A search for axion-like particles in light-by-light scattering at the CLIC

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Abstract

The virtual production of axion-like particles (ALPs) in the light-by-light scattering at the CLIC collider is studied. Both differential and total cross sections are calculated, assuming interaction of the ALP with photons via CP-odd term in the Lagrangian. The 95% C.L. exclusion regions for the ALP mass and its coupling constant are given. By comparing our results with existing collider bounds, we see that the ALP search at the CLIC has a great physics potential of searching for the ALPs, especially, in the mass region 1 TeV – 2.4 TeV, with the collision energy $\sqrt{s} = 3000$ GeV and integrated luminosity $L = 5000$ fb$^{-1}$ for the Compton backscattered initial photons. In particular, our limits are stronger than recently obtained bounds for the ALP production in the light-by-light scattering at the LHC.
I. INTRODUCTION

The notion of the QCD axion is closely related to the strong CP problem, which means the absence of the CP violation in the strong interactions. In its turn, the CP problem arises as a possible solution to the $U(1)$ problem. The QCD Lagrangian in the limit of vanishing masses of $u$ and $d$ quarks has a global symmetry $U(2)_V \times U(2)_A = SU(2)_I \times U(1)_Y \times SU(2)_A \times U(1)_A$. The non-zero quark condensates $\langle \bar{u}u \rangle$ and $\langle \bar{d}d \rangle$ break down the axial symmetry $SU(2)_A \times U(1)_A$ spontaneously. As a result, four Nambu-Goldstone bosons should appear. But besides light pions, no another light state is present in the hadronic spectrum, since $m_\eta \gg m_\pi$. It is called the $U(1)$ problem.

The $U(1)_A$ symmetry is connected with a transformation of the fermion fields $\psi \rightarrow e^{i\alpha\gamma_5}\psi$, $\bar{\psi} \rightarrow \bar{\psi}e^{i\alpha\gamma_5}$. One possible resolution of the $U(1)$ problem is provided by the Adler-Bell-Jackiw chiral anomaly for the axial current $J_5^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$. In the limit of vanishing values of quark masses, it gives

$$\partial^\mu J_5^\mu = \frac{g^2 N_f}{16\pi^2} G_{a\mu\nu} \bar{G}_a^{\mu\nu},$$

where $G_{a\mu\nu}$ is a gluon tensor, $\bar{G}_a^{\mu\nu} = (1/2)\varepsilon_{\mu\nu\rho\sigma} G_\rho^\sigma$ its dual, and $N_f$ is a flavor number. However, the problem is not so simple, since the term $G_{a\mu\nu} \bar{G}_a^{\mu\nu}$ is a total divergence,

$$G_{a\mu\nu} \bar{G}_a^{\mu\nu} = \partial^\mu K_\mu,$$

where

$$K_\mu = 2\varepsilon_{\mu\nu\lambda\rho} A_\mu^\nu \left[ G^\lambda^\rho - \frac{g}{3} f_{abc} A_\rho^b A_c^c \right],$$

$A_{a\mu}$ is a gluon field, $f_{abc}$ is a structure constant of the QCD group. A new current $\tilde{J}_5^\mu = J_5^\mu - K_\mu$ can be introduced which is conserved in the limit $m_q \rightarrow 0$, but is not gauge-invariant.

The chiral anomaly introduce a pure surface integral to the QCD action $S_{QCD}$

$$\Delta S_{QCD} = \alpha \frac{g^2 N_f}{16\pi^2} \int ds_\mu K_\mu,$$

where $\alpha$ is a parameter of the chiral transformations. If $A_{a\mu} = 0$ at spatial infinity, then $\int ds_\mu K_\mu = 0$, $\Delta S = 0$, and $U(1)_A$ is an unbroken global symmetry. However, $A_{a\mu} = 0$ can be a pure gage at spatial infinity, if

$$A_{\mu} \big|_{r \rightarrow \infty} \rightarrow -\frac{i}{g} \partial_{\mu} \omega \omega^{-1},$$
\( A_\mu = A_{\mu} t^a \). For such a configuration, \( \int ds_\mu K^\mu \neq 0 \), and consequently \( U(1)_A \) is not a symmetry of the strong interactions. In the \( SU(2) \) QCD \( \omega \) are classified by the integer \( n \), \( \omega_n \to e^{i2\pi n} \) as \( r \to \infty \), and condition (4) is a map of three-dimensional sphere \( S^3_\infty \) on the sphere \( S^3_1 \) of the \( SU(2) \) group. The winding number \( n \) is given by

\[
n = \frac{g^2}{32\pi^2} \int ds_\mu K^\mu .
\]  

(5)

Note that

\[
K_\mu = \frac{4}{3g^2} \varepsilon_{\mu\nu\rho\sigma} \text{tr}(\omega \partial_\nu \omega^{-1} \omega \partial_\rho \omega^{-1} \omega \partial_\sigma \omega^{-1}) .
\]  

(6)

