Fractal analysis in rural domestic wastewater quality under dissolved oxygen stability in wetlands planted with *Chrysopogon zizanioides*

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**Abstract.** *Chrysopogon zizanioides* is a plant with adaptability characteristics in a variety of soils and climatic conditions, improves quality of wastewater and industrial water, due to the root system, allows the consumption of organic matter and capture pollutants. One of the water quality indicators is the amount of dissolved oxygen present. To analyze the recovery behavior of dissolved oxygen in wastewater, an artificial subsurface flow wetland was designed with stone as a filter material with dimensions 0.9 m wide, 2.5 m long and 0.60 m deep and 30 Vetiver plants. These dimensions and quantity of plants were considered appropriate in terms of space and low cost in order to replicate in rural and semi-urban areas, because in some of these they do not have an appropriate aqueduct, then this strategy becomes an option for people residing in these places. The water treated in this wetland is domestic wastewater from a rural house in the municipality of Floridablanca, Colombia. For 4 weeks the data of the dissolved oxygen present in said water was recorded using a multiparameter probe, to observe how effective the process is in a shorter time and using fractal dimension the volatility of this time series was observed.

1. Introduction

Water is one of the most important natural resources for the survival of any species. Due to domestic and industrial use, this resource has been affected in such a way that the recovery of the same, in the cycle of use, is very slow and sometimes too expensive, for example, when there is interaction with
heavy metals, with crude oil by some type of spill, with the final disposition of some electronic devices, with mining among others. Generating alternatives to recover this important resource is one of the main objectives that as human beings we should impose. In Colombia, mining practices, final disposal of waste and other activities that pollute our water are developed. The oxygen level in a body of water is determined by the oxidation of organic and inorganic waste discharges and interactions with aquatic plants. The effects on aquatic life due to a certain concentration of dissolved oxygen is a function of each species, the duration of this concentration and its state of maturity [1]. Several studies have been conducted in order to evaluate the concentrations of heavy metals using various types of plants such as Colocasia esculenta, Heliconia, Psittacorum y Gynerium sagittatum planted in horizontal flow surface wetlands for the treatment of landfill leachates, in which it has been found that these plants have potential for the phytoremediation of leachates and also behave as heavy metal accumulators [2].

The biological treatment of leachates is viable when the content of biodegradable organic material is high; however, high concentrations of nitrogen and other refractory compounds may be inhibitory or toxic. A management strategy has been its incorporation into anaerobic domestic wastewater treatment (ARD), due to its simplicity and lower operating costs. The scale evaluation of the starting of a Upflow Anaerobic Sludge Blanket reactor (UASB) to treat ARD Reactor 1 (R1), compared with two reactors with pH conditioning, to treat mixtures of leached with ARD in volume proportions of 5% Reactor 2(R2) and 10% Reactor 3 (R3) leached, showed that with a hydraulic retention time (HRT) of eight hours, R1 and R2 reached average efficiencies of 66% reduction and 73% of chemical oxygen demand (COD) and 90% and 84% of total suspended solids (TSS), respectively. In R3 the optimal HRT was higher, and the efficiencies of reduction of COD and SST were lower. The results confirm the feasibility of the joint treatment of ARD and leached in proportions up to 10%; nevertheless, the increase in the leachate fraction up to 10% affected the reactor performance in a greater proportion by only treating ARD [3]. Approximately 80% of the wastewater generated in developing countries is discharged without treatment because of the lack of infrastructure capacity [4,5], for which many systems have been proposed for the decontamination of domestic, industrial and leachate wastewater. Currently it is expected to treat the wastewater and derive an economic-environmental benefit. It is in this context that high-rate algal systems were initially proposed for the treatment of domestic wastewater. However, research conducted around this technology has shown the advantages and benefits that were not initially contemplated. Currently there are implementations in the treatment of wastewater, seeking the production of biofuels, algal biomass for food (animal and human), obtaining pharmaceutical products and the bioprospecting of algae [6]. With the implementation of Vetiver in residential wastewater, it is expected to provide a natural economic strategy for the recovery of dissolved oxygen in these liquids, so that when they are discharged into the open field, they do not generate a negative environmental impact. The main objective of this paper is to socialize the findings given with the use of this plant for the purpose of phytoremediation and water recovery in sectors that do not have an aqueduct with the minimum conditions that guarantee an optimal use of the resource. The fractal dimension and volatility are coefficients that provide a direct relationship with the time series of data recorded in various phenomena, with the behavior that this phenomenon may have in the future. Due to the self-similarity of natural time series such as the recovery of dissolved oxygen, the fractal dimension becomes a suitable tool for the treatment of collected data.

