Resonant spin–flavour precession of neutrinos and pulsar velocities

E.Kh. Akhmedov

International Centre for Theoretical Physics
Strada Costiera 11, I-34100 Trieste, Italy

A. Lanza and D.W. Sciama

Scuola Internazionale Superiore di Studi Avanzati
Via Beirut 2–4, I-34014 Trieste, Italy

Abstract

Young pulsars are known to exhibit large space velocities, up to $10^3$ km/s. We propose a new mechanism for the generation of these large velocities based on an asymmetric emission of neutrinos during the supernova explosion. The mechanism involves the resonant spin–flavour precession of neutrinos with a transition magnetic moment in the magnetic field of the supernova. The asymmetric emission of neutrinos is due to the distortion of the resonance surface by matter polarisation effects in the supernova magnetic field. The requisite values of the field strengths and neutrino parameters are estimated for various neutrino conversions caused by their Dirac or Majorana–type transition magnetic moments.

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1 Introduction

In this paper we propose a new mechanism for generating large birth velocities of pulsars which is related to the possible existence of transition magnetic moments of neutrinos. Observations imply that pulsars have rapid proper motions with mean space velocity of $450 \pm 90$ km/s \cite{1}. In particular, observations in young supernova remnants have identified pulsars with velocities up to 900 km/s \cite{2}. Such high velocities of young pulsars are most probably associated with the supernova event in which the pulsar is born. Many different models have been formulated to explain the origin of these velocities: asymmetric collapse \cite{3}, asymmetry due to misalignment of dipole magnetic field and rotation axis \cite{4}, a “runaway star” produced by a supernova explosion in a close binary star \cite{5}, and recoil momentum from the asymmetric production of neutrinos \cite{6} in the supernova’s magnetic field. The latter possibility is especially interesting since neutrinos carry away more than 99% of the supernova’s gravitational binding energy and so even a $\sim 1\%$ asymmetry in the neutrino emission could lead to the observed recoil velocities of pulsars. However, this mechanism requires a magnetic field $B \gtrsim 10^{16}$ G which may be somewhat too strong for supernovae. It is also possible that the asymmetry in the neutrino momenta will be washed out by multiple scattering, absorption and re-emission of the neutrinos in the core of the supernova \cite{7}.

Recently, a very interesting new mechanism for the generation of the pulsar velocities has been proposed by Kusenko and Segrè (KS) \cite{8}. The idea is that the neutrino emission from a cooling proto–neutron star can be asymmetric due to resonant neutrino oscillations (MSW effect \cite{9}) in the supernova’s magnetic field. The anisotropy of the neutrino emission is driven by the polarization of the medium in a strong magnetic field, which distorts the resonance surface. According to KS, this mechanism requires magnetic fields $B \gtrsim 3 \times 10^{14}$ G. However, this estimate was recently criticised by Qian who showed that in fact magnetic fields $B \gtrsim 10^{16}$ G are necessary \cite{10}. We will come back to this question later on. An advantage of the KS scenario is that it is free from the above–mentioned shortcoming of the asymmetric production mechanism: the neutrinos carrying the momentum asymmetry are free–streaming from the supernova and therefore the asymmetry is hardly affected by the interaction of neutrinos with matter. This mechanism is very attractive since it predicts a well defined correlation between the strength of the magnetic field and the observed space velocities of pulsars \cite{11}. As a follow-up, in ref. \cite{12} the same authors have considered the effects of sterile-to-active neutrino oscillations, where now neutral–current effects are important. Thus, as was stressed in \cite{8, 11, 12}, pulsar motions may be a valuable source of information on neutrino properties. One should therefore study all possible mechanisms of asymmetric neutrino conversions in supernovae that might be the cause of the observed velocities of pulsars.

The mechanism that we propose here is similar to the KS one – it is based on the observation that matter polarisation in the strong magnetic field of a supernova distorts the resonance surface of the neutrino conversion. The difference from the KS scenario is in the nature of the neutrino transition involved: in our case it is the resonant spin–
flavour precession of neutrinos due to their transition magnetic moments \[^{13, 14}\] rather than neutrino oscillations\[^{1}\]. We show here that our proposed mechanism can account for the observed large space velocities of pulsars provided that the neutrino transition magnetic moment satisfies \(\mu_\nu \gtrsim (10^{-15} - 10^{-14}) \mu_B\), the neutrino mass is less than or about 16 keV and the supernova magnetic magnetic field \(B \gtrsim 4 \times 10^{15} \text{ G}\). Notice that the required magnetic field is slightly weaker than the one necessary for the KS mechanism \[^{10}\]. Such large fields are presently considered possible in supernovae \[^{16}\].

If the supernova magnetic field has a noticeable “up–down” asymmetry, neutrino spin or spin–flavour precession can affect differently neutrinos emitted in the upper and lower hemispheres. As was pointed out by Voloshin \[^{7}\], this could also be the reason for the observed birth velocities of pulsars. In this paper we restrict ourselves to the case of symmetric magnetic fields; possible effects of neutrino conversions in asymmetric magnetic fields will be considered elsewhere \[^{17}\].

### 2 Neutrino potentials in polarised media

In a medium containing a magnetic field the particles of matter have in general nonzero average spin. This spin polarisation contributes to the neutrino potential energy in matter through the neutrino coupling with the axial–vector currents of the matter constituents \[^{18–26}\]. For a very lucid and detailed discussion of matter polarisation effects on neutrino propagation in a medium we refer the reader to ref. \[^{26}\]; here we summarize some results that will be relevant for our discussion and elaborate on some of them.

