Peripherical processes $2 \rightarrow 3$ and $2 \rightarrow 4$ in QED and QCD in $p(\bar{p})p$ high energy collisions

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Differential cross section of processes with high energy $p(\bar{p})p$ collisions in frames of QED: creation of scalar, pseudoscalar and lepton pair - are considered in Weizsäcker - Williams approximation. In frames of QCD processes with conversion of initial proton (antiproton) to fermionic jets accompanied with one gluon jet as well as the state of two gluons and quark-antiquark pair (without rapidity gap) are considered in frames of effective Reggeon action of theory of Lipatov. Process of creation of a Higgs boson accompanied with two fermionic jets is considered. The azimuthal correlation in process of two gluon jet separated by rapidity gap is investigated. Effects of gluon reggeization are taken into account. Some distributions are illustrated by numerical calculations.

I. INTRODUCTION

Motivation of this paper is the construction of realistic formulae and the estimation of the cross section with creation of two jets in the proton (anti-proton) fragmentation regions and one or two additional jets in multi-regge kinematics. Application of QCD methods to describe the peripherical processes in high energy proton (antiproton)- proton scattering is based on the proof of gluon reggeization phenomenon, which was done in papers [1] in
For this purpose we use the effective Regge action of conversion of two reggeized gluons $R$ to some set of real particles $P, Q$-to one and two gluons separated by rapidity gap, two gluon or quark-anti quark pair, without rapidity gap and a scalar (Higgs) meson.

Construction of paper is as follows. After short review of QED processes in Section II, we consider processes of a single and two gluon production and production in Section III and lepton pair in Section IV, assuming the absence of rapidity gap between the couple of particles created in pionization region. Measuring these processes provide the possibility to check the $RRP, RRPP, RRq\bar{q}$ vertices of an effective Regge action theory.

In Section V the azimuthal correlation between two gluon jets separated by some rapidity gap is considered. In Section VI the production of quark-antiquark is considered. In Section VII the Higgs boson production is considered. In Conclusion we discuss the main topics of our approach and give the results of numerical calculations.

II. QED PROCESSES

In the early seventies of the last century the processes of creation of some set of particles were intensively studied. There was considered a different mechanism of pair production in electron-positron collisions. The relevant formulae can in principle be applied to proton-proton (antiproton) collisions. Production of some set of particles in pionization region at high energy $p(p)p$ collision

$$p(p) + p \rightarrow p(p) + p + F$$

$$\left( \frac{s_1 ds}{ds_1} \right)^{pp \rightarrow ppF} = \left( \frac{\alpha}{2\pi} \right)^2 \ln^2 \left( \frac{s}{M_p^2} \right) \cdot f \left( \frac{s_1}{s} \right) \cdot \sigma^{\gamma\gamma \rightarrow F}_{tot} (s_1) \left( 1 + O \left( \frac{1}{\ln s/M_p^2} \right) \right),$$

$$f(z) = (2 + z)^2 \ln \frac{1}{z} - 2(1 - z)(3 + z).$$

with $s_1$ - invariant mass square of the produced set of particles $F$.

In the case when the lepton pair is created outside the fragmentation regions of protons the process cross section (see Fig. II a)

$$p(p_1) + p(p_2) \rightarrow p(p'_1) + p(p'_2) + \mu^+(q_+) + \mu^-(q_-),$$
Figure 1: Feynman diagrams for creation of 2 jet (a), 1 jet (b) by two reggeized gluons; (c) — creation of $P$, $S$-mesons by two reggeized gluons

has the form

$$d\sigma^\gamma \to \bar{u}pp = \frac{2\alpha^4}{\pi} \frac{d^2q_1d^2q_2d^2k_1dx}{\pi^3} \frac{d\beta_1}{(q_1^2 + M^2\beta_1^2)^2(q_2^2 + M^2\alpha^2)^2} \cdot F;$$

(4)

where $M$ is the proton mass,

$$sa = \frac{-c}{\beta_1x(1-x)}, \quad 0 < x = \frac{\beta_2}{\beta_1} < 1; \quad \beta_1 \ll 1,$$

(5)

$$c = m^2 + q_2^2 + q_1^2x + 2\bar{q}_1\bar{q}_2x,$$

(6)

$$c_1 = m^2 + (\vec{k}_2 - \vec{q}_2)^2 + \vec{q}_1^2x + 2\bar{q}_1(\bar{q}_2 - \vec{k}_2)x;$$

(7)

$$\vec{q}_1\vec{q}_2F = \frac{\vec{q}_2\vec{q}_1}{cc_1} x\frac{x}{c^2c_1} + \frac{1}{\beta_1^2} (\vec{q}_1^2 + 2\bar{q}_1\bar{q}_2)(\vec{q}_2^2 - 2\vec{k}_2\bar{q}_2) + 2(\bar{q}_2\vec{q}_1)(m^2 + \vec{k}_2^2)^2.$$

(8)

Here $m$ is the lepton mass, $x_1 = 1 - \beta_1 \approx 1$, $-\bar{q}_1$ is the energy fraction of the scattered proton and its momentum transversal to the initial proton direction $\vec{p}_1$ (center of mass of initial particles implied); $1 + \alpha \approx 1$; $\vec{q}_2$ are similar quantities for the scattered proton (anti-proton); $x\beta_1 + \frac{m^2 + \vec{k}_1^2}{s\beta_1x} = -\vec{k}_1$ and $(1 - x)\beta_1 + \frac{m^2 + \vec{k}_2^2}{s\beta_1(1-x)} = \vec{k}_2 = \bar{q}_1 - \bar{q}_2 - \vec{k}_1$ are the corresponding quantities for negative and positive charged leptons from the pair created; $m$ is the mass of the created particle.

