Quantum mechanics: Myths and facts

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January 17, 2022

Abstract

A common understanding of quantum mechanics (QM) among students and practical users is often plagued by a number of “myths”, that is, widely accepted claims on which there is not really a general consensus among experts in foundations of QM. These myths include wave-particle duality, time-energy uncertainty relation, fundamental randomness, the absence of measurement-independent reality, locality of QM, nonlocality of QM, the existence of well-defined relativistic QM, the claims that quantum field theory (QFT) solves the problems of relativistic QM or that QFT is a theory of particles, as well as myths on black-hole entropy. The fact is that the existence of various theoretical and interpretational ambiguities underlying these myths does not yet allow us to accept them as proven facts. I review the main arguments and counterarguments lying behind these myths and conclude that QM is still a not-yet-completely-understood theory open to further fundamental research.

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1 Introduction

On the technical level, quantum mechanics (QM) is a set of mathematically formulated prescriptions that serve for calculations of probabilities of different measurement outcomes. The calculated probabilities agree with experiments. This is the fact! From a pragmatic point of view, this is also enough. Pragmatic physicists are interested only in these pragmatic aspects of QM, which is fine. Nevertheless, many physicists are not only interested in the pragmatic aspects, but also want to understand nature on a deeper conceptual level. Besides, a deeper understanding of nature on the conceptual level may also induce a new development of pragmatic aspects. Thus, the conceptual understanding of physical phenomena is also an important aspect of physics. Unfortunately, the conceptual issues turn out to be particularly difficult in the most fundamental physical theory currently known – quantum theory.

Textbooks on QM usually emphasize the pragmatic technical aspects, while the discussions of the conceptual issues are usually avoided or reduced to simple authoritative claims without a detailed discussion. This causes a common (but wrong!) impression among physicists that all conceptual problems of QM are already solved or that the unsolved problems are not really physical (but rather “philosophical”). The purpose of the present paper is to warn students, teachers, and practitioners that some of the authoritative claims on conceptual aspects of QM that they often heard or read may be actually wrong, that a certain number of serious physicists still copes with these foundational aspects of QM, and that there is not yet a general consensus among experts on answers to some of the most fundamental questions. To emphasize that some widely accepted authoritative claims on QM are not really proven, I refer to them as “myths”. In the paper, I review the main facts that support these myths, but also explain why these facts do not really prove the myths and review the main alternatives. The paper is organized such that each section is devoted to another myth, while the title of each section carries the basic claim of the corresponding myth. (An exception is the concluding section where I attempt to identify the common origin of all these myths.) The sections are roughly organized from more elementary myths towards more advanced ones, but they do not necessarily need to be read in that order. The style of presentation is adjusted to readers who are already familiar with the technical aspects of QM, but want to complete their knowledge with a better understanding of the conceptual issues. Nevertheless, the paper is attempted to be very pedagogical and readable by a wide nonexpert audience. However, to keep the balance and readability by a wide physics audience, with a risk of making the paper less pedagogical, in some places I am forced to omit some technical details (especially in the most advanced sections, Secs. 9 and 10), keeping only those conceptual and technical details that are essential for understanding why some myths are believed to be true and why they may not be so. Readers interested in more technical details will find them in more specialized cited references, many of which are pedagogically oriented reviews.

As a disclaimer, it is also fair to stress that although this paper is intended to be a review of different views and interpretations of various aspects of QM, it is certainly not completely neutral and unbiased. Some points of view are certainly emphasized more than the others, while some are not even mentioned, reflecting subjective preferences of the author. Moreover, the reader does not necessarily need to agree with all conclusions
and suggestions presented in this paper, as they also express a subjective opinion of
the author, which, of course, is also open to further criticism. By dissolving various
myths in QM, it is certainly not intention of the author to create new ones. The main
intention of the author is to provoke new thinking of the reader about various aspects
of QM that previously might have been taken by him/her for granted, not necessarily to
convince the reader that views presented here are the right ones. Even the claims that
are proclaimed as “facts” in this paper may be questioned by a critical reader. It also
cannot be overemphasized that “myths” in this paper do not necessarily refer to claims
that are wrong, but merely to claims about which there is not yet a true consensus.

2 In QM, there is a wave-particle duality

2.1 Wave-particle duality as a myth

In introductory textbooks on QM, as well as in popular texts on QM, a conceptually
strange character of QM is often verbalized in terms of wave-particle duality. According
to this duality, fundamental microscopic objects such as electrons and photons are neither
pure particles nor pure waves, but both waves and particles. Or more precisely, in some
conditions they behave as waves, while in other conditions they behave as particles. How-
ever, in more advanced and technical textbooks on QM, the wave-particle duality is rarely
mentioned. Instead, such serious textbooks talk only about waves, i.e., wave functions
ψ(x, t). The waves do not need to be plane waves of the form ψ(x, t) = e^{i(kx−ωt)}, but, in
general, may have an arbitrary dependence on x and t. At time t, the wave can be said
to behave as a particle if, at that time, the wave is localized around a single value of x.
In the ideal case, if

\[ \psi(x) = \sqrt{\delta}(x - x'), \]  

then the position x of the particle has a definite value x'. The state (1) is the eigenstate
of the position operator, with the eigenvalue x'. Typically, the wave attains such a
localized-particle shape through a wave-function collapse associated with a measurement
of a particle position. Moreover, the wave may appear as a pointlike particle for a long
time if the particle position is measured many times in sequence with a small time interval
between two measurements. This makes the wave to appear as a classical particle with
a trajectory, which occurs, e.g., in cloud chambers. However, the position operator is
just one of many (actually, infinitely many) hermitian operators in QM. Each hermitian
operator corresponds to an observable, and it is widely accepted (which, as we shall see
later, is also one of the myths) that the position operator does not enjoy any privileged
role. From that, widely accepted, point of view, there is nothing dual about QM; electrons
and photons always behave as waves, while a particlelike behavior corresponds only to a
special case (1). In this sense, the wave-particle duality is nothing but a myth.

But why then the wave-particle duality is so often mentioned? One reason is philo-
sophical; the word “duality” sounds very “deep” and “mysterious” from a philosophical
point of view, and some physicists obviously like it, despite the fact that a dual picture
is not supported by the usual technical formulation of QM. Another reason is historical;
in early days of QM, it was an experimental fact that electrons and photons sometimes
behave as particles and sometimes as waves, so a dual interpretation was perhaps natural at that time when quantum theory was not yet well understood.

From above, one may conclude that the notion of “wave-particle duality” should be completely removed from a modern talk on QM. However, this is not necessarily so. Such a concept may still make sense if interpreted in a significantly different way. One way is purely linguistic; it is actually common to say that electrons and photons are “particles”, having in mind that the word “particle” has a very different meaning than the same word in classical physics. In this sense, electrons and photons are both “particles” (because we call them so) and “waves” (because that is what, according to the usual interpretation, they really are).

Another meaningful way of retaining the notion of “wave-particle duality” is to understand it as a quantum-classical duality, because each classical theory has the corresponding quantum theory, and vice versa. However, the word “duality” is not the best word for this correspondence, because the corresponding quantum and classical theories do not enjoy the same rights. Instead, the classical theories are merely approximations of the quantum ones.

2.2 Can wave-particle duality be taken seriously?

However, is it possible that the “wave-particle duality” has a literal meaning; that, in some sense, electrons and photons really are both particles and waves? Most experts for foundations of QM will probably say – no! Nevertheless, such a definite “no” is also an unproved myth. Of course, such a definite “no” is correct if it refers only to the usual formulation of QM. But who says that the usual formulation of QM is the ultimate theory that will never be superseded by an even better theory? (A good scientist will never say that for any theory.) In fact, such a modification of the usual quantum theory already exists. I will refer to it as the Bohmian interpretation of QM [1], but it is also known under the names “de Broglie-Bohm” interpretation and “pilot-wave” interpretation. (For recent pedagogic expositions of this interpretation, see [2, 3], for a pedagogic comparison with other formulations of QM, see [4], and for an unbiassed review of advantages and disadvantages of this interpretation, see [5].) This interpretation consists of two equations. One is the standard Schrödinger equation that describes the wave-aspect of the theory, while the other is a classical-like equation that describes a particle trajectory. The equation for the trajectory is such that the force on the particle depends on the wave function, so that the motion of the particle differs from that in classical physics, which, in turn, can be used to explain all (otherwise strange) quantum phenomena. In this interpretation, both the wave function and the particle position are fundamental entities. If any known interpretation of QM respects a kind of wave-particle duality, then it is the Bohmian interpretation. More on this interpretation (which also provides a counterexample to some other myths of QM) will be presented in subsequent sections.
3 In QM, there is a time-energy uncertainty relation

3.1 The origin of a time-energy uncertainty relation

For simplicity, consider a particle moving in one dimension. In QM, operators corresponding to the position \( x \) and the momentum \( p \) satisfy the commutation relation

\[
[\hat{x}, \hat{p}] = i\hbar,
\]

where \([A, B] \equiv AB - BA\). As is well known, this commutation relation implies the position-momentum Heisenberg uncertainty relation

\[
\Delta x \Delta p \geq \frac{\hbar}{2}.
\]

It means that one cannot measure both the particle momentum and the particle position with arbitrary accuracy. For example, the wave function corresponding to a definite momentum is an eigenstate of the momentum operator

\[
\hat{p} = -i\hbar \frac{\partial}{\partial x}.
\]

It is easy to see that such a wave function must be proportional to a plane wave \( e^{ipx/\hbar} \). On the other hand, the wave function corresponding to an eigenstate of the position operator is essentially a \( \delta \)-function (see (1)). It is clear that a wave function cannot be both a plane wave and a \( \delta \)-function, which, in the usual formulation of QM, explains why one cannot measure both the momentum and the position with perfect accuracy.

There is a certain analogy between the couple position-momentum and the couple time-energy. In particular, a wave function that describes a particle with a definite energy \( E \) is proportional to a plane wave \( e^{-iEt/\hbar} \). Analogously, one may imagine that a wave function corresponding to a definite time is essentially a \( \delta \)-function in time. In analogy with (3), this represents an essence of the reason for writing the time-energy uncertainty relation

\[
\Delta t \Delta E \geq \frac{\hbar}{2}.
\]

In introductory textbooks on QM, as well as in popular texts on QM, the time-energy uncertainty relation (5) is often presented as a fact enjoying the same rights as the position-momentum uncertainty relation (3). Nevertheless, there is a great difference between these two uncertainty relations. Whereas the position-momentum uncertainty relation (3) is a fact, the time-energy uncertainty relation (5) is a myth!

3.2 The time-energy uncertainty relation is not fundamental

Where does this difference come from? The main difference lies in the fact that energy is not represented by an operator analogous to (4), i.e., energy is not represented by the operator \( i\hbar \partial / \partial t \). Instead, energy is represented by a much more complicated operator called Hamiltonian, usually having the form

\[
\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x}).
\]
Nothing forbids the state $\psi(x,t)$ to be an eigenstate of $\hat{H}$ at a definite value of $t$. This difference has a deeper origin in the fundamental postulates of QM, according to which quantum operators are operators on the space of functions depending on $x$, not on the space of functions depending on $t$. Thus, space and time have very different roles in nonrelativistic QM. While $x$ is an operator, $t$ is only a classical-like parameter. A total probability that must be equal to 1 is an integral of the form
\[
\int_{-\infty}^{\infty} dx \psi^*(x,t)\psi(x,t),
\]
not an integral of the form
\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dt \psi^*(x,t)\psi(x,t).
\]
In fact, if $\psi(x,t)$ is a solution of the Schrödinger equation, then, when the integral (7) is finite, the integral (8) is not finite. An analogous statement is also true for one or more particles moving in 3 dimensions; the probability density $\psi^*\psi$ is not to be integrated over time.

As the time $t$ is not an operator in QM, a commutation relation analogous to (2) but with the replacements $x \rightarrow t$, $p \rightarrow H$, does not make sense. This is another reason why the time-energy uncertainty relation (5) is not really valid in QM. Nevertheless, there are attempts to replace the parameter $t$ with another quantity $T$, so that an analog of (2)
\[
[\hat{T},\hat{H}] = -i\hbar
\]
is valid. However, there is a theorem due to Pauli that says that this is impossible [6]. The two main assumptions of the Pauli theorem are that $T$ and $H$ must be hermitian operators (because only hermitian operators have real eigenvalues corresponding to real physical quantities) and that the spectrum of $H$ must be bounded from below (which corresponds to the physical requirement that energy should not have the possibility to become arbitrarily negative, because otherwise such a system would not be physically stable). Note that $p$, unlike $H$, does not need to be bounded from below. For a simple proof of the Pauli theorem, consider the operator
\[
\hat{H}' \equiv e^{-i\epsilon \hat{T}/\hbar}\hat{H}e^{i\epsilon \hat{T}/\hbar},
\]
where $\epsilon$ is a positive parameter with the dimension of energy. It is sufficient to consider the case of small $\epsilon$, so, by expanding the exponential functions and using (9), one finds
\[
\hat{H}' \approx \hat{H} - \epsilon.
\]
Now assume that the spectrum of $\hat{H}$ is bounded from below, i.e., that there exists a ground state $|\psi_0\rangle$ with the property $\hat{H}|\psi_0\rangle = E_0|\psi_0\rangle$, where $E_0$ is the minimal possible energy. Consider the state
\[
|\psi\rangle = e^{i\epsilon \hat{T}/\hbar}|\psi_0\rangle.
\]
Assuming that $\hat{T}$ is hermitian (i.e., that $\hat{T}^\dagger = \hat{T}$) and using (10) and (11), one finds
\[
\langle \psi|\hat{H}|\psi\rangle = \langle \psi_0|\hat{H}'|\psi_0\rangle \approx E_0 - \epsilon < E_0.
\]
This shows that there exists a state $|\psi\rangle$ with the energy smaller than $E_0$. This is in contradiction with the assumption that $E_0$ is the minimal energy, which proves the theorem! There are attempts to modify some of the axioms of the standard form of quantum theory so that the commutation relation (9) can be consistently introduced (see, e.g., [7, 8] and references therein), but the viability of such modified axioms of QM is not widely accepted among experts.

Although (5) is not a fundamental relation, in most practical situations it is still true that the uncertainty $\Delta E$ and the duration of the measurement process $\Delta t$ roughly satisfy the inequality (5). However, there exists also an explicit counterexample that demonstrates that it is possible in principle to measure energy with arbitrary accuracy during an arbitrarily short time-interval [9]. It remains true that the characteristic evolution time of quantum states is given by an uncertainty relation (5), but this evolution time is not to be unequivocally identified with the duration of a quantum measurement. In this sense, the time-energy uncertainty relation (5) is not equally fundamental as the position-momentum uncertainty relation (3).

While different roles of space and time should not be surprising in nonrelativistic QM, one may expect that space and time should play a more symmetrical role in relativistic QM. More on relativistic QM will be said in Sec. 7, but here I only note that even in relativistic QM space and time do not play completely symmetric roles, because even there integrals similar to (8) have not a physical meaning, while those similar to (7) have. Thus, even in relativistic QM, a time-energy uncertainty relation does not play a fundamental role.

