SIX NEW GALACTIC ORBITS OF GLOBULAR CLUSTERS IN A MILKY WAY–LIKE GALAXY

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ABSTRACT

Absolute proper motions for six new globular clusters have recently been determined. This motivated us to obtain the Galactic orbits of these six clusters both in an axisymmetric Galactic potential and in a barred potential, such as that of our Galaxy. Orbits are also obtained for a Galactic potential that includes spiral arms. The orbital characteristics are compared and discussed for these three cases. Tidal radii and destruction rates are also computed and discussed.

Subject headings: Galaxy: halo — Galaxy: kinematics and dynamics — globular clusters: general

1. INTRODUCTION

Absolute proper motions have become available for six new globular clusters (Casetti-Dinescu et al. 2007), thus increasing to 54 the number of globular clusters in our Galaxy for which full space velocities and Galactic orbits can now be calculated. In a previous paper (Allen et al. 2006, hereafter Paper I), Galactic orbits were computed for 48 globular clusters, using both an axisymmetric and a barred Milky Way–like potential. We found that the effect of the bar was greatest for clusters with orbits residing mostly within the bar, and was negligible for the outermost clusters in our sample.

Since then, Casetti-Dinescu et al. (2007) have obtained absolute proper motions for six additional clusters, and computed their Galactic orbits in an axisymmetric potential. Since most of their cluster orbits appear to reside well within the region of influence of the Galactic bar, it seems worthwhile to compute the orbits in a barred potential. Also, by computing orbits using different axisymmetric Galactic models, some insight might be gained into the sensitivity of the results to the potential model adopted.

In particular, Casetti-Dinescu et al. (2007) comment on the apparent “pairing” of the orbital parameters of NGC 2808 and NGC 4372, and NGC 4833 and NGC 5986. Since these pairings, or kinematic groups, may have important implications for the dynamical and merger history of our Galaxy (Kepley et al. 2007; Allen et al. 2007, and references therein), it is interesting to assess whether or not these similarities are sensitive to the Galactic model potential used.

In the present paper, we compute Galactic orbits for the six new clusters in the axisymmetric potential of Allen & Santillán (1991), in the barred model of Pichardo et al. (2004), and in a potential model that includes spiral arms (Pichardo et al. 2003). As before, we find that the orbits in the barred potential generally do not show secular changes in the total energy, $E$, or in the $z$-component of the angular momentum, $\mathbf{\alpha}$, which are both computed in the inertial Galactic frame.

Since the permanence of barlike structures in galaxies is a matter of debate, we cannot be sure that the bar of the Milky Way has existed throughout the Galactic lifetime. Therefore, we also ask ourselves how the orbits would look if the bar had existed for only about a third of this lifetime.

To investigate the possible effects of spiral structure on the orbital characteristics of the clusters, we also include computations of the orbits of the six globular clusters in a Galactic potential that incorporates spiral arms. Among the six clusters selected for this computation, there is at least one cluster that clearly belongs to the thick disk, for which the effect of spiral perturbations is expected to be large (NGC 5927).

This paper is organized as follows. In § 2, the Galactic potentials used to compute the cluster orbits are briefly described, and the initial conditions are presented. In § 3, the orbits obtained with both the axisymmetric and the barred potential are presented and compared with the ones obtained by Casetti-Dinescu et al. (2007). In particular, we show that the effect of the bar is only negligible for one cluster NGC 3201, the most energetic one. We also discuss the effects on the orbits of a bar that has existed for only a fraction of the Galactic lifetime. In § 4, the effect of spiral structure is presented for all six orbits. In § 5, tidal radii are computed for orbits in the barred potential and compared with the axisymmetric case. Destruction rates for these clusters are also computed and discussed. The final section (§ 6) presents a brief discussion and our conclusions.

2. THE GALACTIC POTENTIALS AND THE INITIAL CONDITIONS

For our study, we will employ the axisymmetric Galactic potential of Allen & Santillán (1991), the barred Galactic potential of Pichardo et al. (2004), and the bar+spiral arms model of Pichardo et al. (2003).

