Oscillation mode analysis considering the interaction between a DFIG-based wind turbine and the grid

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Abstract. Sub-synchronous interactions between wind farms and transmission networks with series compensation have drawn great attention. As most large wind farms in Europe and Asia employ doubly fed induction generator turbines, there has recently been a growing interest in studying this phenomenon. To study the stability of wind turbine with doubly fed induction generator after a small disturbance, a complete small signal system is built in this paper. By using eigenvalue and participation factor analysis, the relation between the modes and state variables can be discovered. Thereafter, the oscillation modes are classified into electrical resonance, sub-synchronous resonance, sub-synchronous oscillation, sub-synchronous control interaction, and low frequency oscillation. To verify the oscillation frequency of each oscillation mode, time-domain simulation based on MATLAB/Simulink is presented. The simulation results justify the effectiveness of the small-signal models.

1. Introduction
With the deterioration of environment and concerns of energy security and fossil energy deficit, wind power becomes the fastest growing power technology [1-3]. Among several wind generation technologies, variable speed wind turbines utilizing doubly fed induction generators (DFIGs) have attracted extraordinary attention because of their advantages over other type of wind turbines [4, 5]. An increasing number of wind farms are expected to be connected to lines with series compensation, which however may produce adverse effects, such as sub-synchronous interactions (SSI), with other power system components [6]. As a result, the system generally experience over-voltages and sub-synchronous oscillations, which may cause equipment damage.

Frequency response and eigenvalue techniques are fundamental tools in analyzing small signal stability of multi-machine power systems [7]. Small signal stability model is derived and an eigenvalue-based objective function is utilized using particle swarm optimization to optimize the controllers’ parameters [8]. [9] develops an approach to analyze the impact of increased penetration of DFIG-based wind turbines on transient and small signal stability of a large power system. In [10], a frequency response-based method is also introduced, as an alternative to the eigen analysis approach, to provide a robust performance evaluation approach. To study the stability of doubly fed induction generators on large power systems, a control strategy to mitigate the impact of reduced inertia is confirmed by time domain and Prony analysis in [11]. However, the references on DFIGs mainly concentrate on certain oscillation modes or optimization of controllers’ parameters, they do not analyze all the oscillation modes utilizing detailed mathematic model.
This study is targeted at studying SSI between the grid and DFIG-based wind turbines. Small signal stability model of a wind turbine to an infinite bus system was developed based on previous work [12]. The eigenvalues and their related state variables are then studied by utilizing participation factors. Accordingly, the oscillation modes are classified into five categories: electrical oscillation, sub-synchronous resonance (SSR), sub-synchronous oscillation (SSO), sub-synchronous control interaction (SSCI), and low frequency oscillation.

The major contribution of this paper is: i) it deduces the small-signal model of a whole system from basic flux linkage, voltage and torque equations, including three-mass shaft, induction generator, rotor side converter, grid side converter, DC-link and transmission line; ii) it analyzes all oscillation modes by eigenvalues and participation factors analysis; iii) time-domain simulation is presented to verify the oscillation frequency obtained by the small-signal model.

The rest of the paper is organized as follows: Section 2 introduces the small-signal model of each part and the whole system. Section 3 analyzes the simulation results of small-signal model to identify the oscillation frequency of each eigenvalue and the corresponding oscillation mode. In Section 4, the time-domain simulation is presented to verify the oscillation modes in the small-signal model. Conclusions are drawn in Section 5.

2. Small-signal Modeling of the DFIG

The schematic diagram of the studied system is presented in figure.1, where the wind turbine driven DFIG is connected to the infinite bus system through a transmission line. The one-wind turbine to infinite bus system consists of a three-mass shaft, a induction generator, a rotor side converter(RSC), a grid side converter (GSC), a GSC side reactance, and a transmission line.

