Dark Energy, Dark Matter and baryogenesis from a model of a complex axion field

Robert Brandenberger\(^a,\)\(^b\) and Jürg Fröhlich\(^b\)

\(^a\)Department of Physics, McGill University, Montréal, QC, H3A 2T8, Canada
\(^b\)Institute of Theoretical Physics, ETH Zürich, CH-8093 Zürich, Switzerland

E-mail: rhb@physics.mcgill.ca, juerg@phys.ethz.ch

Received September 19, 2020
Revised March 3, 2021
Accepted March 4, 2021
Published April 12, 2021

Abstract. We introduce and study a model designed to simultaneously shed light on the mysteries connected with Baryogenesis, Dark Matter and Dark Energy. The model describes a self-interacting complex axion field whose imaginary part, a pseudo-scalar axion, couples to the instanton density of gauge fields including the hypercharge field. This coupling may give rise to baryogenesis in the early universe. After tracing out the gauge and matter degrees of freedom, a non-trivial effective potential for the angular component of the axion field is obtained. It is proposed that oscillations of this component around a minimum of its effective potential can be interpreted as Dark Matter. The absolute value of the axion field rolls slowly towards 0. At late times, it can give rise to Dark Energy.

Keywords: dark energy theory, dark matter theory, axions

ArXiv ePrint: 2004.10025
1 Introduction

In this paper we describe theoretical ideas on the origin of Dark Matter, Dark Energy and baryogenesis. The thrust of our efforts is to identify a unified mechanism that may explain the presence of Dark Matter and Dark Energy in the universe and, at the same time, illuminate the origin of baryogenesis. At the present time, such ideas are necessarily rather speculative, and ours are no exception. Our guiding principle is to describe a model that is as economical and conventional as possible and, in particular, involves only few degrees of freedom not already present in the standard model and in general relativity.

The dark sector is known to make up about 95% of the energy density of the universe. Roughly 70% of the total energy density corresponds to Dark Energy, while approximately 25% originates in Dark Matter; see, e.g., [1]. Dark Matter has an equation of state given by \( w \simeq 0 \), where \( w = p/\rho \), \( p \) and \( \rho \) being pressure and energy density, respectively, whereas the equation of state of Dark Energy is known to be \( w \simeq -1 \).

A conventional candidate for a Dark Matter particle is a WIMP (=weakly interacting massive particle, see [2] for a review), and Dark Energy is usually described by a small cosmological constant. However, these simple descriptions of Dark Matter and Dark Energy appear to meet with increasing difficulties. The WIMP model of Dark Matter faces the problem that WIMP’s have not been observed in any direct detection experiments, which rules out part of the preferred parameter space [3]. For what concerns Dark Energy, there is increasing evidence that a positive cosmological constant cannot appear in current theories of quantum gravity [4–10]. There are thus good reasons — see e.g. the discussion in [11, 12] — to imagine that Dark Energy is described by dynamical degrees of freedom, such as the slowly rolling scalar field introduced in Quintessence models [13–15]. Oscillating pseudo-scalar fields with a small mass, such as an axion field, have long been envisaged as candidate degrees of freedom describing Dark Matter; see, e.g., [16] for a review.

From a theorist’s point of view it would be attractive if Dark Matter and Dark Energy turned out to have a common origin. This is the theme developed in this paper. We introduce a model of a complex scalar field (with non-canonical kinetic term), \( Z = e^{-(\varphi + i\theta)/f} \), whose radial component, \( \varphi \), gives rise to Dark Energy, while the angular component, \( \theta \), is supposed to describe Dark Matter, and \( f > m_{\text{pl}} \) is a constant of nature rendering \( (\varphi + i\theta)/f \).
dimensionless; see \cite{17, 18} for earlier attempts, and \cite{19} for a review of various unified dark sector models. For positive values of $\varphi$, the self-interaction potential of the field $Z$ is assumed to be proportional to $\bar{Z}Z = e^{-2\varphi/f}$; (it is exponential in the scalar field $\varphi$, as in Quintessence models \cite{13–15}). We further assume that the imaginary part of $Z$ is coupled to the instanton density of some gauge fields in a way analogous to how the QCD axion is coupled to the color gauge field. When tracing out the gauge and matter degrees of freedom, this coupling generates a potential for the angular component, $\theta$, of $Z$, which gives rise to oscillations of the pseudo-scalar axion field $\theta$ around a minimum. These oscillations are a source of Dark Matter. The radial part, $\varphi$, of $Z$ slowly grows towards very large values, and hence the potential, $\propto e^{-2\varphi/f}$, slowly approaches 0. This potential is a source of (dynamical) Dark Energy.

An intriguing feature of our model is that it also naturally incorporates a mechanism for baryogenesis. The imaginary part of $\partial_\mu Z$ can be coupled to the anomalous axial baryon current, $j_\mu_B$. During an era when the time derivative of the imaginary part, $\Im Z$, of $Z$ (or of $\theta$) has a fixed sign this coupling gives rise to a matter-antimatter asymmetry; see also \cite{20–23}.

The organization of this paper is as follows. In the next section we introduce the model studied afterwards. In section 3 we discuss constraints on the parameters of the model and show that we can satisfy all the known constraints derived from the requirement that one wants to obtain the right amount of Dark Matter and Dark Energy. We discuss baryogenesis in sections 4 and 5. Section 6 contains some conclusions. In an appendix we discuss possible roots of our model in more fundamental physical theories.

Throughout this paper we employ natural units in which the speed of light, Planck’s constant and Boltzmann’s constant are set to 1. The cosmological scale factor appearing in the equations of the Friedman-Lemaître universe is denoted by $a(t)$, where $t$ denotes time. The radiation temperature, $T$, is related to time $t$ via the Friedmann equation and the Stefan-Boltzmann law. The Hubble expansion rate is denoted by $H(t)$, and the Planck mass by $m_{pl}$.

There are various times which play a role in our analysis: the current time is denoted by $t_0$, the time of equal matter and radiation is $t_{eq}$, and the time after which the dynamics of the universe starts to be described by our model is denoted by $t_c$. The corresponding radiation temperatures are $T_0$, $T_{eq}$ and $T_c$, respectively. We sometimes express time in terms of the cosmological redshift, $z$. The redshift at time $t$ is defined by

\begin{equation}
  z(t) + 1 \equiv \frac{a(t_0)}{a(t)}.
\end{equation}

2 The model

As announced in the Introduction, the model studied in this paper describes a (dimensionless) complex scalar field

\begin{equation}
  Z = e^{-(\varphi + i\theta)/f},
\end{equation}

where $\varphi$ is a real scalar field, called the “radial component” of $Z$, $\theta$ is a real pseudo-scalar axion field, called “angular component” of $Z$, and $f$ is the field range over which the potential, $\bar{Z}Z$, of $Z$ varies appreciably. We introduce the one-form

\begin{equation}
  j := Z^{-1}dZ, \quad \text{i.e.,} \quad j_\mu = Z^{-1}\partial_\mu Z = -\partial_\mu(\varphi + i\theta)/f.
\end{equation}
Let \( CS(G) \) denote the Chern-Simons 3-form of a (non-abelian) gauge field \( G \). An example of a plausible action functional is given by

\[
S(\bar{Z}, Z, G) := \int d^4x \sqrt{-g} \left( \frac{1}{2} f^2 \partial_\mu j^\mu - U(|Z|^2) \right) - \lambda \int d^4Z \wedge CS(G) + \ldots ,
\]

(2.3)

where \( g \) is the determinant of the space-time metric (with components \( g_{\mu\nu} \)),

\[
U(\varphi) \equiv U(|Z|^2) \sim \Lambda \bar{Z} Z, \quad \text{as } |Z| \downarrow 0,
\]

(2.4)

is a selfinteraction potential, \( \Lambda \) is a constant of (mass) dimension 4, \( \lambda \) is a dimensionless coupling constant, and the dots stand for a possible coupling of \( \theta \) to the instanton density of the hypercharge \( U(1)_Y \) gauge field. For \( \lambda = 0 \), the action thus takes the form

\[
S_0 = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{1}{2} \partial_\mu \theta \partial^\mu \theta - U(\varphi) \right\},
\]

(2.5)

or, in terms of the component fields

\[
S_0 = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{1}{2} \partial_\mu \theta \partial^\mu \theta - U(\varphi) - V(\varphi, \theta) + \ldots \right\},
\]

(2.6)

**Remarks:** (i) The potential \( U(\varphi) \) can take a fairly arbitrary value at \( \varphi \approx 0 \), corresponding to early times. But at late times, \( \varphi \) grows very large, and we henceforth set \( U(\varphi) = \Lambda e^{-2\varphi/f} \).

