HELAC: a package to compute electroweak helicity amplitudes

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ABSTRACT

HELAC is a FORTRAN based package that is able to compute efficiently helicity amplitudes for arbitrary scattering processes within the standard electroweak theory. The algorithm exploits the virtues of the Dyson-Schwinger equations as compared to the traditional Feynman graph approach. All electroweak vertices are included in both the unitary and Feynman gauges, and computations including all mass effects are available. A version performing multi-precision computations with arbitrary - user defined - accuracy is also included, allowing access to any phase space point for arbitrary high energies.

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PROGRAM SUMMARY

Title of the program:
HELAC.

Catalogue number:

Program obtainable from:
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Also at ftp://alice.nuclear.demokritos.gr/dist/pub/papadopo/helac.

Licensing provisions: none

Computer for which the program is designed and others on which it has been tested:
ALPHA, HP, IBM workstations.

Operating system under which the program has been tested:
Unix.

Programming language:
FORTRAN 77 and FORTRAN 90

Keywords:
Automatic evaluation of helicity amplitudes, Dyson-Schwinger equations, recursive algorithms.

Nature of physical problem:
A substantial part of particle phenomenology nowadays is based upon our ability to study efficiently processes involving a relatively large number of particles. This requires efficient algorithms for matrix element calculation and phase space generation and integration. As far as the matrix element calculation is concerned, the traditional approach utilizes standard Feynman graph representation of the scattering amplitude, resulting to a computational cost that grows asymptotically as $n!$, where $n$ is the number of particles involved in the process.

Method of solution:
As an alternative recursive algorithms based on Schwinger-Dyson equations lead asymptotically to a much lower growth of the computational cost, namely $a^n$, where $a \sim 3$. It is the aim of this paper to present such an algorithm as well as the corresponding FORTRAN code which allows the computation of any tree-order electroweak amplitude.
1 Introduction

A substantial part of current particle phenomenology is based upon our ability to study efficiently processes involving a relatively large number of particles. This requires efficient algorithms for matrix element calculation and phase space generation and integration. As far as the matrix element calculation is concerned, the traditional approach utilizes standard Feynman graph representation of the scattering amplitude, resulting to a computational cost that grows asymptotically as \( n! \), where \( n \) is the number of particles involved in the process. As an alternative recursive algorithms based on Schwinger-Dyson equations lead asymptotically to a much lower growth of the computational cost, namely \( a^n \), where \( a \sim 3 \). It is the aim of this paper to present such an algorithm as well as the corresponding FORTRAN code which allows the computation of any tree-order electroweak amplitude.

2 The algorithm

As advertised, the algorithm is based upon the well-known Dyson-Schwinger equations. These equations are equivalent to the field equations derived from the classical lagrangian and express recursively the \( n \)-point Green’s functions in terms of the \( 1-, 2-, \ldots, (n-1) \)-point functions.

In order to illustrate this point let us consider a simple example of a QED-like theory possessing minimal interactions of a spinor field to a gauge boson. Let \( p_1, p_2, \ldots, p_n \) represent the external momenta involved in the scattering process under consideration, taken to be incoming. For a vector field we define

\[
b_\mu(P) = \begin{array}{c}
\end{array}
\]

a four vector, which describes any sub-amplitude from which a boson \( V \) with momentum \( P \) can be constructed. The momentum \( P \) is given as a sum of momenta of external particles, namely \( P = \sum_{i \in I} p_i \) with \( I \subset \{1, \ldots, n\} \). Accordingly, we define by

\[
\psi(P) = \begin{array}{c}
\end{array}
\]

a four-dimensional spinor, which describes any sub-amplitude from which a fermion with momentum \( P \) can be constructed and by

\[
\bar{\psi}(P) = \begin{array}{c}
\end{array}
\]
a four-dimensional antispinor, which describes any sub-amplitude from which an antifermion with momentum $P$ can be constructed.

