The interaction time of a packet with a potential barrier

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Abstract

We study the evolution of a wave packet impinging onto a one dimensional potential barrier. The transmission and reflection times discussed in the literature for stationary states do not correspond to the times required for the emergence of a transmitted or a reflected packet. We propose new definitions for the interaction (dwell) time and the transmission and reflection times which are suitable for packets and fit better the actual time evolution of the packet.

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I. INTRODUCTION

The tunneling of a particle beyond a potential barrier is one of the simplest effects predicted by Quantum Mechanics where the conflict between classical and quantum pictures is most striking. Although it has been studied since the early days of Quantum Mechanics [1], the debate is still open as to what should be meant for tunneling time (see refs. [2], [3] for extensive reviews).

It could be objected that in a proper quantum formulation of the problem there is no room for such a concept, all that we can ask being the probability of detecting the particle beyond the barrier or of having it reflected by the potential. We feel, however, that this is an extreme view. It is legitimate, for example, to think of an ensemble of systems prepared in a given initial state, for which the times are measured when a detector located in front or beyond the barrier reveals the arrival of the particle (in order not to disturb the state, we think of separate experiments for measuring the arrival and the exit times). Knowledge of the initial state (i.e. the wave function at time $t = 0$) should enable us to determine which is the difference between the average exit time and the average arrival time. This difference could be regarded as the average tunneling time. While we do not advocate the above definition as the definition of the tunneling time, we insist that the wave function contains implicitly the information as to the time that in the average the particle spends in the potential barrier. The problem with the determination of this average time is that, the process being intrinsically non-classical, it is not possible to look for a quantum counterpart of a classical observable, whose average value on the state of the particle should be interpreted as the tunneling time. On the other hand, a clearer view about the tunneling time is urged also by current experiments on semi-conductor devices [4] and evanescent waves [5], where observation of superluminal velocity has sometimes been claimed.

Most of the approaches to the problem of tunneling time deal with stationary states. The particle is in an eigenstate of the Hamiltonian, and the tunneling time $\tau_T$, as well as the reflection time $\tau_R$ and the dwell time $\tau_D$, are functions of its energy $E$, or momentum $k = \sqrt{2mE/\hbar}$. Also those who envisage wave packets (see [6], [7], [8]) use packets which are so narrow in energy as to allow the monochromatic approximation to hold. We have preferred to investigate the time evolution of a wave packet with a spread in energy which forbids this approximation. The packet impinges onto the barrier and is partially reflected and partially transmitted. It is constructed as a Gaussian superposition of eigenstates centered around a value $E$ lower than the height $V_0$ of the barrier. We observe the evolution of the packet at times $\tau_T(E)$, $\tau_R(E)$ and $\tau_D(E)$ after it has reached the barrier.
Although the time $t_{in}$ when the packet "reaches the barrier" is not sharply defined, due to the interference of the higher energy components, which reach the barrier earlier and are partially reflected, with the lower energy incoming components, we find that the above mentioned times $\tau_T(E)$, $\tau_R(E)$ and $\tau_D(E)$ definitely do not correspond to the times required for the packet to emerge from the barrier. The uncertainty in $t_{in}$ is not such as to alter this conclusion. It can be argued that the transmitted packet is centered around an energy $E_T$ higher than $E$, whereas the reflected packet is centered around an energy $E_R$ lower than $E$, but also the times $\tau_T(E_T)$ or $\tau_R(E_R)$ are by no means representative of the times required to see the reflected packet or the transmitted packet. The conclusion seems to be that the tunneling times defined for stationary states with energy $E$ are not meaningful for the evolution of a wave packet having $E$ as average energy.

This has prompted us to look for another determination of the times $\tau_T$, $\tau_D$ and $\tau_R$ which take into account the actual behaviour of the wave packet. As for $\tau_D$, we define it as the time integral of the probability $P_2(t)$ of finding the particle within the barrier region,

$$\tau_D = \int_{t_2}^{+\infty} P_2(t)dt$$

(1)

this definition being unsensitive to the time $t_2$ when the particle begins to interact with the barrier, provided it is earlier than the time when the packet impinges onto the barrier. Thus, $\tau_D$ can be regarded as the total interaction time. As for $\tau_T$, we define it as a weighted sum of every time interval $\Delta t$, the weight being the fraction of the transmitted packet which at time $t$ has not yet been transmitted. The definition for $\tau_R$ is similar. The trouble with these definitions is that there is still a problem with the time when the integral over $t$ begins. In principle, the packet is interacting with the barrier since time $t = 0$, but the contribution of the earlier times, when the particle has not yet arrived onto the barrier, to $\tau_T$ should be negligible.