The bar model (for details see Pichardo et al. 2004) includes a bar of 3.13 kpc scale length, with axial ratios of 1.7:0.64:0.44 and a conservatively estimated mass of $\sim 10^{10} M_\odot$, which replaces 70% of the spherical bulge mass. The bar moves with an angular velocity of $60 \text{ km s}^{-1} \text{kpc}^{-1}$, and closely approximates model S of Freudenreich (1998). Again, we use the superposition model of Pichardo et al. (2004).

To model the spiral perturbation, we proceed as Pichardo et al. (2003), who refined their model until self-consistent orbital solutions were found. The spiral arms are constructed using a three-dimensional mass distribution obtained by superposing oblate spheroidal potentials as building blocks of the global spiral potential. For an extensive description of the model, see Pichardo et al. (2003). The adopted parameters are given in § 4.

To calculate the initial conditions, we take the absolute proper motions provided by Casetti-Dinescu et al. (2007). Other relevant data are taken from the compilation by Harris (1996). Once the space velocities are obtained, we integrate the orbits backward in time using a fourth-order Runge-Kutta method.
time for $1.5 \times 10^{10}$ yr. As in Paper I, we analyze the time-reversed orbits. For the integration, we use the Bulirsch-Stoer algorithm of Press et al. (1992). In the axisymmetric case, the relative errors in the total energy were of about $10^{-14}$ at the end of the integration. In the barred and spiral potentials, the orbits are computed in the noninertial reference frame, in which the bar is at rest. In the barred potential, the precision of the calculations can be checked using Jacobi’s constant. The relative errors in this quantity turn out to be, typically, $10^{-10}$ to $10^{-11}$.

3. THE GALACTIC ORBITS

Figure 1 shows the meridional orbits of the six clusters in both the axisymmetric and the barred potentials. Tables 1 and 2 summarize our results for both cases. In Table 1, which summarizes our results for the axisymmetric potential, successive columns contain the name of the cluster, the minimum perigalactic distance reached in the course of the complete orbit, the average perigalactic distance, the maximum apogalactic distance, the average apogalactic distance, the maximum distance from the Galactic plane reached throughout the entire orbit, the average orbital eccentricity, the orbital energy per unit mass, the $z$-component of angular momentum per unit mass, two values for the computed tidal radii (see discussion in §5), and the observed limiting radius, given in Harris (1996).

In Tables 1 and 2, two additional rows are included for each cluster. These correspond to “extreme” orbits, which take into account observational uncertainties. The meaning of these extreme orbits is discussed later in this section. Table 2, which summarizes our results for the barred potential, is similar, but since neither the orbital energy, $E$, nor the $z$-component of angular momentum, $h$ (both computed in the inertial Galactic frame), are constants of motion, we give in columns (9) and (10) the minimum and maximum values attained by $h$ in the course of the complete orbit.

Only the orbit of NGC 3201, the least bound of the clusters, is not noticeably affected by the bar. The orbit remains entirely outside the bar region, and closely resembles that of NGC 4590, presented in Paper I. As can be seen in Figure 1, the orbits of the five remaining clusters are significantly affected by the bar. Their orbital parameters show fairly large changes compared to the axisymmetric case, tending to reach higher values of apogalactic distance and distance to the Galactic plane, as well as becoming noticeably more irregular. They resemble the class of “inner” clusters studied in Paper I.

The orbits of NGC 4833 and NGC 5986 are irregular both with and without the bar. The orbit of NGC 2808 is near resonant in the axisymmetric case, but seems more irregular in the barred potential. The bar also causes the orbit to reach radii and $z$-distances larger than in the axisymmetric case. This orbit resembles that of NGC 6382, presented in our earlier study. NGC 5927, a thick-disk cluster, has a tightly confined box-type orbit in the axisymmetric case. The bar causes the orbit to extend to larger radial distances, reaching almost 7 kpc. Finally, the orbit of NGC 4372, clearly near resonant in the axisymmetric case, becomes irregular with the bar, and reaches larger $z$-distances.

