Consistent histories, quantum truth functionals, and hidden variables

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Abstract

A central principle of consistent histories quantum theory, the requirement that quantum descriptions be based upon a single framework (or family), is employed to show that there is no conflict between consistent histories and a no-hidden-variables theorem of Bell, and Kochen and Specker, contrary to a recent claim by Bassi and Ghirardi. The argument makes use of “truth functionals” defined on a Boolean algebra of classical or quantum properties.

1 Introduction

Textbook quantum theory is beset by numerous conceptual difficulties when it comes to providing a physical interpretation of the mathematical formalism of quantum theory. The unsatisfactory nature of the usual treatments results in a variety of paradoxes: Schrödinger’s cat, Einstein-Podolsky-Rosen, and the like [1]. The consistent histories (sometimes called decoherent histories) approach [2, 3, 4] to quantum mechanics disposes of these difficulties by combining probability theory with the standard Hilbert space formalism in a coherent way through the use of histories which satisfy certain consistency conditions.

A central principle of the more recent formulations of consistent history (CH) ideas [5, 6, 7, 8] is the single framework (also known as the single family, single logic, or single set) rule, which plays an essential role in ensuring the logical consistency of quantum theory, resolving various quantum paradoxes, and getting rid of the mysterious superluminal influences which are sometimes thought to be a consequence of the quantum formalism. The purpose of this Letter is to explain how the single framework rule renders the CH approach fully compatible with a well-known result on the impossibility of hidden variables in a Hilbert space of dimension three or greater due to Bell [9], and Kochen and Specker [10]. Following recent work by Bassi and Ghirardi [11], we shall explore the problem using the notion of a truth functional. While this is not absolutely essential—the ideas of ordinary probability theory suffice, when they are properly used—it is convenient for the conceptual issues we

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will be discussing, and assists in explaining why our key conclusions are directly contrary to those of \[11\].

Since we will be concerned with the Hilbert space description of a system at a single time, most of the formal machinery of the CH approach—histories, consistency, and assignment of probabilities—is not needed for the following discussion. This will allow us to concentrate on the most essential point, which is that a quantum Hilbert space differs in crucial respects from a classical phase space, and this mathematical difference must be reflected in a physical interpretation of the theory. The idea of a truth functional, which may be unfamiliar to some readers, is developed in Sec. 2 using a classical phase space, where classical intuition is a reliable guide, and then applied in Sec. 3 to a quantum Hilbert space, where certain classical ideas run into difficulty. It is at this point that results on no-hidden-variables are used, illustrated with a two-spin paradox due to Mermin [12], based on an idea of Peres [13]. The CH approach to truth functionals and the no-hidden-variables theorem is the subject of Sec. 4, while the claims found in [11] are discussed in Sec. 5.

2 Classical Truth Functionals

Consider a classical mechanical system, such as a simple harmonic oscillator, described by a phase space \( \Gamma \), with \( \gamma \) a representative point. Any physical property \( P \) of the system corresponds to some set of points \( \mathcal{P} \) in the phase space for which this property is true, and we shall define the corresponding indicator function \( P(\gamma) \) to be 1 whenever \( \gamma \) is in \( \mathcal{P} \), and 0 otherwise. For example, let \( P \) be the property that the total energy of the oscillator is less than some constant \( E_0 \). Then \( \mathcal{P} \) is the the region inside an ellipse in the \( x, p \) plane (\( x \) the position, \( p \) the momentum), and \( P(\gamma) \) is 1 for \( \gamma \) inside and 0 for \( \gamma \) outside this ellipse. If \( I \) is the function equal to 1 everywhere on the phase space, the indicator of the negation \( \tilde{P} \) of \( P \), “energy greater than or equal to \( E_0 \)” in our example, is the function \( I - P \): it is 0 wherever \( P \) is 1, and 1 wherever \( P \) is 0. Likewise, if \( P \) and \( Q \) are any two properties, the product \( PQ \) of the two indicators is the indicator for the conjunction \( P \land Q \) of \( P \) and \( Q \), the property “\( P \) AND \( Q \)” . Similarly, the disjunction \( P \lor Q \), “\( P \) OR \( Q \)” , corresponds to the indicator \( P + Q - PQ \).

