1. INTRODUCTION

The expansion of the universe can be quantitatively studied using the results from a variety of cosmological observations, for example, the mapping of the cosmic microwave background (CMB) anisotropies (Spergel et al. 2007; Komatsu et al. 2010), the measurement of baryon acoustic oscillation (BAO) peaks (Eisenstein et al. 2005; Percival et al. 2010), the linear growth of large-scale structures (Wang & Tegmark 2004), strong gravitational lensing (Yang & Chen 2009), and measurements of “standard candles” such as the redshift–distance relationship of Type Ia supernovae (SNIa) data, observations of BAO peaks have also been used to determine the cosmic scale factor and its rate of change with respect to the cosmic time. In practice, the Hubble parameter is usually measured as a function of the redshift, and the expansion history of the universe by its definition: $H = \dot{a}/a$, where $a$ denotes the cosmic scale factor and $\dot{a}$ is its rate of change with respect to the cosmic time. In the rest of this paper, we will use the abbreviation “OHD” for observational $H(z)$ data.

This interest of ours is expressed by the following two questions: Can future observational determinations of the Hubble parameter be used as a viable alternative to current SNIa data? If so, how many more data points are needed so that the cosmological parameter constraints obtained from $H(z)$ data are as good as those obtained from SNIa distance-redshift relations?

We attempt to illustrate possible answers to these two questions via an exploratory, statistical approach. This paper is organized as follows. We first briefly summarize the current status of available OHD results in Section 2. Next, we show how the simulated $H(z)$ data sets are used in our exploration in Section 3, and present the results from the simulated data in Section 4. In Section 5, we turn to the data expected from future observation programs. Finally, in Section 6 we discuss the limitation and implications of our results.

2. AVAILABLE HUBBLE PARAMETER DATA SETS

Currently the amount of available $H(z)$ data is scarce compared with SNIa luminosity distance data. Jimenez et al. (2003) first obtained one $H(z)$ data point at $z \approx 0.1$ from observations of galaxy ages (henceforward the “JVT03” data set). Simon et al. (2005) further obtained eight additional $H(z)$ values up to $z = 1.75$ from the relative ages of passively evolving galaxies (henceforward “SVJ05”) and used it to constrain the redshift dependency of dark energy potential. Stern et al. (2010a) obtained an expanded data set (henceforward “SVJKS10”) and combined it with CMB data to constrain dark energy parameters and the spatial curvature. Besides $H(z)$ determinations from galaxy ages, observations of BAO peaks have also been used to extract $H(z)$ values at low redshift (Gaztaña et al. 2009, henceforward “GCH09”). These data sets are summarized in Table 1 and displayed in Figure 1.

These data sets have seen wide application in cosmological research. In addition to those mentioned above, Yi & Zhang (2007) first used the SVJ05 data set to constrain cosmological model parameters. Samushia & Ratra (2006) also used the data to constrain parameters in various dark energy models. Their results are in consistence with other observational data, in particular the SNIa. Besides parameter constraints, OHD can...
Figure 1. Full OHD set and best-fit \( \Lambda \)CDM models. The top panel shows the data set and fit results. The bottom panel shows the residuals with respect to the best-fit model with Gaussian prior on \( H_0 \). The \( H_0 \) priors are described in Section 3.2.

also be used as an auxiliary model selection criterion (Li et al. 2009).

In this paper, the OHD sets used are taken from the union of JVTS03, GCH09, and SJVKS10. The SJV05 data set, having been replaced by SJVKS10, is no longer used, and is listed in Table 1 for reference only. Using these data sets, we find the parameter constraints for a non-flat \( \Lambda \)CDM universe, \( \Omega_m = 0.37^{+0.15}_{-0.16} \) and \( \Omega_{\Lambda} = 0.93^{+0.25}_{-0.29} \), assuming the Gaussian prior \( H_0 = 74.2 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1} \) suggested by Riess et al. (2009). We also use a conservative, “top-hat” prior on \( H_0 \), namely, a uniform distribution in the range [50, 100], and obtained \( \Omega_m = 0.34^{+0.20}_{-0.27} \) and \( \Omega_{\Lambda} = 0.86^{+0.64}_{-0.57} \). These \( H_0 \) priors are discussed in detail in Section 3.2, and their effects on the parameter constraints are illustrated in Section 4.

