Determination of the Biot coefficient of limestones from Perm Krai, Russia

A E Kukhtinskiy, D V Shustov, A A Efimov, Yu A Kashnikov and S G Ashikhmin
Perm National Research Polytechnic University, Perm, 614990, Russia
artyom@pstu.ru

Abstract. The study presents results of Biot coefficient determination for limestones from Perm krai, Russia. To the date there is no standard method to determine the poroelastic parameters. That’s why different researchers use different approaches to perform these tests. In this research an attempt was made to compare how some aspects of Biot coefficient determination influence its values. The tests were performed on a triaxial test system PIK-UIDK/PL where deformations are measured by LVDT sensors. The Biot coefficient was calculated using two different equations. The results from three types of tests were used for these calculations. These tests are: 1) test when both confining and pore pressures are changed for the same values (similar to what is called unjacketed test), 2) drained test (confined pressure is changed while pore pressure is maintained at a constant value, 3) pore pressure change (while confining pressure is maintained at a constant value). The variation of the calculated Biot coefficient values in relation to the amplitude of pressure variation in each test was analyzed. The tests for each sample were performed with small pressure steps in the beginning and then the tests were repeated with increased maximum amplitude and larger pressure steps. The results show significant nonlinearity in the deformation of samples with larger pressure amplitude. This results in significantly higher Biot coefficient values when the full load cycle is used for calculation. The obtained Biot coefficient values were then correlated with the P-wave velocity determined for these samples in the reservoir conditions on the test system in the beginning of each test.

1. Introduction
Nowadays Biot coefficient is widely used to calculate effective stress in rock and its value variation can have a dramatic effect on results. So it is a very important poroelastic parameter in geomechanics. Despite this, many researchers still use the value of 1 in their calculations due to the lack of data on the Biot coefficient variation. The concept of effective stress was introduced by Terzaghi [1]. It was later modified considering advances in poroelasticity. The fundamental principles of the poroelastic theory were presented in the paper by Biot [2] and later advanced in his following papers and papers by other authors. In 1971 Nur and Byerlee [3] proposed the now well-known equation for the effective stress \((\sigma_{ij})\) defined using the coefficient \(\alpha\) which is now called the Biot (or Biot’s or poroelastic) coefficient.

\[
(\sigma_{ij}) = \sigma_{ij} - \alpha P_p \cdot \delta_{ij}
\]
where $\sigma_{ij}$ is the total stress tensor, $P_p$ is pore pressure and $\delta_{ij}$ is the Kronecker delta.

The Biot coefficient is defined in several ways. The most common is [4]

$$\alpha_1 = 1 - \frac{K}{K_s}$$

(2)

where $K$ is the specimen bulk modulus in drained conditions obtained by changing the confining pressure while keeping the pore pressure constant, $K_s$ is the grain bulk modulus.

Another way to define the Biot coefficient is through the bulk modulus $K_P$ obtained by changing the pore pressure while keeping the confining pressure constant:

$$\alpha_2 = \frac{K}{K_P}$$

(3)

These two ways to determine the Biot coefficient were compared in the paper by Lion et al. [5] along with a third method which requires the determination of the undrained bulk modulus. They tested 6 similar specimens and the mean values from the two formulas presented here were about 6% apart. Although the maximum difference between the two values for one specimen was more than 10%. In the presented work results from undrained tests gave significantly different results (up to 30% difference). It is assumed to be connected with technical difficulties in performing truly undrained tests.

Another way to determine the Biot coefficient is through the “null” test which is demonstrated in the paper by da Silva et al. [6]. In this test they use Eq. (1) to calculate the confining pressure for different values of the Biot coefficient for the fixed pore pressure change. They apply this pore pressure change and the calculated confining pressure change in steps, going back to initial conditions every step and measure the resulting strains. Then it can be possible to plot the “preassigned” Biot coefficient vs strains and find the actual Biot coefficient by finding the intersection of the approximated line with the y-axis.

Different dependences of the Biot coefficient on porosity are presented in [7], [8], [9]. In [10] the variation of the Biot coefficient on effective stress and different types of specimen behavior under loading are analyzed.

