$X(1835)$ and the New Resonances $X(2120)$ and $X(2370)$ Observed by the BES Collaboration

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Abstract

We calculate the decay widths of both the second and the third radial excitations of $\eta$ and $\eta'$ within the framework of $^3P_0$ model. After comparing the theoretical decay widths and decay patterns with the available experimental data of $\eta(1760)$, $X(1835)$, $X(2120)$ and $X(2370)$, we find that the interpretation of $\eta(1760)$ and $X(1835)$ as the second radial excitation of $\eta$ and $\eta'$ crucially depends on the measured mass and width of $\eta(1760)$, which is still controversial experimentally. We suggest that there may be sizable $p\bar{p}$ content in $X(1835)$. $X(2120)$ and $X(2370)$ can not be understood as the third radial excitations of $\eta$ and $\eta'$, $X(2370)$ probably is a mixture of $\eta'(4^1S_0)$ and glueball.

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I. INTRODUCTION

$X(1835)$ was first observed by BESII in the $\eta^\prime\pi\pi$ invariant mass spectrum in the process $J/\psi \to \gamma\pi^+\pi^-\eta^\prime$ with a statistical significance of $7.7\sigma$. The fit with the Breit-Wigner function yields mass $M = 1833.7\pm 6.1(stat)\pm 2.7(syst)$ MeV/c$^2$, width $\Gamma = 67.7\pm 20.3(stat)\pm 7.7(syst)$ MeV/c$^2$ and the product branching fraction $Br(J/\psi \to \gamma X(1835))Br(X(1835) \to \pi^+\pi^-\eta^\prime) = (2.2 \pm 0.4(stat) \pm 0.4(syst)) \times 10^{-4}$ [1]. Recently $X(1835)$ has been confirmed by BESIII collaboration in the same process with statistical significance larger than $25\sigma$, and its mass and width are fitted to be $M = 1838.1 \pm 2.8$ MeV and $\Gamma = 179.5 \pm 9.1$ MeV. Moreover two new resonances are reported, which are denoted as $X(2120)$ and $X(2370)$ respectively. Their masses and widths are determined to be $M_{X(2120)} = 2124.8 \pm 5.6$ MeV, $\Gamma_{X(2120)} = 101 \pm 14$ MeV, $M_{X(2370)} = 2371.0 \pm 6.4$ MeV and $\Gamma_{X(2370)} = 108 \pm 15$ MeV [2, 3].

The experimental observation of $X(1835)$ stimulated a number of theoretical speculations about its underlying structure. Some interpret $X(1835)$ as a $p\bar{p}$ bound state [4–8], a glueball candidate [9–12] or the radial excitation of $\eta^\prime$ [13, 14], and some others interpret it as final state interaction or a rescattering effect [15–17]. Naively the observation of $X(2120)$ and $X(2370)$ seems to indicate that all the three resonances $X(1835)$, $X(2120)$ and $X(2370)$ are possibly the radial excitations of $\eta$ or $\eta^\prime$, they jump to the ground state $\eta^\prime$ through emitting two $\pi$ [35]. Moreover, we note that before we consider the exotic structure hypothesis for some newly observed resonance, it is very necessary to study whether the assignment of conventional hadron is possible. Consequently, we shall investigate in the following whether $X(1835)$, $X(2120)$ and $X(2370)$ could be canonical $q\bar{q}$ pseudoscalar mesons.

It is well-known that there are nine pseudoscalar mesons $\pi$, $K$, $\eta$ and $\eta^\prime$, which form a good nonet in the limit of SU(3) flavor symmetry. From Particle Data Group(PDG) [18], we see that the first radial excitations of these pseudoscalars have been well established, concretely they are $\pi(1300)$, $K(1460)$, $\eta(1295)$ and $\eta(1475)$. As a result, if the three resonances $X(1835)$, $X(2120)$ and $X(2370)$ are canonical $q\bar{q}$ pseudoscalar mesons, the natural assignment would be $\eta(1760)$ and $X(1835)$ as the second radial excitation of $\eta$ and $\eta^\prime$, $X(2120)$ and $X(2370)$ as the third radial excitation of $\eta$ and $\eta^\prime$ respectively. In this work, we shall study the decays of these four resonances under the above assignment within the framework of $^3P_0$ model. Our goal is to shed some light on the nature of these structures by comparing the predictions for the hadronic decay widths with the available experimental data.
This paper is organized as follows. Firstly we review the $^3P_0$ model briefly in sections II. The flavor mixing between the $\eta$ and $\eta'$ radial excitation and the allowed decay modes are presented in section III. The OZI (Okubo, Zweig and Iizuka) allowed strong decays of $\eta(1760)$, $X(1835)$, $X(2120)$ and $X(2370)$ are studied in section IV. Finally we present in section V our conclusions and some discussions.

