Study of Branes with Variable Tension

Rodrigo Aros$^{1,4}$ and Milko Estrada$^{2,3,4}$

$^1$Departamento de Ciencias Fisicas, Universidad Andres Bello, Av. Republica 252, Santiago, Chile
$^2$Department of Physics, Universidad de Antofagasta, postal address 1240000 Antofagasta, Chile
$^3$Instituto de Matematica, Fisica y Estadistica, Universidad de las Americas, Manuel Montt 948, Providencia, Santiago, Chile

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Abstract In this work we study a brane world model with variable tension, which gives rise to four-dimensional cosmologies. The brane worlds obtained correspond to E"otv"os branes whose (internal) geometry can be casted as either a four-dimensional (A)dS$_5$ or a standard radiation period cosmology. The matter dominated period is discussed as well.

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1 Introduction

A largely open question in physics has been the huge difference between the values of the Higgs mass, $m_H \approx 1$ TeV, and the Planck mass $m_P \approx 10^{19}$ GeV, also called the hierarchy problem. To address this in 1999 Randall and Sundrum[1] proposed a model of two statics 3-branes with constant tensions of equal magnitude but opposite signs embedded in an AdS$_5$ space. Our observed universe corresponds in this model to the positive tension brane. The geometry considered in that model can be described by the metric

$$ds_5^2 = e^{-2kr|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu + r^2d\phi^2,$$  

where $e^{-2kr|\phi|}$ is called the warp factor. The coordinate system $x^M = (x^\mu, \phi)$, with $x^\mu = (t, x, y, z)$, satisfies $-\infty < x^\mu < \infty$ and $\phi \in [-\pi, \pi]$. Our universe is located at $\phi = 0$ and the secondary brane, which is called the strong brane, at $\phi = \pi = -\pi$. The hierarchy problem is solved in this model due to the masses of the branes are related by $m_{\phi=0} = e^{-kr\pi}m_{\phi=\pi}$. This allows the Planck mass at strong brane to be of same order of Higgs mass at our universe provided $e^{kr\pi} \approx 10^{15}$.

In RS model both branes have constant tension and the geometry is static. This last, however, does not describe an evolving universe.\cite{2,3,4} To do that the brane section should be promoted at least to (a flat-FLRW)

$$\eta_{\mu\nu}dx^\mu dx^\nu \rightarrow -dt^2 + a(t)^2dx \cdot dx.$$  

From a geometrical point of view Eq. (2) is not a minor change. A solution is to consider a brane world scenario with variable tension branes. This implies that the branes considered cannot be fundamental branes of the standard fashion. However, this not all, and another aspect to be addressed is establishing the dependency on the temperature of the evolution of the brane-universe or viceversa. Reference [5] was proposed that the tension of our brane universe should depend on the temperature of the universe according to the E"otv"os law:

$$T = K(x_c - x),$$  

where $K$ is a positive constant, $x$ stands for the temperature and $x_c$ is the initial temperature of our universe. Motivated by Stefan–Boltzmann law, where the energy density of the CMB is proportional to $x^4$, it can be proposed that $x \propto a^{-1}$ and therefore Eq. (3) can be rewritten as:

$$T = Kx_c \left(1 - \frac{a_{\text{min}}}{a(t)}\right),$$  

where $a_{\text{min}}$ is the initial value of scale factor on our universe. This model is called E"otv"os branes, see Ref. [5], and predicts that as our universe expands, and the temperature decreases, the tension of our brane universe increases. In fact, this model is completely compatible with the current observations. It must be stressed that, at least in principle, this model does not constraint the strong brane dependency on time or temperature.

In Ref. [6] Abdalla, da Silva and da Rocha proposed a modification of the E"otv"os branes where the brane tension becomes the linear function of $t$, the FLRW time,

$$T = \pm \lambda t \pm \beta,$$  

where $\lambda$ and $\beta$ are positive constants. In this model as universe expands it cools and the brane tension increases.
This model reproduces most of the features of the Eötvös branes mentioned above. Unfortunately this model does not provide a direct solution for the scale factor $a(t)$ in the FLRW metric.

