Distributed Consensus Control Protocols for Heterogeneous Multi-Agent Systems With Time-Varying Topologies

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ABSTRACT Distributed group consensus control problems of heterogeneous multi-agent systems with time-varying topologies and communication delay are investigated in this paper. Two different types of control protocols without relative velocity information are designed. By reformulating the original problem and utilizing the property of Metzler matrix, sufficient group consensus criteria under some assumptions for the two proposed protocols are developed, respectively. Simulation examples are exploited to illustrate the results.

INDEX TERMS Group consensus control, heterogeneous multi-agent systems, time-varying topologies, communication delay.

I. INTRODUCTION
In recent decades, the distributed cooperative control of multi-agent systems (MASs) has been widely studied due to the potential applications on sensor networks, rendezvous, satellites cluster, and large-scale scientific installations [1]-[11]. As a basic problem of MASs, consensus is the intersection of graph theory and control system theory, which involves a series of agents to achieve common goals through information exchange and sharing.

In the literature, most works concentrate more on homogeneous systems, which means all agents are in a same dynamic state [12]-[18]. For example, in [12], Olfati initially designed three groups of consensus protocols for networked first-order MASs, and showed that balanced directed graphs are important for achieving average-consensus. Based on [12], in [13], Ren investigated second-order MASs and found a necessary consensus seeking criterion is that the corresponding graph is connected. In [14], Yu considered second-order consensus problems under directed topology and communication delay, and established an explicit equation between the maximum tolerable delay and the eigenvalues of the corresponding Laplacian matrix. In [16], Tian studied MASs with both communication and input delays in directed networks, and found communication delays have no effect whether the systems can reach consensus. In [17], Lin considered the second-order networks with and without time delay, respectively, and obtained the sufficient and necessary criteria to reach consensus for each system. In [18], Hou investigated a class of general second-order delay MASs, and got an explicit equation for maximum tolerable delay.

In real world applications, agents in complex control systems inevitably have different dynamic characteristics. Compared with homogeneous MASs, heterogeneous MASs are more realistic. Therefore, the consensus control of heterogeneous MASs has been intensively studied. As one of the key extended issues of consensus, group consensus requires agents in the same subgroup can ultimately achieve a same athletic behavior and there is no consensus in different subgroups when information exchange not only exists between one subgroup but also between different subgroups. In [19], Yu firstly introduced a group consensus protocol with undirected topology, and so far, there are many literatures on group consensus. In [20], Mojeed investigated a general first-order system and presented some new group consensus conditions based on the relationships between each subgroup and subgroups. In [21], Xie investigated the group consensus control problems of second-order MASs with identical...
time delays. In [22], Feng investigated two group consensus protocols for delay-free discrete-time heterogeneous MASs. In [23], Ji proposed a novel couple-group consensus protocol and showed that the consensus of the system is independent of the communication delay. In [24]–[26], group consensus control of heterogeneous MASs with communication delays and input delays were investigated. As far as we know, there are rare works on group consensus control of heterogeneous MASs with time-varying topologies, especially with time-varying communication delay.

Motivated by this fact, this paper investigates the group consensus problems for heterogeneous MASs with time-varying topologies and communication delay. Comparing with the works in [21] and [22], this paper investigates the heterogeneous systems with time delay, which makes the systems more realistic and more challenging. In addition, different from [23]–[26] in which the delays are time-invariant, time-varying communication delay, directed topologies and control parameters are considered.

The main contributions of this paper are summarized as follows. Firstly, two novel group consensus protocols for heterogeneous MASs with time-varying delays are proposed. Particularly, the second protocol is more general than the first one. Secondly, we not only consider the fixed topology with uniform control parameters, but also consider the time-varying topologies and multiple control parameters, which is more realistic in engineering practice. Last but not least, under some mild assumptions and by using the property of Metzler matrix, we show that the group consensus can be achieved with arbitrary bounded communication delays even if the digraph has no spanning tree.

