Manipulating the scattering length of a Bose-Einstein condensate in an amplitude-modulated optical lattice.

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Abstract

The scattering length in a BEC confined in an amplitude-modulated optical lattice can be manipulated by the modulation strength. Two standing laser waves of main and sideband frequencies of an optical lattice induce a Raman transition; due to resonant Raman driving the effective scattering length depends on the modulation strength and the detuning from resonance and can be tuned to a given value.

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The recent experimental observation of Bose-Einstein condensation in a dilute gas of ultracold trapped atoms [1-3] has generated much interest in studying the properties of a Bose-Einstein condensate (BEC) and manipulating such coherent matter by external electromagnetic fields. The experimental observation of Bose-Einstein condensation in a dilute gas of ultracold trapped atoms [1-3] has generated much interest in manipulating such coherent matter by external electromagnetic fields. Recently atoms have been confined in optical potentials created by standing light waves [4, 5], with the wavelength of the optical potential much smaller than the dimensions of the trap. The unique prospects of this new class of system (the correlated bosons on a lattice) became evident. The possibility of creating optical lattice in trapped Bose-condensed gases has provided an opportunity to study superfluids in novel situations. The presence of the lattice leads to a variety of solid-state effects. For example, a transition to a Mott insulator becomes possible when a superfluid like a BEC is placed in a periodic potential [4]; the oscillation frequency of the center-of-mass motion of the condensate is reduced [6] as a result of the enhanced effective mass of the atoms tunneling between potential wells. By accelerating the optical lattice [7] Bloch oscillations of the condensate have been observed, and reducing the amplitude of the lattice optical potential leads to a breakdown of these oscillations as a result of Landau-Zener tunneling between bands.

Many physical properties of a dilute gas are determined by the atom-atom interaction. In a dilute and cold gas only binary collisions at low energy are relevant and these collisions are characterized by a single parameter, the $s$-wave scattering length. The confinement of a gas in one or more spatial dimensions can strongly modify the collisional properties (two-body scattering in an optical lattice) of atoms [8-12]. In Ref.[12] was shown that in an 3D optical lattice a dilute gas is described by the effective atomic scattering length $a_{eff}$, modified and tunable by the lattice parameters,

$$a_{eff} = \frac{\pi ln2}{4} - d a + l_0 \sqrt{D_0},$$

$$l_0 = \frac{ln2}{4} l_0 \sqrt{D_0},$$

(1)
where \( a \) is the free-space scattering length, the optical lattice potential is determined by a \( V_0 \) and lattice spacing \( d \), and \( l_0 \) is the size of the ground state wave function in an individual well of an optical lattice. The wave functions of the relative motion of two particles of mass \( m \) are characterized by the quasimomentum \( q \), the band index \( s \), and the energy \( \varepsilon_{sq} = \varepsilon_s - t_s \cos(qd) \), where \( t_s = \sqrt{D_s \hbar \omega / \pi} \), \( D_s \ll 1 \) is WKB tunneling exponent between the neighboring wells, \( \omega^2 = V_0 / (md^2) \), \( \varepsilon_s \) are the energies of states \( s \) in isolated wells. The effective scattering length \( a_{eff} \) is modified by the optical lattice if the tunneling between the neighboring lattice sites is sufficiently small and the new length scale \( l_* \) contains the tunneling amplitude (for the band index \( s = 0 \) the amplitude \( D_0 \)) and describes the influence of particle states extending over several lattice sites on the two-body scattering in an optical lattice. In Ref. [12] was shown that in the case of attractive interaction, for the free-space length \( a < 0 \), the effective scattering length exhibits resonant behavior and leads to a geometric resonance at \( l_* = |a| \), and for \( l_* > |a| \) the effective scattering becomes repulsive \( a_{eff} > 0 \).

The goal of this paper is manipulating of the effective scattering length by a time-dependent action of the lattice parameters. We consider a manipulation of the scattering length in a BEC in an amplitude-modulated optical lattice. The such optical lattice can be created by interfering pair of amplitude-modulated beams. In general, there are the two generic types (a resonant and a nonresonant cases) of the manipulation. The effect of the modulation is to modulate the optical lattice potential so that it becomes a time-dependent optical lattice potential which the depth of the optical potential is

\[
V_0(t) = V_0(1 + \nu \cos(\omega_M t)), \nu < 1,
\]

where \( \nu \) is the modulation strength, \( \omega_M \) is the modulation frequency. If the period of modulation \( 2\pi / \omega_M \) is larger the nonequilibrium dynamic time \( \tau_{noneq} \), then the time behavior of a BEC in an amplitude-modulated optical lattice is quasistatic. The nonequilibrium dynamic time \( \tau_{noneq} \) for trapped bosonic atoms in an optical lattice potential was considered in Ref.[11] and was shown that the characteristic time of the dynamical restoration of the
phase coherence is a Josephson time $h/J$.