The Dyson-Schwinger equations take the following simple form. For a boson with momentum $P^\mu$,

\[ b^\mu(P) = \sum_{i=1}^{n} \delta_{P=p_i} b^\mu(p_i) + \sum_{P=P_1+P_2} (ig) \Pi^\mu_\nu \bar{\psi}(P_2) \gamma^\nu \psi(P_1) \epsilon(P_1, P_2) \]  

(4)

where the propagator $\Pi$ is given by

\[ \Pi^\mu_\nu = \frac{i}{P^2 - m^2} \left( -g^\mu_\nu + \frac{P^\mu P_\nu}{P^2 - \xi m^2} \left( 1 - \xi \right) \right) \]

and $\epsilon(P_1, P_2)$ is a sign factor, that takes the value $\pm 1$ and whose definition will be described below. For a fermion with momentum $P$

\[ \psi(P) = \sum_{i=1}^{n} \delta_{P=p_i} \psi(p_i) + \sum_{P=P_1+P_2} (ig) \mathcal{P} \bar{\psi}(P_2) \psi(P_1) \epsilon(P_1, P_2) \]  

(5)

where $\mathcal{P}$ is given by

\[ \mathcal{P} = \frac{i}{\bar{P} - m} \]

and for an antifermion with momentum $P$,

\[ \bar{\psi}(P) = \sum_{i=1}^{n} \delta_{P=p_i} \bar{\psi}(p_i) + \sum_{P=P_1+P_2} (ig) \bar{\psi}(P_1) \bar{\mathcal{P}} \psi(P_2) \epsilon(P_1, P_2) \]  

(6)

where

\[ \bar{\mathcal{P}} = \frac{i}{-\bar{P} - m} \].
The scattering amplitude can be calculated by any of the following relations, 
\[ \mathcal{A}(p_1, \ldots, p_n) = \begin{cases} 
  b_0^\mu(P_i) b_\mu(p_i) & \text{where } i \text{ corresponds to a photon} \\
  \bar{\psi}_0(P_i) \psi(p_i) & \text{where } i \text{ corresponds to an incoming fermion line} \\
  \bar{\psi}(p_i) \psi_0(P_i) & \text{where } i \text{ corresponds to an outgoing fermion line} 
\end{cases} \]

where 
\[ P_i = \sum_{j \neq i} p_j, \]
so that \( P_i + p_i = 0 \). The functions with subscript 0 are given by the previous expressions, except for the propagator term. This is because the outgoing momentum \( P_i \) being on shell the propagator factor is removed by the amputation procedure. The initial conditions are given by
\[ b_\mu(p_i) = \epsilon_\lambda^\mu(p_i), \lambda = \pm 1, 0 \]
\[ \psi(p_i) = \begin{cases} 
  u_\lambda(p_i) & \text{if } p_i^0 \geq 0 \\
  v_\lambda(-p_i) & \text{if } p_i^0 \leq 0 
\end{cases} \]
\[ \bar{\psi}(p_i) = \begin{cases} 
  \bar{u}_\lambda(p_i) & \text{if } p_i^0 \geq 0 \\
  \bar{v}_\lambda(-p_i) & \text{if } p_i^0 \leq 0 
\end{cases} \]

where the explicit form of \( \epsilon_\lambda^\mu, u_\lambda, v_\lambda, \bar{u}_\lambda, \bar{v}_\lambda \) are given in the Appendix.

