APPLICATIONS OF EFFECTIVE LAGRANGIANS

JOSÉ WUDKA
Physics Department, University of California, Riverside
Riverside, CA 92521-0413, U.S.A.
E-mail: wudka@ucrph0.ucr.edu

ABSTRACT

The applications of effective lagrangians to the determination of the effects of physics beyond the Standard Model are briefly described. Emphasis is given to those effective operators which generate the largest deviations from the Standard Model; some applications are described.

1. Introduction

The Standard Model is a theory in perfect agreement with all experimental data. Its predictive power is very large and is accepted as a correct description of nature at all scales which have been probed (up to energies ~ 100GeV). Nonetheless the model has various theoretical problems (such as the possible triviality of the scalar sector \( \mathcal{L} \)) and it is the consensus of the community that it represents the low energy limit of a more fundamental theory.

There have been many proposed extensions of the Standard Model, mainly based on specific kinds of new physics imagined to be apparent at energies significantly above the ones currently probed. Typical examples of these approaches are the supersymmetric \( \mathcal{L} \) and technicolor \( \mathcal{L} \) pictures of new physics. Should a future experiments discover a techni-rho, or a slepton or wino, the guessing game would be over and all efforts will be concentrated in elucidating the specific technicolor or supersymmetric model realized in nature.

There is, however, the possibility that we will not be able to directly observe the effects of the new dynamics (via the creation of new particles); then only virtual effects are available as probes into the physics beyond the Standard Model. For this situations the most concise approach available is based on the use of effective lagrangians \( \mathcal{L} \).

Suppose for the moment that the lagrangian \( \mathcal{L}_{\text{new}} \), which describes the physics beyond the Standard Model, is known. Suppose that non-standard effects become directly observable at center of mass energies of the order of a scale \( \Lambda \). If we wish to obtain a description of the non-standard effects generated by \( \mathcal{L}_{\text{new}} \) at low energies, one should integrate out all heavy fields (of mass ~ \( \Lambda \) or higher) and determine the corresponding effective action (which, by construction, contains only Standard Model fields). This effective action will also contain the scale \( \Lambda \) as a parameter.

\*Talk given at Beyond the Standard Model IV, Lake Tahoe, CA, Dec. 13-18, 1994
Since we are interested in physics at energies significantly below \( \Lambda \), a large-\( \Lambda \) expansion is appropriate. The result of performing this expansion on the effective action obtained above is a tower of \textit{gauge-invariant} local operators (which I denote by \( \mathcal{O}_i \)) containing the Standard Model fields multiplied by some function of \( \Lambda \) and the couplings present in \( \mathcal{L}_{\text{new}} \), namely

\[
S_{\text{eff}} = \int d^4 x \mathcal{L}_{\text{eff}}; \quad \mathcal{L}_{\text{eff}} = \sum_i f_i(\Lambda; \text{couplings}) \mathcal{O}_i. \tag{1}
\]

The dependence on \( \Lambda \) of each term is determined by the dimension of the operator in question (up to factors of \( \ln \Lambda \)):

\[
f_i \propto \Lambda^{4 - \text{dim} \mathcal{O}_i}. \tag{2}
\]

All operators will be generated (in general) irrespective of the specific details of \( \mathcal{L}_{\text{new}} \). In contrast the \( \alpha_i \) are model specific and summarize all the information that can be gathered from \( \mathcal{L}_{\text{new}} \) at low energies. One can then use these parameters to parametrize all new physics effects without model prejudices.

\section*{2. Symmetries of the effective lagrangian}

It was stated above that the local symmetries of the Standard Model are preserved by the effective lagrangian. An effective lagrangian which does not satisfy this condition presents severe self-consistency problems. Suppose for example that a gauge-variant three-gauge-boson vertex is introduced into an otherwise gauge-invariant model. We can then consider the corrections to the vector boson masses generated by this term

\[
\delta m^2 \sim m^2 \left( \frac{g_H \Lambda^3}{4\pi m^3} \right)^2 \tag{3}
\]

where \( m \) denotes the vector-boson-mass, \( \Lambda \) is the scale of the physics generating the gauge-variant term and \( g_H \) the corresponding coupling constant. For the Standard Model the corrections to \( m \) must be small (due to the agreement with the data) hence \( \Lambda \ll m(4\pi/g_H)^{1/3} \). For \( m_W \) we know that \( \delta m_W^2 / m_W^2 \lesssim 0.0064 \) which implies \( \Lambda \lesssim m_W / 2 \). If no restrictions are placed on \( \Lambda \) radiative corrections shift \( m \rightarrow \mathcal{O} (\Lambda) \).

