Colored Pseudo-Goldstone Bosons and Gauge Boson Pairs

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ABSTRACT

If the electroweak symmetry breaking sector contains colored particles weighing a few hundred GeV, then they will be copiously produced at a hadron supercollider. Colored technipions can rescatter into pairs of gauge bosons. As proposed by Bagger, Dawson, and Valencia, this leads to gauge boson pair rates far larger than in the standard model. In this note we reconsider this mechanism, and illustrate it in a model in which the rates can be reliably calculated. The observation of both an enhanced rate of gauge-boson-pair events and colored particles would be a signal that the colored particles were pseudo-Goldstone bosons of symmetry breaking.
Technicolor remains an intriguing possibility for electroweak symmetry breaking. Typically, the technihadrons which are lowest in mass are the technipions. A technicolor model must have at least three technipions, because they become the longitudinal components of the $W$ and $Z$. In general there may be others. In models where there are colored technifermions, there are colored technipions. The one-family model, for example, has 63 technipions, including some which transform as $(8,3)$ under $SU(3)_{\text{color}} \times SU(2)_{\text{weak}}$.

Colored technipions are easy to produce at a hadron supercollider. Interestingly, in technicolor models a pair of colored technipions can rescatter into the colorless ones. For example, in the one-family model the reaction $(8,3)(8,3) \rightarrow (1,3)(1,3)$ can occur. Since the former particles are easy to produce, and the latter are the longitudinal components of the gauge bosons, there will be a large rate for gauge boson pair events at the SSC or LHC. This mechanism was recently proposed by Bagger, Dawson, and Valencia to test electroweak symmetry breaking at a hadron supercollider.

We are left with an interesting possibility. The colored technipions are produced in large numbers at the SSC and LHC, and should be observable even in the worst case scenario in which they decay exclusively into light quarks and flavor tagging is useless. However, because they are produced strongly, there is no obvious way to connect them to the symmetry breaking sector, and indeed a skeptic might argue that the colored scalars are unrelated to it. It is the combination of their discovery with the observation of a large number of gauge-boson-pair events which permits us to argue that the colored scalars are pseudo-Goldstone bosons of the symmetry breaking sector.

Since they are approximate Goldstone bosons, the low-energy behavior of the technipions can be described by chiral Lagrangian techniques. Consider the lowest order, two-derivative, chiral Lagrangian for the one-family model: $L_2 = (f^2/4) \text{tr}[(D^\mu \Sigma)^\dagger(D_\mu \Sigma)]$, where $\Sigma = \exp(2i\pi^a T^a/f)$. Here $T^a$ are generators of $SU(8)$, and $\pi^a$ are the 63 technipion fields. The derivatives above are gauge covariant, using the proper imbedding of the color and electroweak groups into the chiral $SU(8)_L \times SU(8)_R$. We may fix $f$ by noting that since there are four electroweak doublets condensing in this model – three quarks and one lepton – we have $M_W^2 = g^2 f^2$. From this we deduce that $f = v/2$, where $v \approx 250\text{GeV}$.

This lowest order chiral Lagrangian contains terms which allow the computation of the $gg \rightarrow ZZ$ process in which we are interested. The computation is facilitated by the

\[1\] On the other hand, the generic expectation is that the technipions will mostly decay to the heaviest fermions. Observation that the colored scalar decayed in this way would be an argument that it had something to do with flavor physics, and hence symmetry breaking.
use of the equivalence theorem \[6\], which states that at energies large compared to the 
\(W\) mass, the amplitude for a process containing external longitudinal gauge bosons is 
equal to the amplitude for the process with the longitudinal gauge bosons replaced by 
their swallowed unphysical Goldstone bosons. It is impossible to construct a gauge and 
chiral invariant four-derivative counterterm for the coupling of the gluons to the uncolored 
swallowed technipions, so the one-loop calculation of the gluon-gluon to the longitudinal 
\(ZZ\) state must be finite. This is the computation presented by Bagger, Dawson, and 
Valencia \[3\].

