Policy Learning Using Weak Supervision

Jingkang Wang$^{1,2}$ Hongyi Guo$^3$ Zhaowei Zhu$^4$ Yang Liu$^4$
University of Toronto$^1$, Vector Institute$^2$, Northwestern University$^3$, UC Santa Cruz$^4$

wangjk@cs.toronto.edu hongyiguo2025@u.northwestern.edu zwzhu@ucsc.edu yangliu@ucsc.edu

Abstract

Most existing policy learning solutions require the learning agents to receive high-quality supervision signals such as well-designed rewards in reinforcement learning (RL) or high-quality expert demonstrations in behavioral cloning (BC). These quality supervisions are usually infeasible or prohibitively expensive to obtain in practice. We aim for a unified framework that leverages the available cheap weak supervisions to perform policy learning efficiently. To handle this problem, we treat the “weak supervision” as imperfect information coming from a peer agent, and evaluate the learning agent’s policy based on a “correlated agreement” with the peer agent’s policy (instead of simple agreements). Our approach explicitly punishes a policy for overfitting to the weak supervision. In addition to theoretical guarantees, extensive evaluations on tasks including RL with noisy rewards, BC with weak demonstrations, and standard policy co-training show that our method leads to substantial performance improvements, especially when the complexity or the noise of the learning environments is high.

1 Introduction

Recent breakthroughs in policy learning (PL) open up the possibility to apply reinforcement learning (RL) or behavioral cloning (BC) in real-world applications such as robotics [1, 2] and self-driving [3, 4]. Most existing works require agents to receive high-quality supervision signals, e.g., reward or expert demonstrations, which are either infeasible or expensive to obtain in practice [5, 6].

The outputs of reward functions in RL are subject to multiple kinds of randomness. For example, the reward collected from sensors on a robot may be biased and have inherent noise due to physical conditions such as temperature and lighting [7–9]. For the human-defined reward, different human instructors might provide drastically different feedback that leads to biased rewards [10]. Besides, the demonstrations by an expert in behavioral cloning (BC) are often imperfect due to limited resources and environment noise [11–13]. Therefore, learning from weak supervision signals such as noisy rewards [7] or low-quality demonstrations produced by untrustworthy expert [12, 14] is one of the outstanding challenges that prevents a wider application of PL.

Although some works have explored these topics separately in their specific domains [7, 15, 14, 16], there lacks a unified solution for robust policy learning in imperfect situations. Moreover, the noise model as well as the corruption level in supervision signals is often required. To handle these challenges, we first formulate a meta-framework to study RL/BC with weak supervision and call it weakly supervised policy learning. Then we propose a theoretically principled solution, PeerPL, to perform efficient policy learning using the available weak supervision without requiring noise rates.

Our solution is inspired by peer loss [17], a recently proposed loss function for learning with noisy labels but does not require the specification of noise rates. In peer loss, the noisy labels are treated as a peer agent’s supervision. This loss function explicitly punishes the classifier from simply agreeing with the noisy labels, but would instead reward it for a “correlated agreement” (CA). We adopt a
similar idea and treat the “weak supervision” as the noisy information coming from an imperfect peer agent, and evaluate the learning agent’s policy based on a “correlated agreement” (CA) with the weak supervision signals. Compared to standard reward and evaluation functions that encourage simple agreements with the supervision, our approach punishes “over-agreement” to avoid overfitting to the weak supervision, which offers us a family of solutions that do not require prior knowledge of the corruption level in supervision signals.

To summarize, the contributions in the paper are: (1) We provide a unified formulation of the weakly supervised policy learning problems; (2) We propose PeerPL, a new way to perform policy evaluation for RL/BC tasks, and demonstrate how it adapts in challenging tasks including RL with noisy rewards and BC from weak demonstrations; (3) PeerPL is theoretically guaranteed to recover the optimal policy, as if the supervision are of high-quality and clean. (4) Experiment results show strong evidence that PeerPL brings significant improvements over state-of-the-art solutions. Code is online available at: https://github.com/wangjksjtu/PeerPL.

1.1 Related Work

**Learning with Noisy Supervision** Learning from noisy supervision is a widely explored topic. The seminal work [18] first proposed an unbiased surrogate loss function to recover the true loss from the noisy label distribution, given the knowledge of the noise rates of labels. Follow-up works offered ways to estimate the noise level from model predictions [19–27] or label consensuses of nearby representations [28]. Recent works also studied this problem in sequential settings including federated bandit [29] and RL [7]. The former work assumes the noise can be offset by averaging rewards from multiple agents. [7] designs a statistics-based estimation algorithm for noise rates in observed rewards, which can be inefficient especially when the state-action space is huge. Moreover, the error in the estimation can accumulate and amplify in sequential problems. Inspired by recent advances of peer loss [17, 30, 31], our solution is able to recover true supervision signals without requiring a priori specification of the noise rates.

**Behavioral Cloning (BC)** Standard BC [32, 33] tackles the sequential decision-making problem by imitating the expert actions using supervised learning. Specifically, it aims to minimize the one-step deviation error over the expert trajectory without reasoning about the sequential consequences of actions. Therefore, the agent suffers from compounding errors when there is a mismatch between demonstrations and real states encountered [33–35]. Recent works introduce data augmentations [36] and value-based regularization [37] or inverse dynamics models [38, 39] to encourage learning long-horizon behaviors. While being simple and straightforward, BC has been widely investigated in a range of application domains [40, 41] and often yields competitive performance [42, 37]. Our framework is complementary to the current BC literature by introducing a learning strategy from weak demonstrations (e.g., noisy or from a poorly-trained agent) and provides theoretical guarantees on how to retrieve clean policy under mild assumptions [43].

**Correlated Agreement** In [44, 45], a correlated agreement (CA) type of mechanism is proposed to evaluate the correlations between agents’ reports. In addition to encouraging a certain agreement between agents’ reports, CA also punishes over-agreement when two agents always report identically. Recently, [17, 30, 25] adapt a similar idea to noisy label learning thus offloading the burdens of estimating noise rates. We consider a more challenging sequential decision-making problem and study the convergence rates under noisy supervision signals.

2 Policy Learning from Weak Supervision

We begin by reviewing conventional reinforcement learning and behavioral cloning with clean supervision signals. Then we introduce the weak supervision problem in policy learning and define two concrete instantiations: (1) RL with noisy reward and (2) BC using weak expert demonstrations.

2.1 Overview of Policy Learning

The goal of policy learning (PL) is to learn a policy \( \pi \) that the agent could follow to perform a series of actions in a stateful environment. For reinforcement learning, the interactive environment is characterized as an MDP \( \mathcal{M} = (\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathcal{P}, \gamma) \). At each time \( t \), the agent in state \( s_t \in \mathcal{S} \) takes an action \( a_t \in \mathcal{A} \) by following the policy \( \pi : \mathcal{S} \times \mathcal{A} \to \mathbb{R} \), and potentially receives a reward \( r(s_t, a_t) \in \mathcal{R} \). Then the agent transfers to the next state \( s_{t+1} \) according to a transition probability
Weak Demonstrations (\(s_i, a_i\)), \(a_i\) as a ‘Y Agent: …, \(s_1^!, a_1^!, \ldots, s_k^!, a_k^!, \ldots\) PeerAgent: …, %\(Y_1^!, \ldots, %\(Y_k^!, \ldots\) CA: \(\text{Eva}(s_0, a_0); \tilde{Y}_E\) – \(\text{Eva}(s_j, a_j); \tilde{Y}_E\)

Figure 1: Illustration of *weakly supervised policy learning* and our PeerPL solution with correlated agreement (CA). We use \(\tilde{Y}\) to denote a weak supervision, be it a noisy reward, or a noisy demonstration. Ev\(\text{a}\) stands for an evaluation function. “Peer Agent” corresponds to weak supervision.

The function \(P\). We denote the generated trajectory \(\tau = ((s_t, a_t, r_t))_{t=0}^{T}\), where \(T\) is a finite or infinite horizon. RL algorithms aim to maximize the expected reward over the trajectory \(\tau\) induced by the policy: 

\[
\text{J}^\text{clean}(\pi) = \mathbb{E}_{(s, a, r) \sim \tau}[\sum_{t=0}^{T} \gamma^t r_t],
\]

where \(\gamma \in (0, 1]\) is the discount factor.

Another popular policy learning method is behavioral cloning. Let \(\pi(\cdot|s)\) denotes the distribution over actions formed by \(\pi\), and \(\pi(a|s)\) is the probability of choosing action \(a\) given state \(s\) and policy \(\pi\). The goal of BC is to mimic the expert policy \(\pi_E\) through a set of demonstrations \(D_E = \{(s_i, a_i)\}_{i=1}^{N}\), drawn from a distribution \(D_E\), where \((s_i, a_i)\) is the sampled state-action pair from the expert trajectory and \(a_i \sim \pi_E(\cdot|s_i)\). Then training a policy with standard BC corresponds to maximizing the following log-likelihood:

\[
\text{J}^\text{clean}(\pi) = \mathbb{E}_{(s, a) \sim D_E}[\log \pi(a|s)].
\]

In both RL and BC, the learning agent receives supervision through either the (clean) reward \(r\) by interacting with environments or the expert policy \(\pi_E\) as observable demonstrations. Consider a particular policy class \(\Pi\), the optimal policy is then defined as \(\pi^* = \arg\max_{\pi \in \Pi} \text{J}^\text{clean}(\pi)\): \(\pi^*\) obtains the maximum expected reward over the horizon \(T\) in RL and \(\pi^*\) corresponds to the clean expert policy \(\pi_E\) in BC. In practice, one can also combine both RL and BC approaches to take advantage of both learning paradigm [46, 47, 15, 43]. Specifically, a recent hybrid framework called policy co-training [43] will be considered in this paper.

### 2.2 Weak Supervision in Policy Learning

The weak supervision signal \(\tilde{Y}\) could be noisy reward \(\tilde{r}\) for RL or noisy action \(\tilde{a}\) from an imperfect expert policy \(\tilde{\pi}_E\) for BC, which are noisy versions of the corresponding high-quality supervision signals. See more details below.

**RL with Noisy Reward** Consider a finite MDP \(\tilde{\mathcal{M}} = (\mathcal{S}, \mathcal{A}, \mathcal{R}, F, \mathcal{P}, \gamma)\) with noisy reward channels [7], where \(\mathcal{R}: \mathcal{S} \times \mathcal{A} \rightarrow \mathcal{R}\), and the noisy reward \(\tilde{r}\) is generated following a certain function \(F: \mathcal{R} \rightarrow \tilde{\mathcal{R}}\). Denote the trajectory a policy \(\tilde{\pi}_E\) generates via interacting with \(\tilde{\mathcal{M}}\) as \(\tilde{\tau}_E\). Assume the reward is discrete and has \(|\mathcal{R}|\) levels. The noisy reward can be characterized via a unknown matrix \(C^\text{RL}_{|\mathcal{R}| \times |\mathcal{R}|}\), where each entry \(c^\text{RL}_{j,k}\) indicates the flipping probability for generating a possibly different outcome: \(c^\text{RL}_{j,k} = \mathbb{P}(\tilde{r}_t = R_k | r_t = R_j)\). We call \(r\) and \(\tilde{r}\) the true reward and noisy reward.

**BC with Weak Demonstration** Instead of observing the true expert demonstration generated according to \(\pi_E\), denote the available weak demonstrations by \(\{(s_i, \tilde{a}_i)\}_{i=1}^{N}\), where \(\tilde{a}_i\) is the noisy expert action drawn according to a random variable \(\tilde{a}_i = \tilde{\pi}_E(s_i) \sim \tilde{\pi}_E(\cdot|s_i)\), each state-action pair \((s_i, \tilde{a}_i)\) is sampled from distribution \(D_E\). Note there may exist two randomness factors in getting \(\tilde{a}_i\): uncertainty in true policy \(\pi_E\) and noise from imperfect policy \(\tilde{\pi}_E\). In particular, we do not consider the former randomness in theoretical analyses: given the output distribution \(\pi_E(\cdot|s_i)\), only one deterministic action \(\pi_E(s_i)\) is taken by expert. This is because with uncertainty in true expert actions, it is hard to distinguish a clean case with true expert actions from the weak supervision case without addition knowledge. Similar assumptions are also adopted in [23, 28]. The noisy action is modeled by a unknown confusion matrix \(C^\text{BC}_{|\mathcal{A}| \times |\mathcal{A}|}\), where each entry \(c^\text{BC}_{j,k}\) indicates the flipping probability for taking a sub-optimal action that differs from \(\pi_E(s)\): \(c^\text{BC}_{j,k} = \mathbb{P}(\tilde{\pi}_E(s) = A_k | \pi_E(s) = A_j)\). \(A_k\) and \(A_j\) denote the \(k\)-th and the \(j\)-th action from the action space \(\mathcal{A}\). In the above definition, we assume the noisy action \(\tilde{a}_i\) is independent of the state \(s\) given the deterministic expert action \(\pi_E(s)\),
i.e., \( P(\hat{a}_i | \pi_E(s_i)) = P(\hat{a}_i | s_i, \pi_E(s_i)) \). We aim to recover \( \pi^* \) as if we were able to access the quality expert demonstration \( \pi_E \) instead of \( \tilde{\pi}_E \).