4 QM implies that nature is fundamentally random

4.1 Fundamental randomness as a myth

QM is a theory that gives predictions on probabilities for different outcomes of measurements. But this is not a privileged property of QM, classical statistical mechanics also does this. Nevertheless, there is an important difference between QM and classical statistical mechanics. The latter is known to be an effective approximative theory useful when not all fundamental degrees of freedom are under experimental or theoretical control, while the underlying more fundamental classical dynamics is completely deterministic. On the other hand, the usual form of QM does not say anything about actual deterministic causes that lie behind the probabilistic quantum phenomena. This fact is often used to claim that QM implies that nature is fundamentally random. Of course, if the usual form of QM is really the ultimate truth, then it is true that nature is fundamentally random. But who says that the usual form of QM really is the ultimate truth? (A serious scientist will never claim that for any current theory.) A priori, one cannot exclude the existence of some hidden variables (not described by the usual form of QM) that provide a deterministic cause for all seemingly random quantum phenomena. Indeed, from the experience with classical pseudorandom phenomena, the existence of such deterministic hidden variables seems a very natural hypothesis. Nevertheless, QM is not that cheap; in QM there exist rigorous no-hidden-variable theorems. These theorems are often used to claim that hidden variables cannot exist and, consequently, that nature is fundamentally
random. However, each theorem has assumptions. The main assumption is that hidden variables must reproduce the statistical predictions of QM. Since these statistical predictions are verified experimentally, one is not allowed to relax this assumption. However, this assumption alone is not sufficient to provide a theorem. In the actual constructions of these theorems, there are also some additional “auxiliary” assumptions, which, however, turn out to be physically crucial! Thus, what these theorems actually prove, is that hidden variables, if exist, cannot have these additional assumed properties. Since there is no independent proof that these additional assumed properties are necessary ingredients of nature, the assumptions of these theorems may not be valid. (I shall discuss one version of these theorems in more detail in Sec. 5.) Therefore, the claim that QM implies fundamental randomness is a myth.

4.2 From analogy with classical statistical mechanics to the Bohmian interpretation

Some physicists, including one winner of the Nobel prize [10], very seriously take the possibility that some sort of deterministic hidden variables may underlie the usual form of QM. In fact, the best known and most successful hidden-variable extension of QM, the Bohmian interpretation, emerges rather naturally from the analogy with classical statistical mechanics. To see this, consider a classical particle the position of which is not known with certainty. Instead, one deals with a statistical ensemble in which only the probability density \( \rho(x, t) \) is known. The probability must be conserved, i.e., \( \int d^3 x \, \rho = 1 \) for each \( t \). Therefore, the probability must satisfy the local conservation law (known also as the continuity equation)

\[
\partial_t \rho + \nabla (\rho v) = 0,
\]

where \( v(x, t) \) is the velocity of the particle at the position \( x \) and the time \( t \). In the Hamilton-Jacobi formulation of classical mechanics, the velocity can be calculated as

\[
v(x, t) = \frac{\nabla S(x, t)}{m},
\]

where \( S(x, t) \) is a solution of the Hamilton-Jacobi equation

\[
\left( \frac{\nabla S}{2m} \right)^2 + V(x, t) = -\partial_t S,
\]

\( V(x, t) \) is an arbitrary potential, and \( m \) is the mass of the particle. The independent real equations (14) and (16) can be written in a more elegant form as a single complex equation. For that purpose, one can introduce a complex function [11]

\[
\psi = \sqrt{\rho} e^{iS/\hbar},
\]

where \( \hbar \) is an arbitrary constant with the dimension of action, so that the exponent in (17) is dimensionless. With this definition of \( \psi \), Eqs. (14) and (16) are equivalent to the equation

\[
\left( -\frac{\hbar^2 \nabla^2}{2m} + V - Q \right) \psi = i\hbar \partial_t \psi,
\]
Indeed, by inserting (17) into (18) and multiplying by $\psi^*$, it is straightforward to check that the real part of the resulting equation leads to (16), while the imaginary part leads to (14) with (15).

The similarity of the classical equation (18) with the quantum Schrödinger equation

\[ \left( -\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi = i\hbar \partial_t \psi \]  \hspace{1cm} (20)

is obvious and remarkable! However, there are also some differences. First, in the quantum case, the constant $\hbar$ is not arbitrary, but equal to the Planck constant divided by $2\pi$. The second difference is the fact that (18) contains the $Q$-term that is absent in the quantum case (20). Nevertheless, the physical interpretations are the same; in both cases, $|\psi(x, t)|^2$ is the probability density of particle positions. On the other hand, we know that classical mechanics is fundamentally deterministic. This is encoded in the fact that Eq. (18) alone does not provide a complete description of classical systems. Instead, one is also allowed to use Eq. (15), which says that the velocity of the particle is determined whenever its position is also determined. The classical interpretation of this is that a particle always has a definite position and velocity and that the initial position and velocity uniquely determine the position and velocity at any time $t$. From this point of view, nothing seems more natural than to assume that an analogous statement is true also in the quantum case. This assumption represents the core of the Bohmian deterministic interpretation of QM. To see the most obvious consequence of such a classical-like interpretation of the Schrödinger equation, note that the Schrödinger equation (20) corresponds to a Hamilton-Jacobi equation in which $V$ in (16) is replaced by $V + Q$. This is why $Q$ is often referred to as the quantum potential. The quantum potential induces a quantum force. Thus, a quantum particle trajectory satisfies a modified Newton equation

\[ m \frac{d^2x}{dt^2} = -\nabla(V + Q). \]  \hspace{1cm} (21)

Such modified trajectories can be used to explain otherwise strange-looking quantum phenomena (see, e.g., [3]), such as a two-slit experiment.

Note that, so far, I have only discussed a single-particle wave function $\psi(x, t)$. When one generalizes this to many-particle wave functions, an additional important feature of the Bohmian interpretation becomes apparent – the nonlocality. However, I delegate the explicit discussion of nonlocality to Sec. 6.

### 4.3 Random or deterministic?

As we have seen above, the analogy between classical statistical mechanics and QM can be used to interpret QM in a deterministic manner. However, this analogy does not prove that such a deterministic interpretation of QM is correct. Indeed, such deterministic quantum trajectories have never been directly observed. On the other hand, the Bohmian interpretation can explain why these trajectories are practically unobservable [12], so
the lack of experimental evidence does not disprove this interpretation. Most experts familiar with the Bohmian interpretation agree that the observable predictions of this interpretation are consistent with those of the standard interpretation, but they often prefer the standard interpretation because the standard interpretation seems simpler to them. This is because the standard interpretation of QM does not contain Eq. (15). I call this technical simplicity. On the other hand, the advocates of the Bohmian interpretation argue that this technical extension of QM makes QM simpler on the conceptual level. Nevertheless, it seems that most contemporary physicists consider technical simplicity more important than conceptual simplicity, which explains why most physicists prefer the standard purely probabilistic interpretation of QM. In fact, by applying a QM-motivated technical criterion of simplicity, it can be argued that even classical statistical mechanics represented by (18) can be considered complete, in which case even classical mechanics can be interpreted as a purely probabilistic theory [13]. But the fact is that nobody knows with certainty whether the fundamental laws of nature are probabilistic or deterministic.

5 QM implies that there is no reality besides the measured reality

This is the central myth in QM and many other myths are based on this one. Therefore, it deserves a particularly careful analysis.

5.1 QM as the ultimate scientific theory?

On one hand, the claim that “there is no reality besides the measured reality” may seem to lie at the heart of the scientific method. All scientists agree that the empirical evidence is the ultimate criterion for acceptance or rejection of any scientific theory, so, from this point of view, such a claim may seem rather natural. On the other hand, most scientists (apart from quantum physicists) do not find such a radical interpretation of the scientific method appealing. In particular, many consider such an interpretation too anthropomorphic (was there any reality before humans or living beings existed?), while the history of science surprised us several times by discovering that we (the human beings) are not so an important part of the universe as we thought we were. Some quantum physicists believe that QM is so deep and fundamental that it is not just a science that merely applies already prescribed scientific methods, but the science that answers the fundamental ontological and epistemological questions on the deepest possible level. But is such a (certainly not modest) belief really founded? What are the true facts from which such a belief emerged? Let us see!

5.2 From a classical variable to a quantumlike representation

Consider a simple real physical classical variable $s$ that can attain only two different values, say $s_1 = 1$ and $s_2 = -1$. By assumption, such a variable cannot change continuously. Nevertheless, a quantity that can still change continuously is the probability $p_n(t)$ that, at
a given time $t$, the variable attains the value $s_n$. The probabilities must satisfy
\[ p_1(t) + p_2(t) = 1. \] (22)

The average value of $s$ is given by
\[ \langle s \rangle = s_1 p_1(t) + s_2 p_2(t). \] (23)

Although $s$ can attain only two values, $s_1 = 1$ and $s_2 = -1$, the average value of $s$ can continuously change with time and attain an arbitrary value between 1 and $-1$.

The probabilities $p_n$ must be real and non-negative. A simple formal way to provide this is to write $p_n = \psi_n^* \psi_n$, where $\psi_n$ are auxiliary quantities that may be negative or even complex. It is also convenient to view the numbers $\psi_n$ (or $\psi_n^*$) as components of a vector. This vector can be represented either as a column
\[ |\psi\rangle \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \] (24)
or a row
\[ \langle \psi | \equiv (\psi_1^*, \psi_2^*). \] (25)

The norm of this vector is
\[ \langle \psi | \psi \rangle = \psi_1^* \psi_1 + \psi_2^* \psi_2 = p_1 + p_2. \] (26)

Thus, the constraint (22) can be viewed as a constraint on the norm of the vector – the norm must be unit. By introducing two special unit vectors
\[ |\phi_1\rangle = |\uparrow\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |\phi_2\rangle = |\downarrow\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \] (27)

one finds that the probabilities can be expressed in terms of vector products as
\[ p_1 = |\langle \phi_1 | \psi \rangle|^2, \quad p_2 = |\langle \phi_2 | \psi \rangle|^2. \] (28)

It is also convenient to introduce a diagonal matrix $\sigma$ that has the values $s_n$ at the diagonal and the zeros at all other places:
\[ \sigma \equiv \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \] (29)

The special vectors (27) have the property
\[ \sigma |\phi_1\rangle = s_1 |\phi_1\rangle, \quad \sigma |\phi_2\rangle = s_2 |\phi_2\rangle, \] (30)

which shows that (i) the vectors (27) are the eigenvectors of the matrix $\sigma$ and (ii) the eigenvalues of $\sigma$ are the allowed values $s_1$ and $s_2$. The average value (23) can then be formally written as
\[ \langle s \rangle = \langle \psi(t) | \sigma | \psi(t) \rangle. \] (31)
What has all this to do with QM? First, a discrete spectrum of the allowed values is typical of quantum systems; after all, discrete spectra are often referred to as “quantized” spectra, which, indeed, is why QM attained its name. (Note, however, that it would be misleading to claim that quantized spectra is the most fundamental property of quantum systems. Some quantum variables, such as the position of a particle, do not have quantized spectra.) A discrete spectrum contradicts some common prejudices on classical physical systems because such a spectrum does not allow a continuous change of the variable. Nevertheless, a discrete spectrum alone does not yet imply quantum physics. The formal representation of probabilities and average values in terms of complex numbers, vectors, and matrices as above is, of course, inspired by the formalism widely used in QM; yet, this representation by itself does not yet imply QM. The formal representation in terms of complex numbers, vectors, and matrices can still be interpreted in a classical manner.

5.3 From the quantumlike representation to quantum variables

The really interesting things that deviate significantly from the classical picture emerge when one recalls the following formal algebraic properties of vector spaces: Consider an arbitrary 2 \times 2 unitary matrix \( U, U^\dagger U = 1 \). (In particular, \( U \) may or may not be time dependent.) Consider a formal transformation

\[
|\psi'\rangle = U|\psi\rangle, \quad \langle \psi'| = \langle \psi|U^\dagger, \quad \sigma' = U\sigma U^\dagger.
\]  

(32)

(This transformation refers to all vectors \( |\psi\rangle \) or \( \langle \psi| \), including the eigenvectors \( |\phi_1\rangle \) and \( |\phi_2\rangle \).) In the theory of vector spaces, such a transformation can be interpreted as a new representation of the same vectors. Indeed, such a transformation does not change the physical properties, such as the norm of the vector (26), the probabilities (28), and the eigenvalues in (30), calculated in terms of the primed quantities \( |\psi'\rangle, \langle \psi'|, \) and \( \sigma' \). This means that the explicit representations, such as those in (24), (25), (27), and (29), are irrelevant. Instead, the only physically relevant properties are abstract, representation-independent quantities, such as scalar products and the spectrum of eigenvalues. What does it mean physically? One possibility is not to take it too seriously, as it is merely an artefact of an artificial vector-space representation of certain physical quantities. However, the history of theoretical physics teaches us that formal mathematical symmetries often have a deeper physical message. So let us try to take it seriously, to see where it will lead us. Since the representation is not relevant, it is natural to ask if there are other matrices (apart from \( \sigma \)) that do not have the form of (29), but still have the same spectrum of eigenvalues as \( \sigma \)? The answer is yes! But then we are in a very strange, if not paradoxical, position; we have started with a consideration of a single physical variable \( s \) and arrived at a result that seems to suggest the existence of some other, equally physical, variables.

As we shall see, this strange result lies at the heart of the (also strange) claim that there is no reality besides the measured reality. But let us not jump to the conclusion too early! Instead, let us first study the mathematical properties of these additional physical variables.

Since an arbitrary 2 \times 2 matrix is defined by 4 independent numbers, each such matrix can be written as a linear combination of 4 independent matrices. One convenient choice
of 4 independent matrices is

\[
1 \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\
\sigma_2 \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\] (33)

The matrix \( \sigma_3 \) is nothing but a renamed matrix \( \sigma \) in (29). The matrices \( \sigma_i \), known also as Pauli matrices, are chosen so that they satisfy the familiar symmetrically looking commutation relations

\[
[\sigma_j, \sigma_k] = 2i \epsilon_{jkl} \sigma_l
\] (34)

(where summation over repeated indices is understood). The matrices \( \sigma_i \) are all hermitian, \( \sigma_i^\dagger = \sigma_i \), which implies that their eigenvalues are real. Moreover, all three \( \sigma_i \) have the eigenvalues 1 and \(-1\). The most explicit way to see this is to construct the corresponding eigenvectors. The eigenvectors of \( \sigma_3 \) are \( | \uparrow_3 \rangle \equiv | \uparrow \rangle \) and \( | \downarrow_3 \rangle \equiv | \downarrow \rangle \) defined in (27), with the eigenvalues 1 and \(-1\), respectively. Analogously, it is easy to check that the eigenvectors of \( \sigma_1 \) are

\[
| \uparrow_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{| \uparrow \rangle + | \downarrow \rangle}{\sqrt{2}}, \\
| \downarrow_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{| \uparrow \rangle - | \downarrow \rangle}{\sqrt{2}},
\] (35)

with the eigenvalues 1 and \(-1\), respectively, while the eigenvectors of \( \sigma_2 \) are

\[
| \uparrow_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{| \uparrow \rangle + i | \downarrow \rangle}{\sqrt{2}}, \\
| \downarrow_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{| \uparrow \rangle - i | \downarrow \rangle}{\sqrt{2}},
\] (36)

with the same eigenvalues 1 and \(-1\), respectively.

