Calculating the parameters of the induced hydraulic fracture that moves in hard rock roof of a seam in the vicinity of the installation chamber

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Abstract. The results of solving the task for tracing the movement of a hydraulic fracture in the top rocks situated in the vicinity of a stope are introduced in the paper. The massif is loaded with equally component gravity field of stresses and is in the state of plane deformation. The bedrocks and top rocks (roof rocks) are rather tough and deform resiliently and when reaching the limit state fall as brittle materials. The task is solved in the framework of the model of the geo-mechanical state of the rock massif with the adjacent in-seam working. It is based on fundamental principles of the deformable solids mechanics and fulfilled applying the boundary element method and basic classical concepts about the fracture state, its stable or unstable growth in a brittle material. According to the results of the experiment, the trajectories of the fracture growth for a range of coordinate values, its initial position and the angle of gradient initial towards the horizon are built. The graphs of critical pressure changes while the growing of a fracture are introduced.

1. Introduction
While developing a coal seam with strong adjoining rocks the area of the basic roof rocks hanging over the goaf reaches significant values. It endangers mining operations brining about different gas-dynamic and geo-dynamic phenomena [1-4]. To decrease the sizes of the roof overhanging a range of methods are used and the most effective method among them is the method of directional hydraulic fracturing [5].

The main problem for the effective application of the hydraulic method in coal industry is in defining optimal parameters of the initiating (primordial) fracture for setting favourable condition of roof caving caused by a growing fracture. By present moment the influence of the working (groove) on the path of the fracture development is not taken into account [5].

2. Setting the task and its solution
The paper presents the results of solving the geo-mechanic task on the development of the fracture loaded with intrinsic pressure (hydrofracture) in the roof of the stope. While solving the task the field of stresses in a coal massif is defined in the framework of previously worked out model of geomechanical condition of the anisotropic massif inclosing mine workings. This model is successfully applied to calculate the massif [6-11].

The task is set as follows (figure 1). In the rock massif modelled by the plane there is a working. It is introduced by a rectangular-sectioned installation chamber – 1 with the dimensions $b \times (h + h_d)$, and
by a stope (2) developed along the thickness of a seam $m$ in the distance of $a_v$ from the installation chamber.

The installation chamber and the working are situated in the depth of $H$ from the surface of the earth. The strength characteristics of the coal seam is less than the strength characteristics of rocks of the adjacent massif but larger than the one of the seam contacts with the rest massif loaded with gravitational pressure above and below $\gamma H$ ($\gamma$ – bulk density of the overlying rocks) and in sides – $\lambda \gamma H$ ($\lambda$ – lateral thrust coefficient). In the selvage of the seam zones of inelastic deformation with the width $L_1$ (3) and $L_2$ (4) are formed. In the roof of the working there is a narrow cut with the length $2l$, which simulates the fracture and is loaded with the pressure $p$ on the area with the length $2l_0$. The length cut is considerably small comparing with the size of the working. The system of coordinates $y_0$, $z$, and the angle of the slope towards the horizon is $\alpha$. Along the $AB$, $CD$ lines curves of the bearing pressure are built.

Figure 1. Computational scheme of the task on the state of the fracture in the roof of the stope.

The value of the critical (fracture driving) pressure $p_{kp}$, that corresponds to the sustainable growth of the induced hydraulic fracture is defined according to the formula [12]

$$p_{kp} = \frac{\pi}{2 \arctg \left[ \frac{l - l_0}{l} \right]} \left[ p + \left( \frac{E \cdot P}{\pi (1 - \mu^2)} - p_{nl}^2 \right)^{\frac{1}{2}} \right],$$

(1)
where \( p_I, p_II \), are normal and tangential loads on the sides of the fracture, respectively, \( E \) is an elastic modulus, \( \mu \) is a Poisson ratio of the massif rocks, \( P \) is a surface energy density, \( l = l_0 + \Delta l \) (\( \Delta l \) – fracture length movement due to its “jump-like” growth).

As for the direction of the fracture development, considering that \( P \) characteristic is constant for an elastic material, its deviation from the initial direction is expected when the values of tangential loads are not equal to zero (\( p_II \neq 0 \)). In this regard, the direction of the fracture development takes place at the angle \( \theta \) to its initial direction and coincides with the direction of the basic area of the tensile stress. It can be defined through the ratio [14]

\[
\left( p_{kp} - p_I \right) \left( \sin \frac{\theta}{2} + \sin \frac{3\theta}{2} \right) + p_II \left( \cos \frac{\theta}{2} + 3 \cos \frac{3\theta}{2} \right) = 0, \tag{2}
\]

and the direction of the fracture coincides with the direction of the basic area of the maximal tensile stress. The components of equation (2) \( p_I \) and \( p_II \) are expressed through the components of the stress field in the vicinity of the working which are known as the functions from the theory of stress state [13, 14].

\[
p_I = \frac{\sigma_z + \sigma_y}{2} + \left( \frac{\sigma_z - \sigma_y}{2} \right) \cos 2\theta + \tau_{yz} \sin 2\theta, \quad p_II = \left( \frac{\sigma_z - \sigma_y}{2} \right) \sin 2\theta + \tau_{yz} \cos 2\theta. \tag{3}
\]

While growing, the fracture length changes in a jump-like manner and the value of this jump depends on the characteristics of the pumping equipment that supports the liquid pressure and its capacity rate in a hydraulic system. In this task the jump of the fracture length \( \Delta l \) is taken as a constant value equal to 5 mm. Then pressure \( p_{kp} \) during the growing of the fracture can be easily defined with equation (1), and its direction, on each following stage of the growth, is defined with equation (2), but herein, every time, it is important to consider the components of the stress field of the inclosing massif together with wedging and rectangular loads at the edges of the growing fracture using equation (3).

