A PROPOSAL FOR A NON-PERTURBATIVE REGULARIZATION OF $\mathcal{N} = 2$ SUSY 4D GAUGE THEORY

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Abstract

In this letter we show that supersymmetry like geometry can be approximated using finite dimensional matrix models and fuzzy manifolds. In particular we propose a non-perturbative regularization of $\mathcal{N} = 2$ supersymmetric $U(n)$ gauge action in 4D. In some planar large $N$ limits we recover exact SUSY together with the smooth geometry of $\mathbb{R}_\theta^4$.

Noncommutative geometry [3] is the only known modification of field theory which preserves supersymmetry. In this note we will go one step further beyond the infinite dimensional matrix algebras of noncommutative Moyal-Weyl spacetimes and show that supersymmetry (like geometry itself) can be approximated using finite dimensional matrix models and fuzzy manifolds [1,2]. In particular we propose a non-perturbative regularization of $\mathcal{N} = 2$ supersymmetric $U(n)$ gauge action in 4 dimensions. In some planar large $N$ limits we will recover exact SUSY action together with the smooth geometry of spacetime $\mathbb{R}_\theta^4$.

Motivated by 1) the IKKT matrix models approach [4,5] to spacetime generation and 2) the noncommutative fuzzy geometry approach [1,2] to i) quantum geometry and to ii) the non-perturbative quantum field theory we are led to the following considerations and a proposal for a non-perturbative regularization of $\mathcal{N} = 2$ SUSY in 4 dimensions using finite dimensional $N \times N$ matrix algebras.

Let us now consider the following bosonic actions [7]

$$S_B^{(X)} = N \left[ -\frac{1}{4} Tr[X_a, X_b]^2 + \frac{2i\alpha}{3} \epsilon_{abc} TrX_aX_bX_c \right].$$

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\[
S_B^{(Y)} = N \left[ -\frac{1}{4} Tr[Y_a, Y_b]^2 + \frac{2i\alpha}{3} \epsilon_{abc} Tr Y_a Y_b Y_c \right].
\]  

(2)

\[
S_B^{(XY)} = -\frac{N}{2} Tr[X_a, Y_b]^2.
\]  

(3)

$X_a$ and $Y_a$ are $N \times N$ matrices where $N = (L + 1)^2 n$. We define $g^2 = 1/(N^2 \alpha^4)$. The two dimensional model given by the matrices $X_a$ alone is studied extensively in [8]. The corresponding action is some modification of $S_B^{(X)}$ which involves the addition of a potential term which is polynomial in $X_a^2$.

The minimum of the model is the solution of the conditions $F_{ab} = i[X_a, X_b] + \alpha \epsilon_{abc} X_c = 0$, $G_{ab} = i[Y_a, Y_b] + \alpha \epsilon_{abc} Y_c = 0$ and $H_{ab} = i[X_a, Y_b] = 0$. This solution is given explicitly by the matrices $X_a = \alpha L_a$ and $Y_a = \alpha K_a$ where $\{L_a\}$ and $\{K_a\}$ are two commuting sets of generators of $SU(2)$ in the irreducible representation $\frac{1}{2}$ which satisfy $[L_a, L_b] = i \epsilon_{abc} L_c$, $[K_a, K_b] = i \epsilon_{abc} K_c$ and $[L_a, K_a] = 0$. These matrices define the fuzzy $S^2 \times S^2$ geometry [7, 9, 10]. Expanding around these matrices by writing $X_a = \alpha (L_a + A_a)$ and $Y_a = \alpha (K_a + B_a)$ and substituting back into the action $S_B^{(X)} + S_B^{(Y)} + S_B^{(XY)}$ we get a $U(n)$ gauge theory on fuzzy $S^2 \times S^2$ with gauge coupling constant equal $g^2$. The $U(n)$ gauge transformations are implemented by $U(N)$ unitary matrices. The $A_a$ are the components of the gauge field in the directions of the first sphere while $B_a$ are the components in the directions of the second sphere. Two of these components are normal to the spheres and hence the true 4–dimensional gauge field is also coupled to two scalar fields (these two normal components) which (by construction) transform in the adjoint representation of the group $U(N)$.

