Research on Centralized Voltage and Effective Inequality Identification Based on Circuit Analysis Method

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Abstract. This paper is based on affine function equation of the grid and OPF problem, discusses the equivalent of some inequality constraints variables optimizing. Further, we propose the model of injection current and set up the constraint sensitivity index of affine characteristics. The index can be used to identify the central point voltage and effective inequality of the system automatically. And then we can know how to compensate reactive power of the corresponding generator node and control the voltage to ensure the quality of the system voltage. When checking the effective inequalities we introduce cross-solving method of power flow. This provide a different idea for solving the power flow. The paper uses the results of the IEEE5 node examples to illustrate the validity and practicality of the proposed method.

1. Introduction
The general form of constrained nonlinear programming problems is
\[
\begin{align*}
\min f(u) \\
\text{s.t. } g_i(u) &\geq 0 \quad i \in I = \{1, \ldots, p\} \\
h_j(u) &= 0 \quad j \in E = \{1, 2, \ldots, q\} \\
u_{\min} \leq u \leq u_{\max}
\end{align*}
\]

(1)

The variables in \(u\) are associated with equality and inequality constraints. For the inequality constraints, by Farkas theorem, it is proved that there exists a set of \(\lambda, \mu\) values can make the KT condition establish. If the constraint specification is not specified, the condition of the extreme point \(\bar{u}\) is
\[
\nabla f(\bar{u}) = \sum_{i=1}^{q} \lambda_i \nabla h_i(\bar{u}) + \sum_{j=1}^{p} \mu_j \nabla g_j(\bar{u})
\]
\[
h_j(\bar{u}) = 0 \quad j \in E = \{1, 2, \ldots, q\}
\]
\[
\mu^*_i g_i(\bar{u}) = 0 \quad i \in I = \{1, \ldots, p\}
\]
\[
\mu_i \geq 0 \quad i \in I = \{1, \ldots, p\}
\]

(2)

The equation (2) requires the solution of the variable \(u, \lambda, \mu\), and must be \(\mu \geq 0\). The equation (2) also shows that the effective inequality (\(\mu_i > 0\)) and the ineffective inequality (\(\mu_i = 0\)) that can be distinguished by solving the \(\mu\) value.

This paper focuses on discussing the equivalence of partial variables for optimization of inequality constraints based on the affine function equality constraints [1,2].
Equivale

c principle of partial variables for optimization of optimization problems is
\[
\inf_{x,y} f(x,y) = \inf_y \tilde{f}(x)
\]  

Which,
\[
\tilde{f}(x) = \inf_y f(x,y)
\]

The equation (3) shows that if we can determine that the partial variable \( y \) can obtain the lower bound function \( \tilde{f}(x) = \inf_y f(x,y) \) for \( f(x,y) \), we can obtain the original problem of the bounds of the variables \( x \) and \( y \), which is equivalent to the partial variables \( x \) can get the lower bound function \( \inf_x \tilde{f}(x) \).

The key to applying the equivalence principle is to make the appropriate variables separate and obtain the new lower bound functions \( \tilde{f} \).

The inequality constraint in the optimal power flow is usually the branch power constraint formed by the constraint of the node voltage or the function of the node voltage, which is expressed as the constraint that the voltage is the desired value. According to the analysis of the power system, it is recognized that the reactive power is most effective for adjusting the voltage amplitude. The active power is most effective for adjusting the voltage phase angle. It is realized that the high voltage is conducive to reduce network loss and improve system voltage stability. Limited to the length and highlight the problems to be solved, this article does not make these cognitive function of the form of argument, but the direct application of these physical laws. In this way, for the voltage target point, the nearest choice of power supply, is to choose the most sensitive part of the variable \( y \). The upper limit of the allowable voltage of the voltage source, which is sensitive to the network loss, constitutes the lower bound function \( \tilde{f} \) which is most conducive to the running level of the system.

