STELLA R ORBITAL STUDIES IN NORMAL SPIRAL GALAXIES. I. RESTRICTIONS TO THE PITCH ANGLE

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ABSTRACT

We built a family of non-axisymmetric potential models for normal non-barred or weakly barred spiral galaxies as defined in the simplest classification of galaxies: the Hubble sequence. For this purpose, a three-dimensional self-gravitating model for the spiral arm PERLAS is superimposed on the galactic axisymmetric potentials. We analyze the stellar dynamics varying only the pitch angle of the spiral arms, from $4^\circ$ to $40^\circ$ for an Sa galaxy, from $8^\circ$ to $45^\circ$ for an Sb galaxy, and from $10^\circ$ to $60^\circ$ for an Sc galaxy. Self-consistency is indirectly tested through periodic orbit analysis and through density response studies for each morphological type. Based on ordered behavior, periodic orbit studies show that, for pitch angles up to approximately $15^\circ$, $18^\circ$, and $20^\circ$ for Sa, Sb, and Sc galaxies, respectively, the density response supports the spiral arms’ potential, a requisite for the existence of a long-lasting large-scale spiral structure. Beyond those limits, the density response tends to “avoid” the potential imposed by maintaining lower pitch angles in the density response; in that case, the spiral arms may be explained as transient features rather than long-lasting large-scale structures. In a second limit, from a phase-space orbital study based on chaotic behavior, we found that for pitch angles larger than $\sim 30^\circ$, $\sim 40^\circ$, and $\sim 50^\circ$ for Sa, Sb, and Sc galaxies, respectively, chaotic orbits dominate the all phase-space prograde region that surrounds the periodic orbits sculpting the spiral arms and even destroying them. This result seems to be in good agreement with observations of pitch angles in typical isolated normal spiral galaxies.

Key words: chaos – galaxies: evolution – galaxies: kinematics and dynamics – galaxies: spiral – galaxies: structure

Online-only material: color figures

1. INTRODUCTION

When Hubble (1926, 1936) introduced his classification scheme of galaxies, he emphasized that the turning-fork diagram he obtained depicted the systematic variation of the morphological characteristics, and the terms “early type” and “late type” referred only to the relative positions of a galaxy in this empirical sequence. The sequence did not imply temporal evolution connections. Some modifications have been introduced to this scheme over the years (de Vaucouleurs 1959; Sandage 1961). In the case of isolated galaxies, astronomers continue to search in this well-ordered sequence of galaxy types for a clue to possible formation and evolutionary processes to explain galactic morphology. It is not surprising then that morphology is frequently an underlying theme in the study of galaxies. A few percent of all galaxies are unclassifiable, and for many of these this is due to their unusual morphology produced by their interacting natures. In this paper, we focus on intrinsic dynamical processes of galaxies, which refers to relatively isolated galaxies.

Spiral galaxies are classified in the Hubble sequence based mainly on two criteria: the pitch angle and the bulge-to-disk luminosity ratio. On Sandage’s classification (Sandage 1975), three parameters to classify spiral galaxies are employed: the bulge-to-disk ratio, the pitch angle of the spiral arms, and the arm fragmentation into stars. These two classifications are very similar. Then, early-type spirals show small pitch angles with smooth structures and conspicuous central bulges. For late spirals, the arms are open and flocculent, and the central bulges are smaller. Finally, Sb galaxies are between the above two types.

Since the Hubble classification was introduced, astronomers have been puzzled and continue to search for correlations or dependences between galactic parameters and morphological types. Holmberg (1958) compiled and analyzed photometric data (integrated magnitudes, colors, and diameters) for hundreds of galaxies and found a correlation between Hubble type and galaxy colors. The mass ratio $M_{H}/M_{B}$ decreases with type from Sa to Sd (Sage 1993; Roberts & Haynes 1994). The total mass decreased as the Hubble type varied from Sa to Sc (Pišníc & Maupomé 1978; Maupomé et al. 1981; Roberts & Haynes 1994). Another interesting correlation is found in the maximum value of rotation velocity: values for Sa galaxies are higher than Sc galaxies (Rubin et al. 1985; Sandage 2000; Sofue & Rubin 2001), although the maximum rotation velocity presents a large scatter. Along the Hubble sequence, spiral galaxy disks tend to be thinner (Ma 2002). Some correlations are also found with the pitch angle, such as that related to the central supermassive black hole in spiral galaxies whose mass seems to decrease with the pitch angle (Seigar et al. 2008; Shields et al. 2010); spiral galaxies with higher rotational velocity also have tighter spirals (Kennicutt 1981; Savchenko & Reshetnikov 2011); in the same manner, it seems that for disks with lower surface densities and lower total mass–luminosity ratios, pitch angles tend to be larger (Ma 2002).

Studies specifically regarding spiral arms morphology began prior to the Hubble classification. Von der Pahlen (1911), Groot (1925), Danver (1942), and Kennicutt (1981) found that spiral arms in galaxies are well fitted with logarithmic helices. Kennicutt (1981) concluded that the pitch angle in early-type galaxies tends to be smaller than in late-type galaxies, but there is a large scatter in the pitch angle within each morphological type, concluding that the ideal Hubble classification is not closely followed by real galaxies (Kennicutt 1981; Ma et al. 2000; Davis et al. 2012).

