Collision strengths and transition probabilities for Co III forbidden lines

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ABSTRACT
In this paper we compute the collision strengths and their thermally-averaged Maxwellian values for electron transitions between the fifteen lowest levels of doubly-ionised cobalt, Co²⁺, which give rise to forbidden emission lines in the visible and infrared region of spectrum. The calculations also include transition probabilities and predicted relative line emissivities. The data are particularly useful for analysing the thermodynamic conditions of supernova ejecta.

Key words: atomic data – atomic processes – radiation mechanisms: non-thermal – supernovae: general – infrared: general.

1 INTRODUCTION
Cobalt is an iron-group element but is the least abundant of this group with a solar abundance of about 300 times less than Fe. However, in supernova (SN) ejecta it is much more abundant. For example, in SN 1987A the ratio of Co to Fe, 255 days after outburst, is approximately 0.2 by number (Varani et al 1990). The spectral lines of Co are therefore valuable investigative tools in analysing the chemical and thermodynamic conditions of supernovae where these emissions are mostly found. These lines are also useful in investigating the evolutionary history and chemical development by nucleosynthesis and decay processes within the SN explosions (Colgate & McKee 1969; Axelrod 1980; Kuchner et al 1994; Bowers et al 1997; Liu et al 1997; Churazov et al 2014; Childress et al 2015). The lines of Cobalt have also been observed in the spectral emissions of astronomical objects with more normal Co abundances such as planetary nebulae (Baluteau et al 1995; Zhang et al 2005; Pottasch & Surendiranath 2005; Wang & Liu 2007; Fang & Liu 2011).

Little computational and experimental work has been done previously to generate essential atomic data for Co III and none of the previous work deals with excitation of Co²⁺ levels by electron impact. Hansen et al (1984) calculated magnetic dipole and electric quadrupole transition probabilities in the 3d⁷ ground configuration of Co III using parametric fitting to the observed energy levels and Hartree-Fock values for the electric quadrupole moments. In their investigation of the forbidden transition probabilities relevant to the analysis of infrared lines from SN 1987A, Nussbaumer & Storey (1988) provided a few transition probabilities for low levels of Co III assuming LS-coupling. Tankosic et al (2003) calculated Stark broadening data for a number of Co III spectral lines as a function of temperature by using a semi-empirical approach. Experimental investigations have also been conducted by Sugar & Corliss (1981, 1985) where atomic data related to Co III transitions, mainly energy levels of Co²⁺, have been collected. Very recently, Fivet et al (2016) calculated radiative probabilities of Co III forbidden transitions between low-lying levels of doubly ionised cobalt as part of a larger investigation of the radiative rates in doubly ionised iron-peak elements.

We have recently reported a calculation of atomic parameters for energetically low-lying levels of Co⁺ (Storey et al 2016). In this paper we present a similar calculation of atomic parameters related to forbidden transitions in Co²⁺, which includes lines ranging from the visible to three mid-infrared lines which arise from transitions within the ground term at 11.88, 16.39 and 24.06 µm. The paper primarily addresses a shortage in collisional atomic data which forced some researchers (Dessart et al 2014; Childress et al 2015) to adopt collision strengths generated for Ni IV (Sunderland et al 2002) as a substitute for corresponding data of Co III justifying this by the fact that the two ions possess similar electronic and term structures. Our principal result is collision strengths and their thermally-averaged Maxwellian values for electron excitation and de-excitation between the fifteen lowest levels of Co²⁺. The study also includes the most important radiative transition probabilities for the same levels. The main tools used in generating...
2 Co$^{2+}$ ATOMIC STRUCTURE

2.1 The scattering target

A schematic diagram of the term structure of Co III up to 1.5 Rydberg is shown in Figure 1. The heavy solid line shows the extent of the close-coupled target states.

These data are the R-matrix atomic scattering code ([Berrington et al. 1974] [1987] [Hummer et al. 1990] [Berrington et al. 1995] and the general purpose AUTOSTRUCTURE code [Eissner et al. 1974] [Nussbaumer & Storey 1978] [Badnell 2011]). The scattering calculations were performed using a 10-configuration atomic target within a Breit-Pauli intermediate coupling approximation, as will be detailed in Section 2.

The paper is structured as follows. In Section 2 the Co$^{2+}$ model is presented and the resulting transition probabilities are given, whereas in Section 3 the Breit-Pauli R-matrix Co$^{2+} + e$ scattering calculation is described. Results and general analysis related to the diagnostic potentials of some transitions appear in Section 4, and section 5 concludes the paper.

