Dalitz plot analysis in the FOCUS experiment

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Abstract. The $K$-matrix formalism has been applied to the charm sector for the first time in the FOCUS Dalitz plot analyses of the $D^+_s$ and $D^+ \rightarrow \pi^+\pi^-\pi^+$ final states. The results, presented here, are extremely encouraging since the same $K$-matrix description provides a coherent picture of both two-body scattering measurements in light-quark experiments as well as charm meson decay. Such a result was not obvious beforehand. Moreover, the non-resonant component of each decay appears to be described by known two-body $S$-wave dynamics, without the need to include any three-body constant amplitude contribution.

1. Dalitz-plot analyses in the heavy-flavor sector

Dalitz-plot analysis is an unique tool to investigate hadronic decay dynamics. It provides the complete set of observables of the decay, (i.e., amplitude coefficients and phases), thus allowing, in principle, for a rich range of investigations: from CP violation to non-spectator process measurements in the decay. This analysis technique is being applied more and more in the heavy-flavor sector; however, strong-dynamic effects in the final states, if not properly accounted for, could confuse the underlying fundamental physics, and compromise future sophisticate analyses for CP-violation measurements and new-physics searches. The FOCUS $D^+$ and $D^+_s \rightarrow \pi^+\pi^-\pi^+$ Dalitz studies represent the first example of a $K$-matrix analysis in the heavy-flavor sector [1]; it is able to include the already measured dynamics of the $S$-wave $\pi\pi$ scattering, characterized by broad/overlapping resonances and large non-resonant background.

2. Formalization problems

The isobar formalism, which has traditionally been applied to charm amplitude analyses, represents the decay amplitude as a sum of relativistic Breit-Wigner propagators multiplied by form factors plus a term describing the angular distribution of the two body decay of each intermediate state of a given spin. Many amplitude analyses require detailed knowledge of the light-meson sector. In particular, the need to model intermediate scalar particles contributing to the charm meson in the decays reported here has caused us to question the validity of the Breit-Wigner approximation for the description of the relevant scalar resonances [2, 3].

A formalism for studying overlapping and many channel resonances has been proposed long ago and is based on the $K$-matrix [4, 5] parametrization. This formalism, originating in the context of two-body scattering, can be generalized to cover the case of production of resonances in more complex reactions [6]. The $K$-matrix approach allows us to include the positions of the poles in the complex plane directly in our analysis, incorporating the results from spectroscopy.

1 See http://www-focus.fnal.gov/authors.html for author information
parameters are coefficients and phases of Eq.1, i.e.,
general complex and describe the initial production process, i.e, the
D\(^{-}\) vector
The
\[ F_i = (I - iK\rho)^{-1} B(abc|r_i) \]
where \( I \) is the identity matrix, \( K \) is the \( K\)-matrix describing the isoscalar \( S\)-wave scattering process, \( \rho \) is the phase-space matrix for the five channels involved in the scattering (i.e., \( \pi\pi, KK, \eta\eta, \eta'\eta' \) and \( 4\pi \)), and \( P \) is the “initial” production vector into the five channels. In this picture, the production process can be viewed as consisting of an initial preparation of several states, which are then propagated by the \((I - iK\rho)^{-1}\) term into the final one. To write our \( F_i \) amplitude we need a self-consistent description of the \( S\)-wave isoscalar scattering, such as that provided by the \( K\)-matrix representation of Anisovich and Sarantsev in reference [8] obtained through a global fit of the available scattering data from \( \pi\pi \) threshold up to 1900 MeV. Their \( K\)-matrix parametrization is:
\[ K_{ij}^{00}(s) = \begin{cases} \sum_{\alpha} g_i^{(\alpha)} m_{\alpha}^2 s + f_{ij}^{\text{scatt}} \frac{1}{s - s_{0}^{\text{scatt}}} & \left( s - s_{A}m_{\pi}^2/2 \right) \\ \left( s - s_{A}(1 - s_{A}) \right) & \end{cases} \]
The \( g_i^{(\alpha)} \) is the coupling constant of the \( K\)-matrix pole \( m_{\alpha} \) to the \( i \) meson channel; the parameters \( f_{ij}^{\text{scatt}} \) and \( s_{0}^{\text{scatt}} \) describe a slowly varying part (which we will call SVP) of the \( K\)-matrix elements; the factor \( \left( s-s_{A}m_{\pi}^2/2 \right) / \left( s-s_{A}(1-s_{A}) \right) \) is to suppress false kinematical singularity in the physical region near the \( \pi\pi \) threshold (Adler zero).

