The differences of the masses of isotopes with atomic numbers between $\sim 10$ and $\sim 30$ can be described within the chiral soliton model in satisfactory agreement with data. The rescaling of the model is necessary for this purpose — decrease of the Skyrme constant by $\sim 30\%$, providing the “nuclear variant” of the model. The asymmetric term in Weizsacker—Bethe—Bacher mass formula for nuclei can be obtained as the isospin dependent quantum correction to the nucleus energy. Some predictions of the binding energies of neutron rich isotopes are made in this way from, e.g. $^{16}$Be, $^{19}$B to $^{31}$Ne or $^{32}$Na. The neutron rich nuclides with high values of isospin are unstable relative to the decay due to strong interactions. The SK4 (Skyrme) variant of the model, as well as SK6 variant (sixth order term in chiral derivatives in the Lagrangian as solitons stabilizer) are considered, the rational map approximation is used to describe multiskyrmions.

1. Introduction

In the absence of the complete theory of strong interactions and nuclear forces the checking of fundamental principles, which are believed to hold in any theoretical model, can be useful and of great importance. The description of some well established and rather general properties of nuclei, as well as searches and studies of unusual forms of matter, in particular the neutron rich nuclides, may provide important source of lacking information and impact for the development of new concepts and ideas.

The effective field theories are the powerful tool for the studies not only of mesons, baryons and their interactions at low energy [1–3], but also baryonic systems (nuclei) which appear as quantized bound states of skyrmions with baryon number $B \geq 2$. The properties of the deuteron and $^3$He, $^3$H and $^4$He were explained semiquantitatively [4–5] starting from few basic principles and ingredients contained in the effective chiral Lagrangian of the Skyrme model [1] or its modifications (short review of early results can be found, e.g. in [6]).

For the $B = 2$ system the quantization of the bound state of skyrmions possessing originally characteristic torus-like form [7] allows to obtain the deuteron and the singlet $NN$-scattering state with isospin $I = 1$ [8]. The binding energy of the deuteron is about $\sim 30$ MeV when zero modes quantum corrections are taken into account. It decreases to $\sim 6$ MeV when some nonzero modes quantum corrections are included [9]. Some states with positive parity and unusual connection between isospin and angular momentum are predicted also, which can reveal themselves as supernarrow radiatively decaying dibaryon [10], as well as dibaryons with negative parity. The experimental indications on the existence of such states have been obtained recently [11, 12]. Another issue is the binding energy of strange hypernuclei with atomic number up to $\sim 16$ which can be described qualitatively under some natural assumptions, and predictions of more bound charmed or beautiful hypernuclei [13].

Here we shall discuss the nuclides with baryon number $A = B \leq 32$: mass splittings of isotopes with different $N, Z$, including neutron rich nuclides with the difference $N - Z$ up to 10 and 11, $N, Z$ are the numbers of neutrons and protons in the nucleus, $N + Z = A$. Some
extrapolation and modification of the approach applied previously [4, 5, 13] allows to make certain predictions and conclusions also in this case. In particular, the symmetry energy term in the Weizsacker—Bethe—Bacher mass formula for nuclei [14], \(E_{\text{sym}} = b_{\text{sym}}(N - Z)^2/2A\) can be described successfully (\(b_{\text{sym}} \simeq 50\) MeV). The presence of this term (more correctly, it should be called the asymmetry energy since it appears due to difference of \(N\) and \(Z\) numbers) combined with the energy of Coulomb repulsion of protons, \(E_C \simeq 0.6Z^2e^2/R_Z\), leads to the observed excess of neutrons in nuclei (see, e.g. [15, 16]). Within the chiral soliton approach this symmetry energy originates from isotopical rotations of multiskyrmions and has the form

\[
E_{\text{sym}} = I(I + 1)/(2\Theta I),
\]

with isospin \(I = (N - Z)/2\) for the states of lowest energy (we shall consider mainly the case \(N > Z\), as it follows from presence of the Coulomb repulsion). Within the conventional approach this term appears due to dependence of the Fermi-motion kinetic energies \(\epsilon_N, \epsilon_Z\) of protons and neutrons on \(N\) and \(Z\) numbers [15]:

\[
b_{\text{sym}} = \frac{2}{3}(\epsilon_Z + \epsilon_N),
\]

\(\epsilon_Z \simeq k_F(p)^2/2M\), with Fermi-momentum of protons in the nucleus \(k_F(p) = (3\pi^2Z/V_p)^{1/3}\), \(V_p\) being the volume occupied by protons in the nucleus, and similarly for \(\epsilon_N\) (\(V_n\) can be different from \(V_p\)). Numerically this gives for heavy nuclei \(b_{\text{sym}} \simeq 24\) MeV if the radius of the nucleus (for protons and neutrons) \(R_A = 1.12A^{1/3}\) fm, and approximately the same contribution give potential terms [15]. The presence of the linear in isospin \(I\) term shifts slightly the balance of energies \(E_{\text{sym}}\) and \(E_C\) towards larger values of \(N\), in better agreement with data, especially for smaller \(A\).

The question, if the neutron rich light nuclides may exist, which we address here as well, seems to be more viable now when the beams of nuclides become available, and the possibilities for production and observation of such states have been increased [17–20]. After short theoretical introduction in the next section we discuss qualitatively situation with the mass splittings of relatively light nuclear isotopes (Section 3), give the results for differences of binding energies of isotopes with integer isospin (Section 4) and half-integer isospin, odd atomic numbers (Section 5). Some predictions for binding energies of neutron rich nuclides are given in Sections 4 and 5. Discussion of our method and results is presented in concluding Section 6.

2. Theoretical background

The brief description of the chiral soliton approach and some basic formulas are necessary. The \(SU(2)\) skyrmions are described by the Lagrangian density of the effective field theory, depending on the \(SU(2)\) matrix \(U = f_0 + i(\tau_1f_1 + \tau_2f_2 + \tau_3f_3)\). The Lagrangian in its “minimal” form, suggested by Skyrme [1], is

\[
L = -\frac{F^2}{16}\text{Tr}L_\mu L^\mu + \frac{1}{32e^2}\text{Tr}[L_\mu L_\nu]^2 + \frac{F^2m^2}{16}\text{Tr}(U + U^\dagger - 2)
\]

(1)

with \(L_\mu = \partial_\mu UU^\dagger = i\lambda_\mu \cdot \tau\).

The sixth order in chiral derivatives term also can be included into consideration [21]:

\[
L_6 = -c_6\text{Tr}\left(\left[L_\mu L^\nu\right]\left[L_\nu L^\gamma\right]\left[L_\gamma L^\mu\right]\right)
\]

1) Probably, one of the first attempts to include the sixth order term was made in [7] where the bound \(B = 2\) torus-like configuration was found, similar to the case of fourth order, or Skyrme term.
with dimensional constant $c_6$. The important property of the Lagrangian (1) is that it is proportional to the number of colors $N_c$ of the underlying quantum chromodynamics — theory of colored quarks and gluons and their interactions [22]. The effective field theory described by Lagrangian density (1) is being built from this QCD.

