Suppression of light nuclei production in collisions of small systems at the Large Hadron Collider

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We show that the recently observed suppression of the yield ratio of deuterons to protons and of helium-3 to protons in p+p collisions compared to those in p+Pb or Pb+Pb collisions by the ALICE Collaboration at the Large Hadron Collider (LHC) can be explained if light nuclei are produced from the coalescence of nucleons at the kinetic freeze-out of these collisions. This suppression is attributed to the non-negligible sizes of deuterons and helium-3 compared to the size of the nucleon emission source in collisions of small systems, which reduces the overlap of their internal wave functions with those of nucleons. The same model is also used to study the production of triton and hypertriton in heavy-ion collisions at the LHC. Compared to helium-3 in events of low charged particle multiplicity, the triton is less suppressed due to its smaller size and the hypertriton is even more suppressed as a result of its much larger size.

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I. INTRODUCTION

Besides the production of the quark-gluon plasma (QGP) [1, 2], relativistic heavy-ion collisions have also led to the production of anti-nuclei [3, 4] and the discovery of anti-hypernuclei [5, 6]. More recently, light nuclei production in relativistic heavy-ion collisions have further been used to search for the possible critical point [10, 11] in the phase diagram of strongly interacting quark matter [12, 13]. However, how and when these light nuclei are produced during relativistic heavy-ion collisions are still under debate because of their small binding energies and finite sizes [14, 15]. On the one hand, they are assumed to be produced at hadronization of the QGP created in these collisions as in the statistical model for particle production [16, 17]. On the other hand, they are described by the coalescence of nucleons and lambda hyperon at the kinetic freeze-out of heavy-ion collisions when the temperature and density of the hadronic matter are low [18, 19].

In recent measurements by the ALICE Collaboration at the LHC, the yield ratios d/p and 3He/p from p+p, p+Pb and Pb+Pb collisions at center-of-mass energies ranging from 900 GeV to 7 TeV have been measured, and they are found to decrease monotonically with decreasing charged particle multiplicity in the collisions [20, 21]. In particular, the ratio d/p is suppressed by more than a factor of 2 in p+p collisions at $\sqrt{s_{NN}} = 7$ TeV [35] compared to that in central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [7]. Because of the high collision energies, the produced matter consists of nearly equal number of particles and antiparticles, and it reaches almost the same temperature of $T_s \approx 154$ MeV [37, 40] at which the initially produced QGP is transformed to the hadronic matter [41]. Therefore, almost the same chemical freeze-out temperature occurs in p+p, p+Pb and Pb+Pb collisions [42, 43] and the only difference between these colliding systems is the size of produced matter or the total number of produced particles. This is confirmed by the two-pion correlation measurements through the Hanbury-Brown Twiss (HBT) interferometry, which gives the Gaussian source radii in p+p and Pb+Pb collisions that are about 2 fm and 10 fm, respectively [44, 45].

In the statistical hadronization approach based on the grand canonical ensemble, all hadrons produced in heavy-ion collisions at the LHC energies are in thermal and chemical equilibrium, and their yield ratios are determined only by the chemical freeze-out temperature, as the baryon chemical potential is nearly zero in collisions at such high energies [42]. For instance, the measured proton to pion ratio is about $5 \times 10^{-2}$ in all p+p, p+Pb and Pb+Pb collisions, which is consistent with the statistical model prediction based on the grand canonical ensemble. For the yield ratios of d/p and 3He/p, the predicted values of about $3.6 \times 10^{-3}$ and $1.0 \times 10^{-5}$ from the statistical model are also in nice agreement with the experimental data from central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. These values are, however, much larger than those from p+p collisions at the LHC [35]. To explain the suppressed production of light nuclei in such collisions, the statistical model has been modified.
to use the canonical ensemble to taken into account the conservation of baryon charges. However, the resulting ratios of light nuclei to proton in collisions of small systems turn out to be too small compared with the experimental data unless the correlation volume for light nuclei is taken to be three times larger than the volume per unity rapidity for abundant particles, such as the pion, or using a higher chemical freeze-out temperature of 170 MeV than the usual value of 155 MeV for collisions of large systems.

