Quantum Illumination with Multiple Entangled Photons

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In this work, a theoretical generalization of Lloyd’s quantum illumination model for signal beams described by two entangled photon states is developed. It is shown that the new protocol is more sensitive than Lloyd’s model with respect to signal-to-noise ratio (SNR) and reduces the required time-bandwidth product to have the same enhancement in the SNR. The protocol is resilient against noise, potentially resilient against losses, and offers a method to find the range of the target. However, the generation of the required three photon states for the protocol remains a technical problem for its practical implementation.

1. Introduction

Lloyd’s quantum illumination is a quantum sensing protocol that has opened new research directions in quantum sensing and quantum communications due to the enhancement in sensitivity that it has over classical single photon illumination and its resilience to noise and entanglement loss. Further developments have also shown that Gaussian quantum illumination offers an enhancement in sensitivity with respect to any analogue classical illumination based upon nonentangled sources of light with the same intensity and frequency characteristics. Furthermore, since the original proposal from Lloyd’s was published, several quantum illumination schemes have been investigated and experimentally implemented. Partially supported by such experiments and further theoretical developments, quantum illumination has raised expectations for its applicability in quantum communications, radar technology and quantum communication, making quantum illumination a promising field for applications in several areas of quantum technology.

Quantum illumination protocols rely on certain nonclassical correlations between the idler and the signal states that survive the annihilation of entanglement due to noise and losses. For quantum sensing applications in the optical domain, the theory based upon Gaussian quantum illumination has been shown to be adequate. The theory presented by Tan et al. overcomes the fundamental result of Shapiro and Lloyd on how coherent light can outperform Lloyd’s quantum illumination when using light of the same energy and intensity characteristics. For radar applications, on the other hand, quantum illumination in the microwave regime has been subjected to extensive study. Indeed, experimental demonstrations in laboratory conditions have shown a relative increase in sensitivity compared to classical illumination protocols.

However, fundamental limitations in the practical implementation of quantum illumination have been recognized and understood, one of which is the range problem. Several proposals of quantum radar protocols that potentially overcome the range problem have been proposed. For instance, Maccone and Ren have suggested the use of N-photon entangled states as signal beams without keeping any of the photons to form the idler beam. Maccone and Ren provided a theoretical method to obtain the target distance and transverse displacement from direct detection of the return state, but the current form of their quantum radar protocol is hampered by the loss of entanglement due to environmental noise and by problems related to losses of the photons describing the signal, since the loss of one of the photons in the signal state make the rest of them useless for target detection. The second method for range determination is based on the concepts of radar distance and coincident photon detection methods. These techniques have been used in recent experiments of quantum illumination in the optical regime, where direct photon detection has been applied in the experiment conducted by England et al., an experiment that follows up the previous experiment of Lopaeva et al.

Another important limitation of quantum illumination, especially for the application of microwave quantum illumination, is the large time-bandwidth product required for a significant sensitivity enhancement, which is difficult to achieve in the microwave frequency domain.

In this paper we discuss a theoretical generalization of Lloyd’s quantum illumination protocol where the signal state is described by two photon states originally entangled with the idler state describing one photon. Although coherent light outperforms Lloyd’s quantum illumination in terms of sensitivity for light of the same characteristics, the approach taken in this paper is to consider the simplest technical model of quantum illumination as discussed originally by Lloyd, leaving the treatment of non-Gaussian squeezed states multiple photons quantum for future work. Note that the protocol for signals described by $N > 2$ photon states follows as analogous scheme as
the one described by the protocol in this work, but to simplify the treatment and due to the difficulties in generating the required multiple entangled photons, more attention is paid in this paper to the case \( N = 2 \).

The protocol of quantum illumination with three entangled photon states discussed in this paper is depicted schematically in Figure 1 and briefly described as follows. A three entangled photon state is generated through four photon nonlinear optical interaction.\(^{[17]}\) The three photon state is then split into an idler state, containing one photon, and a signal state, containing two photons. Note that already by such splitting the initial quantum entanglement of the triplet is lost, but the three photons of the quantum state remain correlated in the time of generation and each mode is correlated in frequency with the pump frequency in the same way. After preparation, the signal beam is sent to explore a given region of the spacetime where a possible target could be located and the idler beam is retained in the laboratory. When the return the signal is received, a joint measurement idler/signal is made where both the time correlations and the frequency correlations are checked.

