Dark radiation and dark matter in supersymmetric axion models
with high reheating temperature

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Recent studies of the cosmic microwave background, large scale structure, and big bang nucleosynthesis (BBN) show trends towards extra radiation. Within the framework of supersymmetric hadronic axion models, we explore two high-reheating-temperature scenarios that can explain consistently extra radiation and cold dark matter (CDM), with the latter residing either in gravitinos or in axions. In the gravitino CDM case, axions from decays of thermal saxions provide extra radiation already prior to BBN and decays of axinos with a cosmologically required TeV-scale mass can produce extra entropy. In the axion CDM case, cosmological constraints are respected with light eV-scale axinos and weak-scale gravitinos that decay into axions and axinos. These decays lead to late extra radiation which can coexist with the early contributions from saxion decays. Recent results of the Planck satellite probe extra radiation at late times and thereby both scenarios. Further tests are the searches for axions at ADMX and for supersymmetric particles at the LHC.

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I. INTRODUCTION

Recent cosmological studies show trends towards a radiation content of the Universe at the onset of big bang nucleosynthesis (BBN) and much later that exceeds expectations for standard three active neutrino species. The obtained limits on non-standard contributions $\Delta N_{\text{eff}}$ to the effective number of light neutrino species $N_{\text{eff}}$ are still consistent with the standard value $N_{\text{eff}} \approx 3$ at the 1–2$\sigma$ level. However, BBN likelihood analyses based on recent studies of the mass fraction $Y_p$ of primordial helium $^4$He, $^3$He [1, 2] find posterior maxima of $\Delta N_{\text{eff}} \approx 0.7–0.8$ [1, 3] and precision cosmology studies of the cosmic microwave background (CMB) and large scale structure (LSS) means of $\Delta N_{\text{eff}} \approx 0.8–1.8$ [3, 4] prior to the announcement of the new Planck results [5]. While the BBN studies are limited by systematic errors (see e.g., [2]), the Planck satellite mission has recently probed $N_{\text{eff}}$ at the CMB decoupling epoch – as expected [3, 4] – with an unprecedented sensitivity of $\Delta N_{\text{eff}} \approx 0.26$ at the 1$\sigma$ level. In fact, the Planck results point to favored values of $\Delta N_{\text{eff}} \approx 0.25–0.6$ and upper limits of $\Delta N_{\text{eff}} \lesssim 1$ at the 2$\sigma$ level [5]. In particular, with the above $\Delta N_{\text{eff}}$ values, a tension between Planck data and direct measurements of the Hubble constant $H_0$ [11] is relieved that is present in the base ΛCDM model that does not allow for the possibility of $\Delta N_{\text{eff}} > 0$. Indeed, new astrophysical data sets on $H_0$ seem crucial to clarify whether there is extra radiation pointing to new physics or a Hubble constant that is considerably below the current values from direct measurements.

Various explanations for $\Delta N_{\text{eff}} \sim 1$ have been explored in the literature invoking, e.g., light sterile neutrinos [2, 12], other light species [13, 14], neutrino asymmetries [15, 16], or decays of heavy particles [4, 17, 50]. Here we study two classes of supersymmetric (SUSY) hadronic axion models which describe consistently extra radiation and cold dark matter (CDM) for a high reheating temperature after inflation of up to $T_R \sim 10^9$ GeV or $10^{11}$ GeV.

In the considered (R-parity-conserving) models, it may thereby be possible to generate the baryon asymmetry, e.g., via thermal leptogenesis with hierarchical heavy Majorana neutrinos [31]. Moreover, SUSY axion models are compelling since both the strong CP problem and the hierarchy problem are solved simultaneously. These models come with new fields including the axion $a$, the saxion $\sigma$, the axino $\tilde{a}$, and the gravitino $\tilde{G}$, which can play important cosmological roles depending on their masses, the Peccei–Quinn (PQ) scale $f_{\text{PQ}}$, and the reheating temperature $T_R$.

As the pseudo-Nambu-Goldstone boson associated with the U(1)$_{\text{PQ}}$ symmetry broken spontaneously at $f_{\text{PQ}}$ [32, 33], the axion has interactions suppressed by $f_{\text{PQ}}$ and a mass of $m_a \sim 6 \text{ meV}(10^9 \text{ GeV}/f_{\text{PQ}})$. With laboratory, astrophysical, and cosmological studies [34, 35] pointing to $f_{\text{PQ}} \gtrsim 6 \times 10^8$ GeV, the axion is predicted to be an extremely weakly interacting particle (EWIP) with a tiny mass of $m_a \lesssim 10$ meV. In SUSY settings, the saxion and the axino appear respectively as the scalar and the fermionic partner of the axion. They are EWIPs as well with masses $m_{\sigma}$ and $m_{\tilde{a}}$ that depend on details of the model and of SUSY breaking. For example, one expects the saxion mass $m_{\sigma}$ to be of the order of the gravitino mass $m_{\tilde{G}}$ in gravity-mediated SUSY breaking. As the gauge field associated with local SUSY transformations, the gravitino is another EWIP with interactions suppressed by the (reduced) Planck scale $M_P = 2.4 \times 10^{18}$ GeV and a mass that depends on the SUSY breaking scale. While we do not assume a specific SUSY breaking model, $m_{\sigma} = m_{\tilde{G}}$ is used in the main part of this work. Other than that, $m_{\tilde{G}}$ (together with $m_{\sigma}$) and $m_{\tilde{a}}$ are treated as free parameters set in a way to evade cosmological constraints. Model building aspects of the considered mass hierarchies will be considered elsewhere.

In the first of the two classes that we consider, the gravitino is the lightest supersymmetric particle (LSP)
that provides CDM. Here decays of thermal saxions into axions can provide $\Delta N_{\text{eff}} \sim 0.5$ prior to BBN \cite{4,17,20,29,30,36,37}; see also \cite{14,29,30,36,37} for extra radiation from late decays of non-thermal saxions. In the second class, a very light axino is the LSP, the gravitino the next-to-LSP (NLSP) and CDM resides in axions from the misalignment mechanism. Again, it is possible to have $\Delta N_{\text{eff}} \sim 0.5$ from decays of thermal saxions into axions already prior to BBN. However, now there can be an additional contribution of $\Delta N_{\text{eff}} \sim 0.5$ but only well after BBN from gravitino decays into the axion and the axino \cite{19,20}. For both classes, we show updated $\Delta N_{\text{eff}}$ contours that point to new limits on $T_R$ accounting for the recent Planck results on $\Delta N_{\text{eff}}$ \cite{8}. Moreover, we devote particular attention to cosmological viability and to the interplay with present and potential future insights from SUSY searches at the LHC.

Some points by which our present study goes beyond directly related existing studies \cite{4,22} are the following. Decays are treated beyond the sudden-decay approximation. In the $\tilde{G}$ LSP case, the resulting $\Delta N_{\text{eff}}$ contours are confronted explicitly with the $T_R$ limit imposed by a gravitino density $\Omega_{\tilde{G}}$ that cannot exceed the dark matter density $\Omega_{\text{CDM}}$. Here cosmological constraints require $m_{\tilde{G}} \gtrsim 2$ TeV such that saxinos decay prior to the decoupling of the lightest ordinary sparticle (LOSP), which denotes the lightest sparticle within the minimal supersymmetric standard model (MSSM). Saxinos can then provide a sizable fraction of the total energy density of the Universe when decaying and thereby produce entropy \cite{38,39,40}. This is included in our calculations, as is the gravitino density $\Omega_{\tilde{G}} \rightarrow a\tilde{G}$ from rare axino decays into axions and gravitinos. Here we apply an updated result for the axino abundance produced thermally in the early Universe, which we obtain by including quartic axino-squark-antiquark-gluino interactions \cite{41} omitted in an earlier calculation \cite{42}. In the $\tilde{G}$ LSP case with the $\tilde{G}$ NLSP, we present $\Delta N_{\text{eff}}$ contours that account for both decays, $\tilde{G} \rightarrow a\tilde{a}$ and $\sigma \rightarrow a\sigma$, explicitly. Moreover, our treatment includes contributions of the gravitino-spin-3/2 components and of electroweak processes to the thermally produced gravitino yield. In both of the considered LSP cases, we account systematically for the possibility that saxion decays into gluon pairs can have a sizable branching ratio and can thereby produce significant amounts of entropy.

The remainder of this paper is organized as follows. In the next section we discuss the observational hints towards extra radiation beyond the SM and possible scenarios in light of the recent Planck results. Section IV is devoted to general aspects of the considered SUSY hadronic axion models in high-$T_R$ scenarios, which apply to the two explored LSP cases. This section contains our updated result for the primordial abundance of thermally produced axinos. The gravitino CDM and the axion CDM scenarios are presented in Sects. IV and V respectively. Here we consider the corresponding contributions to $\Omega_{\text{CDM}}$, $\Delta N_{\text{eff}}$, and entropy, provide resulting $T_R$ limits, and address the testability of these scenarios. We summarize our conclusions in Sect. VI. Appendix A provides details on our updated calculation of the thermally produced axino abundance, where hard thermal loop (HTL) resummation \cite{13,44} is used to treat screening effects of the primordial plasma as in Ref. \cite{42}. In Appendix B approximate expressions for the numerical results obtained in Sects. IV and V are given that allow for a qualitative understanding of those results. While $m_\sigma = m_{\tilde{G}}$ is assumed throughout the main part of this work, we briefly describe the changes that occur for $m_\sigma \neq m_{\tilde{G}}$ in Appendix C.

II. EXTRA RADIATION

One of our key motivations for the studies presented in this work is the trend towards extra radiation inferred from current cosmological investigations as summarized briefly in the Introduction. In this section we expand slightly on the description of the current situation and outline different possible perspectives accounting for the new Planck results on $\Delta N_{\text{eff}}$.

The standard model (SM) predictions of the total relativistic energy density,

$$\rho_{\text{rad}}^\text{tot}(T) = \left[1 + \frac{7}{8} N_{\text{eff}} \left(\frac{T_{\nu}}{T}\right)^4\right] \rho_\gamma(T),$$  \hspace{1cm} \text{(1)}$$

are given by $N_{\text{eff}} = 3$ and $T_{\nu} = T$ at $T \sim 1$ MeV (before neutrino decoupling and $e^+e^-$ annihilation) and by $N_{\text{eff}} = 3.046$ and $T_{\nu} = (4/11)^{1/3}T$ after neutrino decoupling. Here $\rho_\gamma$ is the photon energy density and $T_{\nu}$ the temperature of photons (neutrinos). The effective number of light neutrino species $N_{\text{eff}}$ increases slightly due to residual neutrino heating by $e^+e^-$ annihilation \cite{43}.

There are various ways to probe $N_{\text{eff}}$ and thereby non-standard contributions $\Delta N_{\text{eff}}$ to which we refer as extra radiation. At the epoch of BBN, a speed-up of the Hubble expansion rate caused by $\Delta N_{\text{eff}} > 0$ leads to a more efficient $^4\text{He}$ output than in standard BBN. Observationally inferred limits on the primordial $^4\text{He}$ mass fraction $Y_p$ can thus be translated into $\Delta N_{\text{eff}}$ limits. Much later, at the epoch of CMB decoupling, $\Delta N_{\text{eff}} > 0$ affects the time of radiation-matter equality, leads to a less efficient early integrated Sachs–Wolfe effect, and reduces the scale of the sound horizon. This affects the CMB power spectrum by increasing the height of the first peak and by shifting the peak positions towards higher multipole moments. Moreover, free-streaming of the relativistic populations associated with $\Delta N_{\text{eff}} > 0$ suppresses power on small scales and thereby affects the matter power spectrum inferred from studies of the LSS. Based on those observables, numerous studies of BBN, CMB, and LSS have explored limits and favored values for $\Delta N_{\text{eff}}$ \cite{38,40,43} with the outcome outlined in the Introduction.
TABLE I. Constraints on $\Delta N_{\text{eff}}$ from BBN and precision cosmology. The first two lines give the posterior maximum (p.m.) and the minimal 99.7% credible interval imposed by BBN as obtained in Ref. [4] using the indicated data sets and the prior $\Delta N_{\text{eff}} \geq 0$. The third line lists the mean and the 95% CL upper limit on $\Delta N_{\text{eff}}$ from the precision cosmology study [4] based on CMB data, the Sloan Digital Sky Survey (SDSS) data-release 7 halo power spectrum (HPS), and data from the Hubble Space Telescope (HST). The last two lines provide the mean and the 95% CL upper limit on $\Delta N_{\text{eff}}$ as obtained by the Planck collaboration [8] when combining Planck CMB data with WMAP polarization data (WP), data from high-l experiments (highL), and data on baryon acoustic oscillations (BAO). The values in the last line emerge when results of Ref. [11] on a direct measurement of the Hubble constant $H_0$ are taken into account.

| Data | p.m./mean upper limit |
|------|-----------------------|
| $Y_p^\text{TT}$ [1] + $D/H_p$ [49] | 0.76 < 1.97 (3$\sigma$) |
| $Y_p^\text{AV}$ [2] + $D/H_p$ [49] | 0.77 < 3.53 (3$\sigma$) |
| CMB + HPS + HST [6] | 1.73 < 3.50 (2$\sigma$) |
| Planck+WP+highL+BAO [8] | 0.25 < 0.79 (2$\sigma$) |
| Planck+WP+highL+H0+BAO [8] | 0.47 < 0.95 (2$\sigma$) |

To motivate the $\Delta N_{\text{eff}}$ values considered in our study, we quote representative current constraints on $\Delta N_{\text{eff}}$ imposed by BBN and precision cosmology in Table I. The first two lines have been obtained in a BBN-likelihood analysis [4] based on the recent $Y_p$ studies of Izotov and Thuan [1] and of Aver et al. [2]. Those studies report primordial $^4$He abundances of $Y_p^\text{TT} = 0.2565 \pm 0.001\text{(stat.)} \pm 0.005\text{(syst.)}$ and $Y_p^\text{AV} = 0.2561 \pm 0.0108$, respectively, with errors referring to 68% intervals. Moreover, a primordial D abundance of $\log[D/H_p] = -4.56 \pm 0.04$ [49] and a free-neutron lifetime of $\tau_n = 880.1 \pm 1.1 \text{ s}$ [38] have been used in the determination of the listed posterior maxima (p.m.) and the 3$\sigma$ upper limits. The third line gives the mean and the 95% confidence level (CL) upper limit on $\Delta N_{\text{eff}}$ obtained in the precision cosmology study of Ref. [6] based on CMB data, the Sloan Digital Sky Survey (SDSS) data-release 7 halo power spectrum (HPS), and data from the Hubble Space Telescope (HST). Compatibility with $\Delta N_{\text{eff}} = 0$ is found at the 1–2$\sigma$ level in both the BBN and that precision cosmology study. While a more decisive compatibility test seems to be difficult for BBN investigations due to significant systematic uncertainties (see e.g. [2]), the new results of the Planck satellite mission have improved the $\Delta N_{\text{eff}}$ accuracy of precision cosmology investigations substantially [8]. Even with the improved accuracy, compatibility with $\Delta N_{\text{eff}} = 0$ is found to hold still at the 1–2$\sigma$ level. In the last two lines of Table I, we provide the mean and the 95% CL upper limit on $\Delta N_{\text{eff}}$ ($= N_{\text{eff}} - 3.046$) obtained by the Planck collaboration [8] when combining CMB data from Planck with WMAP polarization data (WP), data from high-l experiments (highL), and data on baryon acoustic oscillations (BAO). The values in the last line emerge with a Gaussian prior on $H_0$ based on the direct measurement of the Hubble constant of Ref. [11].

The Planck results quoted in Table I still allow for (or even favor) a relatively small amount of extra radiation, e.g., from saxion decays and/or gravitino decays. With the current BBN limits, the following scenarios are possible: (i) this small amount was already present at the onset of BBN with no additional contribution after BBN, (ii) this small amount was generated only well after BBN, or (iii) part of this small amount was generated already prior to BBN and the remaining part well after BBN.