In the coalescence model, the formation probability of a light nucleus in a heavy-ion collision depends not only on the thermal properties and volume of the nucleon and hyperon emission source but also on the internal wave function of the light nucleus. The small size of the emission source in p+p collisions is expected to significantly reduce the phase-space volume in which a light nucleus can be formed, leading to a suppression of its production. Using a schematic coalescence model based on nucleons from the UrQMD model by allowing a deuteron to be formed from a pair of proton and neutron when their separation in phase-space is less than certain value, it is found in Ref. [17] that this model can give a good description of the experimental data on the d/p ratio in p+p, p+Λ, and A+A collisions at the LHC.

In this Letter, we use a more realistic coalescence model to study the system size or charged particle multi-

city dependence of the d/p and He/p ratios as well as their momenta k1 and k2 to their center-of-mass,

\[ \begin{align*}
X &= \frac{x_1 + x_2}{2}, \quad x = x_1 - x_2, \\
K &= k_1 + k_2, \quad k = \frac{k_1 - k_2}{2},
\end{align*} \]

The integrals in Eq. (1) can then be straightforwardly evaluated, leading to

\[ \begin{align*}
N_d &= \frac{8 g_d N_p N_n}{(2\pi)^6 (m_T K R^2)^3} \int d^3 X e^{-\frac{\kappa^2}{4mTK}} \int d^3 k e^{-\frac{k^2}{4mTK}} \\
&= \frac{3N_n N_p}{4(m_T K R^2)^{3/2}} \frac{1}{\left(1 + \frac{mTK}{\sigma^2}\right)^{3/2}} \frac{1}{\left(1 + \frac{\sigma^2}{4\pi^2}\right)^{3/2}}.
\end{align*} \]

The parameter σ in Eq. (4) is related to the root-mean-square matter radius \( r_d = 1.96 \text{ fm} \) of deuteron [59] by \( \sigma = \sqrt{8/3} r_d \approx 3.2 \text{ fm} \). For the kinetic freeze-out temperature \( T_k \) of nucleons, it is typically of the order of 100 MeV. We therefore have \( m_T K \gg 1/\sigma^2 \), and the yield ratio d/p is then approximately given by

\[ \frac{N_d}{N_p} \approx \frac{3N_n}{4(m_T K R^2)^{3/2}} \left(1 + \frac{1.6 \text{ fm}}{R}\right)^{3/2}. \]
The last factor in the above equation describes the suppression of deuteron production due to its finite size relative to that of the nucleon emission source. Its value approaches unity as the source radius \( R \) becomes much larger than the size of deuteron, while it is significantly smaller than unity when \( R \) is close to or less than 1.6 fm. The factor \( C_1 = \frac{3N_\text{eff}}{4mT_KR^2} \) in Eq. (6) corresponds to the \( d/p \) ratio in the limit of large nucleon emission source when the suppression effect due to finite deuteron size is negligible, and it is directly related to the entropy per nucleon in a nuclear collision, which remains essentially unchanged after chemical freeze-out \[60\]. Therefore, the value of \( C_1 \) is the same in p+p, p+Pb and Pb+Pb collisions at the LHC.

The radius of the Gaussian emission source for nucleons at kinetic freeze-out is known to be proportional to the charged particle multiplicity \( (dN_{ch}/d\eta) \) in a collision \[61\],

\[
R = \lambda(dN_{ch}/d\eta)^{1/3}. \tag{7}
\]

The proportional constant \( \lambda \) in the above equation can be determined from the value of \( R \) in central Pb+Pb collisions. In terms of the Gaussian radii \( R_{\text{side}} \) and \( R_{\text{long}} \) in the side and longitudinal directions as extracted from the two-pion interferometry measurements, see Ref. \[62\] for an overview, an effective radius can be defined by \( R_{\text{eff}} = (R_{\text{side}}^2 R_{\text{long}})^{1/3} \). Because of the expansion of the emission source, the values of \( R_{\text{side}} \) and \( R_{\text{long}} \) depend on the momentum \( k_T \) in the analysis. Also, there is a reduction of about 20-25\% when \( R_{\text{side}} \) is used to estimate the transverse size, suggesting that there is an additional factor of about 1.3 when we use \( R_{\text{eff}} \) for the value of \( R \) \[61, 62\]. From the measured values of \( R_{\text{side}} = 6.9 \pm 0.47 \) fm and \( R_{\text{long}} = 8.26 \pm 0.44 \) fm at \( k_T = 0.25 \) GeV in central Pb+Pb collisions with \( dN_{ch}/d\eta \) of about 1500 at \( \sqrt{s_{NN}} = 2.76 \) TeV \[7, 14\], we obtain \( R_{\text{eff}} = 7.3 \pm 0.46 \) fm and \( R \approx 1.3R_{\text{eff}} = 9.5 \pm 0.6 \) fm, corresponding to a volume of \( V_f = (2\pi)^{3/2}R^3 \approx 1.4 \times 10^4 \) fm\(^3\), which is about three times of the chemical freeze-out volume of 5.38 \( \times \) 10\(^4\) fm\(^3\) \[12\]. This gives a value of \( \lambda = 0.83 \pm 0.005 \) fm in Eq. (7). With the factor \( C_1 = 4.0 \times 10^{-3} \) determined from the measured \( d/p \) ratio of 3.8 \( \times \) 10\(^{-3}\) in these collisions at \( dN_{ch}/d\eta = 1000 \), which corresponds to \( R \approx 8.3 \) fm, Eq. (6) can be rewritten as

