Gravitational lensing by stars with angular momentum

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ABSTRACT

Gravitational lensing by spinning stars, approximated as homogeneous spheres, is discussed in the weak field limit. Dragging of inertial frames, induced by angular momentum of the deflector, breaks spherical symmetry. We examine how the gravito-magnetic field affects image positions, caustics and critical curves. Distortion in microlensing-induced light curves is also considered.

Key words: gravitation – gravitational lensing – relativity – astrometry – stars: rotation – white dwarfs.

1 INTRODUCTION

Gravitational lensing is one of the most extensively investigated phenomena of gravitation and it is becoming a more and more important tool for experimental astrophysics and cosmology.

The impressive development of technical capabilities will make it possible to obtain observational evidences of higher-order effects in gravitational lensing phenomena in the future, so that a full treatment of lensing theory to any order of approximation is necessary. In particular, observations of gravitational lensing phenomena could demonstrate the inertia-influencing effect of masses in motion. In fact, the gravito-magnetic field, predicted in 1896–1918, has not yet had a firm experimental detection.

Mass–energy currents relative to other masses generate space–time curvature. This phenomenon, known as intrinsic gravitomagnetism, is a new feature of general relativity and other conceivable alternative theories of gravity, and cannot be deduced by a motion on a static background; for a detailed discussion on gravitomagnetism we refer to Ciufolini & Wheeler (1995). Gravity induced by moving matter is related to the dragging of inertial frames and the effects of mass currents on the propagation of light signals must be considered. Lensing of light rays by stars with angular momentum has been addressed by several authors with very different approaches. Epstein & Shapiro (1980) performed a calculation based on the post-Newtonian expansion. Ibáñez & Martín (1982) and Ibáñez (1983) resolved the motion equation for two spinning point-like particles, when the spin and the mass of one of the particles were zero, by expanding the Kerr metric in a power series of gravitational constant $G$. Dymnikova (1986) studied the time delay of a signal in a gravitational field of a rotating body by integrating the null geodesics of the Kerr metric. Glicenstein (1999) applied an argument based on Fermat’s principle to the Lense–Thirring metric to study the lowest-order effects of rotation of the deflector. The listed results give a deep insight into some peculiar aspects of spinning lenses but are very difficult to generalize. Capozziello et al. (1999) discussed the gravito-magnetic correction to the deflection angle caused by a point-like, shifting lens in the weak field regime and slow motion approximation. Asada & Kasai (2000) considered the light deflection angle caused by an extended, slowly rotating lens.

In a series of papers, Sereno and co-workers developed a new approach to address the effect of the lens spin. In Sereno (2002a), the action of the gravito-magnetic field has been considered in the usual framework of lensing theory: that is, (i) weak field and slow motion approximation for the lens; (ii) thin lens hypothesis (Schneider, Ehlers & Falco 1992; Petters, Levine & Wambsganss 2001). Sereno & Cardone (2002) applied this formalism to consider the gravito-magnetic contribution to the deflection angle for extended, spherically symmetric, gravitational lenses in rigid rotation. Then, the gravito-magnetic correction to the deflection angle has been considered for spiral galaxy models of astrophysical interest (Capozziello et al. 2003). Finally, Sereno (2003) extended the formalism of gravitational lensing in metric theories of gravity, up to the post-post-Newtonian order.

The simplest lens model, the point-like Schwarzschild lens, cannot produce an intrinsic gravito-magnetic field because the local Lorentz invariance on a static background does not account for the dragging of inertial frames (Ciufolini & Wheeler 1995). Viable theories of gravity, such as general relativity, are classical non-quantized theories where the classical angular momentum of a particle goes to zero as its size goes to zero. To treat the gravito-magnetic field, we need a further step after the point mass as a lens model: extended lens models have to be considered.

