Universal scaling dependence of QCD energy loss from data driven studies

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In this paper we study the energy loss of jets in the QGP via the nuclear modification factor $R_{AA}$ for unidentified particles at high $p_T (> 10 \text{ GeV/c})$ in and out of the reaction plane of the collision. We argue that at such a high $p_T$ there are no genuine flow effects and, assuming that the energy loss is only sensitive to initial characteristics such as the density and geometry, find that $R_{AA}$ depends linearly on the (RMS) length extracted from Glauber simulations. Furthermore we observe that for different centrality classes the density dependence of the energy loss enters as the square root of the charged particle multiplicity normalized to the initial overlap area. The energy loss extracted for RHIC and LHC data from the $R_{AA}$ is found to exhibit a universal behavior.

I. INTRODUCTION

One of the most stunning results from the heavy ion programs at RHIC and LHC is the quenching of jets and single-inclusive hadron spectra [1–3]. Being perturbative probes for which we can calculate the vacuum baseline to high precision, jets are potentially excellent probes of the medium created in heavy ion collisions. Modifications, arising due to interactions with the hot and dense matter, are indeed expected to arise at timescales comparable to the lifetime of the medium are typically characterized in terms of elastic and radiative energy losses [4, 5], for recent reviews see, e.g., [6–8]. Presently our theoretical control of the jet fragmentation is however limited. In particular, the importance of modifications of the jet substructures due to the transverse medium resolution was only recently pointed out [9]. Recent results from the LHC on the suppression of single-inclusive hadrons and jets are in this context challenging to reconcile with the corresponding observations at RHIC [10] and call for the refinement of present theoretical
tools. Furthermore, at RHIC it is challenging to reconcile both the data on the nuclear modification factor, $R_{AA}$, and azimuthal flow, characterized by $v_2$, at high $p_T$ within models based on radiative QCD mechanisms [11, 12]. We take this uncertainty at the theoretical level as an opportunity to make a data driven study that we present here. Similar studies have also been carried out previously in [13–15], see also [16] for more theoretically driven studies, and we will return to how they differ from the present work in Section IV.

One of the challenges of modeling the energy loss is that the medium created in heavy ion collisions behaves as a perfect liquid. There are at least 2 major issues. First of all, both the geometry and the dynamical expansion of the medium introduce a complication for the clean extraction of the transport properties of the medium. The longitudinal expansion of the medium causes the energy density to decrease quickly with time (as the inverse of the proper time in the Bjorken model [17]), and this could clearly affect the path length dependence of the energy loss. Furthermore, the dynamics of the medium in the transverse plane signifies that in non-central collisions there is an asymmetric expansion of the medium, where the expansion in the reaction plane is larger than out-of-plane. To first order the latter effect is supposed to be negligible, but various studies have documented significant effects [18].

Since we wish to pursue a data driven study, these effects cannot be handled without recourse to modeling and so we will focus on characterizing the energy loss in terms of initial state observables. It is quite remarkable that this seems to work very well.

Secondly, the convincing signals of collective behavior in A-A collisions hint at the existence of a strongly coupled system. This, in turn, challenges the paradigm of using perturbative methods to calculate the relevant degrees of freedom for the jet-medium interactions. Our present study avoids these conceptual difficulties.

One could worry that the measured $R_{AA}$ in and out of the reaction plane is significantly affected by flow. Let us try to argue here that for $p_T > 8$ GeV/c this is in our opinion not very likely. Flow is typically characterized by introducing a mass dependence. Both measurements of $v_2$ [19] and the $R_{AA}$ [20] have shown that for $p_T > 8$ GeV/c there is little or no difference between results for pions and protons. The triangular flow, characterized by the coefficient $v_3$, also seems to disappear in this $p_T$ region [19]. As the baryon to meson ratios are rather similar from RHIC energy ($\sqrt{s_{NN}} = 200$ GeV) to LHC energies ($\sqrt{s_{NN}} = 2.76$ TeV) [21] this indicates that also for RHIC energies we need to have data for $p_T > 8$ GeV/c to eliminate
flow effects. In our opinion, this allows us safely to assume that both $R_{AA}$ and $v_2$ at high $p_T$ are dominated by energy loss. The effect of residual flow would be an underestimate (overestimate) of the quenching contribution in (out) of plane. There are no indications for such an effect in Fig. 2.

