Heavy hypernuclei with $A = 3$ in a relativistic quark-gluon model

S.M. Gerasyuta and E.E. Matskevich

Department of Physics, St. Petersburg State Forest Technical University, Institutski Per. 5, St. Petersburg 194021, Russia

We generalized our approach to the hypernuclei with $A = B = 3$ containing one charm or one bottom quark. We derive the relativistic nine-quark equations using the dispersion relation technique. The hypernuclei as the system of interacting quarks and gluons are considered. The relativistic nine-quark amplitudes of hypernuclei, including the constituent quarks with the charm or bottom are calculated. The approximate solutions of these equations are obtained using a method based on the extraction of leading singularities of the amplitudes. The poles of the multiquark amplitudes allow us to determine the masses and the binding energy of hypernuclei with the $A = 3$. We predict the mass spectrum of hypernuclei with $A = 3$, which is valuable to further experimental study of the hypernuclei with charm and bottom.

PACS numbers: 11.55.Fv, 11.80.Jy, 12.39.Ki, 14.20.Pt.

I. INTRODUCTION.

Due to the increasing computational resources becoming available to the field, progress is being made toward a direct connection between QCD and nuclear physics using the numerical technique of lattice QCD (LQCD). Steady progress is being made toward this objective, but calculations at the physical light-quark masses have not yet been performed, and essentially only one lattice spacing has been used in calculation.

One reason that there are presently few LQCD calculations of $NN$ interacting is the significantly greater complexity of multi-nucleon systems as compared with systems of single mesons and baryons. A second reason is that significant computational resources are required to generate high-quality ensembles of gauge field configurations at or near the physical light-quark masses, an effort that has only become practical with the availability of petascale computers, and as yet these ensembles are not at sufficiently large volume to be of use in nuclear physics [1–14].

In our paper [15] the $^3$He also as the system of interacting quarks and gluons is considered. The relativistic nine-quark equations are found in the framework of the dispersion relation technique. The relativistic nine-quark amplitudes of $^3$He, including the $u, d$ quarks are calculated. The approximate solutions of these equations using the method based on the extraction of leading singularities of the amplitudes were obtained. The pole of the nonaquark amplitudes determined the mass of $^3$He.

Hypernuclei spectroscopy is enjoying an experimental renaissance with ongoing and planned program at DAΦNE, FAIR, Jefferson Lab, J-PARC and Mainz providing motivation for enhanced theoretical efforts (for a recent review, see Ref. [16]).

There are a number of theoretical predictions using various models: the chiral $SU(3)$ quark model [17], the flavor $SU(3)$ skyrmion model [18], the quark-delocation model [19, 20] and the quark cluster model [21, 22].

Lomon predicted a deuteron-like dibaryon resonance using R-matrix theory [23]. During the last several years, there has been a substantial effort to determine the $NN$ interactions directly from QCD using the numeral technique of LQCD. However, there is still little understanding of the connection to the underlying theory of the strong interactions, QCD, and the basic building blocks of nature, quarks and gluons. The main element in connecting QCD to nuclei is determined by the properties of hadrons, and can be described and predicted in terms of quark and gluon distribution. The calculated scattering lengths and effective ranges indicate that the pion is not the dominant contribution to the long range part of the nuclear force at these large light-quark masses, as anticipated from the single-hadron spectrum. This suggests that the form of the nuclear interactions, and the effective potentials that will reproduce the scattering amplitude below the inelastic threshold, is qualitatively similar to be phenomenological potentials describing the experimental scattering data at the physical pion mass.

In the recent paper [24] the relativistic six-quark equations are found in the framework of coupled-channel formalism. The dynamical mixing between the subamplitudes of hexaquark is considered. The six-quark amplitudes of dibaryons...
are calculated. The poles of these amplitudes determine the masses of dibaryons. We calculated the contributions of six-quark amplitudes to the hexaquark amplitudes.

In the previous paper \cite{25} the lowest hypernuclei are considered in the framework of the dispersion relation technique. The approximate solutions of the nine-quark equations using the method based on the extraction of leading singularities of the amplitude are obtained. The relativistic nine-quark amplitudes of low-lying hypernuclei, including the three flavors \((u, d, s)\) are calculated. The poles of these amplitudes determine the masses of the hypernuclei with the atomic (baryon) number \(A = B = 3\).