In this paper, we consider the lensing signatures of the homogeneous spinning sphere. The homogeneous sphere is a usual approximation to model several astrophysical systems, such as stars. A quite complete theoretical analysis of the lensing properties, both inside and outside the lens radius, is given. All the relevant lensing quantities will be corrected for the gravito-magnetic effect. In
general, the inversion of the lens equation becomes a mathematically demanding problem. However, the gravito-magnetic effect is a higher-order correction and interesting gravitational lens systems can be studied in some detail despite their complexity using a perturbative approach. This procedure is quite usual in gravitational lensing problems (Konno & Kojima 1999; Bozza 2000). By perturbing the non-rotating case, critical curves, caustics and image positions for a rotating system will be calculated. Our results extend previous analyses (Ibáñez 1983; Glicenstein 1999). In Section 2, basic notations for spherically symmetric lenses in rigid rotation will be introduced. In Section 3, the deflection angle and the deflection potential for the homogeneous sphere will be derived. Critical curves and caustics are discussed in Section 4. In Section 5 we turn to the inversion of the lens mapping and present a perturbative analysis to locate the image positions. The distortion in microlensing induced light curves is discussed in Section 6. In Section 7 we give some estimates of the gravito-magnetic effect in the most promising gravitational lensing system: a white dwarf as deflector. In Section 8, we consider the gravitational Faraday rotation. Section 9 is devoted to a summary and to some final considerations.

2 BASICS

Let us consider a class of matter distributions with a spherically symmetric mass density in rigid rotation. The components of the deflection angle are (Sereno & Cardone 2002)

\[
\hat{\alpha}_1(\xi, \theta) = \frac{4G}{c^2} \left\{ \frac{M(\xi)}{\xi} \cos \theta + \frac{I_1(\xi)}{\xi^2} \left( \frac{\omega_1}{c} \cos 2\theta - \frac{\omega_1}{c} \sin 2\theta \right) \right\} - M(\xi) \frac{\omega_2}{c},
\]

(1)

\[
\hat{\alpha}_2(\xi, \theta) = \frac{4G}{c^2} \left\{ \frac{M(\xi)}{\xi} \sin \theta + \frac{I_1(\xi)}{\xi^2} \left( \frac{\omega_1}{c} \cos 2\theta + \frac{\omega_1}{c} \sin 2\theta \right) \right\} + M(\xi) \frac{\omega_1}{c}
\]

(2)

where \( c \) is the vacuum speed of light, \( \xi \) and \( \theta \) are polar coordinates in the lens plane, and \( \omega_1 \) and \( \omega_2 \) are the components of the angular velocity along the \( \xi_1 \)-axis and \( \xi_2 \)-axis, respectively. \( M(\xi) \) is the mass of the lens within \( \xi \) (Schneider et al. 1992)

\[
M(\xi) \equiv 2\pi \int_0^\xi \Sigma(\xi') \xi' \, d\xi',
\]

(3)

where \( \Sigma \) is the projected surface mass density, \( M(\xi) \) is the lens mass outside \( \xi \), \( M(\xi) \equiv M(\xi) = M(\xi) \), and

\[
I_1(\xi) \equiv 2\pi \int_0^\xi \Sigma(\xi') \xi'^2 \, d\xi'.
\]

(4)

is the momentum of inertia of the mass within \( \xi \) about a central axis (Sereno & Cardone 2002). \( I_1 \times \omega_1 \) is the component of the angular momentum along the \( \xi_1 \)-axis.

Let us change to a dimensionless variable \( x \equiv \xi / \xi_0 \). We introduce the dimensionless mass \( m(x) \) within a circle of radius \( x \equiv |x| \) (Schneider et al. 1992)

\[
m(x) \equiv 2 \int_0^x k(x') x' \, dx',
\]

(5)

where \( k(x) \) is the convergence,

\[
k(x) \equiv \frac{\Sigma(x)}{\Sigma_c},
\]

(6)

and \( \Sigma_c \) is the critical surface mass density:

\[
\Sigma_c \equiv \frac{c^2}{4\pi G} \frac{D_s}{D_L D_A}.
\]

(7)

Here, \( D_\odot \) is the angular diameter distance from the lens to the source, \( D_s \) is the angular diameter distance from the observer to the source, and \( D_A \) is the angular diameter distance from the observer to the deflector. We can also define a dimensionless momentum of inertia within \( x \) (Sereno 2002b)

\[
i_n(x) \equiv 2 \int_0^x k(x') x'^3 \, dx'.
\]

(8)

With these notations, the scaled deflection angle

\[
\alpha(x) \equiv \frac{D_A D_\odot}{\xi_0^2 D_s} \hat{\alpha}(\xi)
\]

(9)

where

\[
v_1 \equiv \frac{\omega_1 \xi_0}{c}, \quad v_2 \equiv \frac{\omega_2 \xi_0}{c}
\]

(10)

are the circular velocities at the scalelength \( \xi_0 \) around \( \xi_1 \) and \( \xi_2 \), respectively, in units of the speed of light.

