Theory of Heavy Quark Energy Loss

Pol Gossiaux, Jörg Aichelin and Thierry Gousset

SUBATECH, Université de Nantes, EMN, IN2P3/CNRS,
4 rue Alfred Kastler, 44307 Nantes cedex 3, France

We briefly review some of the models and theoretical schemes established to describe heavy quark quenching in ultrarelativistic heavy ions collisions. Some lessons are derived from RHIC and early LHC data, especially as for the contraints they impose on those models.

§1. Introduction

Jet-quenching of light hadrons is one of the key features observed in ultrarelativistic heavy ions collisions (URHIC) performed at RHIC ($\sqrt{s} = 200$ AGeV). It has been recently confirmed in $\sqrt{s} = 2.76$ AGeV lead-lead collisions at the LHC (see 1 and references therein). This observation is usually interpreted as the consequence of the energy loss which affects the leading parton during its passage through a hot deconfined medium and is therefore crucial for scrutinizing its properties. In recent years, several theoretical schemes, based on the eikonal approximation, have been developed in order to describe this energy loss.²⁻⁵ Numerical evaluations of these approaches turned out to be able to reproduce the rather flat momentum dependence of the nuclear modification factor ($R_{AA}$) observed at RHIC for pions at large $p_T$. The driving parameters needed to obtain quantitative agreement.⁶ however, overshoot pQCD predictions, sometimes by a factor 10.⁷ This might be taken as an indication that the fundamental theory is not able yet to describe the experimental results without introducing ad hoc parameters.

This is one of the motivations for addressing on the same footing the quenching of jets consisting of leading heavy quarks (HQ), as they might help to better constrain the models. In this respect, the guiding concept is the so-called mass-hierarchy $\Delta E(g) > \Delta E(q) > \Delta E(c) > \Delta E(b)$. The first inequality stems from the respective $SU(3)$ Casimirs of the gluons and quarks. The second is generally attributed to the dead cone effect⁸ in the context of radiative energy loss but is present whenever the higher mass of the parton implies a reduction of the formation time and hence of the radiated field. It is also found in collisional energy loss.⁹ Before the advent of the HQ data at RHIC, it was even advocated¹⁰ that HQ jets might be unquenched, but early $R_{AA}$ and elliptic flow ($v_2$) data of non-photonic single-electrons (NPSE) revealed that also HQ were strongly quenched in those collisions, nearly as much as light ones.¹¹ Since then, some schemes designed for light quarks have been extended to heavy quarks¹²⁻¹⁴ with the main conclusion that a common quantitative agreement between $R_{AA}(\pi)$ and $R_{AA}(\text{NPSE})$ can only be achieved if those NPSE

¹) This large discrepancy should be taken with a grain of salt, as the optimal parameter vastly depends on the way the underlying medium is described in the model (see 7), for a recent discussion, as well as the contribution of S. Bass to these proceedings).
stem exclusively from $c$-quarks. This is, however, incompatible with recent STAR measurements in $p - p$ and FONLL calculations which indicate a significant component of $b$-quarks for $p_T > p_{T,cross} \approx 5$ GeV/c. Faced with this puzzle of lack of coupling of HQ with the QGP, several dedicated models have flourished in the literature. In this contribution, we will review them shortly. Then we will provide a tentative explanation why several models with various physical inputs are able to cope with data. Finally, we address the benefits of LHC and conclude.

