Analysis of Maxwell bioconvective nanofluids with surface suction and slip conditions in the presence of solar radiations

Naseer M Khan, Naeem Ullah, Jahan Zeb Khan, Dania Qaiser and M Riaz Khan
1 School of Mathematics and Statistics, Central South University Changsha 410083, Hunan, People’s Republic of China
2 Department of Mathematics, Quaid-I-Azam University 45320, Islamabad, 44000, Pakistan
3 Department of Physics, Quaid-I-Azam University 45320, Islamabad, 44000, Pakistan
4 LSEC and ICMSEC, Academy of Mathematics and Systems Science, Chinese Academy of Sciences; School of Mathematical Science, University of Chinese Academy of Sciences, Beijing 100190, People’s Republic of China

E-mail: mrkhan@math.qau.edu.pk

Keywords: Maxwell nanofluid, solar radiations, velocity slip, surface suction, numerical solutions

Abstract
Several researchers have studied nanofluids over the past several decades and tried to identify potential agents that are added to nanofluids (nanoparticle suspensions) with tremendous thermal conductivity. In such suspensions, the Brownian motion of nanoparticles is the only means expected to be associated with the improved thermal conductivity of nanofluids, and the sections that may add to this are the subject of main conversation and discussion. In the current evaluation, the effect of Brownian motion has been investigated by injecting nanoparticles into the base fluid, and the existing fundamental information is available at creation. Propagation results show that this mixing effect can significantly increase the thermal conductivity of nanofluids. One of the interesting features of this model is that the temperature can be increased by the energy of sunlight, which is required for some industrial processes. The stretching property of the sheet is more conducive to the temperature rise. This model contains features that have not been previously studied, which is driving demand for this model in a variety of industries, now and in future generations.

1. Introduction
Energy drives an advanced world. Given its critical importance, energy security is a major issue facing today’s reality. This is a direct result of a greater reliance on unsustainable energy sources such as oil and petroleum gas. Once used, these assets cannot be renewed: this creates a security problem for energy assets. To solve this serious problem, the world must move to economical clean energy that guarantees practical transformation. Solar energy is one such asset. It is harmless to the ecosystem and endless, supplying the Earth with radiation at an unknown time. In light of these qualities, modern culture is more inclined to capture the energy of sunlight. Solar radiation received by the Earth is equivalent to possible energy of 6,000 GWh per year. In many cases, productivity exceeds human interest. So the benefits of solar power are really reasonable. While the basic cost of creating it can be high, it is the most appropriate solution given its biological aspects, sustainability, and effectiveness. Previously, people used the emerging achievements to accumulate scattered light, sunlight energy, and heat.

Nanoparticles are the preferred source of clean energy. The size of nanoparticles ranges from 1 nm to 100 nm, that is, their size is equal to or comparable to the de Broglie wavelength. Due to their small size, nanoparticles can completely absorb the light energy they enter. Nanoparticles can transfer energy from one particle to another through Brownian motion (the constant irregular movement of nanoparticles), which causes the thermal conductivity of the base fluid to expand. It is seen that an increase in temperature and a decrease in the particle size increase the Brownian motion of nanoparticles, thereby increasing the thermal conductivity of the nanofluid. Due to the radiation characteristics and Brownian motion of nanoparticles, the use of nanofluids in solar thermal systems presents a new challenge for the study of access points.
The viscosity of the Maxwell fluid model is independent of the shear rate, so a quadratic equation is generated at the position of the Poiseuille fluid flow field. It should be noted that the UCM flow corresponds to solid-like behavior at high Deborah numbers and liquid-like behavior at low Deborah numbers. Readers may refer to a previously published article [1] for a deeper look at the impact of Deborah’s number on UCM flow. The boundary layer approximation is more appropriate for this model than for other models. The results of the Maxwell fluid model are interesting and are of vital importance in industries. Therefore, this model should be used to study the physical aspects of nanofluids. Aliakbar [2] proposed a model of a Maxwellian fluid moving along a stretched sheet under the action of solar radiation, and investigated the influence of a number of parameters on heat and mass transfer.

Today, nanofluid research is a powerful logical field that can be derived from a wide variety of applications in the fields of solar energy, mineral oil, microelectronics, and water. Nanoparticles are also used in chemotherapy to kill damaged cancer cells. The typical properties of nanoparticles, which are sometimes used in heat emitting devices, are generating interest in energy-saving proposals. Nanofluids have improved thermophysical properties such as thermal conductivity and diffusion, which are vital for many mechanical applications such as nuclear reactors, transportation, thermosyphons, biomedicine, and pulsed thermal conductivity. Nanomaterials contain innovative components that can make them useful for various types of heat and mass transfer, such as pharmaceutical processes, engine cooling, fuel cells, and domestic cooling. Nanofluids play an important role in the formation of fiberglass, paper, wire thickening and thinning, polymer production, and more. During the formation of glass fiber sheets, the solidification process of the liquid polymers is followed by collection on a winding roll as soon as it leaves the slotted die. In this process, a large amount of heat exchange is carried out between the plate and the surrounding fluid. By controlling the cooling rate, the mechanical properties of the sheet can be enhanced. Crane [3] was the first to build a model of the boundary layer of a non-Newtonian fluid flowing over a continuously stretched plate and analyzed the heat and material transport in it. Then Gupta [4] and Dutta [5] followed Crane’s research to study other physical aspects in various situations.