We will see below that composition (i) is the only one that can be realized in the considered gravitino LSP case, whereas the alternative axino LSP case allows for all three compositions. Contours of $\Delta N_{\text{eff}} = 0.25$, 0.47, 0.79, and 0.95 will be explored in the respective parameter regions corresponding to the means and the 2$\sigma$ upper limits obtained by the Planck collaboration [8] as quoted in the last two lines of Table I.

III. HIGH-REHEATING-TEMPERATURE SCENARIOS

Throughout this work it is assumed that inflation has governed the earliest moments of the Universe, as suggested by its flatness, isotropy, and homogeneity. Accordingly, any initial EWIP population was diluted away by the exponential expansion during the slow-roll phase of the inflaton field. A radiation-dominated epoch with an initial temperature of $T_R$ emerged from the subsequent reheating phase in which inflaton decays repopulate the Universe. While inflation models may point to $T_R$ well above $10^{10}$ GeV, we limit our studies to the case $T_R < f_{\text{PQ}}$ in which no PQ symmetry restoration takes place after inflation. Focussing on high-reheating temperature scenarios with $T_R > 10^7$ GeV, axions, saxions, axinos, and gravitinos can be produced efficiently in thermal scattering of MSSM fields in the hot plasma. Depending on the PQ scale $f_{\text{PQ}}$ and on $T_R$, even scenarios in which the fields of the axion supermultiplet were in thermal equilibrium are conceivable.

For the axion and the saxion, our estimate for the decoupling temperature reads [4]

$$T_D^{a,\sigma} \approx 1.4 \times 10^9 \text{ GeV} \left( \frac{f_{\text{PQ}}}{10^{11} \text{ GeV}} \right)^2.$$  (2)

Following the approach of Ref. [4] and using our results for thermal axino production presented below and in Ap-
pendix A we estimate the axino decoupling temperature as

\[ T_D^a \approx 5.2 \times 10^8 \text{ GeV} \left( \frac{f_{\text{PQ}}}{10^{11} \text{ GeV}} \right)^2. \]  

(3)

In cosmological scenarios with \( T_R > T_D^a, \), axinos (together with axions/saxions) were in thermal equilibrium before decoupling as a relativistic species provided \( m_{\tilde{a}} \ll T_D^a \) and \( m_{\sigma} \ll T_D^a \). Then the yield of those thermal relic axions/saxions and axinos after decoupling is given respectively by

\[ Y_{a,\sigma}^{\text{eq}} = \frac{n_{a,\sigma}^{\text{eq}}}{s} \approx 1.2 \times 10^{-3}, \]  

(4)

and

\[ Y_{a,\tilde{a}}^{\text{eq}} = \frac{n_{a,\tilde{a}}^{\text{eq}}}{s} \approx 1.8 \times 10^{-3}. \]  

(5)

Here \( n_j^{(\text{eq})} \) denotes the corresponding (equilibrium) number density of species \( j \) and \( s \) the entropy density. For the latter, we use \( s(T) = 2\pi^2 g_s T^3/45 \) with an effective number of relativistic degrees of freedom of \( g_s, s(T_D) \approx 232.5 \) that accounts for the MSSM and the axion multiplet fields, which can all be considered as relativistic at \( T_D \) for \( m_{\tilde{a},\sigma} \ll T_D \).

In scenarios with \( T_R < T_D^{a,\tilde{a}} \), the axion multiplet fields can still be thermally produced (TP) via scattering of colored (s)particles in the primordial plasma. The resulting yields are given by [4]

\[ Y_{a,\tilde{a}}^{\text{TP}} = 1.33 \times 10^{-3} g_s^6 \ln \left( \frac{1.01}{g_s} \right) \left( \frac{10^{11} \text{ GeV}}{f_{\text{PQ}}} \right)^2 \left( \frac{T_R}{10^8 \text{ GeV}} \right)^4, \]  

(6)

and, as derived by updating the result of Ref. [42] in Appendix A by

\[ Y_{a,\tilde{a}}^{\text{TP}} = 1.98 \times 10^{-3} g_s^6 \ln \left( \frac{1.27}{g_s} \right) \left( \frac{10^{11} \text{ GeV}}{f_{\text{PQ}}} \right)^2 \left( \frac{T_R}{10^8 \text{ GeV}} \right)^4. \]  

(7)

Here the strong gauge coupling is understood to be evaluated at \( T_R \), i.e., \( g_s \equiv g_s(T_R) = \sqrt{4\pi\alpha_s(T_R)} \), which we calculate according to its 1-loop renormalization group running within the MSSM from \( \alpha_s(m_{\text{Z}}) = 0.1176 \) at the Z-boson mass \( m_Z = 91.1876 \) GeV.

Note that our focus is on hadronic or KSVZ axion models [42–5] in a SUSY setting, with \( N_Q = 1 \) heavy KSVZ (s)quark multiplets \( Q_L \) and \( Q_R \). After integrating out the KSVZ fields, we obtain the effective Lagrangian [4]

\[ L_{\text{PQ}}^{\text{int}} = \frac{\alpha_s}{8\pi f_{\text{PQ}}} \left[ \sigma \left( G^{\mu\nu} G^b_{\mu\nu} - 2D^b \gamma^\mu D^b \gamma^\nu g_M^b \right) \right. \]

\[ + \alpha \left( G^{\mu\nu} \tilde{G}^b_{\mu\nu} + 2\tilde{g}^b_M \gamma^\mu \gamma^\nu D^b \gamma_M^M \right) \]

\[ - \frac{i}{2} \bar{a}_M \gamma^\mu \gamma^\nu g_M^b G^b_{\mu\nu} + 2\tilde{a}_M D^b \gamma_M^M, \]  

(8)

where \( b \) is a color index, \( D^b \) the corresponding color-gauge covariant derivative, \( G^{\mu\nu}_{\mu\nu} \) the gluon-field-strength tensor, \( \tilde{G}^{\mu\nu}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} G^{b,\rho\sigma}/2 \) its dual, \( \tilde{g}^b \) the gluino field, and \( D^b = -g_s \sum_q \bar{q} \gamma_{i,j} T^b_{ij} \tilde{q} \) with a sum over all squark fields \( \tilde{q} \) and the \( SU(3)_c \) generators \( T^b_{ij} \) in their fundamental representation; the subscript \( M \) indicates 4-component Majorana spinors [5]. In the considered framework, the Lagrangian [5] describes the relevant saxion/axion/axino interactions even in a conceivable very hot early stage of the primordial plasma with temperatures \( T \) not too far below \( f_{\text{PQ}} \). Based on \( \alpha \) the presented results [2, 3, 6, 7] and \( \gamma \) are obtained. In particular, as outlined in more detail in Appendix A our result for the thermally produced axino yield [7] accounts for the second term in the third line of \( \sigma \) that describes the quartic axino-squark-antisquark-gluino interaction [11], whereas the corresponding result of Ref. [42] was based on only the first term in that line.

Gravitinos with mass values of \( m_\tilde{G} \geq 1 \) GeV, which are the ones considered in this work, have never been in thermal equilibrium with the primordial plasma. Nevertheless, they can be produced efficiently in thermal scattering of MSSM fields in the hot plasma. Derived in a gauge-invariant treatment, the resulting thermally produced gravitino yield reads [58–60]

\[ Y_{\tilde{G}}^{\text{TP}} = \sum_{i=1}^{3} y_i g_i^2 \left( 1 + \frac{M^2_{\tilde{G}}}{3m_{\tilde{G}}^2} \right) \ln \left( \frac{k_i}{g_i} \right) \left( \frac{T_R}{10^8 \text{ GeV}} \right)^4, \]  

(9)

with \( y_i \), the gauge couplings \( g_i \), the gaugino mass parameters \( M_i \), and \( k_i \) as given in Table II. Here \( M_i \) and \( g_i \) are understood to be evaluated at \( T_R \).

In the following we consider universal gaugino masses, \( m_{1/2} = M_1(m_{\text{GUT}}) \), at the grand unification scale \( m_{\text{GUT}} \approx 2 \times 10^{16} \) GeV. We do not specify a SUSY model.

| gauge group | \( i \) | \( g_i \) | \( M_i \) | \( k_i \) | \( (g_i/10^{-14}) \) | \( \omega_i \) |
|------------|-----|-----|-----|-----|-----------------|
| U(1)_{\text{Y}} | 1 | \( g' \) | \( M_1 \) | 1.266 | 0.653 | 0.018 |
| SU(2)_L | 2 | \( g \) | \( M_2 \) | 1.312 | 1.604 | 0.044 |
| SU(3)_c | 3 | \( g_s \) | \( M_3 \) | 1.271 | 4.276 | 0.117 |

TABLE II. Assignments of the index \( i \), the gauge coupling \( g_i \), and the gaugino mass parameter \( M_i \), to the gauge groups U(1)_Y, SU(2)_L, and SU(3)_c, and the constants \( k_i \), \( g_i \), and \( \omega_i \).

3 Slightly different expressions for \( L_{\text{PQ}}^{\text{int}} \) can be found in [11, 54]. We use the space-time metric \( g_{\mu\nu} = g^{\mu\nu} = \text{diag}(+1, -1, -1, -1) \) and other conventions and notations of Ref. [53] and, except for a different sign of the Levi-Civita tensor \( \varepsilon^{\mu\nu\rho\sigma} = +1 \), of Ref. [54].

4 We do not consider scenarios with a radiation-dominated epoch with \( T \) above the masses of the heavy KSVZ (s)quarks \( m_{Q,Q} \) such as those considered in Ref. [53].
Nevertheless, we use certain pairs of $m_{1/2}$ and the weak-scale gluino mass $m_{\tilde{g}}$ keeping in mind that these values are related via renormalization group evolution. In particular, we will associate $m_{\tilde{g}} \approx 1, 1.25, 1.5$ TeV with $m_{1/2} = 400, 500, 600$ GeV, respectively. Computing the renormalization group evolution with the spectrum generator SPHENO [61, 62], these relations are obtained within the Constrained MSSM (CMSSM) with a universal scalar mass parameter of $m_0 = 1.7$ TeV, the trilinear coupling $A_0 = 0$, a positive higgsino mass parameter, $\mu > 0$, and a mixing angle in the Higgs sector of $\tan \beta = 10$. The above combinations are still allowed by current SUSY searches at the LHC but are well within reach of the ongoing experiments; see e.g. Ref. [63].

Note that the field-theoretical methods [43, 44] applied in the derivations of (6), (7), and (9) require weak couplings $g_t \ll 1$ and thus $T \gg 10^6$ GeV. Moreover, in those derivations, a hot thermal plasma consisting of the particle content of the MSSM is considered in the high-temperature limit. In fact, it is assumed that radiation governs the energy density of the Universe as long as thermal production of the respective EWIP is efficient, i.e., for $T$ down to at least $T \sim 0.01$ $T_R$. This is assumed in this work also. However, we will encounter situations with significant entropy production at smaller temperatures generated by decays of by then non-relativistic saxions and/or axinos from thermal processes. Then this can dilute the yield of a stable or long-lived EWIP from thermal processes in the earliest epoch correspondingly with dilution factors of $\Delta > 1$:

$$Y_{\text{EWIP}}^{\text{eq/TP}} \rightarrow \frac{1}{\Delta} Y_{\text{EWIP}}^{\text{eq/TP}}.$$  

(10)

Abundances of decoupled species that emerge from decays of thermally produced EWIPs prior to the entropy producing event are equally affected.

In high-reheating temperature scenarios, the LOSP usually freezes-out as a weakly interacting massive particle (WIMP) at a decoupling temperature of $T_D^{\text{LOSP}} \approx m_{\text{LOSP}}/25$ with an abundance $Y_{\text{LOSP}}$ that can be determined by solving the corresponding Boltzmann equations. In the case of entropy production after LOSP decoupling, this abundance will be diluted

$$Y_{\text{LOSP}} \rightarrow \frac{1}{\Delta} Y_{\text{LOSP}}$$  

(11)

as well [39, 60, 69]. However, in situations in which the entropy producing event ends well before LOSP decoupling, $Y_{\text{LOSP}}$ is not affected. Here we assume in both cases that LOSP decoupling takes place in a radiation-dominated epoch. This is justified in the settings considered below where the contribution of long-lived non-relativistic species to the total energy density (that enters the Friedmann equation) is negligible during LOSP freeze-out.

In high-reheating temperature scenarios, thermal leptogenesis with hierarchical heavy Majorana neutrinos can explain the baryon asymmetry of the Universe [31]. Without late-time entropy production, $M_{R1} \sim T_R$ of at least about $10^9$ GeV is then required to generate the observed baryon asymmetry $\eta$, where $M_{R1}$ denotes the mass of the lightest among the heavy right-handed Majorana neutrinos. With late-time entropy production, a baryon asymmetry generated prior to the entropy-producing events must have been larger by the associated dilution factor $\Delta$. In the framework of thermal leptogenesis, this can be realized for up to $\Delta \sim 10^4$ with $M_{R1} \sim T_R \sim 10^{13}$ GeV, as can be seen in Fig. 7(a) of Ref. [70] and in Fig. 2 of Ref. [71]; see also [39, 60]. In fact, with a dilution factor of $\Delta$, the required minimum temperature for successful leptogenesis has to be larger by that factor:

$$T_R \gtrsim 10^9 \text{GeV} \rightarrow \frac{1}{\Delta} T_R \gtrsim 10^9 \text{GeV}. $$  

(12)

Together with (10) and (11), this motivates us to carefully calculate $\Delta$ and to monitor the results for the two scenarios discussed in the following.

### IV. GRAVITINO CDM CASE

In this section we look at the R-parity conserving SUSY scenario in which a gravitino with mass $m_{\tilde{G}} \gtrsim 1$ GeV is the stable LSP whose thermally produced density parameter

$$\Omega_{\tilde{G}} h^2 = m_{\tilde{G}} Y_{\tilde{G}}^{\text{TP}} (T_0) s(T_0) h^2 / \rho_c$$  

(13)

provides a substantial part of the CDM density $\Omega_{\text{CDM}} h^2$, where $T_0 = 0.235$ meV is the present photon temperature, $h$ the Hubble constant in units of 100 km Mpc$^{-1}$ s$^{-1}$, and $\rho_c / s(T_0) h^2 = 3.6$ eV. Motivated by the recent finding of the Planck collaboration [8]

$$\Omega_{\text{CDM}} h^2 = 0.1187 \pm 0.0017 \text{ (1}\sigma)$$  

(14)

obtained from the Planck+WP+highL+BAO data set for the base $\Lambda$CDM model we will consider a nominal $3\sigma$ upper limit of

$$\Omega_{\text{CDM}} h^2 \leq 0.124.$$  

(15)

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5 The methods developed in Refs. [43, 44] are compelling since they allow for a gauge-invariant treatment of plasma screening effects in calculations of thermal EWIP production [4, 72, 55, 56, 60]. For alternative approaches, see [41, 54, 61, 68].

