Reconciling anomalous measurements in $B_s - \bar{B}_s$ mixing: the role of CPT-conserving and CPT-violating new physics

Amol Dighe,$^1$ Diptimoy Ghosh,$^1$ Anirban Kundu,$^2$ and Sunando Kumar Patra$^2$

$^1$Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Mumbai 400005, India
$^2$University of Calcutta, 92, Acharya Prafulla Chandra Road, Kolkata 700 009, India.

(Dated: January 20, 2013)

Recently observed anomalies in the $B_s \to J/\psi \phi$ decay and the like-sign dimuon asymmetry $A_{sl}$ hint at possible new physics (NP) in the $B_s - \bar{B}_s$ sector, and it is quite possible that the NP can affect the $B_s - \bar{B}_s$ system while keeping the $B_d - \bar{B}_d$ system untouched. Indeed, for most of the flavor-dependent NP models, the couplings relevant for the second and third generations of SM fermions are much less constrained than those for the first generation fermions, allowing the NP to play a significant role in the $B_s - \bar{B}_s$ mixing, in principle.

Over the last few years, the Tevatron experiments CDF and DØ, and to a smaller extent the B factories Belle and BaBar, have provided a lot of data on the $B_s$ meson, most of which are consistent with the SM. There are some measurements, though, which show a significant deviation from the SM expectations, and hence point towards new physics (NP). The major ones among these are the following. (i) Measurements in the decay mode $B_s \to J/\psi \phi$ yield a large CP-violating phase $\beta_{J/\psi \phi}$ [1]. In addition, though the difference $\Delta \Gamma_s$ between the decay widths of the mass eigenstates measured in this decay is consistent with the SM, it allows $\Delta \Gamma_s$ values that are almost twice the SM prediction, and also opposite in sign [2]. (ii) The like-sign dimuon asymmetry $A_{sl}$, in the combined $B$ data at DØ [3] is almost 4$\sigma$ away from the SM expectation.

The resolutions of the above anomalies, separately or simultaneously, have been discussed in the context of specific NP models: a scalar leptoquark model [4,5], models with an extra flavor-changing neutral gauge boson $Z'$ or R-parity violating supersymmetry [6,7], two-Higgs doublet model [8,9], models with a fourth generation of fermions [10,11], supersymmetric grand unified models [12], supersymmetric models with split sfermion generations [13] or models with a very light spin-1 particle [14]. Possible four-fermion effective interactions that are consistent with the data have been analyzed by [13] and the results are consistent with [5]. Similar studies, based on the minimal flavor violating (MFV) models [15] and the Randall-Sundrum model [17], have been carried out.

In this paper, we try to determine, in a model-independent way, which kind of NP would be able to account for both the above anomalies simultaneously. We take a somewhat different approach than the references cited above. Rather than confining ourselves to specific models, we assume that the NP responsible for the anomalies contributes entirely through the $B_s - \bar{B}_s$ mixing, and parameterize it in a model independent manner through the effective Hamiltonian for the $B_s - \bar{B}_s$ mixing. This effective Hamiltonian $\mathcal{H}$ is a $2 \times 2$ matrix in the flavor basis, and the relevant NP contribution appears in its off-diagonal elements. The NP can then be parameterized by using four parameters: the magnitudes and phases of the dispersive part and the absorptive part of the NP contribution to $\mathcal{H}$. A “scatter-plot” analysis that constrained these four new parameters using only $A_{sl}$ has been carried out in [17]. We perform a $\chi^2$ fit to the $B_s - \bar{B}_s$ mixing observables and obtain a quantitative measure for which kind of NP is preferred by the data. This would lead us to shortlist specific NP models that have the desired properties, which can give testable predictions for other experiments. It is found that the NP needs to contribute to both the dispersive as well as

PACS numbers: 11.30.Er, 14.40.Nd

I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa (CKM) paradigm of quark mixing in the standard model (SM) is yet to be accurately tested in the $B_s - \bar{B}_s$ sector, and it is quite possible that the NP can affect the $B_s - \bar{B}_s$ system while keeping the $B_d - \bar{B}_d$ system untouched. Indeed, for most of the flavor-dependent NP models, the couplings relevant for the second and third generations of SM fermions are much less constrained than those for the first generation fermions, allowing the NP to play a significant role in the $B_s - \bar{B}_s$ mixing, in principle.

Over the last few years, the Tevatron experiments CDF and DØ, and to a smaller extent the B factories Belle and BaBar, have provided a lot of data on the $B_s$ meson, most of which are consistent with the SM. There are some measurements, though, which show a significant deviation from the SM expectations, and hence point towards new physics (NP). The major ones among these are the following. (i) Measurements in the decay mode $B_s \to J/\psi \phi$ yield a large CP-violating phase $\beta_{J/\psi \phi}$ [1]. In addition, though the difference $\Delta \Gamma_s$ between the decay widths of the mass eigenstates measured in this decay is consistent with the SM, it allows $\Delta \Gamma_s$ values that are almost twice the SM prediction, and also opposite in sign [2]. (ii) The like-sign dimuon asymmetry $A_{sl}$, in the combined $B$ data at DØ [3] is almost 4$\sigma$ away from the SM expectation.

The resolutions of the above anomalies, separately or simultaneously, have been discussed in the context of specific NP models: a scalar leptoquark model [4,5], models with an extra flavor-changing neutral gauge boson $Z'$ or R-parity violating supersymmetry [6,7], two-Higgs doublet model [8,9], models with a fourth generation of fermions [10,11], supersymmetric grand unified models [12], supersymmetric models with split sfermion generations [13] or models with a very light spin-1 particle [14]. Possible four-fermion effective interactions that are consistent with the data have been analyzed by [13] and the results are consistent with [5]. Similar studies, based on the minimal flavor violating (MFV) models [15] and the Randall-Sundrum model [17], have been carried out.

In this paper, we try to determine, in a model-independent way, which kind of NP would be able to account for both the above anomalies simultaneously. We take a somewhat different approach than the references cited above. Rather than confining ourselves to specific models, we assume that the NP responsible for the anomalies contributes entirely through the $B_s - \bar{B}_s$ mixing, and parameterize it in a model independent manner through the effective Hamiltonian for the $B_s - \bar{B}_s$ mixing. This effective Hamiltonian $\mathcal{H}$ is a $2 \times 2$ matrix in the flavor basis, and the relevant NP contribution appears in its off-diagonal elements. The NP can then be parameterized by using four parameters: the magnitudes and phases of the dispersive part and the absorptive part of the NP contribution to $\mathcal{H}$. A “scatter-plot” analysis that constrained these four new parameters using only $A_{sl}$ has been carried out in [17]. We perform a $\chi^2$ fit to the $B_s - \bar{B}_s$ mixing observables and obtain a quantitative measure for which kind of NP is preferred by the data. This would lead us to shortlist specific NP models that have the desired properties, which can give testable predictions for other experiments. It is found that the NP needs to contribute to both the dispersive as well as
absorptive part of the Hamiltonian in order to avoid any tension with the data.

