Beamforming Feedback-based Model-driven Angle of Departure Estimation Toward Firmware-Agnostic WiFi Sensing

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Abstract—This paper proves that the angle of departure (AoD) estimation using the multiple signal classification (MUSIC) with only WiFi control frames for beamforming feedback (BFF), defined in IEEE 802.11ac/ax, is possible. Although channel state information (CSI) enables model-driven AoD estimation, most BFF-based sensing techniques are data-driven because they contain the right singular vectors of CSI and subcarrier-averaged stream gain. Specifically, we find that right singular vectors with a subcarrier-averaged stream gain of zero have the same role as the noise subspace vectors in the CSI-based MUSIC algorithm. Numerical evaluations confirm that the proposed BFF-based MUSIC successfully estimates the AoDs and gains for all propagation paths. Meanwhile, this result implies a potential privacy risk; a malicious sniffer can carry out AoD estimation only with unencrypted BFF frames.

Index Terms—Wireless sensing, channel state information, angle of departure, beamforming feedback.

I. INTRODUCTION

CHANNEL state information (CSI)-based WiFi sensing [1] has attracted both academic and industrial interests, as a technique that generates additional value to wireless local area network (WLAN) systems. Based on the multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) system, CSI-based sensing uses the attenuation and phase between each transmit and receive antenna pair and each OFDM subcarrier. However, there is a critical restriction on CSI-based sensing regarding the underlying WLAN system. Because the CSI is processed and discarded in physical (PHY) layer components, CSI must be extracted from wireless chipsets. This procedure requires additional firmware implementation, and such firmware performs on a few wireless chipsets and protocols [2], [3], which limits the applicability of CSI-based WiFi sensing to several WiFi devices.

To alleviate this restriction, a new WiFi sensing framework, called beamforming feedback (BFF)-based sensing, has been proposed [4], [5]. BFF-based sensing performs without explicitly extracting CSI from the PHY layer components but leveraged BFF frames [6], which contain a compressed version of the CSI. Specifically, the BFF frame contains the right singular vectors of the CSI matrix for each subcarrier and subcarrier-averaged stream gain. In the IEEE 802.11ac/ax standard [6], [7], WiFi access points (APs) and stations (STAs) exchange the BFF without encryption to perform efficient MIMO-OFDM communications. This indicates that the BFF is extracted from the PHY-layer components by default and is stored in a corresponding field in medium access control (MAC)-layer frames, which are exchanged among APs or STAs. This enables us to obtain BFFs using MAC frame-capturing tools, as shown in Fig. 1, where the arbitral sniffer can perform WiFi sensing regardless of the lack of access to the PHY layer components of the AP and STA.

Instead of the BFF-based sensing’s benefits of wide applicability, a critical issue remains; to the best of our knowledge, there are no model-driven algorithms, which geometrically estimate the surrounding environment based on mathematical modeling. The existing BFF-based sensing methods [4], [5] are referred to as data-driven methods. The sensing tasks are conducted via pattern matching to a database consisting of a pre-obtained training dataset, which comprises the BFF and corresponding actual-measured target labels (e.g., human locations or device locations). Such training dataset generation procedure requires tremendous human costs. Moreover, the procedure should be performed whenever the surrounding environment is changed (e.g., the locations of the furniture or the STAs are changed), increasing the cost of system maintenance.

This motivated us to develop a BFF-based model-driven sensing algorithm that estimates the surrounding environment...
without any dataset. To this end, we revisit model-based sensing in CSI-based sensing literature. In the CSI-based sensing literature, a fundamental technique, called the multiple signal classification (MUSIC) algorithm \([8]\), is used to estimate the angle of departure (AoD) and gains for each of the multiple propagation paths. Based on the original MUSIC algorithm \([8]\), which imposes some restrictions, there are vast of its expansion works on CSI-based sensing literature, for example, alleviation of restrictions regarding antenna array \([9]\) and propagation environment \([10]\), and expansion of addressing sensing tasks \([11]\). These works construct high capacity and widely applicable sensing frameworks. However, it is unknown whether the original MUSIC algorithm applies to BFF-based sensing literature.

