Squark Decays in MSSM Under the Cosmological Bounds

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Abstract

We present the numerical investigation of the fermionic two-body decays of squarks in the Minimal Supersymmetric Standard Model with complex parameters. In the analysis we particularly take into account the cosmological bounds imposed by WMAP data. We find that the phase dependences of the decay widths of the third family squarks, as well as those of the first and second families, are quite significant which can provide viable probes of additional CP sources. We plot the CP phase dependences for each fermionic two-body decay channel of squarks $\tilde{q}_i$ (i=1,2, q=u,d;c,s;t,b) and speculate about the branching ratios and total (two-body) decay widths.

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1 Introduction

The experimental HEP frontier is soon reaching TeV energies and most of physicists expect that just there theoretically proposed Higgs bosons and super partners are waiting to be discovered. There are many reasons to be so optimistic. First of all, in spite of its

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remarkable successes, the Standard Model has to be extended into a more complete theory which should solve the hierarchy problem and stabilize the Higgs boson mass against radiative corrections. The most attractive extension to realize these objectives is supersymmetry (SUSY) [1]. Its minimal version (MSSM) requires a non-standard Higgs sector [2] which introduces additional sources of CP-violation [3, 4] beyond the $\delta_{CKM}$ phase [5]. The plethora of CP-phases also influences the decays and mixings of B mesons (as well as D and K mesons). The present experiments at BABAR, Tevatron and KEK and the one to start at the LHC will be able to measure various decay channels to determine if there are supersymmetric sources of CP violation. In particular, CP-asymmetry and decay rate of $B \rightarrow X_s \gamma$ form a good testing ground for low-energy supersymmetry with CP violation [6].

The above-mentioned additional CP-phases explain the cosmological baryon asymmetry of the universe and the lightest SUSY particle could be an excellent candidate for cold dark matter in the universe [7, 8].

In the case of exact supersymmetry, all scalar particles would have to have same masses with their associated SM partners. Since none of the superpartners has been discovered, supersymmetry must be broken. But in order to preserve the hierarchy problem solved the supersymmetry must be broken softly. This leads to a reasonable mass splittings between known particles and their superpartners, i.e. to the superpartners masses around 1 TeV.

The precision experiments by Wilkinson Microwave Anisotropy Probe (WMAP) [9] have put the following constraint on the relic density of cold dark matter:

$$0.0945 < \Omega_{CDM} h^2 < 0.1287$$

(1.1)

Recently, in the light of this cosmological constraint an extensive analysis of the neutralino relic density in the presence of SUSY-CP phases has been given by Belanger et al. [10].

In this study we present the numerical investigation of the fermionic two-body decays of squarks in MSSM with complex SUSY parameters. Actually, we had performed a

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3 In our calculation, we have used WMAP-allowed bands in the plane $M_1 - \varphi$ which are based on 1st year data. Now the WMAP 3rd year data is also available [11], but the new WMAP + SDSS combined value for relic density of dark matter does not change the numerical results in Ref. [10], namely the WMAP-allowed bands. See "Note added" section of Ref. [10].
short study in this direction for the third family squarks [12] incorporating all the existing
bounds on the SUSY parameter space by utilizing the study by Belanger et al. [10]. This
investigation showed us that the effects of $M_1$ and its phase $\varphi_{U(1)}$ on the decay widths of
$\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ are quite significant. Now we extend it to all three families. Although the
SUSY parameters $\mu, M_1, M_2$ and $A_f$ are in general complex, we assume that $\mu, M_2$ and
$A_f$ are real, but $M_1$ and its phase $\varphi_{U(1)}$ take values on the WMAP-allowed bands. These
bands also satisfy the EDM bounds [13]. The experimental upper limits on the EDMs of
electron, neutron and the $^{299}Hg$ and the $^{205}Tl$ atoms may impose constraints on the size
of the SUSY CP-phases [14, 15]. However, these constraints are highly model dependent.
This means that it is possible to suppress the EDMs without requiring the various SUSY
CP-phases be small. For example, in the MSSM assuming strong cancelations between
different contributions [16], the phase of $\mu$ is restricted to $|\varphi_{\mu}| < \pi/10$, but there is no such
restriction on the phases of $M_1$ and $A_f$. In addition, we evaluate the parameter $M_2$
in the relation $M_2 = (3/5)|M_1|(\tan \theta_W)^{-2}$ which can be derived by assuming gaugino mass
unification purely in the electroweak sector of MSSM. It is very important to insert the
WMAP-allowed band in the plane $M_1 - \varphi$ into the numerical calculations instead of taking
one fixed $M_1$ value for all $\varphi$-phases, because, for example, on the allowed band for $\mu = 200$
GeV, $M_1$ starts from 140 GeV for $\varphi = 0$ and increasing monotonously it becomes 165 GeV
for $\varphi = \pi$. In Ref. [10] two WMAP-allowed band plots are given, one for $\mu = 200$ GeV and
the other for $\mu = 350$ GeV. For both plots the other parameters are fixed to be $\tan \beta = 10$,
$m_{H^+} = 1$ TeV, $A_f = 1.2$ TeV, $\varphi_{\mu} = \varphi_{A_f} = 0$. We here simply choose generic masses for
squarks as $m_{\tilde{q}_2} = 1000$ GeV and $m_{\tilde{q}_1} = 750$ GeV ($q = u, d, c, s, t, b$), although we had chosen
more reasonable mass values in our previous note [12]. Taking different sets of values for
squark masses around 1 TeV does not affect much neither the phase dependences nor the
decay widths. As a second possibility, we take masses of 10 TeV for the first and second
family squarks.
2 Squark Masses, Mixing and Two-body Decay Widths

