Stripes and superconductivity in the two-dimensional self-consistent model

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(Dated: November 18, 2011)

We found solutions of the Bogoliubov-de Gennes equations for the two-dimensional self-consistent model of superconductors with $d_{x^2−y^2}$ symmetry of the order parameter, taking into account spin and charge distributions. Analytical solutions for spin-charge density wave phases in the absence of the superconductivity ("stripe" and "checkerboard" structures) are presented. Analytical solutions for coexisting superconductivity and stripes are found.

PACS numbers: PACS numbers: 71.10.Fd, 74.72.-h, 64.60.-i, 71.27.+a

I. INTRODUCTION

Stripe order, e.g., coupled spin- and charge-density periodic superstructure, is found in the under-doped superconducting cuprates and competes/coexists with superconducting order. Charge and spin stripe order have been observed experimentally in a some cuprate compounds, specifically $La_{2−x}Ba_xCuO_4$ and $La_{1.6−x}Nd_{0.4}Sr_{x}CuO_4$. A recent study of $La_{1.875}BaO_{0.125}CuO_4$ with angle-resolved photoemission and scanning tunneling spectroscopies has found evidence for a d-wave-type gap at low temperature, well within the stripe-ordered phase but above the bulk $Tc$. An earlier inelastic neutron scattering data shows field-induced fluctuating magnetic order with space periodicity $a_0$ and wave vector pointing along Cu-O bond direction in the ab-plane of the optimally doped $La_{1.84}Sr_{0.16}CuO_4$ in external magnetic field of 7.5 T below 10 K. The applied magnetic field ($\sim 2 − 7$ T) imposes the vortex lattice and induces "checkerboard" local density of electronic states (LDOS) seen in the STM experiments in high-$T_c$ superconductor $Bi_2Sr_2CaCu_2O_{8+δ}$. The pattern originating in the Abrikosov’s vortex cores has $4a_0$ periodicity, is oriented along Cu-O bonds, and has decay length $\sim 30$ angstroms reaching well outside the vortex core. The existence of antiferromagnetic spin fluctuations well outside the vortex cores is also discovered by NMR in superconducting YBCO in a 13 T external magnetic field. Theoretical predictions had also been made of the magnetic field induced coexistence of antiferromagnetic ordering phenomena and superconductivity in high-$T_c$ cuprates due to assumed proximity of pure superconducting state to a phase with coexisting superconductivity and spin density wave order. In these works effective Ginzburg-Landau theories of coupled superconducting-, spin- and charge-order fields were used. Alternatively, the fermionic quasi-particle weak-coupling approaches were focused on the theoretical predictions arising from the model of BCS superconductor with $d_{x^2−y^2}$ symmetry. An effect of the nodal fermions on the zero bias conductance peak in tunneling studies was predicted. However STM experiments of the vortices in high-$T_c$ compounds revealed a very different structure of LDOS. In this paper we make an effort to combine both theoretical approaches and present analytical mean-field solutions of coexisting spin-, charge- and superconducting orders derived form microscopic Hubbard model in the weak-coupling approximation. The previous analytical results obtained in the quasi 1D cases are now extended for two real space dimensions. Different analytical solutions for collinear and checkerboard stripe-phases, as well as for spin-charge density modulation inside Abrikosov’s vortex core are obtained. Simultaneously, our theory provides wave-functions of the fermionic states in all considered cases.

II. EFFECTIVE HAMITONIAN.

BOGOLIUBOV-DE GENNES EQUATIONS

Consider the Hamiltonian $H = H_0 + H_{sc}$ consisting of two parts: the first part is the Hubbard Hamiltonian with on-site repulsion $U > 0$

$$H_0 = -t \sum_{(i,j),\sigma} c_{i,\sigma}^\dagger c_{j,\sigma} + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} - \mu \sum_i \hat{n}_{i,\sigma}, \quad (1)$$

and the interaction part including superconducting correlations

$$H_{sc} = \sum_{<i,j>,\sigma} \Delta(i,j;\sigma) c_{i,\sigma}^\dagger c_{j,−\sigma}^\dagger + h.c., \quad (2)$$

where $\sum_{<i,j>,\sigma}$ is a summation over nearest neighboring sites $r_i$, $r_j$ of the square lattice, and spin components $\sigma = 2s_\sigma = \pm 1$. 

