The Modeling of Preference Degree based on Asymmetric Effect

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Abstract. The concept of preference is related favor or support on some opinions or objects. It has potential application values in many social or economic management fields. This paper aims to evolutionary problems of preference of general social economic systems. Preference degree evolutionary dynamics model is proposed by employing ordinary differential equations. The interplay of individuals’ preference is described by this model. Furthermore, numerical results reveal that influence matrix of characterizing individual interactions will conduct essential effects on final distribution of preference.

1. Introduction

The preference refers to specific favor or supporting of behavioral entities on some opinions, objects, which displays a variety of application values on many disciplines. For example, in marketing research discipline, consumers' preference for a certain brand will have a positive impact on their purchase intention and behavior. In recent years, accurate description of customers' preferences and ranking of online real-time data streams have become hot issues (1-3). In the field of management complexity research, interactional effects between individual heterogeneity and collective behavior are investigated in complex organizational systems (4). A compartment dynamic model considering actual transmission path of the symptomatic and asymptomatic infected simultaneously is proposed to discuss evolutionary trend of epidemic situation under different initial population distribution structure (5). In the field of group decision-making, intuitionistic fuzzy sets are employed to define the preference degree of decision-makers' intuitionistic fuzzy values, and a more objective and reasonable scheme ranking algorithm is given (6). In the field of operation management, it is usually necessary to select machines with multiple objective attributes under the streamlined production environment. A multi-attribute decision-making model based on interval preference degree is proposed by developing the VIKOR method with both interval target values of attributes and interval ratings of alternatives on attributes (7). By calculating the ratio of the recommended target score to the maximum goal score, a recommendation algorithm based on customer preference from the aspects of accuracy and customer satisfaction is presented (8). A game theoretical model of one-shot network public goods formalizing the ‘closure argument’ that cooperation is more frequent in denser groups or networks is discussed (9). Further, the model is used to examine analytically the hypothesis that cooperation rates are higher in more dense networks or in more dense parts of networks. In transport research literature, the relationship between information use, social networks and cooperation consciousness in travel behavior is explored with empirical data (10). Also, to construct optimal route planning of public transport network, a novel fuzzy
model which also is taken into account preferences such as the stop’s activity, and count of transit lines passing through the stop besides the walking distance is proposed (11). In the field of tourism marketing, different price designs have different impacts on customers’ travel preferences (12). In medical decision research, a novel multi granularity sequential three-way group conflict decision with preference degree regarding the increment of decision makers and the conflicting characteristics of the decision group is discussed (13).

It can be said that the description and modeling of preference entails a wide range of disciplines, and has attracted the attention of a large number of researchers. Most of the existing researches focus on the static characteristics of preference degree, however, there is no sufficient discussion on the dynamic evolution of preference degree among systems composed of multiple individuals. Based on the system level, this paper focuses on the general preference degree. The dynamic model of preference degree evolution is proposed. The model is used to describe the interaction of preference degrees. Through numerical simulation, it is revealed that the initial correlation matrix among individuals plays an important role in the final distribution of preference degree.

2. Methodology and Model

Suppose there are \( n \) components / objects in the system, and each object has a preference degree, which indicates the preference or support degree of the object. The preference of \( i \) th is denoted as \( x_i(t) \), \( 0 \leq x_i(t) \leq 1 \). The preference degree of whole system is described with an influence matrix, which is denoted as \( A(t) = (a_{ij}(t))_{n \times n} \).

![Figure 1. A schematic of three object preference relations.](image)

2.1. Definitions of Notation

In this paper, we only consider the case that the influence matrix is a constant with constant coefficients, that is, \( a_{ij}(t) \) does not change with time. In this setting, we have \( A(t) = A=(a_{ij})_{n \times n} \). The element \( a_{ij} \) represents the influence of the \( i \) th object on the \( j \) th object, which is called influence coefficients. The value range of \( a_{ij} \) is \( 0 \leq a_{ij} \leq 1 \), and the distribution of initial preference degree is \( x_i(0) = p_i, \sum_{i} p_i = 1, 0 \leq p_i \leq 1 \).

It is noted that there may be a situation in which the influence of the \( i \) th object on the \( j \) th object is not equal to that of the \( j \) th object on the \( i \) th object, which is the so-called asymmetric effect. For example, in reality, the influence between two people A and B is different, and the influence of A on B is much greater than that of B on A. The model is based on the following assumptions:

H1: The change rate of each preference degree is determined by the difference between the weighted sum of preference degrees entering the object and the weighted sum of preference degrees leaving the object. The weight \( \omega_{ij} \) represents the weight of the \( i \) th object's influence on the \( j \) th object, which is directly proportional to the influence coefficient \( a_{ij} \) and inversely proportional to the influence coefficient \( a_{ji} \), that is, the weight \( \omega_{ij} = \frac{a_{ij}}{a_{ji}} \).
2.2. Model Formulation
We first derive formulation from Figure 1. to elaborate the modeling process. Then general description will be conducted after three objects case. Based on the above assumptions H1 and symbols, the situation described in Figure 1. could be formulated as:

\[
\begin{align*}
\frac{dx_1}{dt} &= \left( \frac{a_{12}}{a_{21}} x_2 + \frac{a_{13}}{a_{31}} x_3 \right) - \left( \frac{a_{21}}{a_{12}} x_1 + \frac{a_{31}}{a_{13}} x_1 \right) \\
\frac{dx_2}{dt} &= \left( \frac{a_{21}}{a_{12}} x_1 + \frac{a_{23}}{a_{32}} x_3 \right) - \left( \frac{a_{32}}{a_{23}} x_2 + \frac{a_{23}}{a_{32}} x_2 \right) \\
\frac{dx_3}{dt} &= \left( \frac{a_{31}}{a_{13}} x_1 + \frac{a_{32}}{a_{23}} x_2 \right) - \left( \frac{a_{32}}{a_{31}} x_3 + \frac{a_{32}}{a_{31}} x_3 \right)
\end{align*}
\]

