Microscopic derivation of the Bekenstein–Hawking entropy formula for non-extremal black holes

Konstadinos Sfetsos

Theory Division, CERN
CH-1211 Geneva 23, Switzerland
sfetsos@mail.cern.ch

Kostas Skenderis

Instituut voor Theoretische Fysica, KU Leuven
Celestijnenlaan 200D, B-3001 Leuven, Belgium
kostas.skenderis@fys.kuleuven.ac.be

Abstract

We derive the Bekenstein–Hawking entropy formula for four- and five-dimensional non-supersymmetric black holes (which include the Schwarzschild ones) by counting microscopic states. This is achieved by first showing that these black holes are U-dual to the three-dimensional black hole of Banados–Teitelboim–Zanelli and then counting microscopic states of the latter following Carlip’s approach. Black holes higher than five-dimensional are also considered. We discuss the connection of our approach to the D-brane picture.
1 Introduction

Black holes are one of the most fascinating objects in general relativity. Their existence has profound implications for gravity in both the classical and the quantum regime. Black hole quantum mechanics provides a window into strong coupling quantum physics by raising a set of puzzles and questions that any consistent quantum theory of gravity should solve. The discovery that the black-hole laws are thermodynamical in nature \[1\] implies that there should be an underlying statistical description of them in terms of some microscopic states. In addition, black holes can evaporate \[2\], which leads to the “information loss paradox”. These questions, to a large extent, remained unanswered for more than twenty years.

String theory claims to provide a consistent theory of gravity. One would therefore expect that string theory provide answers to these questions. The strong coupling nature of black-hole physics, however, requires an understanding of non-perturbative string theory that was not available until recently. The situation has changed dramatically during the last few years. The duality symmetries have led to a new unified picture and provided a handle into strong coupling physics \[3\]. The discovery of D-branes \[4, 5\] has led to remarkable progress in the understanding of the physics of extremal black holes. In particular, it led to identification and counting of microstates for this subset of black holes \[4, 5\]. The result was in exact agreement with the Bekenstein–Hawking entropy formula. The idea behind these computations was to construct a D-brane configuration with the same quantum numbers as the corresponding black hole we are interested in. The counting of states is then performed at weak coupling, where the D-brane description is valid. The BPS property of these configurations implies that the number of states remains unchanged as the string coupling grows. One, therefore, can extrapolate these results to the black-hole phase. In this way states were counted for extremal 4d and 5d black holes. Near-extremal black holes were also studied \[6, 8\]. The absence of supersymmetry, however, makes these results less rigorous. For the same reason (i.e. absence of supersymmetry) the physically most interesting case, namely the case of non-extremal black holes, is untractable in this framework. Let us mention, however, that a natural extension of these ideas, as formulated in the correspondence principle of Polchinski and Horowitz \[9\] (for earlier ideas see \[10\]) does yield the correct dependence of the entropy on the mass and the charges, even if it does not provide the numerical coefficient. Recently, similar results for non-extremal black holes were obtained in \[11\] using the M(atrix) formulation \[12\] of M-theory.

Another important development in the understanding of the statistical origin of the black-hole entropy (that actually preceded the D-brane developments) was Carlip’s derivation \[13\] of the Bekenstein–Hawking entropy formula for the three-dimensional black hole of Banados–Teitelboim–Zanelli (BTZ) \[14\]. The latter solves Einstein’s equa-
tions in the presence of a negative cosmological constant and is, therefore, asymptotically anti-de Sitter. Soon after its discovery it was shown that the BTZ black hole is actually an exact solution of string theory \[13, 16\], namely that there is an exact conformal field theory (CFT) associated to it. Physics in three dimensions is significantly simpler than in higher dimensions. In particular, three-dimensional gravity can be recast as a Chern–Simons theory \[17, 18\]. If the space has a boundary then the Chern–Simons theory induces a WZW action in this boundary. The latter describes would-be degrees of freedom that become dynamical because certain gauge transformations become inadmissible due to boundary conditions. Carlip has shown that these degrees of freedom correctly account for the Bekenstein–Hawking entropy of the BTZ black hole.\[1]\] It is important to realize that this result is valid both at extremality and away from it. However, the method used seems very particular to three dimensions (see, however, \[21\]). Notice also that all D-brane results are for black holes of dimension higher than three. The main reason for this is that in constructing a solution out of D-branes one usually restricts oneself to at least three overall transverse directions, and three-dimensional space-time has two transverse directions. If the overall transverse directions are less than three, the harmonic functions appearing in the D-brane configuration are not bounded at infinity.

To summarize: the D-branes techniques can be used to derive the Bekenstein–Hawking entropy for 4\(d\) and 5\(d\) supersymmetric black holes, whereas Carlip’s approach is not restricted to supersymmetric black holes, but it seems to apply only to 3\(d\) ones. We shall show in this article that one can use the latter to study non-extremal 4\(d\) and 5\(d\) black holes and, in particular, we will derive the Bekenstein–Hawking entropy formula associated to them with the correct numerical coefficient. Our considerations also apply to higher-dimensional black holes, although we have no derivation of the Bekenstein–Hawking entropy formula in these cases.

Over the last few years a new unifying picture of all five string theories and eleven-dimensional supergravity has emerged \[3\]. A central rôle in these developments has been played by the various duality symmetries. It is now believed that there exist an underlying master theory, the M-theory, that has all string theories and eleven-dimensional supergravity \[22\] as special limits. The dualities symmetries can be viewed as some kind of gauge symmetry of this theory. Physical quantities should be “gauge-invariant”, i.e. U-duality-invariant. Choosing one configuration among all its U-duals to describe a physical system corresponds to choosing a particular “gauge”. As in usual gauge theories, some gauges are preferable for answering certain questions than others. We shall show below that the 4\(d\) and 5\(d\) black holes are U-dual to the BTZ black hole (for related

\[1\] The idea that only physical degrees of freedom defined in a “stretched” horizon may account for the black hole entropy has been advocated in \[13\]. In a string theory context it was put forward by A. Sen \[20\], in order to reconcile the Bekenstein–Hawking entropy for extremal electric black holes, with the entropy of elementary superstring excitations.
work, see [23]). One may, therefore, choose the “BTZ gauge” in order to answer certain physical questions. In particular, we shall address in detail the question of the statistical origin of the entropy.

The BTZ black hole (for $J \neq 0$) is non-singular. One may, therefore, argue that the singularities in the $4d$ and $5d$ black holes are “gauge” artefacts. In addition, the fact that the BTZ black hole is asymptotically anti-de Sitter and the $4d$ and $5d$ black holes are asymptotically flat implies that the cosmological constant is a “gauge-dependent” notion. Furthermore, the simplicity of the “BTZ gauge” may make tractable the study of the final state of black holes.

One may wonder at this point how is it possible to connect objects of different dimensionality using the U-duality group. Consider, for concreteness, type-II string theory on a torus. Then, the U-dual group is considered to be the (discretized) version of the global symmetry group of the various maximal supergravity theories obtained from eleven-dimensional supergravity by toroidal compactification in dimensions $d \leq 10$. Therefore, almost by definition, the dualities do not change the number of non-compact dimensions. For static backgrounds, however, one has, in addition to the isometries corresponding to toroidal directions, an extra time-like non-compact isometry. This leads to a larger group. Consider, for instance, the case of $d$ compact directions. The T-duality group is $O(d,d)$. Suppose for a moment that the time is compact with radius $R$. Then the symmetry group would be enlarged to $O(d+1,d+1)$. To see what happens in the decompactification we let $R$ become larger and larger while restricting the elements of $O(d+1,d+1)$ to the ones that do not mix the coordinates with finite radii with the time coordinate. In the limit $R \to \infty$ the time becomes non-compact and we are left with a subgroup of $O(d+1,d+1)$. The latter is basically a combination of diffeomorphisms of the time coordinate, which involve the compact coordinates and the $O(d,d)$ transformations of the compact coordinates themselves. In particular, this group contains elements that correspond to isometries that are space-like everywhere except at spatial infinity, where they become null. T-dualizing with respect to these isometries changes the asymptotic geometry of space-time [13, 24]. There seems to be a widespread belief that string theory admits only Ricci-flat compactifications. This is, however, not true. We shall exhibit below exact string solutions that correspond to compactifications on $S^2$ and $S^3$ times some torus. T-dualities along the above-mentioned isometries precisely bring us to these compactifications. These, at low energies, reduce to compactifications of $10d$ supergravity on $S^2$ and $S^3$ times some torus, and therefore connect Poincaré supergravities to anti-de Sitter (adS) supergravities. Compactifying eleven-dimensional supergravity on spheres instead of tori yields the latter. A famous example is the compactification of eleven-dimensional supergravity on $S^7$, which yields [25] $N = 8$ adS$_4$ gauged supergravity [26]. In other words, these transformations connect solutions of $10d$ (or of $11d$) supergravity that correspond to different compactifications. As such, they may connect solutions with
different number of non-compact dimensions. From the point of view of M-theory, one may argue that all compactifications of eleven-dimensional supergravity should be on an equal footing. This suggests that the symmetry group of M-theory is actually larger than what is usually assumed. To obtain the full symmetry group one should also consider the various gauged supergravities. The consistent picture that emerges from our discussion of black-hole entropy strongly supports this point of view. We shall, from now on in this article, use the term U-duality transformation to denote the transformation that results from a combination of the usual \((R \leftrightarrow 1/R)\) T-duality, of the S-duality of type-IIB string theory and of the extra transformations that we mentioned above. We shall also freely uplift 10\(d\) results to eleven dimensions.

We shall argue that certain branes, and intersections thereof, are U-dual to supersingleton representations of various anti-de Sitter groups. In this way we have a connection between our considerations and the usual D-brane picture. In particular the branes \(M2\), \(M5\) and \(D3\) are dual to the supersingleton representation of \(adS_4\), \(adS_7\), and \(adS_5\), respectively. A complete list is given in Table 1 (see section 4). All the configurations listed there (with the addition of a wave, in some cases) are dual to black holes in \(4 \leq d \leq 9\). Essentially, the duality transformations map the black hole into the near-horizon geometry (with some global identifications). In the present context, however, this is not an approximation.

