VIABLE SUPERSYMMETRIC MODELS WITH AN INVERTED SCALAR MASS HIERARCHY
AT THE GUT SCALE

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Abstract

Supersymmetric models with an inverted mass hierarchy (IMH: multi-TeV first and second generation matter scalars, and sub-TeV third generation and Higgs scalars) have been proposed to ameliorate phenomenological problems arising from flavor changing neutral currents (FCNCs) and CP violating processes, while satisfying conditions of naturalness. Models with an IMH already in place at the GUT scale have been shown to be constrained in that for many model parameter choices, the top squark squared mass is driven to negative values. We delineate regions of parameter space where viable models with a GUT scale IMH can be generated. We find that larger values of GUT scale first and second generation scalar masses act to suppress third generation scalars, leading to acceptable solutions if GUT scale gaugino masses are large enough. We show examples of viable models and comment on their characteristic features. For example, in these models the gluino mass is bounded from below, and effectively decouples, whilst third generation scalars remain at sub-TeV levels. While possibly fulfilling criteria of naturalness, these models present challenges for detection at future \textit{pp} and \textit{e^+e^-} collider experiments.

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Supersymmetry offers an elegant solution to the problem of quadratically divergent scalar masses in the Standard Model (SM), provided supersymmetric matter exists at or near the weak scale \(^1\). A Minimal Supersymmetric Standard Model (MSSM) can be constructed, with 124 free parameters, most of which occur in the soft SUSY breaking (SSB) sector of the model \(^2\) and reflect our ignorance about how SUSY is broken. Taking arbitrary weak scale SSB parameter choices generally leads to conflict with various low energy constraints associated with flavor changing neutral currents (FCNCs), and CP violating processes such as the electric dipole moments of the proton and neutron \(^3\). Of course, SUSY model builders have to explain the origin of SSB terms while at the same time satisfying constraints imposed by low energy processes.

Three possibilities have emerged for building models consistent with low energy constraints: 1. universality (degeneracy) of scalar masses \(^4\), 2. alignment of fermion and sfermion mass matrices \(^4\) and 3. decoupling, which basically involves setting sparticle masses to such high values that SUSY loop effects are suppressed relative to SM loops \(^5\). Models with gauge mediation \(^7\), anomaly mediation \(^8\) or gaugino mediation \(^9\) of SUSY breaking naturally lead to universality of particles with the same gauge quantum numbers. Supersymmetric models with SUSY breaking communicated via gravity (supergravity models) in general lead to non-universal scalar masses \(^10\). The minimal supergravity model (mSUGRA) adopts universality as an ad hoc assumption \(^1\). In this paper, we explore a class of models which potentially solve the SUSY flavor and CP problems via the decoupling solution.

It is important to notice that “naturalness” arguments \(^11\), which generally require sub-TeV sparticle masses, most directly apply to third generation superpartners, owing to their large Yukawa couplings. In contrast, the constraints from flavor physics mentioned above apply (mainly) to scalar masses of just the first two generations. This observation has motivated the construction of a variety of models, collectively known as inverted mass hierarchy (IMH) models \(^12\), where the first and second generation squarks and sleptons have multi-TeV masses, while third generation scalars have sub-TeV masses. For models in which the IMH occurs at or near the GUT scale (GSIMH models), it has been emphasized \(^13\) that two loop contributions to renormalization group (RG) running can cause tachyonic third generation squark masses to occur, unless these masses are beyond \(\sim 1\) TeV, which again pushes the model towards the “unnatural”.

Recently, it has been pointed out that models with a weak scale IMH can be generated radiatively by starting with multi-TeV scalar masses for all scalars at \(M_{GUT}\) \(^14\). For certain choices of GUT scale SSB boundary conditions, and assuming Yukawa coupling unification, the Higgs and third generation SSB masses then evolve rapidly towards zero, whilst first and second generation scalars remain heavy. However, requiring realistic third generation fermion masses and also a consistent radiative breakdown in electroweak symmetry (REWSB), only a rather small IMH can be generated \(^15\), which is not sufficient by itself to solve the SUSY flavor and CP problems.

