Electrodynamics of s-wave superconductors

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In this paper we give a derivation of a system of equations to describe the electrodynamics of s-wave superconductors. First, we consider a relativistically covariant theory in terms of gauge four-vector electromagnetic potential and scalar complex field. We use the first-order formalism to obtain the supplemented Maxwell equations for gauge invariant electric, magnetic, four-vector fields and the modulus of the superconducting order parameter. The new four-vector field appears in some of the equations as a gauge invariant super-current and in other ones, while gauge invariant, as a four-vector electromagnetic potential. This dual contribution of the new four-vector field is the basis of the electrodynamics of superconductors. We focus on the system of equations with time-independent fields. The qualitative analysis shows that the applied magnetic field suppresses the superconductivity, while the applied electric field impacts appositely, supporting it. Second we consider time-dependent non-relativistic Ginzburg-Landau theory.

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I. INTRODUCTION

The earliest study of the electrodynamics of s-wave superconductors is attributed to London brothers [1]. They supplemented the Maxwell system of equations with a set of equations to explain the electrodynamics of superconductors and more particularly the Meissner-Ochsenfeld effect. The quantum-mechanical foundation of these equations was discussed in phenomenological Ginzburg-Landau theory [2]. There is also an attempt to explain the Meissner-Ochsenfeld effect in a purely classical way [3].

The unsuccessful experiments to detect an electric field in superconductors [4] led to the lack of interest. There is at present no general understanding of the interplay between applied electric, magnetic fields and superconductivity. In [5] a microscopic justification is given that a superconductor may have an electric field in its interior. The phenomenon is considered as a consequence of hole superconductivity [6].

Our main goal is to show that a system of equations which describes the electrodynamics of s-wave superconductors can be derived from time dependent Ginzburg-Landau theory. First, we consider a relativistically covariant theory in terms of gauge four-vector electromagnetic potentials and scalar complex field. The electrodynamics is a Lorentz covariant theory and one expects that the model under consideration will help to get deeper insight for the interplay between electric, magnetic fields and superconductivity. We want also to compare our results with the results in [1] and [7] where relativistically covariant theory of superconductivity is discussed. We focus on the system of equations with time-independent fields. The qualitative analysis shows that the applied magnetic field suppresses the superconductivity, while the applied electric field impacts appositely, supporting it.

The system of equations, derived from a relativistically non-covariant theory, shows that the effect of the applied electric field depends on the direction of the field.

The paper is organized as follows: In Section II, we derive the system of equations to describe the electrodynamics of s-wave superconductors from relativistically covariant theory of superconductivity. For the case when density of Cooper pairs is a constant the system of equations reduces to the London brothers equations. The system of equations derived in the present paper includes an equation for the density of Cooper pairs which shows the different impact on superconductivity of applied electric and magnetic fields. In Section III, we use the same technique of calculations to consider time-dependent relativistically non-covariant Ginzburg-Landau theory. The main results are reported and commented in Section IV.

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II. RELATIVISTICALLY COVARIANT THEORY OF SUPERCONDUCTIVITY

We begin with a well-known field-theory action \[ S \] for relativistically covariant theory of superconductivity

\[
S = \int d^4x \left[ -\frac{1}{4} (\partial_\lambda A_\nu - \partial_\nu A_\lambda) (\partial^\lambda A^\nu - \partial^\nu A^\lambda) + \frac{e^2}{2} \rho^2 (\partial_\lambda \theta + A_\lambda) (\partial^\lambda \theta + A^\lambda) + \partial_\lambda \rho \partial^\lambda \rho + \alpha \rho^2 - \frac{g}{2} \rho^4 \right].
\]

written in terms of gauge four-vector electromagnetic potential "A" and complex scalar field "\( \psi \)", the superconducting order parameter. The parameter

\[
\alpha = \alpha_0 (T_c - T)
\]

where \( T \) is the temperature and \( T_c \) is the critical temperature, is positive when the system is superconductor. We use the standard, for the relativistically covariant systems, notations:

\[
\begin{align*}
\psi(x) & = \rho(x) \exp \left[ i e \phi(x) \right], \\
A'_\nu & = A_\nu - \partial_\nu \phi(x),
\end{align*}
\]

where \( \phi(x) \) is a real function.

