THE HANDBAG CONTRIBUTION TO TWO-PHOTON 
ANNIHILATION INTO MESON PAIRS

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We report on the handbag contribution to two-photon annihilation into pion and kaon pairs at large energy and momentum transfer. The underlying physics of the mechanism is outlined and characteristic features and predictions are presented. (Talk given at workshops “QCD-N’02”, Ferrara, Italy, April 3-6, and “Exclusive Processes at High Momentum Transfer”, Jefferson Lab, Newport News, VA, USA, May 15-18, 2002.)

1. Introduction
Meson pair production in the collision of two real photons at asymptotically large energies can be described in the hard scattering approach, where to leading-twist the transition amplitude factorizes into a perturbatively calculable $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$ subprocess and single-meson distribution amplitudes for the hadronization of each of the $q\bar{q}$ pairs. The perturbative contribution to the cross section for $\gamma\gamma \rightarrow \pi^+\pi^-$, however, turns out to be well below the experimental data if single-pion distribution amplitudes consistent with other data are employed.

In the following, we discuss a complementary approach for large values of $s$, $t$, $u$, where the process amplitude factorizes into a hard subprocess for the production of a single $q\bar{q}$ pair, and a subsequent soft transition $q\bar{q} \rightarrow \pi\pi$ (see Fig.1). The latter is described in terms of a new annihilation form factor which is given by a moment of the two-pion distribution amplitude. This mechanism is analogous to the handbag contribution to wide-angle Compton scattering.

2. The Physics of the Handbag Mechanism
We consider the process $\gamma\gamma \rightarrow \pi^+\pi^-$ in the kinematical region where $s \sim -t \sim -u$. The condition for the transition $q\bar{q} \rightarrow \pi^+\pi^-$ to be soft is
that there be no large invariants at the parton-hadron vertices, cf. Fig. 1b. In particular, all virtualities occurring at these vertices are to be of $O(\Lambda^2)$, where $\Lambda$ is a typical hadronic scale. Moreover, the momenta of the additional $q\bar{q}$ pair and possibly other partons are required to be soft. This implies that up to corrections of $O(\Lambda^2/s)$ the initial quark and antiquark approximately carry the momenta of their respective parent pions, and we have the condition $k \simeq p$ or $k' \simeq p$.

Note that configurations where the blob in Fig. 1a contains hard gluon exchange are part of the leading-twist contribution and not included in the soft handbag amplitude. There are also diagrams of the cat’s-ears topology, where the photons couple to different quark lines. At large $s$, $t$, $u$, these diagrams however require the presence of a large virtuality in one of the quark lines or hard gluon exchange.

In order to display the factorization it is advantageous to choose a symmetrical c.m. frame where the pions carry the same light-cone plus momentum, i.e., the skewness, defined by $\zeta = p^+/(p + p')^+$, has the value $1/2$. From the collinearity condition it follows that the initial quark and antiquark also have approximately equal light-cone plus momenta. Thus, we have $z = k^+/(p + p')^+ = 1/2 + O(\Lambda^2/s)$. Furthermore, corrections from partonic off-shell effects and partonic transverse momenta are of $O(\Lambda^2/s)$.

Exploiting the on-shell and collinearity conditions one can show that the helicity amplitude for the process $\gamma\gamma \to \pi^+\pi^-$ can be written in the simple form

$$A_{\mu\mu'} = -4\pi\alpha_0 \delta_{\mu,-\mu'} \frac{s^2}{lu} R_2(s),$$

where $\mu$, $\mu'$ are the photon helicities and the soft part is encoded in the
annihilation form factor defined by
\[ R_{2\pi}(s) = \sum_q e_q^2 R_{2\pi}^q(s), \quad R_{2\pi}^q(s) = \frac{1}{2} \int_0^1 dz \left( 2z - 1 \right) \Phi_{2\pi}^q(z, 1/2, s). \] (2)

Here the summation is over u, d, s quarks and \( \Phi_{2\pi}^q(z, 1/2, s) \) is the two-pion distribution amplitude in light-cone gauge at \( \zeta = 1/2 \). We remark that the operator corresponding to this form factor is the quark part of the energy-momentum tensor, and that the form factor is \( C \) even due to the weight \( (2z - 1) \), as one could expect for a pion pair produced in two-photon annihilation. The differential cross section of the process is given by
\[ \frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^+\pi^-) = \frac{8\pi\alpha_0^2}{s^2} \frac{1}{\sin^4\theta} \left| R_{2\pi}(s) \right|^2. \] (3)

