Abstract— Increasing numbers of customers intend to install batteries with PV to manage their power bill or even go off-grid. To encourage the sharing of locally generated renewable energy and suburban customers to stay on grid, a novel grid friendly neighborhood energy trading mechanism (NET) has been developed. The NET is intuitive as it uses the existing network topology as the trading medium for scalability with low computation and communication requirements. The NET is designed to account for three technical goals: reflecting losses, meeting network flow constraints, and regulating voltages. A MV-LV distribution network case study shows that NET effectively achieves the three goals with low communication and fast convergence. This NET mechanism helps to enable grid friendly neighborhoods to share locally generated renewable energy in a market context.

Index Terms— transdisciplinary research, power economics, peer to peer, local energy market, demand and supply, electricity price, distributed energy resources, peak demand

NOMENCLATURE

A. Set of indices

\( j \) Node in the network

\( k \) Iteration number

\( t \) Time interval

\( n \) Node number

B. Set of parameters

\( M_P(j,k,t) \) Market price for real power \( P \)

\( m_p \) Nodal market price vector for \( P \)

\( \alpha(j) \) Intercept of the demand/supply curve with \( P \)

\( \beta(j) \) Gradient of \( P \) demand curve with market price \( M_P \)

\( P(j,k,t) \) Real power

\( Q(j,k,t) \) Reactive power

\( BIBC \) Bus injection to branch-current matrix of the network [1]

\( DLF \) Direct load flow matrix, obtained from multiplication of \( BIBC \) and \( BCBV \) [1]

\( M_P(j,k,t) \) Total market price for \( P \)

\( M_Q(j,k,t) \) Total market price for \( Q \)

\( I_j(k,t) \) Nodal current injection (or consumption)

\( B(i,j,k,t) \) Branch currents for Branch \( i \)

\( VD(j,k,t) \) Voltage change

\( V(j,k,t) \) Voltage

\( \Delta I(1,k,t) \) Flow mismatch on the head node

\( \Delta V(j,k,t) \) Voltage mismatch

\( M_P(j,k,t) \) Market price for voltage regulation

\( \beta_Q(j) \) Gradient of \( Q \) demand curve

\( l_{ref} \) Distribution transformer current limit

\( V_{ref} \) Reference voltage

\( V_{ref, max} \) Maximum allowable reference voltage

\( V_{ref, min} \) Minimum allowable reference voltage

\( \delta V(n) \) Voltage sensitivity of node \( n \) with respect to the \( P \) changes at node \( j \) [2]

\( \delta V(n) \) Voltage sensitivity of node \( n \) with respect to the \( Q \) changes at node \( j \) [2]

\( \eta \) Coefficient for updating \( M_p \) to manage flow constraint

\( \Gamma \) Coefficient for updating \( M_p \) to regulate voltage

\( \alpha'(j,k,t) \) Coefficient related to \( M_p \) for updating nodal current real component; calculated from \( \alpha(j) \) divided by nodal voltage

\( \beta'(j,k,t) \) Coefficient related to \( M_p \) for updating nodal current real component; calculated from \( \beta(j) \) divided by nodal voltage

\( \beta'_Q(j,k,t) \) Coefficient related to \( M_q \) for updating nodal current reactive component; calculated from product of \( \beta(j) \) and voltage sensitivity divided by nodal voltage

\( \beta''_Q(j,k,t) \) Coefficient related to \( M_q \) for updating nodal current reactive component; calculated from product of \( \beta(j) \) and voltage sensitivity divided by nodal voltage

\( \delta_j \) Distribution loss factor from head node to Node \( j \)

\( \Delta \) LV distribution loss factor vector

I. INTRODUCTION

Prosumers in distribution networks may have surplus renewable generation at times and some households may remain as pure consumers all the time. In this situation with information and communication technologies, the local surplus renewable energy can be shared among prosumers and consumers using local energy markets [3].

In general, there are three forms of neighborhood energy market designs: peer to peer (P2P), centralized model and a hybrid of P2P and centralized model [4]–[6]. In a full P2P market (a) in Fig. 1, each participant directly communicates with all other participants and there will be N(N-1) communication connections inherent in the structure. This form of energy trading mechanism is consumer-centric and enables

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prosumers to have free choices [4]. There are innovative designs for P2P markets price clearing mechanism, such as the consensus-based approach [7] and an adaptive segmentation method [8]. In those research, prosumers’ demand and supply limits are often considered however network constraints are often not included. Few recent publications have considered P2P trading with network constraints as they play important role in high PV penetration situation [9]. Decentralized P2P trading has been proposed with the consideration of line constraint in [10], flow limits in [11], voltage regulation and losses in [12]. Nodal flow and voltage limits are considered in a two level P2P configuration in [13]. An innovative blockchain enabled P2P trading approach has considered network losses [14]. When a distribution network has a large number of participants, P2P may take many iterations to reach convergence because the computation and communication overhead can be demanding for these market designs [4], [9], [14].

