Acoustic analog of Hall effect in superconductive films

E. D. Gutliansky

(Dated: 14 January 2011)

Longitudinal electric field of a surface acoustic wave (SAW) drags vortex structure of a superconductive film, deposited on a piezoelectric substrate, and generates longitudinal DC component of an acoustoelectric field, which does not depend on direction of an external magnetic field. The contra-directional vortices are dragged by SAW in opposite directions. This phenomenon represents an acoustic analog of Hall effect, where vortices are an analog of current carriers, and the SAW Pointing vector acts as an impressed electric field. The calculation of the acoustoelectric field for a YBCO film with niobate lithium substrate coincides well with experimental data.

Ultrasonic waves (UW), spreading in superconductors with dynamic vortex structure (VS), drag it and induce constant component of an electric field. This effect was observed during the experiments \cite{1,2,3}, and was researched in theory \cite{4,5,6}. We name it an acoustoelectric effect on the analogy of electron-phonon drag effect in semiconductors and metals \cite{7}.

This effect is interesting enough from the experimental point of view. The effect makes it possible to introduce new measuring ways of VS parameters. Namely, we are able to measure VS density derivative and VS viscosity coefficient. There is another reason for our interest. The effect gives us a way of controlling vortices movement by means of ultrasonic waves. It enables us to create a new class of devices using back reaction effects (changing SAW parameters at the expense of its interaction with moving VS \cite{10,11,12,13}). We may call this science area 'acoustovihronics' by analogy with 'acoustoelectronics' \cite{8}.

Acoustoelectric effect in YBCO films with niobate lithium substrate was observed in the works \cite{1,2}. There was a standard acoustoelectric effect above the transition point in superconductive state: entrainment of normal carriers by UW. Acoustoelectric field sign and Hall effect sign proved that there was a hole conductivity type. But the authors discovered a sign reversal (disappearance of effect was expected) in lower temperatures during the transition through the transition point in superconductive transition.

A theory able to present qualitative explanation of the experiment \cite{1,2} was proposed in \cite{3}. The appearance of longitudinal electric field was explained as transversal movement of vortex structure, induced by UW. The effects value was uniquely determined with \( \alpha \) parameter which is proportionate to Hall coefficient in superconductors with VS. In this theory the factor acting on VS was UW deformation vector. The theory gave right qualitative description of the effect. Nonetheless, the usability of the theory for explanation of the discussed experiment is doubtful, because it is difficult to consider the films which were used in the experiment to be clean, namely,

\[ \lambda (L - \text{London penetration depth}), \]

\[ d \leq \lambda \text{ thickness}. \]

Longitudinal SAW is accompanied by longitudinal electric field was not taken into account \cite{3}. In a superconductor at least a longitudinal alternating electric field is known to penetrate on a depth \( \lambda \) (\( \lambda - \text{London penetration depth} \)), and we may consider that the field coincides with SAW field in the films with \( d \leq \lambda \) thickness. This field will induce longitudinal superconductive currents which create a new interaction mechanism between SAW and vortices. This mechanism was not discussed in \cite{3}. The mechanism must work always, regardless of Hall effect in a superconductive film. Therefore it is reasonably to believe, that the interaction would result in an entrainment of VS in the transversal direction and appearance of a longitudinal electric field in the dirty films.

Moreover, SAW electric field creates another interesting possibility of controlling VS. It separates vortices of opposite orientation, since generated by SAW superconductive currents exert forces of opposite directions to vortices opposite orientations. SAW drags different oriented vortices along the line perpendicular to SAW propagating in the opposite directions (see Fig.1), some of them accumulate on the one side of the substrate, others on the other. Above described effect is similar to Hall effect, where a flux of SAW energy acts on opposite directed vortices as electric current on carriers with different signs.

