FATIGUE LIFETIME EVALUATION OF NOTCHED COMPONENTS: IMPLEMENTATION OF THE CONTROL VOLUME CONCEPT IN A STRAIN-BASED LCF CRITERION

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Abstract

The goal of the present paper is to discuss the reliability of a strain-based multiaxial Low-Cycle Fatigue (LCF) criterion, recently proposed by some of the present authors, in estimating the fatigue lifetime of metallic structural components weakened by sharp notches. Such a criterion, based on the critical plane approach, is formulated according to the control volume concept related to the Strain Energy Density (SED) criterion: a material point located at a certain distance from the notch tip is assumed to be the verification point where to perform the fatigue assessment. The above distance is assumed to be a function of both the biaxiality ratio (applied shear stress amplitude over normal stress amplitude)
and the control volume radii under pure Mode I and pure Mode III loading conditions. Once the position of the verification point and the orientation of the critical plane are determined, the fatigue lifetime is theoretically evaluated through an equivalent normal strain amplitude acting on the critical plane, together with the tensile Manson-Coffin curve. Some uniaxial and multiaxial LCF data, recently published in the literature for V-notched round bars made of Ti-6Al-4V titanium alloy, are analysed through the present criterion.

**KEYWORDS:** critical plane approach; control volume concept; multiaxial low-cycle fatigue; notched components; strain-based criterion
NOMENCLATURE

$e_1$ notch geometry parameter related to Mode I

$e_3$ notch geometry parameter related to Mode III

$I$ error index

$N_a$ reference number of loading cycles to failure

$N_{f,\text{cal}}$ theoretical fatigue life

$N_{f,\text{exp}}$ experimental fatigue life

$p$ verification point

$P_{uvw}$ reference system attached to the critical plane

$P_{rTz}$ fixed reference system

$P_{123}$ principal strain axes reference system at time instant for which the maximum principal strain, $\varepsilon_1$, attains its peak value over the loading cycle

$r$ distance of the verification point $P$ from the notch tip

$R_m$ mean control volume radius

$R_1$ control volume radius related to Mode I

$R_3$ control volume radius related to Mode III

$t$ time

$T_{\text{RMS}}$ mean square error

$w$ unit vector normal to the critical plane

$\alpha$ phase angle between transversal normal strain $\varepsilon_t$ and axial normal strain $\varepsilon_z$

$\alpha_{ur}$, $\alpha_{ut}$, $\alpha_{uz}$ direction cosines of $u$-axis

$\alpha_{vr}$, $\alpha_{vt}$, $\alpha_{vz}$ direction cosines of $v$-axis

$\alpha_{wr}$, $\alpha_{wt}$, $\alpha_{wz}$ direction cosines of $w$-axis

$\beta$ phase angle between shear strain $\gamma_\beta$ and axial normal strain $\varepsilon_z$

$\gamma_a$ Manson-Coffin shear strain amplitude

$\gamma_{\beta}$ shear strain

$\delta$ angle between the averaged principal strain direction $\hat{1}$ and the normal $w$ to the critical plane
$\Delta K_{1A}$ Notch Stress Intensity Factor range under Mode I
$\Delta K_{3A}$ Notch Stress Intensity Factor range under Mode III
$\Delta \sigma_{1A}$ High-Cycle Fatigue strength of smooth specimens under Mode I
$\Delta \tau_{3A}$ High-Cycle Fatigue strength of smooth specimens under Mode III
$\varepsilon$ strain tensor at verification point $P$
$\varepsilon_{eq,a}$ equivalent normal strain amplitude
$\varepsilon_a$ Manson-Coffin normal strain amplitude
$\varepsilon_r, \varepsilon_t$ transversal normal strains
$\varepsilon_z$ axial normal strain
$\eta_w$ displacement vector at verification point $P$, related to the critical plane
$\eta_N$ normal displacement vector component of $\eta_w$, related to the critical plane
$\eta_C$ tangential displacement vector component of $\eta_w$, related to the critical plane
$\kappa$ phase angle between transversal normal strain $\varepsilon_r$ and axial normal strain $\varepsilon_z$
$\lambda$ biaxiality ratio, defined as the ratio between the applied shear stress amplitude and the normal stress amplitude
$\lambda_1$ eigenvalues for Mode I
$\lambda_3$ eigenvalues for Mode III
$\sigma_a$ applied normal stress amplitude
$\tau_a$ applied shear stress amplitude
$\nu$ elastic Poisson ratio
$\nu_{eff}$ effective Poisson ratio
$\Phi$ phase angle between tension and torsion loading
$\omega$ pulsation

Subscripts
$\text{a}$ amplitude
$\text{m}$ mean value
1. INTRODUCTION

The state of the art clearly shows that the fatigue problem of mechanical components characterised by geometrical irregularities (such as notches, fillets and key-sets) has widely been examined in order to propose reliable methodologies for estimating the fatigue strength/lifetime under both uniaxial and multiaxial loadings [1-8].

Dealing with the fatigue failures of metallic structural components weakened by notches, the average Strain Energy Density (SED) criterion [9-12], originally proposed by Lazzarin and Zambardi [13], can be considered one of the most powerful engineering tool suitable for accurately performing the fatigue assessment of the above components.