\[
\frac{N_3}{N_p} \approx \frac{4.0 \times 10^{-3}}{1 + (\frac{2.6 \text{ fm}}{R})^{3/2}}. \tag{8}
\]

FIG. 1: charged particle multiplicity dependence of the yield ratios \( d/p \), \(^3\)He/p and \(^3\)H/\(^3\)He. The lines denote the predictions of coalescence model with theoretical uncertainties on the emission source radius given by the shadow region around the lines. Experimental data from the ALICE Collaboration are shown by symbols with error bars \[2, 33, 64, 65\].

In panel (a) of Fig. 1 we plot the \( d/p \) ratio as a function of \( dN_{ch}/d\eta \). Compared with the measured ratio in central Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV \[2, 64\] and in p+Pb collisions at \( \sqrt{s_{NN}} = 900 \) GeV, 2.76 TeV and 7 TeV \[35, 62\], the theoretical results are in nice agreement with the data for all charged particle multiplicities. Our results are consistent with those from a schematic coalescence model based on kinetic freeze-out nucleons from the UrQMD model \[17\]. We note that the finite deuteron size suppresses not only the total yield ratio of deuteron to proton as studied here but also their ratio as a function of transverse momentum \[23\].

Similarly, we can calculate the charged particle multiplicity dependence of the \(^3\)He/p ratio by extending the formalism for deuteron production from proton and neutron coalescence to the production of helium-3 from the coalescence of two protons and one neutron as in Refs. \[24, 33\]. The resulting yield ratio \(^3\)He/p is given by

\[
\frac{N_3}{N_p} \approx \frac{N_n}{4(mT_KR^2)^3} \frac{1}{1 + (\frac{r_3^{\text{He}}}{R})^3}, \tag{9}
\]

where \( r_3^{\text{He}} = 1.76 \) fm is the radius of helium-3 \[59\].
In obtaining the above equation, we have included the statistical factor of 1/4 for forming a spin 1/2 helium-3 from three spin 1/2 nucleons and used the condition \( nTK \gg 1/r_{3H}^2 \). With the factor \( C_2 = \frac{N_{3H}N_{Λ}}{4(mTK R^2)^2} = 4C_1/9 = 7.1 \times 10^{-6} \), determined from the value of \( C_1 \), Eq. (9) becomes

\[
\frac{N_{3H}}{N_p} \approx \frac{7.1 \times 10^{-6}}{1 + (1.24 \text{ fm} R)^2} \left[ 1 + \left( \frac{4.6 \text{ fm} R}{R} \right)^2 \right]^3/2.
\] (10)

We also consider \(^3\text{He}\) production from the coalescence of a deuteron and a proton. In this case, the root-mean-square radius of \(^3\text{He}\) can be estimated as \( r_{3H} \approx (3/8)^{1/2} \sqrt{(r_{pd})^2} = 1.15 \text{ fm} \) with \( \sqrt{(r_{pd})^2} \approx 2.6 \text{ fm} \) being the distance between proton and the center of mass of the deuteron inside the helium-3. The \(^3\text{He}/p\) ratio in this case is

\[
\frac{N_{3H}}{N_p} \approx \frac{7.1 \times 10^{-6}}{1 + (1.15 \text{ fm} R)^2} \left[ 1 + \left( \frac{4.6 \text{ fm} R}{R} \right)^2 \right]^3/2.
\] (11)

where the suppression factor for deuteron production has been included. As shown in panel (b) of Fig. 1, the contribution from the coalescence of deuteron and proton is stronger than that from the coalescence of two protons and one neutron in collisions of small charged particle multiplicities, although the two processes give similar contributions to \(^3\text{He}\) production in collisions of large charged particle multiplicities. Besides, the theoretical results are found in nice agreement with the data at \( dN_{\text{ch}}/d\eta < 1000 \), while they are smaller than the data in the most central Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \).

