Optimized phase switching using a single atom nonlinearity

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We show that a nonlinear phase shift of $\pi$ can be obtained by using a single two level atom in a one sided cavity with negligible losses. This result implies that the use of a one sided cavity can significantly improve the $\pi/18$ phase shift previously observed by Turchette et al. [Phys. Rev. Lett. 75, 4710 (1995)].

One of the most significant achievements in the field of quantum optics is the realization of strong nonlinear effects by enhancing the coupling between single atoms and the light field. In particular, the possibility of obtaining large conditional phase shifts has attracted much attention because of its potential usefulness in the realization of phase gates for optical quantum computation and similar manipulations of quantum states at the few photon level [1, 2, 3, 4, 5, 6]. In order to optimize controlled phase shifts, it is desirable to avoid losses while moving close to the resonance of the two level atom causing the nonlinear phase shift. In the experiment by Turchette et al. [2], the nonlinearity of the atom was observed in the phase change of the light transmitted through the atom-cavity system. However, the transmission of a two sided cavity at the atomic resonance is very low, so the experiment was conducted at frequencies significantly detuned from this resonance. As a result, the phase shift observed was limited to only about $\pi/18$. In order to improve this phase shift, it is necessary to move closer to resonance while avoiding dissipative losses. In this paper, we therefore propose the use of a one sided cavity with negligible losses to non-cavity modes. In such a geometry, the total reflection is always close to one and all dissipation is suppressed. The nonlinearity then has the maximal effect on the phase of the light field while leaving the intensity unchanged. This makes it possible to realize a nonlinear phase shift of $\pi$ at the atomic resonance. Figure 1 shows an illustration of this dissipation free setup. Effectively, the cavity confines the light field interacting with the two level atom to a single beam with a well defined transversal profile. The suppression of losses to non-cavity modes can be achieved by covering a large solid angle of emission with the confocal cavity mirrors. Further improvements may be possible by using dielectric materials, e.g. in a photonic crystal geometry.

The most simple case of this dissipation free setup is obtained in the bad cavity regime, where the cavity lifetime is so short that the cavity field can be adiabatically eliminated. In terms of the conventional cavity quantum electrodynamics parameters this regime is characterized by $\kappa \gg g$, where $\kappa$ is the cavity damping rate and $g$ is the dipole coupling between the atom and the cavity. The effective dipole damping rate caused by emissions through the cavity

\[
\begin{pmatrix}
    |E\rangle \\
    |G\rangle
\end{pmatrix}
\]

\[
T_F \gg T_B
\]

FIG. 1: Illustration of the dissipation free geometry for the realization of an optimized nonlinear phase shift at resonance. $T_F$ and $T_B$ denote the transmission coefficients of the two mirrors. Ideally, $T_B = 0$. 
is then given by $\Gamma = g^2/\kappa$ and the rate of spontaneous emission through the cavity is equal to $2\Gamma$. Note that this emission rate also includes any effects of cavity enhancement of spontaneous emission. If the transverse atomic decay rate to noncavity modes $\gamma$ is negligible ($\gamma \ll 2g^2/\kappa$), nearly all emissions from the atom can be confined to the cavity and the total spontaneous emission rate of the excited atom in the cavity will be given by $2\Gamma$. With these parameters, the relevant dynamics can be described by the well known Bloch equations of the driven two level atom. For a two level system with a ground state $|G\rangle$ and an excited state $|E\rangle$, the Bloch equations can be expressed in terms of the complex dipole operator $\hat{\sigma}_- = |G\rangle\langle E|$ and the inversion operator $\hat{\sigma}_+ = 1/2(|E\rangle\langle E| - |G\rangle\langle G|)$. For a coherent driving field $b_{\text{in}}(t)$, the Bloch equations then read

$$\frac{d}{dt} \langle \hat{\sigma}_- \rangle = -\Gamma \langle \hat{\sigma}_- \rangle + 2\sqrt{2\Gamma} b_{\text{in}}(t) \langle \hat{\sigma}_z \rangle$$

$$\frac{d}{dt} \langle \hat{\sigma}_z \rangle = -2\Gamma \left( \langle \hat{\sigma}_z \rangle + \frac{1}{2} \right) - \sqrt{2\Gamma} \left( b_{\text{in}}^*(t) \langle \hat{\sigma}_- \rangle + b_{\text{in}}(t) \langle \hat{\sigma}_- \rangle^* \right). \quad (1)$$

Note that these equations of motion correctly describe the open system quantum dynamics of the two level atom. In particular, the effects of quantum fluctuations in the coherent driving field are fully included in the relaxation dynamics of the atom. This can be seen clearly in the case of $b_{\text{in}} = 0$, where the incoming field is in the vacuum state. The correct quantum mechanical description of this situation is then given by spontaneous emission dynamics that automatically include the dynamics of vacuum fluctuations $\hat{\sigma}_-$ and $\hat{\sigma}_+$. Since the noise properties of a coherent state are no different from the vacuum, the full quantum mechanical effect of a coherent driving field is indeed properly represented by equations (1).