While there is no objective way of determining the time when the interaction between the particle and the potential starts, we observe that a shifting of $t_2$ in eq. (3) affects $\tau_D$, whose value decreases with increasing $t_2$. On the other hand any determination of $\tau_D$ entails a given accuracy $\epsilon$. For the lower integration limit in the definitions of $\tau_T$ and $\tau_R$ we choose the value $t_L$ such that $\int_{t_2}^{t_L} P_2(t)dt$ equals $\epsilon$. The meaning of this choice is that we have a finite time resolution $\epsilon$, and we neglect those time intervals which contribute to the interaction time less than $\epsilon$. We note however that the time dependence of the probability $P_2(t)$ and of the analogous probabilities of finding the particle beyond the barrier ($P_3(t)$) and in front of it ($P_1(t)$) when the packet impinges onto the barrier is sufficiently steep so that the choice of $t_L$ does not really affect $\tau_T$ and $\tau_R$ in a substantial way. The values
we find (with $\epsilon \simeq 0.01$) for $\tau_T$, $\tau_R$ and $\tau_D$ are definitely different from the values of $\tau_T(E)$, $\tau_R(E)$ and $\tau_D(E)$ and are in agreement with the actual time evolution of the packet.

We have also examined the behaviour of the probabilities $P_i(t)$ for large $t$. These probabilities have an exponential tail $e^{-t/\tau}$, with the same time constant $\tau$ (depletion time) for each probability. The value of $\tau$ is determined by the position of the poles of the transmission coefficient $D(k)$ in the complex $k$ plane, and is independent of the details of the wave packet.

In conclusion, we find that the tunneling times proposed in the literature for stationary states are not meaningful for the actual time evolution of a wave packet. The time lapse from the first contact of the packet with the barrier to its emergence beyond the barrier, although not so sharply definable, is definitely different from the tunneling times proposed for stationary states.

In section 2 we define the problem and build the packet whose time evolution is discussed in section 3. In section 4 we present the definitions of $\tau_D$, $\tau_T$ and $\tau_R$ and in section 5 we discuss the depletion time. Section 6 is devoted to the conclusions.

II. THE WAVE PACKET

We consider a one dimensional problem, the Hamiltonian being

$$H = \frac{p^2}{2m} + V(x)$$

$$V(x) = \begin{cases} V_0 & (|x| < d) \\ 0 & (|x| > d) \end{cases}$$

We use units such that $\hbar = 1$. The eigenfunctions $\psi_k$ of the Hamiltonian (2) are well known (see Appendix). We consider the evolution of a wave packet $\psi$ built as a Gaussian superposition of the functions $\psi_k$, impinging onto the barrier:

$$\psi(x,t) = \int a(k) \psi_k(x) e^{-ik^2t/2m} dk$$

where the coefficients $a(k)$ are

$$a(k) = \left(\frac{2\delta^2}{4\pi^3}\right)^{1/4} e^{-(k-k_{av})^2} e^{-ikx_0}$$

$k_{av}$ is the average momentum, and $x_0$ is the coordinate of the peak of the packet at $t = 0$. We choose the parameters involved in the problem as follows:
\[ m = 1 \quad k_{av} = 9.9 \quad \delta = \sqrt{2} \quad x_0 = -15 \quad d = 2 \quad V_0 = 50 \ (k_0 = 10) \quad (6) \]

The packet \([4]\) is Gaussian also in \(x\) (fig. 1). The reason why the peak of the packet is located so far from the left edge of the barrier at time \(t = 0\) is to have an identification of \(t_{in}\) as sharp as possible. With a packet starting nearer to the barrier, the Gaussian form of the packet would be immediately lost, due to the interference of the incoming and reflected components of the packet.

