Semi-supervised Ranking Pursuit

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Abstract. We propose a novel sparse preference learning/ranking algorithm. Our algorithm approximates the true utility function by a weighted sum of basis functions using the squared loss on pairs of data points, and is a generalization of the kernel matching pursuit method. It can operate both in a supervised and a semi-supervised setting and allows efficient search for multiple, near-optimal solutions. Furthermore, we describe the extension of the algorithm suitable for combined ranking and regression tasks. In our experiments we demonstrate that the proposed algorithm outperforms several state-of-the-art learning methods when taking into account unlabeled data and performs comparably in a supervised learning scenario, while providing sparser solutions.

1 Introduction

Recently, preference learning\textsuperscript{3} has received significant attention in machine learning community. Informally, the main goal of this task is prediction of ordering of the data points rather than prediction of a numerical score as in the case of regression or a class label as in the case of a classification task. The ranking problem can be considered as a special case of preference learning when a strict order is defined over all data points. The applications of algorithms that learn to rank\textsuperscript{3} are widespread, including information retrieval (collaborative filtering, web search e.g. [2]), natural language processing (parse ranking e.g. [3]), bioinformatics (protein ranking e.g. [4]), and many others.

Despite notable progress in the development and application of preference learning/ranking algorithms (e.g. [1]), so far the emphasis was mainly on improving the learning performance of the methods. Much less is known about models that focus in addition on interpretability and sparseness of the ranking solution. In this work we propose a novel preference learning/ranking algorithm that besides state-of-the-art performance can also lead to sparse models and notably faster prediction times (that is an absolute necessity for a wide range of applications such as e.g. search engines), compared to the non-sparse counterparts.

\textsuperscript{3} See e.g. \url{http://research.microsoft.com/en-us/um/beijing/projects/letor/paper.aspx}
Sparse modeling is a rapidly developing area of machine learning motivated by the statistical problem of variable selection in high-dimensional datasets. The aim is to obtain a highly predictive (small) set of variables that can help to enhance our understanding of underlying phenomena. This objective constitutes a crucial difference between sparse modeling and other machine learning approaches. Recent developments in theory and algorithms for sparse modeling mainly concern $L_1$-regularization and convex relaxation of the subset selection problem. Examples of such algorithms include sparse regression (e.g. Lasso [5]) and its various extensions (Elastic Net [6], group Lasso [7,8], simultaneous/multi-task Lasso [9]) as well as sparse dimensionality reduction (sparse PCA [9], NMF [10]) algorithms. Applications of these methods are wide-ranging, including computational biology, neuroscience, image processing, information retrieval, and social network analysis, to name a few. The sparse ranking algorithm we propose here is not tied to a particular domain and can be applied to various problems where it is necessary to estimate preference relations/ranking of the objects as well as to obtain a compact and representative model.

RankSVM [11] is a ranking method that can lead to sparse solutions. However, in RankSVM sparsity control is not explicit and the produced models are usually far from being interpretable. Also note that frequently ranking algorithms are not directly applicable to more general preference learning tasks or can become computationally expensive. Our method is a generalization of the (kernel) matching pursuit algorithm [12] and it approximates the true utility function by a weighted sum of basis functions using squared loss on pairs of data points. Unlike existing methods our algorithm allows explicit control over sparsity of the model and can be applied to ranking and preference learning problems.

Furthermore, an extension of the algorithm allows us to efficiently search for several near-optimal solutions instead of a single one. For example, some of the problems that arise during the sparse modeling include possible existence of multiple nearly-optimal solutions (e.g. due to the lack of a single sparse ground truth). This situation is common for many biological problems when, for example, finding a few highly predictive proteins does not exclude the possibility of finding some other group of genes/proteins with similar properties. The same situation can occur in many other domains, e.g. information retrieval (various groups of highly descriptive document/queries in document ranking task), natural language processing (parse re-ranking), etc. Therefore, it is an important issue to explore and include search for multiple nearly-optimal sparse solutions rather than a single solution. In our empirical evaluation we show that our algorithm can operate in supervised and semi-supervised settings, leads to sparse solutions, and improved performance compared to several baseline methods.

2 Problem Setting

Let $\mathcal{X}$ be a set of instances and $\mathcal{Y}$ be a set of labels. We consider the label ranking task [11,13] namely, we want to predict for any instance $x \in \mathcal{X}$ a preference
relation \( \mathcal{P}_x \subseteq Y \times Y \) among the set of labels \( Y \). We assume that the true preference relation \( \mathcal{P}_x \) is transitive and asymmetric for each instance \( x \in \mathcal{X} \).