Table 1. The ten target configuration basis where the core electronic structure ([Ar]) is suppressed. The bar indicates a correlation orbital.

| Term          | Energy (Ryd) |
|---------------|-------------|
| 3d$^7$        |             |
| 3d$^6$ 4s, 4p, 3d |     |
| 3d$^5$ 4s$^2$, 4p$^2$, 3d$^2$, 4s4p, 4s3d, 4p3d |   |

Table 2. Potential scaling parameters. The bar over the principal quantum number signifies a correlation orbital.

| Term          | Parameter |
|---------------|-----------|
| 1s            | 1.42912   |
| 2s            | 1.13799   |
| 3s            | 1.06915   |
| 4s            | 1.03440   |
| 2p            | 1.08143   |
| 3p            | 1.05203   |
| 3d            | 1.09462   |
| 4p            | 1.02977   |
| 3d            | 1.51187   |

were used to expand the target states. The target wavefunctions were generated with the AUTOSTRUCTURE program, [Eissner et al. 1974] [Nussbaumer & Storey 1978] [Badnell 2011] using radial functions computed within scaled Thomas-Fermi-Dirac statistical model potentials. The scaling parameters were determined by minimising the sum of the energies of all the target terms, computed in LS-coupling, i.e. by neglecting all relativistic effects. The resulting scaling parameters, $\lambda_{nl}$, are given in Table 2.

In Table 3, a comparison is made between the term energies calculated using our scattering target with experimental values for the 36 terms of the target. The term energies are computed with the inclusion of one-body relativistic effects, the Darwin and mass terms, and the spin-orbit interaction. This is the type of approximation that we applied for the scattering calculations in the R-matrix code. In Table 4, the calculated energies of the 15 lowest levels are compared with the corresponding experimental values. The table also shows the values obtained by including the two-body fine structure interactions as described by [Eissner et al. 1974]. The calculated fine-structure splittings of these levels are improved by this inclusion. For the total fine-structure splitting of the six terms, the average absolute difference from experiment drops from 7.3% to 4.6%.

A widely-accepted measure for the quality of the scattering calculations is the degree of agreement between weighted oscillator strengths, $gf$, calculated in the velocity and length formulations, where good agreement is regarded as necessary but not sufficient condition for the quality of the target wavefunctions. Table 5 provides this comparison where it shows an average difference in the absolute values of $gf$ of about 5.8% between the two formulations, which in our view is acceptable for an open d-shell atomic system.

2.2 Transition probabilities

The forbidden transition probabilities between the even-parity low-lying terms are calculated using the afore-described target wavefunctions, with empirical adjustments to the computed energies to ensure more reliable calculation of the fine-structure interactions and accurate energy factors connecting the \textit{ab initio} calculated line strengths to the transition probabilities. The results for the lowest 15 levels are given in Table 10 where the values represent the sum of

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1 See Badnell: R-matrix write-up on WWW. URL: amdpp.phys.strath.ac.uk/UK_RmaX/codes/
2 See Badnell: AUTOSTRUCTURE write-up on WWW. URL: amdpp.phys.strath.ac.uk/autos/
the electric quadrupole and magnetic dipole contributions for each transition. This table includes only those probabilities from a given upper level which exceed 1% of the total probability from that level.

The infrared lines of principal interest are due to transitions between the levels of the ground 3d\(^7\) term and are predominantly of magnetic dipole type. There is therefore a stepwise decay through the levels and only three relevant transition probabilities are known, for 3d\(^7\) - 3d\(^4\)s, 3d\(^7\) - 3d\(^4\)p, and 3d\(^7\) - 3d\(^4\)d. We are aware of only two previous calculations of transition probabilities for Co III, one by Hansen et al. (1984) and one by Nussbaumer & Storey (1988), as well as one preliminary calculation by Fivet et al. (2016). Nussbaumer & Storey (1988) only give values for these three probabilities and these differ by less than 1% from our values. Hansen et al. (1984) give more extensive results which we compare with the present values in Table 10. We find excellent agreement with Hansen et al. (1984) for the magnetic dipole transitions between the levels of individual terms with differences of a few percent or less. There are larger differences for the electric quadrupole transition probabilities between terms. For example the probabilities for the principal transitions between the 3d\(^7\) and 3d\(^4\)s configurations are 5.77, 4.83 for the G term and 5.77, 4.83 for the 4p term of the 15 lowest levels of Co III, ordered according to the experimental energy, where the configuration of all levels is [Ar] 3d\(^7\).

