Atmospheric Pressure and Snowball Earth Deglaciation

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Abstract When a large amount of CO\textsubscript{2} is added to the atmosphere, both the mixing ratio and the surface pressure increase. This causes the troposphere to expand in two directions. The greenhouse effect from the increased mixing ratio causes the tropopause pressure to decrease, so the troposphere expands upward. The increased surface pressure causes the troposphere to expand downward. The first effect is radiative and well known, while the second is nonradiative and unexplored. Here, a method is presented to compare the effect of tropospheric expansion in each direction on the surface temperature of a Snowball Earth. A series of models, from a gray model to a spectral model with realistic Snowball parameters, are used to illustrate this concept. It is shown that near the deglaciation threshold, most of the increase in surface temperature that follows an increase in CO\textsubscript{2} is due to the nonradiative downward tropospheric expansion at the surface, not the radiative upward expansion at the tropopause. The increased atmospheric mass due to the CO\textsubscript{2} entirely apart from its radiative effect, causes an additional increase in surface temperature and therefore aids Snowball deglaciation.

Plain Language Summary The Earth was previously in a frozen state known as a Snowball Earth. There have been several explanations proposed for how the ice melted. All of these explanations require a large amount of carbon dioxide in the atmosphere, sometimes an unrealistically large amount. When a large amount of carbon dioxide is added to the atmosphere, the surface pressure increases. This enhances the greenhouse effect, which further warms the planet. But it also means that convection reaches higher into the atmosphere, which also warms the surface. This last effect has not been previously studied, and it means that the ice covering a Snowball Earth can melt with a lower amount of carbon dioxide than previously thought.

1. Introduction

The Neoproterozoic Era contained at least two global glaciations: the Sturtian, ending 660 Ma, and the Marinoan, ending 635 Ma (Rooney et al., 2015). Despite differences between the two, each featured ice extending to or near to the equator at sea level. Such a scenario is known as a Snowball Earth. The simplest type of Snowball Earth is a Hard Snowball, in which thick ice covers the entire planet.

The basic explanation for the planet’s escape from a Snowball state was proposed by Kirschvink (1992) and elaborated upon by Hoffman et al. (1998) and Hoffman and Schrag (2002).

Ice cover separated the ocean and the atmosphere, meaning the ocean no longer absorbed excess atmospheric CO\textsubscript{2}. Carbonate–silicate chemical weathering also reduced on land due to the colder temperatures and reduced availability of liquid water. Volcanic outgassing of CO\textsubscript{2} continued unabated, allowing a very large amount of CO\textsubscript{2} to build up in the atmosphere. The resultant greenhouse effect sufficed to raise the equatorial temperature to above 273 K, melting the ice at the equator. Positive ice-albedo feedback then ensured that the ice-line retreated to high latitudes.

The problem with this basic scenario was that the greenhouse effect alone is not sufficient, in most models, to ensure deglaciation. For example, Pfreundt and Schmittbuhl (2004), using FOAM (Fast Ocean Atmosphere Model), was unable to observe a large enough greenhouse effect to deglaciate a Hard Snowball with a reasonable CO\textsubscript{2} inventory (although Abbot et al. [2012] later attributed this to the small amount of cloud condensate produced by FOAM compared with other GCMs). The model was far from deglaciation with 0.2 bar of CO\textsubscript{2}, beyond which FOAM could not be reliably used, and extrapolating his results to an unrealistically high 3.2 bar of CO\textsubscript{2} was still insufficient. A high deglaciation threshold was also found by Hu et al. (2011), who found that roughly 0.2–0.4 bar were required to deglaciate a Hard Snowball, depending on the surface albedo and whether pressure broadening and collision-induced absorption (CIA) were considered. However, clouds could lower the deglaciation threshold for a Hard Snowball to around 0.1 bar of CO\textsubscript{2} (Abbot et al., 2012).
This difficulty in deglaciating a Hard Snowball with a reasonable amount of CO$_2$, along with the problem of explaining how photosynthetic life survived through a Hard Snowball, led others to propose alternative scenarios that would lower the amount of CO$_2$ required for deglaciation (the deglaciation threshold) and/or provide a way to explain the survival of life.

Multiple types of Snowball Earth have been proposed. These include the Waterbelt (also known as a Soft Snowball or Slushball), which contains areas of open ocean at low latitudes (Hyde et al., 2000); the Jormungand state, which is similar to the Waterbelt but with an area of open ocean which is narrower and maintained for different physical reasons (Abbot et al., 2011); a “thin ice” state, which allows some sunlight to penetrate through regions of thin ice to provide energy for photosynthetic life beneath the ice (McKay, 2000; Pollard & Kasting, 2005); and the Mudball, in which the deposition of dust on the ice lowers the planet’s albedo sufficiently to trigger deglaciation with a smaller amount of CO$_2$ than required for a Hard Snowball (Abbot & Pierrehumbert, 2010).

The common thread of these scenarios is that they all still require a significant amount CO$_2$ to deglaciate. In many atmospheric models, the radiatively significant gases are trace gases or, at most, minor contributors to the total mass of the atmosphere. In these Snowball deglaciation scenarios, however, CO$_2$ can constitute a large fraction of the mass of the atmosphere and can therefore substantially increase the surface pressure.

