Comparison of optical potential for nucleons and $\Delta$ resonances

In electron scattering from nuclear targets

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Abstract. Precise modeling of neutrino interactions on nuclear targets is essential for neutrino oscillations experiments. The modeling of the energy of final state particles in quasielastic (QE) scattering and resonance production on bound nucleons requires knowledge of both the removal energy of the initial state bound nucleon as well as the Coulomb and nuclear optical potentials for final state leptons and hadrons. We extract the values of the nuclear optical potential for final state nucleons ($U_{opt}^{\Delta}$) from inclusive electron scattering data on nuclear targets in the QE region and compare to theoretical calculations by Cooper et.al. We also extract for the first time values of the nuclear optical potential for a $\Delta(1232)$ resonance in the final state ($U_{opt}^{\Delta(1232)}$). We find that $U_{opt}^{\Delta(1232)}$ is more negative than $U_{opt}^{\Delta}$.

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1 Introduction

Precise modeling of neutrino interactions on nuclear targets is essential for neutrino oscillations experiments. The modeling of the energy of final state particles in quasielastic (QE) scattering and resonance production on bound nucleons requires knowledge of both the removal energy of the initial state bound nucleon as well as the Coulomb and nuclear optical potentials for final state leptons and hadrons. In this communication we extract the values of the nuclear optical potential for final state nucleons ($U_{opt}^{\Delta}$) from inclusive electron scattering data on nuclear targets in the QE region and compare to theoretical calculations by Cooper et.al. We also extract for the first time values of the nuclear optical potential for a $\Delta(1232)$ resonance in the final state ($U_{opt}^{\Delta(1232)}$). We find that $U_{opt}^{\Delta(1232)}$ is more negative than $U_{opt}^{\Delta}$.

First we summarize some of the results of our previous publication on removal energies and the nuclear optical potential for final state nucleons extracted from inclusive quasielastic (QE) electron scattering on a variety of nuclei. The analysis was done within the framework of the impulse approximation.

The diagrams on the top two panels of Fig. 1 show the 1p1h process (one final state proton and one hole) for electron QE scattering from an off-shell bound proton (left) and neutron (right). The diagrams on the bottom two panels show antineutrino (\bar{\nu}) QE scattering from an off-shell bound proton producing a final state neutron (left), and neutrino (\nu) scattering from an off-shell bound neutron producing a final state proton (right). The electrons scatter from an off-shell nucleon of momentum $p_i = k$ bound in a nucleus of mass $A$. For electrons of incident energy $E_0$ and final state energy $E'$, the energy transfer to the target is $\nu = E_0 - E'$. The square of the 4-momentum transfer ($Q^2$), and 3-momentum transfer ($q_3$) to a proton bound in the nucleus are:

\begin{equation}
Q^2 = 4(E_0 + |V_{eff}|)(E_0 - \nu + |V_{eff}|)\sin^2\frac{\theta}{2} \tag{1}
\end{equation}

\begin{equation}
q_3^2 = Q^2 + \nu^2
\end{equation}

We include the effects of the interaction of initial and final state electrons with the Coulomb field of the nucleus by using published values of the average Coulomb energy at the interaction vertex $V_{eff}$ extracted from a comparison of electron and positron inclusive QE differential cross sections [2].

For electron scattering from protons, the Coulomb energies at the interaction vertex for the final state proton (in QE scattering), final state $\Delta^+1232$ (in resonance production), and final state of mass $W^+$ (in inelastic scattering) are defined below.

\begin{equation}
|V_{eff}^p| = |V_{eff}^{\Delta^+1232}| = |V_{eff}^{W^+}| = \frac{Z-1}{Z}|V_{eff}|
\end{equation}