In this study, we mathematically show that the original MUSIC applies to the BFF-based sensing literature. Specifically, the AoDs are estimated using the original MUSIC from the BFF frame, by adding some assumptions regarding the angle of arrival (AoA) (refer to Section \([1]\)). Mathematically, we find that the right singular vectors with a subcarrier-averaged stream gain of zero have the same role as the noise subspace vector in the original MUSIC algorithm. Note that in IEEE 802.11ac standards \([6]\), the BFF frame contains the right singular vectors of the CSI matrix and the subcarrier-averaged stream gain. Based on this finding, we propose a BFF-based MUSIC (B-MUSIC) algorithm, which estimates the AoD and the gain for each of the multiple propagation paths. Moreover, B-MUSIC was validated in both mathematical proof and numerical evaluation.

In parallel with the benefit of the B-MUSIC, B-MUSIC reveals a potential privacy risk in the BFF exchange framework in IEEE 802.11ac/ax. While the CSI-based sensing requires to access authorization to the AP or STA, the BFF-based sensing, in principle, can be conducted without any database generation by arbitral sniffers, including malicious sniffers, because BFF frames are transmitted without encryption over the air. Note that malicious sniffers’ sensing is infeasible through the existing data-driven BFF-based sensing frameworks because they require database generation procedures.

The contributions of this study are summarized as follows:

- We prove that the MUSIC algorithm can be performed using only the BFF frame (Proposition \([1]\)). Specifically, we find that the right singular vectors with a subcarrier-averaged stream gain of zero have the same role as the signal subspace vector in the original MUSIC algorithm. Moreover, based on this finding, various expansions will be realized in the BFF-sensing literature as a variety of expansions of the MUSIC algorithm have been proposed in the CSI-based sensing literature (e.g., expand applicability regarding antenna arrays, propagation models, and addressing sensing tasks).

- Based on the above findings, we propose the B-MUSIC algorithm, which estimates the AoDs and gains for each of the propagation paths. To the best of our knowledge, B-MUSIC is the first model-driven sensing algorithm in the BFF-based sensing literature. In contrast to existing BFF-based sensing schemes, which are referred to as data-driven methods and include the drawbacks discussed above, B-MUSIC is a model-driven method that performs without such drawbacks.

- The feasibility of the model-driven BFF-based sensing implies the potential privacy risk; malicious sniffers can carry out WiFi sensing by only capturing BFF frames.

A. Notations

We denote the transpose of a matrix \(H\) as \(H^T\), conjugate as \(H^H\), Hermitian transpose as \(H^H\), Moore-Penrose inverse as \(H^+\), and \((i, j)\) element as \(H_{i,j}\). We denote the \(i\)th element of a vector \(a\) as \(a_i\) and the Euclidian norm as \(|a|\). The identity matrix is represented as \(E\). The diagonal matrix, whose \(i\)th diagonal element is \(a_i\), is represented as \(\text{diag}(a)\). The \(M \times N\) zero matrix is denoted as \(0_{M \times N}\).

B. Preliminary

1) Beamforming Feedback Scheme in 802.11ac/ax: We consider a MIMO communication system in which a transmitter (e.g., AP) transmits signals to a receiver (e.g., STA). Because in this study we aim to prove the feasibility of the MUSIC using only BFF, under a simplified situation, we assume that the number of antenna elements in the transmitter and receiver are the same, which is denoted as \(M\). Let us denote the CSI matrix from the transmitter to the receiver at the \(k\)th subcarrier as \(H_k \in \mathbb{C}^{M \times M}\). In IEEE 802.11ac/ax standards, to provide eigenbeam-space division multiplexing \([12]\), the receiver feedbacks the BFF frame to the transmitter \([6]\), which contains a compressed version of the CSI matrix. Because the BFF frame is exchanged over the air without encryption, BFFs can be obtained using the MAC frame-capturing tools, where an arbitral sniffer can perform WiFi sensing without any access to the PHY layer components of the transmitter and receiver \([4]\), as shown in Fig. \([1]\).

The BFF contains the right singular vectors \(V_k\) of the CSI matrix \(H_k\) for each subcarrier and a subcarrier-averaged stream gain \([6]\). The right singular vector \(V_k\) is calculated using singular value decomposition as:

\[
H_k = U_k \Sigma_k V_k^H, \tag{1}
\]

where \(U_k\) and \(V_k\) are unitary matrices, and \(\Sigma_k\) is a diagonal matrix with singular values \([13]\). Moreover, the \(m\)th row vector of \(V_k\) is the eigenvector of the correlation matrix \(H_k^H H_k\), whose corresponding eigenvalue is \(\sigma_m^2\), where \(\sigma_m\) is the \(m\)th singular value. The subcarrier-averaged stream gain is represented by a diagonal matrix \(\bar{A}\), where:

\[
\bar{A} = \frac{1}{K} \sum_{k=1}^{K} \Sigma_k^2. \tag{2}
\]

Note that the diagonal elements of \(\Sigma_k\) are generally real and positive and are listed in descending order. In the following, the subcarrier index \(k\) is omitted unless otherwise noted.

