 MSGUTs from Germ to Bloom: towards Falsifiability and Beyond \(^1\)

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Abstract

We review the development of renormalizable SO(10) SUSY GUTs based on the \(210 \oplus 126 \oplus 126 \oplus 10\) Higgs system. These GUTs are minimal by parameter counting. Using the \(SO(10) \rightarrow SU(4) \times SU(2)\(_L\) \times SU(2)\(_R\)\) label decomposition developed by us we calculated the complete GUT scale spectra and couplings and the threshold effects therefrom. The corrections to \(\alpha_G, \sin^2 \theta_W\) and \(M_X\) are sensitive functions of the single parameter \(\xi\) that controls symmetry breaking and slow functions of the other parameters. Scans of the parameter space to identify regions compatible with gauge unification are shown.

The tight connection between the phenomenology of neutrino oscillations and exotic \((\Delta B \neq 0)\) processes predicted by SUSY SO(10) GUTs is discussed in the context of the recent successful fits of all available fermion mass/mixing data using the \(10 \oplus 126\) Higgs representations and Type I/II seesaw mechanisms for neutrino mass. We emphasize that the true output of these calculations should be regarded as the unitary matrices that specify the orientation of the embedding of the MSSM within the MSGUT. The \(\Delta B \neq 0, d=5\) effective Lagrangian is written down using these embedding matrices. We show how Fermion Mass fitting constraints can be combined with GUT spectra to falsify/constrain the MSGUT and its near relatives. An initial survey indicates that Type I Seesaw neutrino masses dominate Type II and both are too small in the perturbative MSGUT even when the mixing and mass squared splitting ratios are as per data. This motivates a detailed study of the MSGUT constraints using the outputs of the fitting of fermion data, as well as consideration of modifications/extensions of the MSGUT scenario.

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1 Introduction

The Supersymmetric SO(10) GUT based on the $126 \oplus \overline{126} \oplus 210$ Higgs multiplets was first introduced over 20 years ago in [1, 2]. At that time the electroweak gauge parameters and fermion mass data were quite incomplete. The next significant development was the realization [3] that R parity $R_p = (-)^{3(B-L)+2S}$ formed a part of the gauge symmetry in LR symmetric models with gauged $U(1)_{B-L}$ [4] and was hence protected by it. In particular it would remain unbroken as long as no field with odd $B - L$ received a vev. SO(10) is now the most favoured GUT gauge group because of the natural way in which it accommodates complete fermion families together with the superheavy right handed neutrinos required by the seesaw mechanism [5]. In 1992, with LEP data in hand and CKM parameters largely known Babu and Mohapatra [6] took up the task of fitting Fermion masses and mixing using the $10 \oplus 126$ FM Higgs system introduced in [1, 2] and the Type I seesaw mechanism. They proposed that once the charged fermion masses in the SM had been fitted the $10 \oplus 126$ matter fermion Yukawa couplings of the GUT would be completely determined and hence the model would predict the allowed neutrino masses. Although, based on the then current notions about neutrino masses/mixings, they concluded that their proposal failed, it would inspire much future work [7, 8, 9, 10, 11].

In the mid-1990’s the Supersymmetric LR model based on a parity odd singlet [12] was found to have [13] a charge breaking vacuum and this was construed as evidence of a low scale linked breaking of $SU(2)_R$, $B - L$ and R-parity [13] driven by the soft supersymmetry breaking terms. However an alternative analysis based on allowing this scale to lie anywhere in the range from $M_S$ and $M_X$ allowed the construction of consistent Minimal Left Right Supersymmetric models (MSLRMs) [14, 15, 16] which were shown to naturally preserve R-parity and thus predict a stable LSP. Moreover these analyses brought clearly to the fore a theme that had been noticed from the beginning [1, 17] of the study of multiscale Susy GUTs: that generically in LR Susy GUTs there are light multiplets whose masses are protected by supersymmetry even though they are submultiplets of a GUT multiplet that breaks gauge symmetry at a high scale. As such they violate the conventional wisdom of the “survival principle” which estimates the masses of such submultiplets to be the same as the large vev. Thus it became clear [18] that Susy GUTs required the use of calculated not estimated masses for RG analyses.

This ambling pace of theoretical development was forced by the epochal discovery of neutrino oscillations by Super-Kamiokande in 1997 and the rapid refinement of our knowledge of the parameters thereof [19]. As is well known, the seesaw mass scale indicated by this discovery was in the range indicated by Grand Unification. Since LR models and GUTs containing them naturally incorporate the Seesaw mechanism, this gave a strong motivation for taking up the detailed study of SO(10) Susy Guts anew, particularly with the understanding provided by the natural class of MSLRMs developed earlier [14, 15, 16]. A completely viable R-parity preserving Susy GUT based on the $10 \oplus 126$ FM (for Fermion Mass )Higgs system and an additional $45 \oplus 54 \oplus 126$ AM (for Adjoint type Multiplet and euphony) system was then developed [20]. The RG analysis carried out in this work already indicated that the use of calculated spectra would force together the various possible intermediate scales into a narrow range close to the GUT scale resulting in an effective “SU(5)
conspiracy”. Moreover the problem of seeing how the various MSLRMs considered by us would fit into Susy GUTs had motivated a review of the various possibilities\cite{[21, 22]} and this had again brought up the model based on the 210-plet as an important and interesting possibility.

It thus became clear that a detailed calculation of the full GUT spectrum and couplings in various SO(10) GUT models would be necessary if further progress was to be made and that this would require complete calculational control on the group theory of SO(10) at the level needed by practical field theory calculations which was still lacking in spite of signal contributions\cite{[23, 24]}. These techniques – based on an explicit decomposition of SO(10) tensor and spinor labels into those of the maximal (“Pati-Salam”) sub-group – were duly if laboriously developed by us\cite{[25]}. At this time the model based on the $126 \oplus \overline{126} \oplus 210$ was again brought to the fore by us\cite{[26]} and its old\cite{[6, 27]} claim to be called the minimal Susy GUT (MSGUT) was buttressed by an analysis of its parameter counting and the simplicity of its structure.

The techniques developed by us\cite{[25]} permitted first a partial calculation of the mass matrices of the MSSM doublets $(1, 2, \pm 1)$ and triplets $(3, 1, \pm \frac{2}{3})$\cite{[25]} and then a complete calculation\cite{[28]} of all the couplings and mass matrices of the MSGUT. With the same motivations two calculations, one in parallel and cross checking with ours\cite{[29]} and another\cite{[30]} quite separate, had commenced after\cite{[25]}. These calculations both used a somewhat abstract\cite{[31]} method that permitted the calculation of the “clebsches” that entered the spectra of MSSM submultiplets of SO(10) tensor (but not – so far – spinor) chiral supermultiplets (but not their couplings). After some controversy concerning the consistency of these three calculations\cite{[25, 29, 32, 33, 34, 35]} a consensus seems to have emerged on their compatibility\cite{[34, 35]} not withstanding notable differences in normalizations and phase conventions. In\cite{[25, 28]} we also provided the complete (chiral and gauge) spectra, neutrino mass matrices, all gauge and chiral couplings and the effective $d = 5$ operators for Baryon violation in terms of GUT parameters. This laid the stage for a completely explicit RG based analysis of the MSGUT: an important example of which was the calculation of threshold effects based on these spectra\cite{[28]} described in Section 3.

Meanwhile the old theme of utilizing the very restricted structure of fermion Higgs couplings in the class of SO(10) GUTs that used only $10 \oplus \overline{126}$ representations and renormalizable couplings to make “predictions” concerning the neutrino mass sector was taken up again. It led\cite{[8]} to a remarkably simple (2 generation) insight into the operation of the Type II mechanism that naturally generated large atmospheric neutrino mixing angles based on the observed approximate $b - \tau$ unification in the MSSM extrapolated to $M_X$. This simple insight then provoked a detailed analysis of the fitting problem for dominant Type II seesaw mechanism and 3 generations\cite{[9]}, which was quite successful. A certain tension in the details of these fits was ameliorated by later work\cite{[10]} and disappeared\cite{[10]} when a $120$-plet was also allowed to perturb the fit of the $10 + \overline{126}$ slightly. Finally, very recently, in fact just in time for PLANCK05 Babu announced that they\cite{[11]} had succeeded in obtaining viable Type I, Type II and mixed fits to the (large mixing angle) neutrino mass data: which possibility had been neglected since the suggestion of\cite{[8]}. These sudden reversals are due to the extreme delicacy and complexity of the task of fitting accurately the masses of particles differing by a factor of upto $10^6$ in a multidimensional complex
parameter space where algorithms “shoot” obliviously past solution points.

Obviously these successful fits of all low energy fermion “quadratic” data (extrapolated up to $M_X$) in generic MSGUTs cry out to be married to a particular simple MSGUT in which all coefficients are specified and testable for viability. This is particularly so because only in a particular MSGUT, where the needed coefficients are fully specified, can one test the viability of a given FM fit (such as the Type II dominant ones). Furthermore, as we shall see in detail below, the true output of the FM fit is really the embedding matrices that specify the relation between MSSM and MSGUT supermultiplets at the GUT scale. Given the phenomenologically expected quasi-identity of flavour mixing in the scalar and fermionic sector down to Electroweak scales, these matrices are the main missing ingredient for performing an informed and realistic calculation of Baryon decay and Lepton Flavour violation in the MSGUT. The combined cuts provided by gauge unification and fermion mass fit viability constrain the MSGUT and render it falsifiable. If it passed these tests with some region of its parameter space unscathed we would be in a position to actually specify the theory at the high scale: a task long thought to be impossible by those despondent at the continuing failure to detect proton decay. The key to this remarkable development is of course the remarkable uroborotic (ουροβορος: the world-snake that swallows its tail) feature of the Seesaw which connects the physics of the ultralight and ultramassive particles.

In Section II we review the structure and spontaneous symmetry breaking of the MSGUT and describe its decomposition to Pati-Salam labels. This is accompanied by an Appendix containing the complete spectrum of MSSM multiplets in the MSGUT -including gauge submultiplets. In Section III we describe the RG analysis of threshold effects and the type of cuts these corrections impose on the parameter space. In Section IV we review the fitting of Fermion masses and mixings with a view to supporting our assertion that the true output of this procedure is the specification of the embedding matrices. In Section V we plot the behaviour of the parameters that control Type I vs Type II dominance in the MSGUT and show that Type II dominance may be quite unlikely. We also use particular examples of solutions provided by one of the groups that have performed the successful fits to plot the Type I and Type II contributions and again find indications that Type II is not easily dominant and that the overall size of neutrino masses tends to be too low except possibly in very special regions of parameter space. In the next section we transform our previously derived formula for the effective $d=5$ Lagrangian for Baryon decay in the MSGUT to the MSSM using the embedding matrices. This operator is then ready for use to calculate Baryon decay. We also briefly comment on the relevance of our considerations to $d=6$ and Lepton Flavour violation. We conclude with a discussion of the outlook and the directions that will be pursued in the forthcoming detailed paper on the application of the techniques indicated here to the pinning down of the MSGUT.

2 Essentials of MSGUTs

There are at present two main types of SO(10) Susy GUT models namely renormalizable GUTs (RGUTs) of the classic type which invoke only gauge symmetry and preserve R-parity by maintaining the sharp distinction between Matter and Higgs Supermultiplets,
and non renormalizable GUTs (nRGUTs)\cite{36} which allow non renormalizable operators - particularly for generating fermion masses - but impose additional symmetries to perform the functions of R-parity and suppress unwanted behaviours allowed by the license granted by non renormalizability. Practitioners of the art are often sharply fixed in their preference for one type or the other and our own is obviously with the former. Many (talks by Babu, Malinsky, Mohapatra..) of the contributions on GUTs to PLANCK05 are of this type but the other type is also well represented (Pati, Raby...).

