We analyze foundational consequences of recently reported loophole free tests of violation of Bell’s inequality. We consider two interpretations of these remarkable experiments. The conventional one is “Einstein was wrong and Bohr was right, there is spooky action at a distance, quantum realism is incompatible with locality.” However, in line with discussions in literature during last decade, we show that it is still possible to treat quantum mechanics without appealing to nonlocality or denying realism. We hope that this note will call the attention of experts in quantum foundations and convince them that the case is not closed, so that they should come with their own comments on the status of the final Bell test.

1 Introduction

It finally happened! In 2015 three world’s leading experimental groups working on foundational aspects of quantum mechanics (QM) announced (practically simultaneously) that they had performed the loophole free tests of violation of the Bell inequality [1]: the groups of Ronald Hanson (Delft University of Technology) [2], Anton Zeilinger (University of Vienna) [3] and Linden Shalm (NIST, Boulder) [4]. This is definitely a great event in quantum foundations; the more so because it took so long to evolve from the pioneering experiments of Stu-art Freedman and John Clauser [5] and Alain Aspect [6] to these final Bell’s tests.

I was surprised to see that this event did not generate a new wave of the quantum foundational enthusiasm, neither in the quantum community nor in general mass-media (even the ones oriented to popularization of science). One of the reasons for this rather mild reaction is that, as was already mentioned, the result was commonly expected. A similar reason is that the Bell inequality has already been widely “sold” and opposing voices have been smothered. In the literature and talks, the Bell argument is typically presented as if everything has already been experimentally proven; scientists working on closing loopholes were considered merely as polishing the famous Aspect’s experiment [6]. Those who are closer to quantum foundations could additionally point that Gregor Weihs [7] contributed to complete Aspect’s experiment by closing the locality loophole. The long-aspired experiment closing the detection loophole performed in 2013 by the groups of Anton Zeilinger [8] and Paul Kwiat [9] did not attract so much attention. (The discussions following this experiment were devoted to technicalities [10, 11].)

Therefore, it would be great if the recent publications [2–4] ignited a serious discussion on the possible impact of this event of the realization of the totally loophole free Bell test. Such a discussion is especially important because conclusions presented in [2–4], see also comments on these tests in [16, 17], present only a part of the wide spectrum of views on Bell’s argument. Although the presented “conventional viewpoint” dominates in the quantum community, it would be natural to represent other,
so to say, singular, parts of this spectrum of viewpoints, see, e.g., the recent comment of M. Kupczynski [18].

We briefly remind the conventional viewpoint presented in [1] and in dozens of articles and monographs, e.g., [2–11, 16, 17, 19–21]. It was “thus” finally confirmed experimentally that

1. CV1: Einstein was wrong and Bohr was right;
2. CV2: there is spooky action at a distance;
3. CV3: quantum realism is incompatible with locality.

The views of those who disagree with the presented “conventional position” are characterized by the high degree of diversity [22–33], see also numerous contributions to proceedings of the Växjö conferences [12, 13, 15, 34]. Therefore I shall not try to elaborate on some common “non-conventional viewpoints”, but present only my own position [32]:

1. NCV1: both Einstein and Bohr were right;
2. NCV2: there is no need in spooky action at a distance;
3. NCV3: quantum realism is compatible with locality.

In section 2 I shall question the CV1-CV3 viewpoint and try to justify the NCV1-NCV3 viewpoint; in particular, I confront the “action at a distance interpretation” with the Copenhagen interpretation of QM. (It is surprising that one may combine without cognitive dissonance these two interpretations.) Then to discuss the issue of realism in QM, I appeal to its ontic-epistemic analysis in the spirit of Atmanspacher and Primas [35]. From this viewpoint, Bell’s argument can be treated as the conjecture that ontic states can be identified with epistemic ones. We also discuss this conjecture by appealing to the old Bild conception (Hertz, Boltzmann, Schrödinger) about the two descriptive levels of nature, theoretical and observational, see, e.g., [36] and chapter 1 of monograph [37]. Our conclusion is that the rejection of Bell’s conjecture as the result of the recent experiments cannot be treated as the impossibility to keep the realist viewpoint. Neither is there need for action at a distance.

Section 3 starts with a presentation of Kolmogorov’s interpretation [38] of classical probability (CP) as the observational theory (describing the epistemic states of nature). Kolmogorov CP is a contextual theory assigning probability spaces to experimental contexts, complexes of experimental physical conditions. This position leads to the contextual representation of the probabilistic structure of Bell’s experimental test [30, 32].