(1)

More generally, we can establish the following preference degree evolutionary dynamics model (PDEDM) described by autonomous ordinary differential equations:

\[
\begin{align*}
\frac{dx_i(t)}{dt} &= \sum_{i \neq j} (a_{ij} / a_{jj}) x_j(t) - \sum_{i \neq j} (a_{ji} / a_{jj}) x_i(t), \\
&= \sum_{i \neq j} \left( \frac{a_{ij}}{a_{jj}} - \frac{a_{ji}}{a_{jj}} \right) x_j(t), \quad i, 1, 2, \ldots, n
\end{align*}
\]

(2)

If formula (2) is written into vector forms, we'll have:

\[
\frac{dx}{dt} = Bx, \quad x_0 = p_0
\]

(3)

Here, the independent variable is the column vector \( x = (x_1, \ldots, x_n)^T \), coefficient matrix \( B \) can be derived by above formula (2). Initial value is \( x_0 = (x_1(0), \ldots, x_n(0))^T = (p_1, \ldots, p_n)^T = p_0 \). According to the basic theory of ODE, for the linear differential equations with constant coefficients, there exits always basic solution matrix \( \Phi(t) = e^{At} \), which is the solution of dynamic model of preference degree evolution PDEDM. Therefore, it can be guaranteed that for any given initial value of preference dynamic model, the final preference distribution results can be obtained.

3. Results
A simple numerical simulation corresponding to Figure 1. will conducted to illustrate how to use the proposed PDEDM. Two different initial preference degrees are given, the first is \( x_0^a = (1/3, 1/3, 1/3) \), the second is \( x_0^b = (2/3, 1/3, 0) \).

3.1. Case 1
Assume the influence matrix is \( A = \begin{pmatrix} 0 & 0.11 & 0.12 \\ 0.21 & 0 & 0.22 \\ 0.31 & 0.32 & 0 \end{pmatrix} \). The numerical simulation results of preference degree evolution are shown in Figure 2. In what follows, we’ll use abbreviation PD to represent preference degree.
4

Figure 2. Preference degree evolution of case 1.

Figure 2 (a) on the left of Figure 2 shows the distribution results of the first group of initial preference degrees, and Figure 2 (b) on the right shows the distribution results of the second group of initial preference degrees. It can be observed that the stable preference degrees obtained by the two groups of initial preference parameters are exactly the same. They all equals $x(t) = (0.08768212, 0.29807011, 0.61424777)$. In the given influence matrix, the influence coefficient of the three objects is $a_{31} > a_{21}, a_{32} > a_{12}, a_{33} > a_{13}$, i.e, the influence coefficient of the third object is largest, the second is the second, and the first is the smallest. Therefore, the final evolution stability preference degree is also satisfied $x_3(t) > x_2(t) > x_1(t)$.

3.2. Case 2

Assume the influence matrix is $A = \begin{pmatrix} 0 & 0.13 & 0.13 \\ 0.22 & 0 & 0.32 \\ 0.22 & 0.32 & 0 \end{pmatrix}$. The numerical simulation results of preference degree evolution are shown in Figure 3.

Figure 3. Preference degree evolution of case 2.

Figure 3 (a) on the left of Figure 3 shows the distribution results of the first group of initial preference degrees, and Figure 3 (b) on the right shows the distribution results of the second group of initial preference degrees. It can be observed that the stable preference degrees obtained by the two groups of initial preference parameters are exactly the same. They all equals $x(t) = (0.14863676, 0.42568162, 0.42568162)$. In the given influence matrix $A$, the influence coefficient of the three objects satisfied with $a_{21} = a_{11}, a_{23} > a_{13}, a_{32} > a_{12}, a_{23} = a_{32}, a_{13} = a_{12}$, i.e., the influences of second and third objects on first object are equal, they are all bigger than the influences of first on second and third objects. Moreover, the influences between second and third object are equal.
The influence of first on third equals to the influence of first on second. Therefore, the final evolution stability preference degree satisfies $x_3(t) > x_2(t) > x_1(t)$.

4. Discussion and Conclusion

(1) For the dynamic model of preference evolution with constant coefficients, the solution of the model is determined by the initial influence matrix $A$, which is independent of the initial value of the object preference degree. That is, regardless of the initial preference degree distribution, the final preference degree is determined by the initial influence matrix $A$, which corresponds to the two numerical results in Section 3.

(2) The structure of the initial influence matrix completely determines the distribution structure of the final preference degree. If two objects have the same influence on other objects, which means the relative value of the influence coefficient of the two objects is the same, the final preference degree of the two objects will tend to be consistent. Corresponding to Figure 3.(b) in Section 3, although initial preference degree of the second and third objects are different (initial preference degree of second object equals $1/3$ and third object equals $0$), while they tend to the same stable value in the end.

As an important concept to describe the individual's psychological and behavioral characteristics, it is of great significance to model and analyze the user's preference. For example, in travel behavior research, understanding user's traveling choice preference accurately will be very beneficial to reach more accurate travel demand forecast. Also, in marking research, how to appropriately understand customer preferences is very important to develop a more reasonable marketing strategy as well. In this paper, a dynamic model of preference evolution is proposed, which is based on the constant coefficient influence matrix. In the future, the evolution of preference degree under the condition of influence matrix with time-varying could be explored further. In addition, the conclusion of this paper can be verified by appropriate experiment approach.

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