A quantitative study on helicity inversion in Majorana neutrino decays at the LHC

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We report an analytical and numerical investigation into the impact of helicity inversion in LHC processes that do not conserve lepton number. As a case study, we focus on the production and decay of Majorana neutrinos through $W$ bosons in the Phenomenological Type I Seesaw model. Using the Monte Carlo event generator MadGraph5_aMC@NLO in conjunction with the HeavyN model libraries, we perform exact matrix element (ME) computations without the narrow width approximation. Despite helicity inversion appearing explicitly in MEs, we report the absence of helicity suppression in collider observables for $1 \to 4$ and $2 \to 4$ processes. We attribute this incongruity to the different scalings of 4-momenta and squared 4-momenta in MEs and squared MEs, with the latter leading to exact cancelations in the on-shell limit. Implications for other scenarios are discussed.

I. INTRODUCTION

Among the outstanding questions in particle physics 1) is whether the light neutrinos observed in nature ($\nu$) are their own antiparticle, i.e., are they Majorana fermions? If so, then the Lagrangian of the Standard Model of particle physics (SM), which stipulates that neutrinos are massless, must be extended by dimensionful operators that violate the SM’s conservation of lepton number ($L$). Gauge invariance and renormalizability, however, require that such operators have ultraviolet completions, and thereby suggests the possibility of new particles [2]. Hence, discovering the Majorana nature of neutrinos may be a stepping-stone to realizing a mechanism for neutrino mass-generation [3–16], new gauge forces [6, 13, 17–20], or even grand unification [4, 7, 12, 15, 21, 22].

Despite this importance, however, direct tests of neutrinos’ Majorana nature, such as through searches for neutrinos’ magnetic dipole moments or through $|\Delta L| = 2$ transitions like neutrinoless $\beta\beta$ decay ($0\nu\beta\beta$), are encumbered by manifestations of the so-called Dirac-Majorana Confusion Theorem [23, 24]. In the absence of new particles, the theorem in its standard formulation [24] shows that an inherent helicity inversion in such processes leads to matrix elements (ME) being proportional to light neutrino masses. This implies that transition probabilities formally vanish in the limit of vanishing neutrino masses, and thus are classified as being helicity suppressed. As such, two complementary approaches to the “Majorana” question are embraced: The first is the development of large-scale facilities that, for example, aim to measure the $0\nu\beta\beta$ decay rate. The second relies on direct searches for $|\Delta L| = 2$ processes in the context of neutrino mass models. For reviews of these approaches, see Refs. [25–28].

In the second approach, processes that do not conserve $L$ are mediated by new particles [3–16] that are typically much heavier than light neutrinos, but possibly lighter than the electroweak (EW) scale. Crucially, the Confusion Theorem follows from rather generic kinematical arguments in the context of chiral gauge theories, e.g., the EW theory, and not on flavor symmetries as considered, for example, in Refs. [29–32]. As such, in scenarios with heavy Majorana neutrinos ($N$), helicity inversion manifests as asymmetries in angular distributions that distinguish $|\Delta L| = 0$ and $|\Delta L| = 2$ channels [33–40]. However, while generalizations of the theorem show [33] that these MEs are consistently proportional to heavy neutrino masses ($m_N$), past studies have not specifically investigated whether the MEs also vanish when $m_N$ do.

In this study, we report an analytical and numerical investigation into the impact of helicity inversion in $L$-violating transition rates involving heavy Majorana neutrinos at the $\sqrt{s} = 13$ TeV Large Hadron Collider (LHC). As a representative case study, we work in the framework of the Phenomenological Type I Seesaw model and focus on the $L$-violating decay and scattering processes [41]

$$W^\pm \to e^\pm N^{(*)} \to e^\pm e^\pm j j, \quad (1.1)$$

$$pp \to W^{\pm(*)} \to e^\pm N^{(*)} \to e^\pm e^\pm j j, \quad (1.2)$$

and their $L$-conserving counterparts, as shown diagrammatically at the parton level in Fig. 1. While Eqs. 1.1 and 1.2 are intimately related, their individual considerations explore subtler polarization and virtuality effects.

By performing exact ME computations with the Monte Carlo (MC) event generator MadGraph5_aMC@NLO [43, 44] in conjunction with the HeavyN model libraries [45, 46], and without invoking the narrow width approximation, we find that the helicity suppression in collider observables is numerically negligible for masses in the range of 1 GeV $< m_N < M_W$. We attribute the seeming incongruity with the presence of helicity inversion to the different scaling of 4-momenta and squared 4-momenta in MEs and squared MEs. In the on-shell limit, this leads to cancelations of the dependence on $m_N$, with corrections proportional to off-shell virtualities and total widths. As the arguments here are kinematical in nature, analogous findings hold for other models with Majorana $N$. 

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The remainder of this study continues in the following order: In Sec. II we summarize the theoretical framework in which we work. In Sec. III we document our computational setup. We then identify analytically in Sec. IV the helicity inversion at the ME level, its propagation to the squared ME level, and finally its cancelation for the processes in Fig. 1. We also comment on implications for other models and $|\Delta L| = 2$ processes. We present our numerical comparisons in Sec. V and conclude in Sec. VI.

II. THEORETICAL FRAMEWORK

To investigate the potential helicity suppression of $L$-violating processes mediated by Majorana neutrinos, we work in the framework of the Phenomenological Type I Seesaw. In this well-documented [25, 47, 48] scenario, the masses and mixing angles of light ($\nu_L$) and heavy ($N_{k'}$) neutrino mass eigenstates are decoupled in order to conduct flavor model-independent studies and searches.

In this model, the SM’s field content is extended by $n_R \geq 3$ right-handed (RH) neutrinos ($\nu_{kR}^i$) that are gauge-singlets, i.e., are chargeless / sterile, under the SM gauge interactions. This property allows the $\nu_{kR}$ to possess RH Majorana masses ($\tilde{\mu}_{kR}$), which violate $L$ conservation and, in principle, can acquire any value. The decoupling of $\tilde{\mu}_{kR}$ subsequently suppresses light neutrino masses [3–9] and is distinct from other neutrino mass mechanisms, e.g., the Type II Seesaw [9–12], where $\nu$ masses are generated via left-handed (LH) Majorana masses.

Accordingly, the Lagrangian of the Phenomenological Type I Seesaw ($\mathcal{L}_{\text{Type I}}$) is characterized by extending the SM Lagrangian ($\mathcal{L}_{\text{SM}}$) at the renormalizable level by kinetic and mass terms for the $\nu_{kR}^i$ ($\mathcal{L}_{\text{Kin.}}$), and by Yukawa couplings ($\mathcal{L}_{Y}$) between the $\nu_{kR}^i$, the SM Higgs field ($\Phi$), and the SM’s LH lepton doublets $L^T = (\nu_L^T, \ell_L^T)$,

$$\mathcal{L}_{\text{Type I}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{Kin.}} + \mathcal{L}_{Y}. \quad (2.1)$$

After EWSB and diagonalizing charged lepton flavor states into their mass eigenstates ($\ell = e, \mu, \tau$), the flavor eigenstates of active, LH neutrinos ($\nu_{LL}$) can be decomposed into mass eigenstates via the rotation [25]

$$\nu_{LL} = \sum_{k=1}^{3} U_{k\ell} \nu_{kR} + \sum_{k'=1}^{n_R} V_{k'\ell} N_{k'}. \quad (2.2)$$

Here the complex-valued mixing elements $U_{k\ell}$ and $V_{k'\ell}$ parametrize the mixing between the flavor state $\nu_{LL}$ with the mass eigenstates $\nu_{kR}$ and $N_{k'}$. For updated measurements and constraints of mixing angles, see Refs. [54–57].

