5-branes, KK-monopoles and T-duality

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Abstract

We construct the explicit worldvolume effective actions of the type IIB NS-5-brane and KK-monopole. These objects are obtained through a T-duality transformation from the IIA KK-monopole and the IIA NS-5-brane respectively. We show that the worldvolume field content of these actions is precisely that necessary to describe their worldvolume solitons. The IIB NS-5-brane effective action is shown to be related to the D-5-brane’s by an S-duality transformation, suggesting the way to construct \((p,q)\) 5-brane multiplets. The IIB KK-monopole is described by a gauged sigma model, in agreement with the general picture for KK-monopoles, and behaves as a singlet under S-duality. We derive the explicit T-duality rules NS-5-brane ↔ KK, which we use for the construction of the previous actions, as well as NS-5 ↔ NS-5, and KK ↔ KK.

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1 Introduction

The worldvolume effective actions that provide the sources for the brane solutions of type II superstring theories have proved to reveal a great deal of information about the dynamics of these objects \[1, 2, 3, 4\]. The Born-Infeld (BI) field present in the worldvolume of D-branes describes the flux of a fundamental string ending on the brane \[5, 6\]. The self-dual 2-form present in the action of the M-5-brane (IIA NS-5-brane) describes the dynamics of the string-like boundary of an M-2-brane (D-2-brane) on the M-5-brane (NS-5-brane) \[7\]. In general, fields propagating on a brane worldvolume describe the dynamics of the boundaries of other branes ending on it. These configurations can be obtained from configurations of intersecting closed branes where one of the branes opens up to lean its boundary on the other brane \[2, 7\]. These intersection regions, as well as the boundaries of branes ending on another branes, are described by worldvolume solitons that preserve 1/4 of the supersymmetry of the bulk. From the point of view of the particular brane they preserve 1/2 of its supersymmetry. Therefore the study of the worldvolume supersymmetry algebra of the brane can be used to reveal its possible soliton solutions \[8, 9\].

In this paper we construct the worldvolume effective actions of the type IIB NS-5-brane and KK-monopole. Six dimensional gauge theories obtained as the weak coupling limit of a system of parallel type II fivebranes have received a lot of attention recently \[10, 11\]. In this limit, interactions on the fivebranes lead to interacting six dimensional gauge theories, and different gauge groups and supersymmetries can be obtained depending on the particular choice of fivebranes that is taken.

The worldvolume field content that we find in the actions for the IIB NS-5-brane and KK-monopole agrees with the field content anticipated in \[12, 13\], based on general arguments following from the representation theory of six dimensional $N = 2$ supersymmetry and $T$-duality. Based on these analysis the field content of the effective actions could be deduced, even though the explicit couplings and therefore the explicit effective actions were not known.

A basic tool in our construction is the $T$-duality symmetry between the type IIA and type IIB superstring theories \[14\]. For D-branes $T$-duality relates the direct dimensional reduction of a given D-$p$-brane with the double dimensional reduction of a D-$(p + 1)$-brane in the dual theory \[8, 15, 16\]. For other solitonic objects, like the NS-5-branes and the KK-monopoles considered in this paper, $T$-duality can work in two different ways, depending on whether we are dualizing with respect to a worldvolume coordinate or with respect to a transverse coordinate. The transformations of the target space fields are of course the same but the way the worldvolume fields trans-
form differ from one case to the other. In this paper we will analyze both possibilities.

The IIB NS-5-brane is related by $T$-duality along a transverse coordinate with the IIA KK-monopole. Therefore, its worldvolume theory must be described by the six dimensional $(1,1)$ vector supermultiplet, which contains 4 scalars and one vector. The IIB KK-monopole is related by $T$-duality (also along a transverse coordinate, in particular its Taub-NUT direction) with the IIA NS-5-brane, which implies that it must be described by a $(2,0)$ vector supermultiplet, which contains a selfdual 2-form and 5 scalars.

The D-5-brane and the NS-5-brane solutions of type IIB supergravity are related by an $S$-duality transformation. Therefore, it is natural to expect that the corresponding effective actions must be related by the same kind of transformation. However, starting from the D-5-brane effective action it is not clear how the worldvolume fields should transform under $S$-duality, in particular whether a worldvolume duality transformation needs to be done, as happens with the relation between F- and D-strings [17]. Our approach in this paper is to construct the action of the IIB NS-5-brane starting from the action of the IIA KK-monopole [18]. At the level of the supergravity solutions one can perform a $T$-duality transformation along the isometry direction of the Taub-NUT space of the monopole in order to obtain the 5-brane supergravity solution. At the level of the worldvolume effective actions we proceed in the same way. Then we check that the resulting action is indeed $S$-dual to the D-5-brane and derive the $S$-duality transformation rules of the corresponding worldvolume fields. An interesting feature is that a worldvolume Poincaré duality is not needed in this case.

On the other hand, $T$-duality also relates both type II NS-5-branes (KK-monopoles). We explicitly work out these duality transformations. They provide a check for the action of the IIB NS-5-brane (IIB KK-monopole) constructed from the IIA KK-monopole (IIA NS-5-brane).

NS-5-branes and KK-monopoles couple to dual spacetime potentials, for which no explicit $T$-duality transformation rules have been derived. We fill this gap and derive as well the explicit $T$-duality rules for the new background fields that couple to the KK-monopole effective actions.

The structure of the paper is as follows. In section 2 we derive the action of the IIB NS-5-brane. We then check that the resulting action is related to the D-5-brane effective action by an $S$-duality transformation.

In section 3 we construct the action of the IIB KK-monopole. In this case we perform a $T$-duality transformation in the worldvolume action of the IIA NS-5-brane [19]. The resulting action is a singlet under $S$-duality, a property that can also be used to derive the field content of the IIB KK-monopole [18].
In section 4 we present the worldvolume $T$-duality rules that map the NS-5-branes of the type IIA and IIB theories onto each other. In this case a further worldvolume duality transformation is required in order to show the equivalence between the two actions. In section 5 we do the same for the KK-monopole effective actions. These T-dualities establish a map between $(1,1)$ and $(2,0)$ six dimensional supersymmetric theories \cite{[12]}. The analysis of the fermionic parts should reveal the reversing of the space-time chirality under the T-duality transformation.

Appendix A contains the $T$-duality rules for the dual background potentials coupled to the NS-5-branes and KK-monopoles, as well as the transformations of the new target space fields present in the KK-monopole effective actions. Appendices B and C summarize the gauge transformation rules of the target space and worldvolume fields considered in the paper.

Finally in section 6 we present our conclusions and open problems.

We finish the introduction by summarizing the target space fields occurring in the IIA and IIB superstring theories in order to set up the notation for the rest of the paper. They can be found in tables 1 and 2.

| Target space Field | Gauge Parameter | Dual Field | Gauge Parameter |
|--------------------|-----------------|------------|-----------------|
| $g_{\mu\nu}$, $\phi$ | $\Lambda_{\mu}$ | $B_{\mu_4...\mu_6}$ | $\Lambda_{\mu_3...\mu_5}$ |
| $B_{\mu\nu}$ | $\Lambda^{(0)}_\mu$ | $C^{(1)}_{\mu\nu}$ | $\Lambda^{(6)}_{\mu_4...\mu_6}$ |
| $C^{(1)}_{\mu}$ | $\Lambda^{(2)}_{\mu\nu}$ | $C^{(3)}_{\mu_4...\mu_5}$ | $\Lambda^{(4)}_{\mu_4...\mu_5}$ |
| $k_{\mu}$ | $N_{\mu_3...\mu_7}$ | $\Sigma^{(6)}_{\mu_4...\mu_5}$ |              |

Table 1: Target space fields of the type IIA superstring. The type IIA background contains the NS-NS sector: $(g_{\mu\nu}, \phi, B_{\mu\nu})$, the RR sector: $(C^{(1)}, C^{(3)})$, and the Poincaré duals of the RR fields and the NS-NS 2-form $B$: $(C^{(5)}, C^{(7)}, \tilde{B})$. The Kaluza-Klein monopole couples to a new field $N$, dual to the Killing vector associated to the Taub-NUT isometry, considered as a 1-form $k_{\mu}$.

2 The IIB NS-5-brane

In this section we construct the action of the IIB NS-5-brane, by means of a $T$-duality transformation in the effective action of the IIA KK-monopole. This duality is performed along the Taub-NUT direction of the monopole.
Table 2: Target space fields of the type IIB superstring. The Type IIB background contains the common sector: \((g_{\mu \nu}, \varphi, B_{\mu \nu})\), the RR sector: \((C^{(0)}, C^{(2)}, C^{(4)})\), and the Poincaré duals of the 2-forms \(C^{(2)}\) and \(B\): \((C^{(6)}, \tilde{B})\). We include as well the Poincaré dual of the Killing vector \(k\), that we denote by \(N\).

The action of the IIA KK-monopole was constructed in [18] and we summarize it in the next subsection. An interesting feature in this T-duality transformation, which makes it different from the more extensively studied D-brane duality [6, 15, 16], is that the number of worldvolume dimensions is the same in the original and dual theories \(^5\). The way it works in this case is explained in detail in subsection 2.2.

Figure 1: In this figure we depict the way we have derived the IIB NS-5-brane action. Applying T-duality in the IIA KK-monopole action along the Taub-NUT direction the type IIB NS-5-brane action is obtained, and it is S-dual to the D-5-brane effective action.

2.1 The IIA KK action

Let us first recall the action of the IIA KK-monopole constructed in [18].

The KK-monopole \([24]\) in \(D\) dimensions can be considered an extended object with \(D - 5\) spatial dimensions and one extra isometry direction transverse to the worldvolume. In order to get the right counting of degrees of freedom, we need to consider the duals of the fields. The duals are listed in Table 2. The action of the IIA KK-monopole was constructed in [18] and we summarize it in the next subsection. An interesting feature in this T-duality transformation, which makes it different from the more extensively studied D-brane duality [6, 15, 16], is that the number of worldvolume dimensions is the same in the original and dual theories \(^5\). The way it works in this case is explained in detail in subsection 2.2.