As for Lloyd’s quantum illumination and Gaussian quantum illumination protocols, the protocol presented in this paper is robust against the loss of entanglement due to interaction with the environment and does not require the preservation of the entanglement between idler and signal states during the round trip detection process: it only makes use of the preserved correlations from the original three photon states encoded by the quantum states,\(^{[18]}\) namely correlations in time and correlations in energy for each mode. We will show that in conditions of low reflectivity of the target, under low environmental noise and for a high time-bandwidth product, the new protocol shows higher sensitivity in terms of SNR than Lloyd’s quantum illumination, in a similar way as Lloyd’s quantum illumination is more sensitive than single photon illumination. Furthermore, the new protocol provides a theoretical solution of the range problem in quantum radar based on the notion of radar distance, a concept already demonstrated in quantum illumination experiments.\(^{[16]}\) With respect to the time-bandwidth problem mentioned above, the protocol proposed in this paper requires a lower time-bandwidth product than Lloyd’s quantum illumination in order to have the same enhancement in the signal to noise ratio.

The theoretical protocol developed in this paper is not free of practical and theoretical difficulties. Besides potential idler losses, which is a common problem in several quantum illumination protocols, the generation of the states for our quantum illumination protocol is an important concern for the practical implementation of our protocol. Another issue is the difficulty of the required joint measurements. The protocol is formulated in a form close to the original Lloyd’s theory, which assumes a joint measurement of the idler/signal photon system and that clearly shows all the non-classical correlations involved. However, the inherent practical difficulty in performing the required joint measurements. For Gaussian quantum illumination, such difficulties have motivated the adoption of direct photon and coincidence detection methods, instead of joint measurements.\(^{[3,16]}\) Therefore, for practical purposes, other versions of the new protocol could be considered where, for instance, the measurements are implemented by direct photon detection and/or coincidence methods, as depicted in Figure 2. Although not all the quantum correlations are used, it has been shown in the case of Gaussian quantum illumination that such protocols still are more sensitive than protocols based on coherent illumination.\(^{[3,16]}\) This suggests that the same happens for quantum illumination with three or multiple entangled photons, providing an alternative practical implementation of quantum illumination with multiple photons.
Figure 2. Quantum illumination with three entangled photons using direct methods of detection. Three photons correlated in time and in frequency are generated. Two photons (signal photons) are sent to explore a region where a target could be located; one photon is either retained or directly detected just after generation (idler photon). Two signal photons are direct detected simultaneously by one detector. Then the temporal correlations between the idler photon and the two signal photons is checked.

2. Quantum Illumination with Multiple Entangled Photons

2.1. Lloyd’s Quantum Illumination using Multiple Entangled Signal Beams

The protocol described in this paper makes use of the assumptions that the background noise $N_B$ is small, the time-bandwidth product $M$ is large and the reflective index of the target $\eta$ is small, as in the original analysis from Lloyd. Note that although signal states describing two photons are considered, the generalization to signal beams describing multiple photon states is straightforward.

Consider a three photon pure quantum state of the form

$$|\psi\rangle = \frac{1}{\sqrt{M}} \sum_{\alpha=1}^{M} \lambda(\alpha) \hat{a}^\dagger(k_1(\alpha)) \hat{a}^\dagger(k_2(\alpha)) \hat{a}^\dagger(k_3(\alpha)) \, |0\rangle$$  \hspace{1cm} (2.1)

where the photons 2 and 3 have parallel momenta, $\bar{k}_i(\alpha)$ and $\bar{k}_i'(\alpha) = \|\bar{k}_i(\alpha)\|^2 \varepsilon_2$, but where the frequencies $\omega_2(\alpha)$ and $\omega_3(\alpha)$ can be different. After this preparation, the beam is split into two beams: the idler beam, which is composed by linear combinations of one photon states with four-momentum $(\hbar \omega_1(\alpha), \hbar \vec{k}_1(\alpha))$, and the signal beam, which is described by states with two photons with four-momentum $(\hbar \omega_2(\alpha) \hbar \vec{k}_2(\alpha))$ and $(\hbar \omega_3(\alpha), \hbar \vec{k}_3(\alpha))$ respectively.