For two-photon processes with creation of pseudo-scalar and scalar particle we use the corresponding subprocess $\gamma(q_1, \mu) + \gamma(q_2, \nu) \to P(S)$ (see Fig. 1 b, c) with matrix elements described in terms of triangle Feynman loop diagrams with quarks as internal fermions:

$$M^{\gamma\gamma P} = \frac{2\alpha N_{F\mu}}{\pi m_q}(q_1e_1q_2e_2)I_P, \quad (q_1e_1q_2e_2) = \epsilon^{\alpha\beta\gamma\sigma} q_{1\alpha}e_{1\beta}q_{2\gamma}e_{2\sigma};$$

$$M^{\gamma\gamma S} = \frac{2\alpha N_{S\mu}}{\pi m_q}[q_1q_2]\epsilon(e_1e_2) - (e_1q_2)(e_2q_1)I_S,$$

(9)

where $\epsilon_{1,2}(q_{1,2})$ are the polarization vectors of photons, $N_{P,S}$ are the color factors

$$N_P = N_c\left(\frac{4}{9} - \frac{1}{9}\right) = 1;$$

$$N_S = N_c\left(\frac{4}{9} + \frac{1}{9}\right) = \frac{5}{3}.$$
Performing the loop momentum integration we obtain

\[ I_{P,S} = \int_0^1 dx \int_0^1 dy \frac{dP, S}{dP, S}(1, 1 - 4y^2x(1 - x)), \]

\[ dP, S = 1 - y^2x(1 - x) \frac{M^2_{P, S}}{m_q^2} - y(1 - y)\left[ x \frac{q_1^2}{m_q^2} + (1 - x) \frac{q_2^2}{m_q^2} \right], \]  

(11)

\[ M_{P, S}, m_q \text{ are the masses of produced particles and quark mass. We can use the Goldberger–Treiman relation } g_P/m_q = 1/F_\pi, \text{ with } F_\pi = 93 MeV \text{ being the decay constant of charged pion, and a similar relation } g_S/m_q = 1/F_\sigma, F_\sigma \approx F_\pi. \]

When inserting these matrix elements into the matrix element of process 2 → 3 the combination is used \[ M_{\gamma\gamma}^F(e_1 \rightarrow p_1, e_2 \rightarrow p_2)/s = m_{\gamma\gamma}^F. \] We obtain

\[ m_{\gamma\gamma}^P = \frac{\alpha N_P}{\pi F_\pi} [\vec{q}_1, \vec{q}_2]_P I_P; \]

\[ m_{\gamma\gamma}^S = \frac{\alpha N_S}{\pi F_\sigma} (\vec{q}_1, \vec{q}_2) I_S, \]  

(12)

where we consider the four-momenta of virtual photons to be essentially transversal two-component Euclidean vectors \( \vec{q}_1, \vec{q}_2 \), \( q_{1,2}^2 = 0 \), \( q_{1,2}^2 = -q_{1,2}^2 < 0 \).

The cross sections of the processes of single meson production in the pionization region are

\[ d\sigma^{pp \rightarrow pp_P} = \frac{2\alpha^4 d\beta_1}{\pi} dN_1 dN_2 C_P \sin^2 \theta; \]

\[ d\sigma^{pp \rightarrow pp_S} = \frac{2\alpha^4 d\beta_1}{\pi} dN_1 dN_2 C_S \cos^2 \theta, \]  

(13)

where \( \theta \) – is the azimuthal angle between two-dimensional vectors \( \vec{q}_1, \vec{q}_2 \),

\[ C_P = \left| \frac{N_P}{F_\pi} I_P \right|^2; C_S = \left| \frac{N_S}{F_\sigma} I_S \right|^2; \]  

(14)

and Weizsäcker–Williams (WW) enhanced factors

\[ dN_1 = \frac{q_1^2 d^2 \vec{q}_1}{(q_1^2 + m_p^2 \beta_1^2)^2}; \]

\[ dN_2 = \frac{q_2^2 d^2 \vec{q}_2}{(q_2^2 + m_p^2 \alpha_2^2)^2}; \]

\[ |s\alpha_2\beta_1| = M^2_{P, S} + (\vec{q}_1 + \vec{q}_2)^2. \]  

(15)

We use the expression of the squared 4-vectors of momenta transferred to a lepton pair:

\[ q_1^2 \approx -(\vec{q}_1^2 + m_p^2 \beta_1^2); \]

\[ q_2^2 \approx -(\vec{q}_2^2 + m_p^2 \alpha_2^2). \]  

(16)
These factors, being integrated, produce the "large logarithmic" factors

\[
\frac{1}{\pi} \int_0^{Q^2} dN_1 = \ln \frac{q^2}{m_p^2 \beta_1^2} - 1, \quad m_p^2 \ll Q^2 \ll s. \tag{17}
\]