![Diagram of DFIG-based wind generator model](image)

**Figure 1.** Block diagram of the DFIG-based wind generator model

2.1. Modeling of three-mass shaft

The wind turbine drive train generally comprises a windmill blade, a low-speed shaft, gearbox, a high-speed shaft and a generator rotor, and it is usually represented by two-mass model. However, the two-mass model is not appropriate for transient stability studies, such as torque oscillations, that may interact with the electrical system. A three-mass model is therefore employed in this study.

Within the three-mass model, the inertia of the gearbox is shared between the low-speed shaft (LSS) and high-speed shaft (HSS), and the inertia of the high-speed shaft aggregates the inertia of the generator. Thus, all three masses correspond to the mass moment of the inertia of blades $J_1$, low-speed shaft $J_2$ and high-speed shaft $J_3$, respectively. By taking into account the stiffness and the damping factors for both shafts, the dynamic equations converted into the high-speed shaft can be written as
\[
\begin{aligned}
\dot{\theta}_1 &= \omega_{turb} \\
\dot{\theta}_2 &= \omega_{gear} \\
\dot{\theta}_3 &= \omega_{gen} \\
J_1\dot{\omega}_{turb} &= T_{aero} - K_1(\theta_1 - \theta_3) - D_2\omega_{turb} \\
&- D_1(\omega_{turb} - \omega_1) \\
J_2\dot{\omega}_{gear} &= -K_2(\theta_2 - \theta_3) - K_3(\theta_1 - \theta_3) - D_2\omega_{gear} \\
&- D_3(\omega_{gear} - \omega_2) \\
J_3\dot{\omega}_{gen} &= -T_{em} - K_1(\theta_1 - \theta_3) - D_3\omega_{gear} \\
&- D_2(\omega_{gen} - \omega_2)
\end{aligned}
\]  

(1)

where \(\omega_{turb}\) is the wind turbine angle speed; \(\omega_{gear}\) is the low-speed shaft angle speed; \(\omega_{gen}\) is the high-speed shaft angle speed; \(\theta_1, \theta_2, \theta_3\) are the twist angles of the wind turbine, low-speed shaft, and high-speed shaft, respectively; \(K_1, K_2\) are the shaft stiffness coefficients; \(D_1, D_2, D_3\) are the self damping coefficients; \(D_{12}, D_{23}\) are the mutual damping coefficients; \(T_{em}\) is the electromagnetic torque; \(T_{aero}\) is the aerodynamic torque.

2.2. Modeling of induction generator

For modeling the induction generator model, machine equations are represented in synchronously rotating frame where the direction of the d-axis is aligned with the rotor’s magnetic flux linkage. The stator, rotor, flux, electromagnetic torque equations are described as follows

\[
\begin{aligned}
\dot{u}_{ds} &= R_1i_{ds} + \dot{\psi}_{ds} - \omega_2\psi_{qs} \\
\dot{u}_{qs} &= R_1i_{qs} + \psi_{ds} + \omega_2\psi_{qs} \\
\dot{u}_{dr} &= R_2i_{dr} + \psi_{dr} - \omega_1\psi_{qr} \\
\dot{u}_{qr} &= R_2i_{qr} + \psi_{qr} + \omega_1\psi_{dr}
\end{aligned}
\]

\[
\begin{aligned}
\psi_{ds} &= L_{ds}i_{ds} + L_{dq}i_{dq} \\
\psi_{qs} &= L_{dq}i_{ds} + L_{qs}i_{dq} \\
\psi_{dr} &= L_{ds}i_{dr} + L_{dq}i_{dq} \\
\psi_{qr} &= L_{dq}^2i_{dq} + L_{qr}i_{dq} \\
T_{em} &= \frac{3}{2}pL_{m}(i_{ds}i_{dq} - i_{ds}i_{dq})
\end{aligned}
\]

(2)

(3)

(4)

where \(u_{ds}\) and \(u_{qs}\) are d and q axis stator voltages; \(u_{dr}\) and \(u_{qr}\) are d and q axis rotor voltage; \(R_1\) and \(R_2\) are stator and rotor resistances; \(\psi_{ds}\) and \(\psi_{qs}\) are the d and q axis stator flux; \(\psi_{dr}\) and \(\psi_{qr}\) are the d and q axis rotor flux; \(i_{ds}\) and \(i_{qs}\) are d and q axis stator currents; \(i_{dr}\) and \(i_{qr}\) are d and q axis rotor currents; \(L_m\) is the mutual inductance; \(L_s\) is the stator self-inductance; \(L_r\) is the d and q axis rotor self-inductance; \(\omega_1\) is the stator flux angular velocity; \(\omega_2\) is the slip angular velocity; \(p\) is the pole pair.

