Coincidence analysis of Stackelberg and Nash equilibria in three-player leader-follower security games

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Abstract

There has been significant recent interest in leader-follower security games, where the leader dominates the decision process with the Stackelberg equilibrium (SE) strategy. However, such a leader-follower scheme may become invalid in practice due to subjective or objective factors, and then the Nash equilibrium (NE) strategy may be an alternative option. In this case, the leader may face a dilemma of choosing an SE strategy or an NE strategy. In this paper, we focus on a unified three-player leader-follower security game and study the coincidence between SE and NE. We first explore a necessary and sufficient condition for the case that each SE is an NE, which can be further presented concisely when
the SE is unique. This condition not only provides access to seek a satisfactory SE strategy but also makes a criterion to verify an obtained SE strategy. Then we provide another appropriate condition for the case that at least one SE is an NE. Moreover, since the coincidence condition may not always be satisfied, we describe the closeness between SE and NE, and give an upper bound of their deviation. Finally, we show the applicability of the obtained theoretical results in several practical security cases, including the secure transmission problem and the cybersecurity defense.

Index Terms

Three-player security game, leader-follower scheme, Stackelberg equilibrium, Nash equilibrium, coincidence analysis.

I. INTRODUCTION

Security games, which usually describe situations that the protected system defends against malicious attacks, have been widely applied in many fields such as secure wireless communications, cyber-physical systems (CPS), and unmanned aerial vehicles (UAV). The three-player security game, as one of the important categories, models the interactive details about defense or attack operations by focusing on three different types of players, with a broad range in many important security scenarios. For instance, [1] investigated a physical layer security issue among a transmitter, a relay, and an eavesdropper, and [2] studied an advanced persistent threat (APT) problem among a defender, an insider, and an attacker, while [3] considered a vehicle formation problem among two vehicles and a jammer.

One classical game model to reflect players’ strategic behaviors in security games is based on leader-follower models [4]–[7]. In the models, the leader dominates the decision process and adopts its optimal strategy by taking account into the followers’ reaction, while the follower chooses the best response (BR) strategy after observing the leader’s strategy. The corresponding equilibrium is the well-known Stackelberg equilibrium (SE) [8]. In the three-player leader-follower game, there is a tri-level hierarchical structure: the top level, the middle level, and the
bottom level. Accordingly, players at high levels are called leaders, while players at low levels are called followers. For example, the source-destination pair at the bottom level is a follower and decides the required transmit power based on the observed strategies of the power station and the jammer [9]. Besides, the defender at the top level is a leader and chooses its defense rate with the consideration of the attacker and the insiders’ strategies [10].

However, such a leader-follower scheme may become invalid in practice, because the low-level player may lose the ability or interest to adopt the BR strategy and even ruin the leader-follower scheme for different reasons, including the limitation of the observation ability, the disturbance of the environment, and the stealthy of the player’s existence. In fact, the jammer may have observation errors due to the uncertainty of the time-variant channel states [11]; the terrorists may choose to directly act in consideration of the expensive surveillance cost of the defense strategy [12]; and the attacker may turn to the stealthy attack scheme instead of the leader-follower scheme to avoid the defender’s fault detection [13].

Hence, when the low-level player does not strictly comply with the leader-follower scheme, the high-level player will lose its corresponding dominant position, since its SE strategy is no longer the optimal one against the low-level player’s non-BR strategy. In this view, a simultaneous-move game model may be another acceptable description, and the best-known solution concept therein is the Nash equilibrium (NE), where players choose their optimal strategies independently without observation and dominance [14]–[16]. Since no one can benefit from changing its strategy unilaterally, it is acceptable for the high-level player to accomplish such an NE when its SE is not available. In some practical security problems, the high-level player may take the NE strategy when the low-level player has the observation barrier [17], and may tolerate an NE to avoid an unsatisfactory outcome [18].

Given the above consideration, a high-level player may have to face a dilemma: which strategy should be adopted, an SE in the leader-follower scheme or an NE in the simultaneous-move scheme? Clearly, the conflict among players’ strategies under different schemes may result in
the failure to achieve either SE or NE and may bring a loss in the utility for the high-level player. However, provided that SE coincide with NE, the high-level player will not suffer from these misgivings anymore. If so, the high-level player can take an SE strategy since its utility is as the same as that of taking an NE strategy. Moreover, when the coincidence relationship is not satisfied, the high-level player can still be fairly reassured of an SE strategy if the SE is quite close to an NE, and the brought gap in the high-level player’s utility is small and tolerated. Such analogous discussions on the relationship between SE and NE have already been a hot topic in security games, and have been analyzed on two-player models such as the radio transmission problem [11] and the security deployment issue [15].

Therefore, this paper focuses on how to help high-level players get rid of the dilemma about the strategy selection in a three-player security game. Specifically, we explore the coincidence condition when an SE is an NE. Moreover, if an effort fails, then we study the deviation between an SE and an NE.

**Contribution:**

We consider a three-player game model established for typical security problems, including secrecy capacity optimization [6], cooperative secure communication [1], and APT defense [10]. Compared with existing literatures, this is the first work that studies the coincidence relationship between SE and NE under a three-player game-theoretical problem. Firstly, we explore a necessary and sufficient condition such that each SE is an NE, and present its concise form when the SE is unique. This coincidence analysis not only develops an approach to seek an SE that exactly meets an NE, but also provides a criterion to verify whether an obtained SE is an NE. When sometimes not all SE are NE, we further focus on whether there exists an SE that coincides with an NE and provide a condition to find that at least one SE is an NE, in which the high-level players can accurately adopt a satisfactory SE strategy. Secondly, considering that the coincide relationship may not exist in all the practical situations, we give an upper bound of the deviation between SE and NE to measure their closeness, in order to reassure the high-level
player for still adopting an acceptable SE strategy. Finally, we show the applicability of the obtained theoretical results in several practical security cases, including the secure transmission and the cybersecurity defense.

**Related work:**

Of particular relevance to this work is the research on three-player security games. Accordingly, wireless communication is one of the most important fields to investigate three-player models. In [6], the macro base station (MBS) employed the jamming SBSs to jam the external eavesdropping for secure transmission, while the jamming SBSs required offloading service from the helping SBSs to satisfy the users. In [9], the source-destination pair at the top level priced the energy transmitted to the middle-level jammer for maximizing the secrecy rate, and the jammer decided the required transmit power to the bottom-level power station for broadcasting energy. Moreover, in [1], the source defended against the eavesdropper with the help of the relay for secure communication by employing a leader-follower scheme. Also, there are other fields involving three-player games. As for CPS security [2], [10], [19], a defender-insider-attacker game model was widely used to study stealthy behaviors and insider threats. In UAV formation [3], a zero-sum game with two vehicles and a jammer was proposed to analyze mobile intruder jamming.

Another highly relevant topic to this study is about relationships between SE and NE, which has been investigated in some two-player leader-follower security games [11], [15], [20], [21]. For instance, [15] considered a security deployment issue and derived a sufficient condition related to the defender’s strategic allocation subset such that the defender’s SE strategy is also an NE strategy. Afterward, [20] extended this condition into a Markov game under the moving target defense background to analyze the optimal strategy for resource placement. Moreover, [11] compared the effectiveness of SE and NE in a power control problem to investigate the impact of the observation accuracy of the jammer, while [21] used the hypergame framework to discuss the robustness of SE strategies and NE strategies with misperception and deception.
II. THREE-PLAYER LEADER-FOLLOWER SECURITY GAME

We begin our study with a three-player leader-follower security game, which refines a unified formulation from several typical security games \[1\], \[6\], \[10\].

Define the three-player security game by

\[
G = \{X \cup Y \cup Z, \Omega_X \times \Omega_Y \times \Omega_Z, U_X \cup U_Y \cup U_Z\},
\]

where \(X\), \(Y\) and \(Z\) are three players. Besides, \(\Omega_X \subseteq \mathbb{R}\), \(\Omega_Y \subseteq \mathbb{R}\), and \(\Omega_Z \subseteq \mathbb{R}\) are the strategy sets of players \(X\), \(Y\), and \(Z\), respectively, where \(\Omega_X = \{x|x_{\text{min}} \leq x \leq x_{\text{max}}\}\), \(\Omega_Y = \{y|y_{\text{min}} \leq y \leq y_{\text{max}}\}\), and \(\Omega_Z = \{z|z_{\text{min}} \leq z \leq z_{\text{max}}\}\). Moreover, \(U_X : \Omega_X \times \Omega_Y \times \Omega_Z \rightarrow \mathbb{R}\), \(U_Y : \Omega_X \times \Omega_Y \times \Omega_Z \rightarrow \mathbb{R}\), and \(U_Z : \Omega_X \times \Omega_Y \times \Omega_Z \rightarrow \mathbb{R}\) are the utility functions of players \(X\), \(Y\), and \(Z\), respectively. Each player aims at maximizing its own utility. Specifically,

\[
U_X(x, y, z) = B(x) + f_x(y, z)x, \tag{1a}
\]

\[
U_Y(x, y, z) = f_{y1}(x, z)y + f_{y2}(x, z), \tag{1b}
\]

\[
U_Z(x, y, z) = f_z(x, y, z). \tag{1c}
\]

One classical game model to reflect players’ strategic behaviors is the leader-follower model \[8\], and this hierarchical interplay reflected in the three-player game \(G\) is a tri-level structure, that is, \(X\) at the top level, \(Y\) at the middle level, and \(Z\) at the bottom level. Players at high levels are called leaders, while players at low levels are called followers. The decision-making order is as follows.