The previous research concluded that the sheet’s stretching velocity (laminar) has the capability to move fluid over a stretching sheet. The study of non-Newtonian fluids was carried out by many researchers in order to study the displacement of mass and heat, taking into account the stretching velocity of the sheet. But as a matter of fact, the flow occurs due to an exponentially stretching sheet having many practical applications in biomedical and engineering processes which is not studied much yet. The flow of the boundary layer past over a stretching sheet was studied by Ali [6]. Partha [7] contributed to the above research by finding a similarity solution for mixed convective boundary layer flows through a continuous exponential sheet. Elbashbeshy [8] studied the effect of surface suction on the heat and mass transfer of nanofluids through continuous stretched sheets. Magyari [9] studied the effect of surface suction on heat and mass transfer in a boundary layer flowing through a continuous exponentially stretched sheet and discussed several aspects that could help improve heat and mass transfer.

The idea of bioconvection can be deciphered by the example of obtaining suspensions of microorganisms, for example, bacteria and algae. Bioconvection is provided by the development of microorganisms. Bioconvection is an interaction that occurs when microorganisms, which are on average 5%–10% thicker than water, float upward. The contribution of these self-propelled mobile microorganisms creates the necessary density of the liquid. Bioconvection is used in a wide range of applications such as pharmaceuticals, biopolymer manufacturing, microsystems, and organic applications, continuous numeric display, practical power plants, green applications, biotechnology and biosensors, and advanced microbial oil extraction. The study of nanofluids flowing in the bustle of an inclined stagnation point along a stretching oscillatory plate was carried out by M Khan [10] and the impact of remarkable parameters on the transfer of mass and heat was investigated. Ayodeji [11] investigated the influence of Brownian motion, viscous dissipation, velocity slip, and thermophoresis on the heat and mass transport of MHD bioconvective nanofluids that flow over an oscillating stretching sheet. M Khan [12] proposed a model of immersion of microorganisms in MHD nanofluids flowing along the rotating upper surface of a paraboloid and presented a numerical solution after studying the effect of important parameters on physical quantities. Firdous [13] developed a bioconvective nanofluid model that takes into account the effects of the Lorentz force and exponential stretching rate and demonstrated the influence of remarkable parameters on this model. Mamatha [14] carried out a study on a suspension of microorganisms with a two-phase liquid and examined the effect of microorganisms and some notable parameters on the mass and heat transfer. Computational solutions for the flow of nanofluids through a porous medium with the introduction of the Lorentz force were found by Sheikholeslami [15] for the analysis of energy and entropy generation. Shafiq [16] analyzed the effects of chemical reactions, bioconvection and buoyancy forces on the boundary layer flow of a second-order nanofluid. This article focuses on the effect of bioconvection and some other important parameters on the mass and heat flowing in the boundary layer.

In some situations, it can be seen that the liquid does not stick to the solid boundary as it passes through it, which is also known as velocity slip. The solid boundary provides resistance to fluid flow. In the case of a viscous
liquid, the resistance is too large to keep the liquid there, while in the case of an inviscid liquid, the resistance is relatively low, and partial slip occurs between the solid interface and the liquid. Joseph [17] introduced the boundary slip condition for the first time. Later, boundary slip conditions attracted the attention of many researchers. For example, Andersson [18] analyzed various aspects of velocity slip in his research. Uddin [19] studied the effect of partial slip on the flow of MHD Newtonian nanofluids through the convective boundary layer of a translating stretching sheet. Considering the variable heat source, Srinivas [20] introduced the characteristics of solar radiation and chemical reactions into the flow of MHD fluid moving on the stretch sheet. Srinivasa’s research makes a significant contribution to blood flow. Wang [21] found exact solutions for the flow in the boundary layer of a non-Newtonian fluid by studying the effect of partial slip on this flow and concluded that partial slip between a solid boundary and a fluid can be observed for some special type of fluid. For example, foams, suspensions, polymer solutions, and emulsions all accept partial slip conditions. The influence of various parameters on the flow in the boundary layer of an MHD nanofluid, together with the slip conditions, was investigated by Ibrahim [22].

Today, research on bioconvective nanofluids is an exciting area for many researchers due to their wide range of applications. Mandal [23] and many other researchers have sought to identify substances added to nanofluids with significantly enhanced thermal conductivity. Inspired by the previous research, we initiated a study aimed at studying the Upper Convected Maxwell Fluid Model associated with the random movement of nanoparticles and motile microorganisms, which is a continuation of the aforementioned article. This model is associated with parameters and boundary conditions that have never been investigated and are subject to new research. The bvpxc code was implemented in MATLAB and found numerical solutions for the flow model of the bioconvective nanofluids. The description of the solution was carried out using tables and graphs.