We also extend our framework to include possible CPT violation in the $B_s - \overline{B_s}$ mixing, parameterized through the difference in diagonal elements of $H$. The motivation is to check if this can obviate the need for an absorptive contribution from the NP. Such an analysis to constrain CPT and Lorentz violating parameters was carried out in [18]. However they have used only $A'^{s}_{th}$ and not $\beta^J_{s/\psi} \equiv \arg(J_{s/\psi})$ in their analysis, and their parameters are only indirectly connected to the elements of $H$. We try to account for the two anomalies above with only CPT violation as the source of NP, and with a combination of CPT violation and the absorptive part of the Hamiltonian in order to avoid any cancellation of the SM unless there is a nonzero absorptive part in the $B_s - \overline{B_s}$ mixing amplitude.

The paper is organized as follows. In Sec. II we introduce our formalism for the four NP parameters. In Sec. III we summarize the experimental measurements and theoretical predictions for the observables relevant for $B_s - \overline{B_s}$ mixing. In Sec. IV we present the results of our fits, and their implications for NP models are discussed in Sec. V. In Sec. VI we introduce the formalism for introducing CPT violation and in Sec. VII we explore the extent to which it can help resolving the anomalies. Sec. VIII summarizes our results and concludes.

II. THE EFFECTIVE HAMILTONIAN

The evolution of a $B_s - \overline{B_s}$ state can be described by the effective Hamiltonian

$$H = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix}$$

(1)

in the flavor basis, where $M_{ij}$ and $\Gamma_{ij}$ are its dispersive and absorptive parts, respectively. When CPT is conserved, $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. The eigenstates of this Hamiltonian are $B_{sH}$ and $B_{sL}$, with masses $M_{sH}$ and $M_{sL}$ respectively, and decay widths $\Gamma_{sH}$ and $\Gamma_{sL}$ respectively. The difference in the masses and decay widths can be written in terms of the elements of the Hamiltonian as

$$\Delta M_s \equiv M_{sH} - M_{sL} \approx 2|M_{12}|, \quad \Delta \Gamma_s \equiv \Gamma_{sL} - \Gamma_{sH} \approx 2|\Gamma_{12}| \cos[\text{Arg}(-M_{12}/\Gamma_{12})].$$

(2)

The above expressions are valid as long as $\Delta \Gamma_s \ll M_s$, which is indeed the case here.

Since CPT is conserved, the effect of NP can be felt only through the off-diagonal elements of $H$. We separate the SM and NP contributions to these terms via

$$M_{12} = M_{12}^{\text{SM}} + M_{12}^{\text{NP}}, \quad \Gamma_{12} = \Gamma_{12}^{\text{SM}} + \Gamma_{12}^{\text{NP}}.$$  

(3)

The NP can then be completely parameterized in terms of four real numbers: $|M_{12}^{\text{NP}}|$, $\text{Arg}(M_{12}^{\text{NP}})$, $|\Gamma_{12}^{\text{NP}}|$ and $\text{Arg}(\Gamma_{12}^{\text{NP}})$. We take the phases $\text{Arg}(M_{12}^{\text{NP}})$ and $\text{Arg}(\Gamma_{12}^{\text{NP}})$ to lie in the range $0-2\pi$.

In a large class of models, including the Minimal Flavor Violation (MFV) models, the NP contribution has no absorptive part, i.e. $\Gamma_{12} = \Gamma_{12}^{\text{SM}}$. This is true for a lot of non-MFV models too. This occurs when NP does not give rise to any new intermediate light states to which $B_s$ or $\overline{B_s}$ can decay. For such models, Eq. (2) implies that $\Delta \Gamma_s \approx 2|\Gamma_{12}^{\text{SM}}|$, i.e. the value of $\Delta \Gamma_s$ is always less than its SM prediction [19]. In such models, the NP is parameterized by only two parameters: $|M_{12}^{\text{NP}}|$ and $\text{Arg}(M_{12}^{\text{NP}})$. An analysis restricted to this class of models was performed in [20].

However there exists a complementary class of viable models where the NP contributes to $\Gamma_{12}$ substantially. These include models with leptoquarks, R-parity violating supersymmetry, a light gauge boson, etc. It has been pointed out in [3] that such a nonzero absorptive part that arises naturally in these class of models can enhance $\Delta \Gamma_s$ significantly above its SM value, contrary to the popular expectations based on [19]. One notes that a new absorptive part in the mixing amplitude necessarily means new final states that can be accessed by both $B_s$ and $\overline{B_s}$. The data from the direct measurements of branching ratios is extremely restrictive [18], apart from that for a few final states like $B_s \rightarrow \tau^+ \tau^-$ [6]. As we shall see later in this paper, such models are favored by $B_s - \overline{B_s}$ mixing data. The importance of $\tau^+ \tau^-$ final states from $B_d$ and $B_s$ decays has also been pointed out in [21].

III. THE MEASUREMENTS

The $B_s - \overline{B_s}$ oscillation and CP violation therein can be quantified by four observables, viz. the mass difference $\Delta M_s$, the decay width difference $\Delta \Gamma_s$, the CP-violating phase $\beta^J_{s/\psi}$, and the semileptonic asymmetry $a^s_L$.

The mass difference is measured to be

$$\Delta M_s = (17.77 \pm 0.10 \pm 0.07) \text{ ps}^{-1},$$

(4)

which is consistent with the SM expectation [21].

$$\Delta M_s^{\text{SM}} = (17.3 \pm 2.6) \text{ ps}^{-1}.$$  

(5)

However measurements in the $B_s \rightarrow J/\psi \phi$ decay mode show a hint of some deviation from the SM. The CP-violating phase $\beta^J_{s/\psi}$ in this decay is

$$\beta^J_{s/\psi} = \frac{1}{2} \text{Arg} \left( -\frac{(V_{cb} V_{cb}^*)^2}{M_{12}} \right),$$

(6)

whose average value measured at the Tevatron experiments [4] is

$$\beta^J_{s/\psi} = (0.41^{+0.18}_{-0.15}) \cup (1.16^{+0.15}_{-0.18}).$$

(7)

In the SM,

$$\beta^J_{s/\psi}^{\text{SM}} = \text{Arg} \left( -\frac{V_{cb} V_{cb}^*}{V_{tb} V_{ts}^*} \right) \approx 0.019 \pm 0.001.$$  

(8)
Thus, the measured value of $\beta_s^{J/\psi}$ is more than 2σ away from the SM expectation. On the other hand, the difference in the decay widths of the mass eigenstates $B_H$ and $B_L$ is measured to be 1]

$$\Delta \Gamma_s = \pm (0.154^{+0.054}_{-0.070}) \text{ ps}^{-1},$$  \hspace{1cm} (9)$$

while the SM expectation is 22

$$\Delta \Gamma_s(\text{SM}) = (0.087 \pm 0.021) \text{ ps}^{-1}. \hspace{1cm} (10)$$

The measurement is consistent with the SM expectation to ~ 1σ, however it allows for $\Delta \Gamma_s$ values that are almost twice the SM prediction. Note that the sign of $\Delta \Gamma_s$ is undetermined experimentally and this gives us more room to play with the NP parameters.

