Diagnostic Power of Future Colliders for $Z'$ Couplings to Quarks and Leptons: $e^+e^-$ versus pp Colliders

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Abstract

We study the diagnostic power of future $e^+e^-$ colliders with $\sqrt{s} = 500$ GeV (the New Large Collider ($NLC$)) for a model independent determination of the $Z'$ gauge couplings to quarks and leptons. The interference of the $Z'$ propagator with the photon and the $Z$ propagator in the two-fermion final state probes are sensitive to the magnitude as well as relative signs of quark and lepton charges. For $Z'$ with $M_{Z'} \sim 1$ TeV all the quark and lepton charges can be determined to around $10 - 20\%$, provided heavy flavor tagging and longitudinal polarization of the electron beam is available. The errors are $2 - 10$ times larger without polarization, and very little information can be obtained about quark charges without heavy flavor tagging. We point out the complementarity of future hadron colliders. At the CERN Large Hadron Collider ($LHC$) primarily the magnitude of three out of four corresponding couplings can be measured; however, their error-bars are typically by a factor of $\sim 2$ smaller than those at the $NLC$. 

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I. INTRODUCTION

If the masses of heavy gauge bosons $Z'$s do not exceed 5 TeV or so, the CERN Large Hadron Collider (LHC), would be an ideal place to discover them [1]. In the last few years a number of diagnostic probes have been proposed [1], allowing for a model independent determination [2] of certain $Z'$ couplings to quarks and leptons provided $M_{Z'} < \sim 2$ TeV.

On the other hand, future $e^+e^-$ colliders with large enough center of mass energy $\sqrt{s}$, e.g., $\sqrt{s} = 2$ TeV, could provide a clean way to discover and study the properties of $Z'$'s. A more likely possibility, however, is the next linear collider (NLC) with $\sqrt{s} = 500$ GeV. Due to the interference of the $Z'$ propagator with the photon and $Z$ propagators, the two-fermion channels yield complementary information on the existence of a $Z'$. An extensive study [3,4] showed that effects of a $Z'$ would be observable at the NLC for a large class of models with $M_{Z'}$ up to $1-3$ TeV. In particular, in Ref. [3] the sensitivity of the NLC to specific classes of extended electroweak models, e.g., different $E_6$ motivated models described by a parameter $\cos \beta$ (the mixing angle between the $Z_X$ and $Z_{\psi}$ defined below) or left-right symmetric models parameterized by the ratio $\kappa = g_R/g_L$ for the $SU(2)_{L,R}$ gauge coupling constants $g_{L,R}$, was explored.

In this paper we explore further the diagnostic power of the NLC for $Z'$ physics. In particular, we investigate a model independent determination of the $Z'$ couplings to quarks and leptons [3]. We take the attitude that at the LHC, which is likely to be built before the NLC, $Z'$ would either be discovered or strong bounds on $M_{Z'}$ (> 5 TeV for typical classes of models) would be achieved. Only in the former case would the NLC provide a testing ground to learn more about the $Z'$. We therefore assume that the $Z'$ has a mass in the range of a few TeV, and thus the NLC has the capability to probe the $Z'$ couplings.

We shall see that heavy flavor ($c,b,t$) tagging would provide a crucial diagnostic tool for the determination of the quark couplings. Based on the success of LEP experiments in measuring quark cross sections for different heavy flavors ($c,b$) [6], we will assume that heavy flavor tagging will be feasible at the NLC. Another crucial tool is the longitudinal polarization of the electron beam, which turns out to be important for an unambiguous determination of the lepton couplings, including their relative signs. Heavy flavor tagging, along with the longitudinal polarization of the electron beam, provide probes in the two-fermion final state channels which are sensitive to the magnitude as well as the relative signs of all the $Z'$ charges to quarks and leptons. It turns out that for $M_{Z'} \sim 1$ TeV, such couplings would be determined to about 10 – 20% at the NLC. If polarization were not available, the determination of the $Z'$ couplings would be marginal, since the error-bars increase by a factor of $2-10$. Similarly, without heavy flavor tagging, very little can be learned about the quark couplings.

Another goal of this paper is to compare the analysis done for the NLC with the one that has been done for the LHC collider [2]. The diagnostic power of the LHC is complementary. It allows primarily for the determination of the magnitude of three out of four normalized couplings, only. However, the corresponding error-bars are typically by a factor of $\sim 2$ smaller than those for the NLC. In addition, the LHC would measure $M_{Z'}$ directly and would allow for a determination of an overall strength of the $Z'$ gauge coupling to fermions. This is in contrast to the NLC which, for fixed c.m. energy, primarily determines only the ratio of an overall $Z'$ gauge coupling strength and $M_{Z'}$. 

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The paper is organized as follows. In Section II we specify the notation and the models used to illustrate the analysis. In Section III we discuss the probes for the two–fermion final state channels at the NLC. In Section IV simulated fits for the $Z'$ charges to quark and leptons are performed for a class of typical models. In Section V we compare results at the NLC with those at the LHC. Conclusions are given in Section VI.

