Neutrinoless double beta decay and pseudo-Dirac neutrino mass predictions through inverse seesaw mechanism

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In the inverse seesaw extension of the standard model, supersymmetric or non-supersymmetric, while the light left-handed neutrinos are Majorana, the heavy right-handed neutrinos are pseudo-Dirac fermions. We show how one of these latter category of particles can contribute quite significantly to neutrinoless double beta decay. The neutrino virtuality momentum is found to play a crucial role in the non-standard contributions leading to the prediction of the pseudo-Dirac fermion mass in the range of 120 MeV – 500 MeV. When the Dirac neutrino mass matrix in the inverse seesaw formula is similar to the up-quark mass matrix, characteristic of high scale quark-lepton symmetric origin, the predicted branching ratios for lepton flavor violating decays are also found to be closer to the accessible range of ongoing experiments.

I. INTRODUCTION: The standard gauge theory of strong, weak, and electromagnetic interactions has confronted numerous experimental tests while the last piece of evidence on the Higgs boson is currently under rigorous scrutiny at the Large Hadron Collider (LHC). In spite of these, neutrino oscillation data uncovering tiny masses of left-handed (LH) neutrinos call for physics beyond the standard model (SM) which is most simply achieved via canonical seesaw mechanism [1, 2] that requires the addition of one heavy right-handed (RH) neutrino per generation provided both LH and RH neutrinos are Majorana fermions [3]. Several other forms of seesaw mechanism [5-7] also require Majorana fermions. Quite interestingly, ongoing experiments on neutrinoless double beta decay (0νββ) [8] is expected to resolve the issue between Majorana [3] or Dirac [4] nature of the neutrino. In contrast to the predicted small contribution to the 0νββ decay rate in the SM, there has been quite significant, or even more dominant predictions if, at the TeV scale, there is left-right (LR) gauge theory [10]1. Even, attempts have been made to predict nonstandard contributions to 0νββ decay rate due to the mediation of pseudo-Dirac neutrinos where each of them is considered to be a pair of Majorana neutrinos [11, 12]. While the possibility of left-handed neutrinos being pseudo-Dirac has been shown to be highly challenging [13], contribution of a fourth generation heavy pseudo-Dirac neutrino to 0νββ has been explored with the condition that its mass should be greater than M_Z/2 [14]. If the Dirac neutrino mass matrix occurring in seesaw formulas has its left-right symmetric or quark-lepton symmetric origin, descending from Pati-Salam symmetry [15] or SO(10) grand unified theory [16] at high scales, then the canonical seesaw scale is too large to be experimentally tested by high energy accelerators including LHC. Alternatively, the inverse seesaw mechanism [17, 18], which requires one RH neutrino as well as an additional sterile fermion per generation, operates at TeV scale and is, therefore, experimentally verifiable. In this framework while the LH light neutrinos are Majorana fermions, the RH neutrinos are pseudo-Dirac by nature having heavier masses.

In this letter we show that the inverse seesaw extension explaining the light neutrino masses and mixings permits the lightest of the three pseudo-Dirac neutrinos in the mass range (120 – 500) MeV leading to new contributions to 0νββ decay comparable to, or much more than, those due to the exchanges of the light left-handed neutrinos. The neutrino virtuality momentum [19-20], |p| ~ 190 MeV, is noted to play a crucial role in such new contributions. The origin of Dirac neutrino mass matrix is also found to be important in our estimations in predicting lepton flavor violating decays accessible to ongoing experimental searches. As our results are also applicable in the inverse seesaw extension of the minimal supersymmetric standard model (MSSM), they are consistent with gauge coupling unification at the MSSM-GUT scale, M_U ~ 2 x 10^{16} GeV.

II. THE INVERSE SEESAW EXTENSION: As is customary to the implementation of inverse seesaw mechanism, we add two fermion singlets to each generation of the SM, with or without supersymmetry. While we call the first type of singlet a RH neutrino (N_R), the second type of singlet is named as a sterile neutrino (S_L) and, in the (ν_L, N_R, S_L) basis, the 9x9 neutrino mass matrix is [18]

\[
\mathcal{M}_\nu = \begin{pmatrix}
0 & M_D & 0 \\
M_D^T & 0 & M^T \\
0 & M & \mu_S
\end{pmatrix},
\]

(1)

where M_D is the Dirac mass term of the neutrino, and M is the heavy Dirac mass matrix relating N_R and S_L. The matrices M_D and M are in general 3x3, and M is the complex symmetric matrix.

