Abstract

In this article we consider some properties of the (0,2) theory using AdS/CFT correspondence. We also consider the "string baryonic state" of the theory. We will show that stable baryon states for $k$ string exist provided $\frac{2}{3} N \leq k \leq N$. The (0,2) theory in the finite temperature is also considered using Schwarzshild geometry, giving information about the five dimensional theory obtained at higher temperature in this background. One can also see that there are two descriptions of a four dimensional gauge theory which become equivalent at strong coupling and approach this five dimensional theory.

1e-mail: alishah@physics.ipm.ac.ir
It has been conjectured\[1\] that four dimensional $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with gauge group $SU(N)$ is equivalent to Type II B superstring theory on $AdS_5 \times S^5$ with $N$ units of five form flux on the five sphere. The effective gauge coupling, $g_{\text{eff}}^2 = g_{YM}^2 N$ where $g_{YM}^2$ is equal to $g_s$, is proportional to the radius of AdS and $S^5$. In the large $N$ limit and large $g_{\text{eff}}$, the string theory can be approximated by supergravity, so one would expect to extract, for example, gauge theory correlation functions, the set of chiral operator, and mass spectrum of the strongly coupled gauge theory. The precise relation between the gauge theory correlation functions and supergravity effective action has been given in[2].

It was also proposed that the SCFT of $N$ M5-branes is dual to M-theory on $AdS_7 \times S^4$ while $N$ M2-branes is dual to M-theory on $AdS_4 \times S^7$. In[3] other field theories with sixteen supercharges, including $U(N)$ Yang-Mills theories in various dimensions, were considered, and it was argued that their large N limits are related to certain supergravity by the Schwarzshild geometry describing a black hole. When the curvature of the space-time is small compared to the string scale (the Plank scale) superstring theory (M-theory) can be approximated by corresponding supergravity. In fact in this background one can study pure QCD$_3$ or QCD$_4$ from supergravity\[4\].

Using the AdS/CFT correspondence, the Wilson loop calculation in the SUSY cases\[5\] was generalized to the non-supersymmetric cases[6]. The 't Hooft vortices and quark-monopole potentials were also considered in\[7\]. The same issue has been studied in\[8\].

It was shown that the supergravity description gives results that are in qualitative agreement with our expectation for QCD$_3$ and QCD$_4$ at strong coupling, including the area law behaviour of Wilson loops, the relation between confinement and monopole condensation, the existence of a mass gap for glueball state and also its mass, the behaviour of Wilson loops for higher representation and construction of heavy quark baryonic states\[9\],[10] (see also\[6\],[7],[8]).

Although there are some difficulties reported in this direction\[11\], one expects that they will be solved if world sheet quantum fluctuations are also considered. There may exist some sort of phase transition (similar to the transition in lattice gauge theory) between the strong coupling phase and a weak coupling phase\[12\]. Evidence for this transition is also considered in[12].

Our aim in this article is to further study this duality for $AdS_7 \times S^4$. We hope that this correspondence will help us to learn about the (0,2) theory which is the decoupled theory on $N$ parallel M5-branes and is dual to M-theory in the above background\[13\]. In[13] (see also[14]), it was proposed that one can study this theory using the Wilson surface and surface equation. Although there is not a well-known Wilson surface for this theory, the correspondence gives us an interesting representation for Wilson surface of the theory\[15\]. In this article using M-theory we be able also to obtain some information about what we call the "string baryonic
states” of (0,2) theory. The baryonic state has been introduced in [13], [9]. In fact
a baryon is a finite energy configuration of N external quarks (or in our theory N
external strings).

