A Multi-Scale Structural Degradation Metric for Perceptual Evaluation of 3D Mesh Simplification*

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SUMMARY  Visual degradation is usually introduced during 3D mesh simplification. The main issue in mesh simplification is to maximize the simplification ratio while minimizing the visual degradation. Therefore, effective and objective evaluation of the visual degradation is essential in order to select the simplification ratio. Some objective geometric and subjective perceptual metrics have been proposed. However, few objective metrics have taken human visual characteristics into consideration. To evaluate the visual degradation introduced by mesh simplification for a 3D triangular object, we integrate the structural degradation with mesh saliency and propose a new objective and multi-scale evaluation metric named Global Perceptual Structural Degradation (GPSD). The proper selection of the simplification ratio under a given distance-to-viewpoint is also discussed in this paper. The accuracy and validity of the proposed metric have been demonstrated through subjective experiments. The experimental results confirm that the GPSD metric shows better 3D model-based multi-scale perceptual evaluation capability.

key words: mesh simplification, structural degradation, visual perception, mesh saliency, entropy

1. Introduction

3D geometry objects are widely used in many computer graphics applications. The most popular representation of geometry object is triangular mesh. Its representation precision and visual quality depend on the total number of vertices and triangles. Much more visual redundancy occurs in 3D object when vast amount of vertices are generated with 3D-scanners, reconstructed from images or other techniques[1]. The visual redundancy increases the complexity of the graphics system to provide the best visual performance but degrades the interactive performance of the graphics system as well. Therefore, the trade-off between complexity and performance is an important issue for interactive graphics systems [2]. In previous perceptual experiments [3], it has been observed that when the number of vertices is larger than a certain number, the extra detail of geometry is imperceptible to the Human Visual System (HVS) because of the limit of its resolution [4]. Hence, many techniques such as simplification, level of detail (LOD) and other multi-resolution technology are usually used as powerful tools to reduce extra detail and complexity during the last decade [2]. They accelerate the rendering process and progressive transmission [5], [6]. The simplification technology based on face merging, incremental decimation or remeshing is the most important tool to reduce those visual redundancy and preserve the same visual shape as the original [1], [2]. To evaluate the perceptual degradation with an objective metric is a challenging issue.

We have adopted the concept of image structural similarity proposed by [7] and extended it to evaluate the visual degradation metric by subdividing the bounding box into 3D cube windows with mesh saliency. Considering the fact that the visual perceptual degradation of a 3D object will differ with the viewing distance from the center of 3D object, a multi-scale perceptual degradation metric with different subdivision parameters is designed and proposed to measure the degradation of all simplified series from the original. The visual degradation degree generated by different simplification algorithms at the same simplification ratio can be computed by our quantitative and objective scheme. We construct a representative corpus with the 3D models and its simplified versions. Subjective experiments were conducted to demonstrate the accuracy and validity of the proposed metric. The main contributions of this article are the following:

1. We propose an entropy metric considering mesh saliency for evaluating visual degradation during mesh simplification to decide the final simplification ratio.
2. We introduce the multi-scale perceptual structural degradation evaluation by subdividing the bounding box of 3D mesh.
3. We present how to apply the GPSD metric to achieve valid mesh simplification ratio selection under visual degradation threshold.

2. Review of Visual Degradation Evaluation

2.1 Visual Degradation Metrics

Several automatic visual degradation metrics based on im-
For a given 3D mesh, either an explicit or an implicit geometric distance metric between the original and the simplified mesh is used to guide the simplification process or decide the final simplification ratio. Luebke et al. have presented a detailed discussion of various mesh simplification algorithms [2], and these algorithms varied greatly in the distortion metrics when measuring the deviation from the original 3D objects. To address this problem, some efficient tools and algorithms have been provided [2], [10], [11]. They can be classified into two main categories: geometric quality evaluation and perceptual quality estimation [10]. Geometric evaluation often only take the geometry property into consideration. Perceptual metric is often used to analyze the impact of simplification on human visual perception of 3D meshes. Currently, the main perceptual approaches are based on subjective test using rating experiments. Few objective and quantitative metrics are available for evaluating the visual perceptual degradation. In most perceptual distance based approaches, the Just-Noticeable-Difference (JND) concept is applied to 2D renderings of the models [6], [21] or directly to 3D models [3], [14], [22].

Although many mesh simplification methods concentrate on the geometric fidelity, it is important that the visual quality is evaluated by people. The comparison of all simplification algorithms and evaluation of the visual perceptual degradation during the simplification are both important issues, and there are few tests about it except the explicit perceptual metric proposed in [3], [10]. However, few general objective and quantitative perceptual degradation evaluation methods for mesh simplification are proposed so far. In particular, many of these existing evaluations fail to provide multi-scale perceptual evaluation of visual degradation for mesh simplification. How to compare mesh simplification algorithms and evaluate visual perceptual degradation of the 3D simplified objects are open issues.

