Benefit of GARCH Multivariate Models: Application to the Energy Market

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Authors’ contributions

This work was carried out in collaboration between both authors. Author MA designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors MA and HZ managed the analyses of the study. Author HZ managed the literature searches. Both authors read and approved the final manuscript.

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Abstract

This article presents the advantages of multivariate GARCH models. Multivariate GARCH models are identified as the best and flexible models in econometrics. Also, the interest of these models is to be able to examine and analyze the various relations which the various series maintain between them. In order to be able to estimate several financial series to analyze their correlations and transfers of volatility. We present an application on the relationship between the existing volatility in the oil market and the energy market, which we found that the assembly performance of the BEKK-GARCH form is better than that of other models.

Keywords: GARCH models; volatility; energy prices; BEKK; CCC and DCCC models.

1 Introduction

A time series is a series of observations over time. Time series analysis is a tool commonly used today to predict future data. This field has many applications in finance, medicine, econometrics, meteorology and many other

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fields. In econometrics, autoregressive conditional heteroskedasticity (ARCH) [1] is a model used for forecasting volatility which captures the conditional heteroskedasticity (serial correlation of volatility) of financial returns. Today's conditional variance is a weighted average of past squared unexpected returns. ARCH is an AR process for the variance.

A generalized autoregressive conditional heteroskedasticity (GARCH) [2] model generalizes the ARCH model. Today's conditional variance is a function of past squared unexpected returns and its own past values. The model is an infinite weighted average of all past squared forecast errors, with weights that are constrained to be geometrically declining. GARCH is an ARMA(p,q) process in the variance, introduced by Box and Jenkins in 1976 [3].

According to Bera and Higgins [4], this ARCH/GARCH modeling and its extensions correspond to a specific representation of nonlinearity, which allows simple modeling. This brief introduction helps to bring forward the various GARCH-multivariate models. We will study the inherent restrictions to these models, and the restrictions motivated by the identification of these models and the positivity of the matrix $H$. GARCH models are tools for forecasting and analyzing volatility of time series when the volatility varies over time. There exist many multivariate GARCH models VEC, BEKK, CCC and DCC.

The goal of this paper, is to add benefits of univariate-multivariate GARCH models whose function contains an application pertaining to the relationship between the existing volatility in the oil market and the energy market. Also, we will investigate the determinants of short-term volatility for oil, natural gas and electricity markets. The structure of the paper is as follows. Section 2 provides definitions and representations of GARCH models. Section 3 displays and discusses real data analysis, while section 4 provides our application and conclusions.

## 2 Forecasting Literature Using GARCH-type Models

As a result of the impact of volatility of financial assets in the economy many studies have been conducted. The ability to forecast financial market volatility is important for portfolio selection and asset management as well as for the pricing of primary and derivative assets [5]. Volatility can’t be observed therefore its difficult to assess which models are better in terms of the estimation of volatility itself. Several studies were conducted in order to forecast volatility using different models. Bollerslev [2] introduced the GARCH model, an extension on the ARCH model introduced by Engle [1], in order to produce better forecasts of conditional volatility and since then several authors introduced additional characteristics to the "traditional" GARCH model with the objective to capture different attributes of returns that have strong influence in the estimation of the conditional volatility. Many authors have used GARCH family models to forecast volatility see [6-12].

In addition, Zeghdoudi et al. [13] used GARCH(1,1) to model the volatility swaps for stock market Forecast using application study on CAC 40 French. Moreover, Bentes [14] found that the FIGARCH (1, d, 1) model was superior compared to others models such as TGARCH, EGARCH and APARCH models. Klein and Walther [15] used MMGARCH's (Mixture memory GARCH model) to forecast the volatility and Value-at-Risk outperforming all other models (ARMA, IGARCH, TGARCH, EGARCH and APARCH). While, Pan et al. [16] found that nonlinear GARCH-MIDAS model performs significantly in the forecasting study. Furthermore, Hung and Thach [17] used EGARCH(1,1) model with Student's distribution which provided the most accurate forecast in short time case. Recently, Zhang et al. [18] used the finite mixture GARCH Approach with EM algorithm for Energy Forecasting Applications.

