A hyperon as helicity analyzer of $s$ quark in $B$ decay

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Abstract

We explore how well one can probe the $s$ quark chirality of the fundamental weak interaction of nonleptonic $B$ decay using the spin-analyzing property of the $\Lambda$ hyperon. We present the prediction of the Standard Model as quantitatively as possible in a perturbative QCD picture avoiding detailed form-factor calculation involving quark mass corrections. A clean test of chirality will be possible with $\overline{B} \to \Lambda X$.

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I. INTRODUCTION

In the Standard Model only the left-chiral quarks enter the fundamental weak interaction. Short-distance loop corrections generate the penguin-type transition $b \to s_L(d_L)$ as an effective decay interaction. This chiral property leads to simple testable constraints on light meson helicities in final states of $B$ decay if the final-state interaction is perturbative. In order to determine chirality of weak interaction, we must measure sign of a meson helicity, $h = +1$ or $h = -1$, more than just transverse ($h = \pm 1$) or longitudinal ($h = 0$). Spin analysis of $B \to 1^-1^-$ through angular correlations was formulated by Dighe et al and the data were analyzed for $B \to J/\psi K^* \to l^+ l^- K\pi$ without lepton spin measurement, however, this analysis is not capable of distinguishing between $h = +1$ and $h = -1$ since it leaves a twofold ambiguity in the transverse helicities. A complete helicity determination can be achieved only with spin and angular correlations, as shown in a more general formulation by Chiang and Wolfenstein.

It has been argued that the penguin transition such as $b \to g^* s$ and $\gamma(s)s$ is more sensitive to a nonstandard weak interaction than the tree interaction. The interaction $\overline{b} \to \gamma s$ leads to $B \to \gamma K^*$ among others. In the Standard Model, $K^* (= \overline{s}uq)$ ought to be produced in the helicity +1 state in this decay in the limit of $m_s = 0$ and zero transverse momentum. If we wish to prove experimentally that $\gamma$ and $K^*$ are emitted with helicity +1 as predicted, we have to make a demanding measurement of lepton spin in $B \to \gamma^* K^* \to l^+ l^- K\pi$. An alternative proposal was made to study the angular distribution of the process $B \to \gamma K_1 \to \gamma K\pi\pi$. In this case the strong phase difference due to the overlapping resonances $\rho K$ and $K^*\pi$ of $K\pi\pi$ will allow us to obtain the $K_1$ spin information. Experimental efforts are being made on $B \to \gamma K\pi\pi$.

Determination of the helicity sign is difficult in the cascade decays so far considered since parity is conserved in the second step of decay. If an intermediate particle of nonzero spin decays into final particles with a parity violating interaction, it is easy to determine the helicity sign through the spin-angular correlation $\langle \mathbf{s} \cdot \mathbf{p} \rangle$. The $B$ decay into a $\Lambda$ hyperon will provide us with such an opportunity since $\Lambda$ decays into $\pi N$ with the well-measured large parity asymmetry. Furthermore, according to hadron spectroscopy, $\Lambda$ has the unique property that its spin is equal to the spin of the valence $s$ quark. Consequently the $s$ quark helicity can be determined by measuring the $\Lambda$ spin through a simple angular correlation of $\Lambda \to \pi N$. To probe a nonstandard weak interaction, therefore, it makes sense to explore the $s$-quark chirality in the QCD penguin interaction with $\Lambda$ as a spin analyzer.

II. $\Lambda$ HELICITY VERSUS STRANGE QUARK HELICITY

Ground-state baryons are made of three valence quarks totally in $s$-wave. Inside $\Lambda$ the $u$ and $d$ quarks form a spin singlet. As it is well known, therefore, the $\Lambda$ spin is made entirely of the $s$-quark spin in the static quark model. Boosting it to a moving frame, the helicity of $\Lambda$ is equal to that of the $s$ quark. The boost does not generate a new helicity component $l_z$ from the orbital motion since the distribution of $s$ quark is spherically symmetric inside $\Lambda$. When this $s$ quark comes directly from weak interaction, the handedness of $\Lambda$ tells us of the $s$-quark chirality in weak interaction. The $s$ quark can also be generated through pair production by gluons.