According to the equivalence principle of optimizing local variables, active identification of the voltage effective inequality is to simplify the analysis and solution without using the Lagrange multiplier \( \mu \) of equation (2). It is the active choice of the desired extreme points to solve the multiplier \( \mu \) solution is not easy to solve the \( \mu \) value to obtain the greatest problem.

After determining the effective voltage inequality, the \( \inf \tilde{f}(x) \) optimization of the remaining optimization variables \( x_{opt} \) is solved along the bounds of the feasible domain, making the optimization variable \( x_{opt} \) a closed convex set and a "non-convex to convex" in the form of the optimal power flow of the equation (1).

2. Affine Function Equation and Optimal Power Flow Problem of Power Network
The nodal voltage equation of the n-node power network is
\[
\dot{I} = Y\dot{V}
\]
Take \( Y = G + jB, \dot{V}_f = e_i + jf_i, \dot{I}_f = I_{s_f} + jI_{y_f} \).

Let node \( n \) be the equilibrium node and take \( \dot{V}_n = e_n + jf_n = e_n \),
\[
G = \begin{bmatrix}
G_{1,1} & \cdots & G_{1,n-1} \\
\vdots & \ddots & \vdots \\
G_{n-1,1} & \cdots & G_{n-1,n-1}
\end{bmatrix}, \quad B = \begin{bmatrix}
B_{1,1} & \cdots & B_{1,n-1} \\
\vdots & \ddots & \vdots \\
B_{n-1,1} & \cdots & B_{n-1,n-1}
\end{bmatrix}
\]
\[
G_{e} = \begin{bmatrix}
G_{e,1} \\
\vdots \\
G_{e,n-1}
\end{bmatrix}, \quad B_{e} = \begin{bmatrix}
B_{e,1} \\
\vdots \\
B_{e,n-1}
\end{bmatrix}
\]
Make \( X = -(B + GB^{-1}G)^{-1}, R = -B^{-1}GX \).
The inverse mapping of the affine function equation (4) can be expressed as
\[
\begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} R & -X \\ X & R \end{bmatrix} \begin{bmatrix} I_x \\ I_y \end{bmatrix} = \begin{bmatrix} R & -X \\ X & R \end{bmatrix} \begin{bmatrix} G_n \\ B_n \end{bmatrix} e_n \quad (5)
\]

Make \( A_k = \sum_{j=1}^{n-1} R_{kj} G_{nk} - \sum_{j=1}^{n-1} X_{kj} B_{nj} \),
\( B_k = \sum_{j=1}^{n-1} X_{kj} G_{nj} + \sum_{j=1}^{n-1} R_{kj} B_{nj} \).

Get \( e_k = \sum_{j=1}^{n-1} R_{kj} I_{kj} - \sum_{j=1}^{n-1} X_{kj} I_{kj} - A_k e_n \) \( (6) \)
\( f_k = \sum_{j=1}^{n-1} X_{kj} I_{kj} + \sum_{j=1}^{n-1} R_{kj} I_{kj} - B_k e_n \) \( (7) \)

The general form of the tidal current problem with operating conditions is

**The Optimization goal is**
\[
\min \quad F(e, f, P_G, Q_G, e_n) \quad (8)
\]

The grid operating conditions are
\( I_x - Ge + Bf - G_n e_n = 0 \) \( (9) \)
\( I_y - Be - Gf - B_n e_n = 0 \) \( (10) \)

The node power condition is
\( P_{Gi} - P_{Bi} - (e_i I_{xi} + f_i I_{yi}) = 0, i = 1, 2, \cdots, n - 1 \) \( (11) \)
\( Q_{Gi} - Q_{Bi} - (f_i I_{xi} - e_i I_{yi}) = 0, i = 1, 2, \cdots, n - 1 \) \( (12) \)

The node voltage condition is
\( V_{imin} \leq V_i = \sqrt{e_i^2 + f_i^2} \leq V_{imax} \quad i = 1, 2, \cdots, n \) \( (13) \)

