AN ELECTROWEAK $SU(2)_L \times U(1)$ GAUGE THEORY OF $J = 0$ MESONS.

B. Machet

Laboratoire de Physique Théorique et Hautes Energies,
Universités Pierre et Marie Curie (Paris 6) et Denis Diderot (Paris 7);
Unité associée au CNRS URA 280.

Extended version of the talk “Custodial symmetry in a $SU(2)_L \times U(1)$ gauge theory of $J = 0$ mesons”
given at the 2nd International Symposium on Symmetries in Subatomic Physics,
Seattle (Washington, USA), June 25th-29th 1997.

Abstract: I display all $J = 0$ scalar and pseudoscalar representations of the standard $SU(2)_L \times U(1)$
group of electroweak interactions which transform like sets of fermion-antifermion composite fields. They
can fit into quadruplets of definite $CP$ quantum numbers. $SU(2)_L \times U(1)$ is embedded in a natural way,
compatible with the Glashow-Salam-Weinberg model for quarks, into the chiral group $U(N)_L \times U(N)_R$.
$N$ being the (even) number of “flavours”. It involves a unitary $N/2 \times N/2$ “mixing matrix” for fermions which
are however only considered here as mathematical objects in the fundamental representation of $U(N)$.
The electroweak gauge Lagrangian for the $J = 0$ particles exhibits a chiral $SU(2)_L \times SU(2)_R$
symmetry at the limit when the hypercharge coupling $g'$ goes to zero. It is spontaneously broken down to its diagonal
$SU(2)_V$ subgroup, which includes the electromagnetic $U(1)$, spanning a bridge to an explanation of electric
charge quantization. Chiral are electroweak spontaneous breaking are identical. The consequences for the
nature of the Goldstone bosons are examined.
Comparison with recent works by Cho et al. unraveling dyon-like solution is an electroweak model with
the same structure suggests that electric-magnetic duality may be realized here, with the occurrence of a
strongly interacting sector.

$SU(2)_L \times U(1)$ allows one mass scale per quadruplet; as each decomposes into one triplet and one singlet
of $SU(2)_V$, the custodial symmetry doubles the amount of masses allowed for the $2N^2$ mesons, from
$N^2/2$ to $N^2$. They bear no special connection with the nature of their “fermionic content” and share with
the leptonic sector the same arbitrariness. No information on the mass of the Higgs boson can be expected
without additional input.

New results about $CP$ violation are obtained: “indirect” $CP$ violation just appears to be the consequence
of $P$ violation (unitarity compels the electroweak mass eigenstates to be $C$ eigenstates), and the role of the
Kobayashi-Maskawa mixing matrix for fermions fades away: mesonic mass eigenstates can still be $CP$
eigenstates despite the presence of a complex phase.

I briefly show how the customary results for the leptonic decays of pseudoscalar mesons are recovered.

The extension to leptons is discussed. It is shown, in the context of previous works linking the effective
$V - A$ structure of weak currents to the non-observation of right-handed neutrinos and the masslessness of
the left-handed ones, how the custodial symmetry constrains them to be Majorana particles.
1 Introduction.

Many difficulties in hadronic physics stem from the fields in the Lagrangian (quarks) not being the observed particles or asymptotic states (mesons, baryons).

I show here that the standard model of electroweak interactions \cite{1} can be straightforwardly extended to $J = 0$ mesons \cite{2}, and I will put a special emphasis on its symmetry properties.

They concern:
- the occurrence of a chiral $SU(2)_L \times SU(2)_R$ chiral symmetry, explicitly broken, when the hypercharge coupling $g'$ is non vanishing, and spontaneously broken, when the Higgs boson gets a non-vanishing vacuum expectation value, to a “custodial” diagonal $SU(2)_V$ symmetry \cite{2}; the latter is local when $g' = 0$. It includes the electric charge generator, such that the quantization of the electric charge is directly linked with the custodial symmetry staying unbroken. It is put in relation with the ideas of electric-magnetic duality \cite{3}, and the occurrence of a strongly interacting sector in the standard model;
- the transformation properties by $CP$: I show \cite{4} that “indirect” $CP$ violation is only a consequence of $P$ violation, and that the existence of a complex phase in the mixing matrix is no longer a sufficient condition for electroweak mass eigenstates to be different from $CP$ eigenstates.

The electroweak group $SU(2)_L \times U(1)$ is embedded into $U(N)_L \times U(N)_R$ in such a way that it acts on fermions like in the Glashow-Salam-Weinberg model; the embedding is accordingly characterized by the Cabibbo-Kobayashi-Maskawa \cite{5} unitary matrix.

This extension of the standard model provides new ideas concerning chiral symmetry in the physics of mesons and also has consequences on the way we interpret their spectrum.

The spontaneous breaking, by $\langle H \rangle \neq 0$, of the chiral $SU(2)_L \times SU(2)_R$ down to $SU(2)_V$ and of $SU(2)_L \times U(1)$ down to the electromagnetic $U(1)_{em}$ are the same phenomenon. The three corresponding Goldstone bosons, that become the longitudinal components of the three massive gauge fields, are linear combinations of the known pseudoscalar mesons (pions, kaons . . .); the latter (pions, kaons . . .) are naturally massive since they only coincide with the Goldstones of the broken chiral symmetry in the case of one generation and, of course, vanishing mixing angle. In the real case of three generations, no $J = 0$ meson which is a flavour eigenstate is to be interpreted as a Goldstone particle.

The chiral breaking relevant to the physics of $J = 0$ mesons thus appears to be that of $SU(2)_L \times SU(2)_R$ into $SU(2)_V$ rather than the one of $U(N)_L \times U(N)_R$ into the diagonal $U(N)$ flavour subgroup \cite{3} ($N$ is the number of flavours). $SU(2)_V$ allows $N^2$ independent mass scales, twice the number shown to be allowed by the chiral $SU(2)_L \times SU(2)_R$ and the electroweak $SU(2)_L \times U(1)$ symmetries. The doubling from $N^2/2$ to $N^2$ can in particular split scalar and pseudoscalar mesons, though the electroweak mass eigenstates do not have in general a definite parity.

The spectrum of $J = 0$ mesons has acquired the same arbitrariness as the one of the leptons. It includes the Higgs boson, the mass of which is now not more but also not less explained than the ones of the other mesons.

A reduction of the number of arbitrary mass parameters would need an additional symmetry.

The quantization of the electric charge for all asymptotic states requires that the same formalism be applied to leptons \cite{6}. Since the hadronic sector is now naturally anomaly-free (no fermionic field is involved any longer), we cannot invoke any longer the usual cancelation \cite{8} between quarks and leptons. The natural candidate is the purely vectorial theory described in \cite{3}, in which the introduction of a composite triplet of scalars links at leading order in $1/N$, the “decoupling”, by an exact “see-saw” mechanism, of an infinitely massive right-handed neutrino, to the effective $V - A$ structure of weak currents. The observed neutrino is then exactly massless. It is shown in \cite{7} that the existence of the same custodial $SU(2)_V$ symmetry compels the neutrino to be a Majorana particle.
2 The chiral group $U(N)_L \times U(N)_R$ and the electroweak subgroup $SU(2)_L \times U(1)$.

Let $N/2$ be the number of generations; the number $N$ of “flavours” is even (the construction below cannot be performed for $N$ odd). The observed $J = 0$ mesons are generally classified according to their parity quantum number $P = \pm 1$ and it is convenient to introduce the parity changing operator $P$ which transforms a scalar into a pseudoscalar and vice-versa; a $P$-even meson will be later written (see subsection 2.3) as the sum “scalar + pseudoscalar”, and a $P$-odd meson as the difference “scalar - pseudoscalar”. The laws of transformation of mesons by the chiral group $U(N)_L \times U(N)_R$ are indeed most simply expressed in terms of $P$-even and $P$-odd particles.

Both types are taken to be $N \times N$ matrices $M$, which will be given an index “odd” or “even”. The quarks are here only considered as mathematical entities \cite{10}, and the mesons fields as objects transforming as composite quark-anti quark operators. The link with physically observed particles is more thoroughly examined in section 6. The link with composite quark-anti quark operators is made by sandwiching $M$ between the $N$-vector of quarks $\Psi$ in the fundamental representation of $U(N)$

$$\Psi = \begin{pmatrix}  a \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix}.$$  \hspace{1cm} (1)

and its hermitian conjugate, and introducing, according to the transformation by parity, the appropriate $\gamma_5$ matrix.

2.1 The action of $U(N)_L \times U(N)_R$.

A generator $A$ of $U(N)_L \times U(N)_R$ is a set of two $N \times N$ matrices $(A_L, A_R)$. A generator of a diagonal subgroup satisfies $A_L = A_R$.

At the level of the algebra, we define the action of the left and right generators by:

$$A^i_L . M_{even} \overset{def}{=} - A^i_L M_{even} = \frac{1}{2} \left( [M_{even}, A^i_L] - \{ M_{even}, A^i_L \} \right),$$

$$A^i_L . M_{odd} \overset{def}{=} + M_{odd} A^i_L = \frac{1}{2} \left( [M_{odd}, A^i_L] + \{ M_{odd}, A^i_L \} \right),$$

$$A^i_R . M_{even} \overset{def}{=} + M_{even} A^i_R = \frac{1}{2} \left( [M_{even}, A^i_R] + \{ M_{even}, A^i_R \} \right),$$

$$A^i_R . M_{odd} \overset{def}{=} - A^i_R M_{odd} = \frac{1}{2} \left( [M_{odd}, A^i_R] - \{ M_{odd}, A^i_R \} \right),$$

which is akin to left- and right- multiplying $N \times N$ matrices.

At the level of the group, let $U_L \times U_R$ be a finite transformation of the chiral group; we have

$$U_L \times U_R . M_{even} = U_R^{-1} M_{even} U_R,$$

$$U_L \times U_R . M_{odd} = U_R^{-1} M_{odd} U_L,$$  \hspace{1cm} (3)

2
reminiscent of the group action in a $\sigma$-model \cite{[1]} \cite{[3]} with a $U(N)_L \times U(N)_R$ group of symmetry. Note that “left” and “right” are swapped in the action on the $P$-odd scalars with respect to the $P$-even ones.

The actions defined above can be derived straightforwardly by acting with the left and right $U(N)$ groups on the fermionic “components” of $\bar{\Psi}(1 \pm \gamma_5)\Gamma\Psi$: the left-handed generators are then given a $(1 - \gamma_5)/2$ projector, and the right ones a $(1 + \gamma_5)/2$. That the $\gamma_5$ of the projectors has to go through the $\gamma_0$ of $\bar{\Psi}$ yields both commutators and anticommutators in eq. (2).

2.2 The electroweak $SU(2)_L \times U(1)$.

The extension of the Glashow-Salam-Weinberg model \cite{[1]} to $J = 0$ mesons proposed in \cite{[2]} is a $SU(2)_L \times U(1)$ gauge theory of matrices. As the action of the gauge group can only be defined if its generators are also $N \times N$ matrices, it is considered as a subgroup of the chiral group. Its orientation within the latter has to be compatible with the customary action of the electroweak group on fermions, and is determined by a unitary $N/2 \times N/2$ matrix which is nothing else than the Cabibbo-Kobayashi-Maskawa mixing matrix $K$ \cite{[5]}.

We hereafter decompose all $N \times N$ matrices into $N/2 \times N/2$ blocks.

The $SU(2)_L$ generators are

\[ T^3_L = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad T^+_L = \begin{pmatrix} 0 & K \\ 0 & 0 \end{pmatrix}, \quad T^-_L = \begin{pmatrix} 0 & 0 \\ K^\dagger & 0 \end{pmatrix}; \quad (4) \]

they act trivially on the $N$-vector of quarks $\Psi$ (they are then given a left $(1 - \gamma_5)/2$ projector) in the same way as in the Glashow-Salam-Weinberg model, ensuring the consistency of our approach with the latter.

$T^+$ and $T^-$ stand respectively for $(T^1 + i T^2)$ and $(T^1 - i T^2)$. $I$ is the $N/2 \times N/2$ identity matrix.

The hypercharge $U(1)$ generator satisfies the Gell-Mann-Nishijima relation \cite{[12]} (written in its “chiral” form)

\[ (Y_L, Y_R) = (Q_L, Q_R) - (T^3_L, 0); \quad (5) \]

it is non-diagonal and commutes with $SU(2)_L$. Taking the customary expression for the electric charge operator

\[ Q = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix}, \quad (6) \]

yields back the usual formula for the “left” and “right” hypercharges

\[ Y_L = \frac{1}{6} I, \quad Y_R = Q_R. \quad (7) \]

The “alignment” of the electroweak subgroup inside the chiral group is controlled by a unitary matrix, $(R, R)$, acting diagonally, with

\[ R = \begin{pmatrix} I & 0 \\ 0 & K \end{pmatrix}, \quad (8) \]
The electroweak group defined by eq. (4) is the one with generators

$$R^\dagger \vec{t}_L R;$$

with

$$t^3_L = \frac{1}{2} \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}, \quad t^+_L = \begin{pmatrix} 0 & \mathbb{1} \\ 0 & 0 \end{pmatrix}, \quad t^-_L = \begin{pmatrix} 0 & 0 \\ \mathbb{1} & 0 \end{pmatrix}. \quad (10)$$

In practice, this rotation only acts on the $t^\pm$ generators (we require $t^- = (t^+)^\dagger$, such that the unit matrices in eqs. (10,4) have the same dimension).