Our training set \( \{(q_i, s_i)\}_{i=1}^n \) contains the data points \((q_i, s_i) = ((x_i, y_i), s_i) \in (\mathcal{X} \times Y) \times \mathbb{R}\) that are an instance-label tuple \( q_i = (x_i, y_i) \in \mathcal{X} \times Y \) and its score \( s_i \in \mathbb{R} \). We define the pair of data points \(((x, y), s)\) and \(((x', y'), s')\) to be relevant, iff \( x = x' \) and irrelevant otherwise.

As an example, consider an information retrieval task where every query is associated with the set of retrieved documents. The intersection of the retrieved documents associated with different queries can be either empty or non-empty.

We are usually interested in ranking the documents that are associated with a single query (the one that has retrieved the documents). Thus, ranks between documents retrieved by different queries are not relevant for this task, whereas those documents retrieved by the same query are relevant.

Given a relevant pair \(((x, y), s)\) and \(((x', y'), s')\), we say that instance \( x \) prefers label \( y \) to \( y' \), iff \( s > s' \). If \( s = s' \), the labels are called tied. Accordingly, we write \( y \succ_x y' \) if \( s > s' \) and \( y \sim_x y' \) if \( s = s' \). Finally, we define our training set \( \mathcal{T} = (Q, s, W) \), where \( Q = (q_1, \ldots, q_n)^t \in (\mathcal{X} \times Y)^n \) is the vector of instance-label training tuples and \( s = (s_1, \ldots, s_n)^t \in \mathbb{R}^n \) is the corresponding vector of scores.

The \( W \) matrix defines a preference graph and incorporates information about relevance of a particular data point to the task, e.g. \([W]_{i,j} = 1\), if \((q_i, q_j)\), \(1 \leq i, j \leq n\), \( i \neq j \), are relevant and 0 otherwise.

Informally, the goal of our ranking task is to find a label ranking function such that the ranking \( \mathcal{P}_{f, x} \subseteq Y \times Y \) induced by the function for any instance \( x \in \mathcal{X} \) is a good “prediction” of the true preference relation \( \mathcal{P}_x \subseteq Y \times Y \). Formally, we search for the function \( f : \mathcal{X} \times Y \rightarrow \mathbb{R} \) mapping each instance-label tuple \((x, y)\) to a real value representing the (predicted) relevance of the label \( y \) with respect to the instance \( x \). To measure how well a hypothesis \( f \) is able to predict the preference relations \( \mathcal{P}_x \) for all instances \( x \in \mathcal{X} \), we consider the following cost function (disagreement error) that captures the amount of incorrectly predicted pairs of relevant training data points:

\[
d(f, \mathcal{T}) = \frac{1}{2} \sum_{i,j=1}^n [W]_{i,j} \left| \text{sign}(s_i - s_j) - \text{sign}(f(q_i) - f(q_j)) \right|, \tag{1}
\]

where \( \text{sign}(\cdot) \) denotes the signum function.

### 3 Ranking Pursuit

In this section we tailor the kernel matching pursuit algorithm [12] to the specific setting of preference learning and ranking. Considering the training set \( \mathcal{T} = (Q, s, W) \) and a dictionary of functions \( \mathcal{D} = \{k_1, \ldots, k_N\} \), where \( N \) is the number of functions in the dictionary. We are interested in finding a sparse approximation of the prediction function \( f_\mathcal{P}(q) = \sum_{p=1}^P a_p k_p(q) \) using the basis functions \( \{k_1, \ldots, k_P\} \subset \mathcal{D} \) and the coefficients \( \{a_1, \ldots, a_P\} \in \mathbb{R}^P \). The order of the dictionary functions as they appear in the expansion is given by a set of indices.
\( \{ \gamma_1, \ldots, \gamma_P \} \), where \( \gamma \in \{1, \ldots, N\} \). Basis functions are chosen to be kernel functions, similar to [12], that is \( k_\gamma(q) = k(q, q) \) with \( k(\cdot, \cdot) \) an appropriate kernel function. We will use the notation \( f_P = (f_P(q_1), \ldots, f_P(q_n))^t \) to represent the \( n \)-dimensional vector that corresponds to the evaluation of the function on the training points and similarly \( k_\gamma = (k_\gamma(q_1), \ldots, k_\gamma(q_n))^t \). We also define \( r = s - f_P \) to be the residue. The basis functions and the corresponding coefficients are to be chosen such that they minimize an approximation of the disagreement error:

\[
\text{Fig. 1. Supervised ranking pursuit algorithm.}
\]

\[
c(f_P, T) = \frac{1}{2} \sum_{i,j=1}^{n} [W]_{i,j} \left( (s_i - s_j) - (f_P(q_i) - f_P(q_j)) \right)^2,
\]

which in matrix form can be written as

\[
c(f_P, T) = (s - f_P)^t L (s - f_P),
\]

where \( L \) is the Laplacian matrix of the graph \( W \).