## 3 Energy Markets Literature Using GARCH-type Models

In this subsection, we provide an overview of recent studies on GARCH-type models in energy markets (oil, gas, electricity).

Liu and Shi [19] applied various autoregressive moving average (ARMA) models with generalized autoregressive conditional heteroskedasticity (GARCH), namely the ARMA -- GARCH models, as well as their modified forms, ARMA -- GARCH-in-mean (ARMA -- GARCH-M), model and forecasted electricity prices in advance, using real-time hourly electricity prices over the period from 1/1/2008 to 31/12/2009.
Lai [20] extended the conventional hedging models by augmenting the copula-GARCH model with realized measures of volatility and co-volatility for effectively managing the risk exposure of portfolios in the market.

Ergen and Rizvanoglu [21] augmented the standard GARCH models with the natural gas market fundamentals in order to isolate the sources of high volatility in natural gas futures prices. Lin, Jiang, Xiao & Zhou [22] proposed a novel hybrid forecast model to forecast crude oil price by considering the long memory, asymmetric, heavy-tail distribution, nonlinear and non-stationary characteristics of crude oil price. The conclusion of this work was as follows:

- Long memory, asymmetric, heavy-tail, nonlinear and non-stationary is included;
- Empirical results showed that the proposed model achieves significant effects;
- The robustness test shows that this hybrid model is superior to traditional models (collection of probability distributions on a set of all possible outcomes of an experiment).

Papaoannou et al. [23], adopted a time series analysis and modeling approach, using average daily Day-Ahead SMP data quoted on ADMIE, the Independent Power Transmission Operator, between January 1, 2004 and December 31, 2014. The authors confirmed the benefit of SARMAX-GARCH and GARCH(1,1) models.

Virginia et al. [24] made an application of GARCH model to forecast data and volatility of share price of energy.

Bouseba and Zeghdoudi [13] used the univariate GARCH models to oil Price Volatility.

Raddant and Wagner [25] used the multivariate GARCH for a large number of stocks.

4 Research process

Three research variables are of interest: the prices of electricity, oil and gas. Data are collected for the period 1/1/2018 to 30/12/2020. There are 36 data points (short term). The process followed included studying the Descriptive statistics, the stationarity of the series, goodness of fit and the forecast for univariate models (GARCH, ARCH, ARMA and MA). As for multivariate GARCH models, a comparative study (goodness of fit) between vec-GARCH, BEKK-GARCH, CCC-GARCH, BEKK-GARCH and DCC-GARCH are performed. Followed by looking for the relationship between the three variables, that is, prices for electricity, gas and oil. Then, we investigated the goodness of forecast of the suggested models on the testing subset using average error in volatility (MAE_v) and average error in correlation (MAE_C).

5 Results and Discussion

The purpose of the analysis is to use a practical application on real data of prices of electricity, gas and oil. And then, applying the GARCH model on monthly price data, for the bivariate and multivariate case, of 36 observations, from 1 January 2018 to 30 December 2020.

The main results of this paper include the modeling and the practical application using the software RATS (Regression Analysis of Time Series) (Editeur estima.com). to the aforementioned is based on the data of the rate of exchange of the oil prices, electricity and gas. The data represents the three monthly prices, the study relates to the period 1/1/2018 to 30/12/2020, a total of 1080 observations given.

5.1 Descriptive statistics

Suitable statistical tools were used whereby, Hejase et al. [26 p. 129] contend that informed objective decisions are based on facts and numbers, real, realistic and timely information. Furthermore, according to Hejase & Hejase [27], “descriptive statistics deals with describing a collection of data by condensing the amounts of data into simple representative numerical quantities or plots that can provide a better understanding of the collected data” (p. 272). Consequently, data tables and figures report different descriptive statistics as shown in Table 1, Table 4 and Figs. 1 to 3.
Table 1. Descriptive statistics for each stationary series

