On the origin of double main-sequence turn-offs in star clusters of the Magellanic Clouds

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ABSTRACT

Recent observational studies of intermediate-age star clusters (SCs) in the Large Magellanic Cloud (LMC) have reported that a significant number of these objects show double main-sequence turn-offs (DMSTOs) in their color-magnitude diagrams (CMDs). One plausible explanation for the origin of these DMSTOs is that the SCs are composed of two different stellar populations with age differences of $\sim 300$ Myr. Based on analytical methods and numerical simulations, we explore a new scenario in which SCs interact and merge with star-forming giant molecular clouds (GMCs) to form new composite SCs with two distinct component populations. In this new scenario, the possible age differences between the two different stellar populations responsible for the DMSTOs are due largely to secondary star formation within GMCs interacting and merging with already-existing SCs in the LMC disk. The total gas masses being converted into new stars (i.e., the second generation of stars) during GMC-SC interaction and merging can be comparable to or larger than the masses of the original SCs (i.e., the first generation of stars) in this scenario. Our simulations show that the spatial distributions of new stars in composite SCs formed from GMC-SC merging are more compact than those of stars initially in the SCs. We discuss both advantages and disadvantages of the new scenario in explaining fundamental properties of SCs with DMSTOs in the LMC and in the Small Magellanic Cloud (SMC). We also discuss the merits of various alternative scenarios for the origin of the DMSTOs.

Key words: Magellanic Clouds – galaxies:star clusters – galaxies:kinematics and dynamics

1 INTRODUCTION

Recent photometric studies of intermediate-age SCs in the LMC have discovered objects exhibiting extremely unusual main-sequence turn-offs (MSTOs) in their CMDs. The first indications that such clusters might exist date back several years to observations obtained with terrestrial facilities – Bertelli et al. (2003) demonstrated that the LMC cluster NGC 2173 apparently possesses an unusually large spread in colour about its MSTO, while Baume et al. (2007) obtained a similar result for NGC 2154.

More recent studies based on deep precision photometry from the Hubble Space Telescope (HST) Advanced Camera for Surveys (ACS) have revealed the truly peculiar nature of many intermediate-age LMC clusters. Most striking are the relatively massive clusters NGC 1846, 1806, and 1783 which possess double MSTOs (hereafter referred to as DMSTOs for convenience) on their CMDs (Mackey & Broby Nielsen 2007, MBN07; Mackey et al. 2008; Milone et al. 2008; Goudfrooij et al. 2008). Despite this, the remaining CMD sequences for these clusters (i.e., their red-giant branches, subgiant branches, main sequences, and red clumps) are all extremely narrow and well-defined. This suggests that none of the clusters possesses a significant line-of-sight depth or internal dispersion in [Fe/H], or suffers from significant differential extinction. The simplest explanation is that the observed DMSTOs in each of these clusters represent two distinct stellar populations with differences in age of $\sim 200 – 300$ Myr. It should be noted, however, that apart from the study of Mucciarelli et al. (2008) who found no significant star-to-star dispersion in [$\alpha$/Fe] for 6 stars in NGC 1783, no constraints on the possibility of internal variations in chemical abundances other than [Fe/H] presently exist for these SCs.

Milone et al. (2008) studied a large sample of 16
intermediate-age LMC clusters, and found that 11 of these possess CMDs exhibiting MSTOs which are not consistent with being simple, single stellar populations. Of these 11, four clearly show DMSTOs (the three described above plus NGC 1751) while the remainder possess more sparsely populated CMDs that make the precise morphologies of their peculiar MSTOs difficult to ascertain. All are consistent with being DMSTOs, or alternatively with possibly being more smoothly distributed intrinsic broadenings. Nonetheless, all 11 clusters again exhibit very narrow sequences across the remainder of their CMDs, suggesting that their peculiar MSTOs are due to internal age dispersions of ~ 200 – 300 Myr. Milone et al. (2008) measured the ratio between the upper and the lower MSTO population in NGC1846, NGC1806 and in the less-populated NGC1751 and found that the inferred young populations may comprise up to ~70 per cent of the stars in the central regions of these SCs.

In addition to the above objects, several younger LMC clusters have been identified as possibly possessing irregular CMDs (e.g., Santiago et al. 2002, Gouliermis et al. 2006). It is further worth noting that one intermediate-age SMC cluster, NGC 419, also likely possesses a DMSTO (Glatt et al. 2008).

The results described above suggest that, contrary to expectation, DMSTOs may be a common feature among intermediate-age SCs of the Magellanic Clouds (MCs). If the origin of such DMSTOs is indeed closely associated with an age difference between a first and second generation of stars in these SCs, the above observations raise the following three key questions: (i) why the second generation of stars in a SC with a DMSTO can be formed about 300 Myr after the formation of the first generation of stars, (ii) why the total mass of the second generation can be comparable or even larger than that of the first, and (iii) how the two different populations can now be in the same SC. Although the origin of the observed multiple stellar populations in the Galactic globular clusters (GCs) has been observationally and theoretically discussed in terms of GC formation scenarios (e.g., Piotto 2008), the above fundamental questions so far have not been discussed by theoretical models of SC formation in the LMC.

The purpose of this paper is to explore a new scenario in terms of the above three fundamental questions on the origin of the observed DMSTOs. In the new scenario, an already formed SC (i.e., the first generation of stars) can merge or interact with a GMC so that a second generation of stars is formed in the GMC. The second generation can then merge and mix dynamically with the SC to form a new SC with two stellar populations of different ages. Thus, the origin of the DMSTOs in SCs of the LMC results from merging between SCs and GMCs in the new “GMC-SC-merger” scenario. The typical age difference ($t_{\text{merg}}$) of the two populations in SCs required for explaining the DMSTOs corresponds to the typical time scale of merging between SCs and GMCs of the LMC in the new scenario. We mainly discuss (i) whether star formation is likely to be enhanced during GMC-SC merging, (ii) the mass fraction of the second generation of stars relative to the first in a SC and its dependence on parameters of GMC-SC merging, and (iii) time scales of GMC-SC merging in the LMC.

