BPS WALL SOLUTIONS IN FIVE DIMENSIONAL SUPERGRAVITY *

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We find an exact solution of BPS wall in five-dimensional supergravity using a gravitational deformation of the massive Eguchi-Hanson nonlinear sigma model. The warp factor decreases for both infinities of the extra dimension. Our solution requires no fine-tuning between boundary and bulk cosmological constants, in contrast to the Randall-Sundrum model. Wall solutions are also obtained with warp factors which are flat or increasing in one side by varying a deformation parameter.

1. Introduction

Randall and Sundrum have proposed one of the most interesting models in the brane-world scenario, which exhibits the localization of four-dimensional graviton by a metric containing a warp factor $e^{2U(y)}$ which decreases exponentially for both infinities of the extra dimension $y \to \pm \infty$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = e^{2U(y)}\eta_{mn}dx^m dx^n + dy^2,$$

where $\mu, \nu = 0, \ldots, 4$, $m, n = 0, 1, 3, 4$ and $y \equiv x^2$. A bulk cosmological constant and a boundary cosmological constant had to be introduced and fine-tuned each other.

This scenario uses an orbifold which may be regarded as a delta-function like domain wall. It is desirable to obtain the domain wall as a classical solution in some field theory from a phenomenological point of view. It has been shown that domain wall solutions in gauged supergravity theories in

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five dimensions require hypermultiplets \(^2\) to obtain warp factors decreasing for both infinities \(y \to \pm \infty\) (infra-red (IR) fixed points in AdS/CFT correspondence). The target space of hypermultiplets in five-dimensional supergravity theory must be quaternionic Kähler (QK) manifolds \(^3\). Domain walls in massive QK nonlinear sigma model (NLSM) in supergravity theories have been studied using homogeneous \(^4,5\) and inhomogeneous manifolds \(^6,7\). Warp factor in the latter models connects between IR fixed points, as is desirable phenomenologically. However, these manifolds do not allow a limit of weak gravitational coupling.

The purpose of this paper is to give an exact BPS domain wall solution in five-dimensional supergravity coupled with hypermultiplets (and vector multiplets). Our strategy to construct the model is to deform the NLSM in SUSY theory having domain wall solution to the model with gravity. Massive hyper-Kähler NSLMs without gravity in four dimensions have been constructed in harmonic superspace as well as \(N=1\) superfield formulation \(^8\), and have yielded the domain wall solution for the Eguchi-Hanson (EH) manifold. Inspired by this solution, we deform this model into five-dimensional supergravity model and we consider the BPS domain wall solution. This paper is based on our original paper \(^9\).

2. Bosonic action of our model in 5D Supergravity

To find a gravitational deformation of the NLSM with EH target manifold, we use the off-shell formulation of five-dimensional supergravity \(^10,11\) combined with the quotient method via a vector multiplet without kinetic term and the massive deformation. We start with the system of a Weyl multiplet, three hypermultiplets and two \(U(1)\) vector multiplets. One of the two vector multiplets has no kinetic term and plays the role of a Lagrange multiplier for hypermultiplets to obtain a curved target manifold. The other vector multiplet, which is referred to as \(U(1)_0\) vector multiplet in the following, serves to give mass terms for hypermultiplets. The deformation parameters are the gravitational coupling constant \(\kappa\) and \(\alpha\) being a constant in generator of \(U(1)_0\) gauge symmetry.

After integrating out the auxiliary fields by their on-shell conditions in the off-shell supergravity action, we obtain the bosonic part of the action for our model with two kinds of constraints. One of constraints comes from the gauge fixing of dilatation, and makes target space of hypermultiplets be a non-compact version of quaternionic projective space, \(\frac{Sp(2,1)}{Sp(2) \times Sp(1)}\), combined with the gauge fixing of \(SU(2)_R\) symmetry. The other is required
by the on-shell condition of auxiliary fields of the $U(1)$ vector multiplet without kinetic term, and corresponds to the constraint for the EH target space in the limit of $\kappa \to 0$.