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\(^5\) This duality was studied in [20] for the heterotic case.
freedom this isometry is gauged, such that the effective number of embedding scalars is 3, fitting in a $D - 4$ dimensional vector multiplet. The effective action of a KK-monopole in $D$ dimensions is then described by a $D - 4$ dimensional gauged sigma model [22, 23, 18]. In particular, the action of the (massless) IIA KK-monopole is given by [18]:

$$S = -T_{\text{AKK}} \int d^6 \xi \, e^{-2\phi} k^2 \sqrt{1 + e^{2\phi} k^{-2} (i_k C^{(1)})^2} \times$$

$$\times \sqrt{\left| \det (D_i X^\mu D_j X^\nu g_{\mu\nu} - (2\pi \alpha')^2 k^{-2} K^{(1)}_i K^{(1)}_j + \frac{(2\pi \alpha')^2 k^{-2} e^{\phi}}{\sqrt{1 + e^{2\phi} k^{-2} (i_k C^{(1)})^2}} K^{(2)}_{ij}) \right|}$$

$$+ \frac{1}{6!} (2\pi \alpha') T_{\text{AKK}} \int d^6 \xi \, \epsilon_{i_1 \ldots i_6} K^{(6)}_{i_1 \ldots i_6}.$$  \hspace{1cm} (2.1)

The covariant derivative is defined by

$$D_i X^\mu = \partial_i X^\mu + A_i k^\mu,$$ \hspace{1cm} (2.2)

where $k^\mu$ is the Killing vector associated to the transverse target space isometry, and the gauge field $A_i$ is a dependent field given by:

$$A_i = k^{-2} \partial_i X^\mu k_\mu,$$ \hspace{1cm} (2.3)

with $k^2 = -k^\mu k^\nu g_{\mu\nu}$. With this choice the metric is effectively nine dimensional since the coordinate adapted to the isometry drops out of the action [22].

$K^{(2)}$ and $K^{(1)}$ are the curvatures of the 1- and 0-forms $\omega^{(1)}$ and $\omega^{(0)}$, respectively:

$$K^{(2)} = 2 \partial \omega^{(1)} + \frac{1}{2\pi \alpha'} (i_k C^{(3)}) - 2 K^{(1)} (DXC^{(1)}),$$

$$K^{(1)} = \partial \omega^{(0)} - \frac{1}{2\pi \alpha'} (i_k B).$$  \hspace{1cm} (2.4)

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6 For some recent work on KK-monopoles see [24].

7 We omit the worldvolume indices with the understanding that they are completely antisymmetrized. We use this notation throughout the paper.
Table 3: Worldvolume field content of the IIA KK-monopole. The field content of the IIA KK-monopole consists on a 1-form $\omega^{(1)}$, a 0-form $\omega^{(0)}$ and 3 embedding scalars $X^\mu$, fitting in a 6 dimensional vector multiplet. One extra degree of freedom has been eliminated through the gauging construction. The 5-form $\omega^{(5)}$ describes the tension of the monopole.

Finally, $\mathcal{K}^{(6)}$ is the WZ curvature of the monopole:

$$
\mathcal{K}^{(6)} = \left\{ 6\partial\omega^{(5)} + \frac{1}{2\pi\alpha'}(i_k N) - 30(i_k C^{(5)})\partial\omega^{(1)} - \frac{15}{2\pi\alpha'}(i_k C^{(5)})(i_k C^{(3)}) \right.
$$

$$
-6(i_k \tilde{B})\mathcal{K}^{(1)} - 120\left(2\pi\alpha'\right)DX^\mu DX^\nu DX^\rho C^{(3)}_{\mu\nu\rho}\mathcal{K}^{(1)}\partial\omega^{(1)}
$$

$$
+\frac{30}{2\pi\alpha'}DX^\mu DX^\nu B_{\mu\nu}(i_k C^{(3)})(i_k C^{(3)})
$$

$$
+\frac{50}{2\pi\alpha'}DX^\mu DX^\nu DX^\rho C^{(3)}_{\mu\nu\rho}(i_k B)(i_k C^{(3)})
$$

$$
-30DX^\mu DX^\nu DX^\rho C^{(3)}_{\mu\nu\rho}(i_k C^{(3)})\partial\omega^{(0)}
$$

$$
-180\left(2\pi\alpha'\right)DX^\mu DX^\nu B_{\mu\nu}\partial\omega^{(1)}\partial\omega^{(1)}
$$

$$
-360(2\pi\alpha')^2 A\partial\omega^{(1)}\partial\omega^{(1)}\partial\omega^{(0)}
$$

$$
+15(2\pi\alpha')^2 \frac{e^{2\alpha_k}}{1 + e^{2\alpha_k}}(i_k C^{(1)})^2 \mathcal{K}^{(2)} \mathcal{K}^{(2)} \mathcal{K}^{(2)} \right\},
$$

(2.5)

where $(i_k L)$ denotes the interior product of the field $L$ with the Killing vector $\mathbf{k}$. The target space fields $N$ and $\tilde{B}$ are the duals of the Killing vector, considered as a 1-form $k_\mu$, and the NS-NS 2-form $B$, respectively.

The worldvolume field content is summarized in Table 3 and the gauge transformations can be found in Appendix C.1.

Worldvolume fields propagating in the effective action of a given brane

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8In our notation: $(i_k L)^\mu_1...\mu_r = k^{\mu_{r+1}}L_{\mu_1...\mu_r}$.
have an interpretation in terms of soliton solutions on the brane. In the IIA KK-monopole worldvolume action there are four soliton solutions preserving 1/4 of the supersymmetry of the bulk [23]. These are a 1-brane, a 2-brane, a 3-brane and a 4-brane solitons. The worldvolume fields present in the KK-monopole effective action are precisely those describing these soliton solutions.

The worldvolume field \( \omega^{(0)} \) couples to the 3-brane soliton (through its dual 4-form) and \( \omega^{(1)} \) couples to the 0- and the 2-brane solitons (to the latter through its dual 3-form). Some of the corresponding intersections are given by: \((3|NS5, KK), (0|D2, KK)\) and \((2|D4, KK)\), where in all these configurations one of the worldvolume directions of the brane is wrapped around the Taub-NUT direction of the monopole.

The intersections: \((3|KK, KK)\) and \((3|KK, KK)\), where either both \(S^1\)'s coincide or one worldvolume direction of a monopole is wrapped around the other's \(S^1\), are also possible. These 3-brane solitons couple to the 4-forms dual to the embedding scalars, and can be obtained from the intersections of two monopoles in M-theory [26] by reducing along a common worldvolume direction.

The worldvolume field \( \omega^{(5)} \) describes the tension of the monopole and couples to the 4-brane soliton, which is a domain wall in the 6 dimensional worldvolume as can be seen from the intersection \((4|D4, KK)\) [26].

### 2.2 T-duality

Our aim now is to perform a T-duality transformation in this action in order to obtain the IIB NS-5-brane. The way T-duality works in this case is as follows: One first reduces the KK-monopole action along its isometry direction, given by the Killing vector \(k\). The result is mapped into the direct dimensional reduction of the IIB NS-5-brane along one transversal coordinate \(Z\), which is T-dual to the worldvolume scalar \(\omega^{(0)}\) present in the KK-monopole action:

\[
\omega^{(0)\prime} = \frac{1}{2\pi\alpha'} Z.
\]

The KK-monopole and the 5-brane couple to target space fields for which the T-duality rules have not been worked out in the literature. In particular, the IIA KK-monopole couples to the 6-form \(i_k N\) and to \(i_k \tilde{B}\), and the IIB NS-5-brane couples to the 6-form \(\tilde{B}\), dual to the NS-NS 2-form \(\mathcal{B}\). We have worked out the explicit T-duality rules for these fields. They can be found in appendix A.
The \( T \)-duality rules for \( \omega^{(1)} \) and \( \omega^{(5)} \) are given by:

\[
\begin{align*}
\omega^{(1)\prime} &= -c^{(1)}, \\
\omega^{(5)\prime} &= \tilde{c}^{(5)} - 60(2\pi \alpha')Z'\partial c^{(1)}\partial c^{(1)}\partial Z. 
\end{align*}
\]

(2.7)

In the last expression \( Z' \) is the adapted coordinate to the Killing isometry of the monopole, and \( \tilde{c}^{(5)} \) is the worldvolume gauge potential that describes the tension of the IIB NS-5-brane. The last identification is crucial in order to eliminate completely the \( Z' \) coordinate associated to the isometry of the monopole. Given that it is not a degree of freedom in the KK-monopole it should not transmit any degree of freedom to the 5-brane.

Considering as well the transformation of the target space field \( (i_k B) \), which can be found in the appendix, we find:

\[
K^{(2)\prime}_{ij} = -\tilde{F}_{ij},
\]

(2.8)

where \( \tilde{F} \) is the field strength of the worldvolume field \( c^{(1)} \), describing the tension of a D-string [27]:

\[
\tilde{F} = 2\partial c^{(1)} + \frac{1}{2\pi \alpha'} C^{(2)}.
\]

(2.9)

Also, under \( T \)-duality the tensions of the two branes must be identified:

\( T_{\text{AKK}}' = T_{B5} \).

### 2.3 The action of the IIB NS-5-brane

Applying the \( T \)-duality as described above, we obtain the following action for the IIB NS-5-brane:

\[
\begin{align*}
S &= -T_{B5} \int d^6\xi \ e^{-2\varphi} \sqrt{1 + e^{2\varphi}(C^{(0)})^2} \left| \det \left( g - \left( 2\pi \alpha' \right)^{e\varphi} \sqrt{1 + e^{2\varphi}(C^{(0)})^2} \right) \tilde{F} \right| \\
&\quad + \frac{1}{6!(2\pi \alpha')^5} T_{B5} \int d^6\xi \ e^{i_1...i_6} \tilde{G}^{(6)}_{i_1...i_6}.
\end{align*}
\]

(2.10)

\( \tilde{F} \) is defined above in (2.9) and \( \tilde{G}^{(6)} \) is the curvature of the 5-form \( \tilde{c}^{(5)} \) describing the tension of the 5-brane. This 6-form is the gauge invariant Wess-
Table 4: **Worldvolume field content of the IIB NS-5-brane.** In the case of the IIB NS-5-brane, there are four embedding coordinates $X^\mu$ and one vector field $c^{(1)}$. This field is the $S$-dual of the BI field $b$, and describes the flux of a $D$-string ending on the NS-5-brane. The 5-form $\tilde{c}^{(5)}$ describes the tension of the 5-brane.