In section 4 I present my personal picture of future development of quantum foundations, in the “after Bell epoch”: from the total rejection of Bell’s conjecture to novel studies on the two descriptive levels approach to QM. In contrast to the rather common opinion (see, e.g., Aspect’s paper [16] entitled “Closing the door on Einstein and Bohr’s quantum debate” and Wiseman’s paper [17] entitled “Quantum physics: Death by experiment for local realism”), for me the final Bell test did not imply the total impossibility to “go beyond quantum” [37]. The main message of this test is that the way to a proper subquantum model is more tricky than it was hypothesized by Bell.

2 Schrödinger, Einstein, Bohr, and Bell: philosophy meets quantum physics

Typically the output of Bell’s argument (nowadays experimentally confirmed) is formulated as *QM is incompatible with local realism*. Thus either one has to reject the possibility of the realist description of quantum phenomena or to imagine nonlocal reality of the Bohmian type which is even more exotic and mystical than quantum surreality. Since Einstein dreamed for the realist interpretation of quantum phenomena and he considered nonlocality as an “absurd alternative” to realism, nowadays it is clear (for everybody, besides a few outsiders, see, e.g., [18, 23–33, 37]) that he was wrong and automatically (as Einstein’s opponent) Bohr was right, see, e.g., [16]. In general in the modern quantum community it is fashionable to be a follower of Bohr. (We remark that in the course of his life, Bohr progressed in his thinking, so that during his career he took various positions …)

2.1 Bohr versus Bell

First of all, I want to point to one of the main interpretational problems of the conventional viewpoint on the output of Bell’s test, see CV1-CV3. This is an attempt to present CV1-CV3 in a single package and consistently with the Copenhagen interpretation of QM. However, none of the fathers of QM, neither Bohr, Heisenberg, Fock, Landau, Pauli nor Einstein, could even imagine that spooky action at a distance would be taken seriously in the quantum foundations. It should be recognized that no such wholesale rejection of Copenhagenist approaches is appropriate.

Thus to be consistent one has to speak about *revolutionary changes in quantum foundations generated by*
Bell's argument and experimental tests confirming it. The possibility of these changes was not even imagined by earlier Copenhagenists. Still, one can argue that “Copenhagenists were against Einsteinian realism” and, hence, Bell’s test can be considered as supporting the Copenhagen position…

2.2 Realism: philosophy meets physics

Realism is a complex issue, both in physics and philosophy. One cannot debate it without taking into account the recent progress in philosophical studies. And I want to analyze Bell’s argument in the light of such studies, namely, the ontic-epistemic viewpoint on physical reality, established in quantum foundations by H. Atmanspacher and Primas [35].

There are ontic states, assigned to physical systems as “they are”, and epistemic states representing knowledge that observers gain from measurements on physical systems. QM is about epistemic states. This is in complete agreement with the Copenhagen interpretation of QM, especially Bohr’s views. This is in complete agreement with the Copenhagen interpretation of QM, especially Bohr’s views.5

It often escapes notice that the EPR-argument [40] was about ontic states; it was directed to show that quantum states are epistemic states. For Einstein, QM was incomplete in the sense that there should exists a finer description of physical processes in the microworld than given by QM. The states of such a subquantum model are ontic states. Einstein and his coauthors did not have any intention to treat the wave function \( \psi \) as the ontic state; for them, it was the epistemic state representing knowledge which can be earn from measurements.6

5 The position of Bohr with respect to existence of ontic states is a more complicated issue. His view on this evolved. Until his exchange with EPR and even in this exchange, he entertained the possibility of ontic states, although they would be not classical, because of the uncertainty relation; only either a position or a momentum could be assigned (ontologically) to quantum objects themselves and only at the time of measurement, but never otherwise. By 1937, he took a position that ontic states of any kind are strictly forbidden in his interpretation either before, during, or after measurements; which, it follows, strictly precludes realism in considering quantum reality, see Plotnitsky [39] for details.

6 We remark that this is also the position taken by Allahverdy dan, Balian and Nieuwenhuizen in their analysis of the dynamics of a quantum measurement [41, 42] in which the degrees of freedom of the apparatus have been accounted for.

This separation between the two descriptive levels is very important for understanding the real tragedy of QM: misunderstanding about the issue of completeness of QM, between Einstein and Bohr, see [40, 43]. As was pointed out, Einstein’s incompleteness is so to say ontic incompleteness of QM, but Bohr’s completeness is epistemic completeness of QM. Bohr in his famous reply to Einstein [43] wanted to show that QM is complete at the epistemic level. However, Einstein did not have doubts in epistemic completeness of QM.