6 Settings beyond the base $\Lambda$CDM model and $\Delta N_{\text{eff}}$ contours obtained from the Planck+WP+highL+$H_0$+BAO data are explored in this work. Nevertheless, for our studies, we consider the upper limit [10] to be sufficiently precise.
In the gravitino LSP case, all heavierparticles including the LOSP and the axino are unstable. In turn, each LOSP and each axino present in the Universe after LOSP decoupling will decay directly or via a cascade into one gravitino. Depending on $Y_{LOSP}$, the contribution to $\Omega_{\tilde{G}}$ from decays of thermal relic LOSPs can be small as will be discussed below in more detail. This is different for long-lived axinos that decay at temperatures below a fiducial $T_{low} < T_{LOSP}$. For settings with $\Gamma^{TP}_{LOSP} \sim \Omega_{CDM}$ and $f_{PQ} < 10^{12}$ GeV, their contribution

$$\Omega^{\tilde{G}}_{\tilde{G}} \sim m_{\tilde{G}} Y_{\tilde{G}}^{eq/TP} (T_{low}) s(T_0) h^2 / \rho_c$$

(16)

exceeds \[15\] by many orders of magnitude. This can be immediately seen when comparing \[5\] and \[7\] with \[9\]. To avoid this excess, we focus in this section on $\tilde{G}$ LSP scenarios in which axinos decay dominantly into gluons and gravitinos well before LOSP decoupling with a rate that can be derived from the effective Lagrangian \[8\].

$$\Gamma_{\tilde{G}} \approx \Gamma_{\tilde{a} \rightarrow gg} \approx \frac{\alpha_s^2 m^3_{\tilde{G}}}{16\pi^3 f_{PQ}} \left( 1 - \frac{m_{\tilde{G}}}{m_{\tilde{a}}} \right)^3.$$  

(17)

While the gluinos will be brought into chemical thermal equilibrium when emitted prior to LOSP decoupling, gravitinos from the rare axino decay $\tilde{a} \rightarrow a\tilde{G}$ will still contribute to the gravitino density

$$\Omega^{\tilde{G}}_{\tilde{a} \rightarrow \tilde{G}} \sim m_{\tilde{G}} \text{BR}(\tilde{a} \rightarrow a\tilde{G}) Y_{\tilde{G}}^{eq/TP} (T_{low}) s(T_0) h^2 / \rho_c$$

(18)

even when axinos decay well before LOSP decoupling, i.e., at temperatures below the fiducial $T_{low}$ but above $T_{LOSP}$. The corresponding partial decay width \[72\] \[73\] governs the branching ratio of that rare decay \[7\]  

$$\text{BR}(\tilde{a} \rightarrow a\tilde{G}) \sim \frac{\Gamma_{\tilde{a} \rightarrow \tilde{G}} \sim m_{\tilde{G}}^5}{6\alpha_s^2 M_P^2 m_{\tilde{G}}^2} \left( 1 - \frac{m_{\tilde{G}}}{m_{\tilde{a}}} \right)^{-3}.$$  

(20)

where $M_P = m_P / \sqrt{8\pi} = 2.44 \times 10^{18}$ GeV is the reduced Planck scale and the limit $m_{\tilde{a}} > m_{\tilde{G}}$ is considered. For example, we find a small branching ratio of $\text{BR}(\tilde{a} \rightarrow a\tilde{G}) \sim 10^{-5}$ for $m_{\tilde{G}} > 1$ GeV, $f_{PQ} \sim 10^{11}$ GeV, and $m_{\tilde{a}} \gg 6$ TeV well above $m_{\tilde{G}} \sim 1$ TeV. For large $Y_{\tilde{G}}^{eq/TP} \sim 10^{-3}$, $\omega_{\tilde{G} \rightarrow \tilde{G}h^2}$ can still contribute significantly to the CDM density. Accordingly, we will consider contours of $\omega_{\tilde{G}h^2} + \omega_{\tilde{G} \rightarrow \tilde{G}h^2} = 0.124$ in this section.

In the $\tilde{G}$ LSP scenarios considered in this section, axions from decays of thermal saxions prior to BBN are the only significant contribution to $\Delta\Omega_{ax}$, as already mentioned in Sects. \[1\] and \[11\]. The Lagrangian that allows for $\tilde{a} \rightarrow gG$ saxion decays into gluinos or axinos is well above the threshold to form hadrons. Accordingly, we will consider settings with $\Omega_{\tilde{a}} \rightarrow aa$ decay reads \[17\]

$$\mathcal{L}_{\tilde{a} \rightarrow aa} = \frac{x^2 m_{\tilde{a}}}{16\pi^3 f_{PQ}}.$$  

(22)

where $x = \sum_i q_i^2 v_i^2 / f_{PQ}$ depends on the axion model with $q_i$ denoting the charges and $v_i$ the vacuum expectation values of the fundamental PQ fields \[17\]. For example, $x = 1$ in a KSVZ axion model with just one PQ scalar $(q = 1)$ and $v = v_{PQ}$ and $x \ll 1$ in such a model with two PQ scalars with $q_1 = q_2 = 1$ and similar vacuum expectation values, $v_1 \approx v_2 \approx v_{PQ} / \sqrt{2}$. The two scales $v_{PQ} = \sqrt{\sum_i v_i^2 q_i^2}$ and $f_{PQ}$ are related via $f_{PQ} = \sqrt{2} v_{PQ} \[13\]$. For $m_{\sigma} > 1$ GeV, the saxion decay into two gluons, $\sigma \rightarrow gg$, can become a competing decay mode towards small values of $x$. The associated rate reads

$$\Gamma_{\sigma \rightarrow gg} = \frac{\alpha_s^2 m_{\tilde{G}}}{16\pi^3 f_{PQ}},$$  

(23)

and is derived from \[8\]. The saxion decay into photons is subdominant whenever the $\sigma \rightarrow gg$ decay is kinematically viable, i.e., for $m_{\sigma}$ above the threshold to form hadrons. Saxion decays into gluinos or axinos are kinematically not possible in the $\tilde{G}$ LSP case with $m_{\sigma} = m_{\tilde{G}}$. Moreover, the lifetime of the saxion and the branching ratio of its decays into axions and into gluons are well described by

$$\tau_{\sigma} = \frac{1}{\Gamma_{\sigma \rightarrow aa} + \Gamma_{\sigma \rightarrow gg}} = \frac{32\pi f_{PQ}^2}{m_{\tilde{G}}^2 [x^2 + 2(\alpha_s / \pi)^2]},$$  

(24)

$$\text{BR}(\sigma \rightarrow aa) \sim \frac{x^2}{x^2 + 2(\alpha_s / \pi)^2},$$  

(25)

$$\text{BR}(\sigma \rightarrow gg) \sim 1 - \text{BR}(\sigma \rightarrow aa) = \left[ 1 + \frac{x^2}{2(\alpha_s / \pi)^2} \right]^{-1},$$  

(26)

respectively, with $\alpha_s \equiv \alpha_s(m_{\sigma})$. For example, for $x \gtrsim 0.2$ and $m_{\sigma} \gtrsim 10$ GeV, one finds BR $(\sigma \rightarrow aa) \gtrsim 0.9$ so that $\tau_{\sigma}$ is governed by the decay into axions. Towards smaller $x$ and/or $m_{\sigma}$, the saxion decay into gluon pairs becomes important with effects discussed below.

When decaying, both the axino and the saxion are non-relativistic. Accordingly, we encounter two types of decays of non-relativistic particles: (i) decays into axions
and gravitinos which are by then decoupled from the thermal plasma and thereby inert relativistic species and (ii) decays into relativistic species that are rapidly thermalized and thereby associated with entropy production. We can indeed face simultaneously situations studied previously for the generic cases of out-of-equilibrium decays of non-relativistic particles into inert radiation and into thermalizing radiation that produce entropy.

Let us now calculate the contribution to $\Delta N_{\text{eff}}$ of the energy density of relativistic axions $\rho_a$ from thermal processes. We see that a discussion of the entropy density of relativistic axions $\rho_a$ is obtained beyond the sudden decay approximation. Nevertheless, we will return to that topic at the end of the axino-decay epoch. It is necessary to include the possibility of entropy production and from decays of thermal axinos, $\Omega_{\text{G}}$ and the relic density of gravitinos from thermal production. By taking into account the possibility of entropy production and from decays of thermal axinos, we generalize and refine our related previous study. Moreover, our numerical results are now obtained beyond the sudden decay approximation. Nevertheless, we will return to that approximation to derive expressions that allow for a qualitative understanding of the behavior of our numerical solutions in Appendix B.

In the epoch when thermal processes involving EWIPs are no longer efficient and when axinos and saxions from these processes are non-relativistic, the time evolution of the energy densities of axinos, saxions, and relativistic axions is described by the following Boltzmann equations

$\dot{\rho}_a + 3H \rho_a = -\Gamma_a \rho_a$,

$\dot{\rho}_\sigma + 3H \rho_\sigma = -\Gamma_\sigma \rho_\sigma$,

$\dot{\rho}_a + 4H \rho_a \approx \text{BR}(\sigma \to aa) \Gamma_\sigma \rho_\sigma$

$+ \text{BR}(\tilde{a} \to a\tilde{G}) \Gamma_\tilde{a} \rho_a/2$, (30)

and the Hubble expansion rate $H \equiv \dot{R}/R$, the dot indicating derivative with respect to cosmic time $t$, and the second term on the right-hand side of (30) providing a valid approximation for $m_\tilde{a} \gg m_\tilde{G}$. The time evolution of the entropy density $S$ is given by

$S^{1/3} \dot{S} \simeq R^4 \left( \frac{2\pi^2}{45} g_{*S} \right)^{1/3} \left\{ [1 - \text{BR}(\tilde{a} \to a\tilde{G})] \Gamma_\tilde{a} \rho_\tilde{a} 

+ [1 - \text{BR}(\sigma \to aa)] \Gamma_\sigma \rho_\sigma \right\}$,

and the one of the cosmic scale factor $R$ by the Friedmann equation for a flat Universe

$H^2 \simeq \frac{8\pi}{3m_p^2} (\rho_\tilde{a} + \rho_\sigma + \rho_a + \rho_{\text{rad}})$ (32)

with the Planck mass $m_p = 1.22 \times 10^{19}$ GeV and the energy density of the thermal MSSM radiation background

$\rho_{\text{rad}} \equiv \frac{\pi^2}{30} g_* T^4 = \frac{3}{4} g_* \left( \frac{45}{2\pi^2 g_{*S}} \right)^{1/3} S^{4/3} R^{-1}$. (33)

where $g_*$ is the effective number of relativistic degrees of freedom within the MSSM only, i.e., without the axion multiplet and the gravitino. In this section, $g_* = g_{*S}$ holds for the interval over which we integrate the Boltzmann equations. (This will be different in Sect. IV)

Equations (28)–(32) form a closed set of differential equations that we solve numerically. We begin our computation at $t_1 = 1.6 \times 10^{-3}$ s corresponding to $T_1 = 1$ TeV with $\rho(0) = gV$ and end at $t_f = 0.7$ s corresponding to $T_f \simeq 1$ MeV. For the initial values of the energy densities, we use

$\rho_\tilde{a}(t_1) = m_\tilde{a} Y_{\tilde{a}}^{\text{eq}/TP} s(T_1)$,

$\rho_\sigma(t_1) = m_\sigma Y_{\sigma}^{\text{eq}/TP} s(T_1)$,

$\rho_a(t_1) = \langle p_{a,i} \rangle Y_a^{\text{eq}/TP} s(T_1)$,

where the average thermal axion momentum is $\langle p_{a,i} \rangle = 2.701 T_{a,i}$ and $T_{a,i} = \left[ g_{*S}(T_1)/228.75 \right]^{1/3} T_1$. Note that saxions can be treated as a non-relativistic species throughout the time interval $[t_1, t_f]$ although a saxion, e.g., with $m_\sigma = 100$ GeV will be relativistic at an initial temperature of $T_1 = 1$ TeV. At times at which saxions are relativistic, their contribution $\rho_\sigma$ on the right-hand side of the Friedmann equation (32) is negligible. Whenever their contribution becomes sizable, they are non-relativistic, which justifies the simplified treatment.

With the initial entropy $S(t_1) = s(T_1) R(t_1)^3$ and after numerical integration, we obtain the dilution factor

$\Delta = \frac{S(t_f)}{S(t_1)}$.

As described already in the previous section, this factor quantifies the dilution due to entropy release which affects the yield of species not in thermal equilibrium such as $Y_{\text{G}}^{TP}$ and thereby $\Omega_{\text{G}}$. The relic gravitino density from axino decays (18) is affected by this dilution as well. In fact, since $\rho_{\tilde{G}}^{TP}$ and $\rho_{\tilde{G}}^{\tilde{a} \to a\tilde{G}}$ can be safely neglected in (32) at the considered times and since the gravitino is stable in the case considered here, it is not necessary to include the Boltzmann equation for the gravitino in the described calculation. While gravitinos from $\tilde{a} \to a\tilde{G}$ decays may still be relativistic at the onset of BBN for $m_{\tilde{G}} \ll m_{\tilde{a}}$, their contribution to $\Delta N_{\text{eff}}$ is negligible in the considered parameter regions. This holds equally for the contribution of the relativistic axions emitted in those decays. In fact, the terms $\propto \text{BR}(\tilde{a} \to a\tilde{G})$ in (30) and (31) can be set to zero as they do not affect the presented results.

Results of our numerical integration are illustrated in Fig. 1 for $m_\tilde{a} = 6$ TeV, $m_{\tilde{G}} = 1$ TeV, and $f_{\text{PO}} = 10^{11}$ GeV. For this setting, $T_{\text{after}} \simeq 10$ GeV is the temperature at the end of the axino-decay epoch, at which $\Gamma_\tilde{a} \simeq 3H$ is satisfied. Therefore, a realistic LOSP with $m_{\text{LOSP}} \lesssim 250$ GeV is compatible with the requirement $T_{\text{after}} > T_{\text{LOSP}} \simeq m_{\text{LOSP}}/25$ that is crucial as discussed at the beginning of this section. As mentioned in Sect. III, the considered gluino mass is still compatible with limits from SUSY searches at the LHC.
Figure 1 (a) shows the time evolution of the energy per comoving volume, $R^3 \rho$, of axinos (dash-dotted), saxions (dashed), axions (dotted) and other radiation (solid) and of entropy $S$ (dash-double-dotted). Here $m_\sigma = 100$ GeV, $m_\tilde{a} = 6$ TeV, $m_{\tilde{g}} = 1$ TeV, $T_R = 10^6$ GeV, and $f_{PQ} = 10^{15}$ GeV, which also becomes manifest in the approximations (B6) and (B7) obtained in Appendix B. This panel of Fig. 1 shows the dilution factor $\Delta$ as a function of the reheating temperature $T_R$ for $x = 1$, 0.2, 0.1, and 0.02 shown by the solid, dashed, dotted, and dash-dotted lines, respectively. Black (gray) lines are obtained with $m_\sigma = 20$ (100) GeV, whereas all other parameter are as in panel (a).

In general, towards small $x$, both the saxion lifetime $\tau_\sigma$ and $\text{BR}(\sigma \rightarrow gg)$ increase which leads to larger values of $\Delta$. This effect becomes even more pronounced towards smaller $m_\sigma$ as long as the decay $\sigma \rightarrow gg$ is not kinematically suppressed. Figure 1(b) illustrates this behavior, which also becomes manifest in the approximations (B6) and (B7) obtained in Appendix B. The $T_R$ dependence of $\Delta$ results from the one of $\Omega_\sigma^{eq}/TP$ and of $Y_\sigma^{eq}/TP$; cf. (B5)–(B7) in Appendix B. The kink in the $\Delta$ contour that results from those decay reactions when calculating the thermally produced yields for $T_R$ near the respective decoupling temperatures, these kinks will disappear. Expecting smoother curves that are close the shown ones, we leave such a treatment for future work.

Let us now explore systematically the amount of extra radiation released by saxion decays and regions in which the constraint $\Omega_\tilde{G}^\text{TP} + \Omega_\tilde{a}^\text{TP} \leq \Omega_\text{CDM}$ is respected. Results for $x = 1$ are presented in Fig. 2 and for $x = 0.1$
and 0.2 in Fig. 3. In both figures, we consider $m_\sigma = m_\tilde{g}$ and $m_{1/2} = M_f (m_{SUSY})$. As already discussed in Sect. 11, there are hints towards the existence of extra radiation. These hints could be an indication for the existence of axions from saxion decay. We investigate this possibility for $f_{\text{PQ}} = 10^{10}, 5 	imes 10^{10}$, and $10^{11}$ GeV. For each of these values, $m_\tilde{a}$ and $m_{\tilde{g}}$ are chosen such that the axino decay can take place before the freeze-out of a not too massive LOSP. We report the considered combinations in Table III together with $T_{\text{after}}$ at which $\Gamma_\tilde{a} = 3H$ and the mass of a LOSP $m_{\text{max, LOSP}}$ for which its decoupling temperature satisfies $T_{\text{LOSP}}^\text{max} \simeq m_{\text{LOSP}}/25 = T_{\text{after}}$. This shows explicitly that the viability of these gravitino LOSP scenarios requires the axino to be quite heavy and the LOSP to be relatively light.