**Zumino term:**

$$\tilde{G}^{(6)} = \left\{ 6\partial \tilde{c}^{(5)} \right. - \frac{1}{2\pi\alpha'} \tilde{B} - \frac{45}{2(2\pi\alpha')} B C^{(2)} C^{(2)} - 15C^{(4)} \tilde{F} - 180(2\pi\alpha') B \partial c^{(1)} \partial c^{(1)} - 90B C^{(2)} \partial c^{(1)}$$

$$+ 15(2\pi\alpha')^2 e^{-\phi} \tilde{C}^{(0)} e^{\frac{\phi}{2}} \tilde{F} \tilde{F} \tilde{F} \} .$$

(2.11)

The worldvolume field content is summarized in Table 4. It consists on a vector and 4 scalars, which is the field content of the six dimensional $(1,1)$ vector supermultiplet [12]. The corresponding gauge transformations can be found in Appendix C.2.

The worldvolume vector field $c^{(1)}$ is associated to the tension of a D-string, therefore it describes the flux of such an object ending on the NS-5-brane [1, 2]. This is the $S$-dual picture of a fundamental string ending on a D-5-brane (see Figure 2).

The action (2.10) is in fact $S$-dual to the D-5-brane action. The following $S$-duality transformation (in the Einstein frame)

$$C^{(0)} \rightarrow \frac{-C^{(0)}(C^{(0)})^2 e^{-2\phi}}{(C^{(0)})^2 e^{-2\phi}} ,$$

$$e^{-\phi} \rightarrow \frac{e^{-\phi}}{(C^{(0)})^2 e^{-2\phi}} ,$$

$$C^{(2)} \rightarrow B ,$$

$$B \rightarrow -C^{(2)} ,$$

$$C^{(6)} \rightarrow \tilde{B} ,$$

$$\tilde{B} \rightarrow -C^{(6)} ,$$

$$C^{(4)} \rightarrow C^{(4)} ,$$

(2.12)

in the D-5-brane action:

$$S = -T_{D5} \int d^5 \xi \left\{ e^{-\phi} \sqrt{\text{det}(g + (2\pi\alpha') F)} + \frac{2\pi\alpha'}{6!} e^{\epsilon_{i_1...i_6}} G^{(6)}_{i_1...i_6} \right\} ,$$

(2.13)
Figure 2: The IIB NS-5-brane worldvolume theory contains a vector $c^{(1)}$, which is the $S$-dual of the BI field $b$ and describes the flux of a D-string ending on the 5-brane. This is the $S$-dual picture of a fundamental string ending on a D-5-brane.

gives the action (2.10) describing the NS-5-brane. In (2.13) $G^{(6)}$ is the WZ curvature:

$$G^{(6)} = \left\{ 6\partial c^{(5)} + \frac{1}{2\pi \alpha'} C^{(6)} + \frac{45}{2(2\pi \alpha')} C^{(2)} B B - \frac{15}{2} C^{(4)} F F + \frac{180}{2\pi \alpha'} C^{(2)} \partial b \partial b + \frac{90}{2\pi \alpha'} C^{(2)} B \partial b - \frac{15}{2} C^{(0)} F F \right\} ,$$

(2.14)

$F = 2\partial b + \frac{1}{2\pi \alpha'} B$ and $b$ is the Born-Infeld field.

The world-volume fields must transform as $SL(2, Z)$ doublets:

$$c^{(1)} \rightarrow b , \quad b \rightarrow -c^{(1)} , \quad (2.15)$$

and:

$$\tilde{c}^{(5)} \rightarrow c^{(5)} , \quad c^{(5)} \rightarrow -\tilde{c}^{(5)} . \quad (2.16)$$

Therefore we have found that the IIB NS-5-brane, defined as the $T$-dual of the IIA KK-monopole, is $S$-dual to the D-5-brane, as it was known already at the level of the corresponding type IIB supergravity solutions. It is interesting to note that the BI field transforms into a 1-form, and not into a 3-form as one would have expected. See the conclusions for a further discussion on this point.
The worldvolume fields present in the IIB NS-5-brane effective action describe the soliton solutions constructed in [25], as we are now going to see. We find the same worldvolume solitons than for the IIA KK-monopole, given that both branes are $T$-dual to each other. Also, they are related by $S$-duality to the worldvolume solitons of the D-5-brane [3, 4, 8].

The worldvolume field $c^{(1)}$ couples to the 0-brane and the 2-brane solitons (to the latter through its worldvolume dual 3-form). Some corresponding intersections are: $(0|D1, NS5)$ and $(2|D3, NS5)$. They are related by $T$-duality to the IIA KK-monopole solitons described by the configurations: $(0|D2, KK)$ and $(2|D4, KK)$, respectively.

There are also two 3-brane solitons corresponding to the intersections: $(3|NS5, NS5)$ and $(3|NS5, KK)$. The first one is obtained by applying $T$-duality to either of the two intersections in IIA: $(3|NS5, KK)$ or $(3|KK, KK)_1$ (here the two $S^1$’s of the monopoles coincide). The second is obtained from the configuration $(3|KK, KK)_2$ in IIA. The 4-forms in the worldvolume of the IIB NS-5-brane that couple to these solitons are the duals of the embedding scalars in the $5 + 1$ dimensional worldvolume.

$\tilde{c}^{(5)}$ describes the tension of the NS-5-brane, and it couples to the domain wall in the six dimensional worldvolume given by the intersection: $(4|D5, NS5)$ [28]. This soliton is $T$-dual to the 4-brane soliton $(4|D4, KK)$ of the IIA KK-monopole.

### 3 The IIB KK-monopole

In this section we derive the action of the IIB KK-monopole through a $T$-duality transformation in the action of the IIA NS-5-brane. In this case $T$-duality is performed along a coordinate transverse to the 5-brane.

![Diagram](image_url)

Figure 3: In this figure we depict how we have obtained the IIB KK-monopole action. We apply $T$-duality on the IIA NS-5-brane action along one coordinate in the transverse space. One check of the action obtained is its invariance under $S$-duality.
3.1 The IIA NS-5 action

Let us first recall the action of the type IIA NS-5-brane. In the quadratic approximation it is given by [19]

\[ S = -T_{A5} \int d^6\xi e^{-2\phi} \sqrt{\det (g - (2\pi\alpha')^2 e^{2\phi} G^{(1)}G^{(1)})} \times \]
\[ \times \left\{ 1 - \frac{1}{4!} (2\pi\alpha')^2 c^{(2)} \right\} + \ldots \} \]
\[ - (2\pi\alpha')^2 T_{A5} \int d^6\xi \epsilon^{i_1 \ldots i_6} \tilde{F}^{(6)}_{i_1 \ldots i_6}, \]

where \( G^{(1)} \) is defined as:
\[ G^{(1)} = \partial c^{(0)} + \frac{1}{2\pi\alpha'} C^{(1)}, \]  
and the selfdual 3-form \( \mathcal{H}^{(3)} \) as:
\[ \mathcal{H}^{(3)} = 3\partial a^{(2)} + \frac{1}{2\pi\alpha'} C^{(3)} + 3\partial c^{(0)} B. \]  

The selfduality condition is inherited from that for the M-5-brane, and takes the form:
\[ \mathcal{H}^{(3)}_{ijk} = \frac{1}{3! \sqrt{\det (g - (2\pi\alpha')^2 e^{2\phi} G^{(1)} G^{(1)})}} \epsilon^{ijklmn} \mathcal{H}^{(3)}_{lmn}. \]  

The 6-form \( \tilde{F}^{(6)} \) is the WZ-curvature associated to the 5-form \( \tilde{b} \), which describes the tension of the IIA NS-5-brane:
\[ \tilde{F}^{(6)} = 6\partial \tilde{b} + \frac{1}{2\pi\alpha'} \tilde{B} - 6C^{(5)} \partial c^{(0)} + 30C^{(3)} B \partial c^{(0)} + 30 \partial a^{(2)} C^{(3)} - 90(2\pi\alpha') B \partial a^{(2)} \partial c^{(0)}. \]  

The NS-5-brane contains a 1-brane, a 3-brane and a 5-brane solitons [25, 19]. These are as well the soliton solutions occurring in the M-5-brane effective action [8, 9], given that both branes are related by dimensional reduction.

The 1-brane, or self-dual string, couples to the self-dual 2-form \( a^{(2)} \), and it describes the boundary of a D-2-brane ending on the NS-5-brane: \( (1|D2, NS5) \). The 3-brane soliton couples to the dual of the worldvolume scalar \( c^{(0)} \) and describes the intersection of two NS-5-branes: \( (3|NS5, NS5) \). Finally, if we considered the NS-5-brane on a massive background [19] we would also find a coupling:
\[ \int d^6\xi m e^{i_1 \ldots i_6} c^{(6)}_{i_1 \ldots i_6}, \]  

13
Table 5: **Worldvolume fields of the IIA NS-5-brane.** In the case of the IIA NS-5-brane, there are four embedding coordinates $X^\mu$, a scalar $c^{(0)}$ and a selfdual 2-form $a^{(2)}$. The 5-form $\tilde{b}$ describes the tension of the 5-brane.

| Worldvolume Field | Field Strength | d.o.f |
|-------------------|----------------|-------|
| $X^\mu$           | $-$            | 4     |
| $c^{(0)}$         | $G^{(1)}$      | 1     |
| $a^{(2)}_{ij}$    | $H^{(3)}$      | 3     |
| $b_{i_1...i_5}$   | $F^{(6)}$      | $\bot$ |

The worldvolume solitons in the IIB NS-5-brane are related to these solitons by a $T$-duality transformation along a worldvolume direction, as shown in [25].

