Attractive and Repulsive Gravity*

Philip D. Mannheim
Department of Physics, University of Connecticut, Storrs, CT 06269
mannheim@uconnvm.uconn.edu

Abstract

We discuss the circumstances under which gravity might be repulsive rather
than attractive. In particular we show why our standard solar system distance
scale gravitational intuition need not be a reliable guide to the behavior of
gravitational phenomena on altogether larger distance scales such as cosmo-
logical, and argue that in fact gravity actually gets to act repulsively on such
distance scales. With such repulsion a variety of current cosmological prob-
lems (the flatness, horizon, dark matter, universe age, cosmic acceleration and
cosmological constant problems) are then all naturally resolved.

I. INTRODUCTION

Few observational facts appear to be as well established in physics as the attractive na-
ture of gravity. However, despite this, recent cosmological observations [1,2] have raised the
possibility that under certain circumstances gravity might actually contain an effective repul-
sive component, to thus invite consideration of the degree to which, and of the specific set of
conditions under which, gravity actually need be strictly attractive in the first place. In fact,
familiar as attractive gravity is, its actual attractiveness stems from the a priori assumption
that Newton’s constant $G$ be chosen (purely by hand) to actually be positive. Then with this
positive $G$ being treated as a fundamental input parameter by both Newtonian gravity and
its relativistic Einstein gravity generalization (and even by their quantum-mechanical string
theory generalization as well for that matter), the universal attractive nature of gravity is
then posited on all distance scales and for all possible gravitational field strengths. As such,
this actually constitutes a quite severe extrapolation of observationally established gravita-
tional information from the kinematic solar system distance scale weak gravity regime where
it was expressly obtained in the first place. And indeed, when the standard Newton-Einstein
gravitational theory is actually extended beyond its solar system origins, notwithstanding
the successes that are then encountered, nonetheless, disturbingly many difficulties are also
encountered, in essentially every single such type of extrapolation that is in in fact made.
Thus its extrapolation to larger distances such as galactic leads to the need for as yet un-
established galactic dark matter, its extrapolation to strong gravity leads to singularities
and the development of event horizons and trapped surfaces in the fabric of spacetime, its

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*gr-qc/0001011, December 31, 1999
extrapolation to the high temperature early universe leads to a cosmology with fine tuning and cosmological dark matter problems as well as to the notorious cosmological constant problem, and its extrapolation to large quantum field theoretic momenta far off the mass shell leads to uncontrollable renormalization infinity problems. Now while all of these issues may ultimately be resolved in favor of the standard theory, it is important to emphasize that all of them essentially arise from using just the first few measured weak, perturbative terms in a series (such as the first few terms in the Schwarzschild metric, the only such terms in that metric which have so far been observationally tested in fact) to try to guess the rest of the series. With there thus being many possible extrapolations of the standard Schwarzschild weak gravity solar system wisdom, extrapolations which can be just as covariant as the standard one, there are thus many possible departures from the standard gravitational intuition when gravity is extended to altogether different conditions, and in this paper we shall explore the possibility that, even as it acts attractively on solar system distance scales, gravity nonetheless gets to act repulsively on the much larger one associated with cosmology.

II. HOW THE STANDARD INTUITION CAME ABOUT

In developing a fundamental theory of planetary motion Newton found that an inverse square force law with a universal gravitational constant $G$ not merely accounted for the Keplerian elliptical orbital motions of the planets around the sun, but equally, it also described the gravitational motions of objects near the surface of the earth, i.e. it described gravity not only on solar system distance scales but also on altogether smaller terrestrial distance scales as well. In this way Newton’s law of gravity thereby acquired a universal character causing it to come to be regarded as a universal law which was then to also be valid on altogether larger distance scales as well even though it had not in fact been tested on them. However, with the advent of galactic astronomy eventually then permitting such testing on these altogether larger distance scales, it was actually found (see e.g. [3,4] for recent reviews) that the detailed orbits of stars and galactic gas in galaxies did not in fact conform to the ones expected on the basis of the known luminous matter content of the galaxies. In fact long ago Zwicky [5] had already noted an analogous problem in (presumed virialized) clusters of galaxies, with the discrepancy he found between the measured mean kinetic energy of the visible galaxies within the cluster and the mean gravitational potential energy generated by those selfsame galaxies leading him to conclude that the continuing applicability of Newton’s law of gravity entailed that there would have to be far more matter in the cluster than he was able to detect. Then, once it became clear that discrepancies such as these were in fact common and even widespread on these large distance scales, a dark matter paradigm was then adopted, a paradigm in which any detected discrepancy between the known luminous matter Newtonian expectation and observation was to then be accounted for by whatever amount of non-luminous matter would then (i.e. only after the fact) specifically be required. Moreover, with there not as yet being a single application or test of Newtonian gravity on these large distance scales (in either galaxies or clusters of galaxies) which does not involve an appeal to this as yet unestablished and still poorly understood dark matter, we thus see the complete circularity of the chain of reasoning which leads to dark matter in the first place, since one assumes the continuing validity of Newton’s law of gravity and then posits
the presence of just the appropriate amount of dark matter needed in order to maintain the validity of the presupposed Newtonian law. Apart from not being a particularly satisfying prescription, this galactic dark matter paradigm does yet even qualify as being a falsifiable theory (the sine qua non of physical theory) since it has not yet been brought to the point where it can actually make definitive and expressly falsifiable galactic predictions, i.e. given a detected amount of luminous matter in a given galaxy, dark matter theory should be able to predict both the amount and spatial distribution of the dark matter in the galaxy in advance of any measurement of a galactic rotation curve. Moreover, even more disturbing than this is the concomitant need of having to now retroactively explain why dark matter is not also required in order to fit solar system planetary orbits, i.e. the need to explain why luminous matter alone should in fact be capable of providing a complete accounting of solar system dynamics in the first place. A model in which dark matter is there when needed and not when not hardly qualifies as being a model at all, with its most conservative characterization actually being that in fact dark matter is nothing more than a phenomenological parameterization of the detected departure of the luminous Newtonian expectation from observation. This then is the galactic dark matter problem, and while it may eventually be resolved by the actual direct detection of dark matter, nonetheless it could equally well be signaling a failure of the standard Newtonian wisdom and intuition on these very large distance scales.

While some Newtonian wisdom was in fact supplanted by the subsequent development of Einstein relativity, it is curious that its development in fact actually served to reinforce the particular Newtonian intuition that we described above. Specifically, even while the general relativistic curved spacetime Einstein gravitational theory replaced the strictly non-relativistic Newtonian gravitational one, nonetheless the Einstein theory still recovered the Newtonian theory in the non-relativistic limit, even as it prescribed relativistic corrections to it. The actual observational confirmation of these relativistic corrections not merely served to establish the validity of Einstein gravity, it also served to reinforce the validity of Newtonian gravity whenever the non-relativistic limit could appropriately be taken. However, since these relativistic corrections were themselves only established on solar system distance scales (cf. the first few terms in a perturbative solar system Schwarzschild metric expansion or the first few perturbative terms in the metric of a binary pulsar, a similarly sized such system), the extrapolation of Einstein gravity (and of its universal character) to galactic distance scales and beyond was then no more secure than had been the extrapolation of Newtonian gravity to those same distance scales (with the Einstein equations thus only being as secure as dark matter). Thus again a new intuition was acquired, one in which the very presence of Newton’s constant as an a priori fundamental coupling constant in the Einstein-Hilbert action then endowed $G$ with an even more fundamental and universal status than it had actually previously possessed. Moreover, in giving Newton’s constant such a status, a dichotomy is immediately set up between gravity and the standard $SU(2) \times U(1)$ electroweak theory, a theory where another dimensionful phenomenological parameter, Fermi’s constant $G_F$, is not in fact elevated into a fundamental parameter at all, a theory which can then be made completely renormalizable precisely because this is not in fact done, with $G_F$ itself then emerging solely as an effective parameter which is only of relevance at low energies.

That Einstein gravity was taken to be universal was hardly surprising given its very general geometric character. However, it is important to distinguish between the geometric
and dynamical aspects of the Einstein theory, and even while these two issues are logically distinct, nonetheless the Einstein theory is ordinarily regarded as being one integral and indivisible package. However, the standard theory both geometrizes gravity by identifying the spacetime metric $g^{\mu\nu}$ as the gravitational field, and then determines its dynamics by imposing the second order Einstein equations

$$R^{\mu\nu} - g^{\mu\nu} R^\alpha_\alpha / 2 = -8\pi G T^{\mu\nu}$$  \hspace{1em} (1)$$

where $R^{\mu\nu}$ is the Ricci tensor associated with the geometry and $T^{\mu\nu}$ is the energy-momentum tensor of its gravitational source. However, even while insisting that the metric is to describe the gravitational field, nonetheless, without giving up general covariance, its dynamical equations of motions could still depart from the above second order Einstein equations, and would in fact readily do so if the gravitational equations were to be obtained from the variation of some equally covariant general coordinate scalar action other than the standard Einstein-Hilbert one ($I_{EH} = - \int d^4x (-g)^{1/2} R^\alpha_\alpha / 16\pi G$) which is ordinarily used. (In fact the fully covariant string gravitational theory, for instance, replaces the Einstein equations by a set of equations which contains an entire, infinite series of derivatives of the Riemann tensor.) One thus has to distinguish between the fact of curvature (viz. geometry) and the amount of curvature (viz. dynamics), and in particular one has to go over both the successes and the problems of Einstein gravity to ascertain which are due to geometry and which to dynamics, and should it turn out that the successes are predominantly due to geometry while the problems arise from a particular assumed dynamics, we would then be able to identify the extrapolation of the standard gravity equations of motion beyond their solar system origin as the root cause of the problems that standard theory currently encounters, while not at the same time needing to give up the underlying geometrical picture.

Indeed, the very cornerstone of standard gravity, viz. the equivalence principle, is completely geometrical and has nothing to do with dynamics at all. Rather it is a statement about the geometric nature of geodesics, with particles which follow such geodesics then having to uniquely couple to an external gravitational field according to

$$\frac{d^2 x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0$$  \hspace{1em} (2)$$

regardless of the particular form of the equation of motion obeyed by the external gravitational field itself. Thus, even while the magnitudes of the Christoffel symbols are of course sensitive to the dynamics associated with the background gravitational field, nonetheless, their very presence in the geodesic equations to begin with is strictly geometric. Gravitational bending, lensing, redshifting and time delaying of light, modifications to Newtonian planetary orbits, and the decay of the orbit of a binary pulsar (a consequence of the retarded nature of the gravitational radiation reaction of each of the two stars in the binary on the other due to the finite limiting velocity with which gravitational information is communicated) will thus all be found to occur in any strictly covariant metric theory of gravity, with the dynamical equations only controlling the magnitude of these effects but not the fact of their existence.¹

¹In passing it is important to note that there is actually a hidden assumption in using geodesic
Moreover, as regards the uniqueness of any possible underlying dynamics, it was noted by Eddington in the very early days of relativity that the familiar standard gravity exterior $R^{\mu\nu} = 0$ Schwarzschild solution (the one used in the standard solar system relativistic tests) is just as equally a solution to higher derivative gravitational theories as well, since the vanishing of the Ricci tensor entails the vanishing of its derivatives as well, with solar system tests thus not in fact being able to definitively exclude gravitational actions other than the Einstein-Hilbert one after all. Moreover, since such higher derivative theories turn out to then have different continuations to larger distances, the possibility then emerges that the need for galactic dark matter is only an artifact of using the Einstein gravity continuation. And should that be the case, the further continuation to cosmology (a regime which is to a good degree controlled by the high symmetry of geometries such as Robertson-Walker and de Sitter rather than by the structure of the explicit dynamical evolution equations themselves) would then potentially become unreliable too.

In order to illustrate the above remarks in an explicit example it is convenient to consider a particular alternate gravitational theory, viz. conformal gravity, a general coordinate invariant pure metric theory of gravity which possesses an explicit additional and highly restrictive symmetry (invariance of the geometry under any and all local conformal stretchings $g_{\mu\nu}(x) \rightarrow \Omega^2(x)g_{\mu\nu}(x)$) as well. In fact, so restrictive is this symmetry that it allows only one unique gravitational action (an action which is to thus replace the standard $I^{EH}$), viz.

$$I_{W} = -\alpha_{g} \int d^{4}x (-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa}$$

(3)

where $C^{\lambda\mu\nu\kappa}$ is the conformal Weyl tensor and where the gravitational coupling constant $\alpha_{g}$ introduced here is a universal dimensionless one, to thereby endow gravity with a structure similar to that found in the electroweak interaction case described above, with such a gravitational action.