The picture emerging from our study is that the microscopic degrees of freedom reside in the intersection region of the various branes, making up the black-hole configuration. This picture is in harmony with results existing in the literature. For \(4d\) and \(5d\) extremal black holes, described by a configuration of D-branes that has a one-dimensional intersection, the entropy can be obtained by treating the degrees of freedom as an ideal gas of bosons and fermions in a one-dimensional compact space. Similar results (but with only qualitative agreement) hold for near-extremal non-dilatonic black holes \([27]\). In that case as well, the microscopic description involves a \(p\)-dimensional theory, where \(p\) is the spatial dimension of the intersection region. Notice, however, that the intersection region is not a U-duality-invariant notion since the same black hole can result from different intersections. For instance, the \(5d\) black holes can be constructed by either the intersection of an M-theory membrane \((M2)\) with an M-theory five-brane \((M5)\) and a wave \((W)\) along the common direction, or from the intersection of three membranes. In the former case the intersection is one-dimensional, i.e. over a string, whereas in the latter it is zero-dimensional, i.e. over a point. Let us emphasize that only U-duality-invariant quantities of the original configuration may be studied in the U-dual formulation. The

---

2 We need not be in the weak string coupling limit for our considerations to be valid. In fact, we will always stay within the black-hole phase, where the string coupling is strong. Hence, by branes we mean the specific solutions of the low energy supergravity. In the case where the latter carry R–R charge, they are the long-distance description of D-branes.
entropy of the black hole is such a quantity and, therefore, can be computed in any dual configuration.

This article is organized as follows. In sections 2 and 3 we concentrate on the 4d and 5d black holes. In particular, in section 2 we show that 5d and 4d non-extremal black holes are U-dual to configurations that contain the BTZ black hole as the only non-compact part. In section 3 we present our microscopic derivation of the Bekenstein–Hawking entropy formula. We follow Carlip’s approach, putting some emphasis on the unitarity issue of the underlying SL(2, R) WZW model. In section 4 we discuss the duality between branes and supersingleton representations. In this way we provide a connection between our considerations and the D-brane picture. In section 5 we briefly discuss higher-dimensional black holes as well as intersections of branes (different from the ones discussed in section 2), which yield 4d and 5d black holes. We conclude in section 5. Appendix A contains the eleven-dimensional supergravity configurations that reduce, upon dimensional reduction along a compact direction, to the ten-dimensional solutions used in section 2. Finally, in appendix B we show that higher than five-dimensional black holes are not U-duals to configurations that contain the BTZ black hole.

2 U-duality between non-extremal and BTZ black holes

We will show in this section that ten-dimensional configurations, which upon dimensional reduction in an appropriate number of dimensions yield a 5d or a 4d black hole, can be mapped by a chain of dualities and a simple coordinate transformation into a configuration that has as the only non-compact part the BTZ black hole. In particular, the configuration that yields the 5d black hole will be mapped to the configuration BTZ × S³ × T⁴, and the one that yields the 4d black hole to BTZ × S² × T⁵. We will show that there is an exact CFT associated to each factor of the final configuration. For this to be true, it is crucial to carry along the gauge fields of the original configuration. After the dualities all the fields acquire their canonical values so that each factor is independently associated to a CFT. In this sense, our considerations also provide exact CFTs associated to 5d and 4d black holes.

The basic mechanism that allows one to map one black hole that is asymptotically flat into other that is asymptotically anti-de Sitter has been discussed in [24]. Here we shall refine this discussion by showing that what was there called shift transformation, is actually a property of the plane-wave solution. Consider the following non-extremal plane-wave solution in (D + 1) dimensions

\[
d s^2 = -K^{-1}(r)f(r)dt^2 + K(r)\left(dx_1 + (K'^{-1}(r) - 1 + \tanh\alpha)dt\right)^2
\]
\[ + f^{-1}(r) dr^2 + r^2 d\Omega_{D-2}^2, \]  

where

\[ K(r) = 1 + \frac{\mu^{D-3} \sinh^2 \alpha}{r^{D-3}}, \quad K^{-1}(r) = 1 - \frac{\mu^{D-3} \sinh \alpha \cosh \alpha}{r^{D-3}} K^{-1}, \]

\[ f(r) = 1 - \frac{\mu^{D-3}}{r^{D-3}}, \quad r^2 = x_2^2 + \cdots + x_D^2. \]  

(1)

The coordinate \( x_1 \) is assumed to be periodic, with radius \( R_1 \), so that \((t, x_1)\) has the topology of a cylinder. The constant of the off-diagonal part is chosen such that this term vanishes at \( r = \mu \). One may T-dualize in the \( x_1 \)-direction to obtain a solution that describes a non-extremal string. In this case, the off-diagonal part of the metric becomes the antisymmetric tensor of the new solution. Our choice of the constant in the off-diagonal part of (1) ensures that the latter is regular at the horizon [28]. We shall call the \( r = \mu \) surface horizon since, as we shall shortly see, the plane wave solution when combined with certain other branes yields 5\( d \) and 4\( d \) black holes solutions with an outer horizon at \( r = \mu \). The area of the latter for the solution (1) is equal to

\[ A = 2\pi R_1 \cosh \alpha \mu^{D-2} \Omega_{D-2}, \]  

(3)

where \( \Omega_{D-2} \) denotes the volume of the unit \((D-2)\)-sphere.

Let us perform the following \( SL(2, \mathbb{R}) \) coordinate transformation that preserves the cylinder:

\[
\begin{pmatrix}
  t \\
  x_1
\end{pmatrix} =
\begin{pmatrix}
  a & b \\
  0 & c
\end{pmatrix}
\begin{pmatrix}
  t' \\
  x_1'
\end{pmatrix}.
\]

(4)

Requiring that the transformed solution still be of the form (1) and have vanishing off-diagonal part at \( r = \mu \) like (1), and that the asymptotics be different, uniquely fixes \( a, b, c \) to

\[ a = \cosh \alpha, \quad b = -\exp(-\alpha), \quad c = \frac{1}{a}. \]  

(5)

One obtains\( ^3 \) (with the primes in \( t' \) and \( x_1' \) dropped)

\[ ds^2 = -\bar{K}^{-1}(r)f(r)dt^2 + \bar{K}(r)\left(dx_1 + (\bar{K}^{-1}(r) - 1)dt\right)^2 + f^{-1}(r)dr^2 + r^2 d\Omega_{D-2}^2, \]  

(6)

where now

\[ \bar{K}(r) = \frac{\mu^{D-3}}{r^{D-3}}. \]  

(7)

\( ^3 \) In the extremal limit the transformation (1) and the metric (1) appear to be singular. In this case, we have to rescale the coordinates \( t' \) and \( x_1' \) as \( t' \rightarrow t'\mu^{D-3} \) and \( x_1' \rightarrow x_1'/\mu^{D-3} \). After taking the limit \( \alpha \rightarrow \infty \) in such a way that the charge \( Q = \mu^{D-3} \sinh^2 \alpha \) is kept fixed, we obtain a well-defined transformation (1) with \( b = -1/(2a) \), \( c = 1/a \) and \( a \) arbitrary. The metric (1) has also a well-defined limit.
Notice that the radius of $x_1$ is now equal to $R_1 \cosh \alpha$. We shall call the transformation (4) the shift transformation.\footnote{The definition of the shift transformation is not the same as the one employed in \cite{24}. There the shift transformation acted on the fundamental string solution and it was a combination of the shift transformation as defined in this article and T-dualities.} One easily checks that the area of the horizon (i.e. of the surface $r = \mu$) is still equal to (3). We therefore conclude that the shift transformation does not change the area of the horizon.

\subsection{5d black holes}

Consider the solution of type-IIA supergravity that describes a non-extremal intersection of a solitonic five-brane ($NS5$) a fundamental string ($F1$) and wave ($W$) along one of the common directions. This configuration can be obtained from a solution of 11d supergravity as described in appendix A. Let us wrap the $NS5$ in ($x_1, x_2, x_3, x_4, x_5$), the fundamental string ($F1$) in $x_1$ and put a wave along $x_1$. The coordinates $x_i, i = 1, \ldots, 5,$ are assumed to be periodic, each with radius $R_i$. The metric, the dilaton and the antisymmetric tensor are given by

$$ds_{10}^2 = H_f^{-1} \left( -K^{-1} f dt^2 + K (dx_1 - (K'-1) dt)^2 \right) + dx_2^2 + \cdots + dx_5^2 + H_{s5}(f^{-1} dr^2 + r^2 d\Omega_3^2),$$

where the various harmonic function are given by (49), with the identifications $H_T \rightarrow H_f$ and $H_F \rightarrow H_{s5}$. One may also express the “magnetic” $NS5$ brane in terms of the dual “electric” field,

$$B_{012345} = \coth H_{s5}(H_{s5}^{-1} - 1) + \tanh \alpha_{s5}.$$

The constant parts of the $B_{01}$ and $B_{012345}$ were chosen (using a constant gauge transformation) such that the antisymmetric tensors are regular at the horizon.

Dimensionally reducing in $x_1, x_2, x_3, x_4, x_5$, one gets a 5d non-extremal black hole, whose metric in the Einstein frame is given by

$$ds_{E,5}^2 = -\lambda^{-2/3} f dt^2 + \lambda^{1/3} (f^{-1} dr^2 + r^2 d\Omega_3^2),$$

\footnote{All configurations studied in this article are built according to the rules of \cite{29}. In the extremal limit they are supersymmetric, and they are constructed according to the intersection rules based on the ‘no-force’ condition \cite{30}.}
where
\[ \lambda = H_{s5} H_f K = \left(1 + \frac{Q_{s5}}{r^2}\right) \left(1 + \frac{Q_f}{r^2}\right) \left(1 + \frac{Q_K}{r^2}\right). \]  
(12)

This black hole is charged with respect to the Kaluza-Klein gauge fields originating from the antisymmetric tensor fields and the metric. When all charges are set equal to zero one obtains the 5d Schwarzschild black hole. The metric (11) has an outer horizon at \( r = \mu \) and an inner horizon at \( r = 0 \). The Bekenstein–Hawking entropy may easily be calculated to be

\[ S = \frac{A_5}{4G^{(5)}_N} = \frac{1}{4} \frac{(2\pi)^5 R_1 R_2 R_3 R_4 R_5}{G^{(10)}_N} \mu^3 \Omega_3 \cosh \alpha_{s5} \cosh \alpha_f \cosh \alpha_K, \]  
(13)

where \( \Omega_3 \) is the volume of the unit 3-sphere and \( G^{(5)}_N \) and \( G^{(10)}_N \) are Newton’s constant in five and ten dimensions, respectively.

