Massive Compact Objects in
a Quantum Theory of Gravity

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ABSTRACT

A massive compact object is that which forms when a sufficiently massive star collapses. This is commonly taken to be a black hole with a singularity surrounded by a horizon and which evolves by emitting Hawking radiation. In a quantum theory of gravity, singularities are expected to be resolved and the evolutions are expected to be unitary. Assuming that such a theory with these properties exists, and with a few more physically motivated assumptions, we argue that a massive compact object has no singularity (by assumption) and must also have no horizon; otherwise, there may be a loss of predictability in the case of a black hole candidate observed today. With no singularity and also with no horizon, the massive compact object will then evolve as a standard quantum system with large number of interacting degrees of freedom.
I. Introduction

A sufficiently massive star will collapse and form what is referred to as a massive compact object (MCO). This is commonly taken to be a black hole which has a singularity surrounded by a horizon and which evolves further by emitting Hawking radiation. The constituents of the collapsed star are taken to have disappeared into the singularity.

In a quantum theory of gravity, singularities are expected to be resolved, and the process of the collapse of a massive star and its further evolution are expected to be unitary. In such a theory, it may be that the MCO formed in the collapse of a massive star has no singularity but has a horizon, and it evolves further by emitting Hawking radiation. Or, it may be that the resulting MCO has no singularity and also has no horizon, and it evolves further as a standard quantum system with large number of interacting degrees of freedom. See [1] – [15] for a sample of possible scenarios.

To deduce the nature of the MCOs, it is necessary to understand the physics of singularities and their resolutions. Although there are many candidates for a quantum theory of gravity, such as string theory, quantum Einstein gravity, loop quantum gravity, and spin network theory, none of them is developed well enough to be applicable for collapsing massive stars. At present, therefore, quantitative approaches to such situations are beyond our reach and only qualitative, physically motivated approaches seem possible [2, 6, 7, 8, 11, 13, 14, 15].

We have found a qualitative, physically motivated approach which enables one to argue that an MCO formed in the collapse of a massive star has no singularity (by assumption) and must also have no horizon; otherwise, there will be a loss of predictability. See [11] for an earlier report. With no horizon, the resulting MCO will evolve further as a standard quantum system with large number of interacting degrees of freedom.

In this paper, we describe our approach, arguments, and their implications leading to the above conclusion. First, we assume that a quantum theory of gravity exists where singularities are resolved and the evolutions are unitary. Next, we estimate the size of the region of singularity resolution, referred to as ‘singularity cloud’. If this size is smaller than the horizon size of a black hole then it can be taken to mean that the MCO formed in the collapse of a massive star has no singularity but has a horizon; if larger then
the resulting MCO has no singularity and also has no horizon.

For the purposes of illustration, we present several plausible scenario of singularity resolution. The estimated size of the singularity cloud is smaller than the horizon size in some of them, and larger in others.

Let the size of the singularity cloud be smaller than the horizon size. We argue that if this is the case then, for an MCO observed today which can be thought of as a black hole candidate, we may not be able to predict when the size of the singularity cloud inside will exceed its horizon size. This is a loss of predictability, and is new. This is absent in general relativity theory where the size of the singularity cloud is always taken to be Planckian. Therefore, if the singularity resolution is expected in a quantum theory of gravity, but not the loss of predictability, then it logically follows that the physics of singularity resolution must be such that the size of singularity cloud is larger than the horizon size.

Note that an MCO having no singularity and no horizon is likely to follow also from Mathur’s fuzz ball proposal [16] in string theory applied to a collapsing massive star. In this paper, we argue that such an MCO must follow in any quantum theory of gravity obeying the present assumptions.

Note also that, over the years, several works have argued that the size of the region of singularity resolution must be much larger than Planck length, see [1, 2, 14] for example. However, in all these works, a horizon is always assumed to exist when an MCO forms. To the best of our knowledge, a further logical step is taken for the first time here which enables one to argue that the size of the singularity cloud is larger than the horizon size, thus an MCO formed has no horizon also.