2. Mathematical modeling

A steady two-dimensional Upper Convected Maxwell (UCM) fluid is believed to move along an expanding sheet that carries tiny particles and motile gyrotactic microorganisms are studied (see figure 1). The contemporary flow model combines the condition of velocity slip and surface suction. The sheet stretching speed is specified as $u_w(x) = U_0 \exp \left( \frac{x}{L} \right)$, pointing to the positive x coordinate, where $l$ and $U_0$ are the characteristic length and velocity of the free flow, respectively. The heat transfer rate $h_f(x)$ and the temperature of the heated surface $T_f$
characterize the convection process and are important factors to compensate for the stretching speed of the sheet. \( B = B_0 \exp \left( \frac{2}{T} \right) \) characterizes the magnetic field of variable strength, acting perpendicular to the fluid flow. We do not deal with the effects of both electric (regardless of whether they are provoked by the external magnetic field or charge polarization) and induced magnetic fields since they can be neglected [24]. Here we assume an optically thick medium in order to apply the Oberbeck-Boussinesq approximation. Taking into account the assumptions made above and using the Oberbeck-Boussinesq approximation, the governing equations, namely the momentum equation, the energy equation, and the equations for the concentration of nanoparticles and microbes can be written as follows:

\[
\begin{align*}
    u_x + v_y &= 0, \\
    uu_x + vv_y &= \nu u_{yy} - \chi [u^2 u_{xx} + v^2 u_{yy} + 2\nu u_{xy}] - \left( \frac{\sigma B_0^2}{\rho} \right) u,
\end{align*}
\]

(1)

\[
\begin{align*}
    uT_x + vT_y &= \alpha T_{yy} + \tau \left[ D_B T_y C_y + \frac{D_T T_{yy}}{T_\infty} \right] - \frac{1}{(\rho c)_f} (q_f)_y,
\end{align*}
\]

(2)

\[
\begin{align*}
    uC_x + vC_y &= D_B C_{yy} + \frac{D_T}{T_\infty} T_{yy},
\end{align*}
\]

(3)

\[
\begin{align*}
    uN_x + vN_y &= D_m N_{yy} - \frac{bW_c}{(C_m - C_{\infty})} [\partial_y (N(C_f))].
\end{align*}
\]

(4)

The above system of equations (6)–(10) is assigned to the boundary conditions as follows.

\[
\begin{align*}
    u &= U(x) + Ky u_y, v = V(x), -kT_y = h_f (T_f - T), D_B \frac{\partial C}{\partial y} + \frac{D_T T}{T_\infty} \frac{\partial T}{\partial y} &= 0, N = N_m, \text{ at } y = 0 \\
    u &\rightarrow 0, C &\rightarrow C_\infty, T &\rightarrow T_\infty = (1 - n) T_0 + n T_w, N &\rightarrow N_\infty \text{ as } y \rightarrow \infty.
\end{align*}
\]

(6)

Where \( x \) and \( y \) are Cartesian coordinates, \( u \) and \( v \) are the velocity components in the \( x \) and \( y \) directions, respectively. \( \nu = \mu / \rho, \chi, \sigma, B_0, \) and \( \rho \) refer to kinematic viscosity, relaxation time, thermal conductivity, magnetic field parameter, and fluid density, respectively. \( \alpha = \frac{1}{\nu c_n} \) and \( T \) respectively represent the thermal diffusivity and temperature of the fluid at any point. \( T_f, T_w, \) and \( T_\infty \) are local temperature, heated surface temperature and free flow temperature respectively. \( K = K_1 \exp \left( \frac{2}{T} \right) \) characterizes the first-order velocity slip, \( k \) describes the thermal conductivity of the fluid and \( h_f = h \exp \left( \frac{2}{T} \right) \) describes the heat transfer coefficient. \( C \) and \( C_\infty \) refer to the local and ambient concentration of nanoparticles (i.e. away from the sheet), and \( N \) is the concentration of microorganisms. \( D_B \) and \( D_T \) are the parameters of Brownian motion and thermophoresis, respectively.

\[
\tau = \frac{\text{concentration of nanoparticles}}{\text{concentration of the base fluid}} = \frac{(\rho C)_f}{(\rho C)_B}.
\]

The radiant heat \( q_r \) can be expressed with the help of famous approximations of Rosseland as:

\[
q_r = -4 \frac{\sigma_1}{3k} \frac{\partial T^4}{\partial y},
\]

(7)

where the parameters \( k^* \) and \( \sigma_1 \) refer to absorption and Stefan Boltzmann constants. The energy equation takes the form after substituting the radiant heat flux value:

\[
\begin{align*}
    uT_x + vT_y &= \alpha T_{yy} + \tau \left[ D_B T_y C_y + \frac{D_T}{T_\infty} T_{yy} \right] + \frac{16 \sigma_1 T_\infty^3}{3(\rho c)_f k^*} T_{yy}.
\end{align*}
\]

(8)

We introduce the similarity variables as:

\[
\begin{align*}
    \eta &= \exp \left( \frac{x}{2L} \right) \sqrt{\frac{U_0}{2
u L}} y, \quad u = \exp \left( \frac{x}{L} \right) U_0 f' (\eta), \quad v = \exp \left( \frac{x}{2L} \right) \sqrt{\frac{U_0 \nu}{2L}}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},
\end{align*}
\]

(9)