**Knowledge of \( C \)** Recall \( C \): the true \( \mathbb{C}^{RL}_{[|R| \times |R|}] \) or \( \mathbb{C}^{BC}_{[|A| \times |A|} \) is unknown in practice. While recent works estimate this matrix \([26, 23, 28]\) in supervised classification problems, it is still challenging to generalize them to a sequential setting \([7]\). When \( C \) is not perfectly estimated, the estimation error of \( C \) may lead to unexpected state-action pairs then the error of reward estimates will be accumulated in sequential learning. Besides, estimating \( C \) involves extra computation burden. In contrast, our method gets rid of the above issues since it is free of any knowledge of \( C \) and leads to more robust policy learning algorithms.

**Learning Goal** With full supervision, both RL and BC can converge to the optimal policy \( \pi^* \). However, when only weak supervision is available, with an over-parameterized model such as a deep neural network, the learning agent will easily memorize the weak supervision and learn a biased policy \([48]\). In our meta framework, instead of converging to any biased policy, we focus on learning the optimal policy \( \pi^* \) with only a weak supervision sequence denoted as \( \{(s_i, a_i), \tilde{Y}_i\}_{i=1}^T \) (RL) or \( \{(s_i, a_i), \tilde{Y}_i\}_{i=1}^N \) (BC).

### 3 PeerPL: Weakly Supervised PL via Correlated Agreement

To deal with weak supervision in PL, we propose a unified and theoretically principled framework PeerPL. We treat the weak supervision as information coming from a “peer agent”, and then evaluate the policy using a certain type of “correlated agreement” function between the learning policy and the peer agent’s information.

#### 3.1 A Unified Evaluation Function

We use an evaluation function \( \text{Eva}_\pi((s_i, a_i), \tilde{Y}_i) \) to evaluate a taken policy \( \pi \) at agent state \((s_i, a_i)\) using the weak supervision \( \tilde{Y}_i \). For RL, \( \text{Eva}_\pi \) is the instance-wise measure (negative loss) for different RL algorithms, which is a function of the noisy reward \( \tilde{r} \) received at \((s_i, a_i)\). In the BC setting, \( \text{Eva}_\pi \) is the loss to evaluate the action \( a_i \) taken by the agent given the expert’s demonstration \( \hat{a}_i \). Note that the larger the \( \text{Eva}_\pi \) is at state \((s_i, a_i)\), the better it follows the supervision \( \tilde{Y}_i \). Specifically, we have

\[
\text{Eva}^\text{RL}_\pi((s, a), \tilde{r}) = -\ell(\pi, (s, a, \tilde{r})) \quad \text{(RL)} \quad \text{and} \quad \text{Eva}^\text{BC}_\pi((s, a), \tilde{a}) = \log \pi(\tilde{a}|s) \quad \text{(BC)},
\]

where the RL loss function \( \ell \) can be temporal difference error \([49, 50]\) or the policy gradient loss \([51]\).

Furthermore, we let \( J(\pi) \) denote the function that evaluates policy \( \pi \) under a set of state action pairs with weak supervision sequence \( \{(s_i, a_i), \tilde{Y}_i\}_{i=1}^N \), i.e.,

\[
J(\pi) = \mathbb{E}_{(s, a) \sim \tau} [\text{Eva}_\pi((s, a), \tilde{Y})].
\]

Then the goal of weakly supervised policy learning is to recover the optimal policy \( \pi^* \) as if we receive clean supervision \( Y \). Note that directly maximizing \( J(\pi) \) might result in sub-optimal performance due to the weak supervisions. The above unified notations are only for better delivery of our framework and we still treat PL as a sequential decision problem.

#### 3.2 Overview of the Idea: Correlated Agreement with Weak supervision

We first present the general idea of our PeerPL framework using a concept named correlated agreement (CA). For each weakly supervised sample \( ((s_i, a_i), \tilde{Y}_i) \), we randomly sample (with replacement) two other peer samples indexed by \( j \) and \( k \). Then we take the state-action pair \((s_j, a_j)\) of sample \( j \) and the supervision signal \( \tilde{Y}_k \) of sample \( k \), and evaluate \( ((s_i, a_i), \til(\tilde{Y}_i)) \) as follows:

\[
\text{CA with Weak Supervision:} \quad \text{Eva}_\pi((s_i, a_i), \tilde{Y}_i) - \text{Eva}_\pi((s_j, a_j), \tilde{Y}_k).
\]

This operation is illustrated in Figure 1. We further show intuitions and a toy example below.

**Intuition** The first term above encourages an “agreement” with the weak supervision (that a policy agrees with the corresponding supervision), while the second term punishes a “blind” and “over” agreement that happens when the agent’s policy always matches with the weak supervision even
on randomly paired traces (noise). The randomly paired instances $j, k$ help us achieve this check. Note our mechanism does not require the knowledge of $C_{\text{RL}}^{R|\times|R}$ nor $C_{\text{BC}}^{A|\times|A}$, and offers a prior-knowledge free way to learn effectively with weak supervision.

Toy Example Consider a toy BC setting where the policy fully memorizes the weak supervision and outputs the same sequence of actions given the same sequence of states, i.e.,

Weak-supervision: $\hat{a}_1 = \hat{a}_2 = \hat{a}_3 = \hat{a}_4 = 0$; Outputs: $a_1 = a_2 = a_3 = 1, a_4 = 0$.

Let $\text{Eva}_π((s_1, a_1), \hat{a}_i) = 1$ if the policy output agrees with the weak demonstration ($a_i = \hat{a}_i$), and 0 otherwise. When the policy fully memorizes weak supervisions, we have:

$$\begin{align*}
\text{Without CA:} & \quad E[\text{Eva}_π((s_j, a_j), \hat{a}_i)] = 1, \\
\text{With CA:} & \quad E[\text{Eva}_π((s_j, a_j), \hat{a}_i) - \text{Eva}_π((s_j, a_j), \hat{a}_k)] = 0.375,
\end{align*}$$

where $0.375 = 1 - (0.75^2 + 0.25^2)$ is obtained by considering the probability of randomly paired $a_j$ and $\hat{a}_k$ matching each other. The above example shows that a full agreement with the weak supervision will instead be punished.

In what follows, we showcase two concrete implementations: PeerRL (peer reinforcement learning) and PeerBC (peer behavioral cloning). We provide algorithms and theoretical guarantees under weak supervisions.

4 PeerRL: Peer Reinforcement Learning

We propose the following objective function to punish the over-agreement of parametric policy $π_θ$ based on CA:

$$J^{\text{RL}}(π_θ) = E\left[\text{Eva}^{\text{RL}}_π((s_i, a_i), \tilde{r}_i)\right] - ξ \cdot E\left[\text{Eva}^{\text{RL}}_π((s_j, a_j), \tilde{r}_k)\right],$$

where $\text{Eva}^{\text{RL}}_π((s, a), \tilde{r}) = -\ell(π_θ, (s, a, \tilde{r}))$. (1)

In (1), the first expectation is taken over $(s_i, a_i, \tilde{r}_i) \sim \tilde{r}$ and second one is taken over $(s_j, a_j, \tilde{r}_j) \sim \tilde{r}, (s_k, a_k, \tilde{r}_k) \sim \tilde{r}$, where $\tilde{r}$ is the trajectory specified by the noisy reward function $\tilde{r}$. Recall $j, k$ denote two randomly and independently sampled instances. Loss function $\ell$ depends on the employed RL algorithms, e.g., temporal difference error [49, 50] or the policy gradient loss [51]. The learning sequence is encoded in $π$. The objective $J^{\text{RL}}(π)$ represents the accumulated peer RL reward. Parameter $ξ ≥ 0$ balances the penalty for blind agreements induced by CA.

4.1 Peer Reward

In what follows, we consider the Q-Learning [52] as the underlying learning algorithm where $\ell(π_θ, (s, a, \tilde{r})) = -\tilde{r}(s, a)$ and demonstrate that the CA mechanism provides strong guarantees for Q-Learning with only observing the noisy reward. For clarity, we define peer RL reward:

Peer Reward: $\tilde{r}_\text{peer}(s, a) = \tilde{r}(s, a) - ξ \cdot \tilde{r}'$, ($s \in S, a \in A) is a reward sampled over all state-action pairs according to a fixed policy $π_{\text{sample}}$. Note the sampling policy $π_{\text{sample}}$ is independent of $π$ and the choice of $π_{\text{sample}}$ does not affect our theoretical results. We adopt a random sampling strategy in practice. Parameter $ξ ≥ 0$ balances the noisy reward and the punishment for blind agreement (with $\tilde{r}'$). We set $ξ = 1$ (for binary case) in the following analysis and treat each $(s, a)$ equally when sampling $\tilde{r}'$. In experiments, we find $\tilde{r}_\text{peer}$ is not sensitive to the choice of $ξ$ and keep $ξ$ constant for each run.

Robustness to Noisy Rewards Now we show peer reward $\tilde{r}_\text{peer}$ offers us an affine transformation of the true reward in expectation, which guarantees that our PeerRL algorithm converges to $π^*$. Consider the binary reward setting $(r_+ \text{ and } r_-)$ and denote the error in $\tilde{r}$ as $e_+ = P(\tilde{r} = r_- | r = r_+), e_- = P(\tilde{r} = r_+ | r = r_-)$ (a simplification of $C_{\text{RL}}^{R|\times|R}$ in the binary setting).

**Lemma 1.** Let $r \in [0, R_{\text{max}}]$ be a bounded reward, $ξ = 1$. Assume $1 - e_- - e_+ > 0$. We have:

$$\begin{align*}
E[\tilde{r}_\text{peer}(s, a)] &= (1 - e_- - e_+) \cdot E[r_\text{peer}(s, a)] = (1 - e_- - e_+) \cdot E[r(s, a)] + \text{const},
\end{align*}$$

where $r_\text{peer}(s, a) = r(s, a) - r'$ is the peer RL reward when observing the true reward $r$, and $r'$ is the true reward corresponding to $r'$. 

5
Lemma 1 shows that by subtracting the peer penalty term \( r' \) from noisy reward \( \tilde{r}(s, a) \), \( \tilde{r}_{\text{peer}}(s, a) \) recovers the clean and true reward \( r(s, a) \) in expectation. Based on Lemma 1, we prove in Theorem A1 that the Q-learning agent will converge to the optimal policy w.p.1 with peer rewards without requiring any knowledge of the corruption in rewards \( (C_{\text{NL}}/|R|) \), as opposed to previous work [7] that requires such knowledge. Moreover, we prove in Theorem A2 that to guarantee the convergence to \( \pi^* \), the number of samples needed for our approach is no more than \( O(1/(1 - e_- - e_+)^2) \) times of the one needed when the RL agent observes true rewards perfectly (see Appendix A).

**Extension** Even though we only present an analysis for the binary case for Q-Learning, our approach is rather generic and is ready to be plugged into modern DRL algorithms. We provide multi-reward extensions, implementations with DQN [49] and policy gradient [51] in Appendix A.

### 4.2 Why does Peer Reward Work?

Compared with noisy reward, proposed peer variant is a less biased estimation of true reward (*Benefit-1*). On the other hand, PeerRL helps break the unstable "tie" states, which might encourage the agent to explore in the early stage [53] (*Benefit-2*).

**Benefit-1: PeerRL reduces the bias** We highlight that the biased noise model considered is rather generic, departing from the previous noise assumption such as zero-mean Gaussian noise [8, 9]. In zero-mean noise models, the major focus is on variance reduction so adding the random term \( \tilde{r}' \) increases the variance thus resulting in worse estimation. However, in the discrete biased noise model [18], bias correction also plays an important role especially the noise rate is high [7].

Similar to peer reward (Lemma 1), the expectation of the noisy reward writes as: \( E[\tilde{r}(s, a)] = (1 - e_- - e_+)E[r(s, a)] + e_-r_+ + e_+r_- = (1 - e_- - e_+)E[r(s, a)] + \text{const.} \). But the constant in peer reward has less effect on the true reward \( r \), especially when the noise rate is high. To see this:

- **noisy reward:** \( E[\tilde{r}(s, a)] = \eta \cdot \left( E[r(s, a)] + \frac{e_+}{1-e_- - e_+}r_- + \frac{e_-}{1-e_- - e_+}r_+ \right) \),
- **peer reward:** \( E[\tilde{r}_{\text{peer}}(s, a)] = \eta \cdot \left( E[r(s, a)] - (1 - p_{\text{peer}})r_- - p_{\text{peer}}r_+ \right) \),

where \( \eta = 1 - e_- - e_+ > 0 \), \( p_{\text{peer}} \in [0, 1] \) denotes the probability that a sample policy sees a reward \( r_+ \) overall. Since the magnitude of noise terms \( \frac{e_+}{1-e_- - e_+} \) and \( \frac{e_-}{1-e_- - e_+} \) can potentially become much larger than \( 1 - p_{\text{peer}} \) and \( p_{\text{peer}} \) in a high-noise regime, \( \frac{e_+}{1-e_- - e_+}r_+ + \frac{e_-}{1-e_- - e_+}r_- \) will dilute the informativeness of \( E[r(s, a)] \). On the contrary, \( E[\tilde{r}_{\text{peer}}(s, a)] \) contains a moderate constant noise thus maintaining more useful training signals of the true reward in practice. In summary, although peer reward (similar to the surrogate reward in previous literature [7]) increases the variance (no free-lunch), it will lead to a better estimation of the true reward due to lower bias.

