The $Q^2$ Dependence of the Sum Rules for Structure Functions of Polarized $e(\mu)N$ Scattering *

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Abstract

The nonperturbative $Q^2$- dependence of the sum rules for the structure functions of polarized $e(\mu)N$ scattering is discussed. The determination of twist-4 corrections to the structure functions at high $Q^2$ by QCD sum rules is reviewed and critically analyzed. It is found that in the case of the Bjorken sum rule the twist-4 correction is small at $Q^2 > 5 GeV^2$ and does not influence the value of $\alpha_s$ determined from this sum rule. However, the accuracy of the today experimental data is insufficient to reliably determine $\alpha_s$ from the Bjorken sum rule. For the singlet sum rule – $p + n$ – the QCD sum rule gives only the order of magnitude of twist-4 correction. At low and intermediate $Q^2$ the model is presented which realizes a smooth connection of the Gerasimov-Drell-Hearn sum rules at $Q^2 = 0$ with the sum rules for $\Gamma_{p,n}(Q^2)$ at high $Q^2$. The model is in a good agreement with the experiment.

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1. Introduction

In the last few years there is a strong interest to the problem of nucleon spin structure: how nucleon spin is distributed among its constituents - quarks and gluons. New experimental data continuously appear and precision increases (for recent reviews see [1], [2]). One of the most important items of the information comes from the measurements of the first moment of the spin-dependent nucleon structure functions $g_1(x)$ which determine the parts of nucleon spin carried by $u$, $d$ and $s$ quarks and gluons. The level of the accuracy of the data is now such that the account of nonperturbative $Q^2$ dependence — the so-called twist-4 terms — is of importance when comparing the data with the Bjorken and Ellis-Jaffe sum rules at high $Q^2$. On the other side, at low and intermediate $Q^2$ a smooth connection of the sum rules for the first moments of $g_1(x)$ with the Gerasimov-Drell-Hearn (GDH) sum rules [3,4] is theoretically expected. This connection can be realized through nonperturbative $Q^2$-dependence only. In this paper I discuss such nonperturbative $Q^2$-dependence of the sum rules.

First, recall the standard definition of the structure functions for scattering of polarized electrons or muons on polarized nucleon. They are characterized by the imaginary part of the forward scattering amplitude of virtual photon on the nucleon (see, e.g., [5])

$$\text{Im } T_{\mu\nu}(q,p) = \frac{2\pi}{m} \epsilon_{\mu\nu\lambda\sigma}q_\lambda \left[s_\sigma G_1(\nu,q^2) + \frac{1}{m^2}(s_\sigma\nu - (sq)p_\sigma)G_2(\nu,q^2)\right]$$ (1)

Here $\nu = pq, q^2 = q_0^2 - q^2 < 0, q$ and $p$ are virtual photon and nucleon momenta, $s_\sigma$ is nucleon spin 4-vector, $G_1(\nu,q^2)$ and $G_2(\nu,q^2)$ are two spin-dependent nucleon structure functions, $m$ is the nucleon mass. Index $a$ in eq.(1) means antisymmetrization in virtual photon polarization indexes $\mu, \nu$. Below I will consider only the structure function $G_1(\nu,q^2)$ related to the scaling structure function $g_1(x,Q^2)$ by

$$\frac{\nu}{m^2}G_1(\nu,q^2) = g_1(x,Q^2),$$ (2)

where $Q^2 = -q^2, x = Q^2/2\nu$. The first moment of the structure function $g_1$ is defined as

$$\Gamma_{p,n}(Q^2) = \int_0^1 dx \ g_{1:p,n}(x,Q^2)$$ (3)

The presentation of the material in the paper is divided into two parts. The first part deals with the case of high $Q^2$. I discuss the determination of twist-4 contributions to $\Gamma_{p,n}$ by QCD sum rules and estimate their influence on the values of $\alpha_s(Q^2)$ as well as on the values of $\Delta u, \Delta d, \Delta s$ — the parts of the proton spin projection, carried by $u, d, s$ quarks. In the second part the case of low and intermediate $Q^2 \lesssim 1 GeV^2$ is considered in the framework of the model which realizes the smooth connection of GDH sum rule at $Q^2 = 0$ with the asymptotic form of $\Gamma_{p,n}(Q^2)$ at high $Q^2$. 
2. High $Q^2$