2.1 Masses and mixing in squark sector

The superpartners of the SM fermions with left and right helicity are the left and right sfermions. In the case of top squark (stop) and bottom squark (sbottom) the left and right states are in general mixed. Therefore, the sfermion mass terms of the Lagrangian are described in the basis \((\bar{\tilde{q}}_L, \bar{\tilde{q}}_R)\) as \([17, 18]\)

\[
\mathcal{L}_M^\tilde{q} = - (\bar{\tilde{q}}_L^\dagger \bar{\tilde{q}}_R^\dagger) \begin{pmatrix} M^2_{LL} & M^2_{LR} \\ M^2_{RL} & M^2_{RR} \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}
\] (2.1)

with

\[
M^2_{LL} = M^2_{\tilde{Q}} + (I_{3L}^q - e_q \sin^2 \theta_W) \cos(2\beta)m^2_z + m^2_q
\] (2.2)

\[
M^2_{RR} = M^2_{\tilde{Q}'} + e_q \sin^2 \theta_W \cos(2\beta)m^2_z + m^2_q
\] (2.3)

\[
M^2_{RL} = (M^2_{LR})^* = m_q(A_q - \mu^* \tan \beta)^{-2I_{3L}^q}
\] (2.4)

where \(m_q, e_q, I_{3L}^q\) and \(\theta_W\) are the mass, electric charge, weak isospin of the quarks and the weak mixing angle, respectively. \(\tan \beta = v_2/v_1\) with \(v_i\) being the vacuum expectation values of the Higgs fields \(H^0_i, i = 1, 2\). The soft-breaking parameters \(M_Q, M_{\tilde{Q}'} = M_U(M_D)\) for up components (down components), \(A_q\) involved in Eqs. \((2.2-2.4)\) can be evaluated for our numerical calculations using the following relations

\[
M^2_{\tilde{Q}} = \frac{1}{2} \left( m^2_{\tilde{q}_1} + m^2_{\tilde{q}_2} \pm \sqrt{(m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1})^2 - 4m^2_q|A_q - \mu^* \cot \beta|^2} \right)
\]

\[
- \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \cos(2\beta)m^2_z - m^2_q
\] (2.5)

\[
M^2_{U} = \frac{1}{2} \left( m^2_{\tilde{q}_1} + m^2_{\tilde{q}_2} \pm \sqrt{(m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1})^2 - 4m^2_q|A_q - \mu^* \cot \beta|^2} \right)
\]

\[- \frac{2}{3} \sin^2 \theta_W \cos(2\beta)m^2_z - m^2_q
\] (2.6)

\[
M^2_{D} = \frac{1}{2} \left( m^2_{\tilde{q}_1} + m^2_{\tilde{q}_2} \pm \sqrt{(m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1})^2 - 4m^2_q|A_q - \mu^* \cot \beta|^2} \right)
\]

\[+ \frac{1}{3} \sin^2 \theta_W \cos(2\beta)m^2_z - m^2_q
\] (2.7)

The squark mass eigenstates \(\tilde{q}_1\) and \(\tilde{q}_2\) can be obtained from the weak states \(\tilde{q}_L\) and \(\tilde{q}_R\)
via the $\tilde{q}$-mixing matrix

$$
\mathcal{R}_{\tilde{q}} = \begin{pmatrix}
    e^{i\varphi_{\tilde{q}}} \cos \theta_{\tilde{q}} & \sin \theta_{\tilde{q}} \\
    -\sin \theta_{\tilde{q}} & e^{-i\varphi_{\tilde{q}}} \cos \theta_{\tilde{q}}
\end{pmatrix}
$$

(2.8)

where

$$
\varphi_{\tilde{q}} = \arg[M_{RL}^2] = \arg[A_{q} - \mu^* (\tan \beta)^{-2I_{3L}}]
$$

