A Study of Accelerating Cosmologies from Superstring/M Theories

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Abstract

We study aspects of the accelerating cosmologies obtained from the compactification of vacuum solution and S2-branes of superstring/M theories. Parameter dependence of the resulting expansion of our universe and internal space is examined for all cases. We find that accelerated expansions are obtained also from spherical internal spaces, albeit the solution enters into contracting phase eventually. The relation between the models of SM2- and SD2-branes are also discussed, and a potential problem with SD2-brane model is noted.

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1 Introduction

Recent discovery of the cosmic acceleration calls for explanation within the framework of fundamental theory of superstring/M theories. It has been known for some time that it is difficult to derive such cosmologies from the compactifications of solutions in superstring/M theories and has been considered that there is a no-go theorem which excludes such possibility if one chooses the internal space compact and static manifolds [1].

Recently it has been shown that this no-go theorem can be overcome if one allows time-dependence, and a solution of the vacuum Einstein equations with compact hyperbolic internal space has been proposed for such a model [2]. The solution turned out [3] to be a special case of the time-dependent solutions discovered before [4]. Moreover it has been found [3] that similar accelerating cosmologies can be obtained for S(pacelike)M2-branes [5, 6, 7, 8, 4] and that not only the hyperbolic but also flat internal spaces give similar behavior. The SM2-brane case is also studied in ref. [9]. Later SD2-brane, which can be obtained from SM2-brane by dimensional reduction, has been discussed in ref. [10]. However, problem with strong string coupling in SD2-brane casts a question on the validity of the results at least for flat internal space. Related cosmologies with accelerated expansion are also discussed in refs. [11, 12].

In ref. [9], the degree of the expansion factor during the period of the accelerated expansion has been studied for SM2-brane, with the small result of order 2. This appears to be too small to solve cosmological issues such as the horizon or flatness problems, which require the magnitude $O(e^{60})$. Since the solutions contain some parameters, it is interesting to examine if there is a possibility of evading this problem.

In this paper, we study aspects of these models of accelerated expansions including parameters for vacuum, SM2-brane and SD2-brane solutions. We also estimate how the expansion factors change by varying the parameters. It turns out that the factors can change but not enough to get around the difficulty mentioned above. We also find that it is possible to have accelerated expansion for spherical internal space too for S2-branes (but not for vacuum solutions) if one chooses parameters appropriately, although the universe enters into contracting phase after some period. We further discuss how SD2-branes are obtained from SM2-branes by dimensional reduction. One can thus see that the features
of the cosmologies from SD2-brane is basically similar to those from SM2-brane. In fact we find that qualitative behavior does not change much, as expected. However, there is a potential problem with the SD2-brane model because it often suffers from the strong string coupling and then it is not clear if we can trust the result in this picture. It is known that the strong string coupling can be understood as the 11-th dimension becoming large, so that the description by M theory becomes more relevant [13].

This paper is organized as follows. In sect. 2, we first discuss the relation of the vacuum and our S-brane solutions, and study the cosmologies from the vacuum solutions. In this case, there is no adjustable parameter and the result does not change. In sect. 3, we examine the cosmologies from SM2-brane. The hyperbolic, flat and spherical internal spaces are discussed in subsections 3.1, 3.2 and 3.3, respectively. We study these cases by changing a parameter, and find that an interesting model can be obtained also from spherical internal space. In sect. 4, the way how SD2-brane is obtained from SM2-brane is explicitly shown. We discuss the cosmologies in this case briefly. We also note that in many cases in this kind of models, the string coupling becomes strong so that the picture becomes obscure. We show how this can be avoided for compactification on hyperbolic manifolds, but there seems to be no way to avoid this for flat and spherical manifolds. Finally sect. 5 is devoted to conclusions and discussions.

In the course of writing this article, a paper appeared with related observations on the spherical internal space for SM2-brane [14].

2 Vacuum solution and S2-brane

2.1 Generality

It is convenient to write the (4 + n)-dimensional solution as

$$ds^2 = \delta^{-n}(t)ds^2_E + \delta^2(t)d\Sigma^2_{n,\sigma},$$

(2.1)

where $n$ is the dimension of the internal spherical ($\sigma = +1$), flat ($\sigma = 0$) or hyperbolic ($\sigma = -1$) spaces, whose line elements are $d\Sigma^2_{n,\sigma}$, and

$$ds^2_E = -S^6(t)dt^2 + S^2(t)dx^2,$$

(2.2)
describes the 4-dimensional spacetime. The form of (2.1) is chosen such that the metric in (2.2) are in the Einstein frame. The solution in refs. [2] is given as

$$\delta(t) = e^{-3t/(n-1)} \left( \frac{\sqrt{3(n+2)/n}}{(n-1) \sinh(\sqrt{3(n+2)/n} |t|)} \right)^{1/(n-1)},$$

$$S(t) = e^{-(n+2)t/2(n-1)} \left( \frac{\sqrt{3(n+2)/n}}{(n-1) \sinh(\sqrt{3(n+2)/n} |t|)} \right)^{n/2(n-1)}.$$ (2.3)

with hyperbolic internal space.

