Eikonal Reaction Theory for Neutron Removal Reaction

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We present an accurate method of treating the one-neutron removal reaction at intermediate incident energies induced by both nuclear and Coulomb interactions. In the method, the nuclear and Coulomb breakup processes are consistently treated by the method of continuum discretized coupled channels without making the adiabatic approximation to the Coulomb interaction, so that the removal cross section calculated never diverges. This method is applied to a recently measured one-neutron removal cross section for $^{31}\text{Ne}+^{12}\text{C}$ scattering at 230 MeV/nucleon and $^{31}\text{Ne}+^{208}\text{Pb}$ scattering at 234 MeV/nucleon. The spectroscopic factor and asymptotic normalization coefficient of the last neutron in $^{31}\text{Ne}$ are evaluated.

Subject Index: 221, 227

Introduction

Unstable nuclei have exotic properties such as the halo structure\(^1\)–\(^3\) and the change of magicity for nuclei in the region called “island of inversion”,\(^4\)–\(^6\) These novel quantum properties have inspired extensive experimental and theoretical studies. An important experimental tool for exploring such exotic properties is the nucleon removal reaction; see for example Ref. 7). The theoretical tool for analyzing such an inclusive reaction is the Glauber model.\(^8\) The theoretical foundation of the model is investigated in Ref. 9). The model is based on the eikonal and adiabatic approximation. It is well known that the adiabatic approximation makes the removal cross section diverge when the Coulomb interaction is included. The Glauber model has thus been applied mainly for lighter targets in which the Coulomb interaction is negligible; see for example Refs. 7), 10)–15)\. Very recently, the Glauber model with the Coulomb correction has been proposed.\(^16\),\(^17\) In the Coulomb-corrected eikonal model (CCE),\(^16\),\(^17\) the E1 (dipole) component of the eikonal Coulomb phase is replaced by that based on the first-order perturbation. For the elastic-breakup scattering of $^{11}\text{Be}$ from a $^{208}\text{Pb}$ target at 69 MeV/nucleon, it was shown in Ref. 17) that CCE well simulates the results of the dynamical eikonal approximation (DEA).\(^18\)

Meanwhile, the method of continuum discretized coupled channels (CDCC)\(^19\),\(^20\) is accurate for treating exclusive reactions such as the elastic scattering and elastic breakup reaction in which the target is not excited, whereas the projectile is broken up into fragments. The theoretical foundation of CDCC is shown in Refs. 21)–23). Actually, CDCC has succeeded in reproducing data on the scattering of not only stable nuclei but also unstable nuclei; see for example Refs. 24)–30) and references therein. The DEA\(^18\) is also an accurate method of treating exclusive reactions at intermediate and high incident energies where the eikonal approximation is reliable. The nucleon removal reaction is composed of the exclusive elastic-breakup component and the inclusive nucleon-stripping component. CDCC and DEA can be used...
to evaluate the elastic-breakup cross section, but not the stripping cross section.

The experimental exploration of halo nuclei is moving from lighter nuclei such as He and C isotopes to relatively heavier nuclei such as Ne isotopes. Very recently, a halo structure of $^{31}\text{Ne}$ has been reported following the experiment on the one-neutron removal reaction $\sigma - n$ at 230 MeV/nucleon not only for a $^{12}\text{C}$ target but also for a $^{208}\text{Pb}$ target.\(^{31}\) This is the heaviest halo nucleus at the present stage confirmed experimentally, which also resides within the region of “island of inversion”. The determination of the spin parity of $^{31}\text{Ne}$ in its ground state is essential to understand the nature of “island of inversion”. The one-neutron removal reaction has been analyzed with the Glauber model\(^{32}\) for a $^{208}\text{Pb}$ target, the elastic breakup component due to the one-step E1 transition is added.

