Non-equilibrium approaches to the pre-thermal and post-hadronisation stages of A + A collisions

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The results related to non-equilibrium phenomena at the very early and late stages of the processes of A + A collisions are presented. A good description of the hadron momentum spectra as well as pion and kaon interferometry data at RHIC is reached within the realistic dynamical picture of A + A collisions: HydroKinetic Model (HKM). The model accumulates the following features: not too early thermalization time; 1 fm/c; a developing of the pre-thermal transverse flows; the effectively more hard, than in the case of chemical equilibrium, equation of state of expanding chemically non-equilibrated multi-hadronic gas; a continuous non-equilibrated emission of hadrons. All these factors lead to a good description of the mentioned RHIC data, in particular, the observed $R_{out}/R_{side}$ ratios, solving, therefore, the HBT puzzle in detailed realistic model.

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I. INTRODUCTION

The typical spacial and temporal scales of the interaction processes in proton-proton collisions are less or of order 1 fm and 1 fm/c correspondingly. This is the result of the femtoscopy analysis and it is agreed with a simple theoretical estimates. The similar experiment measurements reveals a typical lifetime of the system created in A + A collisions to be at least one order of the magnitude larger. The femtoscopy, or intensity interferometry measurements are associated with a set of the points of particle last scattering. Therefore the above result mean that, even if one tries to consider the collision of nuclei as a some kind of superposition of the individual collisions of nucleons of nuclei, the secondary hadrons, produced in these "elementary" collisions, continue to interact during the time interval which is much larger than the time scale related to the individual nucleon-nucleon collisions. Then one can conclude that in ultrarelativistic A + A collisions we
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have to face a phenomenon of the space-time evolution of the strongly interacting matter produced in such processes, and can rise the question as for a nature of this matter at the early collision stage. The other crucial points are: whether this matter becomes them all in the processes of A + A collisions, and if yes, how does it evolves and also how are observed particle momentum spectra formed: in other words, how to describe a particle liberation process, which gradually destroys the local chemical and themal equilibrium in expanding system.

II. THERMALIZATION AND COLLECTIVE FLOWS

Whether the interaction in the systems formed in A + A collisions are strong enough to result in a thermalization and collective effects such as hydrodynamic flows. A compatibility of the form of pion rapidity (y) spectra with that is predicted by hypothesis of the longitudinal hydrodynamic ow was stressed rst in the pioneer Landau paper [1]. The interferometry signature of the longitudinal ows, expressed in the spectral $c m_T$ and y behavior of the long-radius: $R_{long} \frac{d^2 \rho}{d^2 p_T} = \text{cosh} y$ at high $p_T$ [2], was confirmed by NA35/NA49 Collaborations (CERN) [3]. The radial transverse ow in A + A collisions reveal itself in Nu Xu’s plots showing a dependence of the transverse spectra slopes $T_{eff}$ on hadron masses $m$ (in the non-relativistic approximation $T_{eff} = T_{f1} + m \frac{\hbar v^2}{2}$ where $T_{f1}$ is the temperature and $\hbar v^2$ is the mean squared transverse collective velocity at freeze-out).

The most direct evidence of thermalization and transverse anisotropic (elliptic) ows at RHIC [4] is related to the behavior of the so-called $v_2$ coefficients describing the anisotropy of transverse spectra in non-central A + A collisions. It is very naturally explained and basically described within hydrodynamic model for perfect uid [5] wherein the initial geometric anisotropy transforms into the collective velocity anisotropy of themal hadronic gas due to different gradients of the pressure in out-and-in directions as for the reaction plane at the initial stage. To reach a quantitative agreement with the experimental data one needs to use very small initial time, 0.5 f m/c, to start the hydro-evolution. If it starts at later times, neither the collective velocities, nor their (and the transverse spectra) anisotropy will be developed to a sufficient degree.