Other distinguishing features of these two types of models are that RGUTs allow large representations such as the $\overline{126}$ Higgs to generate charged and neutral (via type I and Type II Seesaw) fermion masses but claim minimality on grounds of maximal economy of parameters. In contrast nRGUTs, allow only small Higgs representations and use non renormalizable operators ($16^2 \overline{16}^2$ etc) to generate effective vevs in the $\overline{126}$, $120$ channels. They achieve simplifications by invoking additional symmetries, whose rationale is however not always appreciated by non adherents. A very large number of works on nRGUTs\cite{36} have appeared and some of these have far advanced fitting programs : with claims to generate sample spectra with specific numbers for most (i.e $\sim 10^2$) low energy MSSM parameters. Moreover they also seek to satisfy additional motivations like workable models of gauge mediated supersymmetry breaking\cite{36}.

The RGUTs, on the other hand are mostly at the stage of seeking a semi quantitative non-disagreement with the data as available at present. For example, since no superpartner has as yet actually been observed, and speculations on their possible masses range all the way from the classic 1 TeV up to almost the GUT scale, the value of the low energy parameters renormalized up to the Susy breaking scale where the Susy RG equations take hold, can, realistically, be considered as known to at best an accuracy of $5 – 10\%$. Invoking the remarkable accuracy of experimental data at the scale $M_Z$ is of little avail since nothing is known of the size of the threshold corrections due to superpartners. It is this approach that we shall adopt in this paper and our quantitative analyses regarding quantities unprotected by any symmetry shall not lay claim to accuracy that may turn out to be quite spurious.

A very important but controversial distinction between the two classes of theories is due to the fact that the one loop beta functions of the $\overline{126}$, $120$ chiral supermultiplets are so large (35 and 28 respectively) that they alone overcome the contribution (-24) of the SO(10) gauge supermultiplet and cause the gauge coupling to explode just above $M_X$ the perturbative unification scale. By analyzing the complete two loop RG equations for the MSGUT above $M_X$ we have shown\cite{38} that the large positive coefficient in the one loop gauge beta function precludes evasion of this difficulty by taking refuge in a weakly coupled fixed point before the gauge coupling explodes. Thus such theories effectively determine their own upper cutoff $\Lambda_X \sim 10^{17} GeV$. This feature is so inescapable that we proposed\cite{39,40} to face it by making it a signal of a deeper non-perturbative feature of such Susy GUTs: namely that they dynamically break the GUT symmetry down to a smaller symmetry (such as $H = SU(5) \times U(1)$ or even $H = G_{123}$) at the scale $\Lambda_X$ via a Supersymmetric condensation of gauginos in the coset $SO(10)/H$ which drives a $G_{123}$ preserving Chiral scalar condensate\cite{40}. A new fundamental length $\Lambda_X^{-1}$ below which SM probes cannot delve thus arises, endowing the particles of the SM with “hearts” i.e impenetrable cores with sizes $\sim 10^{-30} cm$. We have implemented\cite{40} the strongly coupled
Supersymmetric dynamics required by this type of scenario in a toy model based on $SU(2)$ which can be easily generalized to at least the simple breaking $SO(10) \rightarrow SU(5) \times U(1)$ [11]. Various fascinating perspectives including dual unification and induced gravity then beckon. However in view of the controversial nature of this proposal we shall not treat of it further here. This talk is aimed at indicating the feasibility of a promotion of the MSGUT to a falsifiable theory using the deep linkages between low and high scale physics inherent in the $SO(10)$ seesaw mechanism in combination with the traditional -relatively permissive- constraints of gauge unification.

The MSGUT is the renormalizable globally supersymmetric $SO(10)$ GUT whose chiral supermultiplets consist of “AM type” totally antisymmetric tensors: $210(\Phi_{ijkl})$, $126(\Sigma_{ijklm})(i,j = 1...10)$ which break the GUT symmetry to the MSSM, together with Fermion mass (FM) Higgs $10$-plet($H_i$). The $126$ plays a dual or AM-FM role since it also enables the generation of realistic charged fermion and neutrino masses and mixings (via the Type I and/or Type II mechanisms); three $16$-plets $\Psi_A (A = 1, 2, 3)$ contain the matter including the three conjugate neutrinos ($\bar{\nu}_i$).

If in addition to the $10, 126$ FM Higgs we also include a $120$ -plet Higgs allowed by the $SO(10)$ multiplication rule $16 \times 16 = 10 + 120 + 126$, the GUT scale symmetry breaking is unchanged since the $120$ contains no SM singlets. However it does contribute two additional pairs of $(1, 2, \pm 1)$ doublets taking the MSSM Higgs doublet mass matrix $H$ from $4 \times 4$ to $6 \times 6$. The resulting theory is thus justly called the next to minimal Susy GUT (nMSGUT).

The superpotential is

$$W = \frac{1}{2} M_H H_i^2 + \frac{m}{4!} \Phi_{ijkl} \Phi_{ijkl} + \frac{\lambda}{4!} \Phi_{ijkl} \Phi_{klmn} \Phi_{mnop} + \frac{M}{5!} \Sigma_{ijklm} \Sigma_{ijklm}$$

$$+ \frac{\eta}{4!} \Phi_{ijkl} \Sigma_{ijmno} \Sigma_{klmno} + \frac{1}{4!} H_i \Phi_{ijklm} (\gamma \Sigma_{ijklm} + \bar{\gamma} \Sigma_{ijklm})$$

$$+ h'_{AB} \psi_A^T C_2^{(5)} \gamma_i \psi_B H_i + \frac{1}{5!} f'_{AB} \psi_A^T C_2^{(5)} \gamma_i \psi_B \Sigma_{i1...is} \Sigma_{i1...is}$$

(1)

The parameter counting is as follows : the 7 complex parameters $m, M, M_H, \lambda, \eta, \gamma$ and $\bar{\gamma}$ can be relieved of 4 phases using the arbitrariness in the phases of the 4 $SO(10)$ multiplets $210, 126, 126, 10$ leaving 10 real parameters. The 12 complex Yukawa contained in the symmetric $3 \times 3$ matrices $h', f'$ can be reduced to 15 real parameters by diagonalizing one linear combination of the two to a real diagonal form. In addition there is the gauge coupling. In all the MSGUT thus has exactly 26 non-soft parameters [26]. Incidentally the MSSM also has 26 non-soft couplings. In [26] we have shown that this number of “hard” parameters is considerably less than any GUT attempting to perform the same tasks via a vis fermion masses that the MSGUT accomplishes. The ‘levity of the cognoscenti’ provoked by the number 26 will recur again below ! Note however that one of these parameters, e.g $M_H$, is traded for the electroweak vev via the fine tuning condition that yields two light doublets and another Susy electro-weak parameter i.e tan $\beta$ must be taken as an additional given of the analysis since it is likely determined by dynamics dependent on the supersymmetry breaking parameters.

The GUT scale vevs that break the gauge symmetry down to the SM symmetry (in the notation $a, b = 1...6; \tilde{a} = 7...10$ of [25]) are [11, 2]: $\langle (15, 1, 1) \rangle_{210}$: $\langle \phi_{abcd} \rangle = \frac{3}{2} \epsilon_{abcdef} \epsilon_{ef}$;
\[ \langle (15,1,3) \rangle_{210} : \langle \phi_{ab\bar{\alpha}} \rangle = \omega \epsilon_{ab} \epsilon_{\bar{\alpha} \beta} ; \langle (1,1,1) \rangle_{210} : \langle \phi_{a\bar{\beta}\bar{\gamma}} \rangle = p \epsilon_{a\bar{\beta}\bar{\gamma}} \delta ; \]

\[ \langle (10,1,3) \rangle_{126} : \langle \Sigma_{1\bar{3}\bar{5}\bar{8}0} \rangle = \bar{\sigma} ; \langle (10,1,3) \rangle_{126} : \langle \Sigma_{2\bar{4}\bar{5}\bar{7}9} \rangle = \sigma. \] The vanishing of the D-terms of the SO(10) gauge sector potential imposes only the condition \(|\sigma| = |\bar{\sigma}|\). Except for the simpler cases corresponding to enhanced unbroken symmetry \((SU(5) \times U(1), SU(5) \ G_{3,2,2,B-L}, G_{3,2,R,B-L} \ etc)\[26, 29\] this system of equations is essentially cubic and can be reduced to the single equation\[26\] for a variable \(x = -\lambda \omega / m\), in terms of which the vevs \(a, \omega, p, \sigma, \bar{\sigma}\) are specified:

\[ 8x^3 - 15x^2 + 14x - 3 = -\xi(1 - x)^2 \]  

(2)

where \(\xi = \frac{\lambda M}{\eta m}\). This exhibits the crucial importance of the parameter \(\xi\).

Using the above vevs and the methods of\[25\] we calculated the complete gauge and chiral multiplet GUT scale spectra and couplings for the 52 different MSSM multiplet sets falling into 26 different MSSM multiplet types of which 18 are unmixed while the other 8 types occur in multiple copies. (On the lighter note: the occurrences yet again of the ‘mystic’ String Theory number 26 demonstrates that one can do just as well without string theory when searching for the fundamental theory!). These spectra may be found in the Appendix.

Among the mass matrices exhibited is the all important 4 x 4 Higgs doublet mass matrix\[25, 28\] \(\mathcal{H}\). To keep one pair of these doublets light one tunes \(M_H\) so that \(\text{Det} \mathcal{H} = 0\). This matrix can then be diagonalized by a bi-unitary transformation yielding thereby the coefficients describing the proportion of the doublet fields in \(10, \overline{126}, 126, 210\) present in the light doublets: which proportions are important for many phenomena.

### 3 RG Analysis

If the serendipity\[42, 43, 44\] of the MSSM gauge coupling unification at \(M_X^0\) is to survive closer examination the MSGUT must answer the query:

Are the one loop values of \(\sin^2 \theta_W\) and \(M_X\) generically stable against superheavy threshold corrections?.

We follow the approach of Hall\[45\] in which the mass of the lightest baryon number violating superheavy gauge bosons is chosen as the common “physical” superheavy matching point \((M_i = M_X)\) in the equations relating the MSSM couplings to the SO(10) coupling\[45\] :

\[ \frac{1}{\alpha_i(M_S)} = \frac{1}{\alpha_G(M_X)} + 8\pi b_i \ln \frac{M_X}{M_S} + 4\pi \sum_j \frac{b_{ij}}{b_j} \ln X_j - 4\pi \lambda_i(M_X) \]  

(3)

See\[28\] for details. In this approach, rather than enforcing unification at a point, it is recognized that above the scale \(M_X\) the effective theory changes to a Susy SO(10) model structured by the complex superheavy spectra which we have computed and which appears as unbroken SO(10) only at the crudest resolution – here surpassed. Thus we compute the corrections to the three parameters \(\log_{10} M_X, \sin^2 \theta_W(M_S), \alpha^{-1}_G(M_X)\) as a function of the MSGUT parameters and the answer to the question of stability of the perturbative unification is determined by the ranges of GUT parameters where these corrections are
consistent with the known or surmised data on $\log_{10}M_X$, $\sin^2\theta_W(M_S)$ and the consistency requirement that the SO(10) theory remain perturbative after correction. We find the corrections

$$\Delta^{(th)}(\log_{10}M_X) = .0217 + .0167(5\tilde{b}'_1 + 3\tilde{b}'_2 - 8\tilde{b}'_3)\log_{10}\frac{M'}{M^0_X}$$

(4)

$$\Delta^{(th)}(\sin^2\theta_W(M_S)) = .00004 - .00024(4\tilde{b}'_1 - 9.6\tilde{b}'_2 + 5.6\tilde{b}'_3)\log_{10}\frac{M'}{M^0_X}$$

(5)

$$\Delta^{(th)}(\alpha^{-1}_G(M_X)) = .1565 + .01832(5\tilde{b}'_1 + 3\tilde{b}'_2 + 12\tilde{b}'_3)\log_{10}\frac{M'}{M^0_X}$$

(6)

Where $\tilde{b}'_i = 16\pi^2b'_i$ are 1-loop $\beta$ function coefficients ($\beta_i = b_ig^3_i$) for multiplets with mass $M'$ (a sum over representations is implicit).

