PHOTOEVAPORATION OF CIRCUMSTEMELLAR DISKS REVISITED: THE DUST-FREE CASE

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ABSTRACT

Photoevaporation by stellar ionizing radiation is believed to play an important role in the dispersal of disks around young stars. The mass-loss model for dust-free disks developed by Hollenbach et al. is currently regarded as the conventional one and has been used in a wide variety of studies. However, the rate in this model was derived using the crude so-called 1+1D approximation of ionizing radiation transfer, which assumes that diffuse radiation propagates in a direction vertical to the disk. In this study, we revisit the photoevaporation of dust-free disks by solving the two-dimensional axisymmetric radiative transfer for steady-state disks. Unlike that solved by the conventional model, we determine that direct stellar radiation is more important than the diffuse field at the disk surface. The radial density distribution at the ionization boundary is represented by a single power law with index $-3/2$ in contrast to the conventional double power law. For this distribution, the photoevaporation rate from the entire disk can be written as a function of the ionizing photon emissivity $\Phi_{\text{EUV}}$ from the central star and the disk outer radius $r_d$ as follows: $\dot{M}_{\text{PE}} = 5.4 \times 10^{-5} (\Phi_{\text{EUV}}/10^{49} \text{ s}^{-1})^{1/2} (r_d/100 \text{ AU})^{1/2} M_{\odot} \text{ yr}^{-1}$. This new rate depends on the outer disk radius rather than on the gravitational radius as in the conventional model, because of the enhanced contribution to the mass loss from the outer disk annuli. In addition, we discuss its applications to present-day as well as primordial star formation.

Key words: accretion, accretion disks – H II regions – protoplanetary disks – radiative transfer – stars: formation – stars: massive – stars: Population III

Online-only material: color figures

1. INTRODUCTION

Stars are formed by the gravitational collapse of pre-stellar cores with nonzero angular momentum. As a natural result, disks are formed around newborn stars, and most of the materials are accreted through them. The final stellar mass at the time of its formation is set when the disk dissipates. Moreover, disk dissipation determines the formation environments of planets, which are formed inside disks at the final stage of low-mass star formation. Photoevaporation is currently considered to be a promising dissipation mechanism by which the disk gas escapes from the gravitational binding of a star as a result of heating by ultraviolet (UV) radiation from the star or external sources. While irradiation by nearby stars can be important in the case of low-mass star formation in a dense cluster (e.g., Johnstone et al. 1998; Adams et al. 2004; Fatuzzo & Adams 2008; Holden et al. 2011; Thompson 2013), the photoevaporation by radiation from the central star should dominate in isolated or massive star formation. In this study, we focus on the latter process.

The photoevaporation of protoplanetary disks around low-mass stars has been studied since the 1990s. Early studies concentrated on photoevaporation by extreme-ultraviolet (EUV), i.e., ionizing, radiation (Shu et al. 1993; Clarke et al. 2001) from the central star. However, photoevaporation by far-ultraviolet (FUV) radiation and/or X-rays is now considered to be a more dominant mechanism for disks around low-mass stars (Gorti & Hollenbach 2009; Owen et al. 2012) on the basis that its timescale to operate, $\sim 3$ Myr, is consistent with the observational disk-dissipation timescale. However, this theory remains in dispute.

EUV photoevaporation has attracted significant attention recently in the context of primordial star formation in the early universe. Although primordial stars were first speculated to be $\sim 1000 M_{\odot}$ because of their natal pre-stellar core masses (Bromm et al. 1999; Omukai & Palla 2003), recent studies that consider stellar feedback onto accretion flows via the disk demonstrated that the EUV effect on the infalling gas becomes significant for masses greater than 10 $M_{\odot}$, and that photoevaporation terminates mass accretion onto the newborn star at a mass of 20–100 $M_{\odot}$ (McKee & Tan 2008; Hosokawa et al. 2011, 2012; Stacy et al. 2012). A similar mechanism can be applied to present-day massive star ($> 20 M_{\odot}$) formation, although this topic has not been studied in depth thus far.

In studies of disk evolution by EUV photoevaporation, the formula derived by Hollenbach et al. (1994, hereafter HILS94) has often been used. By calculating the approximate radiative transfer (RT) for assumed steady-state density distributions around circumstellar disks, HILS94 derived the mass-loss rate from dust-free disks as

$$\dot{M}_{\text{PE},\text{H94}} = 1.3 \times 10^{-5} \left( \frac{\Phi_{\text{EUV}}}{10^{49} \text{ s}^{-1}} \right)^{1/2} \left( \frac{M_*}{10 M_{\odot}} \right)^{1/2} M_{\odot} \text{ yr}^{-1},$$

where $\Phi_{\text{EUV}}$ is the EUV photon emission rate, and $M_*$ is the mass of the central star. Although HILS94 emphasized the importance of diffuse radiation, i.e., the re-emitted radiation from the ionized atmosphere above the disk, in ionizing the disk surface, such radiation was not adequately treated in their calculation. To save computational expense, they adopted the 1+1D approximation for RT, where diffuse radiation is assumed to propagate in a direction vertical to the disk. In reality, of course, diffuse radiation also has radial components, because the density and ionization degree have radial gradients in the
atmosphere. In this study, to accurately treat diffuse radiation, we calculate the axisymmetric two-dimensional (2D) RT and re-examine the results of HJLS94.

