Color Superconductivity and Signs of its Formation

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We study finite density QCD in an approximation in which the interaction between quarks is modelled on that induced by instantons. We sketch the mechanism by which chiral symmetry restoration at finite density occurs in this model. At all densities high enough that the chirally symmetric phase fills space, we find that color symmetry is broken by the formation of a $\langle qq \rangle$ condensate of quark Cooper pairs. The formation of this color superconductor condensate lowers the energy of the system most if the up and down quark chemical potentials are equal. This suggests that the formation of such a condensate in a heavy ion collision may be accompanied by radiation of negative pions, and its decay may yield more protons than were present in the incident nuclei.

1. Introduction

In his talk at Quark Matter ’97, K.R. described our recent work on QCD at finite baryon density presented in Ref. \cite{1}, and this paper should be consulted by those seeking a description of the talk. Here, we focus on a speculation concerning signatures of the formation of a color superconducting state in heavy ion collisions.

Asymptotic freedom leads us to expect that at high density quarks behave nearly freely and form large Fermi surfaces, with interactions between the quasiparticles at the Fermi surfaces which become weak at asymptotically high density. Since the quark-quark interaction is attractive in the color $\bar{3}$ channel, BCS pairing of quarks will occur no matter how weak the interaction. Pairs of quarks cannot be color singlets, and so a $\langle qq \rangle$ condensate inevitably breaks color symmetry. This breaking is analogous to the breaking of electromagnetic gauge invariance in superconductivity. Color superconductivity implies that five of the eight gluons are massive and implies that the $U(1)$ gauge boson which remains massless is a linear combination of the photon and a gluon. Our goal in \cite{1} was to explore this phase in a context that is definite, qualitatively reasonable, and yet sufficiently tractable that likely patterns of symmetry breaking and rough magnitudes of their effects can be identified at non-asymptotic densities where interactions are not weak.

\textsuperscript{*}These ideas were first broached in discussions (in particular with R. Jaffe) at a November workshop at the RIKEN-BNL center. The fact that they are now seeing the light of day owes much to conversations K.R. had at QM97 (in particular with R. Seto.) We are grateful to the organizers of QM97 and the RIKEN-BNL workshop, as both were fruitful. The research of M.A. and F.W. is supported in part by DOE grant DE-FG02-90ER40542; that of K.R. is supported in part by DOE cooperative research agreement DE-FC02-94ER40818.
Color superconductivity requires high densities and low temperatures, so the most favorable experimental conditions are likely those at the AGS. Conditions at the centers of neutron stars are even more favorable, but that is not our subject here. Based on present analyses, it seems unlikely that color superconductivity can arise at temperatures above 100 MeV. Therefore, one must select events from the total data set in an experiment in a way that focuses on a subset of events which happen to have unusually high density and unusually low temperature. The strategy for doing this would vary in different experiments, but one might choose events with little energy at zero degrees (suggesting central events in which most of the baryons were stopped at central rapidity creating high densities) and with protons at central rapidity with unusually soft transverse momentum (suggesting that the effective temperature experienced by the baryons in the dense region was lower than in a typical event.) Signatures of the formation of a color superconductor phase can then be sought in the subset of events relative to the entire set. What, then, should we look for in this sort of an event-by-event analysis? We propose one answer to this question in Section 3 below, although we expect other answers are also possible.

2. A Model and Chiral Symmetry Breaking Therein

We sketch here our variational treatment of a two-parameter class of models having two flavors and three colors of massless quarks. The kinetic part of the Hamiltonian is that for free quarks, while the interaction Hamiltonian is a four fermion interaction (with coupling $K$) which is an idealization of the instanton vertex from QCD. In order to mimic the effects of asymptotic freedom, we multiply the momentum space interaction by a product of form factors each of the form $F(p) = \left[\frac{\Lambda^2}{p^2 + \Lambda^2}\right]^{\nu}$, one for each of the momenta of the four fermions. $\Lambda$, of course, is some effective QCD cutoff scale, which one might anticipate should be in the range $300 – 1000$ MeV. $\nu$ parametrizes the shape of the form factor; we consider $\nu = 1/2$ and $\nu = 1$.