These corrections are to be added to the one loop values corresponding to the successful gauge unification of the MSSM:

Using the values

$$\alpha^{-1}_G(M_X) = 25.6 ; \ M^0_X = 10^{16.25}\ GeV ; \ M_S = 1\ TeV$$

$$\alpha^{-1}_1(M_S) = 57.45 ; \ \alpha^{-1}_2(M_S) = 30.8 ; \ \alpha^{-1}_3(M_S) = 11.04$$

(7)

the two loop corrections are

$$\Delta^{2\text{-loop}}(\log_{10}\frac{M_X}{M_S}) = -.08 ; \ \Delta^{2\text{-loop}}(\sin^2\theta_W(M_S)) = .0026$$

$$\Delta^{2\text{-loop}}\alpha^{-1}_G(M_X) = -.546$$

(8)

We see that in comparison with the large threshold effects to be expected in view of the number of heavy fields and their beta functions\[^{[46, 28]}\] the 2 loop corrections are quite small.

A few remarks on the role of the parameters are in order. The parameter $\xi = \lambda M/\eta m$ is the only numerical parameter that enters into the cubic eqn.\[^{[2]}\] that determines the parameter $x$ in terms of which all the superheavy vevs are given. It is thus the most crucial determinant of the mass spectrum. The dependence of the threshold corrections on the parameters $\lambda, \eta, \gamma, \bar{\gamma}$ is comparatively mild except when coherent e.g when many masses are lowered together leading to $\alpha_G$ explosion. The parameter ratio $m/\lambda$ can be extracted as the overall scale of the vevs. Since the threshold corrections we calculate are dependent only on (logarithms of) ratios of masses, the parameter $m$ does not play any crucial role in our scan of the parameter space: it is simply fixed in terms of the mass $M_V = M_X$ of the lightest superheavy vector particles mediating proton decay.

With these formulae in hand we can explore the dependence of the threshold corrections on the “fast” parameter $\xi$ in response to which the vevs and thus all the threshold corrections can gyrate wildly (see plots for the real solution for real $\xi$ below), and the “slow diagonal” parameters $\lambda, \eta$ whose lowering tends to make fields light and thus give large negative corrections to $\alpha^{-1}_G$. In addition there are the “slow off diagonal” parameters $\gamma, \bar{\gamma}$ whose effect seems quite mild.
Sin²θ_W is now very accurately known\(^{[45]}\) at M_Z: \(s_Z^2 = 0.23120 \pm 0.00015\) or an error of less than 0.065%. Similarly the value \(\alpha_\text{em} = 127.906 \pm 0.019\) has an error of only 0.015%. The main uncertainty in the data (besides \(M_H\)) at M_Z is in \(\alpha_3(M_Z) \sim 1.5\%\). However the same is obviously not true of the overall sparticle mass scale or the intra sparticle mass splittings: with values in current speculation ranging from \(\sim 1\) TeV to \(10^{10}\) TeV! Thus at least until the first superpartners are observed the assumption that we know the values of the \(\alpha_i(M_S)\) with anything like the precision at M_Z is quite unjustified. A rough estimate of the uncertainty in Sin²θ_W(M_S) gives numbers in the ball park of 1% to 10% and we shall not pretend to have any license to impose stronger constraints on our parameters. For the uncertainty in Log\(_{10}\)M_X however there is a quite stringent \(M_S\) independent constraint coming from the requirement that gauge mediated proton decay in channels like \(p \rightarrow e^+\pi^0\) obey the current bounds \(\tau_{p \rightarrow e^+\pi^0} > 10^{33}\) yr. Thus \(\Delta\text{Log}_{10}\)M_X > -1 is a very firm constraint that we may rely upon. Finally it is clear that the applicability of the perturbative formulae used in our treatment will need detailed scrutiny if the fractional change in \(\alpha_G(M_X)^{-1}\) is greater than about 25% (particularly if the change is negative). So \(|\Delta\alpha_G(M_X)^{-1}| < 10\) (say) is a limitation we tentatively observe as a limit on the range of validity of our calculation (rather than the theory itself: since a detailed examination may permit one to handle larger changes – particularly positive ones – in \(\alpha_G(M_X)^{-1}\)). In view of these relatively loose bounds on the acceptable changes associated with threshold effects we cannot expect that gauge constraints can limit the allowed parameter space very precisely. Nevertheless, these constraints used in conjunction with additional very significant information arising from the fitting of Fermion masses and (in more model dependent ways) \(d = 5\) Baryon violation and Lepton flavour violation could allow us to obtain a complete semi-quantitative picture of viable regions, if any, of the MSGUT parameter space.

We now present examples showing how such a semi-quantitative mapping of the RG constrained topography of the GUT scale parameter space may be obtained using the formulae given above. An attempt at an exhaustive mapping would be premature till the recent available FM fits\(^{[10],[11]}\) have been digested. Later we shall present our program of further constraining the parameter space using the constraints of the fit\(^{[8],[9],[10],[11]}\) of Fermion masses and also due to the consequent specification of Baryon decay operators. It should be kept in mind that at any \(\xi\) the cubic equation (2) has three solutions any of which is in principle exploitable for defining a vacuum.

Consider first the plots of the threshold changes \(\Delta^{(th)}(\alpha_G^{-1}(M_X))\) and \(\Delta^{(th)}(\text{Log}_{10}M_X)\) vs \(\xi\) after (arbitrarily) setting \(\{\lambda, \eta, \gamma, \bar{\gamma}\} = \{0.7, 0.5, 0.3, 0.2\}\) which are shown as Fig. 1 for the real solution of the eqn (2) for real \(\xi\).

The “twin towers” due to singularities at \(\xi = -5, 10\) in the plot of \(\alpha_G(M_X)^{-1}\) arise from the SU(5) invariant real vevs at these values of \(\xi\). Similarly the negative spike at \(\xi = -2/3\) corresponds to a real solution with SU(5)flipped × U(1) symmetry\(^{[26],[29]}\). As emphasized by us in\(^{[28]}\) the plots of the threshold changes show very sharply defined features corresponding to the special behaviour near points of enhanced symmetry and thus provide a rapid way of scanning the topography of the parameter space. For these moderately large values of \(\lambda, \eta\) the lower cut at \(\Delta\alpha_G(M_X)^{-1} = -10\) essentially allows all possible values of \(\xi\) though the upper cut does advise caution around the special symmetry points with SU(5) symmetry. However the graph of \(\Delta\text{Log}_{10}M_X\) with a cut at \(\Delta\text{Log}_{10}M_X =\)
change in $\alpha$ immediately rules out most of the region $\xi \in (.25, 8.6)$ due to the ultrarapid gauge boson mediated Proton decay in that range of $\xi$.

When we lower the value of $\lambda$ or $\eta$ many particles became light so that the entire $\alpha_G(M_X)^{-1}$ vs $\xi$ plot shifts downward. This is seen in Fig. 2 where we repeat the same Plot as Fig 1. but now with $\lambda = .1$. As a result larger $\xi$ values are allowed on grounds of limiting the change in $\alpha_G(M_X)^{-1}$. On the other hand the behaviour of $\Delta \log_{10} M_X$ in response to a decrease in the diagonal slow parameters $\lambda, \eta$ is quite mild. Thus although lowering $\lambda$ does tend to make $\Delta \log_{10} M_X$ less negative in $\xi > 0$ region, the large positive change in $\alpha_G(M_X)$ would rule out the small $\lambda, \eta$ region. The condition $\Delta \log_{10} M_X > -1$ required by proton stability rules out the region $0.25 < \xi < 8.6$ for the real case (see Figs. 1., 2.).

Next we plot $\Delta \sin^2 \theta_W(M_S)$ versus $\xi$ for moderate and very small $\lambda$ (Fig 3.). The change in $\lambda$ has practically no effect. We also see that there are ranges of $\xi$ where the change in $\sin^2 \theta_W(M_S)$ is less than 10% and that these ranges are only slightly affected by the change in $\lambda$. However the region $2.8 < \xi < 8$ which was excluded by $\Delta \log_{10} M_X > -1$ is also excluded by the large change $\Delta \sin^2 \theta_W(M_S)$.

The further evolution of $\Delta^{(th)}(\alpha_G^{-1}(M_X))$ and $\Delta^{(th)}(\log_{10} M_X)$ as $\lambda$ is decreased to .01 is similar as can be seen in Fig.4. Lowering $\eta$ has effects very similar to those of lowering $\lambda$ since the effect of smaller values for these diagonal couplings is to lower the masses of whole sets of multiplets and therefore raise $\alpha_G(M_X)$ sharply. The effects of the off diagonal couplings $\gamma$ and $\bar{\gamma}$ are much milder.

With $\xi$ real there are also two complex (mutually conjugate) solutions of the basic cubic equation. Examples of these are displayed as Fig. 5,6 for moderate and small $\lambda$. We observe that the behaviour of the complex solution is smoother than the real one and that apart from the spikes observed at $\xi = -5$ there is hardly any sharp feature to be seen. The effect of decreasing $\lambda$ or $\eta$ on $\Delta^{(th)}(\alpha_G^{-1}(M_X))$ is however even stronger than in the case of real $x$ as may be seen in Fig 6, thus restricting the viable magnitudes of $\lambda, \eta$ to moderately large values again. The corrections to $\sin^2 \theta_W(M_S)$ for the complex solution are shown for moderate and very small values of $\lambda$ as Fig. 7. One can also consider complex values of the parameter $\xi$ and the three solutions in that case. The behaviour is quite similar to that shown for the complex solutions for real $\xi$ but needs detailed and comprehensive examination.

We have exhibited these graphs to give a sense of the structure visible once one ramps up the resolution of analysis to reveal the finestructure hidden within the bland impressiveness of “supersymmetric unification at a point”. As already noted long since the ambiguities associated with superheavy thresholds would not allow one to predict the effective scale of superpartner masses or the unification scale even if the low energy values $\alpha_i(M_Z)$ were known exactly. In fact, following Hall we have chosen not to treat the unification beyond leading order by imposing unification at a point but rather in terms of quantifying the ambiguities in $\alpha_G^{-1}(M_X)$, $\sin^2 \theta_W(M_S)$ and $\log_{10} M_X$ caused by the finestructure of unification scale mass spectra. The range of behaviours exhibited make it unlikely that the constraints of gauge unification alone will rule out this minimal Susy SO(10) GUT or fix its parameters. However when taken together with the rest of the low energy data, the MSGUT provides a well defined and calculable framework within which significant ques-
tions regarding the viable GUT scale structures can be posed and answered. Furthermore
the process of fitting the highly structured fermion data to the relatively few
parameters of the MSGUT excavates crucially important information regarding the
embedding of the MSSM within the MSGUT. This information (concerning mixing angles
of various sorts) is critical for determining the precise predictions of the MSGUT for both
d = 5, 6 baryon number violating operators in the effective MSSM as well as the predictions
for Lepton Flavour violation.

