A SLACKS-BASED MODEL FOR DYNAMIC DATA ENVELPMENT ANALYSIS

MOHAMMAD AFZALINEJAD * AND ZAHRA ABBASI

Department of Mathematics, Tafresh University
Tafresh, 3951879611, Iran

(Communicated by Gerhard-Wilhelm Weber)

Abstract. Dynamic Data Envelopment Analysis (DDEA) deals with efficiency analysis of decision making units in time dependent situations. A finite number of time periods and some carry-over activities between each two consecutive periods are assumed in DDEA. There are many models in DEA for efficiency evaluation of decision making units over time periods. One important class of dynamic models is the class of slacks-based models. By using a numerical example we show that some slacks-based DDEA models, especially ones proposed by Tone and Tsutsui, suffer from efficiency overestimation. A new dynamic slacks-based DEA model is proposed to overcome the deficiencies of the available slacks-based models. The model proposed in this paper is capable of revealing all sources of inefficiencies and providing more discrimination between decision making units. The theoretical and practical examinations demonstrate the merits of the new model.

1. Introduction. Data envelopment analysis deals with efficiency evaluation of a set of decision making units (DMUs) with homogeneous structures (see Charnes et al. [1] as a pioneer work and Cooper et al. [2] for more information). The classical DEA concerns efficiency analysis over a single period of time and assumes that efficiency is constant over this period. But in many applications, changes in efficiency over time should be accounted in performance analysis. Consider the case where efficiency evaluation of a set of DMUs over several consecutive time periods is desired. Each DMU has external inputs and external outputs in each period. If intermediate links between two consecutive periods do not exist, there are still classical DEA tools such as Malmquist productivity index to handle the situation. In many real world applications, however, intermediate links such as capital and savable outputs exist. Dynamic DEA (DDEA) extends the application of DEA to such situations. A link which is also referred to as a carry-over activity can be interpreted as an output from one period and an input to its next period. This connection between periods should be considered in the efficiency estimation. Therefore the classical DEA models (static DEA) may not be appropriate tools to deal with such situation. For example, the efficiency estimation resulted from applying static DEA models to each period separately and calculating the average efficiency, does not fully portray the nature of the problem. DDEA models compare the performance of DMUs over

2010 Mathematics Subject Classification. Primary: 90C05, 90B50; Secondary: 91B06.

Key words and phrases. Data envelopment analysis, dynamic efficiency, DDEA, slacks-based model, carry-over activity.

* Corresponding author: Mohammad Afzalinejad.
all periods together and evaluate overall and period efficiency. Note that the dynamic system defined in DDEA is a general time-related system and it may differ from the more formal concept of dynamic systems defined in Mathematics or Operations Research. It should also be mentioned that there is some similarities between dynamic DEA and multi-stage DEA. The latter one considers systems with production through a series of stages. In multi-stage DEA, each stage has inputs both from its previous stage (if available) and outside world. Outputs of a stage are two kinds: external outputs and intermediate links. Although, two dynamic and multi-stage problems have completely different definitions, there are some basic similarities in their problem formulations. Therefore, it may be constructive to study one of these problems to learn about the other problem (for example see [11, 13, 3]).

There are many studies in DEA literature about dynamic efficiency. Although several works of research like G. Klopp [12] and Färe et al. [7] had studied the time dependency of efficiency, Färe and Grosskopf [6] and Sengupta [20] firstly proposed dynamic models based on the above-mentioned perspective about carry-over activities. Here, we point to some important studies in DDEA among others. Färe and Grosskopf’s [6] work was based on network DEA and Sengupta [20] discussed about the first order condition in modeling DDEA. Nemoto and Goto [15, 16] classified inputs to two categories of variable and quasi-fixed inputs. They showed how the dynamic overall efficiency can be measured considering investment behavior of the firms and decomposed overall efficiency to allocative and technical efficiencies. Soleimani-damaneh [22] introduced a method with less computational efforts for DDEA and Sueyoshi and Sekitani [23] studied the concept of returns to scale in DDEA. Park and Park [18] described extensions of Debreu-Farrell technical efficiency to the case of multi-period production systems in the absence of carry-over activities. Dynamic network DEA is another subject that has attracted attention of several researchers such as Tone and Tsutsui [26] and Omrani and Soltanzadeh [17]. In this approach, the internal network structure of DMUs is considered in dynamic efficiency evaluations. A special case of dynamic network DEA is dynamic two-stage network DEA. A model of this kind was formulated by Moreno and Lozano [14] to evaluate the efficiency of the public finance of European governments during economic crisis period 2008-2012. Fukuyama and Weber [8] applied a dynamic two-stage network model for measuring Japanese banks performance. Kao [10] studied dynamic efficiency from the multiplicative DEA perspective. He defined the overall system efficiency as the weighted sum of period efficiencies. Emrouznejad and Thanassoulis [4] proposed a radial dynamic efficiency measure, while Tone and Tsutsui [25] suggested a slacks-based efficiency model for DDEA.

Tone and Tsutsui’s model [25] which is an extension of slacks-based measure of efficiency [24] to the case of dynamic DEA, imposes some constraints to connect consecutive periods. Although their formulation is very interesting, there are some drawbacks to the resulting model. Tone and Tsutsui do not consider any slacks for link constraints which leads to efficiency overestimation. On the other hand, these constraints cannot be interpreted mathematically. This problem is also observed in some other works in dynamic DEA like the dynamic network DEA model proposed by Tone and Tsutsui [26] and the model employed by Hung et al. [9] for evaluating Taiwanese business groups. Imposing initial condition on efficiency estimation is another problem with the Tone and Tsutsui’s DDEA formulation. In this paper, these issues with Tone and Tsutsui’s model are discussed in detail and a new formulation for the slacks-based dynamic efficiency measurement model is
proposed. In the proposed model, the connecting constraints have slack variables. Including the corresponding slacks in the objective function yields a full-efficiency measure for DDEA. Furthermore, the formulation of constraints in the new model is in accordance with the input or output orientation of the model. This differs from the classical slacks-based DEA models in which the constraints are the same in both input-oriented and output-oriented models. The Tone and Tsutsui’s model and the proposed model are compared using data from a real-world application. The results show that the new model is more rigorous in efficiency evaluation and provides more discrimination between DMUs.

The rest of the paper is organized as follows. Section 2 reviews the slacks-based dynamic models of Tone and Tsutsui. The proposed model is introduced and discussed in Section 3. The practical capability of the proposed model is investigated in Section 4. Finally, Section 5 concludes the paper.

