Analysis of current-voltage characteristics of two-dimensional superconductors: finite-size scaling behavior in the vicinity of the Kosterlitz Thouless transition

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It has been suggested [Pierson et al., Phys. Rev. B 60, 1309 (1999); Ammirata et al., Physica C 313, 225 (1999)] that for two-dimensional (2D) superconductors there exists a phase transition with the dynamic critical exponent \( z \approx 5.6 \). We perform simulations for the 2D resistively-shunted-junction model and compare the results with the experimental data in Repaci et al. obtained for an ultrathin YBCO sample [Phys. Rev. B 54, R674 (1996)]. We then use a different method of analyzing dynamic scaling than in Pierson et al., and conclude that both the simulations and the experiments are consistent with a conventional Kosterlitz Thouless (KT) transition in the thermodynamic limit for which \( z = 2 \). For finite systems, however, we find both in simulations and experiments that the change in the current-voltage \( (I-V) \) characteristics caused by the finite size shows a scaling property with an exponent \( \alpha \approx 1/6 \), seemingly suggesting a vanishing resistance at a temperature for which \( z = \alpha^{-1} \). It is pointed out that the dynamic critical exponent found in Pierson et al. corresponds to the exponent \( \alpha^{-1} \). It is emphasized that this scaling property does not represent any true phase transition since in reality the resistance vanishes only at zero temperature. Nevertheless, the observed scaling behavior associated with \( \alpha \approx 1/6 \) appears to be a common and intriguing feature for the finite size caused change in the \( I-V \) characteristics around the KT transition.

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I. INTRODUCTION

The transition from the superconducting to the resistive state is of the Kosterlitz Thouless (KT) type for an ideal two-dimensional (2D) superconductor in zero magnetic field. Below the KT transition the current-voltage \( (I-V) \) characteristics is nonlinear, \( V \propto I^a \) (or equivalently, \( E \propto J^a \) with the electric field \( E \) and the current density \( J \), characterized by the exponent \( a \). According to the conventional extension of the KT theory, \( a = 3 \) precisely at the transition and \( a \) becomes larger as the temperature is decreased. The scaling argument by Fisher, Fisher, and Huse (FFH) in Ref. 4 leads to the connection \( a = 1 + z \) at the KT transition from which the value \( z = 2 \) is deduced for the dynamic critical exponent \( z \). These predictions of \( z = 2 \) and \( a = 3 \) at the KT transition have been confirmed through numerical simulations for the lattice Coulomb gas with Monte Carlo (MC) dynamics, the Langevin-type molecular dynamics of Coulomb gas particles, and the 2D XY model with resistively-shunted-junction (RSJ) dynamics. The value \( a = 3 \) has also been found to be consistent with numerous measurements.

From the above perspective, the recent claim by Pierson et al. in Ref. 11 that \( z \) has a much larger value at the resistive transition is very intriguing and is in apparent contradiction to the established conventional view. More specifically, Pierson et al. in Ref. 11 have applied a dynamic FFH scaling approach (we call it Pierson scaling in this work; see Sec. II) to existing experimental data and obtained \( z \approx 6 \) in many 2D systems including theoretical models like the lattice Coulomb gas (Ref. 3), \( ^3 \)He films, and an ultrathin YBCO sample (Repacei et al. in Ref. 2). We in our analysis focus on the Repaci et al. data for which a particularly good scaling collapse was obtained by Pierson et al.

Our approach is, as directly as possible, to compare simulation data for the 2D RSJ model obtained in this work with the experimental data obtained for a ultrathin YBCO film by Repaci et al. in Ref. 12 (For brevity, we from now on call the former the RSJ data and the latter the Repaci data, respectively.) In case of the RSJ data we conclude that the established conventional view holds with \( z = 2 \) at the KT transition. We then compare the RSJ data with the Repaci data and show that the latter exhibit a similar evidence of \( z = 2 \) at the KT transition. Both the RSJ and the Repaci data show nonvanishing resistances below the KT transition. In the former case this is due to finite-size effects and in the latter it is due to some length scale which introduces an effective cutoff in the logarithmic vortex interaction, e.g., the finite size of the sample, the perpendicular penetration depth, or a residual weak perpendicular magnetic field (see Ref. 1 for details); We in the following use the term finite-size effects to cover all these possibilities. In the original analysis of the data in Ref. 12 the authors attributed the finite resistance below the KT transition to such finite-size effects, which is in accord with the striking similarity we find to the RSJ data.

We also find that the Pierson scaling can be made to work with \( z \approx 6 \) for the RSJ data which is closely the same \( z \) as found by Pierson et al. for the Repaci data. In addition, we find, by analyzing the resistive tails of both data sets through the use of a new scaling method, that the finite-size caused resistance tails show strikingly similar scaling behaviors with an exponent \( \alpha \approx 1/6 \). How-
however, our conclusion is that this scaling does not signal any real resistive transition, but reflects finite-size effects which will eventually disappear in the thermodynamic limit.

Strachan et al. (Ref. 12) have, from a thorough analysis of the Repaci data, shown that the Pierson method is too flexible to allow any firm conclusion and that the apparent vanishing of the resistance is really a consequence of the finite-voltage resolution threshold in the experiment. Somewhat related conclusions have been reached by Brown in Ref. 14 in the context of the vortex glass transition. This is also in accord with our findings. Nevertheless our conclusion is that the Pierson scaling result is, at least partially, a reflection of an actual scaling like behavior associated with the finite-size caused resistance tail in the vicinity of the KT transition. However, since the resistance tail vanishes only at zero temperature for a finite system, we conclude that there exists no resistive transition at a finite temperature and hence no new transition. A possible underlying reason for the scaling behavior is still an enigma which remains to be resolved in the future.

The present paper is organized as follows: In Sec. II the scaling methods used to analyze the data are presented, and in Sec. III we describe and analyze the RSJ data we obtain from the simulations. The Repaci’s YBCO data is reanalyzed and compared with the RSJ data in Sec. IV. Finally we summarize and discuss our findings in Sec. V.

II. SCALING METHODS

The dynamic scaling relation proposed by FFH (Ref. 3) for a d-dimensional superconductor in the vicinity of the transition is given by

\[ E = J \xi^{d-2} \chi_\pm (J \xi^{d-1}/T), \]  

(1)

where \( E, J, \xi, z, \) and \( T \) are the electric field, the applied current density, the correlation length, the dynamic critical exponent, and the temperature, respectively, and \( \chi_\pm \) denotes the scaling function above (+) and below (−) the transition. It should be kept in mind that Eq. (1) can be directly used only in a system whose linear size \( L \) is much larger than \( \xi (L \gg \xi) \).