arXiv:1111.4139v1 [cond-mat.supr-con] 17 Nov 2011.
In the self-consistent approximation the Hamiltonian acquires the form

\[ H = -t \sum_{\langle ij \rangle, \sigma} c_{i, \sigma}^\dagger c_{j, \sigma} + \frac{U}{2} \sum_{i, \sigma} (\rho_i c_{i, \sigma}^\dagger c_{i, \sigma} - \frac{\rho_i^2}{2}) - U \sum_{i, \sigma} \langle \hat{S}_z (r_i) \rangle c_{i, \sigma}^\dagger c_{i, \sigma} + U \langle \hat{S}_z (r_i) \rangle^2 - \mu \sum_{i, \sigma} c_{i, \sigma}^\dagger c_{i, \sigma} \]

\[ + \sum_{\alpha > \sigma} \Delta(i, j; \sigma) c_{i, \sigma}^\dagger c_{j, -\sigma} + \text{h.c.} + \frac{|\Delta|^2}{g}, \] (3)

where we introduce similar slowly varying functions for spin order parameter \( m(r_i) \) and the charge density \( \rho(r_i) \) defined as

\[ \rho(r) = \langle \hat{n}(r) \rangle, \quad (1)^{x_i+y_i} m(r_i) = U \langle \hat{S}_z (r_i) \rangle, \]

\[ \Delta(i, j; \sigma) = -g(c_{i, -\sigma} c_{j, \sigma}). \] (4)

We can diagonalize the total Hamiltonian \( H = H_0 + H_{sc} \) by performing a unitary Bogoliubov transformation

\[ \hat{c}_\sigma(r) = \sum_n \gamma_n,\sigma u_{n, \sigma}(r) - \sigma \gamma_n^+,\sigma v_{n, -\sigma}(r) \] (5)

New operators \( \gamma, \gamma^+ \) satisfy the fermionic commutative relations \( \{\gamma_n,\sigma, \gamma_m^+,\sigma\} = \delta_{nm} \delta_{\sigma,\sigma'} \). The transformations \( \gamma \) must diagonalize the Hamiltonian \( H \):

\[ H = E_g + \sum_{\epsilon_n > 0} \epsilon_n \gamma_{n, \sigma}^\dagger \gamma_{n, \sigma}, \] (6)

where \( E_g \) is the ground state energy and \( \epsilon_n > 0 \) is the energy of the \( n \)-th excitation. Following \( \gamma \) we obtain the eigenvalue equations

\[ -t \sum_{\delta} u_\sigma(r + \delta) + \left( \frac{U}{2} \rho(r) - \mu \right) u_\sigma + m(r)(-1)^{x_i+y_i} \sigma u_\sigma(r) \]

\[ + \sum_{\delta} \Delta(r, r + \delta; \sigma) \sigma v_\sigma(r + \delta) = \epsilon_\sigma u_\sigma(r), \] (7)

\[ - \sum_{\delta} \Delta^*(r, r + \delta; -\sigma) \sigma u_\sigma(r + \delta) + t \sum_{\delta} v_\sigma(r + \delta) \]

\[ - \left( \frac{U}{2} \rho(r) - \mu \right) v_\sigma + m(r)(-1)^{x_i+y_i} \sigma v_\sigma(r) = \epsilon_\sigma v_\sigma(r), \] (8)

where \( \delta = \pm \hat{x}, \pm \hat{y} \).

