Nuclear Structure and Energy Levels of $^{158}$Er, $^{160}$Yb and $^{162}$Hf Isotones

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Abstract

The properties of even-even $^{158}$Er, $^{160}$Yb and $^{162}$Hf isotones are studied and its energy states calculated. To identify the properties of each isotope, the values of the first excited states $E_{21}^+$ and the ratio of the second to the first excited states $R_4/2 = E_{41}^+/E_{21}^+$ were adopted. The phenomenon of back-bending, the E-GOS curve and the odd-even staggering were studied. Examining the algebraic structure of the Bohr model (BM) clarified the relationship with the interacting vector boson model (IVBM) and the interacting boson model (IBM) which it has related solvable limits and corresponding dynamical symmetries. BM, IVBM and IBM were used to calculate the energy states for each isotope and compared with the experimental data. The results showed that the BM and IVBM are more comfort than the calculation of IBM-1. The contour plots of the potential energy surface (PES) to the IBM Hamiltonian for Er–Hf with $N = 90$ have been obtained using the intrinsic coherent state and evolve from deformed shapes to $\gamma$- unstable with decreasing the boson number.

Keywords: Back bending, BM, IBM, Ground band, IVBM, potential energy surface.

Introduction

The ground states band GSB with ($I_1^+ = 0, 2, 4, 6, ...$) is the first band in even-even nuclei, followed by two other bands which are the $\beta$-band with ($I_{2\sigma r}^+ = 0, 2, 4, 6, ...$) and the $\gamma$-band with ($I_{2\sigma r}^{-} = 1, 2, 3, 4, 5, 6, ...$). The negative parity band NPB with an odd-angular momentum; ($I_1^- = 1, 3, 5, ...$) also occur in even-even nuclei [1-5].
Initial indication of the characteristics of the even-even nuclei can be get from the first excited states $E_{2^+}^1$ and the ratios $R_{4/2} = E_{4^+}^1/E_{2^+}^1$ has been widely used as an indicator of collectivity, for the vibrational limit having the value 2, in the rotational limit 3.33 and 2.5 values for $\gamma$-unstable nuclei[6-10].

To indicate the properties of even-even nuclei at high energy states many methods were introduced. The phase change of the nucleus can be indicated when the relation between the moment of inertia $2\theta/\hbar^2$ and the energy of the emitted photon $\hbar\omega$ are plotted and a back-bending or up-bending occur [11]. Regan and his colleagues presented a good method to determine the properties of the nuclei along their yrast states E-GOS [12], were a relation between gamma-energy over spin ($E_\gamma/I$) and the spin $I$ is plotted, and the shape of the curves indicate the characteristics of the nucleus. Interwove between GSB and NPB forms the octupole band [13], this odd-even staggering or $\Delta I = \pm 1$ staggering occurs in a zigzag mode. Different models were introduced to study the nuclei. The collective model introduced by Bohr and Mottelson [1] is important to determine the spin and the energy of different states and different sort of nuclei. The first version of the interacting boson model IBM-1 [14] is so suitable to calculate the eigenvalues of medium and heavy even-even nuclei. IBM-1 depending on the unitary group U(6) reduced to three dynamical symmetry limits which are the harmonic oscillator U(5), deformed rigid rotor SU(3) and $\gamma$-soft rotor [15-18]. Ganev and his colleagues presented the interacting vector boson model IVBM which distinguish between proton bosons and neutron bosons [19], IVBM is suitable to calculate the eigenvalues of vibrational and rotational nuclei.

In this paper, we study the properties of $^{158}$Er, $^{160}$Yb and $^{162}$Hf isotones by applying back or up-bending, E-GOS and staggering methods. The calculation of the energy states for GSB of $^{158}$Er, $^{160}$Yb and $^{162}$Hf isotones were done using BM, IBM-1 and IVBM, while the calculations of NPB states depend on the BM and IVBM. The results were compared with the measured values for these isotones.

