Some comments on
Models of hadronic interactions at air shower energies

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Abstract
Several models of minimum-bias hadronic interactions at ultra-high energy that have been used for calculations of air showers share essential common features. In this talk I review these common elements and discuss some consequences. I concentrate on properties of hadron-nucleus interactions, and I use mean depth of shower maximum as a function of primary energy to illustrate my main points. I will contrast these models with models that use a more naive treatment of hadronic interactions in nuclei but which have been successfully used to interpret measurements of depth of shower maximum.

1 Astrophysical motivation
The spectrum of cosmic rays extends more than five orders of magnitude beyond the highest energy at which it has been possible so far to observe the primaries directly with experiments on balloons or spacecraft. In this energy range the spectrum has at least two features, the knee between $10^{15}$ and $10^{16}$ eV and the ankle between $10^{18}$ and $10^{19}$ eV. The knee may be associated with a transition between different classes of galactic cosmic rays (or with a feature of propagation of galactic cosmic rays) and the ankle with a transition from galactic to extra-galactic origin of the particles—but these are only plausible conjectures.

In order to understand the implications of these features for the origin of the high-energy particles it is necessary to measure the relative contribution of the different groups of nuclei. This must be done with large ground-based air-shower experiments in order to achieve sufficient exposure to collect a large sample of high-energy events. Because of the indirect nature of air shower experiments and the problems of fluctuations superimposed on a steeply falling energy spectrum, progress toward the goal of measuring the primary composition at high energy has been slow and difficult. New experiments with better resolution, coupled with a better understanding of the hadronic interaction models used to interpret the data, show promise for improving the situation.

1.1 The knee region
The current status is summarized very nicely in the review of Kalmykov and Khristiansen. Concerning the knee region, they conclude that measurements of fluctuations in the muon to electron ratio suggest a gradual change toward a larger fraction of heavy nuclei in the energy range from $10^{15}$ to $10^{17}$ eV. They consider two quite different models which share this feature. One is a diffusion model in which a single underlying composition and source spectrum are modified by
propagation in the galactic magnetic field. The other is a model\cite{3, 4, 5} in which there are two different classes of sources.

The diffusion model is a modern realization of the classic suggestion of Peters\cite{6} and Zatsepin\cite{7}. Its characteristic feature is a steepening of the spectrum at a certain value of magnetic rigidity so that there are relatively more heavy particles at high energy when the classification is by total energy per nucleus. To fit the data the break has to occur at a rigidity of $\geq 10^6$ GV.

The two-component model is motivated by the observation\cite{8} that the proton spectrum as measured by JACEE\cite{9} appears to steepen around 100 TeV or somewhat below while there is no steepening of helium or heavier nuclei at the corresponding rigidity. The models are further stimulated by the fact that 100 TeV is the maximum energy expected for shock acceleration by supernova blast waves expanding into the typical interstellar medium\cite{10}. In this kind of model higher energy particles come from acceleration by supernovae whose environments are richer in heavy nuclei, so the high-energy component contains fewer protons and relatively more heavy nuclei. The specific version of this model described in Ref.\cite{4} has less than 10% protons already for $E > 300$ TeV. According to Kalmykov and Khristiansen the muon/electron measurements rule out such a low fraction of protons below the knee, presumably because without the light nuclei one would see smaller fluctuations in the muon/electron ratio than observed.

Most likely, the real situation combines features of both diffusion and different types of sources. In addition, one would expect supernovae in the same general class to have somewhat different environments (magnetic fields and composition) and thus to exhibit a gradation of cutoffs and compositions of the cosmic-ray spectra they produce. Nevertheless it is interesting to look at the mass fractions in these specific models just to illustrate how well experiments will need to determine the mass composition in order to distinguish among models. If the composition is 64% protons and helium and 36% heavier nuclei below the knee ("normal composition") and all components have a differential spectral index of -2.7, then the corresponding fractions above the knee (e.g. around $10^{17}$ eV) would be 44% and 56% if the spectral index changes by -0.3 to -3.0. The corresponding numbers given in Ref.\cite{1} for the Hall diffusion model of Ptuskin et al.\cite{2} are 55/45 below the knee and 28/72 at $10^{17}$ eV. The light/heavy ratio is similar to this at high energy in the two-source model of Ref.\cite{4}, but the light component is reduced before the knee as well.

