Numerical study of the vortex structure influence on heat transfer in the supersonic flow past a plate and a blunt fin junction

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Abstract. Viscous-inviscid interaction and heat transfer in supersonic 3D laminar flow past a fin-body junction is investigated by means of numerical simulation. A laminar flow regimes at the freestream Mach number equal to 4, 5 and 6 are considered. Specifics of vortex structures and shock wave pattern in the interaction region are discussed. The Mach number effect on the predicted size of separation zone upstream of the fin and the plate Stanton number distribution are analysed. For the Mach number of 5, the temperature factor influence on heat transfer is investigated.

1. Introduction
Shock wave/boundary layer interaction is an influential phenomenon in many practical applications, such as development of high-speed vehicles. It creates complex flow structure with extreme localized pressure and heat transfer regions. Aerodynamic heating is one of the critical issue in supersonic flight, in such flow the peak heating can achieve values several-orders higher than values in undisturbed boundary layer. Understanding this phenomenon and gaining the ability to predict the flow parameters in such configurations is very crucial. According to [1], a substantial improvement of research accuracy for complex supersonic laminar viscous-inviscid interactions is achieved in last decade. One of the canonical problems in which shock wave/boundary layer interaction has significant effect is a supersonic flow past a plate and a blunt fin junction. The basic knowledge of the aerodynamic heating for this case was obtained in [2-6]. At the same time, in three-dimensional flows involving shock-boundary layer interactions the flow features are complex and difficult to predict a priori, particularly the pattern of heating and maximum heat flux in the interaction region. Consequently, it remains necessary to carry out numerical studies in order to determine the exact nature of the flow under given geometric and flow conditions and influence of different parameters on aerodynamic heating. In this study, three-dimensional shock wave boundary layer interaction in a laminar supersonic flow past an isothermal blunt fin body mounted on a flat plate of same temperature is numerically investigated and the effect of Mach number and temperature factor is analyzed.

A schematic illustration of the general problem is shown in figure 1, this type of geometry can be considered as a simplified representation of a wing-body or fin-body junction on a high-speed vehicle. The flow structure is determined by several parameters, such as: geometric factor, which is the ratio of
blunt fin diameter $D$ and incoming boundary-layer thickness $\delta$, freestream Mach number ($M$) and Reynolds number based on the diameter of the cylindrical fin ($Re_D$), Prandtl number ($Pr$), temperature factor ($T_\infty/T_w$) and the ratio of the specific heats ($\gamma$). In this study the following values of parameters were chosen: $Re_D = 4 \times 10^3$, $D/\delta = 1$, $Pr = 0.7$, $\gamma = 1.4$, Mach number $M$ is ranged from 4 to 6, assuming that wall temperature ($T_w$) is equal to half of adiabatic wall temperature $T_{aw}$. For Mach number of 5 different temperature factors, ranged from 1.5 to 4, are considered. The flow is assumed to be symmetric, therefore only a half of the configuration is considered (figure 1 b). The fin is assumed to have infinite height, isothermal no-slip conditions were applied at the surface of the plate and the fin, incoming boundary-layer profile with a given thickness was applied at the inlet boundary.

![Diagram](image)

**Figure 1.** Flow structure scheme (a) and computational domain (b).

2. **Computational method**

Three-dimensional Navier-Stokes equations for compressible viscous gas flow were solved using finite volume method. Balance equations in a finite-volume formulation can be written as:

$$ \int \frac{\partial \bar{w}}{\partial t} d\Omega + \sum_{m} \sum_{S_m} \bar{F} dS = 0, $$

where $\Omega$ is the control (finite) volume, $M$ – number of finite-volume faces, $S_m$ – area of the current face, $m=1,M$, $\bar{w} = [\rho, \rho u, \rho v, \rho w, \rho H]$ – vector of variables. Flux vector $\bar{F}$ can be define as a sum of inviscid and viscous vectors $\bar{F} = \bar{F}^{inv} + \bar{F}^{visc}$:

$$ \bar{F}^{inv} = \left[ \rho V_n, \rho u V_n + pn, \rho v V_n + pn, \rho w V_n + pn, \rho H V_n \right]^T $$

$$ \bar{F}^{visc} = \left[ 0, \bar{n} \cdot \bar{\tau} \cdot \bar{i}, \bar{n} \cdot \bar{\tau} \cdot \bar{j}, \bar{n} \cdot \bar{\tau} \cdot \bar{k}, \bar{n} \cdot \left( \bar{\tau} \cdot \bar{\nabla} + \bar{q} \right) \right]^T $$

where $\bar{n}$ –normal vector to the face, $\bar{i}, \bar{j}, \bar{k}$ – unit vectors in Cartesian coordinate system.