**Calculations**

Increasing the angular momentum $I$, a dramatic change in the moment of inertia $\theta$ may occur in certain states of some nuclei, this increasing of $\theta$ cause a drop in the energy of this state yield a reduction in $\gamma$ energy $\hbar\omega$, which is given by [11, 20, 21]:

$$\hbar\omega = \frac{E_\gamma}{\sqrt{I(I+1)} - \sqrt{(I-2)(I-1)}}$$

and the moment of inertia is given by [20, 22]:

$$\frac{2\theta}{\hbar^2} = \frac{4I - 2}{E(I) - E(I - 2)} = \frac{4I - 2}{E_\gamma}$$

(2)
The (E-GOS) method is important to identify the characteristics of the nucleus along their yrast states [12, 23, 24]. Researchers gave the relations for different kind of nuclei between \( R = \frac{E_\gamma}{I} \) and the angular momentum \( I \) [25, 26] as:

\[
U(5): \quad R = \frac{\hbar \omega}{I} \rightarrow 0 \quad \text{when} \quad I \rightarrow \infty \quad (3)
\]

\[
O(6): \quad R = \frac{E_{21}^+}{4} \left(1 + \frac{2}{I}\right) \rightarrow \frac{E_{21}^+}{4} \quad \text{when} \quad I \rightarrow \infty \quad (4)
\]

\[
SU(3): \quad R = \frac{\hbar^2}{2\bar{\theta}} \left(4 - \frac{2}{I}\right) \rightarrow \frac{4\hbar^2}{2\bar{\theta}} \quad \text{when} \quad I \rightarrow \infty \quad (5)
\]

Odd-even staggering patterns between GSB and NPB have been investigated [27-29]:

\[
\Delta E_{1,\gamma}(I) = \frac{1}{16} \left(6E_{1,\gamma}(I) - 4E_{1,\gamma}(I - 1) - 4E_{1,\gamma}(I + 1) + E_{1,\gamma}(I - 2) + E_{1,\gamma}(I - 2)\right) \quad (6)
\]

where \( E_{1,\gamma}(I) = E(I + 1) - E(I) \). The odd-parity states are raised (or lowered) by an amount of energy with respect to the even-parity states. Alternative positive and negative of zigzag values in the difference of GSB states and NPB states occur and may reach zero, followed by other increases again.

Bohr-Mottelson model (BM), interacting bosons model (IBM-1) and interacting vector bosons model (IVBM) were used to calculate the energy states of GSB for \(^{158}\text{Er}, ^{160}\text{Yb}\) and \(^{162}\text{Hf}\) isotones.

The energy of GSB states and NPB states in BM model are given by\([1, 13, 30]\):

\[
E(I) = AI(I + 1) + BI^2(I + 1)^2 + CI^3(I + 1)^3 \quad (7)
\]

\[
E(I) = E_0 + \tilde{A}I(I + 1) + \tilde{B}I^2(I + 1)^2 + \tilde{C}I^3(I + 1)^3 \quad (8)
\]

where \( E_0 \) is the band head energy of the NPB and from a fit to the available energy levels, the coefficients \( A > A', B > B' \) and \( C > C' \) can be determined.

In IBM-1, the general Hamiltonian is given by \([6, 31, 32] \):

\[
H = \sum_{i=1}^{N} \epsilon_i + \sum_{i<j}^{N} V_{ij} \quad (9)
\]

where \( \epsilon_i \) is the intrinsic boson energy and \( V_{ij} \) is the interaction strength between bosons \( i \) and \( j \); the multipole form of the Hamiltonian is \([33, 34] \):

\[
H = \epsilon n_d + a_0 P^2 + a_1 L^2 + a_2 Q^2 + a_3 T_3^2 + a_4 T_4^2 \quad (10)
\]

where \( a_0, a_1, a_2, a_3 \) and \( a_4 \) are the strengths of pairing, angular momentum, quadrupole, octupole and hexadecupole interactions of each terms in the equation (10).

