I. INTRODUCTION

The standard six-parameter Λ cold dark matter (ΛCDM) cosmological model describes the temperature fluctuations in the cosmic microwave background (CMB) radiation spectacularly well, as demonstrated by the WMAP satellite [1], the Atacama Cosmology Telescope [2], the South Pole Telescope [3], and, especially, the Planck satellite [4]. Central assumptions in the ΛCDM model are that the fluctuations are Gaussian and statistically homogeneous and isotropic. Despite the success of the standard model, several “anomalies” have been noticed in the CMB, which apparently violate these assumptions (for reviews, see Refs. [5–8]). The statistical significance of these anomalies is not very high, and is weakened substantially with a posteriori choices. Nevertheless, a number of models have been proposed that produce a scale-dependent asymmetry. We confront several such models for a physical, position-space modulation with CMB temperature observations. We find that, while some models that maintain the standard isotropic power spectrum are allowed, others, such as those with modulated tensor or uncorrelated isocurvature modes, can be ruled out on the basis of the overproduction of isotropic power. This remains the case even when an extra isocurvature mode fully anti-correlated with the adiabatic perturbations is added to suppress power on large scales.

Measurements of the cosmic microwave background (CMB) temperature anisotropies have revealed a dipolar asymmetry in power at the largest scales, in apparent contradiction with the statistical isotropy of standard cosmological models. The significance of the effect is not very high, and is dependent on a posteriori choices. Nevertheless, a number of models have been proposed that produce a scale-dependent asymmetry. We confront several such models for a physical, position-space modulation with CMB temperature observations. We find that, while some models that maintain the standard isotropic power spectrum are allowed, others, such as those with modulated tensor or uncorrelated isocurvature modes, can be ruled out on the basis of the overproduction of isotropic power. This remains the case even when an extra isocurvature mode fully anti-correlated with the adiabatic perturbations is added to suppress power on large scales.

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modes were studied in [13], and, in the context of a particular inflationary model [22], in Ref. [23]. In the process we also provide constraints on unmodulated tilted tensor and isocurvature modes using the latest data.

II. FORMALISM

Our goal is to construct physical, position-space models for a temperature dipolar asymmetry, which is confined mostly to large scales. We apply the formalism developed in Refs. [18, 19], which captures scale dependence by employing two fluctuation components. The first, $\tilde{Q}^{lo}(x)$, is restricted mainly to large scales (low $k$) and is maximally linearly spatially modulated:

$$\tilde{Q}^{lo}(x) = Q^{lo}(x) \left(1 + \frac{x \cdot d}{r_{LS}} \right),$$

where $Q^{lo}(x)$ is statistically isotropic with power spectrum $P^{lo}(k)$, $d$ is the direction of modulation, and $r_{LS}$ is the comoving distance to last scattering. The total modulation amplitude will be set by a multiplicative factor inside $P^{lo}(k)$ [36]. The second component, $Q^{hi}(x)$, is statistically isotropic with spectrum $P^{hi}(k)$. The two fields are taken to be uncorrelated, i.e., $\langle Q^{lo}(k)Q^{hi}(k') \rangle = 0$. We attempt to be agnostic as to the origin of the modulation; the isotropic $Q^{hi}$ component is adiabatic, while for the modulated component, $Q^{lo}$, we consider adiabatic, CDM isocurvature, and tensor fluctuations.

The total temperature anisotropies due to these two fields will be to a very good approximation [18]

$$\delta T(n) = \delta T^{lo}(n) + \delta T^{hi}(n),$$

where $\delta T^{lo}$, with power spectrum $C^{lo}_\ell$ (called the “asymmetry spectrum”), is produced by $P^{lo}(k)$, while $\delta T^{hi}$, with spectrum $C^{hi}_\ell$, is produced by $P^{hi}(k)$. These anisotropies lead to the lowest-order spherical harmonic multipole covariance [7, 18, 20, 24]

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_{\ell'} \delta_{\ell \ell'} \delta_{m m'} + \frac{\delta C_{\ell'}^{\text{int}}}{2} \sum_{M} d_{M} \xi_{\ell m \ell' m'}^{M},$$

where $\delta C_{\ell'} \equiv 2(C^{lo}_{\ell'} + C^{hi}_{\ell'})$ and $d_{M}$ is the spherical harmonic decomposition of $n \cdot d$. The coefficients $\xi_{\ell m \ell' m'}^{M}$ couple modes $\ell$ to $\ell' + 1$:

$$\xi_{\ell m \ell' m'}^{M} \equiv \sqrt{\frac{4\pi}{3}} \int Y_{\ell' m'}(\hat{n}) Y_{1 M}(\hat{n}) Y_{\ell m}(\hat{n}) d\Omega.$$  

Crucially, the modulated component will also contribute to the total isotropic power, via

$$C_{\ell} = C^{lo}_{\ell} + C^{hi}_{\ell}.$$  

Therefore a model that produces sufficient asymmetry to fit the temperature data may overproduce isotropic power at large scales and hence be inconsistent with experiments such as Planck.