The crucial point to note here is the fact that without the Chern–Simons–like terms given by $i \text{Tr} X_1 X_2 X_3$ and $i \text{Tr} Y_1 Y_2 Y_3$ in $S_B^{(X)}$ and $S_B^{(Y)}$ we will get no finite dimensional useful geometry. By solving the equations of motion which are given in this case by $F_{ab} = i[X_a, X_b] = 0$, $G_{ab} = i[Y_a, Y_b] = 0$ and $H_{ab} = i[X_a, Y_b] = 0$ we find that the minimum is given by diagonal matrices, in other words the geometry is trivial which is that of a single point. We will also be able to get the Moyal–Weyl geometry in the large $N$ limit in this model (i.e the model without the Chern–Simons–like term). This is because the Moyal–Weyl space $R^2_\theta \times R^2_\theta$ can not be realized in terms of finite dimensional matrices. In the presence of the Chern–Simons–like terms the Moyal–Weyl geometry can still be obtained in large $N$ planar limits.

We introduce the following new variables $D_\mu = (D_1 \equiv X_1, D_2 \equiv X_2, D_3 \equiv Y_1, D_4 \equiv Y_2)$ and $X_3 = \frac{\phi+\phi^+}{\sqrt{2}}$, $iY_3 = \frac{\phi^-\phi^+}{\sqrt{2}}$. Then we can write the bosonic action in the form

\[
S_B^{(X)} + S_B^{(Y)} + S_B^{(XY)} = \frac{N}{4} \text{Tr} F^2_{\mu\nu} + N \text{Tr} [D_\mu, \phi]^+ [D_\mu, \phi] + N \text{Tr} [\phi, \phi^+]^2 + \sqrt{2\alpha} N \text{Tr} \phi (F_{12} - iF_{34}) + \sqrt{2\alpha} N \text{Tr} \phi^+ (F_{12} + iF_{34}).
\]  

(4)

In above the curvature tensor is defined now by $F_{\mu\nu} = i [D_\mu, D_\nu]$. The Chern–Simons–like couplings (which are strictly real here) are given in the second line. The term $N \text{Tr} [\phi, \phi^+]^2$ can be replaced by $-\frac{N}{2} \text{Tr} D^2 + N \text{Tr} [\phi, \phi^+] D$ where we have now to do an extra integral over
the hermitian $N \times N$ matrix $D$. The action becomes

$$S_B^{(X)} + S_B^{(Y)} + S_B^{(XY)} = \frac{N}{4} Tr F^2_{\mu\nu} + N Tr[D_\mu, \phi]^+[D_\mu, \phi] - \frac{N}{2} Tr D^2 + N Tr[\phi, \phi^+]D$$

$$+ \sqrt{2} \alpha N Tr \phi (F_{12} - i F_{34}) + \sqrt{2} \alpha N Tr \phi^+ (F_{12} + i F_{34}).$$ \hfill (5)

We recognize this action (modulo the second line) to be the bosonic action of $\mathcal{N} = 2$ SUSY 4D $U(n)$ gauge theory where the $\mathbb{R}^4$ geometry is reduced to a point. Thus we know (more or less) what are the fermionic terms to be added to have full supersymmetry with the first line of this action. The Chern-Simons-like term will not be supersymmetrized. It is exactly these Chern-Simons-like terms which will provide in some approximate sense a non-trivial geometry which will resemble in some large $N$ limit the geometry of $\mathbb{R}^4$. It is in this large $N$ limit (to be thought of as a continuum limit) that we recover exact SUSY (since this term becomes vanishingly small) and also recover smooth $\mathbb{R}^4$. As it turns out we can implement full $\mathcal{N} = 2$ SUSY with the first line even for finite matrix size $N$ by including the correct fermionic degrees of freedom of the $\mathcal{N} = 2$ theory with the canonical "gauge covariant" supersymmetry transformations. We will follow the notation and convention of Weinberg with one exception which we will indicate at the end.