§2. Models at RHIC: fragility and robustness

2.1. The two faces of heavy quark energy loss

The first approach of HQ propagation in a hot medium was based on the Fokker-Planck equation with drag and diffusion coefficients evaluated from collisional energy loss only. This type of approach has survived up to now, although the physical content of the basic interaction has been extended by several authors (presented by increasing order of “strong coupling” content): A. Peshier underlined the role of genuine running $\alpha_s$ in collisional energy loss. Calculations implementing such running feature indeed find a smaller discrepancy with the $R_{AA}(NPSE)$, sometimes at the price of an additional cranking factor of the order of $2 - 4$. Van Hees and Rapp were the first who have investigated the effect of possible heavy-light quark bound states in the s-channel. Later, they have proposed a $T$-matrix description of HQ diffusion based on a bona-fide generalization of the static $Q - \bar{Q}$ potential evaluated through lattice calculations. In both cases they obtain a good agreement with $R_{AA}$ as well as with $v_2$ data. Akamatsu et al. have implemented the Langevin evolution of HQ in a hydrodynamical medium resorting to drag and diffusion coefficient evaluated in the strong coupling limit through AdS/CFT correspondence. Strictly speaking, all these FP/Langevin treatments applies for $p_T \ll m$. In this regime, HQ indeed behave as heavy particles surrounded by light degrees of freedom and their thermalization time $\propto m$ differ significantly from that of light quarks.

For ultra-relativistic HQ ($p_T \gg m$), the radiative energy loss becomes the dominant mechanism. The results of most of the eikonal schemes mentioned above is the probability of radiation which presents significant fluctuations. In principle, this feature invalidates the FP treatment of HQ propagation at high energy. In this regime, the mass of the quark acts mostly as a collinear regulator. For the most energetic case ($E \gg m$), in-medium formation time of the high energy gluons ($\sqrt{\omega/\hat{q}}$) exceeds the path length $L$. Then this scale regulates the radiation spectrum. As a consequence the average energy loss $\Delta E \propto L^2$ and the mass merely appears (if at all) through a logarithmic factor. It should be noted that the conversion of heavy quarks into heavy mesons differ as well in these two regimes: for $p_T \ll m$, coalescence dominates because the probability to pick up a light quark from the medium at small relative velocity is large. For $p_T \gg m$, this probability is close to 0 and fragmentation becomes the dominant mechanism, with, however non-trivial consequences on the $R_{AA}$ due to the presence of possible in-medium bound states.$^1$
2.2. Model fragility

Although the approaches presented above differ vastly w.r.t. their physical assumptions they are all able to cope with the NPSE $R_{AA}$ data measured by RHIC experiments, at the price of a rescaling of the coupling parameter. This questions our ability to achieve robust conclusions w.r.t. the basic mechanism at hand and hence the QGP properties. In 30) we have implemented collisional as well as radiative energy loss in the same numerical framework (MC$\alpha_s$HQ) and confirmed this “model fragility” (see Fig. 1). A finer analysis reveals that the $R_{AA}$ observable is mostly sensitive to the energy loss spectrum $P(\omega)$ at low values of the energy loss $\omega$. As pointed out by Baier et al.,\textsuperscript{31) this is due to the rather stiff initial $p_T$-distribution which makes the sequence of many small losses more probable than a single process involving large energy loss. This explains why both spectra (see 32) for an illustration) lead to similar quenching although the average $\Delta E_{\text{rad}} > \Delta E_{\text{col}}$.

2.3. “Robust” contact with lattice calculations

According to our analysis, all HQ observables at RHIC can be explained with a simple $\Delta E \propto L$ law, an observation in favor of local processes. In Fig. 2, we show the drag coefficient $A$ from the different microscopic energy-loss models implemented in our MC$\alpha_s$HQ framework. Although these models vary largely, their values for $A$ nicely approach a rather unique value for $p_T \gtrsim 10$ GeV/c once the coupling parameter (here the interaction rate) is rescaled in order to match RHIC NPSE data. From this we extract “robust” values of the relaxation coefficient $\eta = \lim_{p \to 0} \frac{A(p)}{p}$ of $0.45 \pm 0.15$ c/fm at $T = 200$ MeV and $0.75 \pm 0.25$ c/fm at $T = 300$ MeV. This yields a spatial diffusion coefficient $D_s$ of $2\pi T D_s = 1.9 \pm 0.5$ at $T = 200$ MeV and of $2\pi T D_s = 2.55 \pm 0.65$ at $T = 300$ MeV, for both $c$ and $b$ quarks, in quite good agreement with recent lattice calculations.\textsuperscript{33) This first successful contact with lattice should encourage us to seek for alternative observables able to discriminate between various models for $p_T \gtrsim 10$ GeV/c.
2.4. The role of the elliptic flow