In work we aim to test a toy model of variable tension branes that reproduce an Eötvös brane for the universe and simultaneously constrains the form $a(t)$ by the five-dimensional Einstein equations.

To consider non constant tension branes requires to propose a model for the brane. Following Ref. [6], and in order to simplify the computations, the only change, with respect to a fundamental brane, is to replace the constant tension by

$$T + t \frac{dT}{k \ dt},$$

where $k$ is a dimensionless parameter. The idea behind this is to introduce a conformal expansion along the FLRW time. This choice allows to obtain different solution for $T$, using the Brane World Sum Rules of the same fashion as Eq. (4) but without the need to impose a priori an Eötvös tension.

It must be stressed that even with the dependency on time of the tensions, this model still provides with a solution the hierarchy problem. To confirm that one can notice that the mass relation depends on the warp factor which, in turns, is only a function of the extra dimension, say $\phi$.

The number of solutions is large, but since this work is not aimed to represent the complete scenario of a variable tension brane world model, the analysis will be restricted to only three cases of physical interest. It is worth noticing that this model is compatible with those discussed in Ref. [7] and references there in.

In the next section, we derive the Brane World Sum Rules method. Next we will find the tensions of both branes and the scale factor.

## 2 Brane World Sum Rules Method

Brane World Sum Rules is a set of consistency conditions derived from the Einstein equations for brane world scenarios with spatially periodic internal space. See Ref. [8-9]. This method allows to find relations between the cosmological constant $\Lambda$, the number of solutions is large, but since this work is not aimed to represent the complete scenario of a variable tension brane world model, the analysis will be restricted to only three cases of physical interest. It is worth noticing that this model is compatible with those discussed in Ref. [7] and references there in.

In the next section, we derive the Brane World Sum Rules method. Next we will find the tensions of both branes and the scale factor.

$$R^{(5D)}_{\mu\nu} = - \frac{4W'^2}{W},$$

where $'$ indicates differentiation with respect to $\phi$. Now we proceed to do some manipulations to extract the physical contain of Eqs. (8) and (9). By contracting Eq. (8) with $g^{(5D)}_{\mu\nu} = W^{-2}g^{(5)}_{\mu\nu}(x^\mu)$ and Eq. (9) by $g^{(5)}_{55} = 1/r^2$ one obtains,

$$R^{(5D)}_{\mu} - R^{(4D)}W^{-2} = - \frac{12}{r^2} (W')^2 W^{-2} - \frac{4}{r^2} W^\prime W^{-1}, \quad (10)$$

$$R^{(5D)}_5 = - \frac{4}{r^2} W^\prime W^{-1}. \quad (11)$$

In the same fashion, multiplying Eq. (10) by $(1-n)W^n$, Eq. (11) by $(n-4)W^n$ and then adding both equations, yields

$$\frac{R^{(5D)}_{\mu} - R^{(4D)}W^{-2}}{12} = \frac{(n-1)(W')^2 + W^n}{r^2}, \quad (12)$$

where it can be recognized

$$W^n \frac{(n-1)(W')^2 + W^n}{r^2} = \frac{(W^n)'^2}{W^2} = \frac{(W^n)^''}{n}, \quad (13)$$

and thus the relation (12) can be written as

$$\frac{(W^n)''}{r^2} = \frac{R^{(5D)}_u - R^{(4D)}W^{-2}}{12} (1-n)W^n$$

$$R^{(5D)}_5 (n-4)W^n. \quad (14)$$

By using the Einstein equation one can obtain

$$R^{(5D)}_u = \frac{8\pi G_5}{3}(T^\mu - 4T^5_\mu), \quad (15)$$

$$R^{(5D)}_5 = \frac{8\pi G_5}{3}(-T^\mu + 2T^5_\mu). \quad (16)$$

Now, by multiplying equations (15) by $(1-n)/12$ and (16) by $(n-4)/12$, it can be obtained

$$\frac{(1-n)}{12} R^{(5D)}_u + \frac{(n-4)}{12} R^{(5D)}_5 = \frac{2}{3} \pi G_5 [T^\mu + (2n-4)T^5_\mu]. \quad (17)$$