There is an infinite number of the vacuum states characterized by the topological index \( n \). The condition \( gA_\mu |_{r\to\infty} \to -i \partial_\mu \omega \omega^{-1} \) is a definition of a classical vacuum of the gauges fields. The topological index \( \nu \) of the instanton solution [4] is equal to the difference of the topological indices of the vacua defined in (5),

\[
\nu = n(t = +\infty) - n(t = -\infty) .
\]  

(7)

It means that the instantons realize vacuum-to-vacuum transition. The true or \( \theta \)-vacuum becomes a superposition of the vacua \( |n\rangle \)

\[
|\theta\rangle = \sum_n e^{-i\theta n} |n\rangle .
\]  

(8)

As a result, an effective QCD action acquires the \( \theta \)-term

\[
S_{\text{eff}} = S_{\text{QCD}} + \theta \frac{g^2}{32\pi^2} \int dx G_{\alpha\mu} \tilde{G}^{\mu\alpha} .
\]  

(9)

Moreover, an account of the weak interactions adds the following term to the QCD Lagrangian

\[
\Delta L_{\text{QCD}} = \tilde{\theta} \frac{g^2}{32\pi^2} \int dx G_{\alpha\mu} \tilde{G}^{\mu\alpha} ,
\]  

(10)

where

\[
\tilde{\theta} = \theta + \text{arg det } M ,
\]  

(11)

and \( M \) is the quark mass matrix. \( \tilde{\theta} \) is invariant under chiral transformation and thus observable. The extra term in (10) breaks P- and T-invariance but conserves C-invariance, so CP-invariance is violated. Thus, it contributes to the neutron electric dipole moment \( d_n \). The current experimental limit \( d_n < 0.021 \times 10^{-23} \text{ e cm} \) [5] requires \( \tilde{\theta} \) to be less than \( 10^{-9} \). The smallness of the angle \( \tilde{\theta} \) is known as strong CP problem.
One possible solution to this problem is a spontaneously broken CP. However, we know that experimental data are in excellent agreement with the CKM-model in which the CP is explicitly broken. The elegant solution of the CP mystery of the SM is provided by the Peccei-Quinn (PQ) mechanism with a new, spontaneously broken approximate global $U(1)_{PQ}$ symmetry \[6\]. As it is shown in \[7, 8\] it leads to a light neutral pseudoscalar particle, the *axion* $a$, which is the Nambu-Goldstone boson of the broken $U(1)_{PQ}$ symmetry. The idea is to replace the CP-violating term $\bar{\theta}$ by the CP-conserving axion. Namely, the axion field can be redefined to absorb the parameter $\bar{\theta}$. In fact, the axion replaces the QCD theta parameter by a dynamical quantity, thereby explaining of non-observation of the strong CP violation. Thus, the PQ mechanism is a compelling solution to the strong CP problem.

In the PQWW scheme \[6\]-\[8\] an extra Higgs doublet is used, and the axion mass is related to the electroweak symmetry breaking scale. There are two models in which the PQ symmetry is decoupled from the electroweak (EW) scale and is spontaneously broken. It results in axions with extremely weak couplings (“invisible” axion). One of the models is the KSVZ model \[9\]-\[10\] with one Higgs doublet in which the axion is introduced as the phase of an EW singlet scalar field. This scalar is coupled to an additional heavy quark, and its coupling is induced by the interaction of the heavy quarks with other fields. In the DFSZ model \[11\]-\[12\] two Higgs doublets are used, as well as an additional EW singlet scalar. The latter is coupled to the SM fields through its interaction with the the Higgs doublets.

The axion also appears in the context of the string theory \[13\]-\[15\]. In the string theory spin-zero particles must couple to a photon field, since all couplings are defined by the expectation value of scalar fields. This implies the existence of the P-odd term in the Lagrangian proportional to

$$\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} = g_{a\gamma\gamma} a \vec{E} \cdot \vec{B},$$

(12)

where $F_{\mu\nu}$ is the electromagnetic tensor, $\tilde{F}_{\mu\nu} = (1/2)\varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$ its dual, and $a$ is the QCD axion or axion-like particle (ALP) \[16\]. APLs can also appear in theories with spontaneously broken symmetries \[17\]-\[18\] or in GUT \[19\]. Lately, a number of new theoretical schemes with the axion as a basic quantity was developed \[20\]-\[27\]. For a review on the axions and APLs, see \[28\]-\[31\] and references therein.

Both theory and phenomenology of the axions were also studied in large \[32\]-\[35\] and warped \[36\]-\[38\] extra dimensions (EDs). In an ED framework, the mass of the axion
becomes independent of the scale associated with the breaking of the PQ symmetry. It means that the axion mass can be treated independently of its couplings to the SM fields.

The very low mass and small coupling axion and/or ALP are a leading dark matter (DM) candidate, since their properties, allow them to be stable and difficult-to-detect. Both axions and ALPs can be produced in the early Universe and therefore constitute the most of the cold DM in the Universe [39]-[40] (see also recent papers [41]-[46]). The relevance of the QCD axion and, more generally, of ALPs in astrophysics and cosmology is of particular interest [47]-[52]. Many axion DM experiments are in progress [53]-[60] (see also [61]).