In order to actually solve the recursive equations it is convenient to use a binary representation of the momenta involved \[ \mathbb{I} \]. For a process involving \( n \) external particles with momenta \( p_i^\mu, i = 1, \ldots, n \) we assign to the momentum \( P_\mu \) defined as
\[ P_\mu = \sum_{i \in I} p_i^\mu \]
where \( I \subset \{1, \ldots, n\} \), a binary vector \( \vec{m} = (m_1, \ldots, m_n) \), where its components take the values 0 or 1 in such a way that
\[ P_\mu = \sum_{i=1}^{n} m_i p_i^\mu. \]

Moreover this binary vector can be uniquely represented by the integer
\[ m = \sum_{i=1}^{n} 2^{i-1} m_i \]
with \( 0 \leq m \leq 2^{n-1} \). Especially, the external momenta \( p_i, i = 1, \ldots, n \) are represented by \( m = 2^{i-1}, i = 1, \ldots, n \). All momenta \( P \) can now be replaced by the corresponding integers
\[ b_\mu(P) \rightarrow b_\mu(m). \]
A very convenient ordering of integers in binary representation relies on the notion of level $l$, defined simply as

$$l = \sum_{i=1}^{n} m_i .$$

As it is easily seen all external momenta are of level 1, whereas the total amplitude corresponds to the unique level $n$ integer $2^n - 1$. This ordering dictates the natural path of the computation; starting with level-1 sub-amplitudes, we compute the level-2 ones using the Dyson-Schwinger equations and so on up to the final expression Eq.(8). As far as the sign factor is concerned

$$\epsilon(P_1, P_2) \rightarrow \epsilon(m_1, m_2)$$

we define

$$\epsilon(m_1, m_2) = (-1)^{\chi(m_1, m_2)}$$

with

$$\chi(m_1, m_2) = \sum_{i=n}^{2} \hat{m}_{1i} \left( \sum_{j=1}^{i-1} \hat{m}_{2j} \right)$$

where hated components are set to 0 if the corresponding external particle is a boson. It is not difficult to see that this sign factor takes properly into account the anti-symmetry of the amplitude with respect to fermionic particles.

As an illuminating example we present here the computation of the amplitude for the process $e^+e^- \rightarrow \mu^+\mu^-\gamma$. The calculation starts with the computation of the level-2 sub-amplitudes, which are all sub-amplitudes produced from any two of the external particles. These are

$$\begin{align*}
b_\mu(12) &= (ig)\Pi_{12\mu}^\nu \bar{\psi}(4)\gamma_\nu\psi(8) \\
\bar{\psi}(18) &= (ig)\bar{\psi}(2)\slashed{b}(16)\slashed{P}_{18} \\
\bar{\psi}(20) &= (ig)\bar{\psi}(4)\slashed{b}(16)\slashed{P}_{20} \\
\psi(24) &= (ig)\slashed{P}_{24}\slashed{b}(16)\psi(8) \\
\end{align*}$$

Then we compute the level-3 sub-amplitudes:

$$\begin{align*}
\bar{\psi}(14) &= (ig)\bar{\psi}(2)\slashed{b}(12)\slashed{P}_{14} \\
b_\mu(28) &= (ig)\Pi_{28\mu}^\nu \left( \bar{\psi}(20)\gamma_\nu\psi(8) + \bar{\psi}(4)\gamma_\nu\psi(24) \right) \\
\end{align*}$$

and then the level-4 sub-amplitude, which is the final level in this case,

$$\bar{\psi}_0(30) = (ig) \left( \bar{\psi}(2)\slashed{b}(28) + \bar{\psi}(14)\slashed{b}(16) + \bar{\psi}(18)\slashed{b}(12) \right)$$
Then the amplitude is simply given by

\[ A = \bar{\psi}_0(30)\psi(1) \]

Note that we have chosen the particle number 1 as our ending point, so we have computed all sub-amplitudes where the momentum \( p_1 \) does not appear: this excludes all odd integers between 1 and \( 2^{n-2} \).

There are two special issues in the computation which go beyond the recursive equations presented above. The first is the generation of all helicity configurations and the second the treatment of the color summation in the case external particles with color are involved.

As far as the helicity configurations are concerned this is done in a rather straightforward way, resulting to an automatic evaluation of all relevant combinations. For any given external particle knowledge of its flavor allows the program to compute all relevant helicity configurations. The total number is given by \( 2^{l_2}3^{l_3} \) where \( l_2 \) is the number of (anti-)fermions and massless gauge bosons and \( l_3 \) the number of massive gauge bosons involved in the scattering process.