### 3. PARAMETER CONSTRAINTS WITH SIMULATED DATA SETS

In the current absence of more OHD, we turn to simulated \( H(z) \) data sets in the attempt to explore the answer to the questions raised in Section 1. To proceed with our exploration, we must prepare ourselves with (1) a way of generating simulated \( H(z) \) data sets, (2) an “evaluation” model (or class of models) of cosmic expansion in which parameter constraint is performed using the simulated data, and (3) a quantified measure of the data sets’ ability of tightening the constraints in the model’s parameter space, i.e., a well-defined “figure of merit” (FoM). These topics are discussed in detail in the rest of this section.

#### 3.1. Generation of Simulated Data Sets

Our simulated data sets are based on a spatially flat \( \Lambda \)CDM model with \( \Omega_m = 0.27 \) and \( \Omega_{\Lambda} = 0.73 \). This fiducial model is consistent with the seven-year Wilkinson Microwave Anisotropy Probe (WMAP; Komatsu et al. 2010), the BAO (Percival et al. 2010), and SNIa (Hicken et al. 2009a) observations. Therefore, it summarizes our current knowledge about the recent history of cosmic expansion fairly well. In this fiducial model, the Hubble parameter is expressible as a function of redshift \( z \) by the simple formula

\[
H(z) = H_0 \sqrt{\Omega_m (1 + z)^3 + \Omega_{\Lambda}},
\]

where \( H_0 \) is the Hubble constant.

The modeling of the observational data’s deviations from the fiducial model, as well as the statistical and systematic uncertainties of the data, can be rather difficult. For SNIa, there are planned projects such as the Wide-Field Infrared Survey Telescope (WFIRST)\(^3\) with well-defined redshift distribution of targets (Aldering et al. 2004) and the uncertainty model (Kim et al. 2004; Huterer 2009) based on which simulated data can be generated. However, this is not true for OHD, as there have not been a formal specification of future observational goals known to the authors. Consequently, we have to work around this difficulty by approaching the problem from a phenomenological point of view.

By inspecting the uncertainties on \( H(z) \) in the SJVKS10 data set (Figure 2), we can see the general trend of the errors increasing with \( z \) despite the two outliers at \( z = 0.48 \) and 0.88. Excluding the outliers from the data set, we find that the uncertainties \( \sigma_z \) are bounded by two straight lines \( \sigma_+ = 16.87 z + 10.48 \) and \( \sigma_- = 4.41 z + 7.25 \) from above and below, respectively. If we believe that future observations would also yield data with uncertainties within the strip bounded by \( \sigma_+ \) and \( \sigma_- \), we can take the midline of the strip \( \sigma_0 = 10.64 z + 8.86 \) as an estimate of the mean uncertainty of future observations. In our code, this is done by drawing a random number \( \tilde{\sigma}(z) \) from the Gaussian distribution \( \tilde{\sigma}(\sigma_0(z), \varepsilon(z)) \), where \( \varepsilon(z) = (\sigma_+ - \sigma_-)/4 \).

\(^3\) http://wfirst.gsfc.nasa.gov/
The parameter $\varepsilon$ is chosen so that the probability of $\tilde{\sigma}(z)$ falling within the strip is 95.4%.

Having found a method of generating the random uncertainty $\tilde{\sigma}(z)$ for a simulated data point, we are able to simulate the deviation from $H_{\text{fid}}$. Namely, we assume that the deviation of the simulated observational value from the fiducial, $\Delta H = H_{\text{sim}}(z) - H_{\text{fid}}(z)$, satisfies the Gaussian distribution $N(0, \tilde{\sigma}(z))$ from which $\Delta H$ can be drawn as a random variable.

Thus, a complete procedure of generating a simulated $H(z)$ value at any given $z$ is formed. First, the fiducial value $H_{\text{fid}}(z)$ is calculated from Equation (1). After that, a random uncertainty $\tilde{\sigma}(z)$ is drawn using the aforementioned method. This uncertainty is in turn used to draw a random deviation $\Delta H$ from the Gaussian distribution $N(0, \tilde{\sigma}(z))$. The final result of this process is a data point $H_{\text{sim}}(z) = H_{\text{fid}}(z) + \Delta H$ with uncertainty $\tilde{\sigma}(z)$.