The plan of the paper is as follows: In the next section, we describe our numerical models for GMC-SC merging in the LMC. In §3, we present numerical results mainly concerning the final distributions of gas and new stars in the remnants of GMC-SC mergers. In §4, we discuss the origin of the DMSTOs of SCs in a more general way. In this section, we also discuss the advantages and disadvantages of two alternative scenarios in explaining the physical properties of SCs with DMSTOs. We summarize our conclusions in §5.

2 THE MODEL

The most important question in the present study is whether the masses of the second generations of stars (i.e., new stars) in the remnants of GMC-SC mergers can be as large as those of the first (i.e., stars initially in SCs). For the masses of the second generations to be comparable to those of the first, a large amount of gas initially in GMCs needs to be accumulated within merging SCs for star formation. Although we do investigate “star formation processes” within GMCs, the adopted model for star formation is rather idealized and phenomenological, and does not allow us to investigate subpc-scale real star formation processes (e.g., the formation of individual stars within the cores of small molecular clouds). We therefore consider that here it is more important for us to (i) investigate how much gas in GMCs can be captured in SCs during GMC-SC merging for various different model parameters for the merging and (ii) suggest that the merger remnants (i.e., new SCs) with large fractions (~0.3–0.5) of gas can finally become SCs with two different populations with comparable masses.

We adopt the same numerical code as adopted in our previous numerical simulations on the hydrodynamical evolution of interstellar gas in interacting and merging galaxies (Bekki et al. 2002) and the formation of star clusters from GMCs in galaxies (Bekki & Couch 2003). Although we can investigate gas dynamics in merging GMCs, the adopted TREESPH code does not allow us to precisely describe long-term dynamical simulations such as two-body relaxation processes in SCs. In future work we will discuss the long-term evolution of the remnants of GMC-SC mergers by using appropriate numerical codes such as NBODY4 adopted in our previous simulations of SC formation (Hurley & Bekki 2008, HB08). In order to understand more clearly the hydrodynamical evolution of GMC-SC mergers, we do not include the external tidal field of the MCs in the present study.

A SC in a GMC-SC merger is represented by a Plummer model with mass and size represented by $M_s$ and $R_s$, respectively. The scale length of the Plummer model is fixed at $0.2R_s$ for all our simulations. A GMC is assumed to be a Bonner-Ebert isothermal sphere (Ebert 1955; Bonner 1956) with a mass of $M_g$, a size of $R_g$, and a temperature of $T_g$. The radial distribution of a Bonner-Ebert isothermal sphere is determined by solving the differential equations for a given $T_g$ of the sphere. Guided by the observed relation between mass densities and sizes of GMCs discovered by Larson’s (1981) relation and the observed typical mass and size of GMCs in the Galaxy (e.g., Solomon et al. 1979), we use the following $R_g - M_g$ relation:

$$R_g = 40 \times \left( \frac{M_g}{5 \times 10^5 M_\odot} \right)^{0.53} \text{pc}$$

(1)
We investigate models with $M_k = 5 \times 10^4 M_\odot$, $R_k = 11.9$ pc, and $T_k = 2.6$ K in the present study. Since the isothermal Bonnor-Ebert sphere is adopted, one of key parameters determining the gas dynamics of GMC-SC merging is the mass-ratio ($s_k = M_k/M_\text{gmc}$) of a GMC to a SC in a given model; if we derive parameter dependences of numerical results on $s_k$ for the above $M_k$, these dependences can be applied for other $M_k$.

The initial locations of the SC and the GMC in the GMC-SC merger are set to be $(x_s, y_s, z_s)$ and $(x_g, y_g, z_g)$, respectively. For convenience, $(x_s, y_s, z_s) = (0, 0, 0)$ so that the initial position of the GMC with respect to the SC can be more clearly understood. The initial velocities of the SC and the GMC in the $x$-, $y$-, and $z$-directions are set to be $(u_s, v_s, w_s)$ and $(u_g, v_g, w_g)$, respectively, and $(u_s, v_s, w_s) = (0, 0, 0)$ for simplicity. The orbital plane of the GMC-SC merger is coincident with the $x$-$y$ plane for all models in the present study (i.e., $z_s = z_g = w_g = 0$). We adopt $x_g = 2 R_k$ for all models so that GMC and SCs are not initially in direct contact. The initial direction of the velocity vector of the GMC is only a parameter for the orbit of the GMC owing to the spherically symmetric distributions of the model GMCs and SCs. We therefore assume that the initial direction is parallel to the $x$-axis toward the negative $x$ (i.e., $u_g < 0$ and $v_g = 0$). Thus $y_g$ and $u_g$ are free parameters that determine the orbit of the GMC with respect to the SC.

To summarize, the key model parameters in the present simulations are $M_k$ (or $s_k$), $R_k$, $y_g$ (i.e., the impact parameter), and $u_g$.

For all models the total number of particles ($N$) is 40,000, half of which is for stars, and the initial gravitational softening length is 0.24 pc. We choose these particle numbers in order to facilitate investigation of the long-term evolution of merger remnants in the external LMC potential in our future numerical studies using NBODY4, as adopted in our previous simulations (HB08): models of SCs using these codes can be undertaken within a reasonable time scale for $N \sim 10^3 - 10^5$ without excessive numerical costs (e.g., D’Ercolono et al. 2008; HB08).