There are radiative mechanisms through which a change in surface pressure can affect the surface temperature, including pressure broadening and CIA. A good summary of the literature on these radiative mechanisms is found in Charnay et al. (2020). On the Early Earth, pressure broadening has been studied for increased N$_2$ (Charnay et al., 2013; Goldblatt et al., 2009) and CO$_2$ (Kasting & Ackerman, 1986), and CIA has been studied for increased N$_2$ (Wordsworth & Pierrehumbert, 2013). Both effects were also investigated for increased CO$_2$ on a Snowball Earth by Hu et al. (2011). These are all radiative mechanisms that operate on the tropopause, changing its temperature, pressure, or both.

Goldblatt et al. (2009) and Charnay et al. (2013) also point out that surface pressure can affect the moist adiabatic lapse rate, which affects the surface temperature.

However, there is another, unexplored, nonradiative consequence of increased surface pressure. In current Earth conditions, when CO$_2$ increases, the tropopause pressure decreases. Since the surface pressure stays constant, the tropopause expands to cover a larger pressure range. This tropospheric expansion is simply another way to conceptualize the enhanced greenhouse effect. In the Snowball scenario, however, it will be shown later that the surface pressure changes much more than the tropopause pressure. To distinguish between these two scenarios, we will refer to tropospheric expansion due to a tropopause pressure decrease as “upward” and that due to a surface pressure increase as “downward.” Figure 1 is a cartoon distinguishing upward and downward tropospheric expansion. Of course, the troposphere can only expand upward relative to the ground, so the terms “upward” and “downward” are simply used for convenience. The effect on the surface temperature of tropospheric expansion is the same whether it is upward or downward.

In Section 2, we explain how an “idealized tropospheric expansion” works under the assumption of a constant tropopause and explore this process in a simple gray radiative–convective model. Section 3 contains a description of our spectral radiative–convective model. In Section 4, we investigate tropospheric expansion using this model in a “Simplified Snowball” scenario, which contains realistic physics but omits certain variables in order to aid understanding of the first-order effect. We find that, unlike with idealized tropospheric expansion, the tropopause temperature and pressure do not remain exactly constant when the amount of CO$_2$ increases. In Section 5, we explain the causes of the changes in the tropopause with the aid of the gray model. In Section 6, we present a method for quantifying the effect of these tropopause changes on the surface temperature, so that they may be compared.
with the warming due to tropospheric expansion at the surface. In Section 7, we rectify the omissions in the Simplified Snowball and examine tropospheric expansion under our best estimate of the Snowball conditions. Finally, in Section 8, we investigate pressure broadening and CIA and show that, under the conditions considered, these radiative consequences of increased surface pressure are not significant when compared to the nonradiative consequence of tropospheric expansion at the surface.

2. Idealized Tropospheric Expansion and a Gray Model

The first case considered is when the surface pressure \( p_s \) increases but the tropopause pressure \( p_t \) and temperature \( T_t \) remain exactly constant. This represents an idealized form of tropospheric expansion.

With a prescribed lapse rate \( \Gamma \), it follows from the equation of state and the assumption of hydrostatic equilibrium that the surface temperature and tropopause temperature are related by a simple formula:

\[
T_s = T_t \left( \frac{p_s}{p_t} \right)^{\frac{n}{\Gamma}}.
\]

If the surface pressure is increased to a new value, \( p'_s \), we can find the new surface temperature, \( T'_s \), and thus the factor by which \( T_s \) has increased (assuming for now that \( p'_t = p_t \) and \( T'_t = T_t \)):

\[
\frac{T'_s}{T_s} = \left( \frac{p'_s}{p_s} \right)^{\frac{n}{\Gamma}}.
\]

This allows the increase in \( T_s \) caused by a given increase in \( p_s \) to be calculated. This is shown in Figure 2, assuming that the initial \( T_s \) is 220 K, \( R = 289.7 \) J kg\(^{-1}\) K\(^{-1}\), and \( \Gamma = 9.8 \) K km\(^{-1}\), which is the dry adiabatic lapse rate.

For an increase in \( p_s \) of just 0.05 bar, in the idealized case with a constant tropopause, Equation 2 predicts a 3 K increase in \( T_s \). For an increase in \( p_s \) of 0.3 bar, near the high end of proposed Snowball Earth deglaciation thresholds, Equation 2 predicts a 20 K increase in \( T_s \).

Given this large potential increase in \( T_s \), it must be determined whether \( p_t \) and \( T_t \) do in fact remain constant.

This investigation is begun with a simple gray atmosphere radiative–convective equilibrium (RCE) model. A gray atmosphere is one in which the absorption coefficient for the radiatively active gas is independent of wavenumber. In this model, we do not assume a priori a constant tropopause temperature or pressure. A convective adjustment is applied when the temperature difference between layers exceeds the critical lapse rate, and the tropopause is defined as the top of the highest layer in which this adjustment is made. Unlike operational definitions that apply to local columns (Hoinka, 1997), this is a global definition that does not assume a temperature inversion, or even a sharp change in temperature gradient. It marks the transition between convective control below and radiative control above. The behavior of the gray model could also be understood with reference to emission layer, as most of the emission to space originates from near an optical depth of one. However, this approach is much less useful in a spectral context, where there is an emission layer for each wavenumber, some of which will be above the tropopause. Since convection dominates radiation in the troposphere, the temperature profile is determined by the location of the convective tropopause as defined here.

Assuming a constant mixing ratio of the gray gas and a critical lapse rate of 9.8 K km\(^{-1}\), the RCE temperature profiles were found for two different surface pressures. These are shown in Figure 3.

