Inflation in the parity-conserving Poincaré gauge cosmology

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Abstract. The general Poincaré gauge cosmology given by a nine-parameter parity-conserving gravitational Lagrangian with ghost- and tachyon-free conditions is studied from the perspective of field theory. By introducing new variables for replacing two (pseudo-) scalar torsions, the Poincaré gauge cosmological system can be recast into a gravitational system coupled to two-scalar fields with a potential up to quartic-order. We discussed the possibility of this system producing inflation without any extra inflatons. The numerical analysis shows that the two-scalar fields system evolved in a potential well processes spontaneously four stages: “pre-inflation”, slow-roll inflation with large enough e-folds, “pre-reheating” and reheating. We also studied the stableness of this system by setting large values of initial kinetic energies. The results show that even if the system evolves past the highest point of the potential well, the scalar fields can still return to the potential well and cause inflation. The Poincaré gauge cosmology provides us with a self-consistent candidate for inflation.

Keywords: inflation, modified gravity

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1 Introduction

The standard model (SM) framework of cosmology based on Einstein’s general relativity (GR) is quite successful in describing the evolution of the Universe on large enough scales [1]. The SM infers that the Universe experienced a significant accelerated expansion in the very early period, which is called the inflation. By introducing the inflation, problems such as the horizon, the flatness, the origin of perturbations, and the monopoles, that have plagued cosmologists before 1980s, can be solved naturally [2]. After years of development, some inflationary models, such as the standard single-field (inflaton) and Starobinsky’s inflation, can match the current observations in very high precision [3, 4]. Unfortunately, those models lack a more essential mechanism for the origin of the inflaton(s), namely the source(s) of inflation. The classical theory of inflation requires the Universe to experience an exponentially accelerated expansion from about $10^{-36}$ s to $10^{-32}$ s in the cosmic chronology [5]. This expansion drove the spatial curvature of the Universe towards extreme flatness, and established the causal correlations on the uniformity of the cosmic microwave background (CMB), and generated the seeds of large-scale structure [6]. At the end of this stage, the expansion decelerated spontaneously and the Universe exited from the adiabatic process. The subsequent reheating led to various particles to be generated [7, 8]. According to the way of exiting the expansion, the inflationary models can be classified into the slow rollover and the first-order phase transition [9]. In addition, an effective correction from the loop quantum cosmology (LQC) enables to push the beginning of the whole process back to the Planck scale, where the big bang singularity have been replaced by the big bounce [10, 11]. The mechanism of inflation from big bounce to reheating is clear phenomenologically.

As a single-field model, Starobinsky’s inflation given by a Lagrangian $\tilde{R} + \frac{\tilde{R}^2}{6M^2}$ plus some small non-local terms (which are crucial for reheating after inflation) is an internally self-consistent cosmological model, which possess a (quasi-)de Sitter stage in the early Universe with slow-roll decay, and a graceful exit to the subsequent radiation-dominated Friedmann-Lemaître-Robertson-Walker (FLRW) stage [12–14]. This is one of the most appealing from both theoretical and observational perspectives among different models of inflation [15].

However, no matter how to modify GR by adding directly higher order curvature invariants or scalar fields to the Einstein-Hilbert (EH) action, GR can’t always reconcile gravity and the microscopic property of matter, i.e. spin angular momentum, in a self-consistent way. GR does not take into account the effect of spin angular momentum on gravity. Not to mention the fusion with the standard model of particle physics built on Yang-Mills gauge framework.
From this perspective, GR is just an effective theory of gravity in macrophysics. In order to incorporate spin into a gravitational theory, Sciama and Kibble developed Einstein-Cartan-Sciama-Kibble (ECSK) theory with respect to the notion of Cartan’s torsion. The structure of ECSK theory can be summarized in figure 1. However, the direct generalization from the EH action will be back to GR, when the spin tensor of matter vanishes because of the algebraic Cartan equation, i.e. torsion cannot propagate. According to quantum mechanics and special relativity theory, all elementary particles can be classified by means of irreducible unitary representations of the Poincare group and can be labeled by mass and spin, ECSK theory can be put into a more general gauge framework of Yang-Mills type with respect to the symmetry of the Poincare group. Mass is connected with the translational part of the Poincare group and spin with the rotational part. This generalization of GR is called Poincare gauge gravity (PGG) or Poincare gauge field theory (PGT). The structure of PGG is summarized in table 1 [16].