Owing to the limited size and mass resolution of the present simulations, subpc-scale physical processes of star formation cannot be investigated in the present study. We do, however, try to investigate whether the gas accumulated in the central regions of SCs after GMC-SC merging can be sufficient to form new stars by adopting a simple prescription for star formation. In the models with “star formation”, a gas particle is converted into a collisionless new stellar one if the gas particle meets the following conditions: (i) the dynamical time scale of the SPH gas particle is shorter than the sound crossing time, and (ii) the gas is converging (i.e., $\nabla v < 0$, where $v$ is the velocity vector of the gas particle). These two conditions mimic the Jeans gravitational instability for gaseous collapse. The adopted model for star formation is rather idealized so that “new stars” need to be interpreted as possible formation sites of stars in GMCs. In the rest of this paper, the stars initially in SCs are just referred to as “stars” to distinguish them from new stars formed from gas.

Although we investigate many models spanning different combinations of the four key parameters, we mainly present the results of the “standard model” which shows the typical behavior of gas accumulation in SCs merging with GMCs. We also briefly describe the results of the “star formation model” in which the simple treatment of star formation is included and which as parameter values exactly the same as those adopted in the standard model.

The parameter values of the standard model and the ranges of parameters investigated for an additional 31 models are summarized in Table 1. The standard model and the star formation model are represented as M1 and M2, respectively. Parameter values in models other than M1 are not described in the table in order to avoid consuming an unnecessarily large space for the description. In the present study we mainly show the dependences of the gas mass fractions within a radius of 10 pc ($f_g$) in merger remnants on $y_g$, $u_g$, $M_k$, and $R_k$.

We note that SC-GMC merging does not occur in some of our models with large $y_g$ and $|u_g|$; however, gas-transfer between SCs and GMCs is still possible in these cases. We also describe the results of these models briefly in the present paper. It should be stressed that the present study is the first step toward understanding the possibly complicated interactions between SCs and GMCs in the MCs: the present models are rather idealized in some respects, in order to grasp the essential ingredients of GMC-SC merging processes. We plan to investigate a more fully-consistent and sophisticated model for GMC-SC merging in our future studies.

| Model no | $M_k$ | $R_k$ | $y_g$ | $u_g$ | Star formation | Comments |
|----------|-------|-------|-------|-------|---------------|----------|
| M1       | 5.0   | 10    | 6.0   | 2.2   | NO            | the standard model |
| M2       | 5.0   | 10    | 6.0   | 2.2   | YES           | the star formation model |
| M3-32    | (1.0, 20.0) | (5, 25) | (0, 23.8) | (0, 12.8) | NO & YES     |          |

$^a$ For models M3 - M32, the ranges of model parameters investigated in the present study are shown in parentheses: the left and the right numbers are the minimum and maximum values.

$^b$ The total mass of stars in a SC in units of $10^4 M_\odot$. The total gas mass in a GMC is $5 \times 10^4 M_\odot$ for all models.

$^c$ The size of a SC in units of pc. The size of a GMC is set to be 11.9 pc for all models in the present study.

$^d$ Initial $y$-position of a GMC with respect to the center of a SC. This corresponds to the impact parameter of the GMC-SC collision.

$^e$ The absolute magnitude of the initial velocity of a GMC along the $x$-axis in units of km s$^{-1}$, with respect to the center of a SC.

$^f$ “YES” (“NO”) means that the star formation model is (is not) included in the simulation.
3 RESULTS

3.1 The standard model

Fig. 1 shows how gas in a GMC evolves during major merging between the GMC and a SC with \( s_g = 1 \) in the standard model M1. During the off-center collision between the GMC and the SC, the central part of the GMC is tidally compressed to form a compact gaseous core. Owing to dynamical friction during merging, the core can sink into the central region of the SC to finally form a flattened gaseous spheroid (or gas disk) within the SC. The SC is not destroyed by this merging and thus retains its initial spherical shape, though it loses about 30% of its initial mass owing to tidal stripping. Hence, the newly formed SC has a very compact, flattened gaseous disk in its central region: further star formation from this central gas can result in two distinct stellar populations in the SC.

About 30% of the gas in the GMC is rapidly stripped during merging to form a very diffuse gaseous halo around the new SC. This stripped gas cannot be accreted onto the

Figure 1. Time evolution of stellar (magenta) and gaseous (cyan) distributions projected onto the \( x-y \) plane (upper four panels) and the \( x-z \) plane (lower four panels) for the standard model M1. The time \( T \) shown in the upper left corner of each panel is given in units of Myr. Note that a gaseous disk can be seen in the inner region of the SC, and the center of the disk seems to deviate from the center of the SC.

Figure 2. The time evolution of stellar (solid) and gaseous (dotted) masses within the central 10pc of the SC in the standard model M1.

Figure 3. The cumulative stellar (solid) and gaseous (dotted) masses within radius \( R \) from the center of the SC at \( T = 11.0 \) Myr in the standard model M1.

Figure 4. Dependences of \( f_g \) (i.e., gas mass fraction within a radius of 10 pc in merger remnants) on \( y_g \) (upper left), \( |u_g| \) (upper right), \( M_s \) (lower left), and \( R_s \) (lower right). The dotted and dashed lines represent \( s_g = 1 \) and 2.5, respectively.
SC after merging even in this isolated model, and thus is likely to be returned back to interstellar medium (ISM) of the LMC if interaction between the stripped GMC gas and the ISM in the LMC is included in future simulations. Fig. 2 shows that the gas mass fraction within the central 10 pc of the new SC (f_g) increases as the SC loses its stellar mass and gas is accreted into the SC. Fig. 2 also shows that f_g can be finally as large as 0.5 at T = 11 Myr. This implies that if all of the gas can be converted into new stars, the mass ratio of the second generation of stars to the first can be as large as 1. We discuss later important implications of this result in terms of the origin of the DMSTOs.

