Local Charge of the $\nu = 5/2$ Fractional Quantum Hall State

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Abstract

Electrons in two dimensions and strong magnetic fields effectively lose their kinetic energy and display exotic behavior dominated by Coulomb forces. When the ratio of electrons to magnetic flux quanta in the system is near $5/2$, the unique correlated phase that emerges is predicted to be gapped with fractionally charged quasiparticles and a ground state degeneracy that grows exponentially as these quasiparticles are introduced. Interestingly, the only way to transform between the many ground states would be to braid the fractional excitations around each other, a property with applications in quantum information processing. Here we present the first observation of localized quasiparticles at $\nu = 5/2$, confined to puddles by disorder. Using a local electrometer to compare how quasiparticles at $\nu = 5/2$ and $\nu = 7/3$ charge these puddles, we are able to extract the ratio of local charges for these states. Averaged over several disorder configurations and samples, we find the ratio to be $4/3$, suggesting that the local charges are $e_{7/3} = e/3$ and $e_{5/2} = e/4$, in agreement with theoretical predictions. This confirmation of localized $e/4$ quasiparticles is necessary for proposed interferometry experiments to test statistics and computational ability of the state at $\nu = 5/2$.

One necessary (but insufficient) condition for exotic braiding statistics at $\nu = 5/2$ is for the ground state to support local excitations with a charge of $e_{5/2} = e/4$, where $e$ is the charge of an electron [2]. Though a charge of $e/4$ had previously been measured using shot noise techniques [5], more recent data from the same group [6] suggest that the value of the measured charge changes continuously as the point contact conductance and temperature are varied, reaching an inferred charge of unity in the weak and strong tunneling limits. Unexpected charges have also been reported for the more conventional fractions at 1/3, 2/3, and 7/3 [7, 6]. Moreover, DC conductance measurements in the weak tunneling regime [8] suggest a quasiparticle charge of $e_{5/2}^* = 0.17e$, in stark contrast to the shot noise results.

Clearly, a better understanding of the tunneling processes that take place between quantum Hall edges in the quantum point contact is needed in order to interpret the shot noise results. Alternatively, one can employ a thermodynamic approach [9] that probes the quasiparticle charge in the bulk of the sample in order to infer quasiparticle charge. Here we use a single electron transistor as a sensitive electrometer to measure the
equilibrium charge distribution in the bulk and its dependence on the average density and magnetic field. Our results provide clear evidence for localized charge e/4 quasiparticles at \( \nu = 5/2 \).

Our measurement employs a fixed single electron transistor (SET) as a gated device capable of sensitively measuring the local incompressibility (\( \kappa^{-1} = \frac{\partial n}{\partial \mu} \)) of a high-mobility 2DES \([10]\). The 2DES has a 200 nm deep, 30 nm wide MBE-grown GaAs/AlGaAs quantum well, with symmetric Si \( \delta \)-doping layers 100 nm on either side. A metallic backgate grown 2 \( \mu \)m below the 2DES allows us to tune the global density, \( n \), in the well over a typical range of \( 2.3 - 2.5 \times 10^{15} \text{m}^{-2} \), with some variation between samples. The SET is fabricated on top of the sample using standard electron beam lithography and shadow-evaporation techniques (Figure 1), creating an island with dimensions 500 nm \( \times \) 80 nm. All measurements were carried out in a dilution refrigerator with an electron temperature of 20 mK, verified using standard Coulomb blockade techniques.

As we adjust the density and magnetic field we expect to see regions of incompressibility when a gap is present, which will only happen precisely when the system is in a QH state. The slope of these incompressible regions in the \( nB \)-plane corresponds to the filling factor of the state \([11]\). Figure 2 shows incompressibility versus density and magnetic field between \( \nu = 2 \) and \( \nu = 3 \), with the two highlighted regions corresponding to FQH states at \( \nu = 5/2 \) and \( \nu = 7/3 \).