The baryon, or winding, number is the fourth component of the Noether current generated by the Wess-Zumino term in the action [22], and in these notations equals to

$$B = -\frac{1}{4\pi^2} \int \text{Tr}(L_1 L_2 L_3) d^3r = -\frac{1}{2\pi^2} \int (l_1 l_2 l_3) d^3r,$$

(2)

$(\mathbf{abc})$ denotes the mixed product of vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. We shall rewrite it in less conventional but transparent form

$$B = -\frac{1}{2\pi^2} \int (\bar{\partial} f_1 \bar{\partial} f_2 \bar{\partial} f_3)/f_0 d^3r =$$

$$= -\frac{1}{\pi^2} \int (f_0^2 + f_1^2 + f_2^2 + f_3^2 - 1)(\bar{\partial} f_1 \bar{\partial} f_2 \bar{\partial} f_3)' f_0 d^3r.$$  

(3)

Three functions are introduced usually to describe $f_i$: $f_0 = c_F$, $f_k = s_F n_k$, where the components of the unit vector $\mathbf{n}$ are $n_1 = s_\alpha c_\beta$, $n_2 = s_\alpha s_\beta$, $n_3 = c_\alpha$, $c_F = \cos F$, $s_F = \sin F$, $s_\alpha = \sin \alpha$, etc. Then $(\bar{\partial} f_1 \bar{\partial} f_2 \bar{\partial} f_3)/f_0 = s_\alpha^2 (\bar{\partial} F \bar{\partial} \alpha \bar{\partial} \beta)$, and the winding number

$$B = -\frac{1}{2\pi^2} \int s_\alpha^2 I[(F, \alpha, \beta)/\{(x, y, z)\}] d^3r.$$  

(4)

with $I[(F, \alpha, \beta)/(x, y, z)]$ — Jacobian of the transformation from variables $\mathbf{r} = x, y, z$ to $F, \alpha, \beta$. Since the element of the unit 3-dimensional sphere $dS^3 = s_\alpha^2 s_\beta dF d\alpha d\beta$, and $2\pi^2$ is the surface of the unit sphere, Eq. (4) shows how many times the sphere $S^3$ (homeomorphic to $SU(2)$) is covered when integration over $d^3r$ is made, so it is the degree of the map $R^3 \to SU(2)$.

The static energy of arbitrary $SU(2)$ skyrmion in these notations is

$$M = \int \left\{ \frac{F^2}{8} \left[ (\bar{\partial} F)^2 + s_F^2 (\bar{\partial} \alpha)^2 + s_\alpha^2 (\bar{\partial} \beta)^2 \right] + \right.$$

$$+ \frac{s_F^2}{2c_\alpha^2} (\bar{\partial} F \times \bar{\partial} \alpha)^2 + s_\alpha^2 (\bar{\partial} F \times \bar{\partial} \beta)^2 + s_F^2 s_\alpha^2 (\bar{\partial} \alpha \times \bar{\partial} \beta)^2 + m_F^2 F^2 (1 - c_F)/4 +$$

$$+ 96c_6 s_F^4 s_\alpha^2 (\bar{\partial} F \bar{\partial} \alpha \bar{\partial} \beta)^2 \right\} d^3r.$$  

(5)

$[\mathbf{a} \times \mathbf{b}]$ means the vector product of $\mathbf{a}$ and $\mathbf{b}$. The mass (5) is proportional to the number of colors of underlying QCD, i.e. it is large at large $N_c$.

The rational map approximation (RM) proposed in [23] is very effective for description of multibaryons with large enough baryon numbers. The direct numerical calculation of configurations of minimal energy [24] provides the results very close to those obtained within the RM approximation. The profile function $F$ within the RM approximation depends on variable $r$ only, and the unit vector $\mathbf{n}$ — on angular variables $\theta, \phi$: $n_1 = 2 \Im R/(1 + |R|^2)$, $n_2 = 2\Im R/(1 + |R|^2)$ and $n_3 = (1 - |R|^2)/(1 + |R|^2)$, $R(\theta, \phi)$ being a rational function of variable $z = \tan(\theta/2) \exp(i\phi)$ defining the map of degree $\mathcal{N}$ of the 2-dimensional spheres $S^2 \to S^2$. We shall use the following notations, introduced in [23]:

$$\mathcal{N} = \frac{1}{8\pi} \int r^2 (\partial_n \mathbf{n})^2 d\Omega = \frac{1}{4\pi} \int \frac{2i dR d\bar{R}}{(1 + |R|^2)^2},$$  

(6)
\[ I = \frac{1}{4\pi} \int r^4 \frac{[\vec{\partial} n_1 \times \vec{\partial} n_2]^2}{n_3^2} d\Omega = \frac{1}{4\pi} \int \left[ \frac{1 + |z|^2}{1 + |R|^2} \frac{|dR|}{d|z|} \right]^4 \frac{2i dz d\bar{z}}{(1 + |z|^2)^2}, \]
\[ d\Omega = \sin \theta d\theta d\phi = 2i dz d\bar{z}/(1 + |z|^2)^2. \]

For configurations of lowest energy \( N = B \) and \( F(0) - F(\infty) = \pi \). The classical mass of the configuration for the RM ansatz can be written as [23, 21]
\[ M_{\text{RM}} = \frac{\pi F_\pi}{e'} \int \left\{ r^2 F r^2 + 2B s_F^2 [1 + (1 - \lambda) F r^2] + \frac{I_{\tau}^4}{r^2} \right\} dr + \text{M.t.}, \]
where the distance \( r \) is measured in units \( 2/(F_\pi e') \), \( e' = e\sqrt{1 - \lambda} \), \( \lambda \) defines the weight of the sixth order term in the Lagrangian according to the relation \( \lambda/(1 - \lambda)^2 = 48c_6 F_\pi^2 e^4 \). If \( \lambda = 0 \), \( c_6 = 0 \) and we obtain original variant of the Skyrme model (SK4 variant), \( \lambda = 1 \) corresponds to the pure SK6 variant. In this case relation takes place: \( e' = 1/(48c_6 F_\pi^2)^{1/4} \)
M.t. denotes the contribution of the mass term.

The energy of the system depending on the angular velocities of rotations in \( SU(2) \) isospin collective coordinate space can be written in such form:
\[ L_{\text{rot}} = \left( \frac{F_\pi^2}{16} + \frac{I_1^2 + I_2^2 + I_3^2}{8e^2} \right) (\bar{\omega}_1^2 + \bar{\omega}_2^2 + \bar{\omega}_3^2) + \frac{\left( \bar{\omega}_1 I_1 + \bar{\omega}_2 I_2 + \bar{\omega}_3 I_3 \right)^2}{8e^2}. \]
The angular velocities of rotation in the isospace are defined in standard way: \( A^i \dot{A} = -\frac{1}{2} \vec{\omega} \cdot \vec{\tau} \).
The functions \( \bar{\omega}_i \) are connected with the body fixed angular velocities of \( SU(2) \) rotations by means of transformation:
\[ \bar{\omega} \cdot \vec{\tau} = U^\dagger \vec{\omega} \cdot \vec{\tau} U - \vec{\omega} \cdot \vec{\tau}, \]
with \( I_1^2 + I_2^2 + I_3^2 = (\bar{\partial} f_0)^2 + (\bar{\partial} f_1)^2 + (\bar{\partial} f_2)^2 + (\bar{\partial} f_3)^2 \) and \( \bar{\omega}^2 = 4[\vec{\omega}^2 \vec{f}^2 - (\vec{\omega} \cdot \vec{f})^2] \). The coefficients in the quadratic form (9) in \( \omega_i \omega_k \) define the \( SU(2) \) (isotopic) moments of inertia of arbitrary systems of \( SU(2) \) skyrmions. In the axially symmetrical case and in RM approximation they are not complicated and were written explicitly in [8, 25, 26].