Let us consider a particular case of states $|\Phi\rangle_{\lambda}$ of the form

$$|\Phi\rangle_{\lambda} = \frac{1}{\sqrt{M}} \sum_{\alpha=1}^{M} \lambda(\alpha) \hat{a}^\dagger(k_1(\alpha)) \hat{a}^\dagger(k_2(\alpha)) \hat{a}^\dagger(k_3(\alpha)) |0\rangle$$  \hspace{1cm} (2.4)

In general, the probability of detection and probability of false positive will depend on the coefficients $\lambda$. Although an indepth analysis has not been provided in the present paper, the enhancement has been illustrated by considering the relevant case of the entangled state (2.4), which is a direct generalization of the original idler-signal state (A12) in Lloyd’s theory. If these values for $\lambda$ are chosen, then the system is described by the density matrix of a pure state

$$\rho_\Phi = |\Phi\rangle\langle\Phi|$$  \hspace{1cm} (2.5)

After the splitting of the beam and the interaction with the background, the photon composing the idler state is not entangled with the photons composing the signal beam; neither are the signal photons entangled. However, the following correlations persist:

- Time correlation in the generation of the three photons,
• The photons on each state exhibit correlations in frequency

\[ \omega_a = \omega_1(a) + \omega_2(a) + \omega_3(a), \quad a = 1, \ldots, M \] (2.6)

The idler photon is retained in the laboratory system while the signal is sent to explore a region in spacetime. One of the effects of noise in the environment is to induce decoherence on the state \( \rho_0 \). Hence the idler system is well described approximately by the total mixed state

\[ \tilde{\rho}_i = \frac{1}{M} \sum_{a=1}^{M} |k_i(a)\rangle\langle k_i(a)| \] (2.7)

while \( \tilde{\rho} \) denotes the state of the idler-signal system, after target scattering. \( \tilde{\rho} \) does not correspond to an entangled state, because decoherence due to interaction with the background has occurred during the roundtrip. However, in the low bright regime that we are considering, \( \tilde{\rho} \) can be approximated by the initial entangled state \( \rho_0 \) as in the case of Lloyd’s quantum illumination.\(^{[1,5]}\)

Let us assume the above mechanism and methodology in the generation and preparation of the idler and signal beams. The signal state is denoted by \( \tilde{\rho}_i \), while the idler state is denoted by \( \tilde{\rho}_i \), both can be mixed states. As discussed in Appendix A, the noise environment is described by a thermal state \( \rho_0 \), that can be approximated by a state of the form

\[ \rho_0 = \left\{ (1 - M_B N_{\beta})|0\rangle\langle0| + \sum_{\beta=1}^{M_B} a^\dagger(k(\beta))|0\rangle\langle0|a(k(\beta)) \right\} \] (2.8)

where the index \( \beta = 1, \ldots, M_B \) indicates the environmental noise spectra

\[ \text{Spect} := \{ \omega(\beta) \in \text{Noise} \} \] (2.9)

\( M_B \) is the number of modes of the noise. This is the noise quantum state used in Lloyd’s theory (see the expression (A3) in Appendix A). Let us remark here that the signal beam is collimated in such a way that the photons 2 and 3 are within the spectrum of the noise

\[ \omega_2(a), \omega_3(a) \in \text{Spect}, \quad a = 1, \ldots, M \] (2.10)

For practical purposes, one can thus assume the index \( a \) and the index \( \beta \) to be identical.

Given the structure of the signal beam and the underlying correlations with the idler beam that undergo the preparation and the whole detection process, the criterion for a positive detection in our protocol is the following:

**Criterion for Positive Detection.** We declare that the target is present if a pair of photons is received simultaneously and a joint measurement of the signal state with the idler state shows time correlation and energy correlation (2.6).

This criterion assumes the measurement of the arrival time correlation and energy correlations (2.6), as originally suggested by Lloyd’s.\(^{[1]}\) However, the measurement of the photon energies in a joint measurement together with the time of arrivals is difficult; only if the sum of energies \( \omega_1 + \omega_2 + \omega_3 \) is measured at the same time, one has the restriction (2.6), which implies the joint measurement of the three energies \( \omega_1, \omega_2, \omega_3 \). Also, the identification of the corresponding idler photon is an issue in the measurement process. For practical implementation reasons, an alternative way of detection based only on the use time correlations is of relevance. In the case of quantum illumination with one signal photon state correlated with an idler photon, direct detection of photons and number counting methods provide a practical implementation of quantum illumination. These methods imply a reduction in the practical sensitivity enhancement, but still offer an advantage in sensitivity over classical strategies for two-mode squeezed vacuum quantum illumination.\(^{[3,16,20]}\) It is reasonable to assume that such an advantage also exists in the present case. It can be thought of as a possible implementation of quantum illumination with two entangled photon signal states.