At high energies the energy loss of a colored parton going through a colored medium is expected to be dominantly radiative. Na"ively one expects that radiative QCD energy loss [22–26] increases quadratic with path length, since this follows from the stimulated emission probability of a single hard gluon [27, 28]. These emissions are however rare and one should also account for multiple soft emissions. This changes the path length dependence of the characteristic $p_T$ shift of the medium-modified spectra so it becomes linear [29], see Appendix B. A crucial point of this paper is that the existing data allows to disentangle more than simply the path length dependence of the suppression. As we will show, this information has to be supplemented by including a dependence on the energy density. While our results will rely on simple estimates of both of these quantities, for details see Section II, the agreement with the data at two, widely separated energies of RHIC and LHC represent a strong argument for the consistency of the interpretation of energy loss in ultrarelativistic heavy ion collisions. Finally we note that there is a significant $p_T$ dependence of the $R_{AA}$. We shall ignore this $p_T$ dependence in our quantitative studies and focus on a common $p_T$ region of $p_T \approx 10$ GeV/$c$ for LHC and RHIC. The scaling plots we show, in particular Fig. 3, does however indicate that the scaling relations we find are also valid at higher $p_T$.

The outline of the paper is as follows. In Section II we describe the data driven set-up and the method for extracting the energy loss (or $p_T$ shift) from the A-A spectra. We go on to present the obtained results and discuss them in Sections III and IV, respectively. Finally, our conclusions are summarized in Section V.

II. DATA DRIVEN SET-UP

Figure 1 illustrates the idea behind the studies presented here. Based on Glauber simulations of the participant distribution, two centrality classes are selected where we can relate some properties in- vs. out-of-plane. In our case the selection was done on the characteristic length, which we define as the root-mean-squared (RMS) of the distribution. We can then compare the in- and out-of-plane $R_{AA}$ data for different centrality classes where these properties will agree. In Fig. 1 we have chosen two centrality classes, 10-20 % and 30-40 %, where the in-plane
FIG. 1. (Color online) The extracted participant distribution for two Glauber samples at $\sqrt{s_{NN}} = 2.76$ TeV (10%-20 % centrality (a) and 30%-40 % centrality (b)) rotated so that the reaction plane coincides with the x-axis. The in-plane RMS of the former equals approximately the out-of-plane of the latter. Comparison of the participant distributions and the $R_{AA}$ for the two cases are shown in (c) and (d), respectively.

RMS width of the former distribution, denoted $L_{in}$, is approximately equal to the out-of-plane RMS, denoted $L_{out}$, of the latter, see the top panels. In the lower-left panel of Fig. 1 we also demonstrate that the participant distribution is quite similar in the two cases.

An important motivation behind such a simplified event selection is the fact that in central collisions we expect the distribution of hard scatterings (binary collisions) to be more narrowly distributed around the origin. In that way the path length of the two samples should on the average be quite similar, but we note most importantly that the density is quite different. Moreover, the transverse expansion could be much more significant in-plane than out-of-plane and could spoil the comparison. In our studies we find that the latter effect can be neglected and this is in fact also, as mentioned above, what one would expect to first order from theoretical arguments.

Once we have fixed the characteristic length to be similar, it remains to include the effect of the difference in energy density. As it is clearly seen in the lower-right panel of Fig. 1, comparing the $R_{AA}$ for our example cases for which the path lengths were equal in- and out-of-plane does not result in the same amount of suppression. The overlapping participant distributions are reasonably described by two-dimensional Gaussian distributions, see lower-left panel of Fig. 1, and so we assign an area as $A \approx 4\pi L_{in}L_{out}$. Then we assume that the characteristic energy density $\rho$ of the sample is given by

$$\rho = K \frac{dN/d\eta}{4\pi L_{in}L_{out}},$$

where $K$ is a constant that is assumed to depend little on centrality and collision energy. In the following we always set $K = 1$ GeV/fm.
such to make $\rho$ have the units $\text{GeV}/\text{fm}^3$. As this density is not normalized in a meaningful way (because of the data driven nature of this study) we will in the following use arbitrary units (arb. units) in the plots. The pseudorapidity distribution, $dN/d\eta$, have been taken from [30]. In Sec. IV where we introduce theoretical estimates for comparison we will discuss how one can normalize this properly to extract meaningful physics parameters. The definition of $\rho$ is inspired by Bjorken’s energy density estimate and the observation that the mean transverse energy per produced particle does not change violently as a function of centrality or collisions energy [31].