\(^1\)This assumption can be easily expanded to the case that the number of antennas is different between the transmitter and the receiver.
2) Propagation Model: We consider a discrete physical propagation model [14], where an uniform linear array is employed at the transmitter and receiver. In the following, for a simple description, we assume that the distances between successive antennas at the transmitter and the receiver are the same, which is denoted as $d$. 

Let $L$ be the number of the propagation paths. In addition, let $\theta_1$ be the AoD and $\theta_2$ be the AoA of the $l$th path. The complex scalar $r_l$ denotes the attenuation from the first antenna of the transmitter to the first antenna of the receiver by the signal traveling along the $l$th propagation path. We denote a complex phase shift $a(\theta)$ as $\exp(2\pi d \sin(\theta)/\lambda)$, where $\lambda$ is the wavelength. For shorthand notation, let $L$-dimensional vectors $\theta$, $\phi$, and $v$ represent $(\theta_1, \ldots, \theta_L)^T$, $(\phi_1, \ldots, \phi_L)^T$, and $(r_1, \ldots, r_L)^T$, respectively. In addition, we denote the steering vector $a(\theta) := (1, a(\theta), \ldots, a(\theta)^{M-1})^T$; $L \times M$ steering matrix $A(\theta) := (a(\theta_1), \ldots, a(\theta_L))^T$; $L \times L$ diagonal matrix $R := \text{diag}(r)$. In the discrete physical propagation model [14], the CSI matrix $H$ is represented as

$$H = A(\theta)^TRA(\phi). \tag{3}$$

II. PROPOSED METHOD: BEAMFORMING FEEDBACK-BASED MULTIPLE SIGNAL CLASSIFICATION

To clarify the difference between B-MUSIC and CSI-based MUSIC, we briefly explain the basic theorem and assumptions in CSI-based MUSIC. The basic theorem of CSI-based MUSIC is as follows. Given that $h$ is an arbitrary row vector of the CSI matrix, if a right singular vector of $h$, whose singular value is zero, exists, such a singular vector is orthogonal to $A(\phi)$. Note that the singular vectors are called noise subspace vectors. To support this theorem, the assumptions are as follows: $a(\phi_l) \neq a(\phi_{l'})$ unless $l = l'$, and the number of transmitter antennas is larger than that of the propagation paths.

We prove that the MUSIC can perform with the right singular vectors $V$ of the CSI matrix and subcarrier-averaged stream gain $\hat{A}$, instead of the right singular vectors and singular values of the row vector of the CSI matrix, which is proved in Proposition 1. More formally, we find that if $m$th diagonal element of $\hat{A}$ is zero, the $m$th row vector of $V$ is orthogonal to $A(\phi)$. To support the theorem, in addition to the aforementioned assumptions, we assume that $a(\theta_l) \neq a(\theta_{l'})$ unless $l = l'$, and the number of receiver antennas is larger than that of the propagation paths.

To the best of our knowledge, no study has been conducted on MUSIC from $V$ of the CSI matrix and $\hat{A}$. Generally, when the CSI matrix can be directly used, each row vector of the CSI matrix is separately used as the original MUSIC. This is because when each row vector of the CSI matrix is used separately, the MUSIC can be conducted with the assumptions only regarding the transmitters and AoDs. However, when only $V$ is available instead of each row vector of the CSI matrix, the assumptions regarding the receivers and AoAs are required to conduct MUSIC. This problem formulation (i.e., conducting MUSIC from $V$ and $\hat{A}$) is unique for BFF-based sensing, where only the BFF flame can be used instead of the CSI matrix.

**Proposition 1.** If the $m$th diagonal element of $\hat{A}$ is zero, the $m$th row vector of $V$ is orthogonal to $A(\phi)$.