(2.9)

and

$$
\cos \theta_{\tilde{q}} = \frac{-|M_{LR}^2|}{\sqrt{|M_{LR}^2|^2 + (m_{\tilde{q}_1}^2 - M_{LL}^2)^2}}, \quad \sin \theta_{\tilde{q}} = \frac{M_{LL}^2 - m_{\tilde{q}_1}^2}{\sqrt{|M_{LR}^2|^2 + (m_{\tilde{q}_1}^2 - M_{LL}^2)^2}}
$$

(2.10)

One can easily get the following squark mass eigenvalues by diagonalizing the mass matrix in Eq. (2.1):

$$
m_{\tilde{q}_1,2} = \frac{1}{2} \left( M_{LL}^2 + M_{RR}^2 + \sqrt{(M_{LL}^2 - M_{RR}^2)^2 + 4|M_{LR}^2|^2} \right), \quad m_{\tilde{q}_1} < m_{\tilde{q}_2}
$$

(2.11)

Note that the left-right mixing is not significant for the first and second generations; i.e., $\tilde{q}_1 \simeq \tilde{q}_R$ and $\tilde{q}_2 \simeq -\tilde{q}_L$ (q=u,d,c,s).

We might add a comment about the possibility of a flavor mixing, for example, between the second and third squark families. In this case, the sfermion mass matrix in Eq. (2.1) becomes a 4x4 matrix in the basis $(\tilde{c}_L, \tilde{c}_R, \tilde{t}_L, \tilde{t}_R)$. Then one obtains squark mass eigenstates $(\tilde{c}_1, \tilde{c}_2, \tilde{t}_1, \tilde{t}_2)$ from these weak states, and analyzes their decays by utilizing procedures similar to the ones indicated in the text. The problem with flavor violation effects is that their inclusion necessarily correlates B, D and K physics with direct sparticle searches at colliders. Moreover, it has been shown that, with sizeable supersymmetric flavor violation, even the Higgs phenomenology at the LHC correlates with that of the rare processes [19]. In this work we have neglected such effects; however, we emphasize that inclusion of such effects can give important information on mechanism that breaks supersymmetry via decay products of squarks.

### 2.2 Fermionic decay widths of squarks

The quark-squark-chargino and quark-squark-neutralino Lagrangians have been first given in Ref. [1]. Here we use them in notations of Ref. [20]:

$$
\mathcal{L}_{\tilde{q}\tilde{q}\chi^\pm} = g\tilde{u}(\ell^d_{ij}P_R + k^d_{ij}P_L)\tilde{\chi}_j^+ \bar{d}_i + g\tilde{d}(\ell^u_{ij}P_R + k^u_{ij}P_L)\tilde{\chi}_j^+ c\bar{u}_i + h.c.
$$

(2.12)
and
\[ \mathcal{L}_{q\tilde{q}h^0} = g\bar{q}(a_{ik}^q P_R + b_{ik}^q P_L)\tilde{q}_k h^0 + h.c. \] (2.13)

where \( u (\tilde{u}) \) stands for up-type (s)quark and \( d (\tilde{d}) \) stands for down-type s(quark). We also borrow the formulas for the partial decay widths of \( \tilde{q}_i \) (\( \tilde{q}_i = \tilde{t}_i, \tilde{c}_i, \tilde{b}_i \) and \( \tilde{d}_i \)) into quark-chargino (or neutralino) from Ref. [20]:

\[
\Gamma(\tilde{q}_i \to q' + \tilde{\chi}_k^\pm) = \frac{g^2\lambda^{1/2}(m_{\tilde{q}_i}^2, m_{\tilde{q}_q}^2, m_{\tilde{\chi}_k^\pm}^2)}{16\pi m_{\tilde{q}_i}^3} \times \left[ \left( |k_{ik}^q|^2 + |\ell_{ik}^q|^2 \right)(m_{\tilde{q}_i}^2 - m_q^2 - m_{\tilde{\chi}_k^\pm}^2) - 4\text{Re}(k_{ik}^q\ell_{ik}^q)m_q m_{\tilde{\chi}_k^\pm} \right] \] (2.14)

and

\[
\Gamma(\tilde{q}_i \to q + \tilde{\chi}_k^0) = \frac{g^2\lambda^{1/2}(m_{\tilde{q}_i}^2, m_{\tilde{q}_q}^2, m_{\tilde{\chi}_k^0}^2)}{16\pi m_{\tilde{q}_i}^3} \times \left[ \left( |a_{ik}^q|^2 + |b_{ik}^q|^2 \right)(m_{\tilde{q}_i}^2 - m_q^2 - m_{\tilde{\chi}_k^0}^2) - 4\text{Re}(a_{ik}^q b_{ik}^q)m_q m_{\tilde{\chi}_k^0} \right] \] (2.15)

with \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz) \).

