Can LHC Test the See-Saw Mechanism?*

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**Abstract.** We discuss the prospects for detecting right-handed neutrinos which are introduced in the see-saw mechanism at future colliders. This requires a very accurate cancellation between contributions from different right-handed neutrinos to the light neutrino mass matrix. We search for possible symmetries behind this cancellation and find that they have to include lepton number conservation. Light neutrino masses can be generated as a result of small symmetry-breaking perturbations. The impact of these perturbations on LHC physics is negligible, so that the mechanism of neutrino mass generation and LHC physics are decoupled in general. In constrained cases, accelerator observables and neutrino masses and mixings can be correlated.

**PACS.** 14.60.Pq Neutrino mass and mixing – 14.60.St Non-standard-model neutrinos, right-handed neutrinos, etc.

1 Introduction

The (type-I) see-saw mechanism\textsuperscript{2,3,4,5,6} generates small neutrino masses in a natural way, introducing right-handed (RH) neutrinos that are singlets under the Standard Model (SM) gauge group and can therefore have large Majorana masses. The light neutrino mass matrix is approximately given by

\[ m_\nu = -m_D m_R^{-1} m_D^T, \]  

(1)

where \( m_D \) is the Dirac mass matrix and \( m_R \) is the Majorana mass matrix of the heavy singlets. A direct test of the see-saw mechanism requires the detection of these heavy neutrinos and the measurement of their Yukawa couplings. Using Eq. (1) in the case of only one generation and \( m_\nu \sim 0.1 \text{eV} \), we obtain the estimate \( m_R \sim 10^{14} \text{GeV} \), if the Dirac neutrino masses are close to the electroweak scale. The singlets may have masses as small as 100 GeV, within the energy reach of the LHC and other future colliders, if the Dirac masses are a bit smaller than the electron mass, which does not appear completely unreasonable either. However, the RH neutrinos interact with the SM particles only via Yukawa couplings\textsuperscript{7} which are tiny in this case. Thus, we expect the RH neutrinos to be either way too heavy or way too weakly coupled to be observable at colliders.

However, this conclusion can be avoided if there are two or more RH neutrinos\textsuperscript{7,8,9,10,11,12,13,14,15,16,17,18,19,20}. Their contributions to the light neutrino masses can cancel, opening up the possibility of rather light singlets with large Yukawa couplings but exactly vanishing light neutrino masses. Non-vanishing masses are generated by small perturbations of the cancellation structure. In this setup, the RH neutrinos may be observable in future collider experiments. This possibility has attracted renewed interest recently, see e.g.\textsuperscript{20,21,22,23,24,25,26}.

In the following, we will discuss the prospects for discovering RH neutrinos at colliders from the point of view of theory. We will consider the cancellation of contributions to the light neutrino mass matrix and possible underlying symmetries in the next section. After briefly discussing small perturbations of the leading-order mass matrices that yield viable masses for the light neutrinos, we will turn to consequences for signatures at colliders. Within the setups relying on a symmetry, lepton number violation is unobservable, while lepton-flavour-violating processes can have sizable amplitudes. Finally, we will comment on the implications a detection of RH neutrinos would have for our understanding of the mechanism of neutrino mass generation.

2 Cancellations and Symmetries

2.1 Vanishing Light Masses

For three generations of left- and right-handed neutrinos, the contributions of the RH neutrinos to the light
mass matrix cancel exactly, if and only if \[ \begin{pmatrix} y_1 & y_2 & y_3 \\ \alpha y_1 & \alpha y_2 & \alpha y_3 \\ \beta y_1 & \beta y_2 & \beta y_3 \end{pmatrix}, \] the Dirac mass matrix has rank 1, \[ m_D = m \begin{pmatrix} y_1 & y_2 & y_3 \\ \alpha y_1 & \alpha y_2 & \alpha y_3 \\ \beta y_1 & \beta y_2 & \beta y_3 \end{pmatrix}, \] and if \[ \frac{y_1^2}{M_1} + \frac{y_2^2}{M_2} + \frac{y_3^2}{M_3} = 0, \] where \( M_i \) are the singlet masses. The mass parameters are defined in the basis where the singlet mass matrix is diagonal. The case of two RH neutrinos is analogous \[ \text{[10],[11],[12]}, \] while for four or more RH neutrinos there are additional possibilities. The cancellation is valid to all orders in \( m_D m_R^{-1} \). The overall scale of the Yukawa couplings is not restricted by the cancellation condition \[ \text{[3]} \] and hence allowed to be large enough to make the detection of RH neutrinos possible. The only relevant constraint is the experimental bound on the mixing \[ V = m_D m_R^{-1} \] between active and singlet neutrinos. \[ \sum_i |V_{\alpha i}|^2 \lesssim 0.01 \quad (\alpha = e, \mu, \tau). \]

