Multi-Objective Hybrid WIPSO–GSA Algorithm-Based DG and Capacitor Planning for Reduction of Power Loss and Voltage Deviation in Distribution System

Arulraj Rajendran and Kumarappan Narayanan
Department of Electrical Engineering, FEAT, Annamalai University, Annamalai Nagar, India

ABSTRACT
In electric distribution system, majority of load connected are inductive in nature and it results in increased system power loss and reduced bus voltage profile. In this study, in order to overcome these issues, optimal planning of DG and capacitor are presented by optimizing multiple objectives such as minimization of total active power loss ($P_{loss_{total}}$) and reduction of voltage deviation (VD). Here, a hybrid configuration of weight improved particle swarm optimization (WIPSO) and gravitational search algorithm (GSA) called hybrid WIPSO–GSA algorithm is proposed to solve the optimization problem in multi-objective problem domain. In order to solve multi-objective optimization problem, the proposed hybrid WIPSO–GSA algorithm is integrated with two components. The first component is fixed-sized archive that is responsible for storing a set of non-dominated Pareto optimal solutions and the second component is a leader selection strategy that helps to update and identify the best compromised solution from the archive. The proposed methodology is tested on standard 33-bus and Indian 85-bus distribution system. Moreover, the total economic benefit due to optimal DG and capacitor planning are established and also the superiority of the proposed technique is illustrated by comparing the results with other existing optimization techniques.

KEYWORDS
Distributed generation; capacitor; optimal planning; multi-objective hybrid WPSO–GSA algorithm; non-dominated Pareto front; leader selection strategy

ARTICLE HISTORY
Received 5 March 2018
Accepted 5 June 2018

1. Introduction
The distribution system is mainly responsible for reliable and quality supply of power to end customers and it is operated at low voltage when compared to high-voltage transmission system. Mostly distribution network is operating at lagging power factor since majority of loads connected to the distribution network are inductive in nature. The lag in load power factor causes wide range of technical issues such as enhanced network power loss and reduced bus voltage profiles. In order to overcome these technical issues compensation devices such as DG and capacitor are installed in distribution network. Generally, DG is a small-scale power generation unit that is usually connected closer to load in the distribution system and it has a significant impact on quality and reliable of power to the end consumers. In power systems, different DG technologies are involved in which some have been in use for a long time, while others are newly emerging. The two main categories of DG technologies are renewable DG technologies (e.g. photovoltaic and wind turbine) and non-renewable
DG technologies (e.g. fuel cells, micro-turbines, and combustion turbines). The review over basic DG definition, potential DG benefits and various DG technologies were presented in [1,2]. The installation of DG and capacitor in distribution network has various technical and economic benefits and their positive impacts on distribution system operation are being analyzed. Even though the integration of DG and capacitor provides various benefits, they may impose certain problems and limitations if they are not optimally planned at appropriate bus location and capacity (i.e. size) in the distribution network. Since installation of compensation devices in distribution system is an optimization problem, several approaches were employed in literature for optimal location and sizing of compensation devices in distribution system. In [3,4], optimal capacitor installation problem was solved using analytical method and in [5,6], analytical methods were also used for optimal installation of DG in distribution network. Even though analytical methods were used to solve capacitor and DG allocation problem, they require complex calculations and formulation of impedance bus matrix. To overcome these problems artificial intelligent techniques are generally used. In recent years, there has been a significant interest among researchers in using artificial intelligent techniques. In [7–9], optimal capacitor allocation problem was addressed using particle swarm optimization (PSO) [7], plant growth simulation algorithm (PGSA) [8], and teaching learning-based optimization (TLBO) [9]. In [10–14], optimal DG allocation problem was solved using genetic algorithm (GA) [10], shuffled frog leap algorithm (SFLA) [11], artificial bee colony (ABC) algorithm [12], intelligent water drop algorithm [13], and combination of improved-PSO and gravitational search algorithm (GSA) [14]. In [15–17], optimal simultaneous DG and capacitor allocation problem was addressed using bacterial foraging optimization algorithm (BFOA) [15], intersect mutation differential evolution (IMDE) algorithm [16], and gest-guided ABC (GABC) algorithm [17].

The studies addressed in [3–17] utilize minimization of \( P_{\text{loss}} \) as main objective and also minimization of total operation cost is another single objective commonly used for allocating compensation devices in distribution network [18,19]. In the single objective DG and capacitor planning problem, the solutions attained optimizes only one objective function, whereas the other important factors involved are left unattended. In order to overcome this drawback, multiple objectives can be solved simultaneously using multi-objective optimization methodologies. In [20–23], different objectives were combined into a single scalar objective optimization problem using weight sum method and the optimization problem was solved using combination of GA and PSO algorithm [20], sequential quadratic programming deterministic technique [21], chaotic ABC (CABC) algorithm [22], and TLBO technique [23]. However, in [20–23], the weights assigned to different objectives are predefined resulting in unequal priority in optimizing different objectives. To overcome this problem multiple objectives are solved simultaneously by attaining non-dominated Pareto optimal solutions using optimization techniques such as non-dominated sorting GA (NSGA) [24], multi-objective PSO (MOPSO) [25], non-dominated sorting modified cuckoo search algorithm (NSMCSA) [26], and peer-enhanced multi-objective TLBO (PeMOTLBO) [27] for optimal planning of compensation devices in distribution network.

In this study, minimization of \( P_{\text{loss}} \) and reduction of total planning period. The effectiveness of the proposed multi-objective hybrid WIPSO–GSA algorithm is also demonstrated by comparing the results with other existing optimization techniques. The remainder of the paper is structured as follows: Section 2 contains problem formulation with necessary technical constraints and expressions to evaluate total economic benefit. Section 3 describes the concept behind hybrid WIPSO–GSA algorithm and implementation of proposed multi-objective hybrid WIPSO–GSA algorithm. Section 4 contains simulation results followed by conclusions.

