Research on Flight Trajectory Optimization Based on Quantum Genetic Algorithm

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Abstract. Taking the passenger aircraft A320 as the research object, the trajectory optimization of the aircraft climbing section under wind disturbance is carried out. Through the analysis of the aircraft's descent process, the kinematics model, fuel consumption model and atmospheric model of the aircraft are established, and the objective function and constraints of the problem are obtained. The quantum genetic algorithm is used to transform the optimal control problem into a nonlinear programming problem. The optimal trajectory of the aircraft's shortest descent time is obtained by simulation. The effects of different cost indices on the fall time and fuel consumption are discussed.

1. Introduction

As an indispensable part in the field of aircraft aviation, trajectory optimization has been widely used in aircraft flight management and traffic management, and has received more and more attention. Trajectory optimization is beneficial to improving the flight efficiency of aircraft and minimizing the cost while meeting the flight requirements; by using optimization techniques, the traffic congestion problem can be alleviated and the fuel-saving performance of aircraft can be significantly improved so that people's travel requirements can be met.

A variety of new methods have been developed based on the early Lev Pontryagin principles, and both direct and indirect methods of aircraft trajectory optimization have been developed. Among them, the Lev Pontryagin principle and variational method are the theoretical basis of the indirect method. Under the influence of the aircraft state variable and the motion equation, the extremum of the performance index function is obtained. The necessary condition of its intermediate optimality can be satisfied by its optimal solution, but at the same time it also has some disadvantages, that is, it has a very complicated derivation process of the optimal solution In the process of solving the two-point boundary value problem, there will be a small convergence region. In recent years, with the rapid development of Trajectory optimization method, the problem of optimal control has been well solved, Wang Yingxun, Chen Zongji, Hu Xiaobing [11, 12] and others used genetic algorithm (GA) to avoid the constrained flight path planning, and Yeonju [13] used genetic algorithm to establish the no-fly zone for several uavs The Path Planning and cooperative task assignment are realized, and then, because of the influence of stochastic search, the search efficiency of genetic algorithm will be reduced under the influence of population degeneration, and then the convergence speed will be reduced Duan Jiajia [8] and others studied the optimization of the lunar soft landing trajectory by
using the ant colony algorithm, but the search time was too long and it was easy to fall into the local optimal solution, Xu ying [6] through the use of Particle swarm optimization to complete the study of the aircraft's climb-cruise and descent of the optimal trajectory, wang Wei [9] used the energy state method to realize and optimize the genetic algorithm and the fuel-saving trajectory of the aircraft. However, because of the simplicity of the aircraft model, safety is not fully considered in civil aviation flight the actual civil aviation can not fully use the climb trajectory of the aircraft.

Quantum Genetic Algorithm (QGA) is a new intelligent Algorithm which combines the traditional Genetic Algorithm (GA) with quantum theory, and is first proposed by Narayanan Han then extends QGA by superposing quantum states and qubits. QGA is widely used in many fields such as path optimization, scheduling optimization and signal processing because of its outstanding advantages such as good population diversity, strong global search ability and fast convergence speed.

2. Basic QGA

With the full integration of GA and quantum computing, a new field of research called Basic QGA has emerged, which is based on quantum computing properties such as quantum entanglement and quantum parallelism. Compared with GA, QGA has stronger parallel processing ability and faster convergence speed by using quantum revolving gate and multi-state gene qubit coding to update the operation.

2.1. Qubit

For a quantum computer, a two-state quantum system acts as the physical medium of the information storage unit, and thus constitutes the qubit. Unlike a classical bit, a qubit can act simultaneously in a superposition of two quantum states, such as $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$, where $(\alpha, \beta)$ is a pair of amplitude constants that satisfy $|\alpha|^2 + |\beta|^2 = 1$ to represent the spin down and up states, respectively. The information in $|0\rangle$ and $|1\rangle$ can be contained in the same quantum bit.

In QGA, a gene can be stored and expressed by quantum bits. The "0" or "1" states can be used in the gene, or they can be superimposed at will. That is, the gene is no longer able to express certain information, but contains all of it. Through the operation of the gene, it can express all kinds of possible information simultaneously. With the use of quantum bit coding, the same chromosome can be simultaneously expressed in multiple superposition states, which greatly increases the diversity of QGA features. We can get better convergence by using quantum bit coding. When $|\alpha|^2$ or $|\beta|^2$ fluctuates at or near 0 or 1, the chromosome can be in a single state of convergence by using quantum bit coding.

