First-forbidden $\beta$ decay of $^{17}$N and $^{17}$Ne

D. J. Millener

*Brookhaven National Laboratory, Upton, New York 11973

and Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195

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Abstract

It is shown that differences, due to charge-dependent effects, in the $^{17}$N and $^{17}$Ne ground-state wave functions account for the fact that the experimentally measured branch for the $\beta^+$ decay of $^{17}$Ne to the first excited state of $^{17}$F is roughly a factor of two larger than expected on the basis of nuclear matrix elements which reproduce the corresponding $\beta^-$ branch in the decay of $^{17}$N.
By measuring positrons in coincidence with 495-keV $\gamma$ rays de-exciting the $1/2^+$ first-excited state of $^{17}\text{F}$, Borge et al. [1] have obtained a branch of 1.65(16)% for the first-forbidden $\beta^+$ decay of $^{17}\text{Ne}$ to the $1/2^+$ state. This is a very interesting result because the measured branch is roughly a factor of two larger than expected on the basis of nuclear matrix elements which reproduce the corresponding $\beta^-$ branch of 3.0(5)% [2,3] in the decay of $^{17}\text{N}$. Recently, Ozawa et al. [4] have confirmed the magnitude of the $\beta$ branch in $^{17}\text{Ne}$ decay, obtaining a value of 1.44(16)% by a method which utilizes a 32 MeV/A radioactive beam of $^{17}\text{Ne}$.

The $\beta$-decay rate is given by $ft = 6170$ sec. For the $1/2^- \rightarrow 1/2^+$ transitions of interest, $f = f^{(0)} + f^{(1)}$ where the superscript refers to the spherical tensor rank of the $\beta$-decay operator. In general, $f^{(0)}$ is much larger than $f^{(1)}$ and, to a very good approximation,

$$f^{(0)} = I_0(\xi'v + \frac{1}{3}W_0w + \xi w')^2$$

(1)

where $\xi = \pm \alpha Z/2R$ for $\beta^\mp$ decay, with $Z$ the charge of the daughter nucleus and $R = 3.499$ fm for $A = 17$, and

$$w = \lambda \sqrt{\frac{3}{2}} \langle J_f | i [\sigma, \nabla] \frac{1}{M} I_T | J_i \rangle C$$

(2)

$$\xi'v = -\lambda \sqrt{\frac{3}{2}} \langle J_f | i [\sigma, \nabla] \frac{1}{M} I_T | J_i \rangle C$$

(3)

with $C$ being the isospin Clebsch-Gordan coefficient and $\lambda = 1.26$. Energies are expressed in units of the electron rest mass and, with $I_0$ the integrated phase-space factor for allowed decays divided by the square of the Compton wave length for the electron, the nuclear matrix elements are in fm. The matrix element $w'$ is closely related to $w$ and takes a value $\sim 0.7w$ [5]. These expressions are based on a systematic expansion of the electron radial wave functions developed by Behrens and Bühring [6], the arcane notation for the nuclear matrix elements in first-forbidden decays being historical (see [3] for details and definitions of the rank-1 matrix elements).

Aside from the use of first-forbidden $\beta$ decay as a spectroscopic tool, there has been great interest in rank-0 decays for two reasons. The first dates back to the suggestion [7]...
that the matrix element $\xi'v$ of the time-like piece of the axial current $\gamma_5$ should be strongly enhanced by meson-exchange currents, largely one-pion-exchange. This enhancement is now well established at $\sim 60\%$ for light nuclei [8] and even larger for heavy nuclei [9]. It is often taken into account, as is done below, by multiplying $\xi'v$ by a factor $\varepsilon_{\text{mec}}$. The second reason relates to the similarity of the operators for parity-mixing and rank-0 first-forbidden $\beta$-decay [10]. As a result of these fundamental interests, a large literature exists on many aspects of first-forbidden $\beta$ decay and parity-mixing in light nuclei. The present treatment of the $^{17}$N and $^{17}$Ne decays, first studied theoretically by Towner and Hardy [11], is based on a systematic study [12] of $J^\pm \to J^\mp$ decays of $^{11}$Be, $^{15}$C, $^{16}$C, $^{16}$N, $^{17}$N, $^{18}$Ne, $^{19}$Ne and $^{20}$F. For the $1h\omega$ basis used in [12], the $1/2^-$ initial-state wave functions have a particularly simple form in a weak-coupling representation, namely that of a $0p$-shell hole coupled to $(1s0d)^2$ eigenstates (notation $J^\pi_n; T$).