Cross section of production of some set of particles in pionization region at high energy \( p(\bar{p})p \) collision

\[
p(\bar{p}) + p \rightarrow p(\bar{p}) + p + F \tag{18}
\]

have a form [3]:

\[
\left( \frac{s_1 d\sigma}{ds_1} \right)^{pp \rightarrow ppF} = \left( \frac{\alpha}{2\pi} \right)^2 \ln^2 \left( \frac{s}{M_p^2} \right) \cdot f \left( \frac{s_1}{s} \right) \cdot \sigma_{\text{tot}}^{\gamma \gamma \rightarrow F} \left( \frac{1}{1 + O \left( \frac{1}{\ln s / M_p^2} \right)} \right)
\]

\[
f(z) = (2 + z)^2 \ln \frac{1}{z} - 2(1 - z)(3 + z), \tag{19}
\]

with \( s_1 \) - invariant mass square of the produced set of particles \( F \).

**III. QCD PROCESSES. CHECK OF RRP VERTEX**

Using V.Gribov's prescription of Green function of exchanging gluon in process \( p(\bar{p})(p_1) + p(p_2) \rightarrow \text{jet}(X_1) + \text{jet}(X_2) \) we put the matrix element in form

\[
M^{pp \rightarrow ji,j2} = \frac{4\pi \alpha_s}{q^2} < X_1 | J_\mu^a | p_1 > < X_2 | J_\nu^a | p_2 > \cdot \cdot \cdot = \frac{8 \pi \alpha_s}{q^2} \cdot s \cdot \Phi_1^a \cdot \Phi_2^a, \tag{20}
\]

\[
\Phi_1^a = \frac{1}{-s_\alpha} < X_1 | J_q^a | p_1 >, \quad \Phi_2^a = \frac{1}{s_\beta} < X_2 | J_q^a | p_2 >, \quad s = (p_1 + p_2)^2 >> M_p^2, \tag{21}
\]

with \( t^a \) - generator of color SU(N) group, where we use the gauge conditions \( q^\mu < X_{1,2} | J_\mu | p_{1,2} > = 0 \) and accept Sudakov parametrization for 4-momentum of exchanged gluon \( q = \alpha p_2 + \beta p_1 + q_\perp \).

Quantities \(-s_\alpha, s_\beta\), can be interpret in terms of invariant mass squared of fermionic jets created by initial protons.

\[
(p_1 - q)^2 \approx M_1^2 \approx -q^2 - s_\alpha; \quad (p_2 + q)^2 \approx M_2^2 \approx -q^2 - s_\beta.
\]

As was shown in papers [6] the phenomenon of "reggeization" of gluon Green function take place in kinematics \( s >> |q|^2 \) - which consist in replacement of ordinary gluon to reggeized
The matrix element of processes with reggeized gluon exchange will acquire the Regge factor
\[ R = \left( \frac{s_0}{s} \right)^{\alpha(q^2)} , \]
with \( \alpha(q^2) = 1 - \alpha' \cdot \vec{q}^2 \) - trajectory of gluon Regge pole (specified below).

Matrix element of creation of an additional gluon have a form (see Fig 2a)
\[ M^{PP \rightarrow j_1 j_2 g} = s \cdot \left( \frac{4\pi \alpha_s}{q_1^2 q_2^2} \right)^{3/2} \cdot \frac{<X_1|\vec{J}_{q_1} t^a|p_1>}{-s \alpha_1} \cdot \frac{<X_2|\vec{J}_{q_2} t^b|p_2>}{s \beta_2} \cdot f^{abc} C_{\mu} C_{\mu}^c(k), \]
\[ k = q_1 - q_2, \]  
(22)

with
\[ C_{\mu} = n_\alpha^a n_\beta^\mu \Gamma_{\alpha \beta \mu}, \quad n_- = \frac{2p_1}{\sqrt{s}}, \quad n_+ = \frac{2p_2}{\sqrt{s}}, \quad n_+^2 = n_-^2 = 0, \quad n_+ n_- = 2. \]
\[ C_{\mu} = 2[(n_-)_\mu (q_1^\dagger + \vec{q}_{1^\dagger}) + (n_+)_\mu (q_2^- + \vec{q}_{2^-}) - (q_1 + q_2)_\mu], \]  
(23)
effective vertex of conversion of two reggeized gluons to a real gluon, with properties
\[ C_{\mu}(q_1, q_2)(q_1 - q_2)_\mu = 0; \quad C_{\mu}^2 = \frac{16q_1^2 q_2^2}{(-q_1^+ q_2^-)}; \quad (q_1 - q_2)^2 = M_g^2 = -q_1^+ q_2^- - (\vec{q}_1 - \vec{q}_2)^2. \]  
(24)

Figure 2: Feynman diagrams of a single gluon jet and Higgs boson (a), quark-antiquark jets (b),
two gluon jets (c), two gluons separated by rapiditi gap (d).

