Quantum-mechanical description of in-medium fragmentation

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We present a quantum-mechanical description of quark-hadron fragmentation in a nuclear environment. It employs the path-integral formulation of quantum mechanics, which takes care of all phases and interferences, and which contains all relevant time scales, like production, coherence, formation, etc. The cross section includes the probability of pre-hadron (colorless dipole) production both inside and outside the medium. Moreover, it also includes inside-outside production, which is a typical quantum-mechanical interference effect (like twin-slit electron propagation). We observe a substantial suppression caused by the medium, even if the pre-hadron is produced outside the medium and no energy loss is involved. This important source of suppression is missed in the usual energy-loss scenario interpreting the effect of jet quenching observed in heavy ion collisions. This may be one of the reasons of a too large gluon density, reported by such analyzes.

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I. INTRODUCTION

Hadronization in a nuclear environment has always been a precious source of information about the space time pattern of hadronization. This process is characterized by the production length of a pre-hadron (a colorless dipole), with the subsequent development of the hadronic wave function. A perturbative description of this process as radiation of a \( \bar{q}q \) pair and creation of colorless dipole evolving to the final pion, as is illustrated in Fig. 1, was proposed in [1, 2]. Although the model was partly probabilistic, it led to rather successful description [2] and even prediction [1] for semi-inclusive hadron production in deep-inelastic scattering (DIS) [3]. A pure classical string model [4–6] fitted to data also leads to a good agreement. No fully quantum mechanical description of in-medium hadronization has been available so far.

The widely debated question whether the hadronization process ends by a leading pre-hadron production "within or without the medium?" [2], strictly speaking does not have a definite answer. In quantum mechanics a pre-hadron may be created both inside and outside the medium, and interference of the corresponding amplitudes is important, as is demonstrated below.

Here we are developing the model [1, 2] of perturbative hadronization and employ the Berger model [7], which we improved in a recent paper [8]. In this model an energetic quark produced in a hard reaction, like \( e^+e^- \) annihilation, DIS, or high-\( p_T \) scattering, creates a leading pion, which carries a major fraction \( z_h \to 1 \) of the quark momentum, via perturbative radiation of a gluon decaying into a \( \bar{q}q \) pair. Then the \( \bar{q} \) fuses with the original quark into the pion, as is illustrated in Fig. 1 for \( \gamma^* \)-nucleus collision. In general, fragmentation is process dependent, due to the higher twist terms calculated in [7, 8], which we neglect here. Therefore our results are applicable to any hard reaction leading to the production of a quark jet.

The main approximations in the Berger model [7] are: (i) the calculation is done to lowest order of \( (1 - z_h) \), which is considered as a small parameter; (ii) the pion is treated as a \( \bar{q}q \) state with no transverse or longitudinal motion of the quarks. New calculations without these approximations were done in [8], where the cross section of the process \( \gamma^*p \to \pi X \) was calculated to all orders of \( (1 - z_h) \) and with a realistic model for the light-cone pion wave function.

The calculations performed in [7, 8] were done in momentum representation, employing the Feynman diagram technique, and for the case of a proton target. Usually for nuclear targets the impact parameter representation is more effective, since at high energies the impact parameters do not vary during propagation through the

![Diagram](https://via.placeholder.com/150)

FIG. 1: Reaction \( \gamma^*p \to \pi X \). The incoming virtual proton is absorbed by a valence quark of a bound nucleon, leading to the production of a quark and gluon, \( \gamma^*q \to q\bar{q}g \). The gluon decays to a \( \bar{q}_2g_3 \), and \( q_1 - \bar{q}_2 \) fuse to the final pion. The intermediate and final partons can experience multiple interactions in the nuclear medium.
nucleus, which allows to apply a Glauber-like eikonalization. If the energy is not sufficiently high to freeze the impact parameters, one should integrate over all possible paths of the propagating partons.

Here we employ the light cone Greens function formalism which is the essential tool for the calculation of the nuclear effects [9–12]. We calculate the ratio of \( p_T \)-integrated cross sections,

\[
R_{A/p}(z_h) = \frac{d\sigma(\gamma^* A \rightarrow \pi X)/dz_h}{\sigma(\gamma^* N \rightarrow \pi X)/dz_h},
\]

as function of pion fractional light-cone momentum \( z_h \).

We consider gluon decay and pre-hadron (\( q_2 q_1 \)) production both inside and outside the nucleus. However, these two possibilities can be clearly separated only in a probabilistic approach. In fact, in quantum mechanics the cross section is related to the square of the process amplitude, and the production of the pre-hadron occurs in the direct and conjugated amplitudes at different points. This also implies the appearance of an interference term between inside-outside production, which we consider in our calculation.

The paper is organized as follows. In section II we introduce the kinematic variables and present the general structure of the amplitude of jet production. The amplitude is written in impact parameter presentation, which is especially convenient for the calculation of multiple interaction effects.

In section III the cross section of inclusive hadron production is expressed via the light-cone Green functions describing the propagation of parton ensembles through a nuclear medium. The composition of these ensembles correlates with the coordinates of gluon-to-\( \bar{q} q \) decays in the two amplitudes, direct and complex conjugated. Different colorless parton ensembles propagating through the nucleus interact with different multi-parton cross sections. These cross sections are derived in Appendix B and expressed in terms of phenomenologically known cross sections of \( \bar{q} q \) dipoles interacting with a proton. Using known multi-parton cross sections one can solve the light-cone Schrödinger equations for the Green function describing propagation of partonic ensembles. These solutions are found in Appendix C.

In section IV the cross section is presented as a sum of three terms, Eq. (32), the first two terms corresponding to the amplitudes for gluon decay both inside or outside the nucleus, and the third term corresponding to inside-outside interference. These three terms are further evaluated employing different models for the pion wave function, which differ in the assumptions about the longitudinal and transverse momentum distributions of valence quarks in the pion. The unrealistic assumption of the Berger model that the quarks have no Fermi motion results in a complete absence of nuclear effects, while more realistic models lead to a considerable nuclear suppression.

Section V presents the main results and observations of this paper, as well as an outlook to future developments.

II. THE PROCESS δ\( \gamma^* p \rightarrow \pi X \) IN IMPACT PARAMETER REPRESENTATION

A. The amplitude

In what follows we consider the reaction \( \gamma^* p \rightarrow \pi X \) as an example of a hard reaction, neglecting the higher-twist terms calculated in [8]. Therefore, the space time development of this process and all the results are valid for any hard process producing a quark jet.

The hard reaction \( \gamma^* A \rightarrow \pi X \) can be considered as a three-step process, as is illustrated in Fig. 1. In the first stage the incoming virtual photon knocks out a quark and a gluon of a bound nucleon, \( \gamma^* + p \rightarrow q_1 + g \), which carry practically all the energy of the photon. We assume that Bjorken \( x \) is sufficiently large to neglect the contribution of the sea. It also allows to treat the gluon radiation process as incoherent. Strictly speaking one should integrate over the longitudinal coordinate of gluon radiation from \(-\infty \) [12], however only a part of this path, \( \Delta z \sim 1/xm_N \), contributes coherently. We neglect \( \Delta z \) assuming that \( x \) is large. Thus, in the light-cone approach one can consider gluon radiation as instantaneous from the point of hard interaction, although the quark-gluon pair loose coherence at longer distances. One can come to the same conclusion analyzing the Feynman graphs. The corresponding space-time structure of DIS is studied in Appendix A.

The second stage is the dissociation of the radiated gluon into a quark pair, \( g \rightarrow q_2 + q_3 \), and the propagation of the colorless dipole \( q_2 q_1 \) through the nucleus.

The third stage is the projection of the colorless dipole into a pion, \( q_2 + q_1 \rightarrow \pi \). The gluon and quarks propagating through the nucleus are assumed to experience only soft final state interactions (soft gluonic exchanges) with other nucleons-spectators, which usually cause attenuation.

The amplitude of this process can be represented as,
\[ M = \int_0^1 d\alpha \int d^2\kappa \lim_{z_3 \to \infty} \int dz_3 e^{i(\Delta + i\kappa z_3)} \Phi_\pi(\alpha, \kappa) \times \int \left[ -i\vec{p}_1 \cdot \vec{r}_2^q - \vec{p}_2 \cdot \vec{r}_2^g \right] \Gamma(\vec{r}_1^q, \vec{r}_2^q) G_{\gamma}(z_2, z_1; \vec{r}_2^q, \vec{r}_1^q; E_g, \{ \vec{R} \}) \times G_{q_1}(z_3, z_1; \vec{r}_2^q, \vec{r}_1^q; E_1; \{ \vec{R} \}) G_{g}(z_3, z_2; \vec{r}_3^g, \vec{r}_2^g; E_2; \{ \vec{R} \}) G_{q_3}(z_3, z_2; \vec{r}_3^q, \vec{r}_2^q; E_3; \{ \vec{R} \}) \]  

Here \( \Phi_\pi \) is the wave function of the \( \bar{q}q \) Fock component of the produced pion:
\[
\vec{p}_1 = \alpha \vec{p}_\pi + \vec{\kappa};
\vec{p}_2 = (1-\alpha) \vec{p}_\pi - \vec{\kappa};
\]
\( \vec{\kappa} \) is the pion transverse momentum; \( \vec{\kappa} = (1-\alpha)\vec{p}_1 - \alpha \vec{p}_2 \) is the relative transverse momentum of the \( q_1 \) and \( \bar{q}_2 \) in the pion;
\[
\alpha = \frac{E_1}{E_\pi};
\Delta = \frac{m_q^2}{2E_2} + \frac{m_\bar{q}^2}{2E_3}.
\]
and \( E_1, E_2, E_3 \) and \( E_\pi \) are the energies of the three quarks and gluon.

The longitudinal coordinates \( z_i \) are defined as follows. \( z_1 \) is the coordinate of the collision between the virtual photon and the nucleon; \( z_2 \) is the longitudinal coordinate of the point of dissociation \( g \to q_3 \bar{q}_2; \) \( \Gamma(\vec{r}_1^q, \vec{r}_1^q) \) is the amplitude of the process \( \gamma^*N \to q_1 \bar{q}_2 X_1, \) with the original impact parameters of the produced quark \( (\vec{r}_1^q) \) and gluon \( (\vec{r}_1^g) \), at the point with coordinate \( z_1. \)

The propagation functions (Green's functions) \( G(z_j, z_{in}; \vec{r}_j, \vec{r}_{in}; E; \{ \vec{R} \}) \) in Eq. (2) describing the propagation of the fast quarks \( q_1, \bar{q}_2, q_3 \) and the gluon in the medium, will be derived in Sect. III. Besides the initial \( (z_{in}, \vec{r}_{in}) \) and final \( (z_f, \vec{r}_f) \) positions, they also depend on the coordinates \( \{ \vec{R} \} \) of the spectator nucleons with whom they interact via soft gluonic exchanges.

The cross section of pion production off a nucleon is given by the amplitude squared and averaged over the positions of all nucleons in the nucleus,
\[
\frac{d\sigma}{d^2p_\pi d^2\kappa dz_3} = A \int d^2b d^2z_1 \rho_\lambda(b, z_1) \langle |M|^2 \rangle_{\{ \vec{R} \}}.
\]

Here \( z_3 \) is the fraction of the photon light-cone momentum carried by the pion. We singled out the integration over the coordinates of the "active" nucleon participating in the hard collision with the virtual photon.

### B. \( Q^2 \)-dependence

At large photon virtuality \( Q^2 \) the relative quark-gluon separation is small, \( |\vec{r}_1^q - \vec{r}_1^g| \sim 1/Q \), and the nuclear effects become independent of \( Q^2 \). Indeed, according to the uncertainty principle the smaller is the quark-gluon separation \( r \), the faster they are expanding with transverse momentum \( k_T \sim 1/r \),
\[
\frac{dr}{dt} = \frac{k_T}{E} \approx \frac{1}{rE}.
\]

Here \( r = \vec{r}_{in} - \vec{r}_f^q; E = E_1 + E_g \) is the total energy of the jet.