II. REVIEW OF THE $^3P_0$ MODEL

The $^3P_0$ model for the decay of a $q\bar{q}$ meson $A$ to mesons $B + C$ was proposed by Micu [19] and developed by Le Yaouanc et al [20–22]. The $^3P_0$ model assumes that strong decay takes place via the creation of a pair of quark and antiquark with $J^{PC} = 0^{++}$ from the vacuum. The created quark pair together with the quark and antiquark in the initial meson recombine to final state mesons in two ways as shown in Fig.1 and the decay amplitude is proportional to wavefunctions (including spatial, spin, flavor and color wavefunctions) overlap between the initial state, created quark pair and the final state. The $^3P_0$ model has been widely applied to meson and baryon strong decays, with considerable success [20–27]. In this work, we shall use the diagrammatic technique developed in Ref. [22] to derive the amplitudes and the $^3P_0$ matrix elements. In this formalism, the $^3P_0$ model describes the strong decay process using a $q\bar{q}$ pair production Hamiltonian, which is the nonrelativistic limit of,

$$H_I = g\int d^3x \bar{\psi}(x) \psi(x) \eqno (1)$$
where $\psi$ is a Dirac quark field, $g$ is the coupling constant. The pair production component of the $^3P_0$ Hamiltonian $H_I$ can be written in terms of creation operators as

$$H_I = \sum_{s\bar{s}} \int d^3k \frac{g m_q}{E_q} [\bar{u}_{k_s} u_{-k\bar{s}}] b_{k_s}^\dagger d_{-k\bar{s}}$$

(2)

where $b_{k_s}^\dagger$ creates a quark with momentum $k$ and spin $s$, $d_{-k\bar{s}}^\dagger$ create a antiquark with momentum $-k$ and spin $\bar{s}$, $m_q$ being the mass of the created quark and antiquark. We note that each effective $^3P_0$ quark pair production vertex is associated with the factor $\frac{g m_q}{E_q} [\bar{u}_{k_s} u_{-k\bar{s}}]$. We assume non-relativistic $q\bar{q}$ wavefunction for the initial and final mesons,

$$| A \rangle = \int d^3p_1 \int d^3p_2 \Psi_{n_AL_A L_A}(\frac{m_2 p_1 - m_1 p_2}{m_1 + m_2}) \delta(P_A - p_1 - p_2) | q_1(p_1) \bar{q}_2(p_2) \rangle$$

(3)

with explicit spin and flavor wave functions which are of the usual non-relativistic quark model forms. $n_A$ denotes the radial quantum number of meson $A$ composed of quark $q_1$ and anti-quark $\bar{q}_2$ with momentum $p_1$ and $p_2$ and mass $m_1$ and $m_2$ respectively, and $P_A$ is the momentum of meson $A$. The wavefunctions of the final state mesons $B$ and $C$ can be written out directly in the same way. The spatial wavefunction $\Psi$ is generally taken to be the simple harmonic oscillator (SHO) wavefunction. The SHO wavefunction enables analytical calculation of the decay amplitude, and it turned out to be a good approximation. Even if we use more realistic wavefunction, the predictions would not be improved systematically due to the inherent uncertainties of the $^3P_0$ model. In momentum-space, the SHO wavefunction reads

$$\Psi_{nLM_L}(p) = \frac{(-1)^n (-i)^L}{\beta^{3/2}} \sqrt{\frac{2n!}{\Gamma(n + L + 3/2)}} \left( \frac{p}{\beta} \right)^L \exp \left( -\frac{p^2}{2\beta^2} \right) L_n^{L+1/2} \left( \frac{p^2}{\beta^2} \right) Y_{LM_L}(\Omega_p)$$

(4)

where $\beta$ is the harmonic oscillator parameter, $Y_{LM_L}(\Omega_p)$ is the spherical harmonic function, and $L_n^{L+1/2}(\frac{p^2}{\beta^2})$ is the Laguerre polynomial.

One can now straightforwardly evaluate the Hamiltonian $H_I$ matrix element for the decay $A \rightarrow B + C$ in terms of overlap integrals,

$$\langle BC | H_I | A \rangle_a = I_{\text{signature}}(a) I_{\text{flavor}}(a) I_{\text{spin+space}}(a) \delta(P_A - P_B - P_C)$$

$$\langle BC | H_I | A \rangle_b = I_{\text{signature}}(b) I_{\text{flavor}}(b) I_{\text{spin+space}}(b) \delta(P_A - P_B - P_C)$$

(5)

where the signature phase $I_{\text{signature}}$ is equal to -1 for both diagrams $(a)$ and $(b)$ due to quark operator anticommutation. Starting from the flavor wavefunctions, we can directly obtain
the flavor overlap factors $I_{\text{flavor}}(a)$ and $I_{\text{flavor}}(b)$ which result from contracting the explicit flavor states corresponding to diagrams Fig.1a and Fig.1b, they are listed in Table III for the decay modes concerned here. In the rest frame of meson $A$, the overlap integral $I_{\text{spin+space}}(a)$ and $I_{\text{spin+space}}(b)$ explicitly are given by

$$I_{\text{spin+space}}(a) = \int d^3k \Psi_{n_A L_AM_A}(k - P_B)\Psi^*_{n_B L_B M_B}(k - \frac{m_3}{m_2 + m_3}P_B)\Psi^*_{n_C L_CM_C}(k - \frac{m_3}{m_1 + m_3}P_B)$$

$$I_{\text{spin+space}}(b) = \int d^3k \Psi_{n_A L_AM_A}(k + P_B)\Psi^*_{n_B L_B M_B}(k + \frac{m_3}{m_1 + m_3}P_B)\Psi^*_{n_C L_CM_C}(k + \frac{m_3}{m_2 + m_3}P_B)$$

(6)

where the relevant spin factor has been omitted, $E_3 = \sqrt{k^2 + m_3^2}$ is the energy of the created quark. We note that the spin factor and the labels $s_q$, $s_{\bar{q}}$ depend on the reaction considered, generally the spin indexes $s_q$ and $s_{\bar{q}}$ associated with diagram Fig.1a and Fig.1b are different. As a result, the amplitude for the meson decay $A \to B + C$ is

$$\mathcal{M}(A \to B+C) = I_{\text{signature}}(a)I_{\text{flavor}}(a)I_{\text{spin+space}}(a) + I_{\text{signature}}(b)I_{\text{flavor}}(b)I_{\text{spin+space}}(b) \equiv h_{fi}$$