CDF has recently announced its new results, based on 5.2 fb$^{-1}$ of data 23:

$$|\Delta \Gamma_s| = (0.075 \pm 0.035 \pm 0.010) \text{ ps}^{-1},$$

$$\beta_s^{J/\psi} = (0.02 - 0.52) \cup (1.08 - 1.55) \hspace{1cm} (11)$$

to 68% C.L.. While we note that the results are consistent with the SM, the final Tevatron averages are still awaited. Therefore, we use the values in Eq. (9) in our analysis.

The other anomalous measurement is the like-sign dimuon asymmetry. Averaging the 9.0 fb$^{-1}$ data of DØ 3 and 1.6 fb$^{-1}$ data of CDF 24, and adding the errors in quadrature and treating them as Gaussian, we get

$$A_{sl}^b = -(7.41 \pm 1.93) \times 10^{-3}, \hspace{1cm} (12)$$

which differs by more than 3σ from its SM prediction

$$A_{sl}^b(\text{SM}) = (-0.23^{+0.055}_{-0.066}) \times 10^{-3}. \hspace{1cm} (13)$$

Note that for $A_{sl}^b$, CDF has a poorer statistics than DØ and therefore the average value is dominated by the DØ data.

Even in the presence of new physics, the SM relationship holds:

$$A_{sl}^b = (0.506 \pm 0.043) a_{sl}^d + (0.494 \pm 0.043) a_{sl}^s,$$

where $a_{sl}^d$ and $a_{sl}^s$ are the semileptonic asymmetries for the $B_s - \bar{B}_s$ and the $B_d - \bar{B}_d$ systems, respectively. The former is related to the $B_s - \bar{B}_s$ mixing observables through

$$a_{sl}^s = \Delta \Gamma_s \tan \phi_s \hspace{1cm} (15)$$

where $\phi_s = \text{Arg}(-M_{12}/\Gamma_{12})$. The latter is defined analogously. The coefficients in Eq. (14) are experimentally measured, and contain information about $\Delta M_{d(s)}$, $\Delta \Gamma_{d(s)}$, and production fractions of $B_d$ and $B_s$ mesons. Using $a_{sl}^d = -(4.7 \pm 4.6) \times 10^{-3}$ 2, this leads to

$$a_{sl}^s = -0.010 \pm 0.006,$$

which is about 1.7σ away from the SM prediction

$$a_{sl}^s(\text{SM}) = (2.06 \pm 0.57) \times 10^{-5}. \hspace{1cm} (17)$$

The value of $a_{sl}^d$ depends on $\Delta M_d$, $\Delta \Gamma_d$ and $\phi_d$, the parameters in the $B_d$ sector analogous to those in Eq. (15). These parameters depend on the NP in the $B_d$ sector, which is independent of the NP parameters in the $B_s$ sector that we are considering. We therefore do not consider the measured values of $a_{sl}^d$ as a direct constraint, but express it in terms of $\Delta M_d$, $\Delta \Gamma_d$, and $\phi_d$, whose experimental values are taken as inputs.

In the SM, we have $\phi_s(\text{SM}) = 0.0041 \pm 0.0007$ 22. Note that if the dominating contribution to $\Gamma_{12}$ were from a pair of intermediate $c$ quarks, $\phi_s(\text{SM})$ would have been equal to $-2\beta_s^{J/\psi}$. Since the intermediate $u - c$ and $u - u$ quark states give comparable contributions to $\Gamma_{12}$, we have $\phi_s(\text{SM}) \neq -2\beta_s^{J/\psi}(\text{SM})$ 22.

IV. THE STATISTICAL ANALYSIS

We perform a $\chi^2$ fit to the observed quantities $\Delta M_s$, $\Delta \Gamma_s$, $\beta_s^{J/\psi}$ and $a_{sl}^s$, using the NP parameters $|M_{12}^{NP}|$, Arg($M_{12}^{NP}$), $|\Gamma_{12}^{NP}|$ and Arg($\Gamma_{12}^{NP}$). We assume all the measurements to be independent for simplicity, though the measurements of $\Delta \Gamma_s$ and $\beta_s^{J/\psi}$ are somewhat correlated. The values of all the observables and their SM values are as given in Sec. III. In order to express them in terms of $M_{12}$, $M_{12}^{SM}$, $\Gamma_{12}$ and $\Gamma_{12}^{SM}$, one has to use Eq. (2) in addition. In order to take into account the errors on the SM parameters, we add the theoretical and experimental errors on our observed quantities in quadrature.

Note that since we have four observable quantities and four parameters, it is not surprising that we obtain the global minimum value of $\chi^2$ as $\chi^2_{\text{min}} = 0$ when all the NP parameters are allowed to vary. The questions we address here are (i) what the preferred values of the NP parameters are, (ii) to what confidence level (C.L.) a given set of NP parameters (or SM, which is a special case of NP with $M_{12}^{NP} = \Gamma_{12}^{NP} = 0$) is allowed. The latter is obtained assuming all errors to be Gaussian. Here we give our results in terms of the goodness-of-fit contours for the joint estimations of two parameters at a time. The $(1\sigma, 2\sigma, 3\sigma, 4\sigma)$ contours, that are equivalent to $p$-values of $(0.3173, 0.0455, 0.0027, 0.0001)$, or confidence levels of $(68.27\%, 95.45\%, 99.73\%, 99.99\%)$, correspond to $\chi^2 = (2.295, 6.18, 11.83, 19.35)$, respectively.

