$N$-photon Correlation Functions in $N$-slit Diffraction Experiments with Thermal Light

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We discuss the analytic presentations of the high-order correlation functions in the $N$-slit diffraction with thermal light in a recent paper [Phys. Rev. Lett. 109, 233603 (2012)]. Our analysis shows that the superresolving fringes in the high-order correlation measurement have two classical counterparts.

In a recent paper, Oppel et al. [1] obtained superresolving fringes in an $N$-slit diffraction experiment with thermal light sources. The analytic results of the high-order correlation functions, with order up to 5, were given. However, the general presentations of arbitrary $N$th-order correlation functions with thermal light were not presented. In this paper, we theoretically reconsider the $N$-slit diffraction experiment, and calculate out the analytic results of the arbitrary $N$th-order correlation functions.

The experimental setup of $N$-photon superresolving interference with thermal light is shown in Fig. 1(a). A uniform thermal light beam passes through $N$ slits of separation $d$, thereafter is registered by $N$ well-arranged detectors in the far-field diffraction plane. The slits are positioned at $(2n - N - 1)d/2$, $n = 1, 2, \ldots, N$, i.e., all the $N$ slits are symmetrical with respective to the origin. Among the $N$ detectors, the first detector scans while the other $N - 1$ ones are fixed at certain magic positions, given in terms of phases $\delta_j = 2\pi d \sin \theta_j/\lambda = (j - 1)2\pi/(N - 1)$, $(j = 2, \ldots, N)$, where $\lambda$ is the light wavelength, and $\theta_j$ is the diffraction angle. The joint measurement of the $N$ detectors gives the $N$th-order intensity correlation function

$$G^{(N)}(\delta, \delta_2, \ldots, \delta_N) = \langle I(\delta)I(\delta_2)\cdots I(\delta_N) \rangle$$

$$= \langle E^*(\delta)E^*(\delta_2)\cdots E^*(\delta_N) \times E(\delta_N)\cdots E(\delta_2)E(\delta) \rangle, \quad (1)$$

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where $\delta = 2\pi d \sin \theta / \lambda$ indicates the position of the scanning detector.

According to the Gaussian moment theorem, one can expand the $N$-th-order correlation function into terms of first-order correlation functions that

$$G^{(N)}(\delta, \delta_2, \cdots, \delta_N) = \sum_{N!} \mathcal{P} \left\langle E^*(\delta)E(\delta) \right\rangle \left\langle E^*(\delta_2)E(\delta_2) \right\rangle \cdots \left\langle E^*(\delta_{N-1})E(\delta_{N-1}) \right\rangle ,$$

where $\mathcal{P}$ represents the $N!$ possible permutations of the underlined fields.

We assume the thermal source satisfies completely spatial incoherence. In the far field diffraction by considering the magic positions, the first-order correlation functions of the detected fields in Eq. (2) are calculated out [5], and the results can be written as

$$\langle E^*(\delta)E(\delta) \rangle = I_0 N,$$

$$\langle E^*(\delta_j)E(\delta_k) \rangle = \begin{cases} I_0 N, & j = k \\ I_0(-1)^{j-k}, & j \neq k \end{cases}$$

$$\langle E^*(\delta)E(\delta_j) \rangle = \frac{I_0}{\sin \left[ \frac{\pi}{2} \left( x - 2\pi \frac{j-1}{N-1} \right) \right]} \sin \left[ \frac{\pi}{2} \left( x - 2\pi \frac{j-2}{N-1} \right) \right],$$

where $j, k = 2, \cdots, N$. Note that $\langle E^*(\delta)E(\delta_j) \rangle$ in Eq. (5) represents the Fourier transform of the $N$-slit. The $N$-th-order intensity correlation function in Eq. (2) can be normalized as

$$g^{(N)}(\delta, \delta_2, \cdots, \delta_N) = \frac{G^{(N)}(\delta, \delta_2, \cdots, \delta_N)}{\langle E^*(\delta)E(\delta) \rangle \langle E^*(\delta_2)E(\delta_2) \rangle \cdots \langle E^*(\delta_{N-1})E(\delta_{N-1}) \rangle}.$$
The denominator in Eq. (6) is
\[
\langle E^*(\delta)E(\delta) \rangle \langle E^*(\delta_2)E(\delta_2) \rangle \cdots \langle E^*(\delta_N)E(\delta_N) \rangle = (I_0N)^N, \tag{7}
\]
in which the results of the first-order correlation functions (3, 4) are taken into account. By categorizing all the \(N!\) possible permutations of the first-order correlation functions, we arrive at
\[
g^{(2)}(\delta, \delta_2) = 1 + |g^{(1)}(\delta, \delta_2)|^2 \tag{8}
\]
for \(N=2\), and
\[
g^{(N)}(\delta, \delta_2, \ldots, \delta_N) = A + B \left| \sum_{j=2}^{N} (-1)^j g^{(1)}(\delta, \delta_j) \right|^2 + C \sum_{j=2}^{N} |g^{(1)}(\delta, \delta_j)|^2, \tag{9}
\]
for \(N \geq 3\). The normalized first-order correlation function is
\[
g^{(1)}(\delta, \delta_j) = \frac{\langle E^*(\delta)E(\delta_j) \rangle}{\sqrt{\langle E^*(\delta)E(\delta) \rangle \langle E^*(\delta_j)E(\delta_j) \rangle}}. \tag{10}
\]

The three coefficients are
\[
A = ND_N(N-1), \tag{11}
\]
\[
B = \sum_{n=0}^{N-3} D_N(N-3-n) \frac{(N-3)!}{(N-3-n)!}, \tag{12}
\]
\[
C = D_N(N-2) - B, \tag{13}
\]
where
\[
D_N(M) = \sum_{n=0}^{M} \frac{N^{M-n-N}M!}{(M-n)!} \sum_{m=0}^{n} \frac{(-1)^m}{m!}. \tag{14}
\]

Let us analyze the three terms in the right side of Eq. (9). The first term contributes a homogeneous background to the \(N\)-photon joint measurement. The last two terms represent the superresolving interference fringes. We note that the last two terms are not the Fourier transform of the source profile (5) when \(N > 2\). By considering Eq. (5), we obtain the summations in Eq. (9)
\[
\left| \sum_{j=2}^{N} (-1)^j g^{(1)}(\delta, \delta_j) \right|^2 = 2(N-1)^2 [1 + \cos(N-1)\delta], \tag{15}
\]
\[
\sum_{j=2}^{N} |g^{(1)}(\delta, \delta_j)|^2 = N(N-1) \left[ 1 + \frac{2}{N} \cos(N-1)\delta \right], \tag{16}
\]
both showing cosinusoidal spatial modulations.

Since the source plane can be regarded as a conjugate mirror in high-order intensity correlation function of thermal light [6], we unfold the experimental setup as shown in Fig. 1 (b). The thermal light correlation in the experiment can become comprehensible that as if the $N - 1$ fixed detectors played the role of optical sources. Imagine the light came from the $N - 1$ fixed detectors, gone back to the thermal source plane, and gone through the $N$-slit, and finally registered by the moving detector. Therefore, the results in Eqs. (15, 16) can be obtained in single-photon diffraction of $N$-slit with coherent (15) and incoherent (16) illuminations, respectively.

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