Specializing to two dimensions, \( d = 2 \), Eq. (1) can be recast into the form

\[ \frac{E}{J} = \xi^{-2} \chi_\pm(J \xi/T), \]  

(2)

which is the starting point in Pierson et al. (Ref. 15) It should be noted that the above FFH dynamic scaling form (2) is valid only for a phase transition when \( \xi \) is finite both above and below the transition temperature. Accordingly, one should note that Eq. (2) is not compatible with a system undergoing a KT transition because the whole low-temperature phase is "quasi" critical with \( \xi_\infty = \infty \) and is characterized by a line of fixed points. This means that only the scaling function \( \chi_\pm \) above the KT transition has any clear justification. 15

In the present paper we study in Sec. III the 2D RSJ model with a finite size \( L \), which, instead of \( \xi \), serves as the large length scale cutoff at and below the KT transition. As the KT transition the finite-size scaling form deduced from Eq. (2) by substituting \( \xi \) by \( L \) takes the form

\[ \frac{E}{J} = L^{-2} f(JL), \]  

(3)

where the scaling function \( f(x) \) satisfies \( f(0) = \text{const.} \) and \( f(x) \propto x^z \) for large \( x \), corresponding to the finite-size induced resistance \( R \propto L^{-z} \) without external current and \( E \propto J^{1+z} \) in the large-current limit, respectively. The conventional view certifies that the nonlinear I-V exponent \( a = 3 \propto J^z \) is equal to 3 precisely at the KT transition, 16 from which \( z = 2 \) follows by the scaling relation \( a = z + 1 \).

Since the low-temperature phase in the KT scenario is described by a line of fixed points, each temperature \( T \) below the KT transition is characterized by its own scaling function \( h_T(x) \):

\[ \frac{E}{JR} = h_T(JLg_L(T)), \]  

(4)

where \( R = \lim_{J\to0}(E/J) \propto L^{-z(T)} \) [compare with Eq. (3)], \( h_T(0) = 1, h(x) \propto x^{z(T)} \) for large \( x \), and \( g_L(T) \) is a function of at most \( T \) and \( L \) such that a finite limit function \( g_\infty(T) \) exists in the large-\( L \) limit.

The temperature-dependent dynamic critical exponent \( z(T) \) can be obtained from a simple scaling argument (See Ref. 10 for more details): The characteristic time \( \tau \), which is related with \( R \) by \( R \sim 1/\tau \), is inversely proportional to the rate of unbinding vortex pairs over the boundary, and is given by the escape-over-barrier rate

\[ \frac{1}{\tau} \propto nL^2 \exp\left( -\frac{1}{\epsilon TCG} \ln L \right), \]  

(5)

where \( n \) is the average number density of vortices and the escape barrier for a pair to unbind over the boundary is determined by the effective logarithmic vortex interaction (1/\( \epsilon \)) \( \ln L \). Here \( \epsilon \) and \( T^{CG} \) are the dielectric constant and the Coulomb gas temperature, respectively (see Ref. 10), and both can be calculated in equilibrium.

From the definition of \( z(T) \) (\( \tau \sim L^z \)) and Eq. (3) we find

\[ z(T) = \frac{1}{\epsilon T^{CG}} - 2. \]  

(6)

Furthermore \( T^{CG} \propto T/\rho_0(T) \), where \( \rho_0(T) \) is the density of Cooper pairs which decreases with increasing \( T \), resulting in \( T^{CG} \) monotonically decreasing to zero as \( T \to 0 \). On the other hand, \( \epsilon(T) \geq 1 \) and remains close to unity in most of the low-temperature phase. It then follows from Eq. (3) that \( z(T) \) increases monotonically as \( T \) is
lowered, reaching \( z(T) = \infty \) at \( T = 0 \). One may note that since the KT transition is characterized by the condition \( 1/T^{CG} = 4 \) (Refs. [3] and [13]) Eq. (1) results in the conventional established value \( z = 2 \) at the KT transition.

Since \( E/JR \propto x(z(T)) \) for large \( x \) [see Eq. (1)] and \( x(T) \) depends on \( T \) [see Eq. (1)], \( E/JR \) cannot be described by a \( T \)-independent scaling function. On the other hand, when the scaling variable \( x = J L g_L(T) \) has small values, the possibility that \( E/JR \) is described by a \( T \)-independent scaling function with the dominant \( T \)-dependence absorbed into \( x \), cannot a priori be excluded. We find from our simulations of the 2D RSJ model that this indeed appears to be the case and one of the main points in the present paper is the apparent existence of the scaling form

\[
E/JR = h(J L g_L(T)) \tag{7}
\]

for smaller values of \( J L g_L(T) \) [compare with Eq. (1)]. What can be concluded about the function \( g_L(T) \) on general grounds? First we note that the resistance \( R \propto n(T)L^{-z(T)} \) for a finite system size vanishes only at zero temperature but has a finite value at any nonzero temperature. This follows since \( z(T) < \infty \) at any nonzero \( T \) and \( n(T) \) is of the thermal activation form, i.e., \( n(T) \propto e^{-[\text{const.}]/T^{CG}} \) (Ref. [1]), which is finite at any nonzero temperature. A finite \( R \) by Eq. (4) implies that \( 0 < g_L(T) < \infty \) and our simulations of the 2D RSJ model are entirely consistent with this expectation. If, on the other hand, \( R \) would vanish at a finite \( T \) this would have implied a diverging \( g_L(T) \) at this temperature.