In Sec. II the relativistic nine-quark equations are derived for the hypernuclei with \(A = B = 3\) containing one charm or one bottom quark. The dynamical mixing between the subamplitudes of hypernuclei is considered. Sec. III is devoted to the calculation results for the masses of the low-lying charmed and bottom hypernuclei (Table I, II).

In conclusion, the status of this model is discussed.

II. NINE-QUARK AMPLITUDES OF HYPERNUCLEI.

In the paper \cite{25} the lowest hypernuclei with the atomic (baryon) number \(A = B = 3\): \(3^+YH, 3^+YHe, nnY\), where \(Y = \Lambda, \Sigma_0, \Sigma_+, \Sigma_-\) are considered in the dispersion relation technique. The approximate solutions of the nine-quark equations using the method based on the extraction of leading singularities of the amplitude are obtained. The relativistic nonaquark amplitudes of low-lying hypernuclei, including the three flavors \((u, d, s)\) are calculated. The poles of these amplitudes determine the masses of the hypernuclei.

In the present paper we generalize our method to the hypernuclei with \(A = B = 3\) containing one charm or one bottom quark. The relativistic nine-quark equations are derived using the dispersion relation technique. These are the relativistic generalization of the Faddeev-Yakubovsky \cite{26, 27} approach.

In constructing the equations for these nine-quark states the two-particle interactions of quarks are considered. We use the results of the paper \cite{28} in which the amplitudes of quark-quark interactions \(qq \rightarrow \bar{q}q a_{n}(s_{ik})\) are calculated in the framework of the dispersion \(N/D\) method with the input four-fermion interaction with quantum numbers of the gluon by using iteration bootstrap procedure:

\[
a_n(s_{ik}) = \frac{G_n^2(s_{ik})}{1 - B_n(s_{ik})} \tag{1}
\]

where, \(s_{ik}\) is the two-particle subenergy squared, \(n\) corresponds to the quantum numbers of channel, \(G_n(s_{ik})\) are the quark-quark vertex functions (Table III), and \(B_n(s_{ik})\) is the Chew-Mandelstam function \cite{20}:

\[
B_n(s_{ik}) = \int_0^{\Lambda_n} \frac{ds'_{ik}}{\pi} \rho_n(s'_{ik}) G_n^2(s'_{ik}) \frac{s'_{ik} - s_{ik}}{s_{ik}}. \tag{2}
\]

\(\rho_n(s_{ik})\) is the phase space:

\[
\rho_n(s_{ik}) = \left(\frac{\alpha_n}{(m_i + m_k)^2} + \beta_n + \delta_n \frac{(m_i - m_k)^2}{s_{ik}}\right) \times \sqrt{(s_{ik} - (m_i + m_k)^2)(s_{ik} - (m_i - m_k)^2)} \tag{3}
\]

The coefficients \(\alpha_n, \beta_n\) and \(\delta_n\) are given in Table III \(n = 1\) corresponds to \(qq\)-pairs with \(J^P = 0^+\), \(n = 2\) describes the \(qq\) pairs with \(J^P = 1^+\).

The current generates a nine-quark system. The correct equations for the amplitude are obtained by taking into account all possible subamplitudes. Then one should represent a nine-particle amplitude as a sum of the following subamplitudes:

\[
A = \sum_i A^1_i + \sum_{i,j} A^2_{ij} + \sum_{i,j,k} A^3_{ijk}. \tag{4}
\]
In this paper, the total amplitude of nine-quark states is represented as the sum of subamplitudes \( A_i \), \( A_j \) and \( A_{ijk} \), where \( i \), \( j \), \( k \) are quantum numbers of diquarks. \( A_i \) is amplitude of diquark and seven quarks. \( A_j \) corresponds to two diquarks and five quarks. Finally, \( A_{ijk} \) is just the three baryon state.