2.3 Mesh Saliency and Its Application

The visual attention mechanism is one of the most often exploited visual perception characteristics in 3D mesh perception or scene analysis applications. Saliency is a very important characteristic in low-level human visual system cues. Koch et al. have suggested a model in which salient image locations are distinct from their surroundings [23]. Itti et al. have maintained that visual attention is saliency-dependent [24]. They first used multi-scale image features according to center-surround mechanisms and applied each scale to different feature maps. Then, each feature map is computed at each different scale. Finally, these different feature maps are combined into a single topographical saliency map that assigns a saliency value to each image pixel. The saliency map can be used to guide the selection of attended locations and mesh simplification. Some other saliency models have been developed in the past decade [25]–[27] and some saliency-based applications are introduced into selective compression [28], shrink images [29] and mesh simplification [30], [31]. However, few visual degradation evaluations of 3D mesh take the saliency into consideration di-
According to the saliency model proposed in [24], Yee et al. have computed a saliency map for the 2D projection rendering of a 3D dynamic scene [32]. Howlett et al. have considered the feature saliency for 3D objects and have demonstrated that the potential value of saliency can be predicted in advance [31]. They found that perceptually weighted simplification leads to significant increase in visual fidelity for the lower-level details of 3D nature objects. Lee et al. have developed a computational model of mesh saliency that can be considered as a perception-inspired measure of regional importance from surface curvatures using center-surround filters with Gaussian-weighted curvatures [30]. It can capture the visually interested regions on a mesh. They have used saliency map successfully in mesh simplification and viewpoint selection. This work has provided a good start in merging perceptual criteria inspired by low-level human visual system cues with mathematical measures based on discrete differential geometry for 3D mesh. Figure 2 shows a saliency map of dinosaur model with varied resolutions. We observe that the regional importance remain basically unchanged in mesh. Feixas et al. have proposed a new definition of mesh saliency based on polygonal mutual information (PMI) and have built a unified information-theoretic framework for viewpoint selection and mesh saliency [33]. Kim et al. have introduced the normalized chance-adjusted saliency mechanism to quantify the correlation between mesh saliency and fixed locations for 3D rendering applications and proposed a new computational model of mesh saliency [34]. Their algorithm can model human eye movements better than that of [30]. Mesh saliency can be also used to guide lighting to emphasize important regions and suppress less important ones [35].

Saliency-based applications mainly focused on viewpoint selection, simplification, scene rendering and so on. Perceptual degradation evaluation based on mesh saliency is rarely mentioned in 3D simplification applications, but it is a potentially promising method for evaluating visual degradation introduced by mesh simplification. Therefore, we will propose an objective, qualitative and multi-scale perceptual degradation evaluation schema in the next section.

3. The Mesh Perceptual Structural Degradation

Many triangles of a 3D object in a specified field of view will be invisible to the viewer. Therefore, when viewpoint becomes farther, some different resolution versions of 3D object generated from the same source may have identical visual quality. On the contrary, they may have different visual degradation when the distance becomes nearer. In this paper, we only consider the factor of distance-to-viewpoint and mesh simplification ratio. So, the global visual perception degradation is a function of those two parameters. We use $PD \in [0, 1)$ to denote the visual perceptual degradation, then it takes the following form

$$PD = f(Dist, Ratio),$$

where $Dist \in [0, +\infty)$ is the viewpoint distance to the center of the bounding-box of the mesh and $Ratio \in [0, 1]$ is the simplification ratio. $PD = 0$ if no visual degradation is appeared and $PD \to 1$ when the degradation becomes bigger. Figure 3 illustrates the multi-scale perceptual degradation effect corresponding to different $Ratio$ and $Dist$.

3.1 Mesh Saliency Computation and Normalization

Let $S = \{s_i\}_{i=1}^{C_v}$ be the saliency map computed as [30] for a 3D mesh $M$ with $C_v$ vertices. In order to generate more comprehensive stimulus than its original saliency status, we have designed the following non-linear normalization operator.

We normalize the saliency into the unit interval $[0, 1]$ before non-linear normalization. Firstly, the mean of saliency is estimated as

$$\mu_s = \frac{1}{C_v} \sum_{i=1}^{C_v} s_i,$$

Secondly, we compute the square root of variance by Eq. (3)

$$\sigma_s = \sqrt{\frac{1}{C_v} \sum_{i=1}^{C_v} (s_i - \mu_s)^2}$$
Finally, we map \( s_i \) from interval \([a, b]\) to \( s'_i \) in the interval \([0, 1]\) using the following non-linear function

\[
s'_i = \frac{1 - e^{-\frac{s_i}{a-b}}}{1 - e^{-\frac{a}{a-b}}}, \quad s \in [a, b]
\]

(4)

### 3.2 Local Perceptual Structural Degradation (LPSD)

Any mesh elements, such as vertex, edge and face, may be changed during mesh simplification. Therefore, each local visual degradation maybe randomly generated during mesh simplification. The global degradation effect will be prominent if all local visual degradation is accumulated to a level and their local distributions satisfy some conditions. In order to compute the global visual degradation, we subdivide the bounding box of the 3D object as shown in Fig. 4 and define each sub-box as a 3D local window. The subdivision style can be used to evaluate the visual degradation between the original mesh and the simplified one and can provide multi-scale visual degradation evaluation. Inspired by the framework of [7], [18], [19], the LPSD is also based on the statistics of 3D local cubic window, including mean, standard deviation and covariance of the saliency over all sub-boxes.

