Low-order dynamic model of a domestic electric oven
Part II: parameter identification and model validation for the static heating mode

Michael Lucchi*, Marco Lorenzini*, Giovanni Roberti*
*Alma Mater Studiorum - Università di Bologna, Forlì, Via Fontanelle 40, Italy
E-mail: michael.lucchi2@unibo.it

Abstract. Appliances are responsible for almost the 25% of the overall environmental impact of buildings, and, among these, electric domestic ovens represent a low-efficiency category when considering flats, with an energy efficiency between 10% and 12%, thus stimulating the development of more efficient technologies. The energy performance of such an appliance is determined through a test according to the EN 60350-1 European standard, which, among others, requires suitable control of the oven centre temperature. Since the development of reliable dynamic models represents a high-potential tool to design better control strategies, the aim of this work is to create a model simulating the transient thermal behaviour during the static, i.e. non-ventilated, heating mode of an electric domestic oven within the cabinet adopted for energy consumption tests; the model must be able to predict the main temperatures within the cavity, with particular attention to the oven centre. A lumped parameter approach based on the thermoelectric analogy was chosen, in order to obtain a low-order dynamic model suitable for control design purposes. Since the model strives to reproduce the transient behaviour of the oven centre temperature, a single lumped parameter simulating the thermal behaviour of the whole oven cavity was unsuitable and a certain level of macroscopic discretization had therefore to be introduced. Thus, the oven cavity was modelled with eight parameters, representing its main parts like the walls, the glass door and the air in the cavity; also, the temperature probe, a Pt500 as is customary for such devices, was included in the model, since this is the only temperature feedback during operation and its presence in the model could lead to an easier implementation of efficient control logics. Three lumped parameters simulating the thermal behaviour of the heaters were also considered, leading to an 11th order model.

1. Introduction
Household appliances contribute to almost 25% of the overall environmental impact of buildings, as reported in [1]; among them, electric ovens represent significant sources of energy consumption when considering apartments, because of their very low energy efficiency (between 10% and 12%), which stimulates the development of new techniques, [2]-[3]. The EN 60350-1 European standard, [4], which regulates the energy consumption characterization of domestic electric ovens, requires a proper control of the oven center temperature, thus making the design of suitable control strategies an important field of investigation. Dynamic models can help designers to obtain control laws suitable for different purposes (e.g. minimizing the energy consumption, increasing the control precision), and this work aims at developing a model to simulate the transient thermal behaviour of an electric domestic oven, reproducing the main temperatures
within the cavity and, in particular, the oven center temperature. Several works can be found in the literature about models of domestic ovens in transient regime, ranging from low to high levels of complexity. As to the former, the work of Abraham and Sparrow, [5], can be mentioned, who developed a simple model based on the first law of thermodynamics for closed systems, in order to predict the temperature of a thermal load placed inside an electric oven, but without giving information about the oven center temperature. Good examples of more complex models are the works where CFD techniques were applied to study the transient behaviour of ovens. Among these works are those of Mistry et al., who investigated the transient natural convection during the broiling and baking cycles in electric ovens, considering the radiation from the heating elements too, [6], and the heat transfer and fluid flow inside a residential gas oven, representing oven burners as wall temperature boundary conditions and trying to find a satisfactory compromise between computational time and accuracy, [7].

When considering models for control design purposes, high-complexity approaches like those adopting CFD are unsuitable: for several thermal systems, like e.g. vapour compression refrigerating machines [8], the best choice is usually represented by low-order dynamic models, often set up through a lumped parameters approach, since it represents a good trade-off between simplicity and accuracy.

On the basis of what was previously presented in [9], Laboreo et al. realized a lumped-parameters dynamic model of a domestic oven with a thermal load (taking into account the water evaporation rate too) using the thermoelectric analogy and investigating both the forced convective and the radiative coking modes, [10]. However, a single lumped parameter was used to represent the whole inner cavity, including metal sheets and air, thus making it impossible to estimate the oven center temperature per se, whose control is required during the energy consumption test. In [11], the dynamic behaviour of a professional electric oven was investigated through the lumped capacitance method, also considering the radiative heat exchange between surfaces and heaters without linearization, approximating the latter with planar surfaces to estimate their view factors. Yet, the validation shown in the paper was carried out at a single temperature set-point, tuning the average convection coefficients between air and walls and the thermal transmittances between air and heaters and between air and thermal masses so as to obtain a good agreement between numerical results and experimental data.

