A Comment on BPS States in F-theory in 8 Dimensions

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ABSTRACT

We study some aspects of enhanced gauge symmetries in F-theory compactified on K3. We find open string configurations connecting various 7-branes which represent stable BPS states. In this approach we recover $D_n$ and $E_n$ gauge groups previously found from an analysis of singularities of the moduli space of elliptically fibered K3 manifolds as well as examples of non-perturbative realizations of $A_n$ groups.

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1. Introduction

Recently great progress has been made in our understanding of enhanced gauge symmetries and matter in compactifications of F-theory [1], [2], [3], [4]. Although the appearance of enhanced gauge symmetries and matter in compactifications of type II [5] [6] or F-theory has been understood in a general geometric framework through an analysis of singularities it is instructive to try to understand it in terms of BPS states produced by open strings with various \((p, q)\) charges [7] connecting (\(SL(2, \mathbb{Z})\) transformed) Dirichlet branes. Such an analysis can provide us with an additional evidence for string-string duality as well as give us a better understanding of quantum field theory applications of the latter [8].

More evidence for the duality between F-theory on an elliptic K3 and the heterotic string compactified on a \(T^2\) has been found in an analysis of the case of K3 corresponding to constant background for dilaton-axion fields [9], [10]. In particular it has been shown [9] that F-theory on \(T^4/\mathbb{Z}_2\) is equivalent to an orientifold of type IIB on \(T^2\), which in turn, is related by a \(T\)-duality transformation to a type I theory on \(T^2\). In this description an enhanced \(SO(8)\) gauge symmetry appears due to twisted sectors of the orientifold theory. In ref. [10] new branches of F-theory on K3 have been found which contain \(T^4/\mathbb{Z}_3\), \(T^4/\mathbb{Z}_4\) and \(T^4/\mathbb{Z}_6\). These special points correspond to \(E_6 \times E_6 \times E_6\), \(E_7 \times E_7 \times SO(8)\) and \(E_8 \times E_6 \times SO(8)\) gauge groups respectively. A rigorous analysis of BPS states responsible for these enhanced gauge symmetries is however very complicated because of the essentially non-perturbative character of the \(D_n\) and \(E_n\) gauge symmetries. More precisely in order to recover \(E_n\) groups in terms of generalized orientifolds it is necessary to take into account the “twisted sector” associated with the element of the orientifold group involving \(S\)-duality transformation [10].

In the present paper we study the structure of BPS states responsible for enhanced gauge symmetries in the worldvolume of 7-branes. While \(A_n\) gauge groups have been already well understood [11], [5] in terms of open strings connecting Dirichlet branes one may wish to have a better insight into an appearance of \(D_n\) and \(E_n\) groups. We formulate the local conditions for stable BPS states in the worldvolume of 7-branes and identify those for \(A_n\), \(D_n\) and \(E_n\) gauge groups with certain non-trivial \((p, q)\) open strings connecting appropriate 7-branes. In general the relevant BPS states are found to correspond to strings connecting various 7-branes along rather complicated trajectories. Since the open strings connecting the 7-branes are, in general, non-perturbative objects our analysis is to some extent phenomenological. Yet it is nice to see how the simple physical arguments fit with the general picture of F-theory compactifications.

2. Conditions for stable BPS states

Consider a compactification of \(F\) theory on an elliptic K3 [1]. The fibration of such a K3 can be described by a Weierstrass model

\[
y^2 = x^3 + f(z)x + g(z) = (x - e_1(z))(x - e_2(z))(x - e_3(z)),
\]

with the discriminant [12] \(\Delta = 4f^3 + 27g^2 = (e_1 - e_2)^2(e_2 - e_3)^2(e_3 - e_1)^2\). Here \(z\) is a
coordinate on $P^1$, and $f$ and $g$ are polynomials of $z$ of degrees 8 and 12 respectively. The 24 points on $P^1$ where the fiber ($T^2$) degenerates, i.e. the discriminant $\Delta = 0$ can be interpreted as 7-branes on $P^1$. Such a single 7-brane can be thought of as a (SL(2, $\mathbb{Z}$) transformed) D-brane. The positions of the 7-branes are mapped into the parameters of Wilson lines in the dual heterotic description [1]. The 7-branes are the magnetic sources of a non-trivial classical background for $\tau = a + ie^{-\phi}$, where $a$ stands for an axion and $\phi$ is a dilaton. The modular parameter $\tau$ is defined by $j(\tau) = 4(24f)^3/\Delta$.

