Quantum fluctuations of the ultracold atom-molecule mixtures

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We investigate evolution of the quantum coherence in the ultracold mixture of fermionic atoms and bosonic dimer molecules. Interactions are there experimentally controlled via tuning the external magnetic field. Consequently, the fermionic atoms and their bosonic counterparts can be driven to a behavior resembling the usual BCS to BEC crossover. We analyze in some detail how this quantum coherence evolves with respect to time upon a smooth and abrupt sweep across the Feshbach resonance inducing the atom-molecule quantum fluctuations.

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ATOM SUPERFLUIDITY

The recent experimental techniques for trapping and cooling of the atomic vapors enabled exploration of the extremely low temperature regions where quantum effects play a crucial role. An example can be the Bose Einstein condensation (BEC) produced out of the bosonic atoms in alkali metals, polarized hydrogen, etc. Phase transition to the BEC state is there triggered purely by the quantum statistical requirements which lead to macroscopic occupancy of the lowest energy level and can occur even in absence of any interactions. Recent activities in the field of ultracold atomic systems focus on application of similar techniques to fermionic atoms like $^6$Li or $^{40}$K (besides even number of nucleons they consist of odd number of electrons). At ultralow temperatures such quantum effects like the Pauli principle play considerable role, but eventual quantum phase transitions would be allowed only if fermionic atoms get correlated via interactions.

Interactions between trapped atoms are routinely induced by applying the magnetic field to fermion atoms prepared in several (two or more) hyperfine configurations. From elementary considerations it turns out that the involved hyperfine states experience the effective scattering described by a potential whose magnitude and sign depend on the applied field $B$. In particular, the various (so called) Feshbach resonances can take place. On this basis there was proposed a mechanism of the resonance superfluidity with a transition occurring near the Fermi temperature $T_F \sim T_F$. Besides the isotropic phase there has already been observed also the exotic $p$-wave superfluidity.

A unique possibility of controlling the effective interactions gives a chance for the experimental realization of the BCS to BEC crossover. The BCS limit corresponds to a case of weakly attracting fermion atoms which get coupled into the large Cooper pairs. In opposite limit, the tightly bound diatomic molecules are formed which ultimately can undergo transition to the BEC. Experimentalists are able to switch between these limits in a controllable manner and change of the interactions can be performed either adiabatically (by slowly varying the field) or in a non-adiabatic way (via the sudden sweep).

In this short paper we investigate the quantum fluctuations induced by the time-dependent change of the interactions. We focus on a situation when the magnetic field is detuned from the resonant value $B_0$ towards the far BCS regime at higher field $B > B_0$. We consider two different processes: the smooth and the sudden switching. The fast sweep has been discussed in the literature but with some ambiguous conclusions concerning evolution of the order parameters with respect to time. From our analysis we find that both parameters do oscillate in a damped way.

HEISENBERG EQUATIONS

In a close proximity to the Feshbach resonance (i.e. when $B \sim B_0$) the ultracold fermion atoms coexist and interact with the diatomic molecules. On a microscopic basis this situation can be described in terms of the two component boson fermion Hamiltonian

$$
H = \sum_{k,\sigma}(\varepsilon^F_k - \mu)c^\dagger_{k\sigma}c_{k\sigma} + \sum_q (\varepsilon^B_q + 2\nu(B) - 2\mu) b^\dagger_q b_q
$$

$$
+ \frac{g}{\sqrt{N}} \sum_{k,q}(b^\dagger_q c_{q-k\uparrow} c_{k\uparrow} + c^\dagger_k c^\dagger_{q-k\downarrow} b_{q\downarrow})
$$

