VACUUM OSCILLATION SOLUTIONS OF THE SOLAR NEUTRINO PROBLEM : A STATUS REPORT

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ABSTRACT

We re-examine vacuum oscillation solutions of the solar neutrino problem taking (a) the results on total rates, electron energy spectrum, and the seasonal variations from the 708 day SuperKamiokande data and (b) those on total rates from the Chlorine and Gallium experiments. Best fit values for the mixing angle and mass splitting are found for oscillations to sequential and sterile neutrinos and the 90\% C.L. allowed regions are determined.
1 Introduction

Within the past year much has taken place in the arena of neutrino physics. The announcement by SuperKamiokande (SK) of the strong evidence in support of a non-zero neutrino mass and oscillation in their atmospheric $\nu$ data \[1\] has catalyzed explorations of the subject from many different angles. The far-reaching impact of a neutrino mass on physics beyond the Standard Model and in astrophysics and cosmology \[2\] is being vigorously examined. Additional details of the neutrino mass and the determination of the complete mass spectrum are therefore awaited with interest.

The results on the arrival rates of solar neutrinos \[3\] from the Chlorine, Kamiokande and Gallium experiments were themselves strong indications of a non-zero neutrino mass. Recent high statistics results on solar neutrino rates, the scattered electron spectrum, and the seasonal variation of the rates from SuperKamiokande have added a new dimension to this effort \[4\]. In this work we consider the vacuum oscillation scenario in the light of this body of data from several directions \[4\]. We utilize the latest 708 day SK data \[6\] and for the neutrino fluxes use the BP98 solar model \[7\] which incorporates the INT normalisation. We consider the two alternatives of oscillation of the $\nu_e$ to a sequential ($\nu_\mu$ or $\nu_\tau$) or a sterile neutrino.

2 Oscillation Probability

In this work we restrict ourselves to the simplest case of mixing between two neutrino flavors. The $\nu_e$ survival probability used by us is

\[
P_{ee}(E_\nu, r, R(t)) = 1 - \sin^2 2\theta \sin^2 \left[ \frac{\pi R(t)}{\lambda} \left( 1 - \frac{r}{R(t)} \right) \right]
\]

where $\theta$ denotes the mixing angle in vacuum and $\lambda$ is the vacuum oscillation wavelength for neutrinos of energy $E_\nu$ given by $4\pi E_\nu/\Delta m^2$, in which $\Delta m^2 = |m_1^2 - m_2^2|$ is the mass

\[4\]Prior to SK, the solar neutrino data have been examined in terms of vacuum oscillation by many authors \[5\].
square splitting. Here \( r \) is the distance of the point of neutrino production from the center of the sun and \( R(t) \) is the sun-earth distance given by,

\[
R(t) = R_0 \left[ 1 - \epsilon \cos(2\pi t/T) \right]
\]

Here, \( R_0 = 1.49 \times 10^{13} \) cm is the mean Sun-Earth distance and \( \epsilon = 0.0167 \) is the ellipticity of the earth’s orbit. \( t \) is the time of the year at which the solar neutrino flux is measured and \( T \) is 1 year.

### 3 Observed rates and neutrino oscillations

After the declaration of the SK results, analysis of the total rates of all the experiments in terms of vacuum oscillation has been considered in \[8\] using the 504 day data. In this section we update this analysis by using the data collected over 708 days \[8\]. The data that we use in our analysis for the total rates are given in Table 1. For the Ga experiments we take the weighted average of the SAGE and Gallex results. Because SK has better statistics we do not include the Kamiokande results. We fit the total rates from the three experiments using a standard \( \chi^2 \)-fitting procedure \[9\].

