Progress in Chiral Perturbation Theory

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Abstract. After a short status report on chiral perturbation theory, I review recent progress in determining some of the low-energy couplings by matching the effective theory to QCD. Consequences for $K_{l3}$ decays and for the extraction of the CKM matrix element $V_{us}$ are reported. Hadronic vacuum polarization at low energies and its impact on the anomalous magnetic moment of the muon are discussed.

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INTRODUCTION

At a time when the LHC is getting ready to open a new era in particle physics, it is legitimate to ask why one should still be interested in QCD (more generally in the Standard Model) at low energies. There are at least two good reasons to pursue the study of QCD in the confinement regime:

• It is a challenge for theoretical particle physics to derive reliable results in the nonperturbative domain. An impressive example is pion-pion scattering, one of the few examples in hadron phenomenology at low energies where theory is ahead of experiment [1]. Important information on the mechanism of spontaneous chiral symmetry breaking can be extracted from pion-pion scattering, especially from S-wave scattering lengths. The study of QCD in the nonperturbative regime may turn out to be relevant even for LHC physics if the simple Higgs mechanism of the Standard Model turns out to be insufficient to describe electroweak symmetry breaking.

• The assessment of physics beyond the Standard Model will remain an important research topic even at much lower than LHC energies. The reduction in energy must be compensated by an increase in precision, both in experiment and in theory. In the long run, lattice gauge theories and low-energy effective field theories will survive as the most comprehensive and reliable approaches in this field.

In this talk, I present a short progress report on chiral perturbation theory (CHPT), which is precisely the effective field theory of the Standard Model at low energies. Green functions and amplitudes are dominated at low energies by the exchange of pseudoscalar mesons, the pseudo-Goldstone bosons of spontaneously broken chiral symmetry, allowing for a systematic expansion in momenta and quark masses. I discuss recent progress in determining some of the a priori unknown coupling constants of CHPT, a recent CHPT analysis of $K_{l3}$ decays to extract the CKM matrix element $V_{us}$ and, finally, very recent developments concerning the determination of hadronic
vacuum polarization, a topic of great importance for comparing the Standard Model prediction of the muon magnetic moment with experiment. The extension of CHPT to the intermediate-energy region dominated by meson resonances is covered by J. Portolés [2].

**STATUS OF CHIRAL PERTURBATION THEORY**

The spontaneously and explicitly broken chiral symmetry of QCD is the key feature of CHPT. The corresponding Lagrangian is organized in an expansion in derivatives (vestige of spontaneous symmetry breaking) and in quark masses (explicit breaking). CHPT is a nonrenormalizable quantum field theory that must nevertheless be renormalized like any respectable quantum field theory. The main difference to renormalizable theories is the rapidly increasing number of low-energy constants (LECs) in higher orders of CHPT. As a low-energy effective field theory, CHPT can be applied to processes with momenta $\ll 1$ GeV.

In the mesonic sector, the original effective chiral Lagrangian of next-to-leading order [3, 4] has been extended to next-to-next-to-leading order [5] or $O(p^6)$ in the standard chiral counting. At this order, diagrams with up to two loops have to be taken into account for a consistent low-energy expansion (see Ref. [6] for a recent review).

Still in the meson sector, the formalism of CHPT has been extended to incorporate the nonleptonic weak interactions and to implement radiative corrections for strong processes as well as for semileptonic and nonleptonic weak decays. The corresponding Lagrangians and the associated number of LECs are displayed in Table 1. As the Table indicates, the state of the art for these extensions is next-to-leading order with at most one-loop amplitudes.

| $\mathcal{L}_{\text{chiral order}}$ | ( # of LECs) | loop order |
|-------------------------------------|--------------|------------|
| $\mathcal{L}_{p^2}(2) + \mathcal{L}_{G_\rho}^{A=1}(2)$ + $\mathcal{L}_{e^2\rho^0}^{\text{em}}(1)$ + $\mathcal{L}_{G_\delta e^2p^0}^{\text{emweak}}(1)$ | $L = 0$ |
| $+ \mathcal{L}_{p^4}(10) + \mathcal{L}_{p^6}^{\text{odd}}(32)$ + $\mathcal{L}_{G_\delta p^4}^{A=1}(22)$ + $\mathcal{L}_{G_\delta p^4}^{A=1}(28)$ + $\mathcal{L}_{e^2p^2}^{\text{em}}(14)$ + $\mathcal{L}_{G_\delta e^2p^2}^{\text{emweak}}(14)$ + $\mathcal{L}_{e^2p}^{\text{leptons}}(5)$ | $L = 1$ |
| $+ \mathcal{L}_{p^6}(90)$ | $L = 2$ |

