A note on M(atrix) theory in seven dimensions with eight supercharges

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Abstract

We consider M(atrix) theory compactifications to seven dimensions with eight unbroken supersymmetries. We conjecture that both M(atrix) theory on $K^3$ and Heterotic M(atrix) theory on $T^3$ are described by the same 5+1 dimensional theory with $\mathcal{N} = 2$ supersymmetry broken to $\mathcal{N} = 1$ by the orbifold projection. The emergence of the extra dimension follows from a recent result of Rozali (hep-th/9702136). We show that the seven dimensional duality between M-theory on $K^3$ and Heterotic string theory on $T^3$ is realised in M(atrix) theory as the exchange of one of the dimensions with this new dimension.

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I. INTRODUCTION

M(atrix) theory has provided new insights into understanding M-theory [1] beyond its description using eleven dimensional supergravity or as the strong coupling limit of IIA superstring theory [2]. Compactifications of M(atrix) theory preserving 16 supersymmetries have been useful in understanding U-dualities in dimensions $d \geq 7$ [3–5]. For example, in $d = 8$ part of the U-duality group is realised as the strong-weak coupling duality of $\mathcal{N} = 4$ Yang-Mills in four dimensions [4]. In the case of $d = 7$, Rozali has shown that the full U-duality group (=SL(5,$\mathbb{Z}$)) is geometrically realised by including an extra dimension (momentum modes in this dimension correspond to zero size instantons on the four-torus) [5]. In addition, string duality predicts interesting properties for each of these Yang-Mills theories [6]. The M(atrix) description of string theory leads to second quantised string theory with the three point interaction vertex being described by the crossing of eigenvalues in M(atrix) theory [8].

Compactification of M(atrix) theory on $T^d$ leads to U(N) Yang-Mills theory in $d + 1$ dimensions on the dual torus [1,3,11]. However, Yang-Mills theory is non-renormalisable in dimensions larger than four. Thus the SYM prescription breaks down when $d$ is larger than three. However there has been progress recently for M(atrix) theory on $T^4$ as well as $T^5$ [3,10]. For $T^4$, the proposed model is a $5 + 1$ dimensional field theory with chiral $\mathcal{N} = 2$ supersymmetry.

The simplest cases where lesser supersymmetry (eight unbroken supersymmetries) occurs is for M(atrix) theory on ALE spaces [11], M(atrix) theory on $K^3$ [12] and Heterotic M(atrix) theory [13–15]. In seven dimensions, it is well known that the latter two theories are dual to each other – this is the M-theoretic extension of the well-established IIA - Heterotic string duality. Both these theories can be obtained from different orbifolds of M(atrix) theory on $T^4$. $K^3$ in an orbifold limit is described by $(T^4/\mathbb{Z}_2)$ while the Heterotic case corresponds to the orbifold $(S^1/\mathbb{Z}_2) \times T^3$. The orbifold group acts on the Yang-Mills theory as: $\tilde{T}^4/\mathbb{Z}_2$ and $\tilde{S}^1 \times (\tilde{T}^3/\mathbb{Z}_2)$ respectively for the K3 and Heterotic theories. Before orbifolding, this is the theory studied by Rozali [8] and one obtains the emergence of an extra dimension. In both cases, the $\mathbb{Z}_2$ orbifold group can be extended to include the extra dimension. By studying the action of the orbifold $\mathbb{Z}_2$ on the coordinates of the underlying 5+1 dimensional theory, we observe that both compactifications can be identified on exchanging/relabeling coordinates! U-duality permits us to treat the five coordinates symmetrically and this leads us to conjecture that the two theories are described by the same 5+1 dimensional theory[1]. The seven dimensional duality mentioned earlier is shown to be a consequence of this conjecture. By using this conjecture, we reproduce the relationship between the Heterotic string tension and string coupling constant to the $K^3$ parameters as predicted by seven dimensional duality. As is clear from the arguments mentioned above, for the conjecture to work we need to assume that orbifolding commutes with part of the U-duality group. Closely related issues have been studied by Sen [17] who has constructed examples where the assumption works as well as those for which the assumption fails. Our case seems to be one where the assumption works.

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1 The emergence of K3 in Heterotic Matrix theory has been observed by P. Hořava [13,14].
II. M(ATRIX) THEORY COMPACTIFICATIONS IN SEVEN DIMENSIONS

In this section, we will first discuss the relevant details of Heterotic M(atrix) theory on $T^3$ followed by M(atrix) theory on $T^4/Z_2$. We then describe the conjecture and show that it implies the seven dimensional duality between Heterotic string theory on $T^3$ and M-theory on $K3$. We shall assume that both theories should be described by some 5+1 dimensional theory just as in the case of $T^4$ compactification of M(atrix) theory [6].

