On the Evolution of Contact Patch Concept

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Abstract. The paper aims to highlight the evolution of the concept of physical contact between two technical bodies (machine parts). The most abstract situation occurs in antiquity when the necessary geometry concepts were created. Engineering applications from the industrial age have demanded higher performances. These have become possible through a continuous refinement of the theoretical models. The hertzian contact between two evolving parts, in general, and between the wheel and the rail, in particular, evolved and became extremely concrete and full of details that were once obscure. The contact patch between the wheel and the rail has today an asymmetric geometry correlated with the state of stress and the dynamics of the railway track. The use of digital methods for describing topologies can be a way to create a model closer to the dynamic state of wheel and rail contact. We also encounter the same contact phenomena in gear power transmission.

1. Introduction

In Materials Resistance area of activity there is an important damage phenomena that come out from a high stress concentration zona as the wheel-rail contact. Therefore the wheel-rail contact has become the main point of any research connected to vehicle improvement. The correct assessment of the contact stress values can be made possible by designing a phenomenological and computational model to follow the true real phenomenon. The wheel-rail contact governs the dynamic performance of rail vehicles through the loads it transmits and, in the same time, the machine parts are subjected to serious damage phenomena.

Starting from Hertz's wheel-rail contact model, various other models were developed, increasingly close to the real wheel-rail contact. Even the Hertz contact solution is yet used since it is of closed-form, some of its underlying assumptions may be violated quite often in wheel-rail contact. The assumption of constant relative curvature which leads to an elliptic contact patch could be much more acceptable.

Some fast non-elliptic contact models are proposed to lift this assumption while avoiding the wearying numerical procedures of calculus. These models are accompanied by a simplified approach to treat tangential tractions.

There are two alternatives for theoretic description of the rolling contact kinematic. A Lagrangian approach, in which the system of coordinates is particle-fixed. This one is mainly used for static or short-displacement applications. The other option, which seems to be more efficient in rolling contact applications, is the so-called arbitrary Lagrangian-Eulerian (ALE) approach in which the total deformation of the rolling wheel is divided into a rigid-body motion and the deformation of the material point.

The formulation of the wheel-rail contact problem is a complex task which has been the subject of several investigations that revealed different solutions for the problem. Some researchers [1-5] studied
the contact forces between the wheel and the rail during the dynamic motion and, as a result of their investigations; several computer routines are now available for the calculations of the tangential forces at the contact point given the normal force and the relative velocities between the bodies.

How can we discover new sources of inspiration for a more refined models and at the same time simple for wheel-rail contact and generally between two convex bodies? This approach can help us get rid of the circle of sterile thought.

The question of contact between the circle and the right line is an ancient dispute between Plato and Protagoras. Plato, the author of the Theory of Forms, had imposed to the mathematicians of his time, the concepts of circle and right line and more the possibility of their relative position in a singular tangent point. On the other hand Protagoras had argued that the contact between circle and right line occurs in a set of distinct points. At their time Plato was the winner and so the geometry of Euclid had developed.

2. Reconsidering the ancient dispute

The mathematicians of ancient Greece operated with ideal geometric figures, these figures had metaphysical properties. For example, the right line consists of an infinity of geometric dots. These are not dimensional and therefore can not be represented graphically. Consequently, the line can be not represented graphically. But the mathematicians of ancient Greece used the graphic representations only from the need to communicate each other. The graphic figures represented merely meant to suggest the metaphysical entities they actually operated with.

The continuity of these geometric figures is difficult to define. If we use a geometrical atomistic conception, the continuity of the circle and the straight line becomes precarious. Salvation of these definitions is made by calling to infinite algebra. The definition of continuity is precarious because we cannot avoid the dot notion. The definition of continuity without dot concept is an aporia. By accepting the continuity made by points and the number of constitutive dots to be infinite, then any cut will be made up of an infinite number of dots (1).

\[ \{N_{AeB}\} = \infty; \{N_{A\psi B}\} = \infty; \{N_{C\xi D}\} = \infty. \]  

(1)

It made a cut on the circle and one on the right line that will be tangent to the circle Figure 1. Each arc consist of an infinite number of dots. The same, the segment CD consist of an infinite number of dots. When the circle and the straight line are tangent, it forced the decrease of the cuts on the circle and the straight line (Figure 2).