In the Lin & Shu (1964) spiral density wave theory, galactic spiral arms are modeled as a periodic perturbation to the axisymmetric background disk’s potential. This is known as the tight-winding approximation (TWA) for small pitch angles...
or WK8 approximation (Wentzel–Kramers–Brillouin approximation of quantum mechanics; Binney & Tremaine 2008). The solution for bisymmetric spiral arms provided by the TW A takes the form

$$\Phi(R, \phi) = f(R) \cos[2\phi + g(R)],$$

(1)

where the function $f(R)$ is the amplitude of the perturbation, and $g(R)$ represents the geometry of the spiral pattern. The amplitude function $f(R)$ given by Contopoulos & Grosbøl (1986) and $g(R)$ of the form presented by Roberts et al. (1979) are commonly used. Most orbital studies on spiral galaxies theory have employed a spiral potential of the form in Equation (1). However, a majority of disk galaxies possess strong spiral structures, making clear that this large-scale structure is an important non-axisymmetric component of galaxies that deserves efforts to model it beyond a simple perturbing term. In this work, we have chosen a more physically and observationally motivated model of the spiral arms for disk galaxies, PERLAS from Pichardo et al. (2003), based on a three-dimensional model mass distribution, which allows a more detailed representation of spiral arms.

Concerning to large-scale structures in galaxies, these seem to be related mainly to ordered orbital behavior (Patsis 2008). This is the case, for example, of grand design galaxies, where non-axisymmetric structures are composed by material librating around the family of periodic orbits known as X1 (Contopoulos 2002). In spiral arms, where X1 periodic orbits dominate, the flow of material through the arms can be described as a “precessing ellipses” flow. This is supported by many observational and theoretical studies (Patsis et al. 1991). Spiral galaxies, with their thin disks and smooth spiral arms, may seem to be dominated by ordered motion, but deeper orbital studies show that chaos may be significant in spiral galaxies (Contopoulos 1983, 1995; Contopoulos et al. 1987; Grosbøl 2003; Voglis et al. 2006; Contopoulos & Patsis 2009; Patsis et al. 2009), and that chaotic orbits surrounding stable periodic orbits could also support the spiral arms. However, spiral arms and bars, for example, are not expected to originate from only chaotic orbits. Patsis & Kalapotharakos (2011) called ordered spirals those that have as a building block a set of stable periodic orbits; on the other hand, they call chaotic spirals those that they believe are constituted mainly from stars in chaotic motion.

Currently, discussion about the nature of spiral arms as a long-lasting or transient structure is ongoing in the field. Theoretical studies have demonstrated that spiral arms might be rather transient structures (Goldreich & Lynden-Bell 1965; Julian & Toomre 1966; Sellwood & Carlberg 1984; Foyle et al. 2011; Sellwood 2011; Pérez-Villegas et al. 2012; Fujii & Baba 2012; Kawata et al. 2012). However, if spiral arms are smooth, weak and/or with small pitch angles, the spiral structure could be long-lasting (Pérez-Villegas et al. 2012; that study refers to late-type spirals).

In a previous paper (Pérez-Villegas et al. 2012), we found two clear limits for the pitch angle in late (Sc) spiral galaxies, one that sets a maximum limit for which steady spiral arms are plausible (based on periodic orbital studies), beyond which a transient nature for the arms is proposed; the second limit is for which spiral arms are so open that orbital chaos dominates completely. In this paper, we extend our studies to early (Sa) and intermediate (Sb) galaxies to see if these limits found in late spirals are also followed by the remainder of spirals in the Hubble sequence and if the values of those limits match with observations. We isolate the effect of the pitch angle, which represents the least restricted parameter (i.e., the one with more spread values going from 4° to 50°), and because of that, one of the most interesting to study from a dynamical point of view. For this purpose we employ realistic values for the rest of the parameters that identify approximately typical Sa, Sb, and Sc galaxies. The rest of the structural parameters such as angular speed, perturbation strength (spiral arms mass), etc., are better restricted in the sense that the ranges for those values are tighter, measured both by observations and/or by self-consistent models. Specific calculations regarding these parameters (angular speed, spiral arms strength, and axisymmetric components) will be discussed in a forthcoming paper.

This paper is organized as follows. The galactic potential and methodology are described in Section 2. Our results, two pitch angle restrictions in galaxies, the first based on ordered orbital behavior, and the second based on chaotic behavior, are presented in Section 3. Finally, we present a discussion and our conclusions in Section 4.

2. METHODOLOGY AND NUMERICAL IMPLEMENTATION

We have constructed a family of models for the potential of normal spiral galaxies (Sa, Sb, and Sc), as classified by Hubble (1926), and also based on recent observational parameters taken from the literature. For this purpose, we numerically solved the equations of motion for stars in an axisymmetric potential plus a spiral arm potential. The main methods that we use in this work to study stellar dynamics are periodic orbital analysis and Poincaré diagrams.

