Abstract—Complex phenomena such as prolonged undamped ultra-low frequency oscillation (ULFO) and eigenmode re-excitation are observed in the simulations of hydroelectric power systems. Emphases are put on nonlinearities and mode interactions, which cannot be analyzed by traditional eigen-analysis methods. In order to investigate the mechanism of the evolvement of the nonlinear dynamic process in ULFO, this paper proposes a method to analyze the mode interactions quantificationally. First, a disturbed trajectory is decoupled into a set of time-varying components. Second, transfer matrices of eigenmodes are extracted along the trajectory. Third, consecutive sequences of eigenvalues and trajectories of components are formed by a proposed technique. Based on the decoupled components and transfer matrices, the mechanisms of mode interactions and inheritance relationships between eigenmodes are analyzed. The causes and developments of the above complex phenomena are revealed by the proposed method in a test two-machine system. Meanwhile, the accuracy of the eigenmode matching technique is verified in the New England system.

Index Terms—Eigenmode matching, hydroelectric power system, mode interaction, trajectory eigenvalue, ultra-low frequency oscillation.

I. INTRODUCTION

In recent years, ultra-low frequency oscillation (ULFO) has been observed in hydroelectric power systems regularly. In 2011, an oscillation with 0.05-0.08 Hz was observed in the Colombian power grid, and 63.8% of the generators are hydropower units [1]. In 2016, an extremely low frequency oscillation (about 0.05 Hz) occurred in China Southern Power Grid during asynchronous testing [2]. Simulation indicates that the parameters of governors may lead to substantial effects [3]. An ultra-low frequency eigenmode with 0.08 Hz was motivated in the second stage of the north Brazil blackout in 2018 [4]. With the enlargement of modern power grids, the integration of renewable energy, and the development of the high-voltage direct current transmission, multiple occurrences of ULFO seriously affect the security and stability of power grids [5].

The dynamic characteristics of water turbines and controllers have been widely concerned in academia since 1992 [6]. In the last decade, ULFOs have been detected and studied by mathematic models at the equilibrium point. Generally, the terrible dynamic performances are attributed to turbine factors, electronic devices, and uncertain loads, such as “water hammer effect” and poorly tuned controllers [7]. Frequency-domain approaches like Nyquist Diagram and Routh Criterion are utilized to analyze the effects of the above factors [8], and the too small values of proportional and integral coefficients in the proportional-integral-derivative (PID) governor are evaluated to be the major reasons of ULFO. Reference [9] denotes that the governor would add a new eigenmode with ultra-low frequency. Addressed to this eigenmode, [10] demonstrates the contributions of governor parameters, load voltages, and turbine inertial based on damping torque analysis and eigen-analysis. Overall, previous researches usually focus on the ULFO eigenmode identification and the PID parameter optimization [11]-[13].

However, the results of the above studies strongly rely on the operation condition of a system. As a consequence, the oscillation characteristics are becoming extremely intricate when the changes in the operation condition are involved [14]. Reference [15] reviews the unusually undamped oscillation in the south-eastern part of the European system that occurred in 2005. The actual frequency of the oscillation is found to be time-varying, influenced by nonlinearities and mode interactions. In the simulation of hydroelectric systems, we observe some complex phenomena in ULFO, which cannot be explained apparently by conventional methods. In the normal range of parameters, it is easy to detect prolonged undamped oscillation in hydroelectric power systems. Meanwhile, some damped eigenmodes are periodically re-excited during the oscillation. Since these special phenomena are related to mode interactions, we explore the mechanisms of the above observations based on trajectory eigenvalue (TE) theory. This paper reports the results of these efforts.

Previous researches on low-frequency oscillation (LFO) and ULFO assume an ideal linear system, which is usually
satisfied in the neighborhood of the equilibrium point. Nonetheless, during an oscillation, the operation point may deviate from the equilibrium point. The effects of the nonlinearities may lead to incredibly complex dynamic behaviors such as mode interactions. The systematical study of the mode interactions originates from the normal forms and the modal series [16], [17]. The oscillation characteristics are described by the solutions of the second-order or higher-order differential equations at the equilibrium point [18], [19]. However, two significant problems need to be addressed: ① the solutions of high-order differential equations are complicated; ② a model-based method can hardly describe the evolutions of the dynamic behavior of a system without trajectories. Alternatively, the methods based on the trajectories given by numerical integration or phasor measurement units may be available. As an attempt, [20]-[22] design an analysis framework based on the mode-energy sequence.

Generally, the eigenmodes of one state matrix are orthogonally decoupled, implying that each eigenmode is independent. Nonetheless, affected by the nonlinearities and mode interactions, eigenmodes may be time-varying so far as to re-excite. In order to reveal the mode interactions and inheritance relationships between eigenmodes along the trajectory, this paper decouples a disturbing trajectory into several components and extracts the transfer matrix of time-varying eigenmodes between two adjacent time sections. The complementary relationships of decoupling components indicate the intrinsic connections of eigenmodes. The elements of the transfer matrix can be taken as indicators of mode interactions, whereas the similarity between the transfer matrix and an identity matrix is utilized to pair the eigenmodes.

The contributions of this paper are twofold. First, this paper proposes a method to quantificationally analyze the mode interactions of power systems based on TEs. The disturbed trajectory is decoupled into n time-varying components, and the transfer matrices are used to evaluate the degree of mode interactions. With the proposed method, the evolutions of two complex phenomena are explained. Two typical indices of normal forms technique are utilized to verify the effectiveness of the proposed method. Second, an eigenmode matching technique is developed to pair the eigenmodes between adjacent state sections. The simulation results in the New England system show that the average error can be reduced by at least one order of magnitude.

The rest of this paper is organized as follows. We give a decoupled expression of a trajectory and propose a method to analyze mode interactions in Section II. In Section III, an eigenmode matching technique is designed. The specific phenomena in ULFO are analyzed in the frequency domain and the time domain in Section IV. Conclusions and discussions are arranged in Sections V and VI, respectively.

II. A METHOD TO ANALYZE MODE INTERACTIONS

In order to describe the evolutions of mode interactions, TE theory is further developed in this section. After a brief review of the theory, the application scope, physical meaning, and mode interactions are discussed, respectively.

