Skin effect with arbitrary specularity in Maxwellian plasma

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The problem of skin effect with arbitrary specularity in maxwellian plasma with specular–diffuse boundary conditions is solved. New analytical method is developed that makes it possible to obtain a solution up to an arbitrary degree of accuracy. The method is based on the idea of symmetric continuation of not only the electric field, but also electron distribution function. The solution is obtained in a form of von Neumann series.

Keywords: skin effect, specular–diffuse boundary conditions, analytical method, von Neumann series.

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I. INTRODUCTION

The skin effect problem is one of the most important problems in plasma kinetic theory (see, for example, in Refs. [1]–[3]). The skin effect in plasma is a response of electron gas to external transverse electromagnetic field. The problem also has great practical importance.

The solution of the skin effect problem with specular reflection boundary conditions is well-known [1], [2]. The analytic solution of the problem with diffuse reflection boundary conditions has been obtained in the middle of the previous century (see, for example, in Ref. [2]).

The skin effect problem with general specular – diffuse reflection boundary conditions is not solved till now. It’s well known that specularity coefficient $q$ is a very important factor in the kinetic skin effect theory [2]–[4]. The limiting cases $q = 0$ (diffuse surface scattering of electrons) and $q = 1$ (specular surface scattering of electrons) are only very special cases. Actually the specularity coefficient $q$ equals neither to zero, nor unit, and takes some intermediate values on.

So, for example, in the work [4] it is shown, that the specularity coefficient $q$ is equal to 0.4 in Na wire. In this connection the skin effect problem with specular–diffuse boundary conditions has exclusively fundamental significance. Its value is great for theories, and for practical applications. So it’s obvious that the solution of the skin effect problem with general specular–diffuse reflection boundary conditions is a very important task.

The method of solution of this problem for degenerate plasma in metal has been developed in Ref. [6]. This method is based on the use of von Neumann series. Authors in Ref. [6] have demonstrated high efficiency of the method developed for computation of the skin effect characteristics. The goal of this work is generalization of the method developed in Ref. [6] in the case of gaseous plasma.

By method of decomposition of the solution by eigenfunctions of the corresponding characteristic equation exact solutions of the skin effect problem in metal for diffuse and specular boundary conditions are received in Refs. [7, 8].

In the last years interest to skin effect problems continues to grow (see, for example, Refs. [11]–[16]). In particular, in Ref. [16] in limiting anomalous skin effect conditions, the oblique electromagnetic wave reflection from the sharp plasma boundary in an assumption of mixed (specular and diffuse) electron reflection from the boundary is considered.

II. PROBLEM STATEMENT

Let’s gaseous (nondegenerate) plasma occupy a half-space $x > 0$. The distribution function of electrons $f = f(t, r, v)$ is normalized by electron numerical density (concentration of electrons):

$$
\int f(t, r, v) d^3v = n(t, r),
$$

where $p = mv$ is the electron momentum, $m$ is the electron mass, $e$ is the electron charge, $d^3v = dv_x dv_y dv_z$.

We consider electromagnetic wave which propagates in direction orthogonal to the plasma surface. Then the external field has only one $y$–component. The internal field inside plasma has only $y$–component

$$
E_y(t, x) = e^{-i\omega t} E(x)
$$
too, where $\omega$ is the field frequency.

To describe the electron distribution function we will use the Vlasov — Boltzmann kinetic equation. The collision integral will be represented in the form of $\tau$–model

$$
\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + eE_y(x, t) \frac{\partial f}{\partial p_y} = \frac{f_0(v) - f(x, v, t)}{\tau},
$$

where $\tau$ is the time between two electrons collisions, $\tau = 1/\nu$, $\nu$ is the effective electron collision frequency, $f_0(v)$ is the equilibrium maxwellian distribution function,

$$
f_0(v) = n \left( \frac{m}{2\pi k_B T} \right)^{3/2} \exp \left( - \frac{mv^2}{2k_B T} \right),
$$
For weak fields this equation may be linearized:

\[ f = f_0(C) \left( 1 + C_y h(x, C_x) e^{-i\omega t} \right). \] (1.1)

Here

\[ f_0(C) = n \left( \frac{\beta}{\pi} \right)^{3/2} \exp \left( -C^2 \right), \]

where \( \beta = m/(2k_B T) \), \( C = \sqrt{\beta} \nu \), \( C \) is the dimensionless electron velocity, \( k_B \) is the Boltzmann constant, \( T \) is the plasma temperature.