II. TYPICAL MODELS AND $Z'$ COUPLINGS

The neutral current gauge interaction term in the presence of an additional $U_1$ is of the form

$$-L_{NC} = eJ_{em}^\mu A_\mu + g_1 J_1^\mu Z_{1\mu} + g_2 J_2^\mu Z_{2\mu},$$

with $Z_1$ the $SU_2 \times U_1$ boson and $Z_2$ the additional boson in the weak eigenstate basis. Here $g_1 \equiv \sqrt{g_{L}^2 + g_{Y}^2} = g/\cos \theta_W$, where $g_L$, $g_Y$ are the gauge couplings of $SU_{2L}$ and $U_{1Y}$, and $g_2$ is the gauge coupling of $Z_2$. The currents are: $J_j^\mu = \frac{1}{2} \sum_i \bar{\psi}_i \gamma^\mu \left[ g_{V}^j - g_{A}^j/\sqrt{2} \right] \psi_i$, $j = 1, 2$, where the sum runs over fermions, and the $g_{V,A}^j$ are the vector and axial vector couplings of $Z_j$ to the $i^{th}$ flavor. Analogously, $\tilde{g}_j^{(L,R)} = \frac{1}{2}(\hat{g}_j^L \pm \hat{g}_j^R)$.

For illustration we consider the following typical GUT, left-right symmetric, and superstring-motivated models.

- \( \chi \) model: $Z_\chi$ occurs in $SO_{10} \rightarrow SU_5 \times U_1$.  
- \( \psi \) model: $Z_\psi$ occurs in $E_6 \rightarrow SO_{10} \times U_1$.  
- \( \eta \) model: $Z_\eta = \sqrt{3/8}Z_\chi - \sqrt{5/8}Z_\psi$ occurs in superstring inspired models in which $E_6$ breaks directly to a rank 5 group.  
- \( LR \) model: $Z_{LR}$ occurs in left-right (LR) symmetric models. Here we consider the special value $\kappa = g_R/g_L = 1$ of the gauge couplings $g_{LR}$ for $SU_{2L,2R}$, respectively.

In the rest of the paper we assume family universality and neglect $Z - Z'$ mixing (as suggested from experiments). We also assume $[Q', T_i] = 0$, where $Q'$ is the $Z'$ charge and $T_i$ are the $SU_{2L}$ generators, which holds for a large class of models, including the above $SU_2 \times U_1 \times U'_1$ and LR models. The relevant quantities to distinguish between different models are then the five charges: $\hat{g}_{L2}^u = \hat{g}_{R2}^d \equiv \hat{g}_{L2}^\nu$, $\hat{g}_{L2}^u$, $\hat{g}_{L2}^d$, $\hat{g}_{L2}^\nu$, and $\hat{g}_{R2}^\nu$, and the gauge coupling strength $g_2$. The overall scale of the charges (and $g_2$) depends on the normalization convention for $\text{Tr}(Q'^2)$, but the ratios characterize particular theories.

Note that one combination of the five charges can always be absorbed in the redefinition of an overall gauge coupling strength. Since the photon couplings are only vector–like and the $\ell$ couplings to $Z$ have the property $\hat{g}_{L1}^\ell \simeq -\hat{g}_{R1}^\ell$, it turns out that the probes in the two–fermion final state channels single out the $Z'$ leptonic couplings primarily in the combinations $\hat{g}_{L2}^\ell \pm \hat{g}_{R2}^\ell$. To trace the combinations of the normalized charges to which the probes are sensitive, it is advantageous to choose either of the two combinations to normalize the charges. We choose the $\hat{g}_{L2}^\ell - \hat{g}_{R2}^\ell$ combination, which turns out to be a convenient choice.
for the typical models used in the analysis. We then define the following four independent "normalized" charges:

\[ P_V^\ell = \frac{g_{L2}^\ell + g_{R2}^\ell}{g_{L2}^\ell - g_{R2}^\ell}, \quad P_L^\eta = \frac{g_{L2}^\eta}{g_{L2}^\ell - g_{R2}^\ell}, \quad P_R^{u,d} = \frac{g_{R2}^{u,d}}{g_{L2}^\ell}. \tag{2} \]

Their values are given for the typical models in Table I. In addition, the probes in the two-fermion final state channels are sensitive to the following ratio of an overall gauge coupling strength divided by the "reduced" \( Z' \) propagator:

\[ \epsilon_A = (\hat{g}_{L2}^\ell - \hat{g}_{R2}^\ell)^2 \frac{g_2^2}{4\pi\alpha M_{Z'}^2} \frac{s}{s}. \tag{3} \]

Here \( \alpha \) is the fine structure constant. Note again that the four normalized charges (Eq.(2)) and \( \epsilon_A \) (Eq.(3)) can be replaced with an equivalent set by choosing \( \hat{g}_{L2}^\ell + \hat{g}_{R2}^\ell \) to normalize the couplings.