To conclude, the equations to describe gravitational phenomena, viz. the assumption that real particles can be treated as classical geodesic test particles in the first place. Now while the geodesic assumption is immediately valid for the massless rays of the eikonal approximation to wave theory (since the rays are already geodesic in flat spacetime, with a covariantizing of their motion then making them geodesic in a background gravitational field as well), the situation regarding material particles is not at all as straightforward, with there actually being no justification for associating real particles (i.e. the excitations of the quantum fields of elementary particle physics) with the classical test particle action $I_{T} = -mc \int d\tau$, even though the variation of this action would enforce geodesic motion. It is thus of interest to note, that in a recent study of the extension of the equivalence principle to the propagation of quantum-mechanical matter waves in a background classical gravitational field (with a curved space Schrodinger equation explicitly being shown to be equivalent to an accelerating coordinate frame flat space one), it was shown that it is only because there is such a quantum extension that the equivalence principle actually gets to hold for real classical particles at all. Thus it was shown that gravity, itself a field theory, couples first and foremost to fields rather than to particles (i.e. first and foremost to wavelength rather than to mass - with the eikonal rays associated with the minimally coupled scalar field wave equation $S^{\mu} \tilde{\mu} - (mc/h)^{2} S = 0$ indeed being geodesic), and then, only upon second quantization ($\lambda = \hbar/mv$) of those rays, to mass. It is thus only because of quantum mechanics that massive classical particles get to be geodesic at all.
tational theory actually then being power counting renormalizable. For this theory variation of the action leads to the equations of motion

\[ (-g)^{-1/2} \delta I_W/\delta g_{\mu\nu} = -2\alpha_g W^{\mu\nu} = -T^{\mu\nu}/2 \]  

(4)

where \( W^{\mu\nu} \) is given by

\[
W^{\mu\nu} = g^{\mu\nu}(R^\alpha_\alpha)^{\beta\gamma}/2 + R^{\mu\alpha\beta\gamma}/\beta - R^\mu\alpha\beta\gamma ;\beta - R^\mu\alpha\beta\gamma ;\beta - 2R^\mu\alpha R^\nu\beta + g^{\mu\nu} R^\alpha\beta R^{\alpha\beta}/2 \\
-2g^{\mu\nu}(R^\alpha_\alpha)^{\beta\gamma}/3 + 2(R^\alpha_\alpha)^{\mu\nu}/3 + 2R^\alpha_\alpha R^{\mu\nu}/3 - g^{\mu\nu}(R^\alpha_\alpha)^2/6,
\]  

(5)

so that we can immediately confirm that the Schwarzschild \( R^{\mu\nu} = 0 \) solution is indeed an exterior solution to the theory just as Eddington had warned us. Standard gravity is thus seen to be only sufficient to give the standard Schwarzschild phenomenology but not at all necessary, with it thus indeed being possible to bypass the Einstein-Hilbert action altogether as far as low energy phenomena are concerned.

Further insight into the structure of this alternate gravity theory is obtained by noting that for a static, spherically symmetric source such as a star, the conformal symmetry allows one to set \( g^{rr} = -1/g_{00} \) without any loss of generality, with the field equations of Eq. (4) then being found to reduce (without any approximation at all, i.e. for strong and weak gravitational fields alike) to

\[ \nabla^4 g_{00} = 3(T^0_0 - T^r_r)/4\alpha_g g_{00} \equiv -f(r), \]

(6)

i.e. to reduce to a fourth order Poisson equation rather than to the second order one familiar in the standard theory. The general solution to Eq. (6) exterior to a star of radius \( R \) is then given by

\[ -g_{00}(r > R) = 1 - 2\beta^*/r + \gamma^*r \]

(7)

with the coefficients being given by

\[ \beta^* = \int_0^R df(r)r^4/12, \quad \gamma^* = -\int_0^R df(r)r^2/2, \]

(8)

i.e. by two different moments of the source. We thus see that the Newtonian potential need not be associated with either the second order Einstein equations or with their non-relativistic second order Poisson equation limit, and that the standard theory is thus only sufficient to give Newton but not at all necessary. (In order to show the lack of necessity of the standard theory it is sufficient to construct just one alternative.) The Newtonian potential can thus just as readily be generated in higher order gravitational theories as well, theories which can then have a very different behavior on altogether larger distance scales. And indeed, through the use of the linear \( \gamma^*r \) potential term, a term which actually dominates over Newton at large enough distances, conformal gravity was actually found capable of providing for a complete accounting of galactic rotation curves without the need to invoke dark matter at all. (In fact, the value for the coefficient \( \gamma^* \) required by galactic data then entailed the numerical irrelevance of the \( \gamma^*r \) term on the much smaller solar system distance scales, to thus yield a solution of the galactic rotation curve problem which naturally leaves standard solar system physics intact.) However, regardless of the specific merits of any
particular alternate gravitational theory such as conformal gravity itself, our analysis here
does serve to underscore the risks inherent in extrapolating solar system wisdom beyond
the confines of the solar system, with the need for galactic dark matter perhaps being
symptomatic of the lack of applicability of one particular such extrapolation.

With regard to this conformal gravity alternative, we note further, that in it, with the
coefficient $\beta^*$ of a star being given as the energy-momentum tensor moment integral of Eq.
(8), an identification of this same moment integral in the case of a single proton or neutron
with one half of the $2\beta_p$ Schwarzschild radius of a single nucleon, then enables us to identify
the stellar coefficient $\beta^*$ in conformal gravity as $N\beta_p$ for a weak gravity star containing
$N$ such nucleons. This relation is completely identical to the one obtained by solving the
standard theory second order Poisson equation for a weak gravity source composed of $N$
fundamental elementary particle sources each contributing a potential $\beta_p/r$, to thus enable
us to recover the familiar extensive property of the Newtonian potential of weak gravity
bulk matter, while also seeing that it need not be tied exclusively to the second order
theory. Moreover, as far as gravitational sources are concerned, the coefficients of their $1/r$
potentials are actually radii, specifically their gravitational Schwarzschild radii, so that an
identification of $\beta_p$ with $Gm_p/c^2$ where $m_p$ is the mass of a proton (to thus define $G$ once
and for all as $c^2\beta_p/m_p$, a purely microscopic quantity) then entails that for a weak gravity
bulk matter star $\beta^*$ is given as $GM^*/c^2$ where $M^* = NM_p$ is the mass of the star. We thus
see (i) that the universality of $G$ need not be tied to the second order Poisson equation, (ii)
that, just like Boltzmann’s constant, Newton’s constant $G$ need not itself be fundamental
(only the product $MG/c^2$ is ever measurable gravitationally and never $G$ itself - just as only
the product $kT$ is measurable in statistical mechanics), and (iii) that $G$ need not have any
applicability at all in the strong gravity limit where the energy-momentum tensor moment
integrals no longer scale linearly with the number of fundamental sources. Thus not only
might our standard gravitational intuition not necessarily be generalizable to large distance
scales, it might also not be generalizable to strong gravity either.

III. WHEN AN ATTRACTIVE POTENTIAL IS REPULSIVE

Even though it is generally thought that the attractive or repulsive nature of a potential
is determined once and for all by its overall sign, in this section we show that this turns out to
not in fact necessarily always be the case, with this then being another piece of the standard
intuition which would appear to require reappraisal. To explicitly illustrate this specific point

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2In a conformal invariant theory dimensionful microscopic parameters such as $\beta_p$ and $m_p$ can only
be explicitly generated when the conformal symmetry is spontaneously broken, with their values
then being fixed by the details of the symmetry breaking mechanism. However, regardless of any
specific dynamics that may be needed to explicitly do this, the dependence of the solution of Eq.
(8) on the radial coordinate $r$ is already the most general allowable one possible.

3In fact, for an appropriate gravitational self-energy contribution, the moment integrals of Eq.
(8) might even have differing signs in the weak and strong gravity limits.
it is convenient to consider the geometry near the surface of a static, spherically symmetric gravitational source of radius $R$ and mass $M$ with exterior geometrical line element of the form

$$d\tau^2 = B(r)c^2dt^2 - dr^2/B(r) - r^2d\Omega$$

(9)

where the metric coefficient $B(r)$ is given not just by the usual Schwarzschild form, but rather by the more general

$$B(r) = 1 - 2MG/c^2r + \gamma r$$

(10)

form we introduced earlier. If we erect a Cartesian coordinate system $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta - R$ at the surface of the source, then, with $z$ being normal to the surface, to lowest order in $x/R$, $y/R$, $z/R$, $MG/c^2R$ ($= gR/c^2$) and $\gamma R$ the line element is then found [6] to take the form

$$d\tau^2 = [1 - a(z)]c^2dt^2 - dx^2 - dy^2 - [1 + a(z)]dz^2 - b(xdx + ydy)dz$$

(11)

where $a(z) = 2g(R - z)/c^2 - \gamma(R + z)$ and $b = 4g/c^2 - 2\gamma$. For trajectories for which the initial velocity $v$ is in the horizontal $x$ direction, the geodesic equations associated with Eq. (11) take the form [6] (the dot denotes differentiation with respect to the proper time $\tau$ for massive particles or with respect to an affine parameter for massless ones)

$$\ddot{t} = 0, \quad \ddot{x} = 0, \quad \ddot{y} = 0, \quad \ddot{z} + 2(g - \gamma c^2/2)v^2/c^2 + g + \gamma c^2/2 = 0.$$  

(12)

Thus in the non-relativistic $v = 0$ case the motion is described by

$$\ddot{z} + g + \gamma c^2/2 = 0,$$

(13)

while in the relativistic $v = c$ case it is described by

$$\ddot{z} + 3g - \gamma c^2/2 = 0$$

(14)

instead. Thus we see that the effect of the $\gamma$ term is actually opposite in these two limits, with positive $\gamma$ leading to attractive bending for slow moving particles but to repulsive deflection for fast moving ones [4]. Thus the fact that a potential may be attractive for non-relativistic motions does not in and of itself mean that it must therefore also be attractive for light, with the $v^2/c^2$ type terms not only modifying the magnitude of the effect of gravity (something already the case even in the standard theory where $\ddot{z} + g(1 + 2v^2/c^2) = 0$), but even being able to modify the sign of the effect as well. Thus in general we see that even after appropriately fixing the sign of the coefficient of a gravitational potential term once and for all, such a potential need not always lead to attraction, with a potential which would ordinarily be considered to be attractive (as defined by non-relativistic binding) still being able to lead to repulsion in other kinematic regimes. Caution thus needs to be exercised

4This deflection of light away from a source also holds for the exact $B(r) = 1 + \gamma r$ geodesic as well [13,16].
before one can conclude that attraction in one kinematic regime entails attraction in all
others as well.

As regards the discussion of the metric given in Eq. (11) some further comment is in
order. Specifically, noting that the transformation
\[
t' = t[1 - g(R - z)/c^2 + \gamma (R + z)/2], \quad x' = x, \quad y' = y, \\
z' = z[1 + g(2R - z)/2c^2 - \gamma(2R + z)/4] + (g + \gamma c^2/2)t^2/2 + (g/c^2 - \gamma/2)(x^2 + y^2)
\]  
(15)
brings the metric of Eq. (11) to the flat coordinate form
\[
d\tau^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2
\]
in the primed Cartesian coordinate system. We thus provide a direct demonstration of the
equivalence principle with gravity indeed being found to act the same way as an acceleration
in flat spacetime, and with the equivalence principle indeed being seen to have validity
beyond the standard second order Einstein theory.

Now while we have just seen that the weak gravity metric of Eq. (11) is equivalent to an
acceleration in flat spacetime, this result is initially somewhat puzzling since the full starting
metric of Eqs. (9) and (10) is not only not at all flat, it even possesses a Riemann tensor
which is explicitly non-zero (and thus explicitly not flat) even in this very same lowest order
in \( g \) (or \( \gamma \)) under which Eq. (11) was actually derived. The answer to this puzzle is that
while the Christoffel symbols are first order derivative functions of the metric, the Riemann
tensor itself is a second order such derivative. Thus to get the lowest non-trivial term in the
Riemann tensor we need to expand the metric to second order in \( x/R, y/R, z/R \). Since a
first order expansion suffices for the Christoffel symbols, we thus see that there is actually a
mismatch between orders of expansion of the Christoffel symbols and the Riemann tensor.
Hence a first order study of the geodesics is simply not sensitive to the curvature, and thus
we not only see why the equivalence principle works for weak gravity near the surface of
a system such as the earth, we even see why it has to do so.\[5\] Moreover, on recognizing
this fact, we immediately realize that a typical non-geodesic but still fully covariant particle
equation of motion such as\[6\]
\[
m \left( \frac{d^2 x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \right) = -\kappa_1 R^\beta_\beta \left( \frac{d^2 x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \right) \\
-\kappa_1 R^\beta_\beta \frac{dx^\lambda}{d\tau} \frac{dx^\alpha}{d\tau}
\]  
(17)

\[5\] With \( g^{\mu\nu} \) vanishing identically, Riemannian geometries possess no non-trivial covariant first
order derivative function of the metric at all. Consequently, lowest order trajectories can only be
described by non-tensors such as the coordinate dependent Christoffel symbols, with only the sum
of the two quantities \( d^2 x^\lambda/d\tau^2 \) and \( \Gamma^\lambda_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau) \) actually transforming as a contravariant
vector. In lowest order then, the coupling of a particle to gravity has to be purely inertial.

\[6\] This (purely illustrative) equation of motion can actually be derived by a stationary variation of
the action \( I = -mc \int d\tau - \kappa_1 \int d\tau R^\beta_\beta \) where \( \kappa_1 \) is an appropriate constant.
will just as readily satisfy the same weak gravity tests as the Eq. (2) geodesic itself. Moreover, since Eq. (2) and Eq. (17) both reduce to the Cartesian coordinate Newtonian law $md^2x^\lambda/d\tau^2 = 0$ in the absence of curvature, both equations represent valid curved space generalizations of Newton’s second law of motion, with Eq. (17) possessing both inertial (the Christoffel symbols) and non-inertial (the Ricci scalar) contributions. With Eqs. (2) and (17) having very different strong gravity continuations, we thus see that the weak gravity successes of the equivalence principle reveal nothing about how Eq. (2) would in fact fare in strong gravity, with an Eotvos experiment near the surface of a black hole not being at all guaranteed to give a null result. Since the presence of such non-inertial terms is actually expected to be the general rule rather than the exception in field theory, it would appear that particle motions may not in fact be controlled by the strong gravity event horizons and trapped surfaces associated with Schwarzschild metric geodesic motion after all, with gravity itself possibly being able to protect particles from such phenomena in the strong gravity limit with the strong gravity extrapolation of standard weak gravity thus being another potentially unreliable extrapolation.

