Schwarzschild black hole as accelerator of accelerated particles

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We consider collision of two particles near the horizon of a nonextremal static black hole. At least one of them is accelerated. We show that the energy in the center of mass can become unbounded in spite of the fact that a black hole is neither rotating nor electrically charged. In particular, this happens even for the Schwarzschild black hole. The key ingredient that makes it possible is the presence of acceleration. This acceleration can be caused by an external force in the case of particles or some engine in the case of a macroscopic body ("rocket").

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I. INTRODUCTION

During last decade, much attention was focused on high energy collisions near black holes. It was stimulated by the observation that, under certain conditions, collisions of free falling particles near the horizon can lead to the unbounded energy $E_{c.m.}$ in the center of mass frame. This is the Bañados-Silk-West (BSW) effect [1] that occurs due to the existence of special fine-tuned (so-called critical) trajectories. For rotating black holes the corresponding relation between the energy at infinity $E$ and angular momentum $L$ reads $E = \omega_L$, where $\omega$ is the metric coefficient responsible for rotation and proportional to the angular momentum of the black hole, subscript "H" refers to the horizon. The counterpart of the BSW effect can happen also near static but electrically charged black holes [2], with the difference that
instead of geodesics, particles move under the action of the electrostatic field. Then, the critical trajectories are characterized by equality \( E = \frac{qQ}{r_+} \), where \( q \) and \( Q \) are the charges of a particle and black hole, respectively, \( r_+ \) being the horizon radius. In other words, for the BSW effect to occur, a black hole should be rotating or/and charged. If one puts \( \omega = 0 \), \( Q = 0 \), this kills the effect in both cases. Correspondingly, the Schwarzschild black hole was unable to serve as particle accelerator to unbounded \( E_{\text{c.m.}} \). (We put aside the case when colliding particles are spinning.) More precisely, if two particles of equal mass \( m \) collide in the Schwarzschild background, \( E_{\text{c.m.}} \leq 2\sqrt{5}m \).

It turns out, however, that modification of the set-up changes the situation drastically, if instead of geodesic motion, we consider motion under the action of some finite force. In contrast to motion of a particle in the Reissner-Nordström space-time, this force can have external source. We will see that for the motion under such a force the analogue of the BSW effect is possible even in the Schwarzschild background, i.e. for a nonrotating and electrically neutral black hole.

The role of a finite force was discussed earlier \([4], [5]\) but just for rotating and/or charged black holes. It was implied in the aforementioned papers that the effect under discussion existed without this force. The question was whether or not the force spoils the effect. It was found that (under some additional weak restrictions on the behavior of the force) the BSW effect survives. Meanwhile, now the existence of such a force is not a potential obstacle but, by contrary, it is the only cause of the effect.

In what follows, we use the geometric system of units in which fundamental constants \( G = c = 1 \).

II. BASIC FORMULAS

Let us consider the black hole metric
\[
\text{ds}^2 = -f \text{dt}^2 + f^{-1} \text{dt}^2 + r^2 (\text{d}\theta^2 + \sin^2 \theta \text{d}\phi^2),
\]
(1)

where the horizon is located at \( r = r_+ \), so \( f(r_+) = 0 \). We will consider pure radial motion with the four-velocity \( u^\mu \) and four-acceleration \( a^\mu \) with
\[
a_\mu a^\mu \equiv a^2,
\]
(2)
where by definition \( a \geq 0 \). Then, it is convenient to use the expression for the acceleration in the simple form (see e.g. eqs. 16, 17 in [6] and [7]):

\[
a^t = f^{-1} u^r \frac{d}{dr} (f u^t),
\]

\[
a^r = fu^t \frac{d}{dr} (fu^t).
\]

In [6], [7], eqs. (3), (4) were exploited for investigation of trajectories with the constant acceleration in the Schwarzschild space-time but they are valid in a more general case as well.

By substitution into (2), one finds for a particle having the mass \( m \):

\[
m u^t = \frac{X}{f},
\]

\[
m u^r = \sigma P,
\]

\[
P = \sqrt{X^2 - m^2 f},
\]

\[
\sigma = \pm 1.
\]

\[
X = m\beta(r) + E.
\]

Here, \( E \) is a constant of integration,

\[
\beta = \delta \int^r dr' a(r'),
\]

\( \delta = \pm 1 \). The forward-in-time condition \( u^t > 0 \) requires \( X \geq 0 \). It follows from (4) that

\[
a^r = \frac{aX}{m} \delta,
\]

so \( \delta = \text{sign} a^r \).

If \( a(r) \) tends to zero at infinity rapidly enough, it is convenient to choose the limit of integration in such a way that

\[
\beta = -\delta \int_r^\infty dr' a(r'),
\]

so \( \text{sign} \beta = -\delta \). If \( a = \text{const} \) we can choose \( \beta = \delta ar \) and we return to the trajectory considered in [7]. Then, \( \text{sign} \beta = +\delta \). If, additionally, \( f = 1 \), we obtain motion along the Rindler trajectory with

\[
r = \frac{\cosh a\tau - E}{a}, \quad t = \frac{\sinh a\tau}{a},
\]

where we put \( \delta = +1 \) to have \( r > 0 \) for large \( \tau \). For the Reissner-Nordström (RN) case

\[
a = \frac{|qQ|}{mr^2},
\]

where \( q \) is the particle charge, \( Q \) is that of a black hole. The BSW effect exists in the RN background if for trajectories with \( E > 0 \) we take \( \delta = +1, \beta < 0, qQ > 0 \) [2]. Then,

\[
a^r > 0,
\]
so we have repulsion between a particle and a black hole.

