Continuous wavelet transform based time-scale and multi-fractal analysis of the nonlinear oscillations in a hollow cathode glow discharge plasma

Md. Nurujjaman, Ramesh Narayanan and A.N. Sekar Iyengar
Plasma Physics Division, Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Kolkata - 700064, India

Continuous wavelet transform (CWT) based time-scale and multi-fractal analyses have been carried out on the anode glow related nonlinear floating potential fluctuations in a hollow cathode glow discharge plasma. CWT has been used to obtain the contour and ridge plots. Scale shift (or inversely frequency shift) which is a typical nonlinear behaviour, has been detected from the undulating contours. From the ridge plots, we have identified the presence of nonlinearity and degree of chaoticity. Using the wavelet transform modulus maxima technique we have obtained the multi-fractal spectrum for the fluctuations at different discharge voltages and the spectrum was observed to become a monofractal for periodic signals. These multi-fractal spectra were also used to estimate different quantities like the correlation and fractal dimension, degree of multi-fractality and complexity parameters. These estimations have been found to be consistent with the nonlinear time series analysis.

I. INTRODUCTION

Glow discharge plasma is a typical complex medium exhibiting a wide variety of nonlinear phenomena such as chaos, noise induced resonances, self organized criticality, frequency entrainment, complex structures, etc. [1-2, 3, 4, 5, 6, 7], which have been studied using a wide variety of analysis methods, such as the nonlinear time series analysis, spectral analysis, etc. Nonlinear time series analysis like the estimation of the correlation dimension, fractal dimension and Lyapunov exponents reflect only the asymptotic and global behavior of the system but cannot identify the degree of chaoticity, local structures, etc. These estimations are also too simplistic to characterize a time series, because the estimation emphasizes regions of the attractor of a time series which is frequently visited by its orbit in the phase space [8, 9, 10, 11, 12]. Further, the estimation of these exponents require very large data sets, and degrade rapidly with noise [8, 13].

Spectral analysis (based on Fourier techniques) has also been used extensively to identify the different routes to chaos, frequency entrainment, and many other nonlinear phenomena in plasma [11, 12, 13, 14]. However, the spectral analysis has a limitation of yielding only the spectral amplitude, neglecting the phase information, as a result of which it can not distinguish between any two time series of the same spectral amplitude [10]. Since the short time scale characteristics of a signal are also overlooked, information on the time history and the local properties like singularity exponents and the scale shift (or inversely frequency shift) are lost [10, 11, 14]. Hence one has to resort to a time-scale based analysis consistent with the nonlinear time series analysis, that can detect the presence of chaos, nonlinearity and other local structures in the system and for this purpose continuous wavelet transform (CWT) is one of the best candidates.

CWT can be used to construct contours, which in turn are useful to study some of the nonlinear properties like the scale shift as well as periodic behaviour. Characterization of the weak and strong chaos is possible by ridge plots constructed from the CWT decomposition of a chaotic time series [11]. Statistical scaling properties of a complex signals can be characterized by the CWT based multi-fractal spectra and these spectra can also be used to estimate several exponents like the fractal dimension, correlation dimension, etc., that are useful to characterize nonlinear and chaotic nature of a time series [16, 17, 18, 19, 20, 21, 22, 23].

In this paper, we have reported some results of the analysis of the floating potential fluctuations in a dc glow discharge plasma using contour and ridge plots based on the CWT decomposition. We have also constructed the multi-fractal spectra using the wavelet transform modulus maxima and from these spectra, the fractal dimension, correlation dimension and other complexity parameters have been estimated. The rest of the article is organized as follows: we present a brief description of the experimental setup and results in Sections [11] and [11], respectively. Analysis of these results have been presented in Sections [11] and [11] respectively. Finally, we discuss and conclude the results in Section [11].