The eigenvalues of the vibrational \( U(5) \), the rotational \( SU(3) \) and the \( \gamma \) -soft \( O(6) \) nuclei are \([2, 33] \):
\[ U(5): \quad E = \varepsilon n_d + K_1 n_d(n_d + 4) + K_4 \nu(\nu + 3) + K_5 L(L + 1) \quad (11) \]

\[ SU(3): \quad E = K_2(\lambda^2 + \mu^2 + 3(\lambda + \mu) + \lambda \mu) + K_5 L(L + 1) \quad (12) \]

\[ O(6): \quad E = K_3\left(N(N + 4) - \sigma(\sigma + 4)\right) + K_4 \tau(\tau + 3) + K_5 L(L + 1) \quad (13) \]

where \( K_1, K_2, K_3, K_4 \) and \( K_5 \) are other forms of the strength parameters and \( N \) is the total boson number. Many nuclei have a transition property of two or three of the above limits.

The eigenvalues in IVBM model for the GSB and NPB states are given by \([19, 35-38]\):

\[ E(I) = \beta I(I + 1) + \gamma I \quad (14) \]

\[ E(I) = \beta I(I + 1) + (\gamma + \eta) I + \zeta \quad (15) \]

The values of \( \beta \) and \( \gamma \) can be determined from a fit to the positive GSB while \( \eta \) and \( \zeta \) are estimated from the NPB.

The IBM energy surface can be created by using coherent state \(|N, \beta, \gamma\rangle\) with the expectation value of the IBM-1 Hamiltonian \([6, 39]\) and which it gives a final shape to the nucleus that corresponds to the function of Hamiltonian as the following forms \([24, 33, 40]\):

\[ E(N_b, \beta, \gamma) = a_2 N_b(N_b - 1)\frac{(1 + 3/4 \beta^4 - \sqrt{2} \beta^3 \cos 3\gamma)/(1 + \beta^2)^2}{(1 + \beta^2)^2}, \ldots O(6), \quad (17) \]

where \( N_b \) is number of bosons, the deformation parameters \( \beta \) and \( \gamma \) (usually, \( \beta \geq 0, 0^\circ \leq \gamma \leq 60^\circ \)) which determine the geometrical shape of the nucleus for \( \gamma = 0^\circ \) (prolate), \( \gamma = 60^\circ \) (oblate), and \( 0^\circ \leq \gamma \leq \pi/3 \) the shape is triaxial shape (triaxial deformation helps to understand the prolate-to-oblate shape transition) and other terms are the same as in the Hamiltonian (10) and \( \beta_{\text{min}} \) for U(5), SU(3), and O(6) = 0, \( \sqrt{2} \), and 1, respectively.

**RESULTS AND DISCUSSION**

\(^{158}\text{Er}, \quad ^{160}\text{Yb}\) and \(^{162}\text{Hf}\) isotones have 68, 70 and 72 protons in the middle of 50 and 82 magic number with number of neutrons for these isotones are 90 which have eight more neutrons than the magic number N=82.

Table 1 shows, in \(^{158}\text{Er}, \quad ^{160}\text{Yb}\) and \(^{162}\text{Hf}\) isotones (with an additional valence neutron pair), centered at \( P = 68 \), \( R_{4/2} \) attains the SU(3)-O(6) value of 2.74, and decreases sharply to the O(6) limit value of 2.5 up to \( P = 72 \) in \(^{162}\text{Hf}\), indicating a shape change from the \( \beta \)-soft deformed to the \( \gamma \)-soft or O(6) with decreasing N.

| Nucleus | \(^{158}\text{Er}\) | \(^{160}\text{Yb}\) | \(^{162}\text{Hf}\) |
|---------|----------------|----------------|----------------|
| \( R_{4/2} \) | 2.74 | 2.61 | 2.55 |
Figure 1 shows that $^{158}\text{Er}$, $^{160}\text{Yb}$, and $^{162}\text{Hf}$ isotones undergo a back-bending which indicate a phase change occur in certain state, this behavior insure the transitional properties for them.