Taking as starting point both the Neuber concept of an 'elementary material volume' [14,15] and the Sih criterion [16,17], Lazzarin and Zambardi adopted the average value of SED, evaluated over a control volume surrounding the notch tip, as a damage parameter for notched structural components. In particular, the radius of the control volume, over which the energy is averaged, was assumed to be a function of the notch geometry, the material fatigue limit evaluated on unnotched specimens, the threshold stress intensity factor range and the Poisson ratio. Moreover, according to such a criterion, fracture of brittle materials is expected to occur when the average value of SED is equal to a critical value, which is a material property [18].

The original version of the SED criterion [13] has been proposed for fatigue strength assessment of components weakened by sharp V-
notches subjected to tensile loading (Mode I), and blunt U and V-notch under Mode I loading have been examined in 2005 [19].

Subsequently, the SED criterion has been extended to notched components under mixed Mode loading [20-23]. Moreover, the aforementioned criterion has successfully been used also to perform the fatigue strength evaluation of both welded joints [24] and notched specimens under high-temperature conditions [25].

Then, an extension of the SED criterion to mechanical and structural components experiencing non-localised creep deformations has been proposed in 2016 [26]. Further, the criterion has recently been applied together with the Equivalent Material Concept (EMC) in order to estimate the failure loads of both U and V-notched aluminum plates characterised by large plastic deformations near the notch tip [27,28].

Note that the overall effectiveness of the above criterion when used to evaluate fatigue failures in notched metallic components has been analysed in Ref.[12]. In that paper, about one thousand experimental data taken from the literature for different materials and notches geometries have been examined, and a satisfactory agreement between experimental and theoretical results has been noticed.

In light of the efficiency and wide applicability of the SED criterion, some of the present authors have recently implemented the concept of the control volume in a strain-based criterion in order to estimate the fatigue life of severely notched specimens under Low-Cycle Fatigue (LCF) [29]. In particular, such a criterion, based on the critical plane approach, is a reformulation of its
counterpart for smooth metallic structural components [30,31] by considering the strain state at a material point $P$ (named verification point in the following) located at a certain distance from the V-notch tip. Such a distance has been proposed to be directly linked to the control volume radius provided by the SED criterion. Then, the fatigue life evaluation is carried out at the verification point by employing an equivalent strain amplitude, related to the critical plane, together with a unique material reference curve (i.e. the tensile Manson-Coffin curve).

The aim of the present paper is to discuss the accuracy and reliability of the strain-based criterion together with the control volume concept (proposed in [29]) in estimating multiaxial LCF lifetime of structural components weakened by notches. Firstly, the analytical main points of such a criterion are outlined in Section 2. Then the fatigue experimental campaign reported in Refs [32,33] for V-notched round bars made of titanium grade 5 alloy (Ti-6Al-4V) is briefly summarised in Section 3. In Section 4, the criterion reported in Ref [29] is applied to the experimental fatigue data and, finally, some conclusions are provided in Section 5.

2. STRAIN-BASED CRITERION FORMULATION FOR SHARP V-NOTCHES

Now the analytical main points of the criterion proposed in Ref.[29] are briefly outlined.

The fatigue life assessment is carried out at point $P$ (verification point), which is distant $r$ from the V-notch tip (Fig. 1). More precisely, such a distance $r$, measured along the notch
bisector line, is assumed to depend on both the biaxiality ratio $\lambda$ (defined as the ratio between the applied shear stress amplitude, $\tau_a$, and the normal stress amplitude, $\sigma_a$) and the control volume radii under pure Mode I and pure Mode III [29]:

$$r = -(0.221)^{2-1.484} \cdot R_m + 11.3R_m$$

(1)

where $R_m$ is the mean control volume radius computed by averaging $R_I$ related to Mode I and $R_3$ related to Mode III, such control volume radii ($R_I$ and $R_3$) being provided by the SED criterion [20,21,32] through the following expressions:

$$R_I = \left(\frac{2c_1 \cdot \Delta K_{1A}}{\Delta \sigma_{1A}}\right)^{1-\lambda_1}$$

(2a)

$$R_3 = \left(\frac{c_3 \cdot \Delta K_{3A}}{\Delta \tau_{3A}}\right)^{1-\lambda_3}$$

(2b)

$c_1$ and $c_3$ being two parameters depending on the V-notch geometry [32]. Further, $\nu$ is the elastic Poisson ratio, and $\lambda_1$ and $\lambda_3$ are the eigenvalues for Mode I and Mode III, respectively, calculated by means of a linear elastic finite element analysis as is reported in Ref. [32]. Moreover, $\Delta K_{1A}$ and $\Delta K_{3A}$ are the mean values of Mode I and Mode III Notch Stress Intensity Factors (NSIFs) ranges, respectively. Finally, the above control volume radii are also function of the High-Cycle Fatigue (HCF) strengths of smooth specimens ($\Delta \sigma_{1A}$ for Mode I, and $\Delta \tau_{3A}$ for Mode III), such strengths
being all referred to the same reference number \( N_a \) of loading cycles to failure (for example, \( N_a = 2 \times 10^6 \)).

**Figure 1.**

Once the position of the verification point \( P \) is determined according to Eq. (1), the strain tensor at the above point is obtained from a finite element analysis by examining a tridimensional model.

