Divergence and Flutter of Multilayered Laminated Structures

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Abstract. The aim of the present paper is to describe and discuss on possible methods used in the divergence and flutter analysis of composite multilayered laminated structures. Dynamic instability is a very dangerous phenomenon and aeroelastic optimization has become an important element of the composite flight vehicle design process. A special attention is focused on the analysis of cylindrical panels and flat plates using numerical (finite element) methods. The structures with different flow angles, fibre orientations, stacking sequences and boundary condition are considered.

1. Introduction

During the process of design composite flight vehicles, special attention should be paid to aeroelastic problems. The complexity of analysis and the number of problems depend on the speed regime, type of structure and the design stages. The durability of flexible construction elements depends on their dynamic stability under specific flow conditions. Loss of stability at a certain speed may consist in the occurrence of self-excited vibrations with rapidly increasing amplitude. Under certain conditions and at a slightly higher speed, the loss of stability may consist of a monotonic (non-oscillating) increase in torsion angles and deflections. Divergence and flutter are very important problems and must be considered in the early stage of the design process.

Flutter can be described as a self-excited dynamic instability in which the structure absorbs energy from the airflow. It is a combination of inertial, elastic and aerodynamic forces where the structure and the flow interact with each other. This problem usually starts from small aerodynamic or mechanical disturbance above a critical flow velocity and it gives large vibration amplitudes. Finally, damage occurs within a short period of time.

Divergence is an instability in which aerodynamic forces produced by structural deformation are large enough to maintain these deformations. Depending on the aerodynamic, geometry and structural properties of the surface, this condition can arise in several ways. This condition is given in terms of dynamic pressure or air speed. At the critical divergence speed, statically stable equilibrium condition does not exists.

A number of vibration problems in aeroelastic engineering arise for elastic systems both of a finite number of degrees of freedom and continuous that are subjected to forces commonly called non-conservative. It means, that forces that are explicitly time independent cannot be represented as the gradients of certain energy functions. The divergence and flutter of thin plates and shells in subsonic or supersonic gas flows just belongs to this group of problems. These phenomena are important and significant problems encountered in the design of turbine blades in the turbomachines [1] or aircraft...
constructions [2,3]. In the divergence and flutter analysis the attention is mainly focused on the
discussion of different problems that can affect the structural behaviour, i.e. the aerodynamic theories,
the form of boundary conditions, the structural geometry (the analysis deals mainly with 2D
structures), the material properties and the effects of aerothermoelastic coupling. Above dynamic
instability problems are discussed by Muc and Flis [4].

A general description of the divergence and flutter problems can be found in literature – see [5-11].
Qiaozhen Sun and Yufeng Xing [12] presented and discussed various aspects related to flutter
characteristics. A broad literature review of divergence and flutter problems is presented by Muc et. all
[13] where authors described fundamental trends in the optimal design not only for multilayered
laminated structures but also for nanocomposites, sandwich structures, functionally graded materials
and piezoelectric materials. We do not intend to repeat a literature review in this paper.

2. Divergence and flutter analysis

The characteristics of the divergence and flutter are determined by different mechanical and
aerodynamic parameters. Aerodynamic reactions depend on the incidence angle, pure and combined
mode shape, Mach number (\(M=\)flow velocity-to-velocity of sound) and reduced frequency.

To evaluate flutter characteristics various methods and theories can be used depending on the flow
velocity (Mach number). For subsonic speed, the flutter characteristics are determined using the
Doublet-Lattice Method, while for supersonic speed, Piston Theory, Navier-Stokes Equation or linear
potential theory are used [14]. For the present study, we described methods, such as: panel methods
(Vortex-Lattice and Doublet-Lattice Method), Piston Theory and Movchan-Krumhaar Method.

The first method, Vortex-Lattice Method (VLM), is using to calculate the pressure distribution on a
lifting surface under steady compressible subsonic flow. To apply VLM, the lifting surface is divided
into trapezoidal small elements (boxes).

Assuming that \(\Gamma_n\) describes the horseshoe vortex strength of the \(n\)th panel, in a uniform flow \(U\), the
downwash at panel control point (3/4th chord) of the wing box is given by:

\[
\left\{ \frac{w_n}{U} \right\} = \frac{1}{4\pi} [A] \{ \Delta C_p \}
\]

For flutter analysis of lifting surfaces (under subsonic speeds) nearly three decades Doublet-Lattice
Method (DLM) is used [15-17]. The complete development of this method is available in Ref. [18],
where authors described a basic flow and numerical calculations.

