Research on aircraft taxiing path optimization based on
digraph model and Dijkstra algorithm

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Abstract: in this paper, the ground structure diagram of an Airport is taken as an example, the specific flight information is substituted, and then the directed graph model is established, and various factors are transformed into weight offspring into the directed graph, and a multi factor constrained aircraft taxiing path optimization scheme is obtained. Through the specific analysis of the influence of each factor on the selection of aircraft taxiing path and the optimization scheme of taxiing path with the corresponding weight of various factors, the Dijkstra algorithm is used to calculate the specific case, and the specific optimal path of aircraft ground taxiing is obtained. The results show that the directed graph model and the optimal path of aircraft taxiing are reasonable, which can provide some reference for the controller's command.

Key word: The Dijkstra algorithm, taxiing path, directed graph model, optimization scheme

1. The establishment of digraph model and case analysis

1.1 Optimization of taxiing path under multi factor constraints
In the process of aircraft operation, there are often a variety of unexpected problems and situations, such as aircraft failure, traffic conflicts, runway replacement, etc., which lead to the aircraft must change the taxiing path, which requires the dynamic of the algorithm. On the other hand, there is a time limit for the controller to command the aircraft to taxi. In some emergencies, the controller needs to respond in time and change the taxi path for the aircraft at the same time, so the optimization algorithm of the taxi path should also reflect the real-time changes. Therefore, the optimization of the aircraft taxi path is the final process constrained by multiple factors.

The allocation of Aircraft Taxiway is a dynamic process. As there are aircraft entering and leaving the airport continuously, when assigning a taxiway to an aircraft, a taxiway with the lowest cost and multi factor constraint should be obtained by step-by-step promotion based on the existing situation of airport taxiway use.

1.2 Building digraph model
Using the method of minimum cost flow, the taxiing route is modeled as a directed graph model, and the initial route has no cost. When substituting flight information, under the influence of many
factors, the path cost will change accordingly. By summing the cost value of the path, we can get a path with the lowest cost. For the establishment of the directed graph model, the mesh structure of all the taxiing paths of an aircraft from the stand to the moving intersection point can be equivalent to the aircraft taxiing path optimization model, that is, the directed graph model. The directions of the directed graph are respectively from the transfer point to the stand for the incoming aircraft and from the stand to the moving intersection point for the departing aircraft.

The flight information is substituted into the static digraph. According to the data of the starting point and the ending point, the static digraph becomes a dynamic process, and the taxiing path will produce the corresponding cost.

The influence of airport operation mode on the optimization of taxiway is not single, but a comprehensive and multi-factor influence mode. The influence degree of each factor is also different. It is assumed that the weight range of the influence of each factor on a taxiway is (0,100), and the sum of the weights of each taxiway is 199 at most.

1.3 Optimization analysis of multi-factor sliding path based on weight

The Airport, 18 kilometers southeast of the airport City, with a runway of 3600 meters long and 45 meters wide, with true azimuth angles of 90° and 270°, and a parallel taxiway of 3600 meters long and 23 meters wide; The Airport has a total of 33 stands, including 17 near stands and 16 far stands. Considering the single runway operation on a certain day, the aircraft entering the site will use runway 09 to land according to the wind direction, and the aircraft leaving the site will use runway 27 to take off. According to the operation specifications of the Airport, medium-sized aircraft depart from the runway from the fast lane R5, and large aircraft depart from the runway from the fast lane R6. The ground structure of the airport is as follows:

![Figure 1. Airport ground traffic network.](image)

Table 1. flight schedule of the Airport one day.

| Call sign | Time  | A/D | Model | Entrance | Exit |
|-----------|-------|-----|-------|----------|------|
| CZ6687    | 10:01 | A   | A333  | R5       | E5   |
| JD5512    | 10:03 | D   | A320  | E10      | R7   |
| MF8324    | 10:10 | D   | B737  | E7       | R1   |
| CZ6355    | 10:17 | A   | A319  | R6       | E6   |
| FU6751    | 10:24 | D   | B737  | E2       | R1   |
| HU7767    | 10:30 | D   | B772  | E9       | R7   |

1.3.1 Distance factor

There are many methods of taxiway classification in airport, which are divided into parallel taxiway, connecting taxiway and apron taxiway. Taxing distance is one of the basic factors affecting aircraft taxiing. In this paper, the taxiway length is weighted, and the longer the taxiway length is, the higher the weight is. Generally speaking, the length of a taxiway in an airport is less than 1000 meters, so this paper uses 10 meters as the unit of evaluation to weigh each taxiway. Take the arrival
of flight CZ6687 as an example, construct a directed graph, as shown below. It can be seen from the figure that there are 7 alternative routes for this flight during the process of taxiing to E5 stand after the separation from R5 lane.