At high $Q^2$ with the account of twist-4 contributions $\Gamma_{p,n}(Q^2)$ have the form

$$\Gamma_{p,n}(Q^2) = \Gamma_{p,n}^{as}(Q^2) + \Gamma_{p,n}^{tw4}(Q^2)$$

$$\Gamma_{p,n}^{as}(Q^2) = \frac{1}{12}\left\{ [1 - a - 3.58a^2 - 20.2a^3 - ca^4][\pm g_A + \frac{1}{3}a_s] + 4\left[ 1 - \frac{1}{3}a - 0.55a^2 - 4.45a^3 \right]\Sigma \right\} - \frac{N_f}{18\pi}\alpha_s(Q^2)\Delta g(Q^2) \tag{5}$$

$$\Gamma_{p,n}^{tw4}(Q^2) = \frac{b_{p,n}}{Q^2} \tag{6}$$

In eq. (5) $a = \alpha_s(Q^2)/\pi$, $g_A$ is the $\beta$-decay axial coupling constant, $g_A = 1.260 \pm 0.002$ [6]

$$g_A = \Delta u - \Delta d \quad a_s = \Delta u + \Delta d - 2\Delta s \quad \Sigma = \Delta u + \Delta d + \Delta s. \tag{7}$$

$\Delta u, \Delta d, \Delta s, \Delta g$ are parts of the nucleon spin projections carried by $u, d, s$ quarks and gluons:

$$\Delta q = \int_0^1 \left[ q_+(x) - q_-(x) \right] \tag{8}$$

where $q_+(x), q_-(x)$ are quark distributions with spin projection parallel (antiparallel) to nucleon spin and a similar definition takes place for $\Delta g$. The coefficients of perturbative series were calculated in [7, 8, 9, 10], the numerical values in (5) correspond to the number of flavours $N_f = 3$, the coefficient $c$ was estimated in [11], $c \approx 130$. In the renormalization scheme chosen in [7, 8, 9, 10] $a_8$ and $\Sigma$ are $Q^2$-independent. In the assumption of the exact $SU(3)$ flavour symmetry of the octet axial current matrix elements over baryon octet states $a_8 = 3F - D = 0.59 \pm 0.02$ [12].

Strictly speaking, in (5) the separation of terms proportional to $\Sigma$ and $\Delta g$ is arbitrary, since the operator product expansion (OPE) has only one singlet in flavour twist-2 operator for the first moment of the polarized structure function – the operator of singlet axial current $j_{\mu5}^{(0)}(x) = \sum_{q} \bar{q}_i(x)\gamma_\mu\gamma_5q, \quad q = u, d, s$. The separation of terms proportional to $\Sigma$ and $\Delta g$ is outside the framework of OPE and depends on the infrared cut-off. The expression used in (5) is based on the physical assumption that the virtualities $p^2$ of gluons in the nucleon are much larger than light quark mass squares, $|p^2| \gg m_r^2$ [13] and that the infrared cut-off is chosen in a way providing the standard form of axial anomaly [14]. (See [14] for review and details).

Let us now discuss twist-4 contributions to $\Gamma_{p,n}$ (4) and estimate the coefficients $b_{p,n}$ in (6). The general theory of twist-4 contributions to the first moment of $g_{1,p,n}$ was formulated by Shuryak and Vainstein [16]. They have found
Here the matrix elements in the double angular brackets mean the matrix elements of the operators (for \( u \)-quarks)

\[
\langle N | U^a_\mu | N \rangle = s_\mu \langle \langle U \rangle \rangle \\
\langle N | V^a_{\mu\nu,\sigma} | N \rangle = S_{\nu,\sigma} A_{\mu,\nu,\sigma} \langle \langle V \rangle \rangle
\]

where \( S \) and \( A \) are symmetrization and antisymmetrization operators. The indices \( S \) (singlet) and \( NS \) (nonsinglet) correspond to \( S \rightarrow u + d + (18/5)s, \ NS \rightarrow u - d \) and

\[
C_S = \frac{5}{18}, \quad C_{NS} = \frac{1}{6}
\]

The last term in (9) is small and can be safely neglected.

The matrix elements \( \langle \langle U \rangle \rangle_{S,NS} \) and \( \langle \langle V \rangle \rangle_{S,NS} \) were calculated by Balitsky, Braun and Koleshichenko (BBK) [17] using the QCD sum rule method in external field. (The explicit form of (9) was taken from [17], where the errors in [16] were corrected - see [17], Errata). In order to perform the calculations one should add to QCD Lagrangian the terms

\[
\delta L = U_\mu A_\mu, \quad \delta L = V_{\mu\nu,\sigma} A_{\mu,\nu,\sigma}
\]

correspondingly, in case of \( \langle N | U^a_\mu | N \rangle \) and \( \langle N | V^a_{\mu\nu,\sigma} | N \rangle \), calculations, where \( A_\mu \) and \( A_{\mu\nu,\sigma} \) are constant external fields. Then the polarization operator

\[
\Pi = i \int dx e^{ipx} \langle 0 | T\{\eta(x), \bar{\eta}(0)\} | 0 \rangle
\]

is considered and the terms in \( \Pi(p) \) linear in external fields are separated. In (14) \( \eta(x) \) is the quark current with nucleon quantum numbers. For proton [18]

\[
\eta_p(x) = \epsilon^{abc} u^a(x) C\gamma_\mu u^b(x)\gamma_5 \gamma_\mu d^c(x),
\]