In Section 2, we describe the basics of photoevaporation and our model for calculation. In Section 3, we present our results of the 2D RT calculation, which significantly differs from the previous 1+1D result. In Section 4, we analytically interpret the numerical results and derive a new formula for the photoevaporation rate for dust-free disks. Section 5 is reserved for a discussion on our model’s implications on star formation and its validity and limitations. Finally, we summarize our study in Section 6.

2. MODEL

Photoevaporation is a mass-loss process from a circumstellar disk due to radiative heating either by its central star or by external objects (Bally & Scoville 1982). We evaluate the mass-loss rate that results from EUV heating by the central star. In this section, we first describe the basics of photoevaporation, and then we present a schematic view of our model in Figure 1. We use the axisymmetric cylindrical coordinates $R$ for the distance from the symmetric axis and $Z$ for the height from the equatorial plane.

2.1. Photoevaporation

We consider photoevaporation from the disk around a star with mass $M_*$. Because the density above the disk is lower than that in the equatorial plane, EUV radiation irradiates the upper surfaces of the disk, and the irradiated thin surface of the disk is ionized. The ionized gas heats to a temperature of $\sim 10^4$ K, which increases the sound speed to $c_{s, \text{H} II} \approx 10$ km s$^{-1}$, and forms the H $\pi$ region above the disk. The characteristic scale length, the so-called gravitational radius, is defined as the radius in which the Keplerian velocity ($\simeq$ escape velocity) is equal to $c_{s, \text{H} II}$:

$$r_g = \frac{G M_*}{c_{s, \text{H} II}^2} \approx 70 \left( \frac{M_*}{10 M_\odot} \right) \text{AU}. \quad (2)$$

Inside the gravitational radius, $r \lesssim r_g$, the ionized gas is gravitationally bound and an ionized atmosphere is created above the un-ionized disk. However, outside the gravitational radius, $r \gtrsim r_g$, the thermal energy of the ionized gas is greater than its gravitational energy; thus, the ionized gas is unbound. Therefore, the ionized gas flows away from the disk surface, and the gas is photoevaporated. Because the photoevaporation flow is driven by thermal pressure, the flow velocity is approximately given by the sound speed $c_{s, \text{H} II}$. Although the definition of the gravitational radius slightly changes if hydrodynamic effects such as gas pressure, radiation pressure, and angular momentum are included (Lifshitz 2003; Font et al. 2004; McKee & Tan 2008), we here use Equation (2) as the gravitational radius for easier comparison with HJLS94. Our results in Section 4 show that the photoevaporation rate does not depend on the gravitational radius. This justifies our approximation for $r_g$.

Because the photoevaporation flow velocity is $\gtrsim c_{s, \text{H} II}$, the mass-loss flux per unit area at radius $R$ is

$$\dot{M}_{\text{PE}}(R) = m_0 c_{s, \text{H} II} n_0(R), \quad (3)$$

where $m_0$ is the proton mass, and $n_0(R)$ is the number density at the bottom of the ionization layers, which are the boundary between the ionized H $\pi$ region and the un-ionized disk. We hereafter refer to $n_0(R)$ as the base density. The evaporation flow launches from the disk outside $r_g$ to form an annulus with $r_g < R < r_d$, where $r_d$ is the outer radius of the disk. By summing the contribution from both the upper and lower surfaces, the total photoevaporation rate from the disk is

$$\dot{M}_{\text{PE}} = 2 \int_{r_g}^{r_d} 2\pi R \dot{M}_{\text{PE}}(R) dR = 4\pi m_0 c_{s, \text{H} II} \int_{r_g}^{r_d} n_0(R) R dR. \quad (4)$$

For a given base-density distribution $n_0(R)$, we can calculate the photoevaporation rate. HJLS94 estimated the base-density distribution from the RT calculation with the 1+1D approximation and derived the photoevaporation rate in Equation (1). In this study, we repeat a similar analysis using the 2D RT calculation.

2.2. Ionized Gas Structure

Following HJLS94, we assume (1) a steady-state density distribution and (2) a thin neutral gas disk. Based on these assumptions, the ionization front is located at $Z = 0$. We confirmed that our results with this thin-disk approximation are consistent with the hydrodynamic simulation for a finite-thickness disk (Section 5.2). Although the exact temperature of the ionized gas slightly depends on detailed heating and cooling processes, we retained the typical value of $T_{\text{H} II} = 10^4$ K.

2.2.1. Vertical Structure

Inside the gravitational radius ($R < r_g$), the bound ionized gas forms an atmosphere above the disk. The gas is vertically hydrostatic in this region:

$$n(R, Z) = n_0(R) \exp \left( -\frac{Z^2}{2H^2} \right), \quad (5)$$
with the scale height expressed as

$$H(R) = \frac{c_{s,HII}}{\Omega K} = r_g \left(\frac{R}{r_g}\right)^{3/2}. \quad (6)$$

On the contrary, the ionized gas will evaporate away in the outer flow region ($R > r_g$). In this region, we assume that the density is vertically constant:

$$n(R, Z) = n_0(R). \quad (7)$$

Although this assumption fails in the upper region of $Z > R$, as shown in the hydrodynamic simulation by Font et al. (2004), we apply the concept for simplicity and for a comparison with HJLS94. In consideration of our result that the density distribution in the lower layers of $Z \ll R$ is more important, this assumption does not significantly affect our conclusion.

