Approximate determination of the plasma potential spatial distribution in the isolated dust particle vicinity

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Abstract. In this paper, results of two numerical models are compared. The main purpose of these models is to determine the self-consistent spatial distributions of plasma (electric potential and space charge) near isolated spherical dust particles. In the first model, the spatial distribution of the self-consistent plasma potential is determined by expanding the plasma space charge spatial distribution in Legendre polynomials; in the second model, it is determined by direct numerical integration of the Poisson equation solution. The results show that the dependences of the system main parameters (wake magnitude and position, dipole moment of the ion cloud) coincide for small values of the external electrostatic field. With an increase in the external field strength, the dependences for two models cease to coincide, which is due to the inapplicability of Legendre polynomial decomposition in the case of strong anisotropy.

1. Introduction
For the past three decades, dusty plasma has been under the scrutiny of many scientific groups [1-3]. This is due to its wide distribution both in the space environment [4-7] and in industrial processes [8,9]. The main interest in dusty plasma is the phenomenon of self-organization of dust particles into strongly bound structures called dust crystals. Dust crystals are studied in microgravity, on the ISS [10-12], and under the laboratory conditions in stratified gas discharges [13, 14].

Dust particles in plasma, in the overwhelming majority of cases, are negatively charged. If a negatively charged particle is placed in a plasma flow, then, as a result of ion focusing, a positive region in the potential, called a wake, will form behind it. It is believed that the self-ordering phenomena is caused by interaction of dust particles through the surrounding plasma. The main elements of this interaction are the appearance of the ion focusing effect and the formation of a wake behind the dust particles [15, 16]. According to numerical studies, the wake charge can reach 10% of the dust particle charge [17, 18]. As a result, it is essential to consider the interaction of an isolated dust particle with a plasma flow for determining the exact mechanisms of the dust particles interaction in a crystal.

Within the [19, 20], a new numerical model was created, where the self-consistent spatial distribution of the plasma potential was determined based on the solution of the Poisson equation, where the space charge density spatial distribution was expanded into Legendre polynomials. One of the main issues in application of any numerical model is determination of the parameter intervals in which this numerical model is applicable. In this paper, the results of the model presented in [19, 20] were compared with the results of the model, where the self-consistent spatial plasma distributions were calculated by the direct solution of the Poisson equation, performed by direct numerical integration.
2. Model

The numerical model presented below was described in great detail in [19, 20]. Therefore, this chapter will be devoted to description of differences between the new model and the model used in [19, 20].

In this model, the area of calculation is chosen as a straight prism, the dimensions of which along the coordinate axes are: \( x, y \in [-20 \lambda_i, 20 \lambda_i], z \in [-20 \lambda_i, 20 - 40 \lambda_i] \), where \( \lambda_i = \left( kT_i / 4\pi e^2 n_\infty \right)^{1/2} \) is the ion Debye length, \( n_\infty \) is the density of the unperturbed plasma and \( T_i \) is the ion temperature. In the center of the computational volume is a sphere \( r_0 \).

It is significantly inconvenient to divide the elongated computational domain into spherical segments \((r, \theta)\), performed in previous modifications. It is much more convenient to subdivide the region into sectors \((\rho, z)\), according to cylindrical symmetry with the volume of each individual segment equal to \( V_{k,l} = 2\pi \rho_k \Delta \rho_k \Delta z_l \). The time \( T_{k,l} \) which the ions spend in total in the random segment \( k, l \) is directly proportional to the ion density \( n_i(r, \theta) \):

\[
T_{k,l} \propto n_i(r, \theta) \quad \text{(1)}
\]

where \( \langle T_{k,l} \rangle_{\text{border}} \) is the average time ions spend at the edge of the system. From the density \( n_i(r, \theta) \) of ions, the spatial distribution of the space charge density \( n(r, \theta) = n_i(r, \theta) - n_e(r, \theta) \) can be obtained if the electron density \( n_e(r, \theta) \) is presented as the Boltzmann distribution.

In both models, the initial potential distribution was specified as a superposition of the Debye-Hückel potential and the potential of the external electrostatic field:

\[
U_0(r, \theta) = -\frac{Q}{r} e^{-r} - \tilde{E} r \cos \theta \quad \text{(2)}
\]

where, \( \tilde{E} = e \lambda_i E / kT_i \) is the dimensionless strength of the external electric field, \( \tilde{Q} = e^2 Z_d \lambda_i kT_i \) is the dimensionless dust particle charge.

The main difference between two models is the method of calculation of the self-consistent potential spatial distribution. In the case of the model presented in [19, 20], it was determined as the following expansion:

\[
U_1(r, \theta) = -\frac{Q}{r} + \sum_k \frac{1}{2k+1} \left[ \frac{1}{r^{k+1}} \int_0^r n_i(r) x^{k+2} dx + r^{k+1} \right] P_k(\cos \theta) - \tilde{E} r \cos \theta, \quad \text{(3)}
\]

where the number of harmonics \( k \) was determined by practicability, proceeding from the strength of the external electrostatic field. In the case of a new model, the self-consistent potential spatial distribution was determined as:

\[
U_2(r, \theta) = -\frac{Q}{r} + \int \frac{n(r', \theta') d^3 r'}{|r - r'|} - \tilde{E} r \cos \theta. \quad \text{(4)}
\]

The algorithm for calculating the self-consistent spatial distributions of the potential and space charge of the plasma for the two models is identical:
1) Ion trajectories are calculated for the initial potential $U_0(r, \theta)$ (2) and ion density is calculated (1).

2) From the space charge spatial distribution according to equations (3) and (4) for the first and second models, respectively, the potential spatial distribution is calculated.

