NUCLEAR DENSITY FUNCTIONAL CONSTRAINED BY LOW-ENERGY QCD*

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We have developed a relativistic point-coupling model of nuclear many-body dynamics constrained by the low-energy sector of QCD. The effective Lagrangian is characterized by density-dependent coupling strengths determined by chiral one- and two-pion exchange (with single and double delta isobar excitations) and by large isoscalar background fields that arise through changes of the quark condensate and the quark density at finite baryon density. The model has been tested in the analysis of nuclear ground-state properties along different isotope chains of medium and heavy nuclei. The agreement with experimental data is comparable with purely phenomenological predictions. The built-in QCD constraints and the explicit treatment of pion exchange restrict the freedom in adjusting parameters and functional forms of density-dependent couplings. It is shown that chiral pionic fluctuations play an important role for nuclear binding and saturation mechanism, whereas background fields of about equal magnitude and opposite sign generate the effective spin-orbit potential in nuclei.

1. Introduction

In this work we would like to investigate the connection between QCD, its symmetry breaking patterns, and the nuclear many-body problem. Usual nuclear structure approaches \(^1\) are consistent with the symmetries of QCD (in particular chiral symmetry) but only functional forms of the interaction terms can be determined. Model parameters cannot be constrained at the level of accuracy required for a quantitative analysis of structure data; they can only be estimated with the Naive Dimensional Analysis \(^2\).

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The approach we propose is based on the following ingredients:

1. The presence of large isoscalar background fields which have their origin in the in-medium changes of the scalar quark condensate and of the quark density.

2. The nucleon-nucleon interaction is described by one- and two-pion exchange (with medium insertions and delta isobar excitations), in combination with Pauli blocking effects.

The first point has a clear connection with QCD sum rules at finite density, in which large nucleon self-energies naturally arise in the presence of a filled Fermi sea of nucleons.

The second point is motivated by the observation that, at the nuclear matter level, the nucleon Fermi momentum \( k_f \), the pion mass \( m_\pi \) and the \( \Delta - N \) mass difference represent comparable scales. Pionic degrees of freedom are therefore included explicitly (through density-dependent couplings), in contrast to the phenomenological relativistic models, in which effects of iterated one-pion and two-pion exchange are treated implicitly through an effective scalar field.

2. The model

The model is defined by the Lagrangian density

\[
\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{4f} + \mathcal{L}_{\text{der}} + \mathcal{L}_{\text{coul}}
\]

with the four terms specified as follows:

\[
\mathcal{L}_{\text{free}} = \bar{\psi}[i\gamma_\mu \partial^\mu - M_N] \psi
\]

\[
\mathcal{L}_{4f} = -\frac{1}{2} G_S(\hat{\rho})(\bar{\psi}\gamma^\mu \psi)(\bar{\psi}\gamma^\mu \psi) - \frac{1}{2} G_V(\hat{\rho})(\bar{\psi}\gamma^\mu \gamma^\rho \psi)(\bar{\psi}\gamma^\rho \gamma^\mu \psi)
\]

\[
\mathcal{L}_{\text{der}} = -\frac{1}{2} D_S(\partial^\nu \bar{\psi}\gamma^\mu \psi)(\partial^\rho \bar{\psi}\gamma^\mu \psi) - \frac{1}{2} D_V(\partial^\nu \bar{\psi}\gamma^\mu \gamma^\rho \psi)(\partial^\rho \bar{\psi}\gamma^\mu \psi)
\]

\[
\mathcal{L}_{\text{coul}} = -eA^\mu \bar{\psi} \left( \frac{1 + \gamma_3}{2} \right) \gamma_\mu \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}
\]

This Lagrangian is understood to be formally used in the mean-field approximation, with fluctuations encoded in density-dependent couplings \( G_i(\hat{\rho}) \).