In [1], we first consider chiral symmetry breaking at zero density. We choose a variational wave function which pairs particles and antiparticles with the same flavor and color but opposite helicity and opposite three-momentum. We derive the gap equation which determines the chiral gap $\Delta_\chi$ as a function of quark number density $n$. We fix the coupling $K$ for each choice of $\Lambda$ and $\nu$ by requiring $\Delta_\chi = 400$ MeV at $n = 0$. $\Delta_\chi$ decreases with increasing $n$, and vanishes at some $n_c$. With wave function in hand, we then evaluate the energy density and pressure as functions of $n$. We find that the phase with broken chiral symmetry is unstable at any nonzero density, with this instability being signalled by negative pressure at all but the lowest densities. At $n = n_c$, we switch over to an essentially free quark phase and the pressure then begins to increase monotonically, reaching zero at a density $n_0 > n_c$ and then becoming positive. In the presence of a chiral condensate the negative pressure associated with increasing vacuum energy overcompensates the increasing Fermi pressure. The uniform, chiral symmetry broken, nonzero density phase is mechanically unstable and breaks up into stable droplets of high density $n = n_0$ in which the pressure is zero and chiral symmetry is restored, surrounded by empty space with chiral symmetry broken. We identify the droplets of chiral symmetric phase with physical nucleons. Nothing within the model tells us that the stable droplets have quark number 3; nucleons are simply the only candidates in
nature which can be identified with droplets within which the quark density is nonzero and the chiral condensate is zero. This physical picture has significant implications for the phase transition, as a function of density, to restored chiral symmetry. The transition should occur by a mechanism analogous to percolation as nucleons, seen as pre-formed bags of symmetric phase, merge.

3. Color Superconductivity with Unequal Numbers of u and d Quarks

We now turn to physics at densities greater than \( n_0 \), at which the model describes a uniform phase with no chiral condensate. At high density, pairing of particles near the Fermi surface as in the original BCS scheme becomes more favorable. Our Hamiltonian supports condensation in quark-quark channels. The condensation is now between fermions with the same helicity, and the Hamiltonian selects antisymmetry in flavor. One can therefore have spin 0 — antisymmetric in spin and therefore in color, forming a \( \bar{3} \), or spin 1 — symmetric in spin and therefore in color, forming a \( 6 \). Here, we only consider the former, because although the spin-symmetric color \( 6 \) can arise, the gap in this channel is very small.\[1\]

In Ref. \[1\], we construct a suitable trial wave function in which the Lorentz scalar \( \langle q^{i \alpha} C^5 q^{j \beta} \epsilon_{ij} \epsilon_{\alpha \beta 3} \rangle \) is nonzero. This chooses a preferred direction in color space and breaks color \( SU(3) \to SU(2) \). Electromagnetism is spontaneously broken but there is a linear combination of electric charge and color hypercharge under which the condensate is neutral, and which therefore generates an unbroken \( U(1) \) gauge symmetry. No flavor symmetries, not even chiral ones, are broken. It is not difficult to redo the derivation of the gap equation satisfied by the superconducting gap parameter \( \Delta \) for the case when down and up quarks have different chemical potentials: \( \mu_+ = \bar{\mu} + \delta \mu \) for the down quarks and \( \mu_- = \bar{\mu} - \delta \mu \) for the up quarks. One finds

\[
1 = \frac{K}{\pi^2} \left\{ \int_{\mu^+}^{\infty} p^2 dp \frac{F^4(p)}{\sqrt{F^4(p) \Delta^2 + (p - \bar{\mu})^2}} + \int_{0}^{\mu_-} p^2 dp \frac{F^4(p)}{\sqrt{F^4(p) \Delta^2 + (\bar{\mu} - p)^2}} + \int_{\mu_-}^{\infty} p^2 dp \frac{F^4(p)}{\sqrt{F^4(p) \Delta^2 + (p - \bar{\mu})^2}} \right\}. \tag{1}
\]

The three terms arise respectively from particles above the Fermi surface, holes below the Fermi surface, and antiparticles. With equal numbers of up and down quarks, \( \mu_+ = \mu_- = \bar{\mu} \) and the particle and hole integrals diverge logarithmically at the Fermi surface as \( \Delta \to 0 \), which signals condensation for arbitrarily weak attraction. However, for \( \delta \mu \neq 0 \) there is no logarithmic divergence, and \( \Delta \) may vanish. For a given \( \bar{\mu} \), as \( \delta \mu \) is increased the domain of integration moves farther and farther from the logarithmic singularity, and \( \Delta \) must decrease in order for the gap equation to be satisfied. Momenta between \( \mu_+ \) and \( \mu_- \) cannot contribute, since neither particle-particle nor hole-hole pairing is possible, given that the condensate necessarily pairs up quarks with down quarks.