$$|1/2^-; 3/2\rangle = 0.967|1/2^- \otimes 0^+_1; 1\rangle - 0.224|3/2^- \otimes 2^+_1; 1\rangle + 0.109|1/2^- \otimes 0^+_2; 1\rangle + ...$$

(4)

In fact, the three components listed account for 99.7% of the wave function. For the dominant component, only the $1s_{1/2}^2$ component contributes to the matrix element $\sigma.r$ and $\sigma.p$, one $s_{1/2}$ nucleon making a transition to fill the $p_{1/2}$ hole with the other forming the single-particle final state. The same is true for the third component, which augments the first (the $0^+_2; 1$ state has a dominant $1s_{1/2}^2$ component). A small $d_{3/2} \to p_{3/2}$ amplitude, arising from the second component of the $1/2^-$ wave function, is important because the single-particle matrix element is large (larger than $s \to p$ by a factor of $\sqrt{5}$ for harmonic oscillator wave functions) and interferes destructively with the dominant $1s_{1/2} \to 0p_{1/2}$ amplitude. This is a common feature of all the transitions studied in [12]. The radial single-particle matrix elements are computed with Woods-Saxon wave functions obtained by adjusting the well depth to match the separation energy from the initial or final state to the appropriate physical core states of the A-1 system [12]. For the $1s_{1/2} \to 0p_{1/2}$ contribution, the only important parent states are the lowest $0^-$ and $1^-$ states of $^{16}$N or $^{16}$F. The separation energies are given in Table [1]
along with the decay energies and the phase-space integrals $I_0$. Since the separation energies are close to the Hartree-Fock energies, the Woods-Saxon wave functions should be a good approximation to one-nucleon overlap functions [13].

For the rank-0 contribution to the $\beta$-decay rates, the calculation gives

$$f^{(0)}(N) = 0.3051(10.971\epsilon_{mec} - 4.216)^2 \quad (5)$$
$$f^{(0)}(\text{Ne}) = 2.380(11.585\epsilon_{mec} - 3.009)^2 \quad (6)$$
$$f^{(0)}(\text{Ne}') = 2.380(15.278\epsilon_{mec} - 3.969)^2 \quad (7)$$

where the first two lines correspond to using identical nuclear structure, the small differences in matrix elements being due to the use of Woods-Saxon wave functions bound at the physical separation energies (note the energy-dependent factors in Eq. (1) for the second term). The resulting $f$ values are compared with experiment in Table [I] for two values of the enhancement due to meson-exchange currents (see Table IV of [8] for theoretical estimates of $\epsilon_{mec}$). Including the calculated $f^{(1)}$ values, it can be seen that the predicted value for the $\beta$ branch in $^{17}\text{Ne}$ is less than $\sim 0.9\%$ for values of $\epsilon_{mec}$ which produce agreement with the $^{17}\text{N}$ data (0.77\% to reproduce the central value).

For the case denoted by $\text{Ne}'$ in Eq. (7) and the last line of Table [I], the $^{17}\text{Ne}$ ground-state wave function has been modified to take into account charge-dependent effects which differ for $1s$ and $0d$ orbits. Now, with a $45 - 50\%$ enhancement from meson-exchange currents, the calculated beta-decay rates are in agreement, within the error bars, for both nuclei.

