A STOCHASTIC PROCESS
FOR THE DYNAMICS
OF THE TURBULENT CASCADE
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Abstract

Velocity increments over a distance r and turbulent energy dissipation on a box of size r are well described by the multifractal models of fully developed turbulence. These quantities and models however, do not involve time-correlations and therefore are not a detailed test of the dynamics of the turbulent cascade.

If the time development of the turbulent cascade, in the inertial range, is related to the lifetime of the eddies at different length scales, the time correlations may be described by a stochastic process on a tree with jumping kernels which are a function of the ultrametric (tree) distance. We obtain the solutions of the Chapman-Kolmogorov equation for such a stochastic process, with jumping kernels depending on the ultrametric distance, but with an arbitrarily specified invariant probability measure. We then show how to use these solutions to compute the time correlations in the turbulent cascade.

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1 Introduction

One of the most interesting phenomena in fully developed turbulence is the occurrence of an energy cascade from the macroscopic length scale $L$ of the experimental apparatus down to smaller and smaller length scales. Length scales $l$ in the range $L \gg l \gg \eta$, $\eta$ being the scale where the fragmentation process is stopped by dissipation, are said to be in the "inertial range". In the inertial range viscosity effects are not important and Kolmogorov proposed long ago a scaling theory with conserved energy transfer between length scales. From scale invariance and the assumption that turbulence is space-filling it follows that the velocity fluctuation $\delta v(l)$ over an active eddy of size $l$ scales as

$$\langle \| \delta v(l) \|^p \rangle \sim l^{\zeta_p}$$

with $\zeta = \frac{p}{3}$.

However turbulence may or may not be space-filling and the volume of the active eddies may change when the energy is transferred from the scale $l_n$ to the scale $l_{n+1}$. This leads naturally to a fractal structure for the cascade with fractal dimension less than 3. For example in the $\beta$-model the rate of energy transfer

$$E_n \sim \frac{\delta v_n^3}{l_n}$$

does not change along the cascade but the total mass of the active eddies is multiplied by $\beta$ at each step. Then the exponent $\zeta_p$ in Eq. (1.1) becomes

$$\zeta_p = p.h + 3 - D$$

with $h = \frac{D-2}{3}$, where $D$ is the fractal dimension related to $\beta$ by

$$\log_2 \beta = D - 3$$

if the length scales are related by $l_n = l_0 2^{-n}$.

The $\beta$-model as well as a log-normal model for the distribution of $E_n$ are however in contradiction with the experimental results on moments of higher order for the velocity structure functions. This fact led to the proposal of a multifractal generalization called the random $\beta$-model where it is assumed that, at each scale $l_n$, there are several distinct $\beta_n(k)$’s which are chosen
according to some probability law. That is, the energy transfer may take place according to several distinct dimensional routes. Requiring a fixed energy transfer rate one obtains

$$\frac{\delta v^3_n(k)}{l_n} = \beta_{n+1}(k) \frac{\delta v^3_{n+1}(k)}{l_{n+1}}$$

(1.2)

Hence at scale $l_n$ the velocity fluctuation in each eddy depends on the fragmentation history which is defined by the product $\beta_1, \beta_2, \ldots, \beta_n$. Then

$$\delta v_n \sim l_n^{\frac{1}{3}} (\prod_{i=1}^{n} \beta_i)^{-\frac{1}{3}}$$

(1.3)

and

$$\langle \| \delta v_n(l_n) \|^p \rangle \sim l_n^{\frac{p}{3}} \int \prod_{i=1}^{n} d\beta_i \beta_i^{1-\frac{p}{3}} P(\beta_1, \ldots, \beta_n)$$

(1.4)

$P(\beta_1, \ldots, \beta_n)$ being the occurrence probability of the sequence $\beta_1, \ldots, \beta_n$.