The goals of the present work are:

1. demonstrate, that SAW electric field drags superconductive film VS in transversal direction and its movements, oscillations in this direction generate constant component of electric field in longitudinal direction (longitudinal acoustoelectric effect);

2. prove, that the effect must be observed in dirty superconductive films with piezoelectric substrate.

3. make certain, that taking into account the electric field of SAW explains the experiment \cite{1,2} both qualitatively and quantitatively without assumption existence of Hall effect of VS in superconductive film. All physical effects named below are novel and have not been discussed in scientific literature before.

We adopt the following picture of current carriers and ion motion in the film induced by SAW (current carriers can be electrons or holes). The current carriers liquid consists of two parts: normal and superconductive. The normal component is moved together with film...
ions dragged by viscous forces responsible for the normal resistance. In this picture the role of normal carriers is reduced only to partial screening of ions. Condition of applicability of this model is given by an inequality \( \omega \tau \ll 1 \) (\( \tau, \omega \) are relaxation time of the normal current carriers, SAW frequency, correspondingly). We describe motion of film VS within the framework of hydrodynamic approach which is valid for SAW wave lengths that are much larger than intervortex distance, so that the VS deformation vector \( \hat{W} \) can be regarded as continuous function of coordinates.

We will consider the following experimental geometry \([\mathbb{2}]\). Let SAW propagates along positive direction of Z axis of the substrate of Y niobate lithium cut. External magnetic field \( \vec{B}_0 \) is directed along positive direction of X axis. The surface is covered with superconductive film with thickness \( d \leq \lambda_L \) and \( d, \lambda_L \ll \lambda_R \) (\( \lambda_R \) is SAW wave length). This condition enables us to assume SAW electric field within the superconductive film \( \vec{E}_{ext} \) homogeneous and we assume it to be equal with the field on the surface of the substrate \( \vec{E}_{ext} = \vec{E}_{ext0} \exp(ikz - i\omega t) \). Deformation vector of ion lattice within the film is equal to substrate surface deformation vector \( \vec{U} = U_0 \exp(ikz - i\omega t) \), where \( k \) is SAW vector.

London equation for superconductive \( J_s \) current generated by field \( \vec{E}_{ext} \) and VS movement in the laboratory coordinate system is:

\[
\lambda_L^2 \mu_0 \frac{\partial J_s}{\partial t} = \vec{E} + \vec{E}_{ext} - \vec{B}_v \times \dot{\vec{W}} \tag{1}
\]

where \( \vec{E} \) - internal electric field, \( \vec{B}_v \) - magnetic induction produced by the VS of the film, \( \vec{W} \) - VS deformation vector, \( \vec{B}_v = \phi_0 n_v \vec{e}, n_v \) - 2D vortex density: number of vortices per square unit in the plane, perpendicular to vortex lines, direction \( \vec{B}_v \) is determined by vector \( \vec{e} \), tangent to vortex line.

Applying the operator \( \nabla \times \) to both sides of Eq. (1), and using Maxwells equation

\[
\nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t} \tag{2}
\]

we obtain:

\[
\lambda_L^2 \mu_0 \nabla \times \frac{\partial J_s}{\partial t} = - \frac{\partial \vec{B}}{\partial t} + \nabla \times \vec{E}_{ext} - \nabla \times (\vec{B}_v \times \dot{\vec{W}}) \tag{3}
\]

Applying the operator \( \nabla \times \) to Eq. (3), and taking into account connection between total current in superconductor and the ionic current with superconductive current \([\mathbb{2}]\)

\[
\dot{J}_s = \vec{J} + qn_s \dot{\vec{U}} \tag{4}
\]

after simple manipulations we obtain the following expression:

\[
\frac{\partial}{\partial t} \left( \lambda_L^2 \nabla^2 \dot{F}_s - \dot{J}_s \right) = -qn_s \dot{\vec{U}} - \frac{1}{\mu_0} \nabla \times \nabla \times \vec{E}_{ext} + \frac{1}{\mu_0} \nabla \times \nabla \times (\vec{B}_v \times \dot{\vec{W}}) \tag{5}
\]