An alternative approach to a decoupling solution is to take very large values of scalar masses in models with intermediate to large values of \(\tan \beta\). The “focus point” behavior of the Higgs SSB masses results in models with all matter scalar above a TeV, but with low values of \(|\mu|\), and possibly low fine-tuning \(^16\). However, even in these models, scalar masses are typically in the 1-3 TeV range, and are again not sufficient to solve the SUSY flavor and
CP problems without some degeneracy or alignment.

In this paper, we examine models with a scalar IMH already in place at the GUT scale. We assume that the MSSM is a valid theory between $M_{GUT}$ and $M_{weak}$, and that REWSB occurs. As noted in Ref. [13], it is crucial to work with two loop RGEs for this class of models. The two loop RGEs for the MSSM have been presented in [17], and have been implemented in ISAJET versions $\geq 7.49$ [18].

We adopt the following parameter space for our studies:

$$m_0(1), \ m_0(3), \ m_{1/2}, \ A_0, \ \tan \beta, \ \text{sign}(\mu), \ (1)$$

where $m_0(1)$ is the common mass of all first generation matter scalars at $M_{GUT}$, and $m_0(3)$ is the common third generation scalar mass. For simplicity, we adopt $m_0(2) = m_0(1)$, although the whole point is that $m_0(2)$ need not equal $m_0(1)$, so long as both are far above the TeV scale. The Higgs scalar SSB masses are set equal to $m_0(3)$, and take values of $\sim 1$ TeV. As usual, $m_{1/2}$ and $A_0$ are common GUT scale gaugino and trilinear masses, $\tan \beta = \frac{v_u}{v_d}$, and $\mu$ is the superpotential Higgs mass term.

For our renormalization group solution to the sparticle and Higgs mass spectrum, we use ISASUGRA (a part of the ISAJET package). Briefly, starting from weak scale values for the gauge and Yukawa couplings, ISASUGRA evolves the couplings up in energy until the GUT scale is determined, where $g_1 = g_2$. At $M_{GUT}$, the various SSB mass parameters are entered, and the set of 26 coupled RGEs for gauge and Yukawa couplings, and SSB masses, are evolved down to scale $M_{weak}$, where the sparticle mass spectrum can be calculated. An iterative procedure is adopted so that the RG improved one loop effective potential can be calculated and minimized at an optimized scale choice $Q = \sqrt{m_{t_L} m_{t_R}}$, where REWSB is required.

The form of the two loop RGEs for SSB masses is given by [17]

$$\frac{d m_i^2}{dt} = \frac{1}{16\pi^2} \beta_{m_i}^{(1)} + \frac{1}{16\pi^2} \beta_{m_i}^{(2)} m_i^2, \ (2)$$

where $t$ is the natural log of the scale, $i = Q_j, U_j, D_j, L_j$ and $E_j$, and $j = 1 - 3$ is a generation index. Two loop terms are suppressed relative to one loop terms by the square of a coupling constant, plus an additional factor of $16\pi^2$ in the denominator. The two loop terms

$$\beta_{m_i}^{(2)} \ni a_i g_3^2 \sigma_3 + b_i g_2^2 \sigma_2 + c_i g_1^2 \sigma_1, \ (3)$$

where

$$\sigma_1 = \frac{1}{5} g_3^2 \{ 3 (m_{H_u}^2 + m_{H_d}^2) + Tr[3m_Q^2 + 3m_L^2 + 8m_U^2 + 2m_D^2 + 6m_E^2] \},$$

$$\sigma_2 = g_2^2 \{ m_{H_u}^2 + m_{H_d}^2 + Tr[3m_Q^2 + m_L^2] \}, \quad \text{and}$$

