The Chromomagnetic Dipole Operator and the $B$ Semileptonic Branching Ratio

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ABSTRACT

We consider the possibility of having a large branching ratio for the decay $b \rightarrow sg$ coming from an enhanced Wilson coefficient of the chromomagnetic dipole operator. We show that values of $BR(b \rightarrow sg)$ up to $\sim 10\%$ or more are compatible with the constraints coming from the CLEO experimental results on $BR(B \rightarrow X_s \gamma)$ and $BR(B \rightarrow X_s \phi)$. Such large values can reconcile the predictions of both the semileptonic branching ratio and the charm counting with the present experimental results. We also discuss a supersymmetric model with gluino-mediated flavour violations, which can account for such large values of $BR(b \rightarrow sg)$.
1 Introduction

The world average of all measurements of the $B$-meson semileptonic branching ratio is \cite{1}

\[ BR_{SL}^{exp} \equiv BR(B \rightarrow Xe\bar{\nu}_e) = (10.4 \pm 0.4)\% . \] (1)

This result is considerably smaller than the theoretical prediction in the parton model, where $BR_{SL} \sim 13\text{–}15\%$ \cite{2}. Furthermore $1/m_Q^2$ non-perturbative corrections cannot reduce the predicted $BR_{SL}$ below 12.5\% \cite{3}. However it has recently been found that charm mass corrections to $\Gamma(b \rightarrow c\bar{c}s)$ are large and can further reduce the theoretical prediction for $BR_{SL}$ \cite{4}:

\[ BR_{SL} = \begin{cases} (12.0 \pm 0.7 \pm 0.5^{+0.9}_{-1.2} \pm 0.2)\% & \text{on-shell scheme} \\ (11.3 \pm 0.6 \pm 0.7^{+0.9}_{-1.7} \pm 0.2)\% & \text{\overline{MS} scheme} \end{cases} \] (2)

Here the errors correspond to the uncertainties on $m_b$, $\alpha_s(M_Z)$, the renormalization scale $\mu$ and the other parameters present in the calculation. It may now seem that the discrepancy in the $B$ semileptonic branching ratio has essentially disappeared. However, as we lower the theoretical prediction for $BR_{SL}$ by increasing $\Gamma(b \rightarrow c\bar{c}s)$, we simultaneously increase the prediction for $n_c$, the average number of charm hadrons produced per $B$ decay. Indeed the theoretical prediction for $n_c$ obtained from eq. (2) is

\[ n_c = \begin{cases} 1.24 \pm 0.05 \pm 0.01 & \text{on-shell scheme} \\ 1.30 \pm 0.03 \pm 0.03 \pm 0.01 & \text{\overline{MS} scheme} \end{cases} \] (3)

where the first error corresponds to the quark mass uncertainties, the second to the $\alpha_s$ variation in $\overline{MS}$ and the last to all the other uncertainties. This prediction is larger than the present world average of the $n_c$ measurements \cite{4}

\[ n_c^{exp} = 1.117 \pm 0.046 . \] (4)

The real problem is therefore to understand the presence of a low $BR_{SL}$ together with a low value of $n_c$. The theoretical difficulty is summarized in fig. 1.

It could well be that this is just an experimental problem; measurements of $n_c$ rely on model assumptions about charm hadron production which have been questioned in ref. \cite{5}. Recently it has been proposed \cite{6} that “spectator effects” can be at the origin of the problem. Although these effects cannot be fully computed without a knowledge of the hadronic parameters, potentially they can bring $BR_{SL}$ close to the experimental value. They also result in a
simultaneous, although modest, decrease of \( n_c \). On the other hand, if “spectator effects” are responsible for the observed low value of the ratio \( \tau(\Lambda_b)/\tau(B_d) = 0.76 \pm 0.05 \), they then tend to increase the prediction for \( BR_{SL} \), making the problem even more pressing. Recently it has also been suggested [7] that the data indicate the presence of \( 1/m_Q \) corrections in non-leptonic decays, which are not present in HQET. This has been attributed to a possible failure of the operator product expansion near the physical cut and a corresponding violation of the local quark–hadron duality. In particular, using the phenomenological recipe of replacing the quark mass by the decaying hadron mass in the \( m^5 \) factor in front of all non-leptonic widths, \( BR_{SL}, n_c, \) and \( \tau(\Lambda_b)/\tau(B_d) \) can be reconciled with their measured values [7].

In this paper we discuss the effect of new physics on \( BR_{SL} \) and \( n_c \). The possibility that large contributions to \( \Gamma(b \to sg) \) can eliminate the disagreement between experiments and Standard Model predictions was first proposed in ref. [8], and then studied in much further detail in ref. [9]. Our goal here is to refine previous analyses and consider new constraints.

In sect. 2 we perform a model-independent analysis of \( \Gamma(b \to sg) \) and the possible constraints coming from \( \Gamma(B \to X_s \gamma) \) and \( \Gamma(B \to X_s \phi) \), extending the results of ref. [9]. In particular we show that \( \Gamma(B \to X_s \phi) \), in spite of its sensitivity to penguin operators, provides a poor probe of New Physics effects. In sect. 3 we discuss a supersymmetric model with gluino-mediated flavour violations, first suggested by Kagan [9], which can explain a large enhancement of \( \Gamma(b \to sg) \).

