Antenna Selection in Full-Duplex Cooperative NOMA Systems

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Abstract—We investigate the problem of antenna selection (AS) in full-duplex (FD) cooperative non-orthogonal multiple access (NOMA) systems, where a multi-antenna FD relay assists transmission from a multi-antenna base station (BS) to a far user, while at the same time, the BS transmits to a near user. Specifically, based on the end-to-end signal-to-interference-plus-noise ratio at the near and far users, two AS schemes to select a single transmit antenna at both the BS and the relay, respectively, as well as a single receive antenna at relay are proposed. In order to study the ergodic sum rate and outage probability of these AS schemes, we have derived closed-form expressions assuming Rayleigh fading channels. The sum rate and outage probability of the AS schemes are also compared with the optimum selection scheme that maximizes the performance as well as with a random AS scheme. Our results show that the proposed AS schemes can deliver a near-optimal performance for near and far users, respectively.

I. INTRODUCTION

Each new generation of wireless communication systems has been designed to support the demands of increased traffic and data throughput with spectral efficiency as a key factor. To this end, non-orthogonal multiple access (NOMA) principle has been recognized as a key technology that can improve the spectral efficiency of 5G wireless systems. In contrast to traditional orthogonal multiple access (OMA) techniques, NOMA multiplexes signals between users with strong/weak channel conditions (a.k.a. near/far users) in the power domain and apply successive interference cancellation (SIC) at the receivers to remove the inter-user interference [11–13].

On the other hand, multiple-input multiple-output (MIMO) technology that offers substantial performance gains has become an integral part of modern communication systems. However, the benefits of MIMO come at the price of increased computational complexity and cost of hardware radio frequency chains that scales with the number of antennas [14]. In this context, antenna selection (AS) schemes with low implementation complexity and perform close to traditional MIMO systems have been touted as a practical solution in the literature. There is a sizable matured body of work on AS for different MIMO systems. To this end, AS in combination with NOMA has received interest in the recent literature [5–7], however the topic is still in infancy. Especially, the complexity of deciding on AS solutions in NOMA systems are exacerbated due to the complicated nature of near/far user performance criterion [5].

In this paper, we analyze the AS performance of a full-duplex (FD) cooperative NOMA system. FD is another promising technology considered for 5G implementation. Furthermore, the marriage between FD and NOMA can boost the performance as confirmed in [8–11] so far. The main bottleneck for FD operation is the self-interference (SI) [12–14]. Thus, AS should consider the effect of SI during the selection of strong channels toward the near/far users. This makes the AS problem in FD NOMA systems a much more complicated affair than in half-duplex (HD) NOMA systems [5]. Specifically, for the considered FD NOMA relay system, we propose AS schemes to achieve near/far user end-to-end (e2e) signal-to-interference noise ratio (SINR) maximization and study the sum rate and outage probability, respectively. Our contributions can be summarized as follows:

- Two low complexity AS schemes, i.e., max-U1 AS scheme and max-U2 AS scheme are proposed to maximize the e2e SINR at the near and far user, respectively. The performance of the FD cooperative NOMA system with the two AS schemes is analyzed by deriving exact ergodic sum rate and outage probability expressions.

- Our findings reveal that max-U1 AS scheme can significantly improve the system sum-rate, while max-U2 AS scheme can provide better user fairness. In particular, max-U1 AS scheme can achieve near optimum rate performance in the entire SNR range.

Notation: We use $\mathbb{E}\{X\}$ to denote the expected value of the random variable (RV) $X$; its probability density function (pdf) and cumulative distribution function (cdf) are $f_X(\cdot)$ and $F_X(\cdot)$ respectively; $\mathcal{CN}(\mu, \sigma^2)$ denotes a circularly symmetric complex Gaussian RV $X$ with mean $\mu$ and variance $\sigma^2$ and $E_i(\cdot) = \int_{0}^{+\infty} \frac{e^{-t}}{t} dt$ is the exponential integral function [15, Eq. (8.211.1)].

