Spinor Dark Energy and Cosmological Coincidence Problem

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ABSTRACT

Recently, the so-called Elko spinor field has been proposed to be a candidate of dark energy. It is a non-standard spinor and has unusual properties. When the Elko spinor field is used in cosmology, its unusual properties could bring some interesting consequences. In the present work, we discuss the cosmological coincidence problem in the spinor dark energy models by using the dynamical system method. Our results show that the cosmological coincidence problem should be taken to heart in the investigations of spinor dark energy models.

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I. INTRODUCTION

Since the discovery of accelerated expansion of our universe, dark energy has been one of the most active fields in modern cosmology [1–3]. The simplest candidate of dark energy is a tiny positive cosmological constant. As an alternative to the cosmological constant, some dynamical field models have been proposed. These dynamical field models can be categorized into three major types: (complex) scalar field models (e.g. quintessence [4, 5], phantom [6], k-essence [7], quintom [8–10], hessence [11, 12]), vector field models (e.g. [13–16]), and spinor field models. Of course, there are also other dark energy models which are not directly described by quantum fields, and so we do not mention them here.

To our knowledge, in the literature there are relatively few works on the dark energy models with spinor fields. In [17], the Bianchi type I cosmology with Dirac spinor fields has been investigated. In [18], it is found that the Dirac spinor fields could be responsible for the cosmic acceleration. In [19], the massive non-linear dark spinors have been discussed. In [20], the spinor quintom has been studied.

It is worth noting that all the spinors considered in the aforementioned works [17–20] are Dirac spinors. In fact, there is a different type of spinor in the literature, namely, the so-called Elko spinor (e.g. [21, 29]), which is similar to Majorana spinor. In the beginning, the Elko spinor was considered as a candidate of dark matter [21]. Subsequently, it has been used to drive inflation [22–26]. Recently, the Elko spinor has been proposed to be a candidate of dark energy [27]. In fact, this type of dark energy model described by the Elko spinor fields is the one we will discuss in the present work.

Following [22–24, 27], here is a brief review of the so-called Elko spinor. It is a spin one half field with mass dimension one [21]. Unlike the standard fields which obey $(CPT)^2 = 1$, the Elko spinor is non-standard spinor according to the Wigner classification [28] and obeys the unusual property $(CPT)^2 = -1$ instead. In fact, the Elko spinor fields (together with Majorana spinor fields) belong to a wider class of spinor fields, i.e., the so-called flagpole spinor fields, according to the Lounesto general classification of all spinor fields [29, 36]. The Elko spinors are defined by [21, 24, 25]

$$\lambda = \begin{pmatrix} \pm \sigma_2 \phi_L^* \\ \phi_L \end{pmatrix},$$  \hspace{1cm} (1)

where subscript $L$ refers to left-handed spinor; $\sigma_2$ denotes the second Pauli matrix; $\phi_L^*$ denotes the complex conjugate of $\phi_L$. Note that the helicities of $\phi_L$ and $\sigma_2 \phi_L^*$ are opposite [21]. Therefore, there are two distinct helicity configurations denoted by $\lambda_{(-, +)}$ and $\lambda_{(+, -)}$. The corresponding action is given by [24, 25]

$$S = \frac{1}{2} \int \left[ g^{\mu\nu} D_\mu \lambda \cdot D_\nu \lambda - V(\lambda \lambda) \right] \sqrt{-g} \, d^4x,$$  \hspace{1cm} (2)

where $V$ is the potential; the round subscript brackets denote symmetrization; $D_\mu$ is covariant derivative and $\lambda_\mu$ is the Elko dual which is different from the standard model spinors (see e.g. [21, 25] for definitions). We consider a spatially flat Friedmann-Robertson-Walker (FRW) universe and assume that the spinor fields are homogeneous. Following [22, 24], one can find that

$$\lambda_{(-, +)} = \phi(t) \frac{\xi}{\sqrt{2}}, \hspace{1cm} \lambda_{(+, -)} = \phi(t) \frac{\zeta}{\sqrt{2}},$$  \hspace{1cm} (3)

where $\phi$ is a homogeneous real scalar; $\xi$ and $\zeta$ are constant spinors satisfying $\xi \xi = \zeta \zeta = +2$. In [22, 24, 27], the effective pressure and energy density of the Elko spinor field are found to be

$$p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) + \frac{1}{8} H^2 \phi^2,$$  \hspace{1cm} (4)