The ranking pursuit starts at stage 0, with \( f_0 \), and recursively appends functions to an initially empty basis, at each stage of training to reduce the approximation of the ranking error. Given \( f_p \), we build \( f_{p+1}(a, \gamma) = f_p + ak_\gamma \), by searching for \( \gamma \in \{1, \ldots, N\} \) and \( a \in \mathbb{R} \) such that at every step (the residue of) the error is minimized:

\[
J(a, \gamma) = c(f_{p+1}(a, \gamma), T) = (s - f_{p+1}(a, \gamma))^t L (s - f_{p+1}(a, \gamma)) = (r_p - ak_\gamma)^t L (r_p - ak_\gamma),
\]
where we define $k_{\gamma} = k(q, q_i)$ and $k_{\gamma} = (k_{\gamma_1}, \ldots, k_{\gamma_n})^t$. By setting the first derivative to zero and solving the resulting system of equations we can obtain the coefficient $a$ that minimizes $J(a, \gamma)$ for a given $\gamma$, that is $a = (k_L k_{\gamma})^{-1} k_L r_p$.

The set of basis functions and coefficients obtained at every iteration of the algorithm is suboptimal. This can be corrected by a back-fitting procedure using a least-squares approximation of the disagreement error. The optimal value of the parameter $P$, that can be considered a “regularization” parameter of the algorithm, is estimated using a cross-validation procedure. The pseudo-code for the algorithm is presented in Figure 1.

3.1 Learning Multiple Near-Optimal Solutions

In this subsection we formulate an extension of the ranking pursuit algorithm that can efficiently use unscored data to improve the performance of the algorithm. The main idea behind our approach is to construct multiple, near-optimal, “sparse” ranking functions that give a small error on the scored data and whose predictions agree on the unscored part.

Semi-supervised learning algorithms have gained more and more attention in recent years as unlabeled data is typically much easier to obtain than labeled data. Multi-view learning algorithms, such as co-training [14], split the attributes into independent sets and an algorithm is learnt based on these different “views”. The goal of the learning process consists of finding a prediction function for every view (for the learning task) that performs well on the labeled data of the designated view such that all prediction functions agree on the unlabeled data. Closely related to this approach is the co-regularization framework described in [15,16], where the same idea of agreement maximization between the predictors is central. Briefly stated, algorithms based upon this approach search for hypotheses from different Reproducing Kernel Hilbert Spaces (RKHS) [17], namely views, such that the training error of each hypothesis on the labeled data is small and, at the same time, the hypotheses give similar predictions for the unlabeled data. Within this framework, the disagreement is taken into account via a co-regularization term. Empirical results show that the co-regularization approach works well for classification [15], regression [18], and clustering tasks [19,20]. Moreover, theoretical investigations demonstrate that the co-regularization approach reduces the Rademacher complexity by an amount that depends on the “distance” between the views [21,22].

Let us consider $M$ different feature spaces $\mathcal{H}_1, \ldots, \mathcal{H}_M$ that can be constructed from different data point descriptions (i.e., different features) or by using different kernel functions. Similar to [12] we consider $\mathcal{H}$ to be a RKHS. In addition to the training set $T = (Q, s, W)$ originating from a set $\{(q_i, s_i)\}_{i=1}^n$ of data points with scoring information. We also have a training set $\overline{T} = (\overline{Q}, \overline{W})$ from a set $\{\overline{q}_i\}_{i=1}^t$ of data points without scoring information, $\overline{Q} = (\overline{q}_1, \ldots, \overline{q}_t)^t \in (X \times Y)^t$, and the corresponding adjacency matrix $\overline{W}$. To avoid misunderstandings with the definition of the label ranking task, we will use the terms “scored” instead of “labeled” and “unscored” instead of “unlabeled”. We search for the
functions $F_P = (f_P^{(1)}, \ldots, f_P^{(M)}) \in \mathcal{H}_1 \times \ldots \times \mathcal{H}_M$, minimizing

$$\bar{c}(F_P, \mathcal{T}, \mathcal{T}) = \sum_{v=1}^{M} c(f_P^{(v)}, \mathcal{T}) + \nu \sum_{v,u=1}^{M} \bar{c}(f_P^{(v)}, f_P^{(u)}, \mathcal{T}),$$

