Analysis of entropy generation for nanofluid flow over a stretching sheet with an inclined magnetic field and uniform heat source/sink

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Abstract. This investigation presents an analytical study of entropy generation of the nanofluid over a stretching sheet with inclined magnetic field and uniform heat generation/absorption. Similarity variables are used to convert the governing PDEs into ODEs. Then it solved analytically by hyper geometric function. The effect of physical parameters such as nanosolid volume fraction, aligned angle, magnetic, slip and suction parameters are discussed for both temperature and velocity profiles. Also, the effects of the same parameters and uniform generation/absorption and dimensionless group parameters on entropy generation are discussed for Ag nanofluid.

Keywords: Nanofluid; Inclined magnetic field; slip; Entropy generation; Stretching sheet

1. Introduction
Entropy generation is linked with thermodynamic irreversibility, which is familiar in all types of heat transfer processes. This method is launched by Bejan [1]. Aboud and Saouli [2] scrutinized the purpose of the viscoelastic MHD flow in the presence of entropy generation by a stretching surface. The second law analysis on nanofluids in dissimilar geometries was agreed in the publications [3-5].

The magnetic field happens to be an applied force for the thermal system that is concerned the heat transfer progression. Ganga et al. [6] calculated the inclined magnetic field on entropy generation analysis for partial slip with nanofluid over a stretching sheet effect. Kalaivanan et al. [7] have analysed the Casson fluid past a stretching sheet with inclined magnetic field and slip effect. Eswaramoorthy et al. [8] had written account of the analytical solution of magnetohydrodynamic flow of viscoelastic fluid past a stretching sheet. Sivasankaran et al. [9] achieved the analytical and numerical solution of magnetohydrodynamic flow over porous medium with heat generation/absorption effects. Karthikeyan et al. [10] examined methodically and in detail the effect of magnetohydrodynamic flow through a vertical plate embedded in a porous medium with uniform heat source/sink.

The aim of the present work is to discuss the effect of entropy analysis for slip magnatohydrodynamic nanofluid flow over a stretching sheet with uniform heat generation/absorption. The effects of various physical parameter results are scrutinized with the help of graphs.

2. Formulation of the problem
We consider a steady state, two-dimensional, laminar, slip flow of an incompressible viscous heat generating/absorbing nanofluid over a stretching sheet. The stretching velocity of the sheet is \( \vec{u}_w = a \hat{x} \). The temperature is deemed to have the constant value \( T_w \) at the stretching surface, while the ambient
value, takes the constant value $T_\infty$ attained as $\bar{y}$ tends to infinity. It is additionally supposed that the induced magnetic field is unimportant in contrast to the applied magnetic field and hence neglected (as the small magnetic Reynolds number). The base fluid is water accommodating silver (Ag) nanoparticles. The thermo physical properties in Table: 1 is given for the nanofluid. Under the above assumptions, the governing equations are written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_{nf} \left( \frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} \right) = \mu_{nf} \frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 \bar{u} \sin^2 \gamma \tag{2}$$

$$\left( \mu C_p \right)_{nf} \left( \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} \right) = k_{nf} \frac{\partial^2 T}{\partial y^2} + q(T - T_\infty) \tag{3}$$

Where $T$ is the confined temperature of the fluid, $\sigma$ is the electric conductivity and $q$ is the rate of volumetric heat source/sink.

$$\rho_{nf} = \left(1 - \phi\right) \rho_f + \phi \rho_s, \quad \mu_{nf} = \frac{\mu_f}{\left(1 - \phi\right)^{2/3}}$$

$$(\mu C_p)_{nf} = \left(1 - \phi\right)(\mu C_p)_f + \phi (\mu C_p)_s, \quad k_{nf} = k_f \frac{k_s + 2k_f - 2\phi (k_f - k_s)}{k_s + 2k_f - \phi (k_f - k_s)} \tag{4}$$

The boundary conditions of Eqs. (1)-(3) are

$$\bar{u} = a\bar{x} + \frac{\partial \bar{u}}{\partial \bar{y}}, \quad \bar{v} = \bar{v}_w, \quad T = T_w, \quad \bar{y} \to 0 \quad \bar{u} \to 0, \quad T \to T_w \quad \text{as} \quad \bar{y} \to \infty \tag{5}$$

