Quasi-degenerate neutrinos and leptogenesis from $L_\mu - L_\tau$

E. J. Chun  
Michigan Center for Theoretical Physics, Department of Physics, University of Michigan, Ann Arbor MI 48109, USA and Korea Institute for Advanced Study, 207-43 Cheongryangri-dong, Dongdaemun-gu, Seoul 130-012, Korea  

K. Turzyński  
Department of Physics, University of Michigan, Ann Arbor MI 48109, USA  
(Dated: March 26, 2022)

We provide a framework for quasi-degenerate neutrinos consistent with a successful leptogenesis, based on the $L_\mu - L_\tau$ flavor symmetry and its breaking pattern. In this scheme, a fine-tuning is needed to arrange the small solar neutrino mass splitting. Once it is ensured, the atmospheric neutrino mass splitting and the deviation from the maximal atmospheric mixing angle $\Delta \theta_{23}$ are driven by the same symmetry breaking parameter $\lambda \sim 0.1$, and the reactor angle $\theta_{13}$ is predicted to be slightly smaller than $\lambda$ while the Dirac CP phase is generically of order one. Given that the pseudo-Dirac nature of right-handed neutrinos is protected from the flavor symmetry breaking, a small mass splitting can be generated radiatively. For moderate values of $\tan \beta \sim 10$, this allows for low-scale supersymmetric leptogenesis, overcoming a strong wash-out effect of the quasi-degenerate light neutrinos and evading the gravitino overproduction.

I. INTRODUCTION

Thanks to an impressive progress made in neutrino oscillation experiments, we have fairly good information of the low-energy observables like neutrino mass differences and mixing angles [1]. The least known parameter is the so-called reactor angle $\theta_{13}$. A measurement of this angle and a study of CP violation in neutrino oscillation is one of major tasks in the next neutrino oscillation experiments. These endeavors cannot, however, reveal what the absolute neutrino mass scale is. A future determination of this feature would be a key element in exploring the origin of neutrino mass, which clearly lies beyond the Standard Model (SM). Among all still allowed possibilities, the scenario of quasi-degenerate neutrinos is interesting, as it can be confirmed or disproved in the future neutrino-less double beta decay experiments or in the cosmological observations of the cosmic microwave radiation and the large scale structure of the universe [1].

One of the most fascinating connections between neutrino physics and cosmology would be a possible explanation of the baryon asymmetry of the Universe, $\eta_B \equiv (n_B - n_\bar{B})/n_\gamma = 6.15(25) \times 10^{-10}$ through leptogenesis [2] (see also [3] for a review of subsequent developments), which is linked with the neutrino masses and mixing originating from the seesaw mechanism [4]. Recent studies of leptogenesis revealed a meaningful constraint on the scale of the heavy right-handed neutrino mass $M$ at which the baryon asymmetry is generated. Under the assumptions of a hierarchy in the masses of the heavy right-handed neutrinos and a CP phase of order one in their decay, $M \gtrsim 10^8 - 10^9$ GeV is required to account for the baryon asymmetry of the Universe, if the inverse-decay of this right-handed neutrino is negligible [5]. In case of the quasi-degeneracy for low-energy neutrinos, the resulting leptonic CP asymmetry is suppressed by a strong inverse-decay effect coming from larger neutrino Yukawa couplings, and as a result, one needs to increase the scale $M$ by a factor of $\sim 10^{2-3}$ compared to the above value. Such a high leptogenesis scale $M$ sets a lower bound on the reheating temperature after inflation, which may endanger the successful prediction of the primordial nucleosynthesis due to gravitino overproduction [7].

Of course, the above-mentioned constraint on $M$ is model-dependent. For instance, nearly mass-degenerate right-handed neutrinos can lead to an increase in the asymmetry [8]. In fact, the quasi-degeneracy of the low-energy neutrinos could be a consequence of that of the high-energy right-handed neutrinos. An extreme possibility along this line is to have the right-handed neutrino mass difference comparable to their decay rate $\Delta M \sim \Gamma$, which leads to the leptonic CP asymmetry resonantly enhanced to its near maximum value and thus the mass scale $M$ can go down to the TeV scale [9]. An interesting way of realizing such a resonant enhancement is to invoke a radiatively induced mass splitting through the renormalization group running from the flavor scale to the mass scale $M$ [10, 11]. Such radiative resonant leptogenesis has also been studied in the context of minimal flavor violation [12] and $\mu$-$\tau$ symmetry [13]. An almost exact degeneracy requires a theoretical justification; in a flavor model of neutrino masses and mixing, an (nearly) exact degeneracy of the singlet right-handed neutrino sector can be a consequence of the flavor symmetry and should also be protected from its breaking effect [11].

In this work, we have taken the viewpoint that, since baryogenesis via leptogenesis is a theoretically elegant explanation of the baryon asymmetry of the Universe, a requirement of successful leptogenesis (with a low reheating temperature to avoid the gravitino problem) can be added to the list of phenomenological constraints that
a neutrino seesaw mass model should observe. We illustrate this point by investigating the properties of a quasi-degenerate neutrino mass model based on the $L_\mu - L_\tau$ flavor symmetry, which also provides a successful realization of the radiative resonant leptogenesis. The $L_\mu - L_\tau$ flavor symmetry is motivated by the fact that the symmetry-preserving right-handed neutrino mass term, $MN_\mu N_\tau$, naturally leads to a maximal mixing required for the atmospheric neutrino oscillation, $\theta_{23} = \pi/4$ [14, 15, 16, 17]. Note also that such a pseudo-Dirac structure of the $\mu - \tau$ sector implies an exact degeneracy for two right-handed neutrinos $M_2 = M_3 = M$. As a consequence of it, the resulting low-energy neutrino mass pattern is required to be quasi-degenerate, and a fine-tuning has to be introduced to arrange a small mass splitting for the solar neutrino oscillation as we will discuss in detail. We will analyze how the atmospheric and solar mass splitting can arise in connection with a small reactor angle and a large solar angle from the flavor symmetry breaking which introduces small complex order parameters $\lambda_i$, suppressed by a factor $\lambda = O(0.1)$ with respect to the symmetry preserving ones. Such a flavor symmetry breaking effect can be exempt in the right-handed neutrino mass matrix by assigning an additional discrete symmetry, as a result of which the resonant leptogenesis can naturally explain the observed baryon asymmetry of the universe for $\tan \beta \sim 10$ (thereby partially compensating the aforementioned fine-tuning). Our scheme predicts generically order-one CP phases for the neutrino oscillation and leptogenesis which are unrelated to each other.