In this paper the laboratory study of Biot coefficients of limestone from a local oil field is presented. The main purpose of the study was to obtain a relationship for the Biot coefficient which can be used for geomechanical models of carbonate reservoirs in Perm krai and especially for the oil field which was studied in the work. Equations (2) and (3) and the corresponding tests were used to determine the Biot coefficient. The study analyses some aspects and options on how to perform the tests.

2. Specimen preparation

Six limestone specimens from an oil field in Perm krai (Russia) were used in this study. The specimens were chosen from a larger group based on their porosity so they would represent the maximum range possible. The specimens were prepared according to Russian GOST standards. They were drilled from full-sized core samples in the direction parallel to bedding planes. Then they were cut and their faces were grinded to be flat. The prepared specimens were 38 mm in diameter and 80 mm in height. These sizes were chosen for the whole group of specimens because most of them were used in triaxial tests and it is the requirement of the corresponding recommendations and standards that specimens have at least 2:1 height to diameter ratio. For these poroelastic tests this also helps to decrease the influence of end effects. Quite often researchers don’t mention specimen size or use 1:1 ratio for these kinds of tests. It is one of the uncertainties that rises from the lack of the common practice for the determination of poroelastic constants. Next, the specimens were cleared of the remaining hydrocarbons using the Soxhlet extractor. Ethanol-benzene mixture (1:2) was used as a solvent. It required 2 weeks to completely clean the specimens from hydrocarbons. Then the
specimens were dried at 105°C temperature until their weight stabilized and after that they were put in desiccators above calcium chloride. The dry specimens were weighed for subsequent porosity calculations. All the specimens were saturated with kerosene. The voids were filled with fluid using vacuum. It was performed in three stages: evacuation of the specimens and saturating liquid separately, capillary impregnation of the specimens, additional saturation of the specimens in the liquid at atmospheric pressure. The time length of individual stages of saturation of the specimens depended on the value of absolute permeability with the minimum time of 12.5 hours. Next the specimens were weighed hydrostatically in kerosene and in the air. The three weights were used to calculate the porosity. The bulk density was also calculated using this data. The results are presented in table 1.

It would have been better to simply drill the specimens just after the core is raised from the well to minimize the distortion of rock characteristics due to various influences. But currently it’s very hard to organize this process in our region and the core usually dries before it makes it to the lab.

| Specimen number | Depth, m | Open porosity (%) | Density of the saturated specimen (g/cc) |
|-----------------|----------|--------------------|------------------------------------------|
| 6               | 1776.68  | 9.81               | 2.52                                     |
| 11              | 1759.38  | 4.93               | 2.61                                     |
| 20              | 1768.21  | 3.78               | 2.64                                     |
| 38              | 1753.48  | 13.68              | 2.45                                     |
| 40              | 1776.44  | 9.76               | 2.52                                     |
| 41              | 1776.93  | 10.18              | 2.51                                     |

3. Poroelastic tests

3.1. Equipment

The tests were performed using Russian triaxial test equipment PIK-UlDK/PL. It is a standard triaxial system which is capable of creating up to 140 MPa confining pressure and 70 MPa pore pressure. The system uses plunger pumps to create confining, pore pressures and pressure in the hydraulic cylinder to create the axial stress. There is also an additional pore pressure line to be able to measure steady state permeability on this equipment. This line is used in the poroelastic tests to speed up pore pressure redistribution around the specimen. Digital manometers are used to control the pressures. All experiment stages are controlled using the software on PC. It is possible to load with a constant stress or strain rate using the built in PID controller. The pumps have a very fine fluid rate control due to the use of stepper motors. The specimens were put inside a PTFE membrane to isolate them from the confining oil. The membrane was made specifically for these tests and had a thickness of only 0.25 mm. Special tests on standard metal specimens were performed to ensure that the membrane has no significant effect on the measurement of lateral strains like some types of membranes have. LVDT-sensors were used for deformation measurements. Two sensors were used to measure the axial deformation of the whole specimen. They were put 180° apart around the specimen and fixed to loading pistons just above and below the specimen. It is done using special mounting rings with spring loaded screws. Also one sensor was used to measure circumferential deformation. It was mounted on a chain which was put around the middle part of the specimen.
3.2. Testing procedure