2.2.2. Radial Structure

The vertical structure of the ionized region, which includes the atmosphere and evaporation flow, is given by Equations (6) and (7). To specify the radial density distribution, we need the base-density distribution, $n_0(R) = n(R, Z = 0)$. Here, we assume that the disk is neutral and geometrically thin and that the ionized gas pervades above the neutral disk. Thus, for a plausible density profile, the ionization front must coincide with the disk surface at $Z = 0$. With densities higher than the plausible value, a neutral zone appears above the disk. This situation is inconsistent, because the vertical structure at $Z > 0$ is only valid for hot ionized gas (Equations (6) and (7)). On the contrary, if the density is very low, ionizing photons can reach the disk and ionize its upper layer, which also conflicts with our assumption of a neutral gas disk. In the following equation, by calculating the ionization structure for various base-density profiles, we search for the plausible density profile, which places the ionization front at $Z = 0$.

In searching for the plausible base-density profile, we assume the piecewise power-law distribution with different exponent indices inside or outside the gravitational radius as in HJLS94:

$$n_0(R) = f n_{g, H94} \left(\frac{R}{r_g}\right)^{-p}, \quad (8)$$

where

$$n_{g, H94} = 1.8 \times 10^7 \left(\frac{\Phi_{EUV}}{10^{49} \text{s}^{-1}}\right)^{1/2} \left(\frac{r_g}{10^{15} \text{cm}}\right)^{-3/2} \text{cm}^{-3}. \quad (9)$$

$$p = \begin{cases} 
  p_{in} & \text{for } R < r_g, \\
  p_{out} & \text{for } R > r_g, 
\end{cases} \quad (10)$$

where $f$, $p_{in}$, and $p_{out}$ are dimensionless parameters. With the parameters set to $f = 0.9$, $p_{in} = 1.5$, and $p_{out} = 2.5$, our base density reproduces that of the HJLS94 model. The normalization density $n_{g, H94} \equiv C_{H94}(3\Phi_{EUV}/4\pi \alpha m_{p} r_{g}^{2})^{1/2}$ is that of HJLS94, which they have derived analytically, where $\alpha m_{p}$ is the recombination coefficient to the excited states (so-called case B) and $C_{H94} \simeq 0.2$ is the correction factor used to reproduce their numerical results. We search for the plausible base-density profile using following process. For a given density profile, we calculate the transfer of ionizing radiation. If the gas at the disk surface ($Z = 0$) is not ionized, we reduce the base density. On the contrary, if the disk surface is well ionized, i.e., the ionizing photons have not been consumed before that point, we elevate the base density. In this manner, we obtain the plausible base-density profile iteratively. In conclusion, we determine the parameters for the plausible base density to be $f = 0.9$ and $p_{in} = p_{out} = 1.5$, which differ from those in the HJLS94 model (Section 3). Because the outer density distribution ($p_{out} = 1.5$) is shallower than that in the HJLS94 model ($p_{out} = 2.5$), the total mass-loss rate by photoevaporation depends on the total area of the evaporating annulus and thus the disk size (Section 4), unlike in HJLS94.

2.3. Radiation Transfer Calculation

We adopt the gray approximation for RT, where the frequency of ionizing photons is represented by the mean value. The equation of RT along a ray is

$$\frac{dI}{ds} = -(1 - x)n\sigma_I I + \frac{\alpha_I x^2 n^2}{4\pi} \epsilon, \quad (11)$$

where $x$ is the ionization degree, $\sigma_I$ is the cross section of a hydrogen atom, $I$ is the irradiance intensity, $\alpha_I$ is the radiative recombination coefficient for the ground state, and $\epsilon$ is the mean energy of the ionization photons. The first term on the right-hand side represents the photon consumption by ionization, and the second term indicates the re-emission by recombinations directly to the ground state. Following HJLS94, we neglect absorption and scattering of dust grains. In the case of primordial star formation, as well as present-day star/planet formation, this may be an appropriate assumption if the grains have significantly settled toward the equatorial plane. However, the dust effects should be examined in future studies.

The ionization degree is obtained from the balance between photoionization and recombination,

$$\frac{4\pi \chi_{HII} n \sigma_{HI} I}{\epsilon} = \alpha_A x_{HI}^2 n^2, \quad (12)$$

where the mean intensity is

$$J = \frac{1}{4\pi} \int I d\Omega \quad (13)$$

and $\alpha_A$ is the radiative recombination coefficient for all levels (so-called case A). Solving the RT equation (Equation (11)) with the photoionization equilibrium (Equation (12)) for a given base-density distribution, we search for the parameter set ($f$, $p_{in}$, $p_{out}$) for the plausible distribution, as described in Section 2.2.2.

