Diffusive equilibration of $N_2$, $O_2$ and $CO_2$ mixing ratios in a 1.5 million years old ice core

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Supplement: Model Sensitivity Tests
Here we show three different model sensitivity tests for the Oldest Ice Core simulations. The first two tests regard uncertainties in two ice parameter, since theses values (Table 1) are not well constrained: the tuning parameter $p$ and the bedrock temperature/geothermal heat flux ($Q_g$). The third test regards the very low total air permeation coefficients of Uchida et al. (2011).

For the Oldest Ice Core simulations shown in section 4 we used a theoretical value for the tuning parameter $p$ (vertical ice flow tuning parameter) calculated according to Parrenin et al. (2007b).
This theoretical value, however, is not applicable for the EDC and the Dome Fuji ice cores (Parrenin et al., 2007b). Realistic $p$ values for these ice cores are in the range of 1-4, which is significantly lower than the theoretical ones (see section 2.2 for more details). In order to test the sensitivity of our results in this regard, we have performed an Oldest Ice Core simulation comparable to the one shown in Fig. 6, but using our $p$ value for the EDC ice core simulations (red lines in Fig. A1). Since the change of the $p$ value results also in different age-depth and temperature-depth distributions, we also adjusted the geothermal heat flux $Q_g$ in addition to the $p$ value in order to keep the ice temperature of the 1.5 Myr old ice (130 m above bedrock) equal to the standard simulation. In this way the difference between the results for the 1.5 Myr old ice of the standard and the $p$-test simulation is a measure for the influence of the ice flow tuning parameter $p$. Due to the weaker thinning of the deep ice in the $p$-test simulation, the result for $O_2/N_2$ in 1.5 Myr old ice shows a 15% weaker dampening compared to the standard run, with a total amplitude dampening of 65%. With respect to the large uncertainties in the gas parameters, the uncertainty in the $p$ value is not critical.
For the bedrock temperature sensitivity test, we have used the standard Oldest Ice Core parameters of Table 1 with the only difference of a geothermal heat flux $Q_g$ of 50 mW/m$^2$ (green lines in Fig. A1). This results in a bedrock ice temperature of about 260 K, roughly 5 K lower than in the standard simulations. The influence of the lower temperature is comparable to the influence of the lower $p$ value and is therefore also not critical for the findings in this work. A lower bedrock temperature is also likely to be found at a location where the ice sheet is significantly thinner than assumed in our Oldest Ice Core simulations. But for a given accumulation rate a thinner ice sheet also implies thinner annual layers on average, with consequently higher diffusion rates. Hence, the gain of lower bedrock temperatures at a place with a thinner ice sheet would to some extent be compensated by thinner annual layers.

For temperatures near the bedrock, where the majority of diffusive dampening takes place, the estimate of total air permeation in ice of Uchida et al. (2011) suggests lower permeabilities than the SS parameters (see Fig. 2). For temperatures below the BCTZ the same estimate suggests higher permeabilities similar to the SS parameters, which is not supported by the results in section 3. Nonetheless, we tested the influence of such low permeabilities. Since the estimate of Uchida et al. (2011) does only give total air permeabilities and not for the single components, we mimicked such low permeabilities by scaling down the SS and FS parameters, respectively, with a constant factor such that they reached the total air permeability of Uchida et al. (2011) at 265 K (bedrock temperature). This approach results also in lower total air permeability compared to Uchida et al. (2011) throughout the ice core with the exception of the very bottom since we kept the temperature sensitivities of the original sets equal. Therefore the results of the corresponding two simulations are actually representing even lower total air permeabilities than suggested by Uchida et al. (2011). The two simulations (one for downscaled SS and one for downscale FS parameters) show very similar results for which reason only the FS result is shown in Fig. A1. The original O$_2$/N$_2$ amplitude is damped by about 50% after 1.5 Myr (30% less as for the standard run) suggesting that also with lower air permeabilities in the range of Uchida et al. (2011) the precessional O$_2$/N$_2$ signal is significantly dampened in this case. However, the precessional signal would not be lost and, hence, orbital tuning of the ice age scale using O$_2$/N$_2$ would still be possible.
Figure A1: Simulated amplitude dampening of CO\textsubscript{2} concentration (dashed lines) and O\textsubscript{2}/N\textsubscript{2} ratio (solid lines) signals with a 20 kyr period. The yellow lines represent the results of the standard Oldest Ice Core parameters set of Table 1 and the SS parameters (also shown in Fig. 8). The magenta line (Uchida-Test) shows the results using the same ice parameters, but downscaled FS parameters (see text). The other coloured lines represent results using the SS parameters as in the standard run but with different sets of Oldest Ice Core parameters: red: p = 3.8, Q\textsubscript{g} = 56 mW/m\textsuperscript{2}; green: Q\textsubscript{g} = 50 mW/m\textsuperscript{2}.