We consider only the planar diagrams, the other diagrams are neglected due to the rules of \( 1/N_c \) expansion [30,32]. The total amplitude can be represented graphically as a sum of diagrams. For example, we use Fig. 1 with the diquarks and five quarks. Finally, the reduced amplitudes are constructed:

\[
\alpha_i \equiv \lambda + 4 \alpha_1^{(1u)} I_{9}(1u)_{1u}^{(1u)} I_{12}(1u)_{1u}^{(1u)} + 3 \alpha_1^{(dd)} I_{12}(0d)_{1d}^{(0d)} + \alpha_0^{(ud)} (4 I_{9}(1u)_{0d}^{(1u)} + 12 I_{12}(1u)_{0d}^{(1u)})
\]

\[
+ 4 \alpha_1^{(0d)} I_{9}(1u)_{0d}^{(1u)} I_{0d}(1u)_{0d}^{(1u)} + 24 \alpha_2^{(0d)} I_{15}(1u)_{0d}^{(1u)} I_{15}(1u)_{0d}^{(1u)}
\]

\[
+ 24 \alpha_2^{(0d)} I_{14}(1u)_{0d}^{(1u)} I_{14}(1u)_{0d}^{(1u)} + 12 \alpha_2^{(0d)} I_{15}(1u)_{0d}^{(1u)} I_{15}(1u)_{0d}^{(1u)} + 6 \alpha_2^{(0d)} I_{15}(1u)_{0d}^{(1u)} I_{15}(1u)_{0d}^{(1u)}
\]

\[
\alpha_3 = \alpha_3^{1u} + \alpha_3^{0d} = \lambda + 4 \alpha_1^{(1u)} I_{9}(1u)_{1u}^{(1u)} I_{12}(1u)_{1u}^{(1u)} + 3 \alpha_1^{(dd)} I_{12}(0d)_{1d}^{(0d)} + \alpha_0^{(ud)} (4 I_{9}(1u)_{0d}^{(1u)} + 12 I_{12}(1u)_{0d}^{(1u)})
\]

\[
+ 4 \alpha_1^{(0d)} I_{9}(1u)_{0d}^{(1u)} I_{0d}(1u)_{0d}^{(1u)} + 24 \alpha_2^{(0d)} I_{15}(1u)_{0d}^{(1u)} I_{15}(1u)_{0d}^{(1u)}
\]

\[
+ 24 \alpha_2^{(0d)} I_{14}(1u)_{0d}^{(1u)} I_{14}(1u)_{0d}^{(1u)} + 12 \alpha_2^{(0d)} I_{15}(1u)_{0d}^{(1u)} I_{15}(1u)_{0d}^{(1u)} + 6 \alpha_2^{(0d)} I_{15}(1u)_{0d}^{(1u)} I_{15}(1u)_{0d}^{(1u)}
\]

\[
+ 12 \alpha_3^{(0d)} I_{18}(0d)_{0d}^{(0d)} + \alpha_3^{(0d)} (24 I_{16}(1u)_{0d}^{(1u)} + 12 I_{18}(0d)_{0d}^{(0d)})
\]

\[
\text{(5)}
\]

We consider the system of 23 equations in the case of \( \frac{3}{2} \) \( He \) (pp\( \Sigma^0 \)) state:

\[
\alpha_1 = 1^{uu}, 1^{dd}, 0^{ud}, 0^{uc}, 0^{dc}
\]

\[
\alpha_2 = 1^{uu}1^{uu}, 1^{uu}1^{dd}, 1^{uu}0^{ud}, 1^{uu}0^{uc}, 1^{uu}0^{dc}, 1^{dd}1^{dd}, 1^{dd}0^{ud}, 1^{dd}0^{uc}, 1^{dd}0^{dc}, 0^{ud}0^{ud}, 0^{ud}0^{uc}, 0^{ud}0^{dc}
\]

\[
\alpha_3 = 1^{uu}1^{uu}1^{dd}, 1^{uu}1^{uu}0^{dc}, 1^{uu}0^{ud}1^{dd}, 1^{uu}0^{ud}0^{dc}, 0^{ud}0^{ud}1^{dd}, 0^{ud}0^{ud}0^{dc}
\]

\[
\text{(6,7,8)}
\]

The coefficients of the coupled equations are determined by the permutation of quarks (Appendix A). The functions \( I_1 - I_18 \) (B1-B18) in the Appendix B) take into account the interactions of quarks and gluons.