(1) The top-level player \(X\) first determines its strategy \(x\) to maximize its utility;
(2) Observing \(x\), the middle-level player \(Y\) then chooses its strategy \(y\) to maximize its utility;
(3) Observing \(x\) and \(y\), the bottom-level player \(Z\) finally adopts \(z\) to maximize its utility.

Many security problems can be modeled by the generalized leader-follower game \(G\). Here we introduce three practical examples, which will be further investigated in Section \[V\].

**Cooperative secure transmission** Consider a secure transmission problem in a downlink heterogeneous network \[6\], \[22\], \[23\]. The macro base station (MBS) within the network em-
ploys some small base stations (SBS) as jamming SBSs to jam the external eavesdropper for maximizing the secrecy rate, and each jamming SBS obtains the offloading service from the rest of the SBSs (called helping SBSs) in the cluster to satisfy the users. Set the number of jamming SBSs as one for simplification, as well as the helping SBSs. In Fig. 1, the MBS is player $\mathcal{X}$, the jamming SBS is player $\mathcal{Y}$, and the helping SBS is player $\mathcal{Z}$.

Fig. 1: Tri-level cooperative secure transmission problem.

**Adversarial cooperative communication** Consider an adversarial cooperative communication system in a wireless network, in which the transmission from the source to the destination is subject to an eavesdropping attacks from an adversary [1], [24], [25]. To achieve cooperative communication and defend against eavesdropping attacks, the source purchases the transmit powers from a selected relay, and this relay provides its relaying service for the source to obtain benefits, while the eavesdropper broadcasts its jamming signal to disrupt the transmission. In Fig. 2, the source, the relay, and the eavesdropper are player $\mathcal{X}$, $\mathcal{Y}$, and $\mathcal{Z}$, respectively.

Fig. 2: Tri-level defending against active eavesdropping attack problem.

**Advanced persistent threat** Consider an advanced persistent threat with advanced attacks and insider threats [2], [10]. The defender and the attacker take actions to gain control of the resource in the system, while an insider with a privileged access to the system can monitor the defender’s action and trade information to the attacker for its own profit. In Fig. 3, the defender is player $\mathcal{X}$, the insider is player $\mathcal{Y}$, and the attacker is player $\mathcal{Z}$.

In the leader-follower scheme of $\mathcal{G}$, the low-level players adopt the best response (BR) strategies based on the observed strategies of the high-level players, while the high-level players
compute their optimal strategies by considering low-level players. Denote $Z$’s best response to $Y$’s strategy $y$ and $X$’s strategy $x$ by
\[ BR_z(x,y) = \{ \omega \in \Omega_Z : U_Z(x,y,\omega) \geq U_Z(x,y,z), \forall z \in \Omega_Z \}. \]

Denote $Y$’s best response to $X$’s strategy $x$ by
\[ BR_y(x) = \{ \xi \in \Omega_Y : \min_{z \in BR_z(x,y)} U_Y(x,\xi,z) \geq \min_{z \in BR_z(x,y)} U_Y(x,y,z), \forall y \in \Omega_Y \}. \]

In this case, we introduce the Stackelberg equilibrium (SE).

**Definition 1** For the three-player leader-follower game $G$, a strategy profile $(x_{SE}, y_{SE}, z_{SE})$ is said to be an SE if
\[
\min_{y \in BR_y(x_{SE})} \min_{z \in BR_z(x_{SE},y)} U_X(x_{SE},y,z) = \max_{x \in \Omega_X} \min_{y \in BR_y(x)} \min_{z \in BR_z(x,y)} U_X(x,y,z),
\]
with $y_{SE} \in BR_y(x_{SE})$ and $z_{SE} \in BR_z(x_{SE},y_{SE})$.

Overall, the conventional decision-making process of $G$ is given as follows, as shown in Fig. 4.

1. The bottom-level player $Z$ solves $BR_z(x,y) = \arg\max_{\omega \in \Omega_Z} U_Z(x,y,\omega)$ for any $x$ and $y$;
2. The middle-level player $Y$ solves $BR_y(x) = \arg\max_{\xi \in \Omega_Y} \min_{z \in BR_z(x,y)} U_Y(x,\xi,z)$ for any $x$;
3. Then $x_{SE} \in \arg\max_{x \in \Omega_X} \min_{y \in BR_y(x)} \min_{z \in BR_z(x,y)} U_X(x,y,z)$ is an SE strategy for $X$;
4. The strategy $y_{SE} \in BR_y(x_{SE})$ is an SE strategy for $Y$;
5. The strategy $z_{SE} \in BR_z(x_{SE},y_{SE})$ is an SE strategy for $Z$;
6. The strategy profile $(x_{SE}, y_{SE}, z_{SE})$ constitutes an SE of $G$.

We give the following assumption for game $G$. 

![Fig. 3: Tri-level APT problem.](image)
Assumption 1

1. $B(x) \in C^1$, $f_z(x, y, z) \in C^1$ in $z$, and $f_x(y, z) \in C^1$ in $y$. For $l = 1, 2$, $f_{yl}(x, y) \in C^1$ in $x$ and $y$. Moreover, $BR_y(x) \in C^1$ and $BR_z(x, y) \in C^1$ in $x$ and $y$.

2. $B(x)$ is concave in $x$ and $f_z(x, y, z)$ is concave in $z$.

Assumption 1 guarantees the existence of SE [8], [26], which was also adopted in many practical security problems such as secure transmission in the physical layer security [22], IoT computational resource trading mechanism [27], APT defense problem [10], and cloud data computing issues [28]. The assumption about the continuous differentiability of $BR_y(x)$ and $BR_z(x, y)$ guarantees that these best responses are single-valued mappings rather than set-valued mappings [8], [10], [22], implying that $y_{SE} = BR_y(x_{SE})$ for any given $x_{SE}$, and $z_{SE} = BR_z(x_{SE}, y_{SE})$ for any given $x_{SE}$ and $y_{SE}$. Moreover, Assumption 1 does not restrict the uniqueness of SE, which is more general than those in some previous works [1], [6], [29].

The following lemma, whose proof is in Appendix A., verifies the existence of an SE.

**Lemma 1** Under Assumption 1, there exists an SE of $\mathcal{G}$. 

Although the leader-follower scheme is indeed used in many security scenarios, it may become
invalid in practice. This is because the low-level player may lose the ability or interest to adopt the BR strategy and even ruin the leader-follower scheme, due to diverse factors such as the disturbance of the transmission environment in cognitive radio network [11], the expensive surveillance cost of the defense strategy in the deployed infrastructure protection [12], and the stealthy of the attack’s existence to avoid the fault detection [13]. Hence, the high-level player may not maintain its dominant position, since its SE strategy is no longer optimal against the low-level player’s non-BR strategy. In this view, the simultaneous-move game model may be an alternative option to reflect the practical situation, and the best-known solution concept is the Nash equilibrium [14].

**Definition 2** For the three-player leader-follower game $G$, a strategy profile $(x_{NE}, y_{NE}, z_{NE})$ is said to be an NE if

$$x_{NE} \in \arg\max_{x \in \Omega_X} U_X(x, y_{NE}, z_{NE}),$$

$$y_{NE} \in \arg\max_{y \in \Omega_Y} U_Y(x_{NE}, y, z_{NE}),$$

$$z_{NE} \in \arg\max_{z \in \Omega_Z} U_Z(x_{NE}, y_{NE}, z).$$

It is acceptable for the high-level player to accomplish such an NE when SE is not available, since no one can benefit from changing its strategy unilaterally. The following lemma verifies the existence of an NE, whose proof is in Appendix A.