That there should be substantial $T_z$-dependent effects is evident from the 376 keV difference in Coulomb energies for the $0d_{5/2}$ and $1s_{1/2}$ orbits at $A = 17$. For $A = 18$, the large shift in the excitation energy of the third $0^+$ state in $^{18}\text{Ne}$ (Table [II]) led to its identification as a largely $1s^{2}_{1/2}$ configuration [14]. The shift in the $s^{2}_{1/2}$ diagonal matrix element relative to $d^{2}_{5/2}$ in going from $^{18}\text{O}$ to $^{18}\text{Ne}$ will also lead to more $s^{2}_{1/2}$ in the $^{18}\text{Ne}$ ground-state wave function and hence, when coupled to a $p_{1/2}$ hole, to an enhancement of the rank-0 matrix element for the $\beta^+$ decay of $^{17}\text{Ne}$. This effect is amplified by the cancellation between the $s_{1/2} \rightarrow p_{1/2}$ and $d_{3/2} \rightarrow p_{3/2}$ contributions.
To make a rough estimate of this effect, the Wildenthal USD interaction [13] is used to obtain \((sd)^2\) wave functions for \(^{18}\)O \((\epsilon_{5/2} = -3.9478, \epsilon_{1/2} = -3.1635, \epsilon_{3/2} = 1.6466,\) upper half diagonal of two-body matrix elements -2.8197, -1.3247, -3.1856, -2.1246, -1.0835, -2.1845). Then, the \(s_{1/2}\) diagonal matrix element is shifted by twice the shift of the \(s_{1/2}\) single-particle energies between \(^{17}\)O and \(^{17}\)F (752 keV) plus 147 keV for the difference between the two-body matrix elements of \(e^2/r\) for \(d^2\) and \(s^2\) configurations [16], and the new matrix is diagonalized to get \((sd)^2\) wave functions for \(^{18}\)Ne. The resulting energies, wave functions, and intensities of \(1s_{1/2}\) are given in Table IV. The \(s_{2/2}\) intensity rises from 15\% to 21.7\%, an increase of 44\% (the squared overlap of the ground-state wave functions is still 0.9925).

The increase in \(\xi'v\) in Eq. (7) by a factor 1.32 rather than 1.20 for the \(s_{1/2} \rightarrow p_{1/2}\) matrix element alone is due to the cancellation effects involving the \(d_{3/2} \rightarrow p_{3/2}\) matrix element.

The above calculation, which does succeed in providing an explanation for the measured \(\beta\)-decay rates, is not a consistent one but clearly indicates the direction in which charge-dependent effects will affect the \(\beta\)-decay branch in \(^{17}\)Ne decay. An explanation of the energy shifts and wave function changes for the \(0^+ T = 1\) states of \(A = 18\) requires that the \(4p2h\) configurations be included. A calculation of the energy shifts without wave function changes [14] does rather well but the \(^{18}\)Ne ground state could do with a “push” of the magnitude (163 keV) shown in Table IV. The \((sd)^2\) calculation is actually more applicable to the \(2p1h\) states of \(^{17}\)N and \(^{17}\)Ne because the \(4p3h\) states are expected [17] to lie above both states obtained by coupling a \(p_{1/2}\) hole to the two lowest \((sd)^2\) \(0^+\) states. The second of these states is known at 3.663 MeV in \(^{17}\)Ne and is lowered from its position in \(^{18}\)O in large part because the spin-average \(p_{1/2}^{-1}s_{1/2} T = 1\) particle-hole interaction is less repulsive by \(\sim 700\) keV than the corresponding \(p_{1/2}^{-1}d_{5/2}\) interaction [18,17] and to a lesser extent because of the removal of the influence of the \(4p2h\) configuration.