**Benefit-2: PeerRL helps break ties** For RL, "tie" states indicate that the rewards for different states are the same, which are less informative as they neither serve as positive nor negative examples. Due to the discrete nature of the noise model, adding a randomly sampled penalty term helps break the tie states and treats them as either positive examples or negative examples such that it can encourage exploration in the early stage, which has similar intuitions to some RL exploration works [53]. It has also been demonstrated that reducing the uncertainty, a.k.a. pushing confident predictions, makes the learning robust to weak-supervisions in supervised learning [17, 54]. On the other hand, it is known that positive examples are sparse yet important in RL. To leverage these useful experiences sufficiently, experience replay [55, 56] is invented to store and up-sample the positive examples for faster convergence. Tie breaking potentially provides an alternative way to access more positive examples. To illustrate tie-breaking phenomenon when using peer reward, we consider a two-state Markov process (no actions) with bounded Gaussian noise and see how well we could infer which state was better by correcting the reward signals. We collect two observations for each state and conduct \( 10^4 \) trials to calculate the success rate of inferring which state has larger returns ("correct" in the Table). As we can see, PeerRL exploits the "discreteness" of the reward thus breaking ties to obtain more examples with good-quality supervision. More examples on varied noise models (bounded continuous noise, discrete noise) are referred to Appendix B.

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2-state Markov process (no actions)

| State | Correct | Tie | Incorrect |
|-------|---------|-----|----------|
| \( r_1 \sim \text{clamp}[\mathcal{N}(0.6, 1)] \), min = 0, max = 1 | 54.6% | 5.6% | 39.8% |
| \( r_2 \sim \text{clamp}[\mathcal{N}(0.4, 1)] \), min = 0, max = 1 | PeerRL 58.0% | 0.3% | 41.7% |
5 PeerBC: Peer Behavioral Cloning

Similarly, we present our CA solution in the setting of behavioral cloning (PeerBC). In BC, the supervision is given by the weak expert’s noisy trajectory. At each iteration, the agent learns under weak supervision, and the training samples are generated from the distribution $D_E$ determined by the weak expert. The $Eva^{BC}_\pi$ function in BC evaluates the agent policy $\pi_\theta$, parametrized by $\theta$, and the weak trajectory $\{(s_i, a_i)\}^N_{i=1}$ using $\ell(\pi_\theta, (s_i, a_i))$, where $\ell$ is an arbitrary classification loss. Taking the cross-entropy for instance, the objective of PeerBC is:

$$J^{BC}(\pi_\theta) = \mathbb{E}\left[Eva^{BC}_\pi((s_i, a_i), 1)\right] - \xi \cdot \mathbb{E}\left[Eva^{BC}_\pi((s_j, a_j), 0)\right],$$

where $Eva^{BC}_\pi(s, a, 1) = -\ell(\pi_\theta, (s, a)) = \log \pi_\theta(a|s)$. (3)

In (3), the first expectation is taken over $(s_i, a_i) \sim D_E, a_i \sim \pi(\cdot|s_i)$ and the second is taken over $(s_j, a_j) \sim D_E, a_j \sim \pi(\cdot|s_j)$. Again, the second $Eva^{BC}_\pi$ term in $J^{BC}$ serves the purpose of punishing over-agreement with the weak demonstration. Similarly, $\xi \geq 0$ is a parameter to balance the penalty for blind agreements.

Robustness to Noisy Demonstrations We prove that the policy learned by PeerBC converges to the expert policy when observing a sufficient amount of weak demonstrations. We focus on the binary action setting for theoretical analyses, where the action space is given by $A = \{A_+, A_-\}$ and the weakness or noise in the weak expert $\tilde{\pi}_E$ is quantified by $e_+ = \mathbb{P}(\tilde{\pi}_E(s) = A_)$ and $e_- = \mathbb{P}(\tilde{\pi}_E(s) = A_-)$. Let $\pi_{\tilde{D}_E}$ be the optimal policy for maximizing the objective in (3) with imperfect demonstrations $\tilde{D}_E$ (a particular set of with $N$ i.i.d. imperfect demonstrations). Note $\ell(\cdot)$ is specified as the 0-1 loss: $\mathbb{I}(\pi(s), a) = 1$ when $\pi(s) \neq a$, otherwise $\mathbb{I}(\pi(s), a) = 0$. We have the following upper bound on the error rate.

**Theorem 1.** Denote by $R_{\tilde{D}_E} := \mathbb{P}(s,a) \sim \tilde{D}_E(\pi_{\tilde{D}_E}(s) \neq a)$ the error rate for PeerBC. When $e_+ + e_- < 1$, with probability at least $1 - \delta$, it is upper-bounded as: $R_{\tilde{D}_E} \leq \frac{1+\xi}{1-e_-e_+} \sqrt{\frac{2\log 2/\delta}{N}}$.

Theorem 1 states that as long as weak demonstrations are observed sufficiently, i.e., $N$ is sufficiently large, the policy learned by PeerBC is able to converge to the clean expert policy $\pi_E(s)$ with a convergence rate of $O(1/\sqrt{N})$.

**Peer Policy Co-Training** Our discussion of BC allows us to study a more challenging co-training task [43]. Given a finite MDP $\mathcal{M}$, there are two agents that receive partial observations and we let $\pi_A$ and $\pi_B$ denote the policies for agent $A$ and $B$. Moreover, two agents are trained jointly to learn with rewards and noisy demonstrations from each other (e.g., at the preliminary training phase). Symmetrically, we consider the case where agent $A$ learns with the demonstrations from $B$ on sampled trajectories, and $\pi_B$ effectively serves as a noisy version of expert policy.

Following [43], we assume a mapping function $f_{A \rightarrow B}$ exists that transforms states under view $A$ into $B$. Denote by $\pi^A = \{(s^A_1, a^A_1, r^A_1)\}^N_{i=1}$ the trajectory that $\pi_A$ generates via interacting with the partial world $\mathcal{M}^A$. Then $\pi_B$ replaces each action $a^A_i$ with its selection $\tilde{a}^B_i = \pi_B(f_{A \rightarrow B}(s^A_i))$ as the weak supervision. To recover the clean expert policy, we adapt the BC peer evaluation term to the...
co-learning objective function:

\[
J^{CT}(\pi_\theta) = \mathbb{E}\left[\text{Eva}^{RL}_\pi((s^A_i, a^A_i, r^A_i)) + \text{Eva}^{BC}_\pi((s^A_i, a^A_i, a^B_i))\right] - \xi \cdot \mathbb{E}\left[\text{Eva}^{BC}_\pi((s^A_j, a^A_j, a^B_i))\right] ,
\]

where the first expectation is taken over \((s^A_i, a^A_i, r^A_i) \sim \tau^A\), and \(\tilde{a}^B_i = \pi_B(f_{A\rightarrow B}(s^A_i))\), and the second is taken over \((s^A_j, a^A_j, r^A_j) \sim \tau^A\), \((s^B_k, a^B_k, r^B_k) \sim \tau^A\), and \(\tilde{a}^B_k = \pi_B(f_{A\rightarrow B}(s^A_k))\), \(\ell\) is the loss function defined in Eqn. (4) to measure the policy difference, and \(\text{Eva}^{RL}_\pi\), \(\text{Eva}^{BC}_\pi\) are defined in Eqn. (2) and (4) respectively. The full algorithm PeerCT is provided in Algorithm 1. We omit detailed discussions on the convergence of PeerCT - it can be viewed as a straightforward extension of Theorem 1 in the context of co-training.

6 Experiments

We evaluate our solution in three challenging weakly supervised RL problems. Experiments on control games and Atari show that, without any prior knowledge of the noise, our approach is able to leverage weak supervision more effectively.

**Experiment Setup & Baselines**

We evaluate PeerPL on a wide variety of control and Atari games. For RL with noisy reward, we add synthetic noise to reward signals and compare with previous work [7], where an unbiased estimator of true reward is constructed by approximating the confusion matrix.

For BC from weak demonstrations, we adopt not fully converged PPO agents as the weak experts and unroll the trajectories. We also consider a standard policy co-training setting [43] without any synthetic noise added and compare PeerCT with single-view training paradigm and CoPiEr [43].

### Algorithm 1 Peer policy co-training (PeerCT)

**Require:** Views \(A, B\), MDPs \(M^A, M^B\), policies \(\pi_A, \pi_B\), mapping functions \(f_{A\rightarrow B}, f_{B\rightarrow A}\) that maps states from one view to the other view, CA coefficient \(\xi\), step size \(\beta\) for policy update.

1. **repeat**
   2. Run \(\pi^A\) to generate trajectories \(\tau^A = \{(s^A_i, a^A_i, r^A_i)^N_{i=1}\}\).
   3. Run \(\pi^B\) to generate trajectories \(\tau^B = \{(s^B_j, a^B_j, r^B_j)^M_{j=1}\}\).
   4. Agents label the trajectories for each other
      \[
      \tilde{\tau}^A \leftarrow \{(s^A_i, \pi_B(f_{B\rightarrow A}(s^A_i)))^N_{i=1}\},
      \tilde{\tau}^B \leftarrow \{(s^B_j, \pi_A(f_{A\rightarrow B}(s^B_j)))^M_{j=1}\}.
      \]
   5. Update policies: \(\pi(A,B) \leftarrow \pi(A,B) + \beta \cdot \nabla J^{CT}(\pi(A,B))\)
5. **until** convergence

6.1 PeerRL with Noisy Reward

**CartPole-v0:** We first evaluate our method in RL with noisy reward setting. Following [7], we consider the binary reward \([-1, 1]\) for Cartpole where the symmetric noise is synthesized with different error rates \(e = e_- = e_+\). We choose DQN [49] and DDQN [50] algorithms and train the models for 10,000 steps. We repeat each experiment 10 times with different random seeds and leave extra results in Appendix D. Figure 2 shows the learning curves for DDQN with different approaches in noisy environments \((\xi = 0.2)\). Since the number of training steps is fixed, the faster the algorithm converges, the fewer total episodes the agent will involve thus the learning curve is on the left side. As a consequence, the proposed peer reward outperforms other baselines significantly even in a high-noise regime (e.g., \(e = 0.4\)). Table 1 provides quantitative results on the average reward \(R_{avg}\) and total episodes \(N_{epi}\). We find the agents with peer reward lead to a larger \(R_{avg}\) (less generalization error) and a smaller \(N_{epi}\) (faster convergence) consistently.

---

1 We analysed the sensitivity of \(\xi\) and found the algorithm performs reasonable when \(\xi \in (0.1, 0.4)\). More insights and experiments with varied \(\xi\) is deferred to Appendix D.
Table 1: Numerical performance of DDQN on CartPole with true reward ($r$), noisy reward ($\hat{r}$), surrogate reward $\hat{r}$ [7], and peer reward $\hat{r}_{\text{peer}}(\xi = 0.2)$. $\bar{R}_{\text{avg}}$ denotes average reward per episode after convergence, the higher ($\uparrow$) the better; $N_{\text{epi}}$ denotes total episodes involved in 10,000 steps, the lower ($\downarrow$) the better. Note $0 \leq e < 0.5$.

| $e$ | $\bar{R}_{\text{avg}} \uparrow$ | $N_{\text{epi}} \downarrow$ |
|-----|-------------------------------|---------------------|
| $0.1$ | 195.6 ± 3.1                     | 101.2 ± 3.2         |
| $0.2$ | 185.2 ± 15.6                  | 114.6 ± 6.0         |
| $0.3$ | 153.0 ± 4.3                   | 113.9 ± 9.6         |
| $0.4$ | 198.5 ± 2.3                   | 86.2 ± 5.0          |

Figure 5: Policy co-training on control/Atari. Single view. [43] PeerCT (ours).

Pendulum: We further conduct experiments on a continuous control task Pendulum, where the goal is to keep a frictionless pendulum standing up. Since the rewards in pendulum are continuous: $r \in (-16.3, 0.0]$, we discretized it into 17 intervals: $(-17, -16], (-16, -15], \cdots, (-1, 0]$, with its value approximated using its maximum point. We test DDPG [57] with uniform noise in this environment following [7]. In Figure 3, the RL agents with the proposed CA objective successfully converge to the optimal policy under different amounts of noise. On the contrary, the agents with noisy rewards suffer from biased noise, especially in a high-noise regime.

Analysis of the benefits in PeerRL: More surprisingly, we observed that the agents on CartPole with peer reward even lead to faster convergence than the ones observing true reward perfectly when the noise rate $e$ is small. This indicates the possibility of other benefits to further promote peer reward, other than the noise reduction one we primarily focused on. We hypothesize this is because (1) the peer penalty term breaks the tie states (Benefit-2 in Section 4.1) and encourages explorations in RL; (2) PeerRL scales the reward signals appropriately for easier learning; (3) the human-specific “true reward” might be also imperfect which leads to a weak supervision scenario. We emphasize that the advantage of recovering from noisy reward signal is non-negligible, especially in a high-noise regime (e.g., $e = 0.4$ in Figure 2 and 3).