Model (b) in Fig. 1 is a centralized energy trading model. A centralized model often has a coordinator or a manager who is responsible for managing energy trading within a community and coordinating with other communities or distribution network [16]. This model may have better energy resilience for the community members and a community can work together to provide service for upper level network operators [4], [16]. However, a main challenge for the centralized model is to ensure a fair and unbiased energy sharing among members [4][17].

There are multiple forms of hybrid P2P and centralized trading models, depending on market design. A form of the hybrid P2P and centralized model is shown in (c) in Fig. 1 which has a system coordinator for energy trading among neighborhoods (microgrids/local energy markets). The system coordinator acts as a mediator for trading and payment transaction among neighborhoods. In the hybrid model in Fig. 1(c), the energy trading mechanism inside a neighborhood can be different and it provides flexibility for neighborhood members to select their own way of trading. However, the hybrid model’s main challenge is the coordination between energy trade at low voltage level and higher voltage level [4],[18].

As more customers become prosumers with energy storage, they may intend to go off-grid. The authors of this paper do not believe that going off-grid is a socially responsible solution for most people in mature electricity markets, because electricity price is impacted by the economy of scale. If more prosumers go off-grid, the electricity price would increase further and quicker, which would result more customers going off-grid (death spiral in [19]). However, in reality, there would always be customers who cannot afford distributed generators or energy storages. Therefore, there is a need to design a grid friendly neighborhood energy trading mechanism to encourage prosumers to stay on grid.

To encourage prosumers to stay connected with grid, the previously discussed energy trading models provide benefits for prosumers who export energy. There are possibly two more strategies to encourage prosumers to stay on grid. The first is to have a reward mechanism to encourage prosumers act to relieve network stress, such as the centralized model providing services to network. The second strategy is to design a trading mechanism which truly reflect the distribution network cost in local energy market such as cost in managing flow constraint and voltage regulation. In this way, other consumers on the network can also benefit from the local energy market by having more affordable, reliable and quality electricity supply.

A grid friendly neighborhood energy trading mechanism (NET) is introduced in this paper to reflect the congestion cost of distribution network delivering electricity and incentivize prosumers to alleviate network stress. The NET mechanism is a directional adjacency-based centralized energy trading mechanism however implemented in a distributed manner. The directional adjacency-based algorithm has been introduced before for voltage, frequency and load control in radial distribution network [20]. The adjacency term indicates that the NET mechanism uses the existing topology of the electric network for communicating pricing and energy information. The pricing information flow and energy information flow are directional (more details in Section II).

The proposed NET mechanism is an iterative process to have one reference price for a feeder, however, each nodal price is corrected for losses, flow constraint and voltage regulation. It has some similarity to a centralized P2P scheme but strictly speaking, this NET mechanism is not a one-to-one energy trading scheme.

The key contributions of this paper include:
- Utilizing the existing network topology/asset as the communication media to transmit pricing and flow information. Therefore, a lower level of communication infrastructure is needed.
- Achieving fast convergence with lower computation cost due to the economic demand and supply approach.
- Attaining scalability and coordination between hierarchical networks with a decentralized NET design.
- Reinforcing technical constraints with price based economic approach. The flow constraint is managed with the price-demand approach. Voltages are regulated with the nodal power and voltage sensitivity approach [2]. The sensitivity-based approach is a technically just approach to reward prosumers who help regulate voltages.

The rest of the paper is structured as follows. Section II.
describes the principles and formulation of the grid friendly NET mechanism. Then the NET mechanism is applied to a two-voltage level MV-LV distribution network case study in Section III. Three scenarios of different network situation and results are presented in that section. The final section is the conclusion, contribution of the paper and possible future development for the NET mechanism.

II. METHODOLOGY

The new NET mechanism is introduced in this section to provide a grid friendly energy trading mechanism for neighborhoods. The principles of the NET mechanism are explained intuitively first in the below section then followed with mathematical formulation for the NET mechanism. The NET mechanism has been designed to clear market energy price and manage three technical constraints: line loss, flow constraint and voltage regulation.

A. Principles of the NET mechanism

In the NET mechanism, the head node functions as a system coordinator (Fig. 2). Each node can have prosumers or consumers. The energy price signals and flow information are passed on like a daisy chain (which is widely used in communication). Customers’ energy management systems provide automated response for demand or generation based on their nodal prices. Energy trading occurs within a feeder, or a neighborhood.