In principle, the $v_2$ observable helps in constraining the models. At low and intermediate $p_T$, it reflects the collectivity acquired by heavy quarks and develops constantly with time up to the end of the transition.\cite{34} It is thus more sensitive to the QGP evolution than the $R_{AA}$ which saturates earlier. Nevertheless, it has been shown recently\cite{36} that two energy loss models with drag factor differing by a factor as large as two were both able to reproduce the experimental $R_{AA}$ and $v_2$ once they are imbedded in different QGP evolution models chosen by the respective authors.\cite{22,24} This clearly points towards the need of performing joint analysis of bulk QGP properties and HQ observables to achieve significant progress, as initiated in \cite{37} for the case of light hadrons. At large $p_T$, one expects some $v_2$ as well, due to the path length difference along both principal directions of the QGP, understood as a source of quenching.\cite{35} Although this observable could be useful to assess the path length dependence of HQ quenching (see \cite{13} for prediction within the ASW model), present $p_T$ range available at RHIC seems to offer little discriminating power for this purpose.

§3. Benefits from early LHC

URHIC performed at LHC offer larger possibilities to discriminate between various models and theoretical schemes, due to wider $p_T$ range and due to the possibility to measure $D$ and $B$ mesons observables separately from the very first runs on. Theoretical predictions\cite{20,38} for the $R_{AA}$ of $D$ mesons range between 0.2 and 0.4 for $p_T \gtrsim 10$ GeV/$c$. First ALICE results on $D$ mesons in Pb-Pb collisions\cite{39} confirmed the HQ quenching found at RHIC, with $R_{AA} \approx 0.3 \pm 0.15$ for $p_T \in [5 \text{ GeV}/c, 10 \text{ GeV}/c]$.

In Fig. 3, we provide the comparison between data and calculations performed with the same elementary reaction cross section as for the RHIC energy. We only adapted the plasma expansion (simulated with the hydrodynamical code of Kolb-Heinz\cite{40}) to obtain $\frac{dN_{ch}}{dy} = 1600$ at the chemical freeze out and the initial $p_T$-distribution (taken according to FONLL 1.3.2), which is harder than at the RHIC

![Fig. 3](https://example.com/fig3.png)  

Fig. 3. (color online) Quenching of $D$ mesons in PbPb collisions at 2.76 TeV: preliminary ALICE data\cite{39} vs collisional + radiative energy loss implemented in MC$\alpha_s$HQ.
Fig. 4. (color online) Left: quenching of $B$ mesons in PbPb collisions at 2.76 TeV (identified to the quenching of non-prompt $J/\psi$ measured by CMS\cite{41}) (preliminary data) vs various model predictions. Right: $R_{CB}$ predictions vs experimental ratio extracted from ALICE data on $D$ mesons and CMS data on non-prompt $J/\psi$ in central PbPb collisions (black disk); caution: the prediction of Greco et al. was made for minimum bias collisions. 

energy\textsuperscript{*}) A satisfactory agreement is obtained although the $p_T$-range of the present data is not sufficient to disambiguate between collisional and radiative energy loss. Apart from the $p_T$ dependence of $R_{AA}$ and $v_2$, the various models exhibit a rather rich mass-dependence which can be probed by addressing simultaneously the light vs heavy or $c$ vs $b$ observables. The quenching of $B$ mesons has not been measured directly yet but can be bona fide assimilated to the quenching of non-prompt $J/\psi$ measured by CMS;\textsuperscript{41} this is legitimate due to the flat shape of $R_{AA}$ at high $p_T$. In Fig. 4, we show the comparison between these data and some collected predictions.\textsuperscript{29,38} Most of the predictions show a lack of quenching as compared to the data. If confirmed, this would indicate that history might repeat itself (large quenching of $c$ quark unpredicted and observed at RHIC; large quenching of $b$ quark unpredicted and observed at LHC) and would question our present “understanding” of HQ propagation in hot media. In Fig. 4 (right), we present various predictions for the $R_{CB}$ ratio (defined as $R_{AA}(D)/R_{AA}(B)$) as a function of $p_T$, together with the experimental point obtained by $R_{AA}(B) = R_{AA}(\text{non-prompt } J/\psi)$ measured by CMS. Although the rather large experimental errors prevent us presently from any firm conclusion, models incorporating rather large mass dependence like AdS/CFT (drag coefficient $\propto m^{-1}$) seem to be disfavored by the data, although the analog of the detailed in-medium evolution of\textsuperscript{26}) is lacking at LHC. 