In addition to the procedures described above, the Hubble constant (in the units of km s$^{-1}$ Mpc$^{-1}$) is also taken as a random variable and is drawn from the Gaussian distribution $N(70.4, 1.4)$ suggested by seven-year WMAP results when we evaluate the right-hand side of Equation (1). We could have fixed $H_0$ at a constant value, but as we shall see in Section 3.2, in our analysis we treat $H_0$ and $\Omega_m$ quite differently. Namely, $\Omega_m$ is a parameter with a posterior distribution to be inferred, but $H_0$ is a nuisance parameter that is marginalized over using some independent measurement results as prior knowledge. This treatment can be found in many works, such as Stern et al. (2010a) and Wei (2010). It can be justified by the need to reduce the dimension of the parameter space given limited data, and we usually prioritize other parameters such as $\Omega_m$ over $H_0$. Therefore, when generating simulated data sets we sample $H_0$ from a random distribution to reflect the uncertainty to be marginalized over. Otherwise, we could have “cheated” by forcing a $\delta$-function prior on $H_0$ centered at its fiducial value in the analysis of simulated data and obtain overly optimistic predictions from the simulated data.

The quality of simulated data thus generated is similar to that of SJVKS10. A snapshot realization of this simulation scheme is displayed in Figure 3, where a total of 128 data points with $z$ evenly spaced within the range $0.1 \leq z \leq 2.0$.

### 3.2. The Evaluation Model

We use a standard non-flat $\Lambda$CDM model with a curvature term $\Omega_k = 1 - \Omega_m - \Omega_\Lambda$ to evaluate the qualities of the simulated data sets. In this model, the Hubble parameter is given by

$$H(z) = H_0 \sqrt{\Omega_m (1 + z)^3 + \Omega_k (1 + z)^2 + \Omega_\Lambda} = H_0 E(z; \Omega_m, \Omega_\Lambda).$$

Our model choice is mainly motivated by our desire to reduce unnecessary distractions arising from the intrinsic complexity of certain cosmological models involving dark energy or modified gravitation.

We perform a standard maximal likelihood analysis using this evaluation model. In our analysis, we intend to marginalize the likelihood function over the Hubble parameter $H_0$, thus obtaining parameter constraints in the $(\Omega_m, \Omega_\Lambda)$ subspace. This marginalization process also allows us to incorporate a priori knowledge about $H_0$ into our analysis.

There is a fair amount of available information from which reasonable priors can be constructed. Samushia & Ratra (2006) used two different Gaussian priors on $H_0$: one with $H_0 = 73 \pm 3$ km s$^{-1}$ Mpc$^{-1}$ from three-year WMAP data (Spergel et al. 2007) and the other with $H_0 = 68 \pm 4$ km s$^{-1}$ Mpc$^{-1}$ by Gott III et al. (2001; for a discussion of the non-Gaussianity of the error distribution in $H_0$ measurements, see Chen et al. 2003). Lin et al. (2009) used $H_0 = 72 \pm 8$ km s$^{-1}$ Mpc$^{-1}$ as suggested by Freedman et al. (2001). In our work, we use a more recent determination of $H_0 = 74.2 \pm 3.6$ km s$^{-1}$ Mpc$^{-1}$ (Riess et al. 2009) as an update to the ones cited above. We also consider a “top-hat” prior, i.e., a uniform distribution in the interval $[50, 100]$. Compared with any of the peaked priors above, this

![Figure 2](image.png)  
Figure 2. Uncertainties of $H(z)$ in the SJVKS10 data set. Solid dots and circles represent non-outliers and outliers, respectively. Our heuristic bounds $\sigma_\varepsilon$ and $\sigma_\sigma$ are plotted as the two dotted lines. The dash-dotted line shows our estimated mean uncertainty $\sigma_0$.

![Figure 3](image.png)  
Figure 3. Snapshot of a simulated data set realized using our method. The uncertainties in the simulated data are modeled phenomenologically after SJVKS10, which is also shown for comparison.
prior shows less preference of a particular central value while still characterizing our belief that any value of \( H_0 \) outside the said range is unlikely to be true. Notice that the intrinsic spread of \( H_0 \) involved in the generation of simulated data sets is smaller than either prior chosen in this step, consistent with the belief that the prior adopted in the estimation of parameters should not be spuriously optimistic.