2.3 The electroweak representations of $J = 0$ mesons.

In the same way (see eq. (2)) as we wrote the action of the chiral group on scalar fields represented by $N \times N$ matrices $\mathbb{M}$, we define the action of its $SU(2)_L$ subgroup, to which we add the action of the electric charge $Q$ according to:

$$Q \cdot \mathbb{M} = [\mathbb{M}, Q]; \quad (11)$$

it acts by commutation because it is a diagonal operator (see subsection 2.1).

The representations of the electroweak group $SU(2)_L \times U(1)$ are also of two types, $\mathcal{P}$-even and $\mathcal{P}$-odd, according to their transformation properties by the parity changing operator $\mathcal{P}$. Only representations transforming alike can be linearly mixed to form another representation of the same type.

We can build a very special type of representations, in the form of quadruplets $(\mathbb{M}^0, \mathbb{M}, \mathbb{M}^+, \mathbb{M}^-)$, where the $\mathbb{M}$’s are still $N \times N$ matrices; $\mathbb{M}$ stands for the sets of complex matrices $\{\mathbb{M}^1, \mathbb{M}^2, \mathbb{M}^3\}$ or $\{\mathbb{M}^3, \mathbb{M}^+, \mathbb{M}^-\}$ with $\mathbb{M}^+ = (\mathbb{M}^1 + i \mathbb{M}^2)/\sqrt{2}$, $\mathbb{M}^- = (\mathbb{M}^1 - i \mathbb{M}^2)/\sqrt{2}$.

Let us consider quadruplets of the form

$$\Phi(\mathbb{D}) = (\mathbb{M}^0, \mathbb{M}^3, \mathbb{M}^+, \mathbb{M}^-)(\mathbb{D})$$

with

$$\mathbb{D} = \begin{pmatrix} \mathbb{1} & 0 & 0 & 0 \\ 0 & \mathbb{1} & 0 & 0 \\ K^\dagger \mathbb{D} K & 0 & \mathbb{1} & 0 \\ 0 & -K^\dagger \mathbb{D} K & 0 & \mathbb{1} \end{pmatrix}.$$

It turns out that it can be rewritten in the form (the Latin indices $i,j,k$ run from 1 to 3):

$$T^i_L \cdot \mathbb{M}^0_{even} = -\frac{i}{2} \left( \epsilon_{ijk} \mathbb{M}^k_{even} + \delta_{ij} \mathbb{M}^0_{even} \right);$$

$$T^i_L \cdot \mathbb{M}^i_{even} = \frac{i}{2} \mathbb{M}^i_{even}; \quad (13)$$
\[ T^i_L \cdot M_{\text{odd}}^j = -\frac{i}{2} \left( \epsilon_{ijk} M_{\text{odd}}^k - \delta_{ij} M_{\text{odd}}^0 \right), \]
\[ T^i_L \cdot M_{\text{odd}}^0 = -\frac{i}{2} M_{\text{odd}}^i. \]  
(14)

The charge operator acts indifferently on \( P \)-even and \( P \)-odd matrices by:
\[ Q \cdot M^i = -i \epsilon_{ij3} M^j, \]
\[ Q \cdot M^0 = 0, \]  
(15)

and the action of the \( U(1) \) generator \( Y \) follows from eq. (8).

Still as a consequence of (2), the action of the “right” group \( SU(2)_R \) is of the same form as displayed in eqs. (13,14) but with the signs in front of the \( M^0 \)'s all swapped.

Taking the hermitian conjugate of any representation \( \Phi \) swaps the relative sign between \( M^0 \) and \( \vec{M} \); as a consequence, \( \Phi^\dagger_{\text{even}} \) transforms by \( SU(2)_L \) as would formally do a \( P \)-odd representation, and \textit{vice-versa}; on the other hand, the quadruplets (12) are also representations of \( SU(2)_R \), the action of which is obtained by swapping eqs. (13) and (14); so, the hermitian conjugate of a given representation of \( SU(2)_L \) is a representation of \( SU(2)_R \) with the same law of transformation, and \textit{vice-versa}. The same result holds for any (complex) linear representation \( U \) of quadruplets transforming alike by the gauge group.

We see that we now deal with 4-dimensional representations of \( SU(2)_L \times U(1) \), which are also, by the above remark, representations of \( SU(2)_R \). In the basis formed by the four entries of any such representation, the generators of the electroweak group can be rewritten as 4 \times 4 matrices. This is also the case for the generators of the diagonal \( SU(2)_V \).

They decompose into “symmetric” representations, corresponding to \( \mathbb{D} = \mathbb{D}^\dagger \), and “antisymmetric” ones for which \( \mathbb{D} = -\mathbb{D}^\dagger \).

There are \( N/2(N/2 + 1)/2 \) independent real symmetric \( \mathbb{D} \) matrices; hence, the sets of “even” and “odd” symmetric quadruplet representations of the type (12) both have dimension \( N/2(N/2 + 1)/2 \). Similarly, the antisymmetric ones form two sets of dimension \( N/2(N/2 - 1)/2 \).

Every representation above is a reducible representation of \( SU(2)_L \) (or \( SU(2)_R \)) and is the sum of two (complex) representations of spin 1/2. This makes it isomorphic to the standard scalar set of the Glashow-Salam-Weinberg model [1].

Now, if we consider the transformation properties by the diagonal \( SU(2)_V \), all \( \vec{M} \)'s are (spin 1) triplets, lying in the adjoint representation, while all \( M^0 \)'s are singlets.

By adding or subtracting eqs. (13) and (14), and defining scalar (\( \mathbb{S} \)) and pseudoscalar (\( \mathbb{P} \)) fields by
\[ (M_{\text{even}} + M_{\text{odd}}) = \mathbb{S}, \]  
(16)
and
\[ (M_{\text{even}} - M_{\text{odd}}) = \mathbb{P}, \]  
(17)
one finds two new types of stable quadruplets which include objects of different parities, but which now correspond to a given \( CP \) quantum number, depending in particular whether \( \mathbb{D} \) is a symmetric or skew-symmetric matrix
\[ (M^0, \vec{M}) = (S^0, \vec{P}), \]  
(18)
and
\[ (M^0, \vec{M}) = (P^0, \vec{S}); \]  
(19)
they both transform by the gauge group like $\mathcal{P}$-even reps, according to eq. (13), and thus can be linearly mixed. As they span the whole space of $J = 0$ mesons too, this last property makes them specially convenient to consider.

By hermitian conjugation, that is charge conjugation, a “symmetric” $(\mathcal{M}^{0}, \mathcal{M})$ representation gives $(\mathcal{M}^{0}, -\mathcal{M})$; an “antisymmetric” representation gives $(-\mathcal{M}^{0}, \mathcal{M})$; the representations (18) and (19) are consequently representations of given $CP$ (charge conjugation $\times$ parity): “symmetric” $(\mathcal{S}^{0}, \mathcal{P})$’s and “antisymmetric” $(\mathcal{P}^{0}, \mathcal{S})$’s are $CP$-even, while “symmetric” $(\mathcal{P}^{0}, \mathcal{P})$’s and “antisymmetric” $(\mathcal{S}^{0}, \mathcal{S})$’s are $CP$-odd.

2.4 “Strong” and electroweak basis for the mesons.

We call “strong” basis the set of flavour (and parity) $U(N)$ eigenstates. They are represented by $N^2$ matrices $\mathcal{F}^{ij}$ for scalars, and $\mathcal{F}^{ij}_{S}, i, j = 1 \cdots N$ for pseudoscalars, in which only one entry, the one at the crossing of the $i$th line and the $j$th column, is non vanishing and has the value 1. This is equivalent, in the quark language, to the set of $\bar{q}_i q_j$ and $\bar{q}_i \gamma_5 q_j$ states. The most general meson $\mathcal{M}$ thus decomposes on the strong basis according to

$$\mathcal{M} = \sum_{i,j=1\ldots N} M_{ij} \mathcal{F}^{ij} + M_{ij}^{S} \mathcal{F}^{ij}_{S}.$$  

(20)

A quadratic expression we call diagonal in the basis of strong eigenstates if it only involves tensor products of the type $\mathcal{F}^{ij} \otimes \mathcal{F}^{ji}$ and $\mathcal{F}^{ij}_{S} \otimes \mathcal{F}^{ji}_{S}$ (we use hereafter the notation $\otimes$ for the tensor product of two fields, not to be mistaken with the ordinary product of matrices).

3 The $SU(2)_L \times U(1)$ Lagrangian.

Having defined the fundamental fields and how they transform by the groups of symmetries involved in the problem, we shall now explicitly write the $SU(2)_L \times U(1)$ gauge Lagrangian for $J = 0$ mesons. It requires knowing which polynomial expressions are invariant by the gauge group (in practice we need only quadratic invariants; the quartic invariants are constructed as products of any two quadratic ones and higher powers are forbidden by the requirement of renormalizability).

It is from the nature of these invariants that the chiral structure of our construction and the role of the group $SU(2)_L \times SU(2)_R$, which will be examined in detail in the next section, spring out.

3.1 The quadratic invariants.

To every representation is associated a unique quadratic expression invariant by the electroweak gauge group $SU(2)_L \times U(1)$

$$\mathcal{I} = (\mathcal{M}^{0}, \mathcal{M}) \otimes (\mathcal{M}^{0}, \mathcal{M}) = \mathcal{M}^{0} \otimes \mathcal{M}^{0} + \mathcal{M} \otimes \mathcal{M};$$

(21)

$\mathcal{M} \otimes \mathcal{M}$ stands for $\sum_{i=1,2,3} \mathcal{M}^i \otimes \mathcal{M}^i$.

Other invariants can be built like tensor products of two representations transforming alike by the gauge group: two $\mathcal{P}$-odd or two $\mathcal{P}$-even, two $(\mathcal{S}^{0}, \mathcal{P})$, two $(\mathcal{P}^{0}, \mathcal{S})$, or one $(\mathcal{S}^{0}, \mathcal{P})$ and one $(\mathcal{P}^{0}, \mathcal{S})$; for example such is

$$\mathcal{I}_{12} = (\mathcal{S}^{0}, \mathcal{P})(\mathcal{D}_1) \otimes (\mathcal{P}^{0}, \mathcal{S})(\mathcal{D}_2) = \mathcal{S}^{0}(\mathcal{D}_1) \otimes \mathcal{P}^{0}(\mathcal{D}_2) + \mathcal{P}(\mathcal{D}_1) \otimes \mathcal{S}(\mathcal{D}_2).$$

(22)

According to the remark made in the previous section, all the above expressions are also invariant by the action of $SU(2)_R$. 

6
3.2 A special combination of invariants.

For the relevant cases $N = 2, 4, 6$, there exists a set of $\mathbb{D}$ matrices such that the algebraic sum (specified below) of invariants extended over all representations defined by $\mathbb{D}$ is diagonal both in the electroweak basis and in the basis of strong eigenstates: in the latter basis, all terms are normalized alike to $(+1)$ (including the sign). Note that two “−” signs occur in eq. (23):

- the first between the $(\mathbb{P}^0, \mathbb{P})$ and $(\mathbb{S}^0, \mathbb{S})$ quadruplets, because, as seen on eq. (12), the $\mathbb{P}^0$ entry of the former has no “$i$” factor, while the $\mathbb{P}$'s of the latter do have one; as we define all pseudoscalars without an “$i$” (like $\pi^+ = \bar{u}d$), a $(\pm i)$ relative factor has to be introduced between the two types of representations, yielding a “−” sign in eq. (23);
- the second for the representations corresponding to skew-symmetric $\mathbb{D}$ matrices, which have an opposite behaviour by charge conjugation (i.e. hermitian conjugation) as compared to the ones with symmetric $\mathbb{D}$’s.

The $SU(2)_L \times U(1)$ kinetic Lagrangian for $J = 0$ mesons is built from the special combination of invariants (23), now used for the covariant derivatives of the fields with respect to the gauge group; its part involving pure derivatives is thus diagonal in both the strong and electroweak basis, too.

The characteristic property of the combination (23) is most simply verified for the “non-rotated” $SU(2)_L \times U(1)$ group and representations $\mathbb{D}$. Explicitly [4]:

3.2.1 $N = 2$.

It is a trivial case: $\mathbb{D}$ is a number.

3.2.2 $N = 4$.

The four $2 \times 2 \mathbb{D}$ matrices (3 symmetric and 1 skew-symmetric) can be taken as

$$\mathbb{D}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbb{D}_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \mathbb{D}_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbb{D}_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (24)$$

3.2.3 $N = 6$.

The nine $3 \times 3 \mathbb{D}$ matrices (6 symmetric and 3 skew-symmetric), can be taken as

Eq. (23) specifies eq. (25) of [2], in which the “−” signs were not explicitly written.
\[ D_1 = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \]
\[ D_2 = \frac{2}{\sqrt{3}} \begin{pmatrix} \sin \alpha & 0 & 0 \\ 0 & \sin(\alpha \pm \frac{2\pi}{3}) & 0 \\ 0 & 0 & \sin(\alpha \mp \frac{2\pi}{3}) \end{pmatrix}, \]
\[ D_3 = \frac{2}{\sqrt{3}} \begin{pmatrix} \cos \alpha & 0 & 0 \\ 0 & \cos(\alpha \pm \frac{2\pi}{3}) & 0 \\ 0 & 0 & \cos(\alpha \mp \frac{2\pi}{3}) \end{pmatrix}, \]
\[ D_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad D_5 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \]
\[ D_6 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad D_7 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad D_8 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad D_9 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \]

where \( \alpha \) is an arbitrary phase.