One might wonder whether it is only the global symmetries associated with the gauge invariance that should be imposed on \( \mathcal{L}_{\text{eff}} \). But in this case there is no cogent reason for lepton universality; moreover, there will be no connection between the cubic and quartic vector-boson couplings and the above problems with the masses reappear.\(^a\)

\(^a\) Of course one can decide to forego gauge invariance completely and fine tune everything. This is possible but quite useless since such a theory has absolutely no predictive power.
These arguments do not apply to global symmetries. Consider for example a term in $\mathcal{L}_{\text{eff}}$ of the form

$$\frac{1}{\Lambda} \alpha_{\epsilon\mu B} \bar{e}_R \sigma_{\mu\nu} \mu_R B^{\mu\nu}$$

(4)

where $\epsilon_R, \mu_R$ denote the corresponding right-handed fermion fields and $B_{\mu\nu}$ is the field-strength for the $U(1)_Y$ gauge field. This interaction generates a non-vanishing branching ratio for the process $\mu \rightarrow e\gamma$ proportional to $|\alpha_{\epsilon\mu B}/\Lambda|^2$; the fact that this transition highly forbidden merely indicates that $\Lambda$ is very large.

3. Low energy particle content

The recipe for constructing the effective lagrangian is to select the light excitations to be studied and the symmetries that they are to respect and then to construct the most general set of local operators involving the corresponding fields and respecting the said symmetries.

For the Standard Model we have the choice of including a scalar sector as low energy excitations or not. If there is a light Higgs and I assume that $\Lambda \neq v$ (where $v$ is the Standard Model vacuum expectation value) then the decoupling theorem insures that all observables appear as a power series in $1/\Lambda$, in particular, all effects from the new physics disappear as $\Lambda \rightarrow \infty$.

If there be no light Higgs then the simplest way of describing the scalar sector (we still need the Goldstone bosons in order generate masses for the $W$ and $Z$) is through a non-linear sigma model. In this case the decoupling theorem is not applicable. Due to time constraints I will not discuss this situation in this talk.

Consider therefore the situation where there is a light Higgs present. Then the effective lagrangian takes the form

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \tilde{\alpha}_i \frac{1}{\Lambda} \mathcal{O}_i^{(\text{dim } 5)} + \sum_i \alpha_i \frac{1}{\Lambda} \mathcal{O}_i^{(\text{dim } 6)} + \cdots.$$  

(5)

It is easy to see that there are no dimension 5 operators satisfying the Standard Model symmetries; there are 81 dimension-six operators (for one family of fermions).

For the case of a light Higgs I’ll require that it’s mass not be shifted to $O(\Lambda)$ by radiative corrections; it follows that the underlying theory should be assumed to be weakly coupled. We can avoid this constraint by either fine-tuning or by modifying the low-energy particle content as in supersymmetry. I will not consider either of these situations, the first is unnatural and the second requires a low-energy sector substantially different from the one in the Standard Model. As indicated in the introduction I assume no direct observation of non-Standard Model physics is available;

\footnote{Note that the condition $\Lambda \neq v$ excludes, for example, a heavy fourth generation.}
dimension-six operators), which are actually generated by a gauge theory. Loop generated operators appear with a characteristic suppression factor $\sim 1/(4\pi)^2$ which significantly decreases the magnitude of their effects. Assuming that the underlying theory is a gauge theory, the only tree level generated operators take the form

$$ (\phi D\phi)^2, \quad \phi^6, \quad (\phi\psi)D(\phi\psi), \quad \phi^3\psi^2 \quad \psi^4, \quad (6) $$

where $\phi$ denotes the Standard Model scalar doublet and $\psi$ a light fermion; $D$ denotes the covariant derivative. The notation used above is, of course symbolical. To see how the list of operators was obtained consider a broken gauge and let $V$ be the corresponding vacuum expectation value. At this level the Standard Model subgroup will be left unbroken. The unbroken subgroup will be $SU(3) \times SU(2) \times U(1)$ and the corresponding (light) generators will be denoted generically by $T^i$; broken (or heavy) generators will be labelled $T^h$: it follows that $T^h V = 0$ while $T^h V \neq 0$. I will denote by $X$ the heavy vector bosons and by $W$ the light ones; light and heavy scalars are denoted by $\phi$ and $\Phi$ respectively (fermions can be treated in the same way). The structure constants of the full theory will be denoted by $f$.

