Lightfront Formalism versus Holography & Chiral Scanning*

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Abstract

The limitations of the approach based on using fields restricted to
the lightfront (Lightfront Quantization or \( p \to \infty \) Frame Approach) which
drive quantum fields towards canonical and ultimately free fields are well
known. Here we propose a new concept which does not suffer from this
limitation. It is based on a procedure which cannot be directly formulated
in terms of pointlike fields but requires “holographic” manipulations of
the algebras generated by those fields. We illustrate the new concepts
in the setting of factorizable \( d=1+1 \) models and show that the known
fact of absence of ultraviolet problems in those models (in the presence of
higher than canonical dimensions) also passes to their holographic images.
In higher spacetime dimensions \( d>1+1 \) the holographic image lacks the
transversal localizability; however this can be remedied by doing hologra-
phy on \( d-2 \) additional lightfronts which share one lightray (Scanning by
d-1 chiral conformal theories).

1 A simple setting of the problem

Lightfront quantum field theory and the closely related \( p \to \infty \) frame method
have a long history. The large number of articles on this subject (which started
at the beginning of the 70ies) may be separated into two groups. On the one
hand there are those papers whose aim is to show that such concepts constitute
a potentially useful enrichment of standard local quantum physics.0000.

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but there are also innumerable attempts to use lightfront concepts as a starting point of more free-floating “effective” approximation ideas whose relations to causal and local quantum physics remained unclear. Here our main interest is to extend the first mentioned results.

The main problem is to overcome the short distance limitations of the old lightfront quantization and the \( p \to \infty \) frame method (which resulted in a severe restriction on the Kallen-Lehmann spectral function \([2]\)) which unfortunately excludes all cases of genuine renormalizable interactions. This restriction is identical to that which canonical commutation relation (or the interpretation of the functional measure in the spirit of a quantum mechanical Feynman-Kac euclidean representation) requires. But whereas the (perturbative) breakdown of canonicity causes no real harm (the canonical setting only serves as a mental starter but plays no role in the Feynman formalism) since the more general spacelike causality/locality structure is all one needs for dealing with renormalization, the restriction for the lightfront approach is a more serious matter. If the whole idea of lightfront-affiliated operators would be limited to free fields as indicated in various rigorous investigations about lightcone restrictions of Wightman fields \([2]\), the subject would have only academic interest.

In a recent paper I have indicated how the field-coordinatization free approach of AQFT can overcome this undesired restriction \([5]\). Its intuitive basis is the radical different nature of the conformal structure one encounters on the lightfront considered as a horizon of the wedge as compared to the massless conformal scaling limit; in fact the correct way to deal with lightfront localization is inexorably linked with the understanding of wedge localized operator algebras. Whereas the scaling limit (which is closely linked to the renormalization group approach) admits a natural formulation in terms of pointlike fields, the present chiral conformal structure on a lightfront horizon of a wedge region associated with interacting fields with nonintegrable Kallen-Lehmann spectral function cannot be obtained simply as a restriction of such fields to the horizon. One rather needs to perform a reprocessing of the field content into the net of algebras and to apply a modular inclusion procedure to the wedge algebra. In fact this process just supplies different spacetime indexed subalgebras; as global algebras the chiral horizon algebra is identical with the wedge algebra. Different from a restriction process, the notion of transversal localization is lost (i.e. the obtained chiral theory is a kind of kinematical coarse grained object) and has to be constructed by tilting the wedge as described in section 4.

To physicists familiar with the Unruh-Hawking effect the special role of the wedge algebra may not come totally unexpected, but the crucial role of operator algebras versus pointlike fields is somewhat surprising in view of the widespread opinion that their use is more a matter of mathematical conciseness and conceptual clarity rather than a matter of principle. Although concepts as Hawking temperature, horizons and holography originated with black hole physics, they have their natural analogs in Minkowski space\([\text{1}]\). Whereas for

\[\text{1}\]
Whereas in flat spacetime these local quantum physical concepts remain hidden and play no important role in particle physics, curved spacetime in special circumstances exposes them
thermal aspects this had been known for some time, the presence of the other concepts is the main issue of this article.

Although our conceptual setting and the mathematical formalism goes beyond what the protagonist of “holographic projections” onto the horizon had in mind \cite{6}, and how this became subsequently related with lightfront quantization ideas \cite{6}, the intuitive physical content of our algebraic approach matches these ideas perfectly. For this reason (and also for the fact that we cannot think of any other way in which this idea could be furnished with a rigorous mathematical and conceptually tight local quantum physical meaning) the excellent name is maintained; in fact we think of our conceptual additions as the liberation of these ideas from their old short distance restrictions which did not leave any room for interactions.