2.1. Models for Normal Spiral Galaxies

The most common method used to model spiral arms is a two-dimensional bisymmetric local potential approximated by a cosine function (based on the solution for the TWA). It assumes that spiral arms are smooth, self-consistent perturbations to the axisymmetric potential. In this regime, self-consistency cannot be assured if pitch angles and/or amplitudes of the spiral arms slightly larger than the ones assumed for the TWA are imposed.

To model typical spiral arms in galaxies (i.e., pitch angles larger than about 6°, or mass of the spiral arms larger than 1% of the disk mass), one goes readily far away from the TWA self-consistency limits. Spiral arms in real galaxies are complicated three-dimensional gigantic structures, far too intricate to be approximated with a function as simple as a cosine. To test self-consistency in these types of potentials, we analyzed ordered and chaotic orbital behavior and specifically the construction of periodic orbits. Between a cosine potential and a potential based on recent observational parameters taken from the literature. For this purpose, we numerically solved the equations of motion for stars in an axisymmetric potential plus a spiral arm potential. The main methods that we use in this work to study stellar dynamics are periodic orbital analysis and Poincaré diagrams.

For this purpose, we employ the spiral arm potential PERLAS from Pichardo et al. (2003); this potential is formed by individual potentials of oblate inhomogeneous spheroids lying along the logarithmic spiral locus given by Roberts et al. (1979),

$$g(R) = -\left(\frac{2}{N \tan \theta} \right) \ln[1 + (R/R_s)^N],$$

(2)

with $\theta$ being the pitch angle. $R_s$ marks the galactocentric position where the spiral arms begin, and $N$ is a constant that gives the shape to the starting region of the spiral arms (we set it to 100; for details, see Pichardo et al. 2003).
This bisymmetric self-gravitating potential is more realistic since it is based on a three-dimensional density distribution, providing a much more complicated function for the gravitational potential, unlike a two-dimensional local arm such as the cosine potential. PERLAS is observationally motivated; comparisons with other theoretical models have been already published (Pichardo et al. 2003; Martos et al. 2004; Antoja et al. 2009, 2011). We approximately tested the self-consistency of the model through reinforcement by the stellar orbits (Patsis et al. 1991; Pichardo et al. 2003).

Parameters used to fit normal spiral galaxies (Sa, Sb, and Sc) with references are presented in Table 1. The spiral arm potential is superimposed on an axisymmetric background, described by a massive halo (Allen & Santillán 1991), and a Miyamoto & Nagai (1975) disk and bulge. We fitted the rotation curve based on observational data from the literature of the masses of the disk and bulge versus disk mass (depending on morphological type). With this information, we derived a halo mass using a typical maximum velocity in the rotation curve for each type. In Figure 1, we show the resulting rotation curves.

Theoretical studies of orbital self-consistency in normal spiral galaxies show that weak spiral arms end at corotation and strong spiral arms end at the \( 4/1 \) resonance (Contopoulos & Grosbøl 1986, 1988; Patsis et al. 1991). Then, the spiral arm limits depend on the position of the inner Lindblad resonance (ILR) for the inner part, and corotation for the outer part of the spirals (in our models we place the spiral arms between these two resonances). Consequently, the angular speed of the spiral arms, \( \Omega_p \), sets the corresponding value of the beginning and the end of these arms. Measuring spiral arm angular velocities observationally is not an easy task. We know, however, that on average, earlier type galaxies tend to be more massive, and therefore Sa galaxies rotate faster than Sc galaxies. On the other hand, considering the few observational studies of angular speeds from literature, these speeds seem to present a small range of difference from Sa to Sc, ranging between 35 and 15 km s\(^{-1}\) kpc\(^{-1}\). For the axisymmetric models we employed mass average values for the different components.
Table 1
Parameters of the Galactic Models