The scaled deflection angle is related to a dimensionless gravitational potential \( \psi \)

\[
\alpha(x) = \nabla \psi(x).
\]

(11)

The lens equation reads

\[
y = x - \alpha(x),
\]

(12)

where \( y \) is the position vector of the source, in units of \( (D_s/D_A) \xi_0 \) in the source plane.

3 THE HOMOGENEOUS SPHERE

Let us consider a homogeneous sphere of radius \( R \) and volume density \( \rho_0 \). The projected surface mass density is

\[
\Sigma_{\text{HOM}}(\xi) = \begin{cases} 2\rho_0 \sqrt{R^2 - \xi^2}, & \text{if } \xi \leq R, \\ 0, & \text{elsewhere.} \end{cases}
\]

(13)

Let us change to dimensionless variables. As a natural length-scale, we introduce the Einstein radius \( R_E \)

\[
\xi_0 = R_E = \sqrt{\frac{4GM_{\text{TOT}}}{c^2}} \frac{D_\odot D_A}{D_s},
\]

(14)

where \( M_{\text{TOT}} \) is the total mass of the sphere, \( M_{\text{TOT}} = \frac{4}{3} \pi \rho_0 R^3 \). The convergence reads, if \( x \leq r \),

\[
k_{\text{HOM}}(x) = \frac{3}{2} \frac{1}{r^2} \left[ 1 - \left( \frac{x}{r} \right)^2 \right]^{1/2}.
\]

(15)

We have

\[
m_{\text{HOM}}(x) = \begin{cases} 1 - \left( 1 - (x/r)^2 \right)^{1/2}, & \text{if } x \leq r, \\ 1, & \text{elsewhere}; \end{cases}
\]

(16)
where \( r \) is the sphere radius in units of \( R_E \), and
\[
i_{\text{N}}^{\text{HOM}}(x) = \frac{2}{5} r^2 \left( 1 - \frac{1}{(x/r)^2} \right)^{3/2}, \quad \text{if } x \leq r,
\]
\[
\text{elsewhere.} \tag{18}
\]

Let us consider a lens rotating about the \( x_2 \)-axis (\( v_1 = 0 \)). The scaled deflection angle becomes (Sereno 2002b)
\[
\begin{align*}
\alpha_1^{\text{HOM}}(x_1, x_2) &= \frac{x_1}{x^2} m_{\text{HOM}}(x) + \frac{5}{2} U \frac{x_1^2 - x_2^2}{x^4} i_{\text{N}}^{\text{HOM}}(x) \\
&\quad - \frac{5}{2} \frac{U}{r^2} m_{\text{HOM}}(\rangle),
\end{align*}
\tag{19}
\]
\[
\alpha_2^{\text{HOM}}(x_1, x_2) = \frac{x_2}{x^2} m_{\text{HOM}}(x) + \frac{U}{r^2} \frac{x_1 x_2}{x^4} i_{\text{HOM}}(x), \tag{20}
\]
where
\[
U = \frac{r}{c M_{\text{TOT}}/R_E}
\]
is the ratio between the total angular momentum of the lens, \( L = (2/5) \omega M_{\text{TOT}} R^2 \), and that of a particle of mass \( M_{\text{TOT}} \) and velocity \( c \) in a circular orbit at the Einstein radius.

Outside the lens radius \((x > r)\), the scaled deflection angle in equations (19) and (20) reduces to (Sereno 2002b)
\[
\begin{align*}
\alpha_1^{\text{OUT}}(x_1, x_2) &= \frac{x_1}{x^2} + U \frac{x_1^2 - x_2^2}{x^4}, \\
\alpha_2^{\text{OUT}}(x_1, x_2) &= \frac{x_2}{x^2} + 2U \frac{x_1 x_2}{x^4}.
\end{align*}
\tag{21}
\tag{22}
\]

We will denote a deflector with the above deflection angle as the nearly point-like, rotating lens. Its deflection angle is the same as that of the two-point mass lens for the close binary system (Konno & Kojima 1999).