gluon with the same quantum numbers except of ”moving” gluon spin-it’s Regge trajectory.
In frameworks of fermion-jet model we replace the set of particles consisting in jet developed by initial proton as on-mass shell proton, and, besides, modify the vertex of it’s interaction with (reggeized) gluon

\[ < X_1 | J_\mu t^a | P_1 > = J_\mu^a = \bar{u}(p_1' + P_1') t^a \bar{V}_\mu u(p_1), \quad \bar{V}_\mu = \gamma_\mu - \frac{p_2^\mu}{q_1 p_2}. \]  

(25)

This vertex function obey the gauge condition \( J_1^\mu q_1^\mu = 0 \). We have, besides

\[ \frac{1}{s} J_\mu p_2^\mu = \frac{1}{-s \alpha_1} \bar{J}_\mu q_1^\mu; \]

\[ \int d(s \alpha_1) d\gamma_1 \sum (\bar{J}_\mu p_2^\mu) \left( \frac{\bar{J}_\mu p_2^\mu}{s} \right) = \frac{1}{2} \delta_{ab} \frac{2 \bar{q}_1^2}{M^2 + \bar{q}_1^2} \]  

(26)

with \( d\gamma_1 \) - phase volume of proton jet defined in (29) and \( \bar{M} \) - the average value of invariant mass of proton jet.

For matrix element squared overaged on final state we obtain in fermion-jet model:

\[ \int d(s \beta_2) d\gamma_2 \int d(s \alpha_1) d\gamma_1 \sum |M|^2 = \frac{s^2 \cdot 2^6 \pi^3 \alpha_1^3}{\bar{q}_1^2 \bar{q}_2^2} \cdot \frac{N(N^2 - 1)}{M_g^2 + (\bar{q}_1 - \bar{q}_2)^2} \cdot \frac{\bar{q}_1^2}{M^2 + \bar{q}_1^2} \cdot \frac{\bar{q}_2^2}{M^2 + \bar{q}_2^2}. \]  

(27)

Considering the phase volume of process \( p(\bar{p}) p \rightarrow j_1 j_2 F \) we introduce two auxiliary variables - the 4-momenta of the exchanged gluons

\[ \int d^4 q_1 d^4 q_2 \delta^4(p_1 - q_1 - p_1' - P_1) \delta(q_2 + p_2 - p_1' - P_2) = 1. \]  

(28)

So we have

\[ d\Gamma_3 = (2\pi)^{-2} d^4 q_1 d^4 q_2 d\gamma_1 d\gamma_2 d\gamma_3, \quad d\gamma_1 = \frac{d^3 p_1'}{2 \varepsilon_1} \prod_{P_1} \frac{d^3 r_i}{2 \varepsilon_i (2\pi)^3} \delta^4(p_1 - q_1 - p_1' - P_1), \quad P_1 = \sum r_i, \]

\[ d\gamma_2 = \frac{d^3 p_2'}{2 \varepsilon_2} \prod_{P_2} \frac{d^3 v_i}{2 \varepsilon_i (2\pi)^3} \delta^4(p_2 + q_2 - p_2' - P_2), \quad P_2 = \sum v_i, \]

\[ d\gamma_3 = \prod_{P_j} \frac{d^3 l_i}{2 \varepsilon_i (2\pi)^3} \delta^4(q_1 - q_2 - P_j), \quad P_j = \sum l_i. \]  

(29)

Using

\[ d^4 q_1 d^4 q_2 = \frac{s}{2} d\alpha_1 d\beta_1 d^2 q_1 \cdot \frac{s}{2} d\alpha_2 d\beta_2 d^2 q_2 = \frac{\pi^2}{4s} d(s \alpha_1) d(s \beta_2) \frac{d\beta_1}{\beta_1} d(s \alpha_2 \beta_1) \frac{d^2 q_1 d^2 q_2}{\pi^2}. \]  

(30)

we write down \( d\Gamma_3 \) in form

\[ d\Gamma_3 = \frac{1}{2^7 \pi^3 s} \frac{d\beta_1}{\beta_1} \cdot d\Phi_1 \cdot d\Phi_2 \cdot d\Phi_g \frac{d^2 q_1 d^2 q_2}{\pi^2}; \quad d\Phi_{1,2} = dM_{1,2}^2 d\gamma_{1,2}; \quad d\Phi_g = dM_g^2 d\gamma_g. \]  

(31)
In fermion - jet model approximation we obtain

\[ d\Gamma^{(3)} = (2\pi)^{-5} \cdot \frac{\pi^2}{4s} \cdot \frac{d\beta_1}{\beta_1} \cdot \frac{d^2q_1 d^2q_2}{\pi^2} \]  

(32)

For cross section of process \( pp \to j_1 j_2 j_g \) we obtain

\[ d\sigma = \frac{\alpha_s^3}{16M_g^2} N(N^2 - 1) R_2 dL_1 \cdot I(\rho), \quad dL_1 = \frac{d\beta_1}{\beta_1} \]  

(33)

\[ R_2 = \left( \frac{s_1}{s_0} \right)^{2(\alpha(\bar{q}_1^2) - 1)} \left( \frac{s_2}{s_0} \right)^{2(\alpha(\bar{q}_2^2) - 1)} \approx \left( \sqrt{s} (GeV) \right)^{-4\alpha_s \bar{q}^2 (GeV^2)}, \]  

