Causality problem in a holographic-dark-energy model

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Abstract – In the model of holographic dark energy, there is a notorious problem of circular reasoning between the introduction of future event horizon and the accelerating expansion of the universe. We examine the problem after dividing it into two parts, the causality problem of the equation of motion and the circular logic on the use of the future event horizon. We specify and isolate the root of the two problems as a boundary condition from the causal equation of motion, which can be determined from the initial data of the universe. We show that the causality will be kept if we define it based on our recognizability and the circular-logic problem can be reduced to imposing an initial condition. We additionally find that the model constant of the holographic dark energy is close to unity from the present data of the universe.

Introduction. – To explain the present accelerating expansion of the universe discovered in 1998 \cite{1}, several dark-energy models have been suggested by introducing new exotic matters or by modifying gravity. For the reviews about the accelerating expansion and dark energy, consult ref. \cite{2}. Of all the models, the holographic-dark-energy (HDE) model takes a unique position because it is an effective theory which introduces an energy density determined by geometric structures of the universe. Interestingly, to explain the present accelerating expansion of the universe, it was shown that the theory requires the future event horizon as an infra-red cutoff of universe rather than the Hubble, the particle, or the apparent horizons \cite{3}. There are several attempts to search for the origin for the HDE from Casimir energy in de Sitter space \cite{4}, quantum uncertainty of transverse position \cite{5}, holographic gas \cite{6}, entanglement entropy from quantum information loss \cite{7}, the Hawking radiation from the future event horizon \cite{8}, or the space-time curvature \cite{9}. However, many physicists may doubt that most of them respect the causality or have a circular reasoning problem since the future event horizon depends on the future globally \cite{5}.

The problems go as follows:

– Circular logic: Given the present data of the universe, we do not know whether the universe will eventually undergo accelerating expansion or not. The future event horizon is determined only after the evolution of the universe finished. On the other hand, once the HDE is introduced the universe is destined to expand with accelerating rate. Then, can we use the future event horizon even if we do not know its existence at present? This composes the heart of the circular logic. The problem becomes worse if we note that the holographic energy based on other horizons than the future event horizon does not accelerate the expansion of the universe.

– Causality problem: Assume that there is a creature which can modify the future event horizon\(^1\). If the creature modifies the horizon, it may affect the present motion of the universe since the equation of motion depends on the distance to the future event horizon. This raises the problem on causality: “Can we predict the next second state of the universe from the present data in the absence of future knowledge?”.

\(^1\)Here we disregard the fact that it is impossible to modify the future event horizon at present because it is determined by the whole evolution of the universe including the creature’s action.
To overcome the problem of circular reasoning, Gong [10] developed the extended holographic-dark-energy model, where the Brans-Dicke theory of gravity is adopted using the Hubble scale in place of the future event horizon. They have shown that there exists a no-go theorem stating that the Hubble scale cannot be an infrared cutoff for the universe with Brans-Dicke gravity. However, in the presence of a potential term for the Brans-Dicke scalar field, they succeed to show that it is possible to generate the HDE from the Hubble horizon [11].

In the present work, we directly analyze the HDE rather than introducing another theory of gravity or matter. In the spatially homogeneous and isotropic universes, the future event horizon is located at a distance

$$R_h = a(\tau) \int_{\tau}^{\infty} \frac{\tau' \, d\tau'}{a(\tau')},$$

where $\tau$ is the comoving time and $a(\tau)$ is the scale factor of the Robertson-Walker metric

$$ds^2 = -d\tau^2 + a^2(\tau) dx^2,$$

where $d\tau^2$ denotes the flat metric for three spatial dimensions. Note that the future horizon satisfies $R_h/H = R_h - H^{-1}$, where $H \equiv \dot{a}/a$ is the Hubble parameter. Therefore, the horizon is always located outside the Hubble (apparent) horizon if $\dot{R}_h > 0$. Any observers in the universe will be surrounded by the future event horizon and their accesses to remote information of the universe are restricted by the horizon. This lack of information is represented by a kind of entropy given by the cosmological horizon area in Planck units similar to the black-hole entropy. The generalized second law of thermodynamics for the Robertson-Walker space-time was proved by Davies [12] and Pollock and Singh [13] showing that the total entropy of gravity+matter does not decrease through physical processes.