The organisation of this paper is as follows.

In section II, we describe a few aspects of singularity resolution as needed here. For the purposes of illustration, we then present several plausible scenario of singularity resolution and estimate the size of the singularity cloud. We state our assumptions at the relevant stages of the paper, but will collect them together in the final section. In section III, we describe the implications of the size of the singularity cloud. In section IV, we describe the loss of predictability that will result if the singularity cloud is smaller than horizon in size, and arrive at our main result. In section V, we give a brief summary, discuss a few points, and close by mentioning two studies that may be pursued fruitfully. In Appendix A, we give a brief explanation regarding the meaning of the word predictability as used in this paper.
II. Aspects of singularity resolution

When a star is sufficiently massive, it begins to collapse and ultimately forms what is referred to as a massive compact object (MCO). It is believed that the resulting MCO is a black hole: that the collapse results in the formation of a singularity surrounded by a horizon and the constituents of the star are taken to have disappeared into the singularity. In a quantum theory of gravity such as string theory, quantum Einstein gravity, loop quantum gravity, and spin network theory, such singularities are expected to be resolved and the entire process of collapse, formation of an MCO, and its further evolution are expected to be unitary.

In this paper, we assume that a quantum theory of gravity exists where singularities are resolved and the evolutions are unitary. By assumption, therefore, an MCO will have no singularity. But it may or may not have a horizon. It will evolve further by emitting Hawking radiation if there is a horizon, or it will evolve further as a standard quantum system with large number of interacting degrees of freedom if there is no horizon.

Let the space be \( d \) dimensional, and the spacetime \( d + 1 \) dimensional, with \( d \) taken to be \( \geq 3 \). Let \( l_f \) and \( m_f = l_f^{-1} \) be the fundamental length and mass scales of the quantum theory of gravity under consideration. In string theory, \( l_f \) is the string length scale. In other theories, \( l_f \) is the Planck length. Also, we assume that the mass \( M \) of the collapsing star is \( \gg m_f \) and that any dimensionless parameters which may be present in the theory, for example string coupling constant \( g_s \) in string theory, take generic values of \( \mathcal{O}(1) \) independently of \( M \).

Quantum gravity effects may be expected to play a role when the constituents of the collapsing massive star reach a density of \( \mathcal{O}(m_f^{d+1}) \) or even earlier, see below. However, these effects are not known in detail. Therefore, as seems physically reasonable, we assume here that the singularities in the collapse of a massive star are resolved due to quantum gravity effects by transforming the star’s constituents into fundamental units, \(^1\) and that

\(^1\)The nature of fundamental units depends on the specific quantum theory of gravity. For example, these units are likely to be highly excited interacting strings, the ‘fuzz’ or, equivalently, the Kaluza – Klein monopoles of the fuzzball picture, or the \( D0 \) branes of the matrix description in string theory; or, loops in the loop quantum gravity; or, spins in the spin network theory [16, 17, 18].
the size of each of these units is \( \gtrsim l_f \). Since the evolutions are assumed to be unitary, it then follows that the information about the star, its collapse, formation of MCO, and its further evolution may not be lost but must all be encoded among these units.

The region of singularity resolution may be visualised as a cloud of fundamental units and, for the sake of convenience here, will be referred to as ‘singularity cloud’. As follows from the above assumptions, its size must depend on mass \( M \) of the collapsing star and must be parametrically much larger than \( l_f \). For the purposes of illustration, we present several plausible scenario for the number of fundamental units and the interactions between them, thereby also estimating the size of the singularity cloud. We will then study its implications in general.

