Limits on the Non-Standard Interactions of Neutrinos
from $e^+e^-$ Colliders

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Abstract

We provide an effective Lagrangian analysis of contact non-standard interactions of neutrinos with electrons, which can be effectively mediated by extra particles, and examine the associated experimental limits. At present, such interactions are strongly constrained only for $\nu_\mu$: the bounds are loose for $\nu_e$ and absent for $\nu_\tau$. We emphasize the unique role played by the reaction $e^+e^- \rightarrow \bar{\nu}\nu\gamma$ in providing direct constraints on such non-standard interactions.

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1 Introduction

Nowadays neutrino physics is very alive thank to the exciting measurements and results reported by atmospheric and solar neutrino experiments. They have been confirming that neutrinos do oscillate and hence they may be mixed and massive, offering us in this way important informations on the ‘flavour’ structure of the Standard Model (SM). In general, extensions of the SM are required to generate non-vanishing neutrino masses and mixing angles, which are supposed to originate at an energy scale much larger than the electroweak scale, for instance the grand unified scale ($\sim 10^{16}$ GeV). Then, as neutrino physics is susceptible to be extended beyond the SM realm, it is also conceivable that some new physics may also predict novel interactions of neutrinos with matter constituents which can be flavour changing as well as flavour conserving. The phenomenological relevance of such non-standard (NS) interactions will depend in general on the scale at which they are presumed to arise: it should be not too far from the electroweak scale. Non-standard flavour changing neutrino interactions have been invoked long ago to explain the solar neutrino anomaly (SNA) [1, 2, 3, 4]. Some time ago [2] the impact of extra $\nu_\tau$ interactions with electrons on the detection cross section has been also investigated in the context of the neutrino long wavelength oscillation as a solution to the SNA. The interest in neutrino NS interactions continues to increase.\(^1\) In particular, at present two points of view can be taken: either to use solar or atmospheric neutrino data to constrain neutrino NS interactions [4, 6], or, on the contrary, to use solar neutrino experiments to detect signatures of neutrino NS interactions [7]. Both approaches cannot leave aside the constraints that emerge from laboratory experiments [8]. The properties of $\nu_e$ and $\nu_\mu$ have been tested and while the accelerator constraints on $\nu_e$ are still loose, those on $\nu_\mu$ are rather severe [8]. On the contrary, until now the $\nu_\tau$ properties have not been directly tested in laboratory. Indeed, the constraints on non-standard interactions rather apply to its $SU(2)_W$-partner, the $\tau$ lepton. In this paper, we point out that neutrino NS interactions with electrons can be constrained at $e^+e^-$ colliders through the reaction $e^+e^- \rightarrow \nu\bar{\nu}\gamma$.\(^2\) It is well-known that the invisible channel reaction with photon emission provides a useful tool for measuring the number of light neutrinos [10] and for revealing new physics. We note that the constraints derived from the cross section measurements performed at LEP are the only laboratory bounds on the $\nu_\tau$ NS neutral current interactions and may be competitive to those obtained from $\nu_\tau e$ scattering as regards the $\nu_e$ NS interactions. (The same bounds when applied to $\nu_\mu$ are not competitive with the existing ones from the $\nu_\mu e$ elastic scattering [11].)

We shall find that $\nu_\tau$ may have sizeable extra-interactions with electrons. This has recently pushed us [7] to investigate in detail the possibility to ‘identify’ the $\nu_\tau$ in solar neutrino elastic scattering experiments like Borexino [12]. Indeed, in view of the atmospheric neutrino data pointing to a quasi-maximal mixing between $\nu_\mu$ and $\nu_\tau$, the solar neutrino anomaly is to be interpreted as the conversion of $\nu_e$ into a state of nearly equal mixture of $\nu_\mu$ and $\nu_\tau$. Therefore the Sun is an abundant source of $\tau$ neutrinos. While in the framework of the Standard Model this observation may sound ‘academic’, it becomes meaningful as soon as $\nu_\tau$ is allowed to have extra (neutral current) interactions and then to be distinguishable from $\nu_\mu$. The elastic scattering $\nu_e$ that will be used by Borexino to detect the mono-energetic $^7$Be neutrinos can

\(^1\)We have to mention that very recently the results by the NuTeV experiment on the determination of electroweak parameters show a discrepancy with the SM expectation that suggests non-standard couplings of neutrinos with quarks [5].

\(^2\)Limits on the magnetic moment of the $\tau$ neutrino have been provided using this reaction [9].
be in this case the right place where to observe neutrino NS interactions with electrons. They could show up in some specific deformations of the electron energy spectrum. The observation of such an effect would simultaneously provide the signature of $\nu_\tau$ interactions and would be a further proof of the large mixing angle i.e. a test of the atmospheric neutrino oscillations.

The content of this paper is organized as follows. In Sec. 2 we present the effective Lagrangian describing neutrino NS interactions with electrons. Sec. 2.1 provides a derivation of the effective $SU(2)_W \times U(1)_Y$ invariant operators (with $d \geq 6$) describing neutrino interactions with electrons. This enables us to find possible theoretical realizations of such interactions without conflicting with the experimental bounds (Sec. 2.2). In Sec. 3 we briefly review the present laboratory bounds on NS leptonic interactions. So we focus on the bounds on $\nu$-$e$ NS interactions achievable with the measurements of the cross section $e^+e^- \to \nu\bar{\nu}\gamma$ (Sec. 4). Our findings are summarised in Sec. 5.

2 Non-standard leptonic interactions

Our discussion is most focused on neutrino interactions with electrons. In the Standard Model, the elastic scattering $\nu_\alpha e \to \nu_\alpha e$ of neutrinos with the electron are described at low energies by the following four-fermion operator ($\nu_\alpha = \nu_\mu, \nu_\tau$):

$$-\mathcal{L}^{\nu}_{\text{SM}} = 2\sqrt{2}G_F(\overline{\nu}_\alpha\gamma^\mu P_L\nu_\alpha)\left[g_R\overline{e}\gamma_\mu P_R e + g_L\overline{e}\gamma_\mu P_L e\right] + 2\sqrt{2}G_F(\overline{\nu}_e\gamma^\mu P_L\nu_\alpha)(\overline{e}\gamma_\mu P_L e) \quad (1)$$

where $G_F$ is the Fermi constant, $P_{L,R} = (1 \mp \gamma^5)/2$ are the chiral projectors, $g_R = \sin^2 \theta_W$ and $g_L = -\frac{1}{2} + \sin^2 \theta_W$ are the electron coupling constant to the $Z$-boson, and the last term is the additional contribution for the electron neutrino arising from the $W$-boson exchange. Alternatively, we can define the vector and axial coupling constant $g_V = g_L + g_R$ and $g_A = g_L - g_R$ and properly express the corresponding four-fermion interaction in terms of them.