It is apparent that \( p_t \) and \( T_t \) do remain constant. This is the expected result, because effectively the only change has been to add new layers to the atmosphere below the emission layer, which means they do not contribute to the top of the atmosphere (TOA) energy balance, while the higher layers remain unchanged. The temperature at the surface is determined by simply extending downward from the constant tropopause to the new surface pressure.
following the critical lapse rate. This behavior is equivalent to idealized tropospheric expansion. We note in passing that if instead we were to increase the surface pressure by the addition of a nonradiative gas, the surface temperature would still increase, but by a lower amount. Because the well-mixed gray gas now has a lower mixing ratio in the stratosphere, the tropopause forms at a higher pressure. The gray model can be used to show that the ratio of new to old tropopause pressures is substantially less than the ratio of new to old surface pressures, explaining the net rise in surface temperature. This scenario is examined in more detail later using the spectral model.

Figure 4 compares the results from the gray model with the predictions made by Equation 2. The gray model results agree well with the theory, which means that idealized tropospheric expansion provides a good description of the behavior of the gray model with a constant gray gas mixing ratio.

However, the gray model has two limitations. First, the gray gas assumption breaks down when dealing with a real gas. Second, in a Snowball deglaciation scenario, the increase in $p_s$ is the result of an increase in CO$_2$ amount, so the CO$_2$ mixing ratio should also increase.

We address both of these limitations with a spectral radiative-convective model.

### 3. Spectral Radiative–Convective Model

We built a radiative–convective model with PRRTM (Planetary Rapid Radiative Transfer Model), the planetary version of RRTM (Rapid Radiative Transfer Model; Mlawer et al., 1997), providing the longwave radiative transfer. PRRTM is designed to cope with conditions outside the range of those typical on Earth in the present day, such as colder temperatures and high concentrations of CO$_2$, and as such does not face the same limitation that Pierrehumbert (2004) encountered with FOAM. PRRTM includes CIA, which becomes important in high-CO$_2$ atmospheres. PRRTM is built using P-LBLRTM, details of which can be found in Eluszkiewicz et al. (2017).

To quantify the amount of a gas in the atmosphere, we use the “gas inventory,” following Pierrehumbert et al. (2011). The inventory of a gas is equal to the pressure that gas would exert on the surface if it was the only gas in the atmosphere. It is directly proportional to the number of molecules of that gas. For example, the CO$_2$ inventory is given by

$$p_{I,CO_2} = \frac{A m_{atm,CO_2} g}{A}$$

(3)

where $m_{atm,CO_2}$ is the total mass of CO$_2$ in the atmosphere, $g$ is the acceleration due to gravity, and $A$ is the surface area of the planet.

For an atmosphere containing only CO$_2$ and dry air (without CO$_2$), the surface pressure is then

$$p_s = p_{I,air} + p_{I,CO_2}$$

(4)

where $p_{I,air}$ is the dry air inventory. The water vapor inventory, $p_{I,H_2O}$, is ignored here as it is small in the cold Snowball climate but could easily be added for a moister climate. For a given $p_{I,CO_2}$ and $p_{I,air}$, the CO$_2$ volume mixing ratio $\chi_{CO_2}$ can be calculated:

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**Figure 3.** Radiative–convective equilibrium (RCE) temperature profiles from the gray model for $p_s = 1$ bar and $p_s = 1.4$ bar with constant gray gas mixing ratio. In each case, the circle marks the tropopause. The red dashed line marks $p = 1$ bar in order to emphasize the increase in $p_s$ and to aid comparison with later figures.

**Figure 4.** Comparison between the predicted increase in $T_s$ as a function of $p_s$ for idealized tropospheric expansion and the results from the gray model.
\( \chi_{CO_2} = \frac{P_{CO_2}}{m_{CO_2}} + \frac{P_{air}}{m_{air}} \), 

(5)

where \( m_{CO_2} \) and \( m_{air} \) are the mean molecular weights of \( CO_2 \) and dry air, respectively.

The \( CO_2 \) partial pressure is then given by

\[ P_{CO_2} = \chi_{CO_2} P_s. \]

(6)

When a given \( CO_2 \) inventory is added to the atmosphere, both \( P_s \) and \( \chi_{CO_2} \) simultaneously increase, as governed by Equations 4 and 5.

4. Simplified Snowball in the Spectral Model

In order to investigate a more realistic tropospheric expansion scenario, we construct a “Simplified Snowball.” This is an atmosphere that contains only \( CO_2 \) and 1 bar of dry air. Only longwave radiative transfer is performed; we assume that the total absorbed shortwave radiation remains constant at 128 W m\(^{-2}\). This assumption is relaxed in Section 7. The solar constant is 1280 W m\(^{-2}\) and the critical lapse rate is the dry adiabatic lapse rate. We assume that \( R \) and \( c_p \) remain constant at 289.7 and 1,003 J kg\(^{-1}\) K\(^{-1}\), respectively.

The purpose of the Simplified Snowball is to show how tropospheric expansion operates with a correctly calculated inventory of a real nongray gas, but without any extra effects (such as shortwave radiative transfer, a temperature-dependent \( H_2O \) inventory, clouds, the dependence of \( R \) and \( c_p \) on atmospheric composition, and the dependence of the moist adiabatic lapse rate on \( R \) and \( c_p \)) to complicate the understanding of the situation. These extra effects are examined in Section 7.

Starting from an initial \( CO_2 \) inventory of 0.0004 bar, which is a typical value at the initiation of a Snowball state, the \( CO_2 \) inventory is quadrupled 5 times to reach an inventory of 0.4096 bar, which is slightly higher than the likely deglaciation threshold. Figure 5 shows the initial and final temperature profiles.