**A. Size of the singularity cloud: a few scenario**

Let \( N \) be the number of fundamental units in the singularity cloud formed when a star of mass \( M \) collapses and the singularities are resolved. This singularity cloud may be taken to have mass \( \sim M \) and an entropy \( S_{cld} \). Note that, in general relativity theory, an MCO is believed to be a black hole with an entropy given by \( S_{bh} \sim (l_f M)^{\frac{d-1}{d-2}} \). On the other hand, the entropy \( S_* \) of the star just before the collapse may be bounded as in [19] by \( S_* \sim (l_f M)^{\frac{d(d-1)}{d+1(d-2)}} \). Thus, it is conceivable that the entropy of the resulting singularity cloud in the MCO is given by \( S_{cld} \simeq S_* \) or, equally conceivably, by \( S_{cld} \simeq S_{bh} \).

Consider the dependence of the number of fundamental units \( N \) on the mass \( M \). With the values of dimensionless parameters, if any, all generically set to be of \( \mathcal{O}(1) \), dimensional analysis implies that the dependence of \( N \) on \( M \) must be of the form

\[
N \sim (l_f M)^\nu.
\]

At least three scenarios now suggest themselves, leading to three different values of the exponent \( \nu \).

1. The mass of each unit equals \( m_f \) and the total mass \( M \) equals the

\[\text{There may be other higher entropic possibilities for } S_{cld}, \text{ as in [12, 20] for example, but they are not necessary for our purposes here.}\]
sum of masses of individual units. Then

\[ N = N_1 \sim (l_f M)^{\nu_1} , \quad \nu_1 = 1 . \]  

(2) In this scenario, each unit must have further internal structure in order to account for the entropy. The number of such internal degrees of freedom \( n_{\text{int}} \) for each unit must be of the order of \( n_{\text{int}} \sim \frac{S_{\text{cld}}}{N_1} \). Thus, if \( S_{\text{cld}} = S_* \) then \( n_{\text{int}} \sim (l_f M)^{\frac{2}{(d+1)(d-2)}} \) and if \( S_{\text{cld}} = S_{\text{bh}} \) then \( n_{\text{int}} \sim (l_f M)^{\frac{1}{d-2}} \).

(2) The \( N \) units account for the entropy \( S_* \) of the star just before collapse. Then

\[ N = N_2 \sim (l_f M)^{\nu_2} , \quad \nu_2 = \frac{d(d-1)}{(d+1)(d-2)} . \]  

(3) In this scenario, the average mass \( m_{\text{av}} \) of each unit is less than \( m_f \) and must be of the order of \( m_{\text{av}} \sim (l_f M)^{-\frac{2}{(d+1)(d-2)}} m_f \).

(3) The \( N \) units account for the entropy \( S_{\text{bh}} \) of the black hole that would have formed in general relativity theory. Then

\[ N = N_3 \sim (l_f M)^{\nu_3} , \quad \nu_3 = \frac{d-1}{d-2} . \]  

(4) In this scenario, the average mass \( m_{\text{av}} \) of each unit is less than \( m_f \) and must be of the order of \( m_{\text{av}} \sim (l_f M)^{-\frac{1}{d-2}} m_f \).

Consider now the size of the singularity cloud. This size, denoted as \( L_{\text{cld}} \), depends on the interactions between the \( N \) fundamental units. The details of these interactions which arise due to quantum gravity effects are not known. But three generic physically reasonable possibilities may be considered: The interactions may be such that the \( N \) units may (a) clump together as densely as possible; or (b) move independently of each other, effectively executing \( N \) units of random walks; or (c) avoid each other, effectively executing \( N \) units of self avoiding random walks [21]. These interactions may be thought of as short range and attractive, neutral, or repulsive. Given that the size of each unit is \( \gtrsim l_f \), it follows that the size of the singularity cloud consisting of \( N \) units is given by

\[ L_{\text{cld}} \gtrsim N^\sigma l_f \]  

