PHASE TRANSITION SIGNAL IN PULSAR TIMING

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August 3, 2018

Invited Paper, Hirschegg '98, Nuclear Astrophysics
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†This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.
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Abstract

A phase transition in the nature of matter in the core of a neutron star, such as quark deconfinement or Bose condensation, can cause the spontaneous spin-up of a solitary millisecond pulsar. The spin-up epoch for our model lasts for $2 \times 10^7$ years or $1/50$ of the spin-down time (Glendenning, Pei and Weber in Ref. [1]). The possibility exists also for future measurements on X-ray neutron stars with low-mass companions for mapping out the tell-tale “backbending” behavior of the moment of inertia. Properties of phase transitions in substances such as neutron star matter, which have more than one conserved charge, are reviewed.

1 Introduction

Neutron stars have a high enough interior density as to make phase transitions in the nature of nuclear matter a distinct possibility. Examples are hyperonization, negative Bose condensation (like $\pi^-$ and $K^-$) and quark deconfinement. According to the QCD property of asymptotic freedom, the most plausible is the quark deconfinement transition. From lattice QCD simulations, this phase transition is expected to occur in very hot ($T \sim 200$ MeV) or cold but dense matter. In this work we will use the deconfinement transition as an example, but in principle, any transition that is accompanied by a sufficient softening of the equation of state and occurs at or near the limiting mass star, can produce a similar signal.

The paper is organized as follows. We discuss first the physical reason why a rapidly rotating pulsar, as it slows down over millions of years because of angular momentum loss through the weak electromagnetic process of magnetic dipole radiation, will change in density due to weakening centrifugal forces and possibly encounter, first at its center, and then in a slowly expanding region, the conditions for a phase transition. Conversely, an accreting star will be spun up from low to high frequency by accretion from a low-mass companion. This too will have a very long time-scale because accretion is regulated by the
radiation pressure of the star’s surface, heated by infalling matter.

After having discussed the reasons why we might see signals of phase changes, both in rapidly rotating stars that are spinning down because of angular momentum loss to radiation and stars that are spinning up due to the input of angular momentum by accretion, we discuss some aspects of phase transitions that are common to all first order transitions in neutron star matter, or more generally in isospin asymmetric matter.

2 Effects of Phase Transitions on Rotating Stars

2.1 Evolutionary Path of Neutron Stars

Since neutron stars are born with almost the highest density that they will have in their lifetime, being very little deformed by centrifugal forces, they will possess cores of the high density phase essentially from birth if the critical density falls in the range of neutron stars. However the global properties, such as mass or size, of a slowly rotating neutron star are little effected by whether or not it has a more compressible phase in the core. In principle, cooling rates should depend on interior composition, but cooling calculations are beset by many uncertainties and competing assumptions about composition can yield similar cooling rates depending on other assumptions about superconductivity and the cooling processes. Moreover, for those stars for which a rate has been measured, not a single mass is known. It is unlikely that these measurements will yield conclusive evidence in the present state of uncertainty [2, 3].

Nevertheless, it may be possible to observe the phase transition in millisecond pulsars by the easiest of measurements—the sign of $\dot{\Omega}$. Normally the sign should be negative corresponding to loss of angular momentum by radiation. However a phase transition that occurs near or at the limiting mass star, can cause spin-up during a substantial era compared to the spin-down time of millisecond pulsars. The transition may be of either first or second order provided that it is to an appreciable more compressible phase. We sketch the conventional evolutionary history [4] of pulsars with the addition of the supposition that the critical density for quark deconfinement falls in the density range spanned by neutron stars.

