Noncommutativity as a Possible Origin of the Ultrahigh Energy Cosmic Ray and the TeV-photon Paradoxes

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Abstract

In this paper, we present a general modified dispersion relation derived from $q$-deformed noncommutative theory and apply it to the ultrahigh energy cosmic ray and the TeV-photon paradoxes—threshold anomalies. Our purpose is not only trying to solve these puzzles by noncommutative theory but also to support noncommutative theory through the coincidence of the region in the parameter space for resolving the threshold anomalies with the one from the $q$-deformed noncommutative theory.

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1 Introduction

Recently there is great interest in the study of the ultrahigh energy cosmic ray (UHECR) and the TeV-photon paradoxes. The first paradox is that some experiments \cite{1}-\cite{7} observed many hundreds of events with energies above $10^{19}$eV and about 20 events above $10^{20}$eV which are above the Greisen-Zatsepin-Kuzmin (GZK) threshold. In principle, photopion production with the microwave background radiation photons should decrease the energies of these protons to the level below the corresponding threshold. The second paradox is the fact
that experiments detected 20TeV photons from Mrk 501 (a BL Lac object at a distance of 150Mpc). Similarly to the first case, due to the interaction with the IR background photons, the 20TeV photons should have disappeared in the ground-based detections. These two puzzles have a common feature that both of them can be seen as some threshold anomalies: energy of an expected threshold is reached but the threshold is not observed. There are numerous solutions such as [8, 9] proposed for the UHECR and the TeV-$\gamma$ paradoxes. In particular, most authors [10]-[17] have suggested that Lorentz-invariance violation can be the origin of these anomalies and have obtained many developments.

In this paper, we try to deal with these paradoxes by a modified dispersion relation which is obtained from $q$-deformed noncommutative theory. It is well-known that noncommutative geometry [18, 19] plays an important role in the trans-Planck physics, therefore, for phenomena related to the ultrahigh energy physics such as these two paradoxes, it is more reasonable to consider the Lorentz-invariance violation is from noncommutative geometry and make some explanations for them in the background of noncommutative theory. The possibility has been mentioned that noncommutative geometry may be responsible for the ultrahigh energy cosmic ray and TeV-photon paradoxes in some papers[20]. Here through a complete process based on a brand-new method which is from $q$-deformed noncommutative theory, we will connect threshold anomalies with noncommutative theory to not only unravel these threshold anomalies but also present a strong support for noncommutative theory in turn.

Our paper is organized as follows: We derive a general modified dispersion relation from $q$-deformed noncommutative theory in section 2. In the following section we will apply this dispersion relation to the ultrahigh energy cosmic ray and the TeV-photon paradoxes, and with the observed energies we get a common region of the $(\omega, |q - 1|)$ parameter space to resolve these two anomalies. In section 4, we will give some implications and remarks for our work.
2 A General Modified Dispersion Relation from \(q\)-deformed Noncommutative Theory

We start our discussion from the general dispersion relation for a massive particle \((m = 0\) for a massless particle) in the realistic case (from then on, we will use the notation \(\hbar = c = 1\)),

\[ E^2 = m^2 + p^2. \tag{1} \]

The Hamiltonian operator for a fermion in the ordinary case is:

\[ \hat{H} = \frac{1}{2}\omega \left( \hat{a}^\dagger \hat{a} - \hat{a}\hat{a}^\dagger \right). \tag{2} \]

The number operator is defined as:

\[ \hat{a}^\dagger \hat{a} = \hat{N}, \quad \hat{a}\hat{a}^\dagger = 1 \hat{N}. \tag{3} \]

So we have:

\[ \hat{H} = \frac{1}{2}\omega \left( \hat{N} - (1 \hat{N}) \right). \tag{4} \]

Then

\[ E = \frac{1}{2}\omega (n - (1 - n)) = \frac{1}{2}\omega (2n - 1). \tag{5} \]

From (1) and (5), we can easily derive:

\[ n = \frac{\sqrt{m^2 + p^2}}{\omega} + \frac{1}{2}. \tag{6} \]

And in the \(q\)-deformed case \([21]\) defined,

\[ \hat{a}^\dagger_q \hat{a}_q = [\hat{N}]_q, \quad \hat{a}_q \hat{a}^\dagger_q = [1 \hat{N}]_q, \quad [x]_q = \frac{q^x - q^{-x}}{q - q^{-1}}, \tag{7} \]

where \(q\) is a complex deformation parameter. The non-deformed case is obtained by setting \(q\) equal to 1.