Some further complication is necessary because one should introduce another set of collective coordinates [8] to take into account the usual space rotations which we shall describe by angular velocities \( \Omega_i, i = 1, 2, 3 \). The part of the Lagrangian describing both kinds of rotations can be written then [8]:
\[ L_{\text{rot}} = \frac{1}{2} U_{ij} \omega_i \omega_j + \frac{1}{2} V_{ij} \Omega_i \Omega_j - W_{ij} \omega_i \Omega_j, \]
where tensors \( U_{ij}, V_{ij}, \) and \( W_{ij} \) are defined by the configuration of lowest energy for each value of \( B \), i.e. they are functions of \( F, \alpha, \) and \( \beta \).

The body fixed isospin and spin angular momentum operators \( K_l = \partial L_{\text{rot}}/\partial \omega_l \) and \( L_l = \partial L_{\text{rot}}/\partial \Omega_l \) are linear combinations of \( \omega_i \) and \( \Omega_i \). Three diagonal moments of inertia and three off-diagonal define each of three rotation terms in (11) in most general case. In the case of axially symmetric systems we obtained four different diagonal moments of inertia: \( \Theta_1 = \Theta_2 = \Theta_N; \Theta_3; \Theta_R \) and \( \Theta_{\text{int}} \) [82]. The static masses of solitons and momenta of inertia were calculated previously for \( B \leq 32 \) starting from rational map ansatz [23, 24] and using the variational minimization program [25, 26], see Table 1. It should be noted that expressions (1)–(9) contain all information which is necessary for the description of arbitrary \( SU(2) \) skyrmions, in such minimal form. It was sufficient for the fitting of the

\(^2\)For the hedgehog-type configurations the isospin and usual space tensors of inertia coincide since isospin and usual space rotations are identical for the hedgehogs, and the configuration is described by one common moment of inertia.
mass differences of the nucleon and $\Delta$ isobar [2, 3] and for the description of many basic properties of light nuclei [4, 5].

The moments of inertia, isotopical and orbital, necessary for our purpose here, are given by the expressions:

$$\Theta_I = \frac{1}{3} \Theta_{I,aa} = \frac{2\pi}{3} \int s^2 F_{\pi}^2 + \frac{4}{e^2} \left[ (1 - \lambda) \left( f'^2 + N^2 \frac{s_I^2}{r^2} \right) + 4\lambda N f'^2 \frac{s_I^2}{(F_\pi e r)^2} \right] r^2 dr;$$

(12)

$$\Theta_J = \frac{1}{3} \Theta_{J,aa} = \frac{2\pi}{3} \int s^2 N F_{\pi}^2 + \frac{4}{e^2} \left[ (1 - \lambda) \left( N f'^2 + T \frac{s_I^2}{r^2} \right) + 4\lambda N f'^2 \frac{s_I^2}{(F_\pi e r)^2} \right] r^2 dr.$$  (13)

The inequalities take place for any value of $\lambda$ [25, 26]:

$$\frac{T}{B} \Theta_I \geq \Theta_J \geq B \Theta_I,$$  (14)

which ensures that the quantum correction due to usual space collective rotations is always smaller (even much smaller at large $A = B$) than isotopical quantum correction. The moments of inertia (12), (13) are proportional to the number of colors $N_c$, similar to the classical mass of solitons, therefore, the rotational quantum corrections are proportional to $\sim 1/N_c$, i.e. they are small, and the whole consideration becomes selfconsistent at large $N_c$. The fact that $N_c = 3$ in our world makes the real life somewhat more complicated.

Considerable simplification takes place because of the symmetry properties of many multiskyrmion configurations, beginning with $B = 3$, 7 and some other configurations found in [23, 24]. It turned out that all three tensors of inertia in (11) at large $B$ approximately are proportional to the unit matrix, i.e. $U_{ij} \simeq u \delta_{ij}, V_{ij} \simeq v \delta_{ij}, W_{ij} \simeq w \delta_{ij}$. Moreover, in many cases of interest the interference tensor of inertia is negligibly small, $w \ll u$ and $w \ll v$ [26, 27]; the relative magnitude of this interference inertia decreases with increasing baryon number, and we shall neglect it for numerical estimates.

The mass splitting between nucleon and $\Delta(1232)$ — usually one of the first quantities fitted within the chiral soliton approach — is described almost equally for different choices of the parameters used in the literature [2, 3, 26] (it scales like $\sim F_\pi e^3$ approximately), but dimensions of solitons are different since they scale like $1/(F_\pi e)$. One of the possible choices of the parameters of the model is [28], $F_\pi = 186$ MeV, $e = 4.12$, and we shall use these numbers for the “nucleon” variant of the model. Dimensions of multiskyrmions, the classical field configurations, are much smaller than dimensions of observable quantized states, i.e. nuclei. The latter are reproduced only when the vibration and breathing motions in the solitons are taken into account, as it was shown recently for the deuteron in [9]. The numerical values of inertia are given in Table 1 for the SK4 ($e = 4.12$ and $e = 3.0$, marked with *) and SK6 variants of the model, $e' = 4.11$ and 2.84, also marked with *. The values $e = 3.0$ and $e' = 2.84$ for the SK6 variant allow to describe well the isotopical splittings of nuclei with atomic numbers between $\sim 10$ and 32, see discussion in the following sections.
### Table 1

