Topological quantum matter with ultracold gases in optical lattices

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Since the discovery of topological insulators, many topological phases have been predicted and realized in a range of different systems, providing both fascinating physics and exciting opportunities for devices. And although new materials are being developed and explored all the time, the prospects for probing exotic topological phases would be greatly enhanced if they could be realized in systems that were easily tuned. The flexibility offered by ultracold atoms could provide such a platform. Here, we review the tools available for creating topological states using ultracold atoms in optical lattices, give an overview of the theoretical and experimental advances and provide an outlook towards realizing strongly correlated topological phases.

The cold-atom toolbox
We start with a very brief overview of the toolbox that has been developed to create and probe synthetic matter with cold atoms in optical lattices.

The main interface to control atoms through light–matter interaction is the optical dipole potential $V(x) = \alpha |E(x)|^2$, where $E(x)$ denotes the electric field associated with the lasers, $\alpha$ is the polarizability, which typically depends on the laser frequency and $x$ denotes the position vector. By interfering several beams, rich spatial patterns of light forming adjustable potential landscapes for atoms can be created. Of great interest are those configurations leading to space-periodic traps, called optical lattices, which can form arbitrary geometries (square, honeycomb, . . .) of various dimensions. These synthetic lattices can be made static, for example, using standing waves $E(x) \sim \cos(qx)$. In the case of a deep lattice, the dynamics of the atoms is well captured by the Hubbard Hamiltonian, a familiar model for a single electronic band,

$$\hat{H} = -J \sum_{\langle m,n \rangle} \hat{a}^\dagger_m \hat{a}_n + U_{\text{int}}$$

where $\hat{a}^\dagger_m (\hat{a}_m)$ creates an atom at lattice site $m$ (annihilates an atom at lattice site $n$), $J$ denotes the constant tunnelling matrix element between nearest-neighbouring sites $(m,n)$, and the interaction term $U_{\text{int}}$ describes on-site (contact) interactions. Such lattice Hamiltonians can be equally realized for Fermi and Bose gases.

Optical lattices can also be made dynamic. For instance, ‘moving’ lattices can be obtained by interfering two laser beams with slightly different frequencies. Optical lattices can be rotated, and even shaken, for example, using piezo-electric actuators. Time-dependent optical lattices constitute a powerful tool for engineering atomic gases with topological properties; this ‘Floquet-engineering’ approach will be presented below. Another important tool is the coherent coupling between different atomic internal states, using laser fields whose frequencies are resonant with specific atomic transitions. Driving controlled transitions between internal states can be exploited both to manipulate single atoms, as well as to generate artificial gauge potentials and so-called synthetic dimensions, as will be explained below (see refs 12,13 for detailed discussions).

Cold atoms can be visualized by imaging the atomic cloud in situ. Momentum distributions and band populations can also

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be obtained through time-of-flight imaging and band-mapping, and the dispersion relation of excitations can be extracted using light scattering. Recently, correlations and entanglement entropy have also been evaluated using single-site-resolved images.

**Probing geometry and topology with cold atoms**

We now briefly review the tools that have been developed to reveal geometrical and topological properties of band structures in the cold-atom context. We focus our discussion on 2D atomic systems exhibiting (integer) QH physics, highlighting those experimental probes that are specific to cold gases, and then mention extensions to other topological atomic states.

The geometrical structure of Bloch bands is captured by the Berry curvature,

\[ \Omega = i \left( \langle \partial_y u_i | \partial_x u_i \rangle - \langle \partial_x u_i | \partial_y u_i \rangle \right) \]  

where \( |u_i(k)\rangle \) is the Bloch state of the band \( \lambda \) at quasi-momentum \( k \). In cold gases, various physical signatures of the Berry curvature can be probed: for instance, the anomalous (transverse) velocity of a wavepacket under the action of a force, or the Aharonov–Bohm phase acquired by a wavepacket performing a loop in \( k \)-space (reflecting that the Berry curvature acts as a ‘magnetic field’ in \( k \)-space). For two-band systems, the Berry curvature can be simply expressed in terms of the momentum distribution, and hence, it can be directly reconstructed from time-of-flight images. These different probing strategies have been successfully implemented in recent experiments (see refs 18, 19, 42, 45).