2. **Tone and Tsutsui’s model.** Consider $n$ DMUs and $T$ periods. In period $t$, $DMU_j$ uses external inputs $x_{ijt}, i = 1, \ldots, m$ to produce external outputs $y_{rjt}, r = 1, \ldots, s, j = 1, \ldots, n, t = 1, \ldots, T$. We also consider two kinds of carry-over activities between periods $t$ and $t+1$ that are good (or desirable) links $z_{ijt}^{\text{good}}, i = 1, \ldots, n_{\text{good}}$ and bad (or undesirable) links $z_{ijt}^{\text{bad}}, i = 1, \ldots, n_{\text{bad}}$. Bad links usually refer to pollutions and waste materials which are by-products of manufacturing good outputs. Undesirable factors like loan losses in a bank system or airplanes delays in an airport are also considered as bad links. Note that, we assume that all external outputs $y_{rjt}$’s are good outputs for ease of notations, however, one can consider the division of external outputs to bad and good ones and treats them like what is done in the classical DEA models. Two other kinds of links are also involved in DDEA which are free and fixed links. Free links can freely be increased or decreased from the observed values while fixed or non-discretionary links are out of the control of DMU and cannot increased or decreased from the observed values. These carry-over activities do not directly appear in the efficiency measure in DEA models. Instead, they have indirect effect on the efficiency evaluation by restricting the production possibility set. Because of similarity and for ease of formulation, we do not include the free and fixed links in our models and prefer instead to give the related constraints separately.

Tone and Tsutsui [25] proposed the following input-oriented dynamic SBM model.

$$
\theta^*_o(TT) = \min \frac{1}{T}\left(\sum_{t=1}^{T} w^t \left(1 - \frac{1}{m + n_{\text{bad}}} \left(\sum_{i=1}^{m} \frac{u_i^t - s_{it}^{\text{b}}}{x_{i^t}} + \sum_{i=1}^{n_{\text{bad}}} \frac{s_{i^t}^{\text{b}}}{z_{i^t}^{\text{b}}} \right) \right)\right)
$$

s.t.

$$
x_{i^t} = \sum_{j=1}^{n} x_{ijt}^{\lambda_j^t} + s_{it}^{\text{b}} \quad i = 1, \ldots, m, \ t = 1, \ldots, T \tag{1a}
$$

$$
y_{i^t} = \sum_{j=1}^{n} y_{ijt}^{\lambda_j^t} - s_{it}^{\text{b}} \quad i = 1, \ldots, s, \ t = 1, \ldots, T \tag{1b}
$$

$$
z_{i^t}^{\text{good}} = \sum_{j=1}^{n} z_{ijt}^{\text{good}} \lambda_j^t - s_{it}^{\text{b}} \quad i = 1, \ldots, n_{\text{good}}, \ t = 1, \ldots, T \tag{1c}
$$
$\bar{z}_{tot}^{bad} = \sum_{j=1}^{n} z_{ijt}^{bad} \lambda_j^t + s_{iit}^{bad} \quad i = 1, \ldots, n_{bad}, \quad t = 1, \ldots, T$ (1d)

$\sum_{j=1}^{n} z_{ijt}^{good} \lambda_j^t = \sum_{j=1}^{n} z_{ijt}^{good} \lambda_j^{t+1} \quad i = 1, \ldots, n_{good}, \quad t = 1, \ldots, T - 1$ (1e)

$\sum_{j=1}^{n} z_{ijt}^{bad} \lambda_j^t = \sum_{j=1}^{n} z_{ijt}^{bad} \lambda_j^{t+1} \quad i = 1, \ldots, n_{bad}, \quad t = 1, \ldots, T - 1$ (1f)

$\lambda_j^t \geq 0 \quad j = 1, \ldots, n, \quad t = 1, \ldots, T$

$s_{iit}^{-}, s_{iit}^{+}, s_{iit}^{good}, s_{iit}^{bad} \geq 0 \quad \forall i, t$

where DMU$_o$ is the DMU under evaluation and constant returns-to-scale is assumed. $w_i$ and $w_t$ are the $i$'th input weight and the period $t$'s weight, respectively and satisfy

$$\sum_{i=1}^{m} w_i^{-} = m$$

$$\sum_{t=1}^{T} w_t = T.$$  

In the above model, it is assumed that there is no initial input from carry-over activities to period 1. If $z_{ij0}^{good}$ and $z_{ij0}^{bad}$ also exist for $t = 0$, some constraints should be imposed for these initial conditions. Tone and Tsutsui proposed the following constraints for this purpose.

$$z_{i00}^{good} \leq \sum_{j=1}^{n} z_{ij0}^{good} \lambda_j^1 \quad i = 1, \ldots, n_{good}, \quad (2)$$

$$z_{i00}^{bad} \geq \sum_{j=1}^{n} z_{ij0}^{bad} \lambda_j^1 \quad i = 1, \ldots, n_{bad}. \quad (3)$$

Fixed and free links constraints can also be imposed on the model as follows.

$$z_{tot}^{free} = \sum_{j=1}^{n} z_{ijt}^{free} \lambda_j^t + s_{iit}^{free} \quad i = 1, \ldots, n_{free}, \quad t = 1, \ldots, T$$

$$z_{tot}^{fix} = \sum_{j=1}^{n} z_{ijt}^{fix} \lambda_j^t \quad i = 1, \ldots, n_{fix}, \quad t = 1, \ldots, T$$

$$\sum_{j=1}^{n} z_{ijt}^{free} \lambda_j^t = \sum_{j=1}^{n} z_{ijt}^{free} \lambda_j^{t+1} \quad i = 1, \ldots, n_{free}, \quad t = 1, \ldots, T - 1$$

$$\sum_{j=1}^{n} z_{ijt}^{fix} \lambda_j^t = \sum_{j=1}^{n} z_{ijt}^{fix} \lambda_j^{t+1} \quad i = 1, \ldots, n_{fix}, \quad t = 1, \ldots, T - 1$$

$s_{iit}^{free}$ in sign $\forall i, t$

In the rest of paper we use TT to refer to the above model and superscript * to indicate optimal values of variables. The efficiency of each period can also be calculated using the results of the TT model. The input-oriented efficiency of DMU$_o$ in period $t$ is evaluated as
A SLACKS-BASED MODEL FOR DYNAMIC DEA

θ∗ ot (TT) = 1 − \frac{1}{m + nbad} \left( \sum_{i=1}^{m} w_i s_i^* x_{iot} + \sum_{i=1}^{nbad} s_{bad i} \right)

where s_i^* and s_{bad i} represent optimal slack values for i'th input and i'th bad link, respectively. Based on the TT model, we define overall and period full efficiencies as follows:

**Definition 2.1.** DMU_o is efficient in period t = 1, . . . , T if θ∗ ot = 1, s + i t = 0 and s good i t = 0 (∀i) in all optimal solutions of model (1).

