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Video-Based Analysis of the Transition from Slipping to Rolling

Álvaro Suárez and Daniel Baccino, CFE, Uruguay
Arturo C. Martí, Universidad de la República, Uruguay

The problem of a disc or cylinder initially rolling with slipping on a surface and subsequently transitioning to rolling without slipping is often cited in textbooks.\textsuperscript{1,2} Students struggle to qualitatively understand the difference between kinetic and static frictional forces—i.e., whereas the magnitude of the former is known, that of the latter can only be described in terms of an inequality while the relative velocity at the point(s) of contact is equal to zero. In addition, students have difficulty understanding that frictional forces can act in the direction of motion—i.e., they can accelerate objects.\textsuperscript{3-6}

Because the time evolution of the linear and angular velocities of a rigid body cannot readily be determined experimentally, the problem is usually addressed from a purely theoretical perspective. One remarkable exception is Ref. 7, where, in a different approach, the authors consider only the evolution of the center-of-mass velocity of an initially rolling cylinder. Clearly, the lack of experimentation may hamper the learning process, as it does not allow students to visualize and internalize facts that challenge preexisting misconceptions formed in earlier stages of learning.

The following experiment serves to clearly demonstrate the transition from rolling with slipping to rolling without slipping. In the experiment, a rotating bicycle wheel was contact with a horizontal surface and the wheel in motion was tracked using Tracker video analysis software.\textsuperscript{8} The software created linear velocity plots for the center of mass and a point on the circumference, as well as a plot of the angular velocity of the rotating wheel. The time evolution plots created by Tracker clearly illustrate the transition between the two types of motion.

**Experimental**

The experimental setup consisted of a bicycle wheel of radius $R = (32.0 \pm 0.2)$ cm initially rotating on its axis at a certain distance above the floor. The wheel was then placed in contact with the floor and released, as shown in Fig. 1. From the moment it came in contact with the floor to the moment it started to rotate without slipping, the wheel in motion was recorded with a Kodak PlaySport video camera mounted on a tripod, with its optical axis perpendicular to the plane of rotation of the wheel.

The results of the analysis are shown in Figs. 2 and 3. The plot of the velocity of the center of mass $v$ vs. time $t$ shows that, after the wheel was contact with the floor, its center of mass displayed different types of motion before and after a point in time denoted by $t_1$, when it transitioned between the two types (Fig. 2). During the first time interval, the center of mass of the wheel traveled in uniformly accelerated rectilinear motion, whereas during the second time interval, starting at approximately $t_2 \sim 0.9$ s, it traveled in uniform rectilinear motion. Results of linear fitting showed that the acceleration of the wheel’s center of mass during the first time interval was

\[ v_F = (1.42 \pm 0.01) \text{ m/s}. \]

\[ \omega_F = (-4.38 \pm 0.06) \text{ s}^{-1}. \]
In the experiment, the results of linear fitting, taking \( g = 9.8 \, \text{m/s}^2 \), led to
\[
\mu = 0.134 \pm 0.004.
\]

In the interval where slipping occurs, the angular velocity of the wheel cannot be related to the velocity of the center of mass. Assuming that both the floor and the wheel are perfectly rigid, the only acting torque is generated by the force of friction. Applying Newton's second law to the rotation of the wheel around its center of mass yields
\[
fR = I\alpha, \quad \text{(6)}
\]
where \( I \) is the moment of inertia of the wheel about an axis passing through the center of mass, such that
\[
I = KmR^2, \quad \text{(7)}
\]
where \( K \) is a dimensionless constant dependent on the distribution of mass, and \( \alpha \) the angular acceleration of the wheel. Combining Eqs. (4), (6), and (7) gives
\[
\alpha = \frac{\mu g}{KR}, \quad \text{(8)}
\]
In the experiment, given the kinetic friction coefficient, the radius of the wheel, and the angular velocity, the distribution of mass of the wheel with respect to the calculated center of mass can be characterized in terms of \( K = 0.56 \pm 0.03 \). Slightly above \( 1/2 \), this value indicates that the wheel's mass is skewed towards the outside of the wheel.

