Latitudinal regionalization of rotating spherical shell convection

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Convection occurs ubiquitously on and in rotating geophysical and astrophysical bodies. Prior spherical shell studies have shown that the convection dynamics in polar regions can differ significantly from the lower latitude, equatorial dynamics. Yet most spherical shell convective scaling laws use globally-averaged quantities that erase latitudinal differences in the physics. Here we quantify those latitudinal differences by analyzing spherical shell simulations in terms of their regionalized convective heat transfer properties. This is done by measuring local Nusselt numbers in two specific, latitudinally separate, portions of the shell, the polar and the equatorial regions, $N_{up}$ and $N_{ue}$, respectively. In rotating spherical shells, convection first sets in outside the tangent cylinder such that equatorial heat transfer dominates at small and moderate supercriticalities. We show that the buoyancy forcing, parameterized by the Rayleigh number $Ra$, must exceed the critical equatorial forcing by a factor of $\approx 20$ to trigger polar convection within the tangent cylinder. Once triggered, $N_{up}$ increases with $Ra$ much faster than does $N_{ue}$. The equatorial and polar heat fluxes then tend to become comparable at sufficiently high $Ra$. Comparisons between the polar convection data and Cartesian numerical simulations reveal quantitative agreement between the two geometries in terms of heat transfer and averaged bulk temperature gradient. This agreement indicates that spherical shell rotating convection dynamics are accessible both through spherical simulations and via reduced investigatory pathways, be they theoretical, numerical or experimental.

Key words: Bénard convection, geostrophic turbulence, rotating flows

1. Introduction

It has long been known that spherical shell rotating convection significantly differs between the low latitudes (e.g., Busse & Cuong 1977; Gillet & Jones 2006) situated outside the axially-aligned cylinder that circumscribes the inner spherical shell boundary (the tangent cylinder, TC) and the higher latitude polar regions lying within the TC (e.g., Aurnou et al. 2003; Sreenivasan & Jones 2006; Aujogue et al. 2018; Cao et al. 2018). Further, in the atmosphere-ocean literature, latitudinal separation into polar, mid-latitude, extra-tropical and tropical zones is essential to accurately model the large-scale dynamics (e.g., Vallis 2017). Yet few scaling studies of spherical shell convection consider the innate regionalization of the dynamics (cf. Wang et al. 2021), and instead mostly focus on globally-averaged quantities (e.g., Gastine et al. 2016; Long et al. 2020).

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In the turbulent rapidly-rotating limit, theory requires the convective heat transport to be independent of the fluid diffusivities regardless of system geometry. This yields (e.g. Julien et al. 2012b; Plumley & Julien 2019)

\[ Nu \sim (Ra/Ra_c)^{3/2} \sim \tilde{Ra}^{-3/2} Pr^{-1/2} \sim Ra^{3/2} E^2 Pr^{-1/2} , \quad (1.1) \]

where, defined explicitly below, the Nusselt number \( Nu \) is the nondimensional heat transfer, \( Ra (Ra_c) \) denotes the (critical) Rayleigh number, \( E \) is the Ekman number, \( Pr \) is the Prandtl number, and \( \tilde{Ra} \equiv Ra E^{4/3} \) expresses the generalized convective supercriticality (Julien et al. 2012b).

Cylindrical laboratory experiments with \( Pr \approx 7 \) and Cartesian (planar) numerical simulations with \( Pr = (1, 7) \) and no-slip boundaries with \( Ra/Ra_c \lesssim 10 \) reveal a steep scaling \( Nu \sim (Ra/Ra_c)^{\beta} \) with \( \beta \approx 3 \) (King et al. 2012; Cheng et al. 2015, 2018). By comparing numerical models with stress-free and no-slip boundaries, Stellmach et al. (2014) showed that the steep \( \beta \approx 3 \) scaling is an Ekman pumping effect (cf. Julien et al. 2016). For larger supercriticalities, \( \beta \) decreases and gradually approaches (1.1). This \( \beta \approx 3 \) regime is expected to hold as long as the thermal boundary layers are in quasi-geostrophic balance, a condition approximated by \( Ra E^{8/5} \lesssim 1 \) (Julien et al. 2012a).

