On a possibility to determine the S–factor of the hep process in experiments with thermal (cold) neutrons

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Abstract

The problem of the magnitude of the S–factor of the hep process $p + ^{3}\text{He} \rightarrow ^{4}\text{He} + e^{+} + \nu_{e}$ is one of the most important unsolved problems in nuclear astrophysics. The magnitude of $S(\text{hep})$ has also an important impact on the interpretation of the Super-Kamiokande and future SNO solar neutrino results in terms of neutrino oscillations. We point out the possibility to determine the major hadronic contribution to $S(\text{hep})$ from the measurement of the total cross section of the process $n + ^{3}\text{H} \rightarrow ^{4}\text{He} + e^{-} + \bar{\nu}_{e}$ in experiments with high intensity beams of thermal or cold neutrons.

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Type\textsuperscript{e}X using REV\textsuperscript{e}X
The reaction

\[ p + ^3\text{He} \rightarrow ^4\text{He} + e^+ + \nu_e \]  

(hep reaction) is the source of solar neutrinos of the highest energies (up to 18.8 MeV). According to the Solar Standard Model (SSM) [1], hep neutrinos constitute only a negligible fraction of the solar neutrino flux: the total flux of the hep neutrinos, predicted by the SSM, is equal to \(2.1 \times 10^3\) \(\nu/\text{cm}^2\text{s}\) whereas the total SSM flux of the main high energy \(^8\text{B}\) neutrinos with energy up to 15 MeV is equal to \(5.15 \times 10^6\) \(\nu/\text{cm}^2\text{s}\) [2].

The recent interest in hep neutrinos (see Refs. [3–5]) was triggered by the results of the Super-Kamiokande experiment [6–8] in which the spectrum of the recoil electrons in the solar neutrino-induced elastic neutrino – electron scattering was measured. The spectrum measured by Super-Kamiokande collaboration agrees with the predicted one in the whole energy region \((\geq 5.5\) MeV) with the exception of the region of highest energies, in which a possible excess of events is observed.

The accuracy of the present data does not allow to make definite conclusions on the distortion of the recoil electron spectrum in the high energy region (see Ref. [8]). However, the existing data can be considered as an indication in favour of such a distortion. It was first shown by Bahcall and Krastev [3] that, if the high-energy enhancement of the Super-Kamiokande spectrum is due to the contribution of the hep neutrinos, then from the fit of the data it follows that the value of the hep astrophysical S–factor \(S(\text{hep})\) is much larger than the SSM value

\[ S_0(\text{hep}) = 2.3 \times 10^{-20}\ \text{keV b}. \]  

If there is no distortion of the \(^8\text{B}\) spectrum, then from the fit of the latest Super-Kamiokande data it was found [8] that \(S(\text{hep})/S_0(\text{hep}) = 16.7\) (\(\chi^2 = 19.5\) at 16 d.o.f.). In the article of Bahcall et al. [9], the Super-Kamiokande data on the recoil electron spectrum were fitted under the assumption of neutrino oscillations with some representative values of neutrino oscillation parameters \(\sin^2 2\theta\) and \(\Delta m^2\), taken in the corresponding allowed regions. The following ranges for \(S(\text{hep})\) were found: for the LMA MSW solution \(27 < S(\text{hep})/S_0(\text{hep}) < 47\), for the SMA MSW solution \(17 < S(\text{hep})/S_0(\text{hep}) < 19\) and for the VO solution \(0 < S(\text{hep})/S_0(\text{hep}) < 40\). A big progress in the investigation of the problem of the solar hep neutrinos is expected in the near future: the accuracy of the measurement of the electron spectrum in the Super-Kamiokande experiment will be increased [3] and the data of the new solar neutrino experiment SNO [10] will appear. In the SNO experiment the spectrum of solar \(\nu_e\)’s will be determined from the measurement of the electron spectrum in the process \(\nu_e + d \rightarrow e^- + p + p\).