The finite-size scaling we are discussing here is on the face of it entirely different from the scaling form given by Eq. (3), which is correct only in the thermodynamic limit \( \xi/L \rightarrow 0 \) and constitutes the basis and theoretical underpinning for the Pierson method in Ref. [1]. We again emphasize that this thermodynamic scaling is only justified as the KT transition is approached from above and has no bearing on the nonzero resistance below the KT transition caused by the finite size of the sample. However, from the practical standpoint of analyzing procedures the Pierson method has a connection to our finite-size scaling form \( E/JR = h(J L g_L(T)) \): The former essentially amounts to assuming that \( \xi \) in Eq. (2) is proportional to \( R^{-\alpha} \), where \( \alpha \) is a \( T \)-independent constant, leading to the scaling form \( E/JR = h(J R^{-\alpha}) \) we have in this consideration ignored an additional weak unimportant \( T \) dependence in Eq. (3). Consequently, if \( g_L(T) = A_L R^{-\alpha} \) where \( A_L \) is a constant which may depend on \( L \), then the Pierson scaling in a practical sense is of the same form as our finite-size scaling, in spite of the totally incompatible theoretical ground. So from this point of view the question is to what extent the relation \( g_L(T) = A_L R^{-\alpha} \) is fulfilled. If we allow \( \alpha \) to be a function of both temperature and size then the relation \( g_L(T) = A_L R^{-\alpha_L(T)} \) just serves as a definition of \( \alpha_L(T) \) because both \( g_L(T) \) and \( R(T) \) are positive functions. Thus, in the present finite-size context and from a practical standpoint, the Pierson scaling amounts to the statement that \( \alpha_L(T) \) extracted from experimental data appears to be a \( T \)-independent constant to a good approximation.

The scaling form Eq. (4) implies that \( R(T) \propto [L g_L(T)]^{-z(T)} \) where the proportionality factor in general depends on temperature. Using the definition of \( \alpha_L(T) \), \( g_L(T) = A_L R^{-\alpha_L(T)} \), we get the connection \( [L g_L(T)]^{-z(L)} = B_L(T) g_L(T) \), where \( B_L(T) \) in general may depend on both \( T \) and \( L \), which means that

\[
\ln g_L(T) = \frac{\alpha_L(T) z(T)}{1 - \alpha_L(T) z(T)} \ln \frac{L}{B_L(T)} - \ln B_L(T). \tag{8}
\]

The point with this relation is that if the dominant \( T \)-dependence of \( R \) comes from the factor \( (L g_L(T))^{-z(T)} \) then the \( T \)-dependence of \( B_L(T) \) is unimportant. Eq. (8) then leads to the requirement \( z(T) < 1/\alpha_L(T) \) where the extreme condition \( z(T) = 1/\alpha_L(T) \) for a \( T > 0 \), if it occurred, would correspond to a diverging \( g_L(T) \) and hence to a vanishing resistance. This means that if \( \alpha_L(T) \) for a fixed size \( L \) to a good approximation was a constant \( \alpha \), then the implication would be that the resistance vanishes at the temperature for which \( z(T) = 1/\alpha \). As discussed above this does not happen within the finite size KT scenario. But it could happen that \( \alpha_L(T) \) is a constant, i.e., \( \alpha_L(T) = \alpha \), over a limited temperature region and if all the available data was within this region one could be tempted to conclude that such a transition did really occur at some smaller \( T \) outside this region. As will be shown in Sec. II we do find such a limited region for the 2D RSJ model.

### III. ANALYSIS OF RSJ DATA

The 2D \( L \times L \) XY model on a square lattice is often used for studies of the KT transition. It is defined in terms of the Hamiltonian

\[
H = -E_J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j - r_{ij} \cdot \Delta), \tag{9}
\]

where \( E_J \) is the Josephson coupling strength, the summation is over nearest-neighbor pairs, \( \theta_i \) is the phase angle at site \( i \) satisfying the periodicity: \( \theta_i = \theta_{i+Lx} = \theta_{i+Ly} \). Here we use the fluctuating twist boundary condition (FTBC) to allow the phase difference across the whole system to fluctuate, and include the twist variable \( \Delta = (\Delta_x, \Delta_y) \) in the Hamiltonian (9), where \( r_{ij} \) is the unit vector from site \( i \) to site \( j \); \( \Delta = 0 \) corresponds to the standard periodic boundary condition. In particular, FTBC is very efficient when the system is driven by an external current since the periodicity of phase variables \( \theta_i \) is preserved. In the RSJ dynamics the local current is conserved at every site, which determines the equations of motion for phase variables.
\[ \dot{\theta}_i = -\sum_j G_{ij} \sum_k \left[ \sin(\theta_j - \theta_k - r_{jk} \cdot \Delta) + \eta_{jk} \right], \] (10)

where the primed summation is over the four nearest neighbors of \( j \), \( G_{ij} \) is the lattice Green function for 2D square lattice, time \( t \) has been normalized in units of \( \hbar/2e r_i \) with the critical current \( i_c \) and the shunt resistance \( r \) of a single junction, and \( \eta_{jk} \) is the thermal noise current in units of \( i_c \) on the link from the site \( j \) to \( k \). The equations of motion for the twist variables in the presence of an external current density \( J \) in the \( x \) direction read

\[ \dot{\Delta}_x = \frac{1}{L^2} \sum_{\langle ij \rangle_y} \sin(\theta_i - \theta_j - \Delta_x) + \eta_{\Delta_x} - J, \] (11)

\[ \dot{\Delta}_y = \frac{1}{L^2} \sum_{\langle ij \rangle_x} \sin(\theta_i - \theta_j - \Delta_y) + \eta_{\Delta_y}, \] (12)

which have been obtained from the condition that the net global current across the system in each direction should vanish (see Refs. [4] and [5] for discussions). Here \( \sum_{\langle ij \rangle_y} \) denotes the summation over all bonds in the \( x \) direction, \( J \) is in units of \( i_c \) (the lattice constant is set to unity). The thermal noise terms \( \eta_{\Delta_i} \) in Eq. (10), \( \eta_{\Delta_x} \) in Eq. (10), and \( \eta_{\Delta_y} \) in Eq. (12) obey the conditions \( \langle \eta_{\Delta_i} \rangle = \langle \eta_{\Delta_x} \rangle = \langle \eta_{\Delta_y} \rangle = \langle \eta_{\Delta_x} \eta_{\Delta_y} \rangle = \langle \eta_{\Delta_y} \eta_{\Delta_x} \rangle = 0 \), \( \langle \eta_{\Delta_x}(t) \eta_{\Delta_x}(0) \rangle = \langle \eta_{\Delta_y}(t) \eta_{\Delta_y}(0) \rangle = (2T/L^2)\delta(t) \), and \( \langle \eta_{\Delta_x}(t) \eta_{\Delta_y}(0) \rangle = 2T\delta(t)\delta_{ij}\delta_{jk} \), where \( T \) is in units of \( E_J/k_B \). The voltage drop \( V \) across the system in the \( x \) direction is written as \( V = -L\Delta_x \) in units of \( i_c r \), and we measure time-averaged (denoted by \( \langle \cdots \rangle \)) electric field \( E = \langle V/L \rangle_t \) to obtain the \( I-V \) characteristics. The set of equations (10), (11), and (12) are integrated by using the second order Runge-Kutta-Helfand-Greenside algorithm\(^\text{[4]}\) with discrete time step \( \Delta t = 0.05 \) for \( L = 4, 6, \) and 8, and the electric fields \( E \)'s are measured from time-averages over \( O(10^4) \) steps for large currents and \( O(10^5) \) steps for small currents.

