Strong squeezing via phonon mediated spontaneous generation of photon pairs

Kenan Qu and G S Agarwal
Department of Physics, Oklahoma State University, Stillwater, Oklahoma, 74078, USA
E-mail: girish.agarwal@okstate.edu

Received 9 May 2014, revised 11 August 2014
Accepted for publication 19 September 2014
Published 31 October 2014
New Journal of Physics 16 (2014) 113004
doi:10.1088/1367-2630/16/11/113004

Abstract
We propose a scheme for generating squeezed light by using a double-cavity optomechanical system driven by a blue detuned laser in one cavity and by a red detuned laser in the other. This double-cavity system is shown to effectively mimic an interaction that is similar to the one for a downconverter, which is known to be a source of strong squeezing for light fields. There are however distinctions, as the phonons which lead to such an interaction can contribute to the quantum noise. We show that squeezing of the output fields, of the order of 10 dB, can be achieved even for an effective mechanical mode occupation number of about 4, which for the chosen parameters corresponds to 10 mK. Our results are generic and applicable to a wide class of electromechanical and optomechanical systems involving the interaction of two electromagnetic modes and one mechanical mode.

Online supplementary data available from stacks.iop.org/njp/16/113004/mmedia

Keywords: optomechanics, squeezed state, quantum noise
1. Introduction

Realization of the quantum regime [1–14], especially the ground state cooling, of optomechanical systems has been of great interest. Many theoretical papers have investigated other interesting possibilities such as the preparation of cat states [15] and Fock states [16] of the mirror. One important signature of the quantum regime is the squeezing which is of fundamental importance in precision measurements [17, 18]. Thus quantum metrology drives the demand not only for higher levels of squeezing but also for the availability of squeezing in a variety of systems. There are several studies demonstrating squeezing in a single-cavity optomechanical system [7, 19–28]. Safavi-Naeini et al [23] first reported the ponderomotive squeezing of light of a few per cent in optomechanics using a strong coherent driving laser in resonance with the cavity frequency and Purdy [24] later pushed the squeezing magnitude to 1.7 dB. Vitali et al [25] theoretically proposed ‘optical spring kicks’ stationary squeezing using a series of optical pulses with quadratic optomechanical interaction, and the potential squeezing magnitude can reach 13 dB. A deeper squeezing in optomechanics can be obtained by closed loop feedback control [29] or injecting presqueezed light directly into the cavity [30], but the proposal itself requires using squeezed light and one has to maintain high fidelity in the state transfer. Kronwald et al [28] recently proposed dissipative optomechanical squeezing of light, which can be directly used to enhance the intrinsic measurement sensitivity of the optomechanical cavity.

In the present work, we use a double-cavity optomechanical system and develop a quantum optical analog of the method of producing two-mode squeezing which is based on the usage of an entangled pair of photons. We effectively mimic the behavior of a downconverter for producing squeezing in an optomechanical system. In a downconverter [31–33], the coupling Hamiltonian which leads to two-mode squeezing is given by $\xi a_1 a_2 + \xi^* a_1^\dagger a_2^\dagger$, where the $a_i$’s are the annihilation operators for the two light fields. The entangled photon pairs $a_i$ are spontaneously produced from the pump. An appropriate linear combination of $a_i$’s produces quadrature squeezing. Any system whose effective interaction can be reduced to this form becomes a good candidate for showing two-mode squeezing. Thus the goal is to find systems where different interactions can be reduced to the form $\xi a_1 a_2 + \xi^* a_1^\dagger a_2^\dagger$. The third-order nonlinearities in optical fibers can also give rise to such an interaction leading to squeezing [34].

We now describe how such an effective Hamiltonian can be realized in the cavity optomechanics. A cavity driven by a blue detuned pump can spontaneously produce a photon and a phonon. Under the usual approximations—an undepleted pump and the rotating wave approximation—this process would be described by the effective Hamiltonian $\xi a_1 b + \xi^* a_1^\dagger b^\dagger$ where $a_1$ stands for the cavity photon and $b$ stands for the phonon. Although this Hamiltonian has the form of a downconverter, it cannot be used to produce squeezing since the phonon frequency is many orders less than the optical frequency. Then one would like to replace the phonon mode with another optical mode. This will be achieved by using another parametric process where a red detuned pump photon combines with a coherent phonon and produces a cavity photon via an upconversion process. In the undepleted pump and rotating wave approximations, this is described by $\xi a_2^\dagger b + \xi^* a_2 b^\dagger$. Effectively, a cavity driven by a red

---

1 For observation of two-mode squeezing, one needs to externally superpose two modes, and this can be done only if the modes are close in frequency. Within the blue cavity the two modes, phonon and photon, are resonant but it is hard to probe the properties of the linear combination of the phonon–photon mode.
detuned pump swaps the cavity photon and phonon. In what follows, we use both of these mechanisms to produce a pair of photons in a double-cavity optomechanical system. Thus we produce a photon pair by using phonons. It should be kept in mind that although we produce a photon pair, the mediating mechanism is an active mechanism which puts a limit on the amount of achievable squeezing. This is in contrast to the situation with a downconverter, where the crystal participates in a passive manner in the sense that it does not contribute to the quantum noise. We show generation of very large two-mode squeezing even at an effective mechanical mode temperature like 10 mK (phonon occupancy $\bar{n}_{th} = 3.7$), which can be obtained either by precooling or by using cooling techniques such as sideband cooling. The large squeezing is a consequence of active phonon nonlinearities which become large due to the resonant nature of the underlying processes. We note that double-cavity optomechanical systems have gained considerable importance because of the many possible applications. These include two-mode electromagnetically induced transparency, electromagnetically induced absorption [35], quantum state conversion [4], optical wavelength conversion [36], enhancing quantum nonlinearities [37], quantum-nondemolition measurement [3] and entangled photon pair generation [5].

