Simulation of the neutron-tagged deep inelastic scattering at EicC

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Measuring the pionic structure function is of high interests as it provides a new area for understanding the strong interaction among quarks and to test the QCD predictions. To this purpose, we investigate the feasibility and the expected impacts of a possible measurement on EicC. We show the simulation result on the statistical precision of an EicC experiment, based on the model of leading neutron tagged DIS process and the dynamical parton distribution functions of pion. The simulation shows that at EicC, the kinematic covers a $x_{\pi}$ range from 0.01 to 1, and a $Q^2$ range from 1 GeV$^2$ to 50 GeV$^2$, within the acceptable statistical uncertainties. Assuming an integrated luminosity of 50 fb$^{-1}$, in the low $Q^2$ region ($< 10$ GeV$^2$), the MC data show that the measurements in the whole $x_{\pi}$ range reach very high precisions ($<3\%$). To perform such an experiment, only the addition of a far-forward neutron calorimeter is needed.

I. INTRODUCTION

Pion, the lightest hadron made of the first-generation quark and antiquark, plays a fundamental role in particle and nuclear physics, as the long-range nuclear force carrier which binds the nucleons together into a nucleus. In theory, it is a good approximation of the Nambu-Goldstone boson from the spontaneous chiral symmetry breaking, however the generation of its small mass (much less than that of the proton) is not yet understood quantitatively and experimentally. Recent progresses from Dyson-Schwinger (DS) equations, which is a nonperturbative quantum chromodynamics (QCD) approach, shows that the dressed quark mass which comes from the dynamical chiral symmetry breaking is greatly cancelled by the attraction interaction between the quark-antiquark pair. Understanding the properties of the simplest hadron from its structure is a remarkable advancement in the field of strong interaction.

With the emergence of the pion mass, DS equations predict a broadening parton distribution amplitude (PDA) compared to the asymptotic form of PDA by the perturbative QCD theory. The width of the pion quark distribution also becomes wider at the hadronic scale $Q_0^2$ (a very low scale where sea quarks and gluons disappear). Using a renormalisation-group-invariant process-independent strong coupling, the valence quark distributions at $Q_0^2$ is connected to the extracted parton distribution functions (PDF) at high $Q^2$ in experiment, and the predicted valence quark distribution from the dynamical chiral symmetry breaking has the similar large-$x$ ($x \to 1$) behavior of the perturbation QCD predictions. To measure the pion structure in the full range of $x$ and a broad range of $Q^2$ provides a promising window to test the dynamical chiral symmetry breaking, which is one of the prominent features of QCD theory, and the related emergence phenomenon from the strong interaction.

Experimentally, the collinear parton distributions (one-dimensional structure) of nucleon are measured very precisely with the helps of the high energy accelerator facilities worldwide, however we have far less experimental data on the pionic structure. Measuring the pionic structure is not easy, since there is no pion target in experiment as it decay quickly. All the data on pion valence quark distributions are measured in the Drell-Yan channel induced by the pion beam, more than thirty years ago. The measurement at small and intermediate $x$ and more data points at different $Q^2$ are needed. Recently, by exploiting the “pion cloud” around the proton beam, the pion structure functions at small $x_{\pi} \lesssim 0.01$ are analyzed from the leading neutron (LN) tagged deep inelastic scattering (DIS) data at HERA collider. Therefore, to fill the data gap in the range $0.01 \lesssim x_{\pi} \lesssim 0.2$ is of the highest interest on the experimental side. Moreover, measuring the pion valence quark distributions at large $x$ using the LN-DIS technique, and comparing it with that from the Drell-Yan process will reinforce our understanding of the perturbative QCD theory on the dynamics when $x_{\pi}$ approaching one. Last but not least, the experimental data from sea quark region to valence quark region will definitely provide an opportunity to differentiate various theoretical approaches, such as DS equations, lattice QCD, holographic QCD, light-front quantization, chiral quark model, constituent quark model, QCD sum rule, and the dynamical parton model predictions with a naive nonperturbative input.

Now, there has been some heating discussions on building a low energy electron-ion collider in China (EicC), by upgrading the under-construction high-intensity heavy
ion accelerator facility (HIAF) \[49, 50\]. By using the same method conducted at HERA, EicC with the center-of-mass (c.m.) energy about 20 GeV provides a competing opportunity to acquire the pionic structure function data in the range $0.01 \lesssim x_\pi \lesssim 1$. Hence in this work, we investigate the opportunity, the feasibility, and the expecting statistical errors for a suggesting LN-DIS experiment at EicC. We focus on the distributions of some invariant kinematics of interests, providing some guidances for detecting the final-state particles, and estimating the statistical precisions of the pionic structure functions that will be measured.

