Study of Balance Equations for Hot-Electron Transport in an Arbitrary Energy Band (III)

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Abstract

By choosing an electron gas resting instead of drifting in the laboratory coordinate system as the initial state, the first order perturbation calculation of the previous paper (Phys. Stat. Sol. (b) 198, 785(1996)) is revised and extended to include the high order field corrections in the expression for the frictional forces and the energy transfer rates. The final expressions are formally the same as those in first order in the electric field, but the distribution functions of electrons appearing in them are defined by different expressions. The problems relative to the distribution function are discussed in detail and a new closed expression for the distribution function is obtained. The nonlinear impurity-limited resistance of a strong degenerate electron gas is computed numerically. The result calculated by using the new expression for the distribution function is quite different from that using the displaced Fermi function when the electric field is sufficiently high.

Subject classification: 72.10 and 72.20
I. INTRODUCTION

In the previous paper [1], we reformulate the balance equation theory of Lei and Ting [2] in \textit{ab initio} manner. In this study, we neither distinguish the degree of freedom of the center of mass from the relative degree of freedom of the electrons nor adopt the effective Hamiltonian. All our calculations are carried out in the laboratory coordinate system and based on the original Hamiltonian (\textit{i.e.} (12) to (21) of [1]). We show that the \textit{S}-matrix is, in fact, also a functional of $H_E$, the electrostatic potential of the electrons. This feature is missing in the Lei-Ting formulation. It implies that the expressions of the frictional forces and the energy transfer rate all depend explicitly on the electric field. In the second paper [3], this electric field dependence is studied perturbatively. To first order in the electric field, the results are given by (45), (65), (68) and (69) of [3].

By choosing a new initial state: both the electron gas and the phonon gas resting in the laboratory coordinate system, we revise and extend in the present paper the first order perturbation calculation given in paper [3] to high order in the electric field. If without a specific statement, we use in this paper the same notations as those in [1] and [3].

II. FRICTIONAL FORCES AND ENERGY TRANSFER RATES: REVISION AND EXTENSION

We recall that in the papers [1] and [3], we follow the current hot-electron transport theory [2] to assume that at time $t = -\infty$ the system is composed of two independent equilibrium subsystems: a drifting electron gas with temperature $T_e$ and a phonon gas with temperature $T$. The corresponding initial density matrix is

\[
\hat{\rho}_0 = \hat{\rho}_e\hat{\rho}_{ph}
\]

with $\hat{\rho}_e = \frac{1}{\Xi_e}e^{-(\hat{H}_e - \hat{\chi}_p - \mu N)/T_e}$ and $\hat{\rho}_{ph} = \frac{1}{Z_{ph}}e^{-\hat{H}_{ph}/T}$. It is pointed out in the Appendix that in order to keep the consistency of the theory, one should abandon such initial state from beginning and adopt instead the initial state: both the electron gas and the phonon gas
resting in the laboratory coordinate system. And so the initial density matrix (1) should be replaced by
\[ \hat{\rho}^0 = \hat{\rho}_e \hat{\rho}_{ph} \] (2)
with \( \hat{\rho}_e^0 = \frac{1}{\Xi_0} e^{-(\hat{H}_e - \mu N)/T_e} \). Keeping this modification in mind, we revise the formulae of [3], and calculate further the high order Feynman diagrams.

In steady state, the balance equations are given by (1) to (5) of [3]. As said above, the density matrix \( \hat{\rho}_0 \) in these equations should be replaced by the new one \( \hat{\rho}^0 \). Taking (67) of [3] into account, we rewrite these equations as follows

\[ eN\mathbf{E} + \mathbf{f}_{ei} + \mathbf{f}_{ep} = 0, \] (3)
\[ eN\mathbf{E} \cdot \mathbf{v}_d + W_{ei} + W_{ep} = 0, \] (4)