2 Model-Independent Analysis

The effective Hamiltonian relevant for \( \Delta B = 1 \) decays is given by [10]

\[
H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \left[ V_{cb}^* V_{cs} (C_1(\mu)Q_1(\mu) + C_2(\mu)Q_2(\mu)) + V_{ub}^* V_{us} (C_1(\mu)Q_1(\mu) + C_2(\mu)Q_2(\mu)) - V_{tb}^* V_{ts} \sum_{i=3}^{12} C_i(\mu)Q_i(\mu) \right],
\]

where \( V_{ij} \) are the CKM matrix elements and \( C_i(\mu) \) are the Wilson coefficients evaluated at the scale \( \mu \) of order \( m_b \). The dimension-six local operator basis \( Q_i \) is given by

\[
\begin{align*}
Q_1 &= (s_\alpha q_\beta)_{V-A} (q_\beta b_\alpha)_{V-A} \\
Q_2 &= (s_\alpha q_\alpha)_{V-A} (q_\beta b_\beta)_{V-A}
\end{align*}
\]
Table 1: Central values of the input parameters used in the analysis

| Parameter                              | Value   |
|----------------------------------------|---------|
| $|V_{ts}^*V_{tb}|/|V_{cb}|$           | 0.95    |
| $\alpha_s(M_Z)$                        | 0.117   |
| $m_t$ (GeV)                            | 174     |
| $m_b$ (GeV)                            | 5       |
| $m_s$ (MeV)                            | 500     |
| $m_c/m_b$                              | 0.316   |
| $M_{\phi}$ (GeV)                       | 1.019   |
| $g_s^2$ (GeV$^4$)                      | 0.0586  |

Here $(\bar{q}_1 q_2)(\bar{q}_3 q_4)$ denotes current–current products, $V \pm A$ indicates the chiral structure and $\alpha$, $\beta$ are colour indices. Moreover, $g_s$ ($e$) is the strong (electromagnetic) coupling constant, $G_{\mu \nu}^A$ ($F_{\mu \nu}$) is the gluon (photon) field strength, and the $t^A$ are the $SU(N)$ colour matrices normalized so that $Tr(t^A t^B) = \delta^{AB}/2$.

This Hamiltonian is known at the next-to-leading order (NLO), as far as one separately considers the current–current, gluon and photon penguin operators $Q_1$-$Q_{10}$ on the one hand [1] and the magnetic-type operators $Q_{11}$-$Q_{12}$ on the other hand [2]. Unfortunately the mixing between these two classes of operators is at present known only at the leading order (LO) [3].

The branching ratio of the decay $b \rightarrow sg$ is given by [4]

$$BR(b \rightarrow sg) = BR_{SL}^{exp} \frac{|V_{ts}^*V_{tb}|^2}{|V_{cb}|^2} \frac{2\alpha_s(\mu)}{\pi g(m_c/m_b)} |C_{11}^{eff}(\mu)|^2, \quad (7)$$

where $BR_{SL}^{exp}$ is the measured semileptonic branching ratio, the phase-space factor $g(z)$ is given
in the Appendix, $\mu = O(m_b)$ and $C_{11}^{\text{eff}}$ is the renormalization-scheme invariant coefficient introduced in ref. [14]. With the input values given in table [1], the LO Standard Model prediction is

$$BR^{SM}(b \rightarrow sg) = (2.3 \pm 0.6) \times 10^{-3},$$

where the error is mainly due to the variation of $\mu$ between $m_b/2$ and $2m_b$ and to the uncertainty on $\alpha_s(M_Z)$. Inclusion of the known part of the NLO corrections reduces the central value and the $\mu$ dependence, but introduces a significant scheme dependence [14].

In the effective Hamiltonian approach, physics beyond the weak scale affects only the initial conditions of the Wilson coefficients, namely $C_i(M_W)$. Therefore these coefficients can be used to parametrize new physics effects without referring to a specific model. In the case at hand, we assume that the initial condition of the Wilson coefficient $C_{11}(M_W)$ is an independent variable and define

$$r_g = \frac{C_{11}(M_W)}{C_{11}^{SM}(M_W)},$$

where

$$C_{11}^{SM}(M_W) = g_1(m_t^2/m_W^2),$$

$$g_1(x) = \frac{x(-x^2 + 5x + 2)}{4(x - 1)^3} - \frac{3x^2 \log x}{2(x - 1)^4}.$$ 

In terms of $r_g$, the branching ratio of $b \rightarrow sg$ is given by

$$BR(b \rightarrow sg) = 2.73 \times 10^{-2}(0.15 + 0.14 r_g)^2,$$

assuming central values for all the relevant parameters.

An enhanced value of $BR(b \rightarrow sg)$ affects the relation between $n_c$ and $BR_{SL}$. We find

$$n_c = 2 - (2 + R_\tau + R_{ud} + 2R_\ell)BR_{SL} - 2BR(b \rightarrow sg),$$

where [16]

$$R_\tau = \frac{\Gamma(b \rightarrow c\tau\nu)}{\Gamma(b \rightarrow ce\nu)} = 0.25,$$

$$R_{ud} = \frac{\Gamma(b \rightarrow c\bar{u}d')}{\Gamma(b \rightarrow ce\nu)} = 4.0 \pm 0.4,$$

$$R_\ell = \frac{\Gamma(b \rightarrow \text{no charm})}{\Gamma(b \rightarrow ce\nu)} = 0.25 \pm 0.10.$$ 

\footnote{Using calculations in the 't Hooft–Veltman scheme, as we do, $C_{11,12}^{\text{eff}} = C_{11,12}^{HV}$. Therefore we denote them $C_{11,12}$ in the following.}
Figure 1: Correlation between the semileptonic branching ratio $BR_{SL}$ and the number of charms per $B$ decay $n_c$. The solid band is the Standard Model prediction, including the theoretical uncertainties, while the dashed one is obtained assuming a $BR(b \to s) = 9\%$. The experimental data point is also shown.