The above calculation for helium-3 can be repeated for triton (\(^3\text{H}\)). Because of its smaller radius, \( r_{3H} = 1.59 \text{ fm} \) [53] than helium-3, triton production in collisions with low multiplicities is expected to be less suppressed than helium-3. Shown in panel (c) of Fig. 1 is the \(^3\text{H}/\text{He}\) yield ratio as a function of charged particle multiplicity. It is seen that this ratio indeed increases with decreasing charged particle multiplicity, particularly for triton and helium-3 production from three-body coalescence. For instance, this ratio in \( p+p \) collisions at \( dN_{\text{ch}}/d\eta = 5 \) is predicted to be 1.1 if triton and helium-3 are produced from two-body coalescence but increases to 1.3 if they are produced from three-body coalescence, suggesting a 10%-30% enhancement in the production of triton than helium-3 in \( p+p \) collisions. Future measurements of the triton yield in \( p+p \) collisions can be used to test this result.

III. \(^3\Lambda\) PRODUCTION IN COALESCENCE MODEL

To study the production of \(^3\Lambda\) in collisions of small systems, we first note that it is the lightest known nucleus with strangeness, and it has a small binding energy of only \( B_\Lambda = 2.35 \text{ MeV} \) and a large root-mean-square radius of \( r_\Lambda \approx 4.9 \text{ fm} \) [66]. Besides being a bound state of proton, neutron and \( \Lambda\)-hyperon, the hypertriton can also be considered as a bound state of a deuteron and a \( \Lambda\)-hyperon with a binding energy \( B_\Lambda = 0.13 \pm 0.05 \text{ MeV} \) [67] and a distance of \( r_{\Lambda d} \approx 10 \text{ fm} \) [68] between deuteron and \( \Lambda\)-hyperon. Because of its large size, the production of \(^3\Lambda\) in collisions of small systems is expected to be much more suppressed than that of helium-3. We note that the study of \(^3\Lambda\) production in relativistic heavy-ion collisions including both the coalescence of \(^3\Lambda\)-n-\( \Lambda\) and of \( d-\Lambda\) has recently been reported in Ref. [68]. According to this study, the process of \( p-\Lambda\) coalescence is more important than the \( d-\Lambda\) coalescence for hypertriton production, and the hypertriton yield in relativistic heavy-ion collisions is essentially determined at the time when nucleons and deuterons freeze out, although it still undergoes reactions with pions.

Similar to helium-3 production, the yield ratio \(^3\Lambda/\Lambda\) is given by

\[
\frac{N_{3\Lambda}}{N_{\Lambda}} \approx \frac{7.1 \times 10^{-6}}{1 + (\frac{4.6 \text{ fm} R}{R})^2} \left[ 1 + \left( \frac{4.6 \text{ fm} R}{R} \right)^2 \right]^3/2.
\] (12)

for hypertriton production from the coalescence of proton, neutron and \( \Lambda\)-hyperon, and

\[
\frac{N_{3\Lambda}}{N_{\Lambda}} \approx \frac{7.1 \times 10^{-6}}{1 + (\frac{4.2 \text{ fm} R}{R})^2} \left[ 1 + \left( \frac{4.6 \text{ fm} R}{R} \right)^2 \right]^3/2.
\] (13)

for hypertriton production from the coalescence of \( d\) and \( \Lambda\). In obtaining Eq. (12) for the three-body coalescence process, we have taken the root-mean-square radius of \(^3\Lambda\) as \( r_{3\Lambda} \approx (3/8)^{1/2} \sqrt{(r_{pd})^2} = 4.2 \text{ fm} \). Also, we have neglected the mass difference of the constituent particles in obtaining above expressions since its effect is small.

