Fitting Atmosphere Temperature of 20th Century in 49 European Capitals using Random Walk Model

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Abstract. The 20th century has been witnessed with the rise of global temperature due to CO₂ emission since the industrial revolution. The global warming is observed around the world, and the governments take measures to reduce CO₂ emission to combat the rise of temperature. The temperature profile has been mainly modelled at global level. Yet, it necessarily models the temperature in cities because each city would have its own feature. Among various models to fit or simulate the temperature, the application of random walk model is rare. In this study, we used the random walk model to fit the temperature of the 21st century in 49 European capitals not only because the industrial revolution first appeared in Europe but also European governments commit and endorse the Paris Agreement. First, the yearly temperature was simplified as increase (+1) or decrease (–1) in comparison with previous year, whose addition in the 20th century in each of 49 European capitals construct a step-typed curve. Second, random walk model was used to fit these step-typed curves with the least squared method. Third, the widely used CRUTEM3 temperature in the 20th century in each of 49 European capitals was directly fitted using random walk. The results show that random walk can fit the step-typed curves and CRUTEM3 temperature.

1. Introduction

The temperature in the 20th century has the clear tendency, i.e. the rise of temperature over time. However, the careful observation reveals that this tendency is not monotonic, but with fluctuation. The fluctuation in temperature can be observed on daily, monthly and yearly bases, that is, it turns out that the temperature in any given year can be higher than that in the previous year, but can also be lower than that in the previous year.

The fluctuation suggests some type of random effects on the temperature, for example, human errors. However, it could also suggest more complicated random effects. Indeed, the similarity between global temperature and the curve generated by random walk was noticed decades ago [1].

Actually, it is very reasonable to expend this type of study to cities because the global temperature is slightly biased on land because there are more land-based observations than sea-based observations. Also, most industrial sectors are located in cities, whose temperature is usually higher than their surroundings.

Of cities around the world, the cities in Europe are particularly important not only because the industrial revolution began in Europe, but also because the governments in Europe commit and endorse
the Paris Agreement. Still, most European cities are located not far from sea, therefore it can avoid land-based and sea-based observation bias to some degree.

In this study, we used the random walk model to fit/simulate the atmosphere temperature in 49 European capitals.

2. Materials and Methods

2.1. Data
Forty-nine European capitals are chosen according to Wikipedia with their latitudes and longitudes [2], which are listed in the first and second columns of Table 1. The temperatures recorded in these 49 European capital cities from 1901 to 1998 based on 0.5° by 0.5° latitude and longitude grid-box basis cross globe are obtained from the website of Oak Ridge National Laboratory [3].

2.2. Simplified Temperature
Temperature is a record over time, so this record is the one-dimensional random walk, which starts at zero and at each step moves by ±1 with equal probability [4]. Therefore, one-dimensional random walk is a sequential result of tossing a fair coin, by which the head is recorded as 1 and the tail is recorded as −1, and their addition along the time course is random walk. Temperature can be simplified in similar way, i.e. the temperature higher than its previous recording time is simplified as +1 whereas the temperature lower than its previous recording time is simplified as −1, and their addition along the time course constructs a curve similar to random walk. The third and fourth columns in Table 2 show the process to convert temperature into the simplified temperature.

2.3. Generation of Random Walk
SigmaPlot [5] was used to generate random sequence with different seeds. Technically, the generation of random walk is quite simple: random numbers ranged from −1 to 1 were generated, and they were classified a random value as +1 if it is larger than its previous one and as −1 if it is smaller than its previous one (the 6th column in Table 2). Thereafter, these values were added together along the time course as random walk (the 7th column in Table 2).

2.4. Searching for Seed
To find a random walk that is very approximate to the simplified temperature is to find a seed that can generate such a random walk. To the best of authors’ knowledge, there is no algorism for searching the seed, which generates the best fit for the observed dataset, therefore the so-called fitting, which traditionally searches the optimum according to various algorisms, becomes to search all possible seeds in order to find out the seed that produces the random walk with the least squares between random walk and simplified temperature.

2.5. Extension of Random Walk
Because the random walk comes from tossing of double-side coin, accordingly this concept can be broadened into tossing of dice, which should not limited to six-faced dice, but can be as many faces as to accommodate the temperature changes along the time course. Thereafter, this multi-faced dice can be tossed to generate a random walk to compare with temperature.