In this paper, we present an accurate method of treating the one-neutron removal reaction at intermediate incident energies such as 200 MeV/nucleon induced by both nuclear and Coulomb interactions. In the method, the nuclear and Coulomb breakup processes are accurately treated using CDCC without making the adiabatic approximation to the latter, so that the removal cross section calculated never diverges and is reliable even in the presence of the Coulomb interaction. Thus, this method is an essential extension of the Glauber theory and CDCC. This method is applied to the one-neutron removal reaction of the $^{31}\text{Ne} + ^{12}\text{C}$ scattering at 230 MeV/nucleon and the $^{31}\text{Ne} + ^{208}\text{Pb}$ scattering at 234 MeV/nucleon. The results of the present method are compared with those of CCE. The spectroscopic factor $S$ and the asymptotic normalization coefficient $C_{\text{ANC}}$\(^{33}\) of the last neutron in $^{31}\text{Ne}$ are evaluated with the new theory.

**Formulation**

We consider the one-neutron halo nucleus as a projectile ($P$) and take a single-particle model for the nucleus; namely, the projectile consists of a core nucleus ($c$) and a neutron ($n$). The scattering of $P$ on a target ($T$) is then described using the three-body ($c + n + T$) Schrödinger equation

$$\left[ -\frac{\hbar^2}{2\mu} \nabla_R^2 + h + U(r_c, r_n) - E \right] \Psi = 0 \quad (1)$$

with the interaction

$$U = U^{(\text{Nucl})}_n(r_n) + U^{(\text{Nucl})}_c(r_c) + U^{(\text{Coul})}_c(r_c), \quad (2)$$

where $\mu$ is the reduced mass between $P$ and $T$. The three-dimensional vector $R = (b, Z)$ stands for the coordinate between $P$ and $T$, whereas $r_x$ ($x = n$ or $c$) is the coordinate between $x$ and $T$, and $r$ means the coordinate between $c$ and $n$. The operator $h = T_r + V(r)$ is the projectile Hamiltonian composed of the kinetic-energy operator $T_r$ and the interaction $V(r)$. The potential $U^{(\text{Nucl})}_x$ is the nuclear part of the optical potential between $x$ and $T$, whereas $U^{(\text{Coul})}_c$ is the Coulomb interaction between $c$ and $T$.

Now, we consider scattering at intermediate incident energies, say 200 MeV/nucleon, for which the eikonal approximation is quite good. The three-body wave function $\Psi$ is first assumed to be

$$\Psi = \hat{O} \psi(R, r), \quad (3)$$
where the operator $\hat{O}$ is defined by

$$\hat{O} = \frac{1}{\sqrt{\hbar}} e^{i\hat{K} \cdot \mathbf{Z}}$$

(4)

with the wave-number operator $\hat{K} = \sqrt{2\mu(E - \hbar)/\hbar}$ and the velocity operator $\hat{v} = \hbar \hat{K}/\mu$ of the relative motion between $P$ and $T$. When (3) is inserted into (1), we have three terms $\hat{O}[\nabla^2_{R\psi}]$, $[\nabla_{R\hat{O}} \hat{O}] \cdot [\nabla_{R\psi}]$ and $[\nabla^2_{R\hat{O}}]\psi$. The first term is much smaller than the others, since $\psi$ is slowly varying with $\mathbf{R}$ compared with $\hat{O}$. Neglecting the first term leads to

$$\frac{id\psi}{dZ} = \hat{O}^\dagger \hat{U} \hat{O} \psi.$$  

(5)

Regarding $Z$ as “time” virtually and solving (5) iteratively, we obtain the formal solution

$$\psi = \exp \left[-i\mathcal{P} \int_{-\infty}^{Z} dZ' \hat{O}^\dagger \hat{U} \hat{O} \right],$$

(6)

where $\mathcal{P}$ is the “time” ordering operator. Taking $Z$ to $\infty$ in (6), we finally obtain the $S$-matrix operator

$$S = \exp \left[-i\mathcal{P} \int_{-\infty}^{\infty} dZ \hat{O}^\dagger \hat{U} \hat{O} \right].$$

(7)

In the Glauber model, the adiabatic approximation is made as the secondary approximation. In the approximation, $h$ is replaced by the ground-state energy $\epsilon_0$, and hence, $\hat{O}^\dagger \hat{U} \hat{O}$ and $\mathcal{P}$ in (7) are reduced to $U/(\hbar v_0)$ and 1, respectively, where $v_0$ is the velocity of $P$ in the ground state relative to $T$. This is nothing but the $S$-matrix in the Glauber model.