The deflection potential can be expressed as (Sereno 2002b)
\[
\psi^{\text{OUT}}(x_1, x_2) = \ln x - U \frac{x_1}{x^2}.
\tag{23}
\]

A point mass \((r = 0)\) at the origin, known as the Schwarzschild lens, has the same lensing quantities of a homogeneous sphere outside the radius.

4 CRITICAL CURVES AND CAUSTICS

The Jacobian matrix of the lens mapping in equation (13) is
\[
A(x) = \frac{\partial y}{\partial x}, \quad A_{ij} = \frac{\partial y_j}{\partial x_i}. \tag{24}
\]

Then, the magnification factor reads
\[
\mu(x) = \frac{1}{\det A(x)}. \tag{25}
\]

Points in the lens plane where the Jacobian is singular, \( \det A = 0 \), form closed curves, the critical curves (Schneider et al. 1992; Petters et al. 2001). They are the locus of all images with formally infinite magnification.

The corresponding locations in the source plane are the caustics; hence, the caustics due to a gravitational lens are the critical values of the associated lensing map. When caustics are curves, the smooth arcs are called folds, while cusps are the points where two abutting fold arcs have coincident tangents with the fold arcs on opposite sides of the double tangent (Schneider et al. 1992; Petters et al. 2001).

Let us first consider the non-rotating case, \( U = 0 \). Multiple images can be produced if \( k(0) > 1 \) (Schneider et al. 1992), i.e. \( r < \sqrt{3} / 2 \).

If \( r < \sqrt{3} / 2 \), there are both a radial critical curve and a tangential critical curve. The position of the radial critical curve verifies
\[
\frac{d}{dx} \left[ \frac{m(x)}{x} \right] = 1
\]
(Schneider et al. 1992); it is
\[
x_r = \frac{r}{\sqrt{2}} \left[ 48 - 32 r^2 + r^4 \right]^{1/2}.
\tag{26}
\]

The radial critical curve is determined by \( m(x) = x^2 \) (Schneider et al. 1992); it is located at
\[
\begin{cases}
(r/\sqrt{2}) \left[ 3 - r^4 - (r^2 - 1)^{1/2} (3 + r^2)^{1/2} \right], & 1 < r \leq \sqrt{3} / 2, \\
1, & r < 1.
\end{cases}
\tag{27}
\]

If \( r < 1 \), the tangential circle is superimposed on to the Einstein ring. The radial circle is inside the tangential one, \( x_t < x_r \). When \( r > 1 \), the two critical curves form inside the radius of the sphere. When \( r = \sqrt{3} / 2 \), they reduce to a point at the centre of the coordinate system. For \( r > \sqrt{3} / 2 \), no critical curve forms.

The tangential critical curve is mapped on a point-like caustic at the centre of the source plane. The radial critical curve is mapped on to a circle. A source inside the radial caustic has three images; a source outside the radial caustic has only one image.

The dragging of the inertial frames distorts the critical curves. In Fig. 1, we plot the critical curves for a rotating sphere. Critical curves are compressed and shifted along the \( x_1 \)-axis. For \( U > 0 \), the geometrical centres move to positive values of \( x_1 \). Given the axial symmetry of the rotating system, critical curves are symmetric for reflection around \( x_1 \).

Let us now consider the critical curves which form outside the lens. Outside the lens radius, the determinant of the Jacobian matrix is (Sereno 2002b)
\[
\det A^{\text{OUT}} = 1 - \frac{1}{x^4} - 4U \frac{x_1}{x^3} - 4U^2 \frac{1}{x^6}.
\tag{28}
\]

In the static case \((U = 0)\), the magnification reduces to
\[
\mu^{\text{STAT}} = \left( 1 - \frac{1}{x^4} \right)^{-1}.
\tag{29}
\]

For a non-rotating lens, only the tangential critical circle, in the case \( r < 1 \), can form outside the lens radius, when it is superimposed on to the Einstein ring. We want to discuss the distortion of the Einstein ring induced by dragging of inertial frames. Let us perform, first, a perturbative analysis in the hypothesis \( U \ll 1 \).