### 2. Problem Formulation
#### 2.1. Distribution Load Flow (DLF)
In any DG and capacitor placement problem, DLF analysis plays a vital role in the solution process. In this study, backward sweep and forward sweep method of DLF [28] is used in order to achieve accurate results.
2.2. Multi-Objective Problem Formulation

In order to find trade-off solutions among different objectives, a Pareto-based multi-objective technique is used. In this study, the objective functions to be optimized are minimization of $P_{\text{loss}}^{\text{total}}$ and reduction of $VD$. Mathematically, the objective functions are formulated as follows:

2.2.1. Minimization of $P_{\text{loss}}^{\text{total}}$

For $N_{\text{bus}}$ radial distribution system, minimization of $P_{\text{loss}}^{\text{total}}$ problem is formulated as follows:

$$f_1 = \text{Min} \ (P_{\text{loss}}^{\text{total}})$$

(1)

where

$$P_{\text{loss}}^{\text{total}} = (P_{\text{SS}}^{\text{total}} + \sum_{a=1}^{ndg} P_{DG(a)} - \sum_{i=2}^{N_{\text{bus}}} P_{\text{load}(i)}$$

(2)

$P_{\text{SS}}^{\text{total}}$ is the injected active power at bus 1 from the substation; $P_{DG(a)}$ is the active power output of $a^{th}$ DG unit; $P_{\text{load}(i)}$ is the active power load connected at bus $i$; $ndg$ is the total number of DG units installed in the distribution network; $N_{\text{bus}}$ is the total number of buses in the distribution network.

2.2.2. Minimization of $VD$

For $N_{\text{bus}}$ radial distribution system, minimization of $VD$ problem is formulated as follows [27]:

$$f_2 = \text{Min} \ (VD)$$

(3)

where

$$VD = \sum_{i=1}^{N_{\text{bus}}} (|V_i| - |V_{\text{rated}}|)^2$$

(4)

$V_i$ is the voltage magnitude of bus $i$ in p.u.; $V_{\text{rated}}$ is the rated voltage magnitude and it is taken as 1 p.u.

2.3. Technical Constraints

The objective functions formulated are optimized subjected to the following technical constraints:

2.3.1. Power Limits of DG and Capacitor

$$P_{\text{DG}}^{\text{min}} \leq \sum_{a=1}^{ndg} P_{DG(a)} \leq P_{\text{DG}}^{\text{max}}$$

(5)

$$Q_{\text{Cap}}^{\text{min}} \leq \sum_{b=1}^{ncap} Q_{\text{Cap}(b)} \leq Q_{\text{Cap}}^{\text{max}}$$

(6)

where $P_{\text{DG}}^{\text{max}}$ and $P_{\text{DG}}^{\text{min}}$ are the lower and upper total active power generation limits of DG units, respectively; $Q_{\text{Cap}}^{\text{min}}$ and $Q_{\text{Cap}}^{\text{max}}$ are the lower and upper total reactive power generation limits of capacitors, respectively; $Q_{\text{Cap}(b)}$ is the reactive power output of $b^{th}$ capacitor; $P_{\text{load}(i)}$ and $Q_{\text{load}(i)}$ are the active and reactive power load connected at bus $i$, respectively; $ncap$ is the total number of capacitors installed in the distribution network. In order to maintain quality and reliable supply of power to end consumers and also to achieve potential economic benefit, minimized power loss, and reduced voltage deviation, 80% of system total active load is taken as the upper limit for active power generation of total installed DG units and 80% of system total reactive power load is taken as the upper limit for reactive power generation of total installed capacitors [12].

2.3.2. Bus Voltage Limits

$$V_{\text{min}} \leq V_i \leq V_{\text{max}}$$

(9)

$$i = 1, 2, \ldots, N_{\text{bus}}$$

where $V_{\text{min}}$ and $V_{\text{max}}$ are the lower and upper limits of bus voltage magnitude, respectively; $V_i$ is the voltage at bus $i$. Here, the lower and upper limits of bus voltage magnitude are taken as 0.90 p.u. and 1.05 p.u., respectively [5].

2.4. Economic Benefit Evaluation of DG and Capacitor

In this study, all the costs are expressed in Indian Rupee (₹). The mathematical formulation involving different DG and capacitor cost terminologies for the total planning period is presented as follows [25]:

2.4.1. DG and Capacitor Investment Cost

Here, the investment cost includes DG unit cost, site for installing DG, equipment, monitoring, and construction. The investment cost of DG ($C_{IDG}$) is evaluated by:

$$C_{IDG} = \sum_{a=1}^{ndg} P_{DG(a)} \times IC_{DG}$$

(10)

where $IC_{DG}$ is the installation cost of DG in ₹/MW.
The investment cost of capacitor \( C_{iCap} \) is evaluated by:

\[
C_{iCap} = \sum_{b=1}^{\text{ncap}} Q_{\text{Cap}(b)} \times IC_{\text{Cap}}
\]  

where \( IC_{\text{Cap}} \) is the installation cost of capacitor in ₹/MVAr.

### 2.4.2. Operational and Maintenance Cost of DG and Capacitor

Here, the operational and maintenance cost includes fuel cost, renovation cost, and electrical and mechanical annual inquiry. The DG operational and maintenance cost \( (C_{OMDG}^{pp}) \) for the total planning period is given by:

\[
C_{OMDG}^{pp} = \sum_{y=1}^{\text{pp}} \sum_{a=1}^{\text{ndg}} \left( P_{DG(a)}^{y} \times (OC_{DG}^{pp} \times T) + MC_{DG}^{pp} \right)
\]

and the capacitor maintenance \( (C_{MCap}^{pp}) \) cost for the total planning period is evaluated as:

\[
C_{MCap}^{pp} = \sum_{y=1}^{\text{pp}} \left( PWF^{y} \times MC_{Cap}^{pp} \right)
\]

where \( PWF \) is the present worth factor for the total planning period and it is formulated as follows:

\[
PWF = \frac{1 + IntR}{1 + IntR}
\]

\( IntR \) is the inflation rate; \( IntR \) is the interest rate; \( P_{DG(a)}^{y} \) is the operating active power output of \( a^{th} \) DG unit (MW); \( OC_{DG}^{pp} \) and \( MC_{DG}^{pp} \) is the operational cost (₹/MWh) and annual maintenance cost (₹/year) of DG, respectively; \( MC_{Cap}^{pp} \) is the annual maintenance cost (₹/year) of capacitor; \( T \) is the total number of operating hours in a year \( (T = 8760) \); \( pp \) is the total planning period (in years).

