Navigating the Small World Web by Textual Cues

Filippo Menczer
Department of Management Sciences
The University of Iowa
Iowa City, IA 52242
Phone: (319) 335-0884
Fax: (319) 335-0297
filippo-menczer@uiowa.edu

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Can a Web crawler efficiently locate an unknown relevant page?
While this question is receiving much empirical attention due to its considerable commercial value in the search engine community, theoretical efforts to bound the performance of focused navigation have only exploited the link structure of the Web graph, neglecting other features. Here I investigate the connection between linkage and a content-induced topology of Web pages, suggesting that efficient paths can be discovered by decentralized navigation algorithms based on textual cues.

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Topic driven crawlers are increasingly seen as a way to address the scalability limitations of universal search engines, by distributing the crawling process across users, queries, or even client computers. The context available to such crawlers can guide the navigation of links with the goal of efficiently locating highly relevant target pages. Given the need to find unknown target pages, we are only interested in decentralized crawling algorithms, which can only use information available locally about a page and its neighborhood. Starting from some source Web page, we aim to visit a target page by navigating a path of length $\ell \ll N$ where $\ell$ is the number of pages visited along the path and $N$ is the total number of pages.

Since the Web is a small-world network, we know that the diameter (at least for the largest connected component) scales logarithmically with $N$, therefore some short path exists between the source and target nodes such that $\ell \sim \log N$. Can a crawler navigate such a short path? If the only local information available is about the hypertext link degree of each node and its neighbors, then simple greedy algorithms that always pick the neighbor with highest degree lead to paths where the number of links traversed $\ell'$ scales sublinearly ($\ell' \sim N^\beta, \beta < 1$) or logarithmically ($\ell' \sim \log N$). However a real Web crawler would have to visit all the neighbors of a node to determine their degree, and moving to high-degree nodes makes such a strategy useless.
for actual Web navigation \( (\ell(\ell') \sim N) \) given the power-law degree sequence distribution.

Kleinberg showed that if information about the geographic location of nodes is available, then a greedy algorithm that always picks the neighbor geographically closest to the target can yield \( \ell \sim (\log N)^2 \) if the link topology follows a \( D \)-dimensional lattice, with \( 2D \) local links to lattice neighbors plus one long range connection per node, linking to another node chosen with probability \( \Pr(r) \sim r^{-\alpha} \) where \( r \) is the lattice distance between the two nodes and \( \alpha \) is a constant clustering exponent. In this model the optimal path length is achieved for a critical clustering exponent dependent on the dimensionality of the lattice \((\alpha = D)\).

Kleinberg’s model is inspired by social small-world networks where geographical knowledge exists, but in the Web hypertext the notion of geography is of little relevance and the lattice model is unrealistic. However, a more relevant topological distance metric can be defined in the Web, namely the distance induced by the lexical similarity between the textual content of pages. Let us define such a lexical distance

\[
    r(p_1, p_2) = \frac{1}{s(p_1, p_2)} - 1
\]

(1)
where \((p_1, p_2)\) is a pair of Web pages and \(s\) is the cosine similarity function traditionally used in information retrieval. The \(r\) distance metric is a natural local cue readily available in the Web, with the target content specified by a query or topic of interest to the user. This metric also does not suffer from the dimensionality bias that makes \(L\)-norms inappropriate in the sparse word vector space.

To investigate the relationship between the lexical topology induced by \(r\) and the link topology, I measured the frequency of linked pairs of pages as a function of the lexical distance, \(Pr(p_1, p_2) = r\):

\[
Pr(r | \lambda) \approx \frac{|\{(p_1, p_2) : r(p_1, p_2) = r \land l(p_1, p_2) > \lambda\}|}{|\{(p_1, p_2) : r(p_1, p_2) = r\}|}
\]

(2)

where the linkage between two pages was approximated by the overlap

\[
l(p_1, p_2) = \frac{|U_{p_1} \cap U_{p_2}|}{|U_{p_1} \cup U_{p_2}|}
\]

(3)

and \(U_p\) is the URL set representing \(p\)’s neighborhood (inlinks, outlinks, and \(p\) itself). The threshold \(\lambda\) models the ratio of local versus long range links, analogous to Kleinberg’s dimensionality \(D\). Larger \(\lambda\) values imply more sparse long range connections.

Figure 4 shows that long range links are indeed distributed according
to a power law $\Pr(\rho|\lambda) \sim \rho^{-\alpha(\lambda)}$, as in Kleinberg’s model, but using lexical distance. A fit of the tails to the power law model reveals that the clustering exponent $\alpha$ grows linearly with the linkage threshold $\lambda$ (inset).

Such a surprising result makes Kleinberg’s analysis applicable to the Web. Crawlers that exploit textual cues may be able to navigate through short paths and locate unknown relevant pages. This is encouraging for the community of crawling algorithm designers. The actual bound on the length of the path $\ell$ depends on whether the clustering exponent is near a critical value; this will have to be determined by extending Kleinberg’s analysis to the lexical topology of the Web.

The observed relationship between link and lexical Web topology has another interesting implication. One of the recent attempts to explain the power law distribution of Web degree sequences is based on a preferential attachment model in which new nodes are linked to existing ones based on a critical mixture of linkage bias (attach to a node within a few links from most other nodes) and geographic bias (attach to a node within a small Euclidean distance), where nodes are given random coordinates in the unit square $[11]$. The present result could lead to a more realistic interpretation whereby authors would link their new pages to sites that are both popular and related in content, i.e., central in link space and nearby in lexical space.
Figure 1: Linkage probability $Pr(\rho)$ as a function of lexical distance $\rho$, for various values of the linkage threshold $\lambda$. The frequency data is based on approximately $6 \times 10^8$ pairs of Web pages sampled from the Open Directory (dmoz.org). The least-squares fit of the tail of each distribution to the power law model $Pr(\rho) \sim \rho^{-\alpha}$ is also shown. The inset plots the relationship between the effective dimensionality induced by the linkage threshold $\lambda$ and the clustering exponent $\alpha$ of the power law tail.
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