Now, let us consider Bell’s project from the philosophic viewpoint, by using the ontic-epistemic approach. In philosophic terms, Bell’s conjecture was that

**O=E**: Ontic states of quantum systems can be identified with epistemic states.

Since the values of observables are assigned directly to the ontic states,

\[
\lambda \rightarrow a(\lambda),
\]

these states also have to interpreted as the epistemic states. Here \( a \) is an observable and the result of a measurement is determined directly by the parameter \( \lambda \). Hence, \( \lambda \) has to be considered as belonging a set of variables describing the epistemic state. At the same time Bell treated \( \lambda \) as representing the state of reality as it is (\( \lambda \) has no relation to context of measurement of \( a \)). Hence, for Bell, a state has to be considered as an ontic state.

The final loophole free Bell tests demonstrated that this conjecture was wrong (if there is no action at a distance, see NCV2).

However, this statement is not the same as to state that realism is incompatible with QM. The impossibility to identify two descriptive levels, ontic and epistemic, does not necessarily imply the rejection of the possibility of a realist description.

2.3 Impact of Bell’s test for Copenhagenists

For Bohr, only epistemic states had physical meaning. Of course, he did not use this terminology, but his statements match it very well; for example [44]:

“This crucial point, which was to become a main theme of the discussions reported in the following, implies the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear. In fact, the individuality of the typical quantum effects finds its proper expression in the circumstance that any attempt of subdividing the phenomena will demand a change in
the experimental arrangement introducing new possibilities of interaction between objects and measuring instruments which in principle cannot be controlled. Consequently, evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as complementary in the sense that only the totality of the phenomena exhausts the possible information about the objects."

Therefore it seems that the problem of identification of ontic and epistemic states was of no interest for Bohr and “old Copenhagenists.”

2.4 Bild concept: Schrödinger versus Bell

We remark that the ontic-epistemic approach can be considered as the modern version of the old Bild-conception tradition (Hertz, Boltzmann and their followers as well as Einstein) [36, 37], chapter 1. In QM this tradition was especially strongly presented in views of Schrödinger who inherited this tradition from his teacher Exner. In the Bild-conception one has to separate two descriptive levels, one is theoretical and another is observational. The latter can be identified with the modernly used epistemic level. The theoretical level, as preceding the observational level, can be coupled to the ontic level of description. However, it seems that these notions cannot be completely identified. The theoretical level was not about “reality as it is”, but about its theoretical mathematical model.

Schrödinger emphasized the impossibility to identify these two descriptive levels: theory and observations are not necessarily related in a term-to-term correspondence and a certain degree of independence exists between them. He also point out that already physicists of 19th century, e.g., Hertz, Boltzmann understood this well.7 As was emphasized by D’Agostino [36], p. 351, Schrödinger called “the classical ideal of uninterrupted continuous description”, at both the observable and the theoretical level, an “old way”, meaning, of course, that this ideal is no longer attainable. He acknowledged that this problem was at the center of the scientific debate in the Nineteenth and Twentieth centuries as well, [45], p.24:

“Very similar declarations...(were) made again and again by competent physicists a long time ago, all through the Nineteenth Century and the early days of our century...they were aware that the desire for having a clear picture necessarily led one to encumber it with unwarranted details.”

During the recent conference “Quantum and Beyond” (Växjö, Sweden, June 13-16) G. Jaeger and A. Plotnitsky pointed my attention to the modal interpretation of QM by van Fraassen [46] and its similarity with the Bild conception. The modal interpretation is based on consideration of two classes of states, the actual states and dynamical states. One can treat the former as ontic states and the latter as epistemic states. Thus the modal interpretation of QM can be considered as a very special application of the Bild concept (“special” - because of the concrete interrelation between actual and dynamical states). Unfortunately, it seems that van Fraassen and other contributors to the modal interpretation were totally unaware about the great philosophic studies of German and Austrian physicists and philosophers on the Bild concept.8

2.5 De Broglie: Hidden probabilities

An excellent presentation of de Broglie’s viewpoint on quantum measurement and its consequences for Bell’s argument can be found in the paper of G. Lochak [24]. We now present this measurement theory briefly by trying to refine it from the presence of the physical guiding waves, de Broglie’s waves.

In his measurement theory de Broglie assigned the special value to the position measurement: every physical quantity can be measured only via the final detection of the position, e.g., localization of a detector produced a click. The unitary equivalence of various representations in the Hilbert space formalism is just a mathematical feature of the theory. To measure the concrete physical quantity \(A\), systems emitted by a source have to pass an analyzer, e.g., a polarization beam splitter (PBS), and then approach the corresponding detector. Probabilities determined by detectors after passing of the analyzer are called the present probabilities; probabilities before

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7 Boltzmann asserted that only one half of our experience is ever experience.