The commutation relations (34) are invariant under the unitary transformations \( \sigma_i \to \sigma_i' = U \sigma_i U^\dagger \). This suggests that the commutation relations themselves are more physical than the explicit representation given by (33). Indeed, the commutation relations (34) can be recognized as the algebra of the generators of the group of rotations in 3 spatial dimensions. There is nothing quantum mechanical about that; in classical physics, matrices represent operators, that is, abstract objects that act on vectors by changing (in this case, rotating) them. However, in the usual formulation of classical physics, there is a clear distinction between operators and physical variables – the latter are not represented by matrices. In contrast, in our formulation, the matrices \( \sigma_i \) have a double role; mathematically, they are operators (because they act on vectors \( | \psi \rangle \)), while physically, they represent physical variables. From symmetry, it is natural to assume that all three \( \sigma_i \) variables are equally physical. This assumption is one of the central assumptions of QM that makes it different from classical mechanics. For example, the spin operator of spin \( \frac{1}{2} \) particles in QM is given by

\[
S_i = \frac{\hbar}{2} \sigma_i,
\] (37)

14
where the 3 labels \( i = 1, 2, 3 \) correspond to 3 space directions \( x, y, z \), respectively. Thus, in the case of spin, it is clear that \( \sigma_3 \equiv \sigma_z \) cannot be more physical than \( \sigma_1 \equiv \sigma_x \) or \( \sigma_2 \equiv \sigma_y \), despite the fact that \( \sigma_3 \) corresponds to the initial physical variable with which we started our considerations. On the other hand, the fact that the new variables \( \sigma_1 \) and \( \sigma_2 \) emerged from the initial variable \( \sigma_3 \) suggests that, in some sense, these 3 variables are not really completely independent. Indeed, a nontrivial relation among them is incoded in the nontrivial commutation relations (34). In QM, two variables are really independent only if their commutator vanishes. (For example, recall that, unlike (34), the position operators \( x_i \) and the momentum operators \( p_i \) in QM satisfy \([x_i, x_j] = [p_i, p_j] = 0\). In fact, this is the ultimate reason why the most peculiar aspects of QM are usually discussed on the example of spin variables, rather than on position or momentum variables.)

5.4 From quantum variables to quantum measurements

Now consider the state

\[ |\psi\rangle = \frac{|\uparrow \rangle + |\downarrow \rangle}{\sqrt{2}}. \tag{38} \]

In our initial picture, this state merely represents a situation in which there are 50 : 50 chances that the system has the value of \( s \) equal to either \( s = 1 \) or \( s = -1 \). Indeed, if one performs a measurement to find out what that value is, one will obtain one and only one of these two values. By doing such a measurement, the observer gains new information about the system. For example, if the value turns out to be \( s = 1 \), this gain of information can be described by a “collapse”

\[ |\psi\rangle \rightarrow |\uparrow \rangle, \tag{39} \]

as the state \( |\uparrow \rangle \) corresponds to a situation in which one is certain that \( s = 1 \). At this level, there is nothing mysterious and nothing intrinsically quantum about this collapse. However, in QM, the state (38) contains more information than said above! (Otherwise, there would be no physical difference between the two different states in (35).) From (35), we see that the “uncertain” state (38) corresponds to a situation in which one is absolutely certain that the value of the variable \( \sigma_1 \) is equal to 1. On the other hand, if one performs the measurement of \( s = \sigma_3 \) and obtains the value as in (39), then the postmeasurement state

\[ |\uparrow \rangle = \frac{|\uparrow_1 \rangle + |\downarrow_1 \rangle}{\sqrt{2}} \tag{40} \]

implies that the value of \( \sigma_1 \) is no longer known with certainty. This means that, in some way, the measurement of \( \sigma_3 \) destroys the information on \( \sigma_1 \). But the crucial question is not whether the information on \( \sigma_1 \) has been destroyed, but rather whether the value itself of \( \sigma_1 \) has been destroyed. In other words, is it possible that all the time, irrespective of the performed measurements, \( \sigma_1 \) has the value 1? The fact is that if this were the case, then it would contradict the predictions of QM! The simplest way to see this is to observe that, after the first measurement with the result (39), one can perform a new measurement, in which one measures \( \sigma_1 \). From (40), one sees that there are 50% chances that the result of the new measurement will give the value \(-1\). That is, there is \( 0.5 \cdot 0.5 = 0.25 \) probability that the sequence of the two measurements will correspond to the collapses

\[ |\uparrow_1 \rangle \rightarrow |\uparrow \rangle \rightarrow |\downarrow_1 \rangle. \tag{41} \]
In (41), the initial value of \( \sigma_1 \) is 1, while the final value of \( \sigma_1 \) is \(-1\). Thus, QM predicts that the value of \( \sigma_1 \) may change during the process of the two measurements. Since the predictions of QM are in agreement with experiments, we are forced to accept this as a fact. This demonstrates that QM is contextual, that is, that the measured values depend on the context, i.e., on the measurement itself. This property by itself is still not intrinsically quantum, in classical physics the result of a measurement may also depend on the measurement. Indeed, in classical mechanics there is nothing mysterious about it; there, a measurement is a physical process that, as any other physical process, may influence the values of the physical variables. But can we talk about the value of the variable irrespective of measurements? From a purely experimental point of view, we certainly cannot. Here, however, we are talking about theoretical physics. So does the theory allow to talk about that? Classical theory certainly does. But what about QM? If all theoretical knowledge about the system is described by the state \( |\psi\rangle \), then quantum theory does not allow that! This is the fact. But, can we be sure that we shall never discover some more complete theory than the current form of QM, so that this more complete theory will talk about theoretical values of variables irrespective of measurements? From the example above, it is not possible to draw such a conclusion. Nevertheless, physicists are trying to construct more clever examples from which such a conclusion could be drawn. Such examples are usually referred to as “no-hidden-variable theorems”. But what do these theorems really prove? Let us see!

5.5 From quantum measurements to no-hidden-variable theorems

To find such an example, consider a system consisting of two independent subsystems, such that each subsystem is characterized by a variable that can attain only two values, 1 and \(-1\). The word “independent” (which will turn out to be the crucial word) means that the corresponding operators commute and that the Hamiltonian does not contain an interaction term between these two variables. For example, this can be a system with two free particles, each having spin \( \frac{1}{2} \). In this case, the state \( |\uparrow\rangle \otimes |\downarrow\rangle \equiv |\uparrow\rangle|\downarrow\rangle \) corresponds to the state in which the first particle is in the state \( |\uparrow\rangle \), while the second particle is in the state \( |\downarrow\rangle \). (The commutativity of the corresponding variables is provided by the operators \( \sigma_j \otimes 1 \) and \( 1 \otimes \sigma_k \) that correspond to the variables of the first and the second subsystem, respectively.) Instead of (38), consider the state

\[
|\psi\rangle = \frac{|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle}{\sqrt{2}}.
\]

This state constitutes the basis for the famous Einstein-Podolsky-Rosen-Bell paradox. This state says that if the first particle is found in the state \( |\uparrow\rangle \), then the second particle will be found in the state \( |\downarrow\rangle \), and vice versa. In other words, the second particle will always take a direction opposite to that of the first particle. In an oversimplified version of the paradox, one can wonder how the second particle knows about the state of the first particle, given the assumption that there is no interaction between the two particles? However, this oversimplified version of the paradox can be easily resolved in classical terms by observing that the particles do not necessarily need to interact, because they could
have their (mutually opposite) values all the time even before the measurement, while the only role of the measurement was to reveal these values. The case of a single particle discussed through Eqs. (38)-(41) suggests that a true paradox can only be obtained when one assumes that the variable corresponding to \( \sigma_1 \) or \( \sigma_2 \) is also a physical variable. Indeed, this is what has been obtained by Bell [14]. The paradox can be expressed in terms of an inequality that the correlation functions among different variables must obey if the measurement is merely a revelation of the values that the noninteracting particles had before the measurement. (For more detailed pedagogic expositions, see [15, 16].) The predictions of QM turn out to be in contradiction with this Bell inequality. The experiments violate the Bell inequality and confirm the predictions of QM (see [17] for a recent review). This is the fact! However, instead of presenting a detailed derivation of the Bell inequality, for pedagogical purposes I shall present a simpler example that does not involve inequalities, but leads to the same physical implications.

The first no-hidden-variable theorem without inequalities has been found by Greenberger, Horne, and Zeilinger [18] (for pedagogic expositions, see [19, 20, 16]), for a system with 3 particles. However, the simplest such theorem is the one discovered by Hardy [21] (see also [22]) that, like the one of Bell, involves only 2 particles. Although pedagogic expositions of the Hardy result also exist [20, 23, 16], since it still seems not to be widely known in the physics community, here I present a very simple exposition of the Hardy result (so simple that one can really wonder why the Hardy result was not discovered earlier). Instead of (42), consider a slightly more complicated state

\[
|\psi\rangle = |\downarrow\rangle|\downarrow\rangle + |\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle \sqrt{3}. \tag{43}
\]

Using (35), we see that this state can also be written in two alternative forms as

\[
|\psi\rangle = \frac{\sqrt{2}|\downarrow\rangle|\uparrow\rangle_1 + |\uparrow\rangle|\downarrow\rangle}{\sqrt{3}}, \tag{44}
\]

\[
|\psi\rangle = \frac{\sqrt{2}|\uparrow\rangle_1|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle}{\sqrt{3}}. \tag{45}
\]

From these 3 forms of \(|\psi\rangle\), we can infer the following:

(i) From (43), at least one of the particles is in the state \(|\downarrow\rangle\).

(ii) From (44), if the first particle is in the state \(|\downarrow\rangle\), then the second particle is in the state \(|\uparrow\rangle_1\).

(iii) From (45), if the second particle is in the state \(|\downarrow\rangle\), then the first particle is in the state \(|\uparrow\rangle_1\).

Now, by classical reasoning, from (i), (ii), and (iii) one infers that

(iv) It is impossible that both particles are in the state \(|\downarrow\rangle\).

But is (iv) consistent with QM? If it is, then \(\langle\downarrow_1|\downarrow_1|\psi\rangle\) must be zero. However, using (44), \(\langle\downarrow_1|\uparrow_1\rangle = 0\), and an immediate consequence of (35) \(\langle\downarrow_1|\uparrow\rangle = -\langle\downarrow_1|\downarrow\rangle = 1/\sqrt{2}\), we see that

\[
\langle\downarrow_1|\downarrow_1|\psi\rangle = \frac{\langle\downarrow_1|\uparrow\rangle|\downarrow_1|\downarrow\rangle}{\sqrt{3}} = -\frac{1}{2\sqrt{3}}, \tag{46}
\]

which is not zero. Therefore, (iv) is wrong in QM; there is a finite probability for both particles to be in the state \(|\downarrow_1\rangle\). This is the fact! But what exactly is wrong with the
reasoning that led to (iv)? The fact is that there are several(!) possibilities. Let us briefly discuss them.

5.6 From no-hidden-variable theorems to physical interpretations

One possibility is that classical logic cannot be used in QM. Indeed, this motivated the development of a branch of QM called quantum logic. However, most physicists (as well as mathematicians) consider a deviation from classical logic too radical.

Another possibility is that only one matrix, say $\sigma_3$, corresponds to a genuine physical variable. In this case, the true state of a particle can be $|\uparrow\rangle$ or $|\downarrow\rangle$, but not a state such as $|\uparrow_1\rangle$ or $|\downarrow_1\rangle$. Indeed, such a possibility corresponds to our starting picture in which there is only one physical variable called $s$ that was later artificially represented by the matrix $\sigma \equiv \sigma_3$. Such an interpretation may seem reasonable, at least for some physical variables. However, if $\sigma_3$ corresponds to the spin in the $z$-direction, then it does not seem reasonable that the spin in the $z$-direction is more physical than that in the $x$-direction or the $y$-direction. Picking up one preferred variable breaks the symmetry, which, at least in some cases, does not seem reasonable.

The third possibility is that one should distinguish between the claims that “the system has a definite value of a variable” and “the system is measured to have a definite value of a variable”. This interpretation of QM is widely accepted. According to this interpretation, the claims (i)-(iii) refer only to the results of measurements. These claims assume that $\sigma = \sigma_3$ is measured for at least one of the particles. Consequently, the claim (iv) is valid only if this assumption is fulfilled. In contrast, if $\sigma_3$ is not measured at all, then it is possible to measure both particles to be in the state $|\downarrow_1\rangle$. Thus, the paradox that (iv) seems to be both correct and incorrect is merely a manifestation of quantum contextuality. In fact, all no-hidden-variable theorems can be viewed as manifestations of quantum contextuality. However, there are at least two drastically different versions of this quantum-contextuality interpretation. In the first version, it does not make sense even to talk about the values that are not measured. I refer to this version as the orthodox interpretation of QM. (The orthodox interpretation can be further divided into a hard version in which it is claimed that such unmeasured values simply do not exist, and a soft version according to which such values perhaps might exist, but one should not talk about them because one cannot know about the existence of something that is not measured.) In the second version, the variables have some values even when they are not measured, but the process of measurement is a physical process that may influence these values. The second version assumes that the standard formalism of QM is not complete, i.e., that even a more accurate description of physical systems is possible than that provided by standard QM. According to this version, “no-hidden-variable” theorems (such as the one of Bell or Hardy) do not really prove that hidden variables cannot exist, because these theorems assume that there are no interactions between particles, while this assumption may be violated at the level of hidden variables. The most frequent argument for the validity of this assumption is the locality principle, which then must be violated by any hidden-variable completion of standard QM. However, since the assumption of the absence of interaction between particles is much more general than the assumption that it is the
locality principle that forbids such an interaction, and, at the same time, since the discussion of the locality principle deserves a separate section, I delegate the detailed discussion of locality to the next section, Sec. [6].

