.2145 and 0.2815 with 97.11% and 79.77% COV, respectively. The identified damping ratio ($\zeta$) is 0.01 with 0.25% COV.

### Table 1. Summary of Cases A and B

| Case | $N_C$ | ($\tilde{K}_L$, $\tilde{K}_R$) | Crack Information |
|------|-------|-------------------------------|-------------------|
| A    | 2     | (0.2, 0.4)                    | $l_1 = 0.14$, $\Delta_1 = 0.05$  
$\quad$                         | $l_2 = 0.04$, $\Delta_2 = 0.03$ |
| B    | 3     | (0.2, 0.4)                    | $l_1 = 0.14$, $\Delta_1 = 0.05$  
$\quad$                         | $l_2 = 0.04$, $\Delta_2 = 0.03$  
$\quad$                         | $l_3 = 0.08$, $\Delta_3 = 0.04$ |

### Table 2. Evidence of different classes of models in Cases A and B

| Case | Class of models | Relative evidence | Logarithm of the Evidence | Likelihood factor | Ockham factor |
|------|-----------------|-------------------|---------------------------|-------------------|---------------|
| A    | $M_0$           | 1.1×10^{-2003}    | 8389                      | 8409              | -20           |
|      | $M_1$           | 6.7×10^{-3}       | 12996                     | 13042             | -46           |
|      | $M_2$           | $I$               | 13001                     | 13057             | -56           |
|      | $M_3$           | 2.5×10^{-3}       | 12995                     | 13059             | -64           |
| B    | $M_0$           | 1.7×10^{-2021}    | 12366                     | 12409             | -43           |
|      | $M_1$           | 5.8×10^{-257}     | 12864                     | 12923             | -59           |
|      | $M_2$           | $I$               | 12956                     | 13020             | -64           |
|      | $M_3$           | 3.4×10^{-4}       | 12948                     | 13023             | -75           |

### 4. Experimental verification

The proposed crack detection methodology was also demonstrated and verified using the cantilever beam. The test sample is an aluminium bar with Young’s modulus $E = 69$ GPa, density $\rho = 2960$ kg/m$^3$, width $b = 12$ mm, height $h = 6$ mm and the length of the aluminium bar is 600 mm. The first 200 mm of the beam is fixed in a rigid clamping system, and the length of the cantilever beam is therefore 400 mm. The cantilever beam is excited at three points 50 ± 1 mm, 200 ± 1 mm and 300 ± 1 mm from the fixed end using a 086D80 PCB Piezotronics impact hammer with a 5 mm thick steel backing mass and a nylon tip together with a 480C02 ICP sensor signal conditioner. The transient transverse vibration response is measured at 220 ± 1 mm from the fixed end using a Polytec laser vibrometer system with an OFV-502 fibre-optic laser head and an OVD-02 velocity demodulator set at 125 mm/s/V measurement resolution. The response signal is collected for 500 ms with an approximately 50 ms pre-trigger with a temporal resolution of 0.2 ms.

The number of cracks, locations, extents and corresponding depth of cracks are summary in Table 3. In the identification process, only the first four modes are considered in the dynamic analysis. Furthermore, the system is assumed to be classically damped with different critical damping ratios for different modes. The uncertain parameter vector is $\mathbf{a} = \{\tilde{K}_L, \zeta_1, \zeta_2, \zeta_3, \zeta_4, l_1, \Delta_1, l_2, \Delta_2\}^T$. 

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Table 3. Summary of all cases in the experimental study.

| Case | $N_c$ | Crack location (mm) | Crack extent | Crack depth (mm) |
|------|------|---------------------|--------------|------------------|
| C    | 1    | $l_1 = 80 \pm 1$   | $\Delta_1 = 0.0407$ | $2.8 \pm 0.15$ |
| D    | 2    | $l_1 = 80 \pm 1$,  | $\Delta_1 = 0.0407$, |
|      |      | $l_2 = 100 \pm 1$  | $\Delta_2 = 0.0028$ | $0.8 \pm 0.15$ |

4.1. Cases C and D

Table 4 shows the results of the first stage of the proposed crack detection methodology for all five cases. It is clear that $M_1$ (one crack) and $M_2$ (two cracks) are selected for Cases C and D, respectively. The proposed methodology successfully identifies the true number of cracks in all cases.

Table 5 summarizes the identified “optimal” crack parameters, and the normalized marginal PDF of the crack parameters in Case C is shown in Figure 2. As there is only one optimal model within the domain of interest, there is only one peak in Figure 2. The figure also shows that the PDF value drops significantly when one moves away from the optimal model $\hat{a}_1$. This is the typical characteristic of an identifiable case [5][7][6]. The uncertainties that are associated with the identified crack detection results are very low. In other words, the identified results are of high degree of confidence. To make the discussion on the uncertainty of the identification results more convenient, the coefficients of variation (COVs) of all uncertain parameters are calculated and summarized in Table 5.