To how high an energy scale can we trust the calculations using \(\mathcal{L}_2\)? Quite gen-
eral considerations \[7\] show that in a theory in which the symmetry breaking pattern is 
\(SU(N)_L \times SU(N)_R \rightarrow SU(N)_V\), the scale at which the calculations fail must fall as \(1/\sqrt{N}\). 
Consider \(\pi^a \pi^b \rightarrow \pi^c \pi^d\) scattering. We can calculate the \(SU(N)_V\)-singlet s-wave amplitude 
using \(\mathcal{L}_2\) \[8\]:

\[
a_{00} = \frac{Ns}{32\pi f^2},
\]

where \(s\) is the usual Mandelstam variable. This amplitude grows bigger than 1 in magni-
tude at a scale \(4\sqrt{2\pi f/\sqrt{N}}\). In the one-family model, this is a mere 440 GeV! Thus, the 
lowest order chiral Lagrangian must fail at a very low energy, at a scale which would be 
easily reachable by the SSC or LHC.

It is now easy to see what goes wrong when we calculate \(gg \rightarrow ZZ\) using \(\mathcal{L}_2\). Consider 
the computation of the imaginary part of the amplitude \[\mathcal{L}_2\]. Only when the two gluons 
produce a pair of on-shell technipions can the diagram be cut in such a way as to give a 
physical intermediate state, and therefore the imaginary part of the amplitude comes from 
the production of on-shell colored technipions which rescatter into the swallowed Goldstone 
bosons. However, as we have noted, this rescattering will violate unitarity. The \(SU(8)_V\)-
singlet spin-0 part fails first, at 440GeV. We therefore conclude that the lowest-order chiral 
Lagrangian calculation of \(gg \rightarrow ZZ\) displays bad high-energy behavior.

The failure of the chiral Lagrangian to address the scattering of technipions in the 
one-family model at the requisite energies leads us to consider a simpler – but calculable – 
model which resembles technicolor. Our toy model of the electroweak symmetry breaking 
sector is based on an \(O(N)\) linear sigma-model solved in the limit of large \(N\) \[9\]. It has

\[2\] The amplitude given in \[3\] has no imaginary part because that paper considered the case of 
massless colored technipions. Colored technipions are expected to have a mass in the hundreds of 
GeV.
both exact Goldstone bosons (which will represent the longitudinal components of the $W$ and $Z$) and colored pseudo-Goldstone bosons. To this end let $N = j + n$ and consider the Lagrangian density \[ \mathcal{L} = \frac{1}{2} (\partial \vec{\phi})^2 + \frac{1}{2} (D \vec{\psi})^2 - \frac{1}{2} \mu_{0\phi}^2 \phi^2 - \frac{1}{2} \mu_{0\psi}^2 \psi^2 - \frac{\lambda_0}{8N} (\vec{\phi}^2 + \vec{\psi}^2)^2, \] (2)

where $\vec{\phi}$ and $\vec{\psi}$ are $j$- and $n$-component real vector fields. This theory has an approximate $O(j+n)$ symmetry which is broken to $O(j) \times O(n)$ so long as $\mu_{0\phi}^2 \neq \mu_{0\psi}^2$. If $\mu_{0\phi}^2$ is negative and less than $\mu_{0\psi}^2$, one of the components of $\vec{\phi}$ gets a vacuum expectation value (VEV), breaking the approximate $O(N)$ symmetry to $O(N-1)$. With this choice of parameters, the exact $O(j)$ symmetry is broken to $O(j-1)$ and the theory has $j-1$ massless Goldstone bosons and, at tree level, one massive Higgs boson. The $O(n)$ symmetry is unbroken, and there are $n$ degenerate massive pseudo-Goldstone bosons. We will consider this model in the limit that $j, n \to \infty$ with $j/n$ held fixed.