The contribution of the polarisation of the medium to the potential energy of a test neutrino \(\nu_j\) is of the order \(\delta V_i(\nu_j) \sim (G_F/\sqrt{2}) g_A^i \langle \lambda_i \rangle \parallel N_i\). Here \(G_F\) is the Fermi constant, \(\langle \lambda_i \rangle \parallel\), \(g_A^i\) and \(N_i\) are the polarisation along the test neutrino momentum, weak axial–vector coupling constant and number density of the particles of the \(i\)th type \((i = e, p, n\) or background neutrinos). Under supernova conditions the average spins of electrons, protons and neutrons are rather small; this means that their polarisations are linear in the magnetic field strength \(B\), i.e. \(\delta V_i = \alpha B \parallel\), where \(B \parallel\) is the component of the magnetic field along the neutrino momentum. The electron spin polarisation affects the potential of electron neutrinos and antineutrinos in matter through both charged–current and neutral–current interactions, whereas for \(\nu_\mu\) and \(\nu_\tau\) and their antineutrinos there are only neutral–current contributions \(C^e_Z\). The polarisations of protons and neutrons contribute to the neutrino potentials in a medium only through the neutral–current interactions; therefore these contributions are the same for neutrinos of all flavours.

Thus, one arrives at the following expressions for the neutrino potentials in a magnetised

\[^{1}\]The resonantly enhanced spin–flavour precession of neutrinos in supernovae has been considered in the literature \[^{13, 14}\], but no implications for pulsar velocities were discussed there.
medium:

\[ V(\nu_e) = -V(\bar{\nu}_e) = \sqrt{2} G_F (N_e - N_n/2 + 2N_{\nu_e}) + (c_W^e + c_Z^e + e^p + e^n)B_\parallel, \]  
\[ V(\nu_{\mu,\tau}) = -V(\bar{\nu}_{\mu,\tau}) = \sqrt{2} G_F (-N_n/2 + N_{\nu_e}) + (c_Z^e + e^p + e^n)B_\parallel. \]  

(1)  
(2)

Here \( N_e \equiv N_{e^-} - N_{e^+} \), etc., and we have taken into account that the number of muon and tauon neutrinos coincides with the number of their antineutrinos in supernovae. The electron neutrino number density is relatively small in the regions of interest to us in supernovae, and from now on we neglect it. Taking it into account would not change our estimates significantly.

The contributions of the electron spin polarisation to the neutrino potentials \( V(\nu_i) \) were calculated in \([18–26]\). The charged–current electron polarisation contribution to \( V(\nu_e) \), which we denoted by \( c_W^e \), turns out to be twice as large as the neutral–current one, and has the opposite sign; as a result,

\[ c^e = c_Z^e + c_W^e = c_W^e - 2c_Z^e = -c_Z^e. \]  

(3)

During the supernova explosion neutrinos and antineutrinos of all flavours are thermally produced in the hot central part of the star. They are trapped in the dense core of the supernova and diffuse out on a time scale of \( \sim 10 \) s. When they reach the regions with density \( \rho \sim (10^{11} - 10^{12}) \) g/cm\(^3\), they are no longer trapped and escape freely from the star. The surface at unit optical depth is called the neutrinosphere. It is located deeper inside the star for \( \nu_\mu, \nu_\tau \) and their antiparticles than for \( \nu_e \) and \( \bar{\nu}_e \) since the medium is more opaque for these latter particles, which have charged–current interactions as well as neutral–current ones. As a result, the non-electron type neutrinos are emitted at a higher temperature, about 6 MeV as against 3 MeV for \( \nu_e \) and slightly higher for \( \bar{\nu}_e \) (because the medium contains more neutrons than protons).

In the supernova environment in the vicinity of the neutrinosphere electrons are relativistic and degenerate. In this case \([20, 23, 24]\)

\[ c_Z^e \simeq \frac{eG_F}{2\sqrt{2}} \left( \frac{3N_e}{\pi^4} \right)^{1/3}. \]  

(4)

The effects of the polarisation of protons have been calculated in \([23]\) in the approximation of Dirac protons, i.e. treating protons essentially as positrons (though with a different mass). This is certainly not a valid approximation since the anomalous magnetic moment of the proton, which is neglected in the Dirac approximation, is even larger than its normal magnetic moment. Also, the strong–interaction renormalisation of the proton axial–vector coupling constant was not taken into account. These shortcomings, however, can be readily removed. As we shall see, the nucleons are non-relativistic and non-degenerate in the hot proto-neutron star during the thermal neutrino emission stage. It is not difficult to calculate
their polarisations directly using the well-known expressions for the Hamiltonian of a non-relativistic fermion in a magnetic field and for the Boltzmann distribution function. This gives

\[ c^p \simeq \frac{G_F}{\sqrt{2}} g_A \frac{\mu_p \mu_N}{T} N_p, \quad c^n \simeq -\frac{G_F}{\sqrt{2}} g_A \frac{\mu_n \mu_N}{T} N_n. \]

(5)

Here \( \mu_p = 2.793 \) and \( \mu_n = -1.913 \) are the proton and neutron magnetic moments in units of the nuclear magneton \( \mu_N = e/2m_p = 3.152 \times 10^{-18} \) MeV/G, and \( g_A = 1.26 \) is the axial–vector renormalisation constant of the nucleon. The expressions in eq. (5) coincide with the results obtained in the recent paper [26]. Notice that \( c^p, c^n \) and \( c^e_Z \) all have the same sign. For non-degenerate particles, thermal fluctuations tend to destroy the polarisation, hence \( c^p \) and \( c^n \) decrease with increasing temperature \( T \). The polarisation of the protons and neutrons in the medium can influence the oscillations between the active and sterile neutrinos as well as the neutrino spin and spin–flavour precession. However, these effects have not been taken into account in most of the previous analyses of neutrino conversions in supernovae (the only exception we are aware of is ref. [26]). Probably, the reason for this was the general idea that heavy nucleons will be polarised to a lesser extent than the light electrons. We shall show now that this is not quite correct: even though the nucleon polarisation contributions to neutrino potentials are typically smaller than that of the polarised electrons in the supernova environment, they are of the same order of magnitude. More importantly, as we shall see, for spin–flavour precession mediated by neutrino transition magnetic moments of Majorana type, the effects of polarised electrons nearly cancel out, and nucleon polarisation constitutes the main magnetic field effect on the resonance conditions.