Consider a coarse graining of the phase space into a collection \( \mathcal{D} \) of \( N \) non-overlapping regions or “cells”. Then we can write

\[
I = \sum_{j=1}^{N} D_j, \tag{1}
\]

where \( D_j \) is the indicator corresponding to the \( j \)’th cell. Since the cells do not overlap it follows that

\[
D_j D_k = \delta_{jk} D_j, \tag{2}
\]

consistent with the obvious fact that \( I^2 = I \). The set of \( 2^N \) indicators of the form

\[
P = \sum_{j=1}^{N} \pi_j D_j, \tag{3}
\]

where \( \pi_j \) is either 0 or 1, form a Boolean algebra \( \mathcal{B} \) of properties, in which \( P \land Q \) is \( P \land Q \), and \( P \lor Q \) is \( P \lor Q \), as defined above.
Given such a Boolean algebra $B$, we define a *truth functional* $\theta$ to be a function which assigns to every property $P$ in $B$ the value 1 (true) or 0 (false) in a way which satisfies the following three conditions:

$$
\theta(I) = 1, \quad \theta(I - P) = 1 - \theta(P), \quad \theta(PQ) = \theta(P)\theta(Q).
$$

These correspond to the rather sensible requirements that something is always true, that $P$ is true if and only if its negation $\bar{P} = I - P$ is false, and both $P$ and $Q$ are true if and only if their conjunction $P \land Q$ is true.

One can show that for a given coarse graining $\mathcal{D}$ with Boolean algebra $B$, there is a one-to-one correspondence between truth functionals on $B$ and the elements of $\mathcal{D}$. That is to say, any function $\theta$ taking only the values 0 and 1 and satisfying (4) must be of the form

$$
\theta_k(P) = \begin{cases} 1 & \text{if } PD_k = D_k, \\ 0 & \text{if } PD_k = 0. \end{cases}
$$

for some $k$. In terms familiar from probability theory, one can regard the non-overlapping cells which constitute the coarse graining $\mathcal{D}$ as a *sample space* of mutually exclusive possibilities, one and only one of which actually occurs, or is “true”, namely the cell which contains the phase point $\gamma$ which represents the actual or true state of the system. From this perspective $\theta_k(P)$ is the probability of the property $P$ conditional upon the property $D_k$, and we have the usual identification of “true” with “probability one” and “false” with “probability zero”.

Notice that it is because we are assuming that $P$ is of the form (3) that the product $PD_k$ must have one of the two forms on the right side of (5): no property of the form (3) can include part but not all of some cell $D_k$. Consequently, (5) defines a truth functional for indicators belonging to this particular algebra $B$, but not for all possible properties; in this sense a truth functional is relative to a particular decomposition $\mathcal{D}$, or Boolean algebra $B$. However, it is also possible to construct a *universal truth functional* which is not limited to a single Boolean algebra, but which will assign 0 or 1 to any indicator on the classical phase space in a manner which satisfies (4). To do this, choose some point $\gamma_0$ in $\Gamma$, and let

$$
\theta_0(P) = P(\gamma_0).
$$

That is, $\theta_0$ assigns the value 1 to any property which contains the point $\gamma_0$, and 0 to any property which does not contain this point, in agreement with how one would normally understand “true” in a case in which the state of the system is correctly described by $\gamma_0$. As we shall see, a key difference between classical and quantum physics is the fact that in the latter there are no universal truth functionals as long as the Hilbert space has a dimension greater than 2.

## 3 Quantum Truth Functionals

The quantum counterpart of a classical phase space is a Hilbert space $\mathcal{H}$. For our purposes it suffices to consider cases in which $\mathcal{H}$ is of finite dimension, thus avoiding the mathematical complications of infinite-dimensional spaces. The counterpart of a classical property is a linear subspace $P$ of $\mathcal{H}$, with a corresponding orthogonal projection operator or *projector* $P$. 
If $I$ is the identity operator, the negation $\tilde{P}$ of the property $P$ corresponds to the projector $I - P$, and the conjunction $P \land Q$ of two properties corresponds to the projector $PQ$ in the case in which $P$ and $Q$ commute with each other. If $PQ \neq QP$, then neither $PQ$ nor $QP$ is a projector, so there is no obvious way to define a property corresponding to the conjunction. We shall return to this later, in Sec. 4.

The quantum counterpart of a coarse graining of a classical phase space is a decomposition $\mathcal{D}$ of the identity which has precisely the form (1), where the $D_j$ are now mutually orthogonal projectors which satisfy (2). This decomposition gives rise to a set of projectors of the form (3), all of which commute with each other, and which form a Boolean algebra $\mathcal{B}$ analogous to the algebra of classical indicator functions.