III. MODELS

We employ the same models as described in Ref. [19] to describe a large-scale modulation. First, we consider the adiabatic tanh model, with $k$-space asymmetry spectrum

$$P^{lo}(k) = \frac{A_{\tan h}}{2} P^{\Lambda\text{CDM}}(k) \left[1 - \tanh \left(\frac{\ln k - \ln k_c}{\Delta \ln k} \right) \right],$$

where

$$P^{\Lambda\text{CDM}}(k) = A_s \left(\frac{k}{k_0} \right)^{n_s-1},$$

describes the usual $\Lambda$CDM power-law primordial comoving curvature perturbation spectrum. The parameters $\Delta \ln k$ and $k_c$ describe the width and position of a small-scale cutoff and $A_{\tan h} \leq 1$ is the amplitude of the modulation.

Next we consider an adiabatic power-law model (abbreviated “ad.-PL”):

$$P^{lo}(k) = A_{PL} P^{\Lambda\text{CDM}}(k) \left(\frac{k}{k_{0}^{\text{lo}}} \right)^{n_{s}^{lo}-1},$$

(5)

Next we consider a single-component adiabatic model with a linear gradient in the tilt, $n_s$, of the primordial power spectrum (“$n_s$-grad” for short). In this case we can directly write the asymmetry spectrum as [18, 19]

$$C^{lo}_{\ell} = -\frac{\Delta n_{s}}{2} \frac{dC^{\Lambda\text{CDM}}_{\ell}}{dn_{s}}.$$  

Here we have used a linear approximation for the effect of the gradient, which will be well justified by our results. The modulation amplitude is specified by the increment in tilt, $\Delta n_{s}$, from modulation equator to pole. Note that this modulation will depend implicitly on the pivot scale for $A_s$.

Finally we consider three models that naturally produce contributions on large scales. The first is a modulation of the standard $\Lambda$CDM integrated Sachs-Wolfe (ISW) contribution with amplitude $A_{\text{ISW}} \leq 1$ [37]. This phenomenological model automatically satisfies isotropic CMB constraints and $C^{lo}_{\ell}$ is simply the contribution of the ISW effect to the total power $C_{\ell}$. The second is a modulated CDM density isocurvature component, via

$$P^{lo}(k) = \frac{\alpha_{s}}{1 - \alpha_{s}} P^{\Lambda\text{CDM}}(k_s) \left(\frac{k}{k_s} \right)^{n_{s}^{lo}-1},$$

and the third is a modulated tensor component,

$$P^{lo}(k) = \gamma_{s} P^{\Lambda\text{CDM}}(k_s) \left(\frac{k}{k_s} \right)^{n_{s}}.$$  

(12)
In these latter two cases the models are described by two parameters, a primordial power ratio \((\alpha_{k_i}, r_{k_i})\), evaluated at scale \(k_i = 0.002\) Mpc\(^{-1}\) and a tilt \((n_\tau, n_s)\). For both isocurvature and tensor models we fix \(\mathcal{P}^{ni}(k) = \mathcal{P}^{\Lambda\text{CDM}}(k)\), so that the additional isotropic power from the modulated component will further constrain these models. For the tensor model we also consider an unmodulated isocurvature component that is fully (anti-)correlated with the adiabatic scalars. Anti-correlated isocurvature modes would decrease power on large scales, potentially allowing for a larger contribution of modulated tensors. This inclusion adds one extra parameter, which is simply the amplitude of perturbations for the new mode.

**IV. MODULATION ESTIMATOR**

For a full-sky, noise-free measurement of the temperature multipoles, we can write down an estimator for the modulation amplitude \(\Delta X_M \equiv A d_M\) as [7, 18, 20]

\[
\Delta X_M = \frac{1}{4A} \sigma_X^2 \sum_{\ell m' m''} \frac{\delta C_{\ell m' m''}}{C_{\ell m' m''}} \delta a_{\ell m' m''} \delta a_{\ell m' m'',},
\]

(13)

where \(A = A_{\text{tanh}}, A_{\text{PL}}, A_{\text{ISW}}, \alpha_{k_i}/(1-\alpha_{k_i}), \) or \(r_{k_i}\), depending on the model, and where the cosmic variance of the estimator is given by

\[
\sigma_X^2 = 12 A^2 \left( \sum_\ell (\ell + 1) \frac{\delta C_{\ell+1}^2}{C_{\ell} C_{\ell+1}} \right)^{-1}.
\]

(14)

The presence of noise and incomplete sky coverage modifies the above relations. We use a C-inverse filter approach that accounts for noise, and, optimally, for the mask (as described in Refs. [25, 26]). Masking and residuals in the data will induce a mean-field value for \(\Delta X_M\) that can be estimated with simulations. Further details of the full estimator we use can be found in Appendix C of Ref. [7].