The scalar sector of the standard one-doublet Higgs model has a global $O(4) \approx SU(2) \times SU(2)$ symmetry, where the 4 of $O(4)$ transforms as one complex scalar doublet of the $SU(2)_W \times U(1)_Y$ electroweak gauge interactions. We will model the $O(4)$ of the standard model by the $O(j)$ of the $O(j+n)$ model solved in the large $j$ and $n$ limit. Of course, $j = 4$ is not particularly large. Nonetheless, the resulting model will have the same qualitative features, and we can investigate the theory at moderate to strong coupling [11].

We have gauged an $SU(3)_c$ subgroup of $O(n)$, so the $\psi$ fields are colored\[.\] We have chosen $\psi$ to be three color octets, analogous to the $(8, 3)$ of the one-family model. Our choice corresponds to $n = 24$.

A simple trick [9] for the solution of this theory to leading order in $1/N$ involves introducing a new field $\chi$, and modifying the Lagrangian to

\[ \mathcal{L} \to \mathcal{L} + \frac{1}{2} \frac{N}{\lambda_0} \left(\chi - \frac{1}{2} \frac{\lambda_0}{N} (\vec{\phi}^2 + \vec{\psi}^2) - \mu_{0\phi}^2 \right)^2. \] (3)

Adding this term has no effect on the dynamics of the theory: since the added term has no space-time derivative, the path integration over the field $\chi - \frac{1}{2} \frac{\lambda_0}{N} (\vec{\phi}^2 + \vec{\psi}^2) - \mu_{0\phi}^2$ will

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3 Technically, this gauging of the color symmetry for $\psi$ and not $\phi$ breaks the $O(N)$ by a dimension-four operator. We are free to ignore this if we regard $\alpha_s$ as small compared to $\lambda$ and accordingly neglect diagrams with loops involving gluons.
yield an irrelevant overall constant. On the other hand, the Feynman rules generated from
the new Lagrangian are different, since

\[ L = \frac{1}{2} (\partial \vec{\phi})^2 + \frac{1}{2} (D \vec{\psi})^2 - \frac{m_\psi^2}{2} \vec{\psi}^2 + \frac{1}{2} \frac{N}{\lambda_0} \chi^2 - \frac{1}{2} \chi(\vec{\phi}^2 + \vec{\psi}^2) - \frac{N \mu_0^2 \phi}{\lambda_0} \chi \] (4)

where \( m_\psi^2 = \mu_0^2 - \mu_0^2 \phi \) is a positive number and where we have ignored an irrelevant
constant. Written in this way, the only nontrivial interactions are the \( \chi \phi^2 \) and \( \chi \psi^2 \) terms.
The advantage of this formalism is that when a diagram is evaluated, the only source of
factors of \( 1/N \) will be the \( \chi \) propagators. To evaluate any process to leading order in \( 1/N \),
therefore, one computes only diagrams with the minimum number of \( \chi \) propagators, i.e. diagrams with no \( \chi \) loops.

The gauge boson production diagrams of this model are shown in fig. [4]. The double
line indicates the \( \chi \) propagator, \( D_{\chi\chi}(s) \), including all radiative corrections coming from
loops of \( \phi \)s and \( \psi \)s. Postponing for the moment the evaluation of \( D_{\chi\chi}(s) \), we see that this
process is very easy to calculate, since the \( g \psi \psi \) vertex is just the ordinary coupling of a
gauge boson to a scalar, and the couplings of the \( \chi \) to \( \phi \phi \) and \( \psi \psi \) are both just \(-i\). We
find that the sum of the diagrams in fig. [4] is

\[ \frac{i}{16\pi^2} \left( g^{\mu\nu} - \frac{2p_2^\mu p_1^\nu}{s} \right) 2n_8 C_8 I(s, m_\psi^2) D_{\chi\chi}(s) . \] (5)

Here \( p_1 \) and \( p_2 \) are the momenta of the two incoming gluons; their polarization vectors are
associated with the indices \( \mu \) and \( \nu \) respectively. The number of octets in \( \psi \) is \( n_8 \); as we
noted above, we have chosen \( n_8 = 3 \). The factor \( C_8 \) denotes the Casimir operator of an
\( SU(3)_c \) octet, which is 3. The variable \( s \) is \( 2p_1 \cdot p_2 \), and the function \( I(s, m_\psi^2) \) is a Feynman
parameter integral

\[ I(s, m_\psi^2) \equiv \int_0^1 dx \int_0^{1-x} dy \frac{xys}{m_\psi^2 - xys - i\epsilon} . \] (6)