|                | Electricity | Gas     | Oil     |
|----------------|-------------|---------|---------|
| No. of obs.    | 1080        | 1080    | 1080    |
| mean           | 4.064173    | 0.345854| 90.18472|
| median         | 4.074471    | 0.342051| 89.18000|
| maximum        | 4.117984    | 0.390691| 113.9300|
| minimum        | 3.819157    | 0.323479| 71.92000|
| std. Dev       | 0.054307    | 0.018800| 10.60613|
| skewness       | -3.207610   | 0.859345| 0.177592|
| kurtosis       | 1407985     | 2.680307| 2.289710|
| Jarque - Bera  | 245.8772    | 4.584149| 0.946000|
| probability    | 0.000000    | 0.101057| 0.623130|

Fig. 1. Gas Series Graph

Fig. 2. Oil Series Graph

Fig. 3. Electricity Series Graph
The first step of this application is to study the stationary of the series. To this end, we used the unit root test of Dickey-Fuller (ADF).

### Table 2. Data statistic results

| Series  | Augmented Dickey-Fuller test statistic | t-Statistic  |
|---------|---------------------------------------|--------------|
| Electricity | α                                      | -5.814064    |
| Test critical values: | 1%                                      | -3.62271     |
| | 5%                                      | -2.94461     |
| | 10%                                     | -2.61050     |
| Gas     | α                                      | 2.80474      |
| Test critical values: | 1%                                      | -3.62271     |
| | 5%                                      | -2.94461     |
| | 10%                                     | -2.61050     |
| Oil     | α                                      | -2.50732     |
| Test critical values: | 1%                                      | -3.62271     |
| | 5%                                      | -2.94461     |
| | 10%                                     | -2.61050     |

Table 2 shows the results generated by using ADF testing. Results show that the series electricity is stationary because $|t_{obs}| > t_{tab}$ for $\alpha=1\%, 5\%, 10\%$ and the gas and oil series not stationary because $|t_{obs}| < t_{tab}$ for $\alpha=1\%, 5\%, 10\%$.

### Table 3. ADF test result

| Series | Test (ADF) | Result |
|--------|------------|--------|
| Gas    |            | 0.03892|
| Dgas   |            | -0.24750|
| Loggas |            | -0.33007|
| Dloggas|            | -3.81963**|
| Oil    |            | 0.17559|
| Doil   |            | -2.74878**|
| Logoil |            | 0.42830|
| Dlogoil|            | -1.00468|

With unit Root test

1% (**) -2.62799
5% (*) -1.95036
10% -1.62063

So the stationary series are Dloggas and Doil.

### Table 4. Descriptive statistics for each stationary series

|          | mean        | std.Dev     | skewness   | kurtosis   | jarque-bera |
|----------|-------------|-------------|------------|------------|-------------|
| Electricity | 4.064173    | 0.054307    | -3.207610  | 14.07985   | 245.8772    |
| Dloggas  | 0.013541    | 0.006358    | -5.399298  | 31.206018  | 1635.637885 |
| Doil     | 2.024722    | 286.991934  | -4.460951  | 23.605170  | 955.206556  |

From Table 4 we find that:

- The kurtosis coefficient (14.07985) is high. It is greater than the kurtosis value of the normal law (3). The value of this resultant coefficient indicates that the curve of the electricity price series is flatter than the normal law curve. This coefficient indicates the high probability of occurrence of extreme points;
- The skewness coefficient (-3.207610) is different from zero (the theoretical value of the Skewness
coefficient for the normal distribution). This coefficient shows the presence of the asymmetry of the curve of the series of the price of electricity. The coefficient of this asymmetry is negative. This allows us to say that the distribution is spread to the left. This negative sign indicates that electricity prices react more to a negative shock than to a positive shock. This asymmetry can be an indicator of non-linearity;

• The Jarque - Bera test rejects the null hypothesis of the normality of the electricity price distribution (JB = 245.8772 > Χ² 0.05 (2) = 5.99).

### 5.2 Goodness of forecast

There are several methods to choose the best models in forecasting like the Mean method, Mean absolute error, Mean absolute percentage error, Root means squared error, Adjusted R-squared, Sum error of regression and the Bayesian information criterion.