In Fig. 1, we show the $1\sigma, 2\sigma, 3\sigma, 4\sigma$ contours in the $|M_{12}| - \text{Arg}(M_{12})$ plane, where the other NP parameters are marginalized over. Clearly, we see a preference towards nonzero $|M_{12}^{NP}|$ as well as nonzero Arg($M_{12}^{NP}$) values. There are two best-fit points with $\chi^2 = 0$, one at $M_{12}^{NP} \approx 6.3 \exp(2.0 \text{ i}) \text{ ps}^{-1}$ and the other at $M_{12}^{NP} \approx 16.2 \exp(2.8 \text{ i}) \text{ ps}^{-1}$, shown with crosses in Fig. 1. Actually, each of these crosses is a superimposed double, with two values of $\Gamma_{12}^{NP}$, as shown in Fig. 2. The points
correspond to the constructive and destructive interference between the SM and NP amplitudes in order to give the measured central values of $\Delta M_s$. The region with $M_{12}^{NP} = 0$, i.e. the $x$-axis, is outside the 2$\sigma$ region, indicating that it will be rather difficult to fit the current data without some NP contribution to the dispersive part of the $B_s - \bar{B}_s$ mixing. The contours also imply that $|M_{12}^{NP}| \lesssim 21.1$ ps$^{-1}$ to 3$\sigma$.

In Fig. 2, we show the goodness-of-fit contours in the $|\Gamma_{12}^{NP}| - \text{Arg}(\Gamma_{12}^{NP})$ plane, marginalizing over other two NP parameters. As the measurements do not determine the sign of $\Delta \Gamma_s$, for any particular value of $|\Delta \Gamma_s|$, we perform the $\chi^2$ fit for both positive and negative values, and keep the minimum $\chi^2$ of the two. This doubles the number of best-fit solutions, and the two best-fit points of Fig. 4 now split into four. For $|M_{12}^{NP}| = 6.3$, the solutions are $\Gamma_{12}^{NP} = 0.18 \exp(6.0 \ i)$ or $0.18 \exp(5.2 \ i)$, and for $|M_{12}^{NP}| = 16.2$, the corresponding solutions are $\Gamma_{12}^{NP} = 0.18 \exp(0.2 \ i)$ or $0.18 \exp(1.1 \ i)$ (both $M_{12}^{NP}$ and $\Gamma_{12}^{NP}$ are in ps$^{-1}$, here, and also later where not mentioned explicitly). Note that there is a reflection symmetry about $\text{Arg}(\Gamma_{12}^{NP}) = \pi$. Again, a preference for nonzero values of $|\Gamma_{12}^{NP}|$ is indicated, though $\text{Arg}(\Gamma_{12}^{NP})$ may vanish. The region with $\Gamma_{12}^{NP} = 0$, i.e. the $x$-axis, is outside the 4$\sigma$ allowed region, indicating that NP contribution to the absorptive part of the effective Hamiltonian is highly favored. The contours also imply that $|\Gamma_{12}^{NP}| \lesssim 0.35$ at 3$\sigma$.

Fig. 3 displays the contours in the $|M_{12}^{NP}| - |\Gamma_{12}^{NP}|$ plane, and the two NP phases are marginalized over. Not only does it show a preference for nonzero values of $M_{12}^{NP}$ and $\Gamma_{12}^{NP}$, but the $M_{12}^{NP} = 0$ axis is outside the 2$\sigma$ allowed region and the $\Gamma_{12}^{NP} = 0$ axis is outside the 4$\sigma$ allowed region. The best fit points are again superimposed doubles, whose values can be read off from the discussion above. The origin in this figure is the SM, which has $\chi^2_{SM} = 25.85$, and lies even outside the 4$\sigma$ allowed region. This dramatically quantifies the failure of the SM to accommodate the current data. The reason is evident from eqs. 1 and 1a; while $B_s \to J/\psi \phi$ prefers $\beta^{J/\psi \phi}_s$ close to $\pi/8$ or $3\pi/8$, with a probability minimum near $\beta^{J/\psi \phi}_s \approx \pi/4$, the measurement of $A_{31}^s$, and hence that of $\alpha^s_3$, prefers large $\phi_s$, forcing $\beta^{J/\psi \phi}_s$ close to $\pi/4$. This creates the tension between these two measurements.

Fig. 4 also tells us that the models for which $\Gamma_{12}^{NP} = 0$, like R-parity conserving supersymmetry, universal extra dimension, and extra scalars, fermions, or gauge bosons, cannot bring the tension down even to the 4$\sigma$ range, un-
less the data moves towards the SM expectations (and unless the new bosons are flavor-changing so as to generate a nonzero $\Gamma_{12}^{NP}$). The best fit point with $\Gamma_{12}^{NP} = 0$ has $\chi^2 = 20.75$ and corresponds to $M_{12}^{NP} = 3.72\exp(1.68\,i)$. This is further emphasized in fig. 4 which shows the $5\sigma$ ($p$ value of $10^{-6}$, $\chi^2 = 27$) contour for those NP models where $\Gamma_{12}^{NP}$ is set to vanish (within the closed contour above and under the open contour below).

One may question the optimistic SM uncertainty for $\Delta\Gamma_s$ as quoted in eq. (10). However, this has an almost negligible effect. For example, if we increase the uncertainty by 50%, neither the best fit points nor the confidence levels change significantly. The best fit point in fig. 4 has a $\chi^2$ minimum of 20.58 instead of 20.75. The reason is the large deviation of $a_0^s$ from its SM value, to explain which we need a significant enhancement in $\tan \phi_s$.

V. PREFERRED NP MODELS

From the results and discussion in the previous section, it appears that:

(i) The SM by itself is strongly disfavored. Either $M_{12}^{NP}$ or $\Gamma_{12}^{NP}$ should be nonzero.

(ii) $M_{12}^{NP} \neq 0$ but $\Gamma_{12}^{NP} = 0$ is also not allowed at 4$\sigma$, but the fit is marginally better than the SM.

(iii) The hypothetical case where $\Gamma_{12}^{NP} \neq 0$ but $M_{12}^{NP} = 0$ is also disfavored to more than 2$\sigma$. (This is a rather natural condition, since any interaction that contributes to $\Gamma_{12}^{NP}$ will necessarily contribute to $M_{12}^{NP}$.)

Most of the NP models can contribute significantly to $M_{12}^{NP}$. Leading examples are the MFV models like minimal supersymmetry, universal extra dimensions, little-Higgs with T-parity, etc. Non-MFV models like a fourth chiral generation, supersymmetry with R-parity violation, two-Higgs doublet models, models with extra $Z'$, etc. can also contribute significantly to $M_{12}^{NP}$.

