Abstract
We study the bulk viscosity of strange quark matter (SQM) in a quasiparticle model at finite chemical potential by extrapolating the previous quasiparticle model of finite temperature lattice QCD. The more proper bulk viscosity coefficient can be given in this model where chemical potential $\mu$ and coupling constant $g$ are interdependent. We also apply our result to determine the critical rotation of strange stars by r-mode instability window. Our model is compatible to the millisecond pulsar data for a wide range of mass and radius of the stars.

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1 INTRODUCTION

According to Witten’s conjecture that the strange quark matter (SQM) composed of roughly equal numbers of up, down and strange quarks might be absolutely stable or metastable phase of nuclear matter in 1984 [1], strange stars might exist in the universe, and their structure and properties have been widely studied [2, 3, 4, 5, 6, 7].

How to distinguish strange stars from neutron stars has been one of the important issues. Madsen pointed out that the r-mode instability in the relativistic stars at all rotating rates may provide a distinguishable signature [8]. The main reason is that the bulk viscosity coefficient of SQM is larger than that of neutron matter, and hence suppresses the r-modal s. Since Wang and Lu [9] found that the non-leptonic weak reaction dominates the bulk viscosity of SQM, some investigations have tried to calculate the relevant viscosity coefficient of SQM. The equation of state (EOS) of strange stars has also been studied as the base of studying the viscosity. Sawyer [2] and Madsen [3] completed the calculations of bulk viscosity for the ideal quark gas. We had investigated the viscosity of interacting SQM in quasiparticle description by regarding the coupling constant as an independent parameter and found the improved result is of importance for astrophysical relevance [10]. However it is desirable to have a more consistent investigation by considering the realistic running coupling constant for SQM in the interior of compact stars. According to the regulations, the realistic coupling constant and EOS need nonperturbative evaluations, i. e. lattice QCD calculations. However, the present lattice QCD calculations are yet restricted to zero chemical potential $\mu$. Attempts to extend the lattice calculations systematically to non-zero chemical potential are underway. Being
aware of the urgent need for the EOS, many works have suggested an approach based on a quasiparticle description of quarks and gluons, to map available lattice data from $\mu = 0$ to finite values of $\mu$ and to small temperatures [11]. Consequently, the running coupling in the asymptotic limit of large chemical potential can be also simulated from that of large temperatures at zero chemical potential, which will also help us to overcome the difficulty of calculating EOS in non-perturbation regime. We thus can consistently evaluate the bulk viscosity in the light of the simulated EOS and running coupling constant. We will see below the coupling among quarks influences remarkably the bulk viscosity in SQM. Our theoretical output is appropriate for a wide range of stellar parameters.

We organize this paper as follows. In Sec.2, we introduce the equation of state and formulate the quasiparticle model at finite chemical. In Sec.3, the bulk viscosity of SQM in the quasiparticle model is derived, which arises from the nonleptonic weak interaction. In Sec.4, we probe the application of our model. Finally, we summarize our conclusion and discussion in Sec.5.

2 THE QUASI-PARTICLE MODEL

Matter in local thermodynamical equilibrium can be described by its EOS which represents an important interrelation of state variables. For the EOS of SQM, non-perturbative methods as lattice QCD should be applied. As a matter of fact, these simulations are presently restricted to vanishing chemical potential $\mu$, and the implementation of physical quark masses is still too expensive numerically. So we often consider phenomenological model. The bag model EOS is popular, but it is in conflict with thermodynamic QCD lattice data. We will apply the quasiparticle model and here repeat a simple extrapolation of finite temperature lattice to nonzero chemical potential relying on thermodynamic consistency [11].