The organization of this paper is as follows. In section 2 we present the basic model, the underlying equations and the different parametric processes in the two cavities. In section 3 we present the calculation of the squeezing spectra for the field which is the linear superposition of the two output fields. In section 4 we present numerical results for squeezing. We explain the origin of squeezing via the phonon-mediated four-wave mixing (FWM) process. We also compare the squeezing in our scheme to that obtained by using other methods. In section 5 we investigate the effect of the Brownian noise of the mirror and its effect on the output state purity. In section 6 we present our conclusions. We would like to emphasize that although the previous works [5] report on the generation of the entangled pairs in a double-cavity optomechanical system, they do not report the reasonable amount of squeezing that one can get in double-cavity optomechanical systems.

2. The model and fluctuating quantum fields

As mentioned in the introduction, we need an optomechanical system with two optical modes interacting with a common mechanical mode. This can be realized in several ways. We mention two possible systems which have already been realized. 1: A double-cavity optomechanical system in which a mechanical membrane coated with perfectly reflecting films on both sides is coupled to two optical cavities. Such a ‘membrane in the middle’ setup [38] is shown in figure 1(a), where we also show all of the fields. 2: A silica microresonator with two preselected optical modes interacting with a mechanical mode as shown in figure 1(b). Such systems have been experimentally studied by Dong et al [39]. In what follows we continue to use the description ‘double-cavity system’, though the discussion would apply to several other two-mode systems.

Let us denote the optical modes using the annihilation operators $a_i$ for the cavity $i$. We denote the mechanical mode of the resonator using the normalized displacement operator $Q = \frac{m_{osc}}{\hbar} q$ and momentum operator $P = \frac{i}{\hbar} \frac{p}{m_{osc}}$ where $q$, $p$, $m$ and $\omega_m$ are the displacement, the momentum, the mass and the oscillation frequency of the mirror, respectively. The interaction between the optical and mechanical modes arises from the radiation pressure of
light, which results in a change in the cavity length and, hence, the cavity frequency. The radiation pressure interaction can be written as \[\omega_i - \frac{\hbar}{L_i} qa_i, a_i \] \[\omega_i - \frac{\hbar}{L_i} qa_i, a_i \]

where \(\omega_i\) and \(L_i\) are the resonance frequency and length of the empty cavity \(i\). We denote the coupling rate by \(\frac{\hbar}{L_i} qa_i, a_i \) \[\omega_i - \frac{\hbar}{L_i} qa_i, a_i \]

In order to enhance the optomechanical coupling, a coherent driving laser with an amplitude \(\alpha_i\) and frequency \(\omega_i\) is applied to each cavity. Then the Hamiltonian for the system can be written as

\[
H = \sum_{i=1,2} \left[ \hbar \omega_i a_i + \hbar \dot{E}_i \right] a_i + \hbar \dot{E}_i a_i e^{-i\omega_i t} - a_i e^{i\omega_i t} \right] \\
+ \frac{1}{2} \hbar \omega_i \left( Q^2 + P^2 \right) - \hbar \left( g_i a_i a_i - g_2 a_i a_2 \right) Q.
\]

(1)

The driving laser amplitude is related to its power \(P_i\) by \(E_i = \sqrt{2 \kappa_i \frac{P_i}{\hbar \omega_i}}\) and \(2\kappa_i\) is the decay rate of the cavity \(i\). The second term in (1) represents how the external coherent fields enter the cavity (chapters 7 and 9 in [32]). It is convenient to rewrite the Hamiltonian in a new picture using the transformation exp \(\left[-i \sum (\omega_i a_i, a_i, t)\right]\); then,

\[
H = \sum_{i=1,2} \left[ \hbar \left( \omega_i - \omega_i \right) b_i + \hbar \dot{E}_i \right] b_i + \hbar \dot{E}_i b_i - b_i \right] \\
+ \frac{1}{2} \hbar \omega_i \left( Q^2 + P^2 \right) - \hbar \left( g_i b_i b_i - g_2 b_i b_2 \right) Q,
\]

(2)

with the \(b_i\)’s defined by \(a_i = b_i e^{-i\omega_i t}\). The derivation of the output optical fields is given in the Supplemental Material and we show the result here:

\[
\tilde{b}_{1\text{out}}(\omega) = E_1(\omega) \tilde{b}_{1\text{in}}(\omega) + F_1(\omega) \tilde{b}_{2\text{in}}^\dagger (-\omega) + V_1(\omega) f^\dagger (-\omega),
\]