If the initial size is small, then after a while its smallness will be forgotten. Indeed, the solution of Eq. (6) reads,
\[
r^2(t) = \frac{2t}{E} + \frac{1}{Q^2}.
\]

At sufficiently long time intervals,
\[
t \gg \frac{1}{4M_N x_BJ},
\]
where \( x_BJ = Q^2/2m_N E \) is the Bjorken scaling variable. At large \( Q^2 \) the second term in (7) can be neglected, and therefore transverse size of the quark-gluon pair does not depend on \( Q^2 \) any more. If this time interval is significantly shorter than the mean free path of partons in the medium, no \( Q^2 \) dependence of nuclear effects should be expected. In cold nuclear matter the typical mean free path is several Fermi, so for \( t \sim 1 \) fm we expect a very weak \( Q^2 \) dependence when \( x_BJ \gg 0.05 \). This condition is well satisfied in the region of \( x_BJ \gtrsim 0.1 \) dominated by valence quarks, in which we are focused. This effect probably explains the very weak dependence on \( Q^2 \) of nuclear ratios observed in the HERMES experiment [13].

In what follows we assume that \( x_BJ \) is sufficiently large to neglect the second term in (7), which is equivalent to the approximation in (2),
\[
\Gamma(\vec{r}_1^q, \vec{r}_1^g) \approx \tilde{\Gamma}(\vec{r}_1^q) \delta(\vec{r}_1^q - \vec{r}_1^g).
\]

### III. Green Function Formalism for Propagation of Partons in a Medium

Although the hard reaction (DIS in Fig. 1) occurs on different nucleons incoherently, the multiple final state interactions of the produced partons proceed further coherently. Indeed, the mean transverse momentum squared
gained by a quark propagating through a heavy nucleus, as measured in the Drell-Yan reaction at 800 GeV, is very small in the order of $\Delta p^2 \sim 0.1 - 0.2 \text{GeV}^2$ [14], and is even several time smaller at lower energies [15, 16]. So a quark of energy $E_0 \sim 10 \text{GeV}$ interacts with coherence length $l_c = 2E_0/\Delta p^2 \sim 40 \text{fm}$, which is quite long compared to the nuclear size.

The cross section for the reaction $\gamma^* p \rightarrow \pi X$ corresponds to the product of the direct and conjugated amplitudes, presented graphically in Fig. 2. The two amplitudes correspond to different impact parameters, $(\vec{r}_i)$ and $(\vec{s}_i)$, of the participating partons, and different longitudinal coordinates, $z_2$ and $z_3$, of $g \rightarrow q\bar{q}$ decay.

One can see that specific partonic ensembles propagate through different intervals of the longitudinal coordinate. It is $\{gq_1q_1\}$ in the interval $z_1 - z_2$, $\{gq_3q_2q_1\}$ in the interval $z_2 - z_3$, $\{q_1q_2\}$ in interval $z_3 - z_4$.

In the case of free propagation in vacuum the Green functions introduced in (2) have a simple form,

$$G \left( z_2, z_1; \vec{r}_2, \vec{r}_1; E \right) = \frac{-iE}{2\pi(z_2 - z_1)} \Theta(z_2 - z_1) \exp \left[ \frac{iE(\vec{r}_2 - \vec{r}_1)^2}{2(z_2 - z_1)} \right] \tag{10}$$

In equation (5) the amplitude (2) squared contains bilinear combinations of Green functions with the same initial and final longitudinal coordinates, but different impact parameters. For instance the product $G_g \left( z_2, z_1; \vec{r}_2^g, \vec{r}_1^g; E_g \right)$ of Green functions with the same initial and final longitudinal coordinates, but different impact parameters, is given by

$$
\begin{align*}
& G_g \left( z_2, z_1; \vec{r}_2^g, \vec{r}_1^g; E_g \right) G_{\bar{q}} \left( z_2, z_1; \vec{r}_2^{\bar{q}}, \vec{r}_1^{\bar{q}}; E_{\bar{q}} \right) \\
& \times G_{q} \left( z_2, z_1; \vec{r}_2^{q}, \vec{r}_1^{q}; E_q \right) G_{\bar{q}} \left( z_2, z_1; \vec{r}_2^{\bar{q}}, \vec{r}_1^{\bar{q}}; E_{\bar{q}} \right) \\
& \times G_{q} \left( z_2, z_1; \vec{r}_2^{q}, \vec{r}_1^{q}; E_q \right) G_{\bar{q}} \left( z_2, z_1; \vec{r}_2^{\bar{q}}, \vec{r}_1^{\bar{q}}; E_{\bar{q}} \right),
\end{align*}
$$

which is a part of the final equation (18) (see below). It satisfies the equations,

$$
\begin{align*}
\frac{i}{2} \rho_A(b, z_2) \Sigma_1 (\vec{r}_1^{q_1}, \vec{s}_1^{q_1}, \vec{r}_1^{\bar{q}_1}, \vec{s}_1^{\bar{q}_1}, \vec{r}_2^{g_1}, \vec{s}_2^{g_1}, \vec{r}_2^{\bar{g}_1}, \vec{s}_2^{\bar{g}_1}, \vec{r}_3^{g_1}, \vec{s}_3^{g_1}, \vec{r}_3^{\bar{g}_1}, \vec{s}_3^{\bar{g}_1}) W_1, \tag{11}
\end{align*}
$$

with initial conditions,

$$
\begin{align*}
W_1 \bigg|_{z_2 < z_1} &= 0 \\
W_1 \bigg|_{z_2 = z_1} &= \delta(\vec{r}_2^{q_1} - \vec{r}_1^{q_1}) \delta(\vec{r}_2^{\bar{q}_1} - \vec{r}_1^{\bar{q}_1}) \\
& \times \delta(\vec{s}_2^{q_1} - \vec{s}_1^{q_1}) \delta(\vec{s}_2^{\bar{q}_1} - \vec{s}_1^{\bar{q}_1}). \tag{12}
\end{align*}
$$

Here $\vec{b}$ is the impact parameter of the virtual photon, and the nuclear density is normalized to one, $\int d^2b dz \rho_A(b, z) = 1$. $\Sigma_1$ is the total cross section of a 4-parton colorless system $gq_1\bar{q}\bar{q}$ interacting with a nucleon target. It is important to notice that $gq_1$ and $\bar{q}\bar{q}$ are in color triplet and anti-triplet states respectively, while $\bar{q}\bar{q}$ and $q_1q_1$ are color singlets.

The 4-body cross section $\Sigma_1$ can be represented as a linear superposition of elementary dipole cross sections of interaction of a colorless $q\bar{q}$ dipole with a nucleon, for which there exists a well developed phenomenology. A derivation presented in Appendix B results in the expression,

$$
\begin{align*}
\Sigma_1 (\vec{r}_1^{q_1}, \vec{s}_1^{q_1}, \vec{r}_1^{\bar{q}_1}, \vec{s}_1^{\bar{q}_1}) &= \frac{9}{8} \left[ \sigma_{qq}(\vec{r}^q - \vec{r}^{q_1}) \\
& + \sigma_{qq}(\vec{s}^q - \vec{s}^{q_1}) - \sigma_{qq}(\vec{r}^q - \vec{s}^{q_1}) - \sigma_{q\bar{q}}(\vec{r}^q - \vec{s}^{\bar{q}_1}) \\
& + \sigma_{\bar{q}q}(\vec{r}^{\bar{q}_1} - \vec{s}^{q_1}) + \frac{9}{4} \sigma_{q\bar{q}}(\vec{r}^q - \vec{s}^{\bar{q}_1}) \right]. \tag{13}
\end{align*}
$$

FIG. 2: Graphical representation for the direct and conjugated amplitudes of the process $\gamma^* p \rightarrow \pi X$. The DIS hard process occurs coherently on the same nucleon with longitudinal coordinate $z_1$. The radiated gluon decays to $q\bar{q}$ coherently in the two amplitudes at the points $z_2$ and $z_3$ respectively. The colorless pre-hadrons (dipoles) $q\bar{q}$ and $q\bar{q}$ created at $z_2$ and $z_3$ respectively, are projected to the pion wave function in each of the two amplitudes.
The solution of Eq. (12) with a realistic dipole cross section, and which is valid for large $\bar{q}q$ separations, can usually be obtained only numerically (see [17]). However, in the reaction under consideration, with a highly virtual photon, the typical separations are small and one may rely on the approximation [18],

$$
\sigma_{\bar{q}q}(\vec{r}_1 - \vec{r}_2) = C(\vec{r}_1 - \vec{r}_2)^2,
$$

where the factor $C$ is known from phenomenology. It depends on energy and should be properly chosen depending on the energy of the photon. With this approximation and a constant nuclear density, $\rho_A(b, z) = \rho_0 \Theta(R_0^2 - b^2 - z^2)$, the equations Eq. (12) can be solved analytically (see Appendix B.B.1).

To progress further, for propagation of the quark $q_1$ we employ the relation,

$$
G(z_3, z_1; \vec{r}_3, \vec{r}_1; E; \{\vec{R}\}) = \int d^2 r_2 G(z_3, z_2; \vec{r}_3, \vec{r}_2; E; \{\vec{R}\})
\times G(z_2, z_1; \vec{r}_2, \vec{r}_1; E; \{\vec{R}\}),
$$

which assumes that $z_1 < z_2 < z_3$. Then the cross section Eq. (5) can be represented as,

$$
\begin{align*}
\frac{d\sigma}{d^2 p_d d^2 p_\alpha d^2 z_h} &= A \frac{F_2(x)}{x} \lim_{z_1 \to \infty} 2 \text{Re} \int d^2 b \int d z_1 d z_2 d z_3 \rho_A(b, z_1) \Theta(z_3 - z_2) \Theta(z_2 - z_1) e^{i(\Delta + i\alpha)z_2 - i(\Delta - i\alpha)z_3} \\
&\times \int d^2 q^i d^2 q^c d^2 s^i d^2 s^c \Gamma(\vec{r}_1^i, \vec{r}_1^c) \\
&\times \int d^2 q^i d^2 q^c d^2 s^i d^2 s^c W_1(z_2, z_1; \vec{r}_2^i, \vec{r}_1^i; \vec{s}_2^i, \vec{s}_1^i; \vec{r}_1^c; \vec{s}_1^c; E_g, \vec{E}_1, \vec{E}_1) \\
&\times \int d^2 q^i d^2 q^c d^2 s^i d^2 s^c d^2 q^i d^2 s^i W_2(z_3, z_2; \vec{r}_3^i, \vec{r}_3^c; \vec{s}_3^i, \vec{s}_3^c; \vec{r}_2^i; \vec{s}_2^i; E_1, E_2, \vec{E}_1, \vec{E}_2) \\
&\times \int d^2 q^i d^2 q^c d^2 s^i d^2 s^c d^2 q^i d^2 s^i d^2 q^i d^2 s^i W_3(z_4, z_3; \vec{r}_4^i, \vec{r}_4^c; \vec{s}_4^i, \vec{s}_4^c; \vec{r}_3^i; \vec{s}_3^c; \vec{r}_2^i; \vec{s}_2^i; E_1, E_2, E_3, \vec{E}_1, \vec{E}_2, \vec{E}_3) \\
&\times \int d^2 k d^2 \kappa' d\alpha \Phi_\pi(\alpha, \vec{\kappa}) \Phi_\pi^*(\bar{\alpha}, \vec{\kappa}') \exp[i\vec{p}_1^i \cdot \vec{s}_1^i + i\vec{p}_2^i \cdot \vec{s}_2^i + i\vec{p}_3^i \cdot \vec{s}_3^i - i\vec{p}_4^i \cdot \vec{s}_4^i - i\vec{p}_4^i \cdot \vec{r}_4^i - i\vec{p}_3^i \cdot \vec{r}_3^i - i\vec{p}_2^i \cdot \vec{r}_2^i - i\vec{p}_1^i \cdot \vec{r}_1^i]
\end{align*}
$$

Here $\vec{s}_j^{q_i}$ and $\vec{s}_j^{\bar{q}_i}$ are the transverse coordinates of the quark $q_i$ ($i = 1, 2, 3$) at the point with longitudinal coordinate $z_j$ ($j = 1, 2, 3, 4$) for the direct and conjugated amplitudes respectively (same for the gluons); $\alpha$ and $\bar{\alpha}$ are the fractional momenta of the quark $q_1$ within the pion in the direct and conjugated amplitudes respectively. Correspondingly, the transverse momenta of the quarks $q_1$ and $\bar{q}_2$ are,

$$
\begin{align*}
\vec{p}_1 &= \alpha \vec{p}_\pi + \vec{\kappa}; & \vec{p}_1' &= \bar{\alpha} \vec{p}_\pi + \vec{\kappa}'; \\
\vec{p}_2 &= (1 - \alpha) \vec{p}_\pi - \vec{\kappa}; & \vec{p}_2' &= (1 - \bar{\alpha}) \vec{p}_\pi - \vec{\kappa}'; \\
\vec{p}_3 &= \vec{\rho}_3.
\end{align*}
$$