A quantum truth functional $\theta$ assigns to every projector $P$ in the Boolean algebra (3) the value 0 or 1 in a way which satisfies the three conditions in (4). Once again, there is a one-to-one correspondence between truth functionals and the elements of $\mathcal{D}$, and each such truth functional is of the form (5) for some $k$. The intuitive interpretation is also similar in the quantum and classical cases. The projectors which enter the decomposition $\mathcal{D}$, or equivalently the corresponding subspaces of $\mathcal{H}$, form a sample space of mutually exclusive possibilities, one and only one of which is a correct description of the system, and thus “true”. Larger subspaces in $\mathcal{B}$ which contain this true space are also true, and the others are false.

A truth functional associated with the decomposition $\mathcal{D}$ can be used to assign a (real) numerical value to an observable corresponding to a Hermitian operator $A$ written in the form

$$A = \sum_j a_j D_j,$$

(7)

where the $a_j$ are, of course, the eigenvalues of $A$. Thus if the truth functional is $\theta_k$, (5), then $D_k$ is true, and any $D_j$ with $j$ unequal to $k$ is false, so $\theta_k$ assigns to $A$ the (eigen)value $a_k$. Similarly, given some collection of observables represented by commuting Hermitian operators $A, B, C, \ldots$, the operators can be simultaneously diagonalized using a single decomposition of the identity. Then a truth functional associated with this decomposition will assign numerical values to each of the operators in a consistent way, so that, for example, if $A$, $B$, and $C$ are assigned the eigenvalues $a$, $b$, and $c$, then, for example, the operator $AB + 2C$—which, of course, can also be represented using the same decomposition of the identity—will be assigned the value $ab + 2c$.

In analogy with the classical case, Sec. 2, let us define a universal quantum truth functional $\theta$ as one which assigns to every projector $P$ (thus every subspace) of $\mathcal{H}$ one of the two values 0 or 1 in a way which satisfies the rules in (1), with, however, the following qualification. If two projectors $P$ and $Q$ do not commute, so that $PQ$ is not a projector, then the third rule in (1) should be ignored; we only require that it hold in cases in which $PQ = QP$. When applied to projectors belonging to the Boolean algebra $\mathcal{B}$ associated with some particular decomposition $\mathcal{D}$ of the identity, a universal truth functional has the same properties as an “ordinary” truth functional; that is, if the decomposition is (1), then $\theta$ coincides, on this Boolean algebra, with $\theta_k$, (5), for some specific $k$. Consequently, a universal truth functional, if it exists, can be used to assign to every observable (Hermitian operator) on $\mathcal{H}$ one of its eigenvalues, and for commuting collections of Hermitian operators these values will satisfy the usual algebraic rules associated with ordinary numbers when one considers products and
sums of operators, as in the example considered above.

Alas, it was shown by Bell [9], and by Kochen and Specker [10] that if $\mathcal{H}$ has a dimension of three or more, universal truth functionals do not exist. A very simple nonexistence proof for a Hilbert space of dimension 4 is provided by Mermin's paradox for two spin-half particles [12, 13], based upon the following “magic square”, which was also used in [11]:

$$
\begin{array}{ccc}
\sigma_x^a & \sigma_x^b & \sigma_x^a \sigma_x^b \\
\sigma_y^b & \sigma_y^a & \sigma_y^a \sigma_y^b \\
\sigma_x^a \sigma_y^b & \sigma_y^a \sigma_x^b & \sigma_x^a \sigma_y^b
\end{array}
$$

Here $\sigma_x^a$ is the Pauli $\sigma_x$ operator for spin $a$, $\sigma_x^b$ the $\sigma_y$ operator for spin $b$, and so forth. Note that the operators for particle $a$ commute with those for particle $b$, whereas the commutator of $\sigma_x^a$ and $\sigma_y^a$ is $2i\sigma_x^a$, etc. Each of the nine operators in the square has eigenvalues $+1$ and $-1$, and each eigenvalue is two-fold degenerate. The three operators in each row in (8) commute with each other, as do the three operators in each column. In addition, it is not hard to show that the product of the three operators in each row is the identity $I$. The product of the three operators in each of the first two columns is $I$, but the product of those in the third column is $-I$.

