Cooperative Precoding with Limited Feedback for MIMO Interference Channels
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Abstract

Multi-antenna precoding effectively mitigates the interference in wireless networks. However, the resultant performance gains can be significantly compromised in practice if the precoder design fails to account for the inaccuracy in the channel state information (CSI) feedback. This paper addresses this issue by considering finite-rate CSI feedback from receivers to their interfering transmitters in the two-user multiple-input-multiple-output (MIMO) interference channel, called cooperative feedback, and proposing a systematic method for designing transceivers comprising linear precoders and equalizers. Specifically, each precoder/equalizer is decomposed into inner and outer components for nulling the cross-link interference and achieving array gain, respectively. The inner precoders/equalizers are further optimized to suppress the residual interference resulting from finite-rate cooperative feedback. Furthermore, the residual interference is regulated by additional scalar cooperative feedback signals that are designed to control transmission power using different criteria including fixed interference margin and maximum sum throughput. Finally, the required number of cooperative precoder feedback bits is derived for limiting the throughput loss due to precoder quantization.

I. INTRODUCTION

In wireless networks, multi-antennas can be employed to effectively mitigate interference between coexisting links by precoding. This paper presents a new precoding design for the two-user multiple-input-multiple-output (MIMO) interference channel based on finite-rate channel-state-information (CSI) exchange between users, called cooperative feedback. Specifically, precoders are designed to suppress interference to interfered receivers based on their quantized CSI feedback, and the residual interference is regulated by additional cooperative feedback of power control signals.

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A. Prior Work

Recently, progress has been made on analyzing the capacity of the multi-antenna interference channel. In particular, interference alignment techniques have been proposed for achieving the channel capacity for high signal-to-noise ratios (SNRs) [1]. Such techniques, however, are impractical due to their complexity, requirement of perfect global CSI, and their sub-optimality for finite SNRs. This prompts the development of linear precoding algorithms for practical decentralized wireless networks. For the time-division duplexing (TDD) multiple-input-single-output (MISO) interference channel, it is proposed in [2], [3] that the forward-link beamformers can be adapted distributively based on reverse-link signal-to-interference-and-noise ratios (SINRs). Targeting the two-user MIMO interference channel, linear transceivers are designed in [4] under the constraint of one data stream per user and using different criteria including zero-forcing and minimum-mean-squared-error. In [5], the achievable rate region for the MISO interference channel is analyzed based on the interference-temperature principle in cognitive radio, yielding a message passing algorithm for enabling distributive beamforming. Assuming perfect transmit CSI, the above prior work does not address the issue of finite-rate CSI feedback though it is widely used in precoding implementation. Neglecting feedback CSI errors in precoder designs can result in over-optimistic network performance.

For MIMO precoding systems, the substantiality of CSI feedback overhead has motivated extensive research on efficient CSI-quantization algorithms, forming a field called limited feedback [6]. Various limited feedback algorithms have been proposed based on different principles such as line packing [7] and Lloyd’s algorithm [8], which were applied to design specific MIMO systems including beamforming [7] and precoded spatial multiplexing [9]. Recent limited feedback research has focused on MIMO downlink systems, where multiuser CSI feedback supports space-division multiple access [10]. It has been found that the number of feedback bits per user has to increase with the transmit SNR so as to bound the throughput loss caused by feedback quantization [11]. Furthermore, such a loss can be reduced by exploiting multiuser diversity [12], [13]. Designing limited feedback algorithms for the interference channel is more challenging due to the decentralized network architecture and the growth of total feedback CSI. Cooperative feedback algorithms are proposed in [14] for a two-user cognitive-radio network, where the secondary transmitter adjusts its beamformer to suppress interference to the primary receiver that cooperates by feedback to the secondary transmitter. This design is tailored for a MISO cognitive radio network and thus unsuitable for the general MIMO interference channel, which motivates this work.
B. Contributions

The precoder design that maximizes the sum throughput of the MIMO interference channel is a non-convex optimization problem and remains open [17]. In practice, sub-optimal linear precoders are commonly used for their simplicity, which are designed assuming perfect transmit CSI and based on various criteria including interference suppression by zero-forcing or minimum transmission power for given received SINRs [17]. However, existing designs fail to exploit the interference-channel realizations for suppressing residual interference due to quantized cooperative feedback. In this work, we consider the two-user MIMO interference channel with limited feedback and propose the decomposed precoder design that makes it possible for precoding to simultaneously regulate residual interference due to precoder-feedback errors and enhance received signal power. For the purpose of exposition, we consider two coexisting MIMO links where each link employs $L$ transmit and $K$ receive antennas to support multiple data streams. Linear precoding is applied at each transmitter and enabled by quantized cooperative feedback. Channels are modeled as i.i.d. Rayleigh block fading.

The main contributions of this work are summarized as follows:

1) A systematic method is proposed for jointly designing the linear precoders and equalizers under the zero-forcing criterion, which decouples the links in the event of perfect feedback. To be specific, precoders and equalizers are decomposed into inner and outer components that are designed to suppress residual interference caused by feedback errors and enhance array gain, respectively.

2) Additional scalar cooperative feedback, called interference power control (IPC) feedback, is proposed for controlling transmission power so as to regulate residual interference. The IPC feedback algorithms are designed using different criteria including fixed interference margin and maximum sum throughput.

3) Consider cooperative feedback of inner precoders of the size $L \times N_p$ with $N_p \leq L$. Under a constraint on the throughput loss caused by precoder quantization, the required number of feedback bits is shown to scale linearly with $N_p(L - N_p)$ and logarithmically with the transmit SNR as it increases.

Despite both addressing cooperative feedback for the two-user interference channel, this work differs from [14] in the following aspects. The current system comprises two MIMO links whereas that in [14] consists of a SISO and a MISO links. Correspondingly, this paper and [14] concern precoding and transmit beamforming, respectively. Furthermore, this work does not consider cognitive radio as in [14], leading to different design principles. In particular, the current system requires CSI exchange between
two links while that in [14] involves only one-way cooperative feedback from the primary receiver to the secondary transmitter.

C. Organization

The remainder of this paper is organized as follows. The system model is discussed in Section II. The transceiver design and IPC feedback algorithms are presented in Section III and IV, respectively. The feedback requirements are analyzed in Section V. Simulation results are presented in Section VI followed by concluding remarks.

