Traffic States and Jamming Transitions Induced by a Slow Car in Two-lane Traffic Flow

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ABSTRACT

We investigate the traffic states transition and jamming induced by a slow car in two-lane traffic of cars by using a generalized optimal velocity model, which is based on the optimal velocity model. We obtain the relationship between current and density under the circumstance when a slow car was introduced into the original high speed moving highway where cars change lanes frequently. It is found that the entire density space is divided to four distinct area, symbolizing different traffic states. The spatiotemporal diagram of each state is presented. Through the diagram, the evolution and development of traffic flow, jamming as well, is demonstrated.  

INTRODUCTION

Car following model have been studied for nearly 50 years, and the earliest car following model was proposed by Pipes et al. in 1953[1]. Later, Bando et al. raised the optimal velocity model (OVM), which can successfully demonstrate the dynamical formation of traffic congestion in 1995[2]. Nevertheless, there appears some different optimal velocity models whose property varies greatly, as the acceleration function they apply differs. For example, the generalized force model (GFM), and the full velocity difference model (FVDM)[3,4]. With the fact that real-time reporting systems are now becoming widely available, raising expectation has been placed on the generalized optimal velocity model (GOVM). As an extension of the OVM, GOVM employs not only the moving state of the current car but also the anterior. Besides, there are many scholars studying the lane changing rules[5,6].

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In this paper, we study the traffic flow transitions induced by lane changing in two-lane traffic, which is guided by the generalized optimal velocity model. What the generalized optimal velocity considers is that the acceleration is controlled by the optimal velocity model of the current car and the former car. Besides, we consider lane changing rules of the safe distance and velocity difference not only in original lane but also target lane[7]. The most important case is that a slow car is introduced to one lane. Finally, we compare the results between optimal velocity model and generalized optimal velocity model about this issue.

In the following sections, models and the lane changing rules is introduced in section 2. Later, in section 3 we present the simulation’s environment and show some results obtained from simulations. The last section 4 is devoted to the conclusion.

MODEL

We study the traffic flow of two-lane highway after the vehicles changing their lane to another lane with the rules we raise. We assume that the slow car does not change its lane and other vehicles change their lanes when their conditions are in accordance with the lane changing rules. In this paper, we consider symmetric lane changing rules. No matter what highways the vehicles drive on, as long as they satisfy the lane changing rules, they suspend their forward movement and change lanes. One of the most famous accomplishment is the OVM model. The OVM model did not consider the optimal velocity function of the former car. So we adopt a generalized optimal velocity model as follow:

\[
V' = k[V(\Delta x_n(t), \Delta x_{n+1}(t)) - v_n(t)]
\]

\[
V(\Delta x_n(t), \Delta x_{n+1}(t)) = (1 - p)V(\Delta x_n(t)) + pV(\Delta x_{n+1}(t))
\]

where \(v_n(t)\) is the velocity of the vehicle with the car number \(n\) at time \(t\), In the Eq.2, \(V'\) indicates the accelerated speed of the vehicle with the number \(n\) at time \(t\), \(k\) is accelerating factor, which is set as 2.5, and \(p\) is scale factor of the \(V(\Delta x_n(t))\) in the generalized optimal velocity model. When it is zero, it turns into optimal velocity model (OVM), and the car is driven without caring about the optimal velocity of the car ahead of it, otherwise, it is GOVM. In reality drivers cannot consider the headway between itself and the car in front. In this paper we let \(p = 0.5\). In forward movement we adopt optimal velocity function as the function (1)

\[
V(\Delta x_n(t)) = V_1 + V_2\tanh[C_1(\Delta x_n(t) - l_c) - C_2]
\]

Where the parameters are defined as \(5 m\). \(V_1 = 6.75 m/s\), \(V_2 = 7.91 m/s\), \(C_1 = 0.13 m^{-1}\) and \(C_2 = 1.57\).
The optimal velocity model of the slow car is described as follows.