The equations (1)–(5) and their associated parameters are converted to dimensionless form using the above similarity variables.
The associated boundary conditions are

\[
(1 - \frac{\lambda}{2}f)_{\eta} + \eta f_{\eta} - 2f_{\eta} + \lambda \left[ 3f_{\eta} f_{\eta} + \frac{\eta}{2} \left( f_{\eta} \right)^2 - 2(f_{\eta})^3 \right] = M_{\eta} = 0, \quad (10)
\]

\[
\left( 1 + R_{D} \right) \theta_{\eta} + Prf_{\eta} + PrN_{\theta}_{\eta} \phi_{\eta} + PrN_{\theta_{\eta}} = 0, \quad (11)
\]

\[
\phi_{\eta} + \frac{\sigma f_{\eta}}{N_{\phi}} + \frac{N_{\phi}}{N_{b}} \theta_{\eta} = 0,
\]

\[
(1 - Pr) \chi_{\eta} + Lp \chi_{\eta} - Pb(\chi + \Omega) \phi_{\eta} = 0. \quad (13)
\]

The associated boundary conditions are

\[
f(0) = S, \quad f_{\eta}(0) = 1 + \gamma f_{\eta}(0), \quad \theta_{\eta}(0) = Bi(\theta(0) - 1), \quad N_{\theta}{\theta}_{\eta}(0) + N_{\phi}{\phi}_{\eta}(0) = 0,
\]

\[
\chi(0) = 1, \quad f_{\eta}(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0, \quad \phi(\eta) \rightarrow 0, \quad \chi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty. \quad (14)
\]

Let’s describe the related parameters with the flow model. The subscript \( \eta \) represents differentiation with respect to \( \eta \) and can be regarded as dimensionless \( y \)-coordinate. \( M = \left( \frac{2 \pi R f_{I}}{U_{I} \eta} \right) \) describes magnetic behavior,

\[ Rd = \frac{16 \alpha T_{o}^2}{Sc^{2}} \] characterizes thermal stratification, \( N_{b} = \frac{D_{n}(\gamma \phi)}{v \eta} \) refers to Brownian motion, and \( N_{p} = \frac{D_{p}(\gamma \phi)}{v \eta} \) characterizes thermophoresis motion. \( Pb = \frac{bW_{p}}{D_{mn}} \) denotes Peclet number, \( Pr = \frac{\nu}{\alpha} \) assigns Prandtl number, \( Lp = \frac{\nu}{D_{n}} \) represents the Lewis number of biological convection and \( \Omega = N_{w_{w}} / (N_{w_{w}} - N_{w_{w}}) \) represents the biological convection constant. \( Sc = \frac{\nu}{D_{n}} \) characterizes Schmidt number and \( Bi = \left( \frac{h_{w} \alpha}{U_{w}^{1/2}} \right) \) denotes Biot number. \( S \) characterizes suction or injection on the surface with \( S > 0 \) describing suction, while \( S < 0 \) injection.

Heat is transferred from one place to another in three different ways: conduction, convection, and radiation. The terms conductivity (diffusion) and advection (fluid movement) are collectively referred to as convection. The ratio of heat conduction to convection at a particular boundary is named the Nusselt number after William Nusselt, who made a significant contribution to heat transfer, especially convective heat transfer. Assuming the fluid is at rest, heat transfer can be measured under the same conditions as convective heat transfer. The Nusselt number measures net thermal conductivity. Laminar flow is characterized by a Nusselt number from 1 to 10, and a turbulent flow is characterized by a Nusselt number from 100 to 1000. Both conduction and convection heat transfer is perpendicular to the boundary surface and parallel to each other.

The Nusselt number in terms of heat transfer coefficient \( h_{f} \) and a characteristic length \( L \) is given by the formula:

\[ \text{Nusselt number} = \frac{\text{Heat transfer through Convection}}{\text{Heat transfer Conduction}} = \frac{h_{f}}{\frac{\nu}{L}} = \frac{h_{f} L}{k} \]

In view of its important physical significance, the research on Nusselt number, skin friction coefficient and Sherwood number cannot be ignored. They are given by the formulae:

\[
\begin{aligned}
\tau_{w} &= \frac{2 \pi r_{w}}{U^{2} \rho \exp \left( \frac{1}{L^{3}} \right)}, \\
N_{u_{w}} &= \frac{x_{u_{w}}}{k(T_{f} - T_{w})},
\end{aligned}
\]

where

\[
\begin{aligned}
\tau_{w} &= \mu \left[ \frac{\partial u}{\partial y} \right]_{y=0}, \\
d_{w} &= -k \left[ \frac{\partial T}{\partial y} \right]_{y=0}.
\end{aligned}
\]

The equation (16) have been replaced in equation (15) to obtain the dimensionless forms of the above physical quantities.
In the current flow model, we see that the dimensionless mass flux, the Sherwood number, is zero.

3. Numerical techniques

Through the first replacement of the similarity variables, the prevailing partial differential equations and boundary conditions are transformed into nonlinear ordinary differential equations. The resulting equations (10)–(13) and the related boundary conditions (14) are converted into first-order ordinary differential equations using appropriate variables. It is then put into a MATLAB scheme called the boundary value problem solver (abbreviated as bvp4c) to solve it. The collocation method is used to find solutions to the problems listed below:

\[ C_f \left[ \frac{1}{2} Re_x \right]^{0.5} = \left[ f''(\eta) \right]_{\eta=0} \]
\[ N_{ix} \left[ \frac{2L}{xRe_x} \right]^{0.5} = -\left[ \theta'(\eta) \right]_{\eta=0} \]  

In the current flow model, we see that the dimensionless mass flux, the Sherwood number, is zero.