In Eq.2, the \( K\)-matrix parameters, which are real by definition, are fixed to values provided by the authors of reference [8]. The free parameters are contained in the \( P\)-vector; they are in general complex and describe the initial production process, i.e, the \( D \) meson decay. Other fit parameters are coefficients and phases of Eq.1, i.e, \( a_0, a_i \) and \( \delta_0, \delta_i \). The squared modulus of the amplitude in Eq.1 gives the probability density in the three-pion Dalitz plot.

3. FOCUS analysis results
The FOCUS samples consist of 1475 ± 50 and 1527 ± 51 signal events for the \( D_\ast^+ \) and \( D^+ \rightarrow \pi^+\pi^-\pi^+ \) respectively. The Dalitz plot analyses are performed on events within \( \sigma \) the nominal \( D_\ast^+ \) or \( D^+ \) mass (Fig. 1). We refer to [1] for a complete and detailed explanation of our analysis and limit ourselves here to some considerations on the final results.
Figure 1. Signal and side-band regions for the $D_s^+$ (a) and $D^+$ (b) three-pion mass distribution. Dalitz plots for the $D_s^+$ (c) and $D^+$ (d) signals.

3.1. Results for the $D_s^+ \rightarrow \pi^+\pi^-\pi^+$ decay
In Table 1 the resulting fit fractions, phases and amplitude coefficients for the $D_s^+$ channel are quoted. Both the three-body non-resonant and $\rho^0(770)\pi^+$ components were not required by the fit. We represent the entire S-wave contribution by a single fit fraction since, as already mentioned, one cannot distinguish the different resonance or SVP S-wave contributions on the real axis; the couplings to T-matrix physical poles are instead computed by continuing the $F_1(s)$ amplitude into the complex s-plane to the position of the poles and evaluating pole residues [1]. The $D_s^+$ Dalitz projections of our data are shown in Fig. 2 superimposed with our final fit projections. The C.L of the fit is 3% and is evaluated with a $\chi^2$ estimator over a Dalitz plot with bin size adaptively chosen to maintain a minimum number of events in each bin. The inclusion of an Adler zero term in the P-vector did not improve our fit quality and was removed.

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2 The quoted fit fractions are defined as the ratio between the intensity for a single amplitude integrated over the Dalitz plot and that of the total amplitude with all the modes and interferences present.

3 We remind the reader that a bit of caution has to be used in comparing C.L. from Dalitz plot analyses from various experiments which use different Dalitz partition and C.L. evaluation method.
Table 1. Fit results from the K-matrix model for $D_s^+$.

| decay channel | fit fraction (%) | phase (deg) | amplitude coefficient |
|---------------|-----------------|-------------|-----------------------|
| $(S$-wave) $\pi^+$ | 87.04 ± 5.60 ± 4.17 | 0 (fixed) | 1 (fixed) |
| $f_2(1270) \pi^+$ | 9.74 ± 4.49 ± 2.63 | 168.0 ± 18.7 ± 2.5 | 0.165 ± 0.033 ± 0.032 |
| $\rho^0(1450) \pi^+$ | 6.56 ± 3.43 ± 3.31 | 234.9 ± 19.5 ± 13.3 | 0.136 ± 0.030 ± 0.035 |

Table 2. Fit results from the $K$-matrix model fit for $D^+$.

| decay channel | fit fraction (%) | phase (deg) | amplitude coefficient |
|---------------|-----------------|-------------|-----------------------|
| $(S$-wave) $\pi^+$ | 56.00 ± 3.24 ± 2.08 | 0 (fixed) | 1 (fixed) |
| $f_2(1270) \pi^+$ | 11.74 ± 1.90 ± 0.23 | -47.5 ± 18.7 ± 11.7 | 1.147 ± 0.291 ± 0.047 |
| $\rho^0(770) \pi^+$ | 30.82 ± 3.14 ± 2.29 | -139.4 ± 16.5 ± 9.9 | 1.858 ± 0.505 ± 0.033 |

Figure 2. $D_s^+$ Dalitz-plot projections with our final fit superimposed. The background shape under the signal is also shown.

3.2. Results for the $D^+\rightarrow \pi^+\pi^-\pi^+$ decay

The $D^+\rightarrow \pi^+\pi^-\pi^+$ Dalitz plot shows an excess of events at low $\pi^+\pi^-$ mass, which could easily be reproduced by an "ad hoc" Breit-Wigner, having mass $m = 442.6 \pm 27.0$ and width $\Gamma = 340.4 \pm 65.4$ MeV/c^2. However this is not enough to claim the existence of a new resonance, especially when its parameters turn out to be very far from the real axis as in this case. We also know that complex structure can be generated by the interplay among the $S$-wave resonances and the underlying non-resonant $S$-wave component that cannot be properly described in the context of a simple isobar model; it is therefore interesting to study this channel with the present formalism, which embeds already a rich available experimental knowledge about the $S$-wave $\pi^+\pi^-$ scattering dynamics.