### 2.4.3. Economic Benefit

The purchased cost of energy from the substation including energy loss before DG and capacitor location \( (C_{SS}^{pp,bef,loc}) \) for the total planning period is given by:

\[
C_{SS}^{pp,bef,loc} = \sum_{y=1}^{\text{pp}} PWF^{y} \times K_{SS} \times P_{SS}^{total,bef,loc} \times T
\]

where \( K_{SS} \) is the grid electricity price in ₹/MWh; \( P_{SS}^{total,bef,loc} \) is the injected active power (in MW) at bus 1 from the substation before DG and capacitor location.

By optimally installing DG and capacitor, the distribution companies can supply portion of system power demand and also compensates system power loss. The purchased cost of energy from the substation including energy loss after DG and capacitor location \( (C_{SS}^{pp,aft,loc}) \) for the total planning period is given by:

\[
C_{SS}^{pp,aft,loc} = \sum_{y=1}^{\text{pp}} PWF^{y} \times K_{SS} \times P_{SS}^{total,aft,loc} \times T
\]

where \( P_{SS}^{total,aft,loc} \) is the injected active power (in MW) at bus 1 from the substation after DG and capacitor location.

After optimal DG and capacitor installation, the cost benefit due to reduction in cost of energy purchased from the substation including energy loss \( (C_{SS}^{pp,benefit}) \) for the total planning period is evaluated by subtracting (16) from (15) and it is expressed as follows:

\[
C_{SS}^{pp,benefit} = \sum_{y=1}^{\text{pp}} PWF^{y} \times K_{SS} \times \left( P_{SS}^{total,bef,loc} - P_{SS}^{total,aft,loc} \right) \times T
\]

The total economic benefit after considering various DG and capacitor cost terminologies for the total planning period \( (Total_{benefit}^{pp}) \) is given by:

\[
Total_{benefit}^{pp} = (C_{SS}^{pp,benefit}) - (C_{IDG} + C_{OMDG}^{pp} + C_{iCap} + C_{MCap}^{pp})
\]

### 3. Solution Technique

In recent years there has been several heuristic evolutionary optimization techniques developed and the main aim of all these techniques is to achieve best solution (i.e. global optimum) amid all possible inputs. In order to achieve global optimum a heuristic technique should have two main features such as exploration and exploitation. For any heuristic technique, the ability to search the whole problem space is termed as exploration and the convergence ability to achieve global optimum near a good solution is termed as exploitation. In order to achieve global optimum, the ultimate aim of any heuristic optimization technique is to find the fine balance between the ability of exploration and exploitation. According to [29], the strengthening of either one ability will weaken the other and vice versa. Thus, the previously mentioned features make the existing heuristic optimization techniques capable of solving only finite set of problems. Merging the strength of optimization techniques is one of the best possible ways to find balance between overall exploration and exploitation abilities. Therefore, in order to maintain good balance between exploration and exploitation abilities, the authors are motivated to hybrid WIPSO and GSA in this study.

#### 3.1. Hybrid WIPSO–GSA Algorithm

PSO proposed by Kennedy [30] has attracted many researchers owing to its simplicity thereby making it one of the most widely used optimization technique in hybrid...
Multi-Objective Hybrid WIPSO–GSA Algorithm

The multi-objective hybrid WIPSO–GSA algorithm almost inherits all the basic features of hybrid WIPSO–GSA algorithm, which means the search space is being explored and exploited by the search agents in a same manner. However, the main difference is that, multi-objective hybrid WIPSO–GSA algorithm searches around a set of non-dominated Pareto optimal solutions stored in the archive, whereas the hybrid WIPSO–GSA algorithm only saves and improves one global optimum. In this study, in order to accomplish multi-objective optimization by hybrid WIPSO–GSA technique, two new components are integrated and they are very similar to

$$c_{2new} = c_{2final} + \left( \frac{c_{2final} - c_{2initial}}{Iter_{max}} \right) \times k$$

$$acc_{ci}^l(k) = \frac{F_i^l(k)}{M_i(k)}$$

where \(x_i^l\) is the current position vector of particle \(l\) in a D-dimensional search space; \(v_{i}^{d}\) is the velocity vector of particle \(l\) in a D-dimensional search space; \(N_p\) is the total number of particles; \(k\) is the current iteration number; \(Iter_{max}\) is the total number of iterations; \(rand_1, rand_2, \text{and} rand_3\) are the random numbers between 0 and 1; \(x_{gbest}^l\) is the gbest of particle group until iteration \(k\); \(acc_{ci}^l(k)\) is the acceleration of particle \(l\) and it is evaluated using expression given in equation (25); \(F_{i}^{l}(k)\) is the resultant force acting on particle \(l\) acquired from every other particles in the search space; \(M_i(k)\) is the inertia mass proportional to the fitness of particle \(l\). The expressions to evaluate \(F_{i}^{l}(k)\) and \(M_i(k)\) are given in [34].

In classical PSO [30] and also in hybrid methods involving PSO [20,31,32], a fixed value (usually fixed to 2) is assigned for acceleration coefficients \(c_1\) (cognitive component) and \(c_2\) (social component). The fixed value of \(c_1\) and \(c_2\) will result in less accurate results and occurrence of premature convergence [33]. Therefore, quality solution is achieved using proposed hybrid WIPSO–GSA algorithm by modifying \(c_1\) and \(c_2\) in an adaptive way such that \(c_1\) is decreased and \(c_2\) is increased as the iteration proceeds [33], so that adaptive weights can be assigned to exploration and exploitation abilities thereby resulting in better global optimum and greater convergence speed. Therefore, the new modified \(c_1\) and \(c_2\) is represented as \(c_{1new}\) and \(c_{2new}\) and are formulated as shown in (23) and (24), respectively, where \(c_{1initial}\) and \(c_{2initial}\) are the initial and final values of cognitive component, respectively; \(c_{1final}\) and \(c_{2final}\) are the initial and final values of social component, respectively.

