All-Versus-Nothing Violation of Local Realism by Swapping Entanglement

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The usual entanglement swapping protocol can entangle two particles that never interact. Here we demonstrate an all-versus-nothing violation of local realism for a partial entanglement swapping process. A Clauser-Horne-Shimony-Holt-type inequality for two particles entangled by entanglement swapping is proposed for a practical experiment and can be violated quantum mechanically up to 4 beyond Cirel’son’s bound $2\sqrt{2}$. Our result uncovers the intriguing nonlocal character of entanglement swapping and may lead to a conclusive (loophole-free) test of local realism versus quantum mechanics. The experimental test of the nonlocality under current technology is also discussed.

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By assuming local realism (LR), Einstein, Podolsky and Rosen (EPR) argued that it seems possible to assign values with certainty to canonically conjugate variables (e.g., position and momentum) if certain perfect quantum correlations are used. EPR’s observation is definitely in conflict with quantum mechanics (QM), but remained for a long time untestable. In 1964, Bell discovered his inequalities, which since then enable quantitative tests of QM versus LR for certain statistical correlations predicted by QM. Strikingly, various versions of Bell’s theorem without inequalities have also been demonstrated. Particularly, the contradiction between QM and local realistic theories arises even for definite predictions, known as all-versus-nothing (AVN) violation of LR. Entanglement is always an unavoidable ingredient in the violation of LR. In various ways of creating entanglement, entanglement swapping is perhaps most intriguing in the sense that it can entangle two particles that never interact. In this paper, we demonstrate an AVN violation of LR for a partial entanglement swapping process, where there is no classical communication between different spatial locations and only partial Bell-state measurement is required. A Clauser-Horne-Shimony-Holt (CHSH) type inequality for two particles entangled by entanglement swapping is proposed for a practical experiment and can be violated, up to 4, by QM beyond the standard Cirel’son bound (One can also surpass the bound by post-selected GHZ-entangled particles). The theorem proved here can lead to an ultimately loophole-free test of LR against QM.

Without loss of generality and for definiteness, in the following we consider polarized photons as example. The four Bell states of photons read

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle |H\rangle)$$

and

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|H\rangle |V\rangle + |V\rangle |H\rangle),$$

where $|H\rangle$ ($|V\rangle$) stands for photons with horizontal (vertical) polarization. They can be created via the spontaneous parametric down-conversion in a nonlinear optical crystal and satisfy (see, e.g., [10])

$$z \otimes z |\phi\rangle = |\phi\rangle, \quad z \otimes z |\psi\rangle = -|\psi\rangle,$$

$$x \otimes x |\phi\rangle = \pm |\phi\rangle, \quad x \otimes x |\psi\rangle = \pm |\psi\rangle.$$

(1)

Here $\sigma_x = |H\rangle \langle V| + |V\rangle \langle H|$ and $\sigma_z = |H\rangle \langle H| - |V\rangle \langle V|$ are the Pauli-type operators for polarizations of photons. Now one can also introduce another set of Bell states $|\chi\rangle = \frac{1}{\sqrt{2}}(|H\rangle \langle H| + |V\rangle \langle V|)$ and $|\omega\rangle = \frac{1}{\sqrt{2}}(|V\rangle \langle H| + |H\rangle \langle V|)$, with $|H\rangle = \sqrt{2}(|V\rangle \langle H| + |V\rangle \langle V|)$ and $|V\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$. Then one has

$$z \otimes x |\chi\rangle = |\chi\rangle, \quad z \otimes x |\omega\rangle = -|\omega\rangle,$$

$$x \otimes z |\chi\rangle = \pm |\chi\rangle, \quad x \otimes z |\omega\rangle = \pm |\omega\rangle.$$

(2)