3. Computational experiment and the analysis of the obtained results

The results of the computational experiment held in the vicinity of the installation chamber and the stope are introduced below. The following parameters of the massif are taken as the initial data: \( \gamma = 25 \) kH/m\(^3\), \( H = 800 \) m, \( \lambda = 1, b = 5 \) m, \( h = m = 3 \) m, \( h_k = 1,6 \) m, \( a_v = 15 \) m, tensile strength of a seam in uniaxial compression \( \sigma_0 = 10 \) MPa, internal friction angle of a seam \( \phi = 20^\circ \), cohesion coefficient and internal friction angle along the contact of the seam with the massif \( K' = 0, \rho' = 10^\circ, P = 0,0087 \) MPa m, \( l_0 = 0,14 \) m, final half-length \( l_k = 5 \) m. Other data were being changed during the experiment.

Figure 5 presents the results of the elasto-plastic solution. Figure 2a presents the stress curves \( \sigma_z \) and \( \tau_{yz} \) built along \( AB \) line of the seam roof (figure 1). Lines 1, 2 are curves \( \sigma_z \), in limit stressed and elastic zones of a seam, respectively, and lines 3, 4 corresponds to curves \( \tau_{yz} \) in the same zones. Maximal value \( \sigma_z \) (maximum of a bearing pressure) is 2,241 \( \gamma H \), and the volume of a limit zone \( L_2 \) equals 5,156 m.

Figure 2b presents the stress curves built along CD lines of a seam roof (figure 1). Graphs 1-4 stand for stress curves similar to curves depicted in figure 2a, the parameters of a bearing pressure are the following: maximum 2,157\( \gamma H \), and the size \( L_1 \) makes 5,057 m.

Figures 3, 5, 7, 9 show the models of the installation chamber – 1, a stope – 2, limit stress zones at the sides of the chamber – 3 and in a working face – 4, initiating fracture – 5 and the fracture path – 6.
till its final growth. The initiating fracture angles are taken such as to support the rectilinear (straight) position of a fracture path at the distance $z_f=7$ m from the axis of a working.

![Stress curves $\sigma_z, \tau_{yz}$ along the seam roof.](image1)

**Figure 2.** Stress curves $\sigma_z, \tau_{yz}$ along the seam roof.

Figures 4, 8, 10 show the graphs of critical (fracture moving) pressures that correspond to rectilinear growth of the fracture under its various positions along the stope trace. It is seen that they have a form of a smoothly changing curves. In figure 6 the graphs of critical pressure correspond to the growth of a horizontal initiating fracture. It is well seen that this graph has an appearance of an abruptly changing curve.

![Fracture path when $y_f=-5$ m, $z_f=7$ m, $\alpha=-45^\circ$.](image2)

**Figure 3.** Fracture path when $y_f=-5$ m, $z_f=7$ m, $\alpha=-45^\circ$. 


Figure 4. The graph of a critical pressure in a hydrofracture when $y_t=-5$ m, $z_t=7$ m, $\alpha=-45^\circ$.

Figure 5. The fracture path when $y_t=-5$ m, $z_t=7$ m, $\alpha=0^\circ$.

Figure 6. The fracture path when $y_t=-5$ m, $z_t=7$ m, $\alpha=-0^\circ$. 
Figure 7. The fracture path when $y_t=5$ m, $z_t=7$ m, $\alpha=-75^\circ$.

Figure 8. The graph of a critical pressure in a hydro-fracture when $y_t=5$ m, $z_t=7$ m, $\alpha=-75^\circ$.

Figure 9. The fracture path when $y_t=15$ m, $z_t=7$ m, $\alpha=65^\circ$. 
The analysis of the presented results of the hydraulic fracture growth and the graphs of the critical pressure show that only inclined positions of the initiating fracture correspond to a rectilinear growth of a flat induced hydraulic fracture. As computational results show, even with horizontal orientation of the initiating fracture, the direction of its growth stays inclined, and the graph of the “fracture moving” pressure represents rather wavy curve with its vertical axes exceeding the graph’s vertical axes that corresponds to the linear growth of the fracture.

4. Conclusions
1. The model of the geo-mechanic state of the coal rock massif inclosing an in-seam working provides the calculation of the fracture path that moves in hard roofs under the influence of the intrinsic pressure under the given characteristics of the media and initial fracture parameters.
2. In the vicinity of the stope along its path, there are such positions of the initiating fracture when it grows strictly rectilinearly.
3. The vertical axes of the critical pressure graphs corresponding to rectilinear growth of the induced hydraulic fracture are minimal among all other graphs, where the fracture-growing path has a wavy character.

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