**II. QUASI-DEGENERATE NEUTRINO MASS MODEL**

**A. General remarks**

Let us write down the Lagrangian with three right-handed neutrinos $N$ as

$$\mathcal{L} = N Y_\nu L H_2 + \frac{1}{2} N M N + h.c.$$  \hspace{1cm} (1)

which leads to the seesaw mechanism explaining the smallness of the neutrino masses:

$$m_\nu = -(H_2)^2 Y_\nu^T M^{-1} Y_\nu$$ \hspace{1cm} (2)

where $m_\nu$ is the mass matrix of the light neutrinos, $M$ is the Majorana mass matrix of the heavy right-handed neutrinos and $Y_\nu$ is the matrix of the neutrino Yukawa coupling. The matrix $m_\nu$ can be diagonalized by a unitary flavor transformation:

$$V_\nu^T m_\nu V_\nu = \text{diag}(m_1, m_2, m_3)$$ \hspace{1cm} (3)

where we take $m_1, m_2, m_3$ to be a priori complex and the neutrino mixing matrix has a CKM-like form:

generated through leptogenesis, and (iii) that the mechanism of supersymmetry breaking predicts the gravitino mass in the range potentially dangerous for primordial nucleosynthesis. There exist models abandoning some of these assumptions.

$$V_\nu = \begin{pmatrix}
c_{12} c_{13} & s_{12} c_{13} & \tilde{s}_{13}^\ast \\
-s_{23} s_{12} - c_{23} c_{12} \tilde{s}_{13} & c_{23} c_{12} - s_{23} s_{12} \tilde{s}_{13} & c_{13} \tilde{s}_{23}
\end{pmatrix}.$$ \hspace{1cm} (4)

Here $s_{13} = s_{13} e^{i\delta}$, and $s_{ij}$, $c_{ij}$ stand for $\sin \theta_{ij}$, $\cos \theta_{ij}$, respectively. The experimental constraints on the dimensionless neutrino observables can be summarized as (see, e.g., [11]):

$$\frac{|m_{23}|^2 - |m_{13}|^2}{|m_{23}|^2} = 0.032(3),$$

$$\sin \theta_{13} = 0.00(5),$$

$$\tan^2 \theta_{12} = 0.45(5),$$

$$\tan^2 \theta_{23} = 1.0(2).$$ \hspace{1cm} (5)

The challenge of building a neutrino flavor model consists in reproducing this peculiar observed pattern of the neutrino mass squared differences and two large and one small mixing angles.

Writing Eqs. (2)-(4), we tacitly assumed that they are valid at the low energy scales at which the neutrino experiments are performed. In order to match these expressions with the neutrino Yukawa couplings and the masses of the right-handed neutrinos defined at the high scale at which the right-handed neutrinos are integrated out, one needs to compute quantum corrections which typically contain large logarithms due to a vast difference in the energy scales. These large logarithms can be conveniently summed up with the use of the renormalization group (RG) technique [18]. The RG corrections to the neutrino masses and mixing angles in the Supersymmetric Standard Model are particularly large for the degenerate mass spectrum with definite CP parities $(\mp, \mp, \pm)$ and for large $\tan \beta$. In particular, for the overall neutrino mass scale
of the right-handed neutrinos, the effects of the contributions from the neutrino Yukawa couplings are negligible, nevertheless, for $M_X = 10^{14}$ GeV we include them into the RG equations, choosing the texture corresponding to $t_{12}^2(M_X)$ and $t_{23}^2(M_X)$ fixed at their best-fit values (2σ deviations) at the low scale, as shown in Fig. [1]. We also chose $m_1 = 0.1$ eV, $s_{13} = 0.075$ and $M_{N_A} = (1.0, 1.1, 1.2) \times 10^6$ GeV. For such low masses of the right-handed neutrinos, the effects of the contributions from the neutrino Yukawa couplings are negligible, nevertheless, for $M_X = 10^{14}$ GeV we include them into the RG equations, choosing the texture corresponding to the Casas-Ibarra matrix $R = 1$ and ‘switching on’ the relevant neutrino Yukawa coupling at the appropriate thresholds. It is convenient to note that the procedure of deriving the RG equations for the neutrino masses and mixing angles allows maintaining an arbitrarily chosen phase convention for the neutrino masses and the neutrino mixing matrix [11], and we shall utilize it to adhere to the phase choice corresponding to Eq. [1] throughout the entire RG evolution.