Figure 2. Velocity curves for different values of physical parameters.

(a) change in velocity at different values of \( \gamma \)

(b) change in velocity at different values of \( S \)

(c) change in velocity at different values of \( M \)

The corresponding boundary conditions can be written as,

\[ bc \left( y(g), y(h) \right) = 0. \]

\( \xi(p) \) is an approximate solution to the above system which is a polynomial of degree three on each sub-interval \([p_j, p_{j+1}]\) of the mesh \( g = p_0 < p_1, \cdots, p_j = h \) satisfies the boundary conditions.
Table 1. Change in the value of $\theta' (0)$ for different Prandtl numbers.

| Pr   | $-\theta' (0)$ | $-\theta' (0)$ | $-\theta' (0)$ |
|------|----------------|----------------|----------------|
| 0.50 | -0.175934      | -0.301698      | -0.175921      |
| 1.00 | -0.512316      | -0.512078      | -0.512320      |
| 3.00 | -1.074528      | -1.074501      | -1.074513      |
| 5.00 | -1.470619      | -1.470600      | -1.470621      |
| 8.00 | -1.939107      | -1.939070      | -1.939103      |
| 10.0 | -2.203880      | -2.203835      | -2.203874      |

Equivalently, $\zeta(p)$ satisfies the system of ODEs (collimates) at both ends of each sub-interval and the midpoint.

$$\zeta'(p_j) = M(p_j, \zeta(q_j)), \quad \zeta'(\frac{p_j + p_{j+1}}{2}) = M\left(\frac{p_j + p_{j+1}}{2}, \zeta\left(\frac{q_j + q_{j+1}}{2}\right)\right),$$

$$\zeta'(p_{j+1}) = M(p_{j+1}, \zeta(p_{j+1})).$$

$\zeta(p)$ is an approximate solution to the above system of nonlinear algebraic equations found by the linearization method. The solution $\zeta(p)$ to $\gamma(p)$ is a fourth-order differential equation i.e. $\|\gamma(p) - \zeta(p)\| \leq k_\delta \Lambda^4$. The constant $k_\delta$ in the preceding relation represents the maximum of $\Lambda_j = p_{j+1} - p_j$. Residual of the solution $\zeta(p)$ is represented by $R(t)$ which is a differential equation defined by,

$$R(t) = \zeta'(q) - M(p, \zeta(p))$$

The error control and choice of mesh points should be made according to the residual while using the bvp4c scheme in MATLAB. The 100 points are equally distributed between 0 and 10, which leads to a step size of 0.1 and a convergence criterion of $1 \times 10^{-6}$. The condition that $\eta$ approaches infinity should be replaced by a suitable numeric. Here, we took $\eta = n_{\infty} = 10$. We take an appropriate initial guess while finding solutions to the problem which best suits equations (10)–(13) and the corresponding boundary conditions. Finding an initial guess for the first solution is easy while a great computational effort is needed to find an initial guess for the second solution; therefore, we randomly choose an initial guess and a set of parameters and reach the accurate values of the parameter through the continuation technique [25].

4. Results and discussions

In addition to the movement of microorganisms, a two-dimensional flow of Maxwells nanofluid developed under the influence of solar radiation. The Upper Convected Maxwell (UCM) model was introduced to incorporate the equations of momentum and energy, and the Rossland approximations were used to account for the effects of thermal radiation. It is assumed that the nanofluid flow over an exponentially stretching surface. The nonlinearity of the PDEs system is reduced by converting it into a coupled nonlinear ordinary differential equation and then using a numerical tool named bvp4c in MATLAB to solve it. The accuracy and convergence of the solutions found with bvp4c were checked by comparing them with already published articles [9, 26] through table 1 and finding similar ones. Because of its high accuracy, excellent convergence, and lower computational cost, we can argue that bvp4c is one of the preferred numerical tools. The physical aspects of some parameters such as $\gamma$, S, M, Pr, Nb, Ni, Rd, Pb, and Lp have been investigated and explored using tables and graphs.

Figure 2(a) manifests the impact of the sheet stretching velocity on the viscous boundary layer and the fluid velocity. The result of this study is that the stretching parameter has the ability to increase the velocity and the viscous boundary layer becomes thicker due to the augmentation in the stretching parameter. Figure 2(b) manifests the impact of the surface suction on the velocity plot. As the suction parameter increases, the speed gradually falls to zero, while the viscous boundary layer becomes thinner due to the augmentation in the suction parameter value. The graphical results are in excellent agreement with physical observations. Suction is the physical movement of a liquid from a high pressure area to a low pressure area, causing the fluid velocity to decrease. Figure 2(c) is intended to describe the impact of the magnetic field parameter on the velocity graph. A decrease in the velocity profile can be observed by amplifying the magnetic field parameter values. The intensified values of the magnetic field parameter lead to a decrease in the penetration depth of the impulse boundary layer. The implanted magnetic field creates a Lorentz force in the fluid, which opposes the fluid flow. The stronger the magnetic field, the more resistance the particles of the liquid experience; therefore, a large Lorentz force leads to a drop in velocity and a thinner boundary layer.
The figure 3(a) is created to investigate the influence of the stretching parameter $\gamma$ on the temperature curve. The estimate shows that the temperature curve increases with escalating values of the stretching parameter. The thermal boundary layer thickens with increasing stretching parameter. By increasing the suction parameter, the increase in Nusselt number and the decrease in temperature distribution are in line with expectations, as shown in figure 3(b). The penetration depth of the thermal boundary layer increases significantly with the increase of $S$. Suction can be mathematically described as the movement of fluid particles from a high pressure region to a low pressure region. In such an application, it may be necessary to select the area of the lowest pressure, which in this case is chosen as $\eta = 0$. A
larger suction value means that more fluid enters the high-pressure zone from the low-pressure zone. It can be seen from the adjacent figure that the volume of the fluid decreases as it passes through the equilibrium zone, thereby reducing the flow rate, where it usually reaches a maximum. In order to observe the influence of Brownian motion Nb parameters on the temperature distribution, a graph is drawn in figure 3(c). It can be seen that as the Brownian motion parameter increases, the temperature distribution decreases. The thickness of the thermal boundary layer decreases with the increase of the Brownian motion parameter. The number of particle collisions increases with increasing temperature values, thereby increasing the translational, rotational, and vibrational kinetic energy of nanoparticles, which, in turn, increases the temperature of the base fluid. On the