Notation: Capitalized and small boldface letters denote matrices and vectors, respectively. The superscript $\dagger$ represents the Hermitian-transpose matrix operation. The operators $[X]_k$ and $[X]_{mn}$ give the $k$-th column and the $(m,n)$-th element of a matrix $X$, respectively. Moreover, $[X]_{m:n}$ with $n \geq m$ represents a matrix formed by columns $m$ to $n$ of the matrix $X$. The operator $(\cdot)^+$ is defined as $(a)^+ = \max(a, 0)$ for $a \in \mathbb{R}$.

II. SYSTEM MODEL

We consider two interfering wireless links as illustrated in Fig. 1. Each transmitter and receiver employ $L$ and $K$ antennas, respectively, to suppress interference as well as supporting spatial multiplexing. These operations require CSI feedback from receivers to their interfering and intended transmitters, called cooperative feedback and data-link feedback, respectively. We assume perfect CSI estimation and data-link feedback, allowing the current design to focus on suppressing interference caused by cooperative feedback quantization. All channels are assumed to follow independent block fading. The channel coefficients are i.i.d. circularly symmetric complex Gaussian random variables with zero mean and unit variance, denoted as $\mathcal{CN}(0,1)$. Let $H_{mn}$ be a $K \times L$ i.i.d. $\mathcal{CN}(0, 1)$ matrix representing fading in the channel from transmitter $n$ to receiver $m$. Then the interference channels are modeled as $\{\nu H_{mn}\}$ with $m \neq n$ and the data channels as $\{H_{mm}\}$. The factor $\nu < 1$ quantifies the path-loss difference between the data and interference links.

Each link supports $M \leq \min(L, K)$ spatial data streams by linear precoding and equalization. To regulate residual interference caused by precoder feedback errors, the total transmission power of each transmitter is controlled by cooperative IPC feedback. For simplicity, the scalar IPC feedback is assumed to be perfect since it requires much less overhead than the precoder feedback. Each transmitter uses

$^1$The errors in data-link feedback decrease received SNRs, which can be compensated by increasing transmission power.
identical transmission power for all spatial streams, represented by $P_n$ for transmitter $n$ with $n = 1, 2$, and its maximum is denoted as $P_{\text{max}}$. Assume that all additive white noise samples are i.i.d. $\mathcal{CN}(0, 1)$ random variables. Let $G_m$ and $F_m$ denote the linear equalizer used by receiver $m$ and the linear precoder applied at transmitter $m$, respectively, which are jointly designed for separating the spatial data streams of user $m$ with $m = 1, 2$. The receive SINR at receiver $m$ for the $\ell$th stream can be written as

$$\text{SINR}^{[\ell]}_m = \frac{P_m [G_m]^H H_{mm} [F_m]^{\ell}}{1 + \nu P_n [G_m]^H H_{mn} F_n^2}, \quad m \neq n. \quad (1)$$

The performance metric is the sum throughput defined as

$$\bar{C} = \sum_{m=1}^{2} \sum_{\ell=1}^{M} \mathbb{E} \left[ \log_2 \left( 1 + \text{SINR}^{[\ell]}_m \right) \right]. \quad (2)$$

### III. Transceiver Design

In this section, we propose a decomposition approach for designing the transceivers (linear precoders and equalizers). Using this approach, the precoder $F_n$ is decomposed into an *inner precoder* $F^i_n$ and an *outer precoder* $F^o_n$. Specifically, $F_n = F^i_n F^o_n$ where $F^i_n$ and $F^o_n$ are $L \times N_p$ and $N_p \times M$ matrices,

$^2$The decomposed precoder designs have been proposed in the literature for other systems such as cellular downlink [18].
respectively, with \( N_p \) being no smaller than the number of data streams \( M \) and \( N_p \leq L \). Similarly, we decompose the equalizer \( G_m \) as \( G_m = G^i_m G^o_m \) where \( G^i_m \) is an \( K \times N_e \) inner equalizer and \( G^o_m \) an \( N_e \times M \) outer equalizer with \( M \leq N_e \leq K \). For simplicity, inner/outer precoders and equalizers are constrained to have orthonormal columns. The inner and outer transceivers are designed to suppress cross-link interference and achieve array gain, respectively. In the following sub-sections, the transceivers, namely the inner/outer precoders and equalizers, are first designed assuming perfect cooperative feedback and then modified to mitigate residual interference caused by feedback quantization.

A. Transceiver Design for Perfect Cooperative Feedback

1) Inner Transceiver Design: A pair of inner precoder and equalizer \((G^i_m, F^i_n)\) with \( m \neq n \) are jointly designed under the following zero-forcing criterion:

\[
(G^i_m)^\dagger H_{mn} F^i_n = 0, \quad m \neq n. \tag{3}
\]

The constraint aims at decoupling the links and requires that \( N_e + N_p \leq \max(L, K) \). Under the constraint in (3), \((G^i_m, F^i_n)\) with \( m \neq n \) are designed by decomposing \( H_{mn} \) using the singular value decomposition (SVD) as

\[
H_{mn} = V_{mn} \Sigma_{mn} U_{mn}^\dagger \tag{4}
\]

where the unitary matrices \( V_{mn} \) and \( U_{mn} \) consist of the left and right singular vectors of \( H_{mn} \) as columns, respectively, and \( \Sigma_{mn} \) is a diagonal matrix with diagonal elements \( \{\sqrt{\lambda_{mn}^{[\ell]}}\} \) arranged in the descending order, namely \( \lambda_{mn}^{[1]} \geq \lambda_{mn}^{[2]} \cdots \geq \lambda_{mn}^{[\min(L,K)]} \). Note that \( \Sigma_{mn} \) is a tall matrix if \( K \geq L \) and a fat one if \( K < L \). Define the index sets \( A \subset \{1, 2, \cdots, K\} \) and \( B \subset \{1, 2, \cdots, L\} \) such that \( |A| = N_e \) and \( |B| = N_p \). The constraint in (3) can be satisfied if \( A \cap B = \emptyset \) and the inner precoder and equalizer are chosen as

\[
G^i_m = [V_{mn}]_A \quad \text{and} \quad F^i_n = [U_{mn}]_B. \tag{5}
\]

Consider the case of \( K \geq N_e + N_p \). Each receiver has sufficiently many antennas for canceling cross-link interference and thus cooperative feedback is unnecessary. Specifically, given an arbitrary fixed precoder \( F^i_n \), the equalizer \( G^i_m \) chosen as in (5) ensures that the zero-forcing criterion in (3) is satisfied. Next, consider the case of \( K < N_e + N_p \). For this case, the receivers have insufficient degrees of freedom (DoF) for canceling cross-link interference and link decoupling relies on inner precoding that is feasible given that \( N_e + N_p \leq L \). Therefore, with \( K < L \),

\(^3\)This condition usually holds for cellular downlink where a base station has more antennas than a mobile terminal.
specific design of inner transceiver for quantized cooperative feedback is discussed in the sequel.