We assume on phenomenological grounds that neutrinos may also have NS interactions with the electron described by the following four-fermion operator:\footnote{For simplicity, here we consider only neutrino flavour-diagonal interactions with electrons, though in general there could also exist the flavour-changing ones.}

$$-\mathcal{L}^{\nu}_{\text{NS}} = 2\sqrt{2}G_F(\overline{\nu}_\alpha\gamma^\mu P_L\nu_\alpha)\left[\varepsilon_{\alpha R}\overline{e}\gamma_\mu P_R e + \varepsilon_{\alpha L}\overline{e}\gamma_\mu P_L e\right], \quad (2)$$

where $\varepsilon_{\alpha R}$ and $\varepsilon_{\alpha L}$ parameterize the strength of the new interactions with respect to $G_F$, and in general they can be dependent on the neutrino flavour ($\alpha = \nu_\mu, \nu_\tau$). The occurrence of these extra interactions entails a ‘coherent’ redefinition of the constants $g_{L,R}$

$$g_R \to \tilde{g}_{\alpha R} = g_R + \varepsilon_{\alpha R}; \quad g_L \to \tilde{g}_{\alpha L} = g_L + \varepsilon_{\alpha L}. \quad (3)$$

so that the effective coupling constants $\tilde{g}_{\alpha R}$ and $\tilde{g}_{\alpha L}$ in general depend on the flavour of the neutrino. Equivalently, we can define the vector and axial constants of the extra interactions, $\varepsilon_{\alpha V} = \varepsilon_{\alpha L} + \varepsilon_{\alpha R}$ and $\varepsilon_{\alpha A} = \varepsilon_{\alpha L} - \varepsilon_{\alpha R}$. Although this presentation may appear redundant, in the next we shall explicitly display the allowed parameter space for both $(\varepsilon_R, \varepsilon_L)$ and $(\varepsilon_V, \varepsilon_A)$ while discussing the bounds on the non-standard neutrino couplings. Indeed, the former parametrization is closer to the theoretical perception, while the latter is somehow more appropriate to the neutrino-propagation phenomenology in matter.
2.1 Theoretical aspects: effective operator picture

In the framework of the Standard Model, neutrino NS interactions (2) with electrons can be derived by $SU(2)_W \times U(1)_Y$-invariant operators of dimension $d \geq 6$:

$$
\frac{h^{\alpha(n)}_R(H, H^\dagger)}{M^2} \left[ (\bar{e}_\alpha \gamma^\mu P_L l_\alpha)(\bar{e}_\gamma^\mu P_R e) \right]_{(n)} + \frac{h^{\alpha(n)}_L(H, H^\dagger)}{M^2} \left[ (\bar{e}_\alpha \gamma^\mu P_L l_\alpha)(\bar{e}_\gamma^\mu P_L e) \right]_{(n)},
$$

(4)

where $l_\alpha$ denotes the $SU(2)_W$ lepton doublets ($\alpha = e, \mu, \tau$), $H$ is the Higgs doublet and $M$ is a cutoff mass scale.\(^4\) In the above expressions all possible $SU(2)_W$ contractions are implicitly understood and counted by the index $n$, while the round brackets merely indicate Lorentz contractions. The field-dependent couplings $h^{\alpha(n)}_{R,L}$ are to be understood as follows ($A = R, L$):

$$
h^{\alpha(n)}_A(H, H^\dagger) = h^{\alpha(n)}_{0,A} + h^{\alpha(n)}_{1,A}(HH^\dagger)_{n} + \cdots .
$$

(5)

The $h^{\alpha(n)}_R$-expansion will give neutrino interactions with right-handed electrons ($\varepsilon_{\alpha R}$-coupling), while the $h^{\alpha(n)}_L$-expansion will give neutrino interactions with left-handed electrons ($\varepsilon_{\alpha L}$-coupling). In eq. (5) the dots stand for higher-dimension operators that we omit for brevity since the explicit discussion of $d = 6$ and $d = 8$ operators is already sufficient to illustrate the main points. We can easily see that the $h^{\alpha(n)}_R$-type expansion (5) produces one invariant ($n = 1$) at lowest order in $1/M^2$, and two invariants ($n = 1, 2$) at next order:

$$
h^{\alpha(1)}_{0,R}(l^\dagger_\alpha l_\alpha)(e^\dagger e),
$$

$$
h^{\alpha(1)}_{1,R}S(l^\dagger_\alpha l_\alpha)(e^\dagger e) + h^{\alpha(2)}_{1,R}T^a(l^\dagger_\alpha \sigma^a l_\alpha)(e^\dagger e).
$$

(6)

Here the Lorentz structure is understood and we have defined $S \equiv (H^\dagger H)/M^2$, $T^a \equiv (H^\dagger \sigma^a H)/M^2$ ($\sigma^a$ are the Pauli matrices, $a = 1, 2, 3$). On the other hand, the $h^{\alpha(n)}_L$-type expansion entails more invariants, namely ($\alpha = e, \mu, \tau$; $\beta = \mu, \tau$):

$$
h^{\alpha(1)}_{0,L}(l^\dagger_\alpha l_\alpha)(l^\dagger_\beta l_\beta),
$$

$$
h^{\alpha(2)}_{0,L}(l^\dagger_\beta \sigma^a l_\beta)(l^\dagger_\alpha \sigma^a l_\alpha),
$$

(7)

$$
h^{\alpha(1)}_{1,L}S(l^\dagger_\alpha l_\alpha)(l^\dagger_\beta l_\beta),
$$

$$
h^{\alpha(2)}_{1,L}S(l^\dagger_\beta \sigma^a l_\beta)(l^\dagger_\alpha \sigma^a l_\alpha),
$$

$$
h^{\alpha(3)}_{1,L}T^a(l^\dagger_\beta \sigma^a l_\beta)(l^\dagger_\alpha l_\alpha),
$$

$$
h^{\alpha(4)}_{1,L}T^a(l^\dagger_\beta \sigma^a l_\beta)(l^\dagger_\alpha \sigma^a l_\alpha),
$$

$$
h^{\alpha(5)}_{1,L}S(l^\dagger_\alpha \sigma^a l_\alpha)(l^\dagger_\beta \sigma^a l_\beta).
$$

(8)

The operators (6-8) form a particular basis in which the singlet and triplet $SU(2)_W$-orientation are manifest, as the symbols $S$ and $T^a$ adopted for the dimensionless Higgs combination suggest, but there are of course other equivalent bases linearly related to this one.\(^5\) Once $SU(2)_W \times$

\(^4\)For an earlier general analysis of $d = 6$ effective operators, see e.g. [13].

