Spin effects in two-particle hadronic
decays of $B_c$ mesons.

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Abstract

We consider spin effects in two–particle hadronic decays of $B_c$ and $B^*_c$ mesons into $J/\psi$ plus $\rho(\pi)$ in the frame work of hard gluon exchange model. It is shown that polarization of the $J/\psi$ meson is very different in decays of $B_c$ and $B^*_c$ mesons as well as their decay widths into $J/\psi$ plus $\rho(\pi)$.

1. Introduction

The start of the experimental study of the $B_c$ mesons, containing heavy quarks of different flavors, was done recently by CDF Collaboration at FNAL [1]. The obtained value of $B_c$ meson mass is $m_{B_c} = 6.40 \pm 0.39 \pm 0.13$ GeV. It doesn’t contradict theoretical predictions, which have been made in nonrelativistic potential models [2] and QCD sum rules method [3]. However, the predicted value for the difference between masses $^3S_1$ state ($B^*_c$) and $^1S_0$ state ($B_c$) of the $(\bar{b}c)$– system is $0.05 - 0.07$ GeV, that now much smaller than experimental uncertainties [1].

The calculation, which was done in $\alpha^4_s$ order of perturbative QCD using nonrelativistic model for heavy quarkonium, shows that the ratio of $B^*_c$ and $B_c$ mesons production cross sections in high energy hadronic collisions is about $2 \sim 3$ [2]. At the same time, the fragmentation model of heavy quarkonium production predicts that $\sigma(B^*_c)/\sigma(B_c) \approx 1.3$ [3]. Such a way, experimental separation of $B^*_c$ and $B_c$ meson production cross section is very important for test of the models, which pretend on description of $B_c$ meson production.

The signal of $B_c$ meson production in [1] is a peak in the invariant mass spectrum of $J/\psi l\nu(l = e, \mu)$, system, which was produced in semileptonic decay

$$B_c \to J/\psi + l + \nu_l,$$

where $J/\psi$ was detected using lepton decay mode ($J/\psi \to l^+l^-$). However, more suitable decay channels from the registration point of view, are two-particle hadronic decays with $J/\psi$ meson in final state:

$$B_c(B^*_c) \to J/\psi\rho(\pi),$$

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It is assumed, that branching ratio of the decays (2) is about 1% [6].

The main feature of the two-particle hadronic $B_c$ decays is the fact that both quarks in the $B_c$ meson ($\bar{b}$ and spectator –quark) are heavy. The necessary condition of $c\bar{c}$ bound state formation creation is large momentum transfer ($k^2 >> \Lambda_{QCD}^2$) to the spectator quark and spectator picture is not valid in the decays under consideration. In fact, the gluon virtuality in the decays (2) is equal to

$$k^2 \approx -\frac{m_2^2}{4m_1}((m_1 - m_2)^2 - m_3^2) \approx -1.2 \text{ GeV}^2,$$

where $m_1$ is $B_c$ or $B_c^*$ meson mass, $m_2$ is $J/\psi$ meson mass and $m_3$ is $\rho$ or $\pi$ meson mass. So, it is needed to use here the hard scattering formalism [7].

2. The hard exchange model

At first time, the exclusive hadronic decays $B_c^+ \to J/\psi\pi^+(\rho^+)$ or $B_c^+ \to \eta_c\pi^+(\rho^+)$ have been under consideration in the frame work of hard exchange model in paper [6]. It was shown, that in contrast to the spectator approach, the hard $t$–channel exchange results in the approximate double enhancement of the decay amplitudes. However the trivial arithmetic error in $\Gamma(B_c^+ \to \eta_c\pi^+)$ calculation was made in [6]. The decay widths of $B_c^*$ mesons, as well as spin asymmetry for the creation of $J/\psi$ mesons in the states with different polarization didn’t were calculated. The polarization of the $J/\psi$ meson in semileptonic decay of $B_c$ meson ($B_c^+ \to J/\psi + \mu + \bar{\nu}_\mu$) was studied recently in [8] using the spectator approach.