### 3.2. Multi-Objective Hybrid WIPSO–GSA Algorithm

The multi-objective hybrid WIPSO–GSA algorithm almost inherits all the basic features of hybrid WIPSO–GSA algorithm, which means the search space is being explored and exploited by the search agents in a same manner. However, the main difference is that, multi-objective hybrid WIPSO–GSA algorithm searches around a set of non-dominated Pareto optimal solutions stored in the archive, whereas the hybrid WIPSO–GSA algorithm only saves and improves one global optimum. In this study, in order to accomplish multi-objective optimization by hybrid WIPSO–GSA technique, two new components are integrated and they are very similar to

$$c_{1new} = c_{1initial} - \left( \frac{c_{1final} - c_{1initial}}{Iter_{max}} \right) \times k$$

$$c_{2new} = c_{2final} + \left( \frac{c_{2final} - c_{2initial}}{Iter_{max}} \right) \times k$$

where \(x_i^l\) is the current position vector of particle \(l\) in a D-dimensional search space; \(v_{i}^{d}\) is the velocity vector of particle \(l\) in a D-dimensional search space; \(N_p\) is the total number of particles; \(k\) is the current iteration number; \(Iter_{max}\) is the total number of iterations; \(rand_1, rand_2, \text{and} rand_3\) are the random numbers between 0 and 1; \(x_{gbest}^l\) is the gbest of particle group until iteration \(k\); \(acc_{ci}^l(k)\) is the acceleration of particle \(l\) and it is evaluated using expression given in equation (25); \(F_{i}^{l}(k)\) is the resultant force acting on particle \(l\) acquired from every other particles in the search space; \(M_i(k)\) is the inertia mass proportional to the fitness of particle \(l\). The expressions to evaluate \(F_{i}^{l}(k)\) and \(M_i(k)\) are given in [34].

In classical PSO [30] and also in hybrid methods involving PSO [20,31,32], a fixed value (usually fixed to 2) is assigned for acceleration coefficients \(c_1\) (cognitive component) and \(c_2\) (social component). The fixed value of \(c_1\) and \(c_2\) will result in less accurate results and occurrence of premature convergence [33]. Therefore, quality solution is achieved using proposed hybrid WIPSO–GSA algorithm by modifying \(c_1\) and \(c_2\) in an adaptive way such that \(c_1\) is decreased and \(c_2\) is increased as the iteration proceeds [33], so that adaptive weights can be assigned to exploration and exploitation abilities thereby resulting in better global optimum and greater convergence speed. Therefore, the new modified \(c_1\) and \(c_2\) is represented as \(c_{1new}\) and \(c_{2new}\) and are formulated as shown in (23) and (24), respectively, where \(c_{1initial}\) and \(c_{2initial}\) are the initial and final values of cognitive component, respectively; \(c_{1final}\) and \(c_{2final}\) are the initial and final values of social component, respectively.

### 3.2. Multi-Objective Hybrid WIPSO–GSA Algorithm

The multi-objective hybrid WIPSO–GSA algorithm almost inherits all the basic features of hybrid WIPSO–GSA algorithm, which means the search space is being explored and exploited by the search agents in a same manner. However, the main difference is that, multi-objective hybrid WIPSO–GSA algorithm searches around a set of non-dominated Pareto optimal solutions stored in the archive, whereas the hybrid WIPSO–GSA algorithm only saves and improves one global optimum. In this study, in order to accomplish multi-objective optimization by hybrid WIPSO–GSA technique, two new components are integrated and they are very similar to
those employed in MOPSO [36]. The first component is an archive which stores the non-dominated Pareto optimal solutions attained so far. The basic concepts and different definitions regarding Pareto dominance were given in [36]. Leader selection strategy is the second component that assists in identifying the leader (i.e. best compromise solution) from the archive.

The key component of the archive is an archive controller. The archive has a fixed number of members and the main feature of the archive controller is to control the archive when the archive is full or when a new solution wants to enter the archive. Once the archive is full, a mechanism called adaptive grid mechanism is triggered. The main role of the grid mechanism is to keep the solutions in the archive as diverse as possible when the archive is full. In the grid mechanism, the objective space is divided into several regions called hypercube (i.e. segment). If a newly attained solution occupies a space outside the grid, then the locations of the grid should be re-evaluated to accommodate the new solution. If a newly attained solution occupies a space within the grid, then it is accommodated to the segment of the grid that have lower number of particles by randomly omitting one of the resident in the most crowded segment. The main advantage of using grid mechanism is low computational cost and it does not require complete grid updating in each and every iteration like in the case of niching [37].

In the archive, the non-dominant solutions obtained so far are compared with new solutions attained during the course of iteration and there would be different cases possible which are given as follows [36]:

- If the new solution is dominated by any one of the archive residents, then in this case the new solution should not be allowed to enter the archive.
- If the new solution dominates one or more residents in the archive, then in this case, the solution(s) in the archive that are dominated by the new solution should be omitted and the new solution will be able to enter the archive.
- If the new solution and the archive residents do not dominate each other, then in this case the new solution should be added to the archive.
- If the new solution dominates a resident in the archive and the archive is full, then in this case the grid mechanism should run in order to accommodate the new solution by randomly omitting one of the resident in the most crowded segment and inserting the new solution in the least crowded segment, so that the diversity of final approximated Pareto optimal front is improved.

The second component that integrated to the multi-objective hybrid WIPSO–GSA algorithm is the leader selection mechanism. In hybrid WIPSO–GSA algorithm, $gbest$ guides the other agents in the search space toward the global optimum. However, in a multi-objective search space, it is difficult to compare solutions owing to the Pareto optimality concepts discussed earlier. Therefore, in order to handle this issue the leader selection mechanism is used. The main feature of leader selection mechanism is to choose a leader from the set of best non-dominated Pareto optimal solutions stored in the archive. For this purpose, the least crowded segment of the archive is chosen by the leader selection component and one of its non-dominated solutions is considered as the leader. Here, the selection is done by the roulette-wheel method with the probability for hypercube $h$ (i.e. $prob_h$) is formulated as follows:

$$prob_h = \frac{const}{N_h}$$

(26)

where $const$ is a constant number greater than one; $N_h$ is the total number of obtained non-dominated Pareto optimal solutions in segment $h$. From (26), it can be seen that segments that are less crowded have higher probability of signifying new leaders. In other words, the probability of picking a segment to select a leader is increased when the number of attained solutions is decreased in the segment.

