Flexible Robust Risk-Constrained Unit Commitment of Power System Incorporating Large Scale Wind Generation and Energy Storage

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ABSTRACT With the increasing penetration of wind power in the power systems, the uncertainties in wind power significantly challenge the reliable and economic operation of power systems. Recently, the worst scenario-based robust optimization approaches have been employed to manage the uncertainties in the unit commitment problem. To further improve the robustness and economic efficiency of power system operation, this article proposes a flexible robust risk-constrained unit commitment formulation, in which flexible reserve capacities of conventional generators and energy storage are allocated to cope with the uncertainty of wind power. The proposed model optimizes the unit commitment and dispatch solutions for the base case while guaranteeing that the flexible reserve capacity can be adaptively adjusted after wind generation realization. In contrast to the predefined uncertainty set in the conventional robust unit commitment, the proposed model constructs an adjustable and flexible uncertainty set via balancing the operational costs and the operational risk. The model establishes worst-case constraints to optimally allocate the flexible reserve capacity. The proposed model can be equivalently transformed into a single-level optimization problem using the strong duality theory. Numerical case studies on a modified standard test system demonstrate the effectiveness and the efficiency of the proposed model.

INDEX TERMS Flexible reserve capacity, energy storage, wind energy generation, robust optimization, unit commitment.

NOMENCLATURE

INDICES

\( i \) Index for conventional units.
\( m \) Index for wind farms.
\( l \) Index for transmission lines.
\( d \) Index for load nodes.
\( t \) Index for time periods.

PARAMETERS

\( N_w \) Number of wind farms.
\( N_g \) Number of conventional units.
\( N_s \) Number of storage units.
\( T \) Number of time periods.
\( F_{l,max} \) Transmission capacity of line \( l \).
\( P_{d,t}^{up} \) Load power of load node \( d \) in period \( t \).
\( c_{l^{up}}^{i} \) Downward flexibility reserve capacity cost coefficient.
\( a_i/b_i/c_i \) Generation cost coefficients of conventional unit \( i \).
\( c_{l^{up}}^{i} \) Start-up cost coefficient of conventional unit \( i \).
\( T_{i,U} \) Minimal on time of conventional unit \( i \).
\( T_{i,D} \) Minimal off time of conventional unit \( i \).
\( P_{d,t}^{up} \) Forecasted output of wind farm \( m \) in period \( t \).
\( P_{i,min} \) Minimum generating capacity of conventional unit \( i \).
\( P_{i,max} \) Maximum generating capacity of conventional unit \( i \).
\( R_{U,i} \) Ramp-up limit of conventional unit \( i \).
\( R_{D,i} \) Ramp-down limit of conventional unit \( i \).
\( E_{j,0} \) Initial stored energy of stored energy \( j \).
\( E_{j,min} \) Minimum storage capacity of stored energy \( j \).
\( E_{j,max} \) Maximum storage capacity of stored energy \( j \).
\( \eta_{C}/\eta_{D} \) Input and output efficiencies of storage unit \( j \).
I. INTRODUCTION

Unit commitment (UC) is a critical scheduling decision process performed by system operators to guide the power system operation in the next dispatching day. The main objective of the UC problem is to determine the on/off schedule and generation plan of generators on the grid to minimize the system comprehensive cost and meet the operation constraints, such as system security and unit-wise constraints.

In recent years, wind power generation has rapidly developed all over the world due to its clean and renewable characteristics. However, wind power is inherently volatile and intermittent. The increasing penetration of wind generation brings significant technical challenges to power system operation. The conventional deterministic optimization method cannot ensure reliable and economic system operation because the power system uncertainty cannot be explicitly captured. In day-ahead scheduling, the UC method should be improved to efficiently account for the uncertainties in wind power generation.

Many inspiring works have been done to address the uncertainty issues. The two main types of common methods for handling uncertain decision problems in power system operation are stochastic optimization (SO) and robust optimization (RO).

In the last decade, the SO have been applied to handle uncertainties in power systems. In [1]–[3], several stochastic unit commitment and stochastic economic dispatch models have been proposed. The SO utilizes a probabilistic manner to ensure security and economy for system operation. However, the SO generates a dispatch strategy that incorporates a large number of selected scenarios and suffers from the computational burden. Only limited sampling scenarios considered in SO approach cannot ensure the feasibility for all the uncertainties.

Recently, many researchers have utilized the RO theory to deal with uncertain factors in power generation [4]–[6]. The uncertainty set is used to depict the stochastic characteristic of renewable energy, making the RO more tractable. Compared with the stochastic UC model, the robust UC model is more reliable as it can guarantee the operational feasibility for all possible scenarios within the uncertainty set. In [4], a two-stage robust UC model has been proposed to handle the net load uncertainties. Another two-stage robust UC model considering the worst scenario of wind power fluctuation has been presented in [5], [6]. Other representative RO-based methods have considered n-k security criterion [7], dispatchable wind power [8], min-max regret concept [9], and

\[
P_{\text{sh}}^{j,\text{max}} \quad \text{Maximum power input of storage unit } j.
\]

\[
P_{d}^{j,\text{max}} \quad \text{Maximum power output of storage unit } j.
\]

\[S_{j} \quad \text{Operation cost coefficient of storage unit } j.
\]

\[
\lambda_{i,t}/\lambda_{m,t}\lambda_{j,i}/\lambda_{d,t} \quad \text{Power flow distribution factors of conventional unit } i/\text{wind farm } m/\text{storage unit } j/\text{load } d \text{ for transmission line } l.
\]

\[\delta_{\text{shed}}/\delta_{\text{spill}} \quad \text{Penalty coefficients of wind power curtailment and load shedding.}
\]

