Jet Quenching in Heavy Ion Collisions from AdS/CFT

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Abstract. The phenomenon of jet suppression observed in highly energetic heavy ion collisions is discussed. The focus is devoted to the stunning applications of the AdS/CFT correspondence to describe these real time processes, hard to be illuminated by other means. In particular, the introduction of as many flavors as colors into the quark-gluon plasma is scrutinized.

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THE PHENOMENON OF JET QUENCHING

One of the most recent surprises in particle physics came from the results of the Relativistic Heavy Ion Collider (RHIC), indicating that hadronic matter at temperatures slightly above the crossover critical temperature, $T_c$, may be a strongly coupled quark-gluon plasma (sQGP). Naive expectations pointed towards a free gas of quarks and gluons or quasi-particles, a picture that can be understood from perturbative calculations of thermodynamic quantities such as the equation of state. These perturbative methods fail, however, at low temperatures, close to $T_c$, where lattice QCD simulations are employed instead. Although the underlying dynamical picture is difficult to infer from these numerical results, the findings at RHIC are in qualitative agreement with the $\sim 20\%$ departure from the ideal gas, Steffan-Boltzmann law, predicted from lattice (see e.g. [2] for a review). This departure is indeed very close to the exact 25% value found from AdS/CFT, which is rigorously valid at infinite coupling (see Igor Klebanov’s article in this volume [3]).

Several observable consequences of the creation of this new state of matter are measured at RHIC as probes to characterize its properties. Among them, we focus in the present article in the suppression of the inclusive cross-section at large transverse momentum, $p_t$, so-called jet quenching. In the absence of any nuclear effect, the cross section to produce a particle with high transverse momentum in heavy ion collisions should scale with the number of elementary interactions. However, a strong suppression is found experimentally, the resulting particle production being about 20% of the expected value. This dramatic departure from the naive expectation can be understood as being due to the energy loss of highly energetic partons traversing the medium created by the collision (see e.g. [4] for a recent review).

Another interesting observation in the high-$p_t$ region of the spectrum is a supression
in the observed back-to-back high-\( p_T \) jets in Au+Au vs. p+p collisions. High-\( p_T \) quarks or gluons are produced predominantly in pairs in elementary, hard collisions, flying in opposite directions in the transverse plane. Correlation of azimuthal angles among high-\( p_T \) particles produced in the same event is measured. In the absence of any final state effect, one is led to expect a peak at \( \phi = 0 \) for partners in the same jet as the trigger parton, and a recoil peak at \( \phi = \pi \). These back-to-back jets are indeed observed in p+p and d+Au collisions. In central Au+Au collisions, though, the recoil jet is greatly distorted (absent) indicating large final state effects on the produced hard quarks and gluons due to the resulting medium.

The observed deficit of high energy jets seems to be the result of a slowing down, damping or quenching of the most energetic partons as they propagate through the quark-gluon plasma (QGP). The rate of energy loss should be spectacular: several GeV per fm, more than one order of magnitude larger than in cold nuclear matter. The energy loss of a hard parton in quantum chromodynamics (QCD) can be parameterized through the transport coefficient known as \( \hat{q} \). If defined at weak coupling, it would measure the average square transverse momentum transferred to the hard parton per mean free path length. Fitting the current data it seems that, with a great degree of confidence, its measured value is generously within the range

\[
\hat{q}_{\text{exp}} = (15 \pm 10) \frac{\text{GeV}^2}{\text{fm}}. \tag{1}
\]

All estimates for this quantity in weak coupling computations lead to much lower values\(^1\) and the obvious question is whether a calculation at strong coupling could result in better agreement. Lattice techniques are not well suited to study this kind of phenomenon as it involves real-time dynamics which is fairly difficult to address in a Euclidean time formulation. Under these conditions, the use of AdS/CFT provides a powerful tool to study this type of physics, providing both valuable insights into the physical aspects of the underlying mechanism of energy loss, as well as on the applicability of these techniques to actual experimental situations.

The calculation of the jet quenching parameter is not the only example of this novel connection: attempts to compute quantities in AdS/CFT which could be of interest for the physics of heavy ion collisions are currently abundant. One of the greatest successes of this approach is the computation of the ratio of the shear viscosity to the entropy density \([6]\), \( \frac{\eta}{s} = \frac{\hbar}{4\pi} \), which agrees with current fits of hydrodynamical models to RHIC data on the elliptic flow (see \([3]\)). Other examples are the computation of thermodynamical quantities such as the free energy, the energy density, the heat capacity, the speed of sound, etc. Further calculations within the AdS/CFT framework aiming at providing a bridge towards experimental data include the drag force coefficient, the relaxation time, diffusion constants, thermal spectral functions, stability of heavy-quark bound states, and the hydrodynamical behavior of the collision.