One should contrast the above choice of the normalized couplings with those chosen for the LHC. There the signs of the couplings are difficult to determine and the following set of four normalized couplings is probed directly [2]:

\[ \gamma_L^\ell \equiv \frac{(\hat{g}_{L2}^\ell)^2}{(\hat{g}_{L2}^\ell)^2 + (\hat{g}_{R2}^\ell)^2}, \quad \gamma_L^\eta \equiv \frac{(\hat{g}_{L2}^\eta)^2}{(\hat{g}_{L2}^\eta)^2 + (\hat{g}_{R2}^\ell)^2}, \quad \tilde{U} \equiv \frac{(\hat{g}_{R2}^u)^2}{(\hat{g}_{L2}^\ell)^2}, \quad \tilde{D} \equiv \frac{(\hat{g}_{R2}^d)^2}{(\hat{g}_{L2}^\ell)^2}. \tag{4} \]

which can be expressed in terms of the couplings (2) as:

\[ \gamma_L^\ell = \frac{(1 + P_V^\ell)^2}{2(1 + P_V^\ell)^2}, \quad \gamma_L^\eta = \frac{2P_L^\eta}{1 + P_V^\ell}, \quad \tilde{U} = (P_R^u)^2, \quad \tilde{D} = (P_R^d)^2. \tag{5} \]

In addition, for \( M_{Z'} \lesssim 5 \) TeV, the LHC would determine \( M_{Z'} \) and the total width \( \Gamma_{Z'} \) directly in the main discovery channel \( pp \rightarrow Z' \rightarrow \ell^+\ell^- (\ell = e, \mu) \). Then the quantity \( \sigma(pp \rightarrow Z')B(Z' \rightarrow \ell^+\ell^-)\Gamma_{Z'} \) would yield the information on an overall strength of the \( Z' \) gauge coupling [3]. Here \( \sigma(pp \rightarrow Z') \) is the total cross-section and \( B(Z' \rightarrow \ell^+\ell^-) \) the branching ratio for the \( \ell^+\ell^- \) final state channel.

The values of the couplings (4) for typical models are given in Table II. Note that the couplings in Eq.(4), probed by the LHC, do not determine the couplings in Eq.(2) uniquely. In particular, determination of \( \gamma_L^\ell, \tilde{U} \) and \( \tilde{D} \) (the three out of four couplings most easily measurable at the LHC) would yield an eight-fold ambiguity for the corresponding three couplings in Eq.(2). Table III exhibits this two-fold ambiguity for each of the \( P_V^\ell \) and \( P_R^{u,d} \) couplings; only the first entry is the actual value of the corresponding coupling in the particular model.

**III. e^+e^- \rightarrow f\bar{f} OBSERVABLES**

At the NLC the cross sections and corresponding asymmetries in the two-fermion final state channels, \( e^+e^- \rightarrow f\bar{f} \), will be measured. Due to the interference of the \( Z' \) propagator with the photon and the \( Z \) propagators such probes are sensitive to the four normalized
TABLE I. The value of the couplings (defined in Eqs.(2) and (3)) and statistical error-bars as determined from the probes defined in Sect. III for the NLC (c.m. energy $\sqrt{s} = 500$ GeV and integrated luminosity $L = \int d\sigma / \int d\sigma$). The models are defined in Sect. II and $M_{Z'} = 1$ TeV. 100% heavy flavor tagging efficiency and 100% longitudinal polarization of the electron beam is assumed for the first set of error-bars, while the error-bars in parentheses are for the probes without polarization.

| Coupling | Value | Error (statistical) |
|----------|-------|---------------------|
| $\chi$  | $2.0 \pm 0.08 (0.26)$ | $0.0 \pm 0.04 (1.5)$ |
| $\psi$  | $-3.0 \pm 0.5 (1.1)$ | $-0.15 \pm 0.018 (0.072)$ |
| $\eta$  | $1.0 \pm 0.15 (0.19)$ | $1.0 \pm 0.11 (0.24)$ |
| $LR$    | $3.0 \pm 0.24 (0.51)$ | $1.0 \pm 0.21 (2.8)$ |
| $\epsilon_A$ | $0.071 \pm 0.005 (0.018)$ | $0.121 \pm 0.017 (0.02)$ |

TABLE II. Values of the couplings (4) probed directly at the LHC. The statistical error-bars indicate how well these couplings can be measured at the LHC (c.m. energy $\sqrt{s} = 16$ TeV and integrated luminosity $L = \int d\sigma / \int d\sigma$) for the typical models with $M_{Z'} = 1$ TeV.

| Coupling | Value | Error (statistical) |
|----------|-------|---------------------|
| $\gamma_L$ | $0.9 \pm 0.018$ | $0.5 \pm 0.03$ |
| $\gamma_R$ | $0.1$ | $0.2 \pm 0.015$ |
| $\eta$ | $1 \pm 0.18$ | $1 \pm 0.27$ |
| $LR$ | $9 \pm 0.61$ | $1 \pm 0.41$ |

TABLE III. Values of three (out of four) couplings (2) which are probed (indirectly) at the LHC (see Eq.(5), which relates the couplings (2) to those directly probed by the LHC). The error-bars indicate how well these couplings can be measured at the LHC (c.m. energy $\sqrt{s} = 16$ TeV and integrated luminosity $L = \int d\sigma / \int d\sigma$) for the typical models with $M_{Z'} = 1$ TeV. There is a two-fold ambiguity for each of the couplings. Only the first number corresponds to the actual value of the coupling of the particular model.

| Coupling | Value | Error (statistical) |
|----------|-------|---------------------|
| $\chi$  | $2 \pm 0.15$ | $0 \pm 0.03$ |
| $\psi$  | $-3 \pm 0.19$ | $-0.333 \pm 0.021$ |
| $\eta$  | $\mp 1 \pm 0.09$ | $\mp 1 \pm 0.14$ |
| $LR$    | $\mp 1 \pm 0.21$ | $\pm 0.5 \pm 0.29$ |
charges in Eq.(2) as well as to the parameter $\epsilon_A$ (Eq.(3)). The tree-level expressions for such probes can be written explicitly in terms of seven generalized charges, which are given in Ref. [3].