We shall now show that this black hole is U-dual to the configuration of the non-extremal BTZ black hole times a 3-sphere. This will be achieved by using the shift transformation and a series of dualities. Since neither dualities\(^6\) nor the shift transformation change the area of the horizon, the Bekenstein–Hawking entropy of the resulting solution is the same as the one of the black hole we started from. The idea is to dualize the fundamental string \( F1 \) and the \( NS5 \) into a wave, apply the shift transformation (4) and then return to the original configuration. One sequence of dualities that achieves that is, first, to perform \( T_1 S \) (\( T_i \) denotes T-duality along the \( x_i \)-direction,\(^7\) and \( S \) is the S-duality transformation of the type-IIB string theory). Then, the \( NS5 \) becomes a \( D5 \)-brane, the wave a \( D1 \)-brane and the fundamental string \( F1 \) a wave. So, we can use the shift transformation (4) in \((t, x_1)\) to change the harmonic function \( H_f \), as in (11). In addition, the radius of \( x_1 \) is now equal to \( R_1 \cosh \alpha_f \). Next, we perform \( T_{1234} ST_1 \). This yields a wave in \( x_5 \), a \( D2 \) in \((x_1, x_5)\) and a \( D4 \) in \((x_2, x_3, x_4, x_5)\). Now, we use the shift transformation (4) in \((t, x_5)\) to change the harmonic function \( H_{s5} \). The radius of \( x_5 \) also changes to \( R_5 \cosh \alpha_{s5} \). Finally, we return to the original configuration with the inverse dualities (no shift transformations). The final result is given by the metric in (8), but with

\[ H_f = \frac{\mu^2}{r^2}, \quad H_{s5} = \frac{\mu^2}{r^2}, \]  
(14)

\(^6\) For T-dualities, this has been shown in [28]. S-duality leaves the Einstein metric invariant and, therefore, it does not change the area either.

\(^7\) T-duality interchanges the type-IIA and type-IIB string theories. When restricted to the fields in the NS–NS sector, the T-duality rules are the same as those of Buscher [31]. For the R–R background fields the corresponding rules can be found in [32].

\(^8\) It is possible to obtain (14) and (15) below in a single step, by combining all preceding transformations into one element of the U-duality group. The same comment applies for the similar considerations in subsection 2.2. Notice that the coordinate transformation (4) and the subsequent \( R \leftrightarrow 1/R \) duality, combine into a single T-duality transformation along an isometry which is space-like everywhere, but at spatial infinity, where it becomes null.
and, in addition,
\[ e^{-2\phi} = 1 \, , \quad B_{01} = H_f^{-1} - 1 \, , \]
\[ H_{ijk} = \frac{1}{2} \varepsilon_{ijkl} \partial_l (H_{s5} - 1) \, , \quad i, j, k, l = 6, \ldots, 9 \, . \] (15)

Notice that the parameters \( \alpha_f \) and \( \alpha_{s5} \) associated to the charges of the original fundamental string \( F1 \) and the solitonic five-brane \( NS5 \) appear only in the compactification radii of \( x_1 \) and \( x_5 \) respectively, and not on the background fields themselves.\(^9\)

Dimensionally reducing along \( x_2, x_3, x_4, x_5 \) we find
\[ ds^2_{E,6} = ds^2_{BTZ} + l^2 d\Omega^2_3 \, , \] (16)

where
\[
d s^2_{BTZ} = - \frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{l^2 \rho^2} dt^2 + \rho^2 (d\varphi - \rho_+ \rho_- dt)^2 + \frac{l^2 \rho^2}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)} dp^2 \] (17)
is the metric of the non-extremal BTZ black hole in a space with cosmological constant \( \Lambda = -1/l^2 \), with inner horizon at \( \rho = \rho_- \) and outer horizon at \( \rho = \rho_+ \). The mass and the angular momentum the BTZ black hole are equal to \( M = (\rho_+^2 + \rho_-^2)/l^2 \) and \( J = 2\rho_+ \rho_- / l \).

In terms of the original variables:
\[ l = \mu \, , \quad \varphi = \frac{x_1}{l} \, , \quad \rho^2 = r^2 + l^2 \sinh^2 \alpha_K \, , \]
\[ \rho_+^2 = l^2 \cosh^2 \alpha_K \, , \quad \rho_-^2 = l^2 \sinh^2 \alpha_K \, . \] (18)

In addition,
\[ \phi = 0 \, , \quad B_{t\varphi} = (\rho^2 - \rho_+^2)/l \, , \quad H = l^2 \epsilon_3 \, , \] (19)

where \( \epsilon_3 \) is the volume form element of the unit 3-sphere. Therefore, the metric (16) describes a space that is a product of a 3-sphere of radius \( l \) and of a non-extremal BTZ black hole. Notice that the BTZ and the sphere part are completely decoupled. Also all fields have their canonical value, so that both are separately exact classical solutions of string theory, i.e. there is an exact CFT associated to each of them. For the BTZ black hole the CFT corresponds to an orbifold of the WZW model based on \( SL(2, \mathbb{R}) \) \([15, 16]\), whereas for \( S^3 \) and the associated antisymmetric tensor with field strength \( H \), given in (19), the appropriate CFT description is in terms of the \( SO(3) \) WZW model.

We can now calculate the entropy of the resulting black hole. The area of the horizon is equal to
\[ A_3 = 2\pi R_1 \cosh \alpha_f \mu \cosh \alpha_K \mu \, , \] (20)
whereas Newton’s constant is given by

\[
G_N^{(3)} = \frac{G_N^{(10)}}{(2\pi)^4 R_2 R_3 R_4 R_5 \cosh \alpha_{s5}) (\mu^3 \Omega_3)}.
\]

It follows that \( S = A_3 / (4G_N^{(3)}) \) equals (13), i.e. the Bekenstein–Hawking entropy of the final configuration is equal to the one of the original 5d black hole. Notice that the Newton constant in (21) contains the parameter \( \alpha_{s5} \), i.e. carries information on the charge of the original NS5 five-brane.

### 2.2 4d black holes

Consider the solution of type-IIA supergravity that describes a non-extremal intersection of a D2 brane in \((x_1, x_2)\), a D6 brane in \((x_1, x_2, x_3, x_4, x_5, x_6)\), a solitonic five-brane NS5 in \((x_1, x_3, x_4, x_5, x_6)\) with a wave along \(x_1\). The eleven-dimensional origin of this solution is described in appendix A. The coordinates \(x_i, i = 1, \ldots, 6\), are assumed to be periodic, each with radius \(R_i\). The metric, the dilaton and the antisymmetric tensors are given by

\[
d s_{10}^2 = (H_6 H_2)^{-1/2} \left(-K^{-1} f dt^2 + K(dx_1 + (K'^{-1} - 1)dt)^2\right)
+ H_{s5}(H_6 H_2)^{-1/2} dx_2^2 + H_6^{-1/2} H_2^{1/2} (dx_3^2 + dx_4^2 + dx_5^2 + dx_6^2)
+ H_{s5}(H_6 H_2)^{1/2} (f^{-1} dr^2 + r^2 d\Omega_2^2),
\]  

and

\[
e^{-2\phi} = H_{s5}^{-1} H_6^{3/2} H_2^{-1/2}, \quad H_{2ij} = \frac{1}{2} \epsilon_{ijk} \partial_k H_{s5}', \quad i, j, k = 7, 8, 9,
\]

\[
(dA)_{ij} = \frac{1}{2} \epsilon_{ijk} \partial_k H_6', \quad C_{012} = \coth \alpha_2 (H_2^{-1} - 1) + \tanh \alpha_2,
\]

where the various harmonic functions are given in (52) of appendix A (but renamed as \(H_{F1} \rightarrow H_{s5}, H_{F2} \rightarrow H_2\) and \(H_{F3} \rightarrow H_6\)).

Upon dimensional reduction in \(x_1, x_2, x_3, x_4, x_5, x_6\), one obtains a charged 4d non-extremal black hole\(^{10}\) with metric in the Einstein frame given by

\[
d s_{E,A}^2 = -\lambda^{-1/2} f dt^2 + \lambda^{1/2} (f^{-1} dr^2 + r^2 d\Omega_2^2),
\]

where

\[
\lambda = H_{s5} H_6 H_2 K = \left(1 + \frac{Q_{s5}}{r}\right) \left(1 + \frac{Q_6}{r}\right) \left(1 + \frac{Q_K}{r}\right) \left(1 + \frac{Q_2}{r}\right).
\]  

\(^{10}\)Extremal 4d black hole solutions embedded in eleven-dimensional supergravity where constructed in [33]. In particular, these authors showed that a configuration of three intersecting five-branes with a wave along a common string and another one of two membranes and two five-branes reduce, upon compactification to four dimensions, to the extremal limit of [24].
The antisymmetric tensor fields and the off-diagonal part of the metric give rise to gauge fields under which this solution is charged. The usual 4d Schwarzchild black hole is obtained by setting all charges equal to zero. The metric (24) has an outer horizon at $r = \mu$ and an inner horizon at $r = 0$. The Bekenstein–Hawking entropy may easily be calculated to be

$$S = \frac{A_4}{4G_N^{(4)}} = \frac{1}{4} \frac{(2\pi)^6 R_1 R_2 R_3 R_4 R_5 R_6}{G_N^{(10)}} \mu^2 \Omega_2 \cosh \alpha_5 \cosh \alpha_6 \cosh \alpha_2 \cosh \alpha_K$$

where $\Omega_2$ is the volume of the unit 2-sphere and $G_N^{(4)}$ is Newton’s constant in four dimensions.