8 It is also the good place to mention the fundamental contribution of Primas to the problem of consistency of realist views and QM [47]. He expected that at the mathematical level the problem can be resolved by using of \(C^*\) algebras and open-systems as opposed to simple Hilbert space vector states for closed systems. However, Primas’ treatment of the problem has a flavor of nonlocality-inseparability (at least as it was presented in [47]) - contrary to the viewpoint presented in this paper. Finally, we remark that the realist articulation of the more modern, structural realist notion of quantum objects can be found in the book by G. Jaeger [48], section 3.4.
the analyzer, corresponding to the prepared pure state, are called the predicted probabilities. The main point of de Broglie’s probabilistic considerations is that it is impossible to combine the predicted and present probabilities straightforwardly, i.e., by using the rules of the classical probability calculus, see section 3. In particular, it is impossible to combine the predicted and present probabilities corresponding to two arbitrary analyzers within classical probability theory, by using the joint probability distribution.

However, this fact (a simple consequence of the quantum formalism leading to “interference of probabilities”, see [32] for details) does not rule out the possibility to introduce hidden variables, because (according to de Broglie) their probability distribution is hidden, i.e., we have to consider the third type of probabilities, hidden probabilities. The main point of de Broglie’s analysis is that there is no reason to identify these hidden probabilities, probabilities for ontic states with probabilities produced by QM, namely, the predicted and present probabilities which both are epistemic probabilities.

Thus similarly to Schrödinger, Einstein, Atmanspacher, Primas and the author of this paper, de Broglie and Lochak were against identification of ontic and epistemic states and more generally their probability distributions (the latter is the main point of a long series of publications of the author of this note, starting with [49]).

Finally, we mention the guiding wave element of de Broglie’s picture. The physical guiding wave appears as the element of de Broglie’s ontic description of micro-phenomena, i.e., as one of the variables representing the ontic state. We emphasize that the above scheme with hidden (ontic) and observational (epistemic) probabilities can be explored in general, i.e., without claiming that a subquantum model has to be of the guiding wave type. Here we will not discuss even briefly the difference in views of L. de Broglie and D. Bohm. We just remark that anybody who read de Broglie personally [50] would never say that de Broglie’s double solution model and the so-called Bohmian mechanics coincide up to a degree of mathematical rigor. In particular, we stress that the double solution model is local (as was emphasized by de Broglie [50]) and Bohmian mechanics is nonlocal.

2.6 Super-determinism of ‘t Hooft: Internal states of cellular automaton

This viewpoint [51–54] on QM matches very well the Bild conception. G. ‘t Hooft tries to reproduce the quantum operational formalism for observations from determin-
Nowadays the latter is practically forgotten, not least because of Bell’s critique. However, we can ask ourselves: Was von Neumann’s theorem really so bad?

Von Neumann did not identify the ontic and epistemic states. He established the special rules of correspondence between the two descriptive levels. In principle, he proceeded in the Bild concept tradition: there are theoretical and observational models and correspondence rules. Bell criticized von Neumann’s image of a possible coupling between the theoretical and observational models and this critique was reasonable. In particular, according to von Neumann the correspondence between ontic and epistemic quantities should be additive,

\[ a + b \rightarrow \hat{a} + \hat{b}, \tag{2} \]

which seems to be unacceptable for incompatible observables.

However, from the philosophic viewpoint the sharp distinguishment between the two descriptive levels present in von Neumann’s theorem (personally he called this statement “Ansatz”) is preferable comparing with simple identification of these levels. A follower of Einstein would say [32] that von Neumann simply was not lucky to find the right correspondence rules. As was emphasized in [32], the “no-go” activity in establishing the correspondence between the two levels of description is totally meaningless, since there is an infinite variety of possibilities to establish the rules of such a correspondence, see, e.g., [37].

2.9 Where-Bell-went-wrong community

It is useful to analyze from the ontic-epistemic viewpoint not only the position of the conventional quantum (and more specifically quantum information) community, see CV1-CV3, but likewise of the “where-Bell-went-wrong community”. It seems that the majority of opponents of the CV1-CV2 interpretation do not understand the philosophic meaning of the Bell conjecture, \( O=E \). Typically one from the anti-Bell opposition tries to show that observational (epistemic) probabilities can violate Bell’s inequality, or to show that the standard proofs of the Bell inequality do not work, or to construct an epistemic (local) model producing correlations violating Bell’s inequality. In more tricky studies there is even a subquantum model and special rules coupling two descriptive levels, but, of course, such a correspondence is more complex than conjectured in \( O=E \). Unfortunately, such studies are not treated from the double descriptive viewpoint; in particular, the rules of correspondence between subquantum and quantum models are not specified, cf., however, with [37].