### 2.2 Underlying Symmetries

Without a symmetry motivation, the cancellation condition \[ \text{[3]} \] amounts to severe fine-tuning and is unstable against radiative corrections. Let us therefore discuss symmetries leading to the cancellation. We will restrict ourselves to the case of three singlets. A well-known possibility is imposing lepton number conservation \[ \text{[17],[18].} \] The assignment \( L(\nu_L) = L(\nu_R^1) = -L(\nu_R^2) = 1, L(\nu_R^3) = 0 \) implies \[ m_R = \begin{pmatrix} 0 & M & 0 \\ M & 0 & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \quad m_D = m \begin{pmatrix} a & 0 & 0 \\ b & 0 & 0 \\ c & 0 & 0 \end{pmatrix}. \] Two singlets form a Dirac neutrino with mass \( M \), while the third one decouples.

An important question is whether lepton number conservation is also a necessary condition for the cancellation of light neutrino masses, i.e. whether the cancellation can result from a symmetry that does not contain \( L \) conservation. One can show that there is always a conserved lepton number, if the cancellation occurs and if all three singlets have equal masses \[ \text{[1].} \] Let us therefore consider the case where the singlets involved in the cancellation, say \( \nu_R^1 \) and \( \nu_R^2 \), have different masses and where the condition \[ \text{[3]} \] is imposed by a symmetry at the energy scale \( M_2 \). Below this scale, the symmetry is broken. The neutrino masses change due to the renormalisation group running. The contributions from the two singlets to \( m_\nu \) run differently between \( M_1 \) and \( M_2 \) in the SM \[ \text{[28],} \] so that the cancellation is destroyed. A rough estimate yields \[ m_\nu(M_1) \sim 10^{-4} \text{ GeV} \ln \frac{M_2}{M_1} \] at \( M_1 \), which is unacceptable unless \( \nu_R^1 \) and \( \nu_R^2 \) are degenerate. Of course, this problem persists if also the third singlet contributes to the cancellation.

Thus, the cancellation of light neutrino masses can only be realised without fine-tuning, if the RH neutrinos involved in the cancellation have equal masses, which implies lepton number conservation. Therefore, any symmetry leading to vanishing neutrino masses via this cancellation has to contain the corresponding \( U(1)_L \) as a subgroup or accidental symmetry.

### 2.3 Small Perturbations

Non-zero masses for the light neutrinos are obtained by introducing small lepton-number-violating entries in the mass matrices \[ \text{[6].} \] In the most general case, \[ m_R = \begin{pmatrix} \epsilon_1 M & \epsilon_{13} M & \epsilon_{13} M \\ M & \epsilon_2 M & \epsilon_{23} M \\ \epsilon_{13} M & \epsilon_{23} M & M_3 \end{pmatrix}, \quad m_D = m \begin{pmatrix} a & \delta_a & \epsilon_a \\ \delta_b & \epsilon_b & \epsilon_b \\ \delta_c & \epsilon_c & \epsilon_c \end{pmatrix}. \] The smallness of the observed neutrino masses leads to the restriction \[ \epsilon_2, \delta_{a,b,c} \lesssim 10^{-10} \] for \( \max(a,b,c) \sim 1, m/M \sim 0.1, M \sim 100 \text{ GeV} \) (as required by observability of RH neutrinos at LHC \[ \text{[21],[22],[24],[26],} \] provided that there are no special relations between the small parameters causing additional cancellations. The perturbations \( \epsilon_2 \) and \( \delta_{a,b,c} \) appear quadratically in \( m_\nu \) and are correspondingly less severely constrained. Finally, \( \epsilon_1 \) and \( \epsilon_{13} \) do not lead to neutrino masses at the tree level at all but do contribute via loop diagrams \[ \text{[12],} \] so that they are only slightly less constrained than the other parameters.

The most general mass matrices of Eq. \[ \text{[3]} \] contain many free parameters, so that there is no clear imprint of the considered setup in the light neutrino mass matrix. A more interesting phenomenology is possible in constrained cases, some of which have been considered earlier in the context of leptogenesis \[ \text{[29],[18].} \] For example, if all small parameters are of the same order of magnitude, \[ m_\nu \approx \frac{m_D^2}{M} \left[ \epsilon_2 v v^T - (v v^T)^T + v s s^T \right], \] where we have abbreviated the first and second column of \( m_D \) by \( v \) and \( v_s \), respectively. The light neutrino masses are strongly hierarchical, since \( m_\nu \) has rank 2 and hence one vanishing eigenvalue. The large Yukawa couplings \( a, b, c \) are determined by the light neutrino masses and mixing parameters, which leads to predictions for correlations between the branching ratios of different lepton-flavour-violating decays in supersymmetric see-saw models \[ \text{[29].} \] Likewise, the amplitudes of LFV processes at colliders are correlated, as we will discuss shortly.
3 Signals at Colliders

A striking signature of RH neutrinos at colliders would be lepton-number-violating (LNV) processes with like-sign charged leptons in the final state \([30]\). However, we have argued that all symmetries guaranteeing the required suppression of the light neutrino masses lead to the conservation of lepton number, so that the amplitudes of such processes vanish. Any \(L\) violation is severely restricted by the smallness of neutrino masses and can therefore not lead to sizable amplitudes. Consequently, in the absence of fine-tuning, LNV signals are expected be unobservable.