If the structural component is subjected to synchronous out-of-phase sinusoidal loadings, the strain state is characterised by two transversal normal strains (\( \varepsilon_r \) and \( \varepsilon_t \)), one axial normal strain (\( \varepsilon_z \)) and one shear strain (\( \gamma_{zt} \)):

\[
\varepsilon_r = \varepsilon_{r,a} \sin (\omega t - \kappa) + \varepsilon_{r,m} \tag{3a}
\]
\[
\varepsilon_t = \varepsilon_{t,a} \sin (\omega t - \alpha) + \varepsilon_{t,m} \tag{3b}
\]
\[
\varepsilon_z = \varepsilon_{z,a} \sin (\omega t) + \varepsilon_{z,m} \tag{3c}
\]
\[
\gamma_{zt} = \gamma_{zt,a} \sin (\omega t - \beta) + \gamma_{zt,m} \tag{3d}
\]

where \( \omega \) = pulsation, \( t \) = time, \( \kappa, \alpha, \beta \) = load phase angles. The subscripts \( a \) and \( m \) refer to amplitude and mean value, respectively. The strain tensor \( \varepsilon(t) \) (at point \( P \)) with respect to the fixed reference system \( Prtz \) (Fig. 2(a)) is hence given by:
\[
\varepsilon(t) = \begin{bmatrix}
\varepsilon_t(t) & 0 & 0 \\
0 & \varepsilon_t(t) & \frac{1}{2} \gamma_\theta(t) \\
0 & \frac{1}{2} \gamma_\theta(t) & \varepsilon_z(t)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\varepsilon_{t,a} \sin(\omega t - \kappa) + \varepsilon_{r,m} & 0 & 0 \\
0 & \varepsilon_{t,a} \sin(\omega t - \alpha) + \varepsilon_{r,m} & \frac{1}{2} \gamma_{\theta,a} \sin(\omega t - \beta) + \frac{1}{2} \gamma_{\theta,m} \\
0 & \frac{1}{2} \gamma_{\theta,a} \sin(\omega t - \beta) + \frac{1}{2} \gamma_{\theta,m} & \varepsilon_{z,a} \sin(\omega t) + \varepsilon_{z,m}
\end{bmatrix}
\]  \hspace{1cm} (4)

**Figure 2.**

Then, the critical plane orientation, which is linked to the averaged principal strain directions, has to be determined by using appropriate weight functions. In particular, the averaged directions of principal strain axes can be deduced on the basis of their instantaneous directions by means of the averaged principal Euler angles. By adopting the weight function \(W(t)\) reported in Ref. [34], the averaged principal strain axes \(\hat{1}, \hat{2}, \hat{3}\) coincide with the instantaneous ones at the time instant for which the maximum principal strain \(\varepsilon_1\) (being \(\varepsilon_1(t) \geq \varepsilon_2(t) \geq \varepsilon_3(t)\)) attains its peak value over the loading cycle.

The orientation of the critical plane, which is the verification plane for fatigue life evaluation, is defined by taking an off-angle \(\delta\) (in the principal averaged plane \(\hat{1}\hat{3}\)) formed by the normal \(\mathbf{w}\) to the critical plane and the averaged direction \(\hat{1}\). The off-angle \(\delta\) is assumed to be expressed as follows:
\[
\delta = \frac{3}{2} \left[ 1 - \left( \frac{1}{2(1 + \nu_{\text{eff}})} \frac{\gamma_a}{\varepsilon_a} \right)^2 \right] 45^\circ
\]  

(5)

\(\nu_{\text{eff}}\) being the effective Poisson ratio. Moreover, \(\varepsilon_a\) and \(\gamma_a\) are determined by means of the tensile and torsional Manson-Coffin equations, respectively:

\[
\varepsilon_a = \frac{a'}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c
\]

(6a)

\[
\gamma_a = \frac{\tau'}{G} (2N_f)^b + \gamma'_f (2N_f)^c_0
\]

(6b)

where \(N_f\) is the number of loading cycles to failure, and \(\sigma'\), \(\varepsilon'\), 
\(b\), \(c\), \(\tau'\), \(\gamma'\), \(b_0\), \(c_0\) are material constants, which can be deduced by running appropriate experimental tests. Note that Eq. (5) can be employed for metals ranging from ductile to extremely brittle fracture behaviour (details can be found in Refs [29-31]).

After defining the normal \(w\) to the critical plane, a local reference system \(Puvw\) is taken into account, where the unit vector \(u\) is on the intersection line between the critical plane and the plane defined by the normal vector \(w\) and the \(z\)-axis; further, \(v\) is normal to \(u\) so that \(Puvw\) forms a right-handed reference system (Fig. 2(b)). The directions cosines of the normal \(w\) can be computed with respect to the \(Prtz\) as a function of two angles, \(\varphi\) and \(\theta\), in a spherical coordinate system \((0 \leq \varphi < 2\pi\) and \(0 \leq \theta \leq \pi\), Fig. 2(c)) [35]:

\[
\alpha_{wr} = \sin \theta \cos \varphi, \quad \alpha_{wt} = \sin \theta \sin \varphi, \quad \alpha_{wc} = \cos \theta
\]

(7)

Furthermore, the direction cosines of the \(u\)- and \(v\)-axis are given by the following expressions [35]:
\[ \alpha_{ur} = \cos \vartheta \cos \varphi, \quad \alpha_{ut} = \cos \vartheta \sin \varphi, \quad \alpha_{uz} = -\sin \vartheta \]
\[ \alpha_{vr} = -\sin \varphi, \quad \alpha_{vt} = \cos \varphi, \quad \alpha_{vz} = 0 \]  
\hspace{1cm} (8a)