If we take the time coordinate $\eta$ defined by

$$d\eta = S^3(t) dt,$$ (2.4)

the metric (2.2) describes a flat homogeneous isotropic universe with scale factor $S(t)$, and $\delta(t)$ gives the measure of the size of internal space. The condition for expanding 4-dimensional universe is that

$$\frac{dS}{d\eta} > 0.$$ (2.5)

Accelerated expansion is obtained if, in addition,

$$\frac{d^2S}{d\eta^2} > 0.$$ (2.6)

It has been shown that these can be satisfied for $n = 7$ and for certain period of negative $t$ (with the convention $t_1 = 0$) which is the period that our universe is evolving ($t < 0$ and $t > 0$ are two disjoint possible universes) [2].

We have noted [3] that the above solution is actually a special case of the solutions derived in ref. [4]. The theory considered there is given by the action for $d$-dimensional gravity coupled to a dilaton $\phi$ and $m$ different $n_A$-form field strengths:

$$I = \frac{1}{16\pi G_d} \int d^d x \sqrt{-g} \left[ R - \frac{1}{2} (\partial \phi)^2 - \sum_{A=1}^{m} \frac{1}{2n_A!} e^{a_A \phi} F^2_{n_A} \right].$$ (2.7)

This action describes the bosonic part of $d = 11$ or $d = 10$ supergravities; we simply drop $\phi$ and put $a_A = 0$ and $n_A = 4$ for $d = 11$, whereas we set $a_A = -1$ for the NS-NS 3-form and $a_A = \frac{1}{2} (5 - n_A)$ for forms coming from the R-R sector. The field strength for an electrically charged $S_q$-brane is given by

$$F_{\alpha_1 \cdots \alpha_{q+1}} = \epsilon_{\alpha_1 \cdots \alpha_{q+1}} \dot{E}, \quad (n_A = q + 2),$$ (2.8)
where $\alpha_1, \ldots, \alpha_{q+1}$ stand for the tangential directions to the $S_q$-brane. The magnetic case is given by

$$F_{\alpha q+2 \cdots a p_1 \cdots a_n} = \frac{1}{\sqrt{-g}} e^{-\alpha_1} \epsilon^{\alpha q+2 \cdots a p_1 \cdots a_n} \hat{E}, \quad (n_A = d - q - 2)$$  \hfill (2.9)

where $a_1, \ldots, a_n$ denote the coordinates of the $n$-dimensional hypersurface $\Sigma_{n,\sigma}$.

In ref. [6] a single $S$-brane solution was given, and in ref. [4] general orthogonally intersecting solutions were derived by solving field equations. Our solutions restricted to a single $S_q$-brane with $(q+1)$-dimensional world-volume in $p$-dimensional space [4] are (hereafter the subscript $A$ is not necessary and is dropped)

$$ds_a^2 = \left[ \cosh \tilde{c}(t - t_2) \right]^{2q+2} \left[ e^{2\gamma(t) + 2c't} \left\{ -e^{2\epsilon g(t)} dt^2 + e^{2g(t)} d\Sigma_{n,\sigma}^2 \right\} \right]^{2q+2} \frac{1}{\Delta} e^{2\gamma(t) + 2c't} (dx^\alpha)^2, \quad (2.10)

E = \sqrt{\frac{2(d-2)}{\Delta}} e^{\tilde{c}(t-t_2) - \epsilon c't/2 + \sum_{\alpha=q} \epsilon' c_{\alpha}}, \quad \tilde{c} = \sum_{\alpha=q} c_{\alpha} - \frac{1}{2} \epsilon \phi \epsilon a,