One expects that the cross section of the hep process at the solar energies \((\lesssim 30\) keV) is very small and cannot be directly measured in an experiment. The most detailed calculations of the cross section of the hep process have been done by Carlson et al. [11] and by Schiavilla et al. [12]. According to these calculations, the value of the hep cross section is \(\sigma(\text{hep}) \simeq 6 \times 10^{-51}\) cm\(^2\) at \(E = 20\) keV, where \(E\) is the kinetic energy of the initial particles in the C.M. system. The value of the astrophysical S–factor obtained in Ref. [12] is adopted in the SSM [1].

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The cross section of the hep process calculated in \[12\] is strongly suppressed by two reasons. The first reason is connected with SU(4) symmetry: in the allowed approximation the matrix element of the process vanishes if only the dominant $s$–states of the $^4\text{He}$ and $^3\text{He}$ wave functions are taken into account \[13\]. The second reason lies in the cancellation between the matrix element of the usual one–body hadronic current and the matrix elements of the (two–body) $\pi$ and $\rho$ exchange currents, which also account for the transition of nucleons to intermediate $\Delta$ isobar states.

The problem of the cross section of the hep process is considered as one of the most important problems of nuclear astrophysics \[14\]. There are many uncertainties in the existing (model-dependent and complicated) calculations of this cross section. It is very desirable to find a way to obtain an experimental information on the astrophysical S–factor of the hep process. The aim of this letter is to propose such an experiment by exploiting the fact that the hadronic matrix element of the process $n + ^3\text{H} \rightarrow ^4\text{He} + e^- + \bar{\nu}_e$ is equal to that of the hep process due to isotopic invariance of strong interactions.

The total cross section of a process with charged initial particles at small energies has the form \[15\]

$$\sigma(E) = \frac{1}{E} S(E) e^{-2\pi\eta}.$$  \hspace{1cm} (3)

Here $E$ is the kinetic energy of the initial particles in the C.M. system and

$$\eta = \frac{Z_1 e Z_2 e^2}{v},$$ \hspace{1cm} (4)

where $Z_1 e$ and $Z_2 e$ are the charges of the initial particles and $v = \sqrt{2E/\mu}$ is the relative velocity with the reduced mass $\mu$. The quantity $P = e^{-2\pi\eta}$ in the relation (3) is the Coloumb penetration factor and the S–factor $S(E)$ is determined mostly by the strong interactions. If there are no resonances at small energies the S–factor depends very weakly on $E$ \[14\] \[15\].

One of the reasons of the smallness of the nuclear cross section at small solar energies is the suppression due to the penetration factor. For example, for the hep process at $E = 15$ keV one obtains $P \simeq 8 \times 10^{-7}$ and at $E = 20$ keV, $P \simeq 5 \times 10^{-6}$.

Let us now compare the process

$$n + ^3\text{H} \rightarrow ^4\text{He} + e^- + \bar{\nu}_e,$$ \hspace{1cm} (5)

for which obviously there is no Gamov penetration factor, with the hep process \[14\]. From isotopic SU(2) invariance of the strong interactions it follows that the hadronic parts of the matrix elements of the hep process and of the process (5) are the same. In fact, with $J^{(\pm)}_\alpha \equiv J^{1\pm i2}_\alpha \equiv J^1_\alpha \pm iJ^2_\alpha$ being the “minus” and “plus” components, respectively, of the isovector hadronic V–A current in the Heisenberg representation, we obtain

$$\langle ^4\text{He} | J^{(-)}_\alpha | ^3\text{He} \rangle = \langle ^4\text{He} | \mathcal{U}^{-1} J^{(-)}_\alpha \mathcal{U}^{-1} \mathcal{U} | ^3\text{He} \rangle = -\langle ^4\text{He} | J^{(+)\alpha} | ^3\text{H} \rangle,$$ \hspace{1cm} (6)

where $\mathcal{U} = \exp(i\pi T_2)$ is the operator of rotation by an angle $\pi$ around the second axis in the isotopic space. In $J^{(\pm)}_\alpha$ all strong interactions are taken into account. With the help of
the relation (6) we can connect the cross section of the process (5) with the astrophysical S–factor of the hep process.