\[ V(\Delta x(t)) = 0.5 \left( V_1 + V_2 \tanh \left[ C_1(\Delta x(t) - l) - C_2 \right] \right) \]  

\[ (4) \]

**SIMULATION**

Firstly, we carry out simulation by varying the number of cars with the safe distance \( x_i \) set as 5m, the maximal velocity of slow car is 7.33m/s and maximal velocity of fast car is 14.66m/s. The traffic flow definitely changes as the number of cars increase, and in order to describe the varieties of traffic flow, we obtain two figures as Figure 2. Figure 2(a) shows the plots of traffic currents against the density from 0 to 0.12 where the density is defined as \( \rho = \frac{1}{\Delta x_{ini}} \). The traffic current is obtained by averaging the current from 400000 steps to 1000000 steps. We all know that there are three states in a lane without lane changing, which are the free traffic, the stop-and-go-wave traffic, and the congested traffic. There are the two transition points from the free traffic, through the stop-and-go-wave traffic to the congested traffic. But in the Figure 1 we could find four states, and they have been marked with area 1, area 2, area 3, and area 4.

We study the spatiotemporal patterns of four distinct traffic states. Figure 2 shows the time-space evolution of cars and the slow car in the area 1 with the density set as 0.01 on the two lanes. Initially, cars are distributed on both lanes. In area 1, the density is low and the headways on lane 2 do not satisfy the incentive criteria, so they do not change their lanes. Thus, all cars on lane 1 change to lane 2 but no car on lane 2 changes to lane 1. So we call this state is free traffic.

We study the spatiotemporal patterns of vehicles at density 0.03. Figure 3 shows the time-space evolution of cars in lane 1 and lane 2. In addition, Figure 3(a) shows the cars changing lanes from lane 1 to lane 2, which followed the slow car before the changing. This situation is very common in reality, where a slow car flow behind a slow car and many drivers energetically change their lanes. In this state, there are many cars changing their lanes, so we call this state as lane changing traffic state.

![Figure 1](image1.png)

**Figure 1.** (a) The plots of traffic current against the initial density ranging from 0 to 0.12.  
(b) The plots of average velocities of cars against the density.
Figure 2. The time–space evolution of cars in lane 1 for the area 1(a) and lane 2 (b) when $\rho$ is 0.01.

Figure 3. The time–space evolution of cars in lane 1(a) and lane 2(b) for the area 2 with $\rho$ =0.03. The red open circles represent the cases changing lane to lane 1 from lane 2, and the red full points represent the cases changing lane to lane 2 from lane 1.

Figure 4 shows the time–space evolution of cars in lane 1 and lane 2. It is shown that there is no longer slow car flow behind the slow car, and the cars are uniform distribution, this also indicate that the slow car has litter effect on lane changing. A new phenomenon is that time–space evolution is a fold line. It indicates that the car is not in uniform motion. It is one obvious feature of the stop-and-go-wave traffic.

Figure 5 shows the time–space evolution of cars in lane1 form the area 4 with the density set as 0.11. From Figure 5, we can see that all cars’ time–space evolution present straight lines. But these straight lines have low gradient. It indicates that they have a low speed. This state corresponds to the congestion in reality.
Figure 4. The time–space evolution of cars in lane 1 (a) and lane 2 (b) for the area 2 with $\rho = 0.07$.

Figure 5. The time–space evolution of cars in lane 1 (a) and lane 2 (b) for the area 4 with $\rho = 0.11$.

CONCLUSIONS

In this paper, we study the traffic states induced by a slow car in a two-lane traffic of cars. The cars comply with the forward movement model called generalized optimal velocity model. We derive a new fundamental (current-density) diagram because slow car is introduced to the original high speed moving highway in which cars change lane frequently. We find that there are four traffic states and the spatiotemporal diagrams of each state are presented. It is clarified that how the traffic flow evolves and how the traffic jams develop in the spatiotemporal diagram.

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