To solve the lifting surface problems (supersonic and subsonic) need to start with the pressure-
downwash integral equation:

\[
w(x, y) = \frac{1}{8\pi\rho U} \int_S K(x, y; \xi, \eta; M_\infty, k) \Delta p(\xi, \eta) d\xi d\eta
\]

where \(K(x, y; \xi, \eta; M_\infty, k)\) is the kernel of the integral equation, different at subsonic and
supersonic flow.

In this method the surface is divided into small trapezoidal boxes arranged in lines parallel to the
surface edges and free stream (hinge lines and fold lines lie on the box boundaries). The pressure
acting on an area is assumed to be uniformly concentrated on a line. The flow singularities used to
model the lifting surface, is a set of line segment of acceleration potential doublets.

The normal velocity induced, at a point \((x, y)\), can be described as a sum of the normal velocities
induced by the \(n\) doublet lines:
\[ w(x, y) = \sum_{n=1}^{N} \Delta p_n \frac{1}{8\pi \rho U} \iint_{S} K(x, y; \xi, \eta; M, k) d\xi d\eta \]  

(3)

The \( \Delta p_n \) can be specified when above equation is used at \( n \) points on the surface and \( w(x, y) \) is known from \( i^{th} \) mode shape. For the discrete panels the pressure-downwash relation are given by:

\[ \{w\} = [D]\{\Delta p\} \]  

(4)

Unsteady pressure is described by below equation:

\[ \{\Delta p\} = [D]^{-1}\{w\} \]  

(5)

Flutter characteristics can be predicts quite accurately using Piston Theory for supersonic flight speeds (Mach number > 1.5). According to this theory the unsteady aerodynamic forces on the wing element \( dxdy \) are influenced by the motion of the element alone. The local isentropic perturbation pressure (in terms of the local downwash) is given by:

\[ p(x, y, t) = p_\infty \left[ 1 - \frac{\gamma - 1}{2} \frac{w(x, y, t)}{c_\infty} \right]^{\gamma/\gamma - 1} \]  

(6)

where \( p \) is undisturbed flow pressure and \( c \) is velocity of sound, \( \gamma \) is the ratio of specific heat of gas. The Binomial Expansion of Equation 5 yields:

\[ p - p_\infty = \rho c_\infty \left\{ \frac{w(x, y, t)}{c_\infty} \right\} + \frac{\gamma + 1}{4} \left\{ \frac{w(x, y, t)}{c_\infty} \right\}^2 + \frac{\gamma + 1}{12} \left\{ \frac{w(x, y, t)}{c_\infty} \right\}^3 + \ldots \]  

(7)

For the general motion of an airfoil of symmetric thickness distribution \( 2Z(x,y) \), without mean camber and mean surface displacement \( w(x, y, t) \) it follows from Equation 7, taking into account the first two terms:

\[ \Delta p(x, y, t) = \frac{4q_\infty}{M} \left[ 1 + \frac{\gamma + 1}{2} \frac{\partial Z}{\partial x} \left( \frac{\partial}{\partial x} + \frac{M^2 - 11}{M^2 - 2V} \frac{\partial}{\partial t} \right) w(x, y, t) \right] \]  

\[ \Delta p = 2\rho_\infty C_\infty \left[ 1 + \frac{\gamma + 1}{2} M \frac{\partial Z}{\partial x} \left( U \frac{\partial}{\partial x} + \frac{M^2 - 1}{M - 2} \frac{\partial}{\partial t} \right) w(x, y) \right] \]  

(8)

The aerodynamic pressure \( \Delta p \) for multilayered laminated panels follows from Equation 8 and is defined as:

\[ \Delta p = A \left( \frac{\partial w}{\partial x} \cos \theta_\infty + \frac{\partial w}{\partial y} \sin \theta_\infty \right) - \mu \frac{\partial w}{\partial t} , A = \frac{\rho_\infty V_\infty^2}{\sqrt{M_\infty^2 - 1}}, \mu = \frac{\rho_\infty V_\infty (M_\infty^2 - 2)}{(M_\infty^2 - 1)^{3/2}} \]  

