Concept of multistage discrete fourier transform without performing multiplications

A Yu Burova
Moscow Aviation Institute (National Research University), Volokolamskoe highway 4, 125993, Moscow, Russia
E-mail: adeliya_yu_burova@mail.ru

Abstract. The issues related to the development and research of digital methods and algorithms for the discrete Fourier transform, which do not require algorithmic multiplication operations, are considered. The aim of the research is to develop and formalize the concept of multistage discrete Fourier transform of complex signals only by algebraic addition of their time samples. The research used the methods of mathematical and hardware-software modeling of the operation of computing algorithms for digital signal processing on programmable logic integrated circuits. The concept of a multistep discrete Fourier transforms without performing multiplications is described. The corresponding formulas for digital signal processing are given. The results of the study showed and confirmed the possibility of this Fourier transform of complex signals only by algebraic addition of their time samples.

1. Introduction
Computational algorithms for discrete Fourier transform (DFT) are successfully used in radio engineering, hydroacoustics and aviation [1-2] The digitalization of these algorithms has actualized the development and research of recurrent differential digital filtering (DDF) methods for a complex signal [3]. Such methods make it possible to reduce hardware costs when implementing digital DFT algorithms based on DDF in software and hardware. Minimization of the number of multiplication operations in digital algorithms of such DFT allows them to be successfully used to reduce hardware costs in the software and hardware implementation of digital signal processing (DSP) algorithms on Programmable Logic Devices (PLD) [4-5].

2. Purpose and methods
The aim of the research is to develop and formalize the concept of multi-stage DFT (MFT) of complex signals only by algebraic addition of their time samples.

There are methods of mathematical and hardware-software modeling of the operation of DSP computational algorithms on PLD [6].

3. Research objective
The objective of the research is the development and formalization of the concept of MFT complex signals by a digital low-pass filter with integer filter coefficients without performing arithmetic multiplication operations using coordinate rotation digital computer (CORDIC) method and the finite difference method in relation to the formation of DDF coefficients with a finite impulse response [7].
4. Theoretical basis
The theoretical basis for the development and formalization of MFT concept of a complex signal is the practical applications of the following scientific theories:

- theory of frequency transfer of band spectra based on CORDIC method, which makes it possible to obtain the vector \( y(nT) = \text{Re}[y(nT)] + i \text{Im}[y(nT)] \), \( n=0,1,2,...N-1, T=2\pi/\omega_D \), when rotating the vector \( x(nT) = \text{Re}[x(nT)] + i \text{Im}[x(nT)] \), \( n=0,1,2,...N-1, T=2\pi/\omega_D \), by an angle \( \Theta \) by formula (1) without performing arithmetic multiplication operations [8]:
  \[
  \begin{align*}
  \text{Re}[y(nT)] &= -\text{Im}[x(nT)], & \text{Im}[y(nT)] &= +\text{Re}[x(nT)], & \text{if } \Theta = \pi/2 \pm 2\pi \\
  \text{Re}[y(nT)] &= -\text{Re}[x(nT)], & \text{Im}[y(nT)] &= -\text{Im}[x(nT)], & \text{if } \Theta = \pi/2 \pm 2\pi \\
  \text{Re}[y(nT)] &= +\text{Im}[x(nT)], & \text{Im}[y(nT)] &= -\text{Re}[x(nT)], & \text{if } \Theta = 3\pi/2 \pm 2\pi.
  \end{align*}
  \]
  
  where \( y(nT) \) is the output parameter of the algorithm by method CORDIC, \( n=0,1,2,...N-1, T=2\pi/\omega_D \);

- adaptive DFT method of a complex signal only by algebraic addition of their time samples [9]:
  \[
  x(nT) = x(nT) - y(nT),
  \]
  
  where \( x(nT) \) is the output parameter of the finite difference algorithm, \( n=0,1,2,...N-1, T=2\pi/\omega_D \);

- \( \text{Re} \) and \( \text{Im} \) - directional search method according to the formula (3):
  \[
  \begin{align*}
  x_{p-1}(nT), & \quad x_{p-1}(nT) < x_p(nT), & x_p(nT) < x_{p+1}(nT), & \quad p=1,2,3,...P-2 \\
  y(nT) = x_p(nT), & \quad x_{p-1}(nT) > x_p(nT), & x_p(nT) < x_{p+1}(nT), & \quad p=1,2,3,...P-2 \\
  x_{p+1}(nT), & \quad x_{p+1}(nT) > x_p(nT), & x_p(nT) > x_{p+1}(nT), & \quad p=1,2,3,...P-2.
  \end{align*}
  \]
  
  where \( y(nT) \) is the output parameter of the directed search algorithm, \( n=0,1,2,...N-1, T=2\pi/\omega_D \);

- theory of construction by formula (2) based on the finite difference method:
  \[
  x(nT) = x(nT) - y(nT),
  \]

5. Practical importance and novelty
The practical significance of the research lies in the definition and formulation of the concept of MFT complex signals only by algebraic addition of their time samples [9-10].