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The first task is to determine the helicity content of \( s_L \) in flight that determines the \( \Lambda \) helicity. If we could ignore the \( s \) quark mass, the whole story would be trivial. In the real world, however, the \( s \) quark carries mass and transverse momentum. When an \( s \) quark is produced with momentum \( p_s \) from the left-chiral field \( \pi_L \), projection of a plane wave shows that it is in the helicity \( h = \pm \frac{1}{2} \) states with the amplitude ratio of

\[
\frac{A_{+\frac{1}{2}}}{A_{-\frac{1}{2}}} = \frac{E_s + m_s - |p_s|}{E_s + m_s + |p_s|} = \frac{m_s}{E_s + |p_s|},
\]

where \( E_s = \sqrt{m_s^2 + p_s^2} \). Inside \( \Lambda \), the transverse quark momentum is part of the constituent quark mass. Therefore it is appropriate to replace \( m_s \) with the constituent mass \( M_s \) when we later express \( A_{+1/2}/A_{-1/2} \) of \( \Lambda \) in \( |p_\Lambda| \). Short-distance QCD interactions can alter the ratio of Eq.(1) by \( O(\alpha_s M_s/\pi E_s) \) for \( M_s \ll E_s \). This is relatively a small correction even for only moderately fast \( \Lambda \); for instance, \( \alpha_s M_s/\pi E_s \approx 0.08 \) for \( \alpha_s = \frac{1}{2} \) and \( \gamma(= E_s/M_s) = 2 \) in the \( \overline{\mathcal{B}} \) rest frame. Our argument would obviously break down when a long-distance interaction plays a role in \( \Lambda \) production, for instance, when \( \Lambda \) is produced by \( \overline{\mathcal{B}} \rightarrow \Sigma(1385)X \rightarrow \pi \Lambda X \). It is easy to remove such a \( \Lambda \) resonance band, if any.

Which reference frame should we choose for Eq.(1)? In a fast-moving frame of \( \overline{\mathcal{B}} \) where the \( s \) quark moves even faster, the \( \pi_L \) field would produce the \( s \) quark almost entirely in the \( h = -\frac{1}{2} \) state since helicity is invariant under the Lorentz boost. On the other hand, if one made the helicity projection in the \( s \)-quark rest frame, the \( \pi_L \) field would lead to \( h = \pm \frac{1}{2} \) in a 50-50 probability. This apparent frame dependence is not physical, of course. The reason is that Eq.(1) is only a projection of the \( s \)-quark plane-wave by \( 1 - \gamma_5 \). The complete decay amplitude is frame independent after the remainder of matrix element is combined. To see the point, we show the frame independence for the hadronic two-body decay \( \overline{\mathcal{B}} \rightarrow \Lambda(p_h)\overline{\pi}(p'h') \) instead of a quark process. The decay amplitude is of the form \( \overline{\pi}_{ph}(A + B\gamma_5)v_{p'h'} \). In the two-component helicity spinors, it can be expressed as

\[
\chi_h^\uparrow \left( -A\sigma_3 \sinh \frac{\eta - \eta'}{2} + B \cosh \frac{\eta - \eta'}{2} \right) \chi_{h'},
\]

where \( \eta(\geq 0) \) and \( \eta'(\leq 0) \) are the rapidities (\( \tanh \eta = p/E \)) of \( \Lambda \) and \( \overline{\pi} \), respectively. (For \( \eta > \eta' \geq 0, \chi_{h'} \rightarrow \chi_{-h'} \)). Therefore the ratio of two \( \Lambda \)-helicity amplitudes is given by

\[
\frac{A_{+\frac{1}{2}}}{A_{-\frac{1}{2}}} = \frac{B - A \tan \frac{1}{2}(\eta - \eta')}{B + A \tan \frac{1}{2}(\eta - \eta')} \quad \text{for} \quad \overline{\mathcal{B}} \rightarrow \Lambda \overline{\pi}.
\]

This is manifestly frame independent since the rapidity difference is invariant under the longitudinal Lorentz boost. In the \( \Lambda \) rest frame, for instance, the \( \Lambda \) helicity is determined by the helicity of the fast moving \( \overline{\pi} \) through overall angular momentum conservation. In fact, Eq.(3) holds more generally. For \( \overline{\mathcal{B}} \rightarrow \Lambda X \), we can lump \( X \) together into a single spinor of general spin and write the decay amplitude as \( \overline{\pi}_{ph}(A + B\gamma_5)\rho_{\mu\nu}p_\mu v_{p'h'} \) since \( \gamma_\mu \), \( \gamma_\mu \gamma_5 \), and \( \sigma_{\mu\nu} \) either reduce to 1 and \( \gamma_5 \) or drop out by the Dirac equation or by the subsidiary conditions on \( v_{p'h'} \). Then Eq.(3) is reproduced. The boost invariance of Eq.(3) is nothing more than
Lorentz invariance of the entire decay amplitude. The frame-independence argument holds likewise at the quark level though the individual emission and absorption vertices of quarks and gluons are not scalars nor pseudoscalars. We shall use the form of Eq. (3) as a guide to make our choice of frame.

