TRAINING NEURAL NETWORKS FOR SEQUENTIAL CHANGE-POINT DETECTION

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ABSTRACT

Detecting an abrupt distributional shift of a data stream, known as change-point detection, is a fundamental problem in statistics and machine learning. We introduce a novel approach for online change-point detection using neural networks. To be specific, our approach is training neural networks to compute the cumulative sum of a detection statistic sequentially, which exhibits a significant change when a change-point occurs. We demonstrated the superiority and potential of the proposed method in detecting change-point using both synthetic and real-world data.

1. INTRODUCTION

Detecting a change-point (CP) is a fundamental problem in statistics and machine learning and has many applications to practical problems across diverse fields such as epidemiology [1], social network analysis [2], and scientific imaging [3] (we refer to a recent survey [4] for further information and background). In particular, CP detection in online data streams has recently gained attention due to the growing amount of online data. For example, [5, 6] used Gaussian process and [7] used kernel two-sample test with generative models for quicker CP detection in online data. While using neural networks for CP detection has several advantages (e.g., high capacity and non-parametric model), it has not been comprehensively explored yet.

In this paper, we present a novel approach for training neural networks to achieve CP detection. The motivation is to convert online CP detection to a binary classification problem. Specifically, trained neural networks with logistic loss converges to a log-likelihood ratio of binary-labeled data, which is expected to show a quick response to CP once it has occurred. Furthermore, we develop an efficient training procedure to quickly detect CP in online streaming data. We empirically demonstrated superior performance of the proposed approach in online CP detection by conducting experiments on synthetic and real-world data.

2. SETUP

Observing a sequence of \(d\)-dimensional data, our objective is to detect an unknown change-point \(k\) where the distribution of data changes. We are interested in detecting change-point as soon as possible after its occurrence with a false-alarm constraint. This can be formulated as a sequential hypothesis testing, which can be expressed as follows:

\[
H_0 : x_1, \ldots, x_t \iid f_0, \\
H_1 : x_1, \ldots, x_k \iid f_0, \quad x_{k+1}, \ldots, x_t \iid f_1.
\]

We assume the availability of ample data from the pre-change distribution \(f_0\), which we refer to pilot sequence. We refer to the sequence we are interested in detecting a change-point as online sequence. This is a common setup in change-point detection problem.

3. BACKGROUND

We start by providing an overview of some key approaches in change-point detection.

**exact-CUSUM.** If pre-change \((f_0)\) and post-change \((f_1)\) distributions are available, log-likelihood ratio can be computed recursively on the data and used as detection statistic. One can compute the log-likelihood ratio

\[
r(x_t) = \log \frac{f_1(x_t)}{f_0(x_t)},
\]

on any data at time \(t\). Starting with an initialization \(S_0 = 0\), cumulative sum of the detection statistic at time \(t\) is updated as

\[
S_t = (S_{t-1} + r(x_t))^+, \quad \text{where } (x)^+ = \max\{x, 0\}.
\]

The stopping time for detection is

\[
\tau_{CUSUM} = \inf\{t : S_t > b\},
\]

where \(b > 0\) is a pre-specified threshold that meets the false alarm constraint. The exact-CUSUM procedure enjoys various asymptotic optimality properties [4]. However, its performance deteriorates rapidly when the specifications of \(f_0\) and \(f_1\) deviate from the true distributions. Due to the difficulty

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1The first author is a student is eligible for student paper competition.
in precise density estimation, especially for high-dimensional data, there is a significant need for robust and non-parametric methods for CP detection in practice.

**Hotelling’s T-squared CUSUM** Instead of using the log-likelihood ratio, Hotelling’s T-squared CUSUM (Hotelling-CUSUM) uses Hotelling’s T-squared statistic for change-point detection. It can be treated as a non-parametric detection statistic (only uses the first and the second order moments). Hotelling’s T-squared statistic is known to be effective in detecting mean shifts, but it may not perform well in detecting other types of changes, such as covariance shifts.

