Penguin contributions in the lifetime difference between $B_s$ and $B_d$, and a possible New Physics

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We consider penguin contributions to the lifetime splitting between the $B_s$ and the $B_d$ meson. In the Standard Model the penguin effects are found to be opposite in sign, but of similar magnitude as the contributions of the current-current operators, despite of the smallness of the penguin coefficients. We predict

$$\frac{\tau(B_s)}{\tau(B_d)} - 1 = (-1.2 \pm 10.0) \cdot 10^{-3} \cdot \left(\frac{f_{B_s}}{190\,\text{MeV}}\right)^2,$$

where the error stems from hadronic uncertainties. Since penguin coefficients are sensitive to new physics and poorly tested experimentally, we analyze the possibility to extract them from a future precision measurement of $\tau(B_s)/\tau(B_d)$. Anticipating progress in the determination of the hadronic parameters $\varepsilon_1, \varepsilon_2$ and $f_{B_s}/f_{B_d}$, we find that the coefficient $C_4$ can be extracted with an uncertainty of order $|\Delta C_4| \simeq 0.1$ from the double ratio $[\tau(B_s) - \tau(B_d)]/[\tau(B_s^+) - \tau(B_d^+)]$, if $|\varepsilon_1 - \varepsilon_2|$ is not too small.

1 Introduction

The theoretical achievement of the Heavy Quark Expansion (HQE) has helped a lot to understand the inclusive properties of $B$-mesons. The comparison of the theoretical predictions with the experimental measurements for the heavy hadron lifetimes and their ratios is an important test of the theory of inclusive decays, and the HQE at the order $(\Lambda_{QCD}/m_b)^3$ which is closely related to the local quark-hadron duality, which is a priori assumption in inclusive non-leptonic decay.

When we neglect higher-order corrections in $1/m_b$, the semileptonic and non-leptonic widths only depend on the CKM-factors and quarks masses. This implies that all the heavy hadrons have the same lifetime and semileptonic width. Lifetime differences arises either from corrections of $O(1/m_b^2)$ originated by heavy-hadron “wave-function” effects, or from $O(1/m_b^3)$ corrections induced by non-spectator corrections; i.e., Pauli-interference and W-exchange diagrams.

$^a$Talk given at Fourth International Workshop on Particle Physics Phenomenology at Kaohsiung, Taiwan, June 18-21, 1998

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In the former case, the corrections to lifetime-universality are due to the heavy-quark kinetic energy $\lambda_1$ and the chromomagnetic term $\lambda_2$ which differ for different hadrons: $\lambda_1(B_s) \neq \lambda_1(B)$ because of $SU(3)_f$ symmetry-breaking effects, the mass of the strange quark is much larger than the mass of the u and d quarks; $\lambda_1(B) \neq \lambda_1(\Lambda_b)$ because mesons and baryons have different wave-functions; for the same reason the chromomagnetic term vanishes for the $\Lambda_b$ but not for the B meson.

The non-spectator contributions, although of $O(1/m_b^3)$, are enhanced by the factor $16\pi^2$ due to the phase factor for $2 \rightarrow 2$ decay and for this reason may give sizeable effects. Note that these corrections may have CKM factors different from those of the leading terms.

By considering lifetime ratios, we can study most conveniently the lifetime difference in B hadrons. In this way, one cancels the dependence on quantities which are poorly known, such as the $b$-quark mass ($\tau \propto m_b^5 |V_{cb}|$, and renormalons.

1.1 Experimental Measurement and Theoretical Prediction

The average experimental results for lifetime ratios are:

$$\frac{\tau(B_s)}{\tau(B^0)} = 0.98 \pm 0.07; \quad \frac{\tau(B^\pm)}{\tau(B^0)} = 1.07 \pm 0.04; \quad \frac{\tau(\Lambda_b)}{\tau(B^0)} = 0.78 \pm 0.04.$$ (1)