The energies of the participating quarks and gluon in the two amplitudes read,

$$
\begin{align*}
E_1 &= \alpha z_h E; \\
E_2 &= (1 - \alpha) z_h E; \\
E_g &= (1 - \alpha z_h) E; \\
\vec{E}_1 &= \bar{\alpha} z_h E; \\
\vec{E}_2 &= (1 - \bar{\alpha}) z_h E; \\
\vec{E}_g &= (1 - \bar{\alpha} z_h) E; \\
\vec{E}_3 &= E_3 = (1 - z_h) E.
\end{align*}
$$

The functions $W_2$ and $W_3$ in (18) describing the propagation of partonic ensembles through the intervals $z_2 - z_3$ and $z_3 - z_4$ respectively, are defined similar to Eq. (11),

$$
W_2 = \left< G_{q_1}[z_3, z_2; \vec{r}_3^{q_1}, \vec{r}_2^{q_1}; E_3; \{\vec{R}\}] G_{q_2}[z_3, z_2; \vec{r}_3^{q_2}, \vec{r}_2^{q_2}; E_2; \{\vec{R}\}] G_{q_1}[z_3, z_2; \vec{r}_3^{q_1}, \vec{r}_2^{q_1}; E_1; \{\vec{R}\}] G_{q_2}[z_3, z_2; \vec{r}_3^{q_2}, \vec{r}_2^{q_2}; E_2; \{\vec{R}\}] \right> \langle \vec{R} \rangle
$$

$$
W_3 = \left< G_{q_1}[z_3, z_2; \vec{s}_3^{q_1}, \vec{s}_2^{q_1}; \vec{E}_1; \{\vec{R}\}] G_{q_2}[z_3, z_2; \vec{s}_3^{q_2}, \vec{s}_2^{q_2}; \vec{E}_2; \{\vec{R}\}] G_{q_1}[z_3, z_2; \vec{s}_3^{q_1}, \vec{s}_2^{q_1}; \vec{E}_1; \{\vec{R}\}] G_{q_2}[z_3, z_2; \vec{s}_3^{q_2}, \vec{s}_2^{q_2}; \vec{E}_2; \{\vec{R}\}] \right> \langle \vec{R} \rangle
$$
\[ W_3 = \left< G_{q_3, z_4, z_3; \vec{r}^q_3, \vec{r}^q_3, E_3; \{ \vec{R} \}} G_{q_2, z_4, z_3; \vec{r}^q_2, \vec{r}^q_2, E_2; \{ \vec{R} \}} G_{q_1, z_4, z_3; \vec{r}^q_2, \vec{r}^q_2, E_1; \{ \vec{R} \}} \right> (22) \]

They are the solutions of the following equations,

\[ i \partial W_2 \over \partial z_3 = \left[ -\frac{\Delta \vec{r}^q_3}{2E_3} - \frac{\Delta \vec{r}^q_2}{2E_2} - \frac{\Delta \vec{r}^q_1}{2E_1} + \frac{\Delta \vec{s}^q_3}{2E_3} + \frac{\Delta \vec{s}^q_2}{2E_2} + \frac{\Delta \vec{s}^q_1}{2E_1} \right] W_2; \quad \text{(23)} \]

\[ i \partial W_2 \over \partial z_2 = \left[ -\frac{\Delta \vec{r}^q_3}{2E_3} - \frac{\Delta \vec{r}^q_2}{2E_2} - \frac{\Delta \vec{r}^q_1}{2E_1} + \frac{\Delta \vec{s}^q_3}{2E_3} + \frac{\Delta \vec{s}^q_2}{2E_2} + \frac{\Delta \vec{s}^q_1}{2E_1} \right] W_2; \quad \text{(24)} \]

\[ i \partial W_3 \over \partial z_4 = \left[ -\frac{\Delta \vec{r}^q_3}{2E_3} - \frac{\Delta \vec{r}^q_2}{2E_2} - \frac{\Delta \vec{r}^q_1}{2E_1} + \frac{\Delta \vec{s}^q_3}{2E_3} + \frac{\Delta \vec{s}^q_2}{2E_2} + \frac{\Delta \vec{s}^q_1}{2E_1} \right] W_3; \quad \text{(25)} \]

\[ i \partial W_3 \over \partial z_3 = \left[ -\frac{\Delta \vec{r}^q_3}{2E_3} - \frac{\Delta \vec{r}^q_2}{2E_2} - \frac{\Delta \vec{r}^q_1}{2E_1} + \frac{\Delta \vec{s}^q_3}{2E_3} + \frac{\Delta \vec{s}^q_2}{2E_2} + \frac{\Delta \vec{s}^q_1}{2E_1} \right] W_3; \quad \text{(26)} \]

with the boundary conditions,

\[ W_2 \big|_{z_3 < z_2} = 0 \]

\[ W_2 \big|_{z_3 = z_2} = \delta(\vec{r}^q_3 - \vec{r}^q_2) \delta(\vec{r}^q_2 - \vec{r}^q_2)\delta(\vec{r}^q_1 - \vec{r}^q_1) \]

\[ \times \delta(s^q_3 - s^q_2) \delta(s^q_2 - s^q_2). \quad \text{(27)} \]

\[ W_3 \big|_{z_4 < z_3} = 0 \]

\[ W_3 \big|_{z_4 = z_3} = \delta(\vec{r}^q_3 - \vec{r}^q_3) \delta(\vec{r}^q_2 - \vec{r}^q_2)\delta(\vec{r}^q_1 - \vec{r}^q_1) \]

\[ \times \delta(s^q_3 - s^q_2) \delta(s^q_2 - s^q_2)\delta(s^q_1 - s^q_1). \quad \text{(28)} \]

The function \( \Sigma_2 \) is the total cross section of interaction of the colorless parton ensemble \( q_1, q_2, q_3, \bar{q}_1, \bar{q}_1, g \) with a nucleon, where the pairs \( q_1 \bar{q}_2 \) and \( q_1, \bar{q}_1 \) are each in colorless states, the pair \( q_2, q_3 \) is a color octet, and the pair \( q_1, g \) is an anti-triplet. Correspondingly, \( \Sigma_3 \) is the total cross section for the ensemble \( q_1, q_2, q_3, \bar{q}_1, \bar{q}_1, q_1, q_2, q_3 \), where each pair \( q_1, q_2, q_1, q_2, q_1, q_2, q_3 \) is colorless, while the pairs \( q_2, q_3 \) and \( \bar{q}_2 \bar{q}_3 \) are color octets. These cross sections are derived in Appendix B and have the form

\[ \Sigma_2 = \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_1 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_2 - \vec{r}^q_1) + \frac{1}{8}[\sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_1) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_1)] \]

\[ + \frac{9}{8}[\sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2)]. \quad \text{(29)} \]

\[ \Sigma_3 = \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) \]

\[ + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \frac{1}{8}[\sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2) + \sigma_{q\bar{q}}(\vec{r}^q_3 - \vec{r}^q_2)]. \quad \text{(30)} \]

The equations (12)-(13) and (23)-(26) have been solved in Appendix C, in the approximation of Eq. (16) and for
a constant nuclear density.

IV. THE THREE PARTS OF THE CROSS SECTION

In order to discriminate between production of the pion within or outside the nucleus, we rely on the approximation of constant nuclear density, \( \rho_A(b, z) = \rho_0 \Theta(L^2 - z^2) \), where \( L = \sqrt{R_A^2 - b^2} \). Then we split the amplitude equation (2) in two parts, \( M = M_1 + M_2 \), corresponding to pion production outside, or inside the nucleus, i.e. \( z_2 \) - integration within intervals \( L < z_2 < z_3 \) and \( z_1 < z_2 < L \) respectively. Correspondingly, the cross section equation (18) contains three terms,

\[
\sigma = \sigma_1 + \sigma_2 + \sigma_3, \tag{31}
\]

which are the amplitudes \( M_1, M_2 \) squared, and their interference, respectively. These terms correspond to the following splitting of the integrations over \( z_2 \) and \( z_3 \) in (18),

\[
\int_{z_1}^{z_2} dz_2 \int_{z_2}^{z_3} dz_3 = \int_{z_1}^{z_2} dz_2 \int_{z_2}^{L} dz_3 + \int_{z_1}^{L} dz_2 \int_{z_2}^{L} dz_3 + \int_{z_1}^{L} dz_2 \int_{z_1}^{L} dz_3. \tag{32}
\]

In what follows we consider cross sections integrated over transverse momenta of the pion and recoil quark,

\[
\frac{d\sigma_i}{dz_h} = \int d^2p_\pi d^2p_3 \frac{d\sigma_i}{d^2p_\pi d^2p_3 dz_h}, \tag{33}
\]

where \( i = 1, 2, 3 \).

Later, the results of numerical calculations will show that the interference term is negative, \( \sigma_3 < 0 \). This can be understood on a much simplified example of an "empty" nucleus, i.e. free propagation of particles. In this case the amplitude of the fragmentation process \( q_1 \to q_1q_2q_3 \) is proportional to the value,

\[
\mathcal{M} = \lim_{z_+ \to \infty} \int dz \exp [-i(\Delta - i\varphi)z] \times G_g(z, z_1; \vec{p}_g)G_{q_1}(z_+, z_1; \vec{p}_1)G_{q_2}(z_+, z_1; \vec{p}_2), \tag{34}
\]

where \( \vec{p}_g = \vec{p}_1 + \vec{p}_2 \);

\[
\Delta = \frac{m^2_q}{2E_1} + \frac{m^2_q}{2E_2};
\]

\[
G_g(z, z_1; \vec{p}_g) = \exp \left[ -i\frac{p^2_g(z - z_1)}{2E_g} \right]; \tag{35}
\]

\[
G_{q_1}(z_+, z_1; \vec{p}_1) = \exp \left[ -i\frac{p^2_1(z_+ - z)}{2E_1} \right];
\]

\[
G_{q_2}(z_+, z_1; \vec{p}_2) = \exp \left[ -i\frac{p^2_2(z_+ - z)}{2E_2} \right].
\]

This amplitude can be represented as,

\[
\mathcal{M} = \lim_{z_+ \to \infty} e^{i\varphi} \int_{z_1}^{z_+} dz e^{-i\varphi z}, \tag{36}
\]

where

\[
\varphi = \frac{m^2_q + p^2_1}{2E_1} + \frac{m^2_q + p^2_2}{2E_2} - \frac{(\vec{p}_1 + \vec{p}_2)^2}{2E_g} - i\vartheta; \tag{37}
\]

\[
\vartheta = \frac{m^2_g}{2E_1} + \frac{m^2_g}{2E_2} - \frac{(\vec{p}_1 + \vec{p}_2)^2}{2E_g} z_1. \tag{38}
\]

Now we can split the amplitude into two terms, \( \mathcal{M} = \mathcal{M}_{in} + \mathcal{M}_{out} \), corresponding to gluon decay inside \( z_1 < z < \bar{z} \) and outside \( \bar{z} < z < z_+ \) the nucleus respectively. Then from (36) we get,

\[
\mathcal{M}_{in} = \frac{e^{i\varphi}}{i\vartheta} \left( e^{-i\varphi z_1} - e^{-i\varphi \bar{z}} \right); \tag{39}
\]

\[
\mathcal{M}_{out} = \frac{e^{i\varphi}}{i\vartheta} \left( e^{-i\varphi z_+} - e^{-i\varphi \bar{z}} \right). \tag{40}
\]

At \( z_+ \to \infty \) the first term in \( \mathcal{M}_{out} \) vanishes, \( e^{-i\varphi z_+} \to 0 \), because of the imaginary term in \( \vartheta \), Eq. (37). So we get,

\[
|\mathcal{M}_{out}|^2 = \frac{1}{\vartheta^2}; \tag{41}
\]

\[
|\mathcal{M}_{in}|^2 = \frac{2}{\vartheta^2} \left[ 1 - \cos \left( \vartheta (\bar{z} - z_1) \right) \right]; \tag{42}
\]

\[
2 \text{Re} (\mathcal{M}_{in}^* \mathcal{M}_{out}) = \frac{2}{\vartheta^2} \left[ 1 - \cos \left( \vartheta (\bar{z} - z_1) \right) \right] = -|\mathcal{M}_{in}|^2. \tag{43}
\]

Thus, we conclude that the interference term (43) is negative and exactly cancels the inside production term. The cross section in this case is given solely by the outside production.