(5)
where the exponent $\sigma$ for the above possible interactions are given by [22]

$$\sigma_a = \frac{1}{d}, \quad \sigma_b = \frac{1}{2}, \quad \sigma_c = \max \left\{ \frac{3}{d+2}, \frac{1}{2} \right\}$$

with $d$ being the number of spatial dimensions, taken to be $\geq 3$ in this paper. Thus, $\sigma_c = \frac{3}{5}$ for $d = 3$ and $\sigma_c = \sigma_b = \frac{1}{2}$ for $d \geq 4$. Using equations (1) and (5), the size of the singularity cloud $L_{cld}$ is then given in terms of its mass $M$ by

$$L_{cld} \gtrsim (l_f M)^\alpha l_f, \quad \alpha = \nu \sigma.$$  \hfill (7)

For the various scenario for $N$ dependence on $M$ and $L_{cld}$ dependence on $N$ considered here, the values of the exponents $\alpha_{ix} = \nu_i \sigma_x$ follow easily where $i = (1, 2, 3)$ and $x = (a, b, c)$ label the scenario. These values are listed in Table 1. Note that $\sigma_c = \sigma_b$ and, hence, $\alpha_{ic} = \alpha_{ib}$ for $d \geq 4$. Therefore, we have listed $\alpha_{ic}$ for $d = 3$ only.

| Values of $\alpha_{ix} = \nu_i \sigma_x$ | a          | b          | c          |
|----------------------------------------|------------|------------|------------|
|                                       | $\frac{1}{d}$ | $\frac{1}{2}$ | $\frac{3}{5}$ |
| 1                                     | $\frac{d-1}{(d+1)(d-2)}$ | $\frac{d(d-1)}{2(d+1)(d-2)}$ | $\frac{9}{10}$ |
| 2                                     | $\frac{d-1}{d(d-2)}$ | $\frac{d-1}{2(d-2)}$ | $\frac{6}{5}$ |

Table 1: The values of the exponents $\alpha_{ix} = \nu_i \sigma_x$ where $i = (1, 2, 3)$ and $x = (a, b, c)$. $\sigma_c = \sigma_b$ and, hence, $\alpha_{ic} = \alpha_{ib}$ for $d \geq 4$. Therefore, in the third column, $\alpha_{ic}$ are listed for $d = 3$ only.
III. Implications of $\alpha < \alpha_h$ and $\alpha > \alpha_h$

Note that, in general relativity theory, the singularity is taken to be of Planckian size, and the constituents of the collapsing massive star are taken to have disappeared into this singularity. One may then say that the size of the singularity cloud is $\mathcal{O}(l_f)$ and, hence, $\alpha = 0$ in this theory. In a quantum theory of gravity, the singularities are assumed to be resolved by transforming the constituents of the collapsing massive star into fundamental units. Therefore, the size of the singularity cloud must depend on its mass and must be parametrically much larger than $l_f$. Hence, the value of the exponent $\alpha$ in equation (7) must be non zero and positive.

Consider now the implications of the values of $\alpha$ being non zero and positive. In general relativity theory, the horizon size $r_h$ of a black hole of mass $M$ is given by

$$r_h \sim (l_f M)^{\alpha_h} l_f, \quad \alpha_h = \frac{1}{d-2}.$$  \hspace{1cm} (8)

Comparing equations (7) and (8), it follows that if $\alpha < \alpha_h$ then the size of the singularity cloud is smaller than the horizon size of the black hole. This case can, therefore, be taken to mean that the MCO formed in the collapse of a massive star has no singularity but has a horizon. Consequently, it evolves further by emitting Hawking radiation. Also, because of the horizon, nothing from the singularity cloud inside the MCO can escape to the outside. The information about the star, its collapse, formation of MCO, and its further evolution is encoded among the fundamental units in the singularity cloud; and, this information is not accessible to an outside observer.

The horizon shrinks due to the emission of Hawking radiation and, eventually, becomes smaller than the singularity cloud in size. This means that the MCO has no horizon now and consists entirely of the singularity cloud. Consequently, it then evolves further as a standard quantum system with large number of interacting degrees of freedom. Objects from the cloud can now escape to the outside and, hence, the information encoded in the singularity cloud becomes accessible to an outside observer. See [2, 7, 8, 14] for a similar scenario.

