The answer is blowing in the wind: Simulating the interaction of jets with dynamic cluster atmospheres

S. Heinz\textsuperscript{1}, M. Brüggen\textsuperscript{2}, A. Young\textsuperscript{1}, E. Levesque\textsuperscript{1}

\textsuperscript{1}Center for Space Research, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139; Chandra Fellow
\textsuperscript{2}International University Bremen, Campus Ring 1, 28759 Bremen, Germany

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ABSTRACT
We present numerical simulations investigating the interaction of AGN jets with galaxy clusters, for the first time taking into account the dynamic nature of the cluster gas and detailed cluster physics. The simulations successfully reproduce the observed morphologies of radio sources in clusters. We find that cluster inhomogeneities and large scale flows have significant impact on the morphology of the radio source and cannot be ignored a-priori when investigating radio source dynamics. Morphological comparison suggests that the gas in the centres of clusters like Virgo and Abell 4059 show significant shear and/or rotation. We find that shear and rotation in the intra-cluster medium move large amounts of cold material back into the path of the jet, ensuring that subsequent jet outbursts encounter a sufficient column density of gas to couple with the inner cluster gas, thus alleviating the problem of evacuated channels discussed in the recent literature. The same effects redistribute the excess energy $\Delta E$ deposited the jet, making the distribution of $\Delta E$ at late times consistent with being isotropic.

Key words: galaxies: clusters: general — galaxies: jets — galaxies: intergalactic medium

1 INTRODUCTION
Recent observations show a multitude of physical effects that occur when active galactic nuclei (AGN) interact with the ambient intra-cluster medium (ICM). While these effects are widely believed to be crucial for the formation of structure in the universe, they are still poorly understood.

The central galaxy in almost every strong cooling core contains an active nucleus and a jet-driven radio galaxy. The radio power of these cooling cores is somewhat correlated with the X-ray luminosity, although the range of the radio power is much greater than the range of the X-ray core power. This is supported by a recently discovered correlation between the Bondi accretion rates and the jet power in nearby, X-ray luminous elliptical galaxies (Allen et al. 2006). These results show that the AGN at the centres of large elliptical galaxies feed back enough energy to quench cooling and star formation, thus providing a possible explanation for the observed cut-off at the bright end of the galaxy luminosity function (Benson et al. 2003; Croton 2003; Bower et al. 2006). The study of AGN-ICM interactions is of great interest in a much broader context. Given the Magorrian relation (Magorrian et al. 1998), clusters hosting a giant cD galaxy should have a massive black hole in their centre. The growth of these black holes is an integral part of feedback-regulated cooling of the ICM.

Radio-loud AGN inflate kpc-sized bubbles of relativistic, underdense plasma (a.k.a. radio lobes) that displace the hot intra-cluster medium and thus appear as depressions in the X-ray surface brightness. High-resolution X-ray observations of cooling flow clusters with Chandra have revealed a multitude of X-ray holes of ten coincident with patches of radio emission, a compiled list of which is presented by Birzan et al. (2004).

Numerical simulations of hot, underdense bubbles in clusters of galaxies have been performed by a number of authors (e.g. Churazov et al. 2001; Brüggen & Kaiser 2002a,b; Reynolds et al. 2001; Ruszkowski et al. 2004; Dalla Vecchia et al. 2004; Omma et al. 2004; Omma & Binney 2004). Common to all of these simulations is that they use a spherically symmetric, analytical profile for the ICM. Quilis et al. (2001) simulated AGN feedback in a cosmologically evolved cluster but did not model the jets. Most recently, Vernaleo & Reynolds (2005) performed three-dimensional simulations of a jet in a hydrostatic, spherically symmetric cluster model. In their simulation, the jet power was modulated by the mass accretion rate across the inner boundary. In all of their models, jet heating failed to prevent the catastrophic cooling of gas at the centre because the jet preferentially heated gas lying along the jet axis but failed to heat matter in the equatorial plane. However, in reality the dynamics of the jets may be strongly affected by the ambient medium. Bulk velocities may advec and distort the jet and the radio lobe. Density inhomogeneities can affect both the jet propagation and the deposition of entropy in the ICM by the jet. For the purpose of studying the extent of cluster heating by AGN, the crucial question is exactly where and how much energy is deposited by the jet in the ICM. The primary aim of this work is therefore to study the dynamics of the jet and the extent of ICM heating subject to the motions and inhomogeneities of the ICM.

