INVERTING THE SEESAW FORMULA

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By inverting the seesaw formula we determine the heavy neutrino mass matrix. The impact on the baryogenesis via leptogenesis and the radiative lepton decays in supersymmetric models is described. Links to neutrinoless double beta decay are also briefly discussed. The analysis leads to two distinct matrix models. One has small mixings while the other has one maximal mixing. Both cannot give a sufficient amount of baryon asymmetry. Then we also comment on a different form of the Dirac neutrino mass matrix, which does provide sufficient baryon asymmetry. In a supersymmetric scenario the branching ratios of radiative lepton decays are enhanced for this model.
I. INTRODUCTION

The seesaw mechanism is a simple framework which can explain the smallness of neutrino mass. It requires only a modest extension of the minimal standard model, namely the inclusion of the heavy right-handed neutrino, but can be well realized within left-right models, partial unified models, and grand unified theories, where the right-handed neutrino does exist. Then the effective neutrino mass matrix $M_L$ is given by the seesaw formula

$$M_L \simeq M_\nu M_R^{-1} M_\nu^T,$$

where $M_R$ is the mass matrix of the right-handed neutrino and $M_\nu$ is the Dirac neutrino mass matrix. The master formula (1) is valid when the eigenvalues of $M_R$ are much larger than the elements of $M_\nu$ and in such a case the eigenvalues of $M_L$ come out very small with respect to those of $M_\nu$. Indeed, unlike $M_\nu$, the generation of $M_R$ is not related to the electroweak symmetry breaking and thus its scale may be very large. Moreover, $M_R$ is a Majorana mass matrix, and as a consequence also $M_L$ is a Majorana mass matrix of left-handed neutrinos (see for example). This fact is related to the violation of total lepton number at high scale, which should produce important phenomena such as the baryogenesis via leptogenesis and the neutrinoless double beta decay. Lepton flavors are also violated, but in the nonsupersymmetric theory, due to the smallness of neutrino mass, such processes are so suppressed to be unobservable, apart from neutrino oscillations. The situation is different in the supersymmetric theory, even with universal soft breaking terms, where some of these processes may be observable.

Both lepton number and lepton flavor violations depend on the mass matrices $M_\nu$ and $M_R$. On the other hand, we have several informations on the effective neutrino mass matrix, coming from neutrino oscillations and more generally from neutrino experiments. Therefore, it is reasonable, relating $M_\nu$ to the charged fermions mass matrices, to obtain informations on $M_R$ by inverting the seesaw formula,

$$M_R \simeq M_\nu^T M_L^{-1} M_\nu.$$

As a consequence, we should be able to determine also the impact on the baryogenesis via leptogenesis, the neutrinoless double beta decay, and for example the radiative lepton decays in some supersymmetric models. The seesaw formula is valid above the $M_R$ scale, so that one should determine $M_L$ at that scale. Although in several case the effect is not relevant, we must take care of the renormalization issue (see the recent paper).

In section II we discuss the Dirac mass matrices of quarks and leptons. In section III we describe the effective neutrino mass matrix and in particular its element $M_{ee}$, related
to neutrinoless double beta decay. In section IV we determine the mass matrix of right-handed neutrinos. In section V and VI, respectively, we study the consequences for the baryogenesis via leptogenesis and the radiative lepton decays in supersymmetry. Finally, we present a discussion.

II. DIRAC MASS MATRICES

A symmetric form of the quark mass matrices, in agreement with the phenomenology of quark masses and mixings, is described in Refs. [12, 13], and given by

\[ M_d \simeq \begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & m_s & \sqrt{m_d m_b} \\ 0 & \sqrt{m_d m_b} & m_b \end{pmatrix}, \]

(3)

\[ M_u \simeq \begin{pmatrix} 0 & \sqrt{m_u m_c} & 0 \\ \sqrt{m_u m_c} & m_c & \sqrt{m_u m_t} \\ 0 & \sqrt{m_u m_t} & m_t \end{pmatrix}. \]

(4)