A. A Brief Description on Trajectory Eigenvalue Theory

Reference [23] proposes a method to linearize the dynamic models of a power system piecewise, which is called the TE theory. The core idea of the theory is to extract oscillation characteristics along the trajectory, step by step. This method is developed in [24], in which the relationship between oscillation stability and transient stability is discussed by estimating the virtual fast end point (FEP) or dynamic saddle point (DSP). Further research works attempt to identify the generator clusters by TEs after large disturbances. This longitudinal study was first carried out in [25]. However, the neglect of the unbalanced power at non-equilibrium points is the central focus of a review reported in [26], which denotes that the TE theory cannot be used to analyze the global stability of linear time-varying systems. A recent study clarifies that the TE theory is ponderable when analyzing the dynamic behavior within one integration step [27], [28].

By far, vague description of the theory has caused controversies on the physical meaning of TEs. Based on the application scope clarified, a mode interaction analysis method is composed of decoupling components and transfer matrix as follows.

B. Application Scope of Trajectory Eigenvalues

The dynamic equations of a power system can be defined as (1).

\[
\begin{aligned}
\dot{X} &= f(X,Y,t) \\
0 &= g(X,Y)
\end{aligned}
\]

where \(X\) is the state vector, and its element \(x_i\) represents the state variables such as rotor angles and angular velocities of generators; \(Y\) is the algebraic vector, and its element \(y_j\) represents the algebraic variables such as node voltages and injecting currents; and \(t\) is time. The trajectories of state and algebraic variables respected to time can be obtained by trapezoidal integration. The algebraic variables are substituted to the differential equations along the trajectory. Besides, the nonlinear components of (1) are expanded in Taylor series (the first-order term) as (2).

\[
\frac{d(X(t_i)+\Delta X_i(\Delta t))}{dt_i+\Delta t}=f(X(t_i),Y(t_i),t_i)+\frac{\partial f}{\partial X} \bigg|_{X(t_i)} \Delta X_i(\Delta t)+
\frac{\partial f}{\partial Y} \bigg|_{Y(t_i)} \Delta Y_i(\Delta t)+\frac{\partial f}{\partial t} \bigg|_{t_i} \Delta t
\]

where \(\Delta t\) is the deviation of time within one integration step; and \(\Delta X_i(\Delta t)\) and \(\Delta Y_i(\Delta t)\) are the changes of state vector and algebraic vector caused by the deviation of time, respectively.

The right part of (2) is made up of four terms. ① \(f(X(t_i),Y(t_i),t_i)\) is a function of \(X(t_i)\) and \(Y(t_i)\), describing the unbalanced power within one integration step, denoted by \(B_c\). ② \(\frac{\partial f}{\partial X} \bigg|_{X(t_i)}\) is the \(n \times n\) Jacobian matrix, also known as the section state matrix, denoted by \(A_s\). The product term \(A_s \Delta X_i(\Delta t)\) represents the influence of state vector on dynam-
ic characteristics. \( \frac{\partial f}{\partial X} \bigg| _{Y(\Delta t)} \) is the partial derivative matrix of algebraic variables, which are constant within one integration step. The product term \( \frac{\partial f}{\partial X} \Delta Y_k(\Delta t) \) is assumed to be a zero vector. \( \frac{\partial f}{\partial t} \bigg| _{t_k} \) is the non-autonomous factors of the equations produced in faults, control measures, or time-varying parameters in \([t_k, t_{k+1})\). Considering that the expressions of \( \frac{\partial f}{\partial t} \bigg| _{t_k} \) can hardly be derived analytically, the product term \( \frac{\partial f}{\partial t} \Delta t \) is denoted by \( \Delta f_t(\Delta t) \). Thus, (2) can be simplified as (3).

\[
\Delta X_k(\Delta t) = A_k \Delta X_k(\Delta t) + \Delta f_t(\Delta t) + B_k
\]

The initial values of state variables and algebraic variables within one integration step are updated along the trajectory, according to the results of numerical integration. To ensure the availability of numerical integration, the number of discontinuity points in the disturbed trajectory should be finite.

The TE theory assumes that \( B_k \) is a constant vector within one integration step. The value of \( B_k \) is determined by the initial values of \( X(t_k) \), \( Y(t_k) \), and \( t_k \), while \( A_k \) has nothing with \( B_k \). In other words, the dynamic characteristics within one step have no concern with the value of \( B_k \). However, \( B_k \) is a pivotal term when solving the differential equations, and it leads to the different motivations of eigenmodes. Although the calculation of section eigenvalues for a time window ignores the unbalanced power, the sequences composed of \( n \) series of section eigenvalues can be used to describe the effects of all factors.

In nonlinear oscillation, the dominant eigenmode may vary with time. Thus, the oscillation characteristics may be significantly different from the equilibrium point. To better investigate the complex dynamic characteristics of power systems, we focus on the evolutions of eigenmodes in this paper. In one integration step, the TE theory can produce the recurrence of the whole trajectory. With continuous trajectories of section eigenvalues, the mode interactions can be analyzed from the perspectives of frequency domain and time domain.

To summarize, the application range of TE theory can be clarified: (1) the disturbed trajectory can be simulated by numerical integration technique, with finite discontinuity points; (2) the section eigenvalues for a time window are adequate to describe the dynamic characteristics of the system in \( t \in [t_k, t_{k+1}) \) only when the unbalanced power can be ignored within one integration step.

C. Decoupling Components

With the time-varying terms neglected, (3) can be rearranged as (4).

\[
\Delta X_k(\Delta t) = A_k \Delta X_k(\Delta t) + B_k
\]

Equation (4) is a set of linear time-invariant differential equations, of which the solution can be analytically deduced by (5).

\[
\Delta X_k(\Delta t) = U_k C_k E_k(\Delta t) - U_k C_k E_k(0)
\]

where \( U_k \) and \( C_k \) are the modal matrix and intermediate matrix, respectively; and \( E_k(\Delta t) = [e^{\lambda_1 \Delta t}, e^{\lambda_2 \Delta t}, \ldots, e^{\lambda_n \Delta t}]^T \) is the complex-exponential components. The eigenvalues of \( A_k \) can be denoted by \([\lambda_1, \lambda_2, \ldots, \lambda_n]^T\). The expressions of the above matrices and vectors can be found in Appendix A.

Assuming the existence of a new system with the parameters updating with time, the state variables within one integration step can be expressed as (6).

\[
X_k^{\text{new}}(t) = U_k C_k E_k(t - t_k)
\]

where \( t = t_k + \Delta t \).