### Table 3. Energies of the 36 target terms in cm\(^{-1}\), ordered according to the experimental energy. The calculated values include only the spin-orbit contribution to the fine-structure energies. Core electronic structure ([Ar]) is suppressed from all configurations.

| Config. Term | Term Energy | Exp. \(^{1}\) | Calc. \(^{1}\) |
|-------------|-------------|--------------|--------------|
| 3d\(^7\) a\(^2\)F | 0 | 0 |
| a\(^4\)F | 14561 | 17891 |
| a\(^4\)G | 16510 | 19120 |
| a\(^4\)I | 19618 | 25103 |
| a\(^2\)H | 22227 | 25265 |
| a\(^4\)D | 22712 | 27597 |
| a\(^2\)F | 36372 | 43416 |
| 3d\(^6\)4s a\(^4\)D | 46239 | 48501 |
| a\(^4\)D | 55448 | 58817 |
| b\(^4\)P | 70965 | 79599 |
| a\(^4\)H | 71096 | 76483 |
| b\(^4\)F | 72717 | 80163 |
| a\(^2\)G | 76219 | 83370 |
| b\(^2\)P | 76521 | 85780 |
| b\(^2\)H | 76900 | 82428 |
| b\(^4\)F | 78323 | 86408 |
| b\(^2\)G | 81793 | 89400 |
| b\(^1\)D | 83031 | 92162 |
| a\(^2\)I | 84076 | 91484 |
| c\(^2\)G | 85485 | 93807 |
| b\(^2\)F | 90897 | 98436 |
| 3d\(^6\)4p x\(^2\)Po | 97807 | 97268 |
| 3d\(^6\)4s \(^2\)S | 100559 |
| 3d\(^6\)4p x\(^2\)Po | 102620 | 102460 |
| 3d\(^6\)4s \(^2\)S | 103690 |
| 3d\(^6\)4p x\(^2\)Po | 104861 | 104906 |
| 3d\(^6\)4p x\(^2\)Po | 106074 | 106802 |
| 3d\(^6\)4p x\(^2\)Po | 106767 | 107272 |
| 3d\(^6\)4p x\(^2\)Po | 109902 | 111225 |
| 3d\(^6\)4s \(^2\)F | 111250 |
| 3d\(^6\)4s \(^2\)F | 119049 |
| 3d\(^6\)4s \(^2\)F | 119600 |
| 3d\(^6\)4s \(^2\)F | 125226 |
| 3d\(^6\)4s \(^2\)F | 125907 |
| 3d\(^6\)4p x\(^2\)Go | 122305 | 129103 |
| 3d\(^6\)4p x\(^2\)Go | 124219 | 127494 |

\(^{1}\)Experimental energies are from NIST (www.nist.gov).

### Table 4. Energies in cm\(^{-1}\) of the 15 lowest levels of Co\(^{2+}\), ordered according to the experimental energy, where the configuration of all levels is [Ar] 3d\(^7\).

| Index | Level | Exp. \(^{1}\) | Calc. \(^{2}\) | Calc. \(^{3}\) |
|-------|-------|--------------|--------------|--------------|
| 1     | a\(^4\)F\(_{3/2}\) | 0.00 | 0.00 |
| 2     | a\(^4\)F\(_{5/2}\) | 841 | 810 |
| 3     | a\(^4\)F\(_{5/2}\) | 1451 | 1408 |
| 4     | a\(^4\)F\(_{3/2}\) | 1867 | 1819 |
| 5     | a\(^5\)P\(_{5/2}\) | 15202 | 18481 |
| 6     | a\(^5\)P\(_{3/2}\) | 15428 | 18770 |
| 7     | a\(^5\)P\(_{1/2}\) | 15811 | 19125 |
| 8     | a\(^2\)G\(_{7/2}\) | 16978 | 19565 |
| 9     | a\(^2\)G\(_{7/2}\) | 17766 | 20348 |
| 10    | a\(^2\)P\(_{3/2}\) | 20195 | 25601 |
| 11    | a\(^2\)P\(_{1/2}\) | 20919 | 26486 |
| 12    | a\(^1\)H\(_{11/2}\) | 22720 | 25690 |
| 13    | a\(^2\)D\(_{5/2}\) | 23059 | 27795 |
| 14    | a\(^2\)H\(_{9/2}\) | 23434 | 26367 |
| 15    | a\(^2\)D\(_{3/2}\) | 24237 | 29058 |

\(^{1}\)Sugar & Corliss (1985).
\(^{2}\)Calculated with only spin-orbit interaction.
\(^{3}\)As 2 plus two-body fine-structure interactions for the first 4 configurations of Table 1.