II. GRAPH THEORY
Let $\mathcal{G} = (\mathcal{V}, \varepsilon, \mathcal{A})$ be a directed graph, where $\mathcal{V} = \{s_1, \ldots, s_n\}$ represents the vertex set, $\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$ represents the edge set, and $\mathcal{A} = [a_{ij}]_{n \times n}$ represents the weighted adjacency matrix ([27]). $\mathcal{L} = \{1, 2, \ldots, n\}$ and $e_{ij} = (s_i, s_j)$ denote the node indexes set and the edge $e_{ij}$ in the digraph, respectively. $\mathcal{N}_i = \{s_j \in \mathcal{V} : (s_i, s_j) \in \varepsilon\}$ represents the neighbours of agent $i$. Moreover, the elements of adjacency matrix $\mathcal{A}$ are denoted as $a_{ii} = 0$ and $a_{ij} > 0$ if $e_{ij} \in \varepsilon$.

In digraph $\mathcal{G}$, if for every pair of $s_i, s_j, i \neq j$, there always exists a path from $s_i$ to $s_j$ and a path from $s_j$ to $s_i$, then digraph $\mathcal{G}$ is said to be strongly connected. A digraph contains a spanning tree if there exists a vertex called root vertex such that there exists a directed path from it to every other vertex. $L_n = [l_{ij}]_{n \times n}$ denotes the Laplacian matrix of digraph $\mathcal{G}$, where

$$l_{ij} = \begin{cases} \sum_{h=1, h \neq i}^{n} a_{ih}, & i = j \\ a_{ij}, & i \neq j \end{cases}$$

According to the definition of $L_n$, we have $L_n \mathbf{1}_n = \mathbf{0}$, where $\mathbf{1}_n = (1, \ldots, 1)^T \in \mathbb{R}^n$.

**Lemma 1:** [27] A matrix $\Psi(t)$ is called to be a Metzler matrix if its off-diagonal elements are nonnegative.

III. PROBLEM FORMULATION
Consider a heterogeneous MAS consisting of $m$ first-order agents and $n - m$ second-order agents. The dynamic equation of the first-order agent is given by

$$\dot{x}_i(t) = u_i(t), \forall i \in \ell_1$$

where $\ell_1 = \{1, 2, \ldots, m\}, x_i(t) \in \mathbb{R}$ is the position state of the $i$th agent and $u_i(t) \in \mathbb{R}$ is its control input, $i \in \ell_1$. The dynamic equation of the second-order agent is given by

$$\begin{align*} 
\dot{x}_i(t) &= v_i(t) \\
\dot{v}_i(t) &= u_i(t), & \forall i \in \ell_2
\end{align*}$$

where $\ell_2 = \{m + 1, m + 2, \ldots, n\}, x_i(t) \in \mathbb{R}$ and $v_i(t) \in \mathbb{R}$ represent the position state and velocity state of $i$th agent, $u_i(t) \in \mathbb{R}$ is its control input, $i \in \ell_2$.

**Definition 1:** Given the heterogeneous systems with $m$ first-order agents (2) and $n - m$ second-order agents (3), the system can achieve group consensus with protocol $u_i(t)$, if and only if, for any initial condition, the following conditions hold:

\begin{enumerate}
\item[(i)] $\lim_{t \to \infty} |x_i(t) - x_j(t)| = 0, \forall i, j \in \ell_1$
\item[(ii)] $\lim_{t \to \infty} |v_i(t) - v_j(t)| = 0, \forall i, j \in \ell_2$
\item[(iii)] $\lim_{t \to \infty} |v_i(t) - v_j(t)| = 0, \forall i, j \in \ell_2$
\end{enumerate}

IV. MAIN Results
In this section, two control protocols are developed. Particularly, the second protocol is more general than the first one in expression.

A. GROUP CONSENSUS PROTOCOL I
In this subsection, the group consensus problem for heterogeneous MAS with time-varying topologies and communication delay will be discussed. In particular, the following control protocol is proposed:

$$
\begin{align*}
\dot{u}_i(t) &= k_1 \sum_{j \in \ell_1} a_{ij} \left( \alpha_1 x_j(t - \tau_{ij}(t)) - \alpha_1 x_i(t) \right) \\
&\quad + k_2 \sum_{j \in \ell_2} a_{ij} \left( \alpha_2 x_j(t - \tau_{ij}(t)) - \alpha_2 x_i(t) \right), & \forall i \in \ell_1
\end{align*}
$$

(4)

$$
\begin{align*}
\dot{u}_i(t) &= k_1 \sum_{j \in \ell_1} a_{ij} \left( \alpha_1 x_j(t - \tau_{ij}(t)) - \alpha_1 x_i(t) \right) \\
&\quad + k_2 \sum_{j \in \ell_2} a_{ij} \left( \alpha_2 x_j(t - \tau_{ij}(t)) - \alpha_2 x_i(t) \right) \\
&\quad - k_3 \alpha_2 v_i(t), & \forall i \in \ell_2
\end{align*}
$$