As \( P_{CO_2} \) increases, both \( P_s \) and \( T_s \) increase. The assumption of a perfectly constant tropopause therefore breaks down, and the reasons for this must be explained.

5. Causes of the Tropopause Changes

An increase in \( P_{CO_2} \) causes simultaneous increases in both \( P_s \) and \( \chi_{CO_2} \). In the previous section, increasing \( P_{CO_2} \) from 0.0004 to 0.4096 bar meant that \( P_s \) increased from 1.0004 to 1.4096 bar, while \( \chi_{CO_2} \) increased from 0.00026 to 0.21. In order to understand the resultant changes in \( P_s \) and \( T_s \), the changes in \( \chi_{CO_2} \) and \( P_s \) can be examined separately.

5.1. Greenhouse Effect

First, the effect of increasing \( \chi_{CO_2} \) while holding \( P_s \) constant is examined. We ran the model with both \( \chi_{CO_2} = 0.00026 \) and \( \chi_{CO_2} = 0.21 \) while holding \( P_s \) constant at 1 bar. Figure 6 shows the temperature profiles.

The surface temperature increase is simply the result of the increased \( CO_2 \) greenhouse effect. Additional model runs confirm that each quadrupling of the \( CO_2 \) amount results in the same increase in \( T_s \), which is the expected logarithmic relationship.
5.2. Window Emission

Second, $p_s$ is increased while keeping $X_{CO_2}$ constant. We chose $X_{CO_2} = 0.1$ as an intermediate value and ran the model for both $p_s = 1.0004$ bar and $p_s = 1.4096$ bar. Figure 7 shows the temperature profiles.

The increase in $T_s$ for a given $p_s$ is less than it would be for idealized tropospheric expansion. We hypothesize that this is due to the presence of an atmospheric window.

Unlike a gray gas, a real gas such as CO$_2$ absorbs strongly in some regions (peaks) and weakly in others (windows), with the absorption coefficient varying by several orders of magnitude between these regions. Upward flux from the surface that lies in a window region is able to escape to the TOA with very little absorption, allowing it to contribute to the outgoing longwave radiation (OLR).

If, when $p_s$ increased, there was idealized tropospheric expansion as in Figure 3, $T_s$ would increase significantly. The upward flux in the window region would increase, and this would be transmitted to the TOA, which would increase the OLR. Therefore, $T_s$ would need to be adjusted downward to compensate, making the surface colder than idealized tropospheric expansion would predict.

This can be tested by modifying the gray model to include a window. Instead of a constant absorption coefficient at all wavenumbers, a window is centered on $\nu_{center}$, with width $\Delta \nu$. At wavenumbers in the range $\nu_{center} - \Delta \nu < \nu < \nu_{center} + \Delta \nu$, the absorption coefficient is multiplied by a factor of $10^{-3}$.

We ran this gray window model with two different surface pressures. We did this for both a wide window ($\Delta \nu = 400$ cm$^{-1}$, with 44% of the upward flux from the surface lying in the window) and a narrow window ($\Delta \nu = 20$ cm$^{-1}$, with 3% of the upward flux from the surface lying in the window). We chose $\nu_{center} = 500$ cm$^{-1}$, near the peak of the emission spectrum from the surface at 250 K. Figure 8 shows the temperature profiles.

With the wide window, $T_s$ is significantly colder than idealized tropospheric expansion predicts. With the narrow window, $T_s$ is essentially the same as idealized tropospheric expansion predicts. This shows that window emission causes the departure from idealized tropospheric expansion.

Given that both the greenhouse effect and the increased window emission can change $T_r$ and $p_r$, a way to quantify the impact this has on $T_s$ is required.

6. Quantifying the Effect of Tropopause Changes on Surface Temperature

Now using tropopause changes diagnosed from the model, we have $p'_r \neq p_r$ and $T'_r \neq T_r$, so from Equation 2 we now have

$$\frac{T'_r}{T_r} = \frac{T'_r}{T_r} \left[ \frac{p'_r}{p_r} \right]^{-\frac{\Gamma}{p'}} \left[ \frac{p'_r}{p_s} \right]^{\frac{\Gamma}{p_s}}. \quad (7)$$

The factor by which $T_r$ changes is the product of three factors, $A$, $B$, and $C$, which represent changes in $T_r$, $p_r$, and $p_s$, respectively. This allows the impact
of each of these three variables on $T_s$ to be separated and compared, which marks an improvement over the approach in Edkins (2016).

The total change in surface temperature is given by

$$ T_s' - T_s = \Delta T_s = T_s(AB - 1). \quad (8) $$

This can be partitioned into a tropopause component (caused by the greenhouse effect and increased emission through the window) and a surface pressure component (caused by tropospheric expansion).

We assume that the change in $T_s$ caused by a change in $p_s$ is given by

$$ \Delta T_s = T_s(C - 1), \quad (9) $$

while the change in $T_s$ caused by a change in the tropopause (i.e., changes in both $p_t$ and $T_t$) is given by

$$ \Delta T_s = T_s(AB - 1). \quad (10) $$

The error, $\epsilon$, that results from the assumption that the total change in $T_s$ is equal to the sum of the surface pressure component and the tropopause component can be found by subtracting Equations (9) and (10) from Equation (8):

$$ \epsilon = T_s[(ABC - 1) - (AB - 1) - (C - 1)], \quad (11) $$

$$ \epsilon = T_s(AB - 1)(C - 1). \quad (12) $$

This error is small when both $AB \approx 1$ and $C \approx 1$, which is approximately true for all the cases in this paper. Indeed, even if both $AB$ and $C$ are larger than unity by 10%, the error is still only 1% of the surface temperature.

