Study on Parameter Calculation of 1000MW Hydro-generator based on Field-circuit Coupled Finite Element Method

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Abstract. This paper mainly presents a field-circuit coupled finite element model (FEM) of a large hydro-generator for the study of generator parameters. In the model, the two-dimensional (2D) moving electromagnetic field finite element model is coupled with the external circuit model, and some influencing factors are taken into account, such as saturation of iron-core, rotation of rotor, distribution of stator windings and end rings of damper bars, as well as end-impedance of stator and rotor windings. Based on this model, the parameters of the 1000MW hydro-generator are calculated, and the steady-state, transient, and sub-transient reactances are obtained. Furthermore, the calculation results are compared with the designed values, and some meaningful conclusions are reached. Finally, the influences of different loads and power factors on saturated synchronous reactance are discussed.

1. Introduction

With the fast development of modern electric power industry, it has become the development trend to install large hydro-generator with unit capacity up to 1000MW. For large-capacity units, the calculation of generator parameters and operation performance is increasingly important. Moreover, the requirement for calculation accuracy of the generator electromagnetic parameters is also high.

The steady-state parameters of the synchronous generator directly affect the voltage regulation and the static stability of the generator, as well as the adjustment of active and reactive power during the parallel operation of the generator and power grid [1]. The transient parameters of the generator have important influence on the transient performance of the generator, such as three-phase sudden short-circuit process, asynchronous closing process and other transient stability [2]. Therefore, it is very important to calculate the steady-state and transient parameters of the generator accurately.

The traditional electric machine theory uses analytical method to calculate the operation performance and parameters of hydro-generator, it is difficult to consider the influence factors such as core saturation, rotor rotation and damper bar eddy current through some assumptions and simplified calculation model, so that the results contain certain limitations. With the emergence of computer technology and numerical calculation method, the operation performance and parameters of generator are mainly determined by calculating the magnetic field distribution recently [2-6]. The finite element numerical calculation method is based on electromagnetic field analysis, avoiding a large number of assumptions and empirical formulas, so the calculation is more accurate.
In this paper, a 1000MW hydro-generator is studied. The steady-state, transient and sub-transient reactance parameters of the hydro-generator are calculated by using the finite element numerical method and the established field-circuit coupled time-stepping finite element model. Finally, the influences of different loads and power factors on saturated synchronous reactance are discussed.

2. Field-circuit coupled time-stepping finite element model

2.1. Boundary value problem of electromagnetic field

The rated data of the 1000MW hydro-generator studied in this paper are shown in Table 1.

| $P_N$/MW | $2P \cos \varphi_N$ | $U_N$/V | $I_N$/A | $q$ | $Y_1$ |
|----------|------------------|--------|--------|-----|-------|
| 1000     | 56               | 0.9    | 26000  | 24670.6 | 4     | 14    |

The following basic assumptions are made for the established mathematical model of hydro-generator:

1. The axial length of the generator is infinite, and the magnetic field distribution remains unchanged along the axis. That is the armature part is treated as a 2D magnetic field.

2. The stator slots as well as the corner and chamfer at the junction between the pole shoe and the pole body are approximated.

3. The outer rim stray fluxes of the stator and rotor core are ignored.

Since the 1000MW hydro-generator adopts integral-slot winding, and the number of slots per pole per phase $q$ is 4, so a pair of poles is taken as the solving region. And the solving region is shown in figure 1 below.

![Figure 1: The 1000MW hydro-generator solving region.](image)

As shown in figure 1, the magnetic potential values meet the first boundary conditions along the outer boundary AB and the inner boundary CD; while along the symmetric boundaries AD and BC, the magnetic potential values confirm to the integer periodic boundary conditions. The complex vector boundary value problem of electromagnetic field in the generator is described as equation (1):

$$\begin{align*}
\left[ \begin{array}{c}
\frac{\partial}{\partial x} \left( \frac{1}{\mu(x, y)} \frac{\partial \hat{A}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu(x, y)} \frac{\partial \hat{A}}{\partial y} \right) \\
\hat{A}_{|_{\text{avg}}} = 0, \hat{A}_{|_{\infty}} = \hat{A}_{|_{BC}}
\end{array} \right] = -\hat{J}_s + j\omega \sigma \hat{A}
\end{align*}$$

(1)

Where, $\hat{A}$ is the magnetic vector potential; $\hat{J}_s$ and $\varphi$ are, respectively, the source current density and angular frequency; $\mu(x, y)$ is the effective magnetic permeability and $\sigma$ is the conductivity.

