Is There A Phase Transition in the Isotropic Heisenberg Antiferromagnet on the Triangular Lattice?

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Abstract

The phase diagram of the classical anisotropic (XXZ) Heisenberg model on the 2-dimensional triangular lattice is investigated using Monte Carlo methods. In the easy-axis limit, two finite temperature vortex unbinding transitions have been observed. In the easy-plane limit, there also appear to be two distinct finite temperature phase transitions which are very close in temperature. The upper transition corresponds to an Ising-like chirality ordering and the lower temperature transition corresponds to a Kosterlitz-Thouless vortex unbinding transition. These phase transition lines all meet at the Heisenberg point and provide strong evidence that the isotropic model undergoes a novel finite temperature phase transition.

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I. INTRODUCTION

The classical Heisenberg model on the two dimensional triangular lattice with antiferromagnetic nearest neighbour coupling is a geometrically frustrated system. Kawamura and Miyashita [1] have previously studied the possibility of a Kosterlitz-Thouless-like [2] transition in this system using Monte Carlo methods and obtained evidence for a finite temperature transition driven by the dissociation of vortices. However, a transition at finite temperature is considered by many to be unlikely for the Heisenberg model in two dimensions. Dombre and Read [3] mapped the continuum version of the model onto a non-linear sigma model (NL$\sigma$) and Azaria et. al. [4] used renormalization group (RG) techniques to study the correlation length and effective long-wavelength spin stiffness of this NL$\sigma$ at low temperatures. Their results were consistent with a phase transition at $T_c = 0$ with an enlarged SO(3) symmetry. Indeed, Monte Carlo simulations by Southern and Young [5] of the spin stiffness seemed to provide partial support for this picture. More recent work by Southern and Xu [6] calculated the rigidity of the isotropic system against the formation of free vortices at low temperatures. The vorticity stiffness was found to be finite at low $T$ and disappeared abruptly near $T = 0.31J$. This behaviour is consistent with a possible Kosterlitz-Thouless defect unbinding transition. It appears that the stiffness and vorticity behave quite differently in the isotropic model. The stiffness is zero on long length scales indicating that the two-spin correlations decay exponentially at all finite $T$. However, the vorticity indicates that an unbinding of topological defects occurs at a finite $T_c$.

II. THE MODEL AND RESULTS

In an attempt to understand this difference in behaviour of the stiffness and vorticity, we have recently studied [7] the following anisotropic model

$$H = +J \sum_{i<j} \left[ S_i^x S_j^x + S_i^y S_j^y + A S_i^z S_j^z \right]$$ (1)

where $S_i^\alpha, \alpha = x, y, z$ represents a classical 3-component spin of unit magnitude located at each site $i$ of a triangular lattice and the exchange interactions are restricted to nearest neighbour pairs of sites. The range $A > 1$ corresponds to an easy axis system. The ground state in this range exhibits two continuous symmetries which can be described in terms of two phase angles. [1,7] We found that the system undergoes two finite temperature phase transitions associated with the unbinding of vortices related to these phase angles.

Information about the rigidity of the anisotropic system against fluctuations can be obtained from the spin wave stiffness and vorticity modulus. The spin stiffness (helicity) tensor is given by the second derivative of the free energy [3,7] with respect to the twist angle about a particular direction in spin space. Similarly, the vorticity [6,7] can be defined as the response of the spin system to an imposed twist about a given axis in spin space along a closed path which encloses a vortex core. This is essentially the response of the system to an isolated vortex and can be calculated as the second derivative of the free energy with respect to the strength of the vortex, or winding number $m$, evaluated at $m = 0$. These two response functions can be calculated using standard Monte Carlo methods for lattices of increasing size. We find that the spin stiffnesses and the corresponding vorticity moduli...
FIG. 1. The transition temperatures $T_{c_1}/J$ (lower) and $T_{c_2}/J$ (upper) as a function of $A$. For $A > 1$ these are determined using the universal value of the stiffness $\rho/T_c = 2/\pi$ and the universal value of $\eta = 1/4$. The value at the Heisenberg point ($A = 1$) was obtained from the vorticity results obtained in reference 6. For $A < 1$ the data points were taken from reference 10.

behave identically for the easy axis case $A > 1$. In contrast, at the isotropic Heisenberg limit ($A = 1$), the spin stiffness vanishes at large length scales but the vorticity moduli are non-zero at low $T$ and vanish abruptly at a finite temperature. Similar work on the $xy$ model also indicated that the vorticity and stiffness behave identically and that there are two phase transitions. These transitions are very close in temperature with the upper transition corresponding to an Ising-like transition and the lower to a Kosterlitz-Thouless transition. In the easy plane range of the present model, $-0.5 < A < 1$, the ground state corresponds to a $120^\circ$ planar arrangement of the spins with the chirality on each triangle aligned along the $z$-axis. Recent Monte Carlo work by Capriotti et al. in the range $0 < A < 1$ has also suggested two closely spaced transitions which are qualitatively the same as those in the $xy$ model.

In figure 1 we show the phase diagram for the anisotropic model as a function of $A$. The data points for $A > 1$ were obtained by studying the finite size behaviour of the structure factor, spin stiffness and vorticity. Both transitions correspond to vortex unbinding transitions and we find that for all $A > 1$ the correlation length exponent $\eta$ and the ratio of the stiffness to the critical temperature $\rho/T_c$ have the universal values $\eta = 1/4$ and $\rho/T_c = 2/\pi$. The data points for $A < 1$ have been taken from the work of Capriotti et
al. [10] who determined the values of $T_c$ by examining the behaviour of the chirality and the structure factor as a function of temperature and lattice size. The value of $T_c$ at the isotropic point ($A = 1$) is that obtained from the vorticity results of Southern and Xu. [6] We have also independently studied the range $-0.5 < A < 1$ and find some evidence for two closely spaced transitions. The upper transition is Ising-like and Monte Carlo snapshots of the chirality clearly indicate Ising-like domains. On the other hand, the spin stiffness exhibits behaviour consistent with a vortex unbinding involving the $xy$ spin components but we are unable to directly observe vortex excitations in the simulations.

The structure of the phase diagram suggests that the Heisenberg point is a multicritical point where possibly four phase transition lines meet. For $A > 1$ there are two KT transition lines whereas as for $A < 1$ there is an Ising and a KT line or possibly a single transition line. The behaviour of these transition lines as $A \to 1$ from both sides strongly suggests that there is a finite temperature phase transition in the Heisenberg model. In particular, the lower transition for $A > 1$ is increasing towards the Heisenberg value in this limit. Since the spin stiffness is zero at large length scales for all finite temperatures when $A = 1$, the spin correlations decay exponentially. Thus the transition would be quite different from the KT transition where spin correlations acquire a power law decay with a temperature dependent exponent. The transition at the Heisenberg point would be purely topological in character. Further theoretical investigations of this possibility are desirable.

III. ACKNOWLEDGEMENTS

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