B. The $L_\mu - L_\tau$ flavor model

Flavor structure of the Yukawa couplings of quarks and leptons are often explained with the use of a Froggatt-Nielsen mechanism with one or more flavons $\Phi_A$, i.e. scalar fields acquiring vacuum expectation values (vevs), spontaneously breaking a beyond-SM flavor symmetry, but coupling to the matter fields in a symmetry-preserving manner. One of numerous attempts to address the empirically determined pattern of the neutrino observables is to postulate an approximate $L_\mu - L_\tau$ global $U(1)$ symmetry in the lepton sector, which has a virtue of predicting almost maximal atmospheric mixing from a pseudo-Dirac structure of the right-handed neutrino mass matrix. In addition, we assume that there is a discrete $Z_n$ symmetry in the lepton-flavon sector. A full field content and charge assignment is given in Table I. The couplings allowed by symmetries give rise to the almost maximal atmospheric mixing, while those arising by the spontaneous breaking of $U(1)$ and thus flavor-scale suppressed allow reproducing the remaining features of the neutrino masses, given that certain constraints are fulfilled. The presence of $Z_n$ symmetry prevents the flavon vevs from contributing to the mass matrix of the right-handed neutrinos, which shall turn out to be important for the possibility of radiative resonant leptogenesis. We would, however, like to stress that our goal consist in exploring a phenomenologically motivated neutrino flavor pattern which may provide successful leptogenesis rather than in pretending that our construction is the ultimate model of leptonic flavor.

Placing the appropriate thresholds $M_{N_A} = (1.0, 1.1, 1.2) \times 10^6$ GeV. The central solid (outer dashed) lines correspond to central values (2σ deviations) of the relevant observables. We also chose $s_{13} = 0.075$.

![FIG. 1: Running $t_{12}(M_X)$ and $t_{23}(M_X)$ for $m_1 = 0.1$ eV at two scales $M_X = 10^6$ GeV and $10^{14}$ GeV. In the latter case, the neutrino Yukawa couplings correspond to Casas-Ibarra matrix $R = 1$ and they are included into the RG equations at appropriate thresholds $M_{N_A} = (1.0, 1.1, 1.2) \times 10^6$ GeV. The central solid (outer dashed) lines correspond to central values (2σ deviations) of the relevant observables. We also chose $s_{13} = 0.075$.](image)

| field | $L_e$ | $L_\mu$ | $L_\tau$ | $N_e$ | $N_\mu$ | $N_\tau$ | $\Phi_{\pm 1}$ | $\Phi_{\pm 2}$ |
|-------|------|--------|----------|------|--------|----------|---------------|---------------|
| $U(1)$ charge | 0    | +1    | −1       | 0    | −1    | +1       | $\pm 1$       | $\pm 2$       |
| $Z_n$ multiplicity | 1    | 1     | 1        | 0    | 0     | 0        | $n - 1$ n $-1$ |

TABLE I: Field content and the charge assignment in the lepton-flavon sector.
The parameters $X$ and $Y$ consistent with the $U(1)$ symmetry are of the same order of magnitude, and the same is true for $a, b, d$. On the other hand, the flavor symmetry breaking parameters $\lambda_i$ are smaller than the latter as they arise from the flavon vacuum expectation values: $\lambda_{1,5} \propto \langle \Phi_{-1} \rangle / M_X$, $\lambda_{2,6} \propto \langle \Phi_{+1} \rangle / M_X$ and $\lambda_{3,4} \propto \langle \Phi_{+2} \rangle / M_X$ where $M_X$ is the flavor symmetry breaking scale. We take the flavor suppression factors $\lambda_i/a$ of the order $\lambda \sim O(10^{-1})$. It follows from the pseudo-Dirac structure the 2-3 sector of $M$ that two right-handed neutrinos are exactly degenerate in masses, while the mass of the third right-handed can be slightly different.

So far, we have not chosen any specific phase convention for the right-handed neutrinos. We can use the transformations $N_i \rightarrow e^{i \varphi_1} N_1$ and $N_{2,3} \rightarrow e^{i \varphi_2} N_{2,3}$ to ensure that $X$ and $Y$ are real and positive. The remaining phase redefinition $N_{2,3} \rightarrow e^{i \varphi_3} N_{2,3}$ leaves $M$ invariant, but it changes phases in the second and third row of the neutrino Yukawa matrix. We can also make the phase redefinitions of the charged lepton doublets, $L_i \rightarrow e^{i \varphi_0} L_i$ ($i = e, \mu, \tau$). First, we can redefine the overall leptonic phase $\varphi_e + \varphi_\mu + \varphi_\tau$ and the phase $\varphi_3$ so that $d$ is real and positive, and $b$ is real and negative (these transformations do not depend on the phase convention imposed by Eq. (1)). The remaining freedom of the phase choice must be then utilized to ensure that the neutrino mass matrix is diagonalized with a matrix of the form $U$. As we shall see, this will introduce some consistency constraints. These unphysical phases correspond to the freedom of $\varphi_e$ (allowing to set an arbitrary phase to $a$) and to the freedom of shifting $\varphi_3$, $-\varphi_\mu$ and $\varphi_\tau$ by the same value; it would be a symmetry of the neutrino Yukawa matrix, if the $L_\mu - L_\tau$ breaking were absent.

Given the form of the neutrino mass matrix from Eqs. (4)-(7), the low-energy observables like neutrino mass splitting and mixing angles can be explicitly calculated perturbatively treating the small symmetry breaking entries as expansion parameters. Using Eqs. (3) and (4), we can expand the neutrino mass matrix around $s_{13} = 0$ and $\theta_{23} = \pi/4$ for an arbitrary $\theta_{12}$ as:

$$m_{\nu} = m_{\nu}^{(0)} + m_{\nu}^{(1)} + \ldots$$  \hspace{1cm} (8)