The color configurations are taken into account as follows. Since only electroweak processes are considered in this version, the only colored particles that can appear in the amplitude are quarks and antiquarks and let us have \( n \) pairs of them. Each color amplitude is proportional to the following color structure

\[ C_i = \delta_{1,\sigma_i(1)}\delta_{2,\sigma_i(2)}\ldots\delta_{n,\sigma_i(n)} \]

where \( \sigma_i \) represents the \( i \)-th permutation of the set \( 1,2,\ldots,n \). The code computes all non-vanishing color amplitudes as well as the corresponding color matrix

\[ M_{ij} = \sum C_iC_j \]

and finally performs the color summation.

## 3 The code HELAC

The computation is split in two major phases. During the first phase, which we call initialization phase, the program selects all the relevant sub-amplitudes for the required process. In the simplest version the program accepts as input the following variables:

- \( n \) the number of particles involved in the scattering
- \( \text{ifl}(1:n) \) the array of flavors of the particles
• **iflag.** If set to 0 sum over all helicity configurations is understood. If set to 1 you must supply also the specific helicity configuration to be computed.

• **iunitary.** If set to 0 the Feynman gauge is considered whereas if set to 1 the unitary gauge is used.

• **ihiggs** denotes the inclusion (1) or not (0) of the Higgs particle as an intermediate state.

• **iwidth** denotes the fixed (0) or complex (1) scheme for the introduction of the width of W and Z.

• **io(1:n)** is used to distinguish among initial (1) and final state particles (-1). By default io(1:2)=1 and io(3:n)=-1.

The first routine called after the input has been read is the routine `physics/physics.f` located in the homonymous file. In this routine all couplings of the standard electroweak theory are defined \cite{3}.

Then the routine `helac_init/master.f` is called. In the beginning the average-over-helicity (avhel), the average-over-color (avcol) and the symmetry(symet) factors are computed. Then, by calling the routine `setncc/intpar.f` the number of color configurations ncc is set up depending on the number of quarks and anti-quarks involved. For each color configuration the program constructs the skeleton of the amplitude starting at level two and proceeding up to level \( n - 1 \). All possible vertices are scanned in this phase, as for instance `vff/pan1.f` which describes the coupling of a fermion anti-fermion to produce a vector boson, in order to select all non-zero sub-amplitudes. A special routine, `redo/pan1.f` is called at the end just to check whether all the selected sub-amplitudes are indeed contributing to the final amplitude under consideration. The program ends this phase when all color configurations have been considered and the color matrix \( \text{rmatrix}(1:\text{ncc}, 1:\text{ncc}) \) has been computed.

During the second phase, which we call the computation phase, the code computes numerically the amplitude for each phase space point introduced. The main calling routine is `helac_master/master.f`. The first step is the automatic setup of the helicity configurations via the routine `sethel/intpar.f`. The next step is the computation of the external particle wave functions by the routine `iniqq/pan1.f`. All relevant routines for this computation can be found in `wavef.f`. The program proceeds by computing the amplitude for every color configuration via `nextq/pan1.f`. The vertex and propagator functions needed are included in `pan2.f`. Moreover the sign factor as described in Eq.(10) is also\footnote{For a recent discussion see reference \cite{2}.}
computed. The routine `helac_master/master.f` returns the squared matrix element \( (\text{smel2}) \) summed and averaged over helicity and color. Finally, a sample `main/main.f` program to run HELAC is provided. In this part of the code a call to a phase-space generator is also included (`rambo/rambo.f`), in order to generate appropriate phase space points for the computation.