\phi = \frac{(d-2)\epsilon a}{\Delta} \ln \cosh \tilde{c}(t - t_2) + \tilde{c}_t + \epsilon \phi, \quad (2.11)

where $d = p + n + 1$ and $\epsilon = +1(-1)$ corresponds to electric (magnetic) fields. The coordinates $x^\alpha, (\alpha = 1, \ldots, p)$ parametrize the $p$-dimensional space, within which $(q+1)$-dimensional world-volume of $S_q$-brane is embedded, and the remaining coordinates of the $d$-dimensional spacetime are the time $t$ and coordinates on compact $n$-dimensional spherical $(\sigma = +1)$, flat $(\sigma = 0)$ or hyperbolic $(\sigma = -1)$ spaces. We have also defined

$$\Delta = (q + 1)(d - q - 3) + \frac{1}{2} a^2(d - 2),

\gamma^{(\alpha)} = \begin{cases} d - 2 & \text{for } x_\alpha \text{ belonging to } q \text{-brane} \\ 0 & \text{otherwise} \end{cases}, \quad (2.12)$$

and

$$g(t) = \begin{cases} \frac{1}{n-1} \ln \frac{\beta}{\cosh[(n-1)\beta(t-t_1)]} & : \sigma = +1, \\ \pm \beta(t-t_1) & : \sigma = 0, \\ \frac{1}{n-1} \ln \frac{\beta}{\sinh[(n-1)\beta(t-t_1)]} & : \sigma = -1, \end{cases} \quad (2.13)$$
\(\beta, t_1, t_2\) and \(c\)'s are integration constants which satisfy

\[
c_0 = \frac{q + 1}{\Delta} \tilde{c} - \sum_{\alpha=1}^p c_\alpha, \quad c'_0 = -\sum_{\alpha=1}^p c'_\alpha,
\]

\[
\tilde{c}_\alpha = c_\alpha - \frac{\gamma^{(a)} - q - 1}{\Delta} \tilde{c}, \quad \tilde{c}_\phi = c_\phi + \frac{(d - 2)\epsilon a}{\Delta} \tilde{c}.
\]

(2.14)

These must further obey the condition

\[
\frac{1}{n - 1} \left( \sum_{\alpha=1}^p c_\alpha \right)^2 + \sum_{\alpha=1}^p c^2_\alpha + \frac{1}{2} c^2_\phi = n(n - 1)\beta^2.
\]

(2.15)

The free parameters in our solutions are \(c_\alpha, c'_\alpha (\alpha = 1, \ldots, p)\), \(c_\phi, c'_\phi, t_1\) and \(t_2\). The time derivative of \(E\) gives the field strengths of antisymmetric tensor and in our convention they are given as

\[
\begin{aligned}
e^\phi \ast F & = \tilde{c} \sqrt{\frac{2(d - 2)}{\Delta}} e^{-\sum_{\alpha=q}^p c'_\alpha + \epsilon a c'_\phi / 2} dx^{\alpha_{q+2}} \wedge \cdots \wedge dx^{\alpha_p} \wedge \text{Vol}(\Sigma_{n,\sigma}), \\
F & = b \ dy^1 \wedge \cdots \wedge dy^{q-n} \wedge \text{Vol}(\Sigma_{n,\sigma}).
\end{aligned}
\]

(2.16)

for electric (first line) and magnetic (second line) fields, where \(\text{Vol}(\Sigma_{n,\sigma})\) is the unit volume form of the hypersurface \(\Sigma_{n,\sigma}\) and \(\ast\) represents dual. We can check that \(\sqrt{\frac{2(d-2)}{\Delta}} = 1\) for SM- and SD-branes. The above solution includes that in ref. [6], and the precise relation is given in [4]. (If we also use the relation \(c'_\phi = \frac{d-2}{q-1} a c'_\gamma + c_2, (\gamma = 1, \ldots, p)\) which was not given in eq. (40) there, we reproduce \(F = b \ dy^1 \wedge \cdots \wedge dy^{q-n} \wedge \text{Vol}(\Sigma_{n,\sigma})\).

For the general S2-brane obtained from the solution (2.10) by putting \(p = q + 1 = 3, c \equiv c_1 = c_2 = c_3, c' \equiv c'_1 = c'_2 = c'_3\), we find that it takes the form (2.1) and (2.2) with

\[
\begin{aligned}
\delta(t) & = [\cosh \tilde{c}(t - t_2)]^{3/\Delta} e^{g(t) + c_0 t + c'_0}, \\
S(t) & = [\cosh \tilde{c}(t - t_2)]^{(n+2)/2} e^{n g(t)/2 + (n+2)(c_0 t + c'_0)/6},
\end{aligned}
\]