Let us consider now the degree of degeneracy of nucleons in the proto-neutron star. The degeneracy parameter of non-relativistic nucleons can be written as

\[ |\kappa_i| \approx \ln \left[ \frac{2}{N_i} \left( \frac{m_N T}{2\pi} \right)^{3/2} \right] \approx \ln \left[ 41.5 Y_i^{-1} \left( \frac{10^{11} g/cm^3}{\rho} \right) \left( \frac{T}{3 \text{ MeV}} \right)^{3/2} \right]. \]

(6)

Here \( m_N \) is the nucleon mass, \( N_i \) and \( \kappa_i \) \((i = p, n)\) are the nucleon number densities and chemical potentials. When \( d_i \equiv \exp (\kappa_i/T) = \exp (-|\kappa_i|/T) \ll 1 \) the nucleons are non-degenerate, i.e. form a classical gas, whereas in the opposite limit \( d_i \gg 1 \) the nucleons will be strongly degenerate. In the vicinity of the neutrinosphere \((\rho \sim (10^{11} - 10^{12}) \) g/cm\(^3\), \( T \sim (3 - 6) \) MeV, \( Y_e \sim 0.1 - 0.2 )\) we have \( d_n \lesssim 0.08, d_p \lesssim 0.02, \) i.e. the nucleons are strongly non-degenerate. Therefore the formulas of eq. (5) are valid there. In the core of the star the densities are \( \rho \gtrsim 10^{14} \) g/cm\(^3\), and the temperatures are higher, too: \( T \gtrsim 20 \) MeV [27]. As a result, nucleons are weakly non-degenerate there with \( d_i \sim 1 \). This means that one can use eq. (5) only for rough estimates of the nucleon polarisation contribution to neutrino dispersion relations in the core of the supernova.

It is instructive to estimate the relative size of the nucleon and electron polarisation contributions to the neutrino potentials. Let \( Y_i \) be the number of particles of the \( i \)th kind per baryon. Then \( N_i = Y_i N \), where \( N \) is the total baryon number density. The electric
neutrality of matter implies that \( Y_p = Y_e \) (we neglect the nuclei since their fraction is very small in the region of interest to us). From eqs. (4) and (5) we get

\[
\frac{c_i}{c_e} Z = g_A \mu_i \left( \frac{\pi}{6} \right)^{1/3} \left( \frac{Y_i}{Y_e} \right)^{1/3} \left[ \frac{N_i}{2} \left( \frac{2\pi}{m_N T} \right)^{3/2} \right]^{2/3} = g_A \mu_i \left( \frac{\pi}{6} \right)^{1/3} \left( \frac{Y_i}{Y_e} \right)^{1/3} d_i^{2/3}. \tag{7}
\]

With increasing degeneracy, the relative contribution of the nucleon polarisation increases. However, even in the non-degenerate case the nucleon polarisation contributions may not be small provided the degeneracy parameter is not too small. To see this, let us rewrite \( c_p/c_e \) and \( c_n/c_e \) in the following form:

\[
\frac{c_p}{c_e} Z \approx 0.24 \left( \frac{\rho}{10^{11} \text{ g cm}^{-3}} \right)^{2/3} \left( \frac{3 \text{ MeV}}{T} \right), \tag{8}
\]

\[
\frac{c_n}{c_e} Z \approx 0.16 \left( \frac{\rho}{10^{11} \text{ g cm}^{-3}} \right)^{2/3} \frac{Y_n}{Y_e^{1/3}} \left( \frac{3 \text{ MeV}}{T} \right). \tag{9}
\]

Taking for an estimate \( Y_e \approx 0.2, Y_n \approx 0.8, \rho \approx 10^{11} \text{ g/cm}^3 \) and \( T \approx 3 \text{ MeV} \), we obtain \( (c^p + c^n)/c^e \approx 0.3 \), i.e. the nucleon polarisation effect is about 30% of the electron polarisation one. The relative contribution of neutrons and protons is \( c^n/c^p \approx 0.68(Y_n/Y_e) \); since \( Y_n \gg Y_e \) in the supernova, \( c^n \) dominates over \( c^p \). Similar conclusions have been reached in [26].

3 Resonant spin–flavor precession in matter and magnetic fields

We shall summarize here the main features of the resonant spin–flavour precession (RSFP) of neutrinos in matter and a magnetic field [13, 14] and compare them with the corresponding characteristics of the MSW effect [9]; for a more detailed discussion of the RSFP see [28].

Neutrinos with non-vanishing flavour-off-diagonal (transition) magnetic moments experience a simultaneous rotation of their spin and flavour in a transverse magnetic field (spin–flavour precession) [29]. In vacuum, such a precession is suppressed because of the kinetic energy difference of neutrinos of different flavours, \( \Delta E_{\text{kin}} \approx \Delta m^2/2E \) for relativistic neutrinos. At the same time, in matter this kinetic energy difference can be cancelled by the potential energy difference \( V(\nu_i) - V(\bar{\nu}_j) \), leading to a resonant enhancement of the spin–flavour precession [13, 14]. The RSFP of neutrinos is similar to resonant neutrino oscillations (MSW effect [8]). For Dirac neutrinos, their transition magnetic moments cause transitions between left–handed neutrinos of a given flavour and right–handed (sterile) neutrinos of a different flavour. For Majorana neutrinos the spin–flavour precession due to their transition magnetic moments induces transitions between left–handed neutrinos of a given flavour and right–handed antineutrinos of a different flavour which are not sterile.
3.1 Resonance conditions

The resonance condition for a transition between left–handed neutrinos of the $i$th flavour and right–handed neutrinos or antineutrinos of the $j$th flavour ($i, j = e, \mu, \tau$ or $s$ where $s$ means the sterile neutrino) is

$$V(\nu_{iL}) + \frac{m_{\nu_i}^2}{2E} = V(\bar{\nu}_{jR}) + \frac{m_{\nu_j}^2}{2E}. \quad (10)$$

For antineutrinos of the $i$th type $V(\bar{\nu}_{iR}) = -V(\nu_{iL})$. Mean potential energies of active neutrinos and antineutrinos including matter polarisation effects are given in eqs. (1) and (2); for sterile neutrinos $V(\nu_s) = 0$. 