Moreover, in Ref. [12], the charged lepton mass matrix has an analogous form,

\[ M_e \simeq \begin{pmatrix} 0 & \sqrt{m_e m_\mu} & 0 \\ \sqrt{m_e m_\mu} & m_\mu & \sqrt{m_e m_\tau} \\ 0 & \sqrt{m_e m_\tau} & m_\tau \end{pmatrix}. \]

(5)

Since the hierarchy and scale of charged lepton masses are similar to the hierarchy and scale of down quark masses (see for example [14]), one has also the relation \( M_e \sim M_d \). Then a natural assumption is \( M_\nu \sim M_u \), in which case the Dirac neutrino mass matrix can be written in the form

\[ M_\nu \simeq \begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & 1 \end{pmatrix} m_t, \]

(6)

where \( a \ll b \sim c \ll 1 \). The relation \( b \simeq c \) in quark mass matrices is discussed in Ref. [12]. We take \( b \) and \( c \) different but of the same order. In fact, also matrices (3) and (5) can be written in the form (6), with overall scales \( m_b \) and \( m_\tau \), respectively.
III. NEUTRINO PHENOMENOLOGY

Neutrino oscillation data imply that the lepton mixing matrix is given by

\[
U \simeq \begin{pmatrix}
\frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \epsilon e^{-i\delta} \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix} \begin{pmatrix}
\epsilon \sqrt{2} \\
\epsilon \sqrt{3} \\
1
\end{pmatrix},
\]

where \(\epsilon < 0.16\), and the square mass differences among effective neutrino masses \(m_1, m_2, m_3\) are

\[
\Delta m_{32}^2 = m_3^2 - m_2^2 \simeq 3 \cdot 10^{-3}\text{eV}^2, \\
\Delta m_{21}^2 = m_2^2 - m_1^2 \simeq 7 \cdot 10^{-5}\text{eV}^2.
\]

In the basis where \(M_e\) is diagonal, \(M_L\) is obtained by the transformation

\[
M_L = U^* D_L U^\dagger,
\]

with \(D_L = \text{diag}(m_1, m_2, m_3)\). The presence of phases \(\varphi_1, \varphi_2\) in the mixing matrix (7) is due to the Majorana nature of effective neutrinos. In the lepton mixing matrix, \(U_{\mu3}\) is maximal, \(U_{e2}\) is large, and \(U_{e3}\) is small. This is in contrast to the small quark mixings.

Since \(\Delta m_{21}^2 \ll \Delta m_{32}^2\), we may consider four kinds of neutrino spectra: the normal hierarchy \(m_1 \ll m_2 \ll m_3\), with \(m_3^2 \simeq \Delta m_{32}^2\) and \(m_2^2 \simeq \Delta m_{21}^2\), the partial degeneracy \(m_1 \simeq m_2 \ll m_3\), with \(m_3^2 \simeq \Delta m_{32}^2\), the inverse hierarchy \(m_1 \simeq m_2 \gg m_3\), with \(m_1^2 \simeq \Delta m_{32}^2\), and the almost degenerate spectrum \(m_1 \simeq m_2 \simeq m_3 \simeq 1\text{eV}\). The elements of \(M_L\) are given by

\[
M_{ee} \simeq \epsilon^2 m_3 + \frac{m_2}{3} + \frac{2m_1}{3}, \\
M_{e\mu} \simeq \epsilon \frac{m_3}{\sqrt{2}} + \frac{m_2}{3} - \frac{m_1}{3}, \\
M_{e\tau} \simeq \epsilon \frac{m_3}{\sqrt{2}} - \frac{m_2}{3} - \frac{m_1}{3}, \\
M_{\mu\tau} \simeq \frac{m_3}{2} - \frac{m_2}{3} + \frac{m_1}{6}, \\
M_{\mu\mu} \simeq M_{\tau\tau} \simeq \frac{m_3}{2} + \frac{m_2}{3} + \frac{m_1}{6},
\]

where phases are inserted by \(\epsilon \rightarrow \epsilon e^{i\delta}\), \(m_1 \rightarrow m_1 e^{i\varphi_1}\), \(m_2 \rightarrow m_2 e^{i\varphi_2}\), and the relation \(M_{\mu\mu} \simeq M_{\tau\tau}\) leads to the nearly maximal mixing \(U_{\mu3}\).

