Design of the Advanced Virgo non-degenerate recycling cavities

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Abstract. Advanced Virgo is the project to upgrade the interferometric gravitational wave detector Virgo, and it foresees the implementation of power and signal non-degenerate recycling cavities. Such cavities suppress the build-up of high order modes of the resonating sidebands, with some advantage for the commissioning of the detector and the build-up of the gravitational signal. Here we present the baseline design of the Advanced Virgo non-degenerate recycling cavities, giving some preliminary results of simulations about the tolerances of this design to astigmatism, mirror figure errors and thermal lensing.

1. Introduction
Interferometric gravitational wave detectors of the first generation like Virgo\textsuperscript{[1]} and LIGO\textsuperscript{[2]} have successfully completed their first long-duration data taking runs, and a new joint run is ongoing presently. Upgrades of the first generation detectors are currently under study, in order to improve the chance of detecting gravitational waves from astronomical sources and open a new observational window on the universe.

Advanced Virgo (adVirgo)\textsuperscript{[3]} is the project to upgrade the present Virgo detector to a second generation instrument. Compared to the optical layout of Virgo, the design of adVirgo features two main changes: the inclusion of the signal recycling technique, and the move from marginally stable recycling cavities to non-degenerate power and signal recycling cavities (NDRC).

In this paper we present the baseline design of the adVirgo NDRC: we first describe the main motivations for having NDRC in adVirgo in section n.2; then, in section n.3, we list the main issues of the NDRC, and the features of the baseline design compliant with these needs. In section n.4 we report on some preliminary simulation results about the tolerances of the NDRC to errors on mirrors’ radii of curvature (RoC) and thermal effects.
2. Motivations

In a marginally stable optical resonator, like the present power recycling cavity of Virgo, the high order modes (HOMs; please refer to Appendix A for the related notation) can resonate as a consequence of misalignment of the mirrors, mirror figure errors or thermal aberrations. If the contribution of HOMs is not negligible, the optical gain of the recycling resonator for the fundamental mode is reduced and the control of the detector becomes more difficult, as it has already been experienced during the commissioning of Virgo [4].

The build-up (optical gain) of the resonator is a function of the Gouy phase, $\Phi_G$, and also of the complex transmissivity $t_i$ and reflectivity $r_i$ of the input ($i = 1$) and the end mirror ($i = 2$), which define the finesse $F$ of the resonator. The optical gain $G_{mn}$ for the TEM$_{mn}$ mode can then be written as follows:

$$G_{mn}(t_1, r_1, r_2, \Delta\Phi_G) = \left| \frac{t_1}{1 - r_1r_2 \cdot exp[-i2(m + n + 1)\Delta\Phi_G]} \right|^2 \quad (1)$$

The normalized optical gain for the TEM$_{01}$ mode ($m = 0$, $n = 1$) is shown in fig.1; this figure clearly illustrates that, given the finesse $F = 70$ and $F = 40$ for power and signal recycling respectively (as stated in ref.[3]), the optical gain of the first high order mode in the NDRC can be reduced of more than two order of magnitude for $\Delta\Phi_G = 20$ degrees.

The NDRC would allow a lower HOMs content in the radio frequency sidebands used for the interferometer signal readout and control, and they would also lower HOMs content in the signal sidebands, reducing gravitational wave signal loss [5]. The HOMs build-up in the resonator would be suppressed with a proper choice of the cumulated Gouy phase, $\Delta\Phi_G$, for the beam resonating in the NDRC.

3. Baseline design

The design concept of the adVirgo NDRC has been inspired by the advanced LIGO NDRC design [6]: both power and signal recycling cavities have a folded design, and a mode matching telescope is placed inside the cavities. PRM1 (SRM1) is the power (signal) recycling mirror, and PRM3 (SRM3) and PRM2 (SRM2) form the fast telescope for the focusing of the large beam ($w = 5.5 \text{ cm}$, see ref.[3]) coming from the Fabry-Perot cavities in the long arms of the interferometer: PRM3 is a strongly converging mirror, and PRM2 is a diverging mirror for the tuning of the beam parameters (see fig.2).

