Nucleon Meson Form Factors and Coupling Constants

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Abstract. The nucleon-nucleon (NN) interaction has been an object of active study for over fifty years. In the past decade high precision potentials based on one boson exchange (OBE) were developed: Argonne 18, Nijmegen '93, CD Bonn, etc. Unfortunately they are highly parametrized ($\approx$ 40 parameters) and less theoretically sound than their predecessors: Bonn B, Nijmegen '78, Paris, etc. Historically OBE potentials parametrize the coupling constants/form factors of an effective Lagrangian. We have calculated these form factors using quark model simple harmonic oscillator wave functions and the $^3P_0$ model of hadronic decays. We present preliminary results for phase shifts of a toy nuclear interaction model, pion ($\pi$) and scalar/isoscalar ($\sigma$) exchange. Prospects for future work, including self-consistently extending this work to non-zero strangeness hadrons, are discussed.

1. Introduction
Effective Lagrangians are widely used to study hadronic interactions. This technique introduces a quantum field for every relevant hadron species and assumes interactions between them that are consistent with known symmetries and conservation laws. It’s most accurate to describe the strength of the interactions with momentum dependent functions, form factors. In the low momentum limit, the hadrons appear pointlike and the form factors become coupling constants. Unfortunately, the form factors usually have many parameters which must be determined from experiment. This approach may be sufficient for the highly constrained NN interaction, but for other, less constrained interactions the form factors are parametrized by plausible values. For these interactions reliable estimates of these parameters is highly desirable.

In this study we examine the relation the between effective Lagrangian form factors and quark model bound state hadron wave functions. First, we briefly present a technique to calculate the form factors from quark model wave functions. We computed a variety of nucleon nucleon meson (NNm) form factors and coupling constants and compare them to the fitted coupling constants from a variety of meson exchange models of the NN force. As a preliminary result, we present Born order, small angle $\pi + \sigma$ exchange phase shifts for the singlet and triplet D-waves both with and without momentum-dependent form factors. Finally, we discuss some possible experimental tests of this technique along with prospects for extending this work into the strange sector.

2. Quark Model Form Factors
In OBE models nucleons interact by exchanging virtual bosons. At present OBE models are considered phenomenological descriptions of the NN interaction. In principle the strength of
those 'exchanges' should be determined by the underlying quark and gluon dynamics of the system, but in practice the non-perturbative nature of low energy QCD requires that we model these interactions. If we focused on the 3 point NNm vertices that are the building blocks of the OBE models, the the 3 point vertex is topologically identical to a decay diagram so studying the simpler 'decay' diagrams should completely characterize the OBE diagrams.

We have chosen to use the successful $^3P_0$ model of hadronic decays [1] to determine the form factors of the three point vertices. This model has one dimensionless parameter, the interaction strength $\gamma$. For matters of convienence we have chosen to use simple harmonic oscillator wave functions which introduce two more parameters, $\alpha$ and $\beta$, the widths of the baryon and meson oscillator wave functions respectively. The Feynman diagrams of the two approaches are shown in figures 1 and 2. By setting the 'decay widths' to be equal, we can solve for the form factor. Further details can be found in [2].

Figure 1. Effective process of a nucleon emitting a meson with form factor $g_{NNm}(\vec{P}_i, \vec{P}_f)$.

Figure 2. Quark Model Process of nucleon emitting a meson by creating a $q\bar{q}$ pair from the vacuum.

We calculated the form factors for NNm where $m$ was: $\pi, \eta, \eta', \omega, \rho, \sigma$ and $a_0$. We confirmed a result for $g_{NN\pi}$ and also $g_{NN\rho}$ with some caveats [2, 3]. In the quark model the $\sigma$ and $a_0$ mesons are $P$ wave while all the other mesons we examined are $S$ wave. Hence, all the form factors are proportional to:

\[ g_{NN\pi}(\vec{P}_i, \vec{P}_f) = \gamma 20\sqrt{3}\pi^{3/4}m_N\sqrt{m_\pi^{\beta^3/2}(4\beta^2 + \alpha^2)} \exp\left\{ -\frac{(\vec{P}_i + \vec{P}_f)^2}{24(3\beta^2 + \alpha^2)} - \frac{(\vec{P}_i - \vec{P}_f)^2}{6\alpha^2} \right\} \]

and

\[ g_{NN\sigma}(\vec{P}_i, \vec{P}_f) = \gamma 9\sqrt{2}\pi^{3/4}\frac{\alpha^2\beta^{5/2}}{(\alpha^2 + 3\beta^2)^{7/2}} \left[ 12\alpha^2(\alpha^2 + 3\beta^2) + (\vec{P}_f - \vec{P}_i)^2(4\beta^2 + \alpha^2) \right] \]

\[ \times \exp\left\{ -\frac{(\vec{P}_i + \vec{P}_f)^2}{24(3\beta^2 + \alpha^2)} - \frac{(\vec{P}_i - \vec{P}_f)^2}{6\alpha^2} \right\} \]

where,

\[ g_{NN\pi} = \frac{5\sqrt{2}}{3} \frac{m_\pi}{m_\eta} g_{NN\eta} = \frac{5\sqrt{2}}{3} \frac{m_\pi}{m_{\eta'}} g_{NN\eta'} = \frac{5}{9} \frac{m_\pi}{m_\omega} g_{NN\omega} = \frac{5}{3} \frac{m_\pi}{m_\rho} g_{NN\rho} \]
Table 1. Compares the coupling constants, zero momentum limit of the form factors, of this approach to fitted values from highly accurate OBE NN potentials. Column 1 present the range of values obtained from the parameter space $\alpha = 0.25 - 0.4 [\text{GeV}]$, $\beta = 0.3 - 0.4 [\text{GeV}]$ and $\gamma = 0.4 - 0.5$. Column 2 assumes $\alpha = \beta = 0.4 [\text{GeV}]$ and $\gamma = 0.4$. Column 3 assumes $g_{NN\pi} = 13.5$.