**Definition 2.2.** DMU_o is overall efficient in model (1) if θ∗ o = 1, s + i t = 0 and s good i t = 0 (∀i, t) in all optimal solutions.

Tone and Tsutsui also suggested the following output-oriented Dynamic model.

\[ \frac{1}{\tau o} = \max T \frac{1}{T} \left( \sum_{t=1}^{T} w^t (1 + s + \frac{1}{s + ngood} \left( \sum_{i=1}^{s} w_{i}^t \frac{s_{i}^*}{y_{i} ot} + \sum_{i=1}^{ngood} s_{good i}^* \right)) \right) \] (4)

s.t.

All the constraints in (1)

where \( \sum_{i=1}^{s} w_{i}^t = s \).

There are some disadvantages to Tone and Tsutsui’s formulation of dynamic DEA. An issue with the TT model is the constraints related to carry-over activities. These constraints do not have any slack variables and this causes overestimation of efficiency in many cases. To demonstrate the problem, consider two DMUs DMU_a and DMU_b in Figure 1. There are two periods t = 1, 2 and the links are good links. Note that DMU_a dominates DMU_b. All external inputs of DMU_a including the input link to the first period are less than or equal to DMU_b and all external outputs of DMU_a including the output link from the last period are more than or equal to DMU_b. The intermediate link between the two periods especially is the same in the two DMUs. This is a necessary condition for overall domination. While DMU_b is not (input-oriented) efficient, it is classified as efficient by the TT model. The efficiency scores resulted from the input-oriented TT model are \( \theta_{a1}^* = \theta_{a2}^* = 1 \) and \( \theta_{b1}^* = \theta_{b2}^* = 1 \) in both variable returns-to-scale (VRS) and constant returns-to-scale (CRS) cases. Another issue with the TT model is the interpretation of link constraints. The link constraints (1c) and (1f) imply that the projection points in periods t and t + 1 have equal components associated to intermediate links. This causes ambiguity since the production possibility sets in periods t and t + 1 are essentially different.

Finally, the way of imposing initial condition in the TT model is unclear. Tone and Tsutsui’s suggestion is dealing with input links of period 1 like output links of this period. In other words, constraint set (2) is the same as (1c) for t = 1 and the constraints set (3) is the same as (1d) for t = 1. It is not clear how these constraints should be interpreted and why these inputs and outputs are treated in the same manner. In the next section a modification to the TT model is proposed, which resolves all the above issues.

3. The proposed dynamic slacks-based model. Consider the notations employed in Section 2. To define a new slacks-based measure of efficiency capable of
detecting all sources of inefficiency, we first define the period production possibility set \((\Gamma_{t}^{CRS})\) and overall static production possibility set \((\Gamma_{CRS})\) as follows:
\[
\Gamma_{t}^{CRS} = \left\{ (x_{t}, z_{t}^{bad}, z_{t-1}^{bad}, z_{t}^{good}, y_{t}) \in \mathbb{R}_{+}^{n + nbad + ngood} \times \mathbb{R}_{+}^{nbad + ngood + s} \mid \sum_{j=1}^{n} x_{jt} \lambda_{t}^{j} \leq x_{t}, \sum_{j=1}^{n} z_{jt}^{bad} \lambda_{t}^{j} \leq z_{t}^{bad}, \sum_{j=1}^{n} z_{jt-1}^{good} \lambda_{t}^{j} \leq z_{t-1}^{good}, \sum_{j=1}^{n} z_{jt-1}^{bad} \lambda_{t}^{j} \geq z_{t-1}^{bad}, \sum_{j=1}^{n} z_{jt}^{good} \lambda_{t}^{j} \geq z_{t}^{good}, \sum_{j=1}^{n} y_{jt} \lambda_{t}^{j} \geq y_{t} \right\} \quad t = 1, \ldots, T,
\]
\[
\Gamma_{CRS} = \Gamma_{1}^{CRS} \times \Gamma_{2}^{CRS} \times \cdots \times \Gamma_{T}^{CRS},
\]
where constant returns-to-scale is assumed. In the above definition, \(z_{t}^{good}\) is considered as output for period \(t - 1\) and as input for period \(t\). Conversely, \(z_{t-1}^{bad}\) is considered as input for period \(t - 1\) and as output for period \(t\) (for information about how bad outputs can be treated in DEA models see Scheel [19]). Based on the above definition for static production possibility set, the following dynamic constraints are suggested to connect consecutive periods:
\[
\sum_{j=1}^{n} z_{jt}^{good} \lambda_{t}^{j} \geq \sum_{j=1}^{n} z_{jt-1}^{good} \lambda_{t}^{j} \quad t = 2, \ldots, T, \quad (5)
\]
\[
\sum_{j=1}^{n} z_{jt}^{bad} \lambda_{t}^{j} \leq \sum_{j=1}^{n} z_{jt-1}^{bad} \lambda_{t}^{j} \quad t = 2, \ldots, T. \quad (6)
\]
The first set of dynamic constraints is resulted from the following static constraints for DMUo:
\[
\sum_{j=1}^{n} z_{jt}^{good} \lambda_{t}^{j} \geq z_{o,t-1}^{good} \quad t = 2, \ldots, T.
\]
\[
\sum_{j=1}^{n} z_{jt}^{bad} \lambda_{t}^{j} \leq z_{o,t-1}^{bad}
\]
Similarly, the second set of dynamic constraints is resulted from

\[
\sum_{j=1}^{n} z_{j,t-1}^b \lambda_{j}^{t-1} \leq z_{o,t-1}^b \\
\sum_{j=1}^{n} z_{j,t-1}^b \lambda_{j}^{t} \geq z_{o,t-1}^b 
\]

\( t = 2, \ldots, T. \)

The link constraints can also be interpreted by duality relationship in DEA models. From multiplicative form of the DEA models, it can be seen that these constraints lead to a unique inputs/outputs weights for identical carry-over activities in the two periods \( t - 1 \) and \( t \). To show this, consider the following constraints for the DEA model in multiplicative form.

\[
\begin{align*}
& \left( p_{t-1}^{g,good} z_{j,t-1}^g + p_{t-2}^{b,good} z_{j,t-2}^b + u_{j,t-1} y_{j,t-1} \right) \leq 1 \quad \text{period}(t-1) \\
& \left( p_{t-2}^{g,good} z_{j,t-2}^g + p_{t-1}^{b,good} z_{j,t-1}^b + v_{t-1} x_{j,t-1} \right) \leq 1 \quad \text{period}(t) \\
& p_{t-1}^{g,good} z_{j,t-1}^g + p_{t-2}^{b,good} z_{j,t-2}^b + u_{i} y_{j,t} \leq 1 \\
& p_{t-1}^{g,good} z_{j,t-1}^g + p_{t-2}^{b,good} z_{j,t-2}^b + v_{i} x_{j,t} \leq 1
\end{align*}
\]