\(^5\)We mention that the invariant formed by the $SU(2)_W$-quintuplet orientation, i.e $1 \subset 5 \times 5$ appears at the successive order, $d = 10$. The $SU(2)_W$-quintuplet representation is the highest one that can be realized, throughout all the expansion. However, this $SU(2)_W$ structure does not give rise to further effective interactions in the broken phase (see below, eq. 9), besides those derived from the $SU(2)_W$ singlet and triplet operator structure in eqs. (6-8).
U(1)_Y is broken by the vacuum expectation value (VEV) of the Higgs field \( H \), \( \langle H^0 \rangle = v \), all the operators above become \( d = 6 \) four-fermion operators of the broken phase. These effective operators can be cast into a parameterization like that in eq. (2), namely

\[
-L_{\text{eff}}' = 2\sqrt{2}G_F(\bar{v}_\alpha\gamma^\mu P_L\nu_\alpha)[\varepsilon_{\alpha R}(\bar{e}\gamma_\mu P_R e) + \varepsilon_{\alpha L}(\bar{e}\gamma_\mu P_L e)],
\]

\[
-L_{\text{eff}}^{(1)} = 2\sqrt{2}G_F(\bar{\tau}_R\gamma^\mu P_L\tau)[\kappa_{\tau R}(\bar{e}\gamma_\mu P_R e) + \kappa_{\tau L}(\bar{e}\gamma_\mu P_L e)] + (\tau \to \mu) + (\tau \to e),
\]

\[
-L_{\text{eff}}^{(2)} = 2\sqrt{2}G_F(\bar{\nu}_R\gamma^\mu P_L\nu_e)[\xi_{\nu R}(\bar{e}\gamma_\mu P_R e) + h.c.] + (\tau \to \mu),
\]

\[
-L_{\text{eff}}^{(3)} = 2\sqrt{2}G_F(\bar{\nu}_\alpha\gamma_\mu P_L\nu_e)[\xi_{\nu L}(\bar{e}\gamma^\mu P_L\nu_e) + (\tau \to \mu) + 2\sqrt{2}G_F\xi_{\nu L}(\bar{e}\gamma_\mu P_L\nu_e)],
\]

where the dimensionless parameters such as \( \varepsilon_{\alpha R(L)}, \kappa_{\alpha R(L)} \) etc. are to be identified as follows (\( \alpha = e, \mu, \tau; \beta = \mu, \tau \)):

\[
\begin{align*}
2\sqrt{2}G_F\varepsilon_{\alpha R} &= \frac{1}{M^2} \left[ h^{(1)}_{\alpha R} + Sh^{(1)}_{\alpha L} + Th^{(2)}_{\alpha L} + \cdots \right], \\
2\sqrt{2}G_F\kappa_{\alpha R} &= \frac{1}{M^2} \left[ h^{(1)}_{\alpha R} + Sh^{(1)}_{\alpha L} - Th^{(2)}_{\alpha L} + \cdots \right], \\
2\sqrt{2}G_F\varepsilon_{\alpha L} &= \frac{1}{M^2} \left[ (h^{(1)}_{\alpha L} - h^{(1)}_{\beta L}) + S(h^{(1)}_{\beta L} - h^{(1)}_{\beta L}) + T(h^{(3)}_{\beta L} - h^{(4)}_{\beta L}) + \cdots \right], \\
2\sqrt{2}G_F\varepsilon_{\alpha L} &= \frac{2}{M^2} \left[ h^{(1)}_{\alpha L} + Sh^{(1)}_{\alpha L} + \cdots \right], \\
2\sqrt{2}G_F\kappa_{\beta L} &= \frac{1}{M^2} \left[ (h^{(1)}_{\beta L} + h^{(2)}_{\beta L}) + S(h^{(1)}_{\beta L} + h^{(2)}_{\beta L}) - T(h^{(3)}_{\beta L} + h^{(4)}_{\beta L}) + \cdots \right], \\
2\sqrt{2}G_F\kappa_{\beta L} &= \frac{1}{M^2} \left[ h^{(1)}_{\beta L} + Sh^{(1)}_{\beta L} - Th^{(2)}_{\beta L} + \cdots \right], \\
2\sqrt{2}G_F\xi_{\beta L} &= \frac{1}{M^2} \left[ (h^{(1)}_{0 L} - h^{(2)}_{0 L}) + S(h^{(1)}_{0 L} - h^{(2)}_{0 L}) - T(h^{(3)}_{0 L} - h^{(4)}_{0 L}) + \cdots \right], \\
2\sqrt{2}G_F\xi_{\nu L} &= \frac{1}{M^2} \left[ (h^{(1)}_{\nu L} + h^{(2)}_{\nu L}) + S(h^{(1)}_{\nu L} + h^{(2)}_{\nu L}) + T(h^{(3)}_{\nu L} + h^{(4)}_{\nu L}) + \cdots \right],
\end{align*}
\]

It is understood that now the symbols \( S \) and \( T \) stand for \( \langle S \rangle = v^2/M^2 \) and \( \langle T^3 \rangle = -2Y_H v^2/M^2 \) (where \( Y_H \) is the \( H \) hypercharge), respectively. Notice that in the triplet-orientation \( T^3 \) only the third component contributes. Also notice that the terms proportional to \( S \) could be reabsorbed in the lowest-order couplings \( h^{(n)}_{\alpha L} \).

We have collected in \( L'_{\text{eff}} \) all the desired operators involving only the neutrino current with either the right-handed or the left-handed electron current, eq. (2). However, we have more interactions which involve also the other components of the \( SU(2)_W \) doublets. Among these we distinguish those which are ‘observable’, contained in the Lagrangians \( L_{\text{eff}}^{(1)}, L_{\text{eff}}^{(2)} \), and all other interactions which instead are ‘unobservable’, collected in \( L_{\text{eff}}^{(3)} \). Here by ‘observable’ we mean operators which can give phenomenologically testable interactions and then are subjected to constraints.\footnote{However, a process such as \( \mu^+ \mu^- \to \nu_\ell \bar{\nu}_e \) described by \( L_{\text{eff}}^{(3)} \) could be studied at the planned muon colliders.}
Some interesting features emerge. First, consider the limit of unbroken $SU(2)_W$. Then only the $d = 6$ operators are relevant. The two types of interactions with right-handed electrons have the same coupling constants ($\varepsilon_{\alpha R} = \kappa_{\alpha R}$). As for the interactions with left-handed electrons, for $\alpha = e$ we have three interactions with the same coupling constant, whereas for $\beta = \mu, \tau$ we have five interactions determined by only two coupling constants. In this limit, therefore, the stringent laboratory bounds on the effective couplings $\kappa_{\alpha L}, \kappa_{\alpha R}$ and $\zeta_{\beta L}$ in $\mathcal{L}_{\text{eff}}^{(1)}$ and $\mathcal{L}_{\text{eff}}^{(2)}$ would imply similar bounds also for the couplings $\varepsilon_{\alpha L}, \varepsilon_{\alpha R}$ in $\mathcal{L}_{\text{eff}}^{(1)}$, which by themselves are more loosely constrained. (For a recent discussion on this issue see [15].) Now consider the effects induced by the breaking of $SU(2)_W$. Then also the $d > 6$ operators play a role and can correct (or even spoil) the strong correlations among couplings described above. These features are manifest in eq. (10). It is then conceivable that the effective couplings in $\mathcal{L}_{\text{eff}}^{(1)}$ and $\mathcal{L}_{\text{eff}}^{(2)}$ be suppressed below the experimental bounds, while at the same time the couplings in $\mathcal{L}_{\text{eff}}^{(1)}$ remain sizeable. Notice also that for example a sizeable $\varepsilon_{eL}$ could be compatible with small $\kappa_{eL}, \zeta_{eL}$ but would also imply that the four-neutrino interaction parameter $\varepsilon_{\nu e}$ be large. This may have interesting consequences for the neutrino decoupling in the Early Universe as well as for the neutrino propagation in dense matter, like that in the supernova core. In conclusion, it is reasonable and worthwhile to explore the possible experimental relevance of neutrino non-standard interactions [7].