In this letter, we present first results from hydrodynamical simulations of AGN heating in a realistic galaxy cluster. The pri-
mary objectives of this work is to study the interaction of the jet with a dynamic, inhomogeneous ICM. The letter is organised as follows: 2 describes the technical setup of the simulations, 3 presents the results and discusses the implications for cluster physics, and 4 summarised and concludes.

2 TECHNICAL SETUP

The initial conditions of our simulation are based on a rerun of the S2 cluster from [Springel et al. 2001], whose properties are sufficiently close to a typical, massive, X-ray bright cluster with a mass of \( M \sim 7 \times 10^{14} M_\odot \) and a central temperature of 6 keV. The output of the GADGET SPH simulation serves as the initial conditions for our simulation. We use the FLASH code [Fryxell et al. 2000], which is a modular block-structured adaptive mesh refinement code, parallelised using the Message Passing Interface. It solves the Riemann problem on a Cartesian grid using the Piecewise-Parabolic Method. Our simulation includes \( 7 \times 10^5 \) dark matter particles. The particles are advanced using a cosmological variable-timestep leapfrog-method. Gravity is computed by solving Poisson’s equation with a multigrid method using isolated boundary conditions. For the relatively short physical time of the jet simulation (160 Myrs), radiative cooling and star formation are neglected, though they were included in the constitutive SPH simulation.

The cluster shows significant signs of anisotropy as well as some net rotation of the central cluster gas and contains a dynamically induced cold-front. We specifically picked a non-relaxed cluster to investigate the possible impact of cluster gas dynamics on jet–cluster interactions, to be compared with hydrostatic, spherical atmospheres previously investigated (e.g. Sommer-Larsen et al. 2003). The computational domain is a \( 2.8 Mpc^3 \) box around the cluster’s centre of mass. The maximum resolution at the grid centre corresponds to a cell size of \( 174 pc \), implying 11 levels of refinement. The total computing time on our 26 processor cluster amounted to 2 processor years. The simulations presented in this letter were performed assuming an adiabatic equation of state with a uniform adiabatic index of \( \gamma = 5/3 \) and no radiative cooling.

The jet is injected through a nozzle placed at the centre of the gravitational potential, coincident with the gas density peak of the central elliptical galaxy. The nozzle is modeled as two circular back-to-back inflow boundaries \( 2pc \) or 12 resolution elements in diameter. The nozzle faces obey inflow boundary conditions fixed by the jet’s mass-, momentum-, and energy fluxes. This treatment avoids the entrainment of cluster gas into the jet which is unavoidable in simpler schemes where the jet is approximated by injecting mass, momentum, and energy into a finite volume of the cluster that contains thermal gas and is part of the active computational grid. We were thus able to cleanly separate jet fluid and cluster fluid and to calculate accurate maps of synchrotron and thermal emission from the two components, respectively, as well as cleanly separating jet fluid and cluster gas. We verified this by running a test case with half the opening angle of \( 20^\circ \) and an internal Mach number of 32.

The jet power of the simulation presented in this letter was chosen to be \( W_{\text{jet}} = 10^{46} \text{ergs s}^{-1} \), corresponding to a rather powerful source, comparable to Cyg A, and in line with the powers now implied by the large scale moderate shocks found around, e.g., Hercules A [Nulsen et al. 2005]. In order to quantitatively study the jet’s effect on the cluster, we ran a simulation without a jet but otherwise identical parameters for the same length of time, which we refer to as the control simulation. Having thus described the numerical setup, we will now proceed to present and discuss the results.

3 RESULTS

3.1 Cluster and Radio Source Morphology

While the jet is active, the jet-inflated cocoon and the surrounding swept up shell generally follow the morphological evolution discussed previously in the literature: The jets inflate two oblong cocoons, which push thermal gas aside. The early expansion is supersonic, shocking the surrounding gas directly, while at later times the expansion slows down and eventually becomes sub-sonic. Fig. 1 shows a time series of density and entropy cuts as well as simulated X-ray maps (using the Chandra ACIS-I response and assuming an APEC thermal model) and low-frequency radio maps (assuming equipartition and neglecting radiative cooling). The aspect ratio of the lobes is roughly 3 and decreases with time, indicating the lateral spreading and shear of the radio plasma.

During the active source evolution, while the jet is on, the source closely resembles the best studied example of a source of comparable power: Cyg A. The fact that aspect ratio and X-ray and radio morphology of the simulated source so closely resemble Cyg A indicates that we properly modeled the effect of the dentist drill effect (which can, in fact, be directly observed in Cyg A, where the jet axis is clearly mis-aligned with the radio and X-ray hot spots). We verified this by running a test case with half the opening angle of the jet cone (\( 10^\circ \) instead of \( 20^\circ \)) which produced too narrow a cocoon (with an aspect ratio of around 7).

