Calculation of the ion current distribution along the cathode in glow discharge under the existence of a periodic relief and an insulating film of varying thickness on its surface

G G Bondarenko\textsuperscript{1} and V I Kristya\textsuperscript{2}

\textsuperscript{1}National Research University Higher School of Economics (HSE), Myasnitskaya st. 20, Moscow, 101000, Russia
\textsuperscript{2}Bauman Moscow State Technical University, Kaluga Branch, Bazhenov st. 2, Kaluga, 248000, Russia

E-mail: kristya@bmstu-kaluga.ru

Abstract. An approximate analytical expression for the ion current density near the cathode in glow discharge is obtained in the presence of a periodic relief of small amplitude and an insulating oxide film of varying thickness on its surface. It is found that ion focusing at the cathode sections with the minimum film thickness, located on any parts of the surface relief, takes place, resulting in an increase of the film thickness non-uniformity with time. Therefore, under the existence of an oxide film on the cathode, its sputtering in glow discharge is determined mainly by the film thickness non-uniformity and not by the surface relief.

1. Introduction

Glow discharge in gases is used in many types of electronic devices (gas lasers, plasma display panels, discharge illuminating lamps) [1-3]. Its feature is a thin layer near the cathode (the cathode sheath (CS)) with the voltage drop of the order of $10^2$ V and a high value of the electric field strength [1]. Therefore, details of the process of glow discharge plasma interaction with the cathode surface are determined by the CS characteristics, such as the CS length and the cathode voltage drop. A reduction of the cathode voltage drop, leading to a decrease of the ion energies at the cathode and intensity of its sputtering in the discharge, can be achieved by formation of a thin insulating oxide film on the cathode surface, which ensures a higher value of its effective ion-electron secondary emission rate $\gamma_{\text{eff}}$ [4, 5].

Because usually a relief exists on the cathode and the oxide film thickness is varied along its surface, the electric field line bending in a near-cathode discharge layer and the focusing of ions at some sections of the cathode surface can take place [6, 7]. This should result in a non-uniformity of its sputtering in the discharge and in a decrease of the cathode service time. In particular, as it was shown in [7-10], the ion current density has a higher value at the metal cathode relief tops, whereas in the case of the cathode with an insulating film of varying thickness the ion flow is focused at its more thin sections. But the combined effect of both these factors on the ion current density distribution and the cathode relief time evolution was not studied before.

In this work, the ion current distribution along the cathode in glow discharge under the presence of a periodic relief of small amplitude and an insulating oxide film of varying thickness on its surface is calculated, and their influence on the cathode sputtering is investigated.
2. Mathematical model

Let the discharge CS be bounded by plane $z=0$ and the metal cathode, on which there is a thin dielectric film, which thickness is periodically varying along the cathode in the direction of $x$-axis (see figure 1). The cathode surface is described by equation $z_c = d_c + h_t \cos(kx)$, equation of the boundary between the film and the cathode metal substrate has the form $z_b = d_c + H_{10} + h_t \cos(kx)$, and the film thickness is determined by the dependence $H_f(x) = z_{b0} - z_c = H_0 + h_t \cos(kx)$, where $h_t = h_{b0} - h_c$, $d_c$ is the CS length, $k = 2\pi/l_s$, $h_{b0}$ and $l_s$ are the amplitude and period of the cathode surface relief, $h_{b0}$ is the amplitude of variation of the film-substrate boundary $z$-coordinate.

We will suppose that the lateral dimension $l_s$ of the cathode surface relief elements greatly exceeds the average characteristic length $\lambda$ of the ion and electron path between collisions with gas atoms and the amplitude of the surface relief elements is rather small, i.e.

$$h_b \ll h \ll \lambda \ll l_s \ll d_c.$$  

(1)

In the discharge, due to bombardment of the cathode by ions, positive charges are accumulated on the film surface. As a result, a strong electric field arises in the film, and when it reaches value of the order of $10^8$-$10^9$ V/m, the field electron emission from the metal cathode substrate into the film starts [11, 12]. The emitted electrons move in the film to its outer boundary and neutralize positive surface charges, preventing further increase of the cathode surface charging, i.e. the discharge becomes stationary [13, 14].