\[\Gamma_{l} \quad \text{Uncertainty budget of uncertainty sets.}
\]

\[
P_{\text{uw}}^{m,t} \quad \text{Upper boundaries of wind generation of wind farm } m \text{ in period } t.
\]

\[
P_{\text{wl}}^{m,t} \quad \text{Lower boundaries of wind generation of wind farm } m \text{ in period } t.
\]

\[d_{l}^{i,t} \quad \text{On/off decisions of conventional unit } i \text{ in period } t.
\]

\[\delta_{s,i,t} \quad \text{Start-up variable of conventional unit } i \text{ in period } t.
\]

\[\delta_{d,i,t} \quad \text{Shut-down variables of conventional unit } i \text{ in period } t.
\]

\[p_{i,t}^{s} \quad \text{Output of conventional unit } i \text{ in period } t.
\]

\[r_{i,t}^{u} \quad \text{Upward flexible reserve capacity of conventional unit } i \text{ in period } t.
\]

\[j_{i,t}^{d} \quad \text{Downward flexible reserve capacity of conventional unit } i \text{ in period } t.
\]

\[p_{j,t}^{c} \quad \text{Charging power of storage unit } j \text{ in period } t.
\]

\[p_{j,t}^{d} \quad \text{Discharging power of storage unit } j \text{ in period } t.
\]

\[c_{j,t} \quad \text{binary variables indicating whether storage unit } j \text{ is in charging state in period } t.
\]

\[d_{j,t} \quad \text{binary variables indicating whether storage unit } j \text{ is in discharging state in period } t.
\]

\[E_{j,t}^{s} \quad \text{Stored energy of storage unit } j \text{ in period } t.
\]

\[f_{j,t}^{\text{adr}} \quad \text{Upward flexible reserve capacity of storage unit } j \text{ in period } t.
\]

\[j_{j,t}^{s,\text{adr}} \quad \text{Downward flexible reserve capacity of storage unit } j \text{ in period } t.
\]

\[f_{j,t}^{\text{up}} \quad \text{Upward flexible reserve capacity of storage unit } j \text{ in period } t \text{ in charging state.}
\]

\[f_{j,t}^{\text{ch,up}} \quad \text{Upward flexible reserve capacity of storage unit } j \text{ in period } t \text{ in charging state.}
\]

\[f_{j,t}^{\text{ch,dr}} \quad \text{Downward flexible reserve capacity of storage unit } j \text{ in period } t \text{ in discharging state.}
\]

\[f_{j,t}^{\text{dr,up}} \quad \text{Upward flexible reserve capacity of storage unit } j \text{ in period } t \text{ in discharging state.}
\]

\[f_{j,t}^{\text{dl,dr}} \quad \text{Downward flexible reserve capacity of storage unit } j \text{ in period } t \text{ in discharging state.}
\]

\[
P_{\text{CUC}} \quad \text{Flexible robust risk-constrained unit commitment}
\]

\[\text{CVaR} \quad \text{Conditional value-at-risk}
\]

\[\text{AIWG} \quad \text{Admissibility interval of wind generation}
\]

\[\text{PDF} \quad \text{Probability distribution function}
\]

\[\text{MILP} \quad \text{Mixed integer linear program}
\]

\[\text{QP} \quad \text{Quadratic programming}
\]

\[\text{ED} \quad \text{Economic dispatch}
\]

\[\text{ACRONYMS}
\]

\[\text{UC} \quad \text{Unit commitment}
\]

\[\text{SO} \quad \text{Stochastic optimization}
\]

\[\text{RO} \quad \text{Robust optimization}
\]

\[\text{ED} \quad \text{Economic dispatch}
\]
the combination of SO and RO [10], [11]. The conventional
two-stage robust UC models immunize solutions against the
worst economic scenario in the prescribed uncertainty set.
However, the solution from the economical aspect is conser-



Third, the boundaries of reserve capacities are allocated in each time period, which
is critical in practical operations. Meanwhile, the proposed method ensures operation security
against wind generation uncertainty scenarios including the
worst-case scenario. The allocation of flexible reserve capac-
ity is considered for uncertainty accommodation. The bound-
aries of the uncertainty set in wind power is modeled as:

\[ \Omega_w = \left\{ \tilde{p}_{m,t}^w, p_{m,t}^w \mid \tilde{p}_{m,t}^w = \tilde{p}_{m,t}^w + v_{m,t}^u - v_{m,t}^l \right\} \]

where \( \tilde{p}_{m,t}^w \) is the power output of wind farm \( m \) in period \( t \); \( p_{m,t}^w \) and \( \tilde{p}_{m,t}^w \) are the upper and lower boundaries of wind
generation of wind farm \( m \) in period \( t \), respectively; \( v_{m,t}^u \) and \( v_{m,t}^l \) are the binary variables indicating the normalized posi-
tive and negative output deviations of wind farm \( m \) in period \( t \), respectively; \( \Gamma_t \) is the uncertainty budget that can adjust
the conservativeness of dispatch plan. When \( \Gamma_t \) is equal to 0, the uncertainty set is reduced to the prediction case without
any uncertainty and the model is degraded to the conventional
UC model. With the increase of \( \Gamma_t \), the proposed model will allocate more flexible reserve capacity to cope with a higher
degree of uncertainty and becomes more conservative. \( \Gamma_t \) can be decided by the feasibility probability \( \alpha \) as follows:

\[ \Gamma_t = \Phi^{-1}(\alpha)\sqrt{\tilde{N}_w} \]

where \( \tilde{N}_w \) is the number of wind farms.

II. MATHEMATICAL FORMULATION

In this section, the uncertainty set and the operational risk
are introduced, and then FRRUC is formulated as a bi-level
optimization problem.