These mathematical properties are incompatible with the existence of a universal truth functional. For suppose that such a functional existed. Then, as explained earlier, it could be used to assign a numerical eigenvalue of $\pm 1$ to each of the nine operators in the square. Since the operators in each row commute with one another, the usual algebraic properties would be preserved for the numerical assignments corresponding to this row. This would mean that the product of the numbers in any given row would have to be $+1$, since a truth functional must assign to $I$ its only eigenvalue, $+1$. Similarly, the product of the numerical values in the first two columns would have to be $+1$, and in the last column it would have to be $-1$. But no such assignment of numerical values exists. For example,

$$
\begin{array}{ccc}
-1 & -1 & +1 \\
+1 & -1 & -1 \\
-1 & -1 & +1
\end{array}
$$

satisfies all the product rules, except that the product of the values in the second column is $-1$, not $+1$. To see that no assignment of $\pm 1$ can satisfy all the rules, find the product of the three numbers in every row, next the product of the three numbers in every column, and, finally, take the product of all six of these products. The result will be the product of the squares of all nine entries in the $3 \times 3$ matrix, thus 1, whereas the rules would require that it be $-1$.

4 Consistent Histories and Truth Functionals

How does the consistent histories approach deal with Mermin’s magic square and similar paradoxes? To understand this, let us return to a problem mentioned earlier, that of making
sense of the conjunction \( P \land Q \), “\( P \ AND \ Q \)”, of two quantum properties when the projectors do not commute, \( PQ \neq QP \). (Notice that this problem never arises in classical physics, since the product of two indicators on the classical phase space is the same in either order.) For example, for a spin-half particle, the projector for the property \( \sigma_y = +1 \) is \( \frac{1}{2}(I + \sigma_y) \), and that for the property \( \sigma_x = +1 \) is \( \frac{1}{2}(I + \sigma_x) \). As these projectors obviously do not commute with each other, can one make sense of the statement \( \sigma_y = +1 \ AND \ \sigma_x = +1 \)?

The answer of the consistent historian is that one cannot make sense of \( \sigma_y = +1 \ AND \ \sigma_x = +1 \): it is a meaningless statement in the sense that the CH interpretation assigns it no meaning. In the CH approach there are no hidden variables, and thus there is a one-to-one correspondence between quantum properties and subspaces of the Hilbert space. Since every one-dimensional subspace of the two-dimensional Hilbert space \( \mathcal{H} \) of a spin-half particle corresponds to a spin in a particular direction, there is none left over which could plausibly represent \( \sigma_y = +1 \ AND \ \sigma_x = +1 \).

To be sure, one might consider assigning to \( \sigma_y = +1 \ AND \ \sigma_x = +1 \) the zero element of \( \mathcal{H} \), which is a zero-dimensional subspace corresponding to the property which is always false, analogous to an indicator which is everywhere zero on a classical phase space. This, in fact, was the proposal (for this situation) of Birkhoff and von Neumann in their discussion of quantum logic [14]. It is important to notice the difference between their approach and the one used in CH. A proposition which is meaningful but false is very different from a meaningless proposition: the negation of a false proposition is a true proposition, whereas the negation of a meaningless proposition is equally meaningless. The Birkhoff and von Neumann approach requires, as they pointed out, a modification of the ordinary rules of propositional logic, whereas the CH approach does not. However, in CH quantum theory it is necessary to exclude meaningless properties from meaningful discussions, which is not a trivial task.

In particular, in CH quantum theory any quantum description of a single system at a particular time must employ a single framework, which is to say a single Boolean algebra of commuting projectors based upon a definite decomposition of the identity. To be sure, alternative descriptions can be constructed using different decompositions of the identity, but these cannot be combined to form a single description, nor can logical reasoning about a quantum system be carried out by combining results from two different Boolean algebras. This is the single framework rule, which is central to a correct understanding of CH quantum theory, and the point most often misunderstood by physicists unfamiliar with the CH approach.

In some cases results for two different Boolean algebras can be combined by using the device of a “common refinement”: a third algebra which includes all the projectors of the first two algebras. If a common refinement exists, the two algebras (or frameworks) are said to be compatible; if not, they are incompatible. If two algebras are compatible, one can use the common refinement instead of the original algebras as the single framework required by the single framework rule. However, such a refinement exists if and only if all the projectors in one of the algebras commutes with all of the projectors in the other algebra. Consequently, it is not possible to produce a consistent quantum description which combines results from two Boolean algebras when some of the projectors in one do not commute with some of the projectors in the other.