For electron scattering from a neutron target we set $|V_{eff}^n| = 0$. The values of $|V_{eff}|$ that we use for various nuclei are given in Table 1.
Electron scattering on proton

\[ E = (E_0, p = E_0) \quad E' = (E_0 - \nu, p' = E') \]

\[ p_{\text{cts}} = p + |V_{\text{eff}}| \]

\[ E_{\text{cts}} = E_0 \]

\[ q = (\nu, q_3) \]

\[ E_i = (M_P - e^P, k) \]

\[ E_{P}\] for proton

\[ E_{N}\] for neutron

Unobserved Removal Energy

\[ \epsilon = S^P + \langle E_{P}^N \rangle + \frac{k^2}{2M_A} \]

Electron scattering on neutron

\[ E = (E_0, p = E_0) \quad E' = (E_0 - \nu, p' = E') \]

\[ p_{\text{cts}} = p + |V_{\text{eff}}| \]

\[ E_{\text{cts}} = E_0 \]

\[ q = (\nu, q_3) \]

\[ E_i = (M_N - e^N, k) \]

\[ E_{P}\] for proton

\[ E_{N}\] for neutron

Unobserved Removal Energy

\[ \epsilon = S^N + \langle E_{P}^N \rangle + \frac{k^2}{2M_A} \]

Fig. 1. The diagrams on the top two panels show electron QE scattering from an off-shell bound proton (left) and neutron (right). The diagrams on the bottom two panels show \( \bar{\nu} \) QE scattering from an off-shell bound proton producing a final state neutron (left), and \( \nu \) scattering from an off-shell bound neutron producing a final state proton (right).

2 Removal energy of initial state nucleons in a nucleus

In our analysis we use the impulse approximation. The nucleon is moving in the mean field (MF) of all the other nucleons in the nucleus. The on-shell recoil excited \([A - 1]^*\) spectator nucleus has a momentum \( p_{(A-1)^*} = -k \) and a mean excitation energy \( \langle E_{P}^{P,N} \rangle \). The off-shell energy of the interacting nucleon is

\[ E_i = M_A - \sqrt{(M_A - 1)^2 + k^2} \]

\[ = M_A - \sqrt{(M_A - 1 + E_{P}^{P,N})^2 + k^2} \]

\[ = M_{P,N} - \epsilon_{P,N} \]

\[ \epsilon_{P,N} = S^P + \langle E_{P}^{P,N} \rangle + \frac{k^2}{2M_{A-1}} \]

Here, \( M_P = 0.938272 \) GeV is the mass of the proton, \( M_N = 0.939565 \) is the mass of the proton, and \( S^P \) the separation energy (obtained from mass differences of the initial and final state nuclei) needed to separate the nucleon from the nucleus. In Ref.\[1\] we extract the mean excitation energy \( \langle E_{P}^{P,N} \rangle \) (or equivalently the removal energy \( \epsilon_{P,N} \)) using exclusive ee'P spectral function measurements. Some of the neutrino MC generators (e.g. current version of GENIE) do not include the effect of the excitation of the spectator nucleus, nor do they include the effects of the interaction of the final state nucleons and hadrons with the Coulomb\[2\] and nuclear optical potential of the nucleus.

3 Nuclear optical potential for final state nucleons in QE scattering \( U_{\text{opt}}^{QE} \)

We model the effect of the interaction of final state nucleons with the nuclear optical potential of the nucleus with a parameter \( U_{\text{opt}}^{QE} (p_{f3}) \), where \( p_{f3} \) is the square of
Fig. 2. Examples of fits for three out of 33 $^{12}$C QE differential cross sections. The solid black curve is the RFG fit with the best value of $U_{opt}^{QE}$ for the final state nucleon. The blue dashed curve is the simple parabolic fit used to estimate the systematic error. The red dashed curve is the RFG model with $U_{opt}^{QE} = 0$ and $|V_p| = 0$.

