A Nonlinear Finite Element Approach to the Economical Design of R.C. Slabs

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Abstract—RC solid slabs form a significant part of the RC structure, which the optimization of its design can lead to substantial savings in the cost of construction. A nonlinear finite element analysis tool can accurately predict the behavior of slabs during the design stage. The high reliability of slab behavior prediction allows the re-evaluation of the provisions of the current design codes, especially regarding the steel detailing in slabs, which leads to substantial savings in the slab construction cost. In this study, the verification of the optimal value of the span / depth ratio of one-way slabs according to the Egyptian Code of practice (ECP-203) is done using nonlinear finite program ANSYS (14.5). This paper presents an implementation of a three-dimensional nonlinear finite element model for a total of sixty-four finite element slab to study the effect of percentage of reinforcement ratio (µ %), the ratio of span length to the slab thickness (L/ts), the bar diameter (Ø) and (spacing) on the behavior of the one way slabs, in additional to the curtailment of steel reinforcement and 50% reduction ratio of reinforcement besides supports on the behavior of one and two way slabs. From the finite element analysis, curtailment reinforcement in the main direction of simply supported one way slab and 50% reduction ratio of reinforcement besides supports at a distance quarter of the span in the secondary direction lead to significant savings in reinforcement up to 14% of total reinforcement value. Curtailment reinforcement in the two directions of the simply supported two way slab at a distance half of the span at maximum tension zone and 50% reduction ratio of reinforcement besides supports at distance quarter of the span lead to significant savings in reinforcement up to 30% of total reinforcement value.

Keywords: Reinforced concrete, Solid slabs, Non-linear, Finite element, Analysis, Economical design.

I. INTRODUCTION

RC slabs constitute an important part of the reinforced concrete (RC) buildings. Normally, the thickness of a slab in a RC structure is very small compared to its own length and width. Slabs are usually supported on beams or columns or bearing walls, to carry the distributed loads primarily by flexure, which are either simple or continuous panels. Solid slabs are classified into two types; one-way slabs if the long to short span ratio is greater than 2.0 (i.e. L/B > 2) and two-way slabs if this ratio become equal or less than 2 (i.e. L/B ≤ 2). Loads on the one way slabs are transferred to the long pair of beams and, the load path is along the short direction of the slab panel, where in a two-way slab, both directions participate in carrying the load as in [1]. The finite element programs became now the bases to solve design and research problems. It can accurately predict the behavior of slabs during the design stage. Sami Mahmoud [2] investigated the validity of the assumption in the yield line theory under concentrated load at the center of two way slab. Experimental specimens tested and analytical analysis were carried on simply and fixed supported two–way slabs with different distribution steel bars in the secondary direction to compare the results with the assumption in the yield line theory. Akinyele J. et. al., [3], investigated the validity of the value of flexural behavior based on different theories and equations, and comparing the difference that may occur between theoretical and experimental results, for the waffle and solid slab structures. Roshini T. et. al., [4], used ANSYS finite element program to simulate the behavior of two way reinforced concrete slab with and without openings for different slab length ratios and different opening ratios with different the boundary conditions (simply and fixed supported). Abdul Ridah S. et. al., [5], studied the effect of strengthening by using CFRP sheets of concrete two-way solid slabs with and without openings in the middle strip under uniformly distributed load with simply supports on the four edges by using high strength concrete. Rouzbeh Khajehdehi [6], used ANSYS (13) finite element program to simulate the behavior of two-way RC solid slabs with and without opening subjected to in plane and out of plane loads. Mustafa Basheer [7] used ANSYS finite element program to analyze one way RC slabs with and without external CFRP strengthening. Ahmed Abdel-Fttah [8] used ANSYS finite element program to simulate the effect of the slab thickness and reinforcement on the punching shear resistance. Most of the above cited research work focus on the behavior of slabs under strengthening, creating openings, etc., not on the design optimization. This study considers one and two way slabs of RC buildings supported by beams and subjected to uniformly distributed loads to study the effect of the two main factors, span to depth ratio and the variation of reinforcement ratio at
different places. The slabs must be deep and reinforcement enough to prevent shear, bending failure and excrescent deflections. The main parameters in this study are the effect of percentage of reinforcement ratio ($\mu\%$), the ratio of span length to the slab thickness ($L/ts$), the bar diameter ($Ø$) and (spacing) on the behavior of the one way slabs, in additional to the curtailment of steel reinforcement and 50% reduction ratio of reinforcement besides supports on the behavior of one and two way slabs. In this study the approaches for designing RC slabs are to make optimization in the design using linear analysis methods and apply the rules of design codes to ensure the satisfactory performance and strength of the slabs, this is leading to substantial savings in the cost of construction. Also, it will be studied the effect of minimum reinforcement ratio and reinforcement details required for different spans on the behavior of RC solid slabs to achieve the safe serviceability limit and ductile failure at the lowest possible costs. In this work, ANSYS parametric design language (APDL) program is used to simulate the behavior of RC slabs by considering constitutive models which are suggested by the theoretical study depend on ECP (203-2007) [9]. Deflection obtained from theoretical results of ECP (203-2007) [9] for one-way and experimental results for two-way slabs are compared with the theoretical results obtained from finite element program by ANSYS.