Fig. 3 shows that the final cumulative radial mass distributions (M(＜ R), where R is the distance from the mass center of the newly-formed SC) of gas and stars at T = 11 Myr are quite different in the sense that the gas shows a more compact distribution than do the stars. The half-mass radius is 3.3 pc for the stars and 1.0 pc for the gas, which reflects that the collapsed central part of the GMC is directly transferred to the center of the SC owing to dynamical friction. Therefore the gas mass fraction is significantly larger in the inner region of the new SC, which means that this region of the new SC would be dominated by young stars if star formation were to subsequently occur in the accumulated gas.

3.2 Parameter dependences

The dependences of the final properties of SCs on a parameter are investigated by changing the values of the parameter and by fixing those of other parameters. Dependences of the present numerical results on the four key parameters (y_g, |u_g|, M_s, and R_s) are described as follows:

(i) As shown in Fig. 4, f_g does not depend strongly on y_g (i.e., impact parameter) as long as GMCs merge with SCs (i.e., y_g ＜ 12 pc). However f_g can be very small (＜ 0.1) for models with large y_g, in which no merging occurs and only small fractions of gas are transferred to SCs during the GMC-SC tidal interaction. For example, the model with y_g = 18 pc and other parameter values being the same as those in the standard model shows f_g = 0.07, corresponding to a gas mass within the central 10 pc of 3.2 × 10^3 M⊙.

(ii) The models with high |u_g| (＞ 7 km s⁻¹) show very small f_g (＜ 0.1), because high-speed encounters between GMCs and SCs can again prevent GMCs from merging with the SCs (see Fig. 4). There is no strong dependence of f_g on |u_g| for models with |u_g| ＜ 7 km s⁻¹ in which GMC-SC merging does occur. Models with |u_g| ＞ 12 km s⁻¹ show very small f_g (＜ 0.01) even if y_g = 0 pc.

(iii) GMCs can either destroy SCs or tidally strip most of their stellar envelopes, if the masses of the GMCs are significantly larger than those of the SCs interacting with the GMCs. Therefore the compact gas cores of GMCs cannot sink into the central regions of SCs in models with large s_g (= M_g/M_s). As a result of this, the models with large s_g (＞ 2.5) show small f_g in the merger remnants for a reasonable set of other model parameters (see Fig. 4). It should be, however, stressed that the models with y_g = 0, small |u_g| (＜ 4 km s⁻¹), and large s_g (＞ 2.5) show large f_g (＞ 0.7), because in these situations the SCs become trapped in the central cores of the GMCs. It might not be appropriate to call these very gas-dominated merger remnants “SCs”.

(iv) In models with smaller s_g (larger M_s), SCs can accumulate a larger amount of gas from merging GMCs. However, in these models it is not possible for s_g to be particularly large, owing to the initially larger stellar masses in these models. Therefore, there is an optimum s_g for which f_g is a maximum for a given set of model parameters. As shown in Fig. 4, the major merger model (i.e., s_g = 1) shows the largest f_g, which implies that GMC-SC mergers with comparable masses can form larger fractions of the second generations of stars after merging.

(v) The models with larger R_s show larger f_g for R_s ＜ 20 pc. This is mainly because the final stellar masses within the central 10 pc for SCs with larger R_s (and thus lower stellar densities) tend to be small owing to the more efficient stripping of stars during GMC-SC merging; the actual mass of gas within the merger remnants do not depend strongly on R_s for R_s ＜ 20 pc. The models with large R_s (e.g., R_s = 25 pc) show very small f_g owing to the total destruction of SCs during GMC-SC merging.

3.3 Star formation

Owing to our adopted prescription for star formation in GMCs, new stars are formed preferentially in rather high-density regions of GMCs. New stars can therefore be formed during the accumulation of compact and flattened gas spheres in the central regions of GMCs due to strong tidal compression of the GMCs by SCs. The total mass of new stars within a merger remnant (i.e., newly-formed SC) depends on how much gas is accumulated into the SC during merging. Therefore the parameter dependences of mass fractions of new stars in merger remnants for models including star formation are very similar to those of f_g for models without star formation. Thus we here describe only the results of the standard star formation model M2.

Fig. 5 shows that the new stars in the merger remnant show a compact, disky distribution and thus have quite different spatial distributions from the original stars: there appears to be a SC within a SC. This “SC-within-SC” appearance is one of key characteristics of the remnants of SC-GMC mergers in the present study. The total masses of stars, gas, and new stars are 3.7 × 10^5 M⊙, 1.1 × 10^5 M⊙, and 2.0 × 10^4 M⊙, respectively, which means that the mass fraction of new stars among all stars is 0.35. The half-mass radii...
for stars, gas, and new stars are 4.0 pc, 3.8 pc, and 1.2 pc, respectively, which clearly demonstrates that the new stars have a much more compact distribution than do the original stars. The simulated flattened structure of new stars is rather similar to the inner flattened "core" observed in M15 (e.g., van den Bosch et al. 2006).