§4. Conclusions and prospects

We have argued that the limited dynamical range of observables at RHIC hinders the discrimination between the various models proposed for HQ evolution in hot media. We have shown that it was nevertheless possible to extract some basic properties of the QGP-HQ interaction — like the drag coefficient at low momentum

\textsuperscript{*} This explains why the medium appears as less opaque to HQ propagation although it is denser.
— in a rather “robust” way, once those models are rescaled in order to match $R_{AA}$ and $v_2$ data. These values are in good agreement with present lattice calculations. We have discussed the recent LHC results pertaining to $D$ mesons and non-prompt $J/\psi$ from $B$ meson decay. In particular, we found that most of the predictions seem to lack quenching for the $b$-quark. Whether it is just a question of fine-tuning of the present models or the sign of a more fundamental misunderstanding of the physics requires refined data and access to the full $p_T$ range of observables like $R_{CB}$. They will come in the next few years. Finding $R_{AA} (q) \approx R_{AA} (c) \approx R_{AA} (b)$ at mid $p_T$ could be the sign that the gluon formation-time which is found to be an increasing function of the radiator’s inverse mass\(^4\)) could be bounded from above by another scale of the problem as for instance the gluon absorption length in the medium.\(^4\))

Acknowledgements

This work was performed under the ANR research program “hadrons @ LHC” (grant ANR-08-BLAN-0093-02) and the PCRD7/I3-HP program TORIC.

