BiHom-Poisson color algebras

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Abstract. The goal of this paper is to introduce BiHom-Poisson color algebras and give various constructions by using some specific maps such as morphisms. We introduce averaging operator and element of centroid for BiHom-Poisson color algebras and point out that BiHom-Poisson color algebras are closed under averaging operators and elements of centroid. We also show that any regular BiHom-associative color algebra leads to BiHom-Poisson color algebra via the commutative bracket. Then we prove that any BiHom-Poisson color algebra together with Rota-Baxter operator or multiplier give rise to another BiHom-Poisson color algebra. Next, we show that tensor product of any BiHom-associative color algebra and any BiHom-Poisson color algebra is also a BiHom-Poisson color algebra.

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1 Introduction

BiHom-algebraic structures were introduced in the first time in 2015 by G. Graziani, A. Makhlouf, C. Menini and F. Panaite in [8] from a categorical approach as an extension of the class of Hom-algebras. Since then, other interesting BiHom-type algebraic structures of many Hom-algebraic structures has been intensively studied as BiHom-Lie colour algebras structures [15], Representations of BiHom-Lie algebras [27], BiHom-Lie superalgebra structures [23], (σ, τ)-Rota-Baxter operators, infinitesimal Hom-bialgebras and the associative (Bi)Hom-Yang-Baxter equation [20], The construction and deformation of BiHom-Novikov algebras [22], On n-ary Generalization of BiHom-Lie algebras and BiHom-Associative Algebras [4], Rota-Baxter operators on BiHom-associative algebras and related structures [16].

The purpose of this paper is to introduce BiHom-Poisson color algebras and give various constructions by using special even linear maps. In section 2, we give basic definition and properties of BiHom-Poisson color algebras. In particular, we show that any regular BiHom-associative color algebra leads to BiHom-Poisson color algebra via the commutative bracket. In section 3, we show
that BiHom-Poisson color algebras are closed under either tensorization with scalar field or commutative BiHom-associative color algebras, averaging operators and elements of centroid. Moreover, we prove that any BiHom-Poisson color algebra endowed with Rota-Baxter operator or multiplier give rises to another BiHom-Poisson color algebra.

Throughout this paper, all graded vector spaces are assumed to be over a field $\mathbb{K}$ of characteristic different from 2.

## 2 Definitions and basic properties

In this section, we recall basic definitions and some elementary properties.

### 2.1 Multipliers

**Definition 2.1.** Let $G$ be an abelian group. A map $\varepsilon: G \times G \to \mathbb{K}^*$ is called a skew-symmetric bicharacter on $G$ if the following identities hold,

(i) $\varepsilon(g, g')\varepsilon(g', g) = 1$,

(ii) $\varepsilon(g, g' + g'') = \varepsilon(g, g')\varepsilon(g, g'')$,

(iii) $\varepsilon(g + g', g'') = \varepsilon(g, g'')\varepsilon(g', g'')$.

If $g, g', g'' \in G$.

**Remark 2.2.** Observe that $\varepsilon(g, e) = \varepsilon(e, g) = 1, \varepsilon(g, g) = \pm 1$ for all $g \in G$, where $e$ is the identity of $G$.

**Example 2.3.** 

1) $G = \mathbb{Z}^n_2 = \{(\alpha_1, \ldots, \alpha_n) | \alpha_i \in \mathbb{Z}_2\}$, $\varepsilon((\alpha_1, \ldots, \alpha_n), (\beta_1, \ldots, \beta_n)) := (-1)^{\alpha_1\beta_1 + \cdots + \alpha_n\beta_n}$,

2) $G = \mathbb{Z} \times \mathbb{Z}$, $\varepsilon((i_1, i_2), (j_1, j_2)) = (-1)^{i_1+j_1}(-1)^{i_2+j_2}$,

3) $G = \{-1, +1\}$, $\varepsilon(i, j) = (-1)^{(i-1)(j-1)/4}$.

**Example 2.4.** Let $\sigma: G \times G \to \mathbb{K}^*$ be any mapping such that

$$\sigma(g, g' + g'')\sigma(g', g'') = \sigma(g, g')\sigma(g + g', g''), \forall g, g', g'' \in G. \tag{2.1}$$

Then, $\delta(g, g') = \sigma(g, g')\sigma(g', g)^{-1}$ is a bicharacter on $G$. In this case, $\sigma$ is called a multiplier on $G$, and $\delta$ the bicharacter associated with $\sigma$.

For instance, let us define the mapping $\sigma: G \times G \to \mathbb{R}$ by

$$\sigma((i_1, i_2), (j_1, j_2)) = (-1)^{i_1j_2}, \forall i_k, j_k \in \mathbb{Z}_2, k = 1, 2.$$ 

It is easy to verify that $\sigma$ is a multiplier on $G$ and $\delta$ a bicharacter on $G$.

**Definition 2.5.** A color BiHom-algebra is a quadruple $(A, \mu, \varepsilon, \alpha)$ in which

a) $A$ is a $G$-graded vector space i.e. $A = \bigoplus_{g \in G} A_g$,

b) $\mu: A \times A \to A$ is a even bilinear map i.e. $\mu(A_g, A_{g'}) \subseteq A_{g+g'}$, for all $g, g' \in G$,

c) $\alpha, \beta: A \to A$ are even linear maps i.e. $\alpha(A_g) \subseteq A_g, \beta(A_g) \subseteq A_g$,

d) $\varepsilon: G \times G \to \mathbb{K}^*$ is a bicharacter.

If $x$ and $y$ are two homogeneous elements of degree $g$ and $g'$ respectively and $\varepsilon$ is a skew-symmetric bicharacter, then we shorten the notation by writing $\varepsilon(x, y)$ instead of $\varepsilon(g, g')$. 

2.2 BiHom-Poisson color algebras

In this subsection, we introduce BiHom-Poisson color algebra and give some basic properties.

Definition 2.6. (12) A Hom-Poisson color algebra consists of a $G$-graded vector space $A$, a multiplication $\mu : A \times A \to A$, an even bilinear bracket $\{\cdot, \cdot\} : A \times A \to A$ and an even linear map $\alpha : A \to A$ such that:

1) $(A, \mu, \varepsilon, \alpha)$ is a Hom-associative color algebra,
2) $(A, \{\cdot, \cdot\}, \varepsilon, \alpha)$ is a Hom-Lie color algebra,
3) the Hom-Leibniz color identity

$$\{\alpha(x), \mu(y, z)\} = \mu([x, y], \alpha(z)) + \varepsilon(x, y)\mu(\alpha(y), [x, z]),$$

is satisfied for any $x, y, z \in \mathcal{H}(A)$.