(2.17)

where

\[
\begin{aligned}
\tilde{c} & = 3c - \frac{1}{2} c_\phi \epsilon a, \quad c_0 = \frac{3}{\Delta} \tilde{c} - \frac{3}{n - 1} c, \quad c'_0 = -\frac{3}{n - 1} c', \\
\beta & = \sqrt{\frac{3(n + 2)}{n(n - 1)^2} c^2 + \frac{1}{2n(n - 1)} c^2_\phi}.
\end{aligned}
\]

(2.18)

We are now going to discuss how this S2-brane solution includes the vacuum solution (2.3) and the resulting cosmologies [2].
2.2 Vacuum solution

The relation between $\tilde{c}$ and $c$, $\alpha$ and $c\phi$ in eq. (2.11) is derived [4] under the assumption that we have the independent field strengths $F$. In the absence of these, we can disregard this relation and should set $\tilde{c}$ to zero. It is then easy to see that the solution (2.10) reproduces (2.1)-(2.3) for $p = q + 1 = 3, \sigma = -1, c = 1, c' = 0$ without dilaton ($c\phi = 0$) [3]. The scale factor is simply (2.17) with $\tilde{c} = 0$ which coincides with (2.3). On the other hand, the S-brane solutions derived in [6] assume nonzero field strengths from the start, and one cannot simply get the vacuum solution.

The condition of the expansion (2.5) for the vacuum solution (2.3) is [2]

$$n_1(t) \equiv -1 - \sqrt{3n/(n + 2)} \coth \left( \sqrt{3(n + 2)/n} c(t - t_1) \right) > 0,$$

where we have also included parameters $c$, $c'$ and $t_1$. The condition (2.6) gives

$$\frac{3(n - 1)}{(n + 2) \sinh^2 [\sqrt{3(n + 2)/n} c(t - t_1)]} - n_1^2(t) > 0.$$  

The parameter $t_1$ and $c$ can be absorbed into the shift and rescaling of the time $t$. Hence without loss of generality, we can set $t_1 = 0$ and $c = 1$ (changing $c$ gives the change in the scale of time). There is a singularity in $S(t)$ at $t = 0$, but the time $\eta$ run from 0 to infinity while $t$ runs from $-\infty$ to 0, which is an infinite future for any event with $t < 0$ and hence the evolution of our universe can be restricted to $t < 0$.

The left hand side of these eqs. for $n = 7$ are shown in Fig. 1, and the behavior of the scale factor $S(t)$ in (2.3) is depicted in Fig. 2. We see that there is a certain period of negative time that the conditions (2.19) and (2.20) are satisfied [2]. The period of the

![Figure 1: The lhs of eq. (2.19) (solid line) and (2.20) (dashed line).]

![Figure 2: The behavior of the scale factor $S(t)$ in (2.3).]
accelerated expansion can be adjusted by changing the constant $c$, but this does not affect the resulting expansion factor. The scale factor vanishes in the infinite past, but diverges in the infinite future.

The expansion factor $A$ during the accelerated expansion is given by the ratio of $S(t)$ at the starting time $T_1$ and ending time $T_2$ of the acceleration. These are read off from Fig. 1 as

$$T_1 \simeq -0.73, \quad T_2 \simeq -0.07. \quad (2.21)$$

If we keep the parameter $c$, $T = ct$ and this simply changes the scale of the time by a constant multiplicative factor. We then find that the expansion factor is

$$A = \frac{S(T_2)}{S(T_1)} \simeq 2.91. \quad (2.22)$$

This value is too small to explain the cosmological problems. Note that there is no parameter to improve $A$ here.

The behavior of the size of the internal space is also shown in Fig. 3. Though it appears as if the internal space rapidly expands in terms of $t$, the actual time coordinate is $\eta$ and the region $t \sim 0^-$ is actually long. When the acceleration starts, $\delta(t)$ shrinks but finally starts expanding, and the ratio of the sizes during the accelerated expansion is 2.18. As observed for SM2-brane case [9], there is no stable point in the size of the internal space, and in the infinite future and past its size goes to infinity. This seems to be the common problem in the hyperbolic compactification [15]. We will find this behavior in other cases.