| Parameter                          | Value                      | Reference |
|-----------------------------------|----------------------------|-----------|
| Spiral arms                       |                            |           |
| Locus                             | Sa Sb Sc                  |           |
| Arms number                       | 2 2 2                     |           |
| Pitch angle (°)                   | 4–40 8–45 10–60           | 3, 7      |
| $M_{\text{arms}}/M_{\text{disk}}$ | 3%                        |           |
| Scale length (disk based, kpc)    | 7 5 3                     | 4, 5      |
| Pattern speed (km s$^{-1}$ kpc$^{-1}$) (clockwise) | $-30 -25 -20$ | 1, 6      |
| ILR position (kpc)                | 3.0 2.29 2.03             |           |
| Corotation position (CR) (kpc)    | 10.6 11.14 8.63           | ~ILR position based |
| Inner limit (kpc)                 | 3.0 2.29 2.03             | ~CR position based |
| Outer limit (kpc)                 | 10.6 11.14 8.63           |           |
| Axisymmetric components           |                            |           |
| $M_{\text{disk}}/M_{\text{halo}}$ | 0.07 0.09 0.1             | 4, 8      |
| $M_{\text{bulge}}/M_{\text{disk}}$ | 0.9 0.4 0.2              | 5, 8      |
| Rot. curve ($V_{\text{max}}$) (km s$^{-1}$) | 320 250 170 | 7         |
| $M_{\text{disk}}$ ($M_\odot$)     | $12.8 \times 10^{10}$ $12.14 \times 10^{10}$ $5.10 \times 10^{10}$ | 4         |
| $M_{\text{bulge}}$ ($M_\odot$)    | $11.6 \times 10^{10}$ $4.45 \times 10^{10}$ $1.02 \times 10^{10}$ $M_{\text{disk}}/M_{\text{halo}}$ based |
| $M_{\text{halo}}$ ($M_\odot$)     | $1.64 \times 10^{12}$ $1.25 \times 10^{12}$ $4.85 \times 10^{11}$ $M_{\text{disk}}/M_{\text{halo}}$ based |
| Disk scale length (kpc)           | 7 5 3                     | 4, 5      |
| Constants of the axisymmetric components$^b$ |                            |           |
| Bulge ($M_{\text{bulge}}, b_1$)  | 5000, 2.5 2094.82, 1.7 440, 1.0 |           |
| Disk ($M_{\text{disk}}, a_2, b_2$) | 5556.03, 7.0, 1.5 5232.75, 5.0, 1.0 2200, 5.3178, 0.25 |           |
| Halo ($M_{\text{halo}}, a_3$)    | 15000, 18.0 10000, 16.0 2800, 12.0 |           |

Notes.

$^a$ Up to 100 kpc halo radius.

$^b$ In galactic units, where a galactic mass unit = $2.32 \times 10^7 M_\odot$ and a galactic distance unit = kpc.

$^c$ $b_1, a_2, b_2,$ and $a_3$ are scale lengths.

References. (1) Grosbøl & Patsis 1998; (2) Drimmel 2000; Grosbøl et al. 2002; (3) Kennicutt 1981; (4) Pizagno et al. 2005; (5) Weinzirl et al. 2009; (6) Patsis et al. 1991; Grosbøl & Dottori 2009; Egusa et al. 2009; Fathi et al. 2009; (7) Brosche 1971; Ma et al. 2000; Sofue & Rubin 2001; (8) Block et al. 2002; (9) Pichardo et al. 2003.

Figure 3. $Q_T$ parameter (Equation (3)) for an Sa (solid line), Sb (dotted line), and Sc (dashed line) galaxy, where the pitch angles are $30^\circ, 40^\circ,$ and $50^\circ$ for an Sa, Sb, and Sc galaxy, respectively.

Figure 4. Maximum value, ($Q_T$)$_{\text{max}}$, of the parameter $Q_T(R)$ (maximum relative torques) vs. pitch angle of the spiral arms. The solid line gives ($Q_T$)$_{\text{max}}$ for an Sa galaxy, the dotted line for an Sb galaxy, and the dashed line for an Sc galaxy.
Figure 5. Periodic orbits, response maxima (filled squares), and the spiral locus (open squares) for the three-dimensional spiral model of an Sa galaxy (Table 1), with pitch angles ranging from $4^\circ$ to $30^\circ$.

(A color version of this figure is available in the online journal.)

With a clockwise rotation for the disk, we assume an angular velocity of $\Omega_p = -30, -25, \text{ and } -20 \text{ km s}^{-1} \text{ kpc}^{-1}$, for the spiral arms in Sa, Sb, and Sc galaxies, respectively (see Table 1 for references). In Figure 2, we show resonance diagrams for our galactic models. We take a fairly good approximation to the spiral arm mass, of 3% of the total disk mass (Pichardo et al. 2003), independently of the Hubble type. Additionally, to assure the spiral mass, we fixed to our models is within observational limits, we employed the parameter $Q_T$ (Combes & Sanders 1981). This parameter has been implemented in studies of bars and spiral arms (Buta & Block 2001; Laurikainen & Salo 2002; Buta et al. 2004; Laurikainen et al. 2004; Block et al. 2004; Vorobyov 2006; Kalapotharakos et al. 2010) to measure the strength of large-scale non-axisymmetric structures in galaxies. The parameter $Q_T$ is defined as

$$Q_T(R) = \frac{F_T^{\text{max}}(R)}{|\langle F_R(R) \rangle|},$$

where $F_T^{\text{max}}(R) = |\left( \frac{1}{R} \frac{\partial \Phi(R, \theta)}{\partial \theta} \right)|_{\text{max}}$ represents the maximum amplitude of the tangential force at radius $R$, and $\langle F_R(R) \rangle$ is the mean axisymmetric radial force at the same radius, derived from the $m = 0$ Fourier component of the gravitational potential. In Figure 3, we show an example of the behavior of the parameter $Q_T(R)$ given by Equation (3) for an Sa (solid line), Sb (dotted line), and Sc (dashed line) galaxies. In these three cases, we set the pitch angles to their maximum values permitted before chaos destroys all the periodic orbits’ support ($30^\circ$, $40^\circ$, and $50^\circ$ for Sa, Sb, and Sc galaxies, respectively). Note that even for cases with high pitch angles, the parameter $Q_T(R)$ is always lower than the observed maximum values in galaxies (Buta et al. 2005). In Figure 4, we present the maximum value of the parameter $Q_T$ for each type of galaxies as we increase the pitch angle from $0^\circ$ to $90^\circ$. Values up to 0.4 for the $Q_T$ parameter are consistent with observed spirals (Buta et al. 2005).