4 FM Fitting Frenzy

We have already reviewed the sequence of developments preceding the current focus of
interest on the fitting of fermion mass and mixing data in the MSGUT. The fitting program itself has used only the form of the fermion mass formulae in the
MSGUT (which follows from the use of only the 10 + 126 representations) rather than
the specific formulae for the coefficients in the fermion masses dictated by the MSGUT
superpotential. Our concern here is not with the actual values of the successful fits but
rather their implications when combined with the structure of the MSGUT. We therefore
review the fitting procedure from our particular viewpoint. To begin with the “Clebsch
” coefficients for the couplings of the 16 × 16 SO(10) chiral spinors to the
10, 126 irreps were calculated as a part of our explicit decomposition of
SO(10) in terms of Pati-Salam labels[25]: One obtains :

\[
W_{FM}^H = h'_{AB} \psi^T C_2 \gamma_i \psi_i H_i
\]

\[
= \sqrt{2} h'_{AB} \left[ H_{\mu \nu} \tilde{\psi}_{\alpha}^\nu \tilde{\psi}_{\mu}^{\alpha} + \tilde{H}_{\mu \nu} \tilde{\psi}_{\mu A} \psi_{\nu B} - H^{\alpha \dagger}(\tilde{\psi}_{A \alpha} \psi_{B B} + \psi_{A B} \tilde{\psi}_{B \alpha}^{\dagger}) \right]
\]

\[
= -2 \sqrt{2} h'_{AB} \left[ \bar{d}_A Q_B + \bar{e}_A L_B \right] + 2 \sqrt{2} h'_{AB} \left[ \bar{u}_A Q_B + \bar{\nu}_A L_B \right] + ..... \tag{9}
\]

\[
W_{FM}^\Sigma = \frac{1}{5!} \psi^T C_2 \gamma_i \ldots \gamma_i \chi_{i_1 \ldots i_5}
\]

\[
= 4 \sqrt{2} \sum_{\nu} \mu \alpha \tilde{\psi}_{\nu}^\mu \chi_{\alpha \mu} + \psi_{\mu A} \chi_{A \mu} + 4 \left( \sum_{\mu \nu} \tilde{\psi}_{\mu A} \chi_{A \nu} + \sum_{\mu \nu} \psi_{\mu A} \chi_{A \nu} \right) + 4 \sqrt{2} f'_{AB} i \frac{\psi}{\sqrt{3}} \left[ h_2(\bar{d}_A Q_B - 3 \bar{e}_A L_B) - h_2(\bar{u}_A Q_B - 3 \bar{\nu}_A L_B) \right]
\]

\[
+ 4 f'_{AB} [-2 i G_5 \tilde{\psi}_{A B} + \sqrt{2} \tilde{O} L A \tilde{L} B] + ..... \tag{10}
\]

where the alphabetical naming convention regarding the subcomponents of the Higgs
multiplets is given in the Appendix and in detail in[28]. From the properties of the SO(10)
Clifford algebra it follows that the Yukawa coupling matrices \( h'_{AB}, f'_{AB} (A, B = 1, 2, 3) \) are
symmetric \( h'_{AB} = h'_{BA}, f'_{AB} = f'_{BA} \) complex matrices. Therefore the freedom to make
unitary changes of basis allows one to chose one linear combination of the matrices \( h', f' \) to
be diagonal. We shall choose the basis where \( f' \) is diagonal since it proves convenient when
analyzing the Seesaw mechanism but our conclusions are independent of any such choice.
To obtain the formulae for the charged fermion masses from the above decomposition\[28\] one first needs to define the \((G_{321}(1, 2, \pm 1))\) multiplets \(H^{(1)}, \bar{H}^{(1)}\) which are the (light) MSSM Higgs doublet pair. This is achieved by imposing the condition

\[\text{Det}\mathcal{H} \sim O(M_W)\]

on the doublet mass matrix \(\mathcal{H}\) which occurs in the quadratic terms of the superpotential when expanding around the superheavy vevs: \(W = \hat{h}\mathcal{H}h + \ldots\). This amounts to a fine tuning of (say) the mass parameter \(M_H\) of the 10-plet Higgs.

The \(4 \times 4\) matrix \(\mathcal{H}\)\[28\] can be diagonalized by a bi-unitary transformation: \[26, 29, 25, 28\] from the 4 pairs of Higgs doublets \(h^{(i)}, \bar{h}^{(i)}\) arising from the SO(10) fields to a new set \(H^{(i)}, \bar{H}^{(i)}\) of fields in terms of which the doublet mass terms are diagonal.

\[
\begin{align*}
U^T \mathcal{H} U &= \text{Diag}(m_H^{(1)}, m_H^{(2)}, \ldots) \\
h^{(i)} &= U_{ij} H^{(j)} ; \quad \bar{h}^{(i)} = \bar{U}_{ij} \bar{H}^{(j)}
\end{align*}
\]

Then it is clear that in the effective theory at low energies the GUT Higgs doublets \(h^{(i)}, \bar{h}^{(i)}\) are present in \(H^{(1)}, \bar{H}^{(1)}\) in a proportion determined by the first columns of the matrices \(U, \bar{U}\):

\[
E << M_X : \quad h^{(i)} \rightarrow \alpha_i H^{(1)} ; \quad \alpha_i = U_{i1} \\
\bar{h}^{(i)} \rightarrow \bar{\alpha}_i \bar{H}^{(1)} ; \quad \bar{\alpha}_i = \bar{U}_{i1}
\]

Thus the formulae for the charged fermion masses in the MSGUT are\[28\] :

\[
\begin{align*}
M^d &= r_1 \hat{h} + r_2 \hat{f} \\
M^l &= r_1 \hat{h} - 3r_2 \hat{f} \\
M^u &= \hat{h} + \hat{f}
\end{align*}
\]

where

\[
\begin{align*}
\hat{h} &= 2\sqrt{2}h'v\alpha_1 \sin\beta \\
\hat{f} &= -4\sqrt{2}3f'\alpha_2 \sin\beta \\
r_1 &= \frac{\bar{\alpha}_1}{\alpha_1} \cot\beta \\
r_2 &= \frac{\alpha_2}{\bar{\alpha}_2} \cot\beta
\end{align*}
\]

here \((h', f')\) are the couplings in the MSGUT superpotential.

Similarly the Majorana mass of the ‘right handed neutrinos ’ i.e of the fields \(\bar{\nu} \equiv \nu^e\) is read off from the decomposition given above\[28\] :

\[
M_{AB}^\nu = -4i\sqrt{2}f'_{AB} < \sum_{44}^{(R+)} > = 4\sqrt{2}f'_{AB} \bar{\sigma}
\]

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and is of the order of the Unification scale or somewhat less. The left handed or SM
neutrinos receive a direct Majorana mass from the so called Type II seesaw mechanism\[49\]
when the left handed triplet $\bar{O}$ contained in the $\mathbf{126}$ field obtains a vev. One obtains\[25\]:

$$M_{\nu AB} = 4\sqrt{2}f'_{AB} < O^{11} >= 8if'_{AB} < O_1 >$$

(16)

The vev $< O_1 >$: $(\mathbf{10},3L,1)_{\Sigma}$ arises from a tadpole following $SU(2)_L$ breaking (see
below).

The final component of the Seesaw is the Neutrino Dirac mass which links the left and
right handed neutrinos:

Dirac mass :

$$m^{\bar{\nu}D}_{AB} = 2\sqrt{2}h'_{AB} < h_1^{(1)} > +4i\sqrt{6}f'_{AB} < h_2^{(2)} >$$

(17)

To determine the Majorana mass terms of the left handed neutrinos in the effective
MSSM we must eliminate the superheavy $\bar{\nu}$ and evaluate the Type II Seesaw tadpole. One
then obtains\[28\] (some factors of $\sqrt{2}$ have been corrected relative to eqns(77-79) of\[28\]).

$$M^{I}_{\nu} = -\frac{1}{4}m^{D}\nu^{-1}m^{D} = -r_4(\hat{h} - 3\hat{f})\hat{f}^{-1}(\hat{h} - 3\hat{f}) \equiv -r_4\hat{n}$$

(18)

$$M^{II}_{\nu} = 8if'_{AB} < O_1 >= r_3\hat{f}$$

(19)

$$r_3 = -2i\sqrt{3}(\alpha_1\gamma + 2\sqrt{3}\eta\alpha_2)(\frac{v}{M_O})Sin\beta$$

$$r_4 = \frac{-i\alpha_2Sin\beta v}{4\sqrt{3}\bar{\sigma}}$$

(20)

$$M_O = 2(M + \eta(3a - p)))$$

Thus we see that the fermion mass formulae are completely determined in terms of the GUT
scale breaking parameters($\xi, \lambda, \eta, \gamma, \bar{\gamma}, m$), the $10 + \mathbf{126}$ Yukawa couplings(15 parameters)
and the low energy parameters $v_{EW} = 174GeV, tan\beta$.

To perform the fit, we must match the fermion masses and mixings of the MSSM RG-
extrapolated to the GUT scale $M_X$:

$$L^{FM}_{MSSM}(M_X) = l^cT\bar{D}l + u^cT\bar{D}u + d^cT\bar{D}d$$

$$+ \bar{l}^{\nu}W + \bar{u}^{\nu}Wd + \nu^{T}(P\bar{D}P^T)\nu + \cdots$$

(21)

with the effective theory derived from the MSGUT by integrating out the heavy fields at
$M_X$. Here the $D_{l,u,d,\nu}$, $C$, $P$ : are the diagonal fermion mass matrices (with mass eigenvalue
components $l_A, u_A, d_A, \nu_A$), the CKM mixing matrix and the PMNS matrix (Majorana
Neutrino mixing) at the scale $M_X$ in some fixed phase convention for the MSSM masses
and mixings (either at $M_Z$ or at $M_X$) and $d' = Cd$. For example the convention that
all the diagonal masses are real and positive. Currently each group of FM fitters uses
idosyncratic phase conventions. A standard format presentation of the data of the MSSM

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at \( M_X \) is needed and is being pursued. The corresponding quadratic terms in the effective theory derived from the GUT are (GUT fields carry carets):

\[
L_{\text{GUT}}^{(2)} = \bar{\hat{u}}^c (\hat{h} + \hat{f}) \hat{u} + \bar{\hat{d}}^c (\hat{h} r_1 + \hat{f} r_2) \hat{d} + \bar{\hat{c}}^c (\hat{h} r_1 - 3 \hat{f} r_2) \hat{l} \\
+ \bar{\hat{\nu}}^T \hat{M}_\nu \hat{\nu} + \bar{\hat{l}}^T \hat{W} \hat{\nu} + \bar{\hat{u}}^T \hat{W} \hat{d}
\]

where

\[
\hat{M}_\nu = r_3 \hat{f} - r_4 (\hat{h} + \hat{f}) \hat{f}^{-1} (\hat{h} + \hat{f}) \equiv r_3 \hat{f} - r_4 \hat{n}
\]

When equating the two quadratic forms we must allow for Unitary transformations between the fields in the two Lagrangians:

\[
L_{\text{GUT}}^{(2)} = L_{\text{MSSM}}^{FM}(M_X)
\]

\[
\begin{pmatrix}
\hat{u} \\
\hat{d}'
\end{pmatrix} = Q \begin{pmatrix}
\hat{\bar{u}} \\
\hat{\bar{d}}
\end{pmatrix}; \quad \begin{pmatrix}
\hat{\nu} \\
\hat{l}
\end{pmatrix} = L \begin{pmatrix}
\hat{\bar{\nu}} \\
\hat{\bar{l}}
\end{pmatrix}
\]

\[
u^c = V_{(\nu^c)} \bar{\nu}^c, \quad d^c = V_{(d^c)} \bar{d}^c \quad l^c = V_{(l^c)} \bar{l}^c.
\]

The unitary matrices \( Q, L, V_{(\nu^c)}, V_{(d^c)}, V_{(l^c)} \) describe the embedding of the extrapolated MSSM within the MSGUT. The \( (d = 5, 6) \) effective lagrangian \( L_{\varepsilon, f}^{AB \neq 0}(\bar{\psi}) \) must be transformed to the extrapolated MSSM basis in order to derive rates for such exotic processes. Thus it is clear that these matrices are neither unphysical nor conventional once the conventions of the MSSM parameters are fixed. In fact we argue that the crucial information given to us by the fitting procedure is not a prediction of the neutrino masses and mixings but rather information on these embedding matrices.