We represent the order parameter \( \psi(x) \) in the form

\[
\psi(x) = \rho(x) \exp \left[ i e \theta(x) \right],
\]

where \( \rho(x) = |\psi(x)| \) is a gauge invariant and the gauge transformation of \( \theta(x) \) is

\[
\theta'(x) = \theta(x) + \phi(x).
\]

The action \([4]\), rewritten in terms of \( \rho \) and \( \theta \), adopts the form

\[
S = \int d^4x \left[ -\frac{1}{4} (\partial_\lambda A_\nu - \partial_\nu A_\lambda) (\partial^\lambda A^\nu - \partial^\nu A^\lambda) + e^2 \rho^2 (\partial_\lambda \theta + A_\lambda) (\partial^\lambda \theta + A^\lambda) + \partial_\lambda \rho \partial^\lambda \rho + \alpha \rho^2 - \frac{g}{2} \rho^4 \right].
\]

It is convenient to use the action in the first-order formalism

\[
S = \int d^4x \left\{ -\frac{1}{2} \left[ (\partial_\lambda A_\nu - \partial_\nu A_\lambda) F^{\lambda \nu} - \frac{1}{2} F_{\lambda \nu} F^{\lambda \nu} \right] + 2e^2 \rho^2 \left[ (\partial_\lambda \theta + A_\lambda) Q^\lambda - \frac{1}{2} Q_\lambda Q^\lambda \right] + \partial_\lambda \rho \partial^\lambda \rho + \alpha \rho^2 - \frac{g}{2} \rho^4 \right\},
\]

where gauge potential \( A^\lambda \), phase \( \theta \), gauge invariant antisymmetric field \( F^{\lambda \nu} = -F^{\nu \lambda} \), gauge invariant four-vector field \( Q^\lambda \) and gauge invariant scalar field \( \rho \) are assumed to be independent degrees of freedom in the theory.

To derive the system of Maxwell equations for s-wave superconductors we vary the action \([7]\) with respect to \( F^{\lambda \nu} \), \( Q^\lambda \), \( A^\lambda \), \( \theta \) and \( \rho \). The resulting system of equation reads:

\[
\begin{align*}
F_{\lambda \nu} &= (\partial_\lambda A_\nu - \partial_\nu A_\lambda), \\
Q_\lambda &= \partial_\lambda \theta + A_\lambda, \\
\partial_\lambda F^{\lambda \nu} + 2e^2 \rho^2 Q^\nu &= 0, \\
\partial_\lambda \left( \rho^2 Q^\lambda \right) &= 0, \\
\partial_\lambda \rho \partial^\lambda \rho - \alpha \rho + g \rho^3 &= 2e^2 \rho \left[ (\partial_\lambda \theta + A_\lambda) Q^\lambda - \frac{1}{2} Q_\lambda Q^\lambda \right].
\end{align*}
\]
If we set in equations (10), (11) and (12) the expressions for $F_{\lambda \nu}$ and $Q^\lambda$ from equations (8) and (9) we obtain the equations of motion following from the action (6). This means that theories with actions (6) and (7) are equivalent.

Alternatively, one eliminates the gauge fields $A_\lambda$ and $\theta$ from equations (8,12) to obtain the system of equations for the gauge invariant fields $F^{\lambda \nu}, Q^\lambda$ and $\rho$:

\[
\begin{align*}
\partial_\lambda F_{\nu \delta} + \partial_\nu F_{\delta \lambda} + \partial_\delta F_{\lambda \nu} &= 0, \\
\partial_\lambda F^{\lambda \nu} + 2e^{*2}\rho^2 Q^\nu &= 0, \\
\partial_\lambda (\rho^2 Q^\lambda) &= 0, \\
\partial_\lambda Q^\nu - \partial_\nu Q_\lambda &= F_{\lambda \nu}, \\
\partial_\lambda \partial^\lambda \rho - \alpha \rho + g\rho^3 - e^{*2}\rho Q^\lambda &= 0. 
\end{align*}
\]

Equation (13) follows from equation (8), while equation (16) from equation (9).