| $B$ | $\Theta_J$(SK4) | $\Theta_J$(SK4) | $\Theta_J$(SK6) | $\Theta_J$(SK6) | $\Theta_J$(SK6)* | $\Theta_J$(SK6)* |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1   | 5.55 | 5.55 | 5.13 | 5.13 | 12.8 | 14.2 |
| 2   | 10.4 | 22.9 | 9.26 | 21.9 | 24.3 | 25.7 |
| 3   | 14.8 | 49.7 | 12.7 | 46.0 | 34.7 | 35.5 |
| 4   | 18.2 | 78.3 | 15.2 | 68.8 | 42.9 | 43.2 |
| 5   | 22.7 | 127 | 18.7 | 111 | 53.5 | 52.9 |
| 6   | 26.6 | 178 | 21.7 | 153 | 62.6 | 61.4 |
| 7   | 29.5 | 221 | 23.9 | 186 | 69.7 | 68.1 |
| 8   | 33.9 | 298 | 27.2 | 251 | 79.9 | 77.4 |
| 9   | 37.8 | 376 | 30.2 | 315 | 89.0 | 85.7 |
| 10  | 41.4 | 455 | 32.9 | 379 | 97.4 | 93.5 |
| 11  | 45.1 | 547 | 35.8 | 455 | 106 | 102 |
| 12  | 48.5 | 637 | 38.4 | 526 | 114 | 109 |
| 13  | 52.1 | 737 | 41.1 | 606 | 122 | 117 |
| 14  | 56.1 | 865 | 44.3 | 712 | 132 | 125 |
| 15  | 59.8 | 987 | 47.0 | 811 | 140 | 133 |
| 16  | 63.2 | 1110 | 49.7 | 908 | 148 | 141 |
| 17  | 66.2 | 1220 | 52.1 | 996 | 155 | 148 |
| 18  | 70.3 | 1380 | 55.3 | 1130 | 164 | 156 |
| 19  | 73.8 | 1540 | 58.0 | 1260 | 173 | 164 |
| 20  | 77.4 | 1700 | 60.9 | 1390 | 181 | 172 |
| 21  | 80.8 | 1860 | 63.5 | 1520 | 189 | 179 |
| 22  | 84.3 | 2030 | 66.2 | 1660 | 197 | 186 |
| 23  | 88.0 | 2220 | 69.0 | 1810 | 205 | 194 |
| 24  | 91.3 | 2400 | 71.5 | 1950 | 213 | 202 |
| 25  | 94.7 | 2590 | 74.3 | 2110 | 221 | 209 |
| 26  | 98.2 | 2800 | 77.0 | 2280 | 229 | 217 |
| 27  | 102 | 3000 | 79.7 | 2440 | 237 | 224 |
| 28  | 105 | 3230 | 82.5 | 2630 | 245 | 232 |
| 29  | 108 | 3430 | 85.0 | 2790 | 252 | 239 |
| 30  | 112 | 3680 | 87.9 | 3000 | 260 | 246 |
| 31  | 115 | 3900 | 90.4 | 3180 | 268 | 254 |
| 32  | 118 | 4140 | 93.0 | 3360 | 275 | 261 |

Table 1. The moments of inertia of multiskyrmions in the SK4 variant of the model $e = 4.12$ and 3 ($\Theta_J$(SK4)*), and for the SK6 variant of the model, $e' = 4.11$ and 2.84 ($\Theta_J$(SK6)*), in GeV$^{-1}$

### 3. Multiplets of nuclear isotopes

To get an idea how the chiral soliton approach can describe the existing data on the mass splittings of the known nuclear isotopes, we consider the mass differences of some known nuclei with atomic number beginning with $A = 8$.

Consider first the mass difference between isosinglet $^8$Be, $J^P = 0^+$, binding energy (b.e.) $\epsilon = 56.5$ MeV, and the components of the isotriplet $^8$Li–$^8$B, $J^P = 2^+$, b.e. 41.28 MeV and 37.74 MeV (the experimental data on total binding energies of nuclei are taken mainly
from paper [29]). Using the above formula, we obtain easily
\[
\Delta M(A = 8)_{0,1} = \frac{1}{\Theta_{I,8}} + \frac{3}{\Theta_{J,8}}.
\] (15)

In the average of masses of isotopes \(^8\)Li and \(^8\)B the number of protons and neutrons is the same as in \(^8\)Be, and the average Coulomb energy is also the same with quite good accuracy. The theoretical value \(\sim 40\) MeV for the SK4 variant of the model should be compared with \(17\) MeV from the data. Only collective motion is taken into account in (15), and peculiarities of nucleon-nucleon interaction are totally neglected, e.g. the phenomenologically introduced so called pairing energy [15, 16] which increases the binding by \(\sim 4\) MeV for \(^8\)Be and decreases by same value for \(^8\)Li and \(^8\)B. As a result we should compare \(40\) MeV and \(\sim 10\) MeV. So, in this case we can speak only about qualitative agreement.

For \(A = 12\) there is isosinglet nucleus \(^{12}\)C with \(J^P = 0^+\) and b.e. \(92.16\) MeV and isotriplet components \(^{12}\)B and \(^{12}\)N with \(J^P = 1^+,\) b.e. \(79.58\) and \(74.04\) MeV,
\[
\Delta M(A = 12)_{0,1} = \frac{1}{\Theta_{I,12}} + \frac{1}{\Theta_{J,12}},
\] (16)
which is \(22.2\) MeV to be compared with \(15.35\) MeV experimentally. The pairing energy in this case also makes the disagreement even worth by \(5–6\) MeV.

For \(A = 16\) isosinglet and scalar nucleus \(^{16}\)O, b.e. \(127.62\) MeV should be compared with nucleus \(^{16}\)N, \(J^P = 2^–\), b.e. \(117.98\) MeV and \(^{16}\)F, b.e. \(111.41\) MeV, if we assume that the last one also has \(J^P = 2^-\). In this case
\[
\Delta M(A = 16)_{0,1} = \frac{1}{\Theta_{I,16}} + \frac{3}{\Theta_{J,16}},
\] (17)
numerically it is \(18.55\) MeV to be compared with \(14.69\) MeV, or with \(\sim 10\) MeV when pairing energy is added and subtracted in proper way.

For the case of \(A = 10\) we have isosinglet \(^{10}\)B, \(J^P = 3^+\), \(\epsilon = 64.75\) MeV and components of isotriplet \(^{10}\)Be and \(^{10}\)C, \(J^P = 0^+\), b.e. \(64.98\) and \(60.32\) MeV. Theoretical difference of masses is
\[
\Delta M(A = 10)_{0,1} = \frac{1}{\Theta_{I,10}} - \frac{6}{\Theta_{J,10}},
\] (18)
which is about \(11\) MeV, to be compared with experimental value \(2.1\) MeV. The pairing energy of the order of \(3–4\) MeV decreases the binding of \(^{10}\)B \((N = Z = 5)\) and increases the binding of \(^{10}\)Be and \(^{10}\)C, so we should compare \(11\) MeV and \(8–10\) MeV.

To summarize, we can state that the agreement between data and theory is only qualitative for small \(A = B\), if we take the value of the model parameter \(\epsilon = 4.12\) which allowed to describe the mass difference between nucleons and \(\Delta(1232)\) isobar (we shall call this as “nucleon” variant of the model), which improves however with increasing atomic number, as can be seen for \(A = 12, 16\).

It is clear that the “nucleon” variant of the model cannot be good in the case of nuclei: the chiral field configurations given by rational map ansatz, although confirmed by direct numerical computation, represent the “skeleton” configurations which differ from realistic distribution of nucleons inside of nuclei. The nonzero mode motion should be taken into account — vibration, breathing, etc. to get more realistic distribution of matter inside of multibaryons. Technically, this problem is very complicated. Up to now, the mathematical analysis of these modes was made for baryonic numbers up to \(A = 7\) [30], and some numerical estimates were made for the deuteron \((A = 2)\) [9], as it was mentioned in Introduction. For each value of \(A\) we can improve the agreement of theoretical estimates with data if we allow
the only parameter of the model, Skyrme constant $e$, to vary — to decrease, in fact, in
to make the dimensions of multiskyrmion and moments of inertia greater. Effectively,
such approach allows to take into account the vibration and breathing of separate parts of
multiskyrmions. Evidently, for each value of atomic number we can get perfect agreement
if $e = e(A)$. It is, however, more instructive to describe a number of nuclei with the same
value of constant $e$. The optimal value of the constant is $e = 3.0$ (SK4 variant) and $e = 2.84$,
as analysis of data shows (next section).