(7)

Taking into account the phase space, we get the differential decay rate

$$\frac{d\Gamma_{A\to BC}}{d\Omega} = 2\pi P E_B E_C M_A |h_{fi}|^2$$

(8)

where $E_B$ and $E_C$ are the energy of the meson $B$ and $C$ respectively, $P$ is the momentum of the final state mesons in the rest frame of meson $A$

$$P = \sqrt{[M_A^2 - (M_B + M_C)^2][M_A^2 - (M_B - M_C)^2]/(2M_A)}$$

(9)

where $M_A$, $M_B$ and $M_C$ are the masses of the meson $A$, $B$ and $C$ respectively. To compare with experiments, we transform the amplitude $h_{fi}$ into the partial wave amplitude $\mathcal{M}_{LS}$ by the recoupling calculation [28], then the decay width is

$$\Gamma(A \to B + C) = 2\pi \frac{P E_B E_C}{M_A} \sum_{LS} |\mathcal{M}_{LS}|^2$$

(10)

The pair production parameter $g$ and the harmonic oscillator parameter $\beta$ are fitted to the strong decay data, and they are found to be roughly flavor independent for decays involving production of $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ pairs. The typical values obtained from computation
of light meson decays are \( g = 0.334 \) GeV and \( \beta = 0.4 \) GeV \[22\]\[24\], assuming simple harmonic oscillator wavefunctions with a global scale, and they are frequently adopted by the literatures. However, different quark models find different values of \( \beta \) (mostly in the range of \( 0.35 \sim 0.45 \) GeV ), so that there is the question of the sensitivity of our results to \( \beta \), we will address this issue below. The masses of constituent quarks are chosen to be \( m_u = m_d = 0.33 \) GeV and \( m_s = 0.55 \) GeV as usual. The masses used are the experimental values of well-established candidates, which are taken from the PDG \[18\]. Moreover, we have ignored the mass difference between the members of the same isospin multiplet. For the isoscalar we assume ideal mixing \( |\varphi_{\text{nonstrange}}\rangle = 1/\sqrt{2}|u\bar{u} + d\bar{d}\rangle \), \( |\varphi_{\text{strange}}\rangle = |s\bar{s}\rangle \), where except for the ground state pseudoscalar, we choose \( |\eta\rangle = \cos \phi_p |u\bar{u} + d\bar{d}\rangle/\sqrt{2} - \sin \phi_p |s\bar{s}\rangle \) and \( |\eta'\rangle = \sin \phi_p |u\bar{u} + d\bar{d}\rangle/\sqrt{2} + \cos \phi_p |s\bar{s}\rangle \) with the mixing angle \( \phi_p = 39.2^\circ \) \[29\]. The kaons and their excitations are not charge conjugation eigenstates so that mixing can occur among states with the same \( J^P \) that are forbidden for neutral states. For example the \( J^P = 1^+ \) axial vector kaon mesons \( K_1(1273) \) and \( K_1(1402) \) are coherent superpositions of quark model \( 3P_1 \) and \( 1P_1 \) states \[24\],

\[
|K_1(1273)\rangle = \sqrt{\frac{2}{3}}|1P_1\rangle + \sqrt{\frac{1}{3}}|3P_1\rangle
\]

\[
|K_1(1402)\rangle = -\sqrt{\frac{1}{3}}|1P_1\rangle + \sqrt{\frac{2}{3}}|3P_1\rangle
\]

(11)

III. MIXING BETWEEN THE \( \eta \) AND \( \eta' \) EXCITATIONS AND THE ALLOWED DECAY MODES

The radial excitation of \( \eta \) and \( \eta' \) are both isoscalar states with the same \( J^{PC} \) so that there will be mixing between them. Consequently the physical states are the mixture of \( SU(3) \) flavor octet and singlet

\[
|\eta(n^1S_0)\rangle = \cos \theta |\eta_8(n^1S_0)\rangle - \sin \theta |\eta_0(n^1S_0)\rangle
\]

\[
|\eta'(n^1S_0)\rangle = \sin \theta |\eta_8(n^1S_0)\rangle + \cos \theta |\eta_0(n^1S_0)\rangle
\]

where \( n \) represents the radial quantum number, \( |\eta_8(n^1S_0)\rangle \) and \( |\eta_0(n^1S_0)\rangle \) are the octet and singlet states respectively,

\[
|\eta_8(n^1S_0)\rangle \equiv \frac{1}{\sqrt{6}}|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle
\]
\[ |\eta_0(n^1S_0)\rangle \equiv \frac{1}{\sqrt{3}} |u\bar{u} + d\bar{d} + s\bar{s}\rangle \] (13)

In order to explicitly exhibit the \( u\bar{u} + d\bar{d} \) and \( s\bar{s} \) components, we shall choose the so-called nonstrange-strange basis in this work

\[
|\eta(n^1S_0)\rangle = \cos\phi|\eta_{NS}(n^1S_0)\rangle - \sin\phi|\eta_S(n^1S_0)\rangle \\
|\eta'(n^1S_0)\rangle = \sin\phi|\eta_{NS}(n^1S_0)\rangle + \cos\phi|\eta_S(n^1S_0)\rangle
\] (14)

where \( |\eta_{NS}(n^1S_0)\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2} \) and \( |\eta_S(n^1S_0)\rangle = |s\bar{s}\rangle \), and mixing angle \( \phi \) is related to \( \theta \) via \( \phi = \theta + \arctan\sqrt{2} \simeq \theta + 54.7^\circ \). We note that the mixing angle \( \phi \) (or \( \theta \)) is less constrained phenomenologically, its concrete value has to be determined experimentally. It is well-known that \( \eta - \eta' \) mixing has been measured by various means, however, there is still large uncertainty. As a result, we shall take the mixing angle \( \phi \) as a undetermined parameter in the following, the dependence of the amplitudes and widths on \( \phi \) would be considered.

