Magnetohydrodynamics of unsteady viscous fluid on boundary layer past a sliced sphere

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Abstract. Magnetohydrodynamics (MHD) is important study in engineering and industrial fields. By study on MHD, we can reach the fluid flow characteristics that can be used to minimize its negative effect to an object. In decades, MHD has been widely studied in various geometry forms and fluid types. The sliced sphere is a geometry form that has not been investigated. In this paper we study magnetohydrodynamics of unsteady viscous fluid on boundary layer past a sliced sphere. Assumed that the fluid is incompressible, there is no magnetic field, there is no electrical voltage, the sliced sphere is fix and there is no barrier around the object. In this paper we focus on velocity profile at stagnation point \((x = 0)\). Mathematical model is governed by continuity and momentum equation. It is converted to non-dimensional, stream function, and similarity equation. Solution of the mathematical model is obtained by using Keller-Box numerical method. By giving various of slicing angle and various of magnetic parameter we get the simulation results. The simulation results show that increasing the slicing angle causes the velocity profile be steeper. Also, increasing the value of magnetic parameter causes the velocity profile be steeper. On the large slicing angle there is no significant effect of magnetic parameter to velocity profile, and on the high the value of magnetic parameter there is no significant effect of slicing angle to velocity profile.

1. Introduction

In decades, MHD has been widely studied in many variations of geometry forms and fluid types. [1] studied MHD viscoelastic fluid with cylinder geometry form. [2] studied MHD fluid with sphere geometry form. [3] studied Viscoelastic Fluid Flow Pass A Porous Circular Cylinder When Magnetic Field Included. On the Mohammad (et al) research about MHD viscous fluid with sphere geometry form, there are some interesting conclusion. One of the conclusion is if the magnetic parameter value \(M\) increases then flow velocity significantly increases and boundary layer thickness is \(\eta = 2\). Sliced sphere is a different geometry form with a sphere, hence it is very interesting to study MHD with sliced sphere geometry form. By giving a sliced sphere geometry form, it will be appear interesting questions, is there any influence of the sliced angle to velocity profile and boundary layer thickness, and how is the influence of the slicing angle to velocity profile, if magnetic parameter variation is given.

A sliced sphere discussed is a sphere which is sliced by pair of symmetric angle \((\theta_s)\) where \(0^\circ \leq \theta_s < 90^\circ\) and sliced surface faces the fluid flow. The study focus on stagnation point \((x = 0)\).
2. Research Methods
Steps on this research is:

(i) Modeling the problem
Governing equation is constructed by physical model and then it is modified to non
dimensional model, stream function and similarity equations.

(ii) Solve the Model
The model is solved by using Keller-Box numerical method. Steps on the method are
modify the similarity equations to first order equation, convert to descret form, linearize,
and elimination block matrix.

(iii) Simulate the Model
The model is simulated by giving various of slicing angle and various of magnetic parameter.

3. Mathematical Model
While construct the mathematical model, we use coordinate system. The coordinate system is
shown on fig 1.

![Figure 1. Coordinate system of the problem.](image)

According to the coordinate system we can construct a governing equation which contains
continuity equation and momentum equation, and also initial and boundary condition.

3.1. Dimensional Model
Continuity equation
\[
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0
\]

x Momentum equation
\[
\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\mu}{\rho} \left( \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \right) - \frac{\sigma}{\rho} B^2 \bar{u}
\]

y Momentum equation
\[
\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\mu}{\rho} \left( \frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} \right) - \frac{\sigma}{\rho} B^2 \bar{v}
\]

initial and boundary condition
\[
\begin{align*}
\bar{t} < 0 & : \bar{u} = \bar{v} = 0 \text{ for any } \bar{x}, \bar{y} \\
\bar{t} \geq 0 & : \bar{u} = \bar{v} = 0 \text{ at } \bar{y} = 0 \\
\bar{u} = u_e(x) & \text{ at } \bar{y} \to \infty \\
\text{where } u_e(x) = \frac{3}{2} U_\infty \sin \left( \frac{x}{b} \right)
\end{align*}
\]
3.2. Non-Dimensional Model