A. THE UNCERTAINTY SET

The uncertainty set in the proposed FRRUC model is variable
and relevant to the availability of system flexible resources. The scale and the position of the variable uncertainty set depend on the system operation state and the flexible reserve capacity. The variable uncertainty set of wind generation is modeled as:

\[ \Omega_w = \left\{ \tilde{p}_{m,t}^w, p_{m,t}^w \mid \tilde{p}_{m,t}^w = p_{m,t}^w + v_{m,t}^u - v_{m,t}^l \right\} \]

where \( \tilde{p}_{m,t}^w \) is the power output of wind farm \( m \) in period \( t \); \( p_{m,t}^w \) are the upper and lower boundaries of wind
generation of wind farm \( m \) in period \( t \), respectively; \( v_{m,t}^u \) and \( v_{m,t}^l \) are the binary variables indicating the normalized posi-
tive and negative output deviations of wind farm \( m \) in period \( t \), respectively; \( \Gamma_t \) is the uncertainty budget that can adjust
the conservativeness of dispatch plan. When \( \Gamma_t \) is equal to 0, the uncertainty set is reduced to the prediction case without
any uncertainty and the model is degraded to the conventional
UC model. With the increase of \( \Gamma_t \), the proposed model will allocate more flexible reserve capacity to cope with a higher
degree of uncertainty and becomes more conservative. \( \Gamma_t \) can be decided by the feasibility probability \( \alpha \) as follows:

\[ \Gamma_t = \Phi^{-1}(\alpha)\sqrt{\tilde{N}_w} \]

where \( \tilde{N}_w \) is the number of wind farms.

B. THE OPERATIONAL RISK

The conditional value-at-risk (CVaR) is used as a risk indicator
due to its coherence in measuring risk [20]. The upper
boundaries \( p_{m,t}^w \) and the lower boundaries \( \tilde{p}_{m,t}^w \) of wind generation constitute the admissibility interval of wind generation
(AIWG). In this article, the expectation of operational loss
due to the wind power variation beyond the AIWG is mea-
sured using the CVaR.

It is assumed that the probability distribution function (PDF) of wind generation forecast error of each wind farm follows a Gaussian distribution. An example of PDF is shown
in Fig. 1 (Note that the other distribution types are also appropriate for the proposed model). The operational risk of the power system under uncertainty is related to the boundaries of AIWG and the wind power forecast error probability distribution.

For a wind farm numbered with \( m \), if the actual wind generation \( \hat{P}_{m,t} \) is within the AIWG, the wind power can be completely accommodated and the system will be reliable and riskless. If \( \hat{P}_{m,t} \) is higher than \( P^\text{min}_{m,t} \), the excessive wind power will be spilled to keep system dispatch feasibility. The operational risk corresponding to wind power curtailment is calculated by (3).

\[
C^\text{up}_{w,t} = \delta^\text{shed} \int_{P^\text{min}_{m,t}}^{\hat{P}_{m,t}} \left( \hat{P}_{m,t} + u_{m,t} - \hat{P}_{m,t} \right) \times \text{pdf}(u_{m,t}) du_{m,t}
\]

If \( \hat{P}_{m,t} \) is lower than \( P^\text{max}_{m,t} \), load shedding may occur in the next scheduling day. The operational risk corresponding to load shedding is stated in (4).

\[
C^\text{dn}_{w,t} = \delta^\text{spill} \int_{\hat{P}_{m,t}}^{P^\text{max}_{m,t}} \left( P^\text{max}_{m,t} - u_{m,t} - \hat{P}_{m,t} \right) \times \text{pdf}(u_{m,t}) du_{m,t}
\]

where \( \delta^\text{shed} \) and \( \delta^\text{spill} \) are the penalty coefficients of wind power curtailment and load shedding, respectively; \( u_{m,t} \) is the wind generation forecast error of wind farm \( m \) in period \( t \).

### C. FORMULATION OF FRRUC MODEL

The proposed FRRUC model considers both operational economy and operational security. The solution in the model is composed of binary on/off commitment decisions, the output decisions of conventional generators, the AIWG of wind farms, the charge/discharge decisions of storage units, and the flexible reserve capacity scheme. The FRRUC model is formulated as follows.

#### 1) OBJECTIVE FUNCTION

The objective function is to minimize the system comprehensive cost, including the generation cost, the start-up cost, the shut-down cost, the flexible reserve capacity supply cost of conventional units, as well as the operation cost and the flexible reserve capacity supply cost of energy storages, operational risk cost.

\[
\min \sum_{t=1}^{T} \sum_{i=1}^{N_i} \left( a_i (P^g_{i,t})^2 + b_i P^g_{i,t} + c_i u^g_{i,t} + c^\text{ur}_{i,t} P^\text{ur}_{i,t} + c^\text{dr}_{i,t} P^\text{dr}_{i,t} \right) + c^\text{up}_{w,t} C^\text{up}_{w,t} + c^\text{dn}_{w,t} C^\text{dn}_{w,t}
\]

#### 2) BASE-CASE CONSTRAINTS

a: POWER BALANCE AND NETWORK POWER FLOW CONSTRAINTS

The system power balance is represented in (6). Constraint (7) describes the network power flow limit by using a direct current mathematical model.

