null
Activation of the network flows from the input layer through the hidden (middle) layer, then on to the output layer (Baylari, A., & Montazer, G. A., 2009). Each link is associated with weight. The weights of a neural network are determined by a training process with empirical data. The performance of neural networks is also sensitive to the number of neurons (Ding et al, 2013).

D. Genetic Algorithms.

A genetic algorithm is usually used as an optimization technique acting simultaneously on a wide set of points. Genetic Algorithms perform a search on the whole area of possible solutions, in order to find the acceptable solution for a given problem. The steps of the algorithm are: initialization, selection, crossover, and mutation (Huang, M. J., Huang, H. S., & Chen, M. Y., 2007).

E. Constraint-Based Modeling

Constraint based model tutors use student’s errors in order to build a student model represented as a set of constraints infringed or not (Ohlsson, 1994). Each constraint consists of three components: 1) a relevance condition indicating when the constraint is applicable, 2) a satisfaction condition and 3) a feedback message for student’s errors (Ma et al, 2014).

III. INTELLIGENT TUTORING SYSTEMS FOR MATHEMATICS

A. Problem Solving

In 2013 D. Arnau et al designed an intelligent tutoring system for the learning of the arithmetical and algebraic way of solving word problems. Hypergraph Based Problem Solver (HBPS) is able to reinforce the translation of a problem into algebraic language or to provide an arithmetical way of solving a problem. The system uses hypergraphs, easy for users to understand, for representation of analytical reading of a given problem, as well as for representation of a solving process, with many alternative resolution paths. HBPS provides also automatic feedback and hints when the user makes a mistake. The results of the experimental study showed that the use of the ITS resulted a significant improvement on the algebraic translation of a word problem.

In 2011 Voskoglou developed a fuzzy AI model for representing several processes in mathematical problem-solving. The construction of this fuzzy model is based on the Multidimensional Problem-Solving Framework (MPSF), including four phases:

- Orientation
- Planning
- Executing
- Checking

In the experiment which took place at the Graduate Technological Educational Institute of Patras, Greece, 35 engineering students were given a list of 10 word problems to solve. In a second experiment the same list was given to 30 students of the School of Management and Economics. The fuzzy model representing mathematical modelling provides a representative view of the behavior of each student when tackling problems due to the student’s individual learning style.

Edwards et al (2010) presented 4MALITY, a web-based mathematics tutoring system, designed to teach mathematical problem-solving skills and test-taking strategies for 4th grade students. 4MALITY adapts to the students’ levels of knowledge, by using an Artificial Intelligence decision system. The system’s mathematics content is based on the Massachusetts Curriculum Framework. There are four categories of learning styles provided:

- Explain questions in terms of language used.
- Mathematical computational operation.
- Test-taking and problem solving strategies.
- Visual approaches to computation.

B. Algebraic and Arithmetic Operations

In 2008, Chien et al evaluated the effect of an intelligent tutoring system on tutoring students on the topic of Algebraic Expression. In the experimental study participated 62 students arranged in two groups, control and experimental. Students of the experimental group Ms Lindquist an ITS based algebra learning software created by Heffernan (2001). After the students choose their current level of algebraic knowledge they are given a problem to solve. The tutoring session of the system uses dialog-based technique to provide students suggestions to perform better in future algebraic problems. The system integrates dynamic scaffolding tutoring strategy, to wit focusing the dialog on the area where the student has difficulty. The teacher can check anytime through the tutoring session the student’s performance. The results of the study revealed that the ITS approach was effective in helping students improve their algebraic skills.

In order to compare learner’s affect associated with an intelligent tutoring environment and a simulation problem solving game, Rodrigo et al (2008) used Aplusix, an intelligent tutoring system for mathematics (Nicaud, 2006). The system contains activities from six topics based on the curriculum:

- Numerical calculation
- Expansion and simplification,
- Factorization,
- Solving equations,
- Solving inequalities, and
- Solving systems of equations

The student is allowed to solve the given problem gradually, receiving immediate feedback from the tutor as well as guidance to reach the final solution, when needed. According to the results of the study, Aplusix had generally better affect on users than the simulation game.