Globally-averaged quantities in spherical shell models present several differences with the planar configuration. In particular, no steep \( \beta \approx 3 \) exponent is observed. Gastine et al. (2016) showed that the globally-averaged heat transfer first follows a \( Nu \sim 1 \sim Ra/Ra_c \sim 1 \) weakly-nonlinear scaling for \( Ra \leq 6 Ra_c \) before transitioning to a scaling close to (1.1) for \( Ra > 6 Ra_c \) and \( Ra E^{8/5} < 0.4 \). Spherical shell models with a radius ratio \( r_i/r_o = 0.35 \) and fixed-flux thermal conditions recover similar global scaling behaviors, though with a slightly larger exponent \( \beta \approx 1.75 \) for \( E = 2 \times 10^{-6} \) (Long et al. 2020). Because the Ekman pumping enhancement of heat transfer is maximized when rotation and gravity are aligned, \( \beta \) is lower in the equatorial regions of spherical shells. This explains why globally-averaged spherical \( \beta \) values cannot attain the \( \beta \approx 3 \) values found in planar (polar-like) studies.

Recently, Wang et al. (2021) analysed heat transfer within the equatorial regions, at mid-latitudes, and inside the entire TC. They argued that the mid-latitude scaling in their models, similar to Gastine et al. (2016)’s global scaling, follows the diffusion-free scaling (1.1), whilst the region inside the TC follows a \( \beta \approx 2.1 \) trend. This TC scaling exponent is significantly smaller than those obtained in planar models, possibly because of the finite inclination angle between gravity and the rotation axis averaged over the volume of the TC.

Following Wang et al. (2021), this study aims to better characterize the latitudinal variations in rotating convection dynamics and quantify the differences between spherical and non-spherical geometries. To do so, we carry out local heat transfer analyses in the polar and equatorial regions over an ensemble of \( Pr = 1 \) rotating spherical shell simulations with \( r_i/r_o = 0.35 \) and \( r_i/r_o = 0.6 \).

2. Hydrodynamical model

We consider a volume of fluid bounded by two spherical surfaces of inner radius \( r_i \) and outer radius \( r_o \) rotating about the \( z \)-axis with a constant rotation rate \( \Omega \). Both boundaries are mechanically no-slip and are held at constant temperatures \( T_o = T(r_o) \) and \( T_i = T(r_i) \). We adopt a dimensionless formulation of the Navier-Stokes equations using the shell gap \( d = r_o - r_i \) as the reference lengthscale, the temperature contrast \( \Delta T = T_o - T_i \) as the temperature unit, and the inverse of the rotation rate \( \Omega^{-1} \) as the time
scale. Under the Boussineq approximation, this yields the following set of dimensionless equations for the velocity $\mathbf{u}$ and temperature $T$ expressed in spherical coordinates

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\mathbf{e}_z \times \mathbf{u} = -\nabla p + \frac{Ra E^2}{Pr} T g(r) \mathbf{e}_r + E \nabla^2 \mathbf{u}, \quad \nabla \cdot \mathbf{u} = 0, \quad (2.1)$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{E}{Pr} \nabla^2 T, \quad (2.2)$$

where $p$ corresponds to the non-hydrostatic pressure, $g$ to gravity and $\mathbf{e}_r$ ($\mathbf{e}_z$) denotes the unit vector in the radial (axial) direction. The above equations are governed by the dimensionless Rayleigh, Ekman and Prandtl numbers, respectively defined by

$$Ra = \frac{\alpha g_0 \Delta T d_0^3}{\nu \kappa}, \quad E = \frac{\nu}{\Omega d^2}, \quad Pr = \frac{\nu}{\kappa}, \quad (2.3)$$

where $\nu$ and $\kappa$ correspond to the constant kinematic viscosity and thermal diffusivity, and $\alpha$ is the thermal expansion coefficient. Two spherical shell configurations are employed: 

(i) a thin shell with $r_i/r_o = 0.6$ under the assumption of a centrally-condensed mass with $g = (r_o/r)^2$ (Gilman & Glatzmaier 1981); 

(ii) a self-gravitating thicker spherical shell model with $r_i/r_o = 0.35$ and $g = r/r_o$. The latter corresponds to the standard configuration employed in numerical models of Earth’s dynamo (e.g. Christensen & Aubert 2006; Schwaiger et al. 2019). We consider numerical simulations with $10^4 \leq Ra \leq 10^{11}$, $10^{-7} \leq E \leq 10^{-2}$ and $Pr = 1$ computed with the open source code MagIC† (Wicht 2002; Gastine & Wicht 2012). We mostly build the current study on existing numerical simulations from Gastine et al. (2016) and Schwaiger et al. (2021) and continue their time integration to gather additional diagnostics when required.