\(^1\) Parametrically, at large \( N_c \), \( \hat{q} \sim \pi N_c^2 \alpha_s^2 T^3 \), with a prefactor of order one which depends on the assumptions in the calculation, this leading to \( \hat{q} \sim 1 \text{GeV}^2/\text{fm} \) (see e.g. \([5]\)).
A PARTON THROUGH THE QUARK-GLUON PLASMA

In order to study the jet quenching phenomenon, we must first provide an appropriate phenomenological description of the relevant physics. The original formulation of the induced emission by an extended medium goes back to the early fifties, when Landau and Pomeranchuk gave a framework in which to consider a charged particle moving through a classical electrodynamical environment [7], and generalized shortly after to the quantum case by Migdal [8]. These results were extended to the case of QCD by Gyulassy, Plumer and Wang [9], Zakharov [10] and by Baier, Dokshitzer, Mueller, Peigné and Schiff [11].

Let us briefly present the basics of medium induced gluon radiation for a highly energetic parton. A quark with energy $E$ emits a gluon with a fraction of momentum $x$ and transverse momentum with respect to the quark direction $k_\perp$. The interaction of this system is depicted by multiple scatterings with the medium which, in some models, can be considered as a collection of static scattering centers. In the eikonal approximation ($E \gg xE \gg k_\perp$), the particle trajectories can be written as Wilson lines in the light-cone coordinate. Using the approximation above, the quark is seen as traveling in a straight line while the gluon is allowed to move in transverse space by interaction with the medium. In the multiple soft scattering approximation, the transverse position of the gluon follows a Brownian motion, and the average transverse momentum after traveling a distance $L$ is characterized by the transport coefficient $\langle k_\perp^2 \rangle \approx \hat{q}L$. The origin of this parameter and the relation with the Wilson lines can be understood as follows: in order to compute the cross-section for gluon emission, a Wilson line ending at transverse position $x$ appears in the amplitude corresponding to the gluon propagation, together with another Wilson line for the gluon in the conjugate amplitude, at transverse position $y$. These Wilson lines then need to be averaged over all possible medium configurations, appearing only in combinations like [10, 12]

$$\frac{1}{N_c^2 - 1} \text{Tr} \langle W^A(x) W^A(y) \rangle \approx \exp \left[ -\frac{(x - y)^2}{4\sqrt{2}} \int dx_\perp \hat{q}(x_\perp) \right].$$  \hspace{1cm} (2)

This approximation is valid, up to logarithmic corrections, in the small distance limit $(x - y)^2 \ll 1/\Lambda_{QCD}^2$. The average $\langle \cdots \rangle$ and the corresponding medium properties are all encoded in a single jet quenching parameter, $\hat{q}$. When the medium does not vary along the light-cone trajectory of the gluon, and assuming the transverse component to be much smaller than the longitudinal one [13],

$$\langle W^A(x) \rangle \approx \exp \left[ -\frac{1}{4\sqrt{2}} \hat{q}L^- L^2 \right],$$  \hspace{1cm} (3)

for a rectangular Wilson loop with a large light-like side $L^-$ and a much smaller space-like separation $L$, $L \ll L^-$. Eq. (3) can be naturally extrapolated to the strong coupling regime and considered as a non-perturbative definition of the transport coefficient $\hat{q}$.

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2 This hypothesis does not seem appropriate in the experimental set up at RHIC. However, it has been shown in [14] that it is always possible to write eq (2), and interpret $\hat{q}$ as a properly weighted average measure of the time dependent transport coefficient.
THE JET QUENCHING PARAMETER IN ADS/CFT

The Wilson loop computation leading to the jet quenching parameter can be readily performed, at large \( N_c \), within the framework of the AdS/CFT correspondence. It amounts to the evaluation of the (regularized) Nambu–Goto action, \( S_{NG}(\mathcal{C}) \), for a string hanging from the curve \( \mathcal{C} \) at the boundary towards the bulk of AdS,

\[
\langle W^A(\mathcal{C}) \rangle \simeq \exp \left[ -2S_{NG}(\mathcal{C}) \right] + \mathcal{O} \left( 1/N_c \right). \tag{4}
\]