(1)

which has been known and studied in the solid state physics by J. Ranninger and coworkers as a phenomenological model for the high temperature superconductivity. In the present context describes the atoms in two hyperfine states denoted symbolically by $\sigma = \uparrow$ and $\downarrow$. The second quantization operators $c^\dagger_{k\sigma}$, $b^\dagger_q$ correspond to fermion atoms with energy $\varepsilon^F_k = \hbar^2 k^2/2m$ and to diatomic molecules with energy $\varepsilon^B_q = \hbar^2 k^2/2(2m)$. The effect of external magnetic field is included via the detuning parameter $\nu$ which shifts the boson energies and hence affects efficiency of the boson-fermion coupling $g$. As usually $\mu$ is the common chemical potential and we use the grand canonical ensemble.
to ensure the conservation of the total particle number
\[ \sum_{\mathbf{k}, \sigma} c_{\mathbf{k} \sigma}^\dagger c_{\mathbf{k} \sigma} + 2 \sum_q b_q^\dagger b_q. \]

We are interested here in studying the time dependent evolution of fermion and boson occupancies together with the corresponding order parameters. For this purpose we derive the Heisenberg equations of motion which for the Hamiltonian \( \hat{H} \) are given by

\[ i \frac{\partial c_{\mathbf{k} \sigma} c_{\mathbf{k} \sigma}^\dagger}{\partial t} = (\xi_k + \xi_{\mathbf{k}^* - \mathbf{k}}) c_{\mathbf{k} \sigma} c_{\mathbf{k}^* \sigma}^\dagger + gb_q - \sum_{q'} b_{q'} \left( c_{\mathbf{q}_{k^*} \mathbf{k}^* - \mathbf{k}^*}^\dagger c_{\mathbf{q'} \mathbf{k}^* - \mathbf{k}^*} + c_{\mathbf{q'} \mathbf{k}^* - \mathbf{k}^*}^\dagger c_{\mathbf{q} \mathbf{k}^* - \mathbf{k}^*} \right) , \]

\[ i \frac{\partial c_{\mathbf{k} \sigma}^\dagger c_{\mathbf{k} \sigma}}{\partial t} = 2g \sum_{\mathbf{q}} \left( b_{\mathbf{q}} c_{\mathbf{q} \mathbf{k}^* - \mathbf{k}^*}^\dagger - b_{\mathbf{q}}^\dagger c_{\mathbf{q} \mathbf{k}^* - \mathbf{k}^*} \right) , \]

\[ i \frac{\partial c_{\mathbf{k} \sigma}}{\partial t} = E_0 b_q + g \sum_{\mathbf{q}} c_{\mathbf{q} \mathbf{k}^* - \mathbf{k}^*}^\dagger , \]

\[ i \frac{\partial b_q}{\partial t} = g \sum_{\mathbf{k}} \left( b_0 c_{\mathbf{k} \sigma}^\dagger - b_0^\dagger c_{\mathbf{k} \sigma} \right) , \]

which are identical with expressions (5) and (6) in the Ref. \[8\]. One next replaces the boson operators by their time-dependent expectation values \( \langle \mathbf{b}(t) = \langle \mathbf{b}_0 \rangle \) and \( \langle \mathbf{b}^*(t) = \langle \mathbf{b}_0^\dagger \rangle \).

In the stationary case when all parameters in \( \mathbf{b} \) are time independent we can derive various expressions for the static expectation values \[10\]. Hamiltonian \( \mathbf{b} \) has formally the following structure

\[ \hat{H} = - \sum_{\mathbf{k}} \hat{h}_k \cdot \hat{\sigma}_k + \text{const}, \]

so the pseudospin \( \hat{\sigma}_k \) behaves as though affected by a fictitious magnetic field \( \hat{h}_k = (-\Delta', \Delta'' \hat{\xi}_k) \) where \( \Delta' + i \Delta'' = g(\langle \mathbf{b}_0 \rangle) \). Following Anderson \[12\] we can solve this problem \[11\] for arbitrary temperature. In analogy to the Weiss theory of ferromagnetism we obtain that a magnitude of the pseudospin expectation value is \( \langle |\hat{\sigma}_k| \rangle = \tgh \left\{ \sqrt{\xi_k^2 + |\Delta|^2} / 2k_B T \right\} \).

