Universe creation on a computer

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Abstract

The purpose of this paper is to provide an account of the epistemology and metaphysics of universe creation on a computer. The paper begins with F.J. Tipler’s argument that our experience is indistinguishable from the experience of someone embedded in a perfect computer simulation of our own universe, hence we cannot know whether or not we are part of such a computer program ourselves. Tipler’s argument is treated as a special case of epistemological scepticism, in a similar vein to ‘brain-in-a-vat’ arguments. It is argued that Tipler’s hypothesis that our universe is a program running on a digital computer in another universe, generates empirical predictions, and is therefore a falsifiable hypothesis. The computer program hypothesis is also treated as a hypothesis about what exists beyond the physical world, and is compared with Kant’s metaphysics of noumena. It is argued that if our universe is a program running on a digital computer, then our universe must have compact spatial topology, and the possibilities of observationally testing this prediction are considered.

The possibility of testing the computer program hypothesis with the value of the density parameter $\Omega_0$ is also analysed. The informational requirements for a computer to represent a universe exactly and completely are considered. Consequent doubt is thrown upon Tipler’s claim that if a hierarchy of computer universes exists, we would not be able to know which ‘level of implementation’ our universe exists at. It is then argued that a digital computer simulation of a universe, or any other physical system, does not provide a realisation of that universe or system. It is argued that a digital computer simulation of a physical system is not objectively related to that physical system, and therefore cannot exist as anything else other than a physical process occurring upon the components of the computer. It is concluded that Tipler’s sceptical hypothesis, and a related hypothesis from Bostrom, cannot be true: it is impossible that our own experience is indistinguishable from the experience of somebody embedded in a digital computer simulation because it is impossible for anybody to be embedded in a digital computer simulation.

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F.J. Tipler has suggested that our universe could be a computer program running on a computer in another universe, (see, for example, p240-244 of Tipler 1989, and p206-209 of Tipler 1995). Tipler imagines a perfect computer simulation of our universe, which precisely matches the evolution in time of our own universe, and precisely represents every property of every entity in our universe. Such a simulation would simulate all the people who exist in our own universe. Such simulated people, suggests Tipler, would reflect upon the fact that they think, would interact with their apparent environment, and would conclude that they exist. Their experience would be indistinguishable from our own experience, and Tipler infers from this that we ourselves cannot know that we are not part of such a computer program. *Ex hypothesi*, there is nothing in our experience which could be evidence that we are not part of such a program, hence, it might be argued, we cannot know that we are not part of a computer program.

This argument is a type of epistemological scepticism, similar to Descartes’ dreaming argument. Descartes raised the possibility that one could experience a dream which is indistinguishable from the experience of a conscious, waking individual. The sceptical argument from this is that, *ex hypothesi*, there is nothing in one’s experience which could be evidence that one is not dreaming, hence one cannot know that one is not dreaming.

A modern version of this is the ‘brain in a vat’ hypothesis. Jonathan Dancy characterises this sceptical hypothesis as follows: “You do not know that you are not a brain in a vat full of liquid in a laboratory, and wired to a computer which is feeding you your current experiences under the control of some ingenious technician/scientist... For if you were such a brain, then, provided that the scientist is successful, nothing in your experience could possibly reveal that you were; for your experience is *ex hypothesi* identical with that of something which is not a brain in a vat. Since you have only your own experience to appeal to, and that experience is the same in either situation, nothing can reveal to you which situation is the actual one,” (Dancy 1985, p10).

One can identify two distinct premises in this argument:

(a). It is possible for a brain in a vat to be fed experience of an illusional world.

(b). It is possible for that experience to be indistinguishable from our own experience.

From these premises, the reasoning is as follows: Because the experience of the illusional world would be indistinguishable from one’s own experience, it is not possible to know whether or not one’s own experience is experience of a real world, or experience of an illusional world fed to a brain in a vat. Hence, it is not possible to know whether or not one is a brain in a vat.

There is, however, a vital ambiguity in the argument. There are two different senses in which real world experience could be indistinguishable from illusional world experience. One could claim either of the following two propositions:
1. The experience of the illusional world would be indistinguishable from the real world in terms of the detailed content of the experience.

2. The experience of the illusional world would be indistinguishable as experience from experience of the real world. In other words, the form of the illusory experience would be indistinguishable from the form of real-world experience.

It is not clear which of these claims Dancy is making. To illustrate the differences between these claims, consider the following scenarios:

Firstly, suppose that an individual is born in the real world, grows-up in the real world, and experiences the real world for 30 years, developing a range of cognitive skills, and accumulating a large collection of memories. Then, one night, whilst he lies asleep, the individual is unknowingly drugged and kidnapped by a scientist. As the victim lies unconscious in the scientist’s laboratory, his brain is removed and wired up to a computer. When the individual is allowed to recover consciousness, he wakes up to experience an illusional world controlled by the computer. Suppose that the individual retains his memories of the real world. To prevent the individual from having a reason to believe that he is a brain in a vat, the experience of the illusional world must be indistinguishable from the individual’s experience of the real world. Both the form and the detailed content of the individual’s illusory experience must be indistinguishable from his experience of the real world. The illusional world must have the same spatial layout and the same apparent history as that part of the real world known to the victim, and the illusional world must evolve according to the same laws that operate in the real world. The victim must feel that he experiences his world, and influences events in his world, with the same body that he possessed before he fell asleep the previous night. The victim must not recognize any difference between the real world and the illusional world that is not explicable by the laws of the real world. The victim must appear to perceive the same world he perceived before he fell asleep the previous night.

If these conditions were satisfied, then the individual would have no justification for believing that he is a brain in a vat. In accordance with conventional definitions of knowledge, if the individual would not be justified in believing that he is a brain in a vat, then he could not know that he is a brain in a vat.

It is possible to imagine other sceptical scenarios which do not require the detailed nature of the illusional world to be indistinguishable from the detailed nature of the real world. If an individual’s memories of the real world are deleted or suppressed, and apparent memories of an illusional world completely different from the real world are added, then experience of the illusional world would not give the individual reason to believe that he experiences an illusional world. The individual could experience an illusional world with a spatial layout and history totally different to the spatial layout and history of the real world. The illusional world could operate according to laws different to those that operate in the real world. Nevertheless, the experience of the illusional world would be indistinguishable, as experience, from experience of the real world. In other
words, the form of the illusory experience, if not the detailed content, would be indistinguishable from real-world experience.

To take another example, if an individual were fed illusory experiences from birth, that individual would have no memories of the real world. Hence, experience of an illusory world completely different from the real world in terms of detailed content, would not give the individual reason to believe that he experiences an illusional world.

It is not necessary to suppose that the individual who experiences an illusional world is an unwilling participant. It is possible, for example, that one's entire lifetime of experience upon 20th/21st century Earth, is part of a virtual reality game, played on a distant planet in the far-future. The technology of the far-future might enable game-players to play any role, in any factual or fictitious world. The game technology might suppress one's real-world memories, and supply the memories of the character one is playing. If the game technology suppressed one's real-world memories, one would be unaware of playing a virtual reality game. The game technology might even suppress one's real-world cognitive skills; one might experience birth, growth and mental development in the game world. Either way, one would have no memory of deciding to enter the game world. Once again, the sceptical argument is that one's own experience is indistinguishable from the experience of someone playing such a virtual reality game, hence one cannot know whether or not one is playing such a game.

Those sceptical arguments which require the detailed nature of the illusional world to be indistinguishable from the detailed nature of the real world, share a common point of vulnerability. It is possible for the hypothesis supporting such sceptical arguments to be false, and it is possible to know that it is false.

If the detailed content of the illusory experience is indistinguishable from the detailed content of real experience, then one can infer facts about the real world from one's experience, irrespective of whether one's experience is illusory or not. This allows one to determine, by scientific investigation, whether the hypothesis which supports the sceptical argument, is true or false.

For example, consider the brain in a vat argument. Recall that this sceptical argument is based upon the premise that it is possible for a brain in a vat to be fed experience of an illusional world. Because the illusional world would be indistinguishable, by hypothesis, from the real world, one's sensory systems and neurophysiology in the illusional world would be the same as one's sensory systems and neurophysiology in the real world. Hence, one could learn about one's real world physiology and neurology from one's experience, irrespective of whether one's experience is experience of the real world, or the illusional experience of a brain in a vat. One could not be led into forming false beliefs about the kind of entity one is without the violation of the indistinguishability condition.

Investigation of the human brain may reveal that it is impossible for it to be stimulated in a way which would produce experience indistinguishable from the experience of a person who is not a brain in a vat. Thus, the hypothesis upon which the sceptical argument is based, could be false. If one knew from neurophysiology that it is not possible for one to be a brain in a vat, then one
would know that one is not a brain in a vat. When Dancy characterises the brain-in-a-vat argument he states that “you have only your own experience to appeal to,” (Dancy 1985, p10). This is false because one can also appeal to one’s scientific understanding, based upon both theory and empirical evidence.

The other sceptical scenarios share this vulnerability: neurophysiological investigation of the brain could reveal that it is not possible for dreams to be indistinguishable from the experiences of a waking individual; research in micro-electronics, computer science, and human physiology, might conclude that totally authentic virtual reality is not possible.

Those sceptical arguments which do not require the detailed nature of the illusional world to be indistinguishable from the detailed nature of the real world, are more robust. If the detailed nature of the illusional world is different from the detailed nature of the real world, then one cannot necessarily learn about real world physiology and neurology from illusory experience. However, these more robust sceptical scenarios are dependent upon the following premise:

- Either it is possible to delete or suppress an individual’s memories of the real world, and to replace them with apparent memories of an illusional world, or it is possible to feed an individual with illusory experience from birth.

If this premise is false, then all the sceptical arguments which concern illusional worlds might be refuted by empirical investigation. It is, however, difficult to establish whether this premise is true or false. If scientific investigation reveals that it is impossible in our world to feed an individual illusory experiences from birth, and that it is impossible in our world to delete or suppress an individual’s memories, and replace them with apparent memories of an illusional world, then this alone does not establish whether the premise is true or false. If our world is an illusional world, and if the detailed nature of the illusional world is different from the real world, then scientific discoveries about our world, the illusional world, do not tell us anything about the real world.

It has been assumed in this section that it is possible to make a distinction between the form and content of experience. If such a distinction is not possible, then the sceptical scenarios must be re-categorised as follows:

1. An individual in our world experiences an illusional world which is indistinguishable from experience of our world. The individual is unaware that his experience is illusional precisely because the illusional experience is indistinguishable from experience of our world.