As already remarked, with the supposition above, the star has a quark core from birth but its properties are so little effected that this fact cannot be discerned in members of the canonical pulsar population. It is born with moderate rotation period, acquired by the conservation of angular momentum during core collapse and with high magnetic field by flux conservation. In a diagram of magnetic field strength $B$ and rotation period $P$ (Fig. 1), it is
Figure 1: The evolutionary track of pulsars are from high magnetic field and moderate rotation period to long period in about $10^7$ to $10^8$ years, to accreting X-ray neutron stars, to millisecond pulsars with low magnetic fields.

injected near the line marked $10^3$ years (since birth) and with $B \sim 10^{13}$ gauss. It evolves quickly at constant $B$ toward longer period for $\sim 10^6$ to $10^8$ years. There it lingers with the bulk of the pulsar population at long periods (because $\dot{P} \sim 1/P$) before the combination of field strength and rotation period are insufficient to accelerate charged particles that produce the radiation. At that time the star has entered the radio silent epoch. Some pulsars will have had a less dense companion or will acquire one from which they accrete matter and angular momentum. Some will be seen as X-ray emitters during the radio silent phase. During spin-up to frequencies much higher than those with which pulsars are born, the neutron star becomes increasingly centrifugally deformed and its interior density falls. Consequently, the radius at which the critical phase transition density occurs moves toward the center of the star—quarks that were deconfined at the birth of the star, recombine to form hadrons. (This era may also be detectable as discussed in section 2.4.) When accretion ceases, and if the neutron star has been spun up to a state in which the combination of reduced field strength (perhaps to ohmic decay) and increased frequency turn the dipole radiation on again, the pulsar recommences spin-down as a radio
visible millisecond pulsar.

During spin-down as a millisecond pulsar, the central density increases with decreasing centrifugal force. First at the center of the star, and then in an expanding region, the highly compressible quark matter will replace the less compressible nuclear matter. The quark core, weighed down by the overlying layers of nuclear matter is compressed to high density, and the increased central concentration of mass acts on the overlying nuclear matter, compressing it further (see Figs. 2 and 3). The resulting decrease in the moment of inertia causes the star to spin up to conserve angular momentum not carried off by radiation. The phenomenon is analogous to that of “backbending” predicted for rotating nuclei by Mottelson and Valatin [5] and discovered in the 1970’s [6, 7] (see Fig. 4). In nuclei, it was established that the change in phase is from a particle spin-aligned state at high nuclear angular momentum to a superfluid state at low angular momentum. The phenomenon is also analogous to an ice skater who commences a spin with arms outstretched. Initially spin decreases because of friction and air resistance, but a period of spin-up is achieved by pulling the arms in. Friction then reestablishes spin-down. In all three examples, spin up is a consequence of a decrease in moment of inertia beyond what would occur in response to decreasing angular velocity.

2.2 Calculation

In our calculation, nuclear matter was described in a relativistically covariant theory [8, 9, 10] and quark matter in the MIT bag model [11]. The phase transition occurs in a substance of two conserved quantities, electric charge and baryon number, and must be found in the way described in Ref. [12] and in Section 3. The moment of inertia must incorporate all effects described above—changes in composition of matter, centrifugal stretching—and frame dragging, all within the framework of General Relativity. The expression derived by Hartle is inadequate because it neglects these effects [13, 14]. Rather we must use the expression derived by us [15, 16].

For fixed baryon number we solve General Relativity for a star rotating at a sequence of angular velocities corresponding to an equation of state that describes the deconfinement phase transition from charge neutral nuclear matter to quark matter. The equation of state is shown in Fig. 5. The moment of inertia as a function of angular velocity does not decrease monotonically as it would for a gravitating fluid of constant composition. Rather, as described above, the epoch over which an enlarging central region of the star enters the more compressible phase is marked by spin-up (Fig. 6). Does the spin-up
Figure 2: Energy density profiles star at three rotation rates. Notice the 5 km radius quark matter core when the star is not rotating (or only slowly as a canonical pulsar). Mixed phase extends to 8 km.

Figure 3: Radial boundaries between different phases for stars of mass indicated on the y axis. Composition consists of quarks, the baryon octet and leptons. The geometric phases will be discussed in Section 3.

epoch endure long enough to provide a reasonable chance of observing it some members of the pulsar population? This question we turn to now.