Now let us consider the standard entanglement swapping protocol for EPR-entangled photon pairs: EPR source 1 (II) independently emits maximally entangled photons 1 and 2 in state $|\psi^-\rangle_{12}$ (3 and 4 in state $|\psi^-\rangle_{34}$): if photons 2 and 3 are subject to a Bell-state measurement, photons 1 and 4 will then be maximally entangled conditioned on the outcomes of the Bell-state measurement. This can easily be seen by writing down the initial state $|\psi^-\rangle_{12} |\psi^-\rangle_{34}$ in terms of the Bell states in either $\{|\phi\rangle, |\psi\rangle\}$ or $\{|\chi\rangle, |\omega\rangle\}$:

$$|\psi^-\rangle_{12} |\psi^-\rangle_{34} = \frac{1}{2}(|\psi^-\rangle_{14} |\psi^-\rangle_{23} - |\psi^+\rangle_{14} |\psi^+\rangle_{23})$$

$$+ |\phi^-\rangle_{14} |\phi^+\rangle_{23} - |\phi^+\rangle_{14} |\phi^-\rangle_{23})$$

$$= \frac{1}{2}( |\omega^-\rangle_{14} |\omega^+\rangle_{23} - |\omega^+\rangle_{14} |\omega^-\rangle_{23})$$

$$- |\chi^+\rangle_{14} |\chi^-\rangle_{23} + |\chi^-\rangle_{14} |\chi^+\rangle_{23}).$$

(3)

Then using the properties of the Bell states in [10] and one has the following eigenequations

$$z_1 \cdot z_2 z_3 \cdot z_4 |\psi^-\rangle_{12} |\psi^-\rangle_{34} = |\psi^-\rangle_{12} |\psi^-\rangle_{34},$$

(4)

$$x_1 \cdot x_2 x_3 \cdot x_4 |\psi^-\rangle_{12} |\psi^-\rangle_{34} = |\psi^-\rangle_{12} |\psi^-\rangle_{34},$$

(5)

$$z_1 \cdot z_2 z_3 \cdot z_4 |\psi^-\rangle_{12} |\psi^-\rangle_{34} = |\psi^-\rangle_{12} |\psi^-\rangle_{34},$$

(6)

$$x_1 \cdot x_2 z_3 \cdot z_4 |\psi^-\rangle_{12} |\psi^-\rangle_{34} = |\psi^-\rangle_{12} |\psi^-\rangle_{34}.$$
Now suppose that the measurements on photon 1, photons 2 and 3, and photon 4 are performed on three regions that are mutually spacelike separated. Then the perfect correlations in Eqs. (4)-(7) allow one to infer that operators separated by (,) can be identified as EPR’s local “elements of reality” (LERs). For instance, by measuring $z_1$ and $z_4$ one can predict with certainty the value of $z_2z_3$ without in any way disturbing photons 2 and 3, and as such $z_2z_3$ can be regarded as an LER according to the very notion of the EPR local realism. Similar reasoning shows that one may establish the following LERs: $(z_1, x_1)$ for photon 1, $(z_2z_3, x_2x_3, z_2z_3, x_2z_3)$ for photons 2 and 3, and $(x_4, z_4)$ for photon 4. A local realistic interpretation of the quantum results (4)-(7) is to assume that the individual value for photon 4. A local realistic interpretation of the quantum results (4)-(7) is to assume that the individual value for photon 4.

Unfortunately, the relations in Eq. (8) do not show any conflict between QM and LR. The missed link for demonstrating the conflict can be uncovered by noting that one also has a quantum identity

\[ z_2z_3 \cdot x_2x_3 = -z_2x_3 \cdot z_2z_3, \tag{9} \]

regardless of the states of photons 2 and 3. To reproduce the above identity local realistic theories must also predict that

\[ v(z_2z_3) v(x_2x_3) = \epsilon = \pm 1, \quad v(z_2x_3) v(x_2z_3) = -\epsilon. \tag{10} \]

Now it is ready to see that Eqs. (8) and (10) are mutually inconsistent: Multiplying the left-hand sides of the relations in Eqs. (8) and (10) gives +1 (Note that $v(V) = \pm 1$, whereas the product of the right-hand sides yields −1. Thus we do have a GHZ-type contradiction for the perfect correlations in the entanglement swapping process. Namely, LR predicts that the above LERs must satisfy five of the six relations in Eqs. (8) and (10), so that the corresponding five quantum predictions can be reproduced. Then on the level of gedanken experiment, every measurement of the LERs related with the sixth relations must give a perfect correlation, which is, however, exactly opposite to the sixth quantum prediction. In other words, certain outcomes predicted to definitely occur by LR are never allowed to occur by QM and vice versa. This completes the demonstration of an AVN violation of LR for the perfect correlations in an ideal entanglement swapping.