where

\[ f_{ei} = -\frac{i}{V} \sum_{kq} v(q) \rho_I(q)(p_{k+q,k+q} - p_{k,k}) \left\langle S^+(t)c_{k+q,\sigma}^\dagger(t)c_{k,\sigma}(t)S(t) \right\rangle, \] (5)
\[ f_{ep} = -\frac{i}{\sqrt{V}} \sum_{kq\sigma} M_{q\lambda}(p_{k+q,k+q} - p_{k,k}) \left\langle S^+(t)\varphi_{q\lambda}(t)c_{k+q,\sigma}^\dagger(t)c_{k,\sigma}(t)S(t) \right\rangle, \] (6)
\[ W_{ei} = -\frac{i}{V} \sum_{kq} v(q) \rho_I(q)(E(k + q) - E(k)) \left\langle S^+(t)c_{k+q,\sigma}^\dagger(t)c_{k,\sigma}(t)S(t) \right\rangle, \] (7)
\[ W_{ep} = -\frac{i}{\sqrt{V}} \sum_{kq\lambda} M_{q\lambda}(E(k + q) - E(k)) \left\langle S^+(t)\varphi_{q\lambda}(t)c_{k+q,\sigma}^\dagger(t)c_{k,\sigma}(t)S(t) \right\rangle. \] (8)

Here \( \langle \cdots \rangle \) denotes the statistical average over the density matrix \( \hat{\rho}^0 \), \( i.e. \)
\[ \langle \cdots \rangle = \text{tr}(\cdots \hat{\rho}^0). \] (9)

In [4], \( \mathbf{v}_d \) only denotes the drift velocity in the final steady state.

As shown in [3], the frictional forces (5) and (6), and the energy transfer rates (7) and (8) can be expressed in terms of the closed time path Green function \( G(kt,k't') \). \( G \) is defined by (17) to (20) of [3] with \( \langle \cdots \rangle \) redefined as (9). For example, the frictional force due to impurity scattering, \( f_{ei} \), is given by (22) of [3], \( i.e. \)
\[ f_{ei} = -\frac{1}{V} \sum_{kk'} (p_{kk'} - p_{kk}) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \text{tr} \left( \hat{L}G'_{kk'}(\omega) \right). \] (10)

The Green function \( G' \) is calculated by the Keldysh diagrammatic technique. The Feynman diagrams of order \( E^0 \) and \( E^1 \) are shown in Fig. 1 of [3]. The solid line \( \rightarrow \rightarrow \rightarrow \rightarrow \) represents the electron propagator

\[ G^0(k\omega) = \begin{pmatrix} G^R_0(k\omega) & G^K_0(k\omega) \\ 0 & G^A_0(k\omega) \end{pmatrix} \] (11)

where

\[ G^R_0(k\omega) = \frac{1}{\omega - E(k) + i\frac{\tau}{2\tau(k)}}; \] (12)

\[ G^A_0(k\omega) = \frac{1}{\omega - E(k) - i\frac{\tau}{2\tau(k)}}; \] (13)

\[ G^K_0(k\omega) = (G^R_0(k\omega) - G^A_0(k\omega))(1 - 2f^0_k). \] (14)

They are just (24) to (27) of [3], except that \( f_k \) in (27) of [3] is replaced by the Fermi function

\[ f^0_k = \frac{1}{e^{(E(k) - \mu)/T_e} + 1}. \] (15)

It is easy to see that this revision is the direct consequence of replacing the initial density matrix \( \hat{\rho}_0 \) by the new one \( \hat{\rho}' \). Repeating the calculation expounded in [3], the revised version of (28), (40) and (41) of [3] are easily obtained. For convenience of latter discussion, we rewrite them in the form

\[ f_{ei}^{(0)} = \frac{2\pi}{V} \sum_{kk'} n_i |v(k' - k)|^2 (p_{kk'} - p_{kk}) L^{(0)}(k, k') \delta(E(k) - E(k')) , \] (16)

\[ f_{ei}^{(1)} = \frac{2\pi}{V} \sum_{kk'} n_i |v(k' - k)|^2 (p_{kk'} - p_{kk}) L^{(1)}(k, k') \delta(E(k) - E(k')) + \]