Asymptotically, the collective behavior of the plasma can be interpreted in terms of quasiparticle excitations with a dispersion relation $\omega_i^2(k) \approx m_i^2 + k^2$ and $m_i^2 = m_0^2 + \Pi_i^*$ depending on the rest mass and the leading order on-shell self-energies [12],

$$\Pi_q^* = 2\omega_{q0}(m_0 + \omega_{q0}), \quad \omega_{q0}^2 = \frac{N_c^2 - 1}{16N_c} \left[T^2 + \frac{\mu^2}{\pi^2}\right] g^2,$$

$$\Pi_g^* = \frac{1}{6} \left[(N_c + \frac{1}{2}N_f)T^2 + \frac{3}{2\pi^2} \sum_q \mu_q^2\right] g^2,$$  

where $\mu_q$ denotes the quark chemical potential, and $N_c=3$. The pressure of quasiparticle system can be decomposed into the contributions of the quasiparticles and their mean-field interaction $B$,

$$p(T, \mu; m_0^2) = \sum_i p_i[T, \mu_i(\mu); m_i^2] - B(\Pi_i^*),$$

where $p_i = \pm d_i T \int d^3k/(2\pi)^3 \ln(1 \pm \exp(-\omega_i - \mu_i)/T)$ are the contributions of the gluons (with vanishing chemical potential) and the quarks (for the anti-quarks, the
chemical potential differs in the sign). And \( d_g = 2(N_c^2 - 1) \) and \( d_g = 2N_c \) count the degrees of freedom \([13]\). The function \( B(\Pi^*_j) \) is determined from a thermodynamical self-consistency condition, via

\[
\frac{\partial B}{\partial \Pi^*_j} = \frac{\partial p_j(T, \mu_j; m^2)}{\partial m^2}.
\] (4)

Furthermore, the stationarity implies that the entropy and the particle densities are given by the sum of the quasiparticle contributions,

\[
s_i = \left. \frac{\partial p_i(T, \mu_i; m^2)}{\partial T} \right|_{m^2}, \quad n_i = \left. \frac{\partial p_i(T, \mu_i; m^2)}{\partial \mu_i} \right|_{m^2},
\] (5)

while the energy density has the form \( e = \sum_i e_i + B \). By comparison with lattice data, at \( \mu = 0 \), Peshier\([11]\] has tested the quasiparticle approach which is an appropriate description even close to the confinement transition, with the effective coupling in Eq (1) and (2) nonperturbatively parameterized by

\[
g^2(T, \mu = 0) = \frac{48\pi^2}{(11N_c - 2N_f) \ln \left(\frac{T + T_s}{T_c/\lambda}\right)^2},
\] (6)

interpolating to the asymptotic limit of QCD.

Encouraged by the successful quasiparticle description of the \( \mu = 0 \) lattice data, the model is now extrapolated to finite chemical potential. In general, the pressure is a function of the state variables \( T \) and \( \mu \). As a direct consequence thereof, the Maxwell relation implies for the quasiparticle model

\[
\sum_i \left[ \frac{\partial n_i}{\partial m^2} \frac{\partial \Pi^*_i}{\partial T} - \frac{\partial s_i}{\partial m^2} \frac{\Pi^*_i}{\partial \mu} \right] = 0,
\] (7)

which is the integrability condition for the function \( B \) defined by Eq (4). Following directly from principles of thermodynamics, we can get a flow equation for the effective coupling with \( \Pi^*_i \) depending on \( g^2 \). This flow equation is a quasilinear partial differential equation of the form

\[
a_T \frac{\partial g^2}{\partial T} + a_\mu \frac{\partial g^2}{\partial \mu} = b,
\] (8)

with the coefficients \( a_T, a_\mu \) and \( b \) depending on \( T, \mu \) and \( g^2 \).

The flow of the effective coupling is elliptic in the nonperturbative regime, thus mapping the \( \mu = 0 \) axis, where \( g^2 \) can be determined from lattice data, into the \( \mu - T \) plane. As an example, the nonperturbative flow of the coupling of the \( N_f = 2 + 1 \) system is shown in Figure.1. Here we use the parameters \( \lambda = 6.6 \) and \( T_s = -0.78T_c \) \([14]\]. Furthermore, the running coupling constant as a function of chemical potential \( g(T = 0, \mu) \) is displayed in Figure.2. For convenience later, we can approximately formulate \( g(T = 0, \mu) \) as

\[
g^2(T = 0, \mu) = \frac{48\pi^2}{(11N_c - 2N_f) \ln \left(\frac{T + T_s}{T_c/\lambda}\right)^2},
\] (9)
Based on the extended coupling constant, the EOS of SQM in the interior of compact stars is immediately obtained from Eqs (1), (2), (3) and (4).