Straightforward calculations show that equation (15) can be obtained from equation (14) and equation (13) from equation (10). The system of independent equations is:

\[
\begin{align*}
\partial_\lambda F^{\lambda \nu} + 2e^{*2}\rho^2 Q^\nu &= 0, \\
\partial_\lambda Q^\nu - \partial_\nu Q_\lambda &= F_{\lambda \nu}, \\
\partial_\lambda \partial^\lambda \rho - \alpha \rho + g\rho^3 - e^{*2}\rho Q^\lambda &= 0. 
\end{align*}
\]

We construct the antisymmetric tensor $F_{\lambda \nu}$ by means of the electric $E$ and magnetic $B$ fields in a standard way: $(F_{01}, F_{02}, F_{03}) = E/\upsilon$, $(F_{32}, F_{33}, F_{23}) = B$ and $(Q^0, Q^1, Q^2, Q^3) = (Q/\upsilon, Q)$. In terms of $E$, $B$, $Q$ and $Q$ the system of equations which describes the electrodynamics of s-wave superconductors is:

\[
\begin{align*}
\nabla \times B &= \mu \epsilon \frac{\partial E}{\partial t} - 2e^{*2}\rho^2 Q, \\
\nabla \times Q &= B, \\
\nabla \cdot E &= -2e^{*2}\rho^2 Q, \\
\nabla \cdot Q + \frac{\partial Q}{\partial t} &= -E, \\
\mu \epsilon \frac{\partial^2 \rho}{\partial t^2} - \Delta \rho - \alpha \rho + g\rho^3 - e^{*2}\rho [\mu \epsilon Q^2 - Q^2] &= 0. 
\end{align*}
\]

It is important to stress that the gauge invariant vector $Q$ and scalar $Q$ fields take part in the equations (22) and (24) as a magnetic vector and electric scalar potentials, while in equation (21) $(-2e^{*2}\rho^2 Q)$ is a supercurrent and in equation (23) $(-2e^{*2}\rho^2 Q)$ is a density of superconducting quasi-particles. This dual contribution of the new fields is the basis of the electrodynamics of superconductors.

We focus on the system of equations with time-independent fields:

\[
\begin{align*}
\nabla \times B &= -2e^{*2}\rho^2 Q, \\
\nabla \times Q &= B, \\
\nabla \cdot E &= -2e^{*2}\rho^2 Q, \\
\nabla \cdot Q &= -E, \\
\Delta \rho + \alpha \rho - g\rho^3 + e^{*2}\rho [\mu \epsilon Q^2 - Q^2] &= 0. 
\end{align*}
\]

To compare our result with the London equations [1] we assume that deep inside the superconductor $Q$ and $Q$ are zero, while $\rho = \rho_0$ is a constant determined from the equation $\alpha \rho_0 - g\rho_0^3 = 0$, which follows from equation (30). The gauge transformation (5) of the phase of the order parameter $\theta$ implies that one can impose the gauge fixing condition $\theta = 0$. In that case the gauge invariant vector is equal to the vector potential $Q = A$, the gauge invariant scalar field is equal to the scalar potential $Q = A^0$ and the system of equations (26,29) adopts the form

\[
\begin{align*}
\nabla \times B &= -2e^{*2}\rho_0^{-2} A, \\
\nabla \times A &= B, \\
\nabla \cdot E &= -2e^{*2}\rho_0^{-2} A^0, \\
\nabla A^0 &= -E. 
\end{align*}
\]
With the London brothers postulates in mind

\[ \frac{J}{c} = -2e^* \rho_0^2 A, \quad (35) \]
\[ \rho = -2e^* \rho_0^2 A_0, \quad (36) \]

we arrived at London equations \[1]\).

Taking the curl of (31), using the equation (32) and the identity \( \vec{\nabla} \cdot \vec{B} = 0 \), which follows from this equation, we obtain

\[ \Delta B = \frac{1}{\lambda_L^2} B, \quad (37) \]

where

\[ \lambda_L = \sqrt{g/(2e^* \alpha)}. \quad (38) \]

Taking the divergence of (34) and using the equation (33) we obtain

\[ \Delta E = \frac{1}{\lambda_L^2} E. \quad (39) \]

The equations (37) and (39) imply that an electric field penetrates a distance \( \lambda_L \) as a magnetic field does \[7\].