Given Eq. 2.2, the relevant interaction Lagrangian describing the charged current interactions of $N_{k'}$ is,

$$\mathcal{L} = -\frac{g_W}{\sqrt{2}} W_\mu^\tau \sum_{\ell} \tau |\mathcal{H}_{\nu_{LL}}|^{2} P_L \ell + \text{H.c.} \quad (2.3)$$

$$= -\frac{g_W}{\sqrt{2}} W_\mu^\tau \sum_{k=1}^{3} \sum_{\ell} \tau |U_{k\ell}|^2 \gamma^\mu P_L \ell$$

$$-\frac{g_W}{\sqrt{2}} W_\mu^\tau \sum_{k'=1}^{n_R} \sum_{\ell} \tau |V_{k'\ell}|^2 \gamma^\mu P_L \ell + \text{H.c.} \quad (2.4)$$

Here, $g_W \equiv e / \sin^2 \theta_W \approx 0.65$ is the usual weak gauge coupling constant in the SM, and $P_L/R = (1/2)(1 \mp \gamma^5)$ are the LH/RH chiral projection operators in four-component notation. Using Eq. 2.2 to make analogous substitutions, interaction Lagrangians involving the $Z$ and Higgs can be built accordingly [25, 47]. Throughout this study we consider the impact of only the lightest heavy mass eigenstate ($N_{k'=1}$), which we relabel as $N \equiv N_{k'=1}$ with $V_{N\ell} \equiv V_{k'\ell=1}$. We do so to isolate the impact of helicity inversion in $L$-violating currents that can otherwise be obfuscated by strong interference.

III. COMPUTATIONAL SETUP

We now briefly document the computational setup of this study. After summarizing the MC setup in Sec. III.1, the numerical inputs for SM and heavy neutrino parameters are respectively provided in Secs. III.2 and III.3.
III.1. Monte Carlo Setup

To perform our numerical computations, we use the MC event generator MadGraph5_aMC@NLO (v2.7.0) [43, 44] (mgamc). The simulation suite [43, 44, 58–62] operates by constructing helicity amplitudes for short-distance decay and scattering processes [43, 59, 63] according formalism of Refs. [63–66] and performs fast numerical integration over phase space through MC sampling [58, 67]. For heavy neutrino interactions governed by the Lagrangian of Eq. 2.4, we import into mgamc the HeavyN [45, 46] FeynRules libraries. This employs the conventions for Majorana currents developed in Refs. [71, 72]. For select calculations, we compute helicity-polarized MEs in mgamc according to the formalism of Ref. [73].

III.2. Standard Model Inputs

For numerical computations we work in the $n_f=5$ massless / active quarks scheme with SM inputs set to

$$m_t(m_t) = 173.3 \text{ GeV}, \quad M_Z = 91.1876 \text{ GeV}, \quad (3.1)$$

$$\alpha_{\text{QED}}^{-1}(M_Z) = 127.94, \quad G_F = 1.174560^{-5} \text{ GeV}^{-2}. \quad (3.2)$$

We take the Cabbibo-Kobayashi-Maskawa matrix equal to the identity matrix. For relevant computations we use the MSTW 2008 leading order parton density functions (ihaid=21000) [74] as evolved by LHAPDF (v6.2.3) [75], and set the collinear factorization scale ($\mu_f$) to

$$\mu_f = M_W \approx 79.95 \text{ GeV}. \quad (3.3)$$

III.3. Heavy Neutrino Inputs

In addition to SM inputs, the (default) HeavyN model libraries [45, 46] consists of three Majorana neutrino mass eigenstates $N_k$ with mass eigenvalues $m_{N_k}$ and active-sterile mixing elements $V_{\ell_k}$ associated with lepton flavor $\ell$. As explained at the end of Sec. II, we decouple two $N_k$ in order to isolate helicity inversion in the absence of interference. To do this numerically, we set

$$m_{N_2}, m_{N_3} = 10^{12} \text{ GeV} \quad \text{and} \quad |V_{\ell_2}|, |V_{\ell_3}| = 10^{-10}. \quad (3.4)$$

As the values of $m_{N_1}, |V_{\ell N_1}|$ are varied, the total width ($\Gamma_{N_1}$) of $N_1$ is reevaluated on-the-fly using MadDecay [62].

IV. HELICITY INVERSION IN MATRIX ELEMENTS WITH MAJORANA NEUTRINOS

For $|\Delta L| = 2$ transitions that are mediated by heavy Majorana neutrinos in the Phenomenological Type I Seesaw, we establish in this section the presence of helicity inversion in MEs and its propagation into squared MEs. While the inversion has far-reaching consequences [23, 24, 33–40], it is essentially a quirk of chiral gauge theories, like the EW theory, and follows from the Charge-Parity-Time (CPT) theorem. We organize this derivation by first considering $L$-conserving, 4-body decays of $W$ bosons in Secs. IV.1, and then $L$-violating decays in Secs. IV.2. We draw special attention to the precise origin of the inversion and the scaling of (squared) momenta in (squared) MEs. In Sec. IV.3 we consider analogous processes in $2 \to 4$ scattering and comment on the implications for other neutrino mass models in Sec. IV.4.

IV.1. $W$ boson decays with $\Delta L = 0$

As a first step to studying helicity inversion in $|\Delta L| = 2$ processes, we consider the following $L$-conserving, 4-body $W$ boson decay mediated by a Majorana neutrino $N$,

$$W^+_{\lambda_N}(p_W) \to \ell_1^+ R(p_1) \quad N_{\lambda_N}(p_{N}) \quad \leftrightarrow \quad \ell_2^+ L(p_2) \quad c_L(p_c) \quad s_R(p_s). \quad (4.1)$$

as shown as a sub-process in Fig. 1(a). Here, the subscripts $\lambda_N = 0$, $L = R$ denote the helicities of $W^+$ and $N$. The helicities of massless fermions $\ell_k, c, s$ are fixed by the $W$ boson’s chiral couplings.