### Table 5. Weighted LS oscillator strengths, gf, in the length and velocity formulations from the two energetically lowest terms of the 3d\(^7\) and 3d\(^6\)4s configurations.

| Transition | \(g_{L}\) | \(g_{V}\) |
|------------|----------|----------|
| 3d\(^7\) - 3d\(^4\)p | 2.34 | 2.48 |
| - 3d\(^4\)d | 1.16 | 1.21 |
| - 3d\(^4\)s | 2.38 | 2.30 |
| 3d\(^6\)4s - 3d\(^4\)p | 9.45 | 9.75 |
| - 3d\(^4\)d | 13.7 | 13.5 |
| - 3d\(^4\)s | 5.77 | 4.83 |

\(^{1}\)Hansen et al. (1984). With this interaction in our calculation, the radial quadrupole integral and the 3d integral which is significantly larger than for the 3d.

Fivet et al. (2016) have made calculations of forbidden transition probabilities for the twice ionised iron-peak elements from Sc to Ni, including Co, and we compare with their results in Table 10. Their calculations were made with two different methods which we label as FQB1 and FQB2. The FQB2 values were computed with AUTOstructure as in the present work. Apart from the magnetic dipole transitions between the levels of the ground term, which agree to all tabulated figures, the FQB2 results for the electric quadrupole transitions between levels of different terms are systematically larger than the present work by 15-20% with half of them differing by the same fixed amount of 19%.
As discussed above in the comparison with the work of Hansen et al. (1984), the systematic nature of the difference suggests that it is due to a different value for the 3d radial quadrupole integral rather than details of the wave function expansions of individual terms. The configuration expansions in the present work and in that of Fivet et al. (2016) are very similar but differ in one key aspect. We use a somewhat contracted 3d orbital to allow for the differences in the 3d orbital between the 3d\(^7\) and 3d\(^6\)4s configurations, while Fivet et al. (2016) employ a spectroscopic 4d orbital but a contracted 5s orbital which provides flexibility to the spectroscopic 4s. These two different expansions give broadly similar energy levels and fine-structure but result in differences in the quadrupole radial integrals. It is not clear that either approach is necessarily superior, so the approximately 15-20% differences are probably a realistic measure of the uncertainty in the results for the electric quadrupole line strengths. We note that the results for the electric quadrupole transition probabilities from the FQB1 calculation of Fivet et al. (2016) agree better with their FQB2 for some transitions and better with the current work for others.

### 3 Scattering Calculations

In this work we used the Breit-Pauli R-matrix method, which is detailed in Hummer et al. (1993); Berrington et al. (1995) and the references therein, to perform the scattering calculations. The calculations were made using the R-matrix codes\(^3\) where the serial version of the codes were used in some stages and the parallel version in others. An R-matrix boundary radius of 11.3 au defining the inner region was applied so that the most extended orbital (4p) of our target is covered. Each one of the partial waves of the scattered electron was expanded over 12 basis functions within the R-matrix boundary, and the expansion extends to a maximum of \(J = 9\).

Collision strengths were computed over two non-overlapping energy meshes: a fine mesh consisting of 20000 evenly-divided intervals which go from zero up to the highest target threshold (about 1.2 Rydberg), and a coarse mesh consisting of 2000 evenly-divided intervals which reach 1 Rydberg above the highest target threshold. The purpose of the first mesh is to cover the main resonance region while the second mesh is intended to cover the region where all scattering channels are open, up to an incident electron energy of about 2.2 Rydberg. Our results demonstrate that these meshes have achieved these purposes. In Figure 2 we illustrate our results with the computed collision strengths between the lowest four levels of the ground 3d\(^7\) 4F term as a function of final electron energy up to 1 Rydberg above threshold. Dense and complex resonance structure can be seen in these plots due to the multiple close lying thresholds. We also show the collision strength averaged over 0.02 Rydberg intervals.