(9)

The Movchan-Krumhaar Method is used to determine the system stability when the aerodynamic (and viscous structural) damping is added. Movchan applied this method for flutter analysis of flat panels and Krumhaar extended it for flutter analysis of cylindrical shells.
For a finite value of $g$, the eigenvalue $Z^2$ in flutter determinant is complex. The main problem is to find a relationship between the eigenvalue $Z_n(Q)$ with external flow variable ($Q$), which will provide the stability boundary criterion. The condition can be graphically represented by a stability parabola whose equation is given by:

$$ (Im Z)^2 = g Re Z $$

For a given value of flow parameter (dynamic pressure parameter $Q$), the stability of the system is determined by, whether $Z_n(Q)$ lies inside the stability parabola, outside it or on it.

Several methods of divergence analysis predicting the effects of aeroelasticity are available in the literature. They can be divided into: Reference Surface Method, Iterative method, force slope method and Modal Approach [19]. In the present paper the force slope method are discussed.

The method of ascribing the system under consideration by a finite number of degrees of freedom is done by dividing the structure into a finite number of discrete elements, or by using generalized coordinates in conjunction with methods akin to those of the Rayleigh-Ritz and Galerkin (RRG). Methods such as RRG are often used in the analysis.

Vortex Lattice Method (VLM) is used to calculate the aerodynamic loads. The pressure-downwash relationship affects the pressure loading on a flexible lifting surface and can be described as:

$$ \{\Delta p\} = [A] - q[S][B]^{-1}[S]{F} $$

where, $[A]$ - aerodynamic influence coefficient matrix, $[S]$ - structural influence coefficient matrix and $\{F\}$ - aerodynamic load vector.

For a three-dimensional wing divergence, a solution for the critical dynamic pressure can be obtained for which infinite flexible pressure loadings would result. Given the Equation 11 this condition will occur for:

$$ ||[A] - q[S][B]|| = 0 $$

This eigenvalue problem is generalized, the divergence dynamic pressure is given by the positive smallest eigenvalue ($q = q_{div}$).

2.1. Optimal Design

For the first time, finite element method (FEM) was introduce by Olson in 1967 to plate flutter analysis [20]. In a review of existing works, it was noted that many of researchers create their own procedures. The commercial FE packages as e.g. Abaqus, Ansys, or Nisa II Family of Programs (Nisa II/Areo) are also used to divergence and flutter analysis. There are many advantages of using the FE packages as e.g.:

- application different boundary conditions,
- investigate arbitrary laminate configurations with no elimination of the $B_{ij}, A_{16}, A_{26}, D_{16}, D_{26}$ terms in the stiffness matrices,
- possibility to use the first order transverse shear shell/plate described by five independent parameters (three displacements and two angles of rotations) instead of classical plate/shell theory – three independent parameters (three components of displacements).

In the present study the Nisa/Areo package is used and first order transverse shear deformation theory is applied.

For analysis of aeroelastic flutter stability the following actions are generally applied:

1. The FE structure model is generated and then DYNAMICS Module of NISA is used to analysis of free vibration and appropriate files are saved for subsequent aeroelastic analysis.

2. The flutter analysis for high flight speed ($M>1.5$) is calculated using the Piston Theory.
To model multilayered laminated structures, NKTP 32 finite element (five degrees of freedom at each node) is used.

The optimization problem formulated in the present research is to find the maximal value of the critical aerodynamic pressure, i.e.:

$$\text{Max } \Lambda(s)$$

(13)

where the vector $$s$$ is the set of possible design variables. The analysis is carried out for two types of possible fibre orientations (stacking sequences) in the laminates - see Muc [21]:

- angle-ply laminates with an even number of layers (the same mechanical properties and thickness of each layer) and with the orthotropic axis of symmetry in each ply alternately oriented at $$+\theta$$ and $$-\theta$$; two terms in the stiffness matrix [B] are not equal to zero ($$B_{16}$$ and $$B_{26}$$) but the coupling effects can be neglected for large number of plies N,