Here $\Gamma(b \to \text{no charm})$ is the sum of all $B$ decay widths into charmless final states different from $sg$. Notice that in eq. (12) we have eliminated the dependence on $\Gamma(b \to c\bar{c}s)$, which is the main source of theoretical uncertainty. If we require that the experimental central values of $BR_{SL}$ and $n_c$, eqs. (11) and (13), lie within the theoretical uncertainty, we need $7\% < BR(b \to s) < 11\%$. This corresponds to $10.2 < r_g < 13.1$ or $-12.3 < r_g < -15.2$. For instance the effect of $BR(b \to s) = 9\%$ in eq. (12) is shown in fig. 1 (see the dashed band).

We now want to perform a model-independent analysis of the constraints on $r_g$. The first constraint comes from the observation of the inclusive decay $B \to X_s\gamma$. The LO expression for $BR(B \to X_s\gamma)$ is [14, 15]

$$BR(B \to X_s\gamma) = BR_{SL}^{\exp} \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{3\alpha_e}{2\pi g(m_c/m_b)} |C_{12}^{\text{eff}}(\mu)|^2.$$  \hspace{1cm} (14)

It is well known that this branching ratio has very large QCD corrections. It also has a significant theoretical uncertainty, mainly due to the scale dependence in the Wilson coefficient,
to be taken into account. Also in this case, the known NLO terms decrease the central value and the \( \mu \) dependence, but introduce a sizeable scheme dependence, to be cancelled by the unknown terms.

The dependence on \( r_g \) in eq. (14) comes from the operator mixing in the QCD renormalization group equations. Analogously to eq. (9), we define

\[
\gamma = \frac{C_{12}(M_W)}{C_{12}^{SM}(M_W)},
\]

where

\[
C_{12}^{SM}(M_W) = f_1(m_t^2/m_W^2),
\]

\[
f_1(x) = \frac{x(-8x^2 - 5x + 7)}{12(x-1)^3} + \frac{x^2(3x - 2) \log x}{2(x-1)^4}.
\]

We can now rewrite eq. (14) in terms of \( r_g \) and \( r_\gamma \) as

\[
BR(B \to X_s\gamma) = 7.10 \times 10^{-4}(0.017 r_g + 0.273 r_\gamma + 0.313)^2,
\]

where the theoretical error is not shown. This formula gives a model-independent parametrization of \( BR(B \to X_s\gamma) \). Figure 2 shows the ranges of \( r_g \) and \( r_\gamma \) allowed by the measurement \( BR(B \to X_s\gamma) = (2.32 \pm 0.67) \times 10^{-4} \) [17]. The existence of two different bands corresponds to the overall sign ambiguity of \( C_{12}^{\text{eff}}(\mu) \) in eq. (14). The size of the bands accounts for both the experimental error at the 1\( \sigma \) level and a (linearly added) theoretical error of 30% [15]. Figure 2 shows that very large enhancements of \( C_{11}(M_W) \) are allowed, since they can be compensated by appropriate, and relatively small, changes of \( C_{12}(M_W) \). A large values of \( C_{11}(M_W) \) can result in a modification of the photon energy spectrum in \( B \to X_s\gamma \) and could be tested in the future if the threshold in the observed \( E_\gamma \) is appropriately lowered\(^2\) [18]. The solution of the \( BR_{SL-n_c} \) problem in terms of \( \Gamma(b \to sg) \) singles out the four disconnected regions in the \( r_g-r_\gamma \) plane shown in fig. 2.

The extraction of analogous constraints from the measured upper limit [19]

\[
BR(B \to X_s\phi) < (1.1 - 2.2) \times 10^{-4} \quad (90\% \text{ CL})
\]

is more involved. Available theoretical estimates [20, 21] rely on some assumptions which are questionable. In particular, hadronic matrix elements are calculated by combining perturbation

\(^2\)We thank A. Ali for pointing out this possible experimental test.
Figure 2: Correlation between the ratios $r_g$ and $r_\gamma$, defined in eqs. (9) and (15). The scale on the right shows the corresponding values of $\text{BR}(b \rightarrow sg)$. The regions outside the two oblique bands are excluded by the measured $\text{BR}(B \rightarrow X_s\gamma)$. The region above the dashed line is excluded by the upper limit on the $\text{BR}(B \rightarrow X_s\phi)$, assuming $\xi = 1/3$. However, the theoretical error is large and not completely under control in this case, so that the limit is plotted only for illustrative purpose. Finally the two oblatted regions delimit the ranges allowed in the supersymmetric model with LL (dotted line) and LR (solid line) flavour mixing obtained for $m_{\tilde{g}} > 200$ GeV and $\tilde{m} > 100$ GeV.

theory with the factorization method. Moreover they made a strong assumption on the quark momenta distribution inside the $\phi$ meson ($p_s = p_\bar{s} = p_\phi/2$). These hypotheses result in a pure two-body decay $b \rightarrow s\phi$, while for example string fragmentation models indicate high multiplicity final states for the $b \rightarrow sg$ decay [22]. The same assumptions are also used to extract the experimental limit from the measurements. Thus, constraints coming from this limit have large theoretical uncertainties and should be taken with caution.