### III. EVOLUTION OF THE PACKET

We have studied the evolution of the packet in order to verify to what extent the definitions of the tunneling time proposed for stationary problems are meaningful for a wave packet. More precisely, we have tested whether the phase times \(\tau^{ph}\) \([6]\) or the times proposed by Buttiker \(\tau^{B}\) \([9]\) (which turn out to be deeply connected with the times proposed within other approaches, see for \([3]\)- \([4]\) a review) do represent the lapse of time which the packet spends in the barrier. To this purpose, it is necessary to mark the time \(t_{in}\) when the packet begins to interact with the barrier, and the time \(t_{fin}\) when a transmitted (reflected) packet appears. The comparison of \(t_{fin} - t_{in}\) with \(\tau^{ph}\) and \(\tau^{B}\) calculated for significant values of the energy will show that \(\tau^{ph}\) and \(\tau^{B}\) are not significant for a wave packet.

We observe the shape of the packet as it moves towards the potential barrier. As long as it is far enough, its shape is quite similar to the initial shape; when it approaches the left edge of the barrier it begins to become blurred (fig. 2) due to the interference between the incoming and the reflected components. There is a time interval in which the interference phenomenon is dominant, but a reflected packet is still absent. The peak of a reflected packet appears a time \(\Delta t\) after the blurring of the incoming packet. A bit later, we see the emergence of a transmitted packet beyond the barrier.

For the time \(t_{in}\) when the packet begins to interact with the barrier we assume the time when the blurred shape can be macroscopically observed. As for the time \(t_{fin}\), we have two possibilities: the time when the reflected packet appears \((t_{fin,R})\) or the time when the peak of the transmitted packet appears \((t_{fin,T})\). It is clear from the above that neither \(t_{in}\) nor \(t_{fin,R}\) \((t_{fin,T})\) are sharply defined. In the units we have chosen, each of them can be determined only within an error \(\Delta t \simeq 0.1\).

By inspection of the graphs representing the reflected and the transmitted packet respectively (see figs. 2a and fig2b), we get

\[ t_{in} \simeq 0.9 \quad t_{fin,R} \simeq 1.9 \quad t_{fin,T} = 2.7 \quad (7) \]
For the times \( \tau_R \) and \( \tau_T \) it follows

\[
\tau_R \simeq 1, \quad \tau_T \simeq 1.8
\]  
(8)

The error on the above values can be assessed to be of order 0.2.

We compare the above times with the times \( \tau^{ph} \) and \( \tau^B \). These are functions of the momentum \( k \), so we must decide which momentum to consider. We consider the average momentum \( k_{av} \) and the average momenta \( k_R \) and \( k_T \) of the reflected and transmitted components respectively. These latter are calculated to be

\[
k_R = 9.696, \quad k_T = 10.327
\]  
(9)

Incidentally, this shows, as previously noted [4], that a potential barrier acts as an accelerator: the transmitted wave packet has an average momentum larger than the incoming one. The opposite holds for the reflected packet.

In table 1 we present the dwell times \( t^{ph,D} \) and \( t^{B,D} \) calculated for \( k = k_{av}, k_R \) and \( k_T \), together with the reflection time \( t^{B,R} \) calculated for \( k_{av} \) and \( k_R \) and the transmission time \( t^{B,T} \) calculated for \( k_{av} \) and \( k_T \). We see that the dwell times \( t^{ph,D} \) and \( t^{B,D} \) are definitely shorter than the tunneling time \( \tau_T \) reported in (8). The same holds for the reflection times \( t^{B,R} \) calculated for \( k_{av} \) and \( k_R \). As for the transmission time \( t^{B,T} \), we see that the value corresponding to \( k_{av} \) is longer, whereas the value corresponding to \( k_T \) is shorter. We conclude that the tunneling times found for stationary problems are not useful to describe the evolution of a packet.

**IV. DEFINITION OF \( \tau_D \), \( \tau_T \) AND \( \tau_R \)**

So far definitions of the time that the particle interacts with the potential (the so called dwell time \( \tau_D \)) and of the transmission (\( \tau_T \)) and reflection (\( \tau_R \)) times have been given mainly for stationary problems. Even the authors who have dealt with wave packets considered packets which were so narrow in energy that the relevant times could be considered to be functions \( \tau(k) \) of the momentum, as in the stationary case. In this section we propose definitions of \( \tau_D \), \( \tau_T \) and \( \tau_R \) which are suitable for a wave packet.