Since the \( 10, \overline{126} \) Yukawas are symmetric it follows that only two (say \( Q, L \)) of these matrices are independent while the others \( (V_{(\nu^c)}, V_{(d^c)}, V_{(l^c)}) \) can be determined in terms of the two chosen to be independent plus arbitrary diagonal unitary matrices \( \Phi_u, \Phi_d, \Phi_l \). Once this is done we obtain:

\[
\begin{align*}
\Phi_u^* V_{(\nu^c)} &= C \Phi_d^* V_{(d^c)} = Q & \Phi_l^* V_{(l^c)} &= L \\
\hat{h} + \hat{f} &= V_{(\nu^c)}^T D_u Q = Q^T D_u' Q \equiv Q^T \Phi_u D_u Q \\
\hat{h} r_1 + r_2 \hat{f} &= V_{(d^c)}^T D_d C_d^\dagger Q = R^T D_d' R \equiv R^T \Phi_d D_d R \\
\hat{h} r_1 - 3 r_2 \hat{f} &= V_{(l^c)}^T D_l L = L^T D_l' L \equiv L^T \Phi_l D_l L \\
r_3 \hat{f} - r_4 \hat{n} &= L^T P D_{(\nu^c)} P^T L
\end{align*}
\]

(24)
where we have defined \( R = C^T Q \). The phase freedoms \( \Phi_u, \Phi_d \) have been found\cite{9,11} to be important for arranging the tunings that underlie the successful Type I and Type II fits of the fermion masses. However the phases \( \Phi_l \) play no important role so far since the phases in the PMNS matrix \( P \) are unknown at present. If we reabsorb the phases in the equation from \( \hat{M}_l \) in \( \mathcal{L} \) then we also need to redefine the PMNS matrix \( P \) to reabsorb them in the neutrino mass equation (last of equations (25)). This ambiguity should be kept in mind when deducing predictions of Leptonic phases from the fit, but does not play any role at this stage.

We remind the reader that all the parameters \( r_i; i = 1, 2, 3, 4 \) are known in terms of the MSGUT parameters but the explicit form is not used or invoked when solving the fitting problem\cite{9,10}. Rather the parameters \( r_i \) and the Yukawa couplings are determined in terms of the extrapolated experimental data. Thus the compatibility of the FM fits with the MSGUT remains to be verified for each prima facie viable fit since the ability of the MSGUT to reach the required values of the \( r_i \) simultaneously while preserving the constraints of perturbative gauge unification is not obvious.

### 4.1 Solution of the fitting problem

The equations for the down fermion and charged lepton masses may be immediately solved to yield

\[
\begin{aligned}
\hat{h} r_1 &= \frac{1}{4}(\mathcal{L}^T D_l \mathcal{L} + 3 \mathcal{R}^T D_d' \mathcal{R}) \\
\hat{f} r_2 &= \frac{1}{4}(-\mathcal{L}^T D_l \mathcal{L} + \mathcal{R}^T D_d' \mathcal{R})
\end{aligned}
\]  

we define

\[
\mathcal{D} = \mathcal{R}^* \mathcal{L}^T, \quad \hat{u} = C^T D_u' C
\]

for later convenience.

Consider first the case\cite{9,10} where the mixing matrices, (hatted) Yukawa couplings and parameters are all assumed real. Moreover the phases \( \Phi_u, \Phi_d \) are simply signs and the CKM phase \( \delta \) is also a sign which in fact is found \cite{9,10} to be minus in “successful” fits. Then solving the Up quark equations and eliminating \( r_1, r_2 \) using 23 component and trace we can put it in the form

\[
X \equiv X_l \equiv \mathcal{D} D_l \mathcal{D}^T = \frac{X_{23}}{\hat{u}_{23}} (\hat{u} - \frac{T_u'}{T_d} D_d') + \frac{T_l}{T_d} D_d'
\]

Here \( T_f' = tr[D_f'] \). Notice that the non diagonality of \( X_l \) is completely driven by the non-diagonality of the matrix \( u' \) which in turn follows from that of the CKM matrix. Following\cite{9} it is convenient to rescale each of the diagonal fermion mass matrices by the mass of the third generation fermion of that type to get a dimensionless (tilde-ed) form of the equations:
\[ D_f \equiv m_{f_i} D_f \]  

(28)

\[ \tilde{X}_i \equiv \mathcal{D} D_f \mathcal{D}^T = \frac{\tilde{X}_{23}}{u_{23}} (\tilde{u} - \frac{T_u}{T_d} D_{d^c}) + \frac{T_{\tilde{f}}}{T_{d^c}} D_{d^c} \]  

(29)

The matrix \( \tilde{X}_i \) has eigenvalues \( \tilde{l}_{1,2} \) and \( \tilde{l}_3 = 1 \) by construction and hence it follows that we must have

\[ \text{det}(\tilde{X}_i - \tilde{l}_i I_3) = 0 \]  

(30)

Thus we obtain three coupled non-linear equations for \( \tilde{X}_{23} \) and two other quantities which are conveniently chosen to be \( \tilde{d}_{1,2} \). We can solve numerically for \( \tilde{X}_{23} \) and \( \tilde{d}_{1,2} \), given any set of up quark and charged lepton masses together with CKM data. Obviously only solutions within the error bars (usually allowed to be 1-\( \sigma \)) for the down quark masses are accepted. Note however that inasmuch as there is a strong inter-generational hierarchy for the charged fermion masses the numerical solution of the three coupled non-linear equations is an extremely delicate operation requiring utmost care and diligence to find the correct solutions. Since brute shooting usually fails in multidimensional problems of this delicacy approximate analytic solutions obtained by expanding in the light fermion masses are used to guide the numerical search for solutions.

Assuming this has been done the matrix \( \tilde{X}_i \) is completely determined so that on diagonalizing it one obtains the crucial matrix \( \mathcal{D} = \mathcal{R} \mathcal{L}^T \). With \( \mathcal{D} \) in hand one can proceed by choosing the convenient \( \hat{f} \)-diagonal basis mentioned earlier. Rescaling

\[ \hat{\tilde{f}} = \hat{f}/m_t = \tilde{r}_2 \mathcal{L}^T X_f \mathcal{L} \]  

(31)

where we have defined \( X_f \) and unitary \( \mathcal{F} \) by

\[ \tilde{X}_f = \mathcal{D}^T D_{d^c} \mathcal{D} - D_f \left( \frac{m_\tau}{m_b} \right) \equiv \mathcal{F} D_f \mathcal{F}^T \]  

(32)

Since \( \hat{\tilde{f}} \) is diagonal by choice of basis it immediately follows that

\[ \mathcal{L} = \mathcal{F} \Rightarrow \mathcal{R} = \mathcal{D} \mathcal{L} \]  

(33)

Now since \( \mathcal{L}, \mathcal{R} \) are known and since \( \tilde{\tilde{r}}_{1,2} = \frac{m_u}{m_t} \tilde{r}_{1,2} \) are calculable in terms of \( X_f \) it is clear that we have also determined the Yukawa coupling \( \hat{h} \):

\[ \hat{\tilde{h}} = \hat{h}/m_t = \tilde{r}_1 \mathcal{L}^T [3 D^T D_{d^c} + D_f \frac{m_\tau}{m_b}] \mathcal{L} \]  

(34)

In other words one finds that the fitting of charged fermion mass ratios requires tuning of the down quark mass ratios \( \tilde{d}_{1,2} \) to less than one part in \( 10^{-3} \) for given precise values of up quark and charged lepton masses together with CKM data and yields the dimensionless matrices \( \hat{\tilde{h}}, \hat{\tilde{f}} \) and the exotic embedding matrices \( \mathcal{L}, \mathcal{R} \) and, given \( m_t, \hat{h}, \hat{f} \).

In the realistic case when the parameters are complex a similar but numerically even more difficult procedure is followed. Like the sign freedom in the real case the phase freedom
of choosing $\Phi_u, \Phi_d$ is found [9, 10, 11] to be crucial to obtaining a successful fit. The $u$ equation still has the same form

$$X_l \equiv DD_lD^T = p\hat{u} + qD_d$$

but now $TrX_l \neq TrD_l$.

To proceed we solve the 23, 33 components to eliminate $p, q$ and get

$$\tilde{X}_l \equiv DD_lD^T = \frac{\tilde{X}_{23}}{\tilde{u}_{23}}\hat{u} + (\tilde{X}_{33} - \frac{\tilde{X}_{23} \tilde{u}_{23}}{\tilde{u}_{23}})D_d$$

Then to determine $D$ we must diagonalize

$$X_lX_l^\dagger \equiv DD_lD^\dagger$$

This matrix has eigenvalues $\tilde{\rho}_1, \tilde{\rho}_2, 1$ and an inspection of its explicit form shows that it requires knowledge of $|X_{23}|, |X_{33}|, \phi_X = Arg(X_{23}) - Arg(X_{33})$ and a choice [9, 10, 11] of the quark mass phases $\Phi_u, \Phi_d$. The numerical results obtained by these authors are then a specification of the right hand side of eqn (35) consistent with some acceptable values of the charged lepton masses. We refer the reader to the original papers for the procedure for fixing the unknown phases. Once this rather horrendous numerical problem has been solved (its trickiness accounts for the fact that even 12 years after it’s solution was first explicitly attempted [6] this system continues to throw out surprises [11]) one can proceed essentially as before: diagonalizing the rhs of eqn (36) numerically thus yields the matrix $D$.

Since $\hat{f}$ is diagonal and real by convention one writes

$$\tilde{f} = \frac{f}{m_t} = \tilde{\rho}_2L^TX_fL$$

where

$$X_f = D^\dagger D_d^* - D(l\frac{m_t}{m_b}) \equiv FD_fF^T$$

Since $X_f$ is symmetric, $F$ is Unitary and $D_f$ is real like $\hat{f}$, it follows that

$$\tilde{f} = |\tilde{\rho}_2|D_f$$

so that

$$L = F^{*}e^{-\frac{i Arg(\tilde{\rho}_2)}{2}}$$

where $F$ is found by diagonalizing $X_fX_f^\dagger = FD_fF^T$.

Finally

$$R = D^*L \quad ; \quad Q = CR$$

In summary: for given up and charged lepton masses and given CKM mixing angles, by tuning $\tilde{d}_{1,2}, \delta_{CKM}$ within their allowed ranges and for a certain choice of the phases $\Phi_u, \Phi_d$, one completely fits the charged fermion masses and determines the FM Yukawa couplings $h', f'$ of the GUT along with the exotic embedding matrices $L, Q, \Phi_u, \Phi_d$. 

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4.2 Fitting Neutrino Masses

The current FM Fitting Furore\[9, 10, 11\] was triggered by the remarkably simple observation of\[8\] regarding naturalness of large atmospheric mixing angles in MSGUTs with dominant Type II seesaw mechanisms given the near equality of the $\tau$ lepton and bottom quark masses at $M_X$. If one assumes Type II domination i.e $r_3 >> r_4$ the Seesaw formula simplifies to just

$$\tilde{f} = \left(\frac{1}{m_t r_3}\right) L^T P D_{\nu} P^T L$$  \hspace{1cm} (43)

Since we chose a $\tilde{f}$-diagonal basis it immediately follows that

$$P = L^* e^{\frac{\text{Arg}(r_3)}{2}} \hspace{0.5cm}; \hspace{0.5cm} D_{\nu} = |r_3| \tilde{f}$$  \hspace{1cm} (44)

Thus the neutrino mixing angles and ratio of mass squared splittings can be determined under these assumptions since we know $\tilde{f}, L$.