II. EXPERIMENTAL SETUP

The experiments were performed in a hollow cathode dc argon discharge plasma [11] whose schematic diagram is shown in Fig 1. The hollow stainless steel tube of diameter 4 cm was the cathode and the central rod of diameter 2 mm was the anode. The whole assembly

---

*Electronic address: jaman.nonlinear@yahoo.co.in, Present address: TIFR Centre For Applicable Mathematics, Sharda Nagar, Chikkabommasandra, Bangalore - 560065, India.
†Electronic address: rams@plasma.inpe.br, Present address: Laboratorio Associado de Plasma, Instituto Nacional de Pesquisas Espaciais, Av. dos Astronautas, 1758 - Jardim da Granja 12227-010 Sao Jose dos Campos, SP, Brazil.
‡Electronic address: ansekar.iyengar@saha.ac.in
was mounted inside a vacuum chamber and was pumped down to 0.001 mbar. Then it was filled with argon gas up to 0.95 m bar and a discharge was struck by a dc discharge voltage. The main diagnostics was the Langmuir probe used to monitor the floating potential fluctuations. The typical plasma density was about $10^7 \text{cm}^{-3}$ and the electron temperature estimated was about 3 – 4 eV.

### III. FLUCTUATION RESULTS

The formation of the plasma depends on two main parameters: the discharge voltage (DV) and the neutral gas pressure. An anode glow was formed around the anode when a discharge was struck above a threshold neutral pressure and DV. It was observed that at the initial stage of the discharge, the anode glow was large in size and decreased with increase in DV, until it got extinguished beyond a certain DV \[1\]. At the instant of formation of the anode glow, the associated fluctuations in the floating potential were chaotic which became quasi-periodic with increase in the DV and finally vanished to a fixed point when the anode glow disappeared. At this time the plasma inside the hollow cathode became brighter and filled up the entire plasma column. Details of the anode glow and the associated instabilities have been discussed in Ref \[1\]. For the present paper, the floating potential fluctuations were obtained keeping pressure constant at 0.95 m bar. At this pressure the discharge was initiated at $\approx 283$ V and simultaneously irregular oscillations in the floating potential were observed [Fig 2(a) (left panel)]. With increasing DV, the regularity of the oscillations increases and final form of the fluctuations just before their disappearance was the relaxation type of oscillation as shown in Fig 2(h–i) [left panel].

### IV. CWT ANALYSIS OF THE FLUCTUATIONS

The continuous wavelet transform (CWT) is used to decompose a signal using wavelets, i.e., into small oscillations that are highly localized in time and for a signal $\phi(t)$ it is defined as \[22\]:

$$W_\Psi(a, \tau) = \int \phi(t) \Psi_a(t - \tau) dt \quad (1)$$

where, $\phi(t)$ is the signal and $\Psi_a(t)$ is an oscillating function that decays rapidly with time and is termed as wavelet. $a$ and $\tau$ are the scale and temporal propagation parameters respectively. Scale ‘$a$’ can be considered to be the inverse of frequency and so one can estimate the frequency ($F_a$) corresponding to a scale ‘$a$’ for a particular wavelet using the relation $F_a = \frac{c}{2a}$ \[24\], where $F_c$ is the center frequency of the analyzing wavelet and $\Delta$ is the sampling period of the signal. So a signal can be decomposed in time-scale plane or time-frequency plane using above scale-frequency relation. In this paper, we have used the time-scale decomposition using the Daubechies orthogonal wavelet to detect the singularities present in the time series as it has a slight asymmetric structure, uses few coefficients and is a good representation of low-order polynomials \[25, 26\]. Fig 2(a’–i’) shows the contours of $W_\Psi^2(a, \tau)$ for the signals shown in Figs 2(a–i), in the time-scale plane. Though one can see that the oscillations go through different types of behaviour with increasing DV, changes are much more clearer in the CWT contour plots shown in Fig 2 (right panel). Undulating nature of the contours in Fig 2(a’–b’) and Fig 2(f’–g’) is due to the scale shift with time which is one of the signatures of the presence of nonlinearity in the system \[27\]. Periodic behaviour of the fluctuating oscillations of Fig 2(e–e) is also prominent from the periodic contours [Fig 2(c’–c’)]. Though the contour plots of the relaxation oscillations [Fig 2(h–i)] have no such distinct structures, still one can find differences from the other two cases. Importance of the CWT based contour plots is that they can be used to identify the nonlinear nature of a system, and this identification is very essential for the nonlinear time series analysis. The CWT coefficients ($W_\Psi(a, \tau)$) are also useful to devise another plot termed as a ridge plot \[11\] which can be exploited to analyze chaotic data, and this has been presented in the next section.