In the above formulas, \( p_{t}^{g,good}, p_{t}^{b,good}, u_{t}, \) and \( v_{t} \) are non-negative weight (price) variables corresponding to \( z_{t}^{good}, z_{t}^{bad}, y_{t}, \) and \( x_{t} \), respectively. Note that \( z^{good} \) and \( z^{bad} \) have the same weight in both periods \( t - 1 \) and \( t \). The above constraints can be stated in the linear form as follows:

\[
\begin{align*}
& p_{t-1}^{g,good} z_{j,t-1}^g + p_{t-2}^{b,good} z_{j,t-2}^b + u_{j,t-1} y_{j,t-1} - p_{t-2}^{g,good} z_{j,t-2}^g - p_{t-1}^{b,good} z_{j,t-1}^b - v_{t-1} x_{j,t-1} \leq 0 \quad \text{(7)} \\
& p_{t-1}^{g,good} z_{j,t-1}^g + p_{t-2}^{b,good} z_{j,t-2}^b + u_{j,t} y_{j,t} - p_{t-1}^{g,good} z_{j,t-1}^g - p_{t-2}^{b,good} z_{j,t-2}^b - v_{j,t} x_{j,t} \leq 0 \quad \text{(8)}
\end{align*}
\]

Consider \( \lambda_{j}^{t-1} \) and \( \lambda_{j}^{t} \) as the dual variables corresponding to (7) and (8), respectively. Assuming that variables \( p_{t}^{g,good} \) and \( p_{t}^{b,good} \) do not directly play a role in the objective function of the associated multiplicative DEA model, we can derive dynamic constraints sets (5) and (6) as the dual envelopment constraints corresponding to variables \( p_{t}^{g,good} \) and \( p_{t}^{b,good} \).

The following input-oriented slacks-based dynamic DEA model is proposed.

\[
\theta^{o} = \min \left( w^{1} \left( 1 - \frac{1}{m + n^{good}} \sum_{i=1}^{m} s_{i}^{o,good} \sum_{t=1}^{T} \frac{g^{good}}{z_{i,t}^{good}} \right) + \sum_{t=2}^{T} w^{t} \left( 1 - \frac{1}{m + \sum_{i=1}^{m} x_{i,t}^{o}} \right) \right)
\]

\[
\left( w^{1} \left( 1 + \frac{1}{n^{bad}} \sum_{i=1}^{n^{bad}} \frac{g^{bad}}{z_{i,t}^{bad}} \right) + \sum_{t=2}^{T} w^{t} \left( 1 + \frac{1}{n^{good} + n^{bad}} \sum_{i=1}^{n^{bad}} \frac{g^{bad}}{z_{i,t}^{bad}} \right) \right)
\]

\[
\left( w^{1} \left( 1 + \frac{1}{n^{bad}} \sum_{i=1}^{n^{bad}} \frac{g^{bad}}{z_{i,t}^{bad}} \right) + \sum_{t=2}^{T} w^{t} \left( 1 + \frac{1}{n^{good} + n^{bad}} \sum_{i=1}^{n^{bad}} \frac{g^{bad}}{z_{i,t}^{bad}} \right) \right)
\]

\[
\left( w^{1} \left( 1 + \frac{1}{n^{bad}} \sum_{i=1}^{n^{bad}} \frac{g^{bad}}{z_{i,t}^{bad}} \right) + \sum_{t=2}^{T} w^{t} \left( 1 + \frac{1}{n^{good} + n^{bad}} \sum_{i=1}^{n^{bad}} \frac{g^{bad}}{z_{i,t}^{bad}} \right) \right)
\]
where \(w^t\) denotes the weight of period \(t\) satisfying the condition \(\sum_{t=1}^T w^t = 1\). Note that initial and final conditions are also imposed in the above model and the selection of constraints (9c) to (9f) is consistent with the input orientation nature of the model. The VRS case can be obtained by imposing additional constraints \(\sum_{j=1}^n \lambda_j^t = 1\) for \(t = 1, \ldots, T\) on model (9).

The main differences between models (1) and (9) lie in the objective function as well as dynamic constraint sets (1e), (1f), (9g) and (9h). The objective function in (9) proposes a new measure for dynamic DEA. The slack variables of all inputs of first period, i.e. \(s_{1}^{1}, s_{1}^{good} - s_{1}^{bad},\) are included in the objective function to consider the initial condition. Furthermore, carry-over activities play an important role in the efficiency measure and the slack variables associated with intermediate links entering to period \(t\), i.e. \(s_{it}^{good}, s_{it}^{bad, t}\), trap inefficiencies related to carry-over activities. Obviously, the objective function of model (9) is monotone decreasing with respect to slacks variables. Since all slacks associated with external inputs, external outputs and carry-over activities are presented in efficiency evaluation we have a full efficiency measure that prevents efficiency overestimation.

It should be noted that strong disposability is assumed in the above model. In fact, it is quite possible that this condition does not hold in some applications. In
such applications, a reduction in bad outputs leads to reductions in good outputs as well. This condition is called weak disposability. To deal with this condition, we should delete slack variables from the constraints related to bad outputs. For more information on weak and strong disposability see Färe et al. [5] and Scheel [19].

Model (9) is always feasible because setting $\lambda_j^t = 0 \ (\forall j \neq o)$, $\lambda_o^t = 1 \ (t = 1, \ldots, T)$ and all slacks equal to zero gives a feasible solution for (9). Since

$$s_{it}^{-} \leq x_{it}^{*}, \quad t = 1, \ldots, T$$

$$s_{it}^{\text{good}}^{-} \leq z_{it}^{\text{good}},$$

the numerator of the fraction in the objective function is not greater than unity. On the other hand, the denominator is not less than unity. Therefore the efficiency measure in model (9) satisfies $0 \leq \theta_o^{*} \leq 1$. It is noteworthy that the slack variables for link constraints are bounded in variable returns-to-scale. For example,

$$s_{it}^{\text{good}}^{*} = \sum_{j=1}^{n} z_{ij,t-1}^{\text{good}} \lambda_j^{t-1} - \sum_{j=1}^{n} z_{ij,t-1}^{\text{good}} \lambda_j^{t} \leq z_{it}^{\text{good}}^{*} - z_{it}^{\text{good}},$$

where, $z_{it}^{\text{good}}^{*} = \max\{z_{ij,t-1}^{\text{good}} | j = 1, \ldots, n \}$ and $z_{it}^{\text{good}} = \min\{z_{ij,t-1}^{\text{good}} | j = 1, \ldots, n \}$.