The axion phenomenology involves such phenomena as stellar evolution, axion mediated forces, dark matter detection, axion decays, axion-photon conversion, so-called “light shining trough the wall”, etc.

There is a broad experimental program aiming to search for the QCD axion via its coupling to the SM. On the other hand, many ALP searches assume their strong couplings to the electromagnetic term $F_{\mu\nu}\tilde{F}^{\mu\nu}$ as in eq. (12). In terrestrial experiments, bounds on very low mass axions and small mass axions were obtained [62]-[67]. The coupling of the ALPs to other gauge bosons are also studied (see for instance, [68]). Note that the ALPs are not directly relevant for the QCD axion. Therefore, heavy APLs can be detected at colliders, in particular, in a light-by-light scattering [69]-[72]. As it was shown in [73], searches at the LHC with the use of the proton tagging technique can constrain the ALP masses in the region 0.5 TeV–2 TeV.

Compact Linear Collider (CLIC) is the linear collider that is planned to accelerate and collide electrons and positrons at maximally 3 TeV center-of-mass energy [74]. In CLIC, it is possible to obtain accelerating gradients of 100 MV/m. Three energy states are considered to operate CLIC at maximum efficiency [75]. The $\sqrt{s} = 380$ GeV is the first one and it is possible to reach the integrated luminosity $L = 1000$ fb$^{-1}$. This energy stage cover Higgs boson, top quark, and gauge sectors. It is planned to examine such SM particles with high precision [76]. The second one has $\sqrt{s} = 1500$ GeV center-of-mass energy and 2500 fb$^{-1}$ integrated luminosity. At this stage, it is enable to investigate beyond the SM physics. Also, a detailed analysis of the Higgs boson can be made such as the Higgs self-coupling and the top-Yukawa coupling and rare Higgs decay channels. [77]. The third stage of the CLIC has a maximum center-of-mass energy value $\sqrt{s} = 3000$ GeV and integrated luminosity value $L = 5000$ fb$^{-1}$. At this stage, the most precise examinations of the SM is possible. Moreover,
it is enabled to discovery beyond the SM heavy particles of mass greater than 1500 GeV \[76\]. The new physics search potential of the CLIC is presented in \[78\].

At the CLIC it is also possible to study photon-induced processes in $\gamma\gamma$ and $e\gamma$ collisions. In this type of processes, the photons are emitted from the incoming electron beams. The photons scatter at tiny angels from the beam pipe. Hence, they have very low virtuality, that is why these photons are called “almost-real”.

The first evidence of the subprocess $\gamma\gamma \rightarrow \gamma\gamma$ was observed by the ATLAS collaboration in high-energy ultra-peripheral PbPb collisions \[79\]. The same process was also reported by the CMS Collaboration \[80\]. Recently, the ATLAS collaboration have published the evidence of the light-by-light scattering with the certainty of 8.2 sigma \[81\]. The analysis of the exclusive and diffractive $\gamma\gamma$ production in PbPb collisions was done in \[82\]. We have examined a possibility to constrain the parameters of the model with a warp ED in the photon-induced process $pp \rightarrow p\gamma\gamma p \rightarrow p'\gamma\gamma p'$ at the LHC \[83\]. Previously, the photon-induced processes in EDs were studied in \[84\]-\[85\].

In the present paper, we propose to search for the ALP $a$ in the exclusive light-by-light scattering at the lepton collider CLIC.

In the next section differential and total cross sections are calculated as functions of the ALP mass $m_a$ and its coupling $f$. It enables us to estimate the CLIC exclusion regions for both types on the initial photons.

II. LIGHT-BY-LIGHT VIRTUAL PRODUCTION OF ALP

The ALP couples to the SM photons via

$$\mathcal{L}_a = \frac{1}{2} (\partial_\mu a)(\partial^\mu a) - \frac{1}{2} m_a^2 a^2 + \frac{a}{f_a^{(+)}} F_{\mu\nu} F^{\mu\nu} + \frac{a}{f_a^{(-)}} F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (13)$$

were $(f_a^{(+)})^{-1}$ and $(f_a^{(-)})^{-1}$ are CP-even and CP-odd ALP-photon couplings, respectively. Note that, in contrast to the true QCD axion, the mass and couplings of the ALP are independent parameters. In what follows, we assume that only the CP-odd interaction term is realized in (13) with $f_a^{(-)} = f$. As for possible contribution from the CP-even term in (13), it is discussed in the section Conclusions.
The cross section of the diphoton production at the CLIC can be found as the integration

\[
d\sigma(e^+e^- \rightarrow e^+\gamma\gamma e^-) = 2 \int \frac{dz}{z_{\text{min}}} \int \frac{dy}{y_{\text{max}}} f_{\gamma/e}(y) f_{\gamma/e}(z^2/y) \, d\sigma(\gamma\gamma \rightarrow \gamma\gamma),
\]

where \( f_{\gamma/e}(y) \) is the photon spectrum, and \( d\sigma(\gamma\gamma \rightarrow \gamma\gamma) \) is the unpolarized differential cross section of the subprocess \( \gamma\gamma \rightarrow \gamma\gamma \). The explicit expressions for the photon spectrum are given below. The differential cross section is the following sum of helicity amplitudes squared

\[
\frac{d\sigma}{d\Omega} = \frac{1}{128\pi^2 s} \left( |M_{++++}|^2 + |M_{++-+}|^2 + |M_{+-++}|^2 + |M_{++-+}|^2 \right).
\]