II. SYSTEM MODEL

Consider a two user NOMA downlink system where U1 (near user) communicates directly with the base station (BS), while U2 (far user) requires the assistance of a multi-antenna FD relay, $R$ as shown in Fig. 1 Both U1 and U2 are equipped with a single antenna each, the BS is equipped with $N_T$ antennas, while $R$ is equipped with two
A group of antennas: $M_R$ receive antennas and $M_T$ transmit antennas.

In order to keep the implementation complexity low, we assume that the BS and R perform single AS [4]. To be precise, the BS selects one (e.g. $i$-th) out of $N_T$ available transmit antennas, while R selects one (e.g. $j$-th) out of $M_R$ available antennas to receive signals. Moreover, one antenna (e.g. $k$-th) out of $M_T$ transmit antennas is selected at R to forward the BS signal to U2.

We assume that all channels experience independent Rayleigh fading and that they remain constant over one transmission slot. The channel between the $j$-th receive and the $i$-th transmit antenna from terminal X to terminal Y, is denoted by $h_{i,j}^{XY} \sim \mathcal{CN}(0, \sigma_{i,j}^2)$ where $X \in \{S, R\}$ and $Y \in \{R, U1, U2\}$.

### A. Transmission Protocol

According to the NOMA concept [2], [3], the BS transmits a combination of intended messages to both users as

$$s[n] = \sqrt{P_R a_1 x_1[n]} + \sqrt{P_R a_2 x_2[n]},$$

(1)

where $x_i$, $i \in \{1, 2\}$ denotes the information symbol intended for $U_i$, and $a_i$ denotes the power allocation coefficient, such that $a_1 + a_2 = 1$ and $a_1 < a_2$.

Since R is FD capable, it transmits the decoded symbol $x_2[n-\tau]$, where $\tau \geq 1$ accounts for the time delay caused by FD processing at R [13]. Therefore, the receive signal at R can be written as [8]

$$y_R[n] = h_{i,j}^{SR} s[n] + \sqrt{P_R h_{i,j}^{SR} x_2[n-\tau]} + n_R[n],$$

(2)

where $P_R$ is the relay transmit power and $n_R[n] \sim \mathcal{CN}(0, \sigma_n^2)$ is the additive white Gaussian noise (AWGN) at R. We assume imperfect SI cancellation due to FD operation at R and model the elements of the $M_R \times M_T$ residual SI channel $H_{RR} = [h_{i,j}^{SR}]$ as independent identically distributed (i.i.d) $\mathcal{CN}(0, \sigma_{i,j}^2)$ RVs [13].

The intention intended for U2 is decoded at R with SIC treating the symbol of U1 as interference [9]. Hence, the SINR at R can be written as

$$\gamma_R = \frac{a_2^2 \gamma_{SU1}}{a_1 \gamma_{SU1} + \gamma_{RU1} + 1},$$

(3)

where $\gamma_{SU1} = \rho_S |h_{i,j}^{SU1}|^2$ and $\gamma_{RU1} = \rho_R |h_{i,j}^{SU1}|^2$ with $\rho_S = \frac{P_R}{\sigma_n^2}$ and $\rho_R = \frac{P_R}{\sigma_n^2}$.

At the same time, U1 receives the following signal

$$y_1[n] = h_{i,j}^{SU1} s[n] + \sqrt{P_R h_{i,j}^{SU1} x_2[n-\tau]} + n_1[n],$$

(4)

where $n_1[n] \sim \mathcal{CN}(0, \sigma_n^2)$ is the AWGN at U1. Based on [9], the SINR of U2 observed at U1 can be written as

$$\gamma_{12} = \frac{a_2^2 \gamma_{SU1}}{a_1 \gamma_{SU1} + \gamma_{RU1} + 1},$$

(5)

where $\gamma_{SU1} = \rho_S |h_{i,j}^{SU1}|^2$ and $\gamma_{RU1} = \rho_R |h_{i,j}^{SU1}|^2$.