These results, in fact, brought the two new ideas: rst, that the quark-gluon plasma (QGP), at least at the temperatures not much higher $T_c$, is the strongly coupled system – sQGP, and so behaviors as an almost perfect uid, and, second, that thermalization happens at very early times of collisions. A necessity for the thermal pressure to be formed as early as possible appears also in
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hydrodynamic description of the central collisions at RHIC. If one starts the hydrodynamic evolution in "conventional time" \( t = 1 \text{ fm/c} \) without transverse ow, the latter will not be developed enough to describe simultaneously the pion, kaon and proton spectra. The crucial problem is, however, that even the most optimistic theoretical estimates give thermalization time \( 1 - 1.5 \text{ fm/c} \), typically, it is \( 2 - 3 \text{ fm/c} \). The discrepancy could be even more at LHC energies.

III. PRE-THERMAL FLOWS, OR IS EARLY THERMALIZATION REALLY NEEDED?

The way to solve the problem was proposed in year 2006, Ref. [7] and developed and exploited in Refs. [8, 9]. It was shown that the initial transverse ow s in them almost matter as well as their anisotropy, leading to asymmetry of the transverse momentum spectra, could be developed at the pre-thermal, either partonic/string or classical eld/Glasma, stage with even more efficiency than in the case of very early perfect hydrodynamics. The illustration for the case of partonic free streaming is presented in Fig. 1.

The results of above mentioned papers show that:

i) The radial and elliptic ow s develop no matter whether a pressure already established. The general reason for them is an essential

\[ \text{FIG. 1: The collective velocities developed in central (b=0) and non-central (b=6.3 fm ) Au+Au collisions at the pre-thermal stage from proper time } t_0 = 0.3 \text{ fm/c by supposed thermalization time } t_1 = 1 \text{ fm/c for scenarios of partonic free streaming in CGC model [9].} \]

ness of the system in transverse direction. Then the particle number or energy-mom entum density ow s directed outward the system cannot be compensated by the inward directed (from periphery to the center) ow s. Just this creates non-zero net ow s no matter how the collective velocity is defined: according to Ekkart or to Landau-Lifshitz.

ii) The specific (linear) dependence of the transverse spectra slopes on the hadron masses in central A+A collisions and an anisotropy of the spectra in non-central ones happen only if, at least, partial thermalization happens. Then an almost isotropic particle emission in the local rest frames of the decoupling uid elements gains Doppler blue shifts in the Lab system. These shifts depend on the collective velocities of uid elements which are various in different directions.
in non-central collisions – it leads to momentum spectra anisotropy. If no thermalization happen, similar as at free streaming, the final transverse momentum spectra will be practically the same as initial transversely isotropic momentum distribution despite anisotropic collective flows indeed develop in the non-equilibrated system. In that case also no specific dependence of the transverse spectra slopes on particle mass arises.

iii) So, the results, rst published in 2006, show that whereas the assumption of (partial) thermalization in relativistic $A + A$ collisions is really crucial to explain soft physics observables, the hypotheses of early thermalization at times less than 1 fm/c is not necessary.

IV. PHENOMENOLOGICAL APPROACH TO THE PRE-THERMAL EVOLUTION

Of course, the free streaming approximation for the processes at the very early pre-thermal stage is too rough and leads to a non-local equilibrium structure of the energy-momentum tensor at the supposed "thermalization" time $t_\text{th}$. In the forthcoming publication [10] we will present in details the phenomenological approach motivated by Boltzmann equations, which accounts for the energy and momentum conservation laws and contains the two parameters: supposed time of thermalization $t_\text{th}$ and initial relaxation time $t_\text{rel}(\theta)$. The result is: if some model or effective QCD theory bring us an energy-momentum tensor at the very initial time of collision $\theta$, then we can estimate the flows and energy densities at expected time of thermalization $t_\text{th}$ using equations for the energy-momentum tensor of ideal uid with (known) source term which is associated with the free evolving initial system when the interaction there is turned off:

$$ @T_{\text{hyd}}(x) = T_{\text{free}}(x)@P_{\text{rel}}(\theta) \quad (1) $$

with

$$ P(\theta) = \frac{\pi}{\tau_\text{g}} \frac{\tau_\text{g}}{\tau_\text{rel}(\theta)} \quad (2) $$

and $T_{\text{hyd}}$ correspond to $T_{\text{hyd}}$ of ideal uid with renormalized energy density and pressure.