Fig. 3. Extracted values of $U_{opt}^{QE}$ for the final state nucleon in QE scattering (small black markers) for 33 $^{12}$C and four $^{16}$O inclusive electron scattering spectra. Also shown are theory prediction for $U_{opt}^{QE}$ calculated by Artur. M. Ankowski[27,28] and Jose Manuel Udias[29] using the theoretical formalisms of Cooper 1993[30], and Cooper 2009[31]. The dashed grey lines are linear fits to the QE data. In addition, the larger markers are the values of $U_{opt}^{\Delta}$ for the final state $\Delta$(1232) (large markers) extracted from a subset of the data (15 $^{12}$C spectra) for which the measurements extend to higher invariant mass. Here, the solid grey lines are linear fits to the $U_{opt}^{\Delta}$ values. The top and bottom panels show the measurements versus $p_{f3}^2 = (k + q_3)^2$, and versus hadron kinetic energy $T$, respectively.
the 3-momentum of the final state nucleon at the vertex. Alternatively, we also extract \( U_{opt}^{QE}(P^F) \) where \( T \) is the kinetic energy of the final state nucleon. In the analysis we make the assumption that \( U_{opt}^{QE} \) for the proton and neutron are the same.

The energy of the final state nucleon in QE electron scattering is given by the final expressions:

\[ \nu + (M_{P,N} - E_{P,N}^{opt}) = E_{f}^{P,N} \]
\[ p_{f3} = (k + q_{3}) \]
\[ E_{f}^{P,N} = \sqrt{p_{f3}^2 + M_{P,N}^2} + U_{opt}^{QE}(p_{f3}^2) + |V_{eff}| \]
\[ T_{P,N} = E_{P,N} - M_{P,N} \]

We extract \( U_{opt}^{QE}(p_{f3}^2) \) and \( U_{opt}^{QE}(T) \) from a comparison of the relativistic Fermi gas (RFG) model to measurements of inclusive QE e-A differential cross sections [3].

The data samples (see references [4]-[21]) include the following elements which are of interest to current neutrino experiments: 33 \(^{12}\)C spectra, five \(^{16}\)O spectra, seven \(^{29}\)\(^{40}\)Ca spectra, and two \(^{32}\)\(^{18}\)Ar spectra. In addition, the data sample includes four \(^{27}\)Al spectra, 30 \(^{56}\)Fe and \(^{23}\)\(^{29}\)\(^{88}\)Ar spectra, and one \(^{79}\)\(^{197}\)Au spectrum. Most of the QE differential cross sections are available on the QE electron scattering archive [3].

Figure 2 shows examples of three of the 33 fits to QE differential cross sections for \(^{12}\)C. The solid black curve is the RFG fit with the best value of \( U_{opt}^{QE} \) for the final state nucleon. The blue dashed curve is a simple parabolic fit used to estimate the systematic error. The red dashed curve is the RFG model with \( U_{opt}^{QE} \) and \( V_{eff} \) set to zero.

In the extraction of the nuclear optical potential for final state nucleons in QE scattering we only fit to the data in the top 1/3 of the QE distribution and extract the best value of \( U_{opt}^{QE}(p_{f3}^2) \) and \( U_{opt}^{QE}(T) \). Here \( p_{f3} \) is evaluated at the peak of the QE distribution. In the fit we let the normalization of the QE cross section float to agree with the measurements of inclusive QE e-A differential cross sections. The energy of the final state nucleon in QE electron scattering is given by the final expressions:

\[ \nu + (M_{P,N} - E_{P,N}^{opt}) = E_{f}^{P,N} \]
\[ p_{f3} = (k + q_{3}) \]
\[ E_{f}^{P,N} = \sqrt{p_{f3}^2 + M_{P,N}^2} + U_{opt}^{QE}(p_{f3}^2) + |V_{eff}| \]
\[ T_{P,N} = E_{P,N} - M_{P,N} \]

The measurements of \( U_{opt}^{QE} \) for \(^{32}\)Fe and \(^{92}\)Mo are in good agreement with the Cooper 1993 and Cooper 2009 calculations. The measurements are more negative than the theory calculations for \(^{6}\)^{13}C/\(^{16}\)O, \(^{27}\)Al, and \(^{29}\)\(^{88}\)Ar, and the measurements are less negative than the theory calculations for \(^{239}\)\(^{208}\)Pb/\(^{197}\)Au. For the \(^{12}\)C nucleus, although both theory calculations of \( U_{opt}^{QE} \) are above the data, the Cooper 1993 calculations are closer to the data than the Cooper 2009 calculations.