It is assumed that the symbol, $x_2[n-\tau]$, is priory known to U1 and thus U1 can be removed [10]. However, by considering realistic imperfect interference cancellation wherein U1 cannot perfectly remove $x_2[n-\tau]$, we model $h_{i,j}^{RU1} \sim \mathcal{CN}(0, k_1 \sigma_{i,j}^2)$ as the inter-user interference channel where the parameter $k_1$ presents the strength of inter-user interference [9]. If U1 perfectly cancels the U2’s signal, the SINR at U1 is given by

$$\gamma_1 = \frac{a_1 \gamma_{SU1}}{\gamma_{RU1} + 1}.$$  

(6)

Moreover, the received signal at U2, from R can be written as

$$y_2[n] = \sqrt{P_R h_{i,j}^{SR} x_2[n-\tau]} + n_2[n],$$

(7)

where $n_2[n] \sim \mathcal{CN}(0, \sigma_n^2)$ denotes the AWGN at U2. Hence, the signal-to-noise ratio (SNR) at U2 is given by

$$\gamma_{RU2} = \frac{P_R}{\sigma_n^2} |h_{i,j}^{SR}|^2.$$

(8)

The e2e SINR at U2 can be expressed as

$$\gamma_{2} = \min \left( \frac{a_2 \gamma_{SU1}^j}{a_1 \gamma_{SU1} + \gamma_{RU1} + 1}, \frac{a_2 \gamma_{SR}^j}{a_1 \gamma_{SR} + \gamma_{SU1} + 1} \right) \gamma_{RU2}^k.$$  

(9)

### B. Antenna Selection Schemes

We now propose two AS schemes for the described FD cooperative NOMA system where joint selection of single transmit and receive antennas at the BS and R based on e2e SINRs at the near user, U1 and far user, U2 is performed.

1) **max-U1 AS Scheme**: This scheme selects antennas first to maximizes the e2e SINR at U1, 0) and next with remaining AS choices tries to maximize also e2e SINR at U2. Therefore, the particular AS scheme can be mathematically expressed as

$$\{i^*, k^*\} = \arg \max_{1 \leq i \leq N_T, 1 \leq k \leq M_T} \frac{a_1 \gamma_{SU1}^i}{a_1 \gamma_{SU1}^i + \gamma_{RU1}^k + 1},$$

(10)

$$j^* = \arg \max_{1 \leq j \leq M_T} \frac{a_2 \gamma_{SR}^j}{a_1 \gamma_{SR}^j + \gamma_{SU1}^i + 1}.$$  

2) **max-U2 AS Scheme**: The max-U2 AS scheme achieves e2e SINR maximization at U2 according to

$$\{i^*, j^*, k^*\} = \arg \min_{1 \leq i \leq N_T, 1 \leq j \leq M_T} \frac{a_2 \gamma_{SU1}^i}{a_1 \gamma_{SU1}^i + \gamma_{RU1}^j + 1},$$

$$\frac{a_2 \gamma_{SR}^j}{a_1 \gamma_{SR}^j + \gamma_{SU1}^i + 1} \gamma_{RU2}^k.$$  

(11)

According to (11) there is no degree-of-freedom (in terms of AS) available for maximizing the e2e SINR at
In the sequel, we present key results for the near and far user (U1 and U2) with criterion such as to maximize the near/far user performance subject to a pre-defined far/near performance subject to a pre-defined far/near performance subject to a pre-defined far/near performance subject to a pre-defined far/near performance subject to a pre-defined far/near performance (12)

In order to implement max-U1 and max-U2 AS schemes, the BS can transmit a pilot signal and R can decide on the best antenna indexes to be used at the BS and its receive side. R can also transmit a pilot signal to U1 and U2 from each of the relay antenna. Upon reception of the pilot, U1 (in max-U1 AS scheme) and U2 (in max-U2 AS scheme) can next feedback the antenna index that R must be deployed in subsequent information transmission. Moreover, in max-U1 AS scheme, upon reception of the BS pilot signal, U1 can decide the best antenna and feedback the corresponding index to the BS to commence information transmission.

