Multiplicity Distribution and Mechanisms of the High-Energy Hadron Collisions

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Abstract

We discuss the multiplicity distribution for highest accessible energies of $pp$- and $\bar{p}p$- interactions from the point of view of the multiparton collisions. The inelastic cross sections for the single, $\sigma_1$, and multiple (double and, presumably, triple, $\sigma_{2+3}$) parton collisions are extracted from the analysis of the experimental data on the multiplicity distribution up to the Tevatron energies. It follows that $\sigma_1$ becomes energy independent while $\sigma_{2+3}$ increases with $\sqrt{s}$ for $\sqrt{s} \geq 200$ GeV. The observed growth of $\langle p_\perp \rangle$ with multiplicity is attributed to the increasing role of multiparton collisions for the high energy $\bar{p}p(pp)$- inelastic interactions.

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I. INTRODUCTION. KNO SCALING AND ITS VIOLATION

The experimental observations of the violation of KNO scaling [1] at the high energy (pp)- and (¯pp)-collisions [2] and of the correlations between the average transverse momentum \( \langle p_{\perp} \rangle \) and the multiplicity \( N \) of the secondaries (a higher \( \langle p_{\perp} \rangle \) for high multiplicity events) [3] indicate that there are at least two (complementary) mechanisms of the high energy multiparticle production revealing the quark-gluon structure of hadrons and their interactions.

That KNO scaling should be violated at very high energies was realized long ago [4], in the same year when KNO scaling was introduced. This deviation from a KNO type distribution of secondaries was attributed to the possibility of the splitting of each of the colliding hadrons into several constituents (valence quarks, partons) pair wise interacting with their counterparts from oppositely moving hadrons. The above picture results in the production of several showers and, in the frame of the Reggeism, is attributed to the contribution of Regge cuts to the elastic scattering amplitude, in addition to the Regge pole (Pomeron) corresponding to the production of the single shower.

Inclusion of such contributions changes the structure of the distribution \( \frac{\sigma_N}{\sigma_{\text{tot}}} \) \( (\sigma_N \) is the cross section of the production of \( N \) secondary hadrons) at large \( \xi = \ln \frac{s}{s_0} \) \( (s_0 = 1 \text{ GeV}^2) \) leading to the appearance of the additional peaks in the distribution with larger \( \langle N_n \rangle \) (here \( n \) denotes the number of pairs of the simultaneously colliding partons involved in the interaction from the different hadrons, or the number of resulting showers; \( n = 1 \), a single interaction, can be attributed to the Regge pole; \( n = 2 \), a double collision of two pairs of partons from different hadrons, can be attributed to the exchange of two Pomerons (Regge cut), etc.).
II. REGGE PICTURE AND MULTIPLE COLLISIONS OF PARTONS

In this picture, \( \langle N_n \rangle = n \langle N_1 \rangle \) where \( \langle N_1 \rangle \) is the mean multiplicity corresponding to the production of one shower in the single partonic collision. We have, from the data where the one Pomeron exchange describes experiments at lower energies \( \langle N_1 \rangle = a + b \ln \frac{S}{s_0} \), with \( a = -3.879; \ b = 2.099 \) (see Appendix A for details). This qualitative picture [4,5] corresponds to the idealized scenario where only “non-enhanced” Regge-type diagrams are included (“quasi-eikonal” approximation). Inclusion of the so-called “enhanced” diagrams where the possibility of the branching of ladders-showers is allowed, thus giving the emission of the smaller average number of emitted particles than the corresponding non-enhanced cut, leads to the smearing out of the distribution between the peaks reducing the effects of KNO violations. Since the distributions of particles in the different ladders are independent, one expects that \( \Delta \langle N_n \rangle \sim \sqrt{n \langle N_1 \rangle} \) giving the broadening of the distribution \( \frac{\sigma_{2N}}{\sigma_{\text{tot}}} \).