In the frame work of hard gluon exchange model, the amplitudes for decays $B_c(B_c^*) \to J/\psi\rho(\pi)$ are described by diagrams in Fig.1. Using nonrelativistic approach, we assume that quarks in a heavy quarkonium move with equal velocity. So, in the case of $B_c$ meson with four–momentum $p_1$ one has

$$p_c = \frac{m_c}{m_b + m_c}p_1, \quad p_{\bar{b}} = \frac{m_b}{m_b + m_c}p_1,$$

and in the case of $J/\psi$ meson with four–momentum $p_2$

$$p_c = \frac{1}{2}p_2, \quad p_{\bar{b}} = \frac{1}{2}p_2.$$  

Assuming that quarkonium binding energies are small, the heavy quark masses may be presented in the terms of $m_1$ and $m_2$ as follows

$$m_c = \frac{m_2}{2}, \quad m_b = m_1 - \frac{m_2}{2}.$$

The transition from the amplitude of free heavy quarks with equal four–velocity $v = p/m$ production (or decay) to the amplitude of the binding state production (or decay) is done using following prescription:

$$V^i(v)\bar{U}^j(v) \to \hat{a} \frac{(1 + \hat{v})}{2} \frac{fm}{\sqrt{3}} \delta^{ij}.$$

2
where \( \hat{a} = \gamma_\mu \varepsilon^\mu \), \( \varepsilon^\mu \) is four-vector of \( B_c^* \) meson polarization, or \( \hat{a} = \gamma^5 \) in the case of pseudoscalar quarkonium, \( f \) is the leptonic constant, which is related with the value of configuration wave function at the origin

\[
f = \sqrt{\frac{12}{m} |\psi(0)|},
\]

\( \delta^{ij}/\sqrt{3} \) is the color factor, which takes into account color singlet state of heavy quarks in quarkonium.

Because of the quark virtuality in the decays \( \bar{b} \to \bar{c}\pi^+(\rho^+) \) is very small in compare with \( W^- \)-boson mass, one can use four-fermion effective theory and write \( \bar{b} \to \bar{c}\pi^+(\rho^+) \) vertex in the terms of effective Fermi–constant \( G_F/\sqrt{2} \), mixing parameter \( V_{bc} \), left quark current and axial–vector current of \( \pi \) meson \( (f_3p_3^\mu, f_3 = f_\pi) \) or \( \rho \) meson \( (f_3^\rho, f_3 = f_\rho) \).

For example, the amplitude of \( B_c^* \to J/\psi\rho \) decay calculated according to the diagrams in Fig.1, take the form

\[
M(B_c^* \to J/\psi\rho) = \frac{G_F V_{bc} 4\pi \alpha_s}{\sqrt{2}} \frac{4}{3} m_3 f_3 f_{1m1} f_{2m2} a_1 \times \\
\text{Tr} \left[ \hat{\varepsilon}_2 (1 + \hat{v}_2) \gamma^\alpha \hat{\varepsilon}_1 (1 - \hat{v}_1) (\hat{\varepsilon}_3 (1 - \gamma_5)(- \hat{x}_1 + m_c)\gamma^\alpha + \frac{m_2}{m_1} \gamma^\alpha (- \hat{x}_2 + m_b)\hat{\varepsilon}_3 (1 - \gamma_5) \right],
\]

where

\[
\hat{x}_1 = m_2 \hat{v}_2 - \frac{m_2}{2} \hat{v}_1, \quad (4) \\
\hat{x}_2 = m_1 \hat{v}_1 - \frac{m_2}{2} \hat{v}_2, \\
k^2 = \frac{m_2^2}{2} (1 - y), \\
y = (v_1 v_2) = \frac{m_1^2 + m_2^2 - m_3^2}{2m_1 m_2}.
\]

The factor \( a_1 \) comes from hard gluon corrections to the four-fermion effective vertex. In the case of pseudoscalar \( B_c \) meson decays it is needed to do substitution \( \hat{\varepsilon}_1 \to \gamma_5 \), and, if one has \( \pi \) meson in the final state, \( \hat{\varepsilon}_3 \to \hat{v}_3 \). There are following cinematic formulae in the rest frame of \( B_c \) meson:

\[
v_1 = (1, 0, 0, 0), \\
v_2 = \frac{1}{m_2} (E_2, 0, 0, |\vec{p}_2|), \\
v_3 = \frac{1}{m_3} (m_1 v_1 - m_2 v_2), \\
E_2 = \frac{m_1^2 + m_2^2 - m_3^2}{2m_1}, \\
|\vec{p}_2| = \sqrt{E_2^2 - m_2^2}.
\]
Put \( m_3 = 0 \) in (5), one finds:

\[
E_2 \approx \frac{m_1^2 + m_2^2}{2m_1}, \quad |\vec{p}_2| \approx \frac{m_1^2 - m_2^2}{2m_1}.
\]

