The gravitational waveforms of white dwarf collisions in globular clusters

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Abstract. In the dense central regions of globular clusters close encounters of two white dwarfs are relatively frequent. The estimated frequency is one or more strong encounters per star in the lifetime of the cluster. Such encounters should be then potential sources of gravitational wave radiation. Thus, it is foreseeable that these collisions could be either individually detected by LISA or they could contribute significantly to the background noise of the detector. We compute the pattern of gravitational wave emission from these encounters for a sufficiently broad range of system parameters, namely the masses, the relative velocities and the distances of the two white dwarfs involved in the encounter.

1. Introduction
The Galactic population of field binary white dwarf is one of the most promising sources of gravitational waves, since it was shown long ago that LISA could detect them individually (Bender et al. 1998). Field binary white dwarfs are the natural outcome of the evolution of many binary systems and it can be shown that due to tidal interactions their orbits are usually circular. Thus, the gravitational wave pattern of these systems is easy to compute since the two white dwarfs can be safely assumed to be point masses. Due to the ensuing emission of gravitational waves, the orbital separation of these systems decreases and the final fate of these systems is to coalesce. Consequently, the gravitational wave emission resulting from the merger of such systems has been the subject of some recent studies (Guerrero et al. 2004; Lorén–Aguilar et al. 2005). These studies show that the resulting pattern of gravitational wave emission is rather simple. In particular, it has been shown that these waveforms do not show a prominent peak and that the gravitational wave signal rapidly fades in a few orbital periods.

The typical stellar densities of globular clusters are \( \sim 10^3 \text{pc}^{-3} \) and, due to their long lifetimes, their central regions contain many collapsed and degenerate objects, such as white dwarfs. In such a dense environment, where stars can pass very close one to another, every star is expected to suffer one or more strong encounters during its lifetime. In fact, it has been predicted (Willems et al. 2007) that dynamical interactions in globular clusters can form double white dwarf systems which could be eventually detectable by LISA. Moreover, in globular clusters, dynamical interactions can form binary white dwarf systems at an enhanced rate compared to...
Figure 1. Temporal evolution of the close encounters of the double white dwarf systems discussed here. The left panels show the evolution of the positions of the particles for the $0.6+0.8 \, M_\odot$ system, when an initial velocity $v_i = 50 \, \text{km/s}$ and an impact parameter of $b = 0.6 \, R_\odot$ are adopted. The right panels show the evolution of the same system when the initial conditions are $v_i = 150 \, \text{km/s}$ and $b = 0.3 \, R_\odot$. The positions of the particles have been projected onto the $xy$ plane. The units of positions are $10^9 \, \text{cm}$. Times are shown in the right upper corner of each panel. These figures have been done using the visualization tool SPLASH (Price 2007).

the field (Shara & Hurley 2002). Additionally, such systems are generally formed with non-zero eccentricities. This is an important issue because tidal interactions in eccentric systems substantially modify the gravitational wave emission pattern, thus providing us with a unique opportunity to use LISA to check our understanding of the interiors of white dwarfs (Willems et al. 2007). Here we study the dynamical interaction of white dwarfs in globular clusters and we compute the associated gravitational wave radiation. Due to the complicated dynamics of the process, numerical simulations of such encounters are essential to obtain realistic sets of waveforms. Consequently, we have performed a series of SPH simulations of strong encounters of white dwarfs in globular clusters, covering a broad enough range of system parameters, which include impact parameter and relative stellar velocities for two representative masses of the white dwarfs involved in the interaction.

2. Input physics and method of calculation

We follow the hydrodynamic evolution of the interacting white dwarfs using Smoothed Particle Hydrodynamics (SPH). This method is totally lagrangian and does not require a grid making it specially suitable for studying an intrinsically three-dimensional problem. We will not describe in detail the most basic equations of our numerical code, since this is a well-known technique. Instead, the reader is referred to the abundant literature for descriptions of this technique, and to Guerrero et al. (2004) where all the relevant input physics for the problem at hand and of our specific implementation of this technique are explained in depth. We compute the gravitational wave emission in the slow-motion, weak-field quadrupole approximation (Misner et al. 1973). In order to compute a large enough set of gravitational wave patterns we have relaxed two initial white dwarf models with masses $0.6 \, M_\odot$ and $0.8 \, M_\odot$, respectively. The mass of the SPH
Figure 2. Gravitational wave emission for the cases shown in Fig. 1. The left panel corresponds to the case in which two white dwarfs of masses 0.6 and 0.8 $M_\odot$, respectively, interact to produce a head-on collision. The right panel corresponds to the case these two white dwarfs interact and a lateral impact occurs. The dimensionless strains $h_+$ and $h_\times$ are measured in units of $10^{-22}$. The source is located at a distance of 10 kpc.