To simplify the expressions it is convenient to redefine $W = e^{-A}\phi$. With this Eq. (14) becomes

$$- \frac{1}{r^2} (A' e^{-A}) = \frac{2}{3} \pi G_5 e^{-nA} (T^\mu + (2n-4)T^5_\mu) \quad (18)$$

Next, by noticing that the integration of the left side of Eq. (18) vanishes for compact internal spaces without boundary (for example RS), it is obtained

$$\int_{-\pi}^{\pi} d\phi e^{-nA}(T^\mu + (2n-4)T^5_\mu) = \frac{1-n}{8\pi G_5} R^{(4D)} \int_{-\pi}^{\pi} d\phi e^{(2-n)A}. \quad (19)$$
This is particular convenient if one has to consider matter fields constrained to the branes. For later convenience it is worth to explicitly mention the case \( n = 0 \),

\[
\int_{-\pi}^{\pi} d \phi (T^a_\mu - 4T^5_\mu) = \frac{R^{(4D)}}{8\pi G_5} \int_{-\pi}^{\pi} d \phi e^{2A}.
\]  

(20)

3 Calculation of the Tensions of the Two Branes

As mentioned above in this work it is considered only a modification of the tension brane, which is replaced by (the conformal expansion on the FLRW time) \( T + (t/k)(dT/dt) \). Therefore, energy momentum tensor proposed including the two branes is given by

\[
T\_{MN} = -\frac{\Lambda_{5D}}{8\pi G_5} g_{MN} - \left( T_1 + \frac{t}{k} \frac{dT_1}{dt} \right) h^{0}_{\mu\nu} \delta^5_{\nu} \delta(\phi) + \left( T_2 + \frac{t}{k} \frac{dT_2}{dt} \right) h^{0}_{\mu\nu} \delta^5_{\nu} \delta(\phi - \pi) + \tilde{T}_{MN}.
\]

(21)

Here \( \tilde{T}_{MN} \) stands for the EM tensor of the rest of the matter fields, either constrained or not to the branes.

It can be noticed that in this toy model are not considered equations of motion to be satisfied by the matter fields. This obviously produces that part of the physics be the discussion below.

In order to solve these equations one can notice that

\[
T^5_\mu = -\frac{\Lambda_{5D}}{8\pi G_5} + \tilde{T}^5_\mu.
\]

(22)

The partial trace, on the other hand, yields the second condition,

\[
T^\mu_\mu = -4\frac{\Lambda_{5D}}{8\pi G_5} - 4 \left( T_1 + \frac{t}{k} \frac{dT_1}{dt} \right) \delta(\phi) - 4 \left( T_2 + \frac{t}{k} \frac{dT_2}{dt} \right) \delta(\phi - \pi) + \tilde{T}^\mu_\mu.
\]

(23)

Now, replacing Eqs. (22) and (23) into Eq. (20) gives rise

\[
T_1 + \frac{t}{k} \frac{dT_1}{dt} + T_2 + \frac{t}{k} \frac{dT_2}{dt} = U + V,
\]

(24)

where

\[
U = -\frac{R^{(4D)}}{32\pi G_5} \int_{-\pi}^{\pi} d \phi e^{2A},
\]

(25)

\[
V = \frac{1}{4} \int_{-\pi}^{\pi} d \phi (\tilde{T}^5_\mu - 4\tilde{T}^5_\mu).
\]

(26)