3. Properties and Predictions

Considerations of isospin of the intermediate \( q\bar{q} \) pair together with charge conjugation of the final state imply that charged and neutral pion pairs are only produced in isospin zero states. Hence, one can relate their corresponding form factors and finds the key result
\[ \frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^0\pi^0) = \frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^+\pi^-), \] (4)
which is a parameter-free prediction of the handbag approach and in striking contrast to the leading-twist approach, where the differential cross sections for \( \pi^0\pi^0 \) production is found about an order of magnitude smaller than that for \( \pi^+\pi^- \) pairs. By using \( U \)-spin symmetry, i.e., the symmetry under the exchange \( d \leftrightarrow s \), one can further relate the form factor for \( K^+K^- \) to that of \( \pi^+\pi^- \) production, which leads to the relation
\[ \frac{d\sigma}{dt}(\gamma\gamma \rightarrow K^+K^-) \simeq \frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^+\pi^-). \] (5)

The approximate symbol indicates that, in general, flavour symmetry breaking effects have to be expected. Note that (5) holds in any dynamical approach respecting SU(3) flavour symmetry. Isospin also provides a link between the form factors for charged and neutral kaon pairs, resulting in a further relation characteristic for the handbag mechanism:
\[ \frac{d\sigma}{dt}(\gamma\gamma \rightarrow K^0\overline{K^0}) \simeq \frac{4}{25} \frac{d\sigma}{dt}(\gamma\gamma \rightarrow K^+K^-), \] (6)
where we have neglected non-valence contributions and the numerical factor stems from the ratio of the corresponding charge factors.

The annihilation form factors and the two-pion distribution amplitude can as yet not be calculated within QCD. They also do not allow for an
Figure 2. The scaled annihilation form factors $s |R_{2\pi}(s)|$ (left) and $s |R_{2K}(s)|$ (right) versus $s$. The preliminary ALEPH and DELPHI data is taken from $^9, ^10$. Dashed lines represent our fitted values (7).

The pion annihilation form factor is comparable in magnitude with the timelike electromagnetic pion form factor. Note that although the handbag contribution formally provides a power correction to the leading-twist contribution, it appears to dominate at experimentally accessible energies.

Another characteristic result of our approach is the angular dependence (see Eq. (3)), which is in good agreement with the preliminary ALEPH data, as can be seen from Fig. 3.

Having determined the normalization of the pion and kaon annihilation form factors from experiment, we can also compare with the CLEO data for the integrated cross section $^11$, where pions and kaons have not been separated. The result in displayed in Fig. 4 and again we find good agreement.

As already mentioned in the introduction, the perturbative contribution is way below the experimental data $^2$. In order to facilitate comparison of the handbag and the leading-twist approach, we make a rather conservative estimate of the latter and employ a fixed coupling $\alpha_s = 0.5$. We further use the asymptotic form for both the pion and kaon distribution amplitudes. The leading-twist prediction thus obtained amounts to about 15% of our
Figure 3. The normalized angular distribution for $\gamma\gamma \rightarrow \pi^+\pi^-$ (left) and $\gamma\gamma \rightarrow K^+K^-$ (right), compared to the preliminary ALEPH data for $4 \text{ GeV}^2 < s < 36 \text{ GeV}^2$.

fitted handbag result as shown in Fig. 4.

4. Conclusions

We have presented a brief discussion of the handbag contribution to $\gamma\gamma \rightarrow \pi\pi, KK$. In this approach the process amplitude factorizes into a hard $\gamma\gamma \rightarrow q\bar{q}$ subamplitude and an annihilation form factor for the soft transition to a meson pair. The form factor is a moment of the two-meson distribution amplitude at skewness $\zeta = 1/2$. In lack of a model for the form factors and the two-meson distribution amplitude in the kinematical range we are interested, we fit the form factors for $\pi^+\pi^-$ and $K^+K^-$ to the data and we find that their scaling is compatible with dimensional counting rule behaviour. Note that in the spacelike case of wide-angle Compton scattering off protons the overlap representation in terms of light-cone wave functions, together with a plausible model for the latter, explicitly shows how the soft part of the Compton form factors can mimic counting rule behaviour.

Key results of the approach are the prediction that the differential cross sections for $\pi^+\pi^-$ and $\pi^0\pi^0$ production are the same, and the angular distribution $d\sigma/(d\cos\theta) \propto 1/\sin^4\theta$, which agrees well with data.

The handbag mechanism has recently also been applied to the production of baryon-antibaryon pairs. In Ref. $p\bar{p}$ annihilation into photon pairs is investigated in an approach based on double distributions.
Figure 4. The CLEO data \(^{11}\) for the cross section \(\sigma(\gamma\gamma \to \pi^+\pi^-) + \sigma(\gamma\gamma \to K^+K^-)\) integrated with \(|\cos\theta| < 0.6\). The solid line is the result of the handbag approach with our fitted annihilation form factors (7). The dashed line is the estimate of the leading-twist contribution described in the text.

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