Optimal Storage and Solar Capacity of a Residential Household under Net Metering and Time-of-Use Pricing

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Outline

Background and System Model

Optimal Investment in Storage Capacity

Optimal Investment in Solar PV Panel area

Simulation Study and Result Analysis
Incentive programs and ongoing reduction in costs are driving installation of solar PV panels and storage systems in residential households. The United States installed a record 6GW of residential PV and 1.537GWh of residential storage in 2022. There is a need for optimal investment decisions to reduce the electricity consumption costs of the residential sector.
Schematic of a single household with solar and energy storage interconnected to the grid
Pricing Scenario

- Each day is divided into two fixed continuous periods; peak \((h)\) and off-peak \((l)\) with ToU pricing and net metering billing mechanisms.
- The homeowner is compensated for the net generation at price \(\mu\).
- Otherwise, the homeowner is required to pay the net consumption at price \(\lambda\) for the energy consumed from the grid.
- Prices satisfy:

\[
\lambda_h > \mu_h > \lambda_l > \mu_l
\]
Daily cost expression of household

- For a random variable $X$, its expectation is written as $\mathbb{E}[X]$. 
- Peak and off-peak consumption random processes denoted by $H_h$ and $H_l$. 
- Let $F_{H_h}(x) = \int_{-\infty}^{x} f_{H_h}(x) \, dx$ and $F_{H_l}(x) = \int_{-\infty}^{x} f_{H_l}(x) \, dx$ be the cumulative distribution functions, where $f_{H_h}(x)$ and $f_{H_l}(x)$ are the probability density function of $H_h$ and $H_l$ respectively. 
- The daily expected electricity consumption cost of the household (without solar and storage) with time-of-use pricing is 

$$C = \lambda_h \mathbb{E}[H_h] + \lambda_l \mathbb{E}[H_l]$$

(1)
Assumptions and considerations for storage investment

- Consider a household with energy storage capacity $B$ and daily capital cost $\lambda_b$
- Selling price at peak period $\mu_h$ is higher than buying price $\lambda_l$ at off-peak period. So, storage is fully discharged during peak period, charged during off-peak period
- We assume $(\mu_h - \lambda_l) \geq \lambda_b$ for viable arbitrage opportunity
- Household uses storage first and purchases any deficit $(H_h - B)$ from utility at peak period price $\lambda_h$
- Excess energy $(B - H_h)$ sold back to utility at peak period price $\mu_h$
- During off-peak period, storage is fully charged and household purchases energy $(H_l + B)$ at the lower price $\lambda_l$
- Ideal operating conditions of the storage unit are assumed
Condition for optimal investment in storage capacity

The expected cost of the household is

\[ J(B) = \lambda_b B + \mathbb{E}[\lambda_h (H_h - B)^+ - \mu_h (B - H_h)^+ + \lambda_l (H_l + B)] \]  \hspace{1cm} (2)

where \((x)^+ = \max\{x, 0\}\) for any real number \(x\). Optimal storage capacity:

\[ B^0 = \arg \min J(B) \]

**Theorem 1.** The optimal storage investment decision \(B^0\) is given by

\[ F_{H_h}(B^0) = \frac{\lambda_h - \lambda_l - \lambda_b}{\lambda_h - \mu_h} \]  \hspace{1cm} (3)

The optimal cost is given by

\[ J(B^0) = \lambda_h \mathbb{E}[H_h | H_h \geq B^0] + \mu_h \mathbb{E}[H_h | H_h < B^0] + \lambda_l \mathbb{E}[H_l] \]  \hspace{1cm} (4)
Assumptions and considerations for investment in solar PV