For fixed modulation parameters the maximum likelihood is

\[
\ln \mathcal{L} = \sum_M \frac{\Delta X_M^2}{2\sigma_X^2}.
\]

(15)

We can then build the rest of the likelihood by sampling on a grid of values for the \(k\)-space parameters (see Ref. [18]). For the tensor and isocurvature models we assign a uniform prior on \(A\), in order to obtain consistency with the isotropic likelihood results. For all other models we use a prior uniform in the individual \(\Delta X_M\).

**V. RESULTS**

Our dipole asymmetry constraints come from Planck TT data using the SMICA solution [27]. The best-fit asymmetry spectra for all of our models are illustrated in Fig. 1, where we see the expected large-scale character of the asymmetry. The corresponding full posteriors for \(\alpha_{0.002}\) and \(r_{0.002}\) and their tilts are shown in Fig. 2 (orange contours), where we can see that large values of \(\alpha_{0.002}\) or \(r_{0.002}\) are needed to explain the asymmetry. (Recall that the power ratios \(\alpha_{0.002}\) and \(r_{0.002}\) also fix the modulation amplitude for the case of maximal modulation in Eq. (1)).

![Fig. 1: ΛCDM temperature spectrum compared to the best-fit asymmetry spectra, \(C^o_\ell\), for the various models. The best fits correspond roughly to a 5–10% asymmetry for \(\ell \lesssim 100\), as expected, with the exception of the ISW modulation, whose maximum amplitude (and shape) is fixed by ΛCDM.](image)

![Fig. 2: Isocurvature and tensor modulation, the joint constraints are also fix.](image)

**TABLE I: Data sets used for the isotropic constraints. BKP refers to the BICEP2/Keck Array-Planck joint analysis [29].**

| Model       | Data set                                      |
|-------------|-----------------------------------------------|
| isocurvature| Planck TT,TE,EE+lowP+lensing+BKP              |
| tensors     | Planck TT,TE,EE+lowP+lensing                 |

For the isocurvature and tensor models we can also obtain constraints from the isotropic power spectra described in Table I; we will refer to these as *isotropic* constraints. These were obtained with a version of CosmoMC [28] modified to accomodate uncorrelated isocurvature modes. For these models Fig. 2 also shows the isotropic posteriors for \(\alpha_{0.002}\) and \(r_{0.002}\) and their tilts (blue contours), as well as the joint constraints, with the assumption that the isotropic and asymmetry likelihoods are independent (recall that they arise from diagonal and off-diagonal elements of the multipole covariance, respectively). Fig. 2 shows that, for both isocurvature and tensor modulation, the joint constraints are inconsistent with the level of modulation preferred by the asymmetry data. In other words, the addition of the independent isotropy data has substantially reduced the “signal” seen in the asymmetry data. Note that in Fig. 2 we have assumed that the isocurvature and tensor contributions are maximally modulated, via Eq. (1). This allows us to directly compare...
In order to express the above graphical results quantitatively, and determine which models are viable for explaining the original asymmetry signal, we will consider two quantities for each model. The first is the probability, \( P_{>3\sigma} \), that the data allow a modulation amplitude \( A \) that is at least 3 times larger than the cosmic variance \( \sigma_X \). Note that the choice of the value 3 is arbitrary; however, if \( P_{>3\sigma} \) is small then the model cannot source significant modulation and can be ruled out, even if \( P_{>3\sigma} \) being large is an insufficient condition to prefer a modulation model over ΛCDM. The second quantity we use is the maximum-likelihood amplitude of modulation compared to the cosmic-variance value, \( A/\sigma_X \). For both quantities \( \sigma_X \) is calculated for asymmetry only [via Eq. (14)].

We present these quantities for the various model and data combinations in Table II. For the asymmetry data, both quantities are large (except for the ISW model), which simply tells us that the models can produce the considerable asymmetry present in the data. However, in all cases the values drop substantially when adding the isotropic data. This implies that even maximally modulated tensor or isocurvature modes cannot source the large asymmetry signal (or can, but with very small probability) due to their respective isotropic constraints. If we attempt to hide the isotropic tensor temperature power by including an anti-correlated isocurvature mode the conclusions remain the same (see the row marked \( n_t = 0^\circ \) in Table II). This is due to the different shapes of the tensor and anti-correlated isocurvature power spectra, and not, for instance, to the nondetection of primordial \( B \)-modes in the BICEP2/Keck Array-Planck data. Therefore we expect that in general, a modulation model for which (like the tensor and isocurvature models) isotropic power is added will be unable to explain the dipolar asymmetry signal. The tanh, ad.-PL, and \( n_r \)-grad models are of course unaffected by the isotropic constraint and are thus still viable modulation models as far as CMB temperature is concerned. For the ISW model both \( P_{>3\sigma} \) and \( A/\sigma_X \) are small: even for maximal modulation the standard ΛCDM ISW contribution cannot explain the observed asymmetry. Note that, via Eq. (15), the ratio \( A/\sigma_X \) is essentially the best-fit \( \chi \) value, which shows that the tanh model (which has the most free parameters) gives the best fit.