Figure 2 shows the \( I-V \) characteristics (\( E \) versus \( J \)) at various temperatures in log scales for the system size \( L = 8 \). In the small-current regime \( E \) is shown to approach the Ohmic behavior \( (E/J = \text{const.}) \) at any temperature used in the simulations. Since it is well established that the 2D XY model on the square lattice has the KT transition at \( T_{KT} \approx 0.892 \) (Ref. [24], the ubiquitous non-vanishing resistance tails in the small-current regime are clearly due to the finite-size effects, and in the thermodynamic limit the resistance \( R \equiv \lim_{L \to 0}(E/J) \) vanishes in the whole low-temperature phase below \( T_{KT} \) giving rise to a nonlinear \( I-V \) characteristics: \( E \propto J^{a(T)} + 1 \) where \( z(T) = 1/(eT^{cg} - 1) \) can be readily obtained from the equilibrium properties\(^\text{[4]}\). At the KT transition \( z(T_{KT}) = 2 \) which corresponds to \( E \propto J^3 \) (denoted by thick full line in Fig. 2) at \( T = 0.90 \) very close to \( T_{KT} \) and crosses over to the Ohmic behavior \( E/J = \text{const.} \). As \( J \) is decreased due to the finite-size effect (thick broken line).

The nonlinear \( I-V \) characteristics for the somewhat larger currents is caused by the breaking of vortex-antivortex pairs due to the finite current. It is this pair breaking which is described by the exponent \( a(T) = z(T) + 1 \); it continuously increases from 1 in the high-temperature limit to infinity at \( T = 0 \) as \( T \) is decreased\(^\text{[2]}\). Consequently one should note that there is no jump in the exponent describing the current induced vortex-antivortex pair breaking at the KT transition. We stress this point because this fact seems sometimes to be misunderstood and to cause confusions.

Suppose now that the \( I-V \) characteristics in Fig. 2 was all information we had. What feature in the data then implies that we are looking at a KT transition broadened by finite-size effects? Figure 2 shows the same data as \( d\ln E/d\ln J \) plotted against \( J \) in log scale. Below the KT transition in the low-temperature phase the slope \( d\ln E/d\ln J \) for smaller \( J \) should be a constant given by \( z(T) + 1 \). However, because of the finite-size effect, the slope instead crosses over to 1 for smaller \( J \). This crossover produces a maximum for the curves in Fig. 2. Since the crossover is a finite-size effect this means that the position of the maximum is controlled by the variable \( JL \) up to additional \( T \)-dependencies. If the additional \( T \)-dependencies are weak this implies that the maximum is occurring at roughly the same \( I (= JL) \) for all \( T \) below the KT transition. This turns out to be true for the 2D RSJ model as indicated by the full drawn lines connecting the maxima in Fig. 2. Above the KT transition the maximum is instead controlled by the variable \( JL \) and since the correlation length \( \xi \) strongly decreases with increasing \( T \) above the KT transition this means that the maximum above the KT transition should move to larger \( J \) with increasing \( T \). The crossover between these two behaviors should occur near the temperature for which \( L \approx \xi, \) i.e., a \( T \) somewhat larger than \( T_{KT} \). In Fig. 2 such a crossover occurs between \( T = 1.00 \) and 1.10 which is consistent with \( T_{KT} \approx 0.90 \). The inset in Fig. 2 compares these maxima (denoted as \( \text{'max'} \)) with the values \( z(T) + 1 \) expected in the thermodynamic limit: the one obtained from a static calculation [see Eq. (1)] (denoted as \( \text{'static'} \)) and the other one from the scaling form \( R \sim L^{-(z(T) - 1)} \) (denoted as \( \text{'scale'} \), data from Ref. [5]) [see Eq. (4)]. As seen from the inset, the maxima in Fig. 2 agree very well with the actual values for the thermodynamic limit. This implies that the KT transition occurs close to the temperature for which the maximum is 3, which according to the inset gives a value close to \( T = 0.90 \). This suggests that the signature of a KT transition for a finite-size system is reflected in the behavior of the positions of the maxima and that the transition is close to the \( T \) for which the maximum is 3.

Next we demonstrate how the usual finite-size scaling given by Eq. (1) works for the case of the 2D RSJ model. First we assume that the \( L \) dependence of \( g_L(T) \) is weak, so that the scaling variable is to a good approximation \( JL \). Figure 3(a) then verifies the expected result due to \( T_{KT} = 2 \). This is done by calculating the \( I-\)
transition should occur. The connection is that \( e^{-R T_{\text{KT}}} \) in the Pierson form using the scaling method and the putative thermodynamic transition. We illustrate this in Fig. 7(a) where the data are shown in Fig. 5 and in Fig. 6 for which the relation \( g_L(T) \approx 1/T^{CG} - 1 \) is well determined since \( g \) is a function only of \( T \). Figure 5 demonstrates that the exponent \( z \) can be determined from the finite-size scaling, depends on temperature, and has the expected value \( z(T) = 1/T^{CG} - 1 \).

As discussed in Sec. II the existence of the standard finite-size scaling, as illustrated in Fig. 3, does not rule out the existence of a scaling of the form Eq. (7) for small values of \( L j g_L(T) \). This scaling form implies that \( E/J R \) is a function of \( L j g_L(T) \). In Fig. 4 \( E/J R \) is plotted against \( L j g_L(T) \) and shows that a function \( g_L(T) \), which makes the curves collapse, can indeed be found. The function \( g_L(T) \) determined from the two sizes \( L = 6 \) and 8 are shown in Fig. 5 and in Fig. 6 \( g_L(T) \) for \( L = 8 \) is plotted against \( z(T) \). We emphasize that the existence of a function \( g_L(T) \), which makes the data collapse, is by no means trivial and that as a consequence the functions are well defined. We again stress that the scaling in Fig. 4 with \( g_L(T) \) in Fig. 6 is compatible with the usual finite-size scaling given by Eq. (9).