When $n$ Dirichlet 7-branes come to the same point on $P^1$ one gets an $SU(n)$ non-abelian gauge symmetry due to open strings connecting these 7-branes [11]. In the F-theory 7-branes are generically mutually non-perturbative objects [1]. As it has been shown in ref. [13] from an analysis of the low energy anomalous equations of motion for $B_{NS,R}$ fields in the presence of 7-branes branes a given 7-brane admits open strings with a unique $(p, q)$ charges to end on it. For example a Dirichlet 7-brane admits only $(1, 0)$ (NS) strings to end on it.

In general the condition of ref.[13] restricts possible enhanced gauge symmetries for a given configuration of 7-branes. For the purpose of this paper it is convenient to reformulate this condition as follows: a monodromy matrix which transforms the periods of a $T^2$ fiber around the positions of the 7-branes has to leave the $(p, q)$ charges of the open string invariant. Thus we essentially reduce the problem to an analysis of the monodromies of the modular parameter $\tau$. In particular for the case of an $A_{n-1}$ singularity [2], [3] it is easy to check that the monodromies around $n$ 7-branes commute with each other.

A very important subtlety is however that a definition of monodromies for a given configuration of 7-branes implies a choice of a base point on a cover of $P^1$. Therefore the same 7-brane on different sheets may be viewed as having different $(p, q)$ charges with respect to a chosen base point. Hence two 7-branes of different charges may be connected by an open string through a complicated path. This observation is crucial for an explanation of an appearance of $E_n$ gauge groups (see below).

In order to understand this point better let us recall that from the analysis of singularities of type II theory on K3 an enhanced gauge symmetry appears due to solitonic states corresponding to vanishing holomorphic 2-cycles of K3. For the $A_n$, $D_n$, $E_n$ singularities (resolved by an ALE space) the cycles which shrink to zero area have intersection matrix corresponding to the Dynkin diagram for $A_n$, $D_n$, $E_n$ [2], [3], [2]. For an elliptically fibered K3 such a vanishing 2-cycle is locally a combination of an appropriate 1-cycle $b$ of the $T^2$ fibre and a 1-cycle on the $P^1$ base. Its projection on $P^1$ is just an open string connecting appropriate branes.

We expect that the BPS states in F-theory on K3 are also associated with such vanishing 2-cycles even though it may be considered as type IIB theory on $P^1$ [1], [2]. A projection of a 2-cycle to $P^1$ is represented by an open string connecting two 7-branes. Therefore when moving along a contour $\gamma$ encircling such an open string together with

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2 A relevance of such string configurations has been independently noted by Sen [14].
the two 7-branes (one in a clockwise direction, another in an anti-clockwise direction) the 1-cycle in the fiber $T^2$ has to go to itself. That means that the total monodromy along $\gamma$ has to be just 1, i.e. $M^{-1}L^{-1}M'L = 1$ where $M, M'$ are the monodromies around two 7-branes and $L$ stands for a monodromy along the corresponding open string.