The equation for the main critical curve, which is a slight modification of the Einstein circle at \( x_1 = 1 \), is
\[
x_2(x_1) = \pm \left\{ -x_1^2 + \left[ 54U(U + x_1) + \sqrt{27} \sqrt{108U(U + x_1)^2} - 1 \right]^{1/3} \\
+ \frac{1}{3} \left[ 54U(U + x_1) + \sqrt{27} \sqrt{108U(U + x_1)^2} - 1 \right]^{1/3} \right\}^{-1/3} \tag{30}
\]
\[
\pm \left\{ \sqrt{1 - x_1^2} + U \frac{x_1}{\sqrt{1 - x_1^2}} + U^2 \frac{1}{(1 - x_1^2)^{3/2}} \right\}.
\tag{31}
\]
where the above approximate solution in equation (30) holds for $|x_1| < 1$. The main critical curve (see Fig. 2) intersects the $x_1$-axis in $x_1 \simeq -1 + U - (3/2)U^2$ and $x_1 \simeq 1 + U - (3/2)U^2$. The gravito-magnetic correction changes the width of the curve from 2 to $2[1 - (3/2)U^2]$. The maximum height is for $x_1 \simeq U$, when $x_2 \simeq \pm[1 + (3/2)U^2]$; the maximum total height changes to $\sim 2[1 + (3/2)U^2]$. So, the main critical curve is slightly compressed and its centre is shifted of $U$ along the $x_1$-axis.

The gravito-magnetic field changes the number of critical curves. Besides the main critical curve, a secondary critical curve forms; see Fig. 2. It is centred at $(x_1, x_2) = (-2U, 0)$, and has a width $\sim O(U^3)$. Because we have considered the Jacobian for image positions outside the lens, this secondary critical curves forms only if $2U > r$.

The main critical curve is mapped in a diamond-shaped caustic with four cusps (see Fig. 3) which substitutes the central point-like caustic. The main caustic is centred in $(y_1, y_2) = (U, 0)$ and its axes, parallel to the coordinate axes, are of semiwidth $\sim 2U^2$.

The secondary critical curve is mapped in a secondary caustic, far away from the central one, centred at $(y_1, y_2) \sim (1/4U - 2U, 0) \sim (1/4U, 0)$.

When the angular momentum of the sphere increases, the two outer critical curves merge (see Fig. 4).

### 5 Image Positions

When the gravito-magnetic correction is considered, the inversion of the lens mapping is not an easy task. However, under the condition

![Figure 2](image-url)
by substituting the expression in equation (31) for the perturbed images in the full vectorial lens equation, equation (13), we obtain the first-order perturbations

\[
x_{1(1)} = \frac{x_{0(1)}^2 - x_{0(2)}^2 - 1}{x_{0(1)}^4 - 1},
\]

\[
x_{1(2)} = -\frac{2x_{0(1)}x_{0(2)}}{x_{0(1)}^4 - 1}.
\]

Let us consider a source on the \(y_1\)-axis (\(y_2 = 0\)). Equations (34) and (35) reduce to

\[
x_{1(1)} = \frac{1}{1 + x_{0(1)}^2},
\]

\[
x_{1(2)} = 0.
\]

When \(U > 0\), i.e. when the angular momentum of the lens is positively oriented along \(\hat{x}_2\), photons with \(x_1 < 0\) (\(x_1 > 0\)) go around the lens in the same (opposite) sense of the deflector; photons which impact the lens plane on the \(x_1\)-axis at \(x_1 < 0\) move closer to the centre. This feature is common to the strong field limit of gravitational lensing (Bozza 2003), when, again, photons in the equatorial plane moving in the same (opposite) sense of rotation of a black hole form closer (farther) images with respect to the non-rotating case. If we do not limit to the equatorial plane, for \(U > 0\), the images...
6 LIGHT CURVES

Let us consider distortion in microlensing-induced light curves. The total magnification of a source is the sum of the absolute values of the magnifications of all the images. For a non-rotating sphere, if we consider a point-like lens, the total magnification can be expressed in terms of the source position as

$$\mu_{\text{tot}} = \mu_{\text{stat}}(x_1) - \mu_{\text{stat}}(x_2) = -\frac{y^2 + 2}{y \sqrt{y^2 + 4}}.$$  

(39)

When the source lies on the Einstein radius ($y = 1$), the total magnification becomes $\mu = 1.34$, corresponding to a brightening by 0.32 magnitudes.