In order to obtain the propagating torsion in the vacuum, the action should be also generalized. The standard PGG Lagrangian has a quadratic field strength form [17]:

\[ L_G \sim \Lambda + \text{curvature} + \text{torsion}^2 + \frac{1}{\varrho} \text{curvature}^2, \]  

(1.1)

where \(\Lambda\) is the cosmological constant, and \(\varrho\) the parameter with certain dimension. The additional quadratic terms are naturally at most second derivative if one regards tetrads and spin-connections as the fundamental variables. It is likely that such terms introduce ghost degrees of freedom, when one considers the particle substance of the gravity. That would be something troublesome even for a simple modified gravity theory. The existence of the...
The translation and rotation of infinitesimal generator, gauge potential, gauge field strength, Bianchi identity, material current, conservation law, field momentum, gauge current, field equation, and ECSK type are summarized in Table 1.

| Translation | Rotation |
|-------------|----------|
| $D_a$ | $f_{ab}$ |
| $T_{\mu a}^a$ | $R_{\mu ab}^a$ |
| $\Sigma_{\mu}^a = \delta L_m/\delta e^a_\mu$ | $\tilde{\tau}_{ab}^\mu = \delta L_m/\delta \omega_{\mu ab}$ |
| $H_a^{\mu \nu} = 2\partial L_G/\partial T_{\nu \mu a}$ | $\epsilon_{ab}^\mu = H_{[ba]}^\mu$ |
| $H_{ab}^{\mu \nu} = \epsilon_{ab}^\mu = H_{[ba]}^\mu$ | $H_{ab}^{\mu \nu} = ee_{[a}^{\mu} e^{b\nu]/i^2}$ |

Table 1. The structure of PGG.

Ghost is closely related to the fact that the modified equation of motion has orders of time-derivative higher than two, for example, scale factor $a$ will be fourth-order over time in the general quadratic curvature case in FLRW cosmology. Due to Ostrogradsky’s theorem [18], a system is not (kinematically) stable if it is described by a non-degenerate higher time-derivative Lagrangian. To avoid the ghosts, a bunch of scalar-tensor theories of gravity was introduced, such as the Horndeski theory and beyond [19, 20]. Another way to evade Ostrogradsky’s theorem is to break Lorentz invariance in the ultraviolet and include only high-order spatial derivative terms in the Lagrangian, while still keeping the time derivative terms to the second order. This is exactly what Hořava did recently [21, 22]. In addition, another recipe to treat the ghosts is not removing them from the action, while focusing on the higher-order instability in the equations of motion [23]. For the general second-order Lagrangian with propagating torsion, a systematical way to remove ghosts and tachyons was introduced in [24, 25] using spin projection operators. The gauge fields ($e, \omega$) can be decomposed irreducibly by $su(2)$ group into different spin modes by means of the weak-field approximation. In addition to the graviton, three classes spin-$0^\pm, 1^\pm, 2^\pm$ modes of torsion were introduced. Sezgin et al. [25], Kuhfuss et al. [26] and Lin et al. [27] studied the parity-conserving quadratic Lagrangian (denoted as PGT$^+$) with nine parameters and obtained the conditions on the parameters for not having ghosts and tachyons at the massive and massless sectors, successively. Generally speaking, the PGG at most quadratic in field strengths should include both parity-conserving and parity-violating pieces. The general quadratic Lagrangian in PGG, including parity-violating pieces, has been investigated for its particle content by Blagojevic et al. [28] and Karananas [29]. Most recently, Lin et al. [30] further studied the power-counting renormalizable of PGT$^+$ on the basis of [27]. The Hamiltonian analysis of PGG for different modes can be found in [31, 32], which tell us that the only safe modes of torsion are spin-$0^\pm$, corresponding to the scalar and pseudo-scalar components of torsion, respectively. In the present work, to develop a good cosmology based on PGG, we adopt the nine-parameter parity-conserving Lagrangian with ghost- and tachyons-free conditions on parameters. We will investigate the impact of the parity-violating pieces on the cosmological results in the followup research.
It’s natural to apply the corresponding Poincaré gauge cosmology (PGC) on understanding the evolution of the Universe. The last decade, a series of work [33–43] (from both analytical and numerical approach) proved that it is possible to reproduce the late-time acceleration in PGC without “dark energy”. In ref. [44], the authors discussed the early-time behaviors of the expanding solution of PGC with a scalar field (inflaton), while in ref. [45], a power-law inflation was studied in a $R + R^2$ model of PGC without inflaton. Recently, a series of interesting new developments in PGC have been made in terms of cosmological aspects. Aoki et al. studied the possibility of inflation in multiple spin-parity modes [46], and Barker et al. proposed a one-parameter extension to ΛCDM and found an effective dark radiation component in the early Universe which is expected to address cosmological tensions [47]. In addition, they have established a relationship with Horndeski theory so that dark energy can be produced in PGC [48].

