THE INTEREST OF LARGE-\( t \) ELASTIC SCATTERING

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Abstract
Existing data for large-\( t \) \( pp \) elastic-scattering differential cross-sections are energy-independent and behave as \( t^{-8} \). This has been explained in terms of triple-gluon exchange, or alternatively through triple-singlet exchange. A discussion is given of the problems raised by each of these explanations, and of the possibility that at RHIC or LHC energies the exchange of three BFKL pomeron might result in a rapid rise with energy.

The differential cross-section for \( pp \) elastic scattering at any fixed value of \( t \) greater than 3 to 4 GeV\(^2\) falls very rapidly with increasing beam momentum, until 400 GeV/c. Then it flattens dramatically, and becomes essentially energy-independent\(^1\). Furthermore, its shape as a function of \( t \) then becomes extremely simple, as is seen in figure 1, where the data at five energies are plotted together with

\[
\frac{d\sigma}{dt} = 0.09 t^{-8}
\]

(in mb GeV\(^{-2}\) units). One expects that it should be valid to apply perturbative QCD to large-\( t \) elastic scattering. In lowest order the dominant diagram is the 3-gluon-exchange diagram of figure 2. For \( s \gg |t| \gg m^2 \) this yields\(^1\)

\[
\frac{d\sigma}{dt} \sim \alpha_s^6 t^{-8}
\]

One power of \( t^{-2} \) arises from external kinematical factors and the remaining \( t^{-6} \) from the three gluon exchanges, each contributing \( \alpha_s^2 t^{-2} \). To obtain the result (1) it appears necessary to assume that the coupling constant \( \alpha_s \) does not run. Because it occurs raised to the sixth power, any variation with either \( s \) or \( t \) would cause a problem, given the data of figure 1. This has long been a puzzle. Our purpose in this paper is to discuss this, together with expectations for measurements at higher values of \( t \) and of \( s \).

The energy of each incoming proton is shared among its constituent quarks. For want of anything better, we shall assume throughout this paper that, on average, it is shared equally. Then the subenergy and momentum transfer associated with each quark-quark scattering is on average

\[
\hat{s} \approx \frac{s}{9} \quad \hat{t} \approx \frac{t}{9}
\]

Over the range of \( t \) for the data shown in figure 1, this is 1.6 GeV\(^2 \geq \hat{|t|} \geq 0.4 \) GeV\(^2 \). Given this, it is clear that non-perturbative effects should be considered. A well-motivated way to handle the non-perturbative region has been given by Cornwall\(^3\), who deduced by solving Schwinger-Dyson equations...
that the contribution to quark-quark scattering from single-gluon exchange can be well approximated by $\alpha_s(-\hat{t}) D(-\hat{t})$ with

$$D^{-1}(q^2) = q^2 + m^2(q^2)$$

$$\alpha_s(q^2) = \frac{12\pi}{(33 - 2N_f) \log \left[\frac{q^2 + 4m^2(q^2)}{\Lambda^2}\right]}$$

where the running gluon mass is given by

$$m^2(q^2) = m_0^2 \left[\frac{\log \frac{q^2 + 4m_0^2}{\Lambda^2}}{\log \frac{4m_0^2}{\Lambda^2}}\right]^{-12/11}$$

The fixed mass $m_0^2$ can be determined from the condition that the simple exchange of a pair of gluons between quarks is the zeroth-order approximation to soft pomeron exchange at $t = 0$. This
requires that the integral
\[ \beta_0^2 = \frac{4}{9} \int d^2q [\alpha_s(q^2) D(q^2)]^2 \] (6)
be about 4 GeV\(^{-2}\). With a choice of \(\Lambda = 200\) MeV this gives \(m_0 = 340\) MeV.

Over the \(\hat{t}\)-range of interest the variation of \(\alpha_s\) as defined by (4) and the departure of \(D(q^2)\) from \(1/q^2\) work in opposite directions and their product still varies approximately as \(1/q^2\). So this modified quark-quark scattering amplitude still provides a good fit to the data. The prediction of energy independence is unaffected. At larger values of \(t\) the effect of the running coupling does become apparent, but only very very slowly: see figure 3.