Nonetheless, in the following, we calculate the constraints in the $r_g$-$r_\gamma$ plane, coming from the limit on $\text{BR}(B \rightarrow X_s\phi)$, under the same assumptions made in refs. [20, 21], which allow a
straightforward calculation of the branching ratio. In terms of \( r_g-r_\gamma \), the following expression is obtained:

\[
BR(B \to X_s\phi) = a_1|C_\phi(m_b)|^2 + a_2\alpha_s(m_b)\text{Re}(C_\phi(m_b))r_g + a_3\alpha_se\text{Re}(C_\phi(m_b))r_\gamma \\
+ a_4\alpha_s(m_b)^2r_g^2 + a_5\alpha_e^2r_\gamma^2 + a_6\alpha_s(m_b)\alpha_se r_g r_\gamma, \tag{19}
\]

where the coefficients \( a_i \) are functions of the masses \( m_b, M_\phi, m_s \). They are explicitly given in the Appendix.

Beside \( C_{11} \) and \( C_{12} \), eq. (19) depends on the coefficient \( C_\phi \). It appears when the matrix elements of the relevant penguin and electropenguin operators are evaluated using the factorization method and it is given by

\[
C_\phi(\mu) = C_{3\,\text{eff}}(\mu) + C_{4\,\text{eff}}(\mu) + C_{5\,\text{eff}}(\mu) + \xi\left(C_{6\,\text{eff}}(\mu) + O(\alpha_s)(O(\alpha_e))\{C_{7\,\text{eff}}(\mu) + C_{8\,\text{eff}}(\mu) + C_{9\,\text{eff}}(\mu) + C_{10\,\text{eff}}(\mu)\}\right), \tag{20}
\]

where \( \xi = 1/N \). Here \( N \) is the number of colours, taken as a free parameter to account for possible deviations from the factorization result. Since the magnetic operators contribute to eq. (19) through \( O(\alpha_s) \) (\( O(\alpha_e) \)) matrix elements, it is mandatory to calculate \( C_\phi \) at NLO. The effective coefficients\(^3\) \( C_3\,\text{eff} - C_{10\,\text{eff}} \) are defined so that they include the NLO contributions coming from the matrix elements at \( O(\alpha_s) \) (\( O(\alpha_e) \)), calculated in perturbation theory. With such a definition, \( C_3\,\text{eff} - C_{10\,\text{eff}} \) become complex functions. We note that the calculations of these coefficients in refs. [20, 23] are not complete. A full NLO perturbative calculation of the matrix elements would allow for a complete cancellation (at the considered order) of the scheme dependence of the NLO Wilson coefficients that appear in \( C_\phi \). In our case, we use NLO coefficients \( C_1 - C_{10} \) calculated in the ’t Hooft–Veltman scheme, including only the largest contribution to the NLO matrix elements coming from \( Q_2 \), as in ref. [20]. We have estimated the residual scheme dependence (\( \sim 15\% \)), by comparing our effective coefficients to those given in ref. [20]. We consider it negligible, compared with the other uncertainties present in the calculation.

\(^3\)Notice that the denomination “effective coefficient” refers to completely different definitions in the case of \( C_3 - C_{10} \) and \( C_{11} - C_{12} \).
Figure 3: The coefficient $|C_\phi|$ as a function of $C_3(M_W)C_{10}(M_W)$, normalized to the Standard Model. The eight initial conditions are parametrized in terms of the four contributions coming from the box and the $g$, $Z$, $\gamma$ penguin diagrams.

$C_\phi$ is quite insensitive to the initial conditions of the penguin and electropenguin operators $Q_3 - Q_{10}$. In fig. 3 we show $|C_\phi|$ as a function of these initial conditions, parametrized in terms of the contributions of the gluon, $Z$, $\gamma$ penguin and the box diagrams, normalized to the Standard Model. The reason of this weak dependence is twofold. On the one hand, the largest terms in $C_\phi$ come from the gluon penguin operators $Q_3 - Q_6$, which are insensitive to their initial conditions because the large mixing with $Q_1 - Q_2$ dominates their renormalization group evolution down to $\mu = O(m_b)$. On the other hand, the effective coefficients $C_{i eff}$ get a large NLO contribution from the $O(\alpha_s)$ matrix elements, which do not depend on the initial conditions of the Wilson coefficients. This implies that $BR(B \to X_s\phi)$ in general gives a poor probe of new physics effects, contrary to what is often stated in the literature.

Therefore, it is reasonable to assume $C_\phi$ as a constant in the analysis, so that the $BR(B \to X_s\phi)$ could be used to put constraints in the $r_g - r_\gamma$ plane. Unfortunately this constraint suffers from large theoretical uncertainties. For example the predicted branching ratio changes
by a factor of 2 as we vary $\xi$ between $1/2$ and $1/3$, which is a popular way to account for the uncertainty of the factorization method \cite{20}. Moreover the assumption on the decay kinematics is not under control and the related uncertainty is not quantifiable. Just for illustrative purposes we show in fig. 4 the constraint coming from eq. (19), assuming $\xi = 1/3$. For larger values of $\xi$, the bound on $r_g$ from $B \to X_s\phi$ becomes more stringent, disfavouring even further the solutions of the $BR_{SL-n_c}$ problems corresponding to positive $r_g$. However if we assign an overall uncertainty of a factor of 3 in the prediction of the branching ratio, any significant bound on $r_g$ disappears.

An enhanced $b \to sg$ decay rate also affects the exclusive decays $B \to K\pi$ \cite{21}. We estimate $BR(B^- \to K^0\pi^-) \simeq 10^{-5}(1 + 0.1\ r_g)^2$, which corresponds to an effect of order 1 for $|r_g| \sim 10$. Again the theoretical uncertainty of this prediction is large and not under control, so we prefer not to show this constraint in fig. 2.