Let \( N_X, N_Y \) and \( N_Z \) be the number of sub-boxes at \( X \), \( Y \) and \( Z \) direction respectively. In order to simplify the subdivision, we set \( N_X = N_Y = N_Z = N_{sub} \) in this paper and \( 1 \leq N_{sub} < \infty \).

Saliency can be used as a computational model of human visual attention. It shows better visual perceptual performance than curvature for analysis of mesh processing. Therefore, we use the non-linear normalized saliency as the visual perceptual attribution of mesh vertex. Firstly, we evaluate the mean saliency \( \mu_{B_i} \) of each sub-box \( B_i \):

\[
\mu_{B_i} = \frac{1}{C_{B_i}} \sum_{j=1}^{C_{B_i}} s'(v_j),
\]

(5)

where \( C_{B_i} \) is the number of vertices lying in the \( i \)th sub-box \( B_i \), and \( s'(v_j) \) is the normalized saliency value of vertex \( v_j \). Secondly, the standard deviation \( \sigma_{B_i} \) for each sub-box can be computed via the following formula (6)

\[
\sigma_{B_i} = \left( \frac{1}{C_{B_i} - 1} \sum_{j=1}^{C_{B_i}} (s'(v_j) - \mu_{B_i})^2 \right)^{1/2}
\]

(6)

Thirdly, the mesh \( M_1 \) based covariance is defined as:

\[
\sigma_{B_i}^{B_1, B_2} = \frac{1}{C_{B_i} - 1} \sum_{j=1}^{C_{B_i}} (s'(v_j) - \mu_{B_1})(s'(v_j) - \mu_{B_2})
\]

(7)

where \( B_i^1 \) and \( B_i^2 \) are the \( i \)th sub-box of the source mesh \( M_1 \) and target mesh \( M_2 \) respectively, \( s'(v_j) \) is the normalized saliency value of each vertex \( v_j \) lying in the \( i \)th sub-box \( B_i^1 \) of \( M_1 \), and \( s'(v_j) \) is the saliency value of the closest vertex to vertex \( v_j \) from \( B_i^2 \) according to Euclidean distance. The \( M_2 \) based covariance \( \sigma_{B_i}^{B_2, B_2} \) is also defined similarly. The covariance between \( B_i^1 \) and \( B_i^2 \) is then expressed as:

\[
\sigma_{B_i}^{B_1, B_2} = \frac{\sigma_{B_i}^{B_1, B_2} + \sigma_{B_i}^{B_2, B_2}}{2}
\]

(8)

Inspired by the framework of [7], three comparison functions between two corresponding local sub-box \( B_i^1 \) and \( B_i^2 \) are introduced: \( L_\text{(sym)} = L(B_i^1, B_i^2) \) (saliency comparison), \( C_\text{(con)} = C(B_i^1, B_i^2) \) (contrast comparison) and \( S_\text{(str)} = S(B_i^1, B_i^2) \) (structure comparison). Two parts of a Mesh with the same comparison function values may have completely different shape, but they will contribute the same visual degradation for evaluating the total degradation between two meshes. They take the following forms:

\[
L_\text{(sym)} = \frac{2 \star \mu_{B_i} \star \mu_{B_i} + K_1}{\mu_{B_i}^2 + \mu_{B_i}^2 + K_1}
\]

\[
C_\text{(con)} = \frac{2 \star \sigma_{B_i} \star \sigma_{B_i} + K_2}{\sigma_{B_i}^2 + \sigma_{B_i}^2 + K_2}
\]

\[
S_\text{(str)} = \frac{\sigma_{B_i} \star \sigma_{B_i}}{\sigma_{B_i}^2 + \sigma_{B_i}^2 + K_3}
\]

(9)

where the constant \( K_1 \) is included to avoid instability when \( \mu_{B_i}^2 + \mu_{B_i}^2 \) is very close to zero. Specifically, we choose

\[
K_1 = \max_{v_j \in B_i^1, v_k \in B_i^2} \{|s'(v_j)|, |s'(v_k)|\} \ast \varepsilon, \varepsilon \to 0,
\]

\[
K_2 = 3 \ast K_1, K_3 = 0.5 \ast K_2
\]

(10)

where \( \varepsilon \) is a very small positive constant. Finally, we combine the three comparison functions \( L_\text{(sym)}, C_\text{(con)} \) and \( S_\text{(str)} \) with Eq. (11), and name the result as \( \text{LPSD}_{D(i)} \):

\[
\text{LPSD}_{D(i)} = \text{LPSD}(B_i^1, B_i^2) = L_\text{(sym)}^{\alpha} \ast C_\text{(con)}^{\beta} \ast S_\text{(str)}^{\gamma}
\]

(11)

The three parameters \( \alpha > 0, \beta > 0, \gamma > 0 \) can be used to adjust relative importance between \( L_\text{(sym)}, C_\text{(con)} \) and \( S_\text{(str)} \). Obviously, the \( \text{LPSD}_{D(i)} \) satisfies the following four conditions:

1. Symmetry: \( \text{LPSD}(B_i^1, B_i^2) = \text{LPSD}(B_i^2, B_i^1) \).
Therefore, we have the number of vertices in each sub-box is not greater than 1.

3. Unique minimum: \( LPS D \left( B_i^1, B_i^2 \right) = 0 \) if and only if \( B_i^1 \) and \( B_i^2 \) are identical;

4. Unique maximum: \( LPS D \left( B_i^1, B_i^2 \right) \rightarrow 1 \) is a theoretical limit when the \( B_i^1 \) and \( B_i^2 \) are very different.

