Equivalent Versions of “Khajuraho” and “Lo-Shu” Magic Squares and the day 1\textsuperscript{st} October 2010 (01.10.2010)

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Abstract
In this short note we shall give connection between the most perfect “Khajuraho” magic square of order $4 \times 4$ discovered in 10th century and the “Lo-Shu” magic square of order $3 \times 3$ with the day October 1, 2010, i.e., 01.10.2010. The day has only three digits 0, 1 and 2. Here we have given an equivalent version of Khajuraho magic square using only three digits 0, 1 and 2. If we write the above date date in two parts, 0110 2010, interestingly, the sum of new magic square is the first part, i.e. 0110, and the numbers appearing in the magic square are from the second part. An equivalent version of “Lo-Shu” magic square of order $3 \times 3$ is also given.

1 History

The study of magic squares is very old in history. The earliest known magic square is Chinese, recorded around 2800 B.C. described as ”Lo-Shu” magic square. (not sure, because many places we can see written as 2200 B.C. or 2500 B.C., etc.). It is a typical $3 \times 3$ magic square, where the numbers were represented by patterns not numerals. See below

(link: http://illuminations.nctm.org/LessonDetail.aspx?id=L263) – accessed on 01.11.2010.

More precisely it is like this
Numerical transcription of above “Lo-Shu” magic square is

| 4 | 9 | 2 |
|---|---|---|
| 3 | 5 | 7 |
| 8 | 1 | 6 |

Magic squares most likely traveled from China to India, then to the Arab countries. From the Arab countries, magic squares journeyed to Europe, then to Japan. Magic squares in India served multiple purposes other than the dissemination of mathematical knowledge. For example, Varahamihira used a fourth-order magic square to specify recipes for making perfumes in his book on seeing into the future, Brhatsamhita (ca. 550 A.D.). The oldest dated third-order magic square in India appeared in Vrnda’s medical work Siddhayoga (ca. 900 A.D.), as a means to ease childbirth. The fourth-order most perfect magic square found in 10th century (945 AD) in Khajuraho Jain Mandir or Parshvanath Jain Temple. See below plate appeared in Parshvanath Jain Temple in Khajuraho.
It is one of the most perfect magic square of order 4x4. It is generally famous as *Khajuraho magic square*. English transcription of above numbers is as follows:

\[
\begin{array}{cccc}
7 & 12 & 1 & 14 \\
2 & 13 & 8 & 11 \\
16 & 3 & 10 & 5 \\
9 & 6 & 15 & 4 \\
\end{array}
\]

It is pan diagonal magic square of sum 34. Each sub-square sum of 2x2 is also 34. Sometimes it is referred to as the “Chautisa Yantra”.

We are now in 21\textsuperscript{st} century in a digital era, where everything is digitalized. The aim of this work is to produce an *equivalent upside down version* of above magic square using only the digits appearing in a day 01.10.2010. In order to do so, we have used the numbers in the digital form:

\[0, 1 \text{ and } 2\]

These digits generally appear in watches, elevators, etc. We observe that the above three digits are rotatable to 180\degree, and remains the same.

Just to remember

(i) A **magic square** is a collection of numbers put as a square matrix, where the sum of elements of each row, sum of elements of each column and the sum of elements of each two principal diagonals are same.

(ii) **Upside down magic square** is a magic square, if we rotate it to 180\degree (degrees) it remains again the magic square.

(iii) **Mirror looking**, i.e., if we put it in front of mirror or see from the other side of the glass, or see on the other side of the paper, it always remains the magic square.

When the magic square is upside down and mirror looking, we call it **universal magic square**.

2 Equivalent Upside Down Magic Squares

In this section, we shall give *equivalent upside down versions* of above two historical magic squares using only the digits 0, 1 and 2 in digital forms.
2.1 Symmetric and Upside Down Version of “Khajuraho” Magic Square

Following Euler’s idea of 1782 [3] of Latin squares, let us consider the following two mutually orthogonal diagonalize Latin squares of order $4 \times 4$.

Let us apply an operation $4 \times (A - 1) + B$ in the above Latin squares, we get the following well known Khajuraho magic square of order $4\times4$ appearing in the above plate:

As we wrote before the above magic square is pan diagonal. Also each sub-square of order $2\times2$ sums to 34.

Again if we apply an operation $10 \times A + B$ over the above two Latin squares of order 4, then we get the following equivalent version of the Khajuraho magic square with sum as 110. See below

The above magic square is not upside down but is symmetric, i.e. if we have $ab$, then $ba$ is also there. Sometimes, we call it base 10 equivalent version.

Making some adjustments and writing the numbers in the digital form, here below is an upside down equivalent version of Khajuraho magic square:
The above magic square has only three digits 0, 1 and 2. If we gave a rotation of 180° degrees it remains the same. Interestingly, the sum 110 writing as 0110 becomes 180° degrees rotatable. We can consider it universal magic square, since looking from the mirror we again get a magic square, where 2 becomes as 5. Naturally in this case, the sum is not the same. We leave it to readers to check it.