At time \( t \) in detecting other types of changes, such as covariance shifts, we can compute

\[
g^H(x_t) := g_0^H(x_t) - \hat{d}_p,
\]

\[
g_0^H(x_t) := \frac{1}{2} (x_t - \hat{\mu}_p)^T (\hat{\Sigma}_p + \lambda I_d)^{-1} (x_t - \hat{\mu}_p),
\]

where \( \hat{\mu}_p \) and \( \hat{\Sigma}_p \) are estimated mean and covariance matrix from the pilot sequence assumed to be drawn from \( f_0 \). \( \hat{d}_p \) is usually set to be \( \mathbb{E}_{x \sim f_0} [g_0^H(x)] + \epsilon \) with a small constant \( \epsilon \) and is computed by sample average on a test stack of the pilot sequence. The positive scalar \( \lambda \) is a regularization parameter in case \( \hat{\Sigma}_p \) is singular (e.g., when the data dimension is high compared to the size of the pilot sequence). Once \( \hat{\mu}_p, \hat{\Sigma}_p, \) and \( \hat{d}_p \) are computed on the pilot stack, cumulative sum of the detection statistic at time \( t \) is updated as

\[
S_t^H = (S_{t-1}^H + g^H(x_t))^+.
\]

The stopping time for detection is

\[
\tau_H = \inf \{ t : S_t^H > b_H \},
\]

where \( b_H \) is pre-specified threshold.

### 4. PROPOSED METHOD: NN-CUSUM

The proposed method is referred to as neural-network CUSUM (NN-CUSUM), as it utilizes neural networks to compute cumulative sum of the detection statistic for change-point detection. Consider a neural network \( g_\theta(x) : \mathbb{R}^d \to \mathbb{R} \), where \( \theta \) denotes its parameters. The neural network takes pilot sequence and online sequence as input. During training, the data from pilot training stack are labeled \( y_i = 0 \) and the data from online training stack are labeled \( y_i = 1 \) when they are sequentially fed into the network. After training, we use the difference of average logistic function values (i.e., logits) of the pilot testing stack and online testing stack as detection statistic. In particular, we split pilot and online sequence into into training and testing stack so that the detection statistic can have the desired property: increasing quickly after change-point with a small negative drift before change-point.

**Fig. 1.** Data processing for neural-network CUSUM (NN-CUSUM). Once a new sliding window of online data (labeled \( y_i = 1 \)) is received, the window is divided by half to be included into training stack and testing stack, respectively. The new data replace the oldest data in the stack. Pilot training and testing stacks are constructed in the same way. Then, the network will be updated using stochastic gradient descent with pilot and online training stack. The updated network is then used to compute test statistic for the data in testing stack. Note that the size of the training and testing stack is an important hyperparameter that can significantly impact the detection performance.

**4.1. Training of NN-CUSUM**

The training of the network is conducted via stochastic gradient descent (SGD) updates of the neural network parameter \( \theta \) on training stack. The pilot and online training stack contains multiple mini-batches of data from pilot and online sequence, respectively (Figure 1). The sliding window moves forward by a stride of length \( s \). The half of the stride is put into the training stack and the remaining half is put into the testing stack. Suppose we have \( m \) data in both pilot testing stack and online testing stack. Then, the training objective is the logistic loss of the binary classification task, given by:

\[
\ell(\theta; \{x_i, \tilde{x}_i\}_{i=1}^m) = \sum_{i=1}^m (\log(1 + e^{g_\theta(x_i)}) + \log(1 + e^{-g_\theta(\tilde{x}_i)})],
\]

where \( x_i \) is from online testing stack and \( \tilde{x}_i \) is from pilot testing stack.

The following proposition provides a motivation for choosing the logistic loss for training NN-CUSUM. It is known that training of neural networks with logistic loss will lead to the log-likelihood ratio at the global minimum.

**Proposition 4.1.** For \( p \) and \( q \), which are probability densities on \( \mathbb{R}^d \), let

\[
\ell[g] = \int \log(1 + e^{g(x)})p(x)dx + \int \log(1 + e^{-g(x)})q(x)dx,
\]
then \( \ell(g) \) is minimized at \( g^* = \log(q/p) \).