We first consider the probability \( P_2(t) \) that at time \( t \) the particle is within the barrier region,

\[
P_2(t) = \int_{-d}^{d} dx \, |\psi(x,t)|^2
\]  
(10)

and the analogous probabilities that at time \( t \) the particle is in front of the barrier (\( P_1(t) \)) or beyond the barrier (\( P_3(t) \)). Obviously, we have \( P_1(t) + P_2(t) + P_3(t) = 1 \).
The values of $P_1(t)$, $P_2(t)$ and $P_3(t)$ are reported in fig. 4. $P_1$ and $P_3$ tend to asymptotic values which we call respectively $R$ and $T$. They represent the probabilities that the particle is reflected or transmitted respectively. Obviously, $R + T = 1$.

Inspection of $P_2(t)$ shows that for $t \leq 0.75$ the packet does not interact with the barrier, and the interaction reaches its maximum at $t \approx 1.5$. After this time the probability of finding the particle in the barrier region decreases with a tail which has an exponential shape. The interaction time should be the total time that the particle spends in the potential region. With this view, we propose the following (already defined as eq. (1)) as a definition of $\tau_D$ (see also ref. [10], [11] where a similar definition is put forth):

$$\tau_D = \int^{+\infty}_{t_2} P_2(t) dt$$

The meaning of eq. (1) is clear: every time interval $dt$ is weighted with the probability $P_2(t)$ of finding the particle within the potential barrier. The dwell time is to be interpreted as the time the particle interacts with the potential regardless its fate. The definition is independent of the choice of time $t_2$, provided it is chosen earlier than the time the packet is significantly present in the barrier region. We can safely take $t_2 = 0$.

We define the transmission time $\tau_T$ as the average time that it takes for the transmitted particles to emerge beyond the potential barrier. This time is given by the integral

$$I = \int_{t_3}^{\infty} \left(1 - \frac{P_3(t)}{T}\right) dt$$  (11)

In the above definition $1 - P_3(t)/T$ is the fraction of the transmitted packet which has not yet been transmitted at time $t$. We interpret this fact viewing this fraction as ”being transmitted” at time $t$, so that any time interval $dt$ contributes to the transmission time with a weight $1 - P_3(t)/T$.

The trouble with this definition is the lower integration limit $t_3$, in that eq. (11) gives weight 1 also to the time intervals when the packet has not yet arrived in the potential region. But these time intervals are not to be considered as contributing to the transmission time: what eq. (11) actually gives is the transmission time starting from time $t_3$.

The problem cannot be circumvented by any choice of time $t_3$. In principle, $t_3$ should be the time when the particle begins to interact with the potential, but this time cannot be determined in any objective way. However, we can determine that time $t_6$ such that the packet has spent in the potential region an amount $\epsilon$ of time starting from the time $t = 0$. This is the time $t_6$ such that the integral of $P_2$ from $t = 0$ to $t = t_6$ equals $\epsilon$. Now, the transmission time $\tau_T(\epsilon)$ reckoned from time $t_6$ (such that the time spent in the barrier
by the packet is equal to $\epsilon$) is defined unambiguously. On the other hand, the calculation of the dwell time $\tau_D$ (as well as any possible time measurement about the particle) is affected by an error. If we choose $\epsilon$ to be the same as this error, the uncertainty on $\tau_T$ due to the choice of the lower integration limit can be considered of the same order as the time resolution we are able to attain. In conclusion, we put

$$\tau_T(\epsilon) = \int_{-\infty}^{\infty} \Theta \left( \int^t P_2(x) dx - \epsilon \right) \left[ 1 - \frac{P_3(t)}{T} \right] dt$$

(12)

where $\Theta(x)$ is the Heavyside step function. The lower integration limit in eq. (12) can be taken the same as in eq. (11).

Along these lines we define also the reflection time $\tau_R(\epsilon)$:

$$\tau_R(\epsilon) = \int_{-\infty}^{\infty} \Theta \left( \int^t P_2(x) dx - \epsilon \right) \left[ 1 - \frac{P_1(t)}{R} \right] dt$$

(13)

The interpretation of eq. (13) is straightforward. $1 - P_1(t)/R$ is the fraction of the reflected packet that at time $t$ has not yet been reflected. This fraction is taken as the weight for any time interval $dt$. We note however that for times near the beginning of the interaction of the particle with the potential this weight can be negative. But the decrease of $P_1(t)$ is very sharp (see fig. 4) and the contribution to the integral of the region where the weight is negative is small indeed.