The resonance conditions for various RSFP transitions can be written in the following generic form:

$$\sqrt{2} G_F N_{\text{eff}} - c_{\text{eff}} B_{\parallel} = \frac{\Delta m^2}{2E}. \quad (11)$$

The resonance condition for neutrino oscillations in matter (MSW effect) has almost the same form, the difference being that the r.h.s. of eq. (11) is multiplied by the cosine of the double vacuum mixing angle, $\cos 2\theta_0$. The effective parameters of the resonance condition depend on the nature of the transition in question. In the following table we summarize the parameters that enter into the resonance condition (11) for the neutrino conversions of interest to us.

| No | transition | $N_{\text{eff}}$ | $c_{\text{eff}}$ | $\Delta m^2$ |
|----|------------|------------------|------------------|---------------|
| 1  | $\nu_e \leftrightarrow \nu_x$ | $N_e - N_x$ | $2(\theta_e + \theta^n)$ | $m_{\nu_e}^2 - m_{\nu_x}^2$ |
| 2  | $\nu_e \leftrightarrow \nu_x$ | $N_e - N_x$ | $2(\theta_e + \theta^n)$ | $m_{\nu_e}^2 - m_{\nu_x}^2$ |
| 3  | $\nu_e \leftrightarrow \bar{\nu}_x$ | $N_e - N_x/2$ | $\theta_Z - \theta^p - \theta^n$ | $m_{\nu_e}^2 - m_{\nu_x}^2$ |
| 4  | $\nu_e \leftrightarrow \bar{\nu}_x$ | $N_e - N_x/2$ | $\theta_Z - \theta^p - \theta^n$ | $m_{\nu_e}^2 - m_{\nu_x}^2$ |
| 5  | $\nu_x \leftrightarrow \bar{\nu}_x$ | $N_x/2$ | $\theta_Z + \theta^p + \theta^n$ | $m_{\nu_x}^2 - m_{\nu_e}^2$ |
| 6  | $\nu_x \leftrightarrow \bar{\nu}_x$ | $N_x/2$ | $\theta_Z + \theta^p + \theta^n$ | $m_{\nu_x}^2 - m_{\nu_e}^2$ |
| 7  | $\nu_e \leftrightarrow \nu_x$ | $N_e$ | $2\theta_Z$ | $m_{\nu_e}^2 - m_{\nu_x}^2$ |
| 8  | $\bar{\nu}_e \leftrightarrow \bar{\nu}_x$ | $N_e$ | $2\theta_Z$ | $m_{\nu_e}^2 - m_{\nu_x}^2$ |
| 9  | $\nu_e \leftrightarrow \nu_s$ | $N_e - N_x/2$ | $\theta_Z - \theta^p - \theta^n$ | $m_{\nu_x}^2 - m_{\nu_s}^2$ |
| 10 | $\bar{\nu}_e \leftrightarrow \bar{\nu}_s$ | $N_e - N_x/2$ | $\theta_Z - \theta^p - \theta^n$ | $m_{\nu_x}^2 - m_{\nu_s}^2$ |
| 11 | $\nu_x \leftrightarrow \nu_s$ | $N_x/2$ | $\theta_Z + \theta^p + \theta^n$ | $m_{\nu_s}^2 - m_{\nu_x}^2$ |
| 12 | $\bar{\nu}_x \leftrightarrow \bar{\nu}_s$ | $N_x/2$ | $\theta_Z + \theta^p + \theta^n$ | $m_{\nu_s}^2 - m_{\nu_x}^2$ |

Here $\nu_s$ is a sterile neutrino which is assumed to be left–handed ($\bar{\nu}_s$ is right–handed), $\nu_s = \nu_\mu$ or $\nu_\tau$, and for the sake of comparison we have also included the parameters for the neutrino conversions due to the MSW effect (lines 7–12). For $c_{\text{eff}} B_{\parallel} < \sqrt{2} G_F N_{\text{eff}}$, the neutrino transitions listed in Table 1 can only be resonantly enhanced if the corresponding $\Delta m^2$
and $N_{\text{eff}}$ are of the same sign. For given signs of $N_{\text{eff}}$, only 6 of the 12 transitions can be resonant, depending on the signs of the respective $\Delta m^2$. We shall comment on the $c_{\text{eff}} B_{||} > \sqrt{2} G_F N_{\text{eff}}$ case later on. It is interesting to notice that the parameters of the resonance conditions for the RSFP transitions involving sterile neutrinos or antineutrinos and those for the corresponding MSW transitions are the same (lines 3–6 and 9–12 respectively). The reason for this is that $V(\nu_s) = 0 = V(\bar{\nu}_s)$.

Several remarks are in order. The authors of [22] pointed out that the electron polarisation contribution cancels out in the resonance condition for the RSFP transitions $\nu_e \leftrightarrow \bar{\nu}_x$ and $\bar{\nu}_e \leftrightarrow \nu_x$. They therefore concluded that magnetic fields have no effect on these resonance conditions. We would like to emphasize that the cancellation noticed in [22] is not exact; it holds only up to the electroweak radiative corrections. In particular, this cancellation is a consequence of eq. (3) which is based on the tree-level relation between the masses of the $W^\pm$ and $Z^0$ gauge bosons, $M^2_W = M^2_Z \cos^2 \theta_W$. This relation is known to receive radiative corrections of the order of 0.5% (mainly due to the heavy top quark). There are other electroweak corrections that would also lead to an incomplete cancellation, e.g. from the anomalous magnetic moment of the electron. They are typically of the same order of magnitude, $\lesssim 0.5\%$. However, for supernovae more important contributions come from the polarisation of nucleons in the magnetised medium. As was demonstrated in sec. 2, they are quite sizable and can be comparable with $c^e$.

