Probing the $R_{K^{(*)}}$ Anomaly at a Muon Collider

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The LHCb measurements of the $\mu/e$ ratio in $B \to K\ell\ell$ decays ($R_K$) indicate a deficit with respect to the Standard Model prediction, supporting earlier hints of lepton universality violation observed in the $R_{K^{(*)}}$ ratio. Possible explanations of these $B$-physics anomalies include heavy $Z'$ bosons or scalar and vector leptoquarks mediating $b \to s\mu^+\mu^-$. We note that a muon collider can directly measure this process via $\mu^+\mu^- \to b\bar{s}$ and can shed light on the lepton non-universality scenario. Investigating currently discussed center-of-mass energies $\sqrt{s} = 3, 6$ and 10 TeV, we show that the parameter space of $Z'$ and leptoquark solutions to the $R_{K^{(*)}}$ anomalies can be mostly covered. Effective operators explaining the anomalies can be probed with the muon collider setup $\sqrt{s} = 6$ TeV and integrated luminosity $L = 4$ ab$^{-1}$.

Introduction.—Rare decays of mesons are sensitive to effects of heavy particles. Precision studies of many such decays have confirmed the CKM matrix as the source of flavor transitions in the Standard Model (SM) [1]. Nevertheless, long-standing hints for physics beyond the CKM paradigm exist. In particular, decay rates of charged and neutral $B$ mesons into kaons plus first and second generation charged leptons are notoriously away from precisely known SM calculations by 3.1$\sigma$ [2–4]. A straightforward solution to these so-called $R_{K^{(*)}}$ puzzles is that there is new physics in the transition $b \to s\mu^+\mu^-$, which can be rewritten as $\mu^+\mu^- \to b\bar{s}$. For energy scales of $B$ decays this physics can be described by effective operators, which may stem from heavy particles mediating the transition. Essentially, there are only two possibilities at tree level. New $Z'$ bosons that couple to $b\bar{s}$ and $\mu^+\mu^-$ or hypothetical leptoquarks that couple to $\mu^-b$ and $\mu^+\bar{s}$.

This paper is about realizing the process $\mu^+\mu^- \to b\bar{s}(b\bar{s})$ at high energy muon colliders. Those are currently under active discussion [5–21] as a possible future collider. While being interesting for Higgs physics, they would in particular be a powerful probe for anything new that likes muons. In particular, the option to test physics solutions for the anomalous magnetic moment of the muon, another long-standing problem involving muons [22–24], has been investigated. It has been shown that any new physics that may be responsible for explaining the $(g-2)_\mu$ results can be tested at future muon colliders [17–21]. Here we discuss the $B$ physics anomalies in the $R_K$ and $R_{K^{(*)}}$ ratios in terms of a $Z'$ and scalar as well as vector leptoquarks. The former mediates the process $\mu^+\mu^- \to b\bar{s}$ in an $s$-channel diagram, the latter in a $t$-channel diagram. Using the currently discussed setups of 3, 6 and 10 TeV center-of-mass energies [5, 6], we show that both scenarios can be mostly covered. Our analysis takes dijet background from SM processes into account, and is independent of whether flavor tagging is included or not. Before turning to the analysis at the muon collider, we will shortly summarize the current situation of the anomalies and their main solutions.

Theoretical interpretations of the $R_{K^{(*)}}$ anomaly.—The ratios $R_K$ and $R_{K^{(*)}}$, relevant for testing the universality of the gauge-interactions in the lepton-sector, are defined as

$$R_K = \frac{\text{BR}(B^+ \to K^+\mu^+\mu^-)}{\text{BR}(B^+ \to K^+e^+e^-)},$$

$$R_{K^{(*)}} = \frac{\text{BR}(B^0 \to K^{(*)}\mu^+\mu^-)}{\text{BR}(B^0 \to K^{(*)}e^+e^-)}.$$  

Due to highly suppressed hadronic uncertainties, such ratios are supposed to be theoretically clean and could thus be a "clean"signal of BSM-physics. Very recently, the LHCb collaboration
reported the results of $R_K$-measurement (in the region $q^2 \in [1.1, 6]$ GeV$^2$) as \[2\]
\[R_{K}^{\text{LHCb}} = 0.846^{+0.042+0.013}_{-0.039-0.012}, \]
which indicates a 3.1$\sigma$ discrepancy from its SM prediction \[25, 26\]
\[R_{K}^{\text{SM}} = 1.0003 \pm 0.0001. \]