The main contributions are calculated using the functions \( I_1 \) and \( I_2 \):

\[
I_1(ij) = \frac{B_1(s_0^{12})}{B_1(s_0^{12})} \int_{(m_1+2)^2} \int_{(m_2+2)^2} \frac{d s_{12} G_2(s_0^{12}) \rho_i(s_0^{12})}{\pi} \frac{1}{s_{12} - s_0^{12}} \int_1^{+1} dz_1(1) \frac{1}{2} \frac{1}{1 - B_j(s_1^{13})}
\]

\[
I_2(ijk) = B_1(s_0^{12}) B_2(s_0^{24}) \int_{(m_1+2)^2} \int_{(m_2+2)^2} \frac{d s_{12} G_2(s_0^{12}) \rho_i(s_0^{12})}{\pi} \frac{1}{s_{12} - s_0^{12}} \int_1^{+1} dz_1(2) \frac{1}{2} \frac{1}{1 - B_j(s_1^{13})}
\]

\[
\times \int_{z_3^{(2)^+}} z_3^{(2)^-} \frac{1}{\sqrt{1 - z_3^{(2)^+} - z_2^{(2)} - z_3^{(2)^-} + 2z_1(2)z_2(2)z_3(2)}} \frac{1}{1 - B_k(s_1^{24})}
\]

\[
\text{(9,10)}
\]

where \( i, j, k \) correspond to the diquarks.
TABLE I: Masses of charmed hypernuclei. Parameters of model: \( \Lambda = 9.0, g = 0.2122, m_{u,d} = 495 \text{ MeV}, m_c = 1655 \text{ MeV}. \)

| hypernuclei   | quark content | \( Q \) | \( I \) | \( J^P \) | mass, MeV | binding energy, MeV |
|---------------|---------------|--------|--------|----------|-----------|---------------------|
| \( pn \Lambda_c \left( \frac{1}{2}^+ \right) \) | uud udd udc | -2 | 0, 1 | \( \frac{1}{2}^+, \frac{3}{2}^+ \) | 4102 | 63 |
| \( pn \Sigma^+_c \left( \frac{3}{2}^+ \right) \) | uud udd udc | +2 | 0, 1, 2 | \( \frac{1}{2}^+, \frac{3}{2}^+ \) | 4029 | 229 |
| \( pn \Xi^+_c \left( \frac{3}{2}^- \right) \) | uud udd udc | +3 | +1 | \( 1, 2 \), \( \frac{3}{2}^+, \frac{1}{2}^+ \) | 4105 | 227 |
| \( pn \Xi^0_c \left( \frac{3}{2}^0 \right) \) | uud udd udc | +1 | -1 | \( 1, 2 \), \( \frac{3}{2}^+, \frac{1}{2}^+ \) | 4105 | 227 |
| \( pp \Lambda_c \left( \frac{1}{2}^+ \right) \) | uud udd udc | +3 | +1 | \( 1 \), \( \frac{3}{2}^+ \) | 4104 | 59 |
| \( nn \Lambda_c \) | udd udd udc | 0 | 0 | \( \frac{1}{2}^+ \) | 4014 | 63 |
| \( pp \Sigma^+_c \left( \frac{3}{2}^+ \right) \) | uud udd udc | +4 | +2 | \( 2 \), \( \frac{1}{2}^+ \) | 4044 | 286 |
| \( nn \Sigma^+_c \) | udd udd udc | -2 | 2 | \( \frac{1}{2}^+ \) | 4044 | 290 |
| \( pp \Sigma^0_c \left( \frac{3}{2}^0 \right) \) | uud udd udc | +2 | 0, 1, 2 | \( \frac{1}{2}^+ \) | 4144 | 186 |
| \( nn \Sigma^0_c \) | udd udd udc | +2 | 0, 1, 2 | \( \frac{1}{2}^+ \) | 4144 | 190 |