In the case we have investigated we have found

$$\tau_D = 0.93$$

(14)

In order to find $\tau_T$ and $\tau_R$ we have fixed $\epsilon = 10^{-2}\tau_D \simeq 0.01$. This yields

$$\tau_T = 3.39 \quad \tau_R = 0.55$$

(15)

Note that we have $T \simeq 0.14$, $R \simeq 0.86$. With this values and the results reported in eqs. 14 and 15 the conditional probability relation (2), (11)

$$\tau_D = T\tau_T + R\tau_R$$

(16)

is fairly satisfied. Condition (13) would be identically satisfied if the lower limits where the integrands in eqs. (1), (12) and (13) start to differ from zero were the same. The fact that eq. (13) holds true with a fair accuracy, to within that value .01 which can be assessed as the accuracy of all our calculations, can be regarded as a support to the correctness of the definitions of $\tau_D$, $\tau_T$ and $\tau_R$. 7
By inspecting table 1 we see that $\tau_D$ is definitely larger than $\tau_{ph,D}$ and $\tau_{B,D}$ evaluated for $k = k_{av}$. The dwell time $\tau_D$ looks a very reliable estimate of the interaction time. The discrepancy with the value found for stationary states confirms that the extrapolation to wave packet is untenable. As for $\tau_T$ and $\tau_R$, a comparison with the previously reported values of the reflection and transmission times derived by inspection of the wave packet evolution shows that $\tau_T$ and $\tau_R$ in eq. (15) are respectively longer and shorter. Thus, the discrepancy between the times in eq. (15) and the times derived within the stationary approach, for example the Buttiker times calculated for $k = k_{av}$, is even larger.

V. THE DEPLETION RATE

The tail of $P_2(t)$ can be described as an exponential curve with a time constant $\tau_{dep}$:

$$P_2(t) = Ae^{-t/\tau_{dep}} \quad \text{for large } t$$  \hspace{1cm} (17)

By considering the values of $P_2$ for $t > 30$, we find that the exponential fit is excellent, with

$$\tau_{dep} = 16.192$$  \hspace{1cm} (18)

and a correlation coefficient $R = -0.9999883$. The same time constant $\tau_{dep}$ rules the asymptotic behaviour of $P_3(t)$ and, due to probability conservation, of $P_1$:

$$\tau_{dep} = 16.192 \quad \quad R = -0.9999435$$

The value of $\tau_{dep}$ is connected with the behaviour of the complex transmission coefficient $D(k)$ in the complex plane. By explicitly writing $P_3(t)$ (see Appendix) we see that the only singularities are in the product $u(k)u^*(p)$ in the denominator, with

$$u(k) = (\kappa^2 - k^2) \sinh(2\kappa d) - 2ik\kappa \cosh(2\kappa d)$$  \hspace{1cm} (19)

It is easy to see that if $u(k) = 0$, then $u^*(k^*) = 0$. Hence, a zero for $u$ in $k = x + iy$, together with the zero in $x - iy$ for $u^*$, will contribute to $P_2(t)$ with a term $\exp(2xyt/m)$. For large values of $t$, the term with $xy$ negative and minimum in absolute value will dominate. The behaviour of $P_2$ will be as in eq. (17), with:

$$\tau_{dep} = m/2|x|$$  \hspace{1cm} (20)

The search for the pole with smallest $|xy|$ value is discussed in the appendix. We find $x = 10.03$, $y = -3.0565 \cdot 10^{-3}$, which yields $\tau_{dep} = 16.3087$, in fair agreement with the value reported in eq. (18).
VI. CONCLUSIONS

We have studied the evolution of a wave packet which approaches a potential barrier. The time requested for the appearance of a transmitted ($\tau_T$) or a reflected ($\tau_R$) packet, reckoned from the moment the incoming packet begins to interact with the barrier, are definitely different from the values of $\tau_T$ and $\tau_R$ reported in the literature for monochromatic packets. We propose new definitions of $\tau_D$, $\tau_T$ and $\tau_R$ which are suitable for a packet and give results that fit better the actual time evolution of the packet.

The probability of finding the particle within the barrier has an exponential tail whose time constant $\tau_{dep}$ is determined by the behaviour of the stationary solutions in the complex momentum plane.