At the level of precision of the existing experiments, agreement with the data is already obtained if one assumes independent fragmentations

$$P(\beta_1, \ldots, \beta_n) = \prod_{i=1}^{n} P(\beta_i)$$

(1.5a)

and a simple binomial process

$$P(\beta) = \gamma \delta_1 + (1 - \gamma) \delta_{\frac{1}{2}}$$

(1.5b)

$\gamma$ is a parameter chosen to fit the data ($\gamma \simeq 0.875$).

The assumptions (1.5) only define the probability distribution $\rho_i$ at each level of the cascade tree (Fig.1). They make no statement concerning the time evolution and the time scales of the eddies in the cascade. In fact it is well known\textsuperscript{6,9} that only a few restrictions are imposed on the form of the velocity fields by the predictions of the statistical models described above. These are essentially the existence of singularities in the derivatives of the velocity field at some points. Aside from that, several distinct velocity fields may be compatible with the spectra and the scaling laws. They range from the superposition of random uncorrelated Gaussian components having only a correct spectrum in the inertial range\textsuperscript{10,11} to simple flow fields in isolation,
such as a vortex sheet wrapping up while being stretched by a large-scale straining motion\textsuperscript{12}.

One way to further the research on the dynamical properties of the velocity field is to study time-correlations for the observables. To make a connection of the models with the time dependence of each observable one should first consider the dynamical aspects of the energy cascade itself. To do this a time hierarchy in the development of the turbulent cascade must be defined. In what follows we deal with this issue.

For a binary cascade tree (Fig.1) we may use a dyadic labelling for the possible states at each level. The state space $V_n$ at level $l_n$ is the set of all products $\beta^{(1)} \cdot \cdots \beta^{(n)}$ with $\beta^{(i)} \in \beta_0, \beta_1$. (In Eq.(1.5b) $\beta_0 = 1$ and $\beta_1 = \frac{1}{2}$). There are $2^n$ elements in $V_n$ and the probability of the state $i$ is

$$\rho_i = P(\beta^{(1)} \cdot \cdots \beta^{(n)}) = \gamma^{n_0(i)} (1 - \gamma)^{n_1(i)}$$

where $n_0(i)$ and $n_1(i)$ are the number of zeros and ones in the dyadic labelling of the state $i$.

From the random $\beta$-model all one obtains is a statement about these probabilities. This suffices to interpret most of the current experimental results which concern mostly velocity increments over a distance $r$ and turbulent energy dissipation over a box of size $r$. These quantities do not involve time-correlations and therefore do not make a detailed test of the dynamics of the cascade, they only test its invariant probability measure. As discussed above to identify the physical mechanisms behind the structure of fully developed turbulence, more information is needed. If one wants, for example, the time correlations at a point moving with the free-stream velocity of the fluid one should explicitly consider models for the dynamics in state space at each level $n$. To the same invariant measure $\rho_i$ correspond many different processes. The most unstructured process corresponds to the statement that, if at time zero one finds the state $i$, then the transition probability to the state $j$ at time $t$ is proportional to $\rho_j$. For the turbulent cascade the unstructured process does not seem to be natural because, if the lifetime of the eddies in the inertial range scales like $\frac{l_n}{\delta v_n}$, then we expect larger eddies to live longer than small eddies. That is, if at time $t$ the fluctuation $\delta v_n(x)$ at the point $x$ is receiving its energy through a fragmentation history leading to the state $i$ then, a short time thereafter, we expect to find a different state which is nearby in the sense of the natural ultrametric distance in the tree.
To characterize a stochastic process on a tree one has to solve the Chapman-Kolmogorov equation for the transition probabilities

\[ \partial_t p(z|y) = \int dx \{ W(z|x)p(x|y) - W(x|z)p(z|y) \} \]  

with kernels \( W(z|x) \) that reflect the (natural) ultrametric distance in the tree. For kernels that depend only on the distance \( W(z|x) = W(|z - x|) \), Ogielski and Stein\(^{13}\) found the solution of Eq.(1.7). Albeverio and Karwowski\(^{14,15}\) have also constructed the stochastic processes on arbitrary p-adic fields \( Q_p \) for the case where the jumping kernels depend only on the distance between p-adic balls (see also Brekke and Olson\(^{16}\)). However it is easy to see from the equation for the probability densities

\[ \partial_t \rho(z) = \int dx \{ W(z|x)\rho(x) - W(x|z)\rho(z) \} \]  

that if \( W(z|x) = W(|z - x|) \) then the invariant density is \( \rho(z) = \text{const.} \). For the stochastic process of the turbulent cascade we require a non-constant invariant density as in Eq.(1.6) and the results of the authors of Refs. 13-16 cannot be used.