**Lemma 2** Under Assumption 1, there exists an NE of game $G$.

On this basis, a high-level player has to decide which strategy should be adopted: an SE under the leader-follower scheme or an NE under the simultaneous-move scheme. Clearly, players may choose strategies with different schemes, and the derived conflict may bring a loss for the high-level player’s utility. Consider two possible cases for an explanation. One is that $\mathcal{X}$ adopts an SE strategy within the leader-follower scheme, while $\mathcal{Y}$ and $\mathcal{Z}$ act under
the simultaneous-move scheme. In this way, the utility of $X$ may be lower than that when $X$ acts under the simultaneous-move scheme, i.e., $U_X(x_{SE}, y_{NE}, z_{NE}) \leq U_X(x_{NE}, y_{NE}, z_{NE})$. The other is that $X$ adopts an NE strategy within the simultaneous-move scheme, while $Y$ and $Z$ act in the leader-follower scheme. This indicates that $X$’s utility may be lower than that when $X$ acts under the leader-follower scheme, i.e., $U_X(x_{NE}, BR_y(x_{NE}), BR_z(x_{NE}, BR_y(x_{NE}))) \leq U_X(x_{SE}, BR_y(x_{SE}), BR_z(x_{SE}, BR_y(x_{SE}))) = U_X(x_{SE}, y_{SE}, z_{SE})$.

However, it is worth mentioning that if an SE is actually an NE, then the high-level player will not meet these misgivings anymore. In such a case, the high-level player can be reassured to adopt an SE strategy since its utility is the same as that of taking an NE strategy. Therefore, we expect to solve the following problem:

• In what conditions, SE coincide with NE?

However, in many practical situations, SE and NE may not be identical. If their difference is little, the high-level player may still adopt an SE strategy. Hence, we further ask the following question:

• If the coincidence condition cannot be guaranteed, how to describe and measure the closeness between SE and NE?

III. SE COINCIDENT WITH NE

In this section, we explore the coincidence relationship between SE and NE in the three-player leader-follower security game $G$.

Let $(x_{SE}, y_{SE}, z_{SE})$ be an SE of $G$. It is clear that $(x_{SE}, y_{SE}, z_{SE})$ can be equivalently described as $(x_{SE}, BR_y(x_{SE}), BR_z(x_{SE}, BR_y(x_{SE})))$. Obviously, player $Z$’s SE strategy $BR_z(x_{SE}, BR_y(x_{SE}))$ becomes an NE strategy when $x_{SE} = x_{NE}$ and $y_{SE} = y_{NE}$. Hereupon, we focus on the SE strategies for $X$ and $Y$ in the sequel.

In the leader-follower decision-making process, by substituting $z$ with $BR_z(x, y)$, the composited utility function of $Y$ is $\hat{U}_Y(x, y) = f_{y1}(x, BR_z(x, y))y + f_{y2}(x, BR_z(x, y))$. The partial
derivative of \( \hat{U}_Y \) with regard to \( y \) is given as
\[
T_1(x, y) = \frac{\partial \hat{U}_Y(x, y)}{\partial y}.
\]

For any given \( x = x_{SE} \), we have \( T_1(x_{SE}, y) \). Moreover, by substituting \( y \) with \( BR_y(x) \), the composite utility function of \( X \) is \( \hat{U}_X(x) = B(x) + f_x(BR_y(x), BR_z(x, BR_y(x)))x \). Obviously, the gradient of \( \hat{U}_X \) is
\[
T_2(x) = \frac{\partial \hat{U}_X(x)}{\partial x}.
\]

On the other hand, under the simultaneous-move scheme, \( Y \) and \( X \) compute their optimal strategies based on the original \( U_Y(x, y, z) \) and \( U_X(x, y, z) \), respectively. Take
\[
T_3(x) = f_{y1}(x, BR_z(x, BR_y(x))).
\]
The value of \( T_3 \) in \( x_{SE} \) is equal to the partial derivative value of \( U_Y \) with regard to \( y \) in \((x_{SE}, y_{SE}, z_{SE})\). Then let us take
\[
T_4(x) = \frac{\partial B(x)}{\partial x} + f_x(y_{SE}, z_{SE}),
\]
which can be regarded as the partial derivative value of \( U_X \) in \( x \) for given \( y = y_{SE}, z = z_{SE} \).

Assume \( \delta(\cdot) \) as the neighbourhood of one point, \( \delta_-(\cdot) \) as the left neighbourhood and \( \delta_+(\cdot) \) as the right neighbourhood. The following assumption is about the local monotonicity of utility functions, which is more relaxed than the global monotonicity and strict monotonicity [30], [31].

**Assumption 2** For \( y \in \Omega_Y \), there exist \( \delta_-(y) \) and \( \delta_(y) \) such that \( \hat{U}_Y \) is monotone in \( \delta_-(y) \cap \Omega_Y \) and \( \delta_+(y) \cap \Omega_Y \). For \( x \in \Omega_X \), there exist \( \delta_-(x) \) and \( \delta_+(x) \) such that \( \hat{U}_X \) is monotone in \( \delta_-(x) \cap \Omega_X \) and \( \delta_+(x) \cap \Omega_X \).

In the following, we provide a necessary and sufficient condition for the case that each SE is an NE in the three-player leader-follower security game \( G \), whose proof is in Appendix B.

**Theorem 1** Under Assumptions 1 and 2, any SE is an NE if and only if there exists \( \delta(x_{SE}) \) and \( \delta(y_{SE}) \) such that
(i) \( T_1(x_{SE}, y) \cdot T_3(x_{SE}) \geq 0 \) for \( y \in \delta(y_{SE}) \cap \Omega_Y \);
(ii) \( T_4(x_{SE}) = 0 \) or \( T_2(x) \cdot T_4(x) > 0 \) for \( x \in \delta(x_{SE}) \cap \text{rint}(\Omega_X) \).

Theorem \([1]\) provides the coincidence condition to connect SE and NE. If the condition is satisfied, then the high-level players can get rid of the strategy selection dilemma, as they can safely adopt SE strategies. From the sufficiency perspective, the condition develops an approach to seek an SE that is exactly an NE. The approach covers all possible cases in which each player’s SE strategy may be the boundary point or interior point of its strategy set, so that we can directly confirm whether the set of SE is a subset of NE. On the other hand, from the necessity perspective, the condition provides a criterion to verify whether an obtained SE is an NE. The computation is not complicated because the partial derivatives therein may usually be zero \([1], [5]\), and are merely related to the local information of strategy sets.

In fact, this equilibrium coincidence analysis is important and can be employed in many practical security scenarios. In adversarial cooperative communication issues \([1], [24]\), the coincidence condition becomes an inequality merely depending on the strategy of the source \( X \) and the parameters of different channel gains. In APT problems with insider threats \([2], [10]\), the condition is embodied as inequalities related to the defense and attack cost parameters. Readers can see Section \([V]\) for more details.

Moreover, in the case when the SE is a unique solution, we have the following result, whose proof is in Appendix C.

**Corollary 1** Under Assumptions 1 and 2 and provided that the SE is unique, the SE is an NE if and only if there exist \( \delta(x_{SE}) \) and \( \delta(y_{SE}) \) such that

(i) \( T_1(x_{SE}, y) \cdot T_3(x_{SE}) \geq 0 \) for \( y \in \delta(y_{SE}) \cap \Omega_Y \);
(ii) \( T_2(x) \cdot T_4(x) \geq 0 \) for \( x \in \delta(x_{SE}) \cap \Omega_X \).

Moreover, when not all SE are NE, we turn our attention to whether there exists an SE that
is an NE, and provide a condition for the case that at least one SE is an NE in the following result, whose proof is shown in Appendix D.

**Theorem 2** Under Assumptions 1 and 2, at least one SE is an NE if and only if there exists $\delta(y_{SE})$ and $\delta(x_{SE})$ such that

(i) $T_1(x_{SE}, y) \cdot T_3(x_{SE}) \geq 0$ for $y \in \delta(y_{SE}) \cap \Omega_Y$;

(ii) $T_4(x) \cdot \text{sgn}(x - x_{SE}) \leq 0$ for $x \in \delta(x_{SE}) \cap \Omega_X$.

Theorem 2 shows a necessary and sufficient condition for the existence of an SE that is an NE. Different from the discussion of the entire SE set in Theorem 1, the analysis in Theorem 2 focuses on the specific SE. In this way, the high-level players can employ this condition to exactly find out a satisfactory equilibrium and adopt the corresponding SE strategy.

**IV. SE CLOSE TO NE**

In reality, the coincidence between SE and NE may not always happen. Therefore, we expect to find a way to measure the difference between SE and NE so as to help high-level players make a reasonable decision.