Here and below \( s, t \) and \( u \) are the Mandelstam variables of the diphoton system. Each of the helicity amplitudes is a sum of the ALP and SM terms,

\[
M = M_a + M_{\text{ew}}.
\]

The explicit expressions of the pure ALP amplitudes can be found in [71]. In particular, \( \mathbf{71} \)

\[
\begin{align*}
\Re M_{++++}^{(a)} &= -4 \frac{s^2(s - m_a^2)}{f_a^2(s - m_a^2)^2 + m_a^2 \Gamma_a^2}, \\
\Im M_{++++}^{(a)} &= \frac{4}{f_a^2(s - m_a^2)^2 + m_a^2 \Gamma_a^2} s^2 m_a \Gamma_a,
\end{align*}
\]

where \( \Gamma_a \) is the total width of the ALP,

\[
\Gamma_a = \frac{\Gamma(a \rightarrow \gamma\gamma)}{\text{Br}(a \rightarrow \gamma\gamma)},
\]

and

\[
\Gamma(a \rightarrow \gamma\gamma) = \frac{m_a^3}{4\pi f_a^2},
\]

is its decay width into two photons. Correspondingly, we have [71]

\[
\begin{align*}
\Re M_{++-+}^{(a)} &= -4 \frac{t^2}{f_a^2 t - m_a^2}, & \Im M_{++-+}^{(a)} &= 0, \\
\Re M_{+--+}^{(a)} &= -4 \frac{u^2}{f_a^2 u - m_a^2}, & \Im M_{+--+}^{(a)} &= 0, \\
\Re M_{++-+}^{(a)} &= \frac{4}{f_a^2} \left( \frac{s^2(s - m_a^2)}{(s - m_a^2)^2 + m_a^2 \Gamma_a^2} + \frac{t^2}{t - m_a^2} + \frac{u^2}{u - m_a^2} \right), \\
\Im M_{++-+}^{(a)} &= -\frac{4}{f_a^2} s^2 m_a \Gamma_a \left( \frac{s^2(s - m_a^2)}{(s - m_a^2)^2 + m_a^2 \Gamma_a^2} \right).
\end{align*}
\]
\[ M^{(a)}_{++-} = 0. \]  

An account of the ALP width \( \Gamma_a \) is mainly important in a vicinity of the point \( s \sim m_a^2 \). That is why, it is omitted in the denominators of the last two terms in the first row of eq. \((22)\).

The SM (electroweak) amplitude is a sum of the fermion and \( W \) boson one-loop amplitudes

\[ M_{\text{ew}} = M^f + M^W. \]  

They amplitudes \( M^f_{++++}(s,t,u) \) and \( M^W_{++++}(s,t,u) \) are calculated in \([86]-[87]\) (see also \([85]\))

\[
\frac{1}{\alpha^2_{em}} \Re M^f_{++++}(s,t,u) = -8 - 8 \left( \frac{u - t}{s} \right) \ln \left( \frac{u}{t} \right) \\
- 4 \left( \frac{t^2 + u^2}{s^2} \right) \left[ \ln^2 \left( \frac{u}{t} \right) + \pi^2 \right],
\]

\[
\Im M^f_{++++}(s,t,u) = 0,
\]

where \( e_f \) is the fermion electric charge in units of the proton charge.

\[
\frac{1}{\alpha^2_{em}} \Re M^W_{++++}(s,t,u) = 12 + 12 \left( \frac{u - t}{s} \right) \ln \left( \frac{u}{t} \right) \\
+ 16 \left( 1 - \frac{3tu}{4s^2} \right) \left[ \ln^2 \left( \frac{u}{t} \right) + \pi^2 \right] \\
+ 16 \left[ \frac{s}{t} \ln \left( \frac{s}{m_w^2} \right) \ln \left( \frac{-t}{m_w^2} \right) + \frac{s}{u} \ln \left( \frac{s}{m_w^2} \right) \ln \left( \frac{-u}{m_w^2} \right) \right] \\
+ \frac{s^2}{tu} \ln \left( \frac{-t}{m_w^2} \right) \ln \left( \frac{-u}{m_w^2} \right),
\]

\[
\frac{1}{\alpha^2_{em}} \Im M^W_{++++}(s,t,u) = -16\pi \left[ \frac{s}{t} \ln \left( \frac{-t}{m_w^2} \right) + \frac{s}{u} \ln \left( \frac{-u}{m_w^2} \right) \right].
\]