Where \( \mu_0, n_s, m, q \) are magnetic susceptibility, superconductive carriers density, their mass and charge. Let us now write the local equation of motion of the VS (neglecting the inertial mass of the vortices). Within the framework of hydrodynamic approximation this equation follows from the condition of balance of two forces:

\[
f_{fr} = \vec{f}_M \tag{6}
\]

Where \( f_{fr}, f_m \) are the friction force of the VS against the crystal lattice of the film and Magnus force densities, respectively. The latter force has a hydrodynamic nature. It is the result of interaction of superconductive currents with the moving vortices and we name it Magnus force, though it is necessary to note, that there are different interpretations of this term, see for example \([\mathbb{2}]\). If VS is stationary this force is called Lorenz force. \( f_{fr} \) is the function of the VS local velocity relative to superconductor \( (\vec{W} - \vec{U}) \) and \( (\vec{W} - \vec{U}) \times \vec{B}_v \). Expanding of \( f_{fr} \) in these terms with accuracy to the second order, we can write it in the form

\[
f_{fr} = \eta (\dot{\vec{W}} - \vec{U}) - \tilde{\eta} (\dot{\vec{W}} - \vec{U}) \times \vec{B}_v \tag{7}
\]

where \( \eta \) and \( \tilde{\eta} \) are longitudinal and transversal VS viscosity, respectively.

Magnus force is equal to \( \vec{f}_m = (\dot{J}_s - qn_s \dot{\vec{W}}) \times \vec{B}_v \).

It is the result of Lorenz force expression \( \vec{f}_L = \vec{J}_s \times \vec{B}_v \) in local coordinate system, connected with moving VS where \( \vec{J}_s \) is superconductive current in this system. Placing (7) and expression of \( f_m \) to (6) one obtains equation of VS

\[
\eta (\dot{\vec{W}} - \vec{U}) - \tilde{\eta} (\dot{\vec{W}} - \vec{U}) \times \vec{B}_v = (\dot{J}_s - qn_s \dot{\vec{W}}) \times \vec{B}_v \tag{8}
\]

It is convenient to represent (8) replacing \( J_s \) on the total current \( J \) (4) as

\[
\eta (\dot{\vec{W}} - \vec{U}) - \alpha (\dot{\vec{W}} - \vec{U}) \times \vec{B}_v = \vec{J} \times \vec{B}_v \tag{9}
\]

where \( \alpha = qn_s - \tilde{\eta} \).

In order to understand the physical meaning of coefficients \( \eta \) and \( \alpha \), let us assume \( U = 0 \) and \( J = const \) (no ultrasonic wave) and find conductivity \( \sigma_s \) and Hall coefficient \( \sigma_H \) of superconductor in a mixed state from (9).

Multiplying (9) vectorial by \( \vec{B}_v \) and taking into account that \( \vec{E} = \vec{B}_v \times \dot{\vec{W}} \) - electric field, induced by VS movement, one obtains:

\[
\vec{J}_s = \eta \frac{\vec{E}}{B_v^2} - \frac{\alpha}{B_v} \vec{e} \times \vec{E} \tag{10}
\]

Consequently, specific conductivity is expressed through the longitudinal viscosity \( \sigma_s = \frac{\eta}{B_v^2} \) and Hall coefficient is uniquely determined by \( \alpha \) coefficient: \( \sigma_H = \frac{\alpha}{B_v} \).
In dirty superconductors Hall effect is negligibly small and therefore we can take $\alpha$ equal zero and hence VS motion equation will have the form

$$
\eta \left( \dot{\vec{W}} - \dot{\vec{U}} \right) = \dot{\vec{J}} \times \vec{B}_v
$$

(11)