Here, we employ the Hausdorff metric to describe the closeness of SE and NE. Define the Hausdorff metric of two sets $A, B \subseteq \mathbb{R}^n$ by

$$H(A, B) = \max\{\sup_{a \in A} \text{dist}(a, B), \sup_{b \in B} \text{dist}(b, A)\}.$$ 

Denote $\Xi_{SE}$ as the set of SE strategy profile $(x_{SE}, y_{SE}, z_{SE})$, and $\Xi_{NE}$ as the set of NE strategy profile $(x_{NE}, y_{NE}, z_{NE})$. For any SE strategy profile $p^* \in \Xi_{SE}$ with $p^* \triangleq (x_{SE}, y_{SE}, z_{SE})$, take the operator $T_2(\cdot)$ on the element $x_{SE}$ from $p^*$ and denote $\Pi_{x_{SE}}$ as the image set of $T_2(x_{SE})$. Also, take the operator $T_1(\cdot)$ on the pair $(x_{SE}, y_{SE})$ from $p^*$ and denote $\Pi_{y_{SE}}$ as the image set of $T_1(x_{SE}, y_{SE})$. Similarly, for any NE strategy profile $q^* \in \Xi_{NE}$ with $q^* \triangleq (x_{NE}, y_{NE}, z_{NE})$, take the operator $T_2(\cdot)$ on the element $x_{NE}$ from $q^*$ and denote $\Pi_{x_{NE}}$ as the image set of $T_2(x_{NE})$. 

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Also, take the operation $T_1(\cdot)$ on the pair $(x_{SE}, y_{NE})$, where $x_{SE}$ is chosen from any given $p^*$ and $y_{NE}$ is chosen from any given $q^*$. Denote $\Pi_{y_{SE}}$ as the image set of $T_1(x_{SE}, y_{NE})$.

Then the closeness of SE and NE is estimated in the following result, whose proof is in Appendix E.

**Theorem 3** Under Assumption 1, if there exist constants $\kappa_1 > 0$, and $\kappa_2 > 0$ such that $\hat{U}_y$ is $\kappa_1$-strongly concave in $y$ and $\hat{U}_x$ is $\kappa_2$-strongly concave in $x$, then with $\max\{H(\Pi_{x_{SE}}, \Pi_{x_{NE}}), H(\Pi_{y_{SE}}, \Pi_{y_{NE}})\} < \eta$, we have $H(\Xi_{SE}, \Xi_{NE}) < \frac{(1+l)(\kappa_1+\kappa_2)}{\kappa_1\kappa_2}\eta$.

Theorem 3 provides the closeness of SE and NE by giving an upper bound of the distance between their corresponding sets. In addition to the Lipschitz constant $l$, the strong concavity constants $\kappa_1$ and $\kappa_2$, the upper bound of the Hausdorff metric $H(\Xi_{SE}, \Xi_{NE})$ is mainly affected by the maximal value between $H(\Pi_{x_{SE}}, \Pi_{x_{NE}})$ and $H(\Pi_{y_{SE}}, \Pi_{y_{NE}})$. Regarding this maximal value as a perturbation, it is clear that a lower perturbation yields a lower bound. If the bound is low enough, then SE can be regarded as close to NE. This indicates that high-level players can still be reassured to adopt the SE strategy, as the brought deviations in their utilities are tolerable.

Additionally, when both the SE and the NE are unique solutions in some security issues, we can obtain the upper bound of the distance between these two equilibrium points in the following result, whose proof can be easily modified from Theorem 3.

**Corollary 2** Under Assumption 1 with that both the SE and the NE are unique, if there exist constants $\kappa_1 > 0$, and $\kappa_2 > 0$ such that $\hat{U}_y$ is $\kappa_1$-strongly concave in $y$ and $\hat{U}_x$ is $\kappa_2$-strongly concave in $x$, then with $\|T_1(x_{SE}, y_{SE}) - T_1(x_{SE}, y_{NE})\| < \eta_1$ and $\|T_2(x_{SE}) - T_2(x_{NE})\| < \eta_2$, we have $\|p^* - q^*\| < (1 + l)(\frac{\eta_1}{\kappa_1} + \frac{\eta_2}{\kappa_2})$.

**V. Applications**

In this section, we demonstrate our theoretical results in several important security games (introduced in Section II), and further illustrate the equilibria relationship for different scenarios.
A. Adversarial cooperative communication with eavesdropping attack

Consider a security issue on defending against eavesdropping attacks in the cooperative communication system, consisting of a primary source (player $\mathcal{X}$), a relay (player $\mathcal{Y}$), and an eavesdropper (player $\mathcal{Z}$) [1], [24], [32]. The source first decides the transmit power purchased from the selected relay to defend against the eavesdropping attacks, and then the relay decides the price of the unit power, while the eavesdropper finally decides its jamming power to disrupt the legitimate transmission based on the channel information and behavioral information of the relay and the source. Denote $x$ as the amount of the purchased transmit power, $y$ as the price set by the relay, and $z$ as the amount of the jamming power. Referring to [1], [24], [32], the three-player game is modeled as

$$\begin{align*}
\max_{x \in [x_{\min}, x_{\max}]} U_X(x, y, z) &= \frac{d_1|h_{rd}|^2 x}{\eta + |h_{ed}|^2 z} - d_4 xy, \\
\max_{y \in [y_{\min}, y_{\max}]} U_Y(x, y, z) &= xy - d_3 x, \\
\max_{z \in [z_{\min}, z_{\max}]} U_Z(x, y, z) &= -\log_2\left(\frac{|h_{rd}|^2 x + \eta + |h_{ed}|^2 z}{\eta + |h_{ed}|^2 z}\right) - d_2 z,
\end{align*}$$

where $h_{rd}$ and $h_{ed}$ are the respective channel gains of the relay-destination link and eavesdropper-destination link with $h_{rd}, h_{ed} \in [0, 1]$, $\eta$ indicates the background noise on the channel, $d_1$ is the gain coefficient, and $d_i$ are the cost coefficients for $i = \{2, 3, 4\}$. In this model, we denote $U_X$ as the benefits of the source from the secure cooperative transmission with $f_x(y, z)x = \frac{d_1|h_{rd}|^2 x}{\eta + |h_{ed}|^2 z} - d_4 xy$, $U_Y$ as the combination of the relaying payment given by the source and the relay transmission cost with $f_{y1}(x, z)y = xy$ and $f_{y2}(x, z) = -d_3 x$, and $U_Z$ as the benefit of the eavesdropper from reducing secrecy capacity.

Due to the expensive cost of eavesdropping or selfish concerns for own benefits [1], [24], the eavesdropper or the relay may lose interest to obtain the whole transmission information and break down the leader-follower scheme. Thus, the cooperative communication may not be guaranteed and the source’s utility may suffer a loss. To reassure the source, we investigate the
coincidence between SE and NE in this three-player game. It can be derived that

\[ T_1(x, y) = x, \quad T_2(x) = \frac{d_1|h_{rd}|^4|h_{ed}|^2}{\phi_x(\ln 2d_2|h_{rd}|^4x + 2|h_{rd}|^2|h_{ed}|^2 - 2\ln 2d_2|h_{rd}|^2\phi_x)} - d_4y_{SE}, \]

\[ T_3(x) = x, \quad T_4(x) = \frac{d_1|h_{rd}|^2}{\eta + |h_{ed}|^2z_{SE}} - d_4y_{SE} \]

where \( \phi_x = \sqrt{\frac{|h_{rd}|^4x^2}{4} + \frac{|h_{rd}|^2|h_{ed}|^2x}{\ln 2d_2}}. \) Obviously, under Assumptions 1 and 2, there exists \( \delta(y_{SE}), \delta(x_{SE}) \) such that \( T_1(x_{SE}, y) \cdot T_3(x_{SE}) \geq 0, \) for \( y \in \delta(y_{SE}) \cap \Omega_Y \) and \( T_4(x) > 0 \) for \( x \in \delta(x_{SE}) \cap \text{rint}(\Omega_X) \). Thus, the coincidence condition in Theorem 1 is simplified to analyze \( T_2(x) \).