First we need to supersymmetrize the action

$$\frac{S_B^{(1)}}{N} = \frac{1}{4} Tr F^2_{\mu\nu} - \frac{1}{2} Tr D^2.$$ \hfill (6)

We add the gaugino action

$$\frac{S_F^{(1)}}{N} = \frac{1}{2} Tr \bar{\lambda} \gamma^\mu [D_\mu, \lambda].$$ \hfill (7)

The gaugino field $\lambda$ is a $4N \times N$ matrix with Grassmann matrix elements. This is a Majorana field. $D_\mu, D$ and $\lambda$ are members of the same supersymmetric multiplet with transformation properties

$$\delta D_\mu = i \bar{\epsilon} \gamma_\mu \lambda$$

$$\delta D = i \bar{\epsilon} \gamma_5 \gamma_\mu [D_\mu, \lambda]$$

$$\delta \lambda = \frac{1}{4} F_{\mu\nu} [\gamma_\mu, \gamma_\nu] \epsilon - i \gamma_5 D \epsilon, \quad \delta \bar{\lambda} \equiv (\delta \lambda)^+ \beta = -\frac{1}{4} \bar{\epsilon} [\gamma_\mu, \gamma_\nu] F_{\mu\nu} - i \bar{\epsilon} \gamma_5 D.$$ \hfill (8)

$\delta D_\mu$ and $\delta D$ are $N \times N$ hermitian matrices while $(\delta \lambda)_\alpha$ and $(\delta \bar{\lambda})_\alpha$ are $N \times N$ matrices with Grassmann matrix elements. The variation of the bosonic action is

$$\frac{\delta S_B^{(1)}}{N} = i Tr \delta D_\mu [D_\nu, F_{\mu\nu}] - Tr D \delta D.$$ \hfill (9)

The variation of the fermionic action is

$$\frac{S_F^{(1)}}{N} = \frac{1}{2} Tr \delta \bar{\lambda} \gamma^\mu [D_\mu, \lambda] + \frac{1}{2} Tr \delta \lambda \gamma^\mu [D_\mu, \delta \lambda]$$

$$= \frac{1}{4} Tr (\bar{\epsilon} [\gamma_\mu, \gamma_\nu] \gamma^\rho \lambda) [D_\rho, F_{\mu\nu}] + i Tr D [D_\rho, \bar{\epsilon} \gamma_5 \gamma^\rho \lambda]$$

$$= \frac{1}{4} Tr (\bar{\epsilon} [\gamma_\mu, \gamma_\nu] \gamma^\rho \lambda) [D_\rho, F_{\mu\nu}] + Tr D \delta D.$$ \hfill (10)
In the first line above we have used the fact that \( Tr\tilde{\lambda}\gamma^\mu[\delta D_\mu, \lambda] = 0 \) which is due to the identity that given any Majorana field \( \lambda \) we have \( \tilde{\lambda}\gamma^\mu\lambda = 0 \). We have the identity \( [\gamma^\mu, \gamma^\nu]\gamma^\rho = -2\eta^{\mu\rho}\gamma^\nu + 2\eta^{\nu\rho} - 2ie^{\mu\rho\sigma}\gamma_\sigma\gamma_5. \) The last term leads to the Jacobi identity \( e^{\mu\nu\rho}[D_\mu, [D_\nu, D_\rho]] = 0 \) whereas the other two terms lead to the result

\[
\frac{\delta S^{(1)}}{N} = -iTr\delta D_\mu[D_\nu, F_{\mu\nu}] + TrD\delta D. \tag{11}
\]

Hence \( S^{(1)}_B + S^{(1)}_F \) is supersymmetric as expected. We need now to add the other members of the \( \mathcal{N} = 2 \) supermultiplet. The fields \( D_\mu, D \) and \( \lambda \) form an \( \mathcal{N} = 1 \) gauge supermultiplet. The \( \mathcal{N} = 2 \) supermultiplet will also contain an \( \mathcal{N} = 1 \) chiral supermultiplet with components \( \phi \) ( the above scalar field ), \( \psi \) ( another Majorana field ) and \( F \) ( the chiral multiplet’s auxiliary field ). Following Weinberg we will impose an extra R-symmetry relating the two Majorana spinors \( \lambda \) and \( \psi \) via the transformation \( \psi \rightarrow \lambda, \lambda \rightarrow -\psi \) and hence the two \( \mathcal{N} = 1 \) supermultiplets will naturally form an \( \mathcal{N} = 2 \) supermultiplet.