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) - \frac{3}{8} H^2 \phi^2,$$  \hspace{1cm} (5)

where $H \equiv \dot{a}/a$ is the Hubble parameter; $a = (1 + z)^{-1}$ is the scale factor (we have set $a_0 = 1$); $z$ is the redshift; a dot denotes the derivatives with respect to cosmic time $t$; the subscript “0” indicates the present value of the corresponding quantity; we use the units $\hbar = c = 1$. Recently, in [27] the Elko spinor...
field has been proposed to be a candidate of dark energy, and we will call it “spinor dark energy” in the present work. However, very recently, it is found that the previous researches (e.g. [22, 24, 27]) overlooked one part of the energy-momentum tensor which arises when the spin connection is varied appropriately with respect to the metric [37, 38]. Therefore, the correct pressure and energy density of spinor dark energy should be [38]

\begin{equation}
 p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) - \frac{3}{8} H^2 \phi^2 - \frac{1}{4} \dot{H} \phi^2 - \frac{1}{2} H \dot{\phi} \phi, \tag{6}
\end{equation}

\begin{equation}
 \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) + \frac{3}{8} H^2 \phi^2. \tag{7}
\end{equation}

Correspondingly, the equation-of-state parameter (EoS) of spinor dark energy reads

\begin{equation}
 w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{1}{2} \frac{\dot{\phi}^2 - V(\phi) - \frac{3}{4} H^2 \phi^2 - \frac{1}{4} \dot{H} \phi^2 - \frac{1}{2} H \dot{\phi} \phi}{\frac{1}{2} \dot{\phi}^2 + V(\phi) + \frac{3}{4} H^2 \phi^2}. \tag{8}
\end{equation}

In this case, it is easy to see that \( w_\phi \geq -1 \) when \( \dot{\phi}^2 \geq (\dot{H} \phi^2)/4 + H \dot{\phi} \phi/2 \), whereas \( w_\phi < -1 \) when \( \dot{\phi}^2 < (\dot{H} \phi^2)/4 + H \dot{\phi} \phi/2 \). The EoS of spinor dark energy crosses the phantom divide \( w_{de} = -1 \) when \( \dot{\phi}^2 = (\dot{H} \phi^2)/4 + H \dot{\phi} \phi/2 \).

This note is organized as follows. In Sec. II we discuss the cosmological coincidence problem in the spinor dark energy models by using the dynamical system method. A brief summary is given in Sec. III. It is worth noting that in the present work, we merely consider the spinor dark energy with the correct pressure and energy density given in Eqs. (6) and (7) [39].

II. SPINOR DARK ENERGY AND COSMOLOGICAL COINCIDENCE PROBLEM

The cosmological coincidence problem [1–3] is asking why are we living in an epoch in which the dark energy density and the matter energy density are comparable? Since their densities scale differently with the expansion of the universe, there should be some fine-tunings. Most dark energy models are plagued with this coincidence problem. However, this problem can be alleviated in these models via the method of scaling solution. If there is a possible interaction between dark energy and matter, their evolution equations could be rewritten as a dynamical system [30] (see also e.g. [5, 9, 12, 14, 31–35]). There might be some scaling attractors in this dynamical system, and both the densities of dark energy and matter are non-vanishing constants over there. The universe will eventually enter these scaling attractors regardless of the initial conditions, and hence the coincidence problem could be alleviated without fine-tuning. This method works fairly well in most of dark energy models (especially in the scalar field models). To our knowledge, there is no attempt to do this in spinor dark energy model. Let us have a try.

A. Dynamical system

We consider a flat FRW universe containing both spinor dark energy and background matter. The background matter is described by a perfect fluid with barotropic EoS, namely

\begin{equation}
 p_m = w_m \rho_m \equiv (\gamma - 1) \rho_m, \tag{9}
\end{equation}

where the so-called barotropic index \( \gamma \) is a positive constant. In particular, \( \gamma = 1 \) and \( 4/3 \) correspond to dust matter and radiation, respectively. Of course, the Friedmann equation and Raychaudhuri equation are given by

\begin{equation}
 H^2 = \frac{\kappa^2}{3} \rho_{tot} = \frac{\kappa^2}{3} (\rho_\phi + \rho_m), \tag{10}
\end{equation}

\begin{equation}
 \dot{H} = -\frac{\kappa^2}{2} (\rho_{tot} + p_{tot}) = -\frac{\kappa^2}{2} (\rho_\phi + p_\phi + \rho_m + p_m), \tag{11}
\end{equation}