$$ T_{\text{hyd}} = T_{\text{hyd}}(\theta) (P(\theta))p(\theta) \quad (1) \quad P(\theta)p(\theta) $$

Such a method will be included in HydroKinetic Model (HKM) [12, 15] that accounts for non-equilibrium effects in similar way; the essence of the model is discussed in what follows.

V. THE EVOLUTION IN THE EQUILIBRATED ZONE AND INITIAL CONDITIONS

At the temperatures higher than the chemical freeze-out temperature $T_{\text{ch}}$ the hydrodynamical evolution is related to locally equilibrated quark-gluon and hadron phases. The evolution is described by the conservation law for the energy-momentum tensor of perfect uid: $@T(x) = 0$
and conservation laws for baryonic and strange net charge \( Q \) \( \Rightarrow \) \( Q(x) = 0 \). In Ref. [11] the equation of state (EoS) at \( T = T_{\text{ch}} \) is taken from Ref. [12], that describes well the QCD lattice data at zero baryonic chemical potentials and is matched with the chemically equilibrated multi-component hadron resonance gas at \( T = 175 \, \text{MeV} \). In this work we adjust that EoS to the parameters at the chemical freeze-out: \( B = 29 \, \text{MeV} \), \( S = 7 \, \text{MeV} \), \( E = -1 \, \text{MeV} \) [13].

For very central \( A+A \) collisions the initial transverse energy density profile for such an evolution is supposed to be the Gaussian as for the variable \( r^2 = R^2 \) and with the maximal energy density \( \rho_{\text{th}}(r=0) = 0 \). We choose the thermalization time to be \( \tau_{\text{th}} = 1 \, \text{fm/c} \). The initial transverse rapidity profile, developed at the pre-thermal stage, is supposed to be linear in radius \( r \): \( y_T = \frac{r}{R} \). Note that the parameters absorb also a correction for underestimated resulting transverse \( Q \) \( \Rightarrow \) \( Q \) since in this work we did not account for the viscosity effects [14] neither at QGP stage nor at hadronic one. In the longitudinal direction the boost-invariance with the Bjorken \( Q \) \( \Rightarrow \) \( Q \) is supposed. From the estimates presented in Ref. [3] \( R = 5.4 \, \text{fm} \) for \( A+\overline{A} \) collisions. The only free parameters \( 0 \) and \( \alpha \) are deduced from the fitting of the spectra and interferometry radii and turn out to be \( \rho_{\text{th}} = 17 \, \text{MeV} / \text{fm}^3 \); \( \alpha = 0.35 \).

VI. THE EVOLUTION IN THE NON-EQUILIBRATED ZONE AND EOS

At \( T < T_{\text{ch}} = 165 \, \text{MeV} \) the system evolves as a chemically and thermally non-equilibrated hadronic matter. To calculate the EoS we approximate it below \( T_{\text{ch}} \) by multi-component hadronic gas where to guarantee the correct particle number ratios for all the quasi-stable particles a composition is changed only due to resonance decays into expanding fluid with possible recombinations. We include 359 hadron states made of \( u,d,s \) quarks with masses up to 2.6 GeV. Thus, in addition to the equations of energy-momentum conservation, the equations accounting for the particle number conservation and resonance decays are added. If one neglects the thermal motion of heavy resonances, the equations for particle densities \( n_i(x) \) take the form:

\[ \partial_t (n_i(x) u(x)) = \sum_{j} b_{ij} j n_i(x) \]