The organization of the paper is as follows. The formulæ of the leading neutron tagged DIS process to study the structure of pion are discussed in Sec. III. The common invariant kinematical distributions and kinematical distributions of the final-state particles are given in Sec. IV. The dynamical structure of pion are discussed in Sec. II. The common used virtuality of the photon probe $Q^2$, Bjorken variable $x_B$, and the inelasticity $y$ of DIS process are defined as,

$$ Q^2 = -q^2, \quad x_B = \frac{Q^2}{2P_p \cdot q}, \quad y = \frac{P_p \cdot q}{P_p \cdot P_e}. $$ \hspace{1cm} (1)

The other kinematical variables that is related to the final-state neutron are the longitudinal momentum fraction $x_L$ and square of the momentum transfer to the virtual pion $t$,

$$ x_L \equiv \frac{P_n \cdot q}{P_p \cdot q}, \quad t = (P_p - P_n)^2 = p_e^2. $$ \hspace{1cm} (2)

$x_L$ is the longitudinal momentum fraction of the final neutron to the incoming proton. In experiment, the LN-DIS process dominates in the large $x_L$ region ($\gtrsim 0.5$) \[33\], hence the cut on the $x_L$ variable is the efficient method to distinguish the events of interests that is sensitive to the pion structure. Viewing the virtual pion as the target of interests, similar to the definition of normal Bjorken variable, the momentum fraction of the parton inside pion is written as,

$$ x_\pi \equiv \frac{Q^2}{2p_\pi \cdot q} = \frac{x_B}{1 - x_L}. $$ \hspace{1cm} (3)

From above definition, we see that the smallest momentum fraction of the parton in pion measured in LN-DIS process is larger than the smallest momentum fraction of the parton in proton that can be measured in DIS.

To estimate the statistic of LN-DIS events and the distributions of kinematical variables at EicC, we need to calculate the differential cross-section of the channel. Integrated the azimuthal angles, the four-fold differential cross-section of LN-DIS process is expressed with the semi-inclusive structure function $F_2^{LN(4)}(Q^2,x_B,x_L,t)$ \[32, 33, 32\],

$$ \frac{d^4 \sigma(ep \rightarrow enX)}{dx_B dq^2 dx_L dt} = \frac{4 \pi \alpha^2}{x_B Q^4} \left(1 - y + \frac{y^2}{2}\right) F_2^{LN(4)}(Q^2,x_B,x_L,t) = \frac{4 \pi \alpha^2}{x_B Q^4} \left(1 - y + \frac{y^2}{2}\right) F_2\left(\frac{x_B}{1 - x_L},Q^2\right) f_{\pi^+/p}(x_L,t) $$ \hspace{1cm} (4)

In the above formula, the leading-neutron structure function $F_2^{LN(4)}$ is then factorized into the pionic structure function $F_2^{\pi}$ and the pion flux around the proton $f_{\pi^+/p}$.
The pion flux is usually evaluated with a pion pole in the effective field theory [32, 33, 52],

\[ f_{\pi^+}/p(x_L, t) = \frac{1}{2\pi} \frac{g^2_{\pi NN}}{4\pi} (1 - x_L) \frac{-t}{(m^2_{\pi} - t)} \exp \left( R_{\pi}^2 \frac{t - m^2_{\pi}}{1 - x_L} \right), \]  

where \( g^2_{\pi NN}/4\pi = 13.6 \) is the \( \pi NN \) effective coupling, and \( R_{\pi} = 0.93 \) GeV\(^{-1} \) is an adjustable parameter describing the radius of \( n-\pi \) Fock state [32]. By integrating over the \( t \) variable, the three-fold LN structure function is also used often,

\[ F_2^{LN(3)}(Q^2, x_B, x_L) = \int_{t_1}^{t_0} F_2^{LN(4)}(Q^2, x_B, x_L, t) dt. \]  

The theoretical framework for pion structure function measurement in e-p process is mature and has been test with the pioneering HERA facility. The shape of the structure function of pion are encoded in the LN structure function. Now, for a quantitative calculation of the cross-section, we only need to seek a reasonable structure function model of pion in a wide kinematical range of \( x_\pi \) and \( Q^2 \).

### III. DYNAMICAL PARTON DISTRIBUTIONS OF PION

In this simulation, we use the dynamical parton distributions of the pion from a recent study. The pion PDF used in this work is called piIMParton [15, 54]. The magnificent feature of the dynamical parton model is that the nonperturbative input consists of only the valence distributions at extremely low \( Q^2 \) scale. The low \( Q^2 \) scale is estimated to be around 0.1 GeV\(^2 \), which is also called the hadronic scale, since the scale is at where only the minimum components (valence) of the hadron can be resolved. In this dynamical parton model, all the sea quarks and gluons are produced from the parton splitting processes governed by the DGLAP equations [55].