We present the selection rules for the two-body decays of \( \eta \) and \( \eta' \) excitations in Table I. For specific final states listed in Table I, all the four states \( \eta(1760), X(1835), X(2120) \) and \( X(2370) \) could decay into them, if the process is not forbidden kinetically. We note that decays into two pseudoscalar or two scalar mesons are forbidden by parity and charge conjugation conservation, Moreover, the \( G- \)parity forbids the decay processes \( X \rightarrow \rho\pi \), \( X \rightarrow \omega\eta \), \( X \rightarrow \rho\sigma_1(1260) \), \( X \rightarrow \rho\sigma_2(1320) \), \( X \rightarrow \omega(\phi)f_1(1285) \), \( X \rightarrow \omega(\phi)f_1(1420) \), \( X \rightarrow \omega(\phi)f_2(1270) \) and \( X \rightarrow \omega(\phi)f'_2(1525) \), where \( X \) denotes \( \eta(1760), X(1835), X(2120) \) or \( X(2370) \).

IV. STRONG DECAYS OF \( \eta(1760), X(1835), X(2120) \) AND \( X(2370) \)

Following the procedures presented in the previous sections, the total decay rate is given by the Hamilton matrix element squared, multiplied by the phase space, and summed over all final spin and charge states. Since we neglect mass splitting within the isospin multiplet, to sum over all channels, one should multiply the partial width into the specific charge channel by the flavor multiplicity factor \( F \) in Table III. This \( F \) factor also incorporates the statistical factor 1/2 if the final state mesons \( B \) and \( C \) are identical.
### Decay modes Final states

| Decay modes | Final states |
|-------------|--------------|
| $X \to 1^1S_0 + 1^3S_1$ | $KK^*$ |
| $X \to 2^1S_0 + 1^3S_1$ | $K(1460)K^*$ |
| $X \to 1^1S_0 + 2^3S_1$ | $KK^*(1410)$ |
| $X \to 1^1S_0 + 1^3P_0$ | $\pi a_0(1450), \, KK_0^*(1430), \, \eta f_0(1370), \, \eta f_0(1710), \, \eta' f_0(1370)$ |
| $X \to 1^1S_0 + 1^3P_2$ | $\pi a_2(1320), \, KK_2^*(1430), \, \eta f_2(1270), \, \eta f_2(1525), \, \eta' f_2(1270)$ |
| $X \to 1^1S_0 + 1^3D_1$ | $KK^*(1680)$ |
| $X \to 1^1S_0 + 1^3D_3$ | $KK_3^*(1780)$ |
| $X \to 1^3S_1 + 1^3S_1$ | $\rho \rho, \, K^* K^*, \, \omega, \, \phi$ |
| $X \to 1^1S_0 + 2^3S_1$ | $\rho \rho(1450), \, K^* K^*(1410), \, \omega \omega(1420)$ |
| $X \to 1^3S_1 + 1^3P_1$ | $\rho b_1(1235), \, K^* K_1(1273), \, \omega h_1(1170), \, \omega h_1(1380), \, \phi h_1(1170), \, \phi h_1(1380)$ |
| $X \to 1^3S_1 + 1^3P_2$ | $K^* K_1(1402)$ |
| $X \to 1^3S_1 + 1^3P_2$ | $K^* K^*_2(1430)$ |

### Table I: Allowed decay modes of $\eta$ and $\eta'$ radial excitations.

#### A. Decays of $\eta(1760)$ and $X(1835)$

The experimental evidence for $\eta(1760)$ is controversial, its existence evidence was first reported by the Mark III Collaboration in the $J/\psi$ radiative decays to $\omega\omega$ \[^{30}\] and $\rho\rho$ \[^{31}\], then it was further studied by the DM2 and BES collaborations. The various experimental results associated with $\eta(1760)$ are summarized in Table \[\|\], it is obviously that there are big differences between different measurements of $\eta(1760)$ width. In this work, both the mass and width are taken to be the world average listed in PDG. For $\eta(1760)$ and $X(1835)$ as the second radial excitation of $\eta$ and $\eta'$, the allowed decay channels, the corresponding decay amplitudes and partial widths are shown in Table \[\|\] and Table \[\|\] respectively. Clearly the decay amplitudes and widths depend strongly on the mixing angle $\phi$, and measurements of any or several of the larger decay modes will provide constrained tests of the hypothesis and measurement of the mixing angle. We believe that the better way to determine the mixing angle is comparing the ratio between $KK^*$ and $\rho\rho$ partial widths with experimental data, if both $\eta(1760)$ and $X(1835)$ are indeed conventional quark model states assumed above.