To ensure that the computed collision strengths have converged in partial wave for all the levels for which data are

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\(^3\) See Badnell: R-matrix write-up on WWW. URL: \texttt{amdpp.phys.strath.ac.uk/UK_RmaX/codes/}

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given, the contribution of partial wave $J = 9$ was compared to the sum for all transitions and energies. This comparison showed that in almost all cases the contribution from $J = 9$ is negligible. Specifically, the largest contribution from $J = 9$ is for the transition 8-12 at about 1% and the next largest is about 0.1% of the total. However, we note that it is certain that the collision strengths from the lower levels to the levels of the 4p configuration are not converged because they are allowed transitions which have significant high partial wave contributions. We therefore do not provide collision strengths for any of these transitions.

4 RESULTS AND DISCUSSION

The thermally-averaged collision strengths between the fifteen lowest energy levels are given in Table 11 as a function of electron temperature. These values were calculated using the full energy range, as described above. In the energy region where all scattering channels are open there are some small irregular features in the collision strengths that are almost certainly non-physical and caused by the correlation orbital in the target representation. We computed thermally-averaged collision strengths for the transitions and temperature range given in Table 11 both including and excluding the contribution from the region of all channels open, and found the largest change for any transition is 0.3% at log$_{10}T = 4.0$, 2.4% at log$_{10}T = 4.2$ and 9.4% at log$_{10}T = 4.4$. The values tabulated in Table 11 were computed using the full energy range.

4.1 Principal spectral lines

We compute the predicted Co$^{2+}$ fractional level populations using the results in Tables 10 and 11 with a fifteen level model atom including electron collisional excitation and de-excitation and radiative decay. In Tables 6 and 7 we show the resulting ten strongest lines of Co$^{3+}$ in this model. We also ensure that the three Co$^{2+}$ transitions and temperature range given in Table 11 both including and excluding the contribution from the region of all channels open, and found the largest change for any transition is 0.3% at log$_{10}T = 4.0$, 2.4% at log$_{10}T = 4.2$ and 9.4% at log$_{10}T = 4.4$. The values tabulated in Table 11 were computed using the full energy range.

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Table 6. The emissivity ratio, $\rho$, of the 10 strongest lines of Co $\Pi$ with $N_e = 10^4$ cm$^{-3}$ typical of PNe. We also add the 15.46 $\mu$m line discussed by Storey et al. (2016). The other details are as in Table 8.

| $j$ | $i$ | Transition | $\lambda$ | $\rho$ |
|-----|-----|------------|-----------|-------|
| 9   | 1   | $3d^7 3s^2 F_4^+ - 3d^8 3p^3 F_4$ | 1.091 $\mu$m | 1.17(+5) |
| 9   | 4   | $3d^7 3s^2 F_4^+ - 3d^7 4s^3 F_5$ | 1.545 $\mu$m | 6.58(+4) |
| 2   | 1   | $3d^8 3p^2 F_3^+$ | 10.52 $\mu$m | 3.81(+4) |
| 5   | 4   | $3d^7 3s^2 F_4^+ - 3d^7 4s^3 F_5$ | 14.74 $\mu$m | 2.86(+4) |
| 12  | 2   | $3d^8 3d^2 F_2$ | 9342.56 $\mu$m | 1.74(+4) |
| 9   | 2   | $3d^7 3s^2 F_4^+ - 3d^8 3p^3 F_4$ | 1.128 $\mu$m | 1.27(+4) |
| 9   | 6   | $3d^7 3s^2 F_4^+ - 3d^7 4s^3 F_4$ | 1.903 $\mu$m | 9.33(+3) |
| 13  | 2   | $3d^8 3p^2 F_4$ | 8121.13 $\mu$m | 9.04(+3) |
| 12  | 3   | $3d^8 3d^2 F_2$ | 9943.60 $\mu$m | 8.13(+3) |
| 10  | 2   | $3d^7 3s^2 F_4^+ - 3d^8 3p^3 F_4$ | 1.025 $\mu$m | 7.27(+3) |
| 3   | 2   | $3d^8 3p^2 F_3$ | 15.46 $\mu$m | 5.00(+3) |

Table 8. The emissivity ratio, $\rho$, of the 10 strongest lines of Co $\Pi$ with $N_e = 10^5$ cm$^{-3}$ typical of SN remnants. We also add the 10.52, 14.74 and 15.46 $\mu$m lines discussed by Storey et al. (2016). The other details are as in Table 8.