2. An individual in our world experiences an illusional world which is distinguishable from experience of our world. The individual is unaware of the difference, either because his memories of our world have been deleted or suppressed, or because he has experienced the illusional world from birth.
In case 1 the sceptical argument is as before, with the reference to the content of experience omitted. In case 2, the sceptical argument is as follows: If an individual in our world could experience an illusional world which is distinguishable from experience of our world, and if that individual could be made unaware that what he experiences is illusional, then our own world experience could be illusional experience, distinguishable from the real world. We cannot know whether or not our experience is experience of the real world, or experience of an illusional world different from the real world.

Tipler’s computer program hypothesis differs in one respect from the brain-in-a-vat type of hypothesis. The latter hypothesis suggests that an individual in a real world could be fed experiences of an illusional world, a world that does not objectively exist. Tipler’s computer program hypothesis suggests that an entire universe could be created as a computer program, and that many individuals could be created as part of the program. This hypothesis does not merely suggest that there is a computer program which is feeding illusory experiences to individuals who exist in a real world. Instead, individuals capable of experience are themselves created by the program, and the world they experience is just as real relative to them, as our world is relative to us. It is not Tipler’s claim that we cannot know whether or not our world is an illusional world. Instead, he claims that “we cannot know if the universe in which we find ourselves is actually ultimate reality,” (Tipler 1995, p208). Tipler’s claim is that we cannot know what level of reality we experience; that we cannot know whether or not the universe we experience has been created on a computer existing in another universe.

However, the hypothesis that our own universe is indistinguishable from a universe created on a computer, may be false. It will be demonstrated in section 3 of this paper that physical predictions follow from the hypothesis that our universe is a program running on a *digital* computer. For example, it follows that the structure of the universe must be discrete, and that the spatial universe must be compact. If these predictions are found to be false, then it is impossible for our universe to be a program running on a digital computer. If the predictions are falsified, then our universe is distinguishable from a universe created on a digital computer. Alternatively, if these predictions are found to be true, then it remains possible for our universe to be a program running on a digital computer. Empirical investigation is necessary to determine if Tipler’s computer program hypothesis is possible.

Nick Bostrom has proposed a distinct computer program hypothesis in which he proposes that future ‘posthuman’ civilizations will have the technological capability to create simulations “that are indistinguishable from physical reality for human minds in the simulation,” (Bostrom 2003, Section III). Bostrom’s simulation hypothesis is more anthropocentric than Tipler’s hypothesis, proposing not that an entire universe could be created as a computer program, and not, as Tipler proposes, that every property of every entity be simulated, but only “whatever is required to ensure that the simulated humans, interacting in
normal human ways with their simulated environment, don’t notice any irregularities,” (ibid.). Bostrom proposes only that “a posthuman simulator would have enough computing power to keep track of the detailed belief-states in all human brains at all times. Therefore, when it saw that a human was about to make an observation of the microscopic world, it could fill in sufficient detail in the simulation in the appropriate domain on an as-needed basis,” (ibid.). With the possible exception of macroscopic objects in inhabited areas, elements of the simulated world are created on-demand for the purposes of perception, and are not, in general, simulated independently of perception.

Whilst this paper does acknowledge the possibility of creating a computer simulation in which merely the experience of the participants in the simulation is indistinguishable from our own experience, the paper concentrates on the possibility of creating a universe on a computer which is indistinguishable from the realist conception of our own universe, i.e. this paper concentrates on the possibility of creating on a computer a universe in which objects, properties and processes are simulated independently of their perception by observers in the simulation. Bostrom’s hypothesis is less amenable to empirical test precisely because it doesn’t assume that empirical observation and measurement is indicative of an independently existing world. However, sections 4 and 5 of this paper, and, in particular, the argument that a digital computer simulation of a system cannot provide a realisation of such a system, carry equal weight against the hypotheses of Tipler and Bostrom.

2 The metaphysics of universe creation on a computer

The hypothesis that our universe is a program running on a computer in another universe is not merely a sceptical epistemological hypothesis, but a metaphysical hypothesis, in the sense defined below.

The term ‘metaphysics’ seems to have at least two different meanings. On the one hand, it is the study of that which possibly exists beyond the physical world. On the other hand, it is a whole group of philosophical subjects, such as the studies of time, causation, substance, and universals. These subjects seem to be united by the fact that they involve very general, foundational study of the nature of things.¹

For the purpose of this paper, metaphysics is defined to be the study of that which possibly exists beyond the empirically detectable world. In contrast, physics is defined to be the study of the empirically detectable world. The

¹The historical reasons for the double-meaning can be traced to Aristotle, as Barry Smith explains: “The books of Aristotle’s Physics deal with material entities. His Metaphysics (literally ‘what comes after the Physics’), on the other hand, deals with what is beyond or behind the physical world - with immaterial entities - and thus contains theology as its most prominent part. At the same time, however, Aristotle conceives this ‘metaphysics’ as having as its subject matter all beings, or rather being as such. Metaphysics is accordingly identified also as ‘first philosophy’, since it deals with the most basic principles upon which all other science rests,” (Smith 1995, p373).
hypothesis that our universe is a program running on a computer in another universe, is clearly a metaphysical hypothesis, in the very specific sense defined here. The hypothesis is that the computer hardware on which the program is running cannot be empirically detected by the beings represented in the software, hence the hypothesis is metaphysical rather than physical.

It is important to distinguish Tipler’s hypothesis from a metaphysically distinct proposal made by J.D. Barrow. Barrow suggests that “If we were to regard the Universe as a vast computer...then we can readily envisage the laws of Nature as some form of software which runs upon the particular forms of matter that form the world of strings and elementary particles,” (Barrow 1991, p160). In Tipler’s computer program hypothesis, the computer hardware is inaccessible to the people represented in the computer program; the constituents of matter, elementary particles or not, are just as much a part of the program software as the laws of physics. Presumably, each different type of particle or field would correspond to a different data type in the program. Each individual particle or field would then correspond to an instance of the relevant data type. In programming parlance, an instance of a data type is called a data object. Hence, the constituents of matter would correspond to data objects defined in the program. The laws of physics would correspond to the algorithms which act upon the data objects defined in the program. In general, entities would correspond to data objects in a computer program, and processes would correspond to algorithms. For example, an individual electron would correspond to a data object, and the Dirac equation would correspond to an algorithm capable of acting upon any electron data object. To give another example, in the geometrodynamical formulation of general relativity, a 3-manifold Σ, and the tensor fields (γ_i, K_i, φ_i) representing the intrinsic geometry γ_i, extrinsic geometry K_i, and matter fields φ_i at time i, would all correspond to data objects. The geometrodynamical evolution process would correspond to an algorithm which calculates (γ_{i+1}, K_{i+1}, φ_{i+1}) from (γ_i, K_i, φ_i).

After suggesting that our universe could be a computer program running on a computer in another universe, Tipler goes one step further, and claims that there is no need for a computer to be running the program. The state of memory of a digital computer can be treated as a long string of binary digits, and this represents a natural number in binary notation. Given that a computer program maps an initial memory state to a final memory state, a computer program can be treated as a mapping on the set of natural numbers. Tipler duly treats a program as an abstract mapping \( \mathbb{N} \rightarrow \mathbb{N} \), and claims that “if time were to exist globally, and if the most basic things in the physical universe and the time steps between one instant and the next were discrete,” (Tipler 1995, p208), then our universe could be in one-to-one correspondence with such an abstract object. Tipler acknowledges that the most basic things in the physical universe could be continuous, hence he proposes a further generalization of what a simulation is: “Let us say that a perfect simulation exists if the physical universe can be put into one-to-one correspondence with some mutually consistent subcollection of all mathematical concepts,” (ibid., p209).

This proposal does not merely suppose that mathematical Platonism is true,
that mathematical objects exist independently of the physical universe, in an abstract realm. Nor does it merely suppose that physical objects possess intrinsic mathematical properties. Instead, it supposes that physical objects can be identified with mathematical objects. As Barrow puts it, “We exist in the Platonic realm,” (Barrow 1992, p282). Whilst this is a fascinating idea, I shall restrict the discussion in this paper to the hypothesis that our universe is a program running on a computer in another universe.

The notion that there is something which exists beyond the empirically detectable world has famous precedents in the history of philosophy. Various types of thing have been postulated to exist beyond the physical world: mental entities, theological entities, and mathematical entities. These types of metaphysical suggestion are of no relevance to this paper. Rather, the focus of attention is the metaphysical hypothesis that there is something non-mental, non-deistic, and non-mathematical, which exists beyond our physical world. For example, Kant proposed that there are things-in-themselves, so-called ‘noumena’, which exist beyond the empirically accessible world. The metaphysics of the computer program hypothesis can be compared with the metaphysics of Kant’s noumena.

To recall, Kant suggested that there is a distinction between noumena and phenomena. The noumena are things in themselves, and the phenomena are the appearances of things in sensory perception. There are three possible ways of defining noumena. The noumena could be things which exist independently of sensory perception, they could be things which exist independently of empirical detectability, or they could be things which exist independently of cognition altogether. Obviously, things which exist independently of empirical detectability also exist independently of sensory perception, and things which exist independently of cognition also exist independently of empirical detectability.

If one merely stipulates that noumena are things which exist independently of sensory perception, then noumena could simply be things which are too small to see, like atoms and electrons. Things which are too small to see are still empirically detectable. As a classic example, an electron leaves a luminescent trail in a Wilson Cloud Chamber. The electron is not directly perceivable, but it is nevertheless detectable. Kant seems to suggest that noumena exist independently of both sense perception and empirical detectability of any kind. Further, Kant seems to hold that noumena are beyond cognition altogether. The computer program hypothesis holds that the states and processes of the computer in another universe, exist beyond both sense perception and empirical detectability, but these states and processes are not beyond cognition. What exists beyond the physical world is conceivable, according to the computer program hypothesis. In contrast, Kant seems to hold that we cannot even conceive what things in themselves are like.

Tipler’s computer program hypothesis is consistent with a threefold distinction between the phenomenal, the physical, and the metaphysical. This corresponds to the distinction between appearance, physical reality, and metaphysical reality. Appearances and phenomena consist of sensory experiences such as colours, sounds, and smells. Physical reality is the world described by physics, the world of atoms, electrons, and space-time. The hypothetical meta-
physical reality consists of the states and processes of a computer in another universe. In this threefold distinction, space and time exist independently of sensory appearances, whereas Kant believed that space and time are merely the format into which sensory experience is arranged. Unlike Kant, Tipler’s proposal does not relegate space-time to the merely phenomenal.

The computer program hypothesis is an interesting case because the global metaphysics is drawn from local physics. The nature of what lies beyond the entire physical universe (global metaphysics) is drawn from the nature of the computer, a part of the physical world (local physics).