A resonant case.

One is a resonant mechanism which operates when the modulation frequency $\omega_M$ is close to the excited internal state of a BEC. A resonant mechanism is determined by change in the population of the ground state due to a resonant driving. In a BEC in a periodic potential the characteristic energetic scale of the band structure [14,15] is the recoil energy $E_R$. And, usually, $E_R$ exceeds $J$, and the quasistatic behavior is not valid. However, in this case is more convenient to use a picture that an amplitude-modulated field can be presented as a set of monochromatic fields (main on frequency $\omega$ and sideband on frequencies $\omega \pm \omega_M$). These fields can induce a Raman transition between the two Bloch bands of a BEC $s = 0$ and $s \neq 0$. A resonant field via a two-photon (Raman) transition changes the population of the lowest Bloch band $s = 0$ of a BEC. In resonant case the system is excited by an off-resonant Raman (two-photon) driving with the Rabi frequency $\Omega_R$. In an amplitude-modulated optical lattice two standing laser waves of frequencies $\omega$ and $\omega - \omega_M (\omega_M \ll \omega)$ drive an atom in a Raman scheme. The effective two-photon Rabi frequency is given in terms of the single-photon Rabi frequencies $\Omega$ and $\Omega'$ of the fields on the main $\omega$ and sideband $\omega' = \omega - \omega_M$ frequencies as $\Omega_R = \frac{\Omega \Omega'}{2\Delta}$. For the amplitude-modulated laser beams with the modulation strength $\nu$ we have for standing waves $\Omega_R(x) = \frac{\nu\Omega^2(x)}{2\Delta} = \frac{\nu V_0(x)}{\hbar}$, here $\Delta$ is the far-detuning of the laser beams from the atomic resonance (the closest neighboring optical dipole transition), and $V_0(x) = \sum_{j=1}^3 V_{jo} \cos 2 k x_j$ is the optical lattice potential with wave vectors $k = \frac{2\pi}{\lambda}$ and $\lambda$ the wavelength of the laser beams on the main frequency, and we are omitted the terms proportional to the small parameter $\Delta k \cdot x = \frac{\omega_M x}{c} \ll 1$, where $\Delta k \equiv k' - k$, for $x < \lambda \sim 10^{-4} cm, \omega_M \leq 10^7 s^{-1}$, this parameter is about $10^{-7}$.

The eigenstates of a BEC are Bloch waves, and an appropriate superposition of Bloch states yields a set of Wannier functions which are localized on the individual lattice sites. Without a resonant driving the wave functions in a tight binding model can be represented [12] as $w_s(x) = Z_s \Psi_s(x)$, where $\Psi_s(x)$ is the wave function of the oscillator in a state $s$, and $Z_s$ is a normalization factor. However, for a resonant driving the wave functions are
$w_{s}^{\text{res}}(x) = \varphi(x)w_{s}(x)$, where the function $\varphi(x)$ is the solution of the Bloch equation of the standard Rabi problem. We consider a steady state driving regime, in which the function $\varphi(x)$ is determined on the parameters of the field (the Rabi frequency and the detuning from resonance $\delta = \omega_{M} - (\omega_{b} - \omega_{a})$). In order to avoid a sufficient population of the excited Bloch band $s \neq 0$, we assume $\Omega_{R} < \delta$, then $\varphi^{2}(x) \simeq 1 - \frac{1}{2}(\frac{\Omega_{R}(x)}{\delta})^{2}$. Using the wave functions $w_{s}^{\text{res}}$, in which the size of the ground state oscillator wave function $l_{0}$ in a lattice is much less than the lattice period $d = \lambda/2$, and following Ref. [12] we find the effective scattering length $a_{\text{eff}}^{\text{res}}$ modified by the resonantly amplitude modulation

$$\frac{a_{\text{eff}}^{\text{res}} - a_{\text{eff}}}{a_{\text{eff}}} \simeq -\left(\frac{\nu V_{0}}{\hbar \delta}\right)^{2}. \quad (3)$$

As a result, we can manipulate the effective scattering length in a BEC in a resonantly amplitude-modulated optical lattice. This tuning is determined by the modulation strength $\nu$ and the detuning $\delta$. By varying the parameter $\nu/\delta$, we can tune $a_{\text{eff}}^{\text{res}}$ to a given value.

A nonresonant case.