- **The day 01.11.10**

Instead, considering 2010, if we consider a year as 10, then the day we write as 01.11.10. It becomes a number 011110 (naturally we are putting 0 in the front to make it symmetric). The palindromic version of above magic square in four algorism is given by

| 1110 | 1110 | 1110 | 1110 | 1110 | 1110 |
|------|------|------|------|------|------|
| 1110 | 2332 | 3443 | 1111 | 4224 | 1110 |
| 1110 | 1221 | 4114 | 2442 | 3333 | 1110 |
| 1110 | 4444 | 1331 | 3223 | 2112 | 1110 |
| 1110 | 3113 | 2222 | 4334 | 1441 | 1110 |
| 1110 | 1110 | 1110 | 1110 | 1110 | 1110 |

Writing in digital form we have the following **upside down magic square** of sum 011110:

| 0110 | 0110 | 0110 | 0110 | 0110 | 0110 |
|------|------|------|------|------|------|
| 0110 | 1224 | 2220 | 0000+ | 2224 | 2024 |
| 0110 | 1224 | 2220 | 0000+ | 2224 | 2024 |
| 0110 | 2224 | 2220 | 0000+ | 2224 | 2024 |
| 0110 | 2024 | 2220 | 0000+ | 2224 | 2024 |
Here all the numbers are palindromic, except two, the one is 220 and another is the sum 11110. To make them symmetric we have written 0 in the front, i.e, 0220 and 011110.

2.2 Symmetric and Upside Down Version of “Lo-Shu” Magic Square

We can get “Lo-Shu” magic square making an operation $3 \times A + B + 1$ on the following two non diagonal orthogonal Latin squares of order $3 \times 3$

\[
\begin{array}{ccc}
1 & 2 & 0 \\
0 & 1 & 2 \\
2 & 0 & 1 \\
\end{array} \quad \begin{array}{ccc}
0 & 2 & 1 \\
2 & 1 & 0 \\
1 & 0 & 2 \\
\end{array}
\]

Again making an operation $10 \times A + B$ on the above two Latin squares, we get a 33 sum magic square of order 3. The sum 33 can be written as 11+22. Writing the numbers in the digital form, we have the following equivalent upside down version of “Lo-Shu” magic square only with the digits 0, 1 and 2.

\[
\begin{array}{ccc}
\# & \# & \# \\
\# & \# & \# \\
\# & \# & \# \\
\end{array} \quad \begin{array}{ccc}
\# & \# & \# \\
\# & \# & \# \\
\# & \# & \# \\
\end{array}
\]

2.3 Curiosities

(i) The Khajuraho magic square was discovered in 10th century, and we are in 21st century. Both these centuries combined have the digits 0, 1 and 2.

(ii) We are in the year 2010. This has only three digits 0, 1 and 2.

(iii) The day of submission of this work, i.e., 01.11.2010 also has these three digits 0, 1 and 2.

(iv) If we consider the day 01.11.10, i.e, 011110. Still we can have palindromic magic square of sum 011110 only with the digits 0, 1 and 2.

(v) The years 2200 BC, 2500 BC, 2800 BC, appearing in the history in different sites of internet regarding the magic square of order $3 \times 3$, have the digits 0-2-5-8. All these four digits written in the digital form 7-2-5-8 are upside down and mirror looking, where 2 becomes 5 and 5 as 2 when we see them in the mirror.
(vi) The magic square $3 \times 3$ is made from the patterns, rather than numbers can be considered as universal, i.e., upside down and mirror looking, because looking any way it is always the same. See the second plate in section 1.

(vii) There are many other days happening during the year 2010, have the digits 0, 1 and 2. Such as 01.01.2010, 02.01.2010, 20.10.2010, 11.11.2010, etc. The day 01.10.2010, we have taken just randomly.

In [9], the author gave a bimagic square of order $9 \times 9$ using only the digits 0, 1 and 2, and bimagic square of order $8 \times 8$ using only the digits 0 and 1. For more work on magic square using digital numbers can be seen in Taneja [7, 8, 10]. Also the sites [1, 2] are good for further studies on magic squares.

The historical figures or plates given above are accessed from the internet, whose links are given in the references [4, 5, 6]. The historical part is also derived from these sites.

References

[1] C. Boyer - Multimagic Squares, http://www.multimagie.com.

[2] H. Heinz - Magic Squares, Magic Stars and Other Patterns http://www.magic-squares.net.

[3] L. Euler - Recherches sur une nouvelle espèce de quarres magiques. Opera Omnia, Ser. I, Vol 7, 291–392, Verhandelingen uitgegeven door het zeeuwsc genootschap der Wetenschappen te Vlissingen 9, 1782, 85–239. Also available online at http://www.eulerarchive.org.

[4] Jain Tample Khajuraho - http://en.wikipedia.org/wiki/Jain_temple_of_Khajuraho

[5] History of Magic Square - http://illuminations.nctm.org/LessonDetail.aspx?id=L263

[6] Magic Square Worksheets - http://www.magicsquaresworksheet.com/history-of-magic-squares

[7] I.J. Taneja - DIGITAL ERA: Magic Squares and 8th May 2010 (08.05.2010), http://arxiv.org/abs/1005.1384.

[8] I.J. Taneja - ERA DIGITAL E 50 ANOS DA UFSC, http://www.50anos.ufsc.br/Homenagem_MTMTM.pdf.

[9] I.J. Taneja - Universal Bimagic Squares and the day 10th October 2010 (10.10.10), http://arxiv.org/abs/1010.2083.

[10] I.J. Taneja - DIGITAL ERA: Universal Bimagic Squares, http://arxiv.org/abs/1010.2541.