Tidally–induced angular momentum transport in disks

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Abstract. We discuss the transport of angular momentum induced by tidal effects in a disk surrounding a star in a pre–main sequence binary system. We consider the effect of both density and bending waves. Although tidal effects are important for truncating protostellar disks and for determining their size, it is unlikely that tidally–induced angular momentum transport plays a dominant role in the evolution of protostellar disks. Where the disk is magnetized, transport of angular momentum is probably governed by MHD turbulence. In a non self–gravitating laminar disk, the amount of transport provided by tidal waves is probably too small to account for the lifetime of protostellar disks. In addition, tidal effects tend to be localized in the disk outer regions.

1. Angular momentum exchange between the disk rotation and the orbital motion

In a binary system where at least one of the stars is surrounded by a circumstellar disk, tidal waves excited by the companion propagate into the disk. If the disk and the orbital plane are coplanar, these waves are called density waves. In a noncoplanar system, both density and bending waves are excited. They are respectively of even and odd symmetry with respect to reflection in the disk midplane. The pattern speed \( \Omega_P \) with which the tidally excited pattern rotates is \( \omega \) and \( 2\omega \) for density and bending waves, respectively, where \( \omega \) is the binary angular speed. The disk is truncated by tidal effects in such a way that its radius is not greater than about one–third of the separation of the system (Papaloizou & Pringle 1977, Paczyński 1977, Larwood et al. 1996 for the non coplanar case). Therefore, \( \Omega_P \) is smaller than the angular speed of the gas in the disk, and the tidal pattern carries negative angular momentum (or, in other words, the torque exerted on the disk by the companion is negative). Through dissipation of the waves, the disk then loses angular momentum and disk material flows inwards, whereas the companion star, which excites the waves, gains this angular momentum.

Since there is no corotation resonance in the disk, secular exchange of angular momentum between the disk rotation and the orbital motion occurs only if the waves dissipate, either in the bulk of the disk or at its boundaries (Goldreich & Nicholson 1989). In a laminar disk, dissipation of tidal waves may arise through shocks. A shock front forms when the group velocity of the wave relative to that of the fluid (or, equivalently, the perturbed velocity) is supersonic. Shocks are very dissipative so the wave amplitude cannot rise much above the
level where the front first forms. In other words, the wave is decelerated at the shock front in such a way as to restore marginally sonic wave motion. Conservation of wave action may tend to cause the amplitude of the (shock–)wave to increase again as it propagates further in, but this effect is balanced by shock dissipation maintaining the amplitude at the marginal level.

Tidally–induced angular momentum transport has been considered as an alternative to turbulent transport (Shu 1976, Sawada et al. 1986, Spruit et al. 1987, Larson 1989, see also Larson in this volume). It is viewed as particularly attractive in disks where the ionization level is too low for the magnetorotational instability to operate (Balbus & Hawley 1998). However, while the presence of spiral waves have been inferred in the accretion disk of the dwarf nova IP Peg (Steeghs et al. 1997), there are no indications that they are associated with significant angular momentum transport. Below we consider successively the case of density and bending waves.

2. Angular momentum transport by density waves

There are two main obstacles to the propagation of density waves down to small radii. First, if the disk is vertically stratified, the wave front tends to be tilted upwards so that the wave action migrates towards the surface of the disk and into any atmosphere it possesses where it can take on high amplitude and be dissipated (Lin et al. 1990a, 1990b). Only under the artificial assumptions of a strict polytropic edge and no dissipation can it be channeled into and remain in a very narrow surface waveguide (Ogilvie & Lubow 1999). Migration of wave action towards the surface (or wave refraction) is more effective for high azimuthal mode number \( m \), but is still efficient for the two–armed spirals which are predominantly excited in binaries. This is because even though these waves have a larger wavelength in the linear regime, if they become nonlinear their profile necessarily distort and develop short wavelength components for which refraction might be important.

The second obstacle to long range wave propagation is a low temperature or, equivalently, a high Mach number (e.g., Spruit 1987, Savonije et al. 1994, Godon et al. 1998, Armitage & Murray 1998, Blondin 2000). This is because the characteristic wavelength of the excited waves decreases with increasing Mach number, so that it becomes very small compared with the scale associated with the forcing potential (Lin & Papaloizou 1993). The torque, which is obtained by integrating over the volume of the disk the perturbed mass density times the tidal force, is then very small.

There have been a number of 3D numerical calculations of tidal shock waves (see Yukawa et al. 1997 and references therein, Haraguchi et al. 1999) but the loss of disk angular momentum has not been computed in these calculations. Spiral shocks were seen in some of these simulations, but they were much less distinct than in 2D. Even in 2D, where refraction is absent and therefore tidal effects are overestimated, the pattern observed in IP Peg can be reproduced only in the outer disk during outburst, when the enhanced viscosity pushes the disk edge into a region of strong gravitational perturbations from the secondary (Armitage & Murray 1998), or for unrealistically hot disks (Godon et al. 1998). Note that observations themselves only show spirality in the outer disk in IP Peg.
In 2D, calculations by Savonije et al. (1994) and Blondin (2000) suggest that wave-driven accretion onto the central star occurs in disks where the Mach number is smaller than about 10, whereas it is inefficient when the Mach number is larger than about 20. Since the Mach number in protostellar disks is thought to be larger than 10 (the disk aspect ratio is around 0.05–0.1), tidally-induced transport is probably not significant in the disk inner parts.