The definition of \( \chi^2 \) used by us is,

\[
\chi^2 = \sum_{i,j=1,3} \left( F_{ti}^{th} - F_{ti}^{exp} \right) \left( \sigma_{ij}^{-2} \right) \left( F_{tj}^{th} - F_{tj}^{exp} \right)
\]

Here \( F_{ti}^{\alpha} = \frac{F_{i}}{T_{i}^{\alpha}} \) where \( \alpha \) is \( th \) (for the theoretical prediction) or \( exp \) (for the experimental value) and \( T_i \) is the total rate in the \( i \)th experiment. \( F_{i}^{exp} \) is taken from Table 1. The error matrix \( \sigma_{ij} \) contains the experimental errors, the theoretical errors and their correlations. For evaluating the error matrix we use the procedure described in \[10\]. In the presence of neutrino conversions, the detection rate on earth for the radiochemical experiments \( ^{37}Cl \) and \( ^{71}Ga \) is predicted to be:

\[
(T)^{th}_i = \sum_k \int_{E_{th}} \phi_k(E_{\nu}) \sigma(E_{\nu}) < P_{ee}(E_{\nu}, r, R(t)) > dE_{\nu}
\]

where \( \sigma(E_{\nu}) \) is the cross-section for neutrino capture \[11\] and \( \phi_k(E_{\nu}) \) is the neutrino spectrum for the \( k \)th source \[11\]. The sum is over all the individual neutrino sources.
\( \langle P_{ee}(E_\nu, r, R(t)) \rangle \) is the average neutrino survival probability over one year for the time averaged total rates and further where a weighted sum over the production point in the sun has been carried out for each source. The theoretical prediction according to the BP98 solar model, \( T_i^{BP98} \), is obtained by setting the survival probability as 1 in the above. For SK, in the case of oscillation to sequential neutrinos one has to take into account the possible contributions from the \( \nu_\mu \) or \( \nu_\tau \) channel,

\[
(T)_i^{th} = \int_{E_{A_{\text{th}}}}^{\infty} dE_A \int_0^{E_T} dE_T \rho(E_A, E_T) \int_{E_{\nu_{\min}}}^{E_{\nu_{\max}}} dE_\nu \phi_t(E_\nu) \\
\left[ \langle P_{ee}(E_\nu, r, R(t)) \rangle \frac{d\sigma_{\nu_e}}{dE_T} + \langle P_{\mu}(E_\nu, r, R(t)) \rangle \frac{d\sigma_{\nu_\mu}}{dE_T} \right]
\]

The second term in the bracket is absent if oscillation to sterile neutrinos is under consideration. \( E_T \) and \( E_A \) denote the true and apparent electron energies respectively. \( \rho(E_A, E_T) \) is the energy resolution function for which we use the expression given in [12]. \( E_{A_{\text{th}}} \) is 6.5 MeV for the calculation of the total rate at SK. \( \frac{d\sigma}{dE_T} \) is the differential cross-section for the production of an electron with true relativistic energy \( E_T \) and can be calculated from standard electroweak theory. Below we summarize our results for the total rates. For the sequential neutrino case the best-fit values of \( \Delta m^2 \), \( \sin^2(2\theta) \) and \( \chi^2_{\text{min}} \) obtained are

- \( \Delta m^2 = 9.5 \times 10^{-11} \text{ eV}^2 \), \( \sin^2(2\theta) = 0.87 \), \( \chi^2_{\text{min}} = 0.52 \)

For 1 degree of freedom (3 experimental data points – 2 parameters) this solution is allowed at 47.08% C.L. The best-fit point that we find is slightly different from that in [8]. We attribute this to the fact that in the above analysis we have not included the detector efficiencies. Incorporating these [13], we get

- \( \Delta m^2 = 7.8 \times 10^{-11} \text{ eV}^2 \), \( \sin^2(2\theta) = 0.74 \), \( \chi^2_{\text{min}} = 2.54 \)

These values are in agreement with [8] and with the 708 day data the quality of the fit is actually better. In fig. 1 we show the 90% C.L. \( (\chi^2 \leq \chi^2_{\text{min}} + 4.61) \) contours for the vacuum oscillation solution. Apart from the global minimum there are several local minima as shown in the figure. The best fit for the sterile neutrino alternative gives \( \chi^2 = 5.74 \), which, for 1 degree of freedom, is ruled out at 98.3% C.L.
4 Observed spectrum and neutrino oscillations