Transformation from flavor to mass basis and diagonaliza-
tion are achieved through

$$\langle \nu \rangle_f = \mathcal{V}^* \langle \nu \rangle_m, \quad (2)$$

$$\mathcal{V}^\dagger M_\nu \mathcal{V}^* = \text{Diag}\{m_{\nu_i} ; M_{\nu_j}\}, \quad (3)$$

where $\langle \nu \rangle_m = (\bar{\nu}_i, \zeta_j)^T$ represents the three light and six heavy mass states, and $i$ and $j$ run over the light and heavy mass eigenstates, respectively. With $\mu_S, M_D \ll M$, the matrix $M_\nu$ can be block diagonalized to light and heavy sectors

$$m_\nu \simeq \left(\begin{array}{c|c}
M_D & M_D \\
\hline
0 & M_T
\end{array}\right),$$

$$M_H \simeq \left(\begin{array}{c|c}
0 & M_T
\hline
M & \mu_S
\end{array}\right), \quad (4)$$

where $m_\nu$ has the well known inverse seesaw formula [18] and $M_H$ is the mass matrix for heavy pseudo-Dirac pairs of comparable masses with splitting of the order of $\mu_S$. The $\mu_S$ term in the Lagrangian breaks the lepton global symmetry, $U(1)_L$, which is otherwise preserved in the standard model in the limit $\mu_S \to 0$ rendering all the LH neutrinos to be massless. Hence the small $\mu_S$ should be a natural parameter in this theory in the ’t Hooft sense [21]. The above block diagonalized matrices are further diagonalized through the PMNS matrix, $U_\nu$, and a $6 \times 6$ unitary matrix $U_H$, respectively, so that

$$\mathcal{V} \simeq \left(\begin{array}{c|c}
1 - \frac{1}{2}B^*B^T & B^* \\
\hline
-\frac{1}{2}B^*B^T & 1 - \frac{1}{2}B^*B^T
\end{array}\right) \left(\begin{array}{c|c}
U_\nu & 0 \\
\hline
0 & U_H
\end{array}\right), \quad (5)$$

where $B^T \simeq \left(\begin{array}{c|c}
-M_S^{-1}M_D(M_D^{-1})^\dagger & (M_D^{-1})^\dagger
\hline
M_D & M_D
\end{array}\right) \sim \left(\begin{array}{c|c}
0 & X^\dagger
\hline
X & 0
\end{array}\right). \quad (6)$

Hence, in the leading order approximation, $\mathcal{V}$ can be written as

$$\mathcal{V} \simeq \left(\begin{array}{c|c|c}
1 - \frac{1}{2}XX^\dagger & 0 & X \\
\hline
0 & 1 & 0 \\
-XX^\dagger & 0 & \frac{1}{2}X^\dagger X
\end{array}\right) \left(\begin{array}{c|c}
U_\nu & 0 \\
\hline
0 & U_H
\end{array}\right), \quad (7)$$

where $X = (M_D^{-1})^\dagger$, and all the elements in the first block are $3 \times 3$ matrices.

(II.A) $\mu_S$ from neutrino oscillation data: The inverse seesaw formula in eqn. (4) predicts light neutrino mass terms in terms of three other matrices, $M_D, M,$ and $\mu_S$. At first we take $M_D \simeq M_\tau$, the charged lepton mass matrix, which may arise if the SM originates from high scale left-right gauge symmetry, $SU(2)_L \times SU(2)_R \times U(1)_{L-B-L} \times SU(3)_C \frac{M_R}{M_\nu}$, where $M_R >> M_\nu$. Assuming the matrix $M$ to be diagonal for the sake of simplicity and using $M_\nu = \text{diag}\{m_e, m_\mu, m_\tau\} = \{0.005, 0.1, 1.7\}$ eV, we obtain $\mu_S$ from global fits to the neutrino oscillation data [22] given in TABLE II.