3 Deriving empirical predictions from the metaphysical hypothesis

This section proposes that Tipler’s metaphysical hypothesis that our universe is a program running on a digital computer, entails that

- The universe is discrete
- The solutions to the fundamental evolution equations of physics must be computable functions
- The spatial universe has compact topology

These predictions are empirically testable, hence Tipler’s metaphysical computer program hypothesis is empirically testable. It will be demonstrated in this section that Tipler’s computer program hypothesis is potentially verifiable or falsifiable by astronomical observation.²

In Bostrom’s computer simulation hypothesis it would be possible for the simulators to create an ‘apparent’ space-time in which the universe appears to be continuous and of non-compact topology, even though the simulated world is actually discrete and of compact topology. The possibility of such illusions prevents Bostrom’s computer simulation hypothesis from having empirically testable predictions. However, as mentioned at the end of the opening section, Tipler hypothesizes a universe simulation which creates every property of every entity, and does not countenance the possibility of creating illusions for the participants in the simulation. Because, in Tipler’s hypothesis, empirical observation and measurement is indicative of objects, properties and processes which are simulated independently of their observation and measurement, Tipler’s hypothesis does have testable implications.

²None of the predictions above will be invalidated by the development of quantum computers. Although quantum computers might be able to perform certain calculations faster than computers based upon the notion of a Turing machine, the collection of uncomputable functions for a quantum computer is the same as the collection of uncomputable functions for a Turing machine. Like existing computers, quantum computers will possess a finite memory. And like existing digital computers, a quantum computer will only be able to represent discrete things.
J.D. Barrow has claimed that if our universe is a computer program, then all the laws of physics must involve computable functions, (Barrow 1991, p205). A computable function is defined to be a function whose value can always be calculated to arbitrary precision by performing a finite sequence of well-defined steps, often called an ‘effective procedure’.\(^3\) Certainly, if a universe unfolds in time on a computer, evolution equations must be used to calculate each time-step from the preceding time-step, and a solution of those evolution equations implemented on a computer must be a computable function. If the solutions of the fundamental evolution equations of physics were found to be non-computable functions, then the computer program hypothesis would be falsified. Whilst the computer program hypothesis therefore predicts that the solutions to the fundamental evolution equations of physics must be computable functions, computability would not be necessary to represent, at once, an entire space-time on a computer. Computability is only a requirement if the representation attempts to calculate one aspect of the universe from another aspect. As Tegmark remarks, “since we can choose to picture our Universe not as a 3D world where things happen, but as a 4D world that merely is, there is no need for the computer to compute anything at all - it could simply store all the 4D data,” (Tegmark 1998, p26).

Note also that algorithmic compressibility is not a necessary condition to represent a universe on a computer. A digital representation of something is defined to be algorithmically compressible if the length, in bits, of the shortest program capable of generating that digital representation, is shorter than the length, in bits, of the digital representation itself. Our universe might not be algorithmically compressible, but might still be digitally representable on a computer. What follows is an attempt to derive more specific empirical predictions from Tipler’s computer program hypothesis.

To represent the entire universe on a computer one must use either:

- A unified theory of everything.

or

- A set of different theories, each with its own limited domain of applicability, such that the set of domains covers the entire universe.

We do not, at present, have a unified theory of everything, but we do have a set of different theories, which grow progressively closer to covering the entire universe, in all its detail. Of these, the only empirically verified theory which is capable of describing the universe as a whole is general relativity. However, although general relativity can represent the universe as a whole, when it does so, it is only concerned with the large scale structure of the universe. It cannot represent detail on all length scales, as a unified theory of everything could be expected to do. Nevertheless, because general relativity has been empirically verified, the predictions of a unified theory of everything would have to converge

\(^3\) i.e for any function value \(f(x)\) and error margin \(\epsilon\), there is an effective procedure which yields a rational number \(r\) such that \(|f(x) - r| < \epsilon\).
to the predictions of general relativity within general relativity’s domain of applicability.

The physical predictions derived from Tipler’s metaphysical computer program hypothesis will be derived from an examination of how to represent a general relativistic universe on a computer. This is perhaps a weak point of the strategy. The universe may not be a 4-dimensional Lorentzian manifold, as it is represented to be in general relativity. We do not know what type of thing a unified theory of everything, incorporating a theory of quantum gravity, would represent the universe to be. It is, therefore, a provisional decision to consider a universe created on a computer to be a general relativistic universe.

In addition, the predictions derived assume that a digital computer is the only type of computer which has the potential to simulate an entire universe. Although it isn’t proven to be impossible for an analog computer to simulate an entire universe, the current evidence suggests that an analog computer cannot have the representational capacity to do so. An analog computer uses concrete (and continuous) physical quantities of one sort, (e.g. electrical quantities or hydraulic quantities), to represent concrete (and continuous) physical quantities of another sort, (e.g. the varying height of tides). In other words, an analog computer uses the concrete physical quantities of its physical components to represent the physical quantities of the system to be simulated. Early analog computers were constructed from levers, cogs, cams, discs and gears, and used mechanical motions to perform calculations. Modern analog computers tend to use electrical quantities, such as voltage levels, to represent the quantities of a simulated system, and specially designed circuits are used to perform arithmetic upon these voltage levels. Whilst an analog computer might use voltage levels to represent the values of quantities on a simulated system, a digital computer uses voltage levels to represent bits, and then sequences of bits encode the values of quantities on a simulated system. Analog computers tend to rely upon a mathematical resemblance between the pattern of quantity-values possessed by the machine and the pattern of quantity-values possessed by the simulated system. Analog computers do not use the versatile, encoded, abstract representation of physical quantities that a digital computer uses, and this limits their representational capacity.

It is often claimed that the variables of an analog computer are, in fact, continuously variable, but this claim can be disputed. Variables such as electrical voltage or fluid pressure are probably discrete when they are reduced to the quantum level. Even if there are other variables which are genuinely continuous, it would still not be possible to precisely control their value. Suppose for the sake of argument that voltage is continuously variable. It would be impossible to set a precise input voltage of, say, 5.34V. The best one could ever do is to set an input voltage within some interval, say 5.34V ± 0.01. This point is better

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4A good example of a mechanical analog computer is an orrery, a clockwork device for simulating the solar system. The actual position and motion of the balls representing the planets, represents the actual position and motion of the planets.

5As an exception, Hava Siegelmann (1999) has proposed neural net analog computers which are abstract encoders, like a digital computer.
illustrated for the case of irrational numbers. It is impossible to set an input voltage of $\pi$, and this is not because of the limitations of current technology, but because infinite precision is not attainable.

In general relativity, space is represented as a 3-dimensional differential manifold, and space-time is represented as a 4-dimensional differential manifold. Whilst every manifold has the cardinality of the continuum, a digital computer, as it is currently understood, can only deal with discrete items of data. The most crucial fact to recognize about a computer program is that the data objects defined in it are built from $\mathbb{Z}$, the set of integers. In contrast, the objects of analytic mathematics are built from $\mathbb{R}$, the set of real numbers. The memory of a classical, (i.e. non-quantum), digital computer consists of electronic circuits which have two possible voltage states. These voltage states are represented by binary digits, otherwise known as ‘bits’. An element of memory is therefore called a ‘bit’. Each bit of memory has two possible states, represented as 0 and 1. The set of possible states of a bit can be represented as $\mathbb{Z}_2 = \mathbb{Z}/2\mathbb{Z} = \{0, 1\}$, the additive group of integers, modulo 2. $\mathbb{Z}_2$ is a realisation of the cyclic group of two elements. Each byte of memory, a string of 8 bits, and the smallest addressable unit of memory, can be represented by $\left(\mathbb{Z}_2\right)^8 = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$, the 8-fold Cartesian product of $\mathbb{Z}_2$. Thus, for a classical computer with $n$ bytes of memory, the entire memory can be represented by $\left(\mathbb{Z}_2\right)^{8n}$, a discrete mathematical structure of $8n$ dimensions. All the data objects defined in a program correspond to regions of memory, hence the data objects defined in a program are built from subsets of the discrete mathematical structure $\left(\mathbb{Z}_2\right)^{8n}$.

The memory of a quantum computer consists of physical systems which possess a quantum state space isomorphic to the 2-dimensional complex Hilbert space $\mathbb{C}^2$. Each such memory element is referred to as a ‘qubit’, or ‘Qbit’. A string of $n$ qubits is represented by the $n$-fold tensor product of $\mathbb{C}^2$. Hence, the state of 8 qubits is represented by a vector in $\left(\mathbb{C}^2\right)^8 = \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$. As a consequence, the state of the $n$ qubits can be quantum mechanically entangled.

Each qubit is considered to have a fixed basis, $\{v_0, v_1\}$. Each vector in the $n$-fold tensor product consists of a complex linear combination of the $2^n$ basis vectors $\{v_{i_1} \otimes \cdots \otimes v_{i_n} : i_1 = 0, 1, \ldots, i_n = 0, 1\}$. The algorithms of a quantum computer correspond to unitary operators upon these complex Hilbert spaces. Because $\mathbb{C}^2$ is built from the set of real numbers, and because each qubit $\mathbb{C}^2$ possesses a continuum of quantum states, it might appear that a quantum computer can store an infinite amount of information. This appearance, however, is deceptive. Whilst there are a continuum of possible unitary operators on a qubit Hilbert space, each quantum computer will only be engineered to implement a finite collection. Moreover, each quantum computation must cease with a measurement of the state of the $n$ qubits, and this collapses the state
from a linear combination of the basis vectors into one of the fixed basis vectors, $v_{i_1} \otimes \cdots \otimes v_{i_n}$. The initial state on which the unitary transformations can operate is also such a state, and as Mermin comments, “the entire role of the state of the Qbits at any stage of a succession of unitary transformations is to encapsulate the probability of the outcomes, should the final measurement be made at that stage of the process,” (2002, p16). Thus, a quantum computer, like a classical computer, possesses a finite number of accessible states. In fact, $n$ qubits of memory possess exactly the same number of accessible states as $n$ bits of memory, namely $2^n$. The data objects defined in a program running on a quantum computer are discrete objects.

Because every manifold has the cardinality of the continuum, and because digital computers can only represent discrete objects, it is impossible to exactly represent a manifold on a digital computer. It is, therefore, impossible on a digital computer to exactly represent space and space-time as they are represented in general relativity.

If space and space-time actually are manifolds, and if a manifold cannot be exactly represented on a digital computer, then space and space-time cannot be exactly represented on a digital computer. If the space and space-time of our universe cannot be exactly represented on a digital computer, then our universe cannot be a computer program running on a digital computer in another universe.