2.4. Numerical Settings

We conduct axisymmetric 2D RT calculations in the range of stellar mass $M_* = 10$–$100 \ M_\odot$ and EUV emissivity $\Phi_{EUV} = 10^{49}$–$10^{51} \text{s}^{-1}$. The computational domain is a cylindrical region with radial and vertical coordinates $R < R_{max}$ and $Z < Z_{max}$, where $R_{max} = Z_{max} = r_g$ and $10^{15} \text{cm}$. We assign an EUV source representing the central star at $(R, Z) = (0, 0)$. Although the source radius $R_s$ in our calculation ($>0.2$ AU) is greater than the actual stellar radius ($<0.1$ AU), we verify that the results are unchanged with $R_s$ varying in the range $0.02 R_{max}$ to $0.003 R_{max}$. In the following section, we show the results with the highest
resolution of \( R_\text{g} = 0.003 R_{\text{max}} \) (0.2 AU for \( M_* = 10 M_\odot \) and \( R_{\text{max}} = r_g \)). Because finer structures are present in the inner region, the radial and vertical grids are set to be spaced logarithmically. The EUV source radius is resolved with 10 grids and the entire computational domain with 70 grids. We solve the transfer equation with the short characteristic method along rays that are tangential to cylinders with \( R = \text{const.} \) (see Section 4 in Stone et al. 1992). The resolution in the zenithal angle is \( \Delta \theta = \pi / 1800 \). Because we solve ray tracing with tangential plane methods, the resolution of the azimuthal angle changes with spatial position. We verify the accuracy of our 2D RT code by conducting the spherical Strömgren test with the same resolution.

3. RESULTS

In this section, we present the 2D calculation results, which indicate that the exponent for the plausible base density is the same inside and outside the gravitational radius: \( p_{\text{in}} = p_{\text{out}} = 1.5 \). While this value is in disagreement with the conventional value reported by HJLS94, it is consistent with the results of the radiative hydrodynamic simulation by Hosokawa et al. (2011; Section 5.2).

First, as in the fiducial case, we show the results with stellar parameters fixed at \( M_* = 10 M_\odot \) and \( \Phi_{\text{EUV}} = 10^{49} \text{ s}^{-1} \). From this result, we determine that the plausible base-density distribution is that with \( f = 0.9 \) and \( p_{\text{in}} = p_{\text{out}} = 1.5 \), for which the ionization front is located at the equatorial plane \( Z = 0 \). Figure 2 shows the spatial profiles of the ionization degree, neutral fraction, and diffuse radiation field in this case. Inside the gravitational radius \( (R < r_g \approx 70 \text{ AU}) \), the density profile nearly agrees with that of the HJLS94 model \( f = 1, p_{\text{in}} = 1.5 \). However, outside the gravitational radius \( (R > r_g) \), the density determined with our method is higher, that is, the disk surface \( (Z = 0) \) can be ionized despite a density higher than that determined by the HJLS94 model.

As assumed in HJLS94, the fraction of diffuse radiation \( J_{\text{diff}} / J \) is somewhat higher at the disk surface because of re-emission in the atmosphere (Figure 2, bottom left panel). However, this fraction reaches a maximum of \( \sim 0.5 \); thus, direct stellar radiation dominates in the entire region. It should be noted that the diffuse radiation flux, \( F_{\text{diff}} \), has a large radial component (Figure 2, bottom right panel). Such a radiation field cannot be properly expressed by 1+1D treatment, which emphasizes the importance of the 2D calculation.

Figure 3 shows the results for the most plausible density distribution and for other cases. Here, to determine the dependence on the density normalization factor \( f \) and the inner density exponent \( p_{\text{in}} \), the computational domain is limited inside the gravitational radius, i.e., \( R_{\text{max}} = Z_{\text{max}} = r_g \). For a density higher than the plausible value \( (f > 0.9 \text{ or } p_{\text{in}} > 1.5) \), the ionization front does not reach the disk surface at \( Z = 0 \), which is inconsistent with our assumption that the gas above the disk is ionized and has a high temperature. These structures cannot be in the steady state.
because the cold neutral gas would settle with the disk with the base density of ionized gas decreasing until the ionization front reaches the disk surface.

Figure 3 shows that, in all three cases, the ionization boundary $q_{\text{f}}$, where the neutral fraction abruptly increases from $x_{\text{HI}} \ll 1$ to $\sim 1$, is located at $\tau \sim 1$ from the central star. This indicates that direct stellar radiation is dominant in ionization, and that the gas is ionized as far as it reaches. For the plausible distribution with $f = 0.9$, $p_{\text{in}} = p_{\text{out}} = 1.5$, the optical depth remains $\sim 1$ in a wide range of the radius, which indicates that the ionization front is located at $Z = 0$ at all radii.

To see the dependence on the outer density exponent $p_{\text{out}}$, Figure 4 shows the neutral fraction and the optical depth for the cases with five different values of $p_{\text{out}} (0.5, 1, 1.5, 2, \text{and} 2.5)$, where the other parameters are fixed at $f = 0.9$, $p_{\text{in}} = 1.5$. Inside the gravitational radius, both $x_{\text{HI}}$ and $\tau$ behave similarly for various $p_{\text{out}}$ values, which indicates that the inner radiation field is not affected by the outer field. For $p_{\text{out}} < 1.5$, the ionization front emerges above the disk when $\tau$ becomes $\sim 1$. For $p_{\text{out}} > 1.5$, the gas is completely ionized at the disk surface. Thus, the case with $p_{\text{out}} = 1.5$ gives the plausible density distribution, which produces the ionization front at the disk surface.