The amplitudes \( M^f_{++-+}(s,t,u) \) and \( M^W_{++-+}(s,t,u) \) can be obtained with the use of the following relations

\[
M_{+++}(s,t,u) = M_{++++}(u,t,s),
\]

\[
M_{++-+}(s,t,u) = M_{++++}(t,s,u) = M_{++++}(t,u,s).
\]

Note that \( M_{++++}(s,t,u) = M_{++++}(s,u,t) \), since it depends only on \( s \). In particular, we get

\[
\frac{1}{\alpha^2_{em}} e_f^2 \Re M^f_{++-+}(s,t,u) = -8 - 8 \left( \frac{s - t}{u} \right) \ln \left( \frac{s}{-t} \right) \\
- 4 \left[ \left( \frac{t^2 + s^2}{u^2} \right) \ln^2 \left( \frac{s}{-t} \right) + \pi^2 \right],
\]

\[
\frac{1}{\alpha^2_{em}} e_f^2 \Im M^f_{++-+}(s,t,u) = 8\pi \left[ \frac{s - t}{u} + \frac{t^2 + s^2}{u^2} \ln \left( \frac{s}{-t} \right) \right],
\]
and

\[
\frac{1}{\alpha^2_{em} e^4_f} \Re M^{f}_{++-+}(s, t, u) = -8 - 8 \left( \frac{u - s}{t} \right) \ln \left( \frac{-u}{s} \right) - 4 \left[ \left( \frac{s^2 + u^2}{t^2} \right) \ln^2 \left( \frac{-u}{s} \right) + \pi^2 \right],
\]

\[
\frac{1}{\alpha^2_{em} e^4_f} \Im M^{f}_{++-+}(s, t, u) = -8\pi \left[ \frac{u - s}{t} + \frac{s^2 + u^2}{t^2} \ln \left( \frac{-u}{s} \right) \right].
\]  

(29)

The explicit formulas for \( M^{W}_{++-+}(s, t, u) \) have been already derived in [85].

\[
\frac{1}{\alpha^2_{em}} \Re M^{W}_{++-+}(s, t, u) = 12 + 12 \left( \frac{s - t}{u} \right) \ln \left( \frac{s}{-t} \right) + 16 \left( 1 - \frac{3ts}{4u^2} \right) \ln^2 \left( \frac{s}{-t} \right) + 16 \left[ \frac{u}{t} \ln \left( \frac{-u}{m^2_W} \right) \ln \left( \frac{-t}{m^2_W} \right) + \frac{u}{s} \ln \left( \frac{-u}{m^2_W} \right) \ln \left( \frac{s}{m^2_W} \right) + \frac{u^2}{ts} \ln \left( \frac{-t}{m^2_W} \right) \ln \left( \frac{s}{m^2_W} \right) \right],
\]

\[
\frac{1}{\alpha^2_{em}} \Im M^{W}_{++-+}(s, t, u) = -\pi \left[ 12 \left( \frac{s - t}{u} \right) + 32 \left( 1 - \frac{3ts}{4u^2} \right) \ln \left( \frac{s}{-t} \right) + 16 \frac{u}{s} \ln \left( \frac{-u}{m^2_W} \right) + 16 \frac{u^2}{ts} \ln \left( \frac{-t}{m^2_W} \right) \right].
\]  

(30)

The explicit expressions for \( M^{f}_{++-+}(s, t, u) \) and \( M^{W}_{++-+}(s, t, u) \) are also known [86]-[87]

\[
\Re M^{f}_{++-+}(s, t, u) = 8\alpha^2_{em} e^4_f, \quad \Im M^{f}_{++-+}(s, t, u) = 0,
\]

\[
\Re M^{W}_{++-+}(s, t, u) = -12\alpha^2_{em}, \quad \Im M^{W}_{++-+}(s, t, u) = 0.
\]  

(31)

Finally, neglecting term \( m^2_f/s, m^2_f/t \) and \( m^2_f/u \), we have

\[
M^{f}_{++-+}(s, t, u) \simeq M^{W}_{++-+}(s, t, u),
\]

(32)

\[
M^{W}_{++-+}(s, t, u) \simeq M^{W}_{++-+}(s, t, u).
\]  

(33)

A. Compton backscattered photons

In addition to \( e^+e^- \) collisions, \( e\gamma \) and \( \gamma\gamma \) interactions with real photons can be examined at the CLIC. For this process, real photons could be constructed by the Compton backscattering of laser photons off linear electron beam. Most of these real photons have high energy. The Compton backscattered (CB) photons give a spectrum which is defined as follows [88]-[89]

\[
f_{\gamma/e}(x) = \frac{1}{g(\zeta)} \left[ 1 - x + \frac{1}{1 - x} - \frac{4x}{\zeta(1 - x)} + \frac{4x^2}{\zeta^2(1 - x)^2} \right],
\]  