2) Outer Transceiver Design: Given \((G^i_m, F^i_m)\), the outer pair \((G^o_m, F^o_m)\) are jointly designed based on the SVD of the \(N_e \times N_p\) effective channel \(H^o_{mm} = (G^i_m)^\dagger H_{mm} F^i_m\) after inner precoding and equalization:

\[
H^o_{mm} = V^o_{mm} \Sigma^o_{mm} (U^o_{mm})^\dagger
\]

where the singular values \(\sqrt{\lambda^{[1]}_{mm}}, \sqrt{\lambda^{[2]}_{mm}}, \ldots, \sqrt{\lambda^{[\min(N_e,N_p)]}_{mm}}\) follow the descending order. Note that the elements of \(H^o_{mm}\) are i.i.d. \(CN(0,1)\) random variables and their distributions are independent of \((G^i_m, F^i_m)\) since \(H_{mm}\) is isotropic. Transmitting data through the strongest eigenmodes of \(H^o_{mm}\) enhances the received SNR. This can be realized by choosing \(G^o_m\) and \(F^o_m\) as

\[
G^o_m = [V^o_{mm}]_{1:M} \quad \text{and} \quad F^o_m = [U^o_{mm}]_{1:M}.
\]

With perfect data-link feedback, the above joint design of precoders and equalizers converts each data link into \(M\) decoupled spatial channels. As a result, the receive SNR of the \(\ell\)-th data stream transmitted from transmitter \(m\) to receiver \(m\) is given by

\[
\text{SNR}^\ell_m = P_m \lambda^\ell_{mm}, \quad \ell = 1, 2, \ldots, M.
\]

Using the maximum transmission power, the sum capacity \(\tilde{C}\) can be written as

\[
\tilde{C} = 2 \sum_{m=1}^M \sum_{\ell=1}^M \mathbb{E} \left[ \log_2 (1 + P_{\text{max}} \lambda^\ell_{mm}) \right].
\]

Last, it is worth mentioning that with \(L, K\) and \(M\) fixed, maximizing \(N_p\) and \(N_e\) enhances the array gain of both links and hence is preferred if perfect CSI is available at the transmitters. However, for the case of quantized cooperative feedback, small \(N_p\) and \(N_e\) allow more DoF to be used for suppressing residual interference due to precoder-quantization errors as discussed in the next section.

B. Transceiver Design for Quantized Cooperative Feedback

Consider the case of \(K < N_e + N_p\) and mitigating cross-link interference relies on inner precoding with quantized feedback. As mentioned earlier, cooperative feedback is unnecessary if \(K \geq N_e + N_p\).

1) Inner Transceiver Design: In this section, the joint design of inner precoders and equalizers in (5) is modified to suppress the residual interference caused by precoder feedback errors.

First, given the inner equalizer \(G^i_m\) in (5), the inner precoder \(F^i_n\) in (5) is particularized under the criterion of minimizing residual interference power. Recall that the precoding at transmitter \(n\) is enabled
by quantized cooperative feedback of $F^i_n$ from receiver $m$ with $m \neq n$. Let $\hat{F}^i_n$ denote the quantized version of $F^i_n$ that is also an orthonormal matrix. Define the resultant quantization error $\epsilon_n$ as

$$
\epsilon_n = 1 - \frac{\| (F^i_n)'^{\dagger} \hat{F}^i_n \|^2}{N_p}, \quad n = 1, 2
$$

(8)

where $0 \leq \epsilon_n \leq 1$. The error $\epsilon_n$ is zero in the case of perfect cooperative feedback, namely $F^i_n = \hat{F}^i_n$. A nonzero error results in the violation of the zero-forcing criterion in (3)

$$
(G^i_m)^\dagger H_{mn} \hat{F}^i_n \neq 0, \quad m \neq n.
$$

(9)

Given that the inner equalizer designed for perfect feedback is applied, the residual interference at the output of the equalizer at receiver $m$ has the power

$$
I_m = \nu P_n \| (G^o_m)^\dagger H_{mn} \hat{F}^i_n \|^2, \quad m \neq n.
$$

(10)

It is difficult to directly optimize $\hat{F}^i_n$ for minimizing $I_m$. Alternatively, $\hat{F}^i_n$ can be designed for minimizing an upper bound on $I_m$ obtained as follows. By rearranging eigenvalues and eigenvectors, the SVD of $H_{mn}$ in (4) for $L > K$ can be rewritten as

$$
H_{mn} = \begin{bmatrix} G^i_m & B_m \end{bmatrix} 
\begin{bmatrix}
\sqrt{\Sigma_{mn}^{(a)}} & 0 & 0 \\
0 & \sqrt{\Sigma_{mn}^{(b)}} & 0 \\
0 & 0 & \sqrt{\Sigma_{mn}^{(c)}}
\end{bmatrix}
\begin{bmatrix} C_n & F^i_n \end{bmatrix}^\dagger
$$

(11)

where the diagonal matrices $\Sigma_{mn}^{(a)}$ and $\Sigma_{mn}^{(b)}$ have the diagonal elements $\{ \sqrt{\lambda_{mn}^{(a)}} | \ell \in A \}$ and $\{ \sqrt{\lambda_{mn}^{(b)}} | 1 \leq \ell \leq K, \ell \notin A \}$, respectively. It follows from (10) and (11) that

$$
I_m = \nu P_n \| (G^o_m)^\dagger \sqrt{\Sigma_{mn}^{(a)}} F^i_n \|^2
$$

\leq \nu P_n \| (G^o_m)^\dagger \sqrt{\Sigma_{mn}^{(a)}} C_n^{\dagger} \hat{F}^i_n \|^2
$$

= \nu P_n \| (G^o_m)^\dagger \sqrt{\Sigma_{mn}^{(a)}} C_n^{\dagger} \hat{F}^i_n \|^2
$$