The novelty of the research lies in the formalization of recurrent MFT methods of a complex signal using a differential digital low-pass filter without performing arithmetic multiplication operations [11].

6. Results of the study
The research results have shown and confirmed the possibility of DFT complex signals only by algebraic addition of their time samples.

To minimize the computational complexity of the selection of harmonic components \{\( x_m(nT) \), \( m=0,1,2,...N-1 \), complex signal \( \{x(nT)\}, n=0,1,2,...N-1, T=2\pi/\omega_D \), with the time-varying frequency resolution \( \Omega_z \leq \Omega \), of the complex spectrum \( \{X(m\Omega)\}, m=0,1,2,...M-1, \Omega = \omega_D/(2M) \), \( M \leq N \), \( l=1,2,3,...L \), \( N \)-point fragment of a time sample of a complex signal \( \{x(nT)\}, n=0,1,2,...N-1, T=2\pi/\omega_D \), adaptive DFT method of a complex signal \( \{x(nT)\}, n=0,1,2,...N-1, T=2\pi/\omega_D \) is offered, \( L \)-step multiband digital filtering (MDF) based on digital difference filtering (DDF).

MFT is MDF-based L-stage DFT, implemented on the basis of a difference digital filter of \( K_p \)-th and \( J \)-th order of the difference with integer values of \( k_J \)-th difference coefficients \( \{h_j(k_J)\}, k_J=0,1,2,...K_p \).

n-th time samples of the \( N \)-point fragment of the complex sample \( \{x(nT)\}, n=0,1,2,...N-1, T=2\pi/\omega_D \) are received at MFT input.
7. MFT concept

The concept of the proposed adaptive MFT method is a gradual (“step by step”) l-step calculation of statistics (estimates) \( \{x(m; \Omega, n; T_l)\} \), \( m=0,1,2...M_l-1, \Omega=\omega_D/(2-M_l), n=0,1,2...N/M_l-1, T_l=2\pi\cdot M_l/\omega_D, M_l=B_{MFT}^l \), instant spectrum \( \{X(m; \Omega)\} \), \( m=0,1,2...M_l-1, \Omega=\omega_D/(2-M_l), M_l=B_{MFT}^l \), complex signal \( \{x(n-T)\}, n=0,1,2...N-1, T=2\pi/\omega_D \), on l-th step MFT, \( l=1,2,3...L \), only by methods CORDIC and total recordable cases frequency (TRCF) without performing arithmetic multiplication operations. With such a solution to the problem posed, adaptation to the time variation of the frequency resolution \( \Omega \leq \Omega_l \) of instant spectra \( \{X(m_l; \Omega)\}, X(m_l; \Omega)=x(m_l; \Omega, n_l; T_l) \), \( m_l=0,1,2...M_l-1, \Omega=\omega_D/(2-M_l), n=0,1,2...N/M_l-1, T_l=2\pi\cdot M_l/\omega_D, M_l=B_{MFT}^l \leq N, l=1,2,3...L \), provided by the transition from the calculation of instantaneous spectra \( \{X(m_l; \Omega)\}, m_l=0,1,2...M_l-1, \) with frequency resolution \( \Omega=\omega_D/(2-M_l) \) at l-th stage of the MFT to the calculation of instantaneous spectra \( \{X(m_{l+i}; \Omega_{l+i})\}, m_{l+i}=0,1,2...M_{l+i}-1, \) with frequency resolution \( \Omega_{l+i}=\omega_D/(2-M_{l+i}) \), on \( (l+1) \)-th step of MFT, when \( l<L \).