The current work is a continuation of our previous one: Late-time acceleration and inflation in a Poincaré gauge cosmological model [49]. In our previous work, we proposed several fundamental assumptions to define the PGC on FLRW level. Then we studied the general nine-parameter PGC Lagrangian with ghost- and tachyon-free constraints on parameters. With specific choice of parameters, we obtained two Friedmann-like analytical solutions by varying the Lagrangian, where the scalar torsion $h$-determined solution is consistent with the Starobinsky cosmology in the early time and the $f$-determined solution contains naturally a constant geometric “dark energy” density, which cover the ΛCDM model in the late-time. We further constrained the magnitudes of parameters using the latest observations. However, we left a problem unsolved that two solutions are mutually exclusive even they are derived from a same Lagrangian. The reason comes probably from that the restraint on $B_1$ (vanishing) is too strong, so that the high-order terms of $f$ are removed. Therefore, in current work, we will investigate the general case at least ghost-free, and use the new results for leading to the slow-roll inflation without any extra fields. This series of work aims to build self-consistent cosmology to solve the problem of SM in describing the evolution of the Universe, where “self-consistent” means without extra hypothesis of inflaton and dark energy.

This paper is organized as follows. In section 2, we start from the nine-parameter Lagrangian with the ghost- and tachyon-free conditions on parameters. Then replacing the scalar and pseudo-scalar torsion by two new variables, we rewrite the cosmological equations obtained in [49] into new forms. In section 3, we study numerically the slow-roll inflation of this system. We conclude and discuss our work in section 4.

2 Cosmological equations

We consider the nine-parameter PGT$^+$ gravitational Lagrangian $L_G$, which reads:

$$I = \int d^4x \sqrt{|g|} \left[ \frac{1}{2\kappa} L_G + L_M \right],$$

$$L_G = \alpha R + L_T + L_R,$$

$$L_T \equiv a_1 T_{\mu\nu\rho} T^{\mu\nu\rho} + a_2 T_{\mu\nu\rho} T^{\nu\rho\mu} + a_3 T^\mu T^\mu,$$

$$L_R \equiv b_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + b_2 R_{\mu\nu\rho\sigma} R^{\rho\sigma\mu\nu} + b_3 R_{\mu\nu} R^{\mu\nu} + b_4 R_{\mu\nu} R^{\nu\mu} + b_5 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma},$$

(2.1)

where $\kappa \equiv 8\pi G = 8\pi m_{pl}^{-2}$ with $m_{pl}$ the Planck mass, and $\alpha, a_1 \sim a_3$ are freely dimensionless Lagrangian parameters, while $b_1 \sim b_5$ are free Lagrangian parameters with dimension $m_{pl}^{-2}$. 

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Table 2. The ghost- and tachyon-free conditions on parameters for six spin modes, respectively.

| spin modes | conditions on parameters |
|------------|--------------------------|
| $2^-$      | $4b_1 + b_5 < 0$          |
|            | $\alpha + 2a_1 + a_2 < 0$|
| $1^-$      | $(\alpha + 2a_1 + a_2)(2a_1 + a_2 + a_3)(-2\alpha + 2a_1 + a_2 + 3a_3) < 0$|
| $0^-$      | $-2b_1 + b_5 > 0$         |
|            | $\alpha - 4a_1 + 4a_2 > 0$|
| $2^+$      | $4b_1 + 4b_2 + b_3 + b_4 + 2b_5 > 0$ |
|            | $\alpha(2a_1 + a_2)(\alpha + 2a_1 + a_2) < 0$ |
| $1^+$      | $-4b_1 + 4b_2 - b_3 + b_4 < 0$ |
|            | $(2a_1 - a_2)(-\alpha + 4a_1 - 4a_2)(\alpha + 2a_1 + a_2) > 0$ |
| $0^+$      | $b_1 + b_2 + b_3 + b_4 + b_5/2 > 0$ |
|            | $\alpha(2a_1 + a_2 + 3a_3)(-2\alpha + 2a_1 + a_2 + 3a_3) > 0$ |

The $R^2$ term need not be included due to the use of the Chern-Gauss-Bonnet theorem [50]:

$$\int d^4x \sqrt{|g|}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2) = 0,$$

for spacetime topologically equivalent to flat space.

For the pair of gauge field $(e, \omega)$ as the dynamical variables, the field equations are up to 2nd-order. However, the gauge fields $(e, \omega)$ can be decomposed irreducibly by $su(2)$ group into different spin modes by means of the weak-field approximation. In addition to the graviton, three classes spin-$0^\pm, 1^\pm, 2^\pm$ modes of torsion were introduced. It is obvious that in such a general quadratic, the ghosts and tachyons are inevitable for certain modes. Fortunately, we can adopt the systematic method of spin projection operators used in [25–27, 29] to extract all the spin-parity modes and to remove the ghosts and tachyons. After a tedious calculation, we get the ghost- and tachyon-free conditions on parameters for our PGT$^+$ Lagrangian (2.1), and summerize in table 2.$^1$ For the massless sector, the ghost-free condition is just: $\alpha > 0$.