$$\sigma_3 = g_3^2 Tr[2m_Q^2 + m_U^2 + m_D^2], \ (5)$$

and the $m_i^2$ are squared mass matrices in generation space. The numerical coefficients $a_i, b_i$ and $c_i$ are related to the quantum numbers of the scalar fields, but are all positive quantities. Thus, incorporation of multi-TeV masses for the first and second generation scalars leads
to an overall positive, possibly dominant, contribution to the slope of SSB mass trajectories versus energy scale. Although formally a two loop effect, the smallness of the couplings is compensated by the much larger values of masses of the first two generations of scalars. In running from $M_{\text{GUT}}$ to $M_{\text{weak}}$, this results in an overall reduction of scalar masses, which is felt most strongly by the sub-TeV third generation and Higgs scalar masses, and indeed leads to the constraints found in Ref. [13]. For values of SSB masses which fall short of the constraints of Ref. [13], a sort of see-saw effect amongst scalar masses occurs: the higher the value of first and second generation scalar masses, the larger will be the two loop suppression of third generation and Higgs scalar masses, until the constraint of Ref. [13] takes effect (or the lightest SUSY particle (LSP) ceases to be charge or color neutral).

An example of this effect is shown in Fig. 1, where we plot in frame (a) the evolution of third generation SSB scalar masses from a common GUT scale value of $m_0(3) = 1000$ GeV, down to the weak scale. We also take $m_0(1) = 15$ TeV, $m_{1/2} = 1400$ GeV, $A_0 = 0$, $\tan \beta = 3$ and $\mu > 0$. The dashed curves represent the case for universal scalar masses, with $m_0 = 1$ TeV for all scalars at $M_{\text{GUT}}$, while the solid lines indicate the GSIMH model. We see that at scales close to $M_{\text{GUT}}$, the trajectories of SSB scalar masses differ radically from the mSUGRA case, due to the dominant two loop RGE contributions, and that furthermore, these contributions overcome the positive one loop contributions from gauge interactions, and actually suppress the scalar masses relative to the mSUGRA case with universality. At lower energies, the one loop gauge contributions to squark masses again become dominant—due to increasing gluino mass and $SU(3)$ gauge coupling—resulting in an upward turn of the corresponding mass parameters. However, the final weak scale values of SSB mass parameters in this case are suppressed by almost a factor of 2 relative to the model with universality. In fact, most of the GSIMH model SSB masses are at sub-TeV levels, as opposed to the multi-TeV scale of SSB masses from the mSUGRA model. This means the GSIMH model could be in accord with naturalness constraints, even though the mSUGRA model is not. A further interesting feature is that, since the two loop RGE contributions to $m_L$ are larger than those to $m_E$, the lightest third generation slepton is most likely to be dominantly left-handed, instead of right-handed, as in the mSUGRA model. In frame (b), we see that the evolution of SSB Higgs masses is also strongly altered by two loop RGE contributions. The absolute values of $m_{H_d}$ and $m_{H_u}$ are diminished relative to the mSUGRA model case, again resulting in the GSIMH model being more natural.

The corresponding weak scale sparticle and Higgs mass spectra are shown in Table I for the two cases shown in Fig. 1. Two additional cases for a high value of $\tan \beta = 35$ are also shown. For the GSIMH model in case 1, the first and second generation scalar masses are all $\sim 15$ TeV, which should be large enough to suppress various flavor changing and CP violating processes, with the possible exception of $K - \bar{K}$ system. Meanwhile, most of the third generation scalar masses are at sub-TeV values, in accord with naturalness considerations. The relatively high gluino mass ($m_{\tilde{g}} \sim 3.3$ TeV) means that the gluino is effectively decoupling as well as first and second generation scalars. It is a general feature of viable GSIMH models that not only the gluino, but the other chargino and neutralino masses are constrained to be heavy. This has important consequences for searches for GSIMH