### Table II: Summary of $\eta(1760)$ measurements.

| Experiment | Mass (MeV) | Width (MeV) | Production |
|------------|------------|-------------|-------------|
| DM2 \[^{32}\] | $1760 \pm 11$ | $60 \pm 16$ | $J/\psi \to \gamma \eta(1760), \, \eta(1760) \to \rho \rho$ |
| BES \[^{33}\] | $1744 \pm 10 \pm 15$ | $244^{+24}_{-21} \pm 25$ | $J/\psi \to \gamma \eta(1760), \, \eta(1760) \to \omega \omega$ |
| PDG \[^{18}\] | $1756 \pm 9$ | $96 \pm 70$ | |
This is because that the pair production parameter \( g \) cancels out in this ratio, consequently there is less systematic uncertainty than in the decay rates. The partial widths of \( \eta(1760) \) and \( X(1835) \) as functions of the flavor mixing angle \( \phi \) for fixed \( \beta = 0.4 \) GeV is shown in Fig. 2. Evidently large couplings of \( \eta(1760) \) to \( \rho \rho \) and \( \omega \omega \) follow from moderate mixing, which could explain the observation of \( \eta(1760) \) in the \( \rho \rho \) and \( \omega \omega \) final states by the DM2 and BES collaborations. Furthermore, we note that \( \eta(1760) \) should have a sizable branching ratio into \( \pi a_2(1320) \). Therefore we urge experimentalist to search for \( \eta(1760) \) in the process \( J/\psi \to \gamma \eta(1760) \to \gamma \pi a_2(1320) \), which is an important test to our scenario. Obviously the partial width of \( X(1835) \to \eta f_2(1270) \) is particularly small. Taking into account the variation of the mixing angle \( \phi \), we find that \( X(1835) \) may have large branching ratio into \( \rho \rho \), \( \pi a_2(1320) \) and \( KK^* \) final states under the assignment of \( \eta'(3^1S_0) \) \( q\bar{q} \) meson, experimental search of \( X(1835) \) in these modes is suggested.

FIG. 2: Partial decay widths of \( \eta(1760) \) and \( X(1835) \) vs. the flavor mixing angle \( \phi \), the left figure for \( \eta(1760) \), and the right figure for \( X(1835) \).

We note that the mixing angle appearing in the \( \eta(1760) \) and \( X(1835) \) flavor wavefunction is the same, so that a large number of decays are correlated, as is demonstrated in Table IV and Table V. It is essential to investigate whether there exists certain region of mixing angle \( \phi \) so that the predicted widths of \( \eta(1760) \) and \( X(1835) \) agree with the experimental observations within errors. Since the masses of \( \eta(1760) \) and \( X(1835) \) are measured precisely enough, their central values are used, the harmonic oscillator parameter \( \beta \) is allowed to vary in the range of \( 0.35 \sim 0.45 \) GeV, the total decay widths of \( \eta(1760) \) and \( X(1835) \) as functions of the mixing angle are shown in Fig. 3. Obviously we see that there is not a value of \( \phi \)
so that the resulting widths of both $\eta(1760)$ and $X(1835)$ lie in the experimentally allowed range. The same conclusion is reached for the $\eta(1760)$ parameters measured by the DM2 collaboration, as is obvious from Fig. 4a. It seems unappropriate to identify $\eta(1760)$ and $X(1835)$ as the second radial excitation of $\eta$ and $\eta'$ simultaneously. However, if we take the $\eta(1760)$ mass and width to be the BES measurement, the corresponding decay widths are shown in Fig. 4b, we find that the theoretical widths of $\eta(1760)$ and $X(1835)$ could be consistent with experimental data for the mixing angle $\phi$ in the range $-31^\circ \sim -24^\circ$ or $30^\circ \sim 40^\circ$. Therefore experimentally resolving the inconsistence between the DM2 and the BES collaboration results for $\eta(1760)$ is important to understand $X(1835)$. Remembering that $X(1835)$ is close to the threshold of proton and antiproton (i.e., $p\bar{p}$), ”dressing” of the $q\bar{q}$ singlet meson $\eta'(3^1S_0)$ with two $q\bar{q}$ pair can create nucleon-antinucleon, and final state interactions enhance the probability of this transition. In this way, the $\eta'(3^1S_0)$ meson can mix with the $p\bar{p}$ final state and its wave function develops a sizable $p\bar{p}$ component. As a result, $X(1835)$ could be a mixture of $\eta'(3^1S_0)$ and $p\bar{p}$ molecule, then all experimental facts related to $X(1835)$ could be understood qualitatively. To shed light on the nature of $X(1835)$, a coupled channel analysis necessary, this topic is beyond the scope of the present work.

B. Decays of $X(2120)$ and $X(2370)$

Under the assignment of $\eta(4^1S_0)$ and $\eta'(4^1S_0)$ $q\bar{q}$ mesons, the decay amplitudes and partial widths of $X(2120)$ and $X(2370)$ in terms of the general mixing angles are shown in Table VI and Table VII respectively. Since $X(2120)$ and $X(2370)$ have larger masses, many strong decay modes are allowable. $X(2120)$ has large partial widths to $\pi a_2(1320)$ and $KK^*(1410)$, and the main decay modes of $X(2370)$ are $\rho\rho(1450)$, $\rho b_1(1235)$, $\omega\omega(1420)$, $\pi a_2(1320)$, $K^*K^*(1410)$ and $KK^*_2(1430)$, the corresponding partial widths as functions of the flavor mixing angle $\phi$ are shown in Fig. 5. It is obvious that the modes $\pi a_2(1320)$ and $KK^*(1410)$ are important to the search for $X(2120)$, this is because that if the signal of $X(2120)$ is accidently suppressed in one mode, it should be evident in the other. The same is true for the $X(2370)$ decay modes $\rho\rho(1450)$ and $K^*K^*(1410)$. We note that the branching ratios of the $KK^*$ and $\rho\rho$ modes in both $X(2120)$ and $X(2370)$ decays are predicted to be smaller, despite their larger phase space, as they are accidentally near the node in the $^3P_0$ de-
FIG. 3: Total decay widths of $\eta(1760)$ and $X(1835)$ as functions of the mixing angle, where the harmonic oscillator parameter $\beta$ varies from 0.35 GeV to 0.45 GeV. The horizontal yellow and pink bands denote the experimental errors of $\eta(1760)$ and $X(1835)$ widths, where the mass and width of $\eta(1760)$ is taken to be the world average.

