Mathematical model for calculating solar radiation on horizontal and inclined surfaces for the conditions of Yakutsk

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Abstract. The purpose of this research is to develop a simple calculation technique for solar flux density on horizontal and inclined surfaces to allow estimation of the amount of heat energy entering a hybrid system of solar heat supply at any time. Based on the suggested method corresponding software is developed for defining solar flux on horizontal and inclined surfaces using measurement data of maximum daily solar radiation. Hourly values of total and diffuse solar radiation on horizontal surface are defined, as well as average daily, monthly and annual values of total solar radiation. The obtained calculation results are compared to the existing reference, experimental and calculation data on total solar radiation falling on horizontal surface in the city of Yakutsk. The calculated solar flux values show sufficient consistency with the reference materials and NASA SSE database. The paper provides description of the method for calculating specific solar heat flux on inclined surface with known heat fluxes of direct and diffuse solar radiation falling on horizontal surface.

1. Introduction
Design and operation of active heat systems [1–2] require development of methods to define energy performance of solar collectors and technological concepts of their use like in [3]. The paper [4] describes automated system of hot water supply implemented in Yakutsk which has three groups of solar collectors and a gas boiler as a heat energy source. In the process of the system operation it was found that automated control not always allows avoiding boiling of water in a heat exchanger during summer period, besides, in morning hours the hot water temperature may drop below the standard. The results obtained in [4] bring to the conclusion that further improvement of the developed system will be related to experimental and theoretical study of hourly operation modes of hot water supply system. The ability to predict variations of hourly values of heat energy incoming from solar collectors is required to design an efficient control algorithm for a dual hot water supply system.

Today, multiple calculation methods for defining energy characteristics of solar radiation are known [5–12]. However, when applied to certain territories local heliometric observation data should always be generalized to provide sufficient accuracy while defining total solar radiation values [5]. The study [6] demonstrates the impact of general climatic factors of Middle Volga region on empirical coefficients of calculated dependencies for defining possible and actual sum of solar radiation. In [7] modeling of solar
radiation in the Republic of Yakutia is performed based on long-term data from earth stations, a software package is developed for calculating total, direct and diffuse solar radiation. Using a similar approach that accounts existing empirical data the authors of [8] built a number of general and particular quasi-periodic models which allow predicting the dynamics of solar radiation in Krasnoyarsk territory, Republics of Khakassia and Tyva. The paper [9] suggests dependencies based on models described in [10] for defining average monthly total daytime solar radiation on horizontal surface in Baghdad. The study [11] introduces technique for calculating direct, diffuse and reflected incident solar radiation on all types of surfaces both inside and outside buildings. Generalization of calculation and prediction method for solar radiation on inclined surfaces is performed in [12]. Paper [13] recommends using the data of NASA SSE program when designing solar hot water stations in Krasnodar territory and Yakutia; such data are traditionally applied for defining average monthly solar radiation values [14], [15]. In [7], the author informs that the data on direct and diffuse solar radiation intensity obtained on meteorological observing stations are more accurate comparing to satellite observation data.

2. Methods of Research

In some cases when modeling heat supply systems that use solar power [2] complex analytical methods are applied for calculating total solar radiation including defining various coefficients related to latitude of the region, season and overcast characteristics. The simplest method providing sufficient accuracy of daily and monthly solar radiation values is the one suggested in [16]. To perform calculation of solar radiation variation during the daytime and to define hourly and daily totals of solar flux this method requires geographical coordinates, time zone of the region, and maximum intensity of direct and diffuse solar radiation during the day. In this paper, maximum values of total and diffuse radiation ($R_{\text{max}}$, $R_{\text{inc}}$, respectively) on horizontal surface predicted for the day number $n$ from the beginning of the year were defined with the account of overcast based on the 5-year monitoring data of heliometric station in Yakutsk.

