Sizing parameters of interior permanent magnet synchronous motor based on torque-speed characteristics

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Abstract. This paper presents a sizing algorithm for an interior permanent magnet synchronous motor based on its torque-speed characteristic and how these characteristic shifts because of changing motor’s parameters. This knowledge must be helpful in designing motor control systems, where there is no need for a precise model. Also, the proposed method is useful for rapid prototyping based on the existing motor model. The methodology for calculating the torque-speed characteristics is given. The resulting calculated graphs are given. PMSM motor control parameters are designed based on the reference model and verified in MATLAB/Simulink software package. Simulation results are given.

1. Introduction
One of the most important part of designing traction systems with an electric motor is to achieve compliance with electric motor characteristics and conditions of its operation [1]. Nowadays, there are many ways to design an electric motor according to necessary electrical, mechanical, weight and dimensional parameters [2–4]. However, using these methods is a long and laborious process that can take a long period of time. If the initial aim is to develop only drive control algorithms [5, 6] – it is not necessary to use the motor’s precise model. For that case, defining the electric motor’s parameters based on adjusting the properties of existing serial electric motors is the fastest solution to perform. In applications, where the motor needs to be operated in a wide speed range applications and a total weight of a traction unit significantly affects a final system performance, it is most advisable to use a synchronous motor with permanent magnets built into the rotor (also known as IPMSM motors) in comparison with an asynchronous electric motor [7, 8]. Also, the IPMSM motor, in comparison with a synchronous motor with permanent magnets mounted on the surface of the rotor (SPMSM motors), has the highest efficiency, the best power to weight ratio and a wider range of speed control [9, 10]. Therefore, it is widely useful in machinery, as a source for high current consumers [11–13] autonomous research units [14] with current limiting [15, 16] devices, hybrid and electric vehicle motion [17, 18], etc. This paper represents a novel IPMSM motor’s design algorithm based on regulating its parameters in order to fit desired speed and torque values with MATLAB/Simulink model verification.

2. Materials and methods
In order to investigate a proposed method, a mathematical model of the anisotropic permanent magnet synchronous motor must be obtained. The main parameters of the IPMSM motor according to the rotating d-q coordinate system are:

1) $R$ – Stator phase resistance (Ohm)
2) $L_d$ – d-axis inductance (H)
3) $L_q$ – q-axis inductance (H)
4) $\psi_{pm}$ – permanent magnets’ flux (Wb)
5) $p$ – number of motor’s pole pairs (-)
6) $U_{max}$ – maximum voltage (V)
7) $I_{max}$ – maximum current (A)

The equations for IPMSM motor with respect to rotating d-q coordinate system, according to [19–20], can be written as:

$$U_d(t) = L_d \frac{dI_d(t)}{dt} - \omega_m(t) \cdot L_q \cdot I_q(t)$$

$$U_q(t) = L_q \frac{dI_q(t)}{dt} - \omega_m(t) \cdot L_d \cdot I_d(t) + \omega_m(t) \cdot \psi_{pm}$$

$$T_r(t) = \frac{3}{2} p \left( \psi_{pm} \right) \left( L_d - L_q \right) \cdot I_d(t) \cdot I_q(t)$$

$$\omega_m(t) = \frac{U_{max}}{\sqrt{(L_q \cdot I_q(t))^2 + (\psi_{pm} + L_q \cdot I_q(t))^2}}$$

where $U_d(t)$ – d-axis voltage; $U_q(t)$ – q-axis voltage; $R$ – stator resistance; $L_d$ – d-axis self-inductance; $L_q$ – q-axis self-inductance; $\omega_m(t)$ – electrical speed; $\psi_{pm}$ – permanent magnets’ flux linkage or field flux linkage; $I_d(t)$ – d-axis current; $I_q(t)$ – q-axis current; $T_r(t)$ – motor developed torque; $p$ – pole pair numbers;

According to [21], depending on the motor’s speed, the operation mode of the IPMSM can be divided into two parts: operation at speeds from zero to its nominal value (nominal mode) and operation at speeds above the nominal values (weakened field mode). According to [22], depending on speed values, motor’s torque can be found as a tangential intersection of voltage and torque curves relative to the currents in the rotating d-q coordinate reference frame:

$$T_{tan} = \left(-\delta + \left(\delta^2 - 4\zeta^2\sigma\right)^{1/2}\right)^{1/2}$$