\[
\sum_{i=1}^{N_i} P^g_{i,t} + \sum_{m=1}^{N_w} \hat{P}_{m,t} + \sum_{j=1}^{N_g} (P^\text{sh}_{j,t} - P^\text{up}_{j,t}) = \sum_{d=1}^{N_d} P^l_{d,t}, \quad \forall t
\]

\[
F_{l,max} \leq \sum_{i=1}^{N_i} \lambda_{i,t} P^l_{i,t} + \sum_{m=1}^{N_w} \lambda_{m,t} \hat{P}_{m,t}
\]

\[
+ \sum_{j=1}^{N_g} \lambda_{j,t} (P^\text{sh}_{j,t} - P^\text{up}_{j,t}) - \sum_{d=1}^{N_d} \lambda_{d,t} P^l_{d,t} \leq F_{l,max}, \quad \forall t, \forall l
\]

b: OPERATION CONSTRAINTS OF CONVENTIONAL UNITS

(8) is the start-up and shut-down constraints. Constraints (9) and (10) express the minimum on/off time limits for generators. Constraints (11) and (12) indicate the generation capacity limits. The ramp-up and ramp-down rates are constrained by (13) and (14). Constraints (15) and (16) depict the flexible reserve capacity limits.

\[
\sum_{\tau = t - T^U_{i,t} + 1}^{t} s_{u,i,\tau} \leq u^g_{i,t}, \quad \forall T^U_t < t < T, \forall i
\]

\[
\sum_{\tau = t - T^D_{i,t} + 1}^{t} s_{d,i,\tau} \leq 1 - u^g_{i,t}, \quad \forall T^D_t < t < T, \forall i
\]

\[
P^g_{i,t} + P^\text{ur}_{i,t} \leq u^g_{i,t} P^\text{max}_{i,t}, \quad \forall t, \forall i
\]

\[
P^g_{i,t} - P^\text{dr}_{i,t} \geq u^g_{i,t} P^\text{min}_{i,t}, \quad \forall t, \forall i
\]

\[
P^g_{i,t} + P^\text{ur}_{i,t} - P^\text{up}_{i,t-1} - P^\text{dr}_{i,t-1} \leq R^U_{U,t}(1 - s_{u,i,t}), \quad \forall t, \forall i
\]

\[
P^g_{i,t} - P^\text{dr}_{i,t} + P^\text{up}_{i,t-1} \leq R^D_{D,t}(1 - s_{d,i,t}), \quad \forall t, \forall i
\]
c: ENERGY STORAGE CONSTRAINTS
The energy storage device has advantages including fast adjustment speed, flexible operation mode, and bidirectional interaction with the power grid. It can store the surplus wind energy in the low load, generate electricity at the peak load to relieve the peak load regulation pressure of the system, and provide flexible reserve capacity for the system in order to stabilize the wind power output fluctuation. Constraints (17) and (18) depict the conversion relation between energy and power for storage units and the energy capacity limits of storage units, respectively. Constraint (19) guarantees that the initial stored energy is equal to the final quantity of electricity. The charge and discharge power of energy storage are constrained by (20) and (21), respectively. Constraint (22) ensures that the storage units do not input and output simultaneously in any time period. Constraints (23)–(24) depict the limits of upward flexible reserve capacity of storage units. Constraints (25)–(26) depict the limits of downward flexible reserve capacity of storage units. Considering the flexible reserve capacity deployment, the upper and lower bounds of energy storage are constrained by (27)–(28). Constraints (29)–(30) depict the overall upward/downward flexible reserve capacities.

\[
E_{j,t} = E_{j,0} + \sum_{\tau=1}^{t} \left( \eta c P_{j,t,\tau}^{ch} \right) / \eta_{D}, \quad \forall t, \forall j \tag{17}
\]

\[
E_{j,\min} \leq E_{j,t} \leq E_{j,\max}, \quad \forall t, \forall j \tag{18}
\]

\[
0 \leq P_{j,t}^{ch} \leq c_{j,t} P_{j,t,\chi,\max}^{ch}, \quad \forall t, \forall j \tag{19}
\]

\[
0 \leq P_{j,t}^{dh} \leq d_{j,t} P_{j,t,\chi,\max}^{dh}, \quad \forall t, \forall j \tag{20}
\]

\[
c_{j,t} + d_{j,t} \leq 1, \quad \forall t, \forall j \tag{21}
\]

\[
0 \leq E_{j,t}^{ch,up} \leq P_{j,t}^{ch}, \quad \forall t, \forall j \tag{22}
\]

\[
0 \leq E_{j,t}^{dh,up} \leq (1 - c_{j,t}) P_{j,t,\chi,\max}^{dh} - P_{j,t}^{dh}, \quad \forall t, \forall j \tag{23}
\]

\[
0 \leq E_{j,t}^{ch,down} \leq (1 - d_{j,t}) P_{j,t,\chi,\max}^{ch} - P_{j,t}^{ch}, \quad \forall t, \forall j \tag{24}
\]

\[
0 \leq E_{j,t}^{dh,down} \leq P_{j,t}^{dh}, \quad \forall t, \forall j \tag{25}
\]

\[
E_{j,\min} \leq E_{j,0} + \sum_{\tau=1}^{t} \left( \eta c P_{j,t,\tau}^{ch} - r_{j,t}^{ch,up} \right) / \eta_{D}, \quad \forall t, \forall j \tag{26}
\]

\[
E_{j,0} + \sum_{\tau=1}^{t} \left( \eta c P_{j,t,\tau}^{ch} + r_{j,t}^{ch,down} \right) / \eta_{D} \leq E_{j,\max}, \quad \forall t, \forall j \tag{27}
\]

\[
\begin{align*}
\Delta p_{l}^{u} &= \min_{p_{l,\max}} \sum_{j=1}^{N_{g}} p_{l,j}^{g} + \sum_{i=1}^{N_{l}} p_{l,i}^{ur} + \sum_{m=1}^{N_{w}} \tilde{p}_{m,1}^{w} + \tag{a} \\
\sum_{j=1}^{N_{g}} (P_{j,t}^{dh} - P_{j,t}^{ch}) + \sum_{j=1}^{N_{g}} \sum_{d=1}^{N_{l}} (E_{j,t}^{ch,up} - E_{j,t}^{dh,up}) &\geq 0 \quad \forall t \tag{31}
\end{align*}
\]