The explicit forms of \( \ell_{ik}^q, k_{ik}^q \) are

\[
\ell_{ik}^q = -\mathcal{R}_{i1}^q V_{k1} + Y_D \mathcal{R}_{i2}^q V_{k2}, \quad k_{ik}^q = \mathcal{R}_{i1}^q Y_D U_{k2}^*, \] (2.16)

for up squarks \( \tilde{u}, \tilde{c}, \tilde{t} \) and

\[
\ell_{ik}^q = -\mathcal{R}_{i1}^q U_{k1} + Y_D \mathcal{R}_{i2}^q U_{k2}, \quad k_{ik}^q = \mathcal{R}_{i1}^q Y_D V_{k2}^*, \] (2.17)

for down squarks \( \tilde{d}, \tilde{s}, \tilde{b} \) (\( U = u, c, t \); \( D = d, s, b \)). And \( a_{ik}^q, b_{ik}^q \) are given as

\[
a_{ik}^q = \sum_{n=1}^2 (\mathcal{R}_{in}^q)^* \mathcal{A}_{kn}^q, \quad b_{ik}^q = \sum_{n=1}^2 (\mathcal{R}_{in}^q)^* \mathcal{B}_{kn}^q, \] (2.18)

where

\[
\mathcal{A}_{k}^q = \begin{pmatrix} f_{Lk}^q \\ h_{Rk}^q \end{pmatrix}, \quad \mathcal{B}_{k}^q = \begin{pmatrix} h_{Lk}^q \\ f_{Rk}^q \end{pmatrix} \] (2.19)

Here, the components for up squarks \( \tilde{u}, \tilde{c}, \tilde{t} \) are

\[
f_{Lk}^q = -\frac{1}{\sqrt{2}} \left( N_{k2} + \frac{1}{3} \tan \theta_W N_{k1} \right) \]

6
\[ f_{Rk}^q = \frac{2\sqrt{2}}{3} \tan \theta_W N_{k1}^* \]
\[ h_{Lk}^q = (h_{Rk}^q)^* = -Y_U N_{k4}^* \] (2.20)

and for down squarks $\tilde{d}$, $\tilde{s}$, $\tilde{b}$
\[ f_{Lk}^q = \frac{1}{\sqrt{2}} (N_{k2} - \frac{1}{3} \tan \theta_W N_{k1}) \]
\[ f_{Rk}^q = -\frac{\sqrt{2}}{3} \tan \theta_W N_{k1}^* \]
\[ h_{Lk}^q = (h_{Rk}^q)^* = -Y_D N_{k3}^* \] (2.21)

We have to point out that although at the loop level the SUSY-QCD corrections could be important, our analysis here are merely at tree level, as can be seen from Eqs. (2.13) and (2.14). In this work we content with tree-level amplitudes as we aim at determining the phase-sensitivities of the decay rates, mainly.

3 Two-body decays of first and second generations squarks

Now we present the dependences of the $\tilde{u}_1$, $\tilde{u}_2$, $\tilde{d}_1$ and $\tilde{d}_2$ two-body decay widths on the phase $\varphi_{U(1)}$ for $\mu = 200$ GeV and for $\mu = 350$ GeV. Here we choose the values for the masses $(m_{\tilde{u}_2} = m_{\tilde{d}_2}, m_{\tilde{u}_1} = m_{\tilde{d}_1}, m_{\tilde{u}_1}^\pm, m_{\tilde{d}_1}^\pm, m_{\tilde{u}_1}^0, m_{\tilde{d}_1}^0) = (1000$ GeV, 750 GeV, 180 GeV, 336 GeV, 150 GeV) for $\mu = 200$ GeV and $(m_{\tilde{u}_2} = m_{\tilde{d}_2}, m_{\tilde{u}_1} = m_{\tilde{d}_1}, m_{\tilde{u}_1}^\pm, m_{\tilde{d}_1}^\pm, m_{\tilde{u}_1}^0) = (1000$ GeV, 750 GeV, 340 GeV, 680 GeV, 290 GeV) for $\mu = 350$ GeV. Note that although the neutralino and chargino masses vary with $\varphi_{U(1)}$, these variations are not large. Therefore, as a final state particle (i.e., on mass-shell), we have chosen fixed (average) mass values for charginos and neutralinos. Fig.1(a) and Fig.1(b) show the partial decay widths of the channels $\tilde{u}_2 \to d\tilde{\chi}_1^+, \tilde{u}_2 \to d\tilde{\chi}_2^+, \tilde{u}_2 \to u\tilde{\chi}_1^0$, $\tilde{u}_1 \to u\tilde{\chi}_1^0$ and $\tilde{d}_2 \to u\tilde{\chi}_1^-, \tilde{d}_2 \to u\tilde{\chi}_2^-, \tilde{d}_2 \to d\tilde{\chi}_1^0$, $\tilde{d}_1 \to d\tilde{\chi}_1^0$, respectively for $\mu = 200$ GeV. [ We plot the same processes for $\mu = 350$ GeV in Fig.2(a) and Fig.2(b) ]. The decay channels $\tilde{u}_1 \to d\tilde{\chi}_1^+$ and $\tilde{u}_1 \to d\tilde{\chi}_2^+$ are absent because the mixing elements ($R_{ij}$) and the Yukawa couplings $Y_u$, $Y_d$ in $\ell_{ij}^u$ and $k_{ij}^d$ parameters are zero, whereas the channel $\tilde{u}_1 \to u\tilde{\chi}_1^0$ is present, because the parameter $b_{11}$ is not zero. Because of a similar reason the phase dependence of $\tilde{u}_1 \to u\tilde{\chi}_1^0$ channel is very little.
The branching ratios for $\tilde{u}_2$ are roughly $B(\tilde{u}_2 \rightarrow d\tilde{\chi}^+_2) : B(\tilde{u}_2 \rightarrow d\tilde{\chi}^+_1) : B(\tilde{u}_2 \rightarrow u\tilde{\chi}^0_1) \approx 5 : 2 : 0.05$ for the case $\mu = 200$ GeV. From Fig.1(b) one can see that $B(\tilde{d}_2 \rightarrow u\tilde{\chi}^-_2) : B(\tilde{d}_2 \rightarrow d\tilde{\chi}^0_1) \approx 6 : 1 : 0.2$ for the case $\mu = 200$ GeV. The width $\Gamma(\tilde{u}_2 \rightarrow d\tilde{\chi}^+_2)$ increases as the phase increases from 0 to $\pi$, but $\Gamma(\tilde{u}_2 \rightarrow d\tilde{\chi}^+_1)$ decreases as $\varphi_{U(1)}$ increases, both showing significant dependence on the phase. The phase dependence is more significant for the decay channel $\tilde{u}_2 \rightarrow u\tilde{\chi}^0_1$ (its decay width increases from 0.0006 GeV to 0.15 GeV as $\varphi_{U(1)}$ increases from 0 to $\pi$) but its branching ratio is too small to observe it.