The estimates for statistical and systematic errors suggested [3] an analysis based on the following probes:

$$\sigma^\ell, \quad R^{\text{had}} = \frac{\sigma^{\text{had}}}{\sigma^\ell}, \quad A_{FB}^\ell.$$ (6)

In the case that longitudinal polarization of the electron beam is available there are additional probes:

$$A_{LR}^{\ell,\text{had}}, \quad A_{LR,FB}^\ell.$$ (7)

Here $\sigma$, $A_{FB}$, $A_{LR}$ and $A_{LR,FB}$ refer to the corresponding cross sections, forward-backward asymmetries, left-right (polarization) asymmetries and left-right-forward-backward asymmetries, respectively. The superscripts $\ell$ and had refer to all three leptonic channels (considering only s-channel exchange for electrons) and to all hadronic final states, respectively. The above quantities help to distinguish among different models [3]; however, they do not yield information on all the $Z'$ couplings. In particular $\sigma^\ell$ and $A_{FB}^\ell$ probe $\epsilon_A$ and the magnitude of $P_{V^\ell}$, but not its sign [3]. On the other hand, $R^{\text{had}}$ provides additional information on one linear combination of the normalized quark couplings. If polarization is available, $A_{FB}^\ell$ and $A_{LR,FB}^\ell$ are excellent probes for $P_{V^\ell}$ (including its sign), while $A_{FB}^{\ell,\text{had}}$ yields information on another linear combination of the quark couplings. See Table IV for the approximate dependence of the above probes on the couplings.

LEP analyses show that $e^+e^-$ colliders allow for an efficient tagging of charm and bottom final states [6]. Eventually, top events will also be easily identifiable at the future $e^+e^-$ colliders. We therefore assume that at the NLC an efficient tagging of the heavy flavors ($c,b,t$) would be available. This in turn provides an additional set of observables:

$$R^f = \frac{\sigma^f}{\sigma^\ell}, \quad A_{FB}^f; \quad f = c,b,t,$$ (8)

and with polarization available:

$$A_{LR}^f, \quad A_{LR,FB}^f; \quad f = c,b,t,$$ (9)

where the superscript refers to the corresponding heavy flavors. These additional probes would in turn allow for a complete determination of the $Z'$ gauge couplings to ordinary fermions, giving the assumptions of family universality, $[Q',T_i] = 0$, and neglect of $Z-Z'$ mixing (see Sect. II).

To illustrate quantitatively the sensitivity of the above probes for the $Z'$ couplings, we display the explicit dependence on the couplings (2) and $\epsilon_A$ (Eq.(3)) in Table IV. (Table II in Ref. [4] provides analogous expressions for the probes at the LHC.) The expressions are at tree-level, evaluated to $O(\epsilon_A)$, only. In Table IV we neglect fermion masses ($m_f^2 \ll s$), implying $\sigma^\ell = \sigma^\mu = \sigma^\tau \equiv \sigma^\ell$ (only s-channel exchange is considered for electrons), $\sigma^\ell = \sigma^u \equiv \sigma^a$, $\sigma^b \equiv \sigma^d$, and similarly for the corresponding asymmetries. (Obviously, neglecting the
TABLE IV. Tree level expressions, correct to $O(\epsilon_A)$, for the total cross sections, $\sigma^f$, the forward-backward asymmetries, $A^f_{FB}$, and the corresponding polarized asymmetries, $A^f_{LR}$ and $A^f_{LR,FB}$ Here, $f = \ell, u, d$ are the final state flavors. We neglect fermion masses, $m_f^2 \ll s$, and take the weak mixing angle $\sin^2\theta_W = 0.23$. The couplings are defined in Eqs.(2) and (3). $\sigma_0 = \frac{4\pi\alpha^2}{\sqrt{s}}$ is the point-like QED cross section for muon pair production, $\alpha = \frac{1}{128}$ the electromagnetic coupling constant, and $N_c = 3$ is the number of colors.