\[
\begin{align*}
\Delta p_{l}^{d} &= \min_{p_{l,\min}} \sum_{j=1}^{N_{g}} p_{l,j}^{d} + \sum_{i=1}^{N_{l}} p_{l,i}^{dr} + \sum_{m=1}^{N_{w}} \tilde{p}_{m,2}^{w} + \tag{a} \\
- \sum_{j=1}^{N_{g}} \sum_{d=1}^{N_{l}} (F_{j,t}^{dh} + F_{j,t}^{ch}) + \sum_{j=1}^{N_{g}} \sum_{d=1}^{N_{l}} (r_{j,t}^{d,dr}) &\geq 0 \quad \forall t \tag{32}
\end{align*}
\]

b: The network power flow robust constraints
(33) and (34) are the positive and the negative network power flow robust constraints under the worst-case scenarios, respectively. From the perspective of the power network transmission security, the transmission line should have a certain transmission capacity margin to avoid the flow violation caused by the random fluctuation of wind power. The dispatch strategy should ensure that the minimum transmission.
flow margins $\Delta l_i^T$ and $\Delta l_i^L$ of the system are positive.

\[
\begin{align*}
\Delta l_i^T &= \max \sum_{\mu_{mt}} N_t \lambda_{i,t} + \sum_{\mu_{mt}} \lambda_{m,t} \hat{P}_{m,t}^w, \quad (a) \\
&= \sum_{j=1}^{N_t} \lambda_{j,t} (P_{j,t}^d - P_{j,t}^h) - \sum_{d=1}^{N_t} \lambda_{d,t} P_{d,t}^l \leq F_{l,\max} \quad (33) \\
\Delta l_i^L &= \min \sum_{\mu_{mt}} N_t \lambda_{i,t} + \sum_{\mu_{mt}} \lambda_{m,t} \hat{P}_{m,t}^w, \quad (a) \\
&= \sum_{j=1}^{N_t} \lambda_{j,t} (P_{j,t}^d - P_{j,t}^h) - \sum_{d=1}^{N_t} \lambda_{d,t} P_{d,t}^l \geq -F_{l,\max} \quad (34)
\end{align*}
\]

Equations (5)–(34) compose a bi-level robust optimization formulation, in which constraints (6)–(30) constitute the upper-level optimization problem for the base-case and constraints (31)–(34) constitute the lower-level optimization problem for the worst-case.

## III. SOLUTION METHOD

The bi-level flexible robust risk-constrained optimization model formulated in the previous section has the uncertain variables and cannot be solved directly. In this article, the proposed model is reformulated based on the strong duality theory as a single-level robust mixed integer linear program (MILP). The strong duality theorem states that if a problem is convex, the objective functions of the primal and the dual problems have the same value at the optimum [21]. The duality theory is utilized to transform the “min” problem to its equivalent “max” formulation, and vice versa. Then the max/min constraints can be reformulated as a common constraint. The duality counterparts of constraints expressed in (31), (32), (33) and (34) can be formulated as follows.

\[
\begin{align*}
\sum_{i=1}^{N_t} \sum_{t=1}^{N_t} P_{i,t}^d + \sum_{t=1}^{N_t} P_{i,t}^h + \sum_{m=1}^{N_w} \hat{P}_{m,t}^w - \sum_{k=1}^{N_t} x_{k,t} &= 0 \quad (a) \\
- \sum_{k=1}^{N_t} y_{k,t} - \sum_{k=1}^{N_t} \mu_{k,t} - \Gamma_t v_t &= 0 \quad (35) \\
+ \sum_{j=1}^{N_t} (P_{j,t}^d - P_{j,t}^h) + \sum_{j=1}^{N_t} P_{j,t}^{d,dr} - \sum_{d=1}^{N_t} P_{d,t}^l &= 0 \quad (a) \\
- \alpha_{k,t} - \beta_{k,t} - \gamma_{k,t} &= 0 \quad (36) \\
- \alpha_{k,t} - \beta_{k,t} - \gamma_{k,t} &= 0 \quad (37) \\
- \Gamma_t \varphi_t - (P_{d,t}^h - P_{d,t}^h) + \sum_{j=1}^{N_t} \varphi_{j,t} &= 0 \quad (b) \\
- \alpha_{k,t} - \beta_{k,t} - \gamma_{k,t} &= 0 \quad (c) \\
\alpha_{k,t} + \beta_{k,t} + \gamma_{k,t} &= 0 \quad (d)
\end{align*}
\]

where $x_{k,t}, y_{k,t}, \mu_{k,t}$ and $v_t$ are the dual variables of (31); $\alpha_{k,t}, \beta_{k,t}$ and $\gamma_{k,t}$ are the dual variables of (32); $\varphi_{j,t}, \delta_{k,t}, \phi_{k,t,l}$ and $\eta_{k,l}$ are the dual variables of (33); $\xi_{k,t}, \eta_{k,t,l}$, $\nu_{k,t,l}$, $\chi_{t,l}$ and $\chi_{t,l}$ are the dual variables of (34).

The FRRUC model can be reformulated into a single-level robust optimization problem as follows:

**Objective:**

\[
\sum_{i=1}^{N_t} \sum_{t=1}^{N_t} P_{i,t}^d + \sum_{t=1}^{N_t} P_{i,t}^h + \sum_{m=1}^{N_w} \hat{P}_{m,t}^w - \sum_{k=1}^{N_t} x_{k,t} - \sum_{k=1}^{N_t} \mu_{k,t} - \Gamma_t v_t 
\]

**s.t.** (1)–(4), (6)–(30), (35)–(38).

By this means, the FRRUC model becomes a quadratic programming (QP) that can be efficiently solved by several QP methods.