Thus far, we have discussed cases with stellar parameters $M_* = 10 \, M_{\odot}$ and $\Phi_{\text{EUV}} = 10^{49}$ s$^{-1}$. The same plausible density distribution is indicated for other combinations of $(M_*, \Phi_{\text{EUV}})$. Figure 5 shows the neutral fraction and optical depth for the density distribution of $f = 0.9$, $p_{\text{in}} = p_{\text{out}} = 1.5$ for various stellar parameters $(M_*, \Phi_{\text{EUV}})$. It should be noted that the horizontal axis is the radial distance normalized by the gravitational radius. Although the neutral fraction depends on the stellar parameters, curves for the optical depth, which remain at $\sim 1$ for a wide range in radius, completely overlap. This result indicates that the density distribution of $f = 0.9$, 

![Figure 3](image_url)

**Figure 3.** Results with application of various $f$ and $p_{\text{in}}$ showing (left panels) the ionization degree in the $R$–$Z$ plane and (right panels) the radial profiles of the neutral fraction ($x_{\text{HI}}$) and the optical depth from the central object ($\tau$) at the near base of $Z = R_*$. The density parameters are $f = 0.9$, $p_{\text{in}} = 1.5$, which is the plausible case, $f = 3$, $p_{\text{in}} = 1.5$, and $f = 0.9$, $p_{\text{in}} = 2$ from top to bottom, respectively. The ionization boundary is always located at $r \simeq 1$.

(A color version of this figure is available in the online journal.)
Figure 4. Results with application of various \( p_{\text{out}} \) values showing (top panel) the radial profiles of the neutral fraction and (bottom panel) the optical depth from the central object at the near base of \( Z = R_\ast \). The dashed line indicates the gravitational radius, \( R = r_g \).
(A color version of this figure is available in the online journal.)

\[ p_{\text{in}} = p_{\text{out}} = 1.5 \] is plausible for any combination of \((M_\ast, \Phi_{\text{EUV}})\).

Substituting \( f = 0.9 \) and \( p_{\text{in}} = p_{\text{out}} = 1.5 \) in Equation (10), we obtain the base-density distribution

\[ n_0(R) = 1.6 \times 10^7 \left( \frac{\Phi_{\text{EUV}}}{10^{49} \text{ s}^{-1}} \right)^{0.5} \left( \frac{R}{10^{15} \text{ cm}} \right)^{-1.5} \text{ cm}^{-3}. \]  

(14)

This single power-law distribution significantly differs from that of the conventional HJLS94 model, where the density distribution follows the broken power law with \( p_{\text{in}} = 1.5 \) and \( p_{\text{out}} = 2.5 \). The stellar mass does not explicitly appear in the expression of Equation (14), and the base density depends on \( M_\ast \) only through \( \Phi_{\text{EUV}}(M_\ast) \).

4. ANALYTIC EXPRESSION FOR THE PHOTOEVAPORATION RATE

As demonstrated in the previous section, stellar radiation dominates the diffuse radiation, and the ionization front always resides near \( \tau \simeq 1 \). On the basis of this fact, we analytically interpret the obtained base-density profile (Equation (14)), and we derive a new analytical expression for the photoevaporation rate for dust-free disks.

Let us consider a ray from a star irradiating onto the disk surface (see Figure 6). In the vicinity of the star, where

\[ R < R_{\text{in}} = (r_g R_\ast)^{1/3} \]  

(15)

the scale height \( H(R) \) of the ionized gas is lower than the height of the ray from the equatorial plane \((\sim R_\ast)\), and the density is low. The contribution of the gas inside \( R_{\text{in}} \) to the optical depth is negligible. Outside \( R_{\text{in}} \), the density along the ray is approximately given by the base density \( n(R, Z) \simeq n_0(R) \),
because the ray travels below the scale height. Then, the optical depth from the star can be written as

\[ \tau(R) = \int_{R_{\text{in}}}^{R} x_{\text{HI}} n_0(R') \sigma_{\text{HI}} dR'. \quad (17) \]

The neutral fraction \( x_{\text{HI}}(r, z = 0) \) can be obtained from the photoionization equilibrium (Equation (12)),

\[ x_{\text{HI}} \simeq \frac{\alpha_A \epsilon n}{4 \pi \sigma_{\text{HI}} J}, \quad (18) \]

where the case A value \( \alpha_A \) is used as the recombination coefficient, because we consider only direct stellar radiation and ignore re-emitted diffuse radiation. In this equation, we assume that the neutral fraction is small \((x_{\text{HI}} \ll 1)\), and the optical depth along the ray is also small \((\tau \ll 1)\). The direct stellar radiation intensity in \( r \gg R_{\text{n}} \) is

\[ J_* = \frac{\epsilon \Phi_{\text{EUV}}}{16 \pi R^2}, \quad (19) \]

which is evaluated with the radiation from half of the star, which considers shielding by the optically thick disk. From these values, we obtain the optical depth as

\[ \tau(R) \simeq \int_{R_{\text{in}}}^{R} \frac{4 \pi \sigma_{\text{HI}} n_0^2 R'^2}{\Phi_{\text{EUV}}} dR'. \quad (20) \]

