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DYNAMICAL FORMATION OF CENTER DOMAINS
IN QUARK-GLUON PLASMA

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gูกลูออนพลาสมาในการชนกันของไอออนหนัก งานวิจัยนี้ได้พัฒนาแบบจำลองพอลศาสตร์สำเร็จโดยใช้ค่าจำกัดของโพลียาคอฟลูป บันทึกฐานของแอนทิโลหิตที่มีผลกระทบและที่ไม่มีผลกระทบโดยคิดเกี่ยวกับการศึกษาวิวัฒนาการของโพลียาคอฟลูป ทำให้สามารถศึกษาได้ในกรณีที่ทรัพยากรของเขตศูนย์กลางในควาร์กกลูออนพลาสมาได้ โดยการจำลองในวิธีที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซึ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซึ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซึ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซึ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซึ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซึ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซึ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซึ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซึ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซึ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซึ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซึ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการก่อตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการกáoตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการกáoตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพอลศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการกáoตัวของเขตศูนย์กลางในระหว่างการชนของไอออนหนัก และการจำลองแบบจำลองพólศาสตร์ที่มีข้อดีที่ทำให้เกิดความเข้าใจอย่างลึกซิ้ง เช่นการรวมปริมาณการกáoตัวของเขตศูนย์กลางในระหว่างการชนของไอออน

Center domains are structures based on spontaneous breakdown of center symmetry as expected in quark-gluon plasma (QGP) from lattice QCD calculations. Each domain is characterized by a finite value of the Polyakov loop, which here serves as an order parameter to distinguish between confined and deconfined phase. Center domains might possibly occur in heavy-ion collision and may have influence on observable like viscosity or elliptic flow. In this work, we develop a fully dynamical model for the Polyakov loop based on an effective potential and a phenomenological kinetic term. Studying the time evolution of the Polyakov loop allows us to study formation and decay of center domains in the QGP. The results of this simulation give us insight into the formation procedure during a heavy-ion collision and help us understand how the domain size is influenced by temperature and the kinetic coefficient in our model. We find that the domain size grows with this coefficient, together with recent data from lattice QCD, where the domain size was calculated as a function of temperature, we can fix the value of the kinetic coefficient as a function of temperature. Finally, we determine the formation time of domains and find it within the lifetime of a QGP at LHC energies,
therefore making the formation of center domains a relevant effect that needs to be considered in future calculations.
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CHAPTER I

INTRODUCTION

In the past, people believed that all elements in nature were composed of indivisible particles called *atoms*. The first group of people who considered the concept of an atom were ancient Greek and Indian philosophers, studying so-called *Atomism*. Atomism comprises the idea that all elements in nature consist of atoms and voids. However, the crucial problem of Atomism was the lack of experimental evidence. Note that, in modern science, any theory requires experimental support to be verified. In the 19th century, the study of atoms became famous again under the name of *Atomic Theory*. This Atomic Theory was developed by John Dalton, an English chemist, physicist and meteorologist, see figure 1.1. The origin of the theory was not well understood. However, the theory was later verified by many experimental results from other works. The idea of his theory can be summarized in four statements:

1. All elements in nature are made of atoms where one atom is indivisible and indestructible,

2. Any single element is composed of atoms which are identical in size, mass and other properties, different elements have different types of atoms,

3. Any two or more elements can form chemical compounds with an integer ratio,

4. A chemical reaction is a rearrangement of atoms.
This was the first time when scientists had gained an empirical understanding about matter. So, with this success, the Atomic Theory of Dalton is considered as a foundation of particle physics and another related fields in a few centuries later.

Later, contrary to some ideas of the Atomic Theory of Dalton, physicists found that atoms are not indivisible, but contain smaller particles such as electrons, protons and neutrons. Moreover, from modern particle physics, we know that even protons and neutrons are composed of smaller particles called quarks, see figure 1.2 for a table of elementary particles as they are known today.

After the Big Bang, our universe started to expand and cool, resulting in several phases and phase changes to the contained matter. All those phases have different thermodynamic properties. In this work, we focus on one of the early phases called quark-gluon plasma (QGP). The QGP is a primordial state of matter created at about
Figure 1.2 Standard model of elementary particles shows various particles that have been observed so far, figure by MissMJ, used under CC BY 3.0.

$10^{-5}$ s after the Big Bang, The term of QGP denotes matter composed of two elementary particles:

**Quarks**, which are considered as fundamental constituents of matter.

**Gluons**, which act as carriers of the strong nuclear force between quarks.

From the QGP phase, hadrons emerged after further cooling, later atomic nuclei, atoms, and finally gravitating large-scale structures such as stars and galaxies. In the QGP phase, quarks and gluons are in no bound state under the extremely hot and dense conditions. The theory describing the interaction of quarks and gluons due to so called color charges (red (R), green (G) and blue (B)) is *Quantum Chromodynamics (QCD)* which was first proposed by Nambu in 1966. One remarkable feature of QCD which distinguishes hadronic matter from a QGP is called *confinement*. In confinement, quarks are bound together in color-neutral particles such as baryons and mesons. Baryons are composite particles made up of three quarks e.g. protons (p), neutrons (n). Mesons are
composite particles made up of one quark and one antiquark e.g. pions ($\pi$), kaons (K). In contrast to that, in QGP the relevant degrees of freedom carry color charge, matter is deconfined. Nevertheless, isolated color charges have never been observed experimentally. Another characteristic property that distinguishes the QGP from the hadron gas is the restoration of chiral symmetry at high temperatures. The QGP can be found or created in 1) The early universe at about $10^{-5}$ s after the Big Bang, 2) Superdense stars such as neutron stars or quark stars, 3) Heavy-ion collisions.

Nowadays, heavy-ion collisions are done by colliding two heavy nuclei at ultra-relativistic energies using large accelerator facilities. At the moment such experiments are performed at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory and the Large Hadron Collider (LHC) at CERN. At RHIC, scientists found evidence for the formation of a strongly interacting quark-gluon plasma (sQGP) (Arsene et al., 2005; Adcox et al., 2005; Back et al., 2005; Adams et al., 2005; Gyulassy and McLerran, 2005; Müller and Nagle, 2006), based on the discoveries of 1) Elliptic Flow 2) Low ratio of shear viscosity to entropy density $\eta/s$ 3) Jet quenching. These evidences show that the sQGP behaves rather like an ideal fluid than a gas of non-interacting particles.

