Investigation of the influence of elastic-mass characteristics of the axle test stand links on its own oscillation frequencies

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Abstract. The reliability of work according to the criteria of vibration stability of the stand for testing the axles of wheelsets for endurance is largely determined by the natural vibration frequencies of the stand. The work considers a stand with a lever structure for loading the tested axles. In order to determine the influence of the parameters of the cross-section of the links on the natural frequencies of the vibration stand, the frequency characteristics were obtained by the method of mathematical modeling for two variants of the design schemes of the stand. Studies have shown that increasing the cross-sectional area of the levers without measuring other linear dimensions leads to a simultaneous increase in their mass and cruelty. The rigidity of the levers is defined as for beams on two supports with a load of a transverse concentrated force. As a result, the calculated values of the natural frequencies of the stand vibrations change insignificantly. A deeper analysis of the Eigen frequency characteristics of the stand with the tools of the Matlab system made it possible to obtain linear regression equations with the coefficients of determination close to one. The obtained research results gave reason to expect a decrease in stresses in the metal structures of the stand with an increase in the cross-sectional area of the levers, however, the values of the natural frequencies of the stand vibrations vary within a few percent. The way to control the natural frequencies of the stand vibrations only by changing the profile parameters is ineffective.

1. Introduction
The organization of the safe operation of transport is a complex task. This task is subordinated to the issues of safe operation, repair and modernization of rolling stock and, first of all, locomotives. [1 – 3].

The operational safety of a railway transportation vehicle is directly related to the strength of wheelset basic part, i.e. its axle. Therefore, latterly numerical studies of theoretic and experimental nature are conducted, both for railway transport vehicles [4 – 8]. A variety of special stands are used to test parts of wheelsets of railway rolling stock. [9, 10]. On the stands, geometrical dimensions, electrical parameters and mechanical characteristics are checked. Axle fatigue test stands create symmetrical or pulsating loads.

Symmetrical loads are more often created by inertial loading devices - vibrators [11]. Pulsating loads during axle testing are implemented by hydraulic and hydromechanical mechanisms. Such mechanisms contain levers necessary for design reasons and to increase the value of the transfer function of force. The critical frequencies of the stand with a lever loading mechanism are determined as a result of mathematical and simulation modeling of oscillations of the stand links [12]. Modeling significantly speeds up the process of choosing a rational scheme and design parameters of the stand according to the reliability requirements, as well as according to the criteria of strength, stiffness and vibration stability [13, 14].

2. Purpose and problem statement
The aim of the work is to assess the influence of the dimensions of the cross-sections of the arms of the stand for testing the axles of railway wheelsets on its natural frequencies.
To achieve this goal, the following tasks were solved:
1) preparation of the initial data necessary to determine the natural frequencies of the stand using the previously developed mathematical models of its oscillations;
2) obtaining the dependences of the natural frequencies of the stand on the total mass of its levers;
3) evaluation of the influence of the dimensions of the sections of the levers of the stand on its natural frequencies.

The methodology for performing the tasks of the work is based on fundamental studies of the dynamics of mechanical systems with several degrees of freedom. The Mathcad engineering calculation system was chosen as a tool for performing studies of natural vibration frequencies.

3. The construction of the test bench
A stand for testing railway axles for fatigue strength (Figure 1) is similar in design to a stand for testing wheels [15]. It consists of an electric motor with a gearbox (stand drive) 1, a spring damper 2, the pressure force of which is controlled by a force sensor, an intermediate lever 3, which rests on a rack 4, a pusher 5 and a main lever 6 with racks 7. The tested axle 8 is mounted on two supports struts 9 and is loaded by the console of the main lever 6. The struts have the ability to move to change the axle load.

![Figure 1. Test bench for axles of railway wheelsets [15].](image)

The lever system of the stand allows increasing the force generated by the electric motor with gearbox 1 by about 20 times. Each arm of the stand can be viewed as a beam on two supports, loaded with a concentrated force. The cross-sections of the levers can be rectangular box-shaped; also the levers can have the shape of a beam of equal resistance. Therefore, the elastic-mass characteristics of the levers will be different and, therefore, the natural frequencies of the stand will also differ.

4. Mathematical model of stand vibrations

4.1 Kinematic scheme of the stand lever system
By its construction, the stand is a spatial mechanism. In order to simplify the calculations, the kinematic scheme of the stand was idealized (Figure 2) and was considered as a flat system of the type of a double rod pendulum with a uniformly distributed mass along the rods and an elastic connection.
between them due to their flexibility [12]. By the type of relative mobility, the beam 1, interacting with the stand drive, is a pendulum, and the beam 3 loads the tested axle of the wheelset and acts as a lever. The beams are interconnected by a short link - pusher 2.

Figure 2. Idealized kinematic scheme of the stand.

4.2 Equations of motion of the links of the stand
In the first approximation, the construction of a mathematical model of the stand vibrations is carried out as for a simple oscillatory system with levers of constant cross-section and concentrated masses \( m_1 \) and \( m_2 \) (Figure 3). Linear displacements of the centers of mass of the levers of the stand corresponded to the generalized coordinates \( q_1 \) and \( q_2 \).