By substituting the non-dimensional variable

\[ x = \frac{\bar{x}}{a}, \quad y = Re^{1/2}\frac{\bar{y}}{a}, \quad r(x) = \frac{r(x)}{a}, \quad u = \frac{u}{u_\infty}, \quad M = \frac{\sigma}{\rho}B_0^2 \]

\[ v = Re^{1/2}\frac{\bar{v}}{v_\infty}, \quad u_e = \frac{u_e(x)}{U_\infty}, \quad t = \frac{t}{U_\infty}, \quad u = \frac{\bar{u}}{U_\infty}, \quad b = \frac{\bar{b}}{a} \]

we obtain the non-dimensional model

\[
\frac{\partial r}{\partial x} + \frac{\partial r}{\partial y} = 0
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re} \frac{\partial^2 u}{\partial y^2} - Mu
\]

\[
1 \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \frac{1}{Re} \frac{\partial^2 v}{\partial x^2} + \frac{1}{2Re} \frac{\partial^2 v}{\partial y^2} - \frac{1}{Re} Mv
\]

initial and boundary condition

\[ t < 0 : u = v = 0 \text{ for any } x, y \]

\[ t \geq 0 : u = v = 0 \text{ at } y = 0 \]

\[ u = u_e(x) \text{ at } y \to \infty \]

where \( u_e(x) = \frac{3}{2} \sin(\frac{x}{2}) \)

By boundary layer approach, when the Reynold number is large such that \( \frac{1}{Re} \to 0 \) we obtain

\[ -\frac{\partial p}{\partial y} = 0 \]

\[ -\frac{\partial p}{\partial x} = u_{e} \frac{\partial u_{e}}{\partial x} + Mu_{e} \]

Then Non-dimensional model is simplified to

\[
\frac{\partial r}{\partial x} + \frac{\partial r}{\partial y} = 0
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_{e} \frac{\partial u_{e}}{\partial x} + \frac{\partial^2 u}{\partial y^2} + M(u - u_{e})
\]

3.3. Stream Function

The Non-dimensional model is converted to stream function by using this equation:

\[ u = \frac{1}{r} \frac{\partial \psi}{\partial y}, \quad \text{and} \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial x}. \]

By substituting the equation to the non-dimensional model we get stream function:

\[
\frac{\partial^2 \psi}{\partial x \partial y} = \frac{\partial^2 \psi}{\partial y \partial x}
\]

\[
\frac{1}{r} \frac{\partial^2 \psi}{\partial y \partial t} + \frac{1}{r^2} \psi \frac{\partial^2 \psi}{\partial y \partial x} - \frac{1}{r^2} \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x^2} - \frac{1}{r^2} \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = u_{e} \frac{\partial u_{e}}{\partial x} + \frac{1}{r^2} \frac{\partial^3 \psi}{\partial y^3} + M(u_e - \frac{1}{r} \frac{\partial \psi}{\partial y})
\]

\[ t < 0 : \psi = \frac{\partial \psi}{\partial y} = 0 \text{ for any } x, y \]

\[ t \geq 0 : \psi = \frac{\partial \psi}{\partial y} = 0 \text{ at } y = 0 \]

\[ \frac{\partial \psi}{\partial y} = u_{e}(x)r(x) \text{ at } y \to \infty \]

where \( u_e(x) = \frac{3}{2} \sin(\frac{x}{2}) \)
3.4. Similarity Equation

In unsteady condition, the function depends on time. It is important to distinguish between small time \((t \leq t^*)\) and large time \((t > t^*)\) where \(t^*\) can be any value, and in this research we define \(t^* = 1\). The governing equations are transformed by using the following similarity variables for small time

\[
\psi = t^{1/2}u_e(x)r(x)f(x, \eta, t), \text{ and } \eta = y/t^{1/2}
\]

for large time

\[
\psi = u_e(x)r(x)F(x, Y, t), \text{ and } \eta = Y
\]

By substituting similarity variable to the stream function we obtain similarity equation. The similarity equation contains two equations that is for small time and for large time.