Working in the unitary gauge and in the HELAS basis [63] for helicity amplitudes, the corresponding ME is

$$-iM^W_{\ell_1} = \epsilon_{\mu}(p_W, \lambda_N) \quad T^W_{\ell_1 \mu}(p_1, p_2, p_{N}) \quad \times \Delta_{\nu\rho}(p_c + p_s) \quad J^\rho(p_c, p_s). \quad (4.2)$$

Here the ($\epsilon\pi$) fermion current $W$ propagator are

$$J^\rho(p_c, p_s) = -i\frac{g_W \delta^{AB}}{\sqrt{2}} \left[ \bar{\pi}\nu_L(p_c) \gamma^\rho P_L v^B_R(p_s) \right], \quad (4.3)$$

and the $L$-conserving ($\ell_1^+ N_{\ell_2}^-$) fermion current is

$$T^\nu_{\ell_1 L}(p_1, p_2, p_{N}) = \left( \frac{-i g_W}{\sqrt{2}} \right)^2 V_{\ell_1 N}^* V_{\ell_2 N} \times N^{\nu\mu}_{L} \times D, \quad (4.4)$$

where

$$N^{\nu\mu}_{L} = \left[ \frac{\pi\nu_L(p_2) \gamma^\nu W_L(p_N + m_{N\ell_4}) \gamma^\mu W_R(p_1)}{i M^W_{\ell_1} \times D} \right], \quad (4.5)$$

and

$$D = \left[ p^0_{\ell_1} - m^2_{\ell_1} + i \Gamma_{\ell_1} m_{\ell_1} \right]. \quad (4.6)$$

In the quark current $J^\nu$, the indices $A, B = 1, \cdots, N_c = 3$ run over the QCD color states, and the Kronecker $\delta_{AB}$ function $\delta_{AB}$ ensures a color-singlet $W^* \to q\bar{q}^*$ splitting.

Importantly, the ($\ell_1^+ N_{\ell_2}^-$) fermion current is initiated/terminated by successive $W$ interactions. These are maximally parity-violating, are oriented in the left chiral direction, and are responsible for the two ($\gamma^\nu W_L$) in $N^{\nu\mu}_{L}$. Due to orthogonality of RH and LH chiral projection operators, the intermediate $N$ is confined to its LH helicity state (the $\rho_N$ term). The transition is helicity conserving
as RH helicity states (the $m_N\bar{u}_4$ term) do not contribute to successive chiral interactions with the same chirality.

After anticommuting the left-most $P_L$ and using naïve power counting to extract the energy dependence from spinors, we obtain for the $(\ell_1^+ N \ell_2^-)$ lepton current:

$$N_{L}^{\nu\mu} = |\pi_L(p_2)\gamma^\nu P_L(\bar{p}_N + m_N\bar{u}_4)\gamma^\mu P_L v_R(p_1)|$$

$$= |\pi_L(p_2)\gamma^\nu P_L(\bar{p}_N)\gamma^\mu P_L v_R(p_1)|$$

$$\sim \sqrt{E_2} E_N \sqrt{E_1} \sim M_W^2.$$ (4.11)

The scaling in the last line shows that the amplitude $M_W^L$ for the $W^+ \rightarrow \ell_1^+ \ell_2^- \pi^-$ decay grows with the energy of $N$, and therefore is not suppressed for vanishing $m_N$.

We now note that due to Lorentz invariance the scaling of 4-momenta ($p^\mu$) and their squares ($p^\nu p^\mu$) differ. Importantly, this leads to different behavior in squared MEs than in Eq. 4.11. In particular, one finds using Ref. [33],

$$\sum |M_W^L|^2 \sim \sum T_{L}^{\nu\mu} \left[T_{L}^{\alpha\beta}\right]^\dagger$$

$$\sim \sum N_{L}^{\nu\mu} \left[N_{L}^{\alpha\beta}\right]^\dagger \times |D(p_N^2)|^2,$$ (4.12)

where the squared and spin-summed current scales as

$$\sum N_{L}^{\nu\mu} \left[N_{L}^{\alpha\beta}\right]^\dagger \sim E_2^2 p_N^2 E_1 \sim M_W^2 p_N^2.$$ (4.13)

Interestingly, Eq. 4.14 shows that the squared ME scales as the virtuality of the intermediate $N$, and can potentially vanish for tiny $m_N$ in the on-shell limit. In this region of phase space however, i.e., when

$$\delta p_N^2 \equiv |p_N^2 - m_N^2| \ll \Gamma_N m_N \ll m_N^2,$$ (4.15)

the pole structure of the propagator $D$ behaves as

$$|D(p_N^2)|^2 = \frac{1}{(p_N^2 - m_N^2)^2 + (\Gamma_N m_N)^2}$$

$$= \frac{1}{(\Gamma_N m_N)^2} \left[1 + \frac{\delta p_N^2}{(\Gamma_N m_N)^2}\right].$$ (4.16)

In combination with the scaling in Eq. 4.14, we obtain

$$\sum |M_W^L|^2 \sim \frac{M_W^2 p_N^2}{(\Gamma_N m_N)^2} \left[1 - \mathcal{O}\left(\frac{\delta p_N^2}{(\Gamma_N m_N)^2}\right)\right],$$ (4.19)

and see that the dependence on $m_N^2$ is actually cancelled in the on-shell limit. Hence, like at the ME level, the leading contribution to the $W^+ \rightarrow \ell_1^+ \ell_2^- \pi^-$ decay rate at the squared ME level does not vanish for vanishing $m_N$.

## IV.2. $W$ boson decays with $|\Delta L| = 2$

Considering now the $L$-violating analogue of the process in Eq. 4.2, we have the 4-body $W$ boson decay chain

$$\begin{align*}
W_{\nu\mu}^+(p_W) &\rightarrow \ell_1^+(p_1) N_\nu(p_N) \\
&\rightarrow \ell_2^-+ p_2) \tau_R(p_c) s_L(p_s),
\end{align*}$$

as shown as a sub-process in Fig. 1(b). Following the same procedure as needed to construct $M_W^L$ in Eq. 4.3, the ME of the $L$-violating decay process is given by

$$-iM_{L}^W = \varepsilon_\mu(p_W, \lambda_W) T_{L}^{\nu\mu} (p_1, p_2, p_1 + p_c + p_s)$$

$$\times \Delta_{\nu\rho}(p_c + p_s) J^\rho(p_s, p_c) + (p_1 \leftrightarrow p_2).$$ (4.22)

Up to external momentum reassignments, the quark current $J^\rho$, polarization vector $\varepsilon_\mu$, and propagator $\Delta_{\nu\rho}$ are the same as in the $L$-conserving case. In the last line is the interference from $\ell_1^+ \leftrightarrow \ell_2$ particle exchange. Due to charge conservation, no second term exists in $M_W^L$.

The key difference from the $L$-conserving ME is the $L$-violating $(\ell_1^+ N \ell_2^-)$ fermion current. To derive this we note that in going from the $W^+ \rightarrow \ell_1^+ \ell_2^- \pi^-$ process to $W^+ \rightarrow \ell_1^+ \ell_2^- \tau^\pm$, one effectively imposes a charge invariance on the electrically neutral $(\ell_2^- \pi^-)$ system. Under CPT, this is the same as a parity-time inversion and, significantly, is expressible as Feynman rules [71, 72].