In Eq. (8), $m_{\nu}^{(0)}$ is the neutrino mass matrix in the limit $\lambda_i, \delta_i^{(n)} \rightarrow 0$ and $m_{\nu}^{(1)}$ accounts for the $O(\lambda)$ correction to the neutrino mass matrix:

$$m_{\nu}^{(1)} = m_{\nu}^{(0)} \delta s_{13}^{(1)} \frac{\partial m_{\nu}}{\partial s_{13}} \bigg|_{s_{13} = 0} + \Delta s_{13}^{(1)} \frac{\partial m_{\nu}}{\partial s_{13}} \bigg|_{s_{13} = 0} + \theta_{23} = \pi/4 \quad \theta_{23} = \pi/4$$

$$m_i = m_i^{(0)} \quad m_{i} = m_i^{(0)}$$

$$+ \Delta \theta_{23}^{(1)} \frac{\partial m_{\nu}}{\partial \theta_{23}} \bigg|_{s_{13} = 0} \theta_{23} = \pi/4 \quad m_i = m_i^{(0)}$$  \hspace{1cm} (9)

where $\Delta s_{13}^{(1)}$ and $\Delta \theta_{23}^{(1)}$ are corrections to the leading pattern of the neutrino mixing, and $m_{\nu}^{(1)}$ are corrections to the eigenvalues of the neutrino mass matrix. It is straightforward to derive higher order terms of this expansion. Now we shall compare the neutrino mass matrix decomposed as described above with the mass matrix resulting from (7) via (2).

At the leading order, $O(\lambda^0)$, we obtain a neutrino mass matrix which has a pseudo-Dirac structure in the 2-3 sector. This picture can be extended to the first generation, predicting an exactly degenerate mass spectrum, given that:

$$\frac{a^2}{X} = \left( -\frac{bd}{Y} + \frac{\sum \delta_n^{(n)}}{6} \right) e^{i \alpha},$$  \hspace{1cm} (10)

where $\delta_n^{(n)} \sim O(\lambda^n)$ are real and $\alpha \sim O(\lambda)$. Then we find that $(m_1^{(0)}, m_2^{(0)}, m_3^{(0)}) = (-1, -1, +1) \times |b|/Y$ the atmospheric mixing is maximal, $s_{13}$ vanishes and the solar mixing remains undetermined. Clearly, Eq. (10) indicates that our model requires a fine-tuning to describe the neutrino masses and mixing correctly. We shall address the issue of actual fine-tuning compared to other neutrino mass models in the following section.

Let us turn to calculating $O(\lambda)$ corrections to this result. From now on, we shall further simplify our model by setting $\lambda_1 = \lambda_5 = 0$, which is ensured by the absence of the flavon field $\Phi_{-1}$. Such an assumption does not change qualitative features of the model, while simplifying the following formulæ. Comparing the sum and the difference of the 12 and 13 entries of $m_{\nu}^{(1)}$ with the appropriate combinations of the $O(\lambda)$ entries of the neutrino mass matrix resulting from (9) and (7) via (2):

$$m_{\nu} = \left( \begin{array}{ccc} \frac{a^2}{X} - \frac{2 \lambda_1 \lambda_6}{X} + \frac{2 \lambda_2}{X} & \frac{\lambda_1}{X} & \frac{\lambda_2}{X} \\ \frac{\lambda_1}{X} & \frac{\lambda_2}{X} & \frac{\lambda_4}{X} \\ \frac{\lambda_2}{X} & \frac{\lambda_4}{X} & \frac{\lambda_6}{X} \end{array} \right),$$  \hspace{1cm} (11)

(entries denoted by * are given by symmetry of $m_{\nu}$) we obtain:

$$- \frac{a^2}{X} - \frac{d\lambda_6}{Y} = 2 \sqrt{2} m_1^{(0)} \cos \delta s_{13}^{(1)}$$  \hspace{1cm} (12)

$$- \frac{a^2}{X} - \frac{d\lambda_6}{Y} = \sqrt{2} (m_1^{(1)} - m_2^{(1)}) s_{12} c_{12}.$$  \hspace{1cm} (13)

Since we know from the data that the solar mass splitting is much smaller than the atmospheric one, the quantities in Eq. (13) should be smaller than naively assumed $O(\lambda)$, which yields $m_1^{(1)} = m_2^{(1)}$ for large $s_{12}$. From the point of view of the flavor model, the two contributions to the left-hand sides of Eqs. (12)-(13) should either interfere destructively or be small. The first option represents another fine-tuning, while the second can be achieved with a small hierarchy among the flavon vevs: $\lambda_{2,6}/\lambda_{3,4} \propto \langle \Phi_{+1} \rangle / \langle \Phi_{+2} \rangle \sim \lambda$. Irrespective of the actual origin of this feature, it seems more appropriate to defer the discussion of the terms proportional to $- \frac{a^2}{X} - \frac{\lambda_2}{X}$ to the analysis.
of the $O(\lambda^2)$ corrections. A similar comparison for the remaining entries of $m_\nu^{(1)}$ gives:

$$-\frac{d\lambda_3}{Y} - \frac{|b|\lambda_4}{Y} = -2m_3^{(0)} \Delta_2^{(1)},$$

$$-\frac{d\lambda_4}{Y} + \frac{|b|\lambda_4}{Y} = m_2^{(1)} \epsilon_{12} + m_1^{(1)} s_{12},$$

$$-\frac{d\lambda_3}{Y} + \frac{|b|\lambda_4}{Y} = m_3^{(1)},$$

$$-\delta_4^{(1)} + \frac{bd}{Y} = m_1^{(2)} c_{12} + m_2^{(2)} s_{12},$$

where we chose such combinations of the 11, 22, 23 and 33 entries that the results are particularly simple. It may appear that the phases of $\lambda_3$ and $\lambda_4$ must be aligned so that $d\lambda_3 + |b|\lambda_4$ is real up to $O(\lambda^3)$ corrections. However, we still have one phase redefinition, which we can use to impose this condition, so it does not represent another fine-tuning. By taking a linear combination of Eqs. (14)-(17), we obtain a consistency condition for a small solar mass splitting:

$$-\frac{d\lambda_3 + |b|\lambda_4 - \alpha b d}{Y} + \delta_4^{(1)} = (c_{12} - s_{12}^2)(m_2^{(1)} - m_1^{(1)}) \approx 0,$$