Now introducing the stream function $\psi$, we get

$$x = \frac{\bar{x}}{\sqrt{a}}, \quad y = \frac{\bar{y}}{\sqrt{a}}, \quad u = \frac{\bar{u}}{\sqrt{a}}, \quad v = \frac{\bar{v}}{\sqrt{a}}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

Then the equations (2) and (3) become

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} = \frac{1}{\left(1 - \phi + \frac{\phi a}{\sqrt{T}}\right)^2} \left\{ \frac{1}{\left(1 - \phi\right)^2} \frac{\partial^2 \psi}{\partial y^2} - M\sin^2 \gamma \frac{\partial \psi}{\partial y} \right\} \tag{7}$$

$$\frac{\partial \eta}{\partial y} = \frac{x + \frac{\partial^2 \psi}{\partial y^2}}{\frac{\partial^2 \psi}{\partial x^2}}, \quad \frac{\partial \eta}{\partial x} = S, \quad \theta = 1 \quad \text{at} \quad y = 0 \quad \frac{\partial \eta}{\partial x} \to 0, \quad \theta \to 0 \quad \text{as} \quad y \to \infty \tag{9}$$

where $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Mn = \frac{\sigma B_0^3}{\nu^2}$ is the magnetic parameter and $\beta = \frac{q}{\alpha(p_c p_f)}$ is the heat source/sink parameter.

Now by means of the simplified appearance of Lie-group transformations that is to say, we get hold of the similarity transformations as,

$$\eta = y, \quad \psi = x F(\eta), \quad \theta = \theta(\eta) \tag{10}$$

### 3. Flow Analysis

At this instant via the similarity transformations (7), we acquire

$$F'' + \left(1 - \phi\right)^{2/3} \left\{ \left[1 - \phi + \phi \frac{a}{\sqrt{T}}\right] \left(FF'' - F^2\right) - M\sin^2 \gamma \right\} = 0 \tag{11}$$

The resultant boundary conditions turn out to be

$$F(0) = S, \quad F'(0) = 1 + LF'(0) \quad \text{and} \quad F'(\infty) = 0 \tag{12}$$

The solution of equation (11) gratifying the boundary condition (12) is attained as

$$F(\eta) = S + X \left(\frac{1 - e^{-m \eta}}{m}\right) \tag{13}$$

Where $m$ is known to be the parameter coupled with the nanosolid volume fraction.

$$m = \frac{1}{L + 0.33333A_0 - (0.41999A_0 + A_0^2)} \tag{14}$$
where $A_1 = L S \left( 1 - \phi \right)^{2.5}$, $A_1 = L M \sin^2 \gamma$, $A_3 = L \left( 1 - \phi \right)^{2.5}$, $A_4 = \frac{\bar{n}}{\rho_t}$, 
$A_5 = \rho_t - A_1 \rho_t + A_1 \phi \rho_t - A_1 \phi \rho_s$,
$A_6 = -2 + 27 A_3 + 3 A_1 - 3 \tilde{A}^2 + 2 A^3 - 27 L A_3 + 3 A_1 \phi - 6 \tilde{A}^2 \phi - 6 A^3 \phi + 3 A^3 \phi^2 + 6 A^3 \phi^2$
$- 2 A^3 \phi^3 + 18 A_3 A_4 + 9 A_1 A_2 A_3 \phi + 27 L A_3 A_4 - 3 A_1 A_4 \phi$
$+ 6 A^3 A_4 \phi + 6 A^3 A_4 \phi - 6 \tilde{A}^2 A_4 \phi^2 - 12 A^3 A_4 \phi^2 - 6 A^3 A_4 \phi^3 + 9 A_1 A_2 A_3 A_4 \phi$
$+ 3 A^3 A_4 \phi^2 - 6 A^3 A_4 \phi^2 + 2 A^3 A_4 \phi^3$
$A_7 = \frac{1}{\rho_t} \left( 3 A_3 ( - S \rho_t + 3 \phi \rho_t - A_3 \rho_t - S \phi \rho_s ) \right)$, 
$A_8 = \sqrt{\left( A_6^2 + 4 \left( A_7 - \frac{1}{\rho_t} A_3^2 \right) \right)^3}$, and
$X = \frac{1}{L m + 1}$.