The choice of frame would not be an issue if the $s$-quark mass were zero ($A_{+1/2}/A_{-1/2} \to 0$). If we approach the problem by computing weak decay form factors, we have to know all relevant form factors including the quark-mass and transverse-momentum corrections. The numerator of Eq. (4) is such a correction term arising from difference of two large form factors. In perturbative QCD and the light-cone description of hadrons, we can obtain it, in principle, by computing higher twist terms with spin-dependent quark distribution functions. In practice, however, it is difficult to reach quantitatively reliable answers even for the $B$ decay into two mesons. No factorization limit exists for $B \to \Lambda X$. Giving up computing the $O(M_s)$ terms of form factors, we shall present alternative semiquantitative results by stretching the perturbative QCD picture to the limit.

There is one basic problem about quark rapidities. While we can determine a hadron rapidity directly from experiment, a quark rapidity inside a hadron has a continuous distribution which we do not know precisely. We circumvent this problem by introducing an approximation. Since $\Lambda$ is an $s$-wave ground state in the rest frame, one reasonable approximation is to substitute $E_s$ and $p_s$ with their average values inside $\Lambda$:

$$\begin{align*}
E_s & \to \langle E_s \rangle \simeq \frac{M_s}{M_u + M_d + M_s} E_\Lambda, \\
p_s & \to \langle p_s \rangle \simeq \frac{M_s}{M_u + M_d + M_s} p_\Lambda,
\end{align*}$$

(4)

where $M_s/(M_u + M_d + M_s) \simeq 0.45$. Eq. (4) means that the $s$ quark moves on average with the same Lorentz factor $\gamma$ as $\Lambda$ does in a moving frame.

Now we choose the frame in which the helicity amplitude ratio is determined with Eqs. (2) and (3). The rest frame of $\overline{B}$ may come to our mind as an obvious choice. But a better alternative is the rest frame of $X$ for $\overline{B} \to \Lambda X$. In this frame, rapidity $\eta' = 0$ in Eq. (2) or its generalization to $\overline{B} \to \Lambda X$ so that the helicity ratio depends only on $\eta$. If we want to express the ratio in terms of the energy-momentum of $\Lambda$ quark alone without involving $X$, therefore, we should choose the rest frame of $X$. The helicity amplitude ratio is given by Eq. (1) with the energy-momentum $E'_\Lambda$ and $p'_\Lambda$ of the $X$ rest frame:

$$\begin{align*}
\frac{A_{+\frac{1}{2}}}{A_{-\frac{1}{2}}} &= \frac{E'_\Lambda + m_\Lambda - |p'_\Lambda|}{E'_\Lambda + m_\Lambda + |p'_\Lambda|} = \frac{m_\Lambda}{E'_\Lambda + |p'_\Lambda|},
\end{align*}$$

(5)

where $M_u + M_d + M_s \simeq m_\Lambda$ has been used for the constituent quark masses. We would

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1 Spin dependence was studied extensively by theorists for $B \to J/\psi K^*$. In this decay the dominant contribution to the $h = \pm 1$ final helicities arises from $m_c \neq 0$, not from $M_s \neq 0$. While theoretical predictions have converged to the experimental values \cite{3,4} with time, one sees how widely theoretical predictions used to spread when no data were available. \cite{3}
obtain Eq. (5) if we simply project the Λ field onto \((1 - \gamma_5)\psi_Λ\). The ratio of Eq. (5) can be expressed in terms of the quantities in \(B\) rest frame as

\[
\frac{A_{+\frac{1}{2}}}{A_{-\frac{1}{2}}} = \left( \frac{m_Λ}{E_Λ + |\mathbf{p}|} \right) \left( \frac{m_X}{E_X + |\mathbf{p}|} \right) \equiv \delta(E_Λ), \tag{6}
\]

where \(|\mathbf{p}|^2 = E_Λ^2 - m_Λ^2, E_X = m_B - E_Λ,\) and \(m_X^2 = m_B^2 - 2m_BE_Λ + m_Λ^2\).