(5)

where $k_1, k_2, k_3, \alpha_1, \alpha_2$ are all positive constants and represent scaling parameters, $\tau_{ij}(t) \geq 0$ is the communication delay from $j$th agent to $i$th agent. By substituting (4) and (5)
into system (2) and system (3), respectively, it follows that
\[
\dot{x}_i(t) = k_1 \sum_{j \in \mathcal{E}_1} a_{ij} \left( \alpha_1 x_j(t - \tau_{ij}(t)) - \alpha_1 x_i(t) \right) \\
+ k_1 \sum_{j \in \mathcal{E}_2} a_{ij} \left( \alpha_2 x_j(t - \tau_{ij}(t)) - \alpha_2 x_i(t) \right), \quad \forall i \in \mathcal{E}_1
\]
(6)
\[
\dot{x}_i(t) = \psi_i(t)
\]
\[
\dot{\psi}_i(t) = k_2 \sum_{j \in \mathcal{N}_i} a_{ij} \left( \alpha_2 \dot{x}_j(t - \tau_{ij}(t)) - \alpha_2 \dot{x}_i(t) \right) + k_2 \sum_{j \in \mathcal{E}_1} a_{ij} \left( \alpha_1 \dot{x}_j(t - \tau_{ij}(t)) - \alpha_1 \dot{x}_i(t) \right) \\
- k_3 a_{2i} \psi_i(t), \quad \forall i \in \mathcal{E}_2
\]  
(7)

To proceed further, we denote
\[
\dot{x}_i(t) = \alpha_1 x_i(t), \quad \forall i \in \mathcal{E}_1
\]  
(8)
\[
\dot{x}_i(t) = \alpha_2 \dot{x}_i(t), \quad \psi_i(t) = \alpha_2 v_i(t), \quad \forall i \in \mathcal{E}_2
\]  
(9)

Then, by taking the derivative of (8) and (9) with respect to \( t \), we obtain
\[
\dot{\xi}_i(t) = \alpha_1 k_1 \sum_{j \in \mathcal{N}_i} a_{ij} \left( \dot{x}_j(t - \tau_{ij}(t)) - \dot{x}_i(t) \right), \quad \forall i \in \mathcal{E}_1
\]  
(10)
and
\[
\dot{\xi}_i(t) = \psi_i(t)
\]
\[
\dot{\psi}_i(t) = \alpha_2 k_2 \sum_{j \in \mathcal{N}_i} a_{ij} \left( \dot{x}_j(t - \tau_{ij}(t)) - \dot{x}_i(t) \right) \\
- \alpha_2 k_3 \dot{\psi}_i(t), \quad \forall i \in \mathcal{E}_2
\]  
(11)

where \( \mathcal{N}_i \) is the neighbors of agent \( i \).

In what follows, the group consensus problem of the heterogeneous MAS will be discussed by examining the properties of Metzler matrix. For this, we denote \( \eta_i(t) = \frac{\alpha_2 k_3}{2} \psi_i(t) + \xi_i(t) \), and system (11) becomes
\[
\dot{\xi}_i(t) = \frac{\alpha_2 k_3}{2} \left( \eta_i(t) - \xi_i(t) \right)
\]
\[
\dot{\eta}_i(t) = \frac{2 k_2}{k_3} \sum_{j \in \mathcal{N}_i} a_{ij} \left( \xi_j(t - \tau_{ij}(t)) - \xi_i(t) \right) \\
- \frac{\alpha_2 k_3}{2} \left( \eta_i(t) - \xi_i(t) \right), \quad \forall i \in \mathcal{E}_2
\]  
(12)

Then, we define
\[
\xi(t) = [\xi_1^T(t), \xi_2^T(t), \ldots, \xi_M^T(t)]^T
\]  
(13)
where
\[
\xi_i(t) = [\xi_{i1}(t), \xi_{i2}(t), \cdots, \xi_{n_i}(t)]^T
\]
\[
\xi_{i}(t) = [\xi_{i1}(t), \xi_{i2}(t), \cdots, \xi_{n_i}(t)]^T
\]
\[
\eta_{i}(t) = [\eta_{i1}(t), \eta_{i2}(t), \cdots, \eta_{n_i}(t)]^T
\]