We have also examined other internal spaces, but without any adjustable parameter we find that neither flat nor spherical spaces give accelerating cosmologies [3].
3 SM2-brane

The SM2-brane in M-theory can be obtained from (2.10) by putting $p = q + 1 = 3$, $\epsilon = +1$, $a = 0$, $c_\phi = 0$ without dilaton and $\Delta = 3(n - 1)$. We also put $c \equiv c_1 = c_2 = c_3, c' \equiv c'_2 = c'_3$ and then $\beta = n - 1 \sqrt{3(n+2)/n} c$ is determined from (2.15). The solution (2.10) then gives

$$d\bar{s}_d^2 = \left[ \cosh 3c(t - t_2) \right]^{2/(n-1)} \left[ - e^{2ng(t) - 6c'/(n-1)} dt^2 + e^{2g(t) - 6c'/(n-1)} d\Sigma_{n,\sigma} \right] + \left[ \cosh 3c(t - t_2) \right]^{-2(n+2)/3(n-1)^2} e^{2c'} d\chi^2. \quad (3.1)$$

This is the universe (2.1)-(2.2) with

$$\delta(t) = \left[ \cosh 3c(t - t_2) \right]^{1/(n-1)} e^{g(t) - 3c'/ (n-1)},$$

$$S(t) = \left[ \cosh 3c(t - t_2) \right]^{(n+2)/(6(n-1))} e^{ng(t) - 2(n+2)c'/2(n-1)}. \quad (3.2)$$

We now discuss three internal spaces (2.13) separately.

3.1 Hyperbolic internal space

The conditions (2.5) and (2.6) for $n = 7$ of our interest and hyperbolic internal space $\sigma = -1$ are (again shifting the time to set $t_1 = 0$)

$$\frac{3}{4} \tanh[3c(t - t_2)] - \frac{\sqrt{21}}{4} \coth(3\sqrt{3/7}ct) > 0, \quad (3.3)$$

$$\frac{9}{8} \left( \frac{1}{\cosh^2[3c(t - t_2)]} + \frac{1}{\sinh^2(3\sqrt{3/7}ct)} \right) - n_2^2(t) > 0. \quad (3.4)$$

Here we can again consider that our universe evolves only for $t < 0$. The left hand side of these eqs. for $c = 1$ and $t_2 = 0$ are shown in Fig. 4, and the behavior of the scale factor $S(t)$ in (3.2) is depicted in Fig. 5. We again see that there is a certain period of negative time that these conditions are satisfied [3].

The starting time $T_1$ and ending time $T_2$ of the acceleration are read off from Fig. 4 as [9]

$$T_1 \simeq -0.78, \quad T_2 \simeq -0.15. \quad (3.5)$$

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We then find that the expansion factor is

$$A \simeq 2.17.$$  \hfill (3.6)

This value is too small to explain the cosmological problems. However, here is a parameter $t_2$ in contrast to the vacuum solution, and it is interesting to check what effect this may have.

We have examined the conditions for various choices of $t_2$ and found that the typical behavior for positive $t_2$ is basically the same as $t_2 = 0$ case, but the period of the accelerated expansion changes slightly. For example, for $t_2 = 1$, we find

$$T_1 \simeq -0.73, \quad T_2 \simeq -0.06.$$  \hfill (3.7)

and the value of the expansion factor during the accelerated expansion improves:

$$A \simeq 3.13.$$  \hfill (3.8)

This is still not enough improvement for cosmological applications. Increasing the value of $t_2$ does not affect the numerical value of $A$ much. The behavior for negative $t_2$ is also basically the same, but the expansion factor is worse; one typically gets $A \sim 1.4$.

The behavior of the size of the internal space for $t_2 = 0$ is also shown in Fig. 6. As observed in [9], there is no stable point in the size of the internal space, and in the infinite future its size goes to infinity. There is no significant change in this behavior if we change the parameter $t_2$. 

Figure 4: The lhs of eq. (3.3) (solid line) and (3.4) (dashed line).

Figure 5: The behavior of the scale factor $S(t)$ in (3.2).
3.2 Flat internal space

The conditions (2.5) and (2.6) for $n = 7$ and flat internal space $\sigma = 0$ are

$$n_3(t) \equiv \frac{3}{4} \tanh [3c(t - t_2)] + \frac{\sqrt{21}}{4} > 0,$$

$$9 \frac{1}{8 \cosh^2 [3c(t - t_2)]} - n_3^2(t) > 0,$$

where we have chosen the plus sign in eq. (2.13) since minus sign cannot give expanding universe.

Here since $S(t)$ does not have any singularity and is positive, the time $t$ runs from $-\infty$ to $+\infty$ while the time $\eta$ runs from 0 to $+\infty$ monotonously. The left hand side of these eqs. for $c = 1$ and $t_2 = 0$ are shown in Fig. 7, and the behavior the scale factor $S(t)$ in (3.2) is depicted in Fig. 8. We again see that there is a certain period of negative time that these conditions are satisfied [3].