Under condition (1) the macroscopic description of charged particles motion in the gas can be used. Therefore, taking into account that the space charge of electrons in the CS and in the film is small [1, 15], the CS equations can be written in the form [1, 7]:

$$\nabla E_c = \alpha_j j_c, \quad \nabla \psi = \alpha_j j_c, \quad j_c = -e\mu_e n_c V_c, \quad j_i = e\mu_i n_i V_i, \quad \Delta \phi = -en_i/e_0, \quad \Delta \psi = 0$$

(2)

with boundary conditions

$$\bar{E}(x,0) = 0, \quad \phi(x,0) = 0, \quad \phi(x,z_c) = \psi(x,z_b), \quad j_c(x,z_c) = -\gamma_{eff} j_i(x,z_b), \quad \psi(x,z_b) = -U_c,$$

(3)

Figure 1. A schematic of the CS geometry near the cathode with a periodic relief and a surface insulating film of varying thickness.
where $j_e$ and $j_i$ are the electron and ion current densities in the discharge, $\alpha$ is the coefficient of gas ionization by electrons, $\varphi$, $\vec{E} = -\nabla\varphi$ and $\psi$, $\vec{E} = -\nabla\psi$ are the potentials and the electric field strengths in the CS and in the film, $n_e$ and $\mu_e$ are the electron density and mobility, $n_i$ and $\mu_i = k_i/\sqrt{E}$ are the ion density and mobility, $k_i$ is the constant for a given type of the gas, $U_c$ is the cathode voltage drop, $\varepsilon_0$ is the dielectric constant. The field electron emission current density from the metal cathode substrate into the film is defined by the Fowler-Nordheim formula [16, 17]

$$j_{FN}(x) = j_F(E_0(x)/E_p)^2 \exp\left(-E_p/E_F(x)\right),$$

in which $j_F$ and $E_p$ are parameters dependent on the substrate and film materials.

3. Ion current at the cathode

Solution of the boundary value problem (2), (3) under condition (1) can be presented as the sum of unperturbed values and their perturbations caused by the cathode surface non-uniformity:

$$n_e(x, z) = n_{e0}(z) + n_{e1}(z) \cos kx, \quad n_i(x, z) = n_{i0}(z) + n_{i1}(z) \cos kx,$$
$$j_e(x, z) = j_{e0}(z) + j_{e1}(z) \cos kx, \quad j_i(x, z) = j_{i0}(z) + j_{i1}(z) \cos kx,$$

$$\varphi(x, z) = \varphi_0(z) + \varphi_1(z) \cos kx, \quad \psi(x, z) = \psi_0(z) + \psi_1(z) \cos kx,$$

where

$$n_{e1} \ll n_{e0}, \quad n_{i1} \ll n_{i0}, \quad j_{i1} \ll j_{i0}, \quad E_{i1} \ll E_{i0}, \quad \varphi_1 \ll \varphi_0, \quad \psi_1 \ll \psi_0.$$

Substitution of expressions (4) in (2), (3) results in two systems of equations for the zero-order quantities and the first-order quantities with the corresponding boundary conditions. From the first system it follows [7]:

$$\varphi_0(z) = -(U_c - E_{i0}H_{i0})(z/d_c)^2, \quad E_{c0} = -\varphi'_0(d_c) = 2(U_c - E_{i0}H_{i0})/d_c., \quad j_{i0}(d_c) = k_i e_0 E_{i0}^{3/2}/e_d,$$

where $E_{i0}$ is determined by equation

$$(1 + \gamma_{eff}) j_{i0}(d_c) = j_F(E_0/E_p)^2 \exp\left(-E_p/E_{i0}\right).$$