As discussed in Sec. II the Pierson scaling analysis, which presumes a thermodynamic limit, is completely incongruent with the finite-size scaling of the 2D XY model discussed here. Nevertheless a connection from a pragmatic standpoint would arise provided \( g_L(T) = A_L R^{-\alpha} \), as also discussed in Sec. II. Figure 6 demonstrates that such a relation does exist over a limited \( T \) region. The exponent \( \alpha \approx 1/6 \) is found both for lattice sizes 6 and 8 and is well determined since \( R \) has a very strong \( T \) dependence. As discussed in Sec. II in connection with Eq. (8), the relation \( g_L(T) = A_L R^{-\alpha} \), if it applied all the way down to the temperature for which \( z(T) = 6 \), would imply that \( R \) vanishes at this \( T \). This is, from our finite-size perspective, the connection to the Pierson scaling method and the putative thermodynamic transition. We illustrate this in Fig. 3(a) where the data for which the relation \( g_L(T) = A_L R^{-\alpha} \) holds is plotted in the Pierson form using the \( T^{CG} = 0.125 \) for which \( z(T) = 1/T^{CG} - 2 = 6 \) which is where the would-be transition should occur. The connection is that \( 1/z \) in Fig. 3(a) is our \( \alpha \) and that the variable \( \xi \) corresponds to \( R^{1/2} \). Furthermore \( R^{-1/2} \) is assumed to have the form \( \exp(h/\sqrt{T^{CG} - T^{CG}}) \) where \( T^{CG} \) is where the would-be transition should occur. As seen from the inset, \( R \) can be represented by the suggested form with the “correct” \( T^{CG} = 0.125 \) and a good scaling fit is obtained for the value \( z = 1/\alpha = 6 \). Again we stress that there is no real phase transition or a vanishing resistance at this \( T^{CG} \), the resistance is caused by the finite size of the system and the actual scaling property is only present over a limited range. Indeed when the lower temperatures \( T = 0.70 \) and 0.65 are included, a Pierson scaling plot with \( \alpha \approx 1/6 \) can no longer be obtained. However, a Pierson scaling plot can again be obtained if \( \alpha \) is changed to \( \alpha \approx 1/12 \) and the two highest temperatures are excluded as shown in Fig. 3(b). This suggests that the Pierson scaling method has too much flexibility.

IV. RE-ANALYSIS OF REPAICI’S YBCO DATA

In this section we compare our analysis of the 2D XY model with the measured \( I-V \) characteristics for an ultrathin YBCO sample by Repaci et al. Figure 8 shows Repaci’s \( I-V \) data for \( 10K \leq T \leq 40K \) (from left to right) which should be compared to Fig. 8 for the 2D RSJ model. The thick lines have the slopes one, three, and seven (from left to right), respectively, where the slope one represents the linear resistive tail, slope three is the value for the KT transition and slope seven the value where the scaling over a limited region for the 2D RSJ model would place a nonexistent “ghost” transition. Repaci et al. attributed the linear resistance below the KT transition slope three with free vortices induced by the finite size of the sample (width 200 \mu m). The similarity between Fig. 8 and Fig. 8 is in accord with this interpretation. Figure 8 shows the Repaci data plotted in the same way as the data for the 2D RSJ model in Fig. 8. The similarity is again striking. As in Fig. 8 the maxima in Fig. 8 occur at approximately the same \( I \) for lower \( T \) (maxima larger than 2). This is consistent with the finite-size interpretation given in connection with Fig. 8 that the position of the maxima is roughly determined by \( J_L \). Consequently, when the size \( L \) is the dominant length, the maxima occur at the same \( I \). However, for higher \( T \) (maxima lower than 2 in Fig. 8) the correlation length \( \xi \) is the dominant length (\( \xi < L \)) and, since \( \xi \) rapidly decreases with increasing \( T \), the maxima are displaced towards higher \( I \). In parallel with Fig. 8 the thermodynamic KT transition is expected to occur close to the maximum value 3 which means \( T \approx 27K \) (compare full drawn line with slope 3 in Fig. 1). Consequently \( \xi \) is expected to become the dominant length for a somewhat higher \( T \) and this is consistent with the fact that the maxima seem to start to move roughly around maximum value 2. The similarities between Figs. 8 and 8 for the 2D RSJ model and Figs. 8 and 8 for the Repaci data support the interpretation of the Repaci data in terms of a finite-size induced resistance below the thermodynamic KT transitions.

The next question is then whether the size scaling Eq. (8) for the 2D RSJ model likewise applies to Re-
capici’s YBCO data. Figure 10(a) corresponds to Fig. 8(a) for the RSJ model. The data in Fig. 10(a) represents the data in the interval for which a linear resistance \( V/I = R \) is clearly present in the measurements (which means 23K \( T \leq 40K \)). The data collapse in Fig. 10(a) shows that \( V/I \) is only a function of the scaling variable \( JLg_L(T) \) when \( JLg_L(T) \) is small enough, while Fig. 10(b) shows that the form of the obtained scaling function is very similar to the one found for the 2D RSJ model. This suggests that the cause of the scaling is the same for the two cases, supporting the finite-size interpretation. The obtained scaling function \( g_L(T) \) is plotted in Fig. 11, and, just as in the 2D RSJ case, it is very well represented by the \( AR^{-\alpha} \) with \( \alpha = 1/6 \). The range over which this is true for the 2D RSJ model is roughly \( 2 \leq z(T) \leq 3.5 \) (see Fig. 6) and the \( g_L(T) \) in Fig. 11 in fact covers the somewhat larger range \( 2 \leq z(T) \leq 4.5 \). As explained in Sec. III, it is the fact that \( g_L(T) \) is well represented by \( AR^{-\alpha} \) with \( \alpha = 1/6 \) over a range around the KT transition which singles out the temperature where \( z(T) \approx 6 \) as the “ghost” transition. The temperature corresponding to the slope \( z(T) + 1 = 7 \) is given by the steepest full drawn line in Fig. 8. However, in analogy with the 2D RSJ model, we infer that there is in reality no vanishing resistance except at \( T = 0 \). In this sense it represents a “ghost” transition.

Since \( g_L(T) \), \( \alpha \), and \( z(T) \) (estimated from the maxima in Fig. 11) are known, we can directly test Eq. (3) by plotting \( \ln g_L(T) \) as a function of \( \alpha z(T) / [1 - \alpha z(T)] \). According to the discussion of Eq. (7) this should roughly be a straight line. As seen from Fig. 12 this is indeed the case for \( T \) below \( z \approx 2 \) (marked by a vertical dashed line in Fig. 12) note that \( z \approx 2 \) is approximately where the finite-size effect sets in according to Figs. 2 and 3. If it continued to be a straight line for lower \( T \) then the condition \( 1 - \alpha z(T) = 0 \) implies \( g_L = \infty \) and \( R = 0 \) signaling the “ghost” transition for \( z(T) = 1/\alpha \).