The fit results for the $D^+$ channel are reported in Table 2. The decay appears to be dominated by the $S$-wave plus $\rho^0(770)$ and $f_2(1270)$ components. In analogy with the $D_s^+$, the direct three-body non-resonant component was not necessary since the SVP of the $S$-wave could reproduce the entire non-resonant portion of the Dalitz plot. The fit did not require an Adler-zero term in the P-vector. The $D^+$ Dalitz projections are shown in Fig. 3; the fit C.L is 7.7 %. The most interesting feature of these results is the fact that the better treatment of the $S$-wave contribution provided by the $K$-matrix model can reproduce the low-mass $\pi^+\pi^-$ structure of the $D^+$ Dalitz plot. This suggests that any $\sigma$-like object in the $D$ decay should be consistent
Figure 3. $D^+$ Dalitz-plot projections with our final fit superimposed. The background shape under the signal is also shown.

with the same $\sigma$-like object measured in the $\pi^+\pi^-$ scattering. We believe that additional studies with higher statistics will be required to completely understand the $\sigma$ puzzle.

4. Conclusions
The $K$-matrix formalism has been applied for the first time to the charm sector in our Dalitz plot analyses of the $D_s^+$ and $D^+ \to \pi^+\pi^-\pi^+$ final states. The same $K$-matrix description gives a coherent picture of both two-body scattering measurements in light-quark experiments as well as charm meson decay. The $K$-matrix treatment of the $S$-wave component of the decay amplitude allows for a direct interpretation of the decay mechanism in terms of the five virtual channels considered: $\pi\pi$, $KK$, $\eta\eta$, $\eta'\eta'$ and $4\pi$. By inserting $KK^{-1}$ in the decay amplitude, $F$,

$$F = (I - iK\rho)^{-1}P = (I - iK\rho)^{-1}KK^{-1}P = TK^{-1}P = TQ$$

(4)

we can view the decay as consisting of an initial production of the five virtual states which then scatter via the physical $T$ into the final state. The $Q$-vector contains the production amplitude of each virtual channel in the decay. Figure 4 shows the ratio of the moduli of the $Q$-vector amplitudes with respect to the $\pi\pi$ modulus for the $D_s^+$ $S$-wave. The last plot in Fig. 4 represents the normalizing $\pi\pi$ modulus. The two peaks of the ratios correspond to the two dips of the $\pi\pi$ normalizing modulus, while the two peaks due to the $K$-matrix singularities, visible in the normalization plot, cancel out in the ratios. Figure 5 shows the analogous plots for $D^+$ $S$-wave.

The resulting picture, for both $D_s^+$ and $D^+$ decay, is that the $S$-wave decay is dominated by an initial production of $\eta\eta$, $\eta'\eta'$ and $KK$ states. Dipion production is always much smaller. This suggests that in both cases the $S$-wave decay amplitude primarily arises from a $s\bar{s}$ contribution such as that produced by the Cabibbo favored weak diagram for the $D_s^+$ and one of the two possible singly Cabibbo suppressed diagrams for the $D^+$. For the $D^+$, the $s\bar{s}$ contribution competes with a $d\bar{d}$ contribution. That the $f_0(980)$ appears as a peak in the $\pi\pi$ mass distribution in $D^+$ decay, as it does in $D_s$ decay, shows that for the $S$-wave component the $s\bar{s}$ contribution dominates [3]. Comparing the relative $S$-wave fit fractions that we observe for $D_s^+$ and $D^+$ reinforces this picture. The $S$-wave decay fraction for the $D_s^+$ (87%) is larger than that for the $D^+$ (56%). Rather than coupling to an $S$-wave dipion, the $d\bar{d}$ piece prefers to couple to a vector state like $\rho^0(770)$ that alone accounts for $\sim 30\%$ of $D^+$ decay. This interpretation also bears on the role of the annihilation diagram in the $D_s^+ \to \pi^+\pi^-\pi^+$ decay. We believe that Fig. 4 suggests that the $S$-wave annihilation contribution is negligible over much of the dipion mass spectrum. It might be interesting to search for annihilation contributions in higher spin channels, such as $\rho^0(1450)\pi$ and $f_2(1270)\pi$. 
Figure 4. Measured $D^+_s$ Q-vector components: the first four plots are the ratios of moduli of the Q-vector amplitudes with respect to the $\pi\pi$ modulus; the last plot is the normalizing $\pi\pi$ modulus.

Figure 5. Measured $D^+_s$ Q-vector components: the first four plots are the ratios of moduli of the Q-vector amplitudes with respect to the $\pi\pi$ modulus; the last plot is the normalizing $\pi\pi$ modulus.

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