Analysis of the algorithm for determining the meshing forces in a spline coupling with misalignment

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Abstract. In modern engine buildings spline couplings have an important role. These connections can have a serious impact on the dynamic of the system, especially in misalignment conditions. Therefore, assessment its influence on the system is an important task. In this paper the algorithm for determining meshing forces in spline couplings and the influence of various parameters of connections on the meshing forces are analysed. Also the possibilities of further development this algorithm are considered.

1. Introduction
A misalignment in spline coupling it's a common problem in the rotor of aircraft engine. This process can lead to an increase the loads on the support, noise level and the vibration level [1]. Therefore, the analysis of a spline coupling with a misalignment is an important task for simulating dynamic system. One of the software packages that allows you to take into the consideration the misalignment in a dynamic system is the DYNAMICS R4. It provides the ability to model a system with angular or radial misalignment. The misalignment effects are modeled as external non-linear loads; the element is a link between two section with 12 degree of freedom. However, in this software package one of the important condition for the simulating of dynamic model is the additional modeling of an equivalent stiffness model of the connection. For the simulating a such equivalent stiffness model the theory and algorithm presented in [2] was used. The main task of this work is to assess the applicability of this algorithm for its further modification and use.

2. Model
An algorithm for determining the meshing forces for a spline coupling is considered. Conditions of the model are presented below:

- The spline connection has rectangular tooth;
- The spline is a cantilever beam with an elastic base;
- The distributed force from the torque is represented as a concentrated force acting at the end of the spline;
- The place of contact is considered as a point.

The geometric parameters of the spline are shown in figure 1.
The total elastic deformation of the spline is found as:

\[ \delta_i = \delta_{Bi} + \delta_{Si} + \delta_{Mi} \]  

(1)

\( \delta_{Bi} \) is a bending deformation, \( \delta_{Si} \) is a shear deformation and \( \delta_{Mi} \) is deformation of elastic foundation, which are defined as follows:

\[ \delta_{Bi} = \frac{F_i L^3}{3E_e I} \]  

(2)

\[ \delta_{Si} = \frac{12 E_e L_i (1+\nu)}{5E_e A} \]  

(3)

\[ \delta_{Mi} = \frac{F_i}{E_e} \left( \frac{5.306}{BH^2} L^2_i + \frac{2(1-\nu-2\nu^2)}{(1-\nu^2)BH} L_i + \frac{1.534}{B} \right) \]  

(4)

where \( I = \frac{BH^3}{12} \), \( Q = \frac{2(1-\nu-2\nu^2)}{(1-\nu^2)BH} \), \( X = \frac{12(1+\nu)}{5A} \), \( A = BH \).

The stiffness of a single spline is defined a follow expression:

\[ K_i = \frac{E_e}{\frac{1}{3}(I_i) + \left( \frac{5.306}{BH^2} \right) L^2_i + (Q+X)L_i + \frac{1.534}{B}} \]  

(5)

When the system doesn’t have misalignment, the centers of the spline coupling (male and female) are superposed and the distance between the splines in mesh is the same for each pair. In condition of misalignment in the X, Y directions, the eccentricity will be determined as follows:

\[ e = \left( (x + e_0 \cos \varphi_0)^2 + (y + e_0 \sin \varphi_0)^2 \right)^{1/2} \]  

(6)

\[ \cos \varphi = \frac{x + e_0 \cos \varphi_0}{e} \]  

(7)

\[ \sin \varphi = \frac{y + e_0 \sin \varphi_0}{e} \]  

(8)
For the simplify assume, that misalignment appear only in X direction. Taking the positive direction of the splines clockwise, the angle between each spline and the positive direction of the X axis is defined as follow expression:

$$\varphi_i = \frac{2\pi(i-1)}{z} \quad (9)$$

In a misalignment state the meshing in the two parts of the spline coupling is also different. The distance between the splines for each pair in misalignment conditions is determined as:

$$L_i = L_0 - e \cos \varphi_i \quad (10)$$

In a real system all splines are rigidly meshed due to the transmission of a large torque:

$$T = \sum_{i=1}^{z} (F_{ri}(R + L_i)) \quad (11)$$

where $R$ is a radius of the root of the teeth, $\varphi$ is the torsion angle (the same for the all splines) and $F_{ri}$ is the force transmitted by each spline.