Most pragmatic physicists seem to (often tacitly) accept the soft-orthodox interpretation. From a pragmatic point of view, such an attitude seems rather reasonable. However, physicists who want to understand QM at the deepest possible level can hardly be satisfied with the soft version of the orthodox interpretation. They are forced either to adopt the hard-orthodox interpretation or to think about the alternatives (like hidden variables, preferred variables, or quantum logic). Among these physicists that cope with the foundations of QM at the deepest level, the hard-orthodox point of view seems to dominate. (If it did not dominate, then I would not call it “orthodox”). However, even the advocates of the hard-orthodox interpretation do not really agree what exactly this interpretation means. Instead, there is a number of subvariants of the hard-orthodox interpretation that differ in the fundamental ontology of nature. Some of them are rather antropomorphic, by attributing a fundamental role to the observers. However, most of them attempt to avoid antropomorphic ontology, for example by proposing that the concept of information on reality is more fundamental than the concept of reality itself [24], or that reality is relative or “relational” [25, 26], or that correlations among variables exist, while the variables themselves do not [27]. Needless to say, all such versions of the hard-orthodox interpretation necessarily involve deep (and dubious) philosophical assumptions and postulates. To avoid philosophy, an alternative is to adopt a softer version of the orthodox interpretation (see, e.g., [28]). The weakness of the soft versions is the fact that they do not even try to answer fundamental questions one may ask, but their advocates often argue that these questions are not physical, but rather metaphysical or philosophical.

Let us also discuss in more detail the possibility that one variable is more physical than the others, that only this preferred variable corresponds to the genuine physical reality. Of course, it does not seem reasonable that spin in the $z$-direction is more physical than that in the $x$- or the $y$-direction. However, it is not so unreasonable that, for example, the particle position is a more fundamental variable than the particle momentum or energy. (After all, most physicists will agree that this is so in classical mechanics, despite the fact that the Hamiltonian formulation of classical mechanics treats position and momentum on an equal footing.) Indeed, in practice, all quantum measurements eventually reduce to an observation of the position of something (such as the needle of the measuring apparatus). In particular, the spin of a particle is measured by a Stern-Gerlach apparatus, in which the magnetic field causes particles with one orientation of the spin to change their direction of motion to one side, and those with the opposite direction to the other. Thus, one does not really observe the spin itself, but rather the position of the particle. In general, assume that one wants to measure the value of the variable described by the operator $A$. It is convenient to introduce an orthonormal basis $\{|\psi_a\rangle\}$ such that each $|\psi_a\rangle$ is an eigenvector of the operator $A$ with the eigenvalue $a$. The quantum state can be expanded in this basis as

$$|\psi\rangle = \sum_a c_a |\psi_a\rangle,$$

where (assuming that the spectrum of $A$ is not degenerate) $|c_a|^2$ is the probability that the variable will be measured to have the value $a$. To perform a measurement, one must introduce the degrees of freedom of the measuring apparatus, which, before the
measurement, is described by some state $|\phi\rangle$. In an ideal measurement, the interaction between the measured degrees of freedom and the degrees of freedom of the measuring apparatus must be such that the total quantum state exhibits entanglement between these two degrees of freedom, so that the total state takes the form

$$|\Psi\rangle = \sum_a c_a |\psi_a\rangle |\phi_a\rangle,$$

(48)

where $|\phi_a\rangle$ are orthonormal states of the measuring apparatus. Thus, whenever the measuring apparatus is found in the state $|\phi_a\rangle$, one can be certain (at least theoretically) that the state of the measured degree of freedom is given by $|\psi_a\rangle$. Moreover, from (15) it is clear that the probability for this to happen is equal to $|c_a|^2$, the same probability as that without introducing the measuring apparatus. Although the description of the quantum measurement as described above is usually not discussed in practical textbooks on QM, it is actually a part of the standard form of quantum theory and does not depend on the interpretation. (For modern practical introductory lectures on QM in which the theory of measurement is included, see, e.g., [29].) What this theory of quantum measurement suggests is that, in order to reproduce the statistical predictions of standard QM, it is not really necessary that all hermitian operators called “observables” correspond to genuine physical variables. Instead, it is sufficient that only one or a few preferred variables that are really measured in practice correspond to genuine physical variables, while the rest of the “observables” are merely hermitian operators that do not correspond to true physical reality [30]. This is actually the reason why the Bohmian interpretation discussed in the preceding section, in which the preferred variables are the particle positions, is able to reproduce the quantum predictions on all quantum observables, such as momentum, energy, spin, etc. Thus, the Bohmian interpretation combines two possibilities discussed above: one is the existence of the preferred variable (the particle position) and the other is the hidden variable (the particle position existing even when it is not measured).

To conclude this section, QM does not prove that there is no reality besides the measured reality. Instead, there are several alternatives to it. In particular, such reality may exist, but then it must be contextual (i.e., must depend on the measurement itself.) The simplest (although not necessary) way to introduce such reality is to postulate it only for one or a few preferred quantum observables.

6 QM is local/nonlocal

6.1 Formal locality of QM

Classical mechanics is local. This means that a physical quantity at some position $x$ and time $t$ may be influenced by another physical quantity only if this other physical quantity is attached to the same $x$ and $t$. For example, two spacially separated local objects cannot communicate directly, but only via a third physical object that can move from one object to the other. In the case of $n$ particles, the requirement of locality can be written as a requirement that the Hamiltonian $H(x_1, \ldots, x_n, p_1, \ldots, p_n)$ should have the form

$$H = \sum_{l=1}^{n} H_l(x_l, p_l).$$

(49)
In particular, a nontrivial 2-particle potential of the form \( V(x_1 - x_2) \) is forbidden by the principle of locality. Note that such a potential is not forbidden in Newtonian classical mechanics. However, known fundamental interactions are relativistic interactions that do not allow such instantaneous communications. At best, such a nonlocal potential can be used as an approximation valid when the particles are sufficiently close to each other and their velocities are sufficiently small.

The quantum Hamiltonian is obtained from the corresponding classical Hamiltonian by a replacement of classical positions and momenta by the corresponding quantum operators. Thus, the quantum Hamiltonian takes the same local form as the classical one. Since the Schrödinger equation

\[
H|\psi(t)\rangle = i\hbar \partial_t |\psi(t)\rangle
\]  

(50)

is based on this local Hamiltonian, any change of the wave function induced by the Schrödinger equation (50) is local. This is the fact. For this reason, it is often claimed that QM is local to the same extent as classical mechanics is.

### 6.2 (Non)locality and hidden variables

The principle of locality is often used as the crucial argument against hidden variables in QM. For example, consider two particles entangled such that their wave function (with the spacial and temporal dependence of wave functions suppressed) takes the form (43). Such a form of the wave function can be kept even when the particles become spacially separated. As we have seen, the fact that (iv) is inconsistent with QM can be interpreted as QM contextuality. However, we have seen that there are two versions of QM contextuality – the orthodox one and the hidden-variable one. The principle of locality excludes the hidden-variable version of QM contextuality, because this version requires interactions between the two particles, which are impossible when the particles are (sufficiently) spacially separated. However, it is important to emphasize that the principle of locality is an assumption. We know that the Schrödinger equation satisfies this principle, but we do not know if this principle must be valid for any physical theory. In particular, subquantum hidden variables might not satisfy this principle. Physicists often object that nonlocal interactions contradict the theory of relativity. However, there are several responses to such objections. First, the theory of relativity is just as any other theory – nobody can be certain that this theory is absolutely correct at all (including the unexplored ones) levels. Second, nonlocality by itself does not necessarily contradict relativity. For example, a local relativistic-covariant field theory (see Sec. 5) can be defined by an action of the form \( \int d^4x L(x) \), where \( L(x) \) is the local Lagrangian density transforming (under arbitrary coordinate transformations) as a scalar density. A nonlocal action may have a form \( \int d^4x \int d^4x' L(x, x') \). If \( L(x, x') \) transforms as a bi-scalar density, then such a nonlocal action is relativistically covariant. Third, the nonlocality needed to explain quantum contextuality requires instantaneous communication, which is often claimed to be excluded by the theory of relativity, as the velocity of light is the maximal possible velocity allowed by the theory of relativity. However, this is actually a myth in the theory of relativity; this theory by itself does not exclude faster-than-light communication. It excludes it only if some additional assumptions on the nature of matter are used.
best known counterexample are \textit{tachyons} \cite{31} – hypothetical particles with negative mass squared that move faster than light and fully respect the theory of relativity. Some physicists argue that faster-than-light communication contradicts the principle of causality, but this is also nothing but a myth \cite{32, 33}. (As shown in \cite{33}, this myth can be traced back to one of the most fundamental myths in physics according to which time fundamentally differs from space by having a property of “lapsing”.) Finally, some physicists find absurd or difficult even to conceive physical laws in which information between distant objects is transferred instantaneously. It is ironic that they probably had not such mental problems many years ago when they did not know about the theory of relativity but \textit{did} know about the Newton instantaneous law of gravitation or the Coulomb instantaneous law of electrostatics. To conclude this paragraph, hidden variables, if exist, must violate the principle of locality, which may or may not violate the theory of relativity.

To illustrate nonlocality of hidden variables, I consider the example of the Bohmian interpretation. For a many-particle wave function $\Psi(x_1, \ldots, x_n, t)$ that describes $n$ particles with the mass $m$, it is straightforward to show that the generalization of (19) is

$$Q(x_1, \ldots, x_n, t) = -\frac{\hbar^2}{2m} \sum_{l=1}^{n} \nabla_l^2 \sqrt{\rho(x_1, \ldots, x_n, t)} \frac{\sqrt{\rho(x_1, \ldots, x_n, t)}}{\sqrt{\rho(x_1, \ldots, x_n, t)}}.$$ \hspace{1cm} (51)

When the wave function exhibits entanglement, i.e., when $\Psi(x_1, \ldots, x_n, t)$ is \textit{not} a local product of the form $\psi_1(x_1, t) \cdots \psi_n(x_n, t)$, then $Q(x_1, \ldots, x_n, t)$ is not of the form $\sum_{l=1}^{n} Q_l(x_l, t)$ (compare with (49)). In the Bohmian interpretation, this means that $Q$ is the quantum potential which (in the case of entanglement) describes a nonlocal interaction. For attempts to formulate the nonlocal Bohmian interaction in a relativistic covariant way, see, e.g., \cite{34, 35, 36, 37, 38, 39}.

### 6.3 (Non)locality without hidden variables?

Concerning the issue of locality, the most difficult question is whether QM itself, without hidden variables, is local or not. The fact is that there is no consensus among experts on that issue. It is known that quantum effects, such as the Einstein-Podolsky-Rosen-Bell effect or the Hardy effect, cannot be used to transmit information. This is because the choice of the state to which the system will collapse is \textit{random} (as we have seen, this randomness may be either fundamental or effective), so one cannot choose to transmit the message one wants. In this sense, QM is local. On the other hand, the correlation among different subsystems is nonlocal, in the sense that one subsystem is correlated with another subsystem, such that this correlation cannot be explained in a local manner in terms of preexisting properties before the measurement. Thus, there are good reasons for the claim that QM is \textit{not} local.

Owing to the nonlocal correlations discussed above, some physicists claim that it is a fact that QM is not local. Nevertheless, many experts do not agree with this claim, so it cannot be regarded as a fact. Of course, at the conceptual level, it is very difficult to conceive how nonlocal correlations can be explained without nonlocality. Nevertheless, hard-orthodox quantum physicists are trying to do that (see, e.g., \cite{24, 25, 26, 27}). In order to save the locality principle, they, in one way or another, deny the existence of...
objective reality. Without objective reality, there is nothing to be objectively nonlocal. What remains is the wave function that satisfies a local Schrödinger equation and does not represent reality, but only the information on reality, while reality itself does not exist in an objective sense. Many physicists (including myself) have problems with thinking about information on reality without objective reality itself, but it does not prove that such thinking is incorrect.

To conclude, the fact is that, so far, there has been no final proof with which most experts would agree that QM is either local or nonlocal. (For the most recent attempt to establish such a consensus see [40].) There is only agreement that if hidden variables (that is, objective physical properties existing even when they are not measured) exist, then they must be nonlocal. Some experts consider this a proof that they do not exist, whereas other experts consider this a proof that QM is nonlocal. They consider these as proofs because they are reluctant to give up either of the principle of locality or of the existence of objective reality. Nevertheless, more open-minded (some will say – too open-minded) people admit that neither of these two “crazy” possibilities (nonlocality and absence of objective reality) should be a priori excluded.

7 There is a well-defined relativistic QM

7.1 Klein-Gordon equation and the problem of probabilistic interpretation

The free Schrödinger equation

\[ -\hbar^2 \nabla^2 \psi(x,t) = i\hbar \partial_t \psi(x,t) \] (52)

is not consistent with the theory of relativity. In particular, it treats space and time in completely different ways, which contradicts the principle of relativistic covariance. Eq. (52) corresponds only to a nonrelativistic approximation of QM. What is the corresponding relativistic equation from which (52) can be derived as an approximation? Clearly, the relativistic equation must treat space and time on an equal footing. For that purpose, it is convenient to choose units in which the velocity of light is \( c = 1 \). To further simplify equations, it is also convenient to further restrict units so that \( \hbar = 1 \). Introducing coordinates \( x^\mu, \mu = 0, 1, 2, 3 \), where \( x^0 = t \), while \( x^1, x^2, x^3 \) are space coordinates, the simplest relativistic generalization of (52) is the Klein-Gordon equation

\[ (\partial^\mu \partial_\mu + m^2) \psi(x) = 0, \] (53)

where \( x = \{ x^\mu \} \), summation over repeated indices is understood, \( \partial^\mu \partial_\mu = \eta^{\mu\nu} \partial_\mu \partial_\nu \), and \( \eta^{\mu\nu} \) is the diagonal metric tensor with \( \eta^{00} = 1, \eta^{11} = \eta^{22} = \eta^{33} = -1 \). However, the existence of this relativistic wave equation does not imply that relativistic QM exists. This is because there are interpretational problems with this equation. In nonrelativistic QM, the quantity \( \psi^* \psi \) is the probability density, having the property

\[ \frac{d}{dt} \int d^3 x \psi^* \psi = 0, \] (54)
which can be easily derived from the Schrödinger equation (52). This property is crucial for the consistency of the probabilistic interpretation, because the integral $\int d^3x \psi^* \psi$ is the sum of all probabilities for the particle to be at all possible places, which must be equal to 1 for each time $t$. If $\psi$ is normalized so that this integral is equal to 1 at $t = 0$, then (54) provides that it is equal to 1 at each $t$. However, when $\psi$ satisfies (53) instead of (52), then the consistency requirement (54) is not fulfilled. Consequently, in relativistic QM based on (53), $\psi^* \psi$ cannot be interpreted as the probability density.