Fig. 2 shows the flowchart of the solution procedure that can be summarized as follows:
FIGURE 2. Flowchart of the solution procedure.

Step 1: Read the key system operation parameters required for optimization.

Step 2: Linearize the risk cost using the method in [22], and obtain a transformed objective function.

Step 3: Construct the uncertainty set and the system operation constraints, and solve the single-level FRRUC model to determine the UC strategy and dispatch scheduling for the following day.

Step 4: Send the dispatch schedule to units and wind farms. End.

IV. NUMERICAL EXPERIMENTS

In this section, the effectiveness and the efficiency of the proposed FRRUC are investigated on the IEEE 39-bus test system. This test system has 10 thermal generators, 2 storage units, 2 wind farms and 46 transmission lines. The operation parameters of thermal generators and transmission lines can be found in [23]. The installed capacities of wind farms #1 and #2 are 400MW, which are connected to the grid at bus 2 and 21, respectively. The forecast curves of system load and wind generation are shown in Fig. 3. The flexible reserve capacity cost coefficients are $1/MW [24]. The penalties for WGC and LS are set at $80/MWh and $200/MWh, respectively.

All the experiments are programmed using YALMIP toolbox in MATLAB on a personal computer with Intel(R) Core(TM) i3 CPU and 8 GB of RAM. CPLEX12.8 is used as a MILP solver.

FIGURE 3. Forecasted values of load and wind generation.

FIGURE 4. Flexible reserve capacities of the system under FRRUC and CUC models.

A. ANALYSIS OF THE NUMERICAL RESULTS

In this section, FRRUC is compared with a conventional UC (CUC) model in terms of operational cost, operational risk and reserve scheme. The result of AIWG and the effect of storage units are analyzed as well. The reserve services level of CUC is 15% predicted wind generation. It should be noted that the flexible reserve capacities should also address several other types of uncertainty issues, such as generator outages. However, these elements are omitted in the experiments for clarity and concentration.

1) COMPARISON WITH CUC MODEL

The comprehensive operational cost is shown in Table 1. The flexible reserve results are shown in Fig. 4. It can be seen from Table 1 that both the total cost and the operational risk of the FRRUC model are lower than that of CUC. Meanwhile,
TABLE 1. System Operation Cost Under FRRUC and CUC Models.

|               | Total Cost ($10^3) | UC Cost ($10^3) | ED Cost ($10^3) | Risk ($10^3) | Time (h) |
|---------------|--------------------|-----------------|-----------------|--------------|----------|
| FRRUC         | 484970.2           | 3080.0          | 480050.4        | 1839.8       | 38.23    |
| CUC           | 485195.9           | 3080.0          | 478190.9        | 3925.0       | 3.50     |

FIGURE 5. Power output plan of storage units.

The ED cost of FRRUC is $480050.4, which is slightly higher than that in CUC. As shown in Fig. 4, the allocated upward and downward flexible reserve capacities of FRRUC are larger than that of CUC in most periods, which reduce the system operational risk. Compared with the CUC, the FRRUC model has a better capacity to allocate flexible capacity of flexible resources as well as mitigate the system operational risk. It should be noted that the total upward reserve is more than the total downward reserve, reflecting the risk attitude of the operators on WGC and LS. The solving time of the FRRUC model is higher than the CUC model.

2) IMPACT OF ENERGY STORAGE UNITS

The power outputs of energy storage units are shown in Fig. 5. The positive output indicates discharging, and the negative output indicates charging. The flexible reserve capacity in each period is listed in Table 2. It can be observed from Fig. 5 that the storage units charge to store electrical energy when the net load is low (e.g., in periods 2-5, 16-17 and 22-24) and discharge to generate power when the net load is high (e.g., in periods 10-13 and 20-21). It means that the storage units work as a power buffer by charging and discharging synchronously with the change of net load, which can reduce the peak-valley difference of the net load and increase the peak regulation capacity of the grid. From Table 2, the storage units provide a certain size of flexible reserve capacity in some periods that can relieve the regulating pressure of the conventional units and increase the system robustness.

3) AIWG RESULTS OF WIND FARMS

The AIWG results of wind farms are shown in Fig. 6. The CUC model cannot obtain the AIWG for the wind farms. It can be seen from Fig. 6 that the AIWG of wind farms #1 and #2 are optimized simultaneously with the dispatch plan, rather than given. Moreover, the boundaries of uncertainty set are asymmetric, denoted as AIWG. Therefore, the uncertainty set in the proposed FRRUC model is variable, which reflects the optimal allocation of flexible reserve capacity for flexible resources as well as the operator’s risk preference.

B. IMPACT OF THE UNCERTAINTY BUDGET

In this section, the impact of the uncertainty budget is analyzed. The size of AIWG can be defined as follows:

$$w_{size} = \sum_{i=1}^{T} \sum_{m=1}^{N_m} (w^u_{m,t} + w^l_{m,t})$$  \hspace{1cm} (40)

Table 3 shows the optimization results of the proposed FRRUC with different values of the uncertainty budget. It can be seen from Table 3 that both the total cost and the risk cost increase simultaneously with the increase in $\Gamma_i$ while the...
TABLE 3. Results of FRRUC Under Different Uncertainty Budget.

| $\Gamma_t$ | Total cost ($) | Risk ($) | Size of AIWG (MW) | Time (s) |
|-----------|---------------|---------|------------------|---------|
| 0.5       | 481815.7      | 191.1   | 3086.1           | 17.36   |
| 1         | 483640.8      | 685.2   | 2789.5           | 19.25   |
| 1.5       | 484458.9      | 1276.14 | 2399.6           | 21.27   |
| 2         | 485470.7      | 1839.8  | 2275.4           | 38.23   |

FIGURE 7. AIWG of wind farms and the total cost of the system under different transmission capacities.