Similarly, the LHCb Collaboration has also reported the results of $R_{K^*}$-measurement in two low-$q^2$ bins \[3\] ($q^2 \in [0.045, 1.1]$ GeV$^2$ and $q^2 \in [1.1, 6]$ GeV$^2$):
\[R_{K^*}^{\text{LHCb}} = \begin{cases} 0.660^{+0.110+0.070}_{-0.070-0.024}, \\ 0.685^{+0.113+0.069}_{-0.069-0.047}, \end{cases} \]
which shows 2.2$\sigma$ and 2.4$\sigma$ deviations, respectively from their corresponding SM-predictions in each $q^2$ bin \[27, 28\]:
\[R_{K^*}^{\text{SM}} = \begin{cases} 0.92 \pm 0.02, \\ 1.00 \pm 0.01. \end{cases} \]

Furthermore, Belle has also presented their results on $R_K$ \[29\] and $R_{K^*}$ \[30\]. However, there are comparatively larger uncertainties than for the LHCb measurements. There are in fact only a few BSM possibilities which could resolve these $R_{K^{(*)}}$-anomalies. Before entering details, it is quite important to mention that an explanation of $R_{K^{(*)}}$ by modifying the $b \to s\mu^+\mu^-$ decay anticipates a better global-fit to other observables, as compared to altering the $b \to se^+e^-$ decay \[31\].

The effective Lagrangian responsible for semi-leptonic $b \to s\mu^+\mu^-$-transitions can be expressed as ($V$ denotes the CKM-matrix)
\[\mathcal{L}_{bs\mu\mu}^{\text{NP}} \supset 4G_F \sqrt{2} V_{tb} V_{ts}^* \left(C_{9}^{\mu} O_{9}^{\mu} + C_{10}^{\mu} O_{10}^{\mu} \right) + \text{h.c.} \tag{7}\]
with the relevant operators
\[O_{9}^{\mu} = \frac{\alpha}{4\pi} \left( \bar{s}_L \gamma_{\mu} b_L \right) \left( \bar{\mu}_L \gamma_{\mu} \mu \right), \]
\[O_{10}^{\mu} = \frac{\alpha}{4\pi} \left( \bar{s}_L \gamma_{\mu} b_L \right) \left( \bar{\mu}_L \gamma_{\mu} \gamma_5 \mu \right). \tag{8}\]

Using these operators to explain the anomalies leads to best-fit values of the Wilson-coefficients $C_9 = -C_{10} = -0.43$, with the 1$\sigma$ range being $[-0.50, -0.36]$ \[31, 32\].

**Models with $Z'$**—Let us now discuss an explicit new-physics realization for explaining the $B$-anomalies in neutral-currents. As a prototypical-model (a partial list of references is \[33–55\]), we consider a $Z'$ which dominantly couples to $bs$ and $\mu^+\mu^-$, via left-handed currents\(^1\). One can achieve this by extending the SM with an extra $U(1)$ gauge group, which brings in a new $Z'$ boson having a non-universal lepton-coupling and a flavor-changing quark-coupling. Here, we concentrate solely on the Lagrangian-part relevant for $b \to s\mu^+\mu^-$-transitions, namely
\[\mathcal{L}_{Z'} \supset \left( \lambda_{ij}^{Q} d_i^L \gamma_{\mu} d_j^L + \lambda_{i\alpha}^{L} \bar{\phi}_L \gamma_{\mu}^{\alpha} e^\beta_L \right) Z_{\mu}, \tag{9}\]
where $\ell^\alpha$ and $d^\alpha$ denote the different generations of charged-lepton and down-type quark states, respectively.