A Hom-Poisson color algebra $(A, \mu, \{\cdot, \cdot\}, \varepsilon, \alpha)$ in which $\mu$ is $\varepsilon$-commutative is said to be a commutative Hom-Poisson color algebra.

The following definition is motivated by the one above.

Definition 2.7. A BiHom-Poisson color algebra consists of a $G$-graded vector space $P$, a multiplication $\mu : P \times P \to P$, a bilinear bracket $\{\cdot, \cdot\} : P \times P \to P$ and even linear maps $\alpha, \beta : P \to P$ such that:

1) $(P, \mu, \varepsilon, \alpha, \beta)$ is a BiHom-associative color algebra of degree zero i.e. :

$$\alpha \circ \beta = \beta \circ \alpha$$

(2.2)

$$as_{\mu}(x, y, z) = \mu(\alpha(x), \mu(y, z)) - \mu(\mu(x, y), \alpha(z)) = 0,$$

(2.3)

for any $x, y, z \in \mathcal{H}(P)$.

2) $(P, \{\cdot, \cdot\}, \varepsilon, \alpha, \beta)$ is a BiHom-Lie color algebra i.e. :

$$\alpha \circ \beta = \beta \circ \alpha$$

(2.4)

$$\alpha([x, y]) = [\alpha(x), \alpha(y)], \quad \beta([x, y]) = [\beta(x), \beta(y)]$$

(2.5)

$$[\beta(x), \alpha(y)] = -\varepsilon(x, y+)[\beta(y), \alpha(x)], \quad \text{(BiHom-$\varepsilon$-skew-simmetry color identity)}$$

(2.6)

$$\mathbf{J}(x, y, z) = \oint_{x, y, z} \varepsilon(z, x+)[\beta^2(x), [\beta(y), \alpha(z)]] = 0, \quad \text{(BiHom-Jacobi color identity)}$$

(2.7)

for any $x, y, z \in \mathcal{H}(P)$, where $\oint_{x, y, z}$ means cyclic summation over $x, y, z$.

3) the BiHom-Leibniz color identity

$$\{\alpha(x), \mu(y, z)\} = \mu([x, y], \alpha(z)) + \varepsilon(x, y)\mu(\alpha(y), [x, z]),$$

(2.8)

is satisfied for any $x, y, z \in \mathcal{H}(P)$.
Definition 2.8.  a) A BiHom-Poisson color algebra \((P, \mu, \{-,\}, \varepsilon, \alpha, \beta)\) in which \(\mu\) is \(\varepsilon\)-commutative i.e.
\[
\mu(x, y) = \varepsilon(x, y)\mu(y, x)
\]
is said to be a commutative BiHom-Poisson color algebra.

b) A BiHom-Poisson color algebra \((P, \mu, \{-,\}, \varepsilon, \alpha, \beta)\) in which \(\alpha\) and \(\beta\) are automorphisms is called regular BiHom-Poisson color algebra.

Example 2.9. Any BiHom-associative color algebra \([2]\) can be seen as a BiHom-Poisson color algebra with the trivial BiHom-Lie color structure, and any BiHom-Lie color algebra \([15]\) can be a) A BiHom-Poisson color algebra (\(4\) or \(\text{Poisson superalgebras}.\) BiHom-Poisson algebras \([26]\) and BiHom-Poisson superalgebras are called regular BiHom-Poisson color algebra.

Example 2.10. Hom-Poisson color algebras \([12]\) or Poisson color algebras \([13]\) are examples of BiHom-Poisson color algebras by setting \(\beta = id\) or \(\alpha = id\) and \(\beta = id\). If, in addition, \(\varepsilon(x, y) = 1\) or \(\varepsilon(x, y) = (-1)^{|x||y|}\), then the BiHom-Poisson color algebra is nothing but classical Poisson algebras \([\] or Poisson superalgebras. BiHom-Poisson algebras \([26]\) and BiHom-Poisson superalgebras are also obtained when \(\varepsilon(x, y) = 1\) and \(\varepsilon(x, y) = (-1)^{|x||y|}\) respectively. If, moreover \(\beta = id\) we get Hom-Poisson algebras \([2]\) and Hom-Poisson superalgebras.

Example 2.11. (\([10]\)) If \((P,\{-,\},\star,*,\varepsilon,\alpha)\) is a Hom-post-Poisson color algebra. Then \((P,\circ,\{-,\},\varepsilon,\alpha)\) is a commutative Hom-Poisson color algebra with
\[
x \circ y = x \star y + \varepsilon(x, y)y \star x + x \ast y \quad \text{and} \quad [x, y] = x \cdot y - \varepsilon(x, y)y \cdot x + [x, y]
\]
for any \(x, y \in P\).

The next result allows to construct a BiHom-Poisson color algebra from a given one and a bijective map.

Theorem 2.12. Let \((P', \cdot', \{-,\}', \varepsilon, \alpha', \beta')\) be a BiHom-Poisson color algebra and \(P\) a graded vector space with a \(\varepsilon\)-skew-symmetric bilinear bracket of degree and even linear maps \(\alpha\) and \(\beta\). Let \(f : L \to L'\) be an even bijective linear map such that \(f \circ \alpha = \alpha' \circ f, f \circ \beta = \beta' \circ f, f(x \cdot y) = f(x) \cdot f(y)\) for all \(x, y \in \mathcal{H}(P)\).

Then \((P', \cdot', \{-,\}', \varepsilon, \alpha, \beta)\) is a BiHom-Poisson color algebra.

Proof. BiHom-associativity and BiHom-Jacobi color identities are checked as in the following BiHom-Leibniz color identity. Then, for any \(x, y, z \in \mathcal{H}(P)\),
\[
[\beta \alpha(x), y \cdot z] = f^{-1}[f(\beta \alpha(x)), f(y \cdot z)]' = f^{-1}[f(\beta \alpha(x)), f(f^{-1}(f(y) \cdot f(z)))'] = f^{-1}[\beta' \alpha'(f(x)), f(y)' f(z)'] = f^{-1}(f(f^{-1}(f(\beta(x)), f(y))' \cdot f(z)))' + \varepsilon(x, y) f^{-1}(f(\beta(y))' f((\alpha(x))' f(z)))
\]
This gives the conclusion. \(\square\)
The following theorem asserts that the commutator of any BiHom-associative color algebra gives rise to BiHom-Poisson color algebra.