It is important to notice that there is actually no compelling reason why we should introduce the field $\tilde{b}$ describing the tension of the IIA NS-5-brane. It has been pointed out [27, 30] that since there is no 4-brane soliton on this brane, it is not necessary to elevate the brane tension to the status of a dynamical variable. This can be argued from the fact that the IIA NS-5-brane is the dimensional reduction of the M-5-brane. The M-5-brane cannot have a boundary on any other brane [2], so there is no need to replace its tension by a dynamical variable. From this point of view the role of $\tilde{b}$ in the IIA NS-5-brane action is to guarantee invariance under all gauge transformations, including total derivatives. It is also necessary in order to obtain the worldvolume 5-form of the IIB NS-5-brane after a $T$-duality transformation [31]. This 5-form couples to the 4-brane soliton of the IIB NS-5-brane [28].

The worldvolume field content is summarized in Table 5 and the gauge symmetries of these fields are given in Appendix C.4.

---

9The M-5-brane 5-brane soliton can be described as well by the intersection [27, 30]: $(5|M_5, KK)$, though in this case it is realized as a domain wall in the seven dimensional worldvolume of the eleven dimensional KK-monopole. This configuration gives the embedding of the NS-5-brane on the KK-monopole: $(5|NS_5, KK)$ after dimensional reduction.
3.2 \( T \)-duality

We apply now a \( T \)-duality transformation along a transverse direction. The worldvolume fields do not change rank after \( T \)-duality since we keep the same number of worldvolume dimensions. Moreover, we have a new scalar field \( Z' \), which is the \( T \)-dual of the coordinate along which we perform the duality transformation. The \( T \)-duality rules for the worldvolume fields are given by:

\[
\begin{align*}
Z' &= (2\pi\alpha')\omega^{(0)}, \\
c^{(0)\nu} &= -\tilde{\omega}^{(0)}, \\
a^{(2)\nu} &= -\omega^{(2)} + 2(2\pi\alpha')Z\partial\omega^{(0)}\partial\tilde{\omega}^{(0)}, \\
\tilde{b}' &= \omega^{(5)} - 30(2\pi\alpha')^2Z\partial\omega^{(2)}\partial\omega^{(0)}\partial\tilde{\omega}^{(0)}.
\end{align*}
\]  

(3.7)

Notice that the transversal coordinate and the scalar \( c^{(0)} \) transform differently than for D-branes. In that case the transversal coordinate \( T \)-dualizes into one of the components of the BI field in the dual \( D \)-brane. Here, since we do not change the number of worldvolume dimensions, this coordinate \( T \)-dualizes into a new scalar field\(^{10} \). The occurrence of \( Z \), the Taub-NUT coordinate of the KK-monopole, in the right hand side of the last two expressions above is required by gauge invariance, and assures the gauged sigma-model structure necessary to describe the KK-monopole.

The worldvolume curvatures transform as follows:

\[
\begin{align*}
\mathcal{H}^{(3)\nu} &= -\mathcal{K}^{(3)}, \\
\tilde{\mathcal{F}}^{(6)\nu} &= \mathcal{K}^{(6)}, \\
\mathcal{G}^{(1)\nu} &= -\left(\tilde{\mathcal{K}}^{(1)} + \mathcal{C}^{(0)}\mathcal{K}^{(1)}\right),
\end{align*}
\]  

(3.8)

where \( \mathcal{K}^{(3)} \) is the curvature of \( \omega^{(2)} \), and \( \mathcal{K}^{(1)} \) and \( \tilde{\mathcal{K}}^{(1)} \) are the curvatures of \( \omega^{(0)} \) and \( \tilde{\omega}^{(0)} \), respectively. These field strengths are defined below.

\(^{10}\) Furthermore, the scalar field \( c^{(0)} \) (describing the tension of a D-0-brane) is mapped into another scalar: \( \omega^{(0)} \), whereas in a D-brane duality mapping it is mapped into the doubly reduced component of the 1-form \( c^{(1)} \): \( c^{(0)\nu} = c^{(1)}_{\sigma} \).
3.3 The action of the IIB KK-monopole

The result of the $T$-duality transformation above is, again in quadratic approximation:

$$ S = -T_{BKK} \int d^6 \xi \, e^{-2\varphi} k^2 \sqrt{\det(DX^\mu DX^\nu g_{\mu\nu} - (2\pi \alpha')^2 k^{-2} e^{-\varphi} K^T M K)} \times $$

$$ \times \left\{ 1 - \frac{e^{2\varphi}}{4} (2\pi \alpha')^2 k^{-2} (K^{(3)})^2 + \ldots \right\} $$

$$ - (2\pi \alpha') \frac{1}{6!} T_{BKK} \int d^6 \xi \epsilon_{i_1 \ldots i_6} \tilde{K}^{(6)}_{i_1 \ldots i_6}. $$

Here $K^T M K$ is the $SL(2, R)$-invariant combination:

$$ K^T M K = \left( \begin{array}{cc} K^{(1)} & \tilde{K}^{(1)} \\ \tilde{K}^{(1)} & 1 \end{array} \right) \left( \begin{array}{c} e^{-2\varphi} + C^{(0)} C^{(0)} \\ C^{(0)} \end{array} \right) \left( \begin{array}{c} K^{(1)} \\ \tilde{K}^{(1)} \end{array} \right), $$

with

$$ K^{(1)} = \partial \omega^{(0)} - \frac{1}{2\pi \alpha'} (i k B), \quad \tilde{K}^{(1)} = \partial \tilde{\omega}^{(0)} + \frac{1}{2\pi \alpha'} (i k C^{(2)}), $$

the gauge invariant curvatures of the worldvolume scalars $\omega^{(0)}$ and $\tilde{\omega}^{(0)}$. The covariant derivative is the usual one for KK-monopoles:

$$ DX^\mu = \partial X^\mu + A k^\mu, $$

with $k^\mu$ the Killing vector describing the Taub-NUT isometry and $A$ the dependent field:

$$ A = k^{-2} \partial X^\mu k_\mu, $$

where $k^2 = -k^\mu k^\nu g_{\mu\nu}$. The curvature of the two form $\omega^{(2)}$ is given by:

$$ K^{(3)} = 3 \partial \omega^{(2)} + \frac{1}{2\pi \alpha'} (i k C^{(4)}) + 6 (2\pi \alpha') A \partial \omega^{(0)} \partial \tilde{\omega}^{(0)} $$

$$ + \frac{3}{2(2\pi \alpha')} (DX^\mu DX^\nu B_{\mu\nu})(i k C^{(2)}) - \frac{3}{2(2\pi \alpha')} (DX^\mu DX^\nu C^{(2)})(i k B) $$

$$ + 3 \partial \tilde{\omega}^{(0)} DX^\mu DX^\nu B_{\mu\nu} + 3 \partial \omega^{(0)} DX^\mu DX^\nu C^{(2)}.$$
Table 6: Worldvolume field content of the IIB KK-monopole. In the case of the IIB KK-monopole, there are three embedding coordinates $X^\mu$, after gauge fixing the Taub-NUT coordinate, two worldvolume scalars $\omega^{(0)}, \tilde{\omega}^{(0)}$, constituting a doublet under S-duality, a self-dual 2-form $\omega^{(2)}$ (S-selfdual) and a 5-form $\tilde{\omega}^{(5)}$, describing the tension of the KK-monopole and also S-selfdual.

The field content of the worldvolume theory is summarized in Table 6. It consists on a selfdual 2-form and 5 scalars, which is the field content of the IIB KK-monopole:

$$\tilde{\mathcal{C}}^{(6)} = 6\partial \tilde{\omega}^{(5)} + \frac{1}{2\pi\alpha'} (i_k N) + 6(i_k \tilde{B}) \partial \omega^{(0)} - 6(i_k C^{(6)}) \partial \tilde{\omega}^{(0)}$$

$$+ 30(DX \ldots DX C^{(4)})(i_k C^{(2)}) \partial \omega^{(0)} + 30(DX \ldots DX C^{(4)})(i_k B) \partial \tilde{\omega}^{(0)}$$

$$+ 30(i_k C^{(4)})(DXDXC^{(2)}) \partial \omega^{(0)} + 30(i_k C^{(4)})(DXDXB) \partial \tilde{\omega}^{(0)}$$

$$+ 45(DXDXC^{(2)})(DXDXB)(i_k C^{(2)}) \partial \omega^{(0)}$$

$$- 45(DXDXB)(DXDXC^{(2)})(i_k B) \partial \tilde{\omega}^{(0)}$$

$$- 30(2\pi\alpha') \partial \omega^{(2)} \left( \mathcal{K}^{(3)} - 6(2\pi\alpha') A \partial \omega^{(0)} \partial \tilde{\omega}^{(0)} \right)$$

$$+ 60(2\pi\alpha')^2 \mathcal{K}^{(3)} A \partial \omega^{(0)} \partial \tilde{\omega}^{(0)} - 30(2\pi\alpha')(DX \ldots DX C^{(4)}) \partial \omega^{(0)} \partial \tilde{\omega}^{(0)}$$

$$- 360(2\pi\alpha')^2 \partial \omega^2 A \partial \omega^{(0)} \partial \tilde{\omega}^{(0)}.$$  

(3.15)

This action is invariant under local transformations in the worldvolume: $\delta X^\mu = -\sigma(\xi) k^\mu$, with $A$ playing the role of gauge field: $\delta A = \partial \sigma$. This symmetry can be gauge fixed by eliminating the coordinate adapted to it, which is the Taub-NUT direction of the monopole. In this way we obtain the right number of degrees of freedom \[22\].

The field content of the worldvolume theory is summarized in Table 6. It consists on a selfdual 2-form and 5 scalars, which is the field content of the
(2, 0) vector supermultiplet \([2]\). The worldvolume symmetries can be found in Appendix C.4.