We have seen how present constraints allow large enhancements of $BR(b \to sg)$. Measurements of $BR(B \to X_s\gamma)$ require, however, a precise correlation between $r_g$ and $r_\gamma$. The main difficulty to solve the $BR_{SL-n_c}$ problem in terms of new physics is to explain this correlation. We now turn to a discussion of models in which this is possible.

### 3 An Illustrative Model

The suggestion that anomalously large $BR(b \to sg)$ can explain a reduction of $BR_{SL}$ was first made in ref. \cite{8}, where it was assumed that the new effective $bsg$ interaction is mediated by virtual charged Higgs boson exchange. This possibility is now ruled out by a combination of the constraints from $B \to X_\tau\bar{\nu}$, $B \to X_s\gamma$, and $Z \to bb$. The main difficulty of models where the $bsg$ interaction arises from charged-Higgs exchange, shared by most other models with weakly-interacting new particles, is that generically $r_g \sim r_\gamma$. Constraints from $BR(B \to X_s\gamma)$ then allow only small enhancements of $r_g$.

The possibility that flavour-changing quark–squark–gluino interactions can generate large coefficients for the chromomagnetic operator was first suggested in ref. \cite{9}. Now the loop generating $O_{11}$ has a large Casimir factor, which is not present in the loop generating $O_{12}$, and one obtains $r_g/r_\gamma \sim 5-7$. This allows a significant enhancement of $b \to sg$, especially if we
consider solutions with negative $r_\gamma$.

We have in mind the case in which the squark mass matrix is not diagonal in the quark mass eigenbasis. This situation is generic in supersymmetric models derived from supergravity with non-minimal Kähler metric. For simplicity we consider separately two possibilities. In the first case, we assume that flavour non-diagonal entries of the squark mass matrix appear only in the left sector and there is no left–right squark mixing. The new contributions to the Wilson coefficients $C_{11}$ and $C_{12}$, evaluated at the scale of supersymmetric particle masses, are

$$C_{11} = \frac{Z}{\tilde{m}_g^2} \sum_{i=1}^{3} U_{ib}^* U_{is} g_2(m_g^2/\tilde{m}_i^2)$$

$$g_2(x) = \frac{x(-11x^2 + 40x + 19)}{36(x - 1)^3} + \frac{x^2(x - 9) \log x}{6(x - 1)^4}$$

$$C_{12} = \frac{Z}{\tilde{m}_g^2} \sum_{i=1}^{3} U_{ib}^* U_{is} f_2(m_g^2/\tilde{m}_i^2)$$

$$f_2(x) = \frac{2x(-2x^2 - 5x + 1)}{27(x - 1)^3} + \frac{4x^3 \log x}{9(x - 1)^4}$$

$$Z \equiv \sqrt{2\pi\alpha_s} V_{tb} V_{ts} G_F$$

Here $m_g$ is the gluino mass and $\tilde{m}_i^2$ are the eigenvalues of the down-squark squared-mass matrix, which is diagonalized by the $3 \times 3$ unitary matrix $U$. If $\tilde{m}_i$ are nearly degenerate, eqs. (21) and (23) can be Taylor-expanded around the common squark mass $\tilde{m}$:

$$C_{11} = Z \frac{\delta \tilde{m}_{bsl}^2}{\tilde{m}^4} g_3(m_g^2/\tilde{m}^2)$$

$$g_3(x) = \frac{(x^2 + 172x + 19)}{36(x - 1)^4} + \frac{x(x^2 - 15x - 18) \log x}{6(x - 1)^5}$$

$$C_{12} = Z \frac{\delta \tilde{m}_{bsl}^2}{\tilde{m}^4} f_3(m_g^2/\tilde{m}^2)$$

$$f_3(x) = \frac{2(-17x^2 - 8x + 1)}{27(x - 1)^4} + \frac{4x^2(x + 3) \log x}{9(x - 1)^5}$$

Here $\delta \tilde{m}_{bsl}^2 \equiv \sum U_{ib}^* U_{is} \tilde{m}_i^2$ corresponds to the flavour non-diagonal mass insertion.

In the second case we consider, the down-squark mass matrices for the left and right sectors are both diagonal, but there are flavour non-diagonal left–right (LR) mixing terms. Since these terms are not generated by supersymmetry breaking, in the limit of vanishing Yukawa coupling
constants, we will assume here that they are proportional to the corresponding Yukawa coupling. In this case, the mass-insertion approximation is always adequate and the new contributions to the Wilson coefficients $C_{11}$ and $C_{12}$ are

$$C_{11} = Z \frac{\delta \tilde{m}_b^2}{m_b \tilde{m}^2} g_4(m_{\tilde{g}}^2/\tilde{m}^2)$$

(30)

$$g_4(x) = \sqrt{x} \left[ -\frac{2(x + 11)}{3(x - 1)^3} + \frac{(-x^2 + 16x + 9) \log x}{3(x - 1)^4} \right]$$

(31)

$$C_{12} = Z \frac{\delta \tilde{m}_b^2}{m_b \tilde{m}^2} f_4(m_{\tilde{g}}^2/\tilde{m}^2)$$

(32)

$$f_4(x) = \sqrt{x} \left[ \frac{4(5x + 1)}{9(x - 1)^3} - \frac{8x(x + 2) \log x}{9(x - 1)^4} \right].$$

(33)

The two cases here considered are the simplest because they generate only operators with the same chiral structure as in the Standard Model. They also describe the generic features of more complicated squark mass matrices.