IV. THE CASE FOR REPULSIVE GRAVITY

Recently, through study of type 1A supernovae at very high redshift, it has become possible to explore the attractive or repulsive character of gravity on cosmological distance scales, with it now being possible to determine whether the universe itself might indeed be slowing down or whether it might perhaps actually be speeding up. In terms of the standard Robertson-Walker cosmological line element

$$d\tau^2 = c^2 dt^2 - R^2(t)[(1 - kr^2)^{-1}dr^2 + r^2 d\Omega]$$

(18)

In passing it is perhaps worth stressing that some relativists regard the equivalence principle as the statement that gravitational effects are strictly inertial, with Eq. (2) then being the only possible allowable coupling of a particle to gravity. However, we take the far more cautious view here that while the Christoffel symbols certainly do possess the nice, purely geometric inertial property of being simulatable by an acceleration in flat spacetime, that does not, and in fact cannot, preclude there being a non-inertial, truly coordinate independent, coupling to gravity as well, with this issue actually being a dynamical rather than a geometrical one which is only decidable by consideration of the curved spacetime field equations with which real (as opposed to test) particles are associated.

The electromagnetic vector potential equation of motion $g^{\alpha\beta}A_{\mu;\alpha;\beta} - A^{\alpha}_{\;;\alpha;\mu} + A^\alpha R_{\mu\alpha} = 0$ contains an explicit non-inertial piece in curved spacetime, with a similar situation being found for both the Dirac field (vierbeins being non-inertial) and for the non-minimally coupled scalar field $S(x)$ with equation of motion $S^{\alpha}_{\;;\alpha;\mu} - (mc/\hbar)^2S + \xi R^\alpha_{\;\alpha}S/6 = 0$ where $\xi$ is a dimensionless parameter.

Not only could non-inertial effects such as those exhibited in Eq. (17) become significant in strong gravitational fields, there could even be a switch over to the effective repulsion associated with $B(r) = 1 + \gamma r$ type metrics as particles are accelerated to high velocities.

7In passing it is perhaps worth stressing that some relativists regard the equivalence principle as the statement that gravitational effects are strictly inertial, with Eq. (2) then being the only possible allowable coupling of a particle to gravity. However, we take the far more cautious view here that while the Christoffel symbols certainly do possess the nice, purely geometric inertial property of being simulatable by an acceleration in flat spacetime, that does not, and in fact cannot, preclude there being a non-inertial, truly coordinate independent, coupling to gravity as well, with this issue actually being a dynamical rather than a geometrical one which is only decidable by consideration of the curved spacetime field equations with which real (as opposed to test) particles are associated.

8The electromagnetic vector potential equation of motion $g^{\alpha\beta}A_{\mu;\alpha;\beta} - A^{\alpha}_{\;;\alpha;\mu} + A^\alpha R_{\mu\alpha} = 0$ contains an explicit non-inertial piece in curved spacetime, with a similar situation being found for both the Dirac field (vierbeins being non-inertial) and for the non-minimally coupled scalar field $S(x)$ with equation of motion $S^{\alpha}_{\;;\alpha;\mu} - (mc/\hbar)^2S + \xi R^\alpha_{\;\alpha}S/6 = 0$ where $\xi$ is a dimensionless parameter.

9Not only could non-inertial effects such as those exhibited in Eq. (17) become significant in strong gravitational fields, there could even be a switch over to the effective repulsion associated with $B(r) = 1 + \gamma r$ type metrics as particles are accelerated to high velocities.
with associated scale factor $R(t)$ and spatial curvature $k$, the new high $z$ data have made it possible to extend Hubble plot measurements of the temporal behavior of $R(t)$ beyond lowest order in time, with the current era value of the $q(t_0) = -\dot{R}(t_0)/R(t_0)/\dot{R}^2(t_0)$ deceleration parameter having now become as amenable to observation as the current value of the $H(t_0) = \dot{R}(t_0)/R(t_0)$ Hubble parameter itself. Moreover, the actual observations themselves have now provided the first direct evidence that gravity might actually contain an explicit repulsive component. Specifically, in terms of the standard Einstein-Friedmann cosmological evolution equations, viz.

$$\dot{R}^2(t) + kc^2 = \dot{H}(t)(\Omega_M(t) + \Omega_\Lambda(t))$$

and

$$q(t) = (n/2 - 1)\Omega_M(t) - \Omega_\Lambda(t) = (n/2 - 1)(1 + kc^2/\dot{R}^2(t)) - n\Omega_\Lambda(t)/2 \quad (19)\quad (20)$$

where $\Omega_M(t) = 8\pi G\rho_M(t)/3c^2H^2(t)$ is due to ordinary matter (viz. matter for which $\rho_M(t) = A/R^n(t)$ where $A > 0$ and $3 \leq n \leq 4$) and where $\Omega_\Lambda(t) = 8\pi G\Lambda/3cH^2(t)$ is due to a possible cosmological constant $\Lambda$, it was found that the data constrained the allowable current $(n=3)$ era values of the parameters $\Omega_M(t_0)$ and $\Omega_\Lambda(t_0)$ to a quite small region in which $\Omega_\Lambda(t_0) \simeq \Omega_M(t_0) + 1/2$ or so, with (the presumed positive) $\Omega_M(t_0)$ being limited to the range $(0, 1)$ or so and with $\Omega_\Lambda(t_0)$ being limited to the range $(1/2, 3/2)$ or so, to thus yield a current era deceleration parameter $q(t_0)$ which had to lie within the expressly negative $(-1/2, -1)$ interval. While these data thus appear to point toward a universe which is actually currently accelerating, systematic effects (such as an apparent evolutionary effect between high and low $z$ supernovae) are actually large enough that the data could still support a positive value for $q(t_0)$, albeit one which would however still have to be substantially smaller than the $q(t_0) = 1/2$ value expected in the standard $\Omega_\Lambda(t_0) = -kc^2/\dot{R}^2(t_0) \simeq 0$ flat inflationary universe paradigm in which $\Omega_M(t_0) = 1$ and $\Omega_\Lambda(t_0) = 0$. Thus even if the universe is not accelerating, there would still have to be some cosmic repulsion with respect to standard inflation (i.e. with respect to the expressly positive $q(t_0) = (n/2 - 1)\Omega_M(t_0) \equiv 1/2$ associated with normal, gravitationally attractive matter in a flat universe), even if this needed cosmic repulsion did not actually dominate over the positive $\Omega_M(t_0)$ contribution and lead to a net overall acceleration. The data thus entail the existence of some form or other of repulsive component to gravity on cosmological distance scales, and it is to the possibility that gravity need not always be strictly attractive which we therefore now turn.

From a purely phenomenological viewpoint, Eq. (20) immediately suggests two fairly straightforward ways in which $q(t_0)$ could in fact be reduced below one half in standard gravity, viz. a positive $\Omega_\Lambda(t_0)$ or a negative spatial curvature $k$. Of these proposals the possibility of a non-vanishing $\Omega_\Lambda(t_0)$ was first raised by Einstein himself as long ago as the very early days of relativity. Specifically, motivated by the desire to have a static universe, he noted that if he modified the Einstein-Hilbert action by the addition of a fundamental cosmological constant term, the associated cosmology would then admit of a static solution (with $\dot{R}(t) = 0$, $\ddot{R}(t) = 0$ and fixed $R(t) = R_0$ in the Robertson-Walker language) provided $\rho = A/R_0^3 = 2\Lambda = kc^4/4\pi G R_0^2$, i.e. provided $\Lambda$ and $k$ were both taken to be positive. In such a solution a positive cosmological constant term would act repulsively to counteract the attraction (i.e. deceleration) associated with positive $\rho$ and positive $k$, to thus yield a (closed) static universe. While such a cosmology quickly fell into disfavor following the
discovery of the cosmic recession of the nebulae shortly thereafter, it did raise for the first time the issue of cosmic repulsion, while also raising a problem that has been with us ever since, the notorious cosmological constant problem, a problem whose modern variant is not so much one of the possible presence of a fundamental macroscopic $\Lambda$ but rather of the possible presence of a (potentially unacceptably large) microscopically induced one instead.

As regards the second way to get cosmic repulsion, viz. negative curvature, we note that such a mechanism is not in fact tied to any specific cosmological evolution equation such as the Einstein-Friedmann one of Eqs. (19) and (20), with it actually turning out to be quite general in nature. Specifically, it was shown [19] that, no matter what the explicit form of the gravitational field equations of motion themselves, the propagation of waves such as Maxwell waves in a general curved spacetime Robertson-Walker background is completely analogous to the propagation of flat spacetime Maxwell waves in a material medium, with vector (and also scalar) curved space waves being found to have a dispersion relation of the form $\omega^2/c^2 = \lambda^2 + k$ in a Robertson-Walker background. Thus we see that spatial curvature acts just like a frequency dependent refractive index of the form $n(\omega) = c\lambda/\omega = (1 - kc^2/\omega^2)^{1/2}$, with the group velocity associated with energy transport then being given by $v_g = d\omega/d\lambda = c(1 - kc^2/\omega^2)^{1/2}$. When $k$ is negative, spatial curvature is then seen to act as a tachyonic mass, to thus effectively give faster than light propagation, with a negative curvature space then acting just like a diverging dispersive medium wherein group velocities are explicitly greater than their values in empty space. Particles propagating in a negative curvature space are thus effectively accelerated (cf. a diverging lens), while those traveling in a positive curvature space are accordingly decelerated (cf. a converging lens). With the standard wisdom regarding the aftermath of the big bang being that the mutual attraction of the galaxies would serve to slow down the expansion of the universe (i.e. $q(t) = \Omega_M(t)/2 > 0$), we now see that this particular wisdom only applies if the galaxies are propagating in an inert, empty space (viz. a $k = 0$ plane lens) which itself has no dynamical consequences. However, once $k$ is non-zero, the galaxies would instead then propagate in a non-trivial geometric medium, a medium which can then explicitly participate dynamically, with the gravitational field itself which is then present in the medium being able to accelerate or decelerate the galaxies according to the sign of its spatial curvature. Thus we again see that the standard notion of purely attractive gravity needs to be reconsidered once the cosmological curvature $k$ is non-zero, with gravity itself (viz. curvature) potentially being able to generate some repulsion all on its own.

While both of the above cosmic repulsion mechanisms can readily be utilized in standard gravity in order to provide a purely phenomenological fit to the supernovae data, nonetheless the specific values explicitly required of the parameters $\Omega_M(t_0)$ and $\Omega_{\Lambda}(t_0)$ actually pose a somewhat severe theoretical problem for the standard theory. Specifically, given the radically differing temporal behaviors of $\Omega_{\Lambda}(t)$ and $\Omega_M(t)$, the apparent current closeness to one of their ratio [14] entails that in the early universe this same $\Omega_{\Lambda}(t)/\Omega_M(t)$ ratio would have had to have been fantastically small (typically of order as small as $10^{-60}$), with a Friedmann universe only being able to evolve into the currently observed one if this ratio had been extremely fine-tuned by fixing the initial conditions in the early universe to incredible accuracy. Moreover, even if we take advantage of the systematic uncertainties identified in [17] which might potentially (but not necessarily) permit us to set the current value of $\Omega_{\Lambda}(t_0)$ to zero, since $\Omega_M(t_0)$ would still be required to be less than one even in such
a case, Eq. (19) would then oblige the current value of $\Omega_k(t_0)$ to be necessarily different from zero. Then, given the different temporal behaviors of $\Omega_M(t)$ and $\Omega_k(t)$, this time it would be the $\Omega_k(t)/\Omega_M(t)$ ratio which would need early universe fine-tuning. With Eq. (19) being writable as $\Omega_M(t) + \Omega_\Lambda(t) + \Omega_k(t) = 1$, we thus see that current era non-vanishing of any two of these three quantities entails some form of early universe fine-tuning problem. Since the current data do not support the one point (viz. the $\Omega_M(t_0) = 1$, $\Omega_\Lambda(t_0) = 0$, $\Omega_k(t_0) = 0$ inflationary universe) which could be reached without a fine-tuning of Eq. (19), we see that, even with the current observational uncertainties, the new high $z$ data will not support a Friedmann cosmology without some form or other of fine-tuning problem, with the non-vanishing of $1 - \Omega_M(t)$ entailing the explicit presence of some cosmic repulsion.