References

1) ALICE Collaboration, Phys. Lett. B 696 (2011), 30.
2) R. Baier, Y. L. Dokshitzer, S. Peigné and D. Schiff, Phys. Lett. B 345 (1995), 277.
   R. Baier, D. Schiff and B. G. Zakharov, Annu. Rev. Nucl. Part. Sci. 50 (2000), 37.
3) M. Gyulassy, P. Levai and I. Vitev, Phys. Rev. Lett. 85 (2000), 5535; Nucl. Phys. B 594 (2001), 371.
4) U. A. Wiedemann, Nucl. Phys. B 588 (2000), 303.
5) P. Arnold, G. D. Moore and L. G. Yaffe, J. High Energy Phys. 12 (2001), 009; J. High Energy Phys. 11 (2001), 057; J. High Energy Phys. 06 (2002), 030.
6) PHENIX Collaboration, Phys. Rev. C 77 (2008), 064907.
7) D. d’Enterria, Landolt-Boernstein Vol. 1-23A (Springer Verlag, 2010), 49 pages.
8) Yu. L. Dokshitzer and D. E. Kharzeev Phys. Lett. B 519 (2001), 199.
9) E. Braaten and M. H. Thoma, Phys. Rev. D 44 (1991), 2625.
10) M. Djordjevic and M. Gyulassy, Phys. Lett. B 560 (2003), 37.
11) PHENIX Collaboration, Phys. Rev. C 84 (2011), 044905.
    STAR Collaboration, Phys. Rev. Lett. 98 (2007), 192301 [Errata; 106 (2012), 159902].
12) N. Armesto, C. A. Salgado and U. A. Wiedemann, Phys. Rev. D 69 (2004), 114003.
13) M. Djordjevic and M. Gyulassy, Nucl. Phys. A 774 (2006), 589.
14) M. Djordjevic and M. Gyulassy, Nucl. Phys. A 733 (2004), 265.
   S. Wicks, W. Horowitz, M. Djordjevic and M. Gyulassy, Nucl. Phys. A 784 (2007), 426.
15) STAR Collaboration, Phys. Rev. Lett. 105 (2010), 202301.
16) M. Cacciari, P. Nason and R. Vogt, Phys. Rev. Lett. 95 (2005), 122001.
17) R. Rapp and H. van Hees, in Quark-Gluon Plasma 4 (World Scientific, 2010), arXiv:0903.1096.
18) B. Svetitsky, Phys. Rev. D 37 (1988), 2484.
   M. G. Mustafa, D. Pal and D. K. Srivastava, Phys. Rev. C 57 (1998), 889.
   P. B. Gossiaux, V. Guiho and J. Aichelin, J. of Phys. G 31 (2005), S1079.
   G. D. Moore and D. Teaney, Phys. Rev. C 71 (2005), 064904.
19) W. M. Alberico et al., Eur. Phys. J. C 71 (2011), 1666.
   S. Cao and S. A. Bass, arXiv:1108.5101.
20) A. Peshier, Phys. Rev. Lett. 97 (2006), 212301.
21) A. Peshier, arXiv:0801.0595.
22) P. B. Gossiaux and J. Aichelin, Phys. Rev. C 78 (2008), 014904.
23) J. Uphoff, O. Fochler, Z. Xu and C. Greiner, Phys. Rev. C 84 (2011), 024908.
24) H. van Hees and R. Rapp, Phys. Rev. C 71 (2005), 034907.
25) H. van Hees, M. Mannarelli, V. Greco and R. Rapp, Phys. Rev. Lett. 100 (2008), 192301.
26) Y. Akamatsu, T. Hatsuda and T. Hirano, Phys. Rev. C 79 (2009), 054907.
27) R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigné and D. Schiff, Nucl. Phys. B 483 (1997), 291.
28) S. Peigné and A. Smilga, Phys. Usp. 52 (2009), 659; arXiv:0810.5702.
29) R. Sharma, I. Vitev and B.-W. Zhang, Phys. Rev. C 80 (2009), 054902.
30) P. B. Gossiaux, J. Aichelin, T. Gousset and V. Guiho, J. of Phys. G 37 (2010), 094019.
31) R. Baier, Y. L. Dokshitzer, A. H. Mueller and D. Schiff, J. High Energy Phys. 09 (2001), 033.
32) S. Vogel, P. B. Gossiaux, K. Werner and J. Aichelin, Phys. Rev. Lett. 107 (2011), 032302; arXiv:1012.0764.
33) H.-T. Ding et al., arXiv:1107.0311.
34) R. Rapp and H. van Hees, arXiv:0803.0901.
35) S. A. Voloshin and A. M. Poskanzer, Phys. Lett. B 474 (2000), 27.
36) P. B. Gossiaux, S. Vogel, H. van Hees, J. Aichelin, R. Rapp, M. He and M. Bluhm, arXiv:1102.1114.
37) T. Renk, H. Holopainen, R. Paatelainen and K. J. Eskola, Phys. Rev. C 84 (2011), 014906.
38) S. Abreu et al., J. of Phys. G 35 (2008), 054001.
39) P. B. Gossiaux, R. Bierkanndt and J. Aichelin, Phys. Rev. C 79 (2009), 044906.
40) A. Dainese, arXiv:1106.4042.
41) P. F. Kolb, J. Sollfrank and U. Heinz, Phys. Rev. C 62 (2000), 054909.
42) P. F. Kolb and U. Heinz, in *Quark-Gluon Plasma 3* (World Scientific, Singapore, 2004), nucl-th/0305084.
43) T. Dahms, arXiv:1107.0252.
44) P. Arnold, Phys. Rev. D 79 (2009), 065025.
45) M. Bluhm, P. B. Gossiaux and J. Aichelin, arXiv:1106.2856.
46) M. Bluhm, T. Gousset, P. B. Gossiaux and J. Aichelin, in preparation.