| Expression | Formula |
|------------|---------|
| $\sigma^f$ | $\sigma_0[1.140 - (0.500P_V^f)^2 + 0.029P_V^f + 0.183\epsilon_A]$ |
| $\sigma^u$ | $\sigma_0N_c[0.614 + (0.253P_L^u) + 0.354P_P^u - 0.112P_L^u P_R^u + 0.324P_V^u P_R^u]\epsilon_A$ |
| $\sigma^d$ | $\sigma_0N_c[0.323 - (0.309P_L^d) + 0.191P_P^d - 0.056P_L^d P_R^d + 0.162P_V^d P_R^d]\epsilon_A$ |
| $A^f_{FB}$ | $0.483 + (0.091P_L^f)^2 - 0.007P_V^f - 0.251\epsilon_A$ |
| $A^u_{FB}$ | $0.614 + (0.179P_L^u) - 0.045P_P^u - 0.284P_L^u P_R^u - 0.187P_V^u P_R^u\epsilon_A$ |
| $A^d_{FB}$ | $0.634 + (0.163P_L^d) - 0.343P_P^d - 0.266P_L^d P_R^d + 0.188P_V^d P_R^d\epsilon_A$ |
| $A^f_{LR}$ | $0.070 + (0.018P_L^f)^2 - 0.598P_V^f - 0.002\epsilon_A$ |
| $A^u_{LR}$ | $0.348 + (0.433P_L^u) + 0.211P_P^u + 0.591P_L^u P_R^u - 0.366P_V^u P_R^u\epsilon_A$ |
| $A^d_{LR}$ | $0.619 + (0.001P_L^d) - 0.591P_P^d + 0.609P_L^d P_R^d + 0.484P_V^d P_R^d\epsilon_A$ |
| $A^f_{LR,FB}$ | $0.053 + (0.013P_L^f)^2 - 0.449P_V^f - 0.001\epsilon_A$ |
| $A^u_{LR,FB}$ | $0.175 + (0.237P_L^u) + 0.332P_P^u + 0.169P_L^u P_R^u - 0.488P_V^u P_R^u\epsilon_A$ |
| $A^d_{LR,FB}$ | $0.476 - (0.262P_L^d) + 0.162P_P^d + 0.213P_L^d P_R^d - 0.615P_V^d P_R^d\epsilon_A$ |

(top mass may be not a good approximation.) For $M_{Z'} \sim 1$ TeV, $\epsilon_A$ is sufficiently small, so that the use of these expressions versus the exact Born approximation expressions changes the numerical results only by a few %. (The numerical results in Tables I, V, and VI and the Figures use the exact Born approximations, including $m_t = 150$ GeV effects.)

IV. DETERMINATION OF Z’ COUPLINGS AT THE NLC

We now study how well one can determine the couplings defined in Sect. III at the NLC. The effects of a heavy $Z'$ far off-shell are expected to be small and comparable to the electro-weak radiative corrections [3]. The latter ones are dominated by initial state radiation, which can be greatly reduced by applying a cut on the maximum photon energy to exclude $Z$ production. With such a cut the tree-level expressions are a reasonably good approximation to the different observables. Since our present goal is to explore the sensitivity of the $Z'$ couplings, it is sufficient to neglect the remaining radiative corrections. Of course, if a new $Z'$ is actually discovered a realistic fit should include full radiative corrections as well as experimental cuts and detector acceptances.

Throughout the paper we take the c.m. energy $\sqrt{s} = 500$ GeV, and the integrated luminosity $L_{int} = 20$ fb$^{-1}$. For the analysis we use the probes defined in Eqs.(6-9). We assume 100% efficiency for heavy flavor tagging (probes (8-9)) and 100% longitudinal polarization of the initial electron beam for probes (7) and (9). We will, however, also address the case in which the polarization and the heavy flavor tagging efficiency are smaller. We include only statistical errors for the observables and neglect error correlations for the input parameters. For this reason, and because we do not include experimental cuts and detector acceptances.
TABLE V. The values and statistical error-bars for the observables (6-9) (defined in Sect. III) at the NLC (c.m. energy $\sqrt{s} = 500$ GeV and integrated luminosity $L_{\text{int}} = 20$ fb$^{-1}$). The models are defined in Sect. II, and $M_{Z'} = 1$ TeV. The first row is the number of events in the $\ell$ channel. The last column is the standard model (SM) prediction.