Assuming a single power-law distribution for the base density, \( n_0 \propto r^{-p} \), the optical depth is approximately evaluated as

\[ \tau(R) \sim \frac{4 \pi \sigma_{\text{HI}} n_0^2 R^3}{\Phi_{\text{EUV}}} \propto R^{3-2p}. \quad (21) \]

For the plausible density distribution, the ionization front is located at \( Z = 0 \) for all radii or \( \tau \sim 1 \propto R^0 \). Thus, from Equation (21), we obtain the exponent \( p = 3/2 \). If \( p < 3/2 \), the optical depth \( \tau \) increases with \( R \), and it does not satisfy the requisite \( \tau \sim R^0 \). Strictly speaking, with \( p = 3/2 \), \( \tau \) increases with \( \log R \); however, it should be noted that \( \log R \) is a much flatter function of \( R \) than \( R^{3-2p} \) with \( p < 3/2 \). In contrast, if \( p > 3/2 \), the base density \( n_0 \) rapidly decreases with \( R \), and the optical depth \( \tau \) is dominated by the density in the inner region \( (R \approx R_{\text{n}}) \). Then, the optical depth in this case would also become constant with respect to the radius, \( \tau(R) \propto R^0 \). However, if the density is lower than the plausible value and \( \tau < 1 \), the base density would increase because of the direct stellar radiation and ionization and would approach the plausible value, which has the exponent \( p = 3/2 \). On the other hand, if the density is higher than the plausible value and \( \tau > 1 \), the base density would decrease because ionizing radiation does not reach that point. Therefore, for \( p > 3/2 \), the density would approach the plausible value.

We can also obtain the absolute value of the base density from \( \tau \approx 1 \) with \( p = 3/2 \) as

\[ n_0(R) = C \left( \frac{\Phi_{\text{EUV}}}{4 \pi \sigma_{\text{A}} R^3} \right)^{1/2} \]

\[ = 1.6 \times 10^7 \left( \frac{\Phi_{\text{EUV}}}{10^{49} \text{ s}^{-1}} \right)^{1/2} \left( \frac{R}{10^{15} \text{ cm}} \right)^{-3/2} \text{ cm}^{-3}, \quad (22) \]

where \( C \simeq 0.4 \) is the correction factor used to match our numerical result (Equation (14)). It is evident that the agreement between the analytical and numerical results is satisfactory.

Finally, we evaluate the photoevaporation rate using the obtained base density from Equation (22). From Equation (3), the mass-loss flux from a unit area is

\[ \dot{\Sigma}_{\text{PE}}(R) = 6.0 \times 10^{-13} \left( \frac{\Phi_{\text{EUV}}}{10^{49} \text{ s}^{-1}} \right)^{1/2} \]

\[ \times \left( \frac{R}{1000 \text{ AU}} \right)^{-3/2} \text{ g cm}^{-2} \text{ s}^{-1}. \quad (23) \]

From Equation (4), the total evaporation rate from the entire disk is

\[ M_{\text{PE}} = 5.4 \times 10^{-5} \left( \frac{\Phi_{\text{EUV}}}{10^{49} \text{ s}^{-1}} \right)^{1/2} \]

\[ \times \left( \frac{r_d}{1000 \text{ AU}} \right)^{1/2} M_\odot \text{ yr}^{-1}. \quad (24) \]

Here, we assume that the disk size is significantly greater than the gravitational radius \( (r_d \gg r_g) \). However, for very massive stars (>20 M_\odot), the gravitational radius defined by Equation (2) can be as large as the disk size. Even in this case, if we also consider effects such as radiation pressure and angular momentum in addition to gravity, the effective gravitational radius will be reduced below the disk radius (Liffman 2003; Font et al. 2004; McKee & Tan 2008). It should be noted that our evaporation rate (Equation (24)) depends on the disk radius rather than the gravitational radius, unlike that given by HJLS94. This discrepancy occurs because, in the HJLS94 model, the base density steeply decreases as \( R^{-5/2} \) outside the gravitational radius, whereas for the flat base-density distribution \( R^{-3/2} \) in our model, annuli near the outer radius \( r_d \) dominate the evaporation rate. Formally, Equation (24) coincides with the results of HJLS94 if we change \( r_d \) to \( r_g \). Although the terms appear to differ in the expression, their numerical difference, \( \sqrt{r_d/r_g} \), generally remains within an order of magnitude.

5. DISCUSSION

Photoevaporation has an important effect on the formation of stars and planets. In this section, we discuss the impact of EUV photoevaporation on primordial star formation as well as present-day star/planetary formation by considering the new photoevaporation rate. In addition, we describe the validity and the limitations of our model.

5.1. Impacts of Photoevaporation in Star/Planet Formation

5.1.1. Primordial Star Formation

Recent studies have demonstrated that the protostellar accretion of primordial stars is terminated by EUV photoevaporation (McKee & Tan 2008; Hosokawa et al. 2011, 2012; Stacy et al. 2012). Once the stellar mass exceeds \( \sim 10 M_\odot \), the stellar surface temperature becomes sufficiently high to emit a copious amount of EUV photons. As a result, the conical \( \text{H} \, \text{ii} \) regions begin to expand in the polar directions in a process known as the \( \text{H} \, \text{ii} \) region breakout, which significantly decreases the infall rate from the envelope to the disk. Disk photoevaporation begins at that moment and increases with the growth of the stellar mass. Finally, accretion ceases when the growing evaporation rate reaches the infall rate such that \( M_{\text{PE}} = M_{\text{infall}} \).