The selfduality of the 2-form \(\omega^{(2)}\) in the linear approximation takes the form:

\[
K_{i_1 \ldots i_3}^{(3)} = \frac{1}{\sqrt{|\det(\Pi^{(B)})|}} \Pi_{i_1 j_1}^{(B)} \cdots \Pi_{i_3 j_3}^{(B)} \varepsilon^{j_1 \ldots j_6} K_{j_4 j_5 j_6}^{(3)},
\]

where we have introduced a shorthand notation for the gauged sigma-model metric: \(\Pi_{ij}^{(B)} = D_i X^\mu D_j X^\nu g_{\mu\nu} \). \(^{11}\)

Notice that we have introduced a new field \(i_k N\). This field is defined from the \(T\)-duality rule for the field \(\tilde{B}\) of type IIA:

\[
\tilde{B}_{\mu_1 \ldots \mu_6} = N_{\mu_1 \ldots \mu_6 z}.
\]

Its gauge transformation rule can be found in the appendix (formula \([3.4]\)), and shows that \(i_k N\) is S-selfdual. The complete action of the IIB KK-monopole is in fact invariant under S-duality. This duality works on the worldvolume fields as follows:

\[
\begin{align*}
\omega^{(2)} &\rightarrow \omega^{(2)}, & \tilde{\omega}^{(5)} &\rightarrow \tilde{\omega}^{(5)}, \\
\omega^{(0)} &\rightarrow \tilde{\omega}^{(0)}, & \tilde{\omega}^{(0)} &\rightarrow -\omega^{(0)},
\end{align*}
\]

i.e. \(\omega^{(0)}\) and \(\tilde{\omega}^{(0)}\) constitute a doublet under \(S\)-duality whereas \(\omega^{(2)}\) and \(\tilde{\omega}^{(5)}\) are S-selfdual.

Intersections in Type IIB can be cast into representations of \(S\)-duality. This has the consequence that the solitons occurring in the branes fit as well into \(S\)-duality representations. This picture emerges naturally in the worldvolume action of the monopole.

The IIB KK-monopole has the same worldvolume soliton solutions as the IIA NS-5-brane, given that both branes are related by \(T\)-duality \([23]\). We show below that the worldvolume field content that we find in the IIB KK-monopole effective action precisely describes the couplings to these solitons.

The field \(\omega^{(2)}\) couples to a 1-brane soliton on the brane, which can be described by the intersection of a D-3-brane with the KK-monopole: \((1|KK, D3)\). Here one of the worldvolume directions of the D-3-brane is wrapped around the \(S^1\) of the monopole. This configuration is \(S\) selfdual.

\(^{11}\)In the kinetic term of \((3.9)\) there is an inverse metric \(\Pi^{(B)}_{ij}\) such that \(\Pi^{(B)}_{ij} \Pi^{(B)}_{jk} = \delta^i_k\). Although the reduced metric \(\Pi^{(B)}_{\mu\nu} = g_{\mu\nu} + k^{-2} k_{\mu} k_{\nu}\) has no inverse, the pull-back of this \(\Pi^{(B)}_{ij} = \partial_i X^\mu \partial_j X^\nu \Pi^{(B)}_{\mu\nu}\) has a well defined inverse \(\Pi^{(B)}_{ij}\) in the 5 + 1 dimensional worldvolume.
We can also have a 3-brane soliton on the KK-monopole, which can be obtained by two different intersections:

\[(3|KK, D5), \quad (3|KK, NS5),\] (3.19)

where in both cases one direction of the brane is wrapped around the $S^1$ of the monopole. These two configurations are $S$-dual to each other, with each 3-brane soliton described by one of the 0-forms forming an $SL(2, Z)$ doublet: $(\omega^{(0)}, \tilde{\omega}^{(0)})$.

We still have the three embedding scalars, which may play a role in the construction of 3-brane solitons. Since they are inert under $S$-duality, they must couple to solitons that appear in $S$-selfdual configurations of branes. These are the two possible intersections of two IIB KK-monopoles on a three brane: $(3|KK, KK)_{1,2}$, which we also encountered in the IIA theory.

The 5-brane soliton can be realized as the intersection of the IIB KK-monopole with a D-7-brane. The KK-monopole is completely embedded in the D-7-brane, which lies transversal to the $S^1$ direction.

There are other intersections over 1-, 3- or 5-branes for which one can find out to which worldvolume field they should couple, in view of the $S$-symmetry of the configuration. As for the IIA NS-5-brane, there are no intersections over a 4-brane. Therefore the same argument presented in the previous section against a dynamical tension for the IIA NS-5-brane \cite{27, 30} applies to the IIB KK-monopole.

\section{T-duality between type II NS-5-branes}

In this section we present the $T$-duality rules between both type II NS-5-branes. The duality is achieved in this case by means of a double dimensional reduction. We restrict ourselves to the kinetic part of the actions.

In this and the next section hatted (unhatted) worldvolume directions are six (five) dimensional.

We split the worldvolume directions into $\hat{i} = (i, \sigma)$, where $\sigma$ is the worldvolume coordinate which is identified with the target space coordinate in the reduction.

The $T$-duality rules of the worldvolume fields present in the IIA NS-5 brane effective action are:
\[ c^{(0)\nu} = -c^{(1)\sigma}, \]
\[ a^{(2)\nu}_{\sigma \sigma} = -c^{(1)\sigma}, \]
\[ a^{(2)\nu}_{\sigma i} = -\tilde{c}^{(3)\sigma}, \]  
(4.1)

where \( \tilde{c}^{(3)} \) is the worldvolume dual of \( c^{(1)} \). This duality can be seen as inherited from the selfduality of \( a^{(2)} \) in the IIA NS-5-brane, and has the following form in the linear approximation:

\[
\tilde{G}^{(4)\hat{i}_1...\hat{i}_4}_{\hat{i}_5...\hat{i}_8} = \frac{1}{2! \sqrt{|g|} \sqrt{1 + e^{2\phi} C^{(0)2}}} g_{\hat{i}_1 \hat{j}_1} \cdots g_{\hat{i}_4 \hat{j}_4} e^{\hat{i}_1...\hat{i}_6} \tilde{F}_{\hat{j}_5...\hat{j}_8},
\]  
(4.2)

where \( \tilde{G}^{(4)} \) is the curvature of \( \tilde{c}^{(3)} \):

\[
\tilde{G}^{(4)} = 4 \partial \tilde{c}^{(3)} + \frac{1}{2 \pi \alpha'} C^{(4)} + \frac{3}{2 \pi \alpha'} BC^{(2)} + 12 B \partial \tilde{c}^{(1)}. \]  
(4.3)

Using the \( T \)-duality transformation rules for the target space fields given in Appendix A we find that the worldvolume field strengths transform as:

\[
\tilde{G}^{(1)\sigma}_{1} = -\frac{1}{2 \pi \alpha'} C^{(0)}, \quad G^{(1)\sigma}_{i} = -\tilde{F}_{i\sigma}, \quad H^{(3)\sigma}_{ij} = -\tilde{F}_{ij}, \quad H^{(3)\sigma}_{ijk} = -\tilde{G}^{(4)\sigma}_{ijk}. \]  
(4.4)

\( \tilde{G}^{(4)\sigma}_{ijkl} \) is related to \( \tilde{F} \) through (4.2).

It is important to stress the fact that in the dualization of \( G^{(1)\sigma} \) a factor \( 2 \pi \alpha' \) enters in the denominator. This implies that higher order terms in \( \alpha' \) in the kinetic part of the IIA NS-5-brane will contribute after duality to a lower order approximation for the IIB NS-5-brane.

The action (3.1) of the IIA NS-5-brane is valid up to quadratic order in \( \alpha' \). If we want to obtain an action for the IIB NS-5-brane up to the same order we need to consider the following, fourth order, expression for the kinetic term of the IIA NS-5-brane:

\[
S = -\int d^6 \xi e^{-2\phi} \sqrt{|\det (g_{ij} - (2 \pi \alpha')^2 e^{2\phi} G^{(1)\sigma} G^{(1)}_{\sigma})|} \times \left\{ 1 - \frac{(2 \pi \alpha')^2}{4!} e^{2\phi} (H^{(3)})^2 \right. \\
\left. - \frac{3(2 \pi \alpha')^4}{4!} e^{4\phi} e^{2\phi} G^{3} G_{ik} H^{ijkl} + \ldots \right\}. \]  
(4.5)

\( \tilde{c}^{(3)} \) is related to the 3-form \( c^{(3)} \) describing the tension of the explicitly \( S \)-selfdual D-3-brane \([12]\) by \( \tilde{c}^{(3)} = c^{(3)} + (2 \pi \alpha')_c^{(1)} \partial b \).
Note that this extra term does not come from considering the next order contribution to the M-5-brane action, but from the inverse metric in the reduction of the $H^2$-term from eleven to ten dimensions.

$T$-duality in this action yields the action of the IIB NS-5-brane with one extra 3-form $\tilde{c}^{(3)}$, i.e. the IIB NS-5-brane in a “1-3-form” formulation. We need to perform the Poincaré duality (4.2) in order to obtain the action (2.10) in the “1-form” formulation. The result is, up to quadratic order in $\alpha'$:

$$S = -T_{BS} \int d^6\xi e^{-2\varphi} \sqrt{1 + e^{2\varphi}(C^{(0)})^2} \sqrt{\det g} \times \left\{ 1 + \frac{(2\pi\alpha')^2}{4} \frac{e^{2\varphi}}{1 + e^{2\varphi}(C^{(0)})^2} \tilde{F}_{ij} \tilde{F}^{ij} + \ldots \right\}. \quad (4.6)$$

This provides a further check of the action that we have presented in section 2.3 for the IIB NS-5-brane.

5 $T$-duality between type II KK-monopoles

We can do the same calculation as in the previous section for the case of the type II KK-monopoles. Also here we concentrate only on the kinetic terms. The duality is achieved again by means of a double dimensional reduction. The gauged isometry direction is kept the same in both monopoles.