Figure 1: The characteristics of the coupling flow equation (8) for the QCD plasma with \( N_f = 2 + 1 \) flavors.

3 BULK VISCOSITY

Viscosity is important for describing the transport property of matter and can be generally calculated from either quantum field theory or kinetic theory [15]. The bulk viscosity of SQM mainly arises from the nonleptonic weak interactions [9]

\[ u + d \leftrightarrow s + u. \]  

(10)

The importance of dissipation due to the reaction (10) was first stressed by Wang and Lu in the case of neutron stars with quark cores. Afterward, a series of investigations have tried to calculate the viscosity coefficient of SQM in MIT bag model and showed a huge bulk viscosity of strange matter relative to nuclear matter. Furthermore, Zheng et al [10] considered the medium effect on the bulk viscosity in quasiparticle description and found the viscosity is few~tens times larger than that of non-interacting quark gas due to the small influence of medium on the equation of state, where coupling constant \( g \) is regarded as an independent parameter of chemical potential. In fact, \( \mu \) and \( g \) should be interdependent according to the results of lattice calculation. We here reevaluate the bulk viscosity coefficient adopting the description in Sec.2.
Figure 2: The coupling constant $g(T = 0, \mu)$ as a function of $\mu$.

In quasiparticle approximation, we can give the formulae of the bulk viscosity in accordance with Eqs (1), (2), (3) and the reactional rate of reaction (10) at finite chemical potential when temperature is small. Zheng et al [10] have made derivations of bulk viscosity in quasiparticle description. We can apply the result, 

$$\zeta = \frac{1}{\pi v_0} \left( \frac{v_0}{\Delta v} \right) \frac{1}{3} \left( \frac{k_{Fd}^2}{C_d} - \frac{k_{Fs}^2}{C_s} \right) \int_0^\infty dt \left[ \int_0^t \frac{dn_d}{dt} \right] \cos \left( \frac{2\pi t}{\tau} \right), \quad (11)$$

where $k_{Fi} = (\mu_i^2 - m_i^2)^{1/2}$, $C_i = \mu_i - \sqrt{\frac{1}{3} \frac{g_{mn}}{\pi}} - \frac{1}{3} \frac{\mu}{\pi} g^2 - \sqrt{\frac{1}{6} \frac{um_i \partial g}{\pi} + \frac{1}{6} \frac{\mu^3}{\pi} g^2}$, the expression of $C_i$ has been reevaluated, differing from that of Zheng et al [10] because $g$ is the function of $\mu$. The effective mass $m_i$ was given in Sec.2, the current masses vanish for up and down quarks. We continue to use the formula of the reactional rate adopted by Madsen [3]

$$\frac{dn_d}{dt} \approx \frac{16}{5\pi^2} g_F^2 \sin^2 \theta_e \cos^2 \theta_e \mu_3^2 \delta \mu^2 + 4\pi^2 T^2 v_0, \quad (12)$$

with [10]

$$\delta \mu = \frac{1}{3} \left( \frac{k_{Fd}^2}{C_d} - \frac{k_{Fs}^2}{C_s} \right) \Delta v \sin \left( \frac{2\pi t}{\tau} \right) - \frac{\pi^2}{3} \frac{1}{v} \left( \frac{1}{k_{Fd} C_d} - \frac{1}{k_{Fs} C_s} \right) \int_0^t \frac{dn_d}{dt}. \quad (13)$$

When the temperature is high enough, i.e. $2\pi T \gg \delta \mu$, the cubic term $\delta \mu^3$ in Eq (12) can be neglected. We can obtain an analytical result which is similar to Madsen’s
expression (3).

\[ \zeta = \frac{\alpha^* T^2}{\omega^2 + \beta^* T^4} \]  