= \nu P_n \sum_{\ell=1}^{M} \sum_{k=1}^{N_p} \| G^o_m \ell \|^2 \| \sqrt{\Sigma_{mn}^{(a)}} C_n^{\dagger} \hat{F}^i_n \|^2
$$

\leq \nu P_n \sum_{\ell=1}^{M} \sum_{k=1}^{N_p} \| G^o_m \ell \|^2 \| \sqrt{\Sigma_{mn}^{(a)}} C_n^{\dagger} \hat{F}^i_n \|^2
$$

= \nu \sum_{\ell=1}^{M} \sum_{k=1}^{N_p} \| G^o_m \ell \|^2 \| \sqrt{\Sigma_{mn}^{(a)}} C_n^{\dagger} \hat{F}^i_n \|^2
$$

\leq M \nu P_n \| \sqrt{\Sigma_{mn}^{(a)}} C_n^{\dagger} \hat{F}^i_n \|^2
$$

\leq M \nu P_n \| C_n^{\dagger} \hat{F}^i_n \|^2 \max_{\ell \in A} \lambda_{mn}^{(a)}
$$

(12)

(13)

(14)

(15)
where (12) holds since the columns of $U_{o,a}^n$ form a basis of the space $C^L$, (13) applies Schwarz’s inequality and (14) follows from that the columns of $G^a_m$ have unit norms. Next, the precoder quantization error $\epsilon_n$ in (8) can be written as

\begin{align}
\epsilon_n &= \frac{1}{N_p} \sum_{\ell=1}^{N_p} \left( 1 - \| \hat{\hat{F}}_{i,n}^i \|^2 F_{i,n} \right) \\
&= \frac{1}{N_p} \sum_{\ell=1}^{N_p} \| [\hat{\hat{F}}_{i,n}^i]_{\ell} C_n \|^2 F_{i,n} \\
&= \frac{1}{N_p} \| C_n \hat{\hat{F}}_{i,n}^i \|^2 F_{i,n} \tag{16}
\end{align}

where (16) holds since $[F_{i,n}, C_n]$ forms a basis of the space $C^L$. Substituting (17) into (15) gives

\begin{align}
I_m \leq \nu MN_p P_n \epsilon_n \max_{\ell \in A} \lambda_{min}^{(\ell)}. \tag{18}
\end{align}

Minimizing the right-hand side of (18) gives that the columns of $G_{i,m}^i$ should be left eigenvectors of $H_{mn}$ corresponding to the $N_e$ smallest singular values. Therefore, the inner transceiver design in (5) for perfect CSI is particularized as

\begin{align}
G_{i,m}^i = [V_{mn}]_A \text{ with } A = \{ K - N_e + 1, \cdots , K \} \quad \text{and} \quad F_{i,n}^i = [U_{mn}]_B \text{ with } B \cap A = \emptyset \tag{19}
\end{align}

and $\hat{\hat{F}}_{i,n}^i$ is obtained by quantizing $F_{i,n}^i$ such that the quantization error $\epsilon_n$ is minimized [19]. Then (18) can be simplified as

\begin{align}
I_m \leq \nu MN_p P_n \lambda_{min}^{[K - N_e + 1]} \epsilon_n, \quad m \neq n. \tag{20}
\end{align}

Next, if $K > N_e$, besides $N_e$ DoF required for inner equalization, a receiver has $(K - N_e)$ extra DoF that can be used to suppress residual interference. This can be realized at receiver $m$ by redesigning the inner equalizer $G_{i,m}^i$ with the resultant design denoted as $\hat{G}_{i,m}^i$. To this end, the matrix $H_{mn}\hat{F}_{i,n}^i$ is decomposed by SVD as

\begin{align}
H_{mn}\hat{F}_{i,n}^i = \hat{V}_{mn}\hat{\Sigma}_{mn}\hat{U}_{mn}^\dagger
\end{align}

where the singular values along the diagonal of $\hat{\Sigma}_{mn}$ are denoted as $\{ \sqrt{\hat{\lambda}_{mn}} \mid \ell \leq \min(K, N_p) \}$ and arranged in the descending order. The inner equalizer $\hat{G}_{i,m}^i$ that minimizes the residual interference should be chosen to comprise the left eigenvectors of $H_{mn}\hat{F}_{i,n}^i$ that correspond to the smallest singular values and hence $G_{i,m}^i = [\hat{V}_{mn}]_{(K - N_e + 1):K}$. Another interpretation of this design is that the inner equalizer $G_{i,m}^i$ is directed towards the null space of $\hat{F}_{i,n}^i$. The resultant residual interference power after inner precoding
can be upper bounded as
\[
\tilde{I}_m = \nu P_n \| \hat{C}_m^i H_{mn} \hat{F}_m^o \|_F^2 \\
\leq \nu P_n \| \hat{G}_m^i H_{mn} \hat{F}_m^o \|_F^2 \\
= \nu P_n \sum_{m=K-N_e+1}^{N_p} \sqrt{\hat{\lambda}_{m}}.
\] (21)

Last, if \( K = N_e \), the design of \( G_m^i \) remains unchanged and \( \hat{G}_m^i = G_m^i \).

2) Outer Transceiver Design: Let \( \hat{G}_m^o \) and \( \hat{F}_m^o \) denote the outer equalizer and precoder for the case of quantized cooperative feedback. The outer transceiver \( (\hat{G}_m^o, \hat{F}_m^o) \) is designed similarly as its perfect-feedback counterpart in Section III-A2. Decompose the effective channel matrix \( (\hat{G}_m^i H_{mn} \hat{F}_m^o) \) after inner precoding/equalization as
\[
\left( \hat{G}_m^i \right)^\dagger H_{mn} \hat{F}_m^i = \hat{V}_m^o \hat{\Sigma}_m^o \left( \hat{U}_m^o \right)^\dagger
\] (22)
where the diagonal matrix \( \hat{\Sigma}_m^o \) contains singular values \( \{ \sqrt{\hat{\lambda}_{m}} \} \) arranged in the descending order. To maximize the received SNRs, the outer equalizer and precoder are chosen as
\[
\hat{G}_m^o = [\hat{V}_m^o]_{1:M} \quad \text{and} \quad \hat{F}_m^o = \hat{U}_m^o.
\]

The corresponding sum throughput follows from (2) as
\[
\bar{C} = 2 \sum_{m=1}^{M} \sum_{\ell=1}^{E} \mathbb{E} \left[ \log_2 \left( 1 + \frac{P_m \hat{\lambda}_{m}}{1 + \nu P_n \| \left[ G_m^i \right]_\ell H_{mn} \hat{F}_n^o \|^2} \right) \right]
\] (23)
where \( G_m = \hat{G}_m^i \hat{G}_m^o \) and \( F_n = \hat{F}_n^i \hat{F}_n^o \).