It needs to be emphasized that the values of \( X_k^{\text{new}}(t) \) and \( X_k(t) \) are not the same. The derivation of them can be transformed as (7).

\[
X_k^{\text{new}}(t) - X_k^{\text{new}}(t_k) = U_k C_k E_k(t - t_k) - U_k C_k E_k(0) = \Delta X_k(\Delta t)
\]

According to (7), the derivation of the new system is consistent with the original system. The state equation of the new system can be described by (8).

\[
X_k^{\text{new}}(t) = U_k C_k A_k E_k(t - t_k) = U_k C_k A_k X_k^{\text{old}}(t)
\]

where \( A_k = U_k^{-1} A_k U_k \).

Thus, the trajectory of the new system can be decoupled by \( n \) components. Equation (6) can be expanded as (9).

\[
\begin{bmatrix}
X_k^{\text{new}}(t_1) \\
X_k^{\text{new}}(t_2) \\
\vdots \\
X_k^{\text{new}}(t_n)
\end{bmatrix} =
\begin{bmatrix}
\varphi_{t_1}(t) + \varphi_{t_2}(t) + \ldots + \varphi_{t_n}(t) \\
\varphi_{t_2}(t) + \varphi_{t_2}(t) + \ldots + \varphi_{t_n}(t) \\
\vdots \\
\varphi_{t_n}(t) + \varphi_{t_n}(t) + \ldots + \varphi_{t_n}(t)
\end{bmatrix}
\]

where \( x_k^{\text{new}}(t) \) is the \( i \)-th element of \( X_k^{\text{new}}(t) \), and \( \varphi_i(t) \) represents the component of the \( i \)-th eigenmode in the \( i \)-th state variable. Generally, \( \varphi_i(t) = \Gamma(\text{det})^\text{th} \) where \( \Gamma(t) \) is an intermediate variable changing with time. In (9), \( \varphi_i(t) \) describes the time-varying characteristics of the \( i \)-th eigenmode. For a pair of conjugated eigenmodes, their trajectories are coincident, whereas their combination would keep continuous in time domain.

D. Transfer Matrix

Let \( V_k^+ \) denote the inverse matrix of \( U_k \), a new state vector can be defined by the linear transform \( Z_k^{\text{new}}(t) = V_k^+ X_k^{\text{new}}(t) \). When \( t = t_k \) we have (10).

\[
Z_k^{\text{new}}(t_k) = V_k^+ X_k^{\text{new}}(t_k) = V_k^+ (X_k^{\text{new}}(t_k) + \xi_{k-1}) = V_k^+ (U_k^{-1} Z_{k-1}^{\text{new}}(t_k) + \xi_{k-1})
\]

where \( \xi_{k-1} \) is the error vector between the solutions of piecewise linearized equations and the numerical integration results. Reference [27] has proved that \( \xi_{k-1} \) tends to be zero once the step size is small enough. Based on this assumption, (10) can be approximated by (11).

\[
Z_k^{\text{new}}(t_k) \approx V_k^+ U_k^{-1} Z_{k-1}^{\text{new}}(t_k)
\]

As can be expected, when \( t = t_k \), the parameters of \( A_k \) would be updated, and the expression of \( Z_k^{\text{new}}(t) \) can be formed from \( Z_{k-1}^{\text{new}}(t) \). In this paper, transfer matrices of eigenmodes are proposed to describe the inheritance relationships and mode interactions of eigenmodes between the adjacent
time sections. \( T_\tau = V \Omega U_{\tau-1} \) is defined to be the transfer matrix of eigenmodes from \( t_\tau-1 \) to \( t_\tau \). In a linear system, the eigenmodes would be time-invariant, where \( Z_{\tau}^{\text{new}}(t) = Z_{\tau-1}^{\text{new}}(t) \) and \( T_\tau \) is an identity matrix. On the contrary, the eigenmodes are always time-varying when the operating point runs away from the equilibrium point in a nonlinear system. According to (11), \( Z_{\tau}^{\text{new}}(t) \) can be rearranged by the product of \( T_\tau \) and \( Z_{\tau-1}^{\text{new}}(t) \) approximately. Thus, the \( j^\text{th} \) element of \( Z_{\tau}^{\text{new}}(t) \) is expressed as (12).

\[
z_{\tau,j}^{\text{new}}(t) = \sum_{k=1}^{n} t_{\tau,k} z_{\tau-1,j,k}(t) \tag{12}
\]

where \( t_{\tau,k} \) is the element of \( T_\tau \) in the \( j^\text{th} \) row and \( k^\text{th} \) column, describing the impacts of the \( k^\text{th} \) eigenmode on the \( j^\text{th} \) eigenmode. The value distribution of \( z_{\tau,j}^{\text{new}}(t) \) can be classified into three categories. The absolute values of \( t_{\tau,k} \) is designed as the indicator, which can imply the degree of mode interactions.

Value distribution 1 (VD1): generally, \(|t_{\tau,k}| \approx 1 \) and \( t_{\tau,k} \approx 0 \) \((j = 1 \rightarrow n, j \neq p)\). The \( j^\text{th} \) element of \( Z_{\tau}^{\text{new}}(t) \) is nearly the same as the \( p^\text{th} \) element of \( Z_{\tau-1}^{\text{new}}(t) \), indicating that the \( p^\text{th} \) eigenmode at \( t_\tau \) is evolved from the \( p^\text{th} \) eigenmode at \( t_{\tau-1} \), irrelevant to other eigenmodes at \( t_{\tau-1} \). When \( t_{\tau,k} \approx -1 \), the referenced signs of the modal matrix of the \( p^\text{th} \) eigenmode is different between \( t_\tau \) and \( t_{\tau-1} \).

Value distribution 2 (VD2): in some cases, \(|t_{\tau,k,p} - t_{\tau,k,j}| \times \zeta \geq |t_{\tau,k,j}| \) \((j = 1 \rightarrow n, j \neq p)\), where \( \zeta \) is the relative coefficient. Under this circumstance, the \( p^\text{th} \) eigenmode at \( t_\tau \) is most affected by the \( p^\text{th} \) eigenmode at \( t_{\tau-1} \). By regulating the value of \( \zeta \), the \( p^\text{th} \) eigenmode at \( t_\tau \) can be matched to the \( p^\text{th} \) eigenmode at \( t_{\tau-1} \), with different precision.

Value distribution 3 (VD3): in other cases, the \( p^\text{th} \) eigenmode at \( t_\tau \) is composed of several eigenmodes at \( t_{\tau-1} \). \|t_{\tau,k,ip}\| reflects the degree of participation of the \( p^\text{th} \) eigenmode.

