Scalar mesons and Adler zeros

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Abstract

A simple unitarized quark-meson model, recently applied with success to light and charmed scalar mesons, is shown to encompass Adler-type zeros in the amplitude, due to the use of relativistic kinematics in the scattering sector. These zeros turn out to be crucial for the description of the $K^*_0(800)$ resonance, as well as the new charmed scalar mesons $D^*_s(2317)$ and $D^*_0(2300)$.

The light scalar mesons are still causing lots of headaches to many theorists as well as experimentalists, despite the growing, yet far from general, consensus that unitarization in some form should represent the necessary setting. For the purpose of the present short note, we limit ourselves to refer to a recent brief review [1], which, albeit voicing mainly our personal viewpoints, contains many references to work in this field.

An important contribution to unraveling the conundrum of the light scalar mesons from a data-analysis perspective was recently made by D. V. Bugg [2], in successfully describing elastic $\pi\pi$ and $K\pi$ scattering as well as the corresponding $\sigma$ ($f_0(600)$) and $\kappa$ ($K^*_0(800)$) resonances. The clue to this achievement was the inclusion of the respective Adler zeros for these processes directly into the energy-dependent widths contained in the relativistic Breit–Wigner amplitudes used in the phase-shift fits.

Such $T$-matrix zeros below threshold were determined by S. Weinberg [3] for elastic $\pi\pi$ scattering, elaborating upon consistency conditions among strong-interaction amplitudes derived by S. L. Adler [4] for $\pi\pi$, $\pi N$, and $\pi\Lambda$ processes, on the basis of PCAC. In the case of elastic $\pi\pi$ and $K\pi$ scattering, the Adler zeros lie at $s_{\pi\pi}^A \approx m_{\pi}^2/2$ and $s_{K\pi}^A \approx m_K^2 - m_{\pi}^2/2$, respectively, which were the values used by D. V. Bugg.

On the other hand, as early as in 1986 two of us co-authored a paper [5] in which, for the first time, a complete light nonet of scalar-meson resonances [6] was predicted with a simultaneous reasonable description of the corresponding elastic $S$-wave meson-meson phase shifts. Although the latter coupled-channel model results were obtained without any fit in the scalar sector, its predictions for the resonance pole positions lied very close to the present-day
world averages. Nevertheless, no attempt was made in this work to account for dynamical Adler zeros from PCAC or (approximate) chiral symmetry, besides the use of a physical input pion mass. Moreover, in a more recent paper [7], a simplified yet less model-dependent version of the mentioned coupled-channel approach was employed to fit the S-wave $K\pi$ phase shifts up to 1.6 GeV, thereby extracting both the established $K^*_0(1430)$ and the now also listed $K^*_0(800)$ resonances \[8\]. Finally, we used the very same modified Breit–Wigner formula, with adjusted quark and meson masses, to successfully describe \[9\] the couple of just discovered charmed scalar mesons consisting of the very narrow $D^*_{s0}(2317)$ \[10\] and the broad $D^*_{s0}(2300)$ \[11\].

In view of these surprising results, we analyze here in more detail the behavior of the latter model amplitude below threshold and in the complex-momentum plane (see also Ref. \[12\]).

The corresponding $K^{-1}$ matrix is, for the $1\times1$ scalar case, simply given by

$$\cotg(\delta(s)) = \frac{n_0(pa)}{j_0(pa)} - \left\{2\lambda^2\mu(s)pa j_0^2(pa) \left[\frac{1.0}{\sqrt{s} - E_1} + \frac{0.2}{\sqrt{s} - E_2} - 1\right]\right\}^{-1}, \quad (1)$$

where $j_0$ and $n_0$ are spherical Bessel and Neumann functions, respectively, $E_1$ and $E_2$ are the energies of the lowest bare $J^{PC} = 0^{++} q\bar{q}$ states, $\lambda$ is the coupling for $3P_0$ quark-pair creation, $a$ is the corresponding interaction radius, $p$ is the relativistic relative momentum of the two-meson system, and $\mu(s)$ is the associated relativistic reduced mass

$$\mu(s) \equiv \frac{1}{2}\frac{dp^2}{d\sqrt{s}} = \frac{\sqrt{s}}{4} \left[1 - \left(\frac{m_1^2 - m_2^2}{s}\right)^2\right]. \quad (2)$$

We immediately see \[12\] from the latter expression that $\mu(s)$ vanishes at $s = \pm(m_1^2 - m_2^2)$. Therefore, at this point below threshold, the factor with $\lambda$ in Eq. \[1\] is squashed to zero, so that the amplitude also vanishes. This property was shared by the model of Ref. \[5\].

Nice illustrations of the effect of these kinematical Adler-type zeros are the $S$-matrix pole trajectories in the complex-$p$ plane for S-wave $DK$ and $D\pi$ scattering (see the Figure). Note that the parametrization here is slightly different from the one in Ref. \[9\], as we now scale $\lambda$ and $a$ with the reduced mass of the $q\bar{q}$ system, so as to guarantee rigorous flavor invariance (see Refs. \[1\], \[12\] for details). The result is that, in the $DK$ case, the two pole trajectories interchange their roles, so that now it is the bare state which gives rise to the $D^*_{s0}(2317)$ instead of the continuum state. However, despite the dramatic change in the trajectories themselves, their end points, corresponding to the value 0.75 (GeV$^{-3/2}$) of the universal coupling $\lambda_0$, only suffer a modest shift, which even produces an improved value for the $D^*_{s0}(2317)$ mass, viz. 2.327 GeV. In the $D\pi$ case, the trajectories remain qualitatively unaltered, but the predicted $D^*_0(2300)$ pole position also improves, i.e., to $2.114 - i 0.118$ GeV. However, the crucial message from the Figure appears to be the large effect of the Adler-type zeros, indicated by the open circles. Namely, while in the $DK$ case the relatively distant zeros, located at $p_A = \pm im_K$, allow the lower pole to travel all the way to the upper imaginary-$p$ axis, for $D\pi$ scattering the nearby zeros at $p_A = \pm im_\pi$ slow down this pole so as to settle above threshold.
FIGURE. Pole trajectories as a function of coupling $\lambda_0$, and kinematical Adler-type zeros ($\circ$), in complex-momentum plane (GeV units), for $S$-wave $DK$ and $D\pi$ scattering.

Note that these kinematical zeros deviate less than 1\% from the theoretical Adler zeros. Thus, we get a (quasi-)bound state in the former case, and a quite broad resonance in the latter, similarly to what happens with the $\kappa$ meson (see Ref. \[12\] for the $\kappa$-pole trajectory).

As a final test, we check what happens to the $\kappa$ by removing the Adler-type zero. So we substitute the relativistic reduced mass $[2]$ by a nonrelativistic one, and then refit $\lambda$, $a$ to the $K\pi$ phase shifts. The result is a somewhat unphysical $\kappa$ pole at $E = 483 - i 274$ MeV. Defining also the relative momentum nonrelativistically then completely kills the $\kappa$, while the $K^*_0(1430)$ still survives. This may provide a clue to the absence of a light $\kappa$ in the analysis of Ref. \[13\], in which a distant, negative Adler zero at $s_A = -0.41$ GeV$^2$ was used, in contrast with e.g. the unitarized chiral approaches of Ref. \[14\].

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