In this design most of the Gouy phase $\Delta\Phi_G$ is cumulated between PRM2 and PRM1, and the size of the beam on PRM1 is greater than 1 mm, in order to avoid a too large power density on the power recycling mirror.
Here we present the main features of the baseline design for the NDRC of adVirgo (shown in fig.2), drawn by the adVirgo Optical Simulation and Design group [7]. In the design process, the main constraints came from the already existing Virgo infrastructures, in particular from the vacuum system, and from interferometer sensing and control (ISC) requirements on sideband resonance conditions.

The total length of the NDRC baseline design will fit the present Virgo vacuum tubes, and three mirrors will be suspended in the beamsplitter tower: the beam tuning mirrors PRM2 and SRM2, and the beamsplitter itself. For this purpose, the present Virgo suspension system, the SuperAttenuator, will be adapted for off-axis and multiple mirror suspension. The power recycling mirror PRM1 will be placed on the injection bench, which will be suspended; the suspension of PRM1 itself from the bench is currently under study, and the same configuration will be adopted for SRM1.

The main parameters of the design can be adapted to achieve any $\Delta \Phi_G$ desired value; for $\Delta \Phi_G = 20$ degrees, these parameters are listed in table 1.

4. Stability, matching and tolerances

For the following analysis, the parameters of the beam resonating in the NDRC and in the arm cavities of adVirgo have been computed using a Matlab based code, developed by two of the authors (MG and MB); a set of ABCD ray matrices (for both $x$ and $y$ directions in the case of the NDRC) [8] have been used to calculate the stable resonating mode of each cavity. The results given in this section are valid for power and signal recycling, and in the following we will

Table 1: List of parameters of the adVirgo NDRC baseline design, for $\Delta \Phi_G = 20$; the total length of both power and signal recycling cavities is 27.5 m. PRMi (SRMi) are the mirrors inside the power (signal) recycling cavity, ITM is the input test mass of the arm cavity.

| RoC (m)   | Lengths (m)          | Angle of incidence (deg) |
|-----------|----------------------|---------------------------|
| PRM3 = 12.80 | ITM to PRM3 = 11.5 | 1.7                       |
| PRM2 = -2.04 | PRM3 to PRM2 = 5.5  | 1.7                       |
| PRM1 = -2.04 | PRM2 to PRM1 = 10.5 | -                         |
refer mainly to the power recycling cavity for simplicity.

4.1. Coupling mismatch analysis

The non-degenerate power recycling cavity is coupled to the beam coming from the injection bench and to the beam of the Fabry-Perot (FP) cavities in the arms; the non-degenerate cavity for signal recycling is coupled to the beam going towards the detection bench and to the beam of the FP cavities. Any coupling mismatch can decrease the power/signal build-up of the fundamental mode in the NDRC themselves or in the FP cavities.

Since in the baseline design the incidence of the laser beam on telescope mirrors is not perpendicular, the beam resonating in the NDRC will be astigmatic. The waist size and the distance from the cavity waist of the NDRC beam will have different values on each of the two transverse directions, \(x\) and \(y\), and the associated complex parameters \(q_x\), \(q_y\) will be different in general from those of the beams from the injection (detection) and from the FP.

We can evaluate the coupling between two beams, \(U(q_1(z))\) and \(U(q_2(z))\), by means of the overlap integral \(\gamma\), defined as the scalar product of the beams over the transversal plane \(x - y\):

\[
\gamma(q_1, q_2) = \langle U(q_2) | U(q_1) \rangle \tag{2}
\]

As a first approximation we will suppose that all the cavities (NDRC and FP) of the detector are decoupled, so that we can neglect the mutual effects of the mismatch on the build-up of the fundamental mode in the coupled cavities. If we do so, then we can consider the overlap integral as a coupling coefficient; if we indicate with \(\gamma_{NDR C}\) the coupling between the NDRC and the input (output) beam, and with \(\gamma_{FP}\) the NDRC coupling with the arm beam, we can write the coupling losses \(L\) as follows:

\[
L = 1 - \gamma^2_{NDR C} \cdot \gamma^2_{FP} \tag{3}
\]

With this hypothesis, the coupling losses of the baseline design due to astigmatism are 6.8%; in any case we would like to stress the fact that this is just a preliminary estimation, and further investigation is presently ongoing to correctly evaluate the impact of the coupling losses on the sensitivity of adVirgo.