Let us succinctly cover the basics of this computation. The family of black brane metrics of interest for us have the following form [18]:

\[
ds^2 = -c_T^2 dt^2 + c_X^2 dx^i dx_i + c_R^2 dr^2 + G_{Mn} dX^M dX^n, \tag{5}
\]

where \( X^M = (t, x^i, r, X^n) \), \( i = 1, \ldots, p \), \( n = 1, \ldots, 8 - p \). We shall use light-cone coordinates \( x^- = \tau \in (0, L^-) \), \( x^2 = \sigma \in (-L/2, L/2) \), and \( r = r(\sigma) \). For a symmetric configuration around \( \sigma = 0 \), \( r'(0) = 0 \), the Nambu-Goto action takes the following form

\[
S_{NG}(\mathcal{C}) = \frac{L^{-}}{\sqrt{2\pi\alpha'}} \int_{0}^{L/2} d\sigma \left( c_X^2 - c_T^2 \right)^{1/2} \left( c_X^2 + c_R^2 r'(\sigma)^2 \right)^{1/2}, \tag{6}
\]

\( \alpha' \) being the inverse of the string tension. The energy is a first integral of motion, from which the profile \( r(\sigma) \) can be extracted by inverting

\[
\sigma(r) = \int_{r_H}^{r} \frac{d\sigma}{c_X \left( k c_X^2 (c_X^2 - c_T^2) - 1 \right)^{1/2}}. \tag{7}
\]

\( k \) is an integration constant fixed by the relation \( \sigma(\infty) = L/2 \). It is more convenient to deal with a dimensionless radial coordinate \( u = r/r_H \), where \( r_H = r(0) \) is the location of the black brane horizon, and perform the rescalings,

\[
\hat{c}_T^2 = \frac{c_T^2}{c_X^2}, \quad \hat{c}_X^2 = \frac{c_X^2}{c_T^2}, \quad \hat{c}_R^2 = \frac{c_R^2}{c_T^2} = \left( \frac{\alpha'}{\alpha} \right)^{5-p} \lambda. \tag{8}
\]

\( \lambda \) being the ’t Hooft coupling in the dual \((p+1)\)-dimensional gauge theory. \( L \) is inversely proportional to \( k \). Thus, we have to explore the limit \( k \to \infty \), and keep the leading term in \( L^{-} L^2 \) [13]. The action has to be regularized by substracting the Nambu-Goto action for a pair of Wilson lines that stretch straight from the boundary to the horizon. Therefore, the jet quenching parameter finally reads [17]:

\[
\hat{q} = \frac{1}{\pi \lambda} \left( \frac{r_H}{\alpha'} \right)^{6-p} \left( \int_{1}^{\infty} \frac{\hat{c}_R du}{\hat{c}_X \left( \hat{c}_X^2 - \hat{c}_T^2 \right)^{1/2}} \right)^{-1}. \tag{9}
\]

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3 This was first established by Rey and Yee [15] and, independently, by Maldacena [16]. The case of partially light-like Wilson loops was first presented by Liu, Rajagopal and Wiedemann [13], while the schematic computation of the present section was discussed in full detail in [17].
This remarkably compact formula is valid for a vast family of gauge/gravity duals, some of which will be presently explored. As it stands, it calls for a translation of gravity parameters in terms of the field theoretical quantities. This is provided by the (nowadays standard) AdS/CFT dictionary.

Jet Quenching Parameter Bestiary

In this subsection we would like to review some of the quantum field theories for whose plasmas the jet quenching parameter has been computed. In order to provide some numbers, we consider as a representative choice of average values, $\lambda = 6\pi$ (that is, $\alpha_s = 1/2$) and $T = 300$ MeV. This is, of course, a crude simplification for several reasons: the real $\hat{q}$ varies with time and, moreover, it is hard to provide both a reliable dependence of the coupling on the temperature, $\lambda(T)$, as well as to choose a representative average value for these quantities. Thus, numbers should be taken as indicative.