However, as already mentioned, the space and space-time of our universe may not actually be manifolds. Space and space-time may not exactly be as they are represented to be in general relativity. Perhaps space and space-time are discrete, and perhaps the manifolds of general relativity only provide an idealisation of a discrete reality. The space and space-time of our universe can only be exactly represented on a digital computer if space and space-time actually are discrete.

Loop quantum gravity offers, perhaps, a mathematically rigorous means to quantize general relativity, and loop quantum gravity suggests that space is discrete in some sense. Using Ashtekar’s ‘new variables’ approach, canonical general relativity can be cast in the form of a canonical gauge theory, albeit a gauge theory with additional constraints to the Gauss constraint. The configuration space is a space of $SU(2)$-connections on a principal fibre bundle over a 3-manifold $\Sigma$. In loop quantum gravity, each closed curve (‘loop’) in the 3-manifold defines a functional on the space of $SU(2)$-connections. This functional is obtained by taking the holonomy of each connection around the loop, representing that group element as an operator on a vector space, and then taking the trace of that operator. Furthermore, each ‘spin network’ embedded in the 3-manifold defines a functional on the space of $SU(2)$-connections. A spin network, treated in isolation, is a discrete mathematical object consisting of a graph, (a collection of vertices and edges), an irreducible representation of $SU(2)$ assigned to each edge, and an ‘intertwining’ operator between such representations assigned to each vertex. Such a graph embedded in the 3-manifold $\Sigma$ defines a functional on the space of $SU(2)$-connections by taking the holonomy of a connection along each edge, using the representations to obtain operators
along each edge, forming the tensor product of all those operators, tensoring that with all the intertwining operators, and then contracting to obtain a number, the number assigned to the connection, (Baez 1995, p19). Such functionals turn out to be eigenvectors of operators which purportedly represent the area of surfaces in the 3-manifold and the volume of regions in the 3-manifold. Furthermore, these operators have discrete spectra.

If one accepts that quantum theory provides a complete description of a physical system, then, arguably, it is not the configurations of the classical system which exist, but the quantum state functional. Hence, in the case of loop quantum gravity, the 3-manifold used to define the classical configuration space does not exist. Rather, it is the state functional defined by the spin network which exists.

Many important questions remain. For example, the dynamics of loop quantum gravity remain intransigent, and there is no obvious classical limit to the theory. Whilst it is claimed that area and volume are discrete, what are they the area and volume of, if a 3-manifold does not exist? Are area and volume re-interpreted as properties of spin networks?

The established means of finding a discrete approximation to a manifold, is to find a cell complex which is homeomorphic to the manifold. In particular, one tries to find a simplicial complex which is homeomorphic to the manifold. The schema of the simplicial complex is a discrete mathematical object, which can be exactly represented on a computer. By representing the schema on a computer, one approximately represents the manifold.

If the schema of a simplicial complex is the natural discrete approximation to a manifold, then, conversely, the manifold can be said to be the natural continuum idealisation of the schema. If space and space-time are actually discrete, but if they can also be represented in a continuum idealisation as a 3-manifold and 4-manifold, respectively, then it is natural to suggest that space is actually a 3-dimensional schema, and space-time is actually a 4-dimensional schema. Regge calculus is generally considered to be the ‘discretized’ version of general relativity, and Regge calculus duly represents space and space-time as a simplicial complex.

Loop quantum gravity demonstrates that, although space and space-time might not be manifolds, they might not be the schema of simplicial complexes either. However, if space and space-time actually are discrete, it may be that

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6This should be distinguished from the Whitehead triangulation of a smooth manifold, which is a functor rather than an isomorphism. In dimension 6 and below, the equivalence classes of smooth manifolds up to diffeomorphism are in one-to-one correspondence with the equivalence classes of piecewise-linear (PL) manifolds up to PL-isomorphism, and every PL-manifold is PL-isomorphic to a simplicial complex, (Pfeiffer 2004, p17-18). This doesn’t entail that every smooth 4-manifold is such that its underlying topological manifold is homeomorphic with a simplicial complex. In dimension 4, there is a significant discrepancy between the category of topological manifolds up to homeomorphism, and the category of smooth manifolds up to diffeomorphism. There are topological 4-manifolds which have no differential structure, and therefore no Whitehead triangulation, and there are families of smooth 4-manifolds which share the same underlying topological manifold, up to homeomorphism, but which are pairwise non-diffeomorphic, and therefore have distinct Whitehead triangulations.
they are best represented by loop quantum gravity on small scales, and best represented by the schema of simplicial complexes on large scales.

Some explanation of the mathematics is in order here. An n-cell is an object which is homeomorphic with the n-ball in n-dimensional Euclidean space, $D^n = \{ x \in \mathbb{R}^n : \| x \| \leq 1 \}$. For example, a 2-ball is a disc, bounded by a circle, while a 3-ball is a solid ball bounded by a 2-sphere. Any polygon is homeomorphic with a 2-ball, and is therefore a 2-cell. Any solid polyhedron is homeomorphic with a 3-ball, and is therefore a 3-cell.

A cell-complex is obtained by pasting together any number of cells, so that the faces of the cells are either disjoint, or so that they coincide completely. A 3-dimensional cell-complex is obtained by pasting together 3-cells in such a way that the faces, edges and vertices of the cells are either disjoint, or they coincide completely.

The most interesting type of cell is a simplex. A 0-simplex is a point, or ‘vertex’; a 1-simplex is a line segment, or ‘edge’; a 2-simplex is a triangle; and a 3-simplex is a solid tetrahedron. By pasting together simplices, one obtains a simplicial complex, (see Stillwell 1992, p23-24). A 3-dimensional simplicial complex is obtained by pasting together solid tetrahedra. The schema of a 3-dimensional simplicial complex can be specified as follows. First, one declares all the vertices in the complex. Next, one can specify which subsets of the set of vertices correspond to simplexes. By specifying a pair of vertices, $\{ P_i, P_j \}$, one indicates that those vertices are connected by an edge. One can then specify which triples $\{ P_i, P_j, P_k \}$ of vertices correspond to the faces, and finally one can list which quadruples $\{ P_i, P_j, P_k, P_l \}$ of vertices correspond to the tetrahedra. One could alternatively give each edge a name, and then specify which triples of adjoining edges are connected by a face. One would then name each face, and specify which quadruples of adjoining faces are connected by a tetrahedron, (see Geroch and Hartle 1986, p546).

Although the manifold models of general relativity may be idealisations, one particular manifold model may eventually be verified by observation, to the exclusion of all others. To be specific, either a Friedmann-Roberston-Walker (FRW) model, a small perturbation of a FRW model, or an exact solution close to a FRW model, may be verified by astronomical observation. If the computer program hypothesis predicts that space or space-time is actually the schema of a simplicial complex on large scales, then the manifold model of the large-scale universe must be homeomorphic with a simplicial complex whose schema can be represented on a computer. It is therefore important to determine which manifold models of general relativity can be discretely represented on a digital computer by the schema of a simplicial complex. If a particular manifold model were to be verified by astronomical observation, but that model could not be represented by a schema on a digital computer, then the hypothesis that our universe is a computer program running on a digital computer would be falsified.

Suppose, then, that one tries to represent space-time on a computer with the schema of a 4-dimensional simplicial complex. Unfortunately, it is not known if every 4-manifold is homeomorphic to a simplicial complex, (Stillwell 1992,
Hence, there may be 4-manifolds which cannot be discretely represented by the schema of a simplicial complex. If the space-time of the universe has a manifold idealisation which does not have a homeomorphic simplicial complex, then the space-time of the universe would not be representable on a computer by the schema of a simplicial complex. If there were no other means of discretely representing such a 4-manifold on a computer, then the space-time of the universe would not be representable on a digital computer.

More seriously, because a computer can only store a finite amount of data, it can only represent the schema of a finite simplicial complex, a simplicial complex which contains a finite number of simplexes. A finite simplicial complex can only be homeomorphic to a compact manifold, hence only a compact 4-manifold is discretely representable by a schema on a computer. Now, a compact four-manifold can only possess a Lorentzian metric if its Euler characteristic is zero. If one hypothesizes that the entire 4-dimensional history of our universe is a representation on a computer, then one derives the potentially testable prediction that the topology of our space-time must be compact, and of Euler characteristic zero.

As an alternative, the geometrodynamical formulation of general relativity employs a so-called ‘3+1’ decomposition of space-time. One chooses a 3-manifold $\Sigma$, and one studies the time-evolution of the geometry and matter fields on $\Sigma$. As the geometry and matter fields evolve, a 4-dimensional space-time unfolds. Such a space-time will, of necessity, have the topology of $\mathbb{R}^1 \times \Sigma$.

The geometrodynamical formulation is advantageous because of Moise’s triangulation theorem for 3-manifolds, (Stillwell 1992, p25 and p242). Moise demonstrated that every 3-manifold is homeomorphic with a simplicial complex; one says that every 3-manifold can be ‘triangulated’. Although it is true that every n-manifold can be triangulated for $n \leq 3$, it is, to reiterate, unknown whether all 4-manifolds can be triangulated.

Moise’s theorem means that any possible topology of the spatial universe can be discretely represented with the schema of a 3-dimensional simplicial complex. Once again, however, a digital computer can only represent the schema of a finite simplicial complex. Whilst a compact 3-manifold is homeomorphic with a finite simplicial complex, a non-compact 3-manifold can only be homeomorphic with an infinite simplicial complex, a complex which contains an infinite number of simplexes.

Only a compact 3-manifold can be homeomorphic with a 3-dimensional simplicial complex whose schema is representable on a digital computer. Hence, if our universe is a program running on a digital computer, then our spatial universe must have a compact spatial topology in a continuum idealisation. Tipler’s

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7Not to be confused with Markov’s demonstration that the homeomorphism problem is unsolvable for triangulated 4-manifolds. In other words, given a pair of 4-manifolds which are homeomorphic to simplicial complexes, there is no algorithm to determine if they are mutually homeomorphic.

8This coincides with the Whitehead triangulation of a smooth 3-manifold. Every topological manifold of dimension 3 or lower has a unique PL-structure, and homeomorphic topological manifolds have PL-isomorphic PL-structures, (Pfeiffer 2004, p37).
hypothesis that our universe is a program on a digital computer, predicts that the spatial universe is discrete, and yields the potentially testable prediction that our universe has compact spatial topology in a continuum idealisation.

The prediction of compact spatial topology means that the Euclidean $\mathbb{R}^3$ and hyperbolic $H^3$ FRW universes are both inconsistent with Tipler’s computer program hypothesis. The only FRW universe which has both simply connected and compact spatial topology, is the $S^3$-universe. Hence, the only simply connected FRW universe which could be discretely represented on a computer, is the $S^3$-universe. There are, however, a host of multiply connected compact FRW universes. The spatial geometry of each such universe is obtained as a quotient $\Sigma/\Gamma$ of a simply connected Riemannian space form $\Sigma$, where $\Gamma$ is a discrete, properly discontinuous, fixed-point free subgroup of the isometry group $I(\Sigma)$, (O’Neill 1983, p243 and Boothby 1986, p406, Theorem 6.5).