The angle between the z and xy components of the vector \( \hat{h}_k \) we finally arrive at the stationary equations \[10\]

\[ \langle \hat{\sigma}_k^z \rangle = \langle c_{\mathbf{k} \sigma} c_{\mathbf{k} \sigma}^\dagger \rangle = \frac{-g \Delta^*}{2 \sqrt{\xi_k^2 + |\Delta|^2}} \text{tgh} \left\{ \frac{\sqrt{\xi_k^2 + |\Delta|^2}}{2k_B T} \right\} \]

and

\[ \langle \hat{\sigma}_k^x \rangle = 1 - \sum_{\sigma} \langle c_{\mathbf{k} \sigma} c_{\mathbf{k} \sigma}^\dagger \rangle = \frac{-\xi_k}{\sqrt{\xi_k^2 + |\Delta|^2}} \text{tgh} \left\{ \frac{\sqrt{\xi_k^2 + |\Delta|^2}}{2k_B T} \right\}. \]

**FLUCTUATIONS OF THE ORDER PARAMETERS**

In the symmetry broken state (for \( T < T_c \)) the two component model \[11\] is characterized by two order parameters: \( b(t) \) and another one of the fermion subsystem defined as \( \chi(T) = \sum_k \langle c_{\mathbf{k} \sigma} c_{\mathbf{k} \sigma}^\dagger \rangle \). These quantities are complex. In the stationary case they are proportional to each other as can be seen from the equation \[12\]. However, this relation is no longer valid when the Hamiltonian \( \mathbf{b} \) depends on time. Evolution of the order parameters \( \langle \mathbf{b}(t) \rangle \) and \( \langle \chi(t) \rangle \) with respect to time must be determined by solving the Heisenberg equations \[13\] subject to some boundary conditions.
We analyze here such dynamics assuming that initially, for $t \leq 0$, the mixed atom molecule system is exactly on the Feshbach resonance $\nu = \mu$. We also assume that the boson order parameter is real and $b(t \leq 0) = 1$ so that the fermion order parameter is real too. Value of $\chi(t \leq 0)$ was determined from the equations (7,8) by means of the Runge-Kutta algorithm. For simplicity we focus on the ground state and set the boson fermion coupling $g$ as a unit or all the energies appearing in our study.

For time $t > 0$ we change the detuning parameter $\nu$ in the following ways: a) via the sudden detuning as previously discussed in the Refs 7, 8, and 9) and b) through gradually increasing $\nu(t) - \mu \propto t$. Avoiding any constraint solutions we solved numerically the Heisenberg equations (7,8) by means of the Runge-Kutta algorithm.

By increasing $\nu$ the boson fermion system is pushed to the far BCS regime. For $\nu = 0.1g$ at $t \to \infty$ the order parameters $b(t)$ and $\chi(t)$ should reach very small values. Figure 1 shows that indeed temporal evolution occurs into such asymptotics and practically is achieved after several oscillations. In both cases the oscillations are clearly damped in agreement with the previous study by K. Burnet and coworkers 8. However, this process of damping is sensitive on a particular profile of the time dependent detuning. This can be seen from the figure 1 and also in the next figure 2, where we plot the phase $\theta$ of the boson order parameter $b(t) = |b(t)| e^{i \theta(t)}$. For a smooth switching the oscillations seem not to look regular at all.

We studied the dynamics of the ultracold fermion atoms upon the sudden and gradual detuning from the Feshbach resonance. Such situation can be experimentally realized by switching the external magnetic field from $B_0$ to the higher values. From the selfconsistent numerical solution of the equations of motion we find that the order parameters start oscillating with the amplitude decaying in time. Such damped oscillations depend on the specific form in which the detuning $\nu(t)$ is carried out. We also remark that the quantum oscillations of the order parameters turn out to be damped even on the level of the single mode approach without taking into account scattering to the finite boson momenta.

SUMMARY

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