Proof. \( \ell(g) \) is convex with respect to the perturbation in \( g \). Therefore,

\[
\frac{\delta \ell}{\delta g}(x) = pe^g - q \frac{1}{1 + e^g}.
\]

\( \delta \ell/\delta g \) vanishes when \( e^g = q/p \) (\( g = g^* \)), which is a global minimum of \( \ell(g) \). \( \Box \)

4.2. Detection statistic of NN-CUSUM

Given the trained neural network \( g_\theta(\cdot) \) at time \( t \), we can compute the average difference of the logits of pilot testing stack (\( X^{te} \)) and online testing stack (\( \tilde{X}^{te} \)) as

\[
\eta_t = \frac{1}{m} \sum_{x \in X^{te}} g_\theta(x) - \frac{1}{m} \sum_{x \in \tilde{X}^{te}} g_\theta(x),
\]

where \( m \) is the number of data in pilot testing stack and online testing stack. Starting with \( S_0^{NN} = 0 \), cumulative sum of the detection statistic at time \( t \) is updated as

\[
S_t^{NN} = \left( S_{t-1}^{NN} + \eta_t \right)^+.
\]

The stopping time for detection is

\[
\tau^{NN} = \inf \{ t : S_t^{NN} > b^{NN} \},
\]

where \( b^{NN} \) is pre-specific threshold.

5. NUMERICAL EXAMPLES

In this section, we conducted numerical examples using synthetic and real-world data. Hotelling-CUSUM and exact-CUSUM were used as baseline methods for performance comparison. We used two evaluation metrics: average run length (ARL) and expected detection delay (EDD). ARL is the expected stopping time under the assumption that there is no change-point: \( \mathbb{E}^{\infty}[\tau] \); here \( \mathbb{E}^{\infty} \) denotes the measure that the change has never occurred (i.e., the distribution under \( H_0 \)). EDD is the expected stopping time after the change-point occurs: \( \mathbb{E}^k[\tau - k|\tau > k] \); often \( k \) is taken to be 1 (i.e., the change-point occurs at the first timepoint). Given a specific ARL, a method having lower EDD is desirable. In the examples with real-world data, we evaluated NN-CUSUM and Hotelling-CUSUM using the behavior of the detection statistic.

The neural network architecture of NN-CUSUM consists of one fully-connected hidden layer with ReLU activation. We set the dimension of the hidden layer to 512. The last layer is one-dimensional fully-connected layer without activation. Training batch size was set to 10 with 5 epochs. Stride of sliding window and the number of training/testing stacks were set to 10 and 100 respectively. We used Adam [8] for training.

![Figure 2](image-url)

**Fig. 2.** Expected detection delay (EDD) versus average run length (ARL) for NN-CUSUM, Hotelling-CUSUM, and exact-CUSUM on 100-dimensional Gaussian sparse mean shift and Gaussian covariance shift.

5.1. Gaussian with Mean and Covariance Shift

We first evaluated the methods on 100-dimensional Gaussian with sparse mean shift and Gaussian with covariance shift.

- In Gaussian with sparse mean shift, pre-change distribution \( f_0 \) is \( \mathcal{N}(0, I_d) \) and post-change distribution \( f_1 \) is \( \mathcal{N}(\mu_q, I_d) \), where \( d = 100 \) and \( \mu_q = (\delta, \delta/2, \delta/3, 0, \ldots, 0) \) with \( \delta = 1 \).

- In Gaussian with covariance shift, pre-change distribution \( f_0 \) is \( \mathcal{N}(0, I_d) \) and post-change distribution \( f_1 \) is \( \mathcal{N}(0, (1 - \rho)I_d + \rho E) \), where \( E \) is \( d \times d \) all-ones matrix and \( \rho = 0.2 \).