From lattice QCD studies (Danzer et al., 2010; Borsanyi et al., 2011), the existence of so called center domains was confirmed. Center domains are structures that have recently claimed to be responsible for some crucial properties of QGP (Asakawa et al., 2013). In (Asakawa et al., 2013), the authors argue that two important properties of QGP, low shear viscosity and jet quenching can be explained by the formation of center domains in the QGP. This was our motivation to study the behavior and dynamics
Figure 1.3 Schematic of center domains where $\nu = 0$, $\nu = 1$ and $\nu = 2$ are different in types of domains, from (Asakawa et al., 2013).

of center domains in QGP. In (Asakawa et al., 2013), it is argued that domain walls act as potential barriers for in-medium particles, thus limiting their free wavelength, which accounts for a small value of $\eta/s$. On the other hand, jets may rapidly lose their energy in the medium via interaction with the walls and subsequent radiation of soft gluons, see figure 1.3.

Our research here focuses on developing a dynamical model for an effective Polyakov loop field in a QGP. We use a phenomenological Lagrangian from a Polyakov loop potential and a phenomenological kinetic term. Then we study the evolution of the Polyakov loop field in (3+1) dimensions. We hereby restrict ourselves to the case of a medium with a homogeneous temperature. We expect to be able to observe the formation of center domains after changing the global temperature from below to above the critical temperature $T_c$. Our goal is to give estimates for the formation and decay time of domains to better understand their possible role in heavy-ion collisions.
CHAPTER II

POLYAKOV LOOP POTENTIAL

Center domains occur due to the spontaneous breakdown of center symmetry $Z(3) \subset SU(3)$ at high temperatures. Therefore, the existence of center domains is expected in a QGP phase. In QCD, one way to distinguish between QGP phase and hadronic phase is using confinement. Confinement can be mathematically described by the Polykov loop potential. The Polyakov loop arises from pure $SU(3)$ gauge theory, where it serves as an order parameter which distinguishes between a center symmetric confined phase and a deconfined phase where this symmetry is broken.

2.1 Polyakov loop potential

The fundamental Polyakov loop is defined as

$$L(x) = \frac{1}{3} \text{tr} P \exp \left[ i g \int_{0}^{1/T} A_4(\tau, x) d\tau \right],$$

(2.1)

where $P$ denotes the path-ordering operator, $g$ is the strong coupling constant, $T$ is the temperature and $A_4$ is the temporal component of a static gluon background field in Euclidean space-time. From fits of lattice QCD data (Boyd et al., 1996) in the pure gluon sector, we obtain a potential for the Polyakov loop (Roessner et al., 2007)

$$U(L) = -bT[54e^{-a/T} |L|^2 + \ln P(L, L^\dagger)],$$

(2.2)

where $L$ is the Polyakov loop, $T$ is the temperature, $a = 0.664 \text{ GeV}$, $b = 0.0075 \text{ GeV}^3$ and $P(z, \bar{z}) = 1 - 6 |z|^2 - 3 |z|^4 + 4(z^3 + \bar{z}^3)$. Figures 2.1 to 2.3 show plots of the Polyakov
loop potential in the complex plane $L = l_1 + il_2$ with temperatures $T = 100$ MeV, $T = 300$ MeV and $T = 500$ MeV respectively. The equilibrium point of the potential is shifted from one point in figure 2.1 to three points in figure 2.3, due to the spontaneous breakdown of center symmetry $Z(3)$ at the critical temperature of $T = T_c = 270$ MeV. Effective potentials for the Polyakov loop are often used in low-energy models such as the Polyakov loop Nambu-Jona-Lasinio (PNJL) model (Fukushima, 2004) or the Polyakov-Quark-Meson (PQM) model (Schaefer et al., 2007; Herbst et al., 2011). In pure gauge theory, the Polyakov loop is related to the free energy of an infinitely heavy static quark $F_Q(T)$ by

$$F_Q(\bar{x}, T) = -T \ln |\langle L(\bar{x}, T) \rangle|,$$

(2.3)

where

In confinement: $F_Q$ is infinite, $\langle L \rangle = 0$.

In deconfinement: $F_Q$ is finite, $\langle L \rangle > 0$.

Figure 2.1 Contour plot of Polyakov loop potential at $T = 100$ MeV.
2.2 Thermodynamic properties

From equations (2.2) and (2.3), we can study thermodynamic properties of pure gluon QCD, such as pressure, energy density, entropy density and free energy density of
test quarks as a function of temperature. By this, we can understand the consistency of
the Polyakov loop potential with pure gauge QCD. We will therefore also see that there
is a first-order phase transition from the Polyakov loop potential. However, from lattice
QCD we know that there is no phase transition for zero chemical potential, but rather
an analytic crossover (Aoki et al., 2006). We might expect a first-order phase transition
for high density or chemical potential (Scavenius et al., 2001; Schaefer and Wambach,
2005).

In figure 2.4, we track a minimum point of the Polyakov loop potential under
the transition from $T = 0$ MeV to $T = 500$ MeV. Above the critical temperature $T_c$,
there are three degenerate ground states according to three minima in the potential. The
absolute value of the Polyakov loop in figure 2.5 is able to identify the phase of our
system, where in the confined phase $L$ equals 0 and in the deconfined phase ranges from
about 0.5 to 1.0. From the data in figures 2.4 and 2.5, we can further find the pressure
as the negative value of the potential, see figure 2.6, the entropy density which is the
derivative of the pressure with respect to temperature, see figure 2.7, the free energy of
a heavy static quark according to equation (2.3), see figure 2.8 and finally, the energy
density which can be found from the relation $e = T s - p$, see figure 2.9.

From these plots we can clearly identify $T_c$ at 270 MeV from the kink in the
pressure and the discontinuity in $L$, $s$ and $e$ as functions of $T$. 
Figure 2.4 Minimum point of the Polyakov loop potential as a function of \( T \).

Figure 2.5 Absolute value of the Polyakov loop in the equilibrium as a function of \( T \).
Figure 2.6 Pressure of the Polyakov loop potential as a function of $T$.

Figure 2.7 Entropy of the Polyakov loop potential as a function of $T$. 
**Figure 2.8** Free energy of a heavy static quark as a function of $T$.

