Research Article

High-Resolution Squinted SAR Imaging Algorithm Applied in the Near-Field Environment

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In the squinted synthetic aperture radar (SAR) imaging of the near-field environment, range-dependent characteristic of squint angle cannot be ignored, which causes azimuth-dependent range cell migration (RCM) after linear range walk correction (LRWC). In this study, an efficient SAR imaging algorithm applied in the near-field environment is proposed. In the processing of the range focusing, LRWC is firstly used to remove the linear RCM. Then, the residual LRCM is expanded into azimuth-invariant and azimuth-variant terms in consideration of the residual LRCM of azimuth-dependent. Range cell migration azimuth scaling (RCMAS) is designed to remove the azimuth-variant term before secondary range compression (SRC) and range compression (RC). In the azimuth focusing, azimuth distortion compensation (ADC) is performed to compensate the azimuth distortion, following which azimuth nonlinear chirp scaling (ANCS) is applied to equalize the frequency modulation (FM) rate for azimuth compression (AC). The simulated results show that more accurate and improved imaging result can be obtained with the proposed algorithm.

1. Introduction

Due to the ability of high-resolution imaging for the observed scene with any time and any weather, synthetic aperture radar (SAR) has become one of the most attractive radar techniques [1, 2]. There are many imaging modes for SAR, such as stripmap [3, 4], spotlight [5, 6], and sliding spotlight [7, 8]. Particularly, the squinted SAR in the stripmap is widely applied in military and civilian applications. By adjusting the squint angle, SAR can flexibility choose the imaging scene to meet diverse needs. However, the squinted SAR also brings new difficulties and challenges. As the squinted angle increases, the signal property varies a lot and the accurate focusing becomes difficult [1, 9, 10]. Therefore, it is necessary to study imaging algorithm for high-resolution squinted SAR.

A lot of algorithms have been proposed in the past years, for example, range-Doppler (RD) algorithm [11–13], chirp scaling (CS) algorithm [14–19], omega-K algorithm [20, 21], and backprojection (BP) algorithm [22, 23]. Theoretically, omega-K algorithm and BP algorithm can be applied in any imaging geometry and any squint angle without any approximation. However, the low efficiency and high computational load limit its practical application. As the squint angle increases, the range cell migration (RCM) and range-azimuth coupling become severe. This degrades the performance of RD algorithm. CS algorithm and its modifications have shown significant potential to process high-resolution squinted SAR data. Sun et al. [15] analyzed the spectrum of squinted data and used the squint minimization processing proposed in [24] to reduce the range-azimuth coupling. Then, an azimuth scaling function was created to equalize the Doppler rate of the targets in the same range cell. Liu et al. [16] first used linear range walk correction (LRWC), secondary range compression (SRC), and range compression (RC) to accomplish the range focusing. Then, in order to solve the problem of the azimuth-dependent frequency modulation (FM) rate caused by LRWC, a filter function and a scaling function,
called the nonlinear chirp scaling (NCS) algorithm, were constructed. Chen et al. [17] built the accurate signal model of the missile-borne SAR which took into account not only the missile velocity but also the acceleration. In addition, the azimuth distortion of image was considered in the modified azimuth NCS algorithm. Li et al. [18] and An et al. [19] analyzed the characteristics of the azimuth phase terms in the case of bistatic and monostatic SAR, respectively. The modified scaling coefficients were derived by eliminating the higher-order phase approximation and azimuth-independent cubic phase term.

All of the above proposed algorithms processed the squinted SAR data acquired in a far-field environment. In this case, the wavefront of the electromagnetic wave transmitted by a radar was usually considered as a plane because of the very large distance between the radar and target. However, in a near-field environment, the spherical wavefront of the electromagnetic wave cannot be ignored. This will result in the squint angle of target varying with the range. Therefore, the imaging algorithm of high-resolution squinted SAR in the near-field environment needs to be redesigned.

The remainder of this paper is organized as follows. The radar signal model is described in Section 2. Section 3 gives the proposed imaging algorithm including seven procedures. Simulation results are provided to verify the effectiveness of the proposed algorithm in Section 4. Finally, this study is concluded in Section 5.

2. Signal Model

The general geometry of squinted SAR imaging in the near-field environment is shown in Figure 1. The radar moves along the direction parallel to the X-axis at a constant velocity \( v \). \( O_r \) is the synthetic aperture center where the azimuth time \( t_a \) is equal to zero. \( \theta \) is the radar squint angle. The imaging scene is located at \( OXZ \) plane. \( L \) is the distance between the radar trajectory and \( OXZ \) plane, and \( H \) is the height of the imaged scene center. \( r_0 \) is the slant range from the target \( T(t_n, R) \) to the radar trajectory along the radar line-of-sight direction. \( R \) is the instantaneous slant range between the radar and the target \( T \) when the radar is at azimuth time \( t_a \). \( O_rF \) is the half of the synthetic aperture length, whose value is \( v t_a \). The target \( T \) is located at the azimuth time \( t_m \), which satisfies \( HT = vt_m \). The instantaneous squint angle of target \( T \) is denoted by \( \theta_q \), which is expressed as \( \theta_q = \sin^{-1}(L \tan \theta/r_0) \). The targets A–E are used in the following simulation.