Consider M-theory on \( \text{AdS}_7 \times S^4 \). The metric is

\[
ds^2 = l_p^2 \left( \frac{V}{R} dx_\|^2 + \frac{R^2}{V^2} dV^2 + R^2 d\Omega_4^2 \right),
\]

where \( V = \frac{r}{l_p} \) and \( R^3 = \pi N \). This theory is dual to the (0,2) theory living in the N
M5-branes in the decoupled limit. Wilson surface operator is defined by requiring
the membrane end at the boundary of \( \text{AdS} \) [5], so we have (in large N limit)

\[
< W > = e^{-S_{M2}}
\]

where \( S_{M2} \) is the action of M2-brane. Consider a pair of parallel infinite strings
corresponding to the membranes ending on the M5-brane. Assume that they are in
opposite orientations but in the same direction on \( S^4 \). So the action which should
be minimized is

\[
S_{M2} = \frac{TL'}{(2\pi)^2} \int dx \sqrt{(\partial V)^2 + V^3/R^3}
\]

here the strings have length \( L' \) and are separated by a distance \( L \) in the direction \( x \).
The energy \( E \) and the distance \( L \) are

\[
L = 4\sqrt{\pi} \frac{R^{3/2} \Gamma(\frac{2}{3})^3}{V_{1/2} \Gamma(\frac{1}{3})^3},
\]

\[
E/L' = -8\sqrt{\pi} N \frac{\Gamma(\frac{2}{3})^3}{L^2 \Gamma(\frac{1}{3})^3}
\]

By this correspondence one can also calculate correlation function of the Wilson
surface \( < W(S_1)W(S_2) > \). This will give us the spectrum of the theory. It has
been argued [13] that the form of this correlator should be \( \frac{1}{N} f\left(\frac{1-2}{L} \right) \). Whether we
have a continuous spectrum is encoded in the behaviour of this function. In fact
a continuous spectrum could be obtained if the function \( f(x) \) contains a piece like
\( f(x) \sim \cdots + e^{-cx} \). Using AdS/CFT duality we can find another way to obtain
the spectrum. The procedure is very similar to what has been done in [3], [4], where
the mass gap has been obtained for \( QCD_{3,4} \). According to [4] a correlation function
of local operators in the theory at the boundary is obtained by computing the
Green functions of the corresponding supergravity on the bulk. We can expand
the supergravitons in Fourier modes, each Fourier mode corresponding to a particle
pole of the correlation function of the theory with mass equal to the momentum
of the particle \( k \) [3]. We could in principal find whether \( k^2 \) is continuous or not.
The duality can also lead us to consider the origin of the surface equation [13] from
supergravity side.
Now consider the baryon configuration as proposed in [13], [9]. The energy and the baryon size and its stability for QCD3 have been studied in [16]. In our model the string baryonic vertex comes from the M5-brane wrapped over $S^4$. As mentioned in [13] there are two contributions to the action of the system of the same order. One from the membranes stretched between the boundary of AdS and the wrapped M5-brane. The second comes from the wrapped M5-brane itself. Since we are considering a static configuration, the action of the wrapped M5-brane is

$$S_{M5} = \frac{1}{(2\pi)^5 l_p^6} \int dx^6 \sqrt{h} = \frac{TL'V_0N}{12\pi^2}$$

(5)

where $V_0$ is the location of the string baryon vertex and $h$ is the induced metric on the M5-brane. The action for membrane is given by (3). So the total action is

$$S_t = S_{M5} + NS_{M2}$$

(6)

Under $V \rightarrow V + \delta V$, this action leads to the following surface term

$$\frac{\partial V_0}{\sqrt{(\partial V_0)^2 + \frac{V_0'}{R^2}}} = \frac{1}{3}$$

(7)

and volume term

$$\frac{V^3}{\sqrt{(\partial V)^2 + \frac{V^3}{R^2}}} = C_0$$

(8)

Using (7) we find $C_0 = \sqrt{\frac{2}{9}} V_0^{3/2} R^{3/2}$. These equations lead us to the following radius for the string baryon

$$L = 2 \frac{R^{3/2}}{V_0^{1/2}} \int_1^\infty \frac{dy}{y^{3/2} \sqrt{\beta^2 y^3 - 1}}$$

(9)

where $\beta = \frac{\sqrt{8}}{9}$. To find the energy of the string baryon, we first calculate the energy of a single membrane. Note that we should regularize the energy, which means that one should subtract the energy of a configuration with M5-brane located at $V_0 = 0$. Since at $V_0 = 0$ the energy of the wrapped M5-brane is zero, so

$$\frac{E_{Single}}{L'} = \frac{V_0}{(2\pi)^2} [\int_1^\infty dy (\frac{\beta y^{3/2}}{\sqrt{\beta^2 y^3 - 1}} - 1) - 1]$$

(10)

From this result and (3), the total energy of the baryon is

$$\frac{E_t}{L} = -\alpha N \frac{R^3}{L^2} = -\alpha N \frac{\pi N}{L^2}$$

(11)

here $\alpha$ is a numerical constant.
One can also have string baryon with the number of string less than $N$. Following [16], calculating the total force which the wrapped M5-brane can feel, it is easy to see that for $\frac{2}{3}N \leq k \leq N$ one can also have stable baryonic configurations.

