Abstract—We consider the problem of compression of two memoryless binary sources, the correlation between which is defined by a Hidden Markov Model (HMM). We propose a Decision Feedback (DF) based scheme which when used with low density parity check codes results in compression close to the Slepian Wolf limits.

I. INTRODUCTION

Consider the classical Slepian Wolf set up where two correlated sources $X$ and $Y$ have to be independently compressed and sent to a destination. It was shown in [1] that the achievable rate region is $R_X \geq H(X|Y)$, $R_Y \geq H(Y|X)$ and $R_X + R_Y \geq H(X,Y)$. Recently, several practical coding schemes have been designed for this problem based on the idea of using the syndrome of a linear block code as the compressed output [2]. When $Y = X \oplus e$, where the sequence $e$ is memoryless, low density parity check (LDPC) codes have been used to achieve performance close to the Slepian-Wolf limit [3].

In this paper we consider the case when $Y = X \oplus e$, where $X$ and $Y$ are binary i.i.d. sequences and $e$ is the output of a Hidden Markov Model (HMM). This problem has been studied before by Garcia-Frias et al [4] and Tian et al [5]. In their scheme, $X$ is compressed to $H(X)$ bits and transmitted. The encoder for $Y$ transmits a portion of the source bits without compression to “synchronize” the HMM. The remaining bits are used as bit nodes in an LDPC code and the corresponding syndrome is transmitted. The decoder employs a message passing algorithm with messages being passed between the HMM nodes, the bit nodes and the check nodes. In [5] Tian et al, considered three HMM’s and optimized the LDPC code ensemble using density evolution for these specific models. The resulting thresholds (the performance of an infinite length LDPC code) were 0.08-0.12 bits away from the Slepian Wolf limits.

Here, we use a different approach. The main differences between the proposed work and that in [4], [5] are that - (i) a decision feedback scheme is used instead of iterating between the HMM model nodes and the LDPC decoder. This also reduces the decoding complexity significantly (ii) The LDPC codes used are optimized for a memoryless channel instead of being optimized for the channel with memory and, hence, the optimization is considerably simpler than in [5]. (iii) The proposed scheme is similar to the scheme in [7] to find the capacity of the Gilbert-Elliott channel and is provably optimal asymptotically in the length.

With the proposed scheme, for the models considered in [5] we are able to design codes that have thresholds within 0.03 bits of the Slepian Wolf limits allowing for a distortion of 1e-5, which is considerably better than those in [5].

II. PROPOSED SYSTEM

Consider two binary sources $X$ and $Y$ such that $Y = X \oplus e$ where $Y$ is independent and uniformly distributed. Typical compression schemes to achieve a corner point in the Slepian Wolf region involve sending $X$ using $H(X)$ bits and sending the syndrome of $Y$ corresponding to a linear code $C$ using $H(Y|X)$ bits. It can be shown that the problem of compression is equivalent to the problem of finding a capacity achieving linear code for the channel shown in Fig. 1 [2].

When $e$ is memoryless, there are tools available to design LDPC codes that achieve capacity on this channel and, hence, achieve the Slepian-Wolf limit. In our case, $e$ is the output of a HMM with three parameters $S$, $P$ and $\mu$. $S$ defines the different states, $P$ is an $|S| \times |S|$ matrix with $P_{i,j}$ representing the probability of transition from state $S_i$ to $S_j$ and $\mu$, $|S| \times 1$, has elements $\mu_i$ which give $P(e = 0|S_i)$. The probability of $e$ being 0 or 1 depends only on the current state. We further assume that when no state information is available, the output of the HMM is equally likely to be zero or one.

In [6] Narayanan et al use a Decision Feedback Equalization (DFE) based scheme for ISI channels that makes the channel appear memoryless to the LDPC decoder. We use the same technique to make the channel appear memoryless and then design codes for this “memoryless” channel. The encoding and decoding operations are explained below.

A. Encoder

We will describe a scheme to achieve a corner point of the Slepian Wolf coding region corresponding to $R_X = H(X)$ and $R_Y = H(Y|X)$. The encoding process is shown in Fig. 2. Let us assume that both sequences $X$ and $Y$ are first arranged in the form of $L \times N$ matrices $X$ and $Y$. The $(i,j)^{th}$ element in $X$ is $y_{(i-1)L+j}$. We will use $Y_{i,j}$ to denote the $(i,j)^{th}$ element in $Y$ and $y_{i,j}$ to denote the $j^{th}$ column of $Y$. The sequence $X$ is compressed using an entropy coder to $H(X)$ bits. For the models considered in this paper, the sequence $X$ contains independent and uniformly distributed bits and, hence, no compression is needed for $X$. The first $M$ columns in $Y$ are transmitted without any compression...
As the check values. With a suitably chosen LDPC code the

$$\lambda$$

BCJR algorithm. From the soft estimates of

$$e_i$$

The receiver has

$$X$$

the estimate for a particular bit is made only from the past

timesteps of the HMM and, hence, in Equation (2)

$$\gamma_{i, M+1} = \log \frac{P(e_{i, M+1} = 1 | e_{i, M}, e_{i, M-1}, \ldots, e_{i, 1})}{P(e_{i, M+1} = 0 | e_{i, M}, e_{i, M-1}, \ldots, e_{i, 1})}$$  (2)

Note that the consecutive values of

$$e$$

If we use a code of rate 1 - \(H(e_{i,j} | e_{i,j-1}, \ldots, e_{i,1})\)

for the first

$$M$$

columns being transmitted without compression, but that rate

loss can be made arbitrarily small by choosing a large enough

$$L$$.