All floating point computations are performed in `real*8` or double precision. Nevertheless the program is written in such a way that one can choose a higher accuracy if needed. There are two alternatives. The first one is to use `real*16` or quadruple precision. For this task several modules to perform quadruple precision computation with complex numbers are supplied in `qprec.f`. This is necessary because in most of the existing platforms no FORTRAN quadruple precision complex numbers are available. The most efficient way to use this precision with complex numbers is to exploit the virtues of FORTRAN 90 and define the appropriate derived types and modules. The second alternative make use of a multi-precision library [4], written also in FORTRAN 90. The precision is now user-defined and is set up by calling the routine `zmset` in the very beginning of the program (`main.f`). All these are automatically driven by the appropriate `make` files included in the package.

## 4 Results

In this section we will try to explain how to read the output of the program and to highlight several aspects of it. First of all let us follow a sample computation of the process

\[ e^- e^+ \rightarrow e^- e^+ \gamma. \]

As an input we have to provide

1. The number of particles involved in the process (5).

2. For each particle its flavor (2 -2 2 -2 31). In the following table we provide the correspondence used in HELAC:

| Particle | HELAC Flavor |
|----------|--------------|
| \( \nu_e, e^-, u, d, \nu_\mu, \mu^-, e, s, \ldots \) | 1, ..., 12 |
| \( \bar{\nu}_e, e^+, \bar{u}, \bar{d}, \bar{\nu}_\mu, \mu^+, \bar{c}, \bar{s}, \ldots \) | -1, ..., -12 |
| \( \gamma, Z, W^+, W^- \) | 31, ..., 34 |
| \( H, \chi, \phi^+, \phi^- \) | 41, ..., 44 |

3. The parameters `iflag`, `iunitary`, `ihiggs`, `iwidth` (0 1 0 0).
where the first lines show the required input data; then the average-helicity, average-color and symmetry factors are shown; the number of contributing sub-amplitudes for each color configuration is given (44) as well as the total number of Feynman graphs (16). This marks the end of the initialization phase, and afterwards the computation of the squared matrix element for a given phase space point is performed. Results for each helicity configuration are printed and at the end the HELAC amplitude squared is given (1.107657409535622E-04)

As far as the efficiency of the code is concerned extensive comparisons have been made to existing calculations. We have chosen among them two popular tools, namely MADGRAPH [5] and EXCALIBUR [6]. Comparisons with EXCALIBUR have been restricted to four-fermion final states. Running under the same conditions the speed ratio was varying between 1 and 2. The same results have been obtained in comparisons with MADGRAPH taken into account that it can produce results for only up to 7 particles involved in the scattering process. As expected HELAC show an exponential (instead of factorial) CPU-time growth. There is no a priori limitation for the number of particles that HELAC can treat,
the only restrictions being that of memory allocation.

In order to have a taste of a multi-precision computation we have computed the squared amplitude for the process

\[ e^- (p_1)e^+ (p_2) \rightarrow e^- (p_3)e^+ (p_4)e^- (p_5)e^+ (p_6) \]

at two phase space points. Phase space point (A) is just a randomly generated one by the phase-space generator RAMBO. Phase space point (B) on the other hand is a very special one, where

\[ \frac{p_0^1}{\text{GeV}} = 100, \quad \vec{p}_1 + \vec{p}_2 = 0, \quad \frac{p_3^0}{p_1^0} = 0.9, \quad \theta_3 = 0, \]
\[ \frac{(p_5 + p_6)^2}{(p_4 + p_5 + p_6)^2} = 0.1, \quad \theta_4 = 0, \quad \phi_4 = 0, \quad \theta_5 = 0, \quad \phi_5 = 0 \]

and \( m_e = 0.511 \times 10^{-3} \text{ GeV} \). In this case all particles are co-linear to the beam and therefore important enhancements are expected. In the following table results are provided for these points, by using the \texttt{real*8} (DP), \texttt{real*16} (QP) and 34-digit multi-precision version of the code.

\[
\begin{array}{cc}
(A) & (B) \\
1.539728523150595E-008 & 1.256276706229023E+023 \\
1.53972852315058854156763002825013D-08 & 3.07162601093710915134136924973089D+22 \\
1.53972852315058854156763002825011853M-8 & 3.07162601093710915127950109241770808M+22 \\
\end{array}
\]

As one can easily see the numerical stability of the DP computation is spoiled in the co-linear region. This is due to huge gauge cancellations occurring in this region; a way out of this problem will be discussed elsewhere.