The components of viscous stress tensor $\bar{\tau}$ and heat flux vector $\bar{q}$ can be written as

$$ \bar{\tau}_{ij} = \mu \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] - 2/3 \left( \frac{\partial u_k}{\partial x_k} \right) \delta_{ij}, q_j = -\lambda \left( \frac{\partial T}{\partial x_j} \right). $$

Here $\mu$ – molecular dynamic viscosity coefficient, $\lambda$ – thermal conductivity. The viscosity $\mu$ is obtained by using Sutherland’s law for the air, the conductivity of the gas is related to the viscosity...
through a constant Prandtl number of 0.7. The relation between pressure, density and internal energy is determined by ideal gas law \( p = (\gamma - 1)\rho e \), where \( \gamma = c_p / c_v = \text{const} \) is the ratio of specific heats.

Convective flux \( \tilde{F}^{mv} \) is evaluated by AUSM scheme [7]. An extension to higher order is obtained through a MUSCL slope-limiting approach [8] with van Albada limiter. In order to apply such approach to unstructured meshes the method suggested in [9, 10] is used.

Finite-volume unstructured in-house code SINF/Flag-S developed at the Department of Fluid Dynamics, Combustion and Heat Transfer of SPbPU was used to perform calculations. The results of the work were obtained using computational resources of Peter the Great Saint-Petersburg Polytechnic University Supercomputing Center (scc.spbstu.ru).

3. Results and discussion

3.1. Flow structure in case \( M=5 \)

Figures 2 and 3 present results of computation for the case \( M=5 \). The bow shock causes the boundary layer separation, which results in a complicated, three-dimensional shock-wave and boundary-layer interaction. Structure of the flow illustrated in figure 2, where Mach number distribution in several slices along the fin and Stanton number distribution on fin and plate are presented. Stanton number is calculated using adiabatic wall temperature:

\[
St = \frac{q_w}{\rho C_p (T_{aw} - T_w)}; \; T_{aw} = T_\infty \left(1 + r \frac{\gamma - 1}{2} M^2\right), \; r = \sqrt{Pr} - \text{recovery factor.}
\]

Streamlines in figure 2 show vortex structure around the fin. One can see that the separated flow upstream of the fin rolls up into a series of horseshoe vortices as it expands around the side of the fin. Within the separated flow region there are zones of subsonic and supersonic flow including shock waves. High pressure gradients near the plate lead to re-separation of the near-wall flow. As a result, an extended separation zone is formed in front of the body.

**Figure 2.** Flow structure: Mach number distribution, surface streamlines and Stanton number distribution in exponential scale.
Detailed structure of the flow in the separation zone is illustrated in figure 3, where a map of density gradient module and Stanton number distributions along both fin body leading edge and the plate centerline are presented.

Vortex structure of the flow in separation region defines heat transfer on a plate. The Stanton number distribution has several local peaks, which occur where skin friction coefficient changes in sign. Heat transfer on the fin is mainly determined by the shock wave pattern. Additional shock waves emanating from the separated-flow region (separation shocks) interact with the bow shock, causing several peaks of heat flux. Peak value of Stanton number on the fin surface is narrower than that on the plate and about two and half times higher.

![Figure 3. Map of density gradient module and Stanton number distribution along leading edge and plate centerline.](image)

3.2. Grid sensitivity
In order to examine quality of calculations, several hexahedral meshes were used (characteristics of the meshes are given in table 1). The main characteristics of the meshes for such type of flow is the ratio $D/(\Delta_x)^x$, where $(\Delta_x)^x$ is average cell size in $X$ direction in the region where the main horseshoe vortex axis intersects $X$-axis, and $(N_x)_S$ total number of cells in $X$-direction related to separation zone. General view of computational Mesh 1 is shown in figure 4a.

Table 1 includes also data for predicted length of separation zone in symmetry plane. Figure 4b shows Stanton number distribution on the plate centerline computed with different meshes. Distribution of $St$ and $C_f$ obtained with Meshes 3 and 4 are very close. The same can be stated for length of separation zone, therefore, it can be concluded that Mesh 3 is fine enough to resolve all important details of the flow. All results presented below are obtained using Mesh 3.
Table 1. Parameters of computational meshes and predicted length of separation zone.

| Mesh   | $N_x \times N_y \times N_z$ | $N_{cells}$ | $D/\Delta x^*$ | $(N_0)S$ | $L_S/D$ |
|--------|-----------------------------|-------------|----------------|-----------|---------|
| Mesh 1 | $100 \times 50 \times 60$   | 0.3 mln     | 20             | 59        | 4.11    |
| Mesh 2 | $200 \times 100 \times 120$ | 2.4 mln     | 43             | 120       | 3.82    |
| Mesh 3 | $300 \times 150 \times 180$ | 8.1 mln     | 72             | 231       | 3.91    |
| Mesh 4 | $400 \times 150 \times 234$ | 14.0 mln    | 100            | 300       | 3.90    |

Figure 4. 3D view of Mesh 1 (a) and Stanton number distribution along plate centerline for different meshes (b).