From (1.8) it follows that with

\[ W(z|x) = \rho(z)f(|z - x|) \]  

the invariant density is \( \rho(z) \) and, at the same time, full account is taken of the dependence of the transition probability on the distance between the points \( z \) and \( x \) in state space. In the next Section we characterize the solutions of the Chapman-Kolmogorov equation for kernels of the form (1.9). In Section 3 we then show how to use these solutions to compute (or parametrize) the time correlations of the turbulent cascade.

2 Random walk on a tree with asymmetric jumping kernels

We rewrite Eq.(1.8) in matrix form

\[ \frac{\partial}{\partial t} \rho(t) = W \rho(t) \]  

4
where $W$ is the matrix

$$
\begin{pmatrix}
W_{11} & \rho_1 e_1 & \rho_1 e_2 & \rho_1 e_3 & \rho_1 e_3 & \rho_1 e_3 & \cdots \\
\rho_2 e_1 & W_{22} & \rho_2 e_2 & \rho_2 e_2 & \rho_2 e_3 & \rho_2 e_3 & \rho_2 e_3 & \cdots \\
\rho_3 e_2 & \rho_3 e_2 & W_{33} & \rho_3 e_1 & \rho_3 e_3 & \rho_3 e_3 & \rho_3 e_3 & \cdots \\
\rho_4 e_2 & \rho_4 e_2 & \rho_4 e_1 & W_{44} & \rho_4 e_3 & \rho_4 e_3 & \rho_4 e_3 & \cdots \\
\rho_5 e_3 & \rho_5 e_3 & \rho_5 e_3 & \rho_5 e_3 & W_{55} & \rho_5 e_1 & \rho_5 e_2 & \rho_5 e_2 & \cdots \\
\rho_6 e_3 & \rho_6 e_3 & \rho_6 e_3 & \rho_6 e_3 & \rho_6 e_1 & W_{66} & \rho_6 e_2 & \rho_6 e_2 & \cdots \\
\rho_7 e_3 & \rho_7 e_3 & \rho_7 e_3 & \rho_7 e_3 & \rho_7 e_2 & \rho_7 e_2 & W_{77} & \rho_7 e_1 & \cdots \\
\rho_8 e_3 & \rho_8 e_3 & \rho_8 e_3 & \rho_8 e_3 & \rho_8 e_2 & \rho_8 e_2 & \rho_8 e_1 & W_{88} & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{pmatrix}
$$

Notice that the matrix has increasingly larger non-diagonal blocks of size $2^i \times 2^i$ which have the common factor $\epsilon_i$. These blocks correspond to jumps to a ultrametric distance $i$. A matrix element in a block of size $2^i \times 2^i$ at line $k$ equals $\rho_k \epsilon_i$, $\epsilon_i$ being the value of $f(|z-x|)$ in Eq. (1.9) for a jump to a distance $i$. The elements $W_{ii}$ in the diagonal are such that the columns add to zero.