Then it is explained “where Bell went wrong” in his considerations. My impression is that this was precisely the way of treatment of the Bell argument by the main anti-Bell gurus: L. Accardi, K. Hess and W. Philipp, M. Kupczynski, H. De Raedt, K., Michielsen T. Nieuwenhuizen and partially myself. The claim “Bell went wrong” (e.g., by using some assumptions), even though being an equally scientific claim as the opposite one, acts as a red muleta used by Spanish matadors to make bulls angry and generates the strong critical feedback from the pro-Bell community.\(^9\) This “Bell went wrong” position was not fruitful for quantum foundations, it induced brutal debates (in particular, during the Växjö series of annual conferences, 2001-2015). By understanding that Bell’s argument is based on the \( O=E \) conjecture and that the activity to test Bell’s inequality is directed to check this conjecture we would escape these debates or least to make them less brutal and more to a scientific point.

In short, it is definitely possible to construct local probabilistic models violating the Bell inequality, but for such models the Bell conjecture \( O=E \) is clearly wrong; in the same way the standard proofs of the Bell inequality work only under the Bell conjecture that \( O=E \).

3 Classical probability meets quantum physics

Bell’s argument is of the probabilistic nature and therefore it is natural to analyze it from the viewpoint of foundations of probability theory. The latter was formalized in the measure-theoretic framework by A. N. Kolmogorov [38] in 1933. As any scientific theory, it was endowed with the corresponding interpretation [38].

By Kolmogorov [38], see also [32] for discussions, any complex of physical conditions \( C \), experimental context\(^10\), determines its own probability space

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9 It seems that, when “attacked”, the human nature often leads to a defence of the own opinion rather than to a discussion of the arguments at the base of the opponent’s position. This holds in particular for defenders of the conventional view on Bell’s argument, who, rightly or not, are supported by the majority.

10 Here, see [32], the notion of “context” started to be in the use. It is important to point out that here context represents “total experimental arrangement”. Thus contextuality is understood wider than just dependence on measurement of a compatible observable. Our viewpoint on contextuality matches
$\mathcal{P}_C$ - the system of events $\mathcal{F} \equiv \mathcal{F}_C$ and the probability $P \equiv P_C$.

Classical probability (CP) theory was designed to serve the epistemic description. Here the positions of Kolmogorov and Bohr are very close. In CP complementarity is expressed in assigning different probability spaces to different contexts.\(^{11}\)

However, often we have to work with probabilistic data collected from a family of contexts. Therefore CP has to provide some mechanism of embedding of multi-contextual data in a single probability space. It is important to emphasize that there are two main CP constructions of such an embedding:

1. **Marginal embedding**: contextual probabilities are represented as marginal probabilities for a single probability measure $P$.
2. **Conditional embedding**: contextual probabilities are represented as conditional probabilities for some probability for a single probability measure $P$.

The first (straightforward) embedding was excellently realized by A. N. Kolmogorov in his famous theorem \([38]\) about existence of the probability measure $P$ (defined on the space of trajectories) for a stochastic process determined by its probability distributions for all finite sequences of instances of time, $P_{t_1, \ldots, t_n}$. However, this construction can work only in special cases, for special families of contexts. (Therefore Kolmogorov’s theorem about the existence of a probability space for a stochastic process is so famous: he found such a special (and very important) case.) It is well known in CP since the work of Boole, that probability distributions for random variables measured for different contexts can be incompatible, i.e., it is impossible reproduce them as marginal probabilities from a single probability $P$ \([56]\).\(^{13}\) In fact, the inequality providing the necessary condition of the existence of such $P$ derived by Boole coincides with Bell’s inequality; therefore some authors even proposed to speak about the *Boole-Bell inequality* \([30]\). The complete description of conditions of existence of the straightforward embedding of contextual probabilities in a single probability space was presented in the paper of Vorobjev \([58]\) which contains all Bell’s type inequalities known in quantum physics (as well as further ones, still unknown).

We remark that the marginal embedding is based on the identification of the ontic and epistemic descriptions. The epistemic probabilities for fixed contexts are identified with the marginal probabilities with respect to the probability distribution $P$ which can be treated as the ontic probability. In particular, the Kolmogorov theorem on the probability space for a stochastic process can be applied only the assumption of the ontic-epistemic identification; without this assumption one has to use theory of quantum stochastic processes \([59]\).