In ref. [12] it has been claimed that the potential of electron neutrinos $V(\nu_e)$ does not receive any matter-polarisation contributions. The authors, following ref. [23], claimed that the electron and proton polarisation effects cancel each other in $V(\nu_e)$ and therefore concluded that the $\nu_e \leftrightarrow \nu_s$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_s$ oscillations in the supernova are not affected by the magnetic field and so cannot be the cause of the pulsar birth velocities. The cancellation was the result of the assumption that protons are strongly degenerate in the proto-neutron star; as we have shown in sec. 2, this assumption is incorrect. Moreover, even for degenerate protons the cancellation takes place only if they are treated as Dirac particles, i.e. when the strong–interaction renormalisation of the proton’s magnetic moment and axial–vector coupling constant are neglected. In addition, the effects of neutron polarisation, which can be comparable with those of polarised electrons, were not considered in [12]. It should be noted, however, that the above shortcomings have no effect on the $\nu_x \leftrightarrow \nu_s$ transitions which were the main topic of ref. [12].

In refs. [25, 22] it was claimed that in strong enough magnetic fields the term $c^e B_{||}$ can overcome the $\sqrt{2} G_F N_{\text{eff}}$ term in the resonance condition of the $\nu_e \leftrightarrow \nu_x$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_x$ oscillations, leading to the possibility of new resonances. However, it has been demonstrated in [26] that this is incorrect: the electron polarisation contribution can never exceed the electron density contribution, essentially because the mean polarisation of electrons cannot exceed unity. The expressions for the polarisation of matter constituents (4) and (5) that are linear in the magnetic field are valid only when the induced polarisations of the particles in matter are small. Nevertheless, as was stressed in [26], for the $\nu_e \leftrightarrow \nu_s$ and $\bar{\nu}_e \leftrightarrow$
$\bar{\nu}_e$ oscillations the $c_{\text{eff}}B_\parallel$ term can indeed overcome the $\sqrt{2}G_FN_{\text{eff}}$ term in the resonance condition, since the effective number density $N_{\text{eff}} = N_e - N_n/2$ can become small provided that there is a compensation between the electron and neutron contributions. In this case new resonances are indeed possible, and the resonant channel will depend on whether the neutrinos are emitted along the magnetic field or in the opposite direction. Obviously, the same argument applies for the RSFP transitions involving sterile neutrinos or antineutrinos (lines 3–6 of Table 1). We would like to point out here that in the case of the RSFP transitions the same is also true even for the $\nu_e \leftrightarrow \bar{\nu}_x$ and $\bar{\nu}_e \leftrightarrow \nu_x$ oscillations (lines 1 and 2 of Table 1) that involve only active neutrinos; the matter polarisation term can overcome the $\sqrt{2}G_FN_{\text{eff}}$ term provided that the effective density $N_{\text{eff}} = N_n - N_e$ becomes small because of the compensation of the electron and neutron number densities. Thus, in the case of the RSFP of neutrinos, the matter polarisation term in the resonance condition can in principle exceed the effective matter density term leading to the possibility of new resonances for all types of neutrino transitions.

### 3.2 Adiabaticity parameters and transition probabilities

The probability of an RSFP–induced neutrino transition depends on the degree of its adiabaticity and is generally large when the adiabaticity parameter $\gamma$ is large enough:

$$\gamma = 4 \left( \frac{\mu_\nu B_\perp}{\sqrt{2}G_FN_{\text{eff}}} \right)^2 L_{\rho r} \approx 0.81 Y_{\text{eff}}^{-1} \left( \frac{10^{11} \text{ g/cm}^3}{\rho} \right) \left[ \frac{\mu_\nu}{10^{-13} \mu_B} \frac{B_\perp}{3 \times 10^{14} \text{ G}} \right]^2 \left( \frac{L_{\rho r}}{10 \text{ km}} \right) > 1.$$  \hspace{1cm} (12)

Here $\mu_\nu$ is the neutrino transition magnetic moment, $B_\perp$ is the strength of the transverse component of the magnetic field at the resonance, $Y_{\text{eff}}$ is defined through $N_{\text{eff}} = Y_{\text{eff}}N$, and $L_{\rho} \equiv (\frac{\rho}{\rho_0})^{-1}$ is the characteristic length over which the matter density varies significantly in the supernova, $L_{\rho r}$ being its value at the resonance.

The spin–flavour precession is typically strongly suppressed far below and far above the resonance point; in this case the transition probability is to a very good accuracy approximated by

$$P_{\text{tr}} \approx (1 - P') , \quad P' \equiv \exp \left( -\frac{\pi}{2} \gamma \right).$$  \hspace{1cm} (13)

The probability of neutrino transitions due to the MSW effect is given by the same expression with the adiabaticity parameter $\gamma$ replaced by $\gamma_{\text{MSW}} = (\sin^2 2\theta_0 / \cos 2\theta_0)(\Delta m^2 / 2E)L_{\rho r}$. For adiabatic transitions ($\gamma \gg 1$) the transition probability is close to one. In the vicinity of the neutrinosphere, $L_{\rho r} \sim 10 \text{ km}$; therefore the RSFP transitions will be adiabatic for

$$B_\perp \gtrsim 3 \times 10^{14} \left( 10^{-13} \mu_B / \mu_\nu \right) \text{ G}.$$  \hspace{1cm} (14)

\[\text{We note in passing that the MSW adiabaticity condition was formulated incorrectly in refs. 8 and 12. The oscillation length at resonance must be compared with the resonance width $\Delta r = 2\tan 2\theta_0 L_{\rho r}$ and not with $L_{\rho r}$ itself.}\]
This constraint can in principle be somewhat relaxed since $Y_{\text{eff}}$ is typically $<1$ and can also be $\ll 1$ in some cases; however, we will need the RSFP adiabaticity condition to be satisfied with some margin and so shall continue to use eq. (14). For the MSW transitions to be adiabatic near the neutrinosphere the vacuum mixing angle should satisfy $\theta_0 \gtrsim 10^{-4}$.