In their study Graff M., Mayer P., and Lebens M. evaluated a web based intelligent tutoring system for mathematics at German lower secondary schools. Efit uses a sub-scale of the Heidelberg Rechentest (HRT) in order to assess children’s knowledge on basic arithmetic operations, and hence classify them into three categories regarding their educational needs. The efit program provides individual learning curriculum based on the mathematical skills and the learning profile of the student. A sample of 194 students of 5th and 6th grade divided in two groups (treatment and no treatment) was used for the study. The results of the study revealed that students who used the efit program performed significantly better on the post-test than students from the non-eftit group.
Walker, Rummel and Koedinger (2009) had the idea to integrate intelligent tutoring and reciprocal peer tutoring for mathematics learning, in order to investigate how collaborative activities could result to further improvement. The intelligent tutoring system used was Cognitive Tutor Algebra (Koedinger et al., 1997). In reciprocal peer tutoring, students could be placed in the role of tutor, as students of similar algebraic skills are able to communicate and support each other in a chat window during solving. The results of the study revealed that both peer tutors and tutees could benefit from the interaction.

In 2013 Waalkens, Alevin and Taagen developed an Intelligent Tutoring System for solving linear equations using example-tracing tutors. The system contains 44 first degree equations divided into different levels with respect to the difficulty. The researchers developed three versions of the same ITS, strict standard strategy, flexible standard strategy and multi strategy, which differed in the level of freedom they allow to learners. The intelligent tutor provides solved examples, explanations, feedback messages, next step hints, as well as instructions at the start of each problem set. According to the results of the evaluation study there was lack of strategy variations, as students seemed to stick to the standard strategy.

C. Fractions and Decimals.

In 2010, Beal Carole R. et al conducted three studies with middle school students in order to evaluate AnimalWatch, an intelligent tutoring system for arithmetic and fractions. The software was based on artificial intelligence algorithms in order to provide personalized support to individual students. AnimalWatch contain word problems, from easier to more challenging, designed to train user’s basic computations and fractions skills. The ITS is able to assess student’s skills on each topic and respectively provide useful hints or move on a new math topic. In general the results of the studies encouraged the hypothesis that the ITS software could be a useful and motivating educational tool.

In 2011 N.M. Noh developed an Intelligent Tutoring System for primary school children in the subject of Mathematics, especially in fractions. The objective of their study was to compare the implementation of two different artificial intelligence techniques, rule-based and case-based in the ITS. Rule Based is a particular type of reasoning which uses “if-then” rule statements. The tutor agent of the rule based ITS provides sets of questions depending on the user’s progress. The training stops when the user reaches the highest level in the system. In the other hand according to case based reasoning, a given problem could be solved based on the solution of a similar past problem. Case based ITS involve four steps: retrieve, reuse, revise and retain. In order to solve composite problems the system can combine partial solutions.

Rau, M. A., Alevin, V., & Rummel, N. (2011) designed an intelligent tutoring system for fractions using multiple graphical representations (MRGs). This intelligent tutoring system performs as a cognitive tutor based on correct and incorrect solution paths. The tutors use circles, rectangles and number lines in order to represent graphically fractions. Students are called to compare fractions, reconstruct the unit from unit fractions as well as identify improper and proper fractions. The system provides also error feedback reminding fraction concepts and hints on all steps. The results of the experimental study with 290 participants revealed that there were significant learning benefits for students who worked with a tutoring system that supports learning with MRGs of fractions.

Isotani s., McLaren B. and Altman M. (2010) proposed the development of an intelligent tutor that will use the taxonomy of decimal misconceptions, related to decimals comparison and basic operations, in order to provide a useful learning tool for middle school math students. The erroneous example-based tutor (ErrEx) will present students an incorrect solution of a problem along with a correct one, so that they can learn through the comparison. The tutor also provides feedback with hints and examples to help students understand the concept of decimals.

D. Geometry

In 2005 Matsuda and VanLehn proposed an intelligent tutoring system to enhance learning geometry theorem proving with construction. Advance Geometry Tutor starts with a given geometry problem statement and the corresponding figure. The user is allowed to draw line segments on the figure in order to prove the statement. The tutor provides feedback and instructions that the student must follow through the message window. Postulate Browser Window contains a list of postulates which could be used from the student for the proof. Finally the student can see the whole procedure that must be followed step by step to reach the proof on the Inference Step Window.

E. Standardized Tests

Arroyo I. et al (2010) evaluated the impact of Math Facts Retrieval Training integrated with an intelligent tutor on students’ math performance. The Math Facts Retrieval Training software provides assessment and training sections, presenting student’s accuracy in progress charts. According to the researchers training speed of math-fact retrieval could improve students’ cognitive skills in order to archive greater scores on the math sections of standardized tests such as SAT. The Wayang Math Tutor is an intelligent tutoring system, using interactive multimedia, designed to prepare students’ for these tests. Wayang provides immediate feedback on students as well as useful hints adapted to student’s individual learning style. According to the experimental results the Math Facts Retrieval Training software combined with the Wayang tutor improved students’ performance on standardized math tests, specifically to those with low mathematical skills.