1.2 Energies above $10^{17}$ eV

An analysis of the Fly’s Eye data\cite{11} on mean depth of shower maximum as a function of primary energy leads to the conclusion that the primaries are mostly heavy nuclei at the low end of the energy range of this experiment ($10^{17}$ eV), with an increasing fraction of protons as energy increases. It was noted\cite{12} that this transition corresponds in energy with the “ankle” feature of the spectrum. If it is correct, this interpretation is circumstantial evidence for a transition to extragalactic origin for the highest energy cosmic rays, and this extragalactic component is primarily light nuclei (protons and helium).

In their review, Kalmykov and Khristiansen reach a similar conclusion, although the evidence for a change of composition is less strong than indicated by the figure of Ref.\cite{11}. Yakutsk data are consistent with those of the Fly’s Eye group\cite{2, 13}. In fact various calculations give a wide range of results for depth of maximum in the Fly’s Eye energy range. After discussing some of the input to the models, I will return to this question in the last section.
2 Interaction models for EAS

Comparison to Fly’s Eye data, with typical energies of $10^{18}$ eV and above, requires extrapolation of hadronic interaction models more than two orders of magnitude in center of mass energy beyond the highest accelerator energies ($\sqrt{s} = 1.8$ TeV). In fact the required extrapolation is much greater than this because the showers involve nuclei as well as single hadrons (both as targets and as projectiles) and because showers depend essentially on how the energy is divided when particles interact (fragmentation region) rather than on multiplicity (central region). Physical models are required to relate the various processes and to extrapolate to high energy. Thus also in the knee region there can be significant model-dependence in air shower simulations.

In preparation for this meeting Knapp et al. [13] have made a very instructive comparison of some results of different event generators, several of which represent theoretical models of hadronic interactions with similar underlying physical assumptions. The models (codes) that I consider here are:

1. Dual parton model [14] (DPMJET [15]).
2. Quark-Gluon String model [16] (QGSjet [17]).
3. Venus [18].
4. Minijet model [19, 20] (SIBYLL [21]).

All of these codes use strings of hadrons coupled to constituent partons of the incident hadrons to represent particle production in high-energy interactions.

2.1 Inelasticity

The common feature of all these models that I want to draw attention to concerns interaction of a single hadron with a target nucleus and, in particular, the propagation of the projectile hadron and its fragments through the target nucleus. In all the models listed above the valence partons in the projectile only radiate one pair of strings of secondary hadrons. In cases with more than one wounded nucleon in the target the extra strings are connected with sea-quarks in the projectile. This means that the inelasticity in hadron-nucleus collisions is not much larger than that in corresponding hadron-nucleon collisions. This is illustrated by comparison of Figs. 14 and 35 in the report of Knapp et al. [13].

One can contrast this, for example, with the KNP model [22], which was favored by the Fly’s Eye group in its comparison [11] of measurements of depth of maximum between $10^{17}$ and $10^{19}$ eV with simulations. Cronin (private communication) has emphasized that other codes, such as SIBYLL and MOCCA [23], give a rather different impression of the interpretation of the measurements than that shown in Fig. 2 of Ref. [11]. In that figure the Fly’s Eye data are compared to mean depth of maximum vs. energy for proton showers and for iron showers as simulated with the KNP model. This choice was made because, of the several models compared with the same data in an earlier paper [24], the KNP model gave the most rapid shower development and hence the best agreement with the observed depth of maximum.

One feature of the KNP model that led to relatively rapid shower development is the treatment of propagation of the projectile nucleon inside the target nucleus. In this model the projectile
nucleon is assumed to lose energy in the same way each time it interacts with a target nucleon in the nucleus. This leads to collisions that are significantly more inelastic.