Topological invariants are global properties and can often be expressed as integrals over local geometric quantities. For example, the genus (that is, number of handles) of a closed two-dimensional (2D) surface is determined by integrating its Gaussian curvature. Similarly, the topology of a Bloch band in 2D can be characterized by the integrated Berry curvature over the entire Brillouin zone (BZ):

\[ C = \frac{1}{2\pi} \int_{BZ} \Omega \, dk \in \mathbb{Z} \]  

This so-called Chern number of the band is at the heart of the integer QH effect in electronic systems: the topologically quantized Hall conductance associated with a completely filled band is given by \( \sigma_y = e^2/h \) (where \( e^2/h \) is the quantum of conductance) if \( C \) is an integer.

When atoms are uniformly loaded into a Bloch band with Chern number \( C \), and subsequently subjected to a force of strength \( F \), the centre-of-mass velocity along the transverse direction is given by \( v_T = C A_{\text{cell}} F / h \), where \( A_{\text{cell}} \) denotes the unit-cell area, and \( h \) is Planck’s constant (see Fig. 1a). This quantized centre-of-mass (COM) response is an unambiguous manifestation of \( C \) in the bulk, as opposed to edge currents detected in Hall bars. This drift can be directly observed in situ (see Fig. 1b). As discussed in ref. 49, such COM observables could even detect quantized electromagnetic responses not captured by conductivity measurements. Furthermore, the Chern number \( C \) can also be observed through the aforementioned Berry-curvature-reconstruction schemes, or by measuring the spin polarization of an atomic cloud at highly symmetric points of the Brillouin zone. Finally, we point out that a many-body Chern number, as defined in interacting systems, may be probed by extending the interferometry scheme of ref. 42 to mobile impurities bound to quasiparticles.

The bulk–edge correspondence guarantees the existence of chiral edge modes whenever a band structure displays Bloch bands with non-zero Chern numbers. In cold gases, the high-resolution imaging techniques offer the possibility of directly loading atoms into these edge states and visualizing their time-evolution in real space, see refs 56–58 for the experimental detection of edge motion. Moreover, a complete reconstruction of the edge-modes could be performed using Bragg spectroscopy. We note that these techniques may also be adapted to investigate the edge modes of strongly correlated states.

**Artificial gauge fields for cold atoms in optical lattices**

In solid-state systems, prominent mechanisms inducing topological Bloch bands include spin–orbit coupling and externally applied magnetic fields. Formally, these gauge fields affect the tunnelling of electrons within the crystal through the Peierls substitution (hereafter we set \( \hbar = c = 1 \) unless otherwise stated). For instance, an external magnetic field \( \mathbf{B} = \nabla \times \mathbf{A} \) modifies the Hubbard Hamiltonian in equation (1) through the Peierls substitution

\[ -J \sum_{\langle \alpha \beta \rangle} \hat{a}^\dagger_{\alpha} \hat{a}_{\beta} X_{\alpha} \rightarrow -J \sum_{\langle \alpha \beta \rangle} \hat{a}^\dagger_{\alpha} \left( -e^{i e A_{\alpha} x} + 1 \right) \hat{a}_{\beta} \]  

where \( J \) denotes the tunnelling matrix element between lattice sites \( x_{\alpha} \) and \( x_{\beta} \) in the absence of a field, and \( \mathbf{A} \) is the electromagnetic gauge potential, see Fig. 2a. The Peierls phase factor in equation (4) directly determines the Aharonov–Bohm phase acquired by a particle encircling a unit cell of the lattice (Fig. 2b). The Peierls substitution in equation (4) can readily be generalized to the case of non-Abelian gauge fields, for example, spin–orbit coupling, by means of a path–ordered integral.