In Figure 7, we show the mass of primordial stars at the termination of accretion, \( M_{\text{PE}} = M_{\text{infall}} \), estimated by our photoevaporation rate for three disk sizes of
The disk radius is approximately 50 $M_\odot$. We use the EUV emissivity $\Phi_{\text{EUV}}$ of the zero-age main sequence star reported by Schaerer (2002).

$r_d$, 100 AU, 1000 AU, and 10,000 AU (Equation (24)). The disk radius is $\sim$100–10,000 AU for stars with mass of 10–1000 $M_\odot$, as determined by the analytical model for primordial star formation developed by Tan & McKee (2004). Here we evaluate EUV emissivity by assuming that the star is in the zero-age main sequence (ZAMS) phase (Schaerer 2002). EUV emissivity for ZAMS stars can be approximated as $\Phi_{\text{EUV}} \approx 1.26 \times 10^{-10} (M_*/M_\odot)^{1.4} \text{s}^{-1}$ in the range $40 M_\odot < M_* < 1000 M_\odot$. Then, we obtain the stellar mass at which the photoevaporation rate balances the accretion rate,

$$M_* \simeq 55 \left( \frac{r_d}{1000 \text{ AU}} \right)^{-0.7} \left( \frac{M_{\text{infall}}}{10^{-4} M_\odot \text{yr}^{-1}} \right)^{1.4} M_\odot \tag{25}$$

Here we use the typical accretion rate of $10^{-4} M_\odot \text{yr}^{-1}$, which is smaller than the conventional rate of $\sim 10^{-3} M_\odot \text{yr}^{-1}$ in primordial star formation without stellar feedback (Omukai & Nishi 1998; Abel et al. 2002; Bromm & Loeb 2004; Yoshida et al. 2006), because the accretion rate is reduced by approximately one order of magnitude before photoevaporation finally quenches the mass supply to the star owing to the $H$ II region breakout (McKee & Tan 2008; Hosokawa et al. 2011). The obtained mass of 55 $M_\odot$ agrees well with the results of hydrodynamic simulations by Hosokawa et al. (2011), which indicate that the reduction of the infall rate through the $H$ II region breakout, along with photoevaporation, is essential in setting the final stellar mass. It should be noted that the final mass is smaller for the larger disk radius, because the evaporation rate is proportional to $r_d^{3/2}$. Therefore, stars formed in pre-stellar cores with larger angular momentum would be smaller because of the greater radius of the protostellar disk, in addition to the possible reduction of the infall rate due to centrifugal force.

The following caveat is to be noted regarding our adoption of the ZAMS EUV emissivity: although stars generally reach ZAMS by the time of accretion termination, this is a result of reduced accretion, i.e., a longer accretion time than the stellar Kelvin–Helmholtz time, through feedback during the preceding phase. In the pre-ZAMS phase, the stellar structure and the accretion rate interact in a manner such that a more rapid infall results in a larger stellar radius. Thus, EUV emissivity is smaller, which in turn results in weaker feedback to the infall (Hosokawa et al. 2011, 2012). To treat the feedback in the pre-ZAMS phase, elaborate modeling of stellar evolution is necessary.

5.1.2. Present-day Star/Planet Formation

Before discussing present-day star/planet formation, it should be noted that our derived EUV photoevaporation rate ignores the scattering and absorption by dust grains, similar to the cases in the conventional HJLS94 model. Richling & Yorke (1997) conducted a hydrodynamical simulation to demonstrate that in some limited cases, the dust scattering process increases the photoevaporation rate by a factor of $\sim 2$. We thus regard our photoevaporation rate as a rough value with a factor of a few degrees of uncertainty. Therefore, comprehensive research about the dust effect in this process is necessary.

In the formation of present-day massive stars, the maximum stellar mass may be determined from the combination of photoevaporation and radiation pressure. Useful studies have demonstrated that the longstanding issue of a radiation pressure barrier in massive star formation can be overcome by the shielding property of disk accretion (Nakano 1989; Jijina & Adams 1996; Krumholz et al. 2009; Kuiper et al. 2010; Tanaka & Nakamoto 2011). However, the radiation pressure still has a strong effect in depleting the infall rate from the dusty envelope (Kuiper et al. 2012). With the depletion of the infall rate by radiation pressure, photoevaporation can terminate mass accretion onto stars. Therefore, even with an observationally claimed high accretion rate of $10^{-4}$–$10^{-3} M_\odot \text{yr}^{-1}$, which is similar to that in the first star formation, the final masses of these stars are expected to be smaller than those of the first stars.

