Undulating Strings and Gauge Theory Waves

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Abstract

We study some dynamical aspects of the correspondence between strings in $AdS$ space and external heavy quarks in $\mathcal{N} = 4$ SYM. Specifically, by examining waves propagating on such strings, we make some plausible (and some surprising) inferences about the time-dependent fields produced by oscillating quarks in the strongly-coupled gauge theory. We point out a puzzle regarding energy conservation in the SYM theory. In addition, we perform a similar analysis of the gauge fields produced by a baryon (represented as a D5-brane with string-like extension in $AdS$ space) and compare and contrast with the gauge fields produced by a quark-antiquark pair (represented as a string looping through $AdS$ space).

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1 Introduction

In the context of Maldacena’s correspondence between gauge theories and gravity [1],
external charges in the gauge theory are dual to macroscopic strings in anti-de Sitter
(AdS) space whose endpoints lie on the boundary. This identification stems from
the general role of strings connecting parallel branes as W-bosons of the correspond-
ing spontaneously broken worldvolume theory [2], and can be confirmed within the
AdS/CFT setting by computing the energy of such strings [3, 4].

For concreteness, we will restrict attention to the duality between $D = 3 + 1$
$\mathcal{N} = 4$ SU($N$) super-Yang-Mills (SYM) and Type IIB string theory on $AdS_5 \times S^5$.
A solitary static quark (transforming in the fundamental of SU($N$)) corresponds to
a Type IIB string which extends solely in the radial direction; a string of opposite
orientation represents an antiquark (transforming in the anti-fundamental of SU($N$)).
The GKPW recipe for extracting gauge theory expectation values from the bulk action
[5, 6] makes it possible to verify directly that a radial string gives rise to the correct
point charge field configuration [7]. We note in passing that expectation values due
to string probes in the bulk of AdS (with no endpoints on the boundary) have also
been computed [8, 9, 7].

A quark-antiquark pair in the gauge theory is naturally identified with a string
with both of its endpoints on the boundary. Expectation values of Wilson loops can
thus be deduced from the bulk theory by evaluating the area of a string worldsheet
which is bounded by the loop [3, 4]. The result of such a calculation encodes in
particular the quark-antiquark potential (see [10] for a review of results on Wilson
loops obtained from the bulk-boundary correspondence).

A defining property of a string is its ability to undulate. The identification of
strings and charges raises an obvious question: what is the gauge theory interpretation
of string oscillations? This is the issue we will address in what follows. The main
tool at our disposal is again the GKPW calculational prescription [5, 6]. A string
is a source for the supergravity fields, so an oscillating string generates fluctuating
fields in the bulk of AdS space. The correspondence then translates the fluctuating
supergravity fields on the boundary into the time-dependent SYM expectation values
associated with an oscillating charge. The analysis thus establishes a correspondence
between string oscillations and gauge theory waves (including, one would hope, the
usual $r^{-1}$ radiation fields produced by an accelerated charge).

In Section 2 we will fill in the details of the procedure outlined in the previous
paragraph. To understand the basic ideas it will suffice to concentrate on waves of
the dilaton field, which is known to couple to the operator

$$\mathcal{O}_{F^2} = \frac{1}{4g_Y^2} \text{Tr} \left\{ F^2 + [X_I, X_J][X^I, X^J] + \text{fermions} \right\}$$

(1)
in the boundary theory [11, 12]. There is much to be learned by studying waves of
other supergravity fields, especially the graviton, but we will leave this more difficult
exercise for another paper. In the above equation $X^I, I = 1, \ldots, 6$, denote the scalar
fields of the $\mathcal{N} = 4$ SYM theory (living in the vector of SO(6)).
The simple case of an oscillating straight radial string will be worked out in Section 3. In Section 4 we will then extend the analysis to the more intricate case of a ‘bent’ string, and discuss some interesting features of the fields of a quark-antiquark pair. We amplify the discussion on the implications of our results for the SYM theory in Section 5, where we point out a puzzle regarding energy conservation in the gauge theory. In Section 6 we apply the same methods to obtain the gauge field profile due to a baryon (represented as a D5-brane appropriately wrapped in AdS space) and compare with the quark-antiquark case. A final section consists of a brief summary of our conclusions. Some aspects of string oscillations and SYM waves have been examined before in [13, 3, 14, 15] and we have attempted to go beyond these efforts in ways about which we will comment as appropriate.

2 String Oscillations Make SYM Waves

We describe the dynamics of a fundamental string through the Nambu-Goto action

\[ S_F = -\frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{-g}, \]  

(2)

where \( g \) is the induced metric on the string worldsheet. We work in Poincaré coordinates for \( AdS_5 \), with the metric

\[ ds^2 = \frac{R^2}{z^2}(-dt^2 + dx^2 + dz^2) + R^2 d\Omega_5^2. \]  

(3)

Making the static gauge choice \( \sigma^1 = t, \sigma^2 = z \), and restricting attention to configurations with the string pointing along a particular \( S^5 \) direction,\(^3\) the action reduces to

\[ S_F = -\frac{R^2}{2\pi\alpha'} \int dt z^2 \sqrt{1 - \partial_t \vec{X}^2 + \partial_z \vec{X}^2 - \partial_t \vec{X} \cdot \partial_z \vec{X}^2} + \left( \partial_t \vec{X} \cdot \partial_z \vec{X}^2 \right)^2, \]  

(4)

where \( \vec{X}(z,t) \) denotes the position of the string in the \( \vec{x} \) directions. The static solutions to (4) can be taken to lie in the \( z - x^1 \) plane without loss of generality. They satisfy

\[ \partial_z X_s = \pm \frac{z^2}{\sqrt{z^4 - z^4}}. \]  

(5)

This equation describes a string lying along a geodesic which starts and ends at \( z = 0 \) and reaches a maximum at \( z = z_m \) (see Fig. 1). The two endpoints of the string are separated by a coordinate distance [3, 4]

\[ L = z_m \left( \frac{2\pi}{\Gamma(1/4)^2} \right)^{3/2}. \]  

(6)

\(^3\)The operator \( O_{F^2} \) couples to the spherically symmetric mode of the ten-dimensional dilaton, so we will focus attention on this mode alone. A string which is localized on the five-sphere will excite also all of the higher Kaluza-Klein harmonics, which are massive fields on \( AdS_5 \). These excitations would give expectation values to dual higher-dimension operators which have been identified in [5, 6, 12].
Figure 1: A Nambu-Goto string lying along a geodesic, with its two endpoints on the boundary.