2.2. Orbital Analysis

For the orbital analysis we employed periodic orbits and Poincaré diagrams. With this extensive phase-space and configuration space orbital study, we are able to set two restrictions to one of the structural parameters of spiral arms: the pitch angle. The motion equations are solved in the non-inertial reference system of the spiral arms and in Cartesian coordinates ($x'$, $y'$, $z'$).
2.2.1. Periodic Orbits

Periodic orbits represent the simplest orbits of potentials in general. They are also the most important orbits because these are followed by sets of non-periodic orbits, and even followed by quasiperiodic orbits, librating around the periodic ones (i.e., forming tubes surrounding the periodic orbits). In self-consistent systems, periodic orbits support large-scale structures, such as bars and spiral arms, and are known as the “dynamic backbone” of potentials.

We computed between 40 and 60 periodic orbits for each Poincaré diagram using the Newton–Raphson method. As a first guess for the initial conditions in the calculation of the periodic orbit, the code provides the periodic orbit at a given radius in the background axisymmetric potential. The orbits are launched from \( y' = 0 \), on the \( x' \)-axis \( > 0 \) with \( v'_{x} = 0 \) and \( v'_{y} = v_{c} \), where \( v_{c} \) is the corresponding circular velocity given by

\[
v_{c} = \left( \frac{\sigma_{0}}{\sqrt{|\frac{d\Phi_{0}}{dx'}|}} \right)^{1/2},
\]

and \( \Phi_{0} \) is the axisymmetric potential.

2.2.2. Density Response

To calculate the density response to a spiral potential, we employ the method of Contopoulos & Grosbøl (1986) that quantifies the support with periodic orbits to the arms. The method assumes that stars in circular orbits in an axisymmetric potential, rotating in the same direction of the spiral perturbation, will be trapped around the corresponding periodic orbit in the presence of the spiral arms. To this purpose, we computed a large set of periodic orbits and calculated the density response along their extension using the conservation of mass flux between any two successive orbits. With this information we search the position of the density response maxima along each periodic orbit. The locus of the obtained positions is compared with the imposed locus of PERLAS.

We calculated the average density response around each one of these response maxima, taking a circular vicinity with a
radius of 500 pc. We then compared the density response with the imposed density. The imposed density is the sum of the axisymmetric disk density on the galactic plane and the central density of the spiral arms.

2.2.3. Poincaré Diagrams

In Poincaré diagrams, ordered orbits appear as invariant one-dimensional curves; periodic orbits, on the other hand, draw a finite set of dots. In phase-space diagrams, chaotic orbits appear as scattered sets of dots.

In the non-inertial frame, the effective potential on the galactic plane is given by

$$\Phi_{\text{eff}}(x', y') = \Phi_0 + \Phi_{sp}(x', y') - \frac{1}{2} \Omega_p^2 (x'^2 + y'^2),$$

where $$\Phi_0$$ is the axisymmetric potential, $$\Phi_{sp}$$ is the potential of the spiral arms, and $$\Omega_p$$ is its angular velocity. The only known analytical integral of stellar motion in the non-inertial system of reference is the Jacobi constant, given by

$$E_J = \frac{1}{2} v'^2 + \Phi_{\text{eff}},$$

Figure 7. Periodic orbits, response maxima (filled squares), and the spiral locus (open squares) for the three-dimensional spiral model of an Sc galaxy (Table 1), with pitch angles ranging from 10° to 60°.

(A color version of this figure is available in the online journal.)

Figure 8. $$\Delta_{\text{imp-resp}}$$ is the phase angular difference between the density response and the imposed spiral potential. The solid line shows $$\Delta_{\text{imp-resp}}$$ for an Sa galaxy, the dotted line corresponds to an Sb galaxy, and the dashed line to an Sc galaxy.
Figure 9. Filled squares represent the density response of the spiral arms for an Sa galaxy, and open squares represent the imposed density, with pitch angles ranging from 4° to 30°. The dotted, dashed, and dot-dashed lines show the inner Linblad resonance (ILR) position, 4/1 resonance position, and corotation resonance (CR) position, respectively.

(A color version of this figure is available in the online journal.)

where \( v' \) is the star velocity. Poincaré diagrams are constructed following the usual procedure. They present two regions, each one containing 50 orbits with 300 points each (corresponding to the number of periods), with a given prograde or retrograde sense of rotation, defined in the galactic non-inertial frame. In our models, the spiral patterns move in the clockwise sense. Therefore, the right side of the diagram (launching orbits with \( x' > 0, \ y' > 0 \)) is the retrograde region, while the left side (with \( x' < 0, \ y' > 0 \)) is the prograde region.