2.2. Quantum Measurements

The goal of quantum measurement can be realized by the measurement of individual population, that is, a group of definite solutions can be calculated $p(t) = \{p'_1, p'_2, \ldots, p'_n\}$, in this case, the J solution in the t population (the measure of the j individual) is represented by $p'_j$, the binary string is its representation, m is its length, 0 or 1 is its representation at each position. Based on the quantum bit probability $|\alpha'_i|^2$ or $|\beta'_i|^2$, $i = 1, 2, \ldots, m$ the measurement process is as follows, the data with interval $[0,1]$ is selected randomly. If its value is above the square of probability amplitude, the measurement result is 1, otherwise the value is 0.

2.3. Quantum Updates

For evolution operation, Quantum Gate is the executive mechanism of evolution operation, which can be selected according to specific problems. Nowadays, there are many kinds of quantum gate, and
quantum gate should be regarded as the executive mechanism of evolution operation according to the characteristics of QGA. The quantum gate \( a \) can be adjusted by the following relation, which can be used to select the suitable quantum revolving gate \( U(\theta) \). That is:

\[
U(\theta) = \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\]

Where the rotation angle is represented by \( \theta \). Its symbol and size need to be preset first, the following relationship for the update process:

\[
\begin{bmatrix}
\alpha'_i \\
\beta'_i
\end{bmatrix} = U(\theta) \begin{bmatrix}
\alpha_i \\
\beta_i
\end{bmatrix} = \begin{bmatrix}
\cos \theta_i & -\sin \theta_i \\
\sin \theta_i & \cos \theta_i
\end{bmatrix} \begin{bmatrix}
\alpha_i \\
\beta_i
\end{bmatrix}
\]

In the formula, \( \begin{bmatrix}
\alpha_i \\
\beta_i
\end{bmatrix} \) and \( \begin{bmatrix}
\alpha'_i \\
\beta'_i
\end{bmatrix} \) represent the probability amplitude of the ith qubit revolving gate of chromosome before and after updating.

From the above formula, \( \alpha'_i \) and \( \beta'_i \) are respectively:

\[
\begin{align*}
\alpha'_i &= \alpha_i \cos \theta_i - \beta_i \sin \theta_i \\
\beta'_i &= \alpha_i \sin \theta_i - \beta_i \cos \theta_i
\end{align*}
\]

So: \( |\alpha'_i|^2 + |\beta'_i|^2 = 1 \) can see that after the transformation \( |\alpha'_i|^2 + |\beta'_i|^2 \) is still 1

3. Problem Description

Kinematic model of aircraft

After considering the mass of the aircraft, the three-degree-of-freedom motion mathematical model of change is established, namely:

\[
(\mathbf{R} + \mathbf{H}) \cos \theta \frac{d\lambda}{dt} = V \cos \lambda \cos \chi + W_x
\]

\[
(\mathbf{R} + \mathbf{H}) \frac{d\theta}{dt} = V \cos \lambda \sin \chi + W_y
\]

\[
\frac{dH}{dt} = V \sin \gamma, \quad \frac{dm}{dt} = -F_{\text{fuel}}
\]

of the change is established, namely:

\[
m \frac{dV}{dt} = T \cos \alpha - D - mg \sin \lambda
\]

\[
mV \cos \gamma \frac{d\chi}{dt} = L \sin \phi
\]

\[
mV \frac{d\gamma}{dt} = T \sin \alpha + L \cos \phi - mg \cos \gamma
\]

In the formula, \( T, L, D \) are used to express the thrust, lift, and drag of the engine respectively, and the fuel consumption per unit, the component of the wind field along the longitude direction and the component of the wind field along the latitude direction are represented by \( F_{\text{fuel}}, W_x \) and \( W_y \).

When the throttle opening is fixed, the engine thrust is also changing during the flight of Mach and the flight altitude, which can be expressed by the following relationship:

\[
T = T_0(H, Ma) \mu
\]

In the upper formula, the throttle opening is indicated by \( \mu \).

3.1. Objective Function

In the case of Flight Management Systems, the implementation of vertical navigation functions requires the calculation of the main parameter, the cost index, which refers to the ratio of the cost incurred under the effect of flight time to the fuel cost, namely:

\[
\text{Cost Index} = \frac{\text{Flight Time}}{\text{Fuel Consumption}}
\]
In the formula: The cost index and the cost index are represented by $C_i$ and $C_{time}$ respectively; the cost of the fuel consumed by the aircraft 1B is represented by $C_{fuel}$; the optimization of the aircraft trajectory is mainly to reduce the direct cost according to different cost index, the optimization is realized, and then the different state information of aircraft at different time is obtained.

