Resummation for the Tevatron and LHC boson production at small $x$

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Abstract

Analysis of small-$x$ semi-inclusive DIS hadroproduction suggests that multiple parton radiation leads to a broadening of transverse momentum ($q_T$) distributions beyond that predicted by a straightforward calculation in the Collins-Soper-Sterman formalism. This effect can be modeled by a modification of the resummed form factor in the small-$x$ region. We discuss the impact of such a modification on the production of electroweak bosons at hadron-hadron colliders. We show that if substantial small-$x$ broadening is observed in forward $Z^0$ boson production in the Tevatron Run-2, it would strongly affect predicted $q_T$ distributions for $W$, $Z$, and Higgs boson production at the Large Hadron Collider.

In the production of electroweak bosons, precise knowledge of the transverse mass $M_T$ and transverse momentum $q_T$ provides detailed information about the production process, including the mass of the boson and associated radiative corrections. At the Tevatron, $q_T$ distributions of $Z^0$ bosons offer insight into soft gluon radiation, and this information is then used for precision extraction of the $W$ boson mass. At the LHC, good knowledge of the transverse distribution of Higgs bosons $H^0$ will be needed to efficiently separate Higgs boson candidates from the large QCD background. Accurate predictions for the small-$q_T$ region are obtained via resummation of large logarithms $\ln^n(q_T/Q)$ arising from unsuppressed soft and collinear radiation in higher orders of perturbation theory.

As we move from the 2 TeV Tevatron to the 14 TeV LHC, typical values of partonic momentum fractions $x$ for producing $W$, $Z^0$, and $H^0$ bosons become smaller, thus enhancing $\ln(1/x)$ terms in higher orders of $\alpha_s$. It is not entirely known how these terms (not included in a fixed-order cross section or conventional $q_T$ resummation) will affect $W$, $Z^0$, and $H^0$ production at the LHC energies, in part because no Drell-Yan $q_T$ data is available yet in the relevant region of $x$ of a few $10^{-3}$ or less.

Studies $[1, 2]$ in the crossed channel of semi-inclusive deep-inelastic scattering (SIDIS) suggest that hadronic $q_T$ distributions at small $x$ cannot be straightforwardly described within the Collins-Soper-Sterman (CSS) resummation framework $[3]$, if the nonperturbative Sudakov function behaves like its large-$x$ counterpart from the Drell-Yan process. A $q_T$ distribution in SIDIS at $x < 10^{-2}$ is substantially broader than the conventional CSS prediction. The broadening effect can be modeled by including an extra $x$-dependent term in the Sudakov exponent. To describe the data, the extra term must grow quickly as $x \to 0$. It noticeably contributes to the resummed form factor at intermediate impact parameters ($b \sim 1/q_T < 1$ GeV$^{-1}$), which hints at its origin from perturbative physics. A possible interpretation of this term is that it mimics higher-order contributions of the form $\alpha_s^n \ln^n(1/x)$, which are not included in the resummed cross section. Due to the two-scale nature of the $q_T$ resummation problem, the non-resummed $\ln(1/x)$ terms may affect the $q_T$ distribution even when they leave no discernible trace in inclusive DIS structure functions. The DIS structure functions depend on one hard scale (of order $Q$), while the CSS resummation formula (cf. Eq. $[1]$) also includes contributions from large impact parameters $b$ (small momentum scales). As $b$ becomes large, the series $\alpha_s^n (1/b) \ln^n(1/x)$ in the CSS formula may begin to diverge at a larger value of $x$ than the series $\alpha_s^n (Q) \ln^n(1/x)$ in the inclusive structure functions. For this reason, transition to $k_T$-unordered (BFKL-like $[4]$) physics may happen at larger $x$ in $q_T$ distributions than in inclusive (one-scale) observables.