The transformations of the worldvolume fields in the IIA KK-monopole are:

$$\omega^{(0)\nu} = \omega^{(0)}, \quad \omega^{(1)\nu} = \tilde{\omega}^{(0)}, \quad \omega^{(1)1} = \omega^{(2)}, \quad \omega^{(3)ij} = \omega^{(2)ij}. \quad (5.1)$$

The 2-form $\omega^{(2)}_{ij}$ in the IIB KK-monopole is obtained from the $T$-dualization of a 3-form, which is the worldvolume dual of the 1-form $\omega^{(1)}$ in the IIA KK-monopole action. The selfduality of the 2-form $\omega^{(2)}$ in the IIB KK-monopole translates into the duality between $\omega^{(1)}$ and $\omega^{(3)}$. Explicitly, the selfduality condition of $\omega^{(2)}$ (eq. (3.16)), becomes after $T$-duality:

$$K^{(4)}_{i_1\ldots i_4} = \frac{1}{2!\sqrt{\det \Pi^{(A)}} \sqrt{1 + e^{2\varphi}k^{-2}(i_kC^{(1)})^2}} \Pi^{(A)}_{i_1j_1} \ldots \Pi^{(A)}_{i_4j_4} \epsilon^{j_1\ldots j_6} \kappa^{(2)}_{j_5j_6}, \quad (5.2)$$

where $K^{(4)}$ is the curvature of the 3-form $\omega^{(3)}$:

$$K^{(4)} = 4\partial \omega^{(3)} + \frac{1}{2\pi\alpha'}(i_kC^{(5)}) + 4(DXDXDXC^{(3)})K^{(1)} + 12(DXDXB)\partial \omega^{(1)} + 24(2\pi\alpha')A\partial \omega^{(0)} \partial \omega^{(1)}, \quad (5.3)$$
and we have defined $\Pi_{ij}^{(A)} = D_i X^\mu D_j X^\nu g_{\mu\nu}$.

The 3-form $\omega^{(3)}$ is therefore associated to the target space field $i_k C^{(5)}$ of the IIA KK-monopole, and it couples to its 2-brane soliton. See the conclusions for a further discussion on this point.

The worldvolume duality transformations above, together with the transformations for the target space fields that can be found in Appendix A, give the action of the IIB KK-monopole in quadratic approximation. Note that in this calculation it is necessary to perform the Poincaré duality transformation (5.2) in order to obtain the selfduality condition (3.16) of the IIB KK-monopole action.

This result provides a further check of the action presented in section 3.3.

6 Conclusions

In this paper we have constructed the worldvolume effective actions of the NS-5-brane and KK-monopole of the type IIB theory. Their worldvolume field content is precisely what is needed in order to explain the soliton configurations of these branes.

It is remarkable that the worldvolume fields that couple to the soliton solutions of a six dimensional brane are those that are needed to construct invariant field strengths for each target space field coupled to the brane. These field strengths take the form: $p\partial c^{(p-1)} + \frac{1}{2\pi\alpha'} C^{(p)} + \ldots$. Consider for instance the IIB NS-5-brane. The target space fields that couple to its worldvolume are: $C^{(2)}, C^{(4)}$ and $B$. The corresponding worldvolume field strengths contain the worldvolume fields: $c^{(1)}, c^{(3)}$ and a 5-form $\tilde{c}^{(5)}$, which couple to 0-brane, 2-brane and 4-brane solitons, respectively. Given that $c^{(3)}$ is the worldvolume dual of $c^{(1)}$ it is possible to construct an action in which only $c^{(1)}$ is present, which is the one that we have constructed in section 2. If $c^{(3)}$ is included then the 5-form that enters the field strength of $B$ (the WZ term) is $\tilde{c}^{(5)}$, which differs from $c^{(5)}$ by a worldvolume field redefinition. One can conclude that a $(p-2)$-brane soliton in the worldvolume of a $q$-brane couples to its worldvolume field $c^{(p-1)}$ and describes the boundary of a $(p-1)$-brane ending on the $q$-brane. The eleven dimensional soliton configurations derived in [9] can also be associated to the worldvolume fields assigned to each target space field occurring in the worldvolume. This idea provides a very simple way of identifying which soliton solutions should be found in a particular brane. That this could be the case was anticipated in [7, 32].

\footnote{That is why we have denoted it $\tilde{c}^{(5)}$ and not $c^{(5)}$, as it appears in the IIB NS-5-brane action of section 2.}
For the KK-monopoles, the worldvolume fields are associated to the contractions of the Killing vector with the target space fields. The field strengths take the form:

\[ p \partial \omega^{(p-1)} + \frac{1}{2\pi \alpha'} (i_k C^{(p+1)}) + \ldots \]

In this case \( \omega^{(p-1)} \) couples to a \((p-2)\)-brane soliton, which describes the boundary of a \(p\)-brane ending on the KK-monopole, with one of its worldvolume directions wrapped around the Taub-NUT direction of the monopole. Let us consider, for instance, the case of the IIA KK-monopole. The target space field associated to \( \omega^{(1)} \) is \( i_k C^{(3)} \). Then \( \omega^{(1)} \) describes a D-2-brane, wrapped around the Taub-NUT direction, ending on the KK-monopole.

The mechanism by which we have constructed the previous actions is \(T\)-duality. The corresponding objects in IIA which the NS-5-brane and the KK-monopole in IIB are dual to are the IIA KK-monopole and the IIA NS-5-brane. These branes contain couplings to dual target space fields in their worldvolumes, as well as to non-trivial worldvolume fields, for which the \(T\)-duality transformation rules were not known. We have constructed the \(T\)-duality rules for all the target space and worldvolume fields coupled to these actions.

One possible generalization of type IIA supergravity contains a cosmological constant term. This is the so-called massive supergravity \([33, 34]\). This theory is related to type IIB supergravity by massive \(T\)-duality rules, which have been derived in the literature for the gauge potentials occurring in the supergravity actions, as well as for the worldvolume fields coupled to some low dimensional D-branes \([33, 35, 36]\). We have left for a future publication the derivation of the complete set of \(T\)-duality rules that map other solitonic objects, where dual gauge potentials and more general worldvolume fields are also present in the effective actions \([31]\).

An interesting feature about the IIB NS-5-brane is the way the worldvolume fields must transform under \(S\)-duality in order to show its connection with the D-5-brane. One would have expected the corresponding actions to be related by an \(S\)-duality transformation for the target space fields plus an additional worldvolume duality for the BI field, with the consequence that the NS-5-brane would have depended on a worldvolume 3-form \([13]\). This is in fact the way the F-string and the D-string are related \([17]\). There, together with the \(S\)-duality transformation of the backgrounds, one needs to perform a worldvolume duality transformation giving rise to the Born-Infeld field of the D-string. However the worldvolume field content of the NS-5-brane reveals that this Poincaré duality does not take place. Instead, we have found that the \(S\)-duality transformation rules of the worldvolume fields reveal that they transform as doublets or singlets depending on the behaviour under \(S\)-duality of the target space field to which they are associated. We believe this information may be relevant for the construction of the worldvolume action.
of the so-called \((p, q)\) 5-brane multiplet \[13\].

Moreover, this result suggests that type IIB solitonic branes which are\n\(S\)-dual partners of D-\(p\)-branes, i.e. IIB \((1,0)\) \(p\)-branes, with \(p\) odd and \(p > 3\), are described by an effective action whose kinetic term contains the 1-form \(c^{(1)}\), and is given by:

\[
\int d^{p+1} \xi \ e^{-\left(\frac{p-1}{2}\right)\phi} (1 + e^{2\phi}C^{(0)}^2) \frac{\sqrt{1 + e^{2\phi}C^{(0)}^2}}{\sqrt{1 + e^{2\phi}C^{(0)}^2}} \det g + \frac{2\pi\alpha^\prime e^{\phi}}{\sqrt{1 + e^{2\phi}C^{(0)}^2}} F \right). \tag{6.1}
\]

This is the \(S\)-dual of the kinetic term of the corresponding D-\(p\)-brane. The \(p = 5\) case corresponds to the 5-brane that we have obtained in this article. For \(p = 7\) \((6.1)\) describes the kinetic term of a \((1,0)\) 7-brane, which can in fact be obtained as \(T\)-dual to the M-KK-monopole of \[18\] reduced along a transversal coordinate \[31\]. This information can be relevant for the construction of the worldvolume actions of other type IIB \((p, q)\) brane multiplets \[14\].

We have found that the type IIB KK-monopole is described by a gauged sigma model, such that the degree of freedom associated to its Taub-NUT isometry is effectively eliminated from the action. Gauged sigma-models have proved very useful in order to describe Kaluza-Klein monopoles and eleven dimensional massive branes \[19, 37\].

It was shown in \[19\] that the definition of a massive eleven dimensional supergravity from which Romans’ massive IIA SUGRA could be derived upon dimensional reduction, requires the existence of a Killing isometry in the eleven dimensional background. Massive IIA D-branes are obtained through direct and double dimensional reduction from massive M-branes, whose effective actions are described by gauged sigma-models in which the Killing isometry is gauged. The M-9-brane is a solution of massive eleven dimensional supergravity, since its double dimensional reduction gives the massive IIA D-8-brane \[38, 34\]. It has been shown recently \[39\] that the M-9-brane is described by the same kind of gauged sigma-model as the KK-monopoles. The IIA NS-9-brane obtained by reduction along the transverse coordinate contains the correct dilaton coupling \[13\]. Also, the IIB D-9-brane obtained from it after \(T\) and \(S\) dualities has the correct scaling with \(e^{-\phi}\). In this paper we have obtained explicit \(T\) and \(S\) duality rules for worldvolume fields that couple as well to the worldvolume actions of 9-branes. For instance, we know from \(S\)-duality in the IIB D-9-brane that the IIB NS-9-brane must contain

---

\[14\] Recently a \((p, q)\) multiplet of 5-brane solutions of type IIB supergravity has been constructed in \[15\].

\[15\] See \[36\] for some recent work in this direction.

\[16\] For the gravitational part.
the worldvolume vector $c^{(1)}$ in its effective action. Many of the results for 5-branes in this article can be generalized to 9-branes. We hope to report progress in this direction in the near future.