(3)

\[
\tilde{b}_{2\text{out}}(\omega) = E_2(\omega) \tilde{b}_{2\text{in}}(\omega) + F_2(\omega) \tilde{b}_{1\text{in}}^\dagger (-\omega) + V_2(\omega) f(\omega),
\]

(4)

where \(b_{1\text{in}}(\omega)\) and \(f(\omega)\) are the optical and mechanical input vacuum noises with correlation fluctuations.
The above equations are valid under the condition that the system is in the stable regime. The sufficient and necessary condition for stability is that the coefficient matrix of the differential equations equation (S7), by dropping fluctuating forces, must have eigenvalues $\lambda$ with negative real parts:

$$
\begin{bmatrix}
-\gamma_m/2 - \lambda & iG_1 & -iG_2^* \\
-iG_1^* & -(\kappa_1 + i\delta_1) - \lambda & 0 \\
-iG_2 & 0 & -(\kappa_2 + i\delta_2) - \lambda
\end{bmatrix} = 0.
$$

By using the Routh–Hurwitz criterion [41], we get the stability condition $|G_1|^2/\kappa_1 - |G_2|^2/\kappa_2 < \gamma_m/2$, when $-\Delta_1 \sim \Delta_2 \sim \omega_m$. If this condition is violated, the system goes into the unstable regime.

We now give the meanings of the coefficients $E_i$, $F_i$ and $V_i$ in (3) and (4). These coefficients are obtained to all orders in the strengths of the blue and red pumps. The $E_i$'s and $F_i$'s to second order in $G_i$ can be given simple physical interpretations. Let us first consider an incoming vacuum photon, from cavity 1. It should be borne in mind that the frequency $\omega$ from the cavities corresponds to $\omega + c$ according to the relation $\delta = \omega - \omega_c$. This produces a vacuum photon of frequency $\omega + c$ in cavity 1 and a photon of frequency $\omega - c$ in cavity 2. The reason for the production of a photon of frequency $\omega - c$ can be understood as follows. A blue detuned photon of frequency $\omega_c + \omega$ produces a phonon of frequency $\omega_m - \omega$ and a photon of frequency $\omega_c + \omega$. The phonon of frequency $\omega_m - \omega$ interacts with the red detuned pump of frequency $\omega_c - \omega_m$. This is shown in the diagram in figure 2. The term $F_2(-\omega)$ in (4) represents the combined effect of these two processes. We can similarly understand $F_1(-\omega)$ in (3) by considering an incoming vacuum photon, now from cavity 2. Note that these are the diagrams which contribute to the lowest order in $G_1 G_2$ in the expression for $F_2(-\omega)$. The term proportional to $|G_1|^2$ in $E_1(\omega)$ arises from the diagram figure 2(a). The $V_i$ terms in (3) and (4)
correspond to the quantum noise which is added by either the thermal phonons or the vacuum phonons. Note that in the lowest order in the $G_i$’s, we can interpret the last term in (4) by saying that a thermal phonon of frequency $\omega_m + \omega$ combines with a red photon of frequency $\omega_c - \omega_m$ to produce a photon of frequency $\omega_c + \omega$ as shown in figure 2(c). Similarly, in (4) a thermal phonon or a vacuum phonon of frequency $\omega_m - \omega$ and a photon of frequency $\omega_c + \omega$ combine to create a blue photon $\omega_c + \omega_m$. This is the reverse of the process shown in figure 2(a). The net result is the production of an entangled pair of photons with frequencies $\omega_c + \omega$ and $\omega_c - \omega$. These two outputs will be combined in the next section to generate two-mode squeezing.

3. Squeezing spectra

For studying the squeezing spectra, we combine the output fields $\delta a_{1\text{out}}$ and $\delta a_{2\text{out}}$ to construct the field $d$ as shown in figure 3. To make it more general, we add a phase difference $\theta$ between the output fields; then $d(t)$ can be written as

$$d(t) = \frac{1}{\sqrt{2}} \left[ \delta a_{1\text{out}}(t) + e^{i\theta} \delta a_{2\text{out}}(t) \right]$$

$$= \frac{1}{\sqrt{2}} \left[ \tilde{b}_{1\text{out}}(t) + e^{i\theta} \tilde{b}_{2\text{out}}(t) \right] e^{-i\omega_c t}.$$  \hspace{1cm} (7)