Having chosen the priors on \( H_0 \), it is straightforward to derive, up to a non-essential multiplicative constant, the posterior probability density function (PDF) of parameters given the data set \( \{ H_i \} \) by means of Bayes’ theorem:

\[
P(\Omega_m, \Omega_\Lambda \mid \{ H_i \}) = \int P(\Omega_m, \Omega_\Lambda, H_0 \mid \{ H_i \}) \, dH_0
\]

\[
= \int \mathcal{L}(\{ H_i \} \mid \Omega_m, \Omega_\Lambda, H_0) \, P(H_0) \, dH_0, \tag{3}
\]

where \( \mathcal{L} \) is the likelihood and \( P(H_0) \) is the prior PDF of \( H_0 \). Assuming that each measurement in \( \{ H_i \} \) has a Gaussian error distribution of \( H(z) \) and is independent from other measurements, the likelihood is then given by

\[
\mathcal{L}(\{ H_i \} \mid \Omega_m, \Omega_\Lambda, H_0) = \left( \prod_i \frac{1}{\sqrt{2\pi \sigma_i^2}} \right) \exp \left( -\frac{\chi^2_i}{2} \right), \tag{4}
\]

where the \( \chi^2 \) statistic is defined by

\[
\chi^2 = \sum_i \frac{(H_0 E(z_i; \Omega_m, \Omega_\Lambda) - H_i)^2}{\sigma_i^2}, \tag{5}
\]

and \( \sigma_i \)'s are the uncertainties quoted from the data set. The posterior PDF can thus be found by inserting Equation (4) and the exact form of \( P(H_0) \) into Equation (3).

We now show that the integral over \( H_0 \) in Equation (3) can be worked out analytically for our \( P(H_0) \) choices.

**Gaussian prior.** Let

\[
P(H_0) = \frac{1}{\sqrt{2\pi \sigma_H^2}} \exp \left[ -\frac{(H_0 - \mu_H)^2}{2\sigma_H^2} \right]. \tag{6}
\]

In this case, Equation (3) reduces to

\[
P(\Omega_m, \Omega_\Lambda \mid \{ H_i \}) = \frac{1}{\sqrt{A}} \left[ \text{erf} \left( \frac{B}{\sqrt{A}} \right) + 1 \right] \exp \left( \frac{B^2}{A} \right), \tag{7}
\]

where

\[
A = \frac{1}{2\sigma_H^2} + \sum_i \frac{E(z_i; \Omega_m, \Omega_\Lambda)}{2\sigma_i^2},
\]

\[
B = \frac{\mu_H}{2\sigma_H^2} + \sum_i \frac{E(z_i; \Omega_m, \Omega_\Lambda) H_i}{2\sigma_i^2},
\]

and \( \text{erf} \) stands for the error function. The form shown in \( \text{Equation (7)} \) is not normalized; all multiplicative constants have been discarded.

**Top-hat prior.** The PDF of a uniform distribution over the interval \( [x, y] \) can be written as \( P(H_0) = \Theta(y - H_0)\Theta(H_0 - x) \begin{pmatrix} y - x \end{pmatrix} \), where \( \Theta \) denotes the Heaviside unit step function. With this prior on \( H_0 \), Equation (3) becomes

\[
P(\Omega_m, \Omega_\Lambda \mid \{ H_i \}) = \frac{U(x, C, D) - U(y, C, D)}{\sqrt{C}} \exp \left( \frac{D^2}{C} \right), \tag{8}
\]

where

\[
C = \sum_i \frac{E^2(z_i; \Omega_m, \Omega_\Lambda)}{2\sigma_i^2}, \quad D = \sum_i \frac{E(z_i; \Omega_m, \Omega_\Lambda) H_i}{2\sigma_i^2},
\]

and

\[
U(x, \alpha, \beta) = \text{erf} \left( \frac{\beta - \alpha x}{\sqrt{\alpha}} \right).
\]

The normalization constant has been dropped from \( \text{formula (8)} \) as well.

### 3.3. Figure of Merit

The posterior PDFs obtained above put statistical constraints on the parameters. As the data set \( \{ H_i \} \) improves in size and quality, the constraints are tightened. To evaluate a data set’s ability of tightening the constraints, a quantified FoM must be established.