### III. Performance Analysis

In this section, we investigate the performance of proposed AS schemes in terms of the achievable ergodic sum rate and outage probability. The derived results will enable us to examine the benefits of the proposed AS schemes.

#### A. Ergodic Sum Rate

The ergodic achievable sum rate of the FD cooperative NOMA system is given by

$$ R_{\text{sum}} = R_{\text{U1,AS}} + R_{\text{U2,AS}}, $$

with

$$ R_{\text{U1,AS}} = \mathbb{E} \{ \log_2(1 + \gamma_1, \text{AS}) \}, $$

$$ R_{\text{U2,AS}} = \mathbb{E} \{ \log_2(1 + \gamma_2, \text{AS}) \}, $$

where $\gamma_{1, \text{AS}}$ and $\gamma_{2, \text{AS}}$ denote the e2e SINR at the U1 and U2, corresponding to the specific AS scheme with AS $\in \{S1, S2\}$. We use S1 to refer to the max-U1 AS scheme and S2 to refer to the max-U2 AS scheme.

Before we proceed, it is useful, to note that for a nonnegative RV $X$, since $\mathbb{E} \{ X \} = \int_0^\infty \Pr(X > x) \, dx$, the ergodic achievable rate for near user ($u = 1$) or far user ($u = 2$) can be written as

$$ R_{\text{Uu,AS}} = \frac{1}{\ln 2} \int_0^\infty \frac{1 - F_{\gamma_{u,AS}}(x)}{1 + x} \, dx, $$

where $F_{\gamma_{u,AS}}(z) = \Pr(\gamma_{u,AS} \leq z)$ is the cdf of the RV $\gamma_{u,AS}$.

In the sequel, we present key results for the near and far user ergodic rates due to the proposed AS schemes.

**Proposition 1.** The ergodic achievable rates of U1 and U2 of max-U1 AS scheme are respectively given by

$$ R_{\gamma_{\text{U1,AS}}}^{\text{U1}} = \frac{N_T}{\ln 2} \sum_{p=0}^{N_T-1} \left( \frac{(-1)^p \binom{N_T-1}{p}}{(p+1) \binom{N_T+1}{p+1} \gamma_{\text{RU1}}} \right), $$

$$ R_{\gamma_{\text{U2,AS}}}^{\text{U2}} = \frac{M_R N_T}{\ln 2} \sum_{q=0}^{M_R-1} \left( \frac{(-1)^q \binom{M_R-1}{q}}{(q+1) \binom{M_R+1}{q+1} \gamma_{\text{RU2}}} \right), $$

and

$$ R_{\gamma_{\text{U1,AS}}}^{\text{U1}} = \frac{M_R N_T}{\ln 2} \int_0^\infty e^{-\frac{a}{\gamma_{\text{SU1}}}} \frac{1}{1 + x} \sum_{p=0}^{N_T-1} \left( \frac{(-1)^p \binom{N_T-1}{p}}{(p+1) \binom{N_T+1}{p+1} \gamma_{\text{SU1}}} \right) \frac{e^{-\frac{a}{\gamma_{\text{SU1}}}}}{(p+1) \binom{N_T}{p+1} \gamma_{\text{SU1}}} \cdot dx, $$

where $\gamma_{\text{SU1}} = \rho_2 \sigma_{\text{SU1}}^2$, $\gamma_{\text{SU2}} = \rho_2 \sigma_{\text{SU2}}^2$, $\gamma_{\text{RU1}} = \rho_2 k_1 \sigma_{\text{RU1}}^2$, $\gamma_{\text{RU2}} = \rho_2 k_2 \sigma_{\text{RU2}}^2$ and $\gamma_{\text{SU1}} = \rho_2 \sigma_{\text{SU1}}^2$.