How can we predict the future of the universe? At present, we are observing the universe inside the past light-cone of us at the present time. At every moments, our range of observation increases with time and new data on the universe reach us. Therefore, in principle, we cannot predict anything for the future. To circumvent this situation, we resort to the observational data for the past. From the observations, it was found that the observable universe is spatially homogeneous and isotropic at large scales. We assume in this work that our universe is spatially homogeneous.

Many dark-energy models appeared in the literature [2] to explain the recent accelerating expansion of the universe. Some of them introduce exotic matters such as Chaplygin gas, phantom matter, quintessence, or others. Some others modify the gravity theory by considering higher curvatures or branes in higher dimensions. The holographic dark energy locates in the middle of the two directions because it introduces an energy density determined by a geometric quantity. There are several versions of holographic-dark-energy models. Li first introduced the future event horizon to give the future acceleration [3]. Some authors explain the origin of the holographic dark energy as the quantum energy fluctuation [14], cosmic Hawking radiation [15], etc. Many variations of the holographic-dark-energy models are also studied including agegraphic-dark-energy model [5,16] and Ricci dark-energy-models [9].

The equation of motion in the presence of the HDE is given by

$$3M_p^2 H^2 = \sum \rho = \rho_h + \rho_{\text{bh}},$$

where $M_p$ denotes the Planck mass. The energy densities are divided into two pieces: the holographic dark energy,

$$\rho_h = \frac{3M_p^2 H^2}{R_h^2},$$

and the sum of all energy densities other other than the HDE, $\rho_{\text{bh}}$. The portion of the HDE in our universe becomes

$$\Omega_h = \frac{\rho_h}{\sum \rho} = \frac{\rho_h}{3M_p^2 H^2} = \frac{d^2}{(H R_h)^2},$$

where $d$ is a model constant of the HDE.

The causality problem of HDE was recognized by Li [3] for the first time when he introduces the future event horizon. He also provided part of the solution by questioning about the usefulness of comoving time. Even though the comoving time is intrinsic to a comoving observer in a time-dependent background, it may not be the best time parameter to use in order to understand the causality. He show that the event horizon is no longer as acausal as in the comoving time if we write the metric (2) in the conformal time, $\eta = \int_{\tau}^{\tau'} \frac{\tau' \, d\tau'}{a(\tau')}$, as

$$ds^2 = a^2(\eta) (-d\eta^2 + dx^2 + r^2 d\Omega^2).$$

The range of the conformal time has a finite upper limit, for instance $\eta \in (-\infty, 0)$. Due to this finite upper limit, a light ray starting from the origin at the time $\eta$ cannot go arbitrarily far but has a horizon at $r = -\eta$. Now, the formula for the horizon distance $R_h = a(\eta)|\eta|$ appears to be causal. He also mentioned about the possibility that the quantized vacuum energy in a box can be interpreted as the holographic dark energy.

Li’s resolution presents a way to avoid the causality problem. In this work, we extend his resolution to general situations without introducing ad hoc conformal time, which is the definite integral of the inverse of scale factor from infinity to $\tau$. In addition, we discuss the conceptual aspect of the future event horizon.

**Does the future event horizon always lead to future acceleration?** – First, we provide a counterexample to the assertion: “If there exists an energy density depending on the future event horizon, then the universe is...
destined to expand with accelerating rates in the future”. Let us assume an energy density $\rho_n = 3M_p^2 p^n R_h^n$ with an integer $n$ and a real number $p$. If $n = -2$, it is nothing but the holographic dark energy, which is known to give future accelerating expansion respecting the assertion. At the present case, let us consider the case with $n = 2$. In the absence of other field (or if $\rho_d$ dominates the universe which will happen for large $R_h$), the equation of motion becomes $H^2 = p^4 R_h^4$. Since we are interested in finding an expanding solution we may reduce the equation

$$\frac{a}{a^2} = p^2 \int^{\infty}_{\tau} \frac{d\tau'}{a(\tau')} ,$$

and then differentiating both sides with respect to $\tau$ once, we get $(\frac{d}{d\tau})^2 \frac{1}{a} = \frac{\rho_d}{a}$. Note that its general solution

$$a(\tau) = (\alpha e^{\rho_d \tau} + \beta e^{-\rho_d \tau})^{-1},$$

where $\alpha$ and $\beta$ are arbitrary integration constants, does not lead to future accelerating universe contrary to the assumption. In this case, the initially expanding universe eventually contracts and there is no accelerating expansion in the future. Therefore, the energy density depending on the future event horizon does not necessarily give the future accelerating expansion of the universe.