It can be noticed that the nature of \( U \) and \( V \) are different. While \( U \) is purely geometric, and thus can be fit to satisfy a particular ansatz geometry, \( V \) depends on the 5d matter fields considered making the \( V \neq 0 \) case harder to be addressed consistently in general. Fortunately, \( V = 0 \) occurs for the standard cosmological cases \( T^{MN} = 0 \) and \( (\tilde{T}^5_\mu = 0, \tilde{T}^5_\mu = 0) \). See below. In either case, Eq. (24) reduces to

\[
T_1 + \frac{t}{k} \frac{dT_1}{dt} + T_2 + \frac{t}{k} \frac{dT_2}{dt} = U.
\]

(27)

Another comment is in place. It must be noticed that unlike RS scenario here it is not necessary that \( T_1 = -T_2 \) or that they even have opposite signs. This is due to the presence of the term \( U \) and \( T_1 \) and \( T_2 \) are not constants.

From now on it will be considered that our universe corresponds to brane with tension \( T_1 \). In this case, it must be stressed, the physics of the other brane is not totally determined. The reason for this, as mentioned above, is that equations of motion of the five-dimensional matter fields, and its boundary conditions, are not considered in this toy model.

In next section it will be studied the cases where \( T_1 \), the tension of the brane that represents our universe, can be cast as

\[
T_1 = Kx_c \left( 1 - \frac{a_{\text{min}}}{a(t)} \right),
\]

(28)

where \( K \) is a positive constant.

It is worth to notice that the brane tension can only be associated to an energy density for isotropic static (or nearly adiabatic) backgrounds. In fact, in our case that connection fails and, according with the idea of the Eötvös branes, \( T_1 \) does not, and could not, represent an energy density. In fact, \( T_1 \) represents the tension of a physical membrane according to Eötvös law which, in our context, has positive sign and increases as the temperature decreases with time. It indeed increases as the scale factor increases. In this way this reproduces the evolution of the universe in expansion.

4 de-Sitter 4 Scenario

To begin discussion we will start with case \( \tilde{T}^{MN} = 0 \). In this case we consider a flat transverse section in the brane, i.e., a metric of the form

\[
d s^2 = e^{-2A(\phi)} (-dt^2 + a(t)^2 dx \cdot dx) + r^2 d\phi^2.
\]

(29)

The generalization to constant curvature transverse section is straightforward and does not provide new physical relevant information.

Remarkably this case contains as solution

\[
a(t) = a H(t-t_0),
\]

(30)

where \( H \) is a constant and \( t_0 \) is the current age of our universe. \( H \) corresponds to the standard Hubble constant (parameter). The tension of the de-Sitter brane is given by

\[
T_1 = U - \frac{C_3}{e^{H(t-t_0)}},
\]

(31)

which satisfies the Eötvös law. By setting \( U = Kx_c \) (since \( U \) is constant in dS4 case) and \( C_3 = a_{\text{min}} U = a_{\text{min}} Kx_c \), the \( T_1 \) can be written as

\[
T_1 = Kx_c \left( 1 - \frac{a_{\text{min}}}{e^{H(t-t_0)}} \right).
\]

(32)

It must be noticed, since \( U > 0 \), \( T_1 \) increases with \( t \) (or with \( a(t) \)) yielding a brane which becomes more rigid as
time evolves. On the other hand, from Eq. (27), the strong brane tension,
\[ T_2 = \sqrt{m \rho_x} e^H (t - t_0), \]  
(33)
is positive. It is direct to observe that \( T_2 \) decreases with the time and therefore it becomes less rigid as time evolves.

It must be noticed that, even though this model is inspired by a generalization of an RS space, to immerse the branes into a negative cosmological space \( \Lambda_{5D} < 0 \) is not strictly necessary. This is due to the fact that dS4 can be immersed into an AdS5 as well as into a dS5 or even into a five-dimensional Minkowski space provided certain conditions are satisfied.