### 3.3 Global Perceptual Structural Degradation (GPSD)

Minkowski sum of local windows distance is adopted for global similarity or distortion evaluation in [7] and [18], [19]. However, local visual perceptual degradation information for the referenced source object has a certain visual uncertainty because the degradation may be randomly distributed over the 3D bounding box space for all simplification methods. The global visual degradation is a perfect accumulation of all visual attention uncertainty from their local content. Therefore, we present an entropy form for global degradation evaluation of 3D mesh simplification.

Let the series \( \{ LPS D_{(i)} \} \) be perceptual structural degradation of all corresponding sub-box. Then \( \{ LPS D_{(i)} \} \) can be taken as random variables which indicate some uncertainty of local visual perceptual structural degradation and have a spatial distribution over the bounding box space. The probability \( p_i \) is defined as the \( i \)th valid visual degradation variable whose corresponding sub-box \( B_i^1 \) and \( B_i^2 \) have one or more vertices. It takes the following expression:

\[
p_i = \frac{LPS D \left( B_i^1, B_i^2 \right)}{\sum_{i=1}^{N_{valid}} LPS D \left( B_i^1, B_i^2 \right)} \tag{12}
\]

where \( N_{valid} \) is the number of valid sub-box. \( \{ p_i \} \) denotes random variables of visual degradation. Then, the Shannon entropy is used to denote the average uncertainty of entire visual degradation between two meshes \( M_1 \) and \( M_2 \). So the global perceptual structural degradation (GPSD) can be defined as

\[
GPS D_{En} \left( M_1, M_2 \right) = - \sum_{i=1}^{N_{valid}} p_i \ast \log p_i, \tag{13}
\]

Obviously, the \( GPS D_{En} \) is greater than zero, and the lower bound is zero when the probability of all random variables is zero and there is no any simplification. So, in this extreme case, the minimum value of visual perceptual degradation entropy \( GPS D_{min} \) is zero.

According to Shannon theory, the maximum value of the \( GPS D_{En} \) may be achieved when the maximum uncertainty of degradation exists. In such case, the probabilities of the visual perceptual degradation are equal for all sub-boxes, and we have \( p_i = 1/N_{valid}, N_{valid} > 0 \). The maximum number of possible valid sub-box is \( N_{valid} \leq N_X \ast N_Y \ast N_Z \). In this paper, \( N_{valid} \leq (N_{sub})^3 \). However, the continuous fine and precise subdivision procedure may not be necessary if the number of vertices in each sub-box is not greater than 1. Therefore, we have

\[
1 \leq N_{valid} = \min \left( N_X \ast N_Y \ast N_Z, \max \left( C_i^1, C_i^2 \right) \right) < \infty \tag{14}
\]

The maximum value of visual degradation entropy \( GPS D_{max} \) achieves only when all vertices are removed, and can be obtained as

\[
GPS D_{max} \left( M_1, M_2 \right) = - \sum_{i=1}^{N_{valid}} \frac{1}{N_{valid}} \ast \log \frac{1}{N_{valid}} = \log N_{valid} \tag{15}
\]

We normalize the \( GPS D_{En} \) value to unit interval \([0, 1]\) by Eq. (16) in order to utilize visual quality assessment:

\[
PD = \frac{GPS D_{En} - GPS D_{min}}{GPS D_{max} - GPS D_{min}} = \frac{GPS D_{En}}{\log N_{valid}} \tag{16}
\]

Obviously, \( 0 \leq PD \leq 1 \). \( PD = 0 \) when the simplification ratio is zero and \( PD = 1 \) when the vertices of the source mesh are all removed.

### 4. Design of Subjective Experiment

As described previously in Fig.3, visual degradation is a function of simplification ratio and viewpoint distance. Therefore, the different subdividing parameter \( N_{sub} \) in our schema GPSD can be remapped as the multi-scale evaluation of visual degradation in HVS. We have designed and carried out this subjective experiment in order to demonstrate the behavior and efficiency of the GPSD metric regarding the simplification ratio and distance-to-viewpoint. We give the details about the experimental corpus setup and the subjective evaluation protocol in the following sections.

#### 4.1 Experimental Corpus Description

Eight 3D models: RockerArm (10000 vertices, 20000 triangles), Dinosaur (56194 vertices, 112384 triangles), Bunny (35286 vertices, 70568 triangles), Skull (20002 vertices, 40000 triangles), Lion head (38728 vertices, 77452 triangles), Maxplanck (49132 vertices, 98260 triangles), Armadillo (179274 vertices, 345944 triangles) and Horse (48485 vertices, 96906 triangles) are used as source objects to construct the experimental corpus. They are widely used in computer graphics applications. Each source has individual geometric and perceptual characteristics and application scenarios. Dinosaur, Armadillo and Lion head which present the video games application have many rough areas and complex shapes. Rocker Arm in Computer-Aided Design is smooth and has one genus. Skull presents cultural heritage application and has many smooth areas. Maxplanck with a majority of smooth areas is rather convex. Bunny is more complex with many smooth and rough areas. Horse is simple with many smooth and few rough areas.