The LHC data on charged particle $R_{AA}$ and $v_2$ used in this publication have been taken from [32, 33]. CMS has published similar data [34, 35] but with coarser segmentation in centrality and $p_T$, while ALICE $v_2$ measurements does not cover centralities above 50 % [19]. The $R_{AA}$ in- and out-of-plane used in our data driven analysis has been obtained as $R_{AA,\text{in}} = R_{AA}(1+2v_2)$ and $R_{AA,\text{out}} = R_{AA}(1-2v_2)$, respectively. The $p_T$ bins for the $R_{AA}$ and $v_2$ results do not match perfectly but the closest $p_T$ points have been used and as both the $R_{AA}$ and $v_2$ are only rather moderately varying at high $p_T$ we consider this a negligible effect. The error bars shown in the figures for $R_{AA,\text{in}}$ and $R_{AA,\text{out}}$ always include the full statistical and systematic uncertainties added in quadrature from both the $R_{AA}$ and $v_2$. Normalization errors for $R_{AA}$ have been ignored as they are expected to be directly correlated across centralities (and to some degree also across beam energies). When $R_{AA,\text{in}}$ and $R_{AA,\text{out}}$ is compared we assume in our interpretation that the relative systematic error is smaller than shown. For the $R_{AA}$ one expects e.g. the efficiency and corrections to have similar systematic errors and so there it seems a common shift of $R_{AA,\text{in}}$ and $R_{AA,\text{out}}$ is expected. On the other hand for $v_2$ a systematic shift would tend to shift $R_{AA,\text{in}}$ and $R_{AA,\text{out}}$ in opposite directions. A better understanding of this aspect can only be obtained by the experiments.

To extract information beyond merely the level of suppression of the spectra, we would like to study the phenomenon of energy loss more directly [12, 36]. To this aim we will assume that the spectra in p-p and A-A collisions can be described by a power-law with a similar exponent and that the difference comes from the fact that the primordial $p_T$ of the partonic A-A spectrum has been shifted to lower values due to energy loss in the medium. Note that the shift itself could be $p_T$ dependent. Explicitly, the $p_T$ shift is defined as $\Delta p_T \equiv p_{T,i} - p_{T,m}$, where $p_{T,i}$ is the momentum of the parton prior to energy loss while $p_{T,m}$ is the momentum of the hadron as
measured in the detector. Then, following a similar method as employed by PHENIX [36], the $p_T$ spectra of particles in a certain centrality class can be compared via

$$\frac{dN_{pp}}{dp_{T,i}}(p_{T,i}) = \left| \frac{dp_{T,m}}{dp_{T,i}} \right| R_{AA}(p_{T,m}) \frac{dN_{pp}}{dp_{T,m}}(p_{T,m}),$$

(2)

where the first term is the Jacobian of the transformation, see Appendix A for further details. Since we a priori cannot predict the dependence of the shift, we explore two extreme relations between $p_{T,i}$ and $p_{T,m}$ in Eq. (2): $p_T$ independent absolute and relative energy losses (see Appendix A for further details). In all figures the central value for the $p_T$ loss is the average of the two estimates and the systematic uncertainty box shows the actual difference. Here we stress that the observed scaling patterns are not affected by the resulting variations in the parameterization of $\Delta p_T$.

One can find several scaling variables from the orientation-dependent $R_{AA}$ alone since, e.g., the squared scaling variable will also align the $R_{AA}$. As an additional criterium we will therefore demand that the extracted energy loss is approximately linear in the scaling variable.