Figure 3. Calculated scheme of the stand.

Small oscillations of such a conservative dynamical system are described by a pair of differential equations [11]

\[
\begin{align*}
a_{11}q_1'' + c_{11}q_1 + c_{12}q_2 &= F' \sin \omega t, \\
a_{22}q_2'' + c_{22}q_1 + c_{22}q_2 &= 0,
\end{align*}
\]

where \( a_{11} = m_1 + J_1/(l_1 \cos \beta) \), \( a_{22} = m_2 + J_2/l_2^2 \) - inertial coefficients of the system; \( c_{11} = c_1 (l_{c1} \cos \beta / l_1) \), \( c_{12} = c_1 (l_{c1} c_{22} \cos \beta / l_1 l_2) \), \( c_{21} = c_{12} \), \( c_{22} = (c_1 + c_2) (l_{c2} / l_2) \) - elastic coefficients of the system; \( m_1, m_2 \) - stand levers masses; \( J_1, J_2 \) - moments of inertia of the drive levers relative to their axes of rotation; \( c_1, c_2 \) - rigidity of the stand levers relative to their axes of rotation; \( l_1, l_2 \) - distance from the axes of rotation of the levers to their centers of mass; \( l_{c1}, l_{c2} \) - distance from the axes of rotation of the levers to the elastic elements; \( \beta \) - angle of inclination of the lever 1 of the stand; \( F' = 2F \) - the amplitude of the loading force reduced to the center of mass of the lever 1; \( \omega \) - system loading frequency; \( t \) - time.

Frequency equation of the system (1)

\[
A p^2 - B p^2 + C = 0,
\]

where \( A = a_{11} a_{22} \), \( B = a_{11} c_{22} + a_{22} c_{11} \), \( C = c_1 c_{22} - c_{12}^2 \).

The second approximation, when constructing a mathematical model, takes into account the actual deviations and rotation of the centers of mass of the levers from the equilibrium position (Figure 4).
Figure 4. Calculated scheme of the stand, taking into account the actual deviations and rotation of the centers of mass of the levers from the equilibrium position.

For this model, the general view of the system of equations describing the movement of the links of the mechanism is the same as for the previous design scheme (Figure 3), but its inertial and elastic coefficients are different:

\[
\begin{align*}
  a_{11} \ddot{q}_1 + c_{11} q_1 + c_{12} q_2 &= F' \sin \omega t, \\
  a_{22} \ddot{q}_2 + c_{22} q_1 + c_{22} q_2 &= 0,
\end{align*}
\]

(3)

where \( a_{11} = m_1 + J_1 k_{\theta_{11}} \), \( a_{22} = m_2 + J_2 k_{\theta_{12}} \) — system inertial coefficients;
\( c_{11} = c_1 (k_{\theta_1}/\cos \beta)^2 \), \( c_{12} = -c_1 (k_{\theta_1}/k_{\theta_2}/\cos \beta) \), \( c_{21} = c_2 \), \( c_{22} = (c_1 + c_2) k_{\theta_2}^2 \) — elastic coefficients of the system; \( k_{\theta_{11}} = (\theta_1/q_1) \text{ m}^{-1} \), \( k_{\theta_{12}} = (\theta_2/q_2) \text{ m}^{-1} \) — the coefficients of influence of the displacements of the centers of mass on the angle of their rotation;
\( k_{F_1} = \Delta c_1/q_1 \), \( k_{F_2} = \Delta c_2/q_2 \) — the coefficients of the influence of displacements of the centers of mass on the deformation of the elastic elements of the system.

The form of the frequency equation for system (3) is the same as for the system (1):

\[
A' p^4 - B' p^2 + C' = 0,
\]

(4)

where \( A' = a_{11} a_{22} \), \( B' = a_{11} c_{22} + a_{22} c_{11} \), \( C' = c_{11} c_{22} - c_{12}^2 \).

5. Influence of the dimensions of the cross-sections of the levers on the natural frequencies of the stand

5.1 The construction of the sections of the levers of the stand

We will assume that the sections of the stand levers have a rectangular profile of a constant length, formed by welded steel sheets (Figure 5, Figure 6). The dimensions of the arm profiles are determined by design considerations and strength criteria [16].
5.2 Elastic-mass characteristics of the levers

The calculation of the parameters of the levers of the stand is made without changing their length. The masses of the levers of the stand changed due to a proportional increase in the thickness of their walls. Such changes in cross-sections led to a change in the masses of the levers and their stiffness’s.

The rigidity of lever 1 (see Figure 2) was determined as the rigidity of the beam on two supports, loaded with a concentrated force $F$ (Figure 7):

$$c_1 = \frac{F}{y_F},$$

where $y_F = Fa^2b^2/3EI$ - deflection of a beam under force $F$ [16].