Compact FRW models exist for any value of sectional curvature $k$. Of the 18 flat, $k = 0$, 3-dimensional Riemannian space forms, 10 are compact. Given that one can only create compact FRW universes on a computer, it follows that one can only create 10 topologically different $k = 0$ FRW universes on a computer.

All of the 3-dimensional Riemannian space forms of constant positive curvature are compact, hence they could all be created on a computer.

Whilst there are compact and non-compact quotients $H^3/\Gamma$, there are an infinite number of such compact quotients. The work of Thurston demonstrates that ‘most’ compact and orientable 3-manifolds can be equipped with a complete Riemannian metric tensor of constant negative sectional curvature. This means that ‘most’ compact, orientable 3-manifolds can be obtained as a quotient $H^3/\Gamma$ of hyperbolic 3-space.\(^9\) One can therefore create an infinite number of possible negative curvature FRW universes on a computer. However, there is no compact $k = -1$ space form which is globally homogeneous. $H^3$ itself is the only globally homogeneous 3-dimensional Riemannian space form of constant negative curvature, and $H^3$ is, of course, non-compact. Given that one can only create a compact universe on a computer, one cannot create a $k = -1$ FRW universe on a computer which is globally homogeneous. Thus, if our own universe is a globally homogeneous $k = -1$ FRW universe, it cannot exist on a computer. However, a locally homogeneous $k = -1$ FRW universe, with compact, multiply connected topology, could exist on a computer, and it is only local homogeneity which our astronomical observations are capable of detecting.

In practice it is difficult to test the prediction of compact spatial topology. Observational evidence currently indicates that our universe is a FRW universe, but there is no observable parameter in a FRW model which determines the spatial topology. Thus, there is no necessary link between the spatial topology of a FRW universe and the value of the density parameter $\Omega_0$; one cannot infer the spatial topology of our universe from $\Omega_0$.

However, in a ‘small’, compact, multiply connected universe, it is possible to see around the entire universe. To understand this, begin by recalling that a

\(^9\)A complete and connected Riemannian manifold of constant sectional curvature is called a Riemannian space form.

\(^{10}\)The meaning of ‘most’ in this context involves Dehn surgery, (Besse 1987, p159-160).
Riemannian manifold $(\Sigma, \gamma)$ has a natural metric space structure. The metric tensor $\gamma$ determines a Riemannian distance $d(p, q)$ between any pair of points $p, q \in \Sigma$. The Riemannian distance $d(p, q)$ is dimensionless, in the sense that it lacks any physical units. In a FRW model, it is the scale factor $R(t)$ which introduces physical units of distance. The physical distance between $p$ and $q$ at time $t$ is $R(t)d(p, q)$. Because $R(t)$ has physical units, so does $R(t)d(p, q)$.

For any FRW universe, one can calculate the maximum Riemannian distance, $d_{\text{max}}$, that light has travelled by a time $t_0$, which is considered to be the present time. The relevant equation is

$$d_{\text{max}}(t_0) = \int_0^{t_0} \frac{c}{R(t)} \, dt$$

A civilization located at some point $p$ in space, will, at time $t_0$, be able to see no further, in any direction, than a Riemannian distance of $d_{\text{max}}(t_0)$. This distance can therefore be referred to as the Riemannian horizon distance. It is, of course, a dimensionless quantity.

Now, recalling that the diameter of a metric space is the supremum of the distances which can separate pairs of points, it is a fact that any compact Riemannian manifold is a metric space of finite diameter. If one created, on a computer, a FRW universe in which $(\Sigma, \gamma)$ were a compact Riemannian manifold of sufficiently small diameter, $\text{diam} \ (\Sigma, \gamma)$, then the Riemannian horizon distance $d_{\text{max}}(t_0)$ could exceed $\text{diam} \ (\Sigma, \gamma)$ by the time $t_0 \sim 10^{10}$. If so, the horizon would have disappeared for the observers in that universe. They would be able to see their entire spatial universe. No point of their universe could be separated from them by a Riemannian distance greater than $\text{diam} \ (\Sigma, \gamma)$, so if $d_{\text{max}}(t_0) \geq \text{diam} \ (\Sigma, \gamma)$, then they would be able to receive light from all regions of their spatial universe.

In such universes, individual galaxies and clusters of galaxies would produce multiple images upon the celestial sphere of planet-bound observers, (see Ellis 1971). Different compact spatial topologies and geometries would produce different patterns of ghost images and multiple images upon the celestial sphere.

However, although compact spatial topology is a necessary condition for the entire spatial universe to be visible, it is not a sufficient condition. Our universe might have compact spatial topology, but if it is a ‘large’ compact universe, then all of space will not be visible. For all of space to be visible when the universe is only $\sim 10^{10}$ yrs old, the Riemannian manifold $(\Sigma, \gamma)$ which represents the spatial universe must be sufficiently small, as well as compact, Even if our spatial universe is small and compact, it would be extremely difficult to identify multiple images of galaxy clusters. Hence, although the presence of multiple images would verify the hypothesis of a small, compact universe, the fact that they have not been identified at the current time does not falsify the hypothesis. A better means of testing the hypothesis is to search for paired circles in the microwave background radiation. Recent research indicates that if such paired circles exist, then one could derive the spatial topology from the specific pattern of paired circles, (see Cornish, Spergel, Starkman 1998).
The CMBR power spectrum can also be used to determine whether our spatial universe is a small compact universe. A small compact universe would affect the CMBR power spectrum on large angular scales. The WMAP satellite has revealed anomalies in the CMBR power spectrum on large angular scales. The quadrupole $l = 2$ mode was found to be about $1/7$ the strength predicted for an infinite flat universe, while the octopole $l = 3$ mode was $72\%$ of the strength predicted for such a non-compact $k = 0$ universe, (Luminet et al 2003, p3).

The presence of paired circles or specific anomalies in the CMBR power spectrum would verify that the universe is spatially compact, and would thereby verify Tipler’s computer program hypothesis. Unfortunately, the absence of paired circles or anomalies in the power spectrum would not entail that the spatial universe is non-compact. Our universe could simply be a large compact universe. Hence, the absence of paired circles or anomalies in the power spectrum would not falsify the computer program hypothesis.

Predictions about the lifetime of our universe are easier to test than predictions about the spatial topology. The lifetime of our universe is determined by parameters such as the Hubble parameter $H_0$ and the density parameter $\Omega_0$, which can be inferred from observation. Hence, if the computer program hypothesis made predictions about the lifetime of our universe, it would be easier to test it. If a universe is represented by a Lorentzian manifold $(M, g)$, then the lifetime of the universe corresponds to the ‘timelike diameter’ of $(M, g)$. The timelike diameter of $(M, g)$ is the supremum of the length of all past-directed timelike curves in $(M, g)$. As Beem and Ehrlich comment, “the timelike diameter represents the supremum of possible proper times any particle could possibly experience in the given space-time,” (Beem and Ehrlich 1980, p329).

If a Lorentzian manifold with an infinite timelike diameter were represented by a numerical solution of the Einstein geometrodynamical equations, and if the size of the time steps in the numerical solution were constant, then an infinite number of time steps would be necessary. An infinite amount of information would have to be processed. The ever-expanding $k \leq 0$ FRW universes are examples of universes with an infinite lifetime. If a computer in a universe with an infinite lifetime could process information at a constant rate, then it could process an infinite amount of information. However, an ever-expanding universe will suffer an entropy ‘heat death’, the amount of free energy available converging to zero as $t \to \infty$. Brillouin’s inequality entails that there is a minimum, positive amount of free energy which must be expended to irreversibly process, or erase, a bit of information. Where $\Delta I$ is the amount of information processed in bits,

$$\Delta I \leq \Delta E / k_B T \ln 2.$$  

$\Delta E$ is the free energy expended, $T$ is the absolute temperature in degrees K, and $k_B$ is Boltzmann’s constant, (Barrow and Tipler 1986, p661). At first sight, this suggests that it is impossible to process an infinite amount of information in an ever-expanding universe because the amount of free energy converges to zero. However, the amount of energy which must be expended per bit of information
processed is temperature dependent. From the inequality above, one can derive the following constraint on the rate at which information can be processed:

\[ \frac{dI}{dt} \leq \frac{dE}{dt} \frac{1}{k_B T \ln 2}. \]

In turn, this entails the following constraint on the total amount of information \( I \) which can be processed between the current time \( t_0 \) and some future time \( t_f \), which might be \( \infty \):

\[ I = \int_{t_0}^{t_f} \frac{dI}{dt} dt \leq \left( \frac{k_B \ln 2}{T} \right) - 1 \int_{t_0}^{t_f} \frac{dE}{dt} dt. \]

If the temperature converges to zero, \( T \to 0 \), as it does in an ever-expanding \( \Omega_0 \leq 1 \) universe, then the amount of free energy which needs to be expended per bit converges to zero. Hence, although the amount of free energy converges to zero, so also does the amount of free energy which needs to be expended per bit. Thus, because the integral \( \int_{t_0}^{\infty} T^{-1} (dE/dt) dt \) can diverge, it may still be possible to process an infinite amount of information in an ever-expanding universe, even if the total free energy expended \( \int_{t_0}^{\infty} (dE/dt) dt \) is finite, (Barrow and Tipler 1986, p663).

In \( \Omega_0 > 1 \) universes, because the temperature diverges near the final singularity, the rate at which free energy is expended \( dE/dt \), and therefore the total energy expended \( \int_{t_0}^{t_f} (dE/dt) dt \), must diverge if the total information processed is to diverge.

If \( \Omega_0 \leq 1 \) in our universe, as current astronomical evidence indicates, then our universe has an infinite timelike diameter. Assuming that the simulation of such a universe would require an infinite amount of information to be processed, the possibility of the computer program hypothesis then rests upon whether it is physically possible for the integral \( \int_{t_0}^{t_f} T^{-1} (dE/dt) dt \) to diverge in either a \( \Omega_0 \leq 1 \) universe or a \( \Omega_0 > 1 \) universe. In both cases this remains a matter of debate. If it is not physically possible for the integral to diverge in either case, and if the observation that \( \Omega_0 \leq 1 \) in our universe is reliable, then could one conclude that our universe is not a program running on a computer in another universe? If it is impossible to process an infinite amount of information, then the only type of universe which could be entirely simulated on a computer would be a finite lifetime universe. However, as Bostrom might perhaps suggest, it remains possible that a partial simulation of a \( \Omega_0 \leq 1 \) universe could be created on a computer in another universe, a finite lifetime subset of the entire \( \Omega_0 \leq 1 \) universe. Hence, even if our universe is a \( \Omega_0 \leq 1 \) universe, and even if it is impossible to process an infinite amount of information, our universe could be a finite lifetime simulation running on a computer in another universe.