**Proof:** According to (13), the ergodic achievable rate can be calculated from the cdf of $\gamma_{\text{SU1}}$ and $\gamma_{\text{SU2}}$. Let us start with the ergodic achievable rate of U1. By invoking (10), the ratio $\frac{\gamma_{\text{SU1}}}{\gamma_{\text{SU2}}}$ is maximized when the strongest BS-U1 channel and the weakest R-U1 channel are selected. Therefore, the cdf of $\gamma_{1.51}$ can be evaluated as

$$ F_{\gamma_{1.51}}(x) = \int_0^\infty F_A((y+1)x/a_1) f_B(y) dy, $$

where $A$ is a RV defined as the maximum out of $N_T$ exponentially distributed independent RVs, while $B$ is the minimum out of $M_T$ exponentially distributed independent RVs. Substituting the required cdf and the pdf and simplifying yields

$$ F_{\gamma_{1.51}}(x) = 1 - N_T \sum_{p=0}^{N_T-1} \left( \frac{(-1)^p \binom{N_T-1}{p}}{(p+1) \binom{N_T+1}{p+1} \gamma_{\text{RU1}}} \right) \frac{\gamma_{\text{RU1}}}{\gamma_{\text{SU1}}} \cdot dx, $$

Next, substituting (17) into (14), then with the help of the integration identity in (15), and after some algebraic manipulations, we arrive at (15).
Moreover, based on \(10\), for the selected transmit antennas at the BS and \(R\), the ratio \(\frac{a_2 \gamma_{SR,1}}{a_1 \gamma_{SU,1}}\) can be maximized when the strongest BS-R channel and weakest SI channel are selected. However, these two channels are coupled with each other through the selected antenna at the \(R\) input. Therefore, it is difficult, if not impossible, to find the cdf of \(\frac{a_2 \gamma_{SR,1}}{a_1 \gamma_{SU,1}}\). Alternatively, we propose to select the receive antenna at \(R\) such that \(\gamma_{SR,1}^*\) is maximized, which will be shown to be a good approximation across the entire SNR range in Section \(\text{IV}\) (cf. Optimum AS (U2)). Therefore, we have

\[
\Pr (\gamma_{R,S1} > x) = 1 - \int_0^\infty f_B \left( \frac{(y + 1)x}{a_2 - a_1 x} \right) f_B (y) dy,
\]

for \(x < \frac{a_2}{a_1}\), where \(A\) is a RV defined as the largest out of \(M_R\) exponentially distributed independent RVs, and since SI link is ignored, \(B\) is an exponentially distributed RV with parameter \(\gamma_{SI}\). Substituting the required cdf and the pdf and simplifying yields

\[
\Pr (\gamma_{R,S1} > x) = M_R \sum_{q=0}^{M_R-1} \left( \frac{M_R-1-q}{q+1} \right) \left( 1 + \frac{a_2}{a_1 \gamma_{SI} (a_2-a_1 x)} \right) x^{q+1}.
\]

Finally, since the \(R\)-U2 link is ignored, we have

\[
\Pr (\gamma_{SU2,S1} > x) = e^{-\frac{x}{\gamma_{SU2}}}.
\]

To this end, pulling everything together, we obtain

\[
F_{\gamma_{SU2}} (x) = 1 - M_T N_T e^{-\frac{x}{\gamma_{SU2}}} - M_T e^{-\frac{x}{\gamma_{SU2}}} - \sum_{p=0}^{N_T-1} \left(-1\right)^p \left( \frac{N_T-1}{p} \right) e^{-\frac{\gamma_{SU2}}{\gamma_{SU1}}} \left( 1 + \frac{a_2}{a_1 \gamma_{SU1} (a_2-a_1 x)} \right) x^{p+1}.
\]

Having obtained the cdf of \(\gamma_{SU2}\), the ergodic rate of U2 can be obtained by employing \(14\). For \(x > \frac{a_2}{a_1}\), it can be readily checked that \(F_{\gamma_{SU2}} (x) = 1\) and hence \(R_{U2}^S = 0\).