2.3 Spin-up Era

To estimate the duration of the spin-up, we solve the deceleration equation for the star with moment of inertia having the behavior shown in Fig. 6. From the energy loss equation

$$\frac{dE}{dt} = \frac{d}{dt} \left( \frac{1}{2} I \Omega^2 \right) = -C \Omega^4$$

(1)

for magnetic dipole radiation we find

$$\dot{\Omega} = -\frac{C}{I(\Omega)} \left[ 1 + \frac{I'(\Omega) \Omega}{2I(\Omega)} \right]^{-1} \Omega^3.$$  

(2)
This expression reduces to the usual braking equation when the moment of inertia is held fixed. The braking index is a dimensionless combination of three quantities that are observable in principle, namely the angular velocity and its first two time derivatives. It is generally thought to have a constant value, namely 3, for magnetic dipole radiation. However constancy would follow only if the star rotated rigidly. Instead it varies with angular velocity and therefore time according to

\[ n(\Omega) \equiv \frac{\Omega \ddot{\Omega}}{\ddot{\Omega}^2} = 3 - \frac{3\dot{I}' + I''\Omega^2}{2I + I'\Omega} \]  \hspace{1cm} (3)

where \( I' \equiv dI/d\Omega \) and \( I'' \equiv d^2I/d\Omega^2 \). This holds in general even for a star whose internal composition does not change with angular velocity (inconceivable). In particular one can see that for very high frequency, the derivatives will be largest and the braking index for any millisecond pulsar near the Kepler frequency will be less than the dipole value of \( n = 3 \).

The braking index is shown as a function of time in Fig. 4. An anomalous value endures for \( 10^8 \) years corresponding to the slow spin down of the pulsar.
Figure 6: Moment of inertia of corresponding to a change of phase. Time flows from large to small $I$.

Figure 7: The time evolution of the braking index plotted over two decades that include the epoch of the phase transition.

and the corresponding slow envelopment of a growing central region by the new phase. The two points that go to infinity correspond to the infinite derivatives of $I$ at which according to (4), the deceleration vanishes. They mark the boundaries of the spin-up era. The actual spin-up lasts for $2 \times 10^7$ years or $1/50$ of the spin-down time for this pulsar. This could be easily observed in a solitary pulsar and would likely signal a phase transition.

2.4 X-Ray Pulsars in Low Mass Binaries

As Lamb has discussed in this volume, neutron stars that are accreting mass that is channeled to their surface from a low-mass companion have been observed to rotate in the millisecond range. One of the goals in those studies is to observe phenomena associated with the last stable orbit. Mass, radius, pulsar frequency and the frame dragging frequency can be determined in principle, and it is the goal of the experiments to do so. From the combination of the frame dragging frequency and radius, the moment of inertia can be obtained
\[ I = \frac{1}{2} \frac{\omega R^3}{\Omega} \]  \text{(gravitational units)}  \tag{4}

where the frame dragging angular velocity \( \omega(R) \) corresponds to \( R \), and \( \Omega \) is the rotational angular velocity of the pulsar. A few such sets of data corresponding to stars of the same mass would represent points on a plot of moment of inertia vs. angular velocity, as in Fig. 6. If indeed quark matter cores exist in slower pulsars, then the possibility exists that some of the X-ray emitters will lie on the upper branch and others on the lower branch. Even if one did not have observations that lay on the backbend, observations that established the existence of two asymptotes would be quite convincing evidence of a different phase on the two branches.

3 Properties of Phase Transitions

We discuss briefly phase transitions of interest in nuclear and nuclear astrophysics. These include pion condensation [17], hyperonization [18], kaon condensation [19], quark deconfinement in stars [20], and recently, H-dibaryon Bose condensation [21].