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A \hspace{.2cm} T\text{-duality}

In this appendix we give the $T$-duality rules for the background fields that couple to the NS-5-branes and KK-monopoles. For completion we also summarize the target space $T$-duality rules that have been constructed previously in the literature \cite{4,16}. In our notation $z$ is the direction along which we perform the duality transformation.

The $T$-duality rules that map $(i_k N)$ and $(i_k \tilde{B})$, coupled to the IIA KK-monopole action, onto type IIB backgrounds are:

\begin{align}
N'{}_{\mu_1...\mu_6} &= -\tilde{B}{}_{\mu_1...\mu_5} + 6\tilde{B}{}_{\mu_1...\mu_5 z} \frac{g_{\mu_6 z}}{g_{zz}} + 20C^{(4)}{}_{[\mu_1...\mu_3 z} C^{(2)}{}_{\mu_4\mu_5]} \frac{g_{\mu_6 z}}{g_{zz}} \\
&\quad - \frac{15}{2} B_{\mu_1\mu_2} C^{(2)}{}_{\mu_3\mu_4} C^{(2)}{}_{\mu_5\mu_6]} - 15B_{[\mu_1 z C^{(2)}{}_{\mu_2\mu_3}} C^{(2)}{}_{\mu_4\mu_5]} \frac{g_{\mu_6 z}}{g_{zz}} \\
&\quad + 60B_{[\mu_1\mu_2} C^{(2)}{}_{\mu_3\mu_4} C^{(2)}{}_{\mu_5 z} \frac{g_{\mu_6 z}}{g_{zz}}, \\
\tilde{B}'{}_{\mu_1...\mu_5 z} &= \tilde{B}{}_{\mu_1...\mu_5} + 5C^{(4)}{}_{[\mu_1...\mu_4} C^{(2)}{}_{\mu_5 z]} + 5C^{(4)}{}_{[\mu_1...\mu_3} C^{(2)}{}_{\mu_4\mu_5]} + 10C^{(4)}{}_{[\mu_1...\mu_3} C^{(2)}{}_{\mu_4 z} \frac{g_{\mu_5 z}}{g_{zz}} \\
&\quad + \frac{15}{2} B_{[\mu_1\mu_2} C^{(2)}{}_{\mu_3\mu_4} C^{(2)}{}_{\mu_5 z]} + 15B_{[\mu_1 z} C^{(2)}{}_{\mu_2} C^{(2)}{}_{\mu_3\mu_4} \frac{g_{\mu_5 z}}{g_{zz}}.
\end{align}

(A.1)

From type IIB to type IIA we have:
\[\begin{align*}
\bar{B}_{\mu_1...\mu_5z} &= \bar{B}_{\mu_1...\mu_5z} - 5 \left( C_{[\mu_1...\mu_4z}^{(5)} - 3 B_{[\mu_1\mu_2} C_{\mu_3\mu_4z]}^{(3)} \right) 
&\quad - 5 \left( C_{[\mu_1\mu_2\mu_3z}^{(3)} - \frac{3}{2} C_{[\mu_1\mu_2z} \frac{g_{\mu_3z]}{g_{zz}}} \right) C_{\mu_4\mu_5z}^{(3)}, \\
\bar{B}_{\mu_1...\mu_6} &= -N_{\mu_1...\mu_6z} - 6 \bar{B}_{[\mu_1...\mu_5z B_{\mu_6]z}} \\
&\quad + 30 \left( C_{[\mu_1...\mu_4z}^{(5)} - 3 C_{[\mu_1\mu_2z B_{\mu_3\mu_4z]}^{(3)} \right) 
&\quad + 10 \left( C_{[\mu_1...\mu_3}^{(3)} - \frac{3}{2} \frac{g_{\mu_1\mu_2z}}{g_{zz}} C_{\mu_3\mu_4z]}^{(3)} \right) C_{\mu_4\mu_5z} B_{\mu_6]z} \\
&\quad - 30 C_{[\mu_1\mu_2z}^{(3)} C_{\mu_3\mu_4z]}^{(3)} \frac{g_{\mu_5z]}{g_{zz}} B_{\mu_6]z} - \frac{15}{2} C_{[\mu_1\mu_2z}^{(3)} C_{\mu_3\mu_4z]}^{(3)} B_{\mu_5\mu_6].}
\end{align*}\]

Now we summarize the $T$-duality rules that have been constructed previously in [14, 16].

From IIA to IIB we have:

\[\begin{align*}
C_{\mu}^{(1)'} &= -C_{\mu}^{(0)}, \\
C_{\mu}^{(2)'} &= -C_{\mu}^{(2)} + C_{\mu}^{(0)} B_{\mu}, \\
C_{\mu\nu}^{(3)'} &= -C_{\mu\nu}^{(2)} + 2C_{[\mu}^{(2)} \frac{g_{\nu]z}}{g_{zz}}, \\
C_{\mu\nu\rho}^{(3)'} &= -C_{\mu\nu\rho}^{(4)} + \frac{3}{2} C_{[\mu}^{(2)} B_{\rho]z} g_{\nu}^{\rho} = \frac{1}{g_{zz}}, \\
&\quad - \frac{3}{2} B_{[\mu} C_{\rho]}^{(2)} z \quad B_{\mu}^{\nu} = B_{\mu}^{\nu} - 2 \frac{g_{zz}}{g_{zz}} B_{[\mu} g_{\nu]z}, \\
&\quad + 6 C_{[\mu}^{(2)} B_{\nu}^{\rho]} z \quad B_{\mu}^{\nu} = - \frac{g_{\rho]}{g_{zz}} z, \\
C_{\mu_1...\mu_4z}^{(5)'} &= -C_{\mu_1...\mu_4z}^{(4)} + 4 C_{[\mu_1\mu_2\mu_3z}^{(4)} \frac{g_{\mu_4]}{g_{zz}} - 3 C_{[\mu_1\mu_2z} B_{\mu_3\mu_4z]}^{(2)} \\
&\quad - 6 C_{[\mu_1z}^{(2)} B_{\mu_2\mu_3] z} \frac{g_{\mu_4]}{g_{zz}} - 6 B_{[\mu_1} C_{\mu_2\mu_3z]^{(2)} \frac{g_{\mu_4]}{g_{zz}}}, \\
C_{\mu_1...\mu_5}^{(5)'} &= -C_{\mu_1...\mu_5z}^{(6)} + 5 \left( C_{[\mu_1...\mu_4z}^{(4)} - 4 C_{[\mu_1...\mu_3z}^{(4)} \frac{g_{\mu_4]}{g_{zz}} \right) B_{\mu_5]z} \\
&\quad - \frac{15}{2} B_{[\mu_1\mu_2z} B_{\mu_3\mu_4z]}^{(2)} C_{\mu_5]z} - 30 C_{[\mu_1z}^{(2)} B_{\mu_2\mu_3z} B_{\mu_4z}^{\rho]} \frac{g_{\rho]}{g_{zz}} .
\end{align*}\]

Similarly, $T$-duality maps the type IIB background onto the type IIA as...
follows:

\[ C^{(0)\nu} = -C^{(1)}_z, \]
\[ C^{(2)\nu}_{\mu z} = -C^{(1)}_{\mu} + C^{(1)}_z \frac{g_{[\mu}}{g_{zz}}, \]
\[ C^{(2)\nu}_{\mu \rho z} = -C^{(3)}_{\mu \rho z} + 2C^{(1)}_{[\mu} B_{\nu] z}, \]
\[ C^{(2)\nu}_{\mu \nu z} = -2C^{(1)}_z \frac{g_{[\mu}}{g_{zz}} B_{\nu] z}, \]
\[ C^{(4)\nu}_{\mu \nu \rho z} = -C^{(3)}_{\mu \nu \rho z} + 3C^{(3)}_{[\mu \nu z} \frac{g_{[\rho]}}{g_{zz}}, \]
\[ C^{(4)\nu}_{\mu_1 \ldots \mu_4} = -C^{(5)}_{\mu_1 \ldots \mu_4 z} + 4 \left( C^{(3)}_{[\mu_1 \mu_2 \mu_3} - 3C^{(3)}_{[\mu_1 \mu_2 z} \frac{g_{\mu_3]}}{g_{zz}} \right) B_{\nu] z}, \]
\[ C^{(4)\nu}_{\mu_1 \ldots \mu_5 z} = -C^{(5)}_{\mu_1 \ldots \mu_5} + 5C^{(5)}_{[\mu_1 \ldots \mu_4 z} \frac{g_{[\mu_5]}}{g_{zz}} \]
\[ -15C^{(3)}_{[\mu_1 \mu_2 z} B_{\mu_3 \mu_4} \frac{g_{[\mu_5]}}{g_{zz}} \]
\[ + \frac{15}{2} \left( C^{(1)}_{[\mu_1} - C^{(1)}_{z} \frac{g_{[\mu_1]}}{g_{zz}} \right) B_{\mu_2 \mu_3} B_{\mu_4 \mu_5]. \]

B Target Space Gauge Symmetries

In this appendix we give the gauge symmetries for the background fields that couple to the six dimensional worldvolume theories considered in this paper.

- Type IIA:
\[\delta C^{(1)} = \partial \Lambda^{(0)}, \]
\[\delta B = 2 \partial \Lambda, \]
\[\delta C^{(3)} = 3 \partial \Lambda^{(2)} + 3 \partial \Lambda^{(0)} B, \]
\[\delta C^{(5)} = 5 \partial \Lambda^{(4)} + 30 \partial \Lambda^{(2)} B + 15 \partial \Lambda^{(0)} BB, \]
\[\delta \tilde{B} = 6 \partial \tilde{\Lambda} - 30 \partial \Lambda^{(2)} C^{(3)} + 6 \partial \Lambda^{(0)} \left( C^{(5)} - 5 C^{(3)} B \right). \]

The KK-monopole couples as well to a new field \((i_k N)\), with transformation rule:

\[\delta (i_k N) = 6 \partial (i_k \Sigma^{(6)}) + 60 (i_k C^{(3)}) \partial (i_k \Lambda^{(4)}) - 30 \partial (i_k \tilde{\Lambda})(i_k B) \]
\[ - 60 (i_k C^{(3)}) (i_k C^{(3)}) \partial \Lambda + 120 \partial \Lambda^{(2)} (i_k C^{(3)}) (i_k B) \]
\[ + 60 (i_k C^{(3)}) \partial (i_k \Lambda^{(2)}) B - 40 C^{(3)} \partial (i_k \Lambda^{(2)}) (i_k B) \]
\[ - 20 C^{(3)} (i_k C^{(3)}) \partial (i_k \Lambda) - \sigma^{(0)} k^\lambda \partial_\lambda (i_k N). \]

When coupled to the KK-monopole all the target space fields transform as well with respect to its Killing isometry, as indicated by the last term in \(\delta (i_k N)\).