We emphasize that the multipomeron exchanges are especially important in the modern treatment of the Pomeron as a pole with intercept \( \alpha(0) \) higher than unity \( (\alpha(0) = 1 + \Delta; \ \text{for “soft” Pomeron} \ \Delta \approx 0.08 \ \text{as experimental data on high energy “soft” collisions show}) \), where the relative role of these exchanges is increased with energy [6] (e.g., Pomeron contribution to the \( \sigma_{\text{tot}} \) is \( \sim e^{\xi \Delta} \), whereas the \( n \) Pomeron exchange is \( \sim e^{n \xi \Delta} \)). Concerning the effective number of exchanges in “quasi-eikonal” approximation, one has \( n_{\text{eff}} \sim e^{\xi_n \Delta} \leq 2.5 \left( \xi_n = \ln(\frac{\sigma_{\text{tot}}}{s_{0n^2}}) \right) \) [6] for \( \sqrt{s} \leq 2 \ \text{TeV} \) (\( n = 2 \)). Thus, from this point of view, only double parton interactions (two effective Pomeron exchanges) in addition to the single collision are expected to be effective at the energies up to the Tevatron energy. At LHC energy (\( \sqrt{s} = 14 \ \text{TeV} \)) this parameter \( n_{\text{eff}} \approx 3.3 \) (\( n = 3 \)), thus indicating the possibility of the appearance of the third maximum in \( \frac{\sigma_{2N}}{\sigma_{\text{tot}}} \) distribution with \( \langle N_3 \rangle \approx 3 \langle N_1 \rangle \) (see Fig. 1 and Appendix B).

\(^{1}\)Since at the highest achievable energies (\( \sqrt{s} \geq 500 \ \text{GeV} \)) KNO scaling (in the whole rapidity interval) is violated strongly, this indicates that at these energies the “enhanced” graphs, probably, are not important. It is not excluded, that at higher energies KNO scaling will be restored.
III. GLUON STRING MODEL AND POMERONS

On the QCD level, the ultra-high energy interaction between hadrons phenomenologically described by Pomeron exchange is treated as a result of a (few) gluons exchange between the constituents of hadrons as they pass close to one another. After a color exchange between partons (valence quarks, gluons, etc.) carried by the colliding hadrons, partons from one hadron are joined by two gluon strings with partons inside the other hadron. This pair of strings stretch when hadrons separate after the collision and break down transforming into the system of hadrons forming a Pomeron-like shower (ladder), giving the asymptotic density of hadrons.

From this point, the above described $n$ Pomeron exchanges are considered as the joining and stretching of $n$ different pairs of strings between hadron constituents resulting, after the hadron separation, the production of $n$ Pomeron-like showers with $n$-fold increased average multiplicity $\langle N_n \rangle$.

This picture using the language of gluon strings can be related to the Pomeron phenomenology, at least in the quasi-eikonal approximation, and, as shown in [8], leads to the same expressions for a variety of characteristics of the high energy strong interactions as the Regge diagram technique, giving also the possibility to determine some free parameters of last one.

IV. TWO COMPONENT PICTURE OF THE HADRON COLLISIONS

From the more phenomenological approach [9], it is possible to argue that there are at least two components in the cross sections for the ultra-fast hadron collisions: soft component corresponding to the single collisions of partons from different hadrons which can (but not necessarily) be described by the exchange of the single Pomeron (or the pair of gluonic strings), and the so-called semi-hard (jet) component which is responsible for the higher $\langle p_\perp \rangle$ of the secondaries than is usually attributed to the single “soft” Pomeron exchange.
At low energies (up to energy of ISR) only soft interactions with scaled multiplicity distribution of the KNO type are important. At higher energies (CERN SPS, Tevatron) the KNO-like shape of the multiplicity distribution is modified by the increasing role of the “hard” multiparton interactions [9] (four parton collisions of [10]) that leads to the increase of the fraction of events with higher multiplicity \(N \gg \langle N_1 \rangle\). This new channel is characterized, at the same time, by the higher \(\langle p_{\perp} \rangle\), in accordance with the experimental observation [11] that the high multiplicity events indicate a jet-like structure.