where \( b_{ij} = B_{ij} N_{ij} \) denote the average number of \( i \)th particles coming from arbitrary decay of \( j \)th resonance, \( B_{ij} = i \to j \) is branching ratio, \( N_{ij} \) is a number of \( i \)th particles produced in \( j \)th decay channel. The EoS in this chemically non-equilibrated system depends now on particle number densities \( n_i \) of all the 359 particle species. So the energy-momentum conservation equations and 359 equations are solving simultaneously with calculation of the EoS,
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FIG. 2: The thermodynamic (p;c)-region occupied by the actual points (grey points) of equation of state (EoS) p(x) = p(x;f\text{nl}(x)g) in chemically non-equilibrated multi-component hadronic gas during its evolution with IC as described in the body text. The different points are related to the various space-time points of 4-volume swept out by the expanding system. The lines corresponding to chemically equilibrated (red) and chemically frozen evolution (green) are marked for a comparison.

\[ p(x) = p(x;f\text{nl}(x)g), \text{ at each point } x. \]

The thermodynamic region occupied by the EoS is presented in Fig. 2. Here the different points are related to the various space-time points of 4-volume swept out by the expanding system (with initial conditions as is described above).

VII. SYSTEM'S DECOUPLING AND SPECTRA FORMATION IN HYDROKINETIC MODEL - HKM

During the matter evolution, in fact, at \( T \rightarrow T_{ch} \), hadrons continuously leave the system. Such a process is described in the HKM by means of the emission function \( S(x;p) \) which is expressed for pions through the term gain, \( G(x;p) \), in Boltzmann equations and the escape probabilities \( P(x;p) = \exp\left( \frac{1}{R} \int ds R_{+h}(s) + P(x,t;p) \right) \), \( S(x;p) = G(x;p)P(x;p) \) [11, 15]. For pion emission in relaxation time approximation \( G \cdot R_{+h} + G_H \), where \( f(x;p) \) is the pion phase-space Bose-Einstein distribution, \( R_{+h}(x;p) \) is the total collision rate of the pion, carrying momentum \( p \), with all the hadrons \( h \) in the system in a vicinity of point \( x \), the term \( G_H \) describes an income of the pions into phase-space point \( (x;p) \) due to the resonance decays. It is calculated according to the kinematics of decays with simplification that the spectral function of the resonance \( \pi \) is \( (p^2 \ 	ext{Im} \ H \cdot i^2) \). The cross-sections in the hadronic gas, that determine via the collision rate \( R_{+h} \) the escape probabilities \( P(x;p) \) and emission function \( S(x;p) \), are calculated in accordance with UrQMD method [16]. The spectra and correlation function are found from the emission function \( S \) in the standard way (see, e.g., [15]).

VIII. THE RESULTS AND CONCLUSION

The system's formed in A+A collisions go through different stages of evolution: from the initial one, which is far from equilibrium, to the thermal and chemically equilibrated phases of
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SQGP and hadronic matter, then system again becomes non-equilibrated, neither chemically nor thermally. At the late non-equilibrium stage the medium approximation for multi-component hadron gas is destroyed and hadrons are gradually liberated. The correct description of both non-equilibrium stages: very initial one and the latest stage are very important since at the first the IC for the reball evolution are generated and at the latest one the hadronic momentum spectra are formed.

It is worthily note that at the first, pre-thermal stage the collective waves and its anisotropy are developed, as it is illustrated in Fig. 1 for the free streaming approximation. To go ahead one needs in the correct description of them alization of the initially non-equilibrated system of either partons/strings or QGP a la elts. The phenomenological approach allows one to describe transformation from an arbitrary initial energy-momentum tensor to the one corresponding to the perfect fluid in agreement with the conservation laws in the generalized relaxation time approximation.