II. FINITE ELEMENT MODELLING.

The finite element procedures are important and recognized as a general method of wide applicability for engineering and physical scientific problems. It is a numerical method of analysis that can deal with problems of various boundary conditions and loadings. This method has been developed to a high degree for practical computations so that it found wide appeal in engineering practice. The RC slabs with tensile reinforcement have been analyzed using a finite element (FE) model in ANSYS. Here, a nonlinear analysis is considered throughout the study by assuming that there is a perfect bonding between concrete and steel reinforcement.

A. Modeling of Concrete

Element geometric modeling of concrete has been done using solid element (Solid 65) which has eight nodes with three degrees of freedom at each node and translations in the nodal x, y, and z directions. This element is capable of plastic deformation, cracking in three orthogonal directions at each integration point in tension and crushing in compression. The geometry and node locations for this element type are shown in the Fig. (1).

![Solid 65 elements, ANSYS (SAS 2012) [10]](image)

B. Modeling of Steel Reinforcement

Reinforcement modeling could be discrete or smeared or embedded [11], [12]. In our work, a discrete modeling of reinforcement has been done. The reinforcement has been modeled using link 8 elements in ANSYS. It is a 3-D spar that is useful in a variety of engineering applications. This element can be used to model trusses, sagging cables, links, springs, and so on. It is a uniaxial tension-compression element with three degrees of freedom at each node: translations in the nodal x, y, and z directions. Tension - only (cable) and compression-only (gap) options are supported. As in a pin- jointed structure, no bending of the element is considered. Plasticity, creep, rotation, large deflection, and large strain capabilities are included as shown in Fig. (2).

![Link180 element, ANSYS (SAS 2012) [10]](image)
C. Modeling of Steel Plates

SOLID 185 element is used to model the Steel Plates, since it has eight nodes with three degrees of freedom at each node and translations in the nodal x, y, and z directions. The geometry and node locations for this element type are shown in the Fig. (3).

![SOLID 185 element diagram](image)

**Fig. (3): Solid 185 elements, ANSYS (SAS 2012) [10]**

D. Material Properties

Equation (1), which was suggested by MacGregor, is used to represent the uniaxial compressive stress-strain relationship for concrete.

\[
\frac{f}{\varepsilon} = \frac{E_o \varepsilon}{1 + \left(\frac{\varepsilon}{E_o}\right)^2}
\]  

(1)

Where: 
- \(f\) = stress at any strain \(\varepsilon\)
- \(\varepsilon_o\) = strain at the ultimate compressive strength, \(f'_c\) (\(\varepsilon_o = 2 f'_c/E_o\))
- \(f'_c\) = ultimate compressive strength for concrete

This equation was used to plot the multi-linear isotropic stress-strain curve for concrete from 0.3\(f'_c\) until ultimate compressive strength, \(f'_c\), Fig. (4-a). Point 1 is defined as 0.30 \(f'_c\), and calculated in the linear range. The stresses strain curves of steel bars were presented in Fig. (4-b).

![Stress-strain curve diagram](image)

**Fig. (4): the stress strain curves used in ANSYS program [9]**
E. Modeling Methodology

All slabs were modeled in ANSYS taking the advantage of the symmetry across the center of the slabs. This plane of symmetry was represented using relevant constraints in the finite element node points. This approach reduced computational time and resources significantly. The meshing of the typical one and two-way slabs are shown in Fig. (5-a) and (5-b) respectively. Fig. (6-a) and Fig. (6-b) show the load and boundary conditions of the typical for one and two-way slabs respectively.