AIWG size decreases monotonously. The trade-off between the price and the worth of robustness can be observed. The price of robustness is the additional operational cost to adjust the generation output scheme of flexible resources and increase the flexible reserve capacity to capture the uncertainties. The worth of robustness means the increase of robustness level that considers more extreme uncertainties. As illustrated by Table 3, the computing efficiency decreases with the increase in $\Gamma_t$. The operator can balance the robustness and conservatism by selecting appropriate $\Gamma_t$.

C. IMPACT OF TRANSMISSION CAPACITY

It is obvious that the AIWG and the dispatch plan will be influenced by the transmission capacity. To illustrate the impact, the transmission capacity of the system is varied from 0.9 to 1.1 of the original capacity. The total cost and the size of AIWG under different transmission capacities are shown in Fig. 7. As the transmission capacity increases from 0.9 to 1.1, the total cost decreases by 6.14% while the size of AIWG increases by 12.79%. This is because the generation scheme is also adjusted, which will influence the uncertainty set. In this case, the load demand and the flexible reserve capacity allocations among flexible resources are influenced by the system transmission capacity, which will affect the total cost and the robustness of system operation.

V. DISCUSSION

In this article, a novel robust UC model with large scale wind generation and energy storage is proposed. The proposed FRRUC model minimizes the system comprehensive cost for the base-case scenario instead of the worst-case scenario to reduce the conservativeness of the solution. In the proposed model, the UC and the ED solutions are co-optimized with variable uncertainty set. The operation plan is utilized as dispatch signals for the flexible resources and the AIWG serves as the operation signals for wind farms. The proposed method optimizes the boundaries of the variable uncertainty set, denoted as AIWG, to achieve a tradeoff between the system comprehensive cost and the operation robustness.

Compared with the conventional UC model, the proposed FRRUC model optimally allocates the flexible reserve capacities of the flexible resources, such as conventional generators and energy storage, to ensure the feasibility of the re-dispatch solution against the wind generation uncertainty. The uncertainty budget $\Gamma_t$ is an important parameter for the proposed model. Table 3 shows that selecting an appropriate $\Gamma_t$ can balance the robustness and the conservatism of the dispatch scheme. Fig. 7 shows the influence of transmission capacity on the optimization results. With the increase of the transmission capacity, the load power and the flexible reserve capacity of the system can be better allocated over the spatial and the temporal domains. This can provide a reference for the system transmission capacity expansion.

Recently, dynamic uncertainty set [12] and variable uncertainty set [17], [18] have been proposed to adjust the conservativeness of UC strategy. This article also considers the variable uncertainty set. Compared with the robust optimization methods in [12], [17], [18], the proposed approach still statically deal with the uncertainty. However, the solution of the proposed approach is more direct and simple that can achieve higher computation efficiency while maintaining its favorable properties. Compared with the practical approach proposed in [25], the generator outages are omitted in this article for clarity. However, the proposed approach can efficiently handle wind power uncertainty. Compared with the stochastic frequency constrained UC [26], the system frequency stability is not included in this article. However, the proposed method also considers the operational risk to ensure system security. More importantly, the proposed model allows an optimal allocation of operational flexibility of multiple flexible resources and operational risk mitigating capability.

VI. CONCLUSION

In this article, a novel FRRUC model considering the flexible reserve capacity of flexible resources and operational risk is proposed. The proposed FRRUC is formulated as a two-layer robust optimization problem. In the proposed model, the unit commitment and the dispatch solutions for the base-case are determined while the system operation security is ensured in the worst-case. The proposed model is transformed into a single-level optimization problem according to the strong duality theory.

The proposed FRRUC model is applied to a 39-bus system. First, the obtained outcomes demonstrate that the proposed approach can optimally allocate the flexible reserve capacities of flexible resources to cope with the wind power uncer-
tainty. In this connection, the risk cost of the FRRUC model is lower than the CUC model. The obtained results highlight the effectiveness of the proposed model in capturing the operational and adjustable flexibilities of storage units to supply the variations of net load. Second, the size of the uncertainty set is optimized to achieve a trade-off between the operational risk and the operational cost. Moreover, the boundaries of the uncertainty set are asymmetric. Third, both the total cost and the risk cost increase simultaneously with the increase in the uncertainty budget while the AIWG size decreases monotonously. The operators can select an appropriate uncertainty budget to balance the robustness and the conservatism.

In future work, the authors plan to expand the proposed method to dispatch approach studies for bulk AC/DC hybrid systems in order to promote the utilization of wind energy.