$$F_{ri} = \phi L_i K_i \quad (12)$$

Based on the above, the angle $\phi$ can be defined as:

$$\phi = \frac{T}{\sum_{i=1}^{z} (L_i K_i (R + L_i))} \quad (13)$$

The meshing force for each spline in misalignment conditions will be determined as follows:

$$F_i = (\phi L_i + e' \sin \varphi_i) K_i \quad (14)$$

$$e' = \sqrt{x^2 + y^2} \quad (15)$$

From the expression above we can conclude that the meshing force in the splines has the biggest values for the splines from 1 to $z/2$ and also cannot be negative. In addition, for the case when it’s absence the torque, the meshing force is 0 (even if the misalignment is). Therefore, the expression for the determining the meshing force for each spline will take the form:

$$F_i = \begin{cases} 
(\phi L_i + e' \sin \varphi_i) K_i, & \phi L_i + e' \sin \varphi_i > 0 \\
0, & \phi L_i + e' \sin \varphi_i \leq 0 
\end{cases} \quad (16)$$

The components of the above force:

$$F_{x_i} = F_i \cos \theta_i \quad (17)$$

$$F_{y_i} = F_i \sin \theta_i \quad (18)$$

$$\theta_i = \varphi_i + \frac{\pi}{2} \quad (19)$$

where $\theta_i$ is the angle between the positive direction of the X-axis and the meshing force.

The above analysis refers to the case of misalignment in only one direction. For real system the meshing forces are defined as follows:
\[ f_x = F_x \cos \varphi - F_y \sin \varphi \]  
\[ f_y = F_x \sin \varphi + F_y \cos \varphi \]  

(20)  
(21)

### 3. Results

To assess the applicability of the algorithm, the spline coupling with the parameters presented in [2] were calculation. Table 1 shows the parameters of the studying spline connection.

| Parameter                     | Value      |
|-------------------------------|------------|
| Number of splines, \( Z \)    | 14         |
| Spline width, \( B \), mm     | 100        |
| Spline thickness, \( H \), mm  | 16         |
| Spline length, \( L_0 \), mm  | 5.07       |
| Poisson's Ratio, \( \nu \)    | 0.3        |
| Elastic modulus, \( E \), Pa  | 2.12 \( \times 10^{11} \) |
| Radius of spline root, \( R \), mm | 69.5 |
| Transmitting torque, \( T \), Nm | 87000 |

Figure 2 shows the dependence of the meshing force from the value of the initial displacement. For comparison, the dashed lines in the graphs represent the results obtained in [1].

**Figure 2.** Dependence of the meshing force from the initial displacement.

Figure 3 shows the dependence of the meshing force from the misalignment.
Figure 3. Meshing force versus misalignment.

The figures below show the dependence of the meshing force from the spline parameters: in figure 4 - from the spline thickness, in figure 5 - from the spline length.

Figure 4. Meshing force versus spline thickness.
4. Conclusions
1. The dependences of the meshing force from the various parameters of spline coupling has been obtained: an increase of the thickness tooth leads to the increase of meshing force and an increase the tooth length leads to the decrease of meshing force; with an increase of the misalignment the meshing force increase too, the dependence of these parameters close to linear.

2. The research results presented in [2] are reproduced only qualitatively. It is necessary to confirm the calculation results by solving the problem using alternative methods (for example, the FE method).

3. The development of this algorithm seems possible in the transition from the model of rectangular tooth to the models of involutes and triangular spline couplings, taking into account such parameters as the contact patch, clearances (like in [3], [4]), and also subsequent use in the DYNAMICS R4 software package.

References
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