Three types of tests were done for each specimen to calculate the Biot coefficient in two ways using equations (2) and (3). The values of pressures representing reservoir conditions were used as a starting point for each type of test. They were calculated for each specimen based on previous research by authors about the stress state in the region. The confining pressure $P_C$ used in this study was calculated as an average between three stresses:

$$P_C = \frac{\sigma_V + \sigma_H + \sigma_h}{3},$$

where $\sigma_V, \sigma_H, \sigma_h$ are the vertical, the maximum horizontal and the minimum horizontal stresses.

The value of confining pressure for different specimens was in the range from 36.5 to 37 MPa. The pore pressure value was taken from the reservoir model of the oil field in the points from where samples were taken: $P_p = 16.1$ MPa. Confining and pore pressures were changed with a rate of 1 MPa/min on all stages of the experiment. The tests were performed under room temperature as it is very close to the actual reservoir conditions for studied rocks.

In the start of the experiment the test cell is filled with oil. Then the confining pressure is raised to 2 MPa and the pore fluid is pumped into the system, through the specimen and all the pipes with the valve on the other end of the specimen open to the fluid container. This is done to drive all the air out of the system. Then confining and pore pressures are increased simultaneously, keeping the difference between them constant, until the value of pore pressure reaches the value of reservoir pore pressure $P_p$. In the beginning, the pore pressure is raised from one end of the specimen and the value of pressure is observed on the sensor on the other end of the specimen. If there is a significant lag then the rate of pressure increase needs to be lowered. If this rate is adequate than the pore pressure line from the second pump to another end of the specimen is opened to speed up the pore pressure redistribution and stabilization in the specimen. Then the confining pressure is raised to the reservoir value with the same loading rate. After that the specimen is left to consolidate overnight for around 15 hours.

The first type of test is performed to determine the matrix bulk modulus $K_S$. In this test both confining and pore pressures are lowered in equal steps and then raised back again to reservoir conditions. The second type of test is done to determine the drained bulk modulus of the specimen $K$. It is performed by changing confining pressure and keeping pore pressure at a constant value. From the third type of test we get $K_p$ by keeping the confining pressure constant and changing pore pressure. In each test the pressure is changed in steps and after each step there is a pause to wait for the volumetric strain stabilization. An example of this pause for one of the steps is presented on figure 1. The wait lasted around six minutes in that case.

![Figure 1. Waiting for volumetric strain stabilization after changing pressure.](image-url)
Each specimen was tested twice with smaller and larger pressure steps, and with smaller and larger total pressure change in an experiment. This was done to compare the difference between these approaches. These six loading stages in the order previously described are demonstrated on figure 2. Each full test for one specimen took a full working day: around 9 hours for disassembly/assembly and performing the loading stages and the rest of the time was spent for specimen consolidation overnight. Also the P-wave velocity was measured for each specimen in reservoir conditions.

4. Results and discussion
A program to process all the results was written in MATLAB. The deformations from the axial and circumferential sensors had significantly different values in many cases. It was most evident in the first type of test where the values of deformations are one order less than in the second and third types of tests. Sometimes only one of two axial sensors showed some deformation. It is probably connected with the fact that slight deviations of the specimen end faces where it contacts loading pistons affect the way the deformations are measured when they are small. In triaxial tests it is usually observed only in the beginning of the test. There may be a skew or some other end effects which make axial deformations unusable for calculation in some cases. For this reason it was decided to use only data from the circumferential sensor to calculate the volumetric strain as it behaves reliably at all times and in processing the results it was seen that this data is consistent.

The loading diagrams for one of the specimens are presented on figure 3. $\Delta \sigma$ here is the change in confining, pore pressure or both pressures between steps depending on the experiment stage. The bulk modulus for each stage was found using the approximation of both forward and backward loading curves. The top row represents the first three loading stages with smaller steps and the bottom row is the last three stages with larger steps.