In Figure 1, the instantaneous range \( R \) and its Taylor expansion at \( t_a = t_a \) are given by

\[
R(t_a, r_0) = \sqrt{v^2(t_a - t_n)^2 + r_0^2 - 2vr_0(t_a - t_n)\sin \theta_q} \\
= r_0 + k_1(t_a - t_n) + k_2(t_a - t_n)^2 + k_3(t_a - t_n)^3 + k_4(t_a - t_n)^4,
\]

where \( k_1, k_2, k_3, \) and \( k_4 \) are specifically expressed as

\[
k_1 = -(v \sin \theta_q),
\]

\[
k_2 = \frac{v^2 \cos^2 \theta_q}{2r_0} - \frac{\nu^2}{2c},
\]

\[
k_3 = \frac{\nu^2 \cos^2 \theta_q \sin \theta_q}{2r_0^2},
\]

\[
k_4 = \frac{\nu^4 \cos^2 \theta_q (4 \sin^2 \theta_q - \cos^2 \theta_q)}{8r_0^4}.
\]

Assuming a Gaussian pulse is transmitted by the radar, the demodulated radar echo signal \( s_0(t_r, t_a) \) received from target \( T(t_n, R) \) is given by

\[
s_0(t_r, t_a) = w(t_r - \frac{2R(t_a, r_0)}{c}) w_r(t_a) p(t_r - \frac{2R(t_a, r_0)}{c}) e^{-j \frac{4\pi f_c}{c} R(t_a, r_0)},
\]

where \( t_r \) is the fast time, \( t_a \) is the slow time, \( w_r(t_a) \) is the range time envelope, \( w_r(t_a) \) is the azimuth time envelope, \( c \) is the speed of light, \( p(t_r) \) is the Gaussian pulse, and \( f_c \) is the carrier frequency.

As shown in Figure 1, the instantaneous squint angle \( \theta_q \) of target \( T \) satisfies \( \theta_q = \sin^{-1}(L \tan \theta/r_0) \). It is obvious that \( \theta_q \) is dependent on slant range \( r_0 \) when \( L \) and \( \theta \) are invariant. In order to clearly explain the influence of range-dependent squint angle on the RCM, simulation with the parameters listed in Table 1 was carried out. Supposed that there were two target points A (1.15 m, 0 m, 0.10 m) and C (1.15 m, 0 m, 1.50 m). As shown in Figure 1, the targets A and C were in the lowest and highest position of the imaging scene, respectively. They were located in the same azimuth cell. The squint angles of A and C were 29.97° and 24.79°, respectively. Figure 2 shows linear RCM (LRCM) and LRCM error.
3. Imaging Algorithm

The proposed algorithm includes the range focusing and the azimuth focusing, as shown in Figure 4. The range focusing consists of four steps. First, LRWC is used to remove most of the LRCM. Second, azimuth-dependent residual LRCM is removed through range cell migration azimuth scaling (RCMAS) processing. Third, SRC is applied to remove the coupling of range and azimuth. Finally, the range focusing is accomplished by RC processing. The azimuth focusing is resolved by three steps. The azimuth distortion compensation (ADC) is firstly performed to compensate the azimuth distortion. Then, the azimuth-dependent FM rate is equalized by azimuth NCS (ANCS). Finally, AC is operated to finish the azimuth focusing. The detailed derivations and explanations are introduced in the following.

3.1. Linear Range Walk Correction (LRWC). Substituting (1) into (3) and applying the range fast Fourier transform (FFT) yields

\[
S_1(f_r, t_a) = W_r(f_r) w_a(t_a) P(f_r) \exp \left( -j \frac{4\pi (f_c + f_r)}{c} r_0 \right) \exp \left( -j \frac{4\pi (f_c + f_r)}{c} k_1 (t_a - t_0) \right) \exp \left( -j \frac{4\pi (f_c + f_r)}{c} k_2 (t_a - t_0)^2 \right) \exp \left( -j \frac{4\pi (f_c + f_r)}{c} k_3 (t_a - t_0)^3 + k_4 (t_a - t_0)^4 \right),
\]

where \( f_r \) is the range frequency, \( W_r(f_r) \) is the range frequency envelope, and \( P(f_r) \) is the frequency response of the Gaussian pulse. The second exponential term in (4) represents the LRCM term, which is the major component of RCM. The third and fourth exponential terms are the QRCM and higher-order RCMs. According to (4), LRWC function is given as follows:

\[
H_{LRWC}(f_r, t_a) = \exp \left( -j \frac{4\pi (f_c + f_r)}{c} v \sin \theta t_a \right). \tag{5}
\]