This approximation is very rough and does not account for the last term in the equation (30) which is responsible for the different impact on superconductivity of applied electric and magnetic fields. If we apply magnetic field \((E = 0, Q = 0)\) the qualitative analysis of equation (30) shows that the magnetic vector potential effectively decreases the \( \alpha \) parameter,

\[ \alpha \rightarrow \alpha - e^* Q^2 \]

where \( Q^2 \) is some average value. Therefore the Ginzburg-Landau coherence length increases \[9\], which means that applied magnetic field destroys superconductivity. On the other hand, when the electric field is applied \((B = 0, Q = 0)\) the electric scalar potential effectively increases the \( \alpha \) parameter

\[ \alpha \rightarrow \alpha + e^* \mu \varepsilon Q^2. \]

Hence, the Ginzburg-Landau coherence length decreases. This qualitative analysis permits us to formulate the hypotheses that applying electric field at very low temperature one increases the critical magnetic field. This result is experimentally testable.

### III. TIME DEPENDENT GINZBURG-LANDAU THEORY

A number of authors have discussed the non-relativistic time-dependent generalization of the Ginzburg-Landau theory \[10–14\]. We investigate a model with field-theory action

\[ S = \int d^4x \left\{ \frac{1}{4} (\partial_\lambda A_\nu - \partial_\nu A_\lambda) \left( \partial^\lambda A^\nu - \partial^\nu A^\lambda \right) \right. \]
\[ + \frac{1}{D} \psi^* (i \partial_t - e^* \varphi) \psi \]
\[ - \frac{1}{2m^*} (\partial_k - ie^* A_k) \psi^* (\partial_k + ie^* A_k) \psi \]
\[ + \alpha \psi^* \psi - g \left( \psi^* \psi \right)^2 \right\}, \quad (40) \]

where \( \varphi = \nu A_0 \) is the electric scalar potential, with gauge transformation \[ \varphi' = \varphi - \partial_t \phi \), \( D \) is the normal-state diffusion constant \[14\], and \( (e^*, m^*) \) are effective charge and mass of superconducting quasi-particles. The index \( k \) runs \( k = x, y, z \).

We follow the same procedure to derive the system of equations which describe the electrodynamics of s-wave superconductors. We represent the order parameter \( \psi \) by means of modulus and phase \[4\] and write the field-theory action, in the first-order formalism, in the form

\[ S = \int d^4x \left\{ -\frac{1}{2} \left( \partial_\lambda A_\nu - \partial_\nu A_\lambda \right) F^{\lambda \nu} - \frac{1}{2} F_{\lambda \nu} F^{\lambda \nu} \right\} \]
\[ - \frac{e^*}{D} \rho^2 (\varphi + \partial_t \theta) \]
\[ - \frac{e^*}{m^*} \rho^2 \left( \partial_k \theta + A_k \right) Q_k - \frac{1}{2} Q_k Q_k \]
\[ - \frac{1}{2m^*} \partial_k \rho \partial_k \rho + \alpha \rho^2 - \frac{g}{2} \rho^2 \right\}. \quad (41) \]
It is important to stress that $Q_k$ is a three component vector field and $k = x, y, z$. Using a variational principle we obtain the system of equations

\begin{align}
F_{\lambda \nu} &= (\partial_\lambda A_\nu - \partial_\nu A_\lambda) \quad (42) \\
Q_k &= \partial_k \theta + A_k \quad (43) \\
\partial_\lambda F^\lambda_k + \frac{e^2}{m^* \rho^2} Q_k &= 0 \quad (44) \\
\partial_k F_{0k} - \frac{ve^*}{D} \rho^2 &= 0 \quad (45) \\
\partial_t \rho^2 + \frac{D}{m^*} \partial_k (\rho^2 Q_k) &= 0 \quad (46) \\
\frac{1}{2m^*} \Delta \rho + \alpha \rho - g \rho^3 - \frac{e^*}{D} \rho (\varphi + \partial_\lambda \theta) &= \frac{e^2}{m^*^2} \rho \left[ (\partial_k \theta + A_k) Q_k - \frac{1}{2} Q_k Q_k \right]. \quad (47)
\end{align}

