Radiative corrections to radiative B decays: Exclusive $B \to V \gamma$ at NLO

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We discuss a model-independent framework for the analysis of the radiative $B$-meson decays $B \to K^* \gamma$ and $B \to \rho \gamma$ based on the heavy-quark limit of QCD. We present a factorization formula for the treatment of $B \to V \gamma$ matrix elements involving charm (or up-quark) loops, which contribute at leading power in $\Lambda_{QCD}/m_B$ to the decay amplitude. Annihilation topologies are power suppressed, but still calculable in some cases. The framework of QCD factorization is necessary to compute exclusive $b \to s(d) \gamma$ decays systematically beyond the leading logarithmic approximation. Results to next-to-leading order in QCD and to leading order in the heavy-quark limit are given and phenomenological implications are discussed.

1. INTRODUCTION

The radiative transitions $b \to s \gamma$, $b \to d \gamma$ are among the most valuable probes of flavour physics. Among the characteristics are the high sensitivity to New Physics and the particularly large impact of short-distance QCD corrections. Considerable efforts have therefore been devoted to achieve a full calculation of the inclusive decay $b \to s \gamma$ at next-to-leading order (NLO) in renormalization group (RG) improved perturbation theory (see the talk by M. Misiak in these proceedings).

Whereas the inclusive mode can be computed perturbatively, using the fact that the $b$-quark mass is large and employing the heavy-quark expansion, the treatment of the exclusive channel $B \to K^* \gamma$ is in general more complicated. In this case bound state effects are essential and need to be described by nonperturbative hadronic quantities (form factors). The basic mechanisms at next-to-leading order were already discussed previously for the $B \to V \gamma$ amplitudes [1, 2]. However, hadronic models were used to evaluate the various contributions, which did not allow a clear separation of short- and long-distance dynamics and a clean distinction of model-dependent and model-independent features.

In this talk we present a systematic analysis of the exclusive radiative decays $B \to V \gamma$ ($V = K^*, \rho$) in QCD, based on the heavy quark limit $m_b \gg \Lambda_{QCD}$ (see also [3]). We shall provide factorization formulas for the evaluation of the relevant hadronic matrix elements of local operators in the weak Hamiltonian. Factorization holds in QCD to leading power in the heavy quark limit. This result relies on arguments similar to those used previously to demonstrate QCD factorization for hadronic two-body modes of the type $B \to \pi \pi$ [4].

Within this approach higher order QCD corrections can be consistently taken into account. We give the $B \to V \gamma$ decay amplitudes at next-to-leading order (NLO). After including NLO corrections the largest uncertainties still come from the $B \to V$ form factors, which are at present known only with limited precision ($\sim \pm 15\%$), mostly from QCD sum rule calculations [5], which we used in the present analysis.

2. BASIC FORMULAS

The effective Hamiltonian for $b \to s \gamma$ transitions reads (see e.g. [3])

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(s)} \left[ \sum_{i=1,2} C_i Q_i + \sum_{i=3}^8 C_i Q_i \right]$$

(Eq. 1)
where \( \lambda_p^{(s)} = V_p^* V_p \). The four-quark operators \( Q_p, Q_s \) and the magnetic penguin operator \( Q_7 \) give the most important contribution. The most difficult step in computing the decay amplitudes is the evaluation of the hadronic matrix elements of the operators in (1). We will argue that in this case the following factorization formula is valid

\[
\langle \gamma(q, \epsilon) \rangle = \sum_i T_i^I(\xi, v) \Phi_B(\xi) \Phi_V(v) \cdot \epsilon \quad (2)
\]

where \( \epsilon \) is the photon polarization 4-vector. Here \( F^{B \to V} \) is a \( B \to V \) transition form factor, and \( \Phi_B, \Phi_V \) are leading twist light-cone distribution amplitudes (LCDA) of the \( B \) meson and the vector meson \( V \), respectively. These quantities describe the long-distance dynamics of the matrix elements, which is factorized from the perturbative, short-distance interactions expressed in the hard-scattering kernels \( T_i^I \) and \( T_i^{II} \). The QCD factorization formula (2) holds up to corrections of relative order \( \Lambda_{QCD}/m_b \).

For \( Q_7 \) the factorization formula (3) is trivial. The matrix element is simply expressed in terms of the standard form factor, \( T_I^I \) is a purely kinematical function and the spectator term \( T_I^{II} \) is absent. An illustration is given in Fig. 1. In the leading logarithmic approximation (LO) and to leading power in the heavy-quark limit, \( Q_7 \) gives the only contribution to the amplitude of \( B \to V \gamma \).