Solving the second system, which is rather cumbersome and is therefore not presented here, with the asymptotic method [7,18] under condition $kd_c \gg 1$, following from (1), it can be found that

$$\varphi_1(z) = (E_{1i} H_{i0} + E_{i0}(h_b - h_s) + E_{c0} h_s) \exp(k(z - d_c)),
$$

$$j_{i1}(z) = -(k j_{i0}(d_c)/E_{c0})(E_{1i} H_{i0} + E_{i0}(h_b - h_s) + E_{c0} h_s) \exp(k(z - d_c)),$$

where

$$E_{1i} = -\left(E_{i0}(h_b - h_s) + E_{c0} h_s\right)/\left(H_{i0} + E_{c0}(2 + E_p/E_{i0})/kE_{i0}\right).$$

It is seen in expressions (7) that perturbations of the electric field strength and the ion current density, caused by the cathode surface non-uniformity, are concentrated in a thin near-cathode layer with the thickness of the order of $l_\nu \ll d_c$ and have negligible values outside of it, i.e. they do not influence the CS macroscopic characteristics noticeably.

It can be found from (2), (4) and (7) that the ion current density, bombarding the cathode, is described by expression

$$j_i(x, d_c) = j_{i0}(d_c) + j_{i1}(d_c) \cos kx,$$

where

$$j_{i1}(d_c) = -(k j_{i0}(d_c)/E_{c0})(1 - \chi)(E_{i0}(h_b - h_s) + E_{c0} h_s), \quad \chi = (1 + (2 + E_p/E_{i0})E_{c0}/kE_{i0}H_{i0})^{-1}.$$  

Therefore, the cathode is sputtered non-uniformly and numerical estimations show that for the oxide films with $H_{i0} \approx 10^{-8}$ m and $l_\nu \approx 10^{-3} - 10^{-4}$ m magnitude of the coefficient $\chi$ is much less than unity.
4. Time evolution of the cathode surface relief

Variation of the cathode surface $z$ - coordinate with time due to ion sputtering is described by expression [7]:

$$\frac{dz_s(x)}{dt} = \frac{M_a}{N}(RJ_i(x,d_c) - J_{sb}(x,d_c))$$

(9)

where $N$ and $M_a$ are the cathode material density and the mass of sputtered atom, $R$ is the cathode sputtering rate equal to its sputtering yield averaged over ion energies, $J_i(x,d_c) = j_i(x,d_c)/e$ and $J_{sb}(x,d_c) = \beta j_0(1 - kh_s \cos kx)$ are the flow densities of ions bombarding the cathode and atoms sputtered from the cathode, which return back due to scattering on background gas atoms [8,9,19], $\beta < 1$ is a fraction of sputtered atoms re-deposited on the cathode, $e$ is the ion charge.

Substitution of expressions for $J_i(x,d_c)$ and $J_{sb}(x,d_c)$ in (9) gives an equation for the cathode surface relief amplitude:

$$\frac{dh_s}{dt} = -\frac{1}{\tau_s} \left[ \frac{E_{io}}{E_{co}}(h_s - h_b) + (1-\beta)h_s \right],$$

(10)

where $\tau_s = Ne/M_aRk j_0(d_c)$ is the characteristic time of relief variation in the discharge.

It follows from (10) that in case of the metal cathode without an oxide film on the surface, i.e. at $h_s = h_b$, a decrease of the relief amplitude proceeds in the discharge due to focusing of the ion flow on the relief tops (because of $j_{i1}(d_c) < 0$). But when an insulating film of varying thickness exists on the cathode, then in the case of its lower thickness at the relief tops (at $h_s < h_b$) their more intensive sputtering, followed by a decrease of $h_s$, goes on, i.e. a reduction of the film thickness on the relief tops, accelerating with time, takes place as a result of an increase of the magnitude in square brackets in the right-hand side of equation (10). However, under a higher film thickness on the relief tops (at $h_s > h_b$), the ion focusing at the relief bottoms exists already at

$$(h_s - h_b)/h_s > (E_{co}/E_{io})(1-\beta)$$

(11)

because of $E_{io} > E_{co}$, which results in an $h_s$ increase and the film thickness decrease on the relief bottoms.