The name holography is often used in a wider sense for an isomorphism of a local (massive) QFT with data which are associated with lower spacetime dimensions (In case of equal spacetime dimensions of the source and target theory the name transplantation has been used \cite{8}). Not every aspect of such an isomorphism requires the full mathematical power of the field coordinateization free formulation of AQFT. For example the direction

$$AdS \rightarrow CQFT$$

in the Maldacena conjecture \cite{9} can be seen by taking the limit of the spacial infinite remote Anti deSitter boundary in correlation functions of fields and is therefore susceptible to standard formulations of QFT. Only the existence of the inverse and the understanding of the detailed properties which one field theory inherits from the other requires a more ambitious mathematical theorem in the AQFT setting\cite{10,11}. In the present case of lightray holography the algebraic setting is crucial for both directions of the isomorphism. Holography in this sense (where localizations becomes scrambled up) is not useful for physical concepts which rely on spacetime localization between operators as e.g. time dependent scattering theory; rather their beneficial effect is expected to show up in the understanding of coarse-grain properties of a localized algebra as e.g. its relative localization entropy between different states where only the overall relative size of degrees of freedom matter, but not the sublocalizations within that algebra. These matters will be discussed in a forthcoming paper.

The content is organized as follows. In the next section the properties of lightray restrictions in d=1+1 are recalled in the present setting. Section 3 illustrates the new construction based on modular wedge localization within the rich family of factorizing d=1+1 models. The additional problems posed in a geometric quasiclassical veil (in which they were discovered). The claims that holography is a characteristic manifestation of quantum gravity should be treated with cautious scepticism. In fact the present paper casts doubts on suggestions that such concepts require new physical principles which go beyond those of local quantum physics; as the situation looks now it appears more and more like a lack of agreement between older conjectures versus more recent theorems.

\footnote{One rigorous result is that the Maldacena conjecture cannot hold between two Lagrangian (pointlike fields) quantum field theories \cite{6,12,11}.}
by higher dimensional QFT including proposals for their solution are then addressed in section 4. The last section attempts to make connections to other fundamental problems of QFT for whose solution the new concepts are also expected to be important. In a mathematical appendix we collect some known mathematical results for the convenience of the reader.

2 Elementary review of d=1+1 lightray restriction

Let us consider the simplest case namely the lightray holography of a two-dimensional massive selfconjugate QFT. We first remind ourselves of the situation for a free massive scalar field

\[ A(x) = \frac{1}{\sqrt{2\pi}} \int (e^{-ip_x a(\theta)} + e^{ip_x a^*(\theta)}) \, d\theta \]

where for convenience we used the momentum space rapidity description. In order to approach the light ray \( x_- = t - x = 0 \) in such a way that \( x_+ = t + x \) remains finite we approach the \( x_+ > 0 \) horizon of the right wedge \( t^2 - x^2 < 0, x > 0 \) by taking the \( r \to 0, \chi = \hat{\chi} - \ln r \to \hat{\chi} + \infty \) in the x-space rapidity parametrization

\[ x = r(\text{sh} \chi, \text{ch} \chi), \quad x \to (x_- = 0, x_+ \geq 0, \text{finite}) \]

where the last formula serves to make manifest that the limiting \( A(x_+) \) field is a chiral conformal (gapless \( P_- \) spectrum) field; the mass in the exponent \( p_- x_+ = m e^\theta e^{-\chi} \) is just a parameter which keeps track of the “engineering dimension” (the physical mass is the gap in the \( P_- \cdot P_+ \) spectrum). Since this limit only effects the numerical factors and not the Fock space operators \( a^\#(\theta) \), we expect that there will be no problem with the horizontal restriction i.e. that the formal method (the last line in 2) agrees with the rigorous result. Up to a fine point which is related to the bad infrared behavior of a scalar \( \text{dim} A = 0 \), field this is indeed the case. Using the limiting \( \chi \)-parametrization we see that for the smeared field with \( \text{supp} \tilde{f} \in W \) one has the identity

\[ \int A(x_+, x_-) \tilde{f}(x) \, d^2x = \int_C a(\theta) f(\theta) = \int_C A_+(x_+) \tilde{g}(x_+) \, dx_+ \]

These formulas warrant some explanation. The onshell character of free fields restricts the Fourier-transformed test function to their mass shell values \( f(p)|_{p^2 = m^2} = \)
$f(\theta)$ and the wedge support property is equivalent to the strip analyticity. The integration path $C$ consists of the upper and lower rim of the strip and corresponds to the negative/positive frequency part of the Fourier transform. By introducing the test function $\tilde{g}(x_+)$ which is supported on the halfline $x_+ \geq 0$ it becomes manifest that the smeared field on the horizon rewritten in terms of the original Fourier transforms must vanish at $p = 0$ since

$$f(p)|_{p^2 = m^2} \frac{dp}{\sqrt{p^2 + m^2}} = f(\theta) d\theta \equiv g(\theta) d\theta = g(p) \frac{dp}{|p|} \sim g(0) = 0$$