- $XWW$ vertices will be proportional to $f_{\ell hh}$. But, since the unbroken generators form a group, the commutator of two of them should also give a light generator; hence $f_{\ell hh} = 0$ and that this type of vertices is absent.
- $XW\Phi$ vertices will be proportional to $VT^h T^f \Phi$; the vector $VT^h$ lies along a Goldstone boson direction. Since the light group is unbroken, the action on a Goldstone boson direction by $T^f$ must also give a Goldstone boson direction (since the Goldstone bosons transform as a representation of the unbroken group). But then $VT^h T^f$ will be orthogonal to all directions corresponding to heavy scalars: this type of vertices is absent also.

Once all the vertices of the heavy theory have been studied along these lines it is straightforward to determine, given all the possible tree-level graphs that can be drawn (and which correspond to dimension-six operators), which are actually generated by a gauge theory.

For the Standard Model the tree-level generated dimension six operators are

$$ O_\phi = \frac{1}{3} (\phi^\dagger \phi)^3 \quad O_{\theta \phi} = \frac{1}{2} \left( \partial (\phi^\dagger \phi) \right)^2 \quad O^{(1)}_\phi = (\phi^\dagger \phi) |D_\mu \phi|^2 \quad O^{(3)}_\phi = |\phi^\dagger D \phi|^2 $$

$$ O^{(1)}_{\phi\ell} = i \left( \phi^\dagger D_\mu \phi \right) \bar{\ell} \gamma^\mu \ell \quad O^{(3)}_{\phi\ell} = i \left( \phi^\dagger T^I D_\mu \phi \right) \bar{\ell} T^I \gamma^\mu \ell \quad O_{\phi e} = i \left( \phi^\dagger D_\mu \phi \right) \bar{e} \gamma^\mu e $$

$$ O^{(1)}_{\phi q} = i \left( \phi^\dagger D_\mu \phi \right) \bar{q} \gamma^\mu q \quad O^{(3)}_{\phi q} = i \left( \phi^\dagger T^I D_\mu \phi \right) \bar{q} T^I \gamma^\mu q $$

$$ O_{\phi u} = i \left( \phi^\dagger D_\mu \phi \right) \bar{u} \gamma^\mu u \quad O_{\phi d} = i \left( \phi^\dagger D_\mu \phi \right) \bar{d} \gamma^\mu d \quad O_{\phi f} = i \left( \phi^\dagger \epsilon D_\mu \phi \right) (\bar{u} \gamma^\mu d) \quad (7) $$

where $\phi =$Standard Model scalar doublet, $D =$covariant derivative, $\ell =$left-handed lepton doublet, $e =$right-handed (charged) lepton singlet, $q =$left-handed quark doublet, $u =$right-handed u-quark singlet, $d =$right-handed d-quark singlet, and $\epsilon = i \tau^2$. This excludes the second case. 4. Tree-level operators.
Note that this list does not contain any terms that modifies the \(WWZ\) and \(WW\gamma\) couplings. This implies that all such modification will be loop generated. In terms of the now standard notation for the parameters describing the anomalous \(WWZ\) and \(WW\gamma\) vertices this implies

\[
|\lambda| \sim \frac{10^{-4}}{\Lambda_{\text{TeV}}} \quad |\kappa - 1| \sim \frac{2 \times 10^{-4}}{\Lambda_{\text{TeV}}}
\]

where \(\Lambda_{\text{TeV}}\) denotes \(\Lambda\) in TeV units. This implies that a bound \(|\lambda| \lesssim 1\) implies \(\Lambda \gtrsim 10\text{GeV}\): for loop generated operators most of the current data lack the precision necessary to probe interesting regions of the \(\Lambda\) axis. The fact that there are no deviations from the Standard Model in this area is far from surprising.