The matrix elements of the four-quark operators \( Q_i \) (and of \( Q_8 \)) start contributing at \( \mathcal{O}(\alpha_s) \). In this case the factorization formula becomes nontrivial. The diagrams for the hard-scattering kernels \( T_i^I \) are shown in Fig. 2 for \( Q_1, \ldots, Q_6 \) and in Fig. 3 for \( Q_8 \). The non-vanishing contributions to \( T_i^{II} \) are shown in Fig. 4.

The first negative moment of the \( B \)-meson LCDA \( \Phi_B(\xi) \), which will be needed below, can be parametrized by a quantity \( \lambda_B = \mathcal{O}(\Lambda_{QCD}) \), i.e., \( \int_0^1 d\xi \Phi_B(\xi)/\xi = m_B/\lambda_B \).

There are further mechanisms that can in principle contribute to the \( B \to V \gamma \) amplitude. One possibility is weak annihilation (Fig. 5), which is suppressed by \( \Lambda_{QCD}/m_b \). Still the dominant annihilation amplitude can be computed within QCD factorization because the colour-transparency argument applies to the emitted, highly energetic vector meson in the heavy-quark limit (6).

3. \( B \to K^*\gamma \)

In the case of \( B \to K^*\gamma \) the component of the Hamiltonian (1) proportional to \( \lambda_a \) is strongly CKM suppressed (\( |\lambda_a/\lambda_c| \approx 0.02 \)) and has only a minor impact on the decay rate. It is essentially negligible, but will be included for completeness. Here we shall neglect the contribution from the QCD penguin operators \( Q_3, \ldots, Q_6 \), which enter at \( \mathcal{O}(\alpha_s) \) and are further suppressed by very
small Wilson coefficients. (The complete results are given in [8].) We note that to \( \mathcal{O}(\alpha_s) \) the matrix element of \( Q_2 \) is zero because of its colour structure. The result for the diagrams in Figs. 2 and 3, which enter the hard-scattering kernels \( T_1^l, T_8^l \), can be inferred from [8]. In these papers the diagrams were computed to obtain the matrix elements for the inclusive mode \( b \to s \gamma \) at next-to-leading order. In this context Figs. 2 and 3 represented the virtual corrections to the inclusive matrix elements of \( Q_1 \) and \( Q_8 \). In our case they determine the kernels \( T_1^l \) and \( T_8^l \). The results from [8] imply

\[
\langle Q_{1,8} \rangle^l = \langle Q_7 \rangle \frac{\alpha_s C_F}{4\pi} G_{1,8}
\]

where \( G_1 = G_1(s_c) \) and \( G_8 \) can be found in [8] \( (s_c = m_c^2/m_b^2) \).

We now turn to the mechanism where the spectator participates in the hard scattering. The first diagram in Fig. 4 yields

\[
\langle Q_1 \rangle^{II} = \langle Q_7 \rangle \frac{\alpha_s(\mu_h) C_F}{4\pi} H_1(s_c)
\]

(4)

The function \( h(\bar{v}, s_c) \) is shown in Fig. 6. The hard-scattering kernel \( h(\bar{v}, s_c) \) as a function of \( \bar{v} \).

with \( (\bar{v} \equiv 1 - v) \)

\[
H_1(s) = \frac{2\pi^2}{3N} \frac{F_V f_B^2}{m_B^2} \int_0^1 d\xi \frac{\Phi_B(\xi)}{\xi} \cdot \int_0^1 dv h(\bar{v}, s) \Phi_+(v)
\]

(5)

The function \( h(\bar{v}, s_c) \) is shown in Fig. 6. The correction to \( \langle Q_8 \rangle \) from the hard spectator interaction comes from the second diagram in Fig. 6.

\[
\langle Q_{8} \rangle^{II} = \langle Q_7 \rangle \frac{\alpha_s(\mu_h) C_F}{4\pi} H_8
\]

(6)

Combining these results gives

\[
A(\bar{B} \to K^*\gamma) = \frac{G_F}{\sqrt{2}} \left[ \sum_p \lambda_p^{(s)} a_p^{(s)}(K^* \gamma) \right] \langle K^* \gamma | Q_7 | \bar{B} \rangle
\]

where, at NLO

\[
a_p^{(s)}(V\gamma) = C_7 + \frac{\alpha_s(\mu) C_F}{4\pi} (C_1(\mu) G_1(s_p) + C_8(\mu) G_8)
\]

\[
+ \frac{\alpha_s(\mu_h) C_F}{4\pi} (C_1(\mu_h) H_1(s_p) + C_8(\mu_h) H_8)
\]

(8)

4. \( B \to \rho \gamma \)

For the decay \( \bar{B} \to \rho \gamma \) both sectors of the effective Hamiltonian have the same order of magnitude. The amplitude is similar to (7) with obvious
replacements. The rate for the CP-conjugated mode $B \to \rho\gamma$ is obtained with $\lambda_{B}^{d} \to \lambda_{B}^{dA}$. We may then consider the CP asymmetry

$$A_{CP}(\rho\gamma) = \frac{\Gamma(B \to \rho\gamma) - \Gamma(\bar{B} \to \rho\gamma)}{\Gamma(B \to \rho\gamma) + \Gamma(\bar{B} \to \rho\gamma)}$$  \hspace{1cm} (9)$$

A non-vanishing CP asymmetry appears at $\mathcal{O}(\alpha_s)$ only.