We suppose the \( d_{x^2-y^2} \) symmetry of the superconducting order parameter \( \Delta(r, r \pm \hat{x}; \sigma) = \sigma \Delta_d(r), \Delta(r, r \pm \hat{y}; \sigma) = -\sigma \Delta_d(r) \). The Fourier transform gives the usual dependence \( \Delta_p(r) = \sigma \sum_{\delta} \Delta_{u_{\delta}}(r, r + \delta; \sigma) \exp[-i\delta] = 2(\cos p_x - \cos p_y) \Delta_d(r) \). The system (7) - (8) can be rewritten in the continuum approximation. Consider states near the Fermi surface (FS) (see Fig.1) and use linear approximation for the quasiparticles spectrum. Since for SDW pairing components with wave vectors \( p \) and \( p - Q \) (or \( p \) and \( p - Q \)) the lattice vector for the pure system without doping, when \((-1)^{x_i+y_i} \equiv e^{\pm iQr} \) are important (see Fig.1), we represent the functions \( u(r) \) and \( v(r) \), similar to the one-dimensional case, as

\[ u_\sigma(r) = \sum_{p \in F, p_x > 0} [u_{p, \sigma} e^{ipr} + \sigma u_{p-Q, \sigma}(r)e^{i(p-Q)r}], \] (9)

where \( Q = Q_+ \) for wave vectors \( p_y > 0 \) and \( Q = Q_- \) for wave vectors \( p_y < 0 \), respectively.

![FIG. 1: The Fermi surface](image)

For the doped case nesting vectors \( Q_\pm \) are no longer equivalent. Therefore in the general case we consider vectors \( Q_\pm \) as independent and make the substitution

\( m_+(r_i) \exp(iQ_+r_i) + m_-(r_i) \exp(iQ_-r_i) + h.c. \)

Eigenvalue equations (7), (8) take form similar to the 1D case: \( \hat{H} \Psi = \epsilon \Psi \), with

\[ \hat{H} = \begin{pmatrix} A_p & B_p \\ B_{-p} & -A_p \end{pmatrix}, \quad B_p = \begin{pmatrix} \Delta_{-p} & 0 \\ 0 & \Delta_{-p+Q} \end{pmatrix}, \]

\[ A_p = \begin{pmatrix} -iV_p \nabla r + \epsilon_p - \eta & m_\pm(r) \\ m_\pm^*(r) & -iV_p Q \nabla r + \epsilon_p - Q \eta \end{pmatrix}, \]

where \( \eta(r) = \mu - \frac{U}{2} \rho(r), \Psi^T = (u_p, v_p-Q, v_p, v_p-Q) = (u_p, v_p, v_p, v_p-Q), \epsilon_p = -2t(\cos p_x + \cos p_y) - \mu, V_p = 2(sin p_x, sin p_y), \) and, as before, \( Q = Q_+ \) for wave vectors \( p_y > 0 \) and \( Q = Q_- \) for wave vectors \( p_y < 0 \). The sign in \( m_\pm \) is taken by the same rule.

For the case \( d_{x^2-y^2} \) symmetry we consider \( \Delta_{-p} = \Delta_p = -\Delta_{p-Q} = 2(\cos p_x - \cos p_y) \Delta_d(r) \), which corresponds to \( \sigma \Delta(r, r \pm \hat{x}; \sigma) = -\sigma \Delta(r, r \pm \hat{y}; \sigma) = \Delta_0 \) in the uniform ground state. We retained the main terms in the
expansion over $\Delta$. In the higher order approximation, in stead of the terms $\Delta_{-\mathbf{p}}$ and $\Delta_{-\mathbf{p}+\mathbf{Q}}$ we would have to write $\Delta_{-\mathbf{p}} - i (\nabla_\mathbf{p} \Delta_{-\mathbf{p}}) \nabla_\mathbf{r}$ and $\Delta_{-\mathbf{p}+\mathbf{Q}} - i (\nabla_\mathbf{p} \Delta_{-\mathbf{p}+\mathbf{Q}}) \nabla_\mathbf{r}$. The continuum approximation is not valid for a band filling very close to the half-filled case (the number of particles per one site $\rho = 1$), where the Fermi velocity tends to zero at points $\mathbf{p} = (0, \pm \pi), (\pm \pi, 0)$.