As a concrete example, we take \( \nu = 1 \) and \( \Lambda = 800 \text{ MeV} \) in the form factor, and we work at a baryon density eight times that in nuclear matter. This density, which corresponds to a quark number density \( n \) of 4.1 per fm\(^3\), is greater than \( n_0 \), and so space is filled by a chirally symmetric color superconductor phase with quark number density \( n \) and energy density \( \varepsilon \) given by:
\begin{align}
n & = \frac{2}{\pi^2} \left\{ \int_{\mu_-}^{\infty} p^2 dp \left( 1 - \frac{p - \bar{\mu}}{\sqrt{F_4(p) \Delta^2 + (p - \bar{\mu})^2}} \right) - \int_{0}^{\mu_-} p^2 dp \left( 1 - \frac{\bar{\mu} - p}{\sqrt{F_4(p) \Delta^2 + (\bar{\mu} - p)^2}} \right) \\ & - \int_{0}^{\infty} p^2 dp \left( 1 - \frac{\bar{\mu} + p}{\sqrt{F_4(p) \Delta^2 + (\bar{\mu} + p)^2}} \right) \right\} + \frac{\mu^3 + \mu^3}{\pi^2}, \quad (2) \\
\varepsilon & = \frac{2}{\pi^2} \left\{ \int_{\mu_-}^{\infty} p^3 dp \left( 1 - \frac{p - \bar{\mu}}{\sqrt{F_4(p) \Delta^2 + (p - \bar{\mu})^2}} \right) - \int_{0}^{\mu_-} p^3 dp \left( 1 - \frac{\bar{\mu} - p}{\sqrt{F_4(p) \Delta^2 + (\bar{\mu} - p)^2}} \right) \\ & + \int_{0}^{\infty} p^3 dp \left( 1 - \frac{\bar{\mu} + p}{\sqrt{F_4(p) \Delta^2 + (\bar{\mu} + p)^2}} \right) \right\} - \frac{\Delta^2}{K} + \frac{3(\mu^4 + \mu^4)}{4\pi^2}. \quad (3)\end{align}

Working first with \( \delta \mu = 0 \), that is with \( \mu_+ = \mu_- = \bar{\mu} \), we find that \( n = 4.1 \text{ fm}^{-3} \) corresponds to \( \bar{\mu} = 0.534 \) and \( \Delta = 0.156 \), both in GeV. If we create an excess of down quarks by increasing \( \delta \mu \) while keeping \( \bar{\mu} \) fixed (this changes \( n \)) we find that \( \Delta \) decreases, and vanishes when \( \delta \mu = .037 \). What is a reasonable value for \( \delta \mu \)? In \(^{197}\text{Au} \) there are 315 down quarks and 276 up quarks. Keeping the density fixed but enforcing this down/up ratio requires \( \bar{\mu} = 0.535 \) and \( \delta \mu = 0.012 \). Under these conditions, \( \Delta \) is decreased to 0.128. This phase, with a down/up ratio appropriate for gold nuclei, has an energy density which exceeds that of the \( \delta \mu = 0 \) phase with the same density by \( \sim 13 \text{ MeV/fm}^3 \). This, then, is the energy density by which the up–down symmetric phase is favored for these parameters. This estimate is model dependent. By varying the form factor \( F \), it is easy to get a result which is twice (or half) as large.

We have demonstrated that the formation of a color superconducting state lowers the energy most if the matter is up–down symmetric. In a heavy ion collision, in which down quarks initially exceed up quarks, as a color superconductor condensate forms in the densest region near the center of the collision it may therefore expel a few down quarks and up antiquarks (one each per \( 7.4 \text{ fm}^3 \) of condensate for the density considered above) into the surrounding less dense regions. These will eventually become negative pions, although the (inhomogeneous, nonequilibrium) dynamics involved are far from simple. When the condensate breaks up late in the collision, it likely yields equal numbers of protons and neutrons. The total number of protons in the final state will therefore be more than twice 79. We now answer the question posed in the introduction. When experimentalists compare AGS events selected to have unusually high density and low temperature with events from an entire data set, they should look for (i) an increase in the number of protons per event, and (ii) an increase in the \( \pi^-/\pi^+ \) ratio. We close by noting that effects of the kind we have sketched here, namely equalization of chemical potentials due to the formation of a color superconductor in high density regions, may prove more dramatic when the strange quark is included. This is work in progress.

REFERENCES

1. M. Alford, K. Rajagopal and F. Wilczek, \texttt{hep-ph/9711395}, Phys. Lett. \textbf{B} to appear.

See this paper for references. See also R. Rapp \textit{et al}, \texttt{hep-ph/9711396}.

2. R. Seto, private communication.