At intermediate incident energies, the adiabatic approximation is good for the short-range nuclear interactions, $U_{n(\text{Nucl})}$ and $U_{c(\text{Nucl})}$, but not for the long-range Coulomb interaction $U_{c(\text{Coul})}$. This can be understood from the matrix element

$$\int_{-\infty}^{\infty} dZ \langle \varphi_k | \hat{O}^\dagger \hat{U} \hat{O} | \varphi_0 \rangle \approx \frac{e^{i(K_0 - K)R_U}}{\hbar v_0} \int_{-\infty}^{\infty} dZ \langle \varphi_k | U | \varphi_0 \rangle$$

(8)

between the ground state $\varphi_0$ of $P$ with the intrinsic energy $\epsilon_0$ and the continuum state $\varphi_k$ of $P$ with the intrinsic momentum and energy, $h\mathbf{k}$ and $\epsilon(k)$. In (8), $hK_0$ ($h\hat{K}$) is the momentum of $P$ in the ground (continuum) state relative to $T$, and $R_U$ is the range of the interaction considered. For the $^{31}\text{Ne}$ breakup reaction at 200 MeV/nucleon, the typical excitation energy $\epsilon(k) \approx 1$ MeV. For the Coulomb interaction, because of its long-range property, $(K_0 - K)R_U$ is large even if $K_0 - K$ is small. This means that we cannot set $K_0 - K$ to zero, that is, the adiabatic approximation does not work. Actually, the elastic-breakup cross section diverges in the adiabatic limit of $K = K_0$ because of the slow decrease in the matrix element (8) in $b^{17,34}$) For the nuclear interactions, meanwhile, $R_U \approx 11$ fm and, hence,
\( (K_0 - K)R_U \approx 0.074 \). Thus, the adiabatic approximation is good for the nuclear interactions. This indicates that \( U_n^{(\text{Nucl})} \) is commutable with \( \hat{O} \) with high accuracy. Therefore, we can take the replacement

\[
\hat{O}^\dagger U_n^{(\text{Nucl})} \hat{O} \to U_n^{(\text{Nucl})}/(h\nu_0).
\]  

(9)

The replacement (9) is accurate as shown later by numerical calculations. Using this replacement, one can obtain the important result

\[
S = S_n S_c
\]

with

\[
S_n = \exp \left[ -\frac{i}{h\nu_0} \int_{-\infty}^{\infty} dZ U_n^{(\text{Nucl})} \right],
\]

(11)

\[
S_c = \exp \left[ -i\mathcal{P} \int_{-\infty}^{\infty} dZ \hat{O}^\dagger \left( U_c^{(\text{Nucl})} + U_c^{(\text{Coul})} \right) \hat{O} \right].
\]

(12)

Equation (10) can be derived as follows. The replacement (9) leads (5) to

\[
\frac{id\psi}{dZ} = \hat{O}^\dagger U_c \hat{O} \psi + \frac{1}{h\nu_0} U_n^{(\text{Nucl})} \psi
\]

(13)

with \( U_c = U_c^{(\text{Nucl})} + U_c^{(\text{Coul})} \). Defining \( \xi = Q \psi \) with

\[
Q = \exp \left[ -\frac{i}{h\nu_0} \int_{-Z}^{\infty} dZ' U_n^{(\text{Nucl})} \right]
\]

(14)

and inserting \( \psi = Q \xi \) into (13) leads to

\[
\frac{id\xi}{dZ} = \hat{O}^\dagger U_c \hat{O} \xi,
\]

(15)

where use has been made of the fact that \( Q \), i.e., \( U_n^{(\text{Nucl})} \), is commutable with \( \hat{O} \) with high accuracy. The formal solution of (15) is \( S_c \), whereas \( S_n \) is obtained from \( Q \) by taking \( Z \to \infty \). Noting that \( \psi = Q \xi \), one can reach (10).