Visual degradation is not easily noticeable when the
4.2 Subjective Evaluation Protocol

The subjective evaluation protocol basically follows those defined by [3], [10], [12], [19], [38]. First, in order to establish a referential range for the rating, two referential models are displayed together with the rating simplified one which is the original model has a ratio between 0 and 1 for comparison. One referential model is the original model that contains 1032 objects [8 originals × 8 × 4 versions decreased by 5% from 95% to 5% of the original vertices +8 × 4 × 4 versions decreased by 1% from 4% to 1% +8 × 9 × 4 versions decreased by 0.1% from 0% to 0.1%]. Figure 5 shows the eight source models and some simplified versions for each simplification algorithm. Four simplification algorithms, QEM [36], Quadric Error Metric with weighted area of the triangle (QEMArea) [36], melax Simplification [37] and salient simplification [30] are implemented and their results are used to construct the simplification corpus. Hence, the subjective experimental corpus contains 1032 objects [8 originals × 8 × 19 × 4 versions with resolutions decreased by 5% from 95% to 5% of the original vertices +8 × 4 × 4 versions decreased by 1% from 4% to 1% +8 × 9 × 4 versions decreased by 0.1% from 0.9% to 0.1%]. Figure 5 shows the eight source models and some corresponding simplified instances of the corpus based on simplification algorithm QEMArea.

4.3 Statistical Analysis

Let \( \{ M_1 \}_{i=1}^{T_C} \) be the eight original 3D meshes in the corpus and \( \{ M_{i,j}^{(k)} \}_{i=1}^{K} \) be the simplified version generated by the \( j \)th simplification algorithm (maybe QEM, QEMArea, Saliency, Melax) with different resolutions \( R^{(k)} \) for the original mesh \( M_r \), where \( M_{i,j}^{(0)} \) is the original mesh \( M_r \), \( 0 = R^{(0)} < R^{(1)} < \cdots < R^{(32)} = 0.999 \). Let \( S = 12 \) be the number of all subjects and \( m_{i,j,k} \in [0, 10] \) be the score of the mesh \( M_{i,j}^{(k)} \) given by the \( s \)th participant. We normalize all \( m_{i,j,k} \) into [0, 1], then the MOS value \( MOS_{i,j,k} \) of the mesh \( M_{i,j}^{(k)} \) can be computed for each rating object of the corpus:

\[
MOS_{i,j,k} = \frac{1}{S} \sum_{s=1}^{S} m_{i,j,k}^{(s)}
\]  

\( i = 1, 2, 3, 4 \)

\( j = 1, 2, \ldots, 12 \)

\( k = 1, 2, \ldots, 32 \)

We have carried out the one-way analysis of variance (ANOVA) for the subjective experiment and the condition: simplification ratio. The Pearson Product Moment Corre-
In order to investigate the significance of simplification ratio in this study, a single-factor ANOVA test on each 3D object’s set with 396 observations is performed. Table 2 shows the results of one-way ANOVA test on the visual degradation with different simplification ratio and fixed distance-to-viewpoint equals to δ. It shows that all results for this study are statistically significant for each of all eight objects (all p-values are almost equal to zero).

In this experiment, we set the subdivision parameter $N_{sub} = 50$ and $\alpha = \beta = \gamma = 1$ for the GPSD. The spearman and pearson correlation coefficient between the normalized MOS and the GPSD values for this study are also calculated. Table 3 presents the correlation results for all simplified models generated by QEMArea. The correlation values of the whole corpus generated by QEMArea are $r_{s} = 0.9887$ and $r_{p} = 0.9940$. However, the correlations over the QEMArea sub-corpus are not really meaningful because we have established the referential rating range for each model. The means and the standard deviations over all eight models are $\text{Mean (GPSD)} = 0.187$, $\text{Mean (MOS)} = 0.212$, $\sigma(MOS) = 0.200$, $\sigma(GPSD) = 0.179$. The results presented in Table 3 are satisfying. This indicates that the GPSD based on global saliency of the objects can successfully capture the visual degradation phenomenon. The main reason is that the visual degradation caused by mesh simplification occurs in the very local part of the object, and the GPSD can effectively reflect the structural degradation generated from all local degradation.

It is interesting to notice that the correlation results are very stable and relevant. These good results are mainly due to two reasons: firstly, it is based on the uncertainty of the visual degradation over the mesh; secondly, it depends on the mesh saliency statistics which is strongly linked with the visual attention on the mesh. In particular, the spearman and pearson correlation of the horse model are lower than those of other models because it is smoother. The reason is probably that visual attention on a simplified series of a smooth object may introduce significant changes.

We select four objects from the corpus to validate the effectiveness of our metric. The relationship between the GPSD and mesh simplification ratio is shown in Fig. 7. The MOS curve for those four models is also shown in Fig. 7. Obviously, Fig. 7 reveals the fact that the visual perceptual degradation increases with the growth of mesh simplification ratio when the distance-to-viewpoint is fixed. The increase speed of visual degradation is slow when simplification ratio is low. In particular, the visual perceptual degradation relative to the source object is almost zero when simplification ratio is small, but the degradation becomes easily noticeable when the simplification ratio increases gradually. Obviously, GPSD is very effective for evaluating visual perceptual degradation introduced by mesh simplification. It might also reflect the characteristics of HVS according to the fitness between the MOS curve and the GPSD curve with parameter $N_{sub} = 50$. The correlation results shown in Table 3 also demonstrate that GPSD is effective and reasonable.