**Proof.** Considering $v_m$ as the $m$th row vector of $V$, $v_m$ is the eigenvector of the correlation matrix $H^H H$, whose eigenvalue is $m$th diagonal element of $\Sigma^2$. Considering that the diagonal elements of $\Sigma$ are listed in a descending order, if the $m$th diagonal element of $\hat{A}$ is zero, then the $m$th diagonal element of $\Sigma$ is zero. Therefore, $v_m$ is the eigenvector of $H^H H$, whose eigenvalue is zero. Using [3], $H^H H$ is represented as

$$H^H H = A(\phi)^H R^* A(\phi)^T R A(\phi). \tag{4}$$

Thus, $v_m$ satisfies

$$A(\phi)^H R^* A(\phi)^T R A(\phi) v_m = 0_{M \times 1}. \tag{5}$$

Based on the assumption; $a(\theta_l) \neq a(\theta_{l'})$ unless $l = l'$, $A(\theta)$ is full-rank [15], which leads that $A(\theta)^* A(\theta)^T$ is regular [16]. Thus, we obtain $R^* A(\theta)^* A(\theta)^T R$ is a regular matrix, because $R$ is regular. Since a slim matrix $A(\phi)^H$ is full-rank and $R^* A(\theta)^* A(\theta)^T R$ is regular, we obtain $A(\phi) v_m = 0_{L \times 1}$ from (5). □

**Remark 1.** Based on Proposition 1, we propose B-MUSIC, which estimates the AoD and gain for each of the multiple propagation paths. We denote $V'$ as $(M - L) \times M$ matrix $(v'_1, \ldots, v'_{M-L})^T$, where $v'_m$ is a row vector of $V$ with a stream gain of zero. The AoDs are estimated from $V'$, in the same way that the MUSIC estimates the AoDs from the noise subspace vectors. Specifically, the AoDs are estimated as the angles that achieve peak of the MUSIC spectrum function

$$g(\phi) := \frac{1}{a(\phi)^H V^H V a(\phi)}, \tag{6}$$

in the range of $-\pi/2 < \phi < \pi/2$.

Using the estimated AoDs $\hat{\phi}$ and $\hat{A}$, the gain of each propagation path is estimated, which is denoted in Proposition 2. In this gain estimation, we used the subcarrier-averaged stream gain $\hat{A}$ instead of the singular values of an arbitrary subcarrier $\Sigma^2$. Moreover, in the following numerical evaluation, we validate that the gain is accurately estimated when $\hat{A}$ is used instead of $\Sigma^2$. Note that, as discussed in Section I, the B-MUSIC implies the potential privacy risk of IEEE 802.11ac/ax, where the BFF frames are exchanged without encryption. This is because, using the B-MUSIC, malicious sniffer can estimate AoDs only capturing BFF frames.

**Proposition 2.** Given the gain of the $l$th propagation path with AoD of $\phi_l$ as $|\phi_l|$, $|\phi_l|$ is obtained as $\sqrt{C_{l,l}/M}$, where

$$C := A(\hat{\phi})^H V\Sigma^2 V^H A(\hat{\phi})^T. \tag{7}$$

**Proof.** Assuming that the estimated AoDs $\hat{\phi}$ are identical to the ground truth AoDs except for the order, $\hat{\phi}$ is represented as $f(\phi)$, where $f$ is a function that reorders the $L$-dimensional
vector. Let us denote $\hat{\theta} = f(\theta)$ and $\hat{R} = \text{diag}(f(r))$. Here, we introduce a Lemma, which is proved the Appendix.

**Lemma 1.** Given an arbitral function $f$ that reorders the $L$-dimensional vector,

$$A(\theta)^T R A(\phi) = A(f(\theta))^T \hat{R} A(f(\phi)), \quad (8)$$

where $\hat{R} = \text{diag}(f(r))$ and $R = \text{diag}(r)$.

Using (11), (4), and Lemma 1 we obtain

$$V \Sigma^2 V^H = A(\hat{\phi})^H \hat{R}^* A(\hat{\theta})^* A(\hat{\theta})^T \hat{R} A(\hat{\phi}). \quad (9)$$

In addition, because a fat matrix $A(\hat{\phi})$ is full-rank [15], [16],

$$A(\hat{\phi})^+ = A(\hat{\phi})^H (A(\hat{\phi}) A(\hat{\phi})^H)^{-1}. \quad (10)$$

Substituting (9) and (10) into (7), we obtain

$$C = A(\hat{\phi})^H A(\hat{\phi})^+ \hat{R}^* A(\hat{\theta})^* A(\hat{\theta})^T \hat{R} A(\hat{\phi}) A(\hat{\phi})^+. \quad (11)$$

The $l$th diagonal element of $A(\hat{\theta})^* A(\hat{\theta})^T$ in (11) is

$$a(\theta_l)^* a(\theta_l)^T = \sum_{m=1}^{M} a(\theta_l)^{-m+1} a(\theta_l)^{m-1} = M. \quad (12)$$

Thus, the $l$th diagonal element of $C$ is $|\hat{r}_l|^2 M$. Therefore, we obtain $|\hat{r}_l| = \sqrt{C_{l,l}/M}$. □

**III. NUMERICAL EVALUATION**

To validate the feasibility of B-MUSIC, we examined B-MUSIC using a simple numerical evaluation. Because this evaluation aims to validate the mathematical proof, we only consider a simple setup, and a more complex and pragmatic scenario is beyond the scope of this study.