Now consider small oscillations about the solution described by (5), letting $\vec{X}(z, t) = \vec{X}_s(z) + \vec{Y}(z, t)$. For simplicity, we take $\vec{Y} \perp \vec{X}_s$. The linearized equation of motion for $\vec{Y}$ is

$$-\partial^2_t \vec{Y} + \left[1 - \frac{z_0^4}{z_m^4}\right] \partial^2_z \vec{Y} - \frac{2}{z} \partial_z \vec{Y} = 0.$$  \hspace{1cm} (7)

In the calculation to follow, we will cut off $AdS_5$ by moving the boundary in to $z = z_0$ and take $z_0 \to 0$ at the end of the calculation. In order to solve (7), we need boundary conditions for the left and right string endpoints which we will impose in the form $\vec{Y}_{L,R}(z_0, t) = \vec{y}_{L,R}(t)$. The interpretation is straightforward: for a given $z_0$, the string is attempting to describe a Higgsed gauge boson of very large mass ($\propto z_0^{-1}$) transforming in the fundamental of the unbroken $SU(N)$ gauge group; this massive object is an extrinsic degree of freedom from the point of view of the $SU(N)$ gauge theory and has its own dynamics; this dynamics is essentially that of a point particle and is thus described by a trajectory function $\vec{y}(t)$. For the moment, we will simply prescribe a trajectory, but the Nambu-Goto action for the string in the $AdS_5$ geometry implies an action for $\vec{y}(t)$ which in turn implies an equation of motion for the trajectory. We will not pursue this line of thought much further in this paper, but it is interesting to note that the kinetic term in this equation of motion implies a quark mass that matches the static total energy of the quark/string.

Since the Nambu-Goto action (2) depends on the background supergravity fields, it is a source for them as well. In particular, it is a source for the dilaton, a fact which is best displayed by writing the action in terms of the Einstein metric $G^E_{MN} = e^{-\phi/2}G_{MN}$:

$$S_F = -\frac{1}{2\pi\alpha'}\int dt dz \; e^{\phi/2}\sqrt{-g_E}.$$ \hspace{1cm} (8)

The same metric rescaling in the bulk supergravity action yields a dilaton kinetic term

$$S_S = -\frac{\Omega_5 R^5}{4\kappa^2} \int d^5x \sqrt{-\;G_{E}} G^{mn}_E \partial_m \phi \partial_n \phi.$$ \hspace{1cm} (9)

Notice that the original ten-dimensional action has been dimensionally reduced to $AdS_5$ over $S^5$: $\phi$ now denotes the projection of the original ten-dimensional $\phi$ onto $S^5$. 

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the constant $S^5$ spherical harmonic and is a function on $AdS_5$ while $m, n$ are $AdS_5$ indices. The combined action $S_{\text{bulk}} = S_S + S_F$ implies a linearized dilaton equation of motion

$$\partial_m \sqrt{-G_E} G_E^{mn} \partial_n \phi = J, \quad J(x) = \frac{2\kappa^2}{4\pi \alpha' \Omega_5 R^8} \delta \left( \vec{x} - \vec{X}(z, t) \right). \quad (10)$$

This equation is solved by by Greens’ function methods as $\phi(x) = \int \delta \vec{x}' D(x, x') J(x')$, where $D(x, x')$ is the retarded dilaton propagator [7]. The propagator is in fact only a function of the invariant distance $v$, defined by

$$\cos v = 1 - \frac{(t - t')^2 - (\vec{x} - \vec{x}')^2 - (z - z')^2}{2zz'}. \quad (11)$$

Explicitly,

$$D(v) = -\frac{1}{4\pi^2 R^3 \sin v} \frac{d}{dv} \left[ \frac{\cos 2v}{\sin v} \theta(1 - |\cos v|) \right]. \quad (12)$$

This is a fairly complicated-looking propagator, but it is just the dimensional reduction of the much simpler, completely algebraic ten-dimensional $AdS_5 \times S^5$ propagator

$$K \sim \frac{(zz')^4}{[(z\hat{n} - z'\hat{n}')^2 + (t - t')^2 + (\vec{x} - \vec{x}')^2]^4} \quad (13)$$

where the unit vectors $\hat{n}, \hat{n}'$ indicate position on $S^5$.

Having obtained $\phi(x)$ in the bulk, the GKPW recipe to extract the expectation value is [5]

$$\langle O_{F^2} \rangle = -\frac{\delta S_{\text{bulk}}}{\delta \phi}. \quad (14)$$

The expectation value would of course vanish in the gauge theory vacuum sector. On the other hand, the string corresponds to the sector of the gauge theory where a heavy quark has been inserted in the vacuum. We do expect a non-zero expectation value of the $\text{Tr} F^2$ operator in that sector and Eq. (14) gives a method for computing it. Carrying out similar steps with higher $S^5$ harmonic modes of the dilaton would yield gauge theory expectation values for operators of the type $\text{Tr}(F^2 X_I \ldots X_J)$, where the $X_I$ are the scalar fields of the $\mathcal{N} = 4$ gauge theory [5, 6, 12]. These higher-dimension operators should give rise to a correspondingly higher power law falloff with $|\vec{x}|$, a result which should emerge naturally from the structure of the Greens’ functions for the higher $S^5$ harmonic modes of the dilaton.

Under $\phi \rightarrow \phi + \delta \phi$ the action varies only by a surface term, because the configuration about which we vary is a solution to the equation of motion:

$$\delta S_{\text{bulk}} = \frac{\Omega_5 R^8}{2\kappa^2} \int dt d^3 \vec{x} \left. \left( \frac{1}{z^3} \partial_z \phi \right) \partial \phi \right|_{z = z_0}. \quad (15)$$

As a shorthand, it will be convenient to define a rescaled dilaton field $\tilde{\phi} = \Omega_5 R^8 \phi / 2\kappa^2$. It follows from the foregoing discussion that

$$\tilde{\phi}(x) = -\frac{1}{16\pi^3 \alpha'} \int dt' dz' \sqrt{-g_E} \frac{1}{\sin v} \frac{d}{dv} \left[ \frac{\cos 2v}{\sin v} \theta(1 - |\cos v|) \right], \quad (16)$$
and

\[ \langle \mathcal{O}_{F^2} \rangle = - \left. \left( \frac{1}{z^3} \partial_z \tilde{\phi} \right) \right|_{z=z_0=0}. \quad (17) \]

Our task, then, is to calculate the dilaton field produced by various string sources and to pick out the \( O(z^4) \) term in its expansion near the boundary of \( AdS_5 \).

## 3 Gauge Fields of an Oscillating Quark

We will examine first the especially simple case \( z_m \to \infty \), where the static solution describes a straight string extending along the radial direction, \( \vec{X}_s(z) = 0 \). Eq. (7) simplifies to

\[ - \partial_t^2 \vec{Y} + \partial_z^2 \vec{Y} - \frac{2}{z} \partial_z \vec{Y} = 0. \quad (18) \]

This equation is most easily solved via Fourier transformation. The solution describing purely outgoing waves is found to be

\[ \vec{Y}(z, t) = \int d\omega e^{-i\omega(t-z+z_0)} \left( \frac{1 - i\omega z}{1 - i\omega z_0} \right) \vec{y}(\omega). \quad (19) \]

For simplicity, we specialize to harmonic boundary data, \( \vec{y}(t) = \vec{A} \exp(-i\omega t) \).