Period efficiency can be defined similar to what is done for the TT model.

**Definition 3.1.** If $(\theta_o^{*}, s_{i}^{*}, s_{i}^{++}, s_{i}^{--}, s_{i}^{\text{good}^{+}}, s_{i}^{\text{good}^{-}}, s_{i}^{\text{bad}^{+}}, s_{i}^{\text{bad}^{-}}, s_{i}^{\text{good}^{+}}, s_{i}^{\text{bad}^{-}}, s_{i}^{\text{good}^{+}}, s_{i}^{\text{bad}^{-}}, s_{i}^{\text{good}^{+}}, s_{i}^{\text{bad}^{-}})$ is an optimal solution of program (9) then the input-oriented period efficiency is defined as follows

$$\theta_o^{*} = \frac{1}{m + n_{\text{good}} \left( \sum_{i=1}^{m} \frac{s_{it}^{++}}{x_{it}^{*}} + \sum_{i=1}^{m} \frac{s_{it}^{\text{good}^{-}}}{z_{it}^{\text{good}^{*}}} \right)},$$

$$\theta_o^{*} = \frac{1}{n_{\text{bad}} + n_{\text{good}} \left( \sum_{i=1}^{m} \frac{s_{it}^{\text{bad}^{+}}}{z_{it}^{\text{bad}^{*}}} + \sum_{i=1}^{m} \frac{s_{it}^{\text{good}^{+}}}{z_{it}^{\text{good}^{*}}} \right)}, \quad t = 2, \ldots, T.$$
The converse of theorem is proved similarly. for no one of the inequalities in (11) and (14) for condition of Definition(3.3) is violated in both cases (a contradiction). Note that slack among to the final period output links should hold strictly which means that at least one (15) or an inequality related to the first period input links or an inequality related a slack \( s \) nonzero slack variable. To be more precise, if an inequality in (10) holds strictly, Consequently, we have found a feasible solution for model (9) with at least one nonzero slack variable. To be more precise, if an inequality in (10) holds strictly, a slack \( s_{it}^- \) is nonzero which means \( \theta^*_o < 1 \). Otherwise, at least an inequality in (15) or an inequality related to the first period input links or an inequality related to the final period output links should hold strictly which means that at least one slack among \( s_{it}^+ \), \( s_{i1}^{good-} \), \( s_{i1}^{good+} \), \( s_{i1T}^{good-} \) or \( s_{i1T}^{good+} \) should be nonzero. Therefore the condition of Definition(3.3) is violated in both cases (a contradiction). Note that no one of the inequalities in (11) and (14) for \( t = 1, \ldots, T - 1 \) and (12) and (13) for \( t = 2, \ldots, T \) can hold strictly because this violates \( z_t^{good} = z_o^{good} \) or \( z_t^{bad} = z_o^{bad} \). The converse of theorem is proved similarly.

Obviously, \( \theta \) is unit invariant. The following theorem states the monotone feature of \( \theta^* \).

**Proposition 2.** If \( DMU_o \) is dominated by \( DMU_b \) then \( \theta^*_o \leq \theta^*_b \).

**Proof.** From the proof of Proposition 1, we have

\[
\begin{align*}
& s_{ita} \geq s_{itb}, \quad t = 1, \ldots, T, i = 1, \ldots, m \\
& s_{i1a}^{good-} \geq s_{i1b}^{good-}, \quad i = 1, \ldots, n_{good} \\
& s_{i1a}^{bad+} \geq s_{i1b}^{bad+}, \quad i = 1, \ldots, n_{bad}
\end{align*}
\]
Therefore,

\[ w^1(1 - \frac{1}{m + n_{\text{good}}}(\sum_{i=1}^{m} \frac{s_{i,t,1}^-}{x_{i,t,1}} + \sum_{i=1}^{n_{\text{good}}} \frac{s_{i,t,1}^-}{z_{i,t,1}^-})) + \sum_{t=2}^{T} w^t(1 - \frac{1}{m + n_{\text{bad}}}(\sum_{i=1}^{m} \frac{s_{i,t,2}^-}{x_{i,t,2}}) \leq w^1(1 - \frac{1}{m + n_{\text{good}}}(\sum_{i=1}^{m} \frac{s_{i,t,1}^-}{x_{i,t,1}} + \sum_{i=1}^{n_{\text{good}}} \frac{s_{i,t,1}^-}{z_{i,t,1}^-})) + \sum_{t=2}^{T} w^t(1 - \frac{1}{m + n_{\text{bad}}}(\sum_{i=1}^{m} \frac{s_{i,t,2}^-}{x_{i,t,2}}) \leq 0 \]


and

\[ w^1(1 + \frac{1}{n_{\text{bad}}}(\sum_{i=1}^{n_{\text{bad}}} \frac{s_{i,t,2}^+}{z_{i,t}^+}) + \sum_{t=2}^{T} w^t(1 + \frac{1}{n_{\text{bad}} + n_{\text{good}}}(\sum_{i=1}^{n_{\text{bad}}} \frac{s_{i,t,2}^+}{z_{i,t,1}^+} + \sum_{i=1}^{n_{\text{good}}} \frac{s_{i,t,2}^+}{z_{i,t,1}^+})) \geq w^1(1 + \frac{1}{n_{\text{bad}}}(\sum_{i=1}^{n_{\text{bad}}} \frac{s_{i,t,2}^+}{z_{i,t}^+}) + \sum_{t=2}^{T} w^t(1 + \frac{1}{n_{\text{bad}} + n_{\text{good}}}(\sum_{i=1}^{n_{\text{bad}}} \frac{s_{i,t,2}^+}{z_{i,t,1}^+} + \sum_{i=1}^{n_{\text{good}}} \frac{s_{i,t,2}^+}{z_{i,t,1}^+})). \]

This implies that \( \theta_o^* \leq \theta_b^* \).