![Directional diagram of CZ6687 sliding path](image)

**Figure 2.** Directional diagram of CZ6687 sliding path.

After the directed graph is constructed, according to the route of flight CZ6687 and the length of each taxiway, divide by 10, and then round, and then get the distance weight value of each taxiway that the flight passes, as shown in the figure below. At this time, there are many and only shortest taxiing paths found through directed graph, including 4, and the sum of the shortest path costs is 168. At the same time, it also shows that there are more than one taxiing route in the case of single factor weighting of distance. To determine an accurate taxiing route, the remaining factors should be included in the discussion.

![CZ6687 taxiing path digraph with distance weight](image)

**Figure 3.** CZ6687 taxiing path digraph with distance weight.

### 1.3.2 Controller command factors

In a large and busy airport, the workload of controllers is very heavy. The complexity of ground traffic and the number of command aircraft in the airport will have a great impact on the work quality of controllers. In general, the operation manual will stipulate the standard taxi path. The standard taxi path will not only consider the single factor of distance, and cannot ignore the safety in order to save resources and improve efficiency. Because there may be traffic conflicts on the taxiway, the standard taxi path must provide the aircraft with the most convenient taxi path under the premise of ensuring safety. On the other hand, for the apron controller, the standard taxiing path can reduce the workload of the controller. With the standard path as a reference, the apron controller can more easily remember and identify the aircraft operating in the control area. Therefore, in the digraph model, after the control command factor is added, the influence of adopting the standard taxiway should be the least. Therefore, the influence weight of the control command factor on all taxiways the aircraft passes when taxiing according to the standard path is 0.
Aircraft can be divided into different types according to weight or wake, and the performance of each type is different. However, the type of aircraft has little impact on the taxiing speed of aircraft on the taxiing path, so this paper will not discuss the impact of different types on the taxiing speed. In addition, due to the different types of taxiways, the limitation of Aircraft Taxiway speed is also different. Generally, the taxiway speed of aircraft on parallel taxiways is larger, and the taxiway speed of aircraft on connecting taxiways is the second, and the taxiway speed of aircraft on apron is the smallest. As the aircraft enters the apron, there are many mobile airport staff and vehicles on the ground, and at the same time, in order to ensure the safety between aircraft and aircraft Distance. Therefore, the weight is set for three taxiways in order of size, 10 for parallel taxiways, 20 for connecting taxiways and 30 for apron taxiways.

Suppose that the standard route from R5 to E5 aircraft taxiing is: R5 → B9 → B8 → B7 → B6 → E5, and then assign values according to the above control factor weights. The taxiway weights of all sections on the standard taxiway remain unchanged, and the rest are weighted according to the corresponding section weights, as shown in the figure.

**Figure 4.** directional diagram of CZ6687 taxiing path after adding control command weight.

After the factor of controller command is added, the optimal taxiing path with the lowest cost obtained from the digraph is the standard route designated by the controller, and the shortest path cost is still 168.

### 1.3.3 Taxiway constraints

In the case that the airport ground is not affected by the continuous operation of the area, the construction of some areas is called non-stop construction. In addition, when there are temporary and sudden conditions on the airport ground, such as temporary obstacles and dangers, some taxiways may not be able to use normally. In this case, we will change the weight of occupied or temporarily closed taxiways due to emergencies to the maximum, and set the cost to 199.

Suppose there is construction near the taxiway of B6-T section in the airport, which causes the taxiway to be temporarily closed. According to the principle of non-stop construction of the airport, at this time, the taxiway weight of other sections is unchanged, only the weight of this section of taxiway is changed to 199, so the taxiway with the lowest cost according to the digraph at this time is R5 → B9 → B8 → B7 → W6 → E5, and the cost of the shortest path is 218.
1.3.4 Special flight factors

Special type of flight refers to a kind of flight that needs special treatment, such as: special aircraft or aircraft with emergency; aircraft for which the controller has arranged a fixed taxiing path due to special reasons. For this kind of flight, it must be guaranteed that it will not be delayed in any case, so the controller will assign a special taxiing path to this kind of aircraft in advance, and will not arrange other aircraft to use this path before this kind of aircraft is used up. In this special case, for this kind of aircraft, the weight previously given will be invalid. The weight of all taxiways involved in this path will be reduced to 0, and the cost of other taxiways will not be changed. At the same time, for other aircrafts, the weight of this special designated path is changed to 199.

Suppose that flight CZ6687 is no longer an ordinary passenger flight, but a special plane carrying important passengers. One day in advance, the controller has received the notice from the superior, and the designated taxiway is R5 → B9 → w9 → W8 → W6 → E5. Therefore, the weight of all taxiways that this taxiway passes is changed to 0, and the shortest taxiway is the designated taxiway, and the weight sum It is 0, and the specific changes are shown in the figure below.