2.3. Modeling of RSC

The rotor side converter aims to control the output active power to track the input of the wind turbine torque. The decoupled control strategy is used for the DFIG system to regulate its output power, as shown in figure 2.
With introducing variables $x_0$, $x_1$, $x_2$, $x_3$ and $x_4$, the rotor side converter model can be expressed by the following equations:

$$
\begin{align*}
\Delta \dot{x}_0 &= \Delta \omega_{s-ref} - \Delta \omega_r \\
\Delta i_d &= \Delta P_{s-ref} - \Delta P_e + K_{p\omega}(\Delta \omega_{s-ref} - \Delta \omega_r) + K_{p\omega}\Delta x_0 + i_{d0}\Delta u_{d0} + u_{d0}\Delta i_{d0} + i_{q0}\Delta u_{q0} + u_{q0}\Delta i_{q0} \\
\Delta x_2 &= K_{p\alpha}(\Delta P_{s-ref} - \Delta P_e) + K_{r\alpha}\Delta x_1 - \Delta i_{d0} \\
\Delta x_3 &= K_{p\alpha}(\Delta Q_{s-ref} - \Delta Q_e) + K_{r\alpha}\Delta x_1 - \Delta i_{q0} \\
\Delta x_4 &= -K_{p\alpha}(i_{d0}\Delta u_{d0} + u_{d0}\Delta i_{d0} - i_{q0}\Delta u_{q0} - u_{q0}\Delta i_{q0}) \\
\Delta i_{q0} &= -K_{p\beta}(i_{d0}\Delta u_{d0} + u_{d0}\Delta i_{d0} - i_{q0}\Delta u_{q0} - u_{q0}\Delta i_{q0}) \\
\Delta i_{d0} &= K_{p\beta}(\Delta Q_{s-ref} - \Delta Q_e) + K_{r\beta}\Delta x_1 - \Delta i_{q0} \\
\Delta i_{q0} &= -K_{p\beta}(i_{d0}\Delta u_{d0} + u_{d0}\Delta i_{d0} - i_{q0}\Delta u_{q0} - u_{q0}\Delta i_{q0}) \\
\end{align*}
$$

The output variables of the grid side converter is given by:

$$
\begin{align*}
\Delta u_{d0} &= K_{p\beta}(K_{p\beta}(\Delta Q_{s-ref} - \Delta Q_e) + K_{r\beta}\Delta x_1 - \Delta i_{q0}) \\
+ K_{r\beta}\Delta x_1 - X_{s0}\Delta i_{d0} - X_{s0}\Delta i_{q0} + X_{s0}\Delta i_{q0} \\
= K_{p\beta}(K_{p\beta}(\Delta Q_{s-ref} - \Delta Q_e) + K_{r\beta}\Delta x_1 - \Delta i_{q0}) \\
+ K_{r\beta}\Delta x_1 - X_{s0}\Delta i_{d0} - X_{s0}\Delta i_{q0} + X_{s0}\Delta i_{q0} \\
\Delta u_{q0} &= K_{p\beta}(K_{p\beta}(\Delta Q_{s-ref} - \Delta Q_e) + K_{r\beta}\Delta x_1 - \Delta i_{q0}) \\
+ K_{r\beta}\Delta x_1 - X_{s0}\Delta i_{d0} - X_{s0}\Delta i_{q0} + X_{s0}\Delta i_{q0} \\
\end{align*}
$$

2.4. Modeling of GSC

The dc-link voltage is maintained by controlling the grid side converter using the decoupled d-q vector control, with the d-q frame oriented along the stator voltage vector position.
Figure 3. Control diagram of the grid side converter