where \( \theta \) is the squint angle of the reference point. Multiplying (4) with (5) yields

\[
S_2(f_r, t_a) = W_r(f_r) w_a(t_a) P(f_r) \exp \left( -j \frac{4\pi (f_c + f_r)}{c} r_{LRWC} \right) \exp \left( -j \frac{4\pi (f_c + f_r)}{c} (k_{res} (t_a - t_0) + k_2 (t_a - t_0)^2) \right) \exp \left( -j \frac{4\pi (f_c + f_r)}{c} (k_3 (t_a - t_0)^3 + k_4 (t_a - t_0)^4) \right),
\]

where \( r_{LRWC} = r_0 + v \sin \theta t_a \) is the new slant range corresponding to \( r_0 \) after LRWC processing. \( k_{res} = (-v \sin \theta) - (-v \sin \theta) \) is the coefficient of residual LRCM.

In (6), the existence of coefficient \( k_{res} \) indicates that LRCM has not been completely removed after LRWC. Figure 5 shows the schematic diagram of LRWC processing. There are five targets \( P_0 \sim P_4 \), and target \( P_0 \) is regarded as the reference point. Targets \( P_1, P_0, \) and \( P_2 \) have the same beam center crossing time \( t_a = 0 \), and targets \( P_1 \) and \( P_3 \) and \( P_2 \) and \( P_4 \) are located in the same range cell, respectively. In Figure 5(b), the black dotted lines represent the target LRCMs. LRCM trajectories of \( P_1 \) and \( P_3 \) have the same slope because these two targets have the same slant angle. This can also be explained for \( P_2 \) and \( P_4 \). Since the target squint angle varies with the slant range, targets \( P_1, P_0, \) and \( P_2 \) have different LRCMs. As shown in Figure 5(c), the red dotted line denotes LRWC function. After LRWC processing, targets \( P_3, P_0, \) and \( P_4 \) are located in the same range cell. However, targets \( P_1 \) and \( P_4 \) still have the residual LRCMs, and the slopes of their residual LRCM trajectories are different. This problem will be discussed in the next section.

3.2. Range Cell Migration Azimuth Scaling (RCMAS). From previous analysis, after LRWC processing, the residual LRCMs of targets \( P_3, P_0, \) and \( P_4 \) located in the same range cell are azimuth-dependent. Not only the residual LRCM but also the QRCM and higher-order RCM vary with the target azimuth position. As introduced in Section 2, azimuth-dependent residual LRCM needs to be considered, and azimuth-dependent QRCM and higher-order RCM are too smaller which are ignored. Therefore, in order to equalize the azimuth-
dependent residual LRCM, $k_{1\text{res}}$ is expanded in terms of $t_n$ as follows:

$$k_{1\text{res}} = k_{1c} + \frac{\partial k_{1\text{res}}}{\partial t_n} t_n = k_{1c} + k_{1a} t_n,$$

(7)

where $k_{1c}$ is the azimuth-invariant term and $k_{1a}$ is the coefficient of the azimuth-variant term. Based on (7), a perturbation function is designed to remove the azimuth-variant term of residual LRCM as follows:

$$H_{\text{RCMAS}}(f_r, t_a) = \exp\left(4\pi \left(\frac{f_c + f_r}{c}\right) p_2 t_a^2\right),$$

(8)

where $p_2$ is the coefficient in the perturbation function and will be determined later. Multiplying (6) with (8) yields

$$S_3(f_r, t_a) = W_r(f_r) w_a(t_a) P(f_r) \exp\left(-j \frac{4\pi (f_c + f_r)}{c} t_{\text{LRCM}}\right) \exp\left(-j \frac{4\pi (f_c + f_r)}{c} (k_{\text{LRC}}(t_a - t_n) + k_2 (t_a - t_n)^2)\right) \exp\left(-j \frac{4\pi (f_c + f_r)}{c} (k_3 (t_a - t_n)^3 + k_4 (t_a - t_n)^4)\right) \exp\left(-j \frac{4\pi (f_c + f_r)}{c} p_2 t_a^2\right).$$

(9)
Then, making $t_n = t_a - t_a$ and substituting (7) into (9) yields

$$S_4(f_r, t_a) = W_r(f_r) w(t_a) P(f_r) \exp \left( -\frac{4\pi(f_c + f_r)}{c} t_n, t_n \right), \quad \text{(10)}$$

where $\Phi(t_n, t_n)$ is expressed as

$$\Phi(t_n, t_n) = r_{LRCM} + k_{1a} t_n + k_{1a} t_n - k_2 t_n^2 + k_3 t_n^3 + k_4 t_n^4 - (p_1 t_n + 2 p_2 t_n^2 + p_3 t_n^3). \quad \text{(11)}$$