From the point of view of the interactions between constituents, one can argue that this second “semihard” component with \(\langle N_2 \rangle \approx 2\langle N_1 \rangle\) is due to the double collisions when in the same inelastic interaction, two different parton pairs from colliding hadrons independently collide. Since the parton densities of the colliding hadrons overlap strongly at small impact parameters (higher \(p_{\perp}\)) the probability of the double parton collision \((n = 2)\) is enhanced, and, as a result, two pairs of soft partons (e.g., valence quarks) \((x_{\perp} = \frac{p_{\perp}}{\sqrt{s}} \ll 1)\) stacked in the collision resulting in the (mini) jet structure of the secondaries with \(N \geq \langle N_2 \rangle = 2\langle N_1 \rangle\).

In [9] it was shown that the above two-component model gives a successful fit for the total non-single diffractive inelastic cross section inclusive spectra and multiplicity distribution in full phase space for the energies \(\sqrt{s} = 200\) and \(546\) GeV. We note that in [9] the ad hoc assumption was made that the inelastic cross section for the soft component (single parton collisions) is energy independent.

We will see below, from our analysis of the multiplicity distribution, that the high energy inelastic cross section corresponding to the single parton collisions indeed, practically does not depend on \(\sqrt{s}\), as it was assumed in [9]. On the other hand, the one Pomeron exchange in its quasi-eikonal form gives an increase of inelastic cross section. We don’t know the answer to that inconsistency, except that the role of the enhanced diagrams is underestimated, and they have to be included in the analysis of the “soft” cross section.
V. \( \langle P_\perp \rangle \) VERSUS MULTIPLICITY

Now, if we again turn to the language of the Pomeron exchanges, it is possible to conjecture that the above described double parton collisions can be attributed to the double effective Pomeron exchange which leads not only to the higher multiplicity (and to the deviation from KNO shape) but also to the increase of the average transverse momentum \( \langle p_\perp \rangle \) in the regime of \( \frac{p_\perp}{\sqrt{s}} \ll 1 \). Indeed, the \( p_\perp \) dependence of \( n \) Pomeron exchange amplitude is given by the expression \( \exp\left(-\frac{\lambda_n p_\perp^2}{n}\right) \), where \( \lambda_n = R^2 + \alpha'(0)\xi_n \). Experimentally, for \( pp \)-collisions, \( \alpha'(0) = 0.25 \text{ GeV}^{-2} \) [8]. That gives for the ratio

\[
\frac{\langle p_\perp \rangle_2}{\langle p_\perp \rangle_1} \approx \sqrt{2} \sqrt{\frac{R^2 + \alpha'_1\xi_1}{R^2 + \alpha'_2\xi_2}} \approx \sqrt{2}.
\]

This ratio is in good correspondence to the experimental data [12]. The further increase of the \( \langle p_\perp \rangle \) can be expected at the effective opening of the triple parton collision, which as we see from Fig. 1 should be clearly displayed at the LHC energy range. An experimentally established increase of \( \langle p_\perp \rangle \) with energy is connected, of course, with the gradual increase of the relative role of the double Pomeron exchange (or double parton collisions) with energy.

There are recent results from the CDF collaboration [13] for the process—-independent and free from theoretical schemes cross section of double parton collision in the 3 jets data:

\[
\sigma_{dp,\text{eff}} = (14.5 \pm 1.7^{+1.7}_{-2.3}) \text{ mb}.
\]

The AFS experiment [14] (\( \sqrt{s} = 63 \text{ GeV} \)) gives for the same quantity \( \sim 5 \text{ mb} \), while UA2 presents a lower limit of \( \sigma_{dp,\text{eff}} > 8.3 \text{ mb} \) [15].

VI. ANALYSIS OF THE MULTIPLICITY DISTRIBUTION DATA

Here we analyze the data on the multiplicity distributions obtained at the energy range \( \sqrt{s} \) from 30 to 1800 GeV including ISR, UA5 and Tevatron (E735 experiment) data. We use as a basis of our analysis the fact that KNO scaling is well satisfied in the experimental data through the ISR energies. The deviation from the simple KNO scaling at higher energies is
due to another process which is incoherently superimposed on the KNO producing process. In Fig. 3, the differential cross sections are displayed for the UA5 and the E735 data. Note that the E735 data is likely to be less reliable at low multiplicities than the UA5 data whereas the E735 data is more statistically reliable at the higher multiplicities.