Table III. The temperature $T_{\text{after}}$ at which $\Gamma_\tilde{a} = 3H$ for different combinations of the PQ scale $f_{\text{PQ}}$, the axino mass $m_\tilde{a}$, and the gluino mass $m_{\tilde{g}}$ together with the LOSP mass $m_{\text{LOSP}}^\text{max}$ for which $T_{D}^\text{LOSP} \simeq m_{\text{LOSP}}/25 = T_{\text{after}}$.

| $f_{\text{PQ}}$ | $m_{\tilde{a}}$ | $m_{\tilde{g}}$ | $T_{\text{after}}$ | $m_{\text{max, LOSP}}$ |
|----------------|----------------|----------------|-------------------|------------------------|
| [GeV]          | [TeV]          | [TeV]          | [GeV]             | [GeV]                  |
| 10^{10}        | 2              | 1 (1.25)       | 13 (9)            | 325 (225)              |
| 5 \times 10^{10}| 3              | 1 (1.25)       | 6 (5)             | 150 (135)              |
| 10^{11}        | 6              | 1 (1.25)       | 10 (9)            | 250 (235)              |

Moreover, in Figs. 2(b) and (c), one can see slight kinks in the $\Delta N_{\text{eff}}$ contours at $T_R$ values below $T_G^R$. Those kinks appear at the same $T_R$ values as the kinks in Fig. 2(d) and indicate the point above which $T_R > T_D$. The dilution factors obtained for $m_{\tilde{g}} = 1$ and 1.25 TeV are also included in the calculation of the $\Omega_G^{\text{TP}} + \Omega_G^{\alpha a\tilde{G}}$ contours for $m_{1/2} = 400$ and 500 GeV, respectively. Indeed, the slight kinks in the $\Omega_G^{\text{TP}} + \Omega_G^{\alpha a\tilde{G}}$ contours visible in the panels (b) and (c) result from the $\Delta$ behavior shown in panel (d). Despite the larger $\Delta$ for larger $m_{\tilde{g}}$, the $T_R$ limit imposed by $\Omega_G^{\text{TP}} + \Omega_G^{\alpha a\tilde{G}} \leq \Omega_{\text{CDM}}$ is still more restrictive for larger $m_{1/2}$ due to the $M_i$ dependence of $\Omega_i$. While $\Omega_{\text{GUT}}^\text{TP}$ governs this limit towards $f_{\text{PQ}} \sim 10^{10}$ GeV and $m_\tilde{a} \sim 2$ TeV for the considered range $m_G \tilde{G} > 0.5$ GeV, $\Omega_G^{\alpha a\tilde{G}}$ becomes more relevant, e.g., for $f_{\text{PQ}} \sim 10^{11}$ GeV and $m_\tilde{a} = 6$ TeV towards small $m_G \tilde{G}$ below 100 GeV; cf. Fig. 2(c).

As one can see from Figs. 2(a)–(c), axions from saxion decay can contribute to the amount of extra radiation. However, for the considered $x = 1$ case, values of $\Delta N_{\text{eff}} \sim 0.8$ are almost completely disfavored by the $\Omega_G^{\text{TP}} + \Omega_G^{\alpha a\tilde{G}} \leq \Omega_{\text{CDM}}$ constraint if $m_\tilde{g} = 1$ TeV and $m_{1/2} = 400$ GeV. In fact, if SUSY searches at the LHC point to minimum $m_\tilde{g}$ and $m_{1/2}$ values of respectively $1.25$ TeV and $500$ GeV, the $\Omega_G^{\text{TP}} + \Omega_G^{\alpha a\tilde{G}} \leq \Omega_{\text{CDM}}$ constraint will clearly disfavor $\Delta N_{\text{eff}} \sim 0.8$ and the BBN-inferred posterior maxima $\Delta N_{\text{eff}} = 0.76$ and 0.77 given in Table IV. Still axions from decays of thermal saxions can then provide a viable explanation of, e.g., $\Delta N_{\text{eff}} \lesssim 0.5$. This includes the means obtained by the Planck collaboration 8 as quoted in Table I.

To explore the simultaneous viability of successful leptogenesis and an explanation of, e.g., $\Delta N_{\text{eff}} \sim 0.25 - 0.47$ by axions from decays of thermal saxions, one has to consider the minimum $T_R$ value together with the dilution factors shown in Fig. 2(d) as described in 12. Indeed, if the minimum $T_R$ is 10$^9$ GeV without the entropy producing axino decays, it will become almost twice as large in the scenarios with $f_{\text{PQ}} \gtrsim 5 \times 10^{10}$ GeV. Accordingly, as can be seen in Figs. 2(b) and (c), experimental insights on $m_\tilde{g}$ and $m_{1/2}$ will decide on such a simultaneous viability for $x = 1$. For the lower $f_{\text{PQ}}$ value considered in Fig. 2(a), that simultaneous viability is excluded already with $m_\tilde{a} \approx 1$ TeV and $m_{1/2} \approx 400$ GeV.

The described pictures changes considerably if $x \ll 1$. This is shown for $x = 0.2$ (black) and 0.1 (gray) in Figs. 3(a) and (b). Here the dashed and dotted lines indicate $\Delta N_{\text{eff}} = 0.79$ and 0.95, respectively. The solid lines show $\Omega_G^{\text{TP}} h^2 + \Omega_G^{\alpha a\tilde{G}} h^2 = 0.124$ contours. In both panels, $m_\tilde{g} = 1$ TeV and $m_{1/2} = 400$ GeV. For both (a) $f_{\text{PQ}} = 10^{10}$ GeV and (b) $10^{11}$ GeV, one finds that the amount of extra radiation $\Delta N_{\text{eff}}$ that is compatible with the $\Omega_G^{\text{TP}} + \Omega_G^{\alpha a\tilde{G}} \leq \Omega_{\text{CDM}}$ constraint is now significantly larger than the corresponding $x = 1$ cases. For example, the posterior maxima inferred from BBN analyses, $\Delta N_{\text{eff}} = 0.79$, or $\Delta N_{\text{eff}} = 0.95$ can be easily explained by...
FIG. 2. (a)–(c) Contours of $\Delta N_{\text{eff}} = 0.25$ (dash-dotted), 0.47 (dotted), and 0.79 (dashed) provided by axions from decays of thermal saxions and of $\Omega_{\tilde{G}} h^2 + \Omega_{\tilde{a}} h^2 = 0.124$ (solid) in the $m_{\tilde{G}} - T_R$ parameter plane for gravitino LSP scenarios with $m_{\sigma} = m_{\tilde{G}}$ and $x = 1$. Black (gray) curves are obtained with $m_{1/2} = 400 (500) \text{ GeV}$ and $m_{\tilde{g}} = 1 (1.25) \text{ TeV}$. The PQ scale and the axino mass are set to (a) $f_{\text{PQ}} = 10^{10} \text{ GeV}$ and $m_{\tilde{a}} = 2 \text{ TeV}$, (b) $f_{\text{PQ}} = 5 \times 10^{10} \text{ GeV}$ and $m_{\tilde{a}} = 3 \text{ TeV}$, and (c) $f_{\text{PQ}} = 10^{11} \text{ GeV}$ and $m_{\tilde{a}} = 6 \text{ TeV}$, respectively. Regions above the solid lines are disfavored by a gravitino density parameter that exceeds $\Omega_{\text{CDM}}$ at the 3$\sigma$ level. (d) The dilution factor $\Delta$ as a function of the reheating temperature $T_R$ for $x = 1$. The black (gray) solid, dashed, and dotted lines are obtained with $m_{\tilde{g}} = 1 (1.25) \text{ TeV}$ for the $f_{\tilde{a}}$ and $m_{\tilde{a}}$ combinations considered in panels (a), (b), and (c), respectively.
axions from thermal saxions in the part of the $m_{\tilde{G}} - T_R$ parameter plane in which $\Omega_{\tilde{G}}^\text{TP} + \Omega_{\tilde{W}}^\text{TP} \leq \Omega_{\text{CDM}}$. Moreover, one sees in both panels that the $2\sigma$ upper limit from the Planck collaboration [8] impose new more restrictive $\Delta N_{\text{eff}}$ limits that can disfavor such a simultaneous viability. For $0.1 \lesssim x \ll 1$, a simultaneous viability of successful leptogenesis working a minimum $T_R \sim 10^9 \text{GeV}$ and of a sizable $\Delta N_{\text{eff}}$ provided by axions from decays of thermal saxions can now be found towards $f_{\text{PQ}} \sim 10^{10} \text{GeV}$. As can be inferred from Fig. 3(a), where $\Delta$ is close to 1, $T_R \sim 10^9 \text{GeV}$ together with $\Delta N_{\text{eff}} \sim 0.7$ is in the allowed region when $x \sim 0.1$, $m_{\tilde{G},\sigma} \sim 30 \text{ GeV}$ and $f_{\text{PQ}} = 10^{10} \text{GeV}$. On the other hand, towards larger $f_{\text{PQ}} \sim 10^{11} \text{GeV}$, the larger $\Delta N_{\text{eff}}$ values obtained for $0.1 \lesssim x \ll 1$ together with the $2\sigma$ upper limits from the Planck collaboration [8] impose new more restrictive $T_R$ limits that can disfavor such a simultaneous viability. This can be seen explicitly in Fig. 3(b) for the shown $m_{\tilde{G},\sigma}$ range. Even for $m_{\tilde{G},\sigma} \sim 200 \text{ GeV}$ and $x = 0.2$, that simultaneous viability is not possible since $\Delta$ is close to 2 for $T_R \gtrsim 10^9 \text{GeV}$. A minimum of $T_R \sim 2 \times 10^9 \text{GeV}$ will then be required for a leptogenesis scenario working otherwise (i.e. for $\Delta = 1$) at a minimum of $T_R \sim 10^9 \text{GeV}$.

At this point, it should be stressed that contributions to the saxion energy density can arise in coherent oscillations of the saxion field. This can give additional and even dominating contributions to $\rho_\sigma$ and thereby to $\Delta N_{\text{eff}}$ [13, 20, 24, 30, 32, 37]. However, these contributions depend on the initial misalignment of the saxion field $\sigma_i$. In fact, for the considered values of $m_\sigma$ and $f_{\text{PQ}}$, the contribution of this non-thermal source is negligible if $\sigma_i \sim f_{\text{PQ}}$, as often assumed in the literature.
As mentioned at the beginning of this section, we focus here on scenarios in which $\Omega_{\chi}^{\text{PQ}} + \Omega_{\tilde{G}}^{\tilde{a}\to a\tilde{G}}$ provides the dominant part of $\Omega_{\text{CDM}}$. In Figs. 2 and 3 this holds in the region close to the solid lines for the respective gaugino masses. Moving away from those solid lines towards smaller $T_R$, there is room for additional contributions to $\Omega_{\text{CDM}}$ when assuming the considered gaugino masses.

There can be a contribution to $\Omega_{\text{CDM}}$ from coherent oscillations of the axion field after it acquires a mass due to instanton effects at $T \lesssim 1$ GeV. The resulting axion relic density from this misalignment mechanism depends on the initial misalignment angle $-\pi < \theta_i \leq \pi$ and $f_{\text{PQ}}$ [32, 33, 70].

$$\Omega_{\chi}^{\text{MIS}} h^2 \sim 0.15 \xi f(\theta_i^2) \theta_i^2 \left( \frac{f_{\text{PQ}}}{10^{12} \text{ GeV}} \right)^{7/6}, \quad (38)$$

where $\xi = O(1)$ parametrizes theoretical uncertainties related, e.g., to details of the quark–hadron transition and of the $T$ dependence of $m_a$. Moreover, $f(\theta_i^2)$ is the anharmonicity factor which satisfies $f(\theta_i^2) \rightarrow 1$ for small $\theta_i^2 \rightarrow 0$ and becomes sizable towards large $\theta_i^2 \rightarrow \pi^2$ [77–79]. Here the possibility of late time entropy production is not included that can lead to a dilution of $\Omega_{\chi}^{\text{MIS}} h^2$. However, already for $\Delta = 1$ and $\theta_i^2 \sim 1$, $\Omega_{\chi}^{\text{MIS}}$ is only a minor fraction of $\Omega_{\text{CDM}}$ for $f_{\text{PQ}} \lesssim 10^{11}$ GeV considered in this section. In fact, with the possibility of $\theta_i^2 \ll 1$, $\Omega_{\chi}^{\text{MIS}}$ can be negligible and then does not tighten the $T_R$ limits indicated by the solid lines in Figs. 2 and 3.

The contribution from decays of the LOSP into the gravitino $\chi$,

$$\Omega_{\tilde{G}}^{\text{LOSP}} \rightarrow \tilde{G} \chi h^2 = m_{\tilde{G}} Y_{\text{LOSP}} s(T_{\text{LOSP}}) h^2 / \rho_c, \quad (39)$$

depends strongly on the LOSP type, its mass and couplings, and other details of the considered point in the SUSY parameter space. For the case in which the lightest neutralino $\tilde{\chi}_1$ is the LOSP, the yield after freeze-out can be sizable [80, 81].

$$Y_{\tilde{\chi}_1} \text{LOSP} \sim (1 - 4) \times 10^{-12} \left( \frac{m_{\tilde{\chi}_1}}{100 \text{ GeV}} \right), \quad (40)$$

and can thus imply $T_R$ constraints that are significantly more restrictive than those shown in Figs. 2 and 3. This holds even with $Y_{\text{LOSP}}$-diluting entropy production in saxion decays leading to the maximum contribution of $\Delta^{\text{thermal saxions}} \sim 2.5$ the total dilution factor $\Delta$ seen in Fig. 1(b). In contrast, for a charged slepton LOSP $\tilde{l}_1$ or a sneutrino LOSP $\tilde{\nu}_1$ respecting the upper limits on $m_{\text{LOSP}}$, given in Table III, $Y_{\text{LOSP}}$ is relatively small [80, 81].

$$Y_{\tilde{l}_1} \text{LOSP} \lesssim (0.7 - 2) \times 10^{-13} \left( \frac{m_{\tilde{l}_1}}{100 \text{ GeV}} \right), \quad (41)$$

$$Y_{\tilde{\nu}_1} \text{LOSP} \sim 2 \times 10^{-14} \left( \frac{m_{\tilde{\nu}_1}}{100 \text{ GeV}} \right), \quad (42)$$

and basically negligible already without a possible dilution [11]. Then the $T_R$ limits imposed by $\Omega_{\tilde{G}}^{\text{LOSP}} \rightarrow \tilde{G} \chi \lesssim \Omega_{\text{CDM}}$ are very similar to the ones indicated by the solid lines in Figs. 2 and 3.

In the considered gravitino LSP scenarios with a LOSP being the NLSP, the LOSP has a long lifetime before decaying into the gravitino. Often such decays are found to take place during and after BBN. For the $\tilde{\chi}_1$ LOSP, decays such as $\tilde{\chi}_1 \rightarrow \tilde{G} q q$ [81] can then reprocess the primordial light elements via electromagnetic and hadronic energy injection. Thereby, the observationally inferred primordial abundances of those elements translate into upper limits on $Y_{\text{LOSP}}$ that depend on the lifetime of the LOSP $\tau_{\text{LOSP}}$. Towards small values of $\tau_{\text{LOSP}}$ which occur towards smaller values of $m_{\tilde{G}}$, the $Y_{\text{LOSP}}$ limits become weaker and disappear. For a $\tilde{\chi}_1$ LOSP with mass $m_{\tilde{\chi}_1} \sim m_{\text{max}}^{\text{LOSP}}$ and the latter given in Table III, BBN constraints exclude $m_{\tilde{G}} \gtrsim 1$ GeV and thereby most of the interesting parameter regions considered above [80, 81].