Not only could Tipler’s computer program hypothesis be falsified by empirical investigation, as considered above, but there are logical constraints upon what it is possible to simulate on a computer.
A computer is a finite volume subsystem of a universe, which is capable of representing the state of other systems. A system can represent, exactly and completely, the state of another system, if and only if the amount of information which can be coded in the first system is greater than or equal to the amount of information which can be coded in the other system. An entire universe is a special type of system. Hence, a subsystem of a universe $A$ can represent, exactly and completely, the state of a universe $B$, if and only if the amount of information which can be coded in the subsystem of $A$ is greater than or equal to the amount of information which can be coded in universe $B$.

As a special case, if the amount of information which can be coded in a subsystem of a universe $A$ is less than the amount of information which can be coded in the entire universe $A$, then it is impossible for the subsystem of universe $A$ to represent, exactly and completely, the entire universe $A$.

The amount of information which can be coded in a system is determined by the number of different possible states of the system. If $N$ denotes the number of possible states, then the amount of information $I$ which can be coded, in bits, is simply $I = \log_2 N$. Hence, if the number of possible states of a subsystem of a universe is less than the number of possible states of the entire universe, then it is impossible for that subsystem to represent, exactly and completely, the entire universe.

However, just because a system is a subsystem of a universe, it does not follow that the number of possible states of the system is less than the number of possible states of the universe. True, if the number of possible states of a subsystem is finite, then by virtue of being a subsystem, that finite number must be smaller than the number of possible states of the entire universe. For every state of a subsystem, there must be multiple states of the entire universe which induce the same state upon that subsystem, hence the number of possible states of the entire universe must be larger. However, if the number of possible states of a subsystem is not finite, then it is possible that it has the same number of possible states as the entire universe. *A priori*, it is quite possible that a subsystem of a universe $A$, and the entire universe $A$, both possess an infinite number of states. If the state space of a subsystem has the same cardinality as the state space of the entire universe, then, by definition, there exists at least one bijective mapping between the two state spaces. Any such bijective mapping would enable the states of the entire universe to be represented by the states of the subsystem.

This argument can be presented in another way. If a subsystem $I$ of our universe represents the entire universe $U$, it must also represent $I$ representing $U$. If it does this, it must also represent $I$ representing $I$ representing $U$. And so on, *ad infinitum*. This is possible only if the subsystem can store an infinite amount of information.

If the entire universe only has a finite number of possible states, then a subsystem will also have a finite number of states, and the number of subsystem states will be smaller than the number of universe states. However, if the entire universe has an infinite number of possible states, then it is possible for a subsystem to have either a finite number or an infinite number of possible
states.

If the entire universe has an infinite number of possible states, then it could conceivably possess either a continuous infinity of possible states, or a discrete infinity. A digital computer could only represent the universe exactly if the universe is discrete, hence the only case of interest here is the case in which the universe has a discrete infinity of possible states. A digital computer could only represent the universe exactly and completely if the entire universe and the computer subsystem of the universe both possess a discrete infinity of possible states. In other words, a digital computer could only represent the universe exactly and completely if both the computer and the universe can code a discrete infinity of information.

A computer is a finite volume subsystem of the universe, hence to determine if a computer could code the same amount of information as the entire universe, it is necessary to determine if a finite volume subsystem can code a finite or infinite amount of information. To answer this question, it is necessary to determine what the physical structure of the universe is.

At present, it appears that there are discrete levels of physical structure in the universe. All macroscopic material objects in our universe are composed of chemical elements and chemical compounds. The latter are composed of atoms in different combinations and organizations. Atoms are composed of electrons and atomic nuclei. The nuclei of atoms are themselves composed of protons and neutrons, which are themselves composed of quarks. The parts of material objects do not appear to lie on a continuum.

Electrons and quarks are purported to be elementary particles, pieces of matter which have no parts. If elementary particles do exist, then our universe could be said to have a finite lower level of structure. There would be no levels of structure below the level of elementary particles.

I propose that a finite volume subsystem is limited to coding a finite amount of information if and only if the following three conditions are satisfied:

- The number of structure levels available in a finite volume of space is finite.
- On each structure level, there is a finite set of parts in a finite volume of space.
- Each of the parts on each level of structure has a finite set of states.

A finite volume subsystem which satisfies these conditions has only a finite number of possible states, and therefore cannot code the same amount of information which can be coded in the entire universe.

To reiterate, a computer could only represent the universe exactly and completely if a finite volume subsystem can code a discrete infinity of information. It seems safe to assume that, on each level of structure, there is a finite set of parts in any finite volume of space. The Bekenstein bound\(^{11}\) and the so-called

\(^{11}\)Otherwise known as the universal entropy bound.
holographic bound of Susskind and ’t Hooft, purportedly entail that the parts on each level of structure have a finite set of states, (Bekenstein 2003). Moreover, the existence of elementary particles would mean that there is a finite set of structure levels in each finite volume of space. It would appear, therefore, at first sight, that all three conditions are satisfied. It would appear that a finite volume subsystem cannot code a discrete infinity of information, and it would appear that a computer cannot represent the universe exactly and completely.

However, further thought raises some doubts. Both the Bekenstein bound and the holographic bound place an upper limit on the entropy within a finite volume of space. Given a finite quantity of weakly self-gravitating energy \(E\) in a spherical volume of radius \(R\), which is isolated from other systems, (Bekenstein 2004), the entropy \(S\) is subject to the following upper bound:

\[
S \leq 2\pi ER/\hbar c.
\]

The holographic bound is independent of the quantity of energy, and places the following limit on the entropy of a spherical volume of radius \(R\), which is isolated from other systems:

\[
S \leq \pi c^3 R^2/\hbar G.
\]

In both cases, it is then assumed that a finite upper limit to the entropy of a finite volume of space entails a finite upper limit to the information storage capacity of that volume. This might be inferred from the following relationship:

\[
\text{Information} = \text{Maximum entropy} - \text{entropy}.
\]

By implication, it is the statistical states or macrostates of a system which are the bearers of entropy and information here. The states which provide a complete, detailed description of a system are referred to as ‘microstates’. A statistical state expresses only partial knowledge of the state of a system, and, in classical mechanics at least, corresponds to a probability distribution \(\rho\) defined upon the space of microstates \(\Gamma\). A macrostate is a set of macroscopically indistinguishable microstates \(\Gamma_M \subset \Gamma\), and corresponds to a special type of statistical state in which the probability distribution is of a constant value \(|\Gamma_M|^{-1}\) on \(\Gamma_M\), and zero elsewhere.\(^{12}\) The microstate of a system inherits the entropy and information of the macrostate to which it belongs. The entropy of an isolated system increases because the microstate of the system moves into macrostates of ever greater entropy. The equation above means that the information possessed by a system at a point in time is the difference between the maximum entropy of the system, and the entropy possessed by the system at that point in time. The maximum information which can be possessed by a system is that which it possesses when the system’s entropy is zero. Hence, according to the relationship above, the maximum information equals the maximum entropy.

Whether this entails that a finite volume of space possesses a finite number of states is a different question. In classical mechanics, a system consisting of n

\(^{12}\)\(|\Gamma_M|\) denotes the volume of \(\Gamma_M\).
particles has a 6n-dimensional continuum state space $\Gamma$, called the phase space. The entropy $S(\rho)$ of a statistical state $\rho$ in classical mechanics is defined to be

$$S(\rho) = -k_B \int_{\Gamma} \rho \log \rho \, d\mu,$$

where $k_B$ is Boltzmann’s constant. In the case of a macrostate $\rho_M$, this reduces to

$$S(\rho_M) = -k_B \int_{\Gamma_M} |\Gamma_M|^{-1} \log |\Gamma_M|^{-1} d\mu = k_B \log |\Gamma_M|.$$ 

Hence, although the entropy of a macrostate of such a system can be finite, it corresponds to a continuum of possible microstates. An upper limit to entropy does not entail a finite number of possible states. I propose that the link between entropy and information storage capacity is only valid for finite state-space systems. When a system has an infinite number of states, but a finite maximum entropy, I propose that it has an infinite information storage capacity. Ultimately, each different state of a system can represent different information, so a system with an infinite number of possible states, but a finite volume state space, and therefore a finite maximum entropy, nevertheless has an infinite information storage capacity.

To argue that a finite volume of space possesses a finite information storage capacity, one might alternatively start from loop quantum gravity, and try to argue that a finite volume of space only possesses a finite number of quantum states. A finite volume of space corresponds to a finite number of spin network nodes, and for a fixed finite number of nodes, there are a finite number of spin network states. For a system with a finite number of microstates, each macrostate $M$ corresponds to an equivalence class containing a finite number of microstates, $\text{Num}(M)$. The entropy of such a macrostate is simply

$$S = k_B \log \text{Num}(M).$$

Hence, a system with a finite number of microstates possesses a finite maximum entropy, and an upper limit on its information storage capacity.

Quantum theory, however, may not be the definitive theory of the physical world. A quantum state may correspond to many, or an infinite number of actual states. Even though there may be only a finite number of quantum states for a finite volume of space, there may be an infinite number of actual states. It may be that quantum theory is only valid for certain levels of structure, and it might merely be that the amount of information which can be coded above a certain length scale, or the amount of information which can be coded in a certain way, is finite.

There is also no decisive evidence that elementary particles exist. If the current candidates for elementary particles, such as quarks, do have parts, then those parts might only be detectable at energies which are not currently available in particle accelerators.
One could also dispute the assumption that, on each level of structure, there is a finite set of parts in any finite volume of space. If each part has a non-zero spatial extension with a well-defined boundary, and if the parts cannot inter-penetrate, then it does indeed follow that there can only be a finite set of parts packed into a finite volume of space. However, parts in quantum theory do seem able to interpenetrate each other to some degree. If there are levels of structure below the levels of the electron and quark, these might reveal very strange things, beyond the imagination even of quantum theory, such as an infinite number of parts interpenetrating each other in a finite volume of space.

Tipler claims that there could be a hierarchy of computer universes, just like the hierarchy of so-called ‘virtual machines’ which can exist on a computer, and he claims that we would not know which level of the hierarchy our own universe exists at. Whilst I have argued that the Bekenstein bound does not entail that a finite volume subsystem has only a finite number of possible states, Tipler accepts this implication. This, I propose, is inconsistent with the claim that we would not know which level of a universe hierarchy our own universe exists at.