The NP models that can contribute significantly to $\Gamma_{12}^{NP}$, however, are rather rare. This is because the NP contribution to the absorptive part needs light particles in the final state, and there are strong limits on the decays of $B_s$ to most of the possible light final state particles. One of the few exceptions is the mode $\tau^+\tau^-$, on which there is no available bound at this moment. Thus, the NP that contributes to $\Gamma_{12}^{NP}$ has to do so via the interaction $b \rightarrow s\tau^+\tau^-$, but without affecting related decays like $b \rightarrow s\gamma\gamma$ or $b \rightarrow s\mu^+\mu^-$. This can be achieved only in a limited subset of models, for example those with second and third generation scalar leptoquarks, R-parity violating supersymmetry [4], or extra $Z'$ bosons [7]. It turns out that the former can provide enough contribution to $\Gamma_{12}^{NP}$ to increase $\Delta\Gamma_s$ up to its current experimental upper bound [4]. The amount of NP required for this is consistent with the difference between the decay widths of $B_d$ and $B_s$ mesons ($\Gamma_s/\Gamma_d - 1 = (3.6 \pm 1.8)\%$ [2]), and the recent measurement of the branching ratio $B^+ \rightarrow K^+\tau^+\tau^-$, which is less than $3.3 \times 10^{-3}$ at 90% C.L. [20].

One should note here that if the DO results on the dimuon charge asymmetry survive the test of time, it will be a clear indication of the presence of a nonzero $\Gamma_{12}^{NP}$. Such models are also favoured from the CDF and DO combined result on the allowed contours for $\Gamma_s/\Gamma^0$ and $\Gamma_s$, but we need to wait for the final Tevatron average.

VI. CPT VIOLATION: THE FORMALISM

The analysis till now is valid only if we assume CPT-invariance. However, the CPT symmetry may be violated in theories that break Lorentz invariance [27]. Indeed for local field theories, CPT violation requires Lorentz violation [28]. (This need not be true for nonlocal field theories as well as for theories with noncommutative space-time geometry, see [29].) In general, CPT violation should result in differences in masses and decay widths between particle-antiparticles pairs. However it may be easier to identify even through oscillation experiments, which typically are sensitive to an interference between the CPT-conserving and CPT-violating interactions.

While CPT violation in the $K$ system is severely constrained through the mass difference between the neutral kaons [31], the bounds on the CPT violating parameters in the $B_d$ and $B_s$ systems are rather weak. In fact, the bounds for the $B_d$ sector are about three orders of magnitude weaker than those for the $K$ sector [31]. The bounds on Lorentz-violating parameters using the data on $B$ mesons can be found in [18] and references therein. Here we use a model-independent parameterization, like the one earlier followed in [32], and recently used by two of us [33], and determine the preferred parameter space using the data on $B_s - {\bar B}_s$ oscillations. Unlike [18], we
take both $A^{b}_{h}$ and $\beta^{J/\psi}_{s}$ data into account.

One should note that as a new physics option, CPT violation is not exactly at the same footing as the models mentioned before. However in the language of the effective Hamiltonian $\mathcal{H}$, the CPT violation manifests itself naturally through in a difference between the diagonal elements of $\mathcal{H}$. It is therefore interesting to see if the constraints on the NP coming from $\Delta M_s$ and $\Delta \Gamma_s$ can be relaxed at all with these additional degrees of freedom. A posteriori, we will justify the discussion on CPT violation by showing that if the new physics indeed turns out to be without an absorptive part, CPT violation might help to explain the $B_s - \bar{B}_s$ mixing data, albeit only marginally.

The CPT violation manifests itself in the effective Hamiltonian through the difference in the diagonal elements. We write the effective Hamiltonian in eq. (1) as

$$\mathcal{H} = \begin{pmatrix} M_0 - \frac{i}{2} \Gamma_0 - \delta' & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12} - \frac{i}{2} \Gamma_{12} & M_0 - \frac{i}{2} \Gamma_0 + \delta' \end{pmatrix},$$

and define the dimensionless CPT-violating complex parameter $\delta$ as

$$\delta = \frac{H_{22} - H_{11}}{\sqrt{H_{12} H_{21}}} = \frac{2 \delta'}{\sqrt{H_{12} H_{21}}},$$

where $H_{ij} \equiv H_{ij} - 4 \Gamma_{ij}$.

The eigenvalues of $\mathcal{H}$ are

$$\lambda = \left( M_0 - i \frac{\Gamma_0}{2} \right) \pm \alpha y H_{12},$$

where $\alpha \equiv \sqrt{H_{21}/H_{12}}$ and $y \equiv \sqrt{1 + \delta^2/4}$. The corresponding mass eigenstates are

$$|B_{sL}\rangle = p_1 |B_s\rangle + q_1 |\bar{B}_s\rangle,$$

$$|B_{sR}\rangle = p_2 |B_s\rangle - q_2 |\bar{B}_s\rangle,$$

with $|p_1|^2 + |q_1|^2 = |p_2|^2 + |q_2|^2 = 1$, and

$$\eta_1 = \frac{q_1}{p_1} = \sqrt{\frac{H_{21}}{H_{12}}} \left( \sqrt{1 + \frac{\delta^2}{4} - \frac{i}{2}} \right),$$

$$\eta_2 = \frac{q_2}{p_2} = \sqrt{\frac{H_{21}}{H_{12}}} \left( \sqrt{1 + \frac{\delta^2}{4} + \frac{i}{2}} \right).$$

Clearly, CPT invariance corresponds to $\eta_1 = \eta_2$.

Let us now determine the dependence of our four observables on the CPT-violating parameters. The differences in masses and decay widths of the eigenstates are related to the difference in eigenvalues as

$$\lambda_1 - \lambda_2 = \Delta M + \frac{i}{2} \Delta \Gamma,$$

where $\lambda_1$ and $\lambda_2$ are ordered such that $\text{Re} \lambda_1 - \lambda_2 > 0$. From Eq. (20),

$$\Delta M = M_1 - M_2 = 2 \text{Re}(\alpha y H_{12}),$$

$$\Delta \Gamma = \Gamma_2 - \Gamma_1 = 4 \text{Im}(\alpha y H_{12}).$$

Since $|\Gamma_{12}| \ll |M_{12}|$, we can write

$$\alpha H_{12} = |M_{12}| \left[ 1 - \frac{1}{4} \frac{|\Gamma_{12}|^2}{|M_{12}|^2} - i \text{Re} \left( \frac{\Gamma_{12}}{M_{12}} \right) \right]^{1/2} \approx |M_{12}| \left[ 1 - \frac{i}{2} \text{Re} \left( \frac{\Gamma_{12}}{M_{12}} \right) \right].$$

Then Eqs. (21) and (26) yield

$$\Delta M \approx |M_{12}| \left[ 2 \text{Re}(y) + \text{Im}(y) \text{Re} \left( \frac{\Gamma_{12}}{M_{12}} \right) \right],$$

$$\Delta \Gamma \approx |M_{12}| \left[ 4 \text{Im}(y) - 2 \text{Re}(y) \text{Re} \left( \frac{\Gamma_{12}}{M_{12}} \right) \right].$$

The dependence on the CPT-violating parameter $\delta$ appears entirely through $y$.