We end this presentation by summarizing the main achievements:

- An algorithm based on Dyson-Schwinger equations has been presented that enables the computation of electroweak amplitudes with high efficiency.

- Based on this algorithm a FORTRAN package HELAC has been developed, which includes full massive computation in both the unitary and the Feynman gauge of the standard electroweak theory.

- A quadruple as well as a multi-precision version of HELAC have been incorporated allowing numerically stable results for any phase space point and for arbitrary high energies.
Appendix

In all calculations we are using the light-cone representation of a four-vector $V^\mu$, defined as

$$V^A = (V^0 + V_z, V^0 - V_z, V_x + iV_y, V_x - iV_y), \quad A = 1, \ldots, 4. \quad (12)$$

Polarization state-vectors are given by

$$\epsilon^A_+ = \left( \frac{-p_T}{\sqrt{2|\vec{p}|}}, \frac{p_T}{\sqrt{2|\vec{p}|}}, \frac{(p_x + ip_y)(|\vec{p}| + p_z)}{\sqrt{2|\vec{p}|p_T}}, \frac{(p_x - ip_y)(-|\vec{p}| + p_z)}{\sqrt{2|\vec{p}|p_T}} \right)$$

$$\epsilon^A_- = \left( \frac{p_T}{\sqrt{2|\vec{p}|}}, \frac{-p_T}{\sqrt{2|\vec{p}|}}, \frac{(p_x + ip_y)(|\vec{p}| - p_z)}{\sqrt{2|\vec{p}|p_T}}, \frac{(p_x - ip_y)(-|\vec{p}| - p_z)}{\sqrt{2|\vec{p}|p_T}} \right)$$

$$\epsilon^A_0 = \left( \frac{|\vec{p}|}{\sqrt{\vec{p}^2}}, \frac{p_x p_0}{|\vec{p}| \sqrt{\vec{p}^2}}, \frac{|\vec{p}| - p_z}{|\vec{p}| \sqrt{\vec{p}^2}}, \frac{(p_x + ip_y)p_0}{|\vec{p}| \sqrt{\vec{p}^2}}, \frac{(p_x - ip_y)p_0}{|\vec{p}| \sqrt{\vec{p}^2}} \right) \quad (13)$$

As for the Dirac matrices we are using the chiral representation. The wave functions which describe massive spinors are given by:

$$u_+(p) = \begin{pmatrix} r/c \\ a(p_x + ip_y)/r \\ -mb/r \\ -m(p_x + ip_y)/r \end{pmatrix} \quad \bar{u}_+(p) = \begin{pmatrix} mb/r \\ m(p_x - ip_y)/r \\ -r/c \\ -a(p_x - ip_y)/r \end{pmatrix}$$

$$u_-(p) = \begin{pmatrix} m(p_x - ip_y)/r \\ -mb/r \\ a(p_x - ip_y)/r \\ r/c \end{pmatrix} \quad \bar{u}_-(p) = \begin{pmatrix} a(p_x + ip_y)/r \\ -m(p_x + ip_y)/r \\ r/c \\ mb/r \end{pmatrix}$$

$$v_+(p) = \begin{pmatrix} -m(p_x - ip_y)/r \\ mb/r \\ -a(p_x - ip_y)/r \\ r/c \end{pmatrix} \quad \bar{v}_+(p) = \begin{pmatrix} a(p_x + ip_y)/r \\ -r/c \\ m(p_x + ip_y)/r \\ -mb/r \end{pmatrix}$$