We will still focus on the FLRW cosmology, where the spatial curvature free metric and non-vanishing components of torsion are, respectively

$$ds^2 = -dt^2 + a^2 dx^2,$$

$$T_{ij0} = a^2 h \delta_{ij}, \quad T_{ijk} = a^2 f \epsilon_{ijk}, \quad i,j,k = 1, 2, 3. \quad (2.4)$$

Where $a$, $h$, $f$ are scale factor, scalar torsion, and pseudo-scalar torsion respectively, and are functions of cosmic time $t$. We assume that the spin effect of matter doesn’t appear on the

$^1$Where we used the Minkowski metric as $\eta = \text{diag.}(-, +, +, +)$. We compared our table 2 and the results in [25, 26], and found that they are consistent after considering the difference of signature for the Minkowski metric and the coefficient conversion. It is worth mentioning that there are subtle errors in the non-diagonal elements of the coefficient matrix in the spin-$1^+$ sector. But according to the calculation procedure of the saturated propagator, the errors do not affect the final results.
cosmological scales in PGC. Therefore, the spin tensor, defined as the variational derivative of the matter Lagrangian with respect to the contorsion $K$:

$$S^\rho_{\mu\nu} := \frac{1}{\sqrt{|g|}} \frac{\delta \left( \sqrt{|g|} L_m \right)}{\delta K^\rho_{\mu\nu}}, \quad (2.5)$$

vanishes. According to the algebraic relation between contorsion and torsion:

$$K^\rho_{\mu\nu} = \frac{1}{2} (T^\rho_{\mu\nu} + T^\rho_{\nu\mu} + T^\mu_{\nu\rho}), \quad (2.6)$$

the variational derivative of the matter Lagrangian with respect to the torsion $T$ (which is associated with the notion “spin energy potential”, see [51]) vanishes either. Nonetheless, in our work, the energy-momentum tensor defined as

$$T^\rho_{\mu\nu} := \frac{1}{\sqrt{|g|}} \frac{\delta \left( \sqrt{|g|} L_m \right)}{\delta g^\rho_{\mu\nu}}, \quad (2.7)$$

contains radiation, baryonic and dark matter. The general cosmological equations corresponding to the action (2.1), on the background can be found in the appendix A, which are cumbersome and the physical meaning lost. While, we noticed that the equations (A.3) and (A.4) can be regarded as the dynamic evolutions of scalar torsion $h$ and pseudo-scalar torsion $f$, and they are second-order of $h$ and $f$, respectively. We would like to recast them into the Klein-Gordon-like form by introducing the following new variables:

$$h \mapsto \phi_h = \alpha (h - H), \quad f \mapsto \phi_f = \alpha f, \quad (2.8)$$

then, the cosmological equations (A.1)–(A.4) can be rewritten as:

$$H^2 = \frac{2}{A_1} \left( \frac{2}{3} \kappa \rho + \frac{1}{3} \kappa \rho_\phi \right), \quad (2.9)$$

$$2 \dot{H} + 3 H^2 = - \frac{2}{A_1} (\kappa \rho + \kappa \rho_\phi), \quad (2.10)$$

$$\dot{\rho} = -3 H (\rho + p), \quad (2.11)$$

$$\dot{\rho}_\phi = -3 H (\rho_\phi + p_\phi), \quad (2.12)$$

$$\kappa \rho_h = \frac{1}{2} \alpha^2 \phi_h^2 - \frac{1}{2} \alpha^2 \phi_f^2 + V(\phi_h, \phi_f), \quad (2.13)$$

$$\kappa \rho_\phi = \frac{1}{2} \alpha^2 \phi_h^2 - 2(\frac{A_1 - 2\alpha}{a^2} \phi_h^2 - \frac{2}{a^2} \phi_f^2 - 4 A_0 - \alpha) \phi_f^2, \quad (2.14)$$

$$\frac{12B_0}{a} \dot{\phi}_h + 12B_0 \dot{H} \phi_h + 12(2B_0 - 2B_1 - B_2) \phi_f + a \phi_h \phi_f + \frac{\partial V(\phi_h, \phi_f)}{\partial \phi_h} + 3(\alpha - 2\alpha) H = 0, \quad (2.15)$$

$$- \frac{12B_1}{a} \dot{\phi}_f - 12B_1 \dot{H} \phi_f - 12(2B_0 - 2B_1 - B_2) \phi_f + a \phi_h \phi_f + \frac{\partial V(\phi_h, \phi_f)}{\partial \phi_f} = 0, \quad (2.16)$$

$$V(\phi_h, \phi_f) = 3 \left( \frac{A_1 - 2\alpha}{a^2} \phi_h^2 - 3(4A_0 - \alpha) \phi_f^2 - 6B_0 a^2 \phi_h^2 - 2B_0 + 2B_1 - B_0 \phi_f^2 \phi_h^2 + \phi_h^4 \right), \quad (2.17)$$