For the charged slepton and sneutrino LOSP cases, hadronic energy injection requires 4-body decays such as $l_1 \rightarrow \tilde{G} q q$ or $\tilde{\nu}_1 \rightarrow \tilde{G} q q$ [81] and is thereby less efficient. However, a long-lived charged slepton can form bound states with the primordial nuclei and thereby catalyze, e.g., the primordial production of lithium-6 substantially. This catalyzed BBN (CBBN) then imposes the upper limit $\tau_{\tilde{l}_1} \lesssim 5 \times 10^4$ s [80]. Together with an upper limit on the slepton mass $m_{\tilde{l}_1} \lesssim m_{\text{max}}^{\text{LOSP}} \lesssim 300$ GeV imposed by axino cosmology, this translates into the constraint $m_{\tilde{G}} \lesssim 4$ GeV [81, 88], which again disfavors most of the interesting parameter regions considered above. Moreover, the current lower limit from searches for long-lived charged sleptons at the LHC, $m_{\tilde{\chi}_1} \gtrsim 800$ GeV [80, 90], is already in conflict with most of the $m_{\text{max}}^{\text{LOSP}}$ values listed in Table III. In fact, the production of a long-lived charged slepton LOSP could leave clear signatures at the LHC and allow for a precise measurement of its mass. For example, with $m_{\tilde{l}_1} \sim 400$ (500) GeV, axino-imposed constraints become difficult to evade and the CBBN limit on $\tau_{\tilde{l}_1}$ implies $m_{\tilde{G}} \lesssim 6$ (10) GeV [81, 88, 89]. Such a discovery will thus not be compatible with the $N_{\text{eff}}$ expansion via decays of thermal saxions for $T_R \gtrsim 10^9$ GeV. It may instead point to smaller $T_R < 10^8$ GeV, smaller $f_{\text{PQ}} \lesssim 10^{10}$ GeV, and larger $m_{\tilde{a}}$ or to the axion CDM scenarios with an $eV$-scale axino LOSP and the gravitino NLSP considered in the next section.

For the sneutrino LOSP case, the presented high-$T_R$ explanations of additional radiation via decays of thermal saxions are still viable. The BBN constraints imposed by hadronic and electromagnetic energy release become relevant only for large $m_{\tilde{\nu}_1} \gtrsim 500$ GeV [81, 87]. At smaller

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8 Scenarios in which long-lived staus have an exceptionally small thermal relic abundance well below [11] have been found in which CBBN limits can be evaded [81, 83]. However, these scenarios often require a relatively light stau mass of $m_{\tilde{\nu}_1} \lesssim 200$ GeV which is in conflict with the mentioned limit from LHC searches [84, 90].
m_{\tilde{\nu}_i} \leq m_{\text{max,LSP}}$, the only bound on the gravitino mass then results from the hierarchy $m_{\tilde{G}} < m_{\tilde{\nu}_i}$ assumed in this section. In comparison to the $\bar{\nu}_1$ LOSP, it will be much more challenging to identify a sneutrino $\tilde{\nu}_i$ as the LOSP and to measure its mass at the LHC [43, 47]. Such a measurement will allow us to test the presented scenarios in two ways: (i) by confronting $m_{\tilde{\nu}_i}$ with the upper limit $m_{\text{max,LSP}}$ imposed by the axino and (ii) by exploring the maximum $T_R$ values for the maximum viable mass of the gravitino LSP which is then $m_{\tilde{G}} = m_{\tilde{\nu}_i}$.

V. AXION CDM CASE

In this section we consider SUSY scenarios in which the axion with a mass of $m_a \sim 6$ meV provides the CDM density $\Omega_{\text{CDM}} h^2$ via the misalignment mechanism. The associated relic density $\Omega_a^\text{MIS} h^2$ that resides in coherent oscillations of the axion field is given by (35) in the absence of late-time entropy production. With entropy production after the QCD phase transition, $T \ll 1$ GeV, the corresponding dilution factor $\Delta$ has to be taken into account that reduces the density parameter by a factor of $1/\Delta$. Accordingly, $\Omega_a^\text{MIS} h^2 = \Omega_{\text{CDM}} h^2$ holds with $f_{PQ} = 10^{12}$ GeV, e.g., for $f(\theta_i^2)\theta_i^2 \sim 1$ and $\Delta \sim 1$ or equally for $f(\theta_i^2)\theta_i^2 \sim 10$ and $\Delta \sim 10$. In fact, also in the situations with a sizable $\Delta \sim 30$ encountered below, the CDM density can be explained fully by the axion condensate provided $\theta_i^2$ and the associated anharmonicity factor $f(\theta_i^2)$ are sufficiently large to compensate for the dilution.

In a setting with $\Omega_{\text{CDM}}$ provided by the axion condensate, the LSP is no longer required to be a CDM particle. In turn, the LSP can be a very light particle such as an axino with $m_{\tilde{a}} \lesssim 37$ eV, which is the scenario considered in this section. Such a light axino can still be produced thermally when $T_R < T_a^0$ or decouple as a thermal relic when $T_R > T_a^0$. The resulting population can contribute to hot dark matter (HDM). In fact, the upper limit of $m_a \lesssim 37$ eV is inferred from LSS constraints on HDM contributions in mixed models with CDM [83]. When relativistic, the axino population from thermal processes contributes a small amount of $(\Delta N_a^\text{eff})_{\tilde{a}}^{\text{eq,TP}} \lesssim 0.017$ [83] to dark radiation, which is not included in our calculations below.

In our considerations the gravitino is the NLSP that is lighter than the LOSP, i.e., than the lightest sparticle in the MSSM. Thereby, the explored scenarios are not subject to the restrictive upper limits on $T_R$ imposed by BBN constraints on hadronic/electromagnetic energy injection in late decays of gravitinos into MSSM particles [83, 97]. In the R-parity conserving settings considered in this section, gravitinos can decay into axions and axinos only. The gravitino lifetime $\tau_{\tilde{G}}$ is then governed by the associated decay rate [83, 99]

$$\Gamma_{\tilde{G} \to a \tilde{a}} = \frac{m_{\tilde{G}}^3}{192\pi M_{\tilde{P}}^2} = \frac{1}{\tau_{\tilde{G}}}. \quad (43)$$

Accordingly, gravitinos can be very long-lived. For example, $\tau_{\tilde{G}} \approx 10^{40}$ s and $10^9$ s for $m_{\tilde{G}} \approx 60$ GeV and 6 TeV, respectively. The axions and axinos emitted in decays of a thermally produced gravitino population can thereby contribute substantially to $\Delta N_a$ at late times well after BBN [4, 10, 22]. In fact, the time at which the smallest observable modes of the CMB reenter the horizon, $t = 5.2 \times 10^{10}$ s, imposes an upper limit on $\tau_{\tilde{G}}$ because of the non-observation of a significant release of extra radiation thereafter [21]. The corresponding mass limit is $m_{\tilde{G}} \gtrsim 35$ GeV. For $m_{\sigma} = m_{\tilde{G}}$, this limit implies that saxions decay before the onset of BBN even when small $x$ values of are considered. With $\tau_{\sigma}^\text{ee} < 0.4$ s for $f_{PQ} = 10^{12}$ GeV and $x \gtrsim 0.01$.

While the mass hierarchy $m_{\sigma} \gg m_{\tilde{a}}$ and the Lagrangian (21) now allow for the additional $\sigma \to a \tilde{a}$ decay channel, the corresponding decay width

$$\Gamma_{\sigma \to a \tilde{a}} = \frac{x^2 m_{\tilde{a}}^2}{\pi f_{PQ}^2} \left[ 1 - \left( \frac{2m_{\tilde{a}}}{m_{\sigma}} \right)^2 \right], \quad (44)$$

is suppressed by a factor of at least $32m_{\tilde{a}}^2/m_{\sigma}^2$ with respect to $\Gamma_{\sigma \to a a}$ given in (22) and thereby negligible for the considered mass splittings. The saxion lifetime and the relevant branching ratios are thus again described by (21), (26), and (29), respectively.

As in the previous section, we encounter the two types of decays of non-relativistic particles. However, in the scenarios in the previous section, $\Delta N_{\text{eff}}$ originates basically from saxion decays only and entropy production at two very different times is possible. Now there are two possibly significant sources of extra radiation, saxion decays and gravitino decays, which proceed at very different times, whereas entropy can be produced in saxion decays only. In the following we thus calculate

$$\Delta N_{\text{eff}}(T) = \frac{120}{7\pi^2 T_R^4} \rho_{\text{dr}}(T), \quad (45)$$

where the energy density of dark radiation $\rho_{\text{dr}}$ includes contributions of axions from thermal processes in the early universe, of axions from decays of thermal saxions, and of axions and axinos from decays of thermally produced gravitinos. As in the previous section, the possibility of entropy production in saxion decays is taken into account and decays are treated beyond the sudden decay approximation. Thereby, we update and generalize existing results presented in Refs. [4, 22]. For a qualitative understanding of our numerical results, we again refer to the expressions obtained in Appendix B.

The following Boltzmann equations describe the time evolution of the energy densities of gravitinos, saxions, and dark radiation

$$\dot{\rho}_{\tilde{G}} + 3H \rho_{\tilde{G}} = -\Gamma_{\tilde{G}} \rho_{\tilde{G}}, \quad (46)$$

$$\dot{\rho}_{\sigma} + 3H \rho_{\sigma} = -\Gamma_{\sigma} \rho_{\sigma}, \quad (47)$$

$$\dot{\rho}_{\text{dr}} + 4H \rho_{\text{dr}} = \text{BR}(\sigma \to a a) \Gamma_{\sigma} \rho_{\sigma} + \Gamma_{\tilde{G}} \dot{\rho}_{\tilde{G}}, \quad (48)$$
in the epoch well after the one in which thermal processes involving EWIPs were efficient and when gravitinos and saxions from such processes are non-relativistic. Here the time evolution of the entropy \( S \) and the scale factor \( R \) are described respectively by

\[
S^{1/3} \dot{S} = R^4 \left( \frac{2\pi^2}{45} g_{*S} S \right)^{1/3} [1 - \beta R(\sigma \to \aa)] \Gamma_\sigma \rho_\sigma. \tag{49}
\]

and the Friedmann equation

\[
H^2 \simeq \frac{8\pi}{3m_P^2} (\rho_{\text{rad}} + \rho_\sigma + \rho_{\tilde{G}} + \rho_{\text{rad}}), \tag{50}
\]

with \( \rho_{\text{rad}} \) as given in \(^{[33]} \). We solve the closed set of differential equations \(^{[10]–[50]} \) numerically. As in the previous section, we start at \( t_1 = 1.6 \times 10^{-13} \text{s} \) corresponding to \( T_1 = 1 \text{ TeV} \) with \( R(t_1) = 1 \text{ GeV}^{-1} \). However, the considered end of the evolution is now set to a much later time of \( t_f = 10^{12} \text{s} \) corresponding to \( T_f \simeq 1 \text{ eV} \). The initial values of the energy densities are given by

\[
\rho_{\tilde{G}}(t_i) = m_{\tilde{G}} Y_{\tilde{G}}^{\text{TP}} s(T_i), \tag{51}
\]

\[
\rho_{\sigma}(t_i) = m_\sigma Y_{\sigma}^{\text{eq}/\text{TP}} s(T_i), \tag{52}
\]

\[
\rho_{\text{rad}}(t_i) = \langle J_{\text{rad}}^{\text{th}} \rangle Y_{\text{eq}/\text{TP}} s(T_i), \tag{53}
\]

and of the entropy by \( S(t_i) = s(T_i) R(t_i)^3 \). Entropy production is quantified by the dilution factor \( \Delta \) given as in \(^{[37]} \). Note that the contribution of the energy density of axinos from thermal processes in the early universe can be neglected in \(^{[50]} \) at the considered times. Also in \(^{[50]} \), this population is neglected, which contributes at most \( (\Delta N_{\text{eff}})_{\tilde{a}}^{\text{eq}/\text{TP}} \sim 0.017 \) \(^{[83]} \), as mentioned above. The Boltzmann equation for cold dark matter axions and the associated contribution in \(^{[50]} \) are not mentioned above. In fact, including this population explicitly leads to at most a 1-2% effect in \( \Delta N_{\text{eff}} \) and only in settings with \( t_{\tilde{G}} \gtrsim 10^{10} \text{s} \). As in the saxion treatment in the previous section, saxions and gravitinos are described as non-relativistic species throughout the time interval \( [t_i, t_f] \) although saxions and gravitinos, e.g., with \( m_{\sigma,\tilde{G}} = 100 \text{ GeV} \) will be relativistic at an initial temperature of \( T_i = 1 \text{ TeV} \). This simplified treatment is justified since the contributions of saxions and gravitinos to the right-hand side of the Friedmann equation \(^{[50]} \) become relevant only when they are non-relativistic.

Figure \(^{[4]a} \) presents the results of the numerical integration for \( m_{\sigma} = m_{\tilde{G}} = 100 \text{ GeV} \), \( T_R = 5 \times 10^{9} \text{GeV} \), \( f_{\tilde{P}Q} = 10^{12} \text{GeV} \), and universal gaugino masses at the GUT scale of \( m_{1/2} = 400 \text{ GeV} \), which is compatible with \( m_{\tilde{g}} = 1 \text{ TeV} \) at collider energies. The time evolution of \( R^2 \rho \) is shown for saxions (dashed), gravitinos (dash-dotted), dark radiation (dotted), and other radiation (solid), where black and gray lines refer to \( x = 1 \) and 0.02, respectively. The evolution of entropy \( S \) is not shown. For \( x = 1 \), it is simply a horizontal line and \( \Delta = 1 \). In the case with \( x = 0.02 \), it shows an increase by a factor of \( \Delta \simeq 1.5 \) when the saxion decay occurs. The latter dilution factor can be inferred also from the difference of the two solid curves. Here one can see that the energy density of the universe can be dominated by non-relativistic saxions just before/during their decay, which indicates an early intermediate matter-dominated epoch. In fact, such an epoch can be even more pronounced towards larger \( T_R \) and/or smaller \( m_{\sigma} \) and thereby lead to significantly larger \( \Delta \) values, as illustrated in Fig. \(^{[2]b} \). In Fig. \(^{[2]b} \) the \( T_R \) dependence of the dilution factor \( \Delta \) is shown for \( x = 1, 0.2, 0.1, \) and 0.02 by the solid, dashed, dotted, and dash-dotted curves, respectively. Black (gray) lines refer to \( m_{\sigma} = m_{\tilde{G}} = 50 \) (100) GeV, whereas all other parameters are as in panel (a). The \( T_R \) dependence results from the one of \( Y_{\sigma}^{\text{eq}/\text{TP}} \), which explains the kinks at \( T_R = T_{\tilde{P}Q} \) encountered already in the previous section. Again there is an increase of \( \Delta \) towards small \( x \) due to larger values of \( t_{\tilde{G}} \) and \( BR(\sigma \to gg) \). For \( x \lesssim 0.1 \) and towards large \( T_R \gtrsim T_{\tilde{P}Q} \), \( \Delta \) can now be much larger than in the previous section because here \( f_{\tilde{P}Q} = 10^{12} \text{GeV} \). The latter implies a larger saxion lifetime \(^{[24]} \), whereas \( Y_{\sigma}^{\tilde{a}} \) is independent of \( f_{\tilde{P}Q} \); see also approximations \(^{[38]} \) and \(^{[17]} \) in Appendix \(^{[1]} \).

