Geometric Transition versus Cascading Solution

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Abstract

We study Vafa’s geometric transition and Klebanov - Strassler solution from various points of view in M-theory. In terms of brane configurations, we show the detailed equivalences between the two models. In some limits, both models have an alternative realization as fourfolds in M-theory with appropriate G-fluxes turned on. We discuss some aspects of the fourfolds including how to see the transition and a possible extension to the non-supersymmetric case.

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1 Introduction and Summary

The large $N$ limit of $\mathcal{N} = 4$ conformally invariant theory has a supergravity dual which is used to study many aspects of this theory [1]. This can be also be extended to theories with lower supersymmetry. The supergravity duals are now on more exotic backgrounds like orbifolds and conifolds. However the situation is more subtle for non-conformal theories with lower supersymmetry. When we have $N$ D3 branes near a conifold singularity, the world volume theory is $\mathcal{N} = 1 \ SU(N) \times SU(N)$ theory with a quartic superpotential. To break conformal invariance in these theory we have to introduce fractional branes [7, 8]. Fractional branes are higher dimensional branes wrapped on vanishing cycles of a manifold and therefore they carry charges of lower dimensional branes. At the conifold points there are vanishing two cycles on which D5-branes of type IIB theory can wrap. The fractional charges of these branes are generated by $\int B_{NSNS}$ fields that thread through the vanishing cycles. In the presence of $M$ such fractional branes and $N$ integer branes (which are of course D3 branes) the world volume theory is a non-conformal $\mathcal{N} = 1 \ SU(N + M) \times SU(N)$ theory in the UV. This is basically the Klebanov-Strassler model [9].

There is yet another way to generate a non-conformal $\mathcal{N} = 1$ theory in four dimensions. This is by wrapping a D5 brane on the finite two-cycle of a resolved conifold. On the world volume of the D5 brane there is an IR theory (on the remaining unwrapped 3+1 dimensions) with a superpotential which can be calculated using geometric engineering. The superpotential breaks the $\mathcal{N} = 2$ theory to $\mathcal{N} = 1$. This is Vafa’s model [3] (see also [4, 5, 6]). In the UV therefore it is six dimensional whereas the Klebanov-Strassler model remains four dimensional at UV.

Both these theories have a dual picture at all scales where we have a deformed conifold with no branes. The branes are replaced by three form fluxes. The three form RR fluxes are of course the remnant of the wrapped D5 branes. The NS fluxes in Klebanov-Strassler have their origin from the $B_{NSNS}$ fluxes through the vanishing two cycle which is related to the gauge coupling of the field theory. For Vafa’s model the origin of NS fluxes is a little subtle. It is related to the size of the resolved $S^2$ which gives the gauge coupling of the field theory [3]. Since we don’t expect the NS fluxes to be constant, therefore the size of the two cycle is also not a constant quantity. In fact as was shown in [3, 4, 11] the size of the two cycle determines the RG flow of the 3+1 dimensional gauge coupling.

In Klebanov-Strassler model the theory flows in the far IR to a system with only fractional branes via consecutive cascades which in field theory are Seiberg dualities. As we discussed above there is, at all scales, a dual non-singular closed string background with only the three form fluxes. In Vafa’s model the corresponding dual can be reached

\footnote{Other reasons for the existence of $H_{NS}$ fluxes in Vafa’s model have been discussed in [3, 4].}
by a conifold transition. Both of these models have yet another M-theory or IIA dual via a T-duality in which we get a picture completely in terms of brane constructions [10, 11]. In this way its very easy to see why we go to a closed string background. A detailed analysis of this will be presented in sec. 3.3.

The brane realization of the closed string background as given in [10, 11] can be used to our advantage to actually derive the supergravity solution. We will see how far we can trust the supergravity solution obtained by this method. This will be shown in sec. 4.2 as example 2. We shall point out that due to delocalisation this in fact doesn’t give us the exact answer which can nevertheless be derived via a different technique.

The technique that we shall use has been discussed earlier in a different context in [13, 20, 21, 22] and in a recent related context in [23, 24, 25, 26] (see also [28] for related issues). Herein we take a fourfold (which we shall assume to be compact for our purpose) and switch on an appropriate G-flux. The fourfold that we take is a non-trivial torus fibration over a base which is a deformed conifold. The G-fluxes have one leg along the fiber and the other legs on the base – deformed conifold. This background in M-theory is related to the closed string background of the above two models in IIB. There are however some subtlety related to the global structure which we will point out in sec. 4.3. We first give a warm up example in sec. 4.2 (as example 1) which will illustrate the basic procedure. In the next section we give the solutions to the model. We shall see that from M-theory we directly get the linear forms of the background supergravity equations of motion. The solutions of these equations will predict the behaviour of these models at all scales.

The study of the Klebanov-Strassler’s and Vafa’s models from fourfold points of view has an inherent advantage. The background can be easily extended to the non-supersymmetric case. Such non-supersymmetric background was recently studied in [10, 17]. However its not yet clear how the dual brane side would look like. We discuss all these issue in sec. 5. We conclude with some comments on other issues like a possible extension to the Type I background and fate of the open strings in sec. 6.

We now begin with a discussion of Vafa and Klebanov-Strassler models.

2 Vafa’s Geometric Transition for $U(N)$ theory
2.1 General Features for the Transition

In [3], based on Chern-Simons/topological strings duality [2], a duality transition was proposed between \( N \) D5 branes wrapped on the finite two cycle of a resolved conifold and a geometrical picture consisting of a deformed conifold without branes but with a \( H_{RR} \) flux through the finite three-cycle of the deformed conifold and \( H_{NS} \) flux through the noncompact three-cycle. As explained in [3] and further clarified in [4], the exact match between the parameters of the field theory on the \( N \) D5 branes and the parameters read from the geometry is:

- The flux of \( H_{RR} \) through the \( S^3 \) cycle is equal to \( N \), the number of the D5 branes which disappear in the geometry.

- The flux of \( H_{NS} \) through the noncompact 3-cycle of the deformed conifold is equal to \( \alpha \), the bare coupling constant of the field theory living on the D5 branes.

- The period \( S \) of holomorphic 3-form on the \( S^3 \) cycle is equal to the gluino condensate on the field theory.

- The quantum corrections (RG flow) of the field theory on the D5 branes are related to the period \( \Pi \) of holomorphic 3-form on the noncompact 3-cycle in the deformed conifold side. Because this period is over a non-compact cycle, [4] have imposed a cutoff in the integral for large distances in the Calabi-Yau manifold (IR cutoff in the Calabi-Yau). This cutoff was identified with a dynamical scale of the \( U(N) \) theory which is an UV cutoff. To do so, one has to remember the usual UV/IR correspondence for branes.

- The superpotential in the \( \mathcal{N} = 1 \) field theory is determined by the fluxes in the geometry:

\[
- \frac{1}{2 \pi i} W_{\text{eff}} = N \Pi + \alpha S
\]

and after the previous identifications becomes

\[
W_{\text{eff}} = -N S \log S + S (3 N \log \Lambda_0 - 2 \pi i \alpha)
\]

Furthermore, after identifying the cutoff \( \Lambda_0 \) with the UV cutoff in the field theory, [3, 4] obtained the usual superpotential for the glueballs

\[
W_{\text{eff}} = N S \left( \log(\Lambda^3/S) + 1 \right)
\]
2.2 Vafa’s Transition in MQCD

In [10, 11], the transition of [3] was discussed in terms of transitions in MQCD. By starting with $N$ D5 branes wrapped on the $\mathbb{P}^1$ cycle of the resolved conifold, the T-dual picture will be a brane configuration with $N$ D4 branes along the interval with two NS branes in the ‘orthogonal’ direction at the ends of the the interval. Here the length of the interval is the same as the size of the rigid $\mathbb{P}^1$. As the rigid $\mathbb{P}^1$ shrinks to zero, the size of the interval goes to zero and the two NS branes approach each other. To ease the next discussion, let us denote the common 4 directions for all the branes as $(x_0, x_1, x_2, x_3)$, the extra direction of the D4 brane by $x^\text{n}$ ($n$ comes from noncompact, it will be clear below why we use this notation) and the extra directions of the two orthogonal NS branes are $(x^4, x^5)$ and $(x^8, x^9)$ respectively.