Figure 2 displays the log ARL-EDD plot for NN-CUSUM, Hotelling-CUSUM, and exact-CUSUM. The results show that NN-CUSUM outperforms Hotelling-CUSUM. exact-CUSUM shows the best performance since we know both pre-change and post-change distributions. However, in most practical settings, the pre-change and post-change distributions are unknown.

5.2. Human Activity Detection

We evaluated NN-CUSUM and Hotelling-CUSUM on Human Activity Sensing Consortium Challenge 2011 Dataset (HASC data). The dataset contains 3-dimensional G-force measurements of several human activities collected by portable 3D accelerometers. We selected four human activities: "walk"; "stair up"; "elevator up"; and "stay". We then took a sequence of 500 timepoint length from each activity by manually investigating the data. This pre-processing was necessary due to noise in the data (e.g., "elevator up" contains some measurements similar to "stay" before the elevator starts moving). We constructed four activity pairs: (1) "elevator up"-"walk"; (2) "stay"-"elevator up"; (3) "walk"-"stay"; and (4) "stair up"-"stay". Figure 3 shows trajectories of detection statistics computed by using NN-CUSUM and Hotelling-CUSUM. NN-CUSUM showed much more reliable detection performance while Hotelling-CUSUM failed to detect the change in a some cases.
6. CONCLUSION

In this paper, we present a novel method for change-point detection using neural network. Our proposed method outperforms baseline method using Hotelling’s T-squared statistic, which is commonly used method for change-point detection. Particularly, the proposed method showed reliable performance in detecting change-points in real-world data, where no information about pre-change and post-change distributions are available. This demonstrates that our proposed method can avoid the issue of model mismatch, which many classic change-point detection methods suffer from.

Future work includes establishing a theoretical performance bound for NN-CUSUM combining the training dynamics of neural networks with analysis of change-point detection procedures. Comprehensive evaluation with more baselines is also required using diverse real-world data to demonstrate the generalizability of the proposed method and its superiority over existing methods.

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7. REFERENCES

[1] Michael Baron, V Antonov, C Huber, M Nikulin, and VJL Polischook, “Early detection of epidemics as a sequential change-point problem,” *Longevity, aging and degradation models in reliability, public health, medicine and biology, LAD*, pp. 7–9, 2004.

[2] Leto Peel and Aaron Clauset, “Detecting change points in the large-scale structure of evolving networks,” in *Twenty-Ninth AAAI Conference on Artificial Intelligence*, 2015.

[3] Ming Qu, Frank Y Shih, Ju Jing, and Haimin Wang, “Automatic solar filament detection using image processing techniques,” *Solar Physics*, vol. 228, no. 1, pp. 119–135, 2005.

[4] Liyan Xie, Shaofeng Zou, Yao Xie, and Venugopal V Veeravalli, “Sequential (quickest) change detection: Classical results and new directions,” *IEEE Journal on Selected Areas in Information Theory*, vol. 2, no. 2, pp. 494–514, 2021.

[5] Edoardo Caldarelli, Philippe Wenk, Stefan Bauer, and Andreas Krause, “Adaptive gaussian process change point detection,” in *International Conference on Machine Learning*, PMLR, 2022, pp. 2542–2571.

[6] Hossein Keshavarz, Clayton Scott, and XuanLong Nguyen, “Optimal change point detection in gaussian
processes,” *Journal of Statistical Planning and Inference*, vol. 193, pp. 151–178, 2018.

[7] Wei-Cheng Chang, Chun-Liang Li, Yiming Yang, and Barnabás Póczos, “Kernel change-point detection with auxiliary deep generative models,” in *International Conference on Learning Representations*, 2018.

[8] Diederik P Kingma and Jimmy Ba, “Adam: A method for stochastic optimization,” *arXiv preprint arXiv:1412.6980*, 2014.

[9] Simon Fothergill, Helena Mentis, Pushmeet Kohli, and Sebastian Nowozin, “Instructing people for training gestural interactive systems,” in *Proceedings of the SIGCHI conference on human factors in computing systems*, 2012, pp. 1737–1746.