(14)

and

\[ \alpha^* = 9.39 \times 10^{22} \mu_d^5 \left( \frac{k_F d}{C_d} - \frac{k_F s}{C_s} \right)^2 (g \ cm^{-1} s^{-1}), \]  

(15)

\[ \beta^* = 7.11 \times 10^{-4} \left[ \frac{\mu_d^5}{2} \left( \frac{1}{k_F d C_d} - \frac{1}{k_F s C_s} \right) \right]^2 (s^{-2}), \]  

(16)

here \( \omega = \frac{2\pi}{\tau} \). \( \alpha^* \) and \( \beta^* \) remarkably differ from Madsen's.

In general, Eqs (11), (12) and (13) must be solved numerically due to the existence of \( \delta \mu^3 \) in rate (12). The results of such calculations are shown in Figure 3 and Figure 4. The magnitude of viscosity coefficient increases with chemical potential for the low-temperature case such as \( T = 10^{-4} \) MeV in Figure 3. It is just contrary to the high-temperature case as \( T = 10^{-1} \) MeV in Figure 3. This implies a shift of the maximal viscosity to the low-temperature when chemical potential increases as shown in Figure 4. By comparison, the viscosity has more complicated dependence on coupling constant than that of ref [10].

Figure 3: Bulk viscosity coefficient of different \( \mu \) and \( T \), \( m_0 = 200 \) MeV, \( \tau = 0.001 \) s. The lower curves denote \( T = 10^{-4} \) MeV, the upper curves denote \( T = 10^{-1} \) MeV.
Figure 4: Bulk viscosity as function of temperature for different $\mu$, $m_{ss}=200$ MeV, $\tau=0.001$ s, $\Delta u = 10^{-4}$.

4 CRITICAL ROTATION OF STRANGE STARS

Given the EOS of SQM, we can get radius-mass characteristic relationship of strange stars through the Tolman-Oppenheimer-Volkov (TOV) equation [16] for energy densities up to several times the nuclear saturation density and at temperatures less than some 10 MeV, the relation $e(p)$ of $\beta$-stable quark matter can be parametrized [11] by $e = 4B + \tilde{\alpha}p$, as estimated numerically by our quasi-particle model, here energy density $e = e_u + e_d + e_s + e_e + B_0$ and pressure $p = p_u + p_d + p_s + p_e - B_0$. For the chosen parameters in Sec.2 and the bag constant $B_0^{1/4} = 145$ Mev, we can get $\tilde{\alpha} = 3.5$ and $\tilde{B}^{1/4} = 152$ MeV in our EOS. Furthermore, we can find corresponding values of the parameter $B$ by changing the bag constant $B_0$. The values of $B$ have a strong impact on the star’s mass and radius, obtained by integrating the TOV equation. In Figure 5, the mass as a function of the radius of strange stars is displayed for several values of the parameter $B$. The maximal mass and radius are essentially consistent with canonical pulsar data for $B_0^{1/4} \sim 145$ Mev and $\tilde{B}^{1/4} \sim 152$ MeV.

Now we focus on the r-mode unstable window for this case. Andersson in 1998 [17] recognized the existence of unstable r-modes in perfect fluid stars at all rates of
rotation due to gravitational wave emission. The r-mode unstable regime of the realistic stars, neutron stars as well as strange stars, depends on the competition between the gravitational radiation and various dissipation mechanisms. To plot the instability window of r-mode or obtain the critical rotation frequency for given stellar model, we need to work out the characteristic timescales, damping and growing timescales of r-mode instability. The time scale for gravity wave emission is

$$\tau_G = \tau_G(\pi G\overline{\rho}/\Omega^2)^3$$

(17)

where, $\tau_G$ is -3.26 s for $n = 1$ polytropic EOS, $\overline{\rho}$ is the mean density of the star. In strange stars, the time scales for shear and bulk viscous dissipations can be respectively given

$$\tau_S = \tau_S(\alpha_s/0.1)^{5/3}T_9^{5/3}$$

(18)