When we lift this configuration to M-theory, the two NS5 branes become M5 branes and are connected together by another M5 brane emanating from the D4 branes. Defining complex coordinates as:

$$x = x^4 + ix^5, \quad y = x^8 + ix^9, \quad t = \exp(-R^{-1}x^n + ix^{10})$$

where $R$ is the radius of the 11th direction, the world volume of the M5 corresponding to the resolved conifold is given by $R^{1,3} \times \Sigma$ and $\Sigma$ is a complex curve defined, up to an undetermined constant $\zeta$, by

$$y = \zeta x^{-1}, \quad t = x^N$$

As explained in [10, 11], when the size of $\mathbb{P}^1$ goes to zero, the $x^n$ direction becomes very small and the value of $t$ on $\Sigma$ must be constant because $\Sigma$ is holomorphic and there is no non-constant holomorphic map into $S^1$. Therefore the M5 curve makes a transition from a “space” curve into a “plane” curve. From (3), we obtain two relation on $t$ and $t^{-1}$

$$t = x^N, \quad t^{-1} = \zeta^{-N}y^N.$$ 

So there are $N$ possible plane curves which the M5 space curve $\Sigma$ can be reduced to:

$$\Sigma_k : \quad t = t_0, \quad xy = \zeta \exp 2\pi ik/N, \quad k = 0, 1, \ldots, N - 1.$$ 

This is thus the way we see Vafa’s duality transformation. After the transition the degenerate M5 branes are no longer considered as the M-theory lift of D4 branes. This is now a closed string background.
3 Klebanov-Strassler Cascading Solution

3.1 Cascading Solution

In [9], an interesting approach was taken to study the conifold with integer $N$ D3 branes and $M$ fractional D3 branes. Using results of [7], the gauge group in the UV is $SU(M + N) \times SU(N)$ and the field theory flows to infrared by an RG flow which will give a scale dependent number of colors

$$N(r) = N_0 + \frac{3}{2\pi} g_s M^2 \ln(r/r_0).$$

where $g_s$ is the string coupling constant and $r$ is the radial direction for the conifold. Along the RG flow the gauge group is $SU(M + N(r)) \times SU(N(r))$ and as $\ln(r/r_0)$ decreases by $\frac{2\pi}{3g_s M}$, $N(r)$ decreases by $M$, so we will have:

$$SU(M + N) \times SU(N) \rightarrow SU(N(M)) \times SU(N - M)$$

and so on until the gauge group becomes $SU(M)$. In terms of branes, this means that by starting with $N$ D3 branes and $M$ fractional D3 branes, along the RG flow the number of integer branes decreases and in the end we remain with only the $M$ fractional D3 branes. This is the cascade of Seiberg dualities.

In the supergravity, the $N$ integer D3 branes are the sources for the RR 5-form which is

$$\tilde{F}_5 = \mathcal{F} + \ast \mathcal{F}$$

where $\mathcal{F}$ is proportional to $N(r)$ multiplied by the volume of $T^{11}$ which is the horizon for the conifold.

The $M$ fractional D3 branes are sources for a RR 3-form through the 3-cycle $\omega_3$ of $T^{11}$

$$F_3 = M \omega_3$$

and the NS field through the 2-cycle $\omega_2$ of $T^{11}$

$$B_2 = 3g_s M \ln(r/r_0) \omega_2$$

3.2 The Transition in the Klebanov-Strassler Case

We now describe the transition corresponding to the Klebanov-Strassler case. We start with the conifold geometry and we consider a vanishing 2-cycle at the apex where we
wrap M D5 branes which are the fractional D3 branes. In this case, we need to have a NS field through the vanishing $P^1$ cycle in order to insure its stability.

To discuss the transition, we use the approach of [10, 11] modified to be fitted for the conifold instead of the resolved conifold. We take a T-duality in the direction $\psi$ of the conifold which we denote by $x_c$ in order to signal the fact that its compact. The integer D3 branes would become D4 branes with one direction on the circle $x_c$. As discussed in [13], the fractional D3 branes become D4 branes on a half-circle. So in this case the D4 branes will have the 4-th spatial direction in a compact direction. The field theory on the $M$ fractional D3 branes is pure $\mathcal{N} = 1$, $U(M)$ super Yang Mills theory.

After lifting the brane configuration to M theory and studying the transition as in [10, 11], we arrive to a geometrical configuration which should be identical to the Klebanov-Strassler case discussed in the previous subsection. One important observation is the following: because we started with a conifold with a vanishing but stable 2-cycle, we have the compact direction $x_c$ which remains compact on the geometrical side.

In order to discuss this in more detail we use the results of [12, 13] where the elliptic model was lifted to M theory. We now define again the complex coordinates $x$, $y$ as in the resolved conifold case and $t = exp(-R^{-1}x_c + ix^{10})$ where $R$ is the radius of the 11th direction and $x_c$ is the compact direction, $t$ is not periodic in $x_c$ so we should use it only for a finite range of values of $x_c$. The form of the single M5-brane is

$$y = \zeta x^{-1}, \quad t = x^M. \tag{13}$$

However before we go in more details we should ask what the distance between the branes represent. In the usual case the coupling constant of $\mathcal{N} = 1$, 3 + 1D gauge theory is determined by the distance between the branes. In the wrapped brane picture the coupling constant comes from the $B_{NSNS}$ field on the vanishing two cycle of the conifold from the coupling

$$\int_{\Sigma} B_{NSNS} \int F \wedge *F + \int_{\Sigma} B_{RR} \int F \wedge F \tag{14}$$

where $\Sigma$ is the vanishing two-cycle. For our case we put

$$B_{RR} = 0, \quad B_{NSNS} = 3g_s M \ln(r/r_0) \omega_2. \tag{15}$$

The above formula implies that now the distance between the branes is actually a function of $r$. Thus the two NS5 branes are now bent along $r$. In the M-theory lift, the curved M5 brane model will now have $x, y$ as functions of $r$.

If we now shrink the distance between the two M5 branes, from the discussion of [10, 11] we expect $t$ to be fixed to a constant value. In terms of type IIA picture there are now two important considerations.
• The two M5 branes are now at a point $t_0$ and satisfying the equation $xy = \zeta$. When we reduce this to IIA the bending of the M5 branes along $x^{10}$ direction will appear as 2-form field. Making a T-duality along $\psi = x_c$ will give us a deformed conifold with $H_{RR}$ on the 3-cycle. Observe that after reducing to the type IIA theory we obtain an single NS5 brane which is wrapped on the $S^2_1 - S^2_2$ of $T^{11}$ [13].

• The M5 along $x$ and $y$ directions are also bent along $r$. After the transition this bending will remain in the planar M5 model. Now under a T-duality the bending will appear as $H_{NS}$ field on a 3-cycle dual to the 3-cycle on which there is $H_{RR}$. We thus see how the RR and NS forms appear in the brane picture. This is also discussed in some detail from supergravity perspective in eqt. (3.5) of [14] and transparency 18 of [15].

Therefore we see that after the transition we reach a non-singular manifold which is a deformed conifold. The argument is the same as discussed in [10, 11].

There are two further indications that this should be the case:

• In [10, 11] we have also considered the reverse transition, from the geometry to the brane configuration and the D4 branes appeared due to the Hanany-Witten effect in the presence of a singularity at the intersection of orthogonal NS branes. We have discussed the transition from the deformed conifold to resolved conifold, where the 4-th spatial direction of the D4 branes is an interval in a noncompact direction. However there is an ambiguity here. At the conifold point – wherein we have the creation of a D4 brane, – there are now two distinct possibilities to stretch the D4 brane. We could either stretch it along $x_n \equiv x^7$ or along $x_c \equiv x^6$. These two possibilities give rise to the Vafa and Klebanov-Strassler models respectively. In the former case we have D5 branes wrapped on the $S^2$ of a resolved conifold. And in the latter case we have D5 wrapped on vanising $S^2$ with $B_{NSNS}$ flux.

• In section 5.3 of [9], the discussion concerned the validity of a description for the pure $\mathcal{N} = 1, U(M)$ super Yang Mills theory. In order to have a reliable dual of the pure glue theory, one needs to take the limit of finite $g_s$ $M$ which is the limit when the $S^3$ at the apex is finite which is exactly what we discussed above – a finite $S^3$ with non zero $H_{RR}$ through it. Considering the fact that we have started with $U(M)$ theory on the fractional D3 branes, we expect to obtain pure $\mathcal{N} = 1, U(M)$ super Yang Mills theory and this is in accordance to the claims of [9].

When we reduce them to type IIA, they will be D4 branes wrapping the $x_c$ cycle. But this is just the result of [14, 15] after taking direct T-dual to the [9] model! The corresponding D4 branes will contribute to the 4-form which is the T-dual to the 5-form in type IIB.
3.3 Brane Realisation of Cascade vs. Geometric Transition

We can also clarify now the differences between the two approaches of studying $\mathcal{N} = 1$ duality. In one case — of Klebanov-Strassler — we have a T-dual model described on a torus parametrized by $z = x^6 + ix^{10}$. The two M5 branes are at two arbitrary points on the $z$ torus. The transition from “curved” M5 to “plane” M5 can be achieved by a spiral motion on $z$ plane. This spiral motion is the brane realization of cascade in this model. In the other case — of Vafa — the T-dual model is defined on a cylinder parametrised by $w = x^7 + ix^{10}$. There is no spiral motion because this description is in the far IR. Therefore, as we discussed in detail in [10, 11], to see the RG flow of $\mathcal{N} = 1$ coupling we make a transition to the “plane” curve and impose a IR cutoff on the integral

$$\int_{\Lambda_0^{3/2} > |x| > |\zeta|^{1/2}} d\log x \wedge *d\log x$$

This takes the form of the NSNS flux on the deformed conifold side under a T-duality.

We could be a little bit more precise here. From the above paragraph one would naively assume that the difference between Vafa and Klebanov-Strassler is because one of the model is on a torus and the other is on a cylinder. However this is not the case. To see this let us first consider Vafa’s geometric transition:

The M-theory curve is

$$t = x^N, \quad t^{-1} = \zeta^{-N} y^N$$

At the transition point — when the two M5 brane ($x$ and $y$) come at a point $x^7 = 0$ — there is a $S^1$ from the M5 between them. As discussed in detail in [10, 11] this is not holomorphic and therefore the only holomorphic curve is when $x, y$ satisfy $xy = \zeta$ at a point ($x^7 = 0, t = t_0$).