The time detection window required to detect two signal photons and to characterize them as time correlated must be small enough to avoid the misinterpretation of uncorrelated photons as being correlated in time. On the other hand, the time detection window must not be too short as well, in order to avoid missing entangled sets of photons. Indeed, the generation process of three entangled photons implies a lower value of precision for the time window of detection: if the generation time of the three photons is within an interval \( \delta t \) and the time detection window is \( \Delta t \), then it is necessary that \( \Delta t > \delta t \). Although the generation of the three photons by down conversion is not strictly a local process, \( \delta t \) is very small in practical terms. Therefore, we assume the condition \( \Delta t > \delta t \). Besides, additional corrections must be taken into account due to the fact that the idler and signal beam propagate in different media.

### 2.2. Enhancement of the Signal to Noise Ratio with Respect to Classical Light Illumination

In the following paragraphs the SNR for non-entangled illumination and for quantum illumination with multiple entangled photon states as signal is evaluated. The proposed procedure is similar to the treatment of Lloyd.\(^{[1]}\)

#### 2.2.1. Illumination with Nonentangled Light

When no target is present and the illumination is made by means of non-entangled light, the quantum state is described by the product density matrix

\[ \tilde{\rho}_0 = \rho_0 \otimes \rho_0 \] (2.11)

The probability of false positive can be read directly from the structure of the state and, due to the criteria of detection discussed above, it is associated with the event of detecting two photons in the same time window. Therefore, in the case of a low bright environment (\( N_0 \ll 1 \)), the probability of false positive is determined by the criteria of detection of two independent photons simultaneously with the required energies \( \omega_2 \),
the expression from the background. It is the probability to detect two photons when

\[ \tilde{\rho} = (1 - \eta) \cdot \rho \cdot + \eta \rho' \]

where \( \rho \) is the density matrix describing the joint system signal photons/idler after decoherence and scattering with the target has happened, processes that induce the transformation from the original state \( |\Phi\rangle \) to the state \( \tilde{\rho} \). When the target is present, the probability of positive detection when using entangled states is of the general form

\[ \tilde{p}^e(t) = (1 - \eta) \frac{(N_B)}{M} + \eta \]

Therefore, the signal to noise ratio for quantum illumination with signal states described by two photons is of the form

\[ \text{SNR}_{QI,2}^e = \frac{\tilde{p}^e(t)}{\tilde{p}^e(1)} \approx \left( \frac{M}{N_B} \right)^2 (1 - \eta) \left( \frac{N_B}{M} \right)^2 + \eta \]

We can now compare the relation between the SNR of our protocol with entangled light \( \text{SNR}_{QI,2}^e \) and the SNR in Lloyd’s theory \( \text{SNR}_{QI}^e \). It turns out that

\[ \text{SNR}_{QI,2}^e = (1 - \eta) + \left( \frac{M}{N_B} \right)^2 \eta \]

\[ > (1 - \eta) + \frac{M}{N_B} \eta = \text{SNR}_{QI}^e \]

Thus we have

\[ \text{SNR}_{QI,2}^e > \text{SNR}_{QI}^e \]

This result shows that in the regime where \( \eta \ll 1, M \gg 1 \) and \( N_B \ll 1 \), quantum illumination where the signal is described by two photon states is more sensitive than Lloyd’s quantum illumination model. The advantage comes from the large factor \( \left( \frac{M}{N_B} \right)^2 \), originated by the positive detection criteria requiring two photons for detection instead of one single photon.

### 2.3. Relative Reduction of the Time-Bandwidth Product

One of the technical requirements to have sensitivity enhanced in quantum illumination is that the photon states must have a high time-bandwidth product. Quantum illumination provides an advantage with respect to classical illumination in the regime of high values of the product \( T \cdot W \), where \( T \) is the pulse duration of the signal and \( W \) the bandwidth. Depending on the frequency range of interest, it could be a problem to achieve a time-bandwidth product high enough for a significant sensitivity enhancement. This is of particular importance in the microwave regime, where high time-bandwidth products are difficult to achieve.[13]