### 5.2 Visual Perceptual Degradation Analysis

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### 5.3 Multi-Scale Visual Perceptual Degradation Analysis

We have found that visual perceptual degradation grows

### Table 1 Hypothesis test of significant rating variation based on one-way ANOVA for our subjective experiment.

| Source | SS | df | MS | F  | P   |
|--------|----|----|----|----|-----|
| Subjects | 15,781 | 11 | 1,435 | 1,341 | 0.217 |
| Error | 89,875 | 84 | 1,070 |     |     |
| Total | 105,656 | 95 |     |     |     |

### Table 2 One-way ANOVA for the subjective experiment.

| Object | F-Ratio | p-Value |
|--------|---------|---------|
| Dinosaur | 143.531 | $<0$ |
| Amadilio | 151.409 | $<0$ |
| Lion | 141.750 | $<0$ |
| Rocker | 156.566 | $<0$ |
| Skull | 111.553 | $<0$ |
| Maxplanc | 103.842 | $<0$ |
| Bunny | 136.922 | $<0$ |
| Horse | 124.910 | $<0$ |

### Table 3 Spearman and Pearson correlation values between MOS and GPSD.

|              | Dinosaur | Amadilio | Lion | Rocker | Skull | Maxplanc | Bunny | Horse |
|--------------|----------|----------|------|--------|-------|----------|-------|-------|
| $r_{s}$     | 0.9867   | 0.9780   | 0.9967 | 0.9898 | 0.9904 | 0.9953   | 0.9955 | 0.9953 |
| $r_{p}$     | 0.9883   | 0.9892   | 0.9969 | 0.9889 | 0.9973 | 0.9968   | 0.9877 | 0.9893 |
Subjective MOS with the distance-to-viewpoint equal to the length of the diagonal of the bounding box versus GPSD metric with parameter $N_{\text{sub}} = 50$.

with the increase of mesh simplification ratio for all different distances-to-viewpoint. Two meshes with different resolutions may have identical visual degradation when we set different distances-to-viewpoint for them. The phenomena can be found in Fig. 3. The multi-scale visual perceptual structural degradation can be obtained by different subdivision parameters. In this paper, we change the subdivision number $N_{\text{sub}}$ of the bounding box to simulate different distance-to-viewpoint approximatively. The subdivision parameter $N_{\text{sub}} = 50, 20, 10$ is set up to verify the multi-scale functionality of GPSD. Figure 8 presents the multi-scale functionality of GPSD for six objects of the corpus.

The degradation with decreasing of the subdivision parameter for armadillo object is not distinct due to its high complexity and roughness. HVS is not easy to distinguish the change of rough parts’ geometric degradation owing to its visual masking effect. Due to its complexity and roughness, the change rate of the visual degradation is very small even if the parameter $N_{\text{sub}} = 50$. However, the rocker arm model is not the same case because of its simplicity and smoothness. The visual degradation of bunny with change of the subdivision parameter is significant because of its complexity, smoothness and roughness.

To further illustrate the multi-scale functionality, we select the bunny, skull, horse, and maxplanck models and show the degradation in detail in Fig. 9.

Figure 9 also reveals the fact that the mesh visual perceptual degradation between different resolution objects may change with the distance-to-viewpoint when the viewpoint orientation is fixed. When we change the distance-to-viewpoint and keep a constant viewpoint orientation, the visual perceptual quality between different simplification ratios may be the same. When the threshold of visual perceptual degradation is set to 0.041 for skull model in the right top of Fig. 9, different simplification ratios can be obtained as the distance-to-viewpoint changes. The $N_{\text{sub}}$ may be equal to 50, 20 and 10, the corresponding appropriate ratio will be less than 0.4, equal to 0.6 and greater than 0.8 respectively. In these cases, we can obtain the same visual perceptual degradation. Similar to skull object, as shown in the right bottom of Fig. 9, the different simplification ratios with the values greater than 0.9, equal to 0.75 and less than 0.4 are permitted to execute for different distance parameter $N_{\text{sub}}$ which are equal to 10, 20 and 50 respectively when the degradation threshold equals to 0.034. The red dashed lines in the right part of Fig. 9 are remarked and illustrated for this multi-scale phenomenon. Therefore, the mesh with larger simplified ratio and distance-to-viewpoint
Table 4  Spearman and Pearson correlation values between MOS and evaluation values. Mean values and standard deviations over all models are also given.