Now we shall argue that something more interesting happens for Klebanov - Strassler model because we also have D3 branes. To delve into it we would need some details

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6This however doesn’t mean that there is no cascade in this model. We shall discuss this in detail in a forthcoming paper. See also [14] where it is discussed in detail how cascades occur in the geometric transition model. In the presence of $N$ D3 branes and $M$ wrapped D5 branes the theory is $SU(N + M) \times SU(N)$. The cascades in this theory are realised by an infinite sequence of flop transitions. At the end — which is in fact far IR — the theory undergoes a geometric transition via the usual conifold transition. The brane construction that we discuss in this paper only describes the IR aspect of Vafa’s geometric transition picture. In the presence of D3 branes we have to consider a pair of $D5 - \bar{D5}$ with fluxes wrapped on the resolved two cycle of the conifold. The T-dual picture would now be on a compact circle. The behaviour of tachyon in this system and other details would be discussed in the forthcoming paper. We thank the referee for his comments.
of M-theory curves. When we have \( N \) D3 branes at the apex of a conifold then the M-theory lift (of the T-dual) is given by a curve

\[
x^N = y^N = 0 \tag{18}
\]

This means that the D4 branes which turn into M5 branes go right through the other two (orthogonal M5). This M5 we call as toroidal M5\(^7\).

Also when we have D5 wrapped on vanishing \( S^2 \) the M-theory lift of it is a MQCD like structure. However when we have both i.e. \( N \) D3 and \( M \) D5 wrapped on \( S^2 \) then the M-theory lift would be a toroidal M5 and a MQCD five brane.

From this construction it’s now easy to see what happens when we have \( x^6 = 0 \). The MQCD five brane comes to a point. But now there is no \( S^1 \) there. Because of the existence of toroidal M5 branes the system remains holomorphic even when we make \( x^6 = 0 \). Therefore in the original model — when we have \( N \) D3 branes — the UV description of the metric in [7] is the same as that of [9]. This is consistent with the predictions of Klebanov-Strassler. Question is what happens at the infrared. Motivated by the analysis done above for the case of Vafa, we expect a similar “transition” when the toroidal M5 can be made to go away. This would happen when we make one of the M5 \( x \) move spirally up on the \( x^6 + ix^{10} \) torus. Everytime the M5 cross the other one there is a Seiberg duality in the theory and the number of D3 reduces. This is the cascade. Ultimately when the D3 branes completely go away then in M-theory making \( x^6 = 0 \) will lead to a holomorphic structure when we also make \( t = t_0 \). This is now the transition.

### 4 Geometric Transitions from G-Flux

#### 4.1 Basic Idea

In the previous sections we have seen how Klebanov-Strassler and Vafa’s transitions are realized from brane constructions in M-theory. The key point in the constructions was the existence of holomorphic structures. Demanding holomorphicity after the conifold transition gives us essentially the closed string background with fluxes and no branes.

There is yet another realization of the transition from M-theory and this is by invoking the idea of G-fluxes. The whole process can be summarized by the following\(^7\)Because of this construction the \( SU(N) \times SU(N) \) theory has a global symmetry of \( SU(2) \times SU(2) \) from the rotation of the two “decoupled” M5 branes.
steps:

- Consider a 4-fold in M-theory which is a non-trivial $T^2$ fiberation over a base $B$.

- Switch on a G-flux which has one component along the $T^2$. Reducing to type IIB, the G-flux will give the NSNS and RR three form fields on the base $B$.

- If we choose $B$ to be a deformed conifold then this will effectively produce one side of the geometric transition, i.e the side with a closed string background with fluxes and no branes.

- The presence of G-fluxes the 4-fold metric will be warped and the warp factor depends essentially upon the Euler characteristics of the 4-fold and is related to a hierarchy of energies in the dual field theory.

- By doing a conifold transition on the base, we should be able to argue that the warped metric now gives the metric of a D5 wrapped on a two cycle of a resolved conifold. This would then signify Vafa’s transition at least in the far IR.

For the case of Klebanov-Strassler the M-theory realization is been studied in some details in [23, 26]. For example in [23] it was shown that a fourfold which is a direct product of a deformed conifold and a torus actually do not realize the required background as the Euler characteristics is zero and, in the absence of any D3 branes, the quantity $f G \wedge G$ also vanishes.

In [26] the Klebanov-Strassler model has been embedded in a F-theory compactification. The Calabi-Yau fourfold $X$ which admits a conifold singularity in its base $B$ is given by specifying a Weierstrass model

$$y^2 = x^3 + x f(z_i) + g(z_i)$$

where the base $B$ is given by a quartic equation in $P^4$ as:

$$P \equiv z_5^2 \left( \sum_{i=1}^{4} z_i^2 \right) - t^2 z_5^4 + \sum_{i=1}^{4} z_i^4 = 0$$

In the above equation $z_i$ are the homogeneous coordinates on $P^4$ and $t$ a real parameter. $f$ and $g$ are polynomials of degree 4 and 6 in the homogeneous coordinates $z_i$. The fourfold is realised here as a non-trivial $T^2$ fiberation over a conifold base. As such its Euler characteristics can be shown to be $\chi = 72$.

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\footnote{The global structure, as we will discuss later, is more complicated when the fiberation is non-trivial. In this section and the next we will not dwell into this subtlety even though we take non-zero Euler-characteristics, $\chi$, of the fourfold.}
To realize the geometric transition of Vafa we need a fourfold which is a non trivial $T^2$ fibration over a deformed conifold base. In the next section we will give an example of a fourfold which is a non trivial $T^2$ fibration over a base $T^6/\mathbb{Z}_2 \times \mathbb{Z}'_2$ where the $\mathbb{Z}'_2$'s are orientifold actions. To start with, let us see whether we can say something about the case when we have a fourfold with base a conifold and the fluxes switched on. Determining the exact background is a difficult exercise but we can use various approximate methods to get a possible solution. The conifold base (in the absence of any fluxes) is given by the familiar equation

\begin{equation}
\frac{dr^2}{r^2} + \sum_{i=1}^{2} (d\theta_i^2 + \sin^2\theta_i d\phi_i^2) + (d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2
\end{equation}

(21)

For the case of $\theta_2 = \phi_2 = \text{constant}$, the above equation is just an ALE space and therefore should be given by a D6 brane when the M-theory radius is very small. In fact as argued in \[29\] two intersecting D6 branes in IIA when lifted to M-theory is actually a conifold\[10\]. However the above discussions are in the absence of any fluxes. In the presence of fluxes the background metric is complicated but can be worked out.

If $r$ is the radial direction of the conifold base, $x^1$ the M-theory direction and the torus $T^2$ parametrised by $x^1, x^2$ then for small $r$ the torus has a very small warp factor given by

\begin{equation}
\frac{c_1^2 + 1}{c_1^2 + H (dx^1)^2} + \frac{c_2^2 + 1}{c_2^2 + H (dx^2)^2}
\end{equation}

(22)

which is effectively 1 near $r = 0$. The factors $c_i, i = 1, 2$ are the value of $C$ fields $C_{1\theta_1 \psi}, C_{2\theta_2 \psi}$ at the origin $r = 0$. And $H$ is a linear function of $r$. The quantities $\theta, \psi$ are defined before and they are coordinates for a conifold.

Therefore, the above metric is an approximate fourfold where the fiberation is trivial and there is a $H_{NSNS} + \tau H_{RR}$ over a three cycle whose cohomology is given by

\begin{equation}
e^\psi \wedge e^{\theta_1} \wedge e^{\phi_1} - e^\psi \wedge e^{\theta_2} \wedge e^{\phi_2}
\end{equation}

(23)

where $e^{\theta_i} = d\theta_i, e^{\phi_i} = \sin\theta_i d\phi_i, i = 1, 2$. The Euler characteristics of this fourfold is zero as the fiberation is trivial. In the next section we shall discuss a fourfold which has a non-trivial $T^2$ fiberation.

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9This is the conformal transformed metric where $\frac{dr}{r} = d\phi$. Using this form of the metric the base of the conifold can be easily shown to be Einstein\[39\].