In the frame rotating with the cavity frequency $\omega_c$,

$$\tilde{d}(t) = d(t)e^{i\omega_c t} = \frac{1}{\sqrt{2}} \left[ \tilde{b}_{1\text{out}}(t) + e^{i\theta} \tilde{b}_{2\text{out}}(t) \right] e^{-i\omega_c t}.$$  \hspace{1cm} (8)

which obeys the commutation relation $[\tilde{d}(t), \tilde{d}^\dagger(t')] = \delta(t - t')$. We define as usual the quadrature variable $X_\phi(t) = [\tilde{d}(t)e^{-i\phi} + \tilde{d}^\dagger(t)e^{i\phi}]/\sqrt{2}$, and hence in the frequency domain, $X_\phi(\omega) = \frac{1}{\sqrt{2}} \left[ \tilde{d}(\omega)e^{-i\phi} + \tilde{d}^\dagger(-\omega)e^{i\phi} \right]$

$$= \frac{1}{2} \left[ \left( \tilde{b}_{1\text{out}}(\omega) + e^{i\theta} \tilde{b}_{2\text{out}}(\omega) \right)e^{-i\phi} + \left( \tilde{b}^\dagger_{1\text{out}}(-\omega) + e^{i\theta} \tilde{b}^\dagger_{2\text{out}}(-\omega) \right)e^{i\phi} \right]$$

$$= \frac{1}{2} \left[ E(\omega)\tilde{b}_{1\text{in}}(\omega) + E^*(\omega)\tilde{b}^\dagger_{1\text{in}}(-\omega) + F(\omega)\tilde{b}_{2\text{in}}(\omega) + F^*(\omega)\tilde{b}^\dagger_{2\text{in}}(-\omega) + V(\omega)f(\omega) + V^*(-\omega)f^\dagger(-\omega) \right].$$  \hspace{1cm} (9)

Figure 3. The combination of the output fields $\delta a_{1\text{out}}$ from two cavities, using a 50/50 beam splitter.
where

\[ E(\omega) = E_1(\omega)e^{-i\phi} + F_2^*(-\omega)e^{i\phi - i\theta}, \]
\[ F(\omega) = E_2(\omega)e^{i\theta - i\phi} + F_1^*(-\omega)e^{i\phi}, \]
\[ V(\omega) = V_1(\omega)e^{-i\phi} + V_2^*(-\omega)e^{i\phi - i\theta}. \] (10)

The squeezing spectrum defined as \( \langle X_\phi(\omega)X_{\phi}(\omega') \rangle = 2\pi S_{\phi}(\omega)\delta(\omega + \omega') \) can then be obtained using the correlation relations (5):

\[ S_{\phi}(\omega) = \frac{1}{2\pi} \int \langle X_\phi(\omega)X_{\phi}(\omega') \rangle \; d\omega' \]
\[ = \frac{1}{4} \left[ |V(\omega)|^2\gamma_m(n_{\text{th}} + 1) + |V(-\omega)|^2\gamma_m n_{\text{th}} + |E(\omega)|^2 + |F(\omega)|^2 \right]. \] (11)

We note that if \( \tilde{a}(t) \) were to be a vacuum field, then we would have

\[ S_{\phi}(\omega) = 1/2. \] (12)

Hence we define the normalized squeezed parameter as \( 2S_{\phi}(\omega) \). The magnitude of the squeezing in dB units is then \( -10 \log_{10}(2S_{\phi}). \)

4. Squeezing in the output fields from double-cavity optomechanics

We have studied the physics of the squeezing process in optomechanics, in analogy to the downconversion process, and we expect (11) to yield squeezing. We illustrate the features of our two-mode squeezing in the output field \( d(t) \) in figure 4(a) with \( \theta = \pi \) and for cooperativities \( C_2 = 2C_1 = 20 \). In the plot, we set \( \kappa_1 = \kappa_2 = \kappa \). We give the complete set of parameters in the caption. To create this map, we used (11) at zero temperature. In the diagram, we observe the largest magnitude of squeezing in the amplitude quadrature \( S_0 \) (see figure 4(b)). The magnitude of squeezing at \( \omega = 0 \) is about 12 dB. As one rotates towards the phase quadrature \( S_{\pi/2} \), the squeezing magnitude decreases, and the squeezing eventually turns into antisqueezing. In
Figure 4(c), we show the spectrum on a larger scale, and we find that the squeezing only occurs in the frequency region where $\omega/\gamma_m$ is small. There is an antisqueezing noise floor with a full width close to $\kappa$. When $\omega$ further increases until it is comparable to $\kappa$, the optomechanical interaction becomes negligible and the spectrum turns into vacuum noise, $S_0(\omega) = 0$ dB.

In figure 5, we show in detail the dependence of the squeezing on the cooperativity parameters $C_1$ and $C_2$. Before we discuss figure 5, we analyze the situation analytically. We find that under the approximation $\gamma_m, \kappa_i \ll \omega_m$, the peak value is given by

$$S_0(0) = \frac{1 + \left(\sqrt{C_1} - \sqrt{C_2}\right)^4}{2(1 - C_1 + C_2)^2} + \frac{\left(\sqrt{C_1} - \sqrt{C_2}\right)^2(2\bar{n}_{th} + 1)}{(1 - C_1 + C_2)^2} \sqrt{\frac{1}{C_2} + \left(1 - \frac{C_1}{C_2}\right)^2} + \left(1 - \frac{C_1}{C_2}\right)^2 \frac{2\bar{n}_{th}}{C_2}. \quad (13)$$