A careful reader might have found that the same operators may appear in different equations in (11)-(17) and (18). For instance, $z_2z_3$ and $x_2x_3$ not only appear separately in Eqs. (11) and (15), but also appear jointly in Eq. (10) [see also (12) and (16)]. In order to validate our AVN nonlocality argument, it is, however, necessary to assign always a single value to the same operator, though it can appear in different equations. Therefore, one either has to assume noncontextuality (e.g., measurement of $z_2z_3$ does not disturb the value of $x_2x_3$ and vice versa) or one has to be able to measure $z_2z_3, x_2x_3$ and $z_2z_3 \cdot x_2x_3$ with the same apparatuses so that they appear only in the same experimental context.

Therefore, to establish the observables in two groups $(z_2z_3, x_2x_3)$ and $(z_2x_3, x_2z_3)$ as LERs, the two observables of each group must be measured by one and the same apparatus (Note that this is possible because the two observables of each group commute with each other). Explicitly, our argument requires two complementary apparatuses $A$ and $B$: Apparatus $A$ (B) measures $z_2z_3$ and $x_2x_3$ ($z_2x_3$ and $x_2z_3$) simultaneously, and by multiplication gets the value of $z_2z_3 \cdot x_2x_3$ ($z_2x_3 \cdot x_2z_3$). This eliminates the necessity of a noncontextuality assumption, similarly to an AVN nonlocality argument for doubly-entangled two particles (11).

However, there is still another problem that might invalidate our argument: Though Eq. (13) is a quantum identity, there is no a priori reason why each individual measurement result of $z_2z_3, x_2x_3$ must be opposite to that of $z_2x_3 \cdot z_2z_3$ (Of course, the mean value of $z_2z_3 \cdot x_2x_3$ must be exactly opposite to that of $z_2x_3 \cdot z_2z_3$). This is because the collapse to any eigenstate of $z_2z_3 \cdot x_2x_3$ (or $z_2x_3 \cdot z_2z_3$) is completely random according to QM. Thus, for the above argument to hold, one must ensure that the individual measurement results of $z_2z_3, x_2x_3$ and $z_2x_3 \cdot z_2z_3$ are exactly opposite.

Surprisingly enough, one can indeed achieve this by introducing

\[ |\epsilon = \pm 1\rangle_{1234} = \frac{1}{\sqrt{2}} \left( |\psi^+\rangle_{14} |\psi^+\rangle_{23} + |\psi^\pm\rangle_{14} |\phi^\pm\rangle_{23} \right) \]

\[ = \frac{1}{\sqrt{2}} \left( |\chi^\mp\rangle_{14} |\chi^\mp\rangle_{23} + |\omega^\mp\rangle_{14} |\omega^\mp\rangle_{23} \right), \tag{11} \]

with $|\psi^+\rangle_{12} |\psi^+\rangle_{34} = \frac{1}{\sqrt{2}} (|\epsilon = +1\rangle_{1234} - |\epsilon = -1\rangle_{1234})$. Importantly, in terms of $|\epsilon = \pm 1\rangle_{1234}$, Eqs. (11)-(17) are still valid, but with $|\psi^\mp\rangle_{12} |\psi^\mp\rangle_{34}$ being replaced by either $|\epsilon = +1\rangle_{1234}$ or $|\epsilon = -1\rangle_{1234}$. Moreover, one also has

\[ z_2z_3 \cdot x_2x_3 |\epsilon = \pm 1\rangle_{1234} = \pm |\epsilon = \pm 1\rangle_{1234} \cdot \]

\[ z_2x_3 \cdot x_2z_3 |\epsilon = \pm 1\rangle_{1234} = \mp |\epsilon = \pm 1\rangle_{1234}. \tag{12} \]

Therefore, if one can select either $|\epsilon = +1\rangle_{1234}$ or $|\epsilon = -1\rangle_{1234}$ out of the initial state $|\psi^\pm\rangle_{12} |\psi^\pm\rangle_{34}$, then the above AVN nonlocality argument remains to be applicable to the selected $|\epsilon = \pm 1\rangle_{1234}$. This selection is
equivalent to the preparation of the state $|\epsilon = +1\rangle_{1234}$ or $|\epsilon = -1\rangle_{1234}$.