We next comment on the issue of power corrections. The annihilation effect from operator $Q_1$ gives a numerically important power correction, because it receives an enhancement of $|C_1/C_7| \sim 3$. This leads to a 30% correction in the amplitude of the charged mode $B^- \to \rho^-\gamma$. A general discussion of isospin-breaking power corrections was given in [10], where a sizeable effect of 11% from penguin annihilation related to $Q_6$ was identified. This contribution is still calculable, while other terms of the same order are much smaller numerically.

Power corrections can also come from the loops with up- and charm quarks, whose leading-power contributions were computed in (4). These power corrections correspond to the region of integration where the gluon becomes soft, that is $\bar{v} = \mathcal{O}(\Lambda_{QCD}/m_b)$. Their contribution is nonperturbative and cannot be calculated in the hard-scattering formalism. Nevertheless, the expression in (4) can be used to read off the scaling behaviour of these power corrections in the heavy-quark limit. For the charm loop the kernel approaches a constant $\sim m_s^2/m_c^2$ in the endpoint region. Taking into account the linear endpoint suppression of the wave function $\Phi_\perp$, the integral in (6) over the region $\bar{v} \sim \Lambda_{QCD}/m_b$ thus contributes a term of order $(\Lambda_{QCD}/m_b)^2 \times (m_b/m_c)^2 = (\Lambda_{QCD}/m_c)^2$. That is, we recover the power behaviour of soft contributions in the charm sector first pointed out in [11]. This was discussed for the inclusive decay $b \to s\gamma$ in [11,12] and for the exclusive mode $B \to K^*\gamma$ in [13]. Numerically this correction is very small ($\sim 3\%$ in the decay rate). A similar consideration applies to the up-quark sector. In this case the endpoint behaviour of the kernel is singular $\sim 1/\bar{v}$, which now leads to a linear power suppression of the form $\Lambda_{QCD}/m_b$. This coincides with the scaling behaviour derived in [12] in the context of the inclusive process.

5. PHENOMENOLOGY

In this section we present some numerical results based on the QCD analysis at NLO. The input parameters used can be found in [3].

We note a sizable enhancement of the leading order value, dominated by the $T^{I}$-type correction. This feature was already observed in the context of the inclusive case in [3]. A complex phase is generated at NLO, where the $T^{I}$-corrections and the hard-spectator interactions ($T^{II}$) yield comparable effects.

The net enhancement of $a_7$ at NLO leads to a corresponding enhancement of the branching ratios, for fixed value of the form factor. This is illustrated in Fig. 6, where we show the residual scale dependence for $B(B^- \to \rho^-\gamma)$ at leading and next-to-leading order. The uncertainty of the branching fractions is currently dominated by the form factors $F_{K^*}, F_{\rho}$. Our estimates are (in com-
6. $B \to \gamma\gamma$

The decays $B_{s,d} \to \gamma\gamma$ have recently been analyzed using QCD factorization based on the heavy-quark limit \cite{14}. The dominant effect arises from the magnetic-moment type transition $b \to s(d)\gamma$ where an additional photon is emitted from the light quark (one-particle reducible diagram). The contributions from one-particle irreducible diagrams (both photons emitted from up- or charm-quark loops) are power suppressed, but still calculable. The dominant effect is very sensitive to the $B$ meson parameter $\lambda_B$ defined in sect. 2.

7. CONCLUSIONS

In this talk we have discussed a systematic, model-independent framework for the exclusive radiative decays $B \to V\gamma$ based on the heavy-quark limit. This enabled the consistent computation of the decay amplitudes at next-to-leading order in QCD. An important conceptual aspect of this analysis is the interpretation of loop contributions with charm and up quarks, which come from leading operators in the effective weak Hamiltonian. We have argued that these effects are calculable in terms of perturbative hard-scattering functions and universal meson light-cone distribution amplitudes. They are $O(\alpha_s)$ corrections, but are leading power contributions in the framework of QCD factorization. This picture is in contrast to the common notion that considers charm and up-quark loop effects as generic, uncalculable long-distance contributions. Non-factorizable long-distance corrections may still exist, but they are power-suppressed. The improved theoretical understanding of $B \to V\gamma$ decays strengthens the motivation for still more detailed experimental investigations, which will contribute significantly to our knowledge of the flavour sector.

Acknowledgements

I thank Stefan Bosch for a most enjoyable collaboration on the subjects of this talk.

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