In order to show that this black hole is dual to the BTZ one, we use the same strategy as before. We dualize the solution in such a way that each brane becomes a wave, then we apply the shift transformation, and we finally return to the original configuration with the inverse dualities. For instance, the chain of dualities $T_1 ST_{3456} ST_1$ converts the $NS_5$ into a wave. In order to convert $D2$ into a wave one may use the dualities (starting from the original configuration) $T_2 ST_1$. Finally, the $D6$ may be converted to $D2$ by $T_{3456}$. Then, one may use the same dualities as in the previous case. The combined effect of these dualities is to change the radius of $x_1$ to $R_1 \cosh \alpha_5$, the radius of $x_2$ to $R_2 \cosh \alpha_6 \cosh \alpha_2$, the harmonic functions to

$$H_{s5} = H_6 = H_2 = \frac{\mu}{r}$$

and the fields to

$$e^{-2\phi} = 1, \quad C_{012} = H_2^{-1} - 1, \quad H_{2ij} = \frac{1}{2} \epsilon_{ijk} \partial_k H_{s5}, \quad (dA)_{ij} = \frac{1}{2} \epsilon_{ijk} \partial_k H_6, \quad i,j,k = 7,8,9.$$  

Similarly to the case of subsection 2.1, the parameters $\alpha_2$, $\alpha_6$ and $\alpha_5$ associated with the charges of the original $D2$, $D6$ and $NS5$ respectively, appear only in the compactification radii of $x_1$ and $x_2$, but not in the background fields themselves.

After dimensional reduction in $x_2, x_3, x_4, x_5, x_6$, one gets

$$ds^2_{E,5} = ds^2_{BTZ} + \mu^2 d\Omega_2^2$$

where

$$l = 2\mu, \quad \varphi = \frac{x_1}{l}, \quad \rho^2 = 2lr + l^2 \sinh^2 \alpha_K$$

$$\rho_+^2 = l^2 \cosh^2 \alpha_K, \quad \rho_-^2 = l^2 \sinh^2 \alpha_K.$$  

In addition,

$$\phi = 0, \quad B_{t\varphi} = (\rho^2 - \rho_+^2)/l, \quad F = \mu \epsilon_2.$$  

11 In the extremal limit we have to perform a contraction similar to the one described in footnote 9.
where $F$ represents the $U(1)$ field strengths and $\epsilon_2$ is the volume form element of the unit 2-sphere. As in the case of the five-dimensional black hole we also see that the BTZ black hole and the 2-sphere decouple completely. We also note that the second term in (29), representing the 2-sphere with the associated gauge field we have mentioned, corresponds to the monopole CFT of [34]. Equivalently, it can also be viewed as a dimensionally reduced $SO(3)$ WZW model along one of the Euler angles parametrizing the $SO(3)$ group element.

One may calculate the entropy of the final configuration. The result is in agreement with (26). As in the five-dimensional case, the volume of the sphere as well as certain parameters associated with charges of the original configuration (22), enter via the three-dimensional Newton constant.

### 3 Counting microscopic states

In this section we briefly review Carlip’s derivation of the Bekenstein–Hawking entropy formula for the BTZ black hole. The basic idea is that only quantum states leaving on the horizon of the black hole are relevant in such computation, whereas those in the bulk are irrelevant. Since the horizon represents the end of the world for an outside observer, it is treated as a surface boundary. This is in principle applicable in any number of dimensions, and the problem is to be able to separate the boundary from the bulk degrees of freedom and subsequently to quantize them. This is a formidable task by itself in more than three space-time dimensions, and we know of no solution to date. However, in (2 + 1) dimensions the problem is trivially solved since there are no bulk degrees of freedom at all. Moreover, as we have seen in section 2 this is enough for our purposes, since we have mapped the problem of counting microscopic states for the 4$d$ and 5$d$ black holes into the corresponding problem for the 3$d$ BTZ black hole. The topological character of (2 + 1)-dimensional gravity is manifest in its Chern–Simons formulation [17, 18]. In the presence of a non-vanishing cosmological constant the action can be written as

$$S = S_{CS}(A) - S_{CS}(\tilde{A}),$$

where

$$S_{CS}(A) = \frac{k}{8\pi} \int_M \text{Tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$

#12 We only give the bosonic part. The full supersymmetric version has also a Chern–Simons form, but in superspace [17]. In principle, one should also keep the fermions in the derivation of the boundary action. The latter, however, at least in the limit of small cosmological constant, have subleading contribution to the entropy. Nevertheless, it will be useful to repeat the computation by including the fermions as well.

#13 Our normalizations are compatible with the representation $T_0 = i\sigma_3/2, T_1 = \sigma_1/2, T_2 = \sigma_2/2$ for the $SL(2, \mathbb{R})$ generators, and $\text{Tr}$ is the matrix trace.
represents the Chern–Simons actions for some manifold $M$ and similarly for $S_{CS}(\tilde{A})$. The gauge connections are in the Lie algebra of $SO(1,2)$ and are given in terms of the spin connection $\omega^a$ and triad $e^a$ 1-forms as

$$A^a = \omega^a - \frac{e^a}{l}, \quad \tilde{A}^a = \omega^a + \frac{e^a}{l},$$

(34)

where $a = 0, 1, 2$. We will denote $A^{\pm}_\mu = A^1_\mu \pm A^0_\mu$ and similarly for $\tilde{A}^{\pm}_\mu$. The constants $k$ and $l$ are related by

$$k = \frac{l}{4G_N^{(3)}}$$

It is well known that, if the manifold $M$ has no boundary, a Chern–Simons theory in (2+1) dimensions is a topological field theory. However, if there is a non-trivial boundary $\partial M$, then the variational problem of pure Chern–Simons is not well defined unless we specify the boundary conditions and add a boundary-action term $S_B$. This is the case of interest to us, where the non-trivial boundary will be identified with the (apparent) horizon of the (2+1)-dimensional BTZ black hole. This, in turn, is a guideline for fixing the appropriate boundary conditions. We will briefly repeat the arguments of [13] (see also [36] for a systematic general discussion of boundary conditions and edge states in gravity). We change coordinates from $(t, \rho, \phi)$ → $(u, v, \phi)$, where $u$ and $v$ are light-cone coordinates (the precise relation can be read off by comparing (17) and eq. (3.1) of the first article in [13]). Consider the boundary, with the topology of a cylinder, parametrized by the angular variable $\phi$ and the non-compact variable $v$. Keeping $A^2_{\phi}$, $A^+_{\phi}$, $A^+_v$, as well as their tilded counterparts, fixed in the boundary, requires that the action $S_B$ be given by

$$\frac{1}{8\pi} \int_{\partial M} dt d\phi \left( A^2_{\phi} A^2_v + \frac{1}{2} (A^+_{\phi} A^-_v - A^-_{\phi} A^+_v) \right)$$

minus a similar term with $A$’s replaced by $\tilde{A}$’s. The total action is given by the sum of (32) and $S_B$ and, as a result, the variational problem is now well defined. The relevant degrees of freedom in the boundary are isolated by parametrizing $A = g^{-1} A_f g + g^{-1} dg$, where $A_f$ is a fixed gauge connection in the boundary, and similarly for $\tilde{A}$. Then, quite generally, it can be shown that the relevant induced action in the boundary is the sum of two WZW actions

$$S_B = kI_0(g) - kI_0(\tilde{g}) ,$$

(35)

As we have already mentioned, since the horizon of the BTZ black hole at $\rho = \rho_+$ (which in the new coordinates is located at $u = 0$) is a null surface, we should demand that the boundary $\partial M$ be a null surface as well. It was shown in [13] that the appropriate boundary conditions that achieve this are $A^+_{\phi} = A^+_{v} = \tilde{A}^+_{\phi} = \tilde{A}^+_{v} = 0$. Moreover, we should demand that the circumference of the boundary be the same as that of the BTZ black hole, namely $2\pi \rho_+$. Then, a natural boundary condition, which is also

Notice that, since the Newton constant depends on various charges (as follows from our discussion in the previous section), so does $k$. This resonates with the idea of the string-tension renormalization employed in [35].

The actual circumference is $2\pi \delta \rho_+$, where $\delta = R_1 \cosh \alpha_f / \mu$ or $\delta = R_1 \cosh \alpha_s / (2\mu)$ depending on whether the BTZ black hole corresponds to the 5d or to the 4d black hole. Rescaling $t \rightarrow t\delta^2$, $l \rightarrow t\delta$, 

13
in agreement with the metric (17), is \( e_\phi^2 = \rho_+ \). What remains is to choose a boundary condition for \( \omega_\phi^2 \). As there is no physical principle that has not been met at this point, we leave its boundary value undetermined for the moment. The aforementioned boundary conditions are not invariant under the full two-dimensional group of diffeomorphisms but only under rigid translations of the angular variable \( \phi \). Hence, we must impose on the Hilbert space of (35) the constraint

\[
L_0^{\text{total}} = L_0 + \tilde{L}_0 = 0 ,
\]

where \( L_0 \) and \( \tilde{L}_0 \) are the zero modes of the Virasoro generators corresponding to the affine algebras for \( A_\phi^a \) and \( \tilde{A}_\phi^a \) in (35). The expectation value of \( L_0^{\text{total}} \), in a Hilbert space state of total level \( N \), assumes the form

\[
L_0^{\text{total}} = N + \frac{C_{\text{sl}(2,\mathbb{R})}}{k - 2} - \frac{\tilde{C}_{\text{sl}(2,\mathbb{R})}}{k + 2} = 0 ,
\]

with the Casimir operators given by

\[
C_{\text{sl}(2,\mathbb{R})} = (A_0^2)^2 + \frac{1}{2}(A_0^+A_0^- + A_0^-A_0^+) = -j(j + 1) ,
\]

\[
\tilde{C}_{\text{sl}(2,\mathbb{R})} = (\tilde{A}_0^2)^2 + \frac{1}{2}(\tilde{A}_0^+\tilde{A}_0^- + \tilde{A}_0^-\tilde{A}_0^+) = -\tilde{j}(\tilde{j} + 1) ,
\]

where \( A_0^a, a = 2, +, - \), are the zero modes in a Fourier series expansion of the gauge connection \( A_\phi^a \), i.e. \( A_\phi^a = \frac{1}{k} \sum_{n=0}^{\infty} A_n^ae^{in\phi} \) and obey the Lie algebra \( \text{sl}(2,\mathbb{R}) \). A similar expression holds for \( \tilde{A}_\phi^a \) as well. Also \( j \) and \( \tilde{j} \) label the representation of \( \text{sl}(2,\mathbb{R}) \otimes \text{sl}(2,\mathbb{R}) \). Recall that, we have imposed on the boundary that \( A_\phi^+ = \tilde{A}_\phi^+ = 0 \). Hence, the Casimir operators in (38) are positive-definite. As we shall see, this fact and simple thermodynamical considerations, allow only for the principal series representation. Using the boundary condition \( e_\phi^2 = \rho_+ \) and the definition (34) we may express the zero modes as

\[
A_0^2 = k\left( \omega - \frac{\rho_+}{l} \right) , \quad \tilde{A}_0^2 = k\left( \omega + \frac{\rho_+}{l} \right) ,
\]

where \( \omega \) denotes the zero mode of \( \omega_\phi^2 \) and encodes the remaining freedom in choosing boundary conditions. Then, using (37), we find that

\[
N = \frac{k^2}{k + 2} \left( \omega + \frac{\rho_+}{l} \right)^2 - \frac{k^2}{k - 2} \left( \omega - \frac{\rho_+}{l} \right)^2 .
\]