(3)

where $\nu \in \mathbb{R}^+$ is a regularization parameter and where $\bar{c}$ is the loss function measuring the disagreement between the prediction functions of the views on the unscored data:

$$\bar{c}(f_P^{(v)}, f_P^{(u)}, \mathcal{T}) = \frac{1}{2} \sum_{i,j=1}^{l} \lvert \mathcal{W} \rvert_{i,j} \left( (f_P^{(v)}(\mathbf{q}_i) - f_P^{(v)}(\mathbf{q}_j)) - (f_P^{(u)}(\mathbf{q}_i) - f_P^{(u)}(\mathbf{q}_j)) \right)^2.$$

Although we have used unscored data in our formulation, we note that the algorithm can also operate in a purely supervised setting. It will then not only minimize the error on the scored data but also enforce agreement among the prediction functions constructed from different views.

The prediction functions $f_P^{(v)} \in \mathcal{H}_v$ of (3) for $v = 1, \ldots, M$ have the form $f_P^{(v)}(\mathbf{q}) = \sum_{i=1}^{p} \alpha_P^{(v)}(\mathbf{q}) \gamma_i$, with corresponding coefficients $\{\alpha_1^{(v)}, \ldots, \alpha_p^{(v)}\} \in \mathbb{R}^p$. Let $L$ denote the Laplacian matrix of the graph $\mathcal{W}$. Using a similar approach as in section 3 we can write the objective function as

$$J(\mathbf{a}, \gamma) = \bar{c}(F_{P+1}(\mathbf{a}, \gamma), \mathcal{T}, \mathcal{T}) =$$

$$\sum_{v=1}^{M} (\mathbf{r}_p - a^{(v)} \mathbf{k}_v)^	op L (\mathbf{r}_p - a^{(v)} \mathbf{k}_v) +$$

$$\nu \sum_{v,u=1}^{M} (a^{(v)} \mathbf{\bar{k}}_v - a^{(u)} \mathbf{\bar{k}}_u)^	op L (a^{(v)} \mathbf{\bar{k}}_v - a^{(u)} \mathbf{\bar{k}}_u),$$

where $\mathbf{a} = (a^{(1)}, \ldots, a^{(M)}) \in \mathbb{R}^M$, $\gamma = (\gamma_1, \ldots, \gamma_M)$ with $\gamma_v \in \{1, \ldots, N\}$, and $\mathbf{k}_v$ is the basis vector expansion on unscored data with $\mathbf{k}_v = \bar{k}(\mathbf{q}_v, \mathbf{q}_u)$.

By taking partial derivatives with respect to the coefficients in each view (for clarity we denote $\mathbf{k}_v$ and $\mathbf{k}_v$ as $\mathbf{k}(v)$ and $\mathbf{k}(v)$, respectively) and defining $g^{(v)} = 2\nu(M - 1)k^{(v)}_v \bar{k}^{(v)}_v$ and $g^{(v)} = k^{(v)}_v \bar{k}^{(v)}_v$, we obtain

$$\frac{\partial}{\partial a^{(v)}} J(\mathbf{a}, \gamma) = 2(g^{(v)} + g^{(v)}) a^{(v)} - 2k^{(v)}_v L \mathbf{r}_p - 4\nu \sum_{u=1, u \neq v}^{M} \bar{k}^{(v)}_v \bar{k}^{(u)}_u a^{(u)}.$$

At the optimum we have $\frac{\partial}{\partial a^{(v)}} J(\mathbf{a}, \gamma) = 0$ for all views, thus, we get the exact solution by solving
unscored data points. The overall complexity of the algorithm is \(O\)\(n^2\), thus, there is no increase in computational time compared to the kernel matching pursuit algorithm in the supervised setting \[12\]. The semi-supervised version of the ranking pursuit algorithm requires \(O\)\((Pn^M(M^3 + M^2l))\) time, which is linear in the number of unscored data points\[4\]. The pseudo-code for the algorithm is presented in Figure 2.

We can also consider a single prediction function that is given, for example, by the average of the functions for all views. The overall complexity of the standard ranking pursuit algorithm is \(O\)\(n^2\), thus, there is no increase in computational time compared to the kernel matching pursuit algorithm in the supervised setting \[12\]. The semi-supervised version of the ranking pursuit algorithm requires \(O\)\((Pn^M(M^3 + M^2l))\) time, which is linear in the number of unscored data points\[4\]. The pseudo-code for the algorithm is presented in Figure 2.