In gases of neutral atoms, analogues of the gauge field \( \mathbf{A} \) appearing in (4) can be generated artificially, using the atom–light interaction. In the following paragraphs, we detail various ways of engineering such artificial gauge fields that allow the realization of topological matter with cold atoms.
in cold gases. It consists in interpreting a set of internal states of an atom, for example, Zeeman sublevels of a hyperfine state\(^ {36,55}\), as fictitious lattice sites; this defines an extra ‘spatial’ dimension, coined synthetic dimension (see Fig. 3a). In this picture, driving transitions between different internal states, for example, using resonant laser fields, corresponds to inducing ‘hopping’ processes along the synthetic dimension. Interestingly, loading atoms into a (real) \(N\)-dimensional optical lattice then potentially allows one to simulate systems of \((N + 1)\) spatial dimensions. Hence, this technique offers a versatile tool to explore quantum effects associated with higher dimensions, such as the 4D QH effect\(^ {41}\).

Specifically, consider the laser coupling between two internal states of an atom, \([1] \leftrightarrow [2]\); the corresponding coupling matrix element is of the form \(\kappa = \Omega \exp[iq \cdot x]\), where \(\Omega\) denotes the coupling strength and \(q\) denotes the wavevector of the coupling field\(^ {12,13}\). In the synthetic-dimension picture, the quantity \(\kappa\) represents the tunnelling matrix element between the fictitious sites ‘1’ and ‘2’ (see Fig. 3a). Interestingly, and similarly to the laser-induced-tunnelling scheme discussed above, the fictitious tunnelling \(\kappa\) contains a complex phase factor, which can then be exploited to simulate artificial gauge fields in a simple and practical manner\(^ {64,65}\) (see Fig. 3b).

Synthetic dimensions were recently investigated in two independent experiments\(^ {56,57}\). Atoms were loaded into a 1D optical lattice, while a laser-coupling scheme was added to drive coherent transitions between three internal atomic states: this set-up effectively reproduced a three-leg ladder, in the synthetic-dimension picture (see Fig. 3b). By adjusting the wavevector of the coupling field \(q\), the experimentalists generated an artificial flux threading the ladder. The corresponding band structure exhibits chiral edge modes\(^ {56}\), reminiscent of the edge states in the QH effect\(^ {51}\). These edge modes are characterized by semi-classical skipping orbits at the edges of the synthetic ladder, which can be directly imaged through state-resolved images of the cloud, as was experimentally demonstrated in refs 56,57 (see Fig. 3c).

To engineer topological band structures using the synthetic-dimension approach, atomic species with many addressable internal states (for example, Yb, Sr) are required. This is because a proper bulk region within the artificial dimension is crucial to limit undesired finite-size effects (for example, the overlap of chiral edge modes associated with opposite edges). An additional coupling to connect the extremal internal states may be used to apply periodic boundary conditions in the synthetic dimension\(^ {56}\). Finally, we point out two qualitative differences between ordinary crystalline systems and systems involving a synthetic dimension. First, the interactions are generically infinite-ranged (that is, spin-dependent interactions are negligible). While the resulting extended-Hubbard model displays interesting phases\(^ {56}\), it is still debated whether such systems could host fractional QH states\(^ {57}\).