In the following analyses overbars denote time averages, triangular brackets denote azimuthal averages and square brackets denote averages about the angular sectors comprised between the colatitudes $\theta_0 - \alpha$ and $\theta_0 + \alpha$ in radians:

$$\bar{f} = \int_{t_0}^{t_0 + \tau} f dt, \quad \langle f \rangle = \frac{1}{2\pi} \int_0^{2\pi} f(r, \theta, \phi, t) d\phi, \quad [f]_{\theta_0}^\alpha = \frac{1}{S_{\theta_0}^\alpha} \int_{S_{\theta_0}^\alpha} f(r, \theta, \phi, t) dS,$$

with $dS = \sin \theta d\theta$ and $S_{\theta_0}^\alpha = \int_{\min(\theta_0 - \alpha, \pi)}^{\max(\theta_0 + \alpha, \pi)} \sin \theta d\theta$.

For the sake of clarity, we introduce the following notations to characterize the time-averaged radial distribution of temperature

$$\vartheta(r) = \langle \bar{T} \rangle^\pi_{-\pi/2}, \quad \vartheta_c(r) = \langle \bar{T} \rangle_{\pi/2}^{\pi/2}, \quad \vartheta_p(r) = \frac{1}{2} \left( [\langle \bar{T} \rangle]_{0}^{\pi/36} + [\langle \bar{T} \rangle]_{\pi/36} \right),$$

where $\vartheta_c$ and $\vartheta_p$ correspond to the averaged radial distribution of temperature in the equatorial and polar regions, respectively, and $\alpha = \pi/36$ rad corresponds to 5° in colatitudinal angle. The schematic shown in Fig. 1(a) highlights the fluid volumes involved in these measures. The value of $\alpha = 5^\circ$ is quite arbitrary and has been adopted to allow a comparison of polar data with local planar Rayleigh-Bénard convection (hereafter RBC) models while keeping a sufficient sampling.

To quantify the differences between the heat transfer in the polar and equatorial regions, we introduce a Nusselt number that depends on colatitude $\theta$ via

$$Nu_i(\theta) = \frac{d\bar{T}_i}{dr}|_{r_i}, \quad Nu_o(\theta) = \frac{d\bar{T}_o}{dr}|_{r_o}, \quad dT_c \frac{dr}{r} = -\frac{r_i r_o}{r^2}, \quad (2.4)$$

† https://github.com/magic-sph/magic
where $T_c$ corresponds to the dimensionless temperature of the conducting state. The corresponding local Nusselt numbers in the equatorial and polar regions are then defined by

$$Nu_e = \left[Nu(\theta)\right]^{\pi/36}_{\pi/2}, \quad Nu_p = \frac{1}{2} \left(\left[Nu(\theta)\right]^{\pi/36}_0 + \left[Nu(\theta)\right]^{\pi/36}_{\pi/2}\right). \quad (2.5)$$

We finally introduce the mid-shell time-averaged temperature gradient in the polar region

$$\partial T = \frac{-d\bar{\theta}}{dr}\bigg|_{r=r_m}, \quad r_m = \frac{1}{2}(r_i + r_o), \quad (2.6)$$

where normalisation by the conductive temperature gradient allows us to compare the scaling behaviour of $\partial T$ between spherical shells of different radius ratio values, $r_i/r_o$, and planar models.