The theoretical predictions is given as follows:

$$\left| \frac{\tau(B_s)}{\tau(B)} - 1 \right| < 1\%$$ (2)

$$\frac{\tau(B^-)}{\tau(B^0)} = 1 + 16\pi^2 f_B^2 M_B \left[ k_1 B_1 + k_2 B_2 + k_3 \epsilon_1 + k_4 \epsilon_2 \right]$$ (3)

$$\frac{\tau(B^-)}{\tau(B^0)} = 1 - \frac{\lambda_1(\Lambda_b) - \lambda_1(B^0)}{2m_b^2} + c_G \frac{\lambda_2(\Lambda_b) - \lambda_2(B^0)}{m_b^2}$$

$$+ 16\pi^2 f_B^2 M_B \left[ p_1 B_1 + p_2 B_2 + p_3 \epsilon_1 + p_4 \epsilon_2 + (p_5 + p_6 \tilde{B}) \right]$$ (4)

Using the experimental values of the hadron masses, hadronic parameters are given:

$$\lambda_1(B) - \lambda_1(\Lambda_b) = -(0.001 \pm 0.03) GeV^2,$$ (5)

$$\lambda_2(B) \simeq 0.12 GeV^2, \quad \lambda_2(\Lambda_b) = 0.$$ (6)
1.2 Hadronic parameters

When we define the Operator basis as follows

\[ O_1 = \bar{b}\gamma_\mu(1 - \gamma_5)q\bar{q}\gamma^\mu(1 - \gamma_5)b; \quad T_1 = \bar{b}\gamma_\mu(1 - \gamma_5)T_\alpha q\bar{q}\gamma^\mu(1 - \gamma_5)T_\alpha b; \quad (7) \]

\[ O_2 = \bar{b}(1 - \gamma_5)q\bar{q}(1 + \gamma_5)b; \quad T_2 = \bar{b}(1 - \gamma_5)T_\alpha q\bar{q}(1 + \gamma_5)T_\alpha b; \quad (8) \]

the matrix elements for B meson are defined in terms of their B-parameters:

\[ \langle B|O_i|B \rangle = f_B^2 M_B^2 B_i(\mu); \quad \langle B|T_i|B \rangle = f_B^2 M_B^2 \epsilon_i(\mu). \quad (9) \]

For B-baryon, the matrix elements are defined by

\[ \langle \Lambda_b|\bar{b}_\alpha\gamma^\mu(1 - \gamma_5)q_\beta\bar{q}_\gamma\gamma_\mu(1 - \gamma_5)b_\alpha \rangle = -\tilde{B}\langle \Lambda_b|O_1|\Lambda_b \rangle; \quad (10) \]

\[ \langle \Lambda_b|O_1|\Lambda_b \rangle = -\frac{f_B^2 m_{B_s}^2}{6} \epsilon; \quad (11) \]

We have to introduce 4 unknown parameters for B meson and 2 unknown parameters for B baryon, which contain non-pertubative property. Now let us consider the penguin contributions to the lifetime splitting between the \( B_s \) and the \( B_d \) meson.
2 Penguin Contributions

For the non-spectator contributions to the $B_s$ decay rate we need the $|\Delta B| = |\Delta S| = 1$-hamiltonian:

$$H = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left[ \sum_{j=1}^{6} C_j Q_j + C_8 Q_8 \right]$$

(12)

with

$$Q_1 = (\bar{s}^\alpha c^\beta)_{V-A} \cdot (\bar{c}^\beta b^\alpha)_{V-A}, \quad Q_2 = (\bar{s} c)_{V-A} \cdot (\bar{c} b)_{V-A}$$

$$Q_3 = \sum_{q=u,d,s,c,b} (\bar{q} b)_{V-A} \cdot (\bar{q} q)_{V-A}, \quad Q_4 = \sum_{q=u,d,s,c,b} (\bar{s}^\alpha b^\beta)_{V-A} \cdot (\bar{q}^\beta q^\alpha)_{V-A}$$

$$Q_5 = \sum_{q=u,d,s,c,b} (\bar{q} b)_{V-A} \cdot (\bar{q} q)_{V+A}, \quad Q_6 = \sum_{q=u,d,s,c,b} (\bar{s}^\alpha b^\beta)_{V-A} \cdot (\bar{q}^\beta q^\alpha)_{V+A}$$

$$Q_8 = -\frac{g}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu} (1 + \gamma_5) T^{a} \cdot G^a_{\mu\nu}.$$  

(13)