(34)

\[ I(\rho) = \int_0^\infty \int_0^\infty \frac{dx_1 d_2}{(x_1 + \rho)(x_2 + \rho)\sqrt{(1 + x_1 + x_2)^2 - 4x_1 x_2}}, \quad \rho = \frac{\bar{M}^2}{M_g^2}. \]  

(35)

Function \( I(\rho) \) is tabulated in Fig. 3.
### IV. QCD PROCESSES. CHECK OF RRPP VERTEX

Let now consider process of creation of two gluons not separated by rapidity gap

\[ p(p_1) + p(p_2) \rightarrow jet_1(X_1) + jet_2(X_2) + g(k_1) + g(k_2). \]  \hspace{1cm} (36)

For the case of production of two particles \( RR \rightarrow a(k_1) + b(k_2) \) (no rapidity gap between \( a, b \) implied) we obtain for phase volume

\[ d\Gamma_4 = \frac{1}{2^{11} \pi^5} \frac{dx}{x(1-x)} \frac{d^2q_1 d^2q_2 d^2k_1}{\pi^3} \]  \hspace{1cm} (37)

with \( k_i = b_ip_1 + a_ip_2 + k_\perp, \ x = \frac{b_1}{b_1}, \ y = \frac{a_1}{a_2}. \)

Differential cross section of pair of particles \( ab \) (\( ab = gg, q\bar{q} \)) production in fermion-jet model can be written as

\[ d\sigma_{pp+jetjet} = \frac{\alpha^4}{26 \pi} dL \cdot R_2 \cdot \frac{dx}{x(1-x)} \frac{d^2k_1 d^2q_1 d^2q_2}{\pi^2} \Phi^{ab}, \quad \Phi^{ab} = \sum |M^{RRab}|^2 \frac{q_1^2 q_2^2}{q_1^2 q_2^2}. \]  \hspace{1cm} (38)

The explicit expression for \( \Phi^{gg} \) is \[3, 8\] \((M^{RRgg} = M^{RRPP})\):

It was obtained \[2, 3, 6, 8\]:

\[ \sum |M^{RRPP}|^2 = G_1(a^{\nu_1 \nu_2}(k_1, k_2))^2 + G_2 \Omega_{\sigma \rho}(k_1) \Omega_{\rho \sigma'}(k_2) a^{\sigma \rho}(k_1, k_2) a^{\rho' \sigma'}(k_2, k_1) + \]  \hspace{1cm} (39)

\[ + (k_1 \leftrightarrow k_2), \]

where

\[ G_1 = (f_{d_1 d_2 r} f_{c d r})^2 = N^2(N^2 - 1); \]
\[ G_2 = f_{d_1 d_2 r} f_{c d r} f_{d_2 c r} f_{d_1 d r} = -\frac{1}{2} N^2(N^2 - 1), \]  \hspace{1cm} (40)

the projection operators

\[ \Omega_{\sigma \rho}(k) = -g_{\sigma \rho}^{+} - \frac{2}{k^2} k_\sigma k_\rho, \]  \hspace{1cm} (41)
and

$$a^{\nu_1 \nu_2}(k_1, k_2) = 4 \left[ \frac{1}{\lambda} q'_\perp q'_{\perp} - \frac{1}{\lambda} q'_{\perp}(k_1 - \frac{x}{\bar{x}} k_2) \nu_2 + \frac{1}{\lambda} q'_{\perp}(k_2 - \frac{\bar{y}}{y} k_2) \nu_1 - \frac{x q'_2}{\chi k'_2} k'_1 \nu_1 k'_2 \nu_2 - \frac{\bar{y} q'_1}{\chi k'_2} k'_2 \nu_2 - \frac{1}{\lambda} (1 + \frac{tx}{\bar{x} k'_2}) k'_1 k'_2 \nu_2 + \frac{1}{\lambda} k'_1 k'_2 \nu_2 - 2 D_{\perp} g'_{\perp} \right],$$

with

$$D = 1 + \frac{t}{\lambda} + \frac{\bar{x} k'_2}{tx} + \frac{1}{\lambda} \left( \frac{\bar{x} k'_2}{x} - \frac{x \bar{k}'_2}{\bar{x} k'_2} \right) + \frac{\bar{q}'_1}{\chi} \bar{y} + \frac{\bar{q}'_2}{\chi} x.$$  

Using the relations

$$s a_i b_i = \bar{k}'_i^2 + m^2;$$

$$q = q_1 - k_1 = q_2 + k_2; t = q^2; \chi = (k_1 + k_2)^2;$$

$$t = - (\bar{q}'_1 - \bar{k}'_1)^2 - \frac{\bar{x} k'_2}{x}; \chi = \frac{1}{x \bar{x}} (\bar{x} k'_1 - x \bar{k}'_2)^2,$$

one can be convinced that the gauge conditions

$$D|_{\bar{q}'_1 \to 0} = D|_{\bar{q}'_2 \to 0} = 0; \quad a^{\nu_1 \nu_2}(k_1, k_2)|_{\bar{q}'_1 \to 0} = 0; \quad a^{\nu_1 \nu_2}(k_1, k_2)|_{\bar{q}'_2 \to 0} = 0,$$

are fulfilled.