Fig. 2 shows the valence quark distribution of pion predicted by the dynamical parton model, compared with the valence quark distribution extracted from the p-nucleus Drell-Yan data by E615 Collaboration [31]. The excellent agreement in the range \( 0.2 < x_\pi < 1 \) is found. The experimental data in the region \( x_\pi < 0.6 \) exhibit big uncertainties, so to have more data and to reduce the uncertainties are the goals of the future experiments. Though there may be some model uncertainties, a few data on pion structure function is obtained from the H1 experiment at HERA [32, 33]. Fig. 3 shows the predictions from the dynamical parton model of pion compared with the H1 data. Note that, for the calculation of the structure function \( F_2 \), only \( u, d \), and \( s \) quark contributions are included. At small \( x_\pi \), the dynamical parton model predictions for pionic structure are consistent with the current experimental measurements.

More experimental observables are calculated based on the pion dynamical PDF from piIMParton, in order to be carefully checked with the recent H1 experiment [33]. The calculated three-fold leading-neutron structure function \( F_2^{LN(3)} \) are shown in Fig. 4 compared with the H1 data. In the e-\( \pi^+ \) DIS region, i.e. \( x_\pi \geq 0.6 \), the model calculation is consistent with the measurement by H1. The experimental values in the small \( x_L \) region are much higher than the calculation is because there are huge contribution from the normal e-p DIS process in experiment. The differential cross-section as a function of \( x_L \) is also calculated and shown in Fig. 5 with the comparison to the H1 data. Similar to the \( F_2^{LN(3)} \) result, the model prediction agree well with the H1 data for DIS process with the leading neutron of a large longitudinal momentum fraction tagged.

The calculations based on the pion PDFs from the dynamical parton model (piIMParton PDFs) [15, 54] are acceptable to interpret the LN-DIS data at the very high energy where the pionic structure around the nucleon plays an important role. Meanwhile, in the large \( x_\pi \) region, the valence quark distributions of piIMParton consist amazingly with the Drell-Yan data. Convincingly, the cross-section model of LN-DIS and the pion PDFs used in this simulation are reliable to give some meaningful projections of a suggesting LN-DIS experiment at EicC.

### IV. DISTRIBUTIONS OF THE INVARIANT AND FINAL-STATE KINEMATICS

According to the models described in Sec. II and Sec. III, we develop a Monte-Carlo (MC) simulation package which can generate numerous events of LN-DIS process efficiently. In the simulation, the electron beam energy is 3.5 GeV and the proton beam energy is 20 GeV, which
is a typical collision energy for the future EicC \cite{49,50}. Inside the phase space, we apply the following kinematical ranges for the MC sampling: \( 0.01 \text{ GeV}^2 < -t < 1 \text{ GeV}^2 \), \( 0.5 < x_L < 1 \), \( x_{B,\text{min}} < x_B < 1 \), \( 1 \text{ GeV}^2 < Q^2 < 50 \text{ GeV}^2 \), and \( W^2 > 4 \text{ GeV}^2 \).

Fig. 5 shows distributions of the angular, energy, and pseudorapidity of the final-state electron and neutron. Note that, in the simulation the \( z \) direction is defined as the momentum of the incoming proton beam. All the scattered electrons go to the central region of the spectrometer (|\( \eta \)| < 3), and they are precisely and efficiently measured with the tracker and calorimeters \cite{50}. The final neutrons go to very small angles with the pseudorapidity around 5. They are suggested to be detected with the far-forward very small angle calorimeters, such as the zero-degree counter and the Roman pot inside beamline. Fig. 6 shows the cross-section weighted distributions of the invariant kinematics of interests. We see that most of the events distributed in the low \( Q^2 \), small \( x_\pi \), small \( y \) and small \( t \) region. The small \( x_\pi \) region is a unique region where EicC can provide the precise data filling the gap of the current data from the facilities decades ago. The broad \( x_\pi \) distribution from 0.01 to 1 and the high luminosity of EicC will provide a great opportunity to...

**FIG. 3.** The comparison between the prediction from piIMParton PDF for the pion (red solid curves) and the experimental data of the pionic structure function by H1 Collaboration (black squares). The pionic structure function extracted by H1 is from an analysis of the LN-DIS data in the kinematical region of \( x_L \) around 0.73 \cite{33}.

**FIG. 4.** The comparison between the predictions of the LN structure function from piIMParton PDFs \cite{51} and the H1 data \cite{33}, at \( Q^2 = 11 \text{ GeV}^2 \).