Causal evolution equation and initial condition.
- Now we specify the root of the causality problem in the evolution equation (3) as a boundary condition. Then, we separate it from a well-posed causal differential equation representing the evolution of the scale factor. It appears that eq. (3) bears the causality problem in the following sense: Suppose that there exists a creature which can modify the future event horizon. Someday in the future, the creature decides to change the horizon. Now, we should ask the following questions:

1) Does this action modify the present evolution of the universe?

2) Does this imply the causality violation for us? In other words, can we recognize the action of the creature in the future?

To the first question, we should answer “yes”. Noting the evolution equation (3), it is sure that the modification of the horizon area actually changes the present evolution of the universe. However, this does not mean that the answer to the second question is also yes.

To answer the second question correctly, we examine eq. (3) in detail. We separate the future-dependent part from the genuine dynamics described by the well-posed second order differential equation. To do this, we rewrite the equation of motion (3) as

$$\int^{\infty}_{\tau} \frac{d\tau'}{a(\tau')} = \frac{d}{a(\tau)} \sqrt{H^2 - \frac{H^2}{3H^2}},$$

where we assume $d > 0$, without loss of generality. Note that the future-dependent part is located in the left-hand side of the equation. We can separate that part from the others by setting $\int^{\infty}_{\tau} \frac{d\tau'}{a(\tau')} = r_\infty = \int^{\infty}_{0} \frac{d\tau'}{a(\tau')}$, where

$$r_\infty = \int^{\infty}_{0} \frac{d\tau}{a(\tau)} (7)$$

appears to depend on the future evolution of the scale factor. If we differentiate eq. (6) once with respect to $\tau$, the term $r_\infty$ disappears and obtain a well-posed second-order differential equation:

$$\dot{H} + H^2 - \frac{1}{3M_p^2} \left( \rho_n + \rho_{nh} \frac{2\dot{H}}{2H} \right) - (H^2 - \frac{\rho_{nh}/(3M_p^2)^{3/2}}{dH}) = 0. (8)$$

In the series of calculations, the evolution equation (3) is divided into two pieces, one is a well-posed evolution equation (8) and the other is the term $r_{\infty}$ which appears to bear the information of the causality violation.

Equation (8) can be used in two folds: First, it determines the evolution of the scale factor from the initial data on the scale factor and its first derivative, where both can be determined from the observations. Second, we can use eq. (8) to determine the model constant $d$ for the holographic dark energy from the initial data including the deceleration parameter, $q = -1 - \dot{H}/H^2$,

$$d = \frac{(1 - \Omega_{nh})^{3/2}}{(1 - \Omega_{nh})(-q) + \frac{\Omega_{nh}}{2H}} \bigg|_{\tau = \tau_0},$$

where $\Omega_{nh} = \frac{\rho_{nh}}{3M_p^2 H^2}$. Assuming that the non-holographic energy density is dominated by the matters, $\rho_{nh} = \rho_0 a_0^3/\alpha(\tau)^3$, we get $\Omega_{nh} = -3H\Omega_{nh}(1 - 2q)$. Inserting the present data $\Omega_{nh} \approx 0.27$ and $q \approx -0.56$, we get $d \approx 0.90$. One may also get $d \approx 1.17$ from the data $\Omega_{nh} \approx 0.27$ and $q \approx -0.4$. Even though the present data on the deceleration parameter is dependent on the analysis and on models [17], the observations indicate $d \approx 1$.

Given $d$, the distance to the future event horizon at present can be shown to be slightly larger than the Hubble radius from eq. (5),

$$R_h = \frac{d}{\sqrt{1 - \Omega_{nh}} H_0} \approx \frac{1.17}{H_0},$$

where we have used $d = 1$.