EUV photoevaporation has less importance in the dissipation of protoplanetary disks around low-mass stars. Because such stars emit few EUV photons, an increased amount of transmission radiation, such as FUV radiation and/or X-rays, is considered to dominate disk photoevaporation (Gorti & Hollenbach 2009; Gorti et al. 2009; Owen et al. 2010, 2012). In fact, although the EUV evaporation rate determined by our model is higher than that by the HJLS94 model, the value falls below the rate determined by FUV radiation and X-rays by more than one order of magnitude for disks around low-mass stars. For the dissipation of protoplanetary disks, the role of UV radiation from nearby massive stars should also be considered, because most stars are born as members of star clusters (Johnstone et al. 1998; Adams et al. 2004; Fatuzzo & Adams 2008; Holden et al. 2011; Thompson 2013).

5.2. Comparison with Numerical Simulations

In this section, we discuss the validity of our model by comparing our steady-state density model with the numerical results reported by Hosokawa et al. (2011), who investigated photoionization feedback in primordial star formation by combining 2D radiative hydrodynamics for the envelope with the stellar evolution calculation. In their simulation, direct stellar radiation is solved by ray tracing, while diffuse radiation is treated with the flux-limited diffusion approximation. The left panel of Figure 8 shows the temperature structure of the photoevaporation disk at the stellar mass $M_* \approx 40 M_\odot$ with $\Phi_{\text{EUV}} \approx 10^{30} \text{s}^{-1}$, as determined by their fiducial model. The disk is in fact in the quasi-steady state in the photoevaporation epoch. The neutral
disk or toroid ($\lesssim 10^4$ K) extends on the equatorial plane and ionized gas ($\gtrsim 10^4$ K) occupies the region above the disk. In their simulation, accretion is terminated by the photoevaporation at a final stellar mass of $43 M_\odot$.

In the right panel of the figure, the density at the ionization front from the numerical simulation of Hosokawa et al. (2011) and our determined base density in the case of $p_{\text{in}} = p_{\text{out}} = 1.5$ are presented as a function of the radius. In addition, the base density determined by the HILS94 model ($p_{\text{in}} = 1.5$, $p_{\text{out}} = 2.5$) is shown. A comparison of these values clearly reveals that our results, including the exponent of 1.5 and the absolute value, agree with the numerical results. Thus, we conclude that our steady-state assumption is valid even when realistic hydrodynamical effects are included. It should be noted that our results derived with the thin-disk approximation agree well with those of the simulation, in which the disk has finite thickness. Therefore, our model is applicable as long as direct stellar radiation irradiates the disk surface in a wide range of radii. Our model is also consistent with results by Hosokawa et al. (2011) in which the direct stellar radiation dominates photoionization.

5.3. Limitations of Our Model

It should be noted that our determined photoevaporation rate is only a rough approximation and carries some uncertainty. In our model, for example, we assume that the ionized-gas temperature is constant at $T_{\text{HII}} = 10^4$ K and that the flow velocity is given by the sound speed at $v_{\text{PEE}} \sim c_{\text{s,HII}} \propto T_{\text{HII}}^{1/2}$. Both of these assumptions require modification in some circumstances. For the former, the ionized gas near primordial stars can exhibit higher temperatures owing to higher stellar surface temperatures and less efficient cooling ($\sim 4 \times 10^4$ K in Hosokawa et al. 2011). For the latter, according to the hydrodynamical simulation conducted by Font et al. (2004), the flows at the ionization boundary are slightly slower than the sound speed, which makes the mass-loss rate smaller by a factor of two. For a more accurate evaluation, sophisticated radiative hydrodynamical modeling is required.

In this study, we ignore the effects of dust grains. For solar metallicity, the scattering of radiation by dust in the disk atmosphere enhances the irradiation EUV flux onto the disk surface and increases the photoevaporation rate by a factor of approximately two, according to radiation hydrodynamical simulation conducted by Richling & Yorke (1997). However, their calculation was limited to several combinations of stellar mass and disk size with the approximated RT calculation. Further calculations employing wide ranges of these parameters are needed to discuss their influence. The dependence on the disk radius is particularly interesting, because in the simulation conducted by Richling & Yorke (1997), the mass loss from the outer region dominated the total evaporation rate.

In this study, we illustrate single star formation. However, binaries or small multiples are also expected to form both in cases of first star formation (Machida et al. 2008; Stacy et al. 2010; Clark et al. 2011) and in present-day massive star formation (Kratter & Matzner 2006; Krumholz et al. 2009; Peters et al. 2010). Because the infalling material is divided into multiple stars, the individual stars would be smaller than that in a single star case. This process is known as fragmentation-induced starvation (Peters et al. 2010). Therefore, the final mass estimated by Equation (25) may be the upper limit; further investigation is necessary for accurate estimation.

6. SUMMARY

The photoevaporation of circumstellar disks by extreme ultraviolet radiation plays an important role in star formation in the present-day as well as early universe. In this study, we revisit the limitation present in the conventional 1+1D approximation model developed by Hollenbach et al. (1994) in a dust-free case by introducing an updated axisymmetric 2D model. Unlike that in the conventional model, the density distribution at the photoionization front located just above the disk, known as the base-density distribution, is represented by a single-exponent power law with an index of $-3/2$. In our model, the total photoevaporation rate depends on the outer disk radius (Equation (24)), in contrast to that in the conventional model, which depends on the gravitational radius. Although we have derived this base-density distribution under the steady-state assumption, we have confirmed that the results of our model are consistent with those of the radiative hydrodynamical simulation conducted by Hosokawa et al. (2011).

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