C. Discussion

In this section, we discuss the robustness of the proposed quantized feedback precoder by comparison with a conventional design without cooperative feedback. To be specific, the baseline design is the well-known single-user transceiver design that provides no cooperative feedback, where interference is treated as noise and all spatial DoF are applied to maximize array gain [20]. Let the data-channel matrix \( H_{mn} \) be decomposed by SVD as \( H_{mn} = V_{mn} \Sigma_{mn} U_{mn}^\dagger \) with \( m \in \{1, 2\} \). The precoder \( \hat{F}_m \) and receiver \( \hat{G}_m \) for the baseline case as given below transmit data through the \( M \) strongest eigenmodes of \( H_{mn} \) [20]
\[
\hat{F}_m = [V_{mn}]_{1:M} \quad \text{and} \quad \hat{G}_m = [U_{mn}]_{1:M}.
\] (24)
By using the maximum transmission power, the corresponding sum throughput is given as

\[
\hat{C} = 2E \sum_{\ell=1}^{M} \log_2 \left( 1 + \frac{P_{\text{max}} |[\Sigma_{11}]_{\ell,\ell}|^2}{1 + P_{\text{max}} \nu |[G_{1\ell}]_{\ell} [H_{12} F_2]|^2} \right)
\]

\[
\leq 2E \sum_{\ell=1}^{M} \log_2 \left( 1 + \frac{|[\Sigma_{11}]_{\ell,\ell}|^2}{\nu |[G_{1\ell}]_{\ell} [H_{12} F_2]|^2} \right).
\]  

(25)

Similarly as (20), it can be proved that the interference power for the \(\ell\)-th data stream, \(I_m^{[\ell]}\), received at receiver \(m\) is upper funded as

\[
I_m^{[\ell]} \leq \nu N_p P_n \lambda_{mn}^{[K-N_e+1]} \epsilon_n, \quad m \neq n.
\]  

(26)

From (23) and (26) and also using the maximum transmission power, the sum throughput \(\hat{C}\) for the proposed design can be lower bounded as

\[
\hat{C} \geq 2E \sum_{\ell=1}^{M} \log_2 \left( 1 + \frac{P_{\text{max}} \lambda_{11}^{[\ell]}}{1 + N_p P_{\text{max}} \nu \lambda_{12}^{[K-N_e+1]} \epsilon_2} \right).
\]  

(27)

By comparing (25) and (27), it can be observed that \(\hat{C}\) is bounded as \(P_{\text{max}}\) increases but \(\bar{C}\) can grow with increasing \(P_{\text{max}}\) if the quantization errors \(\{\epsilon_m\}\) are regulated by adjusting the number of feedback bits based on \(P_{\text{max}}\) (see Section V for details).

IV. INTERFERENCE POWER CONTROL FEEDBACK

In this section, we consider the case of \(K = N_e\) where receivers have no extra DoF for suppressing residual interference. An alternative solution is to adjust transmission power for increasing the sum throughput. Two IPC feedback algorithms for implementing power control are discussed in the following sub-sections.

A. Fixed Interference Margin

Receiver \(m\) sends the IPC signal, denoted as \(\eta_m\), to transmitter/interferer \(n\) for controlling its transmission power as

\[
P_n = \min(\eta_n, P_{\text{max}}), \quad n = 1, 2.
\]  

(28)

The scalar \(\eta_n\) is designed to prevent the per-stream interference power at receiver \(m\) from exceeding a fixed margin \(\tau\) with \(\tau > 0\), namely \(I_m^{[\ell]} \leq \tau\) for all \(0 \leq \ell \leq M\). A sufficient condition for satisfying this constraint is to bound the right hand side of (26) by \(\tau\). It follows that

\[
\eta_n = \frac{\tau}{N_p \nu \lambda_{mn}^{[K-N_e+1]} \epsilon_n}, \quad m \neq n.
\]  

(29)
Given \( \tau \), a lower bound \( A_{IM} \) on the sum throughput \( \bar{C} \), called achievable throughput, is obtained from (23) as

\[
A_{IM} = 2 \sum_{m=1}^{2} \sum_{\ell=1}^{M} \log_2 \left( 1 + \frac{\min(\eta_m, P_{max}) \lambda_{m\ell}}{1 + \tau} \right).
\]  

(30)

It is infeasible to derive the optimal value of \( \tau \) for maximizing \( A_{IM} \) in (30). However, for \( P_{max} \) being either large or small, simple insight into choosing \( \tau \) can be derived as follows. The residual interference power decreases continuously with reducing \( P_{max} \). Intuitively, \( \tau \) should be kept small for small \( P_{max} \). For large \( P_{max} \), the choice of \( \tau \) is less intuitive since large \( \tau \) lifts the constraints on the transmission power but causes stronger interference and vice versa. We show below that large \( \tau \) is preferred for large \( P_{max} \). This requires the result in \([21, \text{Theorem 1}]\) paraphrased as follows.

**Lemma 1** (\([21]\)). Let \( H \) denote a \( Q_1 \times Q_2 \) matrix of i.i.d. \( \mathcal{CN}(0,1) \) elements with \( Q_1 \geq Q_2 \). The cumulative distribution function of the \( k \)-th eigenvalue \( \varphi_k \) of the Wishart matrix \( H^\dagger H \) can be expanded as

\[
\Pr(\varphi_k < x) = a_k x^{d_k} + o(x^{d_k}), \quad k = 1, \cdots, Q_2
\]  

(31)

where \( d_k = (Q_1 - k + 1)(Q_2 - k + 1) \) and \( a_k = \frac{U^{-1} [A(k)] [B(k)]}{d_k} \) with \( U = \prod_{m=1}^{Q_2} (Q_1 - m)!(Q_2 - m)! \).