In (12) we have set $V_{ub} V_{us}^* = O(\lambda^4)$ to zero. The diagram of Fig. 1 has been calculated in 4,6 and yields contributions to the non-spectator part $\Gamma_{\text{non-spec}}$ of the $B_s$ decay rate proportional to $C_2^2, C_1 \cdot C_2$ and $C_1^2$. $\tau(B_s)/\tau(B_d) - 1$ is proportional to $\Gamma_{\text{non-spec}}(B_d) - \Gamma_{\text{non-spec}}(B_s)$. The main differences between the result of Fig. 1 for these two rates are due to the different mass of $u$ and $c$ and the difference between $f_{B_d}$ and $f_{B_s}$. Hence the current-current parts of $\tau(B_s)/\tau(B_d) - 1$ proportional to $C_2^2, C_1 \cdot C_2$ or $C_1^2$ are suppressed by a factor of $z$ or $\Delta$ with

$$z = \frac{m_c^2}{m_b^2} = 0.085 \pm 0.023, \quad \Delta = 1 - \frac{f_{B_s}}{f_{B_d}} M_{B_d} = 0.23 \pm 0.11.$$  

(14)

The result for $\Delta$ in (14) is the present world average of lattice calculations. There are also $SU(3)_F$ violations in the $B$-factors, but they are expected to be small from the experience with those appearing in $B^0 - \bar{B}^0$-mixing. In this analysis we use the same $B_1, B_2, \epsilon_1$, and $\epsilon_2$ in $\tau(B_s)$ and $\tau(B_d)$.

Our result for the non-spectator part of the $B_c$ decay rates reads:

$$\Gamma_{\text{non-spec}}(B_s) = -\frac{G_F^2 m_b^2}{12\pi} |V_{cb} V_{cs}|^2 \sqrt{1 - 4 z f_{B_s}^2 M_{B_s}} \left[ a_1 \epsilon_1 + a_2 \epsilon_2 + b_1 B_1 + b_2 B_2 \right]$$

(15)

$$4$$
Figure 3: Contribution of $Q_8$ to $\Gamma^{\text{non-spect}}(B_s)$. In the Standard Model the diagram is of the same order of magnitude as radiative corrections to Fig. 3 and therefore negligible. Yet in models in which quark helicity flips occur in flavour-changing vertices $|C_8|$ can easily be ten times larger than in the Standard Model. The contribution of $Q_1$ vanishes.

Figure 4: Penguin diagram contribution to $\Gamma^{\text{non-spect}}(B_s)$. The final state corresponds to a cut through either of the $(\tau,c)$-loops. The contributions of $Q_1$ vanish by colour. This is the only NLO contribution to $\tau(B_s)/\tau(B_d) - 1$ involving $Q_{1,2}$ without suppression factors of $\Delta$ or $z$.

\begin{align*}
&\text{with}
\begin{align*}
a_1 &= \left[2C_2^2 + 4C_2C_4'\right] [1 - z] + 12zC_2C_6' + [1 + 2z] \frac{\alpha_s}{\pi} C_2 C_8 \\
a_2 &= -[1 + 2z] \left[2C_2^2 + 4C_2C_4' + \frac{\alpha_s}{\pi} C_2 C_8\right] \\
b_1 &= \left[ C_2 + N_c C_1 \right] \left\{ (1 - z) \left[ \frac{C_2}{N_c} + C_1 + 2C_3' + 2\frac{C_4'}{N_c} \right] + 6z \left[ C_5' + \frac{C_6'}{N_c} \right] \right\} \\
b_2 &= -[1 + 2z] \left[ C_2 + N_c C_1 \right] \left\{ \frac{1}{N_c} \left[ C_2 + N_c C_1 \right] + 2 \left[ C_5' + \frac{C_6'}{N_c} \right] \right\}
\end{align*}
\end{align*}

Here $N_c = 3$ is the number of colours. By setting $C_j'$, $j = 3, \ldots, 6$, and $C_8$ in (16) to zero one recovers the result of Ref.[4]. The result for the non-spectator contributions to the $B_d$ decay rate reads:

\begin{align*}
\Gamma^{\text{non-spect}}(B_d) &= \frac{G_F^2 m_b^2}{12\pi} |V_{cb}V_{ub}|^2 (1 - z)^2 f_B^2 M_B (\Delta - 1) \\
& \quad \left[ a_1^d \varepsilon_1 + a_2^d \varepsilon_2 + b_1^d B_1 + b_2^d B_2 \right]
\end{align*}

with\[]
\begin{align*}
a_1^d &= 2C_2^2 \left( 1 + \frac{z}{2} \right), & a_2^d &= -2C_2^2 \left( 1 + 2z \right), \\
b_1^d &= \frac{1}{N_c} \left( C_2 + N_c C_1 \right)^2 \left( 1 + \frac{z}{2} \right), & b_2^d &= -\frac{1}{N_c} \left( C_2 + N_c C_1 \right)^2 \left( 1 + 2z \right)
\end{align*}