The Nambu-Goto square root in (16) can be expanded as

\[ \sqrt{-g_E} \simeq \frac{R^2}{z^2} \left[ 1 - \frac{1}{2} \left( \partial_t \vec{Y} \right)^2 + \frac{1}{2} \left( \partial_z \vec{Y} \right)^2 \right]. \quad (20) \]

Keeping only the leading term, (16) reads

\[ \tilde{\phi}(\vec{x}, z, t) = - \frac{R^2 z}{16\pi^3\alpha'} \int \frac{dt'}{\sin v} \frac{dz'}{z'} d\frac{\cos 2v}{\sin v} \theta(1 - |\cos v|). \quad (21) \]

Next, change variables of integration \( t' \to v \), using (11), and integrate by parts on \( v \), to be left with

\[ \tilde{\phi} = \frac{R^2 z}{16\pi^3\alpha'} \int \frac{dz'}{z'} I, \]

\[ I = \int_0^\pi dv \frac{\cos 2v}{\sin v} \frac{d}{dv} \left[ \frac{1}{\sqrt{z^2 + z'^2 + (\vec{x} - \vec{Y})^2 - 2zz' \cos v}} \right]. \quad (22) \]

Now expand the square root in powers of \( Y \). The leading \( (Y\text{-independent}) \) term gives rise to a static component of the dilaton field, \( \tilde{\phi}_s(\vec{x}, z) \). Its contribution to the gauge theory expectation value has been computed in [7] and found to be

\[ \langle \mathcal{O}_{F^2} \rangle_s = \frac{\sqrt{2}}{32\pi^2} \left( \frac{g_{YM}^2 N}{|\vec{x}|^4} \right). \quad (23) \]

\(^{4}\)Henceforth it is understood that one should take the real part of expressions like this.
This is as expected for a point charge of magnitude proportional to \((g_Y^2 N)^{1/4}\), which is the effective strength of the coupling as inferred from the quark-antiquark potential [3, 4].

The next term in expansion of (22) in powers of \(Y\), the term linear in \(Y\), gives the leading dynamical contribution to \(\mathcal{O}_{F^2}\):

\[
\tilde{\phi}_{(1)} = \frac{R^2 z}{16\pi^3 \alpha'} \int \frac{dz'}{z'} I_{(1)},
\]

\[
I_{(1)} = \int_0^\pi dv \frac{\cos 2v}{\sin v} dv \left[ \frac{(\vec{x} \cdot \vec{A}) e^{-i\omega(t-z')} (1 - i\omega z')}{\sqrt{z^2 + z'^2 + |\vec{x}|^2 - 2zz' \cos v}} \right], \tag{24}
\]

where \(t'\) is understood to be a function of \(v\),

\[
t' = t - \sqrt{z^2 + z'^2 + |\vec{x}|^2 - 2zz' \cos v}.
\]

Since, according to (17), we eventually only need the \(O(z^4)\) terms in \(\tilde{\phi}\), we have set \(z_0 \to 0\) in (24).

If one expands the integrand of \(I_{(1)}\) in powers of \(\eta = zz' \cos v/(z^2 + z'^2 + |\vec{x}|^2)\), the first non-vanishing term is found to be \(O(\eta^3)\), and higher-order terms will not contribute to (17). Keeping only the relevant terms one obtains

\[
\tilde{\phi}_{(1)} = -\frac{R^2 (\vec{x} \cdot \vec{A}) z^4}{128\pi^2 \alpha'} \int_0^\infty dz' z'^2 (1 - i\omega z') e^{-i\omega(t - \sqrt{z'^2 + |\vec{x}|^2 - z'})} f(\sqrt{z'^2 + |\vec{x}|^2}),
\]

\[
f(u) = \frac{i\omega^3}{u^6} - \frac{12\omega^2}{u^7} - \frac{57i\omega}{u^8} + \frac{105}{u^9}. \tag{25}
\]

The bulk dilaton field is evidently a superposition of waves radiated from each point along the string. The phase delay \(z' + \sqrt{z'^2 + |\vec{x}|^2}\) is simply the time needed for a null signal to propagate up along the string to the point \(z = z'\), and then travel down diagonally to reach the boundary at \(\vec{x}\) (see Fig. 2).

To understand this result from the viewpoint of the boundary theory, it is advantageous to change the variable of integration to \(\zeta = \sqrt{1 + z'^2/|\vec{x}|^2 + z'/|\vec{x}|}\). Using (25) in (17) one finds, after some integration by parts,

\[
\langle \mathcal{O}_{F^2} \rangle_{(1)} = \frac{\sqrt{2g_Y^2 N}}{2\pi^2 |\vec{x}|^4} \left\{ \int_1^\infty d\zeta i\omega e^{-i\omega(t - \zeta|\vec{x}|)} \chi(\zeta) + \frac{59}{32|\vec{x}|} e^{-i\omega(t - |\vec{x}|)} \right\}
\]

\[
\chi(\zeta) = \frac{210\zeta^{10} - 258\zeta^8 + 267\zeta^6 + 69\zeta^4 + 55\zeta^2 + 1}{2(\zeta^2 + 1)^t}. \tag{26}
\]

The expectation value has been expressed solely in terms of gauge theory quantities through use of the relation \(R^2/\alpha' = \sqrt{2g_Y^2 N}\), as must be possible for a proper gauge theory interpretation. It should be noted that the dependence of the integrand on \(\omega|\vec{x}|\) can be shifted from the phase factor to the envelope function \(\chi\) through an integration by parts.
Figure 2: Any given point on the boundary receives radiation from each point along the string. As a result, the gauge theory wave is a superposition of components with all possible time delays. See text for discussion.

Eq. (26) displays the gauge theory disturbance as a superposition of components propagating at speeds $v = 1/\zeta$, for all $1 \leq \zeta \leq \infty$. Notice that the weight factor $\chi \to 0$ as $\zeta \to \infty$, so low velocity components are evidently suppressed. It should be noted from (19) that the string oscillations actually become large (and our approximations fail) for large $A\omega z'$, so the detailed shape of $\chi$ cannot be trusted at arbitrarily large $\zeta$. Nonetheless, it is clear from the geometric setup (summarized in Fig. 2) that the gauge theory wave should indeed include components propagating at arbitrarily low velocities.