Unless the lens is very massive ($M > 10^6 M_\odot$ for a cosmologically distant source), the angular separation of the two images is too small to be resolved and it is not possible to see the multiple images. However, a lensing event by a point mass can still be detected if the lens and the source move relative to each other, giving rise to lensing-induced time variability of the source. This type of variability, when induced by a stellar mass lens, is referred to as microlensing. The corresponding light curve, known as the Paczyński curve, is described by the last term of equation (38); see Fig. 8.

Let us consider how the gravito-magnetic field perturbs the Paczyński curve. Numerically, the maximum relative variation, for a source moving parallel to the $y_1$-axis, turns out to be $\sim (1/2) [(U/y_2)]$. In Fig. 9, the relative variation in the total magnification is plotted. An asymmetry is induced by rotation. The source is brightened (dimmed) when $y_1 > 0 (y_1 < 0)$.

7 ASTROPHYSICAL SYSTEMS

The homogeneous sphere is the standard lens model to describe lensing by stars. As shown in Sereno & Cardone (2002), the most significant case for the gravito-magnetic effect is a fast rotating white dwarf acting as a deflector. A convenient measure of the importance of rotation is the ratio $t$ of the rotational kinetic energy to the body’s self-gravitating potential energy. For a white dwarf, a range of values $t \sim 0.14–0.26$ can be obtained as maximum bounds in different cases (Padmanabhan 2001). If we take $t \sim 0.2$, it is $L \sim \sqrt{0.2 G M_{\text{TOT}}^3 R}$. 

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Figure 7. The track of a source, with $y_2 = 0.2$, and the corresponding images produced by a homogeneous rotating sphere. Filled circles indicates successive source positions. For every source position, two images (filled boxes) are counterclockwise rotated, around the centre, with respect to the line passing through the near unperturbed images (empty boxes); the third image, which forms near the centre, is nearly stable. The main critical curve is also plotted. It is $U = 10^{-5}$.

The weak field limit holds when the impact parameter of the photon is much larger than the Schwarzschild radius, $R_{sch} = 2GM/c^2$, of the lens. Typically, for a white dwarf $M_{WD} \sim M_\odot$, $R_{WD} \sim 10^{-2} R_\odot$, so that $R_{WD} \sim 2.4 \times 10^5 R_{sch}$, for a photon trajectory that just grazes the star limb, $\xi \sim R$, the weak field condition holds. For $\xi \sim R (5R)$, the rotation induces a correction of $\sim 1.14$ arcsec (0.05 arcsec) to the deflection angle, i.e. a correction of $\sim 0.65$ (0.13) percent. Astrometric accuracy of future interferometric missions, such as the Space Interferometry Mission (SIM) by NASA (scheduled for launch in 2009), should detect this correction.

Let us consider Galactic bulge–bulge lensing, when $D_s \sim 8.5$ Kpc and $D_d \sim 8$ Kpc; we have $U \sim 2 \times 10^{-7}$ and $r \sim 2 \times 10^{-5} \gg U$. So, the third image is eclipsed by the lens. For lensing of sources in the Magellanic Clouds by white dwarfs in the halo, the situation is even worse.

The secondary critical curve and the third image form at nearly $U$, in units of the scalelength $R_e$. The weak field limit still holds if $U \gg R_e/R_s$, i.e. $L \gg GM/c$. For a white dwarf, the angular momentum is $\sim 3 \times 10^{50}$ cm$^2$ g s$^{-1}$, whereas $GM/c \sim 10^{49}$ cm$^2$ g s$^{-1}$; the weak field condition is marginally verified.