In the homogeneous case $\rho(x) = \text{const}$, $m, \Delta_d = \text{const}$ for coexisting spin- and superconducting order parameters, so the eigenvalue spectrum has the form

$$E^2 = (\sqrt{m^2 + \epsilon^2(p)} \pm \eta)^2 + \Delta_p^2,$$

with $\Delta_p = \Delta_d (\cos p_x - \cos p_y)$.

The self-consistent conditions are derived by substitution of functions $u$, $v$ into (14), similar to the one-dimensional case. In the continuum approximation they read:

$$\rho(r) = 2 \sum_\epsilon [(u_+^* u_+ + u_-^* u_-) f + (v_+^* v_+ + v_-^* v_-)(1 - f)]$$

$$(-1)^{x+y}m(r) = 4U(\sum_\epsilon u_+^* u_+ f - \sum_\epsilon v_+^* v_+ (1 - f))$$

$$\Delta_{\mathbf{q}}(r) = 2g \sum_\epsilon [(v_+^* u_+ - v_-^* u_-)((1 - f)(\cos(p_x - q_x)$$

$$+ \cos(p_y - q_y)) - f((\cos(p_x + q_x) + \cos(p_y + q_y)))](15)$$

where $f = 1/(\exp[\epsilon / T] + 1)$. We omitted spin indices since in our representation for wave functions all equations are diagonal over spin.

III. SPIN-CHARGE DENSITY WAVE STRUCTURES

In the low doping limit the ground state of the model is the periodic charge-spin superstructure with the absence of superconductivity: $\Delta \equiv 0$. Consider different structures, having close ground state energies. In real systems the exact ground state must be determined by taking into account real long-distance 3D interactions.

A. Diagonal stripes

For diagonal stripes we search the solution in the form:

$$u_p(r) = u_p(r_+), \quad v_p(r) = v_p(r_+),$$

where $r_\pm = (x \pm y) / \sqrt{2}, p \in FS$. Substituting into Eqn. (14) we obtain a one-dimensional eigenvalue equation

$$-i V_p \frac{\partial}{\partial r_+} u_+ + \frac{U}{2} \rho(r_+) u_+ + m(r_+) u_- = E u_+,$$

$$m(r_+) u_+ + i V_p \frac{\partial}{\partial r_+} u_- + \frac{U}{2} \rho(r_+) u_- = E u_-,$$  

where $V_p = 2t \sin p_x$. The only difference from the considered one-dimensional model is the dispersion of the velocity $V_p$. This system is exactly solvable. In the ground state, at $\rho = 1$, we have $m(r_+) = m_0$. Increased doping leads to the stripe structure. The one stripe solution has the form

$$m(r_+) = m_0 \tanh \frac{r_+}{\xi},$$

where the width $\xi$ is defined from the minimum of the total energy. Solution (17) corresponds to $\rho = 1$ in the thermodynamic limit for number of holes per lattice site. In our case (14) is valid only in the vicinity of each single stripe that enters a periodic superstructure called stripe-phase (compare 16). Distinct from the Peierls model, where $\xi = V_F / m_0$, $V_F = \text{const}$, the present model has a more complicated spectrum. Besides continuum bands $E^2 = V_F^2 k_x^2 + m_0^2$ we find some discrete levels (for a given $p_x$) inside the gap:

$$E_n^2 = m_0^2 \lambda n (2 - \lambda n),$$

where $n$ is integer number, $0 \leq n \leq 1 / \lambda$, and $\lambda = \lambda(p_x) = V_p / (\xi m_0), \lambda = V_p / V$ with $\xi = V / m_0$. Each level inside a gap forms a band due to dispersion of the coefficient $\lambda(p_x)$. For $\lambda > 1$ we obtain only one level, $E = 0$, with wave function $\psi_{\pm} = (u_+ \pm u_-) / \sqrt{2}$:

$$\psi_{\pm} \propto \frac{1}{(\cosh x / \xi)^{1/\lambda}}.$$  

The wave functions of all states are described in terms of the hypergeometric function $F(a, b|c|z)$, and for local levels they have polynomial form: $\psi_{\pm,n} \sim$

$$\frac{1}{(\cosh x / \xi)^{1/\lambda}} F\left[2, 3 \lambda \begin{array}{c} n \end{array}, \begin{array}{c} -n, 1 \end{array}, 1 \right] / \xi.$$ 

