Variable Stiffness Iterative Learning Control for Flexible-Joint Robot

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Abstract. Flexible-Joint (FJ) robots have attracted researchers’ attention in these years for their good compliance and safety. But as a matter of fact, it is hard to design a suitable controller with stiffness adaptation for FJ robots due to the model uncertainties and the nonlinear systems when using elastic components. In this paper, an iterative learning control method for flexible-joint robot with model uncertainties in both robot dynamics and actuator dynamics is proposed, which involves the use of mechanical elastic elements with adjustable stiffness. Adjusting the stiffness of the joints by using the positions of the roller installed inside of the actuator, the joints generate fixed periodic motions and reduced the tracking error. We show the theory of the variable stiffness and the adaptive law, which involve the iterative learning controller. Simulation results also demonstrate that we achieve a perfect result while generating the desired motions.

Index Terms: iterative learning; stiffness adaptation; flexible-joint robot; motion tracking.

1. Introduction
Over the past three decades, FJ robots have been the focus of numerous researchers because of their good compliance and safety. It is safer to interact with the outside world whether it is the FJ robot itself or the person interacting with it. Even if it is subjected to external shocks, it still proves robust to damage, which makes FJ robots better able at adapting to outside environments and enhances their work safety and reliability.

The FJ robot is a very complex dynamic system whose dynamic equations are characterized by nonlinearity, strong coupling, and consolidation. For the study of the dynamics of flexible arms, the establishment of the model is extremely important. The FJ robot is not only a rigid-flexible nonlinear system, but also a nonlinear system in which the system dynamics and control characteristics are coupled. The parametric uncertainties, nonlinear dynamics, and other disturbances will arise in the control procedure. These inaccuracies may lead to poor performance when controlling the FJ robots, thus any practical controller design should not ignore these influences.

A lot of researchers have developed controllers for robots using different control methods. E. Gurkan [1] and C. W. Park [2] may use fuzzy control to deal with the uncertainties of the FJ robots. C. J. B. Macnab [3], C. Kwan [4] and F. Abdollahi [5] used neural networks (NN) control to solve this problem. As a matter of fact, many controller parameters are considered while using neural network learning and some large networks may require much more training data, which would lead to cumbersome calculations [8][9]. In this case, it is of great need to obtain a highly configured computer or other processors, which limits application scope. And it is difficult to guarantee the fast convergence of the NN controllers.
Moreover, the robotic manipulator with active adjustment capability can select the optimal stiffness according to the environment in actual work. If the space is narrow or there are many obstacles, the robot should actively reduce the stiffness and reduce the damage caused by accidental collision [13][14]. If it is in a relatively open work space, the robot should increase the stiffness to meet the operational accuracy requirements. In the future, the manipulator or biped robot should have self-adjusting ability to work in various environment, so as to meet the needs of different working conditions or scenes. Therefore, we try to meet the various conditions by actively controlling the flexibility of the manipulator.

This paper introduces an iterative learning control method [10][12] for FJ robot with model uncertainties in both robot dynamics and actuator dynamics, which involves the use of mechanical elastic elements with adjustable stiffness. Adjusting the stiffness of the joints by using the positions of the roller installed inside of the actuator, the joints generate fixed periodic motions and reduced the tracking error. We show the theory of the variable stiffness and the adaptive law, which involve the iterative learning controller. Simulation results also demonstrate that we achieve a perfect result while generating the desired motions.

2. Problem Formulation

This section formulates the problem of the FJ robot and introduces our control objective.

2.1. Dynamics Model of Flexible-Joint Robots

The dynamic model of FJ robots, can be given by

\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + Dq\dot{q} + g(q) + K(q - \theta) = 0, \tag{1}
\]

\[
B\ddot{\theta} + D_{\theta}\dot{\theta} + K(\theta - q) = \tau, \tag{2}
\]

where \(\theta \in \mathbb{R}^n\) is the vector of actuator with elastic components, \(M(q) \in \mathbb{R}^{n \times n}\) is the inertia matrix, \(C(q, \dot{q})\dot{q} \in \mathbb{R}^n\) denotes the Coriolis and centripetal forces, and \(g(q) \in \mathbb{R}^n\) represents the gravity vector of robot. \(K \in \mathbb{R}^{n \times n}\) is a diagonal stiffness matrix of actuator, \(B \in \mathbb{R}^{n \times n}\) is a diagonal inertia matrix of actuator, \(D_{\theta} \in \mathbb{R}^n\) and \(D_{\theta} \in \mathbb{R}^n\) represent the viscous frictions for the manipulator and the actuator, and \(\tau \in \mathbb{R}^n\) is a vector of the input torque exerted on the actuator. Equation (1) represents the rigid robot dynamics, and (2) describes the dynamic model of actuator.