Two principles for the NET mechanism are: first, energy market reference price from a head node ripples down a radial feeder with adjustments for loss correction, flow constraint and voltage regulation. Second, flow information ripples up the feeder and voltages are calculated accordingly with the direct load flow approach (DLF) [1]. Fig. 2 provides a visual explanation for the NET mechanism. Iterations of these price’s ripple down and flow’s ripple up continue until a convergence is reached without violation to technical constraints. There are multiple benefits for this NET mechanism, including scalable in hierarchical networks, using the existing electric network assets with less requirements for communication infrastructure, locational management of technical constraint etc.

The NET mechanism can be applied to a single voltage level feeder (Fig. 2) or a MV-LV network (Fig. 3). A MV-LV network functions as one entity for energy trading when there is no potential violation to technical constraints. Once a technical constraint is being reached, such as MV-LV transformer flow limit, the MV head node and lower level coordinator node operate in a decentralized manner (more details in Section 0).

Overall, this NET mechanism is a centralized design but implemented in a distributed manner. Details of the mathematical formulation and results are in the following sections.

Step 1: flat start the NET with a market reference price rippling from a head node to downstream. The reference price \( M_p(1, k, t) \) can be determined by a local energy market coordinator or aggregator. All downstream nodal prices are based on the same reference price and corrected with a cumulative distribution loss factor. Because the cumulative distribution loss factor for a network up to Node \( j \) is \( \delta_j \), thus

\[
\Delta = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_j \end{bmatrix}
\]

Then, the head node price \( M_p(1, k, t) \) is rippled downstream with loss correction using (2). \( M_p \) is a market price vector for all nodes on the feeder.

\[
M_p = \Delta \cdot M_p(1, k, t)
\]

Step 2: calculate customers’ nodal current injection. Automated responses for real power \( P \) are expressed by customers’ energy management systems after receiving market price \( M_p \). The real power demand \( P(j, k, t) \) is assumed to be a linear function of energy market price \( M_p(j, k, t) \), coefficients \( \alpha(j) \) and \( \beta(j) \) [21]. The \( M_p(j, k, t) \) to \( P(j, k, t) \) relationship is defined in (3). This relationship is a consolidated demand and supply curve for prosumers (more information in Section VI. A. ). The nodal injection can be calculated using (4). \( V_1 \) is the head node reference voltage.

\[
P(j, k, t) = \alpha(j) + \beta(j) \times M_p(j, k, t)
\]

\[
|P| = \frac{|V|}{V_1}
\]

Step 3: use BIBC and BCBV matrices to calculate branch currents [B] using (5) and calculate nodal voltages [V] using (6) and (7). The idea behind (5) is to directly use network topology to calculate branch currents from nodal injection. Voltage drops can be calculated using (6). With the voltage drops, nodal voltages can be obtained using (7). BIBC and BCBV are formed with the direct load flow approach. Formulation for BIBC and BCBV can be found in [1].

\[
[B] = [BIBC][|I|]
\]

\[
[V] = [BIBC][BCBV][|I|]
\]

\[
V = V_1 + VD
\]
Step 4: calculate the mismatch values for distribution transformer flow violation and voltage violation for all nodes on the feeder. If the distribution transformer flow constraint is violated, the flow mismatch value can be calculated in (8). In (9), Node J has the largest voltage deviation to the nominal voltage, and it is taken to calculate the voltage mismatch \( \Delta V(J, k, t) \).

\[
\Delta I(1, k, t) = \begin{cases} 
|I(1, k, t)| - I_{ref}, & |I(1, k, t)| > I_{ref} \\
0, & |I(1, k, t)| \leq I_{ref}
\end{cases} 
\]

\[
\Delta V(J, k, t) = \begin{cases} 
V(J, k, t) - V_{ref}, & V(J, k, t) \in [V_{ref, min}, V_{ref, max}] \\
V_{ref, max}, & V(J, k, t) > V_{ref, max} \\
V_{ref, min}, & V(J, k, t) < V_{ref, min}
\end{cases}
\]

Step 5: update neighborhood energy market prices based on flow mismatch or voltage mismatch. Use (10) to update \( M_p \) and use (11) to update \( M_q \). For consideration to select \( \eta \) and \( \Gamma \), please refer to Part B. in the Appendix.

\[
M_p(1, k + 1, t) = M_p(1, k, t) + \eta \cdot \Delta I(1, k, t) 
\]

\[
M_q(j, k + 1, t) = M_q(j, k, t) + \Gamma \cdot \Delta V(J, k, t) 
\]

