Charge Form Factors of Quark-Model Pions II

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Abstract

Experimental data of the pion charge form factor are well represented by Poincaré invariant constituent-quark phenomenology depending on two parameters, a confinement scale and an effective constituent–quark mass. Pion states are represented by eigenfunctions of mass and spin operators and of the light-front momenta. An effective current density is generated by the dynamics from a null-plane impulse current density. A simple shape of the wave function depending only on the confinement scale is sufficient.

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In a previous letter [1] we demonstrated that simple constituent-quark models of the pion yielded charge form factors in agreement with data for both low $Q^2$ and $Q^2 > 1\text{GeV}^2$. Recently new measurements [3] provided more precise data for $Q^2 < 2\text{GeV}^2$ and new data at higher values of $Q^2$ are expected. The purpose of this paper is to show that simple relativistic constituent-quark models are consistent with all existing data and with a relatively narrow range of form factor values for larger momentum transfers.

The unitary Poincaré representations of confined quark states are specified by mass and spin operators together with the choice of a kinematic subgroup [4, 5]. The Poincaré covariant, conserved, effective current operator $I^\mu(x)$,

$$U(\Lambda, a)I^\mu(x)U^\dagger(\Lambda, a) = I^\nu(\Lambda x + a)\Lambda_\nu^\mu,$$

$$i[P_\mu, I^\mu(x)] = \partial_\mu I^\mu(x) = 0,$$

is generated by the dynamics from an input that is covariant under the kinematic subgroup. The results of [1] were based on the use of null-plane kinematics for the generation of an effective current density. With that choice the kinematic subgroup leaves the null-plane $n \cdot x$, with $n^2 = 0$, invariant. With a convenient choice of the axes the components of $n$ are given by $n = \{1, 0, 0, 1\}$. It is an important feature of null-plane kinematics that the charge form factor does not depend on the pion mass. This feature is essential for the empirical success of constituent-quark phenomenology applied to pion form factors. With point-form kinematics [6] form factors obtain as functions of $\eta := Q^2/4m_\pi^2$, which is large for moderate values of $Q^2$. Thus, with simple wave functions, form factors are much too small for a realistic representation. There is no intent to approximate features of quantum field theory in the construction of such quark models. In particular the representations of constituent-quark states are not meant to approximate Fock-space amplitudes and/or satisfy features of perturbative QCD [8]. Meson mass operators of constituent-quark models may be defined by simple spectral representations

$$\langle \mu, \bar{\mu}, \xi, k_\perp | M | k_\perp', \xi, \bar{\mu}', \mu' \rangle = \sum_{n,j,\mu} \phi_{n,j,\sigma}(\mu, \bar{\mu}, \xi, k_\perp) M_{n,j} \phi_{n,j,\sigma}(\mu, \bar{\mu}, \xi, k_\perp)^*.$$  

Neither the wave function representing the pion state nor the explicit representation, $\langle \mu', \bar{\mu}', \xi', k_\perp' | I^\nu(0) | k_\perp, \xi, \bar{\mu}, \mu \rangle$, of the current density are observable. The observable form factor is invariant under simultaneous unitary transformations, which may preserve the kinematic subgroup [4, 5].
As in ref. the models are specified by input currents with the representation

\[ \langle \xi', k'_\perp, \mu', \bar{\mu}' | \mathbf{n} \cdot I(0) | \xi, k_\perp, \mu, \bar{\mu} \rangle = \delta_{\mu', \mu} \delta_{\bar{\mu}', \bar{\mu}} \delta(\xi' - \xi) \delta(k'_\perp - k_\perp - (1 - \xi)Q_\perp), \]  