By varying the relevant supersymmetric parameters under the requirement that all squark masses are larger than 100 GeV and $m_{\tilde{g}} > 200$ GeV, we find that the new interactions can generate values of $r_g$ and $r_\gamma$ within the regions illustrated in fig. 2. The prediction has a strong correlation in the $r_g$–$r_\gamma$ plane because the ratio $r_g/r_\gamma$ depends on the supersymmetric parameters only weakly, through the loop functions. Thus, in spite of its generic features, the model can make a rather precise prediction of this ratio. It is interesting that it is indeed possible to reach a region in which $b \to s g$ has the correct enhancement to solve the $BR_{SL}$–$n_c$ puzzle. The solution always corresponds to the case in which the signs of both $C_{11}$ and $C_{12}$ are opposite to the Standard Model results. In the case of left–left (LL) mixings this region is achieved when the mixing between the $s$ and $b$ squark is maximal\(^4\) and when the lightest squark mass\(^5\) is around 100 GeV and $m_{\tilde{g}}$ around 200 GeV. This means that supersymmetry could be soon discovered at the Tevatron, but no light squark has to be expected at LEP2. Similar conclusions apply to the case of LR mixing. However if the LR mixing were not proportional to $m_b$, the same effects could be obtained for much heavier squarks and gluinos, although the existence of colour-breaking minima could impose significant constraints on the parameter

\(^4\)Indeed the mixing is so large that the mass-insertion approximation is not valid. The region in fig. 2 was obtained by using the complete expressions in eqs. (21) and (23).

\(^5\)Here we refer to the mass of the lightest squark, which can be considerably lighter than the other squarks, because of the large mixing. Tevatron bounds on squark masses\(^2\) do not directly apply here, since they are obtained under the assumption of three families of degenerate squarks.
space. Finally notice that, in the case of a very light gluino ($m_{\tilde{g}} \sim 1$ GeV) the ratio $r_g/r_\gamma$ can become much larger than that shown in fig. 2, especially for LR mixings. In this case it is possible to have $r_g$ as small as $-10$ for positive values of $r_\gamma$, and almost reach the shaded region in fig. 2 corresponding to positive $r_\gamma$ and negative $r_g$.

Given a definite model of flavour violation, we can consider more tests on its consistency than in the case of the model-independent analysis of sec. 4. We now assume that gluino-mediated flavour violations occur only between the second and third generation of the down quark–squark sector and consider different processes with $|\Delta B| \neq 0$, which can be of experimental interest.

The $|\Delta B| = 2$ transitions will affect the $B_s - \bar{B}_s$ mixing. For LL mass insertions, the new contribution to $\Delta m_{B_s}$ is \[ \Delta m_{B_s} = \frac{4}{3} B_{B_s} f_{B_s}^2 m_{B_s} \frac{G}{m^2} \left( \frac{\delta \tilde{m}^2_{b,s_L}}{\tilde{m}^2} \right)^2 \] (34)

$$G(x) = \frac{1}{216} \left[ \frac{-2x^3 + 27x^2 + 144x + 11}{(x - 1)^4} - \frac{2x(51x + 39) \log x}{(x - 1)^5} \right].$$ (35)

This contribution is much larger than the Standard Model one, making the discovery of $B_s - \bar{B}_s$ mixing even more arduous. However, for $x \simeq 2.4$, the function $G(x)$ has a zero and therefore, for particular values of the squark and gluino masses, we can reduce the total contribution to $\Delta m_{B_s}$. Nevertheless, the generic prediction of the model is that $B_s - \bar{B}_s$ mixing will not be discovered soon.

The $|\Delta B| = 1$ transitions can induce the flavour-changing decay $Z \rightarrow \bar{b}s$. For LL transitions \[ BR(Z \rightarrow \bar{b}s) = \frac{\alpha \alpha^2 s}{18\pi^2 \sin^2\theta_W \cos^2\theta_W} \left( 1 - \frac{2}{3} \sin^2\theta_W \right)^2 \left| U^*_{ib} U_{is} F(\tilde{m}^2_i, m^2_{\tilde{g}}, m^2_Z) \right|^2 \] (36)

$$F(\tilde{m}^2_i, m^2_{\tilde{g}}, m^2_Z) = \int_0^1 dx \int_{1-x}^{1-x-y} dy \log \left[ \frac{\tilde{m}^2_i(x + y) + m^2_{\tilde{g}}(1 - x - y) - m^2_Z x y}{\tilde{m}^2_i(1 - x) + m^2_{\tilde{g}}x} \right].$$ (37)

Assuming the validity of the mass insertion approximation, eq. (36) can be simply expressed in terms of $r_g$. For $\tilde{m}^2 = m^2_{\tilde{g}} > m^2_Z$, we find

$$BR(Z \rightarrow \bar{b}s) = 5 \times 10^{-8} r_g^2.$$ (38)