Remark: as \( D_1 \) is the only matrix with a non-vanishing trace, \( S^0(D_1) \) is the only neutral scalar matrix with the same property; we take it as the Higgs boson.

Considering that it is the only scalar with a non-vanishing vacuum expectation value prevents the occurrence of a hierarchy problem \([13]\).

This last property is tantamount, in the “quark language”, to taking the same value for all condensates \( \langle \bar{q}_i q_i \rangle, i = 1 \cdots N \), in agreement with the flavour independence of “strong interactions” between fermions, supposedly at the origin of this phenomenon in the traditional framework.

As the spectrum of mesons is, in the present model, disconnected from a hierarchy between quark condensates (see below), it is not affected by our choice of a single Higgs boson.

### 3.3 The basic property of the quadratic invariants.

The quadratic \( SU(2)_L \) invariants are not \textit{a priori} self conjugate expressions and have consequently no definite property by hermitian conjugation; in particular, the one associated with a most general representation \( U \) is \( U \otimes U \) and not \( U \otimes U^\dagger \) (we have seen in the previous section that \( U \) and \( U^\dagger \) do not transform alike by the gauge group).

As far as one only deals with representations of the type of eqs. \([18,19]\), like in the special combination \([23]\), it has no consequence since each of their entries has a well defined behaviour by hermitian conjugation and the associated quadratic invariants are then always hermitian.

But electroweak mass eigenstates are in general (complex) linear combinations of reps \([18,19]\) and have, consequently, no definite behaviour by hermitian (charge) conjugation. This has consequences, in particular as far as the transformation properties by \( CP \) are concerned (see section \([5]\)).

### 3.4 First remarks on the spectrum of \( J = 0 \) mesons.

The quadratic invariants are used to build the gauge invariant mass terms in the Lagrangian.
As long as \( SU(2)_L \times U(1) \) is unbroken, there are \textit{a priori} as many \( (N^2/2) \) independent mass scales as there are independent representations. They share with the leptonic case the same arbitrariness.

Since there are eleven pseudoscalar mesons which “include” the quark top, and since they cannot all be fitted in a unique representation, it should not be a surprise if different mass scales are found to correspond to “topped” mesons; one should be aware not to misinterpret them, like by advocating for the occurrence of a new generation.

The number of mass scales could be reduced if the theory has additional symmetries.

Note that, from the diagonalization property of eq. (23), identical mass terms for the \((S^0, \overline{P})(D), (P^0, S)(D)\) multiplets correspond to the same property for the flavour eigenstates.

Whatever convenient be the distinction between scalars and pseudoscalars, one must keep in mind that, in a parity violating theory such as ours, the most general electroweak mass eigenstates do not have a definite parity.

4   The chiral symmetry and its breaking to the custodial SU(2)_V.

This section is devoted to the study of the symmetries of the \( SU(2)_L \times U(1) \) Lagrangian for \( J = 0 \) mesons. The origin of the “custodial” \( SU(2)_V \) symmetry as the result of the breaking of a chiral \( SU(2)_L \times SU(2)_R \) symmetry is made explicit, together with the similarity of the chiral and electroweak breaking.

Further consequences on the spectrum of mesons and the nature of the Goldstones bosons are emphasized.

Finally, the tight link between the custodial symmetry and the quantization of the electric charge is examined in the light of electric-magnetic duality and the recent works by Cho, Maison and Kimm [14][15].

4.1   The chiral \( SU(2)_L \times SU(2)_R \) symmetry.

All \( SU(2)_L \times U(1) \) quadratic invariants that are used to build the Lagrangian are also invariant by \( SU(2)_R \). The scalar potential is thus \( SU(2)_L \times SU(2)_R \) chirally invariant.

Because the coupling constant \( g' \) of the hypercharge \( U(1) \) is different from the \( SU(2)_L \) coupling \( g \), only the covariant derivatives of the fields with respect to \( SU(2)_L \) have definite transformation properties with respect to \( SU(2)_R \), which are the same as the fields themselves, \textit{when the right-handed \( W \)’s are identified with the left-handed ones}. This can be done since the laws of transformations for the adjoint representations of both groups are identical.

The weak hypercharge group breaks this symmetry, as expected by the Gell-Mann-Nishijima relation which shows that it is “polarized”.

We thus conclude that \textit{the \( SU(2)_L \times U(1) \) Lagrangian for \( J = 0 \) mesons has a chiral \( SU(2)_L \times SU(2)_R \) symmetry at the limit \( g' \rightarrow 0 \).}

4.2   The chiral and electroweak breaking.

While the electroweak symmetry is only spontaneously broken by the Higgs boson \( H = S^0(D_1) \) (see the remark at the end of the paragraph 3.2.3) getting a non vanishing vacuum expectation value \( \langle H \rangle = v/\sqrt{2} \), the chiral \( SU(2)_L \times SU(2)_R \) symmetry is both explicitly broken by \( g' \neq 0 \) and spontaneously by \( \langle H \rangle \neq 0 \).

The electroweak symmetry is broken down to the electromagnetic \( U(1)_{em} \), and the chiral symmetry down to its diagonal subgroup, the “custodial” \( SU(2)_V \). We have indeed seen that all
quadruplets decompose into a triplet plus a singlet of \( SU(2)_V \), and that the Higgs is precisely a singlet.

The electromagnetic \( U(1)_{em} \) is a subgroup of \( SU(2)_V \), as will be studied in detail in the next subsection, and the electroweak spontaneous breaking is identical to the chiral breaking.

This has consequences on the nature of the Goldstone bosons, since there are only three of them, which become the longitudinal components of the massive gauge fields. They are the pseudoscalar triplet \( \mathbb{P}(D_1) \). This means in particular that:

- they are not aligned with any “strong” eigenstate (pion, kaon \ldots), but they are linear combinations of them;
- that the strong (or flavour) eigenstates are experimentally massive (and can be very massive) is no longer a contradiction with the spontaneous breaking of chiral symmetry; the pion triplet would in particular only be a triplet of Goldstone bosons if there was only one generation (meaning of course that the mixing angles do not exist);
- the two “scales” of spontaneous breaking are identical to the mass of the \( W \)’s; since in the real case of three generations, the Goldstones “include” the quark top, this explains why these two phenomenological mass scales are not very different; the scale of the top quark appears as “normal” (with the restriction mentioned in subsection 3.4).

The traditional picture of chiral symmetry breaking is altered since the relevant breaking is now that of \( SU(2)_L \times SU(2)_R \) down to \( SU(2)_V \) and not that of \( U(N)_L \times U(N)_R \) into the diagonal \( U(N) \); the \( U(N)_L \times U(N)_R \) chiral symmetry is explicitly broken by the mass terms that are introduced in a \( SU(2)_L \times U(1) \) and \( SU(2)_L \times SU(2)_R \) invariant way and the \( N^2 \) pseudoscalar \( J = 0 \) mesons do not play anymore the role of Goldstone bosons.

After the breaking of this last symmetry, there exists \( a \ priori \) two different mass scales for each multiplet, and the total number of (arbitrary) mass scales has doubled from \( N^2/2 \) to \( N^2 \). In the hypothesis when the eigenstates can be split into scalars and pseudoscalars, this means a scalar-pseudoscalar splitting within each \( (S^0, \mathbb{P}) \) or \( (P^0, S) \) quadruplet. One expects, as observed, a different spectrum for scalars and pseudoscalars.

### 4.3 The custodial \( SU(2)_V \) symmetry.

I demonstrate explicitly that the present theory has a “custodial” \( SU(2)_V \) symmetry; it is a global symmetry, which becomes local when \( g' \rightarrow 0 \). A local vectorial symmetry having no anomaly can be preserved at the quantum level.

This symmetry is \textit{not} the strong isospin symmetry, because, in particular, of the mixing angles; this means that large violation of the strong isospin symmetry can be expected due to electroweak interactions: the masses of mesons occurring in internal lines of the relevant diagrams can be very different, and are likely to provide very different decay rates for apparently similar decays if only the isospin symmetry is considered (like \( K \rightarrow 2\pi \) decays and the \( \Delta I = 1/2 \) rule). This is currently under investigation.

One has always to keep in mind that all perturbative calculations are now to be done with internal lines which are the \( J = 0 \) mesons and not the quarks; this means a different “filter” with which experimental data are to be analyzed. The suggestion is consequently that the custodial symmetry might then be found unbroken, as suggested by the extreme precision with which the electric charge is quantized (see the next subsection). The very small deviation found from the value 1 for the \( \rho \) parameter could very well be due to the fact that the data have been analyzed and computations done up to now with a theory where fields (quarks) are not particles, and could disappear with a new analysis. Of course, a really good one would require to have a field theory
for (at least) all mesons of arbitrary spin, which is far from being achieved here.

Another consequence concerns the “screening” theorem \[16\]: as the Higgs mass can be made arbitrary without breaking the custodial symmetry, the decoupling becomes exact at the limit where this symmetry is unbroken.

The 4-dimensional representations \([12]\) of \(SU(2)_L \times U(1)\) have already been mentioned to be representations of \(SU(2)_R\). They are thus naturally representations of the diagonal \(SU(2)_V\), that we study in more detail.

When acting in the 4-dimensional vector space of which \([12]\) form a basis, its generators \(T^3, T^\pm\) can be represented as \(4 \times 4\) matrices \(\tilde{T}^3, \tilde{T}^\pm\); explicitly:

\[
\tilde{T}^+ = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{2} & 0 \\
0 & 0 & 0 & 0 \\
0 & -\sqrt{2} & 0 & 0
\end{pmatrix}, \quad \tilde{T}^- = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\sqrt{2} \\
0 & \sqrt{2} & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad \tilde{T}^3 = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix}.
\]

That the first line in any of the three above matrices identically vanishes is the translation of the already mentioned fact that the first entry \(M_0^0\) of the representations \([12]\) are singlets by the diagonal \(SU(2)_V\), while the three other entries \(\vec{M}\) form a triplet in the adjoint representation.

The global \(SU(2)_V\) symmetry occurs when the gauge fields \(W^\pm_\mu\) and \(\tilde{Z}_\mu = Z_\mu / \cos \theta_W\), with \(\theta_W\) the Weinberg angle, transform like a vector in the adjoint representation of \(SU(2)_V\). This is not a surprise since those precisely absorb the \(\vec{P}(D_1)\) triplet of eq. \((18)\), also in the adjoint, to become massive, when the gauge symmetry is broken down from \(SU(2)_L \times U(1)_Y\) to \(U(1)_{em}\). The normalization of the last one ensures that the resulting mass term for the gauge fields \(M_2^2 (2W^+_\mu W^-_\mu + Z_\mu Z^\mu / \cos^2 \theta_W)\) satisfies \(\rho = 1\), where \(\rho = M_W / (M_Z \cos \theta_W)\). We recover the usual link between the custodial \(SU(2)_V\) and the value of \(\rho [17]\).

More precisely, we consider the Lagrangian built with covariant with respect to \(SU(2)_L \times U(1)\) derivatives from the special combination of invariant \((23)\)

\[
\mathcal{L} = \frac{1}{4} \left( \sum_{symmetric \mathcal{D}} - \sum_{skew-symmetric \mathcal{D}} \right) \left( D_\mu \Phi_{even}(\mathcal{D}) \otimes D^\mu \Phi_{even}^+(\mathcal{D}) + D_\mu \Phi_{odd}(\mathcal{D}) \otimes D^\mu \Phi_{odd}^+(\mathcal{D}) \right)
\]

\[(27)\]

The potential, being trivially invariant by \(SU(2)_V\) from what has been said in the construction of the invariants, has been omitted.