Let us consider in particular the element \(M_{ee}\), which is related to neutrinoless double beta decay. For the normal hierarchy we obtain (values in eV) \(10^{-3} < M_{ee} \sim \sqrt{\Delta m_{21}^2} <\)
10^{-2}, for the partial degeneracy $10^{-3} < M_{ee} \sim 10^{-1} \sqrt{\Delta m_{32}^2} < 10^{-2}$, for the inverse hierarchy $10^{-2} < M_{ee} \sim \sqrt{\Delta m_{32}^2} < 10^{-1}$, and for the degenerate spectrum $10^{-1} < M_{ee} < 1$. Hence, different spectra give quite distinct prediction for $M_{ee}$. There is a claim of evidence for the process [16], with $M_{ee} = 0.05 - 0.86$ eV, in agreement with the degenerate spectrum and also the inverse hierarchy. However, this result is controversial.

IV. THE HEAVY NEUTRINO MASS MATRIX

In this section we determine the right-handed neutrino mass matrix by means of the inverse seesaw formula (2). We need $M_L^{-1}$, which is easily achieved, since $M_L^{-1} = U D_L^{-1} U^T$. We stress that the difference of $U_{e2}$ from the maximal mixing could be ascribed to $M_\nu$ [17] and/or to renormalization [18]. Therefore, at the high scale and in the basis where $M_\nu$ is given by Eqn.(5), we use the nearly bimaximal mixing in the seesaw,

$$U \simeq \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \epsilon e^{-i\delta} \\
-\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}}
\end{pmatrix} \text{diag}(e^{i\varphi_1/2}, e^{i\varphi_2/2}, 1), \quad (11)$$

with $\epsilon \simeq 0$. Then the elements of $M_L^{-1}$ are given by

$$M_{ee}^{-1} \simeq \frac{1}{2m_1} + \frac{1}{2m_2} + \frac{\epsilon^2}{m_3}$$

$$M_{e\mu}^{-1} \simeq -\frac{1}{2\sqrt{2}m_1} + \frac{1}{2\sqrt{2}m_2} + \frac{1}{\sqrt{2}m_3}$$

$$M_{e\tau}^{-1} \simeq \frac{1}{2\sqrt{2}m_1} - \frac{1}{2\sqrt{2}m_2} + \frac{1}{\sqrt{2}m_3}$$

$$M_{\mu\tau}^{-1} \simeq -\frac{1}{4m_1} - \frac{1}{4m_2} + \frac{1}{2m_3}$$

$$M_{\mu\mu}^{-1} \simeq M_{\tau\tau}^{-1} \simeq \frac{1}{4m_1} + \frac{1}{4m_2} + \frac{1}{2m_3}$$

where phases are inserted by $m_1 \to m_1 e^{-i\varphi_1}$, $m_2 \to m_2 e^{-i\varphi_2}$, $\epsilon \to \epsilon e^{-i\delta}$. Now, we determine the forms of $M_R$ according to the four kinds of mass spectra for the effective neutrinos. We consider two extreme cases, that is $\varphi_2 \simeq \varphi_1$ and $\varphi_2 \simeq \varphi_1 + \pi$. The other cases should be intermediate between those two.
For the normal hierarchy we obtain

\[ M_R \simeq \begin{pmatrix} a^2 & a(b-c) & -a \\ a(b-c) & (b-c)^2 & -(b-c) \\ -a & -(b-c) & 1 \end{pmatrix} \frac{m_i^2}{4m_1}. \tag{12} \]

An overall phase \( e^{i\varphi_1} \) will be always absorbed. The corresponding approximate form of \( M_L \) at the low scale is given by

\[ M_L \sim \begin{pmatrix} m_2 & m_2 & m_2 \\ m_2 & m_3 & m_3 \\ m_2 & m_3 & m_3 \end{pmatrix}. \]