TABLE II: Masses of bottom hypernuclei. Parameters of model: \( \Lambda = 9.0, \Lambda_b = 4.43, g = 0.1682, m_{u,d} = 495 \text{ MeV}, m_b = 4840 \text{ MeV}. \)

| hypernuclei   | quark content | \( Q \) | \( I \) | \( J^P \) | mass, MeV | binding energy, MeV |
|---------------|---------------|--------|--------|----------|-----------|---------------------|
| \( \Lambda_b \left( \frac{1}{2}^- \right) \) | uud udd udc | +1 | 0 | \( \frac{1}{2}^- \) | 7340 | 168 |
| \( \Xi^+_b \left( \frac{3}{2}^+ \right) \) | uud udd udc | +2 | 0, 1, 2 | \( \frac{3}{2}^+, \frac{5}{2}^+ \) | 7366 | 323 |
| \( \Xi^-_b \left( \frac{3}{2}^- \right) \) | uud udd udc | 0 | -1 | \( 1, 2 \), \( \frac{3}{2}^+, \frac{3}{2}^- \) | 7366 | 328 |
| \( \Lambda_b \left( \frac{1}{2}^- \right) \) | uud udd udc | +2 | 1 | \( \frac{3}{2}^- \) | 7366 | 130 |
| \( \Xi^0_b \left( \frac{3}{2}^0 \right) \) | uud udd udc | 0 | -1 | \( 1 \), \( \frac{3}{2}^- \) | 7366 | 134 |
| \( \Xi^-_b \left( \frac{3}{2}^- \right) \) | uud udd udc | +2 | 0, 1, 2 | \( \frac{3}{2}^- \) | 7366 | 323 |
| \( \Xi^0_b \left( \frac{3}{2}^0 \right) \) | uud udd udc | +2 | 0, 1, 2 | \( \frac{1}{2}^- \) | 7400 | 292 |
| \( \Xi^-_b \left( \frac{3}{2}^- \right) \) | uud udd udc | +1 | 0, 1, 2 | \( \frac{3}{2}^- \) | 7400 | 291 |

III. CALCULATION RESULTS.

The poles of the reduced amplitudes \( \alpha_1, \alpha_2, \alpha_3 \) correspond to the bound states and determine the masses of hypernuclei with the atomic number \( A = 3 \) (Tables I, II).

The values of quark masses in our calculations are the usual for the heavy baryons in our model: \( m_{u,d} = 495 \text{ MeV}, m_c = 1655 \text{ MeV} \) and \( m_b = 4840 \text{ MeV}. \) For the charmed hypernuclei we use the two parameters: the gluon coupling constant \( g = 0.2122 \) and the cutoff \( \Lambda = 9 \) which are similar to the case of light hypernuclei.

The experimental data for the bottom hypernuclei are absent, therefore the mass of the heaviest bottom state \( \frac{3}{2}^- \) \( He \) (pp\( \Sigma^-_c \)) is described to be 7400 MeV. This differs from threshold value for this state by more than 200 MeV. This
TABLE III: The vertex functions and coefficients of Chew-Mandelstam functions.

| \( n \) | \( J^P \) | \( G^a_n(s_{kl}) \) | \( \alpha_n \) | \( \beta_n \) | \( \delta_n \) |
|---|---|---|---|---|---|
| 1 | 0^+ | \( \frac{4g}{3} - \frac{8m_f^2}{3s_{kl}} \) | \( \frac{1}{2} \) | \( \frac{1}{2} - \frac{4}{3} \left( m_k - m_l \right)^2 \) | 0 |
| 2 | 1^+ | \( \frac{2g}{3} \) | \( \frac{1}{2} \) | \( \frac{4m_k m_l}{s(m_k + m_l)} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) |

allows us to determine the value of gluon coupling constant \( g = 0.1682 \) for the case of bottom hypernuclei. Another two parameters \( \Lambda = 9 \) and \( \Lambda_b = 4.43 \) are taken similar to the ones of previous papers [25, 33]. The calculation results are represented in the Tables [II] [III].