In order to remove the azimuth-dependent residual LRCM, the azimuth-variant coefficients $k_{1a}$ should be eliminated. In other words, the azimuth-variant terms should be zero in (11), and it satisfies the following equation:

$$k_{1a} t_n^t - 2 p_2 t_n^t = 0. \quad \text{(12)}$$

Through solving (12), the coefficient $p_2$ in the perturbation function is obtained as

$$p_2 = \frac{k_{1a}}{2}. \quad \text{(13)}$$

Substitute (13) into (10) and rewrite (10) as

$$S_5(f_r, t_a) = W_r(f_r) w(t_a) P(f_r) \exp \left( \frac{j 4\pi(f_c + f_r)}{c} p_2 t_n \right) \exp \left( -\frac{4\pi(f_c + f_r)}{c} \left( r_{LRCM} + k_{1a} t_n - t_n \right) \right) \exp \left( -\frac{4\pi(f_c + f_r)}{c} \left( k_{2a} t_n^2 + k_{3a} t_n^3 \right) \right) \exp \left( -\frac{4\pi(f_c + f_r)}{c} k_{4a} t_n - t_n \right). \quad \text{(14)}$$

where the coefficients $k_{1a}$, $k_{2a}$, $k_{3a}$, and $k_{4a}$ are expressed as

$$k_{1a} = k_{1c}, \quad k_{2a} = k_2 - p_2, \quad k_{3a} = k_3, \quad k_{4a} = k_4. \quad \text{(15)}$$

In (14), the first exponential term is a constant which depends on target position. This term has no effect on the focusing process. It can be ignored if a magnitude image is the final product [14, 25]. Therefore, this term is ignored in the following algorithm.

3.3. Secondary Range Compression (SRC). With the principle of stationary phase (POSP) and the method of series reversion (MSR) [26, 27], (14) is transformed into the 2D frequency domain as follows:

$$S_6(f_r, f_a) = W_r(f_r) W_a(f_a) P(f_r) \exp(j \Phi(f_r, f_a)) \exp(-2\pi f_a t_n), \quad \text{(16)}$$

where $\Phi(f_r, f_a)$ is expressed as (17), and the second exponential term represents the target azimuth position.

$$\Phi(f_r, f_a) = -2\pi \frac{2(f_c + f_r)}{c} r_{LRCM}$$

$$+ 2\pi \frac{1}{4k_{2a}} \frac{c}{2(f_c + f_r)} \left( f_a + \frac{2(f_c + f_r)}{c} k_{1a} \right)^2$$

$$+ 2\pi \frac{k_{3a}}{8k_{2a}} \left( \frac{c}{2(f_c + f_r)} \right)^2 \left( f_a + \frac{2(f_c + f_r)}{c} k_{1a} \right)^3$$

$$+ 2\pi \frac{9k_{2a}^2 - 4k_{2a} k_{4a}}{64k_{2a}^2} \left( \frac{c}{2(f_c + f_r)} \right)^3$$

$$\cdot \left( f_a + \frac{2(f_c + f_r)}{c} k_{1a} \right)^4.$$

Expanding (17) into a power series of $f_r$ yields

$$\Phi(f_r, f_a) = \Phi_0(f_a) + \Phi_1(f_a) f_r + \Phi_2(f_a) f_r^2 \quad + \sum_{n=3}^{\infty} \Phi_n(f_a) f_r^n,$$

where $\Phi_0(f_a)$, $\Phi_1(f_a)$, $\Phi_2(f_a)$, and $\Phi_n(f_a)$ can be found in Appendix. In (18), $\Phi_0(f_a)$ is the azimuth modulated term, $\Phi_1(f_a)$ contains the information of residual RCM, $\Phi_2(f_a)$ represents the coupling of range and azimuth, also called secondary SRC term, and $\Phi_n(f_a)$ ($n \geq 3$) shows the higher-order cross coupling terms. In this paper, the range of reference point was used to realize the range compensation, and SRC function is given as follows:
\[ H_{SRC}(f_r, f_a) = \exp(-j(\phi - \phi_0)) \exp\left(-j\frac{4\pi r_{RCM}}{c} f_r\right). \]  

Multiplying (19) with (16) yields
\[ S_r(f_r, f_a) = W_r(f_r) W_a(f_a) P(f_a) \exp(j \phi_0(f_a)) \]
\[ \exp\left(-j\frac{4\pi r_{RCM}}{c} f_r\right) \exp(-j2\pi f_a t_n). \]  

3.4. Range Compression (RC). RC is a procedure in which the echo signal is multiplied by the conjugate of the radar transmitting signal to improve the range resolution. In this study, RC function is given as follows:
\[ H_{RC}(f_r) = \overline{P(f_r)}. \]

where \( \overline{P(f_r)} \) is the conjugate of the Gaussian pulse. Multiplying (21) with (20) and transforming the result into the range time domain by range inverse FFT (iFFT) yields
\[ S_a(t_r, f_a) = W_a(f_a) \sin\left(\pi f_a t_n \left(1 - \frac{2r_{RCM}}{c}\right)\right) \exp(j \phi_0(f_a)) \exp(-j2\pi f_a t_n), \]

where \( B_r \) is the signal bandwidth.