Let us pause here for a moment and find what the above two equations tell us about the allowed parameter space. Let us first focus on the best constraint, $\Delta M_s$, and work in the limit where $\Gamma_{12}/M_{12}$ is negligible. $|M_{12}|$, and hence $M_{12}^\text{NP}$, can be arbitrarily large, as $\text{Re}(y)$ can be made arbitrarily small by an appropriate choice of $\delta$. Similarly, $\text{Re}(y)$ can be quite large (albeit compatible with other constraints) as long as there is a near-perfect cancellation between the SM and NP mixing amplitudes, making $|M_{12}|$ small. However, the smallness of $\Delta \Gamma/\Delta M$ constrains $\text{Im}(y)/\text{Re}(y)$ to be small, thus indicating that $y$ is almost real. Since $y = \sqrt{1 + \delta^2/4}$, this implies that $\delta^2$ is almost real and $\text{Re}(\delta^2) \gtrsim -4$. Therefore, one would expect that $\delta$ is either almost real, or it is almost imaginary, but with $|\text{Im}(\delta)| < 2$.

Now let us consider the CP-violating observables $\beta_s^{J/\psi}$ and $\alpha_s^{*}$. The effective value of the former may be obtained in the presence of CPT violation by considering the decay rates of $B_s$ and $\bar{B}_s$ to a final CP eigenstate $f_{CP}$ as

$$\Gamma(B_s(t) \to f_{CP}) = |A_f|^2 \left[ |f_+(t)|^2 + |\xi_f|^2 |f_-(t)|^2 \right] + 2 \text{Re}(\xi_f f_-(t) f_+^\dagger(t)),$$

$$\Gamma(\bar{B}_s(t) \to f_{CP}) = \frac{|A_f|^2}{\eta_2} \left[ |f_-(t)|^2 + |\xi_f|^2 |f_+(t)|^2 \right] + 2 \text{Re}(\xi_f f_+^\dagger(t) f_-^\dagger(t)),$$

with

$$\xi_f \equiv \frac{\eta_1}{A_f}, \quad \xi_{f_\perp} \equiv \frac{\eta_2}{\eta_1 A_f}, \quad \omega \equiv \frac{\eta_1}{\eta_2}.\quad (31)$$

Here $A_f$ and $A_{f_\perp}$ are the amplitudes for the processes $B_s \to f_{CP}$ and $\bar{B}_s \to f_{CP}$, respectively. The time evolutions are given by

$$f_-(t) = \frac{1}{1 + \omega} (e^{-i \lambda_1 t} - e^{-i \lambda_2 t}),$$

$$f_+(t) = \frac{1}{1 + \omega} (e^{-i \lambda_1 t} + \omega e^{-i \lambda_2 t}),$$

$$f_+(t) = \frac{1}{1 + \omega} (e^{-i \lambda_1 t} + e^{-i \lambda_2 t}).$$

(32)
The final state in $B_s \to J/\psi \phi$ is not a CP eigenstate, but a combination of CP-even and CP-odd final states, which may be separated using angular distributions. With the transversity angle distribution \cite{24}, the time-dependent decay rate to the CP-eigenstate is given by the coefficient of $(1+\cos^2\theta)$, while the time-dependent decay rate to the CP-odd state is given by the coefficient of $\sin^2\theta$.

The value of effective $\beta_s^{J/\psi \phi}$ in this process is determined by writing the time evolutions \eqref{eq:29} and \eqref{eq:30} in the form

$$
\Gamma(B_s(t) \to f_{CP}) = c_1 \cosh(\Delta M_s t/2) + c_2 \sinh(\Delta M_s t/2) + c_3 \cos(\Delta M_s t) + c_4 \sin(\Delta M_s t), \quad (33)
$$

$$
\Gamma(B_s(t) \to \bar{f}_{CP}) = c_1 \cosh(\Delta M_s t/2) + c_2 \sinh(\Delta M_s t/2) + c_3 \cos(\Delta M_s t) + c_4 \sin(\Delta M_s t). \quad (34)
$$

The direct CP violation in $B_s \to J/\psi \phi$ is negligible; i.e. $|\mathcal{A}_f/\mathcal{A}_\bar{f}| \approx 1$. Also, $|\Gamma_{12}/M_{12}| \ll 1$, so that in the absence of CP violation, $|\eta_1| = |\eta_2| = 1$. Then in terms of $\xi_f \equiv \xi_f = \xi_{f2} = a\mathcal{A}_f/\mathcal{A}_\bar{f}$, one can write

$$
\frac{c_4}{c_1} = \frac{\bar{c}_4}{\bar{c}_1} = \frac{2\text{Im}(\xi_f)}{1 + |\xi_f|^2} \approx -\eta_{CP} \sin(2\beta_s^{J/\psi \phi}), \quad (35)
$$

where $\eta_{CP}$ is the CP eigenvalue of $f_{CP}$.

When CPT is violated, the effective phases $\beta_s^{J/\psi \phi}$ and $\beta_s^{J/\psi \phi}$ measured through $B_s(t)$ and $\bar{B}_s(t)$ decays, respectively, will turn out to be different. Indeed, the difference between these effective phases will be a clean signal of CPT violation.

The semileptonic CP asymmetry $a_{sl}^s$ is measured through the “wrong-sign” lepton signal:

$$
a_{sl}^s = \frac{\Gamma(B_s(t) \to \mu^+ X) - \Gamma(B_s(t) \to \mu^- X)}{\Gamma(B_s(t) \to \mu^+ X) + \Gamma(B_s(t) \to \mu^- X)}. \quad (38)
$$

Here,

$$
\Gamma(B_s(t) \to \mu^- X) = |\eta f_- A(B_s(t) \to \mu^+ X)|^2, \quad (39)
$$

$$
\Gamma(B_s(t) \to \mu^+ X) = |(f_-/\eta_2) A(B_s(t) \to \mu^+ X)|^2, \quad (40)
$$

and since $|A(B_s \to \mu^+ X)| = |A(B_s \to \mu^+ X)|$, we have

$$
a_{sl}^s = \frac{1}{|\eta|^2} \frac{|\eta|^2}{|\eta|^2 + |\eta|^2} = \frac{1 - |\alpha|^4}{1 + |\alpha|^4}, \quad (41)
$$

which is independent of the CPT-violating parameter $\delta$. That the semileptonic asymmetry does not contain a CPT-violating term in the leading order was also noted earlier \cite{33}.