For the protocols considered in this paper, the time-bandwidth product is equal to the number of modes \( M \) of each quantum state. It is shown that in the regime where \( \eta \ll 1, M \gg 1 \) and
N_b \ll 1 quantum illumination with signals whose states describe two photons implies a reduction in the time bandwidth product needed to reach the same advantage in SNR than in Lloyd’s protocol of quantum illumination. Then let us compare the $SNR_{QI2}$ of two entangled photon illumination (2.20) with the $SNR_{QI}$ in Lloyd’s quantum illumination (A23). For both SNR to be approximately equal, the following relations must hold

$$SNR_{QI2}^e = \left( \frac{M}{N_B} \right)^2 \left( 1 - \eta \right) \left( \frac{N_B}{M} \right)^2 + \eta$$

$$= \frac{M'}{N_B} \left( 1 - \eta \right) \frac{N_B}{M'} + \eta = SNR_{QI}^e$$

(2.23)

where $M$ and $M'$ are a priori different. In the regime when $\eta \ll 1$, $M \gg 1$, $M' \gg 1$ and $N_b \ll 1$, the condition $SNR_{QI2}^e \approx SNR_{QI}^e$ requires that

$$M \approx \sqrt{N_B M'}$$

(2.24)

The relation (2.24) shows a reduction of the time bandwidth product, here associated to the number of modes $M$ and $M'$, such that $SNR_{QI2}^e$ and $SNR_{QI}^e$ are approximately equal.

The condition $M \gg 1$ is easily satisfied in the optical regime, where time-bandwidth products of order $M \approx 10^5$ or larger are feasible. In the optical regime, typical values of $N_b$ at $\lambda = 1.55 \mu m$ is of order $10^3$. Although in the microwave regime the condition $N_b \ll 1$ is not satisfied under sky’s daytime conditions, the required time bandwidth product is still reduced as long as $N_b$ is not excessively large, because of the dependence on $\sqrt{M'}$ in Equation (2.24).

### 2.4. Determination of the Target Range using Quantum Illumination with Two Photons Signal States

Let us consider the situation where two photons are detected after the scattering with the target and the idler photon has been tracked from the generation until the detection of the signal photons. In this case, the target range is given by the radar distance

$$r_s = c \left( \frac{1}{2} (T - t_0) \right)$$

(2.26)

where $T$ is the arrival time of the two signal photons and $t_0$ is the time of generation of the idler photon.

The procedure to determine the range of the target described above has, however, important limitations. One of them is that the time of arrival of the signal photons must be known. This difficulty suggests the use of quantum illumination in combination with other detection devices, as recognized already by Lloyd.$^{[1]}$ On the other hand, coincidence detection methods allow to obtain the range while performing quantum illumination experiments.$^{[16]}$ This suggests that such a methodology may be extended to quantum illumination with multiple photons. Another issue is related to the loss of signal photons. After being scattered with the target, they can have different directions of propagation and do not necessarily return to the receiver, producing losses. A similar problem arises in the theory developed by Maccone and Ren,$^{[15]}$ where some of the photons in each quantum state can be lost after the scattering. Finally, the tracking method for the idler photons required to apply this methodology should be mentioned. One possibility is to use matched filtering techniques, similarly to the experiments made by various research groups in microwave quantum illumination,$^{[9,10]}$ although this changes rather radically the theory of the protocol presented in this paper.

### 3. Discussion

In the present work we have developed a protocol for quantum illumination with three entangled photon states as source such that the signal beam consists of two photon states and the idler beam of one photon states. In the case of low intensity signals, low background noise (which can be high compared to the signal level) and low reflectivity of the target, the protocol proposed has a higher SNR than Lloyd’s quantum illumination. The rationale behind this effect is that, according to Lloyd’s theory, the signal photons can be distinguished from noise photons with lower ambiguity if the signal photon and the idler photon share nonclassical correlations, persistent after the loss of the initial entanglement.$^{[1]}$ This property is stronger when the signal states are described by two photons originally entangled with a third idler photon. Indeed, direct inspection shows that in the regime $N_b \ll 1$ and $M \gg 1$ we have $P_{QI}^e(\lambda) \approx P_{QI2}^e(\lambda)$. There are two additional benefits of the new protocol proposed. The first one is the reduction of the required time-bandwidth product to give the same SNR as Lloyd’s quantum illumination. The second additional benefit is that the proposed protocol provides a way to obtain the target range. Since the method is based on the concept of radar distance measurements, it shares the same difficulties as the analogue methods in Lloyd’s quantum illumination or Gaussian quantum illumination with one photon signal states. However, the implementation of such methods to obtain the target range could follow implementations similar to in practical quantum illumination experiments.$^{[16]}$