An alternative way of demonstrating the existence of the scaling Eq. (7) with the scaling variable \( IR^{-\alpha} \) is to note that this scaling implies that \( V/I \) for a given \( IR^{-\alpha} \) is independent of \( T \) (see Ref. 21 where a similar method has been used). Thus we can pick a given \( V/I \) and mark the value \( I \) and \( RI \) for which it occurs. As seen in Fig. 12 this construction gives rise to a straight line for a given value of \( V/I \) in the plot of \( V \) against \( I \) in log-log scale and the slope of such a line is according to the scaling assumption \( 1/\alpha + 1 \). As apparent from Fig. 13 this alternative test of the scaling is also borne out and gives values in the vicinity of \( 1/\alpha \approx 6 \).

Finally in Fig. 14 we plot the result from the Pierson scaling method for comparison.

V. DISCUSSION AND CONCLUSIONS

We have shown from simulations that the finite-size induced features of the I-V characteristics for the finite-size 2D RSJ model obey a scaling of the form given by Eq. (6) over a limited temperature range in the vicinity of the KT transition. The \( T \)-dependence of this scaling is absorbed in a function \( g_L(T) \) for each fixed size \( L \). Furthermore it was shown that this function \( g_L(T) \) over a limited region in the vicinity of the KT transition is to a good approximation proportional to \( R^{-\alpha} \). For the two sizes we studied \( (L = 6 \) and \( 8) \) we found that \( g_L(T) \) was different, yet the exponent \( \alpha \approx 1/6 \) was found in both cases. We noted that if the scaling form Eq. (6) was valid down to low enough \( T \) and that at the same time the proportionality \( g_L(T) \propto R^{-\alpha} \) was valid to low enough \( T \), then this would imply that the resistance vanishes at the \( T \) for which \( z(T) = 1/\alpha \). However, we stressed that in practice there is no such transition for the finite-size 2D RSJ model and introduced the term “ghost” transition for the transition which is not there. The fact that the transition does not occur is linked to the fact that the scaling in practice breaks down for lower \( T \), as clearly shown in Fig. 12. For the finite-size 2D RSJ model the resistance vanishes only for \( T = 0 \).

We also stressed that the theoretical basis for the Pierson scaling method is incompatible with the finite-size effect we study for the 2D RSJ model. However, in practice the Pierson method assumes that for the high-temperature branch of the scaling form Eq. (2) one has \( \xi \propto R^{-\alpha} \). Thus the high-temperature branch of the Pierson scaling is, from a pragmatic point of view, connected to our finite-size scaling. We explicitly demonstrated this connection in Fig. 6 for the finite-size 2D RSJ model.

We compared the data obtained for the finite-size 2D RSJ model with the YBCO data by Repaci et al. From the strong similarity manifested by Figs. 1, 2, 8, and 9 we concluded that Repaci’s data correspond to a KT transition around 27 K and that the resistance below this \( T \) is finite-size induced. This is in accord with the original conclusion by Repaci et al. Here we want to emphasize that finite size essentially means a cutoff in the logarithmic vortex interaction. Apart from the actual finite size of the system this can be caused by a small remnant magnetic field or by a finite perpendicular penetration depth.

We demonstrated the existence of a scaling of the form Eq. (6) also for Repaci’s YBCO data and furthermore demonstrated that the shape of the scaling function appeared to be the same as for the finite-size 2D RSJ model. This we interpreted as that the scaling for the YBCO data and for the 2D RSJ model have a common origin. This further supports the finite-size interpretation of the YBCO data. The function \( g_L(T) \) was just as in the 2D RSJ model well represented by \( g_L(T) \propto R^{-\alpha} \) with the same exponent \( \alpha \approx 1/6 \) as for the 2D RSJ model. The remarkable similarity between the results for the finite-size 2D RSJ model and Repaci’s YBCO data clearly suggests that the same physics is reflected in the two systems. Our conclusion is then that, since it reflects a finite-size induced feature for the 2D RSJ model, it also reflects a finite-size induced feature for the YBCO data.
quently, we conclude that the transition to a vanishing resistance implied by the condition $z(T) = 1/\alpha = 6$ is also for the YBCO data a “ghost” transition.

Pierson et al. in Ref. 11, when analyzing the Repaci’s YBCO data with the Pierson scaling method, inferred a transition to a vanishing resistance for $z(T) = 5.9$. This is closely the same value as where our finite-size scaling places the "ghost" transition both for the same experimental data and for the 2D RSJ model. This we believe is because the high-temperature branch of the Pierson scaling from a pragmatic point reflects the same behavior as our finite-size scaling. The low-temperature branch of the Pierson scaling is as seen in Fig. 14 less convincing and it does not have any relation to our finite-size scaling. The fact that Pierson et al. for very many cases have found that their scaling suggests a transition at $z(T) \approx 6$ from our perspective suggest a common origin. The fact that the same result is consistent with the finite-size 2D RSJ model suggests to us that this common origin is linked to the finite-size induced features of the $I$-$V$ characteristics in the vicinity of the KT transition.

Is the scaling property for the finite-size induced feature of the $I$-$V$ characteristics found in the present paper a real phenomena or just some accidental coincidence? Such accidental coincidences can of course never be completely ruled out. However, our analysis really leaves little room for any such accidental coincidence. Yet, we admit that we do not have a plausible explanation at the moment. Thus the enigma with the scaling and the "ghost" transition calls for further theoretical, experimental and computational efforts. We have in the present paper described how the scaling property is reflected in the $I$-$V$ data from a practical point of view and how to analyze the data from such a perspective.

Finally we stress that our interpretation in terms of a finite-size induced feature of the $I$-$V$ characteristics is entirely compatible with the established view of the KT transition with $z = 2$ and the usual finite-size scaling at a fixed $T$ in the low-temperature phase, as we explicitly showed for the 2D RSJ model. In our finite-size interpretation there is no real phase transition but only a reflection of a "ghost" transition. This also agrees with the view by Strachan et al. in Ref. 3 that the apparent vanishing of the resistance in Repaci’s data is not real but an artifact of the finite-voltage threshold in the measurements. On the other hand our conclusions are entirely incompatible with the interpretation put forward by Pierson et al. 11 that there is a real resistive transition and that this is a KT transition with $z \approx 6$. Yet our analysis from a pragmatic point of view supports the original finding by Pierson et al. 11 that there is after all an intriguing scaling feature in the $I$-$V$ characteristics associated with 2D vortex fluctuations and that this scaling phenomena appears to be quite general.