#### Table 5. Comparison of several models –Doilgas

| Models      | Adj. R² | SEE   | BIC    | RMSE  | MAE   | MAPE  |
|-------------|---------|-------|--------|-------|-------|-------|
| MA(20)      | 0.3486  | 0.0023| -6.1255| 0.0027| 0.0018| 1.2732|
| ARMA(9,20)  | 0.3477  | 0.0021| -6.0345| 0.0026| 0.0015| 1.5625|
| ARCH(3)     | 0.8877  | 4.83E-13| -52.6945| 4.95E-13| 3.62E-13| 2.42E-11|
| GARCH(2,1)  | 0.9835  | 1.32E-15| -63.8563| 1.43E-15| 9.24E-15| 5.48E-13|
| GARCH(1,1)  | 0.9952  | 1.08E-15| -65.3022| 1.28E-15| 9.11E-15| 2.30E-14|

#### Table 6. Comparison of several models –Doil

| Models      | Adj. R² | SEE   | BIC    | RMSE  | MAE   | MAPE  |
|-------------|---------|-------|--------|-------|-------|-------|
| MA(20)      | 0.4575  | 3.44E-5| -28.3641| 4.88E-5| 2.24E-5| 3.15E-5|
| ARMA(9,20)  | 0.5875  | 2.55E-6| -28.8344| 3.25E-6| 2.43E-6| 3.46E-6|
| ARCH(3)     | 0.9995  | 3.54E-16| -70.5812| 4.78E-16| 3.35E-16| 2.32E-15|
| GARCH(2,1)  | 1.0000  | 1.23E-18| -83.6451| 1.45E-18| 6.07E-18| 1.15E-18|
| GARCH(1,1)  | 1.0000  | 1.85E-20| -85.7562| 1.86E-20| 5.33E-21| 4.22E-20|

#### Table 7. Comparison of several models –Electricity

| Models      | Adj. R² | SEE   | BIC    | RMSE  | MAE   | MAPE  |
|-------------|---------|-------|--------|-------|-------|-------|
| MA(20)      | 0.4455  | 0.00323| -8.2264| 0.0038| 0.0026| 7.1673|
| ARMA(9,20)  | 0.4845  | 0.00314| -8.1112| 0.0035| 0.0023| 8.0467|
| ARCH(3)     | 0.9205  | 4.83E-13| -52.6945| 4.95E-13| 3.62E-13| 2.42E-11|
| GARCH(2,1)  | 0.9951  | 1.32E-15| -63.8563| 1.43E-15| 9.24E-15| 5.48E-13|
| GARCH(1,1)  | 0.9995  | 1.08E-15| -65.3022| 1.28E-15| 9.11E-15| 2.30E-14|

Where,

- Mean absolute error (MAE):

\[
MAE = \frac{1}{N} \sum_{i=1}^{N} |y_i - \hat{y}_i|
\]

- Mean absolute percentage error (MAPE):

\[
MAPE = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{y_i - \hat{y}_i}{y_i} \right|
\]

- Root means squared error (RMSE):

\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2}
\]
\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{n} (y_i - \hat{y_i})^2}
\]  

(3)

- Adjusted R-squared (adjust R²)
- Sum error of regression (SEE)
- The Bayesian information criterion (BIC) or Schwarz information criterion: is measured by:

\[
n \ln(SEE) + k \ln(n)
\]

Tables 5 to 7 focused on the prediction ability of volatility of the GARCH family models. In fact, five GARCH models were compared in order to study which model performs the best in forecasting volatility for the real data of prices of electricity, gas and oil. Tables 5 to 7 provide the descriptive statistics of prices, whereby:

- The average prices are positive
- Skewness of Electricity, Dlogas and Doil are the following -3.348790, 5.399298 and -4.460951, respectively where the three are different from 0. Therefore, these coefficients show the presence of the asymmetry of the curves.
- The kurtosis of Electricity, Dlogas and Doil are the following 12.975406, 31.206556 and 23.605170, respectively which are superior to 3 (standard), therefore (thick tails), the distribution is rather sharp (leptokurtic distribution)
- Jarque-Bera of Electricity, Dlogas and Doil are the following ones 319.828116, 1635.637885 and 955.206556, respectively. Thus one rejects the hypothesis of normality H₀:GARCH(1,1) are clearly the best performing models as these receive the lowest score on fitting metrics whilst representing the lowest MAE, RMSE, MAPE, SEE and BIC among all models.