![Figure 1: Back (or up)-bending of the ground-states band for $^{158}\text{Er}$, $^{160}\text{Yb}$, and $^{162}\text{Hf}$ isotones](image1)

The E-GOS curves in the Figure 2 show the $\gamma$-soft properties for the nuclei, where the curve drop slowly from the highest values close to $0.15 \text{ MeV}/\hbar$ to the lowest values.

The staggering curves for $^{158}\text{Er}$, $^{160}\text{Yb}$, and $^{162}\text{Hf}$ isotones shown in Figure 3 indicate that the staggering of GSB and NPB for $^{158}\text{Er}$ begins from zero or close to zero and increases, this close value from zero means the phase change for this nucleus. The staggering of $^{160}\text{Yb}$ is close to zero between $10^1$ and $15^+$ states which again indicate the phase change. The values of staggering of $^{162}\text{Hf}$ nucleus do not close to zero which means that this nucleus has the same properties along their yrast states.

![Figure 2: (a) E-GOS curves for a perfect harmonic vibrator, $\gamma$-soft and axially symmetric rotor. (b) E-GOS curves of the GSB of rare-earth $^{158}\text{Er}$, $^{160}\text{Yb}$, and $^{162}\text{Hf}$](image2)
Fig. 3. Staggering for $^{158}$Er, $^{160}$Yb, and $^{162}$Hf Isotones

The energy states of GSB were calculated using BM, IBM-1 and IVBM models, and the calculations of NPB states were performed using BM and IVBM for $^{158}$Er, $^{160}$Yb and $^{162}$Hf isotones. The parameters of each model were determined by fitting their equations with the measured energy values for these nuclei of each band, these parameters and the boson numbers for the GSB were shown in Table 2, while Table 3 Shows the NPB parameters. The values of the parameters A in Table 2. of BM model are comparable with the values of $\hbar^2/2\eta$ of the rotational energy which are equal 0.032, 0.045 and 0.047 for $^{158}$Er, $^{160}$Yb and $^{162}$Hf nuclei respectively, and this is because of the effect of the rotational properties on these nuclei, while the values of B and C parameters are random values to complete the fitting. We can also observe in Table 1., that the $\beta$ values of the IVBM for $^{158}$Er and $^{160}$Yb are close but this value is smaller for $^{162}$Hf while the $\gamma$ values of $^{158}$Er, $^{160}$Yb and $^{162}$Hf increases gradually, this behavior indicate the exceed of the $\gamma$-soft properties. The $a_0$ parameters for $^{160}$Yb and $^{162}$Hf nuclei are equal and too greater than this value for $^{158}$Er which indicate the greater effect of the rotational properties on $^{158}$Er nucleus, while the properties of $\gamma$-soft is more obvious for $^{160}$Yb and $^{162}$Hf and the zero values of $a_2$ parameters for both nuclei support these behavior.

Using the values of BM, IVBM and IBM-1 parameters, the calculations of the energy states of GSB and NPB for $^{158}$Er, $^{160}$Yb and $^{162}$Hf nuclei were reproduced and compared with the experimental data, and these are shown in Fig. 4. The results of GSB showed that the agreement of BM and IVBM calculations are better than the IBM-1 calculation for all nuclei under consideration, and this is because the high effect of the rotational properties on these nuclei. The BM and IVBM calculations of NPB for $^{158}$Er, $^{160}$Yb and $^{162}$Hf nuclei are not exactly match the experimental data of this band.
Table 2: The BM, IVBM and IBM-1 parameters of GSB in MeV except $N_b$ and CHQ for $^{158}$Er, $^{160}$Yb and $^{162}$Hf nuclei.