III. RESULTS

Figure 2 shows a summary of the main results from our studies of LHC data. In the left column we plot the $R_{AA}$, while in the right one the $p_T$ shift divided by the primordial momentum, $\Delta p_T/p_{T,i}$. Both quantities are plotted vs. the respective scaling variable, for which we explore three possibilities:

FIG. 2. (Color online) Example of scaling relations for LHC data in arbitrary units. $R_{AA}$ vs. $L$ (a), $\rho^{1/2}L$ (c), $\rho^{3/4}L^2$ (e) and extracted energy loss $\Delta p_T/p_T$ vs. the same scaling variables (b, d, f) are shown for $p_T \approx 13$ GeV/c. We have included the uncertainty arising from the unknown functional form of $\Delta p_T$ as shaded boxes on the points in the right column, see Appendix A for details.
the path length, $L$, in the uppermost row, then $\rho^{1/2}L$ in the center and finally $\rho^{3/4}L^2$ in the lower column. The motivation behind these choices will be discussed further in Sec. IV. The plots in the left column illustrate that it is possible to find several scaling variables for the $R_{AA}$, but that the energy loss is only approximately linear for the scaling variable in the middle panel. Extrapolating down, it even seems to vanish for $L = 0$, as expected. We thus find that all $R_{AA}$ and $v_2$ values for a given $p_T$ can be described in terms of a linear energy loss relation.

Furthermore, in Fig. 3 we demonstrate that the proposed scaling variable, $\rho^{1/2}L$, seems to work reasonably well for all $p_T$. Whereas the agreement is good for central collisions, one observes some tension for the 70–80% centrality class. In the most peripheral collisions it is known that the difference between the reaction plane and the impact parameter plane is the largest so that one is more sensitive to the description of individual collisions in the model. The impact of hard scatterings on the experimental measurement of $v_2$ could also be significant due to the smaller number of participants.

In the remainder of this section we will show that the scaling variable found above also works surprisingly well both at RHIC and LHC. Recently PHENIX has published the $R_{AA}$ vs. the event plane at very high $p_T$ for $\pi^0$ [12]. One should note that $p_T$ spectra in p-p at RHIC and LHC are power law-like for $p_T > 5$ GeV/c, but that the power law exponent is quite different in the two cases. The relationship between $R_{AA}$ and energy loss at LHC and RHIC is therefore different even if the $R_{AA}$ are quite compatible for each of the centralities. The main change in the scaling variable going from LHC down to RHIC energies is an almost centrality independent decrease of particle density $dN/d\eta$ of a factor 0.48 [30]. In our picture one therefore expects the energy loss to be approximately 40% larger at LHC than at RHIC for simi-
FIG. 4. (Color online) $R_{AA}$ in- and out-of-plane for $p_T \sim 10 - 13$ GeV/$c$ at $\sqrt{s_{NN}} = 2.76$ TeV (red points) and for $p_T > 10$ GeV/$c$ at $\sqrt{s_{NN}} = 200$ GeV (blue points) as a function of $\rho^{1/2}L$ (a). The corresponding $p_T$ shifts as a function of the same scaling variable are shown in (b). Due to the different shape of the p-p spectrum the energy loss is the same in our model even if the $R_{AA}$ is different.

The two fits are a linear, $\Delta p_T/p_T = C\xi$, and a non-linear relation, found by solving $d\Delta p_T/p_T = C d\xi$, where $\xi = \rho^{1/2}L$ and $C$ is the slope parameter. The latter parameterization illustrates that the deviation from the linear dependence on the scaling variable $\xi$ is consistent with a constant relative energy loss.

IV. DISCUSSION

In the sections above, we have extracted a quite robust scaling law relating the characteristic $p_T$ shift of high $p_T$ hadronic spectra in A-A collisions to generic properties of the collision, such as the multiplicity density and the RMS of its distribution, that seems to work over an order of magnitude in collision energy. Despite the fact that these properties are quite inclusive and do not take account

1 The two fits are a linear, $\Delta p_T/p_T = C\xi$, and a non-linear relation, found by solving $d\Delta p_T/p_T = C d\xi$, where $\xi = \rho^{1/2}L$ and $C$ is the slope parameter.
of the dynamical evolution of the system created in these collisions, the observed scaling suggests a dominant and consistent mechanism underlying the physics of jet quenching from RHIC to LHC.