4.2. Stability and tolerances

The mode-matching telescope of the NDRC is very sensitive to errors on the RoC of the mirrors: this can be seen in fig.3a, where a colourscale plot shows the stability of the NDRC as a function of the percentage error on the RoC of the telescope mirrors.

For any point in the coloured region of fig.3a, the NDRC is stable on both \(x\) and \(y\) transverse directions, and the colourscale indicates the percentage coupling losses due to astigmatism and RoC errors; in the white regions of the plot the NDRC is unstable at least on one transverse direction. The working point of the baseline design is the \((0, 0)\) point at the centre of the plot, indicated by the red arrow.

The RoC of PRM3 is a critical parameter: with the present technology the manufacturers can realize mirrors of \(\sim 10\) m RoC like PRM3 with an accuracy of \(\pm 0.1\) %, while in this case an accuracy of \(\pm 0.02\) % is needed. Manufacturer’s errors on RoC could be overcome by tuning the distance \(L_2\) that separates the two mirrors of the telescope, as shown in fig.3b: a percentage variation of \(L_2\) of the same order of the error on the mirror RoC is needed. Given the parameters listed in table 1, this means that in presence of an error of \(+1\) cm on the RoC of PRM3 (corresponding to an error of about \(+0.1\) %), \(L_2\) should be elongated of about 5.5 mm (corresponding to a variation of about \(+0.1\) %).

The tolerances for the NDRC stability of the other parameters of the design are listed in table 2, together with the tolerances for the Gouy phase to be within the following boundaries: \(10\) deg \(\leq\) \(\Delta\Phi_G\) \(< 30\) deg. The RoC of the other mirrors are not so critical as for PRM3, since manufacturers can realize optics with an accuracy of \(0.04\) % for mirrors whose RoC is \(\sim 1\) m.
(a) NDRC stability as a function of the RoC of the telescope mirrors.

(b) NDRC stability as a function of the distance between the telescope mirrors.

Figure 3: Stability plots of the NDRC baseline design: the cavity is stable only in the coloured region. The colourscale indicates the coupling losses, in percentage; the working point of the detector is (0,0), and the corresponding coupling losses are shown.

4.3. Thermal effects

According to the baseline design of adVirgo [3], the power resonating in the non degenerate power recycling cavity will be of the order of 3 kW, while in the arms the power will be 760 kW. The absorption in the input test masses (ITM) of a small fraction of the circulating power will induce thermal lensing effects, changing the optical path of the light beam; we can then include in our simulations an equivalent lens, giving the same optical path variation, in order to study the 'hot' state of the interferometer for increasing circulating power inside its cavities.

Our simulations indicate that the NDRC baseline design will become unstable for an equivalent lens with focal length $f$ smaller than 7 km, corresponding to an absorption in the ITM of more than 0.45 W. Another preliminary simulation, run by the adVirgo thermal compensation system (TCS) group, shows that in the full-power 'hot' state 0.50 W will be absorbed in the ITM, giving an equivalent thermal lens of $f = 4.5$ km.

Fig.4 shows the results of our simulation for this last value of absorption: it clearly illustrates that NDRC design (conceived for a cold configuration where only a fraction of the nominal power is circulating and thermal lensing is negligible) will be no more stable at full power.

Table 2: List of stability and Gouy phase ($10 \text{ deg} < \Delta \Phi_G < 30 \text{ deg}$) tolerances of the advanced Virgo non-degenerate recycling cavity baseline design.

|            | Stability tol. | Gouy phase tol. |
|------------|----------------|-----------------|
| RM3        | ± 0.03 %       | ± 0.02 %        |
| RM2        | ± 0.3 %        | ± 0.1 %         |
| RM1        | ± 20 %         | ± 11 %          |
| L3         | ± 100 %        | ± 100 %         |
| L2         | ± 0.04 %       | ± 0.02 %        |
| L1         | ± 4.0 %        | ± 1.0 %         |
This result indicates that a TCS will be needed, once adVirgo will be operating at full power. For example, the TCS may include ring heaters placed around the lateral surface of the ITM, and compensation plates placed in front of the ITM [3].