As shown in figure 3, the dc-link is controlled by $i_{dg}$ while the reactive power is controlled by $i_{qg}$. With introducing variables $x_7$, $x_8$ and $x_9$, the grid side converter model can be expressed by the following equations

\[
\begin{align*}
\Delta i_s &= \Delta V_{DC_{ref}} - \Delta V_{DC} \\
\Delta x_q &= \Delta i_{q_{ref}} - \Delta i_q = K_{pg} (\Delta V_{DC_{ref}} - \Delta V_{DC}) + K_{dq0} \Delta x_5 - \Delta i_d \\
\Delta x_7 &= \Delta i_{q_{ref}} - \Delta i_q
\end{align*}
\] (7)

The output variables of the grid side converter is given by

\[
\begin{align*}
\Delta u_{dg} &= K_{pg} (\Delta V_{DC_{ref}} - \Delta V_{DC}) + K_{dq0} \Delta x_5 - \Delta i_d + K_{iq0} \Delta x_7 \\
\Delta u_{qg} &= K_{pg} (\Delta i_{q_{ref}} - \Delta i_q) + K_{iq0} \Delta x_7
\end{align*}
\] (8)

2.5. Modeling of DC-link

The dc-link model can be obtained according to that the active power flow through the converters is balanced, given as follows

\[
Cu_{DC} \frac{du_{DC}}{dt} = \frac{3}{2} (u_{dg} i_{dg} + u_{qg} i_{qg} - u_{d2} i_{d2} - u_{q2} i_{q2})
\] (9)

where $C$ is the capacitance of the capacitor; $u_{DC}$ is the dc-link voltage, defining its positive direction as discharge direction.

2.6. Modeling of Grounded Capacitor

The differential equations of grounded capacitor are given by

\[
\begin{align*}
\frac{du_{gc,x}}{dt} &= \omega u_{gc,x} - \omega X_{gc} i_{gc,x} \\
\frac{du_{gc,y}}{dt} &= -\omega u_{gc,y} - \omega X_{gc} i_{gc,y}
\end{align*}
\] (10)

where $u_{gc,x}$ and $u_{gc,y}$ are the x and y axis voltages; $i_{gc,x}$ and $i_{gc,y}$ are the x and y axis currents; $X_{gc}$ is the value of the grounded capacitor.

2.7. Modeling of GSC side Reactance and Transformer

The differential equations of GSC side reactance and transformer are given by

\[
\begin{align*}
\frac{di_{gs,x}}{dt} &= -\frac{R}{L} i_{gs,x} + \frac{1}{L} u_{gs,x} \\
\frac{di_{gs,y}}{dt} &= -\frac{R}{L} i_{gs,y} + \frac{1}{L} u_{gs,y}
\end{align*}
\] (11)
where \( u_{gx} \) and \( u_{gy} \) are the x and y axis voltages; \( i_{gx} \) and \( i_{gy} \) are the x and y axis currents; \( R \) and \( L \) are the equivalent resistance and reactance of GSC side reactance and transformer.

### 2.8. Modeling of Transmission Line

Based on the analysis above, the differential equations of transmission line are given as

\[
\begin{align*}
\frac{di_{lx}}{dt} &= -\omega R i_{lx} + \omega X_c i_{nx}, \\
\frac{di_{lx}}{dt} &= -\omega R i_{lx} - \omega X_c i_{nx}, \\
\frac{du_{nx}}{dt} &= \omega u_{nx} - \omega X_c i_{lx}, \\
\frac{du_{nx}}{dt} &= -\omega u_{nx} - \omega X_c i_{lx},
\end{align*}
\]

where \( u_{sc,x} \) and \( u_{sc,y} \) are the x and y axis voltages; \( i_{lx} \) and \( i_{ly} \) are the x and y axis currents; \( X_{sc} \) is the reactance of the series capacitor, \( r \) and \( X_L \) are the resistance and reactance of the transmission line.