$$\mu_S (\text{GeV}) = X^{-1} \mathcal{N} \bar{m}_\nu N^T X^{T-1}$$

$$= \left(\begin{array}{cccc}
6.71 \times 10^{-7} + 1.96 \times 10^{-7} i & -1.17 \times 10^{-8} - 3.22 \times 10^{-8} i & -3.71 \times 10^{-8} - 2.03 \times 10^{-8} i \\
-1.17 \times 10^{-8} - 3.22 \times 10^{-8} i & 1.53 \times 10^{-9} + 2.22 \times 10^{-10} i & 7.0 \times 10^{-9} + 2.83 \times 10^{-9} i \\
-3.71 \times 10^{-9} - 2.03 \times 10^{-8} i & 7.0 \times 10^{-9} + 2.83 \times 10^{-9} i & -5.50 \times 10^{-9} + 5.26 \times 10^{-11} i
\end{array}\right), \quad (8)$$

where $\mathcal{N} = (1 - \eta) U_\nu$ and $\eta = \frac{1}{2}XX^\dagger$ is a measure of unitarity violation. This particular structure of $\mu_S$ has been derived using, as an example, the normal hierarchical (NH) light neutrino masses $\bar{m}_\nu^{\text{diag}} = \text{diag}(0.00127 \text{ eV, } 0.00885 \text{ eV, } 0.0495 \text{ eV})$ and non-degenerate eigenvalues of $M = \text{diag}\{0.2, 2.6, 23.7\} \text{ GeV}$. Similar analysis predicts somewhat different structures of $\mu_S$ for inverted hierarchical (IH) and quasi-degenerate (QD) pattern of the light neutrinos and can further be easily obtained for degenerate $M_1 = M_2 = M_3$ or, partially-degenerate $M_1 = M_2 \ll M_3$ after taking care of the phenomenological bounds $|\delta_{ee}| < 2.0 \times 10^{-3}$, $|\eta_{\mu\tau}| < 8.0 \times 10^{-4}$, and $|\eta_{\tau\tau}| < 2.7 \times 10^{-3}$. Our ansatz with $M = \text{diag}(M_1, M_2, M_3)$

![Table 1: Mass squared differences, mixing angles, and CP-phase from global fits to neutrino oscillation data](image)
are current Lagrangian can be expressed as exchanges to the flavor eigenstates as linear combination of light and heavy mass eigenstates

\[ \nu_\alpha = N_{\alpha i} \nu_i + U_{\alpha j} \xi_j, \]

where all masses on the right hand side are in GeV.

### III. NEUTRINOLESS DOUBLE BETA DECAY PREDICTIONS

Two separate contributions due to light and heavy neutrino exchanges to $0\nu\beta\beta$ transition become transparent by writing the flavor eigenstates as linear combination of light and heavy mass eigenstates

\[ \nu_\alpha = N_{\alpha i} \nu_i + U_{\alpha j} \xi_j, \]

where $U \simeq (0, X)U_H$ is a $3 \times 6$ matrix. Then the weak charge-current Lagrangian can be expressed as

\[ \mathcal{L}_{CC} = \frac{g}{\sqrt{2}} W^\mu \ell \gamma^\mu P_L \nu_\alpha + \text{h.c.} \]

resulting in two different categories of Feynman amplitudes:

- $A^e_{\nu,LL}$ which arises from the Feynman diagram of Fig. 1(a) due to only light neutrino exchanges

\[ A^e_{\nu,LL} = G_F^2 N_{\alpha i} \frac{m_{\nu_\alpha}}{p^2}, \]

where $(p) \simeq 190$ MeV represents neutrino virtuality momentum [19, 20].

- $A^e_{\xi,LL}$ which arises from the Feynman diagram of Fig. 1(b) due to heavy pseudo-Dirac neutrinos,

\[ A^e_{\xi,LL} = G_F^2 (U)_{ej} \frac{M_{\xi_j}}{p^2 - M_{\xi_j}^2}. \]

![Feynman diagrams](image)

**FIG. 1**: Feynman diagrams contributing to neutrinoless double beta decay due to light neutrino exchanges (left-panel) and heavy pseudo-Dirac neutrino exchanges (right-panel).