For the partial degeneracy, the case \( \varphi_2 \simeq \varphi_1 \) leads to \( M_R \) the double of that in Eqn.(12). Instead, \( \varphi_2 \simeq \varphi_1 + \pi \) leads to the special form

\[ M_R \simeq \begin{pmatrix} 0 & a & 0 \\ a & 2(c-b) & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{am_i^2}{2\sqrt{2}m_1}. \tag{13} \]

The corresponding approximate forms of \( M_L \) at the low scale are given by

\[ M_L \sim \begin{pmatrix} m_{1,2} & m_2 - m_1 & m_2 - m_1 \\ m_2 - m_1 & m_3 & m_3 \\ m_2 - m_1 & m_3 & m_3 \end{pmatrix}, \]

with \( m_2 - m_1 \sim \Delta m_{21}^2/m_{1,2} \), and

\[ M_L \sim \begin{pmatrix} m_{1,2} & m_{1,2} & m_{1,2} \\ m_{1,2} & m_3 & m_3 \\ m_{1,2} & m_3 & m_3 \end{pmatrix}. \]

For the inverse hierarchy both cases \( \varphi_2 \simeq \varphi_1 \) and \( \varphi_2 \simeq \varphi_1 + \pi \) give

\[ M_R \simeq \begin{pmatrix} a^2 & a(b+c) & a \\ a(b+c) & (b+c)^2 & (b+c) \\ a & (b+c) & 1 \end{pmatrix} \frac{m_i^2}{2m_3}. \tag{14} \]
Note that while for the normal hierarchy the difference \((b - c)\) appears, for the inverse hierarchy, instead, the sum \((b + c)\) appears. At the low scale we have

\[
M_L \sim \begin{pmatrix}
m_{1,2} & m_2 - m_1 & m_2 - m_1 \\
m_2 - m_1 & m_{1,2} & m_{1,2} \\
m_2 - m_1 & m_{1,2} & m_{1,2}
\end{pmatrix},
\]

with \(m_2 - m_1 \sim \Delta m_{2,1}^2/m_{1,2}\), and a form of \(M_L\) with all entries of the order of \(m_{1,2}\).

For the degenerate spectrum we get in the case \(\varphi_2 \simeq \varphi_1\)

\[
M_R \simeq \begin{pmatrix}
a^2 & ab & ac \\
ab & b^2 + c^2 & c \\
ac & c & 1
\end{pmatrix} \frac{m_t^2}{m_3},
\]

(15)

For \(\varphi_2 \simeq \varphi_1 + \pi\) we have the same form as Eqn.(14). At the low scale \(M_L\) is of the same kind as the inverse hierarchy case.

In the following sections we will consider, in a simplified approach, the impact of \(M_\nu\) and \(M_R\) on the baryogenesis via leptogenesis and the radiative lepton decays in some supersymmetric models. We first take \(M_\nu \sim M_u\), so that

\[
M_\nu \sim \begin{pmatrix}
0 & \lambda^6 & 0 \\
\lambda^6 & \lambda^4 & \lambda^4 \\
0 & \lambda^4 & 1
\end{pmatrix} m_t,
\]

(16)

where \(\lambda = 0.22\) is the Cabibbo parameter. Since \(b \sim c\), we take only two forms for \(M_R\), one for the normal, inverse and degenerate case, and the other for the partial degenerate case (13), that is

\[
M_R \sim \begin{pmatrix}
\lambda^{12} & \lambda^{10} & \lambda^6 \\
\lambda^{10} & \lambda^8 & \lambda^4 \\
\lambda^6 & \lambda^4 & 1
\end{pmatrix} \frac{m_t^2}{m_k},
\]

(17)

with eigenvalues \(M_1/M_2 \sim \lambda^4\), \(M_1/M_3 \sim \lambda^{12}\), and

\[
M_R \sim \begin{pmatrix}
0 & \lambda^6 & 0 \\
\lambda^6 & \lambda^4 & 1 \\
0 & 1 & 0
\end{pmatrix} \frac{m_t^2}{m_1},
\]