Integrating out the $Z'$ field, one can obtain the effective-Lagrangian as:
\[\mathcal{L}_{Z'}^{\text{eff}} = -\frac{1}{2M_{Z'}^2} \left( \lambda_{ij}^{Q} \bar{d}_i^L \gamma_{\mu} d_j^L + \lambda_{i\alpha}^{L} \bar{\phi}_L \gamma_{\mu}^{\alpha} e^\beta_L \right)^2 + \frac{\lambda_{Q}^{23}}{2M_{Z'}^2} \left[ \left( \lambda_{Q}^{23} \right)^2 \left( \bar{s}_L \gamma_{\mu} b_L \right)^2 
+ 2\lambda_{Q}^{23} \lambda_{22}^{L} \left( \bar{s}_L \gamma_{\mu} b_L \right) \left( \bar{\mu}_L \gamma_{\mu} \gamma_5 \mu \right) + \text{h.c.} \right]. \tag{10}\]

Now one can find the relevant Wilson-coefficients at tree-level [cf. left-panel of Fig.\ 1] by matching

\(^1\) Right-handed currents in the lepton-sector actually worsen the compatibility of $R_{K^{(*)}}$ explanation with the $\Delta M_s$ (mass-differences of neutral $B$-mesons) measurement \[56\], since they demand a larger Wilson-coefficient.
onto the effective-Lagrangians for the low-energy observables at the scale \((\mu = M_{Z'})\) as

\[
C^\mu_9 = -C^\mu_{10} = -\frac{\pi}{\sqrt{2}G_F M_{Z'}^2} \left( \frac{\alpha_{23} \alpha_{22}}{V_{tb} V_{ts}^*} \right). \tag{11}
\]

However, as shown e.g. in Refs. [56, 57], this \(Z'\)-explanation of \(R_{K^{(*)}}\) anomaly is under tight constraints from several theoretical and experimental limits allowing a narrow mass-range for \(Z'\) boson [cf. the yellow band in Fig. 2]. There are several dedicated \(Z'\)-searches at the LHC looking at dimuon or dijet [58, 59] signatures. The reliance on parton distribution functions of bottom-quarks for production in our scenario dilutes the impact of current LHC-searches. On the other hand, a very stringent bound on our \(Z'\) originates from its flavor-changing coupling, which generates an additional contribution to \(B_s - B_s\) mixing [56, 57]. Note that other constraints, such as \(\text{BR}(B \to K \ell \nu)\) [60] or muon \(g - 2\) [61, 62], are much weaker. In addition, there will be constraints from the measurement of neutrino-trident production [63]. All these constraints are summarized in Fig. 2.

**Models with leptoquarks.**—In order to address the \(R_{K^{(*)}}\)-anomaly, there is another popular class of models (a partial list of references is [64–86]) in which leptoquarks are applied. Here we briefly review these simplified models that can accommodate the \(R_{K^{(*)}}\)-anomaly. There are only four scalar leptoquarks which can interact with the SM-fermions at renormalizable level. Interestingly, \(S_3 \sim (3, 3, -1/3)\) can simultaneously address \(R_K\) and \(R_{K^*}\) and whose constraints are not in conflict with the experimental data [87, 88]. Similarly, the vector leptoquark \(U_1 \sim (3, 1, 2/3)\) can also provide a good fit for the \(R_{K^{(*)}}\)-anomaly. Note that it requires a proper UV-completion for theoretical consistency. Here we focus mainly on the scalar case, delegating details of the vector leptoquark case to the supplemental material.

The relevant Lagrangian for \(S_3\) can be written as:

\[
\mathcal{L}_{S_3} = -M_{S_3}^2 |S_3|^2 + y_{i\alpha}^{LQ} Q^{i\alpha} (\epsilon \sigma^a) L^a S_3^a + h.c., \tag{12}
\]

with lepton and quark-doublets \(L^a = (\nu^a_L, \ell^a_L)\)^T and \(Q^i = (V^*_j u^j_L, d^i_L)^T\), and Pauli-matrices \(\sigma^a\) \((a = 1, 2, 3; \epsilon = i\sigma^2)\). The leptoquark contributes to the Wilson-coefficients at tree-level [cf. Fig. 1] and one can identify:

\[
C^\mu_9 = -C^\mu_{10} = \frac{\pi}{\sqrt{2}G_F M_{S_3}^2} \left( \frac{y_{22} \bar{y}_{22}^{LQ} \bar{y}_{22}^{LQ*}}{V_{tb} V_{ts}^*} \right). \tag{13}
\]