Inelasticity is the fraction of energy in a hadron-nucleus interaction not carried off by the fragment of the projectile hadron. Because there is no way to distinguish a “fragment” hadron from a “produced” hadron of the same type, this is only a theoretical definition (but still a useful one for comparison of theoretical models). The following definition of inelasticity, \( K \), in high-energy proton-nucleus collisions in terms of momentum-weighted integrals of inclusive cross sections is one that could in principle be realized experimentally:

\[
1 - K \approx \frac{\int p \frac{d\sigma_N}{dp} d^3p - \int p \frac{d\bar{\sigma}_N}{dp} d^3p}{(\sigma \times E_0)}.
\]

Here \( \sigma_N \) is the cross section for production of a proton or neutron and \( \sigma_{\bar{N}} \) the cross section for producing an antinucleon. The definition becomes more accurate as energy increases and the target fragments become relatively less important. An experimental definition is more difficult for pions, but what counts is the inclusive cross section in the fragmentation region (e.g. for \( x > 0.05 \)).

Hüfner and Klar [25] compared a simple model with data on p-nucleus interactions in the 100 GeV region [26, 27]. Within the context of a Glauber multiple scattering treatment of hadron-nucleus collisions the mean number of wounded nucleons in the target is given by

\[
\langle N_W \rangle_{pA} = \frac{A \sigma_{pp}^{\text{inel}}}{\sigma_{pA}^{\text{inel}}},
\]

and the probability, \( P_n \) of having exactly \( n \) wounded nucleons is calculated from the nuclear profile functions [28] in the standard way [21]. Hüfner and Klar write the energy of the projectile fragment nucleon after encounters with exactly \( n \) wounded nucleons on average as

\[
E_N(n) = (1 - I_n) \times E_N(n-1).
\]

The total inelasticity is then

\[
1 - K \approx \sum_{n=1}^{A} P_n \prod_{k=1}^{n} (1 - I_k).
\]

In this relation \( I_1 \approx \frac{1}{2} \) is the average inelasticity for proton-proton collisions. Within the string models listed above one expects \( I_1 > I_2 \approx I_3 \approx \ldots \). Comparison of this simple formulation to the data in Ref. [25] gives \( I_n \approx 0.2 \) for \( n \geq 2 \). The KNP model has instead,

\[
I_1 = I_2 = I_3 \ldots \approx 0.5
\]

and hence a larger value of inelasticity.

Figure 1 shows a plot of inelasticity as a function of \( \log(E_0) \) in eV for the two sets of parameters calculated from Eq. [4]. The solid line corresponds to \( \{I_n = 0.5 \text{ for all } n\} \); the dashed line to \( \{I_1 = 0.5, \ I_n = 0.2, \ n \geq 2\} \). In both cases the inelasticity increases with energy. This is a consequence of the increasing cross section, which, according to Eq. [2], drives an increase in the number of wounded nucleons. The same cross sections [21] and hence the same distributions of wounded nucleons have been used for both curves in Fig. 1. The difference is in the assumption for \( I_n, \ n \geq 2 \). The upper curve represents the KNP model, and the lower curve is a good approximation to the inelasticity in the full SIBYLL model.
2.2 SIBYLL model

This model is based on the idea that the increase in cross section is driven by the production of minijets [29, 30, 31]. The emphasis is on the fragmentation region and on collisions of hadrons with light nuclei. Depth of shower maximum and TeV muons are the principal intended applications. With one significant exception, all the ideas of the dual parton model are incorporated. The program uses Lund techniques [32] and is tailored for efficient operation up to at least $10^{20}$ eV.

The exception is that in SIBYLL the soft part of the eikonal function is taken as a constant, fitted to low energy data. The non-jet part of every event is represented by exchange of a single pair of strings. In the other string-type models listed above, non-jet contributions to events can involve exchange of multiple pairs of soft strings. To the extent that the parameters in the two approaches are tuned to reproduce the same increasing cross section, the resulting multiplicity should be similar—in one case it comes from short pairs of multiple soft strings and in the other from minijets. Since the fragmentation of the minijet pair is treated as a loop of string, the multiplicity should be similar. In principle, the soft part of the eikonal should also be energy-dependent, and multiple exchanges of both hard and soft kinds should occur. The question of sensitivity of the parameters of the fits to data when both multiple soft and multiple hard exchanges are allowed is an interesting one.