In the following we shall disregard the second-generation ($\nu_\mu, \mu$)-couplings as these are already severely bounded, $\varepsilon_{\mu R,L} < 0.1$ and $\kappa_{\mu R,L}, \zeta_{\mu L} < 0.01$. So we shall focus mostly on the extra interactions of $\nu_e$ and $\nu_\tau$ for which the parameters $\varepsilon_{eR,L}, \varepsilon_{eL}$ and $\varepsilon_{\tau R,L}$ are so far weakly bounded or unbounded, respectively.

### 2.2 Explicit examples for neutrino non-standard interactions

We shall briefly give two examples of interactions in the fundamental theory, at energy-scale larger than $\Lambda$, to show how the $SU(2)_W$-universality can be concretely broken, specializing, for the sake of simplicity, to the third generation case.

- By exchanging an additional scalar $SU(2)_W$ doublet $\phi$ according to the following coupling:

$$\bar{l}_\tau P R e \phi + \text{h.c.}$$ (11)

we generate the leading $d = 6$ operators and so the first term in the expressions of $\varepsilon_{\tau R}$ and $\kappa_{\tau R}$ in $\mathcal{L}_{\text{eff}}^{(1)}$ and $\mathcal{L}_{\text{eff}}^{(2)}$, respectively. If $H$-legs are inserted in the $\phi$-propagator, higher-dimension operators ($d = 8$ etc.) are effectively generated. The non-vanishing VEV of $H$ introduces $SU(2)_W$ breaking effects, as already outlined, and so generates the successive terms in the $\varepsilon_{\tau R}$ and $\kappa_{\tau R}$-series. This amounts to saying that $SU(2)_W$ breaking effects split the masses of the components of the $\phi$ doublet, $M_{\phi^+} \neq M_{\phi}$, and thus generate different effective couplings for the interactions of $\nu_\tau$ and its partner, the $\tau$ lepton, with the electron. However, this mass

As for the $\xi_{\nu e}$-like neutrino interactions in $\mathcal{L}_{\text{eff}}^{(3)}$, they can contribute to the decay $Z \rightarrow \nu \bar{\nu}$ at one-loop and thus can be constrained by the invisible $Z$-width determination, $\xi_{\nu e} \lesssim 1$ [14]. Analogous considerations for the decays $Z \rightarrow \ell \ell$ constrain the couplings $\varepsilon_{R,L}, \kappa_{R,L}$ as $\varepsilon_{R,L}, \kappa_{R,L} < 0.5$ or so. These bounds are only meant as estimates, since such loop effects are cut-off dependent.

7 Needless to say that the emergence of large $SU(2)_W$-breaking effects means that in the expansion (4) all higher order terms can become comparable in magnitude to the leading one, in which case a proper re-summation has to be implemented. Alternatively, once the fundamental theory is given, one could include $SU(2)_W$-breaking effects in an exact, rather than approximate, way.
splitting would contribute to the Standard Model $\rho$ parameter and the maximally allowed ratio $(M_{\rho^0}/M_{\phi^0})^2$ can be $\sim 7$ (at 90% C.L.) in the most conservative case with $M_{\phi^0} \sim 50$ GeV [15, 16]. Therefore the effective parameter $\varepsilon_{\tau R}$ of $\mathcal{L}_{\text{eff}}^\nu$ can be at most a factor 7 larger than the present experimental constraint on the effective $\kappa_{\tau R}$ coupling: $\kappa_{\tau R} \lesssim 0.1$ (see next Section).

- We can exchange some extra singlet fermion $N$ according to these interactions:

\[ h\tilde{t}_P R NH + M_N \bar{N} N + f \bar{N} P R e S + h.c., \quad (12) \]

where $S$ is some charged scalar $SU(2)_W$ singlet, $M_N$ is the Dirac mass of the fermion $N$ and $h, f$ are dimensionless coupling constants. In this case, after decoupling the fields $N$ and $S$, we obtain the $d = 8$ operator $(\tilde{l}_L H)(H^\dagger l_r)(e^\dagger e)$, which is a linear combination of the two ones shown in (6). This induces the interaction $\mathcal{L}_{\text{eff}}^\nu$ with $\Lambda \sim \sqrt{M_N M_S}$. However, the decoupling of $N$ also generates the $d = 6$ operator $(h/M_N)^2(\tilde{l}_L H)\gamma^\mu\partial_\mu(H^\dagger l_r)$, which induces a correction $(h v/M_N)^2 \nu^\dagger_\tau \gamma^\mu \partial_\mu \nu_\tau$ to the neutrino kinetic term. The consequent redefinition of the $\nu_\tau$ field gives rise to extra (charged current as well as neutral current) neutrino interactions of relative strength $\sim (h v/M_N)^2$. As we will see below, the strongest bound comes from the $\tau$-decay which now imposes $M_N/h \gtrsim 10 v$. Notice that the interactions in (12) cannot induce Majorana neutrino mass terms and therefore the mass scales $M_S$ and $M_N$ involved are not forced to be much larger than the electroweak scale. We can even have $M$ not so far from the weak scale and so $\varepsilon_{\tau R}$ can be $\sim 0.1 \div 1$.