Plots of the run of energy, $E$, and the $z$-component of angular momentum, $h$, were obtained for all clusters. They are shown in Figures 2 and 3. These quantities are, of course, conserved in the axisymmetric case. In the presence of the bar, the orbits generally do not show large secular changes in $E$ or $h$. Indeed, these quantities are conserved, on average, within better than 10%. But
periodic or quasi-periodic changes are seen to occur in all cases, reflecting the interactions with the bar. In Figures 2 and 3, we can see that NGC 3201 and NGC 5927 show small periodic or quasi-periodic changes. The occasional sudden changes we found in our previous study were found again for four of the six orbits examined here, namely those of NGC 2808, NGC 4372, NGC 4833, and NGC 5986. For this last cluster, the one with the smallest angular momentum, the abrupt changes in the angular momentum occasionally reverse the sense of revolution of the cluster around the Galactic center.

Figures 4 and 5 show plots of \( r_{\text{min}} \) and \( r_{\text{max}} \), the minimum and maximum galactocentric distances attained by the six clusters during their time-reversed orbits in the barred potential. The figures show that these distances oscillate, particularly \( r_{\text{max}} \). Clusters showing abrupt energy changes, such as NGC 2808, NGC 4372, NGC 4833, and NGC 5986, also show changes in the oscillation of \( r_{\text{max}} \) and \( r_{\text{min}} \), and these occur at about the same times as the abrupt energy changes. Figure 6 shows a blowup of these oscillations for a short time at the beginning of the orbit computation of NGC 5927. The figure shows that the cluster attains the smaller \( r_{\text{max}} \) twice as often as the larger \( r_{\text{max}} \), and the smallest \( r_{\text{min}} \) twice as often as the larger \( r_{\text{min}} \). This regular behavior is established right from the beginning of the orbit computation.

To obtain a very rough idea of the changes in the orbits that would appear if the Galactic bar were not a permanent feature, we computed the orbits backward in time, but only for the last 5.0 Gyr, i.e., approximately a third of the Galactic lifetime. Figure 7 shows the orbits we obtained. A comparison of Figure 1 and Figure 7 clearly shows that the effects of the bar are quite

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| Cluster       | \( (r_{\text{min}})_{\text{min}} \) (kpc) | \( (r_{\text{min}})_{\text{max}} \) (kpc) | \( (r_{\text{max}})_{\text{min}} \) (kpc) | \( (r_{\text{max}})_{\text{max}} \) (kpc) | \( \langle z_{\text{max}} \rangle_{\text{max}} \) (kpc) | \( \langle z_{\text{max}} \rangle_{\text{max}} \) (kpc) | \( \langle z_{\text{max}} \rangle \) (kpc) | \( \langle z_{\text{max}} \rangle \) (kpc) | \( \langle E \rangle \) (10 km s\(^{-1}\))^2 | \( h \) (10 km s\(^{-1}\) kpc) | \( r_{\kappa} \) (pc) | \( r_c \) (pc) |
|--------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|-------------------------------|------------------------------|-----------------|---------------|
| NGC 2808...... | 2.65                            | 2.76                            | 12.86                           | 12.72                           | 4.42                            | 2.60                            | 0.643                           | -1192.93                        | 87.32                          | 56.3                        | 54.3           |
| NGC 3201...... | 4.28                            | 4.47                            | 6.64                            | 5.98                            | 0.90                            | 0.79                            | 0.144                           | 103.19                          | 112.55                        | 72.0                        | 73.4           |
| NGC 4833...... | 0.19                            | 0.68                            | 8.92                            | 8.08                            | 6.02                            | 1.08                            | 0.847                           | -1134.84                        | 262.22                         | 72.7                        | 73.0           |
| NGC 9836...... | 0.01                            | 0.76                            | 8.13                            | 6.71                            | 3.61                            | 3.32                            | 0.808                           | -30.57                          | 20.2                          | 17.5            | 33.8          |
| NGC 5927...... | 4.70                            | 4.72                            | 5.76                            | 5.74                            | 0.84                            | 0.80                            | 0.098                           | -1422.14                        | 107.14                         | 52.1                        | 61.5           |
| NGC 4372...... | 3.85                            | 3.85                            | 5.46                            | 5.46                            | 0.67                            | 0.62                            | 0.172                           | -1479.97                        | 93.17                          | 45.5                        | 52.9           |
| NGC 4833...... | 0.68                            | 0.82                            | 5.67                            | 4.44                            | 3.97                            | 1.98                            | 0.694                           | -1622.48                        | 4.04                           | 15.1                        | 14.5           |
| NGC 9836...... | 0.00                            | 0.67                            | 4.98                            | 3.83                            | 3.61                            | 1.92                            | 0.714                           | -1681.18                        | 0.00                           | 16.5                        | 16.1           |
| NGC 5927...... | 0.99                            | 1.07                            | 5.93                            | 5.79                            | 2.88                            | 1.74                            | 0.688                           | -1545.02                        | 17.96                          | 21.0                        | 20.8           |

