Bragg scattering of light in vacuum structured by strong periodic fields

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Elastic scattering of laser radiation due to vacuum polarization by spatially modulated strong electromagnetic fields is considered. The Bragg interference arising at a specific impinging direction of the probe wave concentrates the scattered light in specular directions. The interference maxima are enhanced with respect to the usual vacuum polarization effect proportional to the square of the number of modulation periods within the interaction region. The Bragg scattering can be employed to detect the vacuum polarization effect in a setup of multiple crossed super-strong laser beams with parameters envisaged in the future Extreme Light Infrastructure.

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Quantum electrodynamical vacuum fluctuates via virtual electron-positron pairs which induces polarization in strong external fields. The characteristic field when the vacuum polarization effects become significant is the so-called critical field $E_{cr} = m^2/e$ at which an electron gains energy equal to its rest mass $m$ within the Compton wavelength $\lambda_c = 1/m$ [1-3], where $e$ is the electron charge, $E_{cr} \equiv E_{cr}/8\pi \approx 2.3 \times 10^{29} W/cm^2$. Recently, strong field laser technique is advancing rapidly, fostered, from one side, by the laser fusion program [4,5], where $E_{cr}$ is the electron charge, $E_{cr} \equiv E_{cr}/8\pi \approx 2.3 \times 10^{29} W/cm^2$. The attempts to observe photon-photon scattering with laser beams were only able to determine the upper limit of the cross-section [6]. The possibility of observation of the photon-photon scattering with modern strong laser beams is analyzed in [21,23]. In the external field nonuniform in space, vacuum behaves as a nonuniform medium and photon scattering (or diffraction) becomes possible. The vacuum polarization effects due to a spatial gradient of magnetic field are considered in [25]. Diffraction effects due to the vacuum nonuniform polarization in strong focused laser beams are investigated in [26,27]. In optics of continuous media it is known that the periodic structure of a medium can significantly enhance the scattering due to interference of the scattered light generated from different layers of the structure (Bragg scattering) [28]. The Bragg concept is quite general and is applied not only in the context of propagation of electromagnetic waves [29] but also in quantum optics [30], atom optics [31] and for matter waves [32].

In this letter, we investigate Bragg scattering of a probe laser beam due to vacuum polarization in a strong spatially modulated external electromagnetic field. At certain scattering angles when the Bragg interference condition is fulfilled, the probability of the photon scattering is enhanced with respect to the usual photon-photon scattering by a factor proportional to the square of the number of periods in the structure. The enhancement is maintained also in the total probability of the scattering, integrated by scattering angle. First, we show the effect on the theoretically more transparent case of a spatially periodic magnetic field of a magnetic undulator. Then, we discuss the Bragg scattering when a probe laser beam penetrates the periodic structure of multiple focused laser beams, see Fig. 1(c). We consider the experimental realization of the Bragg scattering in the future ELI facility and its advantage with respect to the photon-photon scattering [22].

We consider a photon scattering in an external field, i.e., the transition $|1(k_1,e_1)\rangle \to |1(k_2,e_2)\rangle$, where $k_{1,2}$ and $e_{1,2}$ are the momentum and polarization of the incoming and outgoing
where \( P \) is vacuum polarization the Euler-Heisenberg Lagrangian \[19\]:

\[
l_{\text{vacuum polarization}} = \frac{\eta \omega^2}{4 \pi} \{ (e_1 B_0)(e_2 B_0) 
+ 4(e_1 (n_1 \times B_0)) \left[ (e_2 (n_1 \times B_0)) - i \left( e_2 \left( \frac{\nabla f^2}{\omega f^2} \times B_0 \right) \right) \right] 
+ 2i B_0^2 \left( e_2 \left( \frac{\nabla f^2}{\omega f^2} \times (n_1 \times e_1) \right) \right) \} f^2(r),
\]