Another condition comes out from the existence of a string of a minimal length connecting appropriate 7-branes. The tension of a $(p, q)$ string reads $T_{p,q} = \frac{1}{\sqrt{\tau_2}}|\tau_{p,q}(z)|$, where $\tau_2 = \text{Im}\tau$ and $\tau_{p,q} = p + q\tau$. The mass of a $(p, q)$ string state along a curve $C$ is given by $\int_C T_{p,q} ds = \int_C |dw_{p,q}|$, where the interval element $ds$ is given by $ds^2 = \tau_2|\eta(\tau)|^4 \prod_i |z - z_i|^{-1/6}|dz|^2$, and

$$dw_{p,q} = \eta(\tau)^2 \prod_i (z - z_i)^{-1/12}(p + q\tau)dz.$$ 

Here $z_i$ stand for positions of 7-branes on $P^1$. The 1-differential $dw_{p,q}$ can be represented as $dw_{p,q} = \int_b \omega$, where $\omega$ is an appropriate closed 2-form on an ALE space. An open string of minimal mass corresponds to a straight line in the (flat) $|dw_{p,q}|^2$ metric. Technically there should exist a geodesic subject to $\int d\omega_{p,q} = \alpha t$ which connects the two 7-branes, where $t$ is a proper time parameter along the open string and $\alpha$ is a constant. This argument is similar to that of ref. [16].

In the simplest example of $n$ Dirichlet 7-branes this condition is trivially satisfied because $dw \sim dz$. It is also worth noting that in the presence of a non-Dirichlet 7-brane there is only one geodesic that connects two Dirichlet 7-branes (see, for example Fig.1a for a result of a numerical computation) while the cases of $D_n$ and $E_n$ are more complicated (see below). In what follows we use the above observations to identify the BPS states corresponding to the $D_n$ and $E_n$ groups.

3. $D_n$ case

The $D_4$ case has been studied in ref. [3] where it has been mapped to a $T^4/Z_2$ orientifold. In this subsection we briefly review this example and consider the general case of $D_n$ groups.

In the $D_4$ case $f$ and $g$ are polynomials of degree 2 and 3 respectively [2]. The discriminant $\Delta$ is a polynomial of order 6. Hence there are six 7-branes. The $SO(8)$ gauge group appears when $f \sim z^2$ and $g \sim z^3$. To resolve the singularity one can take $f = z^2 - \frac{6}{7}z^3 + \frac{2}{3}q$, $g = 2z^3 + 2qz + 2$, with $q^3 = -27/2$, so that $\Delta \sim z^4(5z^2 - 6z - 1)$. This resolution corresponds to an unbroken $SU(4)$ subgroup of $SO(8)$ due to four 7-branes at $z = 0$ (the type $I_4$ singularity [3]).

The monodromy at infinity is given by $M_\infty = -1$. We have $A^4 = S^{-1}T^4S$ for a monodromy around $z = 0$, $B = A^{-3}TA^3$ and $C = A^{-1}TA$ for monodromies around the roots of the polynomial $5z^2 - 6z - 1$. One can easily check that $-1 = A^4BC$. The charges of the open strings corresponding to the 7-branes with $A$, $B$ and $C$ monodromies are $(0,1)$, $(1,3)$ and $(1,1)$ respectively.
The maximal $A_n$-type subgroup is $SU(4)$ which is due to the 4 mutually perturbative 7-branes of the same type $A$ at $z = 0$. The monodromy around any of these four 7-branes does not commute with those around each 7-branes of types $B$ and $C$ separately. However it commutes with the monodromy along a path encircling both $B$- and one of the $C$-type 7-branes. The Chan-Paton factors for the open strings stretched along such a non-trivial path are antisymmetric with respect to these 7-branes. In particular this is clear in the $T^4/Z_2$ orbifold limit, see [9]. In the F-theory we observe that because of $BC = -A^{-4}$ the charges change from $(0,1)$ to $(0,-1)$ when one moves around $B$ and $C$ 7-branes. On the other hand the signs of the charges are related to the gauge charges of the ends of the open string on the 7-branes and, hence to the orientation of the string [17], [13]. Therefore locally at each of the 7-branes the string has effectively different orientations. This leads us to an identification of states similar to the case of a IIB theory on a $T^2$ orientifold.