**Figure 2.9** Energy density of the Polyakov loop potential as a function of $T$. 
CHAPTER III

EQUATIONS OF MOTION

Our goal is to study the dynamics of center domains in QGP via the Polyakov loop. As the Polyakov loop has no explicit time dependence, we have to pursue a phenomenological ansatz similar to what has been proposed in (Dumitru and Pisarski, 2001; Herold et al., 2013; Fraga et al., 2007) to study its dynamics. We write down the Lagrangian density as

$$\mathcal{L} = \frac{\sigma}{2} T^2 \partial_\mu L \partial^\mu L - U(L, \bar{L}), \quad (3.1)$$

where $\sigma$ is a dimensionless parameter playing the role of a surface tension that we have to determine later by comparison with the domain size from lattice QCD data (Borsanyi et al., 2011). Splitting the complex-valued Polyakov loop into its imaginary and real part $L = l_1 + il_2$ and $\bar{L} = l_1 - il_2$, equation (2.2) becomes

$$U(l_1 + il_2) = -bT[54e^{-a/T} |l_1 + il_2|^2 + \ln P(l_1 + il_2, (l_1 + il_2)^\dagger)]$$

$$= -bT[54e^{-a/T}(l_1^2 + l_2^2) + \ln P(l_1 + il_2, l_1 - il_2)]. \quad (3.2)$$

For the polynomial function $P$ we obtain

$$P(l_1 + il_2, l_1 - il_2) = 1 - 6|l_1 + il_2|^2 - 3|l_1 + il_2|^4 + 4((l_1 + il_2)^3 + (l_1 - il_2)^3)$$

$$= 1 - 6(l_1^2 + l_2^2) - 3(l_1^2 + l_2^2)^2 + 4(l_1^3 + 3il_1^2l_2 - 3l_1l_2^2 + l_2^3)$$

$$+ 4(l_1^3 - 3il_1^2l_2 - 3l_1l_2^2 - l_2^3)$$

$$= 1 - 6(l_1^2 + l_2^2) - 3(l_1^2 + l_2^2)^2 + 4(2l_1^3 - 6l_1l_2^2).$$
In equation (3.1) $\partial_\mu L \partial^\mu L$ can be written as

$$\partial_\mu L \partial^\mu L = \partial_\mu (l_1 + il_2) \partial^\mu (l_1 - il_2)$$

$$= (\partial_\mu l_1 + i\partial_\mu l_2) (\partial^\mu l_1 - i\partial^\mu l_2)$$

$$= \partial_\mu l_1 \partial^\mu l_1 + \partial_\mu l_2 \partial^\mu l_2.$$  (3.3)

From the general Euler–Lagrange equation:

$$\partial_\mu \left( \frac{\partial L}{\partial (\partial_\mu \phi)} \right) - \frac{\partial L}{\partial \phi} = 0,$$  (3.4)

and equation (3.3), we have the first Euler–Lagrange equation for the real part of the Polyakov loop

$$\partial_\mu \left( \frac{\partial L}{\partial (\partial_\mu l_1)} \right) = \frac{\partial L}{\partial l_1},$$  (3.5)

and the second Euler–Lagrange equation for the imaginary-part of the Polyakov loop

$$\partial_\mu \left( \frac{\partial L}{\partial (\partial_\mu l_2)} \right) = \frac{\partial L}{\partial l_2}.$$  (3.6)

From the Euler-Lagrange equations (3.5) and (3.6), the phenomenological Lagrangian (3.1) and the complex valued Polyakov loop potential (3.2), we can now find the equations of motion. We begin with the equation of motion for the real part of the Polyakov loop $l_1$. 
\[
\frac{\partial L}{\partial l_1} = \frac{\partial}{\partial l_1} \left[ \frac{\sigma}{2} T^2 (\partial_\mu l_1 \partial^\mu l_1 + \partial_\mu l_2 \partial^\mu l_2) - U(l_1, l_2) \right]
\]
\[
= \frac{\partial}{\partial l_1} U(l_1, l_2)
\]
\[
= \frac{\partial}{\partial l_1} \left[ -bT [54 e^{-a/T} (l_1^2 + il_2^2) + \ln P(l_1 + il_2, l_1 - il_2) \right]
\]
\[
= -108bT e^{-a/T} l_1 + \left[ \frac{1}{P(l_1 + il_2, l_1 - il_2)} \frac{\partial}{\partial l_1} P(l_1 + il_2, l_1 - il_2) \right]
\]
\[
\frac{\partial}{\partial x^\mu} \frac{\partial L}{\partial (\partial_\mu l_1)} = \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial (\partial_\mu l_1)} \left[ \frac{\sigma}{2} T^2 (\partial_\mu l_1 \partial^\mu l_1 + \partial_\mu l_2 \partial^\mu l_2) - U(l_1, l_2) \right]
\]
\[
= \frac{\partial}{\partial x^\mu} \sigma T^2 \partial^\mu l_1.
\]

From \( \frac{\partial l_1}{\partial x^\mu} = \partial_\mu l_1 \) and assuming that \( T \) does not depend on \( x^\mu \), we get

\[
\frac{\partial}{\partial x^\mu} \frac{\partial L}{\partial (\partial_\mu l_1)} = \sigma T^2 \partial^\mu l_1.
\]

Thus, for the equation of motion for the real part of the Polyakov loop we get

\[
\sigma T^2 \partial_\mu \partial^\mu l_1 + \frac{\partial}{\partial l_1} U(l_1, l_2) = 0.
\]

The equation of motion for the imaginary part of the Polyakov loop \( l_2 \) can be derived in the same way as for \( l_1 \).
\[ \frac{\partial L}{\partial l_2} = \frac{\partial}{\partial l_2} \left[ \frac{\sigma T^2}{2} (\partial_\mu l_1 \partial^\mu l_1 + \partial_\mu l_2 \partial^\mu l_2) - U(l_1, l_2) \right] \]

\[ = \frac{\partial}{\partial l_2} U(l_1, l_2) \]

\[ = \frac{\partial}{\partial l_2} \left[ - bT [54e^{-a/T}(l_1^2 + i l_2^2) + \ln P(l_1 + il_2, l_1 - il_2)] \right] \]

\[ = -108bTe^{-a/T}l_2 + \left[ \frac{1}{P(l_1 + il_2, l_1 - il_2)} \frac{\partial}{\partial l_2} P(l_1 + il_2, l_1 - il_2) \right] \]

\[ = -108bTe^{-a/T}l_2 + \left[ \frac{1}{P(l_1 + il_2, l_1 - il_2)} \frac{\partial}{\partial l_2} (1 - 6(l_1^2 + l_2^2) - 3(l_1^4 + l_2^4) + 4(2l_1^3 - 6l_1l_2^2)) \right] \]

\[ = -108bTe^{-a/T}l_2 + \left[ \frac{(-12l_2 - 12(l_1^2 + l_2^2)l_2 - 48l_1l_2)}{P(l_1 + il_2, l_1 - il_2)} \right] \]

\[ = -108bTe^{-a/T}l_2 + \left[ \frac{(-12l_2 - 12(l_1^2 + l_2^2)l_2 - 48l_1l_2)}{P(l_1 + il_2, l_1 - il_2)} \right], \]

\[ \frac{\partial}{\partial x^\mu} \frac{\partial L}{\partial (\partial_\mu l_2)} = \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial (\partial_\mu l_2)} \left[ \frac{\sigma T^2}{2} (\partial_\mu l_1 \partial^\mu l_1 + \partial_\mu l_2 \partial^\mu l_2) - U(l_1, l_2) \right] \]

\[ = \frac{\partial}{\partial x^\mu} \sigma T^2 \partial^\mu l_2. \]