If $\alpha > \alpha_h$ then it follows that the size of the singularity cloud is larger than
the horizon size of the black hole. Taken literally, this gives, for example,

$$L_{cld} \sim (l_f M)^{\alpha - \alpha_h} r_h \sim 10^{7.6} \text{ (3 km)}$$

for a singularity cloud of one solar mass in four dimensional spacetime if $\alpha = \frac{6}{5}$. The size of the MCO is also likely to be of this order. But this is not the case for a black hole candidate of one solar mass in our four dimensional universe. (Its expected size is less than 10 km.)

In the collapse of a star in general relativity theory where a black hole forms, its horizon size is believed to grow from zero and finally reach $r_h$ at the end of collapse. With the above assumptions about the singularity resolution and about the value of $\alpha$ being $> \alpha_h$, what may likely happen in such a collapse in a quantum theory of gravity is that the singularity cloud will also grow from zero size, but always remaining larger than the corresponding horizon only by a factor of order unity, and finally reach a size larger than the predicted horizon size by a similar factor of order unity. ³ Clearly, a detailed theory is needed to predict dynamically how a collapse proceeds, how the fundamental units are produced and generate the singularity cloud, and finally to predict the size of the resulting MCO.

Throughout in this paper, therefore, we take the $\alpha > \alpha_h$ case to mean only that $L_{cld} > r_h$ and that the MCO formed in the collapse of a massive star has no singularity and also has no horizon, and consists entirely of the singularity cloud. Consequently, it evolves further as a standard quantum system with large number of interacting degrees of freedom. Objects from the cloud can escape to the outside and, hence, the information about the star, its collapse, formation of MCO, and its further evolution encoded among the fundamental units in the singularity cloud remains accessible to an outside observer at all times.

If $\alpha = \alpha_h$ then $L_{cld} \sim r_h$. The exact coefficient depends on the details of the quantum theory of gravity. Hence we will not consider this case and, instead, take that the $L_{cld} < r_h$ case is covered by $\alpha < \alpha_h$, and that the $L_{cld} > r_h$ case is covered by $\alpha > \alpha_h$. See also footnote 3.

Returning to the values of $\alpha$ listed in Table 1 for various scenario, note that $\nu_1 < \nu_2 < \nu_3$ and, hence, $\alpha_{1x} < \alpha_{2x} < \alpha_{3x}$ for $x = (a, b, c)$. Also,

³This may be the case if, for example, $\alpha = \alpha_h + \epsilon$ where $\epsilon$ is a small positive number. In the limit $\epsilon \to 0_+$, the size of the singularity cloud may be given by $L_{cld} \sim (l_f M)^{\alpha_h} \ln(l_f M) l_f \sim r_h \ln(l_f M)$.  

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\( \alpha_h = \frac{1}{d-2} \). Hence if \( x = a \), namely if the interactions are such that the \( N \) fundamental units clump together as densely as possible, then

\[
\alpha_{1a} < \alpha_{2a} < \alpha_{3a} < \alpha_h
\]

(10)

for \( d \geq 3 \). If \( x = b \), namely if the interactions are such that the \( N \) fundamental units move independently of each other and effectively execute \( N \) units of random walks, then

\[
\alpha_{1b} < \alpha_{2b} < \alpha_{3b} = \alpha_h
\]

(11)

for \( d = 3 \) and

\[
\alpha_h \leq \alpha_{1b} < \alpha_{2b} < \alpha_{3b}
\]

(12)

for \( d \geq 4 \). If \( x = c \), namely if the interactions are such that the \( N \) fundamental units avoid each other and effectively execute \( N \) units of self avoiding random walks, then

\[
\alpha_{1c} < \alpha_{2c} < \alpha_h < \alpha_{3c}
\]

(13)

for \( d = 3 \). For \( d \geq 4 \), we have \( \alpha_{ic} = \alpha_{ib} \) and equation (12) applies.