A quadrupole moment in the mass distribution of the lens also breaks the circular symmetry. The predicted deflection due to the second zonal harmonic, for a lens symmetric about its angular momentum and with no angular momentum along the $\xi_1$-axis, reads (Epstein & Shapiro 1980; Sereno 2003)

$$
\hat{\alpha}^2_1(\xi) \simeq 4 \frac{GM}{c^2} J_2 R^2 \left\{ 1 - 4 \left( \frac{\xi_2 L_2}{\xi L} \right)^2 - \left( \frac{L_{1+1}}{L} \right)^2 \right\} \left( \frac{\xi_1}{\xi} \right)^3,
$$

(40)
where $J_2$ is the dimensionless coefficient of the second zonal harmonic and $L_{1,0,s}$ is the component of the angular momentum along the line of sight. The angular dependence of the gravito-magnetic correction differs from the quadrupole effect (Sereno 2003). The sign of the quadrupole correction does not change in the equatorial plane, whereas the gravito-magnetic correction varies on opposite sides of the lens, so that it can be separated experimentally from other terms. Furthermore, for a deflector with angular momentum with generic direction in the space, the gravito-magnetic effect depends only on the component of $L$ in the lens plane (Sereno 2002a), whereas $L_{1,0,s}$ enters the quadrupole correction. To evaluate the relative contribution to the deflection angle, let us consider photon trajectories which impact the lens plane on the $\xi_1$-axis:

$$\alpha_2^\xi(\xi) \simeq \frac{4GM}{c^2} J_2 R^2 \left\{ 1 - 4 \left( \frac{\xi_2}{\xi} \right)^2 - \left( \frac{L_{1,0,s}}{L} \right)^2 \right\} \frac{\xi_2}{\xi} + 2 \left( \frac{\xi_2}{\xi} \right)^2 \left( \frac{L_{1,0,s}}{L} \right)^2 \frac{1}{\xi^2}. \quad (41)$$

We have to compare the above expression to the strength of the gravito-magnetic correction

$$\alpha_{GRM} = \frac{4G L_2}{c^2 \xi^2}.$$  

For light rays lying in the equatorial plane ($L_{1,0,s} = 0$), it is

$$\left| \frac{\alpha_2^\xi}{\alpha_{GRM}} \right| = J_2 \frac{cMR}{L} \frac{r}{\xi} = J_2 \left( \frac{r}{U} \right) \left( \frac{r}{\xi} \right). \quad (43)$$

For the Sun, $J_2$ is of the order of $10^{-7}$, so that the above ratio is 0.25$R/\xi$. To evaluate $J_2$ for a white dwarf, let us model the lens as a Maclaurin spheroid (Chandrasekhar 1969; Binney & Tremaine 1987). Then, to the ratio $\sim 0.2$, it corresponds to an eccentricity $\sim 0.9$. $J_2^{WD}$ turns out to be $\sim 0.16$ and we find a ratio of the quadrupole correction to the gravito-magnetic effect of $\sim 24.7R/\xi$. In our best candidate for detecting a gravito-magnetic field, the strength of the quadrupole correction is significant, so that the gravito-magnetic contribution can be separated only due to its peculiar signatures.

8 THE GRAVITATIONAL FARADAY ROTATION

The plane of polarization of light rays passing close to a lens may undergo a rotation. This is the well-known gravitational Faraday rotation (Plebansky 1960; Ishihara, Takahashi & Tomimatsu 1988; Nouri-Zonoz 1999). In a Kerr space–time, the rotation angle, $\Omega_{\text{FAR}}$, is proportional to the mass and the line-of-sight component of the angular momentum of the deflector (Ishihara et al. 1988; Nouri-Zonoz 1999):

$$\Omega_{\text{FAR}} \sim \frac{G}{c^2} \left( \frac{R_{\text{Sch}}}{\xi} \right) \left( \frac{L_{1,0,s}}{\xi^2} \right). \quad (44)$$

Contrary to the Faraday effect, the gravito-magnetic contribution to the deflection angle depends only on the component of the angular momentum in the lens plane. For equatorial trajectories of the photons, the rotation of the polarization plane is null whereas the gravito-magnetic contribution to the deflection angle is maximum. In practice, however, the Faraday rotation is really small. Let us compare its magnitude with $\alpha_{GRM}$.

$$\Omega_{\text{FAR}} \sim \left( \frac{L_{1,0,s}}{L_2} \right) \left( \frac{R_{\text{Sch}}}{\xi} \right). \quad (45)$$

For a light ray grazing a white dwarf limb, it is $\Omega_{\text{FAR}}/\alpha_{GRM} \sim 10^{-4}$.