In this work, a control-oriented low-order lumped parameters model of an electric domestic oven without thermal load has been developed in MATLAB/Simulink®, and its transient behaviour in natural convective (i.e. static) heating mode was investigated. Different elements of the cavity (eg. walls, door, heaters) were included in the model, but particular attention was paid to the model ability to reproduce the transients of the oven center temperature. Also the radiative heat exchanges within the cavity were taken into account, without adopting linearization procedures, as a consequence of the benefits highlighted by the same authors in [12]. Moreover, the transient behaviour of the Pt500 sensor was included within the model, since it represents the only temperature feedback available for the control system during normal operating conditions; in this way, the actuation of simulated control logics in a real oven can be facilitated. Model parameters were identified via optimization from experimental data at a given temperature set-point and model validation was then carried out considering data acquired at different set-points.

2. The oven

The oven considered is a commercial model with a 72 dm³ inner cavity volume placed within a typical cabinet for energy consumption tests. The oven is equipped with three heating elements: a 1900 W ring heater (RH), a 2300 W top heater (TH) and a 1000 W bottom heater (BH). In the static heating mode considered here, TH and BH are the active heaters. The details about the measurements setup and the experimental campaign are reported in Part I of this paper, [13].
3. Model

The oven model was developed on the basis of the thermoelectric analogy, thus interconnecting a set of capacitors by means of thermal conductances. While CFD models give a more detailed physical insight of the problem usually requiring long simulation times, the lumped-parameter approach based on the thermoelectric analogy allows to significantly reduce the computational cost, which has a fundamental importance when developing control-oriented models and must be as low as possible, and at the same time it offers a good level of accuracy when applied to transient heating and cooling problems, [10, 14]. Since the main aim was to reproduce the transients of the oven center and the temperatures of the Pt500, a minimum level of macroscopic discretization was necessary. Thus, the oven cavity has been divided into seven main parts, representing five walls (bottom (BW), right (RW), left (LW), rear (PW), top (TW)), the glass door (D) and the air cavity (OC) and three parameters simulating the thermal behaviour of the heaters were considered (bottom (BH), top (TH) and, although not activated, ring heater (RH), in order to be able to investigate in the future also the dynamics of the forced convection mode).

Considering the lumped capacitance introduced for the Pt500 sensor, a 11th order model was obtained. A scheme of the thermal network is shown in Fig. 1. In the static heating mode, since the cavity walls see high temperature elements like the top heater and because of the relatively low temperature characterizing the door, significant radiative heat fluxes can occur and influence the thermal phenomena of the whole cavity, especially during the transients. The influence of radiative terms in the thermal dynamics of the oven has been considered through fourth-power terms, as explained just below.

Equations (1)-(11) describe the energy balances for the 11 parameters constituting the model. $\dot{Q}_{ij}$ are linear terms representing the heat transfer between two generic nodes $i$ and $j$, expressed as $G_{ij} \cdot (T_i - T_j)$, where $G_{ij}$ is the thermal conductance between nodes $i$ and $j$, while $T_i$ and $T_j$ are the temperatures of the corresponding nodes. The radiative heat transfer is expressed through the terms $\dot{Q}_{ij}^{\text{rad}}$, calculated as $G_{ij}^{\text{rad}} \cdot \sigma \cdot (T_i^4 - T_j^4)$, where $G_{ij}^{\text{rad}}$ is the radiative thermal conductance.
conductance between nodes $i$ and $j$, $\sigma$ is the Stefan-Boltzman constant and $T_i$ and $T_j$ are the node temperatures expressed in kelvins. Both $G_{i,j}$ and $G^\text{rad}_{i,j}$ obey a reciprocity law, such that $G_{i,j} = G_{j,i}$ and $G^\text{rad}_{i,j} = G^\text{rad}_{j,i}$. $P_{TH}$ and $P_{BH}$ represent the electric power absorbed by the top and bottom heaters respectively and are model inputs, whilst the ambient temperature $T_{\text{amb}}$, which influences energy dispersions, is kept constant at $23^\circ C$ and is treated as a perturbation to the model. The ambient temperature is at $23^\circ C$, which can be considered reasonable, since it lays between the temperature set-points usually considered for domestic environments, namely $20^\circ C$ in the winter and $26^\circ C$ in the summer, [15].