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**TABLE 1**
Orbits with the Axisymmetric Potential

**TABLE 2**
Orbits with the Barred Potential
similar in both cases. The sole exception is NGC 2808, whose orbit, when integrated backward for only 5 Gyr, remains close to a resonance. We conclude that even if the Galactic bar has existed only for the last third of the Galactic lifetime, its effect on the orbits of the inner clusters is quite as noticeable as that of a bar present throughout the history of the Galaxy.

Of course, the bar is not expected to have suddenly come into existence 5 Gyr ago. A more realistic simulation would have to take into account the gradual formation of the bar (and perhaps even its gradual disappearance and reformation). However, this kind of simulation is well beyond the scope of the present work. The issue of the origin and persistence of bars in galaxies is currently a subject of lively debate.

To assess the effects of observational uncertainties on the orbital parameters, we computed, as in Paper 1, two additional orbits for each cluster. The initial conditions for these orbits were chosen so as to maximize and minimize the orbital energy. In other words, we combined the observational uncertainties in such a way as to obtain two extreme orbits. Errors in the orbital parameters resulting from observational uncertainties are expected to be bounded by these extreme orbits, the real errors being expected to be much smaller. Indeed, note that the errors in the Galactic parameters estimated by Casetti-Dinescu et al. (2007) for their orbits are much smaller than the ones that could be inferred from our extreme orbits.

A comparison of the values in Table 1 with the Galactic parameters obtained by Casetti-Dinescu et al. (2007) shows
satisfactory overall agreement. In general, the uncertainties resulting from observational errors are larger than the ones resulting from using different axisymmetric Galactic potential models. However, our results on the orbits of these authors’ cluster “pairings” (NGC 5986-NGC 4833 and NGC 2808-NGC 4372) are not sufficiently similar to make a convincing case for them. The orbital parameters in the barred potential differ even more. Therefore, we have to regard the pairings as not supported by the orbital parameters we obtain here.

4. THE EFFECTS OF SPIRAL-ARM PERTURBATIONS

To study the possible effects of spiral structure on the orbital characteristics of the clusters, we computed some of the orbits using a Galactic potential that includes spiral arms. A priori, one would expect the effect of spiral structure to be small for two reasons: (1) the mass of the spiral features is small compared to that of the disk, and (2) the great majority of the globular clusters spend most of their lifetimes away from the Galactic plane, and hence from the region of influence of the spiral arms. However, this is not the case for the thick-disk globular clusters. In this section, we calculate the effect of spiral arms on the orbits of the six new clusters.

4.1. The Spiral Model

For the orbit computations we have used the semi-analytical model of Pichardo et al. (2003). This model is based on a three-dimensional mass distribution composed by the superposition of oblate spheroids as building blocks of the global spiral potential, rather than on a simple cosine law, as is customary. It provides a realistic representation of spiral features, particularly of flocculent arms.

To obtain a model that matches current observational data, we have updated the parameters used in Pichardo et al. (2003) according to recent observational work. In Pichardo et al. (2003), information on parameters not directly given by observations was completed by performing orbital calculations and searching...
for the most orbitally self-consistent model. The adopted features of the model we employ here are summarized as follows.