In panel (a) of Fig. 2, we show the charged particle multiplicity dependence of the yield ratio \(^3\Lambda/\Lambda\) in Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \). The dashed and solid lines represent results from the two-body and the three-body coalescence, respectively. It is visible that there is no significant difference between these two processes when \( dN_{\text{ch}}/d\eta > 100 \). For \( dN_{\text{ch}}/d\eta \sim 10 \), both production processes give a yield ratio \(^3\Lambda/\Lambda\) that is two-order of magnitude less than in central Pb+Pb collisions.

We further investigate the strangeness population factor \( S_3 \), which is a double ratio defined by \( S_3 = \frac{N_{3\Lambda}/(\text{He} \times \Lambda/p)}{N_{\text{ch}}/P_{\text{ch}}} \) [69]. It was suggested in Ref. [69] that the value of \( S_3 \) should be about one in the coalescence model for particle production. It was also argued in Ref. [70] that this factor might be a good signal for the local correlation between baryon number and strangeness in a quark-gluon plasma [71], providing thus a valuable probe to the onset of deconfinement in relativistic heavy-ion collisions. The system size dependence of \( S_3 \) can be calculated from Eqs. (10) and (12), for \(^3\text{He}\) and \(^3\Lambda\) production from three-body coalescence and from Eqs. (11) and (13) for their production from two-body coalescence. Results for Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \) will be presented in a forthcoming paper.
helium-3, and triton production in nuclear collisions at energies available from the LHC on the charged particle multiplicity of the collisions. For the nucleon distributions, they are assumed to come from a thermalized hadronic matter at the kinetic freeze-out of heavy-ion collisions with its radius taken to be proportional to the charged particle multiplicity, as indicated by the pion interferometry measurements. We have found that the yield ratios \( \frac{d}{p} \) and \( \frac{\text{He}}{p} \) are significantly reduced once the charged particle multiplicity is below about 100 as a result of the non-negligible deuteron and \( \text{He} \) sizes compared to the nucleon emission source. Our results thus provide a natural explanation for the observed suppression of deuteron and \( \text{He} \) production in p+p collisions by the ALICE Collaboration at the LHC. They also demonstrate the importance of the internal structure of light nuclei on their production in collisions of small systems. We have further found that the production of triton is 10%-30% more than that of helium-3 in p+p collisions because of its smaller matter radius. This enhancement of \( \frac{\text{H}}{\text{He}} \) ratio can be tested in future measurements.

We have also used this model to study the charged particle multiplicity dependence of hypertriton production in Pb+Pb collisions at the LHC by considering both the three-body process of p-n-\( \Lambda \) coalescence and the two-body process of d-\( \Lambda \) coalescence. Because of the much larger \( \frac{3}{2} \text{H} \) radius than those of deuteron and \( \text{He} \), the yield ratio \( \frac{3}{2} \text{H}/\Lambda \) is found to be much more suppressed in collisions with low charged-particle multiplicity, particularly for the three-body coalescence process. Future experimental measurements of the yield ratio \( \frac{3}{2} \text{H}/\Lambda \) in collisions of low charged particle multiplicity will be of great interest because it not only can check the prediction of the present study but also provide the possibility to improve our knowledge on the internal structure of \( \frac{3}{2} \text{H} \).

**IV. CONCLUSIONS**

In summary, based on the coalescence model in full phase space, we have studied the dependence of deuteron, \( \text{He} \), and triton production in nuclear collisions at energies available from the LHC on the charged particle multiplicity of the collisions. For the nucleon distributions, they are assumed to come from a thermalized hadronic matter at the kinetic freeze-out of heavy-ion collisions with its radius taken to be proportional to the charged particle multiplicity, as indicated by the pion interferometry measurements. We have found that the yield ratios \( \frac{d}{p} \) and \( \frac{\text{He}}{p} \) are significantly reduced once the charged particle multiplicity is below about 100 as a result of the non-negligible deuteron and \( \text{He} \) sizes compared to the nucleon emission source. Our results thus provide a natural explanation for the observed suppression of deuteron and \( \text{He} \) production in p+p collisions by the ALICE Collaboration at the LHC. They also demonstrate the importance of the internal structure of light nuclei on their production in collisions of small systems. We have further found that the production of triton is 10%-30% more than that of helium-3 in p+p collisions because of its smaller matter radius. This enhancement of \( \frac{\text{H}}{\text{He}} \) ratio can be tested in future measurements.

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