This infrared restriction is typical for $\text{dim} A = 0$ fields and would not occur for free fields with a nontrivial L-spin. The equality of the $f$-smeared $A(x)$ fields with the $g$-smeared $A_+(x_+)$ leads to (after clarification of some domain problems of these unbounded operators) the equality of the affiliated (weakly closed) operator algebras

$$\mathcal{A}(W) = A(R_+) = A(R_-)$$

Here the last equality expresses the fact we could have taken the lower horizon with the same result. This equality is the quantum version of the classical propagation property of characteristic data on the upper or lower lightfront of a wedge. With the exception of $d=1+1$ $m=0$ the amplitudes inside the causal shadow $W$ of $R_+$ are uniquely determined by the lightfront data. Note that a finite interval on $R_+$ does not cast a 2-dimensional causal shadow; this is only the case for the full characteristic data. Related to this is fact that the opposite lightray translation

$$\text{Ad} U_- (a) A(R_+) \subset A(R_+)$$

$$U_- (a) = e^{-ip^- a}$$

is a totally fuzzy endomorphism of the $A(R_+)$ net whereas in the setting of the $\mathcal{A}(W)$ net spacetime indexing it is a geometric map.

It is very important to notice that even in the free case the horizontal limit is different from the scale invariant massless limit. The latter cannot be performed in the same Hilbert space since the $m \rightarrow 0$ limit needs a compensating $\ln m$ term in the momentum space rapidity $\theta$ of the operators $a^\#(\theta)$ whereas in the horizon limit it was only effecting the c-number factors. There is no problem of taking this massless limit in correlation functions if one uses smearing functions whose integral vanishes (or $f(p = 0) = 0$). The limiting correlation functions define via the GNS construction a new Hilbert space which contains two chiral copies of the conformal $\text{dim} A = 0$ field corresponding to the right/left movers. In the case of interacting theories the appropriately defined horizontal holographic projection is different from the scaling limit by much more than just multiplicities as will be seen below.

For interacting fields a necessary condition for the above lightray restriction to work is the finiteness of the wave function renormalization constant which
in terms of two-point function of the correctly normalized 
\( \langle p | A(0) | \Omega \rangle = \frac{1}{\sqrt{2\pi}} \)
interpolating field is the convergence of the following integral over the Kallen-Lehmann spectral function
\[
\int \rho(k^2) dk^2 < 0 \tag{4}
\]
This condition is violated for all genuinely renormalizable (i.e. not superrenormalizable) theories. In this case the holographic reprocessing has to be done on the level of algebras according to the formal scheme
\[
A(x) \to A(W) \xrightarrow{\text{holography}} A(R_+) \to A_+(x_+) \tag{5}
\]
Here \( A(x) \) stands symbolically for a complete set of local fields which fulfill \( d=1+1 \) locality and \( A_+(x_+) \) stand for possible \( R_+ \)-local chiral fields.

3 Holography for factorizable models

The holography in the presence of renormalizable interactions which violate the finiteness condition \( (4) \) can be nicely illustrated for \( d=1+1 \) factorizable models. The most appropriate construction of these models starts from free fields with nonlocally modified commutations relations for the momentum space creation and annihilation operators \( [13] \)

\[
A(x) = \frac{1}{\sqrt{2\pi}} \int (e^{-ipz} Z(\theta) + e^{ipz} Z^*(\theta)) \, d\theta, \quad p = m(ch\theta, sh\theta)
\]

\[
A(\tilde{f}) = \frac{1}{\sqrt{2\pi}} \int_C Z(\theta)f(\theta) \, d\theta
\]

where the \( Z \)'s are defined on the incoming \( n \)-particle vectors by the following formula for the action of \( Z^*(\theta) \) for the rapidity-ordering \( \theta_i > \theta > \theta_i+1, \theta_1 > \theta_2 > ... > \theta_n \)

\[
Z^*(\theta)a^*(\theta_1)...a^*(\theta_i)...a^*(\theta_n)\Omega = \tag{5}
S(\theta - \theta_1)...S(\theta - \theta_i)a^*(\theta_1)...a^*(\theta_i)a^*(\theta)\Omega
+ \text{contr. from bound states}
\]

In the absence of bound states this amounts to the commutation relations

\[
Z^*(\theta)Z^*(\theta') = S(\theta - \theta')Z^*(\theta')Z^*(\theta), \quad \theta < \theta' \tag{6}
\]

\[
Z(\theta)Z^*(\theta') = S(\theta' - \theta)Z^*(\theta')Z(\theta) + \delta(\theta - \theta')
\]

A smeared field \( A(\tilde{f}) \) applied to the vacuum creates a one-particle vector

\[
A(\tilde{f}) |0 \rangle = \int f(\theta - i\pi) |p(\theta)\rangle \, d\theta
\]
Localized operators which applied to the vacuum create one-particle vectors without admixture of multiparticle vacuum polarization clouds are called polarization-free generators (PFG's). Their structure is restricted by the following theorems

**Theorem 1** PFG's which are localized in regions whose causal completion is genuinely smaller than a wedge (e.g. spacelike cones, double cones) lead to interaction-free theories. On the other hand wedge-localized PFG's always exist even in the presence of interactions. They have Fourier transforms (tempered distributions) only in the case of $d=1+1$ with purely elastic scattering.