### V. RIDGE ANALYSIS

The line joining the maximum of the CWT coefficients along the scales ‘$a$’ is called a ridge plot which can be used to detect the presence of chaos and degree of chaoticity or periodicity in a system \[11\]. Here, we have considered only the prominent ridges to construct the plots. In Fig 3(a) the presence of the broken ridges at the higher scale of 200 and also at the lower scale of about 30 indicates that at the initial stage of DV (283 V) the system
very chaotic [11]. At 284 V [Fig 3(b)] the system is still chaotic but the discontinuous ridges are seen only at the lower scales and chaos is not so strong and these results are consistent with the nonlinear time series analysis [1]. With increase in the DV, the fluctuations [Fig 3(c–e)] become almost regular and their ridges are almost constant with time. The ridge plots of the fluctuations shown in Figs 2(f) and (g) also show broken ridges indicating reappearance of the chaos. Appearance of a chaotic window after the periodic case is always possible and is consistent with previous analysis presented in Ref [1]. Fig 3(h) shows discontinuous ridges but of slightly longer durations and finally Fig 3(i) shows a continuous ridge indicating periodic signals with breaks at the fall phase of the relaxation oscillations at the lower scale of about 300. Here it also shifts between two scales one at the top and the other at the bottom indicating the nonlinear frequency shift.

VI. MULTIFRACTAL SPECTRUM

In order to understand the scaling properties of the signals we have constructed multi-fractal spectrum using the wavelet coefficients of the CWT decomposition. The multi-fractal spectrum ($D(\alpha)$) of a signal can be estimated using the wavelet transform modulus maxima method (WTMM) [20, 21, 22, 28] as follows:

The partition function $Z(a, q)$ of moment $q$ for a particular scale ‘$a$’ is defined as a sum of the wavelet transform modulus maxima at that particular scale is given as

$$Z(a, q) = \sum_{\{t_i(a)\}} |W_\Psi(a, t_i(a))|^q \sim a^{\tau(q)} \quad (2)$$

where, $W_\Psi(a, t_i(a))$ is the wavelet modulus maxima and $q$ is the moment considered. From $\tau(q)$ we can estimate the singularity spectrum using following relations [22]

$$\alpha = \frac{d\tau(q)}{dq} - \frac{1}{2}, \quad (3a)$$

$$D(\alpha) = \min_{q \in \mathbb{N}} q((\alpha + 1/2) - \tau(q)), \quad (3b)$$

We have estimated the multi-fractal spectra for all the nine signals shown in Fig 2(left panel) at different DV. In Fig 4 black dots (−•−) show the typical multi-fractal spectra ($D(\alpha)$ vs $\alpha$) at 283V for the initial stage of discharge and uptriangle (−Δ−) for the fluctuations at 292 V just before the stable state. These plots show that the signals at the initial stage of discharge is multi-fractal in nature and tends to become a mono-fractal with increase in DV for quasi-periodic signal as the width and height of the spectrum have decreased considerably. For the periodic signals [Figs 2(d, e and i)], we obtained a mono-fractal behaviour.

The correlation dimension ($D_{corr}$) has been estimated from their multi-fractal spectra using the relation $D_{corr} = 2 - \lambda$.
FIG. 3: Ridge plots along the scales of the contour plots shown in Fig. 2 (right panel).

FIG. 4: •— shows the multi-fractal spectra at the initial stage of discharge (283 V). △— at 292 V, i.e., just before the steady fixed point is reached and mono fractal in nature.

\[ 2\alpha_q = 2 - f(\alpha_q = 2) \] has been shown in Fig 5(a) by black dots (•—). It is seen that the \( D_{corr} \) is high for the initial stage of discharge and decreases with DV except for 290 V and 293 V. The increase in \( D_{corr} \) at 290 V is probably due to the appearance of a chaotic feature at an intermediate stage of the periodic signal.

For comparison, \( D_{corr} \) using Grassberger-Procaccia algorithm [1], has been shown in the same plot (open circle) and both exhibit a good agreement. There are slight discrepancies observed at 283 V, 286 V and 293 V. The discrepancies at the first two values can be possibility a result of the presence of modes at significantly different scales. This would mean that since wavelet transform is based on the multi-resolution analysis of varying frequency and time resolutions at differing scales, the WTMM estimates joining maxima at different scales might incur errors in the estimated values of \( D_{corr} \). However, the general trend of decrease in \( D_{corr} \), from the two techniques, is quite similar in nature. The increase in \( D_{corr} \) at 293 V may be due to nonlinear nature of the relaxation oscillations. The degree of multi-fractality (\( \beta \)) defined as the difference between the maximum and minimum values of \( \alpha \) and which also statistically gives the range of scale invariance [29] is shown in Fig 5(c).