\footnote{More details about density-dependent hadron field theory can be found in \cite{6}}
Their functional dependence will be determined from finite-density QCD sum rules and in-medium chiral perturbation theory

\[ G_i(\hat{\rho}) = G^{(0)}(\hat{\rho}) + G^{(\pi)}(\hat{\rho}), \] (8)

where the index \( i \) labels all isospin-Lorentz structures of Eq. (1). The variation of the Lagrangian with respect to \( \bar{\psi} \), leads to the single-nucleon Dirac equation

\[ \left[ \gamma^\mu (i \partial^\mu - V^\mu - V^\mu_R) - (M + S) \right] \psi = 0. \] (9)

In addition to the usual self-energies \( V^\mu \) and \( S \), the density-dependence of the vertex functions produces the rearrangement contribution

\[ V^\mu_R = u^\mu \left( \frac{1}{2} \frac{\partial G_S}{\partial \rho} \rho_s^2 + \frac{1}{2} \frac{\partial G_{TS}}{\partial \rho} \rho_s \bar{\rho}_s + \frac{1}{2} \frac{\partial G_V}{\partial \rho} j^\nu j^\nu + \frac{1}{2} \frac{\partial G_{TV}}{\partial \rho} \bar{j}^\nu \cdot j^\nu \right). \] (10)

The inclusion of this additional term ensures energy-momentum conservation and thermodynamical consistency. For a complete treatment of the density dependent point-coupling model, the reader is referred to 7.

3. Interactions

3.1. Self-energies from QCD sum rules

In leading order, which should be valid below and around saturated nuclear matter, the condensate part of the scalar self-energy

\[ \Sigma^{(0)}_S = -\frac{8\pi^2}{\Lambda_B^2} \left[ \langle \bar{q}q \rangle_{\rho} - \langle \bar{q}q \rangle_{\rho} \right] = -\frac{8\pi^2}{\Lambda_B^2} \frac{\sigma_N}{m_n + m_d} \rho_s, \] (11)

is expressed in terms of nucleon sigma-mass term \( \sigma_N = \langle \bar{N}m_q\bar{q}q|N \rangle \) and the quark masses. At the same order, the time-component of the vector self-energy reads

\[ \Sigma^{(0)}_V = \frac{64\pi^2}{3\Lambda_B^2} \langle q^\dagger q \rangle_{\rho} = \frac{32\pi^2}{\Lambda_B^2} \rho, \] (12)

where the quark baryon density is related to that of the nucleons by \( \langle q^\dagger q \rangle_{\rho} = \frac{4}{3} \rho \). In both cases, \( \Lambda_B \approx 1 \text{ GeV} \) is the characteristic scale (the Borel mass), which approximately separates the perturbative and non-perturbative energy domains.

\[ b \] the four-velocity \( u^\mu \) is defined as \( (1 - v^2)^{-1/2}(1, v) \) (\( v = 0 \) in the rest-frame of the nuclear system)
For typical values of the nucleon sigma mass term \( \sigma_N \) (\( \simeq 50 \) MeV) and \( m_u + m_d \) (\( \simeq 12 \) MeV at a renormalization scale of 1 GeV), the in-medium QCD sum rules predict large scalar and vector self energies of about equal magnitude (\( \simeq 300 - 400 \) MeV in agreement with relativistic phenomenological models), and opposite in sign. Neglecting corrections from higher order condensates, these estimates have large uncertainties and the error in the ratio \( \Sigma^{(0)}_S / \Sigma^{(0)}_V \simeq -1 \) is about 20%.

Given the self-energies arising from the condensate background, the corresponding equivalent point-coupling strengths \( G_{S,V}^{(0)} \) are simply determined by

\[
G_{S}^{(0)} = \frac{\Sigma_{S}^{(0)}}{\rho_s} \quad \text{and} \quad G_{V}^{(0)} = \frac{\Sigma_{V}^{(0)}}{\rho}.
\]

At leading order the condensate terms of the couplings are constant and do not contribute to the rearrangement self-energy.

3.2. Self-energies from in-medium chiral perturbation theory

In recent years the nuclear matter problem has been extensively studied in the framework of in-medium chiral perturbation theory. The calculations have been performed to three-loop order in the energy density and include one-pion exchange Fock term, one-pion iterated exchange and irreducible two-pion exchange terms with medium insertions and delta isobar excitations. By adjusting the coupling constants of few \( NN \) contact terms, and encoding short-range effects not resolved at relevant scales, to the properties of the empirical saturation point of isospin-symmetric nuclear matter \( (\tilde{E}_0 = -16 \text{ MeV} \text{ and } \rho_0 = 0.16 \text{ fm}^{-1}) \), several aspects of the problem have been successfully investigated: the symmetric and asymmetric nuclear matter equation of state (EOS), single-particle properties, the low-temperature behaviour, and the connection with non-relativistic nuclear energy density functionals.