Equations (9) and (10) can be used to find the change in $T_s$ caused by changes in both the tropopause and the surface pressure for each quadrupling of the CO$_2$ inventory from 0.0004 to 0.4096 bar. This is shown in Figure 9a. For example, when $p_{1,CO_2}$ is quadrupled from 0.0256 to 0.1024 bar, $T_s$ increases by 6 K. Of this 6 K warming, changes in the tropopause are responsible for 2 K, while changes in $p_s$ are responsible for 4 K.

It is clear that in the Simplified Snowball the increase in $T_s$ is largely caused by changes in the tropopause for $p_{1,CO_2} < 0.0256$ bar. The increase in $T_s$ quickly becomes dominated by changes in $p_s$ for $p_{1,CO_2} \gtrsim 0.1$ bar.

**Figure 8.** Temperature profiles from the gray window model for both wide and narrow windows, each with two different surface pressures and with constant gray gas mixing ratio.

**Figure 9.** The increase in $T_s$ for each quadrupling of $p_{1,CO_2}$ in the spectral radiative–convective model, partitioned into the component caused by the change in surface pressure and the component caused by the change in the tropopause according to Equations (9) and (10), respectively. The components are approximately additive (with the small error given by Equation (11)), so the total increase in $T_s$ for a given quadrupling of $p_{1,CO_2}$ is approximately equal to the sum of the surface pressure component and the tropopause component. Left: Simplified Snowball; middle: Full Snowball with clear sky; and right: Full Snowball with cloudy sky.
The nearly constant tropopause component of the surface warming for the first four quadruplings of $p_{1\text{CO}_2}$ is the expected result of the logarithmic CO$_2$ greenhouse effect. For the fifth quadrupling, increased window emission becomes an important factor because window emission increases at higher surface temperatures. It acts to cool the surface, in opposition to the greenhouse effect, so the net tropopause influence on $T_s$ is small for this interval. However, this does not affect the surface pressure component due to tropospheric expansion, which is large for this interval.

7. Full Snowball in the Spectral Model

Finally, we examine tropospheric expansion in our best estimate of Snowball Earth conditions.

In addition to CO$_2$, the Snowball Earth atmosphere contained H$_2$O and O$_3$. In our model, the vertical H$_2$O distribution is calculated using the relative humidity formula of Manabe and Wetherald (1967), assuming a surface relative humidity of 80%, following Hu et al. (2011). We neglect the contribution of the H$_2$O inventory to the surface pressure. The O$_3$ vertical distribution is given by the formula of Green (1964), incorporating the changes suggested by Yang et al. (2012) (the total ozone amount is halved and the peak of the distribution is lowered). The longwave radiative transfer for all of these gases is calculated using PRRTM. The shortwave radiative transfer is also calculated for O$_3$ and H$_2$O using the parameterization of Lacis and Hansen (1974). The surface albedo is set to 0.663, following Hu et al. (2011).

The specific gas constant $R$ and specific heat at constant pressure $c_p$ depend on both atmospheric composition and, in the case of CO$_2$, particularly, on temperature. It is important to calculate these correctly because they enter into Equation 7. For the Full Snowball, we calculate linear fits to the data in Span and Wagner (2009) for $c_p$ and $c_v$ as a function of temperature for CO$_2$ for the 0.101325 MPa isobar and use these to calculate $R$ for CO$_2$. The total $R$ and $c_v$ for each layer are calculated by adding the contributions from each gas, weighted by their volume mixing ratios.

Under present Earth conditions, horizontal transport means that the global average lapse rate is lower than the moist adiabatic lapse rate (for global average relative humidity) by a factor of approximately 0.88. Therefore, we multiply the moist adiabatic lapse rate by this factor to give an estimate of the critical lapse rate for the Full Snowball.

As in Section 4, the model is initially run with $p_{1\text{CO}_2} = 0.0004$ bar, to represent the conditions at the initiation of the Snowball; $p_{1\text{CO}_2}$ is then quadrupled 5 times to reach $p_{1\text{CO}_2} = 0.4096$ bar. Figure 9b shows the change in $T_s$ for each quadrupling, divided into tropopause and surface pressure components.

The clearest discrepancy between the Simplified Snowball in Figure 9a and the Full Snowball in Figure 9b is that the tropopause component does not sharply drop in the final interval from 0.1024 to 0.4096 bar. The reason for this is that the H$_2$O greenhouse effect compensates for the increased window emission, while also reducing the impact of the increased window emission (since H$_2$O partially blocks the window). This was confirmed by running the model again with no H$_2$O, which produced similar results to Figure 9a.

Aside from this, the pattern holds from the Simplified Snowball that, for $p_{1\text{CO}_2} > 0.1$ bar, the increase in $T_s$ is chiefly caused by the increase in $p_s$. This is confirmed with another experiment. A baseline equilibrium is found with $p_{1\text{CO}_2} = 0.1$ bar. Another 0.1 bar of CO$_2$ is added, which causes a warming of 2.8 K. Next, instead of the extra 0.1 bar of CO$_2$, an additional 0.1 bar of N$_2$ is added. This causes a warming of 2.1 K. Therefore, the majority (73%) of the warming caused by adding 0.1 bar of CO$_2$ is simply due to the fact that $p_{surf}$ has increased by 0.1 bar.