As in the symmetric case studied by Ogielski and Stein\(^{13}\) we find the complete set of eigenvectors of the matrix $W$. For a matrix of dimension $2^n$, which describes the stochastic process at the nth level of the tree, the eigenvectors are:

(i) The eigenvector

$$
\begin{pmatrix}
\rho_1 \\
\rho_2 \\
\vdots \\
\rho_{2^n}
\end{pmatrix}
$$

with eigenvalue $\lambda_0 = 0$;

(ii) $n$ classes with $2^{n-k}$ ($k=1 \ldots n$) eigenvectors each, where each eigenvector has only $2^k$ non-zero elements. The first $2^{k-1}$ elements are positive and the others are negative. The $2^k$ non-zero elements of an eigenvector have a common ancestor, in the tree, at the level $n-k$. The non-zero elements of an (unnormalized) eigenvector are formed by multiplying the corresponding $\rho_i$ by the sum of the $\rho_j$’s of the complementary group in the non-zero set of elements. The formation rule is easier to understand from an example.
Let $n=3$ (Fig. 2). There are then three classes of eigenvectors in the group (ii), typical examples of which are:

a) 
\[
\begin{pmatrix}
\rho_1 \rho_2 \\
-\rho_2 \rho_1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{pmatrix}
\]
\[
\lambda_1 = -(\epsilon_1(\rho_1 + \rho_2) + \epsilon_2(\rho_3 + \rho_4) + \epsilon_3(\rho_5 + \rho_6 + \rho_7 + \rho_8))
\]
Four eigenvectors of this type corresponding to the independent groups of two elements with a common ancestor at level 2.

b) 
\[
\begin{pmatrix}
\rho_1(\rho_3 + \rho_4) \\
\rho_2(\rho_3 + \rho_4) \\
-\rho_3(\rho_1 + \rho_2) \\
-\rho_4(\rho_1 + \rho_2) \\
0 \\
0 \\
0 \\
\end{pmatrix}
\]
\[
\lambda_5 = -(\epsilon_2(\rho_1 + \rho_2 + \rho_3 + \rho_4) + \epsilon_3(\rho_5 + \rho_6 + \rho_7 + \rho_8))
\]
Two eigenvectors of this type.

c) 
\[
\begin{pmatrix}
\rho_1(\rho_5 + \rho_6 + \rho_7 + \rho_8) \\
\rho_2(\rho_5 + \rho_6 + \rho_7 + \rho_8) \\
\rho_3(\rho_5 + \rho_6 + \rho_7 + \rho_8) \\
\rho_4(\rho_5 + \rho_6 + \rho_7 + \rho_8) \\
-\rho_5(\rho_1 + \rho_2 + \rho_3 + \rho_4) \\
-\rho_6(\rho_1 + \rho_2 + \rho_3 + \rho_4) \\
-\rho_7(\rho_1 + \rho_2 + \rho_3 + \rho_4) \\
-\rho_8(\rho_1 + \rho_2 + \rho_3 + \rho_4) \\
\end{pmatrix}
\]
\[
\lambda_7 = -\epsilon_3(\rho_1 + \rho_2 + \rho_3 + \rho_4 + \rho_5 + \rho_6 + \rho_7 + \rho_8) = -\epsilon_3
\]
One eigenvector of this type.
The rule of formation for the eigenvalues is clear from the example above. Each eigenvalue is a sum of terms

$$\lambda_i = - \sum_{j=i(i)}^n \epsilon_j \sum_k \rho_k$$

(2.2)

where $\epsilon_{i(i)}$ is the first $\epsilon$ which covers all the non-zero elements of the vector and the sum $\sum_k \rho_k$ contains all the probability densities of the states reached by an $\epsilon_j$-jump.

A solution of Eq.(2.1) is an arbitrary superposition

$$\rho(t) = \sum c_i e^{-\lambda_i t} v_i$$

(2.3)

of the eigenvectors above. From (2.3) it is now easy to construct the general solution of the Chapman-Kolmogorov equation. To obtain the transition probability $p(jt|00)$ from the state $i$ at time zero to the state $j$ at time $t$, one chooses the coefficients $c_i$ in (2.3) in such a way that, at time zero, only the $i$-th component is non-zero and then read the value of the $j$-th component at time $t$. Before stating the general result we illustrate it by writing the transition probabilities for transitions between typical states in each group for the case $n=3$.