4. Solution of the Thermal Transport

Replacing in the company of similarity variable in Eq.(8), we find

$$0^2 + \frac{Pr_k}{k_{af}} \left( - \phi + \frac{(\tilde{\nu} \nu)}{\nu} \right) \frac{F0'}{k_{af} \beta_0} = 0$$

(15)

And the boundary conditions become $0(0) = 1$ and $0(\infty) = 0$ (16)

Introducing the new variable,

$$\xi = \frac{m^2 k_{af}}{\nu \eta} - \frac{\bar{n}}{\bar{m}} e^{mn}$$

(17)

And inserting (17) in (15), we obtain

$$\xi \theta \eta + (1 - a_0 - \xi) \theta \eta + \frac{Pr_k}{k_{af} m c T} \beta_0(\eta) = 0$$

(18)

and (16) transforms to

$$0 \left( \frac{pr}{\omega \eta} \right) = 1$$

(19)

The solution of the equation (18) with the equivalent boundary conditions (19), we get

$$0(\eta) = e^{-mn a_0 + b_0} M_{a_0 + b_0}^{a_0 + b_0} \left[ \frac{\nu_\nu}{\nu} \eta \right]$$

(20)

Where $M_{a_0 + b_0}^{a_0 + b_0} \left[ \frac{\nu_\nu}{\nu} \eta \right]$ is the Kummer’s function, $\alpha = \frac{k_{af}}{k_{af} 1 - \phi + \frac{\tilde{\nu} \nu}{\nu} \left( \frac{a_0 + b_0}{a_0 + b_0} \right)}$.

$$a_0 = \frac{Pr_k}{\omega \eta} \left( \frac{\eta}{\eta} \right)$$

(21)

And $b_0 = \sqrt{\frac{3}{a_0^2} - 4 \frac{Pr_k}{k_{af}}} m^2 T$.

5. Entropy Generation Analysis

As stated by Bejar [1], the entropy generation equation in dimensional form as

$$S_0 = \frac{k_{af}}{T_e} \left[ \frac{\phi}{\phi} \right]^2 + \frac{\phi}{\phi} \left( \frac{\phi}{\phi} \right)^2 + k_{af} \left[ \frac{\phi}{\phi} \right]^2 + \frac{\eta B_0}{T_e} \tilde{u}^2 \sin^2 \gamma$$

(22)

The right hand side of the exceeding equation is composed of three divisions. The initial part symbolizes the entropy generation owing to heat transfer crosswise a finite temperature variation, the second part corresponds to the local entropy generation owed to viscous dissipation and the third part characterize the local entropy generation appropriate to the effect of the magnetic field. For a prescribed boundary condition the characteristic entropy generation rate is

$$S_0 = \frac{k_{af} (\phi T_e)}{x_0^2 \phi}$$

(23)

For that reason, the entropy generation number is

$$N_s = \frac{S_0}{S_{0b}}$$
By means of Eqs. (20), (21), (22) and (23), the entropy generation number is specified by

$$N_s = \theta^2 \left( \eta \right) \text{Re}_x + \frac{B_r}{\Omega} F^2 \left( \eta \right) \text{Re}_x + \frac{B_h \text{Ha}^2}{\Omega} F^2 \left( \eta \right) \sin^2 \gamma$$

(24)

where $B_r$ is the Brinkman number. $Ha$ is the Hartmann number.

$$Br = \frac{\mu \nu f \bar{u} \bar{w}^2}{k_{ef} \Delta T}, \quad \Omega = \frac{\Delta T}{T_\infty}, \quad Ha = B_0 X \sqrt{\frac{\sigma}{\nu}}$$

(25)

6. Results and Discussion

The physical exhibition of a range of non-dimensionless parameters on the entropy generation, temperature and velocity profiles are observed from the graphical illustration for Ag-water.