The single framework rule as applied to Boolean algebras of properties refers to a single
system at a single instant of time. Given two nominally identical systems, there is no reason why one cannot use one framework for the first and a different framework for the second. For instance, in the case of the two spin-half particles in (8), there is no problem using $\sigma_x$ for one and $\sigma_y$ for the other. Thus, whereas $\sigma_y = +1$ AND $\sigma_x = +1$ is a meaningless expression for a single particle, $\sigma^a_y = +1$ AND $\sigma^b_x = +1$ makes perfectly good sense. Conversely, when incompatible frameworks turn up in some quantum discussion, it is best to think of them as referring to two different systems, or to a single system at two different times. (The latter is an example of a history, and when histories involve three or more times, the CH approach imposes additional rules—but these are outside the scope of the present discussion.)

In view of the preceding remarks, the reader will not be surprised to learn that truth functionals are meaningful constructions within CH quantum theory provided they refer to a single framework or Boolean algebra. Thus a universal truth functional makes no sense as soon as the Hilbert space is of dimension two, since one already has projectors which do not commute with each other, and simultaneously (i.e., with a single truth functional) assigning truth values to properties which cannot simultaneously enter the same quantum description is meaningless. The same objection applies to universal truth functionals in higher-dimensional Hilbert spaces, but, as already noted, there are no such things in Hilbert spaces of dimension three or more!

As the single framework rule may seem a bit abstract, let us see how it applies to the case of the magic square (8). As long as one considers a single row, the operators commute with each other, and hence they are of the general form (7) using a single decomposition of the identity. There is, therefore, no problem with introducing a truth functional which assigns to each of these operators one of its eigenvalues; for example, those in the top row of (9). Of course, the same remark applies to the operators in the first column of (8): they commute with each other, and can all be expressed in terms of a single decomposition of the identity. However, this decomposition of the identity is different from the one used for the first row, and the two are incompatible—they have no common refinement—as is immediately obvious from the fact that $\sigma^a_x$ in the first row does not commute with $\sigma^b_y$ in the first column. Consequently, it makes no sense to simultaneously assign values to all the operators in the top row and those in the first column, much less values to all nine operators in the square. For this reason it is impossible to construct a paradox if one pays attention to the CH rules.

5 The Argument of Bassi and Ghirardi

In [11] Bassi and Ghirardi use the magic square (8) to argue that CH ideas when combined with three assumptions which they regard as reasonable, and even necessary for a sound interpretation, when combined with a fourth assumption usually made by proponents of the CH interpretation (but which they themselves find questionable) lead to a logical contradiction. The four assumptions can be stated briefly as follows, where the language has been changed so that it refers to a system at a single time (rather than to histories), as this is all that is needed for the present discussion:

(a) Ordinary rules of classical reasoning apply to a single Boolean algebra of projectors.
(b) It is possible to assign a truth value to every projector which occurs in a particular Boolean algebra.

(c) A projector should be assigned the same truth value in all Boolean algebras which contain it.

(d) Any Boolean algebra of projectors can be used to construct a legitimate quantum description.

From these assumptions Bassi and Ghirardi deduce the existence of a universal truth functional, to use the terminology employed in this Letter, and then show that such an object is inconsistent with the mathematical properties of the operators in $\mathcal{B}$.

Assumptions (a) and (d) are part of the standard CH approach. As for (b), we noted in Sec. 3 that if a decomposition of the identity contains $N$ projectors, there are $N$ distinct truth functionals associated with the corresponding Boolean algebra $\mathcal{B}$, so it is always possible to choose one of them and use it to assign a truth value to the projectors in $\mathcal{B}$. Thus (b) agrees with the CH interpretation.

Assumption (c) is ambiguous. As noted in Sec. 3, any quantum truth functional is associated with a particular Boolean algebra. One could understand (c) to mean that for a given projector $P$, we should consider all Boolean algebras which contain it, and in each of these algebras we should pick a truth functional which assigns the same value, say 1, to $P$. This can certainly be done, as a mathematical exercise, for a particular projector $P$. However, the authors of [11] mean something quite a bit stronger [15]. Namely, that every projector on $\mathcal{H}$—or, if one does not make assumption (d), every projector belong to some collection of acceptable properties—is assigned a definite truth value, 1 or 0, independent of the Boolean algebra which contains it. Understood in this way, (c) combined with (a), (b), and (d) is equivalent to the assumption of a universal truth functional, an impossibility for a Hilbert space of dimension 3 or more, as explained in Sec. 3, and contrary to CH quantum mechanics for the reasons indicated in Sec. 4.