For function \( h(x, C_x) \) we have the following kinetic equation:

\[ \mu \frac{\partial h(x_1, \mu)}{\partial x_1} + z_0 h(x_1, \mu) = e(x_1), \] (1.2)

where

\[ z_0 = 1 - i \frac{\omega}{\nu}. \]

In the equation (1.2) \( x_1 \) is the dimensionless coordinate,

\[ \mu = v_x \sqrt{\beta}, \quad x_1 = \frac{x}{\gamma} = \nu \sqrt{\beta} x, \quad t_1 = \nu t, \]

\( t_1 \) is the dimensionless time, \( l \) is electron free path and \( e(x_1) \) is the dimensionless electric field:

\[ e(x_1) = \frac{\sqrt{2} e}{\nu \sqrt{mk_B T}} E(x_1). \]

We will neglect displacement current. Then the equation for electric field may be written in the form:

\[ e''(x_1) = -\frac{4\pi i \omega}{\nu^2} j_y(x_1), \] (1.3)

where \( j_y(x_1) \) is the electric current,

\[ j_y(x_1) = e \int v_y \left[ f_0(v) + \sqrt{\beta} v_y e^{-i\omega t} h(x, v_x) \right] dv. \] (1.4)

Let’s extend the electric field and the electric distribution function on the "negative" half-space \( x < 0 \) in symmetric manner. For the functions \( E(x) \) and \( h(x, \mu) \) we will then write:

\[ e(x_1) = e(-x_1), \quad h(x_1, \mu) = h(-x_1, -\mu). \] (1.5)

We may rewrite the equation (1.4) with use of dimensional parameter \( \alpha \):

\[ \frac{d^2 e(x_1)}{dx_1^2} = -i \frac{\alpha}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-\mu'^2) h(x_1, \mu') d\mu'. \] (1.6)

Here

\[ \alpha = \frac{4\pi e^2 \nu \omega}{c^2 \beta \nu^4 m} = 2 \left( \frac{1}{\delta} \right)^2, \quad \delta^2 = \frac{c^2}{2\pi \omega \sigma}, \]

\[ \sigma = \frac{e n}{m \nu}, \quad l = v_T \tau, \quad v_T = \frac{1}{\sqrt{\beta}}. \]

\( v_T \) is the electron thermal velocity, \( \delta \) is the penetration length of external electric field for normal skin effect, \( \sigma \) is the plasma conductivity.

By extension procedure (1.5) on the half-space \( x < 0 \) we may include the surface conditions in the equation for skin effect problem.

Specular – diffuse boundary conditions on the boundaries of positive and negative half-spaces may be written in the form:

\[ h(+0, \mu) = q h(+0, -\mu), \quad 0 < \mu < 1, \]

\[ h(-0, \mu) = q h(-0, -\mu), \quad -1 < \mu < 0, \]

where \( q \) is the specularity coefficient, \( 0 \leq q \leq 1 \).

In accordance with (1.5) we obtain:

\[ h(+0, \mu) = q h(-0, \mu), \quad 0 < \mu < 1, \] (1.7)

\[ h(-0, \mu) = q h(+0, \mu), \quad -1 < \mu < 0. \] (1.8)

The required function \( h(x_1, \mu) \) and the electric field must decay away from the surface:

\[ h(+\infty, \mu) = 0, \quad e(+\infty) = 0. \] (1.9)

We assume that the gradient of the electric field is finite and known at the plasma boundary:

\[ e'(0) = e'_s, \quad |e'_s| < +\infty. \] (1.10)

Here, the gradient of the electric field on the plasma boundary \( e'_s \) is given.

**III. CHARACTERISTIC SYSTEM**

The variable \( x_1 \) will be denoted again by \( x \).

Let’s include boundary conditions (1.7) and (1.8) in the kinetic equation (1.2), and boundary condition (1.10) include in the electric field equation (1.6).