In order to solve this problem, one can introduce the Klein-Gordon current

$$j_\mu = i \psi^* \not{\partial}_\mu \psi,$$

where $a \not{\partial}_\mu b \equiv a(\partial_\mu b) - (\partial_\mu a)b$. Using (53), one can show that this current satisfies the local conservation law

$$\partial_\mu j^\mu = 0,$$

which implies that

$$\frac{d}{dt} \int d^3x j^0 = 0.$$

Eq. (57) suggests that, in the relativistic case, it is $j^0$ that should be interpreted as the probability density. More generally, if $\psi_1(x)$ and $\psi_2(x)$ are two solutions of (53), then the scalar product defined as

$$(\psi_1, \psi_2) = i \int d^3x \psi_1^*(x) \not{\partial}_0 \psi_2(x)$$

does not depend on time. The scalar product (58) does not look relativistic covariant, but there is a way to write it in a relativistic covariant form. The constant-time spacelike hypersurface with the infinitesimal volume $d^3x$ can be generalized to an arbitrarily curved spacelike hypersurface $\Sigma$ with the infinitesimal volume $dS^\mu$ oriented in a timelike direction normal to $\Sigma$. Eq. (58) then generalizes to

$$(\psi_1, \psi_2) = i \int_{\Sigma} dS^\mu \psi_1^*(x) \not{\partial}_\mu \psi_2(x),$$

which, owing to the 4-dimensional Gauss law, does not depend on $\Sigma$ when $\psi_1(x)$ and $\psi_2(x)$ satisfy (53). However, there is a problem again. The general solution of (53) can be written as

$$\psi(x) = \psi^+(x) + \psi^-(x),$$

where

$$\psi^+(x) = \sum_k c_k e^{-i(\omega_k t - kx)},$$

$$\psi^-(x) = \sum_k d_k e^{i(\omega_k t - kx)}.$$

Here $c_k$ and $d_k$ are arbitrary complex coefficients, and

$$\omega_k \equiv \sqrt{k^2 + m^2}$$

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is the frequency. For obvious reasons, $\psi^+$ is called a positive-frequency solution, while $\psi^-$ is called a negative-frequency solution. (The positive- and negative-frequency solutions are often referred to as positive- and negative-energy solutions, respectively. However, such a terminology is misleading because in field theory, which will be discussed in the next section, energy cannot be negative, so it is better to speak of positive and negative frequency.) The nonrelativistic Schrödinger equation contains only the positive-frequency solutions, which can be traced back to the fact that the Schrödinger equation contains a first time derivative, instead of a second time derivative that appears in the Klein-Gordon equation [53]. For a positive-frequency solution the quantity $\int d^3x \, j^0$ is positive, whereas for a negative-frequency solution this quantity is negative. Since the sum of all probabilities must be positive, the negative-frequency solutions represent a problem for the probabilistic interpretation. One may propose that only positive-frequency solutions are physical, but even this does not solve the problem. Although the integral $\int d^3x \, j^0$ is strictly positive in that case, the local density $j^0(x)$ may still be negative at some regions of spacetime, provided that the superposition $\psi^+$ in (61) contains terms with two or more different positive frequencies. Thus, even with strictly positive-frequency solutions, the quantity $j^0$ cannot be interpreted as a probability density.

7.2 Some attempts to solve the problem

Physicists sometimes claim that there are no interpretational problems with the Klein-Gordon equation because the coefficients $c_k$ and $d_k$ in (61) (which are the Fourier transforms of $\psi^+$ and $\psi^-$, respectively) are time independent, so the quantities $c_k^* c_k$ and $d_k^* d_k$ can be consistently interpreted as probability densities in the momentum space. (More precisely, if $c_k$ and $d_k$ are independent, then these two probability densities refer to particles and antiparticles, respectively.) Indeed, in practical applications of relativistic QM, one is often interested only in scattering processes, in which the probabilities of different momenta contain all the information that can be compared with actual experiments. From a practical point of view, this is usually enough. Nevertheless, in principle, it is possible to envisage an experiment in which one measures the probabilities in the position (i.e., configuration) space, rather than that in the momentum space. A complete theory should have predictions on all quantities that can be measured in principle. Besides, if the standard interpretation of the nonrelativistic wave function in terms of the probability density in the position space is correct (which, indeed, is experimentally confirmed), then this interpretation must be derivable from a more accurate theory – relativistic QM. Thus, the existence of the probabilistic interpretation in the momentum space does not really solve the problem.

It is often claimed that the problem of relativistic probabilistic interpretation in the position space is solved by the Dirac equation. As we have seen, the problems with the Klein-Gordon equation can be traced back to the fact that it contains a second time derivative, instead of a first one. The relativistic-covariant wave equation that contains only first derivatives with respect to time and space is the Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0. \tag{63}$$

Here $\gamma^\mu$ are the $4 \times 4$ Dirac matrices that satisfy the anticommutation relations

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}, \tag{64}$$

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where \( \{ A, B \} \equiv AB + BA \). The Dirac matrices are related to the Pauli matrices \( \sigma_i \) discussed in Sec. 5, which satisfy \( \{ \sigma_i, \sigma_j \} = 2\delta_{ij} \). (For more details, see, e.g., [41].) It turns out that \( \psi \) in (63) is a 4-component wave function called spinor that describes particles with spin \( \frac{1}{2} \). The conserved current associated with (63) is
\[
j^\mu = \bar{\psi} \gamma^\mu \psi, \tag{65}\]
where \( \bar{\psi} \equiv \psi^\dagger \gamma_0 \). In particular, (64) implies \( \gamma^0 \gamma^0 = 1 \), so (65) implies
\[
j^0 = \bar{\psi} \psi, \tag{66}\]
which cannot be negative. Thus, the Dirac equation does not have problems with the probabilistic interpretation. However, this still does not mean that the problems of relativistic QM are solved. This is because the Dirac equation describes only particles with spin \( \frac{1}{2} \). Particles with different spins also exist in nature. In particular, the Klein-Gordon equation describes particles with spin 0, while the wave equation for spin 1 particles are essentially the Maxwell equations, which are second-order differential equations for the electromagnetic potential \( A^\mu \) and lead to the same interpretational problems as the Klein-Gordon equation.

There are various proposals for a more direct solution to the problem of probabilistic interpretation of the Klein-Gordon equation (see, e.g., [42, 43, 44, 37, 45]). However, all these proposed solutions have certain disadvantages and none of these proposals is widely accepted as the correct solution. Therefore, without this problem being definitely solved, it cannot be said that there exists a well-defined relativistic QM.

8 Quantum field theory solves the problems of relativistic QM

It is often claimed that the interpretational problems with relativistic QM discussed in the preceding section are solved by a more advanced theory – quantum field theory (QFT). To see how QFT solves these problems and whether this solution is really satisfactory, let me briefly review what QFT is and why it was introduced.

8.1 Second quantization of particles

A theoretical concept closely related to QFT is the method of second quantization. It was introduced to formulate in a more elegant way the fact that many-particle wave functions should be either completely symmetric or completely antisymmetric under exchange of any two particles, which comprises the principle that identical particles cannot be distinguished. Let
\[
\psi(x, t) = \sum_k a_k f_k(x, t) \tag{67}\]
be the wave function expanded in terms of some complete orthonormal set of solutions \( f_k(x, t) \). (For free particles, \( f_k(x, t) \) are usually taken to be the plane waves \( f_k(x, t) \propto e^{-i(\omega t - kx)} \).) Unlike the particle position \( x \), the wave function \( \psi \) does not correspond to an operator. Instead, it is just an ordinary number that determines the probability density.
\( \psi^* \psi \). This is so in the ordinary “first” quantization of particles. The method of second quantization promotes the wave function \( \psi \) to an operator \( \hat{\psi} \). (To avoid confusion, from now on, the operators are always denoted by a hat above it.) Thus, instead of (67), we have the operator
\[
\hat{\psi}(x, t) = \sum_k \hat{a}_k f_k(x, t),
\]
where the coefficients \( a_k \) are also promoted to the operators \( \hat{a}_k \). Similarly, instead of the complex conjugated wave function \( \psi^* \), we have the hermitian conjugated operator
\[
\hat{\psi}^\dagger(x, t) = \sum_k \hat{a}_k^\dagger f_k^*(x, t).
\]
(69)

The orthonormal solutions \( f_k(x, t) \) are still ordinary functions as before, so that the operator \( \hat{\psi} \) satisfies the same equation of motion (e.g., the Schrödinger equation in the nonrelativistic case) as \( \psi \). In the case of bosons, the operators \( \hat{a}_k, \hat{a}_k^\dagger \) are postulated to satisfy the commutation relations
\[
[\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta_{kk'},
\]
\[
[\hat{a}_k, \hat{a}_{k'}] = [\hat{a}_k^\dagger, \hat{a}_{k'}^\dagger] = 0.
\]
(70)

These commutation relations are postulated because, as is well-known from the case of first-quantized harmonic oscillator (discussed also in more detail in the next section), such commutation relations lead to a representation in which \( \hat{a}_k^\dagger \) and \( \hat{a}_k \) are raising and lowering operators, respectively. Thus, an \( n \)-particle state with the wave function \( f(k_1, \ldots, k_n) \) in the \( k \)-space can be abstractly represented as
\[
|n_f\rangle = \sum_{k_1, \ldots, k_n} f(k_1, \ldots, k_n) \hat{a}_{k_1}^\dagger \cdots \hat{a}_{k_n}^\dagger |0\rangle,
\]
(71)
where \( |0\rangle \) is the ground state of second quantization, i.e., the vacuum state containing no particles. Introducing the operator
\[
\hat{N} = \sum_k \hat{a}_k^\dagger \hat{a}_k,
\]
and using (70), one can show that
\[
\hat{N}|n_f\rangle = n|n_f\rangle.
\]
(73)

Since \( n \) is the number of particles in the state \( |n_f\rangle \), Eq. (73) shows that \( \hat{N} \) is the operator of the number of particles. The \( n \)-particle wave function in the configuration space can then be written as
\[
\psi(x_1, \ldots, x_n, t) = \langle 0|\hat{\psi}(x_1, t) \cdots \hat{\psi}(x_n, t)|n_f\rangle.
\]
(74)

From (70) and (68) we see that \( \hat{\psi}(x, t)\hat{\psi}(x', t) = \hat{\psi}(x', t)\hat{\psi}(x, t) \), which implies that the ordering of the \( \hat{\psi} \)-operators in (74) is irrelevant. This means that (74) automatically represents a bosonic wave function completely symmetric under any two exchanges of the arguments \( x_a, a = 1, \ldots, n \). For the fermionic case, one replaces the commutation relations (70) with similar anticommutation relations
\[
\{\hat{a}_k, \hat{a}_{k'}^\dagger\} = \delta_{kk'},
\]
\[
\{\hat{a}_k, \hat{a}_{k'}\} = \{\hat{a}_k^\dagger, \hat{a}_{k'}^\dagger\} = 0,
\]
(75)
which, in a similar way, leads to completely antisymmetric wave functions.
8.2 Quantum fields

The method of second quantization outlined above is nothing but a convenient mathematical trick. It does not bring any new physical information. However, the mathematical formalism used in this trick can be reinterpreted in the following way: The fundamental quantum object is neither the particle with the position-operator $\hat{x}$ nor the wave function $\psi$, but a new hermitian operator

$$\hat{\phi}(x, t) = \hat{\psi}(x, t) + \hat{\psi}^\dagger(x, t).$$

This hermitian operator is called field and the resulting theory is called quantum field theory (QFT). It is a quantum-operator version of a classical field $\phi(x, t)$. (A prototype of classical fields is the electromagnetic field satisfying Maxwell equations. Here, for pedagogical purposes, we do not study the electromagnetic field, but only the simplest scalar field $\phi$.) Using (68), (69), and (70), one obtains

$$[\hat{\phi}(x, t), \hat{\phi}(x', t)] = \sum_k f_k(x, t)f_k^*(x', t) - \sum_k f_k^*(x, t)f_k(x', t).$$

(77)

Thus, by using the completeness relations

$$\sum_k f_k(x, t)f_k^*(x', t) = \sum_k f_k^*(x, t)f_k(x', t) = \delta^3(x - x'),$$

(78)

one finally obtains

$$[\hat{\phi}(x, t), \hat{\phi}(x', t)] = 0.$$

(79)

Thus, from (68), (69), (70), and (76) one finds that (74) can also be written as

$$\psi(x_1, \ldots, x_n, t) = \langle 0|\hat{\phi}(x_1, t)\cdots\hat{\phi}(x_n, t)|n_f\rangle,$$

(80)

which, owing to (79), provides the complete symmetry of $\psi$.

The field equations of motion are derived from their own actions. For example, the Klein-Gordon equation (53) for $\phi(x)$ (instead of $\psi(x)$) can be obtained from the classical action

$$A = \int d^4x L,$$

(81)

where

$$L(\phi, \partial_\alpha \phi) = \frac{1}{2}[(\partial^\mu \phi)(\partial_\mu \phi) - m^2\phi^2]$$

(82)

is the Lagrangian density. The canonical momentum associated with this action is a fieldlike quantity

$$\pi(x) = \frac{\partial L}{\partial(\partial_0 \phi(x))} = \partial^0 \phi(x).$$

(83)

The associated Hamiltonian density is

$$\mathcal{H} = \pi \partial_0 \phi - L = \frac{1}{2}[\pi^2 + (\nabla \phi)^2 + m^2\phi^2].$$

(84)
This shows that the field energy
\[ H[\pi, \phi] = \int d^3x H(\pi(x), \phi(x), \nabla \phi(x)) \] (85)
(where the time-dependence is suppressed) cannot be negative. This is why, in relativistic QM, it is better to speak of negative frequencies than of negative energies. (In (85), the notation \( H[\pi, \phi] \) denotes that \( H \) is not a function of \( \pi(x) \) and \( \phi(x) \) at some particular values of \( x \), but a functional, i.e., an object that depends on the whole functions \( \pi \) and \( \phi \) at all values of \( x \).) By analogy with the particle commutation relations \([\hat{x}_i, \hat{p}_m] = i\delta_{im}, \ [\hat{x}_i, \hat{x}_j] = [\hat{p}_i, \hat{p}_j] = 0\), the fundamental field-operator commutation relations are postulated to be
\[
[\hat{\phi}(x), \hat{\pi}(x')] = i\delta^3(x - x'),
[\hat{\phi}(x), \hat{\phi}(x')] = [\hat{\pi}(x), \hat{\pi}(x')] = 0.
\] (86)

Here it is understood that all fields are evaluated at the same time \( t \), so the \( t \) dependence is not written explicitly. Thus, now (79) is one of the fundamental (not derived) commutation relations. Since \( \hat{\phi}(x) \) is an operator in the Heisenberg picture that satisfies the Klein-Gordon equation, the expansion (76) with (68) and (69) can be used. One of the most important things gained from quantization of fields is the fact that now the commutation relations (70) do not need to be postulated. Instead, they can be derived from the fundamental field-operator commutation relations (86). (The fermionic field-operators satisfy similar fundamental relations with commutators replaced by anticommutators, from which (75) can be derived.) The existence of the Hamiltonian (85) allows us to introduce the functional Schrödinger equation
\[ H[\hat{\pi}, \phi]\Psi[\phi; t] = i\partial_t \Psi[\phi; t], \] (87)
where \( \Psi[\phi; t] \) is a functional of \( \phi(x) \) and a function of \( t \), while
\[ \hat{\pi}(x) = -i\frac{\delta}{\delta \phi(x)} \] (88)
is the field analog of the particle-momentum operator \( \hat{p}_j = -i\partial/\partial x_j \). (For a more careful definition of the functional derivative \( \delta/\delta \phi(x) \) see, e.g., [46].) Unlike the Klein-Gordon equation, the functional Schrödinger equation (87) is a first-order differential equation in the time derivative. Consequently, the quantity
\[ \rho[\phi; t] = \Psi^*[\phi; t]\Psi[\phi; t] \] (89)
can be consistently interpreted as a conserved probability density. It represents the probability that the field has the configuration \( \phi(x) \) at the time \( t \).