The summation over polarization states of vector particles may be done using formula

\[
\sum_{L,T} \varepsilon^\mu_L(v)\varepsilon^\nu_L(v) = -g_{\mu\nu} + v_\mu v_\nu.
\]

(6)

The longitudinal polarization four–vector \( \varepsilon_L(p) \) is usually written explicitly as

\[
\varepsilon^\mu_L(p) = \left( \frac{|\vec{p}|}{m}, \frac{E\vec{p}}{m|\vec{p}|} \right).
\]

(7)

Let us define an auxiliary four–vector \( n^\mu = (1, -\vec{p}/|\vec{p}|) \) such that \( n^2 = 0 \), \( (np) = E + |\vec{p}| \). With the help of four–vector \( n^\mu \) one can rewrite \( \varepsilon^\mu_L(p) \) in the following covariant form

\[
\varepsilon^\mu_L(p) = \frac{n^\mu}{m} - \frac{mn^\mu}{(np)}.
\]

(8)

We can then obtain following covariant expressions for the longitudinal and transverse polarization sum:

\[
\varepsilon^\mu_L(v)\varepsilon^\nu_L(v) = \frac{\varepsilon^\mu\varepsilon^\nu - \frac{n^\mu n^\nu + \varepsilon^\mu n^\nu}{(vn)}}{(vn)^2} + \frac{n^\mu n^\nu}{(vn)^2},
\]

(9)

\[
\sum_{T} \varepsilon^\mu_T(v)\varepsilon^\nu_T(v) = -g_{\mu\nu} + \frac{\varepsilon^\mu\varepsilon^\nu - \frac{n^\mu n^\nu + \varepsilon^\mu n^\nu}{(vn)}}{(vn)^2} - \frac{n^\mu n^\nu}{(vn)^2}.
\]

(10)

3. The results

The unpolarized width of \( B_c^+ \rightarrow J/\psi \rho \) decay is written in the following form

\[
\Gamma(B_c^+ \rightarrow J/\psi \rho) = \frac{\bar{p}_2}{8\pi m_1^2} |M|^2.
\]

(11)

where \( |M|^2 \) is squared matrix element after summation over polarizations of \( J/\psi, \rho^+ \) and \( B_c^+ \) mesons, divided by 3 (the number of the initial \( B_c^+ \) meson polarization states). From Eqs. (3) – (11) one gets:

\[
\Gamma(B_c^+ \rightarrow J/\psi \rho) = G_F^2 |V_{bc}|^2 \frac{4\pi\alpha_s^2}{81} \frac{f_1 f_2 f_3^2}{m_1^4 m_2^6} \frac{|\vec{p}_2|}{(1-y)^3} a_1^2 |F(B_c^+)|,
\]

(12)

\[
\Gamma(B_c \rightarrow J/\psi \rho) = G_F^2 |V_{bc}|^2 \frac{4\pi\alpha_s^2}{81} \frac{f_2 f_3^2}{m_1^4 m_2^6} \frac{|\vec{p}_2|}{(1-y)^3} a_1^2 |F(B_c)|,
\]