Note, in particular, that \( \alpha_{ia} < \alpha_h \) for all \( i \) and that \( \alpha_{3c} > \alpha_h \). Thus, if we assume that the \( N \) fundamental units clump together as densely as possible \((x = a)\) then we have \( \alpha < \alpha_h \). In these scenario, therefore, the collapse of a massive star is likely to result in the formation of an MCO which has no singularity but has a horizon. If we assume that the \( N \) fundamental units account for the entropy of the black hole that would have formed in general relativity theory \((i = 3)\) and that these \( N \) units execute self avoiding random walks \((x = c)\) then we have \( \alpha > \alpha_h \). In these scenario, therefore, the collapse of a massive star is likely to result in the formation of an MCO which has no singularity and also has no horizon, see the comments below equation (9).

**IV. Consequence of** \( 0 < \alpha < \alpha_h \): **Loss of predictability**

Among the values of \( \alpha \) listed in Table 1, \( \alpha_{1a} \) is the smallest. This corresponds to the most conservative assumptions, namely that the mass of each fundamental unit is \( m_f \), which leads to the smallest number \( N \sim l_f M \), and that these \( N \) units clump together as densely as possible, which leads to the
smallest size $L_{\text{cld}} \sim (l_f M)^{\frac{\alpha}{2}} l_f$. This size has also been obtained by others. See [1, 2, 14] for example, and the references therein.

We now assume only that $\alpha$ is strictly positive and study what happens if $\alpha < \alpha_h$, see [11] for an earlier report. The collapse of a massive star in this case is likely to result in the formation of an MCO which has no singularity but has a horizon. Let $M_{\text{init}}$ be its mass at the time of formation. Then the size of the singularity cloud $\sim (l_f M_{\text{init}})^{\alpha} l_f$ and, since $\alpha > 0$, it is parametrically much larger than $l_f$. The singularity cloud consists of $N_{\text{init}}$ number of fundamental units, among which the information about the star and its collapse is encoded.

Consider the evolution of this MCO after formation. It may accrete more mass which will increase its mass and its horizon size. The accreted mass will also increase the size of the singularity cloud and the number of fundamental units in it.

Since the MCO has a horizon, it evolves further by emitting Hawking radiation: a pair of photons is created at the horizon; one of them, the out-photon, goes outside the horizon and the other one, the in-photon, falls inside; the in-photon has negative energy and, hence, it reduces the mass of the MCO and shrinks its horizon size.

In the present case, where the singularities are assumed to be resolved, the in-photons will fall into the singularity cloud. They carry negative energy, so the nett mass of the singularity cloud should decrease. Interacting with the fundamental units in the singularity cloud, the in-photons are likely to generate some ‘decay products’ and form a kind of ‘plasma’ in equilibrium.\(^4\)

In particular, the in-photons must not decrease the number of fundamental units by, for example, disappearing completely together with some fundamental units leaving nothing behind. On the contrary, the in-photons must increase the number of fundamental units and, thereby, increase the size of the singularity cloud. This is because the fundamental units encode information about the star, its collapse, formation of MCO, and its further evolution. If the in-photons can decrease the number $N$ of fundamental units by disappearing completely together with some of them then, in principle, $N$

\(^4\)This is analogous to negative charges falling into a collection of positive charges and producing decay products, all confined in a region. The negative charges annihilate some positive charges and produce photons which are also confined in the region. These photons, in turn, produce pairs of negative and positive charges. In equilibrium then, there will be a plasma of negative and positive charges together with photons, the decay products.
can decrease by arbitrary amounts if, for example, Hawking radiation lasts for arbitrarily long time with accretions suitably compensating the resulting mass loss. In such cases, there will not be sufficient number of fundamental units available in the singularity cloud to encode all the information about the star, its collapse, formation of MCO, and its further evolution through Hawking radiation and more accretions. This will then contradict the assumption of unitary evolution since information can not be lost in such an evolution.

It therefore follows that the in-photons of the Hawking radiation must increase the number of fundamental units in the singularity cloud and, thereby, increase its size.  