Require: Training set with scored and unscored data - \(\mathcal{T}, \overline{\mathcal{T}}\), dictionary of functions - \(\mathcal{D}\), number of basis functions - \(P\), co-regularization parameter - \(\nu\).

1. Initialize: Set residue vector \(r_1 = s\)
2. for \(p = 1, \ldots, P\) (or until performance on the validation set stops improving) do
   3. for each possible labeling \(\gamma\) do
      4. Compute \(a^*(\gamma) = (B + C)^{-1}e\) using the matrices:

\[
B = \begin{pmatrix}
g^{(1)} & 0 & \cdots & \cdot \\
0 & g^{(2)} & \cdots & \cdot \\
\vdots & \vdots & \ddots & \vdots \\
\cdot & \cdot & \cdot & \cdot
\end{pmatrix}
\]

\[
e = \begin{pmatrix}
k^{(1)t}_v Lr_p \\
k^{(2)t}_v Lr_p \\
\vdots \\
\cdot
\end{pmatrix}
\]

\[
C = \begin{pmatrix}
g^{(1)} & -2\nu k^{(1)t}_v \bar{L}k^{(2)} & \cdots \\
-2\nu k^{(2)} \bar{L}k^{(1)} & g^{(2)} & \cdots \\
\vdots & \vdots & \ddots \\
\cdot & \cdot & \cdot
\end{pmatrix}
\]

5. Set \(J(\gamma) = J(a^*(\gamma), \gamma)\)
6. end for
7. Pick \(\gamma_p = \arg\min_J J(\gamma)\)
8. Set \(a_p = a^*(\gamma_p)\) and compute the new residuals \(r_{p+1} = r_p - \frac{1}{M} \sum_{v=1}^{M} a_p^{(v)} k^{(v)}_v\)
9. end for
10. Compute prediction:

\[
f_p(q) = \frac{1}{M} \sum_{v=1}^{M} \sum_{j=1}^{P} a_p^{(v)} k^{(v)}_{\gamma_p}(q)
\]

Fig. 2. Semi-supervised ranking pursuit algorithm.

\[4\] In semi-supervised learning usually \(n \ll l\), thus, linear complexity in the number of unscored data points is beneficial. We note that complexity of the algorithm can be further reduced to \(O\)\((PM^n l)\) by forcing the indices of the nonzero coefficients in the different views to be the same.
4 Combined Ranking and Regression Pursuit

Recently a method on combined ranking and regression has been proposed in [23]. The authors suggest that in many circumstances it is beneficial to minimize the combined objective function simultaneously due to the fact that the algorithm can avoid learning degenerate models suited only for some particular set of performance metrics. Furthermore, such objective can help to improve regression performance in some circumstances e.g. when there is a large class imbalance situation. Empirically, the combined approach gives the “best of both” performance, performing as well at regression as a regression-only method, and as well at ranking as a ranking-only only method. However, despite the efficient stochastic gradient descent algorithm described in [23] the objective function to be minimized still consists of two separate parts, namely regression and ranking with the appropriate weight coefficients attached to both.

Motivated by the above approach and strong empirical results presented in [23] we propose a framework for joint ranking and regression optimization based on our ranking pursuit algorithm. We argue that our approach is slightly more elegant and simpler compared to [23] due to the fact that we employ a generalization of kernel matching pursuit – a genuine regression algorithm, thus, we do not have to consider two separate objective functions when learning joint ranking and regression models.

Compared to the kernel matching pursuit algorithm which minimizes least-squares error function

\[ c(f_P, T) = \frac{1}{2} \sum_{i=1}^{n} \left( s_i - f_P(q_i) \right)^2, \]

recall that the supervised ranking pursuit chooses the basis functions and the corresponding coefficients such that they minimize an approximation of the disagreement error:

\[ c(f_P, T) = \frac{1}{2} \sum_{i,j=1}^{n} [W]_{i,j} \left( (s_i - s_j) - (f_P(q_i) - f_P(q_j)) \right)^2, \]

which in matrix form can be written as

\[ c(f_P, T) = (s - f_P)^t L (s - f_P), \]  \hspace{1cm} (4)