3.1 Maxwell Construction

Depending on the model or strength of coupling, the above transitions may be first or second order. Until recently [22] first order phase transitions in nuclear matter or neutron star matter (nuclear matter in equilibrium under the constraint of charge neutrality), where implemented with some variant of the Maxwell construction (eg. tangent slope \( \mu = \frac{d\epsilon}{d\rho} \)). Obviously the Maxwell construction can make only one chemical potential common to phases in equilibrium, and is a valid construction for simple substances with one independent component, such as water. However, nuclear systems have two or more conserved charges (baryon and electric charge in the case of neutron star matter or low density nuclear matter, and strangeness in addition on the time-scale of high-energy reactions). For such substances, a little reflection reveals that the chemical potential that is rendered common in the Maxwell construction is neither the baryon nor charge chemical potential but a varying combination according to the varying composition of matter as a function of baryon density.

The Maxwell construction causes the pressure to be constant in the mixed phase. (This is obvious from the tangent construction in which the energy is
a linear function of volume, \( E = -pV + \mu N \) (with \( p \) and \( \mu \) constants of the construction) and hence the pressure, \(-dE/dV\), is constant.) The consequence for stellar structure is striking: the mixed phase is absent in the monotonic pressure environment of a star and there is a large density discontinuity at the radial point corresponding to the constant pressure of the mixed phase. Inside this radial point the dense quark matter resides; outside is the less dense nuclear matter.

In some work, phase transitions were implemented in a different but equivalent fashion. Charge neutrality was enforced by requiring the charge density to vanish identically. This is valid in either pure uniform phase. However when applied to the mixed phase it is too stringent a way of enforcing charge neutrality. All that is required by the balance of Coulomb and gravitational forces is that the star be charge neutral to a high degree \((Z_{\text{net}}/A \leq (m/e)^2 \approx 10^{-36})\); not that the charge density vanishes identically. Global neutrality \(\int q(r) dV = 0\) is all that is required. Indeed, when the details are examined, it will be seen that local neutrality is incompatible with Gibbs criteria for phase equilibrium. Gibbs conditions and the conservation laws can be satisfied simultaneously only when the conservation laws are imposed in a global sense.

3.2 Gibbs Equilibrium

We briefly review how to find the conditions for phase equilibrium in substances of more than one conserved charge that are in accord with Gibbs criteria for chemical, mechanical and thermal equilibrium [22]. For definiteness we consider a system with two conserved charges (or independent components), namely, baryon number and electric charge number and refer to the phases as 1 and 2. Gibbs conditions are summarized in

\[
p_1(\mu_n, \mu_e) = p_2(\mu_n, \mu_e)
\]

where \(\mu_n\) and \(\mu_e\) are the chemical potentials corresponding to baryon number and electric charge. We understand that temperature \(T\) is held fixed. (It is small on the nuclear scale within several seconds of birth of a neutron star, and can set it to zero.) The above equation must hold in conjunction with expressions for the conservation of baryon and charge number, \(B\) and \(Q\). The unknowns are the two chemical potentials and the volume \(V\) of the sample containing the charges. (\(V\) is not the volume of the star but any locally inertial volume in the star, one in which the laws of special relativity hold to high precision [23].) For a volume fraction \(\chi = V_2/V\) of phase 2, the conditions
of global conservation can be expressed (for a uniform region) as

\[
\frac{1}{V} \int_V \rho(r) dr = (1 - \chi) \rho_1(\mu_n, \mu_e) + \chi \rho_2(\mu_n, \mu_e) \equiv \frac{B_1 + B_2}{V} = \frac{B}{V} \tag{6}
\]

\[
\frac{1}{V} \int_V q(r) dr = (1 - \chi) q_1(\mu_n, \mu_e) + \chi q_2(\mu_n, \mu_e) \equiv \frac{Q_1 + Q_2}{V} = \frac{Q}{V} \tag{7}
\]