- Homeowner with energy storage \( B^0 \) now considering solar PV panels with area \( a \).
- \( H_h, H_l, S_h, S_l \): random variables of household consumption and irradiance during a day.
- Peak period: uses solar and storage first, purchases deficit \( (H_h - aS_h - B^0) \) at peak price \( \lambda_h \).
- Excess energy \( (aS_h + B^0 - H_h) \) sold back to utility at peak price \( \mu_h \).
- Off-peak period: storage charged to full. Uses solar first, purchases deficit \( (H_l + B^0 - aS_l) \) at lower price \( \lambda_l \).
- Excess solar energy \( (aS_l - B^0 - H_l) \) sold back to utility at lower off-peak price \( \mu_l \).
- \( \lambda_a \): daily capital cost of panel area, amortized over lifespan.
- PV panels operate under ideal conditions.
Condition for optimal investment in solar PV

The expected cost of the homeowner is:

\[ J(a) = \lambda_b B^0 + \lambda_a a + \mathbb{E}[\lambda_h (H_h - aS_h - B^0)^+] - \mu_h (aS_h + B^0 - H_h)^+ + \lambda_i (H_l + B^0 - aS_l)^+ - \mu_l (aS_l - B^0 - H_l)^+] \]  \hspace{0.5cm} (5)

The homeowner will invest in a solar panel area,

\[ a^0 = \arg \min J(a) \text{ subject to } 0 \leq a \leq a_{\text{max}} \]

**Theorem 2.** The optimal investment decision \( a^0 \) area of solar PV panel under the condition \( \lambda_h = \mu_h \) and \( \lambda_l = \mu_l \) is given by:

\[ a^0 = \begin{cases} a_{\text{max}} & \text{if } \lambda_h \mathbb{E}[S_h] + \lambda_l \mathbb{E}[S_l] \geq \lambda_a \\ 0 & \text{else} \end{cases} \]  \hspace{0.5cm} (6)
For the case of $\lambda_h > \mu_h$ and $\lambda_l > \mu_l$, finding an analytical solution seems to be harder.

We can still observe that the cost is convex.

The peak and off-peak distribution functions need to be approximated from the data and the numerical methods are needed to compute the solution.
Model limitations

- Our model is designed for mathematical tractability and simplicity, ignoring complex pricing mechanisms, storage efficiency, device degradation, and operating costs.
- In reality, there can be multiple time periods with different prices, storage and solar devices can degrade, and storage devices can have different charging and discharging efficiencies.
- While analytical solutions can be arrived at for some of these conditions separately, a numerical simulation-based approach is needed for a comprehensive analysis that includes all conditions.
Simulation case study

- We study a single household located in a residential area of Austin, Texas, planning to invest in PV rooftop panels with a storage unit.
- Data is sourced from the Pecan Street project with the prosumer code of the household being 26.
- The study covers the entire year of 2016, with the average monthly consumption of the household being 1000 kWh.
- Household consumption and solar irradiance data are divided into peak (8 hrs to 22 hrs) and off-peak (22 hrs to 8 hrs) periods.
- A standard rooftop solar PV panel produces 183 W at an irradiance of 1000 W/m² for a panel area of 1 m².
- Considering a 93% PV system efficiency, the panel output reduces to 170 W.
Results

- Without storage and solar, the total expected cost for the household is $4,021.49.
- After investing in optimal storage capacity $B^0$, the cost reduces to $2,993.50, leading to 25.56% in savings for one year.
- Every 0.5 kW of solar PV panel investment (3 m$^2$ panel area) results in an additional cost reduction of $134, or 3.3% savings.
Conclusion

▶ We studied the impact of optimal investment decisions for storage and solar PV panels under net metering billing mechanism with time-of-use pricing and obtained some analytical results.
▶ We presented a case study using load consumption and solar irradiance data and investigated how optimal investment decisions affect the net electricity consumption cost.
▶ We showed that through optimal investments in storage and solar, significant cost benefits can be attained.
▶ This work provides useful information for the prosumer to consider before investing in solar and storage resources under the new billing mechanism.
Future Work

▶ We plan to derive joint investment decision of a prosumer who would invest in both solar PV and storage unit simultaneously.
▶ We also plan to consider scenarios like more prices in a day, price uncertainties, battery degradation etc. in our analysis.
▶ Since an analytical solution is not likely, we plan to derive a solution using numerical methods.