2.6. Comparison
Least squares between simplified temperature and random walk, and between temperature and random walk were used to evaluate fitting.
Table.1  Conversion of temperature into simplified temperature and generation of random walk for temperature of the 20th century in Tirana (The random number is generated by SigmaPlot with the seed of 2.49125).

| Year | Recorded Temperature | Simplified Temperature | Addition of Simplified Temperature | Generated Random Number | Random Number Step | Random Walk |
|------|----------------------|------------------------|------------------------------------|-------------------------|--------------------|-------------|
| 1901 | 14.242               | 0                      | 0.601                              | 0                       | 0                  | 0           |
| 1902 | 14.550               | 1                      | 0.067                              | 1                       | 1                  | 1           |
| 1903 | 14.667               | 2                      | 0.314                              | 1                       | –1                 | 0           |
| 1904 | 14.633               | –1                     | 0.810                              | 1                       | 1                  | 1           |
| 1905 | 14.250               | –1                     | 0.690                              | 1                       | 1                  | 1           |
| 1906 | 14.225               | –1                     | –0.826                             | –1                      | –1                 | –1          |
| 1907 | 14.233               | 0                      | 0.424                              | 1                       | 0                  | 0           |
| 1908 | 14.183               | –1                     | –0.997                             | –1                      | –1                 | –1          |
| 1909 | 14.608               | 0                      | 0.983                              | 1                       | 0                  | 0           |
| …    | …                    | …                      | …                                  | …                       | …                  | …           |
| 1991 | 13.783               | –1                     | –0.301                             | –1                      | –6                 | –6          |
| 1992 | 14.642               | –4                     | 0.546                              | 1                       | –5                 | –5          |
| 1993 | 14.358               | –5                     | 0.622                              | 1                       | –4                 | –4          |
| 1994 | 15.458               | –4                     | –0.728                             | –1                      | –5                 | –5          |
| 1995 | 14.408               | –5                     | 0.808                              | 1                       | –4                 | –4          |
| 1996 | 14.117               | –6                     | –0.494                             | –1                      | –5                 | –5          |
| 1997 | 14.442               | –5                     | 0.944                              | 1                       | –4                 | –4          |
| 1998 | 14.992               | –4                     | –0.209                             | –1                      | –5                 | –5          |

Table.2  Seeds for random walk and fitted results for fitting of temperature of the 20th century in 49 European capitals.

| Country         | Capital Fitting of Simplified Temperature | Fitting of Temperature |
|-----------------|------------------------------------------|------------------------|
| Albania         | 2.49125                                  | 0.55555                | 27.76106               |
| Andorra         | 1.61302                                  | 1.51786                | 38.91886               |
| Armenia         | 4.46721                                  | 0.57023                | 106.15862              |
| Austria         | 1.35958                                  | 2.48474                | 62.35353               |
| Azerbaijan      | 3.74341                                  | 4.04701                | 57.01943               |
| Belarus         | 5.73442                                  | 1.65197                | 107.08036              |
| Belgium         | 2.23930                                  | 0.76371                | 48.08131               |
| Bosnia & Herzegovina | 1.20596                              | 0.55555                | 50.46580               |
| Bulgaria        | 4.13267                                  | 2.48474                | 43.00761               |
| Croatia         | 6.69004                                  | 2.23985                | 51.44529               |
| Cyprus          | 1.37166                                  | 2.01257                | 31.81375               |
| Country            | Capital   | Latitude  | Longitude | Altitude  | Population |
|--------------------|-----------|-----------|-----------|-----------|------------|
| Czech              | Prague    | 50.0754   | 14.4068   | 634.00     | 1.2827     |
| Denmark            | Copenhagen| 55.6758   | 12.5167   | 32.00      | 0.8713     |
| Estonia            | Tallinn   | 59.3883   | 24.8962   | 1.00       | 1.00       |
| Finland            | Helsinki  | 60.1667   | 24.9333   | 0.50       | 0.50       |
| France             | Paris     | 48.8571   | 2.3522    | 0.25       | 0.25       |
| Georgia            | Tbilisi   | 41.4967   | 44.7833   | 0.50       | 0.50       |
| Germany            | Berlin    | 52.5278   | 13.4333   | 1.00       | 0.8713     |
| Greece             | Athens    | 37.9667   | 23.7333   | 0.50       | 0.50       |
| Hungary            | Budapest  | 47.4967   | 16.9833   | 1.00       | 1.00       |
| Iceland            | Reykjavik | 64.1667   | -21.8333  | 0.50       | 0.50       |
| Ireland            | Dublin    | 53.3833   | -16.8333  | 1.00       | 1.00       |
| Italy              | Rome      | 41.9067   | 12.4967   | 0.25       | 0.25       |
| Latvia             | Riga      | 54.3833   | 23.5833   | 0.50       | 0.50       |
| Liechtenstein      | Vaduz     | 47.3833   | 11.5833   | 1.00       | 1.00       |
| Lithuania          | Vilnius   | 54.6278   | 25.2333   | 0.50       | 0.50       |
| Luxembourg         | Luxembourg| 49.9667   | 10.8333   | 1.00       | 1.00       |
| North Macedonia    | Skopje    | 41.8333   | 20.2333   | 0.50       | 0.50       |
| Malta              | Valletta  | 39.3833   | 14.4967   | 0.25       | 0.25       |
| Moldova            | Chișinău  | 47.1667   | 29.4967   | 0.50       | 0.50       |
| Monaco             | Monaco    | 43.7333   | 7.4967    | 1.00       | 1.00       |
| Montenegro         | Podgorica | 46.4967   | 18.2333   | 1.00       | 1.00       |
| Netherlands        | Amsterdam | 52.3833   | 4.9667    | 1.00       | 1.00       |
| Norway             | Oslo      | 59.9667   | 10.7333   | 1.00       | 1.00       |
| Poland             | Warsaw    | 52.3833   | 20.9667   | 0.50       | 0.50       |
| Portugal           | Lisbon    | 38.5833   | 8.9667    | 0.25       | 0.25       |
| Romania            | Bucharest | 44.5833   | 26.5833   | 1.00       | 1.00       |
| Russia             | Moscow    | 55.7667   | 37.6333   | 1.00       | 1.00       |
| San Marino         | San Marino| 43.0833   | 12.3333   | 0.50       | 0.50       |
| Serbia             | Belgrade  | 44.1667   | 20.9667   | 1.00       | 1.00       |
| Slovakia           | Bratislava| 48.5833   | 17.4667   | 0.50       | 0.50       |
| Slovenia           | Ljubljana | 45.8333   | 14.5833   | 0.25       | 0.25       |
| Spain              | Madrid    | 40.4667   | 3.4667    | 1.00       | 1.00       |
| Sweden             | Stockholm | 59.3833   | 18.0833   | 1.00       | 1.00       |
| Switzerland        | Bern      | 47.3833   | 8.5833    | 0.50       | 0.50       |
| Turkey             | Ankara    | 39.9667   | 32.8333   | 0.25       | 0.25       |
| Ukraine            | Kiev      | 50.4667   | 30.8333   | 1.00       | 1.00       |
| United Kingdom     | London    | 51.5833   | 0.1667    | 1.00       | 1.00       |
| Vatican City       | Vatican City | 41.9067   | 12.4967   | 0.25       | 0.25       |
3. Results and discussion
Due to the limitation of space, we only show the fittings in five European capitals instead of in all 49 capitals. In fact, all fittings can be made using the recorded temperatures in each capital with the seeds...
shown in Table 2. But the fittings in terms of simplified temperature can be done with conversion of recorded temperature into simplified temperature according to Table 1 meanwhile the fitted curves can be found using the seed in Table 1.