Of course, these simple results are valid only for hadronization in vacuum ("empty" nucleus). Presence of a medium breaks down these simple relations and makes the calculation of different terms in the cross section Eq. (31), performed below, much more complicated. Nevertheless, the negative sign of the interference term will be preserved.

A. Pion production outside the nucleus

We start with the first term \( \sigma_1 \), which dominates at high energy \( E \), and is the easiest one to calculate, since in this case the functions \( W_{1,2} \) contain just products of Green functions for free propagation of quarks in vacuum. So the integration over longitudinal coordinates in (18) and transverse momenta in (33) can be performed analytically.
\[
\frac{d\sigma_1}{dz_h db} = A\rho_0 E^2 F_2^N(x, Q^2) \frac{z^2(1-z)^2}{x} \int d\alpha \int d^2\vec{r_1}^q d^2\vec{r_2}^q d^2\vec{s}_1^q d^2\vec{s}_2^q d^2\vec{r}_1 d^2\vec{r}_2 d^2\vec{s}_1 d^2\vec{s}_2 \frac{d\sigma}{d\alpha d\vec{r}_2},
\]
with the new notation,

\[
\Phi(\vec{r}_2^q; \vec{r}_1^q; \vec{s}_1^q; \vec{s}_2^q) = \delta(\vec{R}_2 - \vec{S}_2) \int d^2\rho d^2\tau K_0(m_q\rho) K_0(m_q\tau) \Phi_\pi \left(\vec{r}_2 + \frac{1 - z_h}{1 - \alpha z_h} \rho \right) \Phi_\pi \left(\vec{s}_2 + \frac{1 - z_h}{1 - \alpha z_h} \tau \right) \times \delta \left(\alpha \vec{r}_2 - \frac{1 - \alpha}{1 - \alpha z_h} \rho - \alpha \vec{S}_2 + \frac{1 - \alpha}{1 - \alpha z_h} \tau \right),
\]

(44)

where \(\vec{r}_i = \vec{r}_i^q - \vec{r}_i^{q_1}; \vec{s}_i = \vec{s}_i^q - \vec{s}_i^{q_1}\), and

\[
\vec{R}_i = \frac{E_\rho \vec{r}_i^q + E_1 \vec{r}_i^{q_1}}{E_\rho + E_1}, \quad \vec{S}_i = \frac{E_\rho \vec{s}_i^q + E_1 \vec{s}_i^{q_1}}{E_\rho + E_1},
\]

(46)

\((i = 1, 2)\) are the intrinsic separations in the \(q_1 - g\) pairs, and the coordinates of their centers of gravity, respectively; \(E_\rho + E_1 = E_\rho + E_1 = E\).

We also introduce the following combinations,

\[
\vec{R}_i^+ = \frac{1}{2} \left(\vec{R}_i + \vec{S}_i\right), \quad \vec{R}_i^- = \vec{R}_i - \vec{S}_i.
\]

(47)

The Jacobian for transition to the new coordinates is one, so,

\[
d^2\vec{r}_2 d^2\vec{r}_1^q d^2\vec{r}_2^q d^2\vec{s}_1^q d^2\vec{s}_2^q = d^2\vec{R}_2^+ d^2\vec{R}_2^- d^2\vec{r}_1 d^2\vec{s}_1.
\]

(48)

It turns out that the function \(W_1\) factorizes in the new coordinates (see Appendix C),

\[
W_1(L, z_1; \vec{r}_2^q; \vec{r}_1^q; \vec{s}_2^q; \vec{s}_1^q; \vec{r}_2^q; \vec{r}_1^q; \vec{s}_2^q; \vec{s}_1^q) = \left[\frac{E}{2\pi(L - z_1)}\right]^2 F(L, z_1; \vec{R}_2^+, \vec{r}_2; \vec{s}_2; \vec{R}_1^-; \vec{r}_1; \vec{s}_1). \times \exp \left[\frac{i E}{L - z_1} \left(\vec{R}_2^+ - \vec{R}_1^-\right) \cdot \left(\vec{R}_2^- - \vec{R}_1^-\right)\right]
\]

(49)

Taking also into account that in (44) \(\vec{r}_1^q = \vec{r}_1^{q_1}\) and \(\vec{s}_1^q = \vec{s}_1^{q_1}\) (i.e. \(\vec{r}_1 = \vec{s}_1 = 0\)), we arrive at the relation,

\[
\int d^2\vec{r}_1^q d^2\vec{s}_1^q d^2\vec{R}_2^+ d^2\vec{R}_2^- \vec{\Gamma}(\vec{r}_1^q) \vec{\Gamma}(\vec{s}_1^q) \times W_1(L, z_1; \vec{r}_2^q; \vec{s}_2^q; \vec{r}_1^q; \vec{s}_1^q; \vec{r}_2^q; \vec{r}_1^q; \vec{s}_2^q; \vec{s}_1^q) \delta \left(\vec{R}_2 - \vec{S}_2\right) = \int d^2\vec{R}_1^+ \vec{\Gamma}(\vec{R}_1^-)^2 \tilde{F}(L, z_1; \vec{r}_2, \vec{s}_2),
\]

(50)

where

\[
\tilde{F}(L, z_1; \vec{r}_2, \vec{s}_2) = F(L, z_1; \vec{R}_2^+, \vec{r}_2, \vec{s}_2; \vec{R}_2^+, \vec{r}_1; \vec{s}_1) \vec{R}_2^- = \vec{R}_1^- = 0
\]

(51)

Thus the cross section equation (44) gets the form,

\[
\frac{d\sigma_1}{dz_h db} = A N z^2_h (1 - z_h)^2 \rho_0 \int d\alpha \int d^2\vec{r}_2 d^2\vec{s}_2 d^2\vec{r}_2 d^2\vec{s}_2 \frac{d\sigma}{d\alpha d\vec{r}_2},
\]

(52)

where

\[
\tilde{F}(\vec{r}_2, \vec{s}_2) = \int d^2\rho d^2\tau K_0(m_q\rho) K_0(m_q\tau) \times \Phi_\pi \left(\vec{r}_2 + \frac{1 - z_h}{1 - \alpha z_h} \rho \right) \Phi_\pi \left(\vec{s}_2 + \frac{1 - z_h}{1 - \alpha z_h} \tau \right) \times \delta \left(\alpha \vec{r}_2 - \frac{1 - \alpha}{1 - \alpha z_h} \rho - \alpha \vec{s}_2 + \frac{1 - \alpha}{1 - \alpha z_h} \tau \right),
\]

(53)

and

\[
N = \frac{F_2(x)}{E^2} \int d^2 r \left|\vec{\Gamma}(\vec{r})\right|^2.
\]

(54)

In order to simplify the calculations we assume a factorized form of the pion light-cone wave function, \(\Phi_\pi(\alpha, \vec{r}) = \phi(\alpha) \phi(\vec{r})\), and a Gaussian dependence on quark separation,

\[
\phi(r) \propto \exp \left(-\frac{\xi}{2} r^2\right),
\]

(55)

where \(\xi\) is related to the mean pion charge radius squared, \(\xi = 3/8(\vec{r}_2^2)\).

Perturbative fragmentation of quarks to pions in e+e− annihilation and DIS was calculated by Berger [7] in the limit of \((1 - z_h) \ll 1\). The pion wave function was maximally simplified assuming that \(\varphi(\alpha) = \delta(\alpha - 1/2)\) and fixing at zero the relative \(\bar{q}q\) momentum. This simplifies the calculations considerably, since the function \(\tilde{F}\) can be obtained analytically,

\[
\tilde{F}(L, z_1; \vec{r}_2, \vec{s}_2; \vec{r}_2, \vec{s}_2) = \left(\frac{\xi}{2\pi(L - z_1)}\right)^2 \exp \left[\frac{i E}{2\pi(L - z_1)} (\vec{r}_2^2 - \vec{s}_2^2)\right] \times \frac{1}{6} \rho_0 C(z_h, \alpha)(\vec{r}_2^2 - \vec{s}_2^2)(L - z_1),
\]

(56)
where \( \varepsilon = \alpha z_h (1 - \alpha z_h) E \);

\[
C(z_h, \alpha) = C \left[ 1 + \alpha^2 z_h^2 + \frac{\alpha z_h}{4} \right].
\]  

(57)

Notice that in this case the expressions for \( W_2 \) and \( W_3 \), equations (21) and (22), also are much simplified.

We can perform the integration over the transverse coordinates and momenta using the integral representation for the modified Bessel functions,

\[
K_0(m_q \rho) K_0(m_q \tau) = \frac{1}{2} \int_{-1}^{1} \frac{dv}{1 - v^2} \int_{0}^{\infty} dw \exp \left[ -\frac{m_q^2}{2w} \left( \frac{\rho^2}{1 + v} + \frac{\tau^2}{1 - v} \right) - w \right]
\]

(58)

Then in the case of equal sharing of longitudinal momentum by the pion quarks we arrive at a simple result,

\[
\frac{d\sigma_1}{dz_h d^2 b} = A N z_h^2 (1 - z_h)^2 \rho_0 \int_{-L}^{L} dz_1 \int_{0}^{\infty} dw \times \int_{-1}^{1} \frac{dv}{m_q^2 + a D(w, v, z_1)},
\]

(59)

where

\[
D(w, v, z_1) = (1 - v^2) w \left( \frac{1 - z_h}{1 - x} \right)^2 + (1 - x)^2 m_q^2 \left( ua + \frac{m_q^2}{w} \right) \left( \frac{L - z_1}{\varepsilon} \right)^2 + \frac{2}{3} C \rho_0 \left( 1 + x^2 + \frac{x}{4} \right) \left( \frac{L - z_1}{\varepsilon} \right)^3 \left( \frac{m_q^2}{\varepsilon} \right)^2 + 2 i v (1 - z_h) m_q^2 \left( \frac{L - z_1}{\varepsilon} \right). 
\]

(60)

Notice that although the function \( D(w, v, z_1) \) is complex, the expression (59) is real.

In the limit of \( \xi \to 0 \) in Eq. (55) the cross section Eq. (59) does not depend any more on the interaction with the medium, which is characterized by the constant \( C \). Thus in the Berger model for fragmentation (\( \alpha = 1/2; \ \kappa^2 = \xi = 0 \)), the interaction of the quark and gluon with the medium does not affect the value of the cross section \( \sigma_1 \), Eq. (59), and only modifies the transverse momentum distribution, which is an effect beyond the scope of this study.