In the discussion of energy loss we have focused on the very high $p_T$ data while in Fig. 3 one clearly observes large differences at lower $p_T$ ($\lesssim 6$ GeV/c). Some of those can be attributed to the typically much larger flow in-plane than out-of-plane. It is important to note that the good agreement at high $p_T$ shows that the density variation seems to be pivotal for the quenching mechanism, see Fig. 2. This might suggest that the transverse expansion of the medium has little effect on jet quenching, i.e., the dilution of the medium is canceled by the longer path length. This important issue certainly deserves further studies.

It is tempting to interpret the results from Sec. III in light of radiative energy loss, see Appendix B for a brief review. Note firstly that the naïve identification of the $p_T$ shift with the mean energy loss taken by one-gluon emission, which would lead to $\Delta p_T \sim \hat{q}L^2 \sim \rho^{3/4}L^2$, cf. Eq. (B4), fails to produce a scaling, see the bottom-right panel of Fig. 2. Accounting for multi-gluon emissions and the bias due to the steeply falling parton spectrum one rather expects $\Delta p_T \sim \rho^{3/4}L$, cf. Eq. (B3), which is close to what we observe in the data.$^2$

Similar studies have, as mentioned before, been carried out by Lacey et al. [13–15]. The main difference from our work is that in their studies they do not take the density effect for different centralities into account and they obtain a single curve for $R_{AA}$ vs. path length $L$. But, as can be seen in the top-left panel of Fig. 2, this relation breaks down when one studies $R_{AA}$ in- and out-of-plane. Therefore their results should be supplemented by the additional information we have extracted here. There are also important differences in the physical pictures one extracts. Based on their findings they assert that jet quenching first sets in after a time of $\approx 1$ fm/$c$ [13]. In our analysis, the intercept in the right panel of Fig. 4 is consistent with zero suggesting that the plasma formation time does not play a role for quenching.

We point out that our improved data driven analysis also allows to extract some information about the centrality dependence of the quenching phenomenon. Presently, we will identify the extracted density $\rho$ from Eq. (1) with the transport parameter for jet quenching averaged over the trajectory of the jets, $\langle \hat{q} \rangle$, in the context of radiative energy

$^2$ To study the expected $p_T$ behavior of the shift from radiative processes, $\Delta p_T \sim \hat{p}_T^{1/2}$ goes beyond the scope of our present study.
FIG. 5. The $\langle \hat{q} \rangle$ as a function of centrality for LHC data extracted using Eq. 3.

loss. Then, from Eq. B3, we find

$$\langle \hat{q} \rangle = \left( \frac{1}{L} \frac{\Delta p_T}{p_T} \right)^2 \frac{n p_T}{4 \pi \bar{\alpha}^2},$$

(3)

where $n$ is the power of the invariant p-p spectrum and $\bar{\alpha} = \alpha_s C_R / \pi$ ($C_R$ being the relevant color factor), and we refer to Appendix B for further details. Figure 5 displays the resulting centrality behavior, with $\bar{\alpha} = 0.3$ and $p_T = 11$ GeV/c. However, we note that this interpretation of the data driven results introduces some conceptual issues. In fact, we expect both $n$ and $\bar{\alpha}$ to vary with the center-of-mass collision energy. The reason for the variation of the latter quantity, comes about since at RHIC (LHC) we expect the high $p_T$ particles to be fragments from dominantly quarks (gluons) implying a different color factor in $\bar{\alpha}$. The similarity between RHIC and LHC in Fig. 4 therefore appears accidental in this context. We recall that the main motivation behind the data driven study was to avoid these conceptual difficulties. In our opinion, the most solid conclusion that can be drawn from Fig. 5 is the decrease of $\langle \hat{q} \rangle$ by roughly a factor 4 from central to peripheral collisions dictated by the $\sqrt{\rho}$ dependence.