2.2. Field-circuit coupled finite element model

In this paper, the field-circuit coupled time-stepping FEM of the unit generator is established, as shown in figure 2, which is realized by the coupling of the 2D electromagnetic field FEM and the external circuits. This model consists of two parts (a) and (b). (a) representing the 2D electromagnetic field finite element region, as shown in figure 1; (b) representing the external circuit model established
by the circuit elements, including the end-impedance of stator and rotor windings, impedance of
damper bars end rings, as well as stator three-phase power supply and excitation power supply, etc.

**Figure 2.** The field-circuit coupled finite element model of the unit hydro-generator.

Considering the integer periodic boundary conditions, a 2D finite element model of generator under
a pair of poles is firstly established, and then coupled with the external circuit model, and finally a
field-circuit coupled time-stepping finite element model of unit generator is established, which greatly
reduces the number of solving elements and nodes. Consequently, the computer memory requirements
are reduced and the calculation time is shortened. In the model, the periodic boundary conditions are
used to couple the relative motion boundary of the generator stator and rotor to realize the free rotation
of the rotor [7]. The actual connection of the stator three-phase windings is considered in establishing
the external circuit model. By adjusting the external impedance at the stator and rotor ends and the
value of power supply, it is convenient to simulate the different working conditions of generator, such
as no-load and short-circuit. With the above model, the steady, transient and sub-transient
electromagnetic field distribution of the generator can be obtained, and then to calculate the reactance
parameters of the generator under different working conditions.

3. Calculation method of hydro-generator synchronous reactance

3.1. Calculation method of steady-state reactance

In order to solve the saturation value of synchronous reactance under a certain working condition, the
2D non-linear constant magnetic field in the generator should be solved first. And the distribution of
field current, as well as the magnetic reluctance in each element of the iron core should be determined
by iteration method. Then, keep the magnetic reluctance constant, set the field current to zero, and add
the pure d-axis and pure q-axis magnetic potential to the stator in turn, so as to calculate the d-axis and
the q-axis synchronous reactance. In this paper, taking into account the stator resistance [6], some
certain parameters of the unit generator under rated load are obtained through iterative calculation,
such as the terminal voltage is 3753.68V, the field current is 3122.39A and the power factor angle is
26.26°. Moreover, the relative errors of terminal voltage and power factor angle are calculated to be
0.0002 and 0.0036 respectively, which meet the operation requirements of rated load.

Keep the rotor stationary, and add the positive sequence three-phase currents to the stator three-
phase windings, as shown in the following equation (2):

\[
\begin{align*}
I_a &= I_n \cos(\theta) \\
I_b &= I_n \cos(\theta - 120^\circ) \\
I_c &= I_n \cos(\theta - 240^\circ)
\end{align*}
\]
Where: $I_m$ is the current amplitude, and $\theta$ is the angle between the axis of the phase A winding and the center line of the magnetic pole. When $\theta$ is $0^\circ$, a d-axis magnetic potential is established on the stator side; when $\theta$ is $90^\circ$, a q-axis magnetic potential is established on the stator side.

Taking the calculation of phase A synchronous reactance $x_d$ as an example, the vector magnetic potential of each element node along the cross section of the generator is obtained by calculating the 2D non-linear constant magnetic field. Then the phase A flux produced by the unit stator current is calculated, and the end-leakage reactance of the stator is taken into account. Finally, the per unit value of the d-axis synchronous reactance $x_d$ can be obtained [6], as shown below.

$$x_d = 2\pi f \beta \psi_\beta \frac{1}{a} \frac{I_{m\alpha}}{U_{\phi N}} + x_{s\alpha e}$$

Where: $\beta$ is the number of phase belts in series per phase; $\psi_\beta$ is the flux linkage in one phase belt of phase A; $I_{m\alpha}$ is the branch current amplitude of phase A; $a$ is the number of parallel branches; $U_{\phi N}$ and $I_{m\alpha}$ are, respectively, the rated phase voltage and phase current; $x_{s\alpha e}$ is the stator end-leakage reactance.

$$\psi_\beta = \sum_{k=1}^{n} (A_{uk} - A_{kk}) \psi_{\beta k}$$

Where: $A_{uk}$ and $A_{kk}$ are respectively, the vector magnetic potential at the center of the upper and lower current-carrying conductors of the $k$th coil in phase A belt; $l_{s\alpha e}$ is the effective length of stator iron core.