We note that the FoM can be defined arbitrarily as long as it reasonably rewards a tight fit while punishing a loose one. Its definition can be motivated purely statistically, for example the reciprocal hypervolume of the 95% confidence region in the parameter space (Albrecht et al. 2006). However, a definition that is sensitive to some physically significant structuring of the parameter space can be preferable if our scientific goal requires it (Linder 2006).

In this paper, we use a statistical FoM definition similar to the ones of Albrecht et al. (2006), Liu et al. (2008), and Bueno Sanchez et al. (2009). Our FoM is defined to be the area enclosed by the contour of \( P(\Omega_m, \Omega_\Lambda \mid \{ H_i \}) = \exp(-\Delta \chi^2/2)P_{\text{max}} \), where \( P_{\text{max}} \) is the maximum value of the posterior PDF, and the constant \( \Delta \chi^2 \) is taken to be 6.17. This value is so chosen that the region enclosed by this contour coincides with the 2\( \sigma \) or 95.4% confidence region if the posterior is Gaussian. In the rest of this paper, we will use the term “2\( \sigma \) region” to refer to the region defined in this way. The 1\( \sigma \) and 3\( \sigma \) regions can be defined in the same manner by setting the constant \( \Delta \chi^2 \) to 2.3 and 11.8, respectively. It is important to bear in mind that our definition of “\( n \sigma \) regions” is motivated by nomenclature brevity rather than by mathematical concreteness, since the posterior PDFs (Equations (7) and (8)) are manifestly non-Gaussian.

We make two further remarks on our FoM definition. First, since our definition of FoM is statistical rather than physical, we do not exclude the unrealistic parts of the confidence regions from the area. Second, the FoM by our definition is obviously invariant under the multiplication of \( P(\Omega_m, \Omega_\Lambda \mid \{ H_i \}) \) by a positive constant, which justifies the omission of normalization constants in \( \text{Section 3.2} \).

### 4. RESULTS FROM THE SIMULATED DATA SETS

Using the method described in \( \text{Section 3.1} \), we generate 500 realizations of the simulated \( H(z) \) data set. Each realization contains 128 data points evenly spaced in the redshift range 0.1 \( \leq z \leq 2.0 \). From each realization, successively shrinking subsamples of 64, 32, and 16 data points are randomly drawn. These subsamples are then used in conjunction with the full OHD set to obtain their respective FoMs.

The FoMs are naturally divided into four groups by the size of the simulated subsample used in the calculation. For each group, median and median absolute deviation (MAD) statistics are calculated. The median and the MAD are used to represent
the central value and spread of FoM data, respectively, and they are used in preference to the customary pair of the mean and the standard deviation, because they are less affected by egregious outliers (see NIST 2003, Chapter 1.3.5.6). The outliers mainly arise from “worst case” realizations of simulated outliers (see NIST 2003). For example, one with a large amount of data points deviating too much (or too little) from the fiducial model.

To compare the parameter constraining abilities of our simulated $H(z)$ data sets with those of SNIa data, we have also fitted our evaluation model (Equation (2)) to the ConstitutionT data set which is a subset of the Constitution redshift–distance data set (Hicken et al. 2009a) deprived of outliers that account for internal tensions (Wei 2010). It is worth pointing out that the prior of $H_0$ used in the SNIa fitting procedure is fundamentally different from either one discussed in Section 3.2. Namely, when SNIa data are used, the parameter $H_0$ and the intrinsic absolute magnitude of SNIa, $M_0$, are combined into one “nuisance parameter” $M = M_0 - 5 \log H_0$, and marginalized over under the assumption of a flat prior over $(-\infty, +\infty)$ (Perlmutter et al. 1999). This discrepancy should be kept in mind when comparing or combining SNIa and $H(z)$ data sets, and we hope that it could be closed in the future by better constraints, either theoretical or observational, on $M_0$.

Our main results are shown in Figure 4. As one may intuitively assume, the median FoM increases with the size of the data set. We find that the data subset with 64 simulated data points leads to an FoM of $8.6 \pm 0.7$ under the Gaussian prior on $H_0$. This median FoM already surpasses that of ConstitutionT. However, the top-hat prior leads to significantly lower FoM. Under the top-hat prior we used, as many as 128 simulated data points are needed to reach the FoM level of ConstitutionT.

The degeneracy of confidence regions obtained from $H(z)$ data in the $(\Omega_m, \Omega_k)$ parameter subspace is shown in Figure 5. Because of this degeneracy, $H(z)$ data sets cannot be used alone to constrain $\Omega_k$ well. This degenerate behavior is similar to that of SNIa data.