| Nuc. | $N_b$ | BM $A \times 10^{-2}$ | BM $B \times 10^{-5}$ | BM $C \times 10^{-8}$ | IVBM $\beta \times 10^{-3}$ | IVBM $\gamma \times 10^{-5}$ | IBM-1 PAI | IBM-1 ELL | IBM-1 QQ | IBM-1 OCT | IBM-1 CHQ |
|------|-------|-----------------------|-----------------------|-----------------------|-------------------------|-------------------------|-----------|-------------|-----------|-----------|----------|
| $^{158}$Er | 11 | 2.5919 | 7.5439 | 11.689 | 8.4704 | 10.706 | 0.0084 | 0.0565 | -0.0199 | 0.0000 | -2.958 |
| $^{160}$Yb | 10 | 3.0757 | 9.8589 | 17.825 | 9.9323 | 12.458 | 0.0490 | 0.0141 | 0.0000 | 0.0300 | 0.0000 |
| $^{162}$Hf | 9 | 3.6107 | 13.212 | 19.634 | 4.9278 | 19.217 | 0.0493 | 0.0194 | 0.0000 | 0.0324 | 0.0000 |

Table 3: The BM and IVBM parameters of NPB in MeV for $^{158}$Er, $^{160}$Yb and $^{162}$Hf Isotones.

| Nucleus | IVBM $\zeta$ | IVBM $\eta \times 10^{-2}$ | BM $E_o$ | BM $A \times 10^{-3}$ | BM $B \times 10^{-6}$ | BM $C \times 10^{-9}$ |
|---------|-------------|-----------------------|-------|-----------------------|-----------------------|-----------------------|
| $^{158}$Er | 1.853 | -14.548 | 1.853 | 8.4088 | 0.37009 | -0.091923 |
| $^{160}$Yb | 1.9267 | -15.847 | 1.9267 | 7.3098 | -5.8212 | -5.5764 |
| $^{162}$Hf | 1.6493 | -2.2654 | 1.6493 | 8.2645 | -4.7522 | -5.0294 |

Fig. 4. (Color online) Comparison the BM, IVBM and IBM-1 energy levels calculations with the available experimental data [41-44] in GSB and NPB for (a) $^{158}$Er, (b) $^{160}$Yb and (c) $^{162}$Hf Isotones.
A very sensitive test for our interpretation of the triplet of isotones states is provided by the B (E2) values. The E2 transition operator has the form \([6, 33, 45]\):

\[
T_{E2} = \alpha_2 [d^\dagger s + s^\dagger d]^{(2)} + \beta_2 [d^\dagger d]^{(2)} = e_B \hat{Q}
\]

(18)

where \((s^\dagger, d^\dagger)\) and \((s, d)\) are creation and annihilation operators for \(s\) and \(d\) bosons, respectively. \(\alpha_2\) and \(\beta_2\) are two parameters \((\beta_2 = \chi \alpha_2, \alpha_2 = e_B\) (effective charge of boson)).

The reduced transition probability for the SU(3) and O(6) limits are given by \([33, 46]\):

\[
\begin{align*}
SU(3) & \quad B(E2; L \rightarrow L - 2) = e_B^2 \frac{3(L+2)(L+1)}{4(2L+3)(2L+5)} (2N - L)(2N + L + 3) \\
O(6) & \quad B(E2; L \rightarrow L - 2) = e_B^2(N - \tau)(N + \tau + 4) \frac{\tau+1}{2\tau+5}
\end{align*}
\]

(19)  (20)

where \(L\) is the angular momentum. The values of the effective charge \((e_B)\) were determined directly from using of the above equations and shown in Table 4. These values are used to calculate the reduced transition probabilities \(B(E2; L \rightarrow L - 2)\) and presented in Table 5 which shown the \(B(E2; L \rightarrow L - 2)\) decreases linearly with decreasing \(N\) in experiment with our calculation. In Table 5 shows, the absolute E2 transition rates in the GSB for 158Er, 160Yb and 162Hf with neutron number \(N= 90\) and the B(E2) values for 2-0, 4-2, 6-4, 8-6 and 10-8 are predicted fairly well between the B(E2) calculated with experimentally in 158Er, 160Yb nuclei, except 162Hf isotope because there's no sufficient experimental data \([41-44]\).