The above result implies in particular that even if the charge is shaken abruptly to generate a sharply defined pulse on the string, the SYM observer at $\vec{x}$ will receive an infinitely broadened pulse, only the leading edge of which travels at the speed of light. The delayed signals presumably arise from rescattering of the original disturbance from the static background (23) by virtue of the nonlinear dynamics of the strongly-coupled gauge theory. This rather complex sequence of events would be difficult to unravel in the gauge theory, but the bulk-boundary correspondence gives a precise and physically plausible account of it. Note that the long time delays originate from string disturbances at large values of the bulk $AdS$ coordinate $z$, as would be expected from the UV/IR connection proposed in [16, 17].

It is tempting to speak of (26) as electromagnetic ‘radiation’, but the rapid falloff with $|\vec{x}|$ indicates that this would not be strictly correct. We are looking at a contribution to $O_{F^2}$ linear in the displacement of the quark and this can only arise from a cross-term between the static field and the fluctuating field. Since the static electric field is radial and the asymptotic radiation gauge fields (the ones that fall off as $|\vec{x}|^{-1}$) are transverse, their scalar product vanishes. Hence there is no unambiguous contribution of electromagnetic radiation to the $O_{F^2}$ expectation value: instead, we see evidence of fluctuations in the near-fields of the moving quarks, fields which do not transport energy to infinity. The unambiguous diagnostic for radiation would be the demonstration of a net energy flux to spatial infinity in the gauge theory. This could be done by determining the expectation value of the gauge theory energy-momentum
tensor, which in the GKPW recipe is dual to the bulk gravitational field produced by the fluctuating string. It would be extremely interesting to carry out this calculation explicitly, for it is not at all obvious how (or even if) the $AdS$ description of waves in the boundary theory incorporates energy conservation. In this connection, we should also remark that our analysis neglects the back-reaction on the string due to the supergravity fields. We will return to these issues in Section 5.

Before closing this section, we note that our external charges are by construction infinitely massive, and consequently immune to the SYM field configuration they help to produce. It is possible to consider instead sources with finite mass, represented by strings which terminate not at the boundary, but on a solitary D3-brane placed at $z_b > 0$. In that case one is really studying an $SU(N + 1)$ gauge theory, broken spontaneously to $SU(N) \times U(1)$ by a Higgs vacuum expectation value $R^2 / z_b \alpha'$ [1, 18].

4 Gauge Fields of Heavy Quark Mesons

We now extend the analysis to the general case $z_m < \infty$, where the string bends along the geodesic (5). Both of its endpoints reach the boundary, so this configuration describes a quark-antiquark pair (see Fig. 1). Notice that now the parametrization $\vec{X}(z, t)$ has the disadvantage of being double-valued: for each value of $z$ there are in fact two points on the string, one on the left and one on the right. When necessary, we will account for this by means of a discrete subindex: $\vec{X}_{L,R}(z, t)$. The need for this awkward notation is compensated by the simple form of the differential equation (7).

The expansion of the Nambu-Goto integrand now yields

$$\sqrt{-g_E} \simeq \frac{R^2}{z^2} \left\{ \Delta + \frac{1}{2\Delta} \left[ \left( \partial_z \vec{Y} \right)^2 - \Delta^2 \left( \partial_t \vec{Y} \right)^2 \right] \right\}, \quad \Delta = \frac{z_m^2}{\sqrt{z_m^4 - z^4}}. \quad (27)$$

The dilaton (16) is again a sum of static and fluctuating components.

It is interesting to determine the gauge field profile due to the static bent string. Inserting the first term of (27) into (16), changing variables $t' \rightarrow v$, and integrating by parts with respect to $v$ one obtains

$$\bar{\phi}_s(\vec{x}, z) = \frac{R^2 z_m^2 z}{16 \pi^3 \alpha'} \int \frac{dz'}{z' \sqrt{z_m^4 - z'^4}} I,$$

$$I = \int_0^\pi dv \frac{\cos 2v}{\sin v} \frac{d}{dv} \left[ \frac{1}{\sqrt{z^2 + z'^2 + (\vec{x} - \vec{X}(z'))^2 - 2zz' \cos v}} \right]. \quad (28)$$

Next, expand the integrand of $I$ in powers of $2zz' \cos v / [z^2 + z'^2 + (\vec{x} - \vec{X}(z'))^2]$, and retain only the leading order term, to find

$$I = -\frac{15 \pi (zz')^3}{8 \left[ z^2 + z'^2 + (\vec{x} - \vec{X}(z'))^2 \right]^{7/2}}. \quad (29)$$
Notice the peculiar dependence on \( L \). Use of this in (28) leads to
\[
\tilde{\phi}_s = -\frac{15R^2z_m^2 \zeta^4}{128\pi^2\alpha'} \int_0^{z_m} \frac{dz' z'^2}{\sqrt{z_m^4 - z'^4}} \times \left\{ \frac{1}{\left[ z'^2 + \left( \vec{x} - \vec{X}_L(z') \right)^2 \right]^{7/2}} + \frac{1}{\left[ z'^2 + \left( \vec{x} - \vec{X}_R(z') \right)^2 \right]^{7/2}} \right\}, \tag{30}
\]
where we have explicitly indicated the contribution from both halves of the string. It is convenient to place the center of the string at the origin, \( \vec{X}(z_m) = 0 \), so that \( X_L(z) = -X_R(z) \), as depicted in Fig. 1.

We wish to extract the leading term in (30) for \( |\vec{x}| \gg L \), which by (6) implies that \( |\vec{x}| \gg z_m \) as well. We find
\[
\tilde{\phi}_s = -\frac{15R^2z_m^2 \zeta^4 L}{128\pi^2\alpha' |\vec{x}|^7}. \tag{31}
\]

The SYM expectation value then follows from (17). We can express the result in terms of quantities in the boundary theory, using (6) and \( R^2/\alpha' = \sqrt{2g_Y^2 N} \):
\[
\langle O_{F^2} \rangle_s = \frac{15\Gamma(1/4)\sqrt{2}L^3 \sqrt{g_Y^2 N}}{8(2\pi)^5 |\vec{x}|^7}. \tag{32}
\]
Notice the peculiar dependence on \( L \) and \( |\vec{x}| \) and the fact that the result is isotropic. This is not what one would expect from a static electric dipole field in a linear gauge theory, but there is nothing obviously inconsistent about it for strongly coupled \( \mathcal{N} = 4 \) SYM. The above result must be regarded as a prediction of the bulk-boundary correspondence for which we have at present no independent test.