In another limiting case \( \Phi_\pi(\alpha, \vec{\kappa}) = \phi(\alpha) \delta(\vec{\kappa}) \) the cross section gets the form,

\[
\frac{d\sigma_1}{dz_h d^2 b} = A N z_h^2 (1 - z_h)^2 \rho_0 \int_{-L}^{L} dz_1 \int_{0}^{\infty} dw \int_{0}^{1} dv \int_{0}^{1} d\alpha \frac{(1 - \alpha z_h) (1 - \bar{\alpha} z_h) \phi(\alpha) \phi(\bar{\alpha})}{D_1 D_2 D_3} 4 \pi \varepsilon e^{-w}. 
\]

(61)

We use here the following notation,

\[
D_1 = (1 - \alpha)^2 t w + m_q^2 (1 - \alpha z_h)^2 u;
D_2 = (1 - \bar{\alpha})^2 t w + m_q^2 (1 - \bar{\alpha} z_h)^2 u;
D_3 = \cos \{ \omega_1 (L - z_1) \} \cos \{ \omega_2 (L - z_1) \};
\]

(62)

\[
\mathcal{J} = \varepsilon \varepsilon \omega_1 \omega_2 \cot \{ \omega_1 (L - z_1) \} \cot \{ \omega_2 (L - z_1) \};
\]

(63)

\[
\mathcal{R} = \frac{i}{2 (1 - \mu_1 \mu_2)} \{ \varepsilon \omega_1 \beta^2 \cot \{ \omega_1 (L - z_1) \} - \varepsilon \omega_1 \gamma^2 \cot \{ \omega_1 (L - z_1) \} \},
\]

(64)

where \( \beta = \alpha - \nu a \); \( \gamma = \bar{\alpha} - \mu a \); \( \varepsilon = E \alpha z_h (1 - \alpha z_h) \);

\( \bar{\varepsilon} = E \bar{\alpha} z_h (1 - \bar{\alpha} z_h) \); \( \omega_1 = \sqrt{-i \lambda_1} \); \( \omega_2 = \sqrt{-i \lambda_2} \);

\[
\lambda_{1,2} = C \rho_0 \sqrt{(a + b) - 4 \varepsilon \varepsilon c + \varepsilon \varepsilon a + \varepsilon \varepsilon b};
\]

\[
\varepsilon \mu = \varepsilon \nu = \frac{\varepsilon a + \varepsilon b - \sqrt{(\varepsilon a + \varepsilon b)^2 - 4 \varepsilon \varepsilon c^2}}{2 c};
\]

(65)

\[
a = (1 - \alpha z_h)^2 + \frac{9}{4} \alpha z_h;
\]

\[
b = (1 - \bar{\alpha} z_h)^2 + \frac{9}{4} \bar{\alpha} z_h;
\]

\[
c = (1 - \alpha z_h)(1 - \bar{\alpha} z_h) + \frac{9}{8} (\alpha z_h + \bar{\alpha} z_h). 
\]

(66)

Nuclear effects for this part of the cross section, \( \sigma_1 \), are shown in Fig. 3 in the form of ratio,

\[
R_1(z_h) = \frac{d\sigma_1 / dz_h}{d\sigma_1(C = 0) / dz_h},
\]

(67)

where both the numerator and denominator are the cross sections on the nucleus integrated over impact parameter, however in the denominator we eliminate the influence of the medium fixing the imaginary part of the light-cone potential \( C = 0 \), so the quark and gluon propagate like in vacuum. The nuclear cross section, here in the numerator and in what follows, is calculated with \( C = 3 \). This value agrees with extrapolation of the saturated cross section [12] down to medium high energies, as well as agrees with data on nuclear broadening of transverse momentum [19].

We performed calculations for four cases:

I. \( \Phi_\pi(\alpha, \vec{\kappa}) \propto \delta(\alpha - 1/2) \delta(\vec{\kappa}) \) (Berger permission);

II. \( \Phi_\pi(\alpha, \vec{\kappa}) = \phi(\alpha) \delta(\vec{\kappa}) \), where \( \phi(\alpha) \propto \alpha(1 - \alpha) \);

III. \( \Phi_\pi(\alpha, \vec{\kappa}) = \delta(\alpha - 1/2) \phi(\vec{\kappa}) \), where \( \phi(\vec{\kappa}) \propto \exp(-\xi \kappa^2/2) \);

IV. \( \Phi_\pi(\alpha, \vec{\kappa}) = \phi(\alpha) \phi(\vec{\kappa}) \), where \( \phi(\vec{\kappa}) \propto \exp(-\xi \kappa^2/2) \).

Comparing curves I, II with III, IV in Fig. 3 one can conclude that the transverse motion of quarks (\( \xi = (\kappa^2) \neq 0 \)) significantly affects the nucleus-to-proton ratio. At the same time, the \( \alpha \)-distribution, i.e. longitudinal motion of quarks in the pion, has almost no influence.
FIG. 3: Comparison of ratios \( R_1(z_h) \), Eq. (67), for pre-hadron production outside the nucleus, for different models for the pion light-cone wave function. The four variants I-IV (see text) differ by absence or presence of longitudinal and transverse motion of the valence quarks in the pion. The calculations are done for lead at \( E = 10 \text{ GeV} \). Since the recoil quark should be ultra-relativistic in order to rely on the Green function method, we restricted the range of \( z_h < 0.9 \).

on the nuclear effects. Indeed, the curves III and IV are nearly very close to each other. In what follows we assume that \( \sigma_{2,3} \) are also insensitive to the form of the \( \alpha \)-distribution, so we will continue our calculations in the approximation III, \( \Phi_\pi(\alpha, \vec{\kappa}) = \delta(\alpha - 1/2) \exp(-\xi \vec{\kappa}^2/2) \).

**B. Pion production inside the nucleus**

The second term in the cross section Eq. (72), which corresponds to gluon decay inside the nucleus, has the form,

\[
\frac{d\sigma_2}{dz_h db} = \pi A N \rho_0 \Re \int_{-L}^{L} dz_1 \int_{z_1}^{L} dz_2 \int_{z_2}^{L} dz_3 \times -i\xi \omega \exp \left[ -\frac{im^2}{2\xi} \left( z_3 - z_2 \right) \right] \frac{1}{E^2 \left( 1 + \frac{\xi}{2} H_2 \right) \sin[\omega(z_3 - z_2)]}.
\]

Here

\[
H_2 = \frac{2}{3} \rho_0 [(c_1 - c_2)(z_2 - z_1)^3 + (c_3 - c_2)(z_3 - z_1)^3 + c_3(L - z_1)^3] + 2i\xi \lambda (2x + \lambda)(z_3 - z_2) - \frac{2i\xi \omega}{\lambda} [(x + \lambda)^2(z_3 - z_1)^2 + \lambda^2(z_2 - z_1)^2] \times \cot[\omega(z_3 - z_2)] + 4\xi(1 - x)^2(L - z_1)^2,
\]

and

\[
\omega = (1 - z_h) \frac{x E}{1 - x}; \\
x = \frac{z_h}{2}; \\
\lambda = \frac{q}{p}; \\
p = \frac{4 - 17x + 22x^2}{4(1 - x)^2}; \\
q = \frac{4 - 8x - 5x^2}{4(1 - x)^2}; \\
c_1 = \left( 1 + \frac{x}{4} + x^2 \right) C; \\
c_2 = \left( 1 + \frac{x}{4} + x^2 - \frac{2}{p} \right) C; \\
c_3 = 4(1 - x)^2 C.
\]

One can see that even in the limit \( \xi \to 0 \) the cross section Eq. (68) is still sensitive to the constant \( C \) due to the presence of \( \sin[\omega(z_3 - z_2)] \) in (69).

The result for the ratio

\[
R_2(z_h) = \frac{d\sigma_2 / dz_h}{d\sigma_2 / dz_h(C = 0) / dz_h},
\]

calculated for lead at \( 10 \text{ GeV} \) is depicted in Fig. 4. As one could expect, this contribution is more suppressed, since the gluon decays inside the nucleus producing the colorless pre-hadron which propagates and attenuates in the medium.

**C. Interference of amplitudes \( M_1 \) and \( M_2 \)**

Eventually, the third term in Eq. (31), corresponding to interference of the two amplitudes, after integration over transverse variables gets the form,

\[
\frac{d\sigma_3}{dz_h db} = \pi A N \rho_0 \Re \int_{-L}^{L} dz_1 \int_{z_1}^{L} dz_2 \int_{z_2}^{L} dw \frac{e^{-w}}{E^2 (1 + \frac{\xi}{2} H_2) A_3 - \xi B^2} \times -i\xi \omega \exp \left[ -\frac{im^2}{2\xi} \left( L - z_2 \right) \right] \frac{1}{\sin[\omega(L - z_2)]},
\]

(72)
Here
\[
H_3 = \frac{m_c^2}{w} (1 - x)^2 (L - z_1)^2 + \frac{2}{3} \rho_0 [(c_1 - c_2)(L - z_2)^3 + c_2 (L - z_1)^3] + 4 \xi (1 - x)^2 (L - z_1)^2 + 2 i \varepsilon (L - z_2) - 2 i \varepsilon \omega \left[ (L - z_1)^2 + (z_2 - z_1)^2 \right] \cot[\omega (L - z_2)] \\
- \frac{(L - z_1)(z_2 - z_1)}{\sin[\omega (L - z_2)]} \right] + 2 i \varepsilon (1 - z_h)(L - z_1);
\]
\[
A_3 = \frac{m_c^2}{w} - 2 i \varepsilon \omega \cot[\omega (L - z_2)];
\]
\[
B_3 = \frac{m_c^2}{w} (1 - x)(L - z_1) + 2 i \left( \frac{1 - z_h}{1 - x} \right) - 2 i \varepsilon \omega \lambda \left[ (L - z_1) \cot[\omega (L - z_2)] \\
- \frac{z_2 - z_1}{\sin[\omega (L - z_2)]} \right].
\]

The values of $R(z_h)$, defined similarly to Eqs. (67), (71), are depicted in Fig. 4. The solid curve in this figure presents the final results for the ratio of all terms in (72),
\[
R(z_h) = \frac{\frac{d \sigma_1}{dz_h} + \frac{d \sigma_2}{dz_h} + \frac{d \sigma_3}{dz_h}}{\left( \frac{d \sigma_1}{dz_h} + \frac{d \sigma_2}{dz_h} + \frac{d \sigma_3}{dz_h} \right)_{C=0}} (76)
\]

Fig. 3 does not contain information about relative contributions of different terms in (76) to the cross section. To show that we depicted the fractions $\sigma_i/(\sigma_1 + \sigma_2 + \sigma_3)$ in Fig. 5. All three terms in the numerator of (76) are of the same order, but the last one presenting interference, is negative. The latter was expected according to the calculation performed in Sect. IV for hadronization in vacuum. We see that at 10 GeV the fractions of the cross section corresponding to production inside and outside the nucleus are about equal, while the former is more suppressed according to Fig. 4. This is, however, a classical interpretation, the inside-outside interference term $\sigma_3$ does not allow to classify events this way.

The nuclear effects represented by the ratio $R(z_h)$ depend on the photon energy, and the higher the energy is, the weaker is the nuclear suppression. This is the obvious manifestation of color transparency [18]: the initially small quark-gluon separation (see Sect. II B) is evolving slower at high energy due to Lorentz time dilation. The energy dependence is illustrated in Fig. 6 by some examples.

V. MAIN RESULTS AND OBSERVATIONS

This paper presents the first attempt to describe hadronization of a parton propagating through a medium on a fully quantum-mechanical basis. For hadronization in vacuum we employ the Berger model [7] of perturbative fragmentation, improved in [8]. This mechanism, imbedded in a nuclear environment, is illustrated in Fig. 1, and the associated space-time development in Fig. 4. We employed the path-integral formulation [22]
of quantum mechanics, which describes propagation of partons and partonic ensembles in terms of the light-cone Green function formalism. This technique properly includes all phases and takes care of all coherence phenomena, including formation of hadronic wave functions and color transparency.

The important observations of the paper can be summarized as follows.

- Contrary to the usual expectation based on classical intuition, even if the radiated gluon always decays outside the medium and the produced $q_2q_3$ pre-hadron has no final state interactions, there is a considerable nuclear suppression for pion production, as is demonstrated by the ratio $R(z_h)$ in Fig. 4. Notice that no energy loss effect or final state absorption are involved in this result. The suppression is caused by multiple interactions of the partons in the medium affecting the overlap of the pre-hadron and pion wave functions, even if the pre-hadron is produced far away from the nucleus. The effect of nuclear suppression is subject to color transparency and is controlled by the size of the effective dipoles. The latter is evolving starting from a very small separation $\sim 1/Q$ in the hard reaction initiating the jet. The magnitude and energy dependence of nuclear suppression is similar to what is known for electroproduction of $\rho$-mesons in the regime of short coherence length [23, 24]. In that case a dipole is also produced in a small-size configuration and then is evolving with a speed dependent on energy.

- Although much more involved, the effect of nuclear suppression of pre-hadrons produced outside the nucleus is in some respect analogous to gluon shadowing (there are also important differences). Indeed, in the nuclear rest frame gluon shadowing looks like suppression of gluon radiation by multiple interactions [11, 12] (Landau-Pomeranchuk effect). In this case there are no colorless objects to be absorbed in the nucleus, yet the production rate of gluons is affected by the medium. In this case gluons radiated inside and outside the nucleus also interfere.