| Coupling          | This Work | This Work | This Work | Paris [4] | Nijmegen [5] | CD Bonn [6] |
|-------------------|-----------|-----------|-----------|-----------|--------------|-------------|
| $g_{NN\pi}$       | 7.1-14.9  | 7.1       | [13.5]    | 13.3      | [13.1]       |             |
| $g_{NN\eta}$      | 6.0-12.6  | 6.0       | 11.5      | –         | 9.8          | –           |
| $g_{NN\eta}'$     | 7.9-16.6  | 7.9       | 15.3      | –         | 10.5         | –           |
| $g_{NN\sigma}$    | 3.0-8.7   | 5.0       | N/A       | –         | 17.9         | (7.3; 14.9) |
| $g_{NNa_0}$       | 1.6-4.7   | 2.7       | N/A       | –         | 3.3          | –           |
| $g_{NN\omega}(\gamma_\mu)$ | 30.2-63.4 | 30.2      | 57.4      | 12.2      | 12.5         | 15.9        |
| $g_{NN\rho}/g_{NN\omega}(\gamma_\mu)$ | +.33      | +.33      | +.33      | –         | 0.22         | 0.20        |
| $\kappa_\omega(\sigma_\mu\nu/\gamma_\mu)$ | -3/2      | -3/2      | -3/2      | -0.12     | 0.66         | 0           |
| $\kappa_\rho(\sigma_\mu\nu/\gamma_\mu)$ | +3/2      | +3/2      | +3/2      | –         | 6.6          | 6.1         |

$$ g_{NN\pi} = 3 \sqrt{\frac{m_\pi}{m_{a_0}}} g_{NNa_0}. $$

Note that these ratios are parameterless and hence form a strict test of the model. Unfortunately many of these coupling constants are usually inferred from a fit to NN scattering data and hence not well established in a model independent way. Nonetheless Table 1 presents comparisons between the coupling constants of this model and a variety of NN potentials.

$g_{NN\pi}$ is by far the most well established strong interaction coupling constant. One can see in Table 1 that the preferred parameter set $\alpha = \beta = 0.4 [\text{GeV}]$ and $\gamma = 0.4$ under predicts $g_{NN\pi}$. This could be explained by several possibilities: because the approach is fundamentally flawed, the preferred parameter space is different for NNm processes, or the pion mass (a known problem for the quark model) should be substituted with a mock mass. Our studies have shown that there is a compatibility between the usual value for $g_{NN\pi}$ and the decay width of $\Delta^{++} \rightarrow p + \pi^+ [2]$. Increased experimental data should differentiate between these various possibilities.

The strength of the $\eta$, $\eta'$, $\omega$ and $\rho$ couplings is much stronger than in most of the OBE potentials. Though its difficult to draw any conclusion from these differences.

An interesting prediction is the relative ratios of the vector to tensor couplings for the $\rho$ and $\omega$ mesons. If $SU(3)$ symmetry holds the ratio of these couplings, $\kappa$, should be equal. The model predicts that $\kappa_\omega = -\kappa_\rho$.

3. Born Order, Small Angle Phase Shifts
Our ultimate goal is to model the NN interaction from the quark model. In order to achieve this goal, we need to be able to compare our model directly to NN scattering data. Typically this is done with phase shifts. A forthcoming paper will contain the details of our phase shift analysis, including phase shifts which have been iterated in the ladder approximation and analytic expressions for Born order phase shifts. In this proceedings we only have Born order phase shifts to present. These phase shifts require that the interaction be weak enough that the first perturbation is sufficient to approximate the full interaction. We have found that D-Wave phase shifts in the NN interaction fit this criteria.
The model used to fit to the D-Wave Phase shifts is one pion exchange and one sigma exchange. This accounts for the much of the phase shift data, but there’s a clear discrepancy in the $^3D_2$ phase shift at Born order. Clearly while the form factors help with the $^3D_2$ phase shift they hurt the fit to the $^3D_1$ wave. A more sophisticated model and iteration may fix this problem and we will explore that possibility in a future paper.

4. Conclusions

We’ve rediscovered a formalism to calculate general BBm form factors from ficticious decays. So far $g_{NN\pi}$ is the most well determined coupling constant to compare with our model. Although the agreement is poor, the parameters needed to reproduce the $\Delta^{++} \rightarrow p + \pi^+$ decay width reproduces $g_{NN\pi} \approx 13$ suggesting more work needs to be done. Several interesting predictions were made regarding the other meson coupling constants and the ratio of vector to tensor couplings for the vector mesons.

Unfortunately there is insufficient data at this time to critically evaluate using this quark model to predict effective couplings. However, current experiments, such as the CLAS experiment at Jefferson Lab or $\pi - N$ scattering may provide many tests of this model. The relevant form factors for these experiments must be calculated to test this model.

One of the virtues of the $^3P_0$ model is that it is highly successful description of many hadronic decays. If it can be established that the approach presented here to calculate BBm form factors from fictitious decays is valid, then this approach can be readily extended to the strange sector where little data is available.

References

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