For SIBYLL at a lab energy of 2 TeV we find [33]

$$I_1 \approx 0.53 \quad I_2 \approx 0.22 \quad I_3 \approx 0.15 \quad I_4 \approx 0.13.$$ 

The overall inelasticity from SIBYLL for proton-nitrogen collisions is shown by the inverted triangles in Fig. 1. It is very close to the simple formula [4] with the partial inelasticities from Ref. [25].

Because the inelasticities tend to converge as energy decreases, it is not clear with what certainty the comparison of Ref. [25] to the data restricts the behavior of the inelasticity and even whether the KNP treatment is completely ruled out. This question needs further investigation.

3 Muon to electron ratio in air showers

Interpretation of measurements of the muon to electron ratio in air showers deep in the atmosphere depend on Monte Carlo simulations. Differences in existing codes at present may make it difficult to use this feature alone to discriminate among light/heavy ratios of the kind illustrated in §1.1 above (see for example Fig. 71 of Ref. [13]. Obvious remedies are to understand and resolve differences among the simulation codes and to measure more components of the showers, as is being done by KASCADE. In the work referred to in Ref. [1], the model-dependence is addressed by considering fluctuations in the muon to electron ratio as well as the mean values.

In comparing production of $\sim$ GeV muons in cascades simulated with different interaction models it may be useful to tabulate the distribution of interactions of various types in the cascade. This diagnostic relates the energy-dependence of low energy muons in showers to fundamental features of the interaction models. Generally it will be possible to express the distribution of hadronic interactions in a proton-induced air shower by an expression of the form [34]

$$\frac{dN_{int}}{dz} \approx \delta(1 - z) + f_p(z) + a_{\pi} \frac{(1-z)^{n_{\pi}}}{z^{1+p_{\pi}}} + a_K \frac{(1-z)^{n_K}}{z^{1+p_K}},$$

(5)
where \( z = E_{\text{int}}/E_0 \), \( f_p(z) \sim 1/z \) and \( E_0 \) is the primary energy. The first two terms on the right side of Eq. 1 represent the interactions of nucleons in the cascade, and the approximate expression for \( f_p \) is exact for a flat inelasticity distribution and neglecting production of \( N\bar{N} \) pairs. The 3rd and 4th terms represent interactions of pions and kaons respectively.

An approximation for the number of low-energy muons in a shower initiated by a proton when \( E_\mu \ll \epsilon_\pi \) is

\[
N_\mu(> E_\mu) \sim a_\pi F_{\pi\pi}(0) \ln \frac{E_0}{E_\mu} \left( \frac{E_0}{\epsilon_\pi} \right)^{p_\pi}
\]  

(plus a similar term for kaons). Here \( \epsilon_\pi \approx 115 \text{ GeV} \) is the critical energy for pions to decay rather than interact in the atmosphere. The quantity \( F_{\pi\pi} \) is approximately the rapidity density in the central region for production of charged pions in interactions of pions with nuclei of the atmosphere evaluated for pion interaction energy of \( \approx \epsilon_\pi \). In a superposition approximation, the \( A \)-dependence of the muon content of a shower will be

\[
N_\mu \approx A^{1-p_\pi}
\]

for \( E_{\text{total}}/A \gg \epsilon_\pi \). The accuracy of these approximations needs to be checked with simulations, but the relations should be a useful guide to investigating the model-dependence of the ratio of low energy muons to electrons in air showers.