6.2 PeerBC from Weak Demonstrations

Atari: In BC setting, we evaluate our approach on four vision-based Atari games. For each environment, we train an imperfect RL model with PPO [58] algorithm. Here, “imperfect” means the training is terminated before convergence when the performance is about 70% ~ 90% as good as the fully converged model. We then collect the imperfect demonstrations using the expert model and generate 100 trajectories for each environment. The results are reported under three random seeds.

Figure 4 shows that our approach outperforms standard BC and even the expert it learns from. Note that during the whole training process, the agent never learns by interacting directly with the environment but only have access to the expert trajectories. Therefore, we owe this performance gain to PeerBC’s strong ability for learning from weak supervision. The peer term we add not only provably eliminates the effects of noise but also extracts useful strategy from the demonstrations. As shown in Table 2, our approach consistently outperforms the expert and standard BC. We provide the sensitivity analysis of $\xi$ in Appendix D.

Comparison with imitation learning baselines: We further extend the empirical study to imitation learning (IL) algorithms on CartPole-v1. To collect weak demonstrations, we train a PPO agent for 50k iterations that are not fully converged. As shown in Figure 6, standard IL algorithms such as BC, AIRL [37], or GAIL [59] cannot handle noisy demonstrations well and lead to sub-optimal performance. Our PeerBC brings 18% improvement over standard BC by penalizing blind agreements with the weak demonstrations. We remark that performance of PeerBC is worse than DAgger due to notorious distribution shift issue. To further improve performance, we train PeerBC in the DAgger fashion (Peer-DAgger) by querying the imperfect expert to augment the training sets. Not surprisingly.
Table 2: BC from weak demonstrations. PeerBC successfully recovers better policies than expert.

| Environment | Pong | Boxing | Enduro | Freeway | Lift (↑) |
|-------------|------|--------|--------|---------|---------|
| Expert      | 15.1 ± 6.6  | 67.5 ± 8.5  | 150.1 ± 23.0  | 21.9 ± 1.7  | -        |
| Standard BC | 14.7 ± 3.2  | 56.2 ± 7.7  | 138.9 ± 14.1  | 22.0 ± 1.3  | -4.6%    |
| PeerBC      | \( \xi = 0.2 \) | 18.8 ± 0.6  | 67.2 ± 8.4  | 177.9 ± 29.3  | 22.5 ± 0.6  | +11.3% |
|             | \( \xi = 0.5 \) | 16.6 ± 4.0  | 75.6 ± 5.4  | 230.9 ± 73.0  | 22.4 ± 1.3  | +19.5% |
|             | \( \xi = 1.0 \) | 16.7 ± 3.1  | 69.7 ± 4.7  | 230.4 ± 61.6  | 8.9 ± 4.9  | +2.0%   |
| Fully converged PPO | 20.9 ± 0.3  | 89.3 ± 5.4  | 389.6 ± 216.9  | 33.3 ± 0.8  | -        |

Table 3: Comparison with single view training and CoPiEr [43] on standard policy co-training.

| Environment | Acrobat | CartPole | Pong | Breakout |
|-------------|---------|----------|------|----------|
| Single View A | −136.6 ± 15.6  | 172.8 ± 5.5  | 17.8 ± 0.6  | 148.0 ± 16.5  |
|             | −126.4 ± 8.0  | 180.7 ± 8.1  | 17.7 ± 0.5  | 137.8 ± 12.5  |
| CoPiEr A    | −136.2 ± 5.2  | 174.1 ± 5.1  | 16.8 ± 0.5  | 107.5 ± 5.8  |
|             | −131.5 ± 4.5  | 174.3 ± 5.4  | 16.5 ± 0.2  | 82.7 ± 6.9  |
| PeerCT A    | −87.0 ± 3.9  | 188.8 ± 2.7  | 20.5 ± 0.4  | 263.6 ± 36.0  |
|             | −87.1 ± 6.3  | 184.7 ± 3.9  | 20.4 ± 0.5  | 268.6 ± 33.6  |

Peer-DAgger surpasses DAgger by a large margin, which indicates that our framework has wide applicability and successfully recovers the true supervision signals. Adapting PeerPL idea to more IL algorithms such as GAIL [59] and DART [35] together with rigorous analysis is left as future works.

Analysis of benefits in PeerBC Similarly, the performance improvement of PeerBC might be also coupled with multiple possible factors. (1) The imperfect expert model might be a noisy version of the fully-converged agent since there are less visited states on which the selected actions of the model contains noise. (2) The improvements might be brought up by biasing against high-entropy policies thus PeerBC is useful when the true policy itself is deterministic. We provide more discussions about the second factor in Appendix D.5.

6.3 PeerCT for Standard Policy Co-training

Continuous Control/Atari: Finally, we verify the effectiveness of the PeerCT algorithm in policy co-training setting [43]. This setting is more challenging since the states are partially observable and each agent needs to imitate another agent’s behavior that is highly biased and imperfect. Note that we adopt the exact same setting as [43] without any synthetic noise included. This implies the potential of our approach to deal with natural noise in real-world applications. Following [43], we mask the first two dimensions respectively in the state vector to create two views for co-training in classic control games (Acrobat and CartPole). Similarly, the agent either removes all even index coordinates (view-A) in the state vector or removing all odd index ones (view-B) on Atari games. As shown in Table 3 and Figure 5, PeerCT algorithm outperforms training from single view, and CoPiEr algorithm consistently on both control games (\( \xi = 0.5 \) in Figure 5a, 5b) and Atari games (\( \xi = 0.2 \) in Figure 5c, 5d). In most cases, our approach leads to a faster convergence and lower generalization error compared to CoPiEr, showing that our ways of leveraging information from peer agent enables recovery of useful knowledge from highly imperfect supervision.

7 Conclusion

We have proposed PeerPL, a weakly supervised policy learning framework to unify a series of RL/BC problems with low-quality supervision signals. In PeerPL, instead of blindly memorizing the weak supervision, we evaluate a learning policy’s correlated agreements with the weak supervision. We demonstrate how our method adapts in RL/BC and the hybrid co-training tasks and provide analysis of the convergence rate and sample complexity. Current theorems focus on the specific discrete noise model. Future work may extend it to more general noise scenarios and evaluate our method on real RL/BC systems, such as robotics and self-driving.
**Broader Impacts**

Weak supervision often encodes biases and noise. Our works aim to improve the robustness of policy learning algorithms which is relevant to applications concerning fairness and training data biases. Our solutions are expected to be of interests to machine learning practitioners and researchers who are interested in applications and theory in RL. We acknowledge that the use of AI technology may bring us an unexpected impact. While we are not aware of any negative social impact, we caution that our theoretical guarantees are mostly for the scenario with a large number of samples. Using our method when the number of weak supervisions is very limiting might lead to unstable performance and unintended consequences, especially when the supervisions are highly noisy.

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Supplementary Material
Policy Learning Using Weak Supervision

Abstract

In this supplementary material, we first provide theoretical analysis of the convergence rate (Sec A.1) and sample complexity (Sec A.2) for Peer $Q$-Learning algorithm. Then we provide the extension to multi-outcome setting with theoretical proofs (Sec A.3). We also show the extensions to other modern DRL algorithms in Sec A.4, and further discussions on the effectiveness of PeerRL in Sec A.5. We then provide more “tie-breaking” examples on varied noise models together with the python-style code snippet in Sec B. In Sec C, we provide the technical proofs for proposed PeerBC approach under mild assumptions. Then, we report the experimental setup details (Sec D.1), the implementation details (Sec D.2), and additional experiments including complete results for Figure 2 and Table 1 (Sec D.3), sensitivity analysis of peer penalty coefficient $\xi$ (Sec D.4), and study of stochasticity for behavioral cloning policy (Sec D.5). The summary of contents in the supplementary is provided in the following.

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A Analysis of PeerRL

We start this section by providing the proof of the convergence of Q-Learning under peer reward \( \hat{r}_{peer} \) (Theorem A1). Moreover, we give the sample complexity of phased value iteration (Theorem A2). In the rest of this section, we show how to extend the proposed method to multi-outcome setting (Section A.3) and modern deep reinforcement learning (DRL) algorithms such as policy gradient [51] and DQN [49, 56] (Section A.4).

A.1 Convergence

Recall that we consider the binary reward case \( \{r_+, r_-\} \), where \( r_+ \) and \( r_- \) are two reward levels. The flipping errors of the reward are defined as \( e_+ = \mathbb{P}(\hat{r}_t = r_-| r_t = r_+) \) and \( e_- = \mathbb{P}(\hat{r}_t = r_+| r_t = r_-) \). The peer reward is defined as \( r_{peer}(s, a) = r(s, a) - r' \), where \( r' \) is randomly sampled reward over all state-action pair \((s, a)\). Note that we treat each \((s, a)\) equally when sampling the \( r' \) due to lack of the knowledge of true transition probability \( P \). In practice, the agent could only noisy observation of peer reward \( \hat{r}_{peer}(s, a) = r(s, a) - r' \). We provide the Q-learning with peer reward in Algorithm A1.

Algorithm A1 Q-Learning with Peer Reward

Require: \( \mathcal{M} = (S, A, \hat{R}, P, \gamma) \), learning rate \( \alpha \in (0, 1) \), initial state distribution \( \beta_0 \).
1: Initialize \( Q : S \times A \to \mathbb{R} \) arbitrarily
2: while \( Q \) is not converged do
3: Start in state \( s \sim \beta_0 \)
4: while \( s \) is not terminal do
5: Calculate \( \pi \) according to \( Q \) and exploration strategy
6: \( a \leftarrow \pi(s); s' \sim P(\cdot| s, a) \)
7: Observe noisy reward \( \hat{r}(s, a) \) and randomly sample another \( \hat{r}' \) from all state-action pairs
8: Calculate peer reward \( \hat{r}_{peer}(s, a) = \hat{r}(s, a) - \hat{r}' \)
9: \( Q(s, a) \leftarrow (1 - \alpha) \cdot Q(s, a) + \alpha \cdot (\hat{r}_{peer}(s, a) + \gamma \cdot \max_{a'} Q(s', a')) \)
10: \( s \leftarrow s' \)
11: end while
12: end while
Ensure: \( Q(s, a) \) and \( \pi(s) \)

We then show the proposed peer reward \( \hat{r}_{peer} \) offers us an affine transformation of true reward in expectation, which is the key to guaranteeing the convergence for RL algorithms.

Lemma 1. Let \( r \in [0, R_{\text{max}}] \) be bounded reward and assume \( 1 - e_- - e_+ > 0 \). Then, if we define the peer reward \( \hat{r}_{peer}(s, a) = r(s, a) - r' \), in which the penalty term \( r' \) is randomly sampled noisy reward over all state-action pair \((s, a)\), we have

\[
\mathbb{E}[\hat{r}_{peer}(s, a)] = (1 - e_- - e_+) \mathbb{E}[r_{peer}(s, a)] = (1 - e_- - e_+) \mathbb{E}[r(s, a)] + \text{const},
\]

where \( r_{peer}(s, a) \) is the clean version of peer reward when observing the true reward.

Proof. With slight notation abuse, we let \( \hat{r}_{peer}, r, \hat{r} \) represent the random variables \( \hat{r}_{peer}(s, a), r(s, a), \hat{r}(s, a) \). Let \( \pi(s, a) \) denotes the RL agent’s policy. Consider the two terms on the RHS of noisy peer reward separately,

\[
\mathbb{E}[\hat{r}] = \mathbb{P}(r = r_+|\pi) \cdot \mathbb{E}_{r=r_+} [\mathbb{P}(\hat{r} = r_-| r = r_+ \cdot r_- + \mathbb{P}(\hat{r} = r_+| r = r_+ \cdot r_+)] + \mathbb{P}(r = r_-|\pi) \cdot \mathbb{E}_{r=r_-} [\mathbb{P}(\hat{r} = r_+| r = r_- \cdot r_- + \mathbb{P}(\hat{r} = r_-| r = r_- \cdot r_-)] + \mathbb{P}(r = r_+|\pi) \cdot \mathbb{E}_{r=r_+} [1 - e_+ r_+]
\]

(6)

\[
\mathbb{P}(r = r_+|\pi) \cdot \mathbb{E}_{r=r_+} [(1 - e_-) r_- + e_- r_+]
\]

(7)

\[
\mathbb{P}(r = r_-|\pi) \cdot \mathbb{E}_{r=r_-} [(1 - e_+) r_+ + e_- r_- + e_- r_+]
\]

(8)

\[
\mathbb{P}(r = r_+|\pi) \cdot \mathbb{E}_{r=r_+} [(1 - e_+) r_+ + e_- r_- + e_- r_+]
\]

(9)

\[
\mathbb{P}(r = r_-|\pi) \cdot \mathbb{E}_{r=r_-} [(1 - e_+) r_+ + e_- r_- + e_- r_+]
\]

(10)

\[
\mathbb{P}(r = r_+|\pi) \cdot \mathbb{E}_{r=r_+} [(1 - e_+) r_+ + e_- r_- + e_- r_+]
\]

(11)

\[
(1 - e_- - e_+) \mathbb{E}[r] + e_- r_+ + e_+ r_-
\]

(12)