\[ + \frac{2\pi}{V} \sum_{kk'} n_i |v(k' - k)|^2 (p_{kk'} - p_{kk}) L^{(0)}(k, k') C(k, k') \delta'(E(k) - E(k')) , \] (17)

in which

\[ L^{(n)}(k, k') = (-e\tau(k)\mathbf{E} \cdot \mathbf{\nabla}_k)^n f^0_k - (-e\tau(k')\mathbf{E} \cdot \mathbf{\nabla}_{k'})^n f^0_{k'}, \] (18)

\[ C(k, k') = -e\mathbf{E} \cdot \langle k | \mathbf{r} | k \rangle + e\mathbf{E} \cdot \langle k' | \mathbf{r} | k' \rangle. \] (19)
The Feynman diagrams of order $E^2$ are given in Fig. 1. Their contributions to the frictional force $f_{ei}$ can be written down by the rules expounded in \[1\] and \[3\]. We obtain

$$f_{ei}^{(2)} = -\frac{1}{V} \sum_{kk'}(p_{kk'} - p_{kk})n_i|v(k' - k)|^2 \int \frac{d\omega}{2\pi} \left\{ \lim_{k_1 \to k_{k'}} \left[ (-eE \cdot \langle k|k_1 \rangle) (-eE \cdot \langle k'|k_1 \rangle) \right] \left[ \hat{L} G^0(k_1 \omega) G^0(k \omega) \right] + \lim_{(k_2,k_1)_k} \left[ (-eE \cdot \langle k|k_2 \rangle) (-eE \cdot \langle k_2|k_1 \rangle) \right] \left[ \hat{L} G^0(k_2 \omega) G^0(k \omega) \right] + \lim_{(k'_1,k_2)_k'} \left[ (-eE \cdot \langle k'|k_2 \rangle) (-eE \cdot \langle k_2|k' \rangle) \right] \left[ \hat{L} G^0(k_2 \omega) G^0(k' \omega) \right] \right\}. \quad (20)$$

The symbol $(k_1,k_2)k_3$ means that $k_1$ and $k_2$ not only approach each other but also approach to $k_3$. It is reminded that the highly singular matrix element $\langle k_1|k_2 \rangle$ should be handled carefully by the trick expounded in \[1\] and \[3\]. After a lengthy but straightforward calculation, we finally obtain

$$f_{ei}^{(2)} = \frac{2\pi}{V} \sum_{kk'}(p_{kk'} - p_{kk})n_i|v(k' - k)|^2 L^{(2)}(k,k')\delta(E(k) - E(k')) + \frac{2\pi}{V} \sum_{kk'}(p_{kk'} - p_{kk})n_i|v(k' - k)|^2 L^{(1)}(k,k')C(k,k')\delta'(E(k) - E(k')) . \quad (21)$$

The same method can be used to calculate any perturbation term. The result is

$$f_{ei}^{(n)} = \frac{2\pi}{V} \sum_{kk'}(p_{kk'} - p_{kk})n_i|v(k' - k)|^2 \left( \sum_{j=0}^{n} \frac{1}{j!} L^{(n-j)}(k,k') [C(k,k')]^j \delta^{(j)}(E(k) - E(k')) \right) . \quad (22)$$

$\delta^{(j)}$ denotes the $j$-th derivative of the delta function. Then the frictional force due to impurity scattering is $\sum_{n=0}^{\infty} f_{ei}^{(n)}$. It can be expressed in the following compact and transparent form:

$$f_{ei} = \frac{2\pi}{V} \sum_{kk'} n_i|v(k' - k)|^2 (p_{kk'} - p_{kk})(\tilde{f}_k - \tilde{f}_{k'})\delta(\tilde{E}(k') - \tilde{E}(k)) . \quad (23)$$

Here,

$$\tilde{E}(k) = E(k) - eE \cdot \langle k|k \rangle , \quad (24)$$

$$\tilde{f}_k = f_0 + \sum_{n=1}^{\infty} (-m v(k) \cdot \nabla_k)^n f_0^n . \quad (25)$$
where \( v(k) = \frac{1}{m} e \mathbf{E} \tau(k) \).