The first term in (13) describes the noise squeezing of the input vacuum field and the second term arises from the noise due to the thermal bath phonons. A short derivation shows that $S_0(0)$ approaches its minimum value:

$$S_{\min}(0) = \frac{\bar{n}_{th} + 1}{2C_2 + 2\bar{n}_{th} + 1}, \quad (14)$$

when $\frac{C_1}{C_2} \rightarrow [(1 + \frac{\bar{n}_{th} + 1}{C_2}) - \frac{1}{C_2} + (\frac{2\bar{n}_{th} + 1}{C_2})^2] \text{ for given } C_2$. For $C_1 \gg 1$, the approximation $S_{\min} = \frac{\bar{n}_{th} + 1}{2C_2 + 2\bar{n}_{th}}$ can be used when $\frac{C_1}{C_2} \rightarrow [1 - \frac{1}{C_2}]^2$, i.e., $\sqrt{C_2} - \sqrt{C_1} \rightarrow 1$. Thus the squeezing reaches its maximum magnitude at this limit and it decreases to 0 as one further makes the increase $C_1/C_2 \rightarrow 1$. In the limit $C_1 = C_2, \kappa_1 = \kappa_2$ and $\delta_1 = \delta_2 = 0$; we see from equation (S7) of the Supplemental Material that the cavity modes couple effectively only to one quadrature of
the mechanical mode. This thus hinders the active participation of the mechanical mode in the squeezing process as the mediating mechanism, and hence there is no squeezing, as seen from equation (13). This is in agreement with a recent result [42] that the system acts more like a phase-insensitive amplifier.

Alternatively, if we fix the ratio $C_1/C_2$, then the squeezing magnitude $S_0(0)$ can be increased by increasing $C_2$. The squeezing magnitude $S_0(0)$ is a monotonically increasing function of $C_2$ and it approaches an asymptotic value $S_0(0) \to \frac{1}{2} \left( \frac{1 - \sqrt{C_1/C_2}}{1 + \sqrt{C_1/C_2}} \right)^2$ at zero temperature. This behavior is shown in detail in figure 5(c). This result holds if $1 - \sqrt{C_1/C_2} \gg 1/\sqrt{C_2}$; otherwise it goes to $1/2$ if $C_1 = C_2$.

We illustrate the dependence of the squeezing magnitude on the cooperativity parameter or on the pump power in figure 5. Figure 5(a) shows the squeezing spectrum for different ratios $C_1/C_2$ and we see that squeezing spectrum gains magnitude but loses width when $C_1/C_2$ increases from 0.3 (dashed) to 0.5 (full) and 0.7 (dotted). This can be roughly understood from the smallest root of the denominator $D(\omega)$, given by equations (S12). The smallest root occurs at $\omega = -i(1 - C_1 + C_2)^{\frac{1}{2}}$. In figure 5(b), we plot the squeezing magnitude at $\omega = 0$ as a function of $C_1/C_2$ when the temperatures are zero and when they are nonzero. We see that when $C_1 = 0$, the vacuum optical inputs only interact with cavity 2, and no squeezing process occurs. The incoherent phonons from the mirror in the thermal bath result in fluctuations in the optical output field; hence $S_0(0) \leq 0$ when $T \geq 0$. The magnitude of the squeezing $S_0(0)$ increases with increasing $C_1/C_2$ until it reaches the maximum squeezing. At $T = 0$ and $C_2 = 20$, the maximum squeezing occurs at $C_1/C_2 \cong 0.67$ and $S_0(0) \cong 0.024 = 13$ dB. The system loses squeezing magnitude after this point if $C_1/C_2$ keeps increasing.

Internal losses in the optomechanical system would degrade the squeezing. We now present a brief discussion of the effect of the internal loss $\kappa_i$. The details are given in Supplemental Material. We define the output coupling ratio $\eta = \kappa_e/\kappa$ with $\kappa = \kappa_e + \kappa_i$, and $\kappa_e$ being the external decay rate. The squeezing magnitude at $\omega = 0$ becomes

$$S_0(0) = \frac{1}{2} (1 - \eta) + \left[ \frac{1 + (\sqrt{C_1} - \sqrt{C_2})^4}{2(1 - C_1 + C_2)^2} + \frac{(\sqrt{C_1} - \sqrt{C_2})^2(2\eta_{th} + 1)}{(1 - C_1 + C_2)^2} \right] \eta. \quad (15)$$

Note that when $\eta = 1$, i.e. the internal losses are zero, $S_0(0)$ reduces to only the terms in the square bracket, which are identical to equation (13). The output state is squeezed, and hence $S_0(\omega) < 1/2$. When the cavities are subject to internal losses, i.e. $\eta < 1$, the squeezing effect is suppressed by a factor of $\eta$ and $S_0(0)$ approaches $1/2$ as $\eta \to 0$. The reason for reducing the squeezing magnitude can be attributed to the vacuum noise coming from the extra decay path $\kappa_i$. Figure 6 shows the effect of the internal losses. The squeezing remains significant even in the presence of 10\%–20\% internal losses.