When one computer is programmed so that it precisely mimics the input-output behaviour of another computer, the latter is said to be emulated by the former. The emulation program, running on the real computer, is said to be a virtual machine. A real machine $T_1$ can be programmed to emulate another, producing a virtual machine $T_2$. The virtual machine $T_2$ can then be programmed to emulate another computer, producing a higher level virtual machine $T_3$. These levels are referred to as levels of implementation.

A universe running on a computer could itself contain computers, upon which other universes are running. The universes would be running at different levels of implementation, and Tipler suggests, (1995, p208), that in this case, the levels should be thought of as levels of reality. Tipler seems to assume that there must be a lowest level of the hierarchy, and refers to this as ‘ultimate reality’. He claims that “we cannot know if the universe in which we find ourselves is actually ultimate reality,” (ibid.).

However, whilst any one computer may be able to emulate the input-output behaviour of another, that does not entail that any one computer has the same representational capacity as another. An actual computer, with a finite memory, does not have the same representational capacity as every other computer. A computer with $N$ bytes of memory does not have the same representational capacity as a computer with $M$ bytes of memory if $M > N$. There may be data structures which the computer with $M$ bytes of memory can represent, but which the computer with $N$ bytes cannot.

It was argued above that a computer with a finite set of states, (and hence a finite memory), cannot perfectly represent the universe to which it belongs. This is because a computer with a finite memory cannot code the same amount of information as the universe to which it belongs. In general, a computer with a finite memory cannot perfectly represent any universe which can code a greater amount of information than the computer. Any universe which can
code a greater amount of information than the universe to which the computer belongs, will code more information than the computer.

If one accepts Tipler’s claim that “complexity is appropriately measured by the number of possible alternative states a system can be in,” (1995, p118), then the complexity of a system can also be measured as the amount of information which that system can code.\textsuperscript{13} If one accepts that a finite volume subsystem has only a finite number of possible states, then a computer can only have a finite memory. If a computer can only have a finite memory, then a computer cannot perfectly represent a universe of the same complexity, or greater complexity, than the universe to which the computer belongs. The complexity of a universe is observable, hence, \textit{contra} Tipler, the levels of implementation are distinguishable. If a finite volume subsystem has only a finite number of possible states, then each higher level of universe implementation is less complex than the level below. A computer with a finite memory cannot perfectly represent a universe unless that universe is simpler than the universe to which the computer belongs. The more complex the universe one belongs to, the lower down the hierarchy that universe is placed. A universe of maximal complexity, if there is such a thing, could be proven to be the universe of ultimate reality.

If our universe is a computer program running on a computer in another universe, then that universe must have a higher level of complexity to our own. This greater complexity might take the form of a higher number of spatial dimensions.

Of course, if a finite volume subsystem has a discrete infinity of possible states, then a computer might be able to perfectly represent a universe with the same complexity as the universe to which the computer belongs. If so, then the levels of universe implementation might all have the same level of complexity. The point is that, if the Bekenstein bound does entail that a finite volume subsystem has only a finite number of possible states, then the Bekenstein bound is inconsistent with the thesis that universes at different levels of implementation are indistinguishable.

\textsuperscript{13}This should not be confused with the \textit{computational complexity} of an algorithm used to calculate the values of a function. This is a measure of the growth in computation time with the growth of the size of the input. For example, those functions which are computable by an algorithm in polynomial time, are referred to as \textit{P} problems. (See Penrose (1989), p181-187, for a good introduction). Tipler’s notion of complexity is also distinct from the \textit{Kolmogorov complexity} of an object, also known as the \textit{algorithmic complexity}. The Kolmogorov complexity of a bit-string is the length, in bits, of the shortest computer program capable of producing that bit-string as output. By extension, if one has a digital representation of an object by a bit-string, one can define the Kolmogorov complexity of that representation to be the length, in bits, of the shortest computer program capable of producing that digital representation as output.
4 Supervenience, identity, and universe creation on a computer

The suggestion that a physical system can be perfectly simulated on a computer is consistent with the principle of supervenience, but suggests that a physical system can be realised on more than one medium. Suppose, for example, that a tornado could be perfectly simulated on a computer. A tornado is described by a solution of the Navier-Stokes equations. To simulate a tornado on a computer, one would define program variables to represent the air pressure, velocity, density etc. in a volume of space, and one would represent the tornado by calculating a solution of the Navier-Stokes equations for these variables. Whilst a ‘real’ tornado is a process running on a collection of air molecules, a simulated tornado is a process running upon the components and circuitry of a computer. Hence, if a tornado could be perfectly simulated on a computer, one might argue that a tornado could be realised upon more than one medium. The processes associated with two completely different lower-level media, appear to be capable of yielding the same higher-level process. In the particular case of the simulation of a mind on a computer, Bostrom describes this as the notion of ‘substrate-independence’, arguing that “mental states can supervene on any of a broad class of physical substrates,” (Bostrom 2003, Section II).

Supervenience basically proposes that the parts of a system, and the way in which the parts are organized and interact, uniquely determine the higher-level states and properties of the system. In other words, the states and properties of the subsystems in a composite system, and the relationships between the subsystems, uniquely determine the higher-level states and properties of the composite system. The idea is that there can be no difference in the higher-level state of a composite system without a difference in the lower-level state, otherwise one would have a one-many correspondence between the lower-level states and higher-level states.

If a physical system could be realised on more than one medium, it would not undermine the principle of supervenience. For example, the properties of a tornado might not determine a unique medium upon which it must be realised, but the properties of air molecules, and the relationships between air molecules, entail that a tornado can be realised on a collection of air molecules. Similarly, if it were possible to realise a tornado on a computer, then it would be the properties of, and relationships between, the components and circuitry of a computer which would entail that a tornado could be realised upon a computer.

Whether or not a physical system can in fact be realised on more than one medium depends upon how one defines the identity of a system. In the case of a tornado there are two possible approaches:

(a). A tornado is a physical system composed of atmospheric molecules, which has the property that it satisfies a tornado-solution of the Navier-Stokes

\[\text{There is, for example, an exact solution of the Navier-Stokes equations called the Sullivan Vortex, which describes the flow in an intense tornado with a central downdraft.}\]
equations. The identity of a tornado is inseparable from being a collection of atmospheric molecules. A tornado is not as much realised upon a collection of atmospheric molecules, as it is composed of atmospheric molecules. A tornado cannot be realised on more than one medium because there is no sense in which a tornado is realised on any medium. It is only if the identity of a tornado could be defined in a formal, mathematical sense, that one could speak of a tornado being realised upon a medium.

(b). The identity of a tornado is defined by a solution of the Navier-Stokes equations, and the identity of a solution of the Navier-Stokes equations is independent of any particular medium, hence the identity of a tornado is independent of any particular medium. The identity of a tornado is independent of its realisation upon a collection of atmospheric molecules. If the components and circuitry of a computer could realise a tornado-solution of the Navier-Stokes equations, then a tornado could be realised on the components and circuitry of a computer.

The identity of a solution to the Navier-Stokes equations is independent of any particular physical medium because a solution of a differential equation is merely a mathematical object. A solution to a differential equation is given physical meaning when the solution variables are given a physical reference i.e. physical units. The solution variables of a differential equation can refer to many different things: consider, for example, the diverse domains in which one can find solutions to the wave equation or the diffusion equation.

When a solution of the Navier-Stokes equations is realised on a medium, the solution variables have physical referents. When a solution of the Navier-Stokes equations is realised on the medium of atmospheric molecules, the solution variables refer to air pressure, velocity, density etc. If, alternatively, a solution could be realised on, say, an economic system, then the solution variables would refer to economic quantities instead.

For a computer to be able to realise a tornado-solution of the Navier-Stokes equations, the computer must possess objective physical properties which could be the referents of the tornado-solution variables. Whilst it is permissible for these properties of the computer to be compound or collective properties, they must be objective physical properties. If a tornado-solution were to be realised on a computer, the solution variables would not refer to properties of the atmosphere, such as pressure, velocity, density etc. Instead, they would refer to properties of the computer components and circuitry, such as, perhaps, the voltage states of the bytes in computer memory. To reiterate, the medium upon which a solution is realised is defined by the referents assigned to the solution variables.

It is possible to accept approach (b), that the identity of a tornado is independent of any particular medium, without accepting that a tornado can be realised on a computer. A computer does not possess objective physical properties which can be the referents of the solution variables for the Navier-Stokes equations. One reason is that the solution variables are continuous, whilst the logical
states of electronic circuits are discrete. A tornado-solution to the Navier-Stokes equations is probably a bad example at this juncture because the Navier-Stokes equations, and fluid mechanics in toto, merely provide a phenomenological approximation.\footnote{Fluid mechanics is able to explain and predict a range of macroscopic phenomena to a certain degree of approximation, but more fundamental theories are required to describe what actually exists and happens.} A tornado-solution of the Navier-Stokes equations is not exactly realised on the medium of air molecules either. However, even if one goes down to the level of fundamental physics, a computer cannot exactly realise solutions to the fundamental equations of physics either. The reason is twofold:

- There is a one-many correspondence between the logical states and the exact electronic states of circuits.
- The logical states of multiple bits in computer memory only represent numbers because they are deemed to do so under a numeric-interpretation.

In contemporary digital computers, each bit of memory corresponds to an electrical circuit, and the two possible logical states of the bit correspond to different possible voltages between fixed points of the circuit. The logical state of 1 is not defined by a single precise voltage value, but by a range of values, and the logical state of 0 is defined by a different range of possible voltages. There is, therefore, a one-many correspondence between logical states and voltage levels. Successive runs of the same program will not produce exactly the same sequence of electronic states in computer memory. The exact voltage levels will be different on successive runs.

This level of electrical noise prevents a contemporary digital computer from exactly realising anything, even discrete objects. Given the one-many correspondence between logical states and exact electronic states, the exact electronic properties of a computer’s components cannot be the referents of the Navier-Stokes solution variables. At best, this suggests that a tornado-solution of the Navier-Stokes equations could only be approximately realised on a digital computer. This is crucial to the question of whether the same physical system can be realised on more than one physical medium. If there cannot be an exact realisation of a tornado on the medium provided by the components and circuitry of a computer, this is presumably because the properties of, and relationships between, the components and circuitry of a computer differ from the properties of, and relationships between, the air molecules in a region of the atmosphere.