$$v_-(p) = \begin{pmatrix} r/c \\ a(p_x + ip_y)/r \\ mb/r \\ m(p_x + ip_y)/r \end{pmatrix} \quad \bar{v}_-(p) = \begin{pmatrix} -mb/r \\ -m(p_x - ip_y)/r \\ -r/c \\ -a(p_x - ip_y)/r \end{pmatrix} \quad (14)$$

\[ \text{For conventions see reference } \number{7}. \]
where:
\[ a = p_0 + |\vec{p}|, \quad b = p_z + |\vec{p}|, \quad c = 2|\vec{p}|, \quad r = \sqrt{abc} \]

For a massless particle the spinors are

\[
\begin{align*}
    u_R(p) &= \begin{pmatrix}
    \frac{\sqrt{p_0 + p_z}}{p_z + ip_y} \\ 0 \\ 0 \\ \frac{0}{p_0 + p_z}
    \end{pmatrix},
    & \quad \bar{u}_R(p) &= \begin{pmatrix}
    0 \\ 0 \\ -\frac{p_0 + p_z}{p_z - ip_y} \\ \frac{0}{p_0 + p_z}
    \end{pmatrix},
    \\
    u_L(p) &= \begin{pmatrix}
    0 \\ 0 \\ \frac{0}{p_0 + p_z}
    \end{pmatrix},
    & \quad \bar{u}_L(p) &= \begin{pmatrix}
    (p_z + ip_y)/\sqrt{p_0 + p_z} \\ \frac{0}{p_0 + p_z} \\ 0
    \end{pmatrix}
\end{align*}
\]

We now proceed to describe the vertex functions. Let us take as an example the Eq.(4).

By using the Dirac matrices in the chiral representation and the light-cone expression of four vectors, the reduced form of this equation becomes very simple, namely the four vector

\[
\begin{align*}
    V_{\mu} &= \bar{\psi}(P_1)\gamma_{\mu}(g_R\omega_R + g_L\omega_L)\psi(P_2)
\end{align*}
\]

turns out to be

\[
V^A = \begin{pmatrix}
    -g_R\psi_1\bar{\psi}_3 - g_L\psi_4\bar{\psi}_2 \\
    -g_R\psi_2\bar{\psi}_4 - g_L\psi_3\bar{\psi}_1 \\
    -g_R\psi_2\bar{\psi}_3 + g_L\psi_4\bar{\psi}_1 \\
    -g_R\psi_1\bar{\psi}_4 + g_L\psi_3\bar{\psi}_2
\end{pmatrix}
\]

where \( \psi_i(\bar{\psi}_i), i = 1, \ldots, 4 \) are the components of the spinor \( \psi(P_2) \) \( (\bar{\psi}(P_1)) \) and

\[
\omega_L = \frac{1}{2} (1 - \gamma_5), \quad \omega_R = \frac{1}{2} (1 + \gamma_5)
\]

On the other hand, the spinor

\[
u = (\sl{P} + m)\psi(P_1)\omega_R\psi(P_2)
\]

used for instance in Eq.(4), can be reduced to

\[
\begin{pmatrix}
    (-b_2p_1 + b_3p_4)\psi_1 + (b_4p_1 - b_1p_4)\psi_2 \\
    (b_3p_2 - b_2p_3)\psi_1 + (-b_1p_2 + b_4p_3)\psi_2 \\
    m(b_2\psi_1 - b_4\psi_2) \\
    m(-b_3\psi_1 + b_1\psi_2)
\end{pmatrix}
\]

\[ (18) \]
where $\psi_i$, $b_i$, $p_i$, $i = 1, \ldots, 4$ are the components of $P$, $b(P_1)$ and $\psi(P_2)$ respectively. In a similar way, for the standard electroweak theory in both the unitary and the Feynman gauge, twenty eight different vertex functions have been implemented in vertices/pan2.f.

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