(ii) Terms proportional to masses of matter degrees of freedom are neglected in the last term on the right side of (2.3).

After a phase transition at some temperature \( T_c \), the non-abelian gauge degrees of freedom acquire a mass and are traced/integrated out. The effective action for \( Z \) then takes the form

\[
S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{1}{2} \partial_\mu \theta \partial^\mu \theta - U(\varphi) - V(\varphi, \theta) + \ldots \right\},
\]

(2.7)

where \( V(\varphi, \theta) = \mathcal{O}(\theta^2) \), for \( \theta \approx 0 \), and the dots have the meaning indicated above. One finds that, for small values of \( \sin(\theta/f)e^{-\varphi/f} \),

\[
V(\varphi, \theta) = \frac{1}{2} \mu^4 \sin^2(\theta/f)e^{-2\varphi/f}
\]

(2.8)

where \( \mu \) is a \( \lambda \)-dependent mass scale set by the phase transition temperature. Note that if \( T_c \) is so large that \( \varphi \) is negative at the time of the phase transition then this transition may be followed by some cosmological “wetting transitions”, as studied in \([23]\).**

**Remark:** Note that when expressed in terms of the fields \( \varphi \) and \( \theta \), the action functional \( S \) has standard derivative terms, which suggests that the theory can be quantized perturbatively in the usual way. The self-interaction terms \( U(\varphi) \) and \( V(\varphi, \theta) \) are, however, not renormalizable, a feature one is used to from theories of axions and gravitational degrees of freedom. (We are really dealing with an effective low-energy description of degrees of freedom not present in the standard model.) For an ultraviolet completion of the theory to become feasible, the theory would have to be embedded in a more fundamental description of interactions between matter and gravitation, such as some string theory.
In the following, we consider a Friedmann-Lemaître cosmology where space is homoge-
neous and isotropic, and, accordingly, we neglect the dependence of the fields $\varphi$ and $\theta$ on the
spatial coordinates. Their time-dependence is then governed by the following equations of
motion.

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial U(\varphi)}{\partial \varphi} + \frac{\mu^4}{f} \sin^2\left(\frac{\theta}{f}\right) e^{-2\varphi/f},$$  \hspace{1cm} (2.9)

with $-\partial U(\varphi)/\partial \varphi \simeq (2\Lambda/f) e^{-2\varphi/f}$ at late times, and

$$\ddot{\theta} + 3H\dot{\theta} = -\frac{\mu^4}{f} \sin\left(\frac{\theta}{f}\right) \cos\left(\frac{\theta}{f}\right) e^{-2\varphi/f},$$  \hspace{1cm} (2.10)

where $V$ is assumed to be given by (2.8), and terms involving the hypercharge gauge field
are neglected.

We propose to explore the possibility that $\varphi$ gives rise to dynamical Dark Energy, while
oscillations of $\theta$ about the minimum of the potential $V$ are a source of Dark Matter. Besides
the dark sector fields $\varphi$ and $\theta$, radiation contributes to the pressure and the energy density
of the early universe. We argue that, in the early universe, after the phase transition at
temperature $T_c$, the contribution of radiation to the energy density dominates over all other
contributions. As a consequence, space is expanding, and the oscillations of $\theta$ are damped.
At late times, $\theta$ approaches 0, and the first term on the right side of eq. (2.9) will become
the dominant term.

The requirement that this model predict the observed energy densities of Dark Energy
and Dark Matter entails that the amplitudes of the two terms on the right side of (2.9) must
have roughly the same mean at redshifts close to $z = 0.33$, when Dark Energy starts to
dominate. We assume that the initial value of $\varphi$ in the very early universe is negative, as is
typically done in Quintessence models, and that $\varphi$ starts out at rest. We further assume that
the potential for $\theta$ is generated at some early time corresponding to a temperature $T \approx T_c$,
and that, at that time, the initial condition for $\theta$ is close to a local maximum of the potential
$V(\varphi, \theta)$, viewed as a function of $\theta$ (with $\varphi$ kept fixed). In accordance with these assumptions,
the model introduced in this paper forces us to consider three time periods in the evolution of
the universe: the late period, when Dark Energy dominates; the intermediate era, when Dark
Matter dominates over Dark Energy; and the early epoch, when $\theta$ is close to a local maximum
of its potential and radiation dominates.\footnote{This assumption could be justified if the corresponding value of $\theta$ is a point of enhanced
symmetry of the classical Lagrangian including other matter fields, the symmetry is broken by quantum effects like in the
Coleman-Weinberg model [24], but thermal effects trap the field at early times.} During the early epoch, both $\varphi$ and $\theta$ are slowly
varying in time, and hence the energy density in $Z$ is approximately constant in time. With
the parameter values in the action functional (2.3) discussed below and the initial conditions
described above, the energy density in $Z$ then starts out much smaller than the energy density
of radiation. However, the contribution of the degrees of freedom described by the field $Z$ to
the energy density of the universe grows relative to the dominant contribution of radiation.
Once the field $\theta$ starts to oscillate around the minimum of $V$, and for an appropriate behavior
of $U$ for small values of $\varphi$, the equation of state of the degrees of freedom described by $Z$ is
$w \approx 0$. We will argue that parameter values in the action functional (2.3) can be chosen in
such a way that the contribution of the field $Z$ to the energy density of the universe becomes
equal to the energy density of radiation at the time $t_{eq}$, which is known from observations.
Appropriate choices of parameter values are discussed in section 3 of this paper.
Next, we present a more precise analysis of the time evolution of the fields \( \varphi \) and \( \theta \) during different periods in the history of the universe, neglecting their space-dependence. We begin by studying the **Dark Energy era**. This era is described, approximately, by an exact solution of the second order differential equation (2.9), with \( \theta \) set to 0, and choosing \( U(\varphi) = \Lambda e^{-2\varphi/f} \), see (2.4). This solution is given by

\[
\varphi(t) = f \ln(\beta t),
\]

where \( \beta \) is a constant that can be determined by inserting the ansatz (2.11) into (2.9), with \( H \) expressed in terms of the Friedmann equation

\[
H^2 = \frac{1}{3} m_{pl}^{-2} [U + \frac{1}{2} \dot{\varphi}^2].
\]

We obtain a quadratic equation for \( \beta^2 \). In the regime where \( f > m_{pl} \) the solution for \( \beta \) is

\[
\beta^2 \simeq \frac{4}{3} \frac{\Lambda f}{f^2} \left( \frac{m_{pl}}{f} \right)^2,
\]

which yields an equation of state

\[
w \simeq -1 + \frac{4}{3} \left( \frac{m_{pl}}{f} \right)^2.
\]

In this regime, the same result is obtained in the slow-roll approximation. As discussed in detail in [25–30], the solution (2.11) is a late-time attractor.