5. FISHER MATRIX ANALYSIS OF FUTURE DATA

The simulated data used in Section 3 are based on the quality of currently available measurements, and we have tried not to be too optimistic about their uncertainties. However, we have reasons to expect an increase in the quality of future $H(z)$ data. First, Crawford et al. (2010) analyzed the observational requirement of measuring $H(z)$ to 3% at intermediate redshifts with age-dating. Second, the Baryon Oscillation Spectroscopic Survey (BOSS) is designed to constrain $H(z)$ with 2% precision at redshifts $z \approx 0.3$ and 0.6 by measuring BAO imprints in the galaxy field, and at $z \approx 2.5$ using the Ly$\alpha$ absorption spectra of quasars.

By incorporating these specifications of future data, we can estimate their expected FoMs using the Fisher matrix forecast technique (Dodelson et al. 1997). The $3 \times 3$ Fisher matrix $F$ is calculated from Equation (5) with the $\sigma_i$’s determined by future data specifications, and the evaluation of matrix elements is made at the fiducial parameter values (see Section 3.1). The matrix elements are the second partial derivatives of $\chi^2$ with respect to the parameters

$$F_{ij} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial \theta_i \partial \theta_j}, \quad (9)$$

where $\theta_i$’s are the parameters, namely $(\Omega_m, \Omega_k, H_0)$. Notice that we use the word “Fisher matrix” only loosely. The second
derivative matrix defined by Equation (9) is not the Fisher matrix in the strict sense, but the ideas are intimately related (see Dodelson 2003).

In order to obtain the FoM in the two-dimensional parameter space of \((\Omega_m, \Omega_\Lambda)\), we must marginalize over \(H_0\). We adopt the Gaussian prior on \(H_0\) with \(\sigma_H = 3.6 \, \text{km s}^{-1} \, \text{Mpc}^{-1}\), which is the same as the one used in Section 3.2. Gaussian marginalization is performed using a straightforward modification of the projection technique from Dodelson (2003) and Press et al. (2007). A more general and detailed analysis of this problem was made by Taylor & Kitching (2010), but for our purpose, simply adding \(1/\sigma_H^2\) to \(F_{13}\) and then projecting onto the first two dimensions will do the job. This is applicable when the mean of the prior \(H_0\) is close to the fiducial value, and we have numerically verified the validity of this approximation in our work.

Denoting the marginalized Fisher matrix by \(\tilde{F}\), the iso-\(\Delta \chi^2\) contour in the parameter space is approximated by the quadratic equation

\[
(\Delta \theta)^T \tilde{F} \Delta \theta = \Delta \chi^2,
\]  

where \(\Delta \theta = \theta - \theta_{\text{fid}}\) is the parameters’ deviation from the fiducial value, and \(\Delta \chi^2\) is our chosen constant 6.17 in Section 3.3. By construction, \(\tilde{F}\) is positive-definite;\(^5\) therefore, the above equation describes an ellipse. Its enclosed area is simply \(A = \pi / \sqrt{\det(\tilde{F} / \Delta \chi^2)}\); therefore, we can estimate the FoM by

\[
\text{FoM} = \frac{1}{A} = \frac{1}{\pi} \sqrt{\det \left( \frac{1}{\Delta \chi^2} \tilde{F} \right)}.
\]

In our setup, we assume that the relative error of \(H(z)\) anticipated by Crawford et al. (2010) could be globally achieved within the redshift interval \(0 \leq z \leq 1.0\). Under this assumption, we can estimate how many data points will be needed to reach the ConstitutionT FoM using the Fisher matrix method discussed above. We chose not to incorporate the available data so that we can work with the simple error model specified by future data.

Our main results are shown in Figures 6 and 7. For relative \(H(z)\) errors of 3\%, 5\%, and 10\%, the required number of measurements are 21, 62, and 256, respectively. This number \(N\) increases steeply as the relative error increases, as shown in Figure 7.

We can also apply the Fisher matrix analysis to future BOSS data. We find that BOSS alone gives FoM \(\approx 15\). This promising result is a combined consequence of its high precision and extended redshift coverage. In Figure 8, we plot the Fisher matrix forecast of the confidence regions.