With rescaled value of the constant $e$ we obtain (see Table 1): $\Delta M(A = 8) = 16.8$ MeV; $\Delta M(A = 12) = 9.5$ MeV and $\Delta M(A = 16) = 7.9$ MeV, and for $A = 12$ and 16
there is now agreement with data within $2-3$ MeV. As we shall see below, quite satisfactory
agreement with data can be obtained when appropriate choice of nuclear isotopes is made,
for $A$ between $\sim 10$ and $\sim 30$.

4. Even baryon numbers, integer isospins

It is possible to obtain more adequate comparison of the model predictions with data,
comparing the differences of masses of nuclei with even isospins, odd isospins, and similar
differences for half-integer isospins (next section). As it was mentioned in previous section,
the nucleons pairing energy should be added to the binding energy, which equals to $\Delta$
when both $N$, $Z$ are even, 0 when $A$ is odd, and $-\Delta$ when both $N$, $Z$ are odd (see, e.g. [15]).
This pairing energy decreases with increasing atomic number; it means that peculiarities
of nucleon interaction become less important for large $A$. In the differences of masses of
isotopes with the same pairing energy this specific contribution is cancelled, and one can
hope that the collective motion effects — which are mainly taken into account within the
chiral soliton approach — play the dominant role.

Formulas for energies differences can be correctly applied to the same nucleus’ quan-
tum states with different isospin. However, experimental data for excited states with higher
isospin are lacking often. Nevertheless, using the isotopical invariance we can obtain binding
energies of this level subtracting the Coulomb energy difference from binding energies of the
components of isotopical multiplet with neutron excess.

In Tables 2–10 the Coulomb energies are calculated according to [16] (finite-range
liquid-drop model):

$$E_C = \frac{3}{5} \frac{e^2 Z^2}{r_0 A^{1/3}} B_3 - \frac{3}{4} \left( \frac{3}{2\pi} \right)^{2/3} \frac{e^2 Z^{4/3}}{r_0 A^{1/3}} - \frac{1}{8} \left( \frac{145}{48} \right) \frac{r_p^2 e^2 Z^2}{r_0^3 A},$$ (19)

where nuclear-radius constant $r_0 = 1.16$ fm, proton root-mean-square radius $r_p = 0.80$ fm.
The first term in (19) is Coulomb energy for a homogeneously charged, diffuse-surface nu-
cleus to all orders in the diffuseness constant $a_{\text{den}}$, the second is Coulomb exchange correc-
tion, and the third is proton formfactor correction to the Coulomb energy. The constant
$B_3 = B_3 \left( r_0 A^{1/3}/a_{\text{den}} \right)$, is normalized so that for zero diffuseness $a_{\text{den}}$ (range of Yukawa
function $\exp(-r/a_{\text{den}})$) $B_3 = 1$. For spherically symmetrical nuclei it can be calculated an-
alytically and presented as a sum $B_3 = 1 - 5 y_0^2 + ..., $ here $y_0 = a_{\text{den}}/(r_0 A^{1/3})$, more details
can be found in [16].

The result provided by (19) for intermediate values of $A$ (practically, between $A \sim 25$
and $A \sim 50$) can be described by simple formula

$$E_C = \frac{3}{5} \frac{Q^2}{r_C A^{1/3}},$$ (20)
where \( Q \) is nucleus’s charge, \( r_C = 1.48 \text{ fm} \) [14]. The effective Coulomb radius in (20) is greater than \( r_C \) given, e.g. in [15], \( r_C = 1.24 \text{ fm} \). The formula (20) is valid for uniformly charged sphere of radius \( R = r_C A^{1/3} \) and total charge \( Q \). For other types of charge distribution inside of nuclei the above formula for \( E_C \) is modified slightly. For example, for thin shell-like distribution suggested by rational map approximation for multiskyrmions, the Coulomb energy equals to

\[
E_C = \frac{1}{2} \frac{Q^2}{r_C A^{1/3}},
\]

which is smaller than in the case of uniformly charged sphere by 17% only.

For the nuclei considered in Tables 2–5 and 7–9, the Coulomb energy increases from zero for nucleus \(^6\text{H}\) to 43.9 MeV for nucleus \(^{32}\text{S}\).

| \( A \)  | \( \epsilon^{\exp}_0 \) | \( \epsilon^{\exp}_2 \) | \( \Delta \epsilon_C \) | \( \Delta \epsilon^{\exp}_{02} \) | \( \Delta \epsilon^{\exp}_{20} \) | \( \Delta \epsilon^{\exp}_{02} \) |
|--------|----------------|-----------------|----------------|----------------|----------------|----------------|
| \(^6\text{Li}−^6\text{H}\) | 32.0 | 5.8 | 1.1 | 27.3 | 112.9 | 47.9 |
| \(^8\text{Be}−^8\text{He}\) | 56.5 | 31.4 | 2.0 | 27.1 | 88.5 | 37.5 |
| \(^{10}\text{B}−^{10}\text{Li}\) | 64.8 | 45.3 | 2.9 | 22.4 | 72.5 | 30.8 |
| \(^{12}\text{C}−^{12}\text{Be}\) | 92.2 | 68.7 | 3.8 | 27.3 | 61.8 | 26.3 |
| \(^{14}\text{N}−^{14}\text{B}\) | 104.7 | 85.4 | 4.6 | 23.9 | 53.5 | 22.8 |
| \(^{16}\text{O}−^{16}\text{C}\) | 127.6 | 110.8 | 5.0 | 22.2 | 47.5 | 20.3 |
| \(^{18}\text{F}−^{18}\text{N}\) | 137.4 | 126.5 | 6.1 | 17.0 | 42.7 | 18.2 |
| \(^{20}\text{Ne}−^{20}\text{O}\) | 160.6 | 151.4 | 6.8 | 16.0 | 38.7 | 16.6 |
| \(^{22}\text{Na}−^{22}\text{F}\) | 174.1 | 167.7 | 7.5 | 13.9 | 35.6 | 15.2 |
| \(^{24}\text{Mg}−^{24}\text{Ne}\) | 198.3 | 191.8 | 8.2 | 14.7 | 32.9 | 14.1 |
| \(^{26}\text{Al}−^{26}\text{Na}\) | 211.9 | 208.2 | 8.9 | 12.6 | 30.5 | 13.1 |
| \(^{28}\text{Si}−^{28}\text{Mg}\) | 236.5 | 231.6 | 9.5 | 14.4 | 28.5 | 12.3 |
| \(^{30}\text{P}−^{30}\text{Al}\) | 250.6 | 247.8 | 10.2 | 13.0 | 26.8 | 11.5 |
| \(^{32}\text{S}−^{32}\text{Si}\) | 271.8 | 271.4 | 10.8 | 11.2 | 25.3 | 10.9 |

**Table 2.** The binding energies differences for isotopes with isospin \( I = 0 \) and \( 2 \) for the original variant, \( e = 4.12 \), and for the variant with rescaled constant, \( e = 3 \) (numbers with the *)

The pairing energy has different sign for nuclei with \( A = 6, 10, 14, ... \) and \( A = 8, 12, 16, ... \), but anyway, it is cancelled in the binding energies differences we consider here.

To obtain the binding energy of isotopes with isospin \( I = 0 \) we take the known binding energy of ground states of nuclei with \( N = Z \) and add the energy of the Coulomb repulsion given by (19), (20). In the case of \( I = 2 \) we take the ground state of nucleus with \( N = Z + 4 \) which is supposed to be the component of \( I = 2 \) isomultiplet with the isospin projection \( I_3 = −2 \), and add the Coulomb repulsion energy as well. The quantity \( \Delta \epsilon^{\exp} \) shown in all Tables includes the Coulomb correction.