The physics in the generation of the squeezed vacuum states can be interpreted using the FWM process via phonons, as shown in figure 7. In cavity 1, a blue detuned driving laser photon ($\omega_1 = \omega_c + \omega_m$), when being scattered by the mechanical oscillator, produces a phonon ($\omega_m - \omega$) and a photon at a lower frequency $\omega + \omega$. At the same time, in cavity 2, a red detuned driving laser photon ($\omega_2 = \omega_c - \omega_m$), by absorbing the phonon ($\omega_m - \omega$), produces a photon at $\omega_c - \omega$. These processes are resonantly enhanced if both the generated photons are close to the cavity resonance frequency. Equivalently, the physics can be described by the
effective Hamiltonian for the FWM process in figure 7: 

\[ \int \Phi(\omega)a_{\omega}^\dagger a_{\omega - \omega + \omega}^\dagger a_{\omega - \omega} d\omega + h.c. \]

\[ \approx a \int \Phi(\omega)a_{\omega - \omega + \omega}^\dagger a_{\omega - \omega}^\dagger a_{\omega + \omega} d\omega + h.c., \]

when the strong driving lasers \( a_{l1}, a_{l2} \) can be approximated classically by a number \( a \). Here \( \Phi(\omega) \) depends on the details of the optomechanical cavities. Such an interaction has been extensively studied in quantum optics [31–33] and is known to lead to the generation of quantum entanglement as well as quantum squeezing. The generation of entangled pairs has been discussed previously in the context of double-cavity optomechanics [5].

Our double-cavity optomechanics proposal is fundamentally different from the ponderomotive squeezing [3, 22–24], which was experimentally realized by Brooks et al [22] in ultracold atoms, by Purdy et al [24] in a membrane setup, and by Safavi-Naeini et al [23] in a waveguide-coupled zipper optomechanical cavity. In their experiments, a coherent input at the cavity resonance frequency is applied and the quantum noise of the coherent light is reduced by using radiation pressure to push the mechanical resonator, which, in turn, feeds back on the light’s phase. The output squeezed light is generated at the sideband of the cavity frequency detuned by \( \omega_m \), which is approximately equal to the cavity linewidth. The degree of noise reduction depends on the optomechanical coupling strength. They did not use the sideband-resolved condition and reported reasonable squeezing (several dB) under experimental conditions. We work in the sideband-resolved limit and by using two different
parametric processes, where the driving lasers are red and blue detuned, produce photon pairs. Such photon pairs are then combined with a beam splitter to produce squeezing. As a benefit of this particular manner of driving, the squeezed output fields are in resonance with the cavity frequency and hence can be made strong. The red detuned driving field, on the other hand, inherently ensures the stability without requiring any extra cooling laser as long as the red detuned pump interaction is stronger than the blue detuned one.

5. The effect of the Brownian noise of the mirror on the squeezing

It is known that the squeezing is degraded by noise effects of any kind. In optomechanics, the Brownian noise of the mirror makes the observation of quantum effects difficult. As we analyzed in the last section, the squeezing mechanism in our scheme relies on the coherent phonons generated by the driving field to actively transfer quantum coherence between two cavity fields. However, at the same time, the mirror is mediated in the thermal reservoir, which excites incoherent phonons and hence limits the purity of the squeezed fields. At a high temperature, the system even loses the squeezing ability. This is illustrated in figure 5(b), where the curve for $\bar{n}_{th} = 20$ shows antisqueezing when $C_1/C_2 < 0.6$.

We now investigate the effect of the $V$ terms in (9) on the possible amount of squeezing. In figure 8(a), we plot the output field amplitude quadrature at a finite temperature $T = 10$ mK and, correspondingly, $\bar{n}_{th} = 3.7$. We assume that such a temperature is obtained either by using a dilution refrigerator [43, 44] or by precooling techniques [1, 24]. Comparing the full curves in figure 5(a) and in figure 8(a), which are plotted using the same parameters other than the different bath temperatures, one can clearly see that the squeezing magnitude decreases from 12 to 6 when the temperature increases from 0 to 10 mK. With a larger phonon occupancy $\bar{n}_{th}$, the second term in equation (13) dominates the spectrum $S_0(0)$. Interestingly, in our system, the decrease of the squeezing due to the rising of the bath temperature can be compensated by increasing the cooperativity, as a way of enhancing the coupling constant or reducing the decaying rates. Now we concentrate on figure 8(a). When $C_2$ is increased from 20 (full) to 40 (dashed) and to 80 (dotted), the squeezing magnitude increases from 6 dB to 8 dB and 10 dB,
successively. The widths of the squeezed spectrum are increased as well. This agrees with (13), from which we find that increasing $C_2$ essentially reduces the effect of $n_{\text{th}}$. Equation (13) also suggests that a larger cooperativity is always preferable for generating a large squeezing magnitude at nonzero temperatures, although it never exceeds the zero-temperature case. One has squeezing as long as the right-hand side of (13) is less than 0.5. For $C_2 = 2C_1 = 20$, $S_0(0)$ has the value $0.030 + 0.028n_{\text{th}}$. Figure 5(b) also indicates that the increasing $n_{\text{th}}$ shifts the optimized ratio of cooperativities $C_1/C_2$ for squeezing closer to 1.