**Proposition 2.** The ergodic achievable rates of U1 and U2 of max-U2 AS scheme, are respectively given by

\[
R_{U1}^S = \frac{1}{\ln 2} \frac{a_1 \gamma_{SU1}}{(\gamma_{SU1} - \gamma_{SU2})} \times \left( e^{-\frac{\gamma_{SU2}}{\gamma_{SU1}}} e^{\frac{\gamma_{SU2}}{\gamma_{SU1}}} \right),
\]

and

\[
R_{U2}^S = \frac{M_T N_T}{\ln 2} \int_0^\infty \left( 1 + \frac{a_2}{a_1 \gamma_{SU1} (a_2-a_1 x)} \right) \left( 1 + x \right) \times \sum_{p=0}^{N_T-1} \left(-1\right)^p \left( \frac{N_T-1}{p} \right) e^{-\frac{\gamma_{SU2}}{\gamma_{SU1}}} \left( 1 + \frac{a_2}{a_1 \gamma_{SU1} (a_2-a_1 x)} \right) x^{p+1} \times \sum_{q=0}^{M_T-1} \left(-1\right)^q \left( \frac{M_T-1}{q} \right) e^{-\frac{\gamma_{SU2}}{\gamma_{SU1}}} x^{q+1} dx.
\]

**Proof:** The proof follows similar steps to the proof of Proposition \(1\) and thus only an outline is presented. According to \(11\) the e2e SINR at U2 is maximized when each term inside in the minimum function is maximized. Therefore, a transmit antenna at \(R\) is selected such that \(\gamma_{RU2}^*\) is maximized, i.e., \(\gamma_{RU2}^*\) is the largest of \(M_T\) exponential RVs with parameter \(\gamma_{SU1}\). Moreover, since SI is the main source of performance degradation in FD mode, for a particular transmit antenna at \(R\), the best receive antenna at the \(R\) is selected such that the SI strength is minimized \(16\). Hence, \(\gamma_{SU1}^*\) is the minimum of \(M_R\) exponential RVs with parameter \(\gamma_{SI}\). Finally, for given \(k^*\) and \(j^*\), a best transmit antenna at the BS is selected such that \(\gamma_{R}\) is maximized. Hence, a single transmit antenna at the BS is selected such that the SINR at BS-R is maximized for the \(i^*\)-th receive antenna at \(R\).

Collecting the required cdfs and pdfs and simplifying yields

\[
F_{\gamma_{SU1}} (x) = 1 - e^{-\frac{x}{\gamma_{SU1}}} \left( 1 + \frac{a_2}{a_1 \gamma_{SU1} (a_2-a_1 x)} \right) \left( q+1 \right)
\]

and

\[
F_{\gamma_{SU2}} (x) = 1 - \sum_{p=0}^{N_T-1} \left(-1\right)^p \left( \frac{N_T-1}{p} \right) e^{-\frac{\gamma_{SU2}}{\gamma_{SU1}}} \left( 1 + \frac{a_2}{a_1 \gamma_{SU1} (a_2-a_1 x)} \right) x^{p+1} \times \sum_{q=0}^{M_T-1} \left(-1\right)^q \left( \frac{M_T-1}{q} \right) e^{-\frac{\gamma_{SU2}}{\gamma_{SU1}}} x^{q+1} \left( \frac{a_2}{a_1 \gamma_{SU1} (a_2-a_1 x)} \right) \left( q+1 \right),
\]

respectively. To this end, by invoking \(14\), \(25\), and \(26\) after some algebraic manipulations, we arrive at the desired result.

**B. Outage Probability**

Outage probability is a key metric used to measure the event that the data rate supported by instantaneous channel realizations is less than a targeted user rate. Therefore, the outage probability is an important performance metric to characterize the performance of NOMA systems \(11\).