**Theorem 2.13.** Let \((A, \cdot, e, \alpha, \beta)\) be a regular BiHom-associative color algebra. Then

\[
P(A) = (A, \cdot, [-, -], e, \alpha, \beta)
\]

is a regular BiHom-Poisson color algebra, where

\[
[x, y] = x \cdot y - e(x, y)\alpha^{-1} \beta(y) \cdot \alpha^{-1} \beta^{-1}(x),
\]

for any \(x, y \in \mathcal{H}(A)\).

**Proof.** It is proved in (Proposition 1.6 [15]) that any BiHom-associative color algebra carries a structure of a BiHom-Lie color algebra. Thus, we only need to check the compatibility condition. For any homogeneous elements \(x, y, z \in A\),

\[
[a\beta(x), y \cdot z] = a\beta(x)(yz) - e(x, y + z)\alpha^{-1} \beta(y) \cdot \alpha^{-1} \beta^{-1} \alpha\beta(x)
\]

\[
= a\beta(x)(yz) - e(x, y + z)(\alpha^{-1} \beta(y) \cdot \alpha^{-1} \beta(z))\alpha^2(x).
\]

\[
[\beta(x), y] \cdot \beta(z) = (\beta(x) \cdot y - e(x, y)\alpha^{-1} \beta(y) \cdot \alpha^{-1} \beta^{-1} \beta(x)) \cdot \beta(z)
\]

\[
= (\beta(x) \cdot y) \cdot \beta(z) - e(x, y)(\alpha^{-1} \beta(y) \cdot \alpha(x)) \cdot \beta(z).
\]

and

\[
\beta(y) \cdot [\alpha(x), z] = \beta(y) \cdot (\alpha(x) \cdot z - e(x, z)\alpha^{-1} \beta(z) \cdot \alpha^{-1} \beta^{-1} \alpha(x))
\]

\[
= \beta(y) \cdot (\alpha(x) \cdot z - e(x, z)\beta(y) \cdot (\alpha^{-1} \beta(z) \cdot \alpha^2 \beta^{-1}(x))
\]

\[
= \alpha(\alpha^{-1} \beta(y)) \cdot (\alpha(x) \cdot z - e(x, z)\alpha^{-1} (\beta(y)) \cdot (\alpha^{-1} \beta(z) \cdot \alpha^2 \beta^{-1}(x))
\]

\[
= (\alpha^{-1} \beta(y) \cdot \alpha(x)) \cdot \beta(z) - e(x, z)(\alpha^{-1} \beta(y) \cdot \alpha^{-1} \beta(z)) \cdot \alpha^2(x).
\]

Therefore,

\[
[a\beta(x), yz] = [\beta(x), y] \cdot \beta(z) + e(x, y)\beta(y) \cdot [\alpha(x), z].
\]

This concludes the proof. \(\square\)

### 3 Main results

This section is devoted to an exposition of various constructions of BiHom-Poisson color algebras.

#### 3.1 BiHom-Poisson color algebras arising from twisting

The following theorem asserts that BiHom-Poisson color algebra turn to another one via morphisms.

**Theorem 3.1.** Let \((P, \mu, [-, -], e, \alpha, \beta)\) be a BiHom-Poisson color algebra and \(\alpha', \beta' : P \to P\) two even morphisms of BiHom-Poisson color such that any two of the maps \(\alpha, \alpha', \beta, \beta'\) commute. Then

\[
P_{(\alpha', \beta')} = (P, * = \mu(\alpha' \otimes \beta'), [-, -] := [-, -](\alpha' \otimes \beta'), e, \alpha', \beta')
\]

is a BiHom-Poisson color algebra.
Corollary 3.2. Let \((P,\mu,[\cdot,\cdot],\varepsilon,\alpha,\beta)\) be a BiHom-Poisson color algebra. Then
\[
(P,\mu \circ (\alpha^n \otimes \beta^n),[\cdot,\cdot] \circ (\alpha^n \otimes \beta^n),\alpha^{n+1},\beta^{n+1})
\]
is also a BiHom-Poisson color algebra.

Proof. It suffices to take \(\alpha' = \alpha^n\) and \(\beta' = \beta^n\) in Theorem 3.1.

By taking \(\alpha' = \beta' = \alpha \neq id\) in Theorem 3.1, we get the following corollary.

Corollary 3.3. Let \((P,\mu,[\cdot,\cdot],\varepsilon,\alpha)\) be a Hom-Poisson color algebra. Then
\[
(P,\mu \circ (\alpha \otimes \beta),[\cdot,\cdot] \circ (\alpha \otimes \beta),\alpha^2,\beta^2)
\]
is also a Hom-Poisson color algebra.

Any regular Hom-Poisson color algebra gives rise to Poisson color algebras as stated in the next corollary.

Corollary 3.4. Let \((P,\mu,[\cdot,\cdot],\varepsilon,\alpha)\) be a regular Hom-Poisson color algebra. Then
\[
(P,\mu \circ (\alpha^{-1} \otimes \beta^{-1}),[\cdot,\cdot] \circ (\alpha^{-1} \otimes \beta^{-1}))
\]
is also a Poisson color algebra.

Corollary 3.5. Let \((P,\mu,[\cdot,\cdot],\varepsilon)\) be a Poisson color algebra. Then
\[
(P,\mu \circ (\alpha \otimes \beta),[\cdot,\cdot] \circ (\alpha \otimes \beta),\alpha,\beta)
\]
is a BiHom-Poisson color algebra.

3.2 Extensions of BiHom-Poisson color algebras

In this subsection, we give extensions of a given BiHom-Poisson color algebra by a field, by commutative associative color algebras or by another BiHom-Poisson color algebra.

Theorem 3.6. Let \((P,\cdot,[\cdot,\cdot],\varepsilon,\alpha)\) be a BiHom-Poisson color algebra over a field \(\mathbb{K}\) and \(\hat{\mathbb{K}}\) an extension of \(\mathbb{K}\). Then, the graded \(\mathbb{K}\)-vector space
\[
\hat{\mathbb{K}} \otimes P = \sum_{g \in G} (\mathbb{K} \otimes P)_g = \sum_{g \in G} \mathbb{K} \otimes P_g
\]
The associative product

\[(r \otimes x) \cdot (s \otimes y) := rs \otimes (x \cdot y),\]

the bracket

\[[r \otimes x, s \otimes y] = rs \otimes [x, y],\]

the even linear maps

\[\alpha'(r \otimes x) := r \otimes \alpha(x) \quad \text{and} \quad \beta'(r \otimes x) := r \otimes \beta(x),\]

and the bicharacter

\[\varepsilon(r + x, s + y) = \varepsilon(x, y), \forall r, s \in \mathbb{H}, \forall x, y \in \mathcal{H}(P).\]

Proof. It is proved by a straightforward computation. \(\square\)

We need the following Lemmas for the next theorem.