### 4 Kick momenta of pulsars

In ref. [8] the following two conditions for generating the pulsar kick velocity through resonant neutrino oscillations were formulated:

1. The neutrino conversion takes place between the neutrinospheres of two different neutrino species;
2. The resonance coordinate depends on the angle between the directions of the magnetic field and the neutrino momentum.

These conditions apply to the RSFP–induced neutrino conversions as well. Consider, e.g., the $\nu_{\tau} \leftrightarrow \bar{\nu}_e$ conversions above the $\nu_{\tau}$–sphere but below the $\bar{\nu}_e$–sphere. A $\nu_{\tau}$ propagates freely until it gets transformed into $\bar{\nu}_e$ through the RSFP conversion. The resulting $\bar{\nu}_e$ cannot escape easily and gets trapped since the resonance point is below its neutrinosphere. At the same time, a $\bar{\nu}_e$ which initially was trapped and diffused out slowly will be converted into $\nu_{\tau}$ when it reaches the resonance surface. The resulting $\nu_{\tau}$ escapes freely since the resonant conversion occurred above its neutrinosphere. Thus, the resonance surface becomes the new “neutrinosphere” for the $\nu_{\tau}$’s. It is, however, not a sphere. The matter polarisation in the supernova magnetic field leads to the resonance taking place at different distances from the core of the star for neutrinos emitted parallel and antiparallel to the magnetic field; as a result, the resonance surface has different temperatures in these two directions leading to an asymmetry of the momenta of the emitted neutrinos.

Let us estimate the magnitudes of $\Delta m^2$ that are necessary for various neutrino conversions to occur in the regions of interest to us. Since the neutrino mean energy is $\langle E \rangle \approx 3.15 T$, from eq. (11) one finds

$$\Delta m^2 \approx 1.4 \times 10^5 Y_{\text{eff}} \left( \frac{\rho}{10^{11} \text{g/cm}^3} \right) \left( \frac{T}{3 \text{MeV}} \right).$$

This depends on $Y_{\text{eff}}$ and therefore on the neutrino transition in question. In the vicinity of the neutrinosphere the electron fraction $Y_e$ is typically of the order of 0.1–0.2 at the time when neutrinos are copiously produced in the supernova (a few seconds after the core bounce). It decreases towards smaller densities and reaches the value of about 0.46. At the same time, in the dense core of the supernova there is still a significant amount of trapped $\nu_e$’s which hinder the neutronization process. Therefore $Y_e$ can be close to 0.4 in the supernova’s core (see [30] for a more detailed discussion).
For our estimates we will assume a hierarchical pattern of neutrino masses. For MSW transitions between active neutrinos or antineutrinos (lines 7 and 8 of Table 1) the required heavier neutrino mass is in general in the range $m_2 \sim (100 - 800) \text{ eV}$. For the RSFP transitions between active neutrinos and antineutrinos one would need $m_2 \sim (300 - 1500) \text{ eV}$, whereas for transitions between muon or tauon neutrinos or antineutrinos and sterile neutrino states (lines 5, 6, 11 and 12 of Table 1) $m_2$ should be in the range $200 \lesssim m_2 \lesssim 1.6 \times 10^4 \text{ eV}$. In all these cases neutrinos do not satisfy the cosmological bounds on the mass of stable neutrinos and would have to decay sufficiently fast. However, for the RSFP and MSW transitions between electron neutrinos or antineutrinos and sterile neutrino states (lines 3, 4, 9 and 10 of Table 1) $m_2$ can be considerably smaller since $Y_{\text{eff}}$ can be very small in this case. Indeed, the parameter $Y_e$ passes through the value $1/3$ somewhere between the supernova core and the neutrinosphere, i.e. $Y_{\text{eff}}$ passes through zero. This means that the required value of $\Delta m^2 \approx m_2^2$ can be very small, too: $0 \leq m_2 \lesssim 10 \text{ keV}$. For example, $m_2$ can well be in the ranges of a few eV or few tens of eV, which are both cosmologically safe and interesting (in particular, the allowed ranges of neutrino mass and transition magnetic moment are consistent with the predictions of the decaying neutrino theory of the ionisation of the interstellar medium [31]). The mass $m_2$ can also be in the range $10^{-3} - 10^{-4} \text{ eV}$ which of interest for the solar neutrino problem; moreover, in this case the transition can be resonant even for massless neutrinos. For $m_2 \lesssim 4 \text{ keV}$ transitions including both electron neutrinos and antineutrinos can be resonantly enhanced; this would lead to an additional factor of two increase of the pulsar velocities for a given value of the supernova magnetic field.

We will now derive the generalized expression for the neutrino momentum asymmetry which will be valid for the RSFP as well as for neutrino oscillation transitions. The relative recoil momentum of a pulsar can be estimated as [8]

$$\frac{\Delta k}{k} \propto \frac{T^4(r_0 - \delta) - T^4(r_0 + \delta)}{T^4(r_0)},$$

where $r_0$ is the position of the resonance in the absence of the magnetic field and $\pm \delta$ is the shift of the resonance coordinate for the neutrinos emitted parallel and antiparallel to the magnetic field. From the generic resonance condition (11) one can estimate the value of $\delta$ as

$$\delta \approx \frac{c_{\text{eff}} B}{\sqrt{2} G_F (dN_{\text{eff}} / dr)},$$

where $B$ is the magnetic field strength. For the neutrino momentum asymmetry we get

$$\frac{\Delta k}{k} \approx \frac{1}{6} \cdot 2 \cdot 4 \cdot R \frac{1}{T} \frac{dT}{dr} \delta \approx \frac{4}{3} R \left( \frac{c_{\text{eff}} B}{\sqrt{2} G_F} \right) \frac{1}{T} \frac{dT}{dN_{\text{eff}}}. \quad (18)$$

This includes the case $\Delta m^2 = 0$ which corresponds to the resonance transition due to the ordinary (flavour–diagonal) magnetic moments of neutrinos [7, 13].