If fixed and free links also exist, the following constraints can be imposed on model (9).

\[ \sum_{j=1}^{n} z_{i,j,t-1}^f = \sum_{j=1}^{n} z_{i,j,t-1}^f \lambda_j^f + s_{i,t}^f \quad i = 1, \ldots, n_{\text{free}}, \quad t = 1, \ldots, T \]

\[ \sum_{j=1}^{n} z_{i,j,t-1}^f = \sum_{j=1}^{n} z_{i,j,t-1}^f \lambda_j^f \quad i = 1, \ldots, n_{\text{fix}}, \quad t = 1, \ldots, T \]

\[ \sum_{j=1}^{n} z_{i,j,t-1}^f = \sum_{j=1}^{n} z_{i,j,t-1}^f \lambda_j^f \quad i = 1, \ldots, n_{\text{free}}, \quad t = 2, \ldots, T \]

\[ \sum_{j=1}^{n} z_{i,j,t-1}^f = \sum_{j=1}^{n} z_{i,j,t-1}^f \lambda_j^f \quad i = 1, \ldots, n_{\text{fix}}, \quad t = 2, \ldots, T \]

The fractional programming model (9) can be transformed into a linear program using the Charnes-Cooper transformation as follows:

\[ \theta_o^* = \min(w^1(\alpha - \frac{1}{m + n_{\text{good}}}(\sum_{i=1}^{m} \frac{S_{i,t,1}^-}{x_{i,t,1}} + \sum_{i=1}^{n_{\text{good}}} \frac{S_{i,t,1}^-}{z_{i,t,1}^-})) + \sum_{t=2}^{T} w^t(\alpha - \frac{1}{m + n_{\text{bad}}}(\sum_{i=1}^{m} \frac{s_{i,t,1}^-}{x_{i,t,1}} + \sum_{i=1}^{n_{\text{bad}}} \frac{s_{i,t,1}^-}{z_{i,t,1}^-}))) \]

s.t.

\[ w^1(\alpha + \frac{1}{n_{\text{bad}}}(\sum_{i=1}^{n_{\text{bad}}} \frac{s_{i,t,2}^+}{z_{i,t}^+}) + \sum_{t=2}^{T} w^t(\alpha + \frac{1}{n_{\text{bad}} + n_{\text{good}}}(\sum_{i=1}^{n_{\text{bad}}} \frac{s_{i,t,2}^+}{z_{i,t,1}^+} + \sum_{i=1}^{n_{\text{good}}} \frac{s_{i,t,2}^+}{z_{i,t,1}^+}))) = 1 \]

\[ \alpha x_{i,t} = \sum_{j=1}^{n} x_{i,j,t} \lambda_j^f + \sum_{i=1}^{m} S_{i,t}^f \quad i = 1, \ldots, n_{\text{free}}, \quad t = 1, \ldots, T \]
\[ \alpha y_{iot} = \sum_{j=1}^{n} y_{ijt} \lambda_j^t - S_{it}^+ \quad i = 1, \ldots, s, \quad t = 1, \ldots, T \]

\[ \alpha z_{good}^{io,t-1} = \sum_{j=1}^{n} z_{ij,t-1}^{good} \lambda_j^t + S_{it}^{good-} \quad i = 1, \ldots, n_{good}, \quad t = 1, \ldots, T \]

\[ \alpha z_{good}^{ioT} = \sum_{j=1}^{n} z_{ijT}^{good} \lambda_j^T - S_{iT}^{good+} \quad i = 1, \ldots, n_{good} \]

\[ \alpha z_{bad}^{io,t-1} = \sum_{j=1}^{n} z_{ij,t-1}^{bad} \lambda_j^t - S_{it}^{bad+} \quad i = 1, \ldots, n_{bad}, \quad t = 1, \ldots, T \]

\[ \alpha z_{bad}^{ioT} = \sum_{j=1}^{n} z_{ijT}^{bad} \lambda_j^T + S_{iT}^{bad-} \quad i = 1, \ldots, n_{bad} \]

\[ \sum_{j=1}^{n} z_{ij,t-1}^{good} \lambda_j^t - \sum_{j=1}^{n} z_{ij,t-1}^{bad} \lambda_j^t \quad i = 1, \ldots, n_{good}, \quad t = 2, \ldots, T \]

\[ \sum_{j=1}^{n} z_{ij,t-1}^{bad} \lambda_j^t - \sum_{j=1}^{n} z_{ij,t-1}^{bad} \lambda_j^t \quad i = 1, \ldots, n_{bad}, \quad t = 2, \ldots, T \]

\[ \lambda_j^t \geq 0 \quad j = 1, \ldots, n, \quad t = 1, \ldots, T \]

\[ S_{it}^+, S_{it}^{good-}, S_{it}^{bad+}, s_{it}^{good-}, s_{it}^{bad+} \geq 0 \quad \forall i, t \]

where \( \alpha > 0 \) in all feasible solutions of the above model and, \( \lambda_j^t \) and all slacks values are \( \alpha \) times corresponding variables in model (9).

The output-oriented slacks-based dynamic model is proposed as follows.

\[ \frac{1}{\varphi_o} = \max \sum_{t=1}^{T-1} w^t \left( \frac{1}{s + n_{good} + n_{bad}} \left( \sum_{i=1}^{s} s_{i}$it$ + \sum_{i=1}^{s_{good}} s_{i}_{good} + \sum_{i=1}^{s_{bad}} s_{i}_{bad} \right) \right) \]

\[ + \sum_{t=1}^{T} \left( \frac{1}{s + n_{good} + n_{bad}} \left( \sum_{i=1}^{s} s_{iT} + \sum_{i=1}^{s_{good}} s_{iT}^{good} + \sum_{i=1}^{s_{bad}} s_{iT}^{bad} \right) \right) \] (16)

\[ s.t. \]

\[ x_{iot} = \sum_{j=1}^{n} x_{ijt} \lambda_j^t + s_{it}^- \quad i = 1, \ldots, m, \quad t = 1, \ldots, T \] (16a)

\[ y_{iot} = \sum_{j=1}^{n} y_{ijt} \lambda_j^t - s_{it}^+ \quad i = 1, \ldots, s, \quad t = 1, \ldots, T \] (16b)

\[ z_{good}^{io} = \sum_{j=1}^{n} z_{ij0} \lambda_j^0 + s_{i1}^{good-} \quad i = 1, \ldots, n_{good} \] (16c)

\[ z_{bad}^{io} = \sum_{j=1}^{n} z_{ij0} \lambda_j^0 + s_{i1}^{bad-} \quad i = 1, \ldots, n_{bad} \] (16d)
which is a linear program. Note that the constraints in the above model are formulated in accordance with the output orientation of the model. This formulation differs from the classical slacks-based models in DEA where the constraints in both input-oriented and output-oriented models are the same.