To put the structure of \(^{17}\)N and \(^{17}\)Ne in a broader context, it should be noted that the four particle-hole matrix elements mentioned above can be deduced directly from the binding energies of the lowest four states of \(^{16}\)N (the charge-dependent shifts of the \(0d_{5/2}\) and \(1s_{1/2}\) orbits, including a dependence on separation energy, can be seen across these \(T = 1\)
multiplets). Within the framework of the same weak-coupling assumption used to deduce the particle-hole matrix elements, the total binding energies and multiplet spacings of the low-lying states of the heavy carbon and nitrogen isotopes which contain one or more \textit{sd}-shell neutrons can be rather nicely accounted for (of course, small components in the wave functions are important for detailed spectroscopic applications such as first-forbidden $\beta$ decay). In consistent shell-model calculations which include charge-dependent interactions, the response to changes in $T_z$ on the one hand and to changes in the number of particles or holes on the other strongly restricts the $d_{5/2}/s_{1/2}$ content of the low-lying states. An interesting case in the context of the present study is $^{16}$C which has a rank-0 $\beta$ decay branch of 0.68\% [19] to the lowest $0^-$ state of $^{16}$N. With an extra $p_{1/2}$ proton hole, the energy of the excited $0^+$ state has been lowered to 3.02 MeV, implying slightly more $1s_{1/2}^2$ in the ground state than for $^{17}$N. The first-forbidden $\beta$-decay rate is well accounted for using the same type of shell-model calculation and meson-exchange enhancement as for $^{17}$N [12].

A unique first-forbidden $\beta$ branch of 1.6(5)\% [20] to the ground state of $^{17}$O is known for the decay of $^{17}$N. This branch corresponds to $f^{(2)} = 24(8)$. With no change in the single nuclear matrix element involved, the expected branch in $^{17}$Ne decay is 0.55(18)\%. Charge-dependent effects should lower this value slightly because of a decrease in the $d_{5/2}^2$ component of the $^{17}$Ne ground state (Table IV), amplified somewhat by cancellation between $d_{5/2} \rightarrow p_{1/2}$ and $d_{5/2} \rightarrow p_{3/2}$ contributions. Shell-model calculations with the basis of Ref. [12] overpredict $f^{(2)}$ by a little more than a factor of two for either harmonic oscillator or Woods-Saxon wave functions. This is quite consistent with a similar overestimate for the unique first-forbidden decay of $^{16}$N for a correspondingly small shell-model basis. This problem is resolved in calculations using a very large shell-model basis with all configurations up to $4h\omega$ [8,21]. The rank-0 matrix elements are also reduced in such calculations [8] but by a lesser amount due to a cancellation between contributions from $2p2h$ admixtures induced by central and tensor forces. The experimental $\beta$-decay rates can then be reproduced using values for $\varepsilon_{mec}$ close to the theoretical value of about 1.6 [8].

In conclusion, the use of realistic (e.g., Woods-Saxon) radial wave functions is essential.
for evaluating first-forbidden $\beta$-decay matrix elements \[8,12\], particularly for $1s_{1/2} \leftrightarrow 0p_{1/2}$ transitions for which the $1s_{1/2}$ nucleon is loosely bound, as is the case for the decay of $^{17}N$ and $^{17}Ne$ to the first-excited $1/2^+$ states of $^{17}O$ and $^{17}F$. However, radial wave function differences do not account for the strong asymmetry observed for these decays. Rather, plausible $T_z$-dependent differences in the $1s_{1/2}$ occupancy for the initial states can account for the asymmetry. Furthermore, the very small separation energy for the $1s_{1/2}$ proton in $^{17}F$ is not germane to the problem since this proton is a spectator in the $\beta$-decay process. In fact, from the way in which the parentage expansion is made and separation energies determined, the spectator $1s_{1/2}$ proton forms part of a $^{16}F$ core where it is unbound for the physical core states (by 535 keV for the $0^-$ state). Substantial asymmetries have also been observed for the allowed decays of $^{17}N$ and $^{17}Ne$ \[22\]. While overlap factors for radial wave functions bound at different energies now play a role because the Gamow-Teller operator has no spatial structure, it again seems likely that the observed asymmetries are largely due to $T_z$-dependent mixing of various shell-model configurations. For the $2p1h$ configurations with $T = 1/2$, the mixing of configurations with $T = 0$ and $T = 1$ for the $(sd)^2$ configurations determines both the overall spatial symmetry and the relative contributions to the Coulomb energy from $p$ and $sd$ orbits. There are also low-lying $4p3h$ configurations (one $1/2^-$ and two $3/2^-$) which have their own Coulomb energy shifts and mix strongly with the $2p1h$ configurations. Thus, there should be significant $T_z$-dependent mixing in both the initial and final states for the Gamow-Teller decays. A beautiful demonstration of this type of $T_z$-dependent mixing is seen in changes of the ratio of Gamow-Teller strengths for the lowest two $2^+; T = 1$ states reached via $(n,p)$, $(p,p')$ and $(p,n)$ reactions on $^{14}N$ \[23\]. Here, the near degeneracy of $2h$ and $2p4h$ configurations \[24\], with Coulomb energies that differ by $\sim 700$ keV across the multiplet, leads to very different wave functions for each nucleus.