According to this theorem we should ask the question whether the above PFG's are wedge-localized. The affirmative answer is contained in the following theorem

**Theorem 2** The PFG's with the above algebraic structure for the $Z$'s are wedge-localized if and only if the structure coefficients $S(\theta)$ are meromorphic functions which fulfill crossing symmetry in the physical $\theta$-strip i.e. the requirement of wedge localization converts the $Z$-algebra into a Zamolodchikov-Faddeev algebra.

In this case the $A(\hat{f})$ are generators affiliated to the wedge algebra

$$A(W) = \text{alg} \{ A(f) | \text{suppf} \subset W \}$$

The most general operator $A$ in $A(W)$ is a LSZ-type power series in the Wick-ordered $Z$'s

$$A = \sum \frac{1}{n!} \int_C \ldots \int_C a_n(\theta_1, \ldots, \theta_n) : Z(\theta_1) \ldots Z(\theta_n) d\theta_1 \ldots d\theta_n :$$

with strip-analytic coefficient functions $a_n$ which are related to the matrix elements of $A$ between incoming ket and outgoing bra multiparticle state vectors. The integration path $C$ consists of the real axis (associated with annihilation operators and the line $\text{Im} \theta = -i\pi$). Writing such power series without paying attention to domains of operators means that we are we are only dealing with bilinear forms whose operator status is still to be settled. The bilinear forms which have their localization in double cones are characterized by their relative commutance (this formulation has to be changed for Fermions or more general objects) with shifted generators $A^{(a)}(f) \equiv U(a)A(f)U^*(a)$

$$[A, A^{(a)}(f)] = 0, \forall f, \text{suppf} \subset W$$

$A \subset A_{\text{bil.}}(C_a)$

where the subscript indicates that we are dealing with spaces of bilinear forms (formfactors of would-be operators localized in $C_a$) and not yet with unbounded operators and their affiliated von Neumann algebras. This relative commutant
relation on the level of bilinear forms is nothing but the famous "kinematical pole relations" which relate the even \( a_n \) to the residuum of a certain pole in the \( a_{n+2} \) meromorphic functions. The structure of these equations is the same as that for the formfactors of pointlike fields; but whereas the latter lead (after splitting off common factors which are independent of the chosen field in the same superselection sector) to polynomial expressions with hard to control asymptotic behavior, the \( a_n \) of the double cone localized bilinear forms are solutions which have better asymptotic behavior which according to the Paley-Wiener-Schwartz theorem. We will not discuss here the problem of how this improvement can be used in order to convert the bilinear forms into genuine operators.

Suppose now that we consider a modular inclusion (see appendix) of wedges which in the above formalism just means that we take \( a \) as a lightlike vector \( a \sim (1,1) \). In that case the relative commutant consists of bilinear forms which are interpreted to be localized in the interval (0,1) on the lightray which according to general theorems are associated to a chiral conformal field theory. It is easily checked that the previous space of double cone localized space is reobtained (as expected) by applying a suitable opposite lightray translation to the interval localized space of bilinear forms. The total spaces of wedge-localized and lightray localized bilinear forms which are the (weak closures of the) unions of the local spaces are the same (as expected from the classical picture that the causal shadow of the characteristic data on the halfline is the wedge i.e. \( \mathcal{A}(R+) = \mathcal{A}(W) \) whereas finite intervals cast no shadow).

According to the construction the net on the horizon is bosonic and since it is also chiral the spectrum of scale dimensions must be integer-valued. The massless limit of the pointlike field generators of the wedge theory on the other hand can have anomalous short distance behavior. This is yet another manifestation of the useful kinematical nature of the lightray algebra (the "scrambled up" wedge algebra). Important dynamical data have been transferred to the action of the opposite lightray translation which plays the role of a kind of hamiltonian and whose action destroys the conformal invariance and recreates the complications of the original massive theory. The fascinating aspects of the free field behavior of pointlike generators of the chiral \( \mathcal{A}(R) \) net is the extreme nonlocality they must have with respect to the generators of the \( \mathcal{A}(W) \) net together with the naturalness of their construction. Despite their simplicity their creation/annihilation operators must be infinite power series in the \( \mathbb{Z} \)-operators. Factorizing models promise to play an important role in the better understanding of this horizon-localized chiral conformal field theory which is quite different from the conformal scaling limit theory (e.g. it has a richer supply of symmetries and maps). Automorphisms which act locally on the original set may become "fuzzy" on the horizon net (example: the opposite lightray translation) and vice versa (example the circular rotation on the conformal horizon).
4 Problems met in higher dimensional cases

When there are transversal spatial dimensions the restriction of fields to a light-front continues to show the same problems in the presence of interactions. For a proof that (under very mild assumptions about operator domains) the fields must have the free canonical structure we refer to the second paper of Driessler. In the first paper the author also shows that the vacuum factorizes transversally (a behavior otherwise met for spatially disjoint localized algebras in theories without vacuum polarization like second quantized QM) although it remains highly entangled with respect to longitudinal (along the light ray) disjointness where one needs the so-called split property in order to meet conditions which are similar to quantum mechanical (type I) algebras.