Table 4. Parameters (in eb) used to reproduce B(E2) values for \(^{158}\text{Er}, ^{160}\text{Yb}\) and \(^{162}\text{Hf}\) nuclei.

| Nuclei | \(N_0\) | \(e_B\) |
|--------|--------|--------|
| \(^{158}\text{Er}\) | 11     | 0.1091 |
| \(^{160}\text{Yb}\) | 10     | 0.1183 |
| \(^{162}\text{Hf}\) | 9      | 0.1074 |

Table 5: The IBM-1 and Experimental \([41-44]\) values of B(E2) (in e\(^2\) b\(^2\)) of some states in the GSB for \(^{158}\text{Er}, ^{160}\text{Yb}\) and \(^{162}\text{Hf}\) nuclei.

| Nuclei | \(2^+_1 \rightarrow 0^+_1\) | \(4^+_1 \rightarrow 2^+_1\) | \(6^+_1 \rightarrow 4^+_1\) | \(8^+_1 \rightarrow 6^+_1\) | \(10^+_1 \rightarrow 8^+_1\) |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \(^{158}\text{Er}\) | Exp. 0.6481 | 1.0660 | -- | 1.4213 | 1.2691 |
|       | Cal. 0.6502  | 0.9120 | 0.9711 | 0.9661 | 0.9244 |
| \(^{160}\text{Yb}\) | Exp. 0.4801 | 0.6659 | 0.7691 | 0.7743 | 0.6195 |
|       | Cal. 0.4798  | 0.6509 | 0.7311 | 0.7415 | 0.7118 |
| \(^{162}\text{Hf}\) | Exp. 0.2676 | --     | --   | --   | --   |
|       | Cal. 0.2674  | 0.3657 | 0.4000 | 0.3989 | 0.3736 |

In Figure 5, the calculated potential energy surfaces, PES are plotted and show, that \(^{158}\text{Er}\) nucleus have the shape phase transition from rotational SU(3) to \(\gamma\)-unstable symmetry O(6) \((\frac{\pi}{3} \geq \gamma \geq \frac{\pi}{6})\). The \(^{160}\text{Yb}\) and \(^{162}\text{Hf}\) nuclei are deformed and have \(\gamma\)-unstable-like characters \((\gamma \approx \frac{\pi}{6})\).
Conclusions

The distribution of protons and neutrons on different shells or subshells and the values of the low-lying energy states of $^{158}\text{Er}$, $^{160}\text{Yb}$ and $^{162}\text{Hf}$ nuclei do not give exact opinion of the properties of these nuclei along the yrast states. The back-bending on the photon energy for these isotones occur at a certain state, give an evidence of the phase change. The transitional properties the SU(3)-O(6) and decreases sharply to the O(6) limit, for $^{158}\text{Er}$, $^{160}\text{Yb}$ and $^{162}\text{Hf}$ nuclei are obvious from the E-GOS curves. The staggering of GSB and NPB for $^{158}\text{Er}$ and $^{160}\text{Yb}$ but not for $^{162}\text{Hf}$ nuclei. The BM and IVBM calculations for the GSB are better than IBM-1 for all nuclei under consideration. The calculations of NPB for $^{158}\text{Er}$, $^{160}\text{Yb}$ and $^{162}\text{Hf}$ nuclei are not axactly match the experimental data. Calculated the reduced transition probabilities B(E2) values by using Interacting Boson Model (IBM) and obtained a good agreement with published experimental data for all isotopes under study, except $^{162}\text{Hf}$ isotope because there's no sufficient experimental data. The potential energy surfaces for isotopes show, that $^{158}\text{Er}$ nucleus has the shape phase transition from rotational SU(3) to $\gamma$-unstable symmetry O(6), while $^{160}\text{Yb}$ and $^{162}\text{Hf}$ nuclei are deformed and have $\gamma$-unstable-like characters ($\gamma \approx \frac{\pi}{6}$), the change from SU(3)-O(6) to described as an evolution from the $\beta$-soft deformed or SU(3) to the $\gamma$-soft or O(6) ($\frac{5\pi}{3} \geq \gamma \geq \frac{\pi}{6}$).