We supplement the system of equations (42-47) with equation

\begin{equation}
Q = \partial_t \theta + \varphi, \quad (48)
\end{equation}

which is a definition of the new gauge invariant field $Q$. In the same way, starting from the system of equations (42-48), we arrive at the Maxwell equations for superconductors in a non-relativistic theory.

\begin{align}
\nabla \times \mathbf{B} &= \mu \varepsilon \frac{\partial \mathbf{E}}{\partial t} - \frac{e^2}{m^* \rho^2} Q \quad (49) \\
\nabla \times \mathbf{Q} &= \mathbf{B} \quad (50) \\
\nabla \cdot \mathbf{E} &= \frac{\mu \varepsilon e^*}{D} \rho^2 \quad (51) \\
\nabla Q + \frac{\partial Q}{\partial t} &= -\mathbf{E} \quad (52) \\
\frac{1}{2m^*} \Delta \rho + \alpha \rho - g \rho^3 - \frac{e^*}{D} \rho Q - \frac{e^*}{2m^*^2} \rho Q^2 &= 0 \quad (53)
\end{align}

The system of equations for time-independent fields is:

\begin{align}
\nabla \times \mathbf{B} &= -\frac{e^2}{m^* \rho^2} Q \quad (54) \\
\nabla \times \mathbf{Q} &= \mathbf{B} \quad (55) \\
\nabla \cdot \mathbf{E} &= \frac{\mu \varepsilon e^*}{D} \rho^2 \quad (56) \\
\nabla Q + &= -\mathbf{E} \quad (57) \\
\frac{1}{2m^*} \Delta \rho + \alpha \rho - g \rho^3 - \frac{e^*}{D} \rho Q - \frac{e^*}{2m^*^2} \rho Q^2 &= 0. \quad (58)
\end{align}

There are two important differences between equations in relativistic theory (26-30) and equations in non relativistic one (54-58). In contrast to equation (28), in equation (56) there is no dependence on gauge invariant scalar field $Q$. The last equation (58) depends on the electric potential $Q$ linearly, which makes the impact of the applied electric field on superconductivity quite nontrivial.

**IV. SUMMARY**

The aim of the present paper was to present the mathematical basis for the exploration of the intricate interplay between superconductivity and applied magnetic and electric fields. We derived a system of equations to describe the electrodynamics of s-wave superconductors.

When the model is relativistically covariant, we obtained that the applied electric field supports the superconductivity. It is very difficult to experimentally test the effects of applied electric field. Right below the superconductor
critical temperature the normal fluid dominates the system. The screening length of the normal fluid is about one Angström. The applied field cannot penetrate into the system more than one Angström from the surface. This is why the electric field cannot affect the system in the interior. To study the effects of the applied electric field one has to do experiments at very low temperatures where there are no normal quasiparticles.

When the model is nonrelativistic, the impact of the applied electric field on superconductivity is more complicated. The equations in relativistic theory are invariant under the discrete transformation of electric field \( E \rightarrow -E \) and independently under the transformation \( E \rightarrow -E, B \rightarrow -B \). In contrast, the system of equations in nonrelativistic theory are invariant under the discrete transformation of magnetic field \( B \) and gauge invariant field \( Q \), but they are not invariant under the discrete transformation of electric field \( E \) and gauge invariant field \( Q \). This means that the effects of the applied electric fields \( E_0 \) and \( -E_0 \) on superconductivity are different.

It is important to underline that the fields \( Q \) and \( Q \) are gauge invariant. This is why they should be measurable as the electric and magnetic fields are measurable. The role of these fields is fundamental in superconductivity but not investigated.

The numerical solution of the system of equations for appropriate geometry of the superconductor, for example slab \( \text{[12]} \), will help us to gain an insight into the impact of the applied electric field on the superconductivity.