We observe that, on a Hubble time scale, the solution (2.11) for \( \varphi \) varies slowly in time. Thus, in eq. (2.10), we may assume \( \varphi \) to be constant (equal to the value it takes at the time \( t_i \) when Dark Energy begins to dominate). In this approximation, (2.10) becomes the equation of motion for a damped harmonic oscillator with frequency

\[
\omega \simeq \frac{\mu^2}{f} e^{-\varphi(t_i)/f}.
\]

A self-consistency condition for the validity of the approximation made above is that the frequency \( \omega \) be large, as compared to the Hubble expansion rate. As we will see later, this condition can be satisfied in our model. Since the potential for \( \theta \) is quadratic in the vicinity of its minimum, the equation of state of the degrees of freedom described by the field \( \theta \) is the one of pressureless Dark Matter. The amplitude, \( A(t) \), of the oscillations of \( \theta(t) \) decreases as

\[
A(t) \sim a(t)^{-3/2} \sim T(t)^{3/2}.
\]

We now turn to an analysis of the evolution of the field \( Z \) in the **intermediate era**: the field \( \theta \) exhibits damped oscillations, as in the Dark Energy era. However, for an appropriate choice of \( U \), the \( \theta \)-dependent term dominates the right side in the equation of motion (2.9), which then takes the form

\[
\ddot{\varphi} + 3H \dot{\varphi} \simeq \frac{\mu^4}{f} \sin^2 \left( \frac{\theta}{f} \right) e^{-2\varphi/f}.
\]

We neglect the time-dependence of \( e^{-2\varphi/f} \), setting \( e^{-2\varphi/f} = \text{const.} \), (and, for simplicity we set \( \text{const.} = 1 \)). Later, we will verify that this assumption is self-consistent. Furthermore,
we replace the $\sin^2$-term by its time average and make the small-angle approximation, with the source term quadratic in the amplitude. When the amplitude, $A$, of $\theta$ is inserted on the right side, eq. (2.17) becomes a first-order inhomogeneous differential equation for $\chi \equiv \dot{\varphi}$; namely
\[
\dot{\chi} + \frac{2}{t} \chi = \frac{1}{2 \beta^2} A^2(t_{eq}) \mu^4 \left( \frac{t_{eq}}{t} \right)^2,
\]
(2.18)
where $t_{eq}$ is the time of equal matter and radiation.

To arrive at the above expression, we approximate the sine by a linear function, normalize the amplitude at time $t = t_{eq}$ and make use of the matter scaling of the energy density of the axion field $\theta$. Furthermore, we have inserted the formula for $H$ during the matter-dominated era. The solution of eq. (2.18) is given by
\[
\chi(t) = \frac{\alpha}{t},
\]
(2.19)
with
\[
\alpha = \frac{\mu^4}{f^3} A^2(t_{eq}) t_{eq}^2,
\]
(2.20)
which implies that
\[
\varphi(t) = \alpha \ln \left( \frac{t}{t_{eq}} \right) + \varphi(t_{eq}).
\]
(2.21)
It is easy to check that $\alpha/f \ll 1$. Hence, the time dependence of $\varphi$ is negligible in this phase.

Note that formula (2.21) is valid for $t > t_{eq}$. A similar analysis of the equations of motion applies at times earlier than the time of equal matter and radiation. All that changes is the coefficient of the Hubble damping term in (2.17), as well as the time dependence of the amplitude of the oscillations of $\theta$. With the same approximations as those made above, the equation of motion for $\chi$ becomes
\[
\dot{\chi} + \frac{3}{2t} \chi = \frac{1}{2f} A^2(t_{eq}) \mu^2 \left( \frac{t_{eq}}{t} \right)^{3/2},
\]
(2.22)
which implies that
\[
\varphi(t) = \beta t^{1/2} + \text{const},
\]
(2.23)
with
\[
\beta = \frac{1}{f^3} A^2(t_{eq}) t_{eq}^{3/2} \mu^4.
\]
(2.24)
Given the parameter values discussed below, it is easy to check that $\varphi$ varies slowly as a function of time.

Since the time dependence of $\varphi$ in this phase is small while $\theta$ is undergoing damped oscillations, the total energy density in $Z$ - which is dominated by $V(\varphi, \theta)$ rather than $U(\varphi)$ (whose precise form is not well known for small values of $\varphi$) scales approximately as matter. For the choice of the potential given by $U(\varphi) \simeq e^{-2\varphi/f}$, this can be seen from the numerical analysis reported below (see figure 3). Hence, before the time $t_{eq}$ of equal matter and radiation, the total energy density is dominated by radiation. Our scenario is therefore consistent with the observation that, during the period of nucleosynthesis, the universe has to be dominated by radiation. If the period of nucleosynthesis takes place when $\theta$ is overdamped (which may happen early in the intermediate phase) then the contribution of $Z$ to the energy density is approximately constant, and radiation would become dominant earlier as we go
back in time. Our conclusion concerning nucleosynthesis would thus be unchanged. Note
that the smallness of the energy density of $Z$ in this phase (and in the early phase discussed
below) is automatically guaranteed if we choose our parameters (including the initial value
of $\varphi$) in such a way that a late-time dark energy phase emerges.

In this intermediate era, before the time of equal matter and radiation, the contribution,
$\rho_T$, of the degrees of freedom described by the field $Z$ to the total energy density does not scale
as radiation, as it might in the absence of the axion field. The scaling of $\rho_T$ is much closer to
the scaling of the energy density of cold matter, i.e., $\rho_T \sim a(t)^{-3}$. (If $\varphi$ were kept constant at
the value $\varphi_*$ corresponding to a local minimum of $U(\varphi)$, with $U(\varphi_*) = 0$, this scaling would
be exact.) Using the above equations of motion, with $U(\varphi) \simeq e^{-2\varphi/f}$, it can be checked that
cold-matter scaling of $\rho_T$ is a good approximation, provided $f \gg m_{pl}$. For values of $f$ close
to $m_{pl}$, there is a sizable deviation from matter scaling. We find that $\rho_T \sim a(t)^{-3-E}$; but the
value of $E$ is smaller than 1; ($E = 1$ would correspond to radiation). This claim is confirmed
by the numerical analysis reported below (see figure 3), which shows that, for a value of
$f = 2m_{pl}$, the exponent $E$ is smaller than 1. For a large range of parameter values in the
action functional (2.3), the time of nucleosynthesis occurs in the intermediate era, as will be
shown in the next section. Since, by the arguments sketched above, we have that $\rho_T \ll \rho_r$ during nucleosynthesis, where $\rho_r$ is the energy density contributed by radiation, our model
is compatible with known bounds from cosmological nucleosynthesis.

Next, we discuss the evolution of the fields $\varphi$ and $\theta$ during the early epoch. We choose
initial conditions for $\varphi$ and $\theta$ at some early time $t_c$, plausibly the time of the phase transition
when a non-trivial effective potential for the axion field $\theta$ is generated. We set

$$\varphi(t_c) = \varphi_0 \leq 0.$$  \hspace{1cm} (2.25)

For small (in particular, negative) values of $\varphi$, the potential $U(\varphi)$ need not have the exponential form it approaches for large values of $\varphi$. In fact, the physics at early times when $\varphi$ is small is presently unknown. Luckily, all we need to assume is that there is some period, after the phase transition, during which $\varphi$ increases slowly, the term $U(\varphi)$ is subdominant as compared to $V(\varphi, \theta)$, and the contribution of $Z$ to the vacuum energy density is negligibly small. For concreteness, we will use the exponential form of the potential $U$ in making an order-of-magnitude estimate of the initial field velocities. This estimate will then be used in our analysis of baryogenesis.