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The $q_T$ broadening discussed above was observed in semi-inclusive DIS processes. In this study, we explore its possible implications for the (crossed) Drell-Yan process. We begin by examining the resummed transverse momentum distribution for the Drell-Yan process [3], following notations from Ref. [5]:

$$\frac{d\sigma}{dy dq_T^2} = \sigma_0 \frac{S}{S} \int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{q}_T \cdot \vec{b}} \tilde{W}(b, Q, x_A, x_B) + Y(q_T, Q, x_A, x_B).$$

(1)

Here $x_{A,B} \equiv Q e^{\pm y}/\sqrt{S}$, the integral is the Fourier transform of a resummed form factor $\tilde{W}$ given in impact parameter ($b$) space, and $Y$ is a regular (finite at $q_T \to 0$) part of the fixed-order cross section. In the small-$b$ limit, the form factor $\tilde{W}$ is given by a product of a perturbative Sudakov exponent $e^{-S_P}$ and generalized parton distributions $\overline{P}(x, b)$:

$$\tilde{W}(b, Q, x_A, x_B) \bigg|_{b^2 \ll \Lambda_{QCD}^2} = e^{-S_P(b, Q)} \overline{P}(x_A, b) \overline{P}(x_B, b).$$

(2)

At moderately small $x$, where the representation [2] for $\tilde{W}$ holds, we write these generalized parton distributions in the form

$$\overline{P}(x, b) \bigg|_{b^2 \ll \Lambda_{QCD}^2} \sim (C \otimes f)(x, b_0/b) e^{-\rho(x) b^2},$$

(3)

where $C(x, b_0/b)$ are coefficient functions, $f(x, \mu)$ are conventional parton distributions, and $b_0 = 2e^{-\gamma_E} = 1.12...$ is a commonly appearing constant factor.

The term $e^{-\rho(x) b^2}$ in $\overline{P}(x, b)$ will provide an additional $q_T$ broadening, with an $x$ dependence specified by $\rho(x)$. For example, it may approximate $x$-dependent higher-order contributions that are not included in the finite-order expression for $(C \otimes f)$. We parametrize $\rho(x)$ in the following functional form:

$$\rho(x) = c_0 \left( \sqrt{\frac{1}{x^2} + \frac{1}{x_0^2}} - \frac{1}{x_0} \right),$$

(4)

such that $\rho(x) \sim c_0/x$ for $x \ll x_0$, and $\rho(x) \sim 0$ for $x \gg x_0$. This parameterization ensures that the formalism reduces to the usual CSS form for large $x (x \gg x_0)$ and introduces an additional source of $q_T$ broadening (growing as $1/x$) at small $x (x \ll x_0)$. The parameter $c_0$ determines the magnitude of the broadening for a given $x$, while $x_0$ specifies the value of $x$ below which the broadening effects become important.

In principle, $c_0$ and $x_0$ may depend on the hard scale $Q$; in this first study, we neglect this dependence. Based on the observed dependence $\rho(x) \sim 0.013/x$ at $x \lesssim 10^{-2}$ in SIDIS energy flow data [2], we choose $c_0 = 0.013$ and $x_0 = 0.005$ as a representative choice for our plots.

As $x \to 0$, the additional broadening term in Eq. (3) affects the form factor $\tilde{W}$ both at perturbative ($b \lesssim 1$ GeV$^{-1}$) and nonperturbative ($b \gtrsim 1$ GeV$^{-1}$) impact parameters. In addition, the resummed cross section contains conventional non-perturbative contributions from power corrections, which become important at large impact parameters ($b \gtrsim 1$ GeV$^{-1}$). We introduce these corrections by replacing the impact parameter $b$ in functions $S_P$ and $(C \otimes f)$ with a variable $b_\ast = b/\sqrt{1 + b^2/(0.25 \text{ GeV}^{-2})}$ [3] and including a nonperturbative Sudakov exponent $\exp\{-S_{NP}(b, Q)\}$. The function $S_{NP}(b, Q)$ is parametrized by a 3-parameter Gaussian form from a recent global fit to low-energy Drell-Yan and Tevatron Run-1 $Z^0$ data [5]. Combining all the terms, we have:

$$\frac{d\sigma}{dy dq_T^2} = \sigma_0 \frac{S}{S} \int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{q}_T \cdot \vec{b}} (C \otimes f)(x_A, b_0/b_\ast) (C \otimes f)(x_B, b_0/b_\ast) \times$$

$$e^{-S_P(b_\ast, Q) - S_{NP}(b, Q) - b^2 \rho(x_A) - b^2 \rho(x_B)} + Y.$$

(5)