Note that the value $r_{\infty}$ does not affect the evolution equation (8). The whole evolution can be solved from the information at some initial time without any future information. Since the evolution can be determined irrespective of $r_{\infty}$, we find that the horizon can be determined from the present information if we solve the equation of motion (8). In this sense, the information about $r_{\infty}$ does not need to be specified to determine the distance to the event horizon. In fact, $r_{\infty}$ is not always well defined. It becomes a finite number only when the scale factor behaves as $a(\tau) \sim \tau^m$ ($m < 1$) for small $a$ and $a(\tau) \sim \tau^n$ ($n > 1$) in the future.
However, even if \( r_\infty \) is ill-defined, the evolution equation (8) works well reproducing the Einstein equation. If someone says that the causality is violated, he/she means that something unexpected from the past data happens at present (due to future action). However, as seen in eq. (8), all future evolutions can be predicted from the past data. If we define the “causality” in a sense that there does not appear what is forbidden from the past data, then we may say that the answer to the second question is “No”, the “causality” is kept.

It is interesting to see the role of \( r_\infty \).

Note that eq. (8) has additional solutions which correspond to the case when the integral in eq. (6) has different upper bounds from \( \infty \). Once the scale factor \( a(\tau_0) \) and the Hubble parameter \( H(\tau_0) \) are specified at a given time \( \tau_0 \), eq. (8) determines the future uniquely. Since the scale factor \( a(\tau_0) \) can be scaled, the only parameter to be specified as an initial condition is the Hubble parameter \( H(\tau_0) \). The Hubble parameter at \( \tau_0 \) is given by

\[
H(\tau_0)^2 = \frac{d^2}{a^2(\tau_0)} \left( r_\infty - \int_0^{\tau_0} \frac{dt'}{a(t')} \right)^2 + \frac{\rho_{\text{rh}}(\tau_0)}{3M_p^2},
\]

which is well defined once \( r_\infty \) is given. Conversely, this equation can be used to fix the value of \( r_\infty \) from the present data on \( \rho_{\text{rh}} \) and the past history of the scale factor. Even though \( r_\infty \) is defined by the integral \( \int_0^\infty a^{-1}d\tau \), we may also obtain its value by using the limit

\[
r_\infty = \lim_{\epsilon \to 0} \int_{\epsilon}^{\infty} \frac{dt'}{a(t')} = \lim_{\epsilon \to 0} \frac{d}{\sqrt{\dot{a}^2 - \frac{\alpha^2 \rho_{\text{rh}}}{3M_p^2}}}. \tag{9}
\]

Therefore, one may simply take the limit to obtain \( r_\infty \) rather than integrating over the whole evolution. From this point of view, the value \( r_\infty \) does the role of initial boundary condition on the Hubble parameter rather than a future-dependent constant.

In fact, eq. (3) plays the role of a Hamiltonian constraint of the Einstein equation. The constraint plays two roles in geometro-dynamics. First, it gives the initial condition for a given spacelike surface. Second, in the present situation of homogeneous space, it gives the evolution of the geometry too. From this constraint, we derived eq. (8) by removing the initial condition part, \( r_\infty \). In this sense, the Hamiltonian constraint is moved to the initial condition at the beginning of the universe. Since the new equation (8) plays the role of equation of motion, one may ask: “what is a Hamiltonian constraint corresponding to this new equation?” In the presence of a local description of the holographic dark energy giving eq. (8), one may find the corresponding constraint. For this subject, refer ref. [18]2. In this work, since we are interested simply in the causal property of the holographic dark energy, we defer the study on the fully covariant description of the holographic dark energy to the forthcoming work.

Summarizing this section, if the creature succeeds in modifying the future event horizon, it means that the creature manages to modify the boundary condition or the initial condition given by \( r_\infty \). Since the universe follows the evolution equation (8) with the modified initial condition from the beginning, any observer including the creature may not notice the change of the initial condition since he or she has been living in the universe with modified initial condition. In this sense, there happens no causality violation from the point of view of an observer in the universe.

Resolution of the circular logic. — The origin of the circular-logic problem comes from the ignorance of the existence of the future event horizon. However, note that the cosmological event horizon cannot be created or be removed by any classical means. The existence or the absence of the horizon is determined from the beginning of the universe. Thus, the cosmological solution can be divided into two separated classes: the universe with future event horizon and the universe without it.

Can we find out which of the two classes we are living in? As shown in the previous calculations, the evolution equation (3) can be modified into a one second-order differential equation (8) supplemented by a boundary condition (9) which can be determined at the beginning of the universe. Given the present data of the universe, we may trace back the universe to know the initial situation by using eq. (8). By doing this procedure, the boundary condition \( r_\infty \) can be obtained from the limiting procedure (9). Note that, in doing this procedure, we do not have to know any future information.

