Shock-Induced Bubble Collapse versus Rayleigh Collapse

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Abstract. This paper compares two physical mechanisms for the collapse of a bubble near a rigid wall: a traveling shock-induced collapse and a Rayleigh-like collapse due to a uniform rise of the pressure around the bubble. A multi-material compressible flow solver capable of handling material interfaces under high pressures is used to investigate these two scenarios for different levels of the driving pressure ranging from 1 MPa to 400 MPa. The two mechanisms are differentiated on the basis of the resulting bubble dynamics, the reentrant jet velocity, and the pressures imparted to the wall.

1. Introduction
Cavitation erosion is a major problem in fluid machinery such as propellers, pumps, turbines, .. etc. Microbubbles grow in low pressure regions then collapse when encountering high pressures to form near boundaries reentrant jets and shock waves [1, 2] leading to material damage and loss of performance. The collapse of these bubbles can also be used for useful purposes such as for cleaning [3] or erosion testing of materials [4]. Under various flow scenarios and applications, the collapse can be of a Rayleigh-type, where all parts of the bubble surface are exposed simultaneously to a high local pressure, or it could be shock-induced [5] where a shock wave (for example generated by a nearby bubble cloud collapse [6] or by a lithotripter) travels to the bubble and collapses it progressively. Depending on the distance of the centre generating the shock wave from the bubble surface, the shock wave can be spherical or planar. In this work we consider planar shock waves to simplify presentation.

2. Solution Method
The governing equations to study the bubble dynamics are Euler equations, described by a set of hyperbolic conservation laws [7]. These equations represent conservation of mass, momentum, and energy. The system of equations is closed with an equation of state for each phase. An ideal gas equation is used for air [8] and Tillotson’s equation is used for water [9]. This model account for high compressibility effects including shock waves, but does not take into account effects due to viscosity, surface tension, and turbulence. These effects will be studied in future work. The details pertaining to the multi-material solver we have developed, 3DYNAFS_COMP©, with validation and verification examples are provided in [1]. The two collapse scenarios described

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above and illustrated in Fig 1 are investigated using 3DYNAFS_COMP© for a set of driving pressures, $P_{\text{drive}}$. A domain size of 1cm by 1cm is used for all the cases. The bubble is resolved using a uniform grid of 10 μm. A reflection boundary condition was imposed on the axis of symmetry and at the bottom boundary, i.e. all physical variables such as density, pressure, velocities and energy are reflected from the axis, while transmission non-reflective boundary conditions (i.e. the flow variables are extrapolated along the characteristic wave direction) were imposed at the far-field boundaries.

3. Rayleigh-like Collapse

In the case of a Rayleigh-like collapse scenario, the bubble is suddenly subjected to a high ambient pressure. As illustrated in Fig. 2, the sudden rise of the pressure (here from 0.1 MPa to 10 MPa) results in the generation and propagation of a shock wave inside the bubble, while an expansion wave travels outward in the water as illustrated by the Schlieren contours in Fig. 2a. The rarefaction wave reflects back at the rigid wall towards the bubble and interacts with the bubble rarefaction wave in an increasingly complex manner as the number of reflections increases. Also, the pressure difference between the inside and the outside of the bubble, compresses the bubble. However, due to presence of the wall, the bubble pole farthest from the wall moves toward the wall faster than the rest of the interface resulting in the formation of a reentrant jet, which is clearly defined in Fig. 2b and very developed in Fig. 2c. As the jet speed increases, the jet tip crosses the full bubble height and impacts the wall generating a high impulsive pressure. This can be seen in Fig. 3, which actually shows a multi peaked pressure. The following peaks are due to the collapse of the remaining toroidal bubble and following shock emissions and reflections [1, 2].