The channels $\tilde{d}_1 \rightarrow u\tilde{\chi}^-_2$ and $\tilde{d}_1 \rightarrow u\tilde{\chi}^0_2$ are absent because the related mixing elements and Yukawa couplings are zero (therefore $\ell_{11}^d$, $k_{11}^d$, $\ell_{12}^d$, $k_{12}^d$ become zero); whereas the channel $\tilde{d}_1 \rightarrow d\tilde{\chi}^0_1$ is present (since $b_{11}^d \neq 0$), but it is little with respect to $\tilde{d}_2$ decay channels. Its phase dependence is also not so strong (from $\varphi_{U(1)}=0$ to $\pi$, its width increases only from 0.264 GeV to 0.271 GeV). Therefore, the $\tilde{d}_1$ decay can be ignored in comparison with the $\tilde{d}_2$ decay.

The decay pattern of the second family squarks coincides with that of the first family, because we can set $m_s \approx 0$ and $m_c \approx 0$ like $m_u \approx 0$ and $m_d \approx 0$ in comparison with the masses $m_{\tilde{q}_2}=1000$ GeV, $m_{\tilde{q}_1}=750$ GeV (The use of $m_s \approx 0.1$ GeV and $m_c \approx 1.5$ GeV does change nothing).

In the analysis of cosmological constraints by Belanger et al. they take masses of $10$ TeV for the first and second generations of squarks. As an academic exercise, we have repeated the above calculations only changing the first and second squarks masses to $m_{\tilde{u}_2} = m_{\tilde{d}_2} = 10$ TeV and $m_{\tilde{u}_1} = m_{\tilde{d}_1} = 9.5$ TeV. We obtained again similar phase dependences, but larger ($\sim$ one order) width values because of huge phase spaces.

4 Two-body decays of third generation squarks

In this section we plot the dependences of the $\tilde{t}_1$, $\tilde{t}_2$, $\tilde{b}_1$ and $\tilde{b}_2$ partial decay widths on $\varphi_{U(1)}$ for $\mu = 200$ GeV and for $\mu = 350$ GeV. Here we simply choose mass values for the third family squarks as $m_{\tilde{t}_2}=m_{\tilde{b}_2}=1000$ GeV and $m_{\tilde{t}_1}=m_{\tilde{b}_1}=750$ GeV in order to compare the results with those of the first two families. We had taken more reasonable mass spectrum for squarks in our previous study [12]. The other masses are $(m_{\tilde{\chi}^+_1}, m_{\tilde{\chi}^+_2}, m_{\tilde{\chi}^0_1}) = (180$
GeV, 336 GeV, 150 GeV) for \( \mu = 200 \) GeV and \((m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}, m_{\tilde{\chi}_3^0}) = (340 \) GeV, 680 GeV, 290 GeV\) for \( \mu = 350 \) GeV.