$$\tau_B = \tau_{BL}(\pi G\overline{\rho}/\Omega^2)T_9^{-2}$$

(19)

for low-$T$ limit,

$$\tau_B = \tau_{BH}(\pi G\overline{\rho}/\Omega^2)^2T_9^2$$

(20)

for high-$T$ limit, where $\tau_s$ is $5.37 \times 10^9$ s for $n=1$, $\tau_{BL} = 2.83 \times 10^3 \alpha^{1/2}(\pi Gm_{100}^4)^{1/2}$, $\alpha_s = \frac{g^2}{4\pi}$ is QCD fine structure constant, $T_9$ denotes temperature in units of $10^9$ K, $m_{100}$ denotes the current mass of strange quark in units of 100 MeV. Obviously, $\tau_B$ is determined with the chemical potential $\mu_d$. For given mean density $\overline{\rho}$, We can find appropriate $\mu_d$ and $g$ via conservation of baryon number,

$$n_B = \frac{1}{3}(n_u + n_d + n_s)$$

(21)

and

$$\overline{\rho} = \left(\frac{E}{A}\right)n_B,$$

(22)

where, $n_B$ represents the baryon number density, and $n_i = \frac{1}{6\pi^2}k_{F_i}^3$ is the flavor number density, $\frac{E}{A} = e_u + e_d + e_s + e_e + B_d$ is the energy per baryon.

We can evaluate the the critical spin frequency as a function of temperature from the equation

$$\frac{1}{\tau_G} + \frac{1}{\tau_S} + \frac{1}{\tau_B} = 0.$$

(23)

Figure.6 shows the regions of r-mode (in)stability in spin frequency-temperature($\nu - T$) plane for a strange star with mass $M = 1.4 M_\odot$ and radius $R = 10$ km. In comparison, the instability window of usual strange stars is also depicted in this figure, which is accomplished by Madsen. We find the medium effect narrows the r-mode instability window. The lowest limiting frequency of upper contour is 541 Hz (the corresponding period is 1.85 msec), which is more close to the most rapidly spinning pulsars with frequencies of 642 Hz and 622 Hz (the periods are 1.56 and 1.61msec). This implies a strange star would slow down by gravitational window and spin around in 1.85 msec instead of the $2.5 \sim 3$ msec expected by Madsen. Figure.7 describes the instability windows for a wide parameters $\tilde{\alpha}$, $\tilde{B}$. The dotted curves denote a strange star with mass $M = 1.0 M_\odot$ and radius $R = 6.6$ km, and the solid curves express a strange star with mass $M = 2.06 M_\odot$ and radius $R = 13.5$ km.
5 SUMMARY AND DISCUSSION

In accordance with the quasiparticle description, we rederive and reevaluate the bulk viscous coefficient of SQM basing on referring $\mu$ dependence of the coupling constant. We find that below few times $10^9$ K, the medium effect is enhancing the viscosity. The dissipation of fluctuations in dense matter is more effective for this situation. We apply our result to study the $r$-mode instability window of strange stars. We find the $r$-mode instability window is narrowed and the lowest limiting frequency is closer to the millisecond pulsars relative to Madsen’s one for stars with mass $M = 1.4 M_\odot$ and radius $R = 10$ km. Our model also allow for wide frequency changes from 400 Hz to 900 Hz. The scenario is compatible with pulsar data for a wide range of mass ($1.0 \sim 2.8 M_\odot$) and radius ($6 \sim 15$ km).

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Figure 5: The dependence of the mass of strange stars on the radius for $\tilde{\alpha} = 3.5$ and several values of parameters $\tilde{B}^{1/4}$.
Figure 6: Critical spin frequencies for strange stars as functions of temperature with $M = 1.4M_\odot$ and $R = 10\text{km}$. The dotted contour stand for Madsen’s model, the solid curves display our result.
Figure 7: Critical spin frequencies for strange stars as functions of temperature. The dotted contour stand for a strange star with $M = 1.0 M_\odot$ and $R = 6.6$ km, the solid curves display a strange star with $M = 2.06 M_\odot$ and $R = 13.5$ km.