The rigidity of lever 3 was determined as the rigidity of a beam on two supports with a console loaded with a concentrated force $F_1$ (Figure 8):

$$c_2 = \frac{F_1}{y_F},$$

where $y_C = F_1c^2(l+c)/3EI$ - deflection of a beam under force $F_1$. 

![Figure 7. Calculated loading scheme for lever 1.](image)

![Figure 8. Calculated loading scheme for lever 3.](image)

The results of determining the parameters of levers with different wall thicknesses, carried out using a PC, are summarized in Table 1.

### Table 1. Elastic-mass characteristics of the levers.

| №  | Lever weight, kg | Lever moment of inertia, kg·m² | Lever stiffness, MN/m | Lever wall thickness, mm | Lever cross-sectional area, cm² |
|----|------------------|-------------------------------|-----------------------|--------------------------|--------------------------------|
| L1 | L3               | L1                            | L3                    | L1                       | L3                            |
| 1  | 136.5            | 388                           | 26.6                  | 98.9                     | 639                           | 204                           | 12                            | 20                            | 114                           | 284                           |
| 2  | 168.3            | 478                           | 32.8                  | 122                      | 772                           | 245                           | 15                            | 25                            | 141                           | 350                           |
| 3  | 188.6            | 531                           | 36.8                  | 136                      | 856                           | 269                           | 17                            | 28                            | 158                           | 389                           |
| 4  | 219.6            | 616                           | 42.8                  | 157                      | 973                           | 305                           | 20                            | 33                            | 184                           | 451                           |
| 5  | 239.9            | 666                           | 46.8                  | 170                      | 1046                          | 326                           | 22                            | 36                            | 201                           | 488                           |

Note: L1 – lever 1; L3 – lever 3

5.3 Own stand frequencies

The roots of the biquadratic frequency equations (2) and (4) are the own frequencies of the stand. After substituting the characteristics of the levers in (2), (4) and solving the equations in the Mathcad package, a tabular dependence of the own frequencies of the stand on the thickness of the walls of the levers was obtained (Table 2).
According to the strength conditions, the levers have different initial wall thicknesses, therefore, it is more convenient to represent the values of the natural vibration frequencies of the stand as a dependence on the total mass of the levers. In order to automate the statistical processing of the results obtained and plotting graphs, the data of Table 2 were loaded into the Matlab system.

**Table 2. Own oscillation frequencies of the stand.**

| №  | Lever wall thickness, mm | Total weight of levers, kg | Scheme Figure 3 | Scheme Figure 4 |
|----|-------------------------|---------------------------|----------------|----------------|
|    | L1                      | L3                        | P₀₁  | P₀₂  | P₀₁  | P₀₂  |
| 1  | 12                      | 20                        | 524.5| 460  | 3053 | 836  | 4234 |
| 2  | 15                      | 25                        | 646.3| 455  | 3022 | 826  | 4190 |
| 3  | 17                      | 28                        | 719.6| 450  | 3013 | 818  | 4177 |
| 4  | 20                      | 33                        | 835.6| 445  | 2983 | 808  | 4135 |
| 5  | 22                      | 36                        | 905.9| 440  | 2970 | 800  | 4116 |

Note: L1 – lever 1; L3 – lever 3; P₀₁, P₀₂ – first and second own frequencies

An increase in the thickness of the profile of the levers of the stand is accompanied by an increase in the moments of resistance of the sections. Therefore, the stresses in the levers of the stand will decrease.

The dependence of the natural frequencies of the stand vibrations on the total mass of the levers (section sizes), firstly, has an almost linear character; secondly - they practically remain unchanged with an increase in the mass of the levers (Figure 9, Figure 10).

![Figure 9](image-url)  
**Figure 9.** Dependence of the own frequencies of the stand vibrations on the total mass levers (section sizes) with a simplified design scheme (Figure 3).
5.4 Analysis of the dependences of the own frequencies of the stand vibrations

A detailed analysis of the dependences of the own frequencies of the stand vibrations on the total mass of the levers (Figure 11, Figure 12) using the Matlab system tools showed a slight decrease in frequencies with an increase in the total mass of the levers.

The linear regression equations reflecting the dependence of the own frequencies of the stand on the total mass of the levers have the form:

$$P_{01} = 487.8 - 0.0521m;$$

$$P_{02} = 3166 - 0.2178m,$$

where $m$ – total weight of stand levers.

The coefficient of determination of equations (7), (8) is 0.99, which indicates a high accuracy of the selection of equations.

An analysis of expressions (7) and (8) showed that with an increase in the mass of the levers by 72.7%, the first natural frequency decreases by 4.3%, and the second decreases only by 2.7%.
6 Conclusions
1. The results obtained make it possible to assert that a change in the thickness of the walls of the stand arms has practically no effect on the values of the own frequencies of the stand. This is due to the simultaneous increase in mass and lateral stiffness of the levers.
2. The dependence of the own frequencies of the stand vibrations on the mass of the levers is linear: with an increase in the total mass of the levers by 72.7%, the first natural frequency decreases by 4.3%, and the second - by 2.7%.

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