$$C_{OC} \frac{dT_{OC}}{d\tau} = Q_{RW} \cdot OC + Q_{LW} \cdot OC + Q_{TW} \cdot OC + Q_{PW} \cdot OC + Q_{D} \cdot OC + Q_{Pt} \cdot OC$$

$$+ Q_{TH} \cdot OC + Q_{BH} \cdot OC + Q_{RH} \cdot OC$$

$$C_{TH} \frac{dT_{TH}}{d\tau} = Q_{OC} \cdot TH + Q_{TW} \cdot TH + Q^\text{rad}_{TW} \cdot TH + Q_{PW} \cdot TH + Q^\text{rad}_{PW} \cdot TH + Q_{BW} \cdot TH + Q^\text{rad}_{BW} \cdot TH$$

$$+ Q_{LW} \cdot TH + Q^\text{rad}_{LW} \cdot TH + Q_{D} \cdot TH + Q^\text{rad}_{D} \cdot TH + Q_{Pt} \cdot TH + P_{TH}$$

$$C_{BH} \frac{dT_{BH}}{d\tau} = Q_{OC} \cdot BH + Q_{BW} \cdot BH + Q^\text{rad}_{BW} \cdot BH + Q_{PW} \cdot BH + Q_{BW} \cdot BH + Q_{TW} \cdot BH + Q_{D} \cdot BH + P_{BH}$$

$$C_{RH} \frac{dT_{RH}}{d\tau} = Q_{OC} \cdot RH + Q_{AMB} \cdot RH$$

$$C_{TW} \frac{dT_{TW}}{d\tau} = Q_{OC} \cdot TW + Q_{TH} \cdot TW + Q^\text{rad}_{TH} \cdot TW + Q_{AMB} \cdot TW + Q_{Pt} \cdot TW + Q^\text{rad}_{Pt} \cdot TW + Q_{PW} \cdot TW$$

$$+ Q_{LW} \cdot TW + Q^\text{rad}_{LW} \cdot TW + Q^\text{rad}_{D} \cdot TW$$

$$C_{BW} \frac{dT_{BW}}{d\tau} = Q_{OC} \cdot BW + Q_{BH} \cdot BW + Q^\text{rad}_{BH} \cdot BW + Q^\text{rad}_{PW} \cdot BW + Q^\text{rad}_{BW} \cdot BW + Q^\text{rad}_{LW} \cdot BW + Q^\text{rad}_{D} \cdot BW + Q_{AMB} \cdot BW$$

$$C_{RW} \frac{dT_{RW}}{d\tau} = Q_{OC} \cdot RW + Q_{TH} \cdot RW + Q^\text{rad}_{TH} \cdot RW + Q_{BH} \cdot RW + Q^\text{rad}_{BW} \cdot RW + Q^\text{rad}_{TW} \cdot RW + Q^\text{rad}_{D} \cdot RW$$

$$+ Q_{AMB} \cdot RW$$

$$C_{LW} \frac{dT_{LW}}{d\tau} = Q_{OC} \cdot LW + Q_{TH} \cdot LW + Q^\text{rad}_{TH} \cdot LW + Q_{BH} \cdot LW + Q^\text{rad}_{BW} \cdot LW + Q^\text{rad}_{TW} \cdot LW + Q^\text{rad}_{D} \cdot LW$$