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TABLES

TABLE I. Parameters governing the decays of $^{17}$N and $^{17}$Ne to the first-excited states of $^{17}$O and $^{17}$F. Separation energies are given for the $0^{-}_1$; 1 core states in $^{16}$N and $^{16}$F; the values for the $1^{-}$ core state are 0.28 MeV and 0.19 MeV higher, respectively.

|       | $W_0$ (MeV) | $I_0$ | $S_{n/p}(s_{1/2})$ (MeV) | $S_{p/n}(p_{1/2})$ (MeV) |
|-------|-------------|-------|-------------------------|-------------------------|
| $^{17}$N | 8.32 MeV    | 0.3051| 6.00 ($n$)              | 13.03 ($p$)             |
| $^{17}$Ne | 13.52 MeV   | 2.380 | 1.48 ($p$)              | 16.80 ($n$)             |

TABLE II. Comparison of theoretical and experimental $\beta$-decay rates via $f$ values. $f_{\text{exp}}$ for $^{17}$Ne decay is derived from the average 1.55(12)% of the two measurements \cite{[1,4]} for the $\beta$ branch.

| $\varepsilon_{\text{mec}}$ | $f(0)$ | $f(1)$ | $f_{\text{exp}}$ |
|---------------------------|--------|--------|------------------|
| $^{17}$N                  | 37.9   | 45.7   | 6.5              | 44.4(74) |
| $^{17}$Ne                 | 415    | 491    | 21               | 873(64) |
| $^{17}$Ne'                | 722    | 854    | 21               | 873(64) |

TABLE III. Excitation energies (MeV) of $0^{+}$ $T = 1$ states relative to the lowest such state. The $0^{+}_2$ states are mainly $4p2h$ in nature. In the case of $^{18}$F, it should be noted that the lowest $0^{+}$ state obtains extra binding energy from the charge-independence breaking $np$ interaction \cite{[16]}.

| $J^p_n$ | $^{18}$O | $^{18}$F | $^{18}$Ne |
|---------|---------|---------|---------|
| $0^+_3$ | 5.336   | 5.094   | 4.590   |
| $0^+_2$ | 3.630   | 3.711   | 3.576   |

TABLE IV. Results of $(sd)^2$ diagonalizations. Wave function amplitudes are given in columns 4 – 6. The binding energy of the $0^+_1$ state of $^{18}$O is chosen as the zero of energy.

| $J^p_n$ | $E_x$  | $d_{5/2}^2$ | $s_{1/2}^2$ | $d_{3/2}^2$ | $\%s_{1/2}^2$ |
|---------|--------|-------------|-------------|-------------|----------------|
| $^{18}$O | $0^+_1$ | 0.000       | 0.8886      | 0.3878      | 0.2448         | 15.0           |
|         | $0^+_2$ | 4.320       | 0.3932      | -0.9190     | 0.0287         | 84.5           |
| $^{18}$Ne | $0^+_1$ | -0.163      | 0.8521      | 0.4654      | 0.2394         | 21.7           |
|         | $0^+_2$ | 3.588       | 0.4667      | -0.8827     | 0.0547         | 77.9           |