APPENDIX:

The stationary solutions of the Schroedinger equation with the Hamiltonian (2) are:

$$\psi_k(x) = \begin{cases} e^{ikx} + A(k)e^{-ikx} & \text{if} \quad x < -d \\ B(k)e^{\kappa x} + C(k)e^{-\kappa x} & \text{if} \quad |x| < d \\ D(k)e^{ikx} & \text{if} \quad x > d \end{cases}$$

where

$$A(k) = -\frac{(\kappa^2 + k^2) \sinh(2\kappa d)}{u(k)} e^{-2ikd}$$

$$B(k) = -\frac{ik(\kappa + ik)}{u(k)} e^{-ikd} e^{-\kappa d}$$

$$C(k) = -\frac{ik(\kappa - ik)}{u(k)} e^{-ikd} e^{\kappa d}$$

$$D(k) = -\frac{2ihk}{u(k)} e^{-2ikd}$$

$$k = \sqrt{k_0^2 - k^2}$$

$$u(k) = (\kappa^2 - k^2) \sinh(2\kappa d) - 2ik\kappa \cosh(2\kappa d)$$

The probability $P_2(t)$ is given by:

$$P_2(t) = \sqrt{\frac{\delta^2}{2\pi^3}} \int dk \int dp \ e^{-(p-k_{av})^2}\delta^2 \ e^{-(k-k_{av})^2}\delta^2 \ e^{-i(k^2-p^2)t/2m} \ kpe^{i(p-k)d} \ [(q^2-p^2) \sinh(2qd) - 2ipq \cosh(2qd)]^* \ [(\kappa^2 - k^2) \sinh(2\kappa d) - 2ik\kappa \cosh(2\kappa d)]^{1/2}.$$
\[
\begin{align*}
&\left[(q + ip)^*(\kappa + ik)\left(\frac{1 - e^{-2(\kappa - \tau)d}}{\kappa - \tau}\right) + (q - ip)^*(\kappa + ik)\left(\frac{1 - e^{-2(\kappa - \tau)d}}{\kappa + \tau}\right) + \\
&- (q + ip)^*(\kappa - ik)\left(\frac{1 - e^{-2(\kappa + \tau)d}}{\kappa - \tau}\right) - (q - ip)^*(\kappa - ik)\left(\frac{1 - e^{-2(\kappa + \tau)d}}{\kappa + \tau}\right)\right]
\end{align*}
\]

where \( q = \sqrt{k_0^2 - p^2} \)

The only singularities are in the factors \( u(k)u^*(p) \) in the denominator. In order to find the zeroes of \( u(k) \) we note that if \( u(k) = 0 \), then

\[
\left[\frac{\sin \left(2ik_0\sqrt{1 - z^2}\right)}{\sqrt{1 - z^2}}\right]^2 = 4z^2
\]

with \( z = k/k_0 \). For an opaque barrier (i.e. \( 2k_0d \gg 1 \)) the solution for \( z \) has to be near \( z = 1 \). The values of the zeroes have been found by solving for \( z \) the relation

\[
0 = u(z) \simeq u(z_0) + (z - z_0) u'(z_0)
\]

and iterating, choosing different values of \( z_0 \) near 1. We have found as many different solutions as predicted by the Cernlib routine nzeros for a neighbourhood of \( z = 1 \). The value of \( \tau_{\text{dep}} \) is given by that solution for which \( xy \) is negative and minimum in absolute value.

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FIGURE CAPTIONS

Figure 1
The Gaussian shape of the incoming packet for $t = 0$.

Figure 2a and 2b
In (a) the wave packet at $t = 0.9$ and in (b) a zoom of the blurred region.

Figure 3a and 3b
In (a) the reflected packet that appears at $t = 1.9$. In (b) the first transmitted peak that appears at $t = 2.7$.

Figure 4
Plots of $P_1(t)$, $P_2(t)$ and $P_3(t)$ as functions of $t$.

TABLE CAPTIONS

Table 1
The phase time and the Büttiker times evaluated in $k = \overline{k}, k_R, k_T$. 
\[
\begin{array}{|c|c|c|c|}
\hline
\text{time} & k = k_{av} & k = k_R & k = k_T \\
\hline
\tau_{ph.d} & 0.143 & 0.0843 & 1.011 \\
\tau_{B.d} & 0.140 & 0.079 & 1.008 \\
\tau_{B.t} & 2.357 & - & 1.248 \\
\tau_{B/R} & 0.140 & 0.079 & - \\
\hline
\end{array}
\]

Table 1
\[ |\psi|^2 \]