3. RESULTS

We carried out an extensive orbital study with periodic orbital analysis, density response, and studies in the phase space to determine whether limit values to different structural parameters of normal spiral galaxies can be established.

We produced axisymmetric potential models to simulate typical Sa, Sb, and Sc spiral galaxies, and superposed a spiral arms potential (PERLAS) to study the stellar orbital behavior in the disk, as we changed the pitch angle for each galaxy type. Normal spiral galaxies present a wide scatter in the pitch angle, ranging from \( \sim 4° \) to \( 50° \). We found two restrictions for the pitch angle in normal spiral galaxies, the first based on ordered behavior and the second based on chaotic behavior.

3.1. Pitch Angle Restriction Based on Ordered Motion: Long-lasting or Transient Spiral Arms

We estimated the self-consistency of each one of our models through the construction of periodic orbits. The existence of periodic orbits makes it more likely in steady conservative potentials, the support to long-lasting large-scale structures. We present a periodic orbital study for each morphological type (see Figures 5–7). In addition, we search the position of the density response maxima along each periodic orbit. We compared these positions with the center of the imposed spiral arms. Figures 5–7 show the response maxima as filled squares, where the orbits crowd producing a density enhancement, and the imposed spiral pattern are represented by open squares. In these figures, we present periodic orbits for Sa, Sb, and Sc galaxies (Figures 5, 6, and 7, respectively). We used the same axisymmetric background potential for each morphological type, based on the parameters presented in Table 1. The pitch angle in these figures ranging from \( 4° \) to \( 40° \) for an Sa galaxy, from \( 8° \) to \( 45° \) for an Sb galaxy, and from \( 10° \) to \( 60° \) for an Sc galaxy.
In Figure 5, we see that for smaller pitch angles ($i \lesssim 15^\circ$), the density response maxima coincides with the imposed spiral arm potential. This means that the filled squares in the figure, which represent the places in the arm where stars would crowd for long times, settle down along the locus of the imposed spiral, making the existence of stable long-lasting spiral arms more likely. On the other hand, for spiral arms with pitch angles larger than $\sim 15^\circ$, the response maxima systematically lag behind the imposed spiral arm potential, i.e., the pitch angles corresponding to the density response are smaller than the imposed ones.

Figure 6 shows similar behavior than the one for Sa galaxies, but in this case, the pitch angle range is slightly wider. For smaller pitch angles ($i \lesssim 18^\circ$), the density response closely supports the imposed spiral arms. For $i > 18^\circ$, the density response produces spiral arms with smaller pitch angles than the imposed locus, avoiding the support to the spiral structure.

Figure 7 shows a similar behavior than Sa and Sb galaxies, but in this case, the pitch angle range is wider. For smaller pitch angles ($i \lesssim 20^\circ$), the density response seems to support the imposed spiral arms. For $i > 20^\circ$, the density response produces spiral arms with smaller pitch angles than the imposed locus, avoiding to support the spiral structure.

In the three types of galaxies, with pitch angles beyond $15^\circ$, $18^\circ$, and $20^\circ$ for Sa, Sb, and Sc galaxies, respectively, the density response seems to “avoid” open long-lasting spiral arms. Long-lasting spiral arms are no longer supported after these limits; in these cases, spiral arms may be rather transient structures. In Figure 8, we show a plot of phase angular difference between the imposed spiral potential and the density response versus pitch angle of the imposed spiral arms.

An additional method used to complement and to reinforce the results given by periodic orbits is the comparison of the spiral arm density response (filled squares in Figures 9–11) with the imposed density (open squares in Figures 9–11). In Figure 9, we present densities (the spiral arms density response and the spiral arms imposed density, i.e., PERLAS) for an Sa galaxy. As the density response maxima in the previous diagrams, this figure shows that for pitch angles up to $\sim 15^\circ$, the density response fits well with the imposed density. In Figure 10, we present densities for an Sb galaxy. In this figure, we see almost the same behavior than in Figure 9, but in this case, the pitch angle limit, where the density response does not fit the imposed arm, is $\sim 18^\circ$. Finally, in Figure 11, we present the same diagrams for an Sc galaxy. Here, the behavior is very similar to the Sa and Sb cases, but the
Figure 11. Filled squares represent the density response of the spiral arms for an Sc galaxy, and open squares represent the imposed density, with pitch angles ranging from 10° to 60°. The dotted, dashed, and dot-dashed lines show the inner Lindblad resonance (ILR) position, 4/1 resonance position, and corotation resonance (CR) position, respectively.

(A color version of this figure is available in the online journal.)

limit for the pitch angle, where the density response is fitted to the imposed density, is ~20°.

With periodic orbits, maxima density response, and comparing the density response and imposed density, in order to obtain support to long-lasting spiral arms, the pitch angle should be smaller than ~15°, 18°, and 20° for Sa, Sb, and Sc galaxies, respectively. However, spiral arms evidently exist with larger pitch angles in galaxies, and we propose that these are rather in a transient form.