For both sets of values by calculating the \( M_{\tilde{Q}} \) and \( M_{\tilde{U}} \) values corresponding to \( m_{\tilde{t}_1} \) and \( m_{\tilde{t}_2} \), we plot the decay widths for \( M_{\tilde{Q}} \geq M_{\tilde{U}} \) and \( M_{\tilde{Q}} < M_{\tilde{U}} \), separately. Fig.3(a) and Fig.3(b) show the partial decay widths of the channels \( \tilde{t}_1 \rightarrow b\tilde{\chi}_1^+, \tilde{t}_1 \rightarrow b\tilde{\chi}_2^+, \tilde{t}_1 \rightarrow t\tilde{\chi}_1^0, \tilde{t}_2 \rightarrow b\tilde{\chi}_1^+, \tilde{t}_2 \rightarrow b\tilde{\chi}_2^+ \) and \( \tilde{t}_2 \rightarrow t\tilde{\chi}_1^0 \) as a function of \( \varphi_{U(1)} \) for \( \mu = 200 \) GeV assuming \( M_{\tilde{Q}} > M_{\tilde{U}} \) and \( M_{\tilde{Q}} < M_{\tilde{U}} \), respectively. In these figures significant dependences on \( \varphi_{U(1)} \) phase are seen. \( \Gamma(\tilde{t}_1 \rightarrow t\tilde{\chi}_1^0) \) and \( \Gamma(\tilde{t}_1 \rightarrow b\tilde{\chi}_1^+) \) decay widths increase as \( \varphi_{U(1)} \) increases from 0 to \( \pi \), but \( \Gamma(\tilde{t}_1 \rightarrow b\tilde{\chi}_2^+) \) width decreases as \( \varphi_{U(1)} \) increases. On the other hand, \( \Gamma(\tilde{t}_2 \rightarrow t\tilde{\chi}_1^0) \) decrease for both \( M_{\tilde{Q}} > M_{\tilde{U}} \) and \( M_{\tilde{Q}} < M_{\tilde{U}} \) cases as \( \varphi_{U(1)} \) increases; \( \Gamma(\tilde{t}_2 \rightarrow b\tilde{\chi}_1^+) \) decreases for \( M_{\tilde{Q}} > M_{\tilde{U}} \) but increases for \( M_{\tilde{Q}} < M_{\tilde{U}} \) and \( \Gamma(\tilde{t}_2 \rightarrow b\tilde{\chi}_2^+) \) increases for \( M_{\tilde{Q}} > M_{\tilde{U}} \) but decreases for \( M_{\tilde{Q}} < M_{\tilde{U}} \).

The branching ratios for \( \tilde{t}_2 \) are roughly \( B(\tilde{t}_2 \rightarrow b\tilde{\chi}_1^+) : B(\tilde{t}_2 \rightarrow t\tilde{\chi}_1^0) : B(\tilde{t}_2 \rightarrow b\tilde{\chi}_2^+) \approx 12 : 2.5 : 1 \). This simply reflects both the large phase space and large Yukawa coupling for the decay \( \tilde{t}_2 \rightarrow b\tilde{\chi}_1^+ \).

In Figs. 4(a) and 4(b) you see the same partial decay widths for \( \mu = 350 \) GeV. They, too, show the significant dependences on CP-violation phase. For both \( M_{\tilde{Q}} > M_{\tilde{U}} \) and \( M_{\tilde{Q}} < M_{\tilde{U}} \) cases; \( \Gamma(\tilde{t}_1 \rightarrow t\tilde{\chi}_1^0) \) and \( \Gamma(\tilde{t}_1 \rightarrow b\tilde{\chi}_1^+) \) decay widths increase as \( \varphi_{U(1)} \) increases from 0 to \( \pi \), but \( \Gamma(\tilde{t}_1 \rightarrow b\tilde{\chi}_2^+) \) width decreases as \( \varphi_{U(1)} \) increases. Besides that \( \Gamma(\tilde{t}_2 \rightarrow t\tilde{\chi}_1^0) \) and \( \Gamma(\tilde{t}_2 \rightarrow b\tilde{\chi}_1^+) \) widths decrease but \( \Gamma(\tilde{t}_2 \rightarrow b\tilde{\chi}_2^+) \) width increases as \( \varphi_{U(1)} \) increases from 0 to \( \pi \) for both \( M_{\tilde{Q}} > M_{\tilde{U}} \) and \( M_{\tilde{Q}} < M_{\tilde{U}} \) cases. The branching ratios for \( \tilde{t}_2 \) are roughly \( B(\tilde{t}_2 \rightarrow b\tilde{\chi}_1^+) : B(\tilde{t}_2 \rightarrow t\tilde{\chi}_1^0) : B(\tilde{t}_2 \rightarrow b\tilde{\chi}_2^+) \approx 8 : 2 : 1 \).