| Model       | MaxGeo α  | MaxGeo β  | MeanGeo α  | MeanGeo β  | Quadric α  | Quadric β  | GPSD (S) α  | GPSD (S) β  | GPSD (20) α  | GPSD (20) β |
|-------------|-----------|-----------|------------|------------|------------|------------|------------|------------|------------|------------|
| Dinosaur    | 0.975     | 0.960     | 0.964      | 0.984      | 0.713      | 0.824      | 0.969      | 0.998      | 0.999      | 0.984      |
| Armadillo   | 0.992     | 0.975     | 0.975      | 0.975      | 0.997      | 0.942      | 0.997      | 0.996      | 0.997      | 0.981      |
| Liten head  | 0.990     | 0.975     | 0.965      | 0.965      | 1.000      | 0.803      | 1.000      | 0.999      | 0.999      | 0.990      |
| RockerArm   | 0.998     | 0.970     | 0.950      | 0.952      | 0.950      | 0.708      | 0.950      | 0.989      | 0.999      | 0.990      |
| Skull       | 0.993     | 0.986     | 0.995      | 0.991      | 1.000      | 0.882      | 1.000      | 0.997      | 1.000      | 0.994      |
| Maxplaneck  | 0.978     | 0.987     | 0.995      | 0.985      | 0.999      | 0.974      | 0.999      | 0.997      | 0.999      | 0.998      |
| Bunny       | 0.996     | 0.974     | 0.999      | 0.958      | 0.713      | 0.783      | 1.000      | 0.998      | 1.000      | 0.981      |
| Horse       | 0.954     | 0.967     | 0.965      | 0.964      | 0.498      | 0.507      | 0.965      | 0.997      | 0.985      | 0.991      |
| WholeCorpus | 0.514     | 0.473     | 0.615      | 0.645      | 0.478      | 0.599      | 0.896      | 0.905      | 0.386      | 0.881      |
| Mean        | 0.965     | 0.933     | 0.996      | 0.972      | 0.965      | 0.950      | 0.998      | 0.996      | 0.999      | 0.986      |

will present the same visual perceptual degradation as the mesh with smaller ratio and distance. This is one kind of multi-scale functionality in our evaluation scheme. If the simplification ratio is fixed, we observe that visual degradation changes with different distance-to-viewpoint. When the simplification ratio is equal to 0.65 for bunny model in the left top of Fig. 9, different visual perceptual degradations will appear with different distance-to-viewpoint. The degradation will be appropriated to 0.083, 0.027 and 0.012 when the subdivision parameter $N_{sub}$ is equal to 50, 20 and 10 respectively. In those cases, we can obtain different visual perceptual degradations with different distance-to-viewpoint for the same 3D object. Similar to the bunny object, given the simplification ratio equal to 0.6 for horse model in the left bottom of Fig. 9, different visual degradations of 0.014, 0.022 and 0.047 can be obtained with different distance parameter $N_{sub}$ which equals to 10, 20 and 50 respectively. The black lines in the left part of Fig. 9 are remarked for this kind of multi-scale phenomenon. We can observe that visual degradation will increase for a fixed simplification ratio when distance-to-viewpoint decreases. This is another kind of multi-scale functionality in our scheme. It reveals the fact that we cannot evaluate the degradation of simplified meshes effectively and correctly if the distance-to-viewpoint is different.

The two types of multi-scale functions in our GPSD scheme demonstrate that GPSD can reflect the multi-scale functionality in our evaluation scheme. This is high for each object (Mean=0.999), moreover its results are very stable ($σ = 0.002$). Its Pearson values which describe the real strength of the relationship are also the best (Mean = 0.996, $σ = 0.003$) among the four metrics over the corpus. So, GPSD outperforms the other evaluation methods on this corpus. This can be explained by the following reasons. Firstly, It is owing to the consideration of low-level human visual attention and mesh saliency with the capability of modeling visually interesting regions on a mesh. Other three metrics is less effective and do not try to model the visual perceptual features and not adequately consider the local context of the mesh. So, they fail to capture perceptual phenomenons. Secondly, the main reason is that we take the local visual attention affect as a random event and use the information entropy to model the uncertainty of visual perceptual degradation. However, the other three measures are based on the global geometric degradation which is not enough to capture the local visual degradation. We also notice that the simpler MeanGeo (for Spearman correlation: Mean = 0.998, $σ = 0.002$; for Pearson correlation: Mean = 0.972, $σ = 0.014$) degradation metric provides better results than the MaxGeo and Quadric.

In particular, GPSD can provide multi-scale visual degradation evaluation. We also notice that the correlation of the GPSD with parameter $N_{sub} = 20$ presented in Table 4 can also capture the visual degradation. Its correlation is poorer than the case of $N_{sub} = 50$, but it is obviously better than the other three metrics. We can configure a larger distance-to-viewpoint in the subjective protocol to improve the correlation. Figure 10 and Fig. 11 further demonstrate the effectiveness of our metrics.
effective and greatly outperforms other three geometric distortion metrics.

5.5 Threshold Driven Simplification Ratio Selection

According to the analysis of multi-scale evaluation of the visual perceptual degradation, we can obtain the qualitative result as shown in Fig. 13.

For a certain visual degradation threshold $\varepsilon$, GPSD can be used to evaluate the distortion introduced by simplification operation and an appropriate simplification ratio can be selected according to the distance-to-viewpoint in each simplification algorithm. For example, as illustrated in Fig. 13, if the distance between the viewpoint and the center of the 3D mesh is $3\delta$, we must ensure that the simplification ratio is less than or equal to $R_1$. Similarly, the maximum ratio should be $R_2$, $R_3$ and $R_4$ if the distance is $6\delta$, $9\delta$ and $12\delta$ respectively. Therefore, the possible simplification ratio may be larger if we move the viewpoint farther from the center of the 3D object. The visual degradation value can be obtained with the simplification procedure when the distance-to-viewpoint is fixed. We can also use this idea to construct LOD series having the best visual shape and the minimize degradation for a 3D object. So GPSD proves to be an effective perceptual method for selecting an optional simplification ratio under a certain visual degradation threshold due to its effectiveness and multi-scale functionality.