| $j$ | $i$ | Transition | $\lambda$ | $\rho$ |
|-----|-----|------------|-----------|-------|
| 12  | 2   | $3d^8 3d^2 F_2$ | 9342.56 $\mu$m | 4.25(+3) |
| 1   | 9   | $3d^7 3s^2 F_4^+ - 3d^8 3p^3 F_4$ | 1.019 $\mu$m | 2.41(+3) |
| 12  | 3   | $3d^8 3d^2 F_2$ | 9943.60 $\mu$m | 1.99(+3) |
| 13  | 2   | $3d^8 3p^2 F_3$ | 8121.13 $\mu$m | 1.72(+3) |
| 9   | 4   | $3d^7 3s^2 F_4^+ - 3d^7 4s^3 F_5$ | 1.547 $\mu$m | 1.35(+3) |
| 10  | 2   | $3d^7 3s^2 F_4^+ - 3d^8 3p^3 F_3$ | 1.025 $\mu$m | 1.07(+3) |
| 13  | 1   | $3d^8 3p^2 F_3$ | 7539.01 $\mu$m | 9.27(+2) |
| 11  | 2   | $3d^7 3s^2 F_4^+ - 3d^8 3p^3 F_3$ | 9639.21 $\mu$m | 8.72(+2) |
| 11  | 3   | $3d^7 3s^2 F_4^+ - 3d^8 3p^3 F_3$ | 1.028 $\mu$m | 8.70(+2) |
| 10  | 1   | $3d^7 3s^2 F_4^+ - 3d^8 3p^3 F_3$ | 9335.84 $\mu$m | 7.67(+2) |
| 2   | 1   | $3d^8 3p^2 F_4$ | 10.52 $\mu$m | 4.46(+2) |
| 5   | 4   | $3d^7 3s^2 F_4^+ - 3d^7 4s^3 F_5$ | 14.74 $\mu$m | 1.41(+2) |
| 3   | 2   | $3d^8 3p^2 F_3$ | 15.46 $\mu$m | 8.48(+1) |

have been computed and reported. The scattering calculations used the R-matrix method in the Breit-Pauli approximation under an intermediate coupling scheme.

The emissivities of the Co $\Pi$ forbidden lines were calculated with a 15-level Co$^{2+}$ model atom and the strongest lines listed with their expected strength relative to H$\beta$ for conditions approximately representative of those in planetary nebulae and supernova remnants. For comparison and completeness we also listed the strongest forbidden lines from Co $\Pi$ in the same conditions based on atomic parameters calculated and presented in a previous paper by Storey et al. (2016).

6 ACKNOWLEDGMENTS & STATEMENT

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Table 10. Transition probabilities in s\(^{-1}\) among the energetically lowest 15 levels of Co\(^{2+}\) as obtained from the current work (CW), from Hansen et al. (1984) (HRU), from Fivet et al. (2016) using HFR (FQB1) and from Fivet et al. (2016) using Autostructure (FQB2). The transition indices \(i\) and \(j\), which refer to the lower and upper levels respectively, are as in Table 4. Only the CW transition probabilities that are at least 1\% of the total probability from a given upper level are listed. The powers of 10 by which the numbers are to be multiplied are given in brackets.