$$\zeta = 4096 \omega_m^2 I_d^2 I_q^2 \left(L_d - L_q\right)^2$$

$$\delta = -144 p^2 \omega_m^4 L_d^2 L_q^2 \left(\psi_{pm}^4 \omega_m^4 L_q^4 + 20 \psi_{pm}^3 \omega_m^2 U_{max}^2 L_d^2 L_q^2 + 8U_{max}^4 L_d^2 \right)$$

$$-32U_{max}^4 L_d^2 L_q^2 - 40 \psi_{pm}^3 \omega_m^2 U_{max}^2 L_q^3 + 20 \psi_{pm}^2 \omega_m^2 U_{max}^2 L_d L_q^4$$

$$\sigma = 81 p^4 U_{max}^2 \left(U_{max} L_d - \omega_m \psi_{pm} L_q - U_{max} L_q\right)^4 \left(U_{max} L_d + \omega_m \psi_{pm} L_q - U_{max} L_q\right)^3$$
Motor’s torque and speed nominal values can be found in the point of intersection MTPA curve with the circle of maximum permissible current restriction in the rotating d-q reference frame. Currents for this point can be expressed according to equations (9-10):

\[
I_d = \frac{4\psi_{pm}(\psi_{pm}^2 + 8I_{max}^2(L_d - L_q)^2)^{1/2} - 4\psi_{pm}^2 + 16I_{max}^2(L_d - L_q)^2)^{1/2}}{4(L_d - L_q)}
\]

(9)

\[
I_q = (I_{max}^2 - I_q^2)^{1/2}.
\]

(10)

After substituting current values obtained from equations (9-10) to equations for torque (3) and speed (4) estimation, one can get its nominal values.

Figures 1-4 shows the results of calculation IPMSM torque-speed plots with motor parameters’ changing. The arrow indicates the direction of increasing the corresponding parameter.

**Figure 1.** Inductance increasing along the d axis. **Figure 2.** Increasing the permanent magnets flux linkage.

It can be seen from figure 2 that inductance along the d axis increasing leads to torque-speed characteristic shifting in direction of a maximum shaft torque decreasing with respect to decreasing the nominal speed at constant torque region. Permanent magnets flux linkage increasing leads both to increasing nominal speed of the IPMSM motor as well as nominal power, which leads to an increase in the resulting power of the electric motor. IPMSM’s pole pairs number increasing leads to boosting motor torque in constant torque region. The maximum allowable current increase leads to boosting maximum motor torque in constant torque region with saving the same performance in the field weakening mode.
Dependencies, which were mentioned above, can be applied for motor sizing when it is necessary to model PMSM motor by focusing only on speed/torque characteristics and if any reference PMSM motor’s parameters are available. Therefore, the sizing algorithm consists of the following steps:

1. Defining the motor’s maximum current and voltage value based on power source limitations.
2. Using the method described in figure 1 or figure 2 in order to achieve the desired constant torque region.
3. Using the method described in figure 3 in order to achieve desired torque values in a field weakening region.

3. Results and discussion

As an example, to confirm the proposed algorithm, PMSM motor sizing for electric vehicle will be described below. Figure 5 shows forces applied to the vehicle during speed increasing, where curves with $0^\circ, 6^\circ, 12^\circ, 18^\circ, 24^\circ, 30^\circ$ degrees – total resistive forces applied to vehicle depending on road inclination grade; $F_{\text{acc}}$ – a force which is necessary to achieve desired acceleration; $F_{\text{des}}$ – desired force, which should be produced by the motor; $F_{\text{ref}}$ – reference motor output force. As seen from figure 5, the total traction force from the motor should exceed the $30^\circ$ inclination angle up to $50 \text{ km/h}$ speed and acceleration force with $2.65 \text{ m/s}^2$ from 0 to $100 \text{ km/h}$ speed. According to [19], grade and acceleration forces can be found from equations (1.11) and (1.12):

\begin{equation}
F_{\text{res}} = \frac{1}{2} \rho_s C_d A_f \left( v_v - v_w \right)^2 + C_r m g \cos(\alpha) + mg \sin(\alpha)
\end{equation}

\begin{equation}
F_{\text{acc}} = ma
\end{equation}

where $\rho_s$ – air density; $C_d$ – aerodynamic resistance coefficient; $A_f$ – vehicle frontal area; $v_v$ – vehicle speed; $v_w$ – wind speed; $C_r$ – rolling resistance coefficient; $m$ – vehicle’s mass; $g$ – gravitational constant; $\alpha$ – road inclination angle; $a$ – desired vehicle acceleration.
Nissan Leaf’s parameters were taken as a reference for the vehicle model. Its parameters according to [23] is presented in table 1. Reference PMSM motors parameters described in [24] and designed motor characteristics are presented in table 2.