Acknowledgments

We thank University of Mosul, College of Education for Pure Science, Department of Physics and University of Kerbala, College of Science, Department of Physics for supporting this work.
References

[1] F. I. Sharrad, I. Hossain, I. M. Ahmed, H. Y. Abdullah, S. T. Ahmad and A. S. Ahmed, Braz J Phys 45, 340 (2015).
[2] F. I. Sharrad, H. Y. Abdullah, N. AL-Dahan, N. M. Umran, A. A. Okhunov and H. Abu-Kassim, Chinese Physics C 37, 034101 (2013).
[3] I. Hossain, I. M. Ahmed, F. I. Sharrad, H. Y. Abdullah, A. D. Salman and N. AL-Dahan, Chiang Mai J. Sci. 42 996 (2015).
[4] A. Shelley, I. Hossain, Fadhil I. Sharrad, Hewa Y Abdullah and M A Saeed, Prob. Atom. Sci. & Tech. 64 38 (2015).
[5] I. Hossain, H. H. Kassim, F. I. Sharrad and A. S. Ahmed, ScienceAsia 42, 22 (2016).
[6] M. A. Al-Jubbori, H. H. Kassim, F. I. Sharrad and I. Hossain, Nucl. Phys. A 955, 101 (2016).
[7] H. H. Khudher, A. K. Hasan and F. I. Sharrad, Ukrainian Journal of Physics 62, 152 (2017).
[8] M. O. Waheed and F. I. Sharrad, Ukrainian Journal of Physics 62, 757 (2017).
[9] R. F. Casten and D. D. Warner, Rev. Mod. Phys. 60, 389 (1988).
[10] I. M. Ahmed, G. N. Flaiyh, H. H. Kassim, H. Y. Abdullah, I. Hossain and F. I. Sharrad, Eur. Phys. J. Plus 132, 84 (2017).
[11] H. H. Kassim and F. I. Sharrad, Nucl. Phys. A 933, 1 (2015).
[12] M. A. Al-Jubbori, F. Sh. Radhi, A. A. Ibrahim, S. A. Abdullah Albakri, H. H. Kassim and F. I. Sharrad, Nuclear Physics A 971, 35 (2018).
[13] O. Scholten, Computer code PHINT, KVI; Groningen, Holland, (1980).
[14] M. A. Al-Jubbori, H. H. Kassim, F. I. Sharrad, A. Attarzadeh and I. Hossain, Nuclear Physics A 970, 438 (2018).
[15] A. Okhunov, F. I. Sharrad, A. A. Al-Sammarea and M. U. Khandaker, Chinese Physics C 39, 084101 (2015).
[16] H. H. Kassim, A. A. Mohammed-Ali, M. Abed Al-Jubburi, F. I. Sharrad, A.S. Ahmed and I. Hossain, J. Natn. Sci. Foundation Sri Lanka 46, 3 (2018).
[17] M. Abed Al-Jubburi, K. A. Al-Mtiuty, K. I. Saeed and F. I. Sharrad, Chinese Physics C 41, 084103 (2017).
[18] I. Hossain, F. I. Sharrad, M. A. Saeed, H. Y. Abdullah and S. A. Mansour, Maejo Int. J. Sci. Technol. 10, 95 (2016).
[19] H. H. Kassim and F. I. Sharrad, International Journal of Modern Physics E 23, (2014) 1450070.
[20] F. Iachello and A Arima The Interacting Boson Model (Cambridge: Cambridge University Press) (1987).
[21] M A Al-Jubbori, H H Kassim, A A Abd-Aljbar, H Y Abdullah, I Hossain, I M Ahmed and Fadhil I Sharrad, Indian Journal of Physics, 94(2020) 379-390
[22] Wisam N. Hussain, I. Hossain and Fadhil I. Sharrad, Journal of Physics: Conf. Series, 1279 (2019) 012027
[23] Mariam O. Waheed and Fadhil I. Sharrad, Journal of Physics: Conf. Series, 1279 (2019) 012077
[24] Mahdi A. Mahdi, Fahmi Sh. Radhi, Huda H. Kassim, Mushtaq AbedAl- Jubbori and Fadhil I. Sharrad, Journal of Physics: Conf. Series, 1279 (2019) 012021
[25] Kahtan A. Hussain, Musa K. Mohsin and Fadhil I. Sharrad, Journal of Physics: Conf. Series, 1279 (2019) 012021
[26] Mushtaq Abed Al-Jubbori, Huda H. Kassim, Fahmi Sh. Radhi, Amin Attarzadeh, I. Hossain, Imad M. Ahmed, and Fadhil I. Sharrad, Physics of Atomic Nuclei, 82 (2019) 201–211

