Shapiro steps as a direct probe of $\pm s$-wave symmetry in multigap superconducting Josephson junctions

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We theoretically study the Shapiro steps in a hetero-Josephson junction made of a single-gap superconductor and a two-gap one. We find that an anomalous dc Josephson current is induced by tuning the frequency of an applied microwave to the Josephson-Leggett mode frequency, which creates an extra step structure in the I-V characteristics besides the conventional Shapiro steps. The step heights at the resonance voltages exhibit an alternate structure of a large and small value reflecting the gap symmetry of the two-gap superconductor. In the $\pm s$-wave case in which the two gaps have opposite signs in the two-gap superconductor the steps with odd index are enhanced, whereas in the $s$-wave case the ones with even index have larger values. The existence of the fractional Shapiro steps is also predicted.

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Since the discovery of iron-pnictide superconductors,17 the symmetry of the superconducting gaps has attracted a great interest. The spin-fluctuation mechanism based on nesting between disconnected multiple Fermi surfaces predicts the $\pm s$-wave symmetry.6 It has been reported that the scanning tunneling microscope exhibits a characteristic field dependence expected from the $\pm s$-wave symmetry, and also the scanning SQUID detects half-fluxon dynamics consistent with the $\pm s$-wave gaps. However, the consensus on the pairing symmetry of the iron-based superconductors has not yet been attained.2 Hence, a more definitive experimental probe is now in great demand.

To clarify the pairing symmetry and associated new physics, various studies have been done for the Josephson effects in the iron-based superconductors both experimentally3–10 and theoretically.11–17 The present authors have shown theoretically that a superconducting-insulator-superconducting (SIS) Josephson junction made of single-gap and multi-gap superconductors reveals Josephson effects depending sensitively on the gap symmetry. In such a hetero Josephson junction the Cooper-pairs can transfer via multiple tunneling channels between the superconducting electrodes, because one of the electrodes has multiple conduction bands available for the tunneling. In the case of the $\pm s$-wave symmetry the two tunneling channels, i.e., “0-junction” and “π-junction” are formed. Therefore, one expects unusual Josephson effects different from the conventional one. Furthermore, a new phase oscillation mode called the “Josephson-Leggett (JL) mode” exists in the low frequency region besides the Josephson plasma. This mode is expected to affect the AC Josephson effect. In this paper, we investigate effects of the JL mode on Shapiro steps in such an SIS hetero Josephson junction under microwave irradiation. We show that resonance between the JL mode and an applied microwave occurs in the phase running states when the microwave frequency is tuned to the frequency of the JL mode. The maximum DC Josephson currents induced at the resonant voltages exhibit remarkable behavior reflecting the gap symmetry. Furthermore, the existence of fractional Shapiro steps is predicted.

Consider an SIS hetero Josephson junction shown schematically in Fig. 1. We assume that the electrodes 1 and 2 in this system are, respectively, single- and two-gap superconductors and the insulating barrier between the two electrodes has width $d$ and dielectric constant $\varepsilon$. To observe the Shapiro steps we irradiate a microwave generating the AC voltage $V_{mw}\cos\Omega_m t$ at the junction site as shown in Fig. 1.

Assuming that the phase difference is uniform along the in-plane direction (parallel to $xy$-plane), one can derive the effective Lagrangian density in the present hetero Josephson junction system under no external magnetic field as follows

\[ \mathcal{L}_{\text{eff}} = \frac{\hbar^2}{8\pi\mu} + \sum_{i=1}^{2} \frac{q_i^2}{8\pi\mu_i} + \sum_{i=1}^{2} \frac{\hbar j_i}{\varepsilon^*} \cos \theta^{(i)} \\
+ \frac{\hbar j_m}{\varepsilon^*} \cos \psi + \frac{d\varepsilon}{8\pi} (E_{21})^2, \tag{1} \]

where $\theta^{(i)}$ and $\psi$ are, respectively, the gauge-invariant phase difference and the relative phase of the superconducting gaps having the phases $\varphi^{(1)}$ and $\varphi^{(2)}$ in the electrode 2 defined as

\[ \theta^{(i)} = \varphi^{(i)} - \varphi - \frac{e^* d}{\hbar c} A_{21}^z, \]

\[ \psi = \varphi^{(1)} - \varphi^{(2)}, \]

with $\varphi$ being the phase of the electrode 1. The quantity $A_{21}^z$ in $\theta^{(i)}$ is given as $A_{21}^z = d^{-1}\int_{-d/2}^{d/2} A^z(z)dz$, where $A^z(z)$ is the $z$-component of the vector potential.