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where \( \kappa^2 \equiv 8\pi G = M_{pl}^{-2} \) and \( M_{pl} \) is the reduced Planck mass. We assume that spinor dark energy and background matter interact through a coupling term \( Q \), according to

\[
\dot{\rho}_\phi + 3H (\rho_\phi + p_\phi) = -Q, \\
\dot{\rho}_m + 3H (\rho_m + p_m) = Q,
\]

which preserves the total energy conservation equation \( \dot{\rho}_{tot} + 3H (\rho_{tot} + p_{tot}) = 0 \). Obviously, \( Q = 0 \) corresponds to no interaction between spinor dark energy and background matter.

Following e.g. [5, 9, 12, 14, 31–35], we introduce following dimensionless variables

\[
x \equiv \frac{\kappa \dot{\phi}}{\sqrt{6H}}, \quad y \equiv \frac{\kappa \sqrt{V}}{\sqrt[3]{3H}}, \quad u \equiv \frac{\kappa \phi}{2\sqrt{2}}, \quad v \equiv \frac{\kappa \sqrt{\rho_m}}{\sqrt[3]{3H}}.
\]

Then, we can recast the Friedmann equation (10) as

\[
x^2 + y^2 + u^2 + v^2 = 1.
\]

From Eqs. (10), (11) and (6), (7), it is easy to find that

\[
s \equiv -\frac{\dot{H}}{H^2} = 3x^2 + su^2 - \sqrt{3} xu + \frac{3}{2}v^2,
\]

in which \( s \) appears in both sides. One can solve Eq. (10) and get

\[
s = \left( 3x^2 - \sqrt{3} xu + \frac{3}{2}v^2 \right) \left( 1 - u^2 \right)^{-1}.
\]

By the help of Eqs. (10), (11) and (6), (7), the evolution equations (12) and (13) can be rewritten as a dynamical system, namely

\[
x' = (s - 3)x + \sqrt{3} u - \frac{\kappa V}{\sqrt{6H}^2} - Q_1,
\]

\[
y' = sy + \frac{x}{\sqrt{2H}} \frac{V}{\sqrt{V}},
\]

\[
u' = \frac{\sqrt{3}}{2} x,
\]

\[
u' = \left( s - \frac{3}{2} \right) v + Q_2,
\]

where

\[
Q_1 \equiv \frac{\kappa Q}{\sqrt{6H}^2}, \quad Q_2 \equiv \frac{vQ}{2H \rho_m},
\]

a prime and the subscript \( \text{“., } \phi \text{”} \) denote derivatives with respect to \( N \equiv \ln a \) and \( \phi \), respectively; we have used the universal relation \( f' = H^{-1} f \) for any function \( f \). On the other hand, the fractional energy densities \( \Omega_i \equiv (\kappa^2 \rho_i)/(3H^2) \) of spinor dark energy and background matter are given by

\[
\Omega_\phi = x^2 + y^2 + u^2, \quad \Omega_m = v^2.
\]

The EoS of spinor dark energy reads

\[
w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{x^2 - y^2 - u^2 + \frac{2}{7} su^2 - \frac{2}{\sqrt{3}} xu}{x^2 + y^2 + u^2}.
\]

Eqs. (18)—(21) could be an autonomous system when the potential \( V(\phi) \) and the interaction term \( Q \) are chosen to be suitable forms. In fact, we will consider the model with a power-law or exponential
potential in the next subsections. In each model with different potential, we consider four cases with various interaction forms between spinor dark energy and background matter. The first case is the one without interaction, i.e., $Q = 0$. The other three cases are taken as the most familiar interaction terms extensively considered in the literature, namely

- Case (I) $Q = 0$,
- Case (II) $Q = \alpha \kappa \rho_m \dot{\phi}$,
- Case (III) $Q = 3\beta H \rho_{tot} = 3\beta H (\rho_\phi + \rho_m)$,
- Case (IV) $Q = 3\eta H \rho_m$,

where $\alpha$, $\beta$ and $\eta$ are dimensionless constants. The interaction form Case (II) arises from, for instance, string theory or scalar-tensor theory (including Brans-Dicke theory) [31–33]. The interaction forms Case (III) [34] and Case (IV) [35] are phenomenally proposed to alleviate the coincidence problem in the other dark energy models.