Step 6: \( M_p(1, k + 1, t) \) ripples downstream in (2) and reflected in customers’ response in (12). \( M_q(j, k + 1, t) \) is populated to network nodes in (12) and (13). The assumption for (13) is that reactive power is fully controllable and it is a linear function of \( \beta_q \) and \( M_q(j, k, t) \) (the price-Q relationship). In (12) and (13), the NET mechanism selects the most efficient nodes for \( P \) and \( Q \) to regulate voltage, based on the \( P \) and \( Q \) sensitivity to Node J voltage of the largest deviation.

\[
P(j, k + 1, t) = \alpha(j) + \beta(j) \cdot (M_p(j, k + 1, t) + \frac{\partial V(J)}{\partial P(j)} \cdot M_q(j, k + 1, t)) 
\]

\[
Q(j, k + 1, t) = \beta_q(j) \cdot \frac{\partial V(J)}{\partial Q(j)} \cdot M_q(j, k + 1, t) 
\]

To view better, prosumers’ injection or consumption (12) and (13) can be written as (14) and (15). In (14), \( M_{pr} \) is the total market price for \( P \) including loss reflection in (2), flow constraining in (10) and voltage regulation in (11). \( M_q(j, k + 1, t) \) in (15) is the total market price for each Node j to regulate voltages, based on voltage to \( Q \) sensitivities.

\[
M_{pr}(j, k + 1, t) = M_p(j, k + 1, t) + \frac{\partial V(J)}{\partial P(j)} \cdot M_q(j, k + 1, t) 
\]

where,

\[
M_p(j, k + 1, t) = \alpha(j) + \beta(j) \cdot M_{pr}(j, k + 1, t) 
\]

\[
M_q(j, k + 1, t) = \frac{\partial V(J)}{\partial Q(j)} \cdot M_q(j, k + 1, t) 
\]

Customers adjust \( P \) or \( Q \) based on the rewards available. The rewards are built within one reward mechanism to reflect losses in (2), flow in (8) (10) and regulate voltages in (9), (11) to (13).

After Step 6, customers’ nodal injection \( I \) can be calculated and Step 3 is iterated. This iterative economic demand and supply process continues until a convergence is reached without violation to the technical constraints. After each trading interval, customers payment or receivables from generation need to be settled based on the energy in/out of each node and corresponding nodal prices. A set of Pseudo-code for the NET mechanism is provided in Algorithm 1.

This section has introduced the principles and the mathematical formulation of the NET mechanism. In the next section, the NET mechanism is applied to a case study of two voltage levels MV-LV network.

### Algorithm 1 NET

1: Initialization
2: Initialize variables \( M_p(1, k, t) \), \( [BIBC] \), \([BCBV] \)
3: for all \( j \) do
4: Establish nodal prices \( M_p \) to reflect loss using (2)
5: Obtain customers’ responses using (3) and (4)
6: While convergence criteria are not satisfied do
7: Calculate network flow and voltages using (5) to (7)
8: if \( \Delta I(1, k, t) > 0 \) using (8) (9) then
9: Update Node 1 \( M_p \), Node J \( M_q \) using (10) (11)
10: for all \( j \) do
11: Update \( M_p \) to reflect loss using (2)
12: Obtain customers’ responses using (12) (13)
13: end
14: end
15: end
16: Calculate flow and voltages using (5) to (7)
17: End

### III. Case Study and Results

This section presents the case study description and results when the NET mechanism is applied to a radial distribution network with multiple LV feeders, MV customers, and voltage levels. The hierarchical structure of the distribution network is shown in Fig. 3. The network parameters are from real suburban feeders [22]. There are three LV feeders in total. LV Feeder 1 and 3 are load dominant feeders, however LV Feeder 2 is a generation dominant feeder. All those three LV feeders have different flow limits for their head node transformers. To save space, the schematic diagram of the LV feeders is not drawn separately however, it can be viewed in Fig. 7 or Fig. 11. Two MV customers are connected directly near the end of the MV network. The two MV customers are mainly consumers. There are 42 nodes across two voltage levels for the studied MV-LV case (more parameters and coefficients are included in the Appendix).
Fig. 3. Case study MV-LV network indicative diagram

To evaluate how the NET performs when customers’ loads vary, reference load scale factors have been considered to scale \( \alpha \) for all demand curves. As shown in Fig. 4, the reference load scale factors are \([0.5 \ 0.6 \ 0.7 \ 0.8 \ 0.9 \ 1.0 \ 1.2 \ 1.4 \ 1.6 \ 1.3 \ 1.0 \ 0.75]\). The twelve intervals can be interpreted as twelve hours across a day from morning 8am to evening 8pm. The scale factor reaches peak on the interval 9.