3. Angular momentum transport by bending waves

Bending waves are more efficient at transporting angular momentum in a disk than density waves, because they have a longer wavelength (Papaloizou & Lin 1995). For the same reason they can also propagate down to smaller radii.

Papaloizou & Terquem (1995) calculated the $m = 1$ bending wave response of an inviscid disk with the rotation axis misaligned with a binary companion’s orbital rotation axis. They assumed that the waves were dissipated by nonlinear interaction with the background flow before reaching the disk inner edge, so that all their angular momentum was deposited into the disk. They found that $m = 1$ bending waves can lead to the accretion of the disk on a timescale in excess of a few times $10^7$ years.

Terquem (1998) calculated the tidal torque in a viscous disk (where the waves are viscously damped) and found that it can be comparable to the torque communicated internally by horizontal viscous stress acting on the background flow when the perturbed velocities in the disk are on the order of the sound speed. The tidal torque can exceed the horizontal viscous torque only if the viscous stress tensor is anisotropic with the parameter $\alpha$ which couples to the vertical shear being larger than that coupled to the horizontal shear. When the perturbed velocities become supersonic, shocks reduce the amplitude of the perturbation such that the disk moves back to a state where these velocities are marginally sonic (Nelson & Papaloizou 1998). When shocks occur, the tidal torque exerted on the disk may become larger than the horizontal viscous torque. Terquem (1998) also found that if the waves are reflected at the center, resonances occur when the frequency of the tidal waves is equal to that of some free normal global bending mode of the disk. If such resonances exist, tidal interactions may then be important even when the binary separation is large. However, it is unlikely that in a realistic accretion disk waves can be reflected at the disk inner edge. Therefore, in a viscous disk, it is unlikely that transport of angular momentum is increased by more than a factor two or so by tidal effects. It was also found that if $\alpha$ is larger than about $10^{-2}$, bending waves are damped before they can reach the disk inner parts.

4. Conclusion

Tidal effects in pre–main sequence binary systems are important for truncating protostellar disks and for determining their size. However, once the disk is truncated, the calculations reviewed above suggest that tidally–induced angular momentum transport does probably not play a dominant role. Where the disk is ionized enough so that the magnetorotational instability can develop, transport of angular momentum is most probably dominated by magnetic turbulence. If
the disk is laminar, the amount of transport provided by tidal waves is unlikely to be large enough to account for the dissipation of the disk on a timescale on the order of a few million years. In addition, tidal effects tend to be exerted mainly at the disk outer edge, where the perturbation is the strongest. Strong tidal effects at the outer edge of the disk would allow mass from the outer region to retract inwards, subsequently weakening the tides at the edge. Some pile up of mass may result at smaller radii, but whether that would affect significantly the whole disk would depend on the disk mass.

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References

Armitage, P. J., & Murray, J. R. 1998, MNRAS, 297, L81
Balbus, S. A., & Hawley, J. F. 1998, Rev. Mod. Phys., 70, 1
Blondin, J. M. 1999, preprint [astro-ph/9909181]
Godon, P., Livio, M., & Lubow, S. 1998, MNRAS, 295, L11
Goldreich, P., & Nicholson P. D. 1989, ApJ, 342, 1075
Haraguchi, K., Boffin, H. M. J., & Matsuda, T. 1999, in Star Formation 1999, ed. T. Nakamoto (Nobeyama Radio Observatory), p. 241
Larson, R. B. 1989, in The Formation and Evolution of Planetary Systems, eds. H. A. Weaver & L. Danly (Cambridge: CUP), p. 31
Larwood, J. D., Nelson, R. P., Papaloizou, J. C. B., & Terquem, C. 1996, MNRAS, 282, 597
Lin, D. N. C., & Papaloizou, J. C. B. 1993, in Protostars and Planets III, ed. E. H. Levy & J. Lunine (Tucson: Univ. Arizona Press), p. 749
Lin, D. N. C., Papaloizou, J. C. B., & Savonije, G. J. 1990a, ApJ, 364, 326
Lin, D. N. C., Papaloizou, J. C. B., & Savonije, G. J. 1990b, ApJ, 365, 748
Nelson, R. P., Papaloizou, J. C. B., 1999, MNRAS, 309, 929
Ogilvie, G. I., & Lubow, S. H. 1999, ApJ, 515, 767
Ogilvie, G. I., & Lubow, S. H. 1999, ApJ, 515, 767
Paczyński, B. 1977, ApJ, 216, 822
Papaloizou, J. C. B., & Lin, D. N. C. 1995, ApJ, 438, 841
Papaloizou, J. C. B., & Pringle, J. E. 1977, MNRAS, 181, 441
Papaloizou, J. C. B., & Terquem, C. 1995, MNRAS, 274, 987
Savonije, G. J., Papaloizou, J. C. B., & Lin, D. N. C. 1994, MNRAS, 268, 13
Sawada, K., Matsuda, T., & Hachisu, I. 1986, MNRAS, 219, 75
Shu, F. H. 1976, in Structure and Evolution of Close Binary Systems, IAU Symp. 73, eds. P. Eggleton, S. Mitton, & J. Whelan (Reidel: Dordrecht), p. 253
Spruit, H. C. 1987, A&A, 184, 173
Spruit, H. C., Matsuda, T., Inoue, M., & Sawada, K. 1987, MNRAS, 229, 517
Steeghs, D., Harlaftis, E. T., & Horne, K. 1997, MNRAS, 290, L28
Terquem, C. E. J. M. L. J. 1998, ApJ, 509, 819
Yukawa, H., Boffin, H. M. J., & Matsuda, T. 1997, MNRAS, 292, 321