| Transition | A-value | Transition | A-value |
|------------|---------|------------|---------|
| \(j\) | \(i\) | CW | HRU | FQB1 | FQB2 | \(j\) | \(i\) | CW | HRU | FQB1 | FQB2 |
| 2 | 1 | 2.00(-2) | 2.0(-2) | 2.01(-2) | 2.00(-2) | 11 | 3 | 2.23(-3) | 2.2(-3) |
| 3 | 2 | 1.31(-2) | 1.3(-2) | 1.32(-2) | 1.31(-2) | 11 | 4 | 2.69(-3) | 2.4(-3) |
| 4 | 3 | 4.63(-3) | 4.7(-3) | 4.65(-3) | 4.63(-3) | 11 | 7 | 1.77(-1) | 2.0(-1) | 1.98(-1) | 2.01(-1) |
| 5 | 1 | 5.55(-2) | 4.8(-2) | 6.59(-2) | 6.65(-2) | 11 | 10 | 6.42(-3) | 6.4(-3) |
| 5 | 2 | 1.51(-2) | 1.35(-2) | 1.74(-2) | 1.78(-2) | 12 | 1 | 6.02(-4) | 6.2(-4) |
| 5 | 3 | 3.14(-3) | 2.68(-3) | 3.94(-2) | 4.2(-2) | 12 | 8 | 4.29(-2) | 4.69(-2) |
| 6 | 2 | 3.14(-2) | 2.7(-2) | 3.69(-2) | 3.73(-2) | 13 | 2 | 7.34(-1) | 7.5(-1) | 7.44(-1) | 8.27(-1) |
| 6 | 3 | 1.58(-2) | 1.63(-2) | 2.18(-2) | 2.21(-2) | 13 | 3 | 7.94(-2) | 8.1(-2) |
| 6 | 4 | 5.14(-3) | 4.43(-3) | 3.65(-2) | 3.5(-2) |
| 7 | 3 | 2.30(-2) | 2.0(-2) | 2.71(-2) | 2.73(-2) | 13 | 5 | 4.74(-2) | 4.7(-2) |
| 7 | 4 | 3.02(-2) | 2.6(-2) | 3.57(-2) | 3.60(-2) | 13 | 6 | 2.38(-2) | 2.4(-2) |
| 7 | 6 | 2.45(-3) | 2.5(-3) | 13 | 10 | 1.87(-2) | 1.8(-2) |
| 8 | 1 | 3.71(-1) | 4.0(-1) | 3.91(-1) | 4.34(-1) | 14 | 1 | 3.61(-3) | 4.32(-3) |
| 8 | 2 | 1.17(-1) | 1.2(-1) | 1.23(-1) | 1.36(-1) | 14 | 2 | 1.90(-3) | 2.24(-3) |
| 9 | 1 | 1.38(-2) | 1.4(-2) | 14 | 8 | 1.23(-1) | 1.3(-1) | 1.33(-1) | 1.46(-1) |
| 9 | 2 | 1.40(-1) | 1.5(-1) | 1.50(-1) | 1.67(-1) | 14 | 9 | 3.70(-2) | 3.9(-2) | 4.03(-2) | 4.41(-2) |
| 9 | 3 | 1.04(-1) | 1.1(-1) | 1.12(-1) | 1.24(-1) | 14 | 12 | 5.26(-3) | 5.3(-3) |
| 9 | 8 | 7.19(-3) | 7.2(-3) | 15 | 3 | 6.93(-1) | 7.3(-1) | 7.34(-1) | 8.02(-1) |
| 10 | 2 | 5.36(-3) | 5.1(-3) | 15 | 4 | 3.67(-1) | 3.9(-1) | 3.86(-1) | 4.19(-1) |
| 10 | 3 | 6.52(-2) | 6.43(-2) | 6.20(-2) | 8.08(-2) | 15 | 6 | 1.52(-2) | 1.4(-2) |
| 10 | 4 | 4.64(-2) | 4.46(-2) | 4.27(-2) | 5.52(-2) | 15 | 10 | 1.49(-1) | 1.5(-1) | 1.41(-1) | 1.67(-1) |
| 10 | 5 | 1.41(-1) | 1.5(-1) | 1.55(-1) | 1.58(-1) | 15 | 11 | 2.71(-2) | 2.7(-2) |
| 10 | 6 | 7.26(-2) | 8.0(-2) | 8.04(-2) | 8.08(-2) | 15 | 13 | 2.43(-2) | 2.5(-2) |
| 10 | 7 | 3.01(-2) | 3.3(-2) |
Table 11: Thermally-averaged collision strengths among the 15 energetically lowest levels of Co^{2+} as a function of \( \log_{10} T \) in Kelvin where \( i \) and \( j \) refer to the index of the lower and upper level respectively (see Table 3 for indexing).

