Fusion of multistatic synthetic aperture radar data to obtain a superresolution image

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Abstract. When using a single emitter and a single receiver, the Synthetic Aperture Radar (SAR) data gives information in the Fourier domain of the scene over a line segment whose width is related to the bandwidth of the emitted signal. The mathematical problem of image reconstruction in SAR then becomes a Fourier Synthesis (FS) inverse problem. When there are more than one emitter or receiver looking at the same scene, the problem becomes fusion and inversion. In this paper we report a Bayesian joint data fusion and inversion method to obtain a super resolution image. The proposed method shows a good performance on real data obtained at ONERA in France.

1. Introduction to SAR imaging

Synthetic Aperture Radar (SAR) has been widely used for remote sensing, geometry detection, and target recognition. Considering the configuration of SAR, the simplest case is “monostatic”, when emitter and receiver are in the same position. In this case, SAR gives information in the Fourier domain of the scene on a set of straight lines. The mathematical problem of image reconstruction becomes an inverse problem of Fourier Synthesis (FS) which is an ill-posed problem. When the emitter and receiver are located differently, as so called “bistatic case”, or if there are more than one receiver or emitter, as so called “multistatic case”, the problem becomes data fusion and inversion. In this paper we employ the Bayesian estimation approach to solve these inverse problems, particularly we are focusing on multi-static case.

2. Forward modeling

In 2D case, after some mathematical simplification, we can obtain a Fourier Transform relation between the spectrum of the observed signals and the 2D spatial F-T of the scene which can be written as:

\[ G(k_x, k_y) = M(k_x, k_y)F(k_x, k_y) \]

where \( F(k_x, k_y) \) is the 2D spatial F-T of the scene \( f(x, y) \), \( M(k_x, k_y) \) is related to the characteristic of radar system, with \( M(k_x, k_y) = 1 \) on the set of points where data is available and \( M(k_x, k_y) = 0 \) on those points where data is lacking, and \( G(k_x, k_y) \) is obtained from the observed signals.
3. Inversion
As we discussed above, the forward problem is going from the target \( f(x, y) \) to the data \( G(k_x, k_y) \) in the Fourier domain. Oppositely, the inverse problem is to obtain \( f(x, y) \) from the data \( G(k_x, k_y) \).

3.1. Bayesian estimation approach
The relation between the observed data \( G(k_x, k_y) \) and the unknown scene \( f(x, y) \) is a linear one, when discretized, we obtain

\[
M : \quad g = Hf + \epsilon
\]

where \( g \) is a vector containing all the values of \( G(k_x, k_y) \) on those points where \( M(k_x, k_y) = 1 \), \( f \) is a vector of the pixel value of the discrete scene \( f(x, y) \) and \( H \) is the DFT matrix and \( \epsilon \) represents the errors.

Using this forward model, we can define \( p(g|f; M) \) and using a prior \( p(f|M) \), we can get

\[
p(g|f; M) = \frac{p(g|f; M)p(f|M)}{p(g|M)}.
\]

In this paper we have limited ourselves to the Maximum A Posterior (MAP) estimation which is:

\[
\hat{f} = \arg \max_f \{ p(f|g) \} = \arg \max_f \{ p(g|f)p(f) \} = \arg \min_f \{ - \ln p(g|f) - \ln p(f) \}
\]

where we choose

\[
p(g|f) \propto \exp \left\{ \frac{1}{2\sigma^2} \| g - Hf \|^2 \right\}
\]

\[
p(f) \propto \exp \left\{ \lambda \sum_j |f_j - f_{j-1}|^\beta \right\} \propto \exp \lambda \sum_j \| Df \|_j^\beta
\]

3.2. Multistatic data fusion methods
For multistatic case, the main objective is to develop different methods to obtain superresolution [1] and [2]. Here, we introduce a joint data fusion and inversion method: we use two sets of data \( G_1(u, v) \) and \( G_2(u, v) \) with their respective relations to \( f(u, v) \):

\[
\begin{align*}
g_1 &= H_1f + \epsilon_1 \\
g_2 &= H_2f + \epsilon_2
\end{align*}
\]

to obtain directly the solution by defining again the MAP estimator:

\[
\hat{f} = \arg \max_f \{ p(f|g_1, g_2) \} = \arg \min_f \{ J(f) \}
\]

where assuming \( \epsilon_1 \) and \( \epsilon_2 \) to be Gaussian and independent and the same priors for \( f \), we have:

\[
J(f) = \frac{1}{\sigma_{\epsilon_1}^2} \| g_1 - H_1f \|^2 + \frac{1}{\sigma_{\epsilon_2}^2} \| g_2 - H_2f \|^2 + \lambda \| g_1 - H_1f \|^\beta
\]

4. Experiments on real data
The proposed method has been applied on a few set of real experimental data measured by ONERA. The details of this experimentation are reported in [3]. There are three target compositions (1, 2 and 4 spheres) and for each set of targets, there are two sets of full polar data corresponding to two bandwidth BF1=(1.16-1.98)GHz and BF2=(1.47-1.68)GHz. In the first experiment, we use only one of the bands to do reconstructions and then, we use both bands to do joint data fusion and inversion. Here, we show a few results we obtained using the proposed method compared to a classical Fourier inversion method.
5. Conclusions and perspectives

In this paper, we proposed a Bayesian estimation MAP method to solve the Fourier Synthesis problem in SAR imaging data. The proposed method can be used in all mono- or bi- (multi-) static case. Particularly, in multistatic case, we employed the proposed method to do data fusion and inversion jointly. The experimental results on real data have shown a good performance of our proposed method. The next step, we will focus on two aspects: (a) to use other prior model as Gauss-Markov-Potts prior models [4], [5], [6] and [7]; (b) for real data, we still need to account for polarization information of the measurement.

6. References

[1] Mohammad-Djafari A, Daout F and Fargette P 2009 ICIP 2009 (Cario, Egypte)
[2] Mohammad-Djafari A, Daout F and Fargette P 2009 GRETSI 2009 (Dijon, France)
[3] Fargette P Novembre 2008 Compte-rendu d’expérimentation dte Rapport dga
[4] Humblot F and Mohammad-Djafari A 2006 EURASIP Journal on Applied Signal Processing vol. 2006 1–16
[5] Mohammad-Djafari A Mar 2008 the Computer Journal 52 126–141
[6] Bali N and Mohammad-Djafari A 2008 IEEE trans. On Image Processing 17 217–225
[7] Féron O and Mohammad-Djafari A 2005 Journal of Electronic Imaging 14 023014