Then, by taking into account the strain tensor \( \varepsilon(t) \) at point \( P \) (Eq.(4)) with respect to the reference system \( Puvw \), the displacement vector \( \eta_w \) related to the critical plane (Fig. 2(b)) can be expressed as follows:

\[ \eta_w = \varepsilon \cdot w \Rightarrow [\eta_w] = \begin{bmatrix} \varepsilon_r & 0 & 0 \\ 0 & \varepsilon_t & 1/2 \gamma_{t\vartheta} \\ 0 & 1/2 \gamma_{t\vartheta} & \varepsilon_z \end{bmatrix} \begin{bmatrix} \alpha_{wr} \\ \alpha_{wt} \\ \alpha_{wz} \end{bmatrix} \]  
\hspace{1cm} (9)

and the scalar value \( \eta_N(t) \) of the normal displacement vector \( \eta_N \) (Fig. 2(b)) is given by:

\[ \eta_N(t) = (w \cdot \eta_w) \Rightarrow \eta_N(t) = \alpha_{wr} \eta_{w,r} + \alpha_{wt} \eta_{w,t} + \alpha_{wz} \eta_{w,z} = \sin^2 \vartheta [\varepsilon_r \cos^2 \varphi + \varepsilon_t \sin^2 \varphi] + \varepsilon_z \cos^2 \vartheta + \gamma_{t\vartheta} \sin \varphi \sin \vartheta \cos \vartheta \]  
\hspace{1cm} (10)

The mean value \( \eta_{N,m} \) and the amplitude \( \eta_{N,a} \) of \( \eta_N(t) \) can be determined by substituting the strain components (Eqs (3)) into Eq. (10):

\[ \eta_{N,m} = \sin^2 \vartheta [\varepsilon_{r,m} \cos^2 \varphi + \varepsilon_{t,m} \sin^2 \varphi] + \varepsilon_{z,m} \cos^2 \vartheta + \gamma_{t\vartheta,m} \sin \varphi \sin \vartheta \cos \vartheta \]  
\hspace{1cm} (11a)

\[ \eta_{N,a} = \sqrt{a^2 + b^2} \]  
\hspace{1cm} (11b)

with:

\[ a = \sin^2 \vartheta [\varepsilon_{r,a} \cos \kappa \cos^2 \varphi + \varepsilon_{t,a} \cos \alpha \sin^2 \varphi] + \varepsilon_{z,a} \cos^2 \vartheta + \gamma_{t\vartheta,a} \cos \beta \sin \varphi \sin \vartheta \cos \vartheta \]  
\hspace{1cm} (12a)
Then, the normal displacement vector \( \eta_N \), perpendicular to the critical plane (Fig. 2(b)), is obtained from Eq. (10):

\[
\eta_N = (w \cdot \eta_w)w \Rightarrow [\eta_N] = \begin{bmatrix} \eta_{N,r} \\ \eta_{N,t} \\ \eta_{N,z} \end{bmatrix}
\]

where

\[
\eta_{N,r} = \sin^3 \theta \cos \phi \left[ \varepsilon_r \cos^2 \phi + \varepsilon_r \sin^2 \phi \right] + \sin \theta \cos \theta \cos \phi \left[ \varepsilon_z \cos \phi \cos \beta \theta \sin \phi \right]
\]

(14a)

\[
\eta_{N,t} = \sin^3 \theta \sin \phi \left[ \varepsilon_r \cos^2 \phi + \varepsilon_r \sin^2 \phi \right] + \sin \theta \cos \theta \sin \phi \left[ \varepsilon_z \cos \phi \cos \beta \theta \sin \phi \right]
\]

(14b)

\[
\eta_{N,z} = \sin^2 \theta \cos \phi \left[ \varepsilon_r \cos^2 \phi + \varepsilon_r \sin^2 \phi \right] + \cos^2 \theta \left[ \varepsilon_z \cos \phi \cos \beta \theta \sin \phi \right]
\]

(14c)

By recalling Eqs (9) and (13), the tangential displacement vector \( \eta_C \) lying on the critical plane (Fig. 2(b)) can be computed according to the following expression:

\[
\eta_C = \eta_w \cdot \eta_N \Rightarrow [\eta_C] = \begin{bmatrix} \eta_{C,r} \\ \eta_{C,t} \\ \eta_{C,z} \end{bmatrix}
\]

where

\[
\eta_{C,r} = \sin \theta \left[ \cos \phi \left( \varepsilon_r \cos^2 \phi + \varepsilon_r \sin^2 \phi - \varepsilon_z \cos^2 \phi \right) \right] - \frac{\gamma_{\beta \theta}}{2} \sin^2 \theta \cos \phi \cos \phi
\]

(16a)

\[
\eta_{C,t} = \sin \theta \left[ \sin \phi \left( \varepsilon_r \cos^2 \phi + \varepsilon_r \sin^2 \phi - \varepsilon_z \cos^2 \phi \right) \right] + \frac{\gamma_{\beta \theta}}{2} \cos \theta \left( 1 - 2 \sin^2 \theta \sin^2 \phi \right)
\]

(16b)

\[
\eta_{C,z} = -\sin^2 \theta \cos \phi \theta \left[ \varepsilon_r \cos^2 \phi + \varepsilon_r \sin^2 \phi - \varepsilon_z \right] + \frac{\gamma_{\beta \theta}}{2} \sin \theta \sin \phi \left( 1 - 2 \cos^2 \phi \right)
\]

(16c)