The HydroKinetic Model allows one to describe all the stages of the system evolution as well as a formation of the particle momentum at the decoupling stage an agreement with the underlying transport equations. The basic hydrokinetic code, proposed in [11], is modified now to include decays of resonances into the expanding hadronic chemically non-equilibrated system. The non-equilibrium process of the spectra formation is evaluated in the first approximation when the back reaction of non-equilibrium hadronic emission on the hydrodynamic evolution is ignored. In form of HKM [11] an account of the back reaction of the particle emission corresponds to an including of the viscosity effects into a description of hadronic hydro evolution. So here we estimate only the main characteristics of the dissipative effects in the most important region corresponding to the maximal pion emission. The results of our analysis show that for above mentioned initial conditions for RHIC energies the maximal emission is achieved at the proper time $13 \text{ fm/c}$ and transverse radii $8.4 \text{ fm}$. At the corresponding space-time vicinity, related to the last stage of the evolution where the spectra are formed, we estimate $\frac{\sigma}{s} = 4.2^\frac{1}{4}_{-1}, \frac{\sigma}{p} = 0.58^\frac{1}{4}_{1}$. The ratio of the total (over all hadrons) shear viscosity to total entropy is $\frac{\sigma}{s} = 6.27^\frac{1}{4}_{-1}$. The corresponding mean free pathes are $\lambda_s = 3 \text{ fm}, \lambda_p = 1.2 \text{ fm}$ and Knudsen numbers, defined as the ratio of the m.f.p. to the hydrodynamic length, are in this region $\text{Kn}_s = 0.5, \text{Kn}_p = 0.2$. These estimates are important to understand the evolution of multi-component hadron gas and applicability of (viscous) hydrodynamic approach to this stage.

The crucial point for any model of space-time evolution is simultaneous description of
The transverse momentum spectra for negative pions, kaons and protons calculated in HKM for the cases of the vacuum cross-sections (solid lines), "transport" cross-sections (dash lines), with account for the resonance recombination processes (dotted line) and attractive for protons mean field (red line). The experimental data are taken from STAR [19] and PHENIX [21] Collaborations (RHIC BNL).

Right panel: The HBT radii for the negative pion and negative + positive kaon pairs calculated in HKM for the same physical conditions as at the left panel. The experimental data: STAR [20] and PHENIX [22,23].

As for the absolute values of the spectra, it is worthy to stress that parameters of the chemical freeze-out are chosen in accordance with STAR RHIC data, while for kaons and protons the PHENIX and STAR results are noticeably different. We analyze an influence of the main physical factors on the spectra and HBT radii.
hadronic gas with different cross-sections: vacuum ones \( i_j \) and with "transport" cross-sections, \( i_j=2 \), and also we analyze the possibility when not only decays of resonances happen but also recombination processes for non-strange resonances takes place, we simulated the latter effect by utilization of the effective resonance widths in Eqs. (3), \( \Gamma_0 \), that corresponds to 50 percent increase of effective life-times of resonances due to the recombination processes. All these modifications shift significantly the time distribution of the hadronic emission but, as it follows from Fig. 3, they do not influence essentially on pion and kaon spectra and the interferometry radii. It illustrates and extend the general analytic results \([17]\) that these quantities do not depend significantly on the freeze-out time at the isentropic evolution. As for the proton transverse spectrum, its best description requires, probably, the mean field contribution, which is attractive for protons \([18]\) and so leads to reduction of their velocities in soft momentum region (central part of the remnant). The red dot line in Fig. 3 (left) corresponds to 14 percent reduction of the proton transverse rapidities in the region \( y_T < 1 \).

Summarizing we would like to emphasize that correct approaches to the non-equilibrated stages in A+A collisions, that are utilized within the Hydrokinetic Model lead to a good simultaneous description of the spectra and space-time scales. The model accumulates the following features: not too early thermalization time, \( 1 \) \( \text{fm/c} \); a developing of the pre-thermal transverse flows; the effectively more hard, than in the case of chemical equilibrium, equation of state in the chemically non-equilibrated multi-hadronic gas; a continuous non-equilibrated emission of hadrons. All these factors in prove the description of the observables at RHIC, in particular, lead to reduction of pion and kaon \( R_{	ext{out}}=R_{	ext{side}} \) ratios, solving, therefore, the HBT puzzle in detailed realistic model.

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