In the same way, the expression for the frictional force \( f_{ep} \) and the energy transfer rates \( W_{ei} \) and \( W_{ep} \) derived in [3] can be revised and extended to take the contribution of the high order Feynman diagram into account. The final results are

\[
\begin{align*}
    f_{ep} & = \frac{4\pi}{V} \sum_{kk'q\lambda} |M_{q\lambda}|^2 (p_{k'} - p_k) \\
           & \times \left[ \tilde{f}_k(1 - \tilde{f}_{k'}) n_{q\lambda} - \tilde{f}_{k'}(1 - \tilde{f}_k)(1 + n_{q\lambda}) \right] \delta(E(k') - E(k) - \Omega_{q\lambda}) , \\
    W_{ei} & = \frac{2\pi}{V} \sum_{kk'} n_i |v(k - k')|^2 (E(k') - E(k)) (\tilde{f}_k - \tilde{f}_{k'}) \delta(\tilde{E}(k') - \tilde{E}(k)) , \\
    W_{ep} & = \frac{4\pi}{V} \sum_{kk'q\lambda} |M_{q\lambda}|^2 (E(k') - E(k)) \\
           & \times \left[ \tilde{f}_k(1 - \tilde{f}_{k'}) n_{q\lambda} - \tilde{f}_{k'}(1 - \tilde{f}_k)(1 + n_{q\lambda}) \right] \delta(\tilde{E}(k') - \tilde{E}(k) - \Omega_{q\lambda}) .
\end{align*}
\]

Comparing the above expressions for \( f_{ei}, f_{ep}, W_{ei} \) and \( W_{ep} \) with the corresponding ones of [3], we find that (23), (26) to (28) are formally the same as those in first order in the electric field but the distribution functions appearing in them are defined by different expressions. We further note that for the special parabolic energy band, the distinction between \( \tilde{E}(k) \) and \( E(k) \) can be neglected for \( \tilde{E}(k') - \tilde{E}(k) = E(k') - E(k) \). As a result, the energy transfer rate due to impurity scattering, \( W_{ei} \), is equal to zero and (23), (26) to (28) are identical to those of Lei and Ting [4] if the concrete expression for the distribution function is not concerned.

**III. DISTRIBUTION FUNCTION: CONSISTENCY AND PARAMETERIZATION**

In the final steady state, the distribution function of the electron gas in a uniform electric field is given by (25).

We first notice the fact that this expression is an unsatisfactory one and requires improving. As an example, we consider the following problem. In weak electric field, we retain only the first two terms of (25), i.e.
\[ \tilde{f}_k = f_k^0 - e\tau(k)E \cdot \nabla_k f_k^0 . \]  

(29)

For simplicity, we consider only impurity scattering and the case where electrons are governed by parabolic energy dispersion. The balance equations of present paper (BEPP), i.e. (3), (4), (23) and (26) to (28), are reduced to \( T = T_e \) [4] and

\[ eNE = -f_{ei} . \]  

(30)

Using (29), one calculates \( f_{ei} \) from (23) and obtains

\[ -f_{ei} = eNE \left< \frac{\tau(k)}{\tau_{tr}(k)} \right> , \]  

(31)

where \( \left< A(k) \right> = \frac{4}{3\pi V} \sum_k \left( -\frac{\partial f_k^0}{\partial E(k)} \right) E(k)A(k) \) and \( \tau_{tr}(k) \) is the transport relaxation time [4]

\[ \frac{1}{\tau_{tr}(k)} = \frac{n_i m}{4\pi k^3} \int_0^{2k} |v(q)|^2 q^3 dq . \]  

(32)

Obviously, both sides of (31) are not equal. It means that expression (23) is inconsistent with BEPP. The origin of this inconsistency is due to the incorrectness of ignoring the renormalization of the electric field vertex in deriving these equations. Now, let the bare electric field vertex (\( \circ \)) in Fig. 1b and c of [3] be replaced by the renormalized one (\( \bullet \)) shown in Fig. 2. We can show that to first order in electric field, BEPP is unchanged, but (29) is modified by replacing \( \tau(k) \) by a function \( \Lambda_k \), i.e.