Therefore, from \([22]\), we can directly get:

\[ \hat{H}_q = \frac{\omega}{2} \left( [\hat{N}]_q - [1 \hat{N}]_q \right). \tag{8} \]
Applying (6) to (8), we can obtain

\[ E = \frac{\omega}{2} \left( \left[ \frac{\sqrt{m^2 + p^2}}{\omega} + \frac{1}{2} \right]_q - \left[ \frac{1}{2} - \frac{\sqrt{m^2 + p^2}}{\omega} \right]_q \right). \]  

(9)

For a boson in the non-deformed case:

\[ \hat{H} = \frac{1}{2} \omega \left( \hat{b}^\dagger \hat{b} + \hat{b} \hat{b}^\dagger \right), \]  

(10)

\[ \hat{b}^\dagger \hat{b} = \hat{N}, \quad \hat{b} \hat{b}^\dagger = \hat{N} + 1. \]  

(11)

The \( q \)-deformed number operator was defined in [23, 24, 25]:

\[ \hat{b}^\dagger_q \hat{b}_q = \left[ \hat{N} \right]_q, \quad \hat{b}_q \hat{b}^\dagger_q = \left[ \hat{N} + 1 \right]_q, \]  

(12)

and the \( q \)-deformed Hamiltonian [23, 26] is:

\[ \hat{H}_q = \frac{\omega}{2} \left( \left[ \hat{N} \right]_q + \left[ \hat{N} + 1 \right]_q \right). \]  

(13)

Similarly to the case of a fermion, we obtain the \( q \)-deformed dispersion relation of a boson:

\[ E = \frac{\omega}{2} \left( \left[ \frac{\sqrt{m^2 + p^2}}{\omega} - \frac{1}{2} \right]_q + \left[ \frac{\sqrt{m^2 + p^2}}{\omega} + \frac{1}{2} \right]_q \right). \]  

(14)

From (9), (14) and according to the definition of \([x]_q\), we have the same dispersion relation for a boson and a fermion in the \( q \)-deformed case.

Expand the right hand side of (9) and (14) in the neighborhood of \( q = 1 \) to the second order, we derive the common approximate deformed dispersion relation for a fermion and a boson:

\[ E = \sqrt{m^2 + p^2} + \sqrt{m^2 + p^2} \left( \frac{4m^2 + 4p^2 - \omega^2}{24\omega^2} \right) (q - 1)^2. \]  

(15)

3 Discussion for Threshold Anomalies in the UHECR and the TeV-photon Paradoxes

Now we will compute the \( q \)-deformed threshold for the UHECRs and the TeV-photons. We consider the head-on collision between a soft photon of energy \( \epsilon \), momentum \( \vec{k} \) and a high energy particle of energy \( E_1 \), momentum \( \vec{p}_1 \), which leads to the production of two particles
with energies $E_2$, $E_3$ and momenta $\vec{p}_2$, $\vec{p}_3$ respectively [8]. From the energy conservation and momentum conservation, we have:

$$E_1 + \epsilon = E_2 + E_3;$$  \hspace{1cm} (16)

$$p_1 - k = p_2 + p_3.$$  \hspace{1cm} (17)

In the C. O. M. frame, $m_2$ and $m_3$ are rest at threshold, so they have the same velocity in the lab frame. It’s easy to give the following equation:

$$\frac{p_2}{p_3} = \frac{m_2}{m_3}.$$  \hspace{1cm} (18)