- **Type IIB:** We work in the basis of fields where the 4-form \(C^{(4)}\) is \(S\)-selfdual. The gauge transformations are given by:

\[\delta B = 2 \partial \Lambda, \]
\[\delta C^{(2)} = 2 \partial \Lambda^{(1)}, \]
\[\delta C^{(4)} = 4 \partial \Lambda^{(3)} + 6 \partial \Lambda^{(1)} B - 6 C^{(2)} \partial \Lambda, \]
\[\delta C^{(6)} = 6 \partial \Lambda^{(5)} + 60 \partial \Lambda^{(3)} B \]
\[+ 45 \partial \Lambda^{(1)} BB - 90 C^{(2)} B \partial \Lambda, \]
\[\delta \tilde{B} = 6 \partial \tilde{\Lambda}^{(5)} - 60 \partial \Lambda^{(3)} C^{(2)} \]
\[+ 45 \partial \Lambda C^{(2)} C^{(2)} - 90 \partial \Lambda^{(1)} BC^{(2)}. \]
We also include here the gauge transformation of the S-selfdual field $i_kN$ which couples to the KK-monopole:

$$
\delta(i_kN) = 6\partial(i_k\tilde{\Sigma}^{(6)}) - 30\partial(i_k\Lambda^{(3)})(i_kC^{(4)}) \\
-6(i_k\tilde{B})\partial(i_k\Lambda) - 6(i_kC^{(6)})\partial(i_k\Lambda^{(1)}) \\
-45BC^{(2)}(i_kC^{(2)})\partial(i_k\Lambda) - 45BC^{(2)}(i_kB)\partial(i_k\Lambda^{(1)}) \\
-30C^{(4)}(i_kC^{(2)})\partial(i_k\Lambda) + 30C^{(4)}(i_kB)\partial(i_k\Lambda^{(1)}) \\
-30(i_kC^{(4)})C^{(2)}\partial(i_k\Lambda) + 30(i_kC^{(4)})B\partial(i_k\Lambda^{(1)}) \\
-45\partial(i_k\Lambda^{(3)})(i_kC^{(2)})B + 45\partial(i_k\Lambda^{(3)})(i_kB)C^{(2)}.
$$

(B.4)

As for the IIA case all the fields transform as well with respect to the Killing isometry of the IIB KK-monopole.

## C Worldvolume Gauge Symmetries

Here we give the gauge transformations of the worldvolume fields present in the different actions that appear in this paper.

### C.1 IIA KK-monopole

The gauge transformations of the worldvolume fields present in this action are [RS]:
\[ \delta \omega^{(0)} = \frac{1}{2\pi\alpha'}(i_k \Lambda), \]
\[ \delta \omega^{(1)} = \partial \mu^{(0)} - \frac{1}{2\pi\alpha'}(i_k \Lambda^{(2)}) + \Lambda^{(0)} \partial \omega^{(0)}, \]
\[ \delta \omega^{(3)} = 3\partial \mu^{(2)} - \frac{1}{2\pi\alpha'}(i_k \Lambda^{(4)}) - 3\Lambda^{(2)} \partial \omega^{(0)} \]
\[ - 6\Lambda \partial \omega^{(1)} - 6(2\pi\alpha')\sigma^{(0)} \partial \omega^{(0)} \partial \omega^{(1)}, \]
\[ \delta \omega^{(5)} = 5\partial \mu^{(4)} - \frac{1}{2\pi\alpha'}(i_k \Sigma^{(6)}) + 5(i_k \tilde{\Lambda}) \partial \omega^{(0)} + 20(i_k \Lambda^{(4)}) \partial \omega^{(1)} \]
\[ + 60(2\pi\alpha')\Lambda^{(2)} \partial \omega^{(1)} \partial \omega^{(0)} + 60(2\pi\alpha') \Lambda \partial \omega^{(1)} \partial \omega^{(1)} \]
\[ + 60(2\pi\alpha')^2 \sigma^{(0)} \partial \omega^{(1)} \partial \omega^{(1)} \partial \omega^{(0)}. \]

\section*{C.2 IIB NS-5-brane}

The worldvolume gauge transformations are given by:
\[ \delta c^{(1)} = \partial \kappa^{(0)} - \frac{1}{2\pi\alpha'} \Lambda^{(1)}, \]
\[ \delta \tilde{c}^{(3)} = \partial \kappa^{(2)} - \frac{1}{2\pi\alpha'} \Lambda^{(3)} - 6\Lambda \partial c^{(1)}, \]
\[ \delta \tilde{c}^{(5)} = 5\partial \tilde{\kappa}^{(4)} + \frac{1}{2\pi\alpha'} \tilde{\Lambda} + 20\Lambda^{(3)} \partial c^{(1)} + 60(2\pi\alpha') \Lambda \partial c^{(1)} \partial c^{(1)}. \]

\section*{C.3 D-5-brane}

The worldvolume symmetries for the D-5-brane are S-dual to those of the NS-5-brane:
\[ \delta b = \partial \rho^{(0)} - \frac{1}{2\pi\alpha'} \Lambda, \]
\[ \delta c^{(5)} = 5\partial \kappa^{(4)} - \frac{1}{2\pi\alpha'} \Lambda^{(5)} + 20\Lambda^{(3)} \partial b - 60(2\pi\alpha') \Lambda^{(1)} \partial b \partial b. \]

\section*{C.4 IIA NS-5-brane}

The worldvolume symmetries are given by \cite{19}: 

\[ \delta \omega^{(0)} = \frac{1}{2\pi\alpha'}(i_k \Lambda), \]
\[ \delta \omega^{(1)} = \partial \mu^{(0)} - \frac{1}{2\pi\alpha'}(i_k \Lambda^{(2)}) + \Lambda^{(0)} \partial \omega^{(0)}, \]
\[ \delta \omega^{(3)} = 3\partial \mu^{(2)} - \frac{1}{2\pi\alpha'}(i_k \Lambda^{(4)}) - 3\Lambda^{(2)} \partial \omega^{(0)} \]
\[ - 6\Lambda \partial \omega^{(1)} - 6(2\pi\alpha')\sigma^{(0)} \partial \omega^{(0)} \partial \omega^{(1)}, \]
\[ \delta \omega^{(5)} = 5\partial \mu^{(4)} - \frac{1}{2\pi\alpha'}(i_k \Sigma^{(6)}) + 5(i_k \tilde{\Lambda}) \partial \omega^{(0)} + 20(i_k \Lambda^{(4)}) \partial \omega^{(1)} \]
\[ + 60(2\pi\alpha')\Lambda^{(2)} \partial \omega^{(1)} \partial \omega^{(0)} + 60(2\pi\alpha') \Lambda \partial \omega^{(1)} \partial \omega^{(1)} \]
\[ + 60(2\pi\alpha')^2 \sigma^{(0)} \partial \omega^{(1)} \partial \omega^{(1)} \partial \omega^{(0)}. \]
\[ \delta c^{(0)} = -\frac{1}{2\pi\alpha'} \Lambda^{(0)}, \]
\[ \delta a^{(2)} = 2\partial\rho^{(1)} - \frac{1}{2\pi\alpha'} \Lambda^{(2)} + 2\partial c^{(0)} \Lambda, \]
\[ \delta \tilde{b} = \delta \tilde{\rho}^{(4)} - \frac{1}{2\pi\alpha'} \tilde{\Lambda} + 5\Lambda^{(4)} \partial c^{(0)} \]
\[ -15\Lambda^{(2)} \partial a^{(2)} - 30(2\pi\alpha')\Lambda \partial a^{(2)} \partial c^{(0)}. \]

(C.4)

C.5 IIB KK-monopole

The worldvolume gauge transformations are:

\[ \delta \omega^{(0)} = \frac{1}{2\pi\alpha'} i_k \Lambda, \]
\[ \delta \tilde{\omega}^{(0)} = -\frac{1}{2\pi\alpha'} i_k \Lambda^{(1)}, \]
\[ \delta \omega^{(2)} = 2\partial \mu^{(1)} - \frac{1}{2\pi\alpha'} i_k \Lambda^{(3)} - 2\Lambda^{(1)} \partial \omega^{(0)} - 2\Lambda \partial \tilde{\omega}^{(0)} - 2(2\pi\alpha')\sigma \partial \omega^{(0)} \partial \tilde{\omega}^{(0)}, \]
\[ \delta \tilde{\omega}^{(5)} = 5\partial \tilde{\mu}^{(4)} - \frac{1}{2\pi\alpha'} i_k \tilde{\Sigma}^{(6)} - 5(i_k \Lambda) \partial \omega^{(0)} + 5(i_k \Lambda^{(5)}) \partial \tilde{\omega}^{(0)} \]
\[ + 20(2\pi\alpha')\Lambda^{(3)} \partial \omega^{(0)} \partial \tilde{\omega}^{(0)} - 15(i_k \Lambda^{(3)}) \partial \omega^{(2)} - 30(2\pi\alpha')\Lambda \partial \omega^{(0)} \partial \omega^{(2)} - 30(2\pi\alpha')\Lambda \partial \tilde{\omega}^{(0)} \partial \omega^{(2)} - 30(2\pi\alpha')^2 \sigma \partial \omega^{(2)} \partial \omega^{(0)} \partial \tilde{\omega}^{(0)}. \]

(C.5)

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