Effective chiral Lagrangians have also been employed for baryonic processes [7] and for light nuclei [8].
LOW-ENERGY CONSTANTS

As Table 1 shows, a major problem of CHPT is the abundance of LECs in higher orders of the chiral expansion. For a phenomenological determination of those constants, two types of LECs can be distinguished.

i. The associated contributions survive in the chiral limit. Such LECs govern the momentum dependence of amplitudes and are at least in principle accessible experimentally.

ii. The couplings are associated with explicit chiral symmetry breaking. Such LECs specify the quark mass dependence of amplitudes. They are difficult if not impossible to extract from experiment but they are accessible in lattice QCD.

However, at the present level of sophistication it is unrealistic to expect a phenomenological determination of all LECs even of type i only. Instead, some progress has been made recently in matching CHPT to QCD by investigating specific Green functions in the limit of large $N_C$. As in every effective field theory, the LECs are sensitive to the “heavy” degrees of freedom not represented by explicit fields in the Lagrangian. Experience shows that truncation of the infinitely many intermediate states (for $N_C \to \infty$) to the lowest-lying resonances is usually sufficient.

Instead of reviewing the matching procedure in general, I discuss two specific examples recently considered that have some impact on topics of current interest.

Radiative semileptonic decays

In the discussion of radiative corrections for semileptonic kaon decays the Lagrangian $\mathcal{L}^{\text{leptons}}_{e^2p}$ entered. In a two-step procedure, the Fermi theory of semileptonic decays was matched to both the Standard Model and CHPT resulting in spectral representations for all five LECs in $\mathcal{L}^{\text{leptons}}_{e^2p}$.

Let me concentrate here on one of those LECs ($X_1$) that will be relevant later on. The authors of Ref. [10] obtain the following representation for $X_1$,

\[ X_1 = \frac{3i}{8} \int \frac{d^4k}{(2\pi)^4} \left( \Gamma_{VV}(k^2) - \Gamma_{AA}(k^2) \right) / k^2, \]  

in terms of vertex functions ($V^a_\mu$ is an $SU(3)$ vector current and $\phi^c$ is a member of the pseudoscalar octet)

\[ \Gamma_{VV}(k^2) \sim \lim_{p \to 0} \int d^4x e^{ikx} \langle 0 | T V^a_\mu(x) V^b_\nu(0) | \phi^c(p) \rangle \]  

and similarly for $\Gamma_{AA}(k^2)$. The integral converges well and, when saturated with the lowest-lying $V,A$ meson resonances, produces a value $X_1 = -0.0037$ [10] to be used for the analysis of $K_{l3}$ decays.
**Strong LECs of O(p^6)**

The second example concerns LECs that appear in the $K_{l3}$ amplitudes at $O(p^6)$. The Green function of interest is the three-point function of scalar and pseudoscalar densities:

$$i^2 \int dx dy e^{i p x + i q y + i r z} \langle 0 | T S^a(x) P^b(y) P^c(z) | 0 \rangle = a^{abc} \Pi_{SPP}(p^2, q^2, r^2).$$  \hspace{1cm} (3)

At low energies, $\Pi_{SPP}$ is given in terms of LECs of $O(p^4)$ and $O(p^6)$ since loop contributions are subdominant for large $N_C$. At high momenta, the operator product expansion (OPE) fixes the behaviour of $\Pi_{SPP}$ that vanishes in QCD perturbation theory as an order parameter of spontaneous chiral symmetry breaking. Additional constraints apply for (transition) form factors at large momentum transfer, with two external momenta on shell.

To interpolate between CHPT and QCD, a large-$N_C$ motivated ansatz can be employed [11]:

$$\Pi_{SPP}^{\mathcal{P}}(s, t, u) = \frac{P_0 + P_1 + P_2 + P_3 + P_4}{[M_S^2 - s][-t][M_P^2 - t][M_P^2 - u]},$$  \hspace{1cm} (4)

with polynomials $P_n$ of degree $n$ in $s, t, u$ (altogether 21 parameters). The OPE limits $n \leq 4$ and lowest-order CHPT fixes the constant $P_0$. The high-energy conditions constrain the polynomials $P_1, P_2$ of direct relevance for the LECs. The final relations for the $O(p^6)$ LECs of interest are [11]

$$C_{12}^{\mathcal{P}} = -\frac{F^2}{8M_S^2}, \quad C_{34}^{\mathcal{P}} = \frac{3F^2}{16M_S^4} + \frac{d_m^2}{2} \left(\frac{1}{M_S^2} - \frac{1}{M_P^2}\right)^2$$  \hspace{1cm} (5)

in terms of the masses $M_S, M_P$ of the lowest-lying (pseudo-)scalar nonets, the pion decay constant $F$ and a resonance coupling $d_m \sim F/(2\sqrt{2})$. All parameters refer to the chiral limit.