The Heterotic string and its compactifications have been the target of recent studies in M(atrix) theory [14,15]. The result which we obtain from these papers is the following, M(atrix) theory on $(S^1/Z_2) \times T^3$ is equivalent to the (orbifold) Yang-Mills theory on $\tilde{S}^1 \times (\tilde{T}^3/Z_2)$. Let us assume that the tori are rectangular and the circles have radii $R_i$, $i = 1, \ldots, 4$ with $i = 1$ corresponding to the circle on which the orbifold $Z_2$ acts. The base space for the Yang-Mills theory is given by the dual torus obtained from circles of radii (in units where the eleven dimensional Planck scale $l_{11} = 1$ and we have ignored some constant factors which do not affect our discussion)

$$\Sigma_i \sim 1/(r R_i) \quad ,$$

where $r$ is the radius of the eleventh dimension. The $Z_2$ group acts by reflection on the coordinates along the 2, 3, 4 directions of the Yang-Mills group. Without the orbifolding, we would be discussing the case of M(atrix) theory on $T^4$. In this case, Rozali has shown that an extra dimension of radius

$$\Sigma_5 \sim 1/(r R_1 R_2 R_3 R_4) \quad (2.2)$$

emerges[2]. The momentum modes around this dimension correspond to zero size instantons with the instanton number being identified with the momentum. Since the $Z_2$ acts as a reflection on odd number of dimensions, the instanton number changes sign under the $Z_2$. The change in the sign of the instanton number corresponds to a reflection in the 5th dimension. Thus we obtain the complete action of $Z_2$ on the five coordinates (i.e., we are including the extra dimension as a fifth coordinate) to be

$$\sigma^1 \rightarrow \sigma^1 \quad \text{and} \quad \sigma^i \rightarrow -\sigma^i \quad \text{for} \quad i = 2, \ldots, 5 \quad (2.3)$$

This implies that heterotic M(atrix) theory on $T^3$ is described by a 5+1 dimensional theory with coordinates $\sigma^i$, $i = 1, \ldots, 5$. [3] From the $Z_2$ action, we see that the last four coordinates form a $K3$ at its $Z_2$ orbifold limit.

We shall next consider M(atrix) compactification on $K3$ (in its orbifold limit $T^4/Z_2$) with the $T^4$ being rectangular with circles of radii $R'_i$. (We shall indicate all parameters on

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[2] This involves making the assumption that the extra dimension emerges even though there are fewer supersymmetries here. This assumption works because the supersymmetry is broken only by the boundary conditions and hence away from the fixed points all the supersymmetries are seen. We thank the referee for this remark.

[3] We shall defer a discussion of this theory to the conclusion.
the $K3$ side by a prime. The unprimed parameters always refer to the Heterotic side.) The M(atrix) theory is obtained from the (orbifold) Yang-Mills theory on the dual torus $\tilde{T}^4/\mathbb{Z}_2$. The radii of the circles of the dual torus are given by a formula similar to eqn. (2.1)

$$\Sigma_i' \sim 1/(rR_i')$$

Again, Rozali’s argument provides us with an extra dimension of radius

$$\Sigma_5' \sim 1/(rR_1'R_2'R_3'R_4')$$

The action of the orbifold $\mathbb{Z}_2$ on this extra dimension is again obtained by considering its action on instanton number. Unlike the heterotic case, there is no change in sign here and hence the $\mathbb{Z}_2$ acts trivially on the fifth dimension. Explicitly, it has the following action on the five coordinates (with the fifth coordinate being the extra dimension)

$$\sigma^i \rightarrow -\sigma^i \text{ for } i = 1, \ldots, 4 \text{ and } \sigma^5 \rightarrow \sigma^5$$

Thus, we obtain that M(atrix) theory on $K3$ is described by a 5+1 dimensional theory with coordinates $\sigma^n, i = 1, \ldots, 5$. From the $\mathbb{Z}_2$ action, we see that the first four coordinates form a $\tilde{K}3$ at its $\mathbb{Z}_2$ orbifold limit.