Moreover, in order to solve the problem of VS entrainment by SAW one needs continuity equation for VS, Maxwell equation and a elasticity equation with term describing coupling between the ionic lattice deformation and the VS motion

$$
\frac{\partial \vec{B}_v}{\partial t} = \nabla \times (\dot{\vec{W}} \times \vec{B}_v)
$$

(12)

$$
\nabla \times \vec{B} = \mu_0 \dot{\vec{J}}
$$

(13)

The elasticity equation was deduced in \[\text{[2]}\]. In this work we neglect back influence of VS on SAW and will assume, that the deformation vector and the electric field of SAW are predetermined. We solve system of equations \[\text{[1]} \; \text{[5]} \; \text{[11]}\] and \[\text{[12]}\] with given SAW parameters using step-by-step approach method assumed that $\vec{B}_v = \vec{B}_0 + \dot{b}_v$, $\dot{\vec{W}} = \dot{\vec{W}}_1 + \dot{\vec{W}}_2$ and find $\dot{\vec{W}}$. Here $\vec{B}_0$ is a homogeneous component of superconductor magnetization equal to external field, $B_0$ is a projection of external field on axis X, $\dot{b}_v$ is VS density fluctuations produced by SAW, $|\dot{b}_v| \ll B_0$, $\dot{\vec{W}}_1$, $\dot{\vec{W}}_2$ are the local velocities of VS in the first and in the second orders of the SAW amplitude, respectively.

The SAW period time averaging of $\dot{\vec{W}}$ shows that in the second order of the SAW amplitude, VS has a constant velocity both in SAW propagating direction and in the perpendicular direction.

$$
\left< \dot{\vec{W}}_2 \right>_z = -\frac{1}{2} k^2 \left( 1 - \frac{\eta_B |B_0|}{\eta_0} \right) \frac{1}{B_0} \frac{X^2}{1 + X^2} U_0 \Phi_0 +
$$

$$
\frac{1}{2} k^2 \left( 1 - \frac{\eta_B |B_0|}{\eta_0} \right) \frac{m \omega}{q B_0} \frac{X^3}{1 + X^2} U_0^2
$$

(14)

$$
\left< \ddot{\vec{W}}_2 \right>_z = \frac{1}{2} k^2 \left( 1 - \frac{\eta_B |B_0|}{\eta_0} \right) \frac{X^2}{1 + X^2} U_0^2
$$

(15)

where the brackets $\left< \ldots \right>$ is an averaging over SAW period, $\dot{E}_{\text{ext}} = -\nabla \Phi$, $\Phi = \Phi_0 \exp (ikz - i\omega t)$ is an electric field potential, attended to SAW, $D = B_0^2 / \mu_0 (1 + \chi^2 k^2)$, $X = D k^2 / \omega \eta_0$, $\eta_B = \eta (|B_0|) / |B_0|$, $\eta_B$ is a zero order in VS viscosity coefficient expansion in fluctuations of its density $b_v$, $\eta_B = (d\eta(B_v)/d\eta_0) B_0$.

The first term in expression \[\text{[15]}\] for the transversal VS velocity appears due to SAW electric field. In case similar experiment is conducted with the films with non-piezoelectric substrate this term equals zero, but transversal entrainment effect still exists, with other sigh. This effect is marked with the second term \[\text{[15]}\]. Its physical reason is connected with oscillations of the superconductors ionic lattice, which induce screen superconductive electronic currents, which compensate magnetic field, induced by these oscillations. These currents create Lorenz force which moves the VS in the transversal direction. This mechanism works in bulk superconductors as well. Therefore a longitudinal ultrasonic wave entrains VS in the transversal direction in bulk superconductors and this effect is described by second term of \[\text{[14]}\].