5.6 Comparison of Mesh Simplification Algorithms

In each mesh simplification algorithm, some visual degradation will be introduced. A common key issue is to find which algorithm is the best. Based on the effectiveness of our GPSD illustrated in previous sections, we will use GPSD to evaluate four mesh simplification algorithms in Sect. 5.6 and 5.7. We set the parameter $N_{sub} = 20$ and $\alpha = \beta = \gamma = 1$ for this experiment, where $N_{sub}$ is a parameter being inversely proportional to the distance between viewpoint and the object. Figure 14 shows the comparison of four mesh simplification algorithms at a fixed distance-to-viewpoint. According to the experiment shown in Fig. 14, the QEM and QEMArea methods can minimize the visual degradation and maximize the similarity of visual shape for almost all simplification ratios. For the dinosaur model shown in Fig. 14, the salient simplification may introduce more visual degradation than the other three algorithms when the sim-
plification ratio is less than 0.7. The same result is also obtained when the ratio is less than 0.65 according to the horse model shown in Fig. 14. We also find that the Melax method will generate more visual distortion than other three methods when the ratio is greater than 0.7 for the dinosaur model and 0.65 for the horse model in Fig. 14. The MPSD value of salient method is the lowest when the ratio approaches to 1. We can conclude that the QEM and QEMArea are the best for almost all simplification cases, the Melax simplification is the least effective in keeping visual shape when the ratio is greater than a certain value, and the salient simplification is more suitable for higher simplification ratio and lower-level detail [31].

5.7 Multi-Scale Comparison of Mesh Simplification Algorithms

We have analyzed the visual perceptual degradation for the four methods at a fixed distance-to-viewpoint in the previous section. The similar analysis is given for different distance-to-viewpoint in this experiment. We set different parameter value \( N_{sub} \) with 10, 20, 50 and 100 to illustrate the multi-scale comparison of those methods. Figure 15 gives the comparison results. If the simplification ratio is fixed to 0.55, we find that the visual degradation introduced by different simplification methods changes with different distance-to-viewpoint. This case has been analyzed in Sect. 5.3. The same result can be shown with the red dashed line marked with (I) in Fig. 15. The visual degradation introduced by salient simplification keeps the maximum value in the case of \( N_{sub} = 100 \) as shown in Fig. 15, the Melax method becomes the least effective. Therefore, the various simplified meshes with the same simplification ratio may have changeable visual degradation order when the distance-to-viewpoint is changed. The red dashed line with (I) in Fig. 15 illustrate this case clearly.

Similarly, if we fix the visual degradation threshold to a certain value 0.05, the Melax method gives the least simplification ratio in the first three cases marked with black bold line and (II) in Fig. 15. However, the salient method has the least simplification ratio when the parameter \( N_{sub} \) is equal to 100. The QEM method has the largest ratio for all four cases. Obviously, the QEM is the best method for this threshold and the QEMArea ranks the second. Especially for \( N_{sub} = 50 \) in Fig. 15, we can find that the Melax can provide the best visual shape when the ratio is less than 0.4, and it also provides the largest simplification ratio if the threshold is under 0.03.

We can conclude that different simplification methods may provide the same visual degradation when the distance-to-viewpoint is different. Therefore, we cannot give the reasonable comparison result if we fail to take the distance-to-viewpoint into consideration. But the QEM and QEMArea can provide the best visual effect for almost all cases in general. The result is also explained in Sect. 5.3.

6. Conclusions and Future Work

We have proposed an objective perceptual metric GPSD based on mesh saliency and information entropy to evaluate the visual degradation introduced by 3D mesh simplification. An analysis of the data collected by a subjective experiment indicated that GPSD is reliable for the visual degradation evaluation of 3D simplified objects. The statistical correlation analysis results between MOS and objective evaluations prove that GPSD is very effective and greatly outperforms all representative geometric distortion metrics in consistency. The capability of multi-scale visual degradation evaluation is helpful to the degradation analysis in Level of Detail. We also point out that, with a fixed distance-to-viewpoint and a certain threshold, the optimal simplification ratio can be computed. We can conclude that GPSD method promotes the capability of the visual degradation evaluation in 3D simplification applications. The proposed GPSD could be used to evaluate the visual perceptual degradation introduced not only by mesh simplification, but also by mesh compression, mesh watermarking and other general mesh processing tasks.

Many research issues are worth exploring. First, only human attention mechanism is considered in this paper. The
degradation metric may be further improved if some other important mechanisms of HVS are introduced. Second, we take the final mesh saliency map as the visual attribution of the vertex in our scheme. The degradation metric may provide more satisfactory results if the saliency maps at each different scale is used for the corresponding distance-to-viewpoint. Third, we will design more subjective experiments and use GPSD to benchmark all mesh simplification algorithms. Fourth, we will extend the GPSD schema to evaluate the visual degradation of 3D scene with multiple polygonal models. Finally, we will analyze the GPSD curves to obtain the simplification ratio value where the visual degradation is just noticeable and apply it to evaluate the degradation introduced by other mesh processing.

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