Another option are events with different leptons such as \(\mu^-\tau^+\) in the final state, since these have a relatively small SM background as well. According to \([20]\), such signals are unlikely to be observable at LHC, however. In the constrained scenarios, the mechanism leading to the cancellation of neutrino masses causes the terms in the corresponding amplitudes to add up constructively, leading to

\[
A_{\alpha\beta} \propto \frac{m^2}{M^2} (a, b, c)_{\alpha} (a^*, b^*, c^*)_{\beta} \tag{11}
\]

for the mass matrices of Eq. 8, where \(\alpha \neq \beta\) denote the flavours of the charged leptons. If the cross sections are large enough for a detection at colliders, flavour-violating decays of charged leptons mediated by the RH neutrinos should be observable in upcoming experiments as well, since their amplitudes depend on the same combination of parameters. In the constrained case that yields Eq. 10, \(a, b, c\) can be determined from the light neutrino mass parameters, as mentioned above, so that the ratios \(A_{e\mu}/A_{e\tau}\) and \(A_{e\mu}/A_{\mu\tau}\) are predicted.

At the ILC, the resonant production of RH neutrinos is possible for \(|V_{ei}| \gtrsim 0.01\) \([31, 22]\). By observing the branching ratios for the subsequent decays into charged leptons, one could then determine the mixings of the heavy neutrinos with the different left-handed doublets directly.

4 Summary and Discussion

We have discussed the prospects for testing the see-saw mechanism of neutrino mass generation in collider experiments. We have assumed the existence of right-handed neutrinos with masses close to the electroweak scale (but no other new particles or interactions). The couplings of these neutrinos to the SM particles can only be large enough to make their observation at colliders possible, if different contributions to the light neutrino masses nearly cancel. This cancellation is then the main reason for the smallness of the observed neutrino masses, while the see-saw mechanism plays only a minor role. Therefore, we have to conclude that a direct test of the see-saw mechanism at the LHC or the ILC is not possible.

If one defines the leading-order mass matrices in such a way that they correspond to exactly vanishing light neutrino masses, non-zero masses appear as a result of small perturbations of this structure. One may then ask whether these perturbations could have consequences for signals at colliders and thus allow for a test of the mechanism of neutrino mass generation. Unfortunately, the smallness of the light neutrino masses immediately tells us that all perturbations are tiny and therefore irrelevant for collider signatures. Thus, the answer to this second question is negative, too. Collider experiments are only sensitive to the leading-order mass matrices which do not lead to neutrino masses.

As a consequence, a connection between collider physics and neutrino masses can only be established, if the perturbations are introduced in such a way that the leading-order parameters are related to the light neutrino masses and mixings. In the most general case, this is not possible because there are too many free parameters. Then collider physics decouples completely from the light neutrino masses and their generation.

However, the situation is better in constrained setups where only some of the perturbations are present or dominant. In the cases we discussed, a strong mass hierarchy is expected. To the extent that the leading-order Yukawa couplings are fixed by the measured neutrino masses and mixings, correlations between the branching ratios of lepton-flavour-violating processes can be obtained. This applies both to reactions at colliders and to LFV decays of charged leptons. Finally, \(e^+\tau^-\) colliders may be able to determine the mixings of RH neutrinos with the different flavours directly. Pursuing all these experimental options provides a chance to test constrained setups of the kind we have described. Of course, even in this optimistic case it is impossible to exclude the existence of additional, very heavy RH neutrinos contributing to neutrino masses via the standard see-saw mechanism.

Without an underlying symmetry, the described cancellation of the light neutrino masses amounts to severe fine-tuning. We have therefore discussed symmetry motivations. We have argued that every symmetry realising the cancellation has to include lepton number conservation. Otherwise, the cancellation is unstable against radiative corrections, so that fine-tuning is still required.

Thus, both lepton number violation and light neutrino masses arise due to small perturbations of the leading-order mass matrices, and their magnitudes are related. Therefore, we expect lepton-number-violating signals at colliders to be unobservable in untuned scenarios. The cross sections for lepton-flavour-violating processes are not suppressed, so that LHC experiments might be able to observe such reactions. If this is the case, lepton flavour violation should also be observable in decays of charged leptons in the near future.

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