Upon compactification from M-theory to Type II A string theory we will find two kinds of descriptions of a theory which become equivalent at strong coupling. The (0,2) theory compactified on a circle becomes at low energy 4+1 dimensional SYM theory living in the N D4-branes of Type II A. For large N limit we have a supergravity description in the region $N^{-1} \ll g^2_{YM} V \ll N^{1/2}$ which is Type II A supergravity on the following background [3]

$$ds^2 = l_s^2 \left( \frac{V^{3/2}}{R^{3/2}} dx^2 + \frac{R^{3/2}}{V^{3/2}} f(V) d\tau^2 + R^{3/2} V^{1/2} d\Omega_4^2 \right),$$

(12)

where $R^3 = g_s \pi N$ and $e^\Phi = g_s V^{3/4}$. In the region $N^{1/2} \ll g^2_{YM} V$ it becomes M-theory on $AdS_7 \times S^4$ with identifications.

The first description (electric description) occurs when we consider strings ending on the boundary and consider the Wilson loop from this string action. The baryonic vertex comes from a D4-brane wrapped over $S^4$. The energy and the size of Wilson loops and baryons can be obtained from M-theory by setting $L' = 2\pi R_{10} \sim g^2_{YM}$ ($R_{10}$ is the radius of the compacted direction) in the equations (4), (11). The second description (magnetic description) occurs when we consider D2-branes ending on the boundary. Now we have Wilson loops for monopoles and thereby the energy will be the potential between monopole and anti-monopole [7], [9]. In this case the magnetic baryon vertex (as one can see from compactification of M-theory) comes from a NS5-brane wrapped over $S^4$. Again one can obtain the energy and L as before. The results are the same as in the electric description divided by $g_s$, as expected. Now when we go to the strong coupling limit (M-theory) both of them flow to the (0,2) theory and we know that the later theory is self dual.

Motivated by the above results in six dimensions we will make an analysis in four dimensional QCD$_D$. In this case as proposed in [4] we must consider the following background

$$ds^2 = l_s^2 \left( \frac{V^{3/2}}{R^{3/2}} (f(V) d\tau^2 + dx_i^2) + \frac{R^{3/2}}{V^{3/2}} f(V)^{-1} dV^2 + R^{3/2} V^{1/2} d\Omega_4^2 \right),$$

(13)

where $f(V) = 1 - \frac{V^3}{V_T^3}$, $V_T = \pi g_s N T^2 = R^3 T^2$. By the same calculation as before we find

$$L = 2\frac{R^{3/2}}{V_0^{1/2}} \int_1^\infty dy \sqrt{\frac{8 - 10 \rho^2 - \rho^6/4}{(y^3 - \rho^3)(9y^3 - 8 + \rho^3 + \rho^6/4)}}$$

(14)

and the total energy of the baryon is

$$E_t = \frac{NV_0}{2\pi} \int_1^\infty dy \sqrt{\frac{9y^3 - 9\rho^3}{9y^3 - 8 + \rho^3 + \rho^6/4} - 1} - 1 + \rho + \frac{1}{3} \sqrt{1 - \rho^3}$$

(15)
here $\rho = \frac{V_T}{V_0}$. To find the four dimensional theory we need to go to IR limit. To calculate the relation between $E_t$ and $L$, we must note that, in this region the main contribution of the integral comes from the contribution around $\rho = 1$, so we find

$$E_t = NT_{YM} L, \quad T_{YM} = \frac{1}{4\pi} R^3 T^3 = \frac{1}{4} g_{YM}^2 NT^2,$$

(16)

where $T_{YM}$ is the tension of the string.