However, the single framework rule of CH quantum theory is already inconsistent with the first (weaker) interpretation of (c) in the preceding paragraph, in the following sense. Suppose that $P$ belongs to two incompatible Boolean algebras $\mathcal{B}'$ and $\mathcal{B}''$ containing projectors which do not commute with each other. Then $\mathcal{B}'$ and $\mathcal{B}''$ cannot be combined in a single quantum description. Therefore, if both are to be employed, they cannot refer to the same physical system at the same time. But if we are dealing with two physical systems, or the same system at two different times, there is, in general, no reason to suppose that two truth functionals should coincide for $P$ or for any other projector in $\mathcal{B}' \cap \mathcal{B}''$.

It may help to consider a specific example. Let $\mathcal{B}'$ be the Boolean algebra corresponding to the operators on the first row of the magic square (8), $\mathcal{B}''$ that of the first column, and choose truth functionals $\theta'$ and $\theta''$ which assign (as discussed in Sec. 3) the values shown in the first row and the first column of (9), respectively. Then both $\theta'$ and $\theta''$ assign to $\sigma^a_x$ the value $-1$, in accordance with the weaker interpretation of (c). Nonetheless, they cannot possibly refer to the same physical situation, because $\theta'$ assigns to $\sigma^b_x$ the value $-1$, and $\theta''$ assigns to it the value $-1$.

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1 A statement in Sec. 5 of [11], could be interpreted to mean that (b) is equivalent to the assumption that every property has a truth value; however the interpretation given here coincides with that intended by the authors [15].
to $\sigma_y^x$ the value $+1$. But a single system at a single time cannot have both a value for $\sigma_y^b$ and a value for $\sigma_y^b$ if one uses the Hilbert space of standard quantum mechanics, for the reasons discussed in Sec. 4 above. Consequently, the fact that $\theta'$ and $\theta'''$ assign the same value, $-1$, to $\sigma_y^x$ is, from the CH point of view, devoid of any particular significance, since if these truth functionals are describing different systems this agreement is fortuitous.

In summary, if (c) is understood in the stronger of the two senses discussed above it leads, in combination with (d), to the existence of a universal truth functional, which conflicts with the Bell-Kochen-Specker results as well as with consistent histories. However, there is already a clear conflict with the single framework rule of CH quantum theory as soon as (c) is interpreted in a weaker way which only requires using two or more incompatible Boolean algebras to refer to a single physical system at the same time.

Unfortunately, the single framework rule, despite the prominence given to it in several of the references cited in their Letter, is not mentioned by Bassi and Ghirardi when they introduce what they consider to be the basic principles of CH quantum theory, nor is it referred to, except somewhat obliquely in their Sec. 2(c), until after they have completed their definitions and their main argument. When at the beginning of Sec. 4 they state the rule in its entirety for the first time, they admit that their argument is, indeed, in conflict with this rule, but then offer the excuse that they are employing a different form of reasoning from that employed in CH quantum theory. While the single framework rule is appropriate for the latter, it cannot, they assert, apply to the former.

There seems to be no reason to debate this point, as Bassi and Ghirardi are surely not obligated to adopt the rules for quantum reasoning which the developers of the CH approach regard as most appropriate. One could wish, however, that they had made plain much earlier in their Letter that it is not “standard” consistent histories quantum theory, but instead an alternative version they themselves invented, with different rules of reasoning, which leads to a logical contradiction. In this regard theirs can be added to a list of work by other authors\textsuperscript{2} which shows that attempting to construct a histories interpretation of quantum theory while omitting the single framework rule generally leads to unsatisfactory results.

Although the claim that (standard) CH leads to a logical contradiction is unfounded for the reasons just noted, there is another aspect of \cite{11} which deserves comment. Translated into the language used in this Letter, the authors make what is, in essence, the claim that one cannot treat quantum properties as part of an “objective reality” if one denies the existence of a universal truth functional. This is the sort of conceptual and philosophical issue which cannot be settled by an appeal to logic and mathematics; instead it requires the application of physical intuition and an exercise of judgment. It is the case that in CH quantum theory, properties at a single time, and histories, which are sequences of quantum properties at successive times, are, under appropriate conditions, considered to be “real” and “objective”. Some issues involved in treating the CH approach as a realistic interpretation of quantum theory are discussed in Sec. V B of \cite{7}, and the conclusion, translated into the terminology of the present Letter, is that there is no reason why one should regard the existence of universal truth functionals as a necessary part of quantum reality. Rather than repeat the argument here, let me simply note that I believe that \cite{7} provides a quite adequate response, even though it was written earlier, to the concerns raised in \cite{11}.

\textsuperscript{2}See Sec. V C and D, and App. A of \cite{7}; also \cite{16, 17}.
Acknowledgments

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