Now, imagine observing today an MCO of mass $M_{\text{today}}$. Thinking of it as a black hole or, more precisely, as a black hole candidate, its horizon size is given by

$$r_{h\ (\text{today})} \sim (l_f M_{\text{today}})^{\frac{1}{d-2}} l_f.$$  

With the singularities assumed to be resolved, there is a singularity cloud inside this black hole candidate. Its size $L_{\text{cld\ (today)}}$, however, depends on (i) the initial mass $M_{\text{init}}$ of the collapsing star, (ii) accreted mass $M_{\text{accrn}}$ till today, and (iii) the amount of Hawking radiation emitted till today. From the above discussions, it follows that the size of the singularity cloud obeys the inequality

$$L_{\text{cld\ (today)}} > (l_f (M_{\text{init}} + M_{\text{accrn}}))^\alpha l_f,$$  

and need not have any relation to the observed mass $M_{\text{today}}$ or to the horizon size $r_{h\ (\text{today})}$. This means that the future evolution of the black hole observed today is uncertain. We cannot predict when the singularity cloud will become larger than the horizon in size, after which objects from it can escape to the outside and, hence, information about the original star, its collapse, and its further evolution encoded among the fundamental units in the singularity cloud will become accessible to an outside observer. Thus there is a loss of predictability. See Appendix A for a brief explanation regarding the meaning of the word predictability as used here.

This loss of predictability is not there in general relativity theory. This is because, in this theory, the size of the singularity is always taken to be

\footnote{A similar result on the increase in size is also arrived at in [7, 8, 14] where the surface of the region of singularity resolution is modelled as an inner horizon.}
\( \mathcal{O}(l_f) \), equivalently \( \alpha = 0 \), and it has no dependence on \( M_{\text{init}} \), or on \( M_{\text{accrn}} \), or on the amount of Hawking radiation emitted. It therefore seems that the resolution of singularity has led us to a worse predicament!

This loss of predictability is, again, not there if \( \alpha > \alpha_h \) because it then follows that the size of the singularity cloud is larger than the horizon size of the black hole. This means that the MCO has no horizon and that all the information about the star, its collapse, formation of MCO, and its further evolution remains accessible to an outside observer at all times.

Thus, if \( 0 < \alpha < \alpha_h \) then there is a loss of predictability described above. The resolution of singularity then seems to lead to a worse predicament than in general relativity theory. The singularity resolution is expected in a quantum theory of gravity, but not the loss of predictability. Therefore, if we further assume that a quantum theory of gravity must not lead to the loss of predictability then it logically follows that the physics of singularity resolution must be such that \( \alpha > \alpha_h \). As explained earlier below equation (9), this inequality is to be taken to mean only that the size of the singularity cloud is larger than the horizon size of the black hole.

\textbf{V. Summary and Conclusion}

In summary, we considered the collapse of a massive star forming an MCO in a quantum theory of gravity. We followed a qualitative, physically motivated approach since a suitable quantitative one is not available. We assumed the following.

- A quantum theory of gravity exists where the singularities are resolved and the evolutions are unitary.
- The mass \( M \) of the collapsing star is \( \gg m_f \). Dimensionless parameters, if any, all take generic values of \( \mathcal{O}(1) \) independently of \( M \).
- The singularities are resolved by transforming the star’s constituents into fundamental units, each of whose size is \( \gtrsim l_f \).
- A quantum theory of gravity must not lead to the loss of predictability.

The above assumptions are physically well motivated. The first three assumptions imply the following.
The information about the star, its collapse, formation of MCO, and its further evolution may not be lost but must all be encoded among the fundamental units.

The size of the singularity cloud must depend on mass $M$ of the collapsing star and must be parametrically much larger than $l_f$. Hence, $\alpha$ in equation (7) must be non zero and positive.

We then argued that the fourth assumption implies the following.

- $\alpha$ must be $> \alpha_h$ in the sense that the size of the singularity cloud is larger than the horizon size of the black hole.