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where the combinations of parameters are:

\[ A_0 \equiv a_1 - a_2, \quad A_1 \equiv 2a_1 + a_2 + 3a_3, \]
\[ B_0 \equiv b_1 + b_2 + b_3 + b_4 + \frac{1}{2}b_5, \]
\[ B_1 \equiv b_1 - \frac{1}{2}b_5, \quad B_2 \equiv 4b_2 + b_3 + b_4 + b_5. \] (2.18)

The degeneracy among these Lagrangian parameters on background makes the inequalities can not be solved completely. However, it is obvious that ghost- and tachyon-free spin-0\(^\pm\) “particles” require:

\[ B_0 > 0, \quad B_1 < 0, \quad \alpha - 4A_0 > 0, \quad \alpha A_1 (A_1 - \alpha) > 0. \] (2.19)

Now, the physical picture is quite clear, that the nine-parameter PGC system is equivalent to a gravitational system coupled two-scalar fields \((\phi_h, \phi_f)\), with a potential up to quartic-order, \(V(\phi_h, \phi_f)\). (2.15) and (2.16) are the equations of motion for \(\phi_h\) and \(\phi_f\), respectively. They look very symmetrical except the last term in (2.15). If \(B_0, B_1\) and \((2B_0 - 2B_1 - B_2)\) don’t vanish, \(|(2B_0 - 2B_1 - B_2)\phi_f/a|\) represents the strength of interaction between two scalar fields. We conclude that \(B_0\) and \(B_1\) must have the opposite sign so that the interaction terms in the equations of motion have opposite sign too. The different between \(A_1\) and \(\alpha\) measures the weight of \(\phi^2_h\) in the potential \(V(\phi_h, \phi_f)\), and analogously, \(A_0\) and \(\alpha\) for \(\phi^2_f\). It will be convenient to overlook the \(1/a\) factor in front of field \(\phi_h\) or \(\phi_f\) because of the inverse factor occurred in (2.8). The ghost- and tachyon-free conditions for spin-0\(^\pm\) ensure that the kinetic energy terms are positive in (2.13), as well as a potential well can be formed from (2.17) when one require \(\alpha > 0\) and \(A_1 > 0\).

The above system is general because we didn’t set any additional assumptions on the parameters yet except the ghost- and tachyon-free conditions (2.19). In the rest of this work, we will focus on the inflationary period of this system, thus the energy densities \(\rho\) and pressures \(p\) of the matters (with equation of state parameters, i.e. EOS: \(w = 0, 1/3\)) will be neglected.

### 3 Slow-roll inflation and numerical analysis

According to our presupposition that \(B_0 > 0\), if the coefficients of the quadratic terms in potential (2.17) are positive with the same magnitude, and \(|B_1|\) has the same magnitude with \(B_0\), we can get a potential well with an effective radius \(r_\phi \lesssim \sqrt{\frac{A_1 - 2\alpha}{4B_0}}\). By choosing special values of parameters, the slow-roll inflation can be obtained in this potential well. In this scenario, the numerical analysis is more clear and convincing than the theoretical analysis.

The dimensions for parameters and quantities read:

\[ \alpha \sim A_0 \sim A_1 \sim 1, \]
\[ B_0 \sim B_1 \sim B_2 \sim m_{Pl}^2, \]
\[ t \sim m_{Pl}^{-1}, \]
\[ H \sim \phi_h \sim \phi_f \sim m_{Pl}, \]
\[ \dot{\phi}_h \sim \dot{\phi}_f \sim m_{Pl}^2. \] (3.1)
In the unit of $m_{Pl} = 1$, and by considering that $a$ can be rescaled, we set the initial data (labeled with “B”) as:

$$
t_B = 0, \quad a_B = 1,
\phi_h(t_B) = \phi_f(t_B) = 0,
\dot{\phi}_h(t_B) = \dot{\phi}_f(t_B) = 1.
$$

(3.2)

To investigate the effect of each parameter on this system, we choose a set of fiducial values:

$$
\beta \equiv 1 - \alpha = 0.99, \quad A_0 = -0.245,
B_0 = 1, \quad B_1 = -1,
B_i = 2B_0 - 2B_1 - B_2 = 0,
$$

(3.3)

then vary a parameter and plot while keeping other parameters to maintain the fiducial value. Figures 2, 4, 6, 8, 10 are the evolution curves of Hubble rate $H$ for every choice of parameter, while figures 3, 5, 7, 9, 11 for the EOS of scalar fields $w_\phi$, where $w_\phi \equiv p_\phi/\rho_\phi$.