![Figure 1](image1.png)

**Figure 1.** Cut on circle and straight line

![Figure 2](image2.png)

**Figure 2.** Cut on circle and tangent straight line

Going to limit it force that
\[ A \rightarrow P; B \rightarrow P; C \rightarrow P; D \rightarrow P \]  \hspace{1cm} (2)

\[ AB \rightarrow 0 \text{ and } CD \rightarrow 0 \]  \hspace{1cm} (3)

\[
N_c = \begin{cases} 
\infty & \text{if } i = 1 \\
1 & \text{if } i \neq 1 \\
0 & \text{if } i = 0 
\end{cases}
\]  \hspace{1cm} (4)

Equation (4) suggests a serious anisotropy of geometric space. It can be admitted that the number of dots of an infinite small cut is a finite number \(N_c\). So equation (4) become equation (5).

\[
N_c = \begin{cases} 
\infty & \text{if } f = 1 \\
N_f & \text{if } f \neq 1 \\
0 & \text{if } f = 0 
\end{cases}
\]  \hspace{1cm} (5)

If “p” is the geometric dot of contact between the circle and the line, according to the Euclidean geometry, 1, 2, 3, 4, ... are a set of dots adjacent to it and all of them are located at infinitely small distances, then all \(R_3, R_i, R_0, R_2, R_4,\ldots\) rays are parallel, that means the angles \(\phi_i\) are equal with 0 (Figure 3).

In this hypothesis there are a finite number of common dots of the circle and the line when they are tangent. From a macroscopic point of view, the contact between the circle and the straight is no longer at a geometric point, but in an exotic geometric entity that we can call it a point of contact. This point tends to lose its metaphysical character and become able to participate in functional models.

This assumption allows proposing \(P_c\) as a dot of contact entity. This point, \(P_c\), will become a patch in the more concrete models.

3. A new wheel–rail contact model for railway dynamics

Coming from a theoretical circle-to-line contact model to a concrete rail contact model, the practical application complicates the model. The wheel-rail contact must be viewed both statically, without traction and dynamic when a creep force occurs. The contact pattern involves to estimate the distribution of contact pressure and friction coefficient.

The deterioration phenomena occurring in the wheel-rail interface involve two phenomena to blame are namely wear and rolling contact fatigue (RCF). The primary step that enables investigating such mechanisms is to clearly know the shape and size of the contact patch, and the distribution of the stresses within it. The contact point detection may be carried out by assuming the wheel and the rail as rigid or accounting for their flexibilities.

The study of rolling contact, or more accurately rolling-sliding contact, started in 1926 by the publication of Carter’s paper [7] on the solution of the two-dimensional rolling contact problem. The three-dimensional rolling contact including sliding was touched upon by Johnson [8] about thirty years later.

Johnson, in particular, attempted to model the three-dimensional rolling. He started investigating circular contact areas. In 1958, he published two papers on the rolling motion of an elastic sphere on a plane. In one of them he investigated, the effect of spin on rolling contact. He found that the spin generates a lateral tangential force as well as a moment around the vertical axis. The spin-generated lateral force is opposed by a lateral force due to lateral creepage. The case of pure creepage without spin was extended to elliptic contact area by Vermeulen and Johnson [10].
In this theory, the stick zone is a smaller ellipse with the same ratio of the axes adjacent to the leading edge as shown in Figure 4. Based on this configuration the traction vanishes only at one point in the leading edge. Pascal and Sauvage [9] realized the existence of possible secondary contact points from the observation of the sudden "jumps"(of the contact stress) in the evolution of contact parameters with respect to wheelset lateral displacement.

These jumps exist when the wheel flange comes into contact with the rail and they are indicative of the secondary contact points that come to be effective due to the elastic deformation of the profiles resulting from the contact pressure.

**Figure 4.** Stick aria along the patch wheel-rail

**Figure 5.** Pascal Model

In detecting process of the contact zones, if profiles are considered rather elastic than rigid ones, several contact zones may be possible to find. Each contact zone indicates a separate contact patch. A Hertzian ellipse is then calculated for each patch using the curvature values at that point. In order to detect the secondary contact point, the profiles in contact are deducted by a semi-elliptic function calculated using the first (main) contact ellipse width, b, and the approach, δ.

The normal forces for each ellipse are determined by taking into account the ratio between $h_1$ and $h_2$ (Figure 5) and the Hertzian relation. The implementation of this method, known as multi-Hertzian, and the coupling with a wheelset dynamic model seemed to be very slow at the time. Therefore, Pascal decided to replace the contact ellipses by an equivalent one (as see in Figure 5).

This equivalent contact ellipse is established in a manner that results in the same creep force as the sum of the ones from two separate ellipses while the direction of the force is determined by weighted averaging.

The approach of equivalent ellipse may be adequate for dynamic simulations but it is not suitable for damage analyses such as wear calculations due to the non-physical contact patch and stress distribution.