10There is an interesting digression. Two intersecting D6, with four common directions, at an angle (in the presence of O6 planes) realize a seven dimensional $G_2$ holonomy manifold when the system is lifted to M-theory. Using this construction and applying the methods of \[31, 32\], one can study chiral matters in $\mathcal{N} = 1$ theory in four-dimensions \[33, 34, 35, 36, 37\]. We shall however not discuss this interesting connection in this paper.
4.2 A First Look: Delocalized Case

Outline of the Setup

Consider a 4-fold given by a non-trivial $T^2$ fibration over a base $\mathcal{B}$. The $T^2$ is parametrised by $x, y$ such that

$$dz = dx + \tau dy, \quad d\bar{z} = dx + \bar{\tau} dy$$

(24)

where $\tau$ is the complex structure of the torus. We now choose a G-flux in the following way:

$$\frac{G}{2\pi} = dz \wedge \omega - d\bar{z} \wedge *\omega$$

(25)

where $\omega \in H^{1,2}(\mathcal{B})$. The flux then lifts to a combination of the NS field strength $H_{NSNS}$ and RR field strength $H_{RR}$. This is given by

$$H_{NSNS} = \omega - *\omega, \quad H_{RR} = \omega \tau - *\omega \bar{\tau}$$

(26)

Before we go any further let us remind ourselves of the following important conditions [18, 19, 20, 21, 22]:

- The 4-fold vacua has a tadpole anomaly given by $\chi/24$ where $\chi$ is the Euler characteristics of the 4-fold. If $\chi/24$ is integral, then the anomaly can be canceled by placing a sufficient number of spacetime filling branes $n$ on points of the compactification manifold. There is also another way of canceling the anomaly and this is through the G-flux. The G-flux contributes a $C$ tadpole through the Chern-Simons coupling $\int C \wedge G \wedge G$. When $\chi/24$ is not integral then we need both the branes and the G-flux to cancel the anomaly. The anomaly cancellation formula becomes

$$\frac{\chi}{24} = \frac{1}{8\pi^2} \int G \wedge G + n$$

(27)

which must be satisfied for type IIA or M-theory.

- If we denote the spacetime coordinates by $x^\mu$ where $\mu = 0, 1, 2$ and the internal space by the complex coordinates $y^a, a = 1, \ldots, 4$ then in the presence of G-flux the metric becomes a warped one

$$ds^2 = e^{-\phi(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{\frac{2\phi(y)}{2}} g_{ab} dy^a dy^b$$

(28)

with the G-flux satisfying the condition

$$G = *G, \quad J \wedge G = 0$$

(29)
where the Hodge star acts on the internal 4-fold with metric \( g \) and \( J \) is the Kahler form of the 4-fold. There is also another non vanishing \( G \) given in terms of the warp factor as 
\[
G_{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\rho\sigma} \partial_\alpha e^{-\frac{3}{2} \phi}.
\] The warp factor satisfy the equation
\[
\Delta e^{\frac{3}{2} \phi} = \ast \left[ 4\pi^2 X_8 - \frac{1}{2} G \wedge G - 4\pi^2 \sum_{i=1}^{n} \delta^8(y - y_i) \right]
\] (30)
where the Laplacian and the Hodge \( \ast \) is defined wrt \( g \), and \( X_8 \) is the 8- form constructed out of curvature tensors.

• The anomaly cancellation condition will now become, in type IIB theory
\[
\frac{\chi}{24} = n + \int H_{RR} \wedge H_{NSNS}
\] (31)
where \( n \) is the number of D3 branes. Observe that if we choose the right background fields satisfying \( n = 0 \) we have the background required for the Vafa and Klebanov-Strassler case.

**Various Notations and Scales**

Before we go any further let us discuss the various limits that we need to impose on the scales to get a supergravity background for our case. Let us denote the eleven dimensional Planck length by \( l_{11} \) and the average volume of the fourfold to be \( l_8 \). We shall assume that \( l_8 >> l_{11} \). The background G-flux and the warp factor \( \omega = e^\phi \) scales as
\[
G = \left[ \frac{l_{11}^3}{l_8^4} \right], \quad \omega = e^{\left[ \frac{l_{11}^6}{l_8^6} \right]}
\] (32)
For very large sized fourfold the metric becomes unwarped and the background G-flux vanishes.

For finite sized fourfold there is a warp factor which is determined by eq. (30). In terms of derivative expansions both the \( X_8 \) and the membrane terms are suppressed by \( \frac{1}{l_{11}^4} \) and therefore can be neglected compared to the \( G \wedge G \) term [21].

The other \( R^4_{MNPQ} \) terms in the low energy M-theory lagrangian are also suppressed by \( \frac{1}{l_{11}^4} \). These terms are written in terms of \( \epsilon \) and \( t_8 \) tensors of [16].

Our definition for the Hodge \( \ast \) in \( d \) dimensions will be:
\[
\ast (dx^{a_1} \wedge .. \wedge dx^{a_q}) = \frac{1}{(d-q)!} \epsilon^{a_1...a_q}_{a_{q+1}...a_d} dx^{a_{q+1}} \wedge .. \wedge dx^{a_d}
\] (33)
where $a_p$ are real coordinates. In terms of complex coordinates $a, b$ the epsilon tensor can be expressed as:

$$\epsilon_{abcd|efgh} = g_{ae} g_{bf} g_{cg} g_{dh} \pm \text{permutations}$$ (34)

We will also assume that the number $M$ of wrapped branes (on either vanishing cycles or finite cycles) to be very large such that even for the case $g_s \to 0$ ($g_s$ being the string coupling) the quantity $g_s M$ is very large. In the UV therefore, for the Klebanov-Strassler model, we assume

$$g_s \to \epsilon, \quad \int_{S^2 \to 0} B_{NSNS} \to \epsilon^{-\beta}, \quad \beta > 1$$ (35)

**Example 1**

As a warm up example consider a 4-fold given by $T^8/G$ where $G$ is the orbifolding group. Therefore type IIB theory will be compactified on the orientifold $T^6/Z_2 \times Z_2'$ and $Z_2$ involves the orientifold group.

In M-theory we define the G-flux to be:

$$G = A dz^1 dz^2 \bar{dz}^3 dz^4 + B dz^1 \bar{dz}^2 dz^3 \bar{dz}^4 + C d\bar{z}^1 dz^2 dz^3 \bar{dz}^4 + D d\bar{z}^1 d\bar{z}^2 d\bar{z}^3 dz^4$$ (36)

where $z^i$ are the complex coordinates of $T^8$ and $z^4$ will be the direction along which we reduce to go to type IIB. The constants $A, B, C, D$ are related by the identity

$$AB + CD = \frac{\chi}{24}$$ (37)

where $\chi$ is the Euler characteristics of the 4-fold $T^8/G$.

Lifting to F-theory (or type IIB) the $H_{NSNS}$ and $H_{RR}$ fields are

$$H_{NSNS} = \frac{1}{2}(Adz^1 dz^2 \bar{dz}^3 + Bd\bar{z}^1 dz^2 \bar{dz}^3 + Cd\bar{z}^1 dz^2 \bar{dz}^3 + Dd\bar{z}^1 d\bar{z}^2 d\bar{z}^3)$$ (38)

$$H_{RR} = \frac{1}{2i}(-Adz^1 dz^2 \bar{dz}^3 + Bd\bar{z}^1 dz^2 \bar{dz}^3 + Cd\bar{z}^1 dz^2 \bar{dz}^3 - Dd\bar{z}^1 d\bar{z}^2 d\bar{z}^3)$$ (39)

Observe that

• All the field components have one of their legs along $z^3$. Here $z^3$ is the complex coordinate of $T^2/Z_2$. 

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• One can show that $H_{NSNS} \wedge *H_{RR} = 0$. From the equation of motion
\[ \nabla^2 \mathcal{L} = -\frac{3}{2} H_{NSNS} \wedge *(H_{RR} - \mathcal{L} H_{NSNS}) \] (40)
would imply that zero axion field, $\mathcal{L} = 0$ is a consistent background. We have also chosen the dilaton $\Phi = 0$.

• $\int H_{RR} \wedge H_{NSNS} = AB + CD = \frac{n}{24}$ which is the required for anomaly cancellation with $n = 0$.

From the above equations its also easy to determine what the $B$ fields would be locally:

\[ B_{NSNS} = \frac{1}{2}(A z^1 d\bar{z}^2 d\bar{z}^3 + B z^1 d\bar{z}^2 d\bar{z}^3 - C z^2 d\bar{z}^1 d\bar{z}^3 - D \bar{z}^2 d z^1 d \bar{z}^3) \] (41)
\[ B_{RR} = \frac{1}{2}(-A \bar{z}^1 d z^2 d z^3 + B z^1 d \bar{z}^2 d \bar{z}^3 - C \bar{z}^2 d z^1 d \bar{z}^3 + D \bar{z}^2 d z^1 d \bar{z}^3) \] (42)

Therefore this is how we get the required background in type IIB. One small subtlety is to see Lorentz invariance in full 3+1 dimensions. From the M-theory compactification it would seem like there is only 2+1 dimensional Lorentz invariance. However this is not the case as can be easily seen from the following arguments:

Compactifying from M-theory to IIA the metric for the system will have the following form:
\[ \begin{pmatrix} \Omega & 0 \\ 0 & \Omega' \eta_{\mu\nu} \end{pmatrix}, \] (43)
where $g_{\mu\nu}$ is the metric of a seven dimensional space and $\eta_{\mu\nu}$ is the 2+1 dimensional Minkowski spacetime. Recall that the M-theory metric is related to IIA metric via some scaling implies that
\[ \Omega = e^{\frac{3}{4} \phi} = \Omega^{-1} \] (44)
where $\phi$ is the dilaton. Now going from IIA to IIB undergoes an inversion of the T-dual direction which, from above equation, restores Lorentz invariance in 3+1 dimensions. On the remaining six dimensions we have the warp factor $\Omega$. The six dimensional manifold in this case is $T^6/Z_2 \times Z_2'$. However the manifold we actually need is a deformed conifold. The case with a trivial fibration over a conifold with fluxes can be worked out in some details as we saw in the previous section. Can that calculation be modified in some aspects so as to get the answer we require?
Example 2

We begin by introducing a circle action on the conifold and extend it to the resolved conifold and the deformed conifold in a compatible manner [10, 11].