3.5. Azimuth Distortion Compensation (ADC). Rewriting \( \Phi_0(f_a) \) as \( \psi(f_a) \) and expanding \( \psi(f_a) \) into Taylor’s series of \( f_a \) at \( f_a = 0 \) yields
\[ \phi_0(f_a) = \psi(f_a) = \pi \psi_0 + \pi \psi_1 f_a + \pi \psi_2 f_a^2 \]
\[ + \pi \psi_3 f_a^3 + \pi \psi_4 f_a^4, \]  

where \( \psi_0, \psi_1, \psi_2, \psi_3, \) and \( \psi_4 \) are expressed in Appendix. The first term is the inherent phase information, the second term represents the azimuth distortion, the third one is the azimuth modulation term, and the fourth and fifth show the cubic and quartic phase terms. The azimuth distortion term may cause the deviation of the target azimuth position. In order to eliminate the azimuth distortion term, the azimuth distortion compensation function is described as follows:
\[ H_{ADC}(f_a) = \exp(-j\pi \psi_4 f_a). \]  

Substituting (23) into (22) and then multiplying with (24) yields
\[ S_a(t_r, f_a) = W_a(f_a) \sin\left(\pi f_a t_n \left(1 - \frac{2r_{RCM}}{c}\right)\right) \exp(-j2\pi f_a t_n) \]
\[ \exp\left(j\pi \psi_0 - \frac{1}{K_a} f_a^2 + \pi \psi_3 f_a^3 + \pi \psi_4 f_a^4\right), \]

where \( K_a \) is the azimuth FM rate expressed as follows:
\[ K_a = -\frac{32k_2^2 f_c}{8k_2 c + 12k_1 k_2^2 k_3 c + 3k_2^2 c (9k_3 - 4k_2 k_4)}. \]  

3.6. Azimuth Nonlinear Chirp Scaling (ANCS). It can be seen that the azimuth FM rate \( K_a \) contains \( k_{1a}, k_{2a}, k_{3a}, \) and \( k_{4a} \), which are expressed in (15). Except \( k_{1a} \), which is a constant, the other three are azimuth-dependent. Therefore, \( K_a \) also varies with the target azimuth position. It is necessary to model the azimuth-dependent \( K_a \). First, \( k_{2a}, k_{3a}, \) and \( k_{4a} \) are expanded in terms of \( t_n \) as follows:
\[ k_{2n} = k_{2ac} + k_{2an} t_n, \]
\[ k_{3n} = k_{3ac} + k_{3an} t_n, \]
\[ k_{4n} = k_{4ac} + k_{4an} t_n, \]  

where \( k_{2ac}, k_{3ac}, \) and \( k_{4ac} \) and \( k_{2an}, k_{3an}, \) and \( k_{4an} \) are the azimuth-invariant terms and \( k_{2an}, k_{3an}, \) and \( k_{4an} \) are the coefficients of the azimuth-variant terms. Then, \( K_a \) is expanded in terms of \( t_n \) and kept up to the second-order as follows:
\[ K_a = K_{a0} + K_{a1} t_n + K_{a2} t_n^2. \]  

It can be seen from (28) that the azimuth-dependent characteristic of the azimuth FM rate is very similar to the range-dependent characteristic of range FM rate. Thus, the range nonlinear chirp scaling algorithm can also be applied to azimuth signal for equalizing the azimuth FM rate, which
has been described in [15–18]. The derivation of ANCS is expressed in the following.

First, the cubic phase term $\psi_3$ is evaluated along the azimuth position as follows:

$$\psi_3 = \psi_{3c} + \psi_{3a} t_n,$$  \hspace{1cm} (29)

where $\psi_{3c}$ is the azimuth-invariant term and $\psi_{3a}$ is the coefficient of the azimuth-variant term. In order to eliminate the azimuth-independent cubic phase term and quartic phase term, the phase compensation factor is given as follows:

$$H_{PA} = \exp(-j\pi\psi_{3c} f_a^3)\exp(-j\pi\psi_4 f_a^4).$$ \hspace{1cm} (30)

Multiplying (30) with (25) yields

$$S_{10}(t_r, f_a) = W_a(f_a)\sin\left(B_a\left(t_r - \frac{2r_{LRCM}}{c}\right)\right)\exp(-j2\pi f_a t_n) \exp\left(j\left(\pi \psi_0 - \pi \frac{1}{K_a} f_a^2 + \pi \psi_{3a} t_n f_a^3\right)\right).$$ \hspace{1cm} (31)