(18)

which determine the unphysical phase $\alpha$ and imposes a constraint on $\delta_4^{(1)}$, thereby increasing the already present fine-tuning (10). A relation of this type seems unavoidable in any neutrino mass model predicting a degenerate spectrum. The atmospheric mass splitting is then:

$$\Delta m_{\text{atm}}^2 = |m_3|^2 - |m_2|^2 = 4m_3^{(0)} \text{Re}[m_3^{(1)}],$$

(19)

which is naturally of the order $O(\lambda)$ with respect to the neutrino mass scale and, for fixed $|\lambda_3|, |\lambda_4|$, it is maximal if $\lambda_3$ and $\lambda_4$ are approximately real.

Using this approach, one can also write the relations between the flavor model and the phenomenological parameterization at the $O(\lambda^2)$ order. These lengthy expressions, which omit here, give 6 independent relations between the flavor model parameters and the variables $m_i^{(2)}, \theta_{12}, \Delta\theta_{23}^{(2)}$ and $s_{13} \Delta s_{13}^{(2)}$ or $c_3 \Delta s_{13}^{(2)}$. Hence, no further fine-tunings appear at this stage and the solar splitting is then given by:

$$\Delta m_{\text{sol}}^2 = |m_2|^2 - |m_1|^2 = -2m_3^{(0)} \left( \text{Re}[m_2^{(2)}] - \text{Re}[m_1^{(2)}] \right).$$

(20)

Finally, we note that the model considered here corresponds in some limiting cases to models already present in the literature. Therefore, the following considerations regarding the viability of our model and, in particular, the amount of fine-tuning necessary to describe the neutrino oscillation data can also be applied to those models. For $\lambda_3 = \lambda_6 = 0$ and real $Y_\nu$, we obtain the model studied previously in Ref. [17]. We also note that for $\lambda_3 = \lambda_6 = 0$ and $X = Y$ the neutrino mass matrix (11) is identical to that considered in an $A_4$-inspired model of Ref. [22].

C. Fine-tuning

In Section III B, we have seen that our model requires a fine-tuning, necessary for arranging a small solar neutrino mass-squared splitting. Here, we shall discuss this issue in more detail and compare our model to other models of neutrino masses and mixing.

Addressing the issue of fine-tuning in a quantitative way is a cumbersome task, since it inevitably requires introducing a probability measure in the parameter space. We shall therefore make a comparative study, checking the performance of our neutrino mass model (with $\lambda_1 = \lambda_5 = 0$) versus another neutrino mass model which also arises from breaking of an $U(1)$ flavor gauge symmetry through Froggatt-Nielsen mechanism and is regarded as rather natural. As the reference model, we chose the $A_3$ model of Altarelli, Feruglio and Masina (AFM) [23], which predicts a hierarchical spectrum of neutrino masses. In order to make a comparison with another model explaining quasi-degenerate neutrino masses, we shall also analyze the model of He, Keum and Volkas (HKV) [24] based on $A_4$ symmetry. For completeness, we shall also compare our model with an anarchical seesaw model, i.e. one exhibiting no structure in $Y_\nu$ or $M$ [25].

The comparison has been performed along the lines of the analysis presented by AFM. Each entry $O(\lambda^n)$ allowed by symmetry was parameterized as $Re^{i\omega} \lambda^n$, where $0.8 \leq f \leq 1.2$ and $0 \leq \omega \leq 2\pi$ were chosen randomly with a constant probability density. We used an optimized value $\lambda = 0.35$ for the AFM model, while we set a suggestive value $\lambda = 0.22$ in our model. For the HKV model, we assumed that all unperturbed entries are $O(\lambda^n)$ and that the perturbations are $O(\lambda)$ with $\lambda = 0.1$. For the anarchical model all the entries in $Y_\nu$ and $M$ were assumed to be $Re^{i\omega}$. We then diagonalized numerically the resulting neutrino mass matrices and calculated four dimensionless observables unambiguously constrained by the present neutrino data: $\Delta m_{\text{atm}}^2/\Delta m_{\text{sol}}^2, s_{13}, t_{12}^2$ and $t_{23}^2$. This procedure was repeated 10^6 times for each model. The resulting probability distributions of the observables are shown in Figure [2]. The overall success rate of each model can be defined as the fraction of points lying in a four-dimensional box whose sides correspond to $3\sigma$ ranges of the observables allowed by the present data. Such a success rate was approximately $3 \times 10^{-3}$ for the AFM model, $2 \times 10^{-2}$ for the HKV model and $4 \times 10^{-4}$ for our model, the actual number depending on the RG corrections (admitting $\lambda_{1,5} \sim \lambda_{3,6}$ does not change this result qualitatively). A purely anarchical model has the success rate twice smaller than our model.

If we consider the success rate an unambiguous measure of naturalness, the AFM model and HKV model are favored over ours by the oscillation data. As regards $\Delta m_{\text{atm}}^2/\Delta m_{\text{sol}}^2$, the AFM distribution, peaked around $10^{-2}$ is rather wide and it could easily account for a wide range of values of this observable, whose experimentally allowed $3\sigma$ range (with RG correction neglected) is as-
FIG. 2: Probability distributions for $\Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}$, $s_{13}$, $\theta_{12}$, and $\theta_{13}$. The lines correspond to predictions of our model for (i) general choice of the parameters and (ii) choice of the parameters with $\Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}$ in the experimentally allowed range. The filled (empty) histograms correspond to the AFM (HKV) model.

sumed with 15% probability. In contrast, the lower value of this observable in our model and in the HKV model, the larger fine-tuning is required, and the probability of obtaining $\Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}$ in the allowed range is $\sim 1\%$ and $\sim 4\%$, respectively. The sign of this observable in our model is positive in more than 95% of cases in our model, which justifies a posteriori the assumptions made in Section II.B. Values of $s_{13}$ come out small in all models but the anarchical one, with $\sim 30\%$ (HKV), $\sim 50\%$ (AFM) and $\sim 60\%$ (our) of the distribution in the allowed range. The atmospheric mixing is peaked around the maximal mixing models, with the HKV distribution being the most narrow.