As a result we will obtain system of equation for skin effect in half-space of the plasma:

\[ \mu \frac{\partial h}{\partial x} + z_0 h(x, \mu) = \]

\[ e(x) - (1 - q) |\mu| h(\mp 0, \mu) \delta(x), \quad \pm \mu > 0, \] (2.1)

\[ \frac{d^2 e}{dx^2} = -i \frac{\alpha}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-\mu'^2) h(x, \mu') d\mu'. \]
The impedance is determined by formula \[1\]:

\[
Z = \frac{4\pi i\omega E_y(0)}{c^2 E'_y(0)}.
\]

With the use of dimensionless field \( e(x) \) this relation may be rewritten in the form:

\[
Z = \frac{4\pi i\omega l e(0)}{c^2 e'_s}.
\]

From equation (2.1) and boundary conditions (1.9) we obtain the following expression for \( x > 0, \mu < 0 \):

\[
h_+(x, \mu) = -\frac{1}{\mu} \exp\left(-\frac{z_0x}{\mu}\right) \int_{-\infty}^{x} \exp\left(-\frac{z_0t}{\mu}\right) e(t) dt.
\]

In the case \( x < 0, \mu > 0 \) we obtain:

\[
h_-(x, \mu) = \frac{1}{\mu} \exp\left(-\frac{z_0x}{\mu}\right) \int_{-\infty}^{x} \exp\left(-\frac{z_0t}{\mu}\right) e(t) dt.
\]

Then we may rewrite the equation (2.1) in the form:

\[
\frac{\partial h}{\partial x} + z_0h(x, \mu) - e(x) = -(1-q)|\mu|h_{\pm}(0, \mu)\delta(x), \pm\mu > 0.
\]

The solution of the equations (2.2) and (2.3) we will seek in the form of Fourier integrals:

\[
e(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} E(k) dk, \quad (2.4)
\]

\[
\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} dk,
\]

\[
h(x, \mu) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} \Phi(k, \mu) dk. \quad (2.5)
\]

Then for the function \( h_+(x, \mu) \) the following expression may be derived:

\[
h_+(x, \mu) = -\frac{\exp(\frac{z_0x}{\mu})}{2\pi \mu} \times
\]

\[
\times \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dt \exp\left(ikt + \frac{z_0t}{\mu}\right) E(k) =
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\exp(ikx)E(k)}{z_0 + ik\mu} dk. \quad (2.6)
\]

It’s may be proved, that the expression for \( h_-(x, \mu) \) coincides with the expression for \( h_+(x, \mu) \). Therefore we have

\[
h_{\pm}(0, \mu) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{E(k) dk}{z_0 + ik\mu}.
\]

We substitute the expressions (2.4), (2.5) and (2.6) into the equations (2.2) and (2.3). This procedure leads to characteristic system of equations:

\[
\Phi(k, \mu)(z_0 + ik\mu) =
\]

\[
E(k) - (1-q)|\mu| \int_{-\infty}^{\infty} \frac{E(k_1) dk_1}{z_0 + ik_1\mu},
\]

\[
-\frac{k^2}{2}E(k) = 2e_s' - i\alpha \int_{-\infty}^{\infty} \exp(-\mu^2)\Phi(k, \mu) d\mu. \quad (2.8)
\]

The function \( e(x) \) is an even function. Then \( E(-k) = E(k) \), and equation (2.7) may be rewritten as

\[
\Phi(k, \mu)(z_0 + ik\mu) =
\]

\[
E(k) - (1-q)|\mu| \int_{-\infty}^{\infty} \frac{E(k_1) dk_1}{z_0 + ik_1\mu^2}.
\]

Let’s substitute the expression (2.9) into the equation (2.8). Then we obtain:

\[
L(k)E(k) = -2e_s' +
\]

\[
(1-q)\frac{\alpha z_0^2}{\pi i} \int_{-\infty}^{\infty} E(k_1) J(k, k_1) dk_1. \quad (2.10)
\]

Here \( L(k) \) is the dispersion function,

\[
L(k) = k^2 - i\frac{\alpha}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-\mu^2) d\mu}{z_0 + ik\mu} =
\]

\[
k^2 - \frac{2iz_0\alpha}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-\mu^2) d\mu}{z_0^2 + k^2\mu^2},
\]
and \( J(k, k_1) \) is the integral

\[
J(k, k_1) = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{\exp(-u^2)}{(z_0^2 + k^2u)(z_0^2 + k_1^2u)}. 
\]

Characteristic system consists of two equations (2.9) and (2.10).