Thus the goal now is to supersymmetrize the following bosonic action

\[
\frac{S^{(2)}_B}{N} = Tr[D_\mu, \phi]^+[D_\mu, \phi] + Tr[\phi, \phi^+]D. \tag{12}
\]

We will set the auxiliary field \( F \) to zero from the start. This is in anyway the value at which the \( \mathcal{N} = 2 \) action is stationary. The variation of the above bosonic action under some SUSY transformations of fields is given by

\[
\frac{\delta S^{(2)}_B}{N} = Tr\delta D_\mu\left([D_\mu, \phi^+] + [D_\mu, \phi], \phi^+\right) + Tr[\phi, \phi^+]\delta D
+ Tr\delta \phi\left[D_\mu, [D_\mu, \phi^+] - Tr[D, \phi^+]\right] + Tr\delta \phi^+\left[D_\mu, [D_\mu, \phi] + Tr[D, \phi]\right]. \tag{13}
\]

Motivated by the canonical \( \mathcal{N} = 2 \) supersymmetry in 4 dimensions we try the following fermionic terms

\[
\frac{S^{(2)}_F}{N} = aTr\bar{\psi}\gamma^\mu[D_\mu, \psi] + b\left(Tr\bar{\psi}_L[\phi, \lambda] - Tr\bar{\lambda}[\phi^+, \psi_L]\right). \tag{14}
\]

This action is real because \( \psi_L \) and \( \lambda \) are Grassmann. Indeed because \( \psi \) is Grassmann and because \( \beta(\gamma^\mu)^+\beta = -\gamma^\mu \) we have \( (\bar{\psi}_L\gamma^\mu[D_\mu, \psi_L])^+ = -[D_\mu, \bar{\psi}_L\gamma^\mu]\psi_L \) and hence \( (Tr\bar{\psi}\gamma^\mu[D_\mu, \psi])^+ = Tr\bar{\psi}\gamma^\mu[D_\mu, \psi] \). Similar argument holds for the other two terms where we will find that we need the above relative minus sign to get a real action. As we have already said \( \lambda \) is a Majorana fermion while \( \psi \) is defined now as the Majorana fermion whose left-handed component is given by \( \psi_L \). Thus the kinetic term should be rewritten

\[
Tr\bar{\psi}\gamma^\mu[D_\mu, \psi] = Tr\bar{\psi}_L\gamma^\mu[D_\mu, \psi_L] - Tr[D_\mu, \bar{\psi}_L]\gamma^\mu\psi_L. \tag{15}
\]

We assume the following extra SUSY transformations
\[ \delta \phi = i \sqrt{2} \psi_L, \quad \delta \phi^+ = i \sqrt{2} \bar{\psi}_L \epsilon \]
\[ \delta \psi_L = i \sqrt{2} [D_\mu, \phi] \gamma^\mu \epsilon_R, \quad \delta \bar{\psi}_L \equiv \delta \psi_L^+ \beta = -i \sqrt{2} [D_\mu, \phi^+] \epsilon_R \gamma^\mu. \] (16)

\( \delta \phi, \delta \phi^+ \) are \( N \times N \) complex matrices while \( (\delta \psi_L)_\alpha (\delta \bar{\psi}_L)_\alpha \) are \( N \times N \) matrices with Grassmann entries. We have the identities

\[ Tr \bar{\psi} \gamma^\mu [D_\mu, \psi] = 0 \]
\[ Tr \bar{\psi} L [\delta \phi, \lambda] - Tr \bar{\lambda} [\delta \phi^+, \psi_L] = 0. \] (17)

Thus the variation of the fermionic action under SUSY transformations is

\[ \frac{\delta S_F^{(2)}}{N} = a Tr \delta \bar{\psi}_L \gamma^\mu [D_\mu, \psi_L] + a Tr \bar{\psi}_L \gamma^\mu [D_\mu, \delta \psi_L] - a Tr [D_\mu, \bar{\psi}_L] \gamma^\mu \delta \psi_L \]
\[ + b Tr \delta \bar{\psi}_L [\phi, \lambda] - b Tr \delta \bar{\lambda} [\phi^+, \psi_L] + b Tr \bar{\psi}_L [\phi, \delta \lambda] - b Tr \bar{\lambda} [\phi^+, \delta \psi_L]. \] (18)