The solution to the problem posed is to apply the following principles of minimizing computational complexity when building digital MFT algorithms by using only arithmetical operations of the least computational complexity of:

- principles of frequency division \( \{X(m_l; \Omega)\}, m_l=0,1,2...M_l-1, \Omega=\omega_D/(2-M_l), M_l=B_{MFT}^l, l=1,2,3...L \), N-point slice of a time sample \( \{x(n-T)\}, n=0,1,2...N-1, T=2\pi/\omega_D \), on \( M_l \) narrowband spectral components \( \{x_l(m_l; \Omega_l, n_l; T_l)\}, m_l=0,1,2...M_l-1, \Omega_l=\omega_D/(2-M_l), n_l=0,1,2...N/M_l-1, T_l=2\pi\cdot M_l/\omega_D \), at \( M_l \leq N, l=1,2,3...L \) and their frequency shift without performing arithmetic multiplication operations;

- principles of low-frequency DDF of l-th spectral components \( \{x_l(m_l; \Omega_l, n_l; T_l)\}, m_l=0,1,2...M_l-1, \Omega_l=\omega_D/(2-M_l), n_l=0,1,2...N/M_l-1, T_l=2\pi\cdot M_l/\omega_D, M_l=B_{MFT}^l, l=1,2,3...L \), complex signal \( \{x(n-T)\}, n=0,1,2...N-1, T=2\pi/\omega_D \), at \( N=M_l \) only arithmetical addition and shift operations.

MDF provides L-step time conversion of a complex signal \( \{x(n-T)\}, n=0,1,2...N-1, T=2\pi/\omega_D \), using the formula (4) into its time-frequency sampling \( \{x_l(m_l; \Omega_l, n_l; T_l)\}, n_l=0,1,2...N/M_l-1, T_l=2\pi\cdot M_l/\omega_D, m_l=0,1,2...M_l-1, \Omega_l=\omega_D/(2-M_l), M_l=B_{MFT}^l \), \( l=1,2,3...L \), using the formula (5) at \( N=B_{MFT}^l \):

\[
M = 1 \quad M_l = 1 \quad M_l - 1
\]

\[
x(n-T) = \{ \sum x_m(0) + x_0(T) + \sum x_m(T) \cdot \sum x_m(N-T-T) + x_0(N-T-T) \}
\]

\[
m = 0 \quad m = 0 \quad m = 0
\]

where \( n=0,1,2...N-1, T=2\pi/\omega_D \), at \( N=B_{MFT}^l \).

\[
\hat{x}_0(0)+\hat{x}_0(0)/M_l, \quad \hat{x}_0(T)+\hat{x}_0(T)/M_l, \quad \hat{x}_0(n_l-T)+\hat{x}_0(n_l-T)/M_l,
\]

\[
\hat{x}_1(0)+\hat{x}_0(0)/M_l, \quad \hat{x}_1(T)+\hat{x}_0(T)/M_l, \quad \hat{x}_1(n_l-T)+\hat{x}_0(n_l-T)/M_l,
\]

\[
\{x(m_l; \Omega_l, n_l; T_l)\} = \hat{x}_0(0)+\hat{x}_0(0)/M_l, \quad \hat{x}_0(T)+\hat{x}_0(T)/M_l, \quad \hat{x}_0(n_l-T)+\hat{x}_0(n_l-T)/M_l
\]

\[
\hat{x}_1(0)+\hat{x}_0(0)/M_l, \quad \hat{x}_1(T)+\hat{x}_0(T)/M_l, \quad \hat{x}_1(n_l-T)+\hat{x}_0(n_l-T)/M_l
\]

where \( n=0,1,2...N/M_l-1, T=2\pi\cdot M_l/\omega_D, m_l=0,1,2...M_l-1, \Omega_l=\omega_D/(2-M_l), M_l=B_{MFT}^l, l=1,2,3...L \), at \( N=B_{MFT}^l \).

The MFT output provides \( m_l \)-th samples of the instantaneous spectrum \( \{X(m_l; \Omega_l)\} \equiv \{x_l(m_l; \Omega_l, n_l; T_l)\}, m_l=0,1,2...M_l-1, \Omega_l=\omega_D/(2-M_l), n_l=0,1,2...N/M_l-1, T_l=2\pi\cdot M_l/\omega_D, M_l=B_{MFT}^l \), complex signal \( \{x(n-T)\}, n=0,1,2...N-1, T=2\pi/\omega_D \), at \( N=M_l, m_l=0,1,2...N-1, \Omega_l=\omega_D/(2-N), n_l=0, T_l=2\pi\cdot N/\omega_D=T-N \).

The proposed adaptive MFT method is intended for constructing computational algorithms for the formation of statistics (estimates) of the numerical values of complex coefficients DFT \( \{x(m_l; \Omega_l, n_l; T_l)\}, n_l=0,1,2...N/M_l-1, T_l=2\pi\cdot M_l/\omega_D, m_l=0,1,2...M_l-1, \Omega_l=\omega_D/(2-M_l), M_l=B_{MFT}^l, l=1,2,3...L \), harmonic
components \{x_w(n-T)\}, m=0,1,2,...N-1, complex signal \{x(n-T)\}, n=0,1,2,...N-1, T=2\pi/\omega_D, L-step MFT with base \(B_{MFT}=3\) against the background of additive uncorrelated interference \(x_s(n_T)\), \(n=0,1,2,...N-1, T_s=2\pi/\omega_D\), such as "white noise", if the size of N-point fragment of its temporal sample \(\{x(n-T)\}, n=0,1,2,..,N-1, T=2\pi/\omega_D\), is a multiple of MFT base MFT, \(N=B_{MFT}^l\).