![Figure 7: The lhs of eq. (3.9) (solid line) and (3.10) (dashed line).](image)

![Figure 8: The behavior of the scale factor $S(t)$ in (3.2).](image)

We again read off the starting time $T_1$ and ending time $T_2$ of the acceleration from
Fig. 7 as

\[ T_1 \simeq -0.48, \quad T_2 \simeq -0.04, \]  

(3.11)

and the expansion factor is

\[ A \simeq 1.35. \]  

(3.12)

This value is again too small to explain the cosmological problems. The conditions (3.9) and (3.10) depend only on \( t-t_2 \), so changing \( t_2 \) simply shifts the evolution of the spacetime and does not give any difference.

The behavior of the size of the internal space is also shown in Fig. 9. There is no

\[ \delta(t) \]

Figure 9: The behavior of the size of the flat internal space \( \delta(t) \) in (3.2).

stable point in the size of the internal space, and in the infinite future its size goes to infinity.

### 3.3 Spherical internal space

The conditions (2.5) and (2.6) for \( n = 7 \) and spherical internal space \( \sigma = +1 \) are (again shifting the time to set \( t_1 = 0 \))

\[ n_4(t) \equiv \frac{3}{4} \tanh[3c(t-t_2)] - \frac{\sqrt{21}}{4} \tanh(3\sqrt{3/7} \, ct) > 0, \]  

(3.13)

\[ \frac{9}{8} \left( \frac{1}{\cosh^2[3c(t-t_2)]} - \frac{1}{\cosh^2(3\sqrt{3/7} \, ct)} \right) - n_4^2(t) > 0. \]  

(3.14)

The time ranges of \( \eta \) and \( t \) are the same as the flat internal space. We have reported the result for \( t_2 = 0 \) in ref. [3]; the result indicates that there is no period of accelerated
expansion. We have examined what happens if we change the parameter $t_2$. The results for $c = 1$ and $t_2 = -1$ are shown in Fig. 10. We find that there is a certain period of negative time that the conditions (3.13) and (3.14) are satisfied, though the universe begin contraction after some positive time. The behavior of the scale factor $S(t)$ in (3.2) is depicted in Fig. 11. It contracts on both ends $t \to \pm \infty$. The acceleration starting time

![Figure 10: The lhs of eq. (3.13) (solid line) and (3.14) (dashed line) for $t_2 = -1$.](image)

![Figure 11: The behavior of the scale factor $S(t)$ in (3.2) for $t_2 = -1$.](image)

$T_1$ and ending time $T_2$ are read off from Fig. 10 as

$$T_1 \simeq -1.48, \quad T_2 \simeq -1.04,$$

(3.15)

and the expansion factor is

$$A \simeq 1.35.$$  

(3.16)

This value is again too small to explain the cosmological problems. We have also checked that the typical behavior for negative $t_2$ is again basically the same as $t_2 = -1$ case, but the period of the accelerated expansion and the value of the expansion factor during the accelerated expansion change.

A different behavior is observed for positive $t_2$. We find the acceleration occurs while the universe is already contracting for $t > 0$, as shown in Fig. 12. We do not know if this case gives any interesting cosmology. The behavior of the size of the internal space is also shown for $t_2 = -1$ in Fig. 13. There is no stable point in the size of the internal space, and in the infinite future its size goes to infinity. There is no significant change in this behavior if we further change the parameter $t_2$. 

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In this section, we discuss accelerating cosmologies from SD2-brane.

4 SD2-brane

4.1 SD2-brane obtained from SM2-brane

We first show how the SD2-brane can be obtained by dimensional reduction from SM2-branes. Though it is possible to examine general dimensions, we restrict ourselves to $d = 11$ here. We will see why the qualitative behavior is similar to SM2-brane.

The 10-dimensional metric in the string frame is obtained from the 11-dimensional one by the relation

$$ds_{11}^2 = e^{-2\phi/3}ds_{10,s}^2 + e^{4\phi/3}dx_{10}^2. \quad (4.1)$$

In our solutions (2.10), we consider SM2-brane by taking $q = 2$, but we set $p = 4$ with one extra coordinate outside SM2-brane, in which direction we make dimensional reduction. We choose $x_1, x_2, x_3$ to be the SD2-brane world-volume and $x_4$ the direction of dimensional reduction. We further set $c \equiv c_1 = c_2 = c_3, c' \equiv c'_1 = c'_2 = c'_3$ but leave $c_4$ and $c'_4$ arbitrary.