Conifold: Consider an action $S_c$ on the conifold $xy - uv = 0$:

$$S_c : (e^{i\theta}, x) \rightarrow x, \quad (e^{i\theta}, y) \rightarrow y, \quad (e^{i\theta}, u) \rightarrow e^{i\theta} u, \quad (e^{i\theta}, v) \rightarrow e^{-i\theta} v, \quad (45)$$

The orbits of the action $S_c$ degenerates along the union of two intersecting complex lines $y = u = v = 0$ and $x = u = v = 0$ on the conifold. Now, if we take a T-dual along the direction of the orbits of the action, there will be NS branes along these degeneracy loci as argued in [38]. So we have two NS branes which are spaced along $x$ (i.e. $y = u = v = 0$) and $y$ directions (i.e. $x = u = v = 0$) together with non-compact direction along the Minkowski space which will be denoted by $NS_x$ and $NS_y$.

Resolved Conifold: This action can be lifted to the resolved conifold. To do that, we consider two copies of $C^3$ with coordinates $Z, X, Y$ (resp. $Z', X', Y'$) for the first (resp. second) $C^3$. Then $O(-1) + O(-1)$ over $P^1$ is obtained by gluing two copies of $C^3$ with the identification:

$$Z' = \frac{1}{Z}, \quad X' = XZ, \quad Y' = YZ. \quad (46)$$

The $Z$ (resp. $Z'$) is a coordinate of $P^1$ in the first (resp. second) $C^3$ and others are the coordinates of the fiber directions. The blown-down map from the resolved conifold $C^3 \cup C^3$ to the conifold $C$ is given by

$$x = X = X'Z', \quad y = ZY = Y', \quad u = ZX = X', \quad v = Y = Z'Y'. \quad (47)$$

From this map, one can see that the following action $S_r$ on the resolved conifold is an extension of the action $S_c$ [15]:

$$S_r : (e^{i\theta}, Z) \rightarrow e^{i\theta} Z, \quad (e^{i\theta}, X) \rightarrow X, \quad (e^{i\theta}, Y) \rightarrow e^{-i\theta} Y$$
$$\quad (e^{i\theta}, Z') \rightarrow e^{-i\theta} Z', \quad (e^{i\theta}, X') \rightarrow e^{i\theta} X', \quad (e^{i\theta}, Y') \rightarrow Y'. \quad (48)$$

The orbits degenerates along the union of two complex lines $Z = Y = 0$ in the first copy of $C^3$ and $Z' = Y' = 0$ in the second copy of $C^3$. Note that these two lines do not intersect and in fact they are separated by the size of $P^1$. Now we take T-dual along the orbits of $S_r$ of type IIB theory. Again there will be two NS branes along the degeneracy loci of the action: one NS brane, denoted by $NS_X$, spaced along $X$ direction (which is defined by $Z = Y = 0$ in the first $C^3$) and the other NS brane, denoted by $NS_{Y'}$ along $Y'$ direction (which is defined by $Z' = X' = 0$ in the second $C^3$). Here the length of the interval is the same as the size of the rigid $P^1$. As the rigid $P^1$ shrinks to zero, the size
of the interval goes to zero and $NS_X$ (resp. $NS_{Y'}$) approaches to $NS_x$ (resp. $NS_y$) of the conifold.

**Deformed Conifold:** Finally we will provide a circle action of the deformed conifold and a T dual picture under this action. Consider the following circle action $S_d$

$$S_d: (e^{i\theta}, x) \rightarrow x, \ (e^{i\theta}, y) \rightarrow y, \ (e^{i\theta}, u) \rightarrow e^{i\theta} u, \ (e^{i\theta}, v) \rightarrow e^{-i\theta} v,$$

on the deformed conifold

$$xy - uv = \mu$$

Then $S_d$ is clearly the extension of $S_c$ and the orbits of the action degenerate along a complex curve $u = v = 0$ on the deformed conifold. If we take a T-dual of the deformed conifold along the orbits of $S_k$, we obtain a NS brane along the curve $u = v = 0$ with non-compact direction in the Minkowski space which is given by

$$xy = \mu$$

in the x-y plane. Topologically, the curve (51) is $\mathbb{R}^1 \times S^1$.

From eq. (50) the metric is determined from Ricci flatness and Kahler potential $K$ as

$$ds^2 = K' tr(W^+dW) + K''|tr(W^+dW)|^2$$

with $W$ satisfying $det W = -\frac{1}{2}\mu^2$. The radial coordinate in $\mathbb{C}^4$ is

$$\rho^2 \equiv tr(W^+W) \equiv \mu^2 \cosh \tau$$

The explicit metric for the deformed conifold is given as:

$$ds^2 = K'\left(\frac{\sinh^3 \tau}{3(\sinh 2\tau - 2\tau)}(dr^2 + ds_1^2) + \frac{\cosh \tau}{4} ds_2^2 + \frac{1}{4} ds_3^2 \right)$$

where the quantities appearing in the above equation are defined in [39, 40, 42]

As discussed in eq. (51) the T-dual of a deformed conifold is given in terms of two intersecting NS5 branes with a “diamond” structure at the center by using the terminology of [41]. However a system of two intersecting NS5 branes are actually delocalized in terms of supergravity solutions. Therefore, in this limit, we expect the following form of the deformed conifold metric:

$$ds^2 = A(r)^2 dr^2 + B(r)^2 ds_1^2 + C(r) ds_2^2 + D(r) ds_3^2$$
As discussed in details in [42] the various coefficients are related as

\[ B(r) = A(r)^{-1} = \frac{1}{4} C(r)^{-1} = \frac{3}{4} D(r)^{-1} = f(r)^{-1} \tag{56} \]

where \(|\delta| < 1\) is an integral constant and \(f(r)\) is a linear function of \(r\).

From the above discussion we conclude that an intersecting NS5 brane configuration gives a conifold when we T-dualise along direction \(\theta \equiv \psi \equiv x^6\). For the deformed conifold we will T-dualise along direction \(S_d\). This will be consistent with the way we discussed the deformed conifold.

Two intersecting NS5 branes intersecting at a point in the presence of a non constant \(B_{NSNS}\) field along direction \(\psi, \theta_2\) shows a change in the metric for the \(\psi\psi\) and the \(\theta_2\theta_2\) components. If the value of the dimensionless \(B_{NSNS}\) field at the origin is given by \(b\) then

\[ g_{\psi\psi} = \frac{(1 + b^2)f^2(r)}{1 + b^2f^3(r)} = f(r)g_{\theta_2\theta_2} \tag{57} \]

where \(f(r)\) is a linear function of \(r\). The \(B\) field in this space is given by:

\[ B = \frac{bf^3(r)}{1 + b^2f^3(r)}(1 + b^2) \left( d\psi + \cos \theta_1 \ d\phi_1 + \cos \theta_2 \ d\phi_2 \right) \wedge d\theta_2 \tag{58} \]

This is basically the change in the background. To proceed further we have to deform the intersection point of the two NS5 system. An approximate form of this has been worked out in [42]. In the absence of any fluxes the intersecting NS5 brane metric has an additional term given by

\[ 2\beta f(r) \left[ A \sin \alpha + B \cos \alpha \right] \tag{59} \]

where \(\alpha\) is a constant, \(\beta\) the size of the deformation and

\[
\begin{align*}
A &= d\phi_1 \ d\theta_2 \ \sin \theta_1 + d\phi_2 \ d\theta_1 \ \sin \theta_2 \\
B &= d\theta_1 \ d\theta_2 - d\phi_1 \ d\phi_2 \ \sin \theta_1 \ \sin \theta_2 
\end{align*}
\tag{60}
\]

This is basically related to \(ds^2_3\) term appearing in eq. (54). For small values of \(r\) the presence of \(B\) field can be easily incorporated as a change in the \(\psi\psi\) and \(\theta_2\theta_2\) components of the new metric. To go to the deformed conifold we have to T-dualise along the compact direction. We now perform the following transformation on the coordinates \(\theta_1, \psi\) as

\[ \psi = \psi' \cos \gamma, \quad \theta_1 = \theta' \sec \gamma + \psi' \sin \gamma \tag{61} \]

Using the known procedure we lift this configuration to M-theory along direction \(x^{10}\) and come down back to type IIA via a different circle, say \(x^7\). This will create a three
form² in type IIA $C_{\psi^2,10}$ [13]. Now making a T-duality along $\psi'$ we essentially get a
deformed conifold with a

$$B_{\theta^2,10}^{RR} = \frac{3}{2} C_{\psi^2,10}, \quad B_{\theta^2,10}^{NSNS} = \frac{g_{\psi^2}}{g_{\psi^2}} \psi$$

(62)

This is thus the required background in the presence of fluxes.