For $1/2 < \lambda < 1$ two levels $n = 0, 1$, can already exist. Allowing for equality $\lambda = V_p / V$ we conclude that both possibilities $\lambda < 1$ and $\lambda > 1$ take place, each one in the proper interval of $p_x$. Similar to the 1D case the appearance of the kink in the spin channel is accompanied by the local charge distribution $\rho(r_+) - \rho \sim 1 / \cosh^2(r_+ / \xi)$. An increase of the doping leads to the periodic spin-charge density superstructure. In the limiting case of ”overdoping“ ($|\rho - 1| \gg 1 / \xi$) the spin-charge structure becomes harmonic

$$m(r_+) \propto \sin(\pi | \rho - 1 | r_+), \rho(r_+) - \rho \propto \cos(2\pi | \rho - 1 | r_+).$$

B. Vertical stripes

Using a similar ansatz for discrete equations

$$u_p(r) = u_p(x), \quad v_p(r) = v_p(x)$$

we obtain a system of equations with $V_p = 2t \sin p_x$, which is similar to the diagonal case. For the same values
where $\Delta_n < \Delta_{0}$ for diagonal stripes. Therefore the condition $n < 1/\lambda$ can be valid for larger values of $n$, resulting in additional bands inside the gap, as it is seen from numerical results.

C. Checkerboard structure

As we have seen the spin-charge density structure may be arranged in vertical (horizontal) or diagonal directions. Consider the solution with square symmetry. In the same approximation as before we find the solution of system (11) in the form $m_{+}(r) = m(r_{\pm})$, $u_{+}(r) = u_{p}(r_{\pm})$, $v_{+}(r) = v_{p}(r_{\pm})$. Equations are decoupled and we obtain

$$-iV_{p}\frac{du_{+}}{dr_{\pm}} + m(r_{\pm})u_{+} = Eu_{+},$$  \hspace{1cm} (19)

$$m^{*}(r_{\pm})u_{+} + iV_{p}\frac{du_{-}}{dr_{\pm}} = Eu_{-},$$  \hspace{1cm} (20)

with $V_{p} = 2t\sin p_{x}, r_{\pm} = (\pm x + y)/\sqrt{2}$. The one "cross" solution has the form

$$m_{+} = m_{0}\tanh \frac{r_{+}}{\xi}, \quad m_{-} = m_{0}\tanh \frac{r_{-}}{\xi},$$  \hspace{1cm} (21)

The spectrum $E$ and wave functions are found as above for the case of stripes. In the case of high doping the one kink solution is transformed to the periodic structure

$$\langle S_{z}(r) \rangle \propto (-1)^{x+y}m_{0}\cos[\pi(\sqrt{p} - 1)x]\cos[\pi(\sqrt{p} - 1)y],$$

in which we considered the squared Fermi surface approximation with electron density $\rho = |Q|^{2}/2\pi^{2}$.

IV. SUPERCONDUCTIVITY AND SPIN-CHARGE MODULATION

A. Vortex solution

Consider pure superconducting state ($\Delta(r) \equiv 0$). The BdG equations are decoupled. The first pair is

$$-iV_{p}\nabla_{r}u_{p}(r) + \Delta_{p}v_{p} = cu_{p},$$  \hspace{1cm} (22)

$$\Delta^{*}u_{p} + iV_{p}\nabla_{r}v_{p}(r) = cv_{p}.$$  \hspace{1cm} (23)