Unfortunately, observation of this decay mode represents a real experimental challenge. LEP1 can at best reach a sensitivity on $BR(Z \rightarrow \bar{b}s)$ of about $10^{-3}$ [27].
Finally we consider FCNC decay processes of $B$ mesons, such as $B \rightarrow X_s \nu \bar{\nu}$ and $B \rightarrow X_s \ell^+ \ell^-$. It is known [24] that gluino-mediated interactions do not affect the $Z$ penguin operator. Indeed, the effective $b$-s-$Z$ vertex turns out to be proportional to the $b$-s-$\gamma$ vertex. QED gauge invariance then implies that the $Z$ penguin is suppressed by a factor $q^2/m_Z^2$, where $q^2$ is the momentum transfer. Because of this suppression we do not expect new contributions to $B \rightarrow X_s \nu \bar{\nu}$. On the other hand, the process $B \rightarrow X_s \ell^+ \ell^-$ receives new contributions from the gluino-mediated $\gamma$ penguin operator and from the electromagnetic dipole operator, which has here a sign opposite to the Standard Model result. This change of sign implies important modifications both in the rate and the lepton asymmetries of $B \rightarrow X_s \ell^+ \ell^-$ [28]. The new contribution to the $\gamma$ penguin is however rather modest. We find that LL transitions give

$$C_e = \frac{Z}{m_g^2} \sum_{i=1}^{3} U_{ib}^* U_{is} f\left(\frac{m_g^2}{\tilde{m}_i^2}\right)$$

$$f(x) = \frac{2x(11x^2 - 7x + 2)}{81(x - 1)^3} - \frac{12x^4 \log x}{81(x - 1)^4},$$

where $C_e$ is the Wilson coefficient at the weak scale of the operator

$$O_e = \frac{\alpha_e}{2\pi} (\bar{s}_a b_a )_V - A(\bar{e}e)_V.$$

Using the mass-insertion approximation and taking for simplicity $m_g^2/\tilde{m}_i^2 = 1$, eq. (39) can be rewritten as

$$\frac{C_e}{C_{eSM}} \simeq 5 \times 10^{-2} \, r_g.$$  

4 Conclusions

In conclusion we have discussed how large $BR(b \rightarrow sg)$ can reconcile the predictions for $BR_{SL}$ and $n_c$ with present experimental measurements. Constraints from $BR(B \rightarrow X_s \gamma)$ require a precise correlation between new physics effects in the coefficients of the chromomagnetic and electromagnetic dipole operators. On the other hand $BR(B \rightarrow X_s \phi)$ does not set any significant constraints on the parameter space, mainly because of the large theoretical uncertainties involved in the calculation. We have also shown that $BR(B \rightarrow X_s \phi)$ does not provide a good probe of new physics effects because the relevant coefficient $C_\phi(m_b)$ is rather insensitive to the initial conditions at the weak scale of the gluon-penguin operators.
Finally we have discussed how a supersymmetric model with gluino-mediated flavour violations can account for a large value of $BR(b \to s g)$, consistently with all other constraints from $\Delta B = 1$ and $\Delta B = 2$ FCNC processes. The theory has a rather precise correlation between the predictions for $r_g$ and $r_\gamma$, which is fairly independent of the specific model assumptions. Although the enhancement of $BR(b \to s g)$ is achieved only for particular values of the parameter space and it is evidently not a general consequence of the model, it is interesting to know that supersymmetry can potentially predict $BR(b \to s g) \simeq 5$–$10\%$, and solve the $BR_{SL-n_c}$ puzzle. Future experiments will certainly be able to test this scenario.

**Note added**

After completion of this work, we received a note by A. Kagan who informed us that their new results on $B \to X_s \phi$ agree with those presented in this paper [29].

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**Appendix**

In this appendix we report on the result of our calculation of $BR(b \to s \phi)$. As in refs. [20, 21], we assume a pure two-body decay and use the factorization method to estimate the relevant matrix elements. We have computed both the contributions of $Q_{11}$ and $Q_{12}$. The amplitude we obtain is

$$A(b \to s \phi) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* g_\phi \epsilon_\mu \left( 2C_\phi(m_b)s_\gamma^\mu Lb + \frac{\alpha_s(m_b)m_b N^2}{8\pi k^2} - \frac{1}{2N^2} C_{11}(m_b) [m_b s_\gamma^\mu Lb + (6 + 4 \frac{m_s^2}{M_\phi^2}) p_\mu^b s Rb - m_s \left( 1 - 2 \frac{m_b^2 + 2M_\phi^2 - m_s^2}{M_\phi^2} \right) s_\gamma^\mu Rb - 4 \frac{m_b m_s}{M_\phi^2} (m_b \bar{s} Rb - m_s) \right] + \frac{\alpha_s m_b}{8\pi k^2} C_{12}(m_b) \left[ 9m_b \bar{s} \gamma^\mu Lb - (10 - 4 \frac{m_s^2}{M_\phi^2}) p_\mu^b \bar{s} Rb - m_s \right].$$
\[-2 \frac{m_b^2 + 2M_\phi^2 - m_s^2}{M_\phi^2} \bar{s} \gamma^\mu R b - 4 \frac{m_b m_s}{M_\phi^2} p_\mu^R \bar{s} L b \right\},
(43)

where \( k^2 = (m_b^2 + m_s^2 - M_\phi^2/2)/2 \), \( C_\phi(m_b) \) has been introduced in eq. (10) and \( g_\phi \) is defined by \( \langle \phi | \bar{s} \gamma^\mu s |0 \rangle = i g_\phi \epsilon^\mu \). This result does not fully agree with those of refs. [20, 21].