4 Nuclear optical potential for a \( \Delta \) resonance in the final state \( U_{opt}^{\Delta} \)

For electron scattering from a bound nucleon the optical potentials for QE electron scattering and \( \Delta \) resonance production, are given below.

\[ \nu + (M_{P,N} - E_{P,N}^{opt}) = E_{f}^{P,N} \]
\[ p_{f3} = (k + q_{3}) \]
\[ E_{f}^{P} = \sqrt{p_{f3}^2 + M_{P,N}^2} + U_{opt}^{QE}(p_{f3}^2) + |V_{eff}| \]
\[ E_{f}^{N} = \sqrt{(k + q_{3})^2 + M_{N}^2} + U_{opt}^{QE} \]
\[ E_{f}^{\Delta} = \sqrt{(k + q_{3})^2 + M_{\Delta} + U_{opt}^{\Delta}} \]

where, \( M_{\Delta} = 1.232 \) GeV is the mass of the \( \Delta \) resonance and \( |V_{eff}| = |V_{eff}|^{2} \). In order to extract the nuclear optical potential for a \( \Delta \) resonance we need to model the cross section between the QE peak and the \( \Delta \) resonance. We use the effective spectral function [23] (which includes a 2p2h contribution) to model the region of the QE peak. In the calculation of the inelastic cross section for the production resonances and the continuum we use Jiab fits [23] to the structure functions for protons and neutrons in the resonance region and continuum. These structure functions were extracted from inclusive electron scattering cross sections on hydrogen and deuterium. The proton and neutron structure functions are combined with the relativistic Fermi gas (RFG) to model the resonance production from nuclei.
Electron scattering on proton

\[
E = (E_0, p = E_0) \quad E' = (E_0 - \nu, p' = E')
\]

\[
electron
\begin{align*}
\nu_{\text{ct}} &= p + |V_{\text{eff}}| \\
E_{\text{ct}} &= E_0 \\
q &= (\nu, q_0)
\end{align*}
\]

\[
P_A
\]

Unobserved energy \( \epsilon_{P,N} = \langle E_P \rangle + \frac{k^2}{2M^*} \)

Proton

\[
E_i = (M_P - \epsilon_P, \mathbf{k})
\]

\[
E_{\text{vtx}} = E_i = T_i^\Delta + M_\Delta,
\]

\[
E'_{\text{vtx}} = E_i = T_i^\Delta + M_\Delta,
\]

\[
E' = (E_0 - \nu, p' = E')
\]

Electron scattering on neutron

\[
E = (E_0, p = E_0) \quad E' = (E_0 - \nu, p' = E')
\]

\[
electron
\begin{align*}
\nu_{\text{ct}} &= p + |V_{\text{eff}}| \\
E_{\text{ct}} &= E_0 \\
q &= (\nu, q_0)
\end{align*}
\]

\[
P_A
\]

Unobserved energy \( \epsilon_{P,N} = \langle E_P \rangle + \frac{k^2}{2M^*} \)

Neutron

\[
E_i = (M_N - \epsilon_P, \mathbf{k})
\]

\[
E_{\text{vtx}} = E_i = T_i^\Delta + M_\Delta,
\]

\[
E'_{\text{vtx}} = E_i = T_i^\Delta + M_\Delta,
\]

\[
E' = (E_0 - \nu, p' = E')
\]