There are several difficulties in the practical implementation of the protocol for quantum illumination with multiple entangled photons. The first is the generation of the required states (2.4). Although three entangled photon states are of fundamental interest for quantum technologies, as for instance in quantum computing$^{[22]}$ or quantum communication,$^{[23]}$ the generation of three entangled photon states is an arduous task.$^{[24,25]}$ For quantum sensing applications, the main difficulty lies in the generation of states with a large time-bandwidth product and with an intensity high enough for target detection. The problem arises due to the typically small value of the third order susceptibility
Quantum illumination experiments have been performed where entangled pairs of photons are generated by four wave mixing generation\(^{(16)}\) and up to 97,000 coincident detection events (idler/signal pair for the idler/signal source) are generated per second. The intensity strongly depends upon the power of the photon pump while, Raman scattering conditions the number of coincident pairs. There have been also advances in three photon generation by means of spontaneous parametric down conversion methods that suggest the possibility of generating brighter photon triplets in the microwave domain. In particular, it has been shown that by means of an intense flux-pumped, superconducting parameter resonator, multiple mode three photon entangled states can be generated\(^{(15)}\) in the microwave domain. The brightness of the triplets obtained reaches up to 60 triplets per second per Hertz over a bandwidth of up to 100 MHz, comparable to two photon down conversion generation.\(^{(26,27)}\) Although such states are not exactly of the form of Equation (2.4), these experimental achievements and underlying theoretical research are of great relevant for quantum illumination with multiple entangled signals, since this possibility of generating bright radiation enables the possibility to be used for quantum illumination with three photons as described in this paper. It should also be mentioned that, using the current methods of generating bright three-photons spontaneous parametric down conversion, the generated states are highly non-Gaussian.\(^{(19)}\) Indeed, the non-Gaussian character of the states by-passes the implications of the optimal probe for quantum illumination discussed in ref. [50]. The detailed investigation of the impact and broadness of three photon generation for quantum illumination should be the subject of future research.

There are alternative methods to generate three photon entangled states. Cascade down conversion generation of three photon states can also be a source for the correlated states required for the protocol. This process has already been observed\(^{(28)}\) and consists of two consecutive spontaneous parametric down conversions to generate three photons final states. For the purpose of quantum illumination, cascade down conversion generation does not constrain the generation time as precise as three photon down conversion generation. If the intermediate energies are under control, however, two energy constraints need to be satisfied. The first down conversion implies the correlation in frequency \(\omega_0 = \omega_1 + \omega'_1\); the second conversion implies \(\omega'_2 = \omega_2 + \omega_3\). This stronger form of the correlations can have additional benefits for sensitivity.

Second, if one of the photons in a signal pair is lost it makes the companion photon useless for a positive detection according to our criteria. In this sense, our protocol shares a similar problem as Maccione–Ren’s quantum radar protocol. To overcome the problem of losses, Maccione and Ren suggested the use of nested entangled photon state systems,\(^{(15,29)}\) which is a more robust protocol against losses. A similar idea can be adapted to the proposed protocol: even if one photon of a pair of signal photons is lost, the arrival of a photon correlated in time with an idler photon can be used to indicate a positive target detection. As long as there are two photon detection events, the protocol will provide an advantage over Lloyd’s quantum illumination.

Third, direct photon detection limits the maximum attainable range. For instance, current avalanche photodiode detectors imply that the maximum range can be up to 300 m. This is not a fundamental limit of the detection method, since other detection methods developed in the literature of quantum illumination and quantum radar\(^{(14)}\) can potentially be adapted to the proposed protocol.

Besides the specific problems and limitations mentioned above there are other issues in common with quantum illumination protocols. Among them is the storage of the idler, which has been recognized as a dramatic constraint for quantum illumination protocols.\(^{(16)}\) Two fundamental solutions have been discussed in the literature. The first is the use of quantum memories\(^{(16)}\) to store the idler. The second is a combination of classical digital recording methods and matched filtering methods as introduced in refs. [9–11]. Both methodologies introduce profound modifications in details of the protocol proposed in this paper but looking at how quantum illumination has evolved in the last decade, are also interesting avenues to explore.