(34)
where
\[
g(\zeta) = \left(1 - \frac{4}{\zeta} - \frac{8}{\zeta^2}\right)\log(\zeta + 1) + \frac{1}{2} + \frac{8}{\zeta} - \frac{1}{2(\zeta + 1)^2},
\]  
(35)

and
\[
x = \frac{E_\gamma}{E_e}, \quad \zeta = \frac{4E_0E_e}{m_e^2}.
\]  
(36)

Note that \(x_{\text{max}} = \zeta/(1 + \zeta)\). Here \(E_\gamma\) is the energy of the backscattered photon, \(E_0\) and \(E_e\) are energies of the incoming laser photon and electron, respectively. \(x_{\text{max}}\) reaches 0.83 when \(\zeta = 4.8\).

We start from the case when the initial photons in the subprocess \(\gamma\gamma \to \gamma\gamma\) are the CB photons, whose spectrum is given by formulas (34)-(36). The Feynman diagrams for this process are shown in Fig. 1. Let us note that in our calculations we take into account \(W\)-loop and fermion-loop contributions as the main SM background. The possible background with fake photons from decays of \(\pi^0\), \(\eta\) and \(\eta'\) is negligible in the signal region.

FIG. 1: The Feynman diagrams describing light-by-light virtual production of the axion-like particle \(a\).

The differential cross sections for the process \(\gamma\gamma \to \gamma\gamma\) for the CB initial photons is shown in Fig. 2 as functions of the transverse momenta of the final photons \(p_t\). The ALP mass \(m_a\) and its coupling \(f\) are chosen to be equal to 1200 GeV and 10 TeV, respectively. In order to reduce the SM background, we have imposed the cut \(W = m_{\gamma\gamma} > 200\) GeV. The curves are presented for two values of the ALP branching into two photons \(\text{Br} = \text{Br}(a \to \gamma\gamma)\). For this differential cross sections, the virtual production of the ALP dominates the SM light-by-light subprocess for \(p_t > 100\) GeV region. The total cross sections \(\sigma(p_t > p_{t,\text{min}})\) as functions of the minimal transverse momenta of the final photons \(p_{t,\text{min}}\) are shown in Fig. 3. It can be seen from this figure that the deviation from the SM gets higher as the \(p_t\)-cut increases. Moreover, while the SM value decreases until the value of \(p_{t,\text{min}} = 500\) GeV, the total cross section value is almost unchanged.
FIG. 2: The differential cross sections for the process $\gamma\gamma \rightarrow \gamma\gamma$ at the CLIC for the CB initial photons with the ALP mass $m_a = 1200$ GeV, coupling constant $f = 10$ TeV, and cut $W > 200$ GeV imposed on the photon invariant mass $W$. The invariant energy is equal to $\sqrt{s} = 1500$ (3000) GeV in the left (right) panel. The curves both for $\text{Br}(a \rightarrow \gamma\gamma) = 1.0$ and $\text{Br}(a \rightarrow \gamma\gamma) = 0.1$ are shown. The dashed lines denote the SM contributions.

FIG. 3: The same as in Fig. [2] but for the total cross sections as functions of the transverse momenta cutoff $p_{t,\text{min}}$ of the final photons.

Fig. [4] demonstrates the dependence of the total cross sections on the ALP mass at two fixed values of the ALP coupling $f = 10$ TeV (in the left panel) and $f = 100$ TeV (in the right panel). As one can see, they are very sensitive to the parameter $m_a$ in the interval $m_a = 1000 - 2500$ GeV, in which it is approximately two orders of magnitude greater than
for $m_a$ outside of this mass range. It is not surprising that this is the region where the value of the ALP coupling constant $f$ is mostly restricted by the light-by-light process, see Figs. 5. In these figures we have applied the cut $p_t > 500$ GeV in order to suppress SM cross sections relative to total cross sections as we analyzed from the Fig. 3. In this analysis, we have used the following statistical significance ($SS$) formula:

$$SS = \sqrt{2[(S + B) \ln(1 + S/B) - S]}.$$  \hspace{1cm} (37)