Figure 4. Concentration profiles for different values of parameters.
other hand, if we increase the temperature of the fluid containing nanoparticles, then the energy supplied from the outside will be spent on increasing the random motion of the nanoparticles, and, as a consequence, lower temperature profiles can be seen for increased values of the Brownian motion parameter. Increasing the values of thermophoresis parameters can significantly increase the temperature and thicken the thermal boundary layer, as can be seen from figure 3(d). Figure 3(e) shows the effect of radiant heat on temperature distribution. As expected, due to an increase in the radiation parameter Rd, the temperature distribution and the thickness of the thermal boundary layer increase. By increased Rd, we mean that more intense sunlight falls on the nanofluid. Under current conditions, it is believed that nanoparticles have the ability to significantly absorb solar energy. The base fluid is considered to be a good absorber of solar energy. After the energy is completely absorbed by the
particles, it is transferred to the base fluid through a convection process. When the base is gently heated, this energy is transferred to the environment.

Figure 4(a) shows the influence of stretching parameters $\gamma$ on the concentration distribution of nanoparticles, indicating that the increase in stretching parameter value does not contribute to the increase in the distribution value, and the concentration of the boundary layer increases with the increase in stretching parameter value. Figure 4(b) shows that the concentration distribution of nanoparticles is significantly affected by the surface suction. An increase in the suction parameter contributes to an increase in the concentration distribution. The concentration boundary layer becomes thinner with an increase in $S$. Figure 4(c) shows that an increase in the value of the Brownian motion parameter $N_b$ will lead to a decrease in the volume fraction of nanoparticles. As the Brownian motion of nanoparticles becomes stronger, the frequency of collisions between nanoparticles increases, and collisions between nanoparticles become more likely. However, stronger collisions can cause cluster fragmentation. Whether a stronger Brownian motion will lead to an increase in the degree of aggregation depends on the interaction between the forces of attraction and repulsion between the nanoparticles. Stronger Brownian motion leads to an increase in the amount of aggregation only if the zeta potential of the system is low. If the zeta potential of the system is high, then the probability of aggregation is very low. In the current situation, it is believed that lower values of the zeta potential and stronger values of the Brownian motion lead to aggregation of nanoparticles, and, as a consequence, the volume fraction of nanoparticles decreases.

The volume fraction of nanoparticles increases and the thickness of the concentration boundary layer decreases due to an increase in the thermophoresis parameter $N_t$, as seen in figure 5(d). Finally, the effect of radiation parameters $R_d$ on the concentration profile of nanoparticles is shown in figure 5(e), with the prevailing fact that the concentration of nanoparticles gradually increases with increasing radiant heat flux $R_d$, as expected. An increase in the values of the radiation parameter makes a significant contribution to the kinetic energy of molecules, which makes the particles move faster, as a consequence, the probability of collisions between particles is higher. Figure 5(a) discusses the effect of stretching parameters $\gamma$ on the concentration of microorganisms in the presence of solar radiation, which indicates that the stretching parameters have little effect on the concentration distribution and thickness of the thermal boundary layer. With large changes in the values of the stretching parameter, small increments in the concentration distribution are observed. The influence of suction parameters $S$ on the concentration distribution of microorganisms can be seen in figure 5(b). This graph is slightly different from other constructed graphs. As the suction parameter increases, a small increase in the concentration distribution of nanoparticles can be observed at a lower $\eta$ value, while a major decrease can be observed at a larger $\eta$ value. As the surface suction $S$ increases, the thickness of the concentration boundary layer decreases. The influence of Brownian motion $N_b$ parameters on the distribution of microbial concentration is shown in figure 5(c), indicating that strong Brownian motion leads to a significant increase in microbial concentration $\chi(\eta)$, and the thickness of the concentration boundary layer decreases with the increase of the $N_b$ value. When the Brownian motion becomes stronger, the nanoparticles move faster, and there are more opportunities for the nanoparticles to collide with the microorganisms, leading to the fragmentation of the microbial clusters and the increase of the volume fraction of the microorganisms.