where \( n_1 = k_1/k_2 \). The term \( T_{21} \) corresponds to the case when the incoming photon is annihilated first, then the final photon is created, while in the cross-term \( T_{12} \) this order is reversed. The photon scattering probability per unit time \( dW = n_1|T_{21}|^2 d^3k_2/\langle 2\pi \rangle^3 \) reads:

\[
\frac{dW}{d\Omega} = \rho_1 \omega^4 |M_{21} + M_{12}^*|^2 \mathcal{P},
\]

with the number and the density of incoming photons \( n_1 \) and \( \rho_1 \), respectively, \( \omega = k \equiv \omega_1 = \omega_2 \), the phase-matching factor

\[
\mathcal{P} = \left| \int_{(\gamma)} e^{i\mathbf{k} \cdot \mathbf{r}} \right|^2,
\]

and

\[
M_{21} = \frac{\eta}{16\pi} \{ 4((n_1 \times e_1)B_0)((n_1 \times e_2)B_0) 
+ 7(e_1 B_0)(e_2 B_0) + \frac{8}{\omega} (n_1 \times e_1)B_0(e_2 B_0) 
+ \frac{4B_0^2 \omega}{\omega} e_2(q \times (n_1 \times e_1)) \},
\]

The transition matrix element \( M_{21} \) simplifies when the incident photon momentum \( \mathbf{k}_1 \) is perpendicular to \( B_0 \):

\[
M_{21}^\perp = \frac{7}{8\pi} \eta B_0^2, \quad M_{21}^\parallel = \frac{\eta B_0^2 \omega}{2\pi} \cos \vartheta,
\]

where \( M_{21}^\perp \) corresponds to the transverse \((B_1 \perp B_0)\) and \( M_{21}^\parallel \) to the longitudinal polarization \((B_1 || B_0)\) and \( \vartheta \) is the angle between \( k_1 \) and \( k_2 \). The space integration in Eq. (7) is carried out over the interaction volume \( \mathcal{V} \), i.e., over the region of the overlap between the impinging laser beam with the undulator field, see the highlighted region in Fig. \( \text{[b]} \). The factor \( \mathcal{P} \) generates the phase-matching Bragg condition \( \Delta \mathbf{k} = \mathbf{k}_3 - \mathbf{k}_1 - \mathbf{nq} = 0 \) (with \( n = 2 \) in the case of magnetic undulator) when the wave scattered from different spatial periods of the structure interfere constructively (Bragg interference). This can take place only at certain impinging angles of the probe wave:

\[
2k \sin \frac{\vartheta}{2} = nq,
\]

The scattered radiation is concentrated in the specular direction. Due to the Bragg interference, \( \mathcal{P} \propto \mathcal{V}^2 \) at exact phase-matching \( \Delta \mathbf{k} = 0 \) and the differential probability is proportional to \( \mathcal{V}^2 \). In particular, if the interaction region is rectangular with corresponding lengths \( L_x, L_y, L_z \),

\[
\mathcal{P} = \left( \sin \frac{\Delta k_x L_x}{2} \sin \frac{\Delta k_y L_y}{2} \sin \frac{\Delta k_z L_z}{2} \right)^2.
\]
The total probability integrated over angular distribution is

$$W = (2\pi)^2 \rho_1 \omega^2 |M_{21}|^2 \int L_{\perp,\perp}(\mathbf{r}_\perp) d^2 \mathbf{r}_\perp,$$

where $L_{\perp,\perp}(\mathbf{r}_\perp)$ is the length of the interaction region along the direction of $k_2$ and $\mathbf{r}_\perp$ the coordinate transverse to $L_{\perp,\perp}$ [28]. Estimating $\int L_{\perp,\perp}(\mathbf{r}_\perp) d^2 \mathbf{r}_\perp \approx L_{\perp,\perp}^2$, with $L_{\perp,\perp}$ being the mean lengths of the interaction region along and transverse to $k_2$, the number of scattered photons per unit time and volume is