Additionally, due to the rough disorder potential created by remote donors, we can expect different points in space to develop gaps at different values of the global density. Because of this, we expect a well-developed QH state to have a percolating incompressible region punctured by small compressible puddles which behave as either dots or anti-dots \([11]\). As the global density is varied, a given compressible puddle will occasionally be populated by quasiparticles or quasiholes of the surrounding incompressible state. This creates a jump in the local chemical potential, \( \mu(n) \), and a spike in the local incompressibility \( \frac{\partial n}{\partial \mu} \). The magnitude and spacing of these spikes is determined by the charging spectrum of the puddle, which in turn is dictated by the quasiparticle charge in the surrounding incompressible region. Namely, if the quasiparticle charge were reduced by a factor of three for a fixed disorder potential, we should see three times as many compressible spikes as a function of global electron density (Figure 1 b,c).

This difference in spike frequencies has previously been used to measure the local charge at \( \nu = 1/3 \) and \( \nu = 2/3 \) \([12]\). Unlike shot noise measurements \([12]\), these local compressibility measurements find a quasiparticle charge of e/3 at both filling factors. Additionally, because of the spatial resolution afforded by the scanning technique in that measurement, it was possible to establish that the disorder potential landscape does not change as the electron system is tuned between Hall states with comparable gaps. Transport measurements confirm that the gap inferred from activation of \( R_{xx} \) minima is comparable for the states at 5/2 and 7/3 \([12, 13]\), so we can expect similar potential landscapes for the two states.

Our procedure begins with obtaining charging spectra (incompressibility versus density) at \( \nu = 5/2 \) and \( \nu = 7/3 \). Because the gap for these states is comparable, and the disorder potential is not altered as we change the magnetic field or density, we expect the spacing between charging features to reflect the quasiparticle charge in each state. In the limit of an isolated compressible puddle surrounded by an incompressible fluid, this relationship is particularly simple - if the ratio of local charges between the two spectra is \( \beta \), the spectra should be identical after one of the density axes is rescaled by a factor of \( \beta \), and shifted by some amount (Figure 3a). To proceed, we choose a value of \( \beta \) and stretch one of the spectra by this factor. We then calculate the correlation \( \left( \frac{C_1(x)C_2(x)}{\sqrt{(C_1(x)^2)(C_2(x)^2)}} \right) \) between the two spectra as a function of density offset and record the highest value. Finally, we repeat this for many scaling factors to obtain quality-of-fit versus \( \beta \), as depicted in Figure 3b.

This procedure was repeated for 20 different disorder configurations, obtained by changing samples, measuring with different SETs, or thermal cycling to change the disorder. A summary of the data is shown in Figure 4a, with an average over the measured ensemble in Figure 4b. The peak observed at \( \beta = 1.31 \) suggests a charge ratio of 4:3 between the two states, and a qualitative inspection of spectra overlap (as in Figure 3a) corroborates this. To determine the significance of the peak value, we repeated our analysis with pairs of spectra from different disorder configurations, which should be less correlated. For each scale, we characterized the distribution of best correlations with a mean and standard deviation. These, in turn, can be simply converted to the expected mean and standard error for our data (if it were uncorrelated).
The $1\sigma$ region around the uncorrelated mean is depicted in red in Figure 3b. Our averaged correlation at $eta = 1.31$ lies 3.8 standard errors above the uncorrelated mean, corresponding to a one-tailed P-value of $7 \times 10^{-5}$. Assuming a charge of $e_{7/3}^* = e/3$, this measured value of $eta$ suggests $e_{5/2}^* = (e/3)/(1.31) = 0.254e$, in agreement with the Moore-Read prediction of $e_{5/2}^* = e/4$ \[2\].

To better understand why some configurations seem to provide weaker (and sometimes different) measurements of $eta$, it helps to abandon the assumption that we are charging and monitoring single puddles, as well as the assumption that quasiparticles in different puddles do not interact. A free energy for our system that takes these into account is given by

$$F = \sum_i (\epsilon_i - V_{BG})Q_i + \frac{1}{2} \sum_i U_i Q_i(Q_i - 1) + \sum_{i<j} V_{ij} Q_i Q_j - \sum_i \Delta \left[ \frac{Q_i}{2} \right].$$

Here, $U_i$ and $\epsilon_i$ are the onsite interaction (self-capacitance) and bare disorder potential for puddle $i$ respectively. $V_{ij}$ is a pairwise interaction, or cross-capacitance, between puddles $i$ and $j$, and $\Delta$ is the energy gained by forming a bound pair of quasiparticles. For now, we will let $\Delta = 0$. We assume that some subset of the puddles is capacitively coupled to and measured by the SET.

