CRITICAL POINT CORRELATION FUNCTION FOR THE 2D RANDOM BOND ISING MODEL

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ABSTRACT

High accuracy Monte Carlo simulation results for 1024×1024 Ising system with ferromagnetic impurity bonds are presented. Spin-spin correlation function at a critical point is found to be numerically very close to that of a pure system. This is not trivial since a critical temperature for the system with impurities is almost two times lower than pure Ising $T_c$. Finite corrections to the correlation function due to combined action of impurities and finite lattice size are described.

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Influence of impurities on a critical behaviour has been a subject of numerous papers. For the simplest possible model – 2D Ising model this problem has been considered theoretically [1-9], experimentally [10,11] and using computer simulations [12-16].

Most of the simulations were devoted to thermodynamic properties such as specific heat, magnetization and magnetic susceptibility. On the other hand, theories give direct predictions for spin-spin correlation function

\[ < S(0)S(r) > \] (1)

where \( r \) is a distance between spins.

We used cluster algorithm special purpose processor (SPP) [17,18] to get accurate values of \( < S(0)S(r) > \) at a critical point. The SPP realizes in hardware Wolff cluster algorithm, and therefore does not suffer of critical slowing down.

The SPP spends 375\( ns \) per one cluster spin. It also has a simple hardware for fast calculation of spin-spin correlation functions. Time necessary to get the correlation function for some \( r \) is equal to \( L^2 * 21 ns \), where \( L \) is a linear lattice size.

We study the following model. Coupling constant \( J \) on each bond can take two positive values: \( J_1 \) with probability \( p \) and \( J_0 \) with probability \( 1 - p \). For \( p = 0.5 \) duality relation [19] shows that \( T_c \) is equal to that of a pure model with all horizontal bonds equal to \( J_1 \) and all vertical bonds equal to \( J_0 \). The known \( T_c \) greatly simplifies simulation data analysis, and therefore we used \( p = 0.5 \).

Theoretical models employ a small parameter

\[ g \sim p(J_0 - J_1)^2 \] (2)

This parameter is connected with impurity induced length \( l_i \)

\[ \log l_i \sim \frac{1}{g} \] (3)

To be able to notice deviations from the pure critical behaviour we should have

\[ l_i << L \] (4)

To satisfy this condition we used large values of \( L \) (\( L = 256, 512 \) and 1024) and quite different values of \( J \): \( J_0 = 1 \) and \( J_1 = 0.25 \). So, in the simulations \( g \) is not very small.

Theories deal with continuum limit and infinite lattice size. Simulations are conducted on a finite lattice with periodic boundary conditions. We calculated \( < S(0)S(r) > \) for spins, located along one lattice row. In this case distance \( r \) can take only integer values.
Our simulations for the pure case [18] showed that discrete lattice effects are significant for \( r < 8 \). Continuum theory can be applied for larger distances. But the finite size corrections for \( r > 8 \) are very significant and should be taken into account explicitly.

Pure Ising correlation function \( c_0(r) \) for \( r/L \to 0 \) has been calculated in [20]

\[
c_0(r) = \frac{0.70338}{r^{1/4}}
\]  

Continuum limit of (1) for the finite lattice with periodic boundary conditions \( c(r, L) \) has been obtained in [21]

\[
c(r, L) \sim \frac{\sum_{\nu=1}^{4} |\theta_{\nu}(\frac{r}{L})|}{|\theta_1(\frac{r}{L})|^{1/4}}
\]  

where \( \theta \) are Jacobi theta functions. For our purposes \( c(r, L) \) can be written in a simpler form

\[
c(r, L) \approx A(L) \frac{1 + e^{-\pi/4} \left[ \sin(\frac{\pi}{2}) + \cos(\frac{\pi}{2}) \right] + e^{-9\pi/4} \left[ \cos(\frac{3\pi}{2}) - \sin(\frac{3\pi}{2}) \right]}{(\sin(\alpha) - e^{-2\pi} \sin(3\alpha))^{1/4}}
\]  

where \( \alpha = \pi r/L \). The coefficient \( A(L) \) can be obtained using the expression for \( c_0(r) \). Formula for \( c(r, L) \) is in excellent agreement with simulation results [18] for the pure system.