$$\frac{dN}{dt d\omega} \approx \mathcal{W}(\theta) \alpha m^4 \frac{E_0^2}{B_0^2} \frac{E_{cr}}{E},$$

where $E_0$ is the amplitude of the probe field, $\mathcal{W}(\theta) \equiv |M_{21}|^2/(\eta B_0^2)$, $\theta = (1/4\pi)^2$, $\alpha \equiv L_{\perp,\perp}/\lambda$ is the enhancement factor due to Bragg interference. The comparison of the Bragg scattering with the stimulated light-by-light scattering, proposed to realize with three strong laser beams [20,22], shows that the Bragg scattering can be larger by a factor $L_{\perp,\perp}/w_0$, with the laser beam waist size $w_0$, which can amount to an order of magnitude. Note that Eq. (13) is valid when the probe beam is rather monochromatic and has a low angular spread. The bandwidth ($\Delta\omega_0$) and the angular spread ($\Delta\theta_0$) of the probe beam should be limited to fulfill the Bragg condition Eq. (10) within the energy uncertainty $\sim 1/\tau$, with the interaction time $\tau$. $\Delta\omega_0 \approx (2\pi/\tau)D_{\omega_0} = (2\pi/\tau)(2/n^2)\sin^2(\theta/2)$ and $\Delta\theta_0 \approx (2\pi/\tau)D_{\theta_0} = (2\pi/\tau)(1/4\cos(\theta/2))$, where $D_{\omega_0} = |\partial(\omega_1 - \omega)/\partial\omega_1|$ and $D_{\theta_0} = |\partial(\omega_1 - \omega)/\partial\theta|$. The latter impose restrictions on the enhancement factor $\epsilon$. The largest interaction length $L_{\perp,\perp}$, equal to the undulator length $L_u$, is possible at $\theta \sim \pi$ when $k \sim q$. Then, the enhancement factor is determined by the number of undulator periods ($N_u$) $\epsilon \sim N_u$ but is restricted by the probe bandwidth $\epsilon \lesssim \omega/\Delta\omega_0$.

Now let us consider Bragg scattering when the periodic structure is formed using a set of $N$ focused laser beams (elliptic Gaussian beams [33]) propagating parallel to each other:

$$E^{(0)} = E_0^{(0)} \sum_{n=1}^{N} f(x,y,z-n\delta)\cos[\omega_n t - k_L x + \varphi(x,y,z-n\delta)],$$

where $f(x,y,z) = (\sqrt{w_x w_z}/\sqrt{w_y(x)w_z(x)})e^{-x^2/w_y^2(x)+z^2/w_z^2(x)}$, $\varphi(x,y,z) = -ky^2/2R_y(x) + z^2/2R_z(x) + \frac{1}{2}\tan^{-1}x/x_0 + \frac{1}{2}\tan^{-1}y/y_0$, $w_y(x) = w_y\sqrt{1+x^2/x_0^2}$, $w_z(x) = w_z\sqrt{1+x^2/x_0^2}$, $R_y(x) = x + x_0^2/x$, $R_z(x) = x + x_0^2/x$, $x_0 = k_0 w_y^2/2$ and $x_0 = k_0 w_z^2/2$. The distance between the adjacent beams $d$ is assumed to be larger than the waist size of a single beam $w_z$, therefore, the superposition of fields of different beams is negligible. It is also assumed that $|Vf(\mathbf{r}) \times E_0| \ll k_L F(r) E_0$ and $|Vf(\mathbf{r})| \ll k_L F(r)$ and in this case $B_0 = n_L \times E_0$, where $n_L = k_L/k$ and $k_L$ is the laser wavevector. The current calculated as

$$j^{(1)} = \frac{n}{4\pi} \left( F^2(r,t) \frac{\partial R_1}{\partial t} + F^2(r,t) \nabla \times R_2 + \nabla F^2(r,t) \times R_2 \right).$$