Thus, \( S \) can be separated into the neutron part \( S_n \) and the core part \( S_c \). We cannot calculate \( S_c \) directly with (12), because it includes the operators \( \hat{O} \) and \( \mathcal{P} \). However, \( S_c \) is the approximate solution of the Schrödinger equation

\[
\left[ -\frac{\hbar^2}{2\mu} \nabla_R^2 + h + U_c^{(\text{Nucl})}(r_c) + U_c^{(\text{Coul})}(r_c) - E \right] \psi_c = 0,
\]

(16)

when the eikonal approximation is made. We can calculate the matrix elements \( \langle \varphi_0 | S_c | \varphi_0 \rangle \) and \( \langle \varphi_k | S_c | \varphi_0 \rangle \) by solving (16) with eikonal-CDCC\(^{34}\) in which the eikonal approximation is made in the framework of CDCC. Noneikonal corrections to \( S_c \) can be easily made by solving (16) with CDCC instead of eikonal-CDCC, although it is not necessary for the present intermediate scattering. As mentioned above, \( S_n \) is obtained with (11).
The reaction theory constructed above is referred to as the eikonal reaction theory (ERT) in this paper. We can test the accuracy of the eikonal approximation by comparing the $S$-matrix elements of CDCC and eikonal-CDCC.\(^{34}\) We confirmed that the approximation is quite good for the present scattering system. This approximation is good even for lower incident energies such as 30 MeV/nucleon.\(^{34}\) Thus, all CDCC calculations in this article are carried out with eikonal-CDCC.

We can derive several cross sections with the product form (10), following the formulation on the cross sections in the Glauber model.\(^{10,12}\) The one-neutron removal cross section $\sigma_{-n}$ is the sum of the total elastic breakup (diffractive) cross section $\sigma_{bu}$ and the total neutron-stripping cross section $\sigma_{str}$ in which $n$ is absorbed by $T$:

$$\sigma_{-n} = \sigma_{bu} + \sigma_{str}$$

with

$$\sigma_{str} = \int d^2b \langle \varphi_0 | S_c |^2 (1 - |S_n|^2) |\varphi_0 \rangle$$

$$= [\sigma_R - \sigma_{bu}] - [\sigma_R(-n) - \sigma_{bu}(-n)],$$

where $\sigma_R$ and $\sigma_{bu}$ are the total reaction and elastic-breakup cross sections, respectively, defined by

$$\sigma_R = \int d^2b [1 - |\langle \varphi_0 | S_c S_n |\varphi_0 \rangle|^2],$$

$$\sigma_{bu} = \int d^2b [\langle \varphi_0 | S_c S_n |^2 |\varphi_0 \rangle - |\langle \varphi_0 | S_c S_n |\varphi_0 \rangle|^2],$$

and $\sigma_R(-n)$ and $\sigma_{bu}(-n)$ correspond to the total reaction and elastic-breakup cross sections, respectively, in which $S_c S_n$ is replaced by $S_c$. The last form of (18) means that $\sigma_{-n}$ can be obtained from $\sigma_R$, $\sigma_{bu}$, $\sigma_R(-n)$ and $\sigma_{bu}(-n)$ by solving (1) and (16) with CDCC.

**Model setting** We take the same three-body model as in Ref. 32) except for a minor difference, since the Glauber model calculation based on the three-body model yields a reasonable value of $\sigma_{-n}$. The nuclear potential $U^{(\text{Nucl})}_x (x = n \text{ or } c)$ is calculated using the folding model in which the effective nucleon-nucleon ($NN$) interaction $t$ is folded with the densities of particles $T$ and $x$. As for $t$, we take the parameter set in Refs. 32) and 35) for $E_{NN} = 240$ MeV, where $E_{NN}$ is the energy of $NN$ scattering.\(^{\ast}\) The folding potential reproduces the reaction cross section of $^{12}\text{C} + ^{12}\text{C}$ scattering in the wide range of incident energies.\(^{35}\) In Ref. 32), the $^{30}\text{Ne}$ density is calculated using the single-particle model in which the separation energies of the last neutron and proton reproduce the experimental values of 3 MeV and 2.4 MeV, respectively; for the potential parameters, see the case of $(r_0, a) = (1.25, 0.75)$ in