3 Laboratory bounds on NS interactions

The most stringent constraints on the ‘observable’ interactions for the first generation come from the collisions $e^+e^- \rightarrow e^+e^-$ at LEP [16] which then constrain the interaction in $\mathcal{L}_{\text{eff}}^{(1)}$ (at 95% C.L.):

\[
\begin{align*}
\kappa_{eR} & \lesssim 0.01, \quad \Lambda > 5.3 \text{ TeV}, \\
\kappa_{eL} & \lesssim 0.03, \quad \Lambda > 3.8 \text{ TeV},
\end{align*}
\quad (13)
\]

where now the cut-off scale is defined, for given parameter $\kappa_{eR(L)}$, according to $2\sqrt{2} G_F \kappa_{eR(L)} \equiv 4\pi/\Lambda^2$. In what follows, the bounds on the cut-off scales will be inferred in analogous way also for other interactions.\(^8\) As for the third generation, analogously, from the collisions $e^+e^- \rightarrow \tau^+\tau^-$ at LEP one also infers the most severe constraints on the couplings $\kappa_{\tau R,L}$ (at 95% C.L.):

\[
\begin{align*}
\kappa_{\tau R} & \lesssim 0.1, \quad \Lambda > 2 \text{ TeV}, \\
\kappa_{\tau L} & \lesssim 0.02, \quad \Lambda > 4 \text{ TeV},
\end{align*}
\quad (14)
\]

On the other hand, the strongest bound on the third-generation is that on the interaction $\mathcal{L}_{\text{eff}}^{(2)}$ (9) that is on the parameter $\zeta_{\tau L}$. We obtain from the measurement of the ratio $\frac{B(\tau \rightarrow \nu_\tau e\bar{\nu}_\mu)}{B(\tau \rightarrow \nu_\tau e\bar{\nu}_\mu)} = 0.974 \pm 0.005$ [16]:

\[ |\zeta_{\tau L}| \lesssim 2.6 \cdot 10^{-3}, \quad \Lambda > 11 \text{ TeV}. \quad (15) \]

As anticipated in the previous Section, the $\nu_\tau$-interaction in $\mathcal{L}_{\text{eff}}^\nu$ is not experimentally constrained and also the $\nu_e$-interaction is poorly tested by low-energy $\nu e$ scattering experiments.

\(^8\) Notice that these $\Lambda$ parameters, defined in order to conform with the common practice, are different from the scale $M$ introduced in the previous section.
Therefore we shall first review the bounds imposed by $\nu_e e$ and $\bar{\nu}_e e$ scattering experiments. In presence of extra interactions, with the effective couplings (3), the cross-section of the $\nu_e e$ elastic scattering becomes (for $m_e \ll E_\nu \ll M_W$):

$$\sigma_{\nu_e e} = \frac{2}{\pi} G_F^2 m_e E_\nu \left( (g_{eL} + 1)^2 + \frac{1}{3} g_{eR}^2 \right).$$  \hspace{1cm} (16)$$

The most accurate measurement of this cross section has been performed by the LSND collaboration [17]:

$$\sigma_{\nu_e e}^{\text{exp}} = (10.1 \pm 1.5) \cdot E_\nu [\text{MeV}] \cdot 10^{-45} \text{ cm}^2. \hspace{1cm} (17)$$

In Fig. 1 we draw the iso-contour of sensitivity to the NS couplings $(\varepsilon_{eR}, \varepsilon_{eL})$ (left panel) or $(\varepsilon_{eA}, \varepsilon_{eV})$ (right panel) from the LSND measurement of the $\nu_e e^- \rightarrow \nu_e e^-$ cross section and the RGS measurement of the $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$ cross section. The present 1-$\sigma$ accuracy is of 15% from LSND experiment and 29% from RGS experiment (see more details in the text).

![Figure 1: Sensitivity contours to non-standard $\nu_e$ interactions in the plane $(\varepsilon_{eR}, \varepsilon_{eL})$ (left panel) or $(\varepsilon_{eA}, \varepsilon_{eV})$ (right panel).](image)

We then observe that the limits on $\varepsilon_{eL}$ are relatively strong, while those on $\varepsilon_{eR}$ are loose enough, $\varepsilon_{eR} \sim 1$ is allowed.

Now we make use of the data from the scattering of reactor $\bar{\nu}_e$ off electrons [18]. The theoretical cross section $\sigma_{\bar{\nu}_e e}$ is obtained just by exchanging $g_{eR} \leftrightarrow g_{eL} + 1$ in eq. (16). The earliest experiment cited in [18], that by Reines, Gurr and Sobel (RGS), still remains the most
accurate\(^9\). The results by the RGS experiment have been later reanalyzed in [20] where a better-understood reactor $\bar{\nu}_e$ spectrum is used to derive the cross section. They found that the RGS cross sections [18] convoluted with the improved $\bar{\nu}_e$ spectrum were $1.35 \pm 0.4$ and $2.0 \pm 0.5$ times the Standard model prediction, for recoil electron energy in the intervals $1.5 - 3$ MeV and $3 - 4.5$ MeV, respectively. By combining the two data, we may conclude that the total cross section (i.e. for recoil electron energy in the range $1.5 - 4.5$ MeV) is $\sigma^{\exp}_{\nu_e e} = (1.7 \pm 0.5) \sigma^{SM}_{\nu_e e}$. The corresponding limits on $\varepsilon_{\nu_e L}$ (or $\varepsilon_{\nu_e A,V}$) are shown in Fig. 1. We see that the bounds on $\varepsilon_{\nu_e L}$ are much worse than those obtained from $\nu_e e$ scattering, however the limits on $\varepsilon_{\nu_e R}$ are somewhat more stringent. Namely, in the neighborhood of the point $(\varepsilon_{\nu_e R}, \varepsilon_{\nu_e L}) = (0, 0)$ we get:\(^{10}\)

\[ \text{for } \varepsilon_{\nu_e L} = 0 : \quad 0.08 \leq \varepsilon_{\nu_e R} \leq 0.34 \ (68\%), \quad -0.95 \leq \varepsilon_{\nu_e R} \leq 0.50 \ (99\%). \] (19)

However, stronger limits on the parameter $\varepsilon_{\nu_e R}$ can be obtained by experiments sensitive to the differential cross section of the $\nu_e e \rightarrow \nu_e e$ scattering at lower energies, with $E_\nu \sim m_e$. As we have discussed in ref. [7] the energy spectrum of the recoil electron can be measured with good precision at the Borexino detector aimed to detect the monoenergetic Beryllium neutrinos from the Sun, with $E_\nu = 0.86$ MeV.

### 4 The relevance of $\sigma(e^+ e^- \rightarrow \nu \bar{\nu} \gamma)$

Now we would like to come to our main point, namely to derive laboratory bounds from the measurements of the $e^+ e^- \rightarrow \nu \bar{\nu} \gamma$ cross-section. This appears of special relevance for $\nu_\tau$ but we shall show that it is relevant also for $\nu_e$. To our knowledge the possibility to extract information on neutrino non-standard interactions with electrons from this process has never been discussed in the past.