Three scenarios are presented in the following sections:
- Low demand scenario: results of a low demand interval are presented when there is no constraint violation across the studied case. In this scenario, the entire network is regarded as one entity. This is used as a simple case to explain the NET mechanism.
- Decentralized MV-LV NET scenario: a LV feeder hits its transformer flow constraint and the NET mechanism is applied in a decentralized manner to the MV network and LV feeders respectively. The decentralized NET iterates between LV NET and MV NET.
- Peak demand scenario: results for peak demand interval 9 are presented. There are flow constraint violations for LV feeder transformers and MV head node transformer, and voltage violation in the LV side as well. In this constrained scenario, the NET mechanism functions in a decentralized way. Converged results of all intervals are summarized at the end of this case study section.

A. Low demand scenario

During low demand intervals, there is no violation to flow or voltage limits, such as in Interval 1 and 2 in Fig. 4. In this situation, the NET mechanism is acting as one coordinator for the whole distribution network. As shown in TABLE I, the MV head node reference pricing \$0.3/kW is passed onto all MV nodes, LV head nodes and other LV downstream nodes with loss corrections using (2). Overall, constant prices are maintained for the whole network. Because suburban MV feeders are not extra-long and their MV loss is minimal, MV network loss factors were neglected in the case examined here.

TABLE I shows the first iteration process for the NET mechanism at Interval 1. After LV customers and MV customers receive their nodal prices, customers’ demand and network flow information would be produced using (3) and (4). This information is then rippled back upstream for branch flow and voltage calculation with (5) to (7). When there is no constraint violation in (8) and (9), there is no need to update energy pricing in (10) and (11). Therefore, the MV head node reference price stays at \$0.3/kW. There is no pricing separation for the MV network and LV feeders’ head nodes. For the following two scenarios, more iterations are needed to adjust price to manage flow or voltages.

B. Managing constrained energy trading with decentralized MV-LV NET (Interval 4)

In the previous scenario, the whole distribution network is treated as one entity with the NET mechanism applied to clear energy prices. In this scenario (Interval 4), there is no violation to MV technical constraints and two of the three LV feeders. However, LV Feeder 1 has a lower flow limit and its flow constraint is violated at the initial stage. It may not be justifiable, nor efficient to run the NET mechanism as one coordinator to iteratively update the MV head node price and populate it to all LV feeders since different LV feeders have different flow limits and customers’ responses.

Therefore under this circumstances, the NET mechanism is applied in a decentralized manner to the MV network and LV feeders respectively. Another way to describe it: there is one coordinator on the MV head node and one coordinator on the head node of each LV feeder. The following paragraphs describe the performance and results of the decentralized NET mechanism.

Because there is no flow mismatch for the MV head node transformer, the MV energy pricing \( (M_p=\$0.3/kW) \) stays unchanged for all the iterations, as shown in the top plot of Fig. 5. However, LV Feeder 1’s pricing changes at every iteration...
when there is a constraint violation (the top plot of Fig. 6). The horizontal axis of Fig. 5 and Fig. 6 is iteration steps.

In Fig. 6, LV Feeder 2 and 3 have no violation to their flow constraints, so they follow the MV pricing and the pricing stays unchanged. However, LV Feeder 1 has hit its transformer flow constraint and its price on the head node has been changed. In the decentralized mode, LV feeders' head node has a function similar to the MV network head node. When the MV head node price is rippled to a LV feeder’s head node, the LV feeder’s head node will act as a coordinator.

At the start of iteration, the MV pricing $M_p$ is passed onto all LV head nodes. For the constrained LV Feeder 1, its head node price will be rippled to its downstream, with loss reflection using (2) and correction to flow using (10) and (12). At each LV NET iteration run, LV Feeder 1 will have updated its nodal prices and flow information in a way that the LV feeder’s flow constraint is alleviated. Then the LV flow information will be fed back to the MV network to evaluate MV flow using (3) to (5). If there is no violation to MV technical constraints, the MV head node pricing will stay unchanged, otherwise it will be updated as per (10).

After each MV NET iteration, there is a LV NET iteration to update LV Feeder 1’s head node price and flows with an aim to satisfy its flow constraint. The decentralized NET mechanism in MV network and LV feeders continues until all the technical constraints are satisfied.

In the top plot of Fig. 6 over the initial ten iterations, LV feeder 1 has gradually got higher prices to regulate flow. When LV Feeder 1 head node price is rippled to its downstream customers, customers respond to reduce loads and limit flows based on (3). The price and flow updates run for a few iterations. By the 11th iteration, a price and flow convergence is reached without any flow mismatch.