We recognize that the electroweak charginos and neutralinos have direct couplings to the Higgs
models at colliders. For instance, a Next Linear Collider (NLC) $e^+e^-$ machine would need at least 1.5 TeV in the CM frame to significantly access the lighter sparticles in this spectrum. The last entry in the Table shows the overlap between $\tilde{\tau}_1$ and $\tilde{\tau}_L$ states, so that the lightest stau in the GSIMH model is predominantly left handed. At the CERN LHC pp collider, top and bottom squark pair production will be the dominant SUSY process. We have generated collider events for all SUSY production processes for case 1 GSIMH model using ISAJET. We examined the various multi-jet plus multi-isolated lepton plus missing $E_T$ signals using the standard cuts and SM backgrounds given in Ref. [19]. In none of the signal channels examined was case 1 GSIMH model visible above SM background at a $5\sigma$ level, assuming 10 fb$^{-1}$ of integrated luminosity. We also looked at possible signals in multijet + $E_T$ events with two tagged $b$-jets. We required the cuts of Ref. [20]: $E_T > 100$ GeV, $p_T(b-jets) > 50$ GeV, and $E_T + \sum E_T(jets) > 1500$ GeV (where the sum runs over non-tagged jets). In addition, the highest $E_T$ b-jet was required to have $p_T > 100$ GeV. Again, for this case, no signal was visible above SM background for 10 fb$^{-1}$. Alternatively, examination of the “effective mass” distribution has been advocated as a means to quickly establish the presence of a SUSY signal against SM backgrounds [21]. The effective mass is defined as

$$M_{eff} = E_T(j1) + E_T(j2) + E_T(j3) + E_T(j4) + E_T,$$  

where $j1$ refers to the highest $E_T$ jet in the event, and so forth. In Fig. 2, we plot the effective mass signal for case 1 (open circles) and backgrounds taken from Ref. [21], after using the cuts of [21]. For the five mSUGRA case studies of Ref. [21], the signal always emerges from the background $M_{eff}$ distribution at a suitably high value of $M_{eff}$; for our GSIMH case 1, this does not happen. For the spectra of the GSIMH model case 1, clearly a more clever dedicated set of cuts will be needed to establish a SUSY signal. Finally, we note that the spectra of GSIMH case 1 may well be excluded by constraints from the cosmological relic density of neutralinos [22], since dominant neutralino annihilation likely takes place via $t$-channel stau, sbottom and stop exchange, and these particles are rather heavy. However, since $m_{\tilde{Z}_1}$ is getting close to $m_{\tilde{\tau}_1}$ and $m_{\tilde{t}_1}$, co-annihilation effects may be important, and could work to reduce the relic density to acceptable levels [23].

In case 2, with $\tan \beta = 35$, we take $m_0(1) = m_0(2) = 10$ TeV, $m_0(3) = 900$ GeV, $m_{1/2} = 1000$ GeV and $A_0 = 0$. Again, the third generation scalars are generally suppressed to sub-TeV values compared to their mSUGRA counterparts. The top squarks and staus are the lightest third generation scalars, with the $\tilde{\tau}_1$ again predominantly left handed. We also examined GSIMH model case 2 for visibility at the LHC just as for case 1. Again, we found no observable signal using the simple cuts of Ref. [19] or for the special $b$-jets cuts listed above. The $M_{eff}$ distribution is shown in Fig. 3. As with case 1, signal is always below background levels. The spectra of GSIMH case 2 may well be allowed by constraints from the cosmological neutralino relic density, since $m_{\tilde{Z}_1} \sim \frac{A_0}{2}$, and the neutralinos can annihilate efficiently through the very wide $s$-channel Higgs graphs to $b\bar{b}$ final states.

To map out the allowable parameter space range for viable GSIMH models, we performed a scan over parameter space, generating random samples of input parameters over the following ranges:

- bosons, and could result in a need for fine-tuning if they are too heavy.
\[ m_0(1) = m_0(2) : 2000 \rightarrow 25,000 \text{ GeV}, \]
\[ m_0(3) : 0 \rightarrow 2000 \text{ GeV}, \]
\[ m_{1/2} : 0 \rightarrow 2000 \text{ GeV}, \]
\[ A_0 : -10,000 \text{ GeV} \rightarrow +10,000 \text{ GeV}, \]
\[ \tan \beta : 3 \rightarrow 55, \quad \text{and} \]
\[ \mu : + \text{ or } -. \]