This explanation of the \(R_{K^{(*)}}\) anomaly also faces several theoretical and experimental constraints. The same combination of Yukawa-couplings leads to \(B_s - B_s\) mixing at one-loop level [57, 89, 90]. This sets an upper bound on the Yukawa-couplings as a function of the leptoquark mass as shown in Fig. 3. Due to the loop-nature of this constraint, it is much weaker compared to the \(Z'\) scenario. There are several relevant direct LHC searches. Pair-production via gluon-gluon fusion processes dominates and the subsequent decay into \(\mu\mu\) can be looked for. A stringent limit from a dedicated LHC search using \(\mu\mujj\) signals exists [91]. Recently, Ref. [92] has worked out in detail the prospect of probing the \(S_3\) leptoquark at current and future runs of the LHC. Based on that analysis masses up to 1.8 TeV are excluded at 95% confidence level from 13 TeV LHC data with an integrated luminosity of \(L = 140 \text{ fb}^{-1}\), whereas HL-LHC (with 3 ab\(^{-1}\) integrated luminosity) can probe up to 2.5 TeV. The minimal constraints without assuming additional flavor structures from indirect high-\(p_T\) searches of \(qq \to \mu^+\mu^-\) are less competitive [93–95]. All these constraints are summarized in Fig. 3.

**Implications of \(R_{K^{(*)}}\) anomaly at a muon collider.**—The transition of \(b \to s\mu^+\mu^-\) in meson decays is directly applicable in a muon collider via \(\mu^+\mu^- \to bs\). This simple two-body scattering allows to directly test any explanation for the anomalous \(R_{K^{(*)}}\) ratios, and we utilize it to study the sensitivity on the representative explanations of the anomalies, i.e. a \(Z'\) and scalar as well as vector leptoquarks.

The Feynman diagrams of the relevant processes are depicted in Fig. 1. For the \(Z'\) model, we have an \(s\)-channel process, and a resonance enhancement is available when the center-of-mass energy \(\sqrt{s}\) is near the \(Z'\) mass \(M_{Z'}\). In
contrast, the $S_3$ leptoquark mediates a $t$-channel process.

Besides the explicit realization of the cross section, we can describe the situation in an effective language. When the $Z'$ or leptoquark mass is larger than the center-of-mass energy, the operators with coefficients $C_{9 \mu}^\mu$ and $C_{10}^{\mu \nu}$ are responsible for the transition. The cross section of $\mu^+ \mu^- \to b \bar{s}$ is then

$$
\sigma(s) = \frac{G_F^2 |V_{tb} V_{ts}^\ast|^2 s}{8\pi^3} \left( |C_{9 \mu}^\mu|^2 + |C_{10}^{\mu \nu}|^2 \right) \cdot (14)
$$

Taking the best-fit scenario of $B$ anomalies, $C_{9 \mu}^\mu = -C_{10}^{\mu \nu} = -0.43$, we obtain the event number of $bs$ final states $\sigma(s) \cdot L$ ($L$ being the luminosity) as

$$
\#\text{signal} \simeq 10^3 \left( \frac{\sqrt{s}}{6 \text{ TeV}} \right)^2 \left( \frac{L}{4 \text{ ab}^{-1}} \right).
$$

As a naive comparison, we obtain the relevant SM background in the form of quark dijets (ignoring flavor tagging, see below), which turns out to be $1.2 \times 10^3 \cdot (6 \text{ TeV}/\sqrt{s})^2 \cdot (L/4 \text{ ab}^{-1})$. The signal exceeds the fluctuation of SM background at around $3\sigma$ level, which is very encouraging. The signal-to-background ratio is roughly proportional to $s^2$; therefore to enhance the sensitivity to the effective operators, larger $\sqrt{s}$ is preferred. With $\sqrt{s} = 10 \text{ TeV}$ and $L = 10 \text{ ab}^{-1}$, values of $|C_{9 \mu}^\mu| = |C_{10}^{\mu \nu}|$ as small as 0.16 can be reached at $3\sigma$ level, which covers the $2\sigma$ range of $|C_{9 \mu}^\mu| = |C_{10}^{\mu \nu}| \in [0.29, 0.57]$ even without the flavor tagging. For comparison, the current LHC (projected HL-LHC) limit on the coefficients of effective operators reads $|C_{9 \mu}^\mu| = |C_{10}^{\mu \nu}| < 100$ (39) [93]. These hadron collider bound on the effective operators is set by searching for the high-$p_T$ tails of the dimuon spectrum, which is not as efficient as a muon collider. In the supplemental material we discuss more details on the muon-collider sensitivity on effective operators. Before discussing the explicit realizations of the process, we consider general background issues.