Figure 5(d) illustrates the effect of thermophoresis parameters $N_t$ on the volume fraction of microorganisms, showing that the volume fraction of microorganisms significantly decreases with an increase in thermophoresis parameters. An increase in the value of the thermophoresis parameter $N_t$ leads to a thinning of the concentration boundary layer. Figure 5(e) discusses the influence of radiant heat flux $R_d$ on the volume fraction of microorganisms. By increasing the value of $R_d$, we mean that more intense sunlight turns to fall on the solution, which leads to an increase in the kinetic energy of nanomaterials, thereby accelerating the movement of microorganisms, so that more collisions between nanoparticles and microorganisms may occur. As a result, the volume fraction of microorganisms increases as the value of $R_d$ increases. Finally, the influence of the Lewis number of biological convection $L_p$ on the volume fraction of nanoparticles is shown in figure 5(f).
can be clearly seen from the figure that the biological convection Lewis number has no inducing effect on the increase of the concentration distribution. By increasing the biological convection Lewis number $L_p$, the concentration boundary becomes thinner.

The influence of various parameters on the surface friction coefficient and the heat transfer rate (Nusselt number) was investigated using tables 2 and 3. It was observed that an increase in the velocity slip value leads to an improvement in the surface friction coefficient, while an increase in the values of the local Deborah number and the magnetic force causes a decrease in the surface coefficient. In the case of a simple Maxwell fluid, an increase in the magnetic force parameter, thermophoresis parameter, and the local Deborah number leads to a decrease in the Nusselt number, while an increase in the slip condition and Prandtl number leads to an improvement in the heat transfer rate.

5. Conclusions

The velocity slip and surface suction are coupled with biological convection, the Maxwell nanofluid model under the action of solar radiation has been established. The basic equations of momentum, temperature, and concentration were solved numerically using the MATLAB bvp4c scheme. The physical aspects of several parameters were investigated in the boundary layer flow of the Maxwell nanofluid model, analyzed using graphs and tables, and discussed quantitatively. It is concluded that the thermal conductivity of the base fluid can be increased due to micro convection arising from the Brownian motion of nanoparticles. This information is very convincing in combination with various studies of the viscosity of nanofluids due to the hydrodynamic connection between nanoparticles of Brownian motion [25, 28–33]. The results, undeniably show the effect of the microconvection achieved by the Brownian motion of nanoparticles in these sorts of suspensions and display that the Brownian motion is one of the basic components behind the saw high convincing thermal conductivity of nanofluids. The industry can improve the quality of the final product by controlling the cooling rate, which can be controlled by surface suction. One of the interesting features of this model is that the temperature can be increased by the energy of sunlight, which is required for some industrial processes. The stretching property of the sheet is more conducive to the temperature rise. The sunlight energy, thermophoresis movement and surface suction undoubtedly contribute to an increase in the volume fraction of nanoparticles, while Brownian motion and stretching of the sheet decrease the volume fraction of nanoparticles. Stronger Brownian motion causes fragmentation of the microbial cluster, increasing the volume fraction of nanoparticles, while the bioconvection Lewis number prevents an increase in the volume fraction of microorganisms.

Data availability statement

The data generated and/or analyzed during the current study are not publicly available for legal/ethical reasons but are available from the corresponding author on reasonable request.

ORCID iDs

Naseer M Khan https://orcid.org/0000-0002-9507-1826
Naeem Ullah https://orcid.org/0000-0002-2238-2279

| Table 3. Change in the coefficient of surface friction $C_f$ and Nusselt number $Nu_x$ at different values of physical parameters. |
|---|---|---|---|---|---|---|
| Rd | $N_t$ | $Pr$ | $\lambda$ | $Nb$ | $f'(0)$ | $-\theta'(0)$ |
| 0.50 | 0.20 | 10.00 | 1.00 | 0.10 | −0.836116 | 0.350514 |
| 0.50 | 0.20 | 11.00 | 1.00 | 0.10 | −0.836116 | 0.354658 |
| 0.50 | 0.20 | 10.00 | 1.00 | 0.50 | −0.836116 | 0.350514 |
| 0.50 | 0.50 | 10.00 | 1.00 | 0.10 | −0.836116 | 0.290279 |
| 0.50 | 0.20 | 10.00 | 0.00 | 0.10 | −0.825793 | 0.354942 |
| 0.50 | 0.20 | 11.00 | 0.00 | 0.10 | −0.825793 | 0.358789 |
| 0.50 | 0.20 | 10.00 | 0.00 | 0.50 | −0.825793 | 0.354929 |
| 0.50 | 0.50 | 10.00 | 0.00 | 0.10 | −0.825793 | 0.297720 |
References

[1] Omowaye A J and Animasaun I L 2016 Upper-convected maxwell fluid flow with variable thermo-physical properties over a melting surface situated in hot environment subject to thermal stratification Journal of Applied Fluid Mechanics 9 1777–90

[2] Aliaakbar V, Alizadeh-Pahlavan A and Sadeghy K 2009 The influence of thermal radiation on mhd flow of Maxwellian fluids above stretching sheets Commun. Nonlinear Sci. Numer. Simul. 14 779–94

[3] Crane L J 1970 Flow past a stretching plate Zeitschrift für Angewandte Mathematik und Physik ZAMP 21 645–7

[4] Gupta P S and Gupta A S 1977 Heat and mass transfer on a stretching sheet with suction or blowing The Canadian Journal of Chemical Engineering 55 744–6