As we already argued in Section II.B there is no point in discussing the solar mixing independently of the solar mass splitting in our model, as the consistency with experimental data introduces some correlation between observables. As shown in Figure 2 (where we also plot the distribution of conditional probability density given that the solar-to-atmospheric ratio lies within the experimentally allowed range), once the fine-tuning required for the solar-to-atmospheric mass ratio is achieved, the distribution of $\theta_{12}$ becomes peaked around values consistent with experiment. Similarly, the distribution of conditional probability density for $s_{13}$ given that the solar-to-atmospheric ratio (empty cyan boxes) is shifted towards smaller values of this observable, pursuant to Eq. (13).

In conclusion, in comparison to the AFM model and the HKV model, our model’s overall performance is worse by a factor of 10 to 100, following mainly from the fine-tuning necessary for the small solar mass splitting. However, if our model can explain the baryon asymmetry of the Universe as resulting from leptogenesis with a low reheating temperature, while the AFM and HKV model cannot, this may be a hint that a quantitatively moderate fine-tuning discussed above allows a glimpse at the structure of a more fundamental physics rather than being an unnatural coincidence.

III. LEPTOGENESIS

In the MSSM, the effects of supersymmetry breaking in leptogenesis can be safely neglected. The CP asymmetries are twice larger than those in the Standard Model and the number of channels through which the lepton asymmetry is generated is also doubled. This is compensated by doubled amplitudes of the washout processes and an almost doubled number of relativistic degrees of freedom after leptogenesis. The conversion factors, relating the generated lepton asymmetry with the final baryon asymmetry, are also very similar. Therefore, the order of magnitude of the baryon asymmetry of the Universe resulting from leptogenesis can be approximated by the nonsupersymmetric formula

$$\eta_B \sim 10^{-2} \sum_{i=1}^3 e^{1-M_i/M_1} \sum_\alpha \varepsilon_{i\alpha} K_{i\alpha} K_{i\alpha}$$

where $\alpha$ runs over distinguishable lepton flavors $\alpha = e, \mu, \tau$ (we assume a reheating temperature $\lesssim 10^{9}$ GeV) and $\varepsilon_{i\alpha}$ are CP asymmetries in the decays of the right-handed neutrinos of mass $M_i$ into flavor $\alpha$. The washout parameters are defined as

$$K_{i\alpha} = \frac{\langle \Gamma(N_i \to L_\alpha H_2) + \Gamma(N_i \to \bar{L}_\alpha H_2^* \rangle}{H(T = M_1)} \approx \frac{\bar{m}_{i\alpha}}{10^{-3} eV},$$

where $\bar{m}_{i\alpha} = |\varepsilon_{i\alpha}|^2 (H_2)^2/M_i$, $K_i = \sum_\alpha K_{i\alpha}$, $K_\alpha = \sum_\alpha e^{1-M_i/M_1} K_{i\alpha}$, and $y$ is the neutrino Yukawa matrix written in the basis of the mass eigenstates for the right-handed neutrinos. It follows from Eq. (22) that for the light neutrinos with masses $0.1 \text{ eV}$, we need $|\varepsilon_{i\alpha}| \sim 10^{-4}$ for successful leptogenesis. However, for $M_i \lesssim 10^9$ GeV which allows avoiding the gravitino problem, a natural scale for the CP asymmetries is

$$|\varepsilon_{i\alpha}| \sim M_i m_\nu (H_2)^2 \sim 10^{-6}$$

(we neglect various $O(1)$ factors), unless, e.g., the right-handed neutrinos are almost degenerate in masses, since
in that case the CP asymmetries in the decays of these right-handed neutrinos are resonantly enhanced.

In our model, the right-handed neutrinos mass matrix $M$ in Eq. (7) has two exactly degenerate eigenvalues, $M_2 = M_3 = Y$, which may become slightly split by RG corrections if the scale $Q_f$ of $U(1)$ breaking is larger than the leptogenesis scale. If the splitting $\delta_N = 1 - M_2 / M_3$ is much smaller than $(yy^\dagger)_{22,33}/8\pi$, the relevant CP asymmetries are given by $^4$ (see also $^8$):

$$\varepsilon_{j\alpha} \approx \frac{16\pi \delta_N \text{Re}[yy^\dagger]_{23} \text{Im}[y_{2\alpha} y^*_{3\alpha}]}{(yy^\dagger)_{22} (yy^\dagger)_{33} (yy^\dagger)_{11}}.$$  

(24)