Lemma 3.7. In any commutative BiHom-associative color algebra, we have

1) \(\beta^2(a)(\beta(b)\alpha(c)) = \beta^2(a)(\beta(b)\beta(c)),\)

2) \(\beta^2(b)(\beta(c)\alpha(a)) = \varepsilon(b + c, a)\beta^2(a)(\beta(b)\beta(c)),\)

3) \(\beta^2(c)(\beta(a)\alpha(b)) = \varepsilon(c, a + b)\beta^2(a)(\beta(b)\beta(c)).\)

Proof. 1) For any \(a, b, c \in \mathcal{H}(A),\) we have :

\[\beta^2(a)(\beta(b)\alpha(c)) = \varepsilon(a, b + c)(\beta(b)\alpha(c))\beta^2(a) = \varepsilon(a, b + c)\alpha\beta(b)(\alpha(c)\beta(a))\]

\[= \varepsilon(a, b + c)\alpha\beta(b)(\beta(a)\alpha(c)) = \varepsilon(a, b)(\beta(b)\alpha(c))\beta\alpha(c)\]

\[= \varepsilon(a, b)\varepsilon(a, b + c)\alpha\beta(c)(\beta(b)\beta(a)) = \varepsilon(a, b)\varepsilon(a, b + c)(\beta(c)\beta(b))\beta^2(a)\]

\[= \beta^2(a)(\beta(b)\beta(c)).\]

2) For any \(a, b, c \in \mathcal{H}(A),\) we have :

\[\beta^2(b)(\beta(c)\alpha(a)) = \varepsilon(b, a + c)(\beta(c)\alpha(a))\beta^2(b) = \varepsilon(b, a + c)\alpha\beta(c)(\alpha(a)\beta(b))\]

\[= \varepsilon(b, a + c)\alpha\beta(c)(\beta(a)\alpha(a)) = \varepsilon(b, c)(\beta(c)\beta(a))\beta\alpha(a)\]

\[= \varepsilon(b, c)\varepsilon(b + c, a)\alpha\beta(a)(\beta(c)\beta(b)) = \varepsilon(b, c)\varepsilon(b + c, a)(\beta(a)\beta(c))\beta^2(b)\]

\[= \varepsilon(b, a + c)(\beta(c)\beta(a))\beta^2(b) = \varepsilon(b, a + c)\alpha\beta(c)(\beta(a)\beta(b))\]

\[= \varepsilon(b, c)\alpha\beta(c)(\beta(b)\beta(a)) = \varepsilon(b, c)(\beta(c)\beta(b))\beta^2(a)\]

\[= \varepsilon(b, c)\varepsilon(b + c, a)\varepsilon(c, b)\beta^2(a)(\beta(b)\beta(c))\]

\[= \varepsilon(b + c, a)\beta^2(a)(\beta(b)\beta(c)).\]

3) For any \(a, b, c \in \mathcal{H}(A),\) we have :

\[\beta^2(c)(\beta(a)\alpha(b)) = \varepsilon(c, a + b)(\beta(a)\alpha(b))\beta^2(c) = \varepsilon(c, a + b)\alpha\beta(a)(\alpha(b)\beta(c))\]

\[= \varepsilon(c, a + b)\varepsilon(b, c)\alpha\beta(a)(\beta(c)\alpha(b)) = \varepsilon(c, a)(\beta(a)\beta(c))\beta\alpha(b)\]

\[= \varepsilon(c, a + b)\varepsilon(c, a)\alpha\beta(b)(\beta(a)\beta(c)) = \varepsilon(c, a + b)\alpha\beta(b)(\beta(c)\beta(a))\]

\[= \varepsilon(c, a + b)(\beta(b)\beta(c))\beta^2(a) = \varepsilon(c, a + b)\varepsilon(b + c, a)\beta^2(a)(\beta(b)\beta(c))\]

\[= \varepsilon(c, a + b)\beta^2(a)(\beta(b)\beta(c)).\]

This ends the proof. \(\square\)
Lemma 3.8. If $(A, \cdot, \varepsilon, \alpha, \beta)$ is commutative BiHom-associative color algebra, we have
\[ a\beta(a)(bc) = \varepsilon(a, b)\beta(b)(\alpha(a)c). \] (3.1)

Proof. For any $a, b, c \in \mathcal{H}(A)$, we have:
\[
\begin{align*}
a\beta(a)(bc) &= \varepsilon(a, b + c)(bc)\beta\varepsilon(a) = \varepsilon(a, b + c)\alpha(b)(c\alpha(a)) = \varepsilon(a, b + c)\varepsilon(c, a)\alpha(b)(\alpha(a)c) \\
&= \varepsilon(a, b)\beta\varepsilon(a)(c) = (a\alpha)\beta\varepsilon(c) = a^2\varepsilon(a)\beta\varepsilon(b) = \varepsilon(b, c)\alpha^2\varepsilon(a)\beta(c) \\
&= \varepsilon(b, c)\alpha\varepsilon(b)(\alpha(a)c) \\
&= \varepsilon(a, b)\beta(b)(\alpha(a)c).
\end{align*}
\]

This finishes the proof. \[\square\]

The below result gives an extension of a BiHom-Poisson color algebra by a commutative BiHom-associative color algebra.

Theorem 3.9. Let $(A, \cdot, \varepsilon, \alpha, \beta)$ be a commutative BiHom-associative color algebra and $(P, *, \{-, -\}, \varepsilon, \alpha_L)$ be a BiHom-Poisson color algebra. Then the tensor product $A \otimes P$ endowed with the even (bi)linear maps
\[
\begin{align*}
\alpha &: = \alpha_A \otimes \alpha_P : A \otimes P \to A \otimes P, \quad a \otimes x \mapsto \alpha_A(a) \otimes \alpha_P(x), \\
\beta &: = \beta_A \otimes \beta_P : A \otimes P \to A \otimes P, \quad a \otimes x \mapsto \beta_A(a) \otimes \beta_P(x), \\
* &: = (A \otimes P) \times (A \otimes P) \to A \otimes P, \quad (a \otimes x, b \otimes y) \mapsto \varepsilon(x, b)a \cdot b \otimes x * y, \\
\{-, -\} &: = (A \otimes P) \times (A \otimes P) \to A \otimes P, \quad (a \otimes x, b \otimes y) \mapsto \varepsilon(x, b)(a \cdot b) \otimes [x, y],
\end{align*}
\]
is a BiHom-Poisson color algebra.