\( u^a(x), d^c(x) \) – are \( u \) and \( d\)– quark fields, \( a, b, c \) – are colour indices. An essential ingredient of the QCD sum rules method in external field is the appearance of induced by external field vacuum condensates in OPE [19]. To determine these condensates some additional sum rules are used. In the case in view there are two such condensates:

\[
\langle 0 | U^a_\mu | 0 \rangle_A = PA_\mu, \quad \langle 0 | V^a_{\mu\nu,\sigma} | 0 \rangle_A = RA_{\mu\nu,\sigma}
\]
Their values were estimated by BBK [17]: 

\[ P = 3.10^{-3}(\pm 30\%) GeV^6, R = 1.10^{-3}(\pm 100\%) GeV^6. \]

BBK accounted in OPE the operators up to dimension 8. The sum rules for nonsinglet matrix elements are

\[
\langle\langle U^{NS}\rangle\rangle + A^{NS}_U M^2 = -\frac{1}{2\lambda^2} e^{m^2/M^2} \left\{ \frac{8}{9} M^2 \alpha_s(M^2) \right\} \times \\
\times \int_0^{W^2} ds e^{-s/M^2} \left( W^2 + s \ln \frac{W^2}{s} \right) - \frac{1}{9} b M^4 E_1 \left( \frac{W^2}{M^2} \right) + \\
\frac{32}{27} M^2 \alpha_s(M^2) \pi a^2 (\ln \frac{W^2}{M^2} + 1.03) + \frac{8}{9} \pi^2 P M^2 - \frac{2}{3} m_0 a^2 \right\}
\]

\[
\langle\langle V^{NS}\rangle\rangle + A^{NS}_V M^2 = -\frac{1}{2\lambda^2} e^{m^2/M^2} \left\{ \frac{52}{135} \alpha_s(M^2) \right\} \times \\
\times \int_0^{W^2} ds e^{-s/M^2} (W^2 + s \ln \frac{W^2}{s}) - \frac{2}{9} b M^4 E_1 \left( \frac{W^2}{M^2} \right) - \\
- \frac{80}{27} \alpha_s(M^2) M^2 a^2 (\ln \frac{W^2}{M^2} + 1.89) + \pi^2 R M^2 - \frac{4}{9} m_0 a^2 \right\}
\]

where \( M^2 \) is the Borel parameter

\[ \tilde{\lambda}^2 = 32\pi^4 \lambda^2, \quad \lambda = \langle 0|\eta|N \rangle, \quad \tilde{\lambda}^2 = 2.1 GeV^6 \]
\[ a = -(2\pi)^2 \langle 0|\bar{q}q|0 \rangle = 0.67 GeV^3 \quad (at \quad M^2 = 1 GeV^2) \]
\[ b = \frac{\alpha_s}{\pi} \langle 0|G_{\mu\nu}|0 \rangle (2\pi)^2 = 0.5 GeV^4, \quad m_0^2 = 0.8 GeV^2 \]

\[ E_1(z) = 1 - (1 + z) e^{-z} \]

and \( W^2 \) is the continuum threshold, \( W^2 = 2.3 GeV^2 \).

The terms \( A^{NS}_U,V \) in the l.h.s. of the sum rules are background contributions corresponding to nondiagonal transitions in the phenomenological parts of the sum rules:

\[ N \rightarrow \text{interaction with external current} \rightarrow N^* \]

They should be separated by studying the \( W^2 \) dependence of the sum rules, e.g., by applying the differential operator \( 1 - M^2 \partial / \partial M^2 \) to the sum rule. Unfortunately, such procedure deteriorates the accuracy of the results.

The sum rules (17),(18) differ from the original ones found by BBK. In the BBK sum rules the first and the third terms in r.h.s. contain the ultraviolet cut-off. As was shown in [20], this is not correct and stems from the fact that BBK used one-variable dispersion relation in \( p^2 \) for the vertex function \( \Gamma(p^2) \). Instead of \( \Gamma(p^2) \) the general vertex function \( \Gamma(p_1^2, p_2^2, Q^2) \) must be considered and for this function double dispersion relation in \( p_1^2 \) and \( p_2^2 \) with subtraction terms must be written. After going to the limit \( p_1^2 \rightarrow p_2^2, \quad q^2 \rightarrow 0 \) the
correct dispersion relation is obtained. In this double dispersion relation representation for $\Gamma(p^2, p^2, 0)$ no ultraviolet cut-off appears, but a more strong assumption about the relation of physical spectrum to $\Gamma(p^2, p^2, 0)$ found in QCD calculation is needed. In expressions (17),(18) the hypothesis of local duality was assumed when the interval $0 < |p_1^2|, |p_2^2| < W^2$ in the dispersion representation for $\Gamma(p^2, p^2)$ corresponds to the nucleon pole while everything outside this interval – to continuum. (See [20] for details).