6 Multivariate GARCH Models

For our comparative study we consider four popular multivariate GARCH approaches for the conditional covariance matrix: the DCC model, vec model, CCC model and the diagonal BEKK model.

| The models     | vec-GARCH | BEKK-GARCH | CCC-GARCH | DCC-GARCH |
|----------------|-----------|------------|-----------|-----------|
| Log Likelihood | -33.9465  | 95.0773    | 10.7313   | -15.5270  |

Table 8 shows that from the log likelihood values of the models vec-GARCH, BEKK-GARCH, CCC-GARCH and DCC-GARCH, one observes that that the model BEKK-GARCH(1,1) is the best model among the selected number of other models.

6.1 The bivariate relationship case

The next section assesses if the gas, oil and electricity are correlated in terms of price?

GARCH (P=1,Q=1,MV=BEKK) / Doil #Electricity

| Log likelihood | -141.5513 |
|----------------|-----------|
| Variable       | Coeff.    | Std Error | T-stat | Signif. |
| mean(Doil)     | -0.79078632 | 0.01910421 | -41.39331 | 0.0000000 |
| C              | 70.85783933 | 0.22354550 | 316.97278 | 0.0000000 |
| A              | -1.20640105 | 0.00236725 | -509.62126 | 0.0000000 |
| B              | 1.00979145  | 0.00170634 | 591.78804 | 0.0000000 |
| Electricity   | -0.50616023 | 0.00599929 | -84.37006 | 0.0000000 |
Table 10. Log likelihood case 2

| Variable        | Coeff.    | Std Error | T-stat  | Signif.             |
|-----------------|-----------|-----------|---------|---------------------|
| mean (Doil)     | -1.489371 | 0.001534  | -970.91367 | 0.0000000          |
| C               | 62.033878 | 0.73513   | 843.84339 | 0.0000000          |
| A               | 0.037402  | 0.000495  | 75.50172  | 0.0000000          |
| B               | 0.045039  | 0.000452  | 99.70954  | 0.0000000          |
| Dloggas         | 1235.65114| 1.136159  | 1087.56899| 0.0000000          |

Table 11. Log likelihood case 3

| Variable        | Coeff.    | Std Error | T-stat  | Signif.             |
|-----------------|-----------|-----------|---------|---------------------|
| mean(electricity)| 4.072271742| 0.015602329| 261.00410| 0.0000000          |
| C               | 0.001584320| 0.000853274| 1.85675  | 0.06334619          |
| A               | 0.082305549| 0.122935302| 1.66950  | 0.50317469          |
| B               | 0.404715865| 0.293927832| 1.37692  | 0.16853619          |
| Dloggas         | -0.005860860| 0.001861090| -3.14915 | 0.00163744          |

Tabled 9 to 10 show that according to the obtained results:

\[ T_{\text{obs}} \text{ Doil/Electricity} = 41.39331 \]
\[ T_{\text{obs}} \text{ Doil/Dlog gas} = 970.91367 \]
\[ T_{\text{obs}} \text{ Electricity/Dlog gas} = 261.00410 \]

There is a very important relationship between electricity and gas because we use gas to produce electricity. But we find that oil affects the production of electricity. The price of gas will no longer be correlated with the price of oil.

In any case, gas, oil and electricity are highly correlated in terms of price: the underlying remain the same.