Even in the absence of the knowledge on \( r_\infty \), we can predict the future from the evolution equation (8) since it is a well-posed differential equation. In principle, we may recognize which of the two classes we live in, if we know the present data of the universe in detail. The above logic holds only when the assumption of space homogeneity is valid in our universe since we predict the future evolution on the basis. Still, we cannot differentiate whether we are living in a universe with or without future event horizon until we have more precise data of the present universe including dark matters and geometries. However, the use of eq. (3) for the next moment evolution can be justified since all of the future evolution is governed by the well-posed second-order differential equation (8) and the future evolution is only related with the boundary condition \( r_\infty \) which can be determined at the beginning of the universe.

As a last example, we derive a formula for the initial condition \( r_\infty \). We divide the time of the universe into three periods, pre-inflationary, inflationary, and post-inflationary one. Even though we do not know the physics for the pre-inflationary period, we assume a decelerating
power law behavior of the scale factor during that time. For the inflationary and post-inflationary periods, we assume the exponential inflation and the matter-dominated universe for simplicity, respectively. Then,

\[ a_0 \int_0^{t_0} \frac{1}{a(\tau)} d\tau = a_0 \int_0^{t_1} \frac{1}{a(\tau)} d\tau + a_0 \int_{t_1}^{t_\text{i}} \frac{1}{a(\tau)} d\tau + a_0 \int_{t_\text{i}}^{t_0} \frac{1}{a(\tau)} d\tau, \]

where the pre-inflationary contribution will be negligible because the period will be short and the inflationary contribution is given by

\[ a_0 \int_{t_\text{i}}^{t_0} \frac{1}{a(\tau)} d\tau = \frac{3}{H_0} \int_{t_\text{i}}^{t_0} e^{-\frac{H(t-t_\text{i})}{H_0}} dt \simeq \left( \frac{t_0}{t_\text{i}} \right)^{2/3} \frac{c N}{H_1}, \]

where we assume \( N \gg 1 \) in the last equality and \( t_1, t_\text{i} \) are the times at which the inflation, respectively, ends and starts and \( H_1 \) are the Hubble parameter during inflation. The contribution from the post-inflationary period is given by

\[ a_0 \int_{t_\text{i}}^{t_0} \frac{1}{a(\tau)} d\tau = t_\text{i}^{2/3} \int_{t_\text{i}}^{t_0} t^{-2/3} dt \simeq 3t_0, \]

where we have ignored the lower bound depending on \( t_\text{i} \). If we consider the continuous variation of the universe from the radiation-dominated era to the dark-energy–dominated era, the factor 3 will be modified a bit. Using these results, we have

\[ r_\infty \simeq \frac{1}{H_0} \left( \frac{H_0}{H_1} \frac{T_R}{T_0} a_0^{3N} + 3H_0 t_0 + \frac{d}{\sqrt{1-4\Omega_{\text{rh}}}} \right), \]

where we have used \( a_0/a_0 = T_0/T_R \), where \( T_R \) and \( T_0 \) are the temperatures at the reheating time and at present, respectively.

Conclusions. – We studied the problem of circular reasoning between the introduction of future event horizon and the accelerating expansion of the universe in HDE models. We divided the problem into the causality of the equation of motion and the circular logic on the use of the future event horizon. The root of the causality problem was identified as a number \( r_\infty \) and was separated from a causal equation of motion (8). The explicit value of \( r_\infty \) can be determined at the beginning of the universe. We have shown that the causality problem is absent if we require the causality in the weak sense that any event prohibited from the present data will not happen in the future. We also reduced the circular-logic problem to the initial condition, \( r_\infty \), by noting the fact that the existence (or absence) of the cosmological event horizon is determined from the initial data of the universe, which can be traced back from the precise present data.

The philosophy of Einstein equation is that “matter determines the geometry and the geometry rules the motion of the matter”. In the present case, \( r_\infty \) has a geometric origin since it is defined from the evolution of the scale factor. Therefore, the holographic dark energy is originated from geometry and also affects the change of the scale factor. As mentioned by Li [3], it still appears rather puzzling why holographic energy is given by the time-dependent horizon size related to this \( r_\infty \). In any cases, the holographic-dark-energy model must be an effective description based on fundamental principles such as the UV/IR connection [19], bulk holography [20], space-time foam [21,22], quantum fluctuation of the space-time itself [23] or others.

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