Proof. To simplify the typography, we make no distinction among $\alpha, \alpha_A, \alpha_P$ or $\beta, \beta_A, \beta_P$. It is easy to prove the color BiHom-associativity. For the BiHom-Jacobi color identity, we have, for any $a, b, c \in \mathcal{H}(A)$ and $x, y, z \in \mathcal{H}(P),
\[
\begin{align*}
[\beta^2(a \otimes x), [\beta(b \otimes y), \alpha(c \otimes z)] &= \varepsilon(y, c)[\beta^2(a \otimes x), \beta(b)\alpha(c) \otimes [\beta(y), \alpha(z)]] \\
&= \varepsilon(y, c)\varepsilon(x, b + c)\beta^2(a)\beta(b)\alpha(c) \otimes [\beta^2(x), [\beta(y), \alpha(z)]]
\end{align*}
\]
By Lemma[3,7]
\[
\int \varepsilon(c + z, a + x)[\beta^2(a \otimes x), [\beta(b \otimes y), \alpha(c \otimes z)] = 0.
\]
Now, for any $a, b, c \in \mathcal{H}(A)$ and $x, y, z \in \mathcal{H}(P),
\[
\begin{align*}
[\beta\alpha(a \otimes x), (b \otimes y) * (c \otimes z)] &= \varepsilon(y, c)[\beta\alpha(a) \otimes \beta\alpha(b) \otimes yz] \\
&= \varepsilon(y, c)\varepsilon(x, b + c)\alpha\beta(a) \otimes [\beta\alpha(x), yz] \\
&= \varepsilon(y, c)\varepsilon(x, b + c)\alpha\beta(a)(bc) \otimes (\beta(x), y)\beta(z) + \varepsilon(x, y)\beta(y)[\alpha(x), z] \\
&= \varepsilon(y, c)\varepsilon(x, b + c)\alpha\beta(a)(bc) \otimes [\beta(x), y]\beta(z) + \varepsilon(y, c)\varepsilon(x, b + c)\varepsilon(x, y)\alpha\beta(a)(bc) \otimes \beta(y)[\alpha(x), z].
\end{align*}
\]
By color BiHom-associativity and Lemma 3.8,
\[
[\beta \alpha(a \otimes x), (b \otimes y) \ast (c \otimes z)] = \varepsilon(x, b)(\beta(a) b \otimes [\beta(x), y])(\beta(c) \otimes \beta(z))
\]
\[
+ \varepsilon(y, c)\varepsilon(x, b + c) \varepsilon(x, y)\varepsilon(a, b) \beta(b)(\alpha(a) c \otimes \beta(y))[\alpha(x), z]
\]
\[
= \varepsilon(x, b)(\beta(a) b \otimes [\beta(x), y])(\beta(c) \otimes \beta(z))
\]
\[
+ \varepsilon(x, c)\varepsilon(x, y)\varepsilon(a, b) \beta(a) \beta(b)(\beta(b) \otimes \beta(y))(\alpha(a) c \otimes [\alpha(x), z])
\]
\[
= [\beta(a) \otimes \beta(x), b \otimes y]\beta(c) \otimes \beta(z)
\]
\[
+ \varepsilon(a + x, b + y)(\beta(b) \otimes \beta(y))(\alpha(a) \otimes \alpha(x), c \otimes z)
\]
\[
= [\beta(a \otimes x), b \otimes y] \beta(c \otimes z) + \varepsilon(a + x, b + y) \beta(b \otimes y)(\alpha(a \otimes x), c \otimes z).
\]
This completes the proof. \(\Box\)

From Theorem 3.9 we obtain Theorem 2.17 [12].

**Corollary 3.10.** Let \((A, \ast, \varepsilon)\) be a commutative Hom-associative color algebra and \((P, \ast, [-,-], \varepsilon, \alpha, \beta)\) be a BiHom-Poisson color algebra. Then the tensor product \(A \otimes P\) is a BiHom-Poisson color algebra.

**Corollary 3.11.** Let \((A, \ast, \varepsilon)\) be a commutative associative color algebra and \((P, \ast, [-,-], \varepsilon, \alpha, \beta)\) be a BiHom-Poisson color algebra. Then the tensor product \(A \otimes P\) is a BiHom-Poisson color algebra.

As the tensor product of two BiHom-Poisson color algebra fails to be a BiHom-Poisson color algebra, we have however the following theorem:

**Proposition 3.12.** Let \((P_1, \ast_1, [-,-], \varepsilon, \alpha_1)\) and \((P_2, \ast_2, [-,-], \varepsilon, \alpha_2)\) be two Hom-Poisson color algebras such that the maps
\[
\alpha(a \otimes x) := \alpha_1(a) \ast_2 \alpha_2(x),
\]
\[
(a \otimes x) \ast (b \otimes y) := \varepsilon(x, b) a \ast_1 b \otimes x \ast_2 y,
\]
\[
[a \otimes x, b \otimes y] := \varepsilon(x, b) (a \ast_1 b \otimes [x, y]_2 + [a, b] \otimes x \ast_2 y),
\]
satisfy the Hom-Jacobi identity on \(P_1 \otimes P_2\). Then, \((P_1 \otimes P_2, \ast, [-,-], \alpha)\) is a Hom-Poisson color algebra.

**Proof.** The proof is long and straightforward but uses the same technics as in Theorem 3.9. \(\Box\)

**Corollary 3.13.** ([5] Proposition 2.5.2) The tensor product of two Poisson color algebras is again a Poisson color algebra.