These states that after assuming a fermion flow (curve in Fig. 1(b)) the $(N \ell_2^- W^-)$ vertex as derived from the Lagrangian in Eq. 2.4 is parity-inverted and becomes

$$-\frac{ig\gamma^\nu}{\sqrt{2}} V_{\ell_2 N} \gamma^\nu P_L \rightarrow (-1)^{\frac{3}{2}} \frac{ig\gamma^\nu}{\sqrt{2}} V_{\ell_2 N} \gamma^\nu P_R.$$ (4.23)

Consistently, as $\ell_2$’s own fermion number flow, which points inward, is antiparallel to the conventional fermion flow, which points outward, its spinor is time-inverted:

$$\pi_R(p_2) \rightarrow \pi_R(p_2).$$ (4.24)

Propagating these modifications and defining for compactness $\tilde{p}_k \equiv p_k + p_c + p_s$, for $k = 1, 2$, we find that both $L$-violating $(\ell_1^+ N \ell_2^-)$ fermion currents are given by

$$T_{L}^{\nu\mu}(p_1, p_2, \tilde{p}_k) =$$

$$-\left(\frac{ig\gamma^\nu}{\sqrt{2}}\right)^2 V_{\ell_1 N} V_{\ell_2 N} \times N_{L}^{\nu\mu} \times D(\tilde{p}_k)^2,$$ (4.25)

$$N_{L}^{\nu\mu} = |\pi_R(p_2)\gamma^\nu P_R(\tilde{p}_k + m_N\bar{u}_4)\gamma^\mu P_L v_R(p_1)|.$$ (4.26)

Importantly, this differs from the $L$-conserving analogue $N_{L}^{\nu\mu}$ in Eq. 4.7 by the replacement of the leftmost chiral projection operator $P_L$ with the RH projector $P_R$, a consequence of Eq. 4.23. Using again the orthogonality of projection operators we see that the intermediate $N$ is confined to its RH helicity state (the $m_N\bar{u}_4$ term). The $L$-violating transition is helicity inverting as LH helicity states (the $p_N$ term) do not contribute to successive chiral interactions of opposite chirality.
After anticommuting the operator $P_R$, we obtain
\[ N''_{\ell J} = \{\gamma^\nu p_2^\nu P_R(\tilde{p}_k + m_N\mathbb{1}_4)\gamma^\mu P_L v_R(p_1)\} \quad (4.27) \]
\[ = m_N \times \{\gamma^\nu p_2^\nu P_R(\tilde{p}_k + m_N\mathbb{1}_4)\gamma^\mu P_L v_R(p_1)\} \quad (4.28) \]
\[ \sim m_N \sqrt{E_2} \sqrt{E_1} \sim m_N M_W. \quad (4.29) \]

In the last line we again employ naïve power counting to find that both $(\ell_1^+ N \ell_2^+)$ currents are proportional to $m_N$, independent of $\tilde{p}_k$. Subsequently, we see that both currents vanish for vanishing Majorana neutrino mass, in line with expectations from the Confusion Theorem.

To address the pole structure in the Majorana neutrino’s propagator ($D$ in Eq. 4.8) as we did for the $L$-conserving decay, we consider again when $N$ is (nearly) on-shell. Without the loss of generality, we assume $\tilde{p}_k^2 = (p_1 + p_\pi + p_s)^2$ satisfies the (near) on-shell condition of Eq. 4.15. By momentum conservation, the non-resonant momentum configuration has the virtuality
\[ \tilde{p}_k^2 = (p_W - p_1)^2 = M_W^2 - 2M_W E_1. \quad (4.30) \]

For these configurations of $\tilde{p}_k$, we obtain the expansions
\[ D(\tilde{p}_1) = \frac{1}{(p_1^2 - m_N^2) + i(\Gamma_N m_N)} \quad (4.31) \]
\[ = \frac{-i}{\Gamma_N m_N} \left[ 1 - \mathcal{O}\left(\frac{\delta p_1^2}{\Gamma_N m_N}\right) \right], \quad (4.32) \]
\[ D(\tilde{p}_2) = \frac{1}{(p_2^2 - m_N^2) + i(\Gamma_N m_N)} \quad (4.33) \]
\[ = \frac{1}{M_W^2 \left(1 - \frac{E_2}{M_W} - \frac{m_N^2}{M_W^2} - i(\Gamma_N m_N)\right)} \quad (4.34) \]
\[ = \frac{1}{M_W^2} \left[1 + \mathcal{O}\left(\frac{E_1}{M_W}\right) + \mathcal{O}\left(\frac{m_N^2}{M_W^2}\right)\right]. \quad (4.35) \]

After combining $D(\tilde{p}_k)$ with $N''_{\ell J}$, we see that the $m_N$ dependence in the non-resonant contribution scales as
\[ N''_{\ell J} \times D(\tilde{p}_2) \sim \frac{m_N}{M_W} \left[1 + \mathcal{O}\left(\frac{E_1}{M_W} + \frac{m_N^2}{M_W^2}\right)\right], \quad (4.36) \]
and thereby vanishes in the limit that $(m_N/M_W) \to 0$. On the other hand, for the resonant contribution, we obtain a qualitatively different behavior, namely that
\[ N''_{\ell J} \times D(\tilde{p}_1) \sim \frac{-iM_W}{\Gamma_N} \left[1 - \mathcal{O}\left(\frac{\delta p_1^2}{\Gamma_N m_N}\right)\right]. \quad (4.37) \]

This shows that the dependence on $N$’s mass cancels in the resonant contribution and hence generates a non-zero ME for $W^+ \to \ell_1^+ \ell_2^+ \gamma s$, even for vanishing $m_N$. While helicity inversion exists at the ME level, its impact is mitigated by the propagator in the on-shell limit, i.e., when $N$ can be approximated as an asymptotic state. Notably, this is independent of active-sterile mixing.

Moreover, since the ME for $W^+ \to \ell_1^+ \ell_2^+ \gamma s$ scales as the $(\ell_1^+ N \ell_2^+)$ current and its crossing interference,
\[ |\mathcal{M}_W^{W}|^2 \sim N''_{\ell J} \times D(\tilde{p}_1) + N''_{\ell J} \times D(\tilde{p}_2), \quad (4.38) \]
we find that the resonant, interference, and non-resonant terms respectively contribute to the squared ME as
\[ \left|\mathcal{M}_W^{W}\right|^2 \sim \mathcal{O}\left(\frac{M_W^2}{\Gamma_N^2}\right) + \mathcal{O}\left(\frac{m_N}{\Gamma_N}\right) + \mathcal{O}\left(\frac{m_N^2}{M_W^2}\right). \quad (4.39) \]

This tells us that while the non-resonant contribution is negligible compared to the (leading) resonant contribution, the (sub-leading) interference is not guaranteed to be negligible if $m_N \sim M_W$. However, for $m_N \ll M_W$, the total width of $N$ scales as $\Gamma_N \sim G_F^2 m_N^5 |V_{1\pi}|^2$, and suggests a numerically insignificant interference term.

Using Eq. 4.37 to keep track of formally sub-leading terms, one finds a more exact scaling of the squared ME:
\[ \sum |\mathcal{M}_W^{W}|^2 \sim \frac{M_W^2}{\Gamma_N} \left[1 - \mathcal{O}\left(\frac{\delta p_1^2}{\Gamma_N m_N^2}\right)\right] \quad (4.40) \]
\[ + \mathcal{O}\left(\frac{m_N}{\Gamma_N}\right) + \mathcal{O}\left(\frac{m_N^2}{M_W^2}\right). \]

In comparison to the squared ME in Eq. 4.19, the above demonstrates that in the limit that the Majorana neutrino goes on-shell, the leading contribution to the squared ME of the $L$-violating decay $W^+ \to \ell_1^+ \ell_2^+ \gamma s$ admits a dependence on the mass $m_N$ that is identical to that found in the $L$-conserving decay $W^+ \to \ell_1^+ \ell_2^+ \gamma s$.