Although the arguments presented above show that this

scheme is optimal as

$$M \rightarrow \infty$$

we do not require this. If we use a code of rate 1 - \(H(e_{i,j} | e_{i,j-1}, \ldots, e_{i,1})\)

for the

$$j$$

column, then we can obtain a compression rate of

$$\frac{1}{L} \sum_j H(e_{i,j} | e_{i,j-1}, \ldots, e_{i,1})$$

which converges to \(H(e) = H(Y | X)\) from above as

$$L \rightarrow \infty$$

for any wide sense stationary

process.

$$e$$

This solution however requires variable rate LDPC
codes for the different columns and, hence, is not used in this

paper.

IV. SIMULATION RESULTS

We compare the performance of the proposed scheme with the

scheme used in [5]. The HMM used in [5] has two

states

$$S_0$$

and

$$S_1$$

and is defined by four parameters

$$P(S_0 \rightarrow S_0)$$, \(P(S_1 \rightarrow S_1)\), \(P(0|S_0)\), \(P(1|S_1)\). The models considered are

M1: (0.01, 0.065, 0.95, 0.925),

M2: (0.97, 0.967, 0.93, 0.973),

M3: (0.99, 0.989, 0.945, 0.9895)

Note that the parameters in the model are chosen so that they satisfy

$$P(e = 0) = 0.5$$.

In Figure 4 we plot \(H(e_{M+1} | e_{M}, \ldots, e_1)\) as a function of

$$M$$

for the models. We observe that for these models the

$$M$$

required to come close to the Slepian Wolf limits is quite

small. We use

$$M = 4$$

for our simulations.
For the three models the pdf of \( \gamma_{i,j} \) conditioned on \( e_{i,j} \) being a 1 and 0 were computed through monte-carlo simulations. From this the distribution of \( \lambda_{i,j} \) conditioned on \( y_{i,j} \) can be computed. Using this, an LDPC code ensemble was designed using density evolution and differential evolution. It was assumed that a Hamming distortion of \( 10^{-5} \) is acceptable. Since the HMM is not symmetric, the pdf of \( \gamma_{i,j} \) conditioned on \( e_{i,j} \) being a 1 or 0 is not symmetric. That is, \( f_{\gamma_{i,j}}(x|e_{i,j} = 1) \neq f_{\gamma_{i,j}}(-x|e_{i,j} = 0) \), for some \( x \). Hence the distribution of \( \lambda_{i,j} \) is also not symmetric. For the density evolution we use the average of these pdf’s similar to the approach in [8], where the correctness of this procedure is proved. Simulations were done with the designed LDPC codes of length 100000 and \( L = 100 \). 2000 such blocks were simulated for each model. A different interleaver was used for each column to avoid repetition of error sequences. The results obtained are compared with those of [5] in Table I. The SW limit column shows the Slepian Wolf compression limit. The THEO column represents the threshold, which is the achievable compression rate with infinite length LDPC codes.

For the DFE scheme simulations were also performed with \( N = 2000 \), \( L = 100 \) and \( M = 4 \). Codes designed for AWGN channel were used in these simulations. The bit filling algorithm [9] was used to reduce error floors. The results are also tabulated in Table I. For each model 5000 blocks were simulated. The Hamming distortion observed was less than \( 2e-7 \). Although the performance in this case seems to be far from the Slepian Wolf limits, it should be noted that this scheme is universal and does not require any optimizations specific to the HMM. Although beyond the scope of this paper, we wish to point out that for small \( L \) and finite lengths, simple improvements to the decoding algorithm can provide significant improvements in the compression rates. For example, allowing for decoding of a particular block using the pilots on both sides.

The loss in rate due to the pilots in the DF Scheme is not included in Table I. If the pilots are sent without compression, then the compression rate would increase by 0.04. However, this loss can be reduced significantly by increasing \( L \) and by compressing the pilots.

### Table I

| Model | SW Limit | Tian et al[5] | THEO | THEO | DF Scheme |
|-------|----------|--------------|------|------|-----------|
|       | \( H(Y|X) \) | \( N = 10^2 \) | \( N = 2000 \) |
| 1     | 0.515    | 0.599        | 0.546 | 0.58 | 0.69      |
| 2     | 0.448    | 0.544        | 0.476 | 0.52 | 0.62      |
| 3     | 0.278    | 0.413        | 0.305 | 0.34 | 0.45      |

With \( L = 100 \) error propagation is a serious problem but it can be overcome by lowering the rate of the LDPC code. In our simulations, no error propagation was observed.

### V. Conclusion

We proposed a low complexity decision feedback based scheme to compress multiterminal sources with hidden Markov correlations. The proposed scheme has thresholds just 0.03 bits away from the Slepian Wolf limits and the simulated performance with designed LDPC codes of length 100000 is within 0.08 bits of the limits which is better than the thresholds of the scheme in [5].

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![Fig. 3. Compression Limit vs. M](image-url)