**Appendix A**

To elucidate the qualitative analysis in section II, we consider the system of equations for fields which depend on \( z \) coordinate only. Then, the system of equations for the fields \( Q(z), Q_y(z) = (0, Q_y(z), 0), E(z) = (0, 0, E_z(z)) \), \( B(z) = (B_x(z), 0, 0) \) and \( \rho(z) \) adopts the form

\[
\begin{align*}
\frac{dB_x}{dz} &= -2e^{\ast 2}\rho^2 Q_y \\
\frac{dQ_y}{dz} &= B_x \\
\frac{dE_z}{dz} &= -2e^{\ast 2}\rho^2 Q \\
\frac{dQ}{dz} &= -E_z \\
\Delta \rho + \alpha \rho - gp^3 + e^{\ast 2} \rho [\mu \varepsilon Q^2 - Q_y^2] &= 0.
\end{align*}
\]

After some calculations one reduces the system to a system of equations for \( Q, Q_y \) and \( \rho \)

\[
\begin{align*}
\frac{d^2Q}{dz^2} &= 2e^{\ast 2}\rho^2 Q \\
\frac{d^2Q_y}{dz^2} &= 2e^{\ast 2}\rho^2 Q_y \\
\Delta \rho + \alpha \rho - gp^3 + e^{\ast 2} \rho [\mu \varepsilon Q^2 - Q_y^2] &= 0.
\end{align*}
\]

It is convenient to introduce dimensionless functions \( f_1(\zeta), f_2(\zeta) \) and \( f_3(\zeta) \) of a dimensionless distance \( \zeta = z/\xi_{GL} \), where

\[
\xi_{GL} = 1/\sqrt{\alpha}
\]

is the Ginzburg-Landau coherence length:

\[
\begin{align*}
Q(\zeta) &= -E_0 \xi_{GL} f_1(\zeta) \\
Q_y(\zeta) &= -B_0 \xi_{GL} f_2(\zeta) \\
\rho(\zeta) &= \rho_0 f_3(\zeta).
\end{align*}
\]

In equations \( \rho_0 = \sqrt{\alpha/g} \), the applied electric field is \( E_0 = (0, 0, E_0) \) and the applied magnetic field is \( B_0 = (B_0, 0, 0) \). The representations of the electric and magnetic fields by means of \( f_1 \) and \( f_2 \) are the following:

\[
\begin{align*}
E_z(\zeta) &= E_0 \frac{df_1(\zeta)}{d\zeta} \\
B_z(\zeta) &= B_0 \frac{df_2(\zeta)}{d\zeta}
\end{align*}
\]
The system of equations (A6–A8), rewritten in terms of the new functions, reads:

\[
\begin{align*}
\frac{d^2 f_1(\zeta)}{d\zeta^2} &= \frac{1}{\kappa^2} f_3(\zeta) f_1(\zeta) \\
\frac{d^2 f_2(\zeta)}{d\zeta^2} &= \frac{1}{\kappa^2} f_3(\zeta) f_2(\zeta) \\
\frac{d^2 f_3(\zeta)}{d\zeta^2} + f_3(\zeta) - f_3^3(\zeta) &= -f_3(\zeta) [\gamma_E f_1^2(\zeta) - \gamma_B f_2^2(\zeta)]
\end{align*}
\] (A12)

In equations (A12) \( \kappa \) is the Ginzburg-Landau parameter

\[
\kappa = \frac{\lambda_L}{\xi_{GL}},
\] (A13)

which satisfies \( \kappa < 1/\sqrt{2} \), for type I superconductors and \( \kappa > 1/\sqrt{2} \) for type II ones, and parameters \( \gamma_E \) and \( \gamma_B \) are

\[
\gamma_E = \frac{e^{*} \mu E_0^2}{\alpha^2}, \quad \gamma_B = \frac{e^{*} B_0^2}{\alpha^2}.
\] (A14)

FIG. 1: (Color online) (a)-function (A17), (b)-function (A18) \( \rho(z/\xi_{GL})/\rho_0 = \rho((\xi_{GL}/\xi^E)z/\xi_{GL}) \) with \( \xi_{GL}/\xi^E = 2 \), (c)-function (A19) \( \rho(z/\xi^B)/\rho_0 = \rho((\xi_{GL}/\xi^B)z/\xi_{GL}) \) with \( \xi_{GL}/\xi^B = 0.6 \).