Table 4. The results of Bayesian model class selection in all cases.

| Case | Class of models | Relative evidence | Logarithm of the Evidence | Likelihood factor | Ockham factor |
|------|-----------------|-------------------|---------------------------|------------------|---------------|
| C    | $M_0$           | $3.1 \times 10^{-4662}$ | $21525$ | $21532$ | -7 |
|      | $M_1$           | $1$               | $31798$ | $31828$ | -39 |
|      | $M_2$           | $2.1 \times 10^{-9}$ | $31778$ | $31830$ | -52 |
| D    | $M_0$           | $1.8 \times 10^{-4714}$ | $16706$ | $16710$ | -4 |
|      | $M_1$           | $2.0 \times 10^{-1159}$ | $25000$ | $25040$ | -40 |
|      | $M_2$           | $1$               | $27668$ | $27728$ | -60 |
|      | $M_3$           | $4.5 \times 10^{-5}$ | $27658$ | $27729$ | -71 |
The identified crack location and crack depth (79.6 mm and 2.92 mm) in the single crack case (Case C) in Table 5 are perfectly matched with the true values (80 ± 1 mm and 2.8 ± 0.15 mm). Furthermore, the COVs of the identified crack location and depth are very small, and the confidence level of the result of crack detection is therefore high. It can be concluded that the proposed crack detection methodology successfully identifies the crack location and depth in Case C.

There are two cracks in Case D, the first crack $\gamma_1$ at 80 ± 1 mm is the same as that in Case C, and the second crack $\gamma_2$ at 100 ± 1 mm is very small (0.8 ± 0.15 mm) at only about 13% of the overall depth of the beam. Hence, this case can be used to test the performance of the proposed methodology in detecting small cracks. The identified crack locations for both $\gamma_1$ and $\gamma_2$ in Case D in Table 5 are very close to the true values. As the COV value of the first crack location is smaller than that of the second crack location, the result shows that the second crack location is relatively more uncertain when compared to the first. This can be explained by the fact that the crack depth of the second crack is much smaller than that of the first crack (i.e., the first crack is more outstanding than the second). When the identified crack depths are considered, the identified crack depth of the first crack $\gamma_1$ (3.02 mm) is closer to the true value (2.8 ± 0.15 mm) when compared to that of the second crack $\gamma_2$. The relatively poor result for the second crack can be explained by the high uncertainty, which is clearly shown by the relatively large COV value of the identified crack depth of $\gamma_2$. This case shows the importance of estimating the uncertainties associated with the identification results.

Table 6 shows the calculated beam properties for the five cases. The normalized rotational spring stiffness of Cases C and D are 378 and 740, respectively. This confirms the well known fact that it is
extremely difficult to experimentally realize a fixed-end condition and to use a semi-rigid end condition in the analytical model is absolutely essential in the case of a “fixed” end. Table 6 also shows a small damping value ($\zeta_1 = 0.0383\%$) with large uncertainty (COV = 16.84%). Mode 2 shows also very little damping and the uncertainties in the calculated values are relatively large. Modes 3 and 4 show higher damping values.

Table 5. The results of crack evaluation in Cases B and C

| Case | Location(s) (mm) | Extent(s)  |
|------|-----------------|------------|
|      | $l_1$ (COV %)  | True location | $\Delta_i$ (COV %) | Crack depth (mm) | True crack depth (mm) |
| C    | 79.6 (0.03)     | 80 ± 1     | 0.0451 (0.12) | 2.92       | 2.8 ± 0.15       |
| D    | 82.2 (0.21)     | 80 ± 1     | 0.0492 (0.34) | 3.02       | 2.8 ± 0.15       |
|      | 102.6 (1.15)    | 100 ± 1    | 0.0091 (5.48) | 1.44       | 0.8 ± 0.15       |

Table 6. The results of beam property identification in all cases.

| Case | $\bar{K}$ (COV %) | Damping Ratio (%) of each mode (C.O.V %) |
|------|------------------|------------------------------------------|
|      |                  | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $\zeta_4$ |
| C    | 428.64 (0.05)    | 0.0383    | 0.0123    | 0.0632    | 0.5367    |
|      |                  | (16.84)   | (4.18)    | (1.72)    | (1.51)    |
| D    | 740.48 (4.90)    | 0         | 0.0076    | 0.0876    | 0.0795    |
|      |                  | (-)       | (11.08)   | (2.52)    | (1.25)    |

5. Conclusions

This paper presented a practical crack detection methodology and its verification through both numerical and experimental case studies. The proposed crack detection methodology consists of two stages. The number of cracks is identified utilizing the Bayesian model class selection method in the first stage. The updated PDF of the crack and model parameters are identified by following the Bayesian statistical framework in the second stage. Very encouraging results were obtained from both the numerical and experimental case studies. The proposed methodology successfully identifies the number of cracks, and the identified crack locations and depths are very close to the true values in all cases. One of the outstanding advantages of the proposed methodology is that the uncertainties associated with the identified results can be quantified. As a result, engineers know the confidence level of the crack detection results.

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