**Locus of the spiral arms.**—The shape of the spiral arms is one of the few characteristics on which almost all observations agree. In general, the spiral arms seem to be approximately logarithmic; for the Milky Way, in particular, this seems to be the case, as shown in the very complete review by Vallée (2005a). We have taken the spiral locus given by Roberts et al. (1979), which combines a straight bar in the center of the Galaxy and a smoothly joined logarithmic region outwards.

**Number of spiral arms and the pitch angle.**—We have taken the bisymmetric fit of Drimmel (2000) to the K band of the COBE satellite, which has a pitch angle of \( i \approx 15.5 \)°. Although observations in H\( \alpha \) (Russell et al. 2005), pulsar rotation measures (Vallée 2005b), radio observations of H\( \iota \) regions (Sewilo et al. 2004; Paladin et al. 2004), O and B stars (Negueruela & Marco 2003), near-IR flux and gas (Bissantz et al. 2003), and ultracompact H\( \iota \) regions seem to show a four-armed locus with a pitch angle \( i \approx 13.5 \)° (Vallée 2005a), we have assumed here that most of the mass is concentrated in the stellar arms, as seen in the K band (which traces the more massive, old stellar population) rather than in the gaseous arms, which have been proposed as being the response of the gas to this two-armed stellar locus (Martos et al. 2004; Drimmel 2000). We expect that taking a two-armed structure instead of a four-armed one will provide a good (and conservative) estimate of the effects of this non-axisymmetric structure on the globular cluster orbits.

**Outer limit for the spiral arms.**—We have based the adopted length of the spiral arms on observations rather than models. We have taken for this parameter a limiting galactocentric distance of \( R_f \approx 12 \) kpc (Drimmel 2000; Caswell & Haynes 1987).

**Radial force produced by arms versus disk.**—Patsis et al. (1991) built a family of self-consistent models and applied it to 12 normal spirals with known rotation curves and photometry. They found a correlation between the pitch angle and the ratio of the radial force of the arms to that of the axisymmetric disk. For a galaxy like the Milky Way, the pitch angles are in the range 11°–16°, from which they obtained a force ratio of 4%–10%. This corresponds in our model to a mass ratio of arms to disk of \( M_\text{arms}/M_\text{disk} = [2\%, 5\%] \). Independently, models based on observations of our own Galaxy find a local force ratio of approximately 4% (Amaral & Lepine 1997). In our model, we adopt a local force ratio of \( \approx 5\% \) and a total mass for the bisymmetric spiral arms of \( M_\text{arms}/M_\text{d} = 0.03 \).

**Angular velocity.**—This is perhaps the most controversial dynamical parameter, due to the intrinsic difficulty of measuring it. Current observations and models, however, seem to show a trend, placing its value in the interval [20, 25] km s\(^{-1}\) kpc\(^{-1}\). Kinematic observations of stars of the local spiral arm, Orion, give \( \Omega = 25.7 \pm 1.2 \) km s\(^{-1}\) kpc\(^{-1}\) (Bobylev et al. 2006), whereas models (Martos et al. 2004; Pichardo et al. 2003) give \( \Omega = 20 \) km s\(^{-1}\) kpc\(^{-1}\). We have taken for this work \( \Omega = 20 \) km s\(^{-1}\) kpc\(^{-1}\).

### 4.2. The Orbits Perturbed by Spiral Arms

Although individual spiral arms may be transient structures in spiral galaxies, the long-term effect of spiral perturbation is likely to be important, since even if the arms were to last only a short time, they would soon be replaced by new ones; this accounts for their ubiquity in the majority of disk galaxies. It has been customary to neglect the dynamical importance of spiral arms based on empirical or intuitive arguments. Here we perform computations to study quantitatively their effects on Galactic orbits, if any.

We integrated the six globular cluster orbits for the last 5 Gyr only, because, as we shall show, the effects of the spiral perturbation are already noticeable in this time span. The orbits are integrated in two variants of the potential, both of which include the axisymmetric potential (bulge, disk, and halo). In one variant, we add the spiral arms to the background axisymmetric potential, and in the other, we add both the spiral arms and the bar.