We now examine the contribution from the fluctuating part. Equation (7) can be solved by Fourier transformation. The general solution is
\[
\tilde{Y}_{L,R}(z, t) = \int d\omega \sqrt{1 + \omega^2 z_m^2} \left\{ \tilde{A}(\omega)e^{-i\omega(t - z_{L,R})} + \tilde{B}(\omega)e^{-i\omega(t + z_{L,R})} \right\},
\]
\[
\tilde{Z}_L(z, \omega) = \sqrt{(\omega z_m)^4 - 1} \int_{z_0}^z \frac{(s/z_m)^2 ds}{(1 + \omega^2 s^2)^2 \sqrt{1 - (s/z_m)^4}} \tag{33}
\]
\[
\tilde{Z}_R(z, \omega) = \tilde{Z}_L(z_m, \omega) + \sqrt{(\omega z_m)^4 - 1} \int_z^{z_m} \frac{(s/z_m)^2 ds}{(1 + \omega^2 s^2)^2 \sqrt{1 - (s/z_m)^4}}.
\]

Notice that component waves with \( \omega z_m < 1 \) are exponentially damped, reflecting a frequency cutoff imposed by the finite size of the string. The oscillations on the left and right halves of the string are related by the requirement that the solution be smooth at the midpoint, \( z = z_m \). The coefficients \( \tilde{A} \) and \( \tilde{B} \) are determined by enforcing boundary conditions at the string endpoints, \( \tilde{Y}_{L,R}(z_0, t) = \tilde{y}_{L,R}(t) \):
\[
\tilde{A}(\omega) = \frac{\tilde{y}_L(\omega) - \Phi \tilde{y}_R(\omega)}{1 - \Phi^2}, \quad \tilde{B}(\omega) = \frac{\tilde{y}_L(\omega) - \Phi^* \tilde{y}_R(\omega)}{1 - \Phi^2}, \quad \Phi(\omega) = e^{i2\omega z_m(\omega)}. \tag{34}
\]
For any given choice of boundary conditions, the string endpoints trace out definite Wilson lines $\vec{y}_{L,R}(t)$ in the gauge theory. The solution (33) can be used in (16) and then (17) to determine the corresponding SYM expectation value. Rather than working out the details of such a calculation, which would not be particularly enlightening, we will point out some interesting general features of the resulting field configurations.

First, it is evident that the SYM waves display a phase delay analogous to the one found for the straight string, although the details are different. To understand this in some detail, imagine that at $t = 0$ a pulse is sent along the string by shaking its left end, which we now take to be located at $\vec{x} = 0$. The induced metric on the bent string is

$$g_{ab}d\sigma^a d\sigma^b = \frac{R^2}{z^2} \left[ -dt^2 + \frac{dz^2}{1 - (z/z_m)^4} \right],$$

so the pulse, following a null trajectory, takes a time

$$\Delta t_1(z') = \int_0^{z'} \frac{dz}{\sqrt{1 - (z/z_m)^4}} \quad \text{or} \quad \left( \int_0^{z_m} + \int_{z'}^{z_m} \right) \frac{dz}{\sqrt{1 - (z/z_m)^4}}$$

(36)

to reach the point $z'$ on the left or right half of the string. In particular, it requires a time [15]

$$T = z_m \frac{\Gamma(1/4)^2}{2\sqrt{2\pi}}$$

(37)
to traverse the entire string and arrive at the right endpoint, where for the time being we assume that it is completely absorbed.

As seen in Fig. 3, a dilaton wave travels from $z'$ down to a boundary point $\vec{x}$ in an additional time $\Delta t_2(z', \vec{x}) = \sqrt{z'^2 + \left(\vec{x} - \vec{X}_s(z')\right)^2}$. As a result, the radiation arriving at $\vec{x}$ has a component with phase lag $\Delta t_1(z') + \Delta t_2(z', \vec{x})$ for each point $z'$ on the string. The net effect is that the SYM observer detects a significantly broadened pulse, whose leading and trailing edges arrive at times $t_f = |\vec{x}|$ and $t_b = T + |\vec{x} - L\hat{x}_1|$, respectively.

The situation is thus similar to the one encoded in (26), in that an oscillating source would ultimately give rise to a superposition of gauge theory waves traveling at different speeds $v \leq 1$. A complicating feature of this situation as compared to the case of an isolated quark is that, because the string now extends only a finite distance into AdS space, a disturbance on the string can only propagate for a finite time before running into the boundary. As has been observed by others [15], the time (37) for a disturbance to propagate from one end of the string to the other corresponds, from the gauge theory point of view, to a subluminal mean speed of propagation of influence $v = L/T \simeq 0.457$. Note, however, that this is not the generic speed of propagation of disturbances in the gauge theory: the whole point of our analysis was to show that disturbances in the expectation value of $O_{F2}$ propagate away from their source in the quark-antiquark system in a conventionally causal fashion: the leading signal arrives on a direct path at the speed of light, followed by indirect signals that arrive later.
As an aside, we remark that disturbances propagating on the strings have bizarre features from the point of view of the boundary gauge theory. For instance, triple-string configurations (describing for example a quark-monopole-dyon system [19]) can be arranged where a signal originating from one charge would give rise to a disturbance which would run along the strings and arrive first at the more distant (from the boundary theory perspective) of the two other charges [20]. This would not violate causality, strictly understood, but is certainly strange. To repeat, the key point is that an oscillation on the string does not translate directly into a wave in the boundary theory. The correct prescription brings the bulk supergravity fields into play, and unequivocally predicts causal SYM propagation, with propagation velocities up to the speed of light.

5 Implications for Gauge Theory Dynamics

In this paper, we have been exploring a picture, derived from Maldacena’s AdS/CFT duality conjecture, of the generation and propagation of disturbances in the $D = 3+1$ $\mathcal{N} = 4$ $SU(N)$ super-Yang-Mills (SYM) gauge theory. In this picture, external sources are described by type IIB strings running from the boundary into the bulk of AdS space; fluctuations in the position of the external sources generate waves on the strings; the string waves generate propagating disturbances in the supergravity fields in AdS space; finally, the fluctuating boundary values of these fields are converted, via the GKPW recipe, into fluctuating expectation values of operators in the gauge theory. Throughout the discussion, we have assumed that the string disturbances propagate according to simple Nambu-Goto dynamics and have treated them as known linear sources for the supergravity fields. In particular, we have not worried about back-reaction of the supergravity fields on disturbances propagating on the string. On the face of it, this seems reasonable because Maldacena’s conjecture includes taking the limit of weak supergravity coupling. On the other hand, as we will now
discuss, this collection of assumptions leads to some surprising, perhaps paradoxical, conclusions about the behavior of the gauge theory that are worth pointing out.

A somewhat perplexing feature of the time-dependent field that can be gleaned from Fig. 3 is that dilaton wavefronts emitted from a point $z'$ on the string describing a quark-antiquark system, give rise to spherical waves in the gauge theory which seem to emanate from neither the quark nor the antiquark, but from the point $\tilde{X}_s(z')$ on the line between them. Imagine an observer situated halfway between the quark and antiquark: if the quark is shaken to produce a pulse on the string, the observer first sees disturbances coming first from the direction of the quark and then (after a time $T/2$) from the opposite direction! Though odd, this feature is in principle consistent with the non-linear character of strongly-coupled SYM: the external sources give rise to propagating disturbances, which propagate through and cause to reradiate, the background gauge field configuration originally set up by the source. This sort of thing would happen in any strongly-coupled theory; what is surprising is the geometrical structure that is inherited from the $AdS$ string.