The signal from the atom will now be observed in the reflected light, where it interferes with the directly reflected component of the input field $b_{\text{in}}$. The coherent amplitude of the output field can then be described according to input-output theory $\hat{\sigma}_-$ as

$$b_{\text{out}}(t) = b_{\text{in}}(t) + \sqrt{2\Gamma} \langle \hat{\sigma}_- \rangle. \quad (2)$$

The normalization of the light field amplitudes $b_{\text{in}}(t)$ and $b_{\text{out}}(t)$ has been chosen so that the squares of the amplitudes correspond to the photon current associated with the fields.

Equations (1) and (2) fully describe the coherent dynamics of the field-atom system. In particular, equation (2) correctly describes the expectation value of the output field, as observed in homodyne or heterodyne detection $\hat{\sigma}_-$. However, this is not sufficient to predict the average emitted photon number, which can only be observed in precise photon detection experiments $\hat{\sigma}_-$. This photon emission rate also includes a noisy component $P_{\text{noise}}$ corresponding to random phase emissions. The total intensity of the emitted field can therefore be separated into a coherent contribution $|b_{\text{out}}|^2$ and an incoherent contribution $P_{\text{noise}}$. Energy conservation requires that any difference between the output intensity and the input intensity $|b_{\text{in}}|^2$ results in a corresponding change of the atomic energy given by $\langle \sigma_z \rangle$. Therefore, the proper value of the emission noise in a dissipation free geometry can be obtained from

$$\frac{d}{dt} \langle \hat{\sigma}_z \rangle = |b_{\text{in}}|^2 - |b_{\text{out}}|^2 - P_{\text{noise}}. \quad (3)$$

In order to identify the physical origin of noisy emissions, it is useful to compare the total change in atomic energy given by $d/dt\langle \hat{\sigma}_z \rangle$ with the change in coherent field energy, $|b_{\text{out}}|^2 - |b_{\text{in}}|^2$,

$$\frac{d}{dt} \langle \hat{\sigma}_z \rangle = 2\Gamma \langle \hat{\sigma}_z \rangle + \frac{1}{2} + \sqrt{2\Gamma} \left( b_{\text{in}}^*(t) \langle \hat{\sigma}_- \rangle + b_{\text{in}}(t) \langle \hat{\sigma}_- \rangle^* \right) \quad (4)$$

The change in coherent field energy $|b_{\text{out}}|^2 - |b_{\text{in}}|^2$ depends only on the coherent dipole expectation value $\langle \hat{\sigma}_- \rangle$ and corresponds exactly to the classical result for dipole emission without fluctuations. In the total change of energy at the atom, the square of the dipole expectation value $\langle \hat{\sigma}_- \rangle^2$ is replaced by $\langle \hat{\sigma}_z \rangle + 1/2$. This replacement suggests that the difference $P_{\text{noise}}$ between the two terms can be interpreted as emission from dipole fluctuations, where $\langle \hat{\sigma}_z \rangle + 1/2$ represents the average of the squared dipole. In terms of the atomic variables, the incoherent emission now reads

$$P_{\text{noise}} = \left( \frac{d}{dt} \langle \hat{\sigma}_z \rangle \right) - (|b_{\text{out}}|^2 - |b_{\text{in}}|^2) = 2\Gamma \left( \langle \hat{\sigma}_z \rangle + \frac{1}{2} - |\langle \hat{\sigma}_- \rangle|^2 \right). \quad (5)$$
It is important to note that this result fundamentally limits the coherence in the emission obtained from a single two level atom, since the dipole expectation value is limited by the maximal length of the Bloch vector, \( |\langle \sigma_- \rangle|^2 + |\langle \sigma_z \rangle|^2 \leq 1/4 \). With this limitation, the minimal incoherent emission is given by

\[
P_{\text{noise}} \geq 2\Gamma \left( |\langle \sigma_z \rangle| + \frac{1}{2} \right)^2.
\]  

(6)

Incoherent spontaneous emission is therefore an unavoidable side effect of the saturation of a two level transition and should be taken into account in the characterization of this kind of nonlinearity.