To be able to estimate non-elliptic contact patches in a fast manner, an approximate contact method should replace the Hertz solution for the normal contact part. In fact, calculation of the elastic deformation, $u_z$, in Equations (6) and (7) involves numerical integration and is not possible analytically for non-Hertzian profiles (where, $C$ denotes the area of contact).

$$d(x, y) = z(x, y) - \delta + u_z(x, y)$$  \hspace{1cm} (6)

$$d(x, y) = 0, \quad p(x, y) > 0 \quad (x, y) \in C$$
$$d(x, y) > 0, \quad p(x, y) = 0 \quad (x, y)(NOT \in)C$$  \hspace{1cm} (7)
Kik and Piotrowski [11] proposed an approximate method based on a concept called virtual penetration. In this concept, the deformations surface are neglected (u_z = 0) and it is assumed that the bodies can rigidly penetrate into each other.

A smaller virtual penetration value d_v than the prescribed one d_0, may result in a penetration zone which is close enough to the real contact area and can be regarded as the contact patch. The virtual penetration value can be set in different ways. As suggested by Kik and Piotrowski, it should be about half the prescribed penetration value. Figure 6 a, b, c, d, illustrates the contact patches resulting from all four virtual penetration strategies for circular wheel and rail profiles with radii 200 mm and 80 mm respectively, and wheel rolling radius 460 mm.

![Figure 6. Different contact patches according to penetration strategies](image)

### 4. Equivalent ellipse adaptation

In order to skip the calculation of no elliptic flexibility parameters and employ the elliptic ones, an imaginary equivalent ellipse may be assigned to the non-elliptic patch. Kik and Piotrowski [11] determined an equivalent ellipse for each separate contact patch by setting the ellipse area equal to the non-elliptic contact area and the ellipse semi-axis ratio equal to the length-to-width ratio of the patch.

By discretizing the contact patch into longitudinal strips, as done in the Linder method [11] and the STRIPES method [11], one may treat each strip differently.

In the Linder model, all local strip ellipses have the same lateral semi-axis. The longitudinal semi-axis is then calculated so that the strip fits into the resulting ellipse. In STRIPES, dissimilarly, the local ellipses are determined by employing relative curvatures at the centre of the strip in Hertz solution.

For engineers and researchers the real physical wheel-rail contact phenomenon is more complicated than the idealized theoretical models. In fact, several other physical phenomena, which themselves are complex in nature, are involved in practical applications: plastic flow, surface roughness and thermo-mechanical coupling and so on.

The friction itself, is an elusive phenomenon which seems to be even more difficult to fully model in the wheel-rail interface subjected to environmental changes. What makes it even more problematic to mathematically model this physical system is the lack of real-life measurements in order to validate the proposed models.

The kinematics of guidance of the wheelsets is based on the wheels and rails geometries. The movement of the wheelsets along the rails is characterized by a complex contact with relative motions on the longitudinal and lateral directions and relative rotations of the wheels with respect to the rails.

If we take the wheel material ER8, according to European standards, with nominal yield stress σ_y = 54 MPa and assume μ = 0.3 the calculated maximum pressure of about p = 1800 MPa is the limit for which the linear elasticity assumption in the contact solution may be valid and the plastic flow in outer rail of tight curves is clearly evident. The contact pressure is reduced and therefore the size of the contact patch increases. Some attempts to include plastic flow rules in wheel-rail contact models has been done.

There are few techniques used for measuring the shape and size of the contact patch and the pressure distribution for the contact between wheel and rail profiles. Some methods utilize ultrasonic approach, while others experiment a kind of special pressure sensitive paper for this purpose. All
researchers studied normal static contact, but there is no one who has done a measuring technique to measure the tangential stress (traction) distribution in wheel-rail applications.

5. Conclusions
Increasing train speeds and increased road safety will require further research into the phenomenon of rolling and the intimacy of the wheel-rail contact. New methods of experimental research will be targeted to reveal accuracy the shapes of contact patch Figure 7, in various transitory phases of the run. In order to advance in shaping the dynamic contact phenomenon of rail-wheel contact it is necessary to remodel the two bodies in contact as a single body. A differential model and a specific new differential equation will emerge.

![Figure 7. Shape of contact patch](image)

Crossing over from an abstract model of circle-line tangential to a concrete wheel-rail contact model requires a strong data enriching act consist of: material structure parameters addition, elastic parameters and stress propagation functions addition. That it means it transforms the contact point into a real power transfer space that we call now the contact patch.

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