The integral \( J(k_1, k_2) \) we will express through the integral exponential function. We will spread out integrand partial fractions:

\[
\frac{1}{(z_0^2 + k_1^2u)(z_0^2 + k_1^2u)} = \frac{A}{z_0^2 + k_1^2u} + \frac{B}{z_0^2 + k_1^2u},
\]

where

\[
A = -\frac{k_1^2}{z_0^2(k_1^2 - k_1^2)}, \quad B = \frac{k_2^2}{z_0^2(k_1^2 - k_1^2)}.
\]

Thus, we receive, that

\[
J(k_1, k_2) = A J_0(k_1) + B J_0(k_2),
\]

where

\[
J_0(k_j) = \int_0^\infty \frac{e^{-u} du}{z_0^2 + k_j^2u} = \frac{1}{k_j^2} \exp\left(\frac{-2}{k_j^2}\right) \int_0^\infty \exp\left(-\frac{x}{k_j^2}\right) dx, \quad j = 1, 2.
\]

**IV. SOLUTION OF THE PROBLEM WITH THE USE OF VON NEUMANN SERIES**

Let’s expand the solution of equations (2.9), (2.10) by the following series:

\[
E(k) = E_0(k) + (1-q) E_1(k) + (1-q)^2 E_2(k) + \cdots, \quad (3.1)
\]

\[
\Phi(k, \mu) = \Phi_0(k, \mu) + (1-q) \Phi_1(k, \mu) + (1-q)^2 \Phi_2(k, \mu) + \cdots. \quad (3.2)
\]

Functions \( E_j(k) \) and \( \Phi_j(k, \mu) \) \((j = 1, 2, 3, \cdots)\) may be obtained from the characteristic system. For zero approximation we have:

\[
E_0(k) = -\frac{2e_s'}{L(k)} \int_0^\infty E_0(k_1) J(k, k_1) dk_1, \quad (3.4)
\]

\[
\Phi_1(k, \mu) = \frac{E_1(k)}{z_0 + i k \mu} - \frac{z_0|\mu|}{(z_0 + i k \mu)\pi} \int_0^\infty E_0(k_1)J(k_1) dk_1. \quad (3.5)
\]

For \( n \)-approximation the following expression may be derived:

\[
E_n(k) = \frac{\alpha z_0^2}{L(k)\pi i} \int_0^\infty E_{n-1}(k_1)J(k, k_1) dk_1, \quad n = 1, 2, \cdots, \quad (3.6)
\]

\[
\Phi_n(k, \mu) = \frac{E_n(k)}{z_0 + i k \mu} - \frac{z_0|\mu|}{(z_0 + i k \mu)\pi} \int_0^\infty E_{n-1}(k_1)J(k_1) dk_1, \quad n = 1, 2, \cdots. \quad (3.7)
\]

We may rewrite the expressions (3.1) in the form

\[
E_1(k) = -2e_s' \frac{\alpha z_0^2}{\pi i} \int_0^\infty \frac{J(k, k_1)}{L(k)L(k_1)} dk_1,
\]

\[
E_2(k) = -2e_s' \left(\frac{\alpha z_0^2}{\pi i}\right)^2 \times
\]

\[
\times \int_0^\infty \int_0^\infty \frac{J(k, k_1)J(k_1, k_2)}{L(k)L(k_1)L(k_2)} dk_1 dk_2, \cdots.
\]

In general case when \( n = 1, 2, 3, \cdots, \) we have:

\[
E_n(k) = -2e_s' \left(\frac{\alpha z_0^2}{\pi i}\right)^n \times
\]

\[
\times \int_0^\infty \cdots \int_0^\infty \frac{J(k, k_1)J(k_1, k_2) \cdots J(k_{n-1}, k_n)}{L(k)L(k_1) \cdots L(k_n)} dk_1 \cdots dk_n. \quad (3.8)
\]