2. Mathematical method and exploratory method
The Chrysopogon zizanioides (Vetiver) it has been used as a soil and sludge improver [7] in different places, within its characteristics is to be a stabilizer in hillside areas to avoid the drag of nutrients by runoff and thus maintain food to the crops [8]. Vetiver is a very versatile plant, adapts easily to different types of soil and climates [9], supports land with a high degree of contaminants and even heavy metals, for this reason and since it is not an invasive plant, it is a very good alternative in the development of environmental projects. The natural environment of Vetiver is the soil, but previous research has proven that the plant adapts and develops very well in the water, also being classified as a
macrophyte; this has led to the Vetiver being used as well as sludge and soil improver, in the decontamination of wastewater, industrial water and leachates [10-14]. The Vetiver can be placed over the water mirror as floating wetlands or planted in treatment ponds on a support medium such as stone or gravel. In our investigation, 30 Vetiver plants were placed in a tank with a volume of 1.3 m$^3$ to which residual water arrives after passing through a septic tank with two chambers; in the first chamber the solids are retained, as the water passes from the first chamber to the second chamber through a bypass; the water from the second chamber is piped to a storage tank and from this tank the water is conducted to three artificial wetlands with stone as support and / or filtering material and it is on this support material that the Vetiver plants have been planted , the filtering material allowed a porosity of 51%; the water after staying for a week in each wetland and after the whole system has made its treatment, is poured into an infiltration field (open field) planted with Vetiver, it is noteworthy that this water meets current regulations regarding the dumps. The whole system works by gravity, thanks to the topography of the site where the research is carried out. The percentage of dissolved oxygen was measured for four weeks. The measurements were made with a Multiparameter probe like the one shown in Figure 1

![Figure 1. Multiparameter probe HI98194 pH/EC/DO.](image)

The hydrologist Harold Hurst studied the fluctuations of the Nile River with the aim of taking advantage of the reservoir capacities and the measures in drought epochs [15-17]. For this purpose, Hurst created a statistical methodology which consists of analysing the persistence of a time series [18] according to the duration of its cycles and determining if that time series is fractal [19]. Fractal geometry to describe irregular patterns and fragmented, which are repeated at different scales, usually in isotropic [20]. Fractal geometry as a branch of mathematics has been allowed to carry out research in relation to the similarity and dynamic systems, using notions of dimension and orbits in various disciplines such as medicine, engineering, psychology, music, biology, among others.

The dimension that is assigned by convention to certain geometric and physical objects is associated with an infinite variable number, for example, the cube is assigned to the triple defined directly by the thickness, width and height of this, and then the size of this object is three. This type of dimension is known as the topological dimension [21,22]. The fractal dimension, as its name suggests, is a fractional dimension and is determined by a rational number. Long-range power law correlations are traditionally measured by a scale parameter or fractal dimension (D). If the time series is self-similar and self-related, the parameter d is related to the Hurst exponent (H) through the expression D = 2-H [23,24]. Therefore, the Hurst exponent is a measure of the long-range correlation in the time
series data and allows distinguishing the persistence (correlation), the persistence (anti-correlation) or the randomness of the data [25]. The original estimate of the Hurst exponent was first made in hydrology by Harold Edwin Hurst in 1951 [26], by introducing an empirical relationship called the Rescaled Range (R/S). Subsequently, this relationship became the starting point to establish the classical method R/S (CR/S) developed by Mandelbrot and Wallis in the context of fractal geometry [25,27,28]. Although CR/S is one of the most popular methods for calculating the Hurst exponent, it has shown some serious limitations to study the long-range correlation when the time series is not large enough [28,29].