5. Some bounds from current data.

The tree level operators considered above have various effects on various observables which have been measured to high precision at LEP1, namely, the couplings of the leptons to the \(Z\) boson. Using the lagrangian \(\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i (\alpha_i \mathcal{O}_i + \text{h.c.}) / \Lambda^2\) the vector and axial couplings of the electron to the \(Z\) and the neutrino couplings to the \(Z\) are modified according to

\[
|\delta g_V(e)| = \frac{v^2}{\Lambda^2} \left| \alpha^{(1)}_{\phi \ell} + \alpha^{(3)}_{\phi \ell} + \alpha_{\phi e} \right| \lesssim 0.0021
\]

\[
|\delta g_A(e)| = \frac{v^2}{\Lambda^2} \left| \alpha^{(1)}_{\phi \ell} + \alpha^{(3)}_{\phi \ell} - \alpha_{\phi e} \right| \lesssim 0.00064
\]

\[
|\delta g(\nu)| = \frac{v^2}{\Lambda^2} \left| \alpha^{(1)}_{\phi \ell} - \alpha^{(3)}_{\phi \ell} \right| \lesssim 0.0018
\]

where the bounds correspond to the 1\(\sigma\) errors given in the particle data book\(^2\). The 3\(\sigma\) limits obtained from these results are\(^c\)

\[
\Lambda_{\text{TeV}} \gtrsim 2.5 \sqrt{\alpha^{(1)}_{\phi \ell}}, \quad 2.5 \sqrt{\alpha^{(3)}_{\phi \ell}}, \quad 2.7 \sqrt{\alpha_{\phi e}}
\]

These bounds obtained above are quite significant; they preempt many new results that could be obtained from LEP2 when considering the couplings of fermions to the gauge bosons.

Just as for the couplings of the leptons to the \(Z\) one can easily derive the modifications to the oblique parameters\(^3\) generated by the effective operators of dimension six. The expressions for these modifications and the contributing operators are\(^3\)

\[
\delta S = 32 \pi \alpha_{WB} (v^2 / \Lambda^2) \quad \mathcal{O}_{WB} = \left( \phi^{\dagger} \tau^I \phi \right) W_{\mu \nu} B^{\mu \nu}
\]

\[
\delta T = -\frac{4 \pi}{\sqrt{2}} \alpha_{\phi}^{(3)} (v^2 / \Lambda^2) \quad \mathcal{O}_{\phi}^{(3)} = \left| \phi^{\dagger} D \phi \right|^2
\]

\[
\delta U = O(v^4 / \Lambda^4)
\]

\(^c\)These values are somewhat different from the ones presented at the talk due to an algebraic error; these are the correct results.
where the natural sizes of the coefficients are $\alpha_{WB} \sim 1/(4\pi)^2$ and $\alpha_{(3)\phi} \sim 1$.

The existing bounds on these quantities imply non-trivial sensitivity to $\Lambda$, namely

$$|\delta S| < 0.4 \Rightarrow \Lambda > 310 \text{GeV}; \quad |\delta T| < 0.4 \Rightarrow \Lambda > 2.9 \text{TeV}; \quad |\delta U| < 1.3 \Rightarrow \Lambda > 430 \text{GeV}. \quad (12)$$

For the case where there is non light Higgs the natural size for the contributions to $U$ are $\sim 1/\pi$. Should the uncertainty in the $W$ mass reach $\sim 40 \text{MeV}$ the corresponding sensitivity to $\Lambda$ reaches $\sim 690 \text{GeV}$ and this measurement can be used to differentiate, albeit indirectly, between the light Higgs and the no-Higgs scenarios.

6. Higgs reactions as probes of new physics.

Most of the measurements of $\Lambda$ to be performed at LEP2 become redundant. It therefore becomes interesting to isolate those processes for which the LEP2 measurements will provide new insights into the physics beyond the Standard Model; the same is true for the proposed New Linear Collider (NLC). I will concentrate the two processes

$$\bullet \quad e^+ e^- \rightarrow \nu \bar{\nu} H \quad \bullet \quad e^+ e^- \rightarrow ZH. \quad (13)$$

and use them as probes for possible deviations form the Standard Model generated by new physics.

The amplitudes for these processes take the (symbolic) form $A = A_{SM} + [\alpha A_1 + \alpha'(s/v^2)A_2](v^2/\Lambda^2)$; obviously this type of expression cannot be used for arbitrary values of $s$ (in fact, we already know that the parametrization used will certainly break down when $s \sim \Lambda^2$). We will only consider values of $s$ for which the new contributions proportional to $\alpha, \alpha'$ are smaller than the Standard Model contributions.

The relevant operators are $O_\phi, O_{\partial \phi}, O_{(1)\phi}, O_{(3)\phi}, O_{(1)\ell}, O_{(3)\ell}$ and $O_{\phi e}$. Using this list and the above definition of the effective lagrangian the cross sections can be easily derived. The results of this calculation are presented in the figures below (imposing the LEP1 constraints on the $\alpha_i$) and show that the reaction $e^+ e^- \rightarrow \nu \bar{\nu} H$ cannot be used as a probe for new physics. This is not the case for $e^+ e^- \rightarrow ZH$.