The following Propositions present exact closed-form expressions for the outage probability of max-U1 and max-U2 AS schemes

**Proposition 3.** The outage probability of U1 with max-U1 and max-U2 AS scheme, are respectively given by

\[
P_{out,1} = 1 - \sum_{p=0}^{N_T-1} \left(-1\right)^p \left( \frac{N_T-1}{p} \right) e^{-\frac{\gamma_{SU2}}{\gamma_{SU1}}} \left( 1 + \frac{a_2}{a_1 \gamma_{SU1} (a_2-a_1 x)} \right) x^{p+1}.
\]

and

\[
P_{out,1} = 1 - e^{-\frac{x}{\gamma_{SU1}}} \left( 1 + \frac{a_2}{a_1 \gamma_{SU1} (a_2-a_1 x)} \right) \left( q+1 \right),
\]

where \(\zeta = \max \left( \frac{\theta_2}{a_2-a_1}, \frac{\theta_1}{a_1} \right)\), \(\theta_1 = 2\R_1 - 1\) and \(\theta_2 = 2\R_2 - 1\) with \(\R_1\) and \(\R_2\) being the transmission rates at U1 and U2.

**Proof:** An outage event at U1 occurs when it cannot decode the intended signal for U2, or when it can decode
it but fails to decode its own signal. Therefore, the outage probability of U1 can be derived as

\[ P_{\text{out},1} = 1 - \Pr (\gamma_{12,\text{AS}} > \theta_2, \gamma_{1,\text{AS}} > \theta_1), \]

\[ = \Pr \left( \frac{\gamma_{1,\text{AS}}}{\theta_1} < \frac{\zeta}{1} \right) \]

\[ = F_{1,\text{AS}}(\theta_1, \zeta). \] \hfill (29)

To this end, the desired result is obtained by evaluating (17) and (25) at \( a_1 \zeta \).

**Proposition 4.** The outage probability of U2 with max-U1 and max-U2 AS scheme, are respectively given by

\[ P_{\text{out},2}^{\text{S1}} = 1 - M_R e^{-\frac{a_2}{N_{\text{SU2}}}} \sum_{q=0}^{M_R - 1} \frac{(-1)^q (M_R - 1) e^{-\frac{(p+1)q}{M_R} \gamma_{\text{RU2}}}}{(q+1) \left( 1 + \frac{p_1 (p+1)q}{\gamma_{\text{SU2}} a_2 - a_1 a_2} \right)}, \] \hfill (30)

and

\[ P_{\text{out},2}^{\text{S2}} = 1 - M_T N_T \sum_{q=0}^{M_T - 1} \frac{(-1)^q (M_T - 1) e^{-\frac{(p+1)q}{M_T} \gamma_{\text{RU2}}}}{(q+1) \left( 1 + \frac{p_1 (p+1)q}{M_T \gamma_{\text{SU2}} (a_2 - a_1 a_2)} \right)}, \] \hfill (31)

**Proof:** An outage event at U2 occurs if R fails to decode the intended message to U2, or R can decode U2 signal but U2 fails to decode its message. Therefore, the outage probability of U2 can be expressed as

\[ P_{\text{out},2}^{\text{AS}} = 1 - \Pr (\gamma_{R,\text{AS}} > \theta_2, \gamma_{\text{RU2,AS}} > \theta_2) \]

\[ = 1 - \Pr (\gamma_{R,\text{AS}} > \theta_2) \Pr (\gamma_{\text{RU2,AS}} > \theta_2). \] \hfill (32)

To this end by substituting the corresponding probabilities presented in subsection III-A into (32), we arrive at (30) and (31).

**IV. NUMERICAL RESULTS AND DISCUSSION**

In this section, we present numerical results to quantify the performance gains when max-U1 and max-U2 AS schemes are adopted in the considered FD cooperative NOMA system. We set \( a_1 = 0.25, a_2 = 0.75 \) and \( k_1 = 0.01 \).