In addition to the total rates, SK has provided the number of events in 17 electron recoil energy bins of width 0.5 MeV in the range 5.5 MeV to 14 MeV and an 18th bin which covers the events in the range 14 to 20 MeV \[3\]. In this section we analyze the spectral data in the light of neutrino oscillations in vacuum. The definition of \(\chi^2\) used by us is,

\[
\chi^2 = \sum_{i,j=1,18} \left( X_n R_i^{th} - R_i^{exp} \right) \sigma_{ij}^{-1} \left( X_n R_j^{th} - R_j^{exp} \right)
\]  

(6)

where \(X_n\) allows for an arbitrary normalisation of the \(^{8}B\) flux with respect to the SSM prediction and \(R_i^\alpha = S_i^\alpha / S_i^{BP98}\) with \(\alpha\) being \(th\) or \(exp\) as before and \(S_i\) standing for the number of neutrinos in the \(i\)th energy bin. The theoretical prediction is given by eq. \[3\] but the integration over the apparent (i.e., measured) energy will now be over each bin. Following SK we include the statistical error, the uncorrelated systematic errors and the energy-bin-correlated experimental errors as well as those from the calculation of the expected spectrum \[14\]. Thus the error matrix \(\sigma_{ij}\) used by us is

\[
\sigma_{ij}^2 = \delta_{ij}(\sigma_{i,stat}^2 + \sigma_{i,uncorr}^2) + \sigma_{i,exp}\sigma_{j,exp} + \sigma_{i,cal}\sigma_{j,cal}
\]  

(7)

Our results are given below:

(a) Oscillation to a sequential neutrino

The best-fit values of parameters and \(\chi^2_{min}\) are

- \(\Delta m^2 = 4.15 \times 10^{-10}\text{eV}^2\), \(\sin^2(2\theta) = 0.89\), \(X_n = 0.75\), \(\chi^2_{min} = 13.07\).

This solution is allowed at 59.68\% C.L. The allowed values of the parameters \(\Delta m^2\) and \(\sin^2(2\theta)\) at 90\% C.L. \((\chi^2 \leq \chi^2_{min} + 6.25)\) are shown in fig. 2a. The best-fit values of \(\Delta m^2\) and \(\sin^2(2\theta)\) that we get are in agreement with those obtained in \[3\] for vacuum oscillations.

(b) Oscillation to a sterile neutrino

For this case we obtain,

- \(\Delta m^2 = 4.16 \times 10^{-10}\text{eV}^2\), \(\sin^2(2\theta) = 0.77\), \(X_n = 0.76\), \(\chi^2_{min} = 12.9\)
This corresponds to a goodness of fit of 61%. Thus, as far as the electron recoil spectrum data is considered the sterile neutrino alternative gives a marginally better fit. In fig. 2b we show the allowed values of parameters for this case at 90% C.L.

5 Combined fits to rates and spectrum

In this section we present the results of the combined fit of the total rates and the spectrum data treating the rates and the electron spectrum data as independent. Thus our definition of \( \chi^2 \) is,

\[
\chi^2 = \sum_{i,j=1,3} (F_{th}^{i} - F_{exp}^{i}) \sigma_{ij}^{-2} (F_{th}^{j} - F_{exp}^{j}) + \sum_{i,j=1,18} (X_n R_{th}^{i} - R_{exp}^{i}) \sigma_{ij}^{-2} (X_n R_{th}^{j} - R_{exp}^{j})
\]

where the first term on the r.h.s is from eq. (3) and the second from eq. (6). We allow the normalisation of the \(^8\)B flux to vary as a free parameter. The \( \chi^2_{min} \) and the best-fit values we obtain are:

(a) Sequential neutrino case

- \( \Delta m^2 = 9.47 \times 10^{-11} \text{ eV}^2 \), \( \sin^2(2\vartheta) = 0.80 \), \( X_n = 0.71 \), \( \chi^2_{min} = 18.66 \)

For 18 (= 21 – 3) degrees of freedom this solution is allowed at 41.3% C.L.. In fig. 3a we show the 90% C.L. allowed regions for the combined analysis of rate and spectrum.