In Figure 8, we show the results of the computations in the form of meridional orbits. The left panels refer to the orbits computed with the first variant of the potential, i.e., the axisymmetric plus two-spiral-arm potential, and the right panels to the orbits in the second variant, i.e., the axisymmetric, plus bar, plus two spiral arms. In all cases we analyze the orbits integrated backward in time.

If we directly compare the orbits in the pure axisymmetric potential (Fig. 7) with those in the potential with spiral arms (Fig. 8), we can clearly see the effect of the spiral perturbation. The orbits change their shape in all cases, although the effect is noticeable only for clusters remaining closest to the Galactic plane (such as NGC 5927, NGC 4372, and NGC 2808). Again, the least affected orbits are those that reach large radial and vertical distances and remain mostly far from the influence of the arms, as is the case with NGC 3201. The orbit of this cluster is practically the same in all model potentials.

The effect of the spiral perturbation is most pronounced in the case of near-resonant orbits (such as that of NGC 2808) or irregular orbits (such as NGC 4833 and NGC 5986). Indeed, NGC 2808 seems to get scattered out of its present near-resonant orbit more readily by the spiral perturbation than by the bar; this is a surprising result, since the mass of the bar is much larger. Another unexpected result is seen in the orbit of NGC 4833, which seems more symmetric with respect to the Galactic plane in the presence of the spiral perturbation.

As far as can be asserted on the basis of the six orbits studied here, the orbital parameters, such as the maximum absolute values of the z-distances and the peri- and apogalactic distances, are not much affected by the presence of the spiral arms. The general tendency seems to be for the cluster orbits to have larger energies in the presence of spiral arms than in the axisymmetric case, and hence to attain larger apogalactic distances. This effect is largest for clusters remaining close to the Galactic plane \( (z \leq 2 \) kpc) and inside the radial zone of strong influence of the spiral arms \( (3 \) kpc \(< R < 8 \) kpc). The combined effects of bar and spiral-arm perturbations seem to be an efficient mechanism for attaining apogalactic distances larger than in the axisymmetric potential.

Even though, in general, the dynamical effects of the bar dominate over those of the spiral arms, it is interesting to see that the spiral perturbations may play a non-negligible role. The clearest example is that of NGC 2808.

### 5. TIDAL RADII AND DESTRUCTION RATES

Ostriker et al. (1989) have emphasized the role played by a Galactic bar in selectively destroying clusters in box orbits passing near the Galactic center. In our present study, as well as in Paper I, we find that the smallest pericenters in the axisymmetric case (e.g., those of NGC 4833 and NGC 5986) tend to be larger in the presence of the bar. This is reflected in the destruction rates, which for these two clusters are sensibly smaller in the presence of the bar. Some possible reasons for this discrepancy are discussed in Paper I.

In the same manner as in Paper I, we have assessed the effects of the bar on the internal dynamics of the clusters, and computed tidal radii and destruction rates. We computed tidal radii for the
six clusters using the King (1962) formula,
\[ r_K = \left( \frac{M_c}{M_g (3 + e)} \right)^{1/3} r_{\text{min}}, \]
where \( M_c \) is the mass of the cluster, \( M_g \) is an effective Galactic mass, \( e \) is the orbital eccentricity, and \( r_{\text{min}} \) is the galactocentric distance of a perigalactic point in the orbit. We have also calculated the tidal radius of each cluster using an alternative approximate formula introduced in Paper I, which has the advantage of not requiring an assumption about an effective Galactic mass, which is difficult to define if the perigalactic point lies in a region with a strong non-axisymmetric potential, as in the case of a bar. Our approximation is given by the equation
\[ r_\ast = \frac{GM_c}{\left( \frac{\partial F}{\partial x'} \right)_{x'=0} + \theta^2 + \dot{\varphi}^2 \sin^2 \theta} \left( \frac{1}{3} \right), \]
where \( F' \) is the component of the Galactic acceleration along the line \( x' \), which joins the cluster with the Galactic center, and its partial derivative is evaluated at the position of the cluster. The angles \( \varphi \) and \( \theta \) are the angular spherical coordinates of the cluster in an inertial Galactic frame. The tidal radius is computed at a perigalactic point in the orbit. In a two-body problem, \( r_\ast \) is reduced to King’s formula. For the derivation of equation (1), see Appendix A of Paper I.