Due to \( y_{SE} = y_{\max} \), any SE is an NE if and only if there exists \( \delta(x_{SE}) \) such that

\[ \phi_x(|h_{rd}|^4d_2x + 2|h_{rd}|^2|h_{ed}|^2 - 2|h_{rd}|^2d_2\phi_x) < \frac{d_1|h_{rd}|^4|h_{ed}|^2}{2d_2d_4y_{\max}}, \quad \forall x \in \delta(x_{SE}) \cap \text{rint}(\Omega_X). \]  \( \text{(2)} \)

It follows from (2) that the coincidence condition in this problem is transformed into an inequality merely depending on the source’s strategy and the parameters of different channel gains. Moreover, the channel gain of relay-destination link \( |h_{rd}|^2 \) in (2) has a large impact on the players’ strategies and their utilities and may vary significantly due to the change of wireless networks [32]. Thus, we set \( |h_{rd}|^2 = 0.2 \) and \( |h_{rd}|^2 = 0.7 \) as two environment settings herein. Then we consider three strategy profiles: each player chooses the SE strategy \( (x_{SE}, y_{SE}, z_{SE}) \); the source takes the SE strategy while the relay and the eavesdropper adopt NE strategies \( (x_{SE}, y_{NE}, z_{NE}) \); each player chooses the NE strategy \( (x_{NE}, y_{NE}, z_{NE}) \). With these strategy profiles, Fig. 5 shows the utilities of the source in different settings. In Fig. 5(a), the SE does not coincide with the NE. If the source insists on the SE strategy, its utility may decrease from ideal case 1 to case 2 since the relay may not forward packets and the eavesdropper may become passive, which makes the leader-follower scheme invalid. Adopting the NE strategy is an acceptable choice for the source, as all players can still reach the equilibrium even when the cooperative communication may not be guaranteed, and the source’s utility in case 2 is higher than that in case 3. Thus, the source needs to make a trade-off between the SE and NE strategies. However, in Fig. 5(b), the SE is indeed the NE. It reflects that the source can be reassured to adopt the SE.
strategy and its utility does not change in each case. Thus, once the coincidence condition \(^2\) is satisfied, there is no strategy selection dilemma for the source. Regardless of whether the relay or the eavesdropper can obtain the whole transmission information, the system security can be guaranteed and the secure transmission performance can be improved \([1], [24], [32]\). Moreover, we establish the coincidence relationship of SE and NE by analyzing the complicated interplay among multiple hierarchies in a three-player problem, which is beyond the consideration of models merely involving two players \([11], [24]\).

![Fig. 5: Utilities of the source under different strategy profiles.](image)

### B. Advanced persistent threats (APT) with insider threats

Consider a three-player APT game with advanced attacks and insider threats in cyber security, consisting of a defender (player \(X\)), an insider (player \(Y\)), and an attacker (player \(Z\)) \([2], [10], [33]\). After the defender first determines its defense rate, the insider determines the amount of the traded inside information to the attacker, and finally, the attacker chooses its attack rate. Denote \(x\) as the defense rate of the defender, \(y\) as the amount of the traded information of the insider, and \(z\) as the attack rate of the attacker. Referring to \([10]\), this game is designed as

\[
\max_{x \in [x_{\min}, x_{\max}]} U_X(x, y, z) = \frac{x}{2z} - C_D x,
\]

\[
\max_{y \in [y_{\min}, y_{\max}]} U_Y(x, y, z) = \rho \frac{x}{2z} + y,
\]

\[
\max_{z \in [z_{\min}, z_{\max}]} U_Z(x, y, z) = 1 - \frac{x}{2z} - C_A (1 - y)^2 z - y,
\]
where $C_D$ is the cost for each defense action, $\rho < 1$ is the constant denoting the insider’s proportion in the system with the upper bound $y_{\text{max}} \leq \rho$ to restrict the capability of the insider, and $C_A$ is the cost for each attack action. The first term in $U_X$ is the gain from the protected system while the second term is the cost of recapturing the compromised resources, where $B(x) = -C_D x$ and $f_x(y, z) = \frac{x}{2z}$. The first term in $U_Y$ represents the profit of selling inside information, while the second term is the profit from the protected system, where $f_{y1}(x, z) = y$ and $f_{y2}(x, z) = \rho \frac{x^2}{2z}$. Moreover, the first two terms in $U_Z$ present the benefit from the compromised system resource, and the third term denotes the cost of launching attacks, while the last term means the cost of purchasing information from the insider.

Accordingly,

$$T_1(x, y) = 1 - \sqrt{\frac{\rho^2 C_A x}{2}}, \quad T_2(x) = (1 - \rho) \sqrt{\frac{C_A}{8x}} - C_D, \quad T_3(x) = 1, \quad T_4(x) = \frac{1}{2z_{\text{SE}}} - C_D.$$  

In this way, we obtain that under Assumptions 1 and 2, any SE is an NE if and only if

$$\frac{C_A}{C_D^2} \geq \frac{8x_{\text{max}}}{(1 - y_{\text{max}})^2} \quad \text{or} \quad \frac{C_A}{C_D^2} \leq \frac{2x_{\text{min}}}{(1 - y_{\text{min}})^2}. \quad (3)$$

From (3), the coincidence between SE and NE is mainly affected by the attack cost parameter $C_A$ and the defense cost parameter $C_D$. The configuration of these two parameters plays an important role in APT issues, and affects the utility of players [2], [10]. Set $C_A \in [0.44, 1.25]$ and $C_D \in [0.15, 0.55]$. Fig. 6(a) first provides the coincidence ratios between SE and NE under different settings of $C_A$ and $C_D$. Clearly, the ratio varies in different ranges with the changes of $C_A$ and $C_D$, and it increases when $C_A/C_D$ becomes large. Moreover, if $C_A$ and $C_D$ correspond to the dark areas, then SE coincide with NE, and the high-level players can safely take SE strategies.

On the other hand, Fig. 6(b) shows the defender’s utilities according to different parameter values in Fig. 6(a). The blue line describes the defender’s utility with SE strategies $(x_{\text{SE}}, y_{\text{SE}}, z_{\text{SE}})$, while the red line describes the defender’s utility with NE strategies $(x_{\text{NE}}, y_{\text{NE}}, z_{\text{NE}})$. As can be seen from each subfigure of Fig. 6(b), a smaller $C_D$ means that the defender can protect the system
with less cost, which corresponds to the higher utility; a larger $C_A$ means that attackers need
to take more cost to compromise the resource system, which also yields the defender’s higher
utility [10]. More importantly, when $C_A$ and $C_D$ satisfy condition [3], the defender’s utility in
the SE strategy is the same as that in the NE strategy. This indicates that the defender can
achieve efficient defense when $C_A$ and $C_D$ are maintained in an acceptable range, even facing
some misgivings brought by stealthy attacks or unknown insider trading in some APT issues
[10], [19], [33], including the three-player problem that only discusses NE [19].

(a) Coincidence ratios between SE and NE

(b) Utilities of the defender

Fig. 6: The relationship between SE and NE with different environment settings.

C. Cooperative secure transmission problems

Consider a secure transmission problem in a downlink heterogeneous network (HetNet),
consisting of an MBS (player $X$), a jamming SBS (player $Y$), and a helping SBS (player $Z$)
[5], [6], [22]. In the leader-follower scheme, the MBS first determines the amount of purchased
jamming power from the jamming SBS, and then the jamming SBS determines the associated
service price, while the helping SBS finally determines the amount of the provided offloading
service for the jamming SBS. Denote $x$ as the purchased jamming power of MBS, $y$ as the price
set by the jamming SBS for jamming service and offloading service, and $z$ as the amount of
offloading service provided by the helping SBS. Denote $R_s$ as the secrecy rate, describing the
difference between the achievable rate of the macrocell users and that of the eavesdropper. It
follows from reference [6] that the expression of $R_s$ is $R_M - \log_2(1 + \frac{P_M |g_{Me}|^2/N_0}{1 + |g_{je}|^2/N_0 + \sigma_{ke}})$, where $R_M$ is the achievable rate at macrocell users, $P_M$ is the MBS’s transmit powers, $\sigma_{ke}$ is the parameter related to transmitting powers of the unemployed SBS, $|g_{je}|$ and $|g_{Me}|$ are channel coefficients from the jamming SBS and the MBS, respectively, and $N_0$ is the variance of the additive white Gaussian noise. Referring to [5], [6], [22], the players’ utility functions are described as

$$
\max_{x \in [x_{\min}, x_{\max}]} U_X(x, y, z) = \lambda_M R_s - |g_{je}|^2 xy,
\max_{y \in [y_{\min}, y_{\max}]} U_Y(x, y, z) = |g_{je}|^2 xy - \theta x + \lambda_j \log(1 + z) - \tau y,
\max_{z \in [z_{\min}, z_{\max}]} U_Z(x, y, z) = y^2 \frac{z}{z + \alpha} - \omega z,
$$

where $\lambda_M$ denotes the unit profit for the secrecy rate, $\theta$ is the unit cost of the power consumption, $\tau$ is the economic incentive parameter, $\lambda_j$ and $\alpha$ are weighting factors, respectively, and $\omega$ is the unit cost. We denote $U_X$ as the benefit gained from the secrecy rate and the payment of employing the jamming SBS with $B(x) = \lambda_M R_s$ and $f_x(y, z)x = - |g_{je}|^2 xy$, $U_Y$ as the reward of providing jamming service and the diminishing benefit of offloading service with $f_{y1}(x, z)y = |g_{je}|^2 xy - \tau y$ and $f_{y2}(x, z) = -\theta x + \lambda_j \log(1 + z)$, and $U_Z$ as the profit of offering offloading service.