Technically, the seed of best fitting was found in searching of one million of seeds ranged from 0 to 10, therefore the computation is quite intensive for fittings in 49 European capitals. In fact, several seeds can generate the same result, which shows the Monte-Carlo random seed generation mechanism quite efficient. As the matter of fact, the chance for the perfect fitting in terms of simplified temperature should be extremely small, that is, \((1/2)^{100}\) because each step in simplified temperature is either up or down, just like tossing a coin, whose face appears randomly.

Figure 1 shows an example of the fittings of simplified temperature with random walk (left-hand side column) and the recorded temperature (right-hand side column) in 5 European capitals. On the left-hand side of Figure 1, the simplified temperature is the conversion from the temperature in terms of steps either up or down, which would be arguably the trend for temperature change over years. Although the simplified temperature is the conversion from temperature, it can indeed be fitted by a random walk no matter of whether the simplified temperature is going up or down because these fittings were done in 49 cities without any exception (the 3rd and 4th columns in Table 1).

On the right-hand side of Figure 1, the random walk was used to fit the recorded temperature. As can be seen, the fittings by random walk go along the general trend of recorded temperature. The fitted results are listed in the 5th and 6th columns of Table 1.

These five European capitals spread almost all European Continent, so they could be treated as representatives of Europe although the detailed fittings in the rest of 44 European cities are available from the authors.

When examining and comparing the simplified temperature (left-hand side) with the recorded temperature (right-hand side), it is clear that trends are different. For example, the recorded temperature in Tbilisi does not reveal an uptrend but its simplified temperature has such an uptrend. This means that Tbilisi has more years increase in temperature but their magnitudes are small, whereas their magnitudes in decrease in temperature are relatively large. Nevertheless, the altitude is high, so the surface of earth may influence its atmosphere temperature less. By contrast, the temperature trends in Berlin and Budapest are clearer in both sides of Figure 1, which is understandable because they have an European continental weather, whereas Athens has a Mediterranean weather.

In conclusion, this study demonstrates that the random walk model can fit the temperature changes in all 49 European capitals. Together with the previous study [6], the authors demonstrate that this model can fit the temperature changes at both global and city levels.

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