Due to gauge properties of $a^{\nu_1 \nu_2}$ the quantity $\Phi^{gg}$ is finite as $\bar{q}'_1, \bar{q}'_2 \to 0$, which provides the convergence of the quantity

$$I^{gg}(\bar{k}'_1) = \int \frac{d^2 \bar{q}'_1 d^2 \bar{q}'_2 (q^2)}{(\bar{q}'_1^2 + M^2)(\bar{q}'_2^2 + M^2)} \Phi^{gg}.$$  

This quantity is presented in Fig. 4.

V. AZIMUTHAL CORRELATION IN PROCESS OF TWO GLUON JETS
CREATION, SEPARATED BY RAPIDITY GAP

Consider now process of two gluon production separated by rapidity gap. Corresponding matrix element contains 3 gluon Regge factors $R_3 = \begin{pmatrix} (\alpha_{q_1}) & (\alpha_{q_2}) & (\alpha_{q_3}) \end{pmatrix}$ with
Figure 4: Value $I_{gg}$ (defined in (47)) as a function of transverse momentum modulus $|\vec{k}_1|$ of one of the gluons in the produced gluon pair in the case of $M_1 = M_2 = 1$ GeV, $x = 0.2$ and $y = 0.3$.

Figure 5: Value $I_{qq}$ (defined in (61)) as a function of transverse momentum modulus $|\vec{k}_1|$ of quark in the produced quark pair in the case of $M_1 = M_2 = 1$ GeV, $x = 0.2$ and $y = 0.3$. 
moments of exchanged gluons $q_i = \alpha_ip_2 + \beta_ip_1 + q_{1\perp}$ and $s_1 \approx -s\alpha_2$, $s_2 \approx -s\alpha_3\beta_1$, $s_3 = s\beta_2$, $1 >> \beta_1 >> \beta_2 >> \beta_3$, $1 >> \alpha_3 >> \alpha_2 >> \alpha_1$.

$$s_1s_2s_3 = [M_1^2 + (\bar{q}_1 - q_2)^2][M_2 + (\bar{q}_2 - \bar{q}_3)^2] \cdot s,$$  \hspace{1cm} (48)

and $M_1^2, M_2^2$ - the invariant mass squared of created gluon jets.

Matrix element have a form

$$M_{pp \to j_1j_2j_3j_4} = \frac{s(4\pi\alpha_s)^2}{2q_1^2q_2^2q_3^2} \frac{<X_1|\bar{q}_1t^a|p_1> \cdot <X_2|\bar{q}_2t^b|p_2>}{(s\alpha_1)} \cdot \frac{s\beta_3}{f_{abc}f_{bde}C^\mu(q_1, q_2)e^{c_1}_\mu \cdot C^\lambda(q_2, q_3)e^{c_2}_\lambda}. \hspace{1cm} (49)$$

Phase volume of process $pp \to j_1j_2j_3j_4$ can be written in form (in fermion-jet model)

$$d\Gamma_4 = (2\pi)^{-s} \frac{\pi^3 d\beta_1 d\beta_2 d^2q_1 d^2q_2 d^2q_3}{\beta_1 \beta_2 \pi^3}. \hspace{1cm} (50)$$

For cross section we obtain

$$d\sigma_{pp \to j_1j_2j_3j_4} = \frac{4\alpha_s^4}{\pi} dL_1 \Delta\phi \frac{d^3q_1 d^3q_2 (d\phi/(2\pi))(d^2q_2/(\pi))R_3 N^2(N^2 - 1)}{(q_1^2 + M^2)(q_2^2 + M^2)(M_{g_1}^2 + (q_1 - q_2)^2)(M_{g_2}^2 + (q_2 - \bar{q}_3)^2)}, \hspace{1cm} (51)$$

with $M_{p_1}^2 = -s\alpha_1$; $M_{p_2}^2 = -s\beta_3$; $M_{j_1}^2 = -s\alpha_2\beta_1$; $M_{j_2}^2 = -s\alpha_3\beta_2$, and $L_1 = \ln \frac{s\alpha_1}{\beta_2}$, $\Delta\phi -$ rapidity gap of gluon jets $\Delta\phi = \ln \frac{s\beta_1}{\beta_2}$.

For azimuthal correlation we obtain (performing integration on $d^2q_2$ and putting $M_{g_1}^2 = M_{g_2}^2 = M_g^2$)

$$\frac{2\pi d\sigma_{pp \to j_1j_2j_3j_4}}{d\phi} = L_1 \sigma_0 \cdot F(\phi), \hspace{1cm} L_1 = \ln \frac{s\beta_3}{M^2}, \hspace{1cm} Y = \ln \frac{\beta_1}{\beta_2}, \hspace{1cm} \sigma_0 = \frac{4\alpha_s^4}{\pi M_g^2} N^2(N^2 - 1). \hspace{1cm} (52)$$

$$F(\phi) = \int_0^\infty \frac{dx_1}{x_1 + \rho} \int_0^\infty \frac{dx_2}{x_2 + \rho} \psi(z), \hspace{1cm} z = x_1 + x_2 - 2\sqrt{x_1x_2} \cos \phi,$$

$$\psi(z) = \frac{2}{\sqrt{z}(4 + z)} \ln \frac{\sqrt{4 + z} + \sqrt{z}}{\sqrt{4 + z} - \sqrt{z}}, \hspace{1cm} \rho = \frac{M_g^2}{M_g^2}. \hspace{1cm} (53)$$