**Small Time**

\[
\frac{\partial^3 f}{\partial \eta^3} + \frac{\eta \partial^2 f}{2 \partial \eta^2} + \frac{\partial u_e}{\partial x}[1 - \left( \frac{\partial f}{\partial \eta} \right)^2 + f \frac{\partial^2 f}{\partial \eta^2}] + Mt(1 - \frac{\partial f}{\partial \eta}) = \frac{\partial^2 f}{\partial \eta \partial t} + tu_e\left[ \frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial \eta \partial x} - \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial \eta^2} - \frac{1}{r} \frac{dr}{dx} f \frac{\partial^2 f}{\partial \eta^2} \right]
\]

\( t < 0 : f = f', \text{ for any } x \text{ dan } \eta \)

\( t \geq 0 : f = f', \text{ at } \eta = 0 \)

\( f' = 1, \text{ at } \eta \to \infty \)

**Large Time**

\[
\frac{\partial^3 F}{\partial Y^3} + \frac{\partial u_e}{\partial x}[1 - \left( \frac{\partial F}{\partial Y} \right)^2 + F \frac{\partial^2 F}{\partial Y^2}] + Mt(1 - \frac{\partial F}{\partial Y}) = \frac{\partial^2 F}{\partial Y \partial t} + u_e\left[ \frac{\partial F}{\partial Y} \frac{\partial^2 F}{\partial Y \partial x} - \frac{\partial F}{\partial x} \frac{\partial^2 F}{\partial Y^2} - \frac{1}{r} \frac{dr}{dx} f \frac{\partial^2 F}{\partial Y^2} \right]
\]

\( F = F', \text{ at } Y = 0 \)

\( F' = 1, \text{ at } Y \to \infty \)

3.5. Initial Condition

We have initial condition for the function when \(t = 0\). By substituting \(t = 0\) to the similarity equation we get

\[
\frac{\partial^3 f}{\partial \eta^3} + \frac{\eta \partial^2 f}{2 \partial \eta^2} = 0
\]

Then, we substitute the boundary condition to the equation, we get:

\[
f = \eta erf(\frac{\eta}{2}) + \frac{2}{\sqrt{\pi}} [e^{-\frac{\eta^2}{4}} - 1]
\]

\[
f' = erf(\frac{\eta}{2})
\]
\[ f'' = \frac{1}{\sqrt{\pi}} e^{-\frac{\eta^2}{4}} \]

4. Results and Discussion

By using Keller-Box numerical method we solve the similarity equations. Simulation results are shown at figure (2), figure (3), figure (4), and figure (5).

**Figure 2.** Velocity Profile on \( M = 0 \) and various slicing angle (\( \theta_s \)).

From figure (2) we can conclude that "the larger the slicing angle, the steeper the velocity profile".

**Figure 3.** Velocity Profile on \( M = 10 \) and various slicing angle (\( \theta_s \)).

From figure (3) we can conclude that "on the high magnetic parameter there is no significant effect of slicing angle on velocity profile".
Figure 4. Velocity Profile on slicing angle ($\theta_s = 30^\circ$) and various magnetic parameter ($M$).

From figure (4) we can conclude that "the higher the magnetic parameter, the steeper the velocity profile".

Figure 5. Velocity Profile on slicing angle ($\theta_s = 70^\circ$) and various magnetic parameter ($M$).

From figure (5) we can conclude that "on the large Slicing angle there is no significant effect of magnetic parameter on velocity profile".

5. Conclusion
Both increasing slicing angle and increasing magnetic parameter cause the velocity profile be steeper and boundary layer thickness be thinner. On the large slicing angle, there is no significant effect of magnetic parameter to velocity profile. And on the high magnetic parameter, there is no significant effect of slicing angle to velocity profile.

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