From \( \frac{\partial l_2}{\partial x^\mu} = \partial_\mu l_2 \) where \( T \) again does not depend on \( x^\mu \), we get

\[ \frac{\partial}{\partial x^\mu} \frac{\partial L}{\partial (\partial_\mu l_2)} = \sigma T^2 \partial^\mu l_2. \]

Thus, for the equation of motion for the real part of the Polyakov loop we get

\[ \sigma T^2 \partial_\mu \partial^\mu l_2 + \frac{\partial}{\partial l_2} U(l_1, l_2) = 0. \] (3.8)

Equations (3.7) and (3.8), will later be used to simulate the dynamics of the Polyakov loop solved by some numerical method in chapter IV. Note that, considering figures 2.1 to 2.3, we can see that \( l_1 \) is fit for distinguishing between confined and
deconfined phase similar to $|< L >|$, whereas $l_2$ is fit for distinguishing between three types of center domains in the deconfined phase.
CHAPTER IV

SIMULATION METHOD

Studying the evolution of the center domains can be done by a time-dependent *Polyakov loop simulation* which numerically solves the equations of motion (3.7) and (3.8) in (3+1)-dimensions. The given space-time lattices are 1) space-time lattice for the real part of the Polyakov loop $l_1$, 2) space-time lattice for the imaginary part of the Polyakov loop $l_2$. Note again, confined and deconfined state of the system can be distinguished by $|< L >|$, on the other hand types of the center domains cannot be distinguished by $|< L >|$ but rather $l_2$, see section 2.1.

4.1 Numerical implementation

Solving equations (3.7) and (3.8) requires some numerical method. In (Cassol-Seewald et al., 2012), a useful method for solving partial differential equations is provided, to apply that method to our problem, we start from writing equations (3.7) and (3.8) into a discrete form

$$\sigma T^2 \left( \frac{\partial^2 l_{n-1}}{\partial t^2} - \nabla^2 l_{n-1} \right) + \frac{\partial}{\partial l_{n-1}} U(l_1, l_2) = 0,$$

(4.1)

$l_n$ here corresponds to either the real part of the Polyakov loop $l_1$ or the imaginary part of the Polyakov loop $l_2$ at time interval $n$ where time $t = n \Delta t$ with $n = 1, 2, 3, \ldots$
can write the time derivatives as
\[ \frac{\partial l_{n-1}}{\partial t} = i_{n-1} = \frac{1}{2}(i_{n-1/2} + i_{n-3/2}), \]
\[ i_{n-1/2} = \frac{1}{\Delta t}(l_n - l_{n-1}), \]
\[ i_{n-3/2} = \frac{1}{\Delta t}(l_{n-1} - l_{n-2}), \]
\[ \frac{\partial^2 l_{n-1}}{\partial t^2} = \ddot{i}_{n-1} = \frac{1}{\Delta t}(i_{n-1/2} - i_{n-3/2}). \]

Equation (4.1) becomes
\[ \sigma T^2 \left( \frac{1}{\Delta t^2}(l_n - 2l_{n-1} + l_{n-2}) - \overrightarrow{\nabla}^2 l_{n-1} \right) + \frac{\partial}{\partial l_{n-1}} U(l_1, l_2) = 0, \]
\[ l_n = 2l_{n-1} - l_{n-2} + \Delta t^2 \left( \overrightarrow{\nabla}^2 l_{n-1} - \frac{1}{\sigma T^2} \frac{\partial}{\partial l_{n-1}} U(l_1, l_2) \right). \quad (4.2) \]

Consider the Laplacian operator
\[ \overrightarrow{\nabla}^2 l_{n-1}^{ijk} = \frac{\partial^2 l_{n-1}^{ijk}}{\partial^2 x} + \frac{\partial^2 l_{n-1}^{ijk}}{\partial^2 y} + \frac{\partial^2 l_{n-1}^{ijk}}{\partial^2 z} = \frac{1}{\Delta x} \left[ \left( \frac{l_{n-1}^{ij+1k} - l_{n-1}^{ij-1k}}{\Delta x} \right) - \left( \frac{l_{n-1}^{ij-1k} - l_{n-1}^{ij-1k}}{\Delta x} \right) \right] \]
\[ + \left( \frac{l_{n-1}^{ij+1k} - l_{n-1}^{ij-1k}}{\Delta x} \right) - \left( \frac{l_{n-1}^{ij+1k} - l_{n-1}^{ij+1k}}{\Delta x} \right) \]
\[ + \left( \frac{l_{n-1}^{ij+1k} - l_{n-1}^{ij-1k}}{\Delta x} \right) - \left( \frac{l_{n-1}^{ij+1k} - l_{n-1}^{ij+1k}}{\Delta x} \right) \]
\[ = \frac{1}{\Delta x^2} \left[ l_{n-1}^{i+1j+1k} + l_{n-1}^{i+1j+1k} + l_{n-1}^{i+1j+1k} - 6l_{n-1}^{ij+1k} + l_{n-1}^{i+1j+1k} + l_{n-1}^{i+1j+1k} + l_{n-1}^{i+1j+1k} \right], \]

where \( i, j \) and \( k \) are positions on the Cartesian grid and \( \Delta x \) is the lattice spacing. We apply periodic boundary conditions for the spatial coordinates
\[ i = \begin{cases} N + i, & \text{if } i < 1, \\ i - N, & \text{if } i > N, \\ i, & \text{otherwise.} \end{cases} \quad (4.3) \]
where \( i \) is a position on the \( x-, y- \) or \( z- \) axis and \( N \) is the maximum size of the space-time lattice in one dimension. In the equations of motion (4.2), the value of the Polyakov loop at any time \( t \) can be calculated from its previous values at times \( n - 1 \) and \( n - 2 \).

### 4.2 Numerical parameters

**Table 4.1** Numerical parameters.

| Variable | Meaning                        | Value      |
|----------|--------------------------------|------------|
| \( \Delta t \) | time step                     | 0.005 (fm/c) |
| \( \Delta x \) | lattice spacing               | 0.5 (fm)   |
| \( N \)   | size of lattice in one dimension | 100        |

Note that the values of \( \Delta t \) and \( \Delta x \) are determined according to the Courant–Friedrichs–Lewy condition where the ratio of \( \Delta t / \Delta x \) is supposed to be small enough to ensure numerical stability. \( N \) has to be chosen large enough for the lattice to be able to contain several larger domains. However, our choice of these numerical parameters is also based on several test runs.