Before we turn to a field-free i.e. algebraic construction of a holographic projection we briefly present the lightfront formalism for free fields. The rapidity parametrization of a scalar free field with $x = r(sh\chi, ch\chi, x_\perp)$, $p = (m_{eff}ch\theta, m_{eff}sh\theta, p_\perp)$, $m_{eff}(p_\perp^2) = \sqrt{m^2 + p_\perp^2}$ leads to the following (upper) horizontal projection formula

$$A(x_+, x_\perp) = \frac{1}{(2\pi)^2} \int (e^{-ip_--x_+ + ip_\perp x_\perp} a(\theta, p_\perp) + e^{ip_--x_+ - ip_\perp x_\perp} a^*(\theta, p_\perp)) d\theta$$

$$p_- x_+ = m_{eff}(p_\perp^2) e^\theta x_+$$

Rewritten formally in terms of the momentum space measure

$$A(x_+, x_\perp) = \frac{1}{(2\pi)^2} \int (e^{-ip_--x_+ + ip_\perp x_\perp} a(p_-, p_\perp) + e^{ip_--x_+ - ip_\perp x_\perp} a^*(p_-, p_\perp)) \frac{dp_-}{|p_-|}$$

we encounter the typical infrared divergence which requires to use again the restricted test functions $f(x_+, x_\perp)$ (again only for $s=0$) with $\int f(x_+, x_\perp) dx_+ = 0$. The above rapidity representation reveals that the lightfront field almost looks like a continuous superposition of chiral fields at different scales set by the continuous values of the magnitude of the transverse momenta $|p_\perp|$.

As in the $d=1+1$ case in section 3, the algebraic holography starts from the modular inclusion of the standard wedge shifted along the $x_+$ lightray into itself (the reason we work with wedge algebras and not with the full algebra is that the modular prerequisites of absence of annihilation operators for the vacuum are violated for the latter). Unlike the lightfront restriction of free fields which together with the action of the opposite lightray translation allows to reconstruct the original $d=1+1$ theory, the modular inclusion in the present case yields a result which has no transverse localization, although the transverse translation $e^{iP_\perp x_\perp}$ acts on $A(R_+)$. Note that our notation is not meant to literally indicate a localization on the $x_+$ lightray but only indicates that the chiral holographic...
projection has no transversal localization concept (otherwise we could not describe it in terms of a chiral theory). This is the prize we have to pay for obtaining such a simple holographic image from a complicated higher dimensional QFT. Knowing the action of the translation into the opposite lightray direction does not help to resolve the lack of transversal localization. Whereas this may be enough in problems of degree of freedom counting (e.g. entropy discussions) for the reconstruction of the original net i.e. for the formulation of a holographic isomorphism one must get a control of transversal localization. This can be achieved by Lorentz tilting the standard wedge around its $x_+$ lightray. This is done by using the “translational” part of the Wigner little group of the light ray. In $d$ dimensions there are $d-2$ such translations inside the homogenous Lorentz group and they act like a transversal Galilei transformation. Together with the longitudinal symmetries the geometric symmetry group of a lightfront is seven-dimensional: the longitudinal dilation (alias L-boost) and translation and the two transversal translations as well as Galilei transformations and a transversal rotation (the wedge allows a natural association with an 8-parametric subgroup of the Poincaré group).

In order to keep things geometrically simple let us choose in the following $d=1+2$. Then there is just one one-parametric group of tilting (Galilean) transformations such that the pair of the original wedge together with the tilted wedge forms a modular intersection in the sense of the appendix. This may be easily rephrased in terms of an additional localization structure on the upper lightfront horizon. The original transversal stripes which correspond to the longitudinal intervals are transformed into sloped stripes which intersect the original ones in parallelograms and form the localization regions of an algebraic net on the horizon. Again this is a fuzzy transformation of the chiral theory. Clearly the parallel translates of the tilted lightfront intersect the original lightfront in lines which are transversally shifted parallels of the original lightray. This can be used to generate a transversal localization on the original lightfront which generates a net structure on this lightfront. Together with the action of the opposite lightray translation this is sufficient to recuperate the original $d=1+2$ net. Instead one may also talk about reconstructing the original theory by scanning with two chiral theories, the second one resulting from the first by the tilting automorphism. The principle of generalization to $d$ dimensions should be clear from our geometric interpretation.