### Table 1. Nissan Leaf parameters.

| Parameter                              | Symbol | Value       |
|----------------------------------------|--------|-------------|
| Aerodynamic resistance coefficient     | $C_d$  | 0.29        |
| Frontal area                           | $A_f$  | 2.19 $m^2$ |
| Rolling resistance coefficient         | $C_r$  | 0.008       |
| Vehicle mass                           | $m$    | 1521 kg     |
| Acceleration (0-100 km/h)              | $a$    | 2.65 $m/s^2$|
| Gear ratio                             | -      | 8.1         |
| Wheel radius                           | $r$    | 0.26 m      |
| Battery’s maximum voltage              | $U_{\text{max}}$ | 370 V        |
| Battery’s maximum current              | $I_{\text{max}}$ | 180A        |

![Figure 5](image1.png)  
**Figure 5.** Forces applied to vehicle during speed increasing.

![Figure 6](image2.png)  
**Figure 6.** Reference motor inductance decreasing along the d axis.

![Figure 7](image3.png)  
**Figure 7.** Increasing numbers of the reference motor pole pairs.
Figures 6 and 7 represent parameter correction for reference PMSM motor in order to achieve the desired dynamic performance, where $T_{\text{des}}$ – desired torque curve which has to be reached according to figure 5. Torque and speed values are measured after gearbox. According to the algorithm, which was presented above, voltage and current limits were taken from the Nissan leaf’s battery characteristics. In order to increase the constant torque speed region, inductance along the d-axis of PMSM motor decreased (figure 6). Then, pole pairs numbers were increased in order to achieve the desired maximum torque values during the whole operating speed region.

Table 2. Reference and designed PMSM motors’ parameters

| Parameter                        | Symbol | Reference PMSM motor | Designed PMSM motor |
|----------------------------------|--------|----------------------|--------------------|
| Armature resistance for each phase | $R_s$  | 0.0116 Ohm           | 0.0116 Ohm         |
| Inductance along d-axis          | $L_d$  | 0.503 mH             | 0.224 mH           |
| Inductance along q-axis          | $L_q$  | 1.39 mH              | 0.618 mH           |
| PM magnets flux                  | $\psi_{pm}$ | 0.0612 Wb            | 0.0612 Wb          |
| Saliency                         | $\frac{L_q}{L_d}$ | 2.76                 | 2.76               |
| Pole pairs number                | $p$    | 4                    | 8                  |
| Maximum current                  | $U_{\text{max}}$ | 220 V                | 370 V              |
| Maximum voltage                  | $I_{\text{max}}$ | 110 A                | 180 A              |

To verify the designed motor’s performance, imitation model, in order to simulate vehicle dynamics with designed motor parameters, was created in Matlab/Simulink software. More detailed view of the designed model presented in figure 8. The results of the simulation are presented in figures 9 and 10.

According to figure 8, the simulation model consists of the following blocks: «IPMSM» – interior permanent magnet model block with three-phase inverter; «Drive controller» – block which forms control signals for inverter IGBT gates based on torque reference input from «Vehicle Controller»; «Gearbox» – gearbox subsystem module; «Vehicle Dynamics» – block for vehicle resistive driving
force simulation; «Vehicle Controller» – block which generates reference torque value based on acceleration and brake commands from «Inputs» block; «Inputs» – setpoints generation block for acceleration, brake, incline and wind parameters; «Scopes» – simulation results displaying block.

According to figure 9, the output torque of the PMSM motor follows in the desired setpoint with the same shape, which is presented in figure 5. The output characteristic of vehicle speed is presented in figure 8. It shows 100 km/h speed reaching at 9.8 seconds which satisfies the acceleration constraint, presented in table 1.

4. Conclusion
Paper represents four of the most useful ways of adjusting the parameters of the serial IPMSM in order to reach the motor’s desired performance in control systems models. This method helps to avoid the long and laborious process of FEA modelling and stay focused on control system design. As seen from figures 2-5, if it is necessary to improve the total power of the motor, the best way is to increase permanent magnets’ flux linkage value as well as increasing pole pairs number. If the main aim is to shift the constant torque region, the best way is to change inductance along d axis. If it is necessary to increase motor’s starting torque, IPMSM’ maximum current has to be increased. To verify the presented sizing algorithm, PMSM motor control parameters were designed based on the reference model in order to satisfy electric vehicle dynamic demands. Output model data, which was generated by MATLAB/Simulink software package, shows that designed PMSM motor parameters are optimal for the selected purpose.

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