**A. Setup**

Fig. 2 shows the system, which comprises an AP, an STA, and two reflection points, resulting in three different propagation paths between an antenna element of the AP and that of the STA: a direct path and two indirect paths caused by the reflection points. We assume free-space propagation, where the indirect paths are decayed by 0.1 of the amplitude, and ignore the effect of the reflection more than once. The AP and STA are equipped with uniform array antennas. The antenna arrays contain four antenna elements, respectively. Because the number of antenna elements in the AP and STA is larger than that of the propagation paths, the B-MUSIC can be applied to this system. The location of the equipment is depicted in Fig. 2. We calculate the CSI matrix for each subcarrier $j$, and then obtain the right eigenvectors $V_j$ for each subcarrier and subcarrier-averaged stream gain $\hat{A}$. Subsequently, B-MUSIC is conducted using $V_j$ and $\hat{A}$. The detailed parameters are as follows: the distance of each antenna element is 3 cm, number of subcarriers is 64, bandwidth is 20 MHz, and center frequency is 5.18 GHz.

**B. Result**

Fig. 3 shows the MUSIC spectrum function $g(\phi)$ of the B-MUSIC. The three peaks of the function indicate three estimated AoDs. The pink and black boxes denote the AoD and its corresponding gain, which are estimated by B-MUSIC and ground-truth, respectively.

**IV. CONCLUSION**

We prove that the MUSIC-based AoD estimation can be conducted using only the BFF frame by adding some assumptions regarding the AoAs and receiver antennas. Based on this finding, B-MUSIC is proposed to estimate the AoD
and gain for each propagation path. Moreover, we pointed out the potential privacy risk of IEEE 802.11ac/ax, where the malicious sniffers can estimate AoDs only by capturing unencrypted BFF frames. In future work, we will expand B-MUSIC by applying the techniques proposed in the CSI-based MUSIC literature. For example, they include expanding applicability to the scenario in which the number of antenna elements is smaller than that of the propagation paths, and address more practical tasks (e.g., human tracking and device localization) using the estimation results of B-MUSIC.

ACKNOWLEDGEMENT
This research and development work was partially supported by the MIC/SCOPE #JP196000002 and JSPS KAKENHI Grant Number JP18H01442.

APPENDIX

Proof of Lemma 1. The matrices \( \mathbf{A}(f(\theta)) \) and \( \mathbf{R} \) are represented as

\[
\mathbf{A}(f(\theta)) = \left( \prod_{i=1}^{N} \mathbf{P}_i \right) \mathbf{A}(\theta),
\]

where \( \mathbf{P}_i \) is the permutation matrix. Because \( \mathbf{P}_i^T = \mathbf{P}_i \) and \( \mathbf{P}_i^T \mathbf{P}_i = \mathbf{E} \), given \( \mathbf{Q} = \prod_{i=1}^{N} \mathbf{P}_i \), we obtain \( \mathbf{Q}^T = \prod_{i=N}^{1} \mathbf{P}_i^T \) and \( \mathbf{QQ}^T = \mathbf{E} \). Thus, (13) and (14) are represented as

\[
\mathbf{A}(f(\theta)) = \mathbf{Q} \mathbf{A}(\theta) \quad \text{and} \quad \mathbf{R} = \mathbf{QQ}^T, \]

respectively. Substituting (13) and (14) to \( \mathbf{A}(f(\theta))^T \mathbf{R} \mathbf{A}(f(\theta)) \), we obtain

\[
\mathbf{A}(f(\theta))^T \mathbf{R} \mathbf{A}(f(\theta)) = (\mathbf{Q} \mathbf{A}(\phi))^T \mathbf{Q} \mathbf{Q}^T \mathbf{QA}(\phi) = \mathbf{A}(\theta)^T \mathbf{R} \mathbf{A}(\phi).
\]

\[\Box\]

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