In Fig. 3 the collider data from different energies is superimposed on one plot. The data have been normalized to the same value of the variable \( x = \frac{N}{\langle N_1 \rangle} \) at the \( x_{\text{max}} \), maximum value \( \frac{1}{\sigma_{\text{int}}} \frac{d\sigma}{dx} \). It turns out that the cross section at the maximum is essentially independent of energy. From the measurement of KNO curve at the ISR energies we know that \( N_{\text{max}} = 0.8\langle N_1 \rangle \). Thus, we can estimate the value of \( \langle N_1 \rangle \), the average multiplicity for the processes with pure KNO scaling from the known value of \( N_{\text{max}} \) at which the cross section is a maximum for higher energies. We made a least square fits to the value of \( N_{\text{max}} \) and \( \langle N_1 \rangle \) over range \( \sqrt{s} \) from 30 GeV to 1800 GeV for the former and \( \sqrt{s} \) from 30 GeV to 200 GeV (where KNO scaling is well satisfied) for the latter. These two quantities are well fitted by the form \( a + b \ln \sqrt{s} \) with the same coefficients over their regions of overlap. As we remarked above, the deviation from KNO scaling is due to another process which is incoherently superimposed on top of the KNO producing process. By subtracting the KNO distribution from the observed data we determine the shape of the competing process as shown in Fig. 4.

The shapes of the multiplicity distribution thus found are rather different from the KNO distribution and is not a simple convolution of the KNO distribution. There must be a correlation between the multiplicities in the two collisions. The main characteristics of the derived distribution is that the most probable value of the distribution occurs at \( x = 2 \) (or at twice the multiplicity corresponding to the initial low energy KNO distribution). The width of the distribution is close to \( \sqrt{2} \) times the width of that KNO shape. This is in quite good agreement with the picture we presented above, which is based on the adding of the double collision of partons of the colliding hadrons by the exchange of the two Pomerons (or two pairs of strings) to the single parton collision which proceeds by the exchange of one effective Pomeron.
Anyway, independently of the concrete realization of the interaction between ultrarelat-
ativistic parton pairs, we can interpret the population of the secondaries with maximum at
$2\langle N_1 \rangle$ as a result of two independent parton-parton collisions occurring in the same encounter.
We denote the inelastic (non-single diffractive) cross section for this process by $\sigma_2$ whereas
$\sigma_1$ denoted the inelastic cross section due to the single parton-parton collision. This $\sigma_1$ is
categorized by the KNO scaling and, as it follows from our analysis, it is nearly energy
independent for $\sqrt{s} \geq 200$ GeV and has a value of $(34 \pm 2)$ mb (Fig. 5).

Some remarks on the experimental data of E735 and UA5 are necessary. In Figs. 3-5 we
do not show error bars on all the points in the derived distributions. The statistical errors
in E735 are relatively small. However, the systematic error in both experiments might be
sizeable. In E735 as well as UA5 the multiplicities in a restricted range of rapidity are
extended to the full range by computer simulation. There are corrections for secondary
interactions in individual events. The derived distributions in Fig. 3 suffer in accuracy from
the fact that one uses subtraction of two different distributions which each have uncertainties.
When the experimental data and the KNO distribution are close in value then the error is
greater. Fortunately, the KNO distribution is only the order of 10% of the experimental
data at $N = 2\langle N_1 \rangle$ $(x = 2)$ and consequently, the position of the peak in the derived
distribution is not strongly affected by errors produced by taking the differences. The
whole distribution might move up and down as a result of errors in either of the KNO or
experimental distrubitions.

The values of the cross sections represented by the curves shown in Figs. 3,4 are given
in Fig. 5.