We conclude this paper by giving a brief discussion of the state of the output field. In particular, we discuss the purity of the generated state. The generated state is determined by several dissipative processes and the effects of thermal noise. Thus the state would in general be mixed. We expect coherent interactions arising from the radiation pressure to make the state more and more pure. The purity of the state is given by the deviation of Tr $\rho^2$ from unity. For the thermal state, Tr $\rho^2 = 1/(2n_{\text{th}} + 1)$. The state of the output field $d(\omega)$ can be obtained from the quantum Langevin equations (S7). These equations imply that the field $d(\omega)$ will have a Gaussian Wigner function. For Gaussian states, the purity can be calculated from the known result for a single mode, namely Tr $\rho^2 = 1/\det \sigma$ where $\sigma$ is the covariance matrix of the state:

$$\sigma = \begin{pmatrix}
2 \langle X_0^2 \rangle & \langle X_0X_{\pi/2} + X_0X_{\pi/2} \rangle \\
\langle X_0X_{\pi/2} + X_0X_{\pi/2} \rangle & 2 \langle X_{\pi/2}^2 \rangle
\end{pmatrix}$$

when $\langle X_0 \rangle = \langle X_{\pi/2} \rangle$, which is zero under vacuum inputs. Here the operators $X$ are defined as in equation (9). For our system, different frequency modes of the output field are uncorrelated, as can be seen from equation (5), with the incoming vacuum fields and the mechanical Brownian noise. Hence, we can effectively use the result for the single mode. In figure 8(b), we plot the purity of the quantum state of the output field $d(\omega)$ for different values of $C_2$. Note that the dissipative processes affect the purity of the state. As the temperature increases, the state purity decreases monotonically. The curves also show that the system with a higher cooperativity $C_2 = 80$ (top dotted curve) loses purity more slowly than one with a lower cooperativity, $C_2 = 20$ (bottom full curve). The state becomes more and pure as $C_2$ increases. This is quite consistent with the result for squeezing in the output fields.

6. Conclusions

In conclusion, we have shown how squeezing of the order of 10 dB or more can be generated in a double-cavity optomechanical system. We presented a detailed discussion of the conditions under which the double-cavity optomechanical system would lead to the generation of strong squeezing as a result of the generation of entangled photon pairs. We show that such photon pair generation can be described through an effective interaction which is used for generating squeezing using parametric downconversion and four-wave mixing. However, there is one significant difference: we generate photon pairs by using the active participation of phonons. The phonon-mediated processes lead to additional noise terms which degrade the squeezing. The purity of the generated squeezed light is stronger with a large cooperativity. In light of the recent progress in optomechanics experiments realizing large cooperativity in [38], our proposal has promising experimental feasibility.
References

[1] Nunnenkamp A, Børkje K and Girvin S M 2011 Phys. Rev. Lett. 107 063602
[2] Rabl P 2011 Phys. Rev. Lett. 107 063601
[3] Heidmann A, Hadjar Y and Pinard M 1997 Appl. Phys. B 64 173
[4] Tian L 2012 Phys. Rev. Lett. 108 153604

Wang Y-D and Clerk A A 2012 Phys. Rev. Lett. 108 153603

Andrews R W, Peterson R W, Purdy T P, Cicak K, Simmonds R W, Regal C A and Lehnert K W 2014 Nat. Phys. 10 321–6
[5] Genes C, Mari A, Vitali D and Tombesi P 2009 Adv. At., Mol. Opt. Phys. 57 33

Wang Y-D and Clerk A A 2013 Phys. Rev. Lett. 110 253601

Yin Z-Q and Han Y-J 2009 Phys. Rev. A 79 024301
[6] Tian L 2013 Phys. Rev. Lett. 110 233602

[7] Huang S and Agarwal G S 2010 Phys. Rev. A 82 033811
[8] Liu Y-C, Xiao Y-F, Chen Y-L, Yu X-C and Gong Q 2013 Phys. Rev. Lett. 111 083601
[9] Leemonde M-A, Didier N and Clerk A A 2013 Phys. Rev. Lett. 111 053602
[10] Børkje K, Nunnenkamp A, D J and Girvin S M 2013 Phys. Rev. Lett. 111 053603

Kronwald A and Marquardt F 2013 Phys. Rev. Lett. 111 133601
[11] Tarabrin S P, Kaufer H, Ya Khalili F, Schnabel R and Hammerer K 2013 Phys. Rev. A 88 023809
[12] Xu G-F and Law C K 2013 New J. Phys. 15 095010
[13] Akram U, Bowen W P and Milburn G J 2013 Phys. Rev. A 89 023849
[14] Jähne K, Genes C, Hammerer K, Wallquist M, Polzik E S and Zoller P 2013 Phys. Rev. A 88 023819
[15] Brookes D W C, Botter T, Schreppler S, Purdy T P, Brahms N and Stamper-Kurn D M 2012 Nature 488 476

Safavi-Naeini A H, Gröblacher S, Hill J T, Chan J, Aspelmeyer M and Painter O 2013 Nature 500 185
[16] Purdy T P, Yu P-L, Peterson R W, Kampel N S and Regal C A 2013 Phys. Rev. X 3 031012
[17] Asjad M, Agarwal G S, Kim M S, Tombesi P, di Giuseppe G and Vitali D 2014 Phys. Rev. A 89 023849