$$+ Q_{AMB} \cdot LW$$
As shown in Eq.(1), the oven center exchanges thermal power with each element of the grid. Equations (2)-(3) describe the energy balance of the top and bottom heaters, which are connected to the side walls (RW, LW, PW) and the door (D), whilst direct connections between bottom heater and top wall, and top heater and bottom wall were neglected. Moreover, radiative terms between the bottom heater and the bottom wall, and between the top heater and each wall in the cavity are considered. Equation (4) shows the energy balance for the ring heater, which receives no electric power in the static mode and exchanges heat with the oven center and the ambient only. Equations (5)-(10) describe the energy balance for the cavity walls and the door, where dispersions towards the ambient are included. As shown in Eqs. (5)-(6), the top and bottom walls are coupled with the other walls and the door through radiative terms, because they take higher temperatures due to heater proximity. Fourth-power terms coupling the door node with those associated to other walls are considered, because of its relatively low temperature, see Eq. (10). The energy balance of the Pt500 sensor is described by Eq.(11): in order to consider the temperature stratification typical of the natural convective heating mode, a linear term coupling the sensor node to the top wall node was introduced in addition to the heat exchange with the oven center. In addition, two non-linear terms coupling the Pt500 sensor to the top wall and the top heater are present to estimate the radiative heat exchange. Finally, the radiative heat exchange between the bottom wall and the top elements (wall and heater) was neglected, since several optimization attempts in the parameters identification process highlighted over-estimations of $T_{BW}$ when these non-linear terms were taken into account, leading to less satisfactory results in the estimation of the oven center temperature.

4. Parameters identification

On the basis of experimental data (temperatures and electric power absorbed by the heaters) obtained in a three-hours test at 200°C (see Part I of this article), which represents the central value among the three temperature set-points at which to carry out the energy consumption test of the oven in non-ventilated mode, as prescribed by the European standard, [4], an optimum set of capacities and conductances was found through an optimization procedure conducted with the Parameter Estimation toolbox available in MATLAB/Simulink®. The Non-Linear-Least-Squares method with the Trust-Region-Reflective algorithm was applied, and the cost function minimized was:

$$f = \sum_{i=1}^{n} w_i \cdot \delta_i = \sum_{i=1}^{n} w_i \cdot \frac{\sum_{k=1}^{N} (T_{model}^{i}(k) - T_{exp}^{i}(k))^2}{N}$$

(12)
The weight $w_i$ for the mean square error $\delta_i$ associated to the i-th lumped parameter, calculated between the temperature evaluated by the model $T_{\text{model}}^i(k)$ and the experimental data $T_{\text{exp}}^i(k)$ at each of the $N$ time-steps of the simulation, was assumed equal to unity for each of the eleven nodes of the thermal grid, with the aim of reproducing equally well almost all the temperature trends within the cavity. The initial estimation of thermal capacities of the walls and the glass door was carried out considering the series of metal sheets, insulation layers and wooden walls capacities for the former and the series of four glass layers and air cavities for the latter. In particular, the relationship $1/C_i = \sum_{l=1}^{L} 1/(m_l \cdot c_l)$ was applied, where $C_i$ is the initial capacity of the i-th parameter and $m_l$ and $c_l$ are the mass and the specific heat capacity of the l-th layer. As for the initial estimation of Pt500 and heaters capacities, the product between their mass and the specific heat capacity of AISI 430 steel was assumed, while the sum of thermal capacities of cavity air $C_{\text{air}}$ and metallic grid $C_{\text{grid}}$ was considered as initial estimation of oven center capacity. Neglecting contact resistances, a series of thermal resistances which took both the insulation layers and the furniture into account was used to calculate the initial values of conductances between the i-th lumped wall and the environment; as for the heat exchanges between cavity air and walls, the first estimation of the conductances was carried out through the correlations for natural convection on a vertical or horizontal surface available in [14]. A manual tuning procedure was carried out to estimate the initial values of conductances between the oven center and the heaters, the glass door and the environment and between the heaters and the side walls, because of the large amount of uncertainty which characterizes these quantities. Since the main aim of the model was to correctly predict the transients of the main temperatures within the cavity in response to a particular input electric power forcing function, neglecting the quantification of the heat exchanges between the grid nodes, a large field of variability was allowed for the model parameters in the optimization procedure, in order to ensure the best possible predictive capacity for the model. A fixed-step Runge-Kutta method (Matlab ode4) with a time-step of $1 \text{ s}$ was chosen as solver for the integration in the time domain for both parameter identification and validation.