4. Conclusion and Outlook

In the present work a new protocol for quantum sensing is proposed that works under the assumptions of low target reflectivity, low environmental brightness \(N_B \ll 1\) (but stronger than the signal beam) and a high time-bandwidth product \(M \ll 1\). It has been shown that in this regime and under the same conditions, the proposed protocol demonstrates higher sensitivity in SNR than Lloyd’s quantum illumination model. The protocol is also resistant against noise and requires a lower time-bandwidth product than Lloyd’s quantum illumination to give the same enhancement in SNR. A methodology to determine the range of the target by means of the radar distance concept has also been presented. In addition, it has been discussed how the required three entangled photon states can potentially be generated by means of current techniques in three photon generation and cascade generation.

The theory developed in the present work leads to new additional questions and research problems. A straightforward next step is the extension of the current model to the case when the environmental noise is large \((N_B \gg 1)\), similar to the extension of the protocol of quantum illumination by using Gaussian states made by Tan et al.\(^{(22)}\) Further problems to be considered in future work are the understanding of the dependence of the enhancement on the choice of the \(\lambda\) coefficients in \(|\Psi\rangle\), as well as the detailed investigation of the preparation of the states.

The results presented in this paper open a new research perspective for quantum illumination, both at the theoretical and practical level, showing how tripartite states can enhance the detection sensitivity of Lloyd’s quantum illumination. For radar purposes, there is also additional evidence that multi-photon squeezed states can give an additional enhancement.\(^{(13)}\) Such additional studies further substantiate the general idea of the present work: the study of entangled multiphoton states to improve the sensitivity of quantum illumination.

Appendix A: Sensitivity Enhancement in Lloyd’s Quantum Illumination: An Illustrative Example

Several aspects of Lloyd’s quantum illumination protocol\(^{(1)}\) are discussed in detail following the exposition of quantum illumination described in ref. [5].
Hypothesis 0 means that a target is not present, while hypothesis 1 refers to the possibility that the target is present. In the models considered, the background noise is described by a thermal state of the form

$$\rho_0 = \bigotimes_{n=1}^{M_2} \rho_{N_B}$$  \hspace{1cm} (A1)

where $\rho_{N_B}$ is the thermal state of a mode with average number $N_B$. For $N_B \ll 1$, the state $\rho_{N_B}$ can be approximated by writing as

$$\rho_{N_B} = N_B |k\rangle_s \langle k| \otimes (1 - N_B) |k\rangle_n \langle k|$$  \hspace{1cm} (A2)

where $|k\rangle_s$ stands for a noise photon $n$-mode corresponding to a photon with four-momentum $\hbar \mathbf{k}$. Under the further hypothesis that $N_B M_B \ll 1$, the state $\rho_0$ is re-written as

$$\rho_0 \approx \left\{ (1 - M_B N_B)|0\rangle \langle 0| + N_B \sum_{n=1}^{M_2} |k\rangle_n \langle k| \right\}$$  \hspace{1cm} (A3)

which is the environment thermal noise state used in Lloyd’s theory.[1]

A.1. Nonentangled Light Illumination. When the light used for illumination is described by nonentangled photons, the density matrix of the system idler-signal-noise, when the target is not there (hypothesis 0) is $\rho_0$. Hence the probability of a false positive is

$$p_0(+) \approx N_B$$  \hspace{1cm} (A4)

while the probability to be correct in the forecast that the target is not there is

$$p_0(-) \approx 1 - p_0(+) \approx 1 - N_B$$  \hspace{1cm} (A5)

If we repeat the experiment $m$ times, the probability of a false positive is

$$p_0(+) \approx (N_B)^m$$  \hspace{1cm} (A6)

If the target is there (hypothesis 1), then the density matrix is given by

$$\rho_1 = (1 - \eta) \rho_0 + \eta \tilde{\rho}$$  \hspace{1cm} (A7)

$$\approx (1 - \eta) \left\{ (1 - MN_B)|0\rangle \langle 0| + N_B \sum_{n=1}^{M_2} |k\rangle_n \langle k| \right\} + \eta |\psi\rangle_s \langle \psi|$$  \hspace{1cm} (A8)

where $|\psi\rangle_s$ stands for the state describing the signal, that one can assume first is a pure state, while $\eta$ is the reflective index. It follows that the probability to measure the arrival of a photon is

$$p_1(+) \approx (1 - \eta) N_B + \eta$$  \hspace{1cm} (A9)

and that consequently, the probability of false negative is

$$p_1(-) \approx 1 - p_1(+) \approx 1 - (1 - \eta) N_B + \eta = (1 - \eta)(1 - N_B)$$  \hspace{1cm} (A10)