In the thermodynamic limit the configurations with maximum number of states dominate. Hence, we should maximize \( N \) with respect to \( \omega \). It can be easily shown that the maximum value of \( N \) is reached for \( \omega = \omega_m \equiv \frac{\rho_+}{2l} \) and that it is given by

\[
N_m = \frac{k^2 \rho_+^2}{l^2} .
\]
Finally, the entropy is computed by using the fact that for a CFT with central charge $c$ the number of states behaves asymptotically at large levels $N$ as

$$n(N) \approx \exp \left( 2\pi \sqrt{\frac{N}{6} c} \right), \quad N \gg 1.$$  \hspace{1cm} (42)

Using the leading order in $k$ value for the central charge, i.e. $c \approx 6$, one computes the entropy to be

$$S = \ln n(N_m) \approx \frac{2\pi \rho_+}{4G_N^{(3)}}.$$  \hspace{1cm} (43)

This is precisely the Bekenstein–Hawking entropy formula for the BTZ black hole.

We next prove that, due to boundary conditions, only principal series representations of $sl(2, \mathbb{R})$ are allowed in the thermodynamic limit, in which $N_m \gg 1$. This limit is what one intuitively expects from a physical point of view, but it can also be established by requiring that for the statistical description to be valid the condition $|\partial T/\partial M|_J \ll 1$ should be fulfilled, where $T = (\rho^2_+ - \rho^2_-)/(2\pi \rho_+l^2)$ is the temperature of the black hole. In our case we have explicitly

$$\left| \frac{\partial T}{\partial M} \right|_J = \frac{1}{\pi} \frac{N_m^2 + 24J^2k^2}{N_m^2 - 8J^2k^2} \frac{1}{\sqrt{N_m}} \ll 1,$$  \hspace{1cm} (44)

which implies that $N_m \gg Jk$. Due to boundary conditions, (38) reduces to $j(j + 1) + (A_0^2)^2 = 0$, where $A_0^2$ is given by (39) (computed for $\omega = \omega_m$). This algebraic equation is solved for $j$ to give

$$j = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - N_m(k - 2)^2},$$  \hspace{1cm} (45)

where we have used (11). It is clear that the discrete series representations for which $j + 1 > 0$ and $j \in \mathbb{Z} + \frac{1}{2}$ (or $j \in \mathbb{R}$ if we consider the universal cover of $sl(2, \mathbb{R})$) and the supplementary series for which $j = -1/2 + s$, $0 < |s| < \frac{1}{2}$, are not allowed if $N_m \gg 1$, since then $j$ becomes complex. However, this is precisely what is needed for the continuous series representation to be allowed, since in this case $j = -1/2 + i\sigma$, $\sigma \in \mathbb{R}$. Identifying the latter expression with the one in (13), after it is rewritten so that it is valid for large $N_m$, we obtain

$$\sigma^2 = N_m(k - 2)^2 - \frac{1}{4}.$$  \hspace{1cm} (46)

For $\tilde{j}$ the corresponding $\tilde{\sigma}$ is given by an expression similar to (13), but with $k$ replaced by $-k$. Clearly, the right-hand side of (13) is positive for a sufficiently large number of states $N_m$, i.e. the principal series representation is allowed.

Our final comment concerns the issue of unitarity in WZW models based on non-compact groups. In general, this is still an unsolved problem (for earlier work on the subject, see [39, 40, 41]), but in the case of the $SL(2, \mathbb{R})$ WZW model it has been argued...
that a consistent, unitary theory, can be obtained by restricting to highest-weight states belonging to the principal series representation \[42\]. In this case the current algebra character formula is the same as that of a theory of three free bosons \[40\]. However, the construction of modular invariants is subtle, essentially because states in the Verma module corresponding to the principal series representation do not form a closed set under the fusion rules \[41\]. In addition, if we try to construct modular invariants by using only principal series representations, we would need to obtain the appropriate measure of integration over all \( j = -\frac{1}{2} + i\sigma \). Notice, however, that since the boundary conditions break the two-dimensional diffeomorphisms it may not be necessary to have a modular invariant formulation; only the norm of the microstates is required to be positive-definite. It would be important to reexamine these and related issues in view of the great relevance of the \( SL(2, \mathbb{R}) \) WZW model in black-hole physics we have uncovered.

4 Connection with D-branes

Since we want to compare our counting of microscopic black-hole states with the counting using D-branes, let us consider the extremal case where the latter is valid. In the D-brane picture, one constructs a configuration of D-branes that carries the same quantum numbers as the corresponding black hole. Counting the degeneracy of this configuration yields the number of microstates. When we uplift it to M-theory it becomes an intersection of membranes \( M_2 \), five-branes \( M_5 \) and plane-wave \( W \) solutions.

The effect of the shift transformation on the M-branes and on intersections of them has been studied in \[24\]. The result is that certain branes and intersections thereof are mapped into spaces that are locally isometric to spaces of the form \( adS_k \times E^l \times S^m \), where \( adS_k \) denotes the \( k \)-dimensional anti-de Sitter space, \( E^l \) denotes the \( l \)-dimensional Euclidean space and \( S^m \) is the \( m \)-dimensional sphere. We tabulate these results below. We also give the result for the \( D3 \)-brane. Similar results hold for the rest of the branes, but only when they are expressed in the “dual \( Dp \)-frame”, i.e. the metric in which the curvature and the \((8-p)\)-form field strength appear in the action with the same power of the dilaton \[43\]. In all cases, in order to arrive at the dual configuration one needs a number of compact isometries. This yields the space indicated in the second column of the table with some global identifications. For instance, the \( adS_3 \) appearing below is more properly viewed as an extremal BTZ black hole (with \( J \neq 0 \) only if a plane wave is added to the corresponding configuration in the left column).
It is rather remarkable that these considerations distinguish branes and intersections that we already know to play a distinguished rôle for other reasons. For instance, from these configurations (with the addition of a wave in some cases) one can obtain black-hole solutions in $4 \leq d \leq 9$ upon dimensional reduction.

Since after the duality the asymptotic geometry has changed, the degrees of freedom should organize themselves into representations of the appropriate anti-de Sitter group. The latter has some representations, the so-called singleton representations, that have no Poincaré analogue.\footnote{These representations, for the case of $adS_4$, were discovered by Dirac \cite{14} and named singletons by C. Fronsdal \cite{15}.} They have appeared in studies of spontaneous compactifications of eleven-dimensional supergravity on spheres. In particular, the fields of the supersingleton representation appear as coefficients in the harmonic expansion of the eleven-dimensional fields on the corresponding sphere. A crucial property is that the singleton multiplets can be gauged away everywhere, except in the boundary of the anti-de Sitter space \cite{16}. In particular, it has been argued in the past that the singleton representations of $adS_4$, $adS_7$, $adS_5$ and $adS_3$ correspond to membranes \cite{17}, five-branes \cite{18,19}, self-dual threebranes \cite{18,19} and strings \cite{20}, respectively. It has actually been shown that, in all cases, the world-volume fields of the corresponding $p$-brane form a supersingleton multiplet. We, therefore, conclude that the membrane $M2$, the five-brane $M5$, the self-dual threebrane $D3$, as well as strings, are U-dual to supersingletons. Looking back to Table 1, we see that the anti-de Sitter spaces appearing there, are precisely the ones we just discussed, with one exception, the $adS_2$ space. The boundary of $adS_2$ is simply a point. Thus, one deals with quantum mechanics instead of quantum field theory. It is very tantalizing to identify the theory on the boundary with $D0$ branes. This might yield a connection with $M(atrix)$ theory. However, the $D0$ solution factorizes as $adS_2 \times S^8$ only in the “dual-8 frame”. So, it is not clear whether or not such an identification is correct.

What is important is that, precisely as in our discussion of the counting of states in section 3, would-be gauge degrees of freedom become dynamical at the boundary. Let us consider, for concreteness, the case of the extremal $5d$ black hole. The M-theory
configuration is the intersection of an $M5$ wrapped in $(x_1, x_2, x_3, x_4, x_5)$, an $M2$ wrapped in $(x_1, x_{10})$ with a wave along $x_1$. The 5d black hole arises after a dimensional reduction along $x_1, x_2, x_3, x_4, x_5, x_{10}$. Let us first consider the effect of the shift transformation to each brane separately (i.e. consider a configuration with only that brane). The $M5$ becomes the singleton representation of $adS_7$. The anti-de Sitter space has coordinates $t, x_1, \ldots, x_5, r$. The coordinate $r$ used to be the radius of the transverse space. The five-brane is represented by gauge degrees of freedom everywhere except at the boundary. Studying the five-brane dynamics is equivalent to studying the supersingleton dynamics of $adS_7$. In a similar fashion, the membrane becomes the singleton representation of $adS_4$ (with coordinates $t, x_1, x_{10}, r$). After superposition the effects of the two branes cancel each other in the relative transverse directions. We end up with $adS_3 \times E^5 \times S^3$, where the $adS_3$ part is along the common world-volume directions. It follows that the latter contains gauge degrees of freedom that become dynamical at the boundary. These correspond to the singleton representation of $adS_3$, which can be interpreted as a string [50]. Thus, we find a string living on the world-volume of the five-brane [51]. Notice that the anti-de Sitter group $SO(d-1,2)$ coincides with the conformal group in one dimension lower. Therefore, one ends up with a conformal field theory on the boundary. Since we are considering extremal black holes, the theory at the boundary is also supersymmetric. After the addition of the wave along $x_1$, the $adS_3$ becomes a massive extremal BTZ black hole. These are precisely the degrees of freedom we have counted in section 3. A similar interpretation holds also for the 4d black hole.