[27] Imad Mamdouh Ahmed, Hewa Y. Abdullah, Mudhafer Mustafa Ameen, Huda H. Kassim and Fadhil I. Sharrad, Physics of Atomic Nuclei, 81 (2018) 695–702

[28] Wisam N. Hussain and Fadhil I. Sharrad, Journal of Physics: Conference Series, 1032 (2018) 012046

[29] Imad Mamdouh Ahmed, Mushtaq Abed Al-Jubbori, Huda H. Kassim, Hewa Y. Abdullah and Fadhil I. Sharrad, Nuclear Physics A, 977 (2018) 34-48

[30] Mushtaq Abed Al-Jubbori, Huda H. Kassim, Fadhil I. Sharrad and I. Hossain, International Journal of Modern Physics E, 27 (2018) 1850035.

[31] Huda H. Kassim, Amir A. Mohammed-Ali, Fadhil I. Sharrad, I. Hossain and Khalid S. Jassim, Iranian Journal of Science and Technology, Transactions A: Science, 42 (2018) 993-999.

[32] Fadhil I. Sharrad, H.Y. Abdullah, N. Al-Dahan, A. A. Mohammed-Ali, A. A. Okhunov and H. Abu Kassim, Romanian Journal of Physics, 57, (2012) 1346-1355

[33] R. F. Casten and D. D. Warner, Rev. Mod. Phys. 60, (1988) 389.

[34] D. Bonatsos, Phys. Lett. B 200, 1(1988).

[35] A. Georgieva, P. Raychev and R. Roussev, J. Phys. G: Nucl. Phys. 8, 1377(1982).

[36] A. Georgieva, P. Raychev and R. Roussev, J. Phys. G: Nucl. Phys. 9, 521(1983).

[37] A. Georgieva, P. Raychev and R. Roussev, Bulg. J. Phys. 12, 147(1985).

[38] P. Raychev, Rev. Roum. Phys. 32, 471(1987).

[39] D. Bonatsos, et al., Phys. Rev. C,62, (2000) 024301.

[40] M. A. Mahdi , F. Sh. Radhi , http://www.nndc.bnl.gov/chart/getENSDFDatasets.jsp.

[42] R. G. Helmer, Nuclear Data Sheets 101 (2004) 325.

[43] C. W. Reich, Nuclear Data Sheets 105 (2005) 557.

[44] R. G. Helmer and C. W. Reich, Nuclear Data Sheets 87, 317 (1999).

[45] A. Arima and F. Iachello, Ann. Phys. (N.Y.)111, (1978).

[46] D. Bonatsos, C. Daskaloyannis, S. B. Drenska, N. Karoussos, N. Minkov, P. P. Raychev and R. P. Roussev, Phys. Rev. C,62, (2000) 024301.