III. Eigenmode Matching Based on Transfer Matrix

Based on the Ostrowski criterion [29], the eigenvalues of a matrix continuously change with the perturbation of the parameters of the matrix. The detailed description of the theorem is listed in Appendix B. As a result, the transfer matrix tends to be an identity matrix with the decreasing step size of integration. In other words, a time section with VD3 can be disassembled into a series of time sections with VD1 and VD2. The value distribution of the transfer matrix approaches VD1 when the step size is deduced. Theoretically, the results of eigenmodes matching can be improved by decreasing the step size.

A. Matching Criteria

Criterion 1: \(|\det(T_\tau) - 1| < \varepsilon_i\), or \(|\det(T_\tau) - 1| > \varepsilon_i\).

Criterion 1 is used to estimate the proximity of \( T_\tau \) to an identity matrix, of which the determinant is equal to 1. Considering that the determinant reflects the influences of all elements of \( T_\tau \), those sections with all eigenmodes satisfying VD1 can be identified by strict criterion 1 with a smaller value of \( \varepsilon_i \). Instead, large error sections can be picked out efficiently by setting \( \varepsilon_i \) to be large enough, and these large error sections should be put in a rematching process.

When \(|\det(T_\tau) - 1| < \varepsilon_i\), \( T_\tau \) can be transformed to an identity matrix \( I \) by elementary transformation, denoted by \( D \).

\[
DT_\tau D^T = I \tag{13}
\]

\( D \) can be easily obtained from (13). Multiplied by the same elementary transformation matrix, the eigenvalues matrix can be rearranged as (14).

\[
A_i' = DA_iD^T \tag{14}
\]

Criterion 2: \(|t_{\tau,ip} - 1| < \varepsilon_i\), \( t_{\tau,ip} \) and \( t_{\tau,ip} \) are the largest and the second-largest elements of \( T_\tau \) in the \( p^\text{th} \) row.

Criterion 3: \(|t_{\tau,ip} - t_{\tau,ip} - 1| < \varepsilon_i\), \( t_{\tau,ip} \) is the largest and the second-largest elements of \( T_\tau \) in the \( p^\text{th} \) row, respectively. If criterion 3 is satisfied, the \( p^\text{th} \) eigenmode is perceived to be the dominant one. Correspondingly, the value distribution can be classified as VD2.

B. Coordination of Matching Criteria

In order to achieve eigenmode matching at any time section, the above criteria are coordinated by a designed module.

As shown in Fig. 1, the inputs are the modal matrices of the section state matrices in two adjacent sections, denoted as \( V_x \) and \( U_y \), which need to be matched.

![Fig. 1. Procedure of eigenmode matching module.](image-url)
After being matched and updated, the eigenvalue matrix and the error are given as outputs. First, $T_i$ is obtained by multiplying $V_i$ and $U_i$. Criterion 1 is preferred to identify the type of value distribution of $T_i$ rapidly. All eigenmodes can be directly matched by (14) when the value of criterion 1 is smaller than $\varepsilon_1$. Alternatively, supposing that the value of criterion 1 is larger than $\varepsilon_2$, the error is considered to be over-size, and the analysis step should be regulated. Second, for those $T_i$ unsatisfied to criterion 1, the module intends to match the eigenmodes individually by criterias 2 and 3 coordinately.

The matching loop runs until all eigenmodes are matched or marked, and then the error of the module is calculated in (15).

$$
\Delta_k = \max \left( \frac{|\lambda_{k,i} - \lambda_{k-1,i}|}{\max_{i=1, \ldots, n} (|\lambda_{k,i}|, 10^{-4})} \right)
$$

$$
\begin{cases} 
\lambda_{k,i}, & \text{if} \; \Delta_k \in \overline{\lambda_{k,u}} \\
\lambda_{k-1,i}, & \text{if} \; \Delta_k \in \overline{\lambda_{k-1,u}}
\end{cases}
$$

(C) Framework with Regulation of Step Size

Based on the matching module, this paper proposes a framework to achieve the eigenmode matching along the trajectory. As shown in Fig. 2, the process of the framework contains two loop layers, i.e. the outer loop and the inner loop. $[h/h']$ is the round down of $h/h'$.

The outer loop: the integration results and state trajectory are obtained at first. The loop runs from 2 to $N$. In every loop, $A_{k-1,i}$ and $A_i$ are assigned to $A_i$ and $A_{k-1,i}$ respectively. The efforts in this loop are made to match the eigenmodes at $t_{k-1}$ and $t_k$.

The inner loop: one integration step is divided into $n'$ parts, and the inner step is denoted as $h'$. $A_i'$ and $A_{k-1,i}'$ are obtained by linear interpolation when $U_i'$ and $V_i'$ are set to be the right modal matrix and left modal matrix of them, respectively. The matching module is called in each inner loop while the matched eigenvalue matrix and the error $A_i$ are obtained. If $A_i$ is less than the threshold, the loop continues until $k'$ is larger than the round down of $h/h'$. Otherwise, too large value of $A_i$ leads to the interruption of the loop. When and only when the number of segments of one integration step is less than 10, we decrease the analysis step to $h''(2n')$ from $h''/n'$, and the inner loop is restarted from $k'=1$.

(D) Accuracy of Eigenmode Matching Technique

For the sake of demonstrating the effectiveness of the proposed mode matching method, an IEEE 10-machine 39-node system is established by Fortran software and analyzed by MATLAB software. The data of the system can be found in [30]. Each generator is equipped with a governor, as listed in Appendix C. The disturbance is three-phase transient faults at the head or the end of lines, with a fault clearing time $t_d$ ranging from 0.05 to 0.40 s with a 0.05 s step size. The simulation time is set to be 10 s with a 0.01 s step size.

One hundred and eighty of the total 232 cases are angularly stable, whereas the others lose angular stability within 10 s. Considering that the system loses angular stability once a DSP appears, the trajectories before DSPs of the 52 unstable cases are extracted. The errors of the above cases are depicted in Fig. 3 and counted in Table I.

![Fig. 3. Mis-matched error of cases.](image)

| Matching error (%) | Raw data | Processed data |
|-------------------|---------|---------------|
| 0.0-0.1           | 0       | 176           |
| 0.1-1.0           | 0       | 34            |
| 1.0-10.0         | 6       | 22            |
| 10.0-20.0       | 157     | 0             |
| 20.0-40.0      | 66      | 0             |
| 40.0-100.0     | 3       | 0             |

As shown in Fig. 3, the number of mis-matched sections...
of TEs have been reduced by at least one order of magnitude compared with the raw data before matching.

The maximum error of the raw data is 40.46%, while the average error is 18.36%. Correspondingly, the proposed matching technique reduces the maximum error and the average error to 3.70% and 0.24%, respectively. It is worth emphasizing that most error time sections locate near the fault occurring time, clearing time, and the DSP, which can be corrected manually with little efforts.