Applying model (9) to the example given by Figure 1 yields 1 and 0.6204 as the input-oriented efficiency scores of DMU \(_A\) and DMU \(_B\), respectively. The result is consistent with the notion of efficiency in DEA. The efficiency scores for VRS case are 1 and 0.7917. The output-oriented model (16) also gives the similar result. The CRS output-oriented efficiency scores for DMU \(_A\) and DMU \(_B\) are 1 and 0.5647, respectively and VRS output-oriented efficiency scores are 1 and 0.7164, respectively.

The next proposition compares the feasible regions in the proposed model and the TT model. Since Tone and Tsutsui models (1) and (4) have been presented without initial conditions, we remove constraints (16d) and (16f) from model (16) in the following comparison.

**Proposition 3.** Every feasible solution of program (4) is feasible to program (16).

**Proof.** All constraints except link constraints are the same in the two models and the link constraints in model (4) are more restricting from the link constraints in model (16). This proves the result. \(\square\)

The following proposition is an immediate consequence of Proposition 3.

**Proposition 4.** If identical efficiency measures are employed as the objective function for models (4) and (16) then, the efficiency score resulting from model (16) is not greater than the efficiency score resulting from model (4).

Based on the input-oriented and output-oriented efficiency scores resulted from models (9) and (16), respectively, the non-oriented combined efficiency score for DMU \(_o\) can be defined as \(\rho^*_o = \theta^*_o \times \varphi^*_o\).

4. **Practical examination of the model.** In this section we employ a real-world application reported by Shafiee et al. [21] to compare Tone and Tsutsui’s model with the proposed model. The application is about the efficiency evaluation of 10 branches of an Iranian bank over three consecutive periods. Each bank branch has
average monthly salary and operating expense as inputs and total value of loans as output in each period. In addition, there are two carry-over activities or links: net profit as a good link and total loan losses as a bad link. This application has constant returns-to-scale. The data is represented in Table 1. The units of measurement of average salary, net profit and loan losses are 1,000,000,000 Rials, of operating expense is 10,000,000 Rials and of total loans is 1,000,000,000 Rials.

The results are summarized in Tables 2 and 3. The input-oriented, output-oriented and non-oriented combined overall efficiency scores obtained from the TT model and the proposed model are compared in Table 2. As can be seen from Table 2, DMU2 and DMU10 are overall efficient in the TT model while they are not efficient in the proposed model. This is because of the ignored slack in the TT model. Two units DMU6 and DMU9 are efficient in both methods. Table 2 also shows the output-efficiency scores \( \varphi^*_o(TT) \) resulted from model (16) with the objective function replaced by the efficiency measure of Tone and Tsutsui’s model \( \varphi^*_o(TT) \). As can be seen, the results are consistent with Proposition 4 and we have \( \varphi^*_o(TT) \leq \tau^*_o(TT) \). This means that even the slack variables common between the two models have different values and the new model gives more freedom to the slack variables to reveal all inefficiencies.

The period efficiency scores are not uniquely evaluated in both methods when alternative optimal solutions exist. Table 3 illustrates the average input-oriented period efficiency scores resulted from the TT model and the proposed model. The difference between the maximum and the minimum period efficiency scores are minor in both models. Since the set of slack variables and the assignment of slack variables to periods are different in the two models, the resulting period efficiency scores are quite different in most of the cases. From Table 3, overall efficient DMUs are also period efficient in all periods. For example, DMU6 is overall efficient and the conditions of Definition 3.2 hold for all periods, that is, DMU6 is period efficient as well.

\[\begin{array}{cccccc}
\text{DMU} & \text{Average monthly salary} & \text{Operating expense} & \text{Total loan} & \text{Net profit} & \text{Loan losses} \\
\hline
1 & 6.06 & 8.84 & 5.72 & 21.58 & 19.81 & 176.84 \\
2 & 7.67 & 10.98 & 12.32 & 21.58 & 21.96 & 212.75 \\
3 & 5.39 & 5.22 & 7.19 & 18.35 & 15.64 & 176.84 \\
4 & 6.71 & 9.18 & 6.59 & 21.58 & 15.64 & 176.84 \\
5 & 7.18 & 8.71 & 6.59 & 21.58 & 15.64 & 176.84 \\
6 & 5.68 & 6.14 & 5.79 & 21.58 & 15.64 & 176.84 \\
7 & 6.84 & 8.92 & 6.59 & 21.58 & 15.64 & 176.84 \\
8 & 7.56 & 9.92 & 6.59 & 21.58 & 15.64 & 176.84 \\
9 & 8.71 & 10.98 & 12.32 & 21.58 & 19.81 & 176.84 \\
10 & 7.67 & 10.98 & 12.32 & 21.58 & 19.81 & 176.84 \\
\end{array}\]

\[\begin{array}{cccccccc}
\text{DMU} & \text{\( \varphi^*_i(TT) \)} & \text{\( \varphi^*_o(TT) \)} & \text{\( \varphi^*_o(TT) \)} & \text{\( \varphi^*_o(TT) \)} & \text{\( \varphi^*_o(TT) \)} & \text{\( \varphi^*_o(TT) \)} & \text{\( \varphi^*_o(TT) \)} & \text{\( \varphi^*_o(TT) \)} & \text{\( \varphi^*_o(TT) \)} \\
\hline
1 & 0.90 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 \\
2 & 0.90 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 \\
3 & 0.90 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 \\
4 & 0.90 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 \\
5 & 0.90 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 \\
6 & 0.90 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 \\
7 & 0.90 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 \\
8 & 0.90 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 \\
9 & 0.90 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 \\
10 & 0.90 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 & 0.87 \\
\end{array}\]
Table 3. The input-oriented period efficiency scores resulted from the TT model and the proposed model

| DMU | Tone and Tsutsui’s model | The proposed model |
|-----|---------------------------|--------------------|
|     | Period 1 efficiency | Period 2 efficiency | Period 3 efficiency | Period 1 efficiency | Period 2 efficiency | Period 3 efficiency |
| DMU1 | 0.9744 | 1 | 1 | 0.8984 | 1 | 1 |
| DMU2 | 0.5217 | 0.9087 | 0.7450 | 0.5862 | 0.4393 | 0.7247 |
| DMU3 | 0.5887 | 0.8334 | 0.8223 | 0.6162 | 0.8324 | 1 |
| DMU4 | 0.5947 | 0.8411 | 0.7477 | 0.5954 | 0.7744 | 1 |
| DMU5 | 1 | 1 | 1 | 1 | 1 | 1 |
| DMU6 | 0.6962 | 0.9718 | 0.8223 | 0.6216 | 0.5221 | 0.9991 |
| DMU7 | 0.5304 | 0.8121 | 0.7477 | 0.5431 | 0.7498 | 1 |
| DMU8 | 1 | 1 | 1 | 1 | 1 | 1 |
| DMU9 | 1 | 1 | 1 | 0.2862 | 0.8350 | 0.5901 |
| DMU10 | 1 | 1 | 1 | 1 | 1 | 1 |

Finally, the comparison of rank order resulted from the two models is represented in Figure 2. As can be seen, only three DMUs have the same rank in the two models: DMU1, DMU6 and DMU9. Changes in ranks are very high in some DMUs, especially rank of DMU10 is degraded from 1 in the TT model to 8 in the proposed model.