From eq. (43), we obtain the branching ratio

\[
BR(b \to s \phi) = \frac{\Gamma(b \to s \phi)}{\Gamma(b \to cev)} BR_{SL}^{exp} = \frac{|V_{ts}|^2}{|V_{tb}|^2} \frac{6 \pi^2 g_\phi^2 BR_{SL}^{exp}}{g(m_c/m_b) \Omega(m_c/m_b, m_b) m_b^4} \lambda(m_b^2, M_\phi^2, m_s^2)^{1/2}
\]

\[
\left[ C_\phi(m_b) \right]^2 \left( 2 + 2 \frac{m_b^2}{M_\phi^2} - 4 \frac{M_\phi^2}{m_b^2} + 2 \frac{m_s^2}{m_b^2 M_\phi^2} + 2 \frac{m_s^2}{m_b^2} - 4 \frac{m_s^2}{M_\phi^2} \right)
\]

\[
+ \frac{\alpha_s(m_b^2)}{8 \pi} 2 \text{Re}(C_\phi(m_b)) \frac{C_{11}(m_b)}{k^2} \left( \frac{N^2 - 1}{2N^2} \right) \left( -5m_b^2 - 20 \frac{m_b^2 m_s^2}{M_\phi^2} + 4 \frac{m_b^4}{M_\phi^2} + M_b^2 \right)
\]

\[
+ 16 \left( \frac{m_b^2}{M_\phi^2} - 23 m_s^2 \right) + \frac{\alpha_s(m_b^2)}{8 \pi} 2 \text{Re}(C_\phi(m_b)) \frac{C_{12}(m_b)}{k^2} \left( 19 m_b^2 - 20 \frac{m_b^2 m_s^2}{M_\phi^2} + 4 \frac{m_b^4}{M_\phi^2} \right)
\]

\[
- 31 \left( \frac{m_b^2}{M_\phi^2} - 23 m_s^2 \right) \left( \frac{\alpha_s(m_b^2)C_{11}(m_b)}{8 \pi k^2} \right) \left( \frac{N^2 - 1}{2N^2} \right) \left( \frac{31}{2} \frac{m_b^2}{M_\phi^2} \right)
\]

\[
- 4 \frac{m_b^2 m_s^2}{M_\phi^2} + 16 \frac{m_b^2 m_s^2}{M_\phi^2} - 54 m_b^2 m_s^2 - 19 m_b^4 - 4 \frac{m_b^4 M_\phi^2}{M_\phi^2} + 4 \frac{m_b^6}{M_\phi^2}
\]

\[
+ 8 \frac{m_b^6}{M_\phi^2} + \frac{15}{2} \frac{M_b^2 m_s^2 - 9}{2} \frac{M_b^4}{M_\phi^2} + 4 \frac{M_b^6}{M_\phi^2} + 8 \frac{m_b^6}{M_\phi^2} + 4 \frac{M_b^6}{M_\phi^2} + 4 \frac{m_b^6}{M_\phi^2} \left( \frac{\alpha_s(m_b^2)}{8 \pi k^2} \right) \left( - \frac{177}{2} \frac{m_b^2}{M_\phi^2} \right)
\]

\[
- 4 \frac{m_b^2 m_s^2}{M_\phi^2} + 112 \frac{m_b^2 m_s^2}{M_\phi^2} - 62 m_b^2 m_s^2 + 93 m_b^4 - 4 \frac{m_b^4 M_\phi^2}{M_\phi^2} - 96 \frac{m_b^4}{M_\phi^2} + 4 \frac{m_b^6}{M_\phi^2} \left( \frac{\alpha_s(m_b)}{8 \pi k^2} \right) \left( - \frac{177}{2} \frac{m_b^2}{M_\phi^2} \right)
\]

\[
+ 8 \frac{m_b^6}{M_\phi^2} + \frac{47}{2} \frac{M_b^2 m_s^2 - 25}{2} \frac{M_b^4}{M_\phi^2} + 4 \frac{M_b^6}{M_\phi^2} - 8 \frac{m_b^6}{M_\phi^2} - 7 m_b^4 \right)
\]

\[
+ \frac{\alpha_s(m_b^2)C_{11}(m_b)}{8 \pi k^2} \frac{\alpha_s(m_b^2)C_{12}(m_b)}{8 \pi k^2} \left( \frac{N^2 - 1}{2N^2} \right) \left( - 41 \frac{m_b^2}{M_\phi^2} - 8 \frac{m_b^2 m_s^2}{M_\phi^2} + 192 \frac{m_b^4 m_s^4}{M_\phi^2} \right)
\]

\[
- 212 m_b^2 m_s^2 + 10 m_b^4 - 8 \frac{m_b^4 M_\phi^2}{M_\phi^2} - 128 \frac{m_b^4}{M_\phi^2} + 8 \frac{m_b^6 m_s^2}{M_\phi^2} + 16 \frac{m_b^6}{M_\phi^2} - 65 M_b^2 m_s^2
\]

\[
+ 15 M_b^4 + 8 \frac{m_b^6}{M_\phi^2} + 48 \frac{m_b^6}{M_\phi^2} + 90 m_b^4 \right) \right],
(44)

where \( N \) is the number of colours, \( \lambda(m_1, m_2, m_3) = (1 - m_2/m_1 - m_3/m_1)^2 - 4m_2m_3/m_1^2 \),

\( g(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln(z) \) is the phase-space correction and \( \Omega(z, \mu) \simeq 1 - \frac{2\alpha_s(\mu)}{3\pi} \left[ (\pi^2 - \frac{3\mu}{4}) (1 - z)^2 + \frac{3}{2} \right] \) is the QCD correction to the semileptonic decay rate.

The coefficients \( a_i \) of eq. (19) can be immediately obtained from eq. (44).

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\(^{6}\) Numerically our branching ratio is \( \sim 25\% \) smaller, including the effect of the residual scheme dependence.
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