Figs. 1 and 2 show the comparison of the resummed cross section [5] with the additional broadening term ($\rho(x) \neq 0$) to the resummed cross section without such a term ($\rho(x) = 0$). We consider cross
sections for the production of $Z^0$ and $H^0$ bosons, calculated according to the procedures in Refs. [6] and [7], respectively. The numerical calculation was realized using the programs Legacy and ResBos [5, 6], and with the CTEQ6M1 parton distribution functions [9]. The perturbative Sudakov factor was included up to $\mathcal{O}(\alpha_s^2)$, and the functions $(C \otimes f)$ up to $\mathcal{O}(\alpha_s)$. The relevant perturbative coefficients can be found in Refs. [3, 8].

Fig. 1(a) shows the differential distribution $d\sigma/dq_T$ for $Z$ boson production in the Tevatron Run-2, integrated over the rapidity $y$ of the $Z$ bosons. We observe that the distribution with the additional small-$x$ term (the dashed curve) essentially coincides with the standard CSS distribution (the solid curve). When $y$ is integrated over the full range, both resummed cross sections are dominated by contributions from $x \sim 0.05 \gg x_0$, where the additional broadening (given by the function $\rho(x)$) is negligible. For this reason, the Tevatron distributions that are inclusive in $y$ (e.g., the Run-1 $Z^0$ boson data) will not be able to distinguish the small-$x$ broadening effects from uncertainties in the nonperturbative Sudakov function $S_{NP}$.

In contrast, the small-$x$ broadening does lead to observable differences in the $q_T$ distributions in the forward rapidity region, where one of the initial-state partons carries a smaller momentum fraction than in the central region. Fig. 1(b) shows the cross section $d\sigma/dq_T$ for $Z$ bosons satisfying $|y| > 2$. The peak of the curve with $\rho(x) \neq 0$ is lower and shifted toward higher $q_T$. While this difference was not large enough to be observed in the Tevatron Run-1, it seems to be measurable in the Run-2 given the improved acceptance and higher luminosity of the upgraded Tevatron collider. The small-$x$ broadening is more pronounced in $W$ boson production due to the smaller mass of the $W$ boson.

We now turn to the LHC, where the small-$x$ broadening may be observed in the whole rapidity range due to the increased center-of-mass energy. Fig. 2(a) displays the distribution $d\sigma/dq_T$ for $Z^0$ production with and without the small-$x$ terms. Here, the difference is striking even if $y$ is integrated out. Effects of a similar magnitude are present in $W$ boson production, and they are further enhanced in the forward regions.

The small-$x$ broadening is less spectacular, but visible, in the production of light Standard Model
Higgs bosons via the effective $ggH$ vertex in the limit of a heavy top quark mass. Fig. 2(b) displays the resummed cross sections for production of Higgs bosons with a mass $M_H = 120$ GeV for several choices of $S_{NP}$ and the broadening term. We first compare cross sections for $\rho(x) = 0$ and $\rho(x) \neq 0$ (thick lines), where the functions $\rho(x)$ and $S_{NP}(b, Q)$ are taken to be the same as in $Z^0$ boson production. The difference between the two cross sections is not large, due to a harder $q_T$ spectrum in the Higgs boson case. The peaking in the $gg$-dominated $H^0$ distribution occurs at $q_T = 10 - 20$ GeV, i.e., beyond the region where the function $\rho(x)b^2$ play its dominant role. This is different from the $q\bar{q}$-dominated $Z^0$ distribution, where the peak is located at $q_T \sim 5 - 10$ GeV and is strongly affected by $\exp\{-\rho(x)b^2\}$.