There is no dilaton which comes from the metric as (4.1) upon dimensional reduction, and we put $a = c_\phi = 0$. This gives

$$\phi = \frac{1}{4} \ln[\cosh 3c(t - t_2)] + \frac{3}{4} (c + 2c_4)t + \frac{3}{2} c'_4,$$

$$ds_{10,s}^2 = e^{2\phi/3} \left[ \cosh 3c(t - t_2) \right]^{1/3} \left[ e^{-(c+2c_4)t/5-2(3c'+c'_4)/5} \left\{ -e^{12g(t)}dt^2 + e^{2g(t)}d\Sigma_{6,\sigma}^2 \right\} + [\cosh 3c(t - t_2)]^{-1} e^{2c'}d\Sigma^2 \right]. \quad (4.2)$$
In the Einstein frame $ds_{10,E}^2 = e^{-\phi/2} ds_{10,s}^2$, this reduces to
\[
ds_{10,E}^2 = \left[ \cosh 3c(t - t_2) \right]^{3/8} \left[ e^{-3(c+2c_4)t/16-3(8c'+c_4')/4} \left\{ -e^{12g(t)} dt^2 + e^{2g(t)} d\Sigma_{6,6}^2 \right\} \right]
+ \left[ \cosh 3c(t - t_2) \right]^{-1} e^{(c+2c_4)t/8+(8c'+c_4')/4} dx^2.
\]
(4.3)
The relation to the notation of [6] is given by
\[
c = \frac{16}{15} \alpha, \quad c_4 = \frac{5}{8} c_1 - \frac{8}{15} \alpha,
\]
which must obey (2.15) or
\[
30 \beta^2 = \frac{128}{25} \alpha^2 + \frac{15}{32} c_1^2.
\]
(4.5)

The scale factor $S(t)$ and $\delta(t)$ are given as
\[
\delta(t) = \left[ \cosh(16\alpha/5)(t - t_2) \right]^{3/16} e^{g(t) - 3c_1 t/64 - 3c'/4},
S(t) = \left[ \cosh(16\alpha/5)(t - t_2) \right]^{1/4} e^{3g(t) - c_1 t/16 - c'},
\phi = \frac{1}{4} \ln[\cosh(16\alpha/5)(t - t_2)] + \frac{15}{16} c_1 t + c'_\phi,
\]
(4.6)
where we have also put
\[
c' = \frac{8c' + c'_4}{10}, \quad c'_\phi = \frac{3}{2} c'_4.
\]
(4.7)

On the other hand, if we consider the SD2-brane with $d = 10, q = 2, p = 3, n = 6, a = \frac{1}{2}, \epsilon = +1$ in eq. (2.10), we get precisely (4.6) with different parametrization
\[
c = \alpha + \frac{5}{64} c_1, \quad c_\phi = \frac{15}{16} c_1 - \frac{4}{5} \alpha, \quad c' = \frac{5}{4} c'.
\]
(4.8)
The scale factors (4.6) almost coincide with those for SM2-brane in (3.2) if we choose $n = 6$, though the power of the cosh is slightly different and there is an additional $c_1 t$ term in the exponent. However, the difference is small and one may expect that qualitative features remain the same. This is the reason why one finds basically the same behavior as SM2-brane. To confirm this, we present some results for SD2-brane below.

Continuing this reduction, one should leave the constants $c_\alpha$ arbitrary in order to get general lower-dimensional solutions; in the SM2-brane solution of ref. [6], homogeneity of the space outside the S-brane is assumed, and then general solutions cannot be obtained by dimensional reduction. If we start with our solutions without putting any relations among $c_\alpha$’s, we can simultaneously discuss all possible cases. This is another virtue of our solutions, in addition to the fact that they cover the vacuum solution.
4.2 Hyperbolic internal space

The conditions (2.5) and (2.6) give

\[ n_5(t) = \frac{4}{5} \alpha \tanh[(16\alpha/5)(t - t_2)] - 3\beta \coth(5\beta t) - \frac{c_1}{16} > 0, \quad (4.9) \]

\[ \frac{32\alpha^2/25}{\cosh^2[(16\alpha/5)(t - t_2)]} + \frac{15\beta^2/2}{\sinh^2(5\beta t)} - n_5^2(t) > 0, \quad (4.10) \]

where we have again set \( t_1 = 0 \) by a shift of time.