RS, UV branch transmission of active and reactive power were
\( P_n = (e_i^2 + f_i^2)(-G_n + g_{ri}) + (e_i e_s + f_i f_s) G_n + (f_i e_s - e_i f_s) B_n \)
\( Q_n = (e_i^2 + f_i^2)(B_n - b_{ri}) + (e_i e_s + f_i f_s) B_n - (f_i e_s - e_i f_s) G_n \)

The branch transmission power conditions are
\( P_{rmin} \leq P_r \leq P_{rmax} \quad r = 1, 2, \cdots, n, s = 1, 2, \cdots, n \) \( (14) \)
\( Q_{vmin} \leq Q_v \leq Q_{vmax} \quad u = 1, 2, \cdots, n, v = 1, 2, \cdots, n \) \( (15) \)

The power operating conditions are
\( P_{Gimin} \leq P_{Gi} \leq P_{Gimax} \quad i = 1, 2, \cdots, n \) \( (16) \)
\( Q_{Gimin} \leq Q_{Gi} \leq Q_{Gimax} \quad i = 1, 2, \cdots, n \) \( (17) \)

**3. Voltage center point and voltage effective inequality**

Power system voltage is generally used reactive power regulation, due to system power and reactive power supply configuration and control of the limitations, the system operating voltage may not be independent of the operating point. To meet the voltage level of the entire network, part of the reactive power can be used to focus on the control of its key point of the voltage, so that the entire network voltage can meet the requirements and reduce the burden of decentralized. This paper refers to the system power control key point for the voltage center.
3.1 Voltage center point
The grid voltage is reflected in the level of power quality, system safety, and economy, so that the voltage must be within the permissible voltage offset, such as \([1.00 \sim 1.05]V_N\) in the transmission network. And the voltage center point reaches the upper limit of its allowable boundary, which is beneficial to reduce the system network loss and improve the voltage stability of the system.

The voltage center point has an adjustable reactive power source so that the point voltage can be controlled to the desired value. The central point voltage should be able to affect the area as large as possible so that the voltage values of the points in the area also meet the required voltage quality requirements. This is the voltage effective control area of the reactive power supply.

The whole network reactive power supply shared the grid voltage, the voltage control area also affect each other. To ensure that the central point voltage for the required value of part of the reactive power supply must be the separation variable \(y\) for \(\tilde{f}(x) = \inf_y f(x, y)\), and the rest of the power variable \(x\) is supported by \(y\) for the optimization of variable \(f\), that is \(\inf_x \tilde{f}(x)\).

3.2 Voltage effective inequality
The central point voltage can reach the boundary limit of its inequality constraint, and the node becomes an effective voltage inequality that works. And the voltage value of other nodes is within the internal limit of the inequality, which is the ineffective voltage inequality that does not work.

By using the equivalent principle of partial variable optimization to identify the voltage effective inequality, the characteristics of the voltage center point and the steps of estimating and analyzing can be analyzed according to the concept of power system analysis which is beneficial to reduce the system network loss and improve the voltage stability of the system.