Hence, for the same function $\rho(x)$ as in the $Z^0$ boson case, the difference between the curves with and without $\rho(x)$ is minimal. The harder $q_T$ spectrum in the Higgs boson case is induced by a larger leading-logarithm coefficient ($C_A$) in $gg$ channels, as compared to the leading-logarithm coefficient $C_F$ in $q\bar{q}$ channels. This suggests that the $Q$-dependent part (and possibly other terms) of the nonperturbative Sudakov function $S_{NP}$ in Higgs boson production is also multiplied by a larger color factor than in the Drell-Yan process. We estimate this effect by multiplying $S_{NP}$ by the ratio of the leading color factors in Higgs and $Z^0$ boson production processes, $C_A/C_F = 9/4$ (the thin solid line). The resulting change turns out to be small because of the reduced sensitivity of the Higgs boson cross section to nonperturbative contributions.

Additional constraints on the small-$x$ behavior of the resummed cross sections in the $gg$ channel could be obtained from examination of photon pair production away from the Higgs signal region. As
the mass of the photon pair decreases, $\gamma\gamma$ production in the gluon fusion channel via a quark box diagram becomes increasingly important. For instance, the subprocess $gg \to \gamma\gamma$ contributes up to 40% of the total cross section at $Q = 80$ GeV \cite{10}. By comparing $q_T$ distributions in $pp \to \gamma\gamma$ and $pp \to Z$ in the same region of $Q$, one may be able to separate the $q\bar{q}$ and $gg$ components of the resummed cross section and learn about the $x$ dependence in the $gg$ channel.

To summarize, we argue that a measurement of transverse momentum distributions of forward $Z$ bosons at the Tevatron will provide important clues about the physics of QCD factorization and possibly discover broadening of $q_T$ distributions associated with the transition to small-$x$ hadronic dynamics. Based upon the analysis of $q_T$ broadening effects observed in semi-inclusive DIS, we have estimated similar effects in the (crossed) processes of electroweak boson production at hadron-hadron colliders. While the estimated impact on the Higgs boson cross section $d\sigma/dq_T$ at high $q_T$ was found to be minimal, much larger effects may occur in $W$ and $Z$ boson production in the forward region at the Tevatron Run-2, and at the LHC throughout the full rapidity range. If present, the small-$x$ broadening will have to be taken into consideration in precision studies of electroweak boson production. Additionally, its observation will provide insights about the transition to $k_T$-unordered (BFKL-like) dynamics in multi-scale distributions at hadron-hadron colliders.

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References

[1] P. Nadolsky, D. R. Stump, and C.-P. Yuan, Phys. Rev. D 61, 014003 (2000) [Erratum-ibid. D 64, 059903 (2001)].

[2] P. M. Nadolsky, D. R. Stump, and C.-P. Yuan, Phys. Rev. D 64, 114011 (2001).

[3] J. C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys. B 250, 199 (1985).

[4] E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Sov. Phys. JETP 44, 443 (1976); I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. 28, 822 (1978).

[5] F. Landry, R. Brock, P. M. Nadolsky, and C.-P. Yuan, Phys. Rev. D 67, 073016 (2003).

[6] C. Balazs and C.-P. Yuan, Phys. Rev. D 56, 5558 (1997).

[7] C. Balazs and C. P. Yuan, Phys. Lett. B 478, 192 (2000).

[8] J. Kodaira and L. Trentadue, Phys. Lett. B 112, 66 (1982); C. T. H. Davies and W. J. Stirling, Nucl. Phys. B 244, 337 (1984); S. Catani, E. D’Emilio and L. Trentadue, Phys. Lett. B 211, 335 (1988); R. P. Kauffman, Phys. Rev. D 44, 1415 (1991); C.-P. Yuan, Phys. Lett. B 283, 395 (1992); D. de Florian and M. Grazzini, Phys. Rev. Lett. 85, 4678 (2000); S. Catani, D. de Florian and M. Grazzini, Nucl. Phys. B 596, 299 (2001).

[9] D. Stump, J. Huston, J. Pumplin, W. K. Tung, H. L. Lai, S. Kuhlmann, and J. F. Owens, JHEP 0310, 046 (2003).

[10] Z. Bern, L. J. Dixon, and C. Schmidt, Phys. Rev. D 66, 074018 (2002).