The resulting nucleon self-energies are expressed as expansions in powers of the Fermi momentum \( k_f \). The expansion coefficients are functions of \( k_f / m_\pi \), the dimensionless ratio of the two relevant small scales:

\[
\Sigma^{CHPT} = f(k_f / m_\pi) \frac{k_f^3}{M_N^2} \quad \text{where} \quad f(k_f / m_\pi) = \sum_k c_k \left( \frac{k_f}{m_\pi} \right)^k.
\]

The density-dependence of the strength parameters is determined by equating the point-coupling self-energies in the single-nucleon Dirac equation (9).
with those calculated using in-medium chiral perturbation theory:

\[ G_S^{(\pi)} \rho_s = \Sigma^{CHPT}_S (k_f) \]  
\[ G_V^{(\pi)} \rho + V_R^{(\pi)} = \Sigma^{CHPT}_V (k_f) \]  
\[ G_{TS}^{(\pi)} \rho^3_s = \Sigma^{CHPT}_TS (k_f) \]  
\[ G_{TV}^{(\pi)} \rho^3 = \Sigma^{CHPT}_TV (k_f). \]

In Fig. (1) the resulting equation of state of isospin-symmetric nuclear matter is compared with the CHPT nuclear matter EOS. The agreement is satisfactory, and the small difference can be attributed to the approximations involved (i.e. the momentum dependence is neglected).

![Graph](image)

Figure 1. Binding energies for symmetric nuclear matter as a function of baryon density. The solid curve (CHPT) is the EOS calculated by using in-medium CHPT. The EOS displayed by the dashed curve (CHPT-mapping) is obtained when the CHPT nucleon self-energies are mapped on the self-energies of the relativistic point-coupling model with density-dependent couplings. The dotted curve denotes the latter EOS when rearrangement terms are neglected.

4. Results

In this section the relativistic point-coupling model will be applied to the description of ground-state properties of finite nuclei. The density-dependent coupling strengths will be determined in a least-squares fit to observables.

\( ^c \)for the definition of \( \rho_s, \rho_s, \rho_3, \rho_{3s} \) the reader is referred to.
(\mathcal{O}) of a rather large set of nuclei along the valley of \( \beta \)-stability (from \(^{16}\text{O}\) to \(^{214}\text{Pb}\)):

\[
\chi^2 = \sum_{\text{nuclei}} \left( \sum_{\text{observ.}} \frac{|\langle \mathcal{O}_{\text{th}} - \mathcal{O}_{\text{exp}} \rangle|^2}{\text{weights}} \right)
\]

(19)

In Fig. 2 the overall agreement is shown in comparison with two standard relativistic meson-exchange parametrization: NL3 \(^{12}\), with explicit higher order self-interaction terms for the \(\sigma\)-meson, and DD-ME1 \(^{13}\), with phenomenological density-dependent couplings strengths. The interaction determined by the \textit{least-squares fit} procedure contains pionic fluctuations on top of the large background fields. Nonetheless, it is interesting to analyze the role of pionic fluctuations, without the presence of background fields, and how these fields modify the single-particle spectra in finite systems. In Fig. 3 we display the single proton levels in \(^{40}\text{Ca}\) without and with background fields. It is obvious that the spin-orbit potential

\[
V_{s.o} \simeq \frac{1}{2M^2} \left( \frac{1}{r} \frac{\partial}{\partial r} (V^0(r) - S(r)) \right) 1 \cdot s
\]

(20)

represents a short-range effect (see also \(^{10}\)), determined by the condensate structure of QCD. The isospin-dependent part of the interaction has been...
studied in the description of the neutron radii. Standard relativistic mean-field calculations systematically overestimate the difference between neutron and proton radii. It turns out that the density dependence of the isovector channel of the interaction is crucial in order to reproduce these observables, as also shown in 13. In Fig. 4 we show the calculated values of $r_n - r_p$ for the Sn isotope chain in comparison with experimental data 14. The agreement is excellent, and it is a clear indication that the isovector channel can be successfully described by pionic fluctuations.
5. Conclusions

It has been demonstrated that an approach to nuclear dynamics constrained by the low-energy sector of QCD provides a quantitative description of properties of nuclear matter and finite nuclei.

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