cay amplitude for the physical masses and $\beta = 0.4$ GeV. The $X(2120)$ decay modes $\rho b_1(1235)$ and $\omega h_1(1170)$ are interesting because the two subamplitudes $^1S_0$ and $^5D_0$ are comparable and individually proportional to $\cos \phi$, thus the $D/S$ amplitude ratio is independent of the mixing angle $\phi$. The measurement of $\rho b_1(1235)$ and $\omega h_1(1170)$ subamplitudes directly access $\cos \phi$, although these modes may be too weak to allow this measurement. Similarly $X(2370)$ can decay into $\rho b_1(1235)$, $\omega h_1(1170)$, $K^*K_1(1273)$, $K^*K_1(1402)$ in both S-wave and D-wave, and the $D/S$ ratio for the latter two modes strongly depends on the flavor mixing angle.

For the harmonic oscillator parameter $\beta$ in the range of 0.35 GeV~0.45 GeV, the total widths of $X(2120)$ and $X(2370)$ against the flavor mixing angle $\phi$ is displayed in Fig. 6. Since $X(2370)$ has many decay modes, its width is predicted to be larger than 300 MeV. Even if the width is overestimated by a factor of 2, it is still larger than the measured value. Obviously there doesn’t exist appropriate value of the mixing angle so that the theoretically predicted widths of $X(2120)$ and $X(2370)$ lie in the experimentally allowed range. Therefore it seems very unlikely that $X(2120)$ and $X(2370)$ can be understood as the third radial
FIG. 4: The same as Fig. 3, where the parameters of $\eta(1760)$ are chosen to be the DM2 and BES collaborations measurements in the left and right figures, respectively.

FIG. 5: Partial decay widths of the leading decay modes of $X(2120)$ and $X(2370)$ vs. the flavor mixing angle $\phi$, the left figure for $X(2120)$, and the right figure for $X(2370)$.

excitation of $\eta$ and $\eta'$ simultaneously. The lattice QCD simulations predict the $0^{-+}$ glueball is about 2.3–2.6 GeV [34], it would mix with the nearby pseudoscalar isoscalar mesons. Consequently $X(2370)$ may be a mixture of $\eta'(4^1S_0)$ and glueball, if its quantum numbers turn out to be $J^{PC} = 0^{-+}$ in future. To understand the nature of $X(2370)$, partial wave analysis is important.
FIG. 6: Total decay widths of $X(2120)$ and $X(2370)$ vs. the flavor mixing angle $\phi$, where the harmonic oscillator parameter $\beta$ varies from 0.35 GeV to 0.45 GeV. The horizontal yellow and pink bands represent the widths of $X(2120)$ and $X(2370)$ fitted by BES collaboration respectively, which are close to each other.

V. SUMMARY AND DISCUSSIONS

In this work, we investigate whether the resonances $X(1835)$, $X(2120)$ and $X(2370)$ newly observed by the BES collaboration could be conventional $q\bar{q}$ mesons. If they are indeed canonical pseudoscalar mesons, the natural assignments are $\eta(1760)$ and $X(1835)$ as the second radial excitation of $\eta$ and $\eta'$ respectively, and $X(2120)$ and $X(2370)$ as the third radial excitation of $\eta$ and $\eta'$. To do so we calculate all kinematically allowed two-body strong decays of $\eta(3^1S_0)$, $\eta'(3^1S_0)$, $\eta(4^1S_0)$ and $\eta'(4^1S_0)$ states within the framework of $^3P_0$ model.

The decay amplitudes and widths turn out to be strongly dependent on the flavor mixing angle. If the mass and width of $\eta(1760)$ are chosen to be the world average listed in PDG or the DM2 measurement, we can not find proper value of the mixing angle so that both the theoretically predicted widths of $\eta(1760)$ and $X(1835)$ lie in the experimentally allowed
range. However, if the BES results for $\eta(1760)$ are taken to be true, the theoretical predictions could be consistent with the experimental data within error for the flavor mixing angle $\phi$ in the range of $-31^\circ \sim -24^\circ$ or $30^\circ \sim 40^\circ$. Further experimental study of $\eta(1760)$ is important to understand the nature of $X(1835)$. Since the $\eta'(3^1S_0)$ $q\bar{q}$ meson would mix with $p\bar{p}$ due to the "dressing" effect and final state interaction, we suggest $X(1835)$ is the mixture of $\eta'(3^1S_0)$ and $p\bar{p}$ molecule, then we can naturally understand all the observations associated with $X(1835)$.