### 2.9. Modeling of the whole system

To study the system stability after suffering a small disturbance, it is necessary to build the complete small-signal model of the system, as shown in figure

\[\text{Figure 4. Small signal model of DFIG to infinite bus}\]

By solving the Equations above, the union model can be written in a compact form in

\[
\Delta \dot{X} = A \Delta X + B \Delta u
\]

### 3. Analysis of the Small-signal Model

The small signal model of the above system is built in MATLAB/Simulink. All parameters of the wind turbine system are provided in Appendix. Based on the small signal model, the eigenvalues of a state matrix can be calculated. The eigenvalues of the system is shown as Table 1.

It can be seen that the system is stable after suffering small disturbance because all of the eigenvalues have negative real parts. There are nine oscillation modes and nine evanescent modes. The participation factors can disclose the relation between the modes and the variables, which are shown in Table 2.
Table 1. Eigenvalues of the small signal system

| No. | Eigenvalues | Oscillation frequency, Hz | Damping |
|-----|-------------|---------------------------|---------|
| 1   | -1715±2.68×10³i | 4.26×10² | 0       |
| 2   | -3189±1.35×10³i | 2.15×10³ | 0       |
| 3   | -942.2       | 0          | 1       |
| 4   | -23.63±498i   | 79.25      | 0.0474  |
| 5   | -51.17±285i   | 43.7    | 0.1767  |
| 6   | -16.75±147i   | 23.40    | 0.1132  |
| 7   | -139.8       | 0          | 1       |
| 14  | -1.477±77.97i | 12.41 | 0.0189  |
| 15  | -8.91±27.45i  | 4.37    | 0.3087  |
| 17  | -11.72±12.08i |         | 1.92    |
| 18  | -0.319±3.177i |         | 0.51    |
| 21  | -2.74        | 0          | 1       |
| 22  | -1.20        | 0          | 1       |
| 23  | -1.00        | 0          | 1       |
| 24  | -0.0538      | 0          | 1       |
| 25  | -0.008       | 0          | 1       |
| 26  | -0.0003      | 0          | 1       |
| 27  | -1.825×10¹⁴  |         | 0       |

Table 2. Participation factors of state variables

|     | \( \Delta l_{1,2} \) | \( \Delta l_{1,4} \) | \( \Delta l_{6,7} \) | \( \Delta l_{9} \) | \( \Delta l_{10,11} \) | \( \Delta l_{17,18} \) | \( \Delta l_{18,20} \) |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \Delta l_{1,2} \) | 0.0000 | 0.0134 | **0.2106** | 0.0008 | **0.1902** | 0.0030 | 0.0098 | 0.0230 | 0.0012 |
| \( \Delta l_{1,4} \) | 0.0034 | 0.0000 | **0.2415** | 0.0007 | **0.1609** | 0.0020 | 0.0169 | 0.0151 | 0.0010 |
| \( \Delta l_{6,7} \) | 0.0000 | 0.0000 | **0.2521** | 0.0050 | **0.2302** | 0.0014 | 0.0077 | 0.0067 | 0.0004 |
| \( \Delta l_{9} \) | 0.0000 | 0.0000 | **0.2605** | 0.0055 | **0.2498** | 0.0018 | 0.0046 | 0.0089 | 0.0005 |
| \( \Delta l_{10,11} \) | 0.0000 | 0.5000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \Delta l_{17,18} \) | 0.0000 | 0.1529 | 0.0000 | 0.0175 | **0.1767** | 0.0875 | 0.0022 | 0.0143 | 0.0312 | 0.0019 |
| \( \Delta l_{18,20} \) | 0.0000 | 0.6032 | 0.0000 | 0.0385 | **0.1661** | 0.0455 | 0.0012 | 0.0023 | 0.0140 | 0.0005 |
| \( \Delta l_{19} \) | 0.0000 | 0.3393 | 0.0000 | 0.0032 | 0.0155 | 0.0320 | 0.0013 | 0.0117 | 0.0310 | 0.0015 |
| \( \Delta l_{20} \) | 0.0000 | 0.1339 | 0.0000 | 0.0160 | 0.0414 | 0.0412 | 0.0014 | 0.0076 | 0.0244 | 0.0012 |
| \( \Delta l_{21} \) | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \Delta \theta_{0} \) | 0.0000 | 0.5000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \Delta \theta_{1} \) | 0.0000 | 0.1499 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \Delta \theta_{2} \) | 0.0000 | 0.1237 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \Delta \theta_{3} \) | 0.0000 | 0.1237 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \Delta \theta_{4} \) | 0.0000 | 0.1237 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \Delta \theta_{5} \) | 0.0000 | 0.1237 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \Delta \theta_{6} \) | 0.0000 | 0.1237 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \Delta \theta_{7} \) | 0.0000 | 0.1237 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \Delta \theta_{8} \) | 0.0000 | 0.1237 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \Delta \theta_{9} \) | 0.0000 | 0.1237 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \Delta \theta_{10} \) | 0.0000 | 0.1237 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \Delta \theta_{11} \) | 0.0000 | 0.1237 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