At this point, all that remains is to evaluate \( D_{\chi\chi}(s) \). The details of its calculation
may be found in [10]. Only one subtlety need concern us here. In the process of solving
the theory (2) to leading order in \( 1/N \), there are divergences which must be regularized
and \( \lambda \) and \( \mu \) must be renormalized. This can be accomplished by defining

\[ \frac{1}{\lambda(M)} \equiv \frac{1}{\lambda_0} - \frac{i}{2} \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{(\ell^2 + i\epsilon)(\ell^2 - M^2 + i\epsilon)} , \] (7)

and

\[ \frac{\mu^2(M)}{\lambda(M)} \equiv \frac{\mu_0^2}{\lambda_0} + \frac{i}{2} \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{\ell^2 + i\epsilon} , \] (8)
where $M$ is an arbitrary renormalization point. These two subtractions are sufficient to render the theory finite to leading order in $1/N$. To leading order in $1/N$, $m_\psi^2$ remains unrenormalized.

Specifying $\lambda(M)$ and $\mu^2(M)$ (as well as $m_\psi^2$) for a particular $M$ specifies the theory completely. We will choose $\mu^2(M)$ negative and $m_\psi^2 > 0$, so that the $O(j)$ symmetry is spontaneously broken and we will orient the VEV of $\vec{\phi}$ so that only $\langle \phi_j \rangle \neq 0$. Instead of $\mu^2(M)$, we will work with the parameter $\langle \phi_j \rangle$ directly, since it has physical meaning and the coefficients in the Lagrangian do not. Consider the $O(N)$ symmetry current $J_\alpha^\nu = i\vec{\phi}^T T^\nu T_\alpha \vec{\phi}/2$ where $T_\alpha$ is a generator of $O(N)$, normalized to $\text{tr} T_\alpha T_\beta = 2\delta_{\alpha\beta}$. When $\phi_j$ gets a VEV, the broken symmetry currents will satisfy $J_\alpha^\nu = i\langle \phi_j \rangle \partial^\nu \phi_a + \ldots$, and so we identify $\langle \phi_j \rangle = v$.

At this point we may trade in the parameter $\lambda$ for the scale $M$. Instead of regarding the renormalization point $M$ as fixed and $\lambda$ as varying, we take

$$\frac{1}{\lambda(M)} = 0$$

(9)

and, therefore, $M$ specifies the strength of the coupling. Of course equation (9) implies that $M$ is the scale at which the renormalized coupling blows up. The fact that the coupling becomes infinite at some finite scale $M$ is a reflection of the triviality of the scalar $O(N)$ theory. For the purposes of this work, however, this need not trouble us. The $O(N)$ model is a consistent effective theory for energies well below $M$ and we will only need results in this regime.

To leading order in $1/N$, we find

$$D_{\chi\chi}(s) = \frac{-is}{\left[v^2 - Ns \left(\frac{1}{\lambda(M)} + \tilde{B}(s; m_\psi, M)\right)\right]} ,$$

(10)

where

$$\tilde{B}(s; m_\psi, M) = \frac{n}{32N\pi^2} \left\{ 1 + \frac{i}{\sqrt{s/(4m_\psi^2 - s)}} \log \frac{i - \sqrt{s/(4m_\psi^2 - s)}}{i + \sqrt{s/(4m_\psi^2 - s)}} - \log \frac{m_\psi^2}{M^2} \right\}$$

$$+ \frac{j}{32N\pi^2} \left\{ 1 + \log \frac{M^2}{s} \right\} .$$

(11)