$\mathcal{N} = 4$ Super Yang–Mills Theory

The original computation in the AdS/CFT framework, performed by Liu, Rajagopal and Wiedemann [13] for $\mathcal{N} = 4$ super Yang–Mills theory, gives

$$\hat{q}_{\text{SYM}} = \frac{\pi^{3/2} \Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})} T^3 \sqrt{\lambda}.$$  \hspace{1cm} (10)

Notice that, contrary to what happens at weak coupling [5], it does not depend explicitly on the number of degrees of freedom. For the representative values, $\hat{q}_{\text{SYM}} = 4.48$ GeV$^2$/fm.

Witten’s QCD

The jet quenching parameter in Witten’s construction [19] of a holographic string dual of 4d SU($N_c$) Yang–Mills theory can be easily computed by applying eq. (9) to a D4–(black) brane background wrapping a Kaluza–Klein circle of radius $\ell$ with antiperiodic boundary conditions [17],

$$\hat{q}_{\text{WQCD}} = \frac{16\pi^{3/2} \Gamma(\frac{2}{3})}{81 \Gamma(\frac{4}{3})} T^4 \ell \lambda,$$  \hspace{1cm} (11)

where $\lambda$ is the 4d (dimensionless) coupling. This background describes the finite temperature physics for $T > T_c$. Introducing representative values, $\ell T \sim 1.7$, we get

\[ ^4 \] The radius $\ell$ gives a further scale that triggers the confinement/deconfinement transition [20]. Thus, it is natural to identify it with the inverse critical temperature, $\ell = T_c^{-1}$. 

Finite 't Hooft Coupling

The AdS/CFT conjecture is a statement which goes beyond the classical limit of string theory, in which it maps classical solutions of supergravity to quantum field theory vacua in the strong coupling limit $\lambda \to \infty$. Corrections in $\lambda^{-1}$ are in direct correspondence with those in powers of $\alpha'$ in the string theory side. Considering the $\alpha'$ corrected near extremal D3–brane [22], it is not difficult to evaluate

$$
\tilde{q}(\lambda) = 1 - \frac{\zeta(3)}{8} \left[ 45 - \frac{30725\pi}{512\sqrt{2}\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{15}{4}\right)} \right] \lambda^{-3/2} + \ldots
$$

(12)

Finite coupling corrections tend to diminish the value of the jet quenching parameter. For the same representative values chosen above, $\tilde{q}(6\pi) = 4.38$ GeV$^2$/fm. The decrease in the jet quenching parameter is suggestive of a smooth interpolation between the strong coupling regime and the perturbative results.

Finite Chemical Potential

$\mathcal{N} = 4$ SYM theory has a global $SO(6)$ R–symmetry. Chemical potentials, $\kappa_i$, for the $U(1)^3 \subset SO(6)_R$, which amount to considering a rotating black D3–brane with maximal number of angular momenta, can be turned on. In spite of the fact that the relevant supergravity solution [23] heavily depends on various angles, the jet quenching parameter reads [17]:

$$
\tilde{q}(\kappa_i) = \frac{8\pi^{1/2}\Gamma\left(\frac{5}{4}\right)\Delta(\kappa_i)}{\Gamma\left(\frac{3}{4}\right)} \left( \int_1^\infty \frac{udu}{\sqrt{u^2 - 1}/\sqrt{u^4 + (1 + \kappa_+)u^2 - \kappa_{123}}} \right)^{-1},
$$

(13)

where $\kappa_+ = \kappa_1 + \kappa_2 + \kappa_3$, $\kappa_{123} = \kappa_1\kappa_2\kappa_3$, and

$$
\Delta(\kappa_i) = \frac{(1 + \kappa_1)^2(1 + \kappa_2)^2(1 + \kappa_3)^2}{(2 + \kappa_1 + \kappa_2 + \kappa_3 - \kappa_1\kappa_2\kappa_3)^3},
$$

(14)

and all the information about the internal coordinates has dissapeared. Instead of performing a detailed analysis of this result, we shall stress its most significant qualitative behaviour: the jet quenching parameter raises its value for nonzero values of the chemical potentials, $\tilde{q}(\kappa_i) > \tilde{q}_{\text{SYM}}$ [17] (see also [24]). The increase is not monotonic across the whole parameter space. It is easy to check that the above ratio tends to one when the chemical potentials are turned off.