3.3. Influence of Mach number and temperature factor

To study effect of Mach number on the plate heat transfer three values of Mach number (equal to 4, 5 and 6) were considered, assuming that wall temperature is equal to half of adiabatic wall temperature. For the case $M = 5$, effect of temperature factor variation was studied as well. Parameters of the cases considered and some results (length of separation zone, and number of vortices occupying the zone) are given in table 2. Stanton number and skin friction coefficient ($C_f$) distributions are shown in figure 5.

As one can see from table 2, separation zone properties differ significantly with changing the Mach number and temperature factor. An increase in Mach number, as well as in temperature factor, leads to decrease of separation zone length. This phenomenon can be attributed to increasing the pre-separation boundary layer temperature (due to viscous dissipation) when Mach number or temperature factor grows up; the latter is considered as a possible reason for the later boundary layer separation. Note however, that there is no monotonic relation between the separation zone length and the number of well-distinguishable vortices: there are only three vortices in the case of $M = 5$ and $T_w/T_\infty = 1.5$ although the separation zone length in this case is larger than in other cases with larger temperature factor, where four vortices are observed. In case of $M = 4$ both number of vortices and length of separation zone are the largest.

Table 2. Flow parameters and separation zone characteristic.

| Mach number $M$ | 4   | 5   | 5   | 5   | 6   |
|-----------------|-----|-----|-----|-----|-----|
| Temperature factor $T_\infty/T_{in}$ | 1.84 | 1.5 | 2.59 | 4   | 3.51 |
| Wall temperature ratio $T_w/T_\infty$ | 0.5 | 0.29 | 0.5 | 0.77 | 0.5 |
| Separation zone length $L_S/D$ | 4.55 | 4.3 | 3.91 | 3.6 | 3.42 |
| Number of vortices | 5   | 3   | 4   | 4   | 4   |

Comparing St and Cf distributions obtained for different Mach numbers (figure 5, left; case of $M = 5$ corresponds to $T_w/T_\infty = 2.59$) one can conclude that Stanton number peaks decreases when Mach number increases, as well as peaks of Cf. Increasing in temperature factor (figure 5, right) leads to the
opposite effect – increasing in peak values. Both (increasing in Mach number and temperature factor) leads to shifting the peaks farther from the fin body.

Figure 5. St and Cf distribution on the plate centerline: Mach number effect (left), temperature factor effect (right).

4. Conclusions
Numerical study of viscous-inviscid interaction and heat transfer in supersonic 3D laminar air flow past a fin-body junction has been carried out varying the freestream Mach number in the range of 4-6. The computations have been performed for the case of isothermal plate and fin surface.

In all the cases considered, the bow shock causes the boundary layer separation, which results in a complicated, three-dimensional shock-wave and boundary-layer interaction. Separated flow upstream of the fin rolls up into a series of horseshoe vortices and an extended separation zone is formed in front of the body. Vortex structure of the flow in separation zone defines remarkable peculiarities of heat transfer on a plate, in particular, the Stanton number has two pronounce local peaks. Heat transfer on the fin, determined mainly by the shock wave pattern, is also characterized by occurrence of several peaks in Stanton number distribution along the fin leading edge.

Separation zone properties depends substantially on Mach number and temperature factor. An increase in Mach number, as well as in temperature factor, leads to decrease of separation zone length and to shifting main peak in Stanton number distribution farther from the fin edge. The peak value of Stanton number is higher for larger Mach numbers, whereas an increase in temperature factor leads to opposite effect.

References
[1] Gaitonde D V 2015 Prog. Aerosp. Sci. 72 80–99
[2] Schuricht P H and Roberts G T 1998 8th AIAA International Space Planes and Hypersonic Systems and Technologies Conference (Norfolk)
[3] Tuttly O R, Roberts G T and Schuricht P H 2013 J. Fluid Mech. 737 19–55
[4] Zhuang Y Q and Lu X Y 2015 Procedia Engineering 126 134–8
[5] Borovoy V, Mosharov V, Radchenko V and Skuratov A 2017 Proceedings of 7th European Conference for Aeronautics and Space Sciences (Milan)
[6] Mortazavi M and Knight D 2017 Proceedings of 7th European Conference for Aeronautics and Space Sciences (Milan)
[7] Liou M S and Steffen C J 1993 J. Comput. Phys. 107 23–39
[8] Van Leer B 1979 J. Comput. Phys. 32 101–36
[9] Le Touze C, Murrone A and Guillard H 2015 J. Comput. Phys. 284 389–418
[10] Bakhvalov P A and Kozubskaya T K 2016 Mathematical Models and Computer Simulations 8 625–37