Digital algorithms based on the proposed MFT method are designed to adapt spectral analysis to changes in frequency resolution over time \(\Omega_l \leq \Omega_c\) of spectrum \(\{x(m_0,\Omega_c)\}, m=0,1,2,...N-1, \Omega=\omega_D/(2N)\), N-point slice of a time sample \(\{x(n-T)\}, n=0,1,2,...N-1, T=2\pi/\omega_D\). At the same time, the economy of the computing resource is ensured by the formation of statistics (estimates) of the numerical values of the complex DFT coefficients \(\{x(m,\Omega_l,n_l{T_l})\}, n_l=0,1,2,...N/M_l-1, T_l=2\pi/M_l/\omega_D\), \(m=0,1,2,...M_l-1, \Omega_l=\omega_D/(2M_l)\), \(M_l=B_{MFT}^l\), \(l=1,2,3,...L\), not for all N harmonic components \(\{x_w(n-T)\}, m=0,1,2,...M_l-1\), complex signal \(\{x(n-T)\}, n=0,1,2,...N-1, T=2\pi/\omega_D\), but only for "useful" or demanded harmonic components \(\{x_w(n-T)\}, m=0,1,2,...M_l-1\), complex signal \(\{x(n-T)\}, n=0,1,2,...N-1, T=2\pi/\omega_D\), in a given frequency range \([0, \omega_D/2]\).

The essence of the proposed adaptive MFT method consists in L-step transformation of the time sampling of the complex signal \(\{x(n-T)\}, n=0,1,2,...,N-1, T=2\pi/\omega_D\), represented by the canonical DFT formula, into its time-frequency sampling \(\{x(m,\Omega_c,n_l{T_l})\}, m=0,1,2,...M_l-1, \Omega=\omega_D/(2M_l)\), \(n_l=0,1,2,...N/M_l-1, T_l=2\pi/M_l/\omega_D\), \(m=0,1,2,...M_l-1, \Omega_l=\omega_D/(2M_l)\), \(M_l=B_{MFT}^l\), \(l=1,2,3,...L\), according to the formula (5) based on multiband DDF at \(N=B_{MFT}^L\) [3]. A statistics (estimates) \(\{x(m,\Omega_c,n_l{T_l})\}, n_l=0,1,2,...N/M_l-1, T_l=2\pi/M_l/\omega_D\), \(m=0,1,2,...M_l-1, \Omega_l=\omega_D/(2M_l)\), \(M_l=B_{MFT}^l\), \(l=1,2,3,...L\), of instant spectrum components \(\{x(m,\Omega_c)\}\), \(m=0,1,2,...M_l-1, \Omega=\omega_D/(2M_l)\), \(M_l=B_{MFT}^l\), \(l=1,2,3,...L\), identical at \(M_l=N\) to the instantaneous spectrum \(\{x(m,\Omega_c)\}, m=0,1,2,...N-1, \Omega=\omega_D/(2N)\), are at \(M_l=N\) statistics (estimates) of the harmonic components \(\{x_w(n-T)\}, m=0,1,2,...N-1\), of this signal.

The essence of building digital algorithms based on the proposed adaptive MFT method is to minimize the computational complexity of \(M_l\)-point spectral analysis of \(N\)-point sampling of a complex signal \(\{x(n-T)\}, n=0,1,2,...N-1, T=2\pi/\omega_D\), with variable frequency resolution. Time variation of the frequency resolution \(\Omega=\omega_D/(2M_l)\) statistics (estimates) \(\{x(m,\Omega_c,n_l{T_l})\}, m=0,1,2,...M_l-1, n_l=0,1,2,...N/M_l-1, T_l=2\pi/\omega_D\), \(M_l=B_{MFT}^l\), \(l=1,2,3,...L\), occurs in the frequency range \([\Omega_l, \Omega_c]\).