$$p(1t|10) = \frac{\rho_1}{\rho_1 + \cdots + \rho_8} + \frac{\rho_1 \rho_2}{\rho_1 (\rho_1 + \rho_2)} e^{-t \left( \epsilon_1 (\rho_1 + \rho_2) + \epsilon_2 (\rho_3 + \rho_4) + \epsilon_3 (\rho_5 + \cdots + \rho_8) \right)}$$

$$+ \frac{\rho_1 (\rho_3 + \rho_4)}{(\rho_1 + \rho_2)(\rho_1 + \rho_2 + \rho_3 + \rho_4)} e^{-t \left( \epsilon_2 (\rho_1 + \epsilon_3 (\rho_5 + \cdots + \rho_8) \right)}$$

$$+ \frac{\rho_1 (\rho_5 + \cdots + \rho_8)}{(\rho_1 + \cdots + \rho_4)(\rho_1 + \cdots + \rho_8)} e^{-t \epsilon_3}$$

$$p(2t|10) = \frac{\rho_2}{\rho_1 + \cdots + \rho_8} - \frac{\rho_2 \rho_1}{\rho_1 (\rho_1 + \rho_2)} e^{-t \lambda_1} + \frac{\rho_2 (\rho_3 + \rho_4)}{(\rho_1 + \rho_2)(\rho_1 + \rho_2 + \rho_3 + \rho_4)} e^{-t \lambda_5}$$

$$+ \frac{\rho_2 (\rho_5 + \cdots + \rho_8)}{(\rho_1 + \cdots + \rho_4)(\rho_1 + \cdots + \rho_8)} e^{-t \lambda_7}$$
\[ p(3t|10) = \frac{\rho_3}{\rho_1 + \cdots + \rho_8} - \frac{\rho_3 (\rho_1 + \rho_2)}{(\rho_1 + \rho_2)(\rho_1 + \rho_2 + \rho_3 + \rho_4)} e^{-t\lambda_3} + \frac{\rho_3 (\rho_5 + \cdots + \rho_8)}{(\rho_1 + \cdots + \rho_4)(\rho_1 + \cdots + \rho_8)} e^{-t\lambda_7} \]

\[ p(5t|10) = \frac{\rho_5}{\rho_1 + \cdots + \rho_8} - \frac{\rho_5 (\rho_1 + \cdots + \rho_4)}{(\rho_1 + \cdots + \rho_4)(\rho_1 + \cdots + \rho_8)} e^{-t\lambda_7} \] (2.4)

Of course in this case \( \rho_1 + \cdots + \rho_8 = 1 \) but we have kept this term to emphasize the rule of formation of the coefficients. The general rule for the transition probability \( p(jt|i0) \) between two states \( i \) and \( j \) at the level \( n \) in the tree is the following:

(i) \( p(jt|i0) \) is a sum of terms, the first of which is \( \rho_j \) (the target probability), and has as many terms as the number of eigenvectors that have non-zero elements in both the \( i \) and the \( j \) positions.

\[ p(jt|i0) = \rho_j + \sum_k c_k e^{-t\lambda_k} \] (2.5)

(ii) The exponential factor in each term contains the eigenvalue of the associated eigenvector.

(iii) The coefficients all contain in the numerator the target probability \( \rho_j \) multiplied by the sum of the probabilities of non-zero elements of the corresponding eigenvectors in the half that does not contain \( j \). The denominator is the sum of the half that contains \( i \) multiplied by the sum of all probabilities associated to the non-zero elements of the eigenvector.

(iv) The sign of the coefficient is the product of the signs of the \( i \) and \( j \) entries in the eigenvector.

3 Time correlations. Application to the turbulent cascade

Once the transition probabilities \( p(zt|y0) \), solutions of the Chapman-Kolmogorov equation are known, the time correlations of the process are obtained from

\[ \langle x(t)x(0) \rangle = \int dydx \ y \ p(yt|x0) \ x \ \rho(x) \] (3.1)
or

$$\langle x(t)x(0) \rangle = \sum_{i,j} x_j p(j|i0)x_i \rho_i$$  \hspace{1cm} (3.2)$$

for a discrete state space.