### B. Spinor dark energy with a power-law potential

In this subsection, we consider the spinor dark energy model with a power-law potential

$$V(\phi) = V_0 (\kappa \phi)^n,$$

where $n$ is a dimensionless constant. Actually, in most of the models with Elko spinor field [21–27], the potential is usually chosen to be, for instance, $\frac{1}{2} m^2 \phi^2$ or $\alpha \phi^4$. It is easy to see that they are the special cases of the power-law potential considered here. In this case, Eqs. (18)–(21) become

$$x' = (s - 3)x + \frac{\sqrt{3}}{2} u - \frac{\sqrt{3}}{4} ny^2 u^{-1} - Q_1,$$  

$$y' = sy + \frac{\sqrt{3}}{4} nxyu^{-1},$$  

$$u' = \frac{\sqrt{3}}{2} x,$$  

$$v' = \left( s - \frac{3}{2} \gamma \right) v + Q_2.$$  

If $Q$ is given, we can obtain the critical points $(\bar{x}, \bar{y}, \bar{u}, \bar{v})$ of the autonomous system by imposing the conditions $\bar{x}' = \bar{y}' = \bar{u}' = \bar{v}' = 0$. Of course, they are subject to the Friedmann constraint Eq. (15), i.e., $\bar{x}^2 + \bar{y}^2 + \bar{u}^2 + \bar{v}^2 = 1$. One the other hand, by definitions in Eq. (14), $\bar{x}$, $\bar{y}$, $\bar{u}$, $\bar{v}$ should be real, and $\bar{y} \geq 0$, $\bar{v} \geq 0$ are required.

Here we consider the interaction forms $Q$ given in the end of Sec. II A one by one. In Case (I) $Q = 0$, the corresponding $Q_1 = Q_2 = 0$. There is only one critical point \(\{ \bar{x} = 0, \bar{y} = \sqrt{2/\alpha}, \bar{u} = \pm \sqrt{2/\alpha}, \bar{v} = 0 \}\). This is not a scaling solution because its $\Omega_m = \bar{v}^2 = 0$. Therefore, the coincidence problem persists. In Case (II) $Q = \alpha \kappa \rho_m \dot{\phi}$, the corresponding $Q_1 = \frac{1}{2} \alpha \bar{v}^2$, and $Q_2 = \frac{1}{2} \alpha \bar{x} \bar{v}$. There are two critical points. The first one is given by

$$\left\{ \bar{x} = 0, \bar{y} = 0, \bar{u} = \frac{-1 + \sqrt{1 + 8\alpha^2}}{2\sqrt{2}\alpha}, \bar{v} = \frac{1}{2} \sqrt{-\frac{1 + \sqrt{1 + 8\alpha^2}}{2\alpha^2}} \right\},$$  

which is a scaling solution. The second one is \(\{ \bar{x} = 0, \bar{y} = \sqrt{2/\alpha}, \bar{u} = \pm \sqrt{2/\alpha}, \bar{v} = 0 \}\), which is not a scaling solution. Finally, we find that in both Cases (III) $Q = 3\beta H \rho_{tot}$ and (IV) $Q = 3\eta H \rho_m$, there is no critical point and hence there is no attractor of course. Therefore, the cosmological evolution trajectory
completely depends on the initial conditions, and the coincidence problem is inevitable. The fine-tuning of initial conditions is required.