\(c\) Notice that our notation of $C_1$ and $C_2$ is opposite to the one in Ref.[4].

\(d\) In the large $N_c$ limit one finds $\Gamma^{\text{non-spect}}$ helicity suppressed in analogy to the leptonic decay rate. This shows that one cannot neglect the $O(1/N_c)$ terms.
When we combine (15-18) in order to predict\( \tau(B_s)/\tau(B_d) - 1 \):
\[
\frac{\tau(B_s)}{\tau(B_d)} - 1 = \frac{\Gamma_{\text{non-spec}}(B_d) - \Gamma_{\text{non-spec}}(B_s)}{\Gamma_{\text{total}}(B_s)} + O(10^{-3})
\]
\[
= 9.0 \cdot 10^{-4} \epsilon_1 - 1.63 \cdot 10^{-2} \epsilon_2 + 2.0 \cdot 10^{-4} B_1 - 5.0 \cdot 10^{-4} B_2
\]

(19)

The experimental world average\( \tau(B_s)/\tau(B_d) = 1.07 \pm 0.04 \) (20)

leads to the following constraint:
\[
\epsilon_1 \simeq (-0.2 \pm 0.1) \left( \frac{0.17 \text{ GeV}}{f_B} \right)^2 \left( \frac{m_b}{4.8 \text{ GeV}} \right)^3 + 0.3 \epsilon_2 + 0.05.
\]

(21)

Here we consider the range \( |\epsilon_1|, |\epsilon_2| \leq 0.3 \), and further obey (20).

From Tab.2 of ref \[8\] we realize that the penguin contributions are comparable in size, but opposite in sign to the current-current part obtained in Ref.\[4\]. This makes the experimental detection of any deviation of \( \tau(B_s)/\tau(B_d) \) from 1 even more difficult, if the penguin coefficients are really dominated by Standard Model physics. From the results of Tab.2 in Ref.\[8\], we obtain
\[
\frac{\tau(B_s)}{\tau(B_d)} - 1 = (-1.2 \pm 8.0 \pm 2.0) \cdot 10^{-3} \cdot \left( \frac{f_B}{190 \text{ MeV}} \right)^2 \left( \frac{4.8 \text{ GeV}}{m_b} \right)^3.
\]

(22)

Here the first error stems from the uncertainty in \( \epsilon_1 \) and \( \epsilon_2 \) and will be reduced once lattice results for the hadronic parameters are available. The second error summarizes the remaining uncertainties.

3 New Physics Effects

Today we have little experimental information on the sizes of the penguin coefficients. Their smallness in the Standard Model allows for the possibility that they are dominated by new physics. The total charmless inclusive branching fraction \( Br(B \rightarrow \text{no charm}) \) is a candidate to detect new physics contributions to \( C_3 \), but it is much less sensitive to \( C_{3-6} \). Eq.(17) of Ref.\[8\] reveals that \( \tau(B_s)/\tau(B_d) \) is a complementary observable mainly sensitive to \( C_4 \), while \( C_8 \) is of minor importance.

New physics contributions \( \Delta C_{3-6}(\mu = 200\text{GeV}) \) affect \( C_4(\mu = 4.8\text{ GeV}) \) by
\[
\Delta C_4(\mu = 4.8 \text{ GeV}) = -0.35 \Delta C_3(200 \text{ GeV}) + 0.99 \Delta C_4(200 \text{ GeV})
\]
\[
-0.03 \Delta C_5(200 \text{ GeV}) - 0.22 \Delta C_6(200 \text{ GeV}).
\]

(23)
Observe that $\Delta C_4(200 \text{ GeV}) = -0.05$ already increases $C_4'(m_b)$ by more than a factor of two.