As a summary, the proposed method achieves accurate eigenmode matching, with the errors of test cases being reduced by at least one order of magnitude.

IV. MODE INTERACTIONS IN ULFO

A. Case Information and Special Phenomena

Special phenomena are observed in a two-machine test system, with the structure shown in Fig. 4.

![Fig. 4. Structure of two-machine test system.](image)

The classical model is used to represent the generators. G1 is a water turbine equipped with a governor, whereas G2 is a steam turbine. The dynamic equations are listed in Appendix D. The line data, generator data, and governor data are listed in Appendix E. The eigenvalues at the equilibrium point and the nonlinear index of normal forms technique are obtained in Table II. The expression of I1 can be found in Appendix F.

TABLE II

| Nomenclature | Real part | Imaginary part | EMCR  | I1  |
|--------------|-----------|----------------|-------|-----|
| \(\lambda_1\) | -16.0361  | +0.0000i       | 0.0095| 0.0011 |
| \(\lambda_2\) | -0.1145   | +3.5487i       | 19.4723| 0.0427 |
| \(\lambda_3\) | -0.1145   | -3.5487i       | 19.4723| 0.0427 |
| \(\lambda_4\) | -2.4511   | +0.0000i       | 0.2699| 0.0479 |
| \(\lambda_5\) | +0.0098   | +0.5414i       | 0.8044| 1.3134 |
| \(\lambda_6\) | +0.0098   | -0.5414i       | 0.8044| 1.3134 |
| \(\lambda_7\) | +0.0000   | +0.0000i       | 0.0860| 0.8993 |
| \(\lambda_8\) | -0.1187   | +0.0000i       | 0.0860| 0.8993 |

Note: EMCR stands for electromechanical mode correlation ratio; I1 is non-linear interaction index 1.

A couple of imaginary roots \((\lambda_5, \lambda_6)\) and two real roots \((\lambda_7, \lambda_8)\) are produced by the hydroelectric power governor. As seen in Table II, the positive values of \((\lambda_5, \lambda_6)\) contribute to negative damping, whereas the other eigenmodes enhance the damping of oscillation. At the equilibrium point, the lowest frequency is lower than 0.1 Hz and reaches 0.08 Hz in the non-electro-mechanical mode \((\lambda_5, \lambda_6)\). The other eigen-modes \((\lambda_7, \lambda_8)\) refer to electro-mechanical oscillation with a common frequency of 0.56 Hz. The damping of \((\lambda_7, \lambda_8)\) are positive, which means that these eigenmodes contribute to negative damping. Besides, \((\lambda_1, \lambda_2)\) are identified as the critical modes by index I1 [16].

The disturbance is set to be a three-phase transient fault at the head of line 2-3, and the duration time is 0.09 s. The equivalent center of inertia (COI) trajectory is obtained by trapezoidal integration.

An undamped oscillation of the relative rotor angle \((\delta_{\text{osc}})\) is depicted in Fig. 5. Two components \((\delta)\) with low frequency and ultra-low frequency are plotted in Fig. 6, distinguished by a Window Fourier Ridges (WFR) filter.

![Fig. 5. COI of rotor angles.](image)

![Fig. 6. Components of signal decomposition. (a) Trajectory of component 1. (b) Trajectory of component 2.](image)

As shown in Fig. 6 (a) and (b), the component with higher frequency naturally decays in 10 s and is re-excited after the 60th s, whereas the other component oscillates with an increasing amplitude before the 60th s. As can be expected, these two components in Fig. 6 can easily correspond with \((\lambda_7, \lambda_8)\) and \((\lambda_5, \lambda_6)\) at the equilibrium point, respectively. However, \((\lambda_7, \lambda_1)\) and \((\lambda_5, \lambda_2)\) can only explain the dynamic behaviors of the components in Fig. 6 before the 50th s. In the view of traditional methods, the mechanism of the undamped oscillation of two components and the re-excitation of component 1 is always implicitly described as the effects of system nonlinearities.

B. Analysis Based on Trajectory Eigenvalues

1) Re-excitation of Low Frequency Component

To demonstrate the evolutions of time-varying eigen-modes, the TEs are obtained and depicted in Fig. 7.

Considering that the real roots \(\lambda_1\) and \(\lambda_4\) represent two syn-
chronous components with exponential decay, the trajectories of these eigenmodes are omitted. Similarly, the trajectory of zero root $\lambda_1$ does not contribute to the complex dynamic behaviors of the oscillation. Therefore, the trajectories of important eigenmodes such as $\lambda_2$, $\lambda_3$, $\lambda_5$, and $\lambda_6$ are depicted in Fig. 7. The instantaneous damping and frequency of an eigenmode are denoted by $\sigma$ and $f$, respectively.

The trajectories of $(\lambda_5, \lambda_6)$ vary with time periodically, whereas $\lambda_8$ keeps a straight line parallel with the horizontal axis during 0-50 s and regularly jumps in the rest of the curve. With the reciprocating movements of the operation point between the dynamic center point (DCP) and the fast end point (FEP), the dynamic characteristics of $(\lambda_5, \lambda_6)$ fall somewhere between two poles of negative damping oscillation and exponential trend. It is worth emphasizing that the changes of $(\lambda_2, \lambda_3)$ and $(\lambda_5, \lambda_6)$, $\lambda_8$ are always synchronized.

With the operation point deviating from the equilibrium point, the characteristics of eigenmodes become more and more different from Table I. From the above discussion, it is clear that the damping characteristics of $(\lambda_2, \lambda_3)$, $(\lambda_5, \lambda_6)$, $\lambda_8$ are reflected by a pair of complementary sequences, which indicate the mode interactions between these oscillation eigenmodes. The motivation of $(\lambda_2, \lambda_3)$ after the 50th s can be ascribed to the mode interactions between $(\lambda_2, \lambda_3)$ and $(\lambda_5, \lambda_6)$.

2) Mode Interactions Between Low-frequency Mode and Ultra-low Frequency Mode

For the sake of illustrating purposes, the parts of the trajectories in Fig. 7(a) and (c) are zoomed and rearranged in Fig. 8.