(13)

where \( f_1 = f_{B_c}, f_2 = f_{J/\psi}, f_3 = f_\rho, m_1 = m_{B_c}, m_2 = m_{J/\psi}, m_3 = m_\rho \) and

\[
3 \times |F(B_c^+)| = m_8^6 + 2m_6^6(13m_2^2 - 2m_1 m_2 - m_2^2)
\]
\[
+2m_3^2(-19m_1^2 - 18m_2^2 - 16m_1^2 m_2 + 2m_1 m_2^2 + 2m_2^4)
\]
\[
+2m_3^2(-3m_1^6 + 18m_1^2 m_2 + m_1^4 m_2^2 - 20m_1^4 m_2^2 + 11m_1^4 m_2^4 + 2m_1 m_2^6 - m_2^6)
\]
\[
+17m_8^2 + 4m_7^2 m_2^2 - 32m_6^2 m_2^2 - 12m_5^6 m_2^6 + 14m_4^4 m_2^4 +
\]
\[
+12m_4^5 m_2^4 - 4m_1 m_2^7 + m_8^2.
\]
\[ F(B_c) = m_3^8 + 2m_3^6m_1(5m_1 + 8m_2) \]
\[ + m_3^6(-19m_1^2 - 56m_1^3m_2 + 6m_1^2m_2^2 - 8m_1m_2^3 - 3m_2^4) \]
\[ + 2m_3^2(2m_1^6 + 20m_1^5m_2 - 3m_1^4m_2^2 - 16m_1^3m_2^3 + 16m_1^2m_2^4 - 4m_1m_2^5 + m_2^6) \]
\[ + 4m_1^4(m_1^4 - 2m_1^2m_2^2 + m_2^4). \]

In the limit of vanishing \( \rho \) meson mass and \( m_{B^*} = m_{B_c} \), one gets:

\[
\frac{\Gamma(B_c^* \to J/\psi\rho)}{\Gamma(B_c^* \to J/\psi\pi)} \approx \frac{\Gamma(B_c \to J/\psi\rho)}{\Gamma(B_c \to J/\psi\pi)} \approx \left( \frac{f_2}{f_\pi} \right)^2 = 2.47
\]

\[
\frac{\Gamma(B_c^* \to J/\psi\rho)}{\Gamma(B_c \to J/\psi\pi)} \approx \frac{\Gamma(B_c \to J/\psi\rho)}{\Gamma(B_c \to J/\psi\pi)} = \frac{1}{12}(17 + 4x + 2x^2 - 4x^3 + x^4) \approx 1.58,
\]

where \( x = m_2/m_1, m_1 = 6.3 \text{ GeV}, m_2 = 3.1 \text{ GeV}. \)

The explicit calculation with \( m_3 = m_\rho = 0.77 \text{ GeV}, m_{B^*} = 6.3 \text{ GeV} \) and \( m_{B_c} = 6.25 \text{ GeV} \), gives following results:

\[
\frac{\Gamma(B_c^* \to J/\psi\rho)}{\Gamma(B_c \to J/\psi\pi)} \approx 1.43,
\]

and

\[
\frac{\Gamma(B_c^* \to J/\psi\rho)}{\Gamma(B_c \to J/\psi\pi)} \approx 2.28, \quad \frac{\Gamma(B_c \to J/\psi\rho)}{\Gamma(B_c \to J/\psi\pi)} \approx 2.54
\]

The polarization of \( J/\psi \) meson in the decays \( [2] \) can be measured using angular distribution of the leptons in the decay \( J/\psi \to l^+l^- \). In the rest frame of \( J/\psi \) meson the angular distribution of the leptons is given by

\[
\frac{d\Gamma}{d\theta}(J/\psi \to l^+l^-) \sim 1 + \alpha \cos^2 \theta,
\]

where \( \theta \) is the angle between the outgoing lepton 3–momentum and the polarization axis of \( J/\psi \) meson and

\[
\alpha = \frac{\Gamma_T - 2\Gamma_L}{\Gamma_T + 2\Gamma_L}.
\]

\( \Gamma_T \) and \( \Gamma_L \) are decay widths of \( B_c^* \) meson into the transverse and longitudinal polarized \( J/\psi \) meson. The exact results, taking into account \( \rho \) meson mass, are following

\[
\alpha(B_c^* \to J/\psi\rho) \approx 0.40\quad (18)
\]
\[
\alpha(B_c \to J/\psi\rho) \approx -0.85\quad (19)
\]