Fig. 6 shows that the star formation rate (SFR) becomes as high as 0.05 $M_\odot$ yr$^{-1}$ in the star formation model M2 when the compact gas sphere of the GMC is sinking into the SC owing to dynamical friction. The SFR is quite peaked so that there is little age dispersion among the new stars formed during GMC-SC merging. The model with larger $y_6$ (=11.9 pc) and thus a weaker tidal field from the SC shows a smaller peak SFR ($\sim 0.02 M_\odot$ yr$^{-1}$), in the later phase of GMC-SC merging. These results clearly suggest that the strength of the tidal field of a SC merging with a GMC is a key factor that can determine the peak SFR and the star formation efficiency of the GMC-SC merger. Fig. 7 confirms that the inner regions of the merger remnant are dominated by new stars, as suggested in Fig. 5.

The derived nested spatial distributions (i.e., SC-within-SC appearance) of the simulated SCs are apparently not observed in the intermediate-age SCs of the LMC which have been tested (e.g., MBN07). Furthermore, observations have not yet discovered inner disky structures of SCs in the LMC, though the SCs themselves have rather flattened shapes in general (e.g., van den Bergh 2000). To resolve these inconsistencies in structural properties between the simulated and observed SCs requires that long-term dynamical relaxation processes play a role in dynamically mixing old and new components and thus in forming the canonical radial density profiles observed in the present SCs (e.g., King-type profiles).

The median relaxation time in a SC like NGC 1846 should be $\sim 2$ Gyr (see for example the models in Mackey et al. 2008b), and a factor of a few shorter than this in the very centre. Therefore, there has conceivably been enough time for dynamical relaxation processes to influence and modify cluster structure. It should also be noted that the second generation of stars in a composite SC formed as described above will initiate a new phase of violent relaxation in the SC as the most massive members evolve quickly and die, and any residual gas is expelled. This can be very good at dynamically mixing the cluster on a relatively short time scale (see e.g., Meylan & Heggie 1997 and references therein). It is likely that these evolutionary processes might alter significantly the centrally concentrated spatial distribution of the second generation of stars, as well as the observable number fraction of the first and second generations of stars in the centres of new SCs.

### 4 DISCUSSION

#### 4.1 Advantages and disadvantages of the scenario

The observed apparently clear distinction between bluer and redder MSTOs in some SCs of the LMC (e.g., MBN07; Mackey et al. 2008; Milone et al. 2008) implies that there were two bursts of star formation rather than prolonged periods of star formation in these SCs. One advantage of the present scenario is therefore that it can naturally explain the possible two burst epochs of star formation: one is during the original formation of SCs and the other is during GMC-SC merging. Furthermore, the new stars formed during the GMC-SC merging are clearly bound to the original SC, which naturally explains how two different stellar populations may coexist in some SCs. The scenario does not need to consider a physical mechanism by which interstellar gas can be accumulated within SCs and then form stars within the SCs.

The present scenario predicts significant age differences between the first and the second generations of stars in SCs owing to the late formation of stars triggered by GMC-SC merging/interaction. A key question here is therefore whether the age difference can be similar to $\sim 3 \times 10^8$ yr ($t_{gap}$), which may be required to explain the origin of the DMSTOs (e.g., MBN07; Mackey et al. 2008; Milone et al. 2008). The time scale of a GMC-SC merger event ($t_m$) in the LMC can be estimated as follows (e.g., Makino & Hut 1997):

$$t_m = \frac{1}{n_g \sigma v},$$

where $n_g$, $\sigma$, and $v$ are the mean number density of the GMCs, the geometrical cross section of a GMC, and the relative velocity between a GMC and a SC. If we consider that the LMC is a uniform disk with a disk radius of $r_d$ ($\sim 5$...
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kpc), a scale height of $z_d$, and the number of GMCs being equal to $N_g$ for simplicity, then

$$n_g = \frac{N_g}{2\pi r_g z_d^2}. \quad (3)$$

We assume here that SCs are more compact than GMCs with comparable masses and thus that \( \sigma \) in the $t_m$ estimation should be the cross section for GMCs. If we consider that GMCs have spherical shapes, then we can use the following:

$$\sigma = \pi r_g^2, \quad (4)$$

where $r_g$ is the size of a GMC (i.e., $r_g = 0.5R_g$).

We are interested in the SCs of the LMC a few Gyr ago when the SCs with DMSTOs were formed. The LMC would have a larger gas mass fraction and thus a larger number of GMCs in the past. Recent NANTEN observations have found 272 GMCs with masses larger than $2 \times 10^4 M_\odot$ in the LMC (Fukui et al. 2008); however, the number of GMCs a few Gyr ago may be significantly larger than that in the present LMC. If we consider (i) that $v$ is similar to the observed velocity dispersion of stars in the LMC (e.g., van der Marel 2002), (ii) that the past $N_g$ is larger by a factor of 2 than that of the present LMC, and (iii) that $z_d$ is about 10% of the observed radial scale length of the LMC ($\sim 1.5$ kpc; van den Bergh 2000), then $t_m$ can be estimated as follows:

$$t_m = 3.2 \left( \frac{N_g}{500} \right)^{-1} \left( \frac{z_d}{150 \text{pc}} \right) \left( \frac{R_g}{15 \text{pc}} \right)^{-2} \left( \frac{v}{20 \text{km/s}} \right)^{-1} \text{Gyr}. \quad (5)$$

It should be stressed here that the above $t_m$ is for the average number density of GMCs with masses larger than $2 \times 10^4 M_\odot$ over the entire LMC region: $t_m$ can be much shorter than the above in some regions with locally high number densities. The above value of $t_m$ is much larger than $t_{gap}$ ($\sim 0.3$ Gyr) required for explaining the DMSTOs. However, $t_m$ can be as small as $t_{gap}$ in the LMC if we adopt a higher $N_g$, an $r_g$ larger than equation (1) describes, and a thinner gas disk as follows:

$$t_m = 0.3 \left( \frac{N_g}{1000} \right)^{-1} \left( \frac{z_d}{150 \text{pc}} \right) \left( \frac{R_g}{30 \text{pc}} \right)^{-2} \left( \frac{v}{20 \text{km/s}} \right)^{-1} \text{Gyr}. \quad (6)$$

It remains unclear which of the above two estimations is more realistic and realistic in the LMC a few Gyr ago. In addition, owing to differences in the structural and kinematical properties of gas and stars between the LMC and the SMC (i.e., $N_g$, $z_d$, $r_g$, $r_d$, and $v$), $t_m$ might well be different between the MCs.