Notice that the non-extremal black holes result from the non-extremal intersection of extremal branes and not from the intersection of non-extremal branes. In other words, they can be viewed as non-extremal “bound-state” configurations [29]. This means that one still has the interpretation of each brane as a singleton representation of the corresponding anti-de Sitter group. Therefore, the above discussion still applies.

5 Higher-dimensional black holes and further comments

Let us briefly discuss higher-dimensional (6 ≤ $d$ ≤ 9) black holes. These cases are more complicated, since they are not connected to three-dimensional black holes. A direct proof that the BTZ black hole cannot appear in U-dual configurations of these black holes is given in appendix B. Already from the discussion of the previous section, however, it follows that the higher than five-dimensional black holes are associated with higher than three-dimensional theories. The 9d black holes can be obtained from the non-extremal intersection of $M2$ with a wave, 7d black holes from the intersection of $D3$ with a wave, and 6d black holes from the intersection of $M5$ with a wave [23]. Hence, these black
holes\textsuperscript{17} are associated with the first three entries of Table 1. It follows that in order to understand them one would need to understand the boundary field theories of $adS_4$, $adS_5$ and $adS_7$, respectively. Our considerations also imply that the metrics (supplied with the appropriate antisymmetric tensor fields), after we remove the part corresponding to the sphere, describe solutions of gauged supergravities in four, five and seven dimensions. Presumably, they are black-hole solutions, but this question deserves further study.

The fourth and fifth entries of Table 1, when supplemented with waves, correspond to the 5$d$ and 4$d$ black holes we discussed in section 2. Closely related are the configurations of the last two entries of Table 1: they also correspond to 5$d$ and 4$d$ black holes. In these cases the non-compact part is a two-dimensional configuration, instead of the three-dimensional BTZ black hole. This can be thought of as the dimensionally reduced BTZ black hole along a compact direction. For both cases, there is an associated exact CFT. In particular, the last entry of Table 1, after dimensional reduction along the directions of $E^7$, corresponds to the 4$d$ configuration $adS_2 \times S^2$. This is the Bertotti–Robertson metric, which (with appropriate gauge fields) corresponds to an exact classical solution of string theory \cite{52}. Notice that the last three entries of Table 1 can be obtained from BTZ $\times S^3$ after we dimensionally reduce along appropriate Euler angles parametrizing the corresponding group elements. Reducing the BTZ part one obtains the $adS_2$ black hole, whereas reducing the $S^3$ part one obtains $S^2$. In all cases the CFT description is in terms of the original one for BTZ $\times S^3$ (the various gauge fields are important for this).

One may also consider the first five entries of Table 1 without the addition of a wave. These are non-dilatonic black branes whose thermodynamic properties were studied in \cite{27}. All of them have zero entropy in the extremal limit\textsuperscript{18} Near-extremality, however, their entropy has the same form as the entropy of an ideal gas of massless particles. For $M5$, $M2$ and $D3$ the entropy behaves, as a function of the temperature $T$, as $S_p \sim T^p$, where $p = 5, 2$ and 3, respectively. This is the scaling behaviour of the entropy of an ideal gas of massless particles in $p$ spatial dimensions. For the fourth and fifth entries one gets $S \sim T^1$, i.e. a string-like form. It is now easy to understand these results. From our previous discussion we know that the degrees of freedom that account for the Bekenstein–Hawking entropy live on the boundary of the corresponding anti-de Sitter space. The latter has precisely the right dimension in each case. Near extremality, the degrees of freedom interact only weakly, and therefore one may associate to them a gas of free particles. Away from extremality, when the various interactions are turned on, full knowledge of the boundary dynamics is required.

\textsuperscript{17} The 8$d$ black hole does not seem to be on an equal footing with the rest. One may obtain 8$d$ black holes from a configuration of an $M2$ brane with a wave that has an extra isometry along which one may dimensionally reduce. This implies, however, that the corresponding sphere does not decouple.

\textsuperscript{18} This can easily be seen from (13) and (26) by first setting $\alpha_K = 0$, i.e. no wave, and then going to the extremal limit $\mu \to 0$ with the charges $(Q_{s5}, Q_f)$ and $(Q_{s5}, Q_2, Q_6)$, respectively, kept fixed.
6 Conclusions

We have presented in this article a microscopic derivation of the Bekenstein–Hawking entropy formula for four- and five-dimensional non-extremal non-supersymmetric black holes. Previous successful attempts to count microscopic black-hole states were based on D-brane techniques, and were confined to extremal (or, at best, infinitesimally away from extremal) configurations, where part of supersymmetry is preserved (strictly in the extremal limit). For non-extremal black holes, the best attempts to date only succeeded in deriving the correct dependence of the Bekenstein–Hawking entropy formula on the charges, but not the precise numerical coefficient. In this article we have computed the entropy of non-extremal, non-supersymmetric 4d and 5d black holes from a microscopic point of view, by embedding these black holes into M-theory and then using its symmetries to map them, via a series of U-dualities, into configurations whose non-compact part is the three-dimensional BTZ black hole. We then performed a counting of microscopic states by following Carlip’s approach. The latter is valid at and away from extremality. We furthermore argued that certain branes are dual to supersingleton representations of various gauged supergravities. In this way we obtained a connection with the D-brane picture.

A crucial step in our approach, that enabled us to relate solutions of Poincaré and anti-de Sitter supergravities, was T-duality transformations with respect to isometries which are space-like everywhere, except at spatial infinity, where they become null. Moreover, since the non-compact time coordinate is involved in these transformations, the orbits of the isometry are non-compact. We have mentioned that a more proper treatment requires that we compactify the time (at some radius $R$) and at the end of the computation we send the compactification radius $R \to \infty$. Notice that in the anti-de Sitter space that we obtain, after all dualities have been performed, the time is naturally compact and taking the infinite radius limit corresponds to considering the covering space. Of course, then, the corresponding winding states decouple since they become infinitely heavy. However, it seems that these winding states should be projected out anyway since they are ghost-like, i.e. the corresponding coordinate has the “wrong” sign in the action. All non-dynamical processes (computation of the entropy is such a process) should be independent of whether or not one uses a non-compact time or keeps the radius $R$ finite until the very end. Probably one should be more careful when it comes to fully dynamical processes, such as scattering. Nevertheless, it will be interesting to understand this point better.

Higher-dimensional black holes also fall into our scheme, with one exception: the eight-dimensional black holes. It will be interesting to understand what distinguishes these black holes from the rest. The higher than five-dimensional black holes are associated to higher than three-dimensional field theories; their analysis is therefore consider-
ably harder. In that respect, it would be interesting to better understand supersingleton field theories. The latter have been analysed in [48]. In that case the corresponding p-brane was considered to lie at the end of the world, where the topology is $S^1 \times S^p$. In the present context we would like to consider the boundary at the horizon of the black hole in a way similar to that used in section 3. Since the anti-de Sitter group coincides with the conformal group in one dimension lower, these field theories should be conformal field theories. The theory on the boundary of the BTZ black hole is indeed a conformal field theory. Having obtained such field theories it would be desirable, as a next step, to further substantiate our assessment that singletons account for the black-hole entropy of the black holes we have considered. For instance, for the case of the 5d black hole, we may try to explicitly construct a $(1 + 1)$ field theoretical action for them. This should arise from the synthesis of the six-dimensional and three-dimensional singleton actions corresponding to $adS_7$ and $adS_4$ spaces into a two-dimensional action corresponding to the one-dimensional intersection of the $M5$ and $M2$ branes. The resulting action should be related to (35). Similar considerations can also be made for the black holes corresponding to the spaces listed in Table 1, although for the higher than five-dimensional black holes we do not know the form of the action in the intersection.

In this article we have only studied black holes that arise from compactifications of type-II string theory. There are also heterotic black holes. One might wonder whether our considerations apply in these as well. Although we do not have a definite answer we remark that a mechanism that changes the asymptotics of $4d$ heterotic solutions (the corresponding non-extremal black-hole solutions have been constructed in [53]) has already been reported in [54] for the gravity–dilaton–axion sector. Extensions of this work that will include the gauge fields should be important.

In our study of $4d$ and $5d$ black holes we have used the “BTZ gauge”. From Table 1 we see that there is also a U-dual configuration that involves, as the only non-compact part, the two-dimensional $adS_2$ black hole. It has been shown [55] that the $adS_2$ gravity can be rewritten as a $BF$ theory, i.e. a topological field theory. Therefore, one would expect that all degrees of freedom reside on the boundary, which is just a point. Thus the computation of the entropy now becomes a quantum mechanical calculation. What is truly remarkable is that, after the U-dualities, all the dependence of the entropy on the various charges resides in the two-dimensional Newton constant. In this sense, the dependence of the Bekenstein–Hawking entropy formula on the mass and the charges is “kinematical”. The precise numerical coefficient becomes a question that requires dynamics. Such a calculation was performed in the last paper in [36]. The authors reported negative results. It view of the relevance of these results, it is definitely worth while to reexamine this calculation.

---

19 This is in accordance with the fact that only qualitative considerations are sufficient to determine the correct dependence of the entropy on the various charges [9, 11].
Perhaps the most interesting application is to study the final state of black holes in our framework. In this respect the most promising “gauge” seems to be the $adS_2$ one. One should find U-duality-invariant quantities that uniquely characterize the final state of the black hole. Furthermore, such calculations should involve the CFTs associated with the spheres and the tori since these carry information about the original black hole. We intend to return to this and other related issues in the future.

Acknowledgements
Each of the authors wishes to thank the home institute of his co-author for financial support and hospitality during crucial stages of this work. K.Sk. is supported by the European Commission HCM program CHBG-CT94-0734 and by European Commission TMR programme ERBFMRX-CT96-0045.

A M-theory configurations
In this appendix we present the M-theory configurations that yield, upon dimensional reduction in one coordinate, the 10d solutions discussed in section 2.