where \( L \) is the Laplacian matrix of the graph \( W \) defined in section 2. Note that we can obtain a standard regression algorithm by using an identity matrix instead of \( L \) in (4). A simple idea behind our combined ranking and regression approach is the appropriate selection of the weights for the matrix \( L \), so that in a special case we can obtain a regression formulation and in another we can recover complete pairwise ranking. For this purpose we consider the weighted Laplacian matrix

\[ \tilde{L} = \beta I + (1 - \beta) L. \]  \hspace{1cm} (5)
By setting the β coefficient equal to zero, we recover the standard ranking pursuit. On the other hand by setting β equal to 1 we obtain kernel matching pursuit (12). By setting the values of the coefficient between these extremes and using such weighted \( \tilde{L} \) in (4) corresponds to minimizing a “combined” ranking and regression objective function. We refer to this algorithm as combined ranking and regression pursuit (CRRP).

5 Subset of Regressors Method for Ranking Algorithms

For comparison with the state-of-the-art, we will compare our algorithm with the sparse RankRLS, recently proposed in [24]. The main idea behind sparse RankRLS is to employ subset selection method (described e.g. in [23]) that is generally used for computational speed up purposes. For example, a popular approach to speed up the algorithm consists in approximating the kernel matrix. However, this in turn leads to solutions that do not depend on all data points present in the training set and, thus, can be considered as sparse.

Let us briefly describe this approach: Consider a setup when instead of selecting a basis function to minimize the disagreement error at every iteration of the algorithm as in section 3 we choose the prediction function to have the following form: \( f(q) = \sum_{p=1}^{n} a_p k(q, q_p) \). Further, given the prediction function that depends on all training data the objective function in matrix form can be written as \( (s - K a)^t L (s - K a) \), where \( K \in \mathbb{R}^{n \times n} \) is a kernel matrix constructed from the training set and \( a = (a_1, \ldots, a_n)^t \in \mathbb{R}^n \) is a corresponding coefficient vector.

Now, let \( R = \{i_1, \ldots, i_r\} \subseteq [n] \) be a subset of indices such that only \( a_{i_1}, \ldots, a_{i_r} \) are nonzero. By randomly selecting a subset of data points, we can approximate the prediction function using \( f(q) = \sum_{j=1}^{r} a_{i_j} k(q, q_{i_j}) \). Similarly we can approximate the kernel matrix and define matrix \( K_{R,R} \in \mathbb{R}^{r \times r} \) that contains both rows and columns indexed by \( R \). This approach for matrix approximation, known as “subset of regressors”, was pioneered in [26] and is frequently applied in practice. Although it may seem over-simplistic (e.g. other methods might appear to be more suitable rather than random selection of the regressors) it is efficient and usually leads to quite good performance. The reason behind this is that the solution obtained using a subset of regressors method can be shown to be equivalent to a “non-sparse” solution obtained with some other kernel function (e.g. [24]).

In our experiments we evaluate the performance of the sparse RankRLS algorithm and compare it to the supervised and semi-supervised ranking pursuit algorithms. We demonstrate that selection of the non-zero coefficients based on iterative minimization of the disagreement error (strategy used by the ranking pursuit algorithm) leads to better results compared to random subset selection.
Table 1. Performance comparison of the learning algorithms in supervised experiment conducted on Jester joke dataset. Normalized version of the disagreement error is used as a performance evaluation measure. Note that despite performance similar to that of ranking algorithms, ranking pursuit leads on average to 30\% sparser solutions.

| Method            | 20 – 40 | 40 – 60 | 60 – 80 |
|-------------------|---------|---------|---------|
| RLS               | 0.425   | 0.419   | 0.383   |
| Matching Pursuit  | 0.428   | 0.417   | 0.381   |
| RankSVM           | 0.412   | 0.404   | 0.372   |
| RankRLS          | 0.409   | 0.407   | 0.374   |
| Sparse RankRLS    | 0.414   | 0.410   | 0.380   |
| Ranking Pursuit   | 0.410   | 0.404   | 0.373   |

6 Experiments

6.1 Jester joke dataset

We perform a set of experiments on the publicly available Jester joke dataset\[^5\]. The task we address is the prediction of the joke preferences of a user based on the preferences of other users. The dataset contains 4.1M ratings in the range from −10.0 to +10.0 of 100 jokes assigned by a group of 73421 users. Our experimental setup is similar to that of \[2\]. We have grouped the users into three groups according to the number of jokes they have rated: 20 – 40 jokes, 40 – 60 jokes, and 60 – 80 jokes. The test users are randomly selected among the users who had rated between 50 and 300 jokes. For each test user half of the preferences is reserved for training and half for testing. The preferences are derived from the differences of the ratings the test user gives to jokes, e.g. a joke with higher score is preferred over the joke with lower score. The features for each test user are generated as follows. A set of 300 reference users is selected at random from one of the three groups and their ratings for the corresponding jokes are used as a feature values. In case a user has not rated the joke the median of his/her ratings is used as the feature value. The experiment is done for 300 different test users and the average performance is recorded. Finally, we repeat the complete experiment ten times with a different set of 300 test users selected at random. We report the average value over the ten runs for each of the three groups.