The last three columns of Table 1 contain \( r_K \), \( r_\ast \), and \( r_t \) for the axisymmetric potential, where \( r_t \) is the observed limiting radius of the cluster. In the last two columns of Table 2, we give \( r_K \) and \( r_\ast \) for the barred potential.

In Paper I, we found that the formulae for \( r_\ast \) and \( r_K \) gave similar values. The same is true for this new set of objects, independent of the potential variant employed (barred, barred+spiral, or axisymmetric). This is shown in Figure 9, where we have taken the barred model as an example. In Figure 9, we show a comparison of the observed limiting radii (\( r_t \)) with the theoretical ones, using King’s formula (\( r_K \)) in particular.

The prograde-retrograde cluster NGC 5986 has a mean perigalactic distance that places it within the bar. Its tidal radius in Figure 10 lies under the line of coincidence, as was the case for...
most of the retrograde or prograde-retrograde clusters with a mean perigalactic distance of less than 3.5 kpc studied in Paper I. The only fully retrograde globular cluster of the new sample, NGC 3201, lies well above the line of coincidence, but it is also the only cluster residing well outside the bar. For a discussion of the importance of the orbital sense (prograde or retrograde) on the tidal radii, we refer the reader to § 4 of Paper I.

Regarding the destruction rates, we have calculated the time of destruction due to components of bulge and disk in the Galaxy based on the results of several papers on the subject (Aguilar et al. 1988; Long et al. 1992; Gnedin & Ostriker 1997, 1999; Spitzer 1987; Kundic & Ostriker 1995; Gnedin et al. 1999a, 1999b). As in Paper I, we define the total destruction rate due to gravitational shocks with the bulge as

\[
\frac{1}{t_{\text{bulge}}} = \frac{1}{t_{\text{bulge,1}}} + \frac{1}{t_{\text{bulge,2}}},
\]

where

\[
t_{\text{bulge,1}} = \left[ \frac{-E_c}{\langle (\Delta E)_b \rangle} \right] P_{\text{orb}},
\]

and

\[
t_{\text{bulge,2}} = \left[ \frac{E_c^2}{\langle (\Delta E)_b^2 \rangle} \right] P_{\text{orb}},
\]

where \(E_c \approx -0.2GM_c/r_b^2\) is the mean binding energy per unit mass of the cluster, and \(P_{\text{orb}}\) is its orbital period in the Galactic radial direction.

In the same manner, we have the total destruction rate due to gravitational shocks for the disk:

\[
\frac{1}{t_{\text{disk}}} = \frac{1}{t_{\text{disk,1}}} + \frac{1}{t_{\text{disk,2}}},
\]

where

\[
t_{\text{disk,1}} = \left[ \frac{-E_c}{\langle (\Delta E)_d \rangle} \right] P_{\text{orb}} n,
\]

and

\[
t_{\text{disk,2}} = \left[ \frac{E_c^2}{\langle (\Delta E)_d^2 \rangle} \right] P_{\text{orb}} n.
\]

Here, \(n\) is the number of crossings with the disk during the radial orbital period \(P_{\text{orb}}\). For a summary of destruction rate formulae, see Paper I.

We have applied these formulae to the six globular clusters and to the specific case including a Galactic bar. It is worth noting that we have taken very conservative values for the mass and radius for the Galactic bar. According to recent estimates, the bar could extend at least 1 kpc beyond the limit we used (López-Corredoira et al. 2007), and it could have at least twice the