### 4.3 Initial conditions

At times \( t = 0.005 \text{ fm/c} \) and \( t = 0.01 \text{ fm/c} \), both real and imaginary part of the Polyakov loop are initialized with Gaussian distributions of mean zero, corresponding to their vacuum expectation value, and standard deviation 0.1. From figure 4.1, we can see small fluctuations of the Polyakov loop correlated over a typical hadron-sized volume of
1 fm³. This corresponds to the initial state in a heavy-ion collision before the creation of the QGP. We can then change the temperature on our lattice to some value above $T_c$ and follow the evolution of the system in the deconfined phase, where the initial fluctuations can amplify and form center domains.

![Figure 4.1 Small fluctuations in Polyakov loop at $z = 0$ fm and time $t = 0.005$ fm/c.](image)

### 4.4 Fixing sigma

According to the equations of motion (3.7) and (3.8), there is one unknown variable called sigma ($\sigma$), a coefficient playing the role of a surface tension which influences the domain size, a large value of sigma gives a large domain size, by contrast, a small value of sigma gives a small domain size, see figure 4.2. In order to develop a realistic model, we need to fix the value of sigma. In figure 4.3 from (Borsanyi et al., 2011), some useful information about the domain size as function of $T$ is provided. The domain size is defined as a physical diameter $D_{\text{phy}}$ which is obtained from fitting the
two-point correlation function \( C(\vec{x} - \vec{y}) \) of \( l_2 \) to the exponential function

\[
C(\vec{x} - \vec{y}) \propto \exp\left(-\frac{|\vec{x} - \vec{y}|}{D_{\text{phy}}}\right). \tag{4.4}
\]

Unfortunately, (Borsanyi et al., 2011) does not give us more data for higher temperatures, so we can only make predictions for these given values and give a rough estimate for sigma as a function of temperature in the regime near \( T_c \). Thus in this work we consider only the temperatures \( T_1 = 1.05 T_c = 283.5 \) MeV, \( T_2 = 1.1 T_c = 297.0 \) MeV and \( T_3 = 1.15 T_c = 310.5 \) MeV.

We fix the coefficient \( \sigma \) using the following procedure:

1. Fix value of \( T \) according to information in figure 4.3,
2. Heuristically fix value of \( \sigma \),
3. Run a simulation,
4. Calculate \( D_{\text{phy}} \),
5. Compare given \( D_{\text{phy}} \) to figure 4.3,
6. If \( D_{\text{phy}} \) does not equal its value in figure 4.3, go back to step 2.

![Figure 4.2](image_url) Space-time lattices with a different sigma values.
Fast fixing of sigma can be done by a following binary search strategy

Searching procedure:

1. Find value of $middle$ from list $l$,

2. Compare target to $middle$,

3. If target less than $middle$ then:
   
   remove $middle$ and its right-hand side values from list $l$ then go to 1;

   else if target greater than $middle$ then:

   remove $middle$ and its left-hand side values from list $l$ then go to 1;

   else: stop.

where $l$ is list of values, $middle$ is a value at middle of list $l$, target is a value that we want to find.

**Example:**
Problem: find a target value \( target = 8 \) in a sorted list \( l = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10] \).

Define \( middle \) as a value at middle of \( l \), \( middle = 5 \). \( (1^{st \ procedure}) \)

Is \( target = 8 \) equal to \( middle = 5 \)? \( (2^{nd \ procedure}) \)

False, then let \( l = [6, 7, 8, 9, 10] \). \( (3^{rd \ procedure}) \)

Define \( middle \) as a value at middle of \( l \), \( middle = 8 \). \( (1^{st \ procedure}) \)

Is \( target = 8 \) equal to \( middle = 8 \)? \( (2^{nd \ procedure}) \)

True, stop. \( (3^{rd \ procedure}) \)

After finishing the iterative process of fixing sigma, we will obtain a reliable coefficient to study the dynamics of domain formation in the range from \( 1 \ T_c \) to \( 1.2 \ T_c \).

4.5 Tracking evolution of center domains

Since \( l_1 \) distinguishes between confined and deconfined phase, \( l_2 \) identifies the type of minimum points in center domains. Both \( l_1 \) and \( l_2 \) are evolve from confined to deconfined phase. In the deconfined phase, the existence of center domains is expected. This leads to the question how to determine the time when center domains are formed. Our solution proposes to solve this problem by using the standard deviation (SD).

The standard deviation at time \( t \) is defined as

\[
SD = \sqrt{\frac{1}{N^3} \sum_{i,j,k} (l_{2ijk} - \bar{l}_2)^2},
\]

(4.5)

where \( \bar{l}_2 \) is volume-averaged value of \( l_2 \). If SD has reached a stable maximum value as a function of time, we can conclude that the domain formation process is finished. Note that, in our research, the standard deviation is then averaged over 100 events with different initial conditions for each time \( t \) to ensure reliable results.
CHAPTER V
RESULTS

5.1 Sigma coefficient

Applying the iterative method introduced in section 4.4, we can obtain a reliable value of sigma. In figure 5.1, we show the result of sigma for temperature $T_1$ where the dots indicate the two-point correlation function of $l_2$ and the solid line is the fitted exponential function (5.1) which is evaluated as

$$C(\vec{x} - \vec{y}) \propto \exp \left( -\frac{1}{2} \frac{|\vec{x} - \vec{y}|}{2.55 \text{ fm}} \right).$$  \hspace{1cm} (5.1)

The exponential function for temperature $T_2$ is

Figure 5.1 Exponential fit of the correlation function for $T = 283.5$ MeV.

$$C(\vec{x} - \vec{y}) \propto \exp \left( -\frac{1}{3} \frac{|\vec{x} - \vec{y}|}{3.75 \text{ fm}} \right),$$  \hspace{1cm} (5.2)
see figure 5.2 for the corresponding graph. For $T_3$ we obtain

$$C(\vec{x} - \vec{y}) \propto \exp \left( -\frac{||\vec{x} - \vec{y}||}{4.75 \text{ fm}} \right).$$  (5.3)

The fitted correlation function of sigma for temperature $T_3$, is shown in figure 5.3. Finally, we put all results of sigma to one graph in figure 5.4 and show sigma as function of $T$ and fit it to a third order polynomial:

$$\sigma(T) = (1.37 \times 10^{-5} (T - T_c)^3)/\text{MeV}^3 + 0.10.$$  (5.4)

![Exponential fit of the correlation function for $T = 297.0$ MeV.](image-url)

**Figure 5.2** Exponential fit of the correlation function for $T = 297.0$ MeV.
**Figure 5.3** Exponential fit of the correlation function for $T = 310.5$ MeV.