In Fig. 4 each dot represents a viable model. Models are ruled out if \( i.) \) REWSB does not occur, \( ii.) \) masses for the physical scalar eigenstates become tachyonic, or \( iii.) \) the \( Z_1 \) is not the LSP. In all frames, the value of \( m_0(1) \) is plotted on the vertical axis. In \( a) \), we plot models versus \( m_0(3) \). It is seen that larger values of \( m_0(1) \) require larger values of \( m_0(3) \). In fact, this is just the result of Ref. [13]. In frame \( b) \), we plot \( m_0(1) \) versus \( m_{1/2} \). Our main point here is that for large enough values of \( m_0(1) \), there is a minimum value of \( m_{1/2} \) that is required: large values of gaugino masses give large negative slope contributions to SSB scalar mass trajectories, which can off-set the large positive two loop contributions referred to earlier. In general, the lower bound on \( m_{1/2} \) leads to rather large gaugino masses at the weak scale, which makes detection of SUSY at colliders more difficult, even though third generation scalars can remain at sub-TeV values. In frame \( c) \), we plot \( m_0(1) \) versus the parameter \( A_0 \), and see that wide ranges of the \( A_0 \) parameter give rise to viable GSIMH models. Finally, in \( d) \), we plot \( m_0(1) \) versus \( \tan \beta \), and see that no range of \( \tan \beta \) is particularly favoured.

In Fig. 5, we show sparticle masses and \( \mu \) contours in the \( m_0(3) \) vs. \( m_0(1) \) plane, where we take \( m_{1/2} = 500 \text{ GeV} \) in frames \( a) \) and \( b) \), and \( m_{1/2} = 1000 \text{ GeV} \) in frames \( c) \) and \( d) \). We take \( A_0 = 0 \) and \( \mu > 0 \) in all frames, and \( \tan \beta = 3 \) in \( a) \) and \( c) \), and \( \tan \beta = 35 \) in \( b) \) and \( d) \). In the dark shaded regions \( \mu^2 < 0 \) so that REWSB fails, while the light shaded regions have a non-neutralino LSP; the intermediate shaded region has tachyonic scalar masses. In unshaded regions viable models may be possible. Some authors may then impose further constraints from fine-tuning, FCNC or other considerations. The constraints that we delineate are the minimal constraints any such model should satisfy. We also show mass contours of 1 TeV for \( \tilde{\tau}_1, \tilde{b}_1, \tilde{t}_1 \) and the \( \mu \) parameter. The labels are located in the regions with mass less than 1 TeV. An exception occurs in frame \( b) \), where \( \mu \) is always less than 1 TeV; there we plot a contour of \( \mu = 300 \text{ GeV} \). In frames \( a) \) and \( b) \), with the smaller values of \( m_{1/2} \), the magnitude of \( m_0(1) \) is generally limited, especially if one requires sub-TeV third generation scalar masses, which occur for lower values of \( m_0(3) \). Moving to higher \( m_{1/2} \) values in \( c) \) and \( d) \), significantly larger values of \( m_0(1) \) are allowed, which translates to greater suppression of FCNC and CP violating processes. Even so, sub-TeV third generation scalar masses are still possible in significant parts of the parameter plane. In the large \( \tan \beta \) plots, we see that \( m_0(3) \) is restricted from above by REWSB.

In Fig. 6, we show potentially viable regions in the \( m_0(3) \) vs. \( m_{1/2} \) plane, for \( m_0(1) = m_0(2) = 10 \text{ TeV}, \ A_0 = 0, \ \mu > 0 \text{ and } a) \tan \beta = 3 \text{ and } b) \tan \beta = 35 \). Points with open circles are excluded by REWSB, those with stars by tachyonic masses and finally those with dots have a non-neutralino LSP. In addition, various sparticle mass contours are shown. All contours show sparticle masses of 1 TeV, except the \( \tilde{g} \) contour shows \( m_{\tilde{g}} = 3 \text{ TeV} \), and the \( Z_1 \) contour shows \( m_{Z_1} = 500 \text{ GeV} \). We see that for a fixed value of \( m_0(1) \), low values of \( m_0(3) \) give rise to sub-TeV stau masses, while low values of \( m_{1/2} \) give rise to sub-TeV top and bottom squark masses. The lightest neutralino is gaugino-like throughout the frame of
Fig. 6a, which in general can lead to large values for the relic density of neutralinos, except for regions where co-annihilation effects are important (along the boundary of the excluded region).