Using the same method, phase B and C are calculated, and the d-axis reactance is obtained separately, and finally the average value of the three phases is taken as $x_d$. Assuring that other conditions remain unchanged, the q-axis synchronous reactance can be obtained by adding the q-axis current to the stator winding. When the magnetic reluctivity of the stator and rotor core is assigned to the value that the core is unsaturated, the result is the synchronous reactance unsaturated value.

### 3.2. Calculation method of transient and sub-transient reactance

Transient and sub-transient reactances are obtained by calculating the frequency characteristics of the operational reactance of the d-axis and the q-axis [2]. An alternating current with a high slip frequency ($s=10^\circ$) is applied to the stator three-phase winding, as shown in equation (5).

$$\begin{align*}
i_n &= I_m \cos(s\alpha) \\
i_q &= I_m \cos(s\alpha - 120^\circ) \\
i_c &= I_m \cos(s\alpha - 240^\circ)
\end{align*}$$

Keep the rotor stationary, and let the d-axis of the rotor coincide with the axis of phase A winding, so that the stator side forms a pure d-axis pulsating magnetic potential. Meanwhile, the field winding is short-circuited and the damper winding is open. Under this condition, the vector magnetic potential of each point is obtained by solving equation (1). After that, the flux linkage of each phase of the stator and the corresponding reactance $x_{d'r}$ are calculated, and the end-leakage reactance $x_{s\alpha e}$ of the stator is added to obtain the d-axis transient reactance $x_d'$ of phase A. Finally, the average value of the three phases A, B, and C is taken as the d-axis transient reactance $x_d'$. While the filed winding and the damper winding are short-circuited, and other conditions remain unchanged, the calculated reactance is d-axis sub-transient reactance $x_d''$. Similarly, let the angle between the d-axis of the rotor and the axis of the phase A winding be $90^\circ$ to form a pure q-axis pulsating magnetic potential on the stator side, the obtained reactance is q-axis sub-transient reactance $x_q'''$.

### 4. Calculation and results of electromagnetic parameters of hydro-generator

#### 4.1. Calculation results of d-axis and q-axis armature reaction magnetic field in steady state

Rotate the rotor until the d-axis of the rotor coincides with the axis of phase A winding, and keep the rotor stationary; let the damper and field windings open, while in actual processing, the infinite resistance are given to the damper and field windings in the model, and then add pure d-axis and pure q-axis currents to the three-phase stator windings in turn. The magnetic field distribution of the d-axis
armature reaction obtained by the finite element method is shown in figure 3. And figure 4 shows the magnetic field distribution of the q-axis armature reaction.

**Figure 3.** Field distribution of the d-axis armature reaction in steady state (unsaturated).

**Figure 4.** Field distribution of the q-axis armature reaction in steady state (unsaturated).

### 4.2. Calculation results of the d-axis armature reaction magnetic field in transient state

It can be known from the electric machine theory that when the damping effect is ignored, the field winding is equivalent to the superconductor region at the moment of the three-phase symmetrical sudden short circuit, and the armature magnetic flux of the d-axis armature reaction cannot pass through the field winding. In actual processing, the region is given a large electrical conductivity ($\rho = 1000\rho_{cu}$)[5]. When solving the d-axis transient reactance, keep the rotor stationary, and the d-axis current is applied to the stator while the field winding is short-circuited and the damper winding is open. Figure 5 shows the magnetic field distribution of the d-axis armature reaction in transient state.