When the filling $\rho$ is close to 1, the Fermi surface has nearly square form, therefore $V_{p}\nabla_{r} \approx V_{p}\partial/\partial r_{\pm}$, depending on signs $p_{x}, p_{y}$ in this case the system of equations (22), (23) has the following vortex solution:

$$\Delta_{p}(r) = \Delta_{p} \frac{\sinh \frac{r_{+}}{\xi} + i\sinh \frac{r_{-}}{\xi}}{\sqrt{\sinh^{2} \frac{r_{+}}{\xi} + \sinh^{2} \frac{r_{-}}{\xi} + 1}},$$  \hspace{1cm} (24)

where $\Delta_{p} = \Delta_{0}(\cos(p_{x}) - \cos(p_{y}))$. For $r_{-} = 0$ the order parameter has a kink form $\Delta_{p}(r) \propto \tanh r_{+}/\xi.$

For the case $r_{+} = 0$ the order parameter acquires the phase: $\Delta \propto \exp(\pi i/2)\tanh r_{-}/\xi$. In the diagonal direction $r_{+} = r_{-}$ the solution $\Delta_{p}(r) \propto \tanh r_{+}/\xi/\sqrt{\tanh^{2} r_{+}/\xi + 1}\exp(\pi i/4)$ has the phase $\pi/4$. It is known that in one-dimensional case finite-band solutions of equations (22), (23) are related to the soliton (kink) solutions of the nonlinear Schrodinger equation (NSE). Note, that along the curve $\tanh r_{+}/\xi = \alpha \cosh r_{+}/\xi$ the order parameter acquires the form of a general kink solution of the NES: $\Delta_{p}(r) \sim (i\alpha + \tanh x/\xi)/\sqrt{\alpha^{2} + 1}$ with the localized state in the gap with the energy $E_{0} = \Delta_{p}\alpha/\sqrt{\alpha^{2} + 1}$.

B. Coexistence of spin-charge structure and superconductivity

Consider solutions of equations (11) in the superconducting region.

By analogy with 1D case\(^{16}\) we use ansatz:

$$v_{\pm} = \gamma_{\pm}u_{\pm}$$

which takes place in the uniform case. The term $U_{p}(r)/2$ in equations can be eliminated by the shift of wave functions $u, v \rightarrow u, v e^{i\Phi}, V_{p}\nabla \Phi = U_{p}(r)/2$. Considering $e(p) - \mu = 0$ on the Fermi surface we obtain for the case $m(r) = |m(r)|e^{i\varphi}$. $\Delta_{p}(r) = |\Delta_{p}(r)|e^{i\varphi}$, $\varphi, \varphi_{s} = \text{const}$, the solution $\gamma_{+} = \pm ie^{i(\varphi - \varphi_{s})}$, $\gamma_{-} = \pm ie^{-i(\varphi + \varphi_{s})}$, and the system (11) acquires the form

$$-iV_{p}\nabla_{r}u_{+} + \tilde{\Delta}(r)u_{+} = Eu_{+},$$  \hspace{1cm} (25)

$$\tilde{\Delta}^{*}(r)u_{+} + iV_{p}\nabla_{r}u_{-} = Eu_{-}$$  \hspace{1cm} (26)

with $\tilde{\Delta}(r) = |m(r)|e^{i\varphi}$, and, as before, $m = m_{\pm}$, depending on the sign of $p_{y}$. Equations (25), (26) are exact provided that phases $\varphi, \varphi_{s}$ are constant or slowly varying in space functions. We show that inhomogeneity of the superconductor order parameter leads to the formation of the antiferromagnetic order parameter. Consider a 1D geometry case: $u = u(r_{+})$, where assumption of constant phases is valid. The solution of Eqs. (25), (26) describing the coexistence of superconductivity and spin-charge density ordering, compatible with self-consistent equations, has the form of two bound solutions of the nonlinear Schrodinger equation, see, for example\(^{20}\)

$$\Delta_{1,2} = \Delta_{p} \frac{\cosh(2\kappa x + c_{1}) + \cosh(c_{2} \pm 2\beta i)/|\lambda|}{\cosh(2\kappa x + c_{1}) + \cosh(c_{2})/|\lambda|},$$  \hspace{1cm} (27)

where $\pm\lambda(p)$ are positions of local levels inside the gap, $\kappa = \sqrt{\Delta_{p} - \lambda^{2}}$, and $\exp i\beta = \lambda + i\kappa$. For superconducting and spin order parameters we obtain