8.3 Does QFT solve the problems of relativistic QM?

After this brief overview of QFT, we are finally ready to cope with the validity of the title of this section. How QFT helps in solving the interpretational problems of relativistic QM? According to QFT, the fundamental objects in nature are not particles, but
fields. Consequently, the fundamental wave function(al) that needs to have a well-defined probabilistic interpretation is not $\psi(x,t)$, but $\Psi(\phi; t)$. Thus, the fact that, in the case of Klein-Gordon equation, $\psi(x,t)$ cannot be interpreted probabilistically, is no longer a problem from this more fundamental point of view. However, does it really solve the problem? If QFT is really a more fundamental theory than the first-quantized quantum theory of particles, then it should be able to reproduce all good results of this less fundamental theory. In particular, from the fundamental axioms of QFT (such as the axiom that $\langle \phi \rangle$ represents the probability in the space of fields), one should be able to deduce that, at least in the nonrelativistic limit, $\psi^*\psi$ represents the probability in the space of particle positions. However, one cannot deduce it solely from the axioms of QFT. One possibility is to completely ignore, or even deny, the validity of the probabilistic interpretation of $\psi$, which indeed is in the spirit of QFT viewed as a fundamental theory, but then the problem is to reconcile it with the fact that such a probabilistic interpretation of $\psi$ is in agreement with experiments. Another possibility is to supplement the axioms of QFT with an additional axiom that says that $\psi$ in the nonrelativistic limit determines the probabilities of particle positions, but then such a set of axioms is not coherent, as it does not specify the meaning of $\psi$ in the relativistic case. Thus, instead of saying that QFT solves the problems of relativistic QM, it is more honest to say that it merely sweeps them under the carpet.

9 Quantum field theory is a theory of particles

What is the world made of? A common answer is that it is made of elementary particles, such as electrons, photons, quarks, gluons, etc. On the other hand, all modern theoretical research in elementary-particle physics is based on quantum field theory (QFT). So, is the world made of particles or fields? A frequent answer given by elementary-particle physicists is that QFT is actually a theory of particles, or more precisely, that particles are actually more fundamental physical objects, while QFT is more like a mathematical tool that describes – the particles. Indeed, the fact that the motivation for introducing QFT partially emerged from the method of second quantization (see Sec. 8) supports this interpretation according to which QFT is nothing but a theory of particles. But is that really so? Is it really a fundamental property of QFT that it describes particles? Let us see!

9.1 A first-quantized analog of particles in QFT

From the conceptual point of view, fields and particles are very different objects. This is particularly clear for classical fields and particles, where all concepts are clear. So, if there exists a relation between quantum fields and particles that does not have an analog in the classical theory of fields and particles, then such a relation must be highly nontrivial. Indeed, this nontrivial relation is related to the nontrivial commutation relations (70) (or (75) for fermionic fields). The classical fields commute, which implies that the classical coefficients $a_k$, $a_k^*$ do not satisfy (70). Without these commutation relations, we could not introduce $n$-particle states (71). However, are the commutation relations (70) sufficient for having a well-defined notion of particles? To answer this question, it is instructive to
study the analogy with the first-quantized theory of particles.

Consider a quantum particle moving in one dimension, having a Hamiltonian

\[ \hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x}). \]  

(90)

We introduce the operators

\[ \hat{a} = \frac{1}{\sqrt{2}} \left( \sqrt{m\omega} \hat{x} + i \frac{\hat{p}}{\sqrt{m\omega}} \right), \]

\[ \hat{a}^\dagger = \frac{1}{\sqrt{2}} \left( \sqrt{m\omega} \hat{x} - i \frac{\hat{p}}{\sqrt{m\omega}} \right), \]  

(91)

where \( \omega \) is some constant of the dimension of energy (or frequency, which, since \( \hbar = 1 \), has the same dimension as energy). Using the commutation relation \([\hat{x}, \hat{p}] = i\), we obtain

\[ [\hat{a}, \hat{a}^\dagger] = 1. \]  

(92)

This, together with the trivial commutation relations \([\hat{a}, \hat{a}^\dagger] = [\hat{a}^\dagger, \hat{a}] = 0\), shows that \( \hat{a}^\dagger \) and \( \hat{a} \) are the raising and lowering operator, respectively. As we speak of one particle, the number operator \( \hat{N} = \hat{a}^\dagger \hat{a} \) now cannot be called the number of particles. Instead, we use a more general terminology (applicable to (72) as well) according to which \( \hat{N} \) is the number of “quanta”. But quanta of what? It is easy to show that (90) can be written as

\[ \hat{H} = \omega \left( \hat{N} + \frac{1}{2} \right) + \left[ V(\hat{x}) - \frac{m\omega^2 \hat{x}^2}{2} \right]. \]  

(93)

In the special case in which \( V(\hat{x}) = m\omega^2 \hat{x}^2 / 2 \), which corresponds to the harmonic oscillator, the square bracket in (93) vanishes, so the Hamiltonian can be expressed in terms of the \( \hat{N} \)-operator only. In this case, the (properly normalized) state

\[ |n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}} |0\rangle \]  

has the energy \( \omega(n+1/2) \), so the energy can be viewed as a sum of the ground-state energy \( \omega/2 \) and the energy of \( n \) quanta with energy \( \omega \). This is why the number operator \( \hat{N} \) plays an important physical role. However, the main point that can be inferred from (93) is the fact that, for a general potential \( V(\hat{x}) \), the number operator \( \hat{N} \) does not play any particular physical role. Although the spectrum of quantum states can often be labeled by a discrete label \( n' = 0, 1, 2, \ldots \), this label, in general, has nothing to do with the operator \( \hat{N} \) (i.e., the eigenstates (94) of \( \hat{N} \) are not the eigenstates of the Hamiltonian (93)). Moreover, in general, the spectrum of energies \( E(n') \) may have a more complicated dependence on \( n' \), so that, unlike the harmonic oscillator, the spectrum of energies is not equidistant. Thus, in general, the state of a system cannot be naturally specified by a number of “quanta” \( n \).

If one insists on representing the system in terms of the states (94), then one can treat the square bracket in (93) as a perturbation \( V_I(\hat{x}) \). (Here “I” stands for “interaction”. ) From (91) one finds

\[ \hat{x} = \frac{\hat{a} + \hat{a}^\dagger}{\sqrt{2m\omega}}, \]  

(95)
so $V_I(\hat{x}) = V_I(\hat{a}, \hat{a}^\dagger)$. Consequently, various terms in the perturbation expansion can be represented in terms of creation and destruction of quanta, owing to the occurrence of $\hat{a}^\dagger$ and $\hat{a}$, respectively. However, treating the square bracket in (93) as a perturbation is completely arbitrary. Such a treatment is nothing but a mathematical convenience and does not make the states (94) more physical. This is particularly clear for the cases in which the original system with the Hamiltonian (91) can be solved analytically, without a perturbation expansion. The creation and destruction of quanta appearing in the perturbation expansion does not correspond to actual physical processes. These “processes” of creation and destruction are nothing but a verbalization of certain mathematical terms appearing only in one particular method of calculation – the perturbation expansion with the square bracket in (93) treated as the perturbation. Last but not least, even if, despite the unnaturalness, one decides to express everything in terms of the operators (91) and the states (94), there still may remain an ambiguity in choosing the constant $\omega$. All this demonstrates that, in general, QM is not a theory of “quanta” attributed to the operator $\hat{N}$.

9.2 Particles in perturbative QFT

The analogy between the notion of “quanta” in the first-quantized theory of particles and the notion of “particles” in QFT is complete. For example, the QFT analog of (95) is the field operator in the Schrödinger picture

$$\hat{\phi}(x) = \sum_k \hat{a}_k f_k(x) + \hat{a}_k^\dagger f_k^*(x),$$

(96)

which corresponds to (76) with (68) and (69), at fixed $t$. If $f_k(x, t)$ are the plane waves proportional to $e^{-i(\omega_k t - kx)}$, then the quantum Hamiltonian obtained from the Lagrangian density (82) turns out to be

$$\hat{H} = \sum_k \omega_k \left( \hat{N}_k + \frac{1}{2} \right),$$

(97)

with $\hat{N}_k \equiv \hat{a}_k^\dagger \hat{a}_k$, which is an analog of the first term in (93). This analogy is related to the fact that (82) represents a relativistic-field generalization of the harmonic oscillator. (The harmonic-oscillator Lagrangian is quadratic in $x$ and its derivative, while (82) is quadratic in $\phi$ and its derivatives). The Hamiltonian (97) has a clear physical interpretation; ignoring the term $1/2$ (which corresponds to an irrelevant ground-state energy $\sum_k \omega_k/2$), for each $\omega_k$ there can be only an integer number $n_k$ of quanta with energy $\omega_k$, so that their total energy sums up to $n_k \omega_k$. (Concerning the irrelevance of the ground-state energy above, it should be noted that it is often claimed that this energy is relevant for the experimentally confirmed Casimir effect, but that the fact is that this effect can be derived even without referring to the ground-state energy [50].) These quanta are naturally interpreted as “particles” with energy $\omega_k$. However, the Lagrangian (82) is only a special case. In general, a Lagrangian describing the field $\phi$ may have a form

$$\mathcal{L} = \frac{1}{2} (\partial^\mu \phi)(\partial_\mu \phi) - V(\phi),$$

(98)

where $V(\phi)$ is an arbitrary potential. Thus, in general, the Hamiltonian contains an additional term analogous to that in (93), which destroys the “particle”-interpretation of the spectrum.
Whereas the formal mathematical analogy between first quantization and QFT (which implies the irrelevance of the number operator $\hat{N}$) is clear, there is one crucial physical difference: Whereas in first quantization there is really no reason to attribute a special meaning to the operator $\hat{N}$, there is an experimental evidence that this is not so for QFT. The existence of particles is an experimental fact! Thus, if one wants to describe the experimentally observed objects, one must either reject QFT (which, indeed, is what many elementary-particle physicists were doing in the early days of elementary-particle physics and some of them are doing it even today [51]), or try to artificially adapt QFT such that it remains a theory of particles even with general interactions (such as those in (98)).

From a pragmatic and phenomenological point of view, the latter strategy turns out to be surprisingly successful! For example, in the case of (98), one artificially defines the interaction part of the Lagrangian as

$$V_I(\phi) = V(\phi) - \frac{1}{2}m^2\phi^2,$$

and treats it as a perturbation of the “free” Lagrangian (82). For that purpose, it is convenient to introduce a mathematical trick called interaction picture, which is a picture that interpolates between the Heisenberg picture (where the time dependence is attributed to fields $\phi$) and the Schrödinger picture (where the time dependence is attributed to states $|\Psi\rangle$). In the interaction picture, the field satisfies the free Klein-Gordon equation of motion, while the time evolution of the state is governed only by the interaction part of the Hamiltonian. This trick allows one to use the free expansion (76) with (68) and (69), despite the fact that the “true” quantum operator $\hat{\phi}(x, t)$ in the Heisenberg picture cannot be expanded in that way. In fact, all operators in the interaction picture satisfy the free equations of motion, so the particle-number operator can also be introduced in the same way as for free fields. Analogously to the case of first quantization discussed after Eq. (95), certain mathematical terms in the perturbation expansion can be pictorially represented by the so-called Feynman diagrams. (For technical details, I refer the reader to [48, 49].) In some cases, the final measurable results obtained in that way turn out to be in excellent agreement with experiments.

### 9.3 Virtual particles?

The calculational tool represented by Feynman diagrams suggests an often abused picture according to which “real particles interact by exchanging virtual particles”. Many physicists, especially nonexperts, take this picture literally, as something that really and objectively happens in nature. In fact, I have never seen a popular text on particle physics in which this picture was not presented as something that really happens. Therefore, this picture of quantum interactions as processes in which virtual particles exchange is one of the most abused myths, not only in quantum physics, but in physics in general. Indeed, there is a consensus among experts for foundations of QFT that such a picture should not be taken literally. The fundamental principles of quantum theory do not even contain a notion of a “virtual” state. The notion of a “virtual particle” originates only from a specific mathematical method of calculation, called perturbative expansion. In fact, perturbative expansion represented by Feynman diagrams can be introduced even in classical physics [52, 53], but nobody attempts to verbalize these classical Feynman diagrams in
terms of classical “virtual” processes. So why such a verbalization is tolerated in quantum physics? The main reason is the fact that the standard interpretation of quantum theory does not offer a clear “canonical” ontological picture of the actual processes in nature, but only provides the probabilities for the final results of measurement outcomes. In the absence of such a “canonical” picture, physicists take the liberty to introduce various auxiliary intuitive pictures that sometimes help them think about otherwise abstract quantum formalism. Such auxiliary pictures, by themselves, are not a sin. However, a potential problem occurs when one forgets why such a picture has been introduced in the first place and starts to think on it too literally.

9.4 Nonperturbative QFT

In some cases, the picture of particles suggested by the “free” part of the Lagrangian does not really correspond to particles observed in nature. The best known example is quantum chromodynamics (QCD), a QFT theory describing strong interactions between quarks and gluons. In nature we do not observe quarks, but rather more complicated particles called hadrons (such as protons, neutrons, and pions). In an oversimplified but often abused picture, hadrons are built of 2 or 3 quarks glued together by gluons. However, since free quarks are never observed in nature, the perturbative expansion, so successful for some other QFT theories, is not very successful in the case of QCD. Physicists are forced to develop other approximative methods to deal with it. The most successful such method is the so-called lattice QCD (for an introductory textbook see [54] and for pedagogic reviews see [55, 56]). In this method, the spacetime continuum is approximated by a finite lattice of spacetime points, allowing the application of brutal-force numerical methods of computation. This method allows to compute the expectation values of products of fields in the ground state, by starting from first principles. However, to extract the information about particles from these purely field-theoretic quantities, one must assume a relation between these expectation values and the particle quantities. This relation is not derived from lattice QCD itself, but rather from the known relation between fields and particles in perturbative QFT. Consequently, although this method reproduces the experimental hadron data more-or-less successfully, the concept of particle in this method is not more clear than that in the perturbative approach. Thus, the notion of real particles is not derived from first principles and nothing in the formalism suggests a picture of the “exchange of virtual particles”.