### III. TREE AND PENGUIN INTERACTIONS

Even in the Standard Model, a hard \(s_R\) can contribute to formation of \(Λ\) through the penguin interaction. To test the Standard Model with the \(Λ\) helicity, therefore, we need to know the \(s_R\) contribution of the penguin interaction. For the purpose of separating this \(s_R\) from \(s_L\), we parametrize relative importance of the penguin interaction to the tree interaction by

\[
p = \frac{dΓ_{\text{penguin}}/dE_Λ}{dΓ_{\text{tree}}/dE_Λ}, \tag{7}
\]

where \(p\) is generally a function of \(E_Λ\). Even in the two-body meson decays, \(B \to K\pi\) and \(B \to ππ\), the relative weight of the two types of interactions has not been well determined from experiment. It is generally agreed among theorists that when \(X\) has net strangeness zero, the dominant interaction is the penguin interaction, \(i.e., p > 1\) though the tree interaction may not be totally negligible. Theoretical uncertainties are smaller for inclusive decays, but the limited accuracy of \(V_{ub}\) at present still makes it difficult to determine the value of \(p\) with certainty; \(p \approx 3 - 10\) for \(|V_{ub}| = 0.0025 - 0.0048\). Fortunately, however, the Standard Model prediction turns out to be insensitive to the value of \(p\). When \(X\) has one unit of net strangeness \((X_π)\), the penguin interaction \((\bar{b}d)(\bar{s}s)\) and the tree interaction \((\bar{b}u)(\bar{u}d)\) followed by \(\bar{u}u \to \bar{s}s\) are responsible for the decay. As for the relative strength between \(X\) of \(X_π\), the smallness of \(|V_{td}/V_{ts}|\) and \(|V_{ub}|\) suppresses \(X_π\) relative to nonstrange \(X\). In the two-body meson decays, this statement suggests \(B(B \to K\bar{K}, ππ) \ll B(B \to Kπ)\); experimentally \([10,11]\), \(B(B \to K^{+}π^-)/B(B \to π^+π^-) \simeq 3.3 \pm 0.5\) and \(B(B \to K^+π^-)/B(B \to K^+K^-) > 15\). It therefore, its reasonable to expect that net strangeness of \(X\) is most often zero in \(\bar{B} \to ΛX\). We proceed with the approximation that \(X\) has net strangeness zero. When a value of \(p\) is relevant, we choose \(p \gg 1\) to reflect the penguin dominance in \(\bar{B} \to ΛX\); more specifically, in the range of

\[
p \approx 6 \pm 3. \tag{8}
\]

That is what we expect since the \(Λ\) spin is equal to the \(s\)-quark spin, and \(Λ\) and \(s\) move with the same Lorentz factor. Such a projection is obviously not valid for the decay \(Λ \to πN\) since the process involves very strong nonperturbative effects, the long-distance \(ΔI = \frac{1}{2}\) enhancement.
IV. HELICITY RATIO FROM ANGULAR ASYMMETRY

The angular distribution of the cascade decay $\bar{B} \rightarrow \Lambda X \rightarrow \pi^- pX$ can be written in the form

$$\frac{d^2\Gamma}{dE_\Lambda d\cos\theta} = \frac{1}{2} \frac{d\Gamma}{dE_\Lambda} (1 + \bar{\alpha}\cos\theta),$$

(9)

where $\theta$ is defined as the emission angle of proton in the $\Lambda$ rest frame that is measured from the direction of the $\Lambda$ momentum $p$ of the $\bar{B}$ rest frame. Knowing the ratio $\delta(E_\Lambda)$ for $\Lambda$ helicity from Eq.(6), we can relate $\alpha$ to the $\Lambda$ decay asymmetry parameter $\alpha_\Lambda = 0.642 \pm 0.013$.[11]. Taking account of coexistence of the tree and penguin interactions, we can express $\alpha$ in terms of $\alpha_\Lambda$ by counting left and right-chiral $s$ fields in $(\bar{b}_L s_L)(u_L q_L + q_R q_R)$ ($q = u, d, s$) of the QCD penguin interaction:

$$\bar{\alpha} = -\left(1 + \frac{4}{5}p\right)\left(1 - \frac{\delta(E_\Lambda)^2}{1 + \delta(E_\Lambda)^2}\right)\alpha_\Lambda.$$

(10)

where we have ignored the electroweak penguin interaction, the interference between the tree and the QCD penguin (an approximation better for inclusive than exclusive decays), and the phase space difference between $u/d$ and $s$. The first factor varies only from 0.9 to 0.8 over the range of $p$ from 1 to $\infty$. The second factor in the right-hand side of Eq.(10) is practically unity over a wide range of $E_\Lambda$ except near the low energy end. The asymmetry $\bar{\alpha}$ approaches zero in the slow limit of $\Lambda$ ($\delta(m_\Lambda) = 1$). This limiting value is a kinematical constraint since no preferential direction exists in space in this limit where all momenta are either zero or integrated over. The asymmetry $\bar{\alpha}$ moves rapidly from 0 to about $-0.4\alpha_\Lambda$ up to the factor $(1 + \frac{4}{5}p)/(1 + p) \simeq 1$. The negative $\bar{\alpha}$ means the $h = -\frac{1}{2}$ dominance for the $s$ quark from the $s_L$ field.