For \( \mu = 350 \) GeV the WMAP-allowed band [10] takes place in larger \( M_1 \) values (\( \sim 305 - 325 \) GeV) leading to larger chargino and neutralino masses. This naturally leads very small decay width for \( \tilde{t}_1 \rightarrow b\tilde{\chi}_2^+ \). The decay width of the process \( \tilde{t}_2 \rightarrow b\tilde{\chi}_1^+ \) is the largest one among the \( \tilde{t}_2 \) channels and the branching ratios are \( B(\tilde{t}_2 \rightarrow b\tilde{\chi}_1^+) : B(\tilde{t}_2 \rightarrow t\tilde{\chi}_1^0) : B(\tilde{t}_2 \rightarrow b\tilde{\chi}_2^+) \approx 8 : 3 : 1 \). The decay \( \tilde{t}_2 \rightarrow t\tilde{\chi}_1^0 \) shows strong phase dependence.

We give sbottom decay widths as a function of \( \varphi_{U(1)} \) in Figs. 5(a)-(b) for \( \mu = 200 \) GeV and in Figs. 6(a)-(b) for \( \mu = 350 \) GeV. \( \Gamma(\tilde{b}_2 \rightarrow b\tilde{\chi}_1^0) \) is smaller than \( \Gamma(\tilde{b}_2 \rightarrow t\tilde{\chi}_1^-) \) in spite of large phase space, because in \( \tilde{b}_2 \rightarrow b\tilde{\chi}_1^0 \) decay only \( Y_b \) coupling enters which is very
small in comparison with $Y_t$. The dependences of the phase $\varphi_{U(1)}$ in sbottom decays are similar to those in stop decays.

For the case $M_Q > M_D$, $\tilde{b}_2$ decay is 7-10 times larger than that of $\tilde{b}_1$, whereas for $M_Q < M_D$ the reverse is true. The branching ratios for $\tilde{b}_2$ decays are $B(\tilde{b}_2 \to t\tilde{\chi}_1^-) : B(\tilde{b}_2 \to t\tilde{\chi}_2^-) : B(\tilde{b}_2 \to b\tilde{\chi}_1^0) \approx 14 : 10 : 0.5$ for $\mu = 200$ GeV and $M_Q > M_D$, and similarly $14 : 3 : 0.5$ for $\mu = 350$ GeV and $M_Q > M_D$. While the process $\tilde{b}_2 \to b\tilde{\chi}_1^0$ is suppressed more than one order its dependence on $\varphi_{U(1)}$ is prominent such that the value of decay width at $\varphi_{U(1)} = 0$ is 2 times larger than that at $\varphi_{U(1)} = \pi$. $\varphi$-dependences of the processes $\tilde{b}_2 \to t\tilde{\chi}_1^-$ and $\tilde{b}_2 \to t\tilde{\chi}_2^-$ can be seen easily in Fig. 5(a).

The branching ratios for $\tilde{b}_1$ decays are $B(\tilde{b}_1 \to t\tilde{\chi}_1^-) : B(\tilde{b}_1 \to t\tilde{\chi}_2^-) : B(\tilde{b}_1 \to b\tilde{\chi}_1^0) \approx 9 : 3 : 0.5$ for $\mu = 200$ GeV and $M_Q < M_D$ and $B(\tilde{b}_1 \to t\tilde{\chi}_1^-) : B(\tilde{b}_1 \to b\tilde{\chi}_1^0) \approx 7 : 0.3$ for $\mu = 350$ GeV and $M_Q < M_D$.

5 Discussion

In this paper, we present the numerical investigation of the fermionic two-body decays of squarks in three families in the Minimal Supersymmetric Standard Model with complex parameters taking into account the cosmological bounds imposed by WMAP data. Numerical calculations of decay widths at tree level show significant dependence on the CP phase $\varphi_{U(1)}$ for the third family squarks, as well as for the first and second families.

We have assumed the same mass of 750 GeV, for all squarks of type 1 and the same mass of 1000 GeV for all squarks of type 2. The decay width values of the third family squarks and those of the first two families differ only two to three times in favor of the third family.

In the case of $\mu = 200$ GeV all the channels of $\tilde{u}_2$ decay ($\tilde{d}_2$ decay), i.e., $\tilde{u}_2 \to d\tilde{\chi}_2^+$, $\tilde{u}_2 \to u\tilde{\chi}_1^+$ and $\tilde{u}_2 \to u\tilde{\chi}_1^-$, $\tilde{d}_2 \to u\tilde{\chi}_1^+$ and $\tilde{d}_2 \to d\tilde{\chi}_1^+$) are present and have very significant phase dependences, but the channel $\tilde{u}_2 \to u\tilde{\chi}_1^0$ ($\tilde{d}_2 \to d\tilde{\chi}_1^0$) has a very small branching ratio. At the $\tilde{u}_1$ decay ($\tilde{d}_1$ decay) only one channel, i.e., $\tilde{u}_1 \to u\tilde{\chi}_1^+$ ($\tilde{d}_1 \to d\tilde{\chi}_1^+$) is open with a small branching ratio. The decay channels $\tilde{u}_1 \to d\tilde{\chi}_1^+$ and $\tilde{u}_1 \to d\tilde{\chi}_2^+$ ($\tilde{d}_1 \to u\tilde{\chi}_1^+$ and $\tilde{d}_1 \to u\tilde{\chi}_2^+$) are absent because of the reason explained in section 3.