If there is no communication delay, i.e., \( \tau_{ij}(t) = 0 \), then, by considering (13), (10) and (12) becomes
\[
\dot{\xi}(t) = \Psi(t) \xi(t)
\]  
(14)

where \( I_n \) is an \( n \)-dimensional identity matrix and \( \Theta \) is a zero matrix with appropriate dimension, and
\[
\Psi(t) = \begin{bmatrix}
-a_1 k_1 L_{11} & a_1 k_1 A_{12} & 0_m \\
0_m & -a_2 k_2 L_{22} & 0_{n-m} \\
L_{21} & a_2 k_2 L_{12} & 2 k_2 L_2 - \frac{2 k_2}{k_3} I_{n-m}
\end{bmatrix}
\]  
(15)

where \( L_{ij}(t) = L_1(t) + D_1(t), \quad \tilde{L}_2(t) = L_2(t) + D_2(t); \quad L_1(t) \) and \( L_2(t) \) are the Laplacian matrices of first-order agents and second-order agents, respectively; \( D_1(t) \) and \( D_2(t) \) are described by
\[
D_1(t) = \text{diag} \left\{ \sum_{j \in \mathcal{E}_2} a_{ij}, i \in \mathcal{E}_1 \right\}
\]
\[
D_2(t) = \text{diag} \left\{ \sum_{j \in \mathcal{E}_1} a_{ij}, i \in \mathcal{E}_2 \right\}
\]

and the adjacency matrix of the heterogeneous MAS is given by
\[
A = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix} \in \mathbb{R}^{n \times n}
\]

where \( A_1 \in \mathbb{R}^{m \times m} \) and \( A_2 \in \mathbb{R}^{(n-m) \times (n-m)} \). In fact, \( \Psi(t) \) is a Metzler matrix, which will be shown later.

Similarly, if the communication delay exists, suppose there are altogether \( M \) different communication delays, which denoted by \( \tau_e(t) \in \{ \tau_{ij}(t), i, j \in \mathcal{E}_1 \cup \mathcal{E}_2 \} \). Then system (14) becomes
\[
\dot{\xi}(t) = \Psi_0(t) \xi(t) + \sum_{\varepsilon=1}^{M} \Psi_{\varepsilon}(t) \xi \left( t - \tau_{\varepsilon}(t) \right)
\]  
(16)

where \( \Psi_0(t) = \text{diag} \left( \Psi(t) \right) \) and the \( j \)th element of \( \Psi_\varepsilon(t) \), \( \varepsilon = 1, 2, \cdots, M \), is either zero or equal to the weight of edge \( e_{ij} \).

Obviously, \( \Psi(t) = \Psi_0(t) + \sum_{\varepsilon=1}^{M} \Psi_{\varepsilon}(t) \).

Then, we shall show that \( \Psi(t) \) is a Metzler matrix with the following assumption.

**Assumption 1:**
\[
\sum_{j \in \mathcal{N}_i} a_{ij} \leq \frac{k_2^2 \alpha_2}{4 k_2}(i \in \mathcal{E}_1 \cup \mathcal{E}_2)
\]

**Theorem 1:** Under Assumption 1, \( \Psi(t) \) is a Metzler matrix with zero row sums.

**Proof:** Since \( k_1 > 0, k_2 > 0, k_3 > 0, \alpha_1 > 0, \alpha_2 > 0 \), then we can verify that the off-diagonal elements of \( \Psi(t) \) are nonnegative under Assumption 1. Note that the Laplacian matrix of the heterogeneous MAS
\[
L(t) = \begin{bmatrix} \tilde{L}_1(t) & -A_{12}(t) \\ -A_{21}(t) & \tilde{L}_2(t) \end{bmatrix}
\]

then we can prove that the summation of each row of \( \Psi(t) \) is equal to zero. This completes the proof. \( \square \)
Remark 1: If there is no communication delay, the heterogeneous system can be described as system (14). From Theorem 1, we can verify that $\Psi(t)$ is a Metzler matrix with zero row sums. Then, by applying Lemma 1 of [28], we conclude that system (14) can reach asymptotic consensus.

Theorem 2: For any given constants $\sigma_1 > 0$ and $\sigma_2 > 0$, suppose that every digraph of the system (16) keep fixed at $\sigma_1$ time, the union of the digraphs has a spanning tree at every bounded time interval $T$, $T < \sigma_2$. Assumption 1 holds for system (16), and time delay is bounded. Then, under protocols (4) and (5), heterogeneous systems (2) and (3) can reach group consensus.