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Here the factor $1/6$ takes into account that only one neutrino or antineutrino species out of 6 acquires a momentum asymmetry, and $R$ is a geometrical factor to be discussed below. The desirable value of $\Delta k/k$ is about $10^{-2}$; this can be achieved for a temperature asymmetry of the order of $10^{-2}$ (we are assuming here the neutrino transition probability $P_{tr} \approx 1$).

For resonant neutrino oscillations, the geometrical factor $R$ in eq. (18) takes into account the fact that for the neutrinos emitted in directions orthogonal to the supernova magnetic field the resonance condition is not affected by the field. Therefore such neutrinos do not contribute to the kick velocity of the pulsar. In order to find this velocity one has to calculate the net momentum of neutrinos emitted in the direction of the magnetic field. KS estimated the resulting geometrical factor as $1/2$; however, their result was criticised by Qian [10] who showed that in fact $R \approx 1/6$.

For spin–flavour precession the situation is somewhat different. The magnetic field plays a dual role in this process: first, its component $B_\perp$ transverse to the neutrino momentum mixes the left–handed and right–handed neutrino states and causes the spin–flavour precession itself; second, the longitudinal component $B_\parallel$ affects the resonance condition, as discussed in sec. 3.1. Both roles are important for the purposes of our discussion. In fact, only the neutrinos emitted in directions different from that of the magnetic field or orthogonal to it can contribute to $\Delta k/k$. For neutrinos emitted exactly along the magnetic field, the RSFP does not occur; for those emitted in the orthogonal plane the field has no effect on the resonance condition and therefore does not lead to any momentum asymmetry. For this reason, in the case of the RSFP, the geometrical factor in eq. (18) can be written as $R = R_1 R_2$, where $R_1 \approx 1/6$ as for neutrino oscillations, while the factor $R_2$ takes into account the reduction of the RSFP transition probability $P_{tr}$ for neutrinos emitted close to the magnetic field directions. Basically, this reduction excludes the zenith angles close to 0 and 180° in the angular integration over the neutrino momenta. The numerical value of $R_2$ depends on the extent of the excluded region, which in turn depends on the magnitude of the adiabaticity parameter $\gamma$. If the adiabaticity condition (12) is satisfied with a large margin (i.e. $\gamma \gg 1$), even a strong reduction of $B_\perp = B \sin \theta$ because of the zenith angle $\theta$ being close to 0 or 180° will not suppress the RSFP transition probability significantly. In this case $R_2 \approx 1$ and $R \approx R_1 \approx 1/6$. In our estimates we will assume that this is the case.

An important parameter that enters into the neutrino momentum asymmetry (18) is the derivative $dT/dN_{\text{eff}} = (dN_{\text{eff}}/dT)^{-1}$. The effective density $N_{\text{eff}}$ is in general a linear combination of $N_e$ and $N_\nu$, depending on the type of the neutrino conversion. The derivative $dN_e/dT$ was estimated by KS as $(\partial N_e/\partial T)_{\kappa_e}$ using the relativistic Fermi distribution function for the electrons. This gave

$$dN_e/dT \approx \frac{2}{3}(3\pi^2 N_e)^{1/3} T.$$  \hfill (19)

However, this approach was criticised by Qian [10] who pointed out that the chemical potential of electrons cannot be considered as temperature independent in the supernova. He suggested to use instead the results of numerical simulations of matter density and
temperature profiles, which typically give \( N \propto T^3 \). We adopt this approach here.

Using \( N_i = Y_i N \ (i = e, n) \) it is easy to show that

\[
\frac{dN_i}{dT} \approx \frac{N_i}{T} \left( 3 + \frac{d \ln Y_i}{d \ln T} \right) .
\] (20)

The electron fraction \( Y_e \) decreases (and therefore \( Y_n \) increases) with increasing \( r \) below the \( \nu_e \)-sphere. For this reason for electrons the expression in the parentheses in eq. (20) is in fact larger than 3 whereas for neutrons it is slightly smaller than 3. Estimates of the logarithmic derivatives using the \( Y_e \) profile from [33] give

\[
\frac{dN_e}{dT} \approx 4 \frac{N_e}{T}, \quad \frac{dN_n}{dT} \approx 2.8 \frac{N_n}{T}
\] (21)

in the neutrinospheric region. Notice that numerically \( dN_e/dT \) in eq. (21) is about an order of magnitude larger than the corresponding KS value (19) [10].

It is instructive to estimate the relative sizes of \( dN_n/dT \) and \( dN_e/dT \) in the supernova environment:

\[
\frac{dN_n}{dN_e} \approx 0.7 \left( \frac{Y_n}{Y_e} \right) .
\] (22)

Thus, \( dN_n/dT \) is typically a factor of 3 to 6 larger than \( dN_e/dT \). Notice that the \( dN_e/dT \) contribution to \( dN_{\text{eff}}/dT \) has the opposite sign compared to the \( dN_n/dT \) one (lines 1–4, 9 and 10 of Table 1) and so will tend to increase the kick, especially for transitions of electron neutrinos and antineutrinos into sterile states.

We shall first estimate the asymmetry \( \Delta k/k \) for the RSFP–induced transitions between active neutrinos \( \nu_e \leftrightarrow \bar{\nu}_x \) and \( \bar{\nu}_e \leftrightarrow \nu_x \) due to the Majorana neutrino transition magnetic moments (lines 1 and 2 of Table 1):

\[
\frac{\Delta k}{k} \approx \frac{2}{9} \left[ \frac{2(c^p + c^n)B}{\sqrt{2}G_F} \right] \frac{1}{T d(N_n - N_e)} \approx 1.2 \times 10^{-4} \left( \frac{B}{3 \times 10^{14} \text{G}} \right) \frac{3 \text{MeV}}{T} .
\] (23)