Accordingly,

$$
T_1(x, y) = |g_{je}|^2 x - \tau + \sqrt{\frac{\alpha}{\omega} - 1 - \alpha + \frac{\alpha y}{\omega}},
T_2(x) = \frac{\lambda_M \sigma_{Me} |g_{je}|^2}{\ln 2 N_0 (\zeta_x^2 + \sigma_{Me} \zeta_x)} - \frac{|g_{je}|^2 \lambda_j}{(\tau - |g_{je}|^2 x)^2},
T_3(x) = |g_{je}|^2 x - \tau,
T_4(x) = \frac{\lambda_M \sigma_{Me} |g_{je}|^2}{\ln 2 N_0 (\zeta_x^2 + \sigma_{Me} \zeta_x)} - |g_{je}|^2 y_{SE},
$$

where $\zeta_x = 1 + |g_{je}|^2 x/N_0 + \sigma_{ke}$ and $\sigma_{Me} = P_M |g_{Me}|^2 / N_0$.

In fact, the information transmission in HetNets is more vulnerable to malicious eavesdropping attacks than traditional single-tier networks, which makes it challenging to obtain a satisfactory equilibrium for the MBS and SBSs [22]. Consider the case that there exists an SE that is an NE. From Theorem [2], at least one SE is an NE of $G$ if and only if there exists $\delta(y_{SE})$ and $\delta(x_{SE})$ such that $y \geq \frac{\lambda_j}{\tau - |g_{je}|^2 x_{SE}} - \frac{\sqrt{\omega(1 - \alpha)}}{\sqrt{\alpha}}$, for $y \in \delta(y_{SE}) \cap \Omega_Y$ and $T_4(x) \cdot \text{sgn}(x - x_{SE}) \leq 0$
Fig. 7: To find that at least one SE is an NE with different environment settings.

for \( x \in \delta(x_{SE}) \cap \Omega_X \). Then we take two different environment parameter settings, and map the strategy spaces of all players on the space \( \Omega_X \times \Omega_Z \) for clarification. The red region represents the set of SE, while the blue region represents the set of NE. In Fig. 7(a), SE are always not NE by verifying the coincidence condition. Furthermore, in Fig. 7(b), there is only one SE that meets an NE. It is usually hard to reach this SE in secure transmission problems [5], [6], [22], since the probability of finding such a singleton is zero. However, by virtue of the derived condition in Theorem 2 we can obtain this SE precisely and conveniently. In this way, the MBS can commit to a satisfactory SE strategy to enhance the security of the macrocell and guarantee user satisfaction, even when the channels may be interfered with external noise and the SBSs’ observability may be lost.

On the other hand, we focus on the closeness of SE and NE. Recalling that Theorem 3 gives an upper bound of \( H(\Xi_{SE}, \Xi_{NE}) \), Fig. 8 reflects the variation trend of this bound under different environment settings. In Fig. 8 the horizontal axis represents the value of \( \eta \) in Theorem 3 while the vertical axis represents the bound of \( H(\Xi_{SE}, \Xi_{NE}) \), expressed as \( \frac{(1+\ell)(\kappa_1+\kappa_2)}{\kappa_1\kappa_2}\eta \). Set \( P_M = 15, 30, 60, 120(dBm) \) in Fig. 8(a) and set \( \lambda_j = 0.1, 0.3, 1, 3 \) in Fig. 8(b), which are involved in \( \kappa_1 \) and \( \kappa_2 \) in Theorem 3 respectively. These two environment parameters are important for secure transmission [6], [22]. As shown in Fig. 8, the smaller value of \( \eta \) leads to the lower bound of \( H(\Xi_{SE}, \Xi_{NE}) \). Also, Fig. 8(a) and Fig. 8(b) show that the performance gaps become small when \( P_M \) and \( \lambda_j \) increase, as they serve as reciprocal terms, respectively, in \( \frac{(1+\ell)(\kappa_1+\kappa_2)}{\kappa_1\kappa_2}\eta \). In a
nutshell, different from [22], the cooperation between the MBS and SBSs is further investigated
from the equilibria relationship view. The decline of the bound gap implies that although the
MBS and the SBSs may not be in the same game scheme due to the vulnerable transmission
channels, the brought conflict can be ignored and the deviation between the SE strategy and the
NE strategy is tolerable. Hence, the MBS and the SBSs can still achieve a win-win cooperation
for the security enhancement.

![Fig. 8: Closeness of SE and NE in different environment settings.](image)

VI. CONCLUSION

In this paper, we have focused on a three-player leader-follower security game and investigated
the coincidence between SE and NE. We have provided a necessary and sufficient condition such
that each SE is an NE and presented the concise form when the SE is unique. Besides, we have
provided another condition such that at least one SE is an NE. Moreover, we have given an
upper bound to measure their closeness once the coincidence condition fails. Finally, we have
shown the validity and applicability of our results in several practical security cases.

In the future, we may explore more deeply to extend the current research, including i)
generalizing the model to \( N \) players; ii) quantitatively analyzing the influence of the uncertainty
to the equilibrium; iii) investigating the equilibrium relationship for other game schemes.

APPENDIX A

**Proof of Lemma** [H] Note that \( \Omega_x, \Omega_y, \) and \( \Omega_z \) are finite sets. Since \( BR_y(x) \) is a subset of \( \Omega_y \)
for \( x \in \Omega_x \) and \( BR_x(x, y) \) is a subset of \( \Omega_z \) for \( x \in \Omega_x \) and \( y \in \Omega_y, \) \( G \) admits a Stackelberg
strategy for player $\mathcal{X}$ [34, Proposition 1]. Therefore, there exists an SE of $\mathcal{G}$. □

**Proof of Lemma 2** Recalling (1h)-(1l), $U_\mathcal{X}(x, y, z)$ is concave in $x$, $U_\mathcal{Y}(x, y, z)$ is linear in $y$, and $U_\mathcal{Y}(x, y, z)$ is concave in $z$, respectively. Together with the compactness and convexity of $\Omega_\mathcal{X}$, $\Omega_\mathcal{Y}$ and $\Omega_\mathcal{Z}$, there exists an NE of $\mathcal{G}$, referring to [35, Theorem 2.1]. □

**APPENDIX B**

**Proof of Theorem 1** We first prove the sufficiency.

Consider $y_{SE}$ and discuss coincidence condition (i) in two cases: $T_3(x_{SE}) = 0$ and $T_3(x_{SE}) \neq 0$.

(1a) For the case that $T_3(x_{SE}) = 0$, it is clear that $y_{SE} \in \text{argmax}_{y \in \Omega_\mathcal{Y}} U_\mathcal{Y}(x_{SE}, y, z_{SE})$ is player $\mathcal{Y}$’s NE strategy due to the concavity of $U_\mathcal{Y}$ in $y$.

(1b) For the case that $T_3(x_{SE}) \neq 0$, consider that $T_3(x_{SE}) > 0$ firstly. Together with condition (i), we obtain $T_1(x_{SE}, y) \geq 0$ for $y \in \delta(y_{SE}) \cap \Omega_\mathcal{Y}$. Suppose that $y_{SE} \in \text{rint}(\Omega_\mathcal{Y})$. On the one hand, when $T_1(x_{SE}, y_{SE}) \neq 0$, we have $T_1(x_{SE}, y_{SE}) > 0$. Then due to the continuity of $T_1(x_{SE}, y) = \frac{\partial U_\mathcal{Y}(x_{SE}, y)}{\partial y}$, there exists another point $y' \in \delta(y_{SE}) \cap \Omega_\mathcal{Y}$ such that $\hat{U}_\mathcal{Y}(x_{SE}, y') > \hat{U}_\mathcal{Y}(x_{SE}, y_{SE})$. This contradicts the definition of $y_{SE}$. On the other hand, when $T_1(x_{SE}, y_{SE}) = 0$, there exists $\delta'(y_{SE})$ such that $\hat{U}_\mathcal{Y}(x_{SE}, y_{SE}) > \hat{U}_\mathcal{Y}(x_{SE}, y)$ for $y \in \delta'(y_{SE}) \cap \Omega_\mathcal{Y}$, since $y_{SE} = BR_y(x_{SE})$ is a singleton. This implies that $T_1(x_{SE}, y) < 0$ for $y \in \delta'(y_{SE}) \cap \Omega_\mathcal{Y}$, which contradicts condition (i). Thus, $y_{SE} \notin \text{rint}(\Omega_\mathcal{Y})$. If $y_{SE} = y_{\min}$, then there exists $\delta''(y_{\min})$ such that $T_1(x_{SE}, y) < 0$ for $y \in \delta''(y_{\min}) \cap \Omega_\mathcal{Y}$. This also contradicts condition (i). Therefore, $y_{SE} = y_{\max}$ is the only possible situation. Moreover, due to the concavity of $U_\mathcal{Y}$ in $y$, it follows from $T_3(x_{SE}) > 0$ that $U_\mathcal{Y}(x_{SE}, y_{\max}, z_{SE}) \geq U_\mathcal{Y}(x_{SE}, y, z_{SE})$ for $y \in \Omega_\mathcal{Y}$. Thus, $y_{\max}$ is an NE strategy. The analysis for the case that $T_3(x_{SE}) < 0$ is similar. Accordingly, we obtain $y_{SE} = y_{\min}$, where $y_{\min} \in \text{argmax}_{y \in \Omega_\mathcal{Y}} U_\mathcal{Y}(x_{SE}, y, z_{SE})$.