where \( B_{1,2} \) and \( Q_{1,2} \) are the baryon and charge numbers, \( \rho_{1,2} \) and \( q_{1,2} \) are the baryon and charge densities in the volumes \( V_1 \) and \( V_2 \) occupied respectively by the two phases. The above three equations (6-7) serve to determine the two independent chemical potentials \( \mu_n, \mu_e \) and volume \( V \) for a specified volume fraction \( \chi \) of phase '2' in equilibrium with phase '1'. Thus the solutions are of the form

\[
\mu_n = \mu_n(\chi), \quad \mu_e = \mu_e(\chi), \quad V = V(\chi). \tag{8}
\]

The equilibrium condition (3) therefore can be rewritten as

\[
p_1(\chi) = p_2(\chi). \tag{9}
\]

This shows, as concerns the bulk properties, that the common pressure and all properties of the phases in equilibrium vary as the proportion \( \chi \) and that the pressure of a multi-component system in the mixed phase is not in general constant. These are fundamentally different properties for phase equilibrium of multi-component substances; they contrast with the properties of single-component substances such as water, in which the properties are independent of the proportion of the phases. For nuclear systems, the only exception to the above conclusion is for symmetric nuclear matter. In that case the system, by preparation, is optimum, and no rearrangement of conserved charges will take place.

### 3.3 Internal Driving Forces

We discuss now the microphysics responsible for variation of all properties of the phases in equilibrium as their proportion varies. It is clear from the above discussion that there is a degree (or degrees) of freedom in a multi-component substance that can be exploited by the internal forces to lower the energy. To see this, consider the concentration of the conserved quantities in both of the pure phases. It is some definite number

\[
c = Q/B \tag{10}
\]
according to the way the system was prepared whether in a test tube by a chemist, or in a neutron star by nature through the partially chaotic processes of a supernova. The degree(s) of freedom that the system can exploit to find the energy minimum in the mixed phase is that of rearranging the concentration of the conserved charges in each phase in equilibrium

$$c_1 = Q_1/B_1, \quad c_2 = Q_2/B_2$$

subject to the overall conservation laws (6,7). More generally, if the system is composed of \(n\) conserved charges, there are \(n - 1\) such degrees of freedom. In particular, a single component substance does not possess any freedom which is why the pressure and all properties of the two phases remain the same for all proportions of the phases in equilibrium (like water and ice).

In nuclear matter the internal force that drives the redistribution of charge allowed by the conservation laws is the isospin symmetry force that is responsible for the valley of beta stability. About half the symmetry energy arises from the Fermi energy and (in our model) the other half to the coupling of isospin to the \(\rho\) meson. Neutron star matter is highly isospin asymmetric. When conditions (say of increasing pressure toward the center of the star) cause a small amount of nuclear matter to transform to the other phase, the isospin driving force will exchange charge between regions of the two phases so as to make the neutron star matter more symmetric (positively charged) to the extent permitted by the conservation laws. The other phase will have a corresponding negative charge. The scope for exchanging charge, changes with the fraction of new phase (hence the non-linearity of energy with volume and the variation of pressure with volume). We show in Fig. 8 how the charge density in each phase varies as their proportion while the total charge is zero. (In the special case of isospin symmetric matter, the concentration is already optimum and the pressure will not vary in the mixed phase.)

The case in which electric charge is one of the conserved quantities is special. Because Coulomb is a long-range force, charges will arrange themselves so as not to create large volumes of like charge. Regions of the two oppositely charged phases will tend to shield each other. The surface interface energy resists breakup into small regions. The competition will define the dimensions of charged regions and their spacing as described next.

### 3.4 Spatial Structure

To calculate the spatial order we use the Wigner-Seitz approximation by choosing a volume \(v\) which is the cell size and contains the rare phase of dimension
Figure 8: The charge density carried by regions of confined and deconfined phases, and on leptons, assumed to be uniformly distributed. Densities times the respective volume fraction add to zero.