Proof: For solving problem (16), we construct the following Lyapunov function

$$
V(\xi_t) = \max_{\sigma \in \{t - \tau_{\max}, t\}} \{\xi_1(\sigma), \xi_2(\sigma), \ldots, \xi_{2n-m}(\sigma)\} - \min_{\sigma \in \{t - \tau_{\max}, t\}} \{\xi_1(\sigma), \xi_2(\sigma), \ldots, \xi_{2n-m}(\sigma)\}
$$

where $\xi_i(\sigma)$, $i = 1, 2, \ldots, 2n - m$, is the $i$th component of $\xi(\sigma)$. $\xi_i$ represents the time function $\xi(\sigma)$ on the interval $[t - \tau_{\max}, t]$, and $\tau_{\max} = \max_{1 \leq i \leq 2n-m} \tau_i(t)$.

Let $\chi_{\max}^i = \max\{f_1(\sigma), f_2(\sigma), \ldots, f_{2n-m}(\sigma)\}$, $\chi_{\min}^i = \min\{f_1(\sigma), f_2(\sigma), \ldots, f_{2n-m}(\sigma)\}$, where $f$ is the solution of (16) defined on the time interval $[\sigma_1, \sigma_2]$. Thus, function (17) at time $\sigma_1$ becomes $V(f(\sigma)) = \chi_{\max}^i - \chi_{\min}^i$.

To proceed, we introduce a new graph $G$ with the node set $N = \{1, 2, \ldots, 2n - m\}$, and the corresponding Laplacian matrix is $P(t) = [p_{ij}(t)]_{(2n-m) \times (2n-m)}$. Consider two nonempty disjoint sets $E$ and $F$, where $E$ is the set of the union digraph $G(\sigma_1, \sigma_2)$ and $F = \hat{L} - E$. Let $h_{\text{EFF}} = \sum_{i \in E, j \in F} p_{ij}(t)dt$, $h_{\text{EFF}} = \sum_{i \in E, j \in F, i \neq j} p_{ij}(t)dt$. Let $\sigma_{\max}^i = \max_{\sigma \in \{t - \tau_{\max}, t\}} \{f_1(\sigma), f_2(\sigma), \ldots, f_{2n-m}(\sigma)\}$ and $\sigma_{\min}^i = \min_{\sigma \in \{t - \tau_{\max}, t\}} \{f_1(\sigma), f_2(\sigma), \ldots, f_{2n-m}(\sigma)\}$ represent the maximum and minimum values associated to the solution $f$ in the set $E$ at time $\sigma_1$, respectively. According to the Lemma 1 in [28], we have

(i) every component of $f_i(\sigma)$, $i \in E$, is in the following interval:

$$
G_1 = \left[\chi_{\min}^1 + (\sigma_{\min}^1 - \chi_{\min}^1)e^{-h_{\text{EFF}}}, \chi_{\max}^1 + (\chi_{\max}^1 - \sigma_{\max}^1)e^{-h_{\text{EFF}}}, \chi_{\max}^1 + (\chi_{\max}^1 - \sigma_{\max}^1)e^{-h_{\text{EFF}}}\right]
$$

(ii) at least one component of $f_j(\sigma)$, $i \in F$, is in the following interval:

$$
G_2 = \left[\chi_{\min}^1 + (\sigma_{\min}^1 - \chi_{\min}^1)e^{-h_{\text{EFF}}}/|F|, 1 + e^{h_{\text{EFF}}}/|F| + e^{h_{\text{EFF}}}/|F|, \chi_{\max}^1 + (\chi_{\max}^1 - \sigma_{\max}^1)e^{-h_{\text{EFF}}}/|F|, 1 + e^{h_{\text{EFF}}}/|F| + e^{h_{\text{EFF}}}/|F|\right]
$$

where $|F|$ represents the number of agents in $F$.