Since we are treating the visitation probability of all state-action pair \((s, a)\) equally while sampling the peer penalty \( r' \), then the probability of true reward \( r \) under this sampling policy \( \pi_{sample} \) is a
constant, denoting as $p_{\text{peer}}$, i.e., $p_{\text{peer}} = \mathbb{P}(r = r_-|\pi_{\text{sample}})$ is a constant. Then we have,

$$
\mathbb{E}[\hat{r}'] = \mathbb{E}(\hat{r} = r_-|\pi_{\text{sample}}) \cdot r_- + \mathbb{P}(\hat{r} = r_+|\pi_{\text{sample}}) \cdot r_+ \tag{13}
$$

$$
= (e_+ p_{\text{peer}} + (1 - e_-)(1 - p_{\text{peer}})) \cdot r_- + ((1 - e_+) p_{\text{peer}} + e_-(1 - p_{\text{peer}})) \cdot r_+ \tag{14}
$$

$$
= (1 - e_- - e_+)[(1 - p_{\text{peer}}) \cdot r_- + p_{\text{peer}} \cdot r_] + e_+ r_- + e_- r_. \tag{15}
$$

As a consequence, we obtain the expectation of peer reward satisfies

$$
\mathbb{E}[\hat{r}_{\text{peer}}] = \mathbb{E}[\hat{r}] - \mathbb{E}[\hat{r}'] \tag{16}
$$

$$
= (1 - e_- - e_+) \mathbb{E}[r] - (1 - e_- - e_+)[(1 - p_{\text{peer}}) \cdot r_- + p_{\text{peer}} \cdot r_] \tag{17}
$$

$$
= (1 - e_- - e_+) \mathbb{E}[r] + \text{const.} \tag{18}
$$

Similarly, it is easy to obtain that $\mathbb{E}[\hat{r}_{\text{peer}}] = \mathbb{E}[r] - [(1 - p_{\text{peer}}) \cdot r_- + p_{\text{peer}} \cdot r_+]$. Therefore, we have $\mathbb{E}[\hat{r}_{\text{peer}}] = (1 - e_- - e_+) \mathbb{E}[r] + \text{const.}$.

Lemma 1 shows the proposed peer reward $\hat{r}_{\text{peer}}$ offers us a “noise-free” positive $(1 - e_- - e_+ > 0)$ linear transformation of true reward $r$ in expectation, which is shown the key to govern the convergence. It is widely known in utility theory and reward shaping literature $[60-62]$ that any positive linear transformations leave the optimal policy unchanged. As a consequence, we consider a “transformed MDP” $\hat{M}$ with reward $\hat{r} = (1 - e_- - e_+) r + \text{const}$, where the const is the same as the constant in Eqn. (18).

In what follows, we provide the formulation of the concept of “transformed MDP” with the policy invariance guarantee.

**Lemma A1.** Given a finite MDP $M = (S, A, R, P, \gamma)$, a transformed MDP $\hat{M} = (\hat{S}, \hat{A}, \hat{R}, \hat{P}, \gamma)$ with positive linear transformation in reward $\hat{r} := a \cdot r + b$, where $a, b$ are constants and $a > 0$, is guaranteed consistency in optimal policy.

**Proof.** The $Q$ function for transformed MDP $\hat{M}$ (denoting as $\hat{Q}$) is given as follows:

$$
\hat{Q}(s, a) = \sum_{t=0}^{\infty} \gamma^t \hat{r}_t = \sum_{t=0}^{\infty} \gamma^t (a \cdot r_t + b)
$$

$$
= a \sum_{t=0}^{\infty} \gamma^t r_t + \sum_{t=0}^{\infty} \gamma^t b
$$

$$
= a \cdot Q(s, a) + B,
$$

where $B = \sum_{t=0}^{\infty} \gamma^t b$ is a constant. Therefore, there is only a positive linear shift $(a > 0)$ in $\hat{Q}(s, a)$ thus resulting in invariance in optimal policy for transformed MDP:

$$
\hat{\pi}^*(s) = \arg \max_{a \in A} \hat{Q}^*(s, a) = \arg \max_{a \in A} [a \cdot Q(s, a) + B]
$$

$$
= \arg \max_{a \in A} Q(s, a) = \pi^*(s).
$$

Lemma A1 states that we only need to analysis the convergence of learned policy $\pi(s)$ to the optimal policy $\hat{\pi}^*(s)$ for transformed MDP $\hat{M}$, which is equivalent to the optimal policy $\pi(s)^*$ for original MDP. This result is relevant to potential-based reward shaping $[60, 61]$ where a specific class of state-dependent transformation is adopted to speed up the convergence speed of Q-Learning meanwhile maintaining the optimal policy invariance. Moreover, a degenerate case for single-step decisions is studied in utility theory $[62]$ which also implies our result.

Finally, we need an auxiliary result (Lemma A2) from stochastic process approximation to analyse the convergence for Q-Learning.

**Lemma A2.** The random process $\{\Delta_t\}$ taking values in $\mathbb{R}^n$ and defined as

$$
\Delta_{t+1}(x) = (1 - \alpha_t(x))\Delta_t(x) + \alpha_t(x)F_t(x)
$$

converges to zero w.p.1 under the following assumptions:
we abbreviate

Theorem A1. In consequence, Subtracting from both sides the quantity \( \hat{M} \) is a constant. From Lemma A1, we know the optimal policy for \( \pi \) is precisely the optimal policy for \( \pi \) as long as \( \sum \alpha = \infty \) and \( \sum \alpha^2 < \infty \).

Proof of Lemma A2. See previous literature \([63, 64]\). □

**Theorem A1.** (Convergence) Given a finite MDP with noisy reward, denoting as \( \hat{M} = (S, A, \hat{r}, F, \hat{P}, \gamma) \), the Q-learning algorithm with peer rewards, given by the update rule,

\[
Q_{t+1}(s_t, a_t) = (1 - \alpha_t)Q_t(s_t, a_t) + \alpha_t \left[ \hat{r}_{\text{peer}}(s_t, a_t) + \gamma \max_{b \in A} Q_t(s_{t+1}, b) \right],
\]

\[
\pi_t(s) = \arg \max_{a \in A} Q_t(s, a)
\]

converges w.p.1 to the optimal policy \( \pi^* \) as long as \( \sum \alpha = \infty \) and \( \sum \alpha^2 < \infty \).

Proof. Firstly, we construct a surrogate MDP \( \hat{M} \) with the positive-linearly transformed reward \( \hat{r} = (1 - e_+ - e_) \cdot r + \text{const} \), where const = \( (1 - e_+ - e_+)((1 - p) \cdot r_+ + p \cdot r_-) \) is a constant. From Lemma A1, we know the optimal policy for \( \hat{M} \) is precisely the optimal policy for \( \hat{M} \). Let \( \hat{Q}^* \) denotes the optimal state-action function for this transformed MDP \( \hat{M} \). For notation brevity, we abbreviate \( s_t, s_{t+1}, \hat{r}_{\text{peer}}(s_t, s_{t+1}), Q_t, \pi_t, \) and \( \alpha_t \) as \( s, s', \hat{r}, Q, \pi, \hat{r}_{\text{peer}} \) and \( \alpha \), respectively.

Subtracting from both sides the quantity \( \hat{Q}^*(s, a) \) in Eqn. (20):

\[
Q'(s, a) - \hat{Q}^*(s, a) = (1 - \alpha) \left( Q(s, a) - \hat{Q}^*(s, a) \right) + \alpha \left[ \hat{r}_{\text{peer}} + \sum_{b \in A} \max_{b \in A} Q(s', b) - \hat{Q}^*(s, a) \right].
\]

Let \( \Delta_t(s, a) = Q(s, a) - \hat{Q}^*(s, a) \) and \( F_t(s, a) = \hat{r}_{\text{peer}} + \sum_{b \in A} \max_{b \in A} Q(s', b) - \hat{Q}^*(s, a) \).

In consequence,

\[
\mathbb{E}[F_t(s, a) | F_t] = \mathbb{E} \left[ \hat{r}_{\text{peer}} + \sum_{b \in A} \max_{b \in A} Q(s', b) - \hat{Q}^*(s, a) \right] \]

\[
= \mathbb{E} \left[ \hat{r}_{\text{peer}} + \sum_{b \in A} \max_{b \in A} Q(s', b) - \hat{r} - \gamma \max_{b \in A} \hat{Q}^*(s', b) \right] \]

\[
= \mathbb{E} \left[ \hat{r}_{\text{peer}} \right] - \mathbb{E} \left[ \hat{r} \right] + \gamma \max_{b \in A} \left[ Q(s', b) - \max_{b \in A} \hat{Q}^*(s', b) \right] \]

\[
\leq \gamma \max_{b \in A, s' \in S} \left[ Q(s', b) - \hat{Q}^*(s', b) \right] \]

\[
= \gamma \mathbb{E} \left[ \| Q - \hat{Q}^* \|_\infty \right] = \gamma \| Q - \hat{Q}^* \|_\infty = \gamma \| \Delta_t \|_\infty.
\]
In above derivations, we utilize the unbiasedness property for peer reward (Lemma 1) and the inequality \(\max_{b \in A} Q(s', b) - \max_{b \in A} \hat{Q}^*(s', b) \leq \max_{b \in A, s' \in S} |Q(s', b) - \hat{Q}^*(s', b)|\).

\[
\text{Var} \left[ F_t(s,a) | F_t \right] = \mathbb{E} \left[ \left( \hat{r}_{\text{peer}} + \gamma \max_{b \in A} Q(s', b) - \hat{Q}^*(s,a) - \mathbb{E} \left[ \hat{r}_{\text{peer}} + \gamma \max_{b \in A} Q(s', b) - \hat{Q}^*(s,a) \right] \right)^2 \right]
\]

\[
= \mathbb{E} \left[ \hat{r}_{\text{peer}} + \gamma \max_{b \in A} Q(s', b) - \mathbb{E} \left[ \hat{r}_{\text{peer}} + \gamma \max_{b \in A} Q(s', b) \right] \right]^2
\]

\[
= \text{Var} \left[ \hat{r}_{\text{peer}} + \gamma \max_{b \in A} Q(s', b) \right].
\]

Since \(\hat{r}_{\text{peer}}\) is bounded, it can be clearly verified that

\[
\text{Var} \left[ F_t(s,a) | F_t \right] \leq C'' (1 + \|\Delta_t(s,a)\|_\infty^2)
\]

for some constant \(C'' > 0\). Then, \(\Delta_t\) converges to zero w.p.1 from Lemma A2, i.e., \(Q(s,a)\) converges to \(\hat{Q}^*(s,a)\). As a consequence, we know the policy \(\pi_t(s)\) converges to the optimal policy \(\hat{\pi}^*(s) = \pi^*(s)\).

\[
\hat{\pi}^*(s) = \pi^*(s)
\]

\section*{A.2 Sample Complexity}

In this section, we establish the sample complexity for Q-Learning with peer reward as discussed in Sec 4. Since the transition probability \(P\) in MDP remains unknown in practice, we firstly introduce a practical sampling model \(G(M)\) following previous literature \([65–67]\), in which the transition can be observed by calling the generative model. Then the sample complexity is analogous to the number of calls for \(G(M)\) to obtain a near optimal policy.

\textbf{Definition A1.} A generative model \(G(M)\) for an MDP \(M\) is a sampling model which takes a state-action pair \((s_t, a_t)\) as input, and outputs the corresponding reward \(r(s_t, a_t)\) and the next state \(s_{t+1}\) randomly with the probability of \(P_{a}(s_t, s_{t+1})\), i.e., \(s_{t+1} \sim P(\cdot | s_t, a_t)\).

It is known that exact value iteration is not feasible when the agent interacts with generative model \(G(M)\) \([7,68]\). For the convenience of analysing sample complexity, we introduce a \textit{phased value iteration} following \([7,65,68]\).

\textbf{Algorithm A2} Phased Value Iteration

\textbf{Require:} \(G(M)\): generative model of \(M = (S,A,R,P,\gamma)\), \(T\): number of iterations.

1: Set \(V_T(s) = 0\)
2: for \(t = T - 1, \ldots, 0\) do
3:  Calling \(G(M)\) \(m\) times for each state-action pair.
4: \(\hat{P}_a(s_t, s_{t+1}) = \frac{\#\{s_t, a_t \rightarrow s_{t+1}\}}{m}\)
5:  Set
6: \(V(s_t) = \max_a \sum_{s_{t+1} \in S} \hat{P}_a(s_t, s_{t+1}) [r_t + \gamma V(s_{t+1})]\)
7: \(\pi(s) = \arg \max_a V(s_t)\)

5: end for
6: return \(V(s)\) and \(\pi(s)\)

Note that \(\hat{P}_a(s_t, s_{t+1})\) is the estimation of transition probability \(P_a(s_t, s_{t+1})\) by calling \(G(M)\) \(m\) times. For the simplicity of notations, the iteration index \(t\) decreases from \(T - 1\) to 0.