With the movement of the operation point from the DCP to FEP, the real parts of $(\lambda_5, \lambda_6)$ significantly reduce to negative (such as in 75-80 s). In the next half-cycle, that process will go into reverse, and the real parts of $(\lambda_5, \lambda_6)$ rise to positive (such as in 80-85 s). When it comes to $(\lambda_2, \lambda_3)$, the trajectories change in opposite position. Notably, once the values of $(\lambda_2, \lambda_3)$ change a lot, the trajectories of $(\lambda_5, \lambda_6)$ would be distorted. In somewhere near FEPs (such as in 90-93 s), the distortions are so severe that $(\lambda_5, \lambda_6)$ transfer to two real roots, which represent exponential trends.

During the undamped oscillation after the 100th s, the trajectories of $(\lambda_2, \lambda_3)$ and $(\lambda_5, \lambda_6)$ become a complementary pair. Responding to the significant changes of the real parts of $(\lambda_2, \lambda_3)$, $(\lambda_5, \lambda_6)$ transfer to two real roots from a couple of imaginary roots, which means the oscillation mode change to a synchronous mode. The damping characteristics of $(\lambda_2, \lambda_3)$ and $(\lambda_5, \lambda_6)$ change periodically between positive and negative in the second part of the trajectory.

C. Analysis Based on Decoupling Components

The decaying of eigenmodes $(\lambda_2, \lambda_3)$ could be attributed to the negative values of their real part sequences. Once re-excited, the real parts of $(\lambda_2, \lambda_3)$ move to the positive direction.
of the vertical axis. The changes of $\phi_3, \phi_4, \phi_5$ are synchronized with $(\lambda_2, \lambda_3)$, which significantly indicate the interactions between $(\lambda_2, \lambda_3)$ and $(\lambda_3, \lambda_5)$. In order to illustrate the time-varying dynamic behaviors of the trajectory, the components of trajectory eigenmodes can be used to analyze the mode interactions.

In the test case, $\delta_{\infty} = \delta_1 - \delta_2$ can be used to describe the time-varying dynamic characteristics. According to (9), the trajectory of $\delta_{\infty}$ can be decoupled by

$$\delta_{\infty}(t) = \phi_1(t) + \phi_2(t) + \ldots + \phi_{10}(t) - (\phi_1(t) + \phi_2(t) + \ldots + \phi_8(t))$$

where $\phi_1(t), \phi_2(t), \ldots, \phi_8(t)$ are the components linearly composed of $\phi_3(t)$ for instance, $\phi_1(t) = \phi_1(t) - \phi_3(t)$ and the rest can be obtained in the same manner. $\phi_1(t)$ and $\phi_2(t)$ represent the components decaying exponentially, manifesting as the lines overlapped with the horizontal axis. $\phi_3(t)$ and $\phi_4(t)$ represent a couple of complex eigenmodes, whose superpositions describe the dynamic behavior of the low-frequency mode. Another couple of complex roots are described by $\phi_5(t)$ and $\phi_6(t)$ in time domain. The superposition of $\phi_3(t)$ and $\phi_4(t)$ describes the dynamic behavior of the ultra-low frequency mode.

Figure 9 depicts the decoupling of eigenmodes in time domain. $\phi_3(t) + \phi_5(t)$, $\phi_4(t) + \phi_6(t)$, and $\phi_3(t)$ are plotted, respectively.

![Fig. 9. Components of TE mode. (a) Trajectory of $\phi_3(t)$+$\phi_5(t)$. (b) Trajectory of $\phi_3(t)$+$\phi_6(t)$. (c) Trajectory of $\phi_3(t)$](image)

In Fig. 9(a), the trajectory of $\phi_3(t) + \phi_5(t)$ mainly exhibits a convergent trend in 0-50 s, consistent with the real parts of $(\lambda_2, \lambda_3)$ in Fig. 7. Near FEPs, short-term impacts can be found in the trajectory after $(\lambda_2, \lambda_3)$ are re-excited.

In Fig. 9(b), the trajectory of $\phi_4(t) + \phi_6(t)$ performs to be a divergent oscillation with gradually increasing amplitude in the first half, whereas high-peaks and short-term impacts are observed near FEPs in the second half. The divergent trend is suppressed, and the undamped oscillation is observed after the 100th s. Moreover, when $(\lambda_3, \lambda_6)$ transfer to two real roots from a couple of complex roots in Fig. 7, the trajectory of $\phi_3(t) + \phi_5(t)$ changes to components decaying exponentially or synchronously unstably. Therefore, the changes of eigenmodes are owing to these peaks, which are affected by mode interactions.

In Fig. 9(c), the trajectory of $\phi_5(t)$ can be described by a straight line overlapped to the horizontal axis near equilibrium points. However, Fig. 7 depicts that $\lambda_5$ is a synchronous eigenmode that strongly interacts with $(\lambda_2, \lambda_3)$. As a result, peaks appear periodically in the trajectory of $\phi_5(t)$, consistent with the distortion of $\phi_5(t) + \phi_6(t)$.

The relationship between $\phi_3(t)$ and $\phi_5(t) + \phi_6(t)$ can be observed by the trajectory of $\phi_3(t) + \phi_5(t) + \phi_6(t)$ in Fig. 10.

![Fig. 10. Recombination of eigenmode trajectories. (a) Trajectory of $\phi_3(t)$+$\phi_5(t)$. (b) Trajectory of $\phi_3(t)$+$\phi_5(t)$+$\phi_6(t)$](image)

Obviously, the peaks of $\phi_5(t) + \phi_6(t)$ and $\phi_6(t)$ are offset each other. The superposition of their trajectories performs to be a complementary component of $\phi_3(t) + \phi_5(t)$. By adding Fig. 10(a) and (b), the original trajectory in Fig. 5 can be reproduced.

As a result, $\phi_3(t)$ is deemed to be interacted with $\phi_3(t) + \phi_5(t)$ at first, while the mode interaction between $\phi_3(t) + \phi_5(t)$ and $\phi_3(t) + \phi_5(t) + \phi_6(t)$ is later.

Comparing with Fig. 6, more accurate approximation can be given by the TE theory.

D. Analysis Based on Transfer Matrices

The sequences of elements of the transfer matrix are depicted in Fig. 11. Without mode interactions, the diagonal figures of Fig. 11 would be horizontal lines equal to 1, while the others are equal to zero. Apparently, the non-one values of diagonal figures and non-zero values of off-diagonal figures show the critical time of mode interactions. From Fig. 11, it can be easily observed that critical points appear highly frequently in the 5th row and the 6th row.

For illustrating purposes, the 5th row of Fig. 11 is zoomed in Fig. 12.