4. A Circuit Analysis Method for Voltage Effective Inequality

4.1 The voltage limit of the rated power mode of operation
Take the initial value \(e^0 = 1, f^0 = 0\), then the node injection current is
\[
I_x^0 = P_G - P_D, \quad I_y^0 = -(Q_{eN} - Q_D) \tag{18}
\]
By the equation (6) and (7) of the network node current equation
\[
I_x - Ge + Bf - G_n e_n = 0 \tag{19}
\]
\[
I_y - Be - Gf - B_n e_n = 0 \tag{20}
\]
Take the estimated value of the input current for the node voltage estimation, but also approximate
\[
\begin{bmatrix}
\bar{e} \\
\bar{f}
\end{bmatrix} = 
\begin{bmatrix}
R & -X \\
X & R
\end{bmatrix} 
\begin{bmatrix}
I_x^0 \\
I_y^0
\end{bmatrix} - 
\begin{bmatrix}
R & -X \\
X & R
\end{bmatrix} 
\begin{bmatrix}
G_n \\
B_n
\end{bmatrix} e_n^0 \tag{21}
\]
If \(R<<X\), then approximate to take
\[
\bar{e} = -XI_x^0 + XG_n e_n^0
\]
\[
\bar{f} = XI_y^0 - XB_n e_n^0 \tag{22}
\]
And then make \(f << e\), approximate \(f=0\)
\[
\bar{V} \approx \bar{e} = -XI_x^0 + XG_n e_n^0 \tag{23}
\]
Different ways to get different accuracy of the node voltage
\[
\bar{V}_i = \sqrt{\bar{e}_i^2 + \bar{f}_i^2}, \quad i = 1, 2, \cdots, n \tag{24}
\]
The equation (24) is the voltage distribution that takes the power supply for the rated power and also represents the upper bound of the power capacity.

When \( \hat{V}_i > V_{\text{imax}} \) or \( \hat{V}_i < V_{\text{imin}} \), then the system will be able to appear voltage limit inequality.

Let \( S_v = \{ \hat{V}_i | \hat{V}_i \geq V_{\text{imax}}, i = 1, 2, \ldots, n-1 \} \) be a set of overvoltage for voltage unrestricted conditions.

4.2 The sensitivity of the power supply to the network loss

Network active power loss \( P_L \) is

\[
P_L = I_x^T R I_x + I_y^T R I_y - 2e_n B^T I_y
\]

\[
- \left[ G_n^T R - B_n^T X - G_n^T X - B_n^T R \right] e_n^2 + G_m e_n^2
\]

\[
\frac{\partial P_L}{\partial I_{si}} = 2 \sum_{j=1}^{n-1} R_{ij} I_{sj}
\]

(25)

\[
\frac{\partial P_L}{\partial I_{yi}} = 2 \sum_{j=1}^{n-1} R_{ij} I_{sy} - 2e_n B_i
\]

(26)

Since the node voltage is estimated, then it can be considered \( f << e \), approximation \( f = 0 \), \( e \approx \hat{V} \). And then according to equation (19) and (20) update \( I_x, I_y \), so as to achieve the purpose of updating the network loss sensitivity.

4.3 Voltage effective control area

Sensitivity of the method employed to determine the effective voltage control region.

4.3.1 Sensitivity. Due to \( V_i = \sqrt{e_i^2 + f_i^2} \), according to equation (6) and (7) on the Iykl complex function relations, available power Iykl on the voltage Vi sensitivity

\[
\frac{\partial V_i}{\partial I_{yk_1}} = -e_i X_{ik_1} + f_i R_{ik_1}
\]

(27)

Then

\[
\frac{\partial V_i}{\partial I_{jk_1}} = \frac{-e_j X_{jk_1} + f_j R_{jk_1}}{\hat{V}_i}
\]

(28)

\[
\frac{\partial V_j}{\partial I_{yk_1}} = \frac{-e_j X_{jk_1} + f_j R_{jk_1}}{\hat{V}_j}
\]

(29)

By the parameter variables of \( I_{yk_1}, V_i \) can be associated with \( V_j \). Relative to the variable \( I_{jk_1} \), the sensitivity of the voltage between nodes \( i, j \) is

\[
\frac{\partial V_j}{\partial V_i} = \frac{\hat{V}_i \left( -e_j X_{jk_1} + f_j R_{jk_1} \right)}{\hat{V}_j \left( -e_j X_{ik_1} + f_j R_{jk_1} \right)}
\]

(30)

When the approximation is taken \( \hat{V}_i \approx \hat{V}_j, \ e_i \approx e_j, R_{ik} << X_{ik} \) for the estimate
\[
\frac{\partial V_j}{\partial I_k} = X_{jk} X_{ik} \tag{31}
\]