The energy of usual space rotation of nucleus as a whole, equal to \( J(J + 1)/2\Theta_J \), is small for large enough nuclei since \( \Theta_J \) is large (see Table 1), and is omitted within this approach. Another reason to omit this correction is that the value of \( J \) is not known in some cases.

It follows from Table 2 that for light nuclei, \( A \) between 6 and 10, there is only qualitative agreement between data and theoretical results, even with rescaled constant \( e \). For atomic numbers between 12 and 32 the agreement is satisfactory and even good in some cases (see column with \( \Delta \epsilon^{\text{th}*} \). By this reason we shall not consider further small values of \( A \).
| $A$       | $\epsilon_{0}^{\exp}$ | $\epsilon_{3}^{\exp}$ | $\epsilon_{4}^{\exp}$ | $\Delta\epsilon_{13}^{\exp}$ | $\Delta\epsilon_{40}^{\exp}$ | $\Delta\epsilon_{40}^{\exp}$ |
|-----------|------------------------|------------------------|------------------------|-------------------------------|-------------------------------|-------------------------------|
| $^{10}$Be–$^{10}$He | 65.0                   | 30.3                   | 2.0                    | 36.7                          | 120.8                         | 51.3                          |
| $^{12}$B–$^{12}$Li  | 79.6                   | 44.4                   | 2.9                    | 38.1                          | 103.0                         | 43.8                          |
| $^{14}$C–$^{14}$Be  | 105.3                  | 70.0                   | 3.7                    | 39.0                          | 89.1                          | 38.0                          |
| $^{16}$N–$^{16}$B   | 118.0                  | 88.2                   | 4.5                    | 34.3                          | 79.1                          | 33.8                          |
| $^{18}$O–$^{18}$C   | 139.8                  | 115.7                  | 5.2                    | 29.3                          | 71.2                          | 30.4                          |
| $^{20}$F–$^{20}$N    | 154.4                  | 134.2                  | 6.0                    | 26.2                          | 64.5                          | 27.6                          |
| $^{22}$Ne–$^{22}$O   | 177.8                  | 162.0                  | 6.7                    | 22.5                          | 59.3                          | 25.4                          |
| $^{23}$Na–$^{24}$F   | 193.5                  | 179.1                  | 7.4                    | 21.8                          | 54.8                          | 23.5                          |
| $^{26}$Mg–$^{26}$Ne  | 216.7                  | 201.6                  | 8.1                    | 23.2                          | 50.9                          | 21.9                          |
| $^{28}$Al–$^{28}$Na  | 232.7                  | 218.4                  | 8.7                    | 23.0                          | 47.5                          | 20.4                          |
| $^{30}$Si–$^{30}$Mg  | 255.6                  | 241.6                  | 9.4                    | 23.4                          | 44.6                          | 19.2                          |
| $^{32}$P–$^{32}$Al   | 270.9                  | 259.2                  | 10.0                   | 21.7                          | 42.2                          | 18.2                          |

Table 3. The binding energies differences for isotopes with isospin $I = 1$ and 3 for the original variant, $e = 4.12$, and for the variant with rescaled constant, $e = 3$

In the case presented in Table 3 we take the binding energies of isotopes with $N = Z + 2$ and $N = Z + 6$ (ground states) which are considered as components of isomultiplets $I = 1$, $I_3 = -1$ and $I = 3$, $I_3 = -3$, Coulomb energy is included into consideration as well. The agreement of theoretical estimates with data is quite satisfactory for $A$ between 14 and 32, qualitative agreement takes place for $A = 10$, 12 as well.

| $A$       | $\epsilon_{0}^{\exp}$ | $\epsilon_{4}^{\exp}$ | $\Delta\epsilon_{13}^{\exp}$ | $\Delta\epsilon_{40}^{\exp}$ | $\Delta\epsilon_{40}^{\exp}$ |
|-----------|------------------------|------------------------|-------------------------------|-------------------------------|-------------------------------|
| $^{16}$O–$^{16}$Be | 127.6                  | —                      | 9.0                           | —                             | 158.3                         | 67.5                          |
| $^{18}$F–$^{18}$B   | 137.4                  | 89.0$^\dagger$         | 10.5                          | 59.4                          | 142.3                         | 60.8                          |
| $^{20}$Ne–$^{20}$C  | 160.6                  | 119.2                  | 12.0                          | 53.4                          | 129.1                         | 55.2                          |
| $^{22}$Na–$^{22}$Ne | 174.1                  | 140.0                  | 13.4                          | 47.5                          | 118.6                         | 50.8                          |
| $^{23}$Mg–$^{23}$O  | 198.3                  | 168.5                  | 14.8                          | 44.6                          | 109.5                         | 47.0                          |
| $^{26}$Al–$^{26}$F  | 211.9                  | 184.5                  | 16.2                          | 43.6                          | 101.8                         | 43.7                          |
| $^{28}$Si–$^{28}$Ne | 236.5                  | 206.9                  | 17.5                          | 47.1                          | 95.1                          | 40.9                          |
| $^{30}$P–$^{30}$Na  | 250.6                  | 224.9                  | 18.8                          | 44.5                          | 89.3                          | 38.4                          |
| $^{32}$S–$^{32}$Mg  | 271.8                  | 249.7                  | 20.1                          | 42.2                          | 84.4                          | 36.3                          |

Table 4. The binding energies differences for isotopes with isospin $I = 0$ and 4 for the original variant, $e = 4.12$, and for the variant with rescaled constant, $e = 3$

For $I = 4$ the ground states of isotopes with $N = Z + 8$ are taken as $I_3 = -4$ components of corresponding isomultiplets. The agreement of data and theoretical values for atomic numbers between 18 and 30 is good or satisfactory, according to Table 4. For the nucleus $^{16}$Be we obtain the b.e. about 60 MeV, so it is unstable: it can decay into isotope $^{14}$Be (b.e. about 70 MeV) and 2 neutrons. This result can be considered as prediction of our approach. For the $^{18}$B isotope our prediction for the b.e. is 87.1 MeV$^3$, in good agreement with extrapolation result 89 MeV of [29], marked in Table 4 with $^\dagger$.

Results presented in Tables 2–4 are illustrated in Fig. 1.

$^3$The value 87.1 MeV = (137.4 − 60.8 + 10.5) MeV, where 60.8 MeV is the difference of quantum corrections to the multiskyrmion, rescaled variant, and 10.5 MeV is difference of Coulomb energies.
**Fig. 1.** The binding energies differences (in MeV) for isotopes with even atomic numbers for the SK4 variant with rescaled constant, $e = 3.0$ (black points connected with solid lines — experimental data, dashed lines — model calculations).