(b) Sterile neutrino case

- \( \Delta m^2 = 8.86 \times 10^{-11} \text{ eV}^2 \), \( \sin^2(2\vartheta) = 0.88 \), \( X_n = 0.82 \), \( \chi^2_{min} = 18.54 \)

\(^5\)In the analysis of the spectrum data we have not included the efficiencies as they are available in [3] as functions of true energy for particular values of the threshold apparent energy. Here, we perform bin by bin integration over the apparent energy and there are different thresholds for each bin for which the appropriate efficiencies are unavailable. We do not have any reason to believe that the inclusion of the efficiencies will change the best-fit values dramatically – see e.g. the fits in Sec. 3 – though the quality of the fit may be affected.
The goodness of fit in this case is 42.06%. In fig. 3b we show the 90% C.L. allowed region for oscillations to a sterile neutrino from the combined analysis of rates and spectrum. We find that only two of the six regions allowed for the sequential case remain for the sterile alternative while for the others either the $\chi^2$ is too high or they merge with these two.

6 Seasonal variation and neutrino oscillations

In [6] the SK collaboration has also presented the preliminary seasonal data which shows a variation apart from that expected from the $1/R^2(t)$ dependence of the neutrino fluxes. If this is confirmed then it can help to distinguish between the MSW [15] and the vacuum oscillation alternatives. To analyze the seasonal data we define our $\chi^2$ as

$$\chi^2 = \sum_{i,j=1,8} \frac{(N_{i}^{\text{exp}} - N_{i}^{\text{theor}})^2}{\sigma_i^2}$$

(9)

$N_i^{\text{th}}$ is as given by eq. (5) but now the integration over the apparent energy is from 11.5 MeV. The time averaged probability for this case is

$$\langle P_{ee}(E_\nu, r, R(t)) \rangle = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} P_{ee}(E_\nu, r, R(t)) dt$$

(10)

where $t_2$ and $t_1$ are determined by the eight bins provided by SK. For each of these bins, the SK collaboration has presented the ratio ($N_i^{\text{exp}}/N_i^{\text{BP98}}$) [6]. The best-fit values for oscillation to sequential neutrinos obtained in this case are

- $\Delta m^2 = 4.26 \times 10^{-10} \text{ eV}^2$, $\sin^2(2\vartheta) = 1.0$, $X_n = 1.12$, $\chi^2_{\text{min}} = 3.96$

The goodness of fit is 55.52%. We find that from the seasonal data almost all of the parameter space in the $\Delta m^2 - \sin^2(2\vartheta)$ plane is allowed at 90% C.L.. In fig. 4 we plot $\chi^2_{\text{min}}$ against one of the three parameters keeping the other two unconstrained to illustrate this point. From fig. 4 it is seen that all values of $\Delta m^2$ in the range $10^{-11} - 10^{-9}$ eV$^2$ and $\sin^2(2\vartheta)$ from very small values to 1.0 are allowed at 90% C.L.
$X_n$ below 0.5 and above 3.5 are not allowed at 90% C.L.. Thus, the preliminary seasonal variation data does not put any strong constraint on the parameters. For oscillation to sterile neutrinos the best-fit values are

- $\Delta m^2 = 4.09 \times 10^{-10} \text{ eV}^2$, $\sin^2(2\vartheta) = 1.0$, $X_n = 0.99$, $\chi^2_{\text{min}} = 3.88$

For 5 (8 – 3) degrees of freedom this solution is allowed at 56.68%.

7 Conclusions

In this paper a detailed $\chi^2$-analysis of the 708 day SK solar neutrino data is performed assuming vacuum oscillation between two neutrino flavors. This includes

- A $\chi^2$-analysis using the total rates from the $^{37}\text{Cl}$, $^{71}\text{Ga}$ and Superkamiokande experiments. We take into account the experimental and the theoretical errors including the correlations among the various theory errors.

- A $\chi^2$-analysis using the SK spectrum data including the uncorrelated as well as the correlated errors among various bins as given by the SK collaboration \[4\]. We float the normalisation of the $^8\text{B}$ flux as a free-parameter and determine its best-fit value.

- A global $\chi^2$-analysis of the combined rates and spectrum data treating them as independent. For this case also we have allowed the $^8\text{B}$ flux to vary as a free-parameter.

- $\chi^2$-analysis of the preliminary seasonal data allowing the $^8\text{B}$ flux normalisation to vary. Here we consider only the experimental statistical errors.