Then, a fourth-order azimuth filter function is designed to filter the phase term, and this function is shown as follows:

$$H_{AB}(f_a) = \exp\left(j\pi Y_3 f_a^3 + j\pi Y_4 f_a^4\right),$$ \hspace{1cm} (32)

where $Y_3$ and $Y_4$ are the coefficients which will be determined later. Multiplying (32) with (31) yields

$$S_{11}(t_r, f_a) = W_a(f_a)\sin\left(B_a\left(t_r - \frac{2r_{LRCM}}{c}\right)\right)\exp(-j2\pi f_a t_n) \exp\left(j\left(\pi \psi_0 - \pi \frac{1}{K_a} f_a^2 + \pi (Y_3 + \psi_{3a} t_n) f_a^3 + \pi Y_4 f_a^4\right)\right).$$ \hspace{1cm} (33)

Transforming (33) into the azimuth time domain yields

$$S_{12}(t_r, t_a) = w_a(t_a)\sin\left(B_a\left(t_r - \frac{2r_{LRCM}}{c}\right)\right) \exp\left(j\pi K_a (t_a - t_n)^2\right) \exp\left(j\pi (Y_3 + \psi_{3a} t_n) K_a^2 (t_a - t_n)^3\right) \exp\left(j\pi Y_4 K_a^4 (t_a - t_n)^4\right).$$ \hspace{1cm} (34)

Then, a fourth-order chirp scaling factor is constructed to eliminate the azimuth-dependent azimuth FM rate, and this factor is given by

$$H_{ANCS}(t_a) = \exp\left(j\pi q_2 t_a^2 + j\pi q_3 t_a^3 + j\pi q_4 t_a^4\right).$$ \hspace{1cm} (35)

Multiplying (35) with (34) yields

$$S_{13}(t_r, t_a) = w_a(t_a)\sin\left(B_a\left(t_r - \frac{2r_{LRCM}}{c}\right)\right) \exp\left(j\pi K_a (t_a - t_n)^3\right) \exp\left(j\pi (Y_3 + \psi_{3a} t_n) K_a^2 (t_a - t_n)^3\right) \exp\left(j\pi Y_4 K_a^4 (t_a - t_n)^3\right).$$ \hspace{1cm} (36)

The POSP is applied to transform (36) into the azimuth frequency domain:

$$S_{14}(t_r, f_a) = W_a\left(\frac{f_a - q_2 t_n}{K_a + q_2}\right) \sin\left(B_a\left(t_r - \frac{2r_{LRCM}}{c}\right)\right) \exp\left(j\pi (f_a - K_a t_n)^2\right) \exp\left(j\pi (Y_3 + \psi_{3a} t_n) K_a^2 (f_a - q_2 t_n)^3\right) \exp\left(j\pi Y_4 K_a^4 (f_a - q_2 t_n)^4\right).$$ \hspace{1cm} (37)

where

$$\Theta(f_a) = \frac{-2\pi f_a}{K_a + q_2} + \frac{\pi}{(K_a + q_2)^2} \left(K_a(f_a - q_2 t_n)^2 + q_2(f_a + K_a t_n)^2\right) + \frac{\pi}{(K_a + q_2)^2} \left((Y_3 + \psi_{3a} t_n) K_a^2 (f_a - q_2 t_n)^3\right) + q_3(f_a + K_a t_n)^3) \exp\left(j\pi (f_a - q_2 t_n)^4 + q_4(f_a + K_a t_n)^4\right).$$ \hspace{1cm} (38)

Substitute (28) into (38); $\Theta(f_a)$ is approximately expressed as a power series of $t_n$ and $f_a$ as follows:

$$\Theta(f_a) = C_0(q_2, q_3, q_4, Y_3, Y_4, f_a, f_a^2, f_a^3, f_a^4) + C_1(q_2, q_3, q_4, Y_3, Y_4, t_n f_a) + C_2(q_2, q_3, q_4, Y_3, Y_4, t_n^2 f_a) + C_3(q_2, q_3, q_4, Y_3, Y_4, t_n^3 f_a) + C_4(q_2, q_3, q_4, Y_3, Y_4, t_n^4 f_a) + C_5(q_2, q_3, q_4, Y_3, Y_4, t_n^5 f_a) + C_6(q_2, q_3, q_4, Y_3, Y_4, t_n^6 f_a) + \Theta_{7m}(q_2, q_3, q_4, Y_3, Y_4, t_n, f_a).$$ \hspace{1cm} (39)

In (39), the first term is the azimuth-independent phase modulation. The second term is the target real azimuth position. The third can cause geometric distortion along target azimuth direction. The fourth, fifth, and sixth are the azimuth-dependent phase modulation terms. The seventh is independent of the azimuth frequency. And the last one,
which represents the summation of other expansion series terms, is so small that it can be neglected.