- symmetrical laminates with an even number of layers (the same mechanical properties and thickness of each layer) and with discrete fibre orientations: $$0, \pm 45, 90$$. For angle-ply laminates fibre orientation angle $$\theta$$ is treated as the design variable $$s$$, whereas for symmetrical laminates the design variable characterizes by the stacking sequences. The nonzero terms in the stiffness matrices ($$A_{11}, A_{12}, A_{22}, A_{66}, D_{11}, D_{12}, D_{22}, D_{66}$$) are functions of $$\cos(2\theta)$$ for angle-ply laminates. For symmetrical laminates (with discrete fibre orientations) the total number of design variables increases to $$3^N/4$$. Muc [22, 23] reduced this number to four by applying of the following values:

$$x_{1A} = \frac{4}{N} \sum_{i=1}^{N/4} i(\theta = 0^0)$$

$$x_{2A} = \frac{4}{N} \sum_{i=1}^{N/4} i(\theta = 90^0)$$

$$x_{1D} = \left(\frac{4}{N}\right)^3 \sum_{i=1}^{N/4} [3i(i-1)+1](\theta = 0^0)$$

$$x_{2D} = \left(\frac{4}{N}\right)^3 \sum_{i=1}^{N/4} [3i(i-1)+1](\theta = 90^0)$$

(14)

Since the terms $$A_{16}, A_{26}, D_{16}, D_{26}$$ are functions of $$\sin(2\theta)$$ only so that they can be expressed by the above variables taking into account the following identities:

$$x_{1A} + x_{2A} + \frac{4}{N} \sum_{i=1}^{N/4} i(\theta = 45^0) = 1$$

$$x_{1D} + x_{2D} + \left(\frac{4}{N}\right)^3 \sum_{i=1}^{N/4} [3i(i-1)+1](\theta = 45^0) = 1$$

(15)

Figure 1 shows the graphical representation of the design variables. To characterize adopted design variables in this work two planes (spaces) are introduced ($$x_{1A}, x_{2A}$$) and ($$x_{1D}, x_{2D}$$). For angle-ply laminates fibre orientations are illustrated by one curve (the parabola), whereas for symmetrical laminates by the parts of the planes cut off by the triangles. The design variables ($$x_{1A}, x_{2A}$$) and ($$x_{1D}, x_{2D}$$) are not independent, however there is no unique mapping between two pairs of variables [22].
3. Aerodynamic Pressures

The aerodynamic pressure $\Delta p$ for multilayered laminated panels is described by the Piston Theory (Eq. 9). The results of the dynamic instability analysis are different for simply-supported angle-ply cylindrical panels and for flat plates. A special attention should be focus on the results of the analysis for structures exposed to different directions of airflow and it is presented at figure 2. If the directions of the orthotropy $E_L (>E_T)$ coincides with the direction of the airflow, the highest values of aerodynamic pressures are obtained. The aerodynamic pressures of cylindrical panels can decrease or increase comparing to the results for flat plates if the flow angle $\theta_{\infty}$ (Eq.13) is varies.

![Figure 1. Graphical representation of design variables](image1)

![Figure 2. Critical aerodynamic pressures of structures with different flow angles and fibre orientations](image2)
Dynamic instability (flutter boundary), is strongly influenced by boundary conditions. The distributions of the aerodynamic pressures for angle-ply cylindrical panels is presented on figure 3. Distributions results are standard, i.e. aerodynamic pressures reaches the highest values for clamped panels and the lowest for cantilever panel. The maximal values for cantilever structures occurred at $\theta = 90^\circ$.

![Graph showing normalized critical pressures for different boundary conditions](image)

**Figure 3.** Distributions of the normalized critical pressures (angle-ply cylindrical panels; $\gamma = 0.1$, $t/L_y = 0.05$, $E_t/E_r = 40$, flow angle $\theta_\infty = 0.0$).

### 4. Conclusion

The present study demonstrates the evaluation of numerical methods used in the divergence and flutter analysis of multilayered laminated structures. Several methods and theories depending on the flow velocity (subsonic and supersonic speed) are presented. Further, supersonic flutter pressure of flat plates and cylindrical panels are studied using commercial FE package. The structures with different flow angles, fibre orientations, stacking sequences and boundary condition are considered. The maximal values of aerodynamic pressures (for flat plates and cylindrical panels) are obtained for angle-ply laminates, and not for discrete (an even number of layers of the same thickness and mechanical properties) stacking sequences. $0^\circ, \pm 45^\circ, 90^\circ$.

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