A more profound set of issues arises from the fact that a disturbance travels from one end of the quark-antiquark string to the other in a finite time (37), forcing us to consider how the string disturbance reflects from the boundary if we wish to account for radiation generated at later times. Since the external sources can be taken to be as massive as we like (by letting $z_0 \to 0$), it seems reasonable to assume that the fluctuating string should be subject to fixed or Dirichlet boundary conditions\(^5\) which reflect any incident disturbance back onto the string (with a change of sign). This would mean that a disturbance, however it was initially generated, would simply reflect back and forth between the quark and antiquark ends of the string without ever dying away. More precisely, the linearized string would have eigenstates of oscillation at a discrete set of frequencies $\omega_n$ running from some lower cutoff on up to infinity. In a WKB approximation, these frequencies would be determined by the requirement that the phase factor $\Phi$ in (33) is real, i.e.

$$\omega_n z_m \sqrt{(\omega_n z_m)^4 - 1} \int_0^1 \frac{d\sigma}{[1 + (\omega_n z_m)^2 \sigma^2] \sqrt{1 - \sigma^2}} = \frac{n\pi}{2}. \tag{38}$$

These oscillations must represent excited states of the dipole field, with a mass gap between states scaled by the dipole separation $L$. These states are quite analogous to the infinite tower of mesons found in the large-$N_c$ limit of ordinary QCD (where the mass gap is set by the confinement scale). On the other hand, it is quite surprising to imagine finding an analogous set of states in a non-confining conformal gauge theory!

At this point we are led back to the questions, first raised in the discussion of the isolated quark in Section 3, of radiation, energy conservation and back-reaction. It is important to realize that a complete treatment of the production of time-varying supergravity fields by a disturbance on the string must include back-reaction on the string disturbance. To the extent that the bulk field includes a net energy flux\(^5\) the boundary conditions appropriate for Wilson loops in the $AdS/CFT$ correspondence have been discussed in [21].
away from the string, the back-reaction should cause the disturbance to damp as it propagates. This process is essential to energy conservation in the supergravity picture. Let us now try to understand how this translates into energy conservation in the gauge theory—we will be led to a paradox.

Focus attention again on the infinite tower of excited states of the SYM dipole system, and consider the following question: are these excited states stable? To answer this question, we will pursue two possible lines of argument. On the one hand, within the supergravity framework we know that, once we take back-reaction into account, the resonances will have finite widths and the notion of resonance will only make sense if there is a limit in which the width becomes small compared to the gap between successive states. The rate at which string disturbances radiate is set by $g_s$, so if we take the usual $AdS/CFT$ limit $g_s \to 0$ (with $g_s N$ fixed), the string will not radiate, and the excitations will be completely stable. This can be seen explicitly in (10): the source term in the dilaton equation of motion vanishes as $g_s \to 0$ with $g_s N$ fixed. We are thus led to conclude that in the $N \to \infty$ limit (with the ’t Hooft coupling $g_{YM}^2 N$ fixed) there exists in the dual gauge theory an infinite tower of stable (i.e., non-radiating) excited states of the gauge field set up by an infinitely massive quark-antiquark dipole. As we have already pointed out, this would be analogous to what happens in conventional QCD in the large-$N$ limit: in the leading approximation, there is a tower of stable states in every sector of the theory (meson, baryon, quarkonium, . . .); beyond leading order, these states acquire finite widths proportional to some power of $1/N$. It would be most remarkable if the same structure of states survived the passage from confining QCD to non-confining $\mathcal{N} = 4$ SYM (with the confinement scale replaced by a variable geometric scale set by the ‘size’ of the configuration).

On the other hand, the central point of this paper is that the GKPW recipe translates a disturbance propagating on the string into waves in the gauge theory. At the end of the calculations one obtains SYM expectation values (Eq. (26), for instance) which depend on $g_{YM}$ only through the ’t Hooft coupling $g_{YM}^2 N$, and consequently do not vanish when $g_s \to 0$. Now, as discussed in Section 3, the result obtained in (26) is a near-field contribution (it involves time-dependent fields which depend on the velocity, but not the acceleration, of the sources), and so does not unambiguously indicate the presence of SYM radiation. Nonetheless, given that the ten-dimensional static, near-, and radiation fields all come in at the same order in $g_s$ (they differ only by their dependence on $|\vec{z}|$), it is natural to expect that a computation of the energy-momentum tensor would reveal a net energy flux away from the external sources, signaling the presence of true radiation. On the face of it, this seems to apply just as much to the solitary oscillating quark as to the quark-antiquark excited states.

We have thus been led to a paradox: if the gauge theory is to conserve energy, a radiating dipole field cannot possibly be stable. To restate the problem in slightly different words, imagine that the quark in the quark-antiquark system is shaken

---

6The reason for this can be seen in (17): the gauge theory expectation value is extracted directly not from the dilaton $\phi$ (which vanishes as $g_s \to 0$), but from the rescaled field $\tilde{\phi} \sim \phi/g_s^2$. 

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abruptly to produce a pulse running along the string, and the external charges are held fixed at all other times. In this process, a definite amount of energy is added to the system. In the $g_s \rightarrow 0$ limit, the pulse on the string will not decay, and so it will endlessly travel back and forth between the quark and antiquark. Through the mechanism analysed in detail in this paper, this disturbance will give rise to time-dependent SYM fields which remain finite as $g_s \rightarrow 0$. If these fields include true radiation (as seems reasonable to expect), they continuously carry energy away from the dipole system, violating energy conservation in the gauge theory. We should remark that, even though the paradox is most evident in the context of the quark-antiquark system, the question of energy conservation must also be addressed in the case of the solitary quark. In that instance, the existence of SYM radiation would not in itself be paradoxical, but it is certainly far from obvious that the infinitely broadened pulse which propagates in the gauge theory after the external charge is shaken properly incorporates energy conservation.

What are we to make of this? In a sense, it is not surprising that we have encountered a problem: given the holographic character of the $AdS/CFT$ correspondence, the interplay between energy conservation in the bulk and on the boundary is bound to be a delicate issue. Notice that the problem would disappear if our assumption regarding the presence of radiation turned out to be erroneous. Since we have not seen direct evidence for the existence of gauge theory radiation in the $g_s \rightarrow 0$ limit, we must bear in mind the possibility that the explicit determination of the energy-momentum tensor will show that there is no net energy flux away from the dipole system. This would undoubtedly be a surprising result. We will leave for future work the more thorough analysis required to reach a definitive conclusion on this important issue.