In order to investigate the predictive capability of the model, the electric power absorbed by the heaters in tests carried out at temperature set-points different from that used for the parameters identification ($200^\circ \text{C}$) was used as a model input and the predicted temperatures were then compared with experimental data. Several simulations of about-three-hours heating cycles were performed: the validations reported here refer to temperature set-points $T_{\text{set}}$ of $180^\circ \text{C}$ and $220^\circ \text{C}$.

5. Results

As anticipated in Sec. 3, the electric power absorbed by the heaters is an input of the model; an example of such an input is shown in Fig. 2, where the temporal profile of the electric power absorbed by the top and bottom heaters during a test at $200^\circ \text{C}$ is plotted.

Figure 3 compares the values of the oven center temperature predicted by the model with the experimental data. In the optimization test-case at $200^\circ \text{C}$, see Fig. 3-(a), the percentage displacement $\Delta$ remains under 2% for almost the whole duration of the simulation, with a peak at the beginning: this spike in the relative error appears for all the nodes of the grid, but can be considered insignificant, as it occurs when temperatures are very low. The trend of $T_{\text{OC}}$ predicted by the model in the validation tests at $180^\circ \text{C}$ and $220^\circ \text{C}$ is shown in Figs. 3 (b)-(c): the model shows good predictive potentiality, since the percentage displacement is confined below 5%, with the mean temperature reached at the stabilized regime which is slightly underestimated at $T_{\text{set}} = 180^\circ \text{C}$ and overestimated at $T_{\text{set}} = 220^\circ \text{C}$. Although it can be considered negligible for control design purposes, a time-delay in achieving the stabilized regime has been highlighted for the test at $180^\circ \text{C}$, probably due to an overestimation of thermal capacities.

The model response is good also when considering the temperature trend predicted for the Pt500
Figure 2: Model power inputs. Static Mode 200°C. (a) Top heater. (b) Bottom heater.

probe, both in the optimization case, Fig. 4 (a), and in the validation procedure, Fig. 4 (b)-(c). A time-delay in reaching the stabilized regime and a slight overestimation of the mean stabilized temperature appear also for the Pt500 in the validation test-cases at 180°C and 220°C, as shown in Fig. 4 (b)-(c): this can be attributed to the oven center dynamics, which directly influences the transient behaviour of the probe. The temperature percentage displacement remains always below 5% in the stabilized regime in both validation tests, a maximum overestimation of the highest temperature sensed of about 5.5% occurring at 180°C set-point.

Figures 5-6 show the results obtained for the top heater and the glass door in the optimization test case (Figs. 5 (a) and 6 (a)) and in the two validation tests (Figs. 5 (b)-(c) and 6 (b)-(c)). The good predictive capability of the model is confirmed: for the top heater, the maximum percentage displacement $\Delta$ appears during the heating phase (about 500 s long) and ranges from 8.7% ($T_{\text{set}} = 200^\circ$C) to 11% ($T_{\text{set}} = 220^\circ$C), while a maximum error in the stabilized regime slightly above 4% is obtained for the door in the validation test at $T_{\text{set}} = 220^\circ$. The underestimation and overestimation highlighted in the predicted temperatures of the oven center and the Pt500 in the validation cases appear also for the door node: this behaviour may be caused by the assumption of temperature independent conductances, whilst natural convection, which together with irradiation represents the main heat transfer mechanism within the cavity when the oven is in non-ventilated mode, is strongly temperature-dependent.