9.5 Particles and the choice of time

As we have seen, although the notion of particles in interacting QFT theories cannot be derived from first principles (or at least we do not know yet how to do that), there are heuristic mathematical procedures that introduce the notion of particles that agrees with experiments. However, there are circumstances in QFT in which the theoretical notion of particles is even more ambiguous, while present experiments are not yet able to resolve these ambiguities. Consider again the free field expanded as in (76) with (68) and (69). The notion of particles rests on a clear separation between the creation operators $a^\dagger_k$ and the destruction operators $a_k$. The definition of these operators is closely related to the choice of the complete orthonormal basis $\{f_k(x,t)\}$ of solutions to the classical Klein-
Gordon equation. However, there are infinitely many different choices of this basis. The plane-wave basis
\[ f_k(x, t) \propto e^{-i(\omega_k t - kx)} \equiv e^{-ik \cdot x} \]  
(100)
is only a particular convenient choice. Different choices may lead to different creation and destruction operators \(a_k^\dagger\) and \(a_k\), and thus to different notions of particles. How to know which choice is the right one? Eq. (100) suggests a physical criterion according to which the modes \(f_k(x, t)\) should be chosen such that they have a positive frequency. However, the notion of frequency assumes the notion of time. On the other hand, according to the theory of relativity, there is not a unique choice of the time coordinate. Therefore, the problem of the right definition of particles reduces to the problem of the right definition of time. Fortunately, the last exponential function in (100) shows that the standard plane waves \(f_k(x)\) are Lorentz invariant, so that different time coordinates related by a Lorentz transformation lead to the same definition of particles. However, Lorentz transformations relate only proper coordinates attributed to inertial observers in flat spacetime. The general theory of relativity allows much more general coordinate transformations, such as those that relate an inertial observer with an accelerating one. (For readers who are not familiar with general theory of relativity there are many excellent introductory textbooks, but my favored one that I highly recommend to the beginners is [57]. For an explicit construction of the coordinate transformations between an inertial observer and an arbitrarily moving one in flat spacetime, see [58, 59], and for instructive applications, see [59, 60, 61]. Nevertheless, to make this paper readable by those who are not familiar with general relativity, in the rest of Sec. 9, as well as in Sec. 10 I omit some technical details that require a better understanding of general relativity, keeping only the details that are really necessary to understand the quantum aspects themselves.) Different choices of time lead to different choices of the positive-frequency bases \(\{f_k(x)\}\), and thus to different creation and destruction operators \(a_k^\dagger\) and \(a_k\), respectively. If
\[ \hat{\phi}(x) = \sum_k \hat{a}_k f_k(x) + \hat{a}_k^\dagger f_k^*(x), \]
\[ \hat{\phi}(x) = \sum_l \hat{a}_l \tilde{f}_l(x) + \hat{a}_l^\dagger \tilde{f}_l^*(x) \]  
(101)
are two such expansions in the bases \(\{f_k(x)\}\) and \(\{\tilde{f}_l(x)\}\), respectively, it is easy to show that the corresponding creation and destruction operators are related by a linear transformation
\[ \hat{a}_l = \sum_k \alpha_{lk} \hat{a}_k + \beta_{lk}^* \hat{a}_k^\dagger, \]
\[ \hat{a}_l^\dagger = \sum_k \alpha_{lk}^* \hat{a}_k^\dagger + \beta_{lk} \hat{a}_k, \]  
(102)
where
\[ \alpha_{lk} \equiv (\tilde{f}_l, f_k), \quad \beta_{lk}^* \equiv (\tilde{f}_l, f_k^*), \]  
(103)
are given by the scalar products defined as in (59). (To derive (102), take the scalar product of both expressions in (101) with \(\tilde{f}_l\) on the left and use the orthonormality relations \((\tilde{f}_l, \tilde{f}_l) = \delta_{ll}, (\tilde{f}_l, \tilde{f}_l^*) = 0\).) The transformation (102) between the two sets of
creation and destruction operators is called \textit{Bogoliubov transformation}. Since both bases are orthonormal, the Bogoliubov coefficients \textbf{(103)} satisfy
\[ \sum_k (\alpha_{lk}\alpha_{l'k}^* - \beta_{lk}^*\beta_{l'k}) = \delta_{ll'}, \] (104)
where the negative sign is a consequence of the fact that negative frequency solutions have negative norms, i.e., \((f_k^*, f_{k'}^*) = -\delta_{kk'}\), \((\bar{f}_l^*, \bar{f}_{l'}^*) = -\delta_{ll'}\). One can show that \textbf{(104)} provides that \(\hat{\bar{a}}_l\) and \(\hat{\bar{a}}_l^\dagger\) also satisfy the same commutation relations \textbf{(70)} as \(\hat{a}_k\) and \(\hat{a}_k^\dagger\) do. A physically nontrivial Bogoliubov transformation is that in which at least some of the \(\beta_{lk}\) coefficients are not zero. Two different definitions of the particle-number operators are
\[ \hat{N} = \sum_k \hat{N}_k, \quad \hat{\bar{N}} = \sum_l \hat{\bar{N}}_l, \] (105)
where
\[ \hat{N}_k = \hat{a}_k\hat{a}_k^\dagger, \quad \hat{\bar{N}}_l = \hat{\bar{a}}_l\hat{\bar{a}}_l^\dagger. \] (106)
In particular, from \textbf{(102)}, it is easy to show that the vacuum \(|0\rangle\) having the property \(\hat{N}|0\rangle = 0\) has the property \(\langle 0|\hat{\bar{N}}_l|0\rangle = \sum_k |\beta_{lk}|^2\). (107)

For a nontrivial Bogoliubov transformation, this means that the average number of particles in the no-particle state \(|0\rangle\) is a state full of particles when the particles are defined by \(\hat{N}\)! Conversely, the no-particle state \(|0\rangle\) having the property \(\hat{N}|0\rangle = 0\) is a state full of particles when the particles are defined with \(\hat{N}\). So, what is the right operator of the number of particles, \(\hat{N}\) or \(\hat{\bar{N}}\)? How to find the right operator of the number of particles? The fact is that, in general, a clear universally accepted answer to this question is not known! Instead, there are several possibilities that we discuss below.

One possibility is that the dependence of the particle concept on the choice of time means that the concept of particles depends on the observer. The best known example of this interpretation is the Unruh effect \cite{62, 63, 64}, according to which a uniformly accelerating observer perceives the standard Minkowski vacuum (defined with respect to time of an inertial observer in Minkowski flat spacetime) as a state with a huge number of particles with a thermal distribution of particle energies, with the temperature proportional to the acceleration. Indeed, this effect (not yet experimentally confirmed!) can be obtained by two independent approaches. The first approach is by a Bogoliubov transformation as indicated above, leading to \cite{62, 64}
\[ \langle 0|\hat{\bar{N}}_l|0\rangle = \frac{1}{e^{2\pi\omega_l/a} - 1}, \] (108)
where \(a\) is the proper acceleration perceived by the accelerating observer, \(\omega_l\) is the frequency associated with the solution \(\bar{f}_l(x)\), and we use units in which \(\hbar = c = 1\). (Coordinates of a uniformly accelerating observer are known as Rindler coordinates \cite{65}, so the quantization based on particles defined with respect to the Rindler time is called Rindler quantization.) We see that the right-hand side of \textbf{(108)} looks just as a Bose-Einstein distribution at the temperature \(T = a/2\pi\) (in units in which the Boltzmann constant

36
is also taken to be unit). The second approach is by studying the response of a theoretical model of an accelerating particle detector, using only the standard Minkowski quantization without the Bogoliubov transformation. However, these two approaches are not equivalent \[67, 66\]. Besides, such a dependence of particles on the observer is not relativistically covariant. In particular, it is not clear which of the definitions of particles, if any, acts as a source for a (covariantly transforming) gravitational field.

An alternative is to describe particles in a unique covariant way in terms of local particle currents \[68\], but such an approach requires a unique choice of a preferred time coordinate. For example, for a hermitian scalar field \(\psi(x)\), the particle current is

\[
\hat{j}_\mu^p(x) = i\hat{\psi}^\dagger(x) \partial_\mu \hat{\psi}(x),
\]

which (unlike \(\hat{j}_\mu(x)\)) requires the identification of the positive- and negative-frequency parts \(\hat{\psi}(x)\) and \(\hat{\psi}^\dagger(x)\), respectively. Noting that the quantization of fields themselves based on the functional Schrödinger equation \(87\) also requires a choice of a preferred time coordinate, it is possible that a preferred time coordinate emerges dynamically from some nonstandard covariant method of quantization, such as that in \(38\).

Another possibility is that the concept of particles as fundamental objects simply does not make sense in QFT \(64, 69\). Instead, all observables should be expressed in terms of local fields that do not require the artificial identification of the positive- and negative-frequency parts. For example, such an observable is the Hamiltonian density \(84\), which represents the \(T_0^0\)-component of the covariant energy-momentum tensor \(T_\mu^\nu(x)\). Whereas such an approach is very natural from the theoretical point of view according to which QFT is nothing but a quantum theory of fields, the problem is to reconcile it with the fact that the objects observed in high-energy experiments are – particles.

### 9.6 Particle creation by a classical field

When the classical metric \(g_{\mu\nu}(x)\) has a nontrivial dependence on \(x\), the Klein-Gordon equation \(53\) for the field \(\phi(x)\) generalizes to

\[
\left( \frac{1}{\sqrt{|g|}} \partial_\mu \sqrt{|g|} g^{\mu\nu} \partial_\nu + m^2 \right) \phi = 0,
\]

where \(g\) is the determinant of the matrix \(g_{\mu\nu}\). In particular, if the metric is time dependent, then a solution \(f_k(x)\) having a positive frequency at some initial time \(t_{\text{in}}\) may behave as a superposition of positive- and negative-frequency solutions at some final time \(t_{\text{fin}}\). At the final time, the solutions that behave as positive-frequency ones are some other solutions \(\bar{f}_l(x)\). In this case, it seems natural to define particles with the operator \(\hat{N}\) at the initial time and with \(\hat{\bar{N}}\) at the final time. If the time-independent state in the Heisenberg picture is given by the “vacuum” \(|0\rangle\), then \(|0\rangle\hat{N}|0\rangle = 0\) denotes that there are no particles at \(t_{\text{in}}\), while \(|0\rangle\hat{\bar{N}}|0\rangle = 0\) can be interpreted as a consequence of an evolution of the particle-number operator, so that \(107\) refers only to \(t_{\text{fin}}\). This is the essence of the mechanism of particle creation by a classical gravitational field. The best known example is particle creation by a collapse of a black hole, known also as Hawking radiation \(70\). (For more details, see
also the classic textbook [64], a review [71], and a pedagogic review [72].) Similarly to the Unruh effect [108], the Hawking particles have the distribution
\[
\langle 0 | \hat{N}_l | 0 \rangle = \frac{1}{e^{8\pi GM\omega_l} - 1},
\]
where \( G \) is the Newton gravitational constant which has a dimension (energy)\(^{-2} \) and \( M \) is the black-hole mass. This is the result obtained by defining particles with respect to a specific time, that is, the time of an observer static with respect to the black hole and staying far from the black-hole horizon. Although (111) looks exactly like a quantum Bose-Einstein thermal distribution at the temperature
\[
T = \frac{1}{8\pi GM},
\]
this distribution is independent of the validity of the bosonic quantum commutation relations (70). Instead, it turns out that the crucial ingredient leading to a thermal distribution is the existence of the horizon [73], which is a classical observer-dependent general-relativistic object existing not only for black holes, but also for accelerating observers in flat spacetime. Thus, the origin of this thermal distribution can be understood even with classical physics [74], while only the mechanism of particle creation itself is intrinsically quantum. In the literature, the existence of thermal Hawking radiation often seems to be widely accepted as a fact. Nevertheless, since it is not yet experimentally confirmed and since it rests on the theoretically ambiguous concept of particles in curved spacetime (the dependence on the choice of time), certain doubts on its existence are still reasonable (see, e.g., [75, 68, 76] and references therein). Thus, the existence of Hawking radiation can also be qualified as a myth.

The classical gravitational field is not the only classical field that seems to be able to cause a production of particles from the vacuum. The classical electric field seems to be able to produce particle-antiparticle pairs [77, 78, 71], by a mechanism similar to the gravitational one. For discussions of theoretical ambiguities lying behind this theoretically predicted effect, see [75, 68, 79].

9.7 Particles, fields, or something else?

Having in mind all these foundational problems with the concept of particle in QFT, it is still impossible to clearly and definitely answer the question whether the world is made of particles or fields. Nevertheless, practically oriented physicists may not find this question disturbing as long as the formalism, no matter how incoherent it may appear to be, gives correct predictions on measurable quantities. Such a practical attitude seems to be justified by the vagueness of the concept of reality inherent to QM itself. Indeed, one can adopt a hard version of orthodox interpretation of QM according to which information about reality is more fundamental than reality itself and use it to justify the noncovariant dependence of particles (as well as some other quantities) on the observer [80]. However, in the standard orthodox QM, where rigorous no-hidden-variable theorems exist (see Sec. 5), at least the operators are defined unambiguously. Thus, even the hard-orthodox interpretation of QM is not sufficient to justify the interpretation of the particle-number-operator ambiguities as different realities perceived by different observers. An alternative
to this orthodox approach is an objective-realism approach in which both particles and fields separately exist, which is a picture that seems to be particularly coherent in the Bohmian interpretation [81].

Finally, there is a possibility that the world is made neither of particles nor of fields, but of strings. (For an excellent pedagogic introduction to string theory see [82]. In particular, this book also breaks one myth in physics – the myth that string theory is mathematically an extremely complicated theory that most other physicists cannot easily understand. For a more concise pedagogic introduction to string theory see also [83].) In fact, many string theorists speak about the existence of strings as a definite fact. Fortunately, there is still a sufficiently large number of authoritative physicists that are highly skeptical about string theory, which does not allow string theory to become a widely accepted myth. Nevertheless, string theory possesses some remarkable theoretical properties that makes it a promising candidate for a more fundamental description of nature. According to this theory, particles are not really pointlike, but extended one-dimensional objects. Their characteristic length, however, is very short, which is why they appear as pointlike with respect to our current experimental abilities to probe short distances. However, just as for particles, there is first quantization of strings, as well as second quantization that leads to string field theory. Thus, even if string theory is correct, there is still a question whether the fundamental objects are strings or string fields. However, while first quantization of strings is well understood, string field theory is not. Moreover, there are indications that string field theory may not be the correct approach to treat strings [84]. Consequently, particles (that represent an approximation of strings) may be more fundamental than fields. Concerning the issue of objective reality, there are indications that the Bohmian interpretation of strings may be even more natural than that of particles [85, 86]. The Bohmian interpretation of strings also breaks some other myths inherent to string theory [87].

10 Black-hole entropy is proportional to its surface

As this claim is not yet a part of standard textbooks, this is not yet a true myth. Nevertheless, in the last 10 or 20 years this claim has been so often repeated by experts in the field (the claim itself is about 30 years old) that it is very likely that it will soon become a true myth. Before it happens, let me warn the wider physics community that this claim is actually very dubious.