3. Results

Figure 1(b) shows $Nu_p$ and $Nu_e$ as a function of $Ra$ for various $E$ at both boundaries, $r_i$ and $r_o$, for spherical shell simulations with $r_i/r_o = 0.6$ and $g = (r_o/r)^2$. Rotation delays the onset of convection such that the critical Rayleigh number required to trigger convective motions increases with decreasing Ekman number, $Ra_c \sim E^{-4/3}$. Convection first sets in outside the tangent cylinder (e.g. Dormy et al. 2004). For each Ekman number, heat transfer behaviour in the equatorial regions (red symbols) first raises slowly following a weakly nonlinear scaling (e.g. Gillet & Jones 2006), before gradually rising in the vicinity of $Nu_e \approx 2$. At $Nu_e \gtrsim 2$, the heat transfer increases more steeply with $Ra$, before gradually tapering off toward the non-rotating RBC trend (e.g. Gastine et al. 2015). For $Ra/Ra_c > O(10)$, convection sets in the polar regions and $Nu_p$ steeply rises with $Ra$ with a much larger exponent than $Nu_e$. At still larger forcings, the slope of $Nu_p$ gradually decreases and comparable amplitudes in polar and equatorial heat transfers are observed. Heat transfer scalings at both spherical shell boundaries $r_i$ and $r_o$ follow similar trends.
that the polar heat transfer onsets imparts a significant regionalization to spherical shell rotating convection right under highly supercritical conditions. This difference in equator versus polar convective onsets is that the lower latitude convection is already 20 times supercritical and is already operating. This means that rotating convection does not typically onset in the polar regions until its plane layer counterpart. However, it is not expected that the polar critical Rayleigh number will exactly agree with plane layer predictions, due to the effects of finite spherical curvature as well as the radial variations of gravity in these $r_i/r_o = 0.6$ simulations. In the rapidly-rotating thin shell limit, in which $r_i/r_o \to 1$ and $E$ is kept asymptotically small, $\tilde{Ra}_c$ will likely approach the planar value. Still, the polar scaling in (3.1) is found to be 51% of the plane layer $E \to 0$ scaling prediction, $Ra_c = 21.9 E^{-4/3}$ (Kunnen 2021), and to be 56% of Nuiler & Bishopp (1965)'s finite Ekman number, no-slip plane layer prediction at $E = 10^{-6}$. In addition to the similarity in critical $\tilde{Ra}$ values, it is found that the polar heat transfer $Nu_p$ rises sharply once polar convection onsets, following

\begin{equation}
\tilde{Ra}_c = 11.2 E^{-4/3}. \tag{3.1}
\end{equation}

Although the polar onset of convection, estimated via $\tilde{Ra}_c E^{4/3}$, remains nearly constant, the global (e.g., low latitude) onset value, estimated by $\tilde{Ra}_c E^{4/3}$, varies by a factor of $\approx 2$ over our $E$ range. Their ratio then yields

\begin{equation}
\frac{\tilde{Ra}_c(E)}{Ra_c(E)} = 20 \pm 5. \tag{3.2}
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This means that rotating convection does not typically onset in the polar regions until the lower latitude convection is already 20 times supercritical and is already operating under highly supercritical conditions. This difference in equator versus polar convective onsets imparts a significant regionalization to spherical shell rotating convection right from the get go.

We find, throughout this investigation, that polar rotating convection compares closely to its plane layer counterpart. However, it is not expected that the polar critical Rayleigh number will exactly agree with plane layer predictions, due to the effects of finite spherical curvature as well as the radial variations of gravity in these $r_i/r_o = 0.6$ simulations. In the rapidly-rotating thin shell limit, in which $r_i/r_o \to 1$ and $E$ is kept asymptotically small, $Ra_c$ will likely approach the planar value. Still, the polar scaling in (3.1) is found to be 51% of the plane layer $E \to 0$ scaling prediction, $Ra_c = 21.9 E^{-4/3}$ (Kunnen 2021), and to be 56% of Nuiler & Bishopp (1965)'s finite Ekman number, no-slip plane layer prediction at $E = 10^{-6}$. In addition to the similarity in critical $\tilde{Ra}$ values, it is found that the polar heat transfer $Nu_p$ rises sharply once polar convection onsets, following

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a $Nu_p \sim \tilde{Ra}^{-3}$ scaling that matches the heat transfer scalings found in no-slip planar simulations carried out over the same $(E, Pr)$ ranges (King et al. 2012; Stellmach et al. 2014; Aurnou et al. 2015).