The calculation of the r.h.s. of (17),(18) in the interval of the Borel parameter $0.8 < M^2 < 1.2 GeV^2$ gives for the l.h.s approximately

$$\langle\langle U^{NS}\rangle\rangle + A^{NS}_U M^2 = 0.15 - 0.23 M^2$$

in $GeV^2$. For the twist-4 correction (6) we have according to (9):

$$b_{p-n} = -\frac{1}{6} \cdot \frac{8}{9} \left[ \langle\langle U^{NS}\rangle\rangle - \frac{1}{4} \langle\langle V^{NS}\rangle\rangle \right] = 0.0015$$

i.e., practically zero. This zero result arises due to strong compensation of two terms in (22). That is why the analysis of uncertainties in the calculation is necessary. A serious uncertainty in the calculation of $\langle\langle U^{NS}\rangle\rangle$ comes from the fact that the main contribution to the r.h.s. of (17) is given by the last term – the operator of dimension 8 and some doubts appear about a possible role of higher dimension operators. Recently Oganesian had calculated the contribution of the dimension-10 operator to the sum rule (17) in the framework of the factorization hypothesis [21]. His result is:

$$\langle\langle U^{NS}\rangle\rangle_{dim10} = \frac{1}{9} \frac{e^{m^2/M^2}}{\lambda^2} m_0 a^2 \left( 1 + \frac{m^2}{2 M^2} \right) \approx 0.05 GeV^2$$

In the case of $\langle\langle V^{NS}\rangle\rangle$ the main contribution comes from the third term in (18) - the operator of dimension 6. So, one may believe, the higher dimension operators are not very important here. Other possible sources of error are: 1) the large background term, proportional to $M^2$ in (20); a much stronger influence of the continuum threshold $W^2$ on the sum rules (17),(18) comparing with usual QCD sum rules, where the $W^2$ dependence appears only through correction terms $\simeq e^{-W^2/M^2}$; 3) the role of anomalous dimensions which are disregarded in (17),(18), but can destroy compensation in (22); 4) even uncertainties in the chosen numerical values of QCD parameters ($\Lambda, a^2$ etc.) can influence this compensation. Adding (23) to (22) and estimating the uncertainties as one half of each term in (22) we have finally

$$b_{p-n} = -0.006 \pm 0.012$$

Adding (24) to (22) and estimating the uncertainties as one half of each term in (22) we have finally

$$\langle\langle U^S\rangle\rangle + A^S_U M^2 = 0.082 - 0.23 M^2$$

(25)
\[ \langle \langle V^S \rangle \rangle + A_S^4 M^2 = -0.22 + 0.05 M^2 \] (26)

The contribution of the dimension-10 operator to \( \langle \langle U^S \rangle \rangle \) was found to be [24]

\[ \langle \langle U^S \rangle \rangle_{dim 10} = \frac{2}{9} \frac{e^{m^2/M^2}}{\lambda^2} m_\rho^4 d^2 \left( 1 + \frac{m^2}{2M^2} \right) \approx -0.10 \] (27)

and is even larger than \( \langle \langle U^S \rangle \rangle \) from (25). For \( \langle \langle V^S \rangle \rangle \) the main contribution to the sum rule comes also from the highest dimension 8 operator.

There are also other very serious drawbacks of the twist-4 matrix elements calculations done by BBK [17] in the singlet case:

1. BBK assumed that \( s \)-quarks do not contribute to the spin structure functions and instead of singlet operator considered the octet one.

2. When determining the induced by external field vacuum condensates, which are very important in the calculation of \( \langle \langle U^S \rangle \rangle \), the corresponding sum rule was saturated by \( \eta \)-meson, what is wrong. (Even saturation by \( \eta' \) meson would not be correct, since \( \eta' \) is not a Goldstone).

3. The calculation of the singlet axial current matrix element over the nucleon state – \( \langle N \mid j^0_{\mu 5} \mid 0 \rangle = s_\mu \Sigma \) – by QCD sum rule fails: it was shown that the OPE series diverges at the scale \( M^2 \sim 1 GeV^2 \) [22]. It is very probably that the same situation takes place in the calculation of \( \langle \langle U^S \rangle \rangle \) and \( \langle \langle V^S \rangle \rangle \).

For all these reasons the results for twist-4 corrections, following from (25),(26)

\[ b_{p+n} = \frac{5}{18} \cdot \frac{8}{9} \left( \langle \langle U^S \rangle \rangle - \frac{1}{4} \langle \langle V^S \rangle \rangle \right) = -0.035 \] (28)

may be considered only as correct by the order of magnitude.

Bearing in mind the values and uncertainties of twist-4 corrections to \( \Gamma_{p,n} \) let us compare the theory with the recent experimental data [1, 2].