In this experiment we compare performance of the ranking pursuit algorithm to several algorithms, namely kernel matching pursuit \[12\], RankSVM \[11\], RLS \[27\] (also known as kernel ridge regression \[28\], proximal-svm \[29\], ls-svm \[30\]), RankRLS and sparse RankRLS \[24\] in terms of the disagreement error \[1\].

In all algorithms we use a Gaussian kernel where the width parameter is chosen from the set \{2^{-15}, 2^{-14}, \ldots, 2^{14}, 2^{15}\} and other parameters (e.g. stopping

\[^5\] Available at http://www.ieor.berkeley.edu/~goldberg/jester-data/
Table 2. Performance comparison of the learning algorithms in semi-supervised experiment conducted on Jester joke dataset. Supervised learning methods are trained only on the scored part of the dataset. Normalized version of the disagreement error is used as a performance evaluation measure. Note that semi-supervised ranking pursuit notably outperforms other methods.

| Method          | 20−40 | 40−60 | 60−80 |
|-----------------|-------|-------|-------|
| RLS             | 0.449 | 0.434 | 0.405 |
| Matching Pursuit| 0.451 | 0.433 | 0.404 |
| RankSVM         | 0.428 | 0.417 | 0.391 |
| RankRLS         | 0.429 | 0.418 | 0.393 |
| Sparse RankRLS  | 0.431 | 0.424 | 0.397 |
| Ranking Pursuit | 0.428 | 0.417 | 0.393 |
| SS Ranking Pursuit | 0.419 | 0.411 | 0.381 |

criteria) are chosen by taking the average over the performances on a holdout set. The hold-out set is created similarly as the corresponding training/test set.

The results of the collaborative filtering experiment are included in Table 1. It can be observed that ranking based approaches in general outperform the regression methods. According to Wilcoxon signed-rank test [31] the differences in performance are statistically significant ($p < 0.05$). However, the differences in performance among the ranking/regression algorithms are not statistically significant. Although performance of the ranking pursuit algorithm is similar to that of the RankSVM and RankRLS algorithms, obtained solutions are on average 30% sparser. To evaluate the performance of the semi-supervised extension of the ranking pursuit algorithm we construct datasets similarly as in the supervised learning experiment with the following modification. To simulate unscored data, for each test user we make only half of his/her preferences from the training set available for learning. Using this training set we construct two views, each containing half of the scored and half of the unscored data points. The rest of the experimental setup follows the previously described supervised learning setting. The results of this experiment are included in Table 2. We observe notable improvement in performance of the semi-supervised ranking pursuit algorithm compared to all baseline methods. This improvement is statistically significant according to Wilcoxon signed-rank test with 0.05 as a significance threshold. The performance of the supervised methods in this experiment is decreased (compared to the supervised learning experiment) as expected, due to the fact that the amount of labeled data is twice smaller.

6.2 MovieLens dataset

The MovieLens dataset consists of approximately 1M ratings by 6,040 users for 3,900 movies. Ratings are integers from 1 to 5. The experiments were set-up
Table 3. Performance comparison of the learning algorithms in supervised experiment conducted on MovieLens dataset. Normalized version of the disagreement error is used as a performance evaluation measure. Note that despite performance similar to that of ranking algorithms, ranking pursuit leads on average to 35% sparser solutions.

| Method            | 20 - 40 | 40 - 60 | 60 - 80 |
|-------------------|---------|---------|---------|
| RLS               | 0.495   | 0.494   | 0.482   |
| Matching Pursuit  | 0.494   | 0.497   | 0.484   |
| RankSVM           | 0.481   | 0.472   | 0.453   |
| RankRLS           | 0.479   | 0.472   | 0.455   |
| Sparse RankRLS    | 0.484   | 0.478   | 0.460   |
| Ranking Pursuit   | 0.480   | 0.472   | 0.453   |

Table 4. Performance comparison of the learning algorithms in semi-supervised experiment conducted on MovieLens dataset. Supervised learning methods are trained only on the scored part of the dataset. Normalized version of the disagreement error is used as a performance evaluation measure. Note that semi-supervised ranking pursuit notably outperforms other methods.