The X-ray maps show a clear increase in the surface brightness in the outgoing shock/compression wave and a wispy, turbulent wake behind the wave (Fig. 2). This is best demonstrated by un-sharp masking of the images. An un-sharp masked movie of the X-ray images can be viewed at [http://space.mit.edu/~heinzs/jetpower/] showing a network of roughly spherical outgoing sound waves that are excited behind the major initial shock wave, consistent with the complex network of sound waves seen in Perseus [Fabian et al. 2003]. They also show bud-like bubbles appear at the edges of the main shell when the jet axis moves due to “dentist-drill”, reminiscent of the budding bubbles in M87 [Forman et al. 2003].

Two distinctions to previously published results are important to point out regarding the morphological evolution of the source:

First, it is clear from the series of snapshots shown in Fig. 1 that the non-sphericity of the atmosphere induces significant asymmetries in the two sides of the cocoon. Given that the difference is so strong, it is clear that the impact on the morphology as a whole must be significant. This justifies our initial motivation for this study: In order to investigate the evolution of radio sources and their impact on galaxy cluster atmospheres, one cannot neglect the dynamic nature of the atmospheres prior to jet injection.
Figure 1. Top-to-bottom: Time series of snapshots at 0Myrs, 5Myrs, 10Myrs, 20Myrs, 40Myrs, 80Myrs, and 160Myrs after jet onset. Left-to-right: (a) density cut through cluster centre, (b) entropy cut through cluster centre. The images are 450 kpc in width and 335 kpc in height.

Figure 2. Time series of snapshots (0Myrs, 5Myrs, 10Myrs, 20Myrs, 40Myrs, 80Myrs, and 160Myrs after jet). Left-to-right: (a) Top panel: velocity map through cluster center; below: low-frequency radio synchrotron map, (b) X-ray maps (red: 0.3-2 keV, green: 2-5 keV, blue: 5-10 keV).
Second, the dynamic nature of the cluster atmosphere significantly alters the late stage evolution, after the jet switches off (between the fourth and the fifth frame at $3.3 \times 10^8$ yrs) and the source evolution becomes sub-sonic. During the sub-sonic stage, the impact of the pressure and density of the surrounding gas become more and more important. In particular, the rotation that is present in the cluster (see Fig. 2) has clearly sheared the plasma away from the jet axis. We will quantify these results below. It is already clear, however, that this effect can solve the dilemma of launching multiple jet episodes into the same cluster: Any evacuated channel created by previous outbursts will have been buffeted around sufficiently to move enough cluster gas back into the path of the new jet to provide a sufficient cross section for interaction and to couple the jet to the inner cluster gas.

The spiral-like morphology of the radio source at late stages is very reminiscent of the large scale ($> 30$ kpc) morphology of M87 and Abell 4059 (Heinz & Sunyaev 2002), where the large scale radio bubbles are clearly mis-aligned with the inner jet. This leads us to suggest that there might be significant rotation or shear present in the gas of these clusters.

Before proceeding to a more quantitative analysis, it is worth pointing out that the episode of jet activity does not disrupt the cluster or blow the central region apart, despite the fact that we are simulating a powerful source (certainly at the upper end of the power range expected from central cluster radio sources). Contrary to what might be expected naively, the expansion shock weakens and does not super-heat the central atmosphere beyond convective stability. This does not mean, however, that the jet does not induce significant turbulent motion in the central cluster.

### 3.2 Quantitative results

As found in earlier investigations of jet-cluster interactions, the impact of the jet on the cluster leads to an increase in the central entropy and a decrease in density, i.e. a net heat input. This is qualitatively apparent from the entropy panels in Fig. 1. Figures 3 and 4 quantify this result. The bottom panel of Fig. 3 shows the cumulative radial mass, i.e., the mass $M(< r)$ contained within radius $r$ from the cluster centre. The initial time step lies above all other curves out to a radius of about 60 kpc, indicating that the cluster has been inflated (i.e., mass moved out).

The bottom panel of Fig. 4 shows the increase in the mean entropy $\Delta s = \Delta \log (p/\rho^{5/3})$ of the cluster gas at different time steps over the control simulation, plotted against the Lagrangian coordinate $M(< r)$ from Fig. 3. This plot clearly shows that the entropy of the central $10^{12}$ $M_\odot$ has been increased by about 0.1, corresponding to an increase in $p/\rho^{5/3}$ by about 25%.