In the general case the neutrino mass fitting equations take the scaled form:

$$\mathcal{N} D_{\tilde{N}} \mathcal{N}^T \equiv \tilde{f} - \left(\frac{r_4}{r_3}\right) \tilde{n} = \left(\frac{1}{m_t r_3}\right) L^T P D_{\nu} P^T L$$  \hspace{1cm} (45)

where

$$\tilde{n} = n/m_t = (\tilde{h} - 3 \tilde{f}) \tilde{f}^{-1} (\tilde{h} - 3 \tilde{f})$$  \hspace{1cm} (46)

Thus the mixing matrix and neutrino masses are also completely determined:

$$P = L^* N e^{\frac{\text{Arg}(r_3)}{2}} \hspace{0.5cm}; \hspace{0.5cm} D_{\nu} = m_t |r_3| D_{\tilde{N}}$$  \hspace{1cm} (47)

These fitting problems have been formulated and solved with increasing refinement by a number of authors\[9, 10, 11\]. It seems\[11\] that Type II dominant as well as Type I and Type I plus Type II combined solutions (which are not perturbations of Type II dominant solutions ) can be found in the complex case. Further solutions probably still remain to be found and the possible solutions definitely still need to be compactly characterized and parameterized. The fits achieved so far already motivate a detailed examination of what type of Seesaw is actually allowed by the MSGUT in various regions of its parameter space. We have argued that in each case the exotic embedding matrices are -like the ‘philosophers stone’ - the so far unregarded true product of the fitting calculation. We emphasize that in practice one accepts solutions which give $\nu$ oscillation parameters which lie within the $1 - \sigma$ ranges around the central values of the fermion mass and mixing parameters so that the true output of the fitting calculation are the previously completely unknown embedding matrices $Q, L, \Phi_u, \Phi_d$ which specify how an MSSM($M_X$) with given conventions lies within the MSGUT. These matrices are crucial for pinning down the prediction of $\Delta B \neq 0$ processes in the MSGUT. Before considering that aspect however we turn to the question of what kinds of solutions are compatible with the MSGUT.
Scanning the MSGUT for Neutrino Masses and Mixings

The possibility of large PMNS mixing angles is well understood in the Type II dominant case [8,9], where it appears as a natural corollary of the approximate unification of the running bottom quark and tau lepton masses at scales $O(M_X)$ due to the 3-fold faster evolution of the bottom quark mass. As is evident from eqn. (25) the parameter which controls the strength of Type I versus Type II Seesaw in the MSGUT is the ratio of the coefficients $r_4$ and $r_3$. Complete Type II dominance requires $r_4 << r_3$. To illustrate how the MSGUT yields information on this ratio we work with an example of a quasi-realistic real Type II fit that ignores the CP violating phase, which was kindly provided to us by S.Bertolini and M. Malinsky. Although Type II and only semi-realistic it should be emphasized that the values of $\hat{h}, \hat{f}$ given will be rather typical since they are fixed by the charged fermion mass data and further selection is then imposed based on compatibility with the neutrino data.

The data of the example solution (at $M_X$ and for $\tan \beta = 10$) is

$$D_u = \{0.785556, -191.546, 70000\} \text{MeV}$$
$$D_l = \{0.3585, 75.7434, 1290.8\} \text{MeV}$$
$$d_3 = m_b = 1138.07 \text{MeV}$$

$\{\sin \theta_{12}, \sin \theta_{23}, \sin \theta_{13}\} = \{0.2229, 0.03652, -0.00319\}$

(48)

Notice the negative sign of $u_3$ and $\theta_{13}$ (this corresponds to taking $\delta_{CKM} = \pi$), moreover the fitting procedure gives

$$\tilde{x}_{23} = -0.14325 ; \quad \tilde{d}_1 = -.001105 ; \quad \tilde{d}_2 = -.02747$$

(49)

so that the sign ambiguity $\Phi_d$ is also fixed.

$$\mathcal{D} = \begin{pmatrix} 0.98953 & -0.136941 & 0.045582 \\ -0.128508 & -0.979735 & -0.153642 \\ -0.0656981 & -0.146176 & 0.987075 \end{pmatrix}$$

(50)

Supplying $m_b = 1138.07 \text{MeV}$ allows one to calculate

$$\mathcal{L} = \mathcal{F} = \begin{pmatrix} 0.794035 & -0.582472 & 0.173881 \\ -0.563923 & -0.599057 & 0.568437 \\ 0.226934 & 0.549415 & 0.804142 \end{pmatrix}$$

(51)

Then the matrices $\mathcal{R}, \mathcal{Q}$ follow:

$$\mathcal{R} = \begin{pmatrix} 0.87329 & -0.469294 & 0.130873 \\ 0.415588 & 0.577356 & -0.702813 \\ 0.254266 & 0.668149 & 0.699233 \end{pmatrix}$$

$$\mathcal{Q} = \begin{pmatrix} 0.943138 & -0.330924 & -0.0313091 \\ 0.219729 & 0.691351 & -0.688297 \\ 0.24942 & 0.642279 & 0.724753 \end{pmatrix}$$

(52)
The scaled Yukawa couplings are

\[ \tilde{f} = \begin{pmatrix} -0.000595629 & 0 & 0 \\ 0 & 0.0020326 & 0 \\ 0 & 0 & -0.00804198 \end{pmatrix} \]

\[ \tilde{h} = \begin{pmatrix} 0.0626836 & 0.159778 & 0.181181 \\ 0.159778 & 0.409183 & 0.466796 \\ 0.181181 & 0.466796 & 0.532013 \end{pmatrix} \] (53)

The matrix \( \hat{n} \) multiplying \(-r_4\) in the Type I mass is

\[ \tilde{n} = \begin{pmatrix} 1.4996 & 3.8747 & 4.55331 \\ 3.8747 & 9.98038 & 11.6875 \\ 4.55331 & 11.6875 & 13.63 \end{pmatrix} \] (54)

From these the Type II mixing angles and the ratio of the 23 and 12 sector mass squared splittings is found to be

\[ \begin{align*} \{ \sin^2(2\theta_{12}^P), \sin^2(2\theta_{23}^P), \sin\theta_{13}^P \} &= \{0.90981, 0.88873, 0.17387\} \\ \Delta m_{12}^2/\Delta m_{23}^2 &= 0.06237 \end{align*} \] (55)

The Yukawa couplings obtained are typical of Type II fits. It is clear that unless \( r_4/r_3 \) is very small the Type I term will dominate completely. To see just how small \( r_4 \) should be we plot the neutrino mixing angles \( \sin^22\theta_{12}, \sin^22\theta_{23} \) versus this ratio as Fig 8.

From Fig. 8 it is evident that a value of \( |r_4/r_3| > 0.0003 \) causes a collapse of the large mixing angles of the Type II dominant solution. Thus we should not expect a Type II solution to work in the MSGUT unless \( R = r_4/r_3 \) (which is completely determined by the GUT as given in the previous section) obeys \( |R| < 10^{-3} \). The MSGUT formulae for \( r_3, r_4 \) are given in eqn(21)

Using these we plot the ratio \( R \) versus \( \xi \) and find that its typical value is \( \sim 10^{-1} \) or more, not \( 10^{-3} \) or less. Illustrative plots for the real and complex solutions of the cubic eqn. [2], for real \( \xi \) and \( \lambda = .7 \) are shown as Fig 9.

From the plots it is evident that in the real case R has a chance of being of the required small size only in the window near \( \xi = -.7 \) and between about \( \xi = 1 \) and \( \xi = 4 \). However in the former region one finds that the corrections to \( \sin^2\theta_W \) diverge, while in the region \( \xi \in (1, 4) \) is not allowed by the requirement that \( \Delta \log_{10}M_X > -1 \). On the other hand, in the complex case, the ratio seems bounded below by 1! Thus we see that if Type II fits were the only allowed ones then the combination of the gauge unification requirements with those imposed by neutrino mass phenomenology tend to rule out the MSGUT based on this class of solutions. This conclusion however still requires a more thorough study of all possibilities. Moreover recent work [11] has shown that in fact large mixing angles can be achieved even in Type I solutions and Type I-II combined solutions (which are far from pure Type II solutions). Our intent, in this talk, is not to provide an exhaustive survey of the possibilities but only to illustrate how the combined requirements of neutrino oscillations and baryon stability can severely constrain the MSGUT over its parameter space. A detailed survey of the MSGUT for each of the three solutions of the basic cubic
equation to find in which regions, if any, the ratio R matches that required by the FM fit is quite feasible using our methods, and will be reported elsewhere.[41]

Note that while the FM fits described in the previous section do not determine the overall mass scale of the neutrinos since the input data does not contain this information, the same is not true in the context of the MSGUT. Using the dimensionless versions \( \tilde{f} \) and \( \tilde{h} \) given by the real Type II FM fitting analysis as a guide to typical values of the Yukawa couplings allows one to compute the magnitudes of all three neutrino masses for each type of fit. When this is done another problem becomes apparent: except possibly for narrow ranges of \( \xi \) the largest neutrino mass (Type I or II) is much smaller than the mass splitting known from atmospheric neutrino oscillations. Thus even if one can find a region where the ratio of mass splittings and mixing angles for neutrinos are in the allowed region the additional consistency constraint that \( |(m_\nu)_{\text{max}}| > |(\Delta m)_{\text{atmos}}| \) alone can exclude most of the parameter space! An illustrative plot of the maximum type I and Type II neutrino masses for the Yukawa coupling matrices that arises in the Bertolini and Malinsky solution used by us for illustration is given for the real solution as Fig. 10. Due to the cut at \( \xi = 8.6 \) imposed by the condition \( \Delta \log_{10} M_X > -1 \) we see that there is no region with large enough values of \( |(m_\nu)_{\text{max}}| \). Similarly in the complex case we get the plots shown in Fig. 11. As expected the Type I dominates completely. However the mass magnitudes for Type I tend to be too small: as can be seen from Fig. 11 and the magnifications shown in Fig 12. Since the shortfall is only a factor of 10 and we have not yet used the complex Type I fits found in [11] it would be premature to rule out Type I fits. Nevertheless a certain tension is apparent.

If the impossibility of large enough overall neutrino mass scale is borne out by a comprehensive analysis[41] it would require a revision of the perturbative MSGUT. Similar arguments were used in [37] (for the Type II FM fit case only) to motivate an extension of the model by introducing an addition \( 54 \)plet so as raise the value of the \( 126 \) vev to a high scale to allow Type II to dominate and yield a large enough neutrino mass. Note however that in that case the minimality of the MSGUT is seriously diluted necessitating a complete reanalysis which has not been performed so far. An alternative to this extension may be to implement a non perturbative mechanism[40, 41] based on dynamical GUT symmetry breaking down to \( SU(5) \times U(1) \) at \( \Lambda_X \sim 10^{17} GeV \) and then use a Type I fit with small \( 126 \) vev \( \sigma \) to achieve a larger value of the overall neutrino mass scale.

6 \( \Delta B \neq 0 \) IN MSGUT

Finally we briefly indicate the \( d = 5 \) baryon decay operators determined by the FM fit. If any FM fit is found to be consistent with all the constraints discussed above then the computation of the actual Baryon Decay predictions will become a worthwhile exercise.

The basis for this is the effective superpotential that arises when the superheavy Higgs triplets are integrated out from the theory[25, 28]

\[
W_{\text{eff}}^{\Delta B \neq 0} = \hat{L}_{ABCD}(\frac{1}{2}\epsilon \hat{Q}_A \hat{Q}_B \hat{Q}_C \hat{L}_D) + \hat{R}_{ABCD}(\epsilon \hat{\bar{e}}_A \hat{f}_B \hat{u}_C \hat{d}_D) \tag{56}
\]

\[
\hat{L}_{ABCD} = S_1^1 h_{AB} h_{CD} + S_1^2 h_{AB} f_{CD} + S_2^1 f_{AB} h_{CD} + S_2^2 f_{AB} f_{CD} \tag{57}
\]
\[ \hat{R}_{ABCD} = S_1^1 h_{AB} h_{CD} - S_1^2 h_{AB} f_{CD} - S_2^1 f_{AB} h_{CD} + S_2^2 f_{AB} f_{CD} \]
\[ - i\sqrt{2} S_1^2 f_{AB} h_{CD} + i\sqrt{2} S_2^4 f_{AB} f_{CD} \]  

(58)

Here \( S = T^{-1} \) where \( T \) is the mass matrix in the \([3, 1, \pm 2/3]\)-sector which is the representation type that mediates \( d = 5 \) baryon decay. The Yukawa coefficients \( h_{AB}, f_{AB} \) are related to those in the superpotential by \( h_{AB} = 2\sqrt{2} h'_{AB}, f_{AB} = 4\sqrt{2} f'_{AB} \).