Let us now turn to \( \Delta N_{\text{eff}} \). The dotted line in Fig. \(^{[2]a} \) illustrates that there are two sizable contributions at very different times, as advertised above: \( \Delta N_{\text{eff}}^{\sigma\rightarrow\aa} \) residing in axions from decays of thermal saxions and \( \Delta N_{\text{eff}}^{\tilde{G}\rightarrow\aa} \) residing in axions and axions from decays of thermally produced gravitinos. For \( x = 1 \) (0.02), there is an early contribution of \( \Delta N_{\text{eff}}^{\sigma\rightarrow\aa} \approx 0.29 \) (0.4) and an additional late contribution of \( \Delta N_{\text{eff}}^{\tilde{G}\rightarrow\aa} = 0.39 \) (0.25) leading to a sum of \( \Delta N_{\text{eff}}^{\sigma\rightarrow\aa} \approx 0.68 \) (0.65). These values are compatible with the 2\( \sigma \) upper limit of \( \Delta N_{\text{eff}} < 0.79 \) (0.95) derived from the Planck+WP+highL+(H0)+BAO data set quoted in Table \(^{[1]} \).

Prior to the announcement of the Planck results, we found it tempting to suggest the substantial difference between the posterior maxima of \( \Delta N_{\text{eff}} \sim 0.8 \) from BBN studies and the mean of \( \Delta N_{\text{eff}} \sim 1.8 \) from pre-Planck precision cosmology as a first indication towards the realization of the considered axion CDM scenario in nature \(^{[4]} \); cf. Table \(^{[1]} \). The Planck results now disfavor such a substantial difference. Nevertheless, a small difference remains viable and the considered scenarios remain attractive with the axion condensate and thermal leptogenesis providing natural explanations of CDM for \( f_{\tilde{P}Q} = 10^{12} \text{GeV} \) and of the baryon asymmetry for \( T_R \gtrsim 10^{8} \text{GeV} \), as already emphasized in Refs. \(^{[22]} \). In the following we systematically explore \( \Delta N_{\text{eff}} \) contributions in settings with \( f_{\tilde{P}Q} = 10^{12} \text{GeV} \) and large \( T_R \). In addition to the latter two features mentioned above, the saxion energy density residing in coherent saxion oscillations with \( \sigma_i \sim f_{\tilde{P}Q} \) is negligible with respect to the one from thermal processes in that parameter region \(^{[20]} \). Our results are presented in Figs. \(^{[5]} \) and \(^{[6]} \) in the \( m_{\tilde{G}}=T_R \) parameter plane for \( m_{\sigma} = m_{\tilde{G}}, m_{\tilde{a}} \lesssim 37 \text{ eV} \), and universal gaugino masses at the GUT scale of \( m_{1/2} = 400 \text{ GeV} \).
or 600 GeV. The latter is compatible with \( m_\tilde{g} = 1.5 \) TeV at collider energies. The region with \( \tau_G < 5.2 \times 10^{10} \) s is not considered and indicated by a vertical gray dotted line at \( m_G \simeq 35 \) GeV.

In Fig. 4(a) the solid black (gray) lines show \( \Delta N_{\rm eff}^{\sigma \to a \tilde{a}} + \Delta N_{\rm eff}^{G \to a \tilde{a}} = 0.95 \) and the dashed lines \( \Delta N_{\rm eff}^{G \to a \tilde{a}} = 0.95 \) for \( m_{1/2} = 400 \) (600) GeV and \( x = 1 \). To allow for a comparison, the diagonal dotted line indicates \( \Delta N_{\rm eff}^{G \to a \tilde{a}} = 0.95 \) as obtained for \( m_G = 1 \) TeV with the existing result of Ref. [22] based on the sudden decay approximation. The difference with respect to the corresponding dashed line is due to the sudden decay approximation, which overestimates \( \Delta N_{\rm eff} \) by about 13%, and the omissions of electroweak and spin-3/2 contributions in the gravitino yield \( Y_{G\tilde{a}}^{\rm TP} \) used in Ref. [22]. Including the electroweak contributions increases \( Y_{G\tilde{a}}^{\rm TP} \) by about 20% at \( m_G \simeq 35 \) GeV, while the importance of the spin-3/2 components becomes much more pronounced towards larger \( m_G \). Comparing the respective dashed and solid lines, we find that \( \Delta N_{\rm eff}^{G \to a \tilde{a}} \) contributions lead to an additional sizable \( \Delta N_{\rm eff} \) increase. In fact, for \( m_\sigma \gtrsim 100 \) GeV, they tighten the upper limit on \( T_R \) imposed by the 2σ upper limit \( \Delta N_{\rm eff} < 0.95 \) derived from the Planck+WP+highL+H0+BAO data set [3] by up to almost one order of magnitude. For further comparison, we refer to Fig. 6 in Ref. [4], where \( \Delta N_{\rm eff}^{\sigma \to a \tilde{a}} \) only is presented as obtained in the sudden decay approximation. Also (B10) and (B13) in Appendix B of this work are approximate analytical expressions respectively for \( \Delta N_{\rm eff}^{\sigma \to a \tilde{a}} \) and \( \Delta N_{\rm eff}^{G \to a \tilde{a}} \) that are based on the sudden decay approximation.

Figure 4(a) demonstrates how the \( \Delta N_{\rm eff} \) contours will move if LHC experiments point to \( m_\tilde{g} \gtrsim 1.5 \) TeV and thereby to \( m_{1/2} \gtrsim 600 \) GeV. These changes are governed fully by \( \Delta N_{\rm eff}^{G \to a \tilde{a}} \) whereas \( \Delta N_{\rm eff}^{\sigma \to a \tilde{a}} \) is not affected. At this point, we should stress that a collider measurement of the LOSP mass \( m_{\rm LOSP} \) will limit \( m_{\tilde{G}} \) from above. While the chosen \( m_{1/2} \) values can imply an \( m_{\rm LOSP} \) value that is well below 1 TeV, we refrain from presenting such an upper limit for \( m_G \) since it will depend strongly on other details of an assumed SUSY model as well.

The \( \Delta N_{\rm eff} = 0.95 \) contours illustrate the impact of the results from the Planck satellite mission. While Planck does not find any statistically significant hints for extra radiation, the contour \( \Delta N_{\rm eff} = 0.95 \) provides the new upper limit on \( T_R \) at the 2σ level as obtained from the Planck+WP+highL+H0+BAO data set [3]. For \( x = 1 \), the viability of \( T_R \gtrsim 10^{9} \) GeV will then depend on \( m_{\rm LOSP} \) and on other LOSP-related cosmological constraints discussed below.

Let us now turn to the case of \( x < 1 \). Figure 4(b) shows \( \Delta N_{\rm eff}^{G \to a \tilde{a}} + \Delta N_{\rm eff}^{G \to a \tilde{a}} = 0.95 \) contours for \( x = 1 \) (solid), 0.2 (dashed), 0.1 (dotted), 0.02 (dash-dotted), and 0.01
FIG. 5. Contours of $\Delta N_{\text{eff}}$ provided by axions by decays of thermal saxions, $\Delta N_{\sigma^{\tau} \rightarrow a a}$, and by axions and axinos from decays of thermally produced gravitinos, $\Delta N_{\tilde{G} \rightarrow a a}$, in the $m_{\tilde{G}}$-$T_R$ parameter plane in axino LSP scenarios with the gravitino NLSP, where $m_{\sigma} = m_{\tilde{G}}$, $m_{a} \lesssim 37$ eV, and $f_{\nu Q} = 10^{12}$ GeV. In panel (a), $x = 1$ and black (gray) contours refer to $m_{1/2} = 400$ (600) GeV. Here we show solid contours of $\Delta N_{\sigma^{\tau} \rightarrow a a} + \Delta N_{\tilde{G} \rightarrow a a} = 0.95$ and dashed contours of $\Delta N_{\tilde{G} \rightarrow a a} = 0.95$. The diagonal dotted line indicates the latter as well but as obtained with the $\Delta N_{\tilde{G} \rightarrow a a}$ estimate from Ref. [22]. In panel (b) we show solid, dashed, dotted, dash-dotted, and dash-double-dotted contours of $\Delta N_{\sigma^{\tau} \rightarrow a a} + \Delta N_{\tilde{G} \rightarrow a a} = 0.95$ for $x = 1, 0.2, 0.1, 0.02, 0.01$, respectively, and $m_{1/2} = 400$ GeV. The regions above these contours are disfavored at the 2σ level by the Planck+WP+highL+$H_0$+BAO data set [8]; cf. Table I. The vertical dotted line indicates the lower limit on $m_{\tilde{G}}$ from the requirement $\tau_{\tilde{G}} \lesssim 5.2 \times 10^{15}$ s in both panels.

(dash-double-dotted) where $m_{1/2} = 400$ GeV. Corresponding dilution factors $\Delta$ have already shown in Fig. III(b) and discussed thereafter. The dilution factor $\Delta$ for $x = 0.01$ has not been shown. It shows a similar behavior but slightly exceeds the one for $x = 0.02$, i.e., it is slightly below 30 (above 20) for $m_{a} = 50$ GeV (100 GeV) and $T_R \gtrsim 10^{11}$ GeV. The $x$ dependence of $\Delta N_{\sigma^{\tau} \rightarrow a a}$ results fully from the one of $\Delta$ so that this contribution decreases towards $x \rightarrow 0$. In contrast, for the same reasons as in the previous section, $\Delta N_{\sigma^{\tau} \rightarrow a a}$ increases towards smaller $x$ in the interval $0.1 \lesssim x < 1$, reaches its maximum at $x \sim 0.1$, and decreases thereafter, i.e., towards smaller $x \lesssim 0.1$. The latter behavior transfers to $\Delta N_{\tilde{G} \rightarrow a a} + \Delta N_{\tilde{G} \rightarrow a a}$, as can be seen in Fig. III(b). Here the most restrictive upper $T_R$ limit is found for $x = 0.1$ and the most relaxed one for $x = 0.01$, where $\Delta N_{\sigma^{\tau} \rightarrow a a} + \Delta N_{\tilde{G} \rightarrow a a} \simeq \Delta N_{\tilde{G} \rightarrow a a}$.

In Fig. III we explore how $\Delta N_{\sigma^{\tau} \rightarrow a a} + \Delta N_{\tilde{G} \rightarrow a a}$ (solid) can emerge as a composition of a late $\Delta N_{\tilde{G} \rightarrow a a}$ (dashed) and an early $\Delta N_{\sigma^{\tau} \rightarrow a a}$ (dotted) for (a) $x = 1$, (b) 0.2, (c) 0.1 and (d) 0.02. In each panel, we consider $m_{1/2} = 400$ GeV and show gray and black contours of $\Delta N_{\text{eff}} = 0.25$ and 0.47, respectively. On the one hand, those $\Delta N_{\text{eff}}$ values are the corresponding means inferred from the Planck+WP+highL+BAO and the Planck+WP+highL+$H_0$+BAO data sets obtained by the Planck collaboration [8]. On the other hand, e.g., a total late $\Delta N_{\text{eff}} = 0.47$ may be composed of an early $\Delta N_{\text{eff}} = 0.25$ from saxion decays and an additional late $\Delta N_{\text{eff}} \simeq 0.22$ from gravitino decays. The parameter points that allow for this composition are the ones at which the gray dotted and the black solid lines intersect. Accordingly, this composition is possible in all four panels, i.e., for $x = 1, 0.2, 0.1, 0.02, 0.01$, respectively, and $m_{1/2} = 400$ GeV. In light of the BBN uncertainties with respect to an early $\Delta N_{\text{eff}}$, it should be emphasized that different compositions are possible as well, as discussed at the end of Sect. III.

In an assessment of the simultaneous viability of thermal leptogenesis and a certain $\Delta N_{\text{eff}}$ composition, the corresponding dilution factor $\Delta$ has to be taken into account in the same way as in the previous section. While this factor can now be much larger, the current upper $T_R$ limit imposed by $\Delta N_{\text{eff}} < 0.95$ still allows for that simultaneous viability even for $x = 0.1$ when $m_{\tilde{G}} \gtrsim 50$ GeV; cf. Figs. III(b) and III(b). This 2σ upper limit from the Planck+WP+highL+$H_0$+BAO data set [8] is a somewhat conservative one. Nevertheless, even with the more restrictive 2σ upper limit from the Planck+WP+highL+BAO data set [8], $\Delta N_{\text{eff}} < 0.79$ or
with the mean $\Delta N_{\text{eff}} = 0.25$ or 0.47, thermal leptogenesis can remain viable for $x = 1$ and also for smaller $x$ provided $m_\tilde{G}$ can be sufficiently large. Because of the assumed hierarchy in this section, $m_\tilde{G} < m_{\text{LOS}}$, a measurement of $m_{\text{LOS}}$ can thus challenge that simultaneous viability, in particular, for $x \gtrsim 0.1$ and larger $m_{1/2}$.

In the considered situation with the axino LSP and the gravitino NLSP, the LOSP is again a long-lived particle. However, in contrast to the gravitino LSP setting in Sect. [IV], it can not only decay into gravitinos, $\text{LOSP} \rightarrow \tilde{G} X$, but also into axinos, $\text{LOSP} \rightarrow \tilde{a} X$, where the relative importance is governed by $m_\tilde{G}$ and $f_{\tilde{P}Q}$. For $f_{\tilde{P}Q} = 10^{12}$ GeV and $m_\tilde{G} \gtrsim 35$ GeV, the decay into the axino is the dominating one, i.e., $\Gamma(\text{LOSP} \rightarrow \tilde{a} X) \gg$
\( \Gamma (\text{LOSP} \to \bar{G}X) \). Thereby, the LOSP lifetime can be significantly shorter than in the previous section so that the (C)BBN constraints related to a late decaying LOSP described at the end of that section can be evaded. In fact, the charged slepton LOSP is now a viable possibility, which is particularly attractive since it could appear as a quasistable charged massive particle in collider experiments. For example, if the LOSP is the lightest stau with \( m_{\tilde{\tau}_1} \gtrsim 300 \text{ GeV} \), there is indeed no limit on the gravitino mass other than \( m_\chi < m_{\tilde{\tau}_1} \) for \( f_{\text{QCD}} \lesssim 5 \times 10^{12} \text{ GeV} \) and already with \( \Delta = 1 \) [83]. Late time entropy production in saxion decays with \( x \ll 1 \) can dilute \( Y_{\text{LOSP}} \) as described by (11) with a sizable \( \Delta \geq 1 \) and thereby imply even more relaxed constraints. The bino-like neutralino LOSP situation was considered in Ref. [100] and found to be viable for \( f_{\text{QCD}} \sim 10^{12} \text{ GeV} \) as well. This work accounted for entropy production in saxion decays also and already with \( \Delta = 1 \) [83]. Late time entropy production in saxion decays with \( x \ll 1 \) can dilute \( Y_{\text{LOSP}} \). A discovery of axions in this search could therefore point towards the realization of one of the settings considered in this section. Further support in favor of those settings (and against the ones considered in Sect. [IV]) would be the discovery of a long-lived charged slepton LOSP at the LHC. It could even be that future cosmological analyses find hints on the time of the release of extra radiation. Such a release may manifest itself in the perturbation spectrum so that precision cosmology might help to assess the lifetime of the gravitino whose late decays produce dark radiation at times before \( 5.2 \times 10^{10} \text{ s} \). Another strong hint for the scenarios considered here would be the confirmation of extra radiation prior to BBN and a significant difference between that amount with respect to the one at much later times. In addition to new astrophysical data sets from improved direct measurements of the Hubble constant \( H_0 \), this would require advances in BBN-related studies. In particular, this calls for new high quality spectra from extragalactic HII regions that should allow for a significantly more precise determination of \( \Delta N_{\text{eff}} \) prior to BBN [102].