Moreover, apart from the above macroscopic fine-tuning problems, microscopic quantum physics presents cosmology with yet more problems, with the current value of the $\Omega_\Lambda(t_0)/\Omega_M(t_0)$ ratio actually being expected to be absolutely enormous - potentially of order $10^{120}$ if generated by quantum gravity, and of typical order $10^{60}$ if generated by elementary particle physics phase transitions such as the electroweak one. Microscopic physics thus leads to an expectation which is nowhere near the current data at all, with this then being the modern variant of the cosmological constant problem to which we referred earlier. In all then we thus identify an uncomfortably large number of problems for current cosmology (even if explicitly less than one, an $\Omega_M(t_0)$ of order one would still require an enormous amount of as yet totally undetected non-luminous, expressly non-baryonic, cosmological dark matter), and see that they all appear to have one common ingredient, namely the use of the evolution equation of Eq. (19) in the first place. Thus it is highly suggestive that the problems that the standard cosmology currently faces might all derive from the lack of reliability of the extrapolation of standard gravity wisdom beyond its solar system origins. In the following then we shall thus relax this assumption, and in particular we shall show that all of the above problems can readily be resolved if gravity acquires one further form of cosmic repulsion, namely that due to an effective cosmological Newton constant which is expressly taken to be negative.

In trying to identify the root cause of the above problems we note that the macroscopic Friedmann equation fine-tuning problem arises because of mismatch between the early and current universes, while the microscopic cosmological constant problem arises because of a clash between elementary particle physics and gravitational physics, with particle physics wanting a large $\Lambda$ and gravity a small one. Since this clash is between different branches of physics, we should not immediately assume that it is the particle physics which is at fault. Rather, the indications of particle physics might well be correct, with its contribution to $\Lambda$ actually being as big as it would appear to be. Indeed, the very failure to date of attempts

\footnote{Associating a typical temperature scale $T_V$ with the electroweak vacuum breaking phase transition and a temperature $T(t)$ with the ordinary matter in the universe leads to $\Omega_\Lambda(t)/\Omega_M(t) = T_V^4/T^4(t)$, a ratio which is of order $10^{60}$ today.}

\footnote{Moreover, should it turn out that there is no phenomenological need for a non-zero $\Omega_\Lambda(t_0)$ after all, even then we would still have to explain why $\Omega_\Lambda(t_0)$ is not as big as its theoretical expectation, with the disappearance of the need not entailing the disappearance of the problem.}
to quench the particle physics $\Lambda$ from so large an expected value might even be an indicator that it is not in fact quenched, with it being reasonable to then ask what the implications for cosmology are if $\Lambda$ really is big, and whether cosmology could actually accommodate a large $\Lambda$. To this end we note that the quantity which is of relevance to cosmological evolution is not in fact $\Lambda$ itself but rather $\Omega_{\Lambda}(t) = \frac{8\pi G \Lambda}{3cH^2(t)}$, i.e. not the energy of the vacuum itself, but rather, its contribution to gravitational evolution, with only this latter quantity being observable. With this latter contribution depending not just on $\Lambda$ but also on $G$, we see that a quenching of $\Omega_{\Lambda}(t)$ could potentially be achieved not by quenching $\Lambda$ but by quenching $G$ instead. Thus again we are led to consider $G$ as being only an effective parameter, one whose cosmological coupling might be altogether smaller than that relevant to the solar system. We shall thus explore the possibility that the effective cosmological $G$ is both small and negative, first as a general effect and then in an explicit solvable model.

As regards explicitly trying to find a solution to the Friedmann universe fine-tuning problem, we note that since the standard cosmology has a big bang, the early universe $\dot{R}(t)$ would have to be divergent at $t = 0$ (or at least be very large), with Eq. (19) then requiring the quantity $\Omega_M(t = 0) + \Omega_{\Lambda}(t = 0)$ to be equal to one at $t = 0$, regardless of what particular value the spatial curvature $k$ might take. Then, given the radically different temporal behaviors of $\Omega_M(t)$, $\Omega_{\Lambda}(t)$ and $\Omega_k(t)$, we see directly that no cosmology which obeys this initial constraint, be it flat or non-flat, could ever evolve into one in which $\Omega_{\Lambda}(t_0) \approx \Omega_M(t_0) \approx O(1)$ today (or into one in which $\Omega_M(t_0) \approx O(1)$ should $\Omega_{\Lambda}(t)$ just happen to be zero) without extreme fine tuning. Hence it is the big bang itself which is creating the fine-tuning problem, with it being very difficult for a Friedmann universe to evolve from a singular early state into the one currently observed. In order to eliminate such an incompatibility we are thus led to consider removing the big bang singularity from cosmology altogether, and have the universe expand from some initial (but still very hot) state characterized by $\dot{R}(t = 0) = 0$ instead. Since the big bang singularity itself derives from the assumption that gravity continues to be attractive even when it is strong, we are thus led to consider the possibility that the effective cosmological $G$ actually be repulsive, with cosmology then being able to protect itself from its own singularities and potentially rid itself of fine-tuning problems.

Continuing in this same vein, we note further that if the universe turns out to ultimately be an accelerating one (either even already, or at some time later in the future), $\dot{R}(t)$ will

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12 Once $\dot{R}(t = 0)$ is non-singular, the natural definition of the initial time in a universe which expands is then the one where $R(t)$ is at a minimum, with its minimum value not needing to be zero in a non-singular cosmology.

13 That the origin of the flatness problem could be traced to the positivity of $G$ was already noted quite some time ago in [21], where it was also pointed out that a negative effective $G$ repulsive cosmology would actually have no fine-tuning flatness problem at all, with quantities which would have had to cancel to very high accuracy in Eq. (19) no longer having to do so.

14 Even if $\Omega_{\Lambda}(t_0)$ turns out to be negligible today, as long as it is greater than zero, then no matter by how little, there will eventually come a time when $\Omega_{\Lambda}(t)$ will ultimately come to dominate the
then become arbitrarily large in the late (rather than the early) universe, with the quantity $\Omega_M(t) + \Omega_\Lambda(t)$ tending to one at late times, again independent of the value of $k$. Then, in such a late universe it would be $\Omega_\Lambda(t)$ which would have to tend to one, no matter what may or may not have happened in the early universe, and regardless of how big $\Lambda$ itself might actually be. Thus at very late times the cosmological constant problem would not only get solved, it would get solved by cosmology itself, with an accelerating universe always being able to quench $\Omega_\Lambda(t)$ once given time enough to do so. Thus we see that the very same problem which is generated by the very existence of a cosmological constant is then made solvable by the very cosmic acceleration which it simultaneously produces; with the key question for cosmology thus being whether the universe is already sufficiently late for this quenching to have already taken place. However, in order for this to actually be the case, it is necessary that the current era contribution of ordinary matter to the evolution of the universe be cosmologically insignificant (i.e. that $\Omega_M(t) = 8\pi G\rho_M(t)/3c^2H^2(t)$ already be close to an asymptotically expected value of zero). Thus again we find ourselves led to considering the possibility that the effective cosmological $G$ be altogether smaller than the one relevant in the solar system, so that, no matter how big $\rho_M(t)$ itself might be, ordinary matter would then no longer be of relevance to the current expansion, though, given its temporal behavior, $\rho_M(t)$ would still be highly significant at altogether earlier times. Thus to conclude, we see that many outstanding cosmological puzzles are readily addressable if gravity were indeed to be describable cosmologically by a repulsive and very small effective $G$, and thus motivated we shall, in the following, present a specific alternate gravitational theory, actually the conformal gravity theory to which we referred above, where this is precisely found to be the case.

V. REPULSIVE GRAVITY

Since we are not currently aware of how it might be possible to implement the above general ideas within standard gravity (a $G$ which evolves with temperature from the early universe until today is certainly conceivable within standard gravity though perhaps not one which might also change sign as it evolves), we shall instead turn to an alternate gravity theory, viz. conformal invariant gravity, a theory which is immediately suggested since gravity then becomes a theory with dimensionless coupling constants and no intrinsic mass scales just like the three other fundamental interactions. Thus gravity becomes a theory which is power counting renormalizable, with the absence of any intrinsic mass scale immediately

\[\text{expansion of the universe.}\]

\[15\text{Thus we do not seek to change the matter content of the universe, but rather only to modify how it impacts on cosmological evolution. Moreover, with the ratio } \Omega_\Lambda(t)/\Omega_M(t) = T_V/T^4(t) \text{ actually being independent of } G, \text{ we see that the requirement that } T_V/T(t_0) \text{ be large and that the current era } \Omega_\Lambda(t_0) \text{ be of order one together entail that } \Omega_M(t_0) \text{ be highly suppressed, something readily achievable if the effective cosmological } G \text{ is then very small.}\]
obliging any fundamental cosmological constant term to be set to zero in it,\textsuperscript{[22]} to thus yield a theory in which the cosmological constant term is actually controlled by an underlying symmetry, an objective long sought in the standard model. With the conformal theory also, as noted earlier, possessing a good Newtonian limit despite the absence of any intrinsic Einstein-Hilbert term (the conformal gravitational action is uniquely given by the conformal $I_W$ action of Eq. (3)), we see that a possible reason why the standard gravity cosmological constant problem has resisted solution for so long could be that was always being sought was a symmetry which would eliminate a fundamental $\Lambda$ but not $I_{EH}$ rather than one which would in fact eliminate both. Since solar system information had never in fact required the presence of $I_{EH}$ in the first place, we see that its removal actually opens up a new line of attack on the cosmological constant problem; and as we shall now see, the conformal symmetry not merely controls any fundamental cosmological constant term, even after the conformal symmetry is spontaneously broken (something needed to generate particle masses in the conformal symmetry), the continuing tracelessness of the energy-momentum tensor sharply constrains the magnitude of any induced one in a manner which is then found to automatically implement the $\Omega_\Lambda(t)$ quenching mechanism we introduced above.

The cosmology associated with conformal gravity was first presented in \cite{21} where it was shown, well in advance of the recent discovery of cosmic repulsion, to be one with an effective repulsive cosmological $G$ just as desired above. To discuss conformal cosmology it is convenient to consider the conformal matter action

$$I_M = -\hbar \int d^4x (-g)^{1/2} [S^\mu S_\mu/2 - S^2 R^\mu_\mu/12 + \lambda S^4 + i\bar{\psi}\gamma^\mu(x)(\partial_\mu + \Gamma_\mu(x))\psi - gS\bar{\psi}\psi] \quad (21)$$

where we take the elementary particle physics matter fields to be generically represented by massless fermions for definitiveness and simplicity, with a conformally coupled massless scalar field being introduced to serve as the order parameter associated with spontaneous breakdown of the scale symmetry. With dynamical symmetry breaking being needed in order to generate elementary particle masses in the scaleless interacting massless fermion and gauge boson theories now standard in particle physics, the scalar field $S(x)$ introduced here should be thought of not as a fundamental scalar field but as the expectation value of an appropriate fermion multilinear in an appropriate coherent fermionic state. $S(x)$ is thus to serve as a cosmological analog of the Cooper pair of superconductivity theory, with the action of Eq. \textsuperscript{(21)} serving as an analog of the Ginzburg-Landau phenomenological superconductivity Lagrangian. In such a case the vacuum energy would be zero above the critical temperature where the order parameter $S(x)$ would then vanish, and would be expressly negative below it. Simulating the vacuum energy below the critical point by the effective $\hbar\lambda S^4(x) \equiv \Lambda$ term, then entails (as noted in \textsuperscript{20}) that in a theory which is scale invariant above the critical point, the effective $\Lambda$ which is induced below the critical point is then expressly negative. Thus unlike the situation in the standard theory, in the conformal case the sign of the induced cosmological constant term is explicitly determined.