It is straightforward to verify that $G_1 \subset [\chi_{\min}^1, \chi_{\max}^1]$ and $G_2 \subset [\chi_{\min}^1, \chi_{\max}^1]$, which means that all the components of $f_i(\sigma)$, $i \in E$ are strictly contained in $[\chi_{\min}^1, \chi_{\max}^1]$ and at least one component of $f_i(\sigma)$, $i \in F$ is strictly contained in $[\chi_{\min}^1, \chi_{\max}^1]$. We may include the component of $f_i(\sigma)$, $i \in E$

which is strictly contained in $[\chi_{\min}^1, \chi_{\max}^1]$ and at the same time remove it from $f_j(\sigma)$, $i \in F$, that means we can denote a new disjoint sets $E_1 = E + m_1$ and $F_1 = F - m_1$, where $m_1 \in F, f_1(\sigma)$ is contained in (19). By repetitive application of this operation, we can finally get $E_\sigma = \hat{L}$ and $F_\sigma = \emptyset$ at time $\sigma$. Since $\sigma_{\max}^1 > \chi_{\min}^1 > \chi_{\max}^1$, Lyapunov function has decreased over the time interval $[\sigma_1, \sigma]$ which means $V(f(\sigma)) = V(f(\sigma_1)) < 0$. According to the analysis approach in [28], we can verify that system (16) can achieve consensus asymptotically, i.e.,

$$
\lim_{t \to \infty} |\xi_i(t) - \xi_j(t)| = \begin{cases} 0, & i, j \in \ell_1 \cup \ell_2 \\ 0, & i, j \in \ell_2 \end{cases}
$$

For that $|\xi_i(t)| = \frac{2}{a_k} \xi_i(t) + \xi_j(t)$, so $\lim \xi_i(t) = 0, i \in \ell_2$. Since $\xi_i(t) = a_k x_i(t), i \in \ell_2 (k = 1, 2)$ and $\varphi_i(t) = a_2 v(t), i \in \ell_2$, then we can get

$$
\lim_{t \to \infty} |a_1 x_i(t) - a_1 y_i(t)| = 0, \quad i, j \in \ell_1
$$

$$
\lim_{t \to \infty} |a_2 x_i(t) - a_2 y_i(t)| = 0, \quad i, j \in \ell_2
$$

$$
\lim_{t \to \infty} |a_1 x_i(t) - a_2 v_i(t)| = 0, \quad i \in \ell_1, j \in \ell_2
$$

Therefore, we can conclude that the heterogeneous systems (2) and (3) can achieve group consensus under protocols (4) and (5). This completes the proof.

Remark 2: Theorem 2 reveals that if the communication delays are bounded, then the weights of the delays do not affect whether the consensus of the heterogeneous systems can be reached. However, the weights of the delays will affect the consensus rate of the heterogeneous systems.

Remark 3: In this paper, we assume the union of the digraphs has a spanning tree at every bounded time interval $T$, then the consensus of the heterogeneous systems can be reached regardless of the existence of the spanning trees for the corresponding graphs. Obviously, if the topology is fixed and connected, the heterogeneous systems is also able to achieve group consensus.

B. GROUP CONSENSUS PROTOCOL II

In this subsection, we shall consider a more general case, and the corresponding control protocol is given by

$$
u_i(t) = \kappa_i \sum_{j \in \ell_1} a_{ij} \left(\alpha_1 x_j(t - \tau_{ij}(t)) - \alpha_1 x_i(t)\right)
$$

$$+ \kappa_i \sum_{j \in \ell_2} a_{ij} \left(\alpha_2 x_j(t - \tau_{ij}(t)) - \alpha_1 x_i(t)\right), \quad \forall i \in \ell_1
$$

$$
u_i(t) = \beta_i \sum_{j \in \ell_2} a_{ij} \left(\alpha_2 x_j(t - \tau_{ij}(t)) - \alpha_2 x_i(t)\right)
$$

$$+ \beta_i \sum_{j \in \ell_1} a_{ij} \left(\alpha_2 x_j(t - \tau_{ij}(t)) - \alpha_2 x_i(t)\right) - \gamma(x_i(t), \forall i \in \ell_2
$$


where $\kappa_i > 0$, $\beta_i > 0$, $\gamma_i > 0$, $\alpha_1 > 0$, $\alpha_2 > 0$ are scaling parameters, $\tau_j(t) \geq 0$ is the communication delay from $j$th agent to $i$th agent. By repeating the same procedure as that in (9), we denote
\[
\tilde{\xi}_i(t) = \alpha_1 \kappa_i \xi_i(t), \forall i \in \ell_1 \\
\tilde{\xi}_i(t) = \alpha_2 \kappa_i \tilde{\xi}_i(t), \forall i \in \ell_2
\] (22)