According to the eigenvalues in Table 1 and the participation factors in Table 2, the oscillation modes can be classified into: electrical resonance, sub-synchronous resonance (SSR), sub-synchronous oscillation (SSO), sub-synchronous control interaction (SSCI), and low frequency oscillation.
3.1. Electrical resonance
As shown in Table 2, $\lambda_{1,2}$ are highly sensitive to voltage of the grounded capacitor $\Delta u_{gc,y}$, generator flux $\Delta \psi_{qs}$ and $\Delta \psi_{qr}$. Similarly, $\lambda_{3,4}$ are highly sensitive to voltage of the grounded capacitor $\Delta u_{gc,x}$, generator flux $\Delta \psi_{ds}$ and $\Delta \psi_{dr}$. So $\lambda_{1,2}$ and $\lambda_{3,4}$ are mainly affected by the generator and grounded capacitor. Therefore, they belong to electrical resonance.

However, the corresponding oscillation frequencies of $\lambda_{1,2}$ and $\lambda_{3,4}$ are very high due to the small value of the grounded capacitance. Generally, the grounded capacitor is equal to open circuit and has no effect to the whole system. Therefore, the two oscillation modes are not the emphasis of the system.

$\lambda_{8,9}$ are highly sensitive to generator flux $\Delta \psi_{qs}$ and $\Delta \psi_{ds}$, and current through GSC side reactance $\Delta i_{gs}$ and $\Delta i_{gs}$, so they are mainly affected by the generator and the GSC side reactance. Therefore, they also belong to electrical resonance.

3.2. Sub-synchronous resonance
SSR can potentially occur in the fixed-series compensated DFIG-based wind farm even at realistic levels of series compensation [13]. In a series-compensated power system, the natural resonance frequency is given by

$$f_r = f_0 \sqrt{\frac{X_{sc}}{X_L}}$$

(14)

where $f_0$ is the synchronous frequency, $X_{sc}$ is the reactance of the series capacitor, and $X_L$ is the total reactance of the transmission line, transformers, and generator.

Therefore, the resonance frequency is influenced by compensation level $k_{con} = \frac{X_{sc}}{X_L}$. In the small signal model simulation, the compensation level is 35%. Therefore, the system resonance frequency is given by

$$f_r = f_0 \sqrt{k_{con}} = 50 \times \sqrt{0.35} = 29.58 \text{Hz}$$

(15)

In the system, $\lambda_{6,7}$ and $\lambda_{10,11}$ are highly sensitive to current and voltage of transmission line, i.e., variables $\Delta i_{Lx}$, $\Delta i_{Ly}$, $\Delta u_{sc,x}$, and $\Delta u_{sc,y}$, so they are mainly affected by the transmission line and the series capacitor. Moreover, the oscillation frequency of $\lambda_{6,7}$ is 79.25Hz, which is approximately equal to $50 + f_r = 79.58$Hz. The oscillation frequency of $\lambda_{10,11}$ is 23.40Hz, which is approximately equal to $50 - f_r = 20.42$Hz.