Variation of $Mn$, $L$ and $S$ on velocity profile is obtainable in Figure 2. It is observed that an increase in $Mn$, $L$ and $S$ trim down the velocity profile. It is for the reason that, Lorentz force acts as a hold back force. Such hold back force boosts the frictional figureh opposing the fluid activity in the momentum boundary layer thickness. That's why the velocity profile decreases for ever-increasing values of $Mn$. It is also observed that both $L$ and $S$ slow down the fluid motion.

Figure 3 expresses the impact of $\phi$ and $\gamma$ on the velocity profile. It is publicized that as these two parameters are greater than before, the fluid velocity is diminished. Figure 4 indicates analysis of the variation of temperature profile via $Mn$ and $L$. We noticed that an increase in $Mn$ and $L$ makes an enrichment of temperature and its associated layer thickness. Impact of $\phi$ and $S$ on temperature profile is considered in Figure 5. It is depicted that the temperature and thermal layer thickness are increasing functions of $\phi$ and increasing value of $S$ decrease the temperature profile. Influence of $\phi$ and $\beta$ on entropy generation is displayed in Figure 6. It is analyzed that an increase in $\phi$ causes the entropy production minimize. It is also clear that an increasing value of heat sink enhances the entropy generation and it minimized the increasing value of heat source. Characteristics of $Br \Omega^{-1} \cdot S$ and $L$ on entropy generation are delineated in Figure 7. It is noticed that for higher values of $Br \Omega^{-1} \cdot S$, the entropy generation enhances. It is also observed that superior values of $L$, the entropy generation minimizes.

7. Conclusion

An analysis has been passed out to study the effects of partial slip nanofluid flow on entropy generation in a stretching sheet with inclined magnetic field and uniform heat generating/absorbing. Key points are listed below:

(1) The $f(\eta)$ reduces in the presence of $Mn$, $\gamma$, $\phi$, partial slip parameters and suction parameter.

(2) The temperature of the nanofluid raises with $\phi$, $\gamma$, $Mn$, partial slip parameters and reduces with suction parameter.

(3) The entropy generation rises with suction, dimensionless group parameters whereas reduces with partial slip, heat source, nanosolid volume fraction parameters.
Figure 1. Physical sketch of the problem

Table 1. Thermo-physical properties.

| Material     | $\rho$ (kg/m$^3$) | $C_p$ (J/kg$^\circ$K) | $k$ (W/m$^\circ$K) | $\beta \times 10^5$ (K$^{-1}$) |
|--------------|-------------------|-----------------------|--------------------|--------------------------------|
| Pure Water   | 997.1             | 4179                  | 0.613              | 21                             |
| Silver(Ag)   | 10500             | 235                   | 429                | 1.89                           |
Figure 2. $f(\eta)$ variation via $S$, $Mn$, $L$ with $Pr=6.2$; $Mn=1.0$; $\phi=0.1$; $L=0.5$; $S=0.5$; $\gamma=45^\circ$.

Figure 3. $f(\eta)$ variation via $\phi$, $\gamma$ with $Pr=6.2$; $Mn=1.0$; $\phi=0.1$; $L=0.5$; $S=0.5$; $\gamma=45^\circ$.

Figure 4. $\theta(\eta)$ variation via $L$, $Mn$, with $Pr=6.2$; $Mn=1.0$; $\phi=0.1$; $L=0.5$; $S=0.5$; $\gamma=45^\circ$; $\beta=0.2$.

Figure 5. $\theta(\eta)$ variation via $S$, $\phi$, with $Pr=6.2$; $Mn=1.0$; $\phi=0.1$; $L=0.5$; $S=0.5$; $\gamma=45^\circ$; $\beta=0.2$.
Figure 6. $N_s$ variation via $\phi$, $\gamma$ with $Pr=6.2; Mn=1.0; \phi=0.1; L=0.01; S=0.5; \beta=0.2; Ha=1.0; Br\Omega^{-1}=1.0; Re_l=1.0; \gamma=45^\circ$;

Figure 7. $N_s$ variation via $Br\Omega^{-1}, S, L$ with $Pr=6.2; Mn=1.0; \phi=0.1; L=0.01; S=0.5; \beta=0.2; Ha=1.0; Br\Omega^{-1}=1.0; Re_l=1.0; \gamma=45^\circ$;

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