Figure 2(b) shows the ratio of polar to equatorial heat transport, which follows a distinct v-shape trend that can be decomposed in three regions. (i) For $Ra > 11.2$, $Nu_p \approx 1$ and the ratio depends directly on $Nu_e = f(Ra)$. (ii) For $11.2 < Ra \lesssim 30$, $Nu_p$ rises much faster than $Nu_e$ hence increasing $Nu_p/Nu_e$. (iii) When rotational effects become less influential, $Nu_p/Nu_e \approx 1$ at $r_i$ and $Nu_p/Nu_e \approx 1.5$ at $r_o$.

Figure 3(a-b) shows the time-averaged temperature profiles in the polar and equatorial regions ($\theta_p$ dashed lines and $\theta_e$ dot-dashed lines) alongside the volume-averaged temperature ($\overline{\theta}$, solid line) for two numerical models with $r_i/r_o = 0.6$, $g = (r_o/r)^2$, $E = 10^{-6}$ and different $Ra$. For the case with $Ra \approx 14.1 Ra_c$ (panel a), low latitude convection is active but has yet to start within the TC. The mean temperature in the polar regions $\theta_p$ thus closely follows the conductive profile $T_c$ (dotted line), while in the equatorial region we observe the formation of a thin thermal boundary layer at $r_i$ and a decrease of the temperature gradient in the fluid bulk. At larger convective forcing ($Ra \approx 69.3 Ra_c$, panel b), convection is space-filling. The temperature profiles in the polar and equatorial regions become comparable and a larger fraction of the temperature contrast is accommodated in the thermal boundary layers.

Figure 3(c) shows the latitudinal variations of the heat flux at both spherical shell boundaries for increasing supercriticalities. These profiles confirm that convection first

\[ \frac{\theta_p}{\theta_e} \approx 1, \text{ while panel (a) corresponds to } r_i/r_o = 0.6, g = (r_o/r)^2, E = 10^{-6}, Ra = 6.5 \times 10^3, Pr = 1, \text{ while panel (b) corresponds to } r_i/r_o = 0.6, g = (r_o/r)^2, E = 10^{-6}, Ra = 3.2 \times 10^5 \text{ and } Pr = 1. (c) Time-averaged local Nusselt number at both spherical shell boundaries as a function of the colatitude for simulations with $r_i/r_o = 0.6, g = (r_o/r)^2, E = 10^{-6}, Pr = 1$ and increasing supercriticalities. Solid (dashed) lines correspond to $r_i$ ($r_o$). The vertical solid lines mark the location of the tangent cylinder. In all panels, the shaded regions correspond to one standard deviation about the time averages.
Importantly, this demonstrates that local, non-spherical models can be used to understand reveals an almost perfect overlap in heat transfer data between the two geometries. Im-

... attaining a relatively large minimum value, \( \partial T \approx 0.5 \) near \( Ra \approx 3 Ra_c^p \), before increasing slightly in the highest supercriticality planar models.

4. Discussion

Globally-averaged heat transfer scalings for rotating convection differ between spherical and planar geometries with the latter yielding steeper \( Nu-Ra \) scaling trends. By introducing regionalized measures of heat transfer, we have shown that this steep scaling can also be recovered in the polar regions of spherical shells. The comparisons in Fig. 4 reveals an almost perfect overlap in heat transfer data between the two geometries. Importantly, this demonstrates that local, non-spherical models can be used to understand...
spherical systems (e.g., Julien et al. 2012b; Horn & Shishkina 2015; Cabanes et al. 2017; Calkins 2018; Cheng et al. 2018; Miquel et al. 2018; Gastine 2019).

Our regional analysis shows that the use of global volume-averaged properties to interpret spherical shell rotating convection can be misleading since such averages are often made over regions with significantly differing convection dynamics (e.g., Ecke & Niemela 2014; Lu et al. 2021; Gramann et al. 2022). As such, it is quite likely that globally-averaged $\beta$ depends on the spherical shell radius ratio, $r_i/r_o$. In higher $r_i/r_o$ shells, more of the fluid will lie within the TC and the globally-averaged $\beta$ will tend towards a polar value near 3. In contrast, lower $r_i/r_o$ shells should trend towards regional $\beta$ values below 2, as found in our $Nu_p$ data. We hypothesize further that the mid latitude $\beta \approx 3/2$ scaling in (Wang et al. 2021) may represent a combination of the low and high latitude scalings, which could also be tested by varying $r_i/r_o$.