Function $F(\phi)$ is presented in Fig. [6]

VI. THE PROCESS $pp \to jjq\bar{q}$. CHECK OF $RRqq$ VERTEX

The matrix element of the subprocess of conversion of two reggeized gluons to the quark-anti-quark pair (see Fig. [2] c)

$$R(a, q_1) + R(b, -q_2) \to q(k_1) + q(k_2), \hspace{1cm} (54)$$
is described by two different mechanisms: direct interaction and production of gluon with its subsequent conversion into the quark pair\[^{13}\]

\[
M^{q\bar{q}} = \bar{u}(k_1)[At_a t_b - Bt_b t_a]v(k_2),
\]

with \(t_a\) being the generator of the color \(SU(N)\) group in the fermion representation,

\[
\sum (t_a^2)^2 = I - \frac{N^2 - 1}{2N} ; \sum Tr t_a t_a t_b = -\frac{N^2 - 1}{4N} ; \sum Tr t_a t_b t_a t_b = \frac{(N^2 - 1)^2}{4N},
\]

and\[^{7,8}\]

\[
A = \gamma^- \frac{\hat{q}_1 - \hat{k}_1 - m}{(q_1 - k_1)^2 - m^2} \gamma^+ - \frac{1}{q^2} \hat{C} ; \quad \gamma_\pm = n_\pm \gamma_\mu,
\]

\[
B = \gamma^+ \frac{\hat{q}_1 - \hat{k}_2 - m}{(q_1 - k_2)^2 - m^2} \gamma^- - \frac{1}{q^2} \hat{C} , \quad q = k_1 + k_2,
\]

where \(m\) is the quark mass and 4-vector \(C_\mu\) describing the conversion of two reggeized gluons into the ordinary gluon which was given above (23). The gauge properties of \(M^{q\bar{q}}\), i.e. turning it to zero in the limit \(\vec{q}_1 \to 0\) as well as in the limit \(\vec{q}_1 \to 0\) can be seen explicitly.

Figure 6: Azimuthal correlation of two gluon jet production separated by rapidity gap (see (53)).
These properties provide convergence of the relevant integrals on $\vec{q}_1, \vec{q}_2$. We obtain

$$\Phi^{q\bar{q}} = \frac{4M^4_{q\bar{q}}}{\vec{q}_1 \vec{q}_2}[N_1(S_A + S_B) - 2N_2S_{AB}], \quad N_1 = \frac{(N^2 - 1)^2}{4N}; \quad N_2 = -\frac{(N^2 - 1)}{4N},$$

$$M^2_{q\bar{q}} = \frac{1}{x \bar{x}}[m^2 + (\vec{k}_1 + x(\vec{q}_2 - \vec{q}_1))^2]$$

with

$$S_A = \frac{1}{4}Sp(\hat{k}_1 + m)A(\hat{k}_2 - m)\tilde{A};$$

$$S_B = \frac{1}{4}Sp(\hat{k}_1 + m)B(\hat{k}_2 - m)\tilde{B};$$

$$S_{AB} = \frac{1}{4}Sp(\hat{k}_1 + m)A(\hat{k}_2 - m)\tilde{B}.$$  

Note that the value $\Phi^{q\bar{q}}$ is finite in both the limits $\vec{q}_1 \to 0$ and $\vec{q}_2 \to 0$.

The result of numerical integration of the quantity

$$I^{qq} = \int \frac{d^2 \vec{q}_1 d^2 \vec{q}_2}{\pi^2} \frac{M^4 \cdot \Phi^{q\bar{q}}}{(M^2 + \vec{q}_1^2)(M^2 + \vec{q}_2^2)}$$

is presented in Fig. 5.

Cross section of process $pp \to j_1 j_2 q\bar{q}$ is:

$$d\sigma^{pp\to j_1 j_2 q\bar{q}} = \frac{\alpha_s^4}{2^6 \pi M^2 \pi M^2 x(1-x)} I^{qq} \cdot \frac{d\beta_1}{\beta_1} \cdot \bar{R}_2.$$  

VII. HIGGS BOSON PRODUCTION

Higgs boson, we assume, to be produced by collision of two reggeized gluons through intermediate state of heavy top quark - antiquark state. By analogy with QED case we have for matrix element (see above)

$$M^{pp\to j_1 j_2 H} = \frac{\pi \alpha_s}{\vec{q}_1 \vec{q}_2} \cdot \frac{2\alpha_s N g_H}{\pi m_t} \cdot \frac{I_s \cdot \vec{q}_1 \vec{q}_2}{-s \alpha_1} \cdot \frac{X_1|\vec{J} \vec{q}_1 t^a|p_1 >}{s |\vec{J} \vec{q}_1 t^a|p_1 >} \cdot \frac{X_2|\vec{J} \vec{q}_2 t^a|p_2 >}{s |\vec{J} \vec{q}_2 t^a|p_2 >}.$$  

For differential cross section we obtain

$$\frac{d\sigma}{dL} = \sigma_0 \gamma, \quad \sigma_0 = \frac{\alpha_s^4 N^2(N^2 - 1)}{2^9 \pi^3 m_t^4} |I_s|^2, \quad \gamma = \int \frac{d^2 \vec{q}_1 d^2 \vec{q}_2 / \pi^2}{(M^2 + \vec{q}_1^2)(M^2 + \vec{q}_2^2)} \bar{R}_2$$

with $I_s\left(\frac{M^2_{m_t}}{m_t^2}, \frac{d^2_{m_t}}{m_t^2}, \frac{d^2_{m_t}}{m_t^2}\right) \approx I_s(0, 0, 0) = \frac{4}{3}$.