Furthermore, for both decay processes, the respective contributions from the polarization vector $\varepsilon_\mu$, the $W^+$ propagator $\Delta_{\mu\nu}$, and the $(\varepsilon_1/\varepsilon_5)$ current $J^\mu$ in Eqs. 4.3 and 4.22 are the same. It follows then that the squared MEs for the two processes do not just have the same scaling dependence on $m_N$ and $\Gamma_N$ but are, in fact, equal in the limit that $N$ goes on-shell, up to off-shell and finite width corrections. Therefore, after phase space integration, one can anticipate highly comparable decay rates despite the relative presence of helicity inversion.

**IV.3. 2 → 4 scattering with $\Delta L = 0$ and $|\Delta L| = 2$**

To extrapolate our findings, i.e., the existence of helicity suppression in $L$-violating decays of $W$ bosons involving (nearly) on-shell Majorana $N$, to other processes, it is helpful to stress that the above arguments are kinematical in nature. They rely on Lorentz invariance, spin correlation, and expansions around leading regions of phase space. They do not rely on strong interference, flavor symmetries, or mixing suppression that one often encounters [29–32, 51–53]. As such, the results are process-dependent and likely do not hold if Eq. 4.15, or an analogous relation, is not satisfied.

With this in mind, one direction where it is possible to extrapolate the above phenomenon is to $2 \to n$ scattering processes. In particular, there is the $L$-conserving
\[ u_L(p_w) \overline{d}_R(p_d) \to W^+_W (p_W) \to \ell_1^+(p_1) N_{\lambda_N} (p_N) \]
\[ \to \ell_1^+(p_1) \ell_2^+(p_2) c_L (p_c) \overline{\tau}_R (p_s), \quad (4.41) \]
as shown in Fig. 1(a), and the \( L\)-violating,

\[
u_L(p_u)a_R(p_d) \rightarrow W^+_W(p_W) \rightarrow \ell^+_1R(p_1)\lambda_{\lambda_L}(p_N) \\
\rightarrow \ell^+_1R(p_1)\ell^+_2R(p_2)\sigma_{\lambda_2}(p_\lambda),
\]

(4.42)
as shown in Fig. 1(b). The novelty of these channels follows from the limitations in the \( W\) decay case study. The first limitation relates to the idealization of working with an unpolarized, on-shell \( W\) boson. This is an object that is never really actualized in nature. By virtue of the \( W\)'s chiral couplings, real \( W\)'s are produced with some degree of polarization \([76–78]\). Likewise, a degree of off-shell virtuality is nearly always present and such contributions are not guaranteed to be negligible if \( m_N \sim M_W\) \([79–82]\).

To check the impact of these matters on the existence of inversion and suppression in Eqs. 4.41–4.42, we again construct the associated MEs. These can be built respectively from the MEs in Eqs. 4.3 and 4.22 by working in the hard scattering frame with momentum assignments

\[
p_u = \frac{Q}{2}(1, 0, 0, 1), \quad p_d = \frac{Q}{2}(1, 0, 0, -1),
\]

(4.43)
\[
p_W = p_1 + p_2, \quad Q^2 = p_W^2 = (p_1 + p_2)^2.
\]

(4.44)

After substituting the \( W\) polarization vector for the current and propagator of the sub-process \( u_La_R \rightarrow W^{+\ast}\),

\[
e_\mu(p_W) \rightarrow J^\sigma(p_u, p_d)\Delta_{\sigma\mu}(p_W = p_u + p_d),
\]

(4.45)

where the propagator \( \Delta\) is the same as in Eq. 4.5 and the \((u_La_R)\) current \( J\) is given by

\[
J^\sigma(p_u, p_d) = \frac{-ig_{W^\ast}\delta^{AB}}{\sqrt{2}} [a^B(p_d)\gamma^\sigma P_L a^A(p_u)],
\]

(4.46)
one obtains the following MEs for the \( L\)-conserving \((M_{L}^{(2\rightarrow4)})\) and \( L\)-violating \((M_{L}^{(2\rightarrow4)})\) scattering processes:

\[
-iM_{L}^{(2\rightarrow4)} = J^\sigma(p_u, p_d)\Delta_{\sigma\mu}(p_u + p_d) \\
\times T^{\mu\nu}_{L}(p_1, p_2, p_N) \\
\times \Delta_{\nu\rho}(p_c + p_s) J^\rho(p_c, p_s),
\]

(4.47)

\[
-iM_{L}^{(2\rightarrow4)} = J^\sigma(p_u, p_d)\Delta_{\sigma\mu}(p_u + p_d) \\
\times T^{\mu\nu}_{L}(p_1, p_2, p_1 + p_c + p_s) \\
\times \Delta_{\nu\rho}(p_c + p_s) J^\rho(p_s, p_c) + (p_1 \leftrightarrow p_2).
\]

(4.48)

To extract the scaling behavior of these two MEs, we exploit the fact that the \( W\)'s longitudinal polarization \((\lambda_W = 0)\), which generates a different mass-energy power counting than transverse polarizations \((\lambda_W = \pm 1)\), does not couple to massless fermions. It does not contribute to the \( ud \rightarrow W^{\ast} \rightarrow N\ell\) sub-process, regardless of external polarizations. Using this and after explicit evaluation of the helicity spinor algebra, we obtain for both cases,

\[
\tilde{J}^\sigma(p_u, p_d)\Delta_{\sigma\mu}(p_u + p_d) = \frac{(\Gamma_{W}W^M)}{\Gamma_{W}W^M} \left( 1 - \mathcal{O} \left( \frac{Q^2 - M_W^2}{\Gamma_{W}W^M} \right) \right),
\]

(4.49)

\[
\tilde{J}^\sigma(p_u, p_d)\Delta_{\sigma\mu}(p_u + p_d) = \frac{Q}{\Gamma_{W}W^M} \left( 1 - \mathcal{O} \left( \frac{Q^2 - M_W^2}{\Gamma_{W}W^M} \right) \right),
\]

(4.50)

For concreteness, we expanded the propagator in the final line around its on-shell limit, i.e., \((Q^2 - M_W^2) \ll \Gamma_{W}W^M\).

It is clear that the substitution in Eq. 4.45 does not introduce any new parity inversion, say via coupling to longitudinal modes, nor any new dependence on \( m_N\). As a consequence, the scaling behavior of the \((\ell^+_1 N \ell^+_2)\) lepton currents and propagators in the scattering process are the same as in the decay process, up to substitutions of the total c.m. energy: \( M_W \rightarrow Q\). Consistently, this causes external momenta to instead scale as \( E_{\text{external}} \sim Q\).

Propagating these modifications, one finds that in the double on-shell limit, the leading contributions to the squared MEs for the \( 2 \rightarrow 4 \) processes scale as

\[
\sum |\mathcal{M}_{L}^{(2\rightarrow4)}|^2 \sim \frac{Q^4p^2_{\tau}}{(\Gamma_{W}W^M)^2}\times \left[ 1 - \mathcal{O} \left( \frac{Q^2 - M_W^2}{\Gamma_{W}W^M} \right) \right],
\]

(4.52)

\[
\sum |\mathcal{M}_{L}^{(2\rightarrow4)}|^2 \sim \frac{Q^4}{(\Gamma_{W}W^M)^2}\times \left[ 1 - \mathcal{O} \left( \frac{Q^2 - M_W^2}{\Gamma_{W}W^M} \right) \right].
\]

(4.53)

As in the \( 1 \rightarrow 4 \) decays, we find that the helicity inversion in \( 2 \rightarrow 4 \) scattering does not manifest as helicity suppression. In fact, we find again that the squared ME for the \( L\)-conserving and \( L\)-violating processes are the same, up to off-shell virtualities. Subsequently, in the absence of phase space cuts one obtains identical cross sections.