6 Gauge Fields of Heavy Quark Baryons

In the preceding two sections, we studied the gauge fields of a color-neutral heavy quark-antiquark pair. Among other interesting things, we found in the static case that the $\mathcal{O}_{r^2}$ operator expectation value falls off with distance like $|\vec{x}|^{-7}$ (as compared to the $|\vec{x}|^{-4}$ falloff of the same quantity around an isolated color fundamental quark). To assess how general this result is, we will now study the state of the gauge field around a color-neutral collection of $N$ quarks: the baryon of this gauge theory.

A baryon in $\mathcal{N} = 4$ SYM is dual to a fivebrane on which $N$ fundamental strings terminate [22, 23]. The precise description of this system was found in Ref. [24] (see also [25]) through a study of the fivebrane worldvolume action. In this approach the strings are faithfully represented by a specific deformation of the flux-carrying fivebrane, in accord with the Born-Infeld string philosophy [27, 28]. The explicit fivebrane embedding that corresponds to a baryon was found to be [24]

$$r(\theta) = \frac{r_0}{\sin \theta} \left[ \frac{3}{2} (\theta - \sin \theta \cos \theta) \right]^{1/3},$$

(39)
where \( r = R^2/z \), \( \theta \) is the \( \mathbb{S}^5 \) polar angle, and \( r_0 = r(\theta = 0) \) is a modulus of the configuration. Since the fivebrane is just as much a source of the dilaton as is the string, we may use the logic of the earlier part of the paper to infer the gauge theory expectation value of \( \mathcal{O}_{F^2} \) in the presence of a baryonic collection of heavy quarks. The interesting question is whether this approach yields the same scaling with \( N \) and \( \|\vec{x}\| \) as would the description of the baryon as a collection of quark strings.

Following [24], the fivebrane action for an embedding of the above type in the presence of a nontrivial dilaton field can be seen to read (in the Einstein frame)

\[
S_{D5} = T_5 \Omega_4 R^4 \int dt d\theta \sin^4 \theta \{ - \frac{e^\phi}{\sqrt{r^2 + r'^2}} - E^2 + 4A_0 \},
\]

from which \( E = F_{0\theta} \) may be eliminated in favor of the displacement field

\[
D = \frac{\sin^4 \theta E}{\sqrt{r^2 + r'^2 - E^2}},
\]

which is known explicitly as a function of \( \theta \):

\[
D(\theta) = \frac{3}{2} \left( \sin \theta \cos \theta - \theta \right) + \sin^3 \theta \cos \theta .
\]

After this replacement, Eq. (40) implies a linearized dilaton source term which can be written in the form

\[
S_{D5\phi} = -\frac{N}{3\pi^2 \alpha'} \int dt d\theta \phi \sqrt{r^2 + r'^2} \sqrt{D^2 + \sin^8 \theta},
\]

where we have made use of the relation \( T_5 \Omega_4 R^4 = N/3\pi^2 \alpha' \).

The embeddings of interest satisfy a BPS condition [25, 24, 26], which can be used to eliminate \( r' \) in favor of \( r \), yielding

\[
S_{D5\phi} = -\frac{N}{3\pi^2 \alpha'} \int dt d\theta r(\theta) \left( \frac{D^2 + \sin^8 \theta}{\sin^4 \theta \cos \theta - D \sin \theta} \right) \phi = -\frac{N}{3\pi^2 \alpha'} \int dt dz f(z) \phi.
\]
the fivebrane, however, it introduces an additional factor of $\sin^4 \theta(z)$. The resulting source for the massless $AdS_5$ dilaton is

$$J(x) = \frac{2\kappa^2}{3\pi^2\alpha'\Omega_5 R^5} f(z) \sin^4 \theta(z) \delta(\vec{x}) \ .$$

(45)

It follows that the (rescaled) dilaton field is now given by

$$\tilde{\phi}(x) = -\frac{N}{12\pi^4\alpha'} \int dt' dz' f(z') \frac{\sin^4 \theta(z')}{\sin v} \frac{d}{dv} \left[ \frac{\cos 2v}{\sin v} \theta(1 - |\cos v|) \right]$$

(46)

where the invariant distance $v$ is given in (11). Through a familiar set of steps, one can extract the leading behavior of $\tilde{\phi}$ in the neighborhood of the $z = 0$ boundary of $AdS_5$:

$$\tilde{\phi} = -\frac{5N z^4}{4(2\pi)^3\alpha'} \int_0^{z_m} dz' z'^4 f(z') \sin^4 \theta(z') \frac{1}{[z'^2 + |\vec{x}|^2]^{7/2}}$$

(47)

where $z_m = R^2/r_0$ is the maximum value of $z$ to which the fivebrane extends.

To obtain information from (47) it is convenient to return to the initial angular parametrization:

$$\tilde{\phi} = -\frac{5N R^8 z^4}{4(2\pi)^3\alpha'} \int_0^\pi \frac{\sin^4 \theta d\theta}{r(\theta)^3 \left[ \frac{R^4}{r(\theta)^2} + |\vec{x}|^2 \right]^{7/2}} \left( \frac{D^2 + \sin^8 \theta}{\sin^4 \theta \cos \theta - D \sin \theta} \right) \ .$$

(48)

where the embedding $r(\theta)$ is given by (39). The complete field profile of the baryon then follows from (17). From the way $R^4/r^2 = z^2$ appears in the denominator of (47) and (48) it is clear that the dilaton field (and consequently the SYM field profile) will have qualitatively different behavior in the regions $|\vec{x}| > z_m$ and $|\vec{x}| < z_m$, so the modulus $z_m$ in fact determines the 'size' of the baryon, as expected from the UV/IR connection [16, 17] (see e.g. the discussion in [29]).

For $|\vec{x}| \gg z_m$, the leading term in (48) is

$$\tilde{\phi} = -\frac{5N R^2 z_m^3 z^4}{9(2\pi)^3\alpha' |\vec{x}|^2} \int_0^\pi d\theta \frac{\sin^6 \theta}{\left[ \theta - \sin \theta \cos \theta \right]^2} (D^2 + \sin^8 \theta) \ .$$

(49)
Letting $c \simeq 2.40$ denote the result of the angular integration and employing (17), we find that the $O_{F^2}$ expectation value at large distance from the baryon is

$$\langle O_{F^2} \rangle = \frac{5c\sqrt{2} z_m^3 N \sqrt{g_Y^2 N}}{18\pi^3 |\vec{x}|^7}.$$  \hspace{1cm} (50)

Notice that the dependence on $|\vec{x}|$ and the scale size of the configuration ($z_m$) is exactly the same as that found for the 'meson', Eq. (32). This is probably a generic feature of color-neutral objects in the $\mathcal{N} = 4$ SYM gauge theory. From the string theory perspective the common origin of this behavior is clear: unlike the quark, the meson and the baryon are represented by brane objects which do not extend all the way to the horizon at $z = \infty$.