6. CONCLUSION AND DISCUSSIONS

We have explored the possibility of using OHD as an alternative to SNIa redshift–distance data in the sense of offering comparable or higher FoM. By using simulated \(H(z)\) data sets with an empirical error model similar to that of current age-dating data, we show that more than 60 future measurements of \(H(z)\) in the redshift range \(0 \leq z \leq 2\) could be needed to acquire parameter constraints comparable with those obtained from SNIa data sets like ConstitutionT. In addition, precise and accurate determination of \(H_0\) is crucial for improving the FoM obtained from OHD, and a broad prior on \(H_0\) leads to lower FoM. When we progressively lower the error of future measurements to 3\% as discussed by Crawford et al. (2010), we estimate that \(\approx 60\) measurements in the shorter redshift interval \([0.1, 1.0]\) will be needed to achieve the same result. In summary, we give an affirmative answer to the first of the two questions raised in Section 1 and a semi-quantitative answer to the second.

---

\(^5\) This can be proved using the positive-definiteness conditions of the Schur complement (Puntanen & Styan 2005).
Our result furthers a conclusion of Lin et al. (2009) and Carvalho et al. (2008), namely, that OHD plays almost the same role as that of SNIa for the joint constraints on the CDM model. We have shown that the OHD set alone is potentially capable of being used in place of current SNIa data sets if it is large enough. We note that our forecast of OHD data requirement is competitive in the sense of observational cost compared with SN observations. Throughout our analysis, we used the Constitution data set used as a standard of FoM. This data set is a subset of the Constitution compilation, a combination of ground-based CfA3 SN observations (Hicken et al. 2009b) and Union, a larger compilation of legacy SNe and space-based spectroscopic instruments (see Sandage 1962 and Loeb 1998 for the foundations of the test).

Finally, we note that future CMB observation programs, such as the Atacama Cosmology Telescope,6 may be able to identify more than 2000 passively evolving galaxies up to $z \approx 1.5$ via the Sunyaev–Zel’dovich effect, and their spectra can be analyzed to yield age measurements that will yield approximately 1000 $H(z)$ determinations with 15% error (Simon et al. 2005). This promises a future data capacity of an order of magnitude more than what we have estimated to be enough to match current SNIa data sets. Combining this prospect with future high-$z$, high-accuracy $H(z)$ determinations from BAO observations, it is reasonable to expect that the OHD will play an increasingly important role in the future study of the expansion history of the universe and cosmological parameters.

We thank the anonymous referee whose suggestions greatly helped us improve this paper. C.M. is grateful to Chen-Tao Yang for useful discussions. This work was supported by the National Science Foundation of China (grant no. 10473002), the Ministry of Science and Technology National Basic Science program (project 973) under grant no. 2009CB24901, and the Fundamental Research Funds for the Central Universities.

REFERENCES

Albrecht, A., et al. 2006, arXiv:astro-ph/0609591
Aldering, G., et al. 2004, arXiv:astro-ph/0405232
Araujo, M. E., & Stoeger, W. R. 2010, Phys. Rev. D, 82, 123513
Blake, C., & Glazebrook, K. 2003, ApJ, 594, 665
Buono Sanchez, J. C., Nesseris, S., & Perivolaropoulos, L. 2009, J. Cosmol. Astropart. Phys., JCAP11(2009)029
Carvalho, F. C., Santos, E. M., Alcaniz, J. S., & Santos, J. 2008, J. Cosmol. Astropart. Phys., JCAP09(2008)008
Chen, G., Gott, J. R., III, & Ratra, B., 2003, PASP, 115, 1269
Corasaniti, P., Huterer, D., & Melchiorri, A. 2007, Phys. Rev. D, 75, 062001
Crawford, S. M., Ratsimbazafy, A. L., Cress, C. M., Olivier, E. A., Blyth, S., & van der Heyden, K. J. 2010, MNRAS, 406, 2569
Dodelson, S. 2003, Modern Cosmology (San Diego, CA: Academic)
Dodelson, S., Kinney, W. H., & Kolb, E. W. 1997, Phys. Rev. D, 56, 3207
Eisenstein, D. J., et al. 2005, ApJ, 633, 560
Freedman, W. L., et al. 2001, ApJ, 553, 47
Gaztañaga, E., Cabrè, A., & Hui, L. 2009, MNRAS, 399, 1663
Ghirlanda, G., Ghisellini, G., Lazzati, D., & Firmani, C. 2004, ApJ, 613, L13
Gott III, J. R., Vogelezang, M. S., Pajari, A., & Ratra, B. 2001, ApJ, 549, 1
Hicken, M., Wood-Vasey, M. W., Blondin, S., Challis, P., Jha, S., Kelly, P. L., Rest, A., & Kirshner, R. P. 2009a, ApJ, 700, 1097
Hicken, M., et al. 2009b, ApJ, 700, 331
Huterer, D. 2009, Nucl. Phys. B, 814, 239
Jimenez, R., & Loeb, A. 2002, ApJ, 573, 37
Jimenez, R., Verde, L., Treu, T., & Stern, D. 2003, ApJ, 593, 622
Kim, A. G., Linder, E. V., Miquel, R., & Mostek, N. 2004, MNRAS, 347, 909
Komatsu, E., et al. 2010, ApJS, 192, 18
Kowalski, M., et al. 2008, ApJ, 686, 749
Li, H., Xia, J.-Q., Liu, J., Zhao, G.-B., Fan, Z.-H., & Zhang, X. 2008, ApJ, 680, 92