3.2. Pitch Angle Restriction Based on Chaotic Motion

Confined chaotic orbits are able to support large-scale structures such as spiral arms (Patsis & Kalapotharakos 2011; Kaufmann & Contopoulos 1996; Contopoulos & Grosbol 1986). However, grand design structures are not expected to arise from systems where chaos fully dominates (Voglis et al. 2006). In this study, we found a restriction based on chaotic behavior. To do this, we produced an extensive study of the Jacobi energy families in the phase space, as a function of the pitch angle in normal spiral galaxies. In this section, we show that there is a limit to the pitch angle, for which chaos becomes pervasive destroying all periodic orbits and the ordered orbits surrounding them in the relevant spiral arm region.

Here we present a set of phase-space diagrams for each morphological type (Figures 12–14). As in the study of periodic orbits (Section 3.1), we employed an axisymmetric background potential for an Sa, Sb, and Sc galaxy, based on the parameters given in Table 1. In these experiments we only varied the pitch angles. Each mosaic has 20 panels that show phase-space diagrams with different Jacobi constant families ranging from $E_J = -4100$ to $-3275 \times 10^2$ km$^2$ s$^{-2}$ for an Sa galaxy (where $E_J = -3290$, $-2550$, $-2442 \times 10^2$ km$^2$ s$^{-2}$ correspond approximately to the positions of ILR, 4/1, and CR resonances, respectively), from $E_J = -3275 \times 10^2$ km$^2$ s$^{-2}$ to $-3150 \times 10^2$ km$^2$ s$^{-2}$ for an Sb galaxy (where $E_J = -3290$, $-2550$, $-2442 \times 10^2$ km$^2$ s$^{-2}$ correspond approximately to the positions of ILR, 4/1, and CR resonances, respectively), and from $E_J = -1080$ to $-1021 \times 10^2$ km$^2$ s$^{-2}$ for an Sc galaxy (where $E_J = -1280$, $-1075$, $-1022 \times 10^2$ km$^2$ s$^{-2}$ correspond approximately to the positions of ILR, 4/1, and CR resonances, respectively), covering the total extension of spiral arms.

Figure 12 shows Poincaré diagrams for an Sa galaxy, going from 4° (top line of diagrams) to 30° (bottom line of diagrams).
For pitch angles between 4° and 10° (first two lines of diagrams), the ordered orbits are dominating and simple, periodic orbits support spiral arms up to corotation, approximately. The onset of chaos is clear at about 10°. As a function of the pitch angle, for 19° (third line of diagrams), the orbital behavior is more complex, and it presents resonant islands. For 30° (bottom line of diagrams), the chaotic region covers most of the regular prograde orbits. For pitch angles beyond ~30°, chaos destroys periodic orbits.

Figure 12. Phase-space diagrams for an Sa galaxy, with $E_J = [-4050, -3278]$, in units of $10^2 \text{ km}^2 \text{s}^{-2}$. From top to bottom, pitch angles go from 4° to 30°.

For pitch angles between 12° and 40° (first two lines of diagrams), the ordered orbits dominate. As we increase the pitch angles, at 12° (first line of diagrams), the orbital behavior is mainly ordered, but close to corotation there is already a small chaotic region. For 21° (second line of diagrams), the onset of chaos is clear, and there is a variety of complicate orbits. For 30° (third line of diagrams), the chaotic region increases, and the orbital structure becomes much more complex. For 40° (bottom line of diagrams), the chaotic region covers almost all regular prograde orbits. For pitch angles beyond ~40°, chaos destroys periodic orbits.

In Figure 13, we present Poincaré diagrams for an Sb galaxy, going from 12° (top line of diagrams) to 40° (bottom line of diagrams). For pitch angles of 10° or less, the ordered orbits dominate. As we increase the pitch angles, at 12° (first line of diagrams), the orbital behavior is mainly ordered, but close to corotation there is already a small chaotic region. For 21° (second line of diagrams), the onset of chaos is clear, and there is a variety of complicate orbits. For 30° (third line of diagrams), the chaotic region increases, and the orbital structure becomes much more complex. For 40° (bottom line of diagrams), the chaotic region covers almost all regular prograde orbits. For pitch angles beyond ~40°, chaos destroys periodic orbits.

Figure 13 shows Poincaré diagrams for an Sb galaxy, going from 12° (top line of diagrams) to 40° (bottom line of diagrams). For pitch angles of 10° or less, the ordered orbits dominate. As we increase the pitch angles, at 12° (first line of diagrams), the orbital behavior is mainly ordered, but close to corotation there is already a small chaotic region. For 21° (second line of diagrams), the onset of chaos is clear, and there is a variety of complicate orbits. For 30° (third line of diagrams), the chaotic region increases, and the orbital structure becomes much more complex. For 40° (bottom line of diagrams), the chaotic region covers almost all regular prograde orbits. For pitch angles beyond ~40°, chaos destroys periodic orbits.