Therefore the series (3.1) constructed may be expressed in the explicit form:

\[
E(k) = -2e_s' \frac{L(k)}{L(k)} \left[ 1 + \sum_{n=1}^\infty (1-q)^n \left(\frac{\alpha z_0^2}{\pi i}\right)^n \times
\]

\[
\times \int_0^\infty \cdots \int_0^\infty \frac{J(k, k_1)J(k_1, k_2) \cdots J(k_{n-1}, k_n)}{L(k_1) \cdots L(k_n)} dk_1 \cdots dk_n \right].
\]
V. ELECTRIC FIELD, DISTRIBUTION FUNCTION AND SURFACE IMPEDANCE

In accordance with (2.4) and (2.5) we will construct expressions for electric field and distribution function. Using the expressions (3.1) and (3.2) we obtain:

\[
e(x) = \frac{1}{\pi} \sum_{n=0}^{\infty} (1 - q)^n \int_{0}^{\infty} E_n(k) \cos kx \, dk, \quad (4.1)
\]

\[
h(x, \mu) = \frac{1}{\pi} \sum_{n=0}^{\infty} (1 - q)^n \int_{-\infty}^{\infty} e^{ikx} \Phi_n(k, \mu) \, dk. \quad (4.2)
\]

We rewrite the expression (4.1) with the use of (3.8) in the following form:

\[
e(x) = -\frac{2e_s}{\pi} \int_{0}^{\infty} \cos kx \, dk \left[ 1 + \sum_{n=1}^{\infty} (1 - q)^n \left( \frac{\alpha z_0^2}{\pi i} \right)^n \times \right.
\]

\[
\times \int_{0}^{\infty} \cdots \int_{0}^{\infty} \frac{J(k, k_1) J(k_1, k_2) \cdots J(k_{n-1}, k_n)}{L(k_1) \cdots L(k_n)} \, dk_1 \cdots dk_n \left. \right].
\]

Function \( h(x, \mu) \) may be written in the form:

\[
h(x, \mu) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ E(k) - (1 - q) \frac{z_0 |\mu|}{\pi} \int_{0}^{\infty} \frac{E(k_1) \, dk_1}{\sqrt{z_0^2 + k_1^2 \mu^2}} \right] e^{ikx} \, dk.
\]

Expression (4.3) may be written also in the next form:

\[
h(x, \mu) = \frac{1}{\pi} \int_{0}^{\infty} \frac{z_0 \cos kx + k \mu \sin kx}{z_0^2 + k^2 \mu^2} \left[ E(k) - (1 - q) \frac{z_0 |\mu|}{\pi^2} \int_{0}^{\infty} \frac{E(k_1) \, dk_1}{\sqrt{z_0^2 + k_1^2 \mu^2}} \right] \, dk.
\]

Expression (4.3) may be written also in the next form:

\[
(1 - q) \frac{z_0 |\mu|}{\pi} \int_{0}^{\infty} \frac{E(k_1) \, dk_1}{\sqrt{z_0^2 + k_1^2 \mu^2}} e^{ikx} \, dk.
\]

If we know function \( h(x, \mu) \) we may write down the electron distribution function \( f \) according to the equation (1.1).

Let us consider now the calculation of impedance:

\[
Z = \frac{4i\omega l}{\epsilon^2 \epsilon_s} \int_{0}^{\infty} E(k) \, dk.
\]

We decompose \( Z \) in the following series:

\[
Z = Z_0 + (1 - q)Z_1 + (1 - q)^2Z_2 + \cdots. \quad (4.4)
\]

Here

\[
Z_n = \frac{4i\omega l}{\epsilon^2 \epsilon_s} \int_{0}^{\infty} E_n(k) \, dk, \quad n = 0, 1, 2, \cdots. \quad (4.5)
\]