IV. CONCLUSIONS.

In the papers [24, 34], we have developed the method of studying of hadrons and hypernuclei. This is the dispersion relations approach using the main postulates of \( S \)-matrix method. The relativistic generalizations of the three-body Faddeev equations in the form of dispersion relations is considered. The mass spectrum of the \( S \)-wave baryons, including \( u, d, s \) quarks was calculated by a method based on isolating the leading singularities in the amplitude. The approximate solution of the integral three-quark equations by taking into account the two-particle and triangle singularities is calculated. If we consider such approximation and define all the smooth functions of the subenergy variables (as compared with the singular part of the amplitude) in the middle point of the physical region of Dalitz-plot, the problem is reduced to the one of solving the system of algebraic equations.

In the present paper, the relativistic nine-quark equations in the framework of dispersion relation technique are derived. We predict the new hypernuclei with the one \( c \)- or \( b \)-quark. The states are calculated using the isospin \( I = 0, 1, 2 \) and the spin-parity \( J^P = \frac{1}{2}^+, \frac{3}{2}^+ \). The masses of nine-quark hypernuclei with \( A = 3 \) are degenerated. The electromagnetic effect is not included.

Acknowledgments

The reported study was partially supported by RFBR, research project No. 13-02-91154.

[1] M. Fukugita, Y. Kuramashi, H. Mino, M. Okawa, and A. Ukawa, Phys. Rev. Lett. 73, 2176 (1994).
[2] M. Fukugita, Y. Kuramashi, M. Okawa, H. Mino, and A. Ukawa, Phys. Rev. D52, 3003 (1995).
[3] S.R. Beane, P.F. Bedaque, K. Orginos, and M.J. Savage, Phys. Rev. Lett. 97, 012001 (2006).
[4] S.R. Beane et al. [NPLQCD Collaboration], Phys. Rev. D81, 054505 (2010).
[5] N. Ishii, S. Aoki, and T. Hatsuda, Phys. Rev. Lett. 99, 002001 (2007).
[6] S. Aoki, T. Hatsuda, and N. Ishii, Comput. Sci. Dis. 1, 015009 (2008).
[7] S. Aoki, T. Hatsuda, and N. Ishii, Prog. Theor. Phys. 123, 89 (2010).
[8] T. Yamazaki, Y. Kuramashi, and A. Ukawa, arXiv: 1105.1418 [hep-lat] (2011).
[9] T. Yamazaki, Y. Kuramashi, and A. Ukawa, Phys. Rev. D81, 111504 (2010).
[10] P. de Forcrand and M. Fromm, Phys. Rev. Lett. 104, 112005 (2010).
[11] S.R. Beane et al. [NPLQCD Collaboration], Phys. Rev. D85, 054511 (2012).
[12] T. Inoue et al. [HAL QCD Collaboration], Nucl. Phys. A881, 28 (2012).
[13] S.R. Beane, E. Chang, S.D. Cohen, W. Detwold, H.W. Liu, I.C. Luu, K. Orgince, A. Parreno et al., arXiv: 1206.5219 [hep-lat] (2012).
[14] T. Yamazaki, K.I. Ishikawa, Y. Kuramashi, and A. Ukawa, Phys. Rev. D86, 074514 (2012).
[15] S.M. Gerasyuta and E.E. Matskevich, arXiv: 1211.0667 [hep-ph] (2012).
[16] J. Pochodzalla, Acta Phys. Polon. B42, 833 (2011).
[17] Z.Y. Zhang, Y.W. Yu, X.Q. Yuan et al., Nucl. Phys. A670, 178 (2000).
[18] V.B. Kopeliovich, Nucl. Phys. A721, 1007 (2005).
[19] F. Wang, G.H. Wu, L.J. Teng and T. Goldman, Phys. Rev. Lett. 69, 2901 (1992).
[20] T. Goldman, K. Maltman, G.J. Stephenson Jr, J.-L. Ping and F. Wang, Mod. Phys. Lett. A13, 59 (1998).
[21] T. Kamae and T. Fujita, Phys. Rev. Lett. 38, 471 (1977).
[22] K. Yazaki, Prog. Theor. Phys. Suppl. 91, 146 (1987).
[23] P. LaFrance and E.L. Lomon, Phys. Rev. D34, 1341 (1986).
[24] S.M. Gerasyuta and E.E. Matskevich, Phys. Rev. D82, 056002 (2010).
[25] S.M. Gerasyuta and E.E. Matskevich, Phys. Rev. D87, 116006 (2013).
[26] O.A. Yakubovsky, Sov. J. Nucl. Phys. 5, 1312 (1967).
[27] S.P. Merkuriev and L.D. Faddeev, Quantum Scattering Theory for System of Few Particles (Nauka, Moscow, 1985) p. 398.
[28] V.V. Anisovich, S.M. Gerasyuta, and A.V. Sarantsev, Int. J. Mod. Phys. A6, 625 (1991).
[29] G. Chew, S. Mandelstam, Phys. Rev. 119, 467 (1960).
[30] G.’t Hooft, Nucl. Phys. B72, 461 (1974).
[31] G. Veneziano, Nucl. Phys. B117, 519 (1976).
[32] E. Witten, Nucl. Phys. B160, 57 (1979).
[33] S.M. Gerasyuta and E.E. Matskevich, Int. J. Mod. Phys. E20, 2443 (2011).
[34] S.M. Gerasyuta, Z. Phys. C60, 683 (1993).
Appendix A