As a brief remark, we caution that at next-to-leading order in EW it may be that differences in the squared MEs for the \( 2 \rightarrow 4 \) decays, we find that the helicity inversion effects are the same, up to substitutions of the total c.m. energy: \( M_W \rightarrow Q\). Consistently, this causes external momenta to instead scale as \( E_{\text{external}} \sim Q\).

\[\text{IV.4. Other Scenarios with } |\Delta L| = 2\]

A second direction where one can apply the above findings is to other new physics scenarios that feature chiral gauge interactions and Majorana fermions. While a systematic survey is beyond the present scope of this work, two concrete examples are: warped extra dimension with RH Majorana neutrinos \((\nu_R)\) \([83–85]\) and the Left-Right Symmetric model (LRSM) \([6, 13, 17–20]\).
The first is characterized by Kaluza-Klein (KK) excitations of SM particles as well as of $\nu_R$. This includes, for example, $W_{KK}^\prime$ gauge bosons, which have the same chiral interaction structure and gauge quantum numbers as the SM W boson. After mass-diagonalization, the resulting Lagrangian that governs interactions between the mass eigenstates $N_{KK}^\prime$, $W_{KK}^\prime$, and $\nu_{KK}^\prime$ is essentially the same as Eq. 2.4, up to an overall rescaling of couplings.

Phenomenologically speaking, this allows processes like those shown in Fig. 1 but with internal particles substituted with their KK excitations. Corresponding MEs and squared MEs are therefore the same as those constructed in Secs. IV.1-IV.3, up to substitutions of mass and coupling constants, implying the presence of helicity inversion. So long as external particles are massless and the near on-shell condition of Eq. 4.15 is satisfied, one should consistently find an absence of helicity suppression, modulo off-shell virtuality and finite width effects.

In the second case, the LRSM is characterized by embedding the SM's $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry into the larger symmetry group, $G_{LRSM} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_B - L \otimes P$. In this model, all of the SM’s RH chiral fields and $\nu_R$ are charged under the $SU(2)_R$ gauge group, just as their LH counter parts are charged under $SU(2)_L$. The $U(1)$ conservation of baryon-minus-lepton numbers ($B - L$) ensures that the theory is anomaly free and the generalized discrete parity $P$ ensures that the LH and RH gauge interactions are identical before spontaneous symmetry breaking.

After LR and EW symmetry breaking, one finds heavy RH gauge bosons $W_R$ that couple to heavy Majorana neutrinos $N$ and charged leptons $\ell$ through RH chiral currents, in analogy to the Lagrangian in Eq. 2.4. This leads to the spectacular $L$-violating scattering process [41]

$$u_R d_L \rightarrow W_R^+ \rightarrow \ell^+_L N \rightarrow \ell^+_L \ell^+_L W_R^- \rightarrow \ell^+_L \ell^+_L \tau_{R,L}. \quad (4.54)$$

This associated diagram is the same as in Fig. 1(b) but with substituting the SM gauge boson W for LRSM gauge boson $W_R$. Explicit computation reveals a ME that is identical to the $L$-violating ME of Eq. 4.48, up to substitutions of masses and couplings as well as the exchange of $P_L$ chiral projection operators for the operator $P_R$.

Importantly, the consistent application of the Feynman rules of Ref. [71, 72] requires the vertex modifications

$$\gamma^\nu P_R \rightarrow (-1)\gamma^\nu P_L \quad (4.55)$$

$$\tau_L(p_2) \rightarrow \tau_L(p_2). \quad (4.56)$$

This leads to an explicit helicity inversion at the ME level as in the Phenomenological Type I Seesaw. Assuming that the near on-shell condition for intermediate resonances is satisfied, one again finds that the explicit dependence on $m_{\nu}^2$ cancels. Again, this leads to an absence of helicity suppression, up to the now-usual caveats.

For completeness, one could also consider the mixed $W_R - W_L$ scattering process given by [33]

$$u_R d_L \rightarrow W_R^+ \rightarrow \ell^+_L N \rightarrow \ell^+_L \ell^+_L W_R^- \rightarrow \ell^+_L \ell^+_L \tau_{R,L}. \quad (4.57)$$

In this case, one finds a second helicity inversion due to inverting the chiral coupling associated with the second charged current. This implies that the roles are now reversed: the $L$-violating process exhibits a net helicity conservation while the $L$-conserving process exhibits a net helicity inversion. Explicit calculation [33] again shows a lack of helicity suppression in the near on-shell limit.

V. NUMERICAL IMPACT OF HELICITY INVERSION IN $|\Delta L| = 2$ LHC PROCESSES

In light of the previous section, the question is not whether there is helicity inversion in $|\Delta L| = 2$ amplitudes mediated by Majorana neutrinos in the Phenomenological Type I Seesaw. It exists and follows from a parity inversion of EW interactions. The more pertinent issue is whether contributions from off-shell virtualities and finite widths, which can potentially give rise to helicity-suppressing behavior, is numerically relevant for standard searches for EW-scale Majorana $N$ at the LHC.

To investigate this, we consider two complementary measures of helicity suppression. The first, presented in section V.3, is based the potential asymmetry that could develop in $L$-conserving and $L$-violating decays of the SM W boson. The second, presented in section V.4, is the analogous asymmetry that can appear in hadronic $2 \rightarrow 4$ cross sections. Before presenting our numerical results, we comment in Sec. V.1 on the preservation of spin-correlation in our computations and then validate the presence of strong helicity inversion in Sec. V.2.

V.1. Numerical preservation of spin-correlation

To undertake our numerical computations we exploit the massive spinor helicity formalism of Refs. [63-66] as implemented in the ALOHA package [44, 59], in the HELAS basis [64]. (For precise details of the computational setup, see section III.) We do so in order to evaluate MEs exactly but at the cost of analytical expressions.

We forego analytical expressions due to the fact that we are dealing with multi-scale, $1 \rightarrow 4$ and $2 \rightarrow 4$ processes. The squared MEs for these processes must be amended with kinematic factors and integrated over phase space to derive total decay widths ($\Gamma$) and cross sections ($\sigma$), i.e., the quantities considered here. In the absence of strong assumptions like the narrow width approximation (NWA), phase space integration usually leaves intractable algebraic expressions for such processes. However, we avoid employing the NWA since its rigorous justification for EW-scale Majorana neutrinos is
not well-established in the literature. On the contrary, studies into the validity of the NWA itself list criteria that may not be satisfied here [60, 79–82], and even show [79] a sizable impact on the spin-propagated by Majorana fermions. While important, such considerations are outside our scope and deferred to later work.

V.2. Numerical validation of helicity inversion

As a first step to quantifying potential helicity suppression in $|\Delta L| = 2$ transitions, we move to establish that our computational setup captures the helicity inversion in such processes. To demonstrate this and in the notation of Sec. IV we consider the simpler $1 \rightarrow 2$ decay

$$W^+ (\lambda_W) \rightarrow e^+ (\lambda_e) N (\lambda_N)^+ ,$$  \label{eq:5.1}

for each helicity permutation ($\lambda_W, \lambda_e, \lambda_N$) in Tab. I.