| $\sigma^f L_{\text{int}}$ | $\chi$ | $\psi$ | $\eta$ | $LR$ | $SM$ |
|---------------------------|--------|--------|--------|------|------|
| $R^\text{had}$ | 8.73 $\pm$ 0.10 | 7.28 $\pm$ 0.08 | 7.69 $\pm$ 0.09 | 7.18 $\pm$ 0.08 | 7.16 $\pm$ 0.08 |
| $A_{LR}^f$ | 0.491 $\pm$ 0.010 | 0.451 $\pm$ 0.009 | 0.490 $\pm$ 0.009 | 0.415 $\pm$ 0.010 | 0.483 $\pm$ 0.009 |
| $A_{LR,FB}^f$ | $-0.018 \pm 0.008$ | $0.070 \pm 0.007$ | $0.094 \pm 0.008$ | $0.092 \pm 0.008$ | $0.070 \pm 0.007$ |
| $R^c$ | 1.83 $\pm$ 0.03 | 1.71 $\pm$ 0.02 | 1.72 $\pm$ 0.02 | 1.58 $\pm$ 0.02 | 1.62 $\pm$ 0.02 |
| $\beta^c$ | 1.17 $\pm$ 0.02 | 0.81 $\pm$ 0.01 | 0.93 $\pm$ 0.01 | 0.88 $\pm$ 0.01 | 0.85 $\pm$ 0.01 |
| $R^d$ | 1.57 $\pm$ 0.02 | 1.43 $\pm$ 0.02 | 1.45 $\pm$ 0.02 | 1.38 $\pm$ 0.02 | 1.37 $\pm$ 0.02 |
| $R^t$ | 1.56 $\pm$ 0.007 | 0.641 $\pm$ 0.006 | 0.614 $\pm$ 0.006 | 0.544 $\pm$ 0.007 | 0.615 $\pm$ 0.007 |
| $A_{FB}^c$ | 0.567 $\pm$ 0.009 | 0.625 $\pm$ 0.009 | 0.655 $\pm$ 0.008 | 0.542 $\pm$ 0.010 | 0.633 $\pm$ 0.009 |
| $A_{FB}^b$ | 0.437 $\pm$ 0.008 | 0.490 $\pm$ 0.008 | 0.465 $\pm$ 0.008 | 0.398 $\pm$ 0.008 | 0.463 $\pm$ 0.008 |
| $A_{LR}^c$ | 0.311 $\pm$ 0.006 | 0.337 $\pm$ 0.005 | 0.300 $\pm$ 0.006 | 0.474 $\pm$ 0.005 | 0.347 $\pm$ 0.005 |
| $A_{LR}^b$ | 0.627 $\pm$ 0.006 | 0.657 $\pm$ 0.006 | 0.633 $\pm$ 0.006 | 0.785 $\pm$ 0.005 | 0.620 $\pm$ 0.006 |
| $A_{LR}^t$ | 0.347 $\pm$ 0.006 | 0.361 $\pm$ 0.006 | 0.339 $\pm$ 0.006 | 0.491 $\pm$ 0.006 | 0.368 $\pm$ 0.006 |
| $A_{LR,FB}^c$ | 0.111 $\pm$ 0.006 | 0.178 $\pm$ 0.006 | 0.117 $\pm$ 0.006 | 0.213 $\pm$ 0.006 | 0.174 $\pm$ 0.006 |
| $A_{LR,FB}^b$ | 0.379 $\pm$ 0.007 | 0.472 $\pm$ 0.007 | 0.458 $\pm$ 0.007 | 0.533 $\pm$ 0.007 | 0.477 $\pm$ 0.007 |
| $A_{LR,FB}^t$ | 0.083 $\pm$ 0.006 | 0.136 $\pm$ 0.006 | 0.089 $\pm$ 0.006 | 0.156 $\pm$ 0.006 | 0.131 $\pm$ 0.006 |

our results should be interpreted as a limit on how precisely the couplings can be determined for each model for the given c.m. energy and the integrated luminosity of the NLC. Realistic fits are expected to give larger uncertainties for the couplings.

In Table V we give the values of the probes (6-9) and their statistical uncertainties at the NLC for the typical models. For comparison, the values in the last column correspond to those of the standard model. The first row is $\sigma^f L_{\text{int}}$, the number of events in one $\ell = (e, \mu, \tau)$ channel.

We perform a simulated $\chi^2$ analysis for the couplings of the typical models given in Table I for $M_{Z'} = 1$ TeV. The resulting $1\sigma$ uncertainties are also given in Table I. The first set of error-bars is with polarization (using probes (6-9)) while the error-bars in parentheses are without polarization (using probes (6) and (8)). The $Z'$ charges can typically be determined to around 10 ― 20%. Without polarization the error-bars increase by a factor 2 ― 10, and thus yield only marginal information about the quark couplings. The poor determination of the couplings for the $\eta$ model is related to the small value of $\epsilon_A$ in this case. The $\psi$ model has particularly poorly determined couplings without polarization.

In Figs. (1a-1e) the 90% confidence level ($\Delta \chi^2 = 4.6$) contours are plotted for the various pairs of parameters in the $\chi$, $\psi$ and $\eta$ models (the $LR$ model is in a different region of parameter space) for $M_{Z'} = 1$ TeV. (They should be compared to analogous contours for the couplings (4) at the LHC in Figs. 1 of Ref. [4].) The contours correspond to 100%
TABLE VI. The value of the couplings (defined in Eqs. (2) and (3)) and $\sigma$ statistical error-bars with decreased heavy flavor tagging efficiency and smaller longitudinal polarization of the electron beam, as determined from the probes in Eqs. (6-9) at the NLC (c.m. energy $\sqrt{s} = 500$ GeV and integrated luminosity $L_{int} = 20$ fb$^{-1}$). The models are defined in Sect. II, and $M_{Z'} = 1$ TeV. The first (second) set of error-bars corresponds to 25% (100%) heavy flavor tagging efficiency and 100% (50%) electron beam polarization.

|   | $\chi$   | $\psi$   | $\eta$   | $LR$   |
|---|----------|----------|----------|--------|
| $P_{V}$ | $2 \pm 0.10(0.12)$ | $0 \pm 0.05(0.08)$ | $-3 \pm 0.55(0.68)$ | $-0.15 \pm 0.022(0.031)$ |
| $P_{R}^q$ | $-0.5 \pm 0.08(0.06)$ | $0.5 \pm 0.16(0.11)$ | $2 \pm 0.56(0.53)$ | $-0.14 \pm 0.07(0.04)$ |
| $P_{R}^u$ | $-1 \pm 0.29(0.17)$ | $-1 \pm 0.19(0.19)$ | $-1 \pm 0.25(0.19)$ | $-6.0 \pm 2.7(1.7)$ |
| $P_{R}^d$ | $3 \pm 0.45(0.35)$ | $-1 \pm 0.37(0.31)$ | $0.5 \pm 0.16(0.15)$ | $8.0 \pm 3.8(2.3)$ |
| $\epsilon_A$ | $0.071 \pm 0.005(0.008)$ | $0.121 \pm 0.018(0.017)$ | $0.012 \pm 0.004(0.005)$ | $0.255 \pm 0.017(0.017)$ |

heavy flavor tagging efficiency as well as 100% electron beam polarization. For the $\eta$ model deviation from Gaussian contours is especially noticeable. Contours in the case without polarization turn out to be unstable, thus indicating marginal diagnostic power of the NLC without polarization.