There is another source of corrections given by world-sheet fluctuations of the string. These go like $\lambda^{-1/2}$. Thus, they are dominant at large $\lambda$ though much harder to compute (see e.g. [21] for a similar problem in the case of the Wilson loop that corresponds to the quark antiquark potential).
There is a generalization of AdS/CFT in which the $S^5$ is replaced by a Sasaki–Einstein manifold $X^5$. The resulting gauge theory ends up having reduced supersymmetry and the field content of an $\mathcal{N} = 1$ superconformal quiver theory (SQT) \[25\]. The jet quenching computation in this case proceeds as before \[18\], the only difference being at the last step where the relation between the radius of the manifold and the ’t Hooft coupling depends on the volume of $X^5$ which, in turn, is inversely proportional to the central charge of the gauge theory \[26\],

$$\hat{q}_{\text{SQT}} = \sqrt{\frac{\text{Vol } S^5}{\text{Vol } X^5}} = \sqrt{\frac{a_{\text{SQT}}}{a_{\text{SYM}}}}. \quad (15)$$

For the prototypical case, $X^5 = T^{1,1}$, i.e. the Klebanov–Witten (KW) model, this equation implies $\hat{q}_{\text{KW}} = \sqrt{27/32} \hat{q}_{\text{SYM}} = 4.12 \text{ GeV}^2/\text{fm}$. A mild version of this result can be extended to further superconformal field theories and, in particular, implies that if two such theories are connected by a renormalization group flow, then $\hat{q}$ for the ultraviolet (UV) theory is always larger than that for the infrared (IR) theory \[26\].

### Breaking of Conformal Invariance

A possible mechanism for conformal symmetry breaking is given by the introduction of fractional branes in a complex deformation of the Calabi–Yau cone over $X^5$ (see e.g. \[27\] for a review). This leads to cascading quiver gauge theories whose archetype is the Klebanov–Strassler (KS) model \[28\]. The jet quenching parameter can be seen to increase its value with respect to the conformal KW case \[18\].

Within the framework of bottom-up approaches like, so-called, AdS/QCD \[29\], a nonconformal gauge/gravity dual pair was studied in \[30\]. The nonconformal deformation is given by a single parameter $c$ that appears in a warp factor in front of the AdS metric. Finite $c$ raises the jet quenching parameter for fixed $\lambda$ and $T$. A detailed study of this behavior was recently performed in \[31\], where it was shown that the enhancement could be as high as 30% of the $\hat{q}_{\text{SYM}}$ value. These two examples suggest that breaking of conformal invariance might be associated to an increase of the jet quenching parameter.

### A CALL FOR MASSLESS DYNAMICAL QUARKS

Quarks are prime ingredients of QCD. Up to this point, however, we have misleadingly used the acronym QGP for theories without quarks. This is quite generic in the literature since gravity duals including quantum field theoretical degrees of freedom in the fundamental representation of the gauge group are scarce. A notable exception is given by the case of quenched flavor, $N_f \ll N_c$, in which quarks can be represented by probe D–branes in the background sourced by a large number of color branes \[32\]. Besides the formal interest of this case, it is evident that, real quark–gluon plasmas demand massless quarks beyond the quenched approximation, i.e., $N_f \sim N_c$. Moreover, we must cope
with finite temperature gauge/gravity duals, with $T > T_c$, which means that we need to scrutinize (non-supersymmetric) black brane solutions which are rather elusive.

There is only one known analytic solution with all these ingredients in critical string theory. A one parameter family of black hole solutions in the background sourced by $N_c$ color wrapped D5–branes and $N_f$ (smeared) flavor D5–branes recently constructed by Casero, Núñez and Paredes (CNP) \cite{33}. This is conjectured to be the thermal deformation of the gauge/gravity dual of an $\mathcal{N} = 1$ SQCD–like theory with quartic superpotential, at the conformal point, $N_f = 2N_c$, coupled to Kaluza–Klein adjoint matter. The temperature of these black holes is independent of the horizon radius and, indeed, coincides with the (Hagedorn) temperature, $T_H$, of Little String Theory (LST). This is possibly related to the fact that the UV completion of this solution involves NS5–branes. We shall comment on this issue while reviewing the computation of $\hat{q}$ performed in \cite{34}.