Quantity $\gamma$ turns out to be small $\gamma \sim 10^{-2}$ for $\alpha_s = 0.1$ $\sqrt{s} \sim 10^3 \div 10^4$ GeV. So the differential on Higgs boson rapidity cross section $\sigma_0$ is rather small $\gamma \cdot \sigma_0 \sim 1$ fb, $\sqrt{s} = 14000$ GeV.
VIII. DISCUSSION

The channel of peripheral processes with creation of some state in $s_0$ - called pionization region in proton (anti-proton) - proton collisions at high energies are considered. We suppose the initial state proton (antiproton) to develop the protonic (anti-protonic) jets resulting from interaction of initial proton (anti-proton) with the reggeized (coloured) gluon.

The fact of ”reggeization” of gluon - the replacement of Green function of the exchanged gluon by the exchanged Regge - pole with gluon quantum numbers except of a spin, which is replaced by gluon Regge trajectory

$$\alpha(q^2) = 1 + \frac{\alpha_s q^2}{\pi} \int \frac{d^2k}{k^2(q-k)^2}$$

was proven in papers [1]. Gluon Regge trajectory suffer from infrared singularities, which can be regularized by introducing the fictitious gluon mass $m$: $k^2 \rightarrow k^2 + m^2$, $(q - k)^2 \rightarrow (q-k)^2 + m^2$. In papers of Lipatov and Balitskii [14] was shown that the singular dependence on gluon mass disappears a experimental set -up with emission of (arbitrary) ”real” gluons, as well, posessing mass. It means taken into account the inelastic processes with creation of so called ”mini-jets” in multi -Regge kinematics. In this way the expression of cross section in terms of Pomeron -pole exchange was developed. The corresponding forward scattering amplitude was shown [1] to obey the so-called BFKL equation.

The statement of gluonic ”mini-jets” is some problematic up to now. Really by the common knowledge the gluon color must manifest itself in developing the hadronic jet, consisting from pions.

In this paper we use the theoretical result about existence of gluon trajectory and use the phenomenologic approach in describing it’s form [12]

$$\alpha(q) = 1 - \frac{\alpha_s \bar{q}^2}{\pi q_0^2} C, \quad q^2 = -\bar{q}^2, < 0, \quad q_0^2 \approx 1 \, GeV^2, \quad C \approx 1.$$  (66)

Besides we use some simplified version of GPD describing the interaction of reggeized gluon with proton (anti-proton) and converging it to proton-jet.

Main feature of this ”fermion-jet” model consist in absence of evolution effects, which is the essential part of Generalized Parton Distribution (GPD) approach. Applying simultaneously BFKL and evolution mechanisms seems to be suffer from the double - counting.
Regge-factor written in form (two and three successive reggeon (R) exchange)

\[ R_2 = \left( \frac{s_1}{s_0} \right)^{2(\alpha(q_1)-1)} \left( \frac{s_2}{s_0} \right)^{2(\alpha(q_2)-1)}; \quad R_3 = \left( \frac{s_1}{s_0} \right)^{2(\alpha(q_1)-1)} \left( \frac{s_2}{s_0} \right)^{2(\alpha(q_2)-1)} \left( \frac{s_3}{s_0} \right)^{2(\alpha(q_3)-1)} \]

(67)

\[ s_1s_2 = M_F^2 s; \quad s_1s_2s_3 = M_{1F}^2 M_{2F}^2 s \]

with \( M_F^2 \)-invariant mass created in pionization regions, turns out to be a rather effective suppression factor. In papers [10] the of gluon reggeization effects was omitted.

In paper [11] the channel \( pp \rightarrow ppH \) aof Higgs boson production was investigated. Here at least the exchange by two (parallel) (reggeized) gluons must be applied to provide colorless \( ppH \) final state.

Introducing the Sudakov - type formfactors as well seems to be illegitimate.

Cross section of Higgs boson production in our approach is rather small \( d\sigma/dL \sim 1 \text{ fb} \), but can be measured at LHC. This result is in agreement with ones obtained in [11, 15]. Tests of effective Regge action theory developed in paper [3] provide by investigation of processes of creation of a single gluonic jet, and production of two gluons and quark-antiquark pair in pionization region (without rapidity gap between created gluon or quarks). In these experiments the form \( RRP, RRPP, RRq\bar{q} \)-vertices of effective Regge action can be tested.

Measuring the azimuthal correlation in process of production of two gluon jets, separated by rapidity gap as a test of theory prediction for multi-regge kinematics as well can be realized at RHIC or LHC.

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