\(^{\ast}\) In Ref. 35), the effective interaction is presented as the profile function $\Gamma(b_{NN})$. It is a function of not the distance $r_{NN}$ between two nucleons but the impact parameter $b_{NN}$. The interaction $t(r_{NN})$ is so constructed that the profile function obtained from $t(r_{NN})$ can agree with that of Ref. 35).
In ERT, the adiabatic approximation is assumed to be good for the nu-


targets is estimated 540 mb from the experimental values of $\sigma_{\text{exp}}(1p3/2)$ in Table II of Ref. 32) for 1p3/2 and the case of $(r_0, a) = (1.25, 0.70)$ for 0f7/2. A large $\sigma_{\text{exp}}^{\text{exp}}$ indicates that $^{31}\text{Ne}$ is a one-neutron halo nucleus. This structure is realized with small angular momenta between $c$ and $n$. However, the possibility of the 2s1/2 orbit is small from the theoretical point of view, since the single-particle energy of 2s1/2 is much higher than those of 1p3/2 and 0f7/2. Hence, we consider the 1p3/2 and 0f7/2 orbits.

Results In ERT, the adiabatic approximation is assumed to be good for the nuclear potential $U_n^{(\text{Nucl})}$. This can be tested by setting $U = U_n^{(\text{Nucl})}(r_n) + U_c^{(\text{Coul})}(R) + U_c^{(\text{Nucl})}(R)$ in the Schrödinger equation (1). In this setup, the projectile breakup is induced only by $U_n^{(\text{Nucl})}(r_n)$, since the argument $r_c$ of $U_c^{(\text{Coul})}$ and $U_c^{(\text{Nucl})}$ is replaced by $R$. Switching the adiabatic approximation on in the Schrödinger equation corresponds to the replacement (9). For the scattering of $^{31}\text{Ne}(1p3/2)$ from $^{208}\text{Pb}$, the error due to the approximation is 0.2% for $\sigma_R$, 1.9% for $\sigma_{\text{bu}}$, 4.1% for $\sigma_{\text{str}}$ and 3.3% for $\sigma_{-n}$. Thus, the error is small.

Table I presents several cross sections and the spectroscopic factor $S = \sigma_{-n}^{\text{exp}}/\sigma_{-n}^{\text{th}}$ for $^{12}\text{C}$ and $^{208}\text{Pb}$ targets. As an important result, $S[1p3/2] = 0.693$ (0.682) for $^{12}\text{C}$ ($^{208}\text{Pb}$), whereas $S[0f7/2] = 2.47$ (5.65) for $^{12}\text{C}$ ($^{208}\text{Pb}$). Thus, $S[1p3/2]$ little depends on the target and is less than 1, but $S[0f7/2]$ does not satisfy these conditions. In Ref. 31), the Coulomb component of $\sigma_{-n}^{[208}\text{Pb}]$ for a $^{208}\text{Pb}$ target is estimated to be 540 mb from the experimental values of $\sigma_{-n}^{[208}\text{Pb}]$ and $\sigma_{-n}^{[12}\text{C}]$. In ERT, the Coulomb component of $\sigma_{-n}^{[208}\text{Pb}]$ agrees with $\sigma_{\text{bu}}^{[208}\text{Pb}]$ with good accuracy. The spectroscopic factor evaluated from the Coulomb component is $S' = 540/\sigma_{\text{bu}}^{[208}\text{Pb}] = 0.675$ for the $1p3/2$ orbit and 7.36 for the $0f7/2$ orbit. Thus, $S'$ is consistent with $S$ only for the $1p3/2$ orbit. Hence, we can infer that the major component of the $^{31}\text{Ne}_g.s.$ wave function is $^{30}\text{Ne}(0^+) \otimes 1p3/2$ ($S \sim 0.69$). We adopt in the following this configuration.