We can also conclude some following important properties of the dynamics:

1). the matrices \(M(q)\) and \(B\) are positive definite;
2). the matrix \(M(q) - 2C(q, \dot{q})\) is skew-symmetric;
3). \(D_{\theta} = \text{diag}(d_{\theta_1}, ..., d_{\theta_2})\) and \(D_{\theta} = \text{diag}(d_{\theta_1}, ..., d_{\theta_2})\), represent the friction matrices, are diagonal and positive definite;

2.2. Control Objective

First of all, we define some pre-specified trajectories, and our control objective is to track them with little error and time, i.e.,

\[
q(t) \to q_d(t), \Delta q(t) \to 0 \text{ as } t \to \infty
\]

where the desired trajectory \(q_d(t)\) and its time derivatives \(\dot{q}_d(t)\) are bounded. We aim to bounded the error between the desired trajectory and the actual trajectory.

Then, during the process, we hope to adjust the stiffness \(K\) to its optimal value, which we will discuss in the next chapter.

\[
K \to K_{opt}, q(t) \to q_d(t), \Delta q(t) \to 0 \text{ as } t \to \infty
\]
3. The Variable Stiffness of the Actuator

3.1. The Theory of Variable Stiffness

Now we discuss the theory of variable stiffness. In Figure 1, one of the spring leaves was fixed to the big motor on the left, and the other was connected with the joint on the right by using the rotator. Between the motor and the joint, there was a roller, which could move from left to right or move back driven by the small motor. The positions of the roller mean different stiffness of the joint. As we can see, the effective length of the spring was designed by the roller position. Obviously, when the roller positions change, different torques are required to achieve the same relative rotation angles, which make sense the different torsional stiffness [11]. Therefore, the torsional rigidity between the input and output can be adjusted by adjusting the position of the roller [6].

![Figure 1. The Internal structure of the actuator.](image1)

![Figure 2. The physical model of the spring.](image2)

In Figure 2, \( L_0 \) is the whole length of the spring from A to C. The positions of A and C cannot be changed, so we just only change the position of B. The length from B to C means the effective length of the leaf spring, which is signed as \( l \). We assume that the force \( F \) is applied at C, which will make a deflection angle \( \theta \) on the spring leaf.

The joint is a rotating joint that transmits flexible torque. The joint stiffness is defined by the rotational stiffness. The stiffness is calculated as:

\[
K = \frac{dT}{d\theta},
\]

where \( T \) is the torque at the free end of the spring piece, and \( \theta \) is the input and output relative torsion angle.

In Figure 2, we can get the bending deflection \( \omega \), and the relationship between \( \theta \) and \( \omega \) is as follows:

\[
\theta = \sin^{-1}\frac{2\omega}{D},
\]

where \( D \) is the diameter of the joint.

We can rewrite the equation of \( \omega \):

\[
\omega = \frac{4 \cdot F \cos \theta \cdot l^2}{Ebh^3} L_0,
\]

where \( E \) is the Young's modulus of the leaf spring, \( b \) is spring leaf width, and \( h \) is the thickness of the spring leaf. From (3)(4)(5), we can obtain \( F \) as:

\[
F = \frac{Ebh^3 D \cdot \tan \theta}{8l^2 L_0},
\]

Then we have,

\[
T = F \cdot D/2,
\]

where \( T \) is the torque at the free end of the spring piece.
So, we can obtain the stiffness $K$:

$$K = \frac{d\tau}{d\theta} = \frac{Ebh^2D^2}{16l^2L_0\cos^2\theta}, \quad (8)$$

### 3.2. Stiffness Adaptation

In this section, we discuss the stiffness adaptation. We hope to adjust the stiffness of our actuator during the process without requiring exact parameter values of the robot system. We obtain the adaptive control law $\dot{K}$ for the stiffness $K$, which means:

$$\dot{K} = \Gamma \cdot \Delta q \cdot y, \quad (9)$$

$$y = \Delta \dot{q} + C \Delta q, \quad (10)$$

where $K = (k_1, ..., k_n)^T$, $C = \text{diag}(b_1, ..., b_n) = K_p K^{-1}_d$, $\Gamma \in \mathbb{R}^{n \times n}$ is a positive definite matrix of adaptive gains, and $\Delta q = (\Delta q_1, ..., \Delta q_n) = q - q_a$.

### 4. Iterative Learning Controller Design

We now design the iterative learning controller for the actuator. The actuator torque is designed by

$$\tau = -K_p \Delta q - K_d \Delta \dot{q} + \Xi_u, \quad (11)$$

where $K_p = \text{diag}(k_{p1}, ..., k_{pn})$ is the angular feedback gain, $K_d = \text{diag}(k_{d1}, ..., k_{dn})$ is the velocity feedback gain, and $\Xi_u$ represents the feedforward torque, which is defined later.