In Fig. 7, the color contours represent the $M_p$ prices. For example, darker colors represent lower prices and brighter colors represent higher prices. The numbers (1 to 12) on Fig. 7 and 11 are node numbers.

There are two types of price differences for the MV-LV network. One difference is between LV Node 1 price and the MV head node price, i.e. 0.416/kW on LV Node 1 in Fig. 7 and 0.3/kW for MV head node price in Fig. 5. The increased LV head node price discourages LV nodes’ consumption and limits flow to reinforce the LV transformer flow constraint. The price difference between MV reference price and LV head nodes can be used to build up a sinking fund for future network expansion when needed. Another price difference is between LV Node 1 price and LV downstream nodal prices, e.g. 0.416/kW on Node 1 compared to 0.433/kW on Node 12 in Fig. 7. This difference is to reflect the losses (2). With the two measures to reinforce flow constraint and reflect losses, the congestion cost of using the distribution network is reflected in the NET pricing mechanism.

C. Peak demand scenario (Interval 9)

This scenario has many challenges to be managed. The challenges are: all the MV and LV transformer flow limits are reached at the initial stage of iterations, and there are also voltage issues on the LV side.
In the top plot of Fig. 8, the MV head node $M_p$ starts with $0.3 \text{ $/kW and it stays unchanged for the following LV iteration when LV NET is running. The middle plot of Fig. 8 shows the two MV customers’ consumption in relation to the MV $M_p$ prices. The bottom plot of Fig. 8 shows the MV flow mismatch calculated from the combined flow of the LV feeders and the MV customers.

Fig. 8 and Fig. 9, there are voltage mismatches at LV feeders as shown in Fig. 10. In this peak demand interval Fig. 10, LV Feeder 1 and 3 have about 0.05 p.u. undervoltage violation at the beginning of iterations (0.05 p.u. lower than the allowable minimum limit). These undervoltage violations are corrected with $M_p$ prices for LV Feeder 1 and 3. In the top plot of Fig. 10, $M_p$ settles at $0.402$ for LV Feeder 3 and $0.637$ for LV Feeder 1. With these $M_p$ terms, LV Feeder 3 voltages have returned to be within the allowable limits from iteration 3 and LV Feeder 1 voltages have returned to be within the limits from iteration 4.

When the MV prices are populated to the LV feeders’ head nodes (the top plot of Fig. 9) and other customer nodes, LV customers respond to the prices. Then, LV’s flow information is generated and fed back to the MV network. Before the 9th step in the bottom plot of Fig. 8, the MV flow mismatch is eliminated with less consumption from the MV & LV customers and more generation from LV prosumers. The MV head node price settles at $1.396 \text{ $/kW (the top plot of Fig. 8). The price and energy flow coordination between this hierarchical network is achieved with the decentralized NET mechanism.

LV Feeder 2 is a generation dominant feeder. When energy price increases, generators would often produce more to respond. In Fig. 9 when the head node of LV Feeder 2 has $0.80 \text{ $/kW price at Iteration Step 2, LV Feeder 2 responds with a 4.5 p.u. flow into the MV network. This exceeds LV Feeder 2’s flow limit. To limit the generation overflow, LV Feeder 2 head node price is reduced. In this scenario, there is a price separation between the MV side and LV Feeder 2 head node. Overall, the NET mechanism ensures the flow constraints are reinforced with the economic approach by rewarding more local generation and disincentivizing consumption.

The converged total nodal energy prices $M_{pf}$ for LV Feeder 1 are shown in Fig. 11. The head node price on LV Feeder 1 is consistent with the MV pricing (in the top plot of Fig. 8). LV Feeder 1’s downstream nodal prices are slightly different, due to loss reflection and voltage regulation.

Voltage issue is often locational and happens in LV feeders. In this challenging scenario, LV feeders’ NET mechanism is able to regulate voltage with less than five iterations as shown in Fig. 10. Therefore, there is no MV voltage regulation needed at the end of LV iterations.

Compared to the previous scenario, this scenario has potential technical constraints on more feeders. The increased price on LV feeders discourages consumption, encourages local generation and manages flow to satisfy network constraints.

D. Overall results

Overall converged results for all the time intervals are presented in Fig. 12. As the reference load scale factor increases, energy market price $M_p$ tends to go up for both MV and LV feeders. Because LV feeders have different transformer current limits, LV feeders’ head nodes start having price separation with the MV pricing at different intervals. For example, LV Feeder 1 starts having the price separation with the MV network from Interval 3 and LV Feeder 3 starts having the price separation from Interval 5. For the generation dominated LV Feeder 2, its price separation from MV starts from Interval 7 which is two intervals away from the peak demand Interval 9. After the peak demand, prices across the network drop to lower levels.
The examined case is quite a constrained case with many challenges, including technical violations to transformer flow limits and voltage limits. For the most challenging scenario in Fig. 8, those challenges along with the loss correction are managed within eight iterations.