Table I. Several cross sections and the spectroscopic factors for $^{31}\text{Ne}+^{12}\text{C}$ scattering at 230 MeV/nucleon and $^{31}\text{Ne}+^{208}\text{Pb}$ scattering at 234 MeV/nucleon. The cross sections are presented in units of mb and the data are taken from Ref. 31).

|          | $^{12}\text{C}$ target |          | $^{208}\text{Pb}$ target |
|----------|-------------------------|----------|---------------------------|
|          | $p_3/2$                | $f_7/2$  | exp                       | $p_3/2$              | $f_7/2$ | exp |
| $\sigma_R$ | 1572.5                 | 1489.9   |                          | 5518.0               | 4589.5  |     |
| $\sigma_{\text{bu}}$ | 23.3                  | 3.3      |                          | 799.5                | 73.0     | (540) |
| $\sigma_R(-n)$ | 1463.5                | 1458.6   |                          | 5151.5               | 4524.2  |     |
| $\sigma_{\text{bu}}(-n)$ | 4.5                  | 1.0      |                          | 677.2                | 60.5     |     |
| $\sigma_{\text{str}}$ | 90                    | 29       |                          | 244                  | 53      |     |
| $\sigma_{-n}$ | 114                   | 32       | 79                       | 1044                 | 126     | 712  |
| $S$       | 0.693                  | 2.47     |                          | 0.682                | 5.65    |     |
In Ref. 32, $S$ is calculated with the Glauber model; for a $^{208}\text{Pb}$ target, the elastic breakup component due to one-step E1 transition is added. The resulting spectroscopic factor for the $1p3/2$ orbit is $S = 0.822$ for a $^{12}\text{C}$ target and 0.624 for a $^{208}\text{Pb}$ target. The difference between the results of the present study and Ref. 32 for $^{12}\text{C}$ mainly comes from the neglect of $\sigma_{\text{bu}}$ in the latter, and that for $^{208}\text{Pb}$ seems to stem from Coulomb higher-order contributions to $\sigma_{\text{bu}}$. Neglect of the Coulomb interaction in $\sigma_{\text{str}}$ may also be responsible for the difference. Thus, CCE is fairly good also for the present system.

The potential $V$ between $c$ and $n$ is not well known. Hence, $S$ has a theoretical error coming from the potential ambiguity. The error is often estimated by changing each of $r_0$ and $a$ by 30%. When the one-neutron separation energy $B_n$ of $^{31}\text{Ne}$ is 0.33 MeV, $S = 0.693 \pm 0.133 \pm 0.061$ for a $^{12}\text{C}$ target and 0.682 $\pm 0.133 \pm 0.062$ for a $^{208}\text{Pb}$ target, where the second and third numbers following the mean value stand for the theoretical and experimental uncertainties, respectively. Thus, $S$ includes a sizable theoretical error. This situation completely changes if we look at the asymptotic normalization coefficient (ANC). For $r$ out of the range of $V$, ANC $C_{\text{ANC}}$ is defined by

$$I_{lj}(r) = C_{\text{ANC}} h_l^{(+)}(i\kappa r)$$

with the radial part of the overlap function $I_{lj}(r) = \langle \phi_c | \phi_n | \phi_P \rangle$, where $\phi_c$ and $\phi_n$ are the intrinsic states of $c$ and $n$, respectively, whereas $\phi_P$ is the ground state of $P$. $h_l^{(+)}$ is a spherical Hankel function and $\kappa$ is the relative wave number between $c$ and $n$ in the ground state of $P$.\(^{33}\) For large $r$, $I_{lj}(r)$ is related to the normalized single-particle wave function $\varphi_{lj}(r)$ that is determined from $V$ as

$$I_{lj}(r) = \sqrt{S} \varphi_{lj}(r) = \sqrt{S} C_{\text{ANC}} h_l^{(+)}(i\kappa r)$$

with the single-particle ANC $C_{\text{ANC}}^{(sp)}$. Hence, we have

$$C_{\text{ANC}} = \sqrt{S} C_{\text{ANC}}^{(sp)}.$$  

When $B_n = 0.33$ MeV, $C_{\text{ANC}} = 0.320 \pm 0.010 \pm 0.028$ fm$^{-1/2}$ for a $^{12}\text{C}$ target and 0.318 $\pm 0.008 \pm 0.029$ fm$^{-1/2}$ for a $^{208}\text{Pb}$ target. Thus, $C_{\text{ANC}}$ has a much smaller theoretical error than $S$. This means that the one-nucleon removal reaction is quite peripheral.