Let us explicitly write the covariant (with respect to \(SU(2)_L \times U(1)\)) derivatives of a quadruplet, and show that they transform like a singlet plus a triplet by the custodial \(SU(2)\). We do it explicitly for a \(\mathcal{P}\)-even quadruplet.

\[2\text{The } J = 1 \text{ mesons can be straightforwardly included in this framework: the quadruplets now split into one triplet and one singlet of the left, right, and diagonal } SU(2) \text{ groups.}\]
\[ D_\mu M^0_{\text{even}} = \partial_\mu M^0_{\text{even}} + \frac{e}{2s_W} (W^1_\mu M^1_{\text{even}} + W^2_\mu M^2_{\text{even}} + (Z_\mu/c_W)M^3_{\text{even}}), \]
\[ = D_\mu M^0_{\text{even}} + \frac{e}{2s_W} (W^1_\mu M^1_{\text{even}} + W^2_\mu M^2_{\text{even}} + (Z_\mu/c_W)M^3_{\text{even}}), \]
\[ D_\mu M^3_{\text{even}} = \partial_\mu M^3_{\text{even}} + \frac{e}{2s_W} (i(W^+_{\mu} M^-_{\text{even}} - W^-_{\mu} M^+_{\text{even}}) - (Z_\mu/c_W)M^0_{\text{even}}), \]
\[ = D_\mu M^3_{\text{even}} - \frac{e}{2s_W} (Z_\mu/c_W)M^0_{\text{even}}, \]
\[ D_\mu M^+_{\text{even}} = \partial_\mu M^+_{\text{even}} - \frac{e}{2s_W} (W^+(\mu) M^0_{\text{even}} + iM^3_{\text{even}}) - i(Z_\mu/c_W)M^+_{\text{even}} + i\frac{e}{c_W} B_{\mu} M^+, \]
\[ = D_\mu M^+_{\text{even}} - \frac{e}{2s_W} W^+_{\mu} M^0_{\text{even}} + i\frac{e}{c_W} B_{\mu} M^+, \]
\[ D_\mu M^-_{\text{even}} = \partial_\mu M^-_{\text{even}} - \frac{e}{2s_W} (W^-_{\mu} M^0_{\text{even}} - iM^3_{\text{even}}) + i(Z_\mu/c_W)M^-_{\text{even}} - i\frac{e}{c_W} B_{\mu} M^-, \]
\[ = D_\mu M^-_{\text{even}} - \frac{e}{2s_W} W^-_{\mu} M^0_{\text{even}} - i\frac{e}{c_W} B_{\mu} M^-. \]  

In eq. (28) above, we noted \( c_W \) and \( s_W \) respectively the cosine and sine of the Weinberg angle. \( A_\mu \) is the photon, \( W^\pm_\mu = (W^1_\mu \pm iW^2_\mu)/\sqrt{2} \), and we have as usual
\[
\begin{align*}
g &= \frac{e}{s_W}, \quad g' = \frac{e}{c_W}, \\
Z_\mu &= c_W W^3_\mu - s_W B_\mu, \quad A_\mu = c_W B_\mu + s_W W^3_\mu. \tag{29}
\end{align*}
\]
\( D_\mu \) is the covariant derivative with respect to the diagonal \( SU(2)_V \) group
\[
D_\mu \tilde{M} = \partial_\mu \tilde{M} - i \frac{e}{s_W} \left( \frac{1}{\sqrt{2}} (W^+_{\mu} \tilde{T}^- + W^-_{\mu} \tilde{T}^+) + \frac{Z_\mu}{c_W} \tilde{T}^3 \right) \tilde{M}. \tag{30}
\]

The normal derivative of \( \tilde{M} \) transforming like \( \tilde{M} \) itself, that \( D_\mu \tilde{M}^0 \) is a singlet of \( SU(2)_V \) is trivial as soon as, as stressed before, \( \tilde{M} \) is a triplet in the adjoint and \( (W^+_\mu, Z_\mu/c_W) \) too, since the scalar product of those two vectors is an invariant;
that the three other covariant derivatives transform like a vector results from the three following facts:
- from the 2 vectors \( \tilde{M} \) and \( (W^+_\mu, Z_\mu/c_W) \) we can form a third one with the \( \epsilon_{ijk} \) tensor
\[
\begin{pmatrix}
M^- W^+_\mu - M^+ W^-_\mu; \\
M^3 W^+_\mu - M^+ (Z_\mu/c_W), \\
M^3 W^-_\mu - M^- (Z_\mu/c_W);
\end{pmatrix}
\tag{31}
\]
- \( \tilde{M}^0 \) being a singlet by \( SU(2)_V \), the terms \( \tilde{M}^0 W^+_\mu \) transform like \( W^+_\mu \) and thus like \( \tilde{M}^+ \), \( (Z_\mu/c_W)\tilde{M}^0 \) like \( (Z_\mu/c_W) \) and thus like \( \tilde{M}^0 \);
- \( B_\mu \) is to be considered as a singlet of \( SU(2)_V \), such that the terms \( (B_\mu/c_W) \tilde{M}^\pm \) transform like \( \tilde{M}^\pm \).

The same argumentation works for \( P \)-odd scalars. Their covariant derivatives are immediately obtained from eqs. (28) above by changing the signs of all \( \tilde{M}^0 \)'s.
This shows the existence of a global \( SU(2)_V \) custodial symmetry for the Lagrangian, independently of the value of the hypercharge coupling \( g' \).
Let us now examine whether this symmetry can be considered as a local symmetry.
Making a space-time dependent \( SU(2)_V \) transformation with parameters \( \tilde{\theta} \) on the scalar fields and transforming the vector fields \( W^\pm_\mu, Z_\mu/c_W \) like the corresponding gauge potentials \( (B_\mu \) being a singlet does not transform), one finds from (28) that the Lagrangian (27) varies, for each
quadruplet, by
\[ \Delta \mathcal{L} = D_\mu \bar{\theta} (\bar{\mathbb{M}} \otimes D^\mu \mathbb{M}^0 - \mathbb{M}^0 \otimes D^\mu \bar{\mathbb{M}}), \]
(32)
such that the existence of a local custodial \( SU(2)_V \) symmetry is linked to the conservation of the triplet of currents \( \vec{V}^\mu \)
\[ D_\mu \vec{V}^\mu = 0, \]
(33)
with
\[ \vec{V}^\mu = \bar{\mathbb{M}} \otimes D^\mu \mathbb{M}^0 - \mathbb{M}^0 \otimes D^\mu \bar{\mathbb{M}}. \]
(34)
\( \vec{V}_\mu \) is an \( SU(2)_V \) triplet. Its “singlet” partner \( V^0_\mu \) identically vanishes by the definition (32).

These currents are automatically covariantly (with respect to \( SU(2)_L \times U(1)_L \)) conserved by the classical equations of motion for the \( \mathbb{M} \) fields, as can be seen from (34), which entails
\[ D^\mu V^i_\mu = \mathbb{M}^i \otimes D^2 \mathbb{M}^0 - \mathbb{M}^0 \otimes D^2 \mathbb{M}^i, \]
(35)
and from the Lagrangian (27) to which we can add any term quadratic in the invariants \( \mathcal{I} \) for any quadruplet.

Now,
\[ D^\mu V^i_\mu = D^\mu V^i_\mu - ig' B_\mu \bar{\mathbb{Q}} V^i_\mu, \]
(36)
where we have used the Gell-Mann-Nishijima relation and the fact that, since \( V^0_\mu \) identically vanishes, the “left” \( SU(2)_L \) acts on \( \vec{V}_\mu \) like the diagonal \( SU(2)_V \).

We can thus conclude that the custodial symmetry, which is a global symmetry, becomes local when the hypercharge coupling \( g' \) goes to zero.

Anomaly-free, it is thus an exact local symmetry of the standard \( SU(2)_L \times U(1)_L \) Lagrangian (27) for \( J = 0 \) fields, with gauge fields \( W^{\pm}_\mu, Z_\mu/c_W \).

### 4.4 Quantization of the electric charge; electric magnetic duality.

The two known ways to explain the quantization of the electric charge are [3]:
- that the corresponding generator is the “\( z \)” component of an angular momentum \( SU(2) \);
- that there exists at least one magnetic “monopole”-like object (Dirac quantization).

The idea of electric-magnetic duality is that these two mechanisms are just two aspects of the same phenomenon and always occur simultaneously.

It is thus suggestive that, at the same time I showed that in the standard model for \( J = 0 \) mesons the electric charge generator is precisely the “\( z \)” component of the custodial \( SU(2)_V \), Cho and Maison [14] showed that the scalar sector of the standard model has, because of the presence of the hypercharge \( U(1) \), the right topological structure \( (CP^1) \) to incorporate dyon-like solutions, which they exhibited numerically. The problem of the infinite zero-point energy of their solutions was later shown [15] to be regularized when the group of symmetry is slightly enlarged and/or new interactions introduced. The fact that the model presented here also incorporates a chiral \( SU(2)_L \times SU(2)_R \) symmetry might also help regularizing their classical solutions.

The model proposed here seems consequently to present the right properties to achieve electric-magnetic duality.
The interest of such a property lies also in the fact that one then expects a strongly interacting sector, in which the fields are monopole-like extended objects. Those skyrmion-like particles [18], built “on top of” mesons, are then natural candidates for baryons.

Starting from an electroweak model of physical particles, we reach the idea that at least a certain aspect of the strong interactions could be included as another sector of the theory (strong interactions of mesons can originate from the high mass limit of the Higgs boson [19]), towards a true unification of non-gravitational interactions.

5 CP violation.

All phenomena of CP violation [20][21] are, up to now, compatible with the so-called “indirect” violation [22], explained by the electroweak mass eigenstates not being CP eigenstates. The stakes are high for the observation of “direct” CP violation, and, in particular, for discovering whether the so-called $\epsilon'$ parameter is vanishing or not.

I show below, that, in the present framework, unitarity requires that the electroweak mass eigenstates are always C eigenstates and that “indirect” CP violation only occurs as a consequence of P violation.

Furthermore, I show, and this is an immediate and very simple consequence of the nature of the $J = 0$ electroweak representations constructed above, that, even when there is a complex phase in the mixing matrix $K$, electroweak mass eigenstates can still be CP eigenstates. So, the existence of a complex mixing matrix at the fermionic level is no longer a sufficient condition for mesonic electroweak mass eigenstates to be different from CP eigenstates.

“Indirect” CP violation consequently fades away, and the true search for CP violation should really be concentrated on that of “direct” CP violation. Phrased in a more provocative way, it seems that we do not know yet if CP is truly violated.

5.1 Electroweak versus CP eigenstates.

The electroweak Lagrangian for $J = 0$ mesons is the one of eq. (27) plus the potential built from quadratic invariants according to section 3.

Unitarity compels this Lagrangian to be hermitian, in particular its quadratic part.

Its diagonalization yields the electroweak mass eigenstates. Let us restrict for the sake of simplicity to a subsystem of two non-degenerate electroweak mass eigenstates $U$ and $V$; they are in general complex linear combinations of quadruplets (18) and (19), and transform by $SU(2)_L$ according to (13). $L$ writes, for example

\[
L = \frac{1}{2} (\partial_\mu U \otimes \partial^\mu U - \partial_\mu V \otimes \partial^\mu V - m_U^2 U \otimes U + m_V^2 V \otimes V + \cdots). \tag{37}
\]

with $m_U^2 \neq m_V^2$, where we have only written above the quadratic part.

Hermiticity yields the two following equations, coming respectively from the kinetic and mass terms

\[
\begin{align*}
(U \otimes U - V \otimes V)^\dagger &= U \otimes U - V \otimes V, \\
(m_U^2 U \otimes U - m_V^2 V \otimes V)^\dagger &= m_U^2 U \otimes U - m_V^2 V \otimes V,
\end{align*}
\]

which, if we reject complex values of the (mass)², entail

\[
U = \pm U^\dagger, \quad V = \pm V^\dagger; \tag{39}
\]
unitarity thus requires that the electroweak mass eigenstates be also \( C \) eigenstates.

Consequence: if electroweak mass eigenstates are observed not to be \( CP \) eigenstates, they can only be mixtures of states with different parities.

We had already mentioned that the most general eigenstates in this \( P \) violating theory do not have, as expected, a definite parity. This transforms the problem of indirect \( CP \) violation into finding an explanation for the smallness of the observed mixture between scalars and pseudoscalars.

5.2 The fading role of the Kobayashi-Maskawa mixing matrix.

Suppose that we have a complex mixing matrix \( K \); the following Lagrangian for \( J = 0 \) mesons, where the sum is extended to all representations defined by eqs. (18,19,12), is nevertheless hermitian, \((D_{\mu} \) is the covariant derivative with respect to \( SU(2)_L \times U(1)\))

\[
\mathcal{L} = \frac{1}{2} \sum_{\text{symmetric} \, \overline{D}} \left( D_{\mu}(S^0, \overline{P})(\overline{D}) \otimes D^{\mu}(S^0, \overline{P})(\overline{D}) - m_D^2(S^0, \overline{P})(\overline{D}) \otimes (S^0, \overline{P})(\overline{D}) \right) - \frac{1}{2} \sum_{\text{skew-symmetric} \, \overline{D}} \left( D_{\mu}(P^0, \overline{S})(\overline{D}) \otimes D^{\mu}(P^0, \overline{S})(\overline{D}) - \tilde{m}_D^2(P^0, \overline{S})(\overline{D}) \otimes (P^0, \overline{S})(\overline{D}) \right) \right)
\]

and its mass eigenstates, being the \((S^0, \overline{P})\) and \((P^0, \overline{S})\) representations given by (18,19), are \( CP \) eigenstates \([2]\). It is of course straightforward to also build hermitian \( SU(2)_L \times U(1) \) invariant quartic terms.

Consequence: The existence of a complex phase in the mixing matrix for quarks is not a sufficient condition for the existence of electroweak mass eigenstates for \( J = 0 \) mesons different from \( CP \) eigenstates.

We have indeed seen that, at the mesonic level, all dependence on the mixing matrix \( K \) can be reabsorbed in the definition of the asymptotic states.

6 The link with observed mesons.

It is shown below that the fields that we have been dealing with are in one-to-one correspondence with the observed scalar and pseudoscalar \( J = 0 \) mesons.

For this purpose we shall study in particular their leptonic decays. After general considerations about which kind are expected to decay into leptons, are which are not, we show that one recovers the standard PCAC result by a simple rescaling of the fields and couplings: the scaling parameter is

\[
a = \frac{f}{\langle H \rangle},
\]

where \( f \) is the leptonic decay constant, supposed here to be the same for all mesons.

We make some general remarks about semi-leptonic decays.

All computations are made at tree-level, with the propagators of the massive gauge bosons taken in the unitary gauge.
6.1 General selection rules.