The matrix \( A(k) \) is defined for \( k \neq 1 \) as

\[
[A(k)]_{mn} = (Q_1 - Q_2 + m + n + 2(Q_2 - k))!, \quad m, n = 1, \cdots, (k - 1)
\]  

(32)

and \( A(1) = I \), \( B(k) \) is defined for \( k \neq Q_2 \) as

\[
[B(k)]_{mn} = \frac{2}{((Q_1 - Q_2 + m + n)^2 - 1)(Q_1 - Q_2 + m + n)}, \quad m, n = 1, \cdots, (Q_2 - k)
\]  

(33)

and \( B(Q_2) = I \).

To simplify notation, we re-denote \((\varphi_k, a_k)\) for \( Q_1 = N_e \) and \( Q_2 = N_p \) as \((\hat{\lambda}_k, \hat{a}_k)\) and those for \( Q_1 = Q_2 = L \) as \((\tilde{\lambda}_k, \tilde{a}_k)\). Using the above result, we obtain the following lemma that is proved in the appendix.

**Lemma 2.** Given finite-rate cooperative feedback and for large \( P_{max} \), the achievable throughput is

\[
A_{IM} = 2 \sum_{\ell=1}^{M} \mathbb{E} \left[ \log_2 \left( 1 + \frac{\tau \hat{\lambda}_\ell}{(1 + \tau) N_p \nu \hat{\lambda}_K - N_e + 1} \right) \right] + o(1).
\]  

(34)

It can be observed from the above result that the first order term of \( A_{IM} \) attains its maximum as \( \tau \to \infty \). However, this term is finite even for asymptotically large \( P_{max} \) and \( \tau \), which is the inherent effect of residual interference.
B. Maximum Achievable Throughput

In this section, an iterative IPC algorithm is designed for increasing the sum throughput $\bar{C}$ in (2). Since $\bar{C}$ is a non-concave function of transmission power, directly maximizing $\bar{C}$ does not yield a simple IPC algorithm. Thus, we resort to maximizing a lower bound $A_{ST}$ (achievable throughput) on $\bar{C}$ instead, obtained from (20) and (23) as $A_{ST} = \mathbb{E}[A]$ with

$$A = \sum_{\ell=1}^{M} \left[ \log_2 \left( 1 + \frac{P_1 \lambda_{11}^{[\ell]}}{1 + P_2 N_p \nu \lambda_{12}^{[K-N_e+1]} \epsilon_2} \right) + \log_2 \left( 1 + \frac{P_2 \lambda_{22}^{[\ell]}}{1 + P_1 N_p \nu \lambda_{21}^{[K-N_e+1]} \epsilon_1} \right) \right].$$  \hspace{1cm} (35)

The corresponding optimal transmission power pair is given as

$$(P_1^*, P_2^*) = \arg \max_{P_1, P_2 \in [0, P_{\text{max}}]} A(P_1, P_2).$$ \hspace{1cm} (36)

The objective function $A$ remains non-concave and its maximum has no known closed-form for $M > 1$. Note that for $M = 1$, it has been shown that the optimal transmission-power pair belongs to the set \{(0, P_{\text{max}}), (P_{\text{max}}, 0), (P_{\text{max}}, P_{\text{max}})\} \cite{22}. For the current case of $M > 1$, inspired by the message passing algorithm in [5], a sub-optimal search for $(P_1^*, P_2^*)$ can be derived using the fact that

$$\frac{\partial A(P_1^*, P_2^*)}{\partial P_m} = 0, \quad \forall \ m = 1, 2.$$

To this end, the slopes of $A$ are obtained using (35) as

$$\frac{\partial A(P_1, P_2)}{\partial P_m} = \mu_m + \psi_m - \rho_m$$  \hspace{1cm} (37)

where

$$\mu_m = \log_2 e \sum_{\ell=1}^{M} \frac{\lambda_{mm}^{[\ell]}}{1 + N_p \nu \lambda_{mm}^{[K-N_e+1]} \epsilon_m P_n + \lambda_{mm}^{[\ell]} P_m}$$

$$\psi_m = \log_2 e \sum_{\ell=1}^{M} \frac{N_p \nu \lambda_{nm}^{[K-N_e+1]} \epsilon_m}{1 + N_p \nu \lambda_{nm}^{[K-N_e+1]} \epsilon_m P_m + \lambda_{mm}^{[\ell]} P_n}$$

$$\rho_m = \log_2 e N_p N^2 \nu \lambda_{mm}^{[K-N_e+1]} \epsilon_m P_m.$$  

Note that based on estimated CSI, $\mu_m$ has to be computed at $R_m$ and $(\psi_m, \rho_m)$ at $R_n$ with $n \neq m$. Therefore, using (37), we propose the following iterative IPC feedback algorithm.

**Algorithm 1:**

1) Transmitter 1 and 2 arbitrarily select the initial values for $P_1$ and $P_2$, respectively.

2) The transmitters broadcast their choices of transmission power to the receivers.
3) Given \((P_1, P_2)\), the receiver 1 computes \((\mu_1, \psi_2, \rho_2)\) and feeds back \(\mu_1\) and \((\psi_2 - \rho_2)\) to transmitter 1 and 2, respectively. Likewise, receiver 2 computes \((\mu_2, \psi_1, \rho_1)\) and feeds back \(\mu_2\) and \((\psi_1 - \rho_1)\) to transmitter 2 and 1, respectively.

4) Transmitter 1 and 2 update \(P_1\) and \(P_2\), respectively, using (37) and the following equation

\[
P_m(k + 1) = \min \left\{ \left[ P_m(k) + \frac{\partial A(P_1, P_2)}{\partial P_m} \Delta \gamma \right]^{+}, P_{\max} \right\}
\]

where \(k\) is the iteration index and \(\Delta \gamma\) a step size.

5) Repeat Steps 2) – 4) till the maximum number of iterations is performed or the changes on \((P_1, P_2)\) are sufficiently small.

Note that the IPC-feedback overhead increases linearly with the number of iterations. By choosing an appropriate step size, the convergence of the above iteration is guaranteed but the converged throughput need not be globally maximum.

V. PRECODER FEEDBACK REQUIREMENTS

In this section, consider the case of \(K = N_e\) as in the preceding section and the number of bits for cooperative precoder feedback is derived under a constraint on the throughput loss due to precoder quantization.