Several notable features can be identified in the MEs of Tab. I. First is that all amplitudes for $e^+ (\lambda_e = L)$ are zero, which is consistent with $W$ bosons only coupling to massless LH particles (RH antiparticles). Second is that amplitudes for $\lambda_W = \pm 1$ and $\lambda_N = L$ feature the characteristic $(1 \pm \cos \theta)$ behavior associated with vector currents. Third, and most relevant, is that amplitudes for $\lambda_N = R$ scale with the mass of $N$, i.e., $-iM \sim m_N$, whereas amplitudes for $\lambda_N = L$ scale with the energy of $N$, i.e., $-iM \sim E_N \sim M_W$, as one would expect for helicity inversion of massive decay products.

Using the definition of the partial decay width for the unpolarized particle $B$ with mass $m_B$ into final-state $f$, 

$$\Gamma(B \rightarrow f_{\lambda_f}) = \frac{1}{2m_B S_B \rho_{\lambda_f}} \sum_{\lambda_f} |\mathcal{M}(B_{\lambda_B} \rightarrow f_{\lambda_f})|^2 ,$$  \label{eq:5.5}

we report in Tab. II the partial width $\Gamma(\lambda_W, \lambda_e, \lambda_N)$ for each permutation of helicities ($\lambda_W, \lambda_e, \lambda_N$). We note that, for consistency, the spin-averaging factor of $S_W = 3$ is not included in $\Gamma(\lambda_W, \lambda_e, \lambda_N)$. This implies that the canonical spin-averaged total is related by

$$\Gamma(W^+ \rightarrow e^+ N) = \frac{1}{S_W} \sum_{\lambda_e} \Gamma(W^+_{\lambda_W} \rightarrow e^+_{\lambda_e} N_{\lambda_N}) .$$  \label{eq:5.6}

Likewise, the partial and total widths of $W$ are related to its branching rate (BR) by the usual definition

$$\text{BR}(W \rightarrow f) \equiv \frac{\Gamma(W \rightarrow f)}{\Gamma_W} = \sum_{X} \frac{\Gamma(W \rightarrow X)}{\Gamma_W} .$$  \label{eq:5.7}

In comparison to the MEs, we observe in the partial widths listed in Tab. II that several kinematic features are washed out after phase space integration. In particular, the characteristic $(1 \pm \cos \theta)$ behavior and sensitivity to the azimuthal angle $\phi_e$ are no longer manifest. What remains, however, is the relative dependence on the heavy neutrino’s mass. For the $\lambda_N = L$ cases, we see that the ME’s linear power dependence on $M_W$ remains linear in the partial widths. The quadratic power one obtains at the squared ME level is canceled by the explicit mass factor in the definition of $\Gamma$ in Eq. 5.5. For the $\lambda_N = R$ cases, the linear power dependence on $m_N$ at the ME level grows at the squared ME level, and leads the partial widths to scale as $\Gamma \sim m_N (m_N / M_W)$. Interestingly, this shows that in the fixed $m_N$ but large $M_W$ limit, the MEs for $\lambda_N = R$ marginally grow and converge, whereas the partial widths vanish. This behavior is consistent with expectations from the Confusion Theorem.

In taking the ratio of the $W^+ \rightarrow e^+ N_{\lambda_N}$ branching rates, we can extract the helicity suppression of $\lambda_N = R$...
5.11 We find that the same behavior holds for all V.2 We plot Fig. 2. The W$^+ \rightarrow e_R^+N_R$ and W$^+ \rightarrow e_L^+N_L$ branching ratio as a function of $m_N$ [GeV], as computed numerically from polarized matrix elements (solid) and analytically (dashed).

Helicity states at small $(m_N/M_W)^2$, and verify the modeling in our setup. Analytically the ratio is given by

$$R = \frac{BR(W^+ \rightarrow e_R^+N_R)}{BR(W^+ \rightarrow e_L^+N_L)} = \frac{\Gamma(W^+ \rightarrow e_R^+N_R)}{\Gamma(W^+ \rightarrow e_L^+N_L)} \approx \frac{1}{2} \left( \frac{m_N}{M_W} \right)^2. \quad (5.8)$$

In Fig. 2 we plot $R$ as a function of heavy neutrino mass $m_N$ [GeV] as computed numerically from polarized matrix elements (solid line) and analytically (dashed line). For heavy neutrino masses in the range of $m_N \in [1\textrm{ GeV}, 75\textrm{ GeV}]$ we find that $R$ spans 3-4 orders of magnitude. Over this entire range we find excellent agreement between our numerical setup and exact analytic expectations. This provides nontrivial checks that (i) helicity inversion for viable values of heavy neutrino masses can be numerically significant, and (ii) our computational setup successfully captures such behavior.

Briefly, we note that we do not consider Majorana neutrinos with masses below $m_N = 1$ GeV. For such states the relevant virtuality scales are comparable to the non-perturbative scale of QCD. Hence, one should treat the decays of lighter sterile neutrinos, i.e., for $m_{N_\ell} \lesssim 1 - 10$ GeV, like decays of $\tau$ leptons and adopt a low-energy, effective field theory, as done for example in Refs. [25, 39, 86]. This introduces additional parity nuances that have been considered elsewhere [39].

V.3. Total Width Asymmetry

As our first measure of helicity suppression in LHC observables for processes that are mediated by heavy Majorana neutrinos, we consider respectively the L-conserving and L-violating, 1 $\rightarrow$ 4-body $W$ boson decay processes,

$$\Delta L = 0 : W^+ \rightarrow e_1^+N \rightarrow e_1^+e_2^+\ell_3\ell_4, \quad (5.11)$$
$$|\Delta L| = 2 : W^+ \rightarrow e_1^+N \rightarrow e_1^+e_2^+\bar{\ell}_s. \quad (5.12)$$

Here we fix final-state flavors for definiteness. Diagrams with $\gamma^*/Z^*$ exchange are removed in a gauge-invariant manner, resulting in those shown in Fig. 1. Interfering diagrams from identical particle exchange are kept.

In Sec. IV, we argued that the ME for these processes exhibit different parametric dependencies on $m_N$ due helicity inversion. At the same time we showed in Sec. V.2 that Lorentz invariance lead to the same parametric dependence in squared MEs, in the on-shell limit for $N$. Differences in decay rates were found to be proportional to the off-shell virtuality of $N$ as well as to its total width.