6 http://www.physics.princeton.edu/act/index.html
Li, M., Li, X.-D., Wang, S., & Zhang, X. 2009, J. Cosmol. Astropart. Phys., JCAP06(2009)036
Liang, N., Wu, P., & Zhang, S. N. 2010, Phys. Rev. D, 81, 083518
Lin, H., Hao, C., Wang, X., Yuan, Q., Yi, Z.-L., Zhang, T.-J., & Wang, B.-Q. 2009, Mod. Phys. Lett. A, 24, 1699
Linder, E. V. 2006, Astropart. Phys., 26, 102
Liu, D.-J., Li, X.-Z., Hao, J., & Jin, X.-H. 2008, MNRAS, 388, 275
Loeb, A. 1998, ApJ, 499, L111
Mignone, C., & Bartelmann, M. 2008, A&A, 481, 295
NIST. 2003, NIST/SEMATECH e-Handbook of Statistical Methods (Gaithersburg, MD: National Institute of Standards and Technology), http://www.itl.nist.gov/div898/handbook/index.htm
Percival, W. J., et al. 2010, MNRAS, 401, 2148
Perlmutter, S., et al. 1999, ApJ, 517, 565
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 2007, Numerical Recipes: the Art of Scientific Computing (3rd ed.; Cambridge: Cambridge Univ. Press)
Puntanen, S., & Styan, G. P. H. 2005, in Numerical Methods and Algorithms, Vol. 4, The Schur Complement and Its Applications, ed. C. Brezinski & F. Zhang (New York: Springer), 163
Riess, A. G., et al. 1998, AJ, 116, 1009
Riess, A. G., et al. 2009, ApJ, 699, 539
Samushia, L., & Ratra, B. 2006, ApJL, 650, L5
Sandage, A. 1962, ApJ, 136, 319
Seo, H.-J., & Eisenstein, D. J. 2005, ApJ, 633, 575
Shafieloo, A., Alam, U., Sahni, V., & Starobinsky, A. A. 2006, MNRAS, 366, 1081
Simon, J., Verde, L., & Jimenez, R. 2005, Phys. Rev. D, 71, 123001
Spergel, D. N., et al. 2007, ApJS, 170, 377
Stern, D., Jimenez, R., Verde, L., Kamionkowski, M., & Stanford, S. A. 2010a, J. Cosmol. Astropart. Phys., JCAP02(2010)008
Stern, D., Jimenez, R., Verde, L., Stanford, S. A., & Kamionkowski, M. 2010b, ApJS, 188, 280
Taylor, A. N., & Kitching, T. D. 2010, MNRAS, 408, 865
Wang, Y., & Tegmark, M. 2004, Phys. Rev. Lett., 92, 241302
Wang, Y., & Tegmark, M. 2005, Phys. Rev. D, 71, 103513
Wei, H. 2010, Phys. Lett. B, 687, 286
Yang, X.-J., & Chen, D.-M. 2009, MNRAS, 394, 1449
Yi, Z.-L., & Zhang, T.-J. 2007, Mod. Phys. Lett. A, 22, 41
Zhang, J., Zhang, L., & Zhang, X. 2010, Phys. Lett. B, 691, 11