Comment on “No enhancement of the localization length for two interacting particles in a random potential”

In a recent letter [1] Römer and Schreiber report on numerical calculations that led them to conclude that the previously observed enhancement [2] of the localization length \( L_2 \) of two interacting particles (TIP) vanishes in the thermodynamic limit. Such a claim (i) is in conflict with the scaling theory of localization, (ii) ignores a consistent picture from a wealth of published numerical data and analytical investigations, and (iii) directly contradicts new numerical results obtained with a Green function method [4] that is well adapted to the problem under study.

(i) Römer and Schreiber considered system sizes \( L \) between 80 and 360 for their extrapolation. For \( L = 360 \) they find \( L_2 \approx 18 \), i.e. \( L_2 \) exceeds the one-particle localization length \( L_1 \approx 12 \) significantly. How can these extended states shrink upon increasing the system size from 20 to, say, 25? According to Thouless’ scaling picture, the energy shift associated with such an increase of \( L \) amounts to a negligible fraction of the mean level spacing, and no significant effect can be expected.

(ii) Let \( n_1 \) and \( n_2 \) be the sites of the two particles. Shepelyansky showed [2] that, for a small fraction \((\approx L_1/(2L))\) of states, the center of mass \((n_1+n_2)/2\) is extended over \( L_2 \), while \(|n_1-n_2| \ll L_1 \). Alternative and complementary derivations are provided by Imry’s variant [5] of the Thouless scaling picture, and by an effective \( \sigma \) model [6], both not restricted to finite \( L \). Three different numerical methods have confirmed this effect.

First, extended states of the type described above were obtained by exact diagonalization [5] of the TIP Hamiltonian in a ring geometry. Second, the delocalization of the center of mass was studied in Ref. [6] by calculating the TIP Green function \( \langle n_2|G(E)|n_1 \rangle \) for \( n_1 = n_2 = 0 \) and \( n_1 = n_2 = L \), i.e. pair transfer from 0 to \( L \). Third, the transport of one particle through a relatively small sample containing the second particle was investigated [7] with the transfer matrix method. They all reach a common conclusion: For an on-site interaction \( U = 1 \) and \( L_1 \approx 11 \), one has \( L_2 \approx 25 > L_1 \). Comparable results were obtained in infinite systems [6] with a prescribed maximum separation \(|n_1-n_2|\) (“bag model”).

(iii) Römer and Schreiber used transfer matrix methods (similar to those in Ref. [6]), and concluded that \( L_2 \to L_1 \) when \( L \to \infty \). In Fig. 1, we compare data from Ref. [1] with results from the Green function method \((U = 1, L_1 \approx 11)\). The latter consistently gives \( L_2 \approx 25 \approx 2L_1 \) up to the maximum size we considered, \( L = 1000 \). Without explaining the data of Ref. [1] in detail, we emphasize the following important problem: The \( L^2 \) entries in the transfer matrix correspond to the \( L^2 \) functions \( \langle 0|n_2|G(E)|L_{n_2} \rangle \). Most of these characterize single particle transport (decay scale \( L_1 \)), only those with \( n_2 \lesssim L_1, n_2' \gtrsim L-L_1 \) describe pair transport (decay scale \( L_2/2 \)), with the conventions of Ref. [1]. To distinguish these two scales one has to either suppress single particle transport (Green function method, bag model) or ensure that \( L_2 \gg 2L_1 \) (as in Ref. [4]). Unfortunately, Ref. [1] meets neither of these requirements, and it is unclear, which scale is actually measured.

In conclusion, the central claim in Ref. [1] is due to a serious misinterpretation, and the localization length \( L_2 \) of the interaction-assisted states is independent of \( L \), as expected on physical grounds.

FIG. 1. Data for \( L_2 \) from Ref. [1] (squares) and the Green function method (circles) versus \( L^{-1/2} \). We thank F. v. Oppen and T. Wettig for their computer program.

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