Then we have
\[
\dot{\tilde{\xi}}_i(t) = \alpha_1 \kappa_i \sum_{j \in N_i} a_{ij} \left( \tilde{\xi}_j(t - \tau_j(t)) - \tilde{\xi}_i(t) \right), \forall i \in \ell_1
\] (23)
and
\[
\dot{\tilde{\xi}}_i(t) = \alpha_2 \kappa_i \sum_{j \in N_i} a_{ij} \left( \tilde{\xi}_j(t - \tau_j(t)) - \tilde{\xi}_i(t) \right) - \alpha_2 \gamma_i \tilde{\phi}_i(t), \forall i \in \ell_2
\] (24)

To proceed, we set $\tilde{\eta}_i(t) = \frac{2}{\alpha_2 \gamma_i} \tilde{\phi}_i(t) + \tilde{\xi}_i(t)$. Then, system (24) becomes
\[
\dot{\tilde{\xi}}_i(t) = \frac{\alpha_2 \gamma_i}{2} \cdot (\tilde{\eta}_i(t) - \tilde{\xi}_i(t))
\]
\[
\dot{\tilde{\eta}}_i(t) = \frac{2}{\alpha_2 \gamma_i} \sum_{j \in N_i} a_{ij} \left( \tilde{\xi}_j(t - \tau_j(t)) - \tilde{\xi}_i(t) \right) - \frac{\alpha_2 \gamma_i}{2} \cdot (\tilde{\eta}_i(t) - \tilde{\xi}_i(t)), \forall i \in \ell_2
\]

Let
\[
\tilde{\zeta}(t) = [\tilde{\xi}_1^T(t), \tilde{\xi}_2^T(t), \tilde{\eta}_1^T(t)]^T
\]
where
\[
\tilde{\xi}_i(t) = \tilde{\xi}_i^T(t), \tilde{\xi}_2(t), \tilde{\xi}_m(t))^T
\]
\[
\tilde{\xi}_i(t) = [\tilde{\xi}_i(t), \tilde{\xi}_i(t), \tilde{\xi}_m(t))^T
\]
\[
\tilde{\eta}_i(t) = [\tilde{\eta}_i(t), \tilde{\eta}_i(t), \tilde{\eta}_m(t))^T
\]
(25)

Therefore, the heterogeneous system under protocols (20) and (21) becomes
\[
\dot{\tilde{\zeta}}(t) = \tilde{\Psi}_0(t) \tilde{\zeta}(t) + \sum_{\varepsilon = 1}^{M} \tilde{\Psi}_{\varepsilon}(t) \tilde{\zeta}(t - \tau_{\varepsilon}(t))
\] (26)

where $\tilde{\Psi}_0(t) = diag(\tilde{\Psi}_0(t))$ and $\tilde{\Psi}_{\varepsilon}(t), \varepsilon = 1, 2, \ldots, M$, is either zero or equal to the weight of edge $e_{ij}$. Obviously, $\tilde{\Psi}(t) = \tilde{\Psi}_0(t) + \sum_{\varepsilon = 1}^{M} \tilde{\Psi}_{\varepsilon}(t)$. $\tilde{\Psi}(t)$ is defined as follows:
\[
\tilde{\Psi}(t) = \\
\begin{bmatrix}
-\alpha_1 \kappa_1 \bar{L}_1 & \alpha_1 \kappa_1 A_{12} & 0_{n-m} \\
0_m & -\alpha_2 \beta_1 I_{n-m} & \alpha_2 \beta_1 I_{n-m} \\
2\beta \gamma^{-1} A_{12} & \frac{\alpha_2}{2} \gamma I_{n-m} - 2\beta \gamma^{-1} \bar{L}_2 & -\frac{\alpha_2}{2} \gamma I_{n-m}
\end{bmatrix}
\]

where $\kappa = diag \{ \kappa_i, i \in \ell_1 \}$, $\beta = diag \{ \beta_j, j \in \ell_2 \}$, $\gamma = diag \{ \gamma_j, j \in \ell_2 \}$.