Since the direction of vector \( \eta_C \) is generally time-varying, the mean value \( \eta_{C,m} \) and the amplitude \( \eta_{C,a} \) are not uniquely defined, and
different methods have been proposed in the literature (for instance, the Prismatic Hull method [36,37]). In the present paper, a min-max procedure to define \( \eta_{C,m} \) and \( \eta_{C,a} \) is applied [38] by examining the components of \( \eta_c \) along both \( u- \) and \( v- \)axis:

\[
\eta_{C,a} = u \cdot \eta_C \Rightarrow \eta_{C,a} = \alpha_u \eta_{C,t} + \alpha_u \eta_{C,z} + \alpha_u \eta_{C,z} \\
\eta_{C,v} = v \cdot \eta_C \Rightarrow \eta_{C,v} = \alpha_v \eta_{C,t} + \alpha_v \eta_{C,z} + \alpha_v \eta_{C,z}
\]

(17a)

(17b)

Recalling Eqs (3) and (16), the above expressions become:

\[
\eta_{C,a} = f \sin(\alpha \theta) + g \cos(\alpha \theta) + \eta_{C,a,m} \\
\eta_{C,v} = p \sin(\alpha \theta) + q \cos(\alpha \theta) + \eta_{C,v,m}
\]

(18a)

(18b)

where the mean values, \( \eta_{C,a,m} \) and \( \eta_{C,v,m} \), of the components of \( \eta_c \) along the \( u- \) and \( v- \)axis are given by:

\[
\eta_{C,a,m} = \frac{1}{2} \sin 2\theta \left[ \varepsilon_{r,m} \cos^2 \varphi + \varepsilon_{t,m} \sin^2 \varphi - \varepsilon_{z,m} \right] + \frac{1}{2} \gamma_{\varphi,m} \cos 2\theta \sin \varphi
\]

(19a)

\[
\eta_{C,v,m} = \sin \theta \left[ \frac{1}{2} \left( \varepsilon_{t,m} - \varepsilon_{r,m} \right) \sin 2\varphi \right] + \frac{1}{2} \gamma_{\varphi,m} \cos \theta \cos \varphi
\]

(19b)

whereas the functions \( f, g, p, q \) are expressed by:

\[
f = \frac{1}{2} \sin 2\theta \left[ \varepsilon_{r,a} \cos \kappa \cos^2 \varphi + \varepsilon_{t,a} \cos \alpha \sin^2 \varphi - \varepsilon_{z,a} \right] + \frac{1}{2} \gamma_{\varphi,a} \cos \beta \cos 2\theta \sin \varphi
\]

(20a)

\[
g = -\frac{1}{2} \sin 2\theta \left[ \varepsilon_{r,a} \sin \kappa \cos^2 \varphi + \varepsilon_{t,a} \sin \alpha \sin^2 \varphi \right] - \frac{1}{2} \gamma_{\varphi,a} \sin \beta \cos 2\theta \sin \varphi
\]

(20b)

\[
p = \sin \theta \left[ \frac{1}{2} \left( \varepsilon_{t,a} \cos \alpha - \varepsilon_{r,a} \cos \kappa \right) \sin 2\varphi \right] + \frac{1}{2} \gamma_{\varphi,a} \cos \beta \cos \theta \cos \varphi
\]

(20c)

\[
q = -\sin \theta \left[ \frac{1}{2} \left( \varepsilon_{t,a} \sin \alpha - \varepsilon_{r,a} \sin \kappa \right) \sin 2\varphi \right] - \frac{1}{2} \gamma_{\varphi,a} \sin \beta \cos \theta \cos \varphi
\]

(20d)

Note that Eqs (18) are the parametric equations of the ellipse described by the tip of the tangential displacement vector \( \eta_c \) on the critical plane during a loading cycle (Fig. 3). This ellipse is
centred at point \( (\eta_{c_{a,m}}; \eta_{c_{v,m}}) \) and its semi-axes can be computed as follows:

\[
\eta_{c,a} = \sqrt{\frac{f^2 + g^2 + p^2 + q^2}{2} + \left(\frac{f^2 + g^2 + p^2 + q^2}{2}\right)^2 (fq - gp)^2} \quad (21a)
\]

\[
\eta_{c,b} = \sqrt{\frac{f^2 + g^2 + p^2 + q^2}{2} - \left(\frac{f^2 + g^2 + p^2 + q^2}{2}\right)^2 (fq - gp)^2} \quad (21b)
\]

The mean value \( \eta_{c,m} \) of the tangential displacement component \( \eta_c \) on the critical plane is obtained from the following expression:

\[
\eta_{c,m} = \sqrt{\eta_{c_{a,m}}^2 + \eta_{c_{v,m}}^2} \quad (22)
\]

whereas the amplitude of vector \( \eta_c \) coincides with the major semi-axis \( \eta_{c,a} \) of the above ellipse.

**Figure 3.**

Then, the fatigue strength is assessed by means of an equivalent strain amplitude, \( \varepsilon_{eq,a} \), together with a unique material reference curve (i.e. the tensile Manson-Coffin curve, Eq. (6a)). More precisely, the above equivalent strain amplitude is expressed by quadratically combining the amplitudes of both the normal \( (\eta_{N,a}, \text{ Eq. (11b)}) \) and the tangential \( (\eta_{C,a}, \text{ Eq. (21a)}) \) displacement vectors acting on the critical plane:

\[
\varepsilon_{eq,a} = \sqrt{\left(\frac{\eta_{N,a}}{\gamma_a}\right)^2 + \left(\frac{\eta_{C,a}}{\gamma_a}\right)^2} \quad (23)
\]
By equating Eq. (23) with Eq. (6a), the number $N_f$ of loading cycles to failure can be worked out through an iterative procedure.