It should be noted that the average rate of photon emission can also be obtained using quantum trajectory theory \(^7\). However, the result of such an analysis is necessarily identical to the one obtained from the energy conservation argument applied above. Effectively, quantum trajectory theory does not modify the averaged results obtained in this paper. A more detailed analysis of the relationship between field and dipole fluctuations in the spontaneous emission of a two level atom mainly provides a time resolved description of the noise dynamics, showing that the minimal noise given by equation \((6)\) is related to unavoidable diffusion effects in the open system dynamics of the fully coherent atomic Bloch vector \(^8\). Quantum trajectory theory thus confirms the identification of \(P_{\text{noise}}\) with time dependent dipole fluctuations in the atomic system suggested by equations \((5)\) and \((6)\).

The nonlinear response of the atomic system at resonance can now be derived using equations \((1)\) and \((2)\). If the input field is given by a constant amplitude \(b_{\text{in}}(t) = b_{\text{in}}\), the stationary solution of the atom-field dynamics reads

\[
\begin{align*}
\langle \sigma_z \rangle &= \frac{-\Gamma}{2\Gamma + 8|b_{\text{in}}|^2} \\
\langle \sigma_- \rangle &= \frac{-\sqrt{2}\Gamma b_{\text{in}}}{\Gamma + 4|b_{\text{in}}|^2} \\
b_{\text{out}} &= \left(1 - \frac{2\Gamma}{\Gamma + 4|b_{\text{in}}|^2}\right)b_{\text{in}}.
\end{align*}
\]  

(7)

The nonlinearity of the response function is given by the ratio of \(b_{\text{out}}\) and \(b_{\text{in}}\),

\[
\frac{b_{\text{out}}}{b_{\text{in}}} = \frac{4|b_{\text{in}}|^2/\Gamma - 1}{4|b_{\text{in}}|^2/\Gamma + 1}.
\]  

(8)

Figure \(2\) illustrates this nonlinearity of the atomic response function. Experimentally, this response function can be directly observed using homodyne detection. Note that this was also done in \(2\), but the data was separated into a transmitted amplitude and a phase. In our case, this separation would create the impression of a sudden jump from a phase shift of \(\pi\) associated with a negative response function to a phase shift of zero associated with a positive response function at a switching intensity of \(|b_{\text{in}}|^2 = \Gamma/4\). The resonant configuration shown in figure \(1\) therefore allows an optimization of the phase shift observed in \(2\) to the ideal case of a seemingly instantaneous phase change of \(\pi\).

However, equation \((8)\) also indicates that the coherent amplitude of the response changes during the switching process. This change of amplitude is due to the incoherent emission \(P_{\text{noise}}\) described by equation \((5)\). As shown by the noise limit \((6)\), this incoherent contribution is unavoidable when the nonlinearity originates from the saturation of a two level atom. The relative contributions of coherent response and incoherent noise to the total output intensity are given by

\[
\frac{\left|\frac{b_{\text{out}}}{b_{\text{in}}}\right|^2}{\frac{P_{\text{noise}}}{|b_{\text{in}}|^2}} = \frac{(4|b_{\text{in}}|^2/\Gamma - 1)^2}{(4|b_{\text{in}}|^2/\Gamma + 1)^2}.
\]  

(9)

Figure \(3\) illustrates this intensity dependence of the noise in the output field. The phase flip at \(|b_{\text{in}}|^2 = \Gamma/4\) is clearly associated with a complete phase randomization in the emitted field. A look at the steady state described by equation \((7)\) shows that the coherent dipole is reduced to one half its linear value at this intensity. As a result, the amplitudes of dipole emission and of directly reflected light are exactly equal and the coherent output component is eliminated by the destructive interference of these two amplitudes. The remaining emission at \(|b_{\text{in}}|^2 = \Gamma/4\) is

\[
\frac{\left|\frac{b_{\text{out}}}{b_{\text{in}}}\right|^2}{\frac{P_{\text{noise}}}{|b_{\text{in}}|^2}} = \frac{16|b_{\text{in}}|^2/\Gamma}{(4|b_{\text{in}}|^2/\Gamma + 1)^2}.
\]  

(10)
therefore a completely incoherent random phase emission originating entirely from quantum fluctuations in the light-atom interaction. The gradual change in amplitude shown in figure 2 is thus associated with a rather drastic increase of noise at the switching point. Note that this kind of noise increase at the switching threshold is also typical for classical bistable systems. The quantum mechanical properties of a single two level system thus appear to be very similar to the noise properties of classical nonlinear systems.