One way to benchmark is to compare the iterations taken for energy price and flow to converge. With the directional adjacency-based approach, the proposed NET mechanism converges within 10 iterations for the case studied. In comparison, P2P market structures may take hundreds of iterations to thousands of iterations for a market of similar size, depending on the design of P2P markets, e.g. hybrid P2P, centralized P2P or full P2P [15].

To further test scalability, all LV feeders of the MV-LV case have been augmented to 130 nodes and the whole MV-LV network has 396 nodes. Out of 2,000 simulations, the mean simulation time for each interval is 0.013 seconds and 0.16 seconds for the original case and the augmented case. For about 10 times more buses, the computing time is increased by 12 times. A reason for this lower than expected computation time increase is: the NET mechanism is mainly matrix-based calculation (e.g. (2), (5) to (7)). Matrices of more rows do not require significantly more time to compute. In comparison with a decentralized bilateral P2P and a similar LV network topology [10], when the number of nodes is expanded to 400, computation time is about 50 seconds and the number of iterations is about 400. A LV circuit of this study is the same to the radial test case in [10]. For the LV circuit, the NET mechanism needs 8 iterations and the computation time is 0.028 seconds; however, [10] needs 176 iterations and 2.56 seconds to compute. When the LV circuit is scaled to 130 nodes, the NET needs 17 iterations and 0.052 seconds, and [10] needs 540 iterations and 17 seconds to compute.

The NET mechanism has presented reductions in iteration numbers and convergence time compared to the radial test case in [10] however, the P2P method in [10] can be more broadly applied to meshed networks. For a generic radial network of N nodes, the NET mechanism has $2(N - 1)$ communication links. A full P2P in Fig. 1(a) has $N(N - 1)$ communication links. As the number of nodes grows, the NET’s communication overhead is expected to be less. The comparison results indicate that the NET mechanism is computationally efficient and scalable for radial feeders. Also in comparison with Australian national energy market’s 5min trading interval, the computing time of NET is a very small. The simulation software is MATLAB 2019b. The computer has an intel i7-7700HQ 2.8GHz CPU and a 16GB installed memory on a x64 Windows 10 environment. In terms of loss factors, the difference between power flow loss calculation and the linearized loss vector approach in (1) is less than 1%. A main reason is that distribution loss is quite small, and thus a linear approximation can be suitable.

IV. CONCLUSION

A grid friendly NET mechanism has been introduced in this paper. This directional adjacency-based mechanism works for both single voltage level networks and hierarchical networks of multiple voltage levels. When the NET mechanism works in a decentralized manner for hierarchical networks, network constraints can be managed locally through the economic approach.

The NET mechanism uses head nodes as the flow constraint items and as the centralized market coordinators, however nodal pricing is determined in a distributed manner following network topology. With the directional adjacency-based design, the existing network topology can be used as communication media to transmit pricing and flow information. Therefore, a lower level of communication infrastructure is needed.

With the economic demand and supply approach, fast convergence is achieved with lower computation cost. The economic approach has pricing signals to reflect network constraints, such as incentivizing local renewable energy generation or disincentivizing consumption.

With the voltage sensitivity-based approach, the NET mechanism provides awards for customers who can help manage voltages. With the financial benefits for prosumers’ providing network services, the NET mechanism can encourage prosumers to stay on grid and share more locally generated renewable energy.

This study has achieved novelty in solving the scalability issue of local energy trading and clearing energy market prices with technical constraints satisfied in a decentralized manner for hierarchical networks. For a generic network of any configurations, P2P may work. In cases of radial feeders, local energy market trading opportunities open up for the NET mechanism. Future research can include further stability studies and detailed financial studies for network expansion or customers’ energy investment.

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VI. APPENDIX

A. Consolidated demand and supply curves

The classical economics theory has one curve for demand and price, and another curve for supply and price [23]. In local
energy market, prosumers can be pure consumers and they can become producers when the incentives are sufficient. Therefore, in this study a consolidated demand and supply curve is proposed to represent prosumers’ demand and supply responses to price signals. To better explain the demand and supply curve \( P = \alpha + \beta \times M_p \), an indicative diagram is provided below:

![Indicative diagram for a demand and supply curve](image)

Fig. 13. Indicative diagram for a demand and supply curve

When a prosumer reaches a limit, the demand-supply curve of this prosumer would be a horizontal line section. Other literature reported on how to obtain customers response model [24][25]. Nonlinear characteristics can also be approximately linearized for a short period (not in the scope of this study).