![FIG. 2: (Color online) The dependence of the scattering probability on the probe impinging angle [the factor $\mathcal{W}(\theta)$]: (solid line) B-field case, the upper curve - transverse polarization, the lower curve - longitudinal polarization; (dashed line) E-field case, the upper curve - longitudinal polarization, the lower curve - transverse polarization.](image-url)
by the wave vectors $k_1$ and $k_2$. The scattering matrix element for different polarizations of the probe wave reads:

\[
M_{21}^{\parallel} = \frac{4\eta E_0^2}{\pi} \sin^4 \frac{\theta}{4} \left( 1 - 4 \cos^2 \frac{\theta}{4} \right),
\]

\[
M_{21}^{\perp} = \frac{7\eta E_0^2}{\pi} \sin^4 \frac{\theta}{4}, \quad M_{21}^{\parallel|} = \frac{4\eta E_0^2}{\pi} \sin^4 \frac{\theta}{4},
\]

\[
M_{21}^{\perp|} = \frac{7\eta E_0^2}{\pi} \sin^4 \frac{\theta}{4} \left( 1 - 4 \cos^2 \frac{\theta}{4} \right),
\]

(18)

where the longitudinal (transverse) polarization means $B_1 \parallel$ ($\perp$) $B_0$ in the case of $B$-field and $E_1 \parallel$ ($\perp$) $E_0$ for $E$-field. The total number of scattered photons per unit volume and time in the laser beam setup is given by the same Eq. (13) with $q = e^{-q^2w_0^2/4}/(\sqrt{2\pi}(360)^2)$ and replacing $B_{cr} \rightarrow E_{cr}, B_0 \rightarrow E_0$. The angular dependence factor $\mathcal{N}(\theta)$ is shown in Fig. 2. The enhancement factor in this case is

\[
\epsilon = \frac{N d}{\lambda}, \quad \text{if} \quad kw_0^2/4 \gtrless N d
\]

\[
\epsilon = 2 \pi \sqrt{N d/\lambda(w_0/\lambda)}, \quad \text{otherwise},
\]

(19)

with the wavelength $\lambda$. When $kw_0^2/4 \lesssim N d$, the emission angular width becomes restricted by the width of the energy uncertainty, as the Bragg condition should be fulfilled simultaneously. We estimate the yield of scattered photons in the setup of multiple laser beams. Employing a laser field with an intensity of $I = 2.3 \times 10^{22}$ W/cm$^2$, $\lambda = 1 \mu$m, pulse duration of $\tau = 100$ fs, $w_0 \sim 5 \lambda$, $w_r \sim \lambda$, $d \sim \pi w_z$, and $N = 10$, the Bragg scattering angle is close to $\pi$ at $n = 2$ resonance, and the number of scattered photons per pulse is

\[
\mathcal{N} = \frac{\alpha(2\pi)^{3/2}}{(360)^2 e^4} \left( \frac{I}{E_r} \right)^3 \frac{w_0^6 \tau}{w_z^4 \lambda^3} \left( \frac{3w_z^2}{d} \right)^2 \mathcal{W}(\theta) \approx 4.8.
\]

(20)

In the Bragg scattering setup, the enhancement factor $\epsilon$ and the interaction volume are larger with respect to those in the stimulated light-by-light scattering, each roughly by an order of magnitude. The requirement for the vacuum background pressure to suppress the competing processes is the same as in the case of photon-phonon scattering \cite{[32]}.

Concluding, we have shown for the first time how the coherence effects arising in spatially structured vacuum in strong periodic fields can enhance the vacuum polarization effects. In particular, the enhancement of the photon-phonon scattering effect is proposed employing Bragg scattering of a probe laser beam by a set of parallel multiple laser beams. We want to underline that the considered coherence effect has a general nature. Similar enhancement effects will exist in all type of inelastic light-by-light scattering and other processes based on spatially modulated vacuum polarization. The latter will be considered elsewhere.

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