VI. CONCLUSION

We have explored two scenarios of hadronic axion models [52, 53] in R-parity conserving SUSY settings: (i) a gravitino LSP scenario with a heavy axino at the TeV scale, and (ii) a scenario with a light axino LSP at the eV scale and the gravitino NLSP. Both scenarios are found to allow for consistent explanations of extra radiation and CDM and for a high reheating temperature \( T_R \) of up to about \( 10^9 \text{ GeV} \), respectively. Testable cases have been outlined that may still allow for the high \( T_R \) values required by successful thermal leptogenesis with hierarchical heavy Majorana neutrinos [51].

In the gravitino LSP scenario, CDM resides dominantly in gravitinos from thermal production and from decays of thermal axions and \( \Delta N_{\text{eff}} \) is explained by thermal saxions which decay into axion pairs prior to BBN. We have shown that up to \( \Delta N_{\text{eff}} \approx 0.8 \) can arise naturally for \( f_{\text{QCD}} \approx 10^{10} \text{ GeV} \) and \( T_R \approx 10^7 \text{ GeV} \). This finding requires that the gluino mass \( m_\chi \) is close to the current experimental limit of about 1 TeV. For a larger \( m_\chi = 1.25 \text{ TeV} \), we have demonstrated that smaller values of \( \Delta N_{\text{eff}} \approx 0.5 \) remain viable for \( 10^{10} \text{ GeV} \lesssim f_{\text{QCD}} \lesssim 10^{11} \text{ GeV} \) and \( 10^7 \text{ GeV} \lesssim T_R \lesssim 10^9 \text{ GeV} \). Viability of larger \( \Delta N_{\text{eff}} \) (i.e., above 0.8 or 0.5) is found to require a more suppressed saxion-axion coupling, \( x \ll 1 \), with a maximum \( \Delta N_{\text{eff}} \) occurring for \( x \sim 0.1 \). There we have
shown that the $2\sigma$ limits of $\Delta N_{\text{eff}} < 0.79$ or 0.95 obtained by the Planck collaboration [8] translate into new upper limits on $T_R$, which can be the most restrictive ones.

For compatibility of the presented gravitino LSP case with cosmological constraints, the axino must be heavy, $m_a \gtrsim 2$ TeV, so that it decays prior to the decoupling of the LOSP from the thermal bath. For the high $T_R$ values considered, such a heavy axino can still be produced very efficiently in thermal processes in the early Universe. Primordial axinos can thereby contribute significantly to the total energy density just before decaying dominantly into gluinos and gluons. Calculating the associated entropy production, we obtain dilution factors of up to $\Delta \tilde{a} \rightarrow gg \sim 2$ that affect the abundances of gravitinos, saxions, and axions produced in thermal processes well before axinos dominate the energy density. Since also a baryon asymmetry generated prior to that epoch is diluted by the same factor, about twice of the observed value is needed prior to that dilution. Within the framework of thermal leptogenesis, this implies that the usually required $T_R \sim 10^{10}$ GeV [31] now has to be basically twice as large [39, 60]. For $x = 1$, we find this to be viable for $f_{\text{PQ}} = 10^{11}$ GeV, $m_{\tilde{g}} \approx 1$ TeV, and $m_a \approx 6$ TeV when $\Delta N_{\text{eff}} \lesssim 0.5$. Towards small $x < 1$, the decay of thermal saxions into gluons can lead to an additional sizable dilution factor of up to $\Delta \sigma \rightarrow gg \sim 3$. This can dilute even the yield of the LOSP after decoupling from the thermal plasma and prior to decay and thereby weaken BBN constraints related to late decaying LOSP [39, 60, 69]. Moreover, for $0.1 \lesssim x \lesssim 1.0$, $f_{\text{PQ}} \sim 10^{10}$ GeV, and $m_{\tilde{a}} \approx 2$ TeV, we have found that a significant part of the parameter space will allow for the simultaneous viability of thermal leptogenesis, a sizable $\Delta N_{\text{eff}}$ provided by axions from decays of thermal saxions, and $\Omega_{\text{CDM}}$ residing almost fully in thermally produced gravitinos. Towards $f_{\text{PQ}} \sim 10^{11}$ GeV, small $m_{\tilde{g}} \lesssim 100$ GeV, and large $m_a \approx 6$ TeV, gravitinos from decays of thermal axinos are found to become an increasingly important component of $\Omega_{\text{CDM}}$, which tightens associated $T_R$ limits considerably.

In the scenario with the light axino LSP and the gravitino NLSP, CDM resides in axions from the misalignment mechanism, which provides naturally $\Omega_{\text{ax}} \simeq \Omega_{\text{CDM}}$ for $f_{\text{PQ}} \approx 10^{12}$ GeV. Remarkably, the ongoing direct axion CDM search by ADMX [101] is sensitive in exactly that $f_{\text{PQ}}$ range and may find signals supporting this CDM explanation in the near future. We have demonstrated that there are now two sources for a possibly substantial $\Delta N_{\text{eff}}$ that work at very different times: thermal saxions that decay into axion pairs prior to BBN and thermally produced gravitinos that decay into axions and axinos well after BBN and before $5.2 \times 10^{10}$ s. Accordingly, within this scenario, we find different possibilities to explain, e.g., the means of $\Delta N_{\text{eff}} = 0.25$ or 0.47 obtained recently by the Planck collaboration [8]. For $\Delta N_{\text{eff}} \approx 0.47$, one natural explanation will be the composition with an early $\Delta N_{\sigma \rightarrow aa} \approx 0.25$ residing in axions from saxion decays and an additional late $\Delta N_{\tilde{G} \rightarrow a\tilde{a}} \approx 0.22$ residing in axions and axinos from gravitino decays. However, without more precise BBN limits for $\Delta N_{\text{eff}}$, which may indeed be difficult to obtain in light of the systematic uncertainties [2], there remains a significant uncertainty with respect to the amount of an early $\Delta N_{\text{eff}}$. Accordingly, $\Delta N_{\text{eff}} \approx 0.47$ can result equally well either dominantly from late $\tilde{G} \rightarrow a\tilde{a}$ decays for $m_{\tilde{G}} \ll 100$ GeV or dominantly from $\sigma \rightarrow aa$ decays prior to BBN for $m_{\tilde{G}} \gg 100$ GeV. In fact, also the amount of the late $\Delta N_{\text{eff}}$ comes with uncertainties that call for new direct $H_0$ measurements.

Our refinements with respect to Refs. [4, 22] have been found to have the following effects. By treating decays beyond the sudden-decay approximation, the resulting $\Delta N_{\text{eff}}$ values decrease by about 10%. Moreover, with the gravitino yield that accounts for the gravitino-spin-3/2 components and for electroweak processes, previously neglected contributions to $\Delta N_{\tilde{G} \rightarrow a\tilde{a}}$ are included which become sizable for $m_{\tilde{G}} \gtrsim 100$ GeV. Together with the contributions from $\sigma \rightarrow aa$ decays, this results in significantly larger $\Delta N_{\text{eff}}$ values in that region. In turn, our new upper bounds on $T_R$ for $m_{\tilde{G}} = O(100)$ GeV are substantially more restrictive than previously expected for $x = 1$ in the axino LSP case with the gravitino NLSP. Towards a more suppressed saxion-axion coupling with $x \sim 0.1$, we find even larger $\Delta N_{\text{eff}}$ values that further tighten the $T_R$ limits significantly. Even smaller values of $x \ll 0.1$ have been found to come with significant dilution factors of up to $\Delta \sigma \rightarrow gg \sim 30$. Those can reduce $\Delta N_{\text{eff}}$ and in turn relax the upper bounds on $T_R$ but have to be included in an assessment of the viability of thermal leptogenesis.

We have discussed ways in which the different explanations of a potentially sizable $\Delta N_{\text{eff}}$ will be narrowed by ongoing SUSY searches at the LHC. Particularly important will be new limits on $m_{\tilde{g}}$ or measurements thereof. The gluino mass governs the thermally produced gravitino yield and thereby limits that relate to this quantity. Upper limits on $T_R$ thus become more restrictive for larger $m_{\tilde{g}}$. In fact, for $m_{\tilde{g}} \gtrsim 1.1$ TeV, we find that a $\Delta N_{\text{eff}} \sim 0.8$ explanation by axions from thermal saxions becomes incompatible with the then too restrictive $\Omega_{\tilde{G}} \lesssim \Omega_{\text{CDM}}$ constraint in the gravitino LOSP case. In the alternative axino LSP case, on the other hand, it is the $\Delta N_{\text{eff}}$-imposed limit that becomes more restrictive towards large $m_{\tilde{g}}$ in the small $m_{\tilde{G}}$ region where the decay $\tilde{G} \rightarrow a\tilde{a}$ contributes significantly to $\Delta N_{\text{eff}}$.

Other relevant LHC findings will be a discovery of the lightest sparticle within the MSSM (i.e., the LOSP), its identification, and a measurement of its mass. In both of the considered cases, the LOSP mass limits $m_{\tilde{g}}$ from above. Moreover, the LOSP is expected to be long-lived so that additional restrictive cosmological constraints can occur depending on the nature of the LOSP. For example, for a long-lived charged slepton LOSP, which has to be heavier than about 300 GeV [83, 88], CBBN constraints can disfavor the presented gravitino LSP case [87, 88].
Remarkably, such an LOSP is found to be compatible with the light axino LSP scenario with the gravitino NLSP [83]. The discovery of such an LOSP could thus become an important additional hint in favor of the latter scenario. In the gravitino LSP scenario, BBN constraints associated with hadronic energy injection disfavor the possibilities of a neutralino LOSP [80] or a colored LOSP [102] as well. Nevertheless, that scenario is found to be viable with a sneutrino LOSP [81, 83]. One will then face the challenge to identify a long-lived sneutrino as the LOSP [91–97], which will be a much more difficult task than the identification of a long-lived charged slepton LOSP.

In summary, we find the presented scenarios appealing from the cosmological point of view and intriguing with respect to their testability. In light of those features, it will be interesting to see ways in which model building can allow for the suggested mass spectra and the large splittings between the axino mass and the masses of the axion and the gravitino. With upcoming new results from the direct axion dark matter search experiment ADMX and the LHC, it will be exciting to see further hints for or against the viability of the considered scenarios soon.

Appendix A: Thermal Axino Production

Let us present some of the details of the calculation that lead to our update of thermally produced axino yield [7]. To obtain a finite result in a gauge-invariant treatment, we rely on systematic field theoretical methods such as HTL resummation [43] and the Braaten-Yuan prescription [44] exactly as applied in Ref. [42]. However, we now include the quartic axino-squark-antisquark-gluino interaction described by the second term in the third line of (8). Process H is also possible with antisquarks, replacing \( \tilde{q}_{i,j} \) by \( \tilde{q}_{i,j} \).

Other entries in that table are not affected. The Mandelstam variables are given by \( s = (P_1 + P_2)^2 \) and \( t = (P_1 - P_3)^2 \), where the particle four-momenta \( P_i \) refer to the particles in the order in which they are written down above and in Fig. 7.

Grouping the processes into different classes (depending on the number of external bosons and fermions) and weighting the matrix elements with their respective multiplicities and statistical factors, we find that the sums of the corresponding squared matrix elements \( |M_{BBF}|^2 \) and \( |M_{BFB}|^2 \) given in (3.7) and (3.8) of Ref. [42] change to:

\[
\begin{align*}
|M_{BBF}|^2 &= \frac{g_s^6(N_f^2-1)}{64\pi^4 f_W^2} \left( s + 2t + \frac{2s^2}{t} \right) (N_c + n_f) + 4sn_f, \quad (A3) \\
|M_{BFB}|^2 &= \frac{g_s^6(N_f^2-1)}{32\pi^4 f_W^2} \left( -t - 2s - \frac{2s^2}{t} \right) (N_c + n_f) - 4tn_f, \quad (A4)
\end{align*}
\]

where the notation of the above reference is adopted with \( N_c = 3 \) denoting the number of colors and \( n_f = 6 \) the number of color triplet and anti-triplet chiral multiplets. Further simplifications lead to

\[
\begin{align*}
|M_{BBF}|^2 &= \frac{g_s^6(N_c^2 - 1)}{32\pi^4 f_W^2} \left[ |M_3|^2 (N_c + n_f) - 2|M_2|^2 n_f \right], \quad (A5) \\
|M_{BFB}|^2 &= \frac{g_s^6(N_c^2 - 1)}{32\pi^4 f_W^2} \left[ |M_1|^2 (N_c + n_f) - 2|M_2|^2 n_f \right], \quad (A6)
\end{align*}
\]

where \( |M_1|^2 = -t - 2s - (2s^2/t), \) \( |M_2|^2 = t^2/s, \) and \( |M_3|^2 = t^2/s. \) Using this instead of Eqs. (3.12) and (3.13) of Ref. [42], our result for the hard part of the thermal production rate shows a prefactor of \(-2n_f\) instead of \(-3n_f/2\) in the fourth line of (E.1) in Ref. [42] but otherwise agrees with that equation.

By adding the soft and hard parts of the thermal production rate and by integrating the resulting total thermal production rate over the energy of the produced axi-
ino, we arrive at the collision term
\[
W_\tilde{a}(T) = \frac{(N_c^2 - 1) 3 \zeta(3) g_s^2 T^6}{4096 \pi^2} \times \left[ \ln \left( \frac{1.647 T^2}{m_{\tilde{a}}^2} \right) (N_c + n_f) + 0.5781 n_f \right]
\]
with \(m_{\tilde{a}}^2 = g_s^2 T^2 (N_c + n_f)/6\) denoting the squared SUSY thermal gluon mass. The collision term enters the Boltzmann equation, \(\dot{n}_a + 3Hn_a = W_\tilde{a}\), that describes the time evolution of the axino number density. Integrating this equation as described in [42], we get for the thermally produced axino yield
\[
Y_{\tilde{a}}^{TP}(T) \approx \frac{W_\tilde{a}(T_R)}{s(T_R)H(T_R)} \tag{A8}
\]
and thereby expression (7) given in Sect. III. In summary, the constant in the logarithm in the expression for \(Y_{\tilde{a}}^{TP}\) changes from 1.211 in (E.3) of [42] to 1.271 in (7) when including the quartic axino-squark-antisquark-gluino vertex. Accordingly, in the R-parity conserving axino LSP scenarios considered in Ref. [42], the density parameter of thermally produced CDM axinos changes to
\[
\Omega_\tilde{a} h^2 = 5.5 g_s^6 \ln \left( \frac{1.271}{g_s} \right) \left( \frac{m_\tilde{a}}{0.1 \text{ GeV}} \right) \times \left( \frac{10^{11} \text{ GeV}}{f_{\tilde{P}Q}} \right)^2 \left( \frac{T_R}{10^4 \text{ GeV}} \right). \tag{A9}
\]
Nevertheless, the qualitative statements and plots of Ref. [42] are only mildly affected by this correction.

**Appendix B: Approximations for \(\Delta\) and \(\Delta N_{\text{eff}}\)**

Here we provide expressions that describe approximately the numerical results obtained in Sects. [IV] and [V]. The presented expressions help to understand the qualitative behavior of those results and their dependences on quantities such as \(f_{\tilde{P}Q}, x, T_R, m_\sigma, m_\tilde{a}, m_\tilde{g},\) and \(m_2\).

We start with the dilution factor \(\Delta\) based on the corresponding considerations in Ref. [104]. The equation describing the change in entropy due to the decay of a single non-relativistic species \(\psi\) into relativistic particles that rapidly thermalize reads
\[
S^{1/3} = R^4 \left( \frac{2\pi^2}{45} g_{*S} \right)^{1/3} \Gamma_\psi \rho_\psi, \tag{B1}
\]
which is the basis for (31) and (49) in the main text. Here \(\rho_\psi\) and \(\Gamma_\psi\) are the energy density and the total decay width of \(\psi\), respectively, and \(\psi\) is assumed to decay fully into rapidly thermalizing particles. By integrating (B1), one arrives at [104]
\[
\left( \frac{S}{S_1} \right)^{4/3} = 1 + \frac{4}{3} \rho_\psi t R^4 \int_{t_1}^t dt' \left( \frac{2\pi^2}{45} g_{*S} \right)^{1/3} \left[ \frac{R(t')}{R_i} \right] e^{-\Gamma_\psi t'}, \tag{B2}
\]
where the subscript \(i\) refers to the respective quantities at the initial time \(t_i\). This time \(t_i\) can differ from the value used in our numerical calculations in Sects. [IV] and [V]. In fact, the main contribution to the integral comes from the time interval around \(\tau_\psi = 1/\Gamma_\psi\) so that, e.g., \(t_i = 0.01\tau_\psi\) is sufficiently early to obtain a good precision.