Now we write down expressions for zero, first and second approximations for impedance (see (4.4) and (4.5))

\[
Z_0 = -\frac{8i\omega l}{\epsilon^2 \pi} \int_{0}^{\infty} \frac{dk}{L(k)} = -\frac{8i\omega l}{\epsilon^2 \pi} \int_{0}^{\infty} \frac{dk}{L(k)},
\]

\[
Z_1 = -\frac{8i\omega l}{\epsilon^2 \pi} \left( \frac{\alpha z_0^2}{\pi i} \right)^2 \int_{0}^{\infty} \cdots \int_{0}^{\infty} \frac{J(k, k_1) \, dk_1}{L(k_1)} \cdots \frac{J(k, k_2) \, dk_2}{L(k_2)}.
\]

Expression for general term of series (4.4) has the form:

\[
Z_n = \frac{8i\omega l}{\epsilon^2} \left( \frac{\alpha z_0^2}{\pi i} \right)^n \times \int_{0}^{\infty} \cdots \int_{0}^{\infty} \frac{J(k, k_1) \, dk_1}{L(k)L(k_1)} \cdots \frac{J(k, k_2) \, dk_2}{L(k_2)}.
\]

\[
\times dk \, dk_1 \cdots dk_n.
\]

VI. ANALYSIS AND DISCUSSION

In the previous sections we have considered the method, leading to exact solution of the skin effect problem with arbitrary specularity coefficient. In the case \( q = 1 \) the method leads to the classical solution (4.6) of the problem with specular surface conditions (see, for example [3, 10, 17]). In [10] this classical solution is represented in the form:

\[
Z_{ref} = \frac{8i\omega l}{\epsilon^2} \int_{0}^{\infty} \frac{d\tau}{\lambda(iz_0 \tau)}, \quad (5.1a)
\]
where

\[ \lambda(i_0 \tau) = 1 - \alpha \tau^3 \int_{-\infty}^{\infty} \frac{\exp(-\mu^2)}{\mu - i_0 \tau} \, d\mu. \]  

(5.1b)

The comparison of the expressions (5.1) and (4.6) gives \( Z_{\text{ref}} = Z_0 \). Indeed, after the change of variables in the integral we have:

\[ \int_0^\infty \frac{d\tau}{\lambda(i_0 \tau)} = \int_0^\infty \frac{d\tau}{\tau^2 \lambda(i_0/\tau)} = \int_0^\infty \frac{dk}{L(k)}. \]

Now let us consider second approximation:

\[ Z = Z_0 + (1 - q)Z_1 + (1 - q)^2Z_2. \]

When \( q = 1 \) this solution is exact. Maximum deviation from exact solution corresponds to the case when \( q = 0 \). The exact solution of the problem in the case \( q = 0 \) is also well known \[3, 10\]:

\[ Z_{\text{dif}} = -\frac{4\pi i\omega}{c^2} \left[ 2 \int_0^\infty \ln \left( \lambda(i_0 \tau) \right) \frac{d\tau}{\tau^2} \right]^{-1}. \]  

(5.2)

It’s convenient to rewrite the expression (5.2) with the use of our notations:

\[ Z_{\text{dif}} = -\frac{4\pi i\omega}{c^2} \left[ 2 \int_0^\infty \ln \left( k^{-2}L(k) \right) \, dk \right]^{-1}. \]

Now consider the ratio of real (and imaginary) parts of the solutions constructed in zero, first and second approximations to the solution in zero approximation \( \Re(Z_0) \) (\( \Im(Z_0) \)) for the case \( q = 0 \). The last solution coincide with the solution of the problem with specular scattering boundary conditions.

We will build two plots (curves 1 and 2):

\[ Y_1 = \frac{\Re(Z_0 + Z_1)}{\Re(Z_0)} = 1 + \frac{\Re(Z_1)}{\Re(Z_0)} \]

and

\[ Y_2 = \frac{\Re(Z_0 + Z_1 + Z_2)}{\Re(Z_0)} = 1 + \frac{\Re(Z_1)}{\Re(Z_0)} + \frac{\Re(Z_2)}{\Re(Z_0)} \]

or

\[ Y_2 = Y_1 + \frac{\Re(Z_2)}{\Re(Z_0)} \]

and also analogous plots for the ratios of the imaginary parts.

Here \( Z_0 \) is the solution for the case of specular surface conditions. Values \( Z_1 \) and \( Z_2 \) correspond to corrections for the first and the second approximations.