\textsuperscript{16}The absence of any intrinsic quantum gravity scale not merely eliminates the Planck length from quantum relevance, it also provides for a theory of gravity in which quantum gravity fluctuations themselves cannot then generate a cosmological constant term at all; with any induced microscopic $\Lambda$ then only being generatable by elementary particle physics phase transitions.
For the above matter action the matter field equations of motion take the form

\[ i\gamma^\mu(x)\left[\partial_\mu + \Gamma_\mu(x)\right]\psi - gS\psi = 0 \]
\[ S^\mu_\nu + S R^\mu_\nu/6 - 4\lambda S^3 + g\bar{\psi}\psi = 0 \]  \hspace{1cm} (22)

with the matter energy-momentum tensor being given by

\[ T^\mu_\nu = h\{i\bar{\psi}\gamma^\mu(x)\left[\partial_\nu + \Gamma_\nu(x)\right]\psi + 2S^\mu_\nu/3 - g^\mu_\nu S^\alpha_\alpha/6 - SS^\mu_\nu/3 \]
\[ + g^\mu_\nu SS^\alpha_\alpha/3 - S^2(R^\mu_\nu - g^\mu_\nu R_\alpha_\alpha/2)/6 - g^\mu_\nu \lambda S^4\}. \]  \hspace{1cm} (23)

Thus, when the scalar field acquires a non-zero vacuum expectation value \( S_0 \), the energy-momentum tensor then takes the form (for a perfect matter fluid \( T^\mu_\nu_{\text{kin}} \) of the fermions)

\[ T^\mu_\nu = T^\mu_\nu_{\text{kin}} - hS_0^2(R^\mu_\nu - g^\mu_\nu R_\alpha_\alpha/2)/6 - g^\mu_\nu h\lambda S_0^4. \]  \hspace{1cm} (24)

Since the Weyl tensor \( C^{\lambda\mu\nu\kappa} \) and the quantity \( W^{\mu\nu} \) of Eq. \((\mathbf{3})\) both vanish identically in the highly symmetric Robertson-Walker geometry, the complete cosmological solution to the joint scalar, fermionic, and gravitational field equations of motion then reduces \[\text{[19]}\] to just one relevant equation, viz.

\[ T^\mu_\nu = 0, \]  \hspace{1cm} (25)

a remarkably simple condition which immediately fixes the zero of energy. Thus in its spontaneously broken phase conformal cosmology is described by the equation

\[ hS_0^2(R^\mu_\nu - g^\mu_\nu R_\alpha_\alpha/2)/6 = T^\mu_\nu_{\text{kin}} - g^\mu_\nu h\lambda S_0^4, \]  \hspace{1cm} (26)

an equation which we recognize as being none other than that of standard gravity save only that the quantity \(-hS_0^2/12\) has replaced the familiar \( c^3/16\pi G \). Conformal cosmology thus acts exactly like a standard gravity theory in which \( G \) is effectively negative, with its magnitude actually becoming smaller the larger \( S_0 \) gets to be, i.e. the same particle physics mechanism which makes the cosmological constant large serves to make the effective cosmological \( G \) small. Thus, just as desired, conformal cosmology is controlled by an effective \( G \) which is both negative and small. Noting further that the Weyl tensor vanishes in high symmetry, cosmologically relevant geometries such as the homogeneous Robertson-Walker one, but not in low symmetry ones such as Schwarzschild (viz. geometries which are generated by the presence of local spatial inhomogeneities in an otherwise homogeneous cosmological background), we thus see that the gravitational coupling constant \( \alpha_g \), viz. the one which according to Eq. \((\mathbf{8})\) explicitly controls the geometry outside of a local static source, simply decouples from cosmology. Thus in conformal gravity (inhomogeneous) locally attractive and (homogeneous) globally repulsive gravity can readily coexist, with solar system physics indeed not then being a good guide to the behavior of gravity in altogether different circumstances.

Given Eq. \((\mathbf{26})\) the conformal cosmology evolution equations immediately take the form

\[ \dot{R}^2(t) + kc^2 = -3\dot{R}^2(t)(\Omega_M(t) + \Omega_\Lambda(t))/4\pi S_0^2 L_{PL}^2 \equiv \dot{R}^2(t)(\bar{\Omega}_M(t) + \bar{\Omega}_\Lambda(t)) \]  \hspace{1cm} (27)

and
\[
q(t) = (n/2 - 1) \Omega_M(t) - \Omega_\Lambda(t)
\]  
(Eq. 27 serves to define \( \Omega_M(t) \) and \( \Omega_\Lambda(t) \)), and are thus remarkably similar to the standard evolution equations of Eqs. (19) and (20); with our entire earlier general discussion on the need to replace the cosmological \( G \) by a small negative effective one now finding explicit realization in a specific alternate gravitational theory. With both the effective \( G \) and the vacuum breaking \( \Lambda \) being negative in conformal gravity, we thus see that \( \Omega_M(t) \) is necessarily negative while \( \Omega_\Lambda(t) \) is positive. Consequently, the ordinary matter energy density and the vacuum energy density both naturally lead to cosmological repulsion, with the conformal \( q(t) \) always having to be less than or equal to zero. Conformal cosmologies thus never decelerate with each epoch seeing some measure of cosmic repulsion no matter what the explicit magnitudes of \( \Omega_M(t) \) and \( \Omega_\Lambda(t) \) might be.

In order to determine how much repulsion there might be in any given epoch it is necessary to determine the value of the spatial curvature \( k \). To this end we note while we cannot immediately fix \( k \) from study of the broken symmetry phase itself (\( \Omega_M(t) \) and \( \Omega_\Lambda(t) \) make contributions of opposite sign in Eq. (27)), it is possible to extract information about \( k \) from the high temperature phase above all phase transitions, a phase where the order parameter vanishes, a phase which can be modeled entirely by the presence of just a perfect fluid, viz. one in which the entire \( T^{\mu\nu} \) is given by \( T_{\text{kin}}^{\mu\nu} \). In such a high temperature Robertson-Walker phase the gravitational equations of motion of Eq. (4) reduce to the condition

\[
T_{\text{kin}}^{\mu\nu} = 0,
\]

an equation which quite remarkably actually has a non-trivial solution in curved space.

To explore such a solution, we note that while we have generically identified the fields in \( T_{\text{kin}}^{\mu\nu} \) to be perfect fluid fermions, for calculational purposes it is more convenient to consider them to be non-interacting scalar fields instead. We thus populate the non-spontaneously broken high temperature universe by a perfect fluid built out of the modes of a normal (i.e. not spontaneously broken) scalar field with equation of motion

\[
S_{\cdot\mu} + SR^{\mu}_{\cdot\mu}/6 = 0
\]

and energy-momentum tensor

\[
T^{\mu\nu} = 2S^\mu S^\nu/3 - g^{\mu\nu} S^\alpha S_\alpha/6 - SS^{\mu\nu}/3 + g^{\mu\nu} SS_{\cdot\alpha}/3 - S^2(R^{\mu\nu} - g^{\mu\nu} R_{\cdot\alpha}/2)/6.
\]

With the Ricci scalar being given by the spatially independent \( R^\alpha_{\cdot\alpha} = -6(k + R(t)\ddot{R}(t) + \dot{R}^2(t))/R^2(t) \), the scalar field equation of motion is found to be separable. Thus on introducing the conformal time \( p = c \int^t dt/R(t) \) and on setting \( S(x) = f(p)g(r, \theta, \phi)/R(t) \), Eq. (30) is found to reduce to (\( \gamma \) denotes the determinant of the spatial metric)

\[
\frac{1}{f(p)} \frac{d^2f}{dp^2} + k f(p) = \frac{1}{g(r, \theta, \phi)} \gamma^{-1/2} \partial_i [\gamma^{1/2} \gamma^{ij} \partial_j g(r, \theta, \phi)] = -\lambda^2,
\]

where we have introduced a separation constant \( -\lambda^2 \). With the dependence on \( f(p) \) on \( p \) being harmonic, the frequencies thus have to satisfy \( \omega^2 = \lambda^2 + k \), a relation we actually presented earlier. For the spatial dependence we can further set \( g(r, \theta, \phi) = g_\lambda^i(r)Y_i^m(\theta, \phi) \) where \( g_\lambda^i(r) \) obeys the radial equation

18
\[
\left[ (1 - kr^2) \frac{\partial^2}{\partial r^2} + \frac{2 - 3kr^2}{r} \frac{\partial}{\partial r} - \frac{\ell(\ell + 1)}{r^2} + \lambda^2 \right] g_\lambda^\ell(r) = 0, \tag{33}
\]
with the radial solutions being given \cite{23} by \( j_\ell(\omega r)/\omega \) Bessel functions when \( k = 0 \), by associated Legendre functions when \( k = -1 \), viz.

\[
g_\lambda^\ell(r) = [\pi \omega^2(\omega^2 + 1) \ldots (\omega^2 + \ell^2)/2]^{-1/2} \sinh^{\ell+1} \chi \left[ \frac{d}{d\cosh \chi} \right] \frac{\cos \omega \chi}{\omega}, \tag{34}
\]

where \( r = \sinh \chi \), and by Gegenbauer polynomials when \( k = 1 \), viz.

\[
g_\lambda^\ell(r) = [\pi \omega^2(\omega^2 - 1) \ldots (\omega^2 - \ell^2)/2]^{-1/2} \sin^{\ell+1} \chi \left[ \frac{d}{d\cos \chi} \right] \frac{\cos \omega \chi}{\omega}, \tag{35}
\]

where \( r = \sin \chi \). With these normalizations (which differ slightly from those given in \cite{23}), an incoherent averaging of the energy-momentum tensor of Eq. (11) over all the available spatial states associated with a given frequency \( \omega \) is then found \cite{23} to lead directly for every allowed \( \omega \) to the traceless kinematic perfect fluid

\[
T^{\mu\nu}_{kin} = \frac{\omega^2 (4U^\mu U^\nu + g^{\mu\nu})}{6\pi^2 R^4(t)} \tag{36}
\]
in all three of the spatial geometries\cite{17}. With the condition \( T^{\mu\nu}_{kin} = 0 \) of Eq. (29) then only permitting the soft \( \omega = 0 \) modes, and with only the \( k = -1 \) radial equation admitting of a

\footnote{On purely general geometric grounds the most general rank two tensor in a Robertson-Walker geometry is writable as \( T^{\mu\nu} = (A(t) + B(t))U^\mu U^\nu + B(t)g^{\mu\nu} \), with the coefficients \( A(t) \) and \( B(t) \) being otherwise unconstrained, so that their ratio \( w(t) = B(t)/A(t) \) need not be time independent in general. However, for a tensor which is both traceless and covariantly conserved, it further follows that \( 3B(t) = A(t) = C/R^4(t) \) where \( C \) is a pure constant; with the calculation leading to Eq. (36) thus being a calculation of the value of this constant. While on this point, it is worth noting in passing that it is not automatically the case that \( w(t) \) is necessarily time independent or that \( A(t) \) and \( B(t) \) necessarily have the same dependence on time in the general Robertson-Walker cosmology case. Relations between \( A(t) \) and \( B(t) \) only follow under specific dynamical assumptions. If, for example, \( A(t) \) and \( B(t) \) contain two or more separate components, then even if the separate components are related via time independent \( w_1 = B_1/A_1, w_2 = B_2/A_2 \), it does not necessarily follow that \( B_1 + B_2 \) is proportional to \( A_1 + A_2 \). Further, even for the restricted case of the kinematic \( T^{\mu\nu}_{kin} \) in which \( A(t) \) and \( B(t) \) are both associated with just one single completely standard perfect fluid, it turns out that even in that case they are still not in fact proportional to each other at all temperatures. To illustrate this point, consider an ideal \( N \) particle classical gas of particles of mass \( m \) in a volume \( V \) at a temperature \( T \). For this system the Helmholtz free energy \( A(V, T) \) is given as \( \exp[-A(V, T)/NkT] = V \int d^3p \exp[-(p^2 + m^2)^{1/2}/kT] \), so that the pressure takes the simple form \( P = -(\partial A/\partial V)_T = NkT/V \), while the internal energy \( U = A - T(\partial A/\partial T)_V \) evaluates in terms of Bessel functions as \( U = 3NkT + NmK_1(m/kT)/K_2(m/kT) \). In the two limits \( m/kT \to 0, m/kT \to \infty \) we then find that \( U \to 3NkT, U \to Nm + 3NkT/2 \). Thus only at these two extreme temperature limits does it follow that the energy density and the pressure are in fact proportional, with their relation in intermediate regimes such as the transition region from the radiation to the matter era being far more complicated.}
non-trivial radial solution in such a case (viz. the entire infinite set\textsuperscript{18} of all \(\ell\) modes built on the \(\ell = 0\) solution \(g_0^\ell(r) \approx \chi / \sinh \chi\), we see that it is possible to satisfy the condition \(T^\mu_\nu = 0\) non-trivially in the negative spatial curvature conformal cosmology case, with the very high temperature universe then being composed of a perfect fluid bath of soft modes which non-trivially support a \(k < 0\) universe. Cosmology thus fixes the curvature itself, and does so before the onset of any symmetry breaking phase transition at all.\textsuperscript{19}