In this study, the multi-objective optimization problem to solve two different objectives is mathematically formulated as follows:

$$Min \ F(X) = \{f_1(X), f_2(X)\}$$

(27)

where

$$X = [x_1, x_2, ..., x_N]$$

(28)

$F$ is the vector of objective functions; $X$ is the decision variable for particles in the search space.

In multi-objective hybrid WIPSO–GSA algorithm, generally there is not one global optimum, but contains a set of so called non-dominated Pareto optimal solutions. Non-dominated solution is the one which is not dominated by any other solution and are located in the archive. A decision vector $x_i$ dominates vector $x_j$ if:

$$\forall i \in \{1, 2, ..., N_{obj}\}: f_i(x_i) \leq f_i(x_j),$$

$$\exists j \in \{1, 2, ..., N_{obj}\}: f_j(x_i) < f_j(x_j)$$

(29)

where $N_{obj}$ is the total objective functions considered in the problem; $x_i$ and $x_j$ are the D-dimensional decision vectors, containing DG location ($DG_{loc}$) and DG size ($DG_{size}$) and also capacitor location ($Cap_{loc}$) and capacitor size ($Cap_{size}$) as shown in (30) and (31):

$$x_i = (x_{i1}, x_{i2}, ..., x_{iD}) = (DG_{1loc}, DG_{1size}, Cap_{1loc}, Cap_{1size})$$

(30)
where

\[ \mathbf{x}_2 = (x_2^1, x_2^2, \ldots, x_2^D) = (DG_{loc}, DG_{size}, Cap_{loc}, Cap_{size}) \]  

\[ DG_{loc} = (bus_{DG(1)}, bus_{DG(2)}, \ldots, bus_{DG(ndg)}) \]  

\[ DG_{size} = (P_{DG(1)}, P_{DG(2)}, \ldots, P_{DG(ndg)}) \]  

\[ Cap_{loc} = (bus_{Cap(1)}, bus_{Cap(2)}, \ldots, bus_{Cap(ncap)}) \]  

\[ Cap_{size} = (Q_{Cap(1)}, Q_{Cap(2)}, \ldots, Q_{Cap(ncap)}) \]

The flowchart describing the computational steps of proposed multi-objective hybrid WIPO–GSA algorithm is shown in Figure 1. The values of various parameters involved in multi-objective hybrid WIPO–GSA algorithm are taken as follows: \( N_p \) (population size) = 50; \( N_{arch} \) (archive size) = 50; \( N_{grid} \) (number of grids per dimension)

---

**Figure 1.** Flowchart for optimal planning of DG and capacitor using multi-objective hybrid WIPO–GSA algorithm.
power load, respectively, thereby resulting in reduced flow of active and reactive power through distribution feeder sections. Here, the reduced flow of both active and reactive current component through distribution feeder sections, respectively, which in turn provides a significant reduction of system power loss when compared to independent DG and capacitor installation cases. Moreover, in Case-3, the additional reactive power support from capacitor installation can be provided.

4. Numerical Results and Discussions

In this study, a standard 33-bus radial distribution system with 32 feeder sections [38] and an Indian 85-bus radial distribution system with 84 feeder sections [39] are considered. For 33-bus system, the total active and reactive power load is 3.72 MW and 2.3 MVar, respectively. For Indian 85-bus system, the total active and reactive power load is 2.55 MW and 2.60 MVar, respectively. The base kVs for standard 33-bus and Indian 85-bus system is 12.66 kV and 11.00 kV, respectively. The optimization process using the proposed multi-objective hybrid WIPSO–GSA algorithm is carried out in MATLAB environment. Here, DG operating at unity power factor (i.e. injecting active power alone) is considered along with capacitor. The different DG and capacitor installation cases are listed as follows:

- Case-1: Multiple installation of capacitor.
- Case-2: Multiple installation of DG.
- Case-3: Simultaneous multiple installation of DG and capacitor.

The non-dominated Pareto optimal solutions of proposed multi-objective hybrid WIPSO–GSA algorithm for different DG and capacitor installation cases are shown in Figures 2 and 3 for standard 33-bus and Indian 85-bus system, respectively. In the set of non-dominated Pareto optimal solutions attained from the proposed multi-objective hybrid WIPSO–GSA algorithm, the best compromise solution is determined using leader selection mechanism and is tabulated in Tables 1 and 2 for standard 33-bus and Indian 85-bus system respectively. The value of $P_{\text{loss\ total}}$ and $V_D$ without installing any DG and capacitor (i.e. base case) is 210.9983 kW and 0.1338 p.u., respectively, for standard 33-bus system and it is 311.4151 kW and 0.8102 p.u., respectively, for Indian 85-bus distribution system. In both standard 33-bus and Indian 85-bus system, among independent DG and capacitor installation cases, it is observed that independent installation of DG (i.e. Case-2) which compensates portion of system active power load alone results in significant reduction of $P_{\text{loss\ total}}$ and $V_D$ when compared to independent installation of capacitor (i.e. Case-1) that supports portion of system reactive power load alone. In both standard 33-bus and Indian 85-bus system, it is observed that, there is considerable amount of reactive power load in addition to active power load. Therefore, in such system conditions, simultaneous installation of DG and capacitor at multiple bus locations (i.e. Case-3) supplies portion of system active and reactive power load, respectively, thereby resulting in reduced flow of active and reactive power through distribution feeder sections. Here, the reduced flow of both active and reactive power results in reduced flow of active and reactive current component through distribution feeder sections, respectively, which in turn provides a significant reduction of system power loss when compared to independent DG and capacitor installation cases. Moreover, in Case-3, the additional reactive power support from capacitor installation can be provided.
only restriction in capacitor installation cases (i.e. Case-1 and Case-3) is that the capacitors are available in fixed sizes only. This restriction can be overcome by cascading the capacitors in parallel in order to attain the required size at appropriate bus locations. The enhancement in bus voltage magnitude for different installation cases are shown in Figures 4 and 5 for standard 33-bus and Indian 85-bus system, respectively. From Figures 4 and 5, it is evident that Case-3 which supplies portion of system active and reactive power load results in enhanced bus voltage profiles at majority of buses when compared to other installation cases.