**Figure 5.** directional diagram of CZ6687 taxiway after adding taxiway restriction weight.

1.3.5 Traffic conflict factors

The ground traffic flow of the airport is also increasing day by day, and the traffic conflicts between the aircraft and the aircraft in the taxiing process are also increasing. There are three kinds of taxiway traffic conflicts of aircrafts: cross collision, head-on collision and rear end collision.
Figure 7. Schematic diagram of Aircraft Taxiway conflict types.

Two aircraft cannot run on the same taxiway at the same time, so it is necessary to arrange the sequence of two aircraft, and then put the sequence priority factor of aircraft into the directed graph as a dynamic factor to analyze the optimization of taxiway. The priority of aircraft taxiing may be different, but under normal circumstances, the priority shall be to meet the principle of first come first serve. In order to improve the efficiency of airport ground operation, flight delay should be reduced as much as possible, and flight normal rate should be improved. Therefore, the status of the two aircraft should be compared first, then the priority should be determined, and then the weight of priority factors should be put into the directed graph.

The weights of different conflict types are: When there is no conflict, the two aircraft are all the same; when there is a cross conflict, the aircraft with high priority are the same, and the weight of the conflict path of the aircraft with low priority is +20; when there is a rear end conflict, the weight of the conflict path of the aircraft with high priority is the same, and the weight of the conflict path of the aircraft with low priority is +10; when there is a normal head to head conflict, it is not allowed to occur, which is very dangerous, so the weight change to 199 maximum.

Taking flight CZ6687 as an example, the shortest taxiing path obtained above through the constraints of distance factor and controller command factor is: R5 → B9 → B8 → B7 → B6 → E5. After that, flight JD5512 is about to leave the site. It is ready to leave from E10 to R7. At this time, there may be potential conflicts between the two aircraft. Therefore, a directed graph is established for JD5512 for analysis. First, the distance factor is added for constraint. The result is shown in the figure below.

Figure 8. JD5512 taxiing path digraph with distance weight.

Then add the control command factors to analyze. The standard path of flight JD5512 from E10
to R7 is E10 → W6 → W8 → B8 → B9 → B10 → B11 → B12 → R7. Then weight it with the above. For the apron taxiway + 30, contact taxiway + 20 and parallel taxiway + 10, the results are as follows.

**Figure 9.** JD5512 taxiing path direction after adding control command weight.

After the restriction of distance factor and control command factor, the shortest taxiing path of JD5512 flight is E10 → W6 → W8 → B8 → B9 → B10 → B11 → B12 → R7, and the total weight is 215. Considering that there may be cross conflict between flight JD5512 leaving from W8 to B8 and flight CZ6687 from B9 to B8, this is a potential cross conflict. According to the above, the priority of the entry aircraft is higher than the departure aircraft, so the weight of the taxiway from W8 to B8 of the departure flight JD5512 is + 20. At this time, the weight sum of the shortest path is 235, which is increased compared with the previous weight sum. At this time, the weight sum of the shortest taxiway of CZ6687 is unchanged. Therefore, flight jd5122 should consider changing the taxiway or delaying the time to solve the problem Cross conflict that is absolutely possible.

1.4 Summary

The taxiing path with the least total weight is constrained by many factors. It can be expressed as follows:

$$w = \min \sum_{N_f^r} (w_D + w_C + w_S + w_T)$$

Among them, \(w\) is the minimum weight sum of the current aircraft \(r\) from the starting node to the final node; \(N_f^r\) is the starting node of aircraft \(R\)'s taxiing path, \(N_t^r\) is the final node of aircraft \(R\)'s taxiing path, \((N_f^r, N_t^r) \in E\), \(V\) is node set, \(e\) is edge set.

\(w_D\) is the weight of Aircraft Taxiway distance factor, \(w_C\) is the weight of control command factor, \(w_S\) is the weight of special circumstances factor, \(w_T\) is the weight of dynamic conflict factor.

2. Solution process based on Dijkstra algorithm

Dijkstra algorithm is a classical algorithm to solve the shortest path of single source point. Its advantages are simple and more efficient calculation, and the digraph with positive cost of calculating weights is widely used. Different from the intelligent algorithm, Dijkstra algorithm can get the shortest path according to the whole problem, which is in line with the optimization calculation of taxiing path in this paper.
2.1 Specific content of Dijkstra algorithm
Label the nodes in the directed graph twice ($P(v_i), x_i$), where label $P(v_i)$ represents the shortest taxi path distance from start $v_1$ to $v_i$, label $x_i$ represents the subscript of the node in front of $v_i$ in the shortest path from $v_1$ to $v_i$, so $x_i$ is used to represent the passing path, so as to achieve the reverse tracking from the end to the start, so as to find the shortest taxi path from $v_1$ to $v_i$. Dijkstra algorithm is suitable for the case that the weight of the edge of the digraph is all positive. The digraph model established in this paper meets this basic condition. Let the digraph simulated by the airport ground system be $G = (V, E, w)$, where $V$ is the node set and $E$ is the edge set of the digraph. $w_{ij}$ is the weight of taxiway, $0 \leq w_{ij}$. If nodes $v_i$ and $v_j$ are not two adjacent nodes, then $w_{ij} = \infty$, find the shortest taxi path from start node $v_1$ to end node in $G$. It is defined as follows:

Set $w_i^{(r)*}$ as the weight of the shortest taxiing path from the initial taxiing node $v_1$ to node $v_i$. If node $v_i$ gets label $w_i^{(r)*}$, then it can be said that $v_i$ obtains P label $w_i^{(r)*}$ (permanent label) in step R, among which $r \geq 0$.

Set $w_i^{(r)}$ as the upper limit of the shortest taxiing path from aircraft starting node $v_1$ to node $v_j$. If $v_j$ obtains label $w_j^{(r)}$, it can be said that $v_j$ obtains t label $w_j^{(r)}$ (temporary label) in step R. Set $P_r = \{v | v \text{ obtain label } p\}$ as the pass through set of step R. Set $T_r = V - P_r$ as the failed set of step R, $r \geq 0$.

2.2 Calculation process of Dijkstra algorithm
At the beginning of the algorithm, $r \leftarrow 0$. The starting node $v_i$ of the aircraft taxiing obtains the T-Mark of $w_i^{(0)*} = 0$, $P_0 = \{v_1\}$, $T_0 = V - \{v_1\}$, $v_j (j \neq 1): w_j^{(0)} = w_{ij}$.

(1) Find the next mark P node: Set $w_{j}^{(r)*} = \min_{v \in T_{r-1}} \{w_{j}^{(r-1)}\}$, $r \geq 1$. Mark $w_j^{(r)*}$ at the corresponding node $v_j$, indicating that $v_j$ obtains the p-mark, and modify the pass set and fail set: $P_r = P_{r-1} \cup \{v_j\}, T_r = T_{r-1} - \{v_j\}$. Then check $T_r$: if $T_r$ is an empty set, end the algorithm and output the shortest taxi path, otherwise step (2) will be performed.

(2) Modify the t-label of each node in $T_r$: $w_j^{(r)} = \min \{w_j^{(r-1)}, w_j^{(r)*} + w_{ij}\}$, $w_i^{(r)*}$ is the P-Label of the node just obtained the P-Label. Let $r \leftarrow r + 1$, go to step (1).

2.3 Solution flow chart
Finally, the shortest taxiing path of the aircraft in the schedule is calculated as follows:
- CZ6687: R5 → B9 → B8 → B7 → B6 → E5
- JD5512: E10 → W6 → W8 → B8 → B9 → B10 → B11 → B12 → R7
- MF8324: E7 → T → B6 → B5 → B4 → B3 → B2 → B1 → R1
- CZ6355: R6 → B10 → B9 → B8 → B7 → W6 → E6
- FU6751: E2 → W5 → B5 → B4 → B2 → B1 → R1
- HU7767: E9 → W6 → B7 → B8 → B9 → B10 → B11 → B12 → R7

According to the results of the shortest taxiing path of the aircraft calculated at last, compared with the optimization scheme of the digraph model added with multi-factor weight in the previous chapter, it can be seen that the shortest taxiing path obtained by the Dijkstra algorithm of CZ6687 and JD5512 entering the site and the Dijkstra algorithm influenced by the distance factor and control command factor without special circumstances is consistent. It is consistent that the analysis of aircraft taxi path optimization in this paper is feasible, which can be applied to other aircraft taxi path optimization analysis, and also can provide some reference and help for the actual apron control command.

3. Summary and Prospect
In this paper, taking the ground traffic network diagram of an Airport as an example, through the establishment of the digraph model for quantitative analysis, various factors that affect the decision-making of aircraft taxiing path optimization are substituted into the digraph by the way of weighting, and the optimization scheme of aircraft taxiing path with the least weight cost under the constraints of multiple factors is obtained. Finally, Dijkstra algorithm is used to solve the specific problems of aviation. The shortest taxiing path is calculated and the result is obtained.

The optimization scheme of aircraft ground sliding proposed in this paper is analyzed from the perspective of apron control department, on the other hand, it can also be analyzed from the
perspective of airport ground support department, and the influence of ground support vehicles managed by ground support department on aircraft sliding path can also be considered. The actual operation of the aircraft has the characteristics of real-time. There are other factors that can be added to the dynamic decision-making of aircraft taxiing path, so as to get a safer and more efficient optimization scheme of aircraft taxiing path.

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