So, the only hope to alleviate the coincidence problem relies on the sole scaling solution given in Eq. (30) for Case (II) \( Q = \alpha \kappa \rho m \phi \). However, its stability is required in order to be an attractor which is necessary to alleviate the coincidence problem. To study the stability of the critical points for the autonomous system Eqs. (26)—(29), we substitute linear perturbations
\[
x \to \bar{x} + \delta x, \quad y \to \bar{y} + \delta y, \quad u \to \bar{u} + \delta u, \quad v \to \bar{v} + \delta v
\]
about the critical point \((\bar{x}, \bar{y}, \bar{u}, \bar{v})\) into the autonomous system Eqs. (26)—(29) and linearize them. Because of the Friedmann constraint (15), there are only three independent evolution equations, namely
\[
\delta x' = (s - 3)\delta x + \bar{x} \delta s + \frac{\sqrt{3}}{2} \delta u - \frac{\sqrt{3}}{4} n (2\bar{y} \bar{u}^{-1} \delta y - \bar{y}^2 \bar{u}^{-2} \delta u) - \delta Q_1, \quad (31)
\]
\[
\delta y' = s \delta y + \bar{y} \delta s + \frac{\sqrt{3}}{4} n [\bar{u}^{-1} (\bar{x} \delta y + \bar{y} \delta x) - \bar{x} \bar{y} \bar{u}^{-2} \delta u], \quad (32)
\]
\[
\delta u' = \frac{\sqrt{3}}{2} \delta x, \quad (33)
\]
where
\[
\bar{s} = \left[ 3\bar{x}^2 - \sqrt{3} \bar{x} \bar{u} + \frac{3}{2} \gamma \left( 1 - \bar{x}^2 - \bar{y}^2 - \bar{u}^2 \right) \right] \left( 1 - \bar{u}^2 \right)^{-1}, \quad (34)
\]
\[
\delta s = \left[ 2u \delta u + 6\bar{x} \delta x - \sqrt{3} (\bar{x} \delta u + \bar{u} \delta x) - 3\gamma (\bar{x} \delta x + \bar{y} \delta y + \bar{u} \delta u) \right] \left( 1 - \bar{u}^2 \right)^{-1}, \quad (35)
\]
and \( \delta Q_1 \) is the linear perturbation coming from \( Q_1 \). The three eigenvalues of the coefficient matrix of Eqs. (31)—(33) determine the stability of the critical point. For Case (II) \( Q = \alpha \kappa \rho m \phi \), the corresponding \( \delta Q_1 = -\sqrt{6} \bar{u} (\bar{x} \delta x + \bar{y} \delta y + \bar{u} \delta u) \). We find that the corresponding eigenvalues for the critical point (30) are given by
\[
\left\{ \frac{3\gamma}{2}, \quad \frac{1}{4} \left[ 3(\gamma - 2) - \sqrt{12 + 8\alpha^2 + [3(\gamma - 2)]^2} \right], \quad \frac{1}{4} \left[ 3(\gamma - 2) + \sqrt{12 + 8\alpha^2 + [3(\gamma - 2)]^2} \right] \right\}. \quad (36)
\]
Note that \( 3\gamma/2 \) is positive, and the second and third eigenvalues are negative and positive, respectively. So, the critical point given in Eq. (30) is unstable, and hence it is not an attractor. Therefore, the hope to alleviate the coincidence problem is shattered. Due to the failure in the models with a power-law potential, we should turn to the models with another potential.

C. Spinor dark energy with an exponential potential

In this subsection, we consider the spinor dark energy models with an exponential potential
\[
V(\phi) = V_0 e^{-\epsilon \kappa \phi}, \quad (37)
\]
where \( \epsilon \) is a dimensionless constant. In this case, Eqs. (18)—(21) become
\[
x' = (s - 3) x + \sqrt{3} u + \sqrt{3} \epsilon y^2 - Q_1, \quad (38)
\]
\[
y' = s y - \sqrt{3} \epsilon x y, \quad (39)
\]
\[
u' = \frac{\sqrt{3}}{2} x, \quad (40)
\]
\[
u' = \left( s - \frac{3}{2} \right) v + Q_2. \quad (41)
\]
If $Q$ is given, we can obtain the critical points $(\bar{x}, \bar{y}, \bar{u}, \bar{v})$ of the autonomous system by imposing the conditions $\bar{x}' = \bar{y}' = \bar{u}' = \bar{v}' = 0$. Of course, they are subject to the Friedmann constraint Eq. (15), i.e., $\bar{x}^2 + \bar{y}^2 + \bar{u}^2 + \bar{v}^2 = 1$. One the other hand, by definitions in Eq. (13), $\bar{x}$, $\bar{y}$, $\bar{u}$, $\bar{v}$ should be real, and $\bar{y} \geq 0$, $\bar{v} \geq 0$ are required.

We consider the four cases of the interaction term $Q$ given in the end of Sec. II A. Unfortunately, similar to the models with a power-law potential, we find that in both cases (III) and (IV) there is no critical point and hence there is no attractor of course. So, the cosmological evolution trajectory completely depends on the initial conditions, and the coincidence problem is inevitable. The fine-tuning of initial conditions is required. In Case (I), there are two critical points, i.e., $\{\bar{x} = 0$, $\bar{y} = 0$, $\bar{u} = 0$, $\bar{v} = 1\}$ and

$$\left\{\begin{array}{l}
\bar{x} = 0, 
\bar{y} = \frac{1}{2}\sqrt{-1 + \sqrt{1 + 8\bar{\epsilon}}}, 
\bar{u} = \frac{1 - \sqrt{1 + 8\bar{\epsilon}}}{2\sqrt{2}\bar{\epsilon}}, 
\bar{v} = 0
\end{array}\right\}. \tag{42}
$$

Unfortunately, these two critical points are not scaling solutions. In Case (II), there are also two critical points. The first one is the same given in Eq. (30), which is a scaling solution. The second one is the same given in Eq. (42), which is not a scaling solution.