Let particles 1 and 2 move from infinity and collide in some point \( r_0 \). The energy in the center of mass frame

\[
E_{\text{c.m.}}^2 = -(m_1u_1^\mu + m_2u_2^\mu)(m_1u_{1\mu} + m_2u_{2\mu}) = m_1^2 + m_2^2 + 2m_1m_2\gamma, \tag{12}
\]

where \( \gamma = -u_{1\mu}u^{2\mu} \) is the Lorentz factor of relative motion. It follows from the above equations that

\[
\gamma = \frac{X_1X_2 - \sigma_1\sigma_2P_1P_2}{m_1m_2f}. \tag{13}
\]

### III. HIGH ENERGY COLLISIONS

Now, the standard classification applies. A particle is called usual if \( X_H > 0 \) if it is separated from zero and it is critical if \( X_H = 0 \). This is possible if \( \delta = +1 \), so \( \beta < 0 \) for a particle coming from infinity, when \( E > 0 \). Then, for the critical particle

\[
E = |\beta(r_+)|. \tag{14}
\]

Assuming at infinity \( f = 1 \), we require \( E > 0 \) where it has the meaning of the Killing energy. We also assume that both particles move towards the horizon, \( \sigma_1 = \sigma_2 = -1 \).

Near \( r_+ \), one has \( X \approx X_H \) for a usual particle. Near the horizon, using the main term in the Taylor expansion, we have for the critical particle,

\[
X(r) \approx b(r - r_+), \; b > 0. \tag{15}
\]

Then, for collisions of the critical particle and a usual one we obtain that near the horizon the expression inside the square root in (6) becomes negative. This is manifestation of the known fact that the critical particle cannot reach the horizon in the nonextremal case. At first, this property was considered as an obstacle against the BSW effect \[8, 9\]. However, the situation changes if instead of exactly critical particle, we consider a near-critical one with small but nonzero \( X_H \). (For the first time, this idea was realized for the Kerr black hole in \[10\]). For such a particle

\[
X \approx X_H + b(r - r_+). \tag{16}
\]

Let \( X_H(r_0) \approx d\sqrt{r_0 - r_+} \), where \( d \) is some constant.
Then, it is the first term in (16) which dominates. For the metric function,

\[ f(r_0) \approx 2\kappa(r_0 - r_+), \quad (17) \]

where \( \kappa \) is the surface gravity. We assume that near-critical particle 1 collides with a usual particle 2. As a result, we find from (13) that

\[ \gamma \approx \frac{D}{\sqrt{(r_0 - r_+)}}, \quad D = \frac{(X_2) \lambda (d - \sqrt{d^2 - 2\kappa})}{2\kappa m_2}, \quad (18) \]

where we also assumed that \( d > \sqrt{2\kappa} \). Then, taking \( r_0 \) as close to \( r_+ \) as one likes, we obtain the unbounded growth of \( \gamma \) and \( E_{c.m.} \). Eq. (18) can be thought of as a counterpart of eq. (19) from [10] where the Kerr metric was considered. Thus there is a close analogy between our case and the BSW effect near nonextremal black holes. In particular, now the same difficulties persist that forbid arrival of the near-extremal particle from infinity because of the potential barrier typical of any nonextremal black hole (for the Kerr metric, see Figure and accompanying discussion in [11]). Therefore, either such a particle is supposed to be created already in the vicinity of the horizon from the very beginning or one is led to exploiting scenarios of multiple scattering [10].

IV. DISCUSSION AND CONCLUSIONS

What is especially interesting is that the effect under discussion is valid for the Schwarzschild black hole. It is also worth mentioning that for the extremal case, both terms inside the square root in (6) have the same order and the result is similar to that for the RN black hole [2]. However, now the acceleration may be caused not by interaction between the particle charge and that of a black hole but by some external force or the engine on a rocket in the macroscopic case. The results are valid both for \( a = \text{const} \) and for \( a \to 0 \) at infinity.

It is worth mentioning that there is one more but a quite different situation when high energy collisions also occur in the Schwarzschild background. This happens if a black hole is immersed in a magnetic field. In doing so, a particle orbiting around a black hole collides with another one coming from infinity. The magnetic field is supposed to be rather big to affect equations of particle motion and ensure the innermost stable circular orbit to be
located near the horizon \[12\]. Meanwhile, the mechanism considered by us works for radial motion and any finite \(a\) and has quite universal character.

Usually, the factor connected with additional forces (like gravitational radiation) are referred to as obstacles to gaining large \(E_{\text{c.m.}}\). To the extent that such influence can be modeled by some force, backreaction does not spoil the effect \[4, 5\]. Meanwhile, as we saw now, in our context the presence of the force not only is compatible with the BSW effect but it can be its origin! It works even in the simplest case of radial motion in the Schwarzschild background.

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