particles were the same in both cases in order to avoid numerical artifacts. Consequently, the 0.6 $M_\odot$ white dwarf was relaxed using $2.0 \times 10^4$ particles, whereas the 0.8 $M_\odot$ white dwarf needed $2.6 \times 10^4$ particles. For the interactions in which two equal-mass white dwarfs were involved we have used the otherwise typical 0.6 $M_\odot$ model, whereas for the other cases we have used the 0.6 $M_\odot$ and the 0.8 $M_\odot$ models. We have fixed the initial distance $x$ between the stars and their spin (fixed to typical values in field white dwarfs) allowing the impact parameter $b$ (distance between stars on the $y$ direction) and the initial velocity $\vec{v}_i = (v_i, 0, 0)$ to be our free parameters. We have performed 11 simulations with impact parameters ranging from 0.3 $R_\odot$ to 0.9 $R_\odot$ and initial velocities from 50 to 150 km/s. These values of the initial relative velocities and impact parameters are typical of globular clusters.

3. Results

We have obtained three different kinds of behavior, depending on the initial parameters adopted for the close encounter. When the initial velocity $v_i$ is relatively small and the impact parameters are not very large the gravitational forces bend the initial trajectories and the result of the interaction is a head-on collision. For increasing initial velocities there is a transition region in which the initial trajectories are less affected by the gravitational interaction and the final outcome turns out to be a lateral collision. Finally, for sufficiently large initial velocities and above a threshold impact parameter which depends on the initial relative velocity of the interacting white dwarfs, the stars do not collide but, instead, form an eccentric binary system.

In this work we pay special attention to those systems in which a collision occurs. The cases in which an eccentric binary system is formed will be discussed at length elsewhere. Fig. 1 shows two typical examples of the temporal evolution of some of the close encounters studied here. The left panels of Fig. 1 show a case in which a head-on impact occurs. The masses of the colliding white dwarfs are, respectively, 0.6 and 0.8 $M_\odot$. The initial conditions adopted in this case were $v_i = 50$ km/s and $b = 0.6 R_\odot$. The right panels of Fig. 1 show the case in which
a lateral collision of the two intervening white dwarfs occurs. In this case the initial conditions were \( v_i = 150 \text{ km/s} \) and \( b = 0.3 R_{\odot} \). The trajectories of the centers of mass of each individual white dwarf are shown using solid lines. These lines can be used as a visual aid to follow the trajectories of the intervening white dwarfs. As can be seen in these panels, once mass transfer starts it occurs on a dynamical timescale and the coalescence occurs in a few seconds. The final outcome of the merger process is in both cases a central object with ellipsoidal shape. The characteristics of these central objects will be discussed in detail in a forthcoming publication and here we concentrate in describing the gravitational wave pattern radiated by these systems. In all the cases studied here, at early times the system does not radiate gravitational waves because the accelerations are very small. Once the stars approach sufficiently each other, the gravitational wave signal rapidly grows. In the case of a head-on collision the signal first grows and then suddenly decreases — see the left panel of Fig. 2. This peak is followed by a series of peaks of smaller amplitude, the so-called ring-down phase. These peaks are caused by the circularization of the orbits of a spiral arm which forms during the merging process. In the case of a lateral collision the gravitational wave emission is more irregular and does present several amplitude peaks — see the right panel of Fig. 2. This occurs because in a lateral collision before the final merger, the two interacting white dwarfs describe orbits of decreasing separation, in which sizeable mass transfer occurs between the two components of the system. Finally, in those cases in which an eccentric binary forms the gravitational wave patterns present a regular, periodical behavior, which we do not show here for the sake of conciseness.

4. Conclusions
We have computed the emission of gravitational waves arising from the dynamical interaction of two white dwarfs in the dense central regions of a globular cluster. These interactions are thought to be quite frequent and thus it is important to characterize the resulting gravitational waveforms. We have done so for a sufficiently broad range of initial relative velocities and impact parameters and for two different component masses. We have presented here a set of calculations corresponding to close encounters in which a typical \( 0.6 M_{\odot} \) white dwarf and a rather massive \( 0.8 M_{\odot} \) companion are involved. We have found that there are three possible different outcomes, depending on the initial relative velocity and impact parameter of the system, which lead to distinctive patterns of gravitational wave radiation. Specifically, we have characterized the range of parameters for which head-on mergers, lateral collisions and formation of an eccentric binary system occur. For the cases in which the outcome is a collision (either a head-on one or a lateral one) we find that signal is a strong burst of gravitational waves, followed in some cases by a series of smaller peaks. Such events will not be detectable by LISA. On the contrary if the initial conditions are such that the dynamical interaction leads to the formation of an eccentric binary system, the signal is periodical and will probably be detectable with LISA.

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