However, in contrast to QM, the “no-go” activity, the impossibility theorems about straightforward marginal embedding was not so popular in CP. And it is clear why. There one has widely explored the second embedding procedure based on the identification of concrete contextual probabilities not with marginal, but with conditional probabilities with respect to a single probability measure $P$. This is so-called *randomization procedure*. It is especially important in statistics. We shall present it for the concrete multi-contextual test - the Bell test.

### 3.1 Contextual probabilistic structure of Bell’s test

The considerations of this section are presented in the form of a formal mathematical model in the article \([60]\).
How would Kolmogorov handle Bell’s argument? For Kolmogorov, it is based on consideration of a few experimental contexts \( C_{\theta_i,\theta'_j} \) corresponding to fixing the pairs of orientations \( \theta_i, \theta'_j \) of the PBSs. There are a few probability spaces corresponding to these measurement contexts. Experimental statistical data collected for each individual context \( C_{\theta_i,\theta'_j} \) has to be handled in its own probability space \( P_{C_{\theta_i,\theta'_j}} \). This raises the questions:

- Can Bell’s type inequalities be derived in such multi-space probabilistic framework? Not!
- Would Kolmogorov be surprised by a violation of Bell’s inequality for multi-space data? Not at all! Instead, he would be surprised if it were not violated.
- Can one construct a single probability space representing all contexts \( C_{\theta_i,\theta'_j} \)? Yes! This construction is known as randomization.

In short, randomization can be described as follows:

- A). Choose the probabilities for the selections of the considered contexts.
- B). Determine probability distributions for fixed contexts, \( P_{C_{\theta_i,\theta'_j}} \).
- C). Construct probability \( P \) serving for all experimental contexts: combine probabilities for fixed contexts \( P_{C_{\theta_i,\theta'_j}} \) with the probabilities for the context-selections.\(^{14}\)

This single probability measure \( P \) represents the multi-context experiment. We stress that, for it, it is impossible to violate Bell’s inequality, since the latter is just a theorem of CP \([32]\).

In this framework the probability distributions for fixed contexts \( P_{C_{\theta_i,\theta'_j}} \) (which are obtained by experimenters as frequencies) appear as conditional probabilities with respect to fixing selections of the parameters \( \theta_i, \theta'_j \) (contexts \( \Theta_{\theta_i,\theta'_j} \)). However, \textit{there is no reason to hope that these conditional probabilities} themselves and the corresponding conditional quantities, e.g., correlations, would satisfy Bell’s inequality, see \([60]\).

In CP the magnitude \( \Delta \) of violation of Bell’s inequality can be treated as the numerical measure of multi-space probabilistic structure of the experiment. Thus \( \Delta \) represents the degree of contextuality (which is understood a la Bohr as taking into account all experimental arrangements). What does such contextuality mean from the QM foundational viewpoint? Contextuality exhibits itself only the existence of incompatible contexts. Therefore \( \Delta \) is also the measure of incompatibility-complementarity. From this viewpoint, \textit{the experimental tests violating Bell’s inequality confirmed complementarity not only for local observables, such as position and momentum or two spin projections of a single electron, but even for nonlocal observables such as projections of spins measured for a pair of electrons.}

3.2 Kolmogorov versus Bell

As we have seen, the CP description of Bell’s test does not lead to revolutionary consequences in the form CV2, CV3. And it is completely clear why. The above CP model of the probabilistic structure of Bell’s test is epistemic - it takes into account randomness of selection of experimental contexts, i.e., random generators used for this aim are also considered as measurement devices. In principle, one can say that this is the end of the CP-story about Bell’s test.

However, we can continue our discussion and represent its output in the hidden variable form. The latter will be not real ontic hidden variables, but so to say “epistemic hidden variables”.

CP teaches us that all random influences involved in the tests have to incorporated into the model. Probabilist would consider \([60]\) not the “Bell’s map” \((1)\), but a map of the form:

\[
\omega \to a(\omega), \quad \omega = (\lambda, \lambda_L, \lambda_R),
\]

where \( \lambda_L, \lambda_R \) are random parameters determining with some probabilities, \( p_L, p_R \) outputs of random generators \( L \) and \( R \) which in turn determine settings.\(^{16}\)

Of course, not only random generators contribute to randomness generated by experimental equipment. There is a lot of randomness in PBSs, in detectors, even theoretically, see \([41, 42]\) and in optical fibers. This randomness also has to be taken into account by modifying \((3)\).

\(^{14}\) As to be discussed soon, they are conditional probabilities.

\(^{15}\) This \( P \) is defined on the system of events \( \mathcal{F} \) related to this randomized experiment; it contains not only events related to observations for fixed contexts \( \Theta_{\theta_i,\theta'_j} \), but also events of selections of these contexts.