If we write

$$\varphi = \varphi_0 + \Delta \varphi, \quad \theta = \theta_0 + \Delta \theta,$$  \hspace{1cm} (2.26)

and assume that $\sin^2(\theta/f) \approx 1$ then $\Delta \varphi$ is approximately given by a solution of the equation

$$\frac{d^2 \Delta \varphi}{dt^2} + 3H \frac{d \Delta \varphi}{dt} = \frac{\mu^4}{f} \kappa,$$  \hspace{1cm} (2.27)

with

$$\kappa = e^{-2\varphi_0/f}.$$  \hspace{1cm} (2.28)

Setting $\dot{\varphi}(t_c) = 0$, the solution has the form

$$\Delta \varphi(t) = A(t) \frac{\mu^4}{2f} \kappa (t - t_c)^2,$$  \hspace{1cm} (2.29)

where the initial value of the amplitude $A$ is 1, and $A$ is slowly decreasing in time, due to Hubble damping.
Assuming that, in the early epoch, $\Delta \varphi \ll f$, the evolution equation for $\Delta \theta$ takes the approximate form

$$\frac{d\Delta \theta}{dt^2} + \frac{3}{2t} \frac{d\Delta \theta}{dt} = \frac{\mu^4}{f^2} \kappa \Delta \theta,$$

where we have inserted the Hubble expansion rate in the radiation phase. Not surprisingly, this is the equation of motion of a damped inverted harmonic oscillator, and the solution is an exponentially growing function multiplied by an amplitude, $B$, that decays in time, due to the Hubble damping:

$$\Delta \theta \equiv B(t) e^{-\frac{\mu^2}{f} \sqrt{\kappa}(t-t_c)}.$$  \hspace{1cm} (2.31)

If $\frac{\mu^2}{f} \sqrt{\kappa} < \frac{3}{4t}$, the amplitude $B(t)$ scales as the inverse scale factor, and hence

$$\Delta \theta \sim \Delta \theta(t_c) \left( \frac{t_c}{t} \right)^{1/2} e^{-\frac{\mu^2}{f} \sqrt{\kappa}(t-t_c)}.$$  \hspace{1cm} (2.32)

To make contact with sections IV and V on baryogenesis we note that, from (2.32), during this early evolution of $\theta$

$$\dot{\theta} \sim \Delta \theta(t) \frac{1}{t} \sim \frac{f}{t},$$  \hspace{1cm} (2.33)

and that, in comparison, $\dot{\varphi}$ makes a negligible contribution to the kinetic energy density.

We have corroborated the heuristic analysis presented above with a numerical study of the system of three coupled differential equations consisting of eqs. (2.9) and (2.10), for the fields $\varphi$ and $\theta$, and of the Friedmann equation

$$H^2 = \frac{m_{pl}^2}{3} \left[ \frac{1}{2} \dot{\varphi}^2 + \frac{1}{2} \dot{\theta}^2 + U(\varphi) + V(\varphi, \theta) \right],$$  \hspace{1cm} (2.34)

for the Hubble parameter $H(t)$, with $U(\varphi)$ set to $\Lambda e^{-2\varphi/f}$, for concreteness. In the simulations, all quantities are expressed in Planck units, including time, the dimensionless time being $\tau = m_{pl} t$. In Figure 1, the evolution of $\Delta \theta$ as a function of time is displayed. Figure 2 shows the evolution of $\varphi$ and of $\Delta \theta$ as functions of time. Figure 3 displays the time evolution of the parameter $w = p/\rho$ appearing in the total equation of state; and figure 4 shows how the ratio of the contributions of Dark Matter and Dark Energy to the potential energy evolves. In our simulation, we have chosen the parameter values $f = 2m_{pl}$, $\mu = 5$, and $\Lambda = 10^{-8}$. The initial conditions have been chosen to be $\varphi = \varphi_0 = 0$, $\dot{\varphi} = m_{pl}^{-1} \ddot{\varphi} = 1$ (the prime denoting a derivative with respect to $\tau$), and $\theta = \theta_0$ displaced from the local maximum at $\theta = -\pi/2$ by $\theta_0 = 10^{-2}$, with $\dot{\theta} = 0$.

The figures show that there are smooth transitions between the three epochs described in the text — the early epoch, when $\theta$ starts to slowly roll from a value close to the one corresponding to a local maximum of the potential $V$, the intermediate era, when $\theta$ oscillates about a minimum of its potential, yielding an era of Dark-Matter domination, and — after the oscillations of $\theta$ have redshifted — the onset of the Dark Energy era, when the ratio, $w$, of pressure to energy density approaches a negative constant.

In our numerical study, the model parameters have not been given realistic values, but have been chosen so as to facilitate the implementation of the numerics.
Figure 1. Time evolution of the displacement, $\Delta \theta$, of the field $\theta$ from its value at the local maximum, for the parameter values and initial conditions chosen in the text. The field and time are in Planck units. After a time period of slow rolling, $\theta$ begins to oscillate about the minimum of its potential. The amplitude of oscillation is damped by the cosmological expansion.

Figure 2. Time evolution of both $\varphi$ and $\Delta \theta$ (in Planck units).

Figure 3. Time evolution of the (total) equation-of-state parameter, $w$. This simulation does not take into account any radiation. Hence, initially, $w$ is negative, since the potential energy dominates over the kinetic energy. Once $\theta$ begins to oscillate about the minimum of its potential, the time average of $w$ has the value typical of Dark Matter ($w \sim 0$). Eventually, the energy stored in the oscillations of $\theta$ has redshifted sufficiently for the $\Lambda$ term in the potential to start to dominate. This signals the onset of the Dark Energy phase.

Figure 4. The ratio $R$ (vertical axis) of the potential energy contributed by $V(\varphi, \theta)$ and the one contributed by the $\Lambda$ term as a function of time (horizontal axis). When this ratio drops below 1 the Dark Energy phase sets in.

3 Physical constraints on parameter values

There are five free parameters in our model, namely $f, U(\varphi_0)$ (where $\varphi_0$ is the initial condition for the field $\varphi$ at time $t = t_c$, see (2.25)), $\Lambda, \mu$ and $T_c$. We propose to estimate the values they must be given for our scenario to work. We start by recalling the various times involved in our analysis: $t_0$, the present time; $t_i$, the time when the Dark Energy era begins; $t_{eq}$, the
time of equal matter and radiation; and $t_c$, the time when the phase transition generating the potential, $V$, for $\theta$ occurs. In the following we are only interested in order-of-magnitude estimates of the different parameters appearing in our equations.

We begin by noting that, in order for the equation of state of $\varphi$ to correspond to the one of Dark Energy at times after $t_i$, we must impose the condition that $f \geq m_{pl}$, which is well-known in Quintessence models with exponential potentials. In the following we set $f = 2m_{pl}^2$ to simplify our estimates of the remaining parameters.

The next condition is that, at late times, the field $\varphi$ contributes an amount to the energy density of the universe sufficient to explain the currently observed Dark Energy. Since, in the Dark Energy era, for large times, the angular field $\theta$ is very small, the first term, $U(\varphi) \simeq \Lambda e^{-2\varphi/f}$, in the potential appearing in the action functional (2.7) dominates over the second term, $V(\varphi, \theta)$, this condition is

$$\Lambda e^{-2\varphi(t_0)/f} \sim T_0^4 z_{eq},$$

(3.1)

where $T_0$ is the current temperature of radiation, and the factor $z_{eq}$ is the redshift at the time $t_{eq}$ of equal matter and radiation. Note that the right hand side of this equation is the current matter energy density which is larger than the current radiation density by the factor of $z_{eq}$ since the radiation and matter energy densities are the same at $t_{eq}$ but the radiation density decreases faster than the matter density by a factor of $z(t)$.

A constraint on the mass scale $\mu$ is obtained by demanding that, at the present time, the oscillations of $\theta$ yield the correct dark matter density. This condition reads

$$\mu^4 A^2(T_0) \frac{T_0^4}{f^2} e^{-2\varphi(t_0)/f} \sim T_0^4 z_{eq}.$$  

(3.2)

In the order-of-magnitude estimates described here we take roughly equal contributions of Dark Energy and of Dark Matter to the current energy density of the universe.

As argued in the previous section, today’s value of $\varphi$ is close to $f$. This allows us to neglect the exponential factors in (3.1) and (3.2). Then (3.1) becomes

$$\Lambda \sim T_0^4 z_{eq}.$$  

(3.3)

Using the fact that the temperature dependence of $A$ is $\propto T_0^{3/2}$, and assuming that the initial amplitude is of the order of $f$, condition (3.2) becomes

$$\mu^4 \left(\frac{T_0}{T_c}\right)^3 \sim T_0^4 z_{eq}.$$  

(3.4)

Note that conditions (3.3) and (3.4) are similar to the tunings required in every dynamical dark sector model known at present: there is no explanation of the fact that Dark Energy, Dark Matter and visible matter yield comparable contributions (within one order of magnitude) to the total energy density of the universe just at the present time. It is important to check that, besides the tuning conditions that guarantee that this fact is properly reproduced in our model, no additional fine-tuning of parameter values is required.