The data shown in figure 3 begin at \(\sqrt{|t|} = 1.9\) GeV. This momentum transfer is shared among the three gluons, and so the momentum transfer \(\hat{t}\) associated with each quark-quark scattering extends down to quite small values. At small \(\hat{t}\), high-energy quark-quark scattering is dominated by soft pomeron exchange. The corresponding amplitude is \([1]\)

\[ i\beta_0^2 \gamma \cdot \gamma (\alpha' s e^{-\frac{1}{2}i\pi})^{\alpha(t)-1} \]

\[ \alpha(t) = 1 + \epsilon_0 + \alpha' t \] (7)

with \(\epsilon = 0.08\), \(\alpha' = 0.25\) GeV\(^{-2}\) and \(\beta_0^2 = 4\) GeV\(^{-2}\). We must discuss whether we should include also contributions where we replace either one of the gluons in figure 2, or all three of them, with a soft pomeron. Thus the amplitude becomes

\[ A(s, t) = A_{ggg}(t) + A_{gP}(s, t) + A_{PP}(s, t) \]
\begin{align}
A_{ggg}(t) & = \frac{N}{t} \frac{5}{3} \left(4\pi\alpha_s(-\hat{t})D(-\hat{t})\right)^3 \\
A_{ggP}(s,t) & = \frac{N}{t} \frac{1}{6} \left(4\pi\alpha_s(-\hat{t})D(-\hat{t})\right)^2 \left(i\beta_0^2(\alpha^\prime s e^{-\frac{1}{2}i\pi})^{\alpha(t)-1}\right) \\
A_{PPP}(s,t) & = \frac{N}{t} \left(i\beta_0^2(\alpha^\prime s e^{-\frac{1}{2}i\pi})^{\alpha(t)-1}\right)^3
\end{align}

where $N$ is a constant, fitted to the data, whose value in principle could be calculated from knowledge of the proton wave function. We are still making the approximation that the energy of each proton is shared equally among its three quarks, so that $\hat{s} = s/9$ and $\hat{t} = t/9$. We can only use the amplitudes $A_{ggP}$ and $A_{PPP}$ in (8) for $|\hat{t}|$ less than about 0.7 GeV$^2$, that is $|t| \text{ less than about 6 GeV}^2$, because for larger values single-soft-pomeron exchange begins to become significantly reduced by double-soft-pomeron exchange. Within this range of $t$-values, including the terms $A_{ggP}$ and $A_{PPP}$ has very little effect, and it decreases as the energy increases because, according to (7), $\alpha(t) - 1$ is negative for $|\hat{t}| > 0.32$ GeV$^2$. At the left-hand side of figure 3 their combined contribution would be less than 10% to the differential cross-section. Thus our fit of figure 3, which includes just triple-gluon exchange, is largely unaffected.

An alternative viewpoint has been put by Sotiropoulos and Sterman, again within the context of the triple quark-scattering model. At the level of quark-quark elastic scattering with multiple-gluon exchange, they find an evolution in $t$ that, in leading log approximation, becomes diagonal in a singlet-octet basis as $s \to \infty$. The octet exchange in the hard scattering is Sudakov-suppressed with the standard reggeized $s^{\alpha_s(t)}$ behaviour. In contrast, the Sudakov suppression for the $t$-channel singlet exchange in the hard scattering is $s$-independent. This difference results in the suppression of the octet exchange relative to the singlet exchange.

The lowest order singlet exchange is simply two-gluon exchange, that is a $C = +1$ exchange. Sotiropoulos and Sterman choose a model in which the large-momentum-transfer quark-quark amplitude is dominated by singlet exchange, which they estimate to approximate to $(\alpha_s^2/\hat{t})$ within the appropriate range of $\hat{t}$. There is an ambiguity in the explicit form of the hard singlet amplitude to lowest order because it is already IR divergent to this order. Different IR subtraction procedures yield different expressions for the singlet hard scattering amplitude, and they can only be fixed by considering the amplitude at higher order in $\alpha_s$. At lowest order it is necessary to introduce an IR cutoff, which is arbitrary at the quark-quark scattering level but which does have a physical meaning when the quarks are embedded in a proton and is related to the transverse size of the proton. Thus the transverse structure of the proton wave function is an essential feature of the calculation, it is intrinsically non-perturbative and introduces an arbitrary parameter. It is possible to reproduce a behaviour close to the $t^{-8}$ of the $pp$ data for particular choices of the non-perturbative proton wave function, the IR cutoff parameter and $\Lambda_{QCD}$. Note, however, that Sotiropoulos and Sterman exchange two gluons between each pair of quarks, and so for them $d\sigma/dt \propto \alpha_s^{12}$, instead of $\alpha_s^6$ as in (2). So any running of $\alpha_s$ would have a particularly notable effect.