\(^{16}\) In Bell’s test for the CHSH inequality each random generator selects a pair of angles with probabilities \( p_L(i), p_R(j), i, j = 1, 2 \).
3.3 Classical probability, operational approach and Bell’s argument

The above CP-analysis of Bell’s test motivates us to comment the representation of observables in the form (3) from the operational viewpoint which is widely used in QM, e.g., [26]. By it, there is a preparation procedure and there is a measurement procedure. From the operational viewpoint, in Bell’s considerations the preparation procedure is performed by the source of entangled systems and the detection procedure by detectors. However, there are, e.g., random generators determining the orientations of PBSs. They exist! One cannot simply ignore them. In a consistent operational approach they have to taken into account. In the model represented by (3) they are included in the preparation procedure. In fact, we follow de Broglie who emphasized the role of analyzers in the production of the present probabilities. In the Bell test for photons PBSs are precisely de Broglie’s analyzers.

3.4 Employing Ockham’s razor against nonlocality

We emphasize that we have not appealed to spooky action at a distance. Contextuality allows a violation of Bell’s inequality. This motivates NCV2 - “there is no need in spooky action at a distance”, although not the rejection of CV2. In principle, one can still play under the assumption CV2 - if one likes spooky action and it helps to create a consistent picture of the world. However, there is no need for it. This is a good case to use Ockham’s razor, see [61] for discussion.

This viewpoint, that by taking into account Kolmogorov probabilistic contextuality of the Bell test one needs not mention nonlocality at all, is often criticized by pointing out that contexts $C_{\theta_i\theta_j}$ are “nonlocal”, since their are based on orientations of two spatially separated PBSs. However, this classical nonlocality of spatial location of experimental equipment has nothing to do with action at a distance.

Another point which was discussed in [62] is that in the Bell conjecture the notion of “locality” is represented in very specific form of action at a distance [1]. Where are there space-time variables? Action at a distance locality is not at all the Einsteinian locality of relativity theory. In principle, one can try to couple “Bellian and Einsteinian localities” by taking into account the space-time dependence of correlations for entangled systems [63]. However, by proceeding this way we lose the simple prequantum-quantum (ontic-epistemic) correspondence given by (1). The real space-time correlations contain the at least contributions of media for signal propagation; then one has to take into account dispersion and losses in these media, finally the temporal structure of functioning of photo-detectors [64]. Thus the model becomes epistemic; violation of Bell’s inequality by such correlations is not surprising, cf. with the positions of de Broglie and ’t Hooft.

Finally, we emphasize that Quantum Field Theory (QFT) do not show any trace of nonlocality, since here the notion of locality is presented in the space-time picture which is basic for relativity theory. The difference between space-time (non)locality and, so to say, measurement (non)locality of Bell is an interesting topic, but we will not discuss it here in very detail. Personally I think that quantum physicists combine the QFT locality with the QM nonlocality by the following reason. QM is an approximative theory (in particular, it is not a relativistic theory). Therefore to come to fundamental conclusions, it would be natural to ignore QM and to proceed in the QFT framework. However, we have measurement theory (at least operationally [55], but cf. [41, 42]) only for QM. There is no consistent measurement theory for QFT. Therefore in the experimental situations, in particular, for the Bell experiment, we use QM and (by ignoring the space-time picture) one comes to the Bell notion of action at a distance.

3.5 Quantum realism

Bell’s argument strongly supported the anti-realist tendency in the quantum community. It was very supporting for the development of a variety of information interpretations of QM - with two bright examples, the Zeilinger-Brukner information interpretation [65] and Fuchs’ QBism [66, 67]. Nowadays, a randomly chosen representative of the quantum community would say with very high probability (I account it as 99%) that realism and QM are incompatible.

I totally disagree with this position, see, e.g., my manifest [68] (even though it may be considered as naive, since at that time I was not so well educated in quantum foundational issues). I repeat that a successful Bell test only rejects the straightforward coupling (1) between the ontic and epistemic or more generally theoretic and

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17 We understand it as the determination of the probability space for the given experimental context.

18 And this is life in the state of cognitive dissonance.
observation levels. Other maps can be constructed and the fact that we still do not have a commonly acceptable map may just mean lack of imagination, cf. with the famous citation of Bell [69], p. 15: "what is proved, by impossibility proofs, is lack of imagination."\(^{19}\)

However, realism discussed in section 2 is merely subquantum realism. What is about, so to say, genuine quantum realism?

As was shown, CP modeling of physical experiments, including multi-contextual experiments such as Bell’s test, does not lead to rejection of realism. This is realism of experimental contexts and observations performed for these contexts. This is precisely “physical realism”, defined as the statement that the goal of physics is to study entities of the natural world, existing independently from any particular observer’s perception, and obeying universal and intelligible rules, see Aufféves and Grangier [70].