Comparing the action of the $\mathbb{Z}_2$ in both the cases of interest as given in eqns. (2.3) and (2.6), we can map the two $\mathbb{Z}_2$’s into one another by exchanging $\sigma^1$ and $\sigma^5$. From the viewpoint of Yang-Mills theory, this is a non-trivial operation since the fifth dimension is not manifest. However, this operation is an element of the U-duality group and if U-duality commutes with orbifolding, this exchange is allowed. Thus, this leads to the following conjecture:

**Heterotic theory on $T^3$ and M(atrix) theory on $K3$ are described by the RG fixed point of the same 5+1 dimensional theory with base $\tilde{S}^1 \times \tilde{K}3$. The orbifold limit of this theory can be obtained by orbifolding the 5+1 dimensional theory with base $\tilde{T}^5$ which describes M(atrix) theory on $T^4$.**

Let us now check whether this conjecture can be true. The conjecture provides a relationship between the heterotic and $K3$ theories. Equating the radius of $\tilde{S}^1$ and volume of the $\tilde{K}3$ in the 5+1 dimensional theory for both theories, we get

$$\Sigma_1 \sim \Sigma_5'$$

$$\Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 \sim \Sigma_1' \Sigma_2' \Sigma_3' \Sigma_4'$$

This leads to the following relationships in target spacetime:

$$R_1 \sim V_{K3}'$$

$$V_{T3} \sim 1$$

where $V_{T3} = R_2 R_3 R_4$ and $V_{K3}' = R_1'R_2'R_3'R_4'$ are volumes measured in units where $l_{11} = 1$. The parameters of the Heterotic string are

$$\alpha' = R_1 \text{ and } 1/\lambda_7^2 = V_{T3}/R_1^{3/2}$$
where $\alpha'$ is the string tension and $\lambda_7$ is the seven dimensional string coupling constant.

Rewriting eqn. (2.11) using (2.3) and (2.6), we obtain that

$$
\alpha' \sim V'_{K3} \quad \text{and} \quad \lambda_7 \sim (V'_{K3})^{3/4}
$$

(2.12)

which is in agreement with seven dimensional duality [2]. The string tension being proportional to the volume of $K3$ agrees with the fact that the heterotic string arises from wrapping an M5-brane on the $K3$. Also, $V_{T3} \sim 1$ shows that in making the correspondence one has to hold the volume of the three torus to be a constant independent of $V_{K3}$. We would like to emphasise that the relationships obtained in eqn. (2.12) are based on the conjecture and hence distinct from (for example) the derivation of Heterotic string tension in ref. [12] for M(atrix) theory on K3.

**III. CONCLUSION**

In this note, we have provided evidence for the conjecture that both seven dimensional theories with eight supercharges are described by the same 5+1 dimensional field theory. The assumption that orbifolding commutes with U-duality seems to be reasonable in this case. For M(atrix) compactification on $K3$ (away from the orbifold limit), the 5+1 dimensional theory has as its base space $\tilde{K}3 \times \tilde{S}1$ and has $\mathcal{N} = 1$ supersymmetry. It would be of interest to work out the precise relationship between the targetspace $K3$ and base $\tilde{K}3$ for generic (non-orbifold) $K3$.

What can we say with regard to the 5+1 dimensional theory? In M-theory on $K3$, the heterotic string is obtained by wrapping the M5-brane around the $K3$. This suggests that the 5 + 1 dimensional theory should be related in some sense to a theory of $N$ coincident M5-branes with worldvolume $K3 \times S^1 \times \mathbb{R}$. The world volume field theory of an M5-brane is described by a chiral $\mathcal{N} = 2$ (16 supersymmetries) with a single tensor multiplet. $N$ coincident M5-branes will have $N$ tensor multiplets, with extra tensor multiplets in the adjoint of $U(N)$ describing M2-branes connecting the M5-branes [20]. The wrapping of the M5-branes on $K3$ breaks half the supersymmetry on the worldvolume leading to $\mathcal{N} = 1$ supersymmetry as required. However at present there does not seem to an understanding of the theory described above. It is thus better to view the theory as the one given by compactifying the M(atrix) theory describing M-theory on $T^4$, the theory proposed in ref. [1], on $S^1 \times K3$.

*Note added*: The results reported here have also been independently derived by P. Hořava [16], S.-J. Rey [21] and Berkooz and Rozali [22].

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4The string tension follows from observing that the heterotic string is obtained by wrapping the M2-brane around the orbifold circle of radius $R_1$. The string coupling is obtained from standard dimensional reduction of the ten dimensional relation $\lambda_{10}^2 = R_1^3$ in units where $l_{11}$ is unity. See [2] for example.
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