3. FATIGUE EXPERIMENTAL CAMPAIGN

The strain-based multiaxial LCF criterion formulated in conjunction with the control volume concept is applied to a set of data recently published in the literature [32,33]. Uniaxial and multiaxial fatigue tests on circumferentially V-notched round bars made of Ti-6Al-4V titanium alloy are briefly described in the present Section.

Each specimen presents a V-notch with depth equal to 6 mm, opening angle equal to 90° and notch root radius equal to about 0.1 mm. All specimens have been polished in order to remove surface scratches before performing the tests.

The experimental uniaxial and multiaxial fatigue tests have been carried out by means of a MTS 809 servo-hydraulic biaxial machine. All tests have been conducted under load control at a frequency value from 5 to 10 Hz, depending on the applied load. Details of the loading conditions being examined are reported in Tables 1-3, where $N_{f,\text{exp}}$ is the experimental fatigue life.

Table 1.

Table 2.

Table 3.
According to the notch geometry and the material properties (that is, the values of the elastic Poisson ratio, the eigenvalues of Mode I and Mode III, the NSIFs ranges, and the HCF strengths) reported in the original papers [32,33], the control volume radius $R_1$ is equal to $0.051 \text{ mm}$ (Eq. (2a)), whereas the control volume radius $R_3$ is equal to $0.837 \text{ mm}$ (Eq. (2b)). The above difference in the values of control volume radii is not only due to the different behaviour in the crack propagation process under either tension or torsion loading, but also to the higher plasticity around the notch tip experienced under torsion with respect to that under tension.

4. CRITERION VALIDATION AND DISCUSSION

The present strain-based multiaxial LCF criterion is here applied to the experimental data described in the previous Section.

All experimental data being examined are characterised by a fatigue life between $10^3$ and $6 \cdot 10^5$ loading cycles and a nominal load ratio equal to $-1$. Six different fatigue test series on V-notched specimens are here analysed (Tables 1-3):

(1) two series of tests under pure tension (specimens No. 1-6 in Table 1) and pure torsion (specimens No. 7-11 in Table 1) fatigue loading;

(2) two series of tests under combined in- ($\Phi=0^\circ$) and out-of-phase ($\Phi=90^\circ$) tension and torsion loading, with constant biaxiality ratio $\lambda$ equal to 0.6 (Table 2);
(3) two series of tests under combined in- ($\Phi=0^\circ$) and out-of-phase ($\Phi=90^\circ$) tension and torsion loading, with constant biaxiality ratio $\lambda$ equal to 2.0 (Table 3).

Different values of the distance $r$ to determine the verification point position are computed according to Eq. (1):

(a) for pure tension loading (that is, $\lambda=0$), $r$ is equal to $1.9 \cdot R_m$;
(b) for pure torsion loading (that is, $\lambda=\infty$), $r$ is equal to $11.3 \cdot R_m$;
(c) for combined in- and out-of-phase tension and torsion loading characterised by $\lambda=0.6$, $r$ is equal to $7.5 \cdot R_m$;
(d) for combined in- and out-of-phase tension and torsion loading characterised by $\lambda=2.0$, $r$ is equal to $10.8 \cdot R_m$.

Note that Eq. (1) has been obtained from a best-fit procedure by considering some values of $\lambda$ related to the experimental data reported in Ref. [32,33]. In particular, the following error index, $I$, has been optimised in order to determine the data-points to be interpolated:

$$I = \sqrt{(\eta_{N,a})^2 + \left(\frac{\epsilon_a}{\gamma_a}ight)^2 - \epsilon_a}$$

where $\epsilon_a$ and $\gamma_a$ are defined by means of Eqs (6a) and (6b), respectively.

As is previously discussed, in order to estimate the fatigue lifetime of the V-notched specimens, the strain tensor at the verification point $P$ has been estimated from a finite element analysis by means of the commercial software Straus7® [39]. More precisely, the strain state at the verification point, for every
investigated fatigue test, has been determined through a tridimensional model by using both 6- and 8-node finite elements. Taking advantage of the geometric symmetry, only one-half of the specimen has been modelled, as is shown in Figure 4. Furthermore, the mesh size has been gradually refined nearing the region containing the notch tip (Fig. 4(b)). The results in terms of strains have been determined by running a series of linear transient dynamic analyses in order to simulate the experimental tests performed by Berto et al. [32,33].

Figure 4.

The Von Mises stress distribution for combined in-phase tension and torsion loading characterised by $\lambda=2.0$ and $\Phi=0^\circ$ is shown in Figure 5, at the time instant corresponding to the maximum value of the both normal and shear stresses applied.

Figure 5.

The values of the material parameters of the Manson-Coffin curves, required for applying the strain-based criterion, are reported in Ref. [29]. The effective Poisson ratio, $\nu_{\text{eff}}$, is assumed to be equal to the elastic Poisson ratio (that is, $\nu=0.3$).