The essence of the digital algorithm built on the basis of the proposed adaptive MFT method consists in the following sequence of computational procedures:

- **frequency shift** of the complex signal \(\{x(n-T)\}, n=0,1,2,...N-1, T=2\pi/\omega_D\), into the low-frequency region at the first stage of the MFT with the base \(B_{MFT}=3\), \(l=1\), and statisticians (estimates) \(x_1(m_1,\Omega_{1,l},n_{1,l}{T_{1,l}})\), \(m_1=0,1,2,...M_{1,l}-1, n_{1,l}=0,1,2,...N/M_{1,l}-1, T_{1,l}=2\pi/M_{1,l}/\omega_D\), instant spectrum \(\{x(m_1,\Omega_{1,l})\}\), \(m_1=0,1,2,...M_{1,l}-1, \Omega_{1,l}=\omega_D/(2M_{1,l})\), \(M_{1,l}=B_{MFT}^l\), of this signal into the low-frequency region at \(l\)-th MFT steps with the base \(B_{MFT}=3\), \(l=2,3,4,...L\);
- **low-frequency difference digital filtering** of time-frequency samples \(\{\exp(-im\cdot\Omega_{c,l}n_l{T_l})\cdot x(n-T)\}, m=0,1,2,...M_{BFT}-1, \Omega_{c,l}=\omega_D/(2M_{BFT})\), \(n=0,1,2,...N-1, T=2\pi/\omega_D\), at the first stage of MFT with base \(B_{MFT}=3\) and time-frequency samples \(\{\exp(-im\cdot\Omega_{c,l}n_{1,l}{T_{1,l}})\cdot x_1(m_1,\Omega_{1,l}n_{1,l}{T_{1,l}})\}, m=0,1,2,...M_{1,l}-1, \Omega_{1,l}=\omega_D/(2M_{1,l})\), \(M_{1,l}=B_{MFT}^l\), \(l=1,2,3,...L\), \(M_{1,l}=B_{MFT}\), \(l=1,2,3,...L\), on \(l\)-th steps of the MFT with the base \(B_{MFT}=3\), \(l=2,3,4,...L\);
- **decimation** of \(n_{1,l}\)-th time samples of \(m_{1,l}\)-th time-frequency samples \(\{x(m_1,\Omega_{1,l}n_{1,l}{T_{1,l}})\}, m=0,1,2,...M_{1,l}-1, \Omega_{1,l}=\omega_D/(2M_{1,l})\), \(M_{1,l}=B_{MFT}\), \(n_{1,l}=0,1,2,...N/M_{1,l}-1, T_{1,l}=2\pi/M_{1,l}/\omega_D\), \(M_{1,l}=B_{MFT}^l\),
\( l = 1, 2, 3 \ldots L-1 \), after TRCF to obtain \( M \) time-frequency samples \( \{x(m_l; \Omega_l, n_l; T_l)\} \), \( m = 0, 1, 2 \ldots M-1 \), \( \Omega_l = \omega_0/(2 \cdot M_l) \), \( n = 0, 1, 2 \ldots N/M_l-1 \), \( T_l = 2 \cdot \pi \cdot M_l / \omega_D \), \( M_l = B_{MFT} l \), at the output of the \( l \)-th stages of the MFT with the base \( B_{MFT} = 3 \), \( l = 1, 2, 3 \ldots L \).

8. Formalization of MFT concept

The proposed adaptive MFT method is assigned a conditional code:

\[
x(m_l; \Omega_l, n_l; T_l) = MFT_1[[x(n-T)]], \ m = 0, 1, 2 \ldots M-1, \ n = 0, 1, 2 \ldots N-1
\]

where \( x(m_l; \Omega_l, n_l; T_l) \) is \( n_l \)-th time count of \( m_l \)-th statistics (estimates) of the instantaneous spectrum \( \{X(m_l; \Omega_l)\} \), \( m = 0, 1, 2 \ldots M-1 \), \( \Omega_l = \omega_0/(2 \cdot M_l) \), \( M_l = B_{MFT} l \) formed as a result of \( l \)-step MFT \( N \)-time samples of the complex signal \( \{x(n-T)\} \), \( n = 0, 1, 2 \ldots N-1 \), \( T = 2 \cdot \pi / \omega_D \), at \( N = B_{MFT} l \), \( l = 1, 2, 3 \ldots L \).