Using the results of Section 2 (Eq.(2.5)) one sees that at large times the
time correlation at level n will be dominated by the largest non-zero eigen-
value $\lambda_{2n-1} = -\epsilon_n$. Assuming that the dynamics of the turbulent cascade
is controlled by the decay of the eddies, the largest non-zero eigenvalue will
always be the same, associated to the mean lifetime of large eddies. However
the asymptotic long-time correlation will be difficult to measure because of
the small values of $\langle x(t)x(0) \rangle$ at large t. Error bars, in numerical or actual
experiments, are likely to be larger than $e^{-\epsilon_n t}$ for $t$ large.

If, as we are proposing, the time correlations in the turbulent cascade are
described by a stochastic process with kernels that depend on tree distances,
a first qualitative prediction is the occurrence of several exponential slopes,
as the time increases, in the time-correlation functions. Notice that the
existence of different time scales, as a consequence of the advection of small-
scale eddies by large-scale motions, was already pointed out by Kolmogorov
(see Ref.9).

Of special interest is the slope of the short-time correlation which is controlled
by the smallest eigenvalue. Using the dyadic expansion to label the points
$x_i$ in state space

$$x_i = \beta_0^{n_0(i)} \beta_1^{n_1(i)}$$  \hspace{1cm} (3.3)$$

$$\rho_i = \gamma^{n_0(i)} (1 - \gamma)^{n_1(i)}$$  \hspace{1cm} (3.4)$$

where $n_0(i)$ and $n_1(i)$ are the number of zeros and ones in the dyadic expan-
sion of $i$. Assuming $\gamma > (1 - \gamma)$ the smallest eigenvalue for the dynamics at
level $n$ is

$$\lambda_1^{(n)} = -\{ \epsilon_1^{(n)} (\rho_1^{(n)} + \rho_2^{(n)}) + \epsilon_2^{(n)} (\rho_3^{(n)} + \rho_4^{(n)}) + \epsilon_3^{(n)} (\rho_5^{(n)} + \ldots + \rho_8^{(n)}) + \epsilon_4^{(n)} (\rho_9^{(n)} + \ldots + \rho_{16}^{(n)}) + \ldots \}$$  \hspace{1cm} (3.5)$$

If the dynamics of the turbulent cascade is associated to the decay of the
eddies of different sizes, it is reasonable to assume that

$$\epsilon_i^{(n-1)} = \epsilon_i^{(n)}$$  \hspace{1cm} (3.6)$$
Using this relation and the relations between the probability densities at the levels \( n \) and \( n-1 \) one obtains

\[
\lambda_1^{(n)} - \lambda_1^{(n-1)} = -(\epsilon_1^{(n)} - \epsilon_1^{(n-1)})\rho_1^{(n-1)} \tag{3.7}
\]

One concludes that the ratio of short-time correlations measures the difference between the lifetimes of the structures at different length scales. If the dynamics of the cascade is controlled by the decay of the eddies and these have different lifetimes at different scales, the ultrametric stochastic model is an appropriate way to parametrize the dynamics and to characterize it in quantitative terms. Other models yield different correlation structures.

Notice that here we are concerned with the time fluctuations of the turbulent cascade itself, not with the changes induced by the overall motion of the fluid. This means that for a fluid in motion with free-stream velocity \( \vec{U} \) the correlations to measure, for an observable \( \Delta \), are

\[
\langle \Delta(x + \vec{U}t, t)\Delta(x, 0) \rangle
\]

The measure of the short-time behaviour of such quantities and the detection of several time scales in the time-correlations will test the usefulness of the turbulent cascade process proposed in this paper. Notice however that, in particular, the accuracy needed to detect different time scales, is a great experimental challenge.

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Figure captions

Fig.1 The state space at level n for a dyadic turbulent cascade

Fig.2 The three types of stochastic transitions associated to three different classes of eigenvectors