A significant difference between (32) and (50) is that the latter includes an additional power of $N$. This is precisely as it should be, since $\text{Tr} F^2 / 4g_Y^2$ should scale with $N$ in the same way as the energy-momentum tensor: at fixed $g_Y^2 N$ it should be $O(1)$ for a meson, and $O(N)$ for an $SU(N)$ baryon [22].

7 Conclusions

We have examined the correspondence between external charges in $\mathcal{N} = 4$ SYM and strings in AdS space. Our principal focus was the connection between string oscillations and gauge theory waves. Specifically, by studying the bulk radiation given off by an undulating string, we determined the time-dependent fields produced by an oscillating quark or a quark-antiquark pair in the strongly-coupled theory. The picture that emerges is one in which the waves are in fact generated not only by the external sources, but also by the non-linear medium supplied by the static background field of the same sources. This is in agreement with our qualitative expectations for strongly-coupled non-Abelian gauge theory. The same considerations also suggest the existence of an infinite tower of excitations in the quark-antiquark system in the extreme Maldacena limit. The status of these excitations is uncertain, pending the resolution of some puzzles regarding energy conservation in the AdS description of the SYM theory, a subject to which we hope to return.

As a side-result, we have determined the static fields produced by a quark-antiquark pair and also by the D-brane representative of the baryon. Both color-neutral systems were found to display the same long-distance behavior, and to have operator expectation values which fall off more rapidly with distance than those of the isolated quark.

Our results provide yet another example of the remarkable way in which the bulk-boundary correspondence manages to relate intricate aspects of the dynamics of strongly-coupled gauge theories to properties of string theory in AdS space. At the same time, we have stressed the need for further work to unravel the precise way in which SYM energy conservation manifests itself in the dual holographic description.

\footnote{We thank Igor Klebanov for a discussion on this point.}
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References

[1] J. Maldacena, “The Large $N$ Limit of Superconformal Field Theories and Supergravity,” Adv. Theor. Math. Phys. 2 (1998) 231, hep-th/9711200.

[2] E. Witten, “Bound States of Strings and $p$-Branes,” Nucl. Phys. B460 (1996) 335, hep-th/9510135.

[3] S.-J. Rey, J. Yee, “Macroscopic Strings as Heavy Quarks of Large $N$ Gauge Theory and Anti-de Sitter Supergravity,” hep-th/9803001.

[4] J. Maldacena, “Wilson Loops in Large $N$ Field Theories,” Phys. Rev. Lett. 80 (1998) 4859, hep-th/9803002.

[5] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, “Gauge Theory Correlators from Noncritical String Theory,” Phys. Lett. B428 (1998) 105, hep-th/9802109.

[6] E. Witten, “Anti-de Sitter Space and Holography,” Adv. Theor. Math. Phys. 2 (1998) 253, hep-th/9802150.

[7] U. H. Danielsson, E. Keski-Vakkuri, and M. Kruczenski, “Vacua, Propagators, and Holographic Probes in AdS/CFT,” hep-th/9812007.

[8] T. Banks, M. R. Douglas, G. T. Horowitz, and E. Martinec, “AdS Dynamics from Conformal Field Theory,” hep-th/9808016.

[9] V. Balasubramanian, P. Kraus, A. Lawrence, and S. Trivedi, “Holographic Probes of Anti-de Sitter Spacetimes,” hep-th/9808017.

[10] H. Dorn and H.-J. Otto, “On Wilson Loops and $Q \bar{Q}$-potentials from the AdS/CFT relation at $T \geq 0$,” hep-th/9812109.

[11] I. R. Klebanov, “World Volume Approach to Absorption by Non-dilatonic Branes,” Nucl. Phys. B496 (1997) 231, hep-th/9702076.

[12] I. R. Klebanov, W. Taylor IV, and M. Van Raamsdonk, “Absorption of Dilaton Partial Waves by D3-Branes,” hep-th/9905174.
[13] S. Lee, A. Peet and L. Thorlacius, “Brane Waves and Strings”, Nucl. Phys. B514 (1998) 161, hep-th/9710097.

[14] S. R. Das, “Brane Waves, Yang-Mills Theories and Causality,” hep-th/9901004.

[15] D. Bak and S.-J. Rey, “Holographic View of Causality and Locality via Branes in AdS/CFT Correspondence,” hep-th/9902101.

[16] L. Susskind and E. Witten, “The Holographic Bound in Anti-de Sitter Space,” hep-th/9805114.

[17] A. W. Peet and J. Polchinski, “UV/IR Relations in AdS Dynamics,” hep-th/9809022.

[18] M. R. Douglas and W. Taylor IV, “Branes in the Bulk of Anti-de Sitter Space,” hep-th/9807225;
A. Bilal and C.-S. Chu, “D3 Brane(s) in AdS\(_5\) \times S^5 and \(\mathcal{N} = 4, 2, 1\) SYM,” hep-th/9810195.

[19] U. H. Danielsson and A. P. Polychronakos, “Quarks, Monopoles and Dyons at Large \(N\),” Phys. Lett. 434B (1998) 294, hep-th/9804141.

[20] A. Güijosa and Ø. Tafjord, unpublished.

[21] N. Drukker, D. J. Gross and H. Ooguri, “Wilson Loops and Minimal Surfaces,” hep-th/9905129.

[22] E. Witten, “Baryons and Branes in Anti de Sitter Space,” J. High Energy Phys. 07 (1998) 006, hep-th/9805112.

[23] D. Gross and H. Ooguri, “Aspects of Large N Gauge Theory Dynamics as seen by String theory,” Phys. Rev. D58 (1998) 106002, hep-th/9805129.

[24] C. G. Callan, A. Güijosa, and K. Savvidy, “Baryons and String Creation from the Fivebrane Worldvolume Action,” Nucl. Phys. B547 (1999) 127, hep-th/9810092.

[25] Y. Imamura, “Supersymmetries and BPS Configurations on Anti-de Sitter Space,” Nucl. Phys. B537 (1999) 184, hep-th/9807179.

[26] B. Craps, J. Gomis, D. Mateos, and A. Van Proeyen, “BPS Solutions of a D5-brane Worldvolume in a D3-brane Background from Superalgebras,” hep-th/9901060.

[27] C. Callan and J. Maldacena, “Brane Dynamics from the Born-Infeld Action,” Nucl. Phys. B513 (1998) 198, hep-th/9708147.

[28] G. Gibbons, “Born-Infeld Particles and Dirichlet p-branes”, Nucl. Phys. B514 (1998) 603, hep-th/9709027.
[29] C. G. Callan, A. Güijosa, K. Savvidy, and Ø. Tafjord, “Baryons and Flux Tubes in Confining Gauge Theories from Brane Actions,” hep-th/9902197.