These evolution curves show that the system starts from a “pre-inflation” stage, then enters into the slow-roll inflation, meanwhile the EOS changes from positive to negative. After a nearly constant stage, the system decay rapidly, which we call it “pre-reheating”. Then the subsequent oscillation indicates that the system has entered a stage of reheating. To match the current observations, the stage of inflation should last long enough, which can be quantified by e-folds $N_{inf}$:

$$
N_{inf} := \ln \frac{a_f}{a_i},
$$

(3.4)

where $a_i$ is the scale factor at the moment $t_i$ of the inflationary onset, which is defined by the time when the Universe begins to accelerate $\ddot{a}(t_i) = 0$, i.e. $\ddot{a}(t)$ first changes its sign right after the bouncing phase [52]. The end of the inflation is defined by the time $t_f$ when the accelerating expansion of the Universe stops, i.e. $w_\phi(t_f) = -1/3$. The current observations require that $N_{inf} > 60$. 

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**Figure 2.** Evolution of Hubble rate $H$ over time. The large dashed (blue), the solid (orange) and the small dashed (red) lines correspond to $\beta = 0.9, 0.99, 0.999$, respectively. To compare with the fiducial value, the smaller $\beta$ makes the decay advanced. The e-folds for three scenarios read $N_{inf} = 18.2, 160.8, 331$. 

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Figure 3. Evolution of EOS $w_\phi$ over time. The large dashed (blue), the solid (orange) and the small dashed (red) lines correspond to $\beta = 0.9, 0.99, 0.999$, respectively. $w_\phi < -1/3$ during the whole inflationary stage, and $w_\phi$ is approximately equal to $-1$ in the deep inflationary stage. The inflation ends when $w_\phi = -1/3$ again, then reheating starts. The e-folds for three scenarios read $N_{\text{inf}} = 18.2, 160.8, 33.1$.

Figure 4. Evolution of Hubble rate $H$ over time. The large dashed (blue), the solid (orange) and the small dashed (red) lines correspond to $A_0 = -0.645, -0.245, -1.045$, respectively. We can see that this parameter has little effect on the Hubble rate. The e-folds for three scenarios read $N_{\text{inf}} = 151.7, 160.8, 151.1$.

According to the numerical analysis, we summarize the influences of every parameter comparing with the fiducial value as following: 1) smaller $\beta$ can lead to the decay advanced; 2) $A_0$ doesn’t influence the decay but causes oscillation before inflation; 3) smaller $B_0$ leads to the curve overall left shift; 4) smaller $|B_1|$ makes $w_\phi$ before inflation; 5) the interaction between two-scalar fields makes the decay advanced and leads to oscillation on $w_\phi$ before inflation.

The 3D phase diagram figure 12 visualizes the evolutions of two-scalar fields $\phi_h$ and $\phi_f$ (over a) on the potential well in the fiducial scenario. The system starts from the on set point where we set the initial data (3.2), where the non-trivial values are the kinetic energies of two fields. Then the system is driven by the kinetic energies and climbs to the high level of the potential well, and prepares for the slow-roll inflation. The slow-roll inflation occurs
Figure 5. Evolution of EOS $w_φ$ over time. The large dashed (blue), the solid (orange) and the small dashed (red) lines correspond to $A_0 = -0.645, -0.245, -1.045$, respectively. $w_φ < -1/3$ during the whole inflationary stage, and $w_φ$ is approximately equal to $-1$ in the deep inflationary stage. The inflation ends when $w_φ = -1/3$ again, then reheating starts. The smaller value of $A_0$ don’t influence the decay and reheating but leads to oscillation before inflation. The e-folds for three scenarios read $N_{\text{inf}} = 151.7, 160.8, 151.1$.

Figure 6. Evolution of Hubble rate $H$ over time. The large dashed (blue), the solid (orange) and the small dashed (red) lines correspond to $B_0 = 0.5, 1.0, 2.0$, respectively. To compare with the fiducial value, the smaller $B_0$ makes the decay advanced. The e-folds for three scenarios read $N_{\text{inf}} = 139.2, 160.8, 82.2$.

on the wall of the potential well, where the deep inflation happened after the inflection point, where $φ_f$ is approximate to 0, and $φ_h$ is almost constant. At last, the system decays and drops down from the wall of the potential well, towards the on set point of the phase space, then leads to the reheating.

It seems that if the initial kinetic energies are large enough, the system may cross the highest point of the potential well, causing it to collapse. To test the stableness of this system, we keep the fiducial values of parameters (3.3) unchanged, but increase the initial kinetic energies (speeds) of two-scalar fields. Figure 13 is the 3D phase diagram of this case, where we set two pairs of very large initial kinetic energies (speeds): $\dot{φ}_h(t_B) = 1000, \dot{φ}_f(t_B) = 500$ and $\dot{φ}_h(t_B) = 1000, \dot{φ}_f(t_B) = -500$. Both trajectories can cross the highest
$B_0 = 0.5$, $B_0 = 1$, $B_0 = 2$.

Figure 7. Evolution of EOS $w_\phi$ over time. The large dashed (blue), the solid (orange) and the small dashed (red) lines correspond to $B_0 = 0.5, 1.0, 2.0$, respectively. $w_\phi < -1/3$ during the whole inflationary stage, and $w_\phi$ is approximately equal to $-1$ in the deep inflationary stage. The inflation ends when $w_\phi = -1/3$ again, then reheating starts. To compare with the fiducial value, the smaller $B_0$ makes the curve overall left shift, while right shift for larger $B_0$. The e-folds for three scenarios read $N_{inf} = 139.2, 160.8, 82.2$.