REFERENCES

[1] Q. Wang, Y. Guan, and J. Wang, “A chance-constrained two-stage stochastic program for unit commitment with uncertain wind power output,” IEEE Trans. Power Syst., vol. 27, no. 1, pp. 206–215, Feb. 2012.
[2] H. Quan, D. Srinivasan, and A. Khosravi, “Integration of renewable generation uncertainties into stochastic unit commitment considering reserve and risk: A comparative study,” Energy, vol. 103, pp. 735–745, May 2016.
[3] F. Liu, Z. Bie, S. Liu, and T. Ding, “Day-ahead optimal dispatch for wind integrated power system considering zonal reserve requirements,” Appl. Energy, vol. 188, pp. 399–408, Feb. 2017.
[4] D. Bertsimas, E. Litvinov, X. A. Sun, J. Zhao, and T. Zheng, “Adaptive robust optimization for the security constrained unit commitment problem,” IEEE Trans. Power Syst., vol. 28, no. 1, pp. 52–63, Feb. 2013.
[5] R. Jiang, J. Wang, and Y. Guan, “Robust unit commitment with wind power and pumped storage hydro,” IEEE Trans. Power Syst., vol. 27, no. 2, pp. 800–810, May 2012.
[6] P. Xiong and P. Jirutitijaroen, “Two-stage adjustable robust optimisation for unit commitment under uncertainty,” IET Gener., Transmiss. Distrib., vol. 8, no. 3, pp. 573–582, Oct. 2013.
[7] A. Street, F. Oliveira, and J. M. Arroyo, “Contingency-constrained unit commitment with n-k security criterion: A robust optimization approach,” IEEE Trans. Power Syst., vol. 26, no. 3, pp. 1581–1590, Aug. 2011.
[8] G. Morales-España, Á. Lorca, and M. M. de Weerdt, “Robust unit commitment with dispatchable wind power,” Electr. Power Syst. Res., vol. 155, pp. 58–66, Feb. 2018.
[9] R. Jiang, J. Wang, M. Zhang, and Y. Guan, “Two-stage minimax regret robust unit commitment,” IEEE Trans. Power Syst., vol. 28, no. 3, pp. 2271–2282, Aug. 2013.
[10] C. Zhao and Y. Guan, “Unified stochastic and robust unit commitment,” IEEE Trans. Power Syst., vol. 28, no. 3, pp. 3353–3361, Aug. 2013.
[11] B. Fanzeres, A. Street, and L. A. Barroso, “Contracting strategies for renewable generators: A hybrid stochastic and robust optimization approach,” IEEE Trans. Power Syst., vol. 30, no. 4, pp. 1825–1837, Jul. 2015.
[12] A. Lorca and X. A. Sun, “Multistage robust unit commitment with dynamic uncertainty sets and energy storage,” IEEE Trans. Power Syst., vol. 32, no. 3, pp. 1678–1688, May 2017.
[13] Z. Li, W. Wu, B. Zhang, and B. Wang, “Robust look-ahead power dispatch with adjustable conservativeness accommodating significant wind power integration,” IEEE Trans. Sustain. Energy, vol. 6, no. 3, pp. 781–790, Jul. 2015.
[14] J. Zhao, T. Zheng, and E. Litvinov, “Variable resource dispatch through do-not-exceed limit,” IEEE Trans. Power Syst., vol. 30, no. 2, pp. 820–828, Mar. 2015.
[15] M. I. Alizadeh, M. P. Moghaddam, and N. Amjady, “Multistage multiresolution robust unit commitment with nondeterministic flexible ramp considering load and wind variabilities,” IEEE Trans. Sustain. Energy, vol. 9, no. 2, pp. 872–883, Apr. 2018.
[16] H. Ye and Z. Li, “Robust security-constrained unit commitment and dispatch with recourse cost requirement,” IEEE Trans. Power Syst., vol. 31, no. 5, pp. 3527–3536, Sep. 2016.
[17] C. Shao, X. Wang, M. Shahidehpour, X. Wang, and B. Wang, “Security-constrained unit commitment with flexible uncertainty set for variable wind power,” IEEE Trans. Sustain. Energy, vol. 8, no. 3, pp. 1257–1264, Jul. 2017.
[18] C. Wang, F. Liu, J. Wang, F. Qiu, W. Wei, S. Mei, and S. Lei, “Robust risk-constrained unit commitment with large-scale wind generation: An adjustable uncertainty set approach,” IEEE Trans. Power Syst., vol. 32, no. 1, pp. 723–733, Jan. 2017.
[19] C. Wang, F. Liu, J. Wang, W. Wei, and S. Mei, “Risk-based admission assessment of wind generation integrated into a bulk power system,” IEEE Trans. Sustain. Energy, vol. 7, no. 1, pp. 325–336, Jan. 2016.
[20] R. T. Rockafellar and S. Uryasev, “Optimization of conditional value-at-risk,” J. Risk, vol. 2, no. 3, pp. 21–41, Oct. 2000.
[21] S. J. Kazempour, A. J. Conejo, and C. Ruiz, “Strategic generation investment using a complementarity approach,” IEEE Trans. Power Syst., vol. 26, no. 2, pp. 940–948, May 2011.
[22] P. Li, D. Yu, M. Yang, and J. Wang, “Flexible look-ahead dispatch realized by robust optimization considering CVaR of wind power,” IEEE Trans. Power Syst., vol. 33, no. 5, pp. 5330–5340, Sep. 2018.
[23] W. Ongsakul and N. Petcharaks, “Unit commitment by enhanced lagrangian relaxation,” IEEE Trans. Power Syst., vol. 19, no. 1, pp. 620–628, Feb. 2004.
[24] Z. Wang, C. Shen, F. Liu, J. Wang, and X. Wu, “An adjustable chance-constrained approach for flexible ramping capacity allocation,” IEEE Trans. Sustain. Energy, vol. 9, no. 4, pp. 1798–1811, Oct. 2018.
[25] H. Narami, A. Azizivahed, E. Naderi, M. Fathi, and M. R. Narami, “A practical approach for reliability-oriented multi-objective unit commitment problem,” Appl. Soft Comput., vol. 85, Dec. 2019, Art. no. 105786.
[26] M. Malekpour, M. Zare, R. Azizipanah-Abarghooee, and V. Terzija, “Stochastic frequency constrained unit commitment incorporating virtual inertial response from variable speed wind turbines,” IET Gener., Transmiss. Distrib., vol. 14, no. 22, pp. 5193–5201, Nov. 2020.

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