Table 1 shows the combined experimental data [1] obtained in SMC [1] and SLAC [2] experiments, transferred to \( Q^2 = 5 GeV^2 \) in comparison with the theoretical values of the Ellis-Jaffe and Bjorken sum rule. The Bjorken sum rule was calculated according to (5), the magnitude of the twist-4 correction was taken from (24), the \( \alpha_s \) value \( \alpha_s(5 GeV^2) = 0.276 \), corresponding to \( \alpha_s(m_z) = 0.117 \) and \( \Lambda^{(3)}_{MS} = 360 MeV \) (the latter in two loops).

| \hline
| \hline
| Combined data | \( \Gamma_p \) | \( \Gamma_n \) | \( \Gamma_p - \Gamma_n \) |
|\hline
| Ellis-Jaffe/Bjorken | 0.168 ± 0.005 | -0.013 ± 0.005 | 0.181 ± 0.002 |
|\hline

The \( \Lambda^{(3)}_{MS} = 200 MeV \) (\( \alpha_s(5 GeV^2) = 0.215, \alpha_s(m_z) = 0.106 \)) would give instead \( \Gamma_p - \Gamma_n = 0.189 \). The Ellis-Jaffe sum rule prediction was calculated according to (5) where \( \Delta s = 0 \), i.e. \( \Sigma = a_8 = 0.59 \) was put, the last - gluonic term in (5) – was omitted and the twist-4
contribution (28) was included into the error. The errors in the second line of Table 1 are only from uncertainties of twist-4 terms. A possible error arising from violation of the $SU(3)$ flavour symmetry only weakly affects the Ellis-Jaffe prediction: 10% variation of $a_8$ results in $\Delta \Gamma_p = \Delta \Gamma_n = 0.008$. Therefore, as follows from Table 1, the Ellis-Jaffe sum rule is in a definite contradiction with experiment - a nonzero value of $\Delta s$ is necessary. From the experimental value of $\Gamma_p - \Gamma_n$ presented in Table 1 and from the Bjorken sum rule

$$\Gamma_p - \Gamma_n = \frac{g_A}{6}[1 - a - 3.58a^2 - 20.2a^3 - 130a^4] + \frac{b_{p-n}}{Q^2}$$

one can determine the coupling constant $\alpha_s(Q^2)$ at $Q^2 = 5GeV^2$. The result is

$$\alpha_s(5GeV^2) = 0.116^{+0.16}_{-0.44} \pm 0.014$$

The first error is experimental, the second comes from the uncertainty in the twist-4 correction (24). The value $\alpha_s(5GeV^2)$ is nonsatisfactory because of large errors and because of that the central point corresponds to $\Lambda(3)^{MS} \approx 15MeV$, what is unacceptable. A serious reduction of experimental errors in $\Gamma_p - \Gamma_n$ - by factors $\approx 3 - 4$ is necessary in order that one could determine QCD coupling constant $\alpha_s(Q^2)$ from the Bjorken sum rule with a reasonable accuracy, say, to distinguish the cases of large and small values of $\Lambda(3) \approx 350MeV$ and $\Lambda(3) \approx 200MeV$, which are now under dispute.

Parts of the proton spin, carried by $u, d$ and $s$-quarks can be calculated from the data of Table 1 and eq.'s (5-7). The results of the overall fit presented in ref.1 are

$$\Delta u = 0.82 \pm 0.02 \quad \Delta d = -0.43 \pm 0.02, \quad \Delta s = -0.10 \pm 0.02$$

$$\Sigma = 0.29 \pm 0.06$$

However, these results are strongly dependent on the values of $\alpha_s$ in the analysis as well as from what set of the data they are determined. For example, at $\alpha_s(5GeV^2) = 0.276$ ($\alpha_s(m_z) = 0.117$) from $\Gamma_p, \Gamma_n$ and $\Gamma_p + \Gamma_n$ it follows correspondingly $\Sigma = 0.48; 0.145; 0.246$ – the values which are partly outside one standard deviation quoted in (31). A selfconsistent value of $\Sigma \approx 0.22$ can be found from all data - $\Gamma_p, \Gamma_n$ and $\Gamma + \Gamma_n$, if we put for $\alpha_s(5GeV^2)$ the central value (30), but this is nonacceptable, as mentioned above. For this reason, I think, that the errors in (31) are underestimated. (The errors from twist-4 corrections $\delta \Sigma = \pm 0.03, \delta \Delta q = \pm 0.01$ are not included in (31), since they are smaller).

Up to now the contribution of gluons – the last term in (5) – was disregarded. The estimation of this term can be done [13], if we assume, that at $1GeV^2$ the quark model is valid and in the relation of the conservation angular momentum

$$\frac{1}{2} \Sigma + \Delta g(Q^2) + L_z(Q^2) = \frac{1}{2}$$

the orbital momentum term $L_z(Q^2)$ can be neglected. Assuming $\Sigma(1GeV^2) \approx 0.4$, we find $\Delta g(1GeV^2) = 0.3$. Then from the evolution equation [23] (see also [15]), it follows that $\Delta g(5GeV^2) \approx 0.6$, what results in increasing of $\Sigma$ by $\delta \Sigma \approx 0.08$ and in corresponding increasing of $\Delta u, \Delta d, \Delta s$ by $\delta q = \delta \Sigma/3 \approx 0.03$. 