Figure 8. Evolution of Hubble rate $H$ over time. The large dashed (blue), the solid (orange) and the small dashed (red) lines correspond to $B_1 = -0.1, -1.0, -1.5$, respectively. To compare with the fiducial value, the larger absolute value of $B_1$ makes the decay advanced. The e-folds for three scenarios read $N_{inf} = 157.2, 160.8, 55.6$.

point of the potential well, but can return to the potential well and cause inflation anyway. Numerical analysis shows that the system has good stability.

4 Conclusion and discussion

PGG as a gauge field gravitational theory is a natural extension of Einstein’s GR to the Poincaré group. It is worth looking forward using PGC, the cosmology of PGG, to solve the problems in the cosmological SM, especially the mechanisms of inflation and late-time acceleration. In this work, we started from the nine-parameter parity-conserving gravitational Lagrangian of PGC, and introduced the ghost- and tachyon-free conditions for this
Figure 9. Evolution of EOS $w_\phi$ over time. The large dashed (blue), the solid (orange) and the small dashed (red) lines correspond to $B_1 = -0.1, -1.0, -1.5$, respectively. $w_\phi < -1/3$ during the whole inflationary stage, and $w_\phi$ is approximately equal to $-1$ in the deep inflationary stage. The inflation ends when $w_\phi = -1/3$ again, then reheating starts. The larger absolute value of $B_1$ makes the decay advanced, while the smaller absolute value of $B_1$ makes $w_\phi$ sinking before inflation. The e-folds for three scenarios read $N_{\text{inf}} = 157.2, 160.8, 55.6$.

Figure 10. Evolution of Hubble rate $H$ over time. The large dashed (blue), the solid (orange) and the small dashed (red) lines correspond to $B_i = -50, 0, 50$, respectively. To compare with the fiducial value, any interaction between two-scalar fields makes the decay advanced. The e-folds for three scenarios read $N_{\text{inf}} = 12.6, 160.8, 105.6$.

Lagrangian. By introducing new variables $\{\phi_h, \phi_f\}$ for replacing the scalar and pseudo-scalar torsion $\{h, f\}$, we found the general PGC on background is equivalent to a gravitational system coupled to two-scalar fields with a potential up to quartic-order. Then by choosing appropriate parameters, we constructed a potential well from the quartic-order potential, and studied the slow-roll inflation numerically. We chose a set of fiducial values for parameters, and investigated the effects of each parameter on this system. All the evolution curves show that this system experiences four different stages: “pre-inflation” (on set), slow-roll inflation, “pre-reheating” (decay) and reheating. Most scenarios possess large enough e-folds which is required by the current theories and observations. The 3D phase diagram of two-scalar fields shows clearly four stages of the evolution in the potential well. At last, we studied the
Figure 11. Evolution of EOS $w_\phi$ over time. The large dashed (blue), the solid (orange) and the small dashed (red) lines correspond to $B_i = -50, 0, 50$, respectively. $w_\phi < -1/3$ during the whole inflationary stage, and $w_\phi$ is approximately equal to $-1$ in the deep inflationary stage. The inflation ends when $w_\phi = -1/3$ again, then reheating starts. The interaction between two-scalar fields leads to oscillation on $w_\phi$ before inflation. The e-folds for three scenarios read $N_{inf} = 12.6, 160.8, 105.6$.

Figure 12. The 3D phase diagram of two-scalar fields $\phi_h$ and $\phi_f$ (over $\sigma$) evolve in the potential well in the fiducial scenario. The shape of potential well is determined by the values of parameters given by (3.3). The initial data is given by (3.2) where the initial kinetic energies (speeds) read $\dot{\phi}_h(t_B) = 1$, $\dot{\phi}_f(t_B) = 1$.

stablence of this system by setting large values of initial kinetic energies (speeds). We found that even if the system evolves past the highest point of the potential well, the scalar fields can still return to the potential well and cause inflation. In short, the numerical analysis for this general PGC system on background indicated that it is a good self-consistent candidate for the slow-roll inflation. Further studies on the aspect of perturbation will be our next work, especially the primordial power spectrum from this system and it’s effects on CMB. It is also worth looking forward to unify the inflation and the late-time acceleration under PGC in the future.
Pre-inflation

Inflation

Pre-reheating

Figure 13. The 3D phase diagram of two-scalar fields $\phi_h$ and $\phi_f$ (over $a$) evolve in the potential well in the fiducial scenario (3.3), but start with very large initial kinetic energies (speeds): $\dot{\phi}_h(t_B) = 1000$, $\dot{\phi}_f(t_B) = 500$ corresponding to the thick trajectory and $\dot{\phi}_h(t_B) = 1000$, $\dot{\phi}_f(t_B) = -500$ to the thin one. The potential well looks shallower than the previous one in figure 12, not because we changed the fiducial parameters, but instead expanded the ranges of $\phi_h$ and $\phi_f$(over $a$).