**FIG. 5.** The comparison between the predictions of one-fold differential cross-section from piIMParton PDFs \cite{51} and the H1 data \cite{33}, integrated in the kinematical range of \( 6 < Q^2 < 100 \text{ GeV}^2 \), \( 1.5 \times 10^{-4} < x_B < 3 \times 10^{-2} \), and \( P_T^{e-n} < 0.2 \text{ GeV}/c. \) In the LN-DIS region (eg. \( x_\pi > 0.75 \)), the cross-section can be explained using the \( e-\pi \) DIS formula combined with the dynamical parton distribution functions of pion.
cross check the large-\(x\) behavior of pion parton distribution when \(x \rightarrow 1\).

V. STATISTICAL ERROR PROJECTIONS OF PIONIC STRUCTURE FUNCTIONS

To give a statistical error estimation of the observable, we assume the integrated luminosity of an EicC experiment to be 50 fb\(^{-1}\), which corresponds to a run of one to two years. To study the pionic structure function, we have performed the following selections: \(x_L > 0.75\), \(P_T^\pi < 0.5\) GeV, \(M_X = (p_e + p_p - p_\pi^\prime - P_n)^2 > 0.5\) GeV, \(W > 2\) GeV, \(x_L > 0.75\) and \(P_T^\pi < 0.5\) GeV makes sure the final neutron is from the Fock state dissociation, which has a large fraction of the longitudinal momentum of the incoming proton and a small transverse momentum, being a spectator in the e-\(\pi\) DIS process. \(M_X > 0.5\) GeV requirement is to get rid of the e-\(\pi\) elastic scattering process, and make sure the struck pion is broken up so as to study the partons inside the pion. \(W > 2\) GeV is the common DIS criterion.

With the above event selections, the LN-DIS events then are divided into different kinematical bins. Fig. 8 shows the binning scheme of \(x_\pi\) and \(-t\), for the low \(Q^2\) (\(\sim 4\) GeV\(^2\)) MC data. The number of events in each bin is calculated with the following formula,

\[
N_i = \epsilon L \sigma_i \Delta x_\pi \Delta Q^2 \Delta x_L \Delta t (1 - x_L),
\]

in which \(\epsilon\) is the detector efficiency, \(L\) is the integrated
The statistical errors are all less than 3\%, starting from 2 GeV
instead of the x/
tical error is estimated to be 1
of events in each bin simulated, then the relative statis-
tions of a conceptual design in the far-forward region,
The right axis shows how large the statistical error is.

FIG. 11. The statistic error projections of the pionic structure
Q
function at Q
0.6
0.4
0.2
−1 (GeV
0.2
0.4
0.6
0.8 1 π
x0
0.2
0.4
0.6
) 2 -t (GeV
X, M2 < 50 GeV2 < Q220 GeV < 0.5 GeV
nT
 > 0.75, PLx-1
EicC 50 fb
−0
20
 (%)
π 2
Statistical error of F

VI. DISCUSSIONS AND SUMMARY

The comparison of a dynamical parton model predic-
tions [48, 54] with the measurement by H1 [33] shows
that the LN-DIS process can be used to study the pion
structure. The LN-DIS process can be understood as the
scattering between the electron and the abundant virtual
pions around the proton at small momentum transfer t.
The pion structure measurement on an electron-ion col-
lider is feasible.

Following the pioneering works of H1 and ZEUS, we
simulate a LN-DIS experiment at EicC to investigate the
pionic structure function in a wide kinematical range.
The simulation implies that EicC machine can provide a
precise measurement of the pionic structure of x
from 0.01 to 0.95, and of Q
from 1 GeV
50 GeV
. Since
neutron is not charged, the very forward neutron can be
separated from the proton beam with a dipole magnet.
Hence measuring the neutrons at very small angles is
not difficult, as long as the space of the tunnel for the
accelerator is long enough to install a neutron calorim-
eter. In the simulation, we choose a conservative neutron
efficiency of 50\% to model the performance of the neu-
tron detector. The low energy EicC of high luminosity
gives us an excellent opportunity to see precisely the one-
dimensional structure of other hadron beyond the proton.

The precise measurement of the pionic structure func-
tions in a broad kinematical domain definitely will flour-
ish our understanding of the strong interaction, to dif-
fentiate the various pictures on hadron structure. The
systematic study of the LN-DIS channel and the precise
extraction of the pionic structure from sea quark region
to valence quark region at EicC will be a critical input
for the database of meson structure. The LN-DIS exper-
iment at EicC has a great potential to reveal pion parton
distributions with a lot of details, leading to a better
understanding of many nonperturbative approaches, the
dynamical symmetry breaking, and why the pion mass is
so small compared to the proton mass.
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