F. Mathematical Logic

Rosa Lanzilotti and Teresa Roselli (2007) presented Logiocando, an intelligent tutoring hypermedia system developed at the Department of Computer Science of the University of Bari. The purpose of this educational software was to help primary school students to understand basic concepts of logic. The educational content of Logiocando was organized in four topics: sets, set operations and diagrams, providing explanations, logic games and tests for each topic. The system contains a Tutor Module which is responsible for assessing the pupil’s knowledge in each topic or show again the theory of the previous. The results of the experimental evaluation revealed that Logiocando could be a substitute of the teacher and be used as a revision tool.

http://www.i-jep.org
G. Multi-Task Math Tutors

Aleven, V., McLaren, B., and Sewall, J. (2009) created an ITS-based open-access web site for middle school mathematics learning. MathTutor site covers five topics for grades 6 to 8:

- Numbers and operations
- Algebra
- Data analysis
- Geometry
- Measurement

The system provides step-by-step hints during problem-solving using example-tracing tutors. These tutors are able to evaluate students’ problem-solving behavior as well as to provide guidance for multi-strategy problem-solving. MathTutor, which is an open access site provides also detailed performance data for each student, available to teachers or parents.

Feng et al. (2008) proposed an intelligent tutoring system designed to predict student math proficiency. The ASSISTment system is a combination of computer-based tutoring and benchmark testing. If the student solves the given problem correctly, a new problem is given. If the student answers wrong, the system provides a small tutoring session, where the student is given the problem broken into sub-problems in order to reach progressively the solution. According to the researchers the model can do as well as a standardized test.

Keleş, A., Ocak, R., Keleş, A., & Gülçü, A. (2009) developed ZOSMAT, a web-based tutoring system for mathematics. ZOSMAT is designed to direct students in each stage of learning process using HTML pages, interactive applications, animations, audios and videos. The system supports the generation of adaptive tests, through the question bank, as well as the analysis of student solutions and errors. The intelligent tutor adapts to individual student’s needs, providing examples of problem solving and hints in the process of problem solving. The student can see where more practice is needed on the general report screen or on the topic based report. According to the researchers ZOSMAT is an attempt to imitate the behavior of a human expert tutor.

In 2012 Cabada et al. presented Fermat, an Intelligent Social Network for Mathematics Learning. Fermat adopts the traditional architecture, with an Expert Module, a Student Module containing a diagnostic test in order to determine the learning level of the student and the appropriate teaching method and a Tutoring Module where the learner can find problems or exercises to solve, while the solutions given are evaluated in terms of difficulty, assistance, errors and time. The system uses a Genie character to give cognitive and affective feedback to students.

Craig, Scotty D. et al (2013) proposed ALEKS, a web-based learning system with artificial intelligence components, as a method of intervention in after-school settings to improve mathematical skills. The Intelligent Tutoring System administers a test in order to assess student’s initial knowledge state. Student can see a summary of his/her learning progress in each topic on a pie chart. Then the student can choose how to continue from a list with problem types he/she is ready to learn. According to the results of the study, 6th grade students who studied using the ALEKS system performed at the same level as students taught by teachers on the Tennessee Comprehensive Assessment Program (TCAP).

IV. CONCLUSIONS

Intelligent Tutoring Systems could be considered as the new trend in mathematics education. As the studies published in the last two decades revealed, educational systems which adopt artificial intelligence techniques, are not meant to be substitutes of the teacher, but useful supportive or revision tools which can sufficiently imitate a human math tutor.

Most ITS are able to assess students’ mathematical skills before they start practice. This process leads to the creation of a personalized intervention program, based on the individual learning profile of each student, in order to achieve the best possible learning outcome. Intelligent tutors have the ability to present the educational material flexibly, using hypertext, audio, video and graphics. Such instruments could be useful in order to encourage children to get involved with mathematics without treat it as an another boring obligation.

One of the most important parameters in the design of an Intelligent Tutoring System is the process in which the user solves a math problem. The answers should be given in detail, so that the intelligent tutor can give proper step by step feedback and hints during the whole process of problem solving. In contrast to Computer-aided tools, ITS are able to guide the resolution of more complex math problems.

ITSs can improve students’ self-efficacy as well as allow them to explore each topic themselves and learn experimentally solving techniques and math formulas. Despite this, the guidance of the teacher is necessary, as in many cases students can fool the system, not following the instructions of the intelligent tutor correctly, and therefore the learning objective of the system is not reached.

However, ITSs are not yet established as educational tools. Complexity in their design and in their development makes them inaccessible to the general public, and their use is currently limited to universities or major educational institutes.

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