Consider the solution of the 11d supergravity that describes a non-extremal intersection of a five-brane ($M_5$), a membrane ($M_2$) with a wave ($W$) in one of the common directions. Let us wrap the $M_5$ in $(x_1, x_2, x_3, x_4, x_5)$, the $M_2$ in $(x_1, x_{10})$ and put a wave along the $x_1$ common direction. The coordinates $x_i, i = 1, \ldots, 5, 10$, are assumed to be periodic, each with radius $R_i$. Explicitly, the solution is given by [29]:

$$ ds_{11}^2 = H_T^{1/3} H_F^{2/3} \left( H_T^{-3} H_F^{-1} \left( -K^{-1} f dt^2 + K \left( dx_1 + (K' - 1) dt \right)^2 \right) + H_F^{-1} \left( dx_2^2 + \cdots + dx_5^2 \right) + f^{-1} dr^2 + r^2 d\Omega_3^2 + H_T^{-1} dx_{10}^2 \right), $$

(47)

with

$$ F_4 = -3 dt \wedge dH_T^{-1} \wedge dx_1 \wedge dx_{10} + 3 * dH_F^{-1} \wedge dx_{10}, $$

(48)

where the various harmonic functions are given by

$$ K = 1 + \frac{Q_K}{r^2}, \quad K' - 1 = 1 - \frac{Q_K}{r^2} K^{-1}, \quad Q_K = \mu^2 \sinh \alpha_K, \quad Q_K = \mu^2 \sinh \alpha_K \cosh \alpha_K $$

$$ H_T = 1 + \frac{Q_T}{r^2}, \quad H_T' - 1 = 1 - \frac{Q_T}{r^2} H_T^{-1}, \quad Q_T = \mu^2 \sinh \alpha_T, \quad Q_T = \mu^2 \sinh \alpha_T \cosh \alpha_T $$

$$ H_F = 1 + \frac{Q_F}{r^2}, \quad H_F' = 1 + \frac{Q_F}{r^2}, \quad Q_F = \mu^2 \sinh \alpha_F, \quad Q_F = \mu^2 \sinh \alpha_F \cosh \alpha_F, $$

(49)

The $*$-duality operation in (48) and (51) is defined with respect to (flat) transverse four-dimensional and three-dimensional spaces, respectively.
and \( f \) is the same as in (2) with \( D = 5 \). The extreme limit is given by \( \mu \to 0, \alpha_K \to \infty, \alpha_T \to \infty, \alpha_F \to \infty \), while \( Q_K, Q_T \) and \( Q_F \) are kept fixed. Upon dimensional reduction along \( x_{10} \), one obtains (5). In (5) we have renamed \( H_T \to H_f \) and \( H_F \to H_{s5} \), and also the charges, so that it is clear to which brane each one of them is associated.

The M-theory configuration that yields (22) involves three five-branes wrapped in \((x_1, x_3, x_4, x_5, x_6)\), \((x_1, x_2, x_5, x_6, x_{10})\) and \((x_1, x_2, x_3, x_4, x_{10})\), each intersecting at a three-brane, with a wave along the string common to all three branes in the direction \( x_1 \). The metric and the four-form are given by (29)

\[
\begin{align*}
\text{ds}_{11}^2 &= (H_{F1}H_{F2}H_{F3})^{2/3}\left(H_{F1}^{-1}H_{F2}^{-1}H_{F3}^{-1}\left(-K^{-1}f dt^2 + K(dx_1 + (K'-1)dt)^2\right)ight. \\
&+H_{F2}^{-1}H_{F3}^{-1}(dx_2^2 + dx_{10}^2) + H_{F1}^{-1}H_{F3}^{-1}(dx_3^2 + dx_4^2) + H_{F1}^{-1}H_{F2}^{-1}(dx_5^2 + dx_6^2) \\
&\left.+f^{-1}dt^2 + r^2d\Omega_2^2\right),
\end{align*}
\]

and

\[
F_4 = 3(*dH_{F1}' \wedge dx_2 \wedge dx_{10} + *dH_{F2}' \wedge dx_3 \wedge dx_4 + *dH_{F3}' \wedge dx_5 \wedge dx_6).
\]

The various harmonic functions are defined as

\[
\begin{align*}
H_{Fi} &= 1 + \frac{Q_{Fi}}{r}, \quad H_{Fi}' = 1 + \frac{Q_{Fi}}{r}, \quad Q_{Fi} = \mu \sinh^2\alpha_{Fi}, \quad Q_{Fi}' = \mu \sinh \alpha_{Fi} \cosh \alpha_{Fi}, \\
K &= 1 + \frac{Q_K}{r}, \quad K' = 1 - \frac{Q_K}{r}K, \quad Q_K = \mu \sinh^2\alpha_K, \quad Q_K' = \mu \sinh \alpha_K \cosh \alpha_K,
\end{align*}
\]

where \( i = 1, 2, 3 \), and \( f \) is the same as in (2) with \( D = 4 \). The extreme limit is given by \( \mu \to 0, \alpha_K \to \infty, \alpha_{Fi} \to \infty \), while \( Q_K \) and \( Q_{Fi}, i = 1, 2, 3 \), are kept fixed. Upon dimensional reduction along \( x_{10} \) one obtains a solitonic five-brane \( NS5 \) in \((x_1, x_3, x_4, x_5, x_6)\), two \( D4 \)-branes in \((x_1, x_2, x_5, x_6)\) and \((x_1, x_2, x_3, x_4)\), with a wave along \( x_1 \). We further T-dualize this solution along \( T_{56} \). This yields the solution (22) used in section 2. There, the harmonic functions are renamed as \( H_{F1} \to H_{s5}, H_{F2} \to H_2 \) and \( H_{F3} \to H_6 \). The charges and the angles are also appropriately renamed.

**B Higher-dimensional black holes and the BTZ black hole**

In this appendix we will show that the BTZ black hole can only be connected with \( 4d \) and \( 5d \) black holes and not with higher-dimensional ones.

Black holes arise from brane intersections after dimensional reduction. The dimensionality \( D \) of the final black hole is equal to the overall transverse dimension plus 1.
The solution depends on a number of harmonic functions with respect to the overall transverse space, i.e. they are of the form

$$H = 1 + \frac{Q}{r^{D-3}}. \quad (53)$$

We consider non-extremal configurations obtained from extremal ones according to the rules discussed in [29].

As a first step we will rewrite the BTZ black hole metric [17] in a way that depends on \((D-1)\)-dimensional harmonic functions. To this end, consider the change of variables

$$\rho^2 = \frac{4}{(D-3)^2} \mu^{D-5} + \rho_+^2, \quad \mu = \frac{D-3}{D-5}. \quad (54)$$

We also make the following identifications

$$l = \frac{2}{(D-3)} \mu, \quad \rho_+ = l \cosh \alpha, \quad \rho_- = l \sinh \alpha, \quad \varphi = \frac{x}{l}, \quad H = \frac{\mu^{D-3}}{r^{D-3}}. \quad (55)$$

The final result is that the BTZ metric (17) takes the form

$$ds^2_{BTZ} = H^{-1} \left( -K^{-1}(r)f(r)dt^2 + K(r) \left( dx + (K'^{-1}(r) - 1)dt \right)^2 \right) + H^{\frac{2}{D-3}} f^{-1}(r)dr^2, \quad (56)$$

where \(f, K\) and \(K'\) are as in (2).

Let us now examine whether or not this metric can result from an intersection of branes. If this is the case, the form of \(H\) implies that the BTZ black hole will emerge after the shift transformation has been applied. The configurations we examine are built from superpositions of single brane solutions. The latter have the form

$$ds^2 = H^{a_p} \left( H^{-1}(-dt^2 + dx_1^2 + \cdots dx_p^2) + (dx_{p+1}^2 + \cdots + dx_{D-1}^2) \right), \quad (57)$$

where \(H\) is a harmonic function and \(a_p\) is a numerical factor that depends on the particular brane (e.g. \(\alpha_2 = 1/3\) for the \(M2\), \(\alpha_5 = 2/3\) for the \(M5\), etc.). What is important for our discussion is that the difference between the power of the harmonic function multiplying the overall transverse coordinates and the power of the harmonic function multiplying the world-volume coordinates is 1. This is true for all branes (and also for the wave). If one superimposes two branes, then the difference of the sums of the powers will be 2. For a solution that depends on \(k\) charges (the charges may be degenerate) the difference will be equal to \(k\). The same results hold for non-extremal intersections built according to the rules of [29]. Notice also that this number is preserved in the process of dimensional reduction. The explicit form of a solution arising from the intersection of \(k\) branes is

$$ds^2 = \left( \prod_{i=1}^{k} H_i \right) \left[ - \left( \prod_{i=1}^{k} H_i^{-1} \right) f dt^2 + ds_w^2 \right] + ds_{RT}^2 + f^{-1}dr^2 + r^2 d\Omega_{D-2}^2, \quad (58)$$
where $ds^2_w$ denotes the metric of the world-volume coordinates apart from the $dt^2$-term shown in (58), and $ds^2_{RT}$ is the metric of the relative transverse coordinates. The precise form of the latter is irrelevant for our argument. Here $H_i$ are harmonic functions (including the one associated with the wave) and $a_i$'s are some numerical constants. Therefore, if the solution (56) originates from an intersection of branes, then we should have

$$\frac{2}{D-3} + 2 = k.$$  \hspace{1cm} (59)

This equation has solutions (for integers $D$ and $k$) only for $D = 4, k = 4$ and $D = 5, k = 3$. These are precisely the cases studied in section 2, namely for a 4$d$ black hole that depends on four charges and a 5$d$ black hole that depends on three charges. Notice that this argument depends crucially on the fact that we build our intersection according to the rules of [29], namely we start from supersymmetric configurations and then add deformation terms appropriately. For a supersymmetric configuration, $k$ is always an integer [56].

References

[1] J.D. Bekenstein, Phys. Rev. D7 (1973) 2333 and D9 (1974) 3292.

[2] S.W. Hawking, Nature 248 (1974) 30; Commun. Math. Phys. 43 (1975) 199.

[3] C.M. Hull and P.K. Townsend, Nucl. Phys. B438 (1995) 109, hep-th/9410167; E. Witten, Nucl. Phys. B443 (1995) 85, hep-th/9503124.

[4] P. Horava, Nucl. Phys. B327 (1989) 461 and Phys. Lett. B231 (1989) 251; J. Dai, R.G. Leigh and J. Polchinski, Mod. Phys. Lett. A4 (1989) 2073; R.G. Leigh, Mod. Phys. Lett. A4 (1989) 2767.

[5] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724, hep-th/9510017.