The Akeno group have published an analysis of low-energy muons in large showers at AGASA [35]. They fit the muon dependence on primary energy to a power 

\[
\langle \rho_\mu(600) \rangle \sim (S_{600})^p
\]

with \( p \approx 0.82 \). Here \( S_{600} \) is the scintillator density at 600 m from the shower core, which is assumed to be a good measure of the primary energy, and \( \rho_\mu \) is the corresponding muon density. A change in composition of the degree suggested by Fly’s Eye would require a value of \( p \) larger by roughly 10%. (The analysis involves a comparison of Eqs. 3 and 6 above.) Because of possible model-dependence of the parameters \( p_\pi \) (and \( p_K \)) there is a corresponding model-dependence of of the \( \mu/e \) ratio which needs further investigation.

### 4 Model-dependence of depth of maximum

In this section I return to the question of the interpretation of the depth of shower maximum as measured by Fly’s Eye. Figure 2 shows the Fly’s Eye data [11] and comparisons to various model calculations. The upper three lines in Fig. 2 are for proton-induced showers calculated in three models: SIBYLL—dotted [21]; QGS—dashed [16]; KNP—highest solid line. The representation of the KNP model [22] used for the Fly’s Eye calculation is described in Refs. [24, 36]. Comparison of these three curves gives an indication of the extent of the uncertainty in interpretation of the depth of maximum measurements arising from uncertainties in the models, such as different inelasticities and different cross sections and interaction lengths.

In addition, an important point is that in the Fly’s Eye analysis of Bird et al. [11] the simulations have been corrected for effects of detector acceptance and other systematics. The lower pair of solid lines in Fig. 2 (taken from Ref. [11]) shows the KNP model for protons and for iron primaries after these corrections have been applied. There are several compensating effects, but the net shift of about \(-20 \) to \(-25 \text{ g/cm}^2 \) is shown by comparison to the unshifted KNP curve for protons.
The recent work of the Utah group \[37\] represents a quite different approach to the calculation of depth of shower maximum. They have analyzed the data in the context of a Chou-Yang model \[38\]. This interaction model features a statistical picture of particle production in the central region according to which the inclusive cross section is of the form

\[
\frac{d^3n}{d^3p} = \frac{C}{E} e^{-\frac{\alpha p_T}{E}} e^{-E/T_p}. \tag{8}
\]

The parameters \(C\) (normalization), \(2/\alpha\) (mean transverse momentum) and \(T_p\) (partition temperature) are adjusted to fit collider data in the central region from \(\sqrt{s} = 53\) GeV to 1800 GeV and extrapolated to higher energies as described in Ref. \[37\]. The mean inelasticity is the integral of Eq. 8, which is approximately

\[
K = \frac{4\pi C}{\alpha^2} \frac{T_p}{\sqrt{s}}. \tag{9}
\]

The striking feature of this model is that, in order to fit the collider data, the inelasticity must decrease as energy increases—from \(\sim 0.5\) at low energy to \(< 0.2\) for \(pp\) interactions in the Fly’s Eye energy range \[37\]. To compensate and obtain a reasonably shallow depth of maximum, they use a naive (KNP) assumption for the inelasticity as well as a rapidly increasing cross section. Since both effects increase with energy, the mean inelasticity is decreased more at high energy. In other words, assumptions which reduce the depth of maximum at one energy also tend to decrease the elongation rate (logarithmic slope of \(X_{\text{max}}\)).

In view of the model dependence of depth of maximum and the significant systematic corrections involved in comparing to the data, it is important to look at the distribution of depth of maximum as well as its mean value. This is illustrated in Fig. 3 \[39\], which shows the contribution of protons and iron separately to the overall distribution for the group of events with \(E > 10^{18}\) eV. The calculations were made with the KNP model. It appears that in this energy range comparable numbers of light and heavy nuclei are needed to fit the data.

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CAPTIONS

Fig. 1. Inelasticity vs. Energy: Points show the SIBYLL interaction model; the lines represent Eq. 4 (see text).

Fig. 2. Mean depth of shower maximum vs. primary energy. Data points from Fly’s Eye. See text for explanation of lines.

Fig. 3. Comparison of the fitted depth of maximum distribution to Fly’s Eye data for $E > 10^{18}$ eV. Contributions of protons (dashed) and iron (dotted) are shown separately.