Neutrino Scattering on Neutron

\[
E = (E_0, p = E_0) \quad E^\mu = (E_0 - \nu_\mu, p^\mu = E^\mu)
\]

\[
\begin{align*}
\nu_{\text{ct}} &= p + |V_{\text{eff}}| \\
E_{\text{ct}} &= E_0 \\
q &= (\nu_\mu, q_0)
\end{align*}
\]

\[
P_A
\]

Unobserved Removal Energy \( \epsilon^N = \langle E^N \rangle + \frac{k^2}{2M^*} \)

Neutron

\[
E_i = (M_N - \epsilon^N, \mathbf{k})
\]

\[
E^\Delta_{\text{vtx}} = E_i = T_i^\Delta + M_\Delta,
\]

\[
E^\Delta_{\text{vtx}} = E_i = T_i^\Delta + M_\Delta,
\]

\[
E^\mu = (E_0 - q_0, p^\mu = E^\mu)
\]

Antineutrino Scattering on Neutron

\[
E = (E_0, p = E_0) \quad E^\nu = (E_0 - \bar{\nu}_\mu, p^\nu = E^\nu)
\]

\[
\begin{align*}
\bar{\nu}_{\text{ct}} &= p + |V_{\text{eff}}| \\
E_{\text{ct}} &= E_0 \\
q &= (\bar{\nu}_\mu, q_0)
\end{align*}
\]

\[
P_A
\]

Unobserved Removal Energy \( \epsilon^N = \langle E^N \rangle + \frac{k^2}{2M^*} \)

Neutron

\[
E_i = (M_N - \epsilon^N, \mathbf{k})
\]

\[
E^\Delta_{\text{vtx}} = E_i = T_i^\Delta + M_\Delta,
\]

\[
E^\Delta_{\text{vtx}} = E_i = T_i^\Delta + M_\Delta,
\]

\[
E^\nu = (E_0 - q_0, p^\nu = E^\nu)
\]

\[\text{Fig. 4.}\] The top two panels show diagrams for electron scattering from an off-shell bound proton producing a \( \Delta^+ \) (left), and scattering from an off shell bound neutron producing a \( \Delta^0 \) (right). The bottom two panels show neutrino scattering from a bound neutron producing a \( \Delta^+ \) (left) and antineutrino scattering on a bound neutron producing a \( \Delta^- \) (right).

\[\text{Fig. 5.}\] Examples of fits for three out of 15 \^12C (1232) production differential cross sections. Here the QE peak is modeled with an effective spectral function (including 2p2h), and \( \Delta \) production is modeled by using RFG to smear fits to \( \Delta \) production structure functions on free nucleons. The solid black curve is the fit with the best value of \( U_{\text{opt}}^\Delta \). The dashed red curve is the prediction with \( U_{\text{opt}}^\Delta = V_{\text{eff}}^\Delta = 0 \).
Table 1. The second column shows values of $|V_{eff}|$ (MeV) for various nuclei. The third column shows the removal energies for protons and neutrons (MeV). The fourth and fifth columns show the intercepts (GeV) at $p^2 f_3 = 0$ and slopes (GeV/GeV$^2$) of linear fits to $U_{opt}^{QE}$ and $U_{opt}^\Delta$ versus $p^2 f_3$. The sixth and seventh columns show the results of a similar analysis versus the final state kinetic energy $T$. The overall systematic error on $U_{opt}^{QE}$ is estimated at ±0.005 GeV. We show the slopes and intercepts for $U_{opt}^{QE}$ and $U_{opt}^\Delta$ on alternate rows. (*The removal energies are (24.1, 27.0) for $^{16}_8 O$ and (30.9, 32.3) for $^{40}_20 Ca$.)
We use a subset of the measured electron scattering cross sections on nuclei that includes measurements of both QE and resonance production. To extract values of the nuclear optical potential for a $\Delta$ (1232) resonance in the final state ($U_{\Delta}^{\text{opt}}$) we compare the data to predictions of the sum of QE and resonance production cross sections. In the fits the normalizations of the QE cross section, resonance cross sections and $U_{\Delta}^{\text{opt}}$ are varied to fit the data. Examples of fits for three out of 15 $\Delta$ (1232) production differential cross sections on $^{12}$C are shown in Fig. 5. The solid black curve is the fit with the best value of $U_{\Delta}^{\text{opt}}$. The dashed red curve is the same fit with $U_{\Delta}^{\text{opt}}$ and $|V_{\text{eff}}|$ set to zero. The extracted values of $U_{\Delta}^{\text{opt}}$ versus $p_f^2$ from $^{12}$C are shown in the top panel of Figure 3. The same values as a function of the $\Delta$ kinetic energy $T_\Delta$ are shown on the bottom panel. The extracted values of $U_{\Delta}^{\text{opt}}(p_f^2)$ versus $p_f^2$ (and $T_\Delta$) from $^{40}$Ca spectra and one $^{40}$Ar spectrum are shown in the top and bottom panels of Fig. 6.