6.2 Multivariate case

According to equation (4), the expression of the matrix \( H_t \) at each instant \( t \) in the BEKK model is given by the following relation.

\[
H_t = CC' + (x + a)^n = \sum_{k=1}^{\infty} A'_{1k} H_{t-1} A_{1k} + \sum_{k=1}^{\infty} B'_{1k} H_{t-1} B_{1k} \tag{4}
\]

\[
A = \begin{bmatrix} 0.3628228 & -0.0042407 & -0.0004405 \\ -23.4830243 & 0.3613381 & 0.0378142 \\ 131.6406958 & 0.5409977 & -0.068014 \end{bmatrix}
\]

\[
B = \begin{bmatrix} -0.2163124 & 0.0012535 & 0.0002825 \\ 1.6158417 & 0.2866741 & -0.0000243 \\ -148.7025596 & 0.4934415 & 0.1910128 \end{bmatrix}
\]

\[
C = \begin{bmatrix} 6.8836290 & 0 & 0 \\ -0.0083230 & 0.0325875 & 0 \\ -0.0255640 & 0.0330037 & 0.0124445 \end{bmatrix}
\]

The estimated model BEKK-GARCH(1,1) can be obtained by substituting matrices A, B and C in the equation (4).
6.3 Goodness of forecast

In this subsection, the different models vec-GARCH, BEKK-GARCH, CCC-GARCH and DCC-GARCH are compared using the mean method. Here, we use only the mean absolute error in volatility (It is also called the mean average error) (MAEν).

Table 12. MAE in volatility of the vec-GARCH model

| Average error in volatility (MAEν) |
|-----------------------------------|
| MAEν (Doil)                      |
| MAEν (Dloggas)                   |
| MAEν (Electricity)               |
| 0.0056                           |
| 0.0132                           |
| 0.0306                           |

Table 13. MAE in volatility of the BEKK-GARCH model

| Average error in volatility (MAEν) |
|-----------------------------------|
| MAEν (Doil)                      |
| MAEν (Dloggas)                   |
| MAEν (Electricity)               |
| 0.00032                          |
| 0.00131                          |
| 0.0026                           |

Table 14. MAE in volatility of the CCC-GARCH model

| Average error in volatility (MAEν) |
|-----------------------------------|
| MAEν (Doil)                      |
| MAEν (Dloggas)                   |
| MAEν (Electricity)               |
| 0.00487                          |
| 0.0149                           |
| 0.0528                           |

Table 15. MAE in volatility of the DCC-GARCH model

| Average error in volatility (MAEν) |
|-----------------------------------|
| MAEν (Doil)                      |
| MAEν (Dloggas)                   |
| MAEν (Electricity)               |
| 0.00034                          |
| 0.00132                          |
| 0.0026                           |

Remark In the case of MAE in correlation we find BEKK-GARCH model is the best.

Table 16. MAE in correlation of the BEKK-GARCH model

| Average error in volatility (MAEν) |
|-----------------------------------|
| MAEν(Doil/Electricity)            |
| MAEν(Doil/Dlog gas)               |
| MAEν(Electricity/Dlog gas)        |
| 0.0146                            |
| 0.0767                            |
| 0.0608                            |

Comparing the goodness of fit through the mean absolute error, results are recorded in Tables 12 to 16. We find that the fitting performance of the BEKK-GARCH form is better than the other different vec-GARCH, BEKK-GARCH, CCC-GARCH and DCC-GARCH models in the energy market. However, the DCC-GARCH and BEKK-GARCH models have an advantage over the BEKK-GARCH model in the area of forecasting as the DCC-GARCH model is more parsimonious than the vec-GARCH and CCC-GARCH models. In this direction, it is extremely important to balance parsimony and flexibility when modeling multivariate GARCH models.

4 Conclusion

This work studied the relationship between the existing volatility in the oil market and the energy market according to the comparison of the multivariate models GARCH (VEC, BEKK, DCC and CCC). By comparing the goodness of fit using the mean absolute error, we find that the fitting performance of the BEKK-GARCH form is better than others models in our case, so there is a very important relationship between electricity and gas because gas is used to produce electricity but also finds that oil influences electricity production therefore the price of gas will no longer be correlated to the price of oil. In any case, gas, oil and electricity are very correlated in terms of prices and the underlying remain the same.
Competing Interests
Authors have declared that no competing interests exist.

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