3. Low and Intermediate $Q^2$.

Connection with Gerasimov-Drell-Hearn Sum Rule.

Consider the forward scattering amplitude of polarized real photon on polarized nucleon. The spin dependent part of the amplitude is expressed through one invariant function. In the lab. system we can write

$$e_i^{(2)} T_{ik}^a e_k^{(1)} = i \frac{\nu}{m^2} \varepsilon_{ikl} e_i^{(2)} e_k^{(1)} s_l S_1(\nu, 0),$$

where $e_i^{(1)}$, $e_i^{(2)}$ ($i, k = 1, 2, 3$) are the polarization vectors of initial and final photons, $s_l$ is the vector of nucleon spin, $\nu = pq = m \omega$ in the l.s. The second, equal to zero argument of the invariant function $S_1(\nu, Q^2)$ means that the photon is real, $Q^2 = 0$. Comparing (33) with the general expression (1) one can easily see, that

$$Im \ S_1(\nu, 0) = 2\pi G_1(\nu, 0)$$

(34)

(the factor in front of the function $G_2(\nu, q^2)$ in (1) vanishes for the case of the real photon).

As follows from Regge theory, the leading Regge pole trajectory, determining the high energy behavior of $S_1(\nu, 0)$ is the trajectory of $a_1$ Regge pole \[24, 3\]

$$S_1(\nu, 0)_{\nu \to \infty} \sim \nu^{\alpha_{a_1}(0)-1},$$

(35)

where $\alpha_{a_1}$ is the intercept of $a_1$ trajectory. The value of $\alpha_{a_1}(0)$ is not completely certain $\alpha_{a_1}(0) \approx -(0.3 \div 0.0)$, but definitely it is negative. Therefore, $S_1(\nu, 0)_{\nu \to \infty} < 1/\nu$ and unsubtracted dispersion relation can be written for it \[3\]:

$$S_1(\nu, 0) = 4 \int_0^\infty d\nu' \frac{G_1(\nu', 0)}{\nu'^2 - \nu^2}$$

(36)

Consider now $S_1(\nu, 0)$ in the limit $\nu \to 0$. According to F.Low theorem $S_1(\nu, 0)$ is expressed through the static nucleon properties and the direct calculation gives

$$S_1(\nu, 0)_{\nu \to 0} = -\kappa^2,$$

(37)

where $\kappa$ is the nucleon anomalous magnetic moment, $\kappa_p = 1.79$, $\kappa_n = -1.91$. The substitution of (37) into (36) gives the Gerasimov-Drell-Hearn (GDH) \[3, 4\] sum rule

$$\int_0^\infty \frac{d\nu}{\nu} G_1(\nu, 0) = -\frac{1}{4}\kappa^2$$

(38)

An important remark: the forward spin dependent photon-nucleon scattering amplitude has no nucleon pole in case of real photon. This means that there is no nucleon contribution in the l.h.s. of (38) – all contributions come from excited states. The GDH sum rule is very nontrivial!

Till now there are no direct experimental checks of GDH sum rule, since the experiments on photoproduction by polarized photon on polarized proton (or neutron) are absent. What was done \[25, 26, 27\] – is the indirect check, when the parameters of
nucleon resonances, determined in nonpolarized photo- or electro production were substi-
tuted into the l.h.s. of (38). In this way with resonances up to  \( W = 1.8 \text{GeV} \) it was 
obtained [25, 26, 27] (the numbers below are taken from [28]):

| Type      | l.h.s. of (38) | r.h.s. of (38) |
|-----------|----------------|----------------|
| proton    | -1.03          | -0.8035        |
| neutron   | -0.83          | -0.9149        |

The errors in (39) are such, that with the account of only resonances up to  \( W = 1.8 \text{GeV} \) the l.h.s. and the r.h.s. of (38) are not in agreement – a nonresonant contribution is needed.

In order to connect the GDH sum rule with  \( \Gamma_{p,n}(Q^2) \) consider the integrals [5]

\[
I_{p,n}(Q^2) = \int_0^\infty \frac{d\nu}{\nu} G_{1,p,n}(\nu, Q^2) 
\]

(40)

Using (2) and changing the integration variable  \( \nu \) to  \( x \), (40) can be also identically written as

\[
I_{p,n}(Q^2) = \frac{2m^2}{Q^2} \int_0^1 dx g_{1,p,n}(x, Q^2) = \frac{2m^2}{Q^2} \Gamma_{p,n}(Q^2)
\]

(41)