**Figure 5.4** Sigma as a function of temperature.
5.2 Formation of center domains

We study the time evolution of center domains via the average value of the standard deviation of $l_2$. Note again, in our simulation, there are only three distinct temperatures that we will consider according to the information in figure 5.4, that are $T_1 = 1.05 T_c = 283.5$ MeV, $T_2 = 1.1 T_c = 297.0$ MeV and $T_3 = 1.15 T_c = 310.5$ MeV.

From the simulation, we obtain three graphs of the standard deviation as a function of time, see figures 5.5, 5.6 and 5.7.

\[ SD \text{ of } l_2 \]

\[ \tau \text{ (fm/c)} \]

\[ 0 \quad 5 \quad 10 \quad 15 \quad 20 \quad 25 \]

\[ 0.05 \quad 0.1 \quad 0.15 \quad 0.2 \quad 0.25 \quad 0.3 \quad 0.35 \]

**Figure 5.5** Evolution of standard deviation from time 0 - 25 fm/c with $T_1$. 


Figure 5.6 Evolution of standard deviation from time 0 - 25 fm/c with $T_2$.

Figure 5.7 Evolution of standard deviation from time 0 - 25 fm/c with $T_3$. 
Finally, we put those three graphs into a comparing graph

![Graph](image)

**Figure 5.8** Evolution of standard deviation from time 0 - 25 fm/c with $T_1$, $T_2$ and $T_3$.

We can clearly identify the center domains by plotting $l_2$ as a function of $x$ and $y$ for a constant $z$ which is shown in figures 5.9 to 5.11. From the plots in figures 5.9 to 5.11, we can follow the formation of the center domains over time starting from small fluctuations at the initial state to the final state where we can clearly see the pattern of the center domains at time $t = 25$ fm/c. When the center domains are fully developed, the border lines between them are more sharp, the expansion of domains has stopped, and only very small fluctuations occur. From figures 5.9 to 5.11, we can also see that size of center domains is influenced by sigma, a higher sigma value of sigma results in a larger size of center domains.
Figure 5.9 Plan plot of $l_2$ from time 0 - 25 fm/c with $T_1$. 
Figure 5.10 Plan plot of $l_2$ from time 0 - 25 fm/c with $T_2$. 
Figure 5.11 Plan plot of $l_2$ from time 0 - 25 fm/c with $T_3$. 
5.3 Decay of center domains

We now study the decay of center domains via the average value of the standard deviation of $l_2$. Here, the standard deviation is defined again as in equation (4.5). After finishing the formation of center domains at $t = 25$ fm/c, we suddenly decrease all temperatures to a new value $T_1 = T_2 = T_3 = 200$ MeV. With this temperature below $T_c$, the center domains will start to decay. Investigation of decay of center domains here is started from $t = 26$ to 50 fm/c. Thus, from the simulation we obtain three graphs of standard deviation versus time, see figures 5.12, 5.13 and 5.14.

![Graph showing evolution of standard deviation from time 26 to 50 fm/c with $T_1$.](image)

**Figure 5.12** Evolution of standard deviation from time 26 - 50 fm/c with $T_1$. 
Figure 5.13 Evolution of standard deviation from time 26 - 50 fm/c with $T_2$.

Figure 5.14 Evolution of standard deviation from time 26 - 50 fm/c with $T_3$. 
Finally, we put those three graphs into a comparing graph

![Graph showing evolution of standard deviation from time 26 - 50 fm/c with $T_1$, $T_2$, and $T_3$.](image)

**Figure 5.15** Evolution of standard deviation from time 26 - 50 fm/c with $T_1$, $T_2$, and $T_3$

We can clearly identify center domains by plotting $l_2$ as a function of $x$ and $y$ for constant $z$ which is shown in figures 5.16 to 5.21. To investigate the decay of center domains, we focus on times when upper and lower peaks in standard deviations occur. Considering times at upper peaks, the center domains slowly decay to some small fluctuation state which looks similar to an initial state of our simulation. These final states have some unique values of the standard deviation depending on temperature. Focusing on lower peaks, center domains decay extremely fast to a fluctuation state which has a value of standard deviation lower than the final state. However, these standard deviation will be increased from peak to peak until the final state is reached.
Figure 5.16 Plan plot of $l_2$ from time 26 - 50 fm/c with $T_1$. 
Figure 5.17 Plan plot of $l_2$ from time 26 - 50 fm/c with $T_1$. 
Figure 5.18 Plan plot of $l_2$ from time 26 - 50 fm/c with $T_2$. 
Figure 5.19 Plan plot of $l_2$ from time 26 - 50 fm/c with $T_2$. 
Figure 5.20 Plan plot of $l_2$ from time 26 - 50 fm/c with $T_3$. 
Figure 5.21 Plan plot of $l_2$ from time 26 - 50 fm/c with $T_3$. 
CHAPTER VI

CONCLUSIONS AND DISCUSSIONS

In pure gauge theory, the Polyakov loop has the ability to distinguish between confined and deconfined phase. By constructing an effective Lagrangian from the Polyakov loop potential and a phenomenological kinetic term we can obtain equations of motion (3.7) and (3.8). Studying the evolution of the center domains in QGP can be done by a time-dependent Polyakov loop simulation which solves the equations of motion for the Polyakov loop value in (3+1) dimensions. In (Borsanyi et al., 2011), the domain size is shown to be dependent on temperature, furthermore, in our work, the domains size is also influenced by the surface tension (\( \sigma \)) which is a coefficient in the equations of motion (3.7) and (3.8). At high temperature or sigma, the domain size is big while at low temperature or sigma, the domain size is small, see figures 4.3 and 5.4. From our simulation, the result shows that, during the formation, small fluctuations around zero in the initial state evolve into domain structures with three different types of domains according to local breakdown of \( Z_3 \) symmetry. From our simulation, the estimated formation time of center domains is about \( 3 - 7 \) fm/c, see figure 5.8, significantly smaller than the estimated QGP lifetime in heavy-ion collision from (Bass and Dumitru, 2000) which is about \( 13 \) fm/c, see figure 6.1. The model that used in (Bass and Dumitru, 2000) is Hydro + UrQMD model. Here, open symbols show distributions for a purely hydrodynamical calculation, solid symbols show the
full calculation with hadronic rescattering. The given formation time is short enough to make center domains possible and necessary to consider in a QGP state. We can see fluctuations in the deviation during the decay of center domains, which takes about $7 - 20 \text{ fm/c}$ before returning to a state with small fluctuations at temperatures below $T_c$. 