- The novel feature related to the quantum-mechanical treatment of the problem, is the production of the pre-hadron both inside and outside the nucleus. This is analogous to the Twin Slit Interference Experiment in quantum mechanics when a particle propagates simultaneously through both slits. Interference of the amplitudes with inside/outside pre-hadron production has a considerable effect on the nuclear absorption. This interference term in the cross section is large and negative, as is explained in Sect. IV on the example of hadronization in vacuum. It is not a surprise that the possibility of pre-hadron production inside the nucleus leads to more suppression due to attenuation of the colorless pre-hadron $q_2q_1$.

- Suppression of hadrons should be much stronger in the case of a dense medium created in heavy ion collisions. This effect is completely missed in calculations based on the energy loss scenario [25]. In fact, it should account for a substantial part of high-$p_T$ hadron suppression observed in heavy ion collisions [26]. This may also explain why the observed suppression, when is related solely to energy loss, demands an unrealistically high density of gluons radiated in heavy ion collisions [27].

While the performed analysis highlights the novel features of in-medium fragmentation brought by a rigorous quantum-mechanical treatment of the process, it is still not sufficiently realistic to be compared with data. Fragmentation was calculated in the Born approximation, and the main lacking element is vacuum energy loss due to gluon radiation caused by the initial hard interaction [28]. Such a modification is expected to shrink the distances $z_2 - z_1$ and $z_3 - z_1$ and make them $Q^2$-dependent. Moreover, vacuum energy loss caused by gluon radiation leads to a distance for pre-hadron production which vanishes in the limit $z_h \rightarrow 1$ [2, 4, 29]. Energy conservation also causes nuclear suppression toward the kinematical limit $z_h = 1$ [30]. These corrections may only enhance the statements listed above. We plan to work on this problem and publish elsewhere.
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Appendix A. SPACE-TIME STRUCTURE OF THE DIS VERTEX

The amplitude of the process $lq_0 \to l'q_1\bar{q}_2q_3$ has the form,

$$M = \frac{j_{\mu}^{(l)}j_{\nu}^{(h)}}{Q^2},$$  \hspace{2cm} (A.1)

where $j_{\mu}^{(l)}$ and $j_{\nu}^{(h)}$ are leptonic and hadronic currents respectively. The latter can be presented as a sum of two terms,

$$J_{\nu}^{(h)} = J_{\nu}^{(a)} + J_{\nu}^{(b)},$$  \hspace{2cm} (A.2)

corresponding to graphs Fig. 7a and b, respectively. The amplitudes have the form,

$$J_{\nu}^{(a)} = \frac{1}{M^2} \bar{u}(p_1)\gamma_\nu \tilde{G}(p_1 + p_2 + p_3)\gamma_\mu u(p_0),$$  \hspace{2cm} (A.3)

$$J_{\nu}^{(b)} = \frac{1}{M^2} \bar{u}(p_1)\gamma_\nu \tilde{G}(p_1 - Q)\gamma_\mu u(p_0),$$  \hspace{2cm} (A.4)

where $M^2 = (p_2 + p_3)^2$.

For a massless quark, $m_q = 0$,

$$\tilde{G}(p) = \frac{\hat{p}}{p^2 + io}.$$  \hspace{2cm} (A.5)

Correspondingly,

$$\tilde{G}(x_2 - x_1) = \frac{1}{(2\pi)^4} \int d^4p \tilde{G}(p) e^{ip(x_2 - x_1)} = \frac{\gamma_\mu x_\mu}{2\pi^2[(x_2 - x_1)^2 - io]^2}. $$  \hspace{2cm} (A.6)

The propagator $\tilde{G}(x_2 - x_1)$ describes propagation of a quark through the interval $x_2 - x_1$ absorbing the virtual photon at one point, and radiating a virtual gluon at another point.

One can obtain an amplitude of the reaction $lq_0 \to l'\pi q_3$ from the amplitude Eq. (A.1) projecting the produced $\bar{q}_2q_1$ pair to the pion wave function. Generally, this is quite a nontrivial problem, which, however, much simplifies in some approximations. In the Berger model of a "frozen" pion [7] one neglects the intrinsic motion of the quarks in the pion, $p_2 = p_1 = \frac{1}{2}p_\pi$, which is certainly not a realistic approximation. In this case the projection is fulfilled using the relation [8],

$$\sum_{\lambda} \bar{u}^{(\lambda)}(p_1) u^{(-\lambda)}(p_2) \sigma_\lambda = \frac{1}{2} \gamma_5 (\hat{p}_\pi + m_\pi).$$  \hspace{2cm} (A.7)

Another approximation is to neglect the quark and pion masses, $m_q = m_\pi = 0$. In this case,

$$\sum_{\lambda} \bar{u}^{(\lambda)}(p_1) u^{(-\lambda)}(p_2) \sigma_\lambda = \sqrt{\alpha(1 - \alpha)} \gamma_5 \hat{p}_\pi,$$  \hspace{2cm} (A.8)

where $\alpha$ is the fractional light-cone momentum of one of the valence quarks in the pion. Then the projections of the components of the hadronic current to the final $\pi q_3$ state have the form,

$$J_{\nu}^{(a)}(p) = \frac{1}{M^2} \int \frac{d\alpha}{\alpha} \frac{\sqrt{2} \phi_\gamma(\alpha)}{M^2} \bar{u}(p_3)\gamma_\gamma \tilde{G}(p_3 + p_\pi)\gamma_\mu u(p_0);$$  \hspace{2cm} (A.9)

$$J_{\nu}^{(b)}(p) = \frac{1}{M^2} \int \frac{d\alpha}{\alpha} \frac{\sqrt{2} \phi_\gamma(\alpha)}{M^2} \bar{u}(p_3)\gamma_\gamma \tilde{G}(\alpha p_\pi - Q)\gamma_\mu u(p_0),$$  \hspace{2cm} (A.10)

where $M^2 = [p_3 + (1 - \alpha)p_\pi]^2$.

Since according to the Dirac equation, $\bar{u}(p_3)\hat{p}_3 = 0$, we can replace the product $\hat{p}_\pi \tilde{G}(p_\pi + p_3)$ in the component $J_{\nu}^{(a)}$, Eq. (A.9), by

$$G(p_\pi + p_3) = (\hat{p}_\pi + \hat{p}_3) \tilde{G}(p_\pi + p_3) \equiv 1.$$  \hspace{2cm} (A.11)

Then the effective propagator,

$$G(x_2 - x_1) = \frac{1}{(2\pi)^4} \int d^4p G(p) e^{ip(x_2 - x_1)} = \delta(x_2 - x_1),$$  \hspace{2cm} (A.12)

is not zero only when $x_2 = x_1$, i.e. the virtual gluon is radiated at the same point where the virtual photon is absorbed.

For the second component of the hadronic current, $J_{\nu}^{(b)}$, Eq. (A.10), it was demonstrated in [8] that its longitudinal-to-transverse contribution ratio is suppressed as $\sigma_L/\sigma_T \sim p^2_{\perp}/(1 - z)Q^2$. Therefore, we will keep only the transverse part of the current $J_{\nu}^{(b)}$.

Then, using the kinematic relation, $p_\pi = z_h p_3/(1 - z_h) + O(p_\perp)$ and applying the Dirac equation, $\bar{u}(p_3)\hat{p}_3 =$
leading to the relations, $a_1 = b_1 + d_1 = \frac{9}{8}$ and $b_1 + c_1 = -\frac{1}{8}$. Otherwise, fixing $\vec{r}^q = \vec{s}^q$, we expect,

$$
\Sigma_1 \Rightarrow \sigma_{gq}(\vec{r}^q - \vec{s}^q) = \frac{9}{4} \sigma_{qg}(\vec{r}^q - \vec{s}^q). \quad (B.3)
$$

This leads to new relations, $a_1 + b_1 = 0$ and $d_1 = \frac{9}{4}$. Thus, we arrive at the coefficients in (B.1), $a_1 = -b_1 = \frac{9}{8}$, $c_1 = 1$ and $d_1 = \frac{9}{4}$ which proves Eq. (15).

### B.2. Cross sections $\Sigma_3(\vec{r}^{q_3}, \vec{r}^{q_2}, \vec{r}^{q_1}, \vec{s}^q, \vec{s}^{q_1} )$ and $\Sigma_3(\vec{r}^{q_3}, \vec{r}^{q_2}, \vec{r}^{q_1}, \vec{s}^q, \vec{s}^{q_2}, \vec{s}^{q_1} )$

Again, relying on the symmetry relative the transmutant $\vec{r} \Rightarrow \vec{s}$ we can write,

$$
\Sigma_3 = a_3 \left[ \sigma_{gq}(\vec{r}^{q_1} - \vec{r}^{q_2}) + \sigma_{qg}(\vec{s}^{q_1} - \vec{s}^{q_2}) \right] \\
+ b_3 \left[ \sigma_{qg}(\vec{r}^{q_1} - \vec{s}^{q_2}) + \sigma_{gq}(\vec{s}^{q_1} - \vec{s}^{q_2}) \right] \\
+ c_3 \left[ \sigma_{gq}(\vec{r}^{q_2} - \vec{s}^{q_2}) + \sigma_{qg}(\vec{s}^{q_2} - \vec{s}^{q_2}) \right] \\
+ d_3 \left[ \sigma_{qg}(\vec{r}^{q_2} - \vec{s}^{q_2}) + \sigma_{gq}(\vec{s}^{q_2} - \vec{s}^{q_2}) \right] \\
+ f_3 \sigma_{qg}(\vec{r}^{q_1} - \vec{s}^{q_1}) + g_3 \sigma_{qg}(\vec{r}^{q_1} - \vec{s}^{q_2}) + h_3 \sigma_{qg}(\vec{r}^{q_2} - \vec{s}^{q_1}) + k_3 \sigma_{qg}(\vec{r}^{q_2} - \vec{s}^{q_2}) \\
+ \frac{9}{8} \sigma_{qg}(\vec{r}^{q_2} - \vec{s}^{q_2}) - \frac{1}{8} \sigma_{qg}(\vec{r}^{q_2} - \vec{s}^{q_2}). \quad (B.4)
$$

We can simplify this expression by considering known limiting combinations. For $\vec{r}^{q_2} = \vec{s}^{q_2} \equiv \vec{s}^q$ and $\vec{s}^{q_2} = \vec{s}^{q_1}$ we have to get $\Sigma_3 \Rightarrow \Sigma_1(\vec{r}^q, \vec{s}^{q_1}, \vec{s}^q, \vec{s}^{q_2})$. This condition leads to the following relations, $a_3 + b_3 = -d_3 = c_3 = \frac{9}{8}, g_3 = -1, 2f_3 + h_3 + k_3 = \frac{9}{8}$. The next possibility is to fix $\vec{s}^{q_2} = \vec{s}^{q_1} = \vec{s}^q$ and $\vec{s}^{q_1} = \vec{s}^{q_2}$. Then we should arrive at the 3-body case,

$$
\Sigma_3 \Rightarrow \sigma_3(\vec{r}^{q_2}, \vec{r}^{q_1}, \vec{s}^q) = \frac{9}{8} \sigma_{qg}(\vec{s}^q - \vec{s}^{q_2}) \\
+ \sigma_{qg}(\vec{s}^q - \vec{s}^{q_1}) - \frac{1}{8} \sigma_{qg}(\vec{r}^{q_2} - \vec{r}^{q_1}). \quad (B.5)
$$

This results in additional relations, $f_3 + h_3 = f_3 + k_3 = \frac{9}{8}, c_3 = -\frac{1}{8}$ and $a_3 + d_3 = b_3 + c_3 = 0$. Eventually, after fixing the coordinates differently, $\vec{s}^{q_2} = \vec{s}^{q_1} \equiv \vec{s}^q$ and $\vec{r}^{q_1} = \vec{r}^{q_2}$, Eq. (B.4) simplifies to,

$$
\Sigma_3 \Rightarrow \sigma_3(\vec{s}^{q_2}, \vec{r}^{q_1}, \vec{s}^q). \quad (B.6)
$$

Correspondingly, new relations emerge, $a_3 + b_3 = k_3 + f_3 = \frac{9}{8}, b_3 + c_3 = d_3 + g_3 = h_3 + f_3 + d_3 + e_3 = 0$, and $c_3 = -\frac{1}{8}$. Solving these sets of relations we get the coefficients in (B.4),

$$
a_3 = -d_3 = g_3 = h_3 = k_3 = 1, \quad b_3 = -e_3 = c_3 = f_3 = \frac{1}{8}, \quad (B.7)
$$

which lead to Eq. (30).