At  \( Q^2 = 0 \)

\[
I_p(0) = -\frac{1}{4} \kappa_p^2 = -0.8035; \quad I_n(0) = -\frac{1}{4} \kappa_n^2 = 0.9149; \quad I_p(0) - I_n(0) = 0.1114 \quad (42)
\]

and  \( \Gamma_{p,n} = 0 \). Sometimes it is convenient to express  \( \Gamma(Q^2) \) in terms of the electroproduction cross sections  \( \sigma_{1/2}(\nu, Q^2) \) and  \( \sigma_{3/2}(\nu, Q^2) \), corresponding to the projections 1/2 and 1/3 of the total photon-nucleon spin upon the photon momentum direction, as well as the quantity  \( \sigma_I(\nu, Q^2) \), describing the interference of transverse and longitudinal virtual photon polarizations [5]:

\[
\Gamma(Q^2) = \frac{Q^2}{16\pi^2\alpha} \int \frac{d\nu}{\nu} \frac{1-x}{1+(m^2Q^2/\nu^2)} \left[ \sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2) + 2\frac{Qm}{\nu} \sigma_I(\nu, Q^2) \right]
\]

(43)

(The normalization of the cross sections is chosen in a way, that the virtual photon flux is assumed to be equal to that of real photons with the energy fixed by condition, that the masses of hadronic states produced by the real and virtual photons are equal.)  \( \sigma_I(\nu, Q^2) \) satisfies the inequality

\[
\sigma_I < \sqrt{R} \sigma_T, \quad \sigma_T = \sigma_{1/2} + \sigma_{3/2}, \quad R = \sigma_L/\sigma_T
\]

(44)

and practically the last term in (43) is small and as a rule can be neglected.
The schematic $Q^2$ dependence of $I_p(Q^2)$, $I_n(Q^2)$ and $I_p(Q^2) - I_n(Q^2)$ is plotted in Fig.1. The case of $I_p(Q^2)$ is especially interesting: $I_p(Q^2)$ is positive, small and decreasing at $Q^2 \gtrsim 3GeV^2$ and negative and relatively large in absolute value at $Q^2 = 0$. With $I_n(Q^2)$ the situation is similar. All this indicates large nonperturbative effects in $I(Q^2)$ at $Q^2 \lesssim 1GeV^2$.

In [29] the model was suggested, which describes $I(Q^2)$ (and $\Gamma(Q^2)$) at low and intermediate $Q^2$, where GDH sum rules and the behaviour of $I(Q^2)$ at large $Q^2$ where fulfilled. The model had been improved in [30, 28]. (Another model with the same goal was suggested by Soffer and Teryaev [31]).

Since it is known, that at small $Q^2$ the contribution of resonances to $I(Q^2)$ is of importance, it is convenient to represent $I(Q^2)$ as a sum of two terms

$$I(Q^2) = I^{res}(Q^2) + I'(Q^2),$$

where $I^{res}(Q^2)$ is the contribution of baryonic resonances. $I^{res}(Q^2)$ can be calculated from the data on electroproduction of resonances. Such calculation was done with the account of resonances up to the mass $W = 1.8GeV$ [27].

In order to construct the model for nonresonant part $I'(Q^2)$ consider the analytical properties of $I(q^2)$ in $q^2$. As is clear from (40),(41) $I(Q^2)$ is the moment of the structure function, i.e. it is a vertex function with two legs, corresponding to ingoing and outgoing photons and one leg with zero momentum. The most convenient way to study of analytical properties of $I(q^2)$ is to consider a more general vertex function $I(q_1^2, q_2^2; p^2)$, where the momenta of the photons are different, and go to the limit $p \to 0$, $q_1^2 \to q_2^2 = q^2$. $I(q_1^2, q_2^2; p^2)$ can be represented by the double dispersion relation:

$$I(q^2) = \lim_{q_1^2 \to q_2^2 = q^2, p^2 \to 0} I(q_1^2, q_2^2; p^2) = \left\{ \int ds_2 \int ds_1 \frac{\rho(s_1, s_2; p^2)}{(s_1 - q_1^2)(s_2 - q_2^2)} + P(q_1^2) \int \frac{\varphi(s, p^2)}{s - q_2^2} ds \right\} + P(q_2^2) \int \frac{\varphi(s, p^2)}{s - q_1^2} ds.$$  

The last two terms in (46) are the subtraction terms in the double dispersion relation, $P(q^2)$ is the polynomial. Since, according to (41), $I(q^2)$ decreases at $q^2 \to \infty$, $P(q^2)$ reduces to a constant, $P(q^2) = Const$ and the constant subtraction term in (46) is absent. We are interesting in $I(Q^2)$ dependence in the domain $Q^2 \lesssim 1GeV^2$. Since after performed subtraction, the integrals in (46) are well converging, one may assume, that at $Q^2 \lesssim 2 - 3GeV^2$ the main contribution comes from vector meson intermediate states. $\varphi$-meson weakly interacts with nucleon, so the general form of $I'(Q^2)$ is

$$I'(Q^2) = \frac{A}{(Q^2 + \mu^2)^2} + \frac{B}{Q^2 + \mu^2},$$

where $A$ and $B$ are constants, $\mu$ is $\rho$ (or $\omega$) mass. The constant $A$ and $B$ are determined from GDH sum rules at $Q^2 = 0$ and from the requirement that at high $Q^2 \gg \mu^2$ takes place the relation