5 Hopes based on modular localization

In these notes we have shown that (in the presence of interactions) modular localization and modular inclusions are essential tools in the formulation of ideas around “holography” as a significant extension of lightcone quantization. A closely related problem is that of modular symmetries beyond the geometric symmetries of Poincaré- or conformal invariance (which are also known to be of modular origin, in the case of the infinite chiral diffeomorphisms group this was shown in [10]). Fuzzy modular groups $\sigma_t$ exist for each standard Reeh-Schlieder
pair \((A(O), \Phi)\), they only depend on the state \(\phi\) and not on its implementing vector \(\Phi\). They are consistent with causality because they keep the causal complement of \(O\) apart from \(O\) in analogy say to the wedge affiliated L-boost which does not mix \(A(W)\) observables with those of \(A(W') = A(W')'\) and as globally defined automorphisms they maintain commutance if applied to originally commuting operators even if the original localization regions suffer a fuzzy \(t\)-dependent “diffusion”. They do not exist in the corresponding classical theory (assuming that the correspondence principle creates such an object) and their concrete form depends much more on quantum dynamical aspects. The intrinsic understanding of interactions is related to the absence of better than wedge localized PFG’s and therefore it seems to be a reasonable conjecture that the action of the modular groups on vectors created by the application of such PFG to the vacuum is related to the shape of their vacuum-polarization clouds. An intrinsic understanding of these interaction-caused clouds is of course an old but never fulfilled dreams of nonperturbative local quantum physics.

Another such dream (which was mentioned in the introduction) is the understanding of “entropy of localization” \[17\]|\[13\]; this is natural problem since “localization temperature” (the Unruh-Hawking effect) has already been successfully explained in terms of modular concepts \[18\]. In particular it would be of great interest to know whether localization entropy preempts the Bekenstein black hole behavior and is proportional to the area of the spatial boundary of the bounding causal horizon and if the analogs to the thermodynamical laws are in some sense quantum renormalized.

The formalism for factorizable models which we used for \(d=1+1\) illustrations of holography in section 2 is of course interesting in its own right, since it offers a constructive approach without facing any ultraviolet problem just as it was expected way back in a pure S-matrix approach. Its pointlike formulation \[19\] has witnessed a quite impressive growth over many years and the present operator algebraic method adds to it a spacetime interpretation of the Smirnov axioms (in particular of the Zamolodchikov-Faddeev nonlocal creation and annihilation operators) and holds the promise of a construction of the operators behind the bilinear forms (formfactors). Operator constructions are easier for extended objects than for pointlike fields.

The insensitivity of modular-based concepts on ultraviolet behavior of pointlike fields calls for the reinvestigation of the perhaps most important question of the post Feynman era: does the standard renormalizability criterion really mark the short distance frontier of the underlying principles of local quantum physics or does it only limit the range of applicability of trying to understand interactions in terms of singular pointlike field coordinatizations? Whereas in the Lagrangian renormalization approach based on power counting massive free fields cannot have dimensions beyond one in \(d=1+3\), the bootstrap-formfactor approach, wherever it works, does not know such bounds.

Unfortunately this onshell approach remains presently limited to factorizing models in \(d=1+1\) where it constructs a tempered (polynomially bounded in momentum space) field theory \[21\] for each admisible (crossing symmetric, unitary) factorizing S-matrix; no a priori restriction on short distance behav-
ior is necessary. This situation was what the S-matrix protagonists dreamed of in the 60ies. Although they extracted most of their S-matrix properties (in particular the important crossing property) from the causality and spectral properties of local quantum physics they had the unfortunate idea that in order to be successful in their pursuit they had to liquidate QFT. One could of course adapt the pessimistic attitude that these factorizing models are too special since they all share the property that whenever they contain a variable coupling parameter (example: massive Thirring model) there is no genuine coupling constant renormalization and the Beta function vanishes.