Figure 9: Size ($S = 2r$) and spacing ($D = 2R$) of geometrical structures. Notation ‘q rods’ means quark rods of negative charge immersed in nuclear matter of positive charge. Charge densities of each phase as a function of proportion $\chi$ of quark phase are shown in Fig. 8.

$r$ and the dominant phase in such amount as makes the cell neutral. Therefore cells do not interact. The Coulomb and surface energies per unit volume can be written in the schematic form that shows their dependence on dimension $r$ of the “geometry” and on the proportion $\chi$.

$$E_C/v = C(\chi)r^2, \quad E_S/v = S(\chi)/r. \quad (12)$$

(The dependence on $r$ can be obtained by dimensional analysis \[22, \text{See end of Section V}\]. Minimizing their sum as a function of $r$ at fixed $\chi$ yields

$$E_S = 2E_C. \quad (13)$$

as always happens in the minimization of a sum of two quantities that vary as $r^2$ and $1/r$ respectively. In the above, $C$ and $S$ are specific functions of proportion...
whose form is dictated by the geometry of the cells (e.g., sphere, rods and slabs) but for brevity we do not write them down. The above equations serve to define the droplet radius and cell size for each proportion of the phases,

\[ r = \left( \frac{S(\chi)}{2C(\chi)} \right)^{1/3}, \quad R = \frac{r}{\chi^{1/3}}, \]  

where for spherical geometry, \( \chi = (r/R)^3 \). As remarked in the introduction, the internal force that drives the charge redistribution between phases in equilibrium is the isospin restoring force. As one can see, the size and spacing of the droplets of rare phase immersed in the dominant will vary as proportion \( \chi \). Other geometries besides spheres may minimize the energy according to the proportion. The functional form of \( S \) and \( C \) is distinct in each case, as is the relation of the dimensions of \( r \) and \( R \) to \( \chi \).

\[ C_d(\chi) = 2\pi \left\{ [q_1(\chi) - q_2(\chi)]e \right\}^2 \chi f_d(x) \]  

\[ S_d(\chi) = \chi \sigma d \]  

where \( d = 1, 2, 3 \) for the idealized geometries of slabs, rods and drops respectively, \( \sigma \) is the surface tension and

\[ x \equiv (r/R)^d, \quad d = 1, 2, 3 \]  

where \( x \) is related to the proportion \( \chi \) by

\[ x = \begin{cases} \chi & \text{, background phase is 1} \\ 1 - \chi & \text{, background phase is 2} \end{cases} \]  

and

\[ f_d(x) = \frac{1}{d + 2} \left[ \frac{1}{(d - 2)} (2 - dx^{1-2/d}) + x \right]. \]  

The above considerations are identical to those encountered in a description of nuclei embedded in an electron gas. At higher relative concentration of nuclear matter to electron gas, the spheres will merge to form rods and so on \([24]\). What is remarkable in the present context is that two phases of one and the same substance, under the action of the isospin symmetry restoring force, are endowed with opposite charge and form a Coulomb lattice. The size and spacing and the geometric form that minimizes the sum of surface and Coulomb energies are shown in Fig. \([3]\).
3.5 Three Theorems

We have thus three theorems concerning the equilibrium configuration of the mixed phase of a first order phase transition of a substance with more than one conserved charge (or independent component in the language of chemistry):

1. All properties of the phases in equilibrium, including common pressure vary as the proportion of phases.

2. If electric charge is one of the conserved charges, the mixed phase will be in the form of a crystalline lattice.

3. Because of theorem 1, the geometry of the crystal and the size and the spacing of the lattice will vary with proportion.

These remarkable properties of first order phase transitions and the role played by the microphysics or internal forces is discussed in detail elsewhere [22, 23, 24]. It will be observed that the above discussion is completely general, and must apply to many systems in physical chemistry, nuclear physics, astrophysics and cosmology. In particular, in nuclear systems it applies to the confined-deconfined phase transition at high density, to the so-called liquid-vapor transition at sub-saturation density as well as to Pion and Kaon condensation if they are first order transitions, and if they occur. (It is understood that if a phase transition is induced by a nuclear collision, complete equilibrium will not be achieved, and in particular, there is insufficient time for formation of spatial structure.)