Applying our resulting dispersion relation (15) and (18) to (16) and (17), we can obtain the expression for $p_{1,th,q}$ in this case:

$$p_{1,th,q} = \frac{(m_2 + m_3)^2 - m_1^2}{4\epsilon} + \frac{p_{1,th,q}^4}{12\epsilon\omega^2} \left( \frac{m_2^3 + m_3^3}{(m_2 + m_3)^3} - 1 \right) (q - 1)^2.$$  \hspace{1cm} (19)

In the course of deriving (19), we have used the simplified form of (15):

$$E_i = p_i + \frac{m_i^2}{2p_i} + \left( p_i + \frac{m_i^2}{2p_i} \right) \left( \frac{1}{6\omega^2} \left( p_i^2 + m_i^2 \right) - \frac{1}{24} \right) (q - 1)^2,$$  \hspace{1cm} (20)

$$\epsilon = k + k \left( \frac{k^2}{6\omega^2} - \frac{1}{24} \right) (q - 1)^2.$$  \hspace{1cm} (21)

While in the non-deformed case, the threshold has the form as below:

$$p_{1,th,nq} = \frac{(m_2 + m_3)^2 - m_1^2}{4\epsilon}.$$  \hspace{1cm} (22)

Hereafter, we will denote the $p_{1,th}$ in the non-deformed case and in the $q$-deformed case by $p_{1,th,nq}$ and $p_{1,th,q}$ separately.

In the UHECR paradox $p + \gamma \rightarrow p + \pi$, we set $m_1 = m_2 = m_p = 940\text{MeV}$, $m_3 = m_\pi = 140\text{MeV}$, in the other one $\gamma + \gamma \rightarrow e^+ + e^-$, we have $m_1 = 0$ and similarly set $m_2 = m_3 = m_e = 0.5\text{MeV}$. Inputting the values of $p_{1,th}$ in [8]

$$p_{1,th,nq}^\text{UHECR} = 5 \times 10^{19}\text{eV}, \quad p_{1,th,q}^\text{UHECR} = 3 \times 10^{20}\text{eV},$$  \hspace{1cm} (23)

and

$$p_{1,th,nq}^\gamma = 10\text{TeV} \quad p_{1,th,q}^\gamma = 20\text{TeV}$$  \hspace{1cm} (24)
to (19) and (22) respectively, we can get two curves for the relation between $\omega$ and $|q - 1|$ as plotted in Fig. 1 (the unit of $\omega$ is MeV).

![Figure 1: the region of parameter space](image)

The region of the $(\omega, |q - 1|)$ parameter space provides a solution to both the UHECR and the TeV-$\gamma$ threshold anomalies. While applying the $q$-deformed dispersion relation to the present observed anomalies, the relation between $\omega$ and $|q - 1|$ can be described by the two curves in Fig. 1 which give the lower bound on the $q$-deformation. In the region below both of the curves, the UHECR and the TeV-photon paradoxes are resolved together. The top left corner is excluded, which is easily understood for at a fixed $\omega$ the conventional dispersion relation can be reproduced if we take $|q - 1| \rightarrow 0$. Under the consideration of $\omega < 10^7$ MeV, the parameters must be within the shaded area. $\omega < 10^7$ MeV is set according to the smallness of $|q - 1|$, which is the requirement of the $q$-deformed noncommutative theory.

4 Implications and Remarks

The relation between the two parameters $\omega$ and $q$ depends strongly on the experimental input. In this paper we just made use of the highest energies of the ultrahigh energy cosmic rays and the TeV-photons arriving on earth observed so far. Because both threshold anomalies of the UHECRs and the Mrk 501 TeV-photons are still under the situation that the data should
be proceeded very cautiously, our resulting relation of the parameters is to be tested. If in
the future better data prove the assumption of these threshold anomalies is incorrect, the
explanation for them in terms of the \( q \)-deformed noncommutative theory has to be excluded,
the same case as other interpretations such as the quantum-gravity-motivated models [8, 27]
will encounter. If, however, the threshold anomalies are confirmed, the enhanced constraints
on the available region of the parameter space can result from future cosmic-ray observations
and experiments performed at very high engines.