### A The original cosmological equations

The ghost- and tachyon-free conditions in table 2 are derived by means of the fundamental variables $(e, \omega)$, but it’s general in any case. Equivalently, it is convenient for us to treat $(g, T)$ as the fundamental variables to derive the field equations. Varying the action (2.1) with respect to $g_{\mu\nu}$ and $T^p_{\mu\nu}$, respectively, as well as considering (2.5) and (2.7), one can get the modified Einstein and the modified Cartan field equations. We do the calculations with the help of xAct: efficient tensor computer algebra for the Wolfram Language. We integrate our calculations in a Wolfram package PGC: symbolic computing package for Poincare Gauge Cosmology, which is available on Github. Since the field equations are too paper-consuming, we don’t intend to copy them here, but feel free to download and install our package if you want to check the field equations and their components on FLRW background. The README file will indicate you how to use it. Here, we just sort out the cosmological equations on FLRW background which read:

$$
\frac{1}{3} \kappa \rho = \alpha (H^2 - h^2 - f^2) + 4 A_0 f^2 + \frac{1}{2} A_1 h^2
$$

$$
+ 2 B_0 [2 (H - h)(\dot{H} - \dot{h}) - (H - h)^2 + 4 H (\dot{H} - \dot{h})(H - h) - 4 f \dot{f} (H - h)]
$$

$$
+ 2 \dot{H} (H - h)^2 + [f^2 - 3 h (H - h)] [f^2 - (H - h)(2 H - h)]
$$

$$
+ 2 B_1 [f^2 (3 H - 2 h)^2 + 2 f \dot{f} (3 H - 2 h) + \dot{f}^2] + 4 B_2 f (f H + \dot{f})(H - h),
$$

(A.1)

$$
- \kappa p = \alpha (2 \dot{H} + 3 H^2 + 3 h^2 - f^2) + 4 A_0 f^2 - \frac{3}{2} A_1 h^2
$$

$$
+ 2 B_0 [2 (H^{(3)} - h^{(3)}) + 12 H (\dot{H} - \dot{h}) + 2 (H - h) \dot{h} - 4 f \dot{f} + 9 H^2 - 10 \dot{H} h]
$$

$$
+ 18 H^2 \dot{H} + 18 h H \dot{H} - 14 h^2 \dot{H} + h^2 - 6 H^2 \dot{h} - 28 h H \dot{h} + 12 h^2 \dot{h} - 4 f^2 \dot{h} - 4 \dot{f}^2
$$

---

[2] Authors: José M. Martín-García et al. Homepage: http://www.xact.es.

[3] PGC version 1.2.1: https://github.com/zhanghc0537/Poincare-Gauge-Cosmology.
\[
-8fh\dot{f} - 12fh\dot{h} + (f^2 + 3h^2 + 9hH)(-f^2 + h^2 - 3hH + 2H^2) \\
+2B_1(Af\ddot{f} + 12f^2H - 8f^2h + 3f^2 + 30fH\dot{f} - 12fh\dot{f} + f^2(27H^2 - 12hH - 4h^2)) \\
+4B_2[f\dddot{f} + f^2H + \dot{f}^2 + 4fh\dot{f} + fh\ddot{f} + 2f^2H^2 + f^2hH],
\]

(A.2)
\[
0 = \alpha h - \frac{1}{2}A_1h \\
+2B_0[(\dot{H} - \dot{h} + 4H\dot{H} - h\dot{H} - 3H\dot{h} - 2f\dot{f} + 4hH^2 - 6h^2H + 2h^3 - 2f^2h)] \\
+4B_1f(\dot{f} + 3fH - 2fh) + 2B_2f(\dot{f} + fH),
\]

(A.3)
\[
0 = f\left\{\alpha - 4A_0 + 4B_0[\dot{H} - \dot{h} + 2H^2 - 3hH + h^2 - f^2] \\
-2B_1[(3H - 2\dot{h}) + 2h(3H - 2\dot{h})] - 2B_2[(\dot{H} - \dot{h}) + H(H - h)] \right\} \\
-2B_1(3H\dot{f} + \dot{f}),
\]

(A.4)

with the combinations of parameters (2.18). (A.1) is the time-component of the modified Einstein field equation, and (A.2) is the trace of its space-component. While (A.3), (A.4) are the non-vanishing components of the modified Cartan field equation corresponding to the evolution of $h$ and $f$, respectively. By introducing the new variables (2.8) the above cosmological equations (A.1)∼(A.4) can be rewritten into the elegant two-scalar fields form.

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