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the electric field. The electric field inside the insulating layer is characterized by the charge screening length \(\alpha\). The phase, that is, the value of the critical current at each step height, that is, the value of the critical current at each step height, depends on the relative phase, that is, the value of the critical current at each step height. The critical current depending on the gap-symmetry also fluctuates. In this paper, \(\psi(t)\) is assumed to be small. Since the total Josephson current in the present system is given as

\[
I_a(t) = \sum_{i=1}^{2} j_i \sin[\theta^{(i)}(t)],
\]

we have a simple expression, \(I_a = j_c(\psi_0)\sin[\theta^{(1)} + \Lambda f(t)]\), for \(\psi(t) = 0\) with \(j_c(\psi_0) = j_1 + j_2 \cos \psi_0\). Then, using the standard technique\(^9\), one finds that the Shapiro steps appear at the voltages which are the same as in the conventional Josephson junctions, i.e., \(V_0 = \omega \Lambda^{-1}(h/e)\Omega_{\text{mw}}\), \(n\) being an integer. We note that the step height \(|j_c(\psi_0)J_a(|Au|)|\) depends on the relative phase, that is, the value of the critical current at each step height is enhanced (suppressed) for the s-wave (±-s-wave) case. The critical current depending on the gap-symmetry also leads to the anomaly of the Reidel peak, which can be used for the identification of the ±-s-wave gap as proposed by Inotani and Ohashi\(^11\).

Next suppose that \(\psi(t)\) is temporally fluctuated. To describe the time evolution of \(\psi\) we utilize the equation of motion obtained from Eq. (1). From the Euler-Lagrange equations with respect to \(\varphi^0\) and \(\varphi^{(i)}\) one can derive the equation as\(^17\),

\[
\frac{1}{\alpha_1} \partial_t^2 \theta^{(1)} - \frac{1}{\alpha_2} \partial_t^2 \theta^{(2)} + \sum_{i=1}^{2} \frac{1}{\alpha_i} \partial_t^2 \theta^{(i)} = -\omega_1^2 \sin \theta^{(1)} + \omega_2^2 \sin \theta^{(2)} - \text{sgn}(J_{\text{in}}) \omega_2^2 \sin \psi.
\]

where \(\omega_i = \sqrt{4\pi e^2 d_j/\hbar \epsilon_j}\), \(\omega_{\text{in}} = \sqrt{4\pi e^2 d_j J_{\text{in}}/\hbar \epsilon_j}\), and 
\(\xi = (\alpha_1 - \alpha_2)/(\alpha_1 + \alpha_2)\). We note that Eq. (3) describes...
the mass of the JL mode comes from, respectively, the charge imbalance

\[ \Delta \Omega_0 = p \Omega_{\text{mw}}, \quad \Omega_{\text{mw}} = q \omega_{\text{JL}}. \]

The frequency \( \Omega_{\text{mw}} \) is then expressed as \( \Omega_{\text{mw}} = (p + k)q \omega_{\text{JL}} \). Close to the resonance we write the coefficients \( A \) and \( B_k \) as

\[ A = -\xi \left( \frac{\omega_1^2 + \omega_2^2}{\omega_1^2 - \omega_2^2} \right) \sin \left( \sin^{-1} \frac{\omega_2}{\omega_1} \right) \],

\[ B_k = -\nu \frac{j_1 - j_2}{2J_{in}} J_k \left( \Lambda u \right) / \omega_{\text{JL}}. \]

with \( \Omega_k = \Lambda \Omega_0 + k \Omega_{\text{mw}} \) and \( J_k \left( \Lambda u \right) \) being the Bessel function of the \( k \)th order. It should be noted that \( \tilde{\psi} \) has poles at \( \Omega_{\text{mw}}^2 = \omega_{\text{JL}}^2 \) and \( \Omega_k^2 = \omega_{\text{JL}}^2 \), that is, \( \tilde{\psi} \) is resonantly enhanced at these frequencies. These resonant conditions can be satisfied by tuning the frequency of the applied microwave. In a region far from the resonance we have \( \tilde{\psi} \approx 0 \). In the region we have conventional Shapiro steps as discussed above. It should be also noted that Eq. (6) has a characteristic oscillating solution corresponding to the JL mode, which is dropped in Eq. (6) because the resonance effects are predominant. An enhancement of the DC tunneling current by this collective mode is pointed out without microwave irradiation as discussed in Ref. 21.