4.3.2 Voltage effective control area. Set the set of reactive power supply to SQG. Let \(\bar{V}_k\) be the highest voltage in the set \(SV\) and select the reactive power source \(I_{yk1}\) that is most sensitive to the voltage control of the node \(k\).

\[
\frac{\partial V_k}{\partial I_{yk}} > \frac{\partial V_k}{\partial I_{yj}} \quad k, j \in S_{QG} \tag{32}
\]

Take the \(k\) point for the voltage center point, because the node \(k\) voltage up to its upper limit, the required voltage adjustment is

\[
\Delta V_k = V_{k_{\text{max}}} - \bar{V}_k \tag{33}
\]

Corresponding to the other node \(j\) the amount of voltage change

\[
\Delta V_j = \frac{\partial V_j}{\partial V_k} \Delta V_k \tag{34}
\]

If \(V_{j_{\text{min}}} \leq \bar{V}_j + \Delta V_j \leq V_{j_{\text{max}}},\) then the \(j\) node voltage in the \(I_{sk1}\) voltage control area, otherwise it belongs to another voltage control area.

4.4 Determine the voltage effective inequality

According to the characteristic of the effective voltage inequality: the voltage inequality takes the boundary value; the voltage of the adjacent node can be the allowable value; the corresponding power variable can maintain the point voltage as the boundary value. The circuit analysis step is

① Estimated grid voltage with rated reactive capacity. Using equation (21), the formation of possible voltage limits may be set. And the higher limit voltage step descending order \(SV\);

② Estimate the effectiveness of each power supply in the \(S_{QG}\) of the reactive power set to reduce the loss of the network. According to equation (26) for the network loss sensitivity of the order, forming a collection \(S'_{k}\) of power nodes;

③ For each voltage point in the set \(SV\), select the most efficient adjustment power supply \(I_{sk1}\) according to equation (27). And arranged in descending order power supply \(S''_{k}\);

④ Select the voltage effective inequality. For a voltage \(\bar{V}_k\) of the highest point \(k\) of the voltage in \(SV\), \(k'\) \(k''\) of \(S'_{k}\) and \(S''_{k}\) are investigated.

If \(k' = k''\), then the best way to reduce the damage and adjust the effect of the best unity, select \(k = k'\).

If \(k' = k''\), and \(\frac{\partial V_k}{\partial I_{yk'}}\) can meet the regulation requirements, preferred \(k = k'\).

If \(k' = k''\), can not meet the regulation requirements, we must choose the technical requirements of \(k = k''\).

⑤ After selecting the power supply \(k\) for the \(k\)-point, it is necessary to reduce the voltage \(\Delta V_k\)
according to the equation (32), and correct the voltage at each point after $V_0$ is not exceeded to form a new over-voltage set.

5 For the $k$ voltage effective control area, the maximum voltage $\bar{V}_k$ point, take $\bar{V}_k = V_{k,\text{max}}$, and select its regulation power $I_{k,1}$.

Therefore, an effective voltage control point set $S_k$ can be formed

$$S_k = \left\{ V_0^k = V_{k,\text{max}} \left| \Delta V_k = \frac{\partial V_k}{\partial I_{k,j}} \Delta I_{k,j} \right. \right\}$$

And the corresponding inequality constraints

$$V_{k,\text{min}} \leq V_k \leq V_{k,\text{max}}$$

Convert to equation

$$V_0^k - V_k = 0 \quad (36)$$

After $V_k = V_0^k$, the new voltage is set to

$$S_v = \left\{ \bar{V}_i + \frac{\partial \bar{V}}{\partial V_k} \Delta V \left| \bar{V}_i \geq V_{\text{max}}, i = 1, 2, \ldots, n-1 \right. \right\} \quad (37)$$

If the new $S_v$, there are still more voltage, then repeat the above set a new adjustment power and adjust the voltage after the adjustment process.