In Fig. 2a 90% confidence level ($\Delta \chi^2 = 6.3$) regions are given in a three-dimensional plot of $P_{R}^q$ versus $P_{R}^d$ versus $P_{V}$ for the $\chi$, $\psi$ and $\eta$ models (the $LR$ model is in a different region of parameter space). The error-bars are again statistical and assume 100% efficiency for the heavy flavor tagging and 100% polarization for the electron beam.

We also checked how the uncertainty for the couplings are affected in the case of smaller, say 25%, heavy flavor tagging efficiency (the error-bars on the probes (8-9) increase by a factor of 2) as well as in the case that the electron beam polarization is reduced to, say, 50% (the error-bars on the probes (7) and (9) increase approximately by a factor of $\sim 2$ for small asymmetries [10]). Increased error-bars on the couplings are given in Table VI: the first (second) set of the error-bars corresponds to 25% (100%) heavy flavor tagging efficiency and 100% (50%) electron beam polarization. In the first case the uncertainties increase primarily on the quark couplings by a factor of $\sim 2$. It is seen that even 25% tagging or 50% polarization efficiency is still very useful.

The diagnostic power of the NLC for the $Z'$ couplings decreases drastically for $M_{Z'} \gtrsim 1$ TeV. E.g., for $M_{Z'} = 2$ TeV, the uncertainties for the couplings in the typical models are 100%, and thus a model-independent determination of such couplings is difficult at the NLC.

V. COMPARISON WITH THE LARGE HADRON COLLIDER

In the previous Section we have seen that at the NLC efficient heavy flavor tagging and electron beam polarization allow for a model independent determination of all of the four normalized $Z'$ couplings to quarks and leptons for a typical class of models, provided $M_{Z'} \lesssim 1$ TeV. It also yields information on the parameter $\epsilon_A$, a ratio of an overall gauge coupling strength and $M_{Z'}$, for fixed c.m. energy $s$. 

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On the other hand, at the LHC $M_{Z'}$ and the total width, $\Gamma_{Z'}$, can be measured well. The magnitude of three ($\gamma^f \ell, \bar{U}, \bar{D}$) out of four $Z'$ couplings to fermions can be well determined \cite{4} at the LHC for a typical class of models and $M_{Z'} \lesssim 2$ TeV. The fourth, $\gamma^f \ell$, requires a measurement of the branching ratio $B(Z' \to q\bar{q})$, which may be possible with appropriate kinematic cuts, excellent dijet mass resolution and detailed knowledge of the QCD background in the $Z' \to \text{jet jet}$ channel \cite{11,12}.

The analysis for the determination of ($\gamma^f \ell, \bar{U}, \bar{D}$) has been done in Ref. \cite{2}. In the main production channel ($pp \to Z' \to \ell^+\ell^-, \ell = e, \mu$) the forward-backward asymmetry and the ratio of cross sections in different rapidity bins were used. In the four-fermion final state channels the rare decays $Z' \to W\ell\nu_\ell$ (with the imposed $m_{T\ell\nu_\ell} > 90$ GeV cut on the transverse mass of the $\ell\nu_\ell$ system) and associated productions $pp \to Z'V$ ($V = Z, W$ and $V = \gamma$, with $p_T\gamma \geq 50$ GeV imposed on the photon transverse momentum) were used. Only statistical error-bars for the probes were incorporated.

The couplings were determined for the CERN LHC (c.m. energy $\sqrt{s} = 16$ TeV, integrated luminosity $L_{\text{int}} = 100$ fb$^{-1}$) \cite{3} for a class of typical models and $M_{Z'} = 1$ TeV. The results are summarized in Table II \cite{2}. In Fig. 3 we also present a three-dimensional plot, where 90% confidence level ($\Delta \chi^2 = 6.3$) regions for $\bar{U}$ versus $\bar{D}$ versus $\gamma^f \ell$ are plotted for $\chi, \psi$ and $\eta$. Note the clear separation between the models.

The couplings in Eq.(4) that are probed directly at the LHC are not sensitive to the relative signs of the $Z'$ charges. This in turn implies that couplings (2), which are observed directly at the NLC, are probed with a few-fold ambiguity at the LHC. In Table III we collect the errors expected at the LHC for the four couplings $P^f_V, P^u_R$ and $P^d_R$. We again choose the typical models and $M_{Z'} = 1$ TeV. There is an eight–fold ambiguity in determination of these couplings; only the first value of $P^f_V, P^u_R$ and $P^d_R$ corresponds to the actual values of the typical models. Note, however, that the error-bars are typically by a factor of $\sim 2$, smaller than those at the NLC (compare Tables I and III).