The experimental value of $B_n$ is not precisely determined: $B_n = 0.29 \pm 1.64$ MeV.\(^{38}\) It is thus better to see the $B_n$ dependences of $C_{\text{ANC}}$ and $S$. When $B_n = 0.1$ MeV, $C_{\text{ANC}} = 0.128 \pm 0.003 \pm 0.011$ fm$^{-1/2}$ and $S = 0.530 \pm 0.084 \pm 0.047$ for a $^{12}\text{C}$ target, and $C_{\text{ANC}} = 0.105 \pm 0.004 \pm 0.010$ fm$^{-1/2}$ and $S = 0.358 \pm 0.057 \pm 0.033$ for a $^{208}\text{Pb}$ target. These values are plotted in Fig. 1. $C_{\text{ANC}}$ and $S$ are sensitive to the value of $B_n$. We can see from the $B_n$ dependence of $S$ for a $^{208}\text{Pb}$ target that $S < 1$ when $B_n \lesssim 0.6$ MeV. It is thus necessary to determine $B_n$ experimentally in the future to evaluate $C_{\text{ANC}}$ and $S$ properly. However, we can say at least that $C_{\text{ANC}}$ has a smaller theoretical error and weaker target dependence than $S$ for any value of $B_n$.  

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\(^{33}\) The $S$ factor is defined by $S = 2\pi \int_0^{\infty} I_{lj}(r) r^2 dr$.

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Summary

We have presented an accurate method of treating the one-neutron removal reaction at intermediate energies induced by both the short-range nuclear and the long-range Coulomb interaction. In the theory called ERT, the nuclear and Coulomb breakup processes are accurately and consistently treated by CDCC without making the adiabatic approximation to the latter, so that the removal cross section never diverges and, hence, is reliable even in the presence of the Coulomb interaction. This method is applicable also for other inclusive reactions such as the two-nucleon removal reaction induced by nuclear and Coulomb interactions.

ERT is a theory of presenting the Schrödinger equation (16) to calculate inclusive cross sections. When these equations are solved, any accurate method is useful. For intermediate and high incident energies, one can use DEA instead of eikonal-CDCC. For lower incident energies where the eikonal approximation is not perfectly accurate, one should make noneikonal corrections to inclusive cross sections. This can be done easily by using CDCC instead of eikonal-CDCC.

$C_{\text{ANC}}$ and $S$ of the last neutron in $^{31}\text{Ne}$ are evaluated from the one-neutron removal reaction of the $^{31}\text{Ne}+^{12}\text{C}$ scattering at 230 MeV/nucleon and the $^{31}\text{Ne}+^{208}\text{Pb}$ scattering at 234 MeV/nucleon. $C_{\text{ANC}}$ has a smaller theoretical error and a weaker target dependence than $S$. Thus, $C_{\text{ANC}}$ is determined more accurately than $S$. This may change the future strategy for the spectroscopy of unstable nuclei.

When the last neutron in $^{31}\text{Ne}$ is in the $1p3/2$ orbit, $S < 1$ for $B_n \lesssim 0.6\ \text{MeV}$, and $S$ and $C_{\text{ANC}}$ have weaker target dependence. When the last neutron is in the $1f7/2$ orbit, meanwhile, $S > 1$ and $S$ and $C_{\text{ANC}}$ have stronger target dependence. These results indicate that the last neutron is mainly in the $1p3/2$ orbit and imply that $^{31}\text{Ne}$ is deformed. This sort of deformation is expected as an origin of the change in magicity.
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