The leptonic and semi-leptonic decays of $J = 0$ mesons occur via the crossed terms in the kinetic terms of the Lagrangian (27) which are proportional to

$$\partial_\mu M \otimes g\bar{W}^\mu T M,$$

(42)

where I have used a shortened and symbolic notation in which the $M$'s are the ingoing and outgoing (if any) meson, $g$ one of the two $SU(2)_L \times U(1)$ coupling constants, $\bar{W}^\mu$ the set of three massive gauge fields (one of them can be the $Z$). The gauge field then couples to the two outgoing leptons. If $T.M$ yields the Higgs boson, then the term proportional to $\langle H \rangle$ triggers a leptonic decay, like described in fig. 1;

![Fig. 1: The leptonic decay of a pseudoscalar meson.](image)

if it yields another meson, then the process is a semi-leptonic decay, as described in fig. 2.

![Fig. 2: The semi-leptonic decay of a meson.](image)

The above mechanisms have immediate and simple consequences, and “selection rules” result:
- only mesons which have a non-vanishing projection on the Higgs boson when acted upon by one of the generators of the electroweak group can decay into leptons;
- hence, if we suppose that the Higgs is unique and is a pure (neutral) scalar (which is not a priori true in a parity violating theory), then scalar mesons never decay in a pure leptonic way: indeed, when acted upon by a generator of the group they can only give a scalar or a pseudoscalar; if it is a scalar, it can only be, as can be seen from eq. (13), a charged one, thus different from the Higgs boson and which does not condensate in the vacuum; if it is a pseudoscalar, it does not condense either;
- all scalars and pseudoscalars can decay semi-leptonically; however, when acted upon by a generator of the electroweek group, any $SU(2)_V$ (neutral) singlet in the representations [18,19] is
transmuted into a particle with opposite parity; a scalar $SU(2)_V$ singlet will consequently only semi-leptonically decay into a pseudoscalar and vice versa; also, the neutral of the $SU(2)_V$ triplet can only give the singlet, with opposite parity, when acted upon by $T^3_L$; as a consequence, the neutral semi-leptonic decays of a neutral $SU(2)_V$ triplet always gives a neutral outgoing particle with opposite parity. We can thus state the rule: the neutral particle produced by the semi-leptonic decay of an incoming neutral $J = 0$ meson has always a parity opposite to that of the incoming particle.

An immediate consequence is that scalars are difficult to detect since they do not have leptonic decays; so are consequently semi-leptonic decays of neutral pseudoscalar mesons which yield scalar neutral mesons.

Decays of neutral mesons are furthermore severely constrained by the absence of flavour changing neutral currents: it can indeed be checked (this is easily understood since the present model has been built in a way compatible with the Glashow-Salam-Weinberg model) that this selection rule is still valid here: no decay is allowed that would require flavour changing neutral currents at the fermionic level. In practice, the semi-leptonic decay of a neutral meson can only yield another neutral meson when the decaying particle is “diagonal in flavour”.

Four outgoing leptons can originate from a neutral scalar decaying semi-leptonically: two leptons come from the gauge field and the two others from the leptonic decay of the produced pseudoscalar.

### 6.2 Explicit representations for $N = 4$.

For the sake of simplicity, we shall work in this section in the case of two generations $N = 4$.

The four types of $SU(2)_L \times U(1)$ quadruplets that now arise, corresponding respectively to the matrices $D_i, i = 1 \cdots 4$ of subsection 3.2 are:

\[
\begin{align*}
\Phi(D_1) &= \begin{pmatrix}
\frac{1}{\sqrt{2}} & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
\frac{1}{\sqrt{2}} & 1 & 1 & 1 \\
1 & 1 & 1 & 1
\end{pmatrix}, \\
\Phi(D_2) &= \begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1
\end{pmatrix}, \\
\Phi(D_3) &= \begin{pmatrix}
c_\theta & s_\theta & 0 & 0 \\
s_\theta & c_\theta & 0 & 0 \\
c_\theta & s_\theta & 0 & 0 \\
s_\theta & c_\theta & 0 & 0
\end{pmatrix}, \\
\Phi(D_4) &= \begin{pmatrix}
c_\theta & -s_\theta & 0 & 0 \\
s_\theta & c_\theta & 0 & 0 \\
c_\theta & -s_\theta & 0 & 0 \\
s_\theta & c_\theta & 0 & 0
\end{pmatrix}.
\end{align*}
\]
$$\Phi(D_2) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ c_\theta^2 - s_\theta^2 & 2 c_\theta s_\theta \\ 2 c_\theta s_\theta & s_\theta^2 - c_\theta^2 \end{pmatrix}, \quad i \sqrt{2} \begin{pmatrix} 1 & -1 \\ s_\theta^2 - c_\theta^2 & -2 c_\theta s_\theta \\ -2 c_\theta s_\theta & c_\theta^2 - s_\theta^2 \end{pmatrix},$$

$$i \begin{pmatrix} c_\theta & s_\theta \\ s_\theta & -c_\theta \end{pmatrix} ;$$

$$\Phi(D_3) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ -2 c_\theta s_\theta & c_\theta^2 - s_\theta^2 \\ c_\theta^2 - s_\theta^2 & 2 c_\theta s_\theta \end{pmatrix}, \quad i \sqrt{2} \begin{pmatrix} 1 & -1 \\ 2 c_\theta s_\theta & s_\theta^2 - c_\theta^2 \\ s_\theta^2 - c_\theta^2 & -2 c_\theta s_\theta \end{pmatrix},$$

$$i \begin{pmatrix} -s_\theta & c_\theta \\ c_\theta & s_\theta \end{pmatrix} ;$$

$$\Phi(D_4) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad i \sqrt{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix},$$

$$i \begin{pmatrix} -s_\theta & c_\theta \\ c_\theta & -s_\theta \end{pmatrix} ;$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} s_\theta & c_\theta \\ -c_\theta & s_\theta \end{pmatrix} ;$$

(44)

(45)

(46)

$c_\theta$ and $s_\theta$ stand respectively for the cosine and sine of the Cabibbo angle $\theta_c$.

We shall also use in the following the notations

$$(S^0, \overline{P})(D_1) = \Phi_1, \quad (S^0, \overline{P})(D_2) = \Phi_2, \quad (S^0, \overline{P})(D_3) = \Phi_3, \quad (S^0, \overline{P})(D_4) = \Phi_4.$$ (47)

According to the remark of subsubsection 3.2.3, we consider the Higgs boson to be the unique
scalar singlet with a non-vanishing trace

\[ H = S^0(\mathbb{D}_1). \]  

(48)

From the general selection rules written above, only the pseudoscalar mesons which have a non-vanishing projection on the three Goldstones \( \vec{P}(\mathbb{D}_1) \) will undergo leptonic decays.

### 6.3 From matrix-fields to observed mesons: the case of leptonic decays.

We call \( \Psi_\ell \) the leptonic equivalent of \( \Psi \) in eq. (1).

Let us rescale the fields according to:

- for mesons and leptons:
  
  \[ \Phi = a\Phi', \quad \Psi_\ell = a\Psi'_\ell; \]
  
  (49)

  in particular one has \( H = aH' \);

- for the gauge fields, generically noted \( \sigma_\mu \):
  
  \[ \sigma_\mu = a\sigma'_\mu; \]
  
  (50)

  and all coupling constants, called generically \( \kappa \), according to

  \[ \kappa = \tilde{\kappa}/a. \]

  (51)

The fields and coupling constants to be considered as physical are the rescaled ones.

The relations between the matrix-valued \( \mathbb{M} \) mesonic fields and the physically observed “strong” eigenstates (kaon, pion . . .) is given by relations like eq. (52) below for \( \mathbb{P}^+(\mathbb{D}_1) \), which can be read off directly from eqs. (43) to (46). The translation is most easily done (see section 2) by sandwiching the \( \mathbb{M} \) matrices between \( \Psi \) and \( \bar{\Psi} \) to find its components on the strong eigenstates.

The Lagrangian we furthermore rescale by \( 1/a^2 \) in order that the kinetic terms are normalized to “1” when expressed in terms of the “primed” fields.

The propagators of the gauge fields are left unchanged, in particular those of the massive \( W \)'s and \( Z \) since \( g^2 \langle H \rangle^2 = \tilde{g}^2 \langle H' \rangle^2 \).

Suppose that the incoming meson in fig. 1 is, for example a “strong” \( K^+ \), that is, in the quark notation, a \( \bar{u}\gamma_5 s \) state created by strong interactions.

As it is the relevant part for leptonic decays, we only rewrite, according to eq. (28), the kinetic terms for \( \Phi_1 \) in terms of the rescaled fields, like for example

\[ \mathbb{P}^+(\mathbb{D}_1) = a\mathbb{P}^+(\mathbb{D}_1) = a \left( c_\theta (\pi^+ + D^+_u) + s_\theta (K^+ + D^+) \right); \]

(52)

One has accordingly

\[ L' = \frac{1}{a^2} L = \frac{1}{a^2} \left( \frac{1}{2} \partial_\mu \mathbb{P}^+ - \frac{g}{2} W^+_\mu H \right) \otimes \left( \partial_\mu \mathbb{P}^- - \frac{g}{2} W^-_\mu H \right) + \cdots \]

\[ = \frac{1}{2} \left( \partial_\mu \mathbb{P}^+ - \frac{\tilde{g}}{2} W^+_\mu H' \right) \otimes \left( \partial_\mu \mathbb{P}^- - \frac{\tilde{g}}{2} W^-_\mu H' \right) + \cdots; \]

(53)

on the leptonic side the coupling of the gauge fields to the electron and the neutrino writes \( (\gamma_L^\mu = \gamma^\mu(1 - \gamma_5)/2) \)

\[ L'_\ell = \frac{1}{a^2} L_\ell = \frac{1}{a^2} \left( g W^+_\mu e^{-\gamma_L^\mu \bar{\nu}_\ell} \right) = \tilde{g} W^+_\mu e^{-\gamma_L^\mu \nu'_\ell}. \]

(54)
The diagram of fig. 1, expressed in terms of the rescaled fields and coupling constant yields
\[
K^+ \frac{1}{2}(s_\theta \bar{H} H)(\frac{4i}{g^2 \langle H'^2 \rangle})(i\bar{g}) e^\gamma \nu^\mu \bar{\nu}^\ell
= \frac{1}{2} s_\theta 4k_\mu \langle H' \rangle K^+ e^{-\gamma^\mu \nu} \bar{\nu}^\ell
= \frac{1}{2} s_\theta \frac{4k_\mu}{\langle H' \rangle^2} K^+ e^{-\gamma^\mu \nu} \bar{\nu}^\ell = \frac{1}{2} s_\theta \frac{k_\mu g^2}{M_W^2} K^+ e^{-\gamma^\mu \nu} \bar{\nu}^\ell
\]
(55)
which is exactly, for the rescaled (physical) fields, the result traditionally obtained by PCAC. We have used eq. (41), the relation
\[
M_W^2 = \frac{g^2 \langle H \rangle^2}{4} = \frac{\bar{g}^2 \langle H' \rangle^2}{4}
\]
and the fact that, in the unitary gauge, the \( W \) propagator \( D^\mu\nu_W \) satisfies
\[
i k_\nu D^\mu\nu_W(k) = -\frac{k_\mu}{M_W^2}, \tag{56}
\]
where \( k_\mu \) is the momentum of the incoming meson.

7 An extension to the leptonic sector.

We have now at our disposal a renormalizable gauge theory for \( J = 0 \) mesons which is anomaly-free, and in which the quantization of the electric charge for asymptotic states has been correlated with a custodial \( SU(2)_V \) symmetry to stay unbroken at the quantum level.

There is now a need to also modify the leptonic sector [7] since:
- charge quantization should also hold for the corresponding asymptotic states; if we suppose that the same mechanism is at work, then the theory that we are looking for should have the same custodial symmetry as the one unraveled above;
- anomalies [23] can now only spring out of fermions, such that this sector should be anomaly-free by itself; we cannot rely anymore on a cancelation between quarks and leptons [8].

It is also well known [24] that there exist problems with Weyl fermions making desirable a vector-like theory of weak interactions.

This is why we propose to start from the purely vectorial theory studied in [9].

We shall not question universality and only deal here with one generation of fermions.

7.1 The custodial symmetry for a vectorial theory.

In the mesonic case, we have seen that each quadruplet (complex doublet) of \( SU(2)_L \) was also the sum of one \( SU(2)_V \) real triplet with electric charges \((-1, 0, +1)\) plus one real chargeless singlet; this made easy and straightforward, in the space spanned by these representations, the connection between the custodial group of symmetry and its electromagnetic subgroup.

Now, in the Glashow-Salam-Weinberg model [1] for leptons, we do not have any more one complex \( SU(2)_L \) doublet, but a set of doublets for left-handed fields and singlets for right-handed ones. Implementing a custodial \( SU(2) \) symmetry is consequently less intuitive here, and make us consider the standard model for leptons as only an effective theory.
7.1.1 Groups and representations.

Because the notion of left and right-handed groups has a very precise meaning when leptons are concerned, it is useful here to change the notation and call $G_1$ and $G_2$ the two $SU(2)$’s which build the equivalent of the chiral $SU(2) \times SU(2)$ group of subsection 4.1. As we shall see later, there however exists a similarity between $G_1$ and the $SU(2)_L$ group of the Glashow-Salam-Weinberg model, in that they act in the same way on the left-handed (neutrino, electron) doublet.