The signal to noise ratio is given by the expression

$$\text{SNR}_{CI} = \frac{p_1(+)}{p_0(+)} \approx \frac{(1 - \eta)N_B + \eta}{N_B}$$  \hspace{1cm} (A11)

A.2. Entangled Light Illumination. Let us now consider the case when the illumination is made using entangled states. In Lloyd’s theory, the initial prepared state is of the form

$$|\Psi_{IS}\rangle = \frac{1}{\sqrt{M}} \sum_{n=1}^{M} |k\rangle_i \otimes |k\rangle_s$$  \hspace{1cm} (A12)

The modes determining the idler/signal $k = 1, \ldots, M$ are selected to coincide with the modes of the noise. For the case when there is no target there, the density matrix is given by the expression

$$\rho_0^e \approx \left\{ (1 - MN_B)|0\rangle \langle 0| + N_B \sum_{n=1}^{M} |k\rangle_i \langle k| \right\} \otimes \left( \frac{1}{M} \sum_{n=1}^{M} |k\rangle_i \langle k| \right)$$  \hspace{1cm} (A13)

where $\frac{1}{M} \sum_{n=1}^{M} |k\rangle_i \langle k|$ is the state of the idler. The state

$$\rho_0 = \left\{ (1 - MN_B)|0\rangle \langle 0| + N_B \sum_{n=1}^{M_2} |k\rangle_n \langle k| \right\}$$  \hspace{1cm} (A14)

is the state describing the absence of the target. It determines the probability distributions to detect one photon due to noise only. In this context, it is remarkable that the false positive probability for one individual detection

$$p_0^e(+) \approx \frac{N_B}{M}$$  \hspace{1cm} (A15)

is dramatically reduced with the number of modes $M$. This was first highlighted by Lloyd in his seminal work.[1] The probability of predicting correctly the absence of the target is given by the probability of the complement

$$p_0^e(-) \approx 1 - \frac{N_B}{M}$$  \hspace{1cm} (A16)

Note that when the experiment is repeated a number $m$ of times in an independent way, the probability of a false positive after detecting $m$ independent photons is

$$p_0^e(+) \approx \left( \frac{N_B}{M} \right)^m$$  \hspace{1cm} (A17)

When the target is present, for entangled states, the system idler-noise-signal is described by a density matrix of the form

$$\rho_1^e \approx (1 - \eta) \cdot \rho_0^e + \eta \cdot \tilde{\rho}_0^e$$  \hspace{1cm} (A18)

where $\rho_0^e$ is the density matrix of the signal-idler system after interaction with the environment. From this expression, one can extract the probability of detecting the target,
\[ p_1^e(+) \approx (1 - \eta) \frac{N_B}{M} + \eta \]  
(A19)

The probability of no detection (interpreted as a false negative) is of the form

\[ p_1^e(-) \approx 1 - p_1^e(+) \approx \left(1 - \frac{N_B}{M}\right) (1 - \eta) \]  
(A20)

When applied \( m \) independent experiments, the probability of right detection is

\[ p_1^e(+, m) \approx \left(1 - \eta\right)^m \]  
(A21)

In the case of false negative

\[ p_1^e(-, m) = 1 - p_1^e(+) = \left(1 - \frac{N_B}{M}\right)^m (1 - \eta)^m \]  
(A22)

The signal to noise ratio for this model of quantum illumination is

\[ \text{SNR}_{QI} = \frac{p_1^e(+) / p_1^e(-) \approx \left(\frac{M}{N_B}\right) \left(1 - \eta\right) \frac{N_B}{M} + \eta}{(A23)} \]

Comparing the probabilities of false positive and detection using quantum enhancement with respect to classical light, one observes a clear enhancement in sensitivity when using quantum illumination.\(^{11}\) Further details of the analysis of the sensitivity enhancement using Lloyd’s protocol can be found in ref. [1], in ref. [5] (Section 5.5.3), and in ref. [14].

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**Conflict of Interest**

The author declares no conflict of interest.

**Data Availability Statement**

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

**Keywords**

quantum correlations, quantum enhancement, quantum entanglement, quantum illumination, quantum radar, triple photon generation

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