As a consequence a natural interpretation of the $SO(8)$ gauge group is that its $SU(4) \times U(1)$ subgroup gets completed into the $SO(8)$ taking into account of states corresponding to open strings which connect the perturbative 7-branes along the path that encircles two non-perturbative $BC$ 7-branes [9]. We then get a $6 + \overline{6}$ representation (for the open strings with both orientations) of $SU(4)$ which is to complete the adjoint representation of the $SO(8)$ gauge group. Equivalently $28 = 15 + 6(-2) + 6(+2) + 1$, where $(\pm 2)$ are the charges with respect to the $U(1)$ gauge group on the two separated 7-branes.

It is now easy to construct a configuration of 7-branes which realizes an $SO(2n + 8)$ group (type $I_n^*$ singularity [2]). Such a configuration consists of $n + 4$ 7-branes with monodromy $A$, one 7-brane with monodromy $B$ and another one of monodromy $C$. In this case the monodromy at infinity is $-A^n$ so that

$$-A^n = A^{n+4}BC.$$ (2)

The perturbative subgroup is in this case $SU(n + 4) \times U(1)$. The adjoint representation of $SO(2n + 8)$ reads as follows in terms of the above subgroup (we omit the $U(1)$ quantum numbers): $(n + 4)(2n + 7) = ((n + 4)^2 - 1) + (n + 4) + \frac{(n+4)(n+3)}{2} + \frac{(n+4)(n+3)}{2}$. The non-perturbative states $\left(\frac{(n+4)(n+3)}{2}\right) + \left(\frac{(n+4)(n+3)}{2}\right)$ are produced by strings connecting two different $A$ 7-branes along a path that encircles $BC$ 7-branes. Because the matrix $BC$ acts with -1 on the $(p,q)$ charges of the open string and moreover we apply similar argument with the $D_4$ case with regard to the Chan-Paton indices the above mentioned states belong to the antisymmetric 2-tensor representation of $SU(n)$. Numerical work with Mathematica confirms the existence of the corresponding geodesics.

In particular the $SO(32)$ group would be realized with $16=12+4$ $A$ 7-branes in the presence of $B$ and $C$ 7-branes. Note however that in terms of elliptic fibration of K3 the configuration (2) should correspond to $n = 3\text{deg } f - \text{deg } \Delta$, $3\text{deg } f = 2\text{deg } g$. Note that for $n > 4$ such a configuration of 7-branes cannot be realized by a compactification of F-theory on K3 because the corresponding singularity destroys the triviality of the canonical bundle on a resolution [2].
4. \( E_n \) case

We first consider a \( E_6 \) gauge group. In this case we have (type IV* singularity [2]) five 7-branes with the monodromy \( A \), one 7-brane with the monodromy \( B \) and two 7-branes with monodromies \( C \), so that

\[
ST = A^5 BC^2,
\]

where \( A = S^{-1}TS \), \( B = A^{-3}TA^3 \), \( C = A^{-1}TA \). In the orientifold limit (F-theory on \( T^4/\mathbb{Z}_6 \)) this case was studied in ref. [10] where it has been noted that in order to have an \( E_6 \) group one needs to take into account non-perturbative twisted sectors. In what follows we identify the corresponding BPS states.