It is also illuminating to derive this result in a slightly different fashion. Since \(T^\mu_\nu\) is a conformal invariant tensor, we could also evaluate it by first making the conformal transformation \(g^\mu_\nu(x) \rightarrow R^{-2}(t)g^\mu_\nu(x)\) on the Robertson-Walker metric, a transformation which brings the geometric line element to the form

\[
d\tau^2 = dp^2 - r^2 (1 - kr^2)(d\theta)^2 - d\Omega = dp^2 - \gamma_{ij}dx^idx^j.
\]

For this transformed metric the Ricci scalar is given by

\[
R^\alpha_\alpha = -6k,
\]

with the incoherently averaged soft mode contribution to the energy density then being given by

\[
T^{00}_{\text{kin}} = \frac{1}{6} \sum_{\ell,m} \left[ 3\gamma_{\ell m} \partial_i(g^\ell_m(\theta,\phi))\right] \Gamma_{\ell m}^0 \left(\partial_i g^\ell_m(\theta,\phi)\right)^2 + k\left| g^\ell_m(\theta,\phi)\right|^2,
\]

(37)

a quantity which can actually vanish non-trivially provided \(k\) is negative. And indeed, through use of the various completeness relations \textsuperscript{22} which these modes obey, explicit evaluation of \(T^{00}_{\text{kin}}\) is then found to yield

\[
T^{00}_{\text{kin}} = \frac{1}{6} \sum_{\ell,m} \frac{(2\ell + 1)(\ell - m)!}{4\pi(\ell + m)!} \left[ (1 - kr^2)P^m_\ell (\cos \theta)^2 \left(\frac{dg^\ell_m}{dr}\right)^2 + \frac{1}{r^2} \left(\frac{dP^m_\ell (\cos \theta)}{d\theta}\right)^2 \left(g^\ell_m\right)^2 \right. \\
+ \frac{m^2}{r^2 \sin^2 \theta}P^m_\ell (\cos \theta)^2 \left(g^\ell_m\right)^2 + kP^m_\ell (\cos \theta)^2 \left(g^\ell_m\right)^2 \right] \\
= \frac{1}{24\pi} \sum_{\ell} \frac{(2\ell + 1)}{(1 - kr^2)} \left(\frac{dg^\ell_m}{dr}\right)^2 + \frac{\ell(\ell + 1)}{r^2} \left(g^\ell_m\right)^2 + k\left(g^\ell_m\right)^2 \\
= \frac{1}{24\pi} \left[ \frac{2k}{3\pi} - \frac{4k}{3\pi} + \frac{2k}{\pi} \right] = 0.
\]

(38)

Thus we see that when the spatial curvature is negative, the negative energy density then present in the gravitational field completely cancels the positive energy density of the matter fields, with gravity itself then being able to fix the spatial curvature of the universe.

Having now shown that \(k\) is uniquely negative in conformal cosmology (we will show below how galactic rotation curve data actually enable us to measure \(k\) to find that it is indeed negative), we can now proceed to study its cosmological implications as the universe

\textsuperscript{18}In passing we note that it might prove interesting should there be a group under which this infinite tower of states transforms irreducibly.

\textsuperscript{19}To show that there actually will be a phase transition as the temperature drops requires the development of a detailed dynamical model for the coherent correlations that are not present in the incoherently averaged perfect fluid limit. However, without knowing the details of how long range order actually sets in, nonetheless, use of the perfect fluid model at temperatures far above the critical region is sufficient to fix the sign of \(k\) once and for all.
cools. Thus, with the signs of \( k \) and \( \lambda \) now being fixed (we simulate the negativity of \( \Lambda = \hbar \lambda S^4_0 \) by a negative \( \lambda \)), Eq. (27) is then found (in the simpler to treat high temperature era where \( cT^0_{\text{kin}} = \rho_M(t) = A/R^4 = \sigma T^4 \)) to admit of the unique solution [19,20]

\[
R^2(t, \alpha > 0, k < 0) = -k(\beta - 1)/2\alpha - k\beta \sinh^2(\alpha^{1/2}ct)/\alpha
\] (39)

where have introduced the positive parameter \( \alpha = -2\lambda S^2_0 \) and the parameter \( \beta = (1 - 16\Lambda/k^2hc)^{1/2} \) which is greater than one. In this solution the deceleration parameter is found to take the requisite non-positive form

\[
q(t, \alpha > 0, k < 0) = -\tanh^2(\alpha^{1/2}ct) + 2(1 - \beta)\cosh(2\alpha^{1/2}ct)/\beta \sinh^2(2\alpha^{1/2}ct). \tag{40}
\]

We thus see that the cosmology is non-singular, having a finite minimum radius and an initial \( \dot{R}(t = 0) \) which is zero rather than infinite. Since the cosmology has a minimum radius, it also has a finite maximum temperature \( T_{\text{max}} \) in terms of which Eq. (39) can be rewritten as

\[
T^2_{\text{max}}(\alpha > 0, k < 0)/T^2(t, \alpha > 0, k < 0) = 1 + 2\beta \sinh^2(\alpha^{1/2}ct)/(\beta - 1), \tag{41}
\]

with the Hubble parameter then being given as

\[
H(t) = \alpha^{1/2}c(1 - T^2(t)/T^2_{\text{max}})/\tanh(\alpha^{1/2}ct). \tag{42}
\]

In terms of the convenient effective temperature \( T_V \) defined via \(-c\Lambda = -c\hbar \lambda S^4_0 = \sigma T^4_V \), we find that the parameter \( \beta \) can be expressed as

\[
\beta = (1 + T^4_V/T^4_{\text{max}})/(1 - T^4_V/T^4_{\text{max}}) \tag{43}
\]

(with \( T_V \) thus being less than the maximum temperature \( T_{\text{max}} \)), with the temporal evolution of the theory then being given by

\[
T^2_{\text{max}}/T^2(t) = 1 + (1 + T^4_{\text{max}}/T^4_V)\sinh^2(\alpha^{1/2}ct) \tag{44}
\]

and with the energy density terms then being given by

\[
\bar{\Omega}_\Lambda(t) = (1 - T^2(t)/T^2_{\text{max}})^{-1}(1 + T^2(t)T^2_{\text{max}}/T^4_V)^{-1}, \\
\bar{\Omega}_M(t) = -(T^4(t)/T^4_V)\bar{\Omega}_\Lambda(t), \\
\Omega_k(t) = -kc^2/\dot{R}^2(t) = 1 - \bar{\Omega}_M(t) - \bar{\Omega}_\Lambda(t). \tag{45}
\]

With \((1 + T^2(t)T^2_{\text{max}}/T^4_V)^{-1} \) being a quantity which is always necessarily bounded between zero and one (no matter what the magnitude of \( T_V \)), we see [20] that the single, simple requirement that \( T_{\text{max}} \) be very much greater than \( T(t_0) \) then entails that \( \bar{\Omega}_\Lambda(t_0) \) must lie somewhere between zero and an upper bound of one today, with its current value then being given by \( \bar{\Omega}_\Lambda(t_0) = (1 + T^2(t_0)T^2_{\text{max}}/T^4_V)^{-1} = \tanh^2(\alpha^{1/2}ct_0) \) according to Eq. (14). Thus, no matter how big \( T_V \) might be, \( \bar{\Omega}_\Lambda(t_0) \) must not only be of order one today, it must also be approaching its asymptotically expected value of one from below; to thus yield a completely natural current era quenching of \( \bar{\Omega}_\Lambda(t_0) \) regardless of the numerical value of any cosmological parameter. Moreover, the larger \( T_V \), the further \( \bar{\Omega}_\Lambda(t_0) \) must lie below its asymptotic bound
of one, with it taking a typical value of one-half in the event that the current temperature is
given by $T(t_0) \simeq T_V^2/T_{\text{max}}$, a condition which is readily realizable if the conditions $T_{\text{max}} \gg T_V$
and $T_V \gg T(t_0)$ both hold. Thus we see that $\Omega(t_0)$ could actually be appreciably below one
today, and that it would actually fall further below one the larger rather than the smaller
the cosmological constant, with conformal cosmology thus having no difficulty handling a
large $T_V/T(t_0)$. Thus it is precisely in the event that the cosmological constant is in fact
large that $\Omega(t_0)$ is then able to be of a phenomenologically acceptable magnitude long
before becoming fully asymptotic. Beyond this, we note additionally, that given the fact
that there is an upper bound on $\Omega(t_0)$, we see that a large $T_V/T(t_0)$ will then completely
quench the current era $\Omega_M(t_0)$ altogether. Thus in conformal gravity a large cosmological
constant naturally leads to current era suppression of $\Omega_M(t_0)$ just as was desired above.
Then, because of this suppression, the current era evolution equation of Eq. (28) reduces to
$1 = \Omega_k(t_0) + \Omega(t_0)$, with negative curvature explicitly forcing $\Omega(t_0)$ to lie below one. It is
thus the negative spatial curvature of the universe which bounds and tames the contribution
of the cosmological constant to cosmology with it thus indeed being possible to construct a

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20There are three possible ways in which the condition $T_{\text{max}} \gg T(t_0)$ can be realized in Eq. (27). If $T_V$ is of order $T(t_0)$, $\sinh^2(\alpha^{1/2}ct_0)$ would be very much less than one. Similarly, if $T_V$ is of order $T_{\text{max}}$, $\sinh^2(\alpha^{1/2}ct_0)$ would be very much greater than one. However, if $T_V$ is of intermediate order $(T(t_0)T_{\text{max}})^{1/2}$, $\sinh^2(\alpha^{1/2}ct_0)$ would then be of order one.

21Since $T_{\text{max}}$ is greater than $T_V$, a large $T_V/T(t_0)$ entails a large $T_{\text{max}}/T(t_0)$, and thus a universe old enough for the current era $\Omega(t_0)$ to be given by the bounded $\tanh^2(\alpha^{1/2}ct_0)$. For large $T_V$ conformal cosmology thus solves the cosmological constant problem simply by living a long time.

22Phenomenologically, this quenching of $\Omega_M(t_0) = -3\Omega_M(t_0)/4\pi S_0^2 L_P^2$ requires that the scale factor $S_0$ be altogether greater than $L_P^{-1}$, a condition which is readily realizable not only by associating a fundamental temperature with a fundamental $S_0$ which is altogether larger than the Planck temperature, but instead, and preferentially, by identifying $S_0$ as a macroscopically occupied order parameter, with $S_0$ then being proportional to the (very large) number, $N$, of occupied positive energy perfect fluid modes, modes whose coherent correlations cause phase transitions to occur in the early universe in the first place.

23In a $\lambda < 0$ cosmology with $k > 0$, the current era $\Omega_M(t_0)$ is still found to be completely suppressed by large $T_V/T(t_0)$, but in such a cosmology $\Omega(t_0)$ is no longer bounded by one from above, but only from below, with the $k > 0$ case $T_{\text{max}}$ being found to be less than $T_V$. Thus we see that in the $k < 0$ case it is precisely the contribution of negative curvature itself which produces a cosmology in which the highest temperature $T_{\text{max}}$ is greater than the vacuum energy temperature $T_V$ (with there thus being an in principle difference between flat space and curved space phase transitions), with curvature itself controlling the value of $T_{\text{max}}$. Moreover, with there being an explicit maximum temperature $T_{\text{max}}^\lambda = -\Lambda S_0^2 c^2/2(\sigma A)^{1/2}$ in a $\lambda < 0$ cosmology even in the absence of any $\lambda S_0^4$ term at all, we see that the magnitudes of $T_{\text{max}}$ and $T_V$ are fixable independently, with the imposition of the condition $T_{\text{max}} \gg T_V$ which we use thus being readily naturally achievable in the $\lambda \neq 0$ case without the need for any fine-tuning of parameters.
phenomenologically acceptable cosmology in which \( \Lambda \) can still be as large as particle physics suggests.

From a phenomenological viewpoint, once \( \Omega_M(t_0) \) is suppressed, the deceleration parameter is then given by \( q(t_0) = -\tanh^2(\alpha^{1/2}ct_0) \) (so that it then has to lie between zero and minus one), while the curvature contribution is given by \( \Omega_k(t_0) = \text{sech}^2(\alpha^{1/2}ct_0) \). Thus in conformal gravity first matter dominates the expansion rate (in the early universe), then curvature, and finally vacuum energy; and even if the current era is not yet vacuum energy dominated, nonetheless \( \Omega_\Lambda(t_0) \) will still be under control no matter what the value of \( \alpha^{1/2}ct_0 \).