A fully coherent output is only obtained in the linear limit given by $|b_{\text{in}}|^2 \ll \Gamma/4$, and in the fully saturated limit, given by $|b_{\text{in}}|^2 \gg \Gamma/4$. It is therefore necessary to consider the sensitivity of various applications to this noise effect. In particular, there is a great difference between an application to interference effects with coherent light and the single photon switching envisioned in [2, 3], since the interference measurements allow an averaging of the signal, whereas single photon experiments do not allow this. The adiabatic elimination used in our model suggests that the average photon number in the cavity is not a useful criterion of the strength of the nonlinearity at the quantum level. Rather, it is more appropriate to compare the rate of incoming photons with the spontaneous emission rate of the excited atom. In the configuration discussed here, the switching intensity $|b_{\text{in}}|^2 = \Gamma/4$ is only one eighth of the spontaneous emission rate. This means that on average, only one eighth of a photon arrives at the atom during the spontaneous emission lifetime $\tau_{\text{sp}} = 1/(2\Gamma)$. Since switching is possible at such a low photon density, it may be possible to use the dissipation free two level nonlinearity to implement a conditional phase shift of $\pi$ for exactly two photons. Specifically, the average intensity $2/T$ of a two photon pulse of pulse duration $T$ would exceed the switching intensity for pulse durations of $T < 8/\Gamma$. This pulse length may indeed be sufficient to justify the analogy between the coherent input field and the two photon state claimed in [2]. However, it should be noted that the switching noise in the two photon interaction will reduce the single photon coherence. A detailed investigation of this effect therefore requires a fully quantum mechanical treatment of the spatiotemporal two photon wavefunction [12, 13].

In general, an experimental realization of the dissipation free nonlinearity described in this paper requires that losses to non-cavity modes are negligible ($\gamma \ll 2\Gamma$). In order to understand how restrictive this limitation is, it may be useful to consider the modification of the linear response of the atom in the presence of losses. These losses can be included in the Bloch equations (1) by increasing the decay rates of the dipole and the inversion to $\Gamma + \gamma/2$ and to $2\Gamma + \gamma$, respectively. This increase can also be represented by the spontaneous emission ratio $\beta = \Gamma/(\Gamma + \gamma/2)$, which is defined as the fraction of spontaneous emission that is emitted through the cavity mode. The total spontaneous emission rate of the atom is then given by $2\Gamma/\beta$. In the linear case, the effect of this increased damping rate in the atomic response simply reduces the dipole response by a factor of $\beta$. According to equation (2), the linear output field amplitude is then given by

$$b_{\text{out}} = (1 - 2\beta)|b_{\text{in}}|.$$  

Thus, spontaneous emission into non-cavity modes gradually convert the phase shift of $\pi$ caused by the resonant dipole response back into absorption losses [14]. The linear phase shift of $\pi$ can be observed as long as more than half of the
FIG. 3: Relative contributions of the coherent signal intensity $|b_{out}|^2$ and the incoherent noise $P_{\text{noise}}$ in the output as a function of scaled input intensity $4|b_{in}|^2/\Gamma$.

spontaneous emission is emitted through the cavity. Since the saturation of the two level atom will eventually result in a phase shift of zero, a nonlinear phase change from phase $\pi$ to phase zero can be obtained whenever the linear phase shift is equal to $\pi$. The condition for an experimental observation of the nonlinear phase change is therefore the realization of optical confinement with $\beta > 1/2$. At present, the most promising technology for the experimental realization of the dissipation free nonlinear phase shift seems to be the microcavity system presented in \[1\,2\]. In particular, the parameters given in \[2\] correspond to a value of $\beta \approx 0.7$. If the setup illustrated in figure \[1\] were to be realized using exactly the same elements as in \[2\], this should already be sufficient to observe the phase flip described in this paper. However, $\beta \approx 0.7$ still implies that more than 80% of the input photons are lost in the linear response case, so further improvements of cavity geometries may be needed for an implementation of a nearly dissipation free device. Moreover, the reflection geometry proposed in this paper requires that special care should be taken to achieve proper mode matching between the cavity mode and the pump beam. In principle, this problem is very similar to the mode matching required to match the local oscillator beam to the cavity output in the transmission geometry. However, the effect of additional reflections of mismatched pump light may increase the effect of mode matching errors. The reflection geometry proposed here could therefore be more challenging to realize than the transmission geometry reported in \[1\,2\]. Nevertheless our theoretical predictions for this geometry indicate that the realization of such a geometry may well be worth the effort.

In conclusion, we have shown that the maximal nonlinear phase shift of $\pi$ can be obtained by using a single two level atom in a one sided cavity with negligible losses. It is then possible to use resonant light to obtain a nonlinearity sensitive to individual excitations of the atom. The reflection geometry illustrated in figure \[1\] thus optimizes the nonlinear phase shifts previously obtained in the experiment by Turchette et al. \[2\].

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Note that the reflected light initially overlaps completely with the input beam. However, the separation of two counter-propagating beams can be achieved by using standard optical elements such as optical directional couplers.

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It may be worth noting that absorption always corresponds to destructive interference between the phase flipped dipole emission stimulated by the input field and the input field itself. In this sense, the phase shift properties of resonant absorption are quite universal.