The perturbative subgroup is just \( SU(5) \times SU(2) \). Let us consider the non-perturbative states. We observe that \( BC \) commutes with \( A \) and \( A^4B \) commutes with \( C \). The matrices \( BC \) and \( A^4B \) act with -1 on the charges of the open strings corresponding to the \( A \)-type and \( C \)-type 7-branes respectively. Therefore one can consider an open string that connects an \( A \) 7-brane with another one along a non-trivial path that encircles \( B \) and one of the \( C \) 7-branes. This implies a representation \((10,2) + (\overline{10},2)\) according to the same argument as in the \( D_n \) case. These states are doublets of \( SU(2) \) because one has to label the string according to which ones of the \( C \) 7-branes is encircled: the Weyl group of \( SU(2) \) exchanges the \( C \)-type 7-branes. Note that this is the only possibility to construct a state with more than 2 Chan-Paton indices. One can also consider an open string connecting the two different \( C \) 7-branes along a non-trivial path encircling \( B \) and four of \( A \) 7-branes. This gives a representation \((5,1) + (\overline{5},1)\). Again the non-trivial quantum numbers with respect to \( SU(5) \) correspond to a choice of four of \( A \) 7-branes to be encircled. It is easy to check by a numerical computation that the corresponding geodesic exists. Thus together with the adjoint representation of the perturbative \( SU(5) \times SU(2) \times U(1) \) gauge group we get an adjoint representation of \( E_6 \) group: \( 78 = (24, 1) + (1, 3) + (1, 1) + (10, 2) + (\overline{10}, 2) + (5, 1) + (\overline{5}, 1) \).

It is interesting to note that the maximal subgroup of \( E_6 \) is \( SU(6) \times SU(2) \). Therefore with the above configuration one realizes the \( SU(6) \) subgroup non-perturbatively. More precisely the \( SU(6) \) adjoint representation consists of the adjoint representation \((24, 1) + (1, 3) + (1, 1)\) of the \( SU(5) \times U(1) \) group and the \( SU(2) \) singlet BPS states \((5, 1) + (\overline{5}, 1)\) which are produced by the open strings connecting two different \( C \) 7-branes! The higgsing of \( E_6 \) to \( SO(10) \times U(1) \) corresponds to a higgsing of the \( SU(5) \) perturbative subgroup to \( SU(4) \times U(1) \). Indeed, the unbroken perturbative subgroup gives now \((15, 1) + (1, 3) + (1, 1)\). The \( AA \) strings encircling \( BC \) give \((6, 2) + (\overline{6}, 2)\), while the \( CC \) strings encircling \( A^4B \) produce \( 2(1, 1) \). All together these representations are perfectly combined into the adjoint representation of \( SO(10) \) which is thus non-perturbative. This realization of \( SO(10) \) is quite different from that of the previous section. By higgsing one of \( C \)-type 7-branes out, the \((5, 1) + (\overline{5}, 1)\) states and half of the \((10, 2) + (\overline{10}, 2)\) ones become heavy and we recover a realization of a \( 45 = 24 + 10 + \overline{10} + 1 \) representation of \( SO(10) \) discussed in the previous section.
The structure of the BPS states for $E_7$ and $E_8$ gauge groups is more complicated. Consider six 7-branes with the monodromy $A$, one 7-brane with the monodromy $B$ and two 7-branes with monodromies $C$, so that

$$S = A^6 BC^2.$$

In the orientifold limit this case corresponds to $T^4/\mathbb{Z}_4$ and has to realize a $E_7$ group (type III$^*$ singularity).

The perturbative gauge group is just $SU(6) \times SU(2) \times U(1)$ with the adjoint representation $(35, 1) + (1, 3) + (1, 1)$. Similar to the above construction we can get additional states $(15, 2) + (\overline{15}, 2)$ due to open strings connecting $A$ 7-branes along a non-trivial path which encircles $B$ and one of the $C$ 7-branes. For strings connecting the $C$-type 7-branes along a path encircling $B$ and four of the $A$ 7-branes one gets $(15, 1) + (\overline{15}, 1)$. The adjoint representation of $SU(6) \times U(1)$ subgroup together with the $(15, 1) + (\overline{15}, 1)$ one gives the adjoint representation of the non-perturbative $SO(12)$. However all of the 129 states listed above do not complete the adjoint representation $133$ of $E_7$. The missed states are $2(1, 2)$. Together with $(15, 2) + (\overline{15}, 2)$ these are combined into $(32', 2)$ of $SO(12) \times SU(2)$.

As it has been anticipated above the additional states can come from string connecting $B$ and $C$ 7-branes along a non-trivial way so that the open string meets these 7-branes on different sheets. The point is that a $B$ 7-brane corresponds to the $(p, q)$ charges of a $C$ 7-brane on another sheet. Numerical computations with Mathematica confirm the existence of the corresponding geodesic schematically shown in Fig.1b.