In the limit of \( m_\rho = 0 \), one gets very simple expressions

\[
\alpha(B_c^* \to J/\psi\rho) \approx \alpha(B_c^* \to J/\psi\pi) \approx \frac{7 - 4x - 2x^2 + 4x^3 - x^4}{9 + 4x + 2x^2 - 4x^3 + x^4} = 0.45
\]

and

\[
\alpha(B_c \to J/\psi\rho) \approx \alpha(B_c \to J/\psi\pi) = -1.
\]
The probability of the spin conservation during $B_c^*$ meson decay into $J/\psi$ meson is described by the parameter
\[
\xi = \frac{\Gamma_{T\to T}}{\Gamma_{T\to T} + \Gamma_{T\to L}},
\] (22)
where $\Gamma_{T\to T}$ is the decay width of the transverse polarized $B_c^*$ meson into transverse polarized $J/\psi$ meson and $\Gamma_{T\to L}$ is the decay width of the transverse polarized $B_c^*$ meson into longitudinal polarized $J/\psi$ meson.

It is obviously that in the limit of vanishing $\rho$ meson mass, one has
\[
\xi(B_c^* \to J/\psi\rho) \approx \xi(B_c^* \to J/\psi\pi) = 1.
\] (23)
Taking into account $\rho$ meson mass, one finds
\[
\xi(B_c^* \to J/\psi\rho) = 0.97.
\]

Using following set of parameters \cite{9}
\[f_{B_c} = f_{B_c^*} = 0.44 \text{ GeV}, \quad f_{J/\psi} = 0.54 \text{ GeV}, \quad f_{\pi} = 0.14 \text{ GeV}, \quad f_{\rho} = 0.22 \text{ GeV}, \quad V_{bc} = 0.04, \quad G_F = 1.166 \cdot 10^{-5} \text{ GeV}^{-2} \text{ and } \alpha_s \approx 0.33,\]
we have obtained (in units $10^{-6} \text{ eV}$)
\[
\Gamma(B_c \to J/\psi\rho) \approx 22.3a_1^2, \quad (24)
\]
\[
\Gamma(B_c^* \to J/\psi\rho) \approx 31.9a_1^2, \quad (25)
\]
\[
\Gamma(B_c \to J/\psi\pi) \approx 8.76a_1^2, \quad (26)
\]
\[
\Gamma(B_c^* \to J/\psi\pi) \approx 13.9a_1^2. \quad (27)
\]

Our results (24) – (27) agree with the values obtained in \cite{10, 11} for $B_c \to J/\psi\rho(\pi)$ decays, if we take into consideration additional factor $a_1^2 \approx 1.21$, which comes from the hard QCD corrections to the effective four–fermion vertex. Note, that the decay width for $B_c^* \to J/\psi\rho(\pi)$ is much smaller than the gamma decay width for $B_c^*$ meson, which is about 60 eV \cite{11}. This one makes impossible experimental study the weak decays of $B_c^*$ mesons.

4. Conclusion

We have shown that the widths of vector $B_c^*$ meson decays $\Gamma(B_c^* \to J/\psi\rho)$ and $\Gamma(B_c^* \to J/\psi\pi)$, in 1.43 and 1.67 times larger then corresponding decay widths of scalar $B_c$ meson.

In decays of $B_c^*$ meson, one has transverse polarized $J/\psi$ meson ($\alpha = 0.40$ for $B_c^* \to J/\psi\rho$ and $\alpha \approx 0.45$ for $B_c^* \to J/\psi\pi$). On the contrary, in decays of $B_c$ meson, one has longitudinal polarized $J/\psi$ meson ($\alpha = -0.85$ for $B_c \to J/\psi\rho$ and $\alpha \approx -1.0$ for $B_c \to J/\psi\pi$).

The $J/\psi$ meson retains initial $B_c^*$ meson polarization in the decay $B_c^* \to J/\psi\pi$ completely and almost completely (97 %) in the decay $B_c \to J/\psi\rho$. 

6
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Figure captions

Fig. 1: The Feynman diagrams for decays $B_c^*(B_c) \rightarrow J/\psi \rho(\pi)$. 
\[ \frac{\pi^+ (\rho^+)}{x_1} k \left( \frac{\pi^+ (\rho^+)}{x_2} \right) \]

Fig. 1, a

\[ \frac{\pi^+ (\rho^+)}{p_1} \left( \frac{\pi^+ (\rho^+)}{p_2} \right) \]

Fig. 1, b