1st line \[ = 2 \sqrt{2} a Tr (\bar{\epsilon} \gamma^\nu \gamma^\mu \psi_L) [D_\mu, [D_\nu, \phi^+]] + 2 \sqrt{2} a Tr (\bar{\psi}_L \gamma^\mu \gamma^\nu \epsilon) [D_\mu, [D_\nu, \phi]] \]
\[ = -\frac{a}{\sqrt{2}} Tr (\bar{\epsilon} [\gamma^\mu, \gamma^\nu] \psi_L) [F_{\mu \nu}, \phi^+] - 2a Tr \delta \phi [D_\mu, [D_\nu, \phi^+]] \]
\[ + a \frac{1}{\sqrt{2}} Tr (\psi_L [\gamma^\mu, \gamma^\nu] \epsilon) [F_{\mu \nu}, \phi] - 2a Tr \delta \phi^+ [D_\mu, [D_\nu, \phi]]. \] (19)

Also (using the fact that \( \bar{\epsilon} \gamma^\mu \lambda = \bar{\epsilon} \gamma^\mu \lambda_R \) and \( \bar{\lambda} \gamma^\mu \epsilon_R = -\bar{\epsilon} \gamma^\mu \lambda_L \))

\[ b Tr \delta \bar{\psi}_L [\phi, \lambda] - b Tr \bar{\lambda} [\phi^+, \delta \psi_L] = -b \frac{2}{\sqrt{2}} Tr \delta D_\mu \left([D_\mu, \phi^+], \phi + [D_\mu, \phi], \phi^+]\right) - b \frac{2}{\sqrt{2}} Tr \delta D [\phi, \phi^+]. \] (20)

\[ -b Tr \delta \bar{\lambda} [\phi^+, \psi_L] + b Tr \bar{\psi}_L [\phi, \delta \lambda] = b \frac{4}{\sqrt{2}} Tr (\bar{\epsilon} [\gamma_\mu, \gamma_\nu] \psi_L) [F_{\mu \nu}, \phi^+] - b \frac{4}{\sqrt{2}} Tr (\bar{\psi}_L [\gamma_\mu, \gamma_\nu] \epsilon) [F_{\mu \nu}, \phi]
\[ + b \frac{4}{\sqrt{2}} Tr \delta \phi [D, \phi^+] - b \frac{2}{\sqrt{2}} Tr \delta \phi^+[D, \phi]. \] (21)

We verify quite easily that with the values \( b = \sqrt{2}, a = 1/2 \) we will have \( \delta S_F^{(2)} = -\delta S_B^{(2)} \).

The full action is

\[ \frac{1}{N} \left( S_B^{(X)} + S_B^{(Y)} + S_B^{(XY)} \right)_{\text{SUSY}} = \frac{1}{4} Tr F_{\mu \nu}^2 + Tr [D_\mu, \phi^+] [D_\mu, \phi] - \frac{1}{2} Tr D^2 + Tr [\phi, \phi^+] D \]
\[ + \frac{1}{2} Tr \bar{\psi}_L \gamma^\mu [D_\mu, \phi] + \frac{1}{2} Tr \bar{\psi} \gamma^\mu [D_\mu, \psi]
\[ + \sqrt{2} \left( Tr \bar{\psi}_L [\phi, \lambda] - Tr \bar{\lambda} [\phi^+, \psi_L] \right)
\[ + \sqrt{2} \alpha Tr \phi (F_{12} - iF_{34}) + \sqrt{2} \alpha Tr \phi^+ (F_{12} + iF_{34}). \] (22)
The first three lines constitute the full $\mathcal{N} = 2$ SUSY $U(n)$ gauge theory on a single point. The last line (although it breaks explicitly SUSY) is added so to be able to have a well defined finite dimensional geometry on which the theory lives. This term will also allow us to have a rigorous continuum limit. In some appropriate ”planar” limit this term will go to zero and hence we recover exact SUSY as well as a smooth geometry. This is another way of getting SUSY on Moyal-Weyl spaces. Let us explain this point a little further. We write the above defined finite dimensional geometry on which the theory lives. This term will also allow us to is kept fixed then we will obtain the noncommutative Moyal-Weyl plane with exact SUSY, viz

\begin{equation}
\frac{1}{N} \left( S_B^{(X)} + S_B^{(Y)} + S_B^{(XY)} \right)_{\text{SUSY}} = \frac{1}{4} Tr \tilde{F}_{\mu\nu}^2 + Tr[D_\mu, \phi]^+[D_\mu, \phi] - \frac{1}{2} Tr D^2 + Tr[\phi, \phi^+] D \\
+ \frac{1}{2} Tr \bar{\lambda} \gamma^\mu [D_\mu, \lambda] + \frac{1}{2} Tr \bar{\psi} \gamma^\mu [D_\mu, \psi] \\
+ \sqrt{2} \left( Tr \bar{\psi}_L [\phi, \lambda] - Tr \bar{\lambda} [\phi^+, \psi_L] \right) \\
- 2\alpha^2 Tr (X_3^2 + Y_3^2).
\end{equation}