In Fig.1 we show the ratio of computed \( \langle S(0) S(r) \rangle \) to \( c(r, L) \) for \( L = 256, 512 \) and 1024.

For each lattice size to get mean values of \( \langle S(0) S(r) \rangle \) and standard deviations we used 1000 samples with different impurities distribution. For each sample all spins initially were pointing in the same direction. Two thousand Wolff clusters were flipped to thermalize spin distribution at critical temperature. Another 8000 clusters were flipped to calculate mean values of \( \langle S(0) S(r) \rangle \) for each sample. For \( L = 1024 \) one cluster flip at \( T_c \) requires about 0.1sec.

For each lattice size correlation functions for different \( r \) were measured for the same spin configurations.

Standard errors are determined mainly by different behaviour of \( \langle S(0) S(r) \rangle \) for different impurity distributions and not by thermal fluctuations for a given sample.

Deviations of \( \langle S(0) S(r) \rangle \) from \( c(r, L) \) at \( r < 8 \) are due to the discrete lattice effects. In fact, at these distances correlation function of the impure system is extremely close to the pure correlation function. For \( r = 1 \) difference between them is of the order of \( 10^{-3} \). At first sight it seems to be quite natural, because \( r << l_i \) and \( \langle S(0) S(0) \rangle = 1 \). But the critical temperatures for pure \((1/T_c \approx 0.44068679)\) and impure \((1/T_c \approx 0.80705186)\)
cases are very different, and continuum theory cannot exclude strong renormalization of 
\[ <S(0)S(1)> \].

On the other hand, for \( r > 8 \) pure correlation function practically coincides with 
\( c(r, L) \). So, Fig.1 shows that impurities decrease spin-spin correlations at large \( r \). Again, 
this is not trivial because of the difference of critical temperatures for pure and impure 
cases. Moreover, we see from Fig.1, that for a given \( r \), the larger the lattice size \( L \), the 
smaller deviations from the pure behaviour.

To investigate this phenomenon more, in Fig.2 we draw the same data for \( L = 256, 512, 1024 \) as a function of \( (r/L) \). We see, that \( L = 256 \) and \( L = 512 \) data prac-
tically coincide. The difference between these data and \( L = 1024 \) data is also within the 
limits of the error bars.

We are forced to conclude that the influence of impurities can be described by some 
function \( F(g, (r/L)) \). The impure system correlation function for any \( L \) is just a product 
of \( F(g, (r/L)) \) and \( c(r, L) \). The behaviour of \( F \) as a function of \( (r/L) \) is given by Fig.2. 
Even in the case of rather strong impurities, which we simulated using the SPP, \( F \) is very 
close to 1 for small \( (r/L) \), and decreases only by 4 percents from 1 at largest possible 
value of \( (r/L) = 0.5 \).

These results are in contradiction with predictions of [22] for the spin–spin correla-
tion function, which would instead show increase in spin correlations at large distances 
due to impurities. First order renormalization group calculations of other authors [2-6] 
lead to the conclusion that there is no influence of impurities on the averaged spin–spin 
correlation function.

Our results show that there exist unexpectedly small finite renormalization of the 
critical point correlation function. Our simulation data cannot exclude third order renor-
malization group corrections, recently found to be nonzero [23]. These corrections should 
 somewhat increase the correlation function at large \( r \).

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Figure Captions

Fig. 1. Ratio of \( \langle S(0)S(r) \rangle \) to \( c(r, L) \). Dashed line shows data for the pure \( L = 1024 \) Ising model \([18]\) at \((1/T_c) = .4406868\). Solid lines connect data points for the systems with impurities at \((1/T_c) = .8070519\). Up triangles show \( L = 1024 \) data, empty squares \( L = 512 \) data, and circles \( L = 256 \) data.

Fig. 2. The same data as in Fig. 1, but shown as a function of \((r/L)\).
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