Therefore, the oscillation mode of $\lambda_{6,7}$ belongs to super-synchronous resonance and that of $\lambda_{10,11}$ belongs to sub-synchronous resonance.

3.3. Sub-synchronous oscillation
Doubly-fed induction generator wind turbines close to series capacitors are prone to subsynchronous oscillations [14].

$\lambda_{13,14}$ are highly sensitive to twist angle of gearbox $\Delta \theta_{gear}$ and $\Delta \omega_{gear}$, and slightly sensitive to twist angle of generator $\Delta \theta_{gen}$ and $\Delta \omega_{gen}$, so they are mainly affected by the three mass shaft. $\lambda_{17,18}$ are highly sensitive to twist angles of generator and gearbox, i.e., variables $\Delta \theta_{pm}$, $\Delta \omega_{pm}$, $\Delta \theta_{gear}$, and
\[ \Delta \omega_{\text{gear}}, \text{ and also sensitive to DC-link voltage } \Delta V_{\text{DC}}, \text{ variables of converter controller such as } \Delta x_0, \Delta x_1, \text{ and } \Delta x_2, \text{ so they are mainly affected by the three mass shaft and the converter controller.} \]

Therefore, the oscillation modes of \( \lambda_{13,14} \) and \( \lambda_{17,18} \) belong to sub-synchronous oscillation.

3.4. Sub-synchronous control interaction
Severe damage to wind turbines due to SSCI has been observed in the case of an interconnection of a wind farm to a series compensated transmission system [15]. The SSCI is not related to mechanical shaft system and may change under different system conditions and converter control algorithms.

In the system, \( \lambda_{15,16} \) are highly sensitive to variable of converter controller \( \Delta x_3 \) and DC-link voltage \( \Delta V_{\text{DC}} \), and also sensitive to generator angle speed \( \Delta \omega_r \), so they are mainly affected by the converter controller and rotor speed of generator. Therefore, the oscillation mode of \( \lambda_{15,16} \) belongs to sub-synchronous control interaction.

3.5. Low-frequency oscillation
Damping low frequency oscillations of power systems is one of the most important determinants of system stability. Due to the increasing growth of wind farms with high penetration levels in power systems as well as their participation and control, it seems important to increase the damping system oscillations [16]. It occurs in the 0.2 to 2.5 Hz frequency range.

In the system, \( \lambda_{19,20} \) are highly sensitive to twist angles of wind turbine and generator \( \Delta \theta_{\text{turb}}, \Delta \theta_r, \Delta \omega_{\text{turb}}, \) and variable of converter controller \( \Delta x_0 \), so they are mainly affected by the three mass shaft and rotor speed control of the generator.

4. Time-domain Simulation Testing
In order to test the oscillation modes of the small signal model, the system shown in figure 1 was implemented in MATLAB/Simulink. The simulation results are shown in figure 5-9.

4.1. Electrical resonance
As analyzed above, the oscillation frequency of \( \lambda_{8,9} \) is 45.37 Hz, and the corresponding oscillation mode is mainly affected by the generator and the GSC side reactance. Therefore, we observe the GSC output current and study the harmonic components through spectral analysis using the FFT analysis built-in the Simulink, as shown in figure 5.

![Figure 5. FFT analysis of GSC output current](image)

It is observed from figure 5 that the fundamental frequency of the GSC output current is 50Hz and there exist harmonics in the waveforms. Apparently, the most obvious harmonic is 45Hz, which is consistent with the simulation result of the small signal model.
4.2. Sub-synchronous resonance
In the proposed system, the compensation level is 35%. Therefore, the network resonance mode has a frequency of 29.58Hz (≈ 30Hz), which induces both 20- and 80-Hz oscillations in the grid side. Therefore, we observe the stator output current and study the harmonic components in the waveform, as shown in figure 6.

![Figure 6. FFT analysis of the stator output current](image)

It is observed from figure 6 that the fundamental frequency of the current is 50 Hz. Harmonics are also observed in the spectrum, most noticeably at 20Hz and 80Hz, which is consistent with the simulation result of the small signal model.