In Fig. 2b we plot 90% confidence level ($\Delta \chi^2 = 6.3$) regions for the $\chi, \psi$ and $\eta$ models as $P^u_R$ versus $P^d_R$ versus $P^f_V$ at the LHC. While the error-bars are small, the figure displays a few-fold ambiguity for the value of the couplings (2) (additional ambiguities are off the scale of the plot). At the NLC the error-bars are on the average larger, but the ambiguity in the value of the couplings is now removed. Thus, the LHC and the NLC are complementary and together have a potential to uniquely determine the couplings with small error-bars.

**VI. CONCLUSIONS**

We have explored the diagnostic power of the NLC (c.m. energy $\sqrt{s} = 500$ GeV, integrated luminosity $L_{\text{int}} = 20$ fb$^{-1}$) for a model independent determination of $Z'$ couplings. The analysis showed that efficient heavy flavor tagging and longitudinal polarization of the electron beam provide probes in the two-fermion final state channels, which are sensitive to the magnitude as well as the relative signs of all the $Z'$ charges to quarks and leptons. For $M_{Z'} \lesssim 1$ TeV, such couplings would be determined to about $10 - 20\%$ for a class of typical models. If the polarization were not available, the determination of the $Z'$ couplings would be marginal, since the error-bars increase by a factor of $2 - 10$. Without heavy flavor tagging very little can be learned about the quark couplings.
We took into account the tree-level expressions for the probes with their statistical errors, only. In addition, we used optimistic, though not unreasonable, assumptions for the heavy flavor tagging efficiency and the electron beam polarization. The analysis is thus useful for gaining qualitative information on the diagnostic power of the NLC for \(Z'\) couplings. If a new \(Z'\) were known to exist, a realistic fit should include full radiative corrections, experimental cuts and detector acceptances, systematic errors and error correlations. It is expected that in this case the error-bars for the couplings would increase.

In the second part of the paper we compared the diagnostic power of the NLC with the LHC. The LHC is complementary in nature; while it primarily allows for the determination of the magnitude of three out of four normalized couplings only, the corresponding errors are typically by a factor of \(\sim 2\) smaller than those for the NLC for typical models with \(M_{Z'} = 1\) TeV. In addition, the LHC would measure \(M_{Z'}\) directly and would allow for a determination of an overall strength of the \(Z'\) gauge coupling to fermions. This is in contrast to the NLC which, for the fixed c.m. energy, primarily determines only the ratio of an overall \(Z'\) gauge coupling strength and \(M_{Z'}\).

In conclusion, the analysis demonstrates the complementarity of the NLC and LHC colliders, which in conjunction allow for determination of \(M_{Z'}\), an overall \(Z'\) gauge coupling strength as well as a unique determination of all the quark and lepton charges with sufficiently small error-bars, provided \(M_{Z'} \lesssim 1\) TeV.

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[10] The error-bars $\Delta A_{LR}$ for the polarization asymmetries $A_{LR}$ are related to the amount of the beam polarization $p$ in the following way: $\Delta A_{LR} = \sqrt{1 - (pA_{LR})^2}/(p\sqrt{N})$. Here $N$ is the number of the events in the particular channel. Since in the standard model most of the asymmetries are small (see Table V), the corresponding error-bars scale approximately as $1/p$.

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Figures 1a-e go here

FIG. 1. 90% confidence level ($\Delta \chi^2 = 4.6$) contours for various pairs of couplings defined in Eqs.(2-3) for the $\chi$, $\psi$ and $\eta$ models (the $LR$ model is in a different region of parameter space) at the $NLC$ (c.m. energy $\sqrt{s} = 500$ GeV and integrated luminosity $L_{\text{int}} = 20$ fb$^{-1}$) and $M_{Z'} = 1$ TeV. Lines correspond to 100% heavy flavor tagging efficiency, and 100% longitudinal polarization of the electron beam. Only statistical error-bars for the probes are used.

Figure 2a goes here

FIG. 2. 90% confidence level ($\Delta \chi^2 = 6.3$) regions for the $\chi, \psi$ and $\eta$ models with $M_{Z'} = 1$ TeV are plotted on Fig. 2a and Fig. 2b for $P^u_R$ versus $P^d_R$ versus $P^\ell_V$ at the $NLC$ (c.m. energy $\sqrt{s} = 500$ GeV, integrated luminosity $L_{\text{int}} = 20$ fb$^{-1}$) and the $LHC$ ($\sqrt{s} = 16$ TeV, $L_{\text{int}} = 100$ fb$^{-1}$), respectively. Only statistical error-bars for the probes are included. Fig. 2b reflects a few-fold ambiguity in the determination of these couplings at the $LHC$.

Figure 3 goes here.

FIG. 3. 90% confidence level ($\Delta \chi^2 = 6.3$) regions for the $\chi, \psi$ and $\eta$ models with $M_{Z'} = 1$ TeV are plotted for $\tilde{U}$ versus $\tilde{D}$ versus $\gamma^\ell_L$ at the $LHC$ (c.m. energy $\sqrt{s} = 16$ TeV and integrated luminosity $L_{\text{int}} = 20$ fb$^{-1}$). Only statistical error-bars are included.