The logs and the square roots have branch cuts, and it is up to us to place them in physically meaningful places. This we do by considering $\phi\phi \rightarrow \phi\phi$ scattering. In (3) this
proceeds entirely by entirely via the $\chi$ exchange; we find that the $O(j - 1)$ singlet spin zero scattering amplitude is
\[ a(s) = \frac{ij}{32\pi} D_{\chi\chi}(s) . \] (12)

This amplitude has two branch cuts just below the real axis \([1 2]\): one starting at $s = 0$ from pairs of massless Goldstone bosons, and the other starting at $s = 4m_{\psi}^2$ from pairs of pseudo-Goldstone bosons. To leading order in $1/N$, there are no other multiparticle states. We have written (11) so that these branch cuts are obtained using the conventional definition of the log and the square root, in which the branch cut is just under the negative real axis. That is, $\log z = \log |z| + i\theta$ and $\sqrt{z} = |z|^{1/2} \exp (i\theta/2)$ where $-\pi < \theta \leq \pi$.

In fig. 2 we show the $ZZ$ cross sections at hadron supercolliders. Our choice of $M = 1800$ GeV, and $m_\psi = 120$ GeV gives a strongly coupled, QCD-like scattering amplitude.\[ Since we have used the equivalence theorem, the $Z$ mass is ignored in the amplitude. However, the $Z$ mass has been retained in the phase space. In this process the cross section for the $W^+W^-$ final state is double that of $ZZ$.

There are two interesting features of these graphs. First, we note that the cross sections fall rapidly at high energies. In the computation using the lowest order chiral Lagrangian the high energy behavior is quite different - the figures in [3] show that the differential cross sections fall by less than a factor 4 between $M_{ZZ} = 200$ and 1000 GeV. As we have noted, this is because the technipion rescattering cannot be as large as given by $L_2$. In contrast, in the toy model the Goldstone boson $S$ matrix is unitary at all energies to leading order in $1/N$. Accordingly, the amplitude ([6] and [10]) for $gg \rightarrow ZZ$ will not display the bad high-energy behavior $L_2$. In fact, fig. 2 shows that at high energies the background cross section exceeds that of the signal. This is not surprising, because the $gg$ luminosity falls considerably faster than that of $q\bar{q}$.

The second interesting feature of these graphs is that the cross sections are quite large, and should easily be observable. These cross sections are much larger than those of non-resonant $ZZ$ production via a top-quark loop. Accordingly, we neglect the top contribution to this process. One should not trust the numbers on these graphs too much – the true calculation is model dependent and one certainly should not take the toy model too seriously. Nonetheless, it is plausible that the computation we have done is conservative,

\[ \text{\footnote{For larger values of } M \text{ the amplitude is weakly coupled, and there is a narrow Higgs resonance. While this is also a plausible scenario, it is not necessarily what we expect in a technicolor theory. See [8] and [10] for a discussion of this point.}} \]
since there may be more colored pseudo-Goldstone bosons than we have assumed, or there may be some in representations with higher Casimirs.

We conclude by noting that the following interesting scenario may be observed at a hadron supercollider. There could be be colored, weakly-decaying particles produced in great numbers. However, there might be no obvious way to connect them to symmetry breaking. Though the process cannot be computed using the chiral Lagrangian, there will be rescattering of these particles into longitudinal Goldstone bosons. If we observe colored particles in conjunction with very large rates of electroweak-gauge-boson pair events, then we may have a hint that they are pseudo-Goldstone bosons.

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Figure Captions

Fig. 1. The Feynman diagrams for $gg \rightarrow \phi\phi$ in the $O(N)$ model.

Fig. 2. The $ZZ$ differential cross section in $nb/GeV$ vs $M_{ZZ}$ computed in the $O(N)$ model. The solid and dashed lines show the $gg$ fusion signal at the SSC and LHC respectively. The $q\bar{q}$ background [13] are the dotdashed and dotted lines. We have put $M = 1800 GeV$ and $m_\psi = 120 GeV$. A rapidity cut of $|y| < 2.5$ is imposed on the final state $Zs$. EHLQ Set II structure functions [14] are used.