A similar argument may also explain Wang et al. (2021)’s higher latitude, tangent cylinder heat transfer scaling of $\beta = 2.1$. We postulate that measuring the rotating heat transfer away from the poles will always yield $\beta < 3$. This may be further exacerbated if the heat transfer is measured across the tangent cylinder, which likely acts as a radial transport barrier (e.g., Guervilly & Cardin 2017; Cao et al. 2018). Thus, Wang et al. (2021)’s $\beta \approx 2.1$ value may arise because their whole tangent cylinder measurements extend to far lower latitudes in comparison to the far tighter, pole-adjacent $Nu_p$ measurements made here that yield $\beta \approx 3$.

The polar heat transfer data in Figure 2 demonstrates a sharp convective onset value, with $Ra_0^p = (11.2 \pm 0.3)E^{-4/3}$ over our range of $r_i/r_o = 0.6$ models and $Ra_0^e/Ra_c = 20 \pm 5$. It is likely that convective turbulence is space-filling in planetary fluid layers. We argue then that realistic geophysical and astrophysical models of rotating convection require $Ra > Ra_0^p$. If the convection is rapidly-rotating as well, this constrains the convective Rossby number $Ro_{conv} = (RaE^2/Pr)^{1/2} \lesssim 0.1$ (e.g., Christensen & Aubert 2006; Aurnou et al. 2020). Thus, space-filling rotating convective turbulence simultaneously requires $Ra \gtrsim 10Ra_0^p$ and $Ro_{conv} \lesssim 1/10$, which then constrains that $E \lesssim 10^{-6}$ in $Pr \simeq 1$ models. Such dynamical constraints are important for building accurate models of $Nu(\theta)$, which are essential to our interpretations of planetary and astrophysical observations. For instance, on the icy satellites, latitudinal changes in ice shell thickness and surface terrain likely reflect the latitudinally-varying convective dynamics in the underlying oceans (e.g., Soderlund et al. 2020). We hypothesize that the broad array of $Nu_p/Nu_e$ solutions found in the models (e.g., Soderlund 2019; Amit et al. 2020; Bire et al. 2022) could possibly arise because convection is not active within the tangent cylinder in some of the models, and is not rapidly-rotating in others. Our results suggest that quantitative comparisons in heat flux profiles can only be made between models having similar latitudinal distributions of convective activity and comparable Rossby number values.

Establishing asymptotically-accurate trends for $Nu_p/Nu_e$ also requires accurate scaling laws for the equatorial heat transfer. A brief inspection of Fig. 2 reveals the complexity of $Nu_e(Ra)$, and its lack of any clear power law trend. To further complicate this task, zonal jets tend to develop in no-slip cases with $E \lesssim 10^{-6}$, which can substantively alter the patterns of convective heat flow. Figure 5 shows (a,b) axial vorticity $\omega_z = e_z \cdot \nabla \times \mathbf{u}$ snapshots and (c) latitudinal heat flux profiles for two $E < 10^{-6}$ simulations with different radius ratios. Convection in the (a) $r_i/r_o = 0.35$ case is sub-critical inside the TC, while it is space-filling in the (b) $r_i/r_o = 0.6$ simulation. In the latter case, polar convection develops as small-scale axially-aligned vortices which do not drive jets within the TC. In contrast, the convective motions outside the TC are already sufficiently turbulent in both cases to trigger the formation of zonal jets. These jet flows manifest via the formation of alternating, concentric rings of positive and negative axial vorticity. These
coherent zonal motions act to reduce the heat transfer efficiency in the regions of intense shear where the zonal velocities become of comparable amplitude to the convective flow (e.g. Aurnou et al. 2008; Yadav et al. 2016; Guervilly & Cardin 2017; Raynaud et al. 2018; Soderlund 2019). Thus, the outer boundary heat flux profile \( \Nu_o(\theta) \) in Fig. 5(c) adopts a strongly undulatory structure exterior to the TC. The asymptotic scaling behaviour of \( \Nu_o \) is hence intimately related to the spatial distribution and amplitude of the zonal jets that develop in the shell, a topic for future investigations of rotating convective turbulence (e.g. Lonner et al. 2022).

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