(3)

and a representation of the pion state by wave functions \( \phi(\xi, k_\perp, \mu, \bar{\mu}) \) which is proportional to a radial wave function \( u(k^2) \) and Melosh rotation matrices

\[ \phi(\xi, k_\perp, \mu, \bar{\mu}) := \sum_{\mu', \bar{\mu}'} \langle \mu | \mathcal{R}^\dagger(\xi, k_\perp) | \mu' \rangle \langle \bar{\mu} | \mathcal{R}^\dagger(1 - \xi, -k_\perp) | \bar{\mu}' \rangle (\frac{1}{2}, \frac{1}{2}, \mu', \bar{\mu}' | 0, 0) u(k^2). \]  

(4)

The argument \( k^2 \) is related to the null-plane momenta by

\[ k^2 + m_q^2 = \frac{k^2_\perp + m^2}{4\xi(1 - \xi)}. \]  

(5)

The pion charge form factor is a functional of the radial wave function, \( u(k^2) \),

\[ F_\pi(Q^2) = \frac{1}{16\pi} \int_0^1 d\xi \int \frac{d^2 k_\perp}{\xi(1 - \xi)} \mathcal{W}(\xi, \bar{k}_\perp) u(k^2) u(k^2), \]  

(6)

with

\[ \bar{k}_\perp = k_\perp - \frac{1}{2}(1 - \xi)Q_\perp = k'_\perp + \frac{1}{2}(1 - \xi)Q_\perp \]  

(7)

and

\[ \mathcal{W} := \sqrt{\frac{\xi(1 - \xi)}{(m_q^2 + k^2_\perp)(m_q^2 + k'^2_\perp)}} \frac{m^2_q + \bar{k}^2_\perp - \frac{1}{4}(1 - \xi)^2Q^2}{\xi(1 - \xi)}. \]  

(8)
FIG. 2: Wave function dependence of pion form factors at high $Q^2$

In [1] a Gaussian shape

$$u(k^2) = \sqrt{\frac{4}{\pi b^3}} \exp\left(-\frac{k^2}{2b^2}\right)$$  \hspace{1cm} (9)

was used for numerical convenience. We expect that a rational shape

$$u(k^2) = \sqrt{\frac{32}{\pi b^3}} \left(\frac{1}{1 + k^2/b^2}\right)^2$$  \hspace{1cm} (10)

may specify a better model. A wave function of essentially the same shape can also be generated by the equation

$$\left(2 \sqrt{m_q^2 + k^2 + \alpha r} - \frac{\beta}{r} + \gamma \bar{s}_q \cdot \vec{s}_q e^{-\lambda r^2} - m_0\right) u(r) = 0$$  \hspace{1cm} (11)

with the parameters adjusted for that purpose. Conventional QCD motivated mass operators [10], designed to fit meson spectra, produce wave functions that require substantial modification of the current [11].

The form factors shown in Figs. 1 and 2 are calculated assuming $m_q = .23$ GeV, $b = .35$ GeV and .43 GeV for the wave functions eq. [9] (dash-dot line) and eq. [10] (solid line) respectively. The dash line is obtained using the ground-state solution of eq. [11] with $\alpha = .1$ GeV$^2$, $\beta = .4$, $\gamma = .229$ GeV and $\lambda = .3$ GeV. The shapes of the three wave functions
are compared in Fig. 3. All three parameterizations are in agreement with existing data. We expect that for larger values of $Q^2$ the form factors obtained with the Gaussian wave function (9) will be in disagreement with future measurements. The results are in agreement with the QCD approximations of Maris and Tandy (12) for $Q^2 < 2\text{GeV}^2$, and with their expectations for larger values of $Q^2$.

For the pion decay constant, (11, 13)

$$f_\pi = \frac{\sqrt{3}g_A^q}{8\pi^2} \int_0^1 d\xi \int \frac{d^2k_\perp}{\xi(1-\xi)} \frac{m_q}{\sqrt{M_0}} u(k^2), \quad M_0^2 := \frac{m_q^2 + k_\perp^2}{\xi(1-\xi)}.$$  \hspace{1cm} (12)

The three wave functions yield the values 92.5 MeV, 101.5 MeV and 101.8 MeV with $g_A^q = 1$.

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