The feedforward torque $\Xi_u$ adopts an iterative learning method, and uses the error correction control information input of the previous or previous (period) operation, so that the repeated task can be better in the next operation. Repeatedly until the output track of the entire time interval tracks the desired trajectory [7].

Therefore, the feedforward $\Xi_u$ of $i \in \mathbb{N}$ ($i \geq 2$) cycle is updated based on the manner of iterative learning control

$$\Xi_u(t) = \Xi_u(t - T) - \beta (y(t - T) - W_i q_a(t - T)), \quad (12)$$

where $\beta \in \mathbb{R}$ is the learning gain, $W_i = \text{diag}(w_{i1}, ..., w_{in})$, $y = (y_1, ..., y_n)^T$. The feedforward torque $\Xi_u$ in the first cycle is initialized as $\Xi_u(t) = 0$.

$$w_{ij} = \int_{t_T}^{T} y_j q_a dt \int_{t_T}^{T} q_{i\dot{a}} dt, \quad (j = 1, ..., n), \quad (13)$$

The coefficients $w_{i1}, ..., w_{in}$ of the $i + 1$ cycle ($iT \leq t < iT + T$) are updated at the beginning of the $i + 1$ cycle ($t = iT$) by using the signals of the $i$ th cycle ($iT - T \leq t < iT$).

### 5. Simulation Results

In this section, to illustrate the validity of the controller, we conduct a simulation for the two-link FJ manipulator. The results of the joint trajectory, error, torque and stiffness $K$ are given in the simulation.

#### 5.1. Parameters Setting

The two links of FJ robot have lengths $l_1 = 0.30m$, $l_2 = 0.30m$, distances between their joints and the respective centre of masses, $l_{c1} = 0.15m$, $l_{c2} = 0.15m$, moments of inertia $I_{c1} = I_{c2} = 0.5kg \cdot m^2$, and their masses are $m_1 = 1.5kg$, $m_2 = 1.0kg$. The inertia matrix $B = 0.2I_2kg \cdot m^2$, where $I_2 \in \mathbb{R}^{2 \times 2}$ is an identity matrix, the friction matrix $D_\theta = l_2 kg \cdot m^2$. The above parameters were the basic elements of the FJ robot, which could be different when using different model.
The desired motion $q_d$ was designed as: $q_{d1} = \cos(1.5 \cdot t + 3/\pi)$, $q_{d2} = 1.2 \cdot \cos(1.5 \cdot t)$. We define tow different desired motions so as to indicate the adaptive ability of our controller. The angel and velocity feedback gains for the rigid robot were set as $k_p = 25$, $k_p = 22.5$, $k_d = 15$, $kd_2 = 14.5$. And the learning parameters were set as $\Gamma = [1.5,0;0,1]$, $\beta = 0.005$.

5.2. Results
Simulation results are shown in Figure.3 - Figure.7. Figure.3 and Figure.4 show the trajectories of each joint; Figure.5 shows the tracking errors of each joint; Figure.6 shows the stiffness $K$. Figure.7 shows the torques that applied in joint 1 and joint 2 using our controller.

![Figure 3. Trajectory for joint 1.](image1)

![Figure 4. Trajectory for joint 2.](image2)

![Figure 5. Tracking error for each joint.](image3)

![Figure 6. The stiffness $K$ for each joint.](image4)

![Figure 7. The torque of each joint.](image5)
From the simulation results, the average trajectory errors for the iterative learning controller are 0.015 rad and 0.011 rad. The optimal stiffness $K$ of joint 1 is 1.75 Nm/rad, and the optimal stiffness $K$ of joint 2 is 0.25 Nm/rad. We set the original stiffness $K$ as 0 Nm/rad because it is easy to discuss the process of the optimal stiffness generation. The torque for joint 1 is 10.5 Nm, and for joint 2 is 2.8 Nm. Obviously, the iterative learning controller increases the tracking accuracy, which represents that the good performances of our control. The iterative learning controller considers the whole dynamic model of the FJ robot, which includes the rigid robot and the actuator. While generating the desired motions, our controller finally find an optimal stiffness for the system without requiring additional parameters of the actuator. Moreover, the iterative learning controller clearly guarantees the convergence of the close-loop system after several period.

The above simulation results showed us the tracking errors became smaller and smaller when the stiffness $K$ reached to its optimal value. Then, we didn’t have to adjust many control parameters, which made it much easier to applied.

6. Conclusion
Adjusting the stiffness of the joints by using the positions of the roller installed inside of the actuator, the joints generate fixed periodic motions and reduced the tracking error. We show the theory of the variable stiffness and the adaptive law, which involve the iterative learning controller. Simulation results demonstrate that our controller gives a perfect performance while generating the desired motions. One of our core future works is to apply our controller to control the real-world manipulators with external disturbance.

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