6. Conclusions
The transient thermal behaviour of a domestic electric oven in natural convective mode has been investigated through a low-order lumped parameter model based on the thermoelectric analogy and suitable for control design purposes. Fourth-power terms were introduced to take into account the radiative heat exchange between the elements of the cavity. In order to facilitate the development of new control laws, the dynamics of the Pt500 probe was also considered in the model, since it represents the only temperature feedback during normal oven operation. The model is of 11th order and includes cavity walls, the glass door, the heaters, the cavity air and the Pt500 probe, and takes the electric power absorbed by the heaters as inputs and the surrounding temperature as disturbance. The set of capacities and conductances which allow to best predict the temperature's temporal trends of the elements in the heated cavity was identified through optimization based on experimental data obtained at a temperature set-point of 200°C and on the Non-Linear-Least-Squares method. Validations were then carried out at different temperature set-points. The model showed good predictive capabilities, with percentage errors with respect to
Figure 3: Oven center temperature prediction in the static heating mode. (a) Optimization at $T_{\text{set}} = 200^\circ\text{C}$. (b) Validation at $T_{\text{set}} = 180^\circ\text{C}$. (c) Validation at $T_{\text{set}} = 220^\circ\text{C}$.

Figure 4: Pt500 temperature prediction in the static heating mode. (a) Optimization at $T_{\text{set}} = 200^\circ\text{C}$. (b) Validation at $T_{\text{set}} = 180^\circ\text{C}$. (c) Validation at $T_{\text{set}} = 220^\circ\text{C}$.
Figure 5: Top heater temperature prediction in the static heating mode. (a) Optimization at $T_{set} = 200^\circ C$. (b) Validation at $T_{set} = 180^\circ C$. (c) Validation at $T_{set} = 220^\circ C$.

Figure 6: Door temperature prediction in the static heating mode. (a) Optimization at $T_{set} = 200^\circ C$. (b) Validation at $T_{set} = 180^\circ C$. (c) Validation at $T_{set} = 220^\circ C$. 

experimental data always below 5% both for the oven center and Pt500 temperatures: since one of the requirements of EN 60350-1 is to correctly control the oven center temperature, the usefulness of the model is confirmed. The model also exhibited good reliability in predicting the temperature of the other components of the oven.

Other forms of the model have been developed with the same numerical approach, in order to carry out transient analysis also for the forced convective heating mode, with and without thermal load: the main results of those analyses will be published at a later stage. Further developments include the investigation of the predictive capability in case of the temperature-dependent thermal conductances, thus leading to a model with non-linear parameters.

**Acknowledgements**

Funding of this research by Electrolux Italia S.p.a. is gratefully acknowledged.

**References**

[1] Hoxha E and Jusselme T 2017 *Science of the Total Environment*, 596–597 pp. 405–416
[2] Amienyo D, Doyle J, Gerola D, Santacatterina G and Azapagic A 2016 *Sustainable Production and Consumption*, 6 67–76
[3] Bansal P, Vineyard E and Abdelaziz O 2011 *Applied Thermal Engineering*, 31 3748–3760
[4] CENELEC 2013 En 60350-1 household electric cooking appliances part 1: Ranges, ovens, steam ovens and grills methods for measuring performance (latest version including all amendments).
[5] Abraham J and Sparrow E 2004 *Journal of Food Engineering*, 62 409–415
[6] Mistry H, Ganapathisubbu S, Dey S, Bishnoi P and Castillo J 2006 *Applied Thermal Engineering*, 26 2448–2456
[7] Mistry H, Ganapathisubbu S, Dey S, Bishnoi P and Castillo J 2011 *Applied Thermal Engineering*, 31 103–111
[8] McKinley T and Alleyne A 2008 *International Journal of Refrigeration* 31 1253–1264
[9] Ramirez-Laboreo E, Sagues C and Llorente S 2014 2014 22nd Mediterranean Conference on Control and Automation, MED 2014 pp 505–510
[10] Ramirez-Laboreo E, Sagues C and Llorente S 2016 *Applied Thermal Engineering*, 93 683–691
[11] Burlon F, Tiberi E, Micheli D, Furlanetto R and Simonato M 2017 *Energy Procedia*, 126 2–9
[12] Lucchi M and Lorenzini M 2019 *Applied Thermal Engineering* 147 438–449
[13] Lucchi M, Lorenzini M and Di Paola V 2018 *Proceedings of 36th UIT Heat Transfer Conference*
[14] Bergman T, Lavine A, Incropera F and Dewitt D 2011 *Fundamentals of Heat and Mass Transfer* (Hoboken: John Wiley & Sons)
[15] Lucchi M, MLorenzini and Valdiserri P 2017 *Journal of Physics: Conference Series*, 796