In the region of very small energies we are interested in, only the s–wave of the initial particles is relevant and the hadronic matrix element \( \langle ^4\text{He}|\bar{J}(-)|p\,^3\text{He} \rangle \) is constant. For the hep cross section one has to take into account the Coulomb interaction of the initial \( p \) and \( ^3\text{He} \) by the usual Coulomb factor

\[
\frac{|\psi_p^{(+)}(0)|^2}{|\psi_p|^2} = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \simeq 2\pi\eta e^{-2\pi\eta},
\]

where the approximate expression holds for small C.M. kinetic energies \( E \). With Eq.(7) we obtain the cross section

\[
\sigma(\text{hep}) = \frac{(2\pi)^7}{E} G_F^2 e^2 m_e^5 \mu \sum_{\text{spins}} \left| \langle ^4\text{He}|\bar{J}(-)|p\,^3\text{He} \rangle \right|^2 f(-2,\varepsilon_0) e^{-4\pi\varepsilon^2/v}.
\]

Here \( m_e \) is the electron mass and

\[
f(Z,\varepsilon) = \int_{1}^{\varepsilon_0} F(Z,\varepsilon)(\varepsilon_0 - \varepsilon)^2 \sqrt{\varepsilon^2 - 1} \varepsilon d\varepsilon
\]

is the phase space factor, \( \varepsilon = k_0/m_e \) and \( \varepsilon_0 = \Delta/m_e \) with [15]

\[
\Delta = m_p + m_3\text{He} - m_4\text{He} \simeq 19.284 \text{ MeV}.
\]

The Fermi function \( F(Z,\varepsilon) \) with \( Z = -2 \) takes into account the Coulomb interaction of the final \( e^+ \) and \( ^4\text{He} \). In our case, this function is practically equal to one due to the large positron energies up to 19 MeV [10]. Notice that in the process (1) the isotopic spin is changed by one and only the Gamov–Teller transition must be taken into account (for the details of the derivation of (8) see Ref. [17]).

In essence, taking into account the relation (3), the difference between the hep process (1) and the process (5) is given by the Coulomb correction factor (7). Therefore, in the region of small energy we have the following expression for the ratio of the cross sections of the process (5) and the hep process:

\[
\frac{\sigma(n\,^3\text{H} \rightarrow ^4\text{He} e^-\bar{\nu}_e)}{\sigma(\text{hep})} = \frac{1}{2\pi\eta e^{-2\pi\eta}} f(2,\varepsilon'_0) f(-2,\varepsilon_0).
\]

In this relation we have also taken into account the difference in the phase space factors. As in the hep process, the Fermi function can be well approximated by one, however, the isospin-breaking effect due to the difference between \( \Delta \) (10) and (see Ref. [18])

\[
\Delta' = m_n + m_3\text{H} - m_4\text{He} \simeq 21.107 \text{ MeV}
\]

cannot be neglected. The reason is that \( f(-2,\varepsilon_0) \simeq \frac{\varepsilon_0}{30} \) and analogously for \( f(2,\varepsilon') \) with \( \varepsilon' = \Delta'/m_e \) and, therefore, the energy releases (10) and (12) enter with the 5th power. Now using the relation (3) which defines the S–factor, we obtain the following relation between the cross section of the process (5) and the astrophysical S–factor of the hep process:
\[
\sigma(n^3H \rightarrow ^4\text{He} e^- \bar{\nu}_e) \simeq \frac{S(\text{hep})}{2\pi e^2 \sqrt{2\mu E}} \left( \frac{\Delta'}{\Delta} \right)^5.
\] (13)

From the above relation we can determine \(S(\text{hep})\) if the cross section of the process \(n^3H \rightarrow ^4\text{He} e^- \bar{\nu}_e\) will be measured. From Eq. (13), using \((\Delta'/\Delta)^5 \simeq 1.57\), we arrive at the numerical estimate

\[
\sigma(n^3H \rightarrow ^4\text{He} e^- \bar{\nu}_e) \simeq 1.3 \times 10^{-47} \frac{S(\text{hep})}{\sqrt{E[\text{MeV}]}} \text{ cm}^2,
\] (14)

where \(S_0(\text{hep}) = 2.3 \times 10^{-20} \text{ keV} \cdot \text{b}\) is the value of \(S(\text{hep})\) adopted in the SSM.