Similarly values extracted of $U_{\Delta}^{\text{opt}}$ versus $p_f^2$ and $T_\Delta$ from two $^6$Li spectra, and three $^{27}$Al spectra are shown in the top two (and bottom two) panels of Fig. 7. As seen in the figures, the values of $U_{\Delta}^{\text{opt}}$ are more negative than the values of $U_{\text{QE}}^{\text{opt}}$. The Values of $U_{\Delta}^{\text{opt}}$ versus $p_f^2$ and $T_\Delta$ shown in Fig. 3 are fit to linear functions which are shown as as solid grey lines. The intercepts at $p_f^2 = 0$ and the slopes of the fits to $U_{\Delta}^{\text{opt}}$ versus $p_f^2$ as well as the intercepts and slopes of the fits to $U_{\text{QE}}^{\text{opt}}$ as a function of $T_\Delta$ are also given in Table 1.
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$^{56}$Fe Fit for $U^\Delta_{\text{opt}}$

$^{208}$Pb Fit for $U^\Delta_{\text{opt}}$

Fig. 8. Same as Fig. 3 for $^{56}$Fe (top two panels) and $^{208}$Pb/$^{197}$Au (bottom two panels).

5 Conclusion

We report on the extraction (from electron scattering data) of the nuclear optical potential for both nucleons and $\Delta$ (1232) resonances in the final state. This is the first measurement of the optical potential for the $\Delta$ (1232) resonance. The result indicate that:

1. The measurements of $U_{\text{opt}}^{QE}$ for $^3$Li and $^{56}$Fe are in good agreement with the Cooper 1993[30] and Cooper 2009[31] calculations. The measurements are more negative than the theory calculations for $^{12}$C/$^{16}$O, $^{27}$Al, and $^{40}$Ca/$^{40}$Ar, and the measurements are less negative than the theory calculations for $^{208}$Pb/$^{197}$Au. For the $^{12}$C nucleus, both theory calculations of $U_{\text{opt}}^{QE}$ are above the data, the Cooper 1993[30] calculations are closer to the data than the Cooper 2009[31] calculations.

2. We find that the optical potential for a $\Delta$ resonance in the final state $U^\Delta_{\text{opt}}$ is more negative than the optical potential for a final state nucleon $U^{QE}_{\text{opt}}$. There are no theory predictions available for $U^\Delta_{\text{opt}}$.

3. Using the measurements of these four parameters $\epsilon^{P,N}$, $U_{\text{opt}}^{QE}$, $U^\Delta_{\text{opt}}$, and $V_{\text{eff}}$, we can model the energy of electrons, nucleons and $\Delta$ (1232) resonance in the final state. For neutrino oscillations experiments these measurements can reduce the systematic uncertainty in the reconstruction of the neutrino energy (originating from uncertainties in the removal energy and nuclear optical potentials) from $\pm 20$ MeV[32] to $\pm 5$ MeV.

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