To estimate the sensitivity of LEP2 and NLC to new physics we evaluate the statistical significance of the deviations from the Standard Model: consider the total number of events $N$ and the corresponding Standard Model prediction $N_{SM}$, then the statistical significance is $N_{sd} = |N - N_{SM}|/\sqrt{N}$. When evaluating $N_{sd}$ we impose the restrictions generated by LEP1, namely, given a value of $\Lambda$ we require that the $\alpha_i$ satisfy the bounds on $\delta g_{V,A}(e)$ and $\delta g(\nu)$ (at the $3\sigma$ level) as given above. In this guise the regions above the curves in the following plot will be inaccessible to LEP2; these curves give the maximum sensitivity for a given $\Lambda$ that can be obtained at LEP2 with the constraints from LEP1 imposed.
As can be seen from this plot non-trivial sensitivity to $\Lambda$ can be attained at LEP2 and, more dramatically, at NLC.

7. Conclusions

I have argued that the effects from the physics beyond the Standard Model can be naturally parametrized using an effective lagrangian. For the case where there is a light Higgs naturality requires the underlying theory to be weakly coupled (alternatively the low energy particle content should be significantly modified) in which case the most sensitive probes into the new interactions correspond to those operators that can be generated at tree level by the new dynamics. For these operators existing LEP1 data imply $\Lambda/\sqrt{\alpha} \gtrsim$ a few TeV.

I have also showed that there are other reactions involving the Higgs boson, that can generate new windows into the physics beyond the Standard Model. These reac-
tions cannot be probed using LEP1 data, but may very well be available at LEP2, provided the Higgs is sufficiently light.

It is important to measure all coefficients of tree level operators as the deviations form the Standard Model can be more significant in one of them than in the rest. But even if no deviations from the Standard Model are observed, their absence will point to the suppression of a large class of interactions (in the underlying theory!) and will restrict the characteristics of the corresponding models.

8. Acknowledgements

Most of the work presented here was developed in collaboration with C. Arzt, M. Einhorn and with B. Grzadkowski.

9. References

1. See for example, D.J.E. Callaway, Phys. Rep. 167 (1988) 241
2. See for example, E. Witten, Nucl. Phys. B188 (1981) 513. S. Dimopoulos et. al., Phys. Rev. D24 (1981) 1681.
3. E. Eichten et. al., Phys. Rev. D34 (1986) 1547 T. Appelquist in the proceedings of the Workshop on Electroweak Symmetry Breaking, Hiroshima, Nov. 12-15 1991. M. Einhorn, in Perspectives on Higgs physics, edited by G.L. Kane (Directions in High Energy Physics, v. 13) (World Scientific, 1992.)
4. S. Weinberg, Physica 96A (1979) 327. H. Georgi, Nucl. Phys. B361 (1991) 339; Nucl. Phys. B363 (1991) 301
5. J. Polchinski, lectures presented at TASI 92, Boulder, CO, Jun 3-28, 1992.
6. For a detailed review see J. Wudka, Int. J. of Mod. Phys. A9 (1994) 2301
7. M. Veltman, Acta Phys. Pol. B12 (1981) 437
8. T. Appelquist and J. Carazzone, Phys. Rev. D11 (1975) 2856. J.C. Collins et. al., Phys. Rev. D18 (1978) 242.
9. S. Coleman et. al., Phys. Rev. 177 (1969) 2239. C.G. Callan et. al., Phys. Rev. 177 (1969) 2247. T. Appelquist, Phys. Rev. D22 (1980) 200. A. Longhitano, Nucl. Phys. B188 (1981) 118. M. Chanowitz et. al., Phys. Rev. D36 (1987) 1490.
10. C.J.C. Burges and H.J. Schnitzer, Nucl. Phys. B228 (1983) 464 C.N. Leung et. al., Z. Phys. C31 (1986) 433 W. Büchmuller and D. Wyler, Nucl. Phys. B268 (1986) 621
11. C. Arzt et. al., UM-TH-94-15 report (to appear in Nucl. Phys. B).
12. Particle Data Book, Phys. Rev. D50 (1994) 1173
13. M. Peskin and T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964. G. Altarelli and R. Barbieri, Phys. Lett. B253 (1991) 161. B. Lynn et. al., CERN report 86-02.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9503423v2
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9503423v2
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9503423v2