### Shaking atoms into topological matter

Complementary to the schemes discussed so far, which rely on the possibility of addressing the internal structure of atoms with light, there exists an even more general strategy to engineer topological band structures in quantum systems, which is commonly called Floquet engineering\(^ {28,30}\). This approach, which is based on applying time-periodic modulations to quantum systems, can be summarized as follows. Consider a static system described by a Hamiltonian \(\mathcal{H}_0\), that is driven by a time-periodic modulation \(\mathcal{V}(t)\), whose period \(T = 2\pi/\omega\) is assumed to be much smaller compared to any characteristic timescale in the problem. In this high-frequency regime, the dynamics is generally well captured by an effective Hamiltonian \(\mathcal{H}_{\text{eff}}\), which stems from a rich interplay between the static and time-dependent parts of the total Hamiltonian \(\hat{H}_0 + \mathcal{V}(t)\) (see refs 28,30). In this way, a target Hamiltonian (for
The occurrence of quasiparticles with non-Abelian statistics, called non-Abelian anyons, has first been predicted in certain fractional QH states\(^{87}\), and later in time-reversal-breaking superconductors with \(p + ip\) pairing in 2D\(^{88}\), as well as \(p\)-wave pairing in 1D\(^{89}\). The specific anyons occurring in these systems are known as Majorana bound states (MBS), due to their algebraic similarities with the real solutions to the relativistic Majorana equation. More recently, it has become clear that \(p + ip\)-wave and \(p\)-wave superconductors can be induced in conventional (proximity-induced) superconductors due to the combination of spin–orbit coupling and Zeeman splitting\(^{90}\). With the recent advances in the experimental realization of synthetic spin–orbit coupling in ultracold quantum gases\(^{91,92}\), all the individual ingredients for synthetic topological superfluids in fermionic quantum gases are in place, and several concrete proposals, both for the 2D \(p + ip\) superfluids\(^{93-95}\) and proximity-induced 1D \(p\)-wave superfluids\(^{96-99}\), have been put forward. Once experimentally realized, the high degree of experimental control over these systems enables new approaches for the direct observation of MBS via braiding\(^{100}\).
Dissipative preparation of topological states

So far we have discussed several tools to engineer Hamiltonians, the ground states of which have topologically non-trivial properties. A complementary approach, in which desired many-body states are directly targeted, is provided by the concept of dissipative state preparation\(^{101,102}\). Intuitively, dissipation is expected to increase the entropy of a system, thus having a detrimental effect on ordering phenomena. However, the flexibility to engineer the interaction of cold-atom systems with their environment allows one to think about dissipative processes as a resource to control quantum many-body systems in a non-equilibrium fashion. That way, dissipation can be harnessed to prepare interesting states of quantum matter as steady states of a master equation governing the open quantum system dynamics. For a weak coupling to a Markovian bath, which in many cases represents a good approximation for atoms coupled to a continuum of radiation modes, the master equation is of Lindblad form and reads as\(^{103}\)

\[
\frac{d\hat{\rho}}{dt} = i [\hat{\rho}, \hat{H}] + \sum_j \left( \hat{L}_j \hat{\rho} \hat{L}_j^\dagger - \frac{1}{2} \{ \hat{L}_j^\dagger \hat{L}_j, \hat{\rho} \} \right)
\]

where \(\hat{\rho}\) denotes the reduced density matrix of the system, and where the incoherently acting Lindblad operators \(\hat{L}_j\) (also called jump operators) account for the system–bath coupling; the dissipative channels are labelled by \(j\), and are often directly related to the degrees of freedom of a lattice system\(^{104,105}\). Steady states \(\hat{\rho}_s\) are defined by \(d\hat{\rho}/dt = 0\). In this dissipative context, the counterpart to an energy gap protecting a Hamiltonian ground state is provided by a damping gap\(^ {104,105}\), defined as the smallest rate at which deviations from \(\hat{\rho}_s\) are damped out. Early works devoted to dissipative state preparation\(^104\) mainly focused on purely dissipative dynamics by assuming the system Hamiltonian \(\hat{H}\) to be zero. Then, pure steady states \(|D\rangle\langle D|\), also referred to as dark states, are simply characterized by \(\hat{L}_j|D\rangle = 0\) for all \(j\). Hence, if the desired topological state can be represented as the (unique) ground state \(|G\rangle\) of the so-called parent Hamiltonian \(\hat{H}_p = \sum_j A_j \hat{L}_j\), realizing \(\hat{L}_j = A_j\), as jump operators, will make \(|G\rangle\) (the unique) dark state. In the context of topological phases, this approach has been pioneered in ref. 104, where a scheme for the dissipative preparation of a 1D topological superconductor\(^{106}\) with a pair of spatially-separated MBS forming a decoherence-free subspace has been proposed.