| \( i \) | \( j \) | \( \log_{10} T \) |
|---|---|---|
| 1 | 2 | 0.3747 4.171 4.321 4.573 |
| 1 | 3 | 1.490 1.471 1.511 1.630 |
| 1 | 4 | 0.429 0.448 0.473 0.502 |
| 1 | 5 | 1.285 1.328 1.379 1.409 |
| 1 | 6 | 0.578 0.611 0.626 0.618 |
| 1 | 7 | 0.232 0.215 0.201 0.188 |
| 1 | 8 | 0.963 0.944 0.937 0.949 |
| 1 | 9 | 0.312 0.322 0.315 0.299 |
| 1 | 10 | 0.361 0.377 0.404 0.422 |
| 1 | 11 | 0.180 0.166 0.147 0.127 |
| 1 | 12 | 4.532 3.989 3.412 2.884 |
| 1 | 13 | 0.375 0.369 0.374 0.393 |
| 1 | 14 | 0.374 0.374 0.364 0.342 |
| 1 | 15 | 0.070 0.066 0.065 0.066 |
| 2 | 3 | 3.301 3.280 3.245 3.264 |
| 2 | 4 | 0.732 0.760 0.831 0.962 |
| 2 | 5 | 1.089 1.064 1.046 1.026 |
| 2 | 6 | 0.658 0.682 0.695 0.690 |
| 2 | 7 | 0.292 0.274 0.254 0.233 |
| 2 | 8 | 0.617 0.610 0.602 0.601 |
| 2 | 9 | 0.490 0.487 0.474 0.461 |
| 2 | 10 | 0.231 0.230 0.240 0.258 |
| 2 | 11 | 0.177 0.188 0.195 0.191 |
| 2 | 12 | 1.319 1.343 1.329 1.257 |
| 2 | 13 | 0.354 0.349 0.340 0.335 |
| 2 | 14 | 0.593 0.590 0.585 0.573 |
| 2 | 15 | 0.163 0.157 0.153 0.150 |
| 3 | 4 | 1.591 1.611 1.692 1.855 |
| 3 | 5 | 1.016 0.953 0.886 0.822 |
| 3 | 6 | 0.544 0.574 0.594 0.592 |
| 3 | 7 | 0.301 0.283 0.264 0.248 |
| 3 | 8 | 0.343 0.344 0.342 0.340 |
| 3 | 9 | 0.509 0.493 0.476 0.466 |
| 3 | 10 | 0.129 0.130 0.140 0.155 |
| 3 | 11 | 0.151 0.159 0.172 0.184 |
| 3 | 12 | 0.312 0.358 0.393 0.401 |
| 3 | 13 | 0.251 0.244 0.232 0.220 |
| 3 | 14 | 0.627 0.625 0.640 0.646 |
| 3 | 15 | 0.179 0.177 0.175 0.175 |
| 4 | 5 | 0.910 0.818 0.721 0.631 |
| 4 | 6 | 0.373 0.394 0.404 0.394 |
| 4 | 7 | 0.261 0.248 0.233 0.220 |
| 4 | 8 | 0.163 0.166 0.166 0.164 |
| 4 | 9 | 0.376 0.365 0.356 0.355 |
| 4 | 10 | 0.059 0.067 0.078 0.080 |
| 5 | 6 | 1.391 1.531 1.673 1.759 |
| 5 | 7 | 0.900 0.835 0.777 0.732 |
| 5 | 8 | 0.518 0.490 0.473 0.473 |
| 5 | 9 | 0.339 0.343 0.330 0.304 |
| 5 | 10 | 0.205 0.197 0.205 0.225 |
| 5 | 11 | 0.143 0.131 0.118 0.106 |

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Table 11: continued.

| i | j | \(T\) (K) | \(\log_{10}T\) |
|---|---|---|---|
| 5 | 12 | 0.265 | 2.0 |
| 5 | 13 | 0.273 | 2.2 |
| 5 | 14 | 0.297 | 2.4 |
| 5 | 15 | 0.322 | 2.6 |
| 5 | 16 | 0.364 | 2.8 |
| 5 | 17 | 0.379 | 3.0 |
| 5 | 18 | 0.376 | 3.2 |
| 5 | 19 | 0.362 | 3.4 |
| 5 | 20 | 0.348 | 3.6 |
| 5 | 21 | 0.345 | 4.0 |
| 5 | 22 | 0.356 | 4.2 |
| 5 | 23 | 0.374 | 4.4 |
| 6 | 7 | 0.160 | 0.000 |
| 6 | 8 | 0.162 | 0.000 |
| 6 | 9 | 0.170 | 0.000 |
| 6 | 10 | 0.185 | 0.000 |
| 6 | 11 | 0.200 | 0.000 |
| 6 | 12 | 0.212 | 0.000 |
| 6 | 13 | 0.222 | 0.000 |
| 6 | 14 | 0.232 | 0.000 |
| 6 | 15 | 0.246 | 0.000 |
| 6 | 16 | 0.261 | 0.000 |
| 6 | 17 | 0.277 | 0.000 |
| 6 | 18 | 0.293 | 0.000 |
| 6 | 19 | 0.301 | 0.000 |

**Atomic data for Co III forbidden lines**

log

2.0 2.2 2.4 2.6 2.8 3.0 3.2 3.4 3.6 3.8 4.0 4.2 4.4

0.000–000