Variation of solar radiation $R_h$ incoming on a horizontal surface was calculated as in [8] and [16] using the following dependency

$$R_h = R_{\text{max}} \sin \left( \frac{\pi}{T_c} \right),$$

where $t$ – daytime counted from sunrise $0 \leq t \leq T_c$, hour.

Length of a certain day can be calculated using a simplified formula [8]

$$T_c' = \frac{2}{15} \arccos(-\tan \varphi \tan \delta),$$

or a more accurate one with the account of atmospheric refraction

$$T_c = \frac{2}{15} \arccos \left( \cos \left( \frac{90,85\pi}{180} \right) - \sin(\varphi) \sin(\delta) \cos(\varphi) \cos(\delta) \right)^{1/2},$$

where $\varphi$, $\delta$ – latitude and sun declination, respectively. Cooper’s equation was used to calculate solar declination:

$$\delta = \arcsin \left( \sin(23,45) \sin \left( \frac{360}{365} (n - 81) \right) \right).$$

True noon $t_m$ local standard time in Yakutsk can be defined with the knowledge of true noon UTC:

$$t_m = UTC + 9,$$

where
\[
\text{UTC} = 12 - \frac{\psi}{15} - \frac{\text{EOT}}{60},
\]

(6)

\(\psi\) – longitude in degrees.

The following ratio can be used to define local sunset and sunrise times:

\[t_s = \frac{1}{2} T_c \pm t_n.\]

(7)

Figure 1 illustrates calculation results for sunrise and sunset time in Yakutsk using the equations (3) – (7). The same graph represents dependencies of sunrise and sunset time on the number of the day \(n\) for the city of Yakutsk obtained by the author [7]. The calculation results are in good consistency with the data from [7].

Based on calculation of \(R_h\) during the day using the equation (1) and involving ratios (2) – (7) one can find solar radiation sums \(R_\Sigma\) for any required time period \(t_\Sigma\) using integral formula

\[R_\Sigma = \int_{t_s}^{t_s} R_h \, dt = \int_{t_s}^{t_s} R_{max} \sin \left( \frac{\pi t}{T_c} \right) \, dt.\]

(8)

The mathematical model (1) – (8) was implemented in the corresponding software enabling to predict solar flux intensity on horizontal surface from maximum solar radiation values for the region.

3. Results and Discussion

Calculation results that demonstrate variation of flux intensity \(R_{hm}\) and hourly integral values \(R_{zh}\) of total solar radiation on an average July day in Yakutsk are presented in Figure 2. Figure 3 illustrate similar data for diffuse solar radiation for July.
Figure 2. Variation of flux intensity $R_{h}^{to}$ (a) and hourly integral values $R_{h}^{to}$ (b) of total solar radiation in Yakutsk depending on local time $t_s$ of an average July day.

Figure 3. Variation of flux intensity $R_{h}^{sc}$ (a) and hourly integral values $R_{h}^{sc}$ (b) of diffuse solar radiation in Yakutsk depending on local time $t_s$ of an average July day.

Figure 4. Daily integral values of $R_{h}^{to}$ total solar radiation for Yakutsk.
Results obtained from using the model (1) – (8) are in good consistency with generalized heliometry data from Yakutsk station.

Daily values of total solar radiation are obtained from modeling of insolation on horizontal surface; corresponding graph is built and presented in Figure 4.

Based on the data from Figure 4 average daily and monthly values of solar radiation for Yakutsk were calculated. Figure 5 illustrates comparison of these values with solar radiation characteristics for Yakutsk given in [17], and with the results of averaging of mean monthly values taken from NASA database for the period of 20 years from 1998 till 2017.

Significant difference between the calculation results for averaged solar radiation characteristics using the model (1) – (8) and NASA SSE data are observed for the period from March till June. For May and June, the calculated average monthly and daily sums of solar flux are on average by 9.8 % and 10.5 % higher than the same values taken from [17] and NASA SSE database, respectively. For March and April, these values are lower than the corresponding characteristics from [17] and NASA SSE database by 6.8 % and 9.2 %.
The suggested method predicts considerably higher total solar radiation intensity for fall and winter period compared to the data of NASA SSE, with the averaged difference for winter season of up to 32%. In [18], the authors had similar difference in winter season (up to 40%) between the heliometry data for Baikal region and NASA SSE data.