It is also worth noting that in the pure CO$_2$ limit, the component of warming due to tropospheric expansion ($C$ in Equation 7) for each doubling of CO$_2$ is given by $2^{0.73 \Gamma}$, which is roughly 1.2 or a warming of 20% of the surface temperature (assuming $\Gamma = 9.8$ in the Snowball case), while of course in the limit where CO$_2$ is a trace gas, tropospheric expansion provides no warming ($C = 1$). The cases presented here are intermediate, but nearer to the pure CO$_2$ limit.

7.1. Clouds

While Abbot and Pierrehumbert (2010) showed that the longwave cloud radiative effect during a Snowball Earth can aid deglaciation, little is known about the properties of Snowball Earth clouds. Abbot (2014) built on this...
earlier work and, based on his results using a less-parameterized cloud resolving model, suggested that the clouds during a Snowball near deglaciation would have been optically thick with a cloud top temperature of around 215 K. He also noted that the fixed anvil temperature hypothesis is a useful guide in thinking about Snowball Earth clouds.

We mapped out a plausible phase space of cloud properties and observed their effect on the likelihood of deglaciation to gain a fuller picture of the importance of clouds in our model. The model is run for cloud fractions between 0 and 1, fixed cloud top temperatures between 215 and 235 K, and CO$_2$ inventories between 0.1 and 0.4 bar. The clouds in all cases have an optical depth of 10. Both the LW and SW radiative transfer schemes include the clouds. The equilibrium surface temperatures for each combination of parameters are shown in Figure 10.

Following Hu et al. (2011), we assume that the equatorial surface temperature is 10 K warmer than the global average surface temperature. This means that a global average temperature of 263 K suffices to begin the melting process at the equator, after which ice-albedo feedback ensures deglaciation. Therefore, the value of $A_{\text{eq}}$ for which $T_s = 263$ K is the deglaciation threshold.

Deglaciation in our model is possible only for very high cloud fractions with $p_{\text{CO}_2} = 0.1$ bar, whereas a wide range of possible cloud properties result in deglaciation with $p_{\text{CO}_2} = 0.4$ bar. However, in the range $p_{\text{CO}_2} = 0.2$–0.3 bar, deglaciation depends strongly on the combination of cloud properties. This highlights the importance of clouds to the deglaciation problem.

We chose to include a cloud with an optical depth of 10, fraction of 0.3, and fixed cloud top temperature of 215 K in the model so that the effect of a cloud on the surface temperature and on the partitioning of surface warming.
into tropopause and surface pressure components could be determined. The model was run as in Figure 9b, but with these cloud properties; Figure 9c shows the results.

The longwave cloud radiative effect means that, for a given \( p_{\text{L,CO}_2} \), the surface temperature in Figure 9c is warmer than the surface temperature in Figure 9b. This means that the \( \text{H}_2\text{O} \) greenhouse effect is stronger, which makes the tropopause component of the surface warming larger for each quadrupling of \( p_{\text{L,CO}_2} \).

However, for the quadrupling of \( p_{\text{L,CO}_2} \) from 0.1024 to 0.4096 bar, the downward tropospheric expansion still causes a much larger surface warming than the tropopause changes cause.

With the chosen cloud properties, the deglaciation threshold in our model is \( p_{\text{L,CO}_2} = 0.32 \) bar. When \( p_{\text{L,CO}_2} \) increases from 0.0004 bar at the initiation of the Snowball state to 0.32 bar at deglaciation, \( T_s \) increases by 34 K. Of this, 18 K is the result of the combined influences on the tropopause of the greenhouse effect and increased window emission, while 16 K is caused by the downward tropospheric expansion that results from the increase in \( p_s \).

However, downward tropospheric expansion dominates as \( p_{\text{L,CO}_2} \) nears the deglaciation threshold. When \( p_{\text{L,CO}_2} \) increases from 0.0004 to 0.16 bar, the change in the tropopause causes \( T_s \) to increase by 14 K, while the change in \( p_s \) causes \( T_s \) to increase by 8 K. By contrast, when \( p_{\text{L,CO}_2} \) increases from 0.16 to 0.32 bar, the change in the tropopause causes \( T_s \) to increase by 4 K, while the change in \( p_s \) causes \( T_s \) to increase by 8 K.

### 8. Pressure Broadening and CIA

Hu et al. (2011) performed a study in which they stated that \( p_s = p_{\text{L,air}} + p_{\text{CO}_2} \), which meant that \( p_s \) increased with \( \chi_{\text{CO}_2} \) in their model. However, they attributed all of the resultant increase in \( T_s \) to pressure broadening and CIA (rather than to tropospheric expansion).

While an increase in \( p_s \) would increase the pressure broadening of \( \text{CO}_2 \) molecules in the lower atmosphere, Abbot and Pierrehumbert (2010) claimed that pressure broadening is not a significant effect for surface pressure variations of \( \text{O}(10\%) \).

We tested this claim in the spectral model. PRRTM uses correlated-\( k \) distributions (Lacis & Oinas, 1991) to determine the absorption coefficients used in the radiative transfer calculations. There is a correlated-\( k \) distribution for each point on a grid of pressure and temperatures.

Therefore, it is possible to isolate the radiative effects of changing pressure broadening and CIA from the nonradiative effect of downward tropospheric expansion. Keeping both \( p_s \) and \( \chi_{\text{CO}_2} \) constant, at each layer we can simply select the correlated-\( k \) distribution for a different pressure.