It can clearly be seen that dropping the pressures for a larger amount leads to a significant nonlinearity of the diagrams. Subsequently all bulk modulus values reduce a lot, sometimes more than 2 times compared to the initial loading. It is most likely connected with the opening of microcracks when the pressure difference is reduced more. The fact that the matrix bulk modulus $K_S$ doesn’t change as much as $K$ or $K_P$ supports this hypothesis. The similar pattern can be seen for other 5 specimens. All determined bulk moduli values are listed in table 2.
Figure 3. Loading diagrams of 6 experiment stages for specimen 40.

Table 2. Bulk moduli values for all specimens (GPa).

| Specimen ID | 6   | 11  | 20  | 38  | 40  | 41  |
|-------------|-----|-----|-----|-----|-----|-----|
| Smaller     |     |     |     |     |     |     |
| pressure    |  \(K_S\) 62.6 | 61.9 | 73.8 | 77.3 | 84.6 | 80.2 |
| change      |  \(K_P\) 44.4 | 59.1 | 120.0 | 31.1 | 40.8 | 24.9 |
| Larger      |     |     |     |     |     |     |
| pressure    |  \(K_S\) 68.3 | 60.4 | 67.8 | 80.7 | 73.9 | 70.2 |
| change      |  \(K_P\) 16.8 | 29.7 | 69.9 | 15.6 | 16.6 | 11.1 |

So, the values obtained with smaller pressure change amplitude were used to calculate the Biot coefficient using Eq. (1) and (2). The values are presented in table 3.

Table 3. Biot coefficient values for all specimens.

| Specimen ID | 6   | 11  | 20  | 38  | 40  | 41  |
|-------------|-----|-----|-----|-----|-----|-----|
| \(\alpha_1\) | 0.587 | 0.522 | 0.378 | 0.728 | 0.714 | 0.781 |
| \(\alpha_2\) | 0.583 | 0.501 | 0.382 | 0.674 | 0.593 | 0.705 |
| Average     | 0.585 | 0.512 | 0.380 | 0.701 | 0.654 | 0.743 |

It can be seen that \(\alpha_1\) and \(\alpha_2\) values are very close for most specimens. The largest the difference between two ways to determine the Biot coefficient is around 20% for specimen 40. The diagrams for this specimen are presented on figure 3 and it’s unclear why there is such a big difference since the diagrams look normal and it doesn’t look like a problem with the experiment. Mostly, \(\alpha_1\) is larger than
\( \alpha_2 \) except in one case where the values differ just by 1%. The average values were used subsequently since it’s not clear which formula and test method gives more accurate estimation of the Biot coefficient.

The Biot coefficient needs to be known for different geomechanical calculations where rock properties can vary across the rock section. For that reason there needs to be a dependence of this parameter on something that is available through well logs or seismic research data to calculate it in all points of the model. P-wave velocity \( V_p \) is used here for this purpose as it can also be measured during these tests on triaxial equipment. The approximation by power law was performed using average values of the Biot coefficient presented in table 3. The approximation was done in a way to satisfy theoretical boundaries of the Biot coefficient, i.e. to approach 1 with very low \( V_p \) values and 0 with very high \( V_p \) values which correspond to extremely tight limestone. Of course, it would be better to have specimens across all this range to get a more accurate dependence. However, testing specimens with high P-wave velocity which have low porosity and permeability is very difficult due to the time needed for the pore pressure to equalize across the specimen. It’s almost impossible to perform such long tests when the work is done under a contract with an industrial partner (like this work) when the results are needed quickly to construct geomechanical models and make decisions on certain field operations. It is important to note that porosity and P-wave velocity didn’t have a straightforward relationship, most likely due to the fact that it was open porosity and it was determined using measurements in atmospheric conditions or due to differences in the mineralogical composition. Therefore, the dependence of the Biot coefficient on this porosity was much worse than the dependence on P-wave velocity.

![Figure 4. The dependence of the Biot coefficient on P-wave velocity.](image)

There is no standard or ISRM suggested method on how to determine the Biot coefficient. There are a number of questions on how to perform the tests.