The mechanisms discussed above determine the VS entrainment direction uniquely. This direction is specified by the sign \[\text{[14]}\]. Separation of vortices resembles that in the Hall effect for example, vortices of one orientation deviate to the right and another to the left. The use of this effect allows separating vortices of different orientation. One type of vortices will accumulate on the right side of the film, and another on the left. For example, in very thin films the vortices of opposite orientation, arising as the result of Kosterlitz Taules transition, can be separated by SAW. Originally not magnetized film under the influence of SAW will get the magnetisation of opposite signs on the its edges.

Obtained results show that SAW wave drags VS both in longitudinal \[\text{[13]}\] and transversal directions \[\text{[14]}\]. Thus, longitudinal UW drags VS angularly to the direction of its spreading. For the first time expression \[\text{[14]}\] was obtained in \[\text{[13]}\].

It is necessary to underline another peculiarity of dragging effect in dirty superconductors, namely: in case of Flux Flow mode, if $\eta$ is proportional to B, then the SAW and longitudinal UW in bulk superconductors does not drag at all, since $\eta_B |B_0|/\eta_0 = 1$.

The main observing quantity in the experiments with SAW influencing on superconductors VS is the electric field, generated by its movement. The expression for this field, induced by VS movement, is

$$
\vec{E} = \vec{B}_v \times (\dot{\vec{W}} - \dot{\vec{U}})
$$

(16)

Now, substituting in \[\text{[16]}\] the solutions of the equation system \[\text{[1]} \; \text{[5]} \; \text{[11]} \; \text{[12]}\] and leaving in the obtained expression the terms only the first and the second order infinitesimal on the SAW amplitude and averaging this expression on the wave period, we obtain a constant component of acoustoelectric fields

$$
\left< \vec{E} \right>_x = \frac{1}{2} k^2 \left( 2 - \frac{\eta_B}{\eta_0} \right) \frac{X^2}{1 + X^2} U_0^2
$$

(17)

$$
\left< \vec{E} \right>_z = -\frac{1}{2} k^2 \left( 1 - \frac{\eta_B}{\eta_0} \right) \frac{X^2}{1 + X^2} U_0 \Phi_0 +
$$

$$
\frac{1}{2} k^2 \frac{m \omega}{q} \left( 2 - \frac{\eta_B}{\eta_0} \right) \frac{X^3}{1 + X^2} U_0^2
$$

(18)

Our first priority in this work is to find the constant longitudinal acoustoelectric field in the direction of axis Z
which was observed in \[1, 2\], but we also provide expression for the transversal electric field \([17]\). It is worth noting, that the latter was observed in YBCO films in \[3\] and \[2\].

The longitudinal acoustoelectric field \([15]\) is independent from the sign of the external magnetic field. This peculiarity allows to measure absolute number of vortices in superconductor regardless of their orientation. Using tables \(\text{a}\) we are able to express \(U_0\) and \(\Phi_0\) with SAW power per film width unit \(P_R\).

\[
\langle \vec{E} \rangle_z = -\frac{1}{2} k^2 \left( 1 - \frac{\eta B_0}{\eta_0} \right) \frac{X^2}{1 + X^2} \frac{P_R}{\omega} 23.75 \times 10^{-5}
\]

(19)

Numerical coefficients in the formula (19) correspond with YZ niobate lithium section. In the formula (19) we neglect the term proportionate to \(m\omega/\eta\), since the working frequency in the experiments \([1-2]\) was 87 Mhz and it is far less than the first term. The expression (19) differs only by the numerical factor for different materials and cuts. All quantity entering into it is presented in SI but the resulting electric field is measured in \(\mu V/cm\).

Fig.2 depicts the result of the calculations of longitudinal acoustoelectric effect for Elisavsky and et. Experiment \([4]\).