One step in the calculation remains to be described. The sum of surface and Coulomb energies, requires a knowledge of certain bulk properties like the charge density of each phase in equilibrium. The bulk energy and pressure can be computed according to ones favorite theory, and phase equilibrium finally has to be computed self-consistently, by minimizing the sum of all three energies, which can conveniently be done by iteration. The surface tension is generally not known and needs to be computed self-consistently for the phases in equilibrium. It will be a function of proportion, just as all other properties are.

Of course we have properly referred to the calculation as approximate: the division of total energy into bulk, surface and Coulomb is approximate. If one has reason to question the range of densities spanned by the structured mixed phase, then the alternative is to compute the energy on a lattice.

There is another important consequence of the existence of degree(s) of freedom for rearranging concentrations of conserved quantities in multi-component
substances according to the energy minimization principle. Whereas in one-component substances the properties of each phase in equilibrium are very unlike (as for example the density of ice and water), in a multi-component substances the rearrangement of charges so as to optimize the energy at each proportion of the phases relaxes their differences. This can be seen in Fig. 7 of Ref. [22]. As a consequence, the transition density from the pure low-density phase to the mixed phase is lower than would be expected were the degree(s) of freedom frozen out (as in the pre-1990 studies of deconfinement in neutron stars) [22, 26, 27, 28].

3.6 Summary

A change in phase during the course of spin-down of a millisecond pulsar occasioned by rising internal density due to weakening centrifugal forces will be reflected in a change in moment of inertia. The moment of inertia will follow different laws as a function of rotational frequency before and after the transition as in Fig. 8. The transition from one law to the other can (and is in our example) pass through an era of spin-up. This era lasts for about 1/50 of the spin-down time of millisecond pulsars, which represents an event rate. Spin up is trivial to detect and would be spectacular in a solitary pulsar which ought to be spinning down because of angular momentum loss to radiation.

For a weaker transition than experienced in our model the spin-up region need not occur: the moment of inertia may simply change smoothly from one trajectory to the other. In that case a sufficient number of observations of X-ray emitters in low-mass binaries could still identify a transition, even though the transition era were not actually observed, provided the observed neutron stars lay some on one branch, some on the other.

For distinct branches to exist, it appears to be necessary that the phase transition occurs near the maximum mass. The transition can be first or second order as long as it is accompanied by a sufficient softening of the equation of state.

References

[1] N. K. Glendenning, S. Pei and F. Weber, Phys. Rev. Lett. 79 (1997) 1603.

[2] D. Page, Thermal Evolution of Isolated Neutron Stars, To be published in the proceedings of the NATO ASI ‘The Many Faces of Neutron Stars’ (Kluwer), Eds A. Alpar, R. Bucheri & J. van Paradijs.
[3] C. Schaab, B. Hermann, F. Weber and M. K. Weigel, Astrophys. J. Lett. 480 (1997) L111.

[4] D. Bhattacharya and E. P. J. van den Heuvel, Physics Reports, 203 (1991) 1.

[5] B. R. Mottelson and J. G. Valatin, Phys. Rev. Lett. 5 (1960) 511.

[6] A. Johnson, H. Ryde and S. A. Hjorth, Nucl. Phys. A179 (1972) 753.

[7] F. S. Stephens and R. S. Simon, Nucl. Phys. A183 (1972) 257.

[8] S. I. A. Garpman, N. K. Glendenning and Y. J. Karant, Nucl. Phys. A322 (1979) 382.

[9] N. K. Glendenning, Astrophys. J. 293 (1985) 470.

[10] N. K. Glendenning and S. A. Moszkowski, Phys. Rev. Lett. 67 (1991) 2414.