It may appear that for $M_2 = M_3$ a transformation $N_2 \rightarrow \cos \zeta N_2 + \sin \zeta N_3$ and $N_2 \rightarrow -\sin \zeta N_2 + \cos \zeta N_3$ is a symmetry of the mass matrix of the right-handed neutrinos, but it allows for rearranging the neutrino Yukawa couplings. However, it has been noted in $^9$ that if we require that the neutrino Yukawa couplings are continuous functions of the renormalization scale then $\zeta$ is fixed at a value corresponding to $\text{Re}[yy^\dagger]_{23} = 0$. Hence, the CP asymmetries vanish at the scale of the exact degeneracy of $N_2$ and $N_3$. They assume nonzero values if $\text{Re}[yy^\dagger]_{23} \neq 0$ is generated through RG corrections. In the leading order, the solutions of the RG equations for degenerate right-handed neutrinos are:

$$\delta_N \approx 4 ((yy^\dagger)_{22} - (yy^\dagger)_{33}) \Delta t = 8(H_2)^{-2} |b\lambda_+^\dagger + d\lambda_3| \Delta t,$$

$$\text{Re}[yy^\dagger]_{23} \approx \text{Re}[y_{23} y_{23}^\dagger] y^2 \Delta t = \langle H_2 \rangle^{-2} |b| \text{d} \text{Im} \lambda_3 | \lambda_3^\dagger | y^2 \Delta t,$$

(25)

(26)

where $y_\tau \sim 10^{-2} \tan \beta$ is the tau Yukawa coupling and $\Delta t = (4\pi)^{-2} \ln(Q_f/Y) \sim 0.1 + 0.006 \ln[Q_f/10^5 Y]$. Other combinations of parameters appearing in $^{24}$ receive negligible RG corrections and can be replaced by their values at the scale $Q_f$. The CP asymmetries can be then expressed as:

$$\varepsilon_{2\alpha} = \varepsilon_{3\alpha} \approx \frac{64\pi y^2 \lambda^2 \lambda_3 \lambda_4 (\Delta t)^2}{(b^2 + d^2)^3} \times \left\{ \frac{O(\lambda^4)}{-|b|^2} \right\} \frac{1}{d^2}$$

(27)

where the upper, middle and lower factors correspond to $\alpha = e, \mu, \tau$, respectively. The other CP asymmetries, $\varepsilon_{1\alpha}$, are much smaller and can be neglected. We estimate from $^{24}$ that $|\varepsilon_{2\alpha}| \lesssim 10^{-6} \tan^2 \beta$, so the CP asymmetries are sufficiently large for successful leptogenesis if $\tan \beta \gtrsim 10$.

The predictions of our neutrino mass model for the CP asymmetries given in Eq. (27) are presented in Figure 3, where we plot the distribution probability for $\varepsilon_{\text{max}} = \max[|\varepsilon_j|;\ j = 1, 2, 3]$, where $\varepsilon_j = \sum_{\alpha} \varepsilon_{j\alpha}$. The numerical procedure is identical to that described in Section IIIC with the exception that we scan over $10^7$ points in the parameter space for the conditional probability distribution. For definiteness, we assume that $\Delta t = 0.1$, $\tan \beta = 10$ and $M_1 = 10^6 \text{GeV}$. The black, solid lines correspond to the general parameter choice in our model, while the blue, dotted lines represent parameter choices satisfying all four phenomenological constraints; the filled (empty) histograms correspond to the AFM (HKV) model. In this Figure, we also show the estimates the baryon asymmetry of the Universe $\eta_B$ given by formula (21). According to the discussion in Section IIIB, a requirement of a small solar neutrino mass splitting favors almost real $\lambda_3$ and $\lambda_4$, which in turn leads to a certain suppression of the CP asymmetry $^{27}$. Moreover, the fact that $\varepsilon_{1\mu}$ and $\varepsilon_{1\tau}$ are proportional to $-|b|^2$ and $d^2$, respectively, is the reason for another slight suppression of the resulting baryon asymmetry. These suppressions can, however, be easily overcome by the enhancement of the tau Yukawa coupling for $\tan \beta \gtrsim 10$, and we conclude that we can easily have the CP asymmetry of the order of $10^{-4}$ which can account for the baryon asymmetry of the Universe. As regards the AFM and HKV models, with an optimistic assumption that the largest values of $\varepsilon_{\text{max}}$ correspond to washout as small as 0.1, we conclude...
that these models can be only marginally consistent with baryogenesis via leptogenesis for a low reheating temperature.

The crucial ingredient of our model which allows for a low-scale leptogenesis is the assumption that the $L_\mu - L_\tau$ flavor symmetry is broken at a scale $M_X$ much larger than the leptogenesis scale $M_N$, and that this breaking is transmitted to the mass matrix of the right-handed neutrinos only through RG corrections. At the leptogenesis scale, the masses of the pseudo-Dirac right-handed neutrinos are then split by a factor proportional to the neutrino Yukawa couplings and it is precisely this small splitting which makes it possible to overcome the naive scaling \[23\] [10]. For comparison, in model described in [17], also based on $L_\mu - L_\tau$ symmetry, the scale of symmetry breaking is identified with the leptogenesis scale, the RG corrections are absent and the pseudo-Dirac right-handed neutrinos are exactly degenerate, which leads to a vanishing CP asymmetry in their decays. (Besides, only one CP violating phase is assumed in this model; although it is straightforward to include other phases in the neutrino sector, this would not change the latter conclusion.) Hence, the scaling \[23\] holds approximately and large values of the right-handed neutrino masses of are necessary for successful leptogenesis. In contrast, in the model of [16] (in which the symmetry breaking scale is also identified with the leptogenesis scale), the $L_\mu - L_\tau$ symmetry is broken in both the neutrino Yukawa matrix and the mass matrix of the right-handed neutrinos which again leads to the approximate scaling \[23\] and large values of the right-handed neutrino masses necessary for successful leptogenesis.

**IV. CONCLUSION**

In this work, we have considered a neutrino mass model where the neutrino Yukawa and mass structures are dictated by the flavor symmetry, $L_\mu - L_\tau$, and its breaking patterns which is controlled by an additional discrete symmetry. The model requires a fine-tuning to correctly predict the smallness of the solar mass splitting. Taking the expansion parameter $\lambda = 0.22$, we have made a quantitative discussion on the fine-tuning in the combined explanation of all the low-energy observables and the demanded baryon asymmetry of the Universe. Once such a fine-tuning is ensured, the bi-large pattern of mixing angles and a successful leptogenesis with a low reheating temperature becomes a natural prediction of this model.