**Table 5.** The binding energies differences for isotopes with isospin $I = 0$ and $2$ for the SK6 variant, $e' = 4.11$, and for the SK6 variant with rescaled constant, $e' = 2.84$

In Table 5 we present the comparison of data for differences of binding energies of isotopes with isospin 0 and 2 (as in Table 2) and results of the SK6 variant of the model, for two values of effective constant $e$. The rescaling of the constant $e'$ to the value $2.84$ allows to make agreement of theoretical prediction and data quite satisfactory, beginning with atomic number $A = 12$, similar to the SK4 variant of the model. We conclude therefore that there is no difference of principle between both variants of the model, and will make further
comparison with the results provided by the SK4 variant of the model. Our predictions for the binding energies of nuclides with highest values of isospin (Tables 6, 10) will be made for both variants of the model.

In view of successful description of binding energies of many isotopes, it is possible to make predictions of binding energies for neutron rich nuclides which have greater isospin than we have considered before, e.g. $I = 5$ or $N = Z + 10$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$A$ & $\frac{14}{\Theta I_{SK4}^A}$ & $\Delta \epsilon C$ & $\epsilon I_{SK4}^A$ & $\epsilon I_{SK6}^A$ & $\epsilon I_{SK6}^+$ \\
\hline
14Be & 85.4 & 8.8 & 63 & 59 & \text{—} \\
20B & 77.3 & 10.3 & 87 & 83 & \text{—} \\
22C & 71.1 & 11.8 & 118 & 114 & \text{—} \\
24N & 65.7 & 13.2 & 141 & 138 & \text{—} \\
26O & 61.1 & 14.6 & 170 & 167 & 168 \\
28F & 57.1 & 15.9 & 191 & 188 & 186 \\
30Ne & 53.8 & 17.2 & 219 & 216 & 212 \\
32Na & 50.9 & 18.5 & 238 & 235 & 231 \\
\hline
\end{tabular}
\caption{Predictions for the binding energies of neutron rich nuclides with $N = Z + 10$ or $I = 5$ for the SK4 and SK6 variants of the model. The values $\epsilon I^+$ are the results of extrapolation of \cite{29}}
\end{table}

The binding energy of neutron rich nuclides with $N = Z + 10$ shown in Table 6 is calculated according to the following formula:

$$
\epsilon_{N=Z+10} = \epsilon_{N=Z+2} - \frac{14}{\Theta A=N+Z} + \Delta \epsilon C, \quad (22)
$$

where $\epsilon_{N=Z+2}$ is the experimentally known binding energy of the isotopes with isospin $I = 1$, the number 14 is half of difference of Casimir operator $I(I + 1)$ for $I = 5$ and $I = 1$, $\Delta \epsilon C$ is the difference of the repulsive Coulomb energies for nuclides with the same $A = N + Z$ and different $Z$, calculated with formula (19). Predictions of two variants of the model do not differ much, the maximal difference is about 4 MeV for lighter nuclides.

5. Odd baryon numbers, half-integer isospin

The consideration similar to that made for integer isospin in previous section can be performed for half-integer isospins, i.e. for nuclei with odd atomic numbers. The phenomenologically introduced pairing energy is absent in this case.
Table 7. The binding energies differences for isotopes with isospin $I = 1/2$ and $5/2$ for the original variant, $e = 4.12$, and for the variant with rescaled constant, $e = 3$.

In Table 7 we take the binding energies of nuclei with $N = Z + 1$ for isospin $I = 1/2$ and $N = Z + 5$ for $I = 5/2$. For the rescaled variant of the model the quantitative agreement with data is quite good for atomic numbers between 13 and 25, qualitative agreement takes place for all values of $A$ presented in Table 7.

| $A$         | $\epsilon_{3/2}^{\text{exp}}$ | $\epsilon_{7/2}^{\text{exp}}$ | $\Delta \epsilon_C$ | $\Delta \epsilon_{3/2,7/2}^{\text{exp}}$ | $\Delta \epsilon_{3/2,7/2}^{\text{th}}$ | $\Delta \epsilon_{3/2,7/2}^{\text{the}}$ |
|-------------|-------------------------------|-------------------------------|----------------------|---------------------------------|---------------------------------|---------------------------------|
| $^{15}$C-$^{15}$Be | 106.5                         | —                             | 3.7                  | 100.4                           | 42.8                           |
| $^{17}$N-$^{17}$B  | 123.9                         | 89.6                          | 5.4                  | 90.6                            | 4.4                             |
| $^{19}$O-$^{19}$C  | 143.8                         | 115.8                         | 5.2                  | 33.2                            | 81.2                           | 34.7                           |
| $^{21}$F-$^{21}$N  | 162.5                         | 138.8                         | 5.9                  | 29.6                            | 74.1                           | 31.8                           |
| $^{23}$Ne-$^{23}$O  | 183.0                         | 164.8                         | 6.6                  | 24.8                            | 68.2                           | 29.2                           |
| $^{25}$Na-$^{25}$F  | 202.5                         | 183.5                         | 7.3                  | 25.3                            | 63.4                           | 27.2                           |
| $^{27}$Mg-$^{27}$Ne | 223.1                         | 203.0                         | 8.0                  | 28.1                            | 59.0                           | 25.4                           |
| $^{29}$Al-$^{29}$Na | 242.1                         | 222.8                         | 8.7                  | 28.0                            | 55.4                           | 23.8                           |
| $^{31}$Si-$^{31}$Mg | 262.2                         | 244.0                         | 9.3                  | 27.5                            | 52.1                           | 22.4                           |

Table 8. The binding energies differences for isotopes with isospin $I = 3/2$ and $7/2$ for the original variant, $e = 4.12$, and for the variant with rescaled constant, $e = 3$.

The nuclei with $N = Z + 3$ and $N = Z + 7$ in Table 8 correspond to the states with isospin $I = 3/2$, $I_3 = -3/2$ and $I = 7/2$, $I_3 = -7/2$. Good agreement between data and theory (rescaled variant) takes place between $A = 17$ and 27. The nucleus $^{15}$Be is not observed yet, so, the b.e. of this nucleus 63.7 MeV is prediction of the model. It is not stable since it can decay into $^{14}$Be (b.e. 70 MeV) plus neutron.

Satisfactory agreement between available data and theoretical result (rescaled variant of the model with Skyrme constant $e = 3.0$) takes place in the case of isotopes with isospin 1/2 and 9/2 as well, see Table 9. The numbers with $^\dagger$ in the 3-d and 5-th column are results of extrapolation made in [29].

The nucleus $^{17}$Be, similar to $^{15}$Be, has not been observed yet, so, the b.e. 63.4 MeV which follows from Table 9 also is prediction of the chiral soliton approach. This isotope is not stable since the b.e. of $^{14}$Be, 70 MeV, is greater. For the isotope $^{19}$B the prediction of our approach is 88.7 MeV, in good agreement with the extrapolation result 90.1 MeV shown in Table 9. Satisfactory agreement between our prediction and extrapolation of paper [29] takes place for isotopes $^{21}$C, $^{23}$N, and $^{25}$O.