Here $S$ and $B$ are the numbers of the signal and background events, respectively. It can be obtained that $SS \simeq S/\sqrt{B}$ for $S \ll B$. It is assumed that the uncertainty of the background is negligible. Our obtained exclusion regions should be compared with the current exclusion regions on the ALP coupling and ALP mass presented in Fig. 4 especially with that obtained for the process $pp \rightarrow p(\gamma\gamma \rightarrow \gamma\gamma)p$ at the LHC [71]. This comparison demonstrates the great potential of the light-by-light scattering at the CLIC. The estimation for the 95% C.L. parameter exclusion region is presented in Fig. 3 for $\sqrt{s} = 1500$ GeV and $L = 2500$ fb$^{-1}$ using $\text{Br}(a \rightarrow \gamma\gamma) = 1, 0.5,$ and $0.1$. The best bounds are obtained for $\text{Br}(a \rightarrow \gamma\gamma) = 1$. This figure shows the exclusion $f^{-1} < 5.5 \times 10^{-2}$ TeV$^{-1}$ for the ALP mass interval 10 GeV–800 GeV, while the light-by-light scattering at the LHC gives the bound $f^{-1} < 4 \times 10^{-1}$ TeV$^{-1}$ for the same mass interval. Moreover, we have obtained the very strong upper bound on $f^{-1}$ which is of order of $10^{-4}$ TeV$^{-1}$ for the mass range $m_a = 1000 - 1200$ GeV. The best limit for the $pp \rightarrow p(\gamma\gamma \rightarrow \gamma\gamma)p$ is of the order of $10^{-2}$ TeV$^{-1}$ for the mass range $m_a = 600 - 800$ GeV, as seen from Fig. 7. Fig. 3 is the same as Fig. 5, but for $\sqrt{s} = 3000$ GeV and $L = 5000$ fb$^{-1}$. It demonstrates the wider exclusion regions. In particular, it shows the exclusion $f^{-1} < 3 \times 10^{-2}$ TeV$^{-1}$ for the ALP mass interval 10 GeV–800 GeV. The stronger bounds on $f^{-1}$ have been obtained which are of the order of $10^{-4}$ TeV$^{-1}$ for the mass range $m_a = 1000 - 2400$ GeV and $\text{Br}(a \rightarrow \gamma\gamma) = 1$.

B. Weizsäcker-Williams photons

The photon-induced process at the CLIC can be also studied in the Weizsäcker-Williams approximation (WWA) [91]-[93]. Numerical calculations can be easily made using this method. The WWA is also useful in experimental studies due to it allows us to find cross sections for the $e^-e^+ \rightarrow e^-Xe^+$ process via subprocess $\gamma\gamma \rightarrow X$ [94]. In the literature, there
FIG. 4: The total cross sections for the process $\gamma\gamma \to \gamma\gamma$ at the CLIC for the CB initial photons as functions of the ALP mass $m_a$ for $f = 10$ TeV and $f = 100$ TeV with two values of $\text{Br}(a \to \gamma\gamma)$.

are many papers, including photon-induced processes, see, for instance \cite{95-103}.

In the WWA, the photons have the following spectrum

$$f_{\gamma/e}(x) = \frac{\alpha}{\pi E_e} \left[ \left( \frac{1 - x + x^2/2}{x} \right) \log \frac{Q^2_{\text{max}}}{Q^2_{\text{min}}} - \frac{m_e^2 x}{Q^2_{\text{min}}} \left( 1 - \frac{Q^2_{\text{min}}}{Q^2_{\text{max}}} \right) \right].$$  \hspace{1cm} (38)

Here $m_e$ shows the electron mass, $Q^2 = -q^2$ is the photon virtuality, $x = E_\gamma/E_e$ is the ratio of the photon energy to the energy of the incoming electron, and $\alpha$ is the fine structure constant.

In our case, $X = \gamma\gamma$ (see Fig. 8), and the WWA spectrum of the photons is given by formula (38). In addition to the backgrounds mentioned in subsection A, possible backgrounds also came from $\gamma\gamma \to e^+e^-\gamma\gamma$ and $ZZ$-induced processes. The first one was estimated in \cite{81} to be below 1%. The second background may not be taken into account since the $ZZ$ luminosity is approximately 100 times smaller than the $\gamma\gamma$ luminosity \cite{104}.

The results of our calculations of the differential and total cross sections are presented in Figs. 9 and 10. They should be compared with the cross sections for the process induced by the CP photons shown in Figs. 2 and 3. The WWA cross sections have appeared to be approximately $10^4$ ($10^2$) times smaller that the CB cross sections for $\sqrt{s} = 1500$ GeV ($\sqrt{s} = 3000$ GeV).

The same one can see in Fig. 11, where the total cross section for the process $e^+e^- \to e^+\gamma\gamma e^- \to e^+a e^- \to e^+\gamma\gamma e^-$ is shown as a function of the ALP mass $m_a$. For the ALP branching ratio $\text{Br}(a \to \gamma\gamma) = 1$, there are big bumps in the curves in the mass region 1000
FIG. 5: The 95% C.L. CLIC exclusion region for the process $\gamma\gamma \rightarrow \gamma\gamma$ for the CB initial photons with the invariant energy $\sqrt{s} = 1500$ GeV, cut $W > 200$ GeV on the photon invariant mass, integrated luminosity $L = 2500$ fb$^{-1}$, and different values of $\text{Br}(a \rightarrow \gamma\gamma)$.

GeV–3000 GeV for both values of the collision energy $\sqrt{s}$.