The dijet signal of the $bs$ final state is contaminated by $\mu^+ \mu^- \to jj$, where $j$ can be $u, d, s, c$ and $b$, due to imperfect flavor reconstruction. The sensitivity depends on the $b$-jet tagging efficiency as well as the mistag rate (identifying a light quark jet as a $b$-jet). In this work, we assume an experimental configuration with a $b$-jet tagging efficiency $\epsilon_t = 70\%$ [8] and mistag rates $\epsilon_{uds} = 1\%$ for light quarks and $\epsilon_c = 10\%$ for $c$ quarks [98–100]. We require in our analysis that one jet is tagged as a $b$ jet, while the other is not. We continue with some comments on the backgrounds:

- $\mu^+ \mu^- \to u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}$: With the tagging requirement, the total cross section for these
processes will be reduced by a factor of \(2\epsilon_{uds,c} \cdot (1 - \epsilon_{uds,c})\), where the factor \(2\) originates from two choices of tagging.

- \(\mu^+\mu^- \rightarrow bb\): To pass our event criteria, one \(b\)-jet is required not to be \(b\)-tagged, and the cross section is reduced by a factor \(2\epsilon_b \cdot (1 - \epsilon_b)\). Note that one could likely further optimize the selection criteria until a higher signal-to-noise ratio is obtained.

In addition, there could be background contributions from top quarks. However, their identification relies crucially on the tagging of a \(b\) quark in their decay \(t \rightarrow Wb\). Above TeV energies, the top-antitop final states are highly boosted, such that multiple final jets may overlap [101]. How-
In this way, the inclusion of \( \mu^+ \mu^- \to b \bar{s} \) helps to clarify that the new physics is indeed what causes the \( B \) anomalies. The \( B_s \) mixing data prefers larger \( \lambda_{22} \) values. If we take \( \lambda_{22} = 1 \), a window between the projection for the HL-LHC and the muon collider setup with \( \sqrt{s} = 3 \) TeV may survive. But this window is expected to be covered by means of radiative return, i.e., \( \mu^+ \mu^- \to bs\gamma \). For the extreme case \( \lambda_{22} = \sqrt{4\pi} \) where more parameter space is valid to explain the \( R_{K^*} \) anomaly, the muon collider with \( \sqrt{s} = 6 \) TeV will rule out most of the favored parameter space. Combining the HL-LHC and the muon collider sensitivities we observe that there is still a corner of the parameter space left.

- For the case of leptoquarks, most of the parameter space will be probed with the muon collider \( \sqrt{s} = 6 \) TeV and \( L = 4 \) ab\(^{-1}\). Only a tiny window around 3 TeV for scalar-leptoquarks may survive, which can be of course covered by a larger integrated luminosity (e.g., with \( L = 16 \) ab\(^{-1}\) for \( \sqrt{s} = 6 \) TeV no space will be left). The parameter space of vector leptoquark and coefficients of effective operators can be fully covered with \( \sqrt{s} = 6 \) TeV and \( L = 4 \) ab\(^{-1}\).

**Conclusion.**—Processes with muons are a reliable source of anomalies which could lead to the discovery of long-awaited new physics beyond the Standard Model. A muon collider is then an ideal machine to probe these effects further. Here we have focused on the highly interesting \( R_K \) and \( R_{K^*} \) ratios, which are object to intense studies in terms of heavy \( Z' \) bosons and leptoquarks. We have demonstrated that the parameter space of such models can be mostly covered at currently discussed muon collider setups, which adds exciting physics potential to these facilities.