At this point, some comparison with the results of Arkani-Hamed and Murayama (AM) and Agashe and Graesser (AG) of Ref. [13] seems worthwhile. As an example, taking $m_0(1) = 10$ TeV, and $m_{1/2} = 500$ GeV, then AM (Fig. 2) require $m_0(3) > 1500$ GeV, AG (Fig. 4a) require $m_0(3) > 1100$ GeV and we require, from Fig. 3 for $\tan \beta = 3$, $m_0(3) > 2400$ GeV. Taking instead $m_{1/2} = 1000$ GeV, AM and AG require $m_0(3) > 300$ GeV, while we require $m_0(3) > 600$ GeV. The work of AM neglects all Yukawa couplings in the RGE evolution, and requires only positive SSB scalar squared masses; the work of AG is similar, but takes into account the top quark Yukawa coupling. In our work, we include the top, bottom and tau Yukawa couplings in the two loop RGEs. Our constraints are somewhat different, also, since we require no tachyonic physical masses, a neutralino LSP, and REWSB, while the authors of Ref. [13] include some fine-tuning constraints. We remark that the exact location of the boundary of the tachyonic excluded region can be very sensitive to the manner in which the superparticle mass spectrum is calculated. For instance, using ISAJET, the tachyonic excluded region occurs during the first pass of RGE evolution, when sparticle masses are calculated at tree level. The top quark Yukawa coupling can be significantly larger during the first iteration than the second, since loop corrections are not yet included. This can cause a greater suppression of top squark squared masses than in a fully consistent one loop treatment, for which the bounds on $m_0(3)$ would be somewhat lower. In a fully consistent one loop calculation of sparticle masses, the excluded region should always come from the LSP and REWSB constraints, instead of from tachyonic masses.

Summary and Conclusions: We have investigated supersymmetric models with a scalar IMH in place at the GUT scale. Two loop RGEs for SSB masses are crucial for this analysis. We map out parameter regions with first and second generation scalar masses in the 5-20 TeV range but with sub-TeV third generation scalars. These models go a long way towards solving the SUSY flavor and CP problems, while remaining on the edge of naturalness. At the very least, all models of this type will have to satisfy the minimal constraints that we have imposed. Finally, since the GUT scale gaugino mass is bounded from below, charginos, neutralinos and gluinos are generally rather heavy, making the resulting sparticle mass spectra challenging to discover at planned future collider facilities.

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TABLE I. Weak scale sparticle masses and parameters (GeV) for two cases of mSUGRA and GSIMH models.