The value of the mass parameter $\mu$ determines the mass, $m_{DM}$, of Dark Matter modes, because this mass is given by the frequency (2.15) of oscillations of $\theta$. Setting the exponential

---

2To obtain an equation of state of Dark Energy sufficiently close to $w = -1$ to agree with current data, we would require a slightly larger value of $f$. 

---
factor in eq. (2.15) to 1, as above, we obtain
\[ m_{DM} \sim \frac{\mu^2}{f}. \] (3.5)
Writing
\[ m_{DM} \equiv m_a 1\text{eV}, \] (3.6)
where \( m_a \) is a dimensionless number, with \( m_a \leq 1 \), the value of the parameter \( \mu \) is determined by \( m_a \). Eq. (3.4) then determines \( T_c \). We find that
\[ \mu \sim m_a^{1/2}10^{5}\text{GeV} \] (3.7)
\[ T_c \sim m_a10^{14}\text{GeV}. \] (3.8)
In order to end up with a Dark Matter mass in the range of a typical axion mass, \( m_{\text{axion}} \sim 1\text{eV} \), the scales \( \mu \) and \( T_c \) are very high energy scales. But to obtain a mass \( m_{DM} \sim 10^{-20}\text{eV} \) corresponding to ultralight Dark Matter, the values of \( \mu \) and \( T_c \) would have to be in the range \( \mu \sim T_c \sim 10^6\text{eV} \). We note that, for \( m_a > 10^{-18} \), the critical temperature \( T_c \) corresponds to a time earlier than the time of nucleosynthesis, and that the period of underdamped oscillations of \( \theta \) does not need to start at the temperature \( T_c \). Without facing inconsistencies we may assume that it starts at a time earlier than \( t_{eq} \).

4 Baryogenesis in the early universe

It is natural to assume that the gradient of the imaginary part, \( \Im Z \), of the complex scalar field \( Z \) couples to the baryon current, \( j_B^\mu \), as described by the term
\[ \delta L = \tilde{\alpha} \partial_\mu (\Im Z) j_B^\mu, \] (4.1)
where \( \tilde{\alpha} \) is a dimensionless coupling constant. The presence of this term in the Lagrangian can be motivated by comparing it with the last term in (2.3) and recalling the chiral anomaly for the baryon current
\[ \partial_\mu j_B^\mu \sim \frac{g^2}{16\pi^2} F \wedge F, \] (4.2)
where the masses of quarks are neglected, and where \( F \) is a non-Abelian gauge field coupling to baryon number. The field \( F \) could be the \( SU(2) \) gauge field of the electroweak interactions; but we leave the question as to what the physical nature of \( F \) is open; it is natural to suppose that it is the gauge field, previously denoted by \( G \), that generates the potential for \( \theta \), discussed at beginning of this paper.

Following [20, 21], we note that, during an era when \( \Im Z \) is rolling uniformly, the above interaction term generates a chemical potential, \( \mu_B \), conjugate to baryon number
\[ \mu_B = \bar{\alpha}(\Im Z) = \frac{\bar{\alpha}}{f} [\dot{\theta} - \frac{\dot{\varphi}}{f} \theta] e^{-\varphi/f}. \] (4.3)
In our cosmological scenario, \( \Im Z \) is rolling uniformly in the initial era, right after the time \( t_c \) of the phase transition generating the effective potential of \( \theta \). Thus, we need to estimate the value of \( \mu_B \) at these early times, making use of the results in eqs. (2.29) and (2.33). We find that
\[ \mu_B \sim \frac{\bar{\alpha} T^2}{m_{pl}}, \] (4.4)
where we have used the Friedmann equation and the Stefan-Boltzmann law to express time \( t \) in terms of temperature \( T \).

As long as baryon-number violating interactions involve degrees of freedom that are in thermal equilibrium during the early phase in the evolution of the universe considered here, a chemical potential \( \mu_B \) corresponds to a baryon number density, \( n_B \), of the order of

\[
    n_B \sim \mu_B T^2, \quad (4.5)
\]

and the induced baryon-number density-to-entropy ratio is found to be given by

\[
    \frac{n_B}{s} \sim \frac{T}{m_{pl}}, \quad (4.6)
\]

which is to be evaluated for values of the temperature \( T \) corresponding to the initial period of the field evolution, i.e., for a value of \( T \) of the order of \( T_c \).

For the baryogenesis scenario described here to work baryon-number violating processes must be in local thermal equilibrium when the temperature of the universe is close to \( T_c \). Moreover, the era of slow roll of \( \theta \) and \( \varphi \) must last at least a Hubble time in order for local thermal equilibrium to be established, which is what would justify introducing the chemical potential \( \mu_B \). It is easy to check that this latter condition is satisfied. The time scale of slow roll can be inferred from (2.32) and is given by \( \gamma^{-1} \sim f/(\mu^2 \sqrt{\kappa}) \). Since \( f > m_{pl} \) and \( \kappa < 1 \), it follows that \( \gamma^{-1} > H^{-1} \). Furthermore, we have to require that the slow roll of \( \theta \) sets in around the time of the electroweak phase transition, when a non-vanishing baryon number is generated. Thus, another condition for the mechanism described here to work is that \( T_c \geq T_{EW} \), where \( T_{EW} \) is the temperature of electroweak symmetry breaking.

We note that the term (4.1) in the Lagrangian of our model violates baryon number conservation. One might then expect that the assumption of local thermal equilibrium during baryogenesis made above is not really necessary. This motivates the discussion of a possible scenario presented in the next section.

5 Baryogenesis from hypercharge helicity

It is well known that the baryon current is anomalous (see [32, 33] for the original articles on the chiral anomaly, [34–36] for reviews of applications of the chiral anomaly to baryogenesis, and [37] for an application of the chiral magnetic effect of electromagnetism to magnetic field generation). In particular, the change in baryon number is proportional to the change of the hypercharge helicity [38, 39] (see [40–43]):

\[
    \Delta N_B = C_y \frac{\alpha_y}{8\pi} \Delta \mathcal{H}, \quad (5.1)
\]

where \( \alpha_y \) is the hypercharge fine structure constant and \( C_y \) is a constant depending on the particle content of the model used to describe visible matter; (see, e.g., [43] for values of \( C_y \)).

The variation in time of the density, \( h \), of the hypercharge helicity is given by

\[
    \dot{h} = -2 < \mathbf{E} \cdot \mathbf{B} >, \quad (5.2)
\]