Conditional coding of the proposed adaptive MFT method is provided for formalizing the transformation of \( n \)-th samples of a complex signal \( \{x(n-T)\} \), \( n = 0, 1, 2 \ldots N-1 \), \( T = 2 \cdot \pi / \omega_D \), into \( n \)-th counts of \( m \)-th statistics (estimates) \( \{x(m_l; \Omega_l, n_l; T_l)\} \), \( m = 0, 1, 2 \ldots M-1 \), \( \Omega_l = \omega_0/(2 \cdot M_l) \), \( n = 0, 1, 2 \ldots N/M_l-1 \), \( T_l = 2 \cdot \pi \cdot M_l / \omega_D \), \( M_l = B_{MFT} l \), \( l = 1, 2, 3 \ldots L \), instant spectrum components \( \{X(m_l; \Omega_l)\} \), \( m = 0, 1, 2 \ldots M-1 \), \( \Omega_l = \omega_0/(2 \cdot M_l) \), \( M_l = B_{MFT} l \), with time-varying frequency resolution \( \Omega_l \leq \Omega_D \) in the frequency range \([0, \omega_D/2]\) at \( M_l = B_{MFT} l \), which correspond to harmonic components \( \{x_n(n-T)\} \), \( m = 0, 1, 2 \ldots N-1 \), this signal at \( N = B_{MFT} l \). For \( N_2 = \omega_0 / \omega_D \), \( \Omega_l = \omega_0/(2 \cdot B_{MFT} l) \), \( l = 1, 2, 3 \ldots L \), at \( B_{MFT} = 3 \) the formalization of the proposed adaptive MFT method is as follows:

\[
x(m_l; \Omega_l, n_l; T_l) = MFT_1[[x(n-T)]] = K_{F-1} \ k_0 \ k_{k-1}
\]

\[
= [\Sigma \Sigma \Sigma \Sigma h_p(J,k_1) \cdot \exp[-im_l \cdot \Omega_l \cdot n_1 \cdot (k_1-k_2) \cdot T_1]; \ x_11((\text{Int}[m/3] \cdot 3 \cdot \Omega_l \cdot (n_1-k_2) \cdot T_1);)]
\]

\[
k_0 = 0 \ k_1 = 0 \ k_2 = 0
\]

at \( m = 0, 1, 2 \ldots 3^j-1 \), \( \Omega_l = \omega_0/(2 \cdot 3^j) \), \( n = \text{Int}[n/3] \), \( T_1 = 3 \cdot T_1 \), \( n_1 = 0, 1, 2 \ldots N/3^{j-1}-1 \), \( T_1 = 2 \cdot \pi \cdot 3^{j-1} / \omega_D \), \( l = 1, 2, 3 \ldots L \);

\[
K_{F-1} \ k_0 \ k_{k-1}
\]

\[
x_i1(m_{l1}; \Omega_{l1}, n_{l1}; T_{l1}) = [\Sigma \Sigma \Sigma \Sigma h_p(J,k_1) \cdot \exp[-im_{l1} \cdot \Omega_{l1} \cdot n_{l1} \cdot (k_1-k_2) \cdot T_{l1};] \times
\]

\[
k_0 = 0 \ k_1 = 0 \ k_2 = 0
\]

\[
\times x_{22}(\text{Int}[m/3] \cdot \Omega_{22}, n_{22}; T_{l2})
\]

at \( m_{l1} = 0, 1, 2 \ldots 3^j-1 \), \( \Omega_{l1} = \omega_0/(2 \cdot 3^j) \), \( n_{l1} = \text{Int}[n/3] \), \( T_{l1} = 3 \cdot T_{l1} \), \( n_{l2} = 0, 1, 2 \ldots N/3^{j-1}-1 \), \( T_{l2} = 2 \cdot \pi \cdot 3^j / \omega_D \), \( l = 3, 4, 5 \ldots L \);

\[
K_{F-1} \ k_0 \ k_{k-1}
\]

\[
x_i1(m_{l1}; \Omega_{l1}, n_{l1}; T_{l1}) = [\Sigma \Sigma \Sigma \Sigma h_p(J,k_1) \cdot \exp[-im_{l1} \cdot \Omega_{l1} \cdot (n-k_2) \cdot T_{l1};] \times
\]

\[
k_0 = 0 \ k_1 = 0 \ k_2 = 0
\]

at \( m_{l1} = 0, 1, 2 \), \( \Omega_{l1} = \omega_0/(2 \cdot 3^j) \), \( n_{l1} = \text{Int}[n/3] \), \( T_{l1} = 3 \cdot T_{l1} \), \( n = 0, 1, 2 \ldots N-1 \), \( T = 2 \cdot \pi / \omega_D \)

\[
W_{MFT}(M_{l1}J_{l1}K_{l1}) \leq W_{DFT}(N), \text{ at } |C_{MFT}(M_{l1}J_{l1}K_{l1}Z) - C_{DFT}(N_{l1}D_{l1}Z)| \leq \omega_0, \ M_{l1} = N
\]
where \( x(n-T) \) are the time samples of \( N \)-point fragment of a time sample of a complex signal, \( n=0,1,2...,N-1, T=2\pi/\omega_D; \)