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**Supplemental Material**

**Signal-to-background Ratio**

In Fig. 4, an illustration of the signal-to-background ratio before (solid) and after (dashed) flavor tagging is given for the leptoquark model with parameter sets explaining the \( B \) anomalies...
\( y_{23}^{LQ} \cdot y_{22}^{LQ} = 0.02 \) and \( M_{S_3} = 5 \) TeV (red curves), as well as \( y_{23}^{LQ} \cdot y_{22}^{LQ} = 0.7 \) and \( M_{S_3} = 30 \) TeV (blue curves). One can observe that with our flavor tagging assumptions, the signal-to-background ratio can be enhanced by one order of magnitude. Considerable variations of the signal-to-background ratio over the polar angle can be noticed for the case with \( M_{S_3} = 5 \) TeV, which helps to preserve the statistical significance against a possibly large systematic uncertainty. However, for the case with \( M_{S_3} = 30 \) TeV, the leptoquark is basically decoupled, and the signal-to-background ratio is nearly a constant if we do not distinguish quark and antiquark. A possible tagging of the \( b \) quark charge [106] will distort the flat signal-to-background ratio.

**Vector Leptoquark**

The Lagrangian describing the \( U_1 \) vector-leptoquark reads

\[
L_{U_1} = -M_{U_1}^2 |U_1|^2 + y_{i\alpha}^{LQ} Q_i \gamma_\mu L^\alpha U_{1\mu} + \text{h.c.} \tag{16}
\]

The corresponding contribution to the Wilson-coefficients at tree-level is similar to the \( S_3 \) leptoquark, namely

\[
C_{y_i}^\mu = -C_{10}^\mu = \frac{\pi}{\sqrt{2} G_F M_{U_1}^2} \left( \frac{y_{i2}^{LQ} y_{i2}^{LQ*}}{V_{tb} V_{ts}^*} \right). \tag{17}
\]

When the leptoquark mass is much larger than the colliding energy, the effects induced by \( S_3 \) and \( U_1 \) leptoquarks at muon colliders will be indistinguishable. When the leptoquark mass is negligible compared to the colliding energy, the \( t \)-channel exchange of \( U_1 \) leptoquark will enhance the cross section significantly by a factor of \( 1/(Q^2 + M_{U_1}^2)^2 \). However, for the \( S_3 \) case, the scalar coupling,
which reverses the chirality, does not feature a $t$-channel enhancement. This can be easily seen: the vertex for the scalar coupling contributes a factor $\text{Tr}(p\tilde{k}) = 4p \cdot k \propto Q^2$ with $p$ and $k$ being the four momentum of initial and final fermions coupled to the leptoquark, and the $t$-channel enhancement when $Q^2 \to 0$ is therefore canceled. As a consequence, in Fig. 3 of the main manuscript, we have better sensitivities at small masses for the vector leptoquark.

**Sensitivity to Effective Operators**

In Fig. 5, we show the 3σ sensitivity of muon colliders to $|C_9^\mu|^2 + |C_{10}^\mu|^2$ as a function of the colliding energy $\sqrt{s}$. The yellow band corresponds to the 2σ range favored by the global analysis, namely $C_9^\mu = -C_{10}^\mu \in [0.29, 0.57]$. The blue region (dashed blue curve) shows the excluded values of $|C_9^\mu|^2 + |C_{10}^\mu|^2$ for a given colliding energy $\sqrt{s}$ assuming only the Standard Model background is observed without (with) flavor tagging. With the setup $\sqrt{s} = 6$ TeV and $L = 4 \text{ ab}^{-1}$, the best-fit point $C_9^\mu = -C_{10}^\mu = -0.43$ can be reached without the flavor tagging. We note that with the colliding energy $\sqrt{s} \gtrsim 6$ TeV and the flavor tagging the entire 2σ range of parameter space favored by the $B$ anomalies can be covered.

![Figure 5](image_url)

Figure 5. The sensitivity of muon colliders to the square sum of effective operator coefficients $|C_9^\mu|^2 + |C_{10}^\mu|^2$ as a function of the colliding energy $\sqrt{s}$. The luminosity has been assumed to satisfy the benchmark value $L = 4 \text{ ab}^{-1} \cdot [\sqrt{s}/(6 \text{ TeV})]^2$. The blue region (the dashed blue curve) is the 3σ exclusion parameter space assuming no excess beyond the Standard Model background has been observed without (with) the flavor tagging, while the yellow band indicates the 2σ range favored by the global fit of $B$ anomalies assuming $C_9^\mu = -C_{10}^\mu$ [31, 32]. The LHC limit and the HL-LHC projection by looking for high-energy dimuon tails, assuming only the $bs\mu\mu$ couplings, are given as black shaded region and dotted line, respectively [93].

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