On the other hand, it has long been recognized that cosmology is a natural laboratory
for testing the possible theories of extremely high energy physics such as string theory and
noncommutative theory, and many authors [26, 28, 29, 30, 31, 32] have tried to search for
some supports for these theories from the cosmological observations and experiments directly
or indirectly. One purpose of this paper is also following this train of thought, and if it
can be verified that the permitted region of the parameter space to resolve the threshold
anomalies is coincident with the parameter range from the noncommutative theory, it will be
more confident for us to consider the noncommutative theory as the suitable theory for the
trans-Planck physics.

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References

[1] M. Takeda et al., Phys. Rev. Lett. 81, 1163 (1998).

[2] M. Takeda et al., arXiv: astro-ph/9902239.

[3] N. Hayashida et al., Phys. Rev. Lett. 73, 3491 (1994).

[4] D. J. Bird et al., Astrophys. J. 441, 144 (1995); D. J. Bird et al., Phys. Rev. Lett. 71,
3401 (1993); D. J. Bird et al., Astrophys. J. 424, 491 (1994).
[5] M. A. Lawrence, R. J. O. Reid and A. A. Watson, J. Phys. G 17, 773 (1991).

[6] N. N. Efimov et al., Ref. Proc. International Symposium on Astrophysical Aspects of the Most Energetic Cosmic Rays, World Scientific, Singapore, pp. 20, (1991).

[7] D. Kieda et al., HiRes Collaboration, Proceeds. 26th ICRC (Salt Lake 1999).

[8] G. Amelino-Camelia and T. Piran, Phys. Rev. D 64, 036005 (2001).

[9] J. M. Carmona, J. L. Cortés, J. Gamboa and F. Méndez, arXiv: hep-th/0207158.

[10] L. G. Mestres, arXiv: physics/9704017.

[11] S. Coleman and S. L. Glashow, Phys. Rev. D 59, 116008 (1999).

[12] R. Aloisio, P. Blasi, P. L. Ghia, and A. F. Grillo, Phys. Rev. D 62, 053010 (2000).

[13] O. Bertolami and C. S. Carvalho, Phys. Rev. D 61, 103002 (2000).

[14] H. Sato, arXiv: astro-ph/0005218.

[15] T. Kifune, Astrophys. J. 518, L21 (1999).

[16] W. Kluzniak, arXiv: astro-ph/9905308.

[17] R. J. Protheroe and H. Meyer, Phys. Lett. B 493, 1 (2000).

[18] A. Connes, Noncommutative Geometry, Academic Press, London (1994).

[19] J. Wess, Int. J. Mod. Phys. A 12, 4997 (1997).

[20] G. Amelino-Camelia, Int. J. Mod. Phys. D 11, 35 (2002).

[21] Y. J. Ng, J. Phys. A 23, 1023 (1990).

[22] M. L. Ge and G. Su, J. Phys. A 24, L721 (1991).

[23] M. A. M. Delgado, J. Phys. A 24, L1285 (1991).

[24] S. Vokos and C. Zachos, Mod. Phys. Lett. A 9, 1 (1994).
[25] J. A. Tuszyński, J. L. Rubin, J. Meyer, and M. Kibler, Phys. Lett. A 175, 173 (1993).

[26] Z. Chang and S. X. Chen, arXiv: cond-mat/0205208.

[27] Y. J. Ng, D. S. Lee, M. C. Oh, and H. van Dam, Phys. Lett. B 507, 236 (2001).

[28] J. A. Gu, P. M. Ho and S. Ramgoolam, arXiv: hep-th/0101058.

[29] S. F. Hassan and M. S. Sloth, arXiv: hep-th/0204110.

[30] C. S. Chu, B. R. Greene and G. Shiu, Mod. Phys. Lett. A 16, 2231 (2001).

[31] F. Lizzi, G. Mangano, G. Miele, and M. Peloso, JHEP 0206, 049 (2002).

[32] R. H. Brandenberger and P. M. Ho, Phys. Rev. D 66, 023517 (2002).