\[ \tilde{f}_k = f_k^0 - e\Lambda_k E \cdot \nabla_k f_k^0 . \]  

(33)

For the electron-impurity scattering, \( \Lambda_k \) is just \( \tau_{tr}(k) \) [5,6]. Then, the consistency between the expression of \( \tilde{f}_k \) and BEPP is recovered. Inspired by this success, we replace the bare electric field vertex in each of the Feynman diagram for \( G' \) by the renormalized one. We have the same balance equations but a different expression for \( \tilde{f}_k \) which is given by

\[ \tilde{f}_k = f_k^0 + \sum_{n=1}^{\infty} (-e\Lambda_k E \cdot \nabla_k)^n f_k^0 . \]  

(34)

\( \Lambda_k \) is defined by an integral equation [5,6]. We do not follow this approach here due to the complexity of the mathematics involved.
We instead follow the approach close to the current hot-electron transport theory \[2,4\] to replace \(v(k)\) by a \(k\)-independent one, \(v\), and therefore write (25) in the form

\[
\tilde{f}_k = f_k^0 + \sum_{n=1}^{\infty} (-m v \cdot \nabla_k)^n f_k^0 .
\] (35)

The point of this alternate approach is to consider \(v\) as an unknown parameter and determine it in the following fashion: combining (35) with BEPP and the equations

\[
2 \sum_k \tilde{f}_k = N ,
\] (36)
\[
2 \sum_k p_{kk} \tilde{f}_k = N m v_d ,
\] (37)

and solving them self-consistently to get \(v\), together with \(T_e, \mu\) and \(v_d\) for given \(N, T\) and \(E\). The consistency between the expression for \(\tilde{f}_k\) and BEPP is obviously automatically satisfied. It implies that if \(v\) is determined in this way, (35) is identical with (34) within the approximation to neglect the \(k\)-dependent of \(\Lambda_k\). So, this alternative approach not only keep the merit of simplicity in mathematics but also take automatically the renormalization of the electric field vertex.

We rewrite the series solution (35) compactly in the form

\[
\tilde{f}_k = \frac{1}{1 + m v \cdot \nabla_k} f_k^0 .
\] (38)

Multiplying both side by the operator \(1 + m v \cdot \nabla_k\), we get

\[
(1 + m v \cdot \nabla_k) \tilde{f}_k = f_k^0 .
\] (39)

Then we obtain the following analytical expression for \(\tilde{f}_k\) through solving the differential equation (39):

\[
\tilde{f}_k = \int_0^{\infty} d\xi e^{-\xi} f_k(\xi) .
\] (40)

It shows that \(\tilde{f}_k\) is equal to a weight average of \(f_k(\xi)\) which is defined as

\[
f_k(\xi) = \frac{1}{\exp[(E_k(\xi) - \mu)/T_e] + 1} ,
\] (41)
with

\[ E_k(\xi) = E(k - m v \xi) . \] (42)

The parameter \( v \) connects with \( v_d \), the drift velocity of the electron gas in the final steady state, by (37). One can prove from (37) that \( v \) is just \( v_d \) only for the special parabolic energy band.

IV. CONCLUDING REMARKS

It is remarkable that (39) is nothing but a simplified form of the Boltzmann equation. In fact, in the relaxation time approximation, the Boltzmann equation of an electron gas in the presence of a uniform electric field is

\[ \frac{\partial \tilde{f}_k}{\partial t} + eE \cdot \nabla_k \tilde{f}_k = -\frac{\tilde{f}_k - f^0_k}{\tau_{tr}} . \] (43)

\( \tau_{tr} \) is the approximate \( k \)-independent relaxation time. In steady state, \( \frac{\partial}{\partial t} \tilde{f}_k = 0 \). We rewrite (43) in the form

\[ \tilde{f}_k + e\tau_{tr}E \cdot \nabla_k \tilde{f}_k = f^0_k . \] (44)

It is just (39) with \( v = eE\tau_{tr}/m \). Thus (40) is a homogeneous steady solution of the Boltzmann equation in constant relaxation time approximation. We recall that in the approach of balance equation, the quantity \( v \) in (40) is considered as an unknown parameter and determined by solving the complete set of the equations: (40) with BEPP, (36) and (37). This trick of parameterization not only provides a method to choose a proper value of the relaxation time for given temperature \( T \) and electric field \( E \), but also ensure (40) consistent with BEPP.