We could also adopt peer reward in phased value iteration by replacing Line 4 in Algorithm A2 by

\[
V(s_t) = \max_a \sum_{s_{t+1} \in S} \hat{P}_a(s_t, s_{t+1}) [\hat{r}_{\text{peer}}(s_t, a) + \gamma V(s_{t+1})].
\]
Then the sample complexity of one variant (phased value iteration) of Q-Learning is given as follows:

**Theorem A2. (Sample Complexity)** Let \( r \in [0, R_{\text{max}}] \) be bounded reward, for an appropriate choice of \( m \), the phased value iteration algorithm with peer reward \( \bar{r}_{\text{peer}} \) calls the generative model \( G(\bar{M}) \) \( O \left( \frac{|S| |A| T}{m(1 - e^{-\delta - \epsilon})} \log \frac{|S| |A| T}{m} \right) \) times in \( T \) epochs, and returns a policy such that for all state \( s \in S \),

\[
\frac{1}{\eta} V^\pi(s) - V^*\left( s \right) \leq \epsilon, \text{ w.p. } \geq 1 - \delta, \ 0 < \delta < 1, \text{ where } \eta = 1 - e_- - e_+ > 0 \text{ is a constant.}
\]

**Proof.** Similar to Theorem A1, we firstly construct a transformed MDP \( \bar{M} \) and the optimal policies for these two MDP are equivalent (Lemma A1). As a result, we could analyse the sample complexity of phased value iteration under \( \bar{M} \).

It is easy to obtain that \( \bar{r}_{\text{peer}} \in [0, R_{\text{max}}] \) and \( V^\pi(s) \in \left[ 0, \frac{R_{\text{max}}}{1 - \gamma} \right] \) are also bounded. Using Hoeffding’s inequality, we have

\[
\Pr \left( \left| \mathbb{E} \left[ \hat{V}^*_t(s_{t+1}) \right] - \sum_{s_{t+1} \in S} \hat{P}_a(s_t, s_{t+1}) \hat{V}^*_t(s_{t+1}) \right| \geq \epsilon \right) \leq 2 \exp \left( \frac{-2m\epsilon^2(1 - \gamma)^2}{R_{\text{max}}^2} \right),
\]

\[
\Pr \left( \left| \mathbb{E} \left[ \bar{r}_{\text{peer}}(s_t, a) - \sum_{s_{t+1} \in S} \hat{P}_a(s_t, s_{t+1}) \bar{r}_{\text{peer}}(s_t, a) \right] \geq \epsilon \right) \leq 2 \exp \left( \frac{-2m\epsilon^2}{R_{\text{max}}^2} \right).
\]

Then the difference between learned value function \( \hat{V}^\pi(s)_t \) and optimal value function \( \hat{V}^*_t(s)_t \) under transformed MDP at iteration \( t \) is given:

\[
\left| \hat{V}^*_t(s) - V_t(s) \right| = \max_{a \in A} \mathbb{E} \left[ r_t + \gamma \hat{V}^*_t(s_{t+1}) \right] - \max_{a \in A} \sum_{s_{t+1} \in S} \hat{P}_a(s_t, s_{t+1}) \left[ \bar{r}_{\text{peer}}(s_t, a) + \gamma V_{t+1}(s_{t+1}) \right]
\]

\[
\leq \max_{a \in A} \left| \mathbb{E} \left[ r_t \right] - \sum_{s_{t+1} \in S} \hat{P}_a(s_t, s_{t+1}) \bar{r}_{\text{peer}}(s_t, a) \right|
\]

\[
+ \max_{a \in A} \left| \mathbb{E} \left[ \hat{V}^*_t(s_{t+1}) \right] - \sum_{s_{t+1} \in S} \hat{P}_a(s_t, s_{t+1}) \hat{V}^*_t(s_{t+1}) \right|
\]

\[
\leq \epsilon_1 + \max_{a \in A} \left| \mathbb{E} \left[ r_t \right] - \mathbb{E} \left[ \bar{r}_{\text{peer}} \right] \right| + \gamma \epsilon_2 + \left| \mathbb{E} \left[ \hat{V}^*_t(s_{t+1}) \right] - \mathbb{E} \left[ \hat{V}^*_t(s_{t+1}) \right] \right|
\]

\[
\leq \max_{s_{t+1} \in S} \left| \hat{V}^*_t(s) - V_t(s) \right| + \epsilon_1 + \gamma \epsilon_2
\]

Recurring above equation, we get

\[
\max_{s \in S} \left| \hat{V}^*_t(s) - V(s) \right| \leq (\epsilon_1 + \gamma \epsilon_2) + (\epsilon_1 + \gamma \epsilon_2) + \cdots + (\gamma T - 1)(\epsilon_1 + \gamma \epsilon_2)
\]

\[
= \frac{(\epsilon_1 + \gamma \epsilon_2)(1 - \gamma^T)}{1 - \gamma}
\]

Let \( \epsilon_1 = \epsilon_2 = \frac{(1 - \gamma)\epsilon}{(1 + \gamma)} \), then \( \max_{s \in S} \left| \hat{V}^*_t(s) - V(s) \right| \leq \epsilon \). In other words, for arbitrarily small \( \epsilon \), by choosing \( m \) appropriately, there always exists \( \epsilon_1, \epsilon_2 \) such that the value function error is bounded within \( \epsilon \). As a consequence the phased value iteration algorithm can converge to the near optimal policy within finite steps using peer reward.

Note that there are in total \( |S| |A| T \) transitions under which these conditions must hold, where \(| \cdot |\) represent the number of elements in a specific set. Using a union bound, the probability of failure in any condition is smaller than

\[
2|S||A|T \cdot \exp \left( -m \frac{2(1 - \gamma)^2}{(1 + \gamma)^2} \cdot \frac{(1 - \gamma)^2}{R_{\text{max}}^2} \right).
\]

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We set above failure probability less than $\delta$, and $m$ should satisfy that
\[ m = O\left(\frac{1}{\epsilon^2} \log \frac{|S||A|T}{\delta}\right). \]

In consequence, after $m|S||A|T$ calls, which is, $O\left(\frac{|S||A|T}{\epsilon^2} \log \frac{|S||A|T}{\delta}\right)$, the value function converges to the optimal value function $\hat{V}^*(s)$ for every $s$ in transformed MDP $\hat{M}$, with probability greater than $1 - \delta$.

From Lemma A1, we know $\hat{V}^*(s) = (1 - e_- - e_+) \cdot V^*(s) + C$, where $C$ is a constant. Let $\epsilon = (1 - e_- - e_+) \cdot \epsilon$ and $V(s) = (1 - e_- - e_+) \cdot \hat{V}(s) + C$, we have
\[
|V^*(s) - V'(s)| = \left| \frac{\hat{V}^*(s) - C}{(1 - e_- - e_+)} - \frac{V(s) - C}{(1 - e_- - e_+)} \right| 
= \frac{1}{(1 - e_- - e_+)} \left| \hat{V}^*(s) - V(s) \right| \leq \epsilon' \tag{21}
\]

This indicates that when the algorithm converges to the optimal value function for transformed MDP $\hat{M}$, it also finds a underlying value function $V'(s) = \frac{1}{\hat{V}} V(s)$ that converges the optimal value function $V^*(s)$ for original MDP $M$.

As a consequence, we know it needs to call $O\left(\frac{|S||A|T}{\epsilon^2(1 - e_- - e_+)^2} \log \frac{|S||A|T}{\delta}\right)$ to achieve an $\epsilon'$ error in value function for original MDP $\mathcal{M}$, which is no more than $O\left(\frac{1}{(1 - e_- - e_+)^2}\right)$ times of the one needed when the RL agent observes true rewards perfectly. When the noise is in high-regime, the algorithm suffers from a large $\frac{1}{(1 - e_- - e_+)^2}$ thus less efficient. Moreover, the sample complexity of phased value iteration with peer reward is less expensive and does not rely on any knowledge of noise rates.

\[ \square \]

### A.3 Multi-outcome Extension

In this section, we show our peer reward is generalizable to multi-class setting. Recall that in Section 2.2 we suppose the reward is discrete and has $|\mathcal{R}|$ levels, and the noise rates are characterized as $C_{\mathcal{R} \times |\mathcal{R}|}^{\text{RL}}$. Here we make further assumptions on the confusion matrix: the reward is misreported to each level with specific probability, e.g.,
\[
C_{|\mathcal{R}| \times |\mathcal{R}|}^{\text{RL}} = \begin{bmatrix}
1 - \sum_{i \neq 1} e_i, & 1 - \sum_{i \neq 2} e_i, & \cdots & e_{|\mathcal{R}|} \\
1 - \sum_{i \neq 1} e_i, & 1 - \sum_{i \neq 2} e_i, & \cdots & e_{|\mathcal{R}|} \\
\vdots & \vdots & \ddots & \vdots \\
e_{|\mathcal{R}|} & \cdots & e_{|\mathcal{R}|} & 1 - \sum_{i \neq |\mathcal{R}|} e_i
\end{bmatrix} \tag{23}
\]

Following the notations in A.1, we define the peer reward in multi-outcome settings as $r(s, a) = \hat{r}(s, a) - r'$, where $r'$ is randomly sampled following a specific sample policy $\pi_{\text{sample}}$ over all state-action pairs. Let $R_{\text{peer}}$, $R$, $\hat{R}$, and $R'$ denote the random variables corresponding to $\hat{r}_{\text{peer}}$, $r$, $\hat{r}$, $r'$, $c_{ij}$ represents the entry of $C_{|\mathcal{R}| \times |\mathcal{R}|}^{\text{RL}}$. Then we have
\[
\mathbb{E}_{\pi} \left[ \hat{R} \right] = \sum_{i=1}^{|\mathcal{R}|} \mathbb{P} \left( R = R_i | \pi \right) \sum_{j=1}^{|\mathcal{R}|} c_{ij} R_j
\]
\[
\quad = \sum_{i=1}^{|\mathcal{R}|} \mathbb{P} \left( R = R_i | \pi \right) \left[ \left( 1 - \sum_{j \neq i} e_j \right) R_i + \sum_{j \neq i} e_j R_j \right]
\]
\[
\quad = \sum_{i=1}^{|\mathcal{R}|} \mathbb{P} \left( R = R_i | \pi \right) \left[ \left( 1 - \sum_{j=1}^{|\mathcal{R}|} e_i \right) R_i + \sum_{j=1}^{|\mathcal{R}|} e_j R_j \right]
\]
\[
\quad = \left( 1 - \sum_{j=1}^{|\mathcal{R}|} e_j \right) \mathbb{E}_{\pi} \left[ R \right] + \sum_{j=1}^{|\mathcal{R}|} e_j R_j,
\]
Then, the peer reward is formulated as

\[ E_{\pi_{\text{sample}}} [\tilde{R}'] = \sum_{i=1}^{\vert R \vert} R_i \cdot P(R = R_i | \pi_{\text{sample}}) \]

\[ = \sum_{j=1}^{\vert R \vert} \sum_{i=1}^{\vert R \vert} P(R = R_i | \pi_{\text{sample}}) e_j \]

\[ = \sum_{j=1}^{\vert R \vert} \sum_{i=1}^{\vert R \vert} P(R = R_i | \pi_{\text{sample}}) e_j + P(R = R_j | \pi_{\text{sample}}) \left( 1 - \sum_{i \neq j}^{\vert R \vert} e_i \right) \]

\[ = \left( 1 - \sum_{i=1}^{\vert R \vert} e_i \right) E_{\pi_{\text{sample}}} [R] + \sum_{j=1}^{\vert R \vert} e_j R_j. \]

Then, the peer reward is formulated as

\[ E \left[ \tilde{R}_{\text{peer}} \right] = E_{\pi} \left[ \tilde{R} \right] - E \left[ \tilde{R}' \right] \]

\[ = \left( 1 - \sum_{j=1}^{\vert R \vert} e_j \right) E_{\pi} [R] - \left( 1 - \sum_{i=1}^{\vert R \vert} e_i \right) E_{\pi_{\text{sample}}} [R] \]

\[ = \left( 1 - \sum_{j=1}^{\vert R \vert} e_j \right) E_{\pi} [R] + \text{const.} \]

### A.4 Extension in Modern DRL algorithms

In this section, we give the following deep reinforcement learning algorithms combined with our peer reward in Algorithm A3 and A4. In Algorithm A3, we give the peer reward aided robust policy gradient algorithm, where the gradient in Equation 24 corresponds to the loss function \( \ell((s, a), q) = q \log \pi_\theta(a|s), \) which is classification calibrated [17]. So the expectation of the gradient in 24 is an unbiased estimation of the policy gradient in corresponding clean MDP. In (A4), we present a robust DQN algorithm with peer sampling, in which the origin loss is \( \ell((s, a), \tilde{y}), \) also classification calibrated. Thus the robustness can be proved via [17].