As shown in [4] the metric (13) comes from Schwarzshild metric in $AdS_7 \times S^4$ compactified on a circle. On the other hand, we can find it by considering $N$ Type II A D4-branes wrapped on a circle and pick a spin structure on the circle that breaks supersymmetry. Now consider the Schwarzshild metric on $AdS_7 \times S^4$ and do not compacting it on a circle, so we find M-theory on the following background

$$ds^2 = l_p^2 \left( \frac{V}{R} (f(V) d\tau^2 + \sum_{i=1}^5 dx_i^2) + \frac{R^2}{V^2} f(V)^{-1} dV^2 + R^2 d\Omega_4^2 \right),$$

(17)

where $f(V) = 1 - \frac{V_T^2}{4V}$, $V_T = R^3 T^2$. Using this background we can study the (0,2) theory in the finite temperature and at higher temperature we can find a theory in five dimensions. Consider the membrane action and the baryonic vertex coming from M5-brane wrapped on $S^4$ in this background, one will find in the low temperature, a correction to the energy given in equations (4) and (11).

In the higher temperature we expect to obtain five dimensional theory. Performing similar calculations, we find that, this theory has the confinement phase, which means that $N$ strings living on the boundary can confine and construct the "string baryon" or, one string with one anti string in the Wilson surface can construct the "string meson". Comparing the theory of Type II A in the background given by (13) (pure gauge theory in four dimensions) with the theory given by M-theory on the background (17) one observes that the string confinement in five dimensional theory leads to quark or monopole confinement in the four dimensional gauge theory depending on which direction is to be compactified. In fact we can find two descriptions of a four dimensional gauge theory; electric or magnetic descriptions. These descriptions are equivalent at strong coupling and approach the five dimensional theory described above. Note that the baryon vertex for monopoles in four dimensions can be obtained by wrapping NS 5-brane on the $S^4$ in the metric (13). As in the previous case the energy of the magnetic description can be obtained by the energy in the electric description dividing by $g_s$.

I would like to thank F. Ardalan, J. H. Brodie, J. Greensite and M. H. Sarmadi for discussion.
References

[1] J. Maldacena, hep-th/9711200.

[2] E. Witten, hep-th/9802150.
  S. S. Gubser, I. R. Klebanov, A. M. Polyakov, hep-th/9802109.

[3] N. Itzhaki, J. Maldacena, J. Sonnenschein, S. Yankilowicz, hep-th/9802042.

[4] E. Witten, hep-th/9803131.

[5] S. J. Rey, J. Yee, hep-th/9803001.
  J. Maldacena, hep-th/9803002.

[6] A. Brandhuber, N. Itzhaki, J. Sonnenschein, S. Yankilowicz, hep-th/9803137;
  hep-th/9803263.
  S. J. Rey, S. Theisen, J. Yee, hep-th/9803135.

[7] M. Li, hep-th/9803252; hep-th/9804173.
  J. A. Minahan, hep-th/9803111.
  U. H. Danielsson, A. Polychronakos, hep-th/9804141.

[8] I. V. Volovich, hep-th/9803174.
  A. Volovich, hep-th/9803220.
  H. Dorn, H. J. Otto, hep-th/9807093.

[9] D. J. Gross, H. Ooguri, hep-th/9805129.

[10] C. Csaki, H. Ooguri, Y. Oz, J. Terning, hep-th/9806021.
  R. de Mello Koch, A. Jevicki, M. Mihaiescu, J. P. Nunes, hep-th/9806127.

[11] H. Ooguri, H. Robins, J. Tannenhauer, hep-th/9806171.
  J. Greensite, P. Olesen, hep-th/9806235.

[12] M. Li, hep-th/9807196.

[13] O. Ganor, Nucl. Phys. B489 (1997) 95, hep-th/9605201.

[14] N. Seiberg, hep-th/9705117.

[15] E. Witten, hep-th/9805112.

[16] A. Brandhuber, N. Itzhaki, J. Sonnenschein, S. Yankilowicz, hep-th/9806158.
  Y. Imamura, hep-th/9806162.