where \( \mathbf{E} \) and \( \mathbf{B} \) are the electric and magnetic fields of \( U(1)_Y \), and the angular brackets indicate spatial averaging. Here and in the following we neglect the expansion of the universe.
At the end of this section we will comment on the effects caused by its expansion. In
a regime where the time derivative of the electric field can be neglected the equations of
magnetohydrodynamics imply that \[44, 45\]
\[
\mathbf{E} \cdot \mathbf{B} = \frac{1}{\sigma} \mathbf{B} \cdot (\nabla \times \mathbf{B}),
\]
(5.3)
where \(\sigma\) is the conductivity, whose order of magnitude is given by the temperature, i.e.,
\[
\sigma \sim T.
\]
(5.4)
The Fourier modes, \(A_k\), of the hypercharge gauge field \(A\) contribute to the spatial average
of \(\mathbf{E} \cdot \mathbf{B}\). As shown in [43], the expression for the spatial average of the right side of (5.3) is
given by
\[
< \mathbf{B} \cdot (\nabla \times \mathbf{B}) > = \int \frac{d^3k}{(2\pi)^3} |k|^2 (|A_{k,+}|^2 - |A_{k,-}|^2),
\]
(5.5)
where the subscripts + and − indicate the helicities of the modes.
We assume that the field \(Z\) also couples to the hypercharge instanton density via a term
\[
\delta \mathcal{L}_2 = \frac{\alpha}{4} \Im Z \tilde{Y}_{\mu\nu} Y^{\mu\nu},
\]
(5.6)
where \(\alpha\) is a dimensionless coupling constant, \(Y_{\mu\nu}\) is the field strength associated with \(A\), and
\(\tilde{Y}_{\mu\nu}\) is its dual; (this term arises from the one in (2.3) by integration by parts, after replacing
\(\theta\) by \(\Im Z\)). The equation of motion for \(A_k\) then becomes \([38, 39, 43]\]
\[
\ddot{A}_{k,+} + (k^2 \pm \alpha \Im Z) A_{k,\pm} = 0,
\]
(5.7)
As shown in [46], the pseudoscalar field \(\Im Z\) can induce growth of the helicity of the hyper-
charge field. As long as the time derivative of \(\Im Z\) has a fixed sign, a property it has in our
model during the initial era of evolution, one helicity mode is enhanced, for small values of
\(k\), while the other one exhibits damped oscillations. In the following we estimate the am-
plification of the growing Fourier modes (the helicity label on the Fourier modes \(A_k\) is now
omitted).
For small values of \(\theta\) we can approximate \(\cos(\theta)\) by 1 and \(\sin(\theta)\) by \(\theta\) and find that
\[
(\Im Z)^{-1} = \frac{1}{f} e^{-\varphi/f} \left(\dot{\theta} - \varphi \frac{\theta}{f}\right).
\]
(5.8)
Inserting (2.29) and (2.33) for \(\varphi\) and \(\theta\) in the initial epoch, we find that the two terms on
the right side of (5.8) coincide, up to a factor of 2. Hence, in (5.7), we can replace \(\Im Z\) by
\(|Z|\dot{\theta}/f\). The equation of motion for \(A_k\) then becomes
\[
\ddot{A}_k + (k^2 \pm \alpha ke^{-\varphi/f} \dot{\theta}) A_k = 0.
\]
(5.9)
An approximate solution of this equation is obtained by making use of
\[
\dot{\theta} \sim \frac{f}{t}.
\]
(5.10)
We also approximate $e^{-\varphi/f}$ by the value it has at the beginning of rolling. Taking into account that $f \sim m_{pl}$ we get

$$\dot{A}_k + [(k^2 - \alpha k k^{-1/2} t^{-1})] A_k = 0.$$ \hspace{1cm} (5.11)

We define the “critical wavenumber” $k_c$ by

$$k_c = \alpha k^{-1/2} t^{-1}.$$ \hspace{1cm} (5.12)

We then find that modes with $|k| < k_c$ are exponentially amplified, whereas modes with $|k| \geq k_c$ oscillate with constant amplitude.

The growth of the modes $A_k$, for $|k| < k_c$, is turned off by back-reaction: the energy density of the field quanta produced by the growth of the unstable Fourier modes of $A$ cannot exceed the one of radiation before non-linear effects become important. (The logic here is similar to the one used to explain the termination of the preheating instability [47, 48] in reheating after inflation; see, e.g., [49, 50] for recent reviews). The energy density of the field quanta of $A$ is given by

$$\rho_A \sim \int d^3 k k^2 A_k^2,$$ \hspace{1cm} (5.13)

an integral dominated by the contribution of the integrand around $k \sim k_c$. The amplitude of the $A_k$-mode at times $t > t_c$, starting from vacuum initial conditions at time $t_c$, is given by

$$A_k(t) = \frac{1}{\sqrt{2k}} e^{(\alpha k k_c)^{1/2} (t-t_c)},$$ \hspace{1cm} (5.14)

where the origin of prefactor $(\sqrt{2k})^{-1}$ is explained by recalling that the harmonic oscillator $A_k$ has been starting in its ground state. Considering the growth rate of the $A_k$ modes described in (5.14), with $k = 0$, we obtain that

$$\rho_A \sim k_c^4 e^{2\alpha^{1/2} k_c (t-t_c)}.$$ \hspace{1cm} (5.15)

The time when the growth of helicity ends is determined, approximately, by equating the energy density $\rho_A$ with the energy density of degrees of freedom contributing to radiation, which is proportional to $T^4$. Since $T \sim T_c$, the length, $\delta t$, of the time interval during which the helicity grows is given by

$$e^{2\alpha^{1/2} k_c \delta t} \sim \left( \frac{T_c}{k_c} \right)^4.$$ \hspace{1cm} (5.16)

Having determined the duration, $\delta t$, of the period over which the hypercharge helicity grows, we return to (5.2) with the purpose of estimating the baryon number density, $\Delta n_B$, produced during that period. We find that

$$\Delta n_B \sim C_y \frac{\alpha y}{8\pi} \frac{1}{\sigma} \frac{1}{\sqrt{\alpha}} k_c^5 \frac{1}{4\pi^2} e^{2\alpha^{1/2} k_c \delta t} \frac{1}{\sqrt{\alpha k_c}},$$ \hspace{1cm} (5.17)

where the last factor on the right hand side comes from integration over time. Inserting the cutoff value (5.16), we find an order-of-magnitude estimate

$$\Delta n_B \sim C_y \frac{\alpha y}{8\pi} \alpha^{1/2}.$$ \hspace{1cm} (5.18)

We thus conclude that the mechanism sketched above has the efficiency to produce the observed baryon number to entropy ratio.
6 Conclusions and discussion

In this paper we have introduced and studied a model of a complex field \( Z = e^{-(\varphi + i\theta)/f} \) describing the presence of plausible amounts of Dark Matter and Dark Energy in the universe. At late times, the energy density stored in the radial part, \( e^{-\varphi/f} \), of \( Z \) can be interpreted as Dark Energy. The gradient of the imaginary part of \( Z \) is coupled to the anomalous baryon current and hence to gauge degrees of freedom. After a phase transition, \( \Im Z \) acquires a periodic effective potential generated by integrating out the matter and gauge degrees of freedom. The field \( \theta \) will then eventually start to oscillate about a minimum of its potential at \( \theta = 0 \), with a frequency (rest mass) that decreases in time like \( e^{-\varphi/f} \). These oscillations yield light Dark Matter. Assuming that, after the phase transition, \( \theta \) exhibits a slow roll starting from an initial value close to a local maximum of its potential, a non-vanishing baryon number is generated during the period of slow roll and before the oscillations of \( \theta \) set in. We conclude that the model discussed in this paper may apparently account for baryogenesis in the very early universe, Dark Matter at intermediate times, and Dark Energy at late times.

Our model should be compared with another model, inspired by a scenario proposed in [31], that has been introduced in a previous paper [23]. In the latter model, Dark Matter and Dark Energy are assumed to originate from the dynamics of a single real scalar field, \( \varphi \). However, an additional scalar field must be introduced to trigger a phase transition, reminiscent of what is known as a “wetting transition”, from a phase where \( \varphi \) produces a high density of Dark Matter to a low-density phase describing Dark Energy. A substantial amount of fine-tuning of the parameters is necessary in order for known model-building- and cosmological constraints to be satisfied. In addition, the model studied in [23] can only describe Dark Matter modes with very tiny masses, corresponding to ultralight Dark Matter. The model discussed in the present paper does not require as much fine-tuning as the model in [23]. Its parameters can be adjusted so as to describe a rather wide range of Dark Matter masses. An additional advantage of the new model, as compared to the one studied in [23], is that its degrees of freedom naturally couple to the anomalous baryon current and to the hypercharge gauge field, and hence it may also describe baryogenesis.

The action of the model studied in this paper appears to satisfy constraints on effective field theories derived from superstring theory; see [11, 12]. But, like other models of dynamical Dark Energy, it does not shed any light on the “coincidence problem”, namely on the question why Dark Energy is becoming dominant precisely at the present time.

In our model, the Dark Matter and Dark Energy fields are coupled. It would be interesting to explore the consequences of this coupling for the evolution of cosmological fluctuations. We leave this for future work.