In Fig. 1 the coefficient of the term $I_9(1^{uu}1^{uu}0^{dc}1^{uu})\alpha_1^{uu}$ is equal to 4, that is the number $4 = 2$ (the permutation of particles 1 and 2) $\times$ (the permutation of particles 3 and 4); the coefficient of the term $I_{12}(0^{dc}1^{uu}1^{uu}1^{dd})\alpha_1^{dd}$ is equal to 3 (we can replace the 7-th $d$-quark with the 8-th and 9-th $d$-quarks); the coefficient of the term $I_{13}(0^{dc}1^{uu}1^{uu}1^{dd}0^{dc})\alpha_2^{dd}$ is equal to 6, that is the number $6 = 3 \times 2$ (we can replace the 7-th $d$-quark with the 8-th and 9-th $d$-quarks, and then replace the 8-th $d$-quark with the 9-th $d$-quark); the coefficient of the term $I_{14}(1^{uu}1^{uu}0^{dc}0^{ud}0^{ud})\alpha_2^{0^{ud}0^{ud}}$ is equal to 24, that is the number $24 = 2$ (the permutation of particles 1 and 2) $\times$ (the permutation of particles 3 and 4) $\times$ 2 (we can replace the 7-th $d$-quark with the 8-th and 9-th $d$-quarks, and then replace the 8-th $d$-quark with the 9-th $d$-quark); the coefficient of the term $I_{15}(0^{dc}1^{uu}1^{uu}0^{ud}0^{dc})\alpha_3^{0^{ud}0^{ud}}$ is equal to 12, that is the number $12 = 2$ (the permutation of particles 1 and 2) $\times$ (the permutation of particles 3 and 4) $\times$ 2 (we can replace the 7-th $d$-quark with the 8-th and 9-th $d$-quarks, and then replace the 8-th $d$-quark with the 9-th $d$-quark) $\times$ 2 (the permutation of pairs (12) and (56)); the coefficient of the term $I_{18}(0^{dc}1^{uu}1^{uu}0^{dc}0^{dc})\alpha_3^{0^{dc}0^{dc}}$ is equal to 12, that is the number $12 = 2$ (the permutation of pairs (34) and (56)) $\times$ 2 (we can replace the 7-th $d$-quark with the 8-th and 9-th $d$-quarks, and then replace the 8-th $d$-quark with the 9-th $d$-quark).

The similar approach allows us to take into account the coefficients in all the diagrams and equations.