The top panel of Fig. 3 shows the ratio of the cooling times with and without jet. At late times ($t \gtrsim 80$ Myrs) the cooling time for the inner few $10^{12}$ $M_\odot$ has been increased by about 50%, more at earlier times. This indicates that, on average, the jet can halt cooling for a time significantly longer than the time the jet is active. Future simulations including radiative cooling will be necessary to investigate whether this increase is sufficient to offset cooling entirely and whether this result is consistent with XMM-Newton and Chandra constraints on cluster cooling rates.

### 3.3 The isotropisation of energy and density

One of the criticisms of models of cluster heating by jets is the non-spherical nature of the energy input. The problem is twofold:

- Firstly, if the energy is predominantly injected into the axial direction, the equatorial cluster gas does not benefit from the energy injection and can cool unimpeded, while the axial gas is super-heated, making AGN heating very inefficient. Secondly, the channel of jet exhaust and super-heated gas along the jet axis would allow subsequent episodes of jet activity to punch easily through the inner cluster and deposit their energy well outside the central region, thus allowing the inner regions to cool unimpeded.

- Secondly, the dynamic nature of the cluster atmosphere significantly alters the late stage evolution, after the jet switches off (between the fourth and the fifth frame at $3.3 \times 10^8$ yrs) and the source evolution becomes sub-sonic. During the sub-sonic stage, the impact of the pressure and density of the surrounding gas become more and more important. In particular, the rotation that is present in the cluster (see Fig. 2) has clearly sheared the plasma away from the jet axis. We will quantify these results below. It is already clear, however, that this effect can solve the dilemma of launching multiple jet episodes into the same cluster: Any evacuated channel created by previous outbursts will have been buffeted around sufficiently to move enough cluster gas back into the path of the new jet to provide a sufficient cross section for interaction and to couple the jet to the inner cluster gas.

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20° angle from the jet axis, relative to that of the control simulation. Clearly, the amount of mass is significantly reduced during the early stages of the simulation. However, at late times, $M_{20}$ increases back up to 80% of the unperturbed value, consistent with the general reduction of density in the central cluster shown in the bottom panel of Fig. 3. Such a small reduction is dynamically insignificant, i.e., the amount of target mass is more than sufficient for a subsequent jet episode to couple to the cluster at any radius. This proves quantitatively that cluster dynamics can solve the problem of evacuated channels raised by Vernaleo & Reynolds (2005).

If cluster mass can be moved around to re-fill the jet channel and to buffet the fossil radio plasma around, it is plausible to assume that the energy input of the jet might be isotropised more rapidly as well. The entropy panel of Fig. 5 shows this qualitatively: there is no clear high entropy channel along the axis of the jet. In fact, the draft of the rising bubbles draws low entropy material out of the central cluster, as previously discussed by Churazov et al. (2003). This is borne out by a quantitative analysis: Fig. 6 shows the excess energy in the thermal cluster gas (once again excluding non-thermal jet exhaust) as a function of a polar angle $\theta$ away from the jet axis (measured around the cluster centre). At early times, the excess energy is clearly peaked around the jet axis (within about 30°), but at late times (160 Myrs, solid black line), the distribution is consistent with being isotropic (dotted black line). This indicates that individual episodes of jet activity can, in fact, distribute their energy rather isotropically into the cluster. More detailed simulations including radiative cooling and multiple jet episodes will be necessary to develop a detailed picture of how heating and cooling in realistic cluster atmospheres proceed.

4 SUMMARY

We have presented simulations of jet-cluster feedback that integrate a correct representation of cluster dynamics, including dark matter, and the collimated input of energy and momentum from an AGN. The simulations accurately reproduce the observed morphological appearance of powerful radio sources like Cyg A, if a jitter is imposed on the jet to account for unresolved dynamical instabilities of the jet commonly referred to as the “dentist drill effect”.

We find that the dynamic structure of the cluster in the form of rotation and shear in the velocity field and the inhomogeneity and anisotropy of the cluster have significant impact on the evolution of the jets and the radio lobes they inflate. At late times, we find that the excess energy deposited into the cluster by the action of the jets is distributed well away from the axis, consistent with roughly spherically symmetric energy input. After long times (>160 Myrs), the inner cluster has been sufficiently rearranged to essentially erase the low density channel blasted out by the jet and move enough material into the way of subsequent jet outbursts to couple efficiently with the inner cluster. This solves the problem of low efficiency feedback found in simulations of spherically symmetric, static atmospheres [Vernaleo & Reynolds, 2005].

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