It is seen there that, in contrast of $\sigma_1$, $\sigma_2$ is increased with $\sqrt{s}$ and equals 16 mb at 1800
GeV. We already have mentioned that CDF collaboration gives the effective double-parton
collision cross section equal $(14.5 \pm 1.7^{+1.7}_{-2.3})$ mb. One has to realize that in the sample of
events with double parton collisions CDF has removed events with possible triple collisions.

We can conclude that the double (and possibly, triple) collisions account for a large
fraction of the increase in the total $p\bar{p}$-cross section and, definitely, are responsible for the
increase of the $p\bar{p}$ inelastic cross section.

As the collision energies are increased to LHC values, it seems likely that double and possibly triple collisions will constitute a larger fraction of the inelastic cross section as is seen in our effective analyses from the previous sections (see Fig. 1).

Another indication of the increasing importance of double collisions comes from an examination of the $\langle p_\perp \rangle$ of the charged particles at high multiplicity events. As we mentioned above there is an increase of $\langle p_\perp \rangle$ as multiplicity increases. We note that $\langle p_\perp \rangle$ increases by close to a factor $\sqrt{2}$ from low to high multiplicity. The picture with the effective double Pomeron exchange in the double-parton collision leads just to this $\sqrt{2}$ extra factor. The increase of the fraction of the multi-collisions with energy will, in general, lead to this well known $\langle p_\perp \rangle$-multiplicity relation. We expect that at LHC energies $\langle p_\perp \rangle$ will undergo the additional increase.

VII. CONCLUSIONS

In this paper we have attempted to isolate and study the double-parton collision mechanism from the analysis of high energy multiplicity data. The main result of our analysis of these data is that the non-single diffractive inelastic cross section consists of two components. The first component corresponding to the single parton collisions is characterized by being practically independent of $\sqrt{s}$ cross section for $\sqrt{s} > 200$ GeV, whereas the second part of the $\sigma_{in}^{NSD}$ increases significantly with energy and achieves the value 16 mb at $\sqrt{s} = 1.8$ TeV. That part was attributed here to the inclusion in the collision process of the double (and, maybe, triple) parton collisions. Thus, the increase of the $\sigma_{in}^{NSD}$ at high $\sqrt{s}$ is almost entirely due to the multi-parton collisions.

On the other hand, we know from the experiment at lower energies where the single parton collisions indeed dominate that $\sigma_{in}^{NSD}$ is increasing with $\sqrt{s}$. This indicates that at higher energies $\sqrt{s} > 200$ GeV the inelastic cross section due to the single collision goes to the saturation, whereas the double collisions give the increasing part of the $\sigma_{in}^{NSD}$. One can
go further and conjecture that the same saturation will occur for the part of $\sigma_{in}$ connected with the double parton collisions at much higher energies whereas the $\sigma_{in}$ due to the triple collisions will still increase with $\sqrt{s}$, etc., until asymptotically the total inelastic cross section (without the diffractive part) will set a constant value $\sigma_{in} = \sigma_{1}^{(sat)} + \sigma_{2}^{(sat)} + \ldots$. Of course, we cannot say anything about the behavior of $\sigma_{el}$ and $\sigma_{dif}$ at this hypothetical limit.

We need to emphasize that in this paper we have often used the notion of the “soft” Pomeron exchange in the process of parton collisions. Of course, this notion should be understood as an effective tool of the description of the ultrarelativistic parton collision and have played only the illustrative role and the trend in our analysis. However, the picture of the multi-parton collisions described here and the corresponding values for $\sigma_{1}$ and $\sigma_{2+3}$ extracted from our above analysis of the multiplicity are independent of the concrete mechanism of the parton-parton interaction.

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FIGURES

FIG. 1. a) Topological cross sections $\sigma_N$ in the quasi-eikonal approximation with exchanges of three effective “soft” Pomerons for $\sqrt{s} = 546, 900, 1800$ and $14.10^3$ GeV. b) Topological cross sections resulting from double and triple parton collisions for $\sqrt{s} = 546, 900, 1800$ and $14.10^3$ GeV.

FIG. 2. Same in Fig. 1 for $\sqrt{s} = 100, 200$ and 300 GeV.