**Corollary 3.14.** ([7] Theorem 2.9) The tensor product of two Hom-Poisson algebras is also a Hom-Poisson algebra.

### 3.3 BiHom-Poisson color algebras induced by Rota-Baxter operators

**Definition 3.15.** Let \((P, \ast, [-,-], \varepsilon, \alpha, \beta)\) be a BiHom-Poisson color algebra. An even linear map \(R : P \to P\) is called a Rota-Baxter operator of weight \(\lambda \in \mathbb{R}\) on \(P\) if
\[
\alpha \circ R = R \circ \alpha, \quad \beta \circ R = R \circ \beta,
\]
\[
R(x) \cdot R(y) \equiv R(R(x) \cdot y + x \cdot R(y) + \lambda x \cdot y),
\]
\[
[R(x), R(y)] = R([R(x), y] + [x, R(y)] + \lambda [x, y]),
\]
for all \(x, y \in \mathcal{H}(A)\).
The below result connects BiHom-Poisson color algebras to Rota-Baxter operator.

**Theorem 3.16.** Let \((P, \cdot, [-,-], e, \alpha, \beta)\) be a BiHom-Poisson color algebra and \(R : P \to P\) be Rota-Baxter operator of weight \(\lambda \in K\) on \(P\). Then \(P\) is also a BiHom-Poisson color algebra with

\[
x \star y = R(x) \cdot y + x \cdot R(y) + \lambda x \cdot y, \quad (3.8)
\]

\[
\{x, y\} = [R(x), y] + [x, R(y)] + \lambda [x, y], \quad (3.9)
\]

for all \(x, y \in \mathcal{H}(P)\).

Moreover, \(R\) is a morphism of BiHom-Poisson color algebra of \((P, \star, [x, y], e, \alpha, \beta)\) into \((P, \cdot, [-,-], e, \alpha, \beta)\).

**Proof.** For any \(x, y, z \in \mathcal{H}(P)\),

\[
[\beta \alpha(x), y \star z] = [\beta \alpha(x), R(y)z + yR(z) + \lambda yz]
\]

\[
= [R \beta(x), R(y)z + yR(z) + \lambda yz] + [\beta(x), R(R(y)z + yR(z) + \lambda yz)]
\]

\[
+ \lambda [\beta(x), R(y)z + yR(z) + \lambda yz]
\]

\[
= [R \beta(x), R(y)z] + [R \beta(x), yR(z)] + [\beta(x), R(y)z] + \lambda [\beta(x), R(y)z] + \lambda [\beta(x), yR(z)] + \lambda^2 [\beta(x), yR(z)].
\]

By BiHom-Leibniz color identity,

\[
[\beta \alpha(x), y \star z] = [R \beta(x), R(y)][\beta(z) + \epsilon(x, y)\beta(R(y))[R \alpha(x), z]] + [R \beta(x), y][\beta(R(z))]
\]

\[
+ \epsilon(x, y)\beta(y)R(y)[R \alpha(x), z] + [\epsilon(x, y)\beta(y)R(y)][R \alpha(x), z] + [\epsilon(x, y)\beta(y)][R \alpha(x), R(z)]
\]

By reorganizing the terms, we have

\[
[\beta \alpha(x), y \star z] = [R \beta(x), R(y)][\beta(z) + \epsilon(x, y)\beta(R(y))[R \alpha(x), z] + [\epsilon(x, y)\beta(y)R(y)][R \alpha(x), z]
\]

\[
+ [\epsilon(x, y)\beta(y)][R \alpha(x), z] + [\epsilon(x, y)\beta(y)][R \alpha(x), R(z)]
\]

\[
+ \lambda [\epsilon(x, y)\beta(y)][R \alpha(x), z] + [\epsilon(x, y)\beta(y)][R \alpha(x), R(z)]
\]

\[
+ \lambda^2 [\epsilon(x, y)\beta(y)][R \alpha(x), z] + [\epsilon(x, y)\beta(y)\lambda^2][R \alpha(x), z] + [\epsilon(x, y)\beta(y)\lambda^2][R \alpha(x), R(z)]
\]

\[
+ [\epsilon(x, y)\beta(y)][R \alpha(x), R(z)] + [\epsilon(x, y)\beta(y)][R \alpha(x), R(z)]
\]

\[
+ [\epsilon(x, y)\beta(y)][R \alpha(x), R(z)] + [\epsilon(x, y)\beta(y)][R \alpha(x), R(z)]
\]

\[
+ [\epsilon(x, y)\beta(y)][R \alpha(x), R(z)] + [\epsilon(x, y)\beta(y)][R \alpha(x), R(z)] = \beta(z).
\]

The Hom-associativity and Hom-Jacobi color identity are proved in a similar way. \(\square\)
Whenever \( \lambda = 0 \), \( \varepsilon = 1 \), \( G = \{ e \} \), we recover

**Corollary 3.17.** [26] Let \((P,\cdot,\{-,-\},\alpha,\beta)\) be a BiHom-Poisson algebra and \( R : P \to P \) be Rota-Baxter operator of weight 0 on \( P \). Then \( P_R = (P,\cdot,\{-,-\},\alpha,\beta) \) is also a BiHom-Poisson algebra. Moreover, \( R \) is a morphism of BiHom-Poisson algebra of \((P,\cdot,\{x,y\},\alpha,\beta)\) into \((P,\cdot,\{-,-\},\alpha,\beta)\).

With \( \beta = \alpha \), we get

**Corollary 3.18.** If \((P,\cdot,\{-,-\},\varepsilon,\alpha)\) be a Hom-Poisson color algebra and \( R : P \to P \) be Rota-Baxter operator of weight \( \lambda \in K \). Then \( P_R = (P,\cdot,\{-,-\},\alpha) \) is also a Hom-Poisson color algebra. Moreover, \( R \) is a morphism of Hom-Poisson color algebra of \((P,\cdot,\{x,y\},\varepsilon,\alpha,\beta)\) into \((P,\cdot,\{-,-\},\varepsilon,\alpha,\beta)\).

### 3.4 BiHom-Poisson color algebras induced by centroid and averaging operators

Now, let us introduce averaging operator for BiHom-Poisson color algebras.

**Definition 3.19.** Let \((P,\cdot,\{-,-\},\varepsilon,\alpha,\beta)\) be a BiHom-Poisson color algebra. For any positive integers \( k, l \), an even linear map \( \theta : P \to P \) is called an \((\alpha^k,\beta^l)\)-averaging operator over \( P \) if

\[
\begin{align*}
\alpha \circ \theta &= \theta \circ \alpha, \quad \beta \circ \theta = \theta \circ \beta, \\
\theta(\theta(x) \cdot \alpha^k \beta^l(y)) &= \theta(x) \cdot \theta(y) = \theta(\alpha^k \beta^l(x) \cdot \theta(y)), \\
\theta([\theta(x), \alpha^k \beta^l(y)]) &= [\theta(x), \theta(y)] = \theta([\alpha^k \beta^l(x), \theta(y)]),
\end{align*}
\]

for all \( x, y \in \mathcal{H}(A) \).