We expand \( f_k(\xi) \) in the series

\[ f_k(\xi) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\partial^n f_k(\xi)}{\partial \xi^n} \bigg|_{\xi=1} (\xi - 1)^n . \] (45)

Substituting it into (40) and carrying out the \( \xi \)-integration, we have
\[
\tilde{f}_k = f_k(1) + \frac{1}{2} \left. \frac{\partial^2 f_k(\xi)}{\partial \xi^2} \right|_{\xi=1} + \frac{1}{3} \left. \frac{\partial^3 f_k(\xi)}{\partial \xi^3} \right|_{\xi=1} + \cdots ,
\]
(46)
in which \(f_k(1)\) is just the displaced Fermi function used by the previous authors [2,4], i.e.
\[
f^d_k = \frac{1}{e^{(E_k(k - mv) - \mu)/T_e} + 1} .
\]
(47)

In view of (41) and (42), we rewrite (46) in the form
\[
\tilde{f}_k - f^d_k = \frac{1}{2} \left[ \left( mv \cdot \nabla E_k(1) \right)^2 \frac{\partial^2 f_k(1)}{\partial E_k(1)^2} + \left( (mv \cdot \nabla k)^2 E_k(1) \right) \frac{\partial f_k(1)}{\partial E_k(1)} \right] + O(v^3) .
\]
(48)
v is the magnitude of \(v\). It is clear that to the accuracy of first order in the electric field, \(\tilde{f}_k\) coincides with \(f^d_k\). Their difference is given by the right hand side of (48) which belongs to the terms of high order in the electric field. It hints that the displaced Fermi function \(f^d_k\) and hence the balance equation theory of Lei and Ting [2] is applicable only when the electric field is so low that the contribution from the terms on the right hand side of (48) are small and can be neglected. As an example, we calculate the nonlinear impurity-limited resistance of an electron gas using either the displaced Fermi function (47) or the new one (40). This problem has been studied by Lei and Ting [7] based on the momentum balance equation and the distribution function (47). We repeat their calculation in the strong degenerate limit and the results are plotted as \(R_i/R_{i0}\) versus \(v/v_F\) curves in Fig. 3. The curve \(a\) (\(b\)) is calculated by using (10) (\(17\)). \(R_{i0}\) is the resistance \(R_i\) at \(v = 0\). \(v_F\) is the Fermi velocity. For \(N/V = 1 \times 10^{16}\)cm\(^{-3}\) and \(m = 0.1m_e\), \(v_F = 7.7 \times 10^4\)m/s. The impurity scattering is assumed due to charged impurities with bare potential \(v(q) \propto q^{-2}\). The Coulomb interaction between carriers is weakened due to the large permittivity \(\epsilon\) of the lattices and takes the form \(\frac{e^2}{\epsilon_0 \epsilon q^2}\) [7]. \(\epsilon = 10\). The two curves shown in Fig. 3 coincide approximately only when \(v < 0.2v_F\). But they are quite different when \(v > 0.2v_F\). Choosing \(\tau \sim 1\) ps, the electric field \(E\) correspond to \(v \sim 0.2v_F\) is estimated to be 90V/cm.

It has been shown by Marchetti and Cai [8] that the momentum and energy balance equations of Lei and Ting can be derived from the conventional Boltzmann equation (if we do not concern of the concrete expressions for the distribution function). The same argument
is applicable to BEPP if the shift of the band energy $E(k)$ in the electric field is taken into account in the collision term of the Boltzmann equation. In view of the achievement of the Boltzmann equation in the condensed matter physics, we believe that the approach based on BEPP and would give better prediction for the hot-electron transport in the higher electric field. The detail of the numerical results will be reported in a separate paper.