4.3. Sub-synchronous oscillation
The results above indicate that there exist two oscillation frequencies in the system, i.e., 12.4Hz corresponding to \( \lambda_{13,14} \), and 1.92Hz corresponding to \( \lambda_{17,18} \), both of which are closely related to the three mass shaft. Therefore, we study the waveform of the electromagnetic torque and the corresponding spectral analysis, as shown in figure 7.

![Figure 7. FFT analysis of the electromagnetic torque](image)

It is observed from figure 7 that the electromagnetic torque exhibits a considerable level of harmonic distortion, most noticeably at 2.5Hz and 14Hz, which are consistent with the SSO modes of \( \lambda_{17,18} \) and \( \lambda_{13,14} \).

4.4. Sub-synchronous control interaction
The SSCI oscillation mode of \( \lambda_{15,16} \) is highly sensitive to the DC-link controller and the oscillation frequency is 4.37Hz. Therefore, we observe the dc-link capacitive voltage and study the harmonic components in the waveform, as shown in figure 8.
It is observed from figure 8 that the dc-link capacitive voltage exhibits a certain level of harmonic distortion, most noticeably at 4.5Hz and 50Hz. While 50Hz is the fundamental frequency of the system, 4.5Hz is consistent with the SSCI oscillation mode of $\lambda_{15,16}$.

4.5. Low-frequency oscillation

The low-frequency oscillation mode of $\lambda_{19,20}$ is mainly affected by the three mass shaft and rotor speed control of the generator. Therefore, we observe the rotor speed of generator and study the harmonic components in the waveform, as shown in figure 9.

It is observed from figure 9 that the generator rotor speed exhibits a considerable level of harmonic distortion, most noticeably at 0.5Hz, 2.5Hz, and 14Hz. While 2.5Hz and 14Hz are corresponding to the SSO modes of $\lambda_{17,18}$ and $\lambda_{13,14}$, 0.5Hz is consistent with the low-frequency oscillation mode of $\lambda_{19,20}$.

5. Conclusions

In this paper, a small signal model of DFIG-based wind turbine connected to power grid is presented. To investigate the vibration modes of the whole system, each sub-model needs to be separately built. Therefore, the dynamic model of three-mass shaft wind turbine, induction generator, and back-to-back converters together with the controllers are derived. Through modal analysis of the union small signal stability model, the model is used to detect and analyze the oscillation modes in the system. Participation factors and eigenvalues analysis reveals that there exist electrical resonance at 45.37Hz, SSR at 79.25Hz and 23.40Hz, SSO at 12.41Hz and 1.92Hz, SSCI at 4.37Hz, and low-frequency oscillation at 0.51Hz. The time-domain simulation results in MATLAB/Simulink verified that oscillation modes and corresponding oscillation frequencies detected in the small signal model are correct.

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Appendix
Wind turbine: rated wind speed = 12 m/s, rated rotor speed = 18.1 r/s, blade length = 40.25 m, blades = 3.
Induction generator: power rating = 2 MW, stator voltage = 690 V, pole pair = 4, frequency = 50 Hz, stator resistance $R_1 = 0.0234 \, \Omega$, rotor resistance $R_2 = 0.0013 \, \Omega$, stator dq axis self-inductances $L_{d_s} = L_{q_s} = L_s = 4.595 \, mH$, rotor dq axis self-inductances $L_{d_r} = L_{q_r} = L_r = 4.575 \, mH$, mutual inductance $L_m = 4.5 \, mH$.

Three-mass model (transferred to high speed side): $J_1 = 2901.8 \, kg \cdot m^2$, $J_2 = 20 \, kg \cdot m^2$, $J_3 = 121 \, kg \cdot m^2$, $K_1 = 6.59 \times 10^4 \, N \cdot m/rad$, $K_2 = 9.22 \times 10^4 \, N \cdot m/rad$, gearbox ratio $N = 47:1$.

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