### VII. CPT Violation: The Statistical Analysis

In this Section, we perform a $\chi^2$-fit to the observables $\Delta M_s$, $\Delta \Gamma_s$, the effective phase $\beta_s^{J/\psi \phi}$, and $a_{sl}^s$. Let us first assume that there is no CP-conserving NP contribution coming from $M_{12}^{NP}$ and $\Gamma_{12}^{NP}$, so that the only relevant NP contribution is CPT violating, and is parameterized by $\text{Re}(\delta)$ and $\text{Im}(\delta)$. The allowed parameter space is shown in Fig. 5. It turns out that in this case, the value of $\chi^2_{\min}$ is $\approx 16.4$ (at $\delta = 0.008+0.958 i$ and $\delta = -0.024+0.958 i$), marginally better than the one obtained in the ($\Gamma_{12}^{NP} = 0, M_{12}^{NP} \neq 0$) case discussed above in Fig. 4. There are some, albeit small, regions in the parameter space that are allowed to $4\sigma$. However a fit good to $3\sigma$ or better is still not possible.

We therefore need to add the CPT-conserving NP to the CPT-violating contribution. However we have already seen in the preceding section that $M_{12}^{NP}$ and $\Gamma_{12}^{NP}$ together are capable of explaining the data by themselves. Therefore the fit using $\delta$, $M_{12}^{NP}$ as well as $\Gamma_{12}^{NP}$ is redundant. With six independent parameters and only four observables, not only is $\chi^2_{\min} = 0$ guaranteed, but no effective limits on CPT-conserving and CPT-violating parameters are generated.

We, therefore, go directly to the possibility where there is CPT-conserving NP, but without an absorptive part: $\Gamma_{12}^{NP} = 0$. We have already observed (Fig. 4) that the entire region in the $|M_{12}^{NP}| - \text{Arg}(M_{12}^{NP})$ is outside the $4\sigma$ region in such a scenario. We would now ask what happens if we enhance the two-parameter NP with two more CPT violating parameters, viz., $\text{Re}(\delta)$ and $\text{Im}(\delta)$. This scenario is interesting because, as we have seen before, only very specific kind of NP can contribute to $\Gamma_{12}^{NP}$, which would be tested severely in near future. In case no evidence for the relevant NP is found (e.g. the branching
ratio of $B_s \to \tau^+\tau^-$ is observed to be the same as its SM prediction), the next step would be to check if CPT violation, along with the NP contribution through $M_{12}^{NP}$, would be able to account for the anomalies. For example, one may want to determine $\beta_s$ and $\tilde{\beta}_s$ of Eqs. (36) and (37) separately and see whether they are different.

Fig. 5 shows the situation in the $|M_{12}^{NP}| - \text{Arg}(M_{12}^{NP})$ plane. As compared to Fig. 4, one can see that once we marginalize over $\delta$, we now have some regions allowed to within 4$\sigma$ (within the closed contour above and below the open contour), but none within 3$\sigma$. Indeed, $\chi^2_{min} = 14.3$ at $M_{12}^{NP} = 3.54 \exp(5.76\, i)$. This clearly does not improve the goodness-of-fit substantially, indicating that there is no good alternative for $\Gamma_{12}^{NP}$.

Fig. 6 shows the situation in the complex $\delta$ plane, when $M_{12}^{NP}$ has been marginalized over. The best-fit point corresponds to $\delta = -0.01 + 1.40\, i$, which gives $\chi^2_{min} = 14.3$ as mentioned earlier. The CPT conserving point ($\delta = 0$) lies outside the 4$\sigma$ region. As expected from the discussion in Sec. VII, the allowed values of $\delta$ are close to the Re($\delta$) or Im($\delta$) axis, with $|\text{Im}(\delta)|$ restricted to 2. One observes that the current data allows rather large ($\sim 1$) positive values of Im($\delta$) at 4$\sigma$.

**VIII. CONCLUSION**

Any flavor-dependent new physics model can in general affect both mass and width differences in the $B_s - \bar{B}_s$ system. It can also affect the CP-violating phase, as well as the dimuon asymmetry, which was found by the DØ collaboration to have an anomalously large value. With these four observables, one can constrain the free parameters of the new physics model. We have used the model independent approach where we consider the effective $B_s - \bar{B}_s$ mixing Hamiltonian $\mathcal{H}$ and parameterize the NP through its contribution to $\mathcal{H}$. We quantify the goodness-of-fit for the SM and NP parameter values by performing a combined $\chi^2$-fit to all the four measurements. The tension of the data with the SM is clear by the high value of $\chi^2$ at the SM. Moreover, it is observed that we need NP to contribute to the dispersive as well as absorptive part of the off-diagonal elements of $\mathcal{H}$ in order for the current data to be explained. The absorptive contribution, in particular, can be obtained from a very limited set of models, which will be severely tested in near future.

We also introduce the possibility of CPT violation by adding unequal NP contributions to the diagonal elements of $\mathcal{H}$. We explicitly show how CPT violation might...
affect the observables, especially dwelling on the effect on $\beta_s^{J/\psi\phi}$. Taken alone, the CPT violation cannot affect the dimuon asymmetry, and it can make the fit to the $B_s - \bar{B}_s$ mixing data only marginally better. In combination with a CPT conserving NP, it can enhance the allowed parameter space for that NP, however it does not seem to be able to obviate the need of an absorptive contribution from NP.

The data on all the observables considered in this paper is still relatively preliminary, the deviations from the SM are only at about 2-3$\sigma$ level, and future data may either confirm these deviations or expose them as statistical fluctuations. If the errors and uncertainties shrink keeping the central values more or less intact, this will mean:

- The SM is strongly disfavored. Moreover, the relevant NP should be flavor-dependent, as we do not see much deviation in the $B_d - \bar{B}_d$ sector.

- The NP models that do not contribute to the absorptive amplitude of the $B_s - \bar{B}_s$ mixing are also strongly disfavored if CPT is conserved. The best bets are those NP models that provide both dispersive and absorptive amplitudes in the $B_s - \bar{B}_s$ mixing. This also gives rise to new decay channels for $B_s$. For example, one might find the branching ratio of $B_s \rightarrow \tau^+\tau^-$ enhanced significantly from its SM expectation.

- Without any CPT-conserving NP, only CPT violation is only of marginal help, as it cannot enhance the semileptonic asymmetry. Even in combination with the CPT-conserving dispersive NP, it cannot allow regions in the parameter space to better than 3$\sigma$.

To summarize, the NP models that contribute an absorptive part to $B_s - \bar{B}_s$ mixing seem to be essential if one wants to explain the data on $\beta_s^{J/\psi\phi}$ and $A_s^{\beta}$ simultaneously. There is only a limited set of such models, and they will be severely tested in near future. In the scenario that such an absorptive NP contribution is ruled out, one may have to resort to CPT violation in order to explain the data. A prominent signature of such a CPT violation would be a difference in $\beta_s^{J/\psi\phi}$ and $\beta_s^{J/\psi}$ as shown in eqs. (36) and (37).