FIG. 3. A comparison of multiplicity distributions at different values $\sqrt{s}$. The distributions are normalized at the maximum value of $\frac{d\sigma}{dx}$ where $x = \frac{n}{\langle n_1 \rangle}$. The solid curve is the KNO distribution from ISR data.

FIG. 4. Multiplicity distributions obtained by taking the difference between the $p\bar{p}$ collider data and the KNO distribution.

FIG. 5. Cross sections for the single ($\sigma_1$) and multiparton (double and triple) ($\sigma_2$) parton collisions as a function of $\sqrt{s}$. 


APPENDIX A

According to our ideology, at high energies ($\sqrt{s} \gg 1$ GeV) the single parton collisions which may be described by the effective single Pomeron exchange and which are characterized by the scaled multiplicity distribution, must be applied only up to the energy range where KNO scaling takes place. That means that the average multiplicity of the hadrons (in $pp$- and $\bar{p}p$-interactions) for the single collisions $\langle N_1 \rangle$ must be obtained only from this energy range. The usually used total range of the existing energies up to highest ones (Tevatron), being important, of course, however, from the developed point of view, disguises the mechanisms of the high energy collisions. Thus, we describe, in the spirit of the Regge picture, the average multiplicity $\langle N_1 \rangle$ as $a + b \ln \frac{s}{s_0}$ and find the coefficients $a$ and $b$ from the data of the high energy $pp$-collisions in the range of $\sqrt{s}$ ($11.5 \div 62.6$) GeV.\footnote{When our paper was written we became aware of the recent similar treatment of the average multiplicity $\langle N_1 \rangle$ as a linear logarithmic function for “soft” component \cite{17}.} As a result of the fit, we have for the coefficients $a$ and $b$: $a = -3.874$, $b = 2.099$ with $\chi^2 = 7.085/7$.

We use the corresponding $\langle N_1 \rangle$ as input in calculation of the multiplicity distributions which takes into account the multiparton ($n = 2, 3$) collisions (see Appendix B).

APPENDIX B

We briefly describe here the results for the cross section of the production of $N$ particle in the quasi-eikonal approximation of the Regge approach (or, equivalently, as we emphasized in the text, in the model of the gluonic strings \cite{3,8}). These results are represented graphically in Fig. 1.

We are interested in the inelastic cross-section $\sigma_{in}(N, s)$ not including the diffraction processes. The ignorance of the “enhanced” graphs allows us to consider that the long distant correlations among particles belonging to one shower-ladder are absent \cite{3}. Similarly,
we suppose that the partons from different showers are uncorrelated also. This permits us to assume that the distributions $P_n(N)$ of the $N$ produced particle (in full phase space) in the $n$ shower events is the Poisson type:

$$P_n(N) = \frac{\langle N_n \rangle^N}{N!} e^{-\langle N_n \rangle} \tag{1}$$

$$\langle N_n \rangle = n \langle N_1 \rangle = n \left( a + b \ln \frac{s}{s_0} \right). \tag{2}$$

We can write for $\sigma_{in}(N, s)$

$$\sigma_{in}(N, s) = \sigma_1 P_1(N) + \sigma_2 P_2(N) + \sigma_3 P_3(N) + \ldots, \tag{3}$$

where $\sigma_n(\xi_n)$ is the cross section for the production of $n$ ladders and $\xi_n = \ln \left( \frac{s}{s_{0n^2}} \right)$. For $\sigma_n(\xi_n)$ we have

$$\sigma_n(\xi_n) = \sigma_p \frac{n Z_n}{n Z_n} \left( 1 - e^{-Z_n} \sum_{k=0}^{n-1} \frac{Z_n^k}{k!} \right) \tag{4}$$

with

$$\sigma_p = 8\pi \gamma \left( \frac{s}{s_0} \right)^\Delta, \quad Z_n = \frac{2C \gamma}{R^2 + \alpha_p' \xi_n} \left( \frac{s}{s_{0n^2}} \right)^\Delta.$$