Consider the quadruplet $Q_L$ of left-handed fields

$$Q_L = (L^0, L^3, L^+, L^-) = \left( -i\frac{\nu - \nu^c}{\sqrt{2}}, \frac{\nu + \nu^c}{\sqrt{2}}, \ell^+, \ell^- \right)_L,$$

(57)

$\ell^+$ and $\ell^-$, $\nu^c$ and $\nu$ are charge conjugate:

$$\ell^+ = C\ell^- T, \quad \nu^c = C\nu T;$$

(58)

the superscript “$T$” means “transposed” and $C$ is the charge-conjugation operator: $C = i\gamma_2\gamma_0$ in the Dirac representation. The convention that $\ell^+ = (\ell^1 + i\ell^2)/\sqrt{2}$ is the charge conjugate of $\ell^- = (\ell^1 - i\ell^2)/\sqrt{2}$ entails that $i$ gives $-i$ by charge conjugation and that the charge conjugate $(Q_L)^c$ of $Q_L$ is its right-handed counterpart $Q_R = (L^0, L^3)\sqrt{2}.$

By analogy with eq. (13), we define the actions of $G_1$ with generators $\vec{T}_1$ and $G_2$ with generators $\vec{T}_2$ on $Q_L$ by

$$T_{1i}^j L^j = \frac{i}{2}(\epsilon_{ijk}L^k + \delta_{ij}L^0),$$

$$T_{1i}^0 L^0 = -\frac{i}{2}L^i,$$

(59)

and

$$T_{2i}^j L^j = \frac{i}{2}(\epsilon_{ijk}L^k - \delta_{ij}L^0),$$

$$T_{2i}^0 L^0 = \frac{i}{2}L^i.$$

(60)

$G_1$ acts on $Q_R$ like $G_2$ does on $Q_L$, according to eq. (60); this clearly shows the difference between $G_1$ and a “left” $SU(2)_L$ since by construction it also acts on right-handed fermions. $G_2$ acts on $Q_R$ like $G_1$ does on $Q_L$, according to eq. (59), and the same remark as above applies to it.

If one changes $Q_L$ into $Q_R$, eqs. (59) and (60) are swapped; the same occurs with $Q_R$.

The last properties just reflect the chiral structure of the symmetry under consideration. $Q_L$ is a reducible representation of each of these two groups and can be decomposed into two spin 1/2 doublets:

- two doublets of $G_1$:

$$l_1 = \begin{pmatrix} \frac{1}{\sqrt{2}}(L^3 + iL^0) \\ L^- \end{pmatrix} = \begin{pmatrix} \nu \\ \ell^- \end{pmatrix}_L, \quad \ell'_1 = \begin{pmatrix} \frac{1}{\sqrt{2}}(L^3 - iL^0) \\ L^- \end{pmatrix} = \begin{pmatrix} \ell^+ \\ \nu^c \end{pmatrix}_L,$$

(61)

with the group action

$$T_{1l}^3\ell^- = \frac{1}{2}\ell^-_L, \quad T_{1l}^3\ell^+ = \frac{1}{2}\ell^+_L, \quad T_{1l}^3\nu_L = \frac{1}{2}\nu_L, \quad T_{1l}^3(\nu^c)_L = -\frac{1}{2}(\nu^c)_L,$$

$$T_{1l}^1\ell^- = \nu_L, \quad T_{1l}^1\ell^+ = 0, \quad T_{1l}^1\nu_L = 0, \quad T_{1l}^1(\nu^c)_L = -\ell^+_L,$$

$$T_{1l}^1\ell^- = 0, \quad T_{1l}^1\ell^+ = -(\nu^c)_L, \quad T_{1l}^1\nu_L = \ell^-_L, \quad T_{1l}^1(\nu^c)_L = 0.$$  

(62)
\( G_1 \) acts on \( l_1 \) like the \( SU(2)_L \) group of the Standard Model; - two doublets of \( G_2 \):

\[
l_2 = \begin{pmatrix} \nu^c \\ \ell^- \end{pmatrix}_L, \quad l_2' = \begin{pmatrix} \ell^+ \\ \nu \end{pmatrix}_L
\]

with the group action

\[
T_3^L, \ell^-_L = -\frac{1}{2} \ell^-_L, \quad T_2^L, \ell^-_L = \frac{1}{2} \ell^-_L, \quad T_2^L, \nu_L = -\frac{1}{2} \nu_L, \quad T_3^L, (\nu^c)_L = \frac{1}{2} (\nu^c)_L,
\]

\[ T_2^L, (\nu^c)_L = 0, \quad T_2^L, \nu_L = -\ell^+_L, \quad T_3^L, (\nu^c)_L = 0, \quad T_2^L, \ell^-_L = 0, \quad T_2^L, \nu_L = 0, \quad T_3^L, (\nu^c)_L = \ell^-_L; \tag{64}\]

With respect to the diagonal \( SU(2)_V \) subgroup \( \tilde{G} \) of the chiral group \( G_1 \times G_2 \) with generators \( \tilde{T}^i = T_1^i + T_2^i \), it decomposes into one spin 1 triplet, \( \tilde{L}_i \), plus one singlet \( \tilde{L}_0 \), with the group action:

\[
\tilde{T}_3^L, \ell^-_L = -\ell^-_L, \quad \tilde{T}_2^L, \ell^-_L = \ell^-_L, \quad \tilde{T}_2^L, \nu_L = 0, \quad \tilde{T}_3^L, (\nu^c)_L = 0,
\]

\[ \tilde{T}_2^L, \nu_L = (\nu^c)_L, \quad \tilde{T}_2^L, \nu_L = 0, \quad \tilde{T}_2^L, (\nu^c)_L = -\ell^+_L, \quad \tilde{T}_3^L, (\nu^c)_L = -\ell^+_L,
\]

\[ \tilde{T}_2^L, (\nu^c)_L = 0, \quad \tilde{T}_2^L, \nu_L = -\ell^-_L, \quad \tilde{T}_2^L, \nu_L = \ell^-_L, \quad \tilde{T}_2^L, (\nu^c)_L = \ell^-_L. \tag{65}\]

\( \tilde{G} \) is the custodial symmetry which occurs in the mesonic sector. The generator of the \( U(1) \) group of electromagnetism is the “\( z \)” generator of this angular momentum.

When operating in the 4-dimensional vector space spanned by the four entries of \( Q_L \), its three generators write as \( 4 \times 4 \) matrices according to (in the basis \( \{L^0, L^3, L^+, L^-\} \)):

\[
\tilde{T}^+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \end{pmatrix}, \quad \tilde{T}^- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2} \\ 0 & -\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \tilde{T}^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{66}\]

The electric charge generator is identical with the third generator of \( SU(2)_V \):

\[
Q = \tilde{T}^3. \tag{67}\]

The decompositions above apply to \( Q_R \) too; eqs. (52) and (54) have to be swapped, but (55) stays unchanged.

We have thus achieved our first goal to define a “chiral” \( SU(2) \) structure which acts on special representations of leptons in such a way that its diagonal subgroup includes the electromagnetic \( U(1) \).

### 7.1.2 Invariants.

They are constructed along the remark made in the previous paragraph that changing \( Q_L \) into \( \overline{Q}_L \) swaps the role of eqs. (59) and (50), and that changing \( Q_L \) into \( Q_R \) has the same effect.

The unique quadratic expression invariant by \( \tilde{G}_1 \) and \( \tilde{G}_2 \) is then

\[
\mathcal{I} = \overline{Q}Q = \overline{Q}_R Q_L + \overline{Q}_L Q_R . \tag{68}\]

It is of course also invariant by the diagonal \( SU(2)_V \).
7.1.3 A vector-like electroweak Lagrangian for leptons.

Let us start, according to [9], from the purely vectorial Lagrangian

\[ \mathcal{L} = \bar{\ell} i \gamma^\mu \partial_\mu \ell - i e \gamma^\mu \partial_\mu \nu \]
+ \( \frac{e}{\sqrt{2} s_W} \left( \bar{\ell} \gamma^\mu W^-_\mu \nu + \nu \gamma^\mu W^+_\mu \ell \right) \]
+ \( \frac{e}{2 s_W} \left( \bar{\ell} \gamma^\mu W^3_\mu \ell - \nu \gamma^\mu W^3_\mu \nu \right) \]
+ \( \frac{e}{2 c_W} \left( \bar{\ell} \gamma^\mu B_\mu \ell - \nu \gamma^\mu B_\mu \nu \right) \), \hspace{1cm} (69)

to which we add the mass term

\[ \mathcal{L}_m = - \frac{m}{2} (\overline{Q_R} Q_L + \overline{Q_L} Q_R). \]

(70)

The quadratic expression (68) being invariant by both \( G_1 \) and \( G_2 \), \( \mathcal{L}_m \) is invariant by the chiral group \( G_1 \times G_2 \), and this invariance is independent of the mass \( m \), which can vary with the leptonic generation.

\( \mathcal{L}_m \) corresponds to a Dirac mass term. It is an important actor in the “see-saw” mechanism evoked in the last section. A Majorana mass term for the neutrino would correspond to the combination (forgetting the charged leptons) \( (\mathbb{R}^3 \mathbb{L}^3 + \mathbb{L}^3 \mathbb{R}^3) - (\mathbb{R}^0 \mathbb{L}^0 + \mathbb{L}^0 \mathbb{R}^0) + \cdots \), (the “-” sign makes the difference), which is not invariant by \( G_1 \).

Using the properties of charge conjugation, it turns out [7] that \( \mathcal{L} \) is the sum \( \mathcal{L} = \mathcal{L}_1 + \mathcal{L}_1' \) (we use the abbreviated notation \( \gamma_{\mu L} = \gamma_{\mu} (1 - \gamma_5)/2, \gamma_{\mu R} = \gamma_{\mu} (1 + \gamma_5)/2 \))

\[ \mathcal{L}_1 = \bar{\ell} i \gamma^\mu \partial_\mu \ell - i e \gamma^\mu \partial_\mu \nu \]
+ \( \frac{e}{\sqrt{2} s_W} \left( \bar{\ell} \gamma^\mu W^-_\mu \nu + \nu \gamma^\mu W^+_\mu \ell \right) \]
+ \( \frac{e}{2 s_W} \left( \bar{\ell} \gamma^\mu W^3_\mu \ell - \nu \gamma^\mu W^3_\mu \nu \right) \]
+ \( \frac{e}{2 c_W} \left( \bar{\ell} \gamma^\mu B_\mu \ell - \nu \gamma^\mu B_\mu \nu \right) \), \hspace{1cm} (71)

\[ \mathcal{L}_1' = \bar{\ell} i \gamma^\mu \partial_\mu \ell - i e \gamma^\mu \partial_\mu \nu \]
+ \( \frac{e}{\sqrt{2} s_W} \left( \bar{\ell} \gamma^\mu W^-_\mu \nu + \nu \gamma^\mu W^+_\mu \ell \right) \]
+ \( \frac{e}{2 s_W} \left( \bar{\ell} \gamma^\mu W^3_\mu \ell - \nu \gamma^\mu W^3_\mu \nu \right) \]
+ \( \frac{e}{2 c_W} \left( \bar{\ell} \gamma^\mu B_\mu \ell + \nu \gamma^\mu B_\mu \nu \right) \). \hspace{1cm} (72)

corresponding respectively to the two doublets \( \ell_1 \) and \( \ell_1' \), and that it can also be rewritten

\[ \mathcal{L} = i \overline{Q_L} \gamma_{\mu} D^\mu_{G_1 \times U(1)} Q_L, \]

(73)

where \( D^\mu_{G_1 \times U(1)} \) is the covariant derivative with respect to \( G_1 \times U(1) \); the \( U(1) \) is defined by a generator \( Y \) satisfying the Gell-Mann-Nishijima relation

\[ Y = Q - T^3_1, \]

(74)

and corresponds to a gauging of the leptonic number; indeed, the leptonic numbers of the entries of \( Q \) are \((-2) \times \) their \( U(1) \) quantum numbers.
While the mass term is trivially $G_1 \times G_2$ chirally invariant, this is not the case for the kinetic term (73) for $Q_L$.

Indeed, as already mentioned, $Q_L$ transforms by $G_1$ according to eq. (59) and by $G_2$ according to eq. (60), but $Q_L$ transforms with eqs. (59) and (60) swapped. So, the kinetic term (73) is neither $G_1$ nor $G_2$-invariant.

This difference with respect to the mesonic case explains why now the custodial $SU(2)_V$ symmetry in particular is not something which is automatically achieved when $g' \to 0$, but requires some constraint to be satisfied.

We can rewrite $L_1 + L'_1$ in the form

$$2(L_1 + L'_1) = \frac{2}{\sqrt{2}} \left( \frac{1}{2} W^- \tilde{T}^+ + W^+ \tilde{T}^- + Z_{cW} \tilde{T}^3 \right) (L^0)$$

$$+ g \left( -\sqrt{2} \gamma_\mu \left( W^- \tilde{T}^+ + W^+ \tilde{T}^- \right) + Z_{cW} \tilde{T}^3 \right) (L_1)$$

$$+ i g \left( -\sqrt{2} \gamma_\mu \left( W^- \tilde{T}^+ + W^+ \tilde{T}^- \right) + Z_{cW} \tilde{T}^3 \right) (L_2)$$

$$+ g' \left( -\sqrt{2} \gamma_\mu \left( W^- \tilde{T}^+ + W^+ \tilde{T}^- \right) + Z_{cW} \tilde{T}^3 \right) (L_3)$$

where we have used the fact that $\tilde{T}^+$ does not act on $L^+$, nor $\tilde{T}^-$ on $L^-$, nor $\tilde{T}^3$ on $L^3$.