The mass eigenstates of heavy pseudo-Dirac neutrinos are $\left( \xi_{1}^+, \xi_{2}^+, \xi_{3}^+, \xi_{1}^-, \xi_{2}^-, \xi_{3}^- \right)$ with almost degenerate pairs $(\xi_{1}^+, \xi_{1}^-; k=1,2,3)$ but having small mass difference $\mu_S$ between the members of the pair and the flavor states are $(N_1, N_2, N_3; S_1, S_2, S_3)$. The mixing matrix for these pseudo-Dirac neutrinos have been discussed in Sec. II. The half-life of $0\nu\beta\beta$ transition is then found to be

\[ T_{1/2}^{0\nu\beta\beta} = K_{0\nu} \left| m_{\nu,LL}^{ee} + M_{\xi,LL}^{ee} \right|^2, \]

where $K_{0\nu}$ contains phase space factors plus nuclear matrix elements and $m_{\nu,LL}^{ee}$, $M_{\xi,LL}^{ee}$ represents the effective neutrino mass derived from light neutrino (heavy pseudo-Dirac neutrino) exchanges in the mass basis. The analytic forms of the two effective masses have been estimated for this model as shown in TABLE. II.

| Effective mass | Analytical expression |
|---------------|-----------------------|
| $m_{\nu,LL}^{ee}$ | $N_{\alpha i}^2 m_{\nu_\alpha}$ |
| $M_{\xi,LL}^{ee}$ | $(U_{ej})^2 \frac{M_{\xi_j}}{p^2 - M_{\xi_j}^2} |(p)|^2$ |

**TABLE II**: Effective mass parameter for standard (non-standard) contributions due to light (heavy pseudo-Dirac) neutrino exchanges for $0\nu\beta\beta$ decay.

We discuss below three different cases:

**III. A** The standard contribution. It is well known that the standard contributions due to light neutrino exchanges are dependent on their allowed mass patterns; normal hierarchical (NH), inverted hierarchical (IH), or quasi-degenerate (QD),

\[ m_{\nu,LL}^{ee} \simeq U_{e1}^2 m_{\nu_1} + U_{e2}^2 e^{2i\alpha} m_2 + U_{e3}^2 e^{2i\beta} m_3 \]

\[ \Rightarrow |m_{\nu,LL}^{ee}| \simeq \begin{cases} 0.004 \text{ eV} & \text{NH}, \\ 0.048 \text{ eV} & \text{IH}, \\ 0.1 \text{ eV} & \text{QD}. \end{cases} \]

In our case, $N_{\alpha i} \simeq U_{ei}$ and light neutrino exchanges in the mass basis gives almost the same contributions which are presented by solid lines shown in Fig. 2, Fig. 3 and Fig. 4.

**III. B** $M_{\xi_j} \gg |p|$: In the inverse see-saw extension under study, in addition to the standard effective mass parameter, the additional effective mass parameter for $|M_{\xi_j}| \gg |p|$ satisfies $(M_{\xi_j})^{-1} = (U^h)^2 \frac{1}{M_{\xi_j}}$. This results in new contribution to $0\nu\beta\beta$ transition half-life

\[ T_{1/2}^{0\nu\beta\beta} = K_{0\nu} \left| (p) |^2 \left( \frac{1}{M_{\xi_j}} - \frac{1}{M_{\xi_j}^{-1}} \right) \right|^2 \]

\[ \simeq K_{0\nu} \left| (p) |^2 (U^h)^2 \frac{\mu_{Skk}}{M_{kk}} \right|^2, \]

where $\mu_{Skk}$ and $M_{kk}$ are the eigenvalues of $\mu_S$ and $M$, respectively. One example of this case has been shown in Fig.
for $M_{\zeta_1} = 0.5$ GeV where the predicted effective mass parameter is nearly 3/2 (4) times larger than the standard prediction for NH (IH) case.