[5] Dutta B K, Roy P and Gupta A S 1985 Temperature field in flow over a stretching sheet with uniform heat flux Int. Commun. Heat Mass Transfer 12 89–94

[6] Ali M E 1995 On thermal boundary layer on a power-law stretched surface with suction or injection Int. J. Heat Fluid Flow 16 280–90

[7] Partha M K, Murthy P V S N and Rajasekhar G P 2005 Effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface Heat Mass Transfer 41 260–6

[8] Elbashbesy E M A 2001 Heat transfer over an exponentially stretching continuous surface with suction Arch. Mech. 53 643–51

[9] Magyari E and Keller B 1999 Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface J. Phys. D: Appl. Phys. 32 577

[10] Choi S U and Eastman J A 1995 Enhancing thermal conductivity of fluids with nanoparticles Technical Report, Argonne National Lab., IL (United States)

[11] Dameshe R A 2006 Thermal boundary layer on an exponentially stretching continuous surface in the presence of magnetic field effect Int. J. of Applied Mechanics and Engineering 11 289–99

[12] Bhattacharyya K and Pop I 2011 Mhd boundary layer flow due to an exponentially shrinking sheet Magneto-hydrodynamics 47 337–44

[13] Nadeem S and Lee C 2012 Boundary layer flow of nanofluid over an exponentially stretching surface Nanoscale Res. Lett. 7 1–6

[14] Nadeem S, Haq R U and Khan Z H 2014 Numerical study of mhd boundary layer flow of a Maxwell fluid past a stretching sheet in the presence of nanoparticles J. Taiwan Inst. Chem. Eng. 45 121–6

[15] Makinde O D, Khan W A and Khan Z H 2013 Buoyancy effects on mhd stagnation point flow and heat transfer of a nanofluid past a convectively heated stretching/shrinking sheet Int. J. Heat Mass Transfer 62 526–33

[16] Akbar N S, Nadeem S, Haq R U and Khan Z H 2013 Numerical solutions of magnetohydrodynamic boundary layer flow of tangent hyperbolic fluid towards a stretching wall Int. J. Phys. 87 1121–4

[17] Beavers G S and Joseph D D 1967 Boundary conditions at a naturally permeable wall J. Fluid Mech. 30 197–207

[18] Anderson H I 2002 Slip flow past a stretching surface Acta Mech. 158 121–5

[19] Uddin M, Anwar Beg O and Amin N 2014 Hydrodynamic transport phenomena from a stretching or shrinking nonlinear nanomaterial sheet with navier slip and convective heating: a model for bio-nano-materials processing J. Magn. Magn. Matter. 368 252–61

[20] Srinivas S, Reddy P B A and Prasad B S R V 2014 Effects of chemical reaction and thermal radiation on mhd flow over an inclined permeable stretching surface with non-uniform heat source/sink: an application to the dynamics of blood flow Journal of Mechanics in Medicine and Biology 14 1450067

[21] Wang C Y 2002 Flow due to a stretching boundary with partial slip: an exact solution of the navier-stokes equations Chem. Eng. Sci. 57 3745–7

[22] Ibrahim W and Shankar B 2013 Mhd boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions Comput. Fluids 75 1–10

[23] Mandal D K, Biswas N, Manna N K, Gorla R S R and Chamkha A J 2021 Role of surface undulation during mixed bioconvective nanofluid flow in porous media in presence of oxytactic bacteria and magnetic fields Int. J. Mech. Sci. 211 106778

[24] Sheikholeslami M and Rokni H B 2017 Nanofluid two phase model analysis in existence of induced magnetic field Int. J. Heat Mass Transfer 107 286–99

[25] Vondermassen K, Bongers J, Mueller A and Versmold H 1994 Brownian motion: a tool to determine the pair potential between colloid particles Langmuir 10 1351–3

[26] Reddy P B, Sumerak S and Reddy N B 2017 Numerical study of magnetohydrodynamics (mhd) boundary layer slip flow of a Maxwell nanofluid over an exponentially stretching surface with convective boundary condition Propulsion and Power Research 6 259–68

[27] Mustafa M, Khan J A, Hayat T and Alsaedi A 2015 Simulations for Maxwell fluid flow past a convectively heated exponentially stretching sheet with nanoparticles AIP Adv. 5 037133

[28] GK406082 1976 Batchelor, Brownian diffusion of particles with hydrodynamic interaction J. Fluid Mech. 74 1–29

[29] Brady J F, Phillips R J, Lester J C and Bossis G 1988 Dynamic simulation of hydrodynamically interacting suspensions J. Fluid Mech. 195 257–80

[30] Brady J F 1993 The rheological behavior of concentrated colloidal dispersions J. Chem. Phys. 99 567–81

[31] Brady J F and Morris J F 1997 Microstructure of strongly sheared suspensions and its impact on rheology and diffusion J. Fluid Mech. 348 103–39

[32] Banchio A J, Gapinski J, Patkowski A, Häußler W, Fluerasu A, Sacanna S, Holmquist P, Meier G, Lettinga M P and Nägele G 2006 Many-body hydrodynamic interactions in charge-stabilized suspensions Phys. Rev. Lett. 96 138303

[33] Petukhov B S et al 1970 Advances in heat Transfer vol 6 (New York: Academic) 503–64