**Constraints**

In Table 1 are shown the different warp factors, the corresponding \( U \) and the constraints for \( U > 0 \), such as \( T_1 \) increases with \( a(t) \), at the scenarios \( \Lambda_{5D} \to 0 \) and \( \Lambda_{5D} = \pm 6/l^2 \) respectively,

| \( \Lambda_{5D} \)  | \( U \) | Constraint |
|------------------|--------|------------|
| \( \to 0 \)      | \( (|\phi| - K_1)^2 \frac{H^2 t^2}{4} \) | \( \frac{3}{4 \pi^2 K_1 G_5 (K_1 - \sigma)} \) | \( K_1 < \pi \) |
| \( < 0 \)       | \( \frac{(\rho)^2}{2} \sinh^2 (K_1 - \frac{1}{2} |\phi|) \) | \( \frac{3 \cosh (K_1) - \cosh (K_1 - \frac{3 \pi}{2})}{4 \pi r G_5} \) | \( K_1 < \frac{\pi}{2} \) |
| \( > 0 \)       | \( \frac{(\rho)^2}{2} \sinh^2 (K_1 \pm \frac{1}{2} |\phi|) \) | \( \frac{3 \cosh (K_1) - \cosh (K_1 + \frac{3 \pi}{2})}{4 \pi r G_5} \) | \( * \) |

where \( K_1 \) is a constant. \( * \) stands for the fact for any value of \( K_1 \) the integral \( U \) is positive.

5 Radiation Domination

The next simplest solution can be obtained by imposing the vanishing of the four-dimensional Ricci scalar, \( R^{(4D)} = 0 \). See Eq. (19). For this let us consider a general energy momentum tensor, only constrained by the symmetries of the space,[10]

\[ \tilde{T}_N^C = \text{diag} (-\rho(t, \phi), p(t, \phi), p(t, \phi), p(t, \phi), p(t, \phi)). \]  
(34)

Firstly, to have a solution with the line element (29) is necessary to be restricted to the case \( \Lambda_{5D} < 0 \). It is direct to prove, as for standard cosmology, that for \( R^{(4D)} = 0 \) the direct solution is \( a(t) = (t/t_0)^{1/2} \) where \( t_0 \) is the current cosmological time. However, and unlike the standard cosmological case, this restricts a set of functions of \( \phi \) and \( t \). The Einstein equations determine that \( p_0(t, \phi) = 0 \) and thus,

\[ A(t) = \left( \frac{1}{a(t)} \right), \]  
(35)

where the negative five-dimensional cosmological constant has been fixed as \( \Lambda_{5D} = -6/l^2 \). Finally, the Einstein equations also determine that

\[ \rho(t, \phi) = \frac{3}{4} \frac{e^2(t, \phi)}{t^2} \text{ and } p(t, \phi) = \frac{1}{4} \frac{e^2(t, \phi)}{t^2}. \]  
(36)

Since the trace of \( \tilde{T}^{MN} \) vanishes (\( \tilde{T}_M^M = 0 \)), as well as its partial trace (\( \tilde{T}_M^\mu = 0 \)), \( \tilde{T}^{MN} \) can be considered to be describing a 5-dimensional generalization of a four-dimensional electromagnetic field. One has to notice that the energy momentum tensor extends into space between both branes. One can argue, by mere observation, that this energy momentum tensor \( \tilde{T}^{MN} \) can be split as a combination of a five-dimensional pressureless fluid, a fluid of string-like objects with endpoints at both branes and a two form field of the form \( B(x^\mu, \phi) = A(x^\mu, \phi) dx^\mu \wedge d\phi \) which couples the string-like fluid. The pullback of \( B \) into the branes, \( A \), can be interpreted as the EM gauge potential.

**Tension of Both Branes**

As \( V = 0 \) and \( \tilde{T}_4^\mu - 4\tilde{T}_5^\mu \) vanishes, then Eq. (27) implies

\[ T_1 + T_2 = -\frac{C}{l^2}, \]  
(37)

where \( C \) and \( k \) are arbitrary. Now, in order to have a consistent set of equations and fix \( T_1 \) into the form of an Eötvös brane, it is necessary that \( k = 1/2 \). This implies that the tensions are respectively,

\[ T_1 = H_1 - \frac{C_4}{l^{1/2}}, \]  
(38)
\[ T_2 = -H_1 + \frac{C_5}{l^{1/2}}, \]  
(39)

where \( C = C_4 - C_5 \) and \( H_1 \) are arbitrary constants. To cast \( T_1 \) into the form of an Eötvös brane form in Eq. (4) it is necessary to fix the constant as \( H_1 = Kx_\infty \) and \( C_4 = m_{\text{min}} l^{1/2} H_1 \).