Under the assignment of $X(2120)$ and $X(2370)$ as $\eta(4^1S_0)$ and $\eta'(4^1S_0) q\bar{q}$ mesons, $X(2120)$ dominantly decays into $\pi a_2(1320)$ and $KK^*(1410)$, the modes $KK^*$ and $\rho\rho$ modes are suppressed by the decay amplitude node. $X(2370)$ is predicted to be rather broad (i.e., its width should be larger than 300 MeV), so it is unlikely that $X(2120)$ and $X(2370)$ can be understood as the third radial excitation of $\eta$ and $\eta'$ simultaneously. Since $X(2370)$ is close to the $0^{-+}$ glueball 2.3~2.6 GeV predicted by lattice QCD, we suggest it may be a mixture of $\eta'(4^1S_0)$ meson and glueball, if its quantum numbers are determined to be $J^{PC} = 0^{-+}$ by future experiments.

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[35] This conjecture is proposed in Ref. [2] as well.
| Generic Decay | Example | $I_{flavor}(a)$ | $I_{flavor}(b)$ | $\mathcal{F}$ |
|---------------|---------|----------------|----------------|-------------|
| $X_0 \to (n \bar{s})(s \bar{n})$ | $X_0 \to K^+ + K^-$  
$X_0 \to K^{*+} + K^-$ | 0 | $-1/\sqrt{2}$ | 2 |
| $X_s \to (n \bar{s})(s \bar{n})$ | $X_s \to K^+ + K^-$  
$X_s \to K^{*+} + K^-$ | $-1$ | 0 | 2 |
| $X_0 \to (u \bar{d})(d \bar{u})$ | $X_0 \to \pi^+ + a^-$  
$X_0 \to \pi^+ + \pi^-$  
$X_0 \to \rho^+ + \rho^-$  
$X_0 \to \rho^+ + \rho^-(1450)$ | $-1/\sqrt{2}$  
$-1/\sqrt{2}$  
$-1/\sqrt{2}$  
$-1/\sqrt{2}$ | $-1/\sqrt{2}$  
$-1/\sqrt{2}$  
$-1/\sqrt{2}$  
$-1/\sqrt{2}$ | 3  
3/2  
3/2  
3 |
| $X_s \to (u \bar{d})(d \bar{u})$ | $X_s \to \pi^+ + a^-$  
$X_s \to \pi^+ + \pi^-$  
$X_s \to \rho^+ + \rho^-$  
$X_s \to \rho^+ + \rho^-(1450)$ | 0 | 0 | 0 |
| $X_0 \to \left(\frac{u \bar{u} + d \bar{d}}{\sqrt{2}}\right)\left(\frac{u \bar{u} + d \bar{d}}{\sqrt{2}}\right)$ | $X_0 \to \eta_0 + f_0$  
$X_0 \to \omega + \omega$ | $1/\sqrt{2}$  
$1/\sqrt{2}$ | $1/\sqrt{2}$  
$1/\sqrt{2}$ | 1  
1/2 |
| $X_s \to \left(\frac{u \bar{u} + d \bar{d}}{\sqrt{2}}\right)\left(\frac{u \bar{u} + d \bar{d}}{\sqrt{2}}\right)$ | $X_s \to \eta_0 + f_0$ | 0 | 0 | 0 |
| $X_0 \to (s \bar{s})(\frac{u \bar{u} + d \bar{d}}{\sqrt{2}})$ | $X_0 \to \eta_s + f_0$ | 0 | 0 | 0 |
| $X_s \to (s \bar{s})(\frac{u \bar{u} + d \bar{d}}{\sqrt{2}})$ | $X_s \to \eta_s + f_0$ | 0 | 0 | 0 |
| $X_0 \to \left(\frac{u \bar{u} + d \bar{d}}{\sqrt{2}}\right)(s \bar{s})$ | $X_0 \to \eta_0 + f_s$ | 0 | 0 | 0 |
| $X_s \to \left(\frac{u \bar{u} + d \bar{d}}{\sqrt{2}}\right)(s \bar{s})$ | $X_s \to \eta_0 + f_s$ | 0 | 0 | 0 |
| $X_0 \to (s \bar{s})(s \bar{s})$ | $X_0 \to \eta_s + f_s$ | 0 | 0 | 0 |
| $X_s \to (s \bar{s})(s \bar{s})$ | $X_s \to \eta_s + f_s$ | 1 | 1 | 1 |
| $X_s \to (s \bar{s})(s \bar{s})$ | $X_s \to \phi + \phi$ | 1 | 1 | 1/2 |

TABLE III: Relevant flavor weight factors for $\eta$ and $\eta'$ excitation decays, where $|X_0\rangle = |u \bar{u} + d \bar{d}/\sqrt{2}$
and $|X_s\rangle = |s \bar{s}\rangle$, $(n \bar{s}) = (u \bar{s})$ or $(d \bar{s})$ for $n$ being up and down quark respectively. $(n \bar{n}')_{I=1} = (u \bar{d})$, $[(u \bar{u}) - (d \bar{d})]/\sqrt{2}$ and $(d \bar{u}), (n \bar{n})_{I=0} = [(u \bar{u}) + (d \bar{d})]/\sqrt{2}$. 
\[ \eta(1760) = \cos \phi_1 |u\bar{u} + d\bar{d}|/\sqrt{2} - \sin \phi_1 |s\bar{s}| \]

| Modes \( \times \) \( \ast \) | \( \Gamma \) (MeV) | Amps. (GeV\(^{-1/2} \)) |
|---|---|---|
| \( K K^* \) | \( 30.85 c^2 + 89.25 c s + 64.56 s^2 \) | \( M_{11} = 0.074 c + 0.11 s \) |
| \( \rho\rho \) | \( 155.36 c^2 \) | \( M_{11} = 0.30 c \) |
| \( \omega\omega \) | \( 69.50 c^2 \) | \( M_{11} = -0.30 c \) |
| \( \pi a_0(1450) \) | \( 44.36 c^2 \) | \( M_{00} = -0.22 c \) |
| \( \pi a_2(1320) \) | \( 60.93 c^2 \) | \( M_{22} = -0.17 c \) |
| Total | \( 341.00 c^2 + 89.25 c s + 64.56 s^2 \) | |