In above $\tilde{F}_{12} = F_{12} + 2\alpha X_3 = -\tilde{F}_{21}, \tilde{F}_{34} = F_{34} + 2\alpha Y_3 = -\tilde{F}_{43}, \tilde{F}_{13} = F_{13} = -\tilde{F}_{31}, \tilde{F}_{14} = F_{14} = -\tilde{F}_{41}, \tilde{F}_{23} = F_{23} = -\tilde{F}_{32}, \tilde{F}_{24} = F_{24} = -\tilde{F}_{42}$. To study the noncommutative planar limit we should consider adding to this action the following potential term

\begin{equation}
V[X_3, Y_3] = -Nm^2 \alpha^2 Tr X_3^2 + \frac{2m^2}{N} Tr (X_3^2)^2 - Nm^2 \alpha^2 Tr Y_3^2 + \frac{2m^2}{N} Tr (Y_3^2)^2.
\end{equation}

This potential is gauge invariant but not rotationally invariant. In above $m = N^p$ with some positive integer power $p$ so in the large $N$ limit we can see that this potential implements the constraints $X_3 = \frac{N\alpha}{2}$ and $Y_3 = \frac{N\alpha}{2}$ which means that on each sphere we are restricted to the north pole in a covariant way. In this large $N$ limit if we also take $\alpha \longrightarrow 0$ such that $N\alpha^2 = 1/\theta^2$ is kept fixed then we will obtain the noncommutative Moyal-Weyl plane with exact SUSY, viz

\begin{equation}
\frac{1}{N} \left( S_B^{(X)} + S_B^{(Y)} + S_B^{(XY)} \right)_{\text{SUSY}} = \frac{1}{4} Tr \tilde{F}_{\mu\nu}^2 + Tr[D_\mu, \phi]^+[D_\mu, \phi] - \frac{1}{2} Tr D^2 + Tr[\phi, \phi^+] D \\
+ \frac{1}{2} Tr \bar{\lambda} \gamma^\mu [D_\mu, \lambda] + \frac{1}{2} Tr \bar{\psi} \gamma^\mu [D_\mu, \psi] \\
+ \sqrt{2} \left( Tr \bar{\psi}_L [\phi, \lambda] - Tr \bar{\lambda} [\phi^+, \psi_L] \right).
\end{equation}

In above we have used the fact that the last line in (23) leads to a constant term in this planar limit. We have also the definitions $\tilde{\phi} = \phi - \frac{N\alpha^2/2}{2}, \tilde{F}_{12} = F_{12} + \frac{1}{\theta^2}, \tilde{F}_{34} = F_{34} + \frac{1}{\theta^2}$. The trace $Tr$ is now infinite dimensional. Under SUSY transformations we will have the variations $\delta \tilde{\phi} = \delta \phi$, $\delta \tilde{F}_{\mu\nu} = \delta F_{\mu\nu}$ and hence this action is still $\mathcal{N} = 2$ supersymmetric.

**Remark:** The factor of $i$ in $\delta D_\mu$, $\delta \phi$ and $\delta \phi^+$ is due to our basic identity which is given any pair of Majorana spinors $s_1$ and $s_2$ (which are here $4N \times N$ matrices) we have

\begin{equation}
(s_1 M s_2)^\dagger = -s_1 M s_2, \quad M = 1, \gamma_\mu, [\gamma_\mu, \gamma_\nu]
\end{equation}
\[(\bar{s}_1 M s_2)^+ = +\bar{s}_1 M s_2, \quad M = \gamma_5, \gamma_\mu \gamma_5.\]  

(27)

The signs in these two equations are opposite to the signs of equations (26.A.20) and (26.A.21) of Weinberg. In our case when we take the hermitian adjoint we include a minus sign (in accordance with the property used in showing the reality of our action) then when we reverse the interchange of \(s_1\) and \(s_2\) we get a second minus sign which cancels the first one.

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