As mentioned above \( T_2 \) is left partially undetermined. The simplest solution is \( T_2 = -Kx_\infty (C_5 = 0) \) leaving a negative, but constant, tension brane.

6 Matter Dominated Era

In the model proposed, for the line element (29), is also possible to obtain a solution with \( A(\phi) \) similar to Eq. (35) and \( a(t) = (t/t_0)^{2/3} \) which corresponds, from the point of view of the universe brane, to a matter dominated era in a standard cosmological model. For this to happen
$p(t, \phi) = 0$ must be satisfied as well. In addition

$$\rho(t, \phi) = \frac{4}{3} \frac{e^{2A(\phi)}}{t^2}. \quad (40)$$

Finally, the rest of the Einstein equations implies that

$$p_5(t, \phi) = -\frac{2}{3t^2} e^{2A(\phi)}, \quad (41)$$

which implies that although there is no pressure along the brane directions it must exist along the fifth dimension for an equilibrium to exist. This can be modeled by a pressure-less (particle) fluid combined with non-fundamental string like fluid with endpoints at the branes.

**Tensions of Both Branes**

In this case Ricci Scalar

$$R^{(4D)} = \frac{4}{3t^2}, \quad (42)$$

and from Eqs. (25) and (26) one can identify that

$$U = \frac{l}{rt^2} \left( \frac{1 - \exp(2\pi(r/l))}{24\pi G_5} \right),$$

$$V = \frac{l}{3rt^2} \left( \exp \left( \frac{2\pi r^2}{l} \right) - 1 \right). \quad (43)$$

Now, by defining $U + V = m/t^2$, where $G_5$ is dimensionless, then it can be shown that $T_1$, the tension of our brane universe, can be shaped up into the form of the tension of an Eötvös brane,

$$T_1(t) = Kx_c \left(1 - \frac{a_{\text{min}}}{a(t)} \right), \quad (44)$$

where $a = (t/t_0)^{2/3}$, and thus $T_1 > 0$.

On the other hand, from Eq. (24), the tension of strong brane is given by

$$T_2(t) = -Kx_c \left(1 - \frac{a_{\text{min}}}{a(t)} \right) - \frac{m}{2t^2}, \quad (45)$$

thus $T_2 < 0$.

**7 Conclusions**

Using the modification of energy momentum tensor of Eq. (21), we have shown that it is possible to study the branes with temporally variable tension using Brane World Sum Rules. Specifically we have studied branes with variable tensions that resemble Eötvös branes. The principal result is that this model reproduces, within its scope, the standard known results of cosmology for our brane world. In fact, the Eötvös brane describing our universe has an induced scale factor describing an expanding universe as the temperature decreases. In addition, for this model, we have displayed an alternative approach to reproduce the standard scale factors for matter, radiation and cosmological constant dominated eras.

It must be noticed that although it is not necessarily to consider the presence of a 4D cosmological constant still an effective one could arise. In particular, at de Sitter scenario, see Sec. 4, by using relations between the Einstein tensor $G^{(5D)}_{\mu\nu}$ and $G^{(4D)}_{\mu\nu}$, one can check that the effective 4D Einstein equation has the form $G^{(4D)}_{\mu\nu} = -3H^2 g^{(4D)}_{\mu\nu}$, see Refs. [11–12], and thus one can cast an effective cosmological constant by defining $H = (\Lambda_{4D}/3)^{1/2}$. This effective cosmological constant could have the accepted value $\Lambda_{4D} \approx 10^{-124}$ Planck units.

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