The idea of persistence of a time series, understood as the tendency to repeat this series (long-term memory), is highly related to the notion of self-sufficiency. Self-splicing is a feature of fractal objects, under which it is possible to observe the irregularity or other properties that maintains the same structure independently of the scale at which it is analysed [21,30].

In addition to self-similarity, fractal objects have another important characteristic, the fractal dimension. The fractal dimension assigns rational values, due to its irregularity, to certain geometric or physical objects. For example, when measuring the perimeter of the coast of an island, the traditional measurement method does not allow a small enough scale to be used, so that it can be measured in the same way, while with the fractal dimension, it is possible to assign a value to such presence of irregularity of the contour of the same island [21]. The Hurst Coefficient is one of the methods which it is possible to calculate the fractal dimension, associated to a time series for statistically self-similar data [31]. Using the accumulated sum and rescaled range method, the coefficient also allows measuring volatility, understand the maximum, and minimum peak values to which data are to be found in a time series with respect to the average value of the same, for the risk analysis of a time series [31]. The persistence of a time series depends on the value of the Hurst coefficient (H). If 0 <H <0.5 the Hurst exponent has a higher fractal dimension, it will exist for a time series with antipersistent behaviors. A larger exponent of H, 0.5 <H <1 has a lower fractal dimension and thus a persistent analysed [18,31,32]. According to the statistical mechanics, if H is equal to 0.5, the series presents a random path [19].

The following is CR/S method the time series described in [33] under consideration X: {x_i} is composed by N values. The full-time series is divided into windows of size M. The number of windows is defined by s ≡ N/M and therefore there are s windows of data Y_j, with j = 1,2,...,s. Defining the vector k = (j − 1)M + 1, (j − 1)M + 2, (j − 1)M + 3,..., (j − 1)M + M, the average of each window is calculated using Equation (1)

\[ \bar{Y}_j = \frac{1}{M} \sum_k x_k \]  

(1)

The profile or sequence of partial summations Z_j: {z_n}, with n = 1,2...M, is defined as the cumulative summation minus the average of the corresponding window and is calculated using the Equation (2)

\[ z_n = \sum_{k}^{n} (x_k - \bar{Y}_j) \]  

(2)

Equation (3) shows the algebraic way of calculating the range R_j, and Equation (4) the standard deviation \sigma_j of each window

\[ R_j \equiv \max \{Z_j\} - \min \{Z_j\} \]  

(3)

\[ \sigma_j = \left[ \frac{1}{M} \sum_{k} (x_k - \bar{Y}_j)^2 \right]^{\frac{1}{2}} \]  

(4)
The rescaled range is described by the quantity \((R/S)_M\) and is calculated using the Equation (5), for the case in which a stochastic process associated to the data sequence under study is rescaled over a certain domain \(M \in \{M_{\text{min}}, M_{\text{max}}\}\), the \(R/S\) follow the power law given by the Equation (6)

\[
(R/S)_M \equiv \text{mean} \left( \frac{R_j}{\sigma_j} \right) \quad (5)
\]

\[
(R/S)_M = aM^H \quad (6)
\]

Herein, \(a\) is a constant and \(H\) is the Hurst exponent which represents a fractal measure of long-range correlations in the analysed released.

### 3. Results

To perform the analysis of the fractality of this research, data were recorded over four weeks which were divided into four groups, one for each week. Given the fractality of the series used, they were analyzed with the objective of observing the persistence or not in each of the series and calculating the percentage of volatility. Each series was partitioned into four other subseries [15]. Table 1 to Table 4 show the data needed for the calculation of the coefficient of hurts for this time-series, as well as compare the natural logarithm of the data of each of the subgroups generated versus the natural logarithm of the ratio between the rescaled range and standard deviation.