In my works this realism was treated as contextual realism [32]. However, this terminology might be misleading, since in quantum foundations contextuality is treated restrictively, as dependence on outcomes of joint measurement of a compatible observable. We can also speak about epistemic realism.

Finally, we remark that physical (contextual) realism matches well with Bohr’s views.

One may even say that my contextual quantum realism (and, similarly, the quantum realism of Grangier)\(^{20}\) is trivially and commonly acceptable. In fact, this is not the case. And even if it were the case, it is very important to emphasize realism of QM. The main problem is wide spread of identifying of ontic and epistemic realisms. As was discussed, the impossibility of this identification was interpreted as the impossibility to keep ontic realism (at least locally). However, what is the most surprising this rejection of (local) ontic realism is often represented as total rejection of realism in QM.

3.6 Växjö interpretation of quantum mechanics

The Växjö interpretation (VI) of QM is a realist statistical local and contextual interpretation of QM [14, 68, 72].

We shall briefly explain the meanings assigned to these terms.

- The easiest for explanation is “statistical”. By VI, probabilities are interpreted statistically, i.e., they are related to ensembles of systems and not individual systems.
- VI is local, there is simply no need to even raise the issue of nonlocality - in complete agreement with Einstein who mentioned that nonlocality is an absurd alternative to realism.
- VI considers both realisms: “subquantum realism” (cf. Schrödinger, Einstein, de Broglie, Lochak, Atmanspacher and Primas) and “contextual realism” (cf. Bohr, Aufféves and Grangier). The first one is more challenging. We all still suffer of the lack of imagination... A clear presentation of the second one was attempted in section 3.5.
- VI is contextual, as any interpretation referring to two descriptive levels, because any epistemic model is contextual as taking into account all experimental arrangement.

4 Future

Recent successful tests for Bell’s inequality can be interpreted in various ways, with two extremes, CV1-CV3 and NC1-NCV3, with numerous intermediate interpretations. We can say that experimenters did their job excellently. However, loophole free Bell tests should not be interpreted as finalizing the century long discussion about the possibility to construct a theoretical subquantum model coupled to the quantum observational model.

My interpretation is that the successful realization of Bell’s test confirmed that

- We have to distinguish sharply two descriptive levels, theoretical and observational (ontic and epistemic).
- The correspondence between them can be very tricky (so, not so straightforward as in the Bell conjecture \(O=E\)).\(^{21}\)
- Contextuality plays the crucial role in coupling of the two descriptive levels.

Thus people looking for new insights on QM have to work hardly to find the proper subquantum theoretical description by relaxing constraints on the rules of correspondence between the two descriptive levels.

\(^{19}\) Which, as we argue, can be held against Bell himself.

\(^{20}\) To reconcile realism with QM, Grangier et al. also proceed in the contextual framework [70, 71]. However, they understood contextuality differently from me and from the conventional viewpoint (as joint measurement with a compatible observable); for them, context is not simply a complex of experimental physical conditions, see [70, 71] for details. Thus in this note we operate with three different types of contextuality.

\(^{21}\) Cf., e.g., with prequantum classical statistical field theory [37].
The final loophole free test says goodbye to the last hopes to identify the observational and ontic descriptions. One has to look for more complex maps

\[ f : \text{SUBQM} \rightarrow \text{QM}. \] (4)

This modeling has to be physically constructive, i.e., from the very beginning one should put efforts not to beat some no-go theorem(s), but to create a physically meaningful theoretical model. \(^\text{22}\) \text{SUBQM} endowed with a physically meaningful map \(f\). A partial experimental verification is a part of this great project which successful realization would finally resolve (as I believe!) the puzzle of QM.

Personally I still believe \(^{[37]}\) in the possibility to construct a subquantum model of the classical random field type, cf. Einstein and Infeld \(^{[73]}\), with nonlinear subquantum dynamics. (For a moment, experimentalists can try to proceed to new measurement technologies which would provide a possibility to measure characteristics of classical fields for quantum systems, e.g., the polarization vector.)

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Key words. Bell inequality, loophole free test, interpretations.

\(^{[22]}\) It may be surprising to hear this statement from a pure mathematician. However, in contrast to physicists who are so excited by the ability of mathematics to model everything in nature, I recognize that sometimes this ability looks like the ability of a prostitution to serve all men’s desires. Thus a purely mathematical subquantum model is practically valueless.

\(^{[23]}\) It was very important for me that both are highly qualified physicists, that they do not simply play with mathematical formulas, but there is a lot of physical intuition behind their positions.

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