Once the strain state at point $P$ is numerically determined and the above material parameters are deduced, the fatigue lifetime can be estimated by using the present strain-based multiaxial LCF criterion together with the control volume concept.
The comparison between experimental, $N_{f,exp}$, and theoretical, $N_{f,cal}$, evaluation in terms of fatigue life for both uniaxial and multiaxial loadings is shown in Figs 6(a) and 6(b), respectively. Note that the solid line indicates $N_{f,cal} = N_{f,exp}$, whereas the dash-dot lines correspond to $N_{f,cal}/N_{f,exp}$ equal to 0.3 and 3.0.

This comparison clearly proves that:

- for uniaxial fatigue loading (that is, pure tension and pure torsion loading, Fig. 6(a)), 71% of fatigue life calculations are included into 3x band;

- for multiaxial fatigue loading (that is, combined tension and torsion loading with $\lambda = 0.6$ and 2.0, Fig. 6(b)), 76% of fatigue life calculations are included into 3x band. Note that better results are obtained for multiaxial fatigue data characterised by biaxiality ratio equal to 0.6. That is due to the fact that the relationship between $r$ and $\lambda$ (Eq. (1)) has been deduced by taking into account only the experimental data with $\lambda = 0.6$ and not those with $\lambda$ equal to 2.0.

**Figure 6.**

The accuracy of the present fatigue lifetime estimation can also be evaluated by means of the root mean square error method [40]. In more detail, the value of the root mean square logarithmic error is computed as follows:
\[ E_{RMS} = \sqrt{\frac{\sum_{i=1}^{j} \log^2 \left( \frac{N_{f,exp}}{N_{f,cal}} \right)_i}{j}} \]  

(25)

where \( j \) is the total number of data, and the mean square error \( T_{RMS} \) is given by:

\[ T_{RMS} = 10^{E_{RMS}} \]  

(26)

If all the computed results fell, for instance, within the 3\( \times \) band, the value of \( T_{RMS} \) would be equal to 3.

Figure 7 shows the mean square error computed for all values of fatigue life, in accordance with the different fatigue test series previously presented.

\[ \text{Figure 7.} \]

The analysis of the results in terms of the mean square error indicates that the agreement between experimental and theoretical fatigue lives is satisfactory, the value of the \( T_{RMS} \) being lower than 3, and that holds true for all the loading conditions being examined.

On the basis of such encouraging results, we can remark that the joined application of the strain-based criterion together with the concept of the control volume radii (provided by the SED criterion) seems to be a promising engineering tool, able to perform the fatigue lifetime evaluation with an adequate accuracy.
5. CONCLUSIONS

In the present paper, the multiaxial fatigue life assessment of components weakened by sharp notches has been performed by employing a strain-based multiaxial LCF criterion in conjunction with the concept of control volume radius, related to the SED criterion originally proposed by Lazzarin and co-workers. In particular, such a radius has been assumed to be a function of both the notch geometry and the material properties.

According to the present criterion, the fatigue assessment has been carried out at a verification point, which is located at a certain distance from the notch tip, depending such a distance on both the biaxiality ratio and the control volume radii under loading conditions of pure Mode I and pure Mode III.

Once the position of the verification point and the orientation of the critical plane have been determined, the fatigue lifetime has theoretically been evaluated through an equivalent normal strain amplitude, acting on the critical plane, together with the tensile Manson-Coffin curve.

Some uniaxial and multiaxial fatigue data, recently published in the literature, have been analysed to evaluate the effectiveness of the present criterion. The agreement between experimental data and theoretical fatigue lives is satisfactory.

Therefore, the present criterion seems to be a promising tool to assess the fatigue lifetime of notched structural components, although both more complex loading configurations (characterised by nominal load ratio different from -1) and different notch geometries
(i.e. blunt notches) need to be processed in order to devise a robust procedure suitable for practical applications.

Acknowledgements
The authors gratefully acknowledge the financial support provided by the Italian Ministry for University and Technological and Scientific Research (MIUR), Research Grant PRIN 2015 No. 2015JW9NJT on “Advanced mechanical modeling of new materials and structures for the solution of 2020 Horizon challenges”.

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FATIGUE LIFETIME EVALUATION OF NOTCHED COMPONENTS: IMPLEMENTATION OF THE CONTROL VOLUME CONCEPT IN A STRAIN-BASED LCF CRITERION

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Table 1. Uniaxial loading conditions of the experimental tests being examined.

| Material                  | No. | $\Phi$ | $\lambda$ | $\sigma_a$ [MPa] | $\tau_a$ [MPa] | $N_{f,exp}$ [cycles] |
|---------------------------|-----|-------|-----------|------------------|----------------|----------------------|
| Ti-6Al-4V Titanium Alloy  | 1   | 0     | 0         | 160.0            | -              | 99067                |
|                           | 2   | 0     | 0         | 160.0            | -              | 107540               |
|                           | 3   | 0     | 0         | 200.0            | -              | 24772                |
|                           | 4   | 0     | 0         | 200.0            | -              | 33670                |
|                           | 5   | 0     | 0         | 230.0            | -              | 11164                |
|                           | 6   | 0     | 0         | 230.0            | -              | 13456                |
|                           | 7   | 0     | $\infty$ | -                | 380.0          | 46496                |
|                           | 8   | 0     | $\infty$ | -                | 380.0          | 65569                |
|                           | 9   | 0     | $\infty$ | -                | 400.0          | 19520                |
|                           | 10  | 0     | $\infty$ | -                | 420.0          | 7518                 |
|                           | 11  | 0     | $\infty$ | -                | 460.0          | 1053                 |
Table 2. Multiaxial loading conditions (with biaxiality ratio $\lambda$ equal to 0.6) of the experimental tests being examined.