**Table 1.** Related data from the four groups depending on the range and standard deviation. Calculation of the natural logarithm of the data number and the natural logarithm of the quotient between the rescaled range and the standard deviation of week 1.

| Subgroup | Number data | Rescaled range | Standard Deviation | Ln (Num) | Ln(R/S) |
|----------|-------------|----------------|--------------------|----------|---------|
| 1        | 535         | 103.86         | 0.55               | 6.28     | 5.50    |
| 2        | 1070        | 184.12         | 0.53               | 6.98     | 6.10    |
| 3        | 1606        | 380.51         | 0.68               | 7.38     | 6.42    |
| 4        | 2142        | 1811.31        | 2.11               | 7.67     | 6.79    |

**Table 2.** Related data from the four groups depending on the range and standard deviation. Calculation of the natural logarithm of the data number and the natural logarithm of the quotient between the rescaled range and the standard deviation of week 2.

| Subgroup | Number data | Rescaled range | Standard Deviation | Ln (Num) | Ln(R/S) |
|----------|-------------|----------------|--------------------|----------|---------|
| 1        | 504         | 145.94         | 0.96               | 6.22     | 5.02    |
| 2        | 1009        | 259.94         | 0.86               | 6.92     | 5.71    |
| 3        | 1514        | 758.95         | 1.26               | 7.32     | 6.40    |
| 4        | 2019        | 2622.06        | 3.37               | 7.61     | 6.66    |

With this data, a graph is constructed in which, comparing the natural logarithm of the data of each of the generated subgroups versus the natural logarithm of the relationship between the rescaled range and the standard deviation. Figure 2 shows the logarithmic linear regression of the data corresponding to the first week, in Figure 3 the regression corresponding to the second week, in Figure 4 the regression corresponding to week 3 and in Figure 5 the corresponding regression at week 4. The slope of the regression line equation coincides with the Hurst coefficient necessary to find the fractal dimension and volatility of each of these time series.
Table 3. Related data from the four groups depending on the range and standard deviation. Calculation of the natural logarithm of the data number and the natural logarithm of the quotient between the rescaled range and the standard deviation of week 3.

| Subgroup | Number data | Rescaled range | Standard Deviation | Ln (Num) | Ln(R/S) |
|----------|-------------|----------------|--------------------|----------|---------|
| 1        | 502         | 246.40         | 1.21               | 6.22     | 5.32    |
| 2        | 1004        | 389.71         | 1.07               | 6.91     | 5.90    |
| 3        | 1506        | 542.35         | 1.09               | 7.32     | 6.21    |
| 4        | 2009        | 614.20         | 0.96               | 7.61     | 6.46    |

Table 4. Related data from the four groups depending on the range and standard deviation. Calculation of the natural logarithm of the data number and the natural logarithm of the quotient between the rescaled range and the standard deviation of week 4.

| Subgroup | Number data | Rescaled range | Standard Deviation | Ln (Num) | Ln(R/S) |
|----------|-------------|----------------|--------------------|----------|---------|
| 1        | 561         | 285.52         | 1.37               | 6.33     | 5.34    |
| 2        | 1123        | 649.11         | 1.48               | 7.02     | 6.08    |
| 3        | 1685        | 1123.52        | 1.69               | 7.43     | 6.50    |
| 4        | 2247        | 1756.05        | 1.94               | 7.72     | 6.81    |

Figure 2. Logarithmic linear regression of the data supplied in Table 1.

Figure 3. Logarithmic linear regression of the data supplied in Table 2.

In this context, the present work reports deviations around the average inherent to environmental data with cyclical behavior. The fractal dimensions corresponding to the four weeks studied were 1.10, 0.78, 1.18 and 0.94 which indicates presence of some type of persistence, in the same way for the volatilities corresponding to the four weeks studied were 55.03%, 39.25%, 59.12% and 47.06%, which reflect a periodic nature in the time series. On the other hand, its magnitude also indicates a low correlation between neighboring concentrations of dissolved oxygen, representing low persistence and the absence of long-term memory in the time series. From the perspective of mathematical modeling, this finding suggests an inadequate alternative to the stochastic approach based on the generation of synthetic time series to predict cause-effect relationships that influence the concentration of dissolved oxygen in the wetland of interest. In this sense, deterministic models of first order, variable order, or mechanistic compartments seem to emerge as the best choice. However, this behavior must be confirmed.