If a nonstandard interaction generates the QCD penguin interaction $\bar{b}(1 - \kappa\gamma_5)s(\bar{q}q)$, the asymmetry is

$$\bar{\alpha} = -\left(1 + \frac{8\kappa p}{5(1 + \kappa^2)}\right)\left(1 - \frac{\delta(E_\Lambda)^2}{1 + \delta(E_\Lambda)^2}\right)\alpha_\Lambda.$$

(11)

If a significant amount of $s_R$ mixes in the QCD penguin interaction, $\bar{\alpha}$ would be close to zero or even positive in contrast to the negative values predicted for the Standard Model. While a precise value of asymmetry depends on the value of $p$, $\bar{\alpha}$ would show a marked departure from the prediction of the Standard Model in this case. This is the helicity test that we propose in this paper.

Plotted in Figure 1 is the asymmetry $\bar{\alpha}$ expected for $\bar{B} \rightarrow \Lambda X$ in the Standard Model. The curve is plotted for $p = 3$ so that it is subject to a small uncertainty of $\pm 2\%$ (for $p = 3$ to 9). The perturbative QCD correction below $m_b$ of $O(\alpha_s M_s/\pi E_s)$ is the main uncertainty, which is $O(10\%)$ at the higher half of the $E_\Lambda$. While the QCD correction is process dependent, a deviation of 20\% or more from the curve in Figure 1 will be a clear warning sign of a wrong helicity $s$ quark in weak interaction, or else, breakdown of perturbative QCD in final-state interactions.
FIG. 1. The asymmetry $\alpha$ in the Standard Model plotted against the $\Lambda$ energy in the $B$ rest frame (for $p = 3$).

A comment is in order on the background from the $b \to c$ transition. The interaction $b \to c_L \bar{u}_L(c_L) s_L$ can only lower $\alpha$, that is, increase the magnitude of $|\alpha|$ since the $s$ quark field is left-chiral. The final state of the lowest mass for $b \to c_L \bar{u}_L(u_L)$ is $\Lambda D\bar{N}$ ($5.26$ GeV vs $m_B = 5.28$ GeV). The phase space suppression virtually eliminates this mode. The cascade weak decay $\bar{B} \to \Lambda_c X \to \Lambda X'$ through $b \to c_L \bar{u}_L d_L \to s_L \bar{u}_L d_L \bar{u}_L d_L$ is more favorable in phase space and in the quark mixing. The branching fraction of $\Lambda_c \to \pi\Lambda$ is about $1\%$ and the inclusive branching to $\Lambda X$ is $\sim 10\%$. If the decay process $c_L \to s_L d_L u_L$ occurs perturbatively, the final $s$ quark is left-chiral so that it tends to lower $\alpha$. In any way we can separate the $\Lambda_c$ band, if necessary. Therefore the $b \to c$ transition will not pose a problem.

V. REMARK AND CONCLUSION

Analysis in the $B$ decay modes feeding $\Lambda$ is still at an early stage. Only an upper bound has been set on the branching to two-body baryonic channels, e.g., $B(B^+ \to p\Lambda) < 2.2 \times 10^{-6}$ [12]. However, the decay into three bodies, $\bar{B}^\pm \to p\bar{p}K^\pm$ has been observed with the branching fraction of $(4.3^{+1.1}_{-0.9} \pm 0.5) \times 10^{-6}$. The decay $B \to p\bar{\Lambda}\pi$ and the conjugate occur presumably at the same level of branching fraction. The inclusive decay events $\bar{B} \to \Lambda X$ will be seen abundantly in near future.

To conclude, measurement of the $\Lambda$ decay asymmetry in $\bar{B} \to \Lambda X$ is a sensible test to probe the chirality structure of the fundamental weak interaction. It will test whether the QCD penguin interaction possibly contains a nonstandard term such as $(\bar{b}s_R)(\bar{q}q)$ or not. Our numerical predictions contain inevitable uncertainties as we have delineated. Nonetheless, experimental determination of helicity with $\Lambda \to \pi^- p$ will be cleaner than that with the radiative $B$ decay. We believe that the decay $\bar{B} \to \Lambda X$ will be competitive with, if not superior to, $B \to \gamma^{(*)} X$ in testing the chiral structure of the penguin interaction at $B$ factories. It also has an advantage over $\Lambda_b \to \gamma \Lambda$ at hadron colliders where the $\Lambda_b$ polarization introduces another uncertainty.
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