Albeit the data-driven analysis and subsequent interpretation both deal with static quantities, and therefore are inherently consistent, a serious caveat of the interpretation in terms of radiative energy loss is the neglecting of the longitudinal expansion of the medium. This can be estimated by making use of the dynamical scaling law for $\hat{q}$ [39, 40]. For a Bjorken-expanding medium the average transport parameter $\langle \hat{q} \rangle$ is related to the initial $\hat{q}_0$ measured at some initial proper time $\tau_0$ as $\langle \hat{q} \rangle \sim \tau_0 \hat{q}_0 / L$. This, in turn, implies that the expected path length dependence due to medium-induced radiative processes would scale as $\sim L^{1/2}$, rendering it incompatible with the extracted scaling behavior. Within our data driven approach, these ideas rather imply that the extracted values of the average transport parameter involves a significantly larger initial $\hat{q}_0$ in the early stages of the collision. A generic theory driven approach to a wide array of energy loss scenar-
ios were presented in [37] in the context of a Monte-Carlo model which also includes realistic nuclear geometry and couples to a hydrodynamical model of the plasma, see also, e.g., [41] for similar efforts.

The extraction of the $p_T$ loss is done for charged particles while the quenching supposedly affects the spectra at the parton level. The charged particle $p_T$ spectrum at high $p_T$ largely reflects leading particles and as we know from measurements at LHC that leading particle fragments in quenched and unquenched jets share similar fractions of the jet $p_T$ [42], this approximation is probably not so bad. Still it would be interesting to make a similar study with jets.

V. CONCLUSIONS

In this study the goal have been to distance ourselves as far as possible from models of jet quenching and rather by selecting samples from different centrality classes with similar path lengths to be able to isolate the density effect and then study the path length dependence. Surprisingly the method works very well and is in fact in reasonable agreement with theoretical considerations. A critical question is how the longitudinal expansion of the medium affects jet quenching and this has tremendous impact on how one would interpret the results in terms of e.g. the path length dependence.

Finally we note that the exact same density dependence observed for different centrality classes for LHC data is consistent with RHIC data indicating that the dense matter at RHIC and LHC has fundamentally similar properties.

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Appendix A: How to estimate the $p_T$ shift

The definition of the nuclear modification factor is

$$R_{AA}(p_T) = \frac{dN_{AA}/dp_T}{N_{coll}dN_{pp}/dp_T},$$

where $N_{coll}$ represents the number of binary collisions (the nuclear overlap function) for the given centrality class estimated from the Glauber model (see, e.g., [43]). Following the standard interpretation of the suppression of hadron spectra in A-A collisions, we assume that it arises due to a $p_T$ shift of the primor-
dial parton spectrum. We will therefore write
\[
\frac{dN_{AA}(p_T)}{dp_T} = N_{\text{coll}} \frac{dN_{pp}(p'_T = p_T + \delta_{p_T})}{dp'_T} \left| \frac{dp'_T}{dp_T} \right|,
\]
where we have made explicit for which \(p_T\) value the spectrum is evaluated at and included the Jacobian of the transformation, which also can be written as \(dp'_T/dp_T = 1 + \delta_{p_T}/dp_T\). Thus, the Jacobian differs from unity if \(\delta_{p_T}\) is a function of \(p_T\). Explicitly, the spectrum on the LHS of Eq. (A2) is measured at a given \(p_T\), while \(p'_T\) on the RHS represents the primordial momentum of the parton prior to energy loss. Thus, the master equation to extract the energy loss via the \(p_T\) shift reads
\[
\frac{dN_{pp}(p'_T)}{dp'_T} = R_{AA}(p_T) \frac{dN_{pp}(p_T)}{dp_T} \left| \frac{dp_T}{dp'_T} \right|.
\]
Having no \textit{a priori} knowledge about the specific form of \(\delta_{p_T}\) that enters the Jacobian, we will parameterize it using two “extreme” cases:

1. Firstly, we assume that \(p_T = kp'_T\), where \(0 < k < 1\) is a constant. This implies that
\[
\frac{dN_{pp}(p'_T)}{dp'_T} = R_{AA}(p_T) p_T \frac{dN_{pp}(p_T)}{dp_T}.
\]

2. Secondly, we assume a constant \(p_T\) shift, \(\delta_{p_T} = \text{const.}\) The Jacobian is simply unity, and we get that
\[
\frac{dN_{pp}(p'_T)}{dp'_T} = R_{AA}(p_T) \frac{dN_{pp}(p_T)}{dp_T}.
\]

\(p_T\) shifts estimated from these two cases will be averaged and the difference will be indicated as a systematic uncertainty of the procedure.