One might argue that the simple exchange of two perturbative gluons is not appropriate as this should be used rather as the input to the BFKL equation, which would convert the energy-independence of the two-gluon exchange to a strong energy dependence and invalidate the argument for this triple $C = +1$ exchange providing the explanation for the existing $pp$ data at $-t \geq 3.5$ GeV$^2$. One can also be more pragmatic and ask whether there is evidence in the $pp$ data which allows one to determine whether $C = -1$ or $C = +1$ exchange dominates in this range of $t$.

It is well established that there is an important $C = -1$ exchange in $pp$ and $p\bar{p}$ scattering at $-t \sim 1.35$ GeV$^2$, as the sharp dip in the differential cross section observed in ISR data for the former process is absent in the latter. It is natural to suppose that this $C = -1$ exchange survives at larger values of $t$, and a consistent picture of $pp$ and $p\bar{p}$ scattering at all values of $t$ can be constructed on the basis that the $C = -1$ exchange that helps to give the $pp$ dip is just the three-gluon-exchange mechanism of figure 2, and that the same mechanism dominates the large-$t$ data. Indeed, this even led us to predict...
the absence of a dip in $\bar{p}p$ scattering\cite{9} before the measurements were made. However the data for $-t \geq 3.5\text{GeV}^2$ alone cannot unambiguously distinguish $C = -1$ exchange from $C = +1$ exchange.

Nevertheless, one can deduce, by considering the data at lower $t$, that the Sotiropoulos-Sterman triple colour-singlet exchange is unlikely to be an adequate replacement for our triple-gluon exchange as the explanation for the existing large-$t$ elastic $pp$ data. In order to describe the large-$t$ data, it would need to be larger than soft-pomeron exchange, in the same way as we have argued above that gluon exchange must be if it is to provide the explanation. The key difference that distinguishes Sotiropoulos-Sterman exchange from gluon exchange is that if it can occur as triple exchange in $pp$ elastic scattering it must also be able to occur as single exchange. The single exchange carries all the momentum transfer. So if in triple exchange at $|t| = 3.5 \text{ GeV}^2$ the Sotiropoulos-Sterman exchange dominates over soft-pomeron exchange, then the same must be true for single exchange at $|t| \approx 0.4 \text{ GeV}^2$. The data at such a small value of $t$ do not support this at all\cite{8}.

Nevertheless, the obvious question is whether the full BFKL pomeron\cite{6}, rather than its Sotiropoulos-Sterman truncated version, will become apparent in large-$t$ elastic scattering at the much higher energies that will be attainable at RHIC or the LHC. After all, the energy $\sqrt{s}$ of each quark-quark scattering is no more than about 20 GeV in the present data, which is surely very far from asymptotic. The trajectory of the BFKL pomeron is surely much flatter than that of the soft pomeron, as well as having a larger intercept. So even at large $t$ the contribution from BFKL exchange will rise rapidly with energy. Tevatron data place severe constraints\cite{10} on the magnitude of the BFKL-exchange contribution to the total cross-section, that is to the amplitude at zero momentum transfer. Also, soft-pomeron phenomenology\cite{8} describes the differential cross-section well at small nonzero momentum transfers. But this does not limit what might be the size of the BFKL contribution at larger momentum transfers. So, while it is unlikely that at RHIC or LHC energies triple-BFKL exchange is significant for $|t|$ as small as 3.5 GeV$^2$, by $|t| = 10 \text{ GeV}^2$, say, it may well have a dramatic effect. On the other hand, the triple-gluon exchange, which we have argued is the explanation for the low-energy data, may well be Sudakov-suppressed\cite{11}\cite{5} at higher energies. So it could be that a dramatic rise with energy at large $t$ is accompanied by a fall at not-so-large $t$.

Whether or not this turns out to be true, further data are needed in order to elucidate the mysteries posed by the existing data: why does the very simple fit (1) work so well?

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