Substituting for the GUT superfields (with carets) in terms of the embedding matrices \( Q, R, L \) determined by the FM fit

\[
\hat{Q}_L = Q^\dagger Q_L \quad \hat{L}_L = L^\dagger L_L \quad \hat{u} = \hat{u}_c = Q^\dagger \Phi^* u_c \quad \hat{d} = \hat{d}_c = R^\dagger \Phi^* d_c \quad \hat{l} = \hat{l}_c = L^\dagger \Phi^* l_c
\]  

(59)

one obtains the coefficients of mass eigenstate MSSM fields in the \( \Delta B \neq 0 \) superpotential to be

\[
L_{ABCD} = Q_{AE}^\dagger Q_{BF}^\dagger Q_{CG}^\dagger L^\dagger DH \hat{L}_{EGFH} \quad R_{ABCD} = L_{AE}^\dagger Q_{BF}^\dagger Q_{CG}^\dagger (R^\dagger)_{DH}(\Phi^*_l)_{EE}(\Phi^*_u)_{FF}(\Phi^*_d)_{GG}(\Phi^*_l)_{HH} \hat{R}_{EGFH}
\]  

(60)

A similar calculation\[41\] can be done to determine the effective operators for \( d = 6 \) baryon violating operators, which will be relevant even if the Supersymmetry breaking scale eventually turns out to be large enough (> 100TeV) to exclude any observable \( d = 5 \) baryon decay. Similar care needs to be exercised when studying lepton flavour violation.

7 Conclusions and Outlook

The MSGUT based on the \( 210 \oplus 126 \oplus 126 \oplus 10 \) Higgs system is the simplest Supersymmetric GUT that elegantly realizes the classic program of Grand Unification and which is prima facie compatible with all known data. Its symmetry breaking structure is so simple as to permit an explicit analysis of its mass spectrum at the GUT scale and an evaluation therefrom of the threshold corrections and mixing matrices relevant to various phenomenologically important quantities. Since it has the least number of parameters of any theory that accomplishes as much this theory currently merits the name of the minimal supersymmetric GUT or MSGUT. The same simplicity and analyzability of GUT scale structure also applies to the theory with an additional \( 120 \)-plet, since it contains no standard model singlets, and thus justifies calling it the nMSGUT.

The small number of Yukawa couplings of the MSGUT makes the fit to the now well characterized fermion mass spectra very tight so that the combined constraints of this fit and the preservation of the gauge unification observed at one loop in the MSSM may well be enough to rule out most or even all of the parameter space of this theory. A corresponding investigation is also possible for the nMSGUT and both are in progress\[41\].

We have emphasized the need for clarity regarding the flavour basis used when performing the FM fit. When this is maintained it becomes obvious that the true outputs of the FM fitting calculation are really the embedding matrices that define how the MSSM, with
fixed phase conventions, lies within the MSGUT. With these matrices determined by the FM fit in hand, one will be in a position to perform a much more reliable calculation of $d = 5, 6$ Baryon decay in the MSGUT with the Susy breaking scale as the chief remaining uncertainty. This realization underlines and emphasizes the organic connection between the physics of neutrino mass and Baryon violation, i.e., between the physics of ultra-heavy and ultra-light particles which is the most intriguing implication of the discoveries of Super-Kamiokande.

Our discussion concerning embedding angles also has implications for a treatment of Lepton Flavour violation in the MSGUT which will be worth exploring in detail if a viable region of the parameter space of the perturbative MGUT emerges. On the other hand if no such candidate region is available then the remaining options that will need to be explored could consist on the one hand of the analogous calculations in the nMSGUT, namely when an additional 120-plet is introduced; which radically enlarges the possibilities as far as FM fitting is concerned. Or the MSGUT could be extended by engineering the models to ensure Type II dominance.

Another possibility is that the symmetry breaking at the GUT scale is primarily determined not by the perturbative superpotential but rather by a non-perturbative mechanism whereby gaugino condensation in the coset SO(10)/H (where H could be, e.g., $SU(5) \times U(1)$ or $SU(5)$) drives a H-singlet Chiral condensate of (say) the 210-plet field at a scale $\sim \Lambda/\lambda^a > M_X$. In that case the spectra given by us in terms of the vevs $a, p, \omega$ are still of use but they are no longer determined by the cubic equation. Rather after breaking the symmetry to the group H at a scale higher than the perturbative scale $M_X$ one would examine the symmetry breaking in the effective $SU(5)$ symmetric theory at lower energies $\sim M_X$. Such a scenario would thus not only utilize the problematic strong coupling regime lying just above the perturbative unification scale for dynamical symmetry breaking of the GUT theory at a scale $\Lambda_X$ (which is determined by the low energy data and the Grand Unified structure and functions as an internally defined upper cutoff for the MSGUT) but also provide an explanation for the “SU(5) conspiracy” that seems to operate within SO(10) Susy GUTs when proper account is taken of neutrino data, superheavy Susy spectra and RG evolution. This kind of scenario would fit naturally with a Type I mechanism which favours small values of the 126 vev. Thus it contrasts sharply with the proposal of and should be distinguishable phenomenologically from it.

We conclude with a tentative proposal to reconcile String theory based models and the 126 based Type I and Type II seesaw mechanisms that occur in RGUTs. Recall that String theory, particularly with level 1 Kac-Moody algebras, does not favour the emergence of effective GUTs containing adjoint and larger representations as its massless sector. Even the use of higher KM levels permits the occurrence of only very restricted numbers and types of higher GUT representations. In particular, in the case of SO(10), one cannot enlist the type of combinations characteristic of RGUTs ($45 \oplus 54 \oplus 126 \oplus 126$ or $210 \oplus 126 \oplus 126$) etc. These difficulties led to a decline in attempts to SO(10) GUTs from string theory. However of late the growing appreciation of the naturalness of SO(10) unification in the light of discovered neutrino mass has motivated renewed effort in this direction. SO(10) type families are generated with gauge symmetry pre-broken to MSSM or somewhat larger. However the issue of implementing the seesaw mechanism
whether Type I or Type II in the way achieved so naturally in RGUTs remains problematic due to the difficulty of building models in which the $\mathbf{126} \oplus \mathbf{126}$ representations remain massless in the string model.

A way out of this difficulty may perhaps be found by appreciating that in RGUTs the $\mathbf{126} \oplus \mathbf{126}$ fields are superheavy and their neutral components have either a superlarge vev (corresponding to $M_{B-L} \sim M_X$) or very small vevs ($\sim M_W$ or $\sim m_\nu$). The former kind of vev is that of the right handed triplets that give rise to the right handed neutrino’s superlarge Majorana mass and thus a small Type I seesaw mass for the MSSM neutrino. Its large size is compatible with the “pre-broken” structure of “string derived GUTs” where the breaking of the SO(10)/GUT gauge symmetry still discernible in the matter super multiplet structure is accomplished already at the level of defining the light sector of the String theory and in an effective description corresponds to a vevs of superheavy fields in the appropriate large chiral representations (such as $\mathbf{45}$, $\mathbf{54}$, $\mathbf{210}$, $\mathbf{126}$, $\mathbf{126}$ etc.). On the other hand the light vev $\sim M_W < M_S \geq 1$ TeV in MSSM doublet channels occurs only after breaking of supersymmetry and its small size is then thought to be naturally compatible with the status of Susy breaking as a tiny correction to the pre-broken String GUT picture which is supersymmetric, conformal etc and therefore its derivation can be legitimately postponed. Furthermore the Type II seesaw mass generating vev of the left handed triplet in the $\mathbf{126}$ is an even higher order effect that arises due to a tadpole in a superheavy field induced once EW symmetry breaking has taken place due to the coupling of this superheavy field with the doublets that get a small EW weak vev.

Thus it appears that the $\mathbf{126} \oplus \mathbf{126}$ fields need no longer be sought in the massless sector of the String theory. Instead it is sufficient to investigate whether the superheavy “Higgs Channel” corresponding to $\mathbf{126}$ or the above mentioned relevant sub-representations in fact couple to the putative light fields in an appropriate way. To put it simply the implementation of the $R_p$ preserving seesaw mechanism in Stringy GUTs may require a small “leakage” connecting the superheavy “field” in the $\mathbf{126}$ channel to the matter fermions and light doublets and the availability of the control parameter $M_W/M_X$ provides a natural way to keep the destabilizing effects of such heavy-light couplings under control: given that some way has somehow been found to break Susy and generate the EW scale in the first place! This refinement of the effective theory paradigm used to extract the low energy theory from string models is both novel and consonant with the characteristic and elegantly consistent tying together of very large and very small mass scales achieved by the seesaw mechanism.

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2005. I am grateful to the G. Senjanovic and High Energy Group of the Abdus Salam ICTP, Trieste, K. Huitu and the Theory Group, Institute of Physics, Helsinki, and the Theory Group, CERN, Geneva for hospitality during the writing of these proceedings and A. Faraggi and H.P. Nilles for discussions concerning the possibility of implementing R-parity preserving seesaw and $SO(10)$ in string theory.
Appendix: Tables of masses and mixings

In this appendix we collect our results for the chiral fermion/gaugino states, masses and mixing matrices for the reader’s convenience. Mixing matrix rows are labelled by barred irreps and columns by unbarred. Unmixed cases(i) are given as Table I.

ii) Chiral Mixed states

\[ [8, 1, 0](R_1, R_2) \equiv (\hat{\phi}_\mu, \hat{\phi}_{\mu(R0)}) \]

\[ \mathcal{R} = 2 \begin{pmatrix} (m - \lambda a) & -\sqrt{2}\lambda \omega \\ -\sqrt{2}\lambda \omega & m + \lambda(p - a) \end{pmatrix} \]

\[ m_{R\pm} = |\mathcal{R}_{\pm}| = |2m[1 + (\frac{\tilde{p}}{2} - \tilde{a}) \pm \sqrt{(\frac{\tilde{p}}{2})^2 + 2\tilde{\omega}^2}]| \]

The corresponding eigenvectors can be found by diagonalizing the matrix \( \mathcal{R} \mathcal{R}^\dagger \).

b) \([1, 2, -1](\bar{h}_1, \bar{h}_2, \bar{h}_3, \bar{h}_4) \oplus [1, 2, 1](h_1, h_2, h_3, h_4) \equiv (H_2^a, \Sigma_{2}^{(15)\alpha}, \Sigma_2^{(15)\alpha}, \frac{\phi_{2\alpha}}{2}) \oplus (H_{a1}, \Sigma_{a1}^{(15)}, \Sigma_{a1}^{(15)}, \frac{\phi_{4\alpha}}{\sqrt{2}}) \]

\[ \mathcal{H} = \begin{pmatrix} -M_H & +\sqrt{3}(\omega - a) & -\gamma \sqrt{3}(\omega + a) & -\gamma \bar{\sigma} \\ -\gamma \sqrt{3}(\omega + a) & 0 & -(2M + 4\eta(a + \omega)) & 0 \\ \gamma \sqrt{3}(\omega - a) & -(2M + 4\eta(a - \omega)) & 0 & -2\eta \bar{\sigma} \sqrt{3} \\ -\sigma \gamma & -2\eta \sigma \sqrt{3} & 0 & -2m + 6\lambda(\omega - a) \end{pmatrix} \]

The above matrix is to be diagonalized after imposing the fine tuning condition \( Det \mathcal{H} = 0 \) to keep one pair of doublets light.

c) \([3, 1, \frac{2}{3}](\bar{t}_1, \bar{t}_2, \bar{t}_3, \bar{t}_4, \bar{t}_5) \oplus [3, 1, -\frac{2}{3}](t_1, t_2, t_3, t_4, t_5) \equiv (H_{\mu 4}, \Sigma_{(a)}^{\mu 4}, \Sigma_4^{\mu 4}, \phi_{4(R+)}^a) \oplus (H_{\mu 4}, \Sigma_{(a)}^{\mu 4}, \Sigma_4^{\mu 4}, \Sigma_4^{\mu 4}, \phi_{\mu(R-)}^4) \]

\[ \mathcal{T} = \begin{pmatrix} M_H & \bar{\gamma}(a + p) & \gamma(p - a) & 2\sqrt{2i}\omega \bar{\gamma} & i\bar{\sigma} \gamma \\ \gamma(p - a) & 0 & 2M & 0 & 0 \\ \gamma(p + a) & 2M & 0 & 4\sqrt{2i}\omega \eta & 2i\eta \bar{\sigma} \\ -2\sqrt{2i}\omega \gamma & -4\sqrt{2i}\omega \eta & 2M + 2\eta p + 2\eta a & -2\sqrt{2i}\eta \bar{\sigma} \\ i\sigma \gamma & 2i\eta \sigma & 2\sqrt{2i}\eta \sigma & -2m - 2\lambda(a + p - 4\omega) \end{pmatrix} \]
Table 1: i) Masses of the unmixted states in terms of the superheavy vevs. The $SU(2)_L$ contraction order is always $F^αF_α$. The primed fields defined for $SU(3)_c$ sextets maintain unit norm. The absolute value of the expressions in the column “Mass” is understood.
iii) Mixed gauge chiral.