(III. C) $M_{\zeta_1} \approx |p|$ In this region where different allowed values of $M_{\zeta_1}$ are of the order of neutrino virtuality momentum $|p| \approx 190$ MeV, the new contribution to neutrinoless double beta decay due to heavy pseudo-Dirac neutrino exchange is found to be more dominant than the standard contribution and the $0\nu\beta\beta$ transition half-life is given below

$$T_{1/2}^{0\nu\beta\beta} \approx K_{0\nu} |\langle p | \hat{U}^\pm \left( M_{\zeta_1} - \frac{1}{2} M_{\zeta_2^+} - \frac{1}{2} M_{\zeta_2^-} \right) |^2$$

The predicted new values of the effective mass parameters arising solely due to pseudo-Dirac neutrino exchanges have been shown in Fig. 2 in the left-panel (right-panel) for NH (IH) patterns of the light LH neutrino masses, respectively, where $M_{\zeta_1} = (0.15 - 0.5)$ GeV. It is quite clear from the plots that even for $M_{\zeta_1} = 0.25$ GeV or, 0.5 GeV, the new contributions are 3-6 times larger than the standard ones. While for the value of $M_{\zeta_1} = 0.18$ GeV, the contribution is nearly 100 times larger shown in Fig. 2. This large enhancement occurs as $M_{\zeta_1}$ approaches the vicinity of the neutrino virtuality momentum, $|p| \approx 190$ MeV. We point out that such important effects of pseudo-Dirac neutrino masses are found for the first time in this work.

IV. LEPTON FLAVOR VIOLATION WITH DOMINANT $0\nu\beta\beta$ DECAY RATE: We have clearly shown that the predicted non-standard contributions to neutrinoless double beta decay rate are dominant for the lightest allowed pseudo-Dirac neutrino mass $M_{\zeta_1} \approx (0.15 - 0.5)$ GeV. However, because of the diagonal nature of $M_D$ and assumed structure of $M$, the branching ratios for lepton flavor violating (LFV) decays, $\mu \to e + \gamma$, $\tau \to e + \gamma$, and $\tau \to \mu + \gamma$ are as small as the SM predictions.

In the next two examples we adopt plausible parametrization predicting significantly larger contribution to these branching ratios while retaining the dominant contributions to $0\nu\beta\beta$ transition.

(IV. A) $M_D \approx M_\ell$ with non-diagonal $M$: We generate non-diagonal matrix $M$ to satisfy the existing phenomenological bounds on the elements of $\eta$ [23].

$$|\eta_{ee}| < 2.0 \times 10^{-3}, \quad |\eta_{e\mu}| < 3.5 \times 10^{-5},$$

$$|\eta_{e\tau}| < 8.0 \times 10^{-3}, \quad |\eta_{\mu\mu}| < 8.0 \times 10^{-4},$$

$$|\eta_{\mu\tau}| < 5.1 \times 10^{-3}, \quad |\eta_{\tau\tau}| < 2.7 \times 10^{-3}.$$  \hspace{1cm} (17)

Using the parametrization of the type used in ref. [24], $M$ can be expressed as

$$M \left( \frac{1}{\sqrt{2\eta}} O^T M_D \right)^{-1} \left( \frac{1}{\sqrt{2\eta}} O^T M_D \right)^{-1} = 1_3 = V^T V$$  \hspace{1cm} (18)

where $O$ is the matrix diagonalizing $|\eta|$ and $V$ is an arbitrary unitary matrix. Choosing, for the sake of simplicity, $V = 1_3$ and we note that that a lightest pair with $M_{\zeta_1} \simeq 0.16$ GeV, in the vicinity of neutrino virtuality momentum, can be achieved by suitable rescaling, e.g. $\eta_{\alpha\beta} \rightarrow \eta_{\alpha\beta}/(1500)$. After this scaling we find

$$M(\text{GeV}) = \left( \frac{1}{\sqrt{2\eta}} O^T M_D \right)$$

where

$$O = \begin{pmatrix}
0.5874i & 0.5446 & 0.5987 \\
0.4284i & -0.8368 & 0.3409 \\
-0.6866i & -0.0562 & 0.7248
\end{pmatrix}.$$
With the allowed mass eigenvalues for the heavy pseudo-
Dirac neutrinos, \( M_\zeta = \text{diag}(0.159, 72.0, 506.4) \text{ GeV} \), the
predicted branching ratios for lepton flavor violating decays are \([25]\)