There are however also encouraging indications from the perturbative aspects of higher spin theories whose couplings violate the orthodox power counting criterion. Consider for example interactions of massive d=1+3 vectormesons. Since the free field dimensions of the vector description is two instead of one (other associations of physical free fields with s=1 cannot lower the dimension), any Lorentz-invariant interaction involves at least the powercounting dimension 5 and hence to the orthodox approach it is nonrenormalizable i.e. the number of counterterm structures increase in each order of perturbation theory which leads also to a nontempered behavior of correlation functions. On the other hand there is the following trick (only available for massive vector-fields) which leads to a renormalizable solution i.e. one for which the operator dimension 2 of the vectormeson field (apart from the expected logarithmic corrections) is maintained after renormalization. The trick consists of representing the Wigner one particle space as a cohomology space in the spirit of a one particle version of the so-called BRST formalism and constitutes a generalization of the approach taken in . The idea behind this trick is the same as in any application of BRST formalism: the combination of lowering the propagator dimensions in the cohomological extension together with the idea of stability of cohomological representations allows a safe return to physics after having renormalized the auxiliary correlation function of the extended correlation functions in the standard way (perturbative renormalization is a linear process which does not require positivity). The particular “one-particle version” of the BRST representation has two additional advantages. On the one hand it keeps the BRST “ghosts” away from self-interactions (their contribution to the action remains bilinear) and maintains the LSZ asymptotic structure throughout. On the other hand it requires the presence of additional interacting physical degrees of freedom which (unlike in the standard approach based on the mass generation by Higgs condensates) was not part of the input. The simplest (and probably only) realization of these additional degrees of freedom is via a standard scalar field i.e. one with vanishing vacuum expectations (i.e. not “Higgs” if this terminology refers to the condensate). There is of course no change of physical content as compared to the standard Higgs mechanism approach, but the perspective and physical interpretation is somewhat different. In particular one returns to the original physical question for renormalizable vectormesons in the pre-gauge spirit of Sakurai and Lewellyn-Smith. In this way the suspicion that there was only one renormalizable coupling involving massive vectormesons is con-
firmed and a gauge principle which selects between different possibilities is not needed. From a particle physics point of view it makes more sense to emphasize the uniqueness of renormalizable vectormeson couplings than the differential geometric esthetical appeal of the gauge principle in the Higgs mechanism. One hopes that the present modular concepts may have will tell us eventually something about the true borders carved out by the underlying physical principles and perhaps show that the widening of renormalizability does not stop at the interaction of spin s=1 particles.

Another more mundane goal where one expects the new concepts to produce explicit answers is the local quantum physics of d=1+2 anyons and of the d=1+3 infinite spin zero mass Wigner representations. In both cases the multiparticle structure does not follow the usual tensor product structure and there exist no PFG’s which have a better than wedge-like localization \[24\]. In fact the maintenance of the spin-statistics connection in the presence of braid group statistics requires even in the “free” case (conservation of real particles, even vanishing of the cross section) the presence of vacuum polarization clouds so that the nonrelativistic limit remains a QFT i.e. cannot be a QM. This connection is in particular not maintained in the QM of the Aharonov-Bohm like topological constructions.

We hope that we were able to convince the reader that there are plenty of deep and physically relevant local quantum physical problems in particle physics which fall into the range of the new method.

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6 Appendix: mathematical aspects of modular theory

For the benefit of the uninitiated reader we briefly collect and comment on some formulas from modular theory \[25\].

Let \((\mathcal{A}, \Omega)\) be a standard pair, i.e. an operator algebra in a Hilbert space \(H\) with a vector \(\Omega\) on which the algebra acts cyclically \((\mathcal{A} \Omega = H)\) and on which there exist no annihilators \((A \Omega = 0, A \in \mathcal{A} \Rightarrow A = 0, “\Omega is separating”)\)

**Definition 3** Tomita’s involution: \(S\mathcal{A} \Omega = A^* \Omega, \sim S\) is closed, antilinear and involutive i.e. \(S^2 \subset 1\) (involutive on the domain). The polar decomposition

\[\text{There are however several couplings involving (semi)classical vector fields. In that case the gauge principle selects the Maxwellian interactions. To the extend that the correspondence principle can be invoked, the classical gauge principle follows from renormalizability of the corresponding local quantum physics.}\]
$S = J\Delta^\frac{1}{2}$ leads to an antiunitary $J$ and a positive $\Delta$ which in turn gives rise to a one parametric unitary group $\Delta^it$, the (Tomita) modular group

**Theorem 4** (Tomita, Takesaki) The Ad-action of $\Delta^it$ defines an automorphism $\sigma_t$ of the operator algebra $\mathcal{A}$ and the Ad -action of $J$ maps $\mathcal{A}$ into its commutant algebra $\mathcal{A}'$

$$\sigma_t(A) : = \text{Ad}\Delta^\text{it}(A) \in \mathcal{A}$$

$$\text{Ad}J(A) = \mathcal{A}'$$

(10)

The modular automorphism group $\sigma_t$ depends only on the state $\omega(\cdot) = \langle \Omega | \cdot | \Omega \rangle$ and not its implementing vector $\Omega$; they are related through the KMS property (strip-analyticity of the function $F(z)$)

$$F(t) = \omega(\sigma_t(A)B)$$

$$F(t + i) = \omega(B\sigma_t(A))$$

The KMS property (which generalizes the Gibbs formula) characterizes the modular automorphism of $\mathcal{A}, \omega$

**Theorem 5** (Bisognano-Wichmann) If we take for $\mathcal{A}$ the wedge algebra $\mathcal{A} = \mathcal{A}(W)$ and for $\Omega$ the vacuum vector, the modular objects have the following physical interpretation

$$\Delta^it = U(\Lambda_W(-2\pi t))$$

$$J = TCP \cdot \text{Rot}_W(\pi)$$

Here $\Lambda_W(\chi)$ is the wedge adapted L-boost, TCP the antiunitary TCP-transformation of local QFT and $\text{Rot}_W(\varphi)$ the rotation group around the spatial axis pointing into $W$.