Fig.1. a) A geodesic connecting two $(1,0)$ 7-branes in the presence of a $(0,1)$ 7-brane; b) A geodesic connecting $B$ and $C$ type 7-branes along a homotopically non-trivial path; c) An open string connecting $A$ and $B$ type 7-branes along a homotopically non-trivial path. The dashed lines correspond to branch cuts in the $z$-plane.

It corresponds to $C^{-1} L^{-1} BL = 1$ where $L = CS$, so that this matrix transform the $(1, 1)$ charges of the $C$ 7-brane into the $(1, 3)$ charges of the $B$ 7-brane. These states
transform as $2(1,2)$. We thus get the $133$ representation of $E_7$. Note that for such a string we need 6 $A$-type 7-branes. Therefore such an additional state was absent in the $A^3BC^2$ case. When one of the $A$-type 7-branes is higgsed out these additional states become heavy and we recover an embedding $E_7 \to E_6 \times U(1)$.

Consider now the configuration corresponding to a $E_8$ group (type $II^*$ singularity) which consists of seven 7-branes with the monodromy $A$, one 7-brane with the monodromy $B$ and two 7-branes with monodromies $C$, obeying

$$ST = A^7BC^2.$$ 

In the orbifold limit this configuration corresponds to $T^4/\mathbb{Z}_6$ as follows from $(ST)^6 = 1$. The perturbative subgroup is just $SU(7) \times SU(2) \times U(1)$ with the gauge bosons in the $(48,1,0) + (1,3,0) + (1,1,0)$ representation. The $AA$ strings encircling the $BC$ 7-branes give $(21,2) + (\overline{21},2)$ representation of $SU(7) \times SU(2)$. The $CC$ strings encircling $A^4B$ give $(35,1) + (\overline{35},1)$. The adjoint representation $248$ of $E_8$ is expanded in terms of representations of the perturbative group $SU(7) \times SU(2)$ as follows

$$248 = (48,1) + (1,3) + (1,1) + (21,2) + (\overline{21},2) + (35,1) + (\overline{35},1) + (7,1) + (\overline{7},1) + (7,2) + (\overline{7},2).$$

The missed representation is thus $(7,1) + (7,1) + (7,2) + (\overline{7},2).$ The representation $(7,2) + (\overline{7},2)$ is due to the string connecting $B$ and $C$ 7-branes along a path that encircles six of seven 7-branes similar to that shown in Fig.1b. The representation $(7,1) + (\overline{7},1)$ seems to correspond to a string connecting $A$ and $B$ 7-branes along a complicated path shown in Fig.1c. This path corresponds to $B = L^{-1}AL$ where $L = S^{-1}C^2A^{-1}S$. This matrix transform the $(1,3)$ charges of the $B$ 7-brane into $(0,1)$ charges of the $A$ 7-brane. Assuming that this representation is correct one can move one of $A$-type 7-branes out. Then the BPS state $(7,1) + (\overline{7},1)$ becomes heavy and only $2(1,2)$ part of $(7,2) + (\overline{7},2)$ remains light. Thus we recover an embedding $E_8 \to E_7 \times U(1)$.

5. Conclusion and Acknowledgements

We have identified stable BPS states relevant for $D_n$ and $E_n$ enhanced gauge symmetries in the F-theory in 8 dimensions with particular open string configurations connecting appropriate 7-branes. As a consistency check of the above picture we have recovered various embeddings $E_8 \to E_7 \times U(1)$, $E_7 \to E_6 \times U(1)$, $SO(12) \times U(1)$, $SU(7) \times U(1)$, $E_6 \to SO(10) \times U(1)$, $SU(6) \times U(1)$.

The simple physical arguments nicely fit with results which follows from the analysis of singularities and duality between F-theory on K3 and heterotic string on $T^2$.

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