Moreover, it is not even possible to contend that a tornado-solution of the Navier-Stokes equations could be approximately realised on a digital computer. A computer simulation provides no type of realisation at all. It is not the logical states of multiple electrical circuits in computer memory, but the numeric interpretation of the logical states which are the candidates to be referents of the tornado solution variables. It is the pattern of numbers represented by a computer which resembles the pattern of values realised by a simulated system’s physical quantities. As explained in the next section, the numbers represented...
by a computer are interpretation-dependent, hence the numbers represented by
a computer cannot be objective physical properties of the computer. If the num-
bers represented by the computer are interpretation-dependent, then the pattern
of numbers represented by the computer must be an interpretation-dependent
pattern. Hence, the resemblance between the pattern of numbers represented
by the computer and the pattern of values possessed by the physical quanti-
ties of a simulated system, must be an interpretation-dependent resemblance.
Change the interpretation of the logical states of the multiple electrical circuits
in computer memory, and there is no resemblance, not even an approximate
one. Even if there was no electrical noise, and even if the simulated system
was discrete itself, (even if there was a bijective correspondence), it would still
be an interpretation-dependent resemblance. The numbers represented by the
computer are not objective physical properties of the computer. To constitute a
realisation of a Navier-Stokes tornado-solution, the referents of the solution vari-
able must be objective physical properties, not interpretation-dependent, hence
a computer cannot realise a tornado-solution of the Navier-Stokes equations, or
any other physical system for that matter.

5 A digital computer simulation of a universe
cannot exist as a universe

A digital computer simulation of a physical system cannot exist as, (does not
possess the properties and relationships of), anything else other than a physical
process occurring upon the components of a computer. In the contemporary
case of an electronic digital computer, a simulation cannot exist as anything else
other than an electronic physical process occurring upon the components and
circuitry of a computer. The following argument will be deployed to establish
this conclusion:

1. A digital computer simulation is a type of representation.

2. There are three types of representation.

3. A digital computer simulation is a special case of the type of represen-
tation in which there is no objective relationship, and in particular no
homomorphy, between the represented thing and the thing which repre-
sents it.

4. If there is no objective relationship between a universe and a digital com-
puter simulation of a universe, then a digital computer simulation of a
universe cannot exist as a universe.

The reasoning that justifies claim 3, outlined at the end of the previous
section, is basically as follows: In a computer simulation, the values of the
physical quantities possessed by the simulated system are represented by the
combined states of multiple bits\textsuperscript{16} in computer memory. However, the combined states of multiple bits in computer memory only represent numbers because they are deemed to do so under a numeric interpretation. There are many different interpretations of the combined states of multiple bits in computer memory. If the numbers represented by a digital computer are interpretation-dependent, they cannot be objective physical properties. Hence, there can be no objective relationship between the changing pattern of multiple bit-states in computer memory, and the changing pattern of quantity-values of a simulated physical system.

Because a digital computer simulation of a universe cannot exist as a universe, it is, \textit{a fortiori}, impossible for anyone to be embedded in a digital computer simulation. It is impossible for our experience to be indistinguishable from the experience of someone embedded in a digital computer simulation because it is impossible for anyone to be embedded in a digital computer simulation.

Tipler and Bostrom both assume that if a universe is simulated on a computer, then the simulation exists as a universe, at a so-called ‘higher level of implementation’. This ontological assumption can be generalized to the following proposition: If a physical system of type $\mathcal{T}$ is simulated on a computer, then the simulation exists as a system of type $\mathcal{T}$, at a higher level of implementation. For example, if a tornado is simulated on a computer, it could be claimed that the simulation exists as a tornado, at a higher level of implementation. In opposition, it will be argued in this section that a digital computer simulation of a physical system, even a perfect simulation, cannot exist as the thing it represents.

A computer simulation is a special type of representation. In general, a representation is defined by a mapping $f$ which specifies the correspondence between the represented thing and the thing which represents it. An object, or the state of an object, can be represented in two different ways:

1. If an object/state is a structured entity $M$, it can provide the entire domain of a mapping $f : M \rightarrow f(M)$ which defines the representation. The range of the mapping, $f(M)$, is also a structured entity. The mapping $f$ is a homomorphism with respect to some level of structure possessed by $M$ and $f(M)$.

2. An object/state can be an element $x \in M$ in the domain of a mapping $f : M \rightarrow f(M)$ which defines the representation.

The representation of a Formula One car by a wind-tunnel model is an example of type-1 representation. There is an approximate homothetic isomorphism\textsuperscript{17} from the exterior surface of the model to the exterior surface of a Formula One car. This notion of structure preservation can be seen in other

\textsuperscript{16}Qubits in the case of quantum computers.

\textsuperscript{17}A transformation which changes only the scale factor.
cases of representation. The notorious map of the London Underground does not preserve geometry, but it does preserve the topology of the network. Hence in this case, there is a homeomorphic isomorphism involved.

Type-2 representation has two sub-types. The mapping \( f : M \rightarrow f(M) \) can be defined by either (2i) an objective, causal physical process, or by (2ii) the decisions of thinking-beings. The three different types of representation are similar to C.S. Peirce’s tripartite division of representational ‘signs’ into ‘icons’, ‘indices’, and ‘symbols’. Peirce held that icons resemble what they represent, indices are causally connected to what they represent, and symbols are arbitrary labels for what they represent, (see Schwartz 1995, p536-537).

The primary example of type-2i representation is the representation of the external world by brain states. Taking the example of visual perception, there is no homomorphism between the spatial geometry of an individual’s visual field, and the state of the neuronal network in that part of the brain which deals with vision. However, the correspondence between brain states and the external world is not an arbitrary mapping. It is a correspondence defined by a causal physical process involving photons of light, the human eye, the retina, and the human brain. The correspondence exists independently of human decision-making.

As an example of type-2ii representation, the state of a light switch could be used to represent things other than itself. One could decide that the On-position of a light switch represents the number 1, and the Off-position represents the number 0. This relationship between the states of the light switch and the set \( \{0, 1\} \) does not exist objectively. In other words, the relationship does not exist independently of the interpretative decisions made by human-beings. Someone else could decide that the On-position represents the number 0, and that the Off-position represents the number 1. One could even decide that the On-position of a light switch represents the colour black, and the Off-position represents the colour white. There is no homomorphism between the On-position of a light switch and either the number 1 or the colour black. The position of the light switch is merely being used as an element in the domain of a mapping which defines the representation.

In the case of a digital computer simulation, the bytes of memory are used to represent numbers and numbers are used to represent the quantities of the simulated system. Hence, the representation of a tornado by the logical states of a current digital computer is an example of type-2ii representation. There is no homomorphism between the electronic states or logical states of a current digital computer and the things those states are chosen to represent.18 The logical states of a computer can be mapped to many different things, (numbers, images, and sounds etc), but in each case a logical state is merely an element in the domain of the mapping which defines the representation. The logical state of a computer is not the domain of a homomorphic mapping, and human decisions,

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18Recall that there is a one-many correspondence between the logical states and the exact electronic states of computer memory. Although there are bijective mappings between numbers and the logical states of computer memory, there are no bijective mappings between numbers and the exact electronic states of memory.
rather than causal processes, determine what things the logical states of a digital computer represent. For these reasons, the states of a digital computer are not objectively related to that which they are deemed to represent.

Whilst the electronic states of a current digital computer do indeed possess a quite intricate structure, that structure is not used for the representational applications of a computer. The state of each bit in the memory of a computer is defined by the 1-dimensional graph topology of an electrical circuit, and by the voltage between specific points of the circuit. Hence, the memory-state of a computer is something which possesses a quite intricate structure. However, this electrical circuit and voltage structure bears no resemblance to the things which the memory of a computer is deemed to represent.

If a digital computer simulation of a universe is a type-2ii representation, then a digital computer simulation of a universe is not objectively related to that universe. This rules out the claim that a digital computer simulation could exist as a universe.

Bostrom’s hypothesis that we could be living in a computer simulation assumes that “it would suffice for the generation of subjective experience [in a computer simulation] that the computational processes of a human brain are structurally replicated in suitably fine-grained detail,” (Bostrom 2003, Section II). The notion of ‘structural replication’ is the same as the notion of isomorphism, so Bostrom’s hypothesis that we could be living in a computer simulation is based upon the false assumption that a computer simulation provides the type of representation in which an isomorphism or homomorphism exists. Hence, even if one endorses the notion of ‘structural realism’, that a thing is completely defined by its structural mathematical relationships, one cannot say that a digital computer simulation realises the thing it represents.

Although the states of a digital computer are not objectively related to the things they are deemed to represent, it is possible that the states of an analog computer could be so related. It is conceivable that there could be a homomorphism between the states of an analog computer and the things those states represent. Whilst an analog computer does not necessarily resemble the system it represents in terms of geometry or topology, a homomorphism between physical objects is not necessarily a homomorphism of spatial geometry or topology. The examples of a wind-tunnel model and the London Underground map are misleading in this respect. The homomorphism could be a non-visual homomorphism. An analog computer could possess objective physical properties which change with the same pattern as the changing pattern of values for the physical quantities on a simulated system. Hence, an analog computer simulation might provide type-1 representation. If this is so, then a more general argument would be required to demonstrate that no type of computer simulation at all could exist as a universe.

In the examples of type-1 representation given above, although there is a physical resemblance in some respects between $M$ and $f(M)$, there is not a total resemblance. For example, although the parts of a wind-tunnel model subtend
the same angles as the actual car, the wind-tunnel model is not the same size as the actual car. Despite such examples, there is no reason in principle why a type-1 representor cannot possess all the properties of the thing it represents. At least, there is no reason why a type-1 representor cannot possess all the ‘intrinsic’ properties of the thing it represents.

If a type-1 representor possesses all the intrinsic properties of the thing it represents, then one might conclude that it exists as the same type of thing as the thing it represents. Accordingly, an analog computer simulation of a universe might exist as a universe. However, to reiterate a point made in section 3, it remains to be proven that an analog computer can possess the representational capacity to represent an entire universe.

Tipler and Bostrom both imagine a computer simulation which would simulate all the people who exist in our own universe. Such simulated people, it is suggested, would reflect upon the fact that they think, would interact with their apparent environment, and would conclude that they exist. The claim that a simulated universe would be real to the simulated people, presupposes that simulated systems provide realisations of those systems, and presupposes that simulated people exist as people. Digital computer simulations of people exist only as physical processes on a computer, not as people. Hence, there are no people in a digital computer simulation to reflect upon the fact that they think, or to interact with their apparent environment.

If a digital computer simulation of a universe cannot exist as a universe, then the sceptical hypotheses of Tipler and Bostrom cannot be true. It is impossible that our own experience is indistinguishable from the experience of somebody embedded in a digital computer simulation because it is impossible for anybody to be embedded in a digital computer simulation. Systems cannot be realised in digital computer simulations, and people cannot exist in digital computer simulations.

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