Some thermal effects will be potentially present also on PRM1, and they have not been considered so far; this issue will require further investigation.

5. Alternative design
In parallel with this preliminary baseline design analysis, an alternative design has also been considered in order to decrease astigmatism coupling losses. This alternative design features longer cavities and smaller angles of incidence on telescope mirrors. The telescopes would be placed between the beamsplitter and the Fabry-Perot input mirrors, i.e. in the small Michelson arms (fig.5). For this design, the simulated coupling losses due to astigmatism are $L = 3.3\%$.

This alternative configuration would also allow to remove the compensation plates of the TCS, which would be only composed of ring heaters and/or CO$_2$ lasers for the actuation on the RoC of the telescope mirrors.

The main drawback of this alternative design is the potential arise of differential noise, eventually introduced by the suspension of two additional mirrors in each arm of the small Michelson: this issue has not been considered so far, and an exhaustive treatment of this topic will be addressed in the near future.

Figure 4: Stability plot with the thermal lensing, $f = 4.5$ km. The working point $(0,0)$ of the ‘hot’ configuration is out of the stability region.

Figure 5: Alternative design for the adVirgo NDRC. The mode-matching telescope is placed in the differential part of the detector; IMx is the input test mass of the Fabry-Perot cavity of the $x$ arm.
6. Conclusions
AdVirgo will have non-degenerate power and signal recycling cavities, and the baseline design of the NDRC has been selected. This baseline design will reduce the HOMs content in the sidebands used for detector control and in the signal sidebands, being also compliant with the requirements of the sideband resonance condition and the infrastructure constraints.

According to the simulation results presented in this paper, the coupling losses of the NDRC baseline design will be 6.8 %, and the impact of this losses on the detector sensitivity is currently under investigation. Our tolerance analysis shows that the stability of the NDRC baseline design can be settled by tuning the distance that separates the telescope mirrors, and that a TCS will be required for compensating the thermal lensing given by the full circulating power.

An alternative NDRC design has been considered, for which the astigmatism coupling losses would be lowered to 3.3 %, but it may introduce some additional source of noise; this issue will require further investigations.

Acknowledgments
The authors would like to thank all the members of the adVirgo OSD and ISC groups, and Viviana Fafone and Alessio Rocchi of the TCS group for their suggestions and comments during the development of this preliminary analysis.

Appendix A. Laser high order modes
On the plane perpendicular to the direction of propagation \( z \), any intensity distribution of a laser beam can be described by a complete set of orthogonal fields [9], the Hermite-Gauss modes \( U_{mn}(x, y, z) \). Their complex amplitude is given by [9, 10]

\[
U_{mn}(x, y, z) = \left( \frac{2}{\pi m! n! 2^{m+n}} \right)^{1/2} \cdot \frac{1}{w(z)} \cdot \exp \left[ -\frac{x^2+y^2}{w^2(z)} \right] \cdot \exp \left[ -i \left( k \frac{x^2+y^2}{2R(z)} - (m + n + 1) \Phi_G \right) \right] \cdot H_m \left( \frac{\sqrt{2} x}{w(z)} \right) H_n \left( \frac{\sqrt{2} y}{w(z)} \right)
\]

In equation (A.1) \( w(z) \) is the beam radius, \( R(z) \) the beam front curvature, \( \Phi_G = \arctan(z/z_0) \) is the Gouy phase, \( z_0 = \pi w_0^2/\lambda \) is the Rayleigh range, \( w_0 \) is the beam waist; the \( H_{m,n} \) are the Hermite polynomials, and \( q(z) = z + iz_0 \) is the beam complex parameter. These modes are also referred to as TEM\(_{mn}\); the TEM\(_{00}\) is the fundamental gaussian mode, while for \( m \neq 0, n \neq 0 \) the TEM\(_{mn}\) are high order modes.

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