To address the importance of these terms and quantify the existence of any such helicity suppression, we consider the following asymmetry $A_T$ in branching rates:

$$A_T = \frac{BR(W^+ \rightarrow e^+e^-\ell^-\bar{\ell}^-) - BR(W^+ \rightarrow e^+e^+\ell^-\bar{\ell}^-)}{BR(W^+ \rightarrow e^+e^-\ell^-\bar{\ell}^-) + BR(W^+ \rightarrow e^+e^+\ell^-\bar{\ell}^-)} \approx \frac{\Gamma(W^+ \rightarrow e^+e^-\ell^-\bar{\ell}^-) - \Gamma(W^+ \rightarrow e^+e^+\ell^-\bar{\ell}^-)}{\Gamma(W^+ \rightarrow e^+e^-\ell^-\bar{\ell}^-) + \Gamma(W^+ \rightarrow e^+e^+\ell^-\bar{\ell}^-)} \approx \frac{\Gamma_{NC} - \Gamma_{LNV}}{\Gamma_{NC} + \Gamma_{LNV}}. \quad (5.13)$$

In Fig. 3(a) we show the decay rate asymmetry $A_T$ between the L-conserving and L-violating $W^+$ boson decays given in Eqs. 5.11-5.12, as a function of $m_N$ [GeV] for representative active-sterile neutrino mixing $|V_{eN}|^2 = 1$ (solid), $10^{-2}$ (dash-dot), and $10^{-4}$ (dash). Also shown is the associated statistical MC uncertainty band $(\delta A_{MC})$. Based on $N = 100k$ events per determination of $\Gamma$ we obtain a statistical MC uncertainty that is nearly uniform and is approximately $\delta A_{MC} \approx 2.2 \times 10^{-3}$.

For heavy neutrino masses in the range of $m_N \in [1\textrm{ GeV}, 75\textrm{ GeV}]$ we report asymmetries consistent with $A_T = 0$, i.e., no asymmetry and hence no helicity suppression. More precisely, we find nonzero $A_T$ that fluctuate above and below zero, reaching at most $|A_T| \sim O(1 \times 10^{-3})$, and are consistent with random, statistical noise$^3$. We find that the same behavior holds for all representative choices of active-sterile mixing.

V.4. Total Cross Section Asymmetry

As our second measure of helicity suppression in LHC observables, we consider the generalization of the $W$ bo-

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3 While we use the uncertainty estimator $\delta O/O = \delta N/N = 1/\sqrt{N}$, for $O = \Gamma, \sigma$, it is actually an upper limit on the MC uncertainty due to the sampling and reweighting routines in mgame [44].
we show the cross section asymmetry would arise if helicity inversion on particle.

\begin{align}
\Delta L = 0 & : \bar{u}d \to W^+ \to e^+_1 N \to e^+_1 d^+_2 \bar{e} s, \\
|\Delta L| = 2 & : \bar{u}d \to W^+ \to e^+_1 N \to e^+_1 d^+_2 \bar{e} s.
\end{align}

We again fix external particle flavors for definiteness and to also avoid interference with the WW scattering process. Diagrams involving $\gamma^*/Z^*$ exchange are removed in a gauge-invariant manner, while interfering diagrams from identical particle exchange are kept. As discussed in Sec. 4.3, the utility of these processes is that they capture polarization and virtuality effects present in real LHC collisions but not in the idealized decays of Sec. V.3.

In analogy to $A_V$, we use the scattering processes above to build an asymmetry $A_\sigma$ that would arise if helicity suppression were to exist. Specifically, we consider

\begin{align}
A_\sigma & \equiv \frac{\sigma(\bar{u}d \to e^+ e^- \bar{c} \bar{s}) - \sigma(\bar{u}d \to e^+ e^+ \bar{c} \bar{s})}{\sigma(\bar{u}d \to e^+ e^- \bar{c} \bar{s}) + \sigma(\bar{u}d \to e^+ e^+ \bar{c} \bar{s})} \\
& \equiv \frac{\sigma_{\text{LNC}} - \sigma_{\text{LNV}}}{\sigma_{\text{LNC}} + \sigma_{\text{LNV}}}.
\end{align}

Here we abuse slightly the conventional notation for hadronic cross sections $\sigma(pp \to B)$ and write explicitly

\begin{equation}
\sigma(\bar{u}d \to B) = f_{u/p} \otimes f_{d/p} \otimes \hat{\sigma}(\bar{u}d \to B), \tag{5.20}
\end{equation}

to denote that we consider only the $\bar{u}d$ partonic contribution to $pp$ scattering, with $f_{i/p}$ representing the PDF for parton $i$ in hadron $p$, and $\hat{\sigma}$ as the parton-level scattering rate. This is given by the standard expression,

\begin{equation}
\hat{\sigma}(ij \to B) = \frac{1}{2Q^2 S_i S_j N_i^N N_j^N} \times \int dP S B \sum_{\text{dof}} |M(ij \to B)|^2. \tag{5.21}
\end{equation}

To avoid potential washout from beam symmetrization, we do not consider the $\bar{d}u$ partonic channel.

In Fig. 3(b) we show the cross section asymmetry $A_\sigma$ between the L-conserving and L-violating scattering processes in Eqs. 5.16-5.17, as a function of heavy neutrino mass $m_N$ [GeV] for representative active-sterile neutrino mixing $|V_{eN}|$.

Remarking, for heavy neutrino masses in the range of $m_N \in [1 \text{ GeV}, 75 \text{ GeV}]$ we report asymmetries that are statistically consistent with $A_\sigma = 0$, i.e., no asymmetry and hence. We find that the same behavior holds for both representative choices of active-sterile mixing.

\section{VI. SUMMARY AND CONCLUSIONS}

Whether or not neutrinos are Majorana fermions remains one of the most pressing open questions in particle physics today. If neutrinos are their own antiparticle, then it is likely that new particles and interactions play a role in generating neutrino masses that are hierarchically smaller than the EW scale. Hence, establishing the Majorana nature of neutrinos is a stepping stone to more fully understanding the fundamental symmetries of nature.

In this study, we report an analytical and numerical investigation into the impact of helicity inversion on par-
tial widths and cross sections of $|\Delta l| = 2$ processes at the LHC. We focus as a case study on $L$-conserving and $L$-violating, 4-body decays of $W$ bosons mediated by a heavy Majorana neutrino $N$ in the Phenomenological Type I Seesaw model. After isolating the relative helicity continuation (inversion) in the $L$-conserving (violating) process at the ME level in Sec. IV.1 (IV.2), we show that up to sub-leading off-shell and finite-width effects an identical dependence on $N$'s mass ($m_N$) emerges at the squared ME level due to the different scaling of 4-momenta and squared 4-momenta. When $N$ goes on-shell, we find that this mass dependence precisely cancels. This renders total decay and scattering rates equal and non-zero, even when $m_N$ vanish. In Sec. IV.3, we show that this is extendable to $2 \rightarrow 4$ scattering processes, and in Sec. IV.4 to other scenarios, so long as consistent propagation of helicity inversion is taken into account.

In Sec. V we perform exact numerical ME computations using the MC event generator MadGraph5_aMC@NLO in conjunction with the HeavyN model libraries. Starting in Sec. V.2, we confirm the strong presence of helicity inversion in the $W \rightarrow N\ell \nu$ decay process using our MC setup. We then move onto the more general $1 \rightarrow 4$ decay and $2 \rightarrow 4$ scattering processes in Secs. V.3 and V.4, respectively. After building asymmetries ($\mathcal{A}$) sensitive to helicity suppression and enhancements in $L$-violating processes, we report the absence of numerically significant helicity suppression despite the presence of helicity inversion, in agreement with theoretical expectations presented in Sec. IV. We find no evidence that off-shell and finite-width contributions are numerically important. Lastly, we note that more could be learned by further investigations into finite width effects as well as into the potential loop-level generation of helicity asymmetries $\mathcal{A}$. We strongly encourage future studies.

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