In the SM, the reaction $e^+ e^- \rightarrow \nu \bar{\nu} \gamma$ proceeds via $Z$-boson exchange ($s$-channel) and $W$-boson exchange ($t$-channel). The $W$ contribution is involved only for the emission of the pair $\nu_e \bar{\nu}_e$. If neutrino non-standard interactions with electrons are present, they contribute coherently to this reaction. The total cross section can be written as $\sigma = \sigma^{SM} + \sigma^{NS}$, where $\sigma^{SM}$ is the SM cross section and $\sigma^{NS}$ includes both the pure NS contribution and SM–NS interference. The constraint $|\sigma - \sigma^{exp}| \leq \delta \sigma^{exp}$, where $\sigma^{exp} \pm \delta \sigma^{exp}$ is the experimental result, can be written in the following form:

\[ \left| 1 + \frac{\sigma^{NS}}{\sigma^{SM}} - \frac{\sigma^{exp}}{\sigma^{SM}} \right| \leq \left( \frac{\sigma^{exp}}{\sigma^{SM}} \right) \left( \frac{\delta \sigma^{exp}}{\sigma^{exp}} \right). \] (20)

The ratio $\sigma^{exp}/\sigma^{SM}$ should be evaluated by combining the latest experimental data with an accurate computation of the SM cross section (see e.g. [21, 22]). On the other hand, as far as NS interactions are concerned, for our purposes it is sufficient to compute the ratio $\sigma^{NS}/\sigma^{SM}$ using some approximation. In particular, we will work at tree level and use the 'radiator' approximation to describe photon emission. Thus we can write [21]:

\[ \sigma(s) = \int dx \int dc_\gamma H(x, s; s) \sigma_0(s), \]

---

\(^9\)Better accuracy is expected in one year or so from MUNU experiment [19].

\(^{10}\)By combining those two data, we implicitly exclude the possibility that the discrepancy of the measurements in the two distinct energy ranges be ascribed to strong spectral deformations. In fact, this assumption is reasonable in view of the LSND limits that prevent $\varepsilon_{\nu_e L}$ to significantly deviate from the SM expectation. Therefore the limits we obtain are certainly reliable along the direction $\varepsilon_{\nu_e L} = 0$. 

---
\[ H(x, s; \gamma) = \frac{\alpha}{s^2 \gamma} \frac{1 + (1 - x)^2}{x}, \]  

(21)

where the cross section \( \sigma_0 \) for \( e^+e^- \rightarrow \nu \bar{\nu} \), evaluated at the energy scale \( s = (1 - x)s \) (\( s \) is the centre-of-mass energy), is dressed with the ‘radiator’ function \( H \) expressing the probability to emit a photon with an energy fraction \( x \) amplitude and \( \nu \) where \( N \) centre-of-mass energy), is dressed with the ‘radiator’ function \( H \) expressing the probability to emit a photon with an energy fraction \( x = 2E_\gamma/\sqrt{s} \) at the angle \( \theta_\gamma \) (\( \gamma \equiv \sin \theta_\gamma, c_\gamma \equiv \cos \theta_\gamma \)).

Using the SM couplings and the contact NS interaction (2), we find:

\[
\sigma_{0}^{SM}(s) = \frac{N_{\nu}G_F^2}{6\pi} M_W^2 (g_L^2 + g_W^2) \left( \frac{s - M_Z^2}{\left[(s - M_Z^2)^2 + (M_\Gamma Z)^2 \right]} \right) + \frac{G_F^2}{\pi} M_W^2 \left\{ \frac{s + 2M_W^2}{2s} - \frac{M_W^2}{s} \left( \frac{(s + M_W^2)^2}{s^2} \right) \log \left( \frac{s + M_W^2}{M_W^2} \right) \right. \\
- \frac{g_L}{(s - M_Z^2)^2 + (M_\Gamma Z)^2} \left[ \left( \frac{(s + M_W^2)^2}{s^2} \right) \log \left( \frac{s + M_W^2}{M_W^2} \right) - \frac{M_W^2}{s} - \frac{3}{2} \right] \right\},
\]

(22)

\[
\sigma_{0}^{NS}(s) = \sum_{\alpha=\epsilon,\mu,\tau} \frac{G_F^2}{8\pi} \left[ \left( \epsilon_{\alpha L}^2 + \epsilon_{\alpha R}^2 \right) - 2(g_L \epsilon_{\alpha L} + g_R \epsilon_{\alpha R}) \frac{M_Z^2(s - M_Z^2)}{(s - M_Z^2)^2 + (M_\Gamma Z)^2} \right] + \frac{G_F^2}{\pi} \epsilon_{\epsilon L} M_W^2 \left[ \left( \frac{(s + M_W^2)^2}{s^2} \right) \log \left( \frac{s + M_W^2}{M_W^2} \right) - \frac{M_W^2}{s} - \frac{3}{2} \right],
\]

(23)

where \( N_\nu \) is the number of neutrinos (fixed to three), \( M_W, M_Z, \Gamma_Z \) are the \( W \)-boson mass, the \( Z \)-boson mass and total decay rate, respectively. The three lines in eq. (22) originate from the square of the \( Z \) amplitude, the square of the \( W \) amplitude and \( W - Z \) interference, respectively. In eq. (23), the terms quadratic in \( \epsilon_{\alpha L}, \epsilon_{\alpha R} \) originate from the square of the NS amplitude, whilst those linear in \( \epsilon_{\alpha L}, \epsilon_{\alpha R} \) originate from the interference of the NS amplitude with the \( Z \) amplitude and (for \( \nu = \nu_e \)) with the \( W \) amplitude. We can now plug eqs. (22) and (23) into eq. (21), integrate over the photon variables and construct the ratio \( \sigma_{0}^{NS}(s)/\sigma_{0}^{SM}(s) \), to be compared with the experimental data through eq. (20). In order to simplify this comparison, we will tentatively assume that the central value \( \sigma_{exp}^{SM} \) coincides with the full \( \sigma_{SM} \). In this case eq. (20) reads

\[
\left| \frac{\sigma_{0}^{NS}(s)}{\sigma_{0}^{SM}(s)} \right| \leq \frac{\delta \sigma_{exp}(s)}{\sigma_{exp}(s)},
\]

(24)

which can be translated into bounds for the NS couplings. Using this procedure, we have obtained bounds for each specific pair \( (\epsilon_{\alpha R}, \epsilon_{\alpha L}) \) (or equivalently \( (\epsilon_{\alpha A}, \epsilon_{\alpha V}) \)), setting to zero the other \( \epsilon \) parameters (barring possible cancellations among the different couplings).