Remarkably, the modification of the bulk–boundary correspondence in open quantum systems\(^{105}\) can lead to phenomena that have no direct analogue in Hamiltonian systems\(^{106}\). For example, unpaired MBS can, in systems with a topologically trivial bulk, form decoherence-free subspaces, the dissipative analogue of degenerate ground states\(^{106}\). The concept of topology by dissipation has formally been extended to higher spatial dimensions and various symmetry classes in ref. 105 for Gaussian, fermionic models. However, there is a fundamental competition between topology and locality\(^ {107,108}\), representing a major caveat for the dissipative preparation of chiral topological phases such as Chern insulators: no exponentially localized set of Lindblad operators \(\hat{L}_j\) can be found that leads to a dark state \(|D\rangle\) with a non-vanishing Chern number. This issue has been addressed in ref. 108, where a generic mechanism to prepare a mixed topological state\(^ {105,108,109}\) corresponding to a Chern insulator at finite temperature has been proposed, based on a local system–bath coupling. In this framework, the topology of the mixed steady state is determined by qualitative features of the system–bath interaction, while going towards a pure steady state, the counterpart of reaching zero temperature in a Hamiltonian system, requires some fine-tuning\(^{108}\).

Towards strongly correlated topological phases

In addition to the possibility of engineering single-particle Hamiltonians, the atomic physics toolbox naturally provides us with the means to flexibly tune complex many-body interactions\(^ {6,7,10}\). The paramount goal of such quantum simulators is the physical realization and control of quantum many-body systems that cannot be efficiently simulated on a classical computer. In the context of topological states of matter, strongly correlated phases such as fractional quantum Hall (FQH) states\(^6\) and spin liquids\(^7\) represent intriguing candidates. Their experimental realization in synthetic material systems could provide new physical insights that are hard to access, both in conventional materials and in numerical simulations of small-size model systems. A primary example along these lines is offered by the possibility to directly observe characteristic entanglement signatures in ultracold atomic gases\(^ {111}\), as has very recently been experimentally demonstrated with quantum gas microscope techniques\(^ {112}\).

FQH physics has been studied for many years, both theoretically and experimentally, in strongly correlated 2D electron gases subjected to a strong perpendicular magnetic field\(^1\). More recently, intense interest has been aroused by the possibility of realizing FQH states in lattice systems: the fractional Chern insulators (see ref. 4 for a recent review). The basic ingredients for lattice FQH states are almost flat (dispersionless) energetically isolated bands with a non-vanishing Chern number that are partially filled with interacting particles. Most interestingly, bands with higher Chern number \(C > 1\) can give rise to FQH states that have no natural analogue in conventional FQH systems based on continuous Landau levels. Recently, numerous proposals\(^ {105,112–117}\) to realize FQH states in Chern bands with ultracold atoms, for example, using optical flux lattices\(^ {117}\) and dipolar spin systems\(^ {113}\), have been reported. The tunability of interactions in such systems opens up the possibility to realize various new FQH states, as indicated by numerical simulations (see for example, ref. 118).

Very recently, ref. 119 reported on the implementation of a minimal toric-code Hamiltonian with cold atoms; this set-up exhibits fractional (anyonic) statistics, an unambiguous signature of topological phases\(^ {120}\).

Towards the implementation of spin liquids, tunable dipole–dipole interactions in Rydberg atoms have recently been employed to develop a flexible toolbox for the synthetic realization of frustrated quantum magnetism\(^ {121,122}\). In particular, an experimentally feasible scheme for the realization of quantum spin ice has been reported\(^ {123}\).

Remarkable progress has been made regarding the quantum engineering of many-body Hamiltonians, which could potentially lead to strongly correlated topological phases\(^ {6,7,11–13}\). However, the preparation of states with sufficiently low temperature or, more generally, states with sufficiently low entropy, is still a major challenge in view of making exotic features (for example, fractionalized excitations and topological entanglement entropies) experimentally accessible. The notion of dissipative state preparation via system–bath engineering discussed above represents a possible direction to overcome these issues.

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