[11] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorne and V. F. Weisskopf, Phys. Rev. D 9 (1974) 3471.

[12] N. K. Glendenning, Nuclear Physics B (Proc. Suppl.) 24B (1991) 110; Phys. Rev. D, 46 (1992) 1274.

[13] J. B. Hartle, Astrophys. J. 150 (1967) 1005.

[14] J. B. Hartle and D. Sharp, Astrophys. J. 147 (1967) 317.

[15] N. K. Glendenning and F. Weber, Astrophys. J. 400 (1992) 647.

[16] N. K. Glendenning and F. Weber, Phys. Rev. D 50 (1994) 3836.

[17] A. B. Migdal, Rev. Mod. Phys. 50 (1978) 107.

[18] N. K. Glendenning, Phys. Lett. 114B (1982) 392; N. K. Glendenning, Astrophys. J. 293 (1985) 470; N. K. Glendenning, Z. Phys. A 326 (1987) 57; N. K. Glendenning, Z. Phys. A 327 (1987) 295.
[19] V. A. Ambartsumyan and G. S. Saakyan, Astron. Zh. 37 (1963) 193 [Soviet Ast. – AJ,4 (1960) 187]; Ya. B. Zel’dovich and I. D. Novikov, Relativistic Astrophysics, Vol. 1, Stars and Relativity (University of Chicago Press, 1971); N. K. Glendenning, Astrophys. J. 293 (1985) 470; D. B. Kaplan and A. Nelson, Phys. Lett. 175 B (1986) 57; H. D. Politzer and M. B. Weise, Phys. Lett. B 273 (1991) 156; G. E. Brown, H. Lee, M. Rho, and V. Thorsson, Nucl. Phys. A 567 (1994) 937; V. Thorsson, M. Prakash and J. M. Lattimer, Nucl. Phys. A 572 (1994) 693; V. Koch, Phys. Lett. B 337 (1994) 7; E. E. Kolomeitsev, D. N. Voskresensky and B. Kampfer, Nucl. Phys. A 588 (1995) 889; N. Kaiser, P. B. Siegel and W. Weise, Nucl. Phys. A 594 (1995) 325; T. Wass, N. Kaiser and W. Weise, Phys. Lett. B 379 (1996) 34; J. Schaffner and I. N. Mishustin, Phys. Rev. C 53 (1996) 1416.

[20] G. Baym and S. A. Chin, Phys. Lett. 62B (1976) 241; G. Chapline and M. Nauenberg, Nature 264 (1976) 235; Phys. Rev. D 16 (1977) 456; B. D. Keister and L. S. Kisslinger, Phys. Lett. 64B (1976) 117.

[21] N. K. Glendenning and J. Schaffner, H-Dibaryon Bose Condensate in Compact Stars (in preparation) 1997.

[22] N. K. Glendenning, Phys. Rev. D, 46 (1992) 1274.

[23] N. K. Glendenning, COMPACT STARS, Nuclear Physics, Particle Physics, and General Relativity (Springer–Verlag New York, 1997).

[24] D. G. Ravenhall, C. J. Pethick and J. R. Wilson, Phys. Rev. Lett. 50 (1983) 2066.

[25] N. K. Glendenning, A Crystalline Quark-Hadron Mixed Phase in Neutron Stars, Physics Reports, 264 (1995) 143.

[26] N. K. Glendenning and S. Pei, Phys. Rev. C 52 (1995) 2250.

[27] H. Heiselberg, C. J. Pethick, and E. F. Staubo, Phys. Rev. Lett. 70 (1993) 1355.
[28] V. R. Pandharipande and E. F. Staubo, in Proc. 2’nd International Conf. of Physics and Astrophysics of Quark-Gluon Plasma, Calcutta, 1993, Eds. B. Sinha, Y. P. Viyogi and S. Raha, (World Scientific, 1994).