When there is no limit voltage, according to equation (41) to estimate the required reactive power, and then according to equation (40) to verify whether the node voltage in line with the requirements of inequality.

4.5 Verification of Voltage Effective Inequality

The voltage effective inequality identified by the sensitivity method is to convert the voltage effective inequality into the voltage equation and set the most efficient regulated power supply. The linearization analysis of the sensitivity method identification is rough and can be checked.

Make the point where the voltage inequality boundary is reached, that is, the effective inequality voltage, be the determined boundary value.

$$V = \sqrt{e_i^2 + f_j^2} = V_0^i, \ i \in S_v \quad (38)$$

Effectively adjust the power supply to the limit voltage $I_{j,l}$, $j \in \{ k_i \}$.

The node voltage in the inequality constraint range is

$$V_l = e_t + f_f, \ l = 1, 2, \ldots, n-1, \ l \in S_v, \ l \notin \{ k_i \} \quad (39)$$

Then the node voltage equation (3-2) can be written as:

$$\begin{bmatrix} e_t \\ f_f \end{bmatrix} = \begin{bmatrix} R & X \\ X & R \end{bmatrix} \begin{bmatrix} G_n \\ B_n \end{bmatrix} e_n \quad (40)$$

Where

$$\begin{bmatrix} e_t \\ f_f \end{bmatrix} = -\begin{bmatrix} R & X \\ X & R \end{bmatrix} \begin{bmatrix} G_n \\ B_n \end{bmatrix} e_n.$$

Since the set elements of $i$ and $j$ are not equal and the position in the voltage equation crosses. In this paper, the known $e_t, I_{j,l}$, solder $e_t, e_j, f, I_{j,l}$ of the current flow is called the position of the intersection of the solution. So the process of verification is actually the algorithm to solve the process.
Let \( f_i \ll e_i \), for the approximate estimate of the estimate can make \( f_i = 0 \), \( e_i = V_i^0 \) for the boundary value. Reset \( I_\gamma \), make \( I_\gamma (f) = 0 \), according to equation (5) can be derived.

\[
I_{\gamma j} = X_{\gamma}^{-1} \left( V_j^0 - e_j - [R(i,:) - X(i,:)] \right) \tag{41}
\]

Using the overlay technique \( I_\gamma (j) = I_\gamma (j) + I_{\gamma j} \) to find the changed \( I_\gamma \). And then \( I_\gamma \) back to the equation (24) can be the corresponding voltage \( e_i, e_j, f_i, f_j \), check the effect of the central point voltage, but also verify the capacity of the reactive power \( I_yk1 \) required.

5. Case and Extreme Value Analysis

5.1 Power Flow Calculation of Fundamental Form of Current Injection Current Flow Model

IEEE 5 system wiring shown in Figure 1.

![IEEE 5 system](image)

Fig .1 IEEE 5 system

The basic mode of the current flow in this paper is used. Take node 5 as the balance node, the voltage \( e_5 = 1.05 \). Node 4 is the PQ node, \( P_{G4} = 2.5 \), \( Q_{G4} = 1.813 \).

The node voltage and the injected power are shown in Table 1.

| Node | V (pu) | I (pu) | P (pu) | Q (pu) |
|------|--------|--------|--------|--------|
| 1    | 0.9543 | 1.8745 | -1.6   | -0.8   |
| 2    | 1.1617 | 1.9248 | -2.0   | -1.0   |
| 3    | 1.0666 | 3.6770 | -3.7   | -1.3   |
| 4    | 1.1300 | 2.7331 | 2.5    | 1.813  |
| 5    | 1.0500 | 1.0000 | 2.5794 | 2.2990 |