**Algorithm A3** Policy Gradient [51] with Peer Reward

**Require:** \( \hat{M} = \{S, A, \tilde{R}, P, \gamma\}, \) learning rate \( \alpha \in (0, 1), \) initial state distribution \( \beta_0, \) weight parameter \( \xi. \)

1: Initialize \( \pi_\theta, S \times A \rightarrow \mathbb{R} \) arbitrarily
2: for \( \text{episode} = 1 \text{ to } M \) do
3: Collect trajectory \( \tau_\theta = \{(s_t, a_t, \tilde{r}_t)\}_{t=0}^{T}, \) where \( s_0 \sim \beta_0, a_t \sim \pi_\theta(\cdot|s_t), s_{t+1} \sim P(\cdot|s_t, a_t). \)
4: Compute \( q_k = \sum_{i=1}^{T} \gamma^{t-i} \tilde{r}_t \) for all \( t \in \{0, 1, \ldots, T\}. \)
5: For each index \( i \in \{0, 1, \ldots, T\}, \) we independently sample another two different indices \( j, k, \)
6: and update policy parameter \( \theta \) following

\[ \theta \leftarrow \theta + \alpha \left( q_k \nabla_\theta \log \pi_\theta(a_1|s_1) - \xi \cdot q_k \nabla_\theta \log \pi_\theta(a_j|s_j) \right) \] (24)

7: end for

**Ensure:** \( \pi_\theta \)
We can also analyze why peer rewards are beneficial from the error upper bound. When $\xi = 1$, define the sample mean of rewards as follows.
\[
\bar{r} := \frac{1}{T} \sum_{t=1}^{T} r(s_t, a_t), \quad \bar{r}_{\text{peer}} := \frac{1}{T} \sum_{t=1}^{T} r_{\text{peer}}(s_t, a_t) = \frac{1}{T} \sum_{t=1}^{T} \left[ \bar{r}(s_t, a_t) + (1 - r'_t) \right] - 1.
\]

By Hoeffding’s inequality, noticing there are $T$ independent random variables in estimating $\bar{r}$ and $2T$ independent random variables in estimating $\bar{r}_{\text{peer}}$, we know w.p. at least $1 - \delta$,
\[
|\bar{r} - \mathbb{E}[\bar{r}]| \leq R_{\max} \sqrt{\frac{\ln 2/\delta}{2T}},
\]
and
\[
|\bar{r}_{\text{peer}} - \mathbb{E}[\bar{r}_{\text{peer}}]| \leq R_{\max} \sqrt{\frac{\ln 2/\delta}{T}}.
\]

We can denote the relationship between reward estimates and the corresponding error rate estimates $\bar{e}_-, \bar{e}_+$ as:
\[
\bar{r} = (1 - \bar{e}_- - \bar{e}_+)\bar{r} + \bar{e}_- r_+ + \bar{e}_+ r_-.
\]

We have
\[
|\bar{r} - \mathbb{E}[\bar{r}]| = |(1 - \bar{e}_- - \bar{e}_+)\bar{r} - (1 - e_- - e_+)\mathbb{E}[r] + (\bar{e}_- - e_-)r_+ + (\bar{e}_+ - e_+)r_-|
\]
\[
= |1 - \bar{e}_- - \bar{e}_+|\bar{r} - (1 - \bar{e}_- - \bar{e}_+)\mathbb{E}[r] + (e_- - \bar{e}_- + e_+ - \bar{e}_+)\mathbb{E}[r] + (\bar{e}_- - e_-)r_+ + (\bar{e}_+ + e_+)r_-|
\]
\[
\geq (1 - \bar{e}_- - \bar{e}_+)\left|\bar{r} - \mathbb{E}[r]\right| - |\bar{e}_- - e_-|(r_+ + \mathbb{E}[r]) - |\bar{e}_+ - e_+|(r_- + \mathbb{E}[r]).
\]

Thus
\[
|\bar{r} - \mathbb{E}[r]| \leq \frac{R_{\max} \sqrt{\ln 2/\delta} + |\bar{e}_- - e_-|(r_+ + \mathbb{E}[r]) + |\bar{e}_+ - e_+|(r_- + \mathbb{E}[r])}{1 - \bar{e}_- - \bar{e}_+}.
\]

Assume $\delta_e = |\bar{e}_- - e_-| = |\bar{e}_+ - e_+|$. We have
\[
|\bar{r} - \mathbb{E}[r]| \leq \frac{R_{\max} \sqrt{\ln 2/\delta} + \delta_e(r_+ + r_- + 2\mathbb{E}[r])}{1 - \bar{e}_- - \bar{e}_+}.
\]

Similarly, for peer rewards, note
\[
\bar{r}_{\text{peer}} = (1 - \bar{e}_- - \bar{e}_+)\left(\bar{r} - (1 - \bar{p}_{\text{peer}})r_- - \bar{p}_{\text{peer}}r_+\right).
\]
We have

\[
|\bar{r}_{\text{peer}} - E[\bar{r}_{\text{peer}}]| = |(1 - \bar{e}_- - \bar{e}_+) (\bar{r} - (1 - \bar{p}_{\text{peer}}) r_- - \bar{p}_{\text{peer}} r_+) - (1 - \bar{e}_- - \bar{e}_+) (E[r] - (1 - p_{\text{peer}}) r_- - p_{\text{peer}} r_+)|
+ (\bar{e}_- - \bar{e}_+ + e_- + e_+) (E[r] - (1 - p_{\text{peer}}) r_- - p_{\text{peer}} r_+)|
\geq (1 - \bar{e}_- - \bar{e}_+) |\bar{r} - E[r]| - (1 - \bar{e}_- - \bar{e}_+) |\bar{p}_{\text{peer}} - p_{\text{peer}}| r_- - r_+ |-
|\bar{e}_- - e_-| \cdot |E[r] - (1 - p_{\text{peer}}) r_- - p_{\text{peer}} r_+| - |\bar{e}_+ - e_+| \cdot |E[r] - (1 - p_{\text{peer}}) r_- - p_{\text{peer}} r_+|.
\]

Thus

\[
|\bar{r} - E[r]| \leq \frac{R_{\text{max}} \sqrt{\ln \frac{2/\delta}{T}} + (|\bar{e}_- - e_-| + |\bar{e}_+ - e_+| \cdot |E[r] - (1 - p_{\text{peer}}) r_- - p_{\text{peer}} r_+| + |\bar{p}_{\text{peer}} - p_{\text{peer}}| r_- - r_+ |)}{1 - \bar{e}_- - \bar{e}_+} + |\bar{p}_{\text{peer}} - p_{\text{peer}}| r_- - r_+ |.
\]

Assume \( \delta_e = |\bar{e}_- - e_-| = |\bar{e}_+ - e_+| = |\bar{p}_{\text{peer}} - p_{\text{peer}}| \). We have

\[
|\bar{r} - E[r]| \leq \frac{R_{\text{max}} \sqrt{\ln \frac{2/\delta}{T}} + 2 \delta_e \cdot |E[r] - (1 - p_{\text{peer}}) r_- - p_{\text{peer}} r_+| + \delta_e r_- - r_+ |}{1 - \bar{e}_- - \bar{e}_+}.
\]  

Comparing Eqn. (25) and Eqn. (26), for the high-noise case, we can infer peer rewards likely have lower sample complexity, i.e. is more sample efficient. For example, when \( p_{\text{peer}} = 0.5 \), \( e_- = e_+ = 0.3 \), \( R_{\text{max}} = 1 \), \( r_+ = 1 \), \( r_- = 0 \), \( \bar{e}_- = \bar{e}_+ = e_- = e_+ = \delta_e \), \( E[r] = 0.5 \), we have

\[
|\bar{r} - E[r]| \leq \frac{\sqrt{\ln \frac{2/\delta}{T}} + 2 \delta_e}{0.4} \quad \text{(Plain Reward)},
\]

\[
|\bar{r} - E[r]| \leq \frac{\sqrt{\ln \frac{2/\delta}{T}} + 1.4 \delta_e}{0.4} \quad \text{(Peer Reward)}.
\]

In this case, we know peer rewards have a lower error upper bound for estimating \( r \) when \( T \) is large.

## B Tie-Breaking: Toy Examples

To illustrate tie-breaking phenomenon when using peer reward, we consider a two-state Markov process (no actions) with varied noise models. An example code segment with stochastic rewards and discrete noise model \( (e_- = e_+ = 0.45) \) is provided below:

```python
def get_rewards(state, num_samples, noise_rate=0.45):
    if state == 0:
        r = np.random.choice([0, 1], p=[0.4, 0.6], size=num_samples) # E[r] = 0.6
    else:
        r = np.random.choice([0, 1], p=[0.6, 0.4], size=num_samples) # E[r] = 0.4
    mask = np.random.choice([0, 1], p=[1 - noise_rate, noise_rate], size=num_samples) # Add noise
    r = (1 - mask) * r + mask * (1 - r)
    return r

num_samples, xi = 1000, 0.1
is_correct_noisy, is_correct_peer = [], []
for _ in tqdm.trange(10000):
    # Baseline
    r_vec = np.stack([get_rewards(0, num_samples), get_rewards(1, num_samples)], axis=1)
    is_correct_noisy.append(r_hat[0] > r_hat[1])
    # PeerRL
    neg_samples = np.concatenate([get_rewards(0, num_samples), get_rewards(1, num_samples)])
    neg_samples0, neg_samples1 = np.split(neg_samples, 2)
    r_vec = np.stack([get_rewards(0, num_samples) - xi * neg_samples0,
                      get_rewards(1, num_samples) - xi * neg_samples1], axis=1)
    r_hat = np.mean(r_vec, axis=0)
    is_correct_peer.append(r_hat[0] > r_hat[1])

print("\nBaseline Success: %")
print("\nPeer RL Success: %")
```

In Table A1, we conducted more experiments with different noise models and reported the absolute accuracy differences between PeerRL and baseline (noisy reward) in the following three cases: (1) "Correct" - successfully inferring the better state \( s_1 \) with larger expected reward, (2) "Tie" - cannot infer which state is better as the means of collected rewards in two states are equal, (3) "Incorrect" - wrongly inferring state \( s_2 \) is better ("Incorrect"). As we can see, PeerRL exploits the "discreteness" of the reward thus breaking ties to obtain more examples with good-quality supervision. This tie-breaking phenomenon also happens for stochastic reward and bounded/discretized continuous reward.
We prove that the policy learned by PeerBC converges to the expert policy when observing a sufficient amount of weak demonstrations in Theorem A3.

**Theorem A3.** With probability at least $1 - \delta$, the error rate is upper-bounded by

$$R^*_{DE} \leq \frac{1 + \xi}{1 - e_- - e_+} \sqrt{\frac{2 \log 2/\delta}{N}},$$

where $N$ is the number of state-action pairs demonstrated by the expert.

**Proof.** Recall $\tilde{D}_E$ denotes the joint distribution of imperfect expert’ state-action pair $(s, \hat{a})$. Assume there is a perfect expert and the corresponding state-action pairs $(s, a) \sim D_E$. The indicator classification loss $\mathbb{I}(\pi(s), a)$ is specified here for a clean presentation, where $\mathbb{I}(\pi(s), a) = 1$ when $\pi(s) \neq a$, otherwise $\mathbb{I}(\pi(s), a) = 0$. Let $\tilde{D}_E := \{(s_i, \hat{a}_i)\}_{i=1}^N$ be the set of imperfect demonstrations, and $D_E := \{(s_i, a_i)\}_{i=1}^N$ be the set of weak demonstrations. Define:

$$R_{DE}(\pi) := \mathbb{E}_{(s, a) \sim D_E} \mathbb{I}(\pi(s), a), \quad \tilde{R}_{DE}(\pi) := \mathbb{E}_{(s, \hat{a}) \sim \tilde{D}_E} \mathbb{I}(\pi(s), \hat{a})$$

$$\tilde{R}_{DE}(\pi) := \frac{1}{N} \sum_{i \in [N]} \mathbb{I}(\pi(s_i), a_i), \quad \tilde{R}_{DE}(\pi) := \frac{1}{N} \sum_{i \in [N]} \mathbb{I}(\pi(s_i), \hat{a}_i).$$

Note we focus on the analyses of loss in this proof. The negative of loss can be seen as a reward. Denote by $\pi_0^\beta_E$ and $\pi_0^\alpha_E$ be the optimal policy obtained with minimizing the indicator loss with dataset $D_E$ and distribution $\tilde{D}_E$. We shorten $\pi_0^\beta_E$ as $\pi^\beta$, which is the best policy we can learn from imperfect demonstration with our algorithm. Let $\pi^*$ be the policy for the perfect expert. We would like to see the performance gap of policy learning between imperfect demonstrations and perfect demonstrations, i.e. $R_{DE}(\pi^*) - R_{DE}(\pi)$. Using Hoeffding’s inequality with probability at least $1 - \delta$, we have

$$|\tilde{R}_{DE}(\pi) - \tilde{R}_{DE}(\pi)| \leq (1 + \xi) \sqrt{\frac{\log 2/\delta}{2N}}.$$
Note we also have

\[
R_{D_E}(\tilde{\pi}^*) - R_{\tilde{D}_E}(\pi_{\tilde{D}_E}) \\
\leq \hat{R}_{D_E}(\tilde{\pi}^*) - \hat{R}_{\tilde{D}_E}(\pi_{\tilde{D}_E}) + \left( R_{\tilde{D}_E}(\tilde{\pi}^*) - \hat{R}_{\tilde{D}_E}(\tilde{\pi}^*) \right) \\
+ \left( \hat{R}_{\tilde{D}_E}(\pi_{\tilde{D}_E}) - R_{\tilde{D}_E}(\pi_{\tilde{D}_E}) \right) \\
\leq 0 + 2 \max_{\pi} \left| \hat{R}_{\tilde{D}_E}(\pi) - R_{\tilde{D}_E}(\pi) \right| \\
\leq (1 + \xi) \sqrt{\frac{2 \log 2/\delta}{N}}.
\]