As regards the actual supernovae data themselves, we note that even though the phenomenological fitting allowed for solutions with \( \Omega_M(t_0) = 0 \) (typically with \( \Omega_\Lambda(t_0) = -q(t_0) = 1/2 \), and even though such solutions would not be expected to occur in the standard theory, we see that in the conformal theory\(^{24} \) solutions with \( \Omega_M(t_0) = 0 \) and \( \Omega_\Lambda(t_0) = 1/2 \) are right in the region allowed by Eq. (15). Moreover, in \(^{24} \) fits with \( \Omega_M(t_0) = 0 \) and \( \Omega_\Lambda(t_0) = 0 \) were found, and even though those fits were of quality comparable with the best reported \( \Omega_M(t_0) \neq 0, \Omega_\Lambda(t_0) \neq 0 \) fits, such fits were not considered further since in standard gravity they would correspond to an empty universe. However, we now see that in conformal gravity not only would such fits (fits in which \( T_V^2/T(t_0)T_{\text{max}} \ll 1 \), so that \( \Omega_M(t_0) \simeq 0 \Omega_\Lambda(t_0) \simeq 0 \) be quite acceptable, such fits would not entail an empty universe since it is \( \Omega_M(t_0) = 0 \) which is suppressed and not \( \rho_M(t_0) \) itself. Thus with no fine-tuning of parameters at all conformal cosmology leads us right into the \( \Omega_M(t_0) = 0, 0 \leq \Omega_\Lambda(t_0) \leq 1 \) region favored by the supernovae data. Moreover, with the suppression of \( \Omega_M(t_0) = 0 \) being achievable without any constraint being put on \( \rho_M(t_0) \), its value is thus not constrained to be of order the critical density \( \rho_c = 3c^2H^2(t_0)/8\pi G \), with conformal cosmology thus being released from the need to contain copious amounts of cosmological dark matter.\(^{25} \) Conformal gravity thus not only gets rid of the need for galactic dark matter, it eliminates the need for cosmological dark matter as well.

With conformal gravity having thus eliminated the need for cosmological dark matter,

\(^{24} \) With \( \Omega_M(t_0) \) and \( \Omega_\Lambda(t_0) \) being treated as free parameters in supernovae data fitting based on the standard Eq. (19), those fits are just as equally phenomenological fits to the conformal Eq. (27).

\(^{25} \) Like \( \Omega_M(t) \), in the conformal case the quantity \( \Omega_M(t) \) itself starts off being infinite at \( t = 0 \) (no matter what the numerical values of the parameters) and finishes up being zero at \( t = \infty \). Thus, \( \Omega_M(t) \) has to pass through one in some epoch without the need for any fine tuning (and even has to pass through one quite slowly since the cosmology associated with \( R(t) \) of Eq. (18) is a very slow coasting one) while being far from one in other epochs, to thus not constrain the value of \( \rho_M(t_0) \) at all. Moreover, if dark matter is not invoked, known explicitly visibly established matter alone leads to an \( \Omega_M(t_0) \) of order \( 10^{-2} \) or so, so \( \Omega_M(t_0) \) may not be so close to one today as to require any particular explanation in the first place (i.e. the value of \( \Omega_M(t_0) \) would not be special unless it was actually equal to one to some incredibly high degree of accuracy); and indeed, even in an \( \Omega_M(t_0) < 1 \) standard theory, ongoing expansion will eventually lead to an \( \Omega_M(t) \) which will be nowhere near to one, with its current closeness to one then being an accidental consequence of the fact that it is this particular epoch in which we just happen to be making observations.
it is of interest to see just exactly how the conformal gravity evolution equation of Eq. (27) itself actually manages to avoid any flatness fine tuning problem.\footnote{The author is indebted to Dr. M. Turner for asking a helpful question in this regard.} Thus in the illustrative $\lambda = 0$ case where the evolution equation is given by

$$\dot{R}^2(t) + 3\dot{R}^2(t)\Omega_M(t)/4\pi S_0^2L_{Pl}^2 = -kc^2$$

(46)

with solution \cite{19}

$$R^2(t, \alpha = 0, k < 0) = -2A/hkS_0^2c - kc^2t^2,$$

(47)

we see the two terms on the left hand side of Eq. (46) have radically different time behaviors, even as their sum remains constant ($= -kc^2$). Specifically, $\dot{R}^2$ begins at zero and slowly goes to $-kc^2$ at late times, while the $3\dot{R}^2(t)\Omega_M(t)/4\pi S_0^2L_{Pl}^2$ term does the precise opposite as it goes to zero from an initial value of $-kc^2$. Moreover, while these two terms turn out to be of the same magnitude at some time in the early universe, simply because the scale factor $S_0$ is so much larger than $L_{Pl}^{-1}$, these two terms are nowhere near the same order of magnitude today, and yet their sum remains constant. As such this behavior differs radically from that found in the standard model, since there the very fact that $L_{Pl}$ is its natural scale forces the analogous two terms (terms which are of opposite sign in the standard model) to be of the same order of magnitude at all times right up to the present and to thus have to cancel to an extraordinary degree. Since such fine-tuning is not required in the conformal case, we see that it is the changing of the effective $G$ which explicitly enables us to resolve the flatness problem.

With the general $\alpha \neq 0$ Hubble parameter obeying Eq. (42), we see that its current value obeys $-q(t_0)H^2(t_0) = \alpha c^2$, with the current age of the universe then being given by $H(t_0)t_0 = \arctanh([-q(t_0)]^{1/2}]/(-q(t_0))^{1/2}$. Thus we see that $t_0$ is necessarily greater or equal to $1/H(t_0)$ ($t_0 = 1/H(t_0)$ when $q(t_0) = 0$, and $t_0 = 1.25/H(t_0)$ when $q(t_0) = -1/2$). Thus we see that conformal cosmology readily resolves [19,21] another problem which has troubled the standard theory, viz. the universe age problem. Further, in conformal cosmology the (dimensionless) ratio of the particle horizon size $d(t)$ to the spatial radius of curvature $R_{\text{curve}}(t)$ (= $(6/R^3)^{1/2}$ where $R^3$ is the modulus of the Ricci scalar of the spatial part of the metric) is given by

$$\frac{d(t)}{R_{\text{curve}}(t)} = (-k)^{1/2}c \int_0^t \frac{dt}{R(t)} = (-k)^{1/2}c \int_0^t \frac{dt}{[-k(\beta - 1)/2\alpha - k\beta \sinh^2(\alpha^{1/2}ct)/\alpha]^{1/2}}.$$  

(48)

Thus, in the illustrative $\lambda = 0$ case where $q(t_0) = 0$ and where $T_{\text{max}}^2 = T^2(t_0)(1 - 1/\Omega_M(t_0))$, Eq. (48) evaluates as

$$\frac{d(t)}{R_{\text{curve}}(t)} = \log \left[ T_{\text{max}}^2 + (T_{\text{max}}^2 - T^2(t))^{1/2} \right],$$

(49)

with the horizon size thus being altogether greater than one at recombination. Similarly in the $q(t_0) = -1/2$, $T_{\text{max}} \gg T_V \gg T(t_0)$ case where the current time obeys $\sinh^2(\alpha^{1/2}ct_0) = 1$
and where $\beta \simeq 1$, we then find for all times up to recombination that $R(t, \alpha > 0, k < 0)$ can be approximated by $-k(\beta - 1)/2\alpha - kc^2t^2$, with the recombination time horizon size then being found to again be given by Eq. (49) (with $T_{\text{max}}$ being the $\alpha > 0$ one this time). Conformal cosmology thus readily resolves [19] the horizon problem, and leads to a naturally causally connected cosmology. Thus, to sum up, we see that simply by modifying the effective cosmological $G$, conformal gravity is then able to resolve a whole variety of current cosmological problems, viz. the flatness, horizon, dark matter, universe age, cosmic acceleration and cosmological constant problems, and should thus be seen as a potentially viable candidate cosmological theory (for its current overall status and for the challenges that it itself still faces see [19, 20]).

With conformal cosmology being rendered singularity free through the negative sign effective Einstein-Hilbert action present in the conformal matter action of Eq. (21), we see that in conformal gravity it is gravity itself which can protect itself from its own singularities. Conformal gravity thus provides a possible (though currently far from guaranteed) mechanism by which the collapse of a star can still possibly be prevented when all conventional mechanisms (such as radiation pressure or Pauli degeneracy) have failed, viz. that a scalar field condensate could be generated inside the star (perhaps when the star reaches nuclear density) which would then induce some repulsive gravity (cf. negative energy density) and arrest the collapse (to then either generate a rebound or produce a stable configuration with radius greater than the Schwarzschild radius of the star). Thus with gravity being able to generate its own repulsion, the whole issue of gravitational singularities needs to be reconsidered, with the standard strong gravity picture possibly being another piece of the

27For comparison, we recall that in the typical $k < 0, \Lambda = 0$ standard gravity case, this same $d(t)/R_{\text{curv}}(t)$ ratio is given by $\log[(T_{\text{ref}}^2 + (T_{\text{ref}}^2 + T^2(t))^{1/2})/T(t)]$ (where $T_{\text{ref}}^2 = T^2(t_0)(1/\Omega_M(t_0) - 1)$) and is thus much smaller than one at recombination. Thus with the conformal gravity $T_{\text{max}}$ being much larger than the standard gravity $T_{\text{ref}}$, simply because the effective conformal $G$ is so much smaller than the standard one, we see that it is the changing of the effective $G$ which explicitly enables us to resolve the horizon problem.

28Such a negative contribution is simply not considered in models in which positivity of the energy density is assumed, and even while the energy density of a standard kinematic perfect fluid is indeed positive, we thus see that the extension of such positivity to the entire energy-momentum tensor which serves as the source of gravity is actually questionable in general (even if absent in the flat space limit, non-inertial explicitly curvature dependent terms are not forbidden in the curved space case), as would then be the gravitational collapse theorems which rely on such positivity and assume that no negative component is ever generated during any such collapse.

29While recent data [24] on the velocities of stars in inner galactic regions have indicated the presence of large (black hole candidate) mass concentrations at the centers of galaxies such as M87 and the Milky Way, those data are currently unable to ascertain whether any such large mass is actually confined to a radius smaller than its Schwarzschild radius or determine whether any event horizon has actually been formed (at 400 km s$^{-1}$ or so the measured velocities, while large by galactic standards, are still well below the velocity of light).
Having thus presented the cosmological case for a negative spatial curvature universe, we now present some additional, quite direct observational evidence in its support, evidence from an at first somewhat unlikely source, namely the systematics of galactic rotation curves. While these curves provided the first clear evidence of the need in standard gravity for dark matter, beyond the fact that these curves show that there actually is a departure from the luminous Newtonian expectation in the outskirts of spiral galaxies, explicit study of the systematics of such departure has revealed the presence of an apparent cosmological imprint on the data, with it being cosmology itself which will actually enable us to eliminate the need for any galactic dark matter at all. Indeed, with the potentials associated with static sources in conformal gravity being ones which actually grow with distance according to Eq. (7), the very fact that they do so entails that in calculating the motions of individual particles within a given galaxy, one is now no longer able to ignore the contributions of the potentials due to distant matter sources outside of that galaxy. Thus in going to a higher order theory such as conformal gravity, we immediately transit into a world where we have to consider effects due to matter not only inside but also outside of individual systems, and thus we are led to look for both local and global imprints on galactic rotation curve data, this being a quite radical (and quite Machian) conceptual departure from the standard second order, purely local Newtonian world view.

To isolate such possible global imprints, it is instructive to look at the centripetal accelerations of the data points farthest from the centers of individual galaxies. In particular, for a set of 11 particular galaxies whose rotation curves are regarded as being particularly characteristic of the pattern of deviation from the luminous Newtonian expectation that has so far been obtained, it was found that the farthest centripetal accelerations in these galaxies could all be parameterized by the universal three component relation

\[
\frac{(v^2/R)_{\text{last}}}{2} = \gamma_0 c^2 / 2 + \gamma^* N^* c^2 / 2 + \beta^* N^* c^2 / R^2
\]

where \(\gamma_0 = 3.06 \times 10^{-30}\) cm\(^{-1}\), \(\gamma^* = 5.42 \times 10^{-41}\) cm\(^{-1}\), \(\beta^* = 1.48 \times 10^5\) cm, and where \(N^*\) is the total amount of visible matter (in solar mass units) in each galaxy. Since the luminous Newtonian contribution is decidedly non-leading at the outskirts of galaxies, we thus uncover the existence of two linear potential terms which together account for the entire measured departure from the luminous Newtonian expectation, with one of these two terms depending on the number, \(N^*\), of stars within each given galaxy, and with the other, the \(\gamma_0 c^2 / 2\) term, not being dependent on the mass content of the individual galaxies at all. Moreover, since numerically \(\gamma_0\) is found to have a magnitude of order the inverse Hubble

\[30\]With it being only for spherically symmetrically distributed matter with \(1/r\) potentials that exterior sources decouple locally, the very detection of any global cosmological imprint within galaxies would then argue against gravitational potentials being of a pure \(1/r\) form.

\[31\]With the luminous Newtonian contribution falling with distance and with the rotation curves of the prominent bright spirals being flat, the departure from the luminous Newtonian expectation must itself thus be growing with distance, and according to Eq. (50) even be growing universally in fact.
radius, we can thus anticipate that it must represent a universal global effect generated by the matter outside of each galaxy (viz. the rest of the matter in the universe), and thus not be associated with any local dynamics within individual galaxies at all.