Acknowledgments

We thank Z. Wang for generating the figures displayed in this paper, and R. Namba for collaboration during initial stages of this project. RB thanks the Pauli Center and the Institutes of Theoretical Physics and of Particle- and Astrophysics of the ETH for hospitality. The research at McGill is supported, in part, by funds from NSERC and from the Canada Research Chair program. JF is a member of the NCCR SwissMAP, which is sponsored by the Swiss National Foundation.
A Possible roots of the model

In this appendix we speculate about possible roots of the model studied in this paper in fundamental theories of Nature. As described in the main text, our model involves a complex scalar field $Z$ given in terms of real component fields by (2.1). The angular variable $\theta$ plays a role similar to the one of the axion in QCD (see e.g. [51] for a review of the coupling of the QCD axion to the QCD gauge fields). The action for $Z$ (in the absence of any couplings to matter and gauge fields) is given by (2.5). Complex scalar fields similar to the field $Z$ are ubiquitous in effective field theories derived from superstring theory. An example encountered in string theory is the axion-dilaton field

$$\hat{\tau} = -e^{-\Phi} + iC_0,$$

(A.1)

where $C_0$ is an axion that originates in the Ramond-Ramond zero form, and $\Phi$ is the dilaton; see, e.g., [52] for a review. At the classical level, the potential is flat in the axionic direction as a consequence of the usual shift symmetry. World sheet- or D-brane instanton effects break this continuous symmetry to a discrete symmetry and generate a potential for the axion $C_0 \equiv a$ of the form

$$V(a) \sim g\mu^4 \sin(a/f),$$

(A.2)

where $\mu$ is a constant of mass dimension 1, and the dimensionless coefficient $g$ is determined by the string coupling constant, i.e.,

$$g \sim e^{-\Phi}.$$  

(A.3)

This is one example of how the potential in eq. (2.8) could arise.

String theories on space-times with six compactified dimensions also tend to give rise to complex scalar fields, with a self-interaction potential of the kind we are considering in this paper. To be specific, we think of an internal space given by a Calabi-Yau manifold. The ten-dimensional metric can then be written as (see, e.g., the discussion in section 3 of [52])

$$ds^2 = e^{-6u(x)}g_{\mu\nu}(x)dx^\mu dx^\nu + e^{2u(x)}\tilde{g}_{ab}(y)dy^a dy^b,$$

(A.4)

where $x^\mu$ are coordinates of the four-dimensional space-time, and $y^a$ are coordinates of the internal Calabi-Yau manifold with metric $\tilde{g}_{ab}(y)$, which we assume to be fixed. The real scalar field $u(x)$ encodes the overall scale of the internal manifold. The field $u$ is related to the real part of a complex modulus field, $T$, namely

$$RT \equiv e^{4u}.$$  

(A.5)

The imaginary part of $T$ arises from the dimensional reduction of the four-form potential. In the low energy (supergravity) limit, dimensional reduction of the ten-dimensional Ricci scalar in the Einstein-Hilbert action yields a canonical kinetic term in the effective action of the field $u(x)$. If the internal manifold has negative curvature, then $u$ acquires a positive potential given by

$$V(u) \sim e^{8u}.$$  

(A.6)

(See also [53] for a derivation of a complex scalar field from compactification of extra dimensions.)

String compactifications in the presence of fluxes [54, 55] exhibit further fields that, after dimensional reduction to a four-dimensional space-time, are complex scalars with an axionic

\[ \hat{\tau} = -e^{-\Phi} + iC_0, \]  

(A.1)
angular variable. For example, in type IIB string theory compactified on a six-dimensional Calabi-Yau manifold $X_6$, axion fields, $a$, with a potential of the form given in (A.2), which are space-time pseudo-scalars, arise naturally. The mass scale $f$ is then given by [52]

$$\frac{f}{m_{pl}} \sim \left(\frac{l_s}{L}\right)^2,$$

where $L$ is a length scale characteristic of the internal manifold $X_6$, and $l_s$ is the string length.

In supersymmetric gauge theories, the gauge coupling constant, $g$, and the vacuum angle, $\Theta$, appear in the combination $\frac{4\pi}{g^2} - \frac{i\Theta}{2\pi}$, see [56]. One may imagine that $g$ and $\Theta$ are related to the expectation value of a dynamical complex scalar field, $\hat{\tau} = \varphi + i\theta$, where $\varphi$ is a real scalar field and $\theta$ is a pseudo-scalar axion field, with

$$\langle \varphi \rangle = \frac{4\pi}{g^2}, \quad \langle \theta \rangle = \frac{\Theta}{2\pi},$$

with $f \approx m_{pl}$, as above; see e.g. [57] for a review. The field $Z$ appearing in the model discussed in this paper could be the exponential of $\hat{\tau}$, i.e.,

$$Z \equiv e^{-\hat{\tau}/f}.$$ (A.9)

One may then argue that the gradient of the imaginary part of $Z$ couples to the Chern-Simons three-form of some non-abelian gauge field or to the anomalous baryon current.

References

[1] Planck collaboration, Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. 641 (2020) A6 [arXiv:1807.06209] [SPIRE].

[2] G. Jungman, M. Kamionkowski and K. Griest, Supersymmetric dark matter, Phys. Rept. 267 (1996) 195 [hep-ph/9506380] [SPIRE].

[3] P.J. Fox, G. Jung, P. Sorensen and N. Weiner, Dark Matter in Light of the LUX Results, Phys. Rev. D 89 (2014) 103526 [arXiv:1401.0216] [SPIRE].

[4] H. Ooguri and C. Vafa, On the Geometry of the String Landscape and the Swampland, Nucl. Phys. B 766 (2007) 21 [hep-th/0605264] [SPIRE].

[5] G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, de Sitter Space and the Swampland, arXiv:1806.08362 [SPIRE].

[6] T.D. Brennan, F. Carta and C. Vafa, The String Landscape, the Swampland, and the Missing Corner, PoS TASI2017 (2017) 015 [arXiv:1711.00864] [SPIRE].

[7] E. Palti, The Swampland: Introduction and Review, Fortsch. Phys. 67 (2019) 1900037 [arXiv:1903.06239] [SPIRE].

[8] G. Dvali and C. Gomez, Quantum Exclusion of Positive Cosmological Constant?, Annalen Phys. 528 (2016) 68 [arXiv:1412.8077] [SPIRE].

[9] G. Dvali and C. Gomez, On Exclusion of Positive Cosmological Constant, Fortsch. Phys. 67 (2019) 1800092 [arXiv:1806.10877] [SPIRE].

[10] G. Dvali, C. Gomez and S. Zell, Quantum Breaking Bound on de Sitter and Swampland, Fortsch. Phys. 67 (2019) 1800004 [arXiv:1810.11002] [SPIRE].

[11] P. Agrawal, G. Obied, P.J. Steinhardt and C. Vafa, On the Cosmological Implications of the String Swampland, Phys. Lett. B 784 (2018) 271 [arXiv:1806.09718] [SPIRE].
[12] L. Heisenberg, M. Bartelmann, R. Brandenberger and A. Refregier, Dark Energy in the Swampland, *Phys. Rev. D* 98 (2018) 123502 [arXiv:1808.02877] [SPIRE].

[13] C. Wetterich, Cosmology and the Fate of Dilatation Symmetry, *Nucl. Phys. B* 302 (1988) 668 [arXiv:1711.03844] [SPIRE].

[14] P.J.E. Peebles and B. Ratra, Cosmology with a Time Variable Cosmological Constant, *Astrophys. J. Lett.* 325 (1988) L17 [SPIRE].

[15] B. Ratra and P.J.E. Peebles, Cosmological Consequences of a Rolling Homogeneous Scalar Field, *Phys. Rev. D* 37 (1988) 3406 [SPIRE].

[16] L.D. Duffy and K. van Bibber, Axions as Dark Matter Particles, *New J. Phys.* 11 (2009) 105008 [arXiv:0904.3346] [SPIRE].

[17] S. Alexander, R. Brandenberger and J. Froehlich, Dark Energy and Dark Matter in a Model of an Axion Coupled to a Non-Abelian Gauge Field, arXiv:1609.06920 [SPIRE].

[18] S. Alexander, R. Brandenberger and J. Froehlich, Tracking Dark Energy from Axion-Gauge Field Couplings, arXiv:1601.00057 [SPIRE].