**TABLE IV:** Partial widths of \( \eta(1760) \) as the second radial excitation of \( \eta \), where \( \phi_1 \) is the flavor mixing angle, \( s \equiv \sin \phi_1 \) and \( c \equiv \cos \phi_1 \). Note that a factor of \( i \) has been suppressed in all odd partial wave amplitudes.

\[ X(1835) = \sin \phi_1 |u\bar{u} + d\bar{d}|/\sqrt{2} + \cos \phi_1 |s\bar{s}| \]

| Modes | \( \Gamma \) (MeV) | Amps. (GeV\(^{-1/2} \)) |
|---|---|---|
| \( K K^* \) | \( 43.18 c^2 - 79.34 c s + 36.45 s^2 \) | \( M_{11} = -0.080 c + 0.074 s \) |
| \( \rho\rho \) | \( 188.24 s^2 \) | \( M_{11} = 0.30 s \) |
| \( K^* K^* \) | \( 29.84 c^2 + 23.08 c s + 4.46 s^2 \) | \( M_{11} = 0.16 c + 0.060 s \) |
| \( \omega\omega \) | \( 62.23 s^2 \) | \( M_{11} = -0.30 s \) |
| \( \pi a_0(1450) \) | \( 47.85 s^2 \) | \( M_{00} = -0.18 s \) |
| \( \pi a_2(1320) \) | \( 136.48 s^2 \) | \( M_{22} = -0.22 s \) |
| \( \eta f_2(1270) \) | \( 0.051 s^2 \) | \( M_{22} = 0.014 s \) |
| Total | \( 73.02 c^2 - 56.26 c s + 475.76 s^2 \) | |

**TABLE V:** Partial widths of \( X(1835) \) as the second radial excitation of \( \eta' \), where \( \phi_1 \) is the mixing angle, \( s \equiv \sin \phi_1 \) and \( c \equiv \cos \phi_1 \), and the factor of \( i \) has been suppressed in all odd partial wave amplitudes.
\[ X(2120) = \cos \phi_2 |u \bar{u} + d \bar{d}|/\sqrt{2} - \sin \phi_2 |s \bar{s}| \]

| Mode | \(\Gamma(\text{MeV})\) | \(\text{Amps.} (\text{GeV}^{-1/2})\) |
|------|----------------|------------------|
| \(K K^*\) | \(2.39c^2 - 9.26cs + 8.98s^2\) | \(\mathcal{M}_{11} = -0.015c + 0.029s\) |
| \(K K^*(1410)\) | \(31.90c^2 + 106.14cs + 88.29s^2\) | \(\mathcal{M}_{11} = 0.082c + 0.14s\) |
| \(\pi a_0(1450)\) | \(0.013c^2\) | \(\mathcal{M}_{00} = -0.0018c\) |
| \(K K^*_0(1430)\) | \(2.98c^2 - 3.70cs + 1.15s^2\) | \(\mathcal{M}_{00} = 0.025c - 0.016s\) |
| \(\eta f_0(1370)\) | \(1.61c^2\) | \(\mathcal{M}_{00} = -0.036c\) |
| \(\pi a_2(1320)\) | \(149.68c^2\) | \(\mathcal{M}_{22} = 0.16c\) |
| \(K K^*_0(1430)\) | \(3.72c^2 - 23.98cs + 38.63s^2\) | \(\mathcal{M}_{22} = 0.028c - 0.092s\) |
| \(\eta f_2(1270)\) | \(21.55c^2\) | \(\mathcal{M}_{22} = -0.12c\) |
| \(\eta f_2^*(1525)\) | \(0.25s^2\) | \(\mathcal{M}_{22} = -0.021s\) |
| \(\rho \rho\) | \(1.09c^2\) | \(\mathcal{M}_{11} = -0.017c\) |
| \(\phi \phi\) | \(4.67s^2\) | \(\mathcal{M}_{11} = -0.097s\) |
| \(\omega \omega\) | \(0.52c^2\) | \(\mathcal{M}_{11} = 0.021c\) |
| \(\rho b_1(1235)\) | \(50.80c^2\) | \(\mathcal{M}_{00} = 0.082c\) |
| \(\omega h_1(1170)\) | \(22.75c^2\) | \(\mathcal{M}_{00} = -0.051c\) |
| Total | \(294.60c^2 + 63.00cs + 143.69s^2\) | \(\mathcal{M}_{00} = -0.093c\) |

**TABLE VI**: Partial widths of \(X(2120)\) as the third radial excitation of \(\eta\), where \(s \equiv \sin \phi_2\) and \(c \equiv \cos \phi_2\), and the factor of \(i\) has been suppressed in all odd partial wave amplitudes.
TABLE VII: Partial widths of $X(2370)$ as the third radial excitation of $\eta'$, where $s \equiv \sin \phi_2$ and $c \equiv \cos \phi_2$, $\phi_2$ is the mixing angle between $X(2120)$ and $X(2370)$, and the factor of $i$ has been suppressed in all odd partial wave amplitudes.