$$I(Q^2) \approx I'(Q^2) \approx \frac{2m^2}{Q^2} \Gamma^{as}(Q^2),$$

(48)
where $\Gamma^{as}(Q^2)$ is given by (5). $(I^{res}(Q^2)$ fastly decreases with $Q^2$ and is very small above $Q^2 = 3 GeV^2)$. These conditions are sufficient to determine in unique way the constant $A$ and $B$ in (47). For $I'(Q^2)$ it follows:

$$I'(Q^2) = 2m^2\Gamma^{as}(Q^2_0)\left[\frac{1}{Q^2 + \mu^2} - \frac{c\mu^2}{(Q^2 + \mu^2)^2}\right],$$

(49)

$$c = 1 + \frac{\mu^2}{2m^2} \frac{1}{\Gamma^{as}(Q^2_0)} \left[\frac{1}{4}\kappa^2 + I^{res}(0)\right],$$

(50)

where $I^{res}(0)$ are given by the left column in (39).

The model and eq.49 cannot be used at high $Q^2 > 5 GeV^2$: one cannot believe, that at such $Q^2$ the saturation of the dispersion relation (46) by the lowest vector meson is a good approximation. For this reason there is no matching of (49) with QCD sum rule calculations of twist-4 terms. (Formally, from (49) it would follow $b_{p-n} \approx -0.15$, $b_{p+n} \approx -0.07$). It is not certain, what value of the matching point $Q^2_0$ should be chosen in (49). This results in 10% uncertainty in the theoretical predictions. Fig.2 shows the predictions of the model in comparison with recent SLAC data [32], obtained at low $Q^2 = 0.5$ and $1.2 GeV^2$ as well as SMC and SLAC data at higher $Q^2$. The chosen parameters are $\Gamma^{as}_p(Q^2_0) = 0.142$, $\Gamma^{as}_n(Q^2_0) = -0.061$, corresponding to $c_p = 0.458$, $c_n = 0.527$ in (49),(50). The agreement with the data, particularly at low $Q^2$, is very good. The change of the parameters only weakly influences $\Gamma_{p,n}(Q^2)$ at low $Q^2$. (For example, if we use instead of $\Gamma^{as}_p$, $\Gamma^{as}_n$, mentioned above, the values $\Gamma_p(Q^2_0) = 0.130$, $\Gamma_n(Q^2_0) = -0.045$, then $\Gamma_p(Q^2)$ at $Q^2 < 1.5 GeV^2$ is shifted by less than 10% and the values of $\Gamma_n(Q^2)$ by less than 20%).

**Conclusion.**

The nonperturbative $Q^2$-dependence of the sum rules for spin dependent $e(\mu)N$ scattering is discussed. Two domains of $Q^2$ are considered. At high $Q^2$ the determination of twist-4 corrections by QCD sum rule approach is analysed. It was found, that for the Bjorken sum rule the twist-4 correction is small and, although its uncertainty is large, about 50%, it does influences too much Bjorken sum rule and the value of $\alpha_s$ determined from this sum rule. However, in order to have the reliable determination of $\alpha_s$ from comparison of the Bjorken sum rule with the data, the accuracy of the latter must be improved by a factor of 3-4. For the singlet sum rule – $p + n$ or proton separately – the QCD sum rule approach gives only the order of magnitude of twist-4 correction. At $Q^2 = 5 GeV^2$ the twist-4 correction in this case is smaller than the today experimental error. At low and intermediate $Q^2$ a model was presented, which realizes a smooth connection of GDH sum rules at $Q^2 = 0$ with the sum rules for $\Gamma_p(Q^2)$, $\Gamma_n(Q^2)$ at high $Q^2$. The agreement of the model with recent data is perfect.
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**Figure Captions**

**Fig. 1** The $Q^2$-dependence of integrals $I_p(Q^2), I_n(Q^2), I_p(Q^2) - I_n(Q^2)$. The vertical axis is broken at negative values.

**Fig. 2** The $Q^2$-dependence of $\Gamma_p = \Gamma'_p + \Gamma_{p}^{\text{res}}$ (solid line), described by eqs.(45,49,50). $\Gamma_{p}^{\text{res}}$ (dash-dotted) and $\Gamma'_p$ (dashed) are the resonance and nonresonance parts. The experimental points are: the dots from E143 (SLAC) 32, the square - from E143 (SLAC) 33, the cross - SMC-SLAC combined data 1, the triangle from SMC cite1.

**Fig. 3** The same as in Fig.2 but for neutron. The experimental points are: the dots from E143 (SLAC) measurements on deuteron 32, the square at $Q^2 = 2GeV^2$ is the E142(SLAC) 34 data from measurements on polarized $^3He$, the square at $Q^2 = 3GeV^2$ is E143(SLAC) 35 deuteron data, the cross is SMC-SLAC combined data 1, the triangle is SMC deuteron data 1.