From the 91th s, the operation point runs near FEP, and the transfer coefficient between $\lambda_3$ and $\lambda_6$ increases in Fig. 12 (d). After the 91.31th s, $\lambda_5$ and $(\lambda_2, \lambda_3)$, $\lambda_5$, $\lambda_3$ have strong interactions. In the 91.31th s time section, the transfer coefficients between $\lambda_5$ and the above four eigenmodes suddenly rise to the local maximum values. As can be expected, $\lambda_5$ is motivated in the 91.31th s time section.
In the next 91.32 th s time section with the self-coefficient and co-coefficients decreased, the mode interactions are smoothed. At the same time, \((\lambda_3, \lambda_5)\) change from a couple of complex roots to two real roots in the 91.31 th s time section. The effects of nonlinearities are reflected by mode interactions significantly in this time section when mode interactions appear again with the reverse direction in the 92.04 th s time section. At that time, \((\lambda_3, \lambda_5)\) change back to a couple of complex roots, and the operation point runs back to DCPs.

Therefore, the mode interactions between \((\lambda_3, \lambda_5)\) and other eigenmodes are the main reasons for the undamped oscillation and re-excitation of \((\lambda_2, \lambda_3)\).

In order to verify the effectiveness of the above analysis, two nonlinear interaction indices of normal forms technique (I1 and I2) are utilized [16]. As is well known, the value of I1 is often used to identify the dominant nonlinear eigenmode, while the module value of I2 can describe the nonlinear effects from other eigenmodes to one eigenmode [31]. The expression of I2 can be found in Appendix F.

In Table II, \(\{\lambda_2, \lambda_5\}\) are identified to be the critical modes with the maximum I1. The nonlinear interaction index I2 of \(\lambda_2\) and \(\lambda_5\) are obtained in Table III. Apparently, mode interactions can be observed between \(\{\lambda_2, \lambda_3\}\) and \(\lambda_3, \lambda_5, \lambda_8\), while \(\{\lambda_2, \lambda_5\}\) are most impacted by \(\lambda_3, \lambda_5, \lambda_2\). The indices of normal form technique verify the effectiveness of the analysis in Section IV.

### Table III

| \(\lambda_2\) | Related eigenmodes | \(\lambda_5\) | Related eigenmodes |
|--------------|--------------------|--------------|--------------------|
| -0.5699+0.1765i | \(\lambda_2, \lambda_5\) | 3.3321+1.6251i | \(\lambda_3, \lambda_5\) |
| -0.5865+0.1097i | \(\lambda_2, \lambda_5\) | -3.4412−1.3791i | \(\lambda_2, \lambda_5\) |
| 0.3928−0.0668i | \(\lambda_2, \lambda_5\) | \(\lambda_2, \lambda_5\) | \(\lambda_2, \lambda_5\) |

Note: I2 is nonlinear interaction index 2.
V. CONCLUSION

Based on the TE theory, this paper proposes a method to analyzes the evolution mechanism of two special phenomena in ULFO. First, the trajectory is decoupled into n time-varying components. Second, the mode interactions and inheritance relationships of eigenmodes are analyzed by the transfer matrices.

The model-based and the trajectory-based paradigm are integrated by the TE theory. The trajectories obtained by numerical integration guarantee the reliability of data, while the piece-wise linearized models make it possible to decouple eigenmodes. Thus, the theory has extensive prospects in the analysis of mode interactions.

1) Decoupling of time-varying eigenmodes

The eigenmodes are decoupled through the proposed eigenmode matching technique. The superposition of n time-varying components can approximate the response of the original system. In the simulation of this paper, the analysis results of the proposed decoupling method are very accordant to that of the WFR method. A trajectory of ULFO is divided into two components according to the frequencies. Unlike the signal analysis approaches, the proposed method can provide a further description of the instantaneous oscillation characteristics, without the limitation of window width. Meanwhile, the time-varying eigenmodes and the parameters of the system can be associated by the TEs.

2) Eigenmode re-excitation and undamped oscillation

Under the influences of nonlinearities, some eigenmodes may be self-motivated or co-motivated. The positive damping characteristics of decayed eigenmodes may transfer to negative damping.

With the operation point deviating from the equilibrium point, nonlinearities always engender strong mode interactions. Decoupled components of eigenmodes are distorted near the equilibrium point, and the characteristics of the oscillation tend to be consistent with the eigen-analysis at the equilibrium point.

During the periodical pendulum movements of the operation point, the increasing amplitude of the relative rotor angle enlarges the proportion of the nonlinear region away from the equilibrium point.

In the first half of the oscillation, the positive damping in the linear region of the low-frequency mode sufficiently eliminates the effects of negative damping in the nonlinear region, making the decoupled component of the low-frequency mode convergent. When the negative damping dominates the dynamic behaviors, the component of the low-frequency mode begins to be re-excited. In the second half of the oscillation, strong mode interactions construct a new balance condition among several eigenmodes. Usually, the transformation between positive damping and negative damping balances out in one swing.

3) Eigenmode matching technique

Typical cases verify the effectiveness and accuracy of the proposed eigenmode matching technique. Compared with the original data, the error is reduced at least one order of magnitude.

VI. DISCUSSION AND FUTURE WORKS

1) Trajectory eigenvalues in practice

Despite many theoretical achievements of TE theory that have been made, the computation amounts of TEs in large power systems prohibit wider acceptance of the novel theory. Hence, the TE theory is still under investigation as well in methodological aspects as in concrete applications. Future explorations can be made by reduction of system order or polymerization of trajectories.

2) Relationship with transient stability analysis (TSA)

Generally, the dynamic behavior of a power system with large disturbances is regarded as a TSA problem. However, the oscillation characteristics before a system losing angular stability are ignored by most TSA approaches. The TE theory can be utilized to study the oscillation characteristics along the trajectory. As for the transient stability problem, multiple mature methods can be nominated, such as extended equal area criterion (EEAC).