Determining suitable location and size of DG and capacitor in radial distribution system is significant for achieving potential technical benefits. However, apart from the technical benefits, the optimal DG and capacitor installations also yields greater economic benefits for distribution companies for the total planning period. The commercial information of DG and capacitor are taken from [40] and all the costs are expressed in Indian Rupee (₹). The values of various cost terminologies involved are: $K_{SS} = 5000$ (₹/MWh); $IC_{DG} = 25 \times 10^6$ (₹/MVar); $OC_{DG} = 2.5 \times 10^3$ (₹/MWh); $MC_{DG} = 10000 + 20\%$ of $IC_{DG}$ (₹/year); $MC_{Cap} = 5000 + 20\%$ of $IC_{Cap}$ (₹/year); $InfR = 9\%$; $IntR = 12.5\%$; $pp = 10$ (years). After optimal DG and capacitor installation, the reduction in cost of energy purchased from the substation including energy loss and the total cost benefit after considering various DG and capacitor cost terminologies that are evaluated using expressions given in (15), (16), (17), and (18) are presented in Tables 3 and 4 for standard 33-bus and Indian 85-bus system, respectively. From Tables 3 and 4, among different DG and capacitor installation cases, it is evident that Case-3 provides greater economic benefit followed by Case-2 and Case-1, respectively, in both standard 33-bus and Indian 85-bus system.

Finally, in order to show the computational supremacy of proposed multi-objective hybrid WIPSO–GSA algorithm over other existing techniques, comparison is made with literature that uses same case study. The best compromised solution of existing techniques and the proposed multi-objective hybrid WIPSO–GSA algorithm for different DG and capacitor installation cases are tabulated in Tables 5 and 6 for standard 33-bus and Indian 85-bus system, respectively. From Tables 5 and 6, it is perceived that, the proposed technique provides

![Figure 3](image-url). Pareto optimal front of different DG and capacitor installation cases for Indian 85-bus distribution system.

| Case   | Cap bus location | Cap size (MVAR) | DG bus location | DG size (MW) | $P_{loss, total}$ (kW) | VD (p.u.) |
|--------|------------------|-----------------|-----------------|--------------|------------------------|----------|
| Base case | –                | –               | –               | –            | 210.9983               | 0.1338   |
| Case-1  | 14,29,30         | 0.6133, 0.6133, 0.6133 | –               | –            | 146.0474               | 0.0004   |
| Case-2  | –                | –               | 13,26,31        | 0.9135, 0.9879, 0.8556 | 83.6003               | 0.0065   |
| Case-3  | 11,29,30         | 0.5003, 0.5630, 0.6133 | 13,24,30        | 0.8830, 0.9827, 0.9907 | 15.8438               | 0.0004   |
exploration and exploitation abilities achieved through merging the strength of local search capability in GSA with the strength of social thinking in WIPSO makes the superior technical benefits over other existing techniques by providing better optimized results and greater diversity among solutions. Thus, the fine balance between exploration and exploitation capabilities of GSA-WIPSO makes it a promising alternative to solve engineering problems. In order to validate the accuracy of the proposed methodology, the application of the GSA-WIPSO optimizer is carried out for two benchmark power systems: a standard 33-bus distribution system and an Indian 85-bus distribution system. The optimal results of DG and capacitor installation cases for the Indian 85-bus distribution system are presented in Table 2.

Table 2. Optimal results of different DG and capacitor installation cases for Indian 85-bus distribution system.

| Case    | Cap bus location | Cap size (MVar) | DG bus location | DG size (MW) | $P_{loss}$ (kW) | $VD$ (p.u.) |
|---------|------------------|----------------|----------------|--------------|-----------------|-------------|
| Base case | –                | –              | –              | –            | 311.4151        | 0.8102      |
| Case-1  | 29,48,67         | 0.6935, 0.6936, 0.6936 | –              | –            | 158.5221        | 0.2793      |
| Case-2  | –                | –              | 28,48,68       | 0.6710, 0.6682, 0.6799 | 153.5574       | 0.1089      |
| Case-3  | 26,48,68         | 0.6936, 0.6936, 0.6899 | 34,64,80       | 0.6782, 0.6799, 0.6760 | 21.7047        | 0.0013      |

Figure 4. Voltage profile at each bus for standard 33-bus distribution system.

Figure 5. Voltage profile at each bus for Indian 85-bus distribution system.

Table 3. Total economic benefit results for standard 33-bus distribution system.

| Case    | $C_{Cap}$ (₹/pp) | $C_{MCap}$ (₹/pp) | $C_{DG}$ (₹/pp) | $C_{MGDG}$ (₹/pp) | $C_{SS}$ (₹/pp) | $C_{SS,benefit}$ (₹/pp) | Total benefit (₹/pp) |
|---------|------------------|------------------|----------------|-----------------|----------------|-----------------------|----------------------|
| Base case | –                | –                | –              | –               | –              | –                     | 1.4512 × 10^9         |
| Case-1  | 1.8399 × 10^3    | 3.5274 × 10^3    | –              | –               | –              | –                     | 1.4512 × 10^9         |
| Case-2  | –                | 6.8925 × 10^7    | 6.2595 × 10^8  | 0.3850 × 10^8   | 1.0662 × 10^9  | 3.7129 × 10^8         | 4.0751 × 10^8         |
| Case-3  | 1.6766 × 10^3    | 3.2517 × 10^7    | 7.1410 × 10^7  | 6.4852 × 10^8   | 0.3232 × 10^9  | 1.1279 × 10^9         | 4.0751 × 10^8         |
Table 4. Total economic benefit results for Indian 85-bus distribution system.

| Case   | $C_{\text{Cap}}$ (₹/pp) | $C_{\text{Rcap}}$ (₹/pp) | $C_{\text{DG}}$ (₹/pp) | $C_{\text{QDG}}$ (₹/pp) | $C_{\text{SS}}$ (₹/pp) | $C_{\text{SS}}$ (₹/pp) | $C_{\text{Total}}$ (₹/pp) |
|--------|--------------------------|---------------------------|-------------------------|-------------------------|------------------------|------------------------|-----------------------------|
| Base case | –                        | –                         | –                       | –                       | 1.0575 x 10^3         | –                       | 210,9983                    |
| Case-1 | 2.0807 x 10^3            | 3.9338 x 10^3             | 5.0477 x 10^3           | 4.5844 x 10^4           | 0.2528 x 10^3         | 0.5631 x 10^5          | 2.9573 x 10^3              |
| Case-2 | 2.0771 x 10^3            | 3.9227 x 10^3             | 5.0582 x 10^3           | 4.6185 x 10^4           | 0.1985 x 10^3         | 8.5895 x 10^3          | 3.4565 x 10^3              |
| Case-3 | –                        | –                         | –                       | –                       | –                      | –                      | 156,284                     |