$$\Delta_{sc} = \Delta_{p} \left(1 - \tanh a \tan(a(r_{+} - \frac{a}{2}) - \tanh(r_{+} - \frac{a}{2}))\right),$$  \hspace{1cm} (28)
where averaged over Fermi surface: \( \Gamma = < \Gamma_p > = \Delta_p / \sqrt{\Delta_p^2 + m_0^2} >_p, \xi = < v_p / (\Delta_p \sqrt{\Gamma_p \tanh a}) >_p, \) and we use the parametrization for the local level \( \lambda. \)

\[
\lambda^2 = \frac{\Delta_p^2}{\Delta_p^2 + m_0^2} \left( m_0^2 + \frac{\Delta_p^2}{\cosh^2 a} \right)
\]

FIG. 2: False colour plot of the coexisting superconducting (downward) and antiferromagnetic stripe-like (upward) orders. The envelope functions are plotted in real space, \( x \) and \( y \) coordinates are measured in units of correlation length \( \xi, a = 5, \Delta_d = 1, m_0 = 0.2. \)

The spin inhomogeneity generates the charge distribution \( \delta \rho(r) \propto m^m(r). \) Note, that two-soliton solution in the similar form was used for describing polaron-bipolaron states in the Peierls dielectrics22.

The correlation length is increased in comparison to clean superconductor case as

\[
\xi = \xi_{sc} \sqrt{1 + \xi_{sc}^2 / \xi_{AF}^2}
\]

V. DISCUSSION

We considered a simple self-consistent 2D model on a squared lattice to describe different states, including charge-spin structures, superconductivity, and their coexistence. The origin of spin-charge periodic state (which is responsible for the pseudogap) is due to the existence of flat parallel segments of the Fermi surface (nesting) at low hole doping concentrations. Effects of commensurability lead to a pinning of stripe structure at rational filling points \( |\rho - 1| = m/n. \) As a result, there is an exponentially small (for large \( n \)) decrease in the total energy of the order \( dE \exp(-\text{const}n) \) at any commensurate point, stabilizing stripes, as in 1D systems. For this reason, we think, stripes are mostly observable near \( n = 8 \) point \( (|\rho - 1| = 1/8). \) An increase of doping leads to the decrease of flat segments of the Fermi surface and attenuation of spin-charge structure.

We found the solution describing the coexistence of superconductivity and stripes (28), (29). The decrease (or a deviation from the homogenous value of the superconducting order parameter generates the spin-charge periodic structure in this region. Note, that due to symmetry of Eqs. (25)-(26) (duality \( \Delta \leftrightarrow im \)) we can write the same equation, describing the origin of superconducting correlations in the region of an inhomogeneity of spin-charge density. The situation is qualitatively similar to the 1D case16. Experimental data in underdoped high-\( T_c \) cuprates LSCO indicates that antiferromagnetic stripe-like spin-density order can be induced by magnetic field perpendicular to the CuO planes in the interval of fields much smaller than upper critical field \( H_{c2} \). The size of the magnetically ordered domains exceeds superconducting vortex’s core size \( \xi_s \) and the inter-vortex distance in the Abricosov’s lattice. Our present theoretical results demonstrate that this is indeed possible in the simple Hubbard \( t-U-V \) model that we consider. In particular, the dimensionless parameter \( a \) in Eqs. (28)-(29) is an independent variational parameter and depends on the magnetic and superconducting coupling strengths16, as well as on the magnitude of the external magnetic field. Hence, the size \( \sim a \times \xi \) of the antiferromagnetic domain (see Fig. 2, upward red plane bump) can exceed the superconducting (and magnetic) Ginzburg-Landau correlation length \( \xi \) when \( a(H) >> 1. \) Previously coexistence of superconducting order and slow antiferromagnetic fluctuations was studied merely on the basis of a phenomenological Ginzburg-Landau free energy functional approach16. We note also, that equations Eqs. (25)-(26) can be simply extended to include d-density waves (DDW).

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