In the context of non-critical string theory, gauge/gravity duals of 4d theories with large $N_c$ and $N_f$, both at zero and high temperature, have been considered in the last few years. We shall focus on two interesting cases: an AdS$_5$ black brane proposed as the non-critical dual of the thermal version of conformal QCD, and an AdS$_5 \times$ S$^1$ black brane solution conjectured to be dual to thermal $\mathcal{N} = 1$ SQCD in the Seiberg conformal window \cite{35}. The dilaton is constant in both models. The zero temperature theories were constructed, respectively, in \cite{36} and \cite{37}. The color degrees of freedom are introduced via $N_c$ D3–brane sources and the back-reaction of $N_f$ flavor branes on the background is taken into account. These flavor branes are, roughly, spacetime filling brane–antibrane pairs (matching the classical $U(N_f) \times U(N_f)$ flavor symmetry). Properties of their quark–gluon plasmas were studied in \cite{34} by means of the gauge/gravity correspondence.\footnote{In order to study the dynamics of hard probes in the quark–gluon plasmas of these theories, it is assumed that the mass of the probes is related to the radial distance of a flavor brane from the center of the space, as it happens in the critical case, at least in some effective way. Therefore, the general results on the drag force and jet quenching parameter extend in a straightforward way to the non-critical setup.}

The relevant gravity solutions are generically strongly curved and $\alpha'$ corrections are not subleading. The optimistic prejudice, driven by the unexpected success of bottom-up approaches like the alluded to AdS/QCD, is that the non-critical solutions might capture at least qualitative information of the dual field theories, insensitive to these corrections.

### Little String Theory Plasmas

A one parameter family of black brane solutions, conjectured to be the finite temperature gauge/gravity dual of an $\mathcal{N} = 1$ SQCD–like theory at the conformal point, $N_f = 2N_c$, was obtained in \cite{33}. Their Hawking temperature is given by the Hagedorn temperature. This suggests that there could be thermodynamical instabilities (negative specific heat) in this solution, in the very same way as happens in the standard LST case \cite{38}. In order to test that the above statement is correct and thermodynamical instabilities cannot be cured by introducing IR cut-offs, we consider a generalization of CNP black branes, parameterized by $\xi \in (0, 4)$, that should be gauge/gravity duals of...
the above LST plasma compactified on $S^3$. Their string frame metric is

$$ds^2 = e^{\Phi_0 + r} \left[ -\mathcal{F} dt^2 + R^2 d\Omega_3^2 + \frac{R^2 N_c \alpha'}{R^2 + N_c \alpha'} \mathcal{F}^{-1} dr^2 + N_c \alpha' \left( \frac{1}{\xi} d\Omega_3^2 \\
+ \frac{1}{4 - \xi} d\Omega_3^2 + \frac{1}{4} (d\psi + \cos \theta d\phi + \cos \tilde{\theta} d\tilde{\phi})^2 \right) \right],$$

where $\mathcal{F} = 1 - e^{2r_H - 2r}$, so the horizon is placed at $r = r_H$. This introduces a new scale into the system, the (quantized \(^3\)) radius $R$ of the $S^3$, that indeed produces a departure in the black hole temperature from Hagedorn’s

$$T(R) = T_H \sqrt{1 + \frac{N_c \alpha'}{R^2}} > T_H,$$

albeit still independent of the horizon radius, seemingly a common feature of black holes obtained from NS5 and D5–brane configurations. These black holes seem to present analogous instabilities as their uncompactified counterparts\(^7\) [40]. It is not clear, thus, if these solutions provide reliable descriptions of 4d finite temperature gauge/gravity duals.

While the energy loss of a probe quark due to drag force in the plasma\(^8\) is found to be non-zero (and formally analogous to the $\mathcal{N} = 4$ SYM theory case\(^8\)), the jet quenching parameter exactly vanishes. These quantities are related to the energy loss of a parton in two opposite regimes of the transverse momentum (large momentum for the quenching parameter and small momentum for the drag coefficient). Still, obtaining such completely different results is puzzling. The jet quenching parameter is definitely dependent on the UV behavior of the dual backgrounds, and so on the 6d LST asymptotics, while the same dependence for the drag force is not apparent. This fact suggests that the result $\hat{q} = 0$ has to be associated more to non-local LST modes than to their claimed local counterparts. These give total screening and, as a consequence, zero jet quenching parameter. Thus, these backgrounds do not seem to be useful in order to study UV properties of realistic plasmas. It is expected that the same should happen for possible $N_f \neq 2N_c$ plasmas in the framework of\(^9\), even though the corresponding gravity backgrounds are not currently known\(^9\).

Interestingly, the hydrodynamic properties of the QGP are not affected by the troublesome thermodynamic behavior mentioned above. This seems to be related to the fact that the ratio $\eta/s$ depends on universal properties of black hole horizons, and is not substantially altered by the presence of flavors.