Fig. 2 shows the ergodic rates of U1 and U2 with the proposed AS schemes. We have also plotted the curves for

i) **Optimum AS scheme:** that performs an exhaustive search of all possible combinations to determine the antenna subset in order to maximize the ergodic sum rate, ii) **Optimum AS (U2) scheme:** This scheme aims to maximize the e2e SINR at U2 in an optimal sense and performs an exhaustive search of all possible combinations to determine the optimum antenna subset in order to maximize the e2e SINR at U2, and iii) **Random AS scheme:** that performs random AS at the BS and relay input/output. It can be observed that the max-U1 AS scheme is able to improve the ergodic rate of both U1 and U2, while max-U2 AS scheme, only increases the ergodic rate of U2. The max-U1 AS scheme provides the best performance for U1, while the performances of the max-U2 AS scheme and random AS scheme for U1 are almost identical. However, both max-U1 AS scheme and max-U2 AS scheme are able to increase the ergodic rate of U2. Moreover, we see that with increasing transmit power at BS and R, max-U2 AS scheme achieves the same performance as Optimum AS scheme.

In Fig. 3 the ergodic sum rate with different AS schemes is illustrated. It is evident that the ergodic achievable sum rate with the proposed AS schemes is higher than that of OMA FD relay system. In all transmit power regimes the performance gap between the max-U1 AS scheme and Optimum AS scheme is negligible. Moreover, the difference between the max-U1 and Optimum AS (U2) enlarges when transmit power at the BS and R is increased and remains constant for medium-to-high transmit power values.

Fig. 4 shows the Jain’s fairness index versus transmit power and for different AS schemes. This index is a
interference at U1 and SI at performance of U2, while it exhibits the same performance scheme provides the best outage performance for U1 and with different residual SI strengths. The max-U1 AS between the ergodic rate and user fairness.

However, max-U2 AS schemes suffer from zero-order diversity. However, the proposed AS schemes can reduce the error floor.

Fig. 5. Outage probability versus transmit power ($N_T = M_R = M_T = 4, \sigma_2^S = 0.3$).

bounded continuous function and is the most used quantitative measure to study the fairness in wireless systems. The Jain’s fairness index for the considered dual-users scenario can be expressed as

$$ J = \frac{(R_{AS}^{U1} + R_{AS}^{U2})^2}{2((R_{AS}^{U1})^2 + (R_{AS}^{U2})^2)$$

which its range is the interval $[\frac{1}{2}, 1]$. In this interval, $J = \frac{1}{2}$ corresponds to the least fair allocation in which only one user receives a non-zero rate, and $J = 1$ corresponds to the fairest allocation in which both near and far user receive the same rate. From Fig. 4 we see that in the low SNR region, Optimum AS scheme achieves the best user fairness out of all AS schemes. However, max-U2 AS scheme can provide a better user fairness in medium-to-high SNR regimes and hence can balance the tradeoff between the ergodic rate and user fairness.

Fig. 5 shows the outage probability of the AS schemes with different residual SI strengths. The max-U1 AS scheme provides the best outage performance for U1 and improves the outage performance of U2. Moreover, the max-U2 AS scheme can significantly improve the outage performance of U2, while it exhibits the same performance as Random AS scheme for U1. Finally, due to inter-user interference at U1 and SI at R our results show that all AS schemes suffer from zero-order diversity. However, the proposed AS schemes can reduce the error floor.

V. CONCLUSION

In this paper, we have studied the AS problem for a FD cooperative NOMA system. Two low complexity AS schemes, namely max-U1 AS scheme and max-U2 AS scheme were proposed to maximize the $e_{2e}$ SINR at the near and far user, respectively. The performance of both AS schemes have been characterized in terms of the ergodic sum rate and outage probability. Our results revealed that the max-U1 AS scheme achieves near optimum sum-rate performance while the max-U2 AS scheme exhibits a better user fairness. Moreover, the outage performance of the near user can be significantly improved by using the max-U1 AS scheme, while the outage probability of the far user can be effectively improved via the max-U2 AS scheme.

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