References
[1] Calvo G and Mora J 2009 Tecnología en marcha 22 1 57
[2] Madera C, Peña E and Solarte J 2004 Ingeniería y Competitividad 16 2 180
[3] Torres P 2010 Ingeniería y Universidad 14 2 315
[4] Qadir M, Wichelns D, Raschid-Sally L, McCormick P G, Drechsel P, Bahri A and Minhas P 2010 Agric. Water Manag. 97 45
[5] Mara D 2013 Domestic wastewater treatment in developing countries (United Kimdong: Earthscan)
[6] Ceron V, Peña M, and Madera C 2015 Ingeniería y Desarrollo 33 1 99
[7] Rosales Y, Acosta Y, Zauahre M, Mogollón P, and Zamora F 2015 Investigación Tecnológica 5 1 38
[8] Tscherninga K, Leihner D, Hilgera T, Müller K, and El Sharkawy M 1995 Field Crops Research 43 2 131
[9] Percy I and Truong P 2005 Proc., Landfill 2005 (Caracas: Proc. Fourth International Vetiver Conference)
[10] Singh S, Fulzele D, and Kaushik C 2016 Ecotox. Env. Safety 132 140
[11] Almeida A, Carvalho F, Imaginário M, Castanheira I, Prazeres A, and Ribeiro C 2017 Ecol. Eng. 99 5 35
[12] Ye M, Sun M, Liu Z, Ni N, Chen Y, Gu C, Orori F, Li H and Jiang X 2014 J. Env. Man. 8 141
[13] Mini M, Claramamma R, Mathukutty S and Cherian S 2016 Proc. Tec. 24 20
[14] Makris K, Shakya K, Datta R, Sarkar D and Pachanoor D 2007 Env. Poll. 146 1
[15] Braun E 2011 Caos, fractales y cosas raras (México: Fondo de Cultura Económica) p 92
[16] Rodríguez E 2012 Lat. Amer. Not. Adm. 3 42
[17] Hurst H 1951 Long-terms storage of reservoirs Transactions of the American Society of Civil Engineers 116
[18] Peters E 1991 Chaos and Order in the Capital Markets (EUA: John Wiley & Sons, Inc.)
[19] Peters E 1994 Fractal market analysis: applying chaos theory to investment and economics (USA: Willey)
[20] Ramírez R, Rodríguez A, and Ordaz A 2017 Min. and Geo. 33 36
[21] Mandelbrot B 2006 Los objetos fractales: forma, azar y dimensión 6 (España: Metatemas)
[22] Barnsley M 2012 Fractals Everywhere (USA: Dover Publications)
[23] Erdf J 1988 Fractals (USA: Plenum Press)
[24] Chen C, Lee Y and Chang Y 2008 Phys. A 387 4643
[25] Mandelbrot B and Wallis J 1968 Wat. Res. Res. 4 909
[26] Hurst H 1951 Trans. Am. Soc. Civ. Eng. 116 770-777
[27] Mandelbrot B and Wallis J 1969 Wat. Res. Res. 5 321
[28] Mandelbrot B and Wallis J 1969 Wat. Res. Res. 5 967
[29] Sanchez M, Trinidad J, and García I 2008 Phys. A 387 5543
[30] Mandelbrot B 2009 La geometría fractal de la naturaleza 2 (España: Metatemas)
[31] Mandelbrot B and Hudson R 2010 Fractales y finanzas, una aproximación matemática a los mercados: arriesgar, perder y ganar (España: Metatemas)
[32] Mandelbrot B 1997 Fractals and Scalind in Finance (EUA: Springer)
[33] Martín L, Aranda N, and Quimbaya C 2015 Phys. A 421 124