Figure 6.1 Heavy-ion collision rates at LHC energy.
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APPENDIX A

RESEARCH WORKFLOW

Polyakov loop potential → Study thermodynamic properties

Numerical methods → Construct Lagrangian

Solve Polyakov loop from equations of motion with time-dependent

Time-dependent Polyakov loop simulation

Results

Polyakov loop Lagrangian → Find equations of motion

Equations of motion for Polyakov loop

Compare formation time

Conclusions

Phenomenological kinetic term
APPENDIX B

NATURAL UNITS

In natural units $\hbar = c = k_B = 1$, we have

$$\hbar = \frac{\hbar}{2\pi} = 6.5821 \times 10^{-25} \text{ GeV} \cdot \text{s} = 1, \quad (0.1)$$

and

$$c = 2.9979 \times 10^8 \text{ m} \cdot \text{s}^{-1} = 1, \quad (0.2)$$

where $\hbar$ is the Planck constant, $\hbar$ is the reduced Planck constant, $c$ is the speed of light, $k_B$ is the Boltzmann constant. From equations 0.1 and 0.2, we obtain a useful relation

$$\hbar c = 1 = (6.5821 \times 10^{-22} \text{ MeV} \cdot \text{s})(2.9979 \times 10^{22} \text{ fm} \cdot \text{s}^{-1}) = 197.3 \text{ MeV} \cdot \text{ fm} \quad (0.3)$$

where 1 fm (femtometer or fermi) = $1 \times 10^{-15}$ m. In high energy physics, energy and mass are frequently measured in a unit of GeV (giga electronvolt), meanwhile length and time are measured in a GeV$^{-1}$ or fm. Note that to make a difference between units of length and time, we can write unit of time as fm/c. Due to the relation in equation 0.3, we can easily convert between GeV$^{-1}$ and fm. In high energy physics, electromagnetic unit can be measured either in Lorentz–Heaviside units or Gaussian units, see table B.1.
| Unit                | Metric value   |
|---------------------|----------------|
| Length              | $1 \text{ eV}^{-1}$ | $1.97 \times 10^{-7} \text{ m}$ |
| Time                | $1 \text{ eV}^{-1}$ | $6.58 \times 10^{-16} \text{ s}$ |
| Mass                | $1 \text{ eV}$     | $1.78 \times 10^{-36} \text{ kg}$ |
| Temperature         | $1 \text{ eV}$     | $1.16 \times 10^{4} \text{ K}$   |
| Electric charge (L-H unit) | 1 unit of electric charge | $5.29 \times 10^{-19} \text{ C}$ |
| Electric charge (Gaussian unit) | 1 unit of electric charge | $1.88 \times 10^{-18} \text{ C}$ |
CODE IMPLEMENTATIONS

Code: Global variables

```fortran
module global_vars
  use, intrinsic :: iso_fortran_env
  implicit none

  ! variables for Polyakov loop potential
  real, parameter, public :: a = 664.0 ! [MeV]
  real, parameter, public :: b = 0.0075e9 ! [MeV^3]
  real, parameter, public :: hc = 197.3 ! [MeV*fm]

  ! sigma and temperatures
  real, public :: Temp=283.5 ! [MeV]
  real, public :: sigma = 0.08 ! []

  ! real, public :: Temp = 297 ! [MeV]
  ! real, public :: sigma = 0.3 ! []

  ! real, public :: Temp = 310.5 ! [MeV]
  ! real, public :: sigma = 0.7 ! []

  ! variables for time–dependent Polyakov loop simulation
  integer, parameter, public :: t_size = 10000 ! [] number of time step
  integer, parameter, public :: lattice_size = 101 ! [] size of lattice
  real, parameter, public :: delta_t = 0.005 ! [fm] time interval
  real, parameter, public :: delta_lattice = 0.5 ! [fm]
  integer, parameter, public :: x_size = lattice_size ! [] size of x
  integer, parameter, public :: y_size = lattice_size ! [] size of y
```
Code (Cont.): Global variables

```fortran
integer, parameter, public :: z_size = lattice_size ! [] size of z

! constants representing time at 1st, 2nd and 3rd time step
integer, parameter, public :: t_at_3 = 3 ! [] constant
integer, parameter, public :: t_at_2 = 2 ! [] constant
integer, parameter, public :: t_at_1 = 1 ! [] constant

! declare array l1 which contains real part Polyakov loop values
! and array l2 contains imaginary part Polyakov loop values
real, dimension(:,:,:,:), allocatable, public :: l1, l2

! array of standard deviations
real, dimension(t_size), public :: deviation

! output array of correlation function
real, dimension(t_size, int(lattice_size - 1)/2), public :: r_list

! dummy variables for handling index shifting
integer, public :: x_plus_1, y_plus_1, z_plus_1
integer, public :: x_minus_1, y_minus_1, z_minus_1

end module global_vars
```

Code: Differentiating Potential Function

```fortran
module misc_func
  use global_vars
  implicit none

  contains
    ! periodic boundary conditions
    integer function period_bound_conds(n)
      implicit none
```
integer, intent (in) :: n

if(n < 1) then
    period_bound_conds = lattice_size + n
else if(n > lattice_size ) then
    period_bound_conds = n - lattice_size
else
    period_bound_conds = n
end if

return
end function period_bound_conds

! apply PBCs
subroutine apply_period_bound_conds(z, y, x)
    implicit none
    integer, intent (in) :: z, y, x
    x_min_1 = period_bound_conds(x - 1)
    y_min_1 = period_bound_conds(y - 1)
    z_min_1 = period_bound_conds(z - 1)
    x_plus_1 = period_bound_conds(x + 1)
    y_plus_1 = period_bound_conds(y + 1)
    z_plus_1 = period_bound_conds(z + 1)
end subroutine apply_period_bound_conds