Fig. 12 gives the 95% C.L. CLIC exclusion region for the $m_a - f^{-1}$ in the case when the subprocess $\gamma\gamma \rightarrow \gamma\gamma$ is induced by the WWA photons with $\sqrt{s} = 1500$ GeV and $L = 2500$ fb$^{-1}$. As one can see from this figure, the bounds are of the order of $10^{-1}$ TeV$^{-1}$ in the mass regions 10 GeV–1000 GeV. In the narrow mass region 1000 GeV–1500 GeV it is obtained to be of the order of $10^{-3}$ TeV$^{-1}$. Similarly, Fig. 13 shows the 95% C.L. exclusion region in the $(m_a - f^{-1})$ plane for $\sqrt{s} = 3000$ GeV and $L = 5000$ fb$^{-1}$. In the mass region 10 GeV-1000 GeV the bounds on $f^{-1}$ are of the order of $10^{-1}$ TeV$^{-1}$. In the mass range 1000 GeV–1500 GeV, these bounds reach the value $1 \times 10^{-3}$ TeV$^{-1}$. For both $\sqrt{s}$, the bounds are much weaker than those for the CB initial photons.
III. CONCLUSIONS

We have studied the possibility to search for heavy axion-like particles in the process $\gamma\gamma \to \gamma\gamma$ with Compton backscattered initial photons and process $e^+e^- \to e^+e^-\gamma\gamma$ induced by light-by-light scattering with Weizsäcker-Williams initial photons at the CLIC. The calculations were made for the collision energy $\sqrt{s} = 1500$ GeV (2nd stage of the CLIC) and integrated luminosity $L = 2500$ fb$^{-1}$, as well as for the energy $\sqrt{s} = 3000$ GeV and integrated luminosity $L = 5000$ fb$^{-1}$ (3rd stage of the CLIC). It was assumed that the ALP interacts with photons via CP-odd term in Lagrangian (13).

We 95% C.L. exclusion regions in the plane $(m_a - f^{-1})$, where $m_a$ is the ALP mass, $f^{-1}$ ALP-photon coupling, are given. The results are presented for two values of $\sqrt{s}$ and $L$ as functions of the ALP branching ratio into photons $\text{Br}(a \to \gamma\gamma)$. The best bounds are obtained for $\text{Br}(a \to \gamma\gamma) = 1$. Our calculations have shown that the numerical results remain almost the same if we take into account the CP-even term instead of the CP-odd...
FIG. 7: The 95% C.L. current exclusion regions for different values of \( \text{Br}(a \to \gamma \gamma) \) \cite{71}.

FIG. 8: The Feynman diagrams describing photon-induced light-by-light virtual production of the axion-like particle \( a \) in \( e^+e^- \) collision.

one in \( \text{[13]} \), with the same coupling \( f^{-1} \).

By comparing our exclusion regions with other collider exclusion regions, we may conclude that the ALP search at the CLIC has the great physics potential of searching for the ALPs, especially, in the mass region 1 TeV – 2.4 TeV, for the collision energy \( \sqrt{s} = 3000 \text{ GeV} \) and integrated luminosity \( L = 5000 \text{ fb}^{-1} \). In particular, our bounds are much stronger that recently obtained bounds for the ALP virtual production in the process \( p(\gamma\gamma \to \gamma\gamma)p \) at the LHC \cite{71}.

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FIG. 9: The differential cross sections for the process $e^+e^- \rightarrow e^+\gamma\gamma e^- \rightarrow e^+a e^- \rightarrow e^+\gamma\gamma e^-$ at the CLIC in the WWA for the initial photons in the subprocess $\gamma\gamma \rightarrow \gamma\gamma$ as functions of the transverse momenta of the final photons $p_t$ for the ALP mass $m_a = 1200$ GeV and coupling constant $f = 10$ TeV. The invariant energy is equal to $\sqrt{s} = 1500$ (3000) GeV on the left (right) panel. The curves for both $\text{Br}(a \rightarrow \gamma\gamma) = 1.0$ and $\text{Br}(a \rightarrow \gamma\gamma) = 0.1$ are shown. The dashed lines denote the SM contributions.

FIG. 10: The same as in Fig. 9, but for the total cross sections as functions of the transverse momenta cutoff $p_{t,\text{min}}$ of the final photons.

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FIG. 11: The total cross sections for the process $e^+e^- \rightarrow e^+\gamma\gamma e^- \rightarrow e^+a e^- \rightarrow e^+\gamma\gamma e^-$ at the CLIC in the WWA for the initial photons as functions of the ALP mass $m_a$ for $f = 10$ TeV and $f = 100$ TeV with two values of $\text{Br}(a \rightarrow \gamma\gamma)$.

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FIG. 12: The 95% C.L. CLIC exclusion region for the process $e^+e^- \rightarrow e^+\gamma\gamma e^- \rightarrow e^+a e^- \rightarrow e^+\gamma\gamma e^-$ with the invariant energy $\sqrt{s} = 1500$ GeV, transverse momentum cut on the final photons $p_t = 500$ GeV, integrated luminosity $L = 2500$ fb$^{-1}$, and different values of Br($a \rightarrow \gamma\gamma$). The WWA for the initial photons in the subprocess $\gamma\gamma \rightarrow \gamma\gamma$ is used.

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