Consider $x_{SE}$ and discuss condition (ii) in two cases: $T_4(x_{SE}) = 0$ and $T_4(x_{SE}) \neq 0$.

(2a) For the case that $T_4(x_{SE}) = \frac{\partial U_\mathcal{X}(x_{SE}, y_{SE}, z_{SE})}{\partial x} |_{x=x_{SE}} = 0$, it is clear that $x_{SE} \in \text{argmax}_{x \in \Omega_\mathcal{X}} U_\mathcal{X}(x, y_{SE}, z_{SE})$ is player $\mathcal{X}$’s NE strategy due to the concavity of $U_\mathcal{X}$ in $x$. DRAFT October 31, 2022
(2b) For the case that \( T_4(x_{SE}) \neq 0 \), with condition (ii), we have \( T_2(x) \cdot T_4(x) > 0 \) for \( x \in \delta(x_{SE}) \cap \text{rint}(\Omega_X) \). If \( x_{SE} \in \text{rint}(\Omega_X) \), then \( T_2(x_{SE}) = 0 \) due to the definition of \( x_{SE} \), which contradicts condition (ii). Thus, \( x_{SE} \notin \text{rint}(\Omega_X) \). If \( x_{SE} = x_{max} \), then there exists \( \delta'(x_{max}) \) such that \( T_2(x) \geq 0 \) for \( x \in \delta'(x_{max}) \cap \Omega_X \). Thus, \( T_2(x) > 0 \) for \( x \in \delta'(x_{max}) \cap \text{rint}(\Omega_X) \). Take \( \delta''(x_{max}) = \delta(x_{max}) \cap \delta'(x_{max}) \). Then we obtain \( T_4(x) > 0 \) for \( x \in \delta''(x_{max}) \cap \text{rint}(\Omega_X) \). Moreover, due to the continuity and monotonicity of \( T_4(x) \), \( T_4(x) > 0 \) for \( x \in \text{rint}(\Omega_X) \). Therefore, \( x_{max} \in \arg\max_{x \in \Omega_X} U_X(x, y_{SE}, z_{SE}) \), which indicates that \( x_{max} \) is an NE strategy. The analysis for \( x_{SE} = x_{min} \) is similar, where \( x_{min} \in \arg\max_{x \in \Omega_X} U_X(x, y_{SE}, z_{SE}) \).

Furthermore, \( z_{SE} = BR_z(x_{SE}, y_{SE}) \) of player \( Z \) becomes an NE strategy when \( x_{SE} = x_{NE} \) and \( y_{SE} = y_{NE} \). Thus, when the coincidence condition (i) and (ii) are satisfied, any SE is an NE.

Next, we prove the necessity. When \((x_{SE}, y_{SE}, z_{SE})\) is an NE, if \( y_{SE} \in \text{rint}(\Omega_Y) \), then \( T_3(x_{SE}) = 0 \). This indicates that there exists \( \delta(y_{SE}) \) such that \( T_1(x_{SE}, y) \cdot T_3(x_{SE}) \geq 0 \) for \( y \in \delta(y_{SE}) \cap \Omega_Y \). If \( y_{SE} = y_{max} \), then \( T_3(x_{SE}) \geq 0 \). Additionally, with the definition of SE, there exists \( \delta'(y_{max}) \) such that \( \hat{U}_Y(x_{SE}, y_{max}) \geq \hat{U}_Y(x_{SE}, y) \) for \( y \in \delta'(y_{max}) \cap \Omega_Y \), which yields \( T_1(x_{SE}, y) = \frac{\partial \hat{U}_Y(x_{SE}, y)}{\partial y} \geq 0 \) for \( y \in \delta'(y_{max}) \cap \Omega_Y \). Thus, \( T_1(x_{SE}, y) \cdot T_3(x_{SE}) \geq 0 \), for \( y \in \delta'(y_{max}) \cap \Omega_Y \). The analysis for the case that \( y_{SE} = y_{min} \) is similar.

On the other hand, if \( x_{SE} \in \text{rint}(\Omega_X) \), then \( T_4(x_{SE}) = 0 \). Moreover, when \( x_{SE} = x_{max} \), if \( T_4(x_{max}) = 0 \), then condition (ii) is satisfied. If not, then there exists \( \delta'(x_{max}) \) such that \( T_4(x) > 0 \) for \( x \in \delta'(x_{max}) \cap \text{rint}(\Omega_X) \). Moreover, recalling the definition of SE, there exists \( \delta''(x_{max}) \) such that \( \hat{U}_X(x_{max}) > \hat{U}_X(x) \) for \( x \in \delta''(x_{max}) \cap \text{rint}(\Omega_X) \), which yields \( T_2(x) = \frac{\partial \hat{U}_X(x)}{\partial x} > 0 \) for \( x \in \delta''(x_{max}) \cap \text{rint}(\Omega_X) \). Thus, by taking \( \delta(x_{max}) = \delta'(x_{max}) \cap \delta''(x_{max}) \), we have \( T_2(x) \cdot T_4(x) > 0 \) for \( x \in \delta(x_{max}) \cap \text{rint}(\Omega_X) \). The analysis for \( x_{SE} = x_{min} \) is similar.

\[ \square \]

**APPENDIX C**

**Proof of Corollary**\[ \] Notice that the coincidence condition for \( y_{SE} \) in Corollary\[ \] is the same as that in Theorem\[ \] so we omit it and focus on \( x_{SE} \).
Consider the sufficiency firstly. If $x_{SE} \in \text{rint}(\Omega_X)$, then $T_2(x_{SE}) = \frac{\partial U_X(x)}{\partial x} \bigg|_{x=x_{SE}} = 0$. Moreover, due to the uniqueness of $x_{SE}$, there exists $\delta'(x_{SE})$ such that $T_2(x) > 0$ for $x \in \delta'_-(x_{SE}) \cap \Omega_X$ and $T_2(x) < 0$ for $x \in \delta'_+(x_{SE}) \cap \Omega_X$. Together with condition (ii), by taking $\delta''(x_{SE}) = \delta(x_{SE}) \cap \delta'(x_{SE})$, we obtain $T_4(x) \geq 0$ for $x \in \delta''_-(x_{SE}) \cap \Omega_X$, $T_4(x) \leq 0$ for $x \in \delta''_+(x_{SE}) \cap \Omega_X$ and $T_4(x_{SE}) = 0$. Based on the concavity and continuity of $U_X$, we further have $T_4(x) \geq 0$ for $x \in [x_{\text{min}}, x_{SE})$ and $T_4(x) \leq 0$ for $x \in (x_{SE}, x_{\text{max}}]$. Thus, it is clear that $x_{SE}$ is player $X$’s NE strategy. If $x_{SE} = x_{\text{max}}$, then $T_2(x_{\text{max}}) \geq 0$, and there exists $\delta(x_{\text{max}})$ such that $T_2(x) > 0$ for $x \in \delta(x_{\text{max}}) \cap \text{rint}(\Omega_X)$. With condition (ii), denote $\delta''(x_{\text{max}}) = \delta(x_{\text{max}}) \cap \delta'(x_{\text{max}})$. Then $T_4(x) \geq 0$ for $x \in \delta''(x_{\text{max}}) \cap \Omega_X$. Obviously, $T_4(x) \leq 0$ for $x \in \Omega_X$, which indicates that $x_{\text{max}}$ is player $X$’s NE strategy. Besides, the analysis for $x_{SE} = x_{\text{min}}$ is similar.

Next, consider the necessity. If $x_{SE}$ is an NE strategy, then $x_{SE} \in \arg\max_{x \in \Omega_X} U_X(x, y_{SE}, z_{SE})$. If $x_{SE} \in \text{rint}(\Omega_X)$, then $T_4(x_{SE}) = 0$, $T_4(x) \geq 0$ for $x \in [x_{\text{min}}, x_{SE})$ and $T_4(x) \leq 0$ for $x \in (x_{SE}, x_{\text{max}}]$ due to the monotonicity of $T_4$. Also, since the SE is unique, it is clear that $T_2(x_{SE}) = 0$, and there exists $\delta(x_{SE})$ such that $T_2(x) > 0$ for $x \in \delta(x_{SE}) \cap \Omega_X$, $T_2(x) < 0$ for $x \in \delta_+(x_{SE}) \cap \Omega_X$. Thus, $T_2(x) \cdot T_4(x) \geq 0$ for $x \in \delta(x_{SE}) \cap \Omega_X$. If $x_{SE} = x_{\text{max}}$, then $T_4(x) \geq 0$ for $x \in \Omega_X$. Moreover, $T_2(x_{\text{max}}) \geq 0$, and there exists $\delta(x_{\text{max}})$ such that $T_2(x) > 0$ for $x \in \delta(x_{\text{max}}) \cap \Omega_X$. Therefore, $T_2(x) \cdot T_4(x) \geq 0$ for $x \in \delta(x_{\text{max}}) \cap \Omega_X$. Moreover, the analysis for $x_{SE} = x_{\text{min}}$ is similar.