Acknowledgements

SKP acknowledges CSIR, Government of India, for a research fellowship. The work of AK was supported by CSIR, Government of India, and the DRS programme of the University Grants Commission.

[1] T. Aaltonen et al. (CDF Collaboration), CDF Note No. CDF/PHYS/BOTTOM/CDFR/9787, 2009; V. M. Abazov et al. (D0 Collaboration), D0 Note No. 5928-CONF, 2009.
[2] D. Asner et al. [Heavy Flavor Averaging Group], arXiv:1010.1589 [hep-ex].
[3] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. D 82, 032001 (2010) [arXiv:1005.2757 [hep-ex]]; Phys. Rev. Lett. 105, 081801 (2010) [arXiv:1007.0395 [hep-ex]]; for the 9.0 fb$^{-1}$ data, see V. M. Abazov et al. [D0 Collaboration], arXiv:1106.6308 [hep-ex].
[4] A. Dighe, A. Kundu and S. Nandi, Phys. Rev. D 76, 054005 (2007) [arXiv:0705.4547 [hep-ph]].
[5] A. Dighe, A. Kundu and S. Nandi, Phys. Rev. D 82, 031502 (2010) [arXiv:1005.4051 [hep-ph]].
[6] N. G. Deshpande, X. G. He and G. Valencia, Phys. Rev. D 82, 056013 (2010) [arXiv:1006.1682 [hep-ph]]; R. -M. Wang, Y. -G. Xu, M. -L. Liu, B. -Z. Li, JHEP 1012, 034 (2010) [arXiv:1007.2944 [hep-ex]].
[7] A. K. Alok, S. Baek and D. London, JHEP 1107, 111 (2011) [arXiv:1010.1333 [hep-ph]].
[8] B. A. Dobrescu, P. J. Fox, A. Martin, Phys. Rev. Lett. 105, 041801 (2010) [arXiv:1006.4238 [hep-ph]].
[9] B. Dutta, S. Khalil, Y. Mimura and Q. Shafi, arXiv:1104.5209 [hep-ph].
[10] D. Choudhury and D. K. Ghosh, JHEP 1102, 033 (2011) [arXiv:1006.2171 [hep-ph]].
[11] S. Nandi and A. Soni, Phys. Rev. D 83, 114510 (2011) [arXiv:1011.6001 [hep-ph]].
[12] B. Dutta, Y. Mimura, Y. Santoso, Phys. Rev. D 82, 055017 (2010) [arXiv:1007.3696 [hep-ph]]; J. K. Parry, Phys. Lett. B694, 363-366 (2011) [arXiv:1006.5331 [hep-ph]].
[13] M. Endo, S. Shirai and T. T. Yanagida, Prog. Theor. Phys. 125, 921 (2011) [arXiv:1009.3369 [hep-ph]].
[14] S. Oh, J. Tandean, Phys. Lett. B697, 41-47 (2011) [arXiv:1008.2153 [hep-ph]].
[15] C. W. Bauer, N. D. Dunn, Phys. Lett. B696, 362-366 (2011) [arXiv:1006.1620 [hep-ph]].
[16] K. Blum, Y. Hochberg, Y. Nir, JHEP 1009, 035 (2010) [arXiv:1007.1872 [hep-ph]].
[17] A. Datta, M. Duraisamy and S. Khalil, Phys. Rev. D 83, 094501 (2011) [arXiv:1011.5979 [hep-ph]].
[18] A. Kostelevsky, R. Van Kooten, Phys. Rev. D82, 101702 (2010) [arXiv:1007.3312 [hep-ph]].
[19] Y. Grossman, Phys. Lett. B 380 (1996) 99 [arXiv:hep-ph/9603244].
[20] Z. Ligeti, M. Papucci, G. Perez, J. Zupan, Phys. Rev. Lett. 105, 131601 (2010) [arXiv:1006.0432 [hep-ph]].
[21] Y. Grossman, Z. Ligeti and E. Nardi, Phys. Rev. D 55, 2768 (1997) [arXiv:hep-ph/9607473].
[22] A. Lenz and U. Nierste, arXiv:1102.4274 [hep-ph]; JHEP 0706 (2007) 072 [arXiv:hep-ph/0612167].
[23] CDF Report No. CDF/ANAL/BOTTOM/PUBLIC/10206, 2010; G. Giurgiu [CDF Collaboration], PoS(ICHEP 2010)236.
[24] T. Aaltonen et al., (CDF Collaboration), CDF Note No. 9015, 2007.
[25] A. Lenz, Nucl. Phys. Proc. Suppl. 177, 81 (2008) [arXiv:0705.3802 [hep-ph]].

[26] K. Trabelsi (Belle Collaboration), talk given at SEL11, TIFR, Mumbai, and available at http://www.tifr.res.in/~sel11.

[27] S. Coleman and S. Glashow, Phys. Rev. D59, 116008 (1999).

[28] O. W. Greenberg, Phys. Rev. Lett. 89, 231602 (2002).

[29] M. Chaichian, A. D. Dolgov, V. A. Novikov, A. Tureanu, Phys. Lett. B699, 177-180 (2011) [arXiv:1103.0168 [hep-th]].

[30] K. Nakamura et al. [Particle Data Group Collaboration], J. Phys. G37, 075021 (2010).

[31] V. A. Kostelecky, N. Russell, Rev. Mod. Phys. 83, 11 (2011) [arXiv:0801.0287 [hep-ph]].

[32] A. Datta, E. A. Paschos and L. P. Singh, Phys. Lett. B 548, 146 (2002); K. R. S. Balaji, W. Horn and E. A. Paschos, Phys. Rev. D 68, 076004 (2003).

[33] A. Kundu, S. Nandi and S. K. Patra, Phys. Rev. D 81, 054024 (2010) [arXiv:0910.5404 [hep-ph]].

[34] A. S. Dighe, I. Dunietz, H. J. Lipkin and J. L. Rosner, Phys. Lett. B 369, 144 (1996) [arXiv:hep-ph/9511363]; A. S. Dighe, I. Dunietz and R. Fleischer, Eur. Phys. J. C 6, 647 (1999) [arXiv:hep-ph/9804253].

[35] A. Pais and S. B. Treiman, Phys. Rev. D 12, 2744 (1975) [Erratum-ibid. D 16, 2390 (1977)]; S. Bar-Shalom, G. Eilam, M. Gronau, J. L. Rosner, Phys. Lett. B694, 374-379 (2011) [arXiv:1008.4351 [hep-ph]].