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16 Below the KT transition the infinite correlation length $\xi = \infty$ leads to the well-known non-linear IV-characteristics $E \propto J^a$ where $a$ is an exponent larger than 3.

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22 Pierson et al. in Ref. [11] argue, on the basis of a crude estimate of the finite size effects, that such effects cannot explain the Repaci data in contrary to the conclusion originally reached by Repaci et al. in Ref. [12]. In the present work we argue, on the basis of the strong similarity with the finite size induced scaling properties of the 2D RSJ model, that the Repaci data for small currents and lower temperatures is finite size induced in agreement with Ref. [22].

FIG. 1. $I$-$V$ characteristics for the 2D RSJ model on a square lattice with the finite size $L = 8$ plotted as $E = V/L$ against $J = I/L$ in log scales at temperatures $T = 2.0$, $1.5$, $1.3$, $1.2$, $1.1$, $1.0$, $0.95$, $0.90$, $0.85$, $0.80$, $0.75$, $0.70$, and $0.65$ (from top to bottom). The thick solid line corresponds to $V \propto I^3$ and has to a good approximation the same slope as the $T = 0.90$ curve over a finite-current region. For smaller currents the system becomes Ohmic, $E = RJ$, due to the finite size, which corresponds to the thick broken line.

FIG. 2. The $I$-$V$ data for the 2D RSJ model with $L = 8$ plotted as $d\ln E/d\ln J$ against $J$ in log scale. From bottom to top the curves correspond to $T = 2.0$, $1.5$, $1.3$, $1.2$, $1.1$, $1.0$, $0.95$, $0.90$, $0.85$, $0.80$, $0.75$, $0.70$, and $0.65$. The full drawn line segments connect the maxima. Down to $d\ln E/d\ln J \approx 2$ the maxima occur at about the same $J$ (between the vertical broken lines) and then move to higher $J$. The inset illustrates that the maxima of $d\ln E/d\ln J$ to a good approximation are given by $z + 1$ where $z$ is determined in two different ways (see text). The horizontal broken line in the inset corresponds to the KT value $a = z + 1 = 3$ and the crossing point gives the estimate $T_{KT} \approx 0.9$. 
FIG. 3. The size scaling given by Eq. (4) for the 2D RSJ model. $L(E/J)^{1/z}$ is plotted against $LJ$ for a fixed $T$ and the three different sizes $L = 4, 6, \text{and} 8$. The exponent $z$ is determined from a scaling collapse of the data: (a) is for $T = 0.90$ and (b) for $T = 0.80$. In (a) and (b) a possible additional small $L$-dependence in Eq. (4) coming from $g_L$ is ignored and the values $z = 2$ and 3.3 are obtained for $T = 0.90$ and 0.80, respectively. In (c) the $L$-dependence from $g_L$ is included for $T = 0.80$ and a good data collapse is obtained for the expected value $z = 1/\epsilon T^{CG} - 2 \approx 3.46$ (see text).

FIG. 4. Demonstration of the existence of a scaling in the form of Eq. (7) for the size-dependent part of the $I-V$ characteristics for the 2D RSJ model. $E/JR$ is plotted against $L J g_L(T)$ and $g_L(T)$ is determined from the condition of a data collapse. In (a) $g_L(T)$ is determined for a fixed $L$ for the two sizes $L = 6$ and 8 within the temperature intervals $0.7 \leq T \leq 1$ and $0.65 \leq T \leq 1$, respectively. A good data collapse is obtained towards smaller values of $L J g_L(T)$ and furthermore the data for $L = 6$ and 8 also fall on top as required by Eq. (4). In more detail, in (b) the same scaling plot is demonstrated for $T = 0.9$ (open squares), 0.8 (open circles), 0.7 (filled squares) and 0.65 (filled squares) in case of $L = 8$.

FIG. 5. The function $g_L(T)$ determined for the 2D RSJ model with size $L = 6$ (open circles) and 8 (open squares). It is shown that the function $g_L$ for both sizes over a limited $T$ interval is well approximated by $g_L(T) \propto R^{-\alpha}$ with $\alpha \approx 1/6$.

FIG. 6. The function $g_L(T)$ as a function of $z(T)$ for the 2D RSJ model with size $L = 8$. The proportionality between $g_L(T)$ and $R^{-1/6}$ holds in the interval $1.3 < z(T) < 3.5$ (see Fig. 5).
that the Pierson construction has too much flexibility.

In the main part, it is shown that a good scaling plot can still be obtained provided the two highest temperatures are excluded but the ensuing

finite-size induced part of the $I\!V$ characteristics for the $2D$ RSJ model with size $L = 8$ can be recast into the Pierson form: $I^{1+\alpha}/(T^CG V^\alpha)$ is plotted against $IR^{-\alpha}/T^CG$ and a good scaling collapse is found with $\alpha = 1/6$ for the data in the interval $0.80 \lesssim T \lesssim 1.2$. Identifying $1/\alpha$ with $z$ and $R^{-\alpha}$ with $\xi$ gives the connection $R^{-1/2} \propto \xi$ and makes the scaling look like the Pierson form. The inset shows that $\ln R \propto 1/\sqrt{T_C CG} - 0.125$ suggesting that the resistance would vanish at $T_C CG = 0.125$ and at this $T$ ($\approx 0.64$) the dynamic critical exponent $z(T)$ is to a good approximation given by $z(T) = 1/T_C CG - 2 = 6$ consistent with the condition $z(T) = 1/\alpha$. (b) Pierson scaling plot including the lower temperatures $T = 0.70$ (open circles) and $0.65$ (asterisks) for which $g L \sim R^{-1/6}$ fails (see Fig. 8). As seen in the upper left inset the same approximation for $R$ can be used provided $\sqrt{T_C CG} - 0.125$ is replaced by $\sqrt{T_C CG} - 0.07$. The lower right inset shows that $\sqrt{T_C CG} - 0.125$ fails for the lowest values of $T$. In the main part, it is shown that a good scaling plot can still be obtained provided the two highest temperatures are excluded but the ensuing $z$ value is now $z \approx 12$. This illustrates that the Pierson construction has too much flexibility.