We remark that the delocalized solution is valid only for the case of [3], which is very
close to the conifold and has a compact direction along which we can take a T-duality. For the case of [3], we cannot use similar arguments because the T-duality is more
complicated and we cannot take a T-duality in a clear space-time direction, although
some discussion in this sense has been made in [14].

4.3 M-Theory with G-Fluxes: Exact Results

Most of the discussions in the previous sections were motivated from the brane con-
structions and T-dualities. Though these techniques give us the back ground geometry,
the fact that we are doing T-dualities introduces delocalization in the picture. This is
a major handicap. Question is can we improve upon this to get the exact background
solution? The answer turns out to be yes and we use the technique of M-theory on
a fourfold with G-fluxes. The fourfold we take is a non-trivial $T^2$ fibration over the
deformed conifold $B$, whose construction will be given in details later. As we discussed
earlier, in this case we expect the metric of the fourfold to be warped.

The warped metric can be written again as:

$$ds^2 = e^{-\phi(g)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{\frac{1}{2}\phi(g)} g_{ab} dy^a dy^b$$

(63)

where $g$ is the metric for the unwarped internal fourfold which is parametrised by com-
plex coordinates $y^a, y^b$ (or real coordinates $y^m$) and $\eta$ is the three dimensional space parametrised by $\mu, \nu$. The eleven dimensional spinor $\kappa_0$ decomposes as

$$\kappa_0 = \epsilon \otimes \zeta$$

(64)

and the gamma matrices decompose as

$$\Gamma_{\mu} = \gamma_{\mu} \otimes \gamma_9, \quad \Gamma_m = 1 \otimes \gamma_m$$

(65)

where $\gamma_9$ is the eight dimensional chirality operator. Here $\epsilon$ is a three dimensional
anticommuting spinor and $\zeta$ is a commuting eight dimensional Majorana-Weyl spinor.

¹¹Recall that because of this background of four form field strength $G$ the susy transformation will pick up an extra contribution of $-\frac{1}{36} g_{\mu\nu} \Gamma_{\rho\sigma\lambda} G^{\rho\sigma\lambda} \eta$ in $\delta_\eta \psi_\mu$. 

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From the susy variation of the gravitino $\delta \psi_\mu = 0$ we expect to find a spinor satisfying
\[
\nabla_\mu \epsilon = 0 \tag{66}
\]
The existence of a covariantly constant spinor puts various constraints on the background.

1. **The Five-form equation**: Eq. \((66)\) imposes the following constraints on the background four form $G = dC$:
\[
e^{3\phi/2}(\gamma_\mu \otimes \gamma^m)e^{\mu\nu\rho}G^{\mu\nu\rho\kappa} - \frac{3}{2}\partial_\mu\phi(\gamma_\mu \otimes \gamma_9 \gamma^n)\kappa = 0
\Rightarrow G^{\mu\nu\rho\kappa} = \epsilon^{\mu\nu\rho\kappa}\partial_\kappa e^{-\frac{3\phi}{2}} \tag{67}
\]
Observe that this G-flux is generated entirely from the warp factor. When we shrink the size of the fibered torus to zero then this four form of M-theory goes to five form $F$ of F-theory (or IIB). The two three form fluxes are respectively the NSNS and RR fluxes which come from the background fluxes switched on as:
\[
\omega - *_B \omega, \quad \omega^\tau - *_B \omega^\bar{\tau} \tag{68}
\]
This can be related to the five form as
\[
F = \frac{3}{4} \epsilon^{ij} * (B^{(i)} \partial B^{(j)}) \tag{69}
\]
with $i, j = 1, 2$ being the two $B$ fields. The Hodge $*$ here is wrt the ten dimensional metric. The above equation can be argued from the self-duality condition of the modified five-form of type IIB theory. Observe that this is precisely one of the linear equation relating the derivative of the warp factor to the background $B$-fields as discussed in \([3]\).

2. **The Donaldson-Uhlenbeck-Yau equation**: Our discussion regarding eq. \((66)\) is however not complete. There is yet another condition \([19, 20, 21, 22]\) on the background four-form. This is the Donaldson-Uhlenbeck-Yau equation for the fourfold which puts a constraint on the G-flux as
\[
G_{a\bar{b}c\bar{d}} g^{\bar{c}d} = 0 \tag{70}
\]
where $g$ is the metric of the fourfold. If $z$ parametrizes the fiber torus and $B$ the deformed conifold base whose complex coordinates are $a, b$ then the above equation gives us the following set of equations:
\[
G_{a\bar{b}z\bar{d}} g^{zd} = 0, \quad G_{z\bar{a}b\bar{d}} g^{zd} = 0 \tag{71}
\]
which is nothing but the self duality relations of the G-fluxes. This further implies
\[
H_{NSNS} = *_B H_{RR} \tag{72}
\]
This equation relates the NSNS and RR three form field strengths linearly giving us the other equations of [9].

**F-theory and Orientifold Limits**

In the previous sections we avoided a subtlety regarding the global structure of the system in IIB and we now turn to clarify that issue. We will construct an elliptically fibered Calabi-Yau fourfold over a compactification of the deformed conifold. For the fourfold which is a non-trivial $T^2$ fiberation over a base $\mathcal{B}$ with the $T^2$ degenerating at some points on the base -- this situation is similar to the case discussed in [21]. Looking from F-theory point of view, as discussed earlier in some details, an F-theory compactification on an elliptically fibered Calabi-Yau fourfold is equivalent to type IIB string theory on the base of the fiberation, where the type IIB coupling is identified with the modular parameter of the elliptic curve.

We begin with the deformed conifold defined in $\mathbb{C}^3$ by

$$x_0^2 + x_1^2 + x_2^2 + x_3^2 = \mu. \quad (73)$$

For non-zero $\mu$, this is smooth and symplectically isomorphic to the cotangent bundle $T^*S^3$ over the three sphere $S^3$. As $\mu \to 0$, the compact cycle $S^3$ will vanish and the conifold singularity will develop. By adding $\mathbb{P}^1 \times \mathbb{P}^1$ to the boundary of (73), we can compactify to a projective variety $\mathcal{B}_\mu$ in $\mathbb{P}^4$ defined by a quadratic equation:

$$\mathcal{B}_\mu : \quad z_0^2 + z_1^2 + z_2^2 + z_3^2 - \mu z_4^2 = 0. \quad (74)$$

The quadric threefold $\mathcal{B}_\mu$ does not develop any new singularities at infinity and thus smooth for $\mu \neq 0$ and has a conifold singularity at $(0, 0, 0, 0, 1) \in \mathbb{P}^4$ when $\mu = 0$. Moreover, the anti-canonical bundle

$$- \mathcal{K}_{\mathcal{B}_\mu} := \wedge^3 T\mathcal{B}_\mu \quad (75)$$

will be the restriction of $\mathcal{O}(3)$ of $\mathbb{P}^4$ to $\mathcal{B}_\mu$ by the adjunction formula, and hence it is very ample. From Kodaira vanishing theorem, one can also show that $H^i(\mathcal{B}_\mu, \mathcal{O}) = 0$ for $i > 0$. We now define a fourfold $Y_\mu$ as a subvariety in the projective bundle $\mathbb{P}(\mathcal{O} \oplus \mathcal{L}^2 \oplus \mathcal{L}^3)$ where $\mathcal{L} := \mathcal{K}_{\mathcal{B}_\mu}^{-1}$, given by the Weierstrass equation

$$y^2z = x^3 + f z^2 x + g z^3, \quad (76)$$

where $z, x, y, f, g$ are the sections of $\mathcal{O}, \mathcal{L}^2, \mathcal{L}^3, \mathcal{L}^4, \mathcal{L}^6$ respectively. Since the anti-canonical bundle $\mathcal{L} = \mathcal{O}(3)|_{\mathcal{B}_\mu}$ is very ample, we may choose $f$ and $g$ so that the fourfold $Y_\mu$ is smooth. By the projection formula, one can see that $Y_\mu$ is Calabi-Yau since $H^i(\mathcal{B}_\mu, \mathcal{O}) = 0, i > 0$. By construction, the natural projection

$$\mathbb{P}(\mathcal{O} \oplus \mathcal{L}^2 \oplus \mathcal{L}^3) \to \mathcal{B}_\mu \quad (77)$$
induces a fibration $Y_\mu \rightarrow B_\mu$ whose fibers are elliptic curves. F-theory on the Calabi-Yau fourfold $Y_\mu$ is by definition type IIB theory compactified on the base $B_\mu$ with background axion-dilaton field $\lambda$ whose $j$-invariant is given by:

$$j(\lambda) = \frac{4 \cdot (24f)^3}{4f^3 + 27g^2}. \quad (78)$$

and various $(p, q)$ seven branes appearing at the loci where the elliptic fibration degenerates, i.e. where the discriminant

$$\Delta = 4f^3 + 27g^2 \quad (79)$$

vanishes. Using Riemann-Roch for $B_\mu$ and integrating over the elliptic fibers, one can evaluate the Euler-Characteristic $\chi$ of $Y_\mu$

$$-Q_3^{D7} = \frac{\chi}{24} = 12 + 15 \int_{B_\mu} c_1(B_\mu)^3 = 822. \quad (80)$$

as in [18]. From the above consideration we expect that the warped F-theory compactification on $Y_\mu$ can be related to M-theory on the Calabi-Yau fourfold with G-fluxes as discussed in details in [19, 21, 23, 26]. In IIB we therefore get a set of $D7$ branes and $O7$ planes along with a deformed conifold background and fluxes. However the issue of fluxes is more subtle now because of the presence of branes and orientifold planes. There are two interesting cases we have to consider from M-theory point of view:

1. The G-fluxes are localised at points where the $T^2$ fiber degenerates.

2. The G-fluxes are spread over the fourfolds but with a normalisable $\int G \wedge G$.

From the first case we will have a decomposition

$$G = \sum_{i=1}^{k} F_i \wedge [\Omega_i] \quad (81)$$

where we take the fiber degenerating at $k$ points and $[\Omega_i]$ are normalisable two forms localized at the singularities. Since the branes are located at those points, we see that the background G-fluxes have actually appeared as gauge fluxes on the branes[12]. The warp factor equation will also be different now since both branes and planes are sources of $tr(R \wedge R)$[14]. The warp factor is given as:

$$\Delta e^{3\phi/2} = *_G \sum_{i=1}^{k} [F_i \wedge F_i + tr(R \wedge R)] \delta^2(l - l_i) \quad (82)$$

[12]Recall that the orientifold planes are $(p, q)$ 7 branes.
where \( l_i \) are the positions of the branes and planes.