To obtain the cross sections \( \sigma_{0}^{NS}(s) \) and \( \sigma_{0}^{SM}(s) \) through eq. (21), the integration in the \( x \) variable has been performed numerically from \( x_{min} = 0.1 \) to \( x_{max} = 1 \). We are aware that a better analysis could be done by cutting at lower \( x_{max} \), below the ‘radiative \( Z \)-peak return’ occurring at \( x = 1 - M_Z^2/s \). In so doing one could study the interplay between an increase in the sensitivity to non-standard couplings and an increase in the statistical error (due to the loss of events).11

11 A better determination of the allowed parameter space would require a proper fit to the experimental data. This is beyond the scope of our work. Nevertheless we urge our experimentalist colleagues to accomplish this analysis.
Figure 2: Sensitivity contours to neutrino non-standard interactions in the plane $(\varepsilon_{eR}, \varepsilon_{eL})$ (left panel) or $(\varepsilon_{eA}, \varepsilon_{eV})$ (right panel) from the reaction $e^+e^-\rightarrow \nu\bar{\nu}\gamma$ for centre-of-mass energy $\sqrt{s} = 207$ GeV. The dotted (solid) contours delimit the parameter space allowed by LEP at 68% (99%) C.L.. For comparison the 99% C.L. parameter space allowed by LSND is also depicted (shaded annulus).

Our ‘sensitivity’ analysis is presented in Fig. 2 and Fig. 3, for $\nu_e$ and $\nu_\tau$, respectively. In both cases in the plane $(\varepsilon_{eR}, \varepsilon_{eL})$ (left panel) or $(\varepsilon_{eA}, \varepsilon_{eV})$ (right panel) we have drawn the iso-contours of the experimental accuracy $\frac{\delta\sigma_{\text{exp}}}{\sigma_{\text{exp}}}$ taking $\sqrt{s} = 207$ GeV. Let us first discuss the allowed ranges for $\nu_e$ non-standard couplings. From the four LEP experiments, we have estimated (perhaps underestimated) the present 1-σ accuracy at the level of $\sim 5\%$ [23]. Then adopting a $\sim 13\%$ accuracy at 99% C.L., we consider as allowed parameter space the annulus bounded by the solid lines in Fig. 2. If we focus on the neighborhood of the Standard Model point, $(\varepsilon_{eR}, \varepsilon_{eL}) = (0, 0)$, we obtain at the 68% or 99% C.L. the following ranges:

for $\varepsilon_{eR} = 0$: $-0.09 \leq \varepsilon_{eL} \leq 0.08$ (68%), $-0.28 \leq \varepsilon_{eL} \leq 0.20$ (99%)

for $\varepsilon_{eL} = 0$: $-0.25 \leq \varepsilon_{eR} \leq 0.45$ (68%), $-0.46 \leq \varepsilon_{eR} \leq 0.65$ (99%) (25)

In Fig. 2 we have also shown together the parameter space for $(\varepsilon_{eR}, \varepsilon_{eL})$ (and $(\varepsilon_{eA}, \varepsilon_{eV})$) allowed at 99% C.L. by the LSND data (coloured annulus) for the sake of comparison. This should help the reader to catch the correlated regions allowed by the two experiments and he/she would notice how the combination of the data restricts the allowed regions. For example, for $\varepsilon_{eR} = 0$ the allowed range for $\varepsilon_{eL}$ is dictated by LSND experiment (see eq. 18), whereas for $\varepsilon_{eL} = 0$ the allowed range for $\varepsilon_{eR}$ is mostly restricted by LEP and RGS data, e.g. $-0.46 \leq \varepsilon_{eR} \leq 0.5$ at 99% C.L.

As for the $\nu_\tau$ non-standard couplings, notice from Fig. 3 that in this case the allowed regions become circles and so and it makes more sense to ignore the parameter correlations in order
Figure 3: Sensitivity contours to neutrino NS interactions in the plane $(\varepsilon_{\tau R}, \varepsilon_{\tau L})$ (left panel) or $(\varepsilon_{\tau A}, \varepsilon_{\tau V})$ (right panel) from the reaction $e^+e^- \to \nu\bar{\nu}\gamma$ for centre-of-mass energy $\sqrt{s} = 207$ GeV.

As is apparent upon comparing the sensitivity contours in Fig. 2 and Fig. 3, for $\nu_\tau$ the allowed ranges are more symmetric around the point $(\varepsilon_{\tau R}, \varepsilon_{\tau L}) = (0, 0)$ than those for $\nu_e$ around $(\varepsilon_{e R}, \varepsilon_{e L}) = (0, 0)$. This comes from the fact that, at $\sqrt{s} > M_Z$, the bounds on $\varepsilon_{\tau R}, \varepsilon_{\tau L}$ mainly originate from the terms proportional to $\varepsilon_{\tau R}^2, \varepsilon_{\tau L}^2$ in eq. (23), which go as $\sim s$. In the case of $\nu_e$, the bounds on $\varepsilon_{e R}, \varepsilon_{e L}$ originate from an interplay between the quadratic terms and the linear term from $W$ interference. The latter does matter, since its milder energy dependence ($\sim \log s$) is compensated by a larger numerical coefficient.

A remark is in order. Our expression (23) and hence our analysis is correct as long as neutrino NS interactions are point-like (see eq. (2)). However, a modified analysis would be needed in case the effective NS interactions originate from the exchange of particles with masses very close to the experimental energy threshold. For instance, in the first example discussed in Sec. 2 (see eq. (11)), the scalar doublet $\phi$ is exchanged in the $t$-channel and therefore the corresponding diagram for $e^+e^- \to \nu\bar{\nu}$ is similar to that with $W$-exchange. In this case for $M_{\phi^+} < 200$ GeV the contribution to $\sigma_0^{NS}(s)$ quadratic in NS couplings would smoothly pass from a ‘contact’ behaviour $\sim s$ for small $s$ to a $1/s$ behaviour at higher energies.\textsuperscript{13} As a result,\textsuperscript{12} we refer as to $\nu_\tau$ non-standard interactions, but needless to say that the same analysis applies as well to $\nu_\mu$ non-standard interactions. However, for $\nu_\mu$, these limits are not competitive with those from $\nu_\mu e$ low-energy elastic scattering [11].

\textsuperscript{12}Notice the different behaviour of the scalar $\phi$-exchange cross section with that from the $W$-exchange: the latter exhibits at high energy an energy independent behaviour due to the vectorial nature of the vertex. Note also that $s$ really means $\hat{s} = (1 - x)s$. 

\textsuperscript{13}As a result, the contribution to $\sigma_0^{NS}(s)$ quadratic in NS couplings would smoothly pass from a ‘contact’ behaviour $\sim s$ for small $s$ to a $1/s$ behaviour at higher energies.
we would obtain looser bounds. On the contrary, stronger bounds would be derived in case of NS interactions arising from the s-channel exchange of some extra particle with mass just above 200 GeV. Indeed, the corresponding contribution would start growing faster than s at high energy. Therefore our results, obtained in the point-like approximation, are representative from this point of view of an intermediate scenario.