! function for finding standard deviation
real function find_deviation (l)
    implicit none
    real, dimension(z_size, y_size, x_size), intent (in) :: l
    integer :: z, y, x
    real :: l_avg

end subroutine find_deviation
real :: sum_val

l_avg = 0.0
sum_val = 0.0

do z = 1, z_size
  do y = 1, y_size
    do x = 1, x_size
      l_avg = l_avg + l(z, y, x)
    end do
  end do
end do

l_avg = l_avg/(real(z_size*y_size*x_size))

l_avg = l_avg/(real(z_size*y_size*x_size))

l_avg = l_avg/(real(z_size*y_size*x_size))

l_avg = l_avg/(real(z_size*y_size*x_size))

l_avg = l_avg/(real(z_size*y_size*x_size))

l_avg = l_avg/(real(z_size*y_size*x_size))

l_avg = l_avg/(real(z_size*y_size*x_size))

l_avg = l_avg/(real(z_size*y_size*x_size))

l_avg = l_avg/(real(z_size*y_size*x_size))

l_avg = l_avg/(real(z_size*y_size*x_size))

l_avg = l_avg/(real(z_size*y_size*x_size))

l_avg = l_avg/(real(z_size*y_size*x_size))

l_avg = l_avg/(real(z_size*y_size*x_size))

find_deviation = sqrt(sum_val/real((z_size*y_size*x_size)))

return
end function find_deviation

end module misc_func

Code: Differentiating Potential Function
module diff_U
use global_vars
implicit none
contains
! function for differentiating ln(P) by l1 or l2
real function diff_ln_P_func(diff_by, l1, l2)
  implicit none
  character(len=2), intent(in) :: diff_by
  real, intent(in) :: l1, l2
  real :: P, diff_P, diff_ln_P
  P = (1.0 - 6.0*(l1**2 + l2**2) &
   3.0*(l1**2 + l2**2)**2 &
   4.0*(2.0*l1**3 - 6.0*l1*l2**2))
  diff_ln_P = 1.0/P
  
  if(diff_by .eq. "l1") then
    diff_P = (-12.0*l1 - 12.0*(l1**2 + l2**2)*l1 &
     + 24.0*l1**2 - 24.0*l2**2)
  else ! or else if(diff_by .eq. "l2") then
    diff_P = (-12.0*l2 - 12.0*(l1**2 + l2**2)*l2 - 48.0*l1*l2)
  end if
  
  diff_ln_P_func = diff_ln_P*diff_P
  
  return
end function diff_ln_P_func

! function for differentiating potential by l1 or l2
real function diff_U_func(diff_by, l, l1, l2)
  implicit none
  character(len=2), intent(in) :: diff_by
  real, intent(in) :: l, l1, l2
diff_U_func = &
−b*Temp*(2.0*54.0*exp(−a/Temp)*l & ! [MeV^3][MeV]
+ diff_ln_P_func (diff_by, l1, l2) & ! [MeV^4]
/hc**3 ! [MeV^3*fm^3]
return
end function diff_U_func

end module diff_U

module solve_partial_diff
use global_vars
use misc_func
use diff_U
implicit none
contains
! function for solving Laplace function
real function laplace_l (l, t, z, y, x)
imPLICIT none
real, dimension(t_at_3, z_size, y_size, x_size), intent(in) :: l
integer, intent(in) :: t, z, y, x

call apply_period_bound_conds(z, y, x)

! operate laplace operator on l
laplace_l = ( &
l(t, z, y, x_plus_1) &
+ l(t, z, y_plus_1, x) &
+ l(t, z_plus_1, y, x) &
− 6.0*l(t, z, y, x) &
)
Code (Cont.): Finding Polyakov loop

```fortran
+ l(t, z, y, x_min_1) &
+ l(t, z, y_min_1, x) &
+ l(t, z_min_1, y, x) &
) / delta_lattice **2 ! [1/fm^2]

return
end function laplace_l

! function for solving diff equation to find Polyakov loop value
real function find_l(diff_by, l, t, z, y, x)
implicit none
character(len=2), intent(in) :: diff_by
real, dimension(t_at_3, z_size, y_size, x_size), intent(in) :: l
integer, intent(in) :: t, z, y, x

find_l = ( &
  2.0*l(t−1, z, y, x) − l(t−2, z, y, x) &
  + laplace_l(t−1, z, y, x) & ! [1/fm^2]
  * delta_t**2 & ! [fm^2]
) &
  &
  diff_U_func(diff_by, l(t−1, z, y, x), & ! [MeV/fm^3]
  l1(t−1, z, y, x), &
  l2(t−1, z, y, x)) &
  *hc & ! [MeV*fm]
  * delta_t**2 & ! [fm^2]
  )/(sigma*270.0**2) ! [MeV^2]

return
end function find_l

end module solve_partial_diff

Code: Two-point correlation function
module two_point_corr
use global_vars
use misc_func
implicit none
integer, parameter :: num_round = 1000
integer :: center_x, center_y, center_z
integer :: start_z, start_y, start_x
contains
! function for generating arbitrary center points.
subroutine rand_center(t)
  implicit none
  integer, intent(in) :: t
  real :: rand_z, rand_y, rand_x
  integer, dimension(12) :: date_time
  integer, dimension(12) :: seed

  call date_and_time(values=date_time)
call random_seed
  seed = date_time(6) * date_time(7) + date_time(8)
call random_seed(put=seed)

  call random_number(rand_z)
center_z = int(ceiling(rand_z* lattice_size ))
call random_number(rand_y)
center_y = int(ceiling(rand_y* lattice_size ))
call random_number(rand_x)
center_x = int(ceiling(rand_x* lattice_size ))

  return
end subroutine rand_center

! function for finding output of correlation function
function find_r(t)
  implicit none
integer, intent(in) :: t
integer :: x, y, z, x_index, y_index, z_index
integer :: start_x, start_y, start_z
integer :: end_x, end_y, end_z
integer :: r, i, j, k, round_loop
integer :: count_num
real, dimension(num_round, int(lattice_size - 1)/2) :: r_list
real, dimension(int(lattice_size - 1)/2) :: find_r

do round_loop = 1, num_round
   call rand_center(t)
do r = 1, (lattice_size - 1)/2
   count_num = 0
   r_list(round_loop, r) = 0
   start_z = center_z - r
   start_y = center_y - r
   start_x = center_x - r
   end_z = center_z + r
   end_y = center_y + r
   end_x = center_x + r
   do z = start_z, end_z
      do y = start_y, end_y
         do x = start_x, end_x
            if(((x - center_x)**2 + (y - center_y)**2 + &
               (z - center_z)**2 >= (r - 0.5)**2 .and. &
               (x - center_x)**2 + (y - center_y)**2 + &
               (z - center_z)**2 < (r + 0.5)**2) then
z_index = period_bound_conds(z)
y_index = period_bound_conds(y)
x_index = period_bound_conds(x)

\[ r_{\text{list}}(\text{round\_loop}, r) = r_{\text{list}}(\text{round\_loop}, r) + & \\
(l2(t, \text{center\_z}, \text{center\_y}, \text{center\_x}) & \\
* l2(t, z_{\text{index}}, y_{\text{index}}, x_{\text{index}})) \]
\[ \text{count\_num} = \text{count\_num} + 1 \]

end if
end do
end do
end do

r_{\text{list}}(\text{round\_loop}, r) = r_{\text{list}}(\text{round\_loop}, r)/\text{count\_num}
end do
end do

\[ \text{return} \]
end function find_r

end module two_point_corr
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