Figures 2 and 3 reveal that during the slipping stage the shapes of the linear acceleration and the angular velocity are, in first approximation, linear:
\[
v_{\text{cm}} = v_0 + at, \quad \text{(9)}
\]
and
\[
\omega = \omega_0 + \alpha t. \quad \text{(10)}
\]

The transition from slipping to pure rolling motion occurs when the velocity of the contact point with the floor first vanishes,
\[
\dot{v}_0 + \ddot{a} t + \tilde{v}_0 \times \tilde{R} = 0. \quad \text{(11)}
\]
Using Eqs. (6)–(11), the transition time \( t_1 \) can be expressed as
\[
t_1 = \frac{\left| \frac{\dot{v}_0}{\omega_0} | R - | v_0 | \right| K}{\mu g (K + 1)}. \quad \text{(12)}
\]
and velocity of the center of mass in the second interval as
\[

\left| v_f \right| = \frac{| v_0 | + | \omega_0 | RK}{(K + 1)}. \quad \text{(13)}
\]
Substituting the corresponding experimental values gives \( t_1 = 0.95 \, \text{s} \) and \( v_f = 1.42 \, \text{m/s} \), which is consistent with the values observed in Figs. 3 and 4. In the experiment, the radius—calculated as the ratio of the angular speed to the linear speed during the second time interval—was \( R = 0.33 \, \text{m} \), also consistent with the measured radius.

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**Discussion and theoretical analysis**

The forces acting on the wheel from the moment of contact and through the interval where slipping occurs (the first time interval) are shown in Fig. 4. In addition to the force of gravity, the wheel is subject to a contact force—i.e., appearing as a result of the interaction between the wheel and the ground—equal to the sum of the normal force \( N \) and the kinetic frictional force \( f \). Since the vertical acceleration is equal to zero, it follows that
\[
N = mg, \quad \text{(1)}
\]
\[
ma = f, \quad \text{(2)}
\]
where \( a \) is the horizontal acceleration along the \( x \)-axis. Meanwhile, the kinetic frictional force is such that
\[
f = \mu N, \quad \text{(3)}
\]
\( \mu \) being the friction coefficient between the wheel and the ground. Combining Eqs. (1), (2), and (3) gives the magnitude of the frictional force and the linear acceleration of the center of mass of the wheel on the \( x \)-axis, according to
\[
f = \mu mg, \quad \text{(4)}
\]
\[
a = \mu g. \quad \text{(5)}
\]
Final remarks

The analysis of the system's dynamics and kinematics is simple, yet conceptually rich as well as highly illustrative. The fact that the force of friction is the only acting force having a horizontal component—as can be clearly seen in the free-body diagram—helps students to visualize and better understand that it must be positive in sign and therefore responsible for accelerating the wheel.

In the plots, the constant increase in the velocity of the center of mass over the time interval where slipping occurred can be readily observed by students. The angular velocity curve plotted by Tracker also clearly illustrates the transition from slipping to rolling, as well as the positive sign of the angular acceleration. Comparing the experimental plots with their reference theoretical equations serves as a visual aid that helps students to reinforce the underlying physical laws.

As bicycle wheels and digital cameras are widely available, this experiment can easily be implemented in almost all the laboratories. Moreover, bicycle wheels are large enough to easily be manipulated and also to mark several points on its rim and track different points in the digital video without the need of high-speed cameras. The experiment can be conducted in small groups or used with Interactive Lecture Demonstrations or other active learning settings. Data acquisition is hardly time consuming and the processing of experimental data is performed automatically by Tracker. In addition, using the appropriate tool of Tracker, the concept of moving frames of reference can be introduced in an intuitive manner.

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References

1. D. Halliday, R. Resnick, and K. S. Krane, *Physics*, Vol. 1 (Wiley, 2001). P. A. Tipler and G. Mosca, *Physics for Scientists and Engineers*, Vol. 1 (Macmillan, 2007).
2. A. Pinto and M. Fiolhais, “Rolling cylinder on a horizontal plane,” *Phys. Educ.* 36, 250 (2001).
3. L. G. Rimoldini and C. Singh, “Student understanding of rotational and rolling motion concepts,” *Phys. Rev. ST Phys. Educ. Res.* 1, 010102 (2005).
4. P. S. Carvalho and A. S. e Sousa, “Rotation in secondary school: Teaching the effects of frictional force,” *Phys. Educ.* 40, 257–265 (2005).
5. A. De Ambrosis et al., “Investigating the role of sliding friction in rolling motion: A teaching sequence based on experiments and simulations,” *Eur. J. Phys.* 36, 035020 (2015).
6. G. C. Adam, B. P. Self, J. M. Widmann, M. George, B. K. Kraw, and L. Chase, “Misconceptions in Rolling Dynamics. A Case Study of an Inquiry Based Learning Activity,” ASEE 123rd Annual Conference, New Orleans (2016).
7. V. L. B. Jesus and D. G. G. Sasaki, “Video-análise de um experimento de baixo custo sobre atrito cinético e atrito de rolamento,” *Rev. Bras. Ensino Fís.* 36 (3), 3503 (2014).
8. D. Brown and A. J. Cox, “Innovative uses of video analysis,” *Phys. Teach.* 47, 145–150 (March 2009).

Universidad de la República, Uruguay; marti@fisica.edu.uy