For semi-infinite superconductors, with a surface of superconductor orthogonal to the \( z \)-axis, the boundary condi-
tions are:
\[
\frac{df_1(0)}{d\zeta} = 1 \quad f_1(\infty) = 0
\]
\[
\frac{df_2(0)}{d\zeta} = 1 \quad f_2(\infty) = 0
\]
\[
f_3(0) = 0 \quad f_3(\infty) = 1. \tag{A15}
\]

If neither electric nor magnetic fields are applied the equation for the dimensionless function \(f_3(\zeta) = \rho(\zeta)/\rho_0\)
\[
\frac{d^2f_3(\zeta)}{d\zeta^2} + f_3(\zeta) - f_3^3(\zeta) = 0 \tag{A16}
\]
is exactly solvable and the solution, for \(z \geq 0\) is
\[
f_3(\zeta) = f_3\left(\frac{z}{\xi_{GL}}\right) = \tanh\left(\frac{z}{\sqrt{2\xi_{GL}}}\right). \tag{A17}
\]

The qualitative analysis in section II shows that applied electric field increases the \(\alpha\) parameter \(\alpha \to \alpha^E = \alpha + e^*e \mu \varepsilon < Q^2 >,\) where \(< Q^2 >\) is an average value of the scalar field. Within this approximation the expression for \(f_3^E\) is
\[
f_3^E(\zeta) = f_3^E\left(\frac{z}{\xi_E}\right) = \tanh\left(\frac{z}{\sqrt{2\xi_E}}\right), \tag{A18}
\]
where \(\xi_E = 1/\sqrt{\alpha^E} < \xi_{GL}.\) When the magnetic field is applied \(\alpha\) decreases, \(\alpha \to \alpha^B = \alpha - e^*e < Q^2 >\) and function \(f_3^B\) reads
\[
f_3^B(\zeta) = f_3^B\left(\frac{z}{\xi_B}\right) = \tanh\left(\frac{z}{\sqrt{2\xi_B}}\right), \tag{A19}
\]
where \(\xi_B = 1/\sqrt{\alpha^B} > \xi_{GL}.\)

The three curves [A17-A19] are depicted in figure [11]. The graph (a) shows the function \(\rho(z/\xi_{GL})/\rho_0\) [A17], when neither electric nor magnetic fields are applied, the graph (b) shows the function [A18], \(\rho(z/\xi_E)/\rho_0 = \rho((\xi_{GL}/\xi_E)z/\xi_{GL})\) with \(\xi_{GL}/\xi_E = 2\) and the graph (c) shows the function [A19] \(\rho(z/\xi_B)/\rho_0 = \rho((\xi_{GL}/\xi_B)z/\xi_{GL})\) with \(\xi_{GL}/\xi_B = 0.6\).

The Ginzburg-Landau coherence length measures the distance over which the superconducting order parameter increases up to the bulk value, measured from the surface of the superconductor (\(z > 0\)). The applied electric field decreases the GL coherence length, which means that the electric field supports the superconductivity, while the applied magnetic field increases the GL coherence length, which means that the magnetic field destroys the superconductivity.

[1] F. London and H. London, Proc. R. Soc. London, Ser. A 149, 71 (1935).
[2] V. L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. 20, 1064 (1950).
[3] W. Farrel Edwards, Phys. Rev. Lett., 47, 1863 (1981).
[4] H. London, Proc. R. Soc. London, Ser. A 155, 102 (1936).
[5] J. E. Hirsch, Phys. Rev. B 68, 184502 (2003).
[6] J. E. Hirsch and F. Marsiglio, Phys. Rev. B 39, 11515 (1989).
[7] J. E. Hirsch, Phys. Rev. B 69, 214515 (2004).
[8] Peter W. Higgs, Phys. Rev. Lett., 13, 508 (1964).
[9] See the Appendix.
[10] E. Abrahams and T. Tsumeto, Phys. Rev. 152, 416 (1966).
[11] C. Caroli and K. Maki, Phys. Rev. 159, 306 (1967).
[12] L. P. Gor’kov and G. M. Eliashberg, Soviet Phys. JETP 27, 328 (1968).
[13] R. S. Thompson, Phys. Rev. B 1, 327 (1970).
[14] Michael Tinkham, Introduction to Superconductivity (McGRAW-HILL, INC, 1975).
[15] Alexander L. Fetter and John Dirk Walecka, Quantum Theory of Many-Particle Systems (McGraw-Hill Book Company, 2003).