Solid line is a theory, squares are the experiment \([4]\). In experiment \([4]\) film width is 1.8 mm, radiation power - 2 watt, thus \(P_R = 1.41 \times 10^4 w/m\). Frequency = 87 Mhz, field = 1, we used Tinkham ansatz for the film specific resistance \([13]\) \(r = r_0 I_0^{-2} (\gamma_0/2), \gamma_0/2 = \delta (1 - t)^{3/2} B^{-1}, t = T/T_c, I_0 - \text{modified Bessel function}, T - \text{absolute temperature}, T_C - \text{temperature of transition in superconductive state}, r_0 = 1.6 \times 10^{-4} \Omega m\), viscosity coefficient was expressed with specific resistance: \(\eta = B^2/r\). Coefficient \(\delta\) was used as an adjustable parameter and was equal to 57.

As follows from equation (19) the sign of the longitudinal acoustoelectric field is negative. Above the superconductive transition point this effect exists as well, though it has positive sign because in this case vortices are absent and the current carriers are normal holes. They entrain by SAW and the relevant acoustoelectric field is positive. It was observed in experiment \([4]\).

Thus we demonstrated that the SAW electric field drags vortices in transversal direction and generates constant component of longitudinal electric field and this field does not depend on the direction of the external magnetic field. This peculiarity of the effect provides us with the opportunity to measure a whole number of vortices in a film regardless of their orientation. The obtained result allows us to explain the experimental data both quantitatively and qualitatively. The theory, as shown in the fig.2, describes the experiment perfectly.

1. Y. Elisavskii, E.Z. Yakkhind, and et al., Fiz. Tverd. Tela (St.Petersburg) \textbf{33}, 824 (1991), [Sov. Phys. Solid State \textbf{33}, 496 (1991)].
2. Y. Elisavskii, E.Z. Yakkhind, and et al., Pis’ma v Zh. Eksp. Teor. Fiz. \textbf{52}, 1138 (1991), [JETP Lett. \textbf{52}, 542 (1991)].
3. N. Zavoritskii, Pis’ma v Zh. Eksp. Teor. Fiz. \textbf{57}, 695 (1993), [JETP Lett. \textbf{57}, 707 (1993)].
4. F. Jachman and C. Hacho, Sol. St. Commun. \textbf{142}, 212 (2007).
5. F. C. Hacho, Mat. Sci. Eng. A \textbf{521-522}, 307 (2009).
6. E. Gutliansky, Pis’ma v Zh. Eksp. Teor. Fiz. \textbf{59}, 459 (1994), [JETP Lett. \textbf{59}, 480 (1994)].
7. E. Gutliansky, Pis’ma v Zh. Eksp. Teor. Fiz. \textbf{67}, 222 (1998), [JETP Lett. \textbf{67}, 245 (1998)].
8. E. Gutliansky, Physica B \textbf{284}, 987 (2001).
9. R. Parmenter, Phys. Rev. \textbf{89}, 990 (1953).
10. E. Gutliansky, Phys. Rev. B \textbf{66}, 52511 (2002).
11. E. Gutliansky, Pis’ma v Zh. Eksp. Teor. Fiz. \textbf{82}, 77 (2005), [JETP Lett. \textbf{82}, 72 (2005)].
12. E. Gutliansky, Zh. Eksp. Teor. Fiz. \textbf{132}, 308 (2007), [JETP \textbf{105}, 272 (2007)].
13. E. G. S. E. Gutliansky, Zh. Eksp. Teor. Fiz. \textbf{132}, 304 (2007), [JETP \textbf{105}, 268 (2007)].
14. Y. Gulyaev, Usp. Fiz. Nauk \textbf{175}, 887 (2005), [Phys. Usp. \textbf{48}, 847 (2005)].
15. E. Gutliansky, *Acoustovertex interaction and acoustoelectrical phenomena in superconductors type II*, Doctor of physical and mathematical sciences dissertation, Rostov State University, Department of Physics (2004), (in Russian).

16. B. A. Auld, *Acoustic field and waves in solids* (New York: John Wiley and Sons, 1973).

17. M. Tinkham, Phys. Rev. Lett. 61, 1658 (1988).

18. E. Sonin, Phys. Rev. B 55, 458 (1997).