The first interpretation of these results is not too encouraging. There are big uncertainties related to the value of $M_S$ in particular and to the rather strong scale dependence of $C_{12}$ and $C_{34}$, which is however inaccessible at leading order in $1/N_C$.

**K_{l3} AND V_{us}**

The analysis of $K_{l3}$ decays allows for the presently most accurate determination of the CKM matrix element $V_{us}$. In general, two form factors characterize the decay matrix element:

$$\langle \pi^- (p_\pi) | \bar{s} \gamma_{\mu} u | K^0 (p_K) \rangle = f_+^{K^0\pi^-} (t) (p_K + p_\pi)_{\mu} + f_+^{K^0\pi^-} (t) (p_K - p_\pi)_{\mu}.$$  \hspace{1cm} (6)

Of special interest for the determination of $V_{us}$ is the quantity $f_+^{K^0\pi^-} (0)$ with the following chiral expansion:

$$f_+^{K^0\pi^-} (0) = 1 + f_4 p^4 + f_2 e^2 p^2 + f_6 p^6 + O[(m_u - m_d) p^4, e^2 p^4].$$  \hspace{1cm} (7)
The present status is as follows:

- $f_{p^4}$: $-0.0227$ (no uncertainty) \[12\]
- $f_{e^2p^2}$: radiative corrections ($X_i$) \[13\]
- $f_{p^6}$: loop contributions \[14, 15\]
- $L_2^5, C_{12} + C_{34}$: tree contributions

For a first comparison with experiment, consider the ratio \[13\]

$$r_{+0} := \left( \frac{2 \Gamma(K^+e^3(\gamma)) M_{K^0}^5 I_{K^0}}{\Gamma(e^3(\gamma)) M_{K^+}^5 I_{K^+}} \right)^{1/2} = \frac{|f_{+0}^{K^0\pi^0}(0)|}{|f_{+0}^{K^0\pi^+}(0)|}.$$ \(8\)

The theoretical prediction for $r_{+0}$ is independent of $f_{p^6}$. The only previously unknown LEC in $r_{+0}$ is $X_1$. With the newly determined value for $X_1$ \[10\] and using quadratic fits for the form factors to extract $f_{+0}(0)$ from the data, one finds \[16, 17\]

$$r_{+0}^{th} = 1.023 \pm 0.003$$

$$r_{+0}^{exp} = 1.036 \pm 0.008.$$ \(9\)

A possible discrepancy between theory and experiment for $r_{+0}$ could be due to several reasons: radiative corrections applied by experimentalists are not always state of the art, the lifetimes of $K^+, K_L$ may still undergo revisions and the error in $r_{+0}^{th}$ due to neglected effects of $O((m_u - m_d)p^4, e^2p^4)$ could be underestimated.

Turning now to $f_{+0}^{K^0\pi^-}(0)$, the uncertainty in the $O(p^6)$ contribution $f_{p^6}$ is mainly due to the LECs. Loop and local contributions are separately scale dependent. The loop contributions at the scale $\mu = M_\rho$ amount to \[15\]

$$f_{p^6}^{L=1,2}(M_\rho) = 0.0093 \pm 0.0005.$$ \(10\)

The local contribution is given by

$$f_{p^6}^{tree}(M_\rho) = \frac{8 (M_K^2 - M_\rho^2)^2}{F_\pi^2} \left[ \frac{(L_5^6(M_\rho))^2}{F_\pi^2} - C_{12}(M_\rho) - C_{34}(M_\rho) \right].$$ \(11\)

The results of large-$N_C$ matching discussed in the previous section can be read off from Fig.\(1\). The separate contributions $L_2^5$ and $C_{12} + C_{34}$ depend strongly both on the uncertain scalar resonance mass $M_S$ and on the renormalization scale. However, as shown in Fig.\(1\) for the $M_S$ dependence, both uncertainties are substantially reduced for the relevant combination entering $f_{p^6}^{tree}(M_\rho)$.