For our purpose however we require the second condition wherein we decompose\( ^3 \) the background G-flux as

\[
\frac{G}{2\pi} = dz \wedge \omega - d\tilde{z} \wedge *_B \omega
\]  

(83)

In the limit when the size of the three cycle of the base goes to zero this fourfold will be related to the one discussed in [26]. Therefore following the discussions of [15], [26], near the conifold point \( \mu \to 0 \), we can study the IIB background as though we are removed far away from the \( O7 \) planes and D7 branes. In this limit the calculations of the previous sections will give us exact results in the local neighbourhood of the singularity [26].

4.4 The Transition From G-fluxes

As discussed in detail in section (3.2) the type IIB metric is given as

\[
\begin{pmatrix}
\Omega & g_{\alpha\beta} & 0 \\
0 & 0 & \Omega' & \eta_{\mu\nu} \\
\end{pmatrix},
\]  

(84)

where \( \mu, \nu \) runs over the 3+1 dimensional spacetime and \( g_{\alpha\beta} \) is the metric of the deformed conifold. \( \Omega \) is the warp factor and we can use it to write the ten dimensional IIB metric explicitly as

\[
ds^2 = e^{-\frac{3\phi}{4}} dx_\mu dx^\mu + e^{\frac{3\phi}{4}} g_{\alpha\beta} dx^\alpha dx^\beta
\]

\[
= H^{-\frac{1}{2}} dx_\mu dx^\mu + H^\frac{1}{2} g_{\alpha\beta} dx^\alpha dx^\beta
\]  

(85)

where we have used eq. (44):

\[
\Omega = e^{\frac{3\phi}{4}} = \Omega'^{-1} \equiv H^{\frac{1}{2}}
\]  

(86)

As discussed in [1] the above form of the metric is in the same category as a D-brane metric. We thus see that eq. (85) can be derived from M-theory. From the above consideration is it possible to see the geometric transition to the wrapped brane picture?

For this first let us consider a small resolution of the compactified conifold \( \tilde{B}_0 \to B_0 \). We can pull back the elliptic fibration \( Y_0 \to B_0 \) to \( \tilde{B}_0 \) which will be denoted by \( \tilde{Y}_0 \). Hence the conifold transition can be lifted to the Calabi-Yau fourfold transition:

\( ^3 \)The two \( B \)-fields survive the orientifold projection.

23
In the fourfold transition, the real five dimensional cycle in $Y_\mu$ will shrink to zero as $\mu$ approaches to zero and the four dimensional cycle will blow up in $\tilde{Y}_0$. How does the background warped metric transform under this?

**Conifold transition of the base:** The base — deformed conifold — metric has a form given in eq. (54) as

$$ds^2 = A(\tau)(d\tau^2 + ds_1^2) + B(\tau)ds_2^2 + C(\tau)ds_3^2$$

where the quantities $A, B, C$ are defined earlier. For this case there are two different limits:

1. $\tau \to 0$, $\mu = \text{fixed}$
2. $\tau \to \infty$, $\mu \to 0$, $\mu \cosh \tau = \text{fixed}$

The first case tells us that the deformed conifold reduces to a $S^3$ and in the second case the metric reduces to the conifold with the radius parameter given by $(\mu \cosh \tau)^{\frac{3}{5}}$. In this limit the equation of the warp factor

$$\nabla_\mu F_{\mu_1\mu_2\mu_3\mu_4} = \frac{3}{5!4} \epsilon^{i_1 i_2 i_3 i_4} H^{(i)}_{\mu_5\mu_6\mu_7} H^{(j)}_{\mu_8\mu_9\mu_10}$$

and eq. (85) will give us the UV metric of [7]. Far in the IR (for the first case) when we make $S^3 \to 0$ there would be source for the $H_{RR}$ signifying the presence of a wrapped D5. A way to argue this is from the background equation of motion for the RR fields:

$$\nabla^\nu H'_{\mu\nu\rho} = (\ast F)_{\nu\rho\sigma\lambda} H'^{\sigma\lambda} \big|_{\text{background}}$$

where we have assumed a constant axion. An alternative way to see that there is a wrapped brane is to go to the T-dual picture of the deformed conifold. At the intersection, in the limit of the vanishing size of the diamond, there is a strong flux which creates a brane by a mechanism similar to the Hanany-Witten effect. We are again assuming that the branes and planes don’t alter the results in any substantial way.

14 We are again assuming that the branes and planes don’t alter the results in any substantial way.

15 We are taking a limit in which $B_{NSNS} \to \epsilon$ and $B_{RR} \to \epsilon^{-\beta}$. Therefore from the above equation we have $\nabla H' \to \epsilon^{1-2\beta}$. The fact that $\beta > 1$ can be argued from the finiteness of the $H'$ flux. Observe that in the far IR $B_{NSNS} \to 0$ so there will be no contribution to the four form charge $\int H_{NSNS} \wedge H_{RR}$. 24
The above analysis is therefore a way to see the transition for the Klebanov-Strassler case. Thus at the conifold point we have a wrapped D5 and a $B_{NSNS}$ field. This $B_{NSNS}$ field will appear in the $3 + 1$ dimensional gauge theory as a coupling constant. Vafa’s case would be to trade the varying $B_{NSNS}$ field with the size $\mathcal{Z}$ of a blown up $P^1$. For our purpose we can define a coupling constant

$$\tilde{\tau} = \int B_{NSNS} + i\mathcal{Z}$$

(92)

whose $Re\,\tilde{\tau}$ and $Im\,\tilde{\tau}$ determine essentially the two model. In fact this is related to the brane construction we had discussed in the earlier sections. For the Klebanov-Strassler case we had two intersecting NS5 brane separated along $x^6$ and a D4 brane stretched between them. For Vafa’s case the system is separated along $x^7$. The most general model would be to stretch the two NS5 on a complex plane $z \equiv x^6 + ix^7$.

We discussed in this section the solution of [9] obtained from F-theory compactified on a non-trivial fiberation over the deformed conifold, where the warp factor is non-constant. We could ask now what happens if we consider Vafa’s model [3]. From the above discussions we see that as long as we take a non-constant size of the $P^1$ in the resolved side we in fact generate an identical back ground after the conifold transition. Therefore we would expect the warp factor to be same as the Klebanov-Strassler case. However if the size of the $P^1$ is a constant then $\mathcal{F} = 0$ resulting in a constant warp factor. In general the transition is to a non-constant warp factor determined mainly by the NSNS field.

5 The Nonsupersymmetric Background

Existence of a supersymmetric background is equivalent to saying that we have a primitive $(2, 2)$ form $f_{(2,2)}$[19, 21]. The generalised primitivity condition is defined on a $(p,q)$ form as

$$J \wedge f_{(p,q)} = 0$$

(93)

When supersymmetry is completely broken then we can have $(2,2)$ form which is not primitive and is given in terms of a closed $(0,0)$ form $f_{(0,0)}$ and the kahler form $J$ as $J \wedge J f_{(0,0)}$[16]. Of course this form is in addition to the primitive $(2,2)$ form and the $(0,4)$ and $(4,0)$ form. The complete four form background that can be switched on for the non supersymmetric case is

$$G = f_{(4,0)} + f_{(0,4)} + f_{(2,2)} + J \wedge J f_{(0,0)}$$

(94)

It was shown in [16] that in these models the three dimensional cosmological constant vanishes.
The non primitive $(2, 2)$ form also receives contributions from $(1, 1)$ form as $J \wedge f_{(1,1)}[21, 22, 17]$. Using the tadpole cancellation condition

$$\int_Y G \wedge G = \frac{X}{24}$$

it was shown in [20, 22, 17] that we generate a potential in three dimensions as

$$V = \int_Y |G|^2 - \frac{X}{6}$$

where a $(3, 1)$ and $(1, 3)$ background is also switched on a Calabi-Yau fourfold $Y$ which has $T^2$ fiberation over a Fano threefold $B$.