Since \( \beta \) is also a measure of complexity, it is seen that the complexity [Fig 5(a)] decreases with increase in DV.

Fractal dimensions, i.e., \( f(\alpha_q = 0) \) of the singularity support of the signals at different the DV have been shown in Fig 5(c) and they decrease with increase in DV. Finally we have estimated another complexity parameter (\( P_c \)) which is high for the complex signal [30], and is defined as \( P_c = \frac{h_{max}}{f(\alpha_q = 0)} H_{FWHM} \), where, \( h_{max} \) is the singularity exponent at the fractal dimension \( f(\alpha_q = 0) \) and \( H_{FWHM} \) is the full width at half maxima of the spectrum. Fig 5(d) shows the behavior of \( P_c \) at different DV. At the initial stage of the discharge \( P_c \) is high and decreases at 288 V and 289 V, which corroborates well with estimated \( D_{corr} \) [Fig 5(a)]. \( P_c \) increases at 290 V where the signal is complex [Fig 5(c)], and for the last case we have high \( P_c \) which may be due to the nonlinear effect of the relaxation oscillation [Fig 2(i)].
VII. DISCUSSION AND CONCLUSION

Glow discharges are simple systems but exhibit exotic features depending on the configuration, discharge parameters, etc. The complexities in the plasma dynamics arise from many degrees of freedom like different sources of free energy, various types of wave particle interaction and many other instabilities. As the frequencies of the instabilities presented in this paper, were within the ion plasma frequency and the presence of anode glow and relaxation oscillations have been attributed to the presence of double layers, these instabilities could be related to ion acoustic waves in the presence of ionization instabilities and also anode glow related double layer \[1\]. Such double layer associated ionization instabilities have also been observed before \[31\]. Different analysis tools have proved to be very useful in exploring the various possible linear and nonlinear features of these instabilities.

Here, we have carried out wavelet based time-scale and multi-fractal analysis of the floating potential fluctuations of the glow discharge plasma for extracting some of the nonlinear features. From the contour plots of the CWT coefficients, we have detected scale shift (or inversely frequency shift) with time in the glow discharge plasma. Though theoretical prediction of such nonlinear phenomenon has been reported in plasmas \[32, 33, 34, 35\], there are very few experimental evidences on this \[36\], and we have identified this phenomenon using wavelet techniques. The scale shifts can occur if the coherent modes like the ion acoustic wave or a double layer propagates through an ion acoustic turbulence \[34, 35\]. The ridge patterns of the plasma signals based on CWT decomposition reveal the chaotic features of the system and these finding agree well with our earlier analysis using the nonlinear time series analysis \[1\]. The estimated correlation dimension, fractal dimension, and complexity parameter from multifractal spectra are also in close agreement with the nonlinear analysis \[1\]. With increase in the DV, the multi-fractal behaviour changes to a monofractal as the irregular oscillations become periodic with increase in the DV.

Finally, we feel, the analysis of chaotic data using the contours, ridge plots and multi-fractal spectra, based on the CWT decomposition, are very useful because these do not require a long data length. However one should be cautious about the choice of the appropriate wavelet. Whereas, the estimation of the correlation dimension and Lyapunov exponents estimated from the nonlinear time series analysis require a long data length, and these estimations depend also on the proper choice of the embedding dimension, time lag, etc.