To eliminate the azimuth-dependent modulation and azimuth distortion, the coefficients $C_1$, $C_2$, $C_3$, $C_4$, and $C_5$ are set as follows:

$$C_1 = \frac{\pi}{\alpha},$$
$$C_2 = 0,$$
$$C_3 = 0,$$
$$C_4 = 0,$$
$$C_5 = 0,$$

where $\alpha$ is the scaling factor ($\alpha \neq 0.5$).

Through solving (39) and (40), the coefficients $q_2$, $q_3$, $q_4$, $Y_3$, and $Y_4$ are obtained:

$$q_2 = (2\alpha - 1)K_{a0},$$
$$q_3 = \frac{(2\alpha - 1)}{3}K_{a1},$$
$$q_4 = \frac{(10\alpha - 5)K_{a2} + 3(2\alpha - 1)\psi_{\delta a}K_{a0}^3}{12K_{a0}},$$

$$Y_3 = \frac{K_{a1}(4\alpha - 1)}{3K_{a0}^2(2\alpha - 1)},$$
$$Y_4 = \frac{(16\alpha - 5)K_{a2} + 3(4\alpha - 1)\psi_{\delta a}K_{a0}^3}{12(2\alpha - 1)K_{a0}^2}.$$  (41)

Substituting (28) and (41) into (37) yields

$$S_{15}(t_r, f_a) = W_a\left(\frac{f_a - q_2 f_n}{K_{a0} + q_2 f_n}\right) \sin c\left( B_a\left( t_r - \frac{2r_{\text{LRCM}}}{c}\right) \right) \exp\left(-j2\pi f_a \frac{t_n}{2\alpha}\right) \exp\left(-j\pi \frac{1}{K_{a0} + q_2 f_n^2}\right) \exp\left(j\pi \frac{Y_3 K_{a0}^3 + q_3 f_n^3}{(K_{a0} + q_2)^3 f_n^3} + j\pi \frac{Y_4 K_{a0}^4 + q_4 f_n^4}{(K_{a0} + q_2)^2 f_n^4}\right).$$  (42)

It can be seen from (42) that the azimuth-dependent modulation and azimuth distortion have been completely eliminated. In (42), the first exponential term contains the azimuth position of target. The residual exponential term is the azimuth compression factor.

3.7. Azimuth Compression (AC). AC is performed after ANCS processing. AC function is given by

$$H_{AC}(f_a) = \exp\left(j\pi \frac{1}{K_{a0} + q_2 f_n^2}\right) \exp\left(-j\pi \frac{Y_3 K_{a0}^3 + q_3 f_n^3}{(K_{a0} + q_2)^3 f_n^3}\right) \exp\left(-j\pi \frac{Y_4 K_{a0}^4 + q_4 f_n^4}{(K_{a0} + q_2)^2 f_n^4}\right).$$  (43)

Multiplying (43) with (42) and transforming the result into the azimuth time domain yields

$$S_{16}(t_r, t_a) = \sin c\left( B_a\left( t_a - \frac{2r_{\text{LRCM}}}{c}\right) \right) \sin c\left( B_a\left( t_a - \frac{t_n}{2\alpha}\right) \right).$$  (44)

where $B_a$ is the Doppler bandwidth. From (44), it is seen that the target is focused at $(t_a/2\alpha, r_{\text{LRCM}})$, which needs to be corrected by geometry correction to obtain the real position using

$$t_{nc} = \frac{t_a}{2\alpha},$$
$$r_{LC} = r_{\text{LRCM}} - \frac{t_a}{2\alpha} v \sin \theta_i.$$  (45)

4. Simulation

To verify the effectiveness of the proposed algorithm, the simulation with parameters in Table 1 was carried out. The Gaussian pulse was defined as (46), and its waveforms in the time and frequency domain are shown in Figure 6.

$$p(t_r) = \exp\left(-\left(\frac{t_r - 3 \times 2.2 \times 10^5}{2.2 \times 10^5}\right)^2\right) \cos\left(2\pi \times 4.3 \times 10^5 \times t_r\right).$$  (46)

There were five targets in the simulation, whose 3D coordinates were A (1.15 m, 0 m, 0.10 m), B (1.15 m, 0 m, 1.04 m), C (1.15 m, 0 m, 1.50 m), D (0.67 m, 0 m, 1.50 m), and E (1.64 m, 0 m, 0.10 m), respectively. The distribution of A–E in the azimuth and range direction is shown in Figure 7. The targets A, B, and C are located in the same azimuth cell. The targets D, B, and E will be lay in the same range cell after LRWC. The target B was chosen as the reference point. The coordinates of these five targets are set so that the performance of the proposed algorithm can be easily shown. The scaling factor $\alpha$ is set to 0.505. In this study, the imaging results are obtained without any processing of the weighting function or side-lobe control.