For instance, in case of the aluminum cathode with an oxide film in the argon normal glow discharge ($p = 1330$ Pa, $U_c = 100$ V, $d_c = 3 \times 10^{-4}$ m, $j_0(d_c) = 2 \times 10^2$ A/m$^2$, $\lambda = 5 \times 10^{-6}$ m, $E_{co} = 6.6 \times 10^5$ V/m, $E_{F} = 1.4 \times 10^{10}$ V/m, $E_{io} = 4 \times 10^8$ V/m, [1, 15]) under $H_{io} = 2 \times 10^{-8}$ m, $h_b = 2 \times 10^{-9}$ m, $l_s = 2 \times 10^{-5}$ m, $R=0.01$ condition (1) is satisfied, $\chi = 0.04$, $E_{co}/E_{io} = 1.6 \times 10^{-3}$ and $\tau_s = 6 \times 10^4$ s. As it follows from (11), ions are focused at the cathode sections with lower film thickness at $h_b/[h_s > 10^{-3}$, i.e. under a very small film thickness non-uniformity, due to the fact that the electric field in the film is three orders of magnitude greater than in the discharge volume.

5. Conclusions

Under the existence of a relief and a thin dielectric film on the cathode surface in glow discharge, ion focusing on its sections with the minimum film thickness, located at any parts of the surface relief, takes place, followed by an increase of the film thickness non-uniformity with time and formation of pores in the film. Therefore, in the presence of an oxide film on the cathode, its sputtering in glow discharge is determined mainly by the film thickness non-uniformity and not by the surface relief.
Acknowledgements
This work was performed in frameworks of the program “Organization of scientific researches” of the Russian Federation Ministry of Education and Science in the Bauman Moscow State Technical University.
Support from the Basic Research Program of the National Research University Higher School of Economics is gratefully acknowledged.

References
[1] Raizer Y P 1991 *Gas discharge physics* (Berlin: Springer)
[2] Lister G G, Lawler J E, Lapatovich W P and Godyak VA 2004 *Rev. Mod. Phys.* **76** 541
[3] Samukawa S. et al 2012 *J. Phys. D: Appl. Phys.* **45** 253001
[4] Boeuf J P 2003 *J. Phys. D: Appl. Phys.* **36** R53
[5] Bondarenko G G and Korzhavyi A P 2007 *Russ. Phys. J.* **50** 125
[6] Kobayashi K and Shimuzu K 1988 *J. Electrochem. Soc.* **135** 908
[7] Kristya V I 2010 *Glow Discharges and Tokamaks* ed S A Murphy (NY: Nova Science Publishers) chapter 7 pp 329-368
[8] Bondarenko G G and Kristya V I 2008 *J. Phys.: Conf. Series* **100** 062009
[9] Kristya V I 2008 *J. Surf. Investig.* **2** 203
[10] Bondarenko G G, Kristya V I and Tun J N 2016 *Russ. Phys. J.* **58** 1313
[11] Moon K S, Lee J and Whang K-W 1999 *J. Appl. Phys.* **86** 4049
[12] Stamenković S N, Marković V Lj, Gocić S R and Jovanović A P 2013 *Vacuum* **89** 62
[13] Riccardi P, Ishimoto M, Barone P and Baragiola R A 2004 *Surf. Sci.* **571** L305
[14] Kristya V I and Tun J N 2014 *Bull. Russ. Acad. Sci. Phys.* **78** 549
[15] Kanter H and Feibelman W A 1962 *J. Appl. Phys.* **33** 3580
[16] Forbes R G 1999 *J. Vac. Sci. Tech. B* **17** 534
[17] Hickmott T W 2000 *J. Appl. Phys.* **87** 7903
[18] Nayfeh A H 1981 *Introduction to Perturbation Techniques* (NY: Wiley-Interscience)
[19] Valles-Abarca J A and Gras-Marti A 1984 *J. Appl. Phys.* **55** 1370