Let us compare eq. (23) with the corresponding expression for the case of the \( \nu_e \leftrightarrow \nu_x \) oscillations derived in [8]. We first notice that the numerical coefficient in eq. (10) of [8] was overestimated (and so the requisite magnetic field underestimated) by about a factor of 40, where a factor \( \sim 3 \) comes from the geometrical factor \( R \) and a factor \( \sim 13 \) from \( dN_e/dT \) [10]. Apart from the different numerical coefficient, their expression for \( \Delta k/k \) falls with increasing temperature as \( T^{-2} \) and not as \( T^{-1} \). Comparing eq. (23) with the corrected eq. (10) of ref. [8] we find that in order to obtain the same effect on the pulsar velocities one would need about a factor of two stronger magnetic field in the case of the RSFP of active neutrinos than in the case of the resonant oscillation of active neutrinos. For example, in order to produce \( \Delta k/k \approx 1\% \) a magnetic field \( B \gtrsim 2.5 \times 10^{16} \text{ G} \) is necessary. This field is of the same order of magnitude as that needed to explain the pulsar birth velocities by asymmetric neutrino
production \[3\]. By contrast, in our case, neutrinos carrying an asymmetric momentum will experience very few interactions with matter, and so the asymmetry is unlikely to be suppressed by such interactions.

Next, we consider the RSFP--induced transitions between active and sterile neutrino states due to the Dirac neutrino transition magnetic moments (lines 3-6 of Table 1). For the transitions $\nu_x \leftrightarrow \bar{\nu}_s$ and $\bar{\nu}_x \leftrightarrow \nu_s$ we obtain

$$\frac{\Delta k}{k} \approx \frac{2}{9} \left\{ \left[ c_e^Z + (c_p + c^n) \right] B/\sqrt{2} \right\} \left( 1/T \right) \left[ dT/d(N_n/2) \right]$$

$$\approx \left[ 3.7 \times 10^{-4} Y_{e}^{1/3} \left( \frac{10^{11} \text{g/cm}^3}{\rho} \right)^{2/3} + 7.6 \times 10^{-5} \left( \frac{3 \text{MeV}}{T} \right) \right] \left( \frac{B}{3 \times 10^{14} \text{G}} \right) . \tag{24}$$

From eq. (24) it follows that in order to get $\Delta k/k \approx 1\%$ one would need $B \gtrsim 9 \times 10^{15} \text{G}$. This field is of the same order of magnitude as the one that is needed in the case of the neutrino flavour oscillations. Moreover, for a hierarchical neutrino mass pattern $m_{\nu_s} \gg m_{\nu_\mu}, m_{\nu_\tau}$ the transitions between both $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ and sterile neutrino states can be resonant and contribute to $\Delta k/k \[3\]$. In this case a factor of two weaker field would be able to produce the desired kick.

For the transitions $\nu_e \leftrightarrow \bar{\nu}_s$ and $\bar{\nu}_e \leftrightarrow \nu_s$ we obtain

$$\frac{\Delta k}{k} \approx \frac{2}{9} \left\{ \left[ c_e^Z - (c_p + c^n) \right] B/\sqrt{2} \right\} \left( 1/T \right) \left[ dT/d(N_n/2 - N_e) \right]$$

$$\approx \left[ 3.7 \times 10^{-4} Y_{e}^{1/3} \left( \frac{10^{11} \text{g/cm}^3}{\rho} \right)^{2/3} - 7.6 \times 10^{-5} \left( \frac{3 \text{MeV}}{T} \right) \right] \left( \frac{Y_n}{Y_n - 2.86 Y_e} \right) \left( \frac{B}{3 \times 10^{14} \text{G}} \right) . \tag{25}$$

For these transitions in order to get $\Delta k/k \approx 1\%$ one would typically need $B \gtrsim 4 \times 10^{15} \text{G}$. This field is about a factor of two weaker than the one that is needed in the case of the KS mechanism. We would like to emphasize, however, that our consideration is rather simplified and can only yield order-of-magnitude estimates for the requisite magnetic field strengths.

It should be noticed that the results in eqs. (24) and (25) apply to the case of resonant oscillations between active and sterile neutrinos as well. This case was studied in \[12\]; however the result obtained in that paper differs from ours. The reason for this is that the authors of \[12\] erroneously considered neutrons as strongly degenerate in the hot proto-neutron star. This resulted in a suppressed value of $\Delta k/k$, and in order to save the situation, they had to assume that the resonant oscillations take place deep in the core of the supernova. However, as follows from our considerations, the non-degeneracy of neutrons increases $\Delta k/k$ so that there is no need to assume that the resonance takes place in the supernova’s core. Moreover, the asymmetry decreases with the resonance density.

\[4\]This has been pointed out for the case of oscillations into sterile neutrinos in \[12\].
The lower bounds on the supernova magnetic fields $B$ were derived here assuming that the RSFP transitions are adiabatic. The adiabaticity condition puts another lower bound on $B$, eq. (14). The bounds obtained in this section are more restrictive provided the neutrino transition magnetic moments satisfy $\mu_\nu \gtrsim 10^{-14} \mu_B (10^{-15} \mu_B)$ for Dirac (Majorana) neutrinos.

5 Conclusion

We have studied the effects of the spin polarisation of matter in a supernova magnetic field on the resonance conditions for spin–flavour precession of Dirac and Majorana neutrinos in supernovae. The magnetic field distorts the resonance surface resulting in an asymmetric neutrino emission, which can explain the observed space velocities of pulsars and their possible correlation with the pulsar magnetic fields. Our estimates for the case of spin and spin–flavour precession of Dirac neutrinos also apply to oscillations into sterile neutrinos and correct the results of ref. [12] where the effect was underestimated. In the case of resonant spin–flavour precession into sterile neutrino states due to Dirac transition magnetic moments of neutrinos, the requisite supernova magnetic field strengths are $B \gtrsim 4 \times 10^{15}$ G. This is about a factor of 2 smaller than the field necessary in the case of neutrino flavour oscillations. Such fields are considered possible in supernovae [16]. For resonant spin–flavour precession between active neutrinos and antineutrinos due to Majorana transition magnetic moments of neutrinos, magnetic field strengths $B \gtrsim 2 \times 10^{16}$ G would be needed.

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