\[
\begin{align*}
\text{Br}(\mu \to e + \gamma) &= 1.56 \times 10^{-26}, \\
\text{Br}(\tau \to e + \gamma) &= 5.79 \times 10^{-27}, \\
\text{Br}(\tau \to \mu + \gamma) &= 1.10 \times 10^{-18}. 
\end{align*}
\]

(20)

Although all the three branching ratios are much smaller than
their corresponding experimental upper limits \([27]\), they are
considerably larger than the SM predictions. However we note below that with \( M_D \) similar to \( M_u \), the up-quark mass
matrix, a phenomenon underlying the possible origin of SM
from Pati-Salam \([15]\) or SO(10) model, LFV decays have
much larger predicted values, accessible to ongoing experi-
mental searches, while similar predictions on dominant \( 0\nu\beta\beta 
\) decay are maintained.

(IV. B) \( M_D \approx M_u \) and GUT connection: In this case
Dirac neutrino mass matrix is approximated to be up-quark mass
matrix, which originates if the high scale symme-
try is Pati-Salam or SO(10) GUT, \( SU(2)_L \times SU(2)_R \times 
SU(4)_C \) or SO(10) \( M_{\text{GUT}} \) SM. Using running masses
\((m_u, m_e, m_\tau) = (0.00233, 1.275, 160) \text{ GeV} \) and Cabbibo-
Kobayashi-Maskawa mixing matrix, \( V_{\text{CKM}} \) \([28]\),

\[
M_D \text{(GeV)} \approx M_u = V_{\text{CKM}} M_u V_{\text{CKM}}^T
= \begin{pmatrix}
0.067 - 0.004i & 0.302 - 0.022i & 0.55 - 0.53i \\
0.302 - 0.022i & 1.48 - 0.0i & 6.534 - 0.001i \\
0.55 - 0.53i & 6.534 - 0.0009i & 159.72 + 0.0i
\end{pmatrix}.
\]

(21)

At first using the phenomenological bounds from eqn. \(17\)
and saturating our ansatz for \( \eta_{\beta\beta} = |\eta|_{\text{max}}/n \), \( n = 1 \ldots 5 \), we
search for matrix \( M \) through eqn. \(18\) which gives \( \mu_S \) from
eqn. \(8\). We obtain for \( n = 4 \)

\[
M \text{(GeV)} = \begin{pmatrix}
-5.9 - 3.45i & 0.2 - 60.72i & 5.15 - 1760i \\
-10.44 + 2.13i & -60.9 - 0.5i & -598.0 - 12.16i \\
7.08 - 5.09i & 70.86 - 0.18i & 1547 - 4.15i
\end{pmatrix},
\]

(22)

\[
\mu_S \text{(eV)} = \begin{pmatrix}
-3.42 - 0.51i & -1.92 + 5.55i & 0.28 - 1.92i \\
-1.92 + 5.55i & 39.1 + 5.68i & -12.1 + 0.11i \\
0.28 - 1.92i & -12.1 + 0.11i & 4.10 - 0.68i
\end{pmatrix}.
\]

(23)

Our predictions on numerical values of the effective mass
parameter for \( 0\nu\beta\beta \) are shown in Fig. 4 for NH, IH and QD
cases. For NH light neutrinos we find that the predicted value
of \( |M_{ee}| \) is increased by a factor 3 for \( \eta = |\eta|_{\text{max}}/3 \),
corresponding to lightest pair \( M_{11} = 131 \text{ MeV} \), while the
increment is 10 times for \( \eta = |\eta|_{\text{max}}/4 \) and \( M_{11} = 152 \text{ MeV} \),
and 30 times for \( \eta = |\eta|_{\text{max}}/5 \) and \( M_{11} = 169 \text{ MeV} \). We
find that the enhancement survives as long as lightest pair
\( M_{11} \sim 120 - 350 \text{ MeV} \). For the IH light neutrino masses the
results are similar as shown on the right panel of Fig. 4. The
branching ratios for lepton flavor violating decays predicted in
this scenario with \( M_\zeta = (0.152, 39.5, 2426) \text{ GeV} \) are