Assumption 2:
\[
\sum_{j \in N_i} a_{ij} < \frac{\gamma_1^2 \alpha_2}{4 \beta_i} (i \in \ell_1 \cup \ell_2)
\]

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Fixed topology of the heterogeneous system.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Position states of all agents for fixed topology.}
\end{figure}

Theorem 3: For any given constants $\sigma_1 > 0$ and $\sigma_2 > 0$, suppose that every digraph of the system (26) keep fixed at $\sigma_1$ time, the union of the digraphs has a spanning tree at every bounded time interval $T$ whose time length is smaller than $\sigma_2$. Assumption 2 holds for system (26), and time delay is bounded. Then under protocols (20) and (21), heterogeneous systems (2) and (3) can reach group consensus.

Proof: The proof of Theorem 3 is omitted, since it is similar to that of Theorem 2.

V. SIMULATION RESULTS

In this section, the effectiveness of the obtained results will be demonstrated by carrying out some numerical simulations. Consider a heterogeneous MAS with random initial conditions, and the weight of each edge is set as 1.

A. EXAMPLE 1: FIXED AND UNDIRECTED TOPOLOGY

In the first example, fixed and undirected topology is considered, which is illustrated in Fig. 1. Two squares represent first-order agents and three circles represent second-order agents, thus it follows that $\sum_{j \in N_i} a_{ij} = 2$. Set $k_1 = k_2 = 1$, $k_3 = 3$, $\alpha_1 = 1$, $\alpha_2 = 2$. Therefore, the conditions $k_1 > 0$, $k_2 > 0$, $k_3 > 0$, $\alpha_1 > 0$, $\alpha_2 > 0$ and Assumption 1 are held. Let $\tau_{15} = 3$, $\tau_{51} = 5$, $\tau_{23} = 1$, $\tau_{32} = 2$, $\tau_{34} = 1$.
8, \tau_{43} = 6, \tau_{45} = 4, \tau_{54} = 7, and the initial states are \(x(0) = [1, 4, -3, 3, 2]^T\), \(v(0) = [1, 0, -1]^T\). Position states of all agents are shown in Fig. 2, velocity states of second-order agents are shown in Fig. 3. As expected, the group consensus is achieved.

**B. EXAMPLE 2: TIME-VARYING DIRECTED TOPOLOGIES**

In the second example, time-varying directed topologies are considered, which is illustrated in Fig. 4. The set \(\{G_a, G_b\}\) represents two network topologies of the heterogeneous system. The heterogeneous system is under topology \(G_a\) at the beginning, and then it switches to the next state after every \(T = 0.5\). Thus, the network topology switching sequence is: \(G_a \rightarrow G_b \rightarrow G_a \rightarrow \cdots\). Note that in every time interval of \(2T\), the union of the digraphs \(G_a \cup G_b\) has a spanning tree. Two squares represent first-order agents and three circles represent second-order agents. Take the control parameters of protocol II as \(\kappa_1 = 2, \kappa_2 = 1, \beta_3 = 1, \beta_4 = 2, \beta_5 = 2, \gamma_3 = 3, \gamma_4 = 2, \gamma_5 = 3, \alpha_1 = 1, \alpha_2 = 2\), which satisfies \(\kappa_i > 0, \beta_i > 0, \gamma_i > 0, \alpha_1 > 0, \alpha_2 > 0\) and Assumption 2. Let \(\tau_{12}(t) = 2 - \cos(t), \tau_{23}(t) = \tau_{34}(t) = 2|\sin(t)|, \tau_{45}(t) = 2, \tau_{51}(t) = 1 + \sin(t),\) and the initial conditions are \(x(0) = [7, 4, -3, 3, 6]^T\), \(v(0) = [1, 0, 2]^T\). Position states of all agents are shown in Fig. 5, velocity states of second-order agents are shown in Fig. 6. As expected, the group consensus is achieved.

To compare the difference between protocol I and protocol II, we take the control parameters of protocol I as \(k_1 = 2, k_2 = 1, k_3 = 2, \alpha_1 = 1, \alpha_2 = 2\), and the initial conditions and communication topologies are same...
as protocol II. Velocity states of second-order agents with protocol II are shown in Fig. 7, it is not hard to see that the consensus speed of protocol II is faster than that of protocol I, and this problem will be discussed in our future work.

VI. CONCLUSION

A group consensus control problem for heterogeneous MAS was investigated in this paper. Different from most works in the existing literature, time-varying topologies and communication delay were considered. Two distributed consensus control protocols were developed. Sufficient group consensus criteria for the heterogeneous MAS were obtained by using the property of Metzler matrix. The main results were verified by carrying out several numerical examples. For future study, the group consensus control of heterogeneous MAS with both of communication and input delays should be addressed.

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