To solve the integral in (B2), one needs to know the evolution of the scale factor. This is described by the Friedmann equation and therefore depends on the energy content of the Universe at the relevant times. For the following two limiting cases, an approximate solution for \(\Delta\), the ratio of the entropy before and after the decay, can be obtained analytically.

When the energy density \(\rho_\psi\) of the non-relativistic particle \(\psi\) prior to its decay dominates the one of the Universe, matter dominates so that \(R \propto t^{2/3}\) and [104]
\[
\Delta_{\text{large}} \approx 1.83 \frac{m_\psi Y_\psi}{(\Gamma_\psi m_{TP})^{1/2}}, \tag{B3}
\]
where \(\langle g_{*S} \rangle\) denotes a suitably averaged value of \(g_{*S}\) over the integration interval. If \(g_{*S}\) does not change significantly around \(t \sim \tau_\psi, \langle g_{*S} \rangle = g_{*S}(\tau_\psi)\) gives a reasonable approximation. The subscript “large” in (B3) is used because of the large dilution factor, \(\Delta \gg 1\), encountered in such situations and to indicate the correspondingly limited applicability range of (B3).

When the energy density in radiation dominates the one of the Universe prior to and during the epoch in which \(\psi\) decays, \(R \propto t^{1/2}\) and [104]
\[
\Delta_{\text{small}} \approx 1 + 1.61 \frac{(g_{*S})^{1/3}}{g_{*S}(t_i)^{1/2}} \frac{m_\psi Y_\psi}{(\Gamma_\psi m_{TP})^{1/2}}. \tag{B4}
\]
Again one obtains a good approximation with \(\langle g_{*S} \rangle = g_{*S}(\tau_\psi)\) if \(g_{*S}\) is (basically) constant in the relevant interval. The subscript “small” in (B4) indicates that its applicability is limited to settings in which \(\Delta\) is not much larger than one.

Let us turn to the case of entropy release from axino decay considered in Sect. [V]. For all of the parameter points examined in this work, \(\rho_\tilde{a} < \rho_{\text{rad}}\) and thus the corresponding dilution factor is \(\Delta_{\tilde{a} \rightarrow gg} = \mathcal{O}(1)\). Consequently, we can use (B4) to approximate the dilution factor from entropy release in axino decays. Indeed, for axinos from thermal processes with \(\text{BR}(\tilde{a} \rightarrow gg) \approx 1\), our numerical results – shown e.g. by the solid line in Fig. [II] – are well approximated by
\[
\Delta_{\tilde{a} \rightarrow gg} \approx 1 + 2.3 \times 10^{-2} \left( \frac{2 \text{ TeV}}{m_\tilde{a}} \right)^{1/2} \left( \frac{f_{\tilde{P}Q}}{10^{10} \text{ GeV}} \right) \times \left( \frac{0.1}{\alpha_s} \right) \left( 1 - \frac{m_\tilde{g}^2}{m_\tilde{a}^2} \right)^{-3/2} \frac{Y_{\text{eq/TP}}}{g_{*S}(T_{\text{eq/TP}})} \frac{g_{*S}(\tau_\tilde{a})^{1/3}}{g_{*S}(0.01 \tau_\tilde{a})^{1/2}} \tag{B5}
\]
where \(t_i = 0.01\tau_\tilde{a}\) is used as suggested above.

Saxions can decay both into inert radiation and into relativistic particles that rapidly thermalize with the respective branching ratios [25] and [26] governed by \(x\).
For \( x \gtrsim 0.1 \), \( \Delta^{\sigma \rightarrow gg} = \mathcal{O}(1) \). Accordingly, after accounting for \( \text{BR}(\sigma \rightarrow gg) \), (B5) can be used to approximate the dilution factor due to entropy release in decays of saxions from thermal processes. Our numerical results – shown e.g. by the dashed and dotted lines in Fig. (b) – are indeed well described by

\[
\Delta^{\sigma \rightarrow gg}_{\text{small}} \approx 1 + 1.03 \times 10^{-2} \left( \frac{100 \text{ GeV}}{m_\sigma} \right)^{1/2} \left( \frac{f_{\text{PQ}}}{10^{10} \text{ GeV}} \right)^{1/2} \frac{\alpha_\sigma}{\sqrt{\alpha_\sigma^2 + 0.5 \pi^2 x^2}} \frac{1}{(10^{-3})} \frac{g_{S}(\tau_\sigma)^{1/3}}{g_{S}(0.01 \tau_\sigma)^{1/3}},
\]

(B6)

where \( t_i \approx 0.01 \tau_\sigma \). For \( x = 0.02 \), \( f_{\text{PQ}} = 10^{12} \text{ GeV} \), and \( T_R \gtrsim 5 \times 10^{10} \text{ GeV} \) in the axion CDM scenario, \( \Delta^{\sigma \rightarrow gg} \gtrsim 10 \) is possible and there best described by using (B5). Setting \( x = 0 \) in \( \Gamma_\sigma \) and \( \text{BR}(\sigma \rightarrow gg) \), we then obtain

\[
\Delta^{\sigma \rightarrow gg}_{\text{large}} \approx 19 \left( \frac{100 \text{ GeV}}{m_\sigma} \right)^{1/2} \left( \frac{f_{\text{PQ}}}{10^{10} \text{ GeV}} \right)^{1/2} \frac{0.1}{\alpha_\sigma} \frac{1}{(10^{-3})} \left[ \frac{g_{S}(\tau_\sigma)^{1/4}}{10.75} \right].
\]

(B7)

which deviates by at most 20% from our numerical results for \( x \leq 0.02 \) – shown e.g. by the dot-dashed lines in Fig. (b) – in that parameter region with large \( \Delta \). For an approximate treatment of entropy production in saxion decays, see also Ref. [29].

In settings in which two non-relativistic species decay at different times and thereby produce entropy at different times, the total dilution factor \( \Delta \) is given by the product of the individual dilution factors \( \Delta_i \). This occurs, e.g., for \( x < 1 \) in Sect. IV where \( \Delta = \Delta^{\sigma \rightarrow gg} \Delta^{\sigma \rightarrow gg} \). There the product of (B5) and (B6) describes approximately the dashed and dotted curves in Fig. (b).

To arrive at approximate expressions for \( \Delta_{\text{eff}}^{\sigma \rightarrow aa} \), we work with the sudden decay approximation as, e.g., in Refs. [2, 22]. The decays that can lead to extra radiation are thus approximated to proceed exactly when cosmic time is equal to the lifetime of the decaying species. The contribution to \( \Delta_{\text{eff}} \) of axions from decays of saxions from thermal processes is then given by

\[
\Delta_{\text{eff}}^{\sigma \rightarrow aa}(T) = \frac{120}{7 \pi^2 T_\sigma^4} \left[ \frac{g_{S}(T)}{g_{S}(T_\sigma)} \right]^{1/3} \left( \frac{T}{T_\sigma} \right)^{4/3} \frac{\text{BR}(\sigma \rightarrow aa) \rho^{eq}/TP(T_\sigma)}{\Delta} \text{[BR]}
\]

(B8)

with a temperature at the decay time \( t = T_\sigma \) of

\[
T_\sigma \approx 10.6 \text{ MeV} \left( x^2 + \frac{2 \alpha_\sigma}{\pi^2} \right)^{1/2} \left( \frac{m_\sigma}{1 \text{ GeV}} \right)^{3/2} \left( \frac{10^{10} \text{ GeV}}{f_{\text{PQ}}} \right) \left[ \frac{10^{-3}}{g_{S}(T_\sigma)} \right]^{1/4}.
\]

(B9)

By going beyond the sudden decay approximation, one finds that \( \Delta_{\text{eff}}^{\sigma \rightarrow aa} \) is overestimated by about 10%. Accounting for this by multiplying (B8) with 0.87 yields

\[
\Delta_{\text{eff}}^{\sigma \rightarrow aa}(T) \approx \frac{0.82}{\Delta} \left( \frac{1 \text{ GeV}}{m_\sigma} \right)^{1/2} \left( \frac{f_{\text{PQ}}}{10^{10} \text{ GeV}} \right) \times \frac{x^2}{\left[ x^2 + 2(\alpha_\sigma/\pi)^2 \right]^{3/2}} \left( \frac{1}{10^{-3}} \right)
\]

(B10)

This equation together with \( \Delta = \Delta^{\sigma \rightarrow gg} \Delta^{\sigma \rightarrow gg} \), as given by (B5) and (B6) for \( x \gtrsim 0.1 \), can allow for a better understanding of the numerical \( \Delta_{\text{eff}}^{\sigma \rightarrow aa} \) results obtained in Sect. IV.

In Sect. V an additional contribution to \( \Delta_{\text{eff}} \) in the form of axions and axinos from late decays of gravitinos is considered. In the sudden decay approximation,

\[
\Delta_{\text{eff}}^{\tilde{G} \rightarrow \tilde{a} \tilde{a}}(T) = \frac{120}{7 \pi^2 T_\tilde{G}^4} \left[ \frac{T}{T_\tilde{G}} \right]^{4/3} \frac{f_{\text{PQ}} + 2(\alpha_\sigma/\pi)^2 \pi^2 x^2}{g_{S}(T_\tilde{G})/g_{S}(T_\tilde{G})^{1/3}}.
\]

(B11)

with a temperature at the decay time \( t = T_\tilde{G} \) of \( \tilde{G} = 22 \) GeV

\[
T_\tilde{G} = 24 \text{ eV} \left( \frac{m_{\tilde{G}}}{100 \text{ GeV}} \right)^{3/2}.
\]

(B12)

Indeed, the gravitino decay happens always after BBN so that \( g_{S}(T) = g_{S}(T_\tilde{G}) = 3.91 \) for the values of \( m_{\tilde{G}} \) considered in Sect. V. Proceeding as above, we obtain

\[
\Delta_{\text{eff}}^{\tilde{G} \rightarrow \tilde{a} \tilde{a}} \approx \frac{0.42}{\Delta} \left( \frac{100 \text{ GeV}}{m_{\tilde{G}}} \right)^{1/2} \left( \frac{1}{10^{-11}} \right)
\]

(B13)

after multiplying (B11) by a factor of 0.87 again to compensate for overestimation by the sudden decay approximation. Without this correction and for \( \Delta = 1 \), the above estimate agrees with the one given in Ref. [22]. To understand better the numerical results of Sect. IV in which \( \Delta_{\text{eff}} \approx \Delta_{\text{eff}}^{\sigma \rightarrow aa} + \Delta_{\text{eff}}^{\tilde{G} \rightarrow \tilde{a} \tilde{a}} \) (B10) and (B13) can be used together with \( \Delta = \Delta^{\sigma \rightarrow gg} \), as given by (B6) for \( x \gtrsim 0.1 \) or by (B7) for \( x \lesssim 0.02 \).

Appendix C: Description of Changes for \( m_\sigma \neq m_{\tilde{G}} \)

Throughout the main part of this work, we use \( m_{\tilde{G}} = m_\sigma \). In this Appendix we describe the differences with respect to the \( m_{\tilde{G}} = m_\sigma \) case that one faces if those two masses differ. Note that we limit our discussion below to the regime with \( \tau_\sigma \lesssim 1 \text{ s} \). In particular, we do not address additional cosmological constraints [20] appearing

9 A similar factor is found in Appendix A of Ref. [28] in a comparison of the exponential decay behavior with the sudden decay approximation; see also Appendix of Ref. [24].
towards small $m_\sigma$ that imply longer lifetimes and thereby saxion decays during/after BBN.

Towards very large $m_\sigma$, additional decay channels into sparticles may open up such as the decay $\sigma \to \tilde{g}\tilde{g}$, whose width can be derived from (3),

$$\Gamma_{\sigma \to \tilde{g}\tilde{g}} = \frac{\alpha_s^2 m_\sigma m_{\tilde{g}}}{4\pi^3 f_{\tilde{P}Q}^2} \left[ 1 - \left( \frac{2m_\sigma}{m_{\tilde{g}}} \right)^2 \right]. \quad (C1)$$

If this decay occurs after the freeze-out of the LOSP, each of the produced gluinos will lead to one LSP and thereby contribute

$$\Omega_{\text{LSP}}^2 \tilde{g}^2 h^2 = 2m_{\text{LSP}} \text{BR}(\sigma \to \tilde{g}\tilde{g}) Y^{\text{eq}/\text{TP}}(T_{\text{low}}) s(T_0) h^2 / \rho_c, \quad (C2)$$

similarly as discussed for axino decays in Sect. IV. Indeed, a significant excess over (15) in the $\tilde{G}$ CDM case can again be avoided when the saxion decays prior to the LOSP decoupling. Because of the additional decay channels into axions and gluinos, this will be easier to realize than for the axino decay in Sect. IV. In the axion CDM case with the very light axino LSP, (C2) can be much smaller because of a much smaller $m_{\text{LSP}} = m_a \lesssim 37$ eV. Here however such decays could lead to additional contributions to $\Delta N_{\text{eff}}$ in the form of relativistic axions. If the saxion decays prior to the LOSP decoupling, again no additional constraints will be expected. A more detailed discussion of effects related to the $\sigma \to \tilde{g}\tilde{g}$ decay is left for future work. In the following description of changes such effects are assumed to be negligible.

In the $\tilde{G}$ CDM case considered in Sect. IV increasing (decreasing) $m_\sigma$ relative to a fixed value of $m_{\tilde{G}}$ as indicated on the horizontal axes moves the $\Delta N_{\text{eff}}$ contours to the left (right) in Figs. 2(a)–(c) and 3. For the $x = 1$ case presented in Figs. 2(a)–(c), there is practically no change of the $\Omega_{\tilde{G}} h^2$ contour since the entropy released in saxion decays is negligible. For $x < 1$, $\Delta$ depends on $m_\sigma$ as can be seen in Fig. 1(b). For a fixed $m_{\tilde{G}}$, increasing (decreasing) $m_\sigma$ reduces (enhances) the dilution due to saxion decay and thus results in more (less) restrictive upper limits on $T_R$ imposed by $\Omega_{\tilde{G}} h^2 < 0.124$, i.e., the respective contours will move downwards (upwards) and show a less (more) pronounced dip.

In the $\sigma$ CDM case considered in Sect. IV increasing (decreasing) $m_\sigma$ relative to a fixed $m_{\tilde{G}}$ value as indicated on the horizontal axes moves the dotted $\Delta N_{\text{eff}} \sigma \rightarrow a a$ contours in Fig. 2(a) to the left (right). The solid contours depicting the sum of both extra radiation components change accordingly in Figs. 5(a) and 6(a), while there is no effect on the dashed $\Delta N_{\text{eff}} \sigma \rightarrow a a$ contours for $x = 1$. For $x < 1$, the entropy release from saxion decays can become sizable. For a fixed $m_{\tilde{G}}$ value, increasing (decreasing) $m_\sigma$ gives a smaller (larger) $\Delta$ and affects the $\Delta N_{\text{eff}} \sigma \rightarrow a a$ contours in a way that is qualitatively comparable to their change observed for increasing (decreasing) $x$. In turn, the dashed, dotted, and dash-dotted curves in Fig. 3(b) that present the sum of both $\Delta N_{\text{eff}}$ components change accordingly as well as all $\Delta N_{\text{eff}}$ contours shown in Figs. 4(b)–(d).

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