A SC can merge with a GMC located within a distance $d_g$ from the SC within the following time scale:

$$t_m = 0.3 \left( \frac{d_g}{60 \text{pc}} \right)^2 \left( \frac{z_d}{150 \text{pc}} \right) \left( \frac{R_g}{15 \text{pc}} \right)^{-2} \left( \frac{v}{20 \text{km/s}} \right)^{-1} \text{Gyr}. \quad (7)$$

This means that $t_m$ can be similar to $t_{gap}$ for some local regions with surface number densities ($\Sigma_g$) of GMCs similar to 73 kpc$^{-2}$. Such high $\Sigma_g$ would be possible in strong tidal arms and in the inner region of the LMC. It is also possible that a group of GMCs (or GMC associations) could have such a high $\Sigma_g$ in the LMC. Thus, $t_m$ can be as small as $\sim 3 \times 10^8$ yr (thus $\sim t_{gap}$), although it is possible that $t_m$ varies significantly between different regions of the LMC a few Gyr ago.

It appears that the observed possible age differences between two populations is an order of $10^8$ yr; no SCs with DMSTOs appear to show implied age differences of an order of $10^9$ yr (see Santiago et al. 2002 for a possibly exceptional case of NGC 1868). Therefore one possible problem in the present scenario is that the time scales of GMC-SC merging can in principle be significantly longer than $10^8$ yr for some SCs: the scenario needs to explain why there are apparently no intermediate-age SCs with implied age differences between the two component stellar populations being as large as 1 Gyr. A possible solution of this potential problem is if the time scales of GMC-SC merging in the LMC grew longer in the more recent past - that is, as the LMC evolved with time GMC-SC merging became less likely because the number density of GMCs and the typical GMC size became lower and smaller, respectively, owing to rapid gas consumption by ongoing star formation in the LMC. Thus it would be possible that intermediate-age SCs cannot merge with GMCs much later ($\sim 1$ Gyr) than their formation in the LMC.

Furthermore, it would be equally possible that the time scales of GMC-SC merging can be significantly shorter than $10^8$ yr owing to locally higher mass densities in the vicinity of some SCs. Although SCs which experience merging with GMCs much less than $10^8$ yr after their formation can still have two distinct component populations, they would not clearly show DMSTOs in their CMDs due to the much smaller age differences between them. Therefore the possible presence of SCs with age differences significantly smaller than $10^8$ yr would be still consistent with observations. Merging of SCs with GMCs less than $10^8$ yr after their formation might well be responsible for the observed binary/multiple SCs with small age differences (e.g., NGC 1850, Leon et al. 1999).

GMC-SC merging is highly likely to occur between SCs and their local neighborhood GMCs in the LMC: cloud complexes (or “superclouds”) with the typical mass of $10^4 M_\odot$ (Elmegreen 1987) would be the progenitors for the GMC-SC mergers. Therefore differences in chemical abundances between merging SCs and GMCs should be quite small unless there exist significant radial and azimuthal abundance gradients in the LMC. Owing to the time lag of $\sim 3 \times 10^8$ yr between SC formation and SC-GMC merging in the GMC-SC merger scenario, GMCs can be chemically polluted by AGB stars in SCs merging with the GMCs. Therefore, it is quite possible that the second generations of stars formed from GMCs could have different abundances in light elements from those in the first generations initially in the SCs. Ongoing spectroscopic observations on the abundance properties of SCs with DMSTOs will soon reveal whether or not abundance inhomogeneities exist in these objects. We thus plan to investigate the chemical evolution of GMC-SC mergers in order to compare between the predicted and the observed degrees of inhomogeneity in light element abundances for SCs with DMSTOs in the LMC.

4.2 Alternative scenarios

4.2.1 Self-pollution by AGB stars

It would be possible that stellar ejecta from AGB stars in a SC can be accumulated in the central region of the SC so that the ejecta can be used for the formation of the second generation of stars. In this “self-pollution” scenario, the second generation of stars needs to form well after the re-
moval of stellar ejecta from Type II supernovae that can cause a significant spread in heavy element abundances. Recent numerical simulations of GC formation based on this self-pollution scenario have shown (i) that star formation for the second generation can start soon after ejection of stellar winds from massive AGB stars, and (ii) that star formation can continue gradually within GCs over a period of \( \sim 100 \) Myr (D’Ercole et al. 2008).

Thus the self-pollution scenario has serious difficulties in explaining (i) why the time difference between the formation epochs of the first and second generations of stars is typically \( \sim 300 \) Myr, and (ii) why most stars in the second generation formed almost simultaneously. It is likely that the numerical simulations by D’Ercole et al. (2008) do not describe so precisely the star formation histories within SCs owing to the adopted rather idealized models for hydrodynamics and star formation within SCs. Therefore, future theoretical studies based on more sophisticated numerical simulations are certainly worthwhile to confirm whether the self-pollution scenario has really the above serious problems.