Let us now study the resonance effects due to the microwave irradiation on the I-V characteristics in the present system. It is convenient to use the dimensionless parameters, \( p \) and \( q \), defined as

\[ p = u \cdot \beta_{\text{nl}} / h, \quad q = \nu \cdot \beta_{\text{nl}} / h. \]

FIG. 3: DC components of \( I_s(t) \) vs. \( \Omega_0 \). (a) \( \Omega_{\text{mw}} = q \omega_{\text{JL}} \) (odd number), (b) \( \Omega_{\text{mw}} = q \omega_{\text{JL}} \) (even number).
the relation $J_n(x) = (-1)^n J_n(x)$ ($n \in \mathbb{Z}$ and $x \in \mathbb{R}$), we find that
\[ I_{s, DC} \approx |j_1 + (-1)^p j_2 \cos \psi_0| J_p(x_1). \]
The coefficient $|j_1 + (-1)^p j_2 \cos \psi_0|$ takes alternating values with respect to $p$, that is, $j_1 + (-1)^p j_2$ in the s-wave case ($\psi_0 = 0$), whereas $j_1 + (-1)^{p+1} j_2$ in the $\pm s$-wave case ($\psi_0 = \pi$), which leads to the fact that a larger DC current appears at even (odd) $p$ in the s-wave ($\pm s$-wave) case. This remarkable feature of the Shapiro steps caused by the resonance with the JL mode induces the fractional Shapiro steps, as well. Figure 3 shows the DC current, which leads to the fact that a larger DC current appears at even (odd) $p$ in the s-wave ($\pm s$-wave) case.

Finally, we examine the case of $q > 1$ ($q \in \mathbb{Z}$). The resonance in this case arises from the poles in the second term on the right hand side of Eq. (6), i.e., $\Omega_k = \omega_{JL}$, which brings about the correction to $\theta^{(i)}(t)$ as $\psi \approx B_k \sin(\omega_{JL}t + \theta_0^{(1)})$. Then, after similar mathematical manipulation to that in the $q = 1$ case one finds the relation $A_{\theta k} + n A_{\text{new}} + \ell_k \omega_{JL} = 0$ with $n, \ell_k \in \mathbb{Z}$, for the DC current to be induced. From this result it follows,
\[ p(\Omega_k = \omega_{JL}, q > 1) = -n - \frac{\ell_k}{q} = -k + \frac{1}{q}. \]

We note that $p$ is generally not an integer, which indicates that the Shapiro steps appear at fractional voltages, that is, the resonance with the JL mode induces the fractional Shapiro steps, as well. Figure 3 shows the DC current, i.e., the step heights at the resonance voltages. The microwave amplitude and the junction parameters are the same as in Fig. 2. Figures (a) and (b) show the cases of $q = 2$ (i.e., $p = m + \frac{1}{2}$ ($m \in \mathbb{Z}$)) and $q = 3$ (i.e., $p = m + \frac{1}{3}$ ($m \in \mathbb{Z}$)), respectively. The numerical results given in these figures proves the existence of the fractional Shapiro steps. Although fractional order steps have been studied in Josephson arrays and highly transmissive junctions with conventional one-gap superconductors, the origin of the present system is an intrinsic feature of multi-gap superconductors, the JL modes.

In conclusion, we revealed that an external microwave applied to the hetero Josephson junction resonantly excites the JL mode and induces DC currents. The voltage dependence of the Shapiro steps is sensitive to the inter-band sign change, i.e., the pairing symmetry of the multi-based superconductor. It is also predicted that the fractional Shapiro steps appear. We suggest that the tuning of the microwave frequency to the JL mode one bring about not only unconventional types of Shapiro steps but also a clear probe to the gap symmetry of a multi-gap superconductor.

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