As a mathematical proof and at the level of gedanken experiment, the above AVN proof works as it is. However, a real experiment always faces imperfect correlations and nonideal detections. To overcome this difficulty, a Bell-type inequality for statistical correlations in the present context is required.

To this end, one should note the following facts:

\[ v(z_2 z_3) \equiv m = \pm 1 \Leftrightarrow v(x_2 x_3) = cm, \]
\[ v(z_2 x_3) \equiv n = \pm 1 \Leftrightarrow v(x_2 z_3) = -cn, \]  \hspace{1cm} (14)

which are the consequences of Eq. (10). Then from Eqs. 4, 7, where $|\psi^-\rangle_{12}|\psi^-\rangle_{34}$ is now replaced by either $|\epsilon = +1\rangle_{1234}$ or $|\epsilon = -1\rangle_{1234}$, one can prove the following CHSH-type inequality, which must be fulfilled by any local realistic theory, for the two entanglement-swapped photons 1 and 4:

\[ |\langle m z_4 + n z_1 x_4 - cn z_1 x_4 + cm z_1 x_4\rangle_{LR}| \leq 2. \]  \hspace{1cm} (15)

Actually,

\[ LR = |\langle m z_4 + n z_1 x_4 - cm z_1 x_4 + cm z_1 x_4\rangle_{LR}| \]
\[ = |\langle z_1 z_4 + mn z_1 x_4 - cm x_1 z_4 + cm x_1 z_4\rangle_{LR}| \]
\[ = |\langle z_1 (z_4 + mn x_4) - c x_1 (mn z_4 - x_1)\rangle_{LR}| \]
\[ = \left\{ \begin{array}{cl} |\langle z_1 (z_4 + x_4) - c x_1 (z_4 - x_1)\rangle_{LR}|, & (mn = 1) \\ |\langle z_1 (z_4 - x_4) + c x_1 (z_4 + x_4)\rangle_{LR}|, & (mn = -1) \end{array} \right. \]  \hspace{1cm} (16)

which are the Bell correlation functions in the present context and satisfy $LR \leq 2$.

It should be noted that Eq. (15) is, in fact, the local realistic prediction for $M = z_1 \cdot z_2 z_3 \cdot z_4 + x_1 \cdot x_2 x_3 \cdot x_4 + z_1 \cdot z_2 x_3 \cdot x_4 + x_1 \cdot x_2 z_3 \cdot z_4$, namely,

\[ |\langle M\rangle_{LR}| \leq 2 \]  \hspace{1cm} (17)

under the restriction of Eq. (11) or (16). However, quantum prediction of $M$ with respect to $|\epsilon = +1\rangle_{1234}$ or $|\epsilon = -1\rangle_{1234}$ can be as large as 4, i.e.,

\[ |\langle M\rangle_{QM}| \leq 4, \]  \hspace{1cm} (18)

which is beyond the Cirel’son bound $2\sqrt{2}$. The upper bound can be reached by either $|\epsilon = +1\rangle_{1234}$ or $|\epsilon = -1\rangle_{1234}$. This much larger violation of the CHSH inequality by the quantum prediction shows more resistance of $\epsilon$ to noise and the maximal visibility to fulfill $\epsilon$ can be reduced to 50%. In this kind of the Bell experiment testing $\lambda_1$ the detection efficiency for photons 2 and 3 does not contribute to the overall detection efficiency because the proposed experiment can be regarded as an “event-ready” Bell test $\lambda_2$.