\section*{Appendix B: Radiative energy loss}

For highly energetic probes the hot and dense medium is parameterized by one characteristic transport coefficient, the so-called \(q\) parameter which encodes the transverse momentum broadening per unit length. Heuristically, this parameter scales with the energy density \(\rho\) as \(q \propto \rho^{3/4}\). The largest energy that can be carried by a medium-induced gluon accumulates momentum along the whole path length of the medium and is usually defined as \(\omega_c \equiv qL^2/2\). The spectrum of induced gluons per unit length reads
\[
\frac{\omega dI}{d\omega dL} = \bar{\alpha} \sqrt{q/\omega}, \quad (B1)
\]
for energies \(\omega < \omega_c\), where \(\bar{\alpha} \equiv \alpha_s C_R/\pi\). It follows that the energy loss caused by the single-gluon emission, given by \(-dE/dL = \bar{\alpha}qL\), is dominated by the hard sector, \(\omega \sim \omega_c\).

\(^3\) To be precise, the spectrum in Eq. (B1) is regularized at a minimal energy marking the onset of the Bethe-Heitler regime.
ωc. One should on the other hand keep in mind that the number of gluons, given by $N(\omega) \sim \sqrt{\alpha^2 \omega_c/\omega}$, becomes large for soft gluons, in particular, when $\omega < \alpha^2 \omega_c$.

The quenching factor, which encodes the partonic spectrum modified in the medium prior to fragmentation,\(^4\) is defined as

$$Q(p_T) \equiv \int_0^\infty d\epsilon D(\epsilon) \frac{d^2\sigma^{\text{vac}}(p_T + \epsilon)/dp_T^2}{d^2\sigma^{\text{vac}}(p_T)/dp_T^2},$$

\[(B2)\]

where $D(\epsilon)$ is the probability distribution of energy loss. Assuming independent gluon emissions it is simply given by a Poisson distribution [29], but this premise can be improved upon by including, e.g., phase-space limitations [40] or energy-momentum conservation, see [44, 45]. These corrected distributions give rise to more complex scaling trends than discussed below, but will be neglected in the following. Presently we assume that the invariant p-p is well described by a power law spectrum with constant exponent $n$. Then, in the large-$n$ approximation we recast the quenching factor as

$$Q(p_T) = \exp(-n\delta_{p_T}/p_T),$$

where $\delta_{p_T}$ is directly related to the $p_T$ shift of the medium-modified parton spectrum as

$$d^2\sigma^{\text{med}}(p_T)/dp_T^2 = \frac{d^2\sigma^{\text{vac}}(p_T + \delta_{p_T})/dp_T^2}{d^2\sigma^{\text{vac}}(p_T)/dp_T^2}.$$

Inserting the latter expression into the formula for $Q(p_T)$ we obtain the so-called “pocket formula” for radiative energy loss [13–15, 29, 46]. Relating to our previous discussion, the shift scales as $\delta_{p_T} \sim p_T^{1/2} \rho^{1/2} L$.

Finally, note that in the special limit of $p_T > n\omega_c$ the $p_T$ shift rather becomes

$$\delta_{p_T} \approx \int_0^\infty d\omega N(\omega) \sim \omega_c,$$

and scales as $\delta_{p_T} \sim \rho^{3/4} L^2$. Thus, only in this particular regime can one identify the mean energy loss with the typical $p_T$ shift due to the dominance of one-gluon emission. The bias due to the steeply falling parton spectrum tend to shift the typical energy loss to smaller values, as given by Eq. (B3).

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\(^4\) See [9] for a discussion of the validity of such an assumption. For our present purposes, the quenching factor serves as a good indicator of the parametric behavior of the nuclear modification factor $R_{AA}$.

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