We find that the simple two-generation vacuum oscillation scenario can well explain the data on total rates and the spectrum but somewhat different best-fit values of $\Delta m^2$ are found in the two cases. If one does a global analysis of the rates and the spectrum then the goodness of fit is poorer. This is because of the fact that the best-fit value
of $\Delta m^2$ for the spectrum data is one order of magnitude higher as compared to the rates value. The sterile neutrino alternative gives a bad fit to the data on total rates but for the SK spectrum data as well as for the combined rate and spectrum data it gives a marginally better fit than the active neutrino scenario. The sterile neutrino also gives a slightly better fit for the preliminary seasonal variation data. Thus we make an important observation that if only the SK spectrum or the seasonal data are considered then the sterile neutrino actually gives a slightly better fit than the sequential neutrino case. It is when the rates in the radiochemical experiments are included that the sterile neutrino fit becomes worse.

Two-generation vacuum oscillation analysis of the 708 days of SK data has also been performed in [16]. However their fitting procedure is somewhat different from ours. We take into account the total SK rate as well as the spectrum data. We also include the theory errors and their correlations in the analysis of the total rates. For the SK spectrum data we have taken the bin by bin correlations into account. Since they have not included the SK rates we cannot compare our results for the total rates with theirs. If we compare our results of only the spectral and seasonal data with their analysis, the best-fit values are more or less in agreement though our $\chi^2_{min}$ are somewhat lower. Because the seasonal data does not put strong constraints on the parameters, this preliminary data have not been included in our global fit. We have not included the data from the measurement of day-night solar neutrino flux [17] as the vacuum oscillation hypothesis does not generate any day-night asymmetry.

In order to explain the energy spectrum observed at SK, in the literature the hep flux has sometimes been allowed to vary apart from the $^8B$ flux [18] and it was found that a hep flux almost 20-30 times larger than the SSM prediction can well explain the high energy part of the spectrum data. In this work we have not considered this possibility. Whether one requires such a higher hep flux than the SSM prediction can be tested in SK. It was concluded in [16] that even if one allows the hep flux to vary, the allowed parameters for the vacuum oscillation hypothesis will not change much.
For our global analysis of the rates and the spectrum we have treated these two data sets as independent. Since the $^8B$ neutrino flux enters the rates as well as the spectrum data there can be some possible correlations among these. In a future study we plan to examine whether the inclusion of these alters the conclusions by a significant amount.

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Table 1: The ratio of the observed solar neutrino rates to the corresponding BP98 SSM predictions used in this analysis. The results are from Refs. [3] and [6]. For Gallium, the weighted average of the SAGE and Gallex results has been used.

| Experiment       | Chlorine     | Gallium     | SuperKamiokande |
|------------------|--------------|-------------|-----------------|
| Observed Rate    | 0.33 ± 0.029 | 0.57 ± 0.054 | 0.471 ± 0.015   |
| BP98 Prediction  |              |             |                 |

Figure Captions

Fig 1. The 90% C.L. allowed region in the $\Delta m^2 - \sin^2(2\theta)$ plane from the analysis of total rates assuming vacuum oscillations to sequential neutrinos. The best-fit point is also indicated.

Fig 2. The 90% C.L. allowed region in the $\Delta m^2 - \sin^2(2\theta)$ plane from the SK recoil electron spectrum data for sequential neutrinos (a) and for sterile neutrinos (b). The best-fit points are also indicated.

Fig. 3. The 90% C.L. allowed region in the $\Delta m^2 - \sin^2(2\theta)$ plane from a global analysis of the rates and the spectrum data for sequential neutrinos (a) and for sterile neutrinos (b). The best-fit points are also indicated.

Fig. 4. Variation of $\chi^2_{\text{min}}$ (solid line) (a) with $\Delta m^2$ keeping $\sin^2(2\theta)$ and $X_n$ unconstrained; (b) with $\sin^2(2\theta)$ keeping $\Delta m^2$ and $X_n$ unconstrained; and (c) with $X_n$ keeping $\sin^2 2\theta$ and $\Delta m^2$ unconstrained. Also shown are the 90% C.L (big-dashed line), 95% C.L. (small-dashed line) and 99% C.L. (dotted-line) limits.
Fig. 2
Fig. 3
Fig. 4