RELATIONSHIP OF PIONIUM LIFETIME WITH PION SCATTERING LENGTHS IN GENERALIZED CHIRAL PERTURBATION THEORY

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The pionium lifetime is calculated in the framework of the quasipotential-constraint theory approach, including the sizable electromagnetic corrections. The framework of generalized chiral perturbation theory allows then an analysis of the lifetime value as a function of the \( \pi\pi \) S-wave scattering lengths with isospin \( I = 0, 2 \), the latter being dependent on the quark condensate value.

The DIRAC experiment at CERN is expected to measure the pionium lifetime with a 10% accuracy. The pionium is an atom made of \( \pi^+\pi^- \), which decays under the effect of strong interactions into \( \pi^0\pi^0 \). The physical interest of the lifetime is that it gives us information about the \( \pi\pi \) scattering lengths. The nonrelativistic formula of the lifetime was first obtained by Deser et al.\(^1\):\(^{1}\)

\[
\frac{1}{\tau_0} = \frac{16\pi}{9} \sqrt{\frac{2\Delta m_\pi}{m_{\pi^+}} \left( a^0_{0} - a^2_{0} \right)^2 \frac{m_{\pi^+}}{m_{\pi^+}^2} |\psi_{++}(0)|^2}, \quad \Delta m_\pi = m_{\pi^+} - m_{\pi^0}, \quad (1)
\]

where \( \psi_{++}(0) \) is the wave function of the pionium at the origin (in \( x \)-space) and \( a^0_{0}, a^2_{0} \), the S-wave scattering lengths with isospin 0 and 2, respectively.

The evaluation of the relativistic corrections to this formula can be done in a systematic way in the framework of chiral perturbation theory (\( \chi PT \)) in the presence of electromagnetism. There arise essentially two types of correction. (i) The pion-photon radiative corrections, which are similar to those met in conventional QED. (ii) The quark-photon radiative corrections, which appear through terms where the photon field is not explicitly present and which are mainly responsible for the pion mass difference at lowest-order.

Three different methods of evaluation have been used for the study of the pionium bound state in the framework of \( \chi PT \). The first method uses a three-dimensionally reduced form of the Bethe–Salpeter equation within the quasipotential–constraint theory approach. The second method uses the Bethe–Salpeter equation with the Coulomb gauge. The third one uses the approach of nonrelativistic effective theory. All the above approaches lead to similar estimates, on the order of 6%, for the relativistic corrections to the nonrelativistic formula of the pionium decay width.
The theoretical interest of the $\pi\pi$ scattering lengths is that they allow us to estimate the value of the quark condensate in QCD. The fundamental order parameter of spontaneous chiral symmetry breaking being $F_\pi$, the pion weak decay constant, other order parameters may eventually vanish in the chiral limit without contradicting chiral symmetry breaking, as long as $F_\pi$ remains different from zero in that limit. Such an issue is intimately dependent on the mechanism of chiral symmetry breaking. In standard $\chi PT$, it is assumed that the quark condensate parameter, defined as $\langle 0 | \bar{q} q | 0 \rangle / F_\pi^2$, is on the order of the hadronic mass scale ($\sim 1$ GeV). This hypothesis is verified in the sigma-model and the Nambu–Jona-Lasinio model. The vacuum state here is similar to a ferromagnetic type medium. On the other hand, in an antiferromagnetic type medium, one would have a vanishing quark condensate and yet chiral symmetry would still be broken. An intermediate possibility, due to an eventual phase transition in QCD for large values of the light quark flavor number, was also advocated recently.

Generalized $\chi PT$ is a framework in which the quark condensate value is left as a free parameter subjected to an experimental evaluation. The Goldstone boson scattering amplitudes are sensitive to the quark condensate value and hence their experimental measurement gives us an estimate of the latter quantity. Thus, in the $\pi\pi$ scattering amplitude relatively small values of the $S$-wave isospin-0 scattering length $a_0^0$, on the order of, say, 0.21-0.22, correspond to the predictions of standard $\chi PT$, while relatively large values of $a_0^0$, on the order of, say, 0.28-0.36, correspond to small values of the quark condensate parameter.

We have redone the analysis of the pionium lifetime in the framework of generalized $\chi PT$. Eliminating the quark condensate parameter in favor of the combination $(a_0^0 - a_2^0)$ we have calculated the pionium lifetime as a function of $(a_0^0 - a_2^0)$. The corresponding curve is presented in Fig. 1.

Values of the lifetime close to 3 fs, lying above 2.9 fs, say, would confirm the scheme of standard $\chi PT$. Values of the lifetime lying below 2.4 fs remain outside the domain of predictions of standard $\chi PT$ and would necessitate an alternative scheme of chiral symmetry breaking. Values of the lifetime lying in the interval 2.4-2.9 fs, because of the possibly existing uncertainties, would be more difficult to interpret and would require a more refined analysis.

References

1. S. Deser, M. L. Goldberger, K. Baumann and W. Thirring, Phys. Rev. 96, 774 (1954).
2. J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) 158, 142 (1984).
Figure 1: The pionium lifetime as a function of the combination $(a_0^S - a_2^S)$ of the S-wave scattering lengths (full line). The band delineated by the dotted lines takes into account the estimated uncertainties (2-2.5%).

3. R. Urech, *Nucl. Phys. B* **433**, 234 (1995); M. Knecht and R. Urech, *Nucl. Phys. B* **519**, 329 (1998).
4. H. Jallouli and H. Sazdjian, *Phys. Rev. D* **58**, 014011 (1998); 099901(E).
5. M. A. Ivanov, V. E. Lyubovitskij, E. Z. Lipartia and A. G. Rusetsky, *Phys. Rev. D* **58**, 094024 (1998).
6. A. Gall, J. Gasser, V. E. Lyubovitskij and A. Rusetsky, *Phys. Lett. B* **462**, 335 (1999); J. Gasser, V. E. Lyubovitskij and A. Rusetsky, *Phys. Lett. B* **471**, 224 (1999).
7. H. Leutwyler, *Phys. Rev. D* **49**, 3033 (1994).
8. J. Stern, preprint [hep-ph/9801282](https://arxiv.org/abs/hep-ph/9801282).
9. S. Descotes, L. Girlanda and J. Stern, *JHEP* **01**, 041 (2000); S. Descotes and J. Stern, *Phys. Rev. D* **62**, 054011 (2000); preprint [hep-ph/0007082](https://arxiv.org/abs/hep-ph/0007082).
10. J. Stern, H. Sazdjian and N. H. Fuchs, *Phys. Rev. D* **47**, 3814 (1993).
11. H. Sazdjian, *Phys. Lett. B* **490**, 203 (2000).