\( x(m;\Omega_n,T_1) \) are the time counts of statistics (estimates) of spectral components equidistant from each other in frequency \( \{x_n(m;\Omega_n)\}, m=0,1,2...,M-1, \Omega=\omega_D/(2M), n=0,1,2...,N/M-1, T_1=2\pi/\omega_D, M=M_{\text{BMT}}^1, B_{\text{MFT}}=3, l=1,2,3...,L \) of complex signal \( \{x(n-T)\}, n=0,1,2...,N-1, T=2\pi/\omega_D, x(m;\Omega_n,n;T_1) = \hat{x}_n(m;T_1)/M_1, m_1=m; \)

\( \exp[-i m \Omega_n T_1/l] \) are rotating (shifting) complex factors of CORDIC algorithm \( \exp[-i m \Omega_n T_1/l] = -1, -j, +1, +j, m_0=0,1,2...,M-1, \Omega=\omega_D/(2M), m_1=0,1,2...,N/M-1, T_1=2\pi/\omega_D, M=3-M_1, M_1=B_{\text{MFT}}^1, l=1,2,3...,L; \)

\( h_0(j,k) \) are difference coefficients of TRCF \( K_p \)-th order and \( J \)-th order of the difference, \( k=0,1,2...,K_p; l=1,2,3...,1 \), \( h_0(j,k) = -2, -1, 0, +1, +2; \)

\( C_{\text{MFT}}(M_1,J,K_p,Z) \) is computational error of the \( L \)-step MFT method based on \( K_p \)-th order TRCF and \( J \)-th order of the difference, \( M_1\leq N; \)

\( C_{\text{DFT}}(N,D,Z) \) is computational error of the \( N \)-point DFT method with the \( D \)-bitness of its quantized coefficients and the \( Z \)-bitness of the representation of real numbers;

\( D \) is the bitness of quantized DFT coefficients;

\( \varepsilon_0 \) is generally accepted criterion (necessary and sufficient tolerance) for assessing the accuracy (computational error) of the \( N \)-point DFT method;

\( W_{\text{MFT}}(M_1,J,K_p) \) is the computational complexity of the \( L \)-step MFT method based on the TRCF of the \( K_p \)-th order and \( J \)-th order of the difference, \( M_1\leq N; \)

\( W_{\text{DFT}}(N) \) is the computational complexity of the \( N \)-point DFT method;

\( N \) is the number of samples of a complex signal \( \{x(n-T)\}, n=0,1,2...,N-1, T=2\pi/\omega_D, N=B_{\text{MFT}}^1, B_{\text{MFT}}=3; \)

\( T \) is the complex signal time sampling period \( \{x(n-T)\}, n=0,1,2...,N-1, T=2\pi/\omega_D; \)

\( \Omega \) is the frequency resolution of the complex spectrum \( \{x_n(m;\Omega_n)\}, m=0,1,2...,3^l-1, \) time sampling \( \{x(n-T)\}, n=0,1,2...,N-1, T=2\pi/\omega_D, \Omega=\omega_D/(2M), M=M_{\text{BMT}}^1, B_{\text{MFT}}=3, l=1,2,3...,L; \)

\( Z \) is the bit representation of real numbers.

9. MFT functionality

Allocation by the proposed adaptive MFT method of a variable number \( M_1\leq M, M=M_{\text{BMT}}^1, B_{\text{MFT}}=3, l=1,2,3...,L \), spectral components \( \{x(m;\Omega_n,n;T_1)\}, m=0,1,2...,M-1, \Omega=\omega_D/(2M), n=0,1,2...,N/M-1, T_1=2\pi/\omega_D, l=1,2,3...,L \), harmonic components \( \{x_n(n-T)\}, m=0,1,2...,M-1, \) complex signal \( \{x(n-T)\}, n=0,1,2...,N-1, T=2\pi/\omega_D, \Omega=\omega_D/(2M), M=M_{\text{BMT}}^1, B_{\text{MFT}}=3, l=1,2,3...,L; \)

The proposed adaptive MFT method provides the ability to select not all \( M_1 \), but only "useful" frequency components \( \{x(m;\Omega_n,n;T_1)\}, m=0,1,2...,M-1, \Omega=\omega_D/(2M), n=0,1,2...,N/M-1, T_1=2\pi/\omega_D, \Omega=\omega_D/(2M), M=M_{\text{BMT}}^1, l=1,2,3...,L \), harmonic components \( \{x_n(n-T)\}, m=0,1,2...,M-1, \) complex signal \( \{x(n-T)\}, n=0,1,2...,N-1, T=2\pi/\omega_D, m=m, M=M; \)