(18)
with eigenvalues \( M_1/M_2 \sim \lambda^6, M_1/M_3 \sim \lambda^6 \). Notice that the scale of matrix (18) is smaller by several orders with respect to the scale of matrix (17). Defining \( M_D = M_\nu U_R \), where \( U_R \) diagonalizes \( M_R \) (\( M_D \) is the Dirac mass matrix in the basis where \( M_R \) is diagonal), we obtain \( M_D^\dagger M_D \), which appears both in the formula for leptogenesis and in that for radiative decays in supersymmetry,

\[
M_D^\dagger M_D \sim \begin{pmatrix}
\lambda^{12} & \lambda^{10} & \lambda^6 \\
\lambda^{10} & \lambda^8 & \lambda^4 \\
\lambda^6 & \lambda^4 & 1
\end{pmatrix} m_i^2, \quad (19)
\]

\[
M_D^\dagger M_D \sim \begin{pmatrix}
\lambda^{12} & \lambda^{10} & \lambda^{10} \\
\lambda^{10} & 1 & 1 \\
\lambda^{10} & 1 & 1
\end{pmatrix} m_i^2. \quad (20)
\]

In the first case, matrix (17), we have \( U_R \) near the identity and \( M_D^\dagger M_D \sim M_R m_k \). In the other case, matrix (18), \( U_R \) is nearly unimaximal. Therefore, in the matrix model made of (16) and (17), \( M_\nu \) and \( M_R \) give small mixings, so that large mixings in \( M_L \) are produced through a matching between \( M_\nu \) and \( M_R \) within the seesaw formula. Instead, in the matrix model made of (16) and (18), the maximal mixing in \( M_L \) comes from \( M_R \).

The structures (17) and (18) agree with the results of Ref. [19], where it was realized that the seesaw enhancement of lepton mixing can be achieved by strong mass hierarchy or large off-diagonal elements in the heavy neutrino mass matrix.

V. BARYOGENESIS VIA LEPTOGENESIS

The baryogenesis via leptogenesis mechanism [7] is a well-known mechanism for baryogenesis, related to the seesaw mechanism, where the decays of heavy right-handed neutrinos produce a lepton asymmetry which is partly transformed in a baryon asymmetry by electroweak sphaleron processes [20]. The amount of baryon asymmetry is then given by the expression

\[
Y_B \simeq \frac{1}{2} g^* \epsilon_1, \quad (21)
\]

where \( \epsilon_1 \) can be written as

\[
\epsilon_1 \simeq \frac{3}{16\pi} \left[ \frac{(Y_D^\dagger Y_D)^2_{12}}{(Y_D^\dagger Y_D)_{11} M_2} + \frac{(Y_D^\dagger Y_D)^2_{13}}{(Y_D^\dagger Y_D)_{11} M_3} \right], \quad (22)
\]
see for instance Ref. [21]. In these formulas $Y_D$ are Yukawa matrices, $g^* \simeq 100$, and $d < 1$ is a dilution factor, which depends especially on the quantity

$$\tilde{m}_1 = \frac{(M_D^T M_D)_{11}}{M_1}. \quad (23)$$

Moderate dilution is present when $\tilde{m}_1$ is in the range of the effective neutrino masses [22]. The allowed value for the baryon asymmetry is $Y_B \simeq 9 \cdot 10^{-11}$, see Ref. [23]. Yukawa matrices are obtained by dividing mass matrices by their overall scale.

For the two matrix models described in the previous section we get, respectively,

$$\epsilon_1 \simeq \frac{3}{16\pi} \left( \frac{\lambda^{20}}{\lambda^{12}} \cdot \lambda^4 + \frac{\lambda^{12}}{\lambda^{12}} \cdot \lambda^{12} \right) \sim \frac{3}{16\pi} \lambda^{12} \sim 10^{-10}, \quad (24)$$

with $\tilde{m}_1 \sim m_k$, and

$$\epsilon_1 \simeq \frac{3}{16\pi} \left( \frac{\lambda^{20}}{\lambda^{12}} \cdot \lambda^6 + \frac{\lambda^{20}}{\lambda^{12}} \cdot \lambda^6 \right) \sim \frac{3}{16\pi} \lambda^{14} \sim 10^{-12}, \quad (25)$$

with $\tilde{m}_1 \sim m_1$. Note that the two terms are comparable. Moreover, it is clear that both models cannot provide a sufficient amount of baryon asymmetry.