Again, the only hope to alleviate the coincidence problem relies on the sole scaling solution given in Eq. (29) for Case (II) $Q = \alpha \kappa m \phi$. To study the stability of the critical points for the autonomous system Eqs. (38)—(41), we substitute linear perturbations $x \rightarrow \bar{x} + \delta x$, $y \rightarrow \bar{y} + \delta y$, $u \rightarrow \bar{u} + \delta u$, and $v \rightarrow \bar{v} + \delta v$ about the critical point $(\bar{x}, \bar{y}, \bar{u}, \bar{v})$ into the autonomous system Eqs. (38)—(41) and linearize them. Because of the Friedmann constraint (15), there are only three independent evolution equations, namely

$$\delta x' = (\bar{s} - 3)\delta x + \bar{x}\delta s + \frac{3}{2}\delta u + \sqrt{6}\epsilon \bar{y}\delta y - \delta Q_1, \tag{43}$$
$$\delta y' = \bar{s}\delta y + \bar{y}\delta s - \frac{3}{2}\epsilon (\bar{x}\delta y + \bar{y}\delta x), \tag{44}$$
$$\delta u' = \frac{3}{2}\delta x, \tag{45}$$

where $\bar{s}$ and $\bar{\delta} s$ are given in Eqs. (31) and (32), respectively. $\delta Q_1$ is the linear perturbation coming from $Q_1$. The three eigenvalues of the coefficient matrix of Eqs. (43)—(45) determine the stability of the critical point. For Case (II) $Q = \alpha \kappa m \phi$, the corresponding $\delta Q_1 = -\sqrt{6}\alpha (\bar{x}\delta x + \bar{y}\delta y + \bar{u}\delta u)$. We find that the corresponding eigenvalues for the critical point (30) are the same given in Eq. (30), in which three eigenvalues are positive, negative and positive, respectively. So, the critical point given in Eq. (30) is also unstable for the models with an exponential potential, and hence it is not an attractor. Therefore, the hope to alleviate the coincidence problem is shattered again.

### III. SUMMARY

Recently, the so-called Elko spinor field has been proposed to be a candidate of dark energy. It is a non-standard spinor and has unusual properties. When the Elko spinor field is used in cosmology, its unusual properties could bring some interesting consequences. In this work, we discussed the cosmological coincidence problem in the spinor dark energy models by using the dynamical system method. According to the results obtained in Sec. II, we should admit that in the spinor dark energy models with $p_\phi$ and $\rho_\phi$ given in Eqs. (6) and (7) coming from (38), it is a hard task to alleviate the coincidence problem (39). Nevertheless, it is still possible to find some suitable potentials $V(\phi)$ and interaction terms $Q$ to obtain the scaling attractors of the most general dynamical system (40), and hence the hope to alleviate the coincidence problem still exists, although this is a fairly hard task. Of course, there might be other smart methods different from the usual method used in most of dark energy models to alleviate the coincidence problem. Anyway, our results obtained in the present work showed that the cosmological coincidence problem should be taken to heart in the investigations of spinor dark energy models.
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[40] Here we would like to briefly discuss the most general case. From the most general Eq. (29), we have \( \bar{x} = 0 \) at the critical points. From Eq. (15), it is easy to see that \( 1 - \bar{u}^2 = \bar{y}^2 + \bar{v}^2 \geq 0 \). Therefore, from Eq. (17), we find that \( \bar{s} > 0 \) when \( \bar{v} \neq 0 \) (i.e., \( \Omega_m \neq 0 \), scaling solution), whereas \( \bar{s} = 0 \) when \( \bar{v} = 0 \) (i.e., \( \Omega_m = 0 \), dark energy dominated). From Eq. (24), we see that \( w_\phi > -1 \) when \( \bar{v} \neq 0 \) (and hence \( \bar{s} > 0 \)), whereas \( w_\phi = -1 \) when \( \bar{v} = 0 \) (and hence \( \bar{s} = 0 \)). Therefore, at the critical points (if any), the big rip can be avoided because \( w_\phi \geq -1 \) (and hence \( w_{\text{eff}} \geq -1 \)).