Therefore, we conclude that in a quantum theory of gravity satisfying the assumptions listed above, the nature of an MCO formed in the collapse of a massive star must be as follows: It has no singularity and also has no horizon. It evolves further as a standard quantum system with large number of interacting degrees of freedom. And, the information about the star, its collapse, etcetera remains accessible to an outside observer at all times.

The arguments presented in this paper and, hence, the resulting conclusions are very general. They are applicable to any quantum theory of gravity satisfying the present assumptions. The flip side of this generality is that the present arguments do not give any hint of the mechanisms responsible, for example, for the singularity resolutions, or for the production of fundamental units, or for the size of the singularity cloud. The nature and the details of such mechanisms are strongly dependent on the theory considered. Clearly, a complete knowledge of the underlying theory is needed to understand them in detail. These details, in turn, are crucial for applications to the actual collapse of stars, and to predict the size and the properties of the resulting MCOs.

We close by mentioning two studies which are fruitful and are likely to provide more insights. In general relativity, the MCOs are believed to be black holes, the simplest ones being described by the Schwarzschild solutions. In Brans–Dicke theory, for example, there are more general Janis–Newman–Winicour–Wyman (JNWW) solutions [23]. They have singularities but no horizon at the Schwarzschild radius $r_{sch}$. Because of these singularities, they are not expected to describe an MCO. However, similar singular solutions generically arise in many contexts including, for example,
in higher dimensional contexts relevant for string/M theory [24]. In a quantum theory of gravity where the singularities are assumed to be resolved, one may expect such JNWW-type solutions to describe MCOs. With singularities resolved, these objects are likely to have no horizon and their sizes are likely to be $\gtrsim r_{sch}$. Indeed, such singularity-free, horizonless solutions, with Kaluza – Klein monopoles decorating a surface of $O(r_{sch})$ radius appear in the fuzz ball descriptions in string theory, see [13, 16] and references therein. It is important to study such JNWW-type solutions with their singularities resolved by quantum gravity effects.

Another aspect that can be fruitfully studied with the presently available techniques is the stability properties of the MCOs which have no singularities and also have no horizon. The arguments of this paper would imply that adding more mass to such an MCO should result in the increase of its size but no collapse should be possible; hence, there should be no instabilities towards a collapse irrespective of how massive an MCO is. See [25] for a preliminary study of this aspect in the context of string/M theory. It is not clear what properties of the constituents of an MCO such as their equations of state, their description as multi perfect fluids, interactions between them, et cetera will ensure the stability against collapse. Nevertheless, such a study seems possible even in the absence of a detailed quantum theory of gravity applicable to a collapsing star, and seems likely to provide useful insights.

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Appendix A. Regarding the meaning of loss of predictability

In this paper, we use the word predictability to mean only the ability to predict the future evolution of a system by making measurements now, and without requiring to know the past history of the system and its constituents. This actually seems to be more a statement about the nature of the theories one constructs and tests in physics. A detailed discussion of these aspects is beyond the scope of the present paper. Hence, instead, we give below two examples of systems which are predictable in the sense used here.

A box of gas is a predictable system. We can measure the values of a suitable set of observables and predict the future evolution of the gas in the box to a specified accuracy. The past histories of gas molecules are not needed.

Schwarzschild black hole in semiclassical gravity is predictable. If its mass is known then Hawking’s calculations tell us that it will emit radiation and evaporate to nothing in a time $\propto (mass)^{3/2}$ in four dimensional spacetime. The past history of the black hole from the time of its formation is not needed. Knowing its present mass alone is sufficient.

In the $0 < \alpha < \alpha_h$ case considered here, there is a loss of predictability in this sense. The past history of the MCO from the time of its formation is needed to predict its future evolution. Knowing its present mass alone is not sufficient. Hence, this system is not predictable in the sense used here.

It is perhaps debatable whether such a loss of predictability is undesirable. In this paper, we assume that such a loss is undesirable since the physical theories we are familiar with are of the predictable type.
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