The pure kinetic terms and the second line of (75) are globally $SU(2)_V$ invariant when the triplet of gauge bosons $W_\mu^\pm$ and $Z_\mu/c_W$ transform (see subsection 4.3) like a vector in the adjoint representation of this group.

The next line of (75) is also globally $SU(2)_V$ invariant, since $L^0$ and $Z_{cW}$ are singlets and are each multiplied by another singlet made by the scalar product of two triplets.

Now, $B_\mu$ being considered (see subsection 4.3) as a singlet of $SU(2)_V$, the last line of (75) only becomes $SU(2)_V$ invariant if, as can be seen by performing an explicit transformation and using (65,66),

$$\nu + \nu^c = 0,$$

i.e. the neutrino has to be a Majorana particle, with only one helicity (or chirality), which can be written (24), in the 4-component notation, either $\gamma_5 \chi$ or $\gamma_5 \omega$ with

$$\chi = \begin{pmatrix} \psi_L \\ -\sigma^2 \psi^*_L \end{pmatrix}, \quad \omega = \begin{pmatrix} \psi_R \\ -\sigma^2 \psi^*_R \end{pmatrix}.$$
ψ_L (resp. ψ_R) is a two-component Weyl spinor transforming like a (1/2, 0) (resp. (0, 1/2)) representation of the Lorentz group; σ^2 is the second Pauli matrix and the superscript “*” means “complex conjugation”; σ^2ψ^*_L (resp. σ^2ψ^*_R) transforms like a (0, 1/2) (resp. (1/2, 0)) representation.

We thus conclude that:

*The leptonic Lagrangian \( \mathcal{L} \) can have a global custodial SU(2)_V symmetry only if the neutrino is a Majorana particle.*

Clearly, this condition is not compatible with the decomposition (61) and the corresponding laws of transformation (59). In particular, it requires that the \( U(1) \) leptonic number be not conserved. We shall see in the next subsection how the necessary modifications can occur dynamically with the introduction of a “hidden” sector, along the lines of [9].

7.2 **From a vectorial theory to an effective \( V − A \) theory with a decoupled right-handed neutrino.**

The goal of going from a fundamental vectorial electroweak theory of leptons to an effective \( V − A \) interaction as we observe it can be achieved [9] by introducing a scalar triplet of composite scalars, “made of” leptons, and the neutral component of which gets a non-vanishing vacuum expectation value. These composite scalars not being independent degrees of freedom, one must introduce, in the quantization process, constraints in the Feynman path integral, which can be exponentiated into an effective Lagrangian. It can be treated at leading order in an expansion in powers of \( 1/N \) [9] and introduces a drastic asymmetry between the two (Majorana) neutrinos: it gives one of them an infinite mass, and this one is consequently unobservable, while the other, by an exact see-saw mechanism, gets a vanishing mass.

The effective Lagrangian of constraint includes four-leptons couplings, but their effective value go to zero in the limit of decoupling neutrino, preserving renormalizability at the approximation that we are working at.

The infinitely massive neutrino conspires with the vanishing effective four-fermion coupling to alter, at the one-loop level, the bare leptonic couplings to reconstruct the well known \( V − A \) effective structure of weak currents.

Furthermore, the composite scalar finally decouple, playing the role of a hidden sector.

So, the two mysterious phenomena of the non-observation of a right-handed neutrino and of the \( V − A \) structure of weak currents (parity violation) have been given the same origin to yield, at the approximation of leading order in \( 1/N \), an effective interaction indistinguishable from those arising from the Glashow-Salam-Weinberg model.

We will only here sketch out the main steps of the demonstration.

7.2.1 **Introducing a composite scalar triplet.**

We rewrite the Lagrangian (59) plus the mass term (70) in terms of the Majorana neutrinos \( \chi \) and \( \omega \) conveniently reexpressed as

\[
\chi = \nu_L + (\nu_L)^c,
\omega = \nu_R + (\nu_R)^c;
\]

this yields
\[
\mathcal{L} + \mathcal{L}_m = \bar{\ell} \gamma^\mu \partial_\mu \ell + \frac{i}{2} \bar{\chi} \gamma^\mu \partial_\mu \chi + \frac{i}{2} \bar{\omega} \gamma^\mu \partial_\mu \omega \\
+ \frac{e}{\sqrt{2} s_W} (\bar{\ell} \gamma_\mu W^\mu_\ell - \bar{\chi} \gamma_\mu W^\mu_\ell) \\
+ \frac{e}{\sqrt{2} s_W} (\bar{\ell} \gamma_\mu R W^\mu_\ell - \bar{\omega} \gamma_\mu R W^\mu_\ell) \\
+ \frac{e}{2 s_W} (\bar{\chi} \gamma_\mu W^3_\mu \chi + \bar{\omega} \gamma_\mu W^3_\mu \omega - \bar{\ell} \gamma_\mu W^3_\ell) \\
- \frac{e}{2 c_W} (\bar{\ell} \gamma_\mu B^\mu_\ell - \bar{\chi} \gamma_\mu B^\mu_\ell + \bar{\omega} \gamma_\mu B^\mu_\ell) \\
- \frac{m}{2} (\bar{\chi} \omega + \bar{\omega} \chi + 2 \bar{\ell} \ell). (79)
\]

(Remark: would we have built the model with the group \( G_2 \times U(1) \), we would have obtained, instead of eq. (79) \( \hat{\mathcal{L}} \), deduced from \( \mathcal{L} \) by the exchange of \( \chi \) and \( \omega \), or, equivalently, by that of the “left” and “right” projectors.)

We introduce a scalar composite triplet \( \Delta \) with leptonic number 2:

\[
\Delta = \\
\begin{pmatrix}
\Delta^0 \\
\Delta^- \\
\Delta^{--}
\end{pmatrix} = \frac{\rho}{\nu^3} \left( \begin{pmatrix}
\bar{\omega}_R \omega_L \\
\frac{1}{\sqrt{2}} (\bar{\ell}_L \omega_R + \bar{\omega}_L \ell_R) \\
\ell_R \ell_R^-
\end{pmatrix} \right) = \frac{\rho}{\nu^3} \left( \begin{pmatrix}
\nu \frac{1+\gamma_5}{2} \\
\frac{1}{\sqrt{2}} (\bar{\ell} \frac{1+\gamma_5}{2} \nu + \bar{\ell} \frac{1+\gamma_5}{2} \ell^-) \\
\bar{\ell} \frac{1+\gamma_5}{2} \ell^-
\end{pmatrix} \right). (80)
\]

It is a triplet of \( G_1 \) but not a representation of \( G_2 \), nor of \( \tilde{G} \).

Its hermitian conjugate is:

\[
\bar{\Delta} = \\
\begin{pmatrix}
\bar{\Delta}^0 \\
\bar{\Delta}^+ \\
\bar{\Delta}^{++}
\end{pmatrix} = \frac{\rho}{\nu^3} \left( \begin{pmatrix}
\bar{\omega}_L \omega_R \\
\frac{1}{\sqrt{2}} (\ell_R \omega_L + \ell_R \ell_L^+) \\
\ell_R^+ \ell_L^-
\end{pmatrix} \right) = \frac{\rho}{\nu^3} \left( \begin{pmatrix}
\nu \frac{1-\gamma_5}{2} \\
\frac{1}{\sqrt{2}} (\ell \frac{1-\gamma_5}{2} \nu^c + \ell \frac{1-\gamma_5}{2} \ell^+) \\
\ell \frac{1-\gamma_5}{2} \ell^+
\end{pmatrix} \right). (81)
\]

As soon as the mass of \( \omega \) is non vanishing, electroweak vacuum fluctuations like described in fig. 3

\[\text{Fig. 3: contributions to the vacuum expectation value of } \Delta^0.\]
can trigger
\[ \langle \Delta^0 \rangle = \langle \Delta^0 \rangle = \rho. \] (82)

The choice of an \( \langle \bar{\omega} \omega \rangle \) condensate, breaking the symmetry between \( \chi \) and \( \omega \), spontaneously breaks the “left-right” symmetry, or, equivalently, parity.

It could be thought arbitrary since the same type of vacuum fluctuations can also \textit{a priori} trigger \( \langle \bar{\chi} \chi \rangle \neq 0 \). However, the diagrams under consideration vanish with the mass of the internal fermion. As, by the see-saw mechanism evoked below, an \( \langle \bar{\omega} \omega \rangle \) condensate pushes the \( \chi \) mass to 0 at the same time that is pushes the \( \omega \) mass to \( \infty \), the \( \langle \bar{\chi} \chi \rangle \) condensate is then automatically suppressed, and vice-versa. This qualitative explanation forbids the coexistence of both condensates.

The proposed mechanism can also be interpreted along the following lines: by expanding \( \gamma_\mu \) into \( (\gamma_\mu L + \gamma_\mu R) \), the vectorial Lagrangian (69) can be considered to be that of an \( SU(2)_L \times SU(2)_R \times U(1) \) gauge model for the doublet \((\nu, \ell^-)\); both \( SU(2)'s \) act the same way, with the “left” and “right” gauge fields identified. The “Higgs” multiplet \( \Delta \) being a triplet of \( SU(2)_R \) and of \( SU(2)_L \) with a non-vanishing leptonic number, the condensation of its neutral component spontaneously breaks both \( SU(2)'s \), and \( U(1) \).

### 7.2.2 The Lagrangian of constraint.

To take into account the non-independence of the leptonic and \( \Delta \) degrees of freedom, we introduce constraints which, once exponentiated, yield the effective Lagrangian (\( \Lambda \) is an arbitrary mass scale):

\[
L_c = \lim_{\beta \to 0} -\frac{\Lambda^2}{\beta^3} \left[ \left( \Delta^0 - \frac{\rho}{\beta^3} \bar{\omega}_L \omega_R \right) \left( \Delta^0 - \frac{\rho}{\beta^3} \bar{\omega}_R \omega_L \right) \right. \\
+ \left. \left( \Delta^- - \frac{1}{\sqrt{2}} \frac{\rho}{\beta^3} \bar{\omega}_L \ell_R + \ell_R^+ \omega_R \right) \right. \\
+ \left. \left( \Delta^+ - \frac{1}{\sqrt{2}} \frac{\rho}{\beta^3} \bar{\omega}_R \ell_L + \ell_L^+ \omega_L \right) \right] \tag{83}
\]

where \( \Lambda \) is an arbitrary mass scale.

### 7.2.3 Effective 4-leptons couplings and mass eigenstates.

The equations (59) and (83) yield a “see-saw” mechanism [25] in the neutrino sector. Indeed, when \( \langle \Delta^0 \rangle = \rho \), \( L_c \) gives the \( \omega \) neutrino an infinite bare Majorana mass,

\[
M_0 = -\frac{\Lambda^2 \rho^2}{\beta^3}, \tag{84}
\]

the \( \chi \) neutrino a vanishing (Majorana) mass, and the finite Dirac mass of the mass Lagrangian (70) connects \( \chi \) and \( \omega \). We have to diagonalize the mass matrix to get the mass eigenstates, (see for example [26]); they are the Majorana neutrinos \( \chi \) and \( \omega \) themselves, and correspond to mass eigenvalues 0 and \( \infty \) respectively. The charged lepton keeps its Dirac mass \( m \).

However, 4-fermions couplings may alter the mass spectrum, together with being an obstacle for renormalizability. We propose to build a reshuffled perturbative expansion based not on the ‘bare’ (infinite when \( \beta \to 0 \)) 4-fermions couplings occurring in \( L_c \), but rather on effective couplings obtained by resumming infinite series of ‘ladder’ diagrams as proposed by Nambu and Jona-Lasinio [27]. This corresponds to only keeping the leading order in an expansion in powers of \( 1/N \). We however differ from them by bare couplings and a bare fermion mass both infinite; this makes the effective 4-fermions couplings vanish with \( \beta \) like \( \beta^2 \), and the “see-saw” mechanism above stay unaltered.
7.2.4 The scalars and their decoupling.

$SU(2)_L$ is broken by the condensation of $\Delta^0$, which thus weakly contributes to the masses of the gauge fields. If $v/\sqrt{2}$ if the vacuum expectation value of the hadronic Higgs boson we impose its role to be dominant, which yields the necessary condition

$$\rho \ll v$$

consistent with an electroweak nature for $\rho$ (see fig. 3), while that of $v$ lies a priori outside the realm of these interactions.

One then has to shifting the usual way the neutral scalar field according to

$$\Delta^0 = \rho + \delta^0.$$  \hspace{1cm} (86)

From the expression of $L_c$, we see that the non-vanishing of $\rho$ yields an infinite mass for $\delta^0, \Delta^+, \Delta^{++}$ and their conjugates.

None of the components of $\Delta$ appears as an asymptotic state and we do not require electric charge quantization for them. It is the same kind of implicit assumption that we made in the mesonic sector where explaining charge quantization for quarks and their underlying gauge theory (Quantum Chromodynamics) was not sought for. So, we can allow a Lagrangian which is not globally $\tilde{G}$ invariant in the hidden sector; this can be the case for the kinetic term for $\Delta$; this non-invariance is responsible for that of the Lagrangian of the Glashow-Salam-Weinberg model.

By the decoupling of the weak hidden sector, the Goldstones of the broken $SU(2)_L$ symmetry align with the customary hadronic ones. This decoupling also motivates non introducing other triplets of composite states, with $\omega$ replaced by $\chi$, since they would not modify the result: as soon as only one type of condensate can occur, the non-condensing additional scalars would simply fade away without any visible effect.