Very roughly speaking, for $\mu = 200$ GeV and $M_Q > M_{\tilde{U}}$ or $M_D$ we find the following
total (two-body) widths:

\[ \Gamma_{\text{total}}(\tilde{u}_2 \text{ or } \tilde{c}_2) \approx \Gamma_{\text{total}}(\tilde{d}_2 \text{ or } \tilde{s}_2) \approx 7 \text{ GeV} \quad (5.1) \]
\[ \Gamma_{\text{total}}(\tilde{u}_1 \text{ or } \tilde{c}_1) \approx 1 \text{ GeV} \quad (5.2) \]
\[ \Gamma_{\text{total}}(\tilde{d}_1 \text{ or } \tilde{s}_1) \approx 0.5 \text{ GeV} \quad (5.3) \]
\[ \Gamma_{\text{total}}(\tilde{t}_2) \approx 15 \text{ GeV} \quad (5.4) \]
\[ \Gamma_{\text{total}}(\tilde{b}_2) \approx 24 \text{ GeV} \quad (5.5) \]
\[ \Gamma_{\text{total}}(\tilde{t}_1) \approx 10 \text{ GeV} \quad (5.6) \]
\[ \Gamma_{\text{total}}(\tilde{b}_1) \approx 3 \text{ GeV} \quad (5.7) \]

From these results we see that although the decays of the third family squarks are more important, the decays of the first two families are not ignorable at all. For example, if we assume that the probability of producing every kind of squarks in a proton-proton collision (LHC) are more or less equal, then the ratio of total decay width of the third family squarks to that of the first two families would be approximately 2.

In the case of the choice of squarks masses of 10 TeV for the first two families and 1 TeV for the third family which we call it as ”an academic exercise” (because there is no hope to produce the first two family squarks in LHC) this ratio even reverses in favor of the first two families.

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Figure 1: (a)-(b) Partial decay widths $\Gamma$ of the $\tilde{u}_{1,2}$ and $\tilde{d}_{1,2}$ decays for $\mu = 200$ GeV, $\tan \beta = 10$, $A_u = A_d = 1.2$ TeV, $\varphi_{\mu} = \varphi_{A_u} = \varphi_{A_d} = 0$, $m_{\tilde{u}_1} = m_{\tilde{d}_1} = 750$ GeV and $m_{\tilde{u}_2} = m_{\tilde{d}_2} = 1000$ GeV.
Figure 2: (a)-(b) Partial decay widths $\Gamma$ of the $\tilde{u}_{1,2}$ and $\tilde{d}_{1,2}$ decays for $\mu = 350$ GeV, $\tan \beta = 10$, $A_u = A_d = 1.2$ TeV, $\varphi_\mu = \varphi_{A_u} = \varphi_{A_d} = 0$, $m_{\tilde{u}_1} = m_{\tilde{d}_1} = 750$ GeV and $m_{\tilde{u}_2} = m_{\tilde{d}_2} = 1000$ GeV.
Figure 3: (a)-(b) Partial decay widths $\Gamma$ of the $\tilde{t}_{1,2}$ decays for $\mu = 200$ GeV , $\tan \beta = 10$, $A_t = 1.2$ TeV, $\varphi_\mu = \varphi_{A_t} = 0$, $m_{\tilde{t}_1} = 750$ GeV and $m_{\tilde{t}_2} = 1000$ GeV.
Figure 4: (a)-(b) Partial decay widths $\Gamma$ of the $\tilde{t}_{1,2}$ decays for $\mu = 350$ GeV, $\tan \beta = 10$, $A_t=1.2$ TeV, $\varphi_\mu=\varphi_{A_t}=0$, $m_{\tilde{t}_1} = 750$ GeV and $m_{\tilde{t}_2} = 1000$ GeV.
Figure 5: (a)-(b) Partial decay widths $\Gamma$ of the $\tilde{b}_1,2$ decays for $\mu = 200$ GeV, $\tan \beta = 10$, $A_b=1.2$ TeV, $\varphi_\mu=\varphi_{A_b}=0$, $m_{\tilde{b}_1} = 750$ GeV and $m_{\tilde{b}_2} = 1000$ GeV.
Figure 6: (a)-(b) Partial decay widths $\Gamma$ of the $\tilde{b}_{1,2}$ decays for $\mu = 350$ GeV, $\tan \beta = 10$, $A_b = 1.2$ TeV, $\varphi_\mu = \varphi_{\lambda_b} = 0$, $m_{\tilde{b}_1} = 750$ GeV and $m_{\tilde{b}_2} = 1000$ GeV.