APPENDIX A

According to the eigen-analysis, in any $t \in [t_k, t_{k+1})$, and its right eigenvector $u_k$, should satisfy:

$$A_k u_k = \lambda_k u_k, \quad i = 1, 2, \ldots, n \quad (A1)$$

The $n \times n$ modal matrix is defined as:

$$U_k = [u_{k,1}, u_{k,2}, \ldots, u_{k,i}, \ldots, u_{k,n}] \quad (A2)$$

Let $V_k = [v_{k,1}, v_{k,2}, \ldots, v_{k,i}, \ldots, v_{k,n}] = U_k^{-1}$, then $A_k U_k = U_k A_k, \quad U_k^{-1} A_k U_k = \Lambda_k$. New variables can be defined as a linear transformation:

$$Z_k(\Delta t) = V_k \Delta X_k(\Delta t) \quad (A3)$$

Both sides of (4) are left-multiplied by $V_k$.

$$Z_k(\Delta t) = A_k Z_k(\Delta t) + V_k B_k \quad (A4)$$

where $A_k = V_k A_k U_k$. As the $i^{th}$ element of $Z_k(\Delta t), z_k(\Delta t)$ can be assumed to be the general solution of (A4).

$$z_k(\Delta t) = c_{k,1} e^{\lambda_{k,1} \Delta t} + d_{k,1} \quad (A5)$$

By substituting (A4) into (A3) we have $d_{k,1} = -(1/\lambda_{k,1}) v_{k,1} B_k$ and $c_{k,1} = -d_{k,1}$, rearranging (A5) as the matrix form (A6).

$$Z_k(\Delta t) = C_k E_k(\Delta t) + D_k \quad (A6)$$

where $C_k = \text{diag}(c_{k,1}, c_{k,2}, \ldots, c_{k,i}, \ldots, c_{k,n}), \quad D_k = [d_{k,1}, d_{k,2}, \ldots, d_{k,1}, \ldots, d_{k,n}]^T, \quad E_k(\Delta t) = [e^{\lambda_{k,1} \Delta t}, e^{\lambda_{k,2} \Delta t}, \ldots, e^{\lambda_{k,i} \Delta t}]$. Both sides of (A6) are left-multiplied by $U_k$.

$$\Delta X_k(\Delta t) = U_k C_k E_k(\Delta t) - U_k C_0 E_k(0) \quad (A7)$$

Thus, the solutions of (4) are given in the time domain.

APPENDIX B

For general matrices, Ostrowski proved that the eigenvalues are continuous with the varying parameters in 1960 and derived the theorems:

Let $A = (a_k) \in C^{n \times n}, \quad B = (b_k) \in C^{n \times n}, \quad \lambda(A) = \{\lambda_i\}, \quad \lambda(B) = \{\rho_i\}$,


then

\[
\begin{align*}
\|B-A\|_a &= \frac{1}{n+1} \sum_{y=1}^n \sum_{y'=1}^n |b_{yy'} - a_{yy'}| \\
n &= (n+2)m_a^{1/n} \|B-A\|_a^{1/n}
\end{align*}
\]

For any \( \rho \in \lambda(B) \), there exists \( \lambda_+(\rho) \in \lambda(A) \) to satisfy:

\[
\left| \lambda_+(\rho) - \rho \right| < \rho \quad (B2)
\]

Moreover, a proper series of 1, 2, ..., \( n \) can be found as \( \pi(1), \pi(2), \ldots, \pi(n) \) to obtain (B3).

\[
\left| \lambda_+(\rho) - \rho \right| < (2n-1)\rho \quad i = 1, 2, \ldots, n
\]

As a result, once \( B \rightarrow A \), the eigenvalues of \( B \) and \( A \) are the same.

**APPENDIX C**

**TABLE CI**

**GOVERNOR DATA FOR IEEE 10-MACHINE 39-BUS SYSTEM**

| Governor No. | \( K_p \) (p.u.) | \( R \) (p.u.) | \( T_d \) (s) | \( T_p \) (s) |
|--------------|-----------------|----------------|--------------|--------------|
| GH30         | 1.0000          | 0.0300         | 0.4000       | 0.0500       |
| GH31         | 1.0000          | 0.0300         | 0.4000       | 0.0500       |
| GH32         | 18.0000         | 0.0300         | 0.4000       | 0.0500       |
| GH33         | 15.0000         | 0.0300         | 0.4000       | 0.0500       |
| GH34         | 13.0000         | 0.0300         | 0.4000       | 0.0500       |
| GH35         | 15.0000         | 0.0300         | 0.4000       | 0.0500       |
| GH36         | 20.0000         | 0.0300         | 0.1000       | 0.0800       |
| GH37         | 20.0000         | 0.0300         | 0.1000       | 0.0800       |
| GH38         | 12.0000         | 0.0300         | 0.4000       | 0.0500       |
| GH39         | 1.0000          | 0.0300         | 0.4000       | 0.0500       |

**Appendix D**

**TABLE EII**

**BUS DATA FOR IEEE 2-MACHINE 4-BUS SYSTEM**

| Type     | Node amplitude | Load active power | Load reactive power | Generator active power | Generator reactive power |
|----------|----------------|-------------------|---------------------|------------------------|-------------------------|
| Bus1     | 1.21 \( \circ \) | 0.00              | 0.00                | 1.00                    | -                       |
| Bus2     | 1.00 \( \circ \) | 1.86              | 0.00                | -                      | -                       |
| Bus3     | 1.00 \( \circ \) | 0.00              | 0.00                | -                      | -                       |
| Bus4     | 1.00 \( \circ \) | 0.00              | 0.00                | -                      | -                       |

**APPENDIX E**

**TABLE EII**

**LINE DATA FOR IEEE 2-MACHINE 4-BUS SYSTEM**

| Type | Start point | Termination point | Resistance (p.u.) | Reactance (p.u.) | Ground capacitance (p.u.) |
|------|-------------|-------------------|------------------|-----------------|--------------------------|
| Line b | 1           | 2                 | 0.0000           | 0.1380          | 0.0000                   |
| Line c | 2           | 3                 | 0.0000           | 0.2430          | 0.0000                   |
| Line d | 3           | 4                 | 0.0000           | 0.1220          | 0.0000                   |

**APPENDIX F**

**TABLE III**

**GENERATOR DATA FOR IEEE 2-MACHINE 4-BUS SYSTEM**

| Generator No. | Inertia (s) | \( X_d' \) (p.u.) | \( X_q' \) (p.u.) | \( D \) (p.u.) |
|---------------|-------------|-------------------|-------------------|--------------|
| G1 (Bus1)    | 44.0000     | 0.295             | 0.295             | 3.0000       |
| G2 (Bus4)    | 44.0000     | 0.295             | 0.295             | 0.0000       |
The definitions of two nonlinear interaction indices are listed as follows:

$$II(j) = \left| \left(y_{j\omega} - z_{j\omega}\right) + \max_{i,j} h_{ij}\frac{\partial z_{j\omega}}{\partial z_{i\omega}} \right| $$  \hspace{1cm} (F1)

$$I2(j) = \max_{i,j} h_{ij}\frac{\partial z_{j\omega}}{\partial z_{i\omega}} $$  \hspace{1cm} (F2)

The definition of the variables in (F1) and (F2) can be found in [16], which are widely recognized in normal form technique.

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