\[
\begin{align*}
\text{Br}(\mu \to e + \gamma) &= 3.6 \times 10^{-13}, \\
\text{Br}(\tau \to e + \gamma) &= 4.2 \times 10^{-14}, \\
\text{Br}(\tau \to \mu + \gamma) &= 3.3 \times 10^{-12},
\end{align*}
\]

(24)

while the present experimental limits at 90% C.L. on these
branching ratios are \( \text{Br}(\mu \to e + \gamma) \leq 1.2 \times 10^{-11}, \)
\( \text{Br}(\tau \to e + \gamma) \leq 3.3 \times 10^{-8}, \) and \( \text{Br}(\tau \to \mu + \gamma) \leq 4.4 \times 
10^{-8} \) \([27]\). The projected reach of sensitivity in the future
is \( \text{Br}(\tau \to e + \gamma), \text{Br}(\tau \to \mu + \gamma) \leq 10^{-9} \) and specifically
\( \text{Br}(\mu \to e + \gamma) \leq 10^{-19} \) \([27]\).

The predicted nonstandard contributions to \( 0\nu\beta\beta \) transition
are shown in the left-panel for NH and in the it right-panel

FIG. 3: Same as Fig. 2 but now with non-diagonal structure of \( M \) and reduced values of nonunitarity matrices \( \eta \) as described in the text.
for IH case of Fig. 4. In view of the $M_{\zeta_1}$ dependent enhancements of $0\nu\beta\beta$ decay rates discussed above it is tempting to search for the possibility of the lightest pseudo-Dirac neutrino mass which we perform by the replacement $M_{\zeta_1}^2 \rightarrow M_{\zeta_1}^2 + i M_{\zeta_1} \Gamma_1$, where $\Gamma_1$ corresponds to plausible value of width of the particle. Using, for example, $\Gamma_1 \approx 0.1$ keV, our predictions are presented by solid curve in Fig. 5 for NH light neutrino masses where the resonant behavior is clearly exhibited around $M_{\zeta_1} = 190$ MeV.

![Variation of effective mass $|M_{ee}|$ prediction as a function of lightest pseudo-Dirac neutrino mass $M_{\zeta_1}$ where $\mu_B$ matrix has been determined for NH light neutrino masses. The resonance peak is at $M_{\zeta_1} \approx 190$ MeV as shown by the solid line. For comparison, the standard contribution with NH masses is shown by the dashed horizontal line.](image)

V. DISCUSSIONS AND CONCLUSION: In this letter we have shown that in the inverse seesaw framework of the standard model, the lightest of the pseudo-Dirac neutrino could be of $O(100)$ MeV in concordance with tiny left handed neutrino masses and the oscillation data. This pseudo-Dirac neutrino mass being in the vicinity of the neutrino virtuality momentum $|p| \approx 190$ MeV, gives very significant non-standard contributions to $0\nu\beta\beta$ decay rates, even far exceeding the standard contributions. The Dirac neutrino mass possibly originating from high scale Pati-Salam symmetry or SO(10) grand unification, plays a crucial role in determining dominant contributions to $0\nu\beta\beta$ decay rates simultaneously with LFV decays with predicted branching ratios accessible to on-going search experiments. The underlying mechanism provides three distinct platforms for its falsifiability (i) $0\nu\beta\beta$ decay rates, (ii) determination of light pseudo-Dirac neutrino mass $M_{\zeta_1} \approx 120-500$ MeV, and (iii) the three predicted branching ratios of eqn. (23). As all our results are applicable in the case of inverse seesaw extended supersymmetric standard model, they are also consistent with gauge coupling unification at the MSSM-GUT scale, $M_U \approx 2 \times 10^{16}$ GeV. The Pati-Salam or SO(10) completion of the model discussed in Sec. IV.B will be reported elsewhere in future publication [26].

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