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\(^7\) Indeed, the same problems are already present in a family of black brane solutions corresponding to finite temperature $\mathcal{N} = 1$ supersymmetric YM theory\(^9\).

\(^8\) In this respect, it seems that the friction that a slow, heavy parton experiences in a strongly coupled plasma is in practice always the same, irrespective of the features of the dual field theory.

\(^9\) The results of the forthcoming subsections, as well as in phenomenological 5d models\(^29\), where both the drag force and the jet quenching parameter are found to be different from zero, indicate that dynamical quarks seem to introduce no specific problem to the evaluation of $\hat{q}$. 

QCD in the Conformal Window

A 5–dimensional model dual to QCD in the conformal window has been constructed in [36] (for $T = 0$) and [35] (for $T \neq 0$). In $\alpha' = 1$ units, the metric reads

$$ds^2 = \left(\frac{r}{R}\right)^2 \left[1 - \frac{H}{r^4}\right] dt^2 + dx_i dx^i + \left(\frac{R}{r}\right)^2 \left[1 - \frac{H}{r^4}\right]^{-1} dr^2,$$

where $r$ is the radius being an increasing function of $\rho \sim N_f/(2\pi N_c)$,

$$R^2 = \frac{200}{50 + 7\rho^2 - \sqrt{200 + 49\rho^2}}.$$

The dilaton is related to the gauge theory coupling by

$$\lambda \equiv g_{\text{QCD}}^2 N_c = e^{\phi_0} N_c = \frac{\pi}{5} \left(\sqrt{200 + 49\rho^2 - 7\rho}\right).$$

It decreases with $\rho$, which is consistent with the known fact that the zero temperature theory should be weakly coupled in the upper part of the conformal window, that is when $\rho$ is the largest. The behavior of the coupling is given by $F(\rho)/N_c$ for a function $F$ whose behavior for large $\rho$ is $F(\rho) \sim 1/\rho$, as expected in the Veneziano limit [42].

The computation of $\hat{q}$ proceeds as in the previous section, and results in a monotonically increasing function of $\rho$. Its asymptotics can be readily computed,

$$\hat{q} \sim \frac{4\pi^{3/2} \Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)} T^3 \begin{cases} 1 + \frac{\sqrt{2}}{2} \rho + O(\rho^2) & \text{for } \rho \to 0, \\ \frac{7}{2} + O\left(\frac{1}{\rho^2}\right) & \text{for } \rho \to \infty. \end{cases}$$

The transport coefficient displays a dependence on the “effective number of massless quarks” $\rho$. A representative value may be obtained by assuming the physically sensible value $\rho \approx 1$, and $T = 300$ MeV, leading to $\hat{q}_{\rho \sim N_c} = 5.29$ GeV$^2$/fm. The variation of $\hat{q}$ is very small in the whole range of $\rho$. It signals the fact that the flavor contribution is not drastically changing the properties of the plasma. This is compatible with (and in a sense gives a reason for) the evidence that the values of plasma properties computed with gravity duals including only adjoint fields are very similar to the experimental ones.

SQCD in Seiberg’s Conformal Window

A 6–dimensional model dual to SQCD in Seiberg’s conformal window has been constructed by Klebanov and Maldacena [37] (for $T = 0$) and [35] (for $T \neq 0$). In $\alpha' = 1$

\[\text{\footnote{These expressions must be taken with a grain of salt: since the background is expected to be corrected by order one terms, the numerical coefficients are not trustworthy. Moreover, since the dual field theory should be QCD in the conformal window for definite, finite values of $\rho$, the limit $\rho \to 0$ ($\rho \to \infty$) is meaningful only as an indication of the behavior for small (large) $\rho$. The strict $\rho = 0$ case is dual to the finite temperature version of a YM theory without flavors first studied by Polyakov [43].}}\]
units, the 6d metric reads
\begin{equation}
 ds^2 = \frac{r^2}{6} \left[ \left( 1 - \frac{r_H^4}{r^4} \right) dt^2 + dx_i dx_j \right] + \frac{6r^2 dr^2}{r^4 - r_H^4} + \frac{2}{3r^2} d\varphi^2 ,
\end{equation}
where the AdS radius is now independent\(^1\) of both \(N_c\) and \(N_f\). The coupling, instead, depends on the quotient \(\rho\),
\begin{equation}
 \lambda = e^{\phi_0} N_c = \frac{2}{3\rho} ,
\end{equation}
and satisfies Veneziano’s asymptotics. The jet quenching parameter also turns out to be independent of \(\rho\),
\begin{equation}
 \hat{q} = \frac{6\pi^{3/2} \Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{3}{4}\right)^2} T^3 ,
\end{equation}
something that looks odd. Strikingly, its value for the representative temperature of the process is \(\hat{q}_{N_f \sim N_c} = 6.19\ \text{GeV}^2/\text{fm}\), slightly higher than the \(\mathcal{N} = 4\ \text{SYM}\) value and so more comfortably within the RHIC range.