A strong destructive interference between the two local contributions is observed. The final result (allowing for a second pseudoscalar multiplet $P'$) is \[11\]

$$f_{p^6}(M_\rho) = -0.002 \pm 0.008 \frac{1}{N_C} \pm 0.002 M_S^{+0.000}_{-0.002} P'$$

$$f_{p^6} = 0.007 \pm 0.012$$

$$f_{+0}^{K^0\pi^-}(0) = 0.984 \pm 0.012.$$ \(12\)
FIGURE 1. \( f_{\rho^6}^{t\text{ree}}(M_P) \) is displayed as a function of \( M_S \) for \( M_P = 1.3 \) GeV (solid line). The dashed line represents the term proportional to \( L_5^2 \), while the dotted line represents the term proportional to \(-(C_{12} + C_{34})\).

We find less \( SU(3) \) breaking in \( f^{K_0^0\pi^-}(0) \) compared to Leutwyler and Roos \cite{18}, with \( f_{\rho^6}^{t\text{ree}} \) being dominated by the loop contribution. From the experimental result \cite{16} \( f^{K_0^0\pi^-}(0) \cdot |V_{us}| = 0.2160(10) \) one obtains

\[
|V_{us}| = 0.2195 \pm 0.0027_{f_+(0)} \pm 0.0010_{\text{exp}}.
\]

Before observing a possible conflict with CKM unitarity (the PDG value \cite{19} for \( V_{ud} \) gives rise to \( |V_{us}|_{\text{unitarity}} = 0.2265 \pm 0.0022 \)), the following remarks are in order.

i. A new result for the neutron lifetime \cite{20} would prefer a value for \( V_{ud} \) in perfect agreement with \( |V_{us}| = 0.2195 \) and unitarity.

ii. A recent analysis of semileptonic hyperon decays \cite{21} yields \( |V_{us}| = 0.2199 \pm 0.0026 \). After the Workshop, the uncertainties of extracting \( V_{us} \) from semileptonic hyperon decays have been reassessed in Ref. \cite{22}.

iii. To achieve an accuracy of better than 1% for \( V_{us} \), the differences between \( K^+ \) and \( K^0 \) results must be straightened out.

An independent check of the theoretical estimate for the LECs of \( O(p^6) \) is provided by the slope \( \lambda_0 \) of the scalar form factor (accessible in \( K_{\mu 3} \) decays) that depends on the same LECs \( C_{12}, C_{34} \) as \( f_+(0) \).

| Ref.          | Cirigliano et al. \cite{11} | KTeV \cite{23} |
|--------------|-----------------------------|----------------|
| \( \lambda_0 \cdot 10^3 \) | 13 \pm 3                   | 13.72 \pm 1.31 |

**HADRONIC VACUUM POLARIZATION AND \((g - 2)_\mu\)**

At present, the biggest uncertainty in the evaluation of the anomalous magnetic moment of the muon \( a_\mu \) in the Standard Model is due to hadronic vacuum polarization at lowest
order in $\alpha$ (shown in Fig. 2) that is directly related to the cross section $\sigma(e^+e^- \rightarrow \text{hadrons})$. About 73% of $a_{\mu}^{\text{vac.pol.}}$ comes from the $\pi^+\pi^-$ final state, the low-energy part being especially important.

**FIGURE 2.** Contribution of lowest-order hadronic vacuum polarization to the muon magnetic moment.

In the isospin limit, the two-pion contribution to hadronic vacuum polarization can also be obtained from the decay $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ [24]. At the level of accuracy needed for a comparison with the measured value of $a_\mu$ [25], isospin violating and electromagnetic corrections must be included [26, 27]. However, until recently the two-pion spectral functions from $e^+e^-$ annihilation and from $\tau$ decays seemed to differ significantly especially above the $\rho$ region, even after accounting for isospin violating effects. The value of $a_{\mu}^{\pi\pi}$ on the basis of the most precise $e^+e^-$ data from the CMD-2 Collaboration [28] was then confirmed by KLOE [29] although the actually measured $\pi\pi$ cross sections are not in very good agreement. The consensus among many experts in the field was spelled out by Höcker at last year’s High Energy Conference in Beijing [30]: until the origin of the discrepancy between $e^+e^-$ and $\tau$ data is understood the $\tau$ data should be ignored for the evaluation of $a_\mu$.

A recent analysis of Maltman [31] suggests a new perspective on this issue. He investigates so-called pinched FESR of the type

$$\int_0^{s_0} w(s)\rho(s)ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} w(s)\Pi(s)ds$$

for current correlators $\Pi(s)$ with associated spectral functions $\rho(s)$. The spectral functions of interest here are the electromagnetic spectral function $\rho_{\text{em}}$ measured in $e^+e^-$ annihilation and the charged $I = 1$ vector current spectral function $\rho_{V=1}$ accessible in $\tau$ decays. The weight function $w(s)$ is a positive definite analytic function in the complex $s$-plane for $|s| \leq s_0$, but otherwise arbitrary except for the constraint $w(s_0) = 0$ to minimize duality violations (pinching).