**Definition 6** (Wiesbrock, Borchers [25]) An inclusion of operator algebras $(\mathcal{A} \subset \mathcal{B}, \Omega)$ is ”modular” if $(\mathcal{A},\Omega)$, $(\mathcal{B},\Omega)$ are standard and $\Delta^B_\mathcal{B}$ acts for ($t<0$) as a compression on $\mathcal{A}$

$$\text{Ad}\Delta^B_\mathcal{B}\mathcal{A} \subset \mathcal{A}$$

(11)

A modular inclusion is standard if the relative commutant $(\mathcal{A}\cap\mathcal{B},\Omega)$ is standard. In that case the conditional expectation $E$.

**Remark 7** In case the equality sign holds (i.e. the compression is an automorphism, a theorem of Takesaki states the equality $\mathcal{B} = \mathcal{A}$. Therefore the modular inclusion situation is a generalization of that studied by Takesaki.
Theorem 8 (Guido, Longo and Wiesbrock [26]) Standard modular inclusions are in correspondence with chiral AQFT

An important physical application used in the main text is obtained by the following adaptation to a massive interacting AQFT

\[ B = \mathcal{A}(W) \]

\[ \mathcal{A} = U(e_+)\mathcal{A}(W)U^*(e_+), \; e_+ = (1, 1) \]

\[ \equiv \mathcal{A}(W_{e_+}) \]

This is the inclusion of the algebra translated via a lightlike translation into itself so the geometrically the relative commutator

\[ \mathcal{A}(W_{e_+})' \cap \mathcal{A}(W) \equiv \mathcal{A}(I(0, 1)) \]

is by causality localized in the upper horizontal interval (0,1). The standardness of this inclusion then leads to a chiral conformal AQFT i.e. a net (more precisely a pre-cosheaf)

\[ I \to \mathcal{A}(I), \; I \subset S^1 \]

\[ \mathcal{A}(R_+) = \bigcup_t \text{Ad}_t \Delta_{it} \mathcal{A}(I(0, 1)) \]

\[ \mathcal{A}(R) = \mathcal{A}(R_+) \lor JA(R_+)J \]

on which the Moebius group which preserves the vacuum vector acts. With the help of an external (i.e. not in Moeb.) automorphism on \( \mathcal{A}(R) \) implemented by the opposite lightray translation \( U_{-}(a) \) we are able to return from the chiral net on the right upper horizon to the original 2-dim. net. We call this chiral theory supplemented by the opposite lightray automorphism the holographic image of the 2-dim. massive net. With this interpretation the holographic projection for massive d=1+1 theories is nothing else than the conceptually and mathematically tightened version of the old lightray/p\to\infty frame quantization including the rules of how to reprocess back the holographic image into the original local d=1+1 theory. For the extension of holographic projection to higher dimensional theory one needs one more mathematical definition and theorem about “modular intersections”

Definition 9 A \((\pm)\) modular intersection is defined in terms of two standard pairs \((N, \Omega)\), \((M, \Omega)\) whose intersection is also standard \((N \cap M, \Omega)\) and which in addition fulfill

\[ J_N( \lim_{t \to \mp \infty} \Delta_{it}^{it} \Delta_{Mi}^{-it})J_N = \lim_{t \to \mp \infty} \Delta_{it}^{it} \Delta_{Mi}^{-it} = J_M( \lim_{t \to \pm \infty} \Delta_{it}^{it} \Delta_{M}^{-it})J_M \]

All limits are in the modular setting are to be understood in the sense of strong convergence on Hilbert space vectors. In the geometric setting of local quantum physics the the modular intersection property is realized par excellence
by the pair of intersecting wedge algebras $\mathcal{M} = \mathcal{A}(W), \mathcal{N} = \text{Ad}U(\Lambda_{e^+}(a))\mathcal{M}$ together with the $\Omega =$vacuum. Here $\Lambda_{e^+}(a)$ denotes a “translation” (transversal Galilei transformation) in the Wigner little group which fixes the lightray vector $e^+$, i.e. the Lorentz transformation which tilts $W$ around this lightray. In fact the limit in the second line is geometrically nothing else but $\Lambda_{e^+}(a)$ and the commutation relation with $J_{\mathcal{N},\mathcal{M}}$ is easily checked as a geometric relation in the extended Lorentz group.

Modular intersections play an analogous role in the construction of 3- and higher-dimensional AQFT starting from a finite set of wedge algebras \[27\] and the related holographic isomorphisms as the modular inclusions used in section 3 for the 2-dimensional case.

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