9 SUMMARY AND DISCUSSION

We have explored gravitational lensing by a spinning homogeneous sphere. Some properties of the dragging of inertial frames are independent of the specific lens model (Sereno 2002b), so that it is useful not to confine the analysis outside the lens. With regard to the effect of the gravito-magnetic field, important results can be easily generalized from the homogeneous sphere to other lens models.

Angular momentum breaks the spherical symmetry of the system leading to interesting effects in the lensing behaviour. The effects of a gravito-magnetic field are peculiar and can be distinguished from other higher-order corrections, such as those induced by a quadrupole moment in the deflection potential.

The ray-trace equation for a non-rotating spherically symmetric lens can be reduced to a one-dimensional equation but the gravito-magnetic field breaks this symmetry. Even for very simple mass distributions, we have to consider the full vectorial equation. However, interesting results can be obtained either numerically or with a perturbative approach.
A perturbative approach allows us to derive simple analytical expressions for image positions and critical curves. For a nearly point-like spinning lens, the deflection angle has the same expression of a close binary system. The tangential critical curve produced by a static homogeneous sphere is distorted and shifted and a secondary critical curve forms. The point-like central caustic changes to an extended diamond-shaped caustic.

Photons which move in the equatorial plane in the same (opposite) sense of the spinning lens are attracted (sent away) by the lens. For a positively oriented angular momentum, images are counter-clockwise shifted in the lens plane.

The total number of images is odd. With respect to the Schwarzschild lens, a third, highly de-magnified image forms nearly behind the lens. When the source moves into a caustic, two additional images appears. Instead of an Einstein ring, a source inside the central caustic produce a cross-shaped pattern made of four images, nearly on the coordinate axes, and a fifth image near the centre.

Microlensing-induced light curves are slightly distorted by the gravito-magnetic field. The asymmetry, caused by dragging of inertial frames, is usually negligible in realistic astrophysical lensing systems on galactic scales. However, space telescopes performing high-precision photometry, such as Eddington from the European Space Agency (ESA), should be able to detect the asymmetry for microlensing events, with very small impact parameters, induced by a fast rotating white dwarf.

The weak field approximation holds whenever the distance of the light ray to the lens is much larger than its Schwarzschild radius. This occurs for main-sequence stars, early-type stars and white dwarfs. For such astrophysical situations, light rays passing close to the Schwarzschild radius are absorbed by the lens and cannot be observed. On the other hand, collapsed objects can probe the gravitational field in the neighbourhood of the lens. A spinning black hole produces the third image outside the lens, but in a region where the weak field condition is only marginally satisfied. A full treatment of this system demands an investigation into the strong field limit of gravitational lensing. Together with the images we have discussed, black holes produce, by strong field lensing, two infinite series of relativistic images, formed by rays winding around the lens at distances comparable to the gravitational radius. Bozza (2003) addressed the quasi-equatorial lensing by a Kerr black hole in the strong field limit. A comparison between the two analyses shows how the rotation of the lens induces some features which are independent of the lensing regime. The shift of images and the topology of critical curves and caustics in the strong field limit are similar. In both regimes, caustics acquire finite extension and drift away from the optical axis. Furthermore, the asymmetry between images formed by photons winding in the same sense of the black hole and those winding in the opposite sense, which appear farther from the lens, is preserved in the strong field limit. A full comparison between lensing by rotating black holes in the weak or strong field limit cannot be performed because the quasi-equatorial approximation misses additional images. The general case is the objective of a future work (Bozza & Sereno, in preparation).

We finally remark how a comparison between general relativity and other viable theories of gravity can be made on the basis of higher-order effects (Sereno 2002b, 2003). An analysis to the lowest order might hide such differences. In this context, the analysis of the gravito-magnetic effect deserves particular attention.

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