3) New dust particle charge $Q$ is derived from the condition of equality of ion and electron fluxes to the surface of the dust grain.

4) Return to step 1. Replacing the potential $U_0(r, \theta)$ with the current potential $U_L(r, \theta)$ or $U_P(r, \theta)$ for the first and second models, respectively.

The calculation by this algorithm is performed until the spatial distributions of the space charge and plasma potential cease to change.

It is obvious that the solution represented by the equation (4) is the most accurate. The reason why it was not used in [19, 20] is the high complexity of obtaining a self-consistent solution by this method. According to our experience, it takes 4-5 times more time to obtain the single self-consistent solution by equation (4) than to calculate it by equation (3).

3. Results

The data presented in the current paragraph are obtained for the following set of parameters: ion temperature $T_i = 273$ K, temperature ratio $T_e/T_i = 100$, ion mean free path for the resonant charge exchange collisions $l_i = 2.5 - 10 \lambda_i$, dust particles radii $r_0 = 1 - 2 \mu m$, and the interval of external electric field values $E = 0 - 12$.

The following is a comparison of the self-consistent spatial distributions of the space charge and potential calculated by two models: the first, where the self-consistent potential spatial distribution is calculated by expanding the spatial distribution of the space charge in terms of Legendre polynomials, and the second, where the potential is calculated by the direct integration method.

Figures 1 and 2 show the functions $n_0(r)r^2$ and $n_1(r)r^3$, calculated by two models, for different $E$, at $l_i = 5 \lambda_i$, $r_0 = 2 \mu m$. Figure 1 shows that functions $n_0(r)r^2$ calculated by two models coincide. Thus, the plasma total charge value, in the computational domain, remains unchanged, regardless of the calculation method. For function $n_1(r)r^3$, differences appear that grow with an increase in $E$. They consist in the fact that the local maximum of functions $n_1(r)r^3$ calculated by the direct integration model proved to be larger than the maximum calculated by the expansion model. That is, the direct integration method takes into account the stronger anisotropy of the ion cloud along the external electrostatic field.

![Figure 1](image1.png)  
**Figure 1.** Comparison of the functions $n_0(r)r^2$, calculated for two models at $l_i = 5 \lambda_i$, $r_0 = 2 \mu m$.  

![Figure 2](image2.png)  
**Figure 2.** Comparison of the functions $n_1(r)r^3$, calculated for two models at $l_i = 5 \lambda_i$, $r_0 = 2 \mu m$.  

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Figure 3 shows the slices of the self-consistent potential spatial distribution $U(\rho = 0, z)$, made in the direction of the external electrostatic field. A comparison of these sections is made for two models, at $l_i = 5 \lambda_i$ and $r_0 = 2 \mu$m. With an increase in $\tilde{E}$, the differences in the wake structure grow. In the method where direct integration is performed, the local maximum of the potential $U_{\text{max}}$ proves to be larger, while its position $Z_{\text{max}}$ shifts downstream from the dust particle. However, it is important to note that, despite the differences in the wake structures, $U(\rho = 0, z)$ coincides for $z < 0$.

In order to evaluate in more detail the changes that the potential calculation method introduces into the wake structure, the dependence of its main characteristics on the plasma parameters, calculated by two methods, is considered. Figures 4 and 5 show a comparison of dependences of $U_{\text{max}}$ and $Z_{\text{max}}$ on $\tilde{E}$, calculated by two models. The dependences shown in Figure 4 are normalized to the same value as those presented in [19]. These dependences demonstrate that, when the strength of the external electrostatic field is low, the wake characteristics calculated by two methods coincide with good accuracy. With an increase in $\tilde{E}$, $U_{\text{max}}$ calculated by the direct integration model proves to be larger than $U_{\text{max}}$ calculated by the expansion method, since the expansion in Legendre polynomials becomes insufficient to describe the space charge spatial distribution anisotropy. The distance from the wake maximum to the dust particle, $Z_{\text{max}}$, also increases for the direct integration method due to the fact that the ion cloud is warped more strongly towards the field.

Figure 6 shows a comparison of the ion cloud dipole moment dependences on $\tilde{E}$, calculated by two methods. The dependences were normalized similarly to that presented in [19]. The dependence in Figure 6 shows that, despite the differences in functions $n_i(r)r^3$ presented in Figure 2, the dipole moment calculated by two models is the same.

As a result of the comparison of the self-consistent spatial distributions, the characteristics of the wake, and the ion cloud dipole moment calculated by two models, it can be concluded that the results obtained by calculating the expansion method have good accuracy in the case when the value of the external electrostatic field is small. With an increase in the strength of the external electrostatic field, the calculation accuracy of the first model decreases significantly, which makes it necessary to use the direct solution of the Poisson equation to calculate the self-consistent spatial distribution of the plasma potential near an isolated dust particle.
4. Conclusion

This paper presents two numerical models in which determination of the self-consistent spatial distribution of the plasma potential around an isolated dust particle is calculated by two methods: the method of direct integration and the method of expansion in Legendre polynomials.

As a result of calculations performed with these models, it is shown that the spatial distributions of the space charge and plasma potential do not depend on the method of calculating the potential for low external electrostatic field strengths, but differ significantly for high strengths.

The influence of the chosen method for calculating the self-consistent potential on the dependence of the wake characteristics (its magnitude and position) is shown. It is shown that in the case of direct numerical integration of the solution to the Poisson equation, for high strengths of the external electrostatic field, the wake turns out to be larger and is located farther from the dust particle. This is due to the fact that the highly anisotropic distribution cannot be described by the method of expansion into Legendre polynomials.
It is shown that, despite the differences in the spatial distributions of the space charge density calculated by two methods, the dependences of the dipole moment coincide.

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