It is always an open question on how accurate the estimate of reservoir conditions is and whether the specimen should be tested starting from reservoir conditions at all. If there is a significant error in these estimate, then the specimen may deform in a different way than needed for the test and our measurements won’t represent what happens in the rock mass. In the next poroelastic tests it is planned to check if the reservoir conditions are well estimated by testing one of the specimen from a lower value of estimated stress difference \( P_C - P_P \). Then by increasing the effective stress it is
possible to find the stress range where bulk modulus stops increasing. If we get a stable linear response from the specimen then we can later use that stress range for testing other specimens. Then it’s possible to test where that linear range will end and pore collapse will begin to limit the maximum amplitude of effective stress change in the poroelastic test. It can be done either by tracking the decrease of the bulk modulus or by measuring the permeability at each step and tracking when it starts to decline rapidly. An example of tracking permeability through the test can be seen in the paper by da Silva et al. [6]

Also it is planned to test if it’s possible to find a low enough loading rate where there will be no need to change the pressures in steps and wait for volume deformation to stabilize after each step. The approximation of the whole loading curve should decrease the error compared with the approach where single points are approximated and there is subjectivity in controlling the stabilization of volume deformation when it’s done in steps. The example on figure 1 is quite clear but often there is no such clear inflection point and it has to be guessed where to stop the step based on the previous steps and general experience with these tests. There is a difficulty with how long potentially the tests could take with a very low loading rate for low permeability specimens where pore pressure takes a long time to equalize in different parts of the specimen and consequently volumetric deformations don’t stabilize for a long time. Another potential problem in this case is that some deformation may be connected with creep and not with poroelastic effects.

5. Conclusions
Six limestone specimens were tested to determine the Biot coefficient. Comparison of different methods and some aspects on how to determine the poroelastic constants was performed. The dependence of the Biot coefficient on P-wave velocity was established which was used in geomechanical modeling.

It would benefit new researchers a lot if there will be a standard testing method for determination of the poroelastic properties. There are a lot of questions on how exactly the test should be done like it is with many other complex tests. Many researchers use different approaches which make it hard comparing their results. For practical purposes, of interest for engineering applications, best practices such as an ISRM suggested methods for the determination of the Biot coefficient would be highly beneficial like it’s done for other tests for important geomechanical parameters.

References
[1] Terzaghi K 1943 *Theoretical Soil Mechanics* (Hoboken, NJ, USA: John Wiley & Sons, Inc.)
[2] Biot M A 1941 General Theory of Three-Dimensional Consolidation *J. Appl. Phys.* 12 155
[3] Nur A and Byerlee J D 1971 An exact effective stress law for elastic deformation of rock with fluids *J. Geophys. Res.* 76 6414–9
[4] Biot M A and Willis D G 1957 The Elastic Coefficients of the Theory of Consolidation *J. Appl. Mech.* 594–601
[5] Lion M, Skoczylas F and Ledésert B 2004 Determination of the main hydraulic and poroelastic properties of a limestone from Bourgogne, France *Int. J. Rock Mech. Min. Sci.* 41 915–25
[6] Ramos da Silva M, Schroeder C and Verbrugge J C 2010 Poroelastic behaviour of a water-saturated limestone *Int. J. Rock Mech. Min. Sci.* 47 797–807
[7] Krief M, Garat J, Stellingwerff J and Vetre J 1991 A petrophysical interpretation using the velocities of P and S waves (full-waveform sonic) *Petrophysics* 31 355–69
[8] Laurent J, Bouteca M J, Sarda J P and Bary D 1993 Pore-pressure influence in the poroelastic behavior of rocks: experimental studies and results *SPE Form. Eval.* 8 117–22
[9] Bailin W 2001 Biot’s Effective Stress Coefficient Evaluation: Static And Dynamic Approaches (CRC Press)
[10] Zhou X, Ghassemi A, Riley S and Roberts J 2017 Biot’s effective stress coefficient of mudstone source rocks *51st US Rock Mech. / Geomech. Symp. 2017* 2 780–90