After solving these constraints, bosonic part of the Lagrangian can be described in terms of independent four real scalar fields $(r, \theta, \Psi, \Phi)$ (eight remnant scalars are eliminated) besides the graviton. The scalar potential can be described by $r$ and $\theta$, and it is found that the theory has two discrete local vacua at $(r, \theta) = (0, 0), (0, \pi)$ in small $\kappa$. We can thus expect domain wall solution connecting them. Note that these local vacua become saddle points as $\kappa$ increases. We consider the case of small $\kappa$ in what follows.

3. BPS equation and the solution

Instead of solving Einstein equations directly, we solve BPS equations to obtain a classical solution conserving a half of SUSY. Since we consider bosonic configurations, we need to examine the on-shell SUSY transformation of gravitino and hyperino. The condition to preserve four SUSY is specified by $\gamma^y \varepsilon_i(y) = i\tau_3^{ij}\varepsilon^j(y)$, where $\tau_3$ is one of the Pauli matrix. Substituting this condition and the metric ansatz (1) into the on-shell SUSY transformation of gravitino and hyperino, we obtain BPS equation. Solving the BPS equation for hyperino, the wall solution interpolating between the two vacua $(r, \theta) = (0, 0), (0, \pi)$ is obtained as

$$r = 0, \quad \cos \theta = \tanh \left(\frac{2g_0M^0(y - y_0)}{\Lambda^3} \right), \quad \Phi = \varphi_0,$$

where $g_0$ and $M^0$ are $U(1)_0$ coupling constant and scalar of $U(1)_0$ vector multiplet fixed as $M_0 = \sqrt{3/2\kappa}$, respectively, and $y_0$ and $\varphi_0$ are constants. Here we take the boundary condition $r = 0$ at $y = -\infty$. Using this solution, we obtain the BPS solution of the warp factor and the Killing spinor from SUSY transformation for gravitino as

$$U(y) = -\frac{\kappa^2\Lambda^3}{3(1 - \kappa^2\Lambda^3)} \left[ \ln \left\{ \cosh \left(2g_0M^0(y - y_0)\right) \right\} + 2ag_0M^0(y - y_0) \right], \quad (2)$$

and $\varepsilon^i(y) \equiv e^{-U(y)/2} \tilde{\varepsilon}^i, \quad (\gamma^y \tilde{\varepsilon}^i = i\tau_3^{ij}\tilde{\varepsilon}^j)$. Here $\Lambda$ is a constant having mass dimension one and $\tilde{\varepsilon}^i$ is a constant spinor.

The warp factor $e^{2U(y)}$ of this solution decreases exponentially for both infinities $y \to \pm \infty$ for $|a| < 1$. In this case, BPS wall solutions interpolate two IR fixed points in boundary field theories. The case of $|a| = 1$ becomes the wall solutions interpolating between AdS and flat Minkowski vacua. On the other hand, warp factor increases exponentially either one of the infinities for $|a| > 1$. The wall solutions interpolate one IR and one UV
fixed points. The family of our BPS solutions contains a parameter \(a\) interpolating between these three classes of field theories.

We find that our BPS solution for \(a = 0\) automatically satisfies the fine-tuning condition. By taking \(g_0 M^0 \to \infty\) and \(\Lambda \to 0\) with \(g_0 M^0 \Lambda^3\), \(\kappa\), and \(a\) fixed, we can obtain thin wall limit. Substituting the BPS wall solutions into the Lagrangian and taking the thin wall limit, we obtain wall tension and bulk cosmological constant as

\[
T_w = 4(\kappa g_0 M^0 \Lambda^3)^2, \quad \Lambda_c = -\frac{8\kappa^2 (g_0 M^0 \Lambda^3)^2}{3},
\]

respectively. These satisfy the fine-tuning condition of the Randall-Sundrum model \(\sqrt{-\Lambda_c} = \kappa \sqrt{\frac{6}{g_0 M^0 \Lambda^3}} T_w\). Therefore we have realized the single-wall Randall-Sundrum model as a thin-wall limit of our solution of the coupled scalar-gravity theory, instead of an artificial boundary cosmological constant put at an orbifold point.

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