In addition, the Dirac CP phase is generically order-one while the reactor mixing angle $\theta_{13}$ is peaked at $\theta_{13} = 0.06$.

Acknowledgments

KT thanks S. Pokorski and P. H. Chankowski for discussions at early stages of this project. This work is supported by the Department of Energy.

[1] For a recent review, see A. Strumia and F. Vissani, \[arXiv:hep-ph/0606054\].
[2] D. N. Spergel et al., \[arXiv:astro-ph/0603449\].
[3] M. Fukugita and T. Yanagida, Phys. Lett. B 174, 45 (1986).
[4] For a review, see W. Buckmuller, P. Di Bari and M. Plümacher, Annals Phys. 315, 305 (2005) [arXiv:hep-ph/0401240].
[5] P. Minkowski, Phys. Lett. B 67 (1977) 421. See also M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, eds. P. van Nieuwenhuizen and D. Freedman, (North-Holland, 1979), p. 315; S.L. Glashow, in Quarks and Leptons, Cargèse, eds. M. Lévy et al., (Plenum, 1980, New-York), p. 707; T. Yanagida, in Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe, eds. O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, 1979), p. 95; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
[6] S. Davidson and A. Ibarra, Phys. Lett. B 535, 25 (2002) [arXiv:hep-ph/0202239].
[7] J. R. Ellis, A. D. Linde and D. V. Nanopoulos, Phys. Lett. B 118, 59 (1982); M. Y. Khlopov and A. D. Linde, Phys. Lett. B 138, 265 (1984); K. Kohri, Phys. Rev. D 64 043515 (2001) [arXiv:astro-ph/0103341]; see also, e.g., M. Kawasaki, K. Kohri and T. Moroi, Phys. Lett. B 625, 7 (2005) [arXiv:hep-ph/0402490]; D. G. Cerdeno, K. Y. Choi, K. Jedamzik, L. Roszkowski and R. Ruiz de Austri, JCAP 0606, 005 (2006) [arXiv:hep-ph/0509275]. F. D. Steffen, JCAP 0609, 001 (2006) [arXiv:hep-ph/0605306]; J. Pradler and F. D. Steffen, Phys. Rev. D 75 023509 (2007) [arXiv:hep-ph/0608344].
[8] M. Flanz, E. A. Paschos and U. Sarkar, Phys. Lett. B 345, 248 (1995) [Erratum-ibid. B 382, 447 (1996)] [arXiv:hep-ph/9411366]; M. Flanz, E. A. Paschos, U. Sarkar and J. Weiss, Phys. Lett. B 389, 693 (1996) [arXiv:hep-ph/9607310]; see also J. R. Ellis, M. Raidal and T. Yanagida, Phys. Lett. B 546, 228 (2002) [arXiv:hep-ph/0206300]; M. Fujii, K. Hamaguchi and T. Yanagida, Phys. Rev. D 65, 115012 (2002) [arXiv:hep-ph/0202210]; A. Pilaftsis and T. E. J. Underwood, Nucl. Phys. B 692, 303 (2004) [arXiv:hep-ph/0309342].
[9] A. Pilaftsis, Phys. Rev. D 56, 5431 (1997) [arXiv:hep-ph/9707235]; A. Pilaftsis and T. E. J. Underwood, Nucl. Phys. B 692, 303 (2004) [arXiv:hep-ph/0309342].
[10] K. Turzynski, Phys. Lett. B 589, 135 (2004) [arXiv:hep-ph/0401219]; R. González Felipe, F. R. Joaquim and B. M. Nobre, Phys. Rev. D 70, 085009 (2004) [arXiv:hep-ph/0311029].
[11] G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim and B. M. Nobre, Phys. Lett. B 633, 336 (2006) [arXiv:hep-ph/0507092].
[12] V. Cirigliano, G. Isidori and V. Porretti, Nucl. Phys. B 763, 228 (2007) [arXiv:hep-ph/0607068].
[19] P. H. Chankowski and S. Pokorski, Nucl. Phys. B 618, 171 (2001) [arXiv:hep-ph/0103065].
[20] A. Pilaftsis and T. E. J. Underwood, Phys. Rev. D 72, 113001 (2005) [arXiv:hep-ph/0506107]. For a more thorough discussion of flavor effects in leptogenesis see, e.g.: A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada and A. Riotto, JCAP 0604, 004 (2006) [arXiv:hep-ph/0601083]; A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada and A. Riotto, JHEP 0609, 010 (2006) [arXiv:hep-ph/0605281]; E. Nardi, Y. Nir, E. Roulet and J. Racker, JHEP 0601, 164 (2006) [arXiv:hep-ph/0601084]; S. Pascoli, S. T. Petcov and A. Riotto, arXiv:hep-ph/0611338.
[21] P. R. Joaquim, I. Masina and A. Riotto, arXiv:hep-ph/0701270.
[22] A. J. Buras, S. Jäger, S. Uhlig and A. Weiler, arXiv:hep-ph/0609067.
[23] G. Altarelli, F. Feruglio and I. Masina, JHEP 0301, 035 (2003) [arXiv:hep-ph/0210342].
[24] X.-G. He, Y.-F. Keum and R. R. Volkas, JHEP 0604, 039 (2006) [arXiv:hep-ph/0601001].
[25] L. J. Hall, H. Murayama and N. Weiner, Phys. Rev. Lett. 84, 2572 (2000) [arXiv:hep-ph/9911341].
[26] F. R. Joaquim, I. Masina and A. Riotto, arXiv:hep-ph/0701270.