The proposed adaptive MFT method allows calculating \( ml \)-th components at each \( l \)-th stage of the MFT \( \{x(m;\Omega_n,n;T_1)\} \) of instant spectrum \( \{X(m;\Omega_n)\}, m=0,1,2...,M-1 \) with fixed frequency step \( \Omega=\omega_D/(2M), M=M_{\text{BMT}}^1, l=1,2,3...,L \), based on the \( ml \)-th components already calculated at the previous \( (l-1) \)-stage MFT \( \{x_{(l-1);m;\Omega_{n,l-1},n;T_{1,l-1}}\} \) of spectrum \( \{X(m_{l-1};\Omega_{n,l-1})\}, m_{l-1}=0,1,2...,M_{l-1}-1, \) with a large frequency step \( \Omega_{l-1}=\omega_D/(2M_{l-1}), M_{l-1}=B_{\text{MFT}}^{l-1}, l=2,3,4...,L \), at \( M=M_{\text{BMT}}^{l-1}, M_{l-1}. \)

10. Discussion

Complete refusal to perform arithmetic multiplication operations with some increase in the number of arithmetic addition operations to reduce the computational complexity of the DFT of a complex signal.
\{x(n-T)\}, n=0,1,2\ldots N-1, T=2\pi/\omega_0, with variable resolution \Omega in frequency N of its m-th spectral components \{X(m,\Omega)\}, m=0,1,2\ldots N, \Omega=\omega_0/(2N), is provided in the proposed adaptive MFT method due to the "stepwise" frequency shift of these spectral components into the low-frequency region and low-frequency digital filtering of K-th order of the time-frequency samples of their statistics (estimates) at each l-th stage of the MFT, \(l=1,2,3\ldots L\), performing only algorithmic addition and shift operations.

The use of CORDIC method in the proposed adaptive MFT method provides hardware implementation on PLD of all procedures for adaptive transfer of spectral components to the low frequency region at \(l\)-th step MFT, \(l=1,2,3\ldots L\), only through the use of elementary adders and hardware shift registers.

Computational algorithms for digital filtering of \(ml\)-th frequency-time samples \(\{\exp[-i\cdot m\cdot\Omega_i\cdot n_i\cdot T_{1,i}]\cdot x_{1,i}([\max\{m/B_{MFT}\cdot\Omega_{1,i},n_i,T_{1,i}\}])\}, m=0,1,2\ldots M_l-1, \Omega_1=\omega_0/(2\cdot M_l), M_l=B_{MFT}\cdot M_l+1, \Omega_{1,i}=B_{MFT}\cdot \Omega_1, n_{i}=0,1,2\ldots N/M_l-1, T_{1,i}=2\pi\cdot M_l/(\omega_0), M_l=2^{j/l}, l=2,3,4\ldots L\), difference digital filter \(K_p\)-th order and \(J\)-th order of difference with binary (binary) values (-2, +2) and (or) trivial values (-1, 0, +1) integer \(k_p\)-th difference coefficients \(\{h_{pr}(J,k)\}, k=0,1,2\ldots K_p-1\), in the proposed adaptive MFT method, it is desirable to implement on the basis of DDF \(K_p\)-th order and first order of the difference, or the 2nd order of the difference, or the third order of the difference.

To select with equal accuracy all \(N\) various \(m\)-th frequency components of the instantaneous spectrum \(\{X(m,\Omega)\}, m=0,1,2\ldots N, \Omega=\omega_0/(2N),\) complex signal \(\{x(n-T)\}, n=0,1,2\ldots N-1, T=2\pi/\omega_0,\) only one differential digital low-pass filter \(K\)-th order in the proposed adaptive MFT method based on DDF \(K_p\)-th order and \(J\)-th order of difference, \(K_p=K_J\), we can and should use the symmetry of the impulse response \(\{h(k)\}, k=0,1,2\ldots K-1,\) equivalent low-pass digital filtering of the \(K\)-th order, \(K=K_pJ\).

11. **Conclusions**

The research results have shown and confirmed the possibility of DFT complex signal without performing arithmetic multiplication operations.

The development and formalization of the concept of MFT complex signal by DDF is the development of the theory and methods of multirate MDF, proposed and developed by V.V. Vityazev and S.V. Vityazev [12-13]. The proposed methods for such a DFT can and should ensure the minimization of hardware costs for DSP when it is implemented in software and hardware on PLD. The reliability of the study results was confirmed by their agreement with the theory and methods of FFT and recurrent DFT. Digital MFT algorithms can be used in DSP to calculate the instantaneous spectra of processed signals similar to digital FFT algorithm and / or for continuous analysis of their spectra similar to the digital recursive DFT algorithms.

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