For example, in the Hom-setting, any even \( \alpha \)-differential operator \( d : A \to A \) (i.e. an \( \alpha \)-derivation \( d \) such that \( d^2 = 0 \)) over a Hom-associative color algebra is an \( \alpha \)-averaging operator.

**Theorem 3.20.** Let \((P,\cdot,\{-,-\},\varepsilon,\alpha)\) be BiHom-Poisson color algebra and \( \theta : P \to P \) be an \( \alpha^0 \)-averaging operator. Then the two new products

\[
x \star y = \theta(x) \cdot \theta(y) \quad \text{and} \quad \{x,y\} = [\theta(x),\theta(y)], \forall x,y \in \mathcal{H}(P)
\]

makes \((P,\cdot,\{-,-\},\varepsilon,\alpha)\) a BiHom-Poisson color algebra.

**Proof.** It is easy to verify the BiHom-associativity and BiHom-Lie color identities. It remains to prove the BiHom-Leibniz color identity. For any \( x,y,z \in \mathcal{H}(P) \),

\[
\{\alpha \beta(x), x \star y\} = [\theta\alpha\beta(x), \theta(y \star z)] \\
= \{\theta\beta\alpha(x), \theta(\theta(y)\theta(z))\} \\
= \{\alpha \beta\theta(x), \theta^2(y)\theta(z)\} \\
= \{\beta \theta(x), \theta^2(y)\} \theta(\theta(z) + \varepsilon(x,y)\theta \theta^2(y)\theta(\theta(\alpha(x), \theta(z)) \\
= \theta([\theta \beta(x), \theta(y)] \theta(\theta(z) + \varepsilon(x,y)\theta \theta^2(y)\theta(\theta(\alpha(x), \theta(z)).
\]

The even linear map being an \( \alpha^0 \)-averaging operator again, it comes

\[
\{\alpha \beta(x), x \star y\} = \theta(\theta \beta(x), \theta(\theta(y)\theta(\theta(\alpha(x), \theta(z)) \\
= \theta([\theta \beta(x), \theta(y)] \theta(\theta(z) + \varepsilon(x,y)\theta \theta(\theta(\alpha(x), \theta(z)) \\
= \theta([\beta(x), y] \theta(\alpha(z) + \varepsilon(x,y)\theta(\theta(\alpha(x), \theta(z)) \\
= \{\beta(x), y\} \star \beta(z) + \varepsilon(x,y)\beta(y) \star \{\alpha(x), z\}.
\]

This finishes the proof. \( \square \)
Whenever, \( \alpha = \beta \), we recover Theorem 2.11 \([12]\).

**Theorem 3.21.** Let \((P, \cdot, [-, -], e, \alpha)\) be a BiHom-Poisson color algebra and \(\theta : P \to P\) be an injective \((\alpha^k, \beta^l)\)-averaging operator. Then the new products

\[
x \ast y = \theta(x) \cdot \alpha^k \beta^l(y) \quad \text{and} \quad \{x, y\} = [\theta(x), \alpha^k \beta^l(y)], \forall x, y \in \mathcal{H}(P) \quad (3.14)
\]

makes \((P, \ast, [-, -], e, \alpha)\) a BiHom-Poisson color algebra.

Moreover, \(\theta\) is a morphism of BiHom-Poisson color algebra of \((P, \ast, [-, -], e, \alpha, \theta)\) onto \((P, \cdot, [-, -], e, \alpha, \theta)\).

**Proof.** We leave the checking of the BiHom-associativity and BiHom-Lie color identities to the reader. Now, let us prove the Hom-Leibniz color identity; for any \(x, y, z \in \mathcal{H}(P)\),

\[
\theta([\alpha \beta(x), y * z]) = \theta([\alpha \beta(x), \alpha^k \beta^l(y + z)]) = \theta([\alpha \beta(x), \alpha^k \beta^l(y) \beta^l(z)])
\]

\[
= [\beta \alpha(x), \theta(y) \alpha^k \beta^l(z)] = [\beta \alpha(x), \theta(y) \theta(z)]
\]

\[
= \theta(\beta \alpha(x), \theta(y) \alpha^k \beta^l(z)] + \theta(\rho(x, y) \theta(y) \cdot \theta(\alpha(x), \alpha^k \beta^l(z)]
\]

\[
= \theta([\beta(x), y] \alpha^k \beta^l)(z) + \theta(y) \beta(\theta(y) \cdot \alpha^k \beta^l \alpha^l(\theta(x), \beta^l)(z))
\]

\[
= \theta([\beta(x), y] \alpha^k \beta^l(\alpha^l(\theta(x), \beta^l)(z)) + \theta(y) \beta(\theta(y) \cdot \alpha^k \beta^l \alpha^l(\theta(x), \beta^l)(z))
\]

The second part comes from the definition of averaging operator. The associativity and Hom-Jacobi identity are proved in the same way. \(\square\)

Whenever, \(\alpha = \beta\), we recover Theorem 2.13 \([12]\).

**Definition 3.22.** Let \((P, \cdot, [-, -], e, \alpha, \beta)\) be a BiHom-Poisson color algebra. For any integers \(k, l\), an even linear map \(\theta : P \to P\) is called an element of \((\alpha^k, \beta^l)\)-centroid on \(P\) if

\[
\alpha \circ \theta = \theta \circ \alpha, \quad \beta \circ \theta = \theta \circ \beta, \quad (3.15)
\]

\[
\theta(x) \cdot \alpha^k \beta^l(y) = \theta(x) \cdot \theta(y) = \alpha^k \beta^l(x) \cdot \theta(y), \quad (3.16)
\]

\[
[\theta(x), \alpha^k \beta^l(y)] = [\theta(x), \theta(y)] = [\alpha^k \beta^l(x), \theta(y)], \quad (3.17)
\]

for all \(x, y \in \mathcal{H}(P)\).

The set of elements of centroid is called centroid.