| Material            | No. | $\Phi$ | $\lambda$ | $\sigma_a$ | $\tau_a$ | $N_{f,exp}$ |
|---------------------|-----|--------|------------|------------|-----------|-------------|
|                     |     | [°]    | [MPa]      | [MPa]      | [cycles]  |             |
| Ti-6Al-4V Titanium Alloy |     |        |            |            |           |             |
| 1                   | 0   | 0.6    | 130.0      | 78.0       | 137540    |             |
| 2                   | 0   | 0.6    | 140.0      | 84.0       | 93785     |             |
| 3                   | 0   | 0.6    | 140.0      | 84.0       | 141768    |             |
| 4                   | 0   | 0.6    | 160.0      | 96.0       | 67500     |             |
| 5                   | 0   | 0.6    | 160.0      | 96.0       | 85000     |             |
| 6                   | 0   | 0.6    | 170.0      | 102.0      | 36177     |             |
| 7                   | 0   | 0.6    | 170.0      | 102.0      | 43400     |             |
| 8                   | 0   | 0.6    | 190.0      | 114.0      | 12668     |             |
| 9                   | 0   | 0.6    | 190.0      | 114.0      | 15346     |             |
| 10                  | 90  | 0.6    | 130.0      | 78.0       | 182284    |             |
| 11                  | 90  | 0.6    | 130.0      | 78.0       | 237488    |             |
| 12                  | 90  | 0.6    | 140.0      | 84.0       | 74178     |             |
| 13                  | 90  | 0.6    | 140.0      | 84.0       | 91230     |             |
| 14                  | 90  | 0.6    | 170.0      | 102.0      | 19789     |             |
| 15                  | 90  | 0.6    | 170.0      | 102.0      | 26540     |             |
| 16                  | 90  | 0.6    | 190.0      | 114.0      | 10698     |             |
Table 3. Multiaxial loading conditions (with biaxiality ratio $\lambda$ equal to 2.0) of the experimental tests being examined.

| Material                  | No. | $\Phi$ | $\lambda$ | $\sigma_a$ [MPa] | $\tau_a$ [MPa] | $N_{f,exp}$ [cycles] |
|---------------------------|-----|--------|-----------|------------------|----------------|----------------------|
| Ti-6Al-4V Titanium Alloy  | 1   | 0      | 2.0       | 90               | 180            | 384500               |
|                           | 2   | 0      | 2.0       | 90               | 180            | 390000               |
|                           | 3   | 0      | 2.0       | 90               | 180            | 455000               |
|                           | 4   | 0      | 2.0       | 90               | 180            | 567300               |
|                           | 5   | 0      | 2.0       | 90               | 180            | 622000               |
|                           | 6   | 0      | 2.0       | 90               | 180            | 750000               |
|                           | 7   | 0      | 2.0       | 90               | 180            | 834000               |
|                           | 8   | 0      | 2.0       | 100              | 200            | 198000               |
|                           | 9   | 0      | 2.0       | 100              | 200            | 206000               |
|                           | 10  | 0      | 2.0       | 100              | 200            | 221000               |
|                           | 11  | 0      | 2.0       | 110              | 220            | 103000               |
|                           | 12  | 0      | 2.0       | 110              | 220            | 108475               |
|                           | 13  | 0      | 2.0       | 110              | 220            | 112000               |
|                           | 14  | 0      | 2.0       | 110              | 220            | 121000               |
|                           | 15  | 0      | 2.0       | 130              | 260            | 27500                |
|                           | 16  | 0      | 2.0       | 130              | 260            | 34500                |
|                           | 17  | 0      | 2.0       | 130              | 260            | 38000                |
|                           | 18  | 0      | 2.0       | 130              | 260            | 54021                |
|                           | 19  | 0      | 2.0       | 130              | 260            | 16500                |
|                           | 20  | 0      | 2.0       | 140              | 280            | 19000                |
|                           | 21  | 0      | 2.0       | 150              | 300            | 10200                |
|                           | 22  | 0      | 2.0       | 150              | 300            | 12000                |
|                           | 23  | 0      | 2.0       | 150              | 300            | 13400                |
|                           | 24  | 90     | 2.0       | 100              | 200            | 506280               |
|                           | 25  | 90     | 2.0       | 100              | 200            | 621000               |
|                           | 26  | 90     | 2.0       | 110              | 220            | 182000               |
|                           | 27  | 90     | 2.0       | 110              | 220            | 225000               |
|                           | 28  | 90     | 2.0       | 110              | 220            | 301000               |
|                           | 29  | 90     | 2.0       | 120              | 240            | 131200               |
|                           | 30  | 90     | 2.0       | 120              | 240            | 154000               |
|                           | 31  | 90     | 2.0       | 130              | 260            | 87500                |
|                           | 32  | 90     | 2.0       | 130              | 260            | 96000                |
|                           | 33  | 90     | 2.0       | 130              | 260            | 118500               |
|                           | 34  | 90     | 2.0       | 150              | 300            | 18900                |
|                           | 35  | 90     | 2.0       | 150              | 300            | 22000                |
|                           | 36  | 90     | 2.0       | 150              | 300            | 26500                |
|                           | 37  | 90     | 2.0       | 150              | 300            | 31500                |
|                           | 38  | 90     | 2.0       | 170              | 340            | 8900                 |
|                           | 39  | 90     | 2.0       | 170              | 340            | 11000                |
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