**APPENDIX D**

**Proof of Theorem 2** The coincidence condition for $y_{SE}$ in Theorem 2 is the same as that in Theorem 1 so we omit it and focus on the analysis of $x_{SE}$.

Consider the sufficiency firstly. If $x_{SE} \in \text{rint}(\Omega_X)$, then $\text{sgn}(x_{SE} - x_{SE}) = 0$, $\text{sgn}(x - x_{SE}) = -1$ for $x \in \delta_-(x_{SE}) \cap \Omega_X$, and $\text{sgn}(x - x_{SE}) = 1$ for $x \in \delta_+(x_{SE}) \cap \Omega_X$. Together with condition (ii) of Theorem 2 it derives that $T_4(x) \geq 0$ for $x \in \delta_-(x_{SE}) \cap \Omega_X$ and $T_4(x) \leq 0$ for $x \in \delta_+(x_{SE}) \cap \Omega_X$. In this way, $T_4(x_{SE}) = 0$ due to the continuity of $T_4$, which implies that
$x_{SE}$ is player $\mathcal{X}$’s NE strategy. If $x_{SE} = x_{\max}$, then $\text{sgn}(x_{\max} - x_{\max}) = 0$, $\text{sgn}(x - x_{\max}) = -1$ for $x \in \delta_{-}(x_{\max}) \cap \Omega_{\mathcal{X}}$. Similarly, we have $T_4(x_{\max}) = 0$, which implies that $x_{\max}$ is player $\mathcal{X}$’s NE strategy. Also, the analysis for $x_{SE} = x_{\min}$ is similar.

Next, consider the necessity. When there exists an SE which is an NE, if $x_{SE} \in \text{rint}(\Omega_{\mathcal{X}})$, then $T_4(x_{SE}) = 0$, and there exists $\delta(x_{SE})$ such that $T_4(x) \geq 0$ for $x \in \delta_{-}(x_{SE}) \cap \Omega_{\mathcal{X}}$, $T_4(x) \leq 0$ for $x \in \delta_{+}(x_{SE}) \cap \Omega_{\mathcal{X}}$. Moreover, we have $\text{sgn}(x_{SE} - x_{SE}) = 0$, $\text{sgn}(x - x_{SE}) = -1$ for $x \in \delta_{-}(x_{SE}) \cap \Omega_{\mathcal{X}}$, and $\text{sgn}(x - x_{SE}) = 1$ for $x \in \delta_{+}(x_{SE}) \cap \Omega_{\mathcal{X}}$. Thus, $T_4(x) \cdot \text{sgn}(x - x_{SE}) \leq 0$ for $x \in \delta(x_{SE}) \cap \Omega_{\mathcal{X}}$. If $x_{SE} = x_{\max}$, then there exists $\delta(x_{\max})$ such that $T_4(x) \geq 0$ for $x \in \delta(x_{\max}) \cap \Omega_{\mathcal{X}}$. Also, $\text{sgn}(x_{\max} - x_{\max}) = 0$, $\text{sgn}(x - x_{\max}) = -1$ for $x \in \delta(x_{\max}) \cap \Omega_{\mathcal{X}}$. Hence, condition (ii) is also satisfied. Moreover, the analysis for $x_{SE} = x_{\min}$ is similar.

\[\square\]

**APPENDIX E**

**Proof of Theorem 3**

Recall that $BR_{2}(x_{SE}, y_{SE}) = z_{SE}$ and $BR_{2}(x_{NE}, y_{NE}) = z_{NE}$. Because $\Omega_{Y}$ and $\Omega_{\mathcal{X}}$ are compact with $BR_{2}(x, y) \in C$ in $x$ and $y$, there exists a constant $l > 0$ such that

$$\|BR_{2}(x_{SE}, y_{SE}) - BR_{2}(x_{NE}, y_{NE})\| \leq l(\|x_{SE} - x_{NE}\| + \|y_{SE} - y_{NE}\|).$$

Since $\hat{U}_{Y}$ is $\kappa_{1}$-strongly concave in $y$, $\kappa_{2}\|y_{SE} - y_{NE}\| \leq \|T_{1}(x_{SE}, y_{SE}) - T_{1}(x_{SE}, y_{NE})\|$. Also, since $\hat{U}_{\mathcal{X}}$ is $\kappa_{2}$-strongly concave in $x$, $\kappa_{2}\|x_{SE} - x_{NE}\| \leq \|T_{2}(x_{SE}) - T_{2}(x_{NE})\|$. Let $p^{*} = (x_{SE}, y_{SE}, z_{SE})$ be an SE and $q^{*} = (x_{SE}, y_{SE}, z_{SE})$ be an NE. Then we have $\|p^{*} - q^{*}\| \leq (1 + l)(\|x_{SE} - x_{NE}\| + \|y_{SE} - y_{NE}\|)$. Following the definition of $H(\Pi_{x_{SE}}, \Pi_{x_{NE}})$, we get

$$H(\Pi_{x_{SE}}, \Pi_{x_{NE}}) = \max\{\sup_{T_{2}(x_{SE}) \in \Pi_{x_{SE}}} \inf_{T_{2}(x_{NE}) \in \Pi_{x_{NE}}} \|T_{2}(x_{SE}) - T_{2}(x_{NE})\|, \sup_{T_{2}(x_{NE}) \in \Pi_{x_{NE}}} \inf_{T_{2}(x_{SE}) \in \Pi_{x_{SE}}} \|T_{2}(x_{SE}) - T_{2}(x_{NE})\|\};$$

$$\geq \max\{\sup_{T_{2}(x_{SE}) \in \Pi_{x_{SE}}} \inf_{T_{2}(x_{NE}) \in \Pi_{x_{NE}}} \kappa_{2}\|x_{SE} - x_{NE}\|, \sup_{T_{2}(x_{NE}) \in \Pi_{x_{NE}}} \inf_{T_{2}(x_{SE}) \in \Pi_{x_{SE}}} \kappa_{2}\|x_{SE} - x_{NE}\|\};$$
Similarly, \( H(\Pi_{ySE}, \Pi_{yNE}) \geq \max \left\{ \sup_{T_1(x_{SE}, ySE) \in \Pi_{ySE}} \inf_{T_1(x_{SE}, yNE) \in \Pi_{yNE}} \kappa_1 \|ySE - yNE\|, \sup_{T_1(x_{SE}, yNE) \in \Pi_{yNE}} \inf_{T_1(x_{SE}, ySE) \in \Pi_{ySE}} \kappa_1 \|ySE - yNE\| \right\} \).

Therefore,
\[
H(\Xi_{SE}, \Xi_{NE}) = \max \left\{ \sup_{\rho^* \in \Xi_{SE}} \inf_{\nu^* \in \Xi_{NE}} \|\rho^* - \nu^*\|, \sup_{\nu^* \in \Xi_{NE}} \inf_{\rho^* \in \Xi_{SE}} \|\rho^* - \nu^*\| \right\} \leq (1 + l)/\kappa_1 \max \left\{ \sup_{T_2(xSE) \in \Pi_{ySE}} \inf_{T_2(xNE) \in \Pi_{yNE}} \|T_1(x_{SE}, ySE) - T_1(x_{SE}, yNE)\|, \right. \\
\left. \sup_{T_2(xNE) \in \Pi_{yNE}} \inf_{T_2(xSE) \in \Pi_{ySE}} \|T_1(x_{SE}, ySE) - T_1(x_{SE}, yNE)\| \right\} \\
+ (1 + l)/\kappa_2 \max \left\{ \sup_{T_2(xSE) \in \Pi_{ySE}} \inf_{T_2(xNE) \in \Pi_{yNE}} \|T_2(xSE) - T_2(xNE)\|, \right. \\
\left. \sup_{T_2(xNE) \in \Pi_{yNE}} \inf_{T_2(xSE) \in \Pi_{ySE}} \|T_2(xSE) - T_2(xNE)\| \right\} \\
= (1 + l)/\kappa_1 H(\Pi_{ySE}, \Pi_{yNE}) + (1 + l)/\kappa_2 H(\Pi_{xSE}, \Pi_{xNE}) \leq \frac{(1 + l)(\kappa_1 + \kappa_2)}{\kappa_1\kappa_2} \eta,
\]

which yields the conclusion.

\[\Box\]

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