To compute charging spectra from this model, we first choose values of $U$, $V$, and $\epsilon$ for each puddle from Gaussian distributions. We then discretize $Q_i$ into units of $e/3$ or $e/4$ and determine how many units of charge to put in each puddle to minimize the above free energy. This is done for each value of $V_{BG}$, and converted into a charging spectrum. Finally, we can take the resulting spectra and repeat the processing performed on data to obtain summary statistics for comparison. The result, with $\epsilon = 0 \pm 0.3U$ and $V_{ij} = 0.3U \pm 0.2U$, is shown in Figure 4c. Results for other parameter choices in a large range are qualitatively similar, with smaller values of $\sigma$, and $V_{ij}$ corresponding to sharper peaks and less spread. As expected, these simulations tell us that both $\epsilon$ and $V_{ij}$ can distort spectra in such a way that the maximum cross-covariance will shift slightly or even dramatically away from 4/3. Still, we should always expect some weight at 4/3, and this can be extracted by averaging over disorder configurations (Figure 4d).

Recently, there has been some suggestion that $e/2$ quasiparticles at the $\nu = 5/2$ edge may be present and relevant to interference measurements \[13\]. In the context of our model, we can consider the effect weak binding of quasiparticles would have on measured spectra. This binding is parameterized by $\Delta$ above, and we only consider the case where pairing affects the $e/4$ quasiparticles. As the strength of pairing is increased relative to the onsite interaction (Figure 4d), we expect weight to shift from the peak at 4/3 to a peak at 2/3 (corresponding to $e/2$ quasiparticles), with considerable weight at 2/3 even when $\Delta = 0.1U$. Our data show no appreciable evidence for a peak at 2/3, suggesting that the only quasiparticles participating in localization are have charge $e/4$.

These measurements constitute the first direct measurement of incompressibility and localized states at $\nu = 5/2$, and provide an equilibrium probe of the local charge that is insensitive to complications that arise from measurements of transport through nanostructures. The measured value, $e_{5/2}^* = e/4$, indicates that the FQH state at $\nu = 5/2$ demonstrates pairing, in agreement with proposed non-Abelian variational wavefunctions and different from other observed FQH states. Finally, the localization of $e/4$ quasiparticles is essential to the development of interferometers capable of detecting and exploiting these exotic braiding properties \[15\] \[16\], and our measurements suggest that $e/4$ localization does indeed occur in a well-behaved way.

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References

[1] Girvin, S. & Prange, R. *The Quantum Hall Effect* (Springer, 1987).

[2] Moore, G. & Read, N. Nonabelions in the fractional quantum hall effect. *Nuclear Physics B* **360**, 362–396 (1991).

[3] Nayak, C., Simon, S. H., Stern, A., Freedman, M. & Sarma, S. D. Non-abelian anyons and topological quantum computation. *Reviews of Modern Physics* **80**, 1083 (2008).

[4] Das Sarma, S., Freedman, M. & Nayak, C. Topologically protected qubits from a possible non-abelian fractional quantum hall state. *Phys. Rev. Lett.* **94**, 166802 (2005).

[5] Dolev, M., Heiblum, M., Umansky, V., Stern, A. & Mahalu, D. Observation of a quarter of an electron charge at the $\nu = 5/2$ quantum hall state. *Nature* **452**, 829–834 (2008).

[6] Dolev, M. *et al.* Dependence of the tunneling quasiparticle charge determined via shot noise measurements on the tunneling barrier and energetics. *Phys. Rev. B* **81**, 161303 (2010).

[7] Bid, A., Ofek, N., Heiblum, M., Umansky, V. & Mahalu, D. Shot noise and charge at the $2/3$ composite fractional quantum hall state. *Phys. Rev. Lett.* **103**, 236802 (2009).

[8] Radu, I. *et al.* Quasi-particle properties from tunneling in the $\nu = 5/2$ fractional quantum hall state. *Science* **320**, 899 (2008).

[9] Martin, J. *et al.* Localization of fractionally charged quasi-particles. *Science* **305**, 980 (2004).