If the upper and lower limits of the voltage are 0.9 to 1.1, the set of the limit voltage under unrestricted conditions is: \( S_V = \{ V_2, V_4 \} \), where \( V_2 = 0.9543 \). The sensitivity of the network loss is sorted by equation (10-2), the formation of reactive power set \( S_k' \), and \( S_k' = \{ S_4 = -0.2604 \} \), \( k_1 = 4 \). The voltage sensitivity is sorted by equation (27), the formation of power adjustment set \( S_k'' \), \( S_k'' = \{ S_4 = 0.9524, S_3 = 0.1461, S_1 = 0.6655 \} \), \( k_1 = 4 \). The voltage \( V_2 = 1.1617 \) for the highest voltage of the voltage in \( S_V \) is satisfied, and \( k_1 = k_1 = 4 \). At this point to reduce the loss and adjust the system of the best results.

Then by equation (31) \( \frac{\partial V_j}{\partial V_i} = X_{ji} \) available

\[
\frac{\partial V_j}{\partial V_i} = X_{ji} = \begin{bmatrix} 0.6655 & 1.0000 & 0.1461 & 0.9524 \end{bmatrix}
\]

From the equation (37) can be a new set of voltage \( S_V = \{ V_1 = 0.9133, V_2 = 1.1000, V_3 = 1.0576, V_4 = 1.0712, V_5 = 1.0500 \} \).

5.2 IEEE 5 system voltage valid inequality verification

As the new voltage set to meet the upper and lower limits of the voltage of 0.9 to 1.1, according to equation (41) can be obtained in the generator node 4 reactive power, and its reactive power is \( I_{\gamma 4} = -1.9691 \). To sum up the following table.
Table 2. IEEE5 system effective inequality and power regulation node

| Node N | Power node N adjusts reactive power | The power supply to be adjusted is reactive (pu) |
|--------|-------------------------------------|-----------------------------------------------|
| 2      | 4                                   | -1.9691                                       |

According to equation (40) on the voltage of each node to verify, the results are as follows.

Table 3. IEEE5 system verification results

| Node  | \( V_{pu} \) | \( e_{pu} \) | \( f_{pu} \) |
|-------|--------------|--------------|-------------|
| node1 | 0.9381       | 0.8564       | -0.3830     |
| node 2| 1.1373       | 1.0900       | -0.2890     |
| node 3| 1.0631       | 1.0528       | -0.1479     |
| node 4| 1.1054       | 1.0772       | -0.2483     |
| node 5| 1.0500       | 1.0500       | 0.0000      |

System net loss consumption is \( P_{Loss} = 0.2118 \). As can be seen from Table 2, the maximum voltage \( V_2 = 1.1373 \) in the calibration result, and some deviations from the expected \( V_2 = 1.1 \), the deviation rate \( \varepsilon = 3.27\% \). Analyze the cause of the error. Since it is estimated, in the case of the value of the effective inequality, let \( f_i = 0 \), and \( e_i = V_i^0 \) be the boundary value. But the check results show \( f_i \neq 0 \), so the value calculated from \( V_i = \sqrt{e_i^2 + f_i^2} \) is slightly larger than the estimated value, and the deviation rate \( \varepsilon = 3.27\% \) is quite reasonable.

The results show that the method can automatically identify the voltage center point 2 of the IEEE 5-node system and realize the control of the voltage of the IEEE 5-node system through the effective adjustment of the load node 2 of the generator node 4. Indicating that the other nodes are in the \( I_{vd} \) voltage control area, to ensure the system voltage quality. The method also provides a new idea for voltage partitioning.

6. Conclusions

Based on the affine function equation and the optimal power flow problem of the power grid, the equivalence of the partial variable of the inequality constraint is discussed in this paper. Based on this model, the constraint sensitivity index of the affine property is established, and the central point voltage and effective inequality of the system are automatically identified by this index, and the reactive power compensation of the corresponding generator node is realized. System voltage control ensures the system voltage quality. In the check of the effective inequality, this paper introduces the cross-solving method of the trend, which provides a way for the solution of the trend. The verification results of the IEEE5-node example show that the method of this method is effective and correct and has practical significance.

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