| $A$         | $\epsilon_{1/2}^{\text{exp}}$ | $\epsilon_{9/2}^{\text{exp}}$ | $\Delta \epsilon_C$ | $\Delta \epsilon_{1/2,9/2}^{\text{exp}}$ | $\Delta \epsilon_{1/2,9/2}^{\text{th}}$ | $\Delta \epsilon_{1/2,9/2}^{\text{the}}$ |
|-------------|-------------------------------|-------------------------------|----------------------|---------------------------------|---------------------------------|---------------------------------|
| $^{17}$O-$^{17}$Be | 131.8                         | —                             | 8.9                  | 181.3                           | 77.3                            |
| $^{19}$F-$^{19}$B  | 147.8                         | 90.1                          | 10.4                 | 68.1                            | 162.4                           | 69.5                            |
| $^{21}$Ne-$^{21}$C  | 167.4                         | 118.8                         | 11.9                 | 60.5                            | 148.3                           | 63.5                            |
| $^{23}$Na-$^{23}$N  | 186.6                         | 142.2                         | 13.3                 | 57.7                            | 136.4                           | 58.5                            |
| $^{25}$Mg-$^{25}$O  | 205.6                         | 168.4                         | 14.7                 | 51.9                            | 126.8                           | 54.4                            |
| $^{27}$Al-$^{27}$F  | 224.9                         | 185.8                         | 16.0                 | 55.1                            | 118.1                           | 50.7                            |
| $^{29}$Si-$^{29}$Ne | 245.0                         | 208.2                         | 17.4                 | 54.2                            | 110.7                           | 47.6                            |
| $^{31}$P-$^{31}$Na | 262.9                         | 228.9                         | 18.7                 | 52.7                            | 104.2                           | 44.8                            |

Table 9. The binding energies differences for isotopes with isospin $I = 1/2$ (i.e. $N = Z + 1$) and $9/2$ ($N = Z + 9$) for the original variant, $e = 4.12$, and for the variant with rescaled
Fig. 2. The binding energies differences (in MeV) for isotopes with odd atomic numbers for the rescaled SK4 variant of the model, $\epsilon = 3.0$ (black points connected with solid lines — experimental data, dashed lines — model calculations)
The constant, \( e = 3.0 \)

The results for odd atomic numbers (half integer isospins, Tables 7–9) are presented also in Fig. 2. Similarity to behavior of data and calculation results shown in Fig. 1 is evident.

In Table 10 we present also predictions of the chiral soliton approach for nuclides with \( N = Z + 11 \), or isospin \( I = 11/2 \) for the SK4 and SK6 variants of the model.

| \( A \) | \( 16/\Theta_I^{SK4} \) | \( \Delta \epsilon_{C}^{SK4} \) | \( \epsilon_{11/2}^{SK4} \) | \( \epsilon_{11/2}^{SK6} \) | \( \epsilon_{11/2}^\dagger \) |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \(^{11}\text{Li}\) | 103.2 | 7.2 | 28 | 23 | — |
| \(^{19}\text{Be}\) | 92.5 | 8.7 | 60 | 56 | — |
| \(^{21}\text{B}\) | 84.7 | 10.2 | 88 | 83 | — |
| \(^{23}\text{C}\) | 78.0 | 11.7 | 117 | 113 | — |
| \(^{25}\text{N}\) | 72.4 | 13.1 | 143 | 139 | — |
| \(^{27}\text{O}\) | 67.5 | 14.4 | 170 | 166 | — |
| \(^{29}\text{F}\) | 63.5 | 15.8 | 194 | 191 | 187 |
| \(^{31}\text{Ne}\) | 59.7 | 17.1 | 219 | 216 | 212 |

Table 10. Predictions for the binding energies of nuclides with \( I = 11/2 \) or \( N = Z + 11 \) calculated from nuclides with \( I = 3/2 \) \( (N = Z + 3) \)

For the nuclides \(^{29}\text{F}\) and \(^{31}\text{Ne}\) we obtain fair agreement with the extrapolation result obtained in [29], and the SK6 variant prediction is more close to it than the SK4 one. Experimental data for this case and \( A \leq 32 \) are still lacking.

6. Conclusions

The estimates of the mass splittings of nuclides with baryon numbers up to 32 are made using the properties of the bound states of skyrmions obtained in the rational map approximation. The results are in impressive agreement with data, provided the rescaling of the Skyrme constant \( e \) is made — decrease by about 25–30% — which effectively takes into account the role of nonzero modes, vibration and breathing. This nonzero modes motion brings the rational map skyrmions, classical configurations of the one-shell type, in better agreement with observed nuclear matter distribution: it increases the dimensions of configuration and, therefore, the moments of inertia. Without rescaling of the constant \( e \) the mass splittings of light nuclear isotopes are overestimated more than twice. It is known that calculations of nonzero modes contribution to the energy (mass) of multiskyrmion are closely related to calculations of the so called Casimir energy and/or loop corrections to the soliton mass [31] which are of the order of \( N_c^0 \sim 1 \). The estimate of this contribution was made for the \( B = 1 \) hedgehog-type skyrmion [31]. Anyway, these unknown contributions cancel out in the differences of energies of quantized states we have considered here.

Since 1983 [2] it was believed that the chiral soliton approach allows to describe the properties of baryons (the \( B = 1 \) sector) with accuracy \( \sim 30\% \) and better. We have shown, that quite good description of the binding energy differences can be obtained also for light nuclei \( B = A \leq 32 \), when appropriate rescaling of the constant \( e \) is made, providing the “nuclear” variant of the model. Taking into account the former results on good qualitative description of binding energies of light hypernuclei [13], we conclude that effective field theories, including the chiral soliton approach, provide good description of certain basic properties of relatively light nuclei.
We can conclude, that besides the $B = 1$ version of the model, one can consider the “nuclear” version of the model with rescaled value of $e$ which is good for the description of nuclear properties for $A < 30$. Observation of the predicted states can provide additional support of the validity of this approach which turned out to be very effective in the understanding of the properties of mesons and baryons. Studies of other properties of nuclei (e.g. magnetic moments, radii of different distributions, etc.) is now one of actual problems.

No stable multineutron nuclides are found within the chiral soliton approach; all states are several tens of MeV above the threshold and can decay strongly into final states consisting of nucleons. The states with isospin $T > B/2$ also can be obtained. They can be interpreted as states consisting of nucleons and $\Delta^-$ isobars, and some of them can be negative — as $n...n\Delta^-$. Estimates show, however, that their energies are higher than thresholds for strong decays.

One of the evident drawbacks of this approach is that one cannot describe the variations of the isotopical content of nuclides in space: in the quantization procedure solitons are rotated in the isospace as a whole. In particular, the description of neutron halo on the periphery of neutron rich nuclides, established experimentally (see, e.g. [20]), is behind the possibilities of our approach in its present form. The studies of nonzero modes quantum fluctuations are necessary for this, since the approach we used corresponds to excitation only of collective rotational modes.

Careful study of the results presented in Tables 2–5 and 7–9 allows to reveal the following general feature: for relatively light nuclei, $A < 20–22$, theoretical predictions for the difference of binding energies of isotopes are greater than the experimentally observed values of this difference. For $A > 22–24$ theoretical predictions for this difference are smaller than data, as a rule. One could reach better agreement with data by decreasing the value of the constant $e$ at smaller $A$ (thus providing the additional increase of dimensions of multiskyrmions), and increasing slightly $e$ at larger $A$. It means, that the relative role of nonzero modes in formation of the size of nuclei slightly decreases with increasing $A$, probably, because for large $A$ the size of multiskyrmion is large enough. This observation shows that further more detailed studies and refinements would be useful and of interest, although they will not change our main conclusions concerning the validity of the chiral soliton approach for description of the binding energies of nuclides.

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