Figure 8 shows the RCM trajectories of targets D, B, and E before and after RCMAS. In Figure 8(a), it was observed that RCM trajectory of target B, as well as targets D and E, was approximated into a straight line, which illustrated LRWC was the major component of RCM. After the reference range LRWC, targets D, B, and E were located in the
same range cell. LRCM of the target B had been eliminated. However, because of the range-dependent characteristic of the squint angle, targets D and E still had different residual LRCM. As shown in Figure 8(c), after RCMAS, RCM trajectories of D and E were azimuth-invariant, which proved that the azimuth-dependent residual LRCM caused by range-dependent squint angle had been eliminated.

In order to evaluate the performance of the proposed algorithm, two methods, the extended NCS (ENCS) algorithm in [19] and the proposed algorithm, were utilized to process the simulated SAR data. Figure 9 shows the imaging results using these two algorithms. The color in Figure 9 represents the scattering intensity of target. In the simulation imaging, the deep red denoted the target point, and the dark blue showed there was no target. Because of the influence of side lobe, there also existed light blue. It can be easily seen that five targets were focused on the true position in Figure 9(b). However, in Figure 9(a), targets A, C, D, and E all had the azimuth shift compared with the true position. Targets A and E shifted along the azimuth positive direction, and C and D shifted along the azimuth negative direction. In the ENCS algorithm, because the range-dependent characteristic of squint angle was not considered, there was no the residual LRCM term after LRWC, that is, \( k_{1res} \) in (6) was equal to zero. This could cause that \( \psi_1 \) was equal to zero and azimuth distortion compensation processing was not operated. Therefore, the focused targets had the azimuth shift using ENCS algorithm. In addition, there was defocus in targets A, C, D, and E because their RCMs were not fully removed. The proposed algorithm improved the image.

![Figure 6: The Gaussian pulse waveform in the (a) time domain and (b) frequency domain.](image-url)
quality as shown in Figure 9(b). The imaging result contained the range and azimuth information of targets. Table 3 shows the target position error in the simulated images using ENCS algorithm and the proposed algorithm. The range errors of D and E with the ENCS algorithm were much larger than those with the proposed algorithm because RCMs of D and E did not make a precise compensation. In the azimuth, because of the impact of the azimuth shift, the errors of
targets A, C, D, and E with ENCS algorithm were much larger than those with the proposed algorithm. This proved that the proposed algorithm performed well in the near-field environment.

5. Conclusion

In the near-field environment, the range-dependent squint angle of the target needs to be considered, which causes that there exists the residual LRCM of azimuth-dependent after reference range LRWC. In order to overcome the azimuth-dependent characteristic of the residual LRCM, the residual LRCM is expanded into azimuth-invariant and azimuth-variant terms. A novel perturbation function in RCMAS is designed to remove the azimuth-variant term. In the azimuth focusing, a filter function and a scaling function are applied to eliminate the high-order Doppler phases and equalize FM rate. The whole procedures only consist of FFT, IFFT, and multiplications operations without interpolation, which means higher efficiency and easier implementation. The simulation proves that more accurate and improved imaging result can be obtained with the proposed algorithm.

Appendix

$\psi_0(f_a)$, $\psi_1(f_a)$, $\psi_2(f_a)$, and $\psi_3(f_a)$ in (18) are expressed as (A.1)–(A.5). $\psi_0, \psi_1, \psi_2, \psi_3,$ and $\psi_4$ in (23) are expressed as (A.5).
\[
\phi_n(f_a) \approx \frac{1}{n!} \frac{d^n \phi(f_r, f_a)}{df_r^n} \bigg|_{f_r = 0}, \quad (A.4)
\]

\[\psi_0 = -4 f_c f_{r, \text{LRCM}} \frac{k_{in}^2 k_{in}}{k_{2n}^2 c} \frac{9 k_{3n}^2 - 4 k_{2n} k_{4n}}{2 k_{2n}^2 c} \frac{k_{1n}^2 k_{3n} f_c k_{2n}}{16 k_{2n}^2 c},\]

\[\psi_1 = \frac{k_{in}^2}{k_{2n} c} + \frac{3 k_{3n}^2 k_{3n} c}{4 k_{2n}^3} + \frac{(9 k_{3n}^2 - 4 k_{2n}^2 k_{4n})^2 k_{1n}^2}{8 k_{2n}^5 c},\]

\[\psi_2 = \frac{c}{4 k_{2n} f_c} + \frac{3 k_{3n}^2 k_{3n} c}{8 k_{2n}^2 f_c} + \frac{(9 k_{3n}^2 - 4 k_{2n}^2 k_{4n})^2 k_{1n}^2}{32 k_{2n}^3 f_c},\]

\[\psi_3 = \frac{k_{3n}^2}{16 k_{2n}^2 f_c^2} + \frac{(9 k_{3n}^2 - 4 k_{2n}^2 k_{4n}) k_{1n}^2}{32 k_{2n}^3 f_c^2},\]

\[\psi_4 = \frac{(9 k_{3n}^2 - 4 k_{2n}^2 k_{4n}) c^3}{256 k_{2n}^5 f_c^3}.\]  

(A.5)

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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