Eventually, one can get Eq. (29) for $\Sigma_2(\vec{r}^{q_3}, \vec{r}^{q_2}, \vec{r}^{q_1}, \vec{s}^q, \vec{s}^{q_1})$ by fixing $\vec{s}^{q_2} = \vec{s}^{q_1} = \vec{s}^q$ in Eq. (30) for $\Sigma_3(\vec{r}^{q_3}, \vec{r}^{q_2}, \vec{r}^{q_1}, \vec{s}^q, \vec{s}^{q_2}, \vec{s}^{q_1})$. 

### Appendix B. Multi-Parton Cross Sections $\Sigma_i$

The effective cross sections $\Sigma_i$, which are linear combinations of dipole $q\bar{q}$ cross sections, can be derived within the Born approximation. The derivation is quite lengthy and not easy. It is much easier to use a set of equations which correspond to different limiting configurations within the multi-parton state. The way how it works is explained further in concrete examples.
Appendix C. FUNCTIONS $W_i$

C.1. Full calculation

As we already mentioned, equations (12)-(13) and (23)-(26) can be solved analytically, provided that the nuclear density is constant, $\rho_A(b, z) = \rho_0$, and the dipole cross section has the simple form $\sigma_{\tilde{q}q} = C r^2$. In this case the equations, which are bilinear in the interaction potential, can be solved following Ref. [22].

We demonstrate here the method for the example of Eq. (12), which can be represented as,

$$\frac{d}{dz} W_i = H_1 W_i. \quad (C.1)$$

The effective Hamiltonian $H_1$ can be written as a sum of the effective kinetic and potential energies,

$$H_1 = T_1 + V_1. \quad (C.2)$$

Then $W_1$ can be presented as,

$$W_1(z_2, z_1) = N_1(z_2, z_1) e^{i S_1}, \quad (C.3)$$

where

$$S_1 = \int_{z_1}^{z_2} dz \, L_1(z). \quad (C.4)$$

The effective Lagrangian has the form,

$$L_1 = T_1(z) - V_1(z), \quad (C.5)$$

where the potential term reads

$$V_1(z) = -i \rho_0 \sum_1 \left( \bar{r}^g(z), \bar{r}^q(z), \bar{s}^g(z), \bar{s}^q(z) \right). \quad (C.6)$$

The kinetic term has the form,

$$T_1(z) = \frac{1}{2} E_g \left( \bar{s}^g(z) \right)^2 + \frac{1}{2} E_1 \left( \bar{s}^q(z) \right)^2$$

$$- \frac{1}{2} E_g \left( \bar{u}^g(z) \right)^2 - \frac{1}{2} E_1 \left( \bar{u}^q(z) \right)^2, \quad (C.7)$$

where

$$\bar{r}^g(z) = \frac{d}{dz} \bar{r}^g(z);$$

$$\bar{r}^q(z) = \frac{d}{dz} \bar{r}^q(z);$$

$$\bar{s}^g(z) = \frac{d}{dz} \bar{s}^g(z);$$

$$\bar{s}^q(z) = \frac{d}{dz} \bar{s}^q(z).$$

The transverse separations as functions of $z$ are the solutions of the Euler-Lagrange differential equations,

$$\frac{d}{dz} \left( \frac{\partial L_1}{\partial \bar{v}^g} \right) - \frac{\partial L_1}{\partial \bar{v}^g} = 0;$$

$$\frac{d}{dz} \left( \frac{\partial L_1}{\partial \bar{v}^q} \right) - \frac{\partial L_1}{\partial \bar{v}^q} = 0;$$

$$\frac{d}{dz} \left( \frac{\partial L_1}{\partial \bar{u}^g} \right) - \frac{\partial L_1}{\partial \bar{u}^g} = 0; \quad (C.9)$$

It is convenient to use the coordinates of the center of mass,

$$\tilde{R} = \frac{E_g \bar{r}^g + E_1 \bar{r}^q}{E_g + E_1} = (1 - x) \bar{r}^g + x \bar{r}^q, \quad (C.10)$$

where $x = \alpha z_h$, and

$$\tilde{S} = \frac{E_g \bar{s}^g + E_1 \bar{s}^q}{E_g + E_1} = (1 - x) \bar{s}^g + x \bar{s}^q, \quad (C.11)$$

The relative separations are given by,

$$\bar{r} = \bar{r}^g - \bar{r}^q;$$

$$\bar{s} = \bar{s}^g - \bar{s}^q. \quad (C.12)$$

The corresponding velocities read,

$$\tilde{V}(z) = \frac{d}{dz} \tilde{R}(z);$$

$$\tilde{U}(z) = \frac{d}{dz} \tilde{S}(z);$$

$$\tilde{v}(z) = \frac{d}{dz} \bar{r}(z);$$

$$\tilde{u}(z) = \frac{d}{dz} \bar{s}(z). \quad (C.13)$$

In the new variables the Lagrangian Eq. (C.5) gets the form,

$$L_1(z) = \frac{1}{2} E \left( \tilde{V}^2 - \tilde{U}^2 \right) + \frac{1}{2} \varepsilon \tilde{v}^2$$

$$- \frac{1}{2} \varepsilon \tilde{u}^2 + \frac{i}{2} \rho_0 \sum_1 \left( \tilde{r}, \tilde{s}, \tilde{R} - \tilde{S} \right), \quad (C.14)$$

where $E = E_g + E_1 = \tilde{E}_g + \tilde{E}_1 = E$.

Then, we make the following combinations of the centers of gravity coordinates,

$$\tilde{R}^+ = \frac{1}{2} (\tilde{R} + \tilde{S});$$

$$\tilde{R}^- = \tilde{R} - \tilde{S}; \quad (C.15)$$

and velocities,

$$\tilde{V}^+ = \frac{1}{2} (\tilde{V} + \tilde{U});$$

$$\tilde{V}^- = \tilde{V} - \tilde{U}; \quad (C.16)$$
Notice that the cross section $\Sigma_1$ Eq. (15), which enters the potential term of the Lagrangian, Eq. (C.6), is independent of $R^+$. Therefore, the Euler-Lagrange equations (C.9) written via new variables $\vec{R}^+, \vec{R}^-, \vec{r}, \vec{s}$ have a simple solution, $\frac{d}{dz} \vec{V}^- = 0$, i.e.,

$$
\vec{V}^- (z) = \text{Const} = \frac{\vec{R}^-_2 - \vec{R}^-_1}{z_2 - z_1};
\vec{R}^- (z) = \vec{R}^-_1 + (z - z_1) \vec{V}^-.
$$

(C.17)

Then, for the first term in the Lagrangian Eq. (C.14) the integral Eq. (C.4) can be calculated as,

$$
\frac{E}{2} \int_{z_1}^{z_2} dz (\vec{V}^2 - \vec{U}^2) = E \int_{z_1}^{z_2} dz \vec{V} + \vec{V}^- = \frac{E}{z_2 - z_1} (\vec{R}^+_2 - \vec{R}^+_1)(\vec{R}^-_2 - \vec{R}^-_1).
$$

(C.18)

Thus, for this part of the integral Eq. (C.4) we did not need to know the explicit form of $\vec{R}^+(z)$, which is rather complicated.

In order to calculate the rest of the integral (C.4), we need to know $\vec{r}(z)$ and $\vec{s}(z)$. The potential Eq. (C.6) can be represented as,

$$
V_1 = \frac{1}{2} \left[ a\vec{r}^2 - 2b \vec{r} \cdot \vec{s} + c\vec{s}^2 + 2d \vec{r} \cdot \vec{R}^- - 2e \vec{s} \cdot \vec{R}^- + f \left( \vec{R}^- \right)^2 \right],
$$

(C.19)

where

$$
a = -i C_0 \left[ 1 + \alpha^2 z^2 + \alpha z h \right] ;
b = -i C_0 \left[ 1 + \alpha h z + (\alpha + \bar{\alpha}) z h \right] ;
c = -i C_0 \left[ \alpha z + \frac{1}{8} \right] ;
d = -i C_0 \left[ \bar{\alpha} z + \frac{1}{8} \right] ;
e = -i C_0 \left[ \alpha h z + \bar{\alpha} z + \frac{1}{8} \right] ;
f = -i C_0 .
$$

(C.20)

Then the Euler-Lagrange equations lead to the following linear equations for $\vec{r}(z)$ and $\vec{s}(z)$,

$$
\varepsilon \left( \frac{d}{dz} \right)^2 \vec{r} = -a \vec{r} + b \vec{s} - d \vec{R}^-;
\varepsilon \left( \frac{d}{dz} \right)^2 \vec{s} = -b \vec{r} + c \vec{s} - e \vec{R}^-,
$$

(C.21)

where $\varepsilon = x(1-x)E$, $\bar{\varepsilon} = \bar{x}(1-\bar{x})E$. To make these equations homogeneous we switch to new variables,

$$
\vec{r}' = \vec{r} + \nu \vec{R}^-; 
\vec{s}' = \vec{s} + \zeta \vec{R}^-,
$$

(C.22)

where $\nu$ and $\zeta$ are solutions of the algebraic equations,

$$
va - \zeta b = d 
v b - \zeta c = e.
$$

(C.23)

Then, $\vec{r}'(z)$ and $\vec{s}'(z)$ satisfy the homogeneous equations,

$$
\varepsilon \frac{d^2 \vec{r}'}{dz^2} = -a \vec{r}' + b \vec{s}';
\varepsilon \frac{d^2 \vec{s}'}{dz^2} = -b \vec{r}' + c \vec{s}' .
$$

(C.24)

The solution of these equation is,

$$
\vec{r}'(z) = \vec{A} \sin(\omega_1 z) + \vec{B} \cos(\omega_1 z) + \nu \vec{C} \sin(\omega_2 z) + \bar{\nu} \vec{D} \cos(\omega_2 z).
$$

(C.25)

Here $\omega_{1,2} = \sqrt{\lambda_{1,2}}$, where $\lambda_{1,2}$ are the solutions of the quadratic equation $(a - \varepsilon \lambda)(c + \varepsilon \lambda) - b^2 = 0$, and $\mu = (a - \varepsilon \lambda_1)/b, \nu = (c + \varepsilon \lambda_2)/b$.

The vectors $\vec{A}, \vec{B}, \vec{C}, \bar{\nu}$ are fixed by the boundary conditions $\vec{r}'(z_2) = \vec{r}'_1, \bar{\nu}$ and $\vec{s}'(z_2) = \vec{s}'_1, \bar{\nu}$,

$$
\vec{A} = \frac{1}{1 - \mu \nu s_{11} c_{12} - s_{12} c_{11}} \vec{\rho}_{1,2} c_{12} - \vec{\rho}_{2,2} c_{11};
\vec{B} = \frac{1}{1 - \mu \nu c_{11} s_{12} - c_{12} s_{11}} \vec{\rho}_{1,2} s_{12} - \vec{\rho}_{2,2} s_{11};
\vec{C} = \frac{1}{1 - \mu \nu s_{21} c_{22} - s_{22} c_{21}} \vec{\tau}_{1,2} c_{22} - \vec{\tau}_{2,2} c_{21};
\vec{D} = \frac{1}{1 - \mu \nu c_{21} s_{22} - c_{22} s_{21}} \vec{\tau}_{1,2} s_{22} - \vec{\tau}_{2,2} s_{21}.
$$

(C.27)

where

$$
\vec{\rho}_{1,2}(z) = \vec{r}'_1(z) - \mu \vec{r}'_2(z); 
\vec{\tau}_{1,2}(z) = \vec{s}'_1(z) - \nu \vec{s}'_2(z).
$$

(C.28)

and $s_{i,j} = \sin(\omega_j z_j), c_{i,j} = \cos(\omega_j z_j), i, j = 1, 2.$

Now we are in a position to perform the rest of integration in Eq. (C.4), and we arrive at the final expression for the action,
where $\Delta z_{12} = z_2 - z_1$; $x = \alpha z_h$; $\bar{x} = \bar{\alpha} z_h$.