5. Conclusion. In this paper, a new model for slacks-based efficiency analysis in DEA was proposed which considers the time dependency of efficiency. This model treats link constraints in a different way than Tone and Tsutsui’s model does. The production possibility set in the proposed model is redefined and we have two more sets of slack variables for the dynamic link constraints. This makes it possible to reveal all sources of inefficiency and obtain sharper discrimination among DMUs. Furthermore, the new link constraints can be interpreted by duality relationship in DEA models. The way of imposing initial condition in the Tone and Tsutsui’s model is a problem with this model. Because of different constraints structure in the proposed model, we can impose the initial condition on the model in a way consistent with other constraints in the model. In contrast with the Tone and Tsutsui’s model, the slack variables of link constraints are included in the objective function of the proposed model. Since all slack variables corresponding to initial conditions, external inputs, external outputs and intermediate links are available in the efficiency evaluation, the efficiency measure evaluated in the proposed method
is a complete measure. This measure scores all sources of efficiency and prevents efficiency overestimation observed in Tone and Tsutsui’s model.

By using real world data, the new model and, Tone and Tsutsui’s model have been compared. The results plainly confirm the theoretical findings. While 4 out of 10 DMUs are efficient by Tone and Tsutsui’s model just two of them are efficient by the proposed model.

The new model can be a subject for further research on other issues like returns-to-scale change over time.

Acknowledgments. The authors would like to thank the two anonymous referees for their valuable comments and suggestions which were helpful in improving the paper.

REFERENCES

[1] A. Charnes, W. W. Cooper and E. Rhodes, Measuring the efficiency of decision making units, European Journal of Operational Research, 2 (1978), 429–444.
[2] W. W. Cooper, L. M. Seiford and K. Tone, Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-solver Software, 2nd edition, Springer-Verlag, New York, 2007.
[3] D. K. Despotis, D. Sotiros and G. Koronakos, A network DEA approach for series multi-stage processes, Omega, 61 (2016), 35–48.
[4] A. Emrouznejad and E. Thanassoulis, A mathematical model for dynamic efficiency using data envelopment analysis, Applied Mathematics and Computation, 160 (2005), 363–378.
[5] R. Färe, S. Grosskopf, C. A. K. Lovell and C. Pasurka, Multilateral productivity comparisons when some outputs are undesirable: A nonparametric approach, The Review of Economics and Statistics, 71 (1989), 90–98.
[6] R. Färe and S. Grosskopf, Intertemporal Production Frontiers: With Dynamic DEA, Springer-Verlag, Netherlands, 1996.
[7] R. Färe, S. Grosskopf, M. Norris and Z. Zhang, Productivity growth, technical progress, and efficiency change in industrialized countries, The American Economic Review, 84 (1994), 66–83.
[8] H. Fukuyama and W. L. Weber, Measuring Japanese bank performance: A dynamic network DEA approach, Journal of Productivity Analysis, 44 (2015), 249–264.
[9] S. Hung, D. He and W. Lu, Evaluating the dynamic performances of business groups from the carry-over perspective: A case study of Taiwan’s semiconductor industry, Omega, 46 (2014), 1–10.
[10] C. Kao, Dynamic data envelopment analysis: A relational analysis, European Journal of Operational Research, 227 (2013), 325–330.
[11] C. Kao and S. Hwang, Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan, European Journal of Operational Research, 185 (2008), 418–429.
[12] G. A. Klopp, The Analysis of the Efficiency of Production System with Multiple Inputs and Outputs, Ph.D thesis, Industrial and System Engineering College, University of Illinois in Chicago, 1985.
[13] L. Liang, W. D. Cook and J. Zhu, DEA models for two-stage processes: Game approach and efficiency decomposition, Naval Research Logistics, 55 (2008), 643–653.
[14] P. Moreno and S. Lozano, Super SBI Dynamic Network DEA approach to measuring efficiency in the provision of public services, International Transactions in Operational Research, 25 (2018), 715–735.
[15] J. Nemoto and M. Goto, Dynamic data envelopment analysis: modeling intertemporal behavior of a firm in the presence of productive inefficiencies, Economic Letters, 64 (1999), 51–56.
[16] J. Nemoto and M. Goto, Measuring dynamic efficiency in production: an application of data envelopment analysis to Japanese electric utilities, Journal of Productivity Analysis, 19 (2003), 191–210.
A SLACKS-BASED MODEL FOR DYNAMIC DEA

[17] H. Omrani and E. Soltanzadeh, Dynamic DEA models with network structure: An application for Iranian airlines, *Journal of Air Transport Management*, 57 (2016), 52–61.

[18] K. S. Park and K. Park, Measurement of multiperiod aggregative efficiency, *European Journal of Operational Research*, 193 (2009), 567–580.

[19] H. Scheel, Undesirable outputs in efficiency valuations, *European Journal of Operational Research*, 132 (2001), 400–410.

[20] J. K. Sengupta, A dynamic efficiency model using data envelopment analysis, *International Journal of Production Economics*, 62 (1999), 209–218.

[21] M. Shafiee, M. Sangi and M. Ghaderi, Bank performance evaluation using dynamic DEA: A slacks-based measure approach, *Journal of Data Envelopment Analysis and Decision Science*, 2013 (2013), 1–12.

[22] M. Soleimani-damaneh, An effective computational attempt in DDEA, *Applied Mathematical Modelling*, 33 (2009), 3943–3948.

[23] T. Sueyoshi and K. Sekitani, Returns to scale in dynamic DEA, *European Journal of Operational Research*, 161 (2005), 536–544.

[24] K. Tone, A slacks-based measure of efficiency in data envelopment analysis, *European Journal of Operational Research*, 130 (2001), 498–509.

[25] K. Tone and M. Tsutsui, Dynamic DEA: A slacks-based measure approach, *Omega*, 38 (2010), 145–156.

[26] K. Tone and M. Tsutsui, Dynamic DEA with network structure: A slacks-based measure approach, *Omega*, 42 (2014), 124–131.

Received May 2017; 1st revision September 2017; 2nd revision October 2017.

E-mail address: afzalinejad@tafreshu.ac.ir
E-mail address: z.abbasi@tafreshu.ac.ir