If we put \(S(\text{hep}) = S_0(\text{hep})\), for the cross section of the process (5) we obtain the following estimates for thermal \((E \approx 2.5 \times 10^{-2} \text{ eV})\), cold \((E \approx 2 \times 10^{-3} \text{ eV})\) and ultracold \((E \approx 2 \times 10^{-4} \text{ eV})\) neutrons [13]:

\[
\sigma(n^3H \rightarrow ^4\text{He} e^- \bar{\nu}_e) \simeq 10^{-43} \text{ cm}^2 \times \begin{cases} 
1.3 & \text{thermal} \\
4.7 & \text{cold} \\
15 & \text{ultracold}
\end{cases}
\] for \(\begin{cases} 
\text{thermal} \\
\text{cold} \\
\text{ultracold}
\end{cases}\) neutrons. (15)

In conclusion, from isotopic SU(2) invariance of strong interactions we have obtained a relation that connects the main hadronic part of the astrophysical \(S\)-factor of the hep process (1) with the total cross section of the process \(n + ^3\text{He} \rightarrow ^4\text{He} + e^- + \bar{\nu}_e\) at low energies. To work with a \(^3\text{H}\) target is a difficult problem. But there are several advantages in the investigation of the \(n^3\text{H}\) process (5):

1. There is no Coloumb penetration factor which strongly suppresses the cross sections of processes with equally charged particles at very small energies.

2. The matrix element of this process determines the main hadronic part of the \(S\)-factor of the hep process. This follows from simple isotopic symmetry of strong interactions. Note that this is not the case for the reaction \(n + ^3\text{He} \rightarrow ^4\text{He} + \gamma\) (see, for example, Ref. [11]).

3. The possibility of using high intensity very low energy thermal, cold and ultracold (?) neutron beams is a unique possibility to increase the cross section of the weak process due to the famous \(1/v\) effect.

4. The signature of the process (5) are electrons with energies up to 20 MeV, which is a much higher energy than the one of the background electrons from the decay of \(^3\text{H}\).

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\(^1\)For a liquid tritium target see the paper of H.H. Grafov et al. [20].
NOTE ADDED

After this paper was finished and sent to the editor, new calculations of the hep cross section appeared. In L.E. Marcucci et al., nucl-th/0003065 and nucl-th/0006005, it is claimed that, inspite of the centrifugal suppression, the $p$–state of the initial $p – ^3\text{He}$ system gives a sizable contribution to the cross section of the hep process (see also C.J. Horowitz, Phys. Rev. C 60, 022801 (1999)). The value of $S(\text{hep})$ that was obtained by L.E. Marcucci et al. is approximately 5 times larger than the value found in Ref. [12] and ascribes to the $p$–wave contribution about 40% of the evaluated cross section. On the other hand, in T.-S. Park et al., nucl-th/0005069, based on an effective field theory of QCD, no explicit role is attributed to the $p$–wave contribution.

Our paper is based on the classical assumption that in the region of small energies (keV) of the proton only the $s$–wave is important (see, e.g., D.D. Clayton, Principles of stellar evolution and nucleosynthesis (McGraw-Hill, New York, 1968)). Should the importance of the $p$–wave be confirmed by further calculations, the method we proposed here allows to determine a lower bound on $S(\text{hep})$ which is given by the major $s$–wave contribution. Finally we want to stress that a measurement of the process $n + ^3\text{H} \rightarrow ^4\text{He} + e^- + \bar{\nu}_e$ at thermal neutron energies would allow to test theories of nuclear reactions, which are relevant for astrophysics.

In conclusion, we would like to stress that, in spite of the fact that the expected cross section of the above $n + ^3\text{H}$ process with thermal neutrons is relatively large, the proposed experiment is a very difficult one. A basic requirement for such an experiment is the availability of a large tritium target. We would like to notice that a 4 kg $^3\text{H}$ target exists and was discussed in the poster session P25 (L.N. Bogdanova et al.) at the recent conference Neutrino 2000 in Sudbury, June 2000. Of course, the problem of background to the process we propose is very severe. Let us notice that in order to reduce the background from cosmic rays, one could conceive to perform the experiment at a powerful underground reactor, like the one in Krasnoyarsk.
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