The conditions (4.9) and (4.10) for \( \alpha = 0.92, c_1 = 1, \beta = \frac{2}{5} \) was examined in [10], and the qualitative behavior is found to be the same as SM2-brane, as expected. We have studied how the behavior changes if we change parameter \( t_2 \). The results for \( t_2 = -1, +1 \) are shown in Figs. 14 and 15, respectively. The expansion factor is also obtained for

\[ A \simeq 3.28. \quad (4.11) \]

Again similarly to the SM2-brane, the positive \( t_2 \) gives bigger \( A \), but not so much change. We thus see that the behavior does not change qualitatively from the case of SM2-brane, as expected.

However, there is a potential subtlety in this picture of SD2-brane. This is because the string coupling is given by the dilaton as \( e^\phi \), and it may diverge for the solution (4.6) for certain range of parameters. If this happens, quantum effects dominate and we cannot simply trust the result. Fortunately there is a safe range of parameters for this compactification on hyperbolic internal space because the time range is restricted to \( t < 0 \). From
eq. (4.6), one finds that if the condition
\[ c_1 > \frac{64}{75} |\alpha| \] (4.12)
is satisfied, we can remain in the weak coupling region and the above picture may be trusted. The above choice of the parameters is within this range so the above result may be correct. However we will see that this may be a more serious problem for other internal spaces.

### 4.3 Flat internal space

The conditions (2.5) and (2.6) give
\[ n_6(t) \equiv \frac{4}{5} \alpha \tanh[(16\alpha/5)(t - t_2)] + 3\beta - \frac{c_1}{16} > 0, \] (4.13)
\[ \frac{32\alpha^2/25}{\cosh^2[(16\alpha/5)(t - t_2)]} - n_6^2(t) > 0. \] (4.14)

This case for \( \alpha = 0.92, c_1 = 1, \beta = \frac{2}{5} \) was examined in [10] with results similar to SM2-brane again, and one can see that the change of \( t_2 \) simply gives the shift in time of the whole behavior without new feature.

However, in this case the time range is from \(-\infty\) to \(+\infty\) so that the dilaton (4.6) always gives strong string coupling somewhere for large \( |t| \) whatever the choice of the parameters are, and it is not clear if this classical analysis is valid. So we cannot conclude whether this model gives a reasonable result. It is actually known that this limit is the one in which 11-th dimension becomes large [13]. This case is better described in the SM2-brane picture, with large 11-th dimension. This is what we have already analyzed in the preceding section.

### 4.4 Spherical internal space

The conditions (2.5) and (2.6) give
\[ n_7(t) \equiv \frac{4}{5} \alpha \tanh[(16\alpha/5)(t - t_2)] - 3\beta \tanh(5\beta t) - \frac{c_1}{16} > 0, \] (4.15)
\[ \frac{32\alpha^2/25}{\cosh^2[(16\alpha/5)(t - t_2)]} - \frac{15\beta^2/2}{\cosh^2(5\beta t)} - n_7^2(t) > 0. \] (4.16)
where we have again set $t_1 = 0$ by a shift of time.

We have examined the behaviors for $\alpha = 1, c_1 = 0, \beta = \frac{8}{5\sqrt{15}}$ for $t_2 = -1, +1$ which are shown in Figs. 16 and 17, respectively. We find again that there is a possibility of getting accelerating cosmologies for negative $t_2$, and also for positive $t_2$ though in this case the universe is contracting. The typical features of the solution is essentially the same as SM2-brane case. Here again this internal space suffers from the problem of strong string coupling. It is not clear if we can trust this result, but the analysis in SM2-brane seems to support the basic behavior.

5 Conclusions and discussions

We have examined accelerating cosmologies obtained from time-dependent solutions in superstring/M theories including parameter dependence, with the hope to get bigger expansion factors. As a new feature, we have found that it is also possible to obtain accelerating phase not only for hyperbolic and flat internal spaces but also for spherical space. In the last case, the universe eventually enters into the contracting phase.

The obtained expansion factors of our universe seem to be rather small to solve the horizon and flatness problems, and this seems to be typical problem in these models. This factor improves slightly if we change parameters of the models, but still not enough.

We have also shown the relation between SM2-brane and SD2-brane, and thus have given the reason why the qualitative behavior is basically the same as SM2-brane. There
is also a problem associated with the strong string coupling in SD2-brane, in which case one must return to the SM2-brane picture.

One of the common features of the above results is that our spacetime starts from very small size while the internal space is fairly large, and our scale factor grows in time while the size of the internal space shrinks for a while (perhaps in our time). It is interesting to further pursue this possibility.

An obvious generalization of our work would be to consider more general case allowing the full parameters in our solution (2.10). Also it would be interesting to examine other possibility of the internal space such as product space. It is extremely important to investigate how one can achieve expansion factor large enough to explain the cosmological problems in this context.

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