The above discussions therefore indicates that we can extend our results to the non susy case also. It remains however to see what happens after the transition. As pointed out to us by S. Kachru [48] this susy breaking is more of a global effect and therefore it mayn’t be possible to argue susy breaking in the dual field theory. However the examples considered above are for compact fourfolds. For non-compact cases the situation will be different. More details on this will be reported elsewhere.

6 Discussions

6.1 Is there a Type I dual also?

In this paper we have given a M-theory compactification which, in some limits, reproduce the Klebanov-Strassler or Vafa’s model. However there are many interesting directions still remain to be explored. Let us go back again to our construction of the fourfold. The deformed conifold has been compactified to a smooth projective threefold in $\mathbb{P}^4$:

$$\mathcal{B}_\mu : \ z_0^2 + z_1^2 + z_2^2 + z_3^2 - \mu z_4^2 = 0.$$  \hspace{1cm} (97)

Now consider a family $\mathcal{F}$ of $\mathbb{P}^2$’s in $\mathbb{P}^4$ which contain a fixed generic projective line $l_0$ at the infinity (i.e. $z_4 = 0$). So here we assume that the line $l_0$ will intersect $\mathcal{B}_\mu$ at two distinct points. The family $\mathcal{F}$ is two dimensional and parametrized by $\mathbb{P}^2$. So after blowing up the line $l_0$ in $\mathbb{P}^4$, we obtain a $\mathbb{P}^2$ fibered space $\tilde{\mathcal{P}}^4$ over the parameter space $\mathbb{P}^2$. Let $\tilde{\mathcal{B}}_\mu$ be the proper transform of $\mathcal{B}_\mu$ under this blow-up. Then we have a family of quadric curves over $\mathbb{P}^2$:

$$\pi : \tilde{\mathcal{B}}_\mu \rightarrow \mathbb{P}^2.$$ \hspace{1cm} (98)

The quadric curves are obtained by intersecting $\mathcal{B}_\mu$ with $\mathbb{P}^2$ and isomorphic to $\mathbb{P}^1$. Now we choose a smooth generic quadric threefold $Q$ in $\mathbb{P}^4$ which will intersect with $\mathcal{B}_\mu$.
smoothly. As a special case of F-theory constructed in section 4.3, we may assume that the discriminant \( \Delta \) is of the form
\[
\Delta = 4f^3 + 27g^2 = h^{18}
\]
with \( h \in \mathcal{O}(2) \). Let \( Q \) be the quadric threefold in \( \mathbb{P}^4 \) defined by \( h \). We take a double covering \( W \) over \( \mathbb{P}^4 \) which is ramified over the smooth divisor \( Q \). Then the fiber product \( \mathbb{P}^4 \times_{\mathbb{P}^4} W \) will be a double cover over \( \mathbb{P}^4 \) ramified over the proper transform \( \tilde{Q} \) of \( Q \). We restrict the double covering
\[
f : \tilde{\mathbb{P}}^4 \times_{\mathbb{P}^4} W \to \tilde{\mathbb{P}}^4
\]
to the inverse image \( E \) of \( \tilde{B}_\mu \) under \( f \). Therefore \( E \) will be a double cover of \( \tilde{B}_\mu \) ramified over \( \tilde{Q} \cap \tilde{B}_\mu \). Hence we have the following situation:

\[
\begin{array}{ccc}
E & \xrightarrow{f|_E} & \tilde{B}_\mu \\
\downarrow \pi & & \downarrow \\
\mathbb{P}^2 & & \\
\end{array}
\]

The general fibers of the composite map \( f|_E \circ \pi \) will be elliptic curves since they are double covering of \( \mathbb{P}^1 \) ramified over 4 points. Thus the map
\[
f|_E \circ \pi : E \to \mathbb{P}^2
\]
will be a \( T^2 \) fibration and there is an involution \( \mathcal{I}_2 \) such that \( E/\mathcal{I}_2 = \tilde{B}_\mu \). Here the 72 branes are grouped into four sets of 18 coincident branes situated on the ramification divisor \( \tilde{Q} \cap \tilde{B}_\mu \) and are located at the fixed points of \( \mathcal{I}_2 \). So as in [49], we have type IIB compactification on \( \tilde{B}_\mu \) such that we go once around each fixed point of \( \mathcal{I}_2 \) the theory comes back to itself transformed by the symmetry \( (-1)^{F_L} \cdot \Omega \). Here \( (-1)^{F_L} \) changes the sign of all the Ramond sector states on the left moving fermions and \( \Omega \) denotes the orientation reversal transformation. Therefore the theory can be identified to type IIB on \( E \), moded out by the \( \mathbb{Z}_2 \) transformation
\[
(-1)^{F_L} \cdot \Omega \cdot \mathcal{I}_2.
\]

By making T-dualities along both the circles of the \( T^2 \), we can map the \( \mathbb{Z}_2 \) transformation \( (-1)^{F_L} \cdot \Omega \cdot \mathcal{I}_2 \) to \( \Omega \). Since modding out the type IIB theory by \( \Omega \) produces type I theory, we obtain type I theory on \( E \) [49]. By pulling back \( H \) fields constructed before on \( \tilde{B}_\mu \) to \( E \), we expect the \( H_{NSNS} \) fields to dissolve in the metric in type I background and the metric will become non-Kähler i.e. the metric will have torsion\(^{\text{16}}\). The RR field will

\(^{\text{16}}\)Those which don’t are actually projected out by the orientifold projection.

\(^{\text{17}}\)A similar case has been noticed earlier in [2].
appear as a three form field in type I. Notice that we have changed a compactification of the deformed conifold from $\mathcal{B}_\mu$ to $\tilde{\mathcal{B}}_\mu$ which is still Fano so that the discussions in the previous sections go through. As discussed in [51, 21] the three forms $H$ are related to the Type I metric as

$$H = \frac{i}{2}(\bar{\partial} - \partial) J$$

(103)

Here $J$ is the $(1, 1)$ form associated with the metric which becomes non-Kähler due to $H_{NSNS}$ after T-dualities. In terms of components the above equation can be recast as

$$H_{abc} = -g_{c[a,b]} , \quad H_{ab\bar{c}} = -g_{\bar{c}[a,b]}$$

(104)

where [ ] denotes antisymmetrisation. The equation (104) can be shown to reproduce the linear equation written earlier as eq. (92) when we suppress the $X_8$ and the membrane terms. In order to see this, one only needs to consider the components of $H_{NSNS}$ and $H_{RR}$ fields which have one leg along the $T^2$ direction as the other components are projected out. Then we can show that the self-duality of G-flux (72) is equivalent to (103). This is not surprising considering the fact that both conditions are derived in order to have supersymmetry. The warp factor for $E$ will descend to $\mathcal{B}_\mu$ because $H$ fields have been lifted from $\mathcal{B}_\mu$. Therefore this serves as another alternative way to see the background equation of motions. However the above method takes into account the regions close to the D7-O7 systems. But for the Klebanov-Strassler case we have restricted ourselves to the regions far away from the D7-O7 system. It would therefore be interesting to see the range of validity of the above technique.

### 6.2 Fate of the open strings

Another interesting issue here is the fate of the open strings. In the final picture we have a complete closed string background without any open strings. In the dual brane picture the open strings on the D4 branes become 2-branes when the system is taken to strong coupling. They then combine with the two branes which determine the dynamics of the MQCD five brane. This system eventually becomes two intersecting M5 with a diamond structure which is a closed string background in IIB.

An interesting question is to see whether we can argue directly the existence of open strings in the background described by eq. (111). To do this we have to determine the zero mode fluctuation of the background three forms. Let us decompose the three form as

$$H_{RR} = H_{RR} \big|_{\text{background}} + h$$

(105)
where \( h \) is the fluctuation. As we argued earlier in eq. (11) the limit when \( S^3 \to 0 \) we have a source of a D5 brane. How does the fluctuation manifest itself now?

To see this we need to recall the conifold equation written suggestively as

\[
x_1^2 + x_2^2 + x_3^2 = -x_4^2
\]

(106)

In this form it describes a \( \mathbb{Z}_2 \) ALE space fibered over a \( x_4 \) plane. We can use this information now to our advantage. Recall that a \( \mathbb{Z}_2 \) ALE space supports a normalisable harmonic two form \( l_2 \). Therefore the small fluctuations of the background three form field can be decomposed as

\[
h = A \otimes l_2
\]

(107)

where \( A \) is a one-form restricted on the plane \( x_4 \) and spacetime. Therefore this fluctuations appear as \( U(1) \) gauge fields on the D5 world-volume. This may be one way to understand the transition from closed string backgrounds to open string backgrounds.

Before we end, observe that the closed string background is a conjectured dual \( 3 \) of the \( \mathcal{N} = 1 \) pure glue theory in the following limits:

\[
g_s \to \epsilon, \quad \int_{S^2 \to 0} B_{NSNS} \to \epsilon
\]

(108)

Observe that the above limit is opposite of the limit discussed in eq. (35) for the validity of the sugra backgrounds. This implies that we really don’t have a rigorous proof of the dual of the pure glue theory from supergravity point of view. However as we saw earlier the brane construction method are at finite \( g_s M \). And therefore they provide a strong argument in the favor of this duality.

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