**CONCLUDING REMARKS**

The AdS/CFT correspondence embodies a powerful device to scrutinize the strong coupling regime of non-Abelian gauge theories. Its application to finite temperature quantum field theories has produced stunning results, mostly connected to the physics above the crossover in the phase diagram of Quantum Chromodynamics. This is the regime of QCD presently being explored at RHIC where increasing evidence points towards the formation of a short lived strongly coupled quark–gluon plasma that behaves like a nearly perfect fluid. Within the framework of the AdS/CFT correspondence, it has been proven that this is a universal behavior of plasmas of fairly generic gauge theories. Several other properties of this plasma have been studied.

We do not have a string dual of QCD at our disposal. And it is not clear if we will have something like this in the future. However, we can try to understand which results are universal, how different quantities depend on supersymmetry, dimensionality, field content, etc. Some attempts to extrapolate results from \(\mathcal{N} = 4\ \text{SYM}\) towards QCD have been explored recently in theories without fundamental degrees of freedom. Gubser argues that the extrapolation should be done by using the energy densities of both theories as an unambiguous quantity to be fixed for comparison \([44]\). This leads to a map between temperatures of the sort \(T_{\text{SYM}} = 3^{-1/4} T_{\text{QCD}}\) and, consequently, to the somehow disturbing conclusion that \(\hat{q}_{\text{QCD}} < \hat{q}_{\text{SYM}}\) \([45]\). Liu, Rajagopal and Wiedemann \([26]\), instead, conjecture that, since QCD’s QGP is approximately conformal at \(T \approx 2T_c\),
\begin{equation}
 \frac{\hat{q}_{\text{QCD}}}{\hat{q}_{\text{SYM}}} \approx \sqrt{\frac{s_{\text{QCD}}}{s_{\text{SYM}}}} \approx 0.63 ,
\end{equation}

\(^1\) The fact that the radius is independent of \(N_c\) and \(N_f\) is probably signaling that this model is incomplete.
the same tendency as before. The results in [34] seem to indicate that the relation between ratios of jet quenching parameter and entropy densities ceases to be valid when quarks are introduced, even if the theory remains conformal. Those in [18, 31, 34] tend to suggest, on the contrary, that $\hat{q}_{QCD}$ may be higher than $\hat{q}_{SYM}$.

A full understanding of the consequences of adding quarks to YM theory at strong coupling is still missing. In this presentation we attempted to give an insight into the behavior of $\hat{q}$ with respect to parameters that are relevant to QCD. However, it seems clear that elaborations on a critical string theory example not involving NS5–branes would be desirable. The dependence of all the observables on $\rho$ is mild, so that the numerical results for $\hat{q}$ (as well as other interesting quantities that are not detailed in this article) are always very similar to $\hat{q}_{SYM}$. This provides an a posteriori explanation of why the latter is so similar to what is observed at RHIC.

The experimental program of the LHC will begin this year, with energies extending the present reach by more than one order of magnitude. The first heavy-ion collisions will start one year after. The use of the same machine to accelerate protons and nuclei implies that, for the first time, the energy frontier is the same for Standard Model (SM) or beyond the SM searches and for hot and dense QCD physics. This increase in energy translates into an increase in the temperature reached in the corresponding Pb+Pb collisions of a little less than a factor of two. A longer–lived plasma is also expected, reducing the hadronic matter effects which could obscure the interpretation of some of the measurements. Hopes exist, also, that this increase in the temperature could presumably be large enough to see a transition from the sQGP to the expected weakly coupled QGP.

String theory seems to have something to say, through the AdS/CFT correspondence, in real sQGP physics. Even if as of today the connection is still preliminary and not on sufficiently firm ground, no doubt this is a very promising avenue for future research in high energy theoretical physics.

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