The parameters in these expressions are fixed by the fits of the experimental data on $\sigma_{tot}$ and $\frac{d\sigma_{el}}{dt}$ for pp- and $\bar{p}p$-collisions:

$$\gamma_p = 3.64 \text{ (GeV)}^{-2}, \quad R^2 = 3.56 \text{ (GeV)}^{-2}, \quad C = 1.5,$$

$$\Delta = \alpha_p(0) - 1 = 0.08, \quad \alpha_p' = 0.25 \text{ (GeV)}^{-2}(c = 1).$$

3Use of the concrete form of (1) is not crucial for obtaining the relation (5). It has a place for normalized $P_n(N)$ with $\langle N_n \rangle = n \langle N_1 \rangle$ and large $\langle N_1 \rangle$. The relation (4), of course, does not depend on (1). Figs. 1-2 show only the trends.
We retain three terms in (3) since the quadruple (or four-parton pair) collision has small effect (see Fig. 1 which shows that even the triple collisions appear noticable only at the energy range of LHC).

Summing (3) over \( N \) from 1 to \( \infty \) with \( \langle N_n \rangle \gg 1 \) we obtain for the total inelastic cross section

\[
\sigma_{in}(s) = \sigma_1 + \sigma_2 + \sigma_3 + O(\sigma_4)
\]  

Fig. 1a) shows the \( N \)-distributions of \( \sigma_{in}(N, s) \) at \( \sqrt{s} = 0.55; 0.9; 1.8 \) and \( 14 \) TeV. At lower \( \sqrt{s} \) the second peak of \( \sigma(N, s) \) is not resolved. Fig. 1b) shows the behavior of the parts of \( \sigma(N, s) \) which can be attributed to the double (and triple) collisions.

In the table we present values of the inelastic “partial” cross sections \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) corresponding to the single, double and triple parton collisions in the Regge quasi-eikonal approach.

We see that in this approach all \( \sigma_i \ (i = 1, 2, 3) \) increase with \( \sqrt{s} \) whereas our analysis in the text above indicates that the “single” collision contribution \( \sigma_1 \) extracted from the experimental data on the multiplicity distribution practically is independent of \( \sqrt{s} \).

Furthermore, \( \sigma_1 + \sigma_2 + \sigma_3 \) are systematically lower than the experimental cross section \( \sigma_{in}^{tot}(s) \) for the non-single diffraction events, whereas the corresponding theoretical values of this cross section resulting from the summation of all quasi-eikonal pomeron graphs [6,8]

\[
\sigma_{in}^{tot}(s) = \sigma_p f(Z)
\]

where

\[
f(Z) = \sum_{v=1}^{\infty} \frac{(-Z)^{v-1}}{vv!} = \frac{1}{Z} \int_0^Z (1 - e^{-x}) \frac{dx}{x} = \frac{1}{Z} (\Gamma(0, Z) + \ln(\gamma_E Z))
\]

with \( \gamma_E = 1.78 \ldots \) Euler constant and \( \Gamma(\alpha, Z) \)-incomplete gamma function, are in excellent agreement with experimental data for \( \sigma_{in}^{tot} \).

Fig. 2a and 2b show for completeness the \( \sigma(N, s) \) for lower \( \sqrt{s} \) (but higher than 62 GeV) \((\sqrt{s} = 100, 200, 300 \text{ GeV})\). On these figures at \( \sqrt{s} = 200 \) GeV the shoulder, corresponding to the double collision is clearly seen.
| $\sqrt{s}$, TeV | $\sigma_1$, mb | $\sigma_2$, mb | $\sigma_3$, mb | $\sigma_1 + \sigma_2 + \sigma_3$, mb |
|-----------------|----------------|----------------|----------------|---------------------------------|
| 0.55            | 21.94          | 9.57           | 5.25           | 36.76                           |
| 0.9             | 22.72          | 10.16          | 5.71           | 38.59                           |
| 1.8             | 23.84          | 10.72          | 6.34           | 40.90                           |
| 14              | 27.19          | 14.70          | 8.43           | 50.32                           |
The diagram shows the cross section (mb) as a function of the number of particles for different energies:

- 546 GeV
- 900 GeV
- 1.8 TeV
- 14 TeV