It is pointed out in that our theory is applicable only when the electric field should not be too higher so that the Wannier levels and the tunneling between the bands can be neglected. In such lower electric field, the intracollision field effect can also be neglected.

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1) For example, the Wannier level is observed only when the electric field is not lower than $10^5$ V/cm for $\tau \sim 1$ps.
In this appendix, we show that there is an inconsistency in the theory presented in [3].

The statistical average of the total momentum of electron gas $\hat{P} = \sum_{k\sigma} p_{kk}^c c_{k\sigma}^+$ is given by

$$\overline{P}(t) = \sum_{k\sigma} p_{kk}^c \langle \tilde{c}_{k\sigma}^c(t) \tilde{c}_{k\sigma}(t) \rangle.$$  \hspace{1cm} (A1)

Here $\langle \cdots \rangle = \text{tr}(\cdots \hat{\rho}_0)$. By the method of [3], we express (A1) in terms of the closed time path Green function as follows:

$$\overline{P}(t) = -i \lim_{t' \to t+0^+} \sum_{k\sigma} p_{kk}^c \text{tr} \left( \hat{L} G(kt, kt') \right).$$  \hspace{1cm} (A2)

Here, $G$ is defined by (17) to (20) of [3]. In steady state, $G(kt, kt')$ depends only on $k$ and $t - t'$ and so $\overline{P}(t)$ is independent on time. Performing Fourier transformation of $G$ with respect to $t - t'$, we obtain

$$\overline{P} = \sum_{k\sigma} p_{kk} \int \frac{d\omega}{2\pi i} e^{i\omega t'0^+} \text{tr} \left( \hat{L} G(k, \omega) \right).$$  \hspace{1cm} (A3)

Calculating $G$ to first order in the electric field [3], we have

$$\overline{P} = \sum_{k\sigma} p_{kk} \int \frac{d\omega}{2\pi i} e^{i\omega t'0^+} \text{tr} \left( \hat{L} G^0(k, \omega) \right) +$$

$$+ \sum_{k\sigma} p_{kk} \int \frac{d\omega}{2\pi i} \text{tr} \left( \hat{L} \lim_{k' \to k} \left[ G^0(k, \omega) U(k, k') G^0(k', \omega) \right] \right) + O(E^2),$$  \hspace{1cm} (A4)

where $G^0$ is defined by (24) to (27) of [3]. One can easily prove the result

$$\overline{P} = N m v_d + \sum_{k\sigma} p_{kk} (-e\tau(k)E) \cdot \nabla_k f_k + O(E^2),$$  \hspace{1cm} (A5)

in which $f_k$ is given by (12) of [3]. The self-consistent condition requires that the drift velocity of the electron gas in the initial state, $v_d$, should be equal to that in the final steady state, $\frac{1}{N m} \overline{P}$. We see from (A5) with disappointment that this self-consistent condition does not hold. It implies that there are some improper things involved in the current balance equation theory.
Examining the theory carefully, we find the inconsistency results from us choosing the drifting electron gas as the initial state. In order to keep the consistency of the theory, we should abandon such initial state and adopt instead the electron gas resting in the laboratory coordinate system as the initial state. In connection with this, the self-consistent condition to identify the drift velocity of the initial state with that in the final state is no longer required. Then the first term on the right hand side of (A5) is equal to zero, and (A5) is nothing but the equation defined the drift velocity of the electron gas in the final steady state, \( \frac{1}{Nm} \mathbf{D} \). The said inconsistency no longer appears.
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FIGURES

Fig.1 Feynman diagrams for $G'$, each with two bare electric field vertices.

Fig.2 (a) Feynman diagrams for $G'$, each with one renormalized electric field vertex.

(b) Renormalized electric field vertex in the ladder approximation.

Fig.3 $R_i/R_{i0}$ versus $v/v_F$ curves.

Curve $a$, using (40); curve $b$, using (47).
Fig. 1
Fig. 2
Fig. 3