\[
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
Cluster (1) & \(M_\star\) (M\(_\odot\)) (2) & \(c\) (3) & \(r_b\) (pc) (4) & \(\langle 1/t_{\text{bulge,1}} \rangle\) (yr\(^{-1}\)) (5) & \(\langle 1/t_{\text{bulge,2}} \rangle\) (yr\(^{-1}\)) (6) & \(\langle 1/t_{\text{disk,1}} \rangle\) (yr\(^{-1}\)) (7) & \(\langle 1/t_{\text{disk,2}} \rangle\) (yr\(^{-1}\)) (8) & \(\langle 1/t_{\text{disk}} \rangle\) (yr\(^{-1}\)) (9) & \(\langle 1/t_{\text{disk}} \rangle\) (yr\(^{-1}\)) (10) \\
\hline
NGC 2808 & 9.7E05 & 1.77 & 2.12 & 9.5E-15 & 6.3E-16 & 1.0E-14 & 4.4E-14 & 3.5E-15 & 4.7E-14 \\
NGC 3201 & 1.6E05 & 1.30 & 3.90 & 8.1E-17 & 8.1E-18 & 8.9E-17 & 8.2E-15 & 9.3E-16 & 3.2E-15 \\
NGC 4372 & 2.2E05 & 1.30 & 6.58 & 3.1E-12 & 7.2E-13 & 3.9E-12 & 2.2E-11 & 6.3E-12 & 2.9E-11 \\
NGC 4833 & 3.1E05 & 1.25 & 4.56 & 5.1E-12 & 1.2E-12 & 6.3E-12 & 2.7E-11 & 7.8E-12 & 3.5E-11 \\
NGC 5927 & 2.2E05 & 1.60 & 2.54 & 6.8E-16 & 5.0E-17 & 7.3E-16 & 7.2E-13 & 7.3E-14 & 8.0E-13 \\
NGC 5986 & 4.1E05 & 1.22 & 3.18 & 4.8E-10 & 1.6E-10 & 6.4E-10 & 1.4E-12 & 3.0E-13 & 1.7E-12 \\
\hline
\end{tabular}
\]
axisymmetric potential, respectively. Asterisks correspond to clusters with the longest total destruction times, taking both disk and bulge. These destruction rates are the averages over the orbit, taking the corresponding peri- and apocentral distances. The non-axisymmetric potential (Fig. 4a) and for the barred potential (Fig 4c). Clusters with perigalactic points close to the Galactic center and large orbital eccentricities (circled squares) have, in general, greater bulge destruction rates. Figures 11b and 11d show the comparison of the destruction rates due to the bulge and disk in the axisymmetric and in the barred potential, respectively. These frames show that bulge shocking dominates in the bar region, as found by Aguilar et al. (1988). We confirm, as in Paper I, that the destruction rates are very similar in both the axisymmetric and the barred Galactic potential.

In this study, we have found that the bar is the most important non-axisymmetric structure in the Galaxy with regard to its orbital effects, tidal radii, and destruction rates of globular clusters. The spiral arms produce small but non-negligible effects in the orbits of the clusters moving closest to the Galactic plane (see § 4.2). Since the changes in destruction rates and tidal radii depend mainly on the orbital parameters, the effects of the spiral arms in a Milky Way–like galaxy are negligible compared to the effects of the bar. However, in the case of grand-design spirals or of galaxies with a weak bar or entirely without a bar, the effects of the spiral arms should not be considered negligible.

6. DISCUSSION AND CONCLUSIONS
We have obtained orbits for six additional globular clusters in both a barred and an axisymmetric Galactic potential, as well as in models including spiral arms. The total number of clusters with available orbits is new. Among the newly calculated cases, only the orbit of NGC 3201, an outer cluster, is unaffected by the bar. The other five clusters have orbits residing within the region of influence of the bar, and their orbits are clearly influenced by it. As in our previous study, we find that the bar causes the largest destruction rates. Although the influence of the spiral arms on the orbits of the clusters closest to the Galactic disk is not negligible, the long-term effects are definitely dominated by the Galactic bar. Tidal radii have been computed with the expression derived in Paper I, as well as with a numerical evaluation of the relevant quantities along the orbit. Again, we find little change due to the bar. When changes did occur, they again make the computed tidal radii somewhat larger in the presence of a bar. With the newly available material, we confirm our earlier finding that the destruction rates due to shocks with the Galactic bulge and disk are not strongly affected when we consider a barred Galactic potential or a barred potential with spiral-arm perturbations.

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