4. Shock-Induced Collapse

To present a shock-induced collapse, we consider here a very thick shock front (finite shock thickness is quite important and will be addressed in another publication) with a high pressure advancing toward the bubble. The bubble initial radius, the standoff to the wall, and the pressure magnitude are the same as in the previous section. The high pressure hits the air/water interface, sending a shock wave inside the bubble and an expansion wave in water as shown in Fig. 4a. The shock front is seen in this figure not having reached the wall. The bubble starts compressing in a gradual fashion starting from the top as the high pressure reaches lower and lower parts of the interface. A reentrant jet then forms and generates a second shock wave in front of it, which propagates inside the bubble (Fig. 4b). This shock is here stronger than the initial wave and catches it. In the liquid, the incident shock wave gets reflected from the wall, while the shock wave travelling inside the bubble also hits the wall and gets reflected. These details affect the history of the pole velocities presented in Fig. 5a. The figure shows
almost no movement for the bottom pole for a long time. Finally, the combined shocks in the bubble hit the bottom pole (12.05 µs, pt. A) and reflect forcing its movement up into the bubble as shown in Fig. 5a (pt. B). The reflected wave from the wall inside the bubble hits the top pole, resulting in the slowing down of top pole (pt. C) before it collapses. Bubble contours showing the development of the reentrant jet and comparing Rayleigh-like collapse and shock-induced collapse are also shown in Fig. 5b and Fig. 5c. Both sets of contours are very much alike with the shock-induced reentrant jet occurring later and being slightly thinner. The motion up of the bottom pole is not shown to avoid overlapping contours.

![Fig. 4](image)

**Fig. 4** Coloured Shadowgraphs showing evolution of solution for shock collapse scenario for a driving pressure of 10 MPa. (a) $t = 1.1$ µs (b) $t = 11.96$ µs (c) $t = 12.41$ µs.

![Fig. 5](image)

**Fig. 5** (a) Pole velocities for both Rayleigh-like collapse and shock-like collapse scenarios at a driving pressure of 10 MPa. (b) Bubble contours at selected times for the Rayleigh-like collapse. (c) Bubble contours at selected times for the shock-induced collapse. An adaptive time stepping is used. The contour plots are shown for every 100 time steps with time step varying from $10^{-8}$ s to $10^{-5}$ s.

5. Effects of the Driving Pressure

The two collapse scenarios are investigated further by analyzing the reentrant jet development (top pole position vs. time). The evolution of the top pole position with time for driving pressures ranging from 10 MPa to 400 MPa is shown in Fig. 6. As the driving pressure is increased the bubble period is reduced as expected. The Rayleigh-like collapse scenario results in a faster collapse compared to the shock-induced collapse scenario for the same driving pressure. The bubble collapse times are shown in Fig. 7a, and follow the expected $P_{\text{drive}}^{1/2}$ trend. The corresponding jet velocities are shown in Fig. 7c. The peak pressures monitored at the wall for both scenarios and for a whole set of driving pressures between 1 MPa and 400 MPa are shown in Fig. 7b. These peaks increase with increasing driving pressures. Moreover, the Rayleigh-like collapse results in a higher wall pressure compared to the shock-driven collapse. Finally the maximum jet velocities for both

![Fig. 6](image)

**Fig. 6** Evolution of top pole position with time for different pressure and different collapsing scenarios, uniform collapse (solid lines) and shock collapse (dashed lines).
scenarios are compared with the sound speed in the liquid and with the characteristic velocity of collapse of an empty sphere in an infinite medium or the Rayleigh time,$\sqrt{P_{\text{drive}}/\rho}$, for different driving pressures and are shown in Fig. 7c. The jet velocity for the Rayleigh-like collapse is much higher compared to the shock induced collapse. The lower speed in the shock induced cases is due to the slowing down of the top pole when the reflected shock from the wall hits the reentrant jet tip. This slowing down of jet is mostly observed at the highest driving pressures observed above 80 MPa, when the jet speed and the sound speed become of the same order. In the case of the Rayleigh-like collapse the jet speed is able to easily exceed the sound speed for the higher driving pressures.

Fig. 7 (a) Bubble collapse time (b) Peak pressure at the wall (c) Jet velocity comparison with sound speed and characteristic speed for both scenarios covering different driving pressures

6. Conclusion
In this work, we compared the characteristics of bubble collapse dynamics near a rigid wall for a Rayleigh-like bubble collapse and a shock-induced bubble collapse. The collapsing mechanisms were studied for different driving pressures and the resulting pole positions, reentrant jet velocities, and impact pressures at the wall were compared. The results show that, for the same driving pressure, the Rayleigh-like collapse is faster than the shock-induced collapse and results in higher impact pressures at wall. The reflected shock waves at the wall play an important role in the shock-induced collapse leading to slowing down of incoming jet for higher driving pressures. These effects are more amplified with thin shock waves, which we did not address here for sake of brevity.

7. Acknowledgements
This work was supported by a DOE SBIR Phase II (DE-SC0006443), by ONR under contract N00014-12-C-0382 monitored by Dr. Ki-Han Kim and by DYNAFLOW IR&D. We are grateful for this support.

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