Table 5. Comparative Study for standard 33-bus distribution system.

| Case   | Technique | Cap bus location | Cap size (MVAr) | DG bus location | DG size (MW) | $P_{\text{loss}}$ total (kW) | $VD$ (p.u.) |
|--------|-----------|------------------|-----------------|-----------------|--------------|-------------------------------|-------------|
| Base case | –         | –                | –               | –               | –            | 210,9983                      | 0.1338      |
| Case-1 | NSMCSA[26] | 30,32            | 0.6000, 0.4500  | –               | –            | 156,284                       | 0.0498      |
| Case-2 | LSF[5]    |                  | –               | –               | –            | 146,047                       | 0.0479      |
| Case-3 | BFOA[15]  | 18,30,33         | 0.1632, 0.5410, 0.3384 | – | – | 154,144                       | 0.0479      |
| Case-4 | BFOA[15]  | 18,30,33         | 0.1632, 0.5410, 0.3384 | – | – | 154,144                       | 0.0479      |
| Case-5 | BFOA[15]  | 18,30,33         | 0.1632, 0.5410, 0.3384 | – | – | 154,144                       | 0.0479      |

Table 6. Comparative study for Indian 85-bus distribution system.

| Case   | Technique | Cap bus location | Cap size (MVAr) | DG bus location | DG size (MW) | $P_{\text{loss}}$ total (kW) | $VD$ (p.u.) |
|--------|-----------|------------------|-----------------|-----------------|--------------|-------------------------------|-------------|
| Base case | –         | –                | –               | –               | –            | 311,4151                      | 0.8102      |
| Case-1 | MINLP[4]  | 7,829,58         | 0.3000, 0.7000, 0.9000, 0.5000 | – | – | 159,41                       | –           |
| Case-2 | PSO[7]    |                  | –               | –               | –            | 163,54                       | –           |
| Case-3 | PGS[8]    | 7,858            | 0.2000, 0.1200, 0.9080 | – | – | 174,66                       | –           |
| Case-4 | Analytical[6] | –                | –               | –               | –            | 158,5221                      | 0.2793      |
| Case-5 | SFLA[11]  | –                | –               | –               | –            | 280,14                        | –           |
| Case-6 | GABC[17]  | 46,53,54         | 0.4500, 0.3000, 0.3000 | – | – | 73,24                        | –           |
| Case-7 | Proposed  | 26,48,68         | 0.6936, 0.6936, 0.6899 | – | – | 21,7047                      | 0.0013      |

The proposed multi-objective hybrid WIPSO–GSA technique highly suitable for solving multi-objective optimization problems, thereby resulting in best compromised optimal planning of DG and capacitor in distribution network.

5. Conclusion

The proposed novel multi-objective hybrid WIPSO–GSA algorithm is employed to determine a set of non-dominated Pareto optimal solutions for optimal planning of DG and capacitor in distribution network and the leader selection strategy has been used to identify the best compromised location and sizing of DG and capacitor in distribution network. From the simulation results, it is concluded that the best compromised optimal DG and capacitor planning using the proposed multi-objective hybrid WIPSO–GSA algorithm results in reduction of $P_{\text{loss}}$ and $VD$. Apart from technical benefits, the economic benefits achieved through optimal DG and capacitor installation are also established. From technical and economic benefit results, it is evident that, among different DG and capacitor installation cases, simultaneous installation of DG and capacitor outperforms all other cases owing to reactive power support from capacitor along with active power compensation by DG. A comparative study has been carried out with other existing techniques and the best compromised solution of proposed multi-objective hybrid WIPSO–GSA algorithm dominates those compared techniques thereby making it highly appropriate for solving multi-objective optimization problems involving optimal DG and capacitor planning in distribution network.

Disclosure Statement

No potential conflict of interest was reported by the authors.
Funding
The work of Arulraj Rajendran was supported by University Grants Commission (UGC) India [grant number RGNF-2014-15-SC-TAM-60538].