Finally, we have to notice that the expression in (23) is also suitable to study the effect of flavour changing neutrino interactions which would lead to the process $e^+e^- \rightarrow \nu_\alpha \overline{\nu}_\beta \gamma$ ($\alpha, \beta = e, \mu, \tau; \alpha \neq \beta$). Analogously to the flavour conserving couplings discussed until now, there are so far no direct experimental bounds on neutrinos flavour-changing couplings with electrons. The existing bounds are derived from flavour universality violation and as such they apply to the charged operators involving the charged leptons. As the process $e^+e^- \rightarrow \nu_\alpha \overline{\nu}_\beta \gamma$ would add incoherently to the SM one, in eq. (23) the interference terms with Z, W-diagrams should be dropped, and only the quadratic terms in $\varepsilon^2_L, \varepsilon^2_R$ would appear. In this case the allowed regions are circles centered in (0,0) (at variance with those in Figs. 2, 3) and the associated most conservative bounds (at 99% C.L.) are:

$$\begin{align*}
|\varepsilon_{\alpha\beta L}| &\leq 0.53, \\
|\varepsilon_{\alpha\beta R}| &\leq 0.53, \\
|\varepsilon_{\alpha\beta V}| &\leq 0.75, \\
|\varepsilon_{\alpha\beta A}| &\leq 0.75,
\end{align*}$$

(27)

To conclude this discussion, we add that the astrophysical bounds from the stellar evolution imply for the strength of neutrino NS interactions with electrons (2) $\varepsilon_{\alpha R}, \varepsilon_{\alpha L} \leq 1$ in the most conservative case. Neutrino non-standard interactions may affect the primordial nucleosynthesis as they would maintain longer these species in equilibrium. For the sake of completeness we mention that phenomenological bounds of NS flavour-changing as well as flavour-diagonal neutrino interactions have been recently obtained by atmospheric neutrino data fitting [6]. However, we have to stress that these bounds apply to the NS vector-coupling $\varepsilon_{\alpha V}$. For the flavour-diagonal coupling with electrons (denoted as $\varepsilon'_e$ by the authors [6]) we have inferred that $\varepsilon_{eV} \lesssim 0.2$ at 90% C.L.. Comparable bounds can be deduced from the atmospheric neutrino analysis performed in [24, 25]. The bounds obtained on the amount $\sin^2\xi$ of sterile neutrino mixed with $\nu_\tau$, namely $\sin^2\xi < 0.25$ [24] and 0.8 [25], can be translated into $\varepsilon_{\tau V} \lesssim 0.125$ and $\lesssim 0.4$ at 90% C.L., respectively. However, the analysis performed in [25] does not rule out the pure sterile oscillation which would imply $\varepsilon_{\tau V} \lesssim 0.5$ at 90% C.L..

5 Conclusions

Extensions of the Standard Model often predict new neutral current interactions that can be flavour changing as well as flavour conserving. As is well-known, scenarios with neutrino non-standard interactions with matter have been invoked to explain the solar and atmospheric neutrino anomalies. In this paper, we have discussed in detail neutrino NS interactions with electrons motivated by the fact that they can be detected in Borexino detector through the measurement of the electron energy spectrum in the $\nu e$ scattering reaction [7]. First in Sec. 2.1 we have presented a general operator analysis of such non-standard interactions. This can help to figure out the features that the underlying theory has to accomplish to fulfill the phenomenological bounds. Then, the latter have been reviewed in Sec. 3. The bounds for both the first and third generation are tight but they apply to the couplings of the operators...
involving the $SU(2)_W$ related charged-lepton. On the other hand, the laboratory limits on $\nu_e$ interactions with electrons have only been extracted from the measurements of the $\nu_e e$ elastic scattering cross section [8]. We have updated this analysis using the most recent data from LSND experiment and found that the limits are still loose enough (see Fig. 1). In this work we have derived complementary bounds by using the measurements of the $\bar{\nu}_e e$ scattering cross section [18]. As for the third generation, there were no direct bounds on $\nu_\tau$ NS couplings with electrons. We have suggested to constrain novel neutrino interactions through the reaction $e^+e^- \to \nu\bar{\nu}\gamma$, measurable at $e^+e^-$ colliders. Our results are shown in Figs. 2 and 3, for $\nu_e$ and $\nu_\tau$ case, respectively. All the results are obtained considering for given flavour $\alpha$ ($\alpha = e, \tau$) only the pair $(\varepsilon_{\alpha R}, \varepsilon_{\alpha L})$ to be non-zero at time. The accuracy reached by LEP experiments helps to further restrict the parameters $\varepsilon_{eR,L}$. By comparing the bounds obtained from LSND, RGS and LEP experiment, we can say that for $\varepsilon_{eL} = 0$, $|\varepsilon_{eR}| \lesssim 0.5$ at 99% C.L., while for $\varepsilon_{eR} = 0$, $-0.15 \lesssim \varepsilon_{eL} \lesssim 0.17$. It would be interesting to perform a more refined statistical analysis (than that tentatively done by us and displayed in Fig.2) of these LEP data with those by LSND and possibly by reactor $\bar{\nu}_e$, from diffusion on electrons, to determine accurately the allowed (correlated) parameter space for $\varepsilon_{eR,L}$. For $\nu_\tau$ the allowed range looks somehow more symmetric: both $|\varepsilon_{\tau R}|$ and $|\varepsilon_{\tau L}|$ can be sizeable $\sim 0.5 - 0.7$. A better accuracy, say $\sim 1\%$, in the measurement of the $e^+e^- \to \nu\bar{\nu}\gamma$ cross section [26], achievable in the planned Linear Collider [27], will allow to further shrink the allowed parameter regions. For example, in Fig. 2 the present annulus allowed by LEP would be replaced by a much thinner one (crossing the point $(0,0)$) and then essentially only the upper overlapping portion would survive, restricting definitely $\varepsilon_{eL}$ in the negative range.\footnote{This holds if no discrepancy with the SM prediction on the cross section is found in the experimental data.}

We have to stress that, at variant with the atmospheric (or solar) neutrino phenomenology where only the ‘vector’ parameter $\varepsilon_{\alpha V}$ can be tested, the measurements of $\sigma(e^+e^- \to \nu\bar{\nu}\gamma)$ has allowed to study $\varepsilon_{\alpha V}, \varepsilon_{\alpha A}$ in a correlated way, and to constrain for the first time non-standard couplings of $\nu_\tau$ with electrons. It would be desirable to exploit further the diagnostic potential of the reaction $e^+e^- \to \nu\bar{\nu}\gamma$ by using polarized electron - positron beams so that to separately disentangle $\varepsilon_{\alpha R}$ and $\varepsilon_{\alpha L}$. This also would be certainly achieved by the Linear Collider.

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