As regards the \(N^*\) dependent contribution of the matter within the individual galaxies, the integration of the non-relativistic stellar potentials \(V^*(r) = -\beta^* c^2 / r + \gamma^* c^2 r / 2\) over an infinitesimally thin galactic optical disk with luminous surface matter distribution \(\Sigma(R) = \Sigma_0 \exp(-R/R_0)\) and total number of stars \(N^* = 2\pi \Sigma_0 R_0^2\) yields \([13]\) the centripetal acceleration

\[
v^2 / R = g^\text{lum}_{gal} = g^\text{lum}_\beta + g^\text{lum}_\gamma \tag{51}
\]

where

\[
g^\text{lum}_\beta = (N^* \beta^* c^2 / 2R_0^3)[I_0(r/2R_0)K_0(r/2R_0) - I_1(r/2R_0)K_1(r/2R_0)] \tag{52}
\]

and where

\[
g^\text{lum}_\gamma = (N^* \gamma^* c^2 / 2R_0)I_1(r/2R_0)K_1(r/2R_0), \tag{53}
\]

to thus yield a net acceleration which behaves asymptotically as \((v^2 / R)_{last} = \gamma^* N^* c^2 / 2 + \beta^* N^* c^2 / R^2\). The conformal gravity local galactic potentials associated with the matter within any given galaxy thus nicely generate the \(N^*\) dependent terms exhibited in Eq. (50).

As regards the remaining \(N^*\) independent \(\gamma_0 c^2 / 2\) term, in order to be able to identify it as being of cosmological origin, we need to rewrite the comoving Hubble flow of the rest of the universe in the rest frame coordinate system of any given galaxy of interest. To determine just how a comoving geometry might look in a Schwarzschild coordinate system, we note \([11]\) that the general coordinate transformation

\[
r = \rho / (1 - \gamma_0 \rho / 4)^2, \quad t = \int d\sigma / R(\sigma) \tag{54}
\]

effects the metric transformation

\[
d\tau^2 = (1 + \gamma_0 r)c^2 dt^2 - \frac{dr^2}{(1 + \gamma_0 r)} - r^2 d\Omega \rightarrow \frac{(1 + \rho \gamma_0 / 4)^2}{R^2(\sigma)(1 - \rho \gamma_0 / 4)^2} \left(c^2 d\sigma^2 - \frac{R^2(\sigma)(d\rho^2 + \rho^2 d\Omega)}{(1 - \rho^2 \gamma_0^2 / 16)^2}\right). \tag{55}
\]

Thus with metrics conformal to a Robertson-Walker one also being allowed cosmological solutions in a conformal invariant theory, we see that in conformal gravity a static, Schwarzschild coordinate linear potential metric is coordinate equivalent to conformal cosmologies in which the spatial curvature \(k = -\gamma_0^2 / 4\) is expressly negative,\(^{32}\) with a universal linear potential thus being the local manifestation of global negative curvature, and with the local \(\gamma_0^\text{and}

\[^{32}\] Positive \(k\) would lead to a complex \(\gamma_0\) in Eq. (55), with it not being possible for the topologically open \(d\tau^2 = (1 + \gamma_0 r)c^2 dt^2 - dr^2/(1 + \gamma_0 r) - r^2 d\Omega\) metric to ever be equivalent to anything other than other topologically open ones.
the global $k$ thus having a common connection. As such, this connection is actually geometrically quite natural, since not only does negative $k$ lead to repulsion, but, as had been noted earlier, so does positive $\gamma_0$. However, while a positive $\gamma_0r$ metric term does indeed lead to gravitational deflection of light, nonetheless, for non-relativistic systems this same positive $\gamma_0r$ term acts attractively, with cosmological negative spatial curvature thus generating an attractive gravitational effect for non-relativistic motions within galaxies, an effect which the standard theory in essence tries to simulate locally by the introduction of local galactic dark matter. Now while we had initially identified the static $\gamma_0$ term of Eq. (50) as being of cosmological origin since its phenomenologically measured value was found to be of order the inverse of the Hubble radius, such an identification could at best have only been heuristic since the Hubble parameter itself is a time dependent one which varies from one epoch to the next. However, cosmology actually possesses a second scale beyond that associated with its expansion rate, namely that associated with its spatial curvature, with it being this latter, epoch independent one, with which $\gamma_0$ is then nicely identified, with the $\gamma_0r$ term thus serving as a time independent universal potential term no matter what the epoch.

Further, for galaxies which have no peculiar velocities with respect to the Hubble flow, i.e. for any galaxy whose center can precisely serve as the coordinate origin for the radial coordinate $r$ in the transformation of Eq. (54), the metrics of Eqs. (51) and (55) can simply be added in the weak gravity limit, to yield as the net weak gravity centripetal acceleration

$$v^2/R = g_{tot} = g_{\beta}^\text{lum} + g_{\gamma}^\text{lum} + \gamma_0c^2/2,$$

an acceleration whose asymptotic limit precisely yields Eq. (50) for $(v^2/R)_{\text{last}}$. Moreover, not only does Eq. (56) yield this requisite asymptotic formula, its use in the sub-asymptotic region as well is then found to provide acceptable parameter free (and thus dark matter free) fitting [13,14] to all of the (in excess of 250) data points in the 11 galaxy sample, with Eq. (56) thus capturing the essence of the data. Thus we identify an explicit imprint of cosmology on galactic rotation curves, recognize that it is its neglect which may have led to the need for dark matter, and for our purposes here confirm that $k$ is indeed negative, just as had been required in the cosmological study which we presented above, with the phenomenological formula of Eq. (50) actually providing an explicit measurement of the curvature of the universe which test particles sample as they orbit in galaxies.

Moreover, not only does Eq. (56) provide for an acceptable accounting of galactic rotation curve data, the magnitude obtained for the stellar $\gamma^*$ is found to be so small (of order $10^{-41}$ cm$^{-1}$) that the linear potential term then makes a completely negligible contribution on solar system distance scales, with the metric of Eq. (5) thus reducing to the standard Schwarzschild one within the solar system. The strength of the linear potential terms in Eq. (56) thus serve as the scale which is to parameterize departures from the luminous Newtonian expectation (for the bright spirals $\gamma^*N^*$ and $\gamma_0$ are of the same order of magnitude), with such a scale nicely explaining why no dark matter is needed on solar system distance scales, with the solar system simply being too small to be sensitive to any cosmologically relevant

\[33\] For comparison, standard dark matter halo fitting uses two free parameters per halo and thus no less than 22 additional free parameters for the same 11 galaxy sample.

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scale. With the linear potential term first becoming competitive with the Newtonian one on none other than galactic distance scales, we thus explain not only why solar system physics is unaffected by conformal gravity, but also, we identify at exactly what point departures from the luminous Newtonian contribution are to first set in. The (negative) spatial curvature of the universe thus sets the scale at which standard gravity needs to introduce dark matter in order to avoid failing to fit data.

Our uncovering of a universal acceleration in conformal gravity immediately recalls the presence of a similar one in the MOND theory \[25\], one also of a cosmologically significant magnitude. Specifically, Milgrom had suggested that if a universal acceleration \(a_0\) did exist, then Newton’s law of gravity could possibly be phenomenologically modified into a form such as \(v^2/R = v(a_0/g_N)g_N\) where \(g_N = g_{\text{lum}}\) in the galactic case. The candidate functional form \(\nu(x) = (1/2 + (4x^2 + 1)^{1/2}/2)^{1/2}\) would then yield

\[
v^2/R = g_N\{1/2 + (g_N^2 + 4a_0^2)/2g_N\}^{1/2},
\]

an expression which, despite the absence so far of any deeper underlying theory, is nonetheless found to perform extremely well phenomenologically. While Eqs. \(56\) and \(57\) have different underlying motivations, it is of interest to note that Eq. \(56\) would in fact fall into the general MOND approach if the MOND formula were to be reinterpreted as

\[
v^2/R = v(\gamma_0c^2/2g_{\text{loc}})g_{\text{loc}}
\]

where \(g_{\text{loc}}\) is the entire local luminous galactic contribution \(g_{\text{lum}}\) given in Eq. \(51\), and if the function \(\nu(x)\) were to instead take the form \(\nu(x) = 1 + x\). Conformal gravity thus not only provides a rationale for why there is in fact a universal acceleration in the first place (something simply assumed in MOND), but also it even yields an explicit form for the function \(\nu(x)\), albeit not the one commonly utilized in the standard MOND studies (where \(g_{\text{loc}}\) is taken to be the purely Newtonian \(g_N\)). However, despite such differences, both theories have in common the recognition that there is a universal scale which is to parameterize departures from the standard luminous Newtonian expectation, and that its magnitude is a cosmologically significant one.

Further support for the existence of such a scale has been presented by McGaugh \[26\] in a study of the behavior of the quantity \(M_{\text{dyn}}(R)/M_{\text{lum}}(R)\) as a function of the measured orbital acceleration \(v^2(R)/R\) at points \(R\) within galaxies. \(M_{\text{dyn}}(R) = Rv^2(R)/G\) is the amount of matter interior to \(R\) as would be required by Newtonian gravity, while \(M_{\text{lum}}(R)\) is the amount of luminous matter detected in the same region. In this study McGaugh found that mass discrepancies (viz \(M_{\text{dyn}}(R)/M_{\text{lum}}(R) > 1\)) systematically occurred in galaxies whenever the measured \(v^2(R)/R\) fell below a universal value of \(10^{-8}\) cm s\(^{-2}\) or so, a value which is immediately recognized as being close to the values of the \(a_0\) and \(\gamma_0c^2/2\) acceleration parameters which were respectively phenomenologically obtained in the MOND and conformal gravity theories. While MOND and conformal gravity might differ as to how the centripetal accelerations should behave in regions where there are measured mass discrepancies, both theories (and the data) thus agree that there is a universal scale which determines when such discrepancies should first set in.

Hence, independent of the merits of alternate theories such as conformal gravity or MOND themselves, it would appear that the data possess a cosmological imprint, an imprint which heralds when dark matter is first needed in the standard theory, with the very
existence of such an imprint enabling Eqs. (56) and (57) to organize the data in an extremely economical fashion. Thus even if one does not want to contemplate going beyond standard gravity, the galactic data themselves seem to be insisting that dark matter theories should be parameter free (i.e. that they should be formulatable without the (extravagant) need for two free parameters per galactic halo), and that there is a cosmological imprint in the data which dark matter theories must be able to produce. Moreover, apart from the fact that dynamical dark matter models have not yet produced such a scale (say by studying the growth of galaxy fluctuations in cosmology), it would appear difficult for them to ever be able to do so in the standard cold dark matter flat inflationary universe cosmological model, since simply by virtue of being flat such a cosmology then lacks the one key ingredient which leads to a universal galactic scale in the conformal theory, namely a non-zero spatial curvature, with the standard flat classical cosmological model simply not possessing any intrinsic such universal scale at all. Given the above observational support for the existence of such a scale, galactic data thus seem to be supporting the notion that the universe has a non-zero spatial curvature, a non-trivial curvature which conformal gravity (and for the moment only conformal gravity apparently) can readily and naturally produce, a curvature which releases gravity from having to always be attractive.

Thus to conclude, we believe that it is not yet justified to assert that gravity is always attractive, and that in fact a repulsive cosmological component immediately allows one to resolve a whole host of problems which currently beset the standard theory. And even if conformal gravity itself should not turn out to be the correct extrapolation of the standard solar system wisdom (as a cosmology conformal gravity is not without challenges of its own [14,19]), that would in no way constitute evidence in favor of the correctness of the standard extrapolation. Since our study of conformal gravity has shown that the problems with which the standard theory is currently afflicted are not in fact generic to cosmology, their very existence could be a warning that the extrapolation of standard gravity beyond the confines of the solar system might be a lot less reliable than is commonly believed, with gravity not always being as attractive as Newton initially took it to be.

VI. AUTHOR’S NOTE

During the last few years I have written articles [12,14] on gravitational theory for special issues of Foundations of Physics in honor of my longtime colleagues and friends Fritz Rohrlich and Larry Horwitz. It is a great pleasure for me to dedicate this third article in that series to another equally close colleague and friend Kurt Haller. This work has been supported in part by the Department of Energy under grant No. DE-FG02-92ER4071400.

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34 See [27] for an interesting attempt to generate such a scale quantum-mechanically as a renormalization group correlation length associated with a quantum gravity fixed point, a point at which scale invariance (such as that in conformal gravity) is realized via anomalous dimensions.
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