10.1 Black-hole “entropy” in classical gravity

The claim in the title of this section is actually a part of a more general belief that there exists a deep relation between black holes and thermodynamics. The first evidence supporting this belief came from certain classical properties of black holes that, on the mathematical level, resemble the laws of thermodynamics [88, 89]. (For general pedagogic overviews, see, e.g., [57, 90] and for an advanced pedagogic review with many technical details, see [91].) Black holes are dynamical objects that can start their evolution from a huge number of different initial states, but eventually end up in a highly-symmetric equilibrium stationary state specified only by a few global conserved physical quantities,
such as their mass $M$ (i.e., energy $E$), electric charge $Q$, and angular momentum $J$. The physical laws governing the behavior of such equilibrium black holes formally resemble the laws governing the behavior of systems in thermodynamic equilibrium. The well-known four laws of thermodynamics have the following black-hole analogues:

- **Zeroth law:** There exists a local quantity called surface gravity $\kappa$ (which can be viewed as the general-relativistic analog of the Newton gravitational field $GM/r^2$) that, in equilibrium, turns out to be constant everywhere on the black-hole horizon. This is an analog of temperature $T$ which is constant in thermodynamic equilibrium.

- **First law:** This is essentially the law of energy conservation, which, both in the black-hole and the thermodynamic case, has an origin in even more fundamental laws. As such, this analogy should not be surprising, but it is interesting that in both cases the conservation of energy takes a mathematically similar form. For black holes it reads
  \[ dM = \frac{\kappa}{8\pi G} dA + \Omega dJ + \Phi dQ, \tag{113} \]
  where $A$ is the surface of the horizon, $\Omega$ is the angular velocity of the horizon, and $\Phi$ is the electrostatic potential at the horizon. This is analogous to the thermodynamic first law
  \[ dE = TdS - pdV + \mu dN, \tag{114} \]
  where $S$ is the entropy, $p$ is the pressure, $V$ is the volume, $\mu$ is the chemical potential, and $N$ is the number of particles. In particular, note that the black-hole analog of the entropy $S$ is a quantity proportional to the black-hole surface $A$. This allows us to introduce the black-hole “entropy”
  \[ S_{\text{bh}} = \alpha A, \tag{115} \]
  where $\alpha$ is an unspecified constant.

- **Second law:** Although the fundamental microscopic physical laws are time reversible, the macroscopic laws are not. Instead, disorder tends to increase with time. In the thermodynamic case, it means that entropy cannot decrease with time, i.e., $dS \geq 0$. In the gravitational case, owing to the attractive nature of the gravitational force, it turns out that the black-hole surface cannot decrease with time, i.e., $dA \geq 0$.

- **Third law:** It turns out that, by a realistic physical process, it is impossible to reach the state with $\kappa = 0$. This is analogous to the third law of thermodynamics according to which, by a realistic physical process, it is impossible to reach the state with $T = 0$.

Although the analogy as presented above is suggestive, it is clear that classical black-hole parameters are conceptually very different from the corresponding thermodynamic parameters. Indeed, the formal analogies above were not taken very seriously at the beginning. In particular, an ingredient that is missing for a full analogy between classical black holes and thermodynamic systems is – radiation with a thermal spectrum. Classical black holes (i.e., black holes described by the classical Einstein equation of gravity) do not produce radiation with a thermal spectrum.
10.2 Black-hole “entropy” in semiclassical gravity

A true surprise happened when Hawking found out [70] that *semiclassical* (i.e., gravity is treated classically while matter is quantized) black holes not only radiate (which, by itself, is not a big surprise), but radiate exactly with a thermal spectrum at a temperature proportional to $\kappa$. In the special case of a black hole with $J = Q = 0$, this temperature is equal to (112). Since $dJ = dQ = 0$, we attempt to write (113) as

$$dS_{bh} = \frac{dM}{T},$$

(116)

which corresponds to (114) with $dV = dN = 0$. From (112), we see that

$$\frac{dM}{T} = 8\pi GM dM.$$

(117)

From the Schwarzschild form of the black-hole metric in the polar spacial coordinates $(r, \vartheta, \varphi)$ (see, e.g., [57])

$$ds^2 = \frac{dt^2}{1 - \frac{2GM}{r}} - \left(1 - \frac{2GM}{r}\right) dr^2 - r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2),$$

(118)

we see that the horizon corresponding to the singular behavior of the metric is at the radius

$$r = 2GM.$$

(119)

Consequently, the surface of the horizon is equal to

$$A = 4\pi r^2 = 16\pi G^2 M^2.$$

(120)

Therefore, (115) implies

$$dS_{bh} = \alpha 32\pi G^2 M dM.$$

(121)

Thus, we see that (117) and (121) are really consistent with (116), provided that $\alpha = 1/4G$. Therefore, (115) becomes

$$S_{bh} = \frac{A}{4G}.$$

(122)

In fact, (122) turns out to be a generally valid relation, for arbitrary $J$ and $Q$.

Now, with the results (112) and (122), the analogy between black holes and thermodynamics seems to be more complete. Nevertheless, it is still only an analogy. Moreover, thermal radiation (which is a kinematical effect depending only on the metric) is not directly logically related to the four laws of classical black-hole “thermodynamics” (for which the validity of the dynamical Einstein equations is crucial) [92]. Still, many physicists believe that such a striking analogy cannot be a pure formal coincidence. Instead, they believe that there is some even deeper meaning of this analogy. In particular, as classical horizons hide information from observers, while the orthodox interpretation of QM suggests a fundamental role of information available to observers, it is believed that this could be a key to a deeper understanding of the relation between relativity and quantum theory [80]. As the correct theory of quantum gravity is not yet known (for
reviews of various approaches to quantum gravity, see [93, 94], there is a belief that this deeper meaning will be revealed one day when we better understand quantum gravity. Although this belief may turn out to be true, at the moment there is no real proof that this necessarily must be so.

A part of this belief is that (122) is not merely a quantity analogous to entropy, but that it really is the entropy. However, in standard statistical physics (from which thermodynamics can be derived), entropy is a quantity proportional to the number of the microscopic physical degrees of freedom. On the other hand, the derivation of (122) as sketched above does not provide a direct answer to the question what, if anything, these microscopic degrees of freedom are. In particular, they cannot be simply the particles forming the black hole, as there is no reason why the number of particles should be proportional to the surface $A$ of the black-hole boundary. Indeed, as entropy is an extensive quantity, one expects that it should be proportional to the black-hole volume, rather than to its surface. It is believed that quantum gravity will provide a more fundamental answer to the question why the black-hole entropy is proportional to its surface, rather than to its volume. Thus, the program of finding a microscopic derivation of Eq. (122) is sometimes referred to as “holly grail” of quantum gravity. (The expression “holly grail” fits nice with my expression “myth”.)

10.3 Other approaches to black-hole entropy

Some results in quantum gravity already suggest a microscopic explanation of the proportionality of the black-hole entropy with its surface. For example, a loop representation of quantum-gravity kinematics (for reviews, see, e.g., [95, 96]) leads to a finite value of the entropy of a surface, which coincides with (122) if one additional free parameter of the theory is adjusted appropriately. However, loop quantum gravity does not provide a new answer to the question why the black-hole entropy should coincide with the entropy of its boundary. Instead, it uses a classical argument for this, based on the observation that the degrees of freedom behind the horizon are invisible to outside observers, so that only the boundary of the black hole is relevant to physics observed by outside observers. (The book [96] contains a nice pedagogic presentation of this classical argument. Besides, it contains an excellent pedagogic presentation of the relational interpretation of general relativity, which, in particular, may serve as a motivation for the conceptually much more dubious relational interpretation of QM [25, 26] mentioned in Sec. 5.) Such an explanation of the black-hole entropy is not what is really searched for, as it does not completely support the four laws of black-hole “thermodynamics”, since the other extensive quantities such as mass $M$ and charge $Q$ contain information about the matter content of the interior. What one wants to obtain is that the entropy of the interior degrees of freedom is proportional to the boundary of the interior.

A theory that is closer to achieving this goal is string theory, which, among other things, also contains a quantum theory of gravity. Strings are one-dimensional objects containing an infinite number of degrees of freedom. However, not all degrees of freedom need to be excited. In low-energy states of strings, only a few degrees of freedom are excited, which corresponds to states that we perceive as standard particles. However, if the black-hole interior consists of one or a few self-gravitating strings in highly excited states, then the entropy associated with the microscopic string degrees of freedom is of the
order of $GM^2$ (for reviews, see [82, 97]). This coincides with the semiclassical black-hole “entropy”, as the latter is also of the order of $GM^2$, which can be seen from (122) and (120). The problem is that strings do not necessarily need to be in highly excited states, so the entropy of strings does not need to be of the order of $GM^2$. Indeed, the black-hole interior may certainly contain a huge number of standard particles, which corresponds to a huge number of strings in low-excited states. It is not clear why the entropy should be proportional to the black-hole surface even then.

A possible reinterpretation of the relation (122) is that it does not necessarily denote the actual value of the black-hole entropy, but only the upper limit of it. This idea evolved into a modern paradigm called holographic principle (see [98] for a review), according to which the boundary of a region of space contains a lot of information about the region itself. However, a clear general physical explanation of the conjectured holographic principle, or that of the conjectured upper limit on the entropy in a region, is still missing.

Finally, let me mention that the famous black-hole entropy paradox that seems to suggest the destruction of entropy owing to the black-hole radiation (for pedagogic reviews, see [99, 100]) is much easier to solve when $S_{bh}$ is not interpreted as true entropy [101]. Nevertheless, I will not further discuss it here as this paper is not about quantum paradoxes (see, e.g., [16]), but about quantum myths.

11 Discussion and conclusion

As we have seen, QM is full of “myths”, that is, claims that are often presented as definite facts, despite the fact that the existing evidence supporting these claims is not sufficient to proclaim them as true facts. To show that they are not true facts, I have also discussed the drawbacks of this evidence, as well as some alternatives. In the paper, I have certainly not mentioned all myths existing in QM, but I hope that I have caught the most famous and most fundamental ones, appearing in several fundamental branches of physics ranging from nonrelativistic quantum mechanics of single particles to quantum gravity and string theory.

The question that I attempt to answer now is – why the myths in QM are so numerous? Of course, one of the reasons is certainly the fact that we still do not completely understand QM at the most fundamental level. However, this fact by itself does not explain why quantum physicists (who are supposed to be exact scientists) are so tolerant and sloppy about arguments that are not really the proofs, thus allowing the myths to form. To find a deeper reason, let me first note that the results collected and reviewed in this paper show that the source of disagreement among physicists on the validity of various myths is not of mathematical origin, but of conceptual one. However, in classical mechanics, which is well-understood not only on the mathematical, but also on the conceptual level, similar disagreement among physicists almost never occur. Thus, the common origin of myths in QM must lie in the fundamental conceptual difference between classical and quantum mechanics. But, in my opinion, the main conceptual difference between classical and quantum mechanics that makes the latter less understood on the conceptual level is the fact that the former introduces a clear notion of objective reality even without measurements. (This is why I referred to the myth of Sec. 5 as the central myth in QM.) Thus, I conclude that the main reason for the existence of myths in QM is the fact that
QM does not give a clear answer to the question what, if anything, objective reality is.

To support the conclusion above, let me illustrate it by a simple model of objective reality. Such a model may seem to be naive and unrealistic, or may be open to further refinements, but here its only purpose is to demonstrate how a model with explicit objective reality immediately gives clear unambiguous answers to the questions whether the myths discussed in this paper are true or not. The simple model of objective reality I discuss is a Bohmian-particle interpretation, according to which particles are objectively existing pointlike objects having deterministic trajectories guided by (also objectively existing) wave functions. To make the notion of particles and their “instantaneous” interactions at a distance unique, I assume that there is a single preferred system of relativistic coordinates, roughly coinciding with the global system of coordinates with respect to which the cosmic microwave background is homogeneous and isotropic. Now let me briefly consider the basic claims of the titles of all sections of the paper. Is there a wave-particle duality? Yes, because both particles and wave functions objectively exist. Is there a time-energy uncertainty relation? No, at least not at the fundamental level, because the theory is deterministic. Is nature fundamentally random? No, in the sense that both waves and particle trajectories satisfy deterministic equations. Is there reality besides the measured reality? Yes, by the central assumption of the model. Is QM local or nonlocal? It is nonlocal, as it is a hidden-variable theory consistent with standard statistical predictions of QM. Is there a well-defined relativistic QM? Yes, because, by assumption, relativistic particle trajectories are well defined with the aid of a preferred system of coordinates. Does quantum field theory (QFT) solve the problems of relativistic QM? No, because particles are not less fundamental than fields. Is QFT a theory of particles? Yes, because, by assumption, particles are fundamental objects. (If the current version of QFT is not completely compatible with the fundamental notion of particles, then it is QFT that needs to be modified.) Is black-hole entropy proportional to its surface? To obtain a definite answer to this last question, I have to further specify my model of objective reality. For simplicity, I assume that gravity is not quantized (currently known facts do not actually exclude this possibility), but determined by a classical-like equation that, at least at sufficiently large distances, has the form of a classical Einstein equation in which “matter” is determined by the actual particle positions and velocities. (For more details of such a model, see [102].) In such a model, the four laws of black-hole “thermodynamics” are a direct consequence of the Einstein equation, and there is nothing to be explained about that. The quantity $S_{bh}$ is only analogous to entropy, so the answer to the last question is – no. Whatever (if anything) the true quantum mechanism of objective particle creation near the black-hole horizon might be (owing to the existence of a preferred time, the mechanism based on the Bogoliubov transformation seems viable), the classical properties of gravity near the horizon imply that the distribution of particle energies will be thermal far from the horizon, which also does not require an additional explanation and is not directly related to the four laws of black-hole thermodynamics [92].

Of course, with a different model of objective reality, the answers to some of the questions above may be different. But the point is that the answers are immediate and obvious. With a clear notion of objective reality, there is not much room for myths and speculations. It does not prove that objective reality exists, but suggests that this is a possibility that should be considered more seriously.

To conclude, the claim that the fundamental principles of quantum theory are today
completely understood, so that it only remains to apply these principles to various practical physical problems – is also a myth. Instead, quantum theory is a theory which is not yet completely understood at the most fundamental level and is open to further fundamental research. Through this paper, I have demonstrated this by discussing various fundamental myths in QM for which a true proof does not yet really exist. I have also demonstrated that all these myths are, in one way or another, related to the central myth in QM according to which objective unmeasured reality does not exist. I hope that this review will contribute to a better general conceptual understanding of quantum theory and make readers more cautious and critical before accepting various claims on QM as definite facts.

Acknowledgments

As this work comprises the foundational background for a large part of my own scientific research in several seemingly different branches of theoretical physics, it is impossible to name all my colleagues specialized in different branches of physics that indirectly influenced this work through numerous discussions and objections that, in particular, helped me become more open minded by understanding how the known physical facts can be viewed and interpreted in many different inequivalent ways, without contradicting the facts themselves. Therefore, I name only J. M. Karimäki who suggested concrete improvements of this paper itself. I am also grateful to the anonymous referees whose constructive critical objections stimulated further improvements and clarifications in the paper. This work was supported by the Ministry of Science and Technology of the Republic of Croatia.

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