Clearly the usefulness of $\tau(B_s)/\tau(B_d)$ to probe $C_{3-6}$ crucially depends on the size of $|\varepsilon_1 - \varepsilon_2|$ and $f_{B_s}$. We now investigate the sensitivity of $\tau(B_s)/\tau(B_d)$ to $\Delta C_4(\mu = m_b)$ in a possible future scenario for the hadronic parameters. We assume

$$\varepsilon_1 = -0.10 \pm 0.05, \quad \varepsilon_2 = 0.20 \pm 0.05, \quad B_1, B_2 = 1.0 \pm 0.1,$$

$$f_{B_s} = (190 \pm 15) \text{ MeV}, \quad \Delta = 0.23 \pm 0.05, \quad m_b = (4.8 \pm 0.1) \text{ GeV}.$$ (24)

A cleaner observable which can see a new physics effects is the double ratio:

$$\frac{\tau(B_s) - \tau(B_d)}{\tau(B^+) - \tau(B_d)} = \frac{B_{SL}(B_s) - B_{SL}(B_d)}{B_{SL}(B^+) - B_{SL}(B_d)},$$

(25)

which depends on $\varepsilon_1, \varepsilon_2$ and $\Delta$, while the dependence on $f_B$ and $m_b$ cancels. The corresponding plot for the parameter set of (24) can be found in Fig. 5.

If $\Delta C_4 < -0.075$ or $\Delta C_4 > 0.140$, we find the allowed range for $[\tau(B_s) - \tau(B_d)]/[\tau(B^+) - \tau(B_d)]$ incompatible with the Standard Model. The experimental detection of a sizeable negative lifetime difference $\tau(B_s) - \tau(B_d)$ may reveal non-standard contributions to $C_4'$ of similar size as its Standard Model value. Fig. 5 shows that e.g. the bound $\tau(B_s) - \tau(B_d) < -0.20[\tau(B^+) - \tau(B_d)]$ would indicate $\Delta C_4 > 0.051$. We conclude that the detection of new physics contributions to $C_4'$ of order 0.1 is possible with precision measurements of $\tau(B_s)/\tau(B_d)$.

4 Conclusions

We have calculated the contributions of the penguin operators $Q_{3-6}$, of the chromomagnetic operator $Q_8$ and of penguin diagrams with insertions of $Q_2$ to the lifetime splitting between the $B_s$ and $B_d$ meson. In the Standard Model the penguin effects are found to be roughly half as big as the contributions from the current-current operators $Q_1$ and $Q_2$, despite of the smallness of the penguin coefficients. Yet they are opposite in sign, so that any deviation of $\tau(B_s) - \tau(B_d)$ from zero is even harder to detect experimentally. Assuming a reasonable progress in the determination of the hadronic parameters a precision measurement of $\tau(B_s)/\tau(B_d)$ can be used to probe the coefficient $C_4$ with an accuracy of $|\Delta C_4| = 0.1$. Hence new physics can only be detected, if $C_4$ is dominated by non-standard contributions. The sensitivity to $C_4$ depends crucially on the difference of the hadronic parameters $\varepsilon_1$ and $\varepsilon_2$. For the extraction of $C_4$ the double ratio $[\tau(B_s) - \tau(B_d)]/[\tau(B^+) - \tau(B_d)]$ turns out to be very useful to see a new physics effects.
Figure 5: Dependence of \(\frac{\tau(B_s) - \tau(B_d)}{\tau(B^+) - \tau(B_d)}\) on \(\Delta C_4\) for the parameter set in (24). This double ratio depends on \(f_{B_s}\) and \(f_{B_d}\) only through \(\Delta\), and the factor of \(m_{b}^{-3}\) cancels.

Acknowledgements

The author would like to thank the organizers of the fourth international workshop on particle physics phenomenology at Kaoshiung. Y.-Y. Keum thanks Chris Sachrajda and Hai-Yang Cheng for helpful discussions and U. Nierste for enjoyable collaboration on this subject. He is grateful to Prof. M. Kobayashi and Prof. W. Buchmüller for their hospitality and encouragement. This work is supported in part by the Basic Science Research Institute Program, Ministry of Education, Project No. BSRI-97-2414, and in part by the Grant-in Aid for Scientific from the Ministry of Education, Science and Culture, Japan.

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