4.2.2 Star cluster merging

It is a well known observational fact that a significant fraction of SCs in the LMC are binary or multiple clusters (e.g., Bhatia & Hatzidimitriou 1988; Bhatia et al. 1991; Dieball et al. 2002). This fact implies that merging of binary SCs can form single SCs with distinct stellar populations, if there are initial differences in age between the two original clusters. This merger scenario needs to explain why age differences between two merging clusters can be similar to \( 3 \times 10^8 \) yr for some binary SCs. Observational studies on possible age differences in binary SCs in the LMC have demonstrated that the components of binary or multiple SC systems typically appear to be small, hence implying that all SCs in a given bound system are generally formed simultaneously or over a very short time scale (e.g., Dieball et al. 2002). Therefore, the merger scenario appears to have a serious problem in explaining the frequently occurring age difference of \( \sim 3 \times 10^8 \) yr observed for the DMSTOs in intermediate-age MC clusters.

It should, however, be stressed that there are some binary SCs in the LMC with age differences up to \( \sim 0.5 \) Gyr (e.g., SL356-357 pair, Leon et al. 1999): it would be possible that merging of the two component SCs in such systems can happen in the LMC. If this is the case, the merger scenario needs to explain (i) why SC merging apparently happens preferentially for those systems which have age differences of \( \sim 300 \) Myr and (ii) why the mass-ratios of the younger SCs to the older ones are comparable to and larger than 1. It is currently unclear whether such preferential SC merging can happen in the history of the LMC.

4.3 DMSTOs only for intermediate-age SCs?

The age distribution of the LMC SCs shows a gap extending from 3 to 13 Gyr – with only one cluster in this age range – suggesting that a second epoch of cluster formation started abruptly in the LMC about 3 Gyr ago (e.g., Da Costa 1991; Geisler et al. 1997; Rich et al. 2001; Piatti et al. 2002). SCs with DMSTOs are observed to be \( \sim 1.5 - 2.5 \) Gyr old (Mackey et al. 2008; Milone et al. 2008) and thus were formed just after the “age-gap” period. So far it remains observationally unclear whether the oldest GCs in the LMC, which formed before the commencement of the age gap, also show DMSTOs. This raises the following two questions: (i) whether the origin of the SCs with DMSTOs is closely associated with some specific formation processes at the reactivation of SC formation a few Gyr ago, and (ii) whether SCs have DMSTOs irrespective of their formation epochs owing to a general physical mechanism responsible for the DMSTO formation.

Recent numerical simulations have shown that tidal interaction between the LMC and the SMC can dramatically increase (by a factor of ten) the cloud-cloud collision rate in the LMC during the interaction (e.g., see Fig. 1 in Bekki et al. 2004a). This result implies that if a high cloud-cloud collision rate means a high GMC-SC collision/merging rate in the interacting MC system, the origin of SCs with DMSTOs might be closely associated with the commencement of strong tidal interaction between the MCs a few Gyr ago in the present GMC-SC merger scenario. Recent numerical simulations have suggested that the MCs might have started their strong tidal interaction about 3 – 4 Gyr ago for a reasonable set of orbital parameters (Bekki et al. 2004b; Bekki & Chiba 2005). Thus it is possible that the origin of SCs with DMSTOs may result from high GMC-SC collision/merging rates in the LMC due to strong tidal interactions between the MCs a few Gyr ago.

Owing to continuous interaction between the MCs after the last dynamical coupling about 3 – 4 Gyr ago (e.g., Bekki & Chiba 2005), the LMC can retain an enhanced cloud-cloud (and thus probably GMC-SC) collision rate. This means that SCs with ages younger than 1.5 Gyr can also show DMSTOs, though strong observational evidence for the presence of DMSTOs in the CMDs of young SCs does not yet exist. Previous numerical simulations (e.g., Yoshizawa & Noguchi 2003) have shown that the LMC and SMC can very strongly interact with each other about 1.5 Gyr ago – in this case it would be highly likely that a larger fraction of SCs with ages of 1.5–0.3 Gyr may show DMSTOs. This would be consistent with the recent observational results of Milone et al. (2008). In addition, the cloud-SC collision rate at the epoch of disk formation in the LMC might well be much higher than that at the present owing to the higher gas mass fraction and the higher degree of random motion at the time of disk formation. Therefore, it is very possible that the old GCs in the LMC may show DMSTOs in the GMC-SC merger scenario.

It should be stressed, however, that recent proper motion measurements of the MCs by the Advanced Camera for Surveys (ACS) on the Hubble Space Telescope (HST) have reported that the LMC and the SMC have significantly high Galactic tangential velocities (367 \( \pm 18 \) km s\(^{-1}\) and 301 \( \pm 52 \) km s\(^{-1}\), respectively), suggesting that the MCs may be unbound from each other (e.g., Kallivayalil et al. 2006). In this case, LMC-SMC interactions about 1.5 and 3 – 4 Gyr ago are unlikely to have occurred, meaning that the formation of SCs with DMSTOs has little to do with any past interaction between the MCs. Given that detailed modeling of the LMC-SMC orbits that includes the new proper motion data as well as factors previously ignored (e.g., a common halo and/or different circular velocity of the Galaxy) has not
yet been completed, it would be fair to say that at present the connection between the interaction histories of the MCs, and the formation of DMSTOs in SCs is not so clear.