The above feature of our CHSH inequality $\lambda_1$ is very helpful to close the detection-efficiency loophole in the Bell-type experiments (The detection-efficiency loophole has been closed in $\lambda_7$ by using entangled ions, which can be detected with nearly perfect efficiency). Hence, photons 2 and 3 does not contribute to the overall detection efficiency because the proposed experiment can be regarded as an “event-ready” Bell test $\lambda_2$.

![FIG. 1: A linear optics apparatus selecting $|\epsilon = +1\rangle_{1234}$ out of $|\psi^-\rangle_{12}|\psi^-\rangle_{34}$. The four quarter wave plates (at $45^\circ$) can affect the transformations $R = 1/2(H + iV) \rightarrow H$ and $L = 1/2(H - iV) \rightarrow V$; the two half wave plates function as $R \leftrightarrow L$; the polarizing beam splitter (PBS) reflects $V$ photons and transmits $H$ photons. Only for $|\epsilon = +1\rangle_{1234}$ there will be a coincidence detection between the two detectors.](image)
tors with arbitrarily low efficiency. It is worthwhile to point out that our AVN argument is valid regardless of whether the two entanglement sources are independent or not. The two apparatuses A and B used to avoid the contextuality assumption are crucial in this aspect.

Finally, let us discuss briefly the practical aspects of implementing the required measurements. Measuring jointly the two observables in \((z_2z_3, x_2x_3)\) \(((z_2x_3, x_2z_3)\) is equivalent to making a complete discrimination between four Bell states \(\{|\psi^\pm\rangle_{23}, |\phi^\pm\rangle_{23}\} \) \((\{|\chi^\pm\rangle_{23}, |\omega^\pm\rangle_{23}\})\). In the case of polarized photons, such a complete Bell-state measurement is impossible with only linear optics and necessitates nonlinear optical interactions at the single-photon level \[23\]. Fortunately, a nondestructive controlled-NOT (CNOT) gate for two independent photons using only linear optical elements is sufficient for the present purpose and has been realized in a very recent experiment \[24\]. This experimentally demonstrated CNOT gate is probabilistic, but “event-ready” in the sense that one can detect when the gate has succeeded by performing some appropriate measurement on ancilla photons. The information gathered in the measurement can then be feed-forwarded for conditional future operations on the photonic qubits. As pointed out by Cabello \[11\], distinguishing between the two results \(z_2z_3 \cdot x_2x_3 = \pm 1\) (or \(z_2x_3 \cdot x_2z_3 = \pm 1\)) corresponds to partial Bell-state measurement, which can be accomplished by only linear optical elements \[23, 25\]. In Fig. 1, we propose a linear optics apparatus which can select \(|\chi^+\rangle_{1234}\) out of \(|\psi^-\rangle_{12}|\psi^-\rangle_{34}\): There will be one photon in each output mode only for \(|\chi^+\rangle_{1234}\), which can then be subject to future measurements. For \(|\chi^-\rangle_{1234}\) photons 2 and 3 will go together to one of the two output modes. In this way, coincidence detection between the two output modes, after all necessary measurements being performed on photons 2 and 3, may distinguish \(|\chi^+\rangle_{1234}\) (corresponding to a coincidence detection) from \(|\chi^-\rangle_{1234}\) (corresponding to no coincidence detection).

In summary, we have demonstrated an AVN violation of LR for a partial entanglement swapping process. A CHSH inequality for two particles entangled by entanglement swapping is proposed for a practical experiment and can be violated, up to 4, by QM. This larger violation is helpful to close the detection-efficiency loophole in the Bell-type experiments for photons. For two purely independent entanglement sources, our argument may lead to a conflict between LR and QM for detectors with any nonzero efficiency. Compared with the usual nonlocality argument, our result uncovers the intriguing nonlocality of entanglement swapping and provides a theoretical base for an ultimately loophole-free refutation of local realism. The experimental test of the nonlocality under current technology is also discussed by using linear optical elements.

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**Note added**—After the completion of our work, we became aware of the beautiful papers by Greenberger, Horne, and Zeilinger \[24\] who obtained a similar result, without touching the contextuality issue. Hail the GHZ factorization theorem!

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