To determine the influence of pressure broadening and CIA on the surface temperature in Snowball Earth conditions at the point of deglaciation, we compared two atmospheric profiles, A and B. In profile A, \( p_{\text{L,CO}_2} = 0.32 \) bar, \( p_s = 1.32 \) bar in the hydrostatic structure subroutine, and \( p_s = 1.32 \) bar in the radiative subroutine. In profile B, \( p_{\text{L,CO}_2} = 0.32 \) bar, \( p_s = 1.32 \) bar in the hydrostatic structure subroutine, but \( p_s = 1 \) bar in the radiative subroutine. This means that the correlated-\( k \) distribution that is selected for each layer in profile B is for the same temperature and \( \text{CO}_2 \) amount as for that layer in profile A, but for a lower pressure; this is equivalent to holding the pressure broadening and CIA constant at their values for \( p_s = 1 \) bar.

We ran our model with each profile and found that \( T_s \) in profile B was cooler by only 2 K. This implies that, in these conditions, pressure broadening and CIA are small effects compared to tropospheric expansion.

### 9. Discussion

There is little agreement in the literature on the value of the oxygen inventory, \( p_{\text{L,O}_2} \), during the Cryogenian, which is the period in the Neoproterozoic Era during which the Sturtian and Marinoan glaciations occurred. As summarized by Blamey et al. (2016), the traditional viewpoint (Kump, 2008) has been that \( p_{\text{L,O}_2} \) was low during the Cryogenian, possibly around 10% of its present level, and that the Neoproterozoic Oxidation Event (NOE) occurred after the Cryogenian. Canfield (2005) and Lyons et al. (2014) argue that the NOE occurred more gradually during the Cryogenian, which implies a value of \( p_{\text{L,O}_2} \) between 10% and 100% of the present value (and different values for the Sturtian and the Marinoan). The measurements presented by Blamey et al. (2016) suggest that the
NOE occurred much earlier than previously thought, and well before the Cryogenian. Without a consensus on the value of $p_{i, CO_2}$, we decided in this study to use the present value of 0.21 bar. It is the fractional change in surface pressure that is relevant to tropospheric expansion, so a smaller initial surface pressure due to a lower $p_{i, CO_2}$ would mean a greater relative contribution from tropospheric expansion to the overall warming, but it also would increase the deglaciation threshold. We ran our model with $p_{i, CO_2} = 0.021$ bar and found that the deglaciation threshold increased from 0.32 to 0.42 bar. This decreased $p_{i, CO_2}$ did not alter the conclusion that the change in $T_s$ is primarily caused by the change in $p_s$ for $p_{i, CO_2} \gtrsim 0.1$ bar. It is also noteworthy that, since $O_2$ is not a greenhouse gas, any change in surface temperature that it causes is a result of the increase in surface pressure that it causes, and therefore due to either CIA or pressure broadening of other gases and/or downward tropospheric expansion.

We know of no proxy measurements of the surface pressure during the Neoproterozoic Era, which means that we must infer the surface pressure from what is known about the inventories of the major atmospheric gases. However, recent paleobarometry studies concerning the Archean Eon (Marty et al., 2013; Som et al., 2016) suggest that the surface pressure was around half of its modern value, which is a larger deviation than anything considered in this paper. In contrast, Goldblatt et al. (2009) found that Earth’s total nitrogen inventory is around 3 times its present value and that in the Archean a larger portion of this was in the atmosphere instead of in the crust and mantle. Krissansen-Totton et al. (2018) also notes that the Archean $p_{CO_2}$ was potentially several bars. Therefore, it is possible that tropospheric expansion also played a role in the Archean climate (which could have been cooling or warming, depending on the surface pressure), but more evidence is required before drawing specific conclusions.

Two studies provide estimates of the maximum value of $p_{i, CO_2}$ during the Neoproterozoic Era. Le Hir et al. (2008) found that if even a small area of open ocean persists, $p_{CO_2}$ is limited to around 0.25 bar (which means $p_{i, CO_2} = 0.35$ bar) due to exchange between the atmosphere and the ocean. Through analysis of the oxygen isotope composition of a sulfate sample, Bao et al. (2009) suggested a value for $p_{CO_2}$ of up to 0.08 bar (which means $p_{i, CO_2} = 0.12$ bar), although Abbot and Pierrehumbert (2010) point out that this was not necessarily the maximum $p_{CO_2}$ reached. With $p_{i, CO_2}$ values around this range, tropospheric expansion aids deglaciation. To determine the deglaciation threshold more precisely would require assumptions about the seasonal and latitudinal temperature profiles, since it may only be necessary for a small region of ice to melt to trigger deglaciation through the ice-albedo feedback. This is beyond the scope of this paper but could be addressed with a more complex model.

While the contribution of $p_{i, H_2O}$ to $p_s$ is small at low temperatures and is therefore neglected here, it may have been important during the Hothouse in the immediate aftermath of the Snowball Earth, and also in other very warm periods in Earth’s history.

In studying the atmosphere of Early Mars, Kasting (1991) showed that Rayleigh scattering can increase the planetary albedo significantly for high CO$_2$ concentrations. However, this is not as important a consideration for a Snowball Earth. The very high surface albedo means that changes in the albedo of the atmosphere have only a small effect on the planetary albedo. A simple calculation indicates that the increased Rayleigh scattering caused by the addition of 0.4 bar of CO$_2$ would only cool the surface by around 1 K, which is insignificant compared to the warming caused by the greenhouse effect and by tropospheric expansion. In addition, the uncertainties in the surface albedo and paleocloudiness outweigh the error caused by the neglect of increased Rayleigh scattering.