For our best-fit parameters, the \( n_r \)-grad model induces a modulation amplitude of roughly 1.6% at \( k = 1 \text{ Mpc}^{-1} \). On such small scales this model should be vulnerable to constraints from large-scale structure surveys [30–34]. Indeed, this modulation amplitude is close to (or in excess of) the 95% upper limit based on quasar data in [35], and so a rigorous joint analysis may already rule this model out.

In order to determine quantitatively the level of modulation allowed by the full data we look at constraints on the \( r \) and \( \alpha \) parameters for the isocurvature and tensor models (where we are able to use power spectra to provide tighter constraints). In Table III we show the 95% CLs (or upper limits where relevant) for \( r_{0.002} \) and

![Graphical results](attachment:graph_results.png)

**FIG. 2:** Posterior for \( \alpha_{0.002} \) or \( r_{0.002} \) and tilt of the isocurvature (top panel) and tensor (bottom) models. Contours enclose 68% and 95% of the posteriors. We have conservatively assumed maximal modulation, so that the vertical axes are also a measure of the level of modulation relative to the isotropic ΛCDM spectrum. We can see that the modulation allowed by the asymmetry constraints is reduced substantially when adding the isotropic constraints.

the asymmetry and isotropic posteriors, but is also a conservative choice, because for less than full modulation the corresponding \( r \) and \( \alpha \) values preferred by the asymmetry constraints would necessarily be larger with larger uncertainties. This would increase the tension we find between asymmetry and isotropic constraints and increase the dominance of the isotropic data in the joint constraints.
Table III: 95% CL (or upper limits) for the parameters $A$ and data combinations. While the general tensor and isocurvature models (where the tilts are free to vary) show no strong detection with the asymmetry constraints alone, we see that the addition of power spectrum data strongly constrains the amount of modulation allowed by the data. For models where the tilt is fixed and not allowed to vary, the modulation signal is more apparent; however, the addition of isotropic constraints removes the signal to a similar degree. Note that the asymmetry constraints in Table III allow much larger values of $r$ than $\alpha$. This is due simply to the fact that identical primordial ratios of tensors and isocurvature-to-adiabatic scalar fluctuations produce much larger isocurvature temperature fluctuations.

VI. DISCUSSION

The models we have examined fall into two general classes. In the first class, the total statistically isotropic temperature power was constrained to match that of standard $\Lambda$CDM. Therefore the degree of modulation could be varied without spoiling the success of $\Lambda$CDM. In the second class, the modulated component contributed extra power to the isotropic spectra. Our main conclusion is that models in this latter class fail to provide sufficient modulation to explain the dipole asymmetry without producing too much large-scale statistically isotropic power. Hence these models, which include modulated tensor and uncorrelated isocurvature, can be ruled out as the source of the large-scale dipolar asymmetry.

Models in the first class, however, can fit the asymmetry while maintaining the success of the $\Lambda$CDM isotropic spectra, and hence some cannot yet be ruled out. One exception is a modulated ISW contribution, which cannot source enough asymmetry to explain the signal in temperature. The scalar tilt gradient model produces substantial modulation on small scales, and so is at risk from survey data. The surviving models are the phenomenological adiabatic modulation models. Of course the contrived nature of such models should mean that $\Lambda$CDM is still preferred: they essentially add parameters to fit features in the data that may simply be random noise. Unfortunately a Bayesian model selection procedure would not provide an unambiguous Bayes factor for these models, since the modulation model evidence is strongly driven by the parameter prior ranges, which are completely undetermined. It will only be possible to confirm or refute these models by comparing their predictions for probes, (such as CMB polarization) which are sensitive to independent fluctuation modes from CMB temperature, with future observations [19].

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[37] This convention differs trivially from that used in [18, 19], in which the parameter $A$ is equivalent to our parameters $A_{\text{tanh}}$ or $A_{\text{PL}}$.
[38] Note that the ISW effect is sourced over a wide range of distances, so it is unlikely that a position-space modulation could result in maximal ISW modulation, i.e., $A_{\text{ISW}} = 1$. Therefore our results will be conservative, in that a realistic ISW modulation would likely produce less asymmetry.