[6] A. Strominger and C. Vafa, Phys. Lett. B379 (1996) 99, hep-th/9601029.

[7] C.G. Callan and J.M. Maldacena, Nucl. Phys. B472 (1996) 591, hep-th/9602043.

[8] S. Das and S. Mathur, Phys. Lett. B375 (1996) 103, hep-th/9601152; G.T. Horowitz and A. Strominger, Phys. Rev. Lett. 77 (1996) 2368, hep-th/9602051.

[9] G.T. Horowitz and J. Polchinski, Phys. Rev. D55 (1997) 6189, hep-th/9612146 and Self gravitating String for Black Holes and Strings, hep-th/9707170.

[10] L. Susskind, Some speculation about black hole entropy in string theory, hep-th/9309145.
[11] T. Banks, W. Fischler, I.R. Klebanov and L. Susskind, *Schwarzchild black holes from Matrix theory*, hep-th/9709091; I.R. Klebanov and L. Susskind, *Schwarzchild black holes in various dimensions from Matrix theory*, hep-th/9709108; E. Halloy, *Six-dimensional Schwarzchild black holes in M(alter) theory*, hep-th/9709227; G.T. Horowitz and E.J. Martinec, *Comments on black holes in Matrix theory*, hep-th/9710217; M. Li, *Matrix Schwarzchild black holes in large N limit*, hep-th/9710226; S.R. Das, S.D. Mathur, S.K. Rama and P. Ramadevi, *Boosts, Schwarzchild black holes and absorption cross-sections in M theory*, hep-th/9711003; T. Banks, W. Fischler, I.R. Klebanov and L. Susskind, *Schwarzchild black holes from Matrix theory 2*, hep-th/9711003.

[12] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, Phys. Rev. D55 (1997) 5112, hep-th/9610043.

[13] S. Carlip, Phys. Rev. D51 (1995) 632, gr-qc/9409052 and D55 (1997) 878, gr-qc/9606043.

[14] M. Banados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. 69 (1992) 1849, hep-th/9204099; M. Banados, M. Henneaux, C. Teitelboim and J. Zanelli, Phys. Rev. D48 (1993) 1506, gr-qc/9302012.

[15] G. Horowitz and D. Welch, Phys. Rev. Lett. 71 (1993) 328, hep-th/9302120.

[16] N. Kaloper, Phys. Rev. D48 (1993) 2598, hep-th/9303007.

[17] A. Achúcarro and P. K. Townsend, Phys. Lett. B180 (1986) 89.

[18] E. Witten, Nucl. Phys. B311 (1988) 46.

[19] G. ’t Hooft, Nucl. Phys. B256 (1985) 727 and B335 (1990) 138; L. Susskind, L. Thorlacius and J. Uglum, Phys. Rev. D48 (1993) 3743, hep-th/9306069; M. Maggiore, Nucl. Phys. B429 (1994) 205, qr-qc/9401027.

[20] A. Sen, Mod. Phys. Lett. A10 (1995) 2081, hep-th/9504147.

[21] A. Ashtekar, J. Baez, A. Corichi and K. Krasnov, *Quantum Geometry and Black Hole Entropy*, gr-qc/9710007, and references therein.

[22] E. Cremmer, B. Julia and J. Scherk, Phys. Lett. B76 (1978) 409.

[23] S. Hyun, *U-duality between Three and Higher Dimensional Black Holes*, hep-th/9704003.

[24] H.J. Boonstra, B. Peeters and K. Skenderis, Phys. Lett. B411 (1997) 59, hep-th/9706192.
[25] F. Englert, Phys. Lett. B119 (1982) 339; B. Biran, F. Englert, B. de Wit and H. Nicolai, Phys. Lett. B124 (1983) 45.

[26] B. de Wit and H. Nicolai, Phys. Lett. B108 (1981) 285 and Nucl. Phys. B208 (1982) 323; M.J. Duff, B.E.W. Nilsson and C.N. Pope, Phys. Rep. 130 (1986) 1.

[27] I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. B475 (1996) 164, hep-th/9604089.

[28] G. Horowitz and D. Welch, Phys. Rev. D49 (1994) 590, hep-th/9308077.

[29] M. Cvetič and A.A. Tseytlin, Nucl. Phys. B478 (1996) 431, hep-th/9606033.

[30] G. Papadopoulos and P.K. Townsend, Phys. Lett. B380 (1996) 273, hep-th/9603087;
A.A. Tseytlin, Nucl. Phys. B475 (1996) 149, hep-th/9604036; A.A. Tseytlin, Nucl. Phys. 487 (1997) 141, hep-th/9609212; R. Argurio, F. Englert and L. Houart, Phys. Lett. B398 (1997) 61, hep-th/9701042.

[31] T. Buscher, Phys. Lett. B194 (1987) 59 and B201 (1988) 466.

[32] E. Bergshoeff, C.M. Hull, T. Ortin, Nucl. Phys. B451 (1995) 547, hep-th/9504081.

[33] I.R. Klebanov and A.A. Tseytlin, Nucl. Phys.B475 (1996) 179, hep-th/9604166.

[34] I. Antoniadis, C. Bachas and A. Sagnotti, Phys. Lett. B235 (1990) 255;
S.B. Giddings, J. Polchinski and A. Strominger, Phys. Rev. D48 (1993) 5784, hep-th/9305083.

[35] F. Larsen and F. Wilczek, Phys. Lett. B375 (1996) 37, hep-th/9511064; M. Cvetic, Phys. Rev. D53 (1996) 5619, hep-th/9512031; A.A. Tseytlin, Nucl. Phys. B477 (1996) 431, hep-th/9605091.

[36] A.P. Balachandran, L. Chandar and A. Momen, Nucl. Phys. B461 (1996) 581, gr-qc/9412013; M. Banados and A. Gomberoff, Phys. Rev. D55 (1997) 6162, gr-qc/9611044; J. Gegenberg, G. Kunstatter and T. Strobl, Phys. Rev. D55 (1997) 7651, gr-qc/9612033.

[37] J.L. Cardy, Nucl. Phys. B270 (1986) 186.

[38] J. Preskill, P. Schwarz, A. Shapere, S. Trivedi and F. Wilczek, Mod. Phys. Lett. A6 (1991) 2353.

[39] J.D. Lykken, Nucl. Phys. B313 (1989) 473; J. Balog, L. O’Raifeartaigh, P. Forgacs and A. Wipf, Nucl. Phys. B325 (1989) 225; I. Bars, Nucl. Phys. B334 (1990) 125; L.J. Dixon, M.E. Peskin and J. Lykken, Nucl. Phys. B325 (1989) 329; P.M.S. Petropoulos, Phys. Lett. B236 (1990) 151; I. Bars and D. Nemeschansky, Nucl. Phys. B348 (1991) 89; J. Distler and P. Nelson, Nucl. Phys. B366 (1991) 255.
[40] K. Sfetsos, Phys. Lett. B271 (1991) 301; I. Bakas and E. Kiritsis, Int. J. Mod. Phys. A7 (1992) 55, \texttt{hep-th/9109029}.

[41] P.A. Griffin and O.F. Hernandez, Nucl. Phys. B356 (1991) 287.

[42] I. Bars, Phys. Rev. D53 (1996) 3308, \texttt{hep-th/9503205}; Y. Satoh, \textit{Ghost-free and modular invariant spectra of a string in SL(2, \mathbb{R}) and three-dimensional black hole geometry}, \texttt{hep-th/9705208}.

[43] H.J. Boonstra, B. Peeters and K. Skenderis, in preparation.

[44] P.A.M. Dirac, J. Math. Phys. 4 (1963) 901.

[45] C. Fronsdal, Phys. Rev. D12 (1975) 3819.

[46] C. Fronsdal, Phys. Rev. D26 (1982) 1988; H. Nicolai and E. Sezgin, Phys. Lett. B143 (1984) 389; M. Günyaydin and N. Marcus, Class. Quant. Grav. 2 (1985) L1; M. Günyaydin, P. van Nieuwenhuizen and N.P. Warner, Nucl. Phys. B255 (1985) 63.

[47] E. Bergshoeff, M.J. Duff, C.N. Pope and E. Sezgin, Phys. Lett. B199 (1987) 69; M.P. Blencowe and M.J. Duff, Phys. Lett. B203 (1988) 229; E. Bergshoeff, A. Salam, E. Sezgin and Y. Tani, Phys. Lett. B205 (1988) 237 and Nucl. Phys. B305 [FS23] (1988) 497.

[48] H. Nicolai, E. Sezgin and Y. Tani, Nucl. Phys. B305 [FS23] (1988) 483.

[49] G.W. Gibbons and P.K. Townsend, Phys. Rev. Lett. 71 (1993) 3754, \texttt{hep-th/9307049}.

[50] M. Günyaydin, B.E.W. Nilsson, G. Sierra and P.K. Townsend, Phys. Lett. B176 (1986) 45.

[51] R. Dijkgraaf, E. Verlinde and H. Verlinde, Nucl. Phys. B486 (1997) 77, \texttt{hep-th/9603126}; B486 (1997) 89, \texttt{hep-th/9604055}; and B484 (1997) 543, \texttt{hep-th/9607026}.

[52] D.A. Lowe and A. Strominger, Phys. Rev. Lett. 73 (1994) 1468, \texttt{hep-th/9403180}.

[53] M. Cvetic and D. Youm, \textit{BPS Saturated and nonextremal states in abelian Kaluza-Klein theory and the effective N = 4 supersymmetric string vacua}, in Strings'95, Future perspectives in string theory, p. 131-147, \texttt{hep-th/9508058}.

[54] I. Bakas, Phys. Lett. B343 (1995) 103, \texttt{hep-th/9410104}.
[55] T. Fukuyama and K. Kamimura, Phys. Lett. B160 (1985) 259; K. Isler and C. Trugenberger, Phys. Rev. Lett. 63 (1989) 843; A. Chamseddine and D. Wyler, Phys. Lett. B228 (1989); P. Schaller and T. Strobl, Phys. Lett. B337 (1994) 266, hep-th/9401110.

[56] H. Lu, C.N. Pope, E. Sezgin and K.S. Stelle, Nucl. Phys. B456 (1995) 669, hep-th/9508042; H. Lu and C.N. Pope, Nucl. Phys. B465 (1996) 127, hep-th/9512012.