Another consideration is the effect of the latitudinal distribution of lapse rates. The magnitude of the warming caused by tropospheric expansion depends directly on the lapse rate. The global average lapse rate may include some regions with shallow lapse rates (or even inversions) and some regions with steeper lapse rates. This could be explored with a radiative-convective model that allows for the input of a latitudinal distribution of atmospheric properties.

In the winter hemisphere of a Snowball Earth, there is the possibility of frequent temperature inversions at the surface. However, this would not affect the conclusions of this paper if the inversion strength remains constant with changes in the gas inventory. We tested this by running the model in winter conditions and adding 0.1 bar of either CO$_2$ or N$_2$ to the baseline equilibrium. This resulted in a decrease in inversion strength of less than 1% for CO$_2$ and less than 5% for N$_2$.

Of the two possible directions of tropospheric expansion, all previous attention has focused on the upward variety. This is because a small increase in a greenhouse gas can significantly decrease the tropopause pressure while having a negligible effect on surface pressure, which is the typical situation. Downward tropospheric expansion is
presented here for the specific case of a Snowball Earth, but it would apply to any atmosphere that meets certain conditions.

The first condition is that the inventory of one or more gases in the atmosphere increases significantly. This is necessary to get the initial expansion of the troposphere.

The second condition is that convection continues. If the convection is driven from the surface, this would mean that the surface energy budget remains positive. However, if convection proceeds by other means, as may be the case for an optically thick atmosphere like that of Venus (Izakov, 2007), the convective depth would be set by higher layers of the atmosphere, and the effect would be the same.

Finally, for downward tropospheric expansion to be dominant over the upward variety, the relative change in tropopause pressure must be smaller than the relative change in surface pressure. This is a necessary consequence of the gray model, since emission to space comes mainly from an optical depth of about one. It carries with it the implication that changes to the (longwave) radiative properties of the lower tropospheric constituents (gases or clouds) below that depth have little effect on lower tropospheric temperature because that region is dominated by convection and the tropopause is fixed. The situation is more complicated when a spectral radiative transfer model is used because the emission depth varies with wavenumber, allowing some influence on the tropopause from lower altitudes. However, in the cases studied here we found that despite some variation in tropopause pressure, the above implication remained useful. More generally, we expect that for all planetary atmospheres that are not optically thin, a tropopause perspective to RCE provides greater intuition to their vertical temperature structure than does a surface perspective. It is thus relevant that Robinson and Catling (2014) found that tropopause pressure stays relatively constant across a range of planetary atmospheres.

10. Conclusions

When the CO$_2$ inventory increases in a Snowball Earth atmosphere, a number of competing factors influence the surface temperature. The increase in $\chi_{CO_2}$ causes the surface to warm via the greenhouse effect (which causes an upward tropospheric expansion). The increase in $p_s$ causes a downward tropospheric expansion, which also warms the surface. This increased surface temperature means that surface emission through the atmospheric window increases, which results in a slight cooling of the surface. Finally, the increased surface pressure means that pressure broadening and CIA increase, which also increases the surface temperature.

The work in this paper allows these effects to be quantified and compared with each other. Changes in surface pressure and in tropopause temperature and pressure can be directly related to their impact on surface temperature.

Our model deglaciates from a Hard Snowball state at $p_{t,CO_2} = 0.32$ bar. Although the deglaciartion threshold provided by a RCE model is not precise due to assumptions that must be made about the way deglaciation starts, this does suggest that the greenhouse effect and tropospheric expansion could be enough to cause deglaciartion at CO$_2$ amounts close to the maximum estimates. However, perhaps a more likely scenario is that tropospheric expansion worked in conjunction with other phenomena such as dust deposition and the persistence of areas of open ocean to collectively enable deglaciartion. A particular advantage of the tropospheric expansion explanation is that it quickly increases with higher CO$_2$ amounts, unlike the CO$_2$ greenhouse effect, and therefore helps to rule out an unrealistic scenario with a very large amount of CO$_2$ on a still-glaciarted planet.

For $p_{t,CO_2} \geq 0.1$ bar, changes in surface pressure have more influence on the surface temperature than changes in the tropopause.

When $p_{t,CO_2}$ is doubled from 0.16 bar to the deglaciartion threshold of 0.32 bar, the change in surface pressure is more than twice as important as the change in the tropopause in determining the surface temperature.

The increase in pressure broadening and CIA caused by increasing the surface pressure from 1 bar to its value at the point of deglaciartion of 1.32 bar was found to cause only a 2 K increase in surface temperature. This suggests that it is mostly due to downward tropospheric expansion, not pressure broadening and CIA, that the surface temperature increases when the surface pressure increases.
While the deglaciation threshold depends on cloud properties, about which there is significant uncertainty, downward tropospheric expansion lowers the deglaciation threshold for any scenario that requires a large inventory of CO$_2$, and therefore aids the explanation of the deglaciation of the Snowball Earth.

Data Availability Statement

The radiative–convective model used in this study can be found at https://auckland.figshare.com/articles/software/Atmospheric_Pressure_and_Snowball_Earth_Deglaciation_Model_and_Input_Data/16799620.

Acknowledgments

This work was funded in part by subcontract 1460339 from the California Institute of Technology/Jet Propulsion Laboratory. We thank Eli Mlawer for providing PRRTM.

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