| Method            | 20 - 40 | 40 - 60 | 60 - 80 |
|-------------------|---------|---------|---------|
| RLS               | 0.497   | 0.495   | 0.487   |
| Matching Pursuit  | 0.498   | 0.495   | 0.485   |
| RankSVM           | 0.487   | 0.479   | 0.464   |
| RankRLS           | 0.486   | 0.479   | 0.463   |
| Sparse RankRLS    | 0.490   | 0.485   | 0.470   |
| Ranking Pursuit   | 0.487   | 0.479   | 0.462   |
| SS Ranking Pursuit| **0.481**| **0.474**| **0.458**|

in the same way as for the Jester joke dataset. The results of the supervised experiment are presented in Table 3. Similarly to the results obtained on Jester joke dataset we observe that the ranking pursuit algorithm leads to much more compact models, about 35% sparser, while having performance comparable to that of the ranking algorithms. The results of the semi-supervised experiment are presented in Table 4. We again observe notable improvement in performance of the semi-supervised ranking pursuit algorithm compared to all baseline methods. The improvement is statistically significant ($p < 0.05$).

6.3 CRRP Algorithm Evaluation

To empirically evaluate our approach, termed combined ranking and regression pursuit (CRRP), we conduct experiments using the CRRP algorithm on the
The experiments are conducted following the supervised learning setup described above. We use the disagreement error and the mean squared error (MSE) to measure performance of the algorithm for ranking and regression setting, respectively. The obtained results are presented in Table 5.

It can be observed that by choosing the weight coefficient appropriately ($\beta = 0.5$) the CRRP algorithm performs almost as well as the specialized algorithms on ranking and regression tasks. Note that when considering the regression setting the CRRP algorithm improves over the MSE performance of the rank-only methods, but does not outperform the regression only methods. Furthermore, the performance differences of the CRRP algorithm to the ranking methods on the regression task as well as to the regression methods on the ranking task are statistically significant according a Wilcoxon signed-rank test ($p < 0.05$). To summarize, while CRRP does not outperform specialized algorithms in regression or ranking, it is able to achieve notably better performance compared to the regression-only methods for the ranking task or ranking-only methods for the regression task.

### Table 5. Performance comparison of the ranking, regression, and CRRP algorithms on the Jester joke dataset.

For the performance evaluation in regression task we use mean squared error (MSE) and for the ranking task we use a normalized version of the disagreement error. Note that CRRP is able to achieve good performance in both ranking and regression settings.

|                  | Ranking task | Regression task |
|------------------|--------------|-----------------|
|                  | 20 − 40 − 60 − 80 | 20 − 40 − 60 − 80 |
| RLS              | 0.425 0.419 0.383 | 21.6 19.3 15.2 |
| Matching Pursuit | 0.428 0.417 0.381 | 20.1 18.7 14.9 |
| RankSVM          | 0.412 0.404 0.372 | 34.2 31.6 28.9 |
| RankRLS          | 0.409 0.407 0.374 | 33.8 31.3 29.2 |
| Sparse RankRLS   | 0.414 0.410 0.380 | 36.5 33.8 32.9 |
| Ranking Pursuit  | 0.410 0.404 0.373 | 34.0 30.9 29.1 |
| CRRP             | 0.413 0.408 0.373 | 23.2 20.4 17.3 |
7 Conclusions

We propose sparse preference learning/ranking algorithm as well as its semi-supervised extension. Our algorithm is a generalization of the kernel matching pursuit algorithm [12] and allows explicit control over sparsity of the solution. It is also naturally applicable in circumstances when one is interested in obtaining multiple near-optimal solutions that frequently arise during the sparse modeling of many problems in biology, information retrieval, natural language processing, etc. Another contribution of this paper is a combined ranking and regression (CRRP) method, formulated within the framework of the proposed ranking pursuit algorithm.

The empirical evaluation demonstrates that in the supervised setting our algorithm outperforms regression methods such as kernel matching pursuit, RLS and performs comparably to the RankRLS, sparse RankRLS and RankSVM algorithms, while having sparser solutions. In its semi-supervised setting our ranking pursuit algorithm notably outperforms all baseline methods. We also show that CRRP algorithm is suitable for learning combined ranking and regression objectives and leads to good performance in both ranking and regression tasks. In the future we aim to apply our algorithm in other domains and will examine different aggregation techniques for multiple sparse solutions.

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