**Theorem 3.23.** Let \((P, \cdot, [-, -], e, \alpha, \beta)\) be a BiHom-Poisson color algebra and \(\theta : P \to P\) an element of \((\alpha^k, \beta^l)\)-centroid. Then, for any \(k, l \geq 0\)

\[
(P, \ast, [-, -], e, \alpha^{k+1} \beta^l, \alpha^k \beta^{l+1})
\]

is a BiHom-Poisson color algebra with the multiplications

\[
x \ast y := \alpha^k \beta^l(x) \cdot \theta(y) \quad \text{and} \quad \{x, y\} := [\alpha^k \beta^l(x), \theta(y)]
\]

for all \(x, y \in \mathcal{H}(P)\).
Proof. Let us check the BiHom-associativity. For any homogeneous elements \( x, y, z \in P \), we have

\[
\alpha^{k+1} \beta^l(x) \ast (y \ast z) = \theta \alpha^{k+1} \beta^l(x) \cdot \theta(\theta(y) \cdot \theta(z)) \\
= \theta \alpha^{k+1} \beta^l(x) \cdot \alpha^k \beta^l(\theta(y) \cdot \theta(z)) \\
= \theta \alpha^{k+1} \beta^l(x) \cdot (\alpha^k \beta^l \theta(y) \cdot \alpha^k \beta^l \theta(z)) \\
= \alpha \theta(\alpha^k \beta^l(x)) \cdot (\alpha^k \beta^l \theta(y) \cdot \alpha^k \beta^l \theta(z)) \\
= (\theta(\alpha^k \beta^l(x)) \cdot \alpha^k \beta^l \theta(y)) \cdot \alpha^k \beta^l \theta(z) \\
= \alpha^k \beta^l(\theta(x) \cdot \theta(y)) \cdot \alpha^k \beta^l \theta(z) \\
= (\theta(x) \cdot \theta(y)) \ast \alpha^k \beta^l \theta(z) \\
= (x \ast y) \ast \alpha^k \beta^l \theta(z).
\]

Now let us prove the BiHom-Jacobi identity,

\[
\{(\alpha^k \beta^l)^2(x), ((\alpha^k \beta^l)(y), (\alpha^k \beta^l)(z)) = [\theta((\alpha^k \beta^l)^2(x), \theta(\alpha^k \beta^l(y), \alpha^k \beta^l(z))]
\]

Thus,

\[
\int e(z,x)\{(\alpha^k \beta^l)^2(x), ((\alpha^k \beta^l)(y), (\alpha^k \beta^l)(z))
\]= \alpha^{2k} \beta^2 \int e(z,x)[\beta^2 \theta(x), [\beta \theta(y), \alpha \theta(z)]]; \tag{3.18}
\]

Finally let us check the BiHom-Leibniz color identity:

\[
\{\alpha^{k+1} \beta^l \alpha^k \beta^l+1(x), y \ast z \} = \alpha^{2k} \beta^2 \alpha \beta(x), y \ast z
\]

This conclude the proof. \( \square \)

### 3.5 BiHom-Poisson color algebras induced by multipliers

Let \( P \) be a BiHom-Poisson color algebra and \( \sigma : G \times G \rightarrow \mathbb{K}^* \) be a symmetric multiplier i.e.

i) \( \sigma(g, g') = \sigma(g', g), \forall x, y \in G \)

ii) \( \sigma(g, g')\sigma(g'', g + g') \) is invariant under cyclic permutation of \( g, g', g'' \in G \).
\textbf{Theorem 3.24.} Let \((P, \ast, [-,-], e, \alpha)\) be a BiHom-Poisson color algebra and \(\delta : G \times G \to \mathbb{K}^*\) be the bicharacter associated with the multiplier \(\sigma\) on \(G\) Then, \((P, \ast^\sigma, [-,-]^\sigma, e\delta, \alpha)\) is also a BiHom-Poisson color algebra with

\[ x \ast^\sigma y = \sigma(x,y) x \ast y, \quad [x,y]^\sigma = \sigma(x,y) [x,y] \quad \text{and} \quad e\delta(x,y) = e(x,y) \sigma(x,y) \sigma(y,x)^{-1}, \]

for any \(x, y \in H(P)\).

Moreover, an endomorphism of \((P, \ast, [-,-], e, \alpha, \beta)\) is also an endomorphism of \((P, \ast^\sigma, [x,y]^\sigma, e, \alpha, \beta)\).

\textbf{Proof.} For any homogeneous elements \(x, y, z \in P\),

\[ [\alpha \beta(x), y \ast^\sigma z]^\sigma = [\alpha \beta(x), \sigma(y,z) y \ast z]^\sigma \]

\[ = \sigma(y,z) \sigma(x,y + z) [\alpha \beta(x), y \ast z] \]

\[ = \sigma(y,z) \sigma(x,y + z) (\beta(y) \ast \epsilon(x,y) \beta(y) \ast [\alpha(x), z]) \]

\[ = \sigma(y,z) \sigma(x,y + z) (\beta(y) \ast \beta(z) + e(x,y) \sigma(y,z) \sigma(x,y + z) \beta(y) \ast [\alpha(x), z]) \]

Using twice the fact that \(\sigma\) is a multiplier, we have

\[ [\alpha \beta(x), y \ast^\sigma z]^\sigma = \sigma(y,z) \sigma(x,y + z) \beta(y) \ast [\alpha(x), z] \]

\[ = \sigma(y,z) \sigma(x,y + z) \beta(y) \ast \beta(z) \]

\[ + e(x,y) \sigma(y,z) \sigma(x,y) \epsilon(x,y) \beta(y) \ast [\alpha(x), z] \]

\[ = \sigma(y,z) \sigma(x,y + z) \beta(y) \ast \beta(z) \]

\[ + e(x,y) \sigma(y,z) \sigma(x,y + z) \sigma(x,y) \beta(y) \ast [\alpha(x), z] \]

\[ = [\beta(x,y)^\sigma, \ast^\sigma, \beta(z) + e(x,y) \beta(y) \ast^\sigma [\alpha(x), z]]^\sigma. \]

This ends the proof.

\textbf{Corollary 3.25.} If \((P, \ast, [-,-], e, \alpha, \beta)\) is a BiHom-Poisson color algebra and \(\sigma\) a symmetric multiplier on \(G\). Then \((P, \ast^\sigma, [-,-]^\sigma, e, \alpha, \beta)\) is a BiHom-Poisson color algebra with

\[ x \ast^\sigma y = \sigma(x,y) x \ast y \quad \text{and} \quad [x,y]^\sigma = \sigma(x,y) [x,y], \]

for any \(x, y \in H(P)\).

Similar constructions may be made for BiHom-Poisson color algebra of arbitrary degree which will include BiHom-Gerstenhaber color algebras.

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