VI. RADIATIVE LEPTON DECAYS

In supersymmetric seesaw models with universality above the heavy neutrino mass scale, lepton flavor violations are produced by running effects from the universality scale $M_U$ to the scale $M_R$ [10]. The branching ratio for radiative lepton decays is given by the approximate formula [24]

$$\text{Br}(l_i \to l_j \gamma) \sim \frac{\alpha^3}{G_F m_S^2} \left( \frac{3m_0^2 + A_0^2}{8\pi^2 \log M_U \log M_R} \right)^2 (Y_D^T Y_D)_{ij} \tan^2 \beta, \quad (26)$$

with $l_1 = e$, $l_2 = \mu$, $l_3 = \tau$. Here, $m_0$ is the universal scalar mass, $A_0$ the universal trilinear coupling, and $m_S$ is the average slepton mass at the weak scale, which can be quite different from $m_0$. The experimental upper bounds are: $\text{Br}(\mu \to e \gamma) < 1.2 \cdot 10^{-11}$, $\text{Br}(\tau \to e \gamma) < 2.7 \cdot 10^{-6}$, $\text{Br}(\tau \to \mu \gamma) < 1.1 \cdot 10^{-6}$. The first and third results are expected to be lowered by almost three orders in the future.

Assuming $m_0 = m_S = 100$ GeV, $A_0 = 0$ and $\tan \beta = 50$, we obtain for the first matrix model the values $10^{-18}$, $10^{-12}$, $10^{-9}$, and for the second matrix model the values $10^{-18}$, $10^{-18}$, $10^{-3}$. Due to large uncertainties in supersymmetric parameters, we cannot make definite predictions, so that previous numbers represent the effect of distinct matrix models, which is our main interest here. However, the element $(Y_D^T Y_D)_{32} \sim 1$ in matrix (20) seems critical.
VII. DISCUSSION

By inverting the seesaw formula we have calculated the heavy neutrino mass matrix, and the implications for baryogenesis via leptogenesis and radiative lepton decays in certain supersymmetric models. The analysis leads to two distinct matrix forms, that is a nearly diagonal model and a nearly offdiagonal model, which cannot provide sufficient baryon asymmetry. For recent related studies, see Ref. [23].

We have assumed $M_\nu \sim M_u$. However, this assumption can be changed. Indeed, the main feature of the Dirac neutrino mass matrix within the seesaw mechanism is that its overall scale is of the order of $m_t$. For example, we can take $M_\nu \simeq M_d m_t / m_b$, which means that it has the same overall scale of $M_u$, but the internal hierarchy of $M_d$,

$$
M_\nu \sim \begin{pmatrix} 0 & \lambda^3 & 0 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix} m_t. \tag{27}
$$

In this case, sufficient baryon asymmetry is achieved, especially for

$$
M_R \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \frac{m_t^2}{m_k}. \tag{28}
$$

The branching ratios of lepton decays are also enhanced to $10^{-10}$, $10^{-7}$, $10^{-6}$. However, these strongly depend on the mechanism of supersymmetry breaking. In fact, in the previous section we have adopted a gravity mediated breaking, where $M_U > M_R$, while for a gauge mediated breaking $M_U < M_R$ and running effects are not induced.

An indication towards the existence of the seesaw mechanism would be the evidence for neutrinoless double beta decay. For the moment we predict (in eV) $10^{-3} < M_{ee} < 0.86$. While the upper part of this range will be checked rather soon, the lower part is more difficult to reach.

In conclusion, assuming baryogenesis from leptogenesis, we are led towards a Dirac neutrino mass hierarchy similar to the down quark and charged lepton mass hierarchy. In some supersymmetric scenarios, this model may be checked by measurements of radiative lepton decays.
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