a) $[1, 1, 0]|(G_1, G_2, G_3, G_4, G_5, G_6) \equiv (\phi, \phi^{(15)}, \phi^{(15)}, \frac{\Sigma^{(44)}_{(R_-)}}{\sqrt{2}}, \frac{\Sigma^{(44)}_{(R_+)}}{\sqrt{2}}, \frac{\sqrt{2}\lambda^{(R_0)}}{\sqrt{2}} - \frac{3\lambda^{(15)}}{\sqrt{2}})\$

$$G = 2 \begin{pmatrix} m & 0 & \sqrt{6}\lambda\omega & \frac{i\eta\sigma}{\sqrt{2}} & \frac{-i\eta\sigma}{\sqrt{2}} & 0 \\ 0 & m + 2\lambda a & 2\sqrt{2}\lambda\omega & \frac{i\eta\sigma\sqrt{2}}{2} & \frac{-i\eta\sigma\sqrt{2}}{2} & 0 \\ \sqrt{6}\lambda\omega & 2\sqrt{2}\lambda\omega & m + \lambda(p + 2a) & -i\eta\sqrt{3}\sigma & i\sqrt{3}\eta\sigma & 0 \\ \frac{i\eta\sigma}{\sqrt{2}} & \frac{i\eta\sigma\sqrt{2}}{2} & -i\eta\sqrt{3}\sigma & M + \eta(p + 3a - 6\omega) & \frac{\sqrt{2}\eta\sigma}{\sqrt{2}} & \frac{\sqrt{2}\eta\sigma}{\sqrt{2}} \\ \frac{-i\eta\sigma}{\sqrt{2}} & \frac{-i\eta\sigma\sqrt{2}}{2} & i\eta\sqrt{3}\sigma & 0 & \frac{\sqrt{2}\eta\sigma}{\sqrt{2}} & \frac{\sqrt{2}\eta\sigma}{\sqrt{2}} \\ 0 & 0 & 0 & \frac{\sqrt{2}\eta\sigma}{\sqrt{2}} & \frac{\sqrt{2}\eta\sigma}{\sqrt{2}} & 0 \end{pmatrix}$$

b) $[3, 2, -\frac{2}{9}]|\tilde{(E_2, E_3, E_4, E_5)} \oplus [3, 2, \frac{2}{3}]|\tilde{(E_2, E_3, E_4, E_5)}$

$$\equiv (\Sigma^{(4\alpha)}_{(R_\alpha)}, \phi^{(a)}_{\alpha, \beta}, \phi^{(a)}_{\alpha, \beta}, \phi^{(a)}_{\alpha, \beta}) \oplus (\Sigma^{(4\alpha)}_{(R_\alpha)}, \phi^{(a)}_{\alpha, \beta}, \phi^{(a)}_{\alpha, \beta}, \lambda^{(2)}_{\alpha, \beta})$$

$$E = \begin{pmatrix} -2(M + \eta(a + 3\omega)) & 2\sqrt{2}i\eta\sigma & 2i\eta\sigma & ig\sqrt{2}\omega^* \\ 2i\sqrt{2}i\eta\sigma & -2(m + \lambda(a - \omega)) & -2\sqrt{2}\lambda\omega & -2g(a^* - \omega^*) \\ -2i\eta\sigma & -2\sqrt{2}\lambda\omega & -2(m - \lambda\omega) & \sqrt{2}g(\omega^* - p^*) \\ -ig\sqrt{2}\omega^* & 2g(a^* - \omega^*) & g\sqrt{2}(\omega^* - p^*) & 0 \end{pmatrix}$$ (63)

c) $[1, 1, 2]|\tilde{(F_1, F_2, F_3)} \oplus [1, 1, 2]|\tilde{(F_1, F_2, F_3)}$

$$\equiv (\Sigma^{(4\alpha)}_{(R_\alpha)}, \phi^{(15)}_{(R_-)}, \lambda^{(R_-)}_{(R_-)} \oplus (\Sigma^{(4\alpha)}_{(R_\alpha)}, \phi^{(15)}_{(R_+)}), \lambda^{(R_+)}_{(R+)})$$

$$F = \begin{pmatrix} 2(M + \eta(p + 3a)) & 2i\sqrt{2}\eta\sigma & -2i\sqrt{2}\eta\sigma & -g\sqrt{2}\omega^* \\ 2i\sqrt{2}\eta\sigma & 2(m + \lambda(p + 2a)) & -2\sqrt{4i}\omega^* & 0 \\ -g\sqrt{2}\sigma^* & -g\sqrt{2}\sigma^* & -2\sqrt{4i}\omega^* & 0 \end{pmatrix}$$ (64)

d) $[3, 1, -\frac{4}{9}]|\tilde{(J_1, J_2, J_3, J_4)} \oplus [3, 1, \frac{4}{9}]|\tilde{(J_1, J_2, J_3, J_4)}$

$$\equiv (\Sigma^{(4\alpha)}_{(R_-)}, \phi^{(a)}_{\alpha, \beta}, \phi^{(a)}_{\alpha, \beta}, \lambda^{(2)}_{\alpha, \beta}) \oplus (\Sigma^{(4\alpha)}_{(R_+)}, \phi^{(a)}_{\alpha, \beta}, \phi^{(a)}_{\alpha, \beta}, \lambda^{(2)}_{\alpha, \beta})$$

$$J = \begin{pmatrix} 2(M + \eta(a + p - 2\omega)) & -2\eta\sigma & 2\sqrt{2}\eta\sigma & -ig\sqrt{2}\sigma^* \\ 2\eta\sigma & -2(m + \lambda a) & -2\sqrt{2}\lambda\omega & -2ig\sqrt{2}a^* \\ -2\sqrt{2}\eta\sigma & -2\sqrt{2}\lambda\omega & -2(m + \lambda(a + p)) & -4ig\omega^* \\ -ig\sqrt{2}\sigma^* & 2\sqrt{2}iga^* & 4ig\omega^* & 0 \end{pmatrix}$$ (65)

e) $[3, 2, \frac{5}{3}]|\tilde{(X_1, X_2, X_3)} \oplus [3, 2, -\frac{5}{3}]|\tilde{(X_1, X_2, X_3)}$

$$\equiv (\phi^{(a)}_{\alpha, \beta}, \phi^{(a)}_{\alpha, \beta}, \lambda^{(2)}_{\alpha, \beta}) \oplus (\phi^{(a)}_{\alpha, \beta}, \phi^{(a)}_{\alpha, \beta}, \lambda^{(2)}_{\alpha, \beta})$$

$$X = \begin{pmatrix} 2(m + \lambda(a + \omega)) & -2\sqrt{2}\lambda\omega & -2g(a^* + \omega^*) \\ -2\sqrt{2}\lambda\omega & 2(m + \lambda\omega) & \sqrt{2}g(\omega^* + p^*) \\ -2g(a^* + \omega^*) & \sqrt{2}g(\omega^* + p^*) & 0 \end{pmatrix}$$ (66)
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Figure 1: Plot of the threshold corrections $\Delta^{(th)}(\alpha^{-1}_G(M_X))$ and $\Delta^{(th)}(\log_{10} M_X)$ vs $\xi$ for real $\xi$: real solution for $x$, $\lambda = .7$. Note the exclusion of $0.25 < \xi < 8.6$ from the requirement that $\Delta^{(th)}(\log_{10} M_X) > -1$. 

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Figure 2: Plot of the corrections $\Delta^{(th)}(\alpha^{-1}_G(M_X))$ and $\Delta^{(th)}(\log_{10}M_X)$ vs $\xi$ for real $\xi$: real solution for $x$, $\lambda = .1$. Note how the region around $\xi = 0$ is excluded by $\Delta^{(th)}(\alpha^{-1}_G(M_X)) > -10$ while $\Delta^{(th)}(\log_{10}M_X) > -1$ continues to rule out the $8.6 > \xi > .25$ region.
Figure 3: Plot of $\Delta^{th}(\sin^2\theta_W(M_S))$ vs $\xi$ for real $\xi$: real solution for $x$, $\lambda = .7$ (upper) and $\lambda = .01$ (lower). Note how the large change in $\sin^2\theta_W(M_S)$ for $2.8 < \xi < 8$ also excludes this region.
Figure 4: Plot of the corrections $\Delta^{(th)}(\alpha^{-1}_G(M_X))$ and $\Delta^{(th)}(\log_{10} M_X)$ vs $\xi$ for real $\xi$ : real solution for $x$, $\lambda = .01$. Only intermediate values of $\xi$ around the two $SU(5)$ singularities are now viable since $\alpha_G$ is too large.
Figure 5: Plot of the corrections $\Delta^{(th)}(\alpha_G^{-1}(M_X))$ and $\Delta^{(th)}(\log_{10} M_X)$ vs $\xi$ for real $\xi$: complex solution for $x$, $\lambda = .7$. Most values of $\xi$ are allowed.
Figure 6: Plot of the corrections $\Delta^{(th)}(\alpha_G^{-1}(M_X))$ and $\Delta^{(th)}(\log_{10} M_X)$ vs $\xi$ for real $\xi$: complex solution for $x$, $\lambda = .01$. Decreasing $\lambda$ further caused a catastrophic increase of $\alpha_G(M_X)$.
Figure 7: Plot of the threshold and two-loop corrections to $\sin^2\theta_W(M_S)$ vs $\xi$ for real $\xi$: complex solution for $x$, $\lambda = .7$ and $\lambda = .01$. Values of $|\xi| < 10$ are allowed.
Figure 8: $\sin^2 2\theta_{12}, \sin^2 2\theta_{23}$ vs $r_4/r_3$ for the sample real solution of Bertolini and Malinsky. Note the rapid collapse of the large Type II angles as $|r_4/r_3|$ increases.
Figure 9: Plots of MSGUT $R = r_4/r_3$ vs $\xi$ for real $\xi$: real solution (upper) and complex solution (lower), $\lambda = .7$. In the real case the candidate Type II region $.8 < \xi < 4$ with small $R$ is disallowed by $\Delta^{(th)}(\text{Log}_{10} M_X) > -1$. In the complex case there is no Type II candidate region.
Figure 10: Plots of the maximum neutrino masses in the MSGUT (obtained using the sample solution of Bertolini and Malinsky) vs $\xi$ for real $\xi$: real solution for $x$, $\lambda = .7$. Type I (upper) and Type II (lower). The candidate region $.25 < \xi < 8.6$ with possibly large $m^\nu_{\text{max}}$ is disallowed by $\Delta^{(\text{th})}(\log_{10} M_X) > -1$. 
Figure 11: Plots of the maximum neutrino masses in the MSGUT (obtained using the sample solution of Bertolini and Malinsky) vs $\xi$ for real $\xi$: complex solution for $x$, $\lambda = 0.7$. Type I (upper) and Type II (lower). Type I solutions are always dominant and may have the right magnitude only in $4 < \xi < 6$ region.
Figure 12: Magnified plots of the maximum neutrino masses in the MSGUT (obtained using the sample solution of Bertolini and Malinsky) vs $\xi$ for real $\xi$: complex solution for $x, \lambda = .7$. Type I (upper) and Type II (lower). Type I solutions achieve about $10^{-1}$ of the required values at their maximum.