Kronwald A, Marquardt F and Clerk A A 2013 Phys. Rev. A 88 063833
[18] Woolley M J and Clerk A A 2014 Phys. Rev. A 89 063805

Kronwald A, Marquardt F and Clerk A A 2014 New J. Phys. 16 063058
[19] Szorkovszky A, Brawley G A, Doherty A C and Bowen W P 2013 Phys. Rev. Lett. 110 184301

Jähne K, Genes C, Hammerer K, Wallquist M, Polzik E S and Zoller P 2009 Phys. Rev. A 79 063819

Huang S and Tsang M 2014 (arXiv:1403.1340)

Tsang M and Caves C M 2010 Phys. Rev. Lett. 105 123601
[20] Barzanjeh Sh, Abdi M, Milburn G J, Tombesi P and Vitali D 2012 Phys. Rev. Lett. 109 130503

Mandel L and Wolf E 1995 Optical Coherence and Quantum Optics (New York: Cambridge University Press) Chapter 21
[21] Walls D F and Milburn G J 1994 Quantum Optics (Berlin: Springer) Chapter 8
[22] Agarwal G S 2012 Quantum Optics (New York: Cambridge University Press) Chapter 3
[23] Szizmann A and Leuch G 1999 Prog. Opt. ed E Wolf vol 39 (Amsterdam: Elsevier) p 373
[24] Kronwald A, Marquardt F and Clerk A A 2013 Phys. Rev. A 88 063805

Kronwald A, Marquardt F and Clerk A A 2014 New J. Phys. 16 063058
[25] Szorkovszky A, Brawley G A, Doherty A C and Bowen W P 2013 Phys. Rev. Lett. 110 184301

Jähne K, Genes C, Hammerer K, Wallquist M, Polzik E S and Zoller P 2009 Phys. Rev. A 79 063819
[26] Mandel L and Wolf E 1995 Optical Coherence and Quantum Optics (New York: Cambridge University Press) Chapter 21

Walls D F and Milburn G J 1994 Quantum Optics (Berlin: Springer) Chapter 8
[27] Agarwal G S 2012 Quantum Optics (New York: Cambridge University Press) Chapter 3
[28] Sizmann A and Leuch G 1999 Prog. Opt. ed E Wolf vol 39 (Amsterdam: Elsevier) p 373
[29] Kronwald A, Marquardt F and Clerk A A 2013 Phys. Rev. A 88 063805

Kronwald A, Marquardt F and Clerk A A 2014 New J. Phys. 16 063058
[30] Szorkovszky A, Brawley G A, Doherty A C and Bowen W P 2013 Phys. Rev. Lett. 110 184301

Jähne K, Genes C, Hammerer K, Wallquist M, Polzik E S and Zoller P 2009 Phys. Rev. A 79 063819
[31] Mandel L and Wolf E 1995 Optical Coherence and Quantum Optics (New York: Cambridge University Press) Chapter 21

Walls D F and Milburn G J 1994 Quantum Optics (Berlin: Springer) Chapter 8
[32] Agarwal G S 2012 Quantum Optics (New York: Cambridge University Press) Chapter 3
[33] Sizmann A and Leuch G 1999 Prog. Opt. ed E Wolf vol 39 (Amsterdam: Elsevier) p 373
[34] K Qu and G S Agarwal 2013 Phys. Rev. A 87 031802

Dong C, Fiore V, Kuzyk M C, Tian L and Wang H 2012 (arXiv:1205.2360)

Barzanjeh Sh, Abdi M, Milburn G J, Tombesi P and Vitali D 2012 Phys. Rev. Lett. 109 130503
[35] Ludwig M, Safavi-Naeini A H, Painter O and Marquardt F 2012 Phys. Rev. Lett. 109 063601
[38] Karuza M, Biancofiore C, Bawaj M, Molinelli C, Galassi M, Natali R, Tombesi P, di Giuseppe G and Vitali D 2013 Phys. Rev. A 88 013804
[39] Dong C, Fiore V, Kuzyk M C and Wang H 2012 Science 338 1609
[40] Aspelmeyer M, Kippenberg T J and Marquardt F 2013 Rev. Mod. Phys. accepted (arXiv:1303.0733)
[41] Hurwitz A 1964 On the conditions under which an equation has only roots with negative real parts Selected Papers on Mathematical Trends in Control Theory (New York: Dover)
[42] Metelmann A and Clerk A A 2014 Phys. Rev. Lett. 112 133904
[43] Teufel J D, Donner T, Li D, Harlow J H, Allman M S, Cicak K, Sirois A J, Whittaker J D, Lehnert K W and Simmonds R W 2011 Nature 475 359–63
[44] Massel F, Cho S U, Pirkkalainen J-M, Hakonen P J, Heikkilä T T and Sillanpää M A 2012 Nat. Commun. 3 987
[45] Cerf N J, Leuchs G and Polzik E S 2007 Quantum Information with Continuous Variables of Atoms and Light (London: Imperial College Press) Chapter 1