FIG. 7. (a) Demonstration that the scaling obeyed by the finite-size induced part of the $I\!V$ characteristics for the 2D RSJ model with size $L = 8$ can be recast into the Pierson form: $I^{1+\alpha}/(T^CG V^\alpha)$ is plotted against $IR^{-\alpha}/T^CG$ and a good scaling collapse is found with $\alpha = 1/6$ for the data in the interval $0.80 \lesssim T \lesssim 1.2$. Identifying $1/\alpha$ with $z$ and $R^{-\alpha}$ with $\xi$ gives the connection $R^{-1/2} \propto \xi$ and makes the scaling look like the Pierson form. The inset shows that $\ln R \propto 1/\sqrt{T_C CG} - 0.125$ suggesting that the resistance would vanish at $T_C CG = 0.125$ and at this $T$ ($\approx 0.64$) the dynamic critical exponent $z(T)$ is to a good approximation given by $z(T) = 1/T_C CG - 2 = 6$ consistent with the condition $z(T) = 1/\alpha$. (b) Pierson scaling plot including the lower temperatures $T = 0.70$ (open circles) and $0.65$ (asterisks) for which $g L \sim R^{-1/6}$ fails (see Fig. 8). As seen in the upper left inset the same approximation for $R$ can be used provided $\sqrt{T_C CG} - 0.125$ is replaced by $\sqrt{T_C CG} - 0.07$. The lower right inset shows that $\sqrt{T_C CG} - 0.125$ fails for the lowest values of $T$. In the main part, it is shown that a good scaling plot can still be obtained provided the two highest temperatures are excluded but the ensuing $z$ value is now $z \approx 12$. This illustrates that the Pierson construction has too much flexibility.

FIG. 8. The $I\!V$ characteristics for Repaci’s YBCO data: $V$ is plotted as a function of $I$ for fixed $T$ where $10K \leq T \leq 40K$ (from left to right). The thick full line in the middle has the slope 3, which is the KT transition value and corresponds to the steepest slope of the data for $T = 27K$. The right thick full line has the slope 7, which corresponds to the "ghost" transition for the finite-size 2D RSJ model and which is the steepest slope of the data for $T = 18K$. The thick broken line has the slope 1 and corresponds to Ohmic resistance $V = RI$. Note the striking similarity with the 2D RSJ data in Fig. 8.

FIG. 9. The $I\!V$ data for Repaci’s YBCO data plotted as $d\ln V/d\ln I$ against $I$ in log scale for $17K \leq T \leq 40K$. The values of the maxima increase with decreasing $T$ and give estimates of $z(T) + 1$. The horizontal broken line corresponds to the KT value $z(T) + 1 = 3$. The vertical broken line gives the approximate position for the maxima when $d\ln V/d\ln I \gtrsim 2$, showing that the maxima occur at approximately the same $I$. The maxima for $d\ln V/d\ln I \lesssim 2$ occur at higher $I$ with increasing $T$. Note the striking similarity with Fig. 8 for the finite-size 2D RSJ model.
FIG. 10. Demonstration of the existence of a scaling in the form of Eq. (7) for Repaci's I-V data. \( V/IR \) is plotted against \( Ig_L(T) \) and \( g_L(T) \) is determined from the condition of a data collapse: (a) shows that a good scaling collapse is obtained for the data in the interval \( 23K \leq T \leq 40K \) (the highest \( T \) corresponds to the lowest curve). Note the striking similarity with the 2D RSJ data in Fig. 4(a). This similarity is further emphasized in (b) where the scaling function for the 2D RSJ model (full lines, note that \( E/J = V/I \)) is directly compared with the scaling function for the Repaci data (dotted lines). The two scaling function appears to fall on to each other when allowing for a constant shift along the \( x \) axis, suggesting that they are closely related.

FIG. 11. The function \( g_L(T) \) determined for Repaci's YBCO data: The filled circles give the values of \( g_L(T) \) determined from the data collapse in Fig. 10(a). The open circles demonstrates that the obtained \( g_L(T) \) is well represented by \( g_L(T) = AR^{-\alpha} \) with \( \alpha = 1/6 \). Compare Fig. 5 for the 2D RSJ model.

FIG. 12. Repaci's data analyzed according to Eq. (8): \( g_L(T) \) in log scale is plotted against \( \alpha z(T)/(1-\alpha z(T)) \) where \( \alpha \) according to Fig. 11 is 1/6 to a good approximation and \( z(T) \) is determined from the maxima in Fig. 9. As seen, the data fall approximately along a straight line from about where \( z(T) = 2 \) (vertical broken line). This corresponds to the \( T \) region where the maxima in Fig. 9 occur for the same current \( I \) suggesting that this is the finite-size dominated part of the I-V characteristics. If the data would continue to fall on the straight line all the way to infinity, this would imply \( g_L(T) = \infty \) and equivalently a vanishing resistance for \( z(T) = 1/\alpha \). Since this does not happen in practice we use the term “ghost” transition.

FIG. 13. Direct determination of the exponent \( \alpha \) for Repaci's YBCO data. Eq. (8) together with \( g_L \propto R^{-\alpha} \) means that \( V/IR \) is a function of \( IR^{-\alpha} \). \( V(=RI) \) is plotted as a function of \( I \) for a given fixed value of \( V/IR \) = const. (open and filled circles correspond to const.\( =1.5 \) and 2.0, respectively). Consequently, the scaling implies that the data points should fall on straight lines given by \( RI \propto I^{-1/\alpha-1} \) and the slope of these lines should be given by \( -1/\alpha-1 \). As seen this prediction is borne out and the obtained values are \( \alpha^{-1} = 6.3 \) and 6.2 in close agreement with the value \( \alpha^{-1} \approx 6 \) obtained from Fig. 11.
FIG. 14. The Pierson scaling plot for Repaci’s YBCO data: $T^{1+1/z}/(TV^1/z)$ is plotted against $I\xi/T$ where it is assumed that $\xi \propto R^{-1/z}$. The data is plotted in log scales and the construction makes the scales very extended which exaggerates the goodness of the data collapse. The high-$T$ branch of the scaling plot corresponds to $18K \leq T \leq 40K$ and is given by the lower scaling curve and should be compared to the scaling of the finite-size 2D RSJ model in Fig. 7. The low-$T$ branch contains data for $10K \leq T \leq 16K$. A transition of a vanishing resistance at $T = 17K$ with $z = 5.9$ was concluded from this scaling construction. We claim that this scaling plot does not reflect any real transition to a vanishing resistance.