| parameter | mSUGRA case 1 | GSIMH case 1 | mSUGRA case 2 | GSIMH case 2 |
|-----------|---------------|--------------|---------------|--------------|
| $m_0(1)$  | 1000.0        | 15000.0      | 900.0         | 10000.0      |
| $m_0(3)$  | 1000.0        | 1000.0       | 900.0         | 900.0        |
| $m_{1/2}$ | 1400.0        | 1400.0       | 1000.0        | 1000.0       |
| $A_0$     | 0.0           | 0.0          | 0.0           | 0.0          |
| $\tan \beta$ | 3             | 3            | 35            | 35           |
| $m_{\tilde{g}}$ | 2965.1        | 3291.6       | 2181.7        | 2395.4       |
| $m_{\tilde{u}}$ | 2745.6        | 14993.0      | 2076.5        | 10014.8      |
| $m_{\tilde{d}}$ | 2639.9        | 15009.0      | 1997.4        | 10024.2      |
| $m_{\tilde{\ell}}$ | 1351.8        | 14983.5      | 1110.7        | 9991.0       |
| $m_{\tilde{\tau}}$ | 1122.3        | 14991.4      | 970.9         | 9994.9       |
| $m_{\tilde{b}}$ | 1349.9        | 14983.4      | 1107.8        | 9990.7       |
| $m_{\tilde{t}}$ | 2131.2        | 658.7        | 1616.2        | 792.6        |
| $m_{\tilde{e}}$ | 2546.4        | 1113.7       | 1877.8        | 1072.2       |
| $m_{\tilde{b}}$ | 2527.2        | 922.3        | 1834.7        | 940.0        |
| $m_{\tilde{t}}$ | 2638.8        | 1316.6       | 1907.6        | 1147.2       |
| $m_{\tilde{\tau}}$ | 1121.0        | 671.6        | 835.4         | 690.0        |
| $m_{\tilde{e}}$ | 1351.4        | 851.0        | 1063.7        | 761.2        |
| $m_{\tilde{\nu}}$ | 1349.4        | 667.9        | 1055.3        | 721.4        |
| $m_{\tilde{W}}$ | 1110.7        | 1097.0       | 785.3         | 745.2        |
| $m_{\tilde{Z}}$ | 1110.6        | 1097.7       | 785.2         | 745.8        |
| $m_{\tilde{Z}}$ | 600.3         | 616.5        | 424.6         | 432.1        |
| $m_h$     | 112.8         | 111.8        | 122.4         | 126.5        |
| $m_A$     | 2302.3        | 1447.7       | 1218.3        | 801.9        |
| $m_{H^+}$ | 2305.2        | 1450.7       | 1222.5        | 807.7        |
| $\mu$     | 1721.8        | 1200.2       | 1070.0        | 792.3        |
| $\langle \tau_L | \tau_1 \rangle$ | 0.02         | 0.99         | 0.15          | 0.71         |
FIG. 1. Evolution of a) third generation SSB masses and b) Higgs SSB masses from $M_{\text{GUT}}$ to $M_{\text{weak}}$ versus scale choice $Q$. Dashed lines indicate mSUGRA model while solid lines indicate the GSIMH model. We take $m_0(3) = 1000 \text{ GeV}$, $m_{1/2} = 1400 \text{ GeV}$, $A_0 = 0$, $\tan \beta = 3$ and $\mu > 0$. In the GSIMH model, $m_0(1) = m_0(2) = 15,000 \text{ GeV}$, while in mSUGRA, $m_0(1) = m_0(2) = m_0(3)$.
FIG. 2. A plot of the effective mass distribution background (histogram), and signal (open circles) for GSIMH model case 1.
FIG. 3. A plot of the effective mass distribution background (histogram), and signal (open circles) for GSIMH model case 2.
FIG. 4. A plot of parameter space where viable GSIMH models are located. The vertical axis is $m_0(1)$, while in a) models are plotted versus $m_0(3)$, in b) versus $m_{1/2}$, in c) versus $A_0$ and in d) versus $\tan \beta$. 
FIG. 5. A plot of the $m_0(3)$ vs. $m_0(1)$ plane where viable GSIMH models are located. In $a)$ and $b)$, we take $m_{1/2} = 500$ GeV, while in $c)$ and $d)$, we take $m_{1/2} = 1000$ GeV. In $a)$ and $c)$, $\tan \beta = 3$, while in $b)$ and $d)$, $\tan \beta = 35$. We take $A_0 = 0$ and $\mu > 0$. The contours show where various sparticle masses and the $\mu$ parameter are equal to 1000 GeV (except frame $b)$, where $\mu = 300$ GeV). The darkest region is excluded by lack of REWSB, while the lightest excluded region has a non-neutralino LSP. Tachyonic sparticle masses are generated in the intermediate shaded regions.
FIG. 6. A plot of parameter space where viable GSIMH models are located in the $m_0(3)$ vs. $m_{1/2}$ plane, where we take $m_0(1) = m_0(2) = 10$ TeV, $A_0 = 0$, $\mu > 0$ and $a) \tan \beta = 3$ and $b) \tan \beta = 35$. Points with circles have no REWSB, points with stars have tachyonic masses and dots represent points with a charged and/or colored LSP. All contours represent sparticle masses of 1 TeV, except $m_{\tilde{g}} < 3$ TeV below the $\tilde{g}$ contour, and $m_{\tilde{Z}_1} < 500$ GeV below the $\tilde{Z}_1$ contour.