The onset of chaos begins in the prograde region of Poincaré diagrams. The main cause for chaotic motion has been attributed to resonance interactions (Martinet 1974; Athanassoula et al. 1983; Pichardo et al. 2003). In the case of retrograde orbital regions, resonances are more widely separated than in the prograde resonances case, which may explain why the onset and well-developed chaos takes place in the prograde region first (and, in general, only). However, it is worth noting that in our experiments we have maintained the same pattern angular speeds, which places resonances at the same radii, increasing only pitch angles, and we have noted that chaos (not originated only by the resonances position) increases dramatically. This means that at a given position of the resonances (posed by the
Figure 13. Phase-space diagrams for an Sb galaxy, with $E_J = [-3150, -2445]$, in units of $10^2$ km$^2$ s$^{-2}$. From top to bottom, pitch angles go from $12^\circ$ to $40^\circ$.

angular speeds), increasing pitch angles will widen the chaotic regions.

4. DISCUSSION AND CONCLUSIONS

We produced a set of models for observationally motivated potentials to simulate typical Sa, Sb, and Sc spiral galaxies with three-dimensional bisymmetric spiral arms. Observed galaxies, classified as Sa, Sb, or Sc, present a wide scatter in pitch angles going from $\sim 4^\circ$ to $25^\circ$ for Sa galaxies, from $\sim 8^\circ$ to $35^\circ$ for Sb galaxies, and from $\sim 10^\circ$ to $50^\circ$ for Sc galaxies. With our models, we extensively studied the stellar dynamical effects of the spiral arm pitch angle on the plane of the disk, ranging from $4^\circ$ to $40^\circ$, for an Sa galaxy, from $8^\circ$ to $45^\circ$, for an Sb galaxy, and from $10^\circ$ to $60^\circ$, for an Sc galaxy.

We found two important restrictions to the pitch angle. The first restriction is based on the orbital ordered behavior. With the study of periodic orbits and density response, we found that there is an abrupt limit for the density response at approximately $15^\circ$ for Sa galaxies, $18^\circ$ for Sb galaxies, and $20^\circ$ for Sc galaxies. This limit denotes the end of orbital support of the density response to the imposed spiral arm potential. In cases where the spiral arm potential is followed by their density response produced by periodic orbital crowding, the spiral arms are more stable and could be explained better as a long-lasting feature. Beyond these limits, the density response produces systematically smaller pitch angles than the imposed spiral arms. Galaxies with spiral arms beyond these limits would rather be explained as transient features.

We found a second restriction, this time based on chaotic orbital behavior. In this case, we find a limit for the very same existence of spiral arms for each morphological type: for an Sa, the limit is $\sim 30^\circ$, for an Sb, the limit is $\sim 40^\circ$, and for an Sc, the limit is $\sim 50^\circ$, for which spiral arms are so open that chaos dominates large regions of phase space, but periodic orbits supporting spiral arms exist. Beyond these limits for each type of galaxies, chaos becomes pervasive destroying all orbital support.

With this orbital study, we are able to pose both a limit for steady long-lasting spiral arms, beyond which spiral arms are better explained as transient features and a limit for maximum pitch angles (no matter their nature) in normal spiral galaxies before the system becomes completely chaotic.

Several structural and dynamical parameters may play an important role in the stellar dynamics behavior, such as the pattern speed, the strength of the spiral arms (spiral arms mass), the density decay along the spiral arms, the axisymmetric components mass ratios, etc. In this paper, we isolated the effect of the pitch angle on stellar dynamics to understand at what extent we could impose restrictions to its properties in models for spiral galaxies based on order and chaos. For this purpose we employ realistic values for the rest of the parameters that identify approximately typical Sa, Sb, and Sc galaxies. A comprehensive
study for other parameters will be presented in a forthcoming paper. Here we provide preliminary results on those studies for three of the relevant parameters within the observational or theoretical values for spiral galaxies: pattern speed, spiral arm mass, and density decay of the spiral arms. For the pattern speed, although the construction and radial extension of periodic orbits (length of the spiral arms) depends sensibly on the pattern speed, the existence of the periodic orbits supporting the spiral structure inside the corotation resonance (or 4/1 resonance) do not seem to be too sensitive, within the observed (20–30 km s\(^{-1}\) kpc\(^{-1}\)) limits for real spiral galaxies. This means that, within the observed limits for spiral angular speeds in galaxies, the effect on the restriction of the angular speed on whether spiral arms are transient or long-lived structures seems to be much less important. The spiral-to-disk mass ratio, as well as the angular speed, is a very restricted quantity by observations and theory. Translating the amplitudes or relative forcing to spiral arms mass, the masses to obtain self-consistent models go up to 5% or 6% of the mass of the disk. As long as we change the spiral arm mass within these limits, the restriction to the pitch angle for the spiral arms to be supported by periodic orbits is only slightly sensitive. Regarding the density fall of the spiral arms, we studied several types of both lineal decay and exponential decay. This specific parameter results of little consequence as long as the linear decay slope goes approximately similar to the average slope of the exponential fall in the first half of the spiral arm extent. In a future work, we will present a detailed stellar dynamical study with these and other parameters of spiral galaxies.

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