**Figure 5.** Field distribution of the d-axis armature reaction in transient state (unsaturated).

### 4.3. Calculation results of d-axis and q-axis armature reaction magnetic field in sub-transient state

When considering the damping effect, the field and damper windings are equivalent to the superconductor region at the moment of three-phase symmetrical sudden short circuit, and the armature magnetic flux of the d-axis armature reaction cannot pass through the field and damper winding. When solving the d-axis sub-transient reactance, keep the rotor stationary, the d-axis current is applied to the stator, and the field winding and damper winding are simultaneously short-circuited.

**Figure 6.** Field distribution of the d-axis armature reaction in sub-transient state (unsaturated).

**Figure 7.** Field distribution of the q-axis armature reaction in sub-transient state (unsaturated).
Similarly, the sub-transient magnetic field of the q-axis is that the armature flux of q-axis armature reaction that cannot pass through the field winding and the damper winding. And the precondition of solving the q-axis sub-transient reactance is the same as solving the d-axis sub-transient reactance, except that the q-axis current is applied to the stator. The unsaturated field distributions of the d-axis and the q-axis armature reaction in sub-transient state are shown in figure 6 and figure 7, respectively.

4.4. Calculation results of synchronous reactance
According to the above calculation results of the magnetic field in the generator solved by ANSYS simulation software, the unsaturated values of synchronous reactance, saturated values under rated load, as well as unsaturated values of transient and sub-transient reactance of the 1000MW hydro-generator are obtained, as shown in table 2 below.

Table 2. The calculation results of the 1000MW hydro-generator

| Parameter | Calculation results (p.u.) | Designed values (p.u.) |
|-----------|----------------------------|------------------------|
|           | Unsaturated | Saturated | Unsaturated | Saturated |
| $x_d$     | 1.177       | 1.079      | 1.118       | 0.987     |
| $x_d'$    | 0.399       | —          | 0.317       | —         |
| $x_d''$   | 0.266       | —          | 0.220       | —         |
| $x_q$     | 0.845       | 0.739      | 0.792       | 0.745     |
| $x_q'$    | 0.272       | —          | 0.236       | —         |

5. Analysis of influencing factors on saturated synchronous reactance

5.1. Synchronous reactance under different loads
Considering that the hydro-generator will run under different loads during actual operation, and in order to study the influence of load magnitude on saturated synchronous reactance, this paper calculates the saturated synchronous reactance $x_d$ and $x_q$ under different loads with the rated terminal voltage and rated power factor. Figure 8 shows the saturated synchronous reactance value curve under different loads. In the figure, $I_n$ is the rated load and $I/I_n$ is the rated load multiple. It can be seen that as the load increases, the saturated synchronous reactances $x_d$ and $x_q$ gradually decrease.

![Figure 8](image)
(a). D-axis saturated reactance $x_d$.  
(b). Q-axis saturated reactance $x_q$.

5.2. Synchronous reactance with different power factors
Due to the diversity of power loads in modern power systems, there are some changes in the power factor [8]. Therefore, this paper analyzed the changing trend of synchronous reactance under rated current and rated voltage. Figure 9 shows the changing trend of saturated synchronous reactance $x_d$ and $x_q$ with different power factors. It can be seen that with the increase of power factor, the d-axis saturated synchronous reactance $x_d$ decreases gradually, which is the same as the reactance changing
trend under different loads; on the contrary, the q-axis saturation synchronous reactance $x_q$ is increasing, which is opposite to the reactance changing trend under different loads.

Figure 9. The saturated reactance of different power factors.

6. Conclusion
In this paper, the established field-circuit coupled finite element model is used to calculate the synchronous reactance of 1000MW hydro-generator, including steady-state saturated synchronous reactance, unsaturated synchronous reactance and the unsaturated values of transient and sub-transient reactance. And the calculated results are compared with the traditional designed values. The comparison results show that the d-axis synchronous reactance $x_d$ saturation value calculated by the finite element method is slightly higher, and the q-axis synchronous reactance $x_q$ saturation value is slightly lower. It shows that the results obtained by the field-circuit coupled time-stepping finite element model established in this paper can meet the engineering accuracy requirements.

Finally, the saturation synchronous reactances of 1000MW hydro-generator with different loads and different power factors are analyzed. The d-axis saturated synchronous reactance decreases as the load and power factor increase. The q-axis saturation synchronous reactance gradually decreases as the load increases, and gradually increases as the power factor increases.

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