Before proceeding, we need to define a constant to show the affect of label noise. When the dimension of action space is 2, the problem is essentially a binary classification with noisy labels [17], where the noise rate (a.k.a confusion matrix) is defined as \( e_+ = P(\tilde{\pi}_E(s) = A_-| \pi^*(s) = A_+) \) and \( e_- = P(\tilde{\pi}_E(s) = A_+| \pi^*(s) = A_-) \). Recall the action space is defined as \( \mathcal{A} = \{A_+, A_-\} \). The noise constant is denoted by \( e = e_+ + e_- \). Accordingly, when the dimension of action space is \(|\mathcal{R}| > 2\), we can also get similar results under uniform noise where

\[
e_u := P(\tilde{\pi}_E(s) = u| \pi^*(s) = u'), u' \neq u.
\]

The noise constant \( e \) is denoted by \( e = \sum_{u=1}^{\mathcal{R}} e_u \). The feature-independent assumption holds thus the properties of peer loss functions [17] can be used, i.e.

\[
R_{D_E}(\tilde{\pi}^*) - R_{D_E}(\pi^*) \\
= \frac{1}{1 - e} \left( R_{\tilde{D}_E}(\pi_{\tilde{D}_E}) - R_{\tilde{D}_E}(\pi_{\tilde{D}_E}) \right) \\
\leq \frac{1 + \xi}{1 - e} \sqrt{\frac{2 \log 2/\delta}{N}}.
\]

From definition and deterministic assumption for \( \pi^* \), we have \( R_{D_E}(\pi^*) = 0 \). Thus the error rate in the \( k \)-th iteration is

\[
R_{D_E}(\tilde{\pi}^*) \leq R_{D_E}(\pi^*) + \frac{1 + \xi}{1 - e} \sqrt{\frac{2 \log 2/\delta}{N}}.
\]

Note \( R_{D_E}(\tilde{\pi}^*) = R_{\tilde{D}_E} \) by definition.

\[\square\]

## D Supplementary Experiments

### D.1 Experimental Setup

We set up our experiments within the popular OpenAI stable-baselines\(^2\) and keras-rl\(^3\) framework. Specifically, three popular RL algorithms including Deep-Q-Network (DQN) [49, 56], Dueling-DQN (DDQN) [50] and Proximal Policy Optimization Algorithms (PPO) are evaluated in a varied of OpenAI Gym environments including classic control games (CartPole, Acrobot) and vision-based Atari-2600 games (Breakout, Boxing, Enduro, Freeway, Pong).

### D.2 Implementation Details

#### RL with noisy reward

Following [7], we consider the binary reward \( \{-1, 1\} \) for Cartpole where the symmetric noise is synthesized with different error rates \( e = e_+ = e_- \). We adopted a five-layer fully connected network and the Adam optimizer. The model is trained for 10,000 steps with the learning rate of \( 1e^{-3} \) and the Boltzmann exploration strategy. The update rate of target model and the memory size are \( 1e^{-2} \) and 50,000. The performance is reported under 10 independent trials with different random seeds.

\(^2\)https://github.com/hill-a/stable-baselines

\(^3\)https://github.com/keras-rl/keras-rl
**BC with weak expert** We train the imperfect expert on the framework stable-baselines with default network architecture for Atari and hyper-parameters from rl-baselines-zoo. The expert model is trained for 1, 400, 000 steps for Pong and 2, 000, 000 steps for Boxing, Enduro and Freeway. For each of those environment, We use the trained model to generate 100 trajectories, and behavior cloning is performed on these trajectories. We adopt cross entropy loss for behavior cloning and add a small constant ($1 \times 10^{-8}$) for each logit after the softmax operation for peer term to avoid this term become too large. In BC experiments, the batchsize is 128, learning rate is $1 \times 10^{-4}$ and the $\epsilon$ value for Adam optimizer is $1 \times 10^{-8}$.

**Policy co-training** For the experiments on Gym (CartPole and Acrobot), we mask the first coordinate in the state vector for one view and the second for the other, same as [43]. Both policies are trained with PPO[58] + PeerBC. In each iteration, we sample 128 steps from each of the 8 parallel environments. These samples are fed to PPO training with a batchsize of 256, a learning rate of $2.5 \times 10^{-4}$ and a clip range of 0.1. Both learning rate and clip range decay to 0 throughout time. We represent the policy by a fully connected network with 2 hidden layers, each has 128 units.

For the experiments on Atari (Pong and Breakout), the input is raw game images. We adopt the preprocess introduced in [49] and mask the pixels in odd columns for one view and even columns for the other. The policy we use adopts a default CNN as in stable-baselines. Batchsize, learning rate, clip range and other hyper-parameters are the same as Gym experiments. Note that we only add PeerBC after 1000 episodes.

### D.3 Supplementary Results for Figure 2 and Table 1

![Learning curves on CartPole game](image)

**Figure A1:** Learning curves on CartPole game with true reward ($r$), noisy reward ($\hat{r}$), surrogate reward [7] ($\hat{r}$), and peer reward ($\hat{r}_{peer}$, $\xi = 0.2$). Each experiment is repeated 10 times with different random seeds.

**Table A2:** Numerical performance of DDQN on CartPole with true reward ($r$), noisy reward ($\hat{r}$), surrogate reward $\hat{r}$ [7], and peer reward $\hat{r}_{peer}$ ($\xi = 0.2$). $R_{avg}$ denotes average reward per episode after convergence, (last five episodes) the higher (↑) the better; $N_{episode}$ denotes total episodes involved in 10,000 steps, the lower (↓) the better.

|        | $\epsilon = 0.1$ | $\epsilon = 0.2$ | $\epsilon = 0.3$ | $\epsilon = 0.4$ |
|--------|------------------|------------------|------------------|------------------|
|        | $R_{avg}$ ↑ | $N_{episode}$ ↓ | $R_{avg}$ ↑ | $N_{episode}$ ↓ | $R_{avg}$ ↑ | $N_{episode}$ ↓ | $R_{avg}$ ↑ | $N_{episode}$ ↓ |
| **DQN** | r | 183.6 ± 7.6 | 101.3 ± 4.8 | 184.0 ± 7.3 | 101.5 ± 4.6 | 184.0 ± 7.3 | 101.5 ± 4.6 | 184.0 ± 7.3 | 101.5 ± 4.6 |
|        | $\hat{r}$ | 189.3 ± 12.7 | 98.2 ± 6.5 | 189.7 ± 7.9 | 110.5 ± 7.1 | 183.2 ± 9.8 | 130.5 ± 7.7 | 169.7 ± 18.6 | 150.2 ± 11.4 |
|        | $\hat{r}$ | 188.3 ± 5.2 | 101.1 ± 6.2 | 192.7 ± 9.2 | 97.9 ± 6.4 | 185.4 ± 15.9 | 116.9 ± 11.0 | 184.8 ± 16.4 | 123.1 ± 8.6 |
| $\hat{r}_{peer}$ | 177.2 ± 19.1 | 91.2 ± 5.9 | 170.0 ± 24.8 | 94.6 ± 8.5 | 190.5 ± 14.3 | 99.4 ± 5.2 | 183.1 ± 13.3 | 118.1 ± 10.7 |
| **DDQN** | r | 195.6 ± 3.1 | 101.2 ± 3.2 | 195.6 ± 3.1 | 101.2 ± 3.2 | 195.6 ± 3.1 | 101.2 ± 3.2 | 195.2 ± 3.0 | 101.2 ± 3.3 |
|        | $\hat{r}$ | 185.2 ± 15.6 | 114.6 ± 6.0 | 168.8 ± 13.6 | 123.9 ± 9.6 | 177.1 ± 11.2 | 133.2 ± 9.1 | 185.5 ± 10.9 | 163.1 ± 11.0 |
|        | $\hat{r}$ | 183.9 ± 10.4 | 110.6 ± 6.7 | 165.1 ± 18.2 | 113.9 ± 9.6 | 192.2 ± 10.9 | 115.5 ± 4.3 | 179.2 ± 6.6 | 125.8 ± 9.6 |
| $\hat{r}_{peer}$ | 198.5 ± 2.3 | 86.2 ± 5.0 | 195.5 ± 9.1 | 85.3 ± 5.4 | 174.1 ± 32.5 | 88.8 ± 6.3 | 191.8 ± 8.5 | 106.9 ± 9.2 |

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[https://github.com/araffin/rl-baselines-zoo/blob/master/hyperparams/ppo2.yml#L1](https://github.com/araffin/rl-baselines-zoo/blob/master/hyperparams/ppo2.yml#L1)
D.4 Sensitivity Analysis of Peer Penalty $\xi$

In this section, we analyze the sensitivity of $\xi$ in RL and BC tasks. Note that we did not tune this hyperparameter extensively in all the experiments presented above since we found our method works robustly in a wide range of $\xi$.

![Learning curves of DQN on CartPole game with peer reward ($\tilde{r}_{\text{peer}}$) under different choices of $\xi$ (from 0.1 to 0.4).](image)

**Figure A2**: Learning curves of DQN on CartPole game with peer reward ($\tilde{r}_{\text{peer}}$) under different choices of $\xi$ (from 0.1 to 0.4).

**RL with noisy reward** We repeat the experiment in Figure A1 for DQN but with a varying $\xi$ from 0.1 to 0.4. As shown in Figure A2, our method works reasonably and leads to faster convergence compared to baselines. However, we found that the late stage of training, a small $\xi$ is necessary since the agent already gains useful knowledge and make reasonable actions, therefore, an over-large penalty might avoid the agent achieving simple agreements with the supervision signals, especially in a low-noise regime (see $\xi = 0.4$, $e = 0.1$). This observation inspires us that a decay schedule of $\xi$ might be helpful in stabilizing the training of PeerRL algorithms. To verify this hypothesis, we repeat the above experiments but with a linear decay $\xi$ that decreases from 0.4 to 0.1. In Figure A3, we found the linear decay schedule is able to stabilize the convergence of PeerRL algorithms compared to static $\xi = 0.4$. The theoretical principles and insights of dynamic peer penalty merit further study.

**BC from weak demonstrations** We conduct experiments on Pong with 12 different $\xi$ values, varying from 0.1 to 1.2. From Figure A4, we can see PeerBC outperforms pure behavior cloning and SQIL[37] when $\xi$ is within $[0.1, 0.7]$, revealing our proposed PeerBC is a superior behavior cloning approach able to better elicit information from imperfect demonstrations.
Figure A3: Learning curves of DQN on CartPole game with peer rewards ($\tilde{r}_{\text{peer}}$). Here, a linear decay $\xi$ is applied during training procedure (initial $\xi = 0.4$). Compared to static $\xi = 0.4$, the linear decay peer penalty stabilizes the convergence of RL algorithms.

Figure A4: Sensitivity analysis of $\xi$ for PeerBC on Pong with behavior cloning $\blacktriangle$. PeerBC $\blacktriangle$ ($\xi$ varies from 0.2 to 0.5 and 1.0) and expert $\blacktriangle$. Each experiment is repeated under 3 different random seeds.

D.5 Stochastic Policy for Behavioral Cloning

In this section, we analyze the stochasticity of the imperfect expert model and fully-converged PPO agent (assumed to be the clean expert), and show that our PeerBC can handle both cases when the clean expert is stochastic and when it’s rather deterministic.

We plot the entropy of the PPO agent during training on four environments from the BC task in Figure A5, and we give the entropy value of the imperfect expert model and the optimal policy in Table A3. We observe that except for Freeway, the entropy of expert policies is always larger than 1. We calculate the mean value of the highest action probability over 1000 steps for the full-converged
Figure A5: The policy entropy of the PPO agent during training. The imperfect expert model is trained for $0.2 \times 10^7$ timesteps as the red line indicates.

| Timesteps ($\times 10^7$) | Pong | Boxing | Enduro | Freeway |
|---------------------------|------|--------|--------|---------|
| 0.2 (Imperfect Expert)    | 1.201| 1.949  | 1.637  | 0.318   |
| 1.0 (Fully converged PPO) | 1.250| 1.168  | 1.126  | 0.171   |

Table A3: The policy entropy of the PPO agent during training.

| Trained timesteps ($\times 10^7$) | Pong | Boxing | Enduro | Freeway |
|-----------------------------------|------|--------|--------|---------|
| 1.0 (Fully converged PPO)         | 0.492| 0.579  | 0.664  | 0.903   |

Table A4: The mean value of the highest action probability over 1000 steps.

PPO agents in Table A4, which again verifies that the true expert policy we aim to recover might not be fully deterministic. These results demonstrate the flexibility of our proposed approach in dealing with both stochastic and deterministic clean expert policies in practice, although a deterministic clean expert policy is assumed in our theoretical analysis.

Also, from Figure A5 and Table A3, we notice that the entropy of imperfect expert models are higher than the fully converged PPO agents, implying that the expert models might contain an amount of noise. That’s because there might be states on which the expert has not seen enough and the selected actions contain much noise. This is consistent with our claim, that the benefits of PeerBC might come from two aspects, both noise reduction of the imperfect expert and inducing a more deterministic policy.