The expected precoder quantization errors is related to the number of feedback bits as follows. Consider the codebook \(\mathcal{F}\) of \(V L \times N_p\) orthonormal matrices that is used by each receiver to quantize the inner precoder for the corresponding interferer. Given \(\mathcal{F}\), the quantization error \(\epsilon_n\) in (8) is minimized by selecting the quantized precoder \(\hat{F}_i^n\) as

\[
\hat{F}_i^n = \arg \min_{W \in \mathcal{F}} \left( 1 - \frac{\|W^\dagger F_i^n\|^2}{N_e} \right), \quad n = 1, 2.
\]

(38)

The above operation is equivalent to minimizing the Chordal distance between \(\hat{F}_i^n\) and \(F_i^n\): [7]

\[
\hat{F}_i^n = \arg \min_{W \in \mathcal{F}} d_c(W, F_i^n)
\]

(39)

where the chordal distance \(d_c\) is defined as

\[
d_c(W, F_i^n) = \frac{1}{\sqrt{2}}\|WW^\dagger - F_i^n(F_i^n)^\dagger\|_F
\]

\[
= \sqrt{N_e - \|W^\dagger F_i^n\|^2_F}.
\]

The codebook selection in (39) motivates the codebook design based on minimizing the maximum chordal distance between every pair of codebook members [9]. For such a design, the expected quantization error
can be upper bounded as [24]
\[
E[\epsilon_n] \leq \frac{\Gamma\left(\frac{1}{2}\right)}{Z} \beta^{-\frac{1}{2}} 2^{-\frac{n}{\beta}} + L e^{-\left(2^n \beta\right)^{1-n}}
\]  
(40)

where \( Z = N_e (L - N_e) \), the number of feedback bits \( B = \log_2 V \), \( \beta = \frac{1}{Z!} \prod_{m=1}^{N_e} \frac{(L-m)!}{(N_e-m)!} \), \( \kappa \in (0, 1) \) is a given constant, and \( \Gamma \) denotes the gamma function.

First, we consider a constraint on the minimum throughput loss due to quantized cooperative precoder feedback, namely
\[
\Delta C = \bar{C} - \max_P \tilde{C}(P)
\]
(41)
with \( \bar{C} \) and \( \tilde{C} \) given in (7) and (23), respectively, and \( P \) denotes a power-control policy. To satisfy the constraint \( \Delta C \leq c \) with \( c > 0 \), it is sufficient to equate the following upper bound on \( \Delta C \) to \( c \):
\[
\Delta C \leq \bar{C} - \tilde{C}(P_{\text{max}}, P_{\text{max}})
\]
(42)
where \( \tilde{C}(P_{\text{max}}, P_{\text{max}}) \) corresponds to a sub-optimal power-control policy that fixes the power of both transmitters at the maximum. The above upper bound has a similar form as the throughput loss for multi-antenna downlink with limited feedback as defined in [19]. Thus, the following result can be proved following a similar procedure as [19, Theorem 2].

**Corollary 1.** For large \( P_{\text{max}} \), choosing the number of bits for cooperative precoder feedback as
\[
B = Z \log_2 (\nu P_{\text{max}}) - Z \log_2 \left(2^{\frac{\nu}{\kappa N_e}} - 1\right) + \omega
\]
(43)
ensures that
\[
\Delta C \leq c + o(1), \quad P_{\text{max}} \to \infty
\]
(44)
where \( \omega = Z \log_2 \frac{N_e \Gamma\left(\frac{1}{2}\right) E[\hat{\lambda}_K - N_e]}{Z \beta^{\frac{1}{2}}} \).

A few remarks are in order:
1) For large \( P_{\text{max}} \), \( B \approx Z \log_2 P_{\text{max}} \). For small \( P_{\text{max}} \), the network is noise limited and the number of precoder feedback bits can be kept small.
2) For the case of fixed interference margin, it can be also proved that \( B \approx Z \log_2 P_{\text{max}} \) for large \( P_{\text{max}} \) following a similar procedure as Corollary 1.
3) The upper bound on the capacity loss approaches \( c \) as \( P_{\text{max}} \) increases.
4) The feedback-bit scaling obtained in [19] for the MIMO downlink system is similar to that in (43) despite the difference in system configuration. Specifically, it is shown in [19, Theorem 2] that the number of precoder-feedback bits per user should scale as \( B \approx \tilde{N} (\tilde{M} - \tilde{N}) \log \tilde{P} \) for large \( \tilde{P} \) so
as to constrain the sum-throughput loss, where $\tilde{M}$ and $\tilde{N}$ are the numbers of antennas at the base station and each mobile, respectively, and $\hat{P}$ is the total transmission power at the base station. The above similarity rises from the fact that both the proposed precoding and the block-diagonalization precoding in [19] are designed using the zero-forcing criterion to null multiuser interference.

VI. Simulation Results

In the simulation, the codebook for quantizing the feedback precoders is randomly generated as in [28] and the system performance is averaged over a larger number of codebook realizations. The simulation parameters are set as follows unless specified otherwise. The numbers of antennas at each transmitter and receiver are $L = 6$ and $K = 3$, respectively. The number of data stream per user is $M = 2$. The size of inner precoder is fixed as $6 \times 3$. The path-loss factor $\nu$ is set as $\nu = 0.5$. Power control based on interference margin uses $\tau = 2$. The number of cooperative-feedback bit is $B = 8$.

A. Performance Evaluation

The size of inner equalizer determines the allocation of DoF at each receiver for mitigating residual interference and for enhancing array gain. The sum throughput for two different inner-equalizer sizes,
Fig. 3. Comparison of achievable sum throughput of the proposed precoding design for different numbers of cooperative-feedback bit $B$. The transmission power is fixed as $P_{\text{max}}$ and the inner equalizer has the size of $N_e \times K = 3 \times 3$.

namely $3 \times 2$ and $3 \times 3$, is compared in Fig. 2 with transmission power of each transmitter fixed as $P_{\text{max}}$. The larger inner precoder ($3 \times 3$) allocates more DoF for achieving array gain and it can be observed to increase the throughput for low SNRs where noise dominates residual interference. However, the smaller inner precoder is preferred for high SNRs since it allows more DoF for mitigating residual interference.

Fig. 3 compares the sum throughput of the proposed transceiver design for the cooperative-feedback bit $B = \{4, 6, 8, 10\}$. The transmission power of each transmitter is fixed as $P_{\text{max}}$ and the inner equalizer has the size of $N_e \times K = 3 \times 3$. It can be observed that the increment of every 2 cooperative-feedback bits increases the sum throughput by about 0.7 bit/s/Hz for high SNRs.