Now we can solve equation (C.1),

$$\left(i \frac{\partial}{\partial z_2} - H_1\right) e^{i S_1} = i \Phi(\Delta z_{12}) e^{i S_1},$$

(C.30)

where

$$\Phi(\Delta z_{12}) = \frac{2}{\Delta z_{12}} + \omega_1 \cot(\omega_1 \Delta z_{12}) + \omega_2 \cot(\omega_2 \Delta z_{12}).$$

(C.31)

Then, from (12) and the boundary condition (14) we find the factor in (C.3),

$$N_1(\Delta z_{12}) = \left(\frac{E}{2\pi \Delta z_{12}}\right)^2 \frac{E^2 x(1-x)\bar{x}(1-\bar{x})\omega_1 \omega_2}{(2\pi)^2 \sin(\omega_1 \Delta z_{12}) \sin(\omega_2 \Delta z_{12})}. $$

(C.32)

The derivation of the functions $W_{2,3}(z_2, z_1)$ is analogous, but rather cumbersome, so we skip it here.

### C.2. Approximations

#### 1. Function $W_1$

The expressions for $W_i(z_2, z_1)$ ($i = 1, 2, 3$) simplify, if $x = \bar{x}$ (i.e., $\alpha = \bar{\alpha}$), and the parameters $a, \ldots, f$ in Eq. (C.19), which are functions of $x$ and $\bar{x}$, are related if $x = \bar{x}$,

$$a(x = \bar{x}) = b(x = \bar{x}) = c(x = \bar{x})$$

and

$$d(x = \bar{x}) = e(x = \bar{x})$$

(C.33)

Besides, for the parameters defined in (C.21) $\varepsilon = \bar{\varepsilon}$, and the parameters $\mu, \nu \to 1$. It turns out that it is more complicated to perform a transition in the found solution for the action $S_1(\alpha - \bar{\alpha} \to 0)$, than to repeat the derivation specifically in this limit.

In this case the Lagrangian (C.5) gets a simple form,

$$L_1(\alpha = \bar{\alpha}) = E \bar{V}^+ \cdot \bar{V}^- + E x(1-x)\bar{x}^+ \cdot \bar{x}^- - 1/2 [a(\bar{x}^-)^2 + f(\bar{r}^-)^2 + 2e\bar{r}^- \bar{R}^-],$$

(C.34)

where $\bar{x}^\pm = \frac{\partial}{\partial z^\pm}, \bar{x}^+ = (\bar{r} + \bar{s})/2, \bar{x}^- = \bar{r} - \bar{s}$.

From the Euler-Lagrangian equations of motion it follows that $\frac{\partial}{\partial z^-} \bar{V}^- = \frac{\partial}{\partial z^-} \bar{v}^- = 0$, so

$$\bar{v}^- = \frac{\bar{R}^- - \bar{R}_1^-}{\Delta z_{12}},$$

(C.35)

and

$$\bar{r}^- = \frac{\bar{r}_2^- - \bar{r}_1^-}{\Delta z_{12}};$$

(C.36)

This is sufficient for calculating the action Eq. (C.4), and we arrive at,

$$S_1 = \frac{E}{\Delta z_{12}} \left[ \left(\bar{R}^+_2 - \bar{R}^+_1\right) \left(\bar{R}^-_2 - \bar{R}^-_1\right) + x(1-x) \left(\bar{r}^+_2 - \bar{r}^+_1\right) \left(\bar{r}^-_2 - \bar{r}^-_1\right) \right] - \frac{a \Delta z_{12}}{6} \left[ (\bar{r}^-_2)^2 + \bar{r}_2^- \bar{r}_1^- + (\bar{r}_1^-)^2 \right]$$

$$- \frac{f \Delta z_{12}}{6} \left[ (\bar{R}^-_2)^2 + \bar{R}_2^- \bar{R}_1^- + (\bar{R}_1^-)^2 \right] - \frac{e \Delta z_{12}}{6} \left[ 2\bar{R}_2^- \bar{r}_2^- + 2\bar{R}_1^- \bar{r}_1^- + \bar{R}_2^- \bar{r}_1^- + \bar{R}_1^- \bar{r}_2^- \right]$$

(C.37)

Then the coefficient $N_1(\Delta z_{12})$ in Eq. (C.3) gets the very simple form,

$$N_1(\Delta z_{12}) = x(1-x) \left(\frac{E}{2\pi \Delta z_{12}}\right)^4.$$
2. Function $W_3$

The next case is $W_3$, which is simple due to the symmetry relative to interchange $\vec{r}^q \leftrightarrow \vec{s}^q$. First we introduce the Jacoby coordinates,

\[
\begin{align*}
\vec{R} &= x_1 \vec{r}^q_1 + x_2 \vec{r}^q_2 + x_3 \vec{r}^q_3; \\
\vec{r} &= \frac{x_2 \vec{r}^q_2 + x_3 \vec{r}^q_3}{x_2 + x_3} - \vec{r}^q_1; \\
\vec{S} &= x_1 \vec{s}^q_1 + x_2 \vec{s}^q_2 + x_3 \vec{s}^q_3; \\
\vec{s} &= \frac{x_2 \vec{s}^q_2 + x_3 \vec{s}^q_3}{x_2 + x_3} - \vec{s}^q_1; \\
\vec{p}^r &= \vec{r}^q - \vec{r}^q_2; \\
\vec{p}^s &= \frac{\vec{s}^q}{x_2 + x_3} - \vec{s}^q_1; \\
\vec{R} &= \vec{r}^a + 2 \vec{s}^a.
\end{align*}
\]

(C.39)

\[\text{Here } x_1 = \alpha z_h, \ x_2 = (1 - \alpha) z_h, \ x_3 = 1 - z_h.\]

We also introduce combinations of the Jacoby coordinates, $\vec{R}^+ = \frac{1}{2}(\vec{R} + \vec{S}), \vec{R}^- = \vec{R} - \vec{S}, \vec{r}^+ = \frac{1}{2}(\vec{r} + \vec{s}), \vec{r}^- = \vec{r} - \vec{s}, \vec{p}^r_+ = \frac{\vec{r} + \vec{s}}{x_2 + x_3}, \vec{p}^r_- = \vec{p}^r - \vec{r}.\]

The Lagrangian $L_3$ can be represented as,

\[
L_3 = E \vec{V}^+ \vec{V}^- + x(1 - x) E \vec{u}^+ \vec{u}^- + \vec{E} \vec{v}^+ \vec{v}^- - \frac{1}{2} \left\{ a \left( \vec{r}^- \right)^2 + b \left( \vec{p}^r_- \right)^2 + c \left( \vec{R}^- \right)^2 \right\} + 2d \vec{r}^- \vec{p}^- + 2e \vec{r}^- \vec{R}^- + 2f \vec{p}^- \vec{R}^-.
\]

(C.40)

Again, the equations of motion lead to the relations,

\[
\frac{\partial}{\partial z} \vec{V}^- = \frac{\partial}{\partial z} \vec{V}^- = \frac{\partial}{\partial z} \vec{V}^- = 0;
\]

\[
\vec{R}^- = \vec{R}^+ + \left( \vec{R}^- - \vec{R}^+ \right) \frac{z - z_3}{\Delta z_{34}};
\]

\[
\vec{r}^- = \vec{r}^+ + \left( \vec{r}^- - \vec{r}^+ \right) \frac{z - z_3}{\Delta z_{34}};
\]

\[
\vec{p}^- = \vec{p}^+ + \left( \vec{p}^- - \vec{p}^+ \right) \frac{z - z_3}{\Delta z_{34}}.
\]

(C.42)

These relations lead to the following action,

\[
S_3 = \int_{z_3}^{z_4} dz L_3
\]

\[
= \frac{1}{\Delta z_{34}} \left\{ E \left( \vec{R}^+_2 - \vec{R}^+_1 \right) \left( \vec{R}^-_2 - \vec{R}^-_1 \right) + E x(1 - x) \left( \vec{r}^+_2 - \vec{r}^+_1 \right) \left( \vec{r}^-_2 - \vec{r}^-_1 \right) + \vec{E} \left( \vec{p}^+_2 - \vec{p}^+_1 \right) \left( \vec{p}^-_2 - \vec{p}^-_1 \right) \right\}
\]

\[
- \frac{\Delta z_{34}}{6} \left\{ a \left[ \left( \vec{r}^-_2 \right)^2 + \vec{r}^+_2 \vec{r}^-_2 + \left( \vec{r}^-_1 \right)^2 \right] + b \left[ \left( \vec{p}^+_2 \right)^2 + \vec{p}_2 \vec{p}^+_1 + \left( \vec{p}^-_1 \right)^2 \right] + c \left[ \left( \vec{R}^-_2 \right)^2 + \vec{R}^+_2 \vec{R}^-_1 + \left( \vec{R}^-_1 \right)^2 \right] + d \left[ 2 \vec{r}^-_2 \vec{p}^-_2 + 2 \vec{r}^-_1 \vec{p}^-_1 + \vec{r}^-_2 \vec{p}^-_1 + \vec{r}^-_1 \vec{p}^-_2 \right] + e \left[ 2 \vec{r}^-_2 \vec{R}^-_2 + 2 \vec{r}^-_1 \vec{R}^-_1 + \vec{r}^-_1 \vec{R}^-_2 + \vec{r}^-_2 \vec{R}^-_1 \right] + f \left[ 2 \vec{p}^-_2 \vec{R}^-_2 + 2 \vec{p}^-_1 \vec{R}^-_1 + \vec{p}^-_2 \vec{R}^-_1 \vec{R}^-_2 \right] \right\}
\]

(C.43)

Eventually, we arrive at,

\[
W_3(z) = N_3(\Delta z_{34}) e^{iS_3},
\]

where

\[
N_3(\Delta z_{34}) = \alpha (1 - \alpha) z_h^2 (1 - z_h) \left( \frac{E}{2\pi \Delta z_{34}} \right)^6 = x_1 x_2 x_3 \left( \frac{E}{2\pi \Delta z_{34}} \right)^6.
\]

(C.44)

3. Function $W_2$

The calculation of $W_2$ is more involved. First we switch to new variables.

\[
\vec{R} = x_1 \vec{r}^q_1 + x_2 \vec{r}^q_2 + x_3 \vec{r}^q_3;
\]

\[
\vec{r} = \frac{x_2 \vec{r}^q_2 + x_3 \vec{r}^q_3}{x_2 + x_3} - \vec{r}^q_1;
\]

\[
\vec{S} = x \vec{s}^q_1 + (1 - x) \vec{s}^q_2;
\]

\[
\vec{s} = \vec{s}^q - \vec{s}^q_1.
\]

(C.46)
\[ \bar{R}^+ = \frac{1}{2} \left( \bar{R} + \bar{S} \right); \quad \bar{R}^- = \bar{R} - \bar{S}; \]
\[ \bar{\tau}^+ = \frac{1}{2} \left( \bar{\tau} + \bar{s} \right); \quad \bar{\tau}^- = \bar{\tau} - \bar{s}. \]  \tag{C.47}

Then the Lagrangian \( L_2 \) gets the form,
\[ L_2 = E V + \bar{V}^{-} + x (1 - x) E v^{+} \bar{v}^{-} + \frac{1}{2} \mathcal{E} \bar{\omega}^2 \]
\[ - \frac{1}{2} \left[ \bar{a} \rho^2 + b \left( \bar{\tau}^+ \right)^2 + c \left( \bar{R}^- \right)^2 + 2 d \bar{\rho} \bar{\tau}^- \right] + 2 f \bar{\rho} \bar{\tau}^- + 2 e \bar{\tau}^- \bar{R}^- \]  \tag{C.48}

where the coefficients \( a, ..., f \) were defined in (C.41).

In this case the equation of motion has the form,
\[ \left( \frac{\partial}{\partial z} \right)^2 \bar{R}^- = \left( \frac{\partial}{\partial z} \right)^2 \bar{\tau}^- = 0; \]
\[ \mathcal{E} \left( \frac{\partial}{\partial z} \right)^2 \bar{\tau}^- + a \bar{\tau}^- = 0, \]  \tag{C.49}

where \( \omega_0 = \sqrt{a / \mathcal{E}}; \Delta z_{23} = z_3 - z_2. \)

Eventually, we get,
\[ W_2 (z) = N_2 (\Delta z_{23}) e^{i S_2}, \]  \tag{C.52}

where
\[ N_2 (\Delta z_{23}) = \frac{- i \mathcal{E} \omega_0}{2 \pi \sin (\omega_0 \Delta z_{23})} \frac{(E \mathcal{E})^2}{(2 \pi \Delta z_{23})^4}. \]  \tag{C.53}

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