3.2. Normalization

In the process of solving the optimization problem, if we want to improve the convergence speed of the algorithm, we need to deal with the aircraft equation by normalization. A dimensionless physical quantity is defined as:

$$\vec{V} = V / k_v, \vec{R} = R / k_R$$

$$\vec{H} = H / k_H, m = m / k_m$$

Thus, under the constraints of aircraft motion model, boundary conditions, state control variables and intermediate processes, the optimization problem of aircraft trajectory can be transformed into an optimal control problem the objective function of the latter is mainly the arrival time and the minimum time cost, namely:

$$\min J(X(t), U(t), t) = \int_{t_0}^{t_f} F_{fuel}(t) dt + C_i \times \int_{t_0}^{t_f} dt$$

$$X = f(X(t), U(t), t)$$

$$X_{min} \leq X \leq X_{max}$$

$$U_{min} \leq U \leq U_{max}$$

$$\phi(X(t_b), t_0, X(t_f), t_f) = 0$$

$$C(X(t), U(t), t) \leq 0$$

4. Trajectory optimization of climbing section

According to the quantum genetic algorithm and the actual trajectory optimization model as shown in the climb trajectory optimization algorithm flow:

(1) Population $Q(t_0)$ is initialized by the coevolutionary strategy of Niche Coevolution, n chromosomes are generated randomly, and all chromosomes are encoded by Qubit.

(2) Measure each individual in the initial population $Q(t_0)$ one by one, so as to obtain the corresponding definite solution $P(t_0)$.

(3) Evaluate the various deterministic solutions.

(4) Keep track of individual fitness and optimization.

(5) To judge whether it can be finished, if it can satisfy the condition, it can be quit, otherwise it needs to be further calculated.

(6) All individuals in population $Q(t)$ are measured one by one and the corresponding definite solution is obtained.

(7) Evaluate the various deterministic solutions.

(8) With the help of the dynamic adjustment mechanism of the rotation angle of the quantum revolving gate $U(t)$, the selection and crossover are used to realize the relative operation to the individual, thus the acquisition of the new population $Q(t + 1)$ can be realized.

(9) Keep track of the optimal individuals and their fitness.

(10) Add the number of iterations to 1 and return to step 5.
4.1. Simulation and Analysis
Taking the Airbus A-320 as an example, this paper completes the ambush optimization of the climbing section by using the algorithm mentioned above. The initial speed of the plane is 10m/S, the plane climbs gradually from the bottom to the altitude of 10000m, the terminal speed is 280m/S, and the constraint condition is $0 \leq 25 \leq \text{Ma} \leq 0.85, 0 \leq n \leq 1.75$, population size and evolutionary Algebra are n 300 and m 150 respectively, crossover probability 0.8, mutation probability 0.05.

Simulation Results:

![Flight trajectories](image1.png)

**Figure 1.** Climb flight path.

![Mach Number-time](image2.png)

**Figure 2.** Mach Number, track angle, flight altitude with time curve.

![Angle of attack-time](image3.png)

**Figure 3.** Curves of angle of attack and overload with time.
4.2. Result Analysis

As you can see from the oil diagram, after the trajectory optimization of the climb phase of the civil aircraft is completed by quantum genetic algorithm, the requirement of two climbs can be met. Fig. 2 illustrates in detail the variation of Mach number, altitude and track angle of the passenger aircraft during the crawling process. The speed of the aircraft can be significantly increased in the range of 110-150s, this area happens to be in the process of the first level flight of the aircraft, thus accumulating climbing power to prepare for the second climb. According to the engine curve in figure 4, the overall characteristics of the engine decrease with the increase of the aircraft altitude, so the first level flight can improve the efficiency of the engine. As can be seen from Fig. 3, the overload and angle of attack control variables of the passenger will change in the range of \([0, 1.5 \, \text{G}]\) when the aircraft is climbing, so as to satisfy the comfort requirements of the passenger. Aircraft generally need to pass through 270 seconds of crawling time, need to consume 200kg of fuel, to meet the relevant fuel consumption standards.

Fig. 5 shows the optimization process based on quantum genetic algorithm, that is, it reflects the change law of the population's optimal individual fitness, and finally the optimal individual fitness tends to be stable and has good convergence, the results show that the algorithm is very effective.

5. Conclusion

In this paper, the application of quantum genetic algorithm (QGA) is used to complete the research on the climb flight path optimization of civil aviation passenger aircraft. Taking Airbus A-320 as an example, the effectiveness of QGA is verified. Quantum genetic algorithm (QGA) can satisfy all the technical constraints, and also can achieve the climb requirements of civil aircraft, which shows that QGA can effectively solve the climb trajectory optimization problem of civil aircraft.
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