It should be noted that solar radiation characteristics cited in [17] were obtained in the USSR from meteorological series over the observation period of more than 10 years mainly belonging to the period from 1960 to 1980, whereas the compared calculated results are based on heliometry data from the station in Yakutsk for the last 5 years. The study [19] emphasizes that using series of data obtained for longer observation periods while analyzing various solar radiation databases not always provides higher accuracy of calculated solar radiation parameters. Moreover, appropriateness of the suggested method is confirmed by comparing the average annual value of total solar radiation falling on horizontal surface with similar values taken from other sources (Table 1).

| Table 1. Comparison of average annual values of total solar radiation. |
|-----------------------------------------------|
| Source | Calculation by (1) – (8) | Data from [17] | Data from NASA SSE | Calculation from [7] |
|--------|-------------------------|----------------|-------------------|---------------------|
| Average annual value, MJ/m² | 3,848 | 3,827 | 3,825 | 3,727 |
| Difference, % | – | 0.6 | 0.6 | 3.2 |

With the known values of direct \( R_d \) and diffuse \( R_{dc} \) solar flux falling on horizontal surface one can define solar power incoming on inclined plate \( R_i \), using ratios [20–22] obtained while defining the position of the plate relative to solar beams at a certain moment of time.

Total solar flux incoming on inclined surface is defined by the formula:

\[
R_i = R_d K_d + R_{dc} K_{sc},
\]

where \( K_d, K_{sc} \) – coefficients of solar collector location relative to direct and diffuse solar radiation, respectively, which can be found from the following ratios:

\[
K_d = \frac{\cos i}{\sin \alpha}, \quad K_{sc} = \frac{\cos^2 \beta}{2},
\]

where \( \cos i \) – cosine of solar incidence angle, \( \alpha \) – elevation angle, \( \beta \) – angle of surface tilt relative to the horizon.

Cosine of solar incidence angle when falling to some oriented surface is calculated by the formula:

\[
\cos i = \sin \beta \left[ \cos \delta \left( \sin \varphi \cos \gamma \cos \omega + \sin \gamma \sin \omega \right) - \sin \delta \cos \varphi \cos \gamma \right] \cos \delta \sin \alpha,
\]

\[
\sin \alpha = \cos \delta \cos \varphi \cos \omega + \sin \delta \sin \varphi,
\]

where \( \gamma \) – azimuth of inclined surface, \( \omega \) – hour angle.

Thus, heat flux intensity of solar radiation for an inclined surface depends both on heat fluxes of direct and diffuse radiation and on the surface orientation with respect to the cardinal directions, latitude of the region, angle of the receiving surface tilt relative to the horizon, sun declination and hour angles.
4. Conclusion
Calculation method for total and diffuse solar radiation is suggested based on the series of maximum daily values of total and diffuse radiation obtained from Yakutsk meteorological station. Hourly, daily, monthly and annual sums of solar radiation are calculated for the city of Yakutsk and compared with the existing experimental and calculation data showing good consistency level. Appropriateness of the suggested calculation method and the mathematical model for calculating integral density of solar energy are confirmed.

Further research will be aimed at developing a software package for automated design of hybrid “solar” heating systems which will integrate the already developed software for solar radiation parameters calculation for a horizontally oriented surface. This will enable to define the amount of heat energy incoming to a heating system at any moment of time during any day of any month, as well as to design reliable pumping and heat exchange equipment with required characteristics and correctly estimate the power of additional heat energy sources. In the end, the required actual input data will be obtained for developing efficient algorithms for automated parameter control and solar heating systems control.

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