Fig. 4 compares the sum throughput of different IPC feedback algorithms where the inner equalizer size is set as $3 \times 3$. Residual interference between links is observed to decrease the throughput dramatically with respect to perfect CSI feedback. For large $P_{\text{max}}$, the IPC feedback Algorithm 1 designed for maximizing the achievable throughput is observed to provide substantial throughput gain over that based on fixed interference margin. Moreover, iterations for the IPC feedback Algorithm 1 are observed to give significant gain only at high SNRs.
B. Comparison with a Conventional Transceiver Design

The proposed precoding algorithm is compared with the conventional interference coordination [17] in terms of sum throughput with quantized cooperative feedback. The interference coordination algorithm attempts to align the interference by precoding such that at each receiver interference is observed only over the last $M$ antennas and the signals received over the first $(L-M)$ antennas are free of interference. To this end, the $L \times M$ precoder of user $m$, denoted as $\hat{F}_m'$, is chosen to be orthogonal to the channel sub-matrix $[H_{nm}]_{\text{row } 1:(L-M)}$ with $n \neq m$ where $[X]_{\text{row } m:n}$ denotes a sub-matrix comprising row $n$ to $m$ of a matrix $X$; the quantized version $\hat{F}_m'$ of $F_m'$ is fed back from receiver $n$ to transmitter $m$ for precoding. Furthermore, the receiver $G'_m$ of user $m$ decouples the data streams by zero-forcing:

$$G'_m = \left( [H_{mm}]_{\text{row } 1:(L-M)} \hat{F}'_m \right) \left( [H_{mm}]_{\text{row } 1:(L-M)} \hat{F}'_m \right)^\dagger \left( [H_{mm}]_{\text{row } 1:(L-M)} \hat{F}'_m \right)^{-1}.$$  \hfill (45)

Also considered in the comparison is the case of no CSIT where the precoder $\{\hat{F}'_m\}$ are arbitrarily chosen to be independent with all channels. The achievable sum throughput for the proposed and conventional algorithms are compared in Fig. 5. The transmission power of each transmitter is $P_{\text{max}}$; for the proposed design, the inner equalizer size is set as $3 \times 3$. It can be observed from Fig. 5 that the
proposed cooperative-feedback design yields dramatic throughput gains over the conventional algorithms. In particular, the gain over interference coordination is as large as about 4 bit/s/Hz for high SNRs. The performance gains of the proposed design result from the joint tuning of precoders and equalizers for simultaneously suppressing residual interference and harvesting diversity gain, which is enabled by the proposed inner/outer transceiver structure.

VII. CONCLUSION

We have proposed a systematic design of linear precoders and equalizers for the two-user MIMO interference channel with finite-rate cooperative precoder feedback. This design suppresses residual interference due to feedback precoder quantization. Building upon the above design, we have further proposed scalar cooperative feedback algorithms for controlling transmission power based on different criteria including fixed interference margin and maximum sum throughput. Finally, we have derived the scaling of the number of cooperative precoder-feedback bits under a the constraint on the sum throughput loss. Possible extensions of the current work include generalizing the proposed algorithms to
the interference channel with an arbitrary number of users and relaxing the current zero-forcing criterion on the precoder design.

**APPENDIX**

Lemma 2 is proved as follows. For convenience, the achievable throughput in (30) can be written as

\[ A_{\text{IM}} = \sum_{m=1}^{2} \sum_{\ell=1}^{M} A_m^{[\ell]} \]  

where

\[ A_m^{[\ell]} = \mathbb{E} \left[ \log_2 \left( 1 + \frac{\min(\eta_m, P_{\text{max}}) \lambda_{mm}^{[\ell]}}{1 + \tau} \right) \right]. \]  

Expand \( A_m^{[\ell]} \) as

\[ A_m^{[\ell]} = \mathbb{E} \left[ \log_2 \left( 1 + \frac{\eta_m \lambda_{mm}^{[\ell]}}{1 + \tau} \right) \mid \eta_m \leq P_{\text{max}} \right] \Pr(\eta_m \leq P_{\text{max}}) + \]

\[ \mathbb{E} \left[ \log_2 \left( 1 + \frac{P_{\text{max}} \lambda_{mm}^{[\ell]}}{1 + \tau} \right) \right] \Pr(\eta_m > P_{\text{max}}). \]  

Using (29), the first term \( \Phi_1 \) of \( A_m^{[\ell]} \) in (48) can be rewritten as

\[ \Phi_1 = \mathbb{E} \left[ \log_2 \left( 1 + \frac{\tau \hat{\lambda}_\ell}{(1 + \tau) N_p \nu \lambda_K - N_e + 1} \right) \right] - \Phi_3 \]  

with \( \Phi_3 \) defined and simplified as

\[ \Phi_3 = \mathbb{E} \left[ \int_0^{\frac{\tau}{N_p \nu \lambda_{K-N_e+1}}} \log_2 \left( 1 + \frac{\tau \hat{\lambda}_\ell}{(1 + \tau) N_p \nu \lambda_K - N_e + 1} \right) f_{\lambda_{K-N_e+1}}(x) dx \right] \]

\[ = \mathbb{E} \left\{ \int_0^{\frac{\tau}{N_p \nu \lambda_{K-N_e+1}}} \left( O(1) + \log_2 \frac{1}{x} \right) f_{\lambda_{K-N_e+1}}(x) dx \right\}, \quad P_{\text{max}} \to \infty \]

\[ \equiv \mathbb{E} \left\{ \int_0^{\frac{\tau}{N_p \nu \lambda_{K-N_e+1}}} o(1) \times \left[ \hat{a}_{K-N_e+1} x^{N_e-2} + o(x^{N_e-2}) \right] dx \right\} \]

\[ = o \left( P_{\text{max}}^{-N_e+1} \right) \]  

where \((a)\) is obtained using Lemma 1 and \( x \log_2 \frac{1}{x} = o(1) \) for \( x \to 0 \). Similarly, we obtain the second term \( \Phi_2 \) of \( A_m^{[\ell]} \) in (48) as

\[ \Phi_2 = o \left( P_{\text{max}}^{-N_e+1} \right), \quad P_{\text{max}} \to \infty. \]  

Substituting (49), (50), and (51) into (48) and then (46) yields (34), completing the proof.
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