References
[1] Ackermann T, Andersson G, Soder L. Distributed generation: a definition. Electr Power Syst Res. 2001;57(3):195–204.
[2] El-Khattam W, Salama MMA. Distributed generation technologies, definitions and benefits. Electr Power Syst Res. 2004;71(2):119–128.
[3] Abul’Wafa AR. Optimal capacitor allocation in radial distribution systems for loss reduction: a two stage method. Electr Power Syst Res. 2013;95:168–174.
[4] Nojavan S, Jalali M, Zare K. Optimal allocation of capacitors in radial/mesh distribution systems using mixed integer nonlinear programming approach. Electr Power Syst Res. 2014;107:119–124.
[5] Hung DQ, Mithulananthan N. Multiple distributed generator placement in primary distribution networks for loss reduction. IEEE Trans Indus Electro. 2013;60(4):1700–1708.
[6] Kumawat P, Sarfaraz, Tandon A. An analytical approach for optimal allocation of DG unit in distribution system. IEEE 7th Power India International Conference (PIICON), Bikaner; 2016. p. 1–6.
[7] Prakash K, Sydulu M. Particle swarm optimization based capacitor placement on radial distribution systems. IEEE Power Engineering Society General Meeting, Tampa; 2007. p. 1–5.
[8] Rao RS, Narasimham SVL, Ramalingaraju M. Optimal capacitor placement in radial distribution system using plant growth simulation algorithm. Int J Elect Power Energy Syst. 2011;33:1133–1139.
[9] Sultana S, Roy PK. Optimal capacitor placement in radial distribution systems using teaching learning based optimization. Int J Elect Power Energy Syst. 2014;54:387–398.
[10] Shukla TN, Singh SP, Srinivasarao V, et al. Optimal sizing of distributed generation placed on radial distribution systems. Elect Power Comp Syst. 2010;39(3):260–274.
[11] Yammani C, Siripurapu N, Maheswarapu S, et al. Optimal placement and sizing of the DER in distribution systems using Shuffled Frog Leap algorithm. IEEE Recent Advances in Intelligent Computational Systems, Trivandrum; 2011. p. 62–67.
[12] Abu-Mouti FS, El-Hawy ME. Optimal distributed generation allocation and sizing in distribution systems via artificial bee colony algorithm. IEEE Trans Power Del. 2011;26(4):2090–2101.
[13] Prabha DR, Jayabarathi T, Umamageswari R, et al. Optimal location and sizing of distributed generation unit using intelligent water drop algorithm. Sustain Energy Techno Assess. 2015;11:106–113.
[14] Arulraj R, Kumarappan N. Optimal installation of different DG types in radial distribution system considering load growth. Elect Power Comp Syst. 2017;45(7):739–751.
[15] Imran AM, Kowsalya M. Optimal Distributed Generation and capacitor placement in power distribution networks for power loss minimization. International Conference on Advances in Electrical Engineering (ICAEE), Vellore; 2014. p. 1–6.
[16] Khodabakhshian A, Andishgar MH. Simultaneous placement and sizing of DGs and shunt capacitors in distribution systems by using IMDE algorithm. Int J Elect Power Energy Syst. 2016;82:599–607.
[17] Dixit M, Kundu P, Jariwala HR. Incorporation of distributed generation and shunt capacitor in radial distribution system for techno-economic benefits. Int J Engg Sci Tech. 2017;20(2):482–493.
[18] Tautiva C, Cadena A. Optimal placement of distributed generation on distribution networks. IEEE/PES Transmission and Distribution Conference and Exposition, Bogota; 2008. p. 1–5.
[19] Arulraj R, Kumarappan N, Vigneyesh T. Optimal location and sizing of DG and capacitor in distribution network using weight-improved particle swarm optimization algorithm (WIPSO). International Multi-Conference on Automation, Computing, Communication, Control and Compressed Sensing (iMac4s), Kottayam; 2013. p. 759–764.
[20] Moradi MH, Abedini M. A combination of genetic algorithm and particle swarm optimization for optimal DG location and sizing in distribution systems. Int J Elect Power Energy Syst. 2012;34(1):66–74.
[21] Darfoun MA, El-Hawry ME. Multi-objective optimization approach for optimal distributed generation sizing and placement. Elect Power Comp Syst. 2015;43(7):828–836.
[22] Mohandas N, Balamurugan R, Lakshminarasimman L. Optimal location and sizing of real power DG units to improve the voltage stability in the distribution system using ABC algorithm united with chaos. Int J Elect Power Energy Syst. 2015;66:41–52.
[23] Mohanty B, Tripathy S. A teaching learning based optimization technique for optimal location and size of DG in distribution network. J Elect Syst Info Tech. 2016;3(1):33–44.
[24] Buayai K. Optimal multi-type DGs placement in primary distribution system by NSGA-II. Res J Appl Sci Engg Tech. 2012;4(19):3610–3617.
[25] Zeinalzadeh A, Mohammadi Y, Moradi MH. Optimal multi objective placement and sizing of multiple DGs and shunt capacitor banks simultaneously considering load uncertainty via MOPSO approach. Int J Elect Power Energy Syst. 2015;67:336–349.
[26] Giridhar MS, Sivanagaraju S, Suresh CV, et al. Analyzing the multi objective analytical aspects of distribution systems with multiple multi-type compensators using modified cuckoo search algorithm. Int J Parallel Emerg Distrib Syst. 2017;32(6):549–571.
[27] Selvam K, Kumar DMV, Siripuram R. Distributed generation planning using peer enhanced multi-objective teaching–learning based optimization in distribution networks. J Instu Engineers (India). 2017;98(2):203–211.
[28] Shirmohammadi D,Hong HW, Selmyen A, et al. A compensation-based power flow method for weakly meshed distribution and transmission networks. IEEE Trans Power Syst. 1988;3(2):753–762.
[29] Eiben AE, Schippers CA. On evolutionary exploration and exploitation. Funda Info. 1998;35(1):35–50.
[30] Kennedy J, Eberhart R. Particle swarm optimization. International Conference on Neural Networks (ICNN’95), Perth; 1995. p. 1942–1948.

[31] Mirjalili S, Hashim SZM. A new hybrid PSOGSA algorithm for function optimization. International Conference on Computer and Information Application, Tianjin; 2010. p. 374–377.

[32] Holden N, Freitas AA. A hybrid PSO/ACO algorithm for discovering classification rules in data mining. J Artific Evolu Applic. 2008;1(1):1–11.

[33] Vu PT, Le DL, Vo ND, et al. A novel weight-improved particle swarm optimization algorithm for optimal power flow and economic load dispatch problems. IEEE PES T&D, New Orleans; 2010. p. 1–7.

[34] Rashedi E, Nezamabadi-pour H, Saryazdi S. GSA: a gravitational search algorithm. Info Sci. 2009;179(13):2232–2248.

[35] Singh A, Deep K, Nagar A. A new improved gravitational search algorithm for function optimization using a novel “best-so-far” update mechanism. 2nd International Conference on Soft Computing and Machine Intelligence (ISCMI), Hong Kong; 2015. p. 35–39.

[36] Coello CAC, Pulido GT, Lechuga MS. Handling multiple objectives with particle swarm optimization. IEEE Trans on Evolu Comput. 2004;8(3):256–279.

[37] Horn J, Nafpliotis N, Goldberg DE. A niched Pareto genetic algorithm for multiobjective optimization. IEEE World Congress on Computational Intelligence, Orlando; 1994. p. 82–87.

[38] Aman MM, Jasmon GB, Bakar AHA, et al. A new approach for optimum simultaneous multi-DG distributed generation Units placement and sizing based on maximization of system loadability using HPSO (hybrid particle swarm optimization) algorithm. Energy. 2014;66:202–215.

[39] Das D, Kothari DP, Kalam A. Simple and efficient method for load flow solution of radial distribution networks. Int J Elect Power Energy Syst. 1995;17(5):335–346.

[40] Kansal S, Tyagi B, Kumar V. Cost–benefit analysis for optimal distributed generation placement in distribution systems. Int J Ambient Energy. 2017;38(1):45–54.