Second is a nonresonant mechanism for which we will use a direct time-dependent description. Note that such potential is created by interfering pairs of amplitude-modulated laser beams with the modulation strength $\nu/2$ of the electric field amplitude, and was omitted the quadratic term for the small parameter $\nu^{2}/4$. A nonresonant low-frequency mechanism of manipulation is in operation for a quasistatic regime $\omega_{M} < 2\pi \tau_{\text{noneq}}^{-1}$, therefore, if $V_{0}$ corresponds to a geometric resonance at $l_{*} = |a|$ for attractive interaction then as the time runs, the system goes from an attractive interaction to a repulsive interaction and back again with frequency $\omega_{M}$. Therefore, it is allow to investigate the nonequilibrium dynamics in a BEC. In a nonresonant case $\Omega_{R}/\delta \to 1$ then $\varphi(x) = 1$ and we have a time-dependent WKB tunneling exponent $D_{0}(t)$. The WKB tunneling exponent is a nonlinear function on the strength of the periodic potential $V_{0}$ which is proportional to the laser intensity. If we have a time-dependent lattice laser intensity, then, although $\overline{V_{0}(t)} = V_{0}$, due to the nonlinearity of a WKB tunneling exponent a time-averaged $\overline{D_{0}(t)}$ will be different from a WKB tunneling exponent $D_{0}$ without a modulation. For a small $\nu$ we have the
time - averaged $D_0(t) = D_0 + \Delta D_0$, where the bar stands for a time average and $\Delta D_0$ is proportional to the time - averaged $\nu^2\cos^2 \omega_M t$ ( the time - averaged of the modulation harmonic oscillation to the lowest even order power ). For a low - frequency nonresonant manipulation we will use a time - dependent potential ( Eq.(2)), and there could be a modulation frequency $\omega_M$ low enough for quasistatic behavior. Using the WKB tunneling exponent $D_0 \simeq \exp(-\frac{\pi d}{\hbar} \sqrt{2mV_0})$ and using $V_0(t)$ (Eq.(2)), we have a time - dependent length scale $l_{\text{non}}^*(t)$ and, correspondingly, a time - dependent effective scattering length $a_{\text{eff}}^*(t)$.

Calculating the time - average $\overline{D_0(t)}$ by expanding in a series about the small parameter $\frac{2mV_0\pi^2d^2}{\hbar^2} < 1$, we find the time - average length scale $\overline{l_{\text{non}}^*}$, which is modified by the nonresonant amplitude modulation

$$\frac{\overline{l_{\text{non}}^*} - l_*}{l_*} \simeq \frac{\nu^2 2mV_0 \pi^2 d^2}{4 \hbar^2}.$$ (4)

If $\frac{\nu^2 2mV_0 \pi^2 d^2}{4 \hbar^2}$ is not small, then we may expand the expression for the time - dependent $D_0(t)$ using Bessel functions $I_n(x)$ and the functions $\cos^n \omega_M t$. In this case instead the parameter $\frac{\nu^2 2mV_0 \pi^2 d^2}{4 \hbar^2}$ we have $I_2(\nu \frac{\pi d}{\hbar} \sqrt{2mV_0})$, because, usually $\frac{\pi d}{\hbar} \sqrt{2mV_0} \gg 1$ and for $\nu < 0.4$ among $I_n(x < 2)$ the largest is $I_2(x)$.

In summary, the effective scattering length in a BEC in an optical lattice can be manipulated by an amplitude - modulated manner. The effect of the modulation is to modulate the optical lattice potential so that it becomes a time - dependent optical lattice potential depth and, correspondingly, the effective scattering length becomes time - dependent. There are two mechanisms of the manipulation. One, a resonant mechanism is in operation for the modulation frequency closed to the excited internal state of a BEC, and two standing laser waves of main and sideband frequencies of an amplitude - modulated optical lattice induce a Raman transition. Due to resonant Raman driving the population of the lowest Bloch band $s = 0$ of a BEC changes, therefore, the effective scattering length is also changed. As a result, the effective scattering length is other than without a modulation, this tuning is determined by the square of the modulation strength and the square of the detuning. Second, a nonresonant low - frequency mechanism, for which we use a direct time - depen-
dent description. Due to the nonlinear dependence of the WKB tunneling amplitude on the optical lattice depth for the harmonic modulation of the optical lattice depth the time - averaged tunneling exponent $D_0(t)$ is different from $D_0$ without a modulation. The proposed mechanism of manipulating the effective scattering length makes it possible to tune it to a given value. As a result, although a time - averaged optical lattice potential with a harmonic modulation is equal to an optical lattice potential without a modulation, the effective scattering length in a BEC confined in an amplitude - modulated optical lattice can be manipulated by the modulation strength. In conclusion, the nonequilibrium dynamics of a BEC can be investigated by manipulating the effective scattering length of a BEC in an amplitude - modulated optical lattice.

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