Appendix B

We used the functions $I_1$, $I_2$, $I_3$, $I_4$, $I_5$, $I_6$, $I_7$, $I_8$, $I_9$, $I_{11}$, $I_{12}$, $I_{13}$, $I_{14}$, $I_{15}$, $I_{16}$, $I_{18}$:

$$I_1(ij) = \frac{B_i(s_0^{13})}{B_i(s_0^{12})} \left( \frac{m_1 + m_2}2 \right) \int_{(m_1 + m_2)^2}^{(m_1 + m_2)^2} d\text{s'}_{12} \frac{G^2(s_0^{12})\rho_i(s'_{12})}{\pi} \frac{sz_1(1)}{2} \frac{dz_2(2)}{2},$$

$$I_2(ijk) = \frac{B_j(s_0^{13})B_k(s_0^{14})}{B_i(s_0^{12})} \left( \frac{m_1 + m_2}2 \right) \int_{(m_1 + m_2)^2}^{(m_1 + m_2)^2} d\text{s'}_{12} \frac{G^2(s_0^{12})\rho_i(s'_{12})}{\pi} \frac{sz_1(1)}{2} \frac{dz_2(2)}{2},$$

$$I_3(ijk) = \frac{B_k(s_0^{13})}{B_i(s_0^{12})B_j(s_0^{14})} \left( \frac{m_1 + m_2}2 \right) \int_{(m_1 + m_2)^2}^{(m_1 + m_2)^2} d\text{s'}_{12} \frac{G^2(s_0^{12})\rho_i(s'_{12})}{\pi} \frac{sz_1(1)}{2} \frac{dz_2(2)}{2}.$$
\[ I_4(ik) = I_1(ik), \tag{B4} \]

\[ I_5(ijkl) = I_2(ikl), \tag{B5} \]

\[ I_6(ijkl) = I_1(ik) \times I_1(jl), \tag{B6} \]

\[ I_7(ijkl) = \frac{B_k(s_0^{23} B_l(s_0^{25})}{B_i(s_0^{12} B_j(s_0^{34})^2 \times \frac{(m_1 + m_2)^2 \Lambda_i}{(m_3 + m_4)^2} \int \frac{ds'_{12} G_i^2(s_0^{12}) \rho_i(s_{12})}{s_{12} - s_0^{12}} \times \int \frac{ds'_{34} G_2^2(s_0^{34}) \rho_j(s_{34})}{s_{34} - s_0^{34}} \times \int \frac{d z_4(7)}{z_4(7)^{-}} \frac{1}{1 - B_k(s_0^{23}) - B_l(s_0^{25})},} \tag{B7} \]

\[ I_8(ijklm) = \frac{B_k(s_0^{25} B_l(s_0^{23} B_m(s_0^{46})}{B_i(s_0^{12} B_j(s_0^{34})^2 \times \frac{(m_1 + m_2)^2 \Lambda_i}{(m_3 + m_4)^2} \int \frac{ds'_{12} G_i^2(s_0^{12}) \rho_i(s_{12})}{s_{12} - s_0^{12}} \times \int \frac{ds'_{34} G_2^2(s_0^{34}) \rho_j(s_{34})}{s_{34} - s_0^{34}} \times \int \frac{d z_4(8)}{z_4(8)^{-}} \frac{1}{1 - B_k(s_0^{25}) - B_l(s_0^{23}) - B_m(s_0^{46})},} \tag{B8} \]

\[ I_9(ijkl) = I_3(ijl), \tag{B9} \]

\[ I_{11}(ijklm) = I_1(ik) \times I_2(ilm), \tag{B10} \]
\[ I_{12}(ijkl) = I_1(il), \]  
\[ I_{13}(ijklm) = I_2(ilm), \]  
\[ I_{14}(ijklm) = I_1(il) \times I_1(jm), \]  
\[ I_{15}(ijklm) = I_7(ijlm), \]  
\[ I_{16}(ijklmn) = I_8(ijlmn), \]  
\[ I_{18}(ijklmn) = I_1(il) \times I_2(jmn). \]

Here \( i, j, k, l, m, n, p, q \) correspond to the diquarks with the spin-parity \( J^P = 0^+, 1^+ \).

The other functions \( I_i \) can be neglected. The contributions of these functions are smaller of few orders as compared the functions (B1) – (B16). We do not take into account these functions in the systems of coupled equations.
Fig. 1. The graphical equations of the reduced amplitude $\alpha_3^{uu1uu0^{dc}}$ for the $\Sigma_c^+He \, pp \Sigma_c^0 \, I_3 = 0 \, J^P = \frac{1}{2}^+$. 