[19] D. Bertacca, N. Bartolo and S. Matarrese, Unified Dark Matter Scalar Field Models, *Adv. Astron.* 2010 (2010) 904379 [arXiv:1008.0614] [SPIRE].

[20] A.G. Cohen and D.B. Kaplan, Thermodynamic Generation of the Baryon Asymmetry, *Phys. Lett. B* 199 (1987) 251 [SPIRE].

[21] A.G. Cohen and D.B. Kaplan, SPONTANEOUS BARYOGENESIS, *Nucl. Phys. B* 308 (1988) 913 [SPIRE].

[22] K. Kamada and A.J. Long, Baryogenesis from decaying magnetic helicity, *Phys. Rev. D* 94 (2016) 063501 [arXiv:1606.08891] [SPIRE].

[23] R. Brandenberger, J. Fröhlich and R. Namba, Unified Dark Matter, Dark Energy and baryogenesis via a “cosmological wetting transition”, *JCAP* 09 (2019) 069 [arXiv:1907.06353] [SPIRE].

[24] S.R. Coleman and E.J. Weinberg, Radiative Corrections as the Origin of Spontaneous Symmetry Breaking, *Phys. Rev. D* 7 (1973) 1888 [SPIRE].

[25] F. Lucchin and S. Matarrese, Power Law Inflation, *Phys. Rev. D* 32 (1985) 1316 [SPIRE].

[26] J.J. Halliwell, Scalar Fields in Cosmology with an Exponential Potential, *Phys. Lett. B* 185 (1987) 341 [SPIRE].

[27] J.D. Barrow, Cosmic No Hair Theorems and Inflation, *Phys. Lett. B* 187 (1987) 12 [SPIRE].

[28] J. Yokoyama and K.-i. Maeda, On the Dynamics of the Power Law Inflation Due to an Exponential Potential, *Phys. Lett. B* 207 (1988) 31 [SPIRE].

[29] D. Wands, E. J. Copeland and A. R. Liddle, Exponential potentials, scaling solutions and inflation, *Ann. New York Acad. Sci.* 688 (1993) 647.

[30] E.J. Copeland, A.R. Liddle and D. Wands, Exponential potentials and cosmological scaling solutions, *Phys. Rev. D* 57 (1998) 4686 [gr-qc/9711068] [SPIRE].

[31] R. Brandenberger, R.R. Cuzinatto, J. Fröhlich and R. Namba, New Scalar Field Quartessence, *JCAP* 02 (2019) 043 [arXiv:1809.07409] [SPIRE].

[32] S.L. Adler, Axial vector vertex in spinor electrodynamics, *Phys. Rev.* 177 (1969) 2426 [SPIRE].

[33] J.S. Bell and R. Jackiw, A PCAC puzzle: $\pi^0 \rightarrow \gamma \gamma$ in the $\sigma$ model, *Nuovo Cim. A* 60 (1969) 47 [SPIRE].
[34] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, *On the Anomalous Electroweak Baryon Number Nonconservation in the Early Universe*, Phys. Lett. B 155 (1985) 36 [inSPIRE].

[35] A.D. Dolgov, *NonGUT baryogenesis*, Phys. Rept. 222 (1992) 309 [inSPIRE].

[36] A. Riotto and M. Trodden, *Recent progress in baryogenesis*, Ann. Rev. Nucl. Part. Sci. 49 (1999) 35 [hep-ph/9901362] [inSPIRE].

[37] J. Fröhlich and B. Pedrini, *New applications of the chiral anomaly*, hep-th/0002195 [inSPIRE].

[38] M.S. Turner and L.M. Widrow, *Inflation Produced, Large Scale Magnetic Fields*, Phys. Rev. D 37 (1988) 2743 [inSPIRE].

[39] W.D. Garretson, G.B. Field and S.M. Carroll, *Primordial magnetic fields from pseudoGoldstone bosons*, Phys. Rev. D 46 (1992) 5346 [hep-ph/9209238] [inSPIRE].

[40] M. Giovannini and M.E. Shaposhnikov, *Primordial hypermagnetic fields and triangle anomaly*, hep-ph/9710234 [inSPIRE].

[41] T. Fujita and K. Kamada, *Large-scale magnetic fields can explain the baryon asymmetry of the Universe*, Phys. Rev. D 93 (2016) 083520 [arXiv:1602.02109] [inSPIRE].

[42] K. Kamada and A.J. Long, *Baryogenesis from decaying magnetic helicity*, Phys. Rev. D 94 (2016) 063501 [arXiv:1606.08891] [inSPIRE].

[43] M.M. Anber and E. Sabancilar, *Hypermagnetic Fields and Baryon Asymmetry from Pseudoscalar Inflation*, Phys. Rev. D 92 (2015) 101501 [arXiv:1507.00744] [inSPIRE].

[44] P.B. Arnold, G.D. Moore and L.G. Yaffe, *Transport coefficients in high temperature gauge theories. 1. Leading log results*, JHEP 11 (2000) 001 [hep-ph/0010177] [inSPIRE].

[45] A. Boyarsky, J. Fröhlich and O. Ruchayskiy, *Magnetohydrodynamics of Chiral Relativistic Fluids*, Phys. Rev. D 92 (2015) 043004 [arXiv:1504.04854] [inSPIRE].

[46] N. Barnaby, E. Pajer and M. Peloso, *Gauge Field Production in Axion Inflation: Consequences for Monodromy, non-Gaussianity in the CMB, and Gravitational Waves at Interferometers*, Phys. Rev. D 85 (2012) 023525 [arXiv:1110.3327] [inSPIRE].

[47] J.H. Traschen and R.H. Brandenberger, *Particle Production During Out-of-equilibrium Phase Transitions*, Phys. Rev. D 42 (1990) 2491 [inSPIRE].

[48] A.D. Dolgov and D.P. Kirilova, *On Particle Creation by a Time Dependent Scalar Field*, Sov. J. Nucl. Phys. 51 (1990) 172 [inSPIRE].

[49] R. Allahverdi, R. Brandenberger, F.-Y. Cyr-Racine and A. Mazumdar, *Reheating in Inflationary Cosmology: Theory and Applications*, Ann. Rev. Nucl. Part. Sci. 60 (2010) 27 [arXiv:1001.2600] [inSPIRE].

[50] M.A. Amin, M.P. Hertzberg, D.I. Kaiser and J. Karouby, *Nonperturbative Dynamics Of Reheating After Inflation: A Review*, Int. J. Mod. Phys. D 24 (2014) 1530003 [arXiv:1410.3808] [inSPIRE].

[51] R.D. Peccei, *The Strong CP problem and axions*, Lect. Notes Phys. 741 (2008) 3 [hep-ph/0607268] [inSPIRE].

[52] D. Baumann and L. McAllister, *Inflation and String Theory*, Cambridge Monographs on Mathematical Physics, Cambridge University Press (5, 2015), 10.1017/CBO9781316105733 [arXiv:1404.2601] [inSPIRE].

[53] A.H. Chamseddine, J. Fröhlich and O. Grandjean, *The Gravitational sector in the Connes-Lott formulation of the standard model*, J. Math. Phys. 36 (1995) 6255 [hep-th/9503093] [inSPIRE].

[54] E. Witten, *Some Properties of O(32) Superstrings*, Phys. Lett. B 149 (1984) 351 [inSPIRE].

[55] K. Becker, M. Becker and A. Strominger, *Five-branes, membranes and nonperturbative string theory*, Nucl. Phys. B 456 (1995) 130 [hep-th/9507158] [inSPIRE].
[56] N. Seiberg and E. Witten, *Electric-magnetic duality, monopole condensation, and confinement in N = 2 supersymmetric Yang-Mills theory*, *Nucl. Phys. B* **426** (1994) 19 (Erratum ibid. *430* (1994) 485) [hep-th/9407087] [inSPIRE].

[57] A. Bilal, *Duality in N = 2 SUSY SU(2) Yang-Mills theory: A Pedagogical introduction to the work of Seiberg and Witten*, in *NATO Advanced Study Institute on Quantum Fields and Quantum Space Time*, 1995 [hep-th/9601007] [inSPIRE].