[10] Ilani, S., Yacoby, A., Mahalu, D. & Shtrikman, H. Microscopic structure of the metal-insulator transition in two dimensions. *Science* **292**, 1354–1357 (2001).

[11] Ilani, S. *et al.* The microscopic nature of localization in the quantum hall effect. *Nature* **427**, 328 (2004).

[12] Dean, C. R. *et al.* Intrinsic gap of the $\nu = 5/2$ fractional quantum hall state. *Phys. Rev. Lett.* **100**, 146803 (2008).

[13] Choi, H. C., Kang, W., Das Sarma, S., Pfeiffer, L. N. & West, K. W. Activation gaps of fractional quantum hall effect in the second landau level. *Phys. Rev. B* **77**, 081301 (2008).

[14] Bishara, W., Bonderson, P., Nayak, C., Shtengel, K. & Slingerland, J. K. Interferometric signature of non-abelian anyons. *Phys. Rev. B* **80**, 155303 (2009).

[15] Stern, A. & Halperin, B. I. Proposed experiments to probe the non-abelian $\nu = 5/2$ quantum hall state. *Phys. Rev. Lett.* **96**, 016802 (2006).

[16] Bonderson, P., Kitaev, A. & Shtengel, K. Detecting non-abelian statistics in the $\nu = 5/2$ fractional quantum hall state. *Phys. Rev. Lett.* **96**, 016803 (2006).
Figure 1: Filling puddles with fractional charge. **a**, The sample well width is 30 nm, with symmetric Si δ-doping layers 100 nm on either side indicated by orange bands. Donors in these layers create a disorder potential in the 2DES, which produce puddles of localized states when the bulk is tuned to an incompressible, percolating Hall state. These puddles have some charging energy associated with adding electrons \( U_i \), and possibly some interaction with surrounding puddles \( V_{ij} \). Incompressibility \( \kappa^{-1} = \partial \mu / \partial n \) is measured using an SET fabricated on the surface. **b**, While the global chemical potential should increase smoothly with density (black dashed line), the local chemical potential will increase in jumps (red line), with charge being added when the global chemical potential aligns with a localized state. **c**, Repeating the charging of an identical puddle with charge \( e/3 \) objects instead of charge \( e \) objects results in three times as many charging events in the same range of global density. Scaling the density axis of the charge \( e \) spectrum by \( 1/3 \) and shifting by some amount (green curve) should result in good overlap of the incompressibility spectra.

Figure 2: Incompressibility and localized states at 5/2. **a**, By varying the magnetic field and the backgate voltage (density), we can identify incompressible phases of the 2DES. Our samples show clear incompressible FQH states at 5/2 and 7/3, with the expected slopes in the \( nB \)-plane. **b**, Zooming in shows repeatable charging events associated with quasiparticles localizing in puddles under the SET, stable on a timescale of days. **c**, A linecut showing the charging spectrum of any puddles coupled to the SET. Downwards spikes correspond to quasiparticles entering puddles beneath the SET.
Relative Scale (β)

Figure 3: Comparison of spectra at 5/2 and 7/3. 

a. To determine the charge, we first choose a relative scale between the two density axes (β), and determine the offset between the two spectra that maximizes the cross-covariance. Here the density for the spectrum at 5/2 is scaled up by a factor of 1.29 and shifted to match up with the spectrum at 7/3. The guide lines show the density change required to add 1 electron to an area of 100 nm x 500 nm, approximately the size of our SET. We would therefore expect, very roughly, 3 e/3 charging events in a window this size.

b. Repeating this for many values of β suggests that a relative scale of 1.29 best describes this data set.

Figure 4: Summary of Data and Model. 

a. Repeating the measurement over many disorder configurations and samples shows that the peak at 4/3 is usually present. 
b. Averaging over all measurements yields a clear peak at β = 1.31, 3.8σ above the uncorrelated background for that scale (P = 7 × 10^{-5}), suggesting a local charge ratio of 4/3. 
c. d. Running our model with parameters ϵ = 0±.3, V = 0.3±0.2, and Δ5/2 = 0.01, 0.1, and 1.0 (all in units of U, the on-site charging energy). We simulated charging of four puddles, of which two were capacitively coupled to the SET.