The left-hand side of the FESR (14) is evaluated with experimental input (CMD-2 [28] and ALEPH [32]) whereas the right-hand side is calculated from QCD with the help of the OPE. The freedom of choosing the weight function $w(s)$ can be employed to eliminate the dimension $D = 6$ OPE contributions altogether. The right-hand side is then mainly sensitive to the $D = 0$ perturbative part known up to $O(\alpha_s^3)$, with weaker dependences on $m_s$ (in the $D = 2$ piece) and on $D = 4$ quark and gluon condensates. Effects with $D \geq 8$ can be kept under control by varying $s_0$. Discarding all low-energy
input for the determination of $\alpha_s(M_Z)$ (such as the $\tau$ data that are to be tested with FESR), Maltman obtains a value

$$\alpha_s(M_Z) = 0.1200 \pm 0.0020$$ \hspace{1cm} (15)

to be used for the right-hand side of (14).

A first test performed in Ref. [31] consists in fitting $\alpha_s(M_Z)$ from the experimentally determined spectral integrals (left-hand side), leaving all other input for the right-hand side unchanged. The results for two typical weight functions $w_1, w_6$ are shown in Table 2 to be compared with the best value from high-energy data in Eq. (15). Taking into account that the weights are positive definite, the results in Table 2 indicate that the electromagnetic spectral density is too low whereas the $\tau$ spectral data are in perfect agreement with the canonical value of $\alpha_s$.

**TABLE 2.** Fitted values of $\alpha_s(M_Z)$ from experimentally determined spectral integrals for two different weight functions [31].

| weight | type | $\alpha_s(M_Z)$  |
|--------|------|------------------|
| $w_1$  | em   | $0.1138 \pm 0.0030$ |
| $w_6$  | em   | $0.1150 \pm 0.0022$ |
| $w_1$  | $\tau$ | $0.1218 \pm 0.0027$ |
| $w_6$  | $\tau$ | $0.1201 \pm 0.0020$ |

A second independent consistency check of the data comes from a comparison of the two sides in Eq. (14) for different values of $s_0$ [31]. When plotting the spectral integrals as functions of $s_0$ one arrives at a similar conclusion as before: the slopes in the electromagnetic case differ by about 2.5 $\sigma$ between data and QCD. On the other hand, the $\tau$ data show perfect consistency both for the slope and in absolute normalization (depending on $\alpha_s$).

The conclusions of Ref. [31] are very convincing even if the statistical weight is not overwhelming: the sum rule tests clearly favour the $\tau$ over the $e^+e^-$ data. The status of $a_\mu$ at the time of the Workshop can be summarized as follows [33]:

$$\left(a_\mu^{\text{exp}} - a_\mu^{\text{SM}}\right) \cdot 10^{10} = \begin{cases} 23.9 \pm 9.9 \ (2.4 \sigma) & [e^+e^-] \\ 7.6 \pm 8.9 \ (0.9 \sigma) & [\tau, e^+e^-] \end{cases} \hspace{1cm} (16)$$

Using the isospin corrected $\tau$ data for the $2\pi$ and $4\pi$ final states thus leads to agreement between theory and experiment to better than 1 $\sigma$.

Two weeks after the Workshop, new $e^+e^- \rightarrow \pi^+\pi^-$ data were released [34] that appear to lie between the CMD-2 and the (isospin corrected) ALEPH data.
CONCLUSIONS

In the meson sector, chiral perturbation theory has been pushed to next-to-next-to-leading order. At this order, the main limitation for further progress is the abundance of coupling constants, an unavoidable feature of a nonrenormalizable effective field theory. Some progress has been made recently in estimating those constants by using large-$N_C$ methods to interpolate between CHPT and QCD.

CHPT is the only reliable approach for calculating electromagnetic and isospin violating corrections for hadronic processes at low energies. This is in particular important for the analysis of $K_{l3}$ decays in order to extract the CKM matrix element $V_{us}$ to better than 1% accuracy.

Recent sum rule tests \[31\] favour $\tau$ over $e^+e^-$ data for evaluating the hadronic vacuum polarization at low and intermediate energies. As a consequence, there is at present no conflict between the Standard Model and experiment for the anomalous magnetic moment of the muon.

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