acknowledgement

We acknowledge the constant support and encouragement from the Director, SINP. We would also like to thank our colleagues of the plasma physics division for their help during the experiments. One of the authors (MN) would like to acknowledge the support of Amit Apte at TIFR.
[1] Md. Nurujjaman, Ramesh Narayanan, and A. N. Sekar Iyengar, Chaos 17, 043121 (2007) and references therein.
[2] Md. Nurujjaman, A.N. Sekar Iyengar, and P. Parmananda, Phys. Rev. E 78, 026406 (2008).
[3] Md. Nurujjaman, and A.N. Sekar Iyengar, Phys Letts A 360, 717 (2007).
[4] Lin I and Jeng-Mei Liu, Phys. Rev. Lett. 74, 3161 (1995).
[5] A. Dinklage, C. Wilke and T. Klinger, Phys. Plasmas 6, 2968 (1999).
[6] T. Klinger, F. Greiner, A. Rohde, A. Piel, and M.E. Koepke, Phys. Rev. E 52, 4316 (1995).
[7] R.H. Abrams. E.J. Yadlowsky, and H. Lashinsky, Phys. Rev. Lett. 22, 275 (1969).
[8] Julien Clinton Sprott, Chaos and Time-Series Analysis (Oxford University Press, Oxford, 2003) p.123.
[9] Holger Kantz and Thomas Schreiber, Nonlinear time series analysis (Cambridge University Press, Cambridge, 2004) Chapter 5-6.
[10] Junfeng Sun, Yi Zhao, Tomonichi Nakamura, and Michael Small, Phys. Rev. E 76, 016220 (2007).
[11] C. Chandre, S. Wiggins, and T. Uzer, Physica D 181, 171 (2003).
[12] Sprott, J. C. and Rowlands, G., International Journal of Bifurcation and Chaos 11, 1861 (2001).
[13] Gabriel B. Mindlin and R. Gilmore, Physica D 58, 229 (1992).
[14] R. W. Boswell, Plasma Physics and Controlled Fusion 27, 405 (1985).
[15] T. Higuchi, Physica D 31, 277 (1988).
[16] A. Arneodo, Y. d’ Aubenton-Carafa, E. Bacry, P.V. Graves, J.F. Muzy, and C. Thermo, Physica D 96, 291 (1996).
[17] Alexander Silchenko and Chin-Kun Hu, Phys. Rev. E 63, 041105 (2001).
[18] Ashvin B. Chhabra, Charles Meneveau, Roderick V. Jensen, and K. R. Sreenivasan, Phys. Rev. A 40, 5284 (1989).
[19] Zbigniew R. Struzik and Arno P.J.M. Siebes, Physica A 309, 388 (2002).
[20] J.F. Muzy, E. Bacry, and A. Arneodo, Phys. Rev. Lett. 67, 3515 (1991).
[21] J.F. Muzy, E. Bacry, and A. Arneodo, Physical Review E 47, 875 (1993).
[22] Stéphane Mallat, A Wavelet Tour of Signal Processing (Academic Press, Burlington,1999) p.79.
[23] H. Grussbach and M. Schreiber, Phys. Rev. B 51, 663 (1995).
[24] scal2frq is a standard MATLAB function to compute frequency from scale.
[25] Odim Mendes Jr, Margarete Oliveira Dominguesb, Aracy Mendes da Costa, and Alicia L. Clúa de Gonzalez, Journal of Atmospheric and Solar-Terrestrial Physics 67, 18271836 (2005).
[26] Margarete Oliveira Domingues, Odim Mendes Jr., and Aracy Mendes da Costa, Advances in Space Research 35, 831842 (2005).
[27] Rick Lind, Kyle Snyder, and Marty Brenner, Mechanical Systems and Signal Processing 15, 337 (2001).
[28] Plamen Ch. Ivanov, Luís A. Nunes Amaral, Ary L. Goldberger, Shlomo Havlin, Michael G. Rosenblum, H. Eugene Stanley, and Zbigniew R. Struzik, Chaos 11, 641 (2001).
[29] Xia Sun, Huiping Chen, Yongzhuan Yuan, and Ziqin Wu, Physica A 301, 473 (2001).
[30] Yu Shimizu, Markus Barth, Christian Windischberger, Ewald Moser, and Stefan Thurner, NeuroImage 22, 1195 (2004).
[31] James C Johnson, Nicola D’Angelo, and Robert L Merlin, J.Phys.D:Appl.Phys 23, 682 (1990).
[32] John M. Dawson, Phys. Rev. 113, 383 (1959).
[33] G. J. Morales and T. M. O’Neil, Phys. Rev. Lett. 28, 417 (1972).
[34] J.Vaclavik and K.Appert, Phys.Fluids 23, 1801 (1980).
[35] O.Ishihara and A.Hirose, Phys.Fluids 10, 2429 (1984).
[36] M. E. Koepke, A. Dinklage, T. Klinger, and C. Wilke, Phys. Plasmas 8, 1432 (2001).