4.4 Relations to the Galactic GCs with abundance inhomogeneities

The origin of the observed star-to-star light element abundance inhomogeneities in the Galactic GCs (GGCs) has been extensively discussed both theoretically and observationally (e.g., Snaed et al. 1992; Norris & Da Costa 1995; Smith et al. 2005; see Gratton et al. 2004 for a recent review). The observed presence of star-to-star abundance inhomogeneities in less evolved stars on the main sequence and subgiant-branch (e.g., Cannon et al. 1998 for 47 Tuc) now strongly suggests that the origin of these abundance inhomogeneities is due to the early chemical evolution of GC-forming gas clouds. Stellar ejecta from AGB stars and massive stars have been considered to play a key role in the early chemical evolution of GCs for the self-pollution scenario (e.g., Karakas et al. 2006; Prantzos & Charbonnel 2006). The early chemical pollution in GC-forming clouds results from stellar ejecta from massive AGB stars in the “AGB scenario” (e.g., Ventura & D’Antona 2005; Bekki et al. 2007) and from the stellar winds from massive stars in the “massive star scenario” (e.g., Prantzos & Charbonnel 2006).

If the origin of the GGCs with abundance inhomogeneities is essentially the same as that of the SCs with DMSTOs in the LMC, then age differences between the first and the second generations of stars in these GGCs may be as large as \(10^8\) yr. This means that the massive star scenario can be ruled out, because the time lag between the first and the second generations is at most an order of \(10^8\) yr (Bekki & Chiba 2007; Decressin et al. 2007). This also means that gaseous ejecta from the first generation of AGB stars in a SC needs to be retained in the SC without star formation for \(3 \times 10^8\) yr and then converted into the second generation in the AGB scenario. Such delayed formation of the second generation of stars implies that the second generation would form from the mixed gas of stellar ejecta from AGB stars with different masses. Thus, if only stellar ejecta from massive AGB stars (\(\sim 6 M_\odot\)) can much better explain the observed C-N, O-Na, and Mg-Al anticorrelations (e.g., Ventura & D’Antona et al. 2008), the AGB scenario would have a serious problem in explaining the possible age different of \(3 \times 10^8\) yr between the first and the second generations.

It would be equally possible that the origin of GCs with abundance inhomogeneities in the Galaxy is different from that of SCs with DMSTOs in the LMC, given that the physical properties are different between these clusters in the two different galaxies (e.g., Mackey & Gilmore 2004). The implied high fraction of He-rich stars with \(Y > 0.3\) in \(\omega\) Cen and NGC 2808 (Piotto et al. 2005; Piotto et al. 2007) cannot be explained simply by the GMC-SC merger scenario: such He-rich stars would need to form from stellar ejecta either from fast-rotating massive stars (e.g., Prantzos & Charbonnel 2006; Decressin et al. 2007) or from massive AGB stars (e.g., D’Antona et al. 2002). Possibly, the time-lags between formation of the first and the second generations of stars in clusters can be different between host galaxies. This might well cause a variety of differences in physical properties between the first and second generations of stars in clusters belonging to separate host galaxies.

5 CONCLUSIONS

We have numerically investigated how GMCs evolve during GMC-SC merging in order to better understand the origin of the DMSTOs observed in intermediate-age SCs in the GCs. We have mainly investigated mass fractions of gas within 10 pc of the remnants of GMC-SC mergers for variously different model parameters. We summarise our principle results as follows:

1. Gas initially in GMCs can be accumulated within SCs during GMC-SC merging so that high-density compact gaseous regions are formed. GMCs (or SCs) may be giant in the central regions of the SCs. The mass fractions of gas \(f_g\) within the central 10 pc of the merger remnants (i.e., the newly formed SCs) can be as large as 0.5 for models with \(s_g \sim 1\) (i.e., major mergers).

2. There is an optimum \(M_m\) (or \(s_g\)) for a given set of model parameters, for which \(f_g\) is a maximum. For example, the model with \(M_m = M_g\) (i.e., \(s_g = 1\)) shows the maximum \(f_g\) in models with \(M_m = 5 \times 10^4 M_\odot\) and \(y_k = 0.5 R_g\) (6 pc). The remnants of major GMC-SC mergers are likely to show lower \(f_g\) in the present study.

3. As long as GMC-SC merging occurs, \(f_g\) does not depend strongly on \(y_k\) and \(|u_y|\). The models with larger \(y_k\) and \(|u_y|\), for which GMCs can interact with SCs without merging, show only very small \(f_g\) (an order of \(10^{-2}\)). These results mean that merging is essential for the formation of new SCs with large \(f_g\).

4. If star formation is included in GMC-SC merging, compact, flattened star clusters composed of new stars can be formed in the central regions of the merger remnants. Since the simulated SCs-within-SCs appearances (i.e., doubly nested SCs) are not observed in the GCs, some later dynamical evolution processes need to transform the doubly nested SCs into normal-looking (well-mixed) objects with standard (King-type) density profiles.

5. The time scale of GMC-SC merging \(t_m\) in the LMC can be similar to the typical age difference of \(3 \times 10^8\) yr between the component stellar populations implied by observations of DMSTOs in SCs in the LMC. However, \(t_m\) depends on the number of GMCs, the sizes of GMCs, and the velocity dispersions of GMCs and stars in the LMC a few Gyr ago, all of which remain observationally unclear.

Based on these results, we have pointed out that the observed possibly large fractions of the second generations of stars in SCs with DMSTOs in the LMC can be due to the past GMC-SC merging. We have also suggested that time lags between SC formation and the subsequent GMC-SC merging can be responsible for the possible age differences between the first and the second generations of stars in the SCs with DMSTOs in the LMC. The adopted numerical code does not allow us to investigate whether the simulated doubly-nested SCs can evolve into normal SCs with canonical radial density profiles due to internal dynamical processes. We plan to investigate this question in our future studies using the appropriate numerical codes (e.g., NBODY4).
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