7.2.5 The effective $V - A$ theory.

The (massless) $\chi$ neutrino has the standard weak $V - A$ couplings and we can identify it with the observed neutrino.

The $\omega$ neutrino is infinitely massive and will never be produced as asymptotic state; we however expect renormalization effects through $\omega$ loops. They drastically affect the neutral weak couplings of the leptons, in a way that rebuilds their “standard” $V - A$ structure. This result, non-trivial if one remembers that the original coupling is purely vectorial, is sketched out below (see [28]).

To the bare (purely vectorial) couplings, we must add the following diagrams (in figures below, the $L, R$’s at the vertices stand for the projectors $(1 - \gamma_5)/2$ and $(1 + \gamma_5)/2$)

![Fig. 4: additional leptonic neutral couplings.](image)

28
In fig. 4, the 4-fermions vertex is the effective \((\ell\ell\omega\omega)\) coupling vanishing like \(\beta^2\). Because of this dependence in \(\beta\), diagrams similar to fig. 4 but with \(\omega\) replaced with \(\chi\) vanish. Those involving \(\omega\) do not because the \(\omega\) loop behaves like \(\beta^{-2}\), and yield a coupling

\[-W_3^{\pm} \gamma^\mu \frac{1 + \gamma_5}{2} \ell,\]

such that \(W_3^{\pm}\) couples finally to

\[-\bar{\ell} \gamma^\mu \ell - \bar{\ell} \gamma^\mu \frac{1 + \gamma_5}{2} \ell = \bar{\ell} \gamma^\mu \frac{1 - \gamma_5}{2} \ell,\]

which is the “standard” \(V - A\) coupling.

The charged couplings do not get modified with respect to their bare values since the diagram equivalent to fig. 4 depicted in fig. 5 involving an infinitely massive \(\omega\) behaves like \(\beta^2 M_0 \ln M_0\) and so vanishes with \(\beta\).

![Fig. 5: the charged couplings do not get altered.](image)

At the approximation that we are working at, the leading order in \(1/N\), ours is thus presently experimentally indistinguishable from the Standard Model.

### 8 Conclusion.

Symmetries have been our main concern in the proposal that we made above for an extension (to mesons) and a modification (for leptons) of the standard electroweak model.

The \(SU(2)_L \times SU(2)_R\) chiral symmetry and its breaking down to the custodial diagonal \(SU(2)_V\) has been seen to play a crucial role in the mesonic sector and in its spectrum; the new picture that arises alters in particular the usual framework in which the pseudoscalar mesons are the Goldstones of the chiral \(U(N)_L \times U(N)_R\) symmetry spontaneously broken down to the diagonal \(U(N)\) of flavour; it furthermore opens the door, via the existence of soliton-like classical solutions, to the existence of a strongly interacting sector which needs to be investigated.

We propose that, in relation with the extreme accuracy with which the electric charge is observed to be quantized, that the custodial symmetry is an exact global symmetry of the physics of mesons.

We have also seen how crucial is the determination of the \(\epsilon'\) parameter in kaon physics to determine whether \(CP\) is violated or not. Our conclusion is indeed that the usual mechanism proposed at the fermionic level to trigger \(CP\) violation may not be operative anymore, which reinforces the mystery of the origin of this phenomenon and leaves the door open for other mechanisms [29]. If \(\epsilon'\) if found compatible with zero, we could conclude that \(CP\) is not violated, and that the observation that some electroweak mass eigenstates are not \(CP\) eigenstates is just a reflection of parity violation.
The quarks, considered here as only mathematical, bear no more connection with leptons. They have always been anyhow totally different types of objects, the former being only fields and not particles.

That the leptonic sector has the same custodial symmetry as the mesonic sector seems mandatory if one wants that the electric charge of leptons is quantized for the same reasons. This is proposed as the unifying link between the two sectors, and it now holds between fields that are also asymptotic states.

To consider the fundamental leptonic electroweak Lagrangian as purely vectorial is not a new idea. It solves many conceptual problems, including the one of anomalies. We could connect above the observed parity violation to another experimental mystery, the absence of a right-handed neutrino and the vanishing (?) mass of the left-handed one [30] and give them a common origin. Of course a justification remains to be found for artificially introducing, as we did, an additional triplet of composite scalars “made of” leptons, and thus to find the true origin of parity violation as we observe it.

The paraphernalia of field theory are also to be used to make phenomenological predictions about electroweak processes involving mesons. As the model has been built to be compatible with the Glashow-Salam-Weinberg for quarks, deviations, if any, should be rather subtle. But the ideas of factorization could for example be tested precisely, and, if one adds a simple model for strong interactions, results concerning $K \to 2\pi$ decays can be expected. The latter are a particular good test field since the problem of the $\Delta I = 1/2$ rule is precisely that of a strong breaking of the isospin symmetry in the final state, which can be easily triggered in our framework. The problems lie more in determining what is the nature of the observed particles, strong or electroweak eigenstates, and which ones occur in the internal lines of the corresponding diagrams. As the latter also involve the Higgs field, it is not excluded that one gets some information about it in this way, though, as the custodial symmetry is more “protective” than ever, any kind of decoupling theorem may well be exact now.

We hope that the reader has found here some new ideas in the pressing hunt for a fundamental theory of the interactions of particles.

Acknowledgments: it is a pleasure to thank the organizing committee of the 2nd International Symposium on Symmetries in Subatomic Physics, and especially Prof. Ernest M. Henley, for their very kind hospitality in Seattle and all their successful efforts to make this meeting most interesting and enjoyable.
Figure captions.

Fig. 1: The leptonic decay of a pseudoscalar meson;
Fig. 2: The semi-leptonic decay of a meson;
Fig. 3: Contributions to the vacuum expectation value of $\Delta^0$;
Fig. 4: Additional leptonic neutral couplings;
Fig. 5: The charged couplings do not get altered.
References

[1] S. L. GLASHOW: Nucl. Phys. 22 (1961) 579;
   A. SALAM: in “Elementary Particle Theory: Relativistic Groups and Analyticity” (Nobel
   symposium No 8), edited by N. Svartholm (Almqvist and Wiksell, Stockholm 1968);
   S. WEINBERG: “A model of leptons”, Phys. Rev. Lett. 19 (1967) 1264.

[2] B. MACHET: “Chiral Scalar Fields, Custodial Symmetry in Electroweak SU(2)_L × U(1),
   and the Quantization of the Electric Charge”, hep-ph/9606239, Phys. Lett. B 385 (1996)
   198-208.

[3] See for example:
   D. I. OLIVE: “Exact electromagnetic duality”, invited talk at the Trieste Conference on
   Recent Developments in Statistical Mechanics and Quantum Field Theory (April 1995),
   preprint SWAT/94-95/81 (1995), and references therein.

[4] B. MACHET: “Two results concerning CP violation for \( J = 0 \) mesons”, preprint PAR-
   LPTHE 97/18, hep-ph/9706308.

[5] N. CABIBBO: “Unitary symmetry and leptonic decays”, Phys. Lett. 10 (1963) 513;
   M. KOBAYASHI and T. MASKAWA: “CP-Violation in the Renormalizable Theory of Weak
   Interactions”, Prog. Theor. Phys. 49 (1973) 652.

[6] See for example:
   S. L. ADLER and R. F. DASHEN: “Current Algebra and Application to Particle Physics”,
   (Benjamin, 1968);
   B. W. LEE: “Chiral Dynamics”, (Gordon Breach, 1972), and references therein.

[7] B. MACHET: “Leptonic custodial symmetry, quantization of the electric charge and the
   neutrino in the Standard Model”, hep-ph/9606308, Mod. Phys. Lett. A 11 (1996) 2297-2307.

[8] C. BOUCHIAT, J. IlioPOULOS and Ph. MEYER: “An anomaly-free version of Weinberg’s
   model”, Phys. Lett. 38 B (1972) 519;
   D. J. GROSS and R. JACKIW: “Effect of Anomalies on Quasi-Renormalizable Theories”,
   Phys. Rev. D 6 (1972) 477.

[9] M. BELLON & B. MACHET: “The Standard Model of leptons as a purely vectorial theory”,
   hep-ph/9305212, Phys. Lett. B 313 (1993) 141.

[10] M. GELL-MANN: “A schematic model of baryons and mesons”, Phys. Lett. 8 (1964) 214.

[11] M. GELL-MANN & M. LEVY: Nuov. Cim. 16 (1960) 705.

[12] M. GELL-MANN: Phys. Rev. 92 (1953) 833;
   K. NISHIJIMA: Prog. Theor. Phys. 13 (1955) 285.

[13] E. GILDENER and S. WEINBERG: “Symmetry breaking and scalar bosons”, Phys. Rev. D
   13 (1976) 3333;
   E. GILDENER: “Gauge-symmetry hierarchies”, Phys. Rev. D 14 (1976) 1667.

[14] Y. M. CHO & D. MAISON: “Monopole configuration in Weinberg-Salam model”, Phys.
   Lett. B 391 (1997) 360-365.

[15] Y. M. CHO & K. KIMM: “Electroweak Monopoles”, hep-th/9705213;
   Y. M. CHO & K. KIMM: “Finite Energy Electroweak Monopoles”, hep-th/9707038.
[16] M. VELTMAN: Acta Phys. Pol. B8 (1977) 475;
    F. ANTONELLI, M. CONSOLI and O. PELLEGRINO: Nucl. Phys. B 183 (1981) 195;
    J. VAN DER BIJ and M. VELTMAN: Nucl. Phys. B 232 (1984) 205;
    J. VAN DER BIJ: Nucl. Phys. B 248 (1984) 141;
    M. B. EINHORN and J. WUDKA: Phys. Rev. D 39 (1989) 2758.

[17] P. SIKIVIE, L. SUSSKIND, M. VOLOSHIN and V. ZAKHAROV: “Isospin breaking in technicolour models”, Nucl. Phys. B 173 (1980) 189.

[18] T. H. R. SKYRME: Proc. Roy. Soc. A260 (1961) 127;
    E. WITTEN: “Global aspects of Current Algebra”, Nucl. Phys. B 223 (1983) 422; “Current Algebra, baryons, and quark confinement”, ibidem 433;
    G. S. ATKINS, C. R. NAPPI and E. WITTEN: “Static properties of the nucleon in the Skyrme model”, Nucl. Phys. B 228 (1983) 552.

[19] B. W. LEE, C. QUIGG & H. B. THACKER: “Weak interactions at very high energies: The role of the Higgs-boson mass”, Phys. Rev. D 16 (1977) 1519.

[20] J. H. CHRISTENSON, J. W. CRONIN, J. W. FITCH and R. TURLAY: “Evidence for the $2\pi$ decay of the $K^0_2$ meson”, Phys. Rev. Lett. 13 (1964) 138;
    V. L. FITCH: “The discovery of charge-conjugation parity asymmetry”, Rev. Mod. Phys. 53 (1981) 367;
    J. W. CRONIN: “CP symmetry violation - the search for its origin”, Rev. Mod. Phys. 53 (1981) 373.

[21] H. ALBRECHT et al. (ARGUS collaboration): “Observation of $B^0 - \overline{B^0}$ mixing”, Phys. Lett. B 245 (1987), 245.

[22] see for example:
    Y. NIR: “CP Violation”, Lectures given at 20th Annual SLAC Summer Institute on Particle Physics: The Third Family and the Physics of Flavor (School: Jul 13-24, Topical Conference: Jul 22-24, Symposium on Tau Physics: Jul 24), Stanford, CA, 13-24 Jul 1992. Published in SLAC Summer Inst.1992:81-136 (QCD161:S76:1992).

[23] S. L. ADLER: “Axial-Vector Vertex in Spinor Electrodynamics”, Phys. Rev. 177 (1969) 2426;
    J. S. BELL and R. JACKIW: Nuovo Cimento 60 (1969) 47;
    W. A. BARDEEN: “Anomalous Ward Identities in Spinor Field Theories”, Phys. Rev. 184 (1969) 1848.

[24] P. RAMOND: “Field Theory; a Modern Primer”, p. 226; Frontiers in Physics, Lecture Notes Series 51 (Benjamin/Cummings 1981), p. 226.

[25] M. GELL-MANN, P. RAMOND and R. SLANSKY: “Supergravity” p. 315 (P. van Nieuwenhuizen and D.Z. Freedman eds, North Holland, Amsterdam 1979).

[26] T.P. CHENG and Ling Fong LI: “Neutrino masses, mixings, and oscillations in $SU(2) \times U(1)$ models of electroweak interactions”, Phys.Rev. D 22 (1980) 2860.

[27] Y. NAMBU and G. JONA-LASINIO: “Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity” Phys. Rev. 122 (1961) 345; ibidem, 124 (1961) 246.

[28] T. APPLEQUIST and J. CARRAZONE: “Infrared singularities and massive fields”, Phys.Rev. D 11 (1975) 2856.
[29] T. D. LEE: “A Theory of Spontaneous T Violation”, Phys. Rev. D 8 (1973) 1226; S. WEINBERG: “Gauge Theory of CP Nonconservation”, Phys. Rev. Lett. 37 (1976) 657.

[30] S. M. BILENKI & S. T. PETCOV: “Massive neutrinos and neutrino oscillations”, Rev. Mod. Phys. 59 (1987) 671.