B. Selecting \( \eta \) and \( \Gamma \)

The following consideration is to assist with the design of \( \eta \) in (10) and \( \Gamma \) in (11). For \( M_{pr} \), (10) can be written as:

\[
M_{pr} (1, k + 1, t) = M_{pr} (1, k, t) + \eta \cdot I (1, k, t) - \eta \cdot V_{ref}
\]

where, \( I (1, k, t) = \text{BIBC}(1,:) \cdot [A (1,:) \cdot [M_{ij} (1,k,t)] + K0(1, :)] \)

\[
\eta = \begin{bmatrix} \beta' \Delta \beta' \end{bmatrix}
\]

\[
K0 = \begin{bmatrix} \alpha' \\ 0 \end{bmatrix}
\]

With (19), (18) can be written as:

\[
M_{pr} (1, k + 1, t) = M_{pr} (1, k, t) + \eta \cdot \text{BIBC}(1,:) \cdot A (1,:) \cdot [M_{ij} (1,k,t)]
\]

Since BIBC(1,:) indicates that neglecting losses, the current in line segment 1 is the sum of the injection currents and the stability of (22) is approximately \( 1 + \eta \cdot [1 \ldots 1][\beta'] \). Therefore, if the voltage is close to one and the losses are low, this becomes \( 1 + \eta \cdot \text{sum}(\beta(j)) \). So if we learn the sum of load \( \beta \), the gain \( \eta \) can be found based on the desired closed loop pole. For a \( \beta \) sum of 50 and a desired pole at 0.5 the gain \( \eta \) becomes 0.5/50. For \( M_{pr} \), (11) can be written as:

\[
M_{pr} (1, k + 1, t) = M_{pr} (1, k, t) + \Gamma \cdot V (1, k, t) - \Gamma \cdot V_{ref}
\]

where, \( V (1, k, t) = \text{DLF}(1,:) \cdot [A (1,:) \cdot [M_{ij} (1,k,t)] + K0(1, :)] \)

With (24), (23) can be written as:

\[
M_{pr} (1, k + 1, t) = M_{pr} (1, k, t) + \Gamma \cdot \text{DLF}(1,:) \cdot A (1,:) \cdot [M_{ij} (1,k,t)]
\]

\[
+ \Gamma \cdot \text{DLF}(1,:) \cdot K0(1,:)
\]

When the reactive power of the load is independent of the real power, the stability of (25) is approximately \( 1 + \Gamma (1 \ldots 1)[Z(j,:)] \beta'' \). Similar to \( \eta \), the gain \( \Gamma \) can be selected with a desired pole and line impedance \( Z(j,:) \) weighted \( \beta'' \) sum. This study considers linear demand and supply curves (Fig.13). If any player reaches consumption or generation limits, their coefficients \( \beta \) no longer count and will make the system more stable but slower to converge.

C. Case study parameters

The below two tables list the parameters used for the studied case in Section III. For LV feeders, a modified IEEE 13-bus system has been used [26].

### TABLE II. CASE STUDY MV NETWORK PARAMETERS

| Description | Value | Unit |
|-------------|-------|------|
| Nominal voltage | 1 | p. u. (11 kV) |
| Base power | 1.15 | kVA or kW |
| Sub-transmission Transf. current limit | 30 | p. u. |
| R | 0.099 | Ohm/km |
| X | 0.103 | Ohm/km |
| \( M_{ij} (1, k, t) \) | 0.3 | SAUD/kW |
| \( \eta \) | 0.0295 | Per unit DI |
| \( \alpha \) (2 MV Customers) | [10000, 25000] | kW |
| \( \beta \) (2 MV Customers) | [-5000, -10000] | kW/SAUD |

### TABLE III. CASE STUDY LV FEEDERS’ PARAMETERS

| Description | Value | Unit |
|-------------|-------|------|
| Nominal voltage | 1 | p. u. (230 V) |
| Base power | 1.15 | kVA or kW |
| Distribution transf. current limit | 3, 2, 4, 4.8 for LV feeder | p. u. |
| Voltage limits | 0.95 to 1.05 | Of nominal voltages |
| \( \eta \) | 0.0008 | Per unit DI |
| \( \Gamma \) | -0.42 | Per unit DI' |
| \( \alpha \) for 30 customers on 3 LV feeders | Between -6000 and 9000 | kW |
| \( \beta \) for 30 customers on 3 LV feeders | Between -9000 and 800 | kW/SAUD |
| \( \beta_q \) | Between -2250 to -200 | kVAR/SAUD |
| \( \delta_n \) | Between 1.005 to 1.047 |

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