The curves 3 on the plots correspond the ratios of impedance for diffuse scattering surface condition to impedance for specular scattering surface condition \( \Re(Z_{\text{dif}})/\Re(Z_{\text{ref}}) \) (\( \Im(Z_{\text{dif}})/\Im(Z_{\text{ref}}) \)).

The derived method has maximum error in the case of extremely anomalous skin effect, when parameter \( \alpha \gg 1 \). In this case ratios defined above are equal to 1.125. So it is obvious from the plots, that in zero approximation the method error is equal to 12.5%.

For the first approximation in this case we have \( Z_{\text{dif}}/Z = 1.03 \). So the first approximation error is equal to 3%. For the second approximation in this case we have \( Z_{\text{dif}}/Z = 1.01 \). And the second approximation error is equal to 1%.

The analysis of plots shows, that the considered impedance ratios in first approximation coincide with the exact solution when \( \alpha < 10^{-1} \). For the second approximation the coincidence is observed when \( \alpha < 1 \).

VII. CONCLUSION

The effective method of the solution of boundary problems of the kinetic theory is developed. This method is based on symmetric continuation of the electric field and distribution function of electrons.

The offered method gives an error less, than 1% in the second approximation already. The method is accurate and allows to construct exact solution in the form of von Neumann series.

The curves 3 on the plots correspond the ratios of impedance for diffuse scattering surface condition to impedance for specular scattering surface condition \( \Re(Z_{\text{dif}})/\Re(Z_{\text{ref}}) \) (\( \Im(Z_{\text{dif}})/\Im(Z_{\text{ref}}) \)).

The derived method has maximum error in the case of extremely anomalous skin effect, when parameter \( \alpha \gg 1 \). In this case ratios defined above are equal to 1.125. So it is obvious from the plots, that in zero approximation the method error is equal to 12.5%.

For the first approximation in this case we have \( Z_{\text{dif}}/Z = 1.03 \). So the first approximation error is equal to 3%. For the second approximation in this case we have \( Z_{\text{dif}}/Z = 1.01 \). And the second approximation error is equal to 1%.

The analysis of plots shows, that the considered impedance ratios in first approximation coincide with the exact solution when \( \alpha < 10^{-1} \). For the second approximation the coincidence is observed when \( \alpha < 1 \).

VII. CONCLUSION

The effective method of the solution of boundary problems of the kinetic theory is developed. This method is based on symmetric continuation of the electric field and distribution function of electrons.

The offered method gives an error less, than 1% in the second approximation already. The method is accurate and allows to construct exact solution in the form of von Neumann series.

Now consider the ratio of real (and imaginary) parts of the solutions constructed in zero, first and second approximations to the solution in zero approximation \( \Re(Z_0) \) (\( \Im(Z_0) \)) for the case \( q = 0 \). The last solution coincide with the solution of the problem with specular scattering boundary conditions.

We will build two plots (curves 1 and 2):

\[ Y_1 = \frac{\Re(Z_0 + Z_1)}{\Re(Z_0)} = 1 + \frac{\Re(Z_1)}{\Re(Z_0)} \]

and

\[ Y_2 = \frac{\Re(Z_0 + Z_1 + Z_2)}{\Re(Z_0)} = 1 + \frac{\Re(Z_1)}{\Re(Z_0)} + \frac{\Re(Z_2)}{\Re(Z_0)} \]

or

\[ Y_2 = Y_1 + \frac{\Re(Z_2)}{\Re(Z_0)} \]

and also analogous plots for the ratios of the imaginary parts.

Here \( Z_0 \) is the solution for the case of specular surface conditions. Values \( Z_1 \) and \( Z_2 \) correspond to corrections for the first and the second approximations.

FIG. 1: Dependence of \( \Re(Z)/\Re(Z_0) \) on parameter \( \alpha \) for the case \( q = 0 \). The curve 3 corresponds to diffuse scattering boundary conditions, the curves 1, 2 correspond to first and second approximations.

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FIG. 2: Dependence of $\Im(Z)/\Im(Z_0)$ on parameter $\alpha$ for the case $q = 0$. The curve 3 corresponds to diffuse scattering boundary conditions, the curves 1, 2 correspond to first and second approximations.

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