III. EVALUATION OF THE ECP 203-2007 LIMITATION ON (L/t) FOR ONE-WAY SLABS

Egyptian code of practice 203-2007 mentions that, for simply supported one-way slabs in normal structures, it is possible to calculate the thickness of slab in the case of deflection negligence by assuming that the slab thickness bigger than or equal to span/25, i.e. \(t_s \geq \text{span}/25\). Different codes like ACI 318-11, [13], UBC1997 [14], CSA (A23.3-04) [15], and ECP 203-2000 [16] take this ratio \(t_s \geq \text{span}/20\), while BS 8110-1:1997 [17], EN 1992-1-1:2004 [18], IS 456: 2000 [19], mention this ratio by \(d \geq \text{span}/20\). Analytical analysis was done on one-way slabs with different spans from 2m to 10m, thickness equal to the length of the span divided by \(25\) or \(20\) to evaluate ECP 203-2007 Limitation on \(L/t\). The concrete compressive strength and the steel yield stress of steel were 25MPa and 400MPa respectively, the live load and floor cover were 2 KN/m² and 1.5KN/m² respectively. Compression steel with ratio 20% of the used tensile steel was used in case of slab thickness more than 160mm but not less than 5Ø8/m. A comparison between the calculated and allowable deflection according to ECP 203-2007 was carried out by using two scenarios in the calculations. The first scenario, the gross moment of inertia \(I_g\) with neglecting the cross-sectional area of steel reinforcement was used in the calculated deflection where, the second one used the virtual moment \(I_v\). It is noted that the cross-sectional area of the steel reinforcement can be neglected in the deflection calculations; this may be causes an error ratio does not exceed than about 15% as shown in Fig. (7). The calculated deflection for the studied one-way slabs with thickness \(t_s \geq \text{span}/25\) according to ECP 203-2007 is not satisfied the allowable deflection requirement of the code for the...
span length more than 5.5m, while for the other studied slabs with thickness ($t_s \geq \frac{span}{20}$) the calculated deflection satisfied the allowable deflection requirement up to span length 10m as shown in Fig. (7), (a & b).

![Graph showing relationship between span length and deflections](image1)

**Fig. (7):** Relationship between the span length and deflections ($\delta$)

The span length to depth ratio in the most of the mentioned codes depends on the residential buildings loads. The Relation between the different slab span lengths and the calculated / allowable deflection ratio with different load values for the studied slabs is shown in Fig. (8). (Noted that slab thickness equal to ($L/20$) - and the virtual moment $I_v$ was used in the calculations)

![Graph showing relationship between slab span length and calculated / allowable deflection ratio](image2)

**Fig. (8):** Relationship between slab span length and calculated / allowable deflection ratio

From this analysis, it is noted that for the studied one way slabs with span to depth ratio equal to 20, the deflection calculations can be neglected for the slabs with span length less than 10m and 7m under live and floor cover loads value equal to 3.5 and 7 KN/m² respectively.

A. **Verification of the Proposed Finite Element Models.**

To verify the theoretical results from ANSYS analysis for the one way slabs, a number of preliminary finite element analysis was attempted and compared with the calculated results by ECO equations for assumed specimen. The assumed specimen is a simply supported one-way slab under uniformly distributed load with clear span length 4000mm and 160mm thickness. The strength of the used materials is 25 MPa and 360 MPa for concrete and steel reinforcement respectively. The area of the steel bar in the main and secondary direction is 113mm² and 48mm² respectively as shown in Fig. (9).

![Typical Details of assumed one way slab specimen](image3)

**Fig. (9):** Typical Details of assumed one way slab specimen
For verification the two way slabs in ANSYS model, the theoretical results are compared with the experimental work by Dina Hassan [20]. The average compressive strength of the experimental specimen after 28 days was 37 MPa and the yield strength of the steel bars was 246 MPa, and 365 MPa for steel bars 8 mm and 10 respectively. The details of the tested specimen are shown in Fig. (10).

Many assumptions for the value of the ANSYS coefficient like shear transfer open and closed crack were done to ensure that the elements, material properties, real constants, and convergence criteria are proportional to the assumed and experimental specimen for one and two way slab respectively. To verify a good agreement for the load deflection curves of the assumed and experimental specimen with the theoretical results by ANSYS analysis, the values of the open crack, closed crack and the tensile strength of the concrete had taken 0.2, 0.7 and 3 Mpa respectively for the analyzed one way slabs. These values had changed to 0.4, 0.8 and 3.5 Mpa respectively, for the analyzed two way slabs. The comparison between the load deflection curve of the assumed and experimental specimen with the theoretical results by ANSYS analysis for one and two way slab is shown in Fig. (11) and (12) respectively.
IV. DISCUSSION AND ANALYSIS OF THE STUDIED PARAMETERS.

In this part, Finite element analysis study by ANSYS the effect of span to depth ratio (L/ts), bar diameter (spacing) (Ø), curtailment of steel reinforcement (RFT) and decreasing the reinforcement (RFT) ratio besides the supports on the behavior of one and two-way slabs. The discussion of the results includes load-deflection curves, un-cracked stiffness (the slope of the load-deflection curves before cracking), cracked stiffness (the slope of the load-deflection curves after the cracking load), and absorbed energy (the area under load-deflection curves) values for one and two-way slabs respectively.

A. Effect of the span to depth ratio (L/ts) on the behavior of one-way slabs.

Three different slab thickness 80mm, 120mm, and 160mm with (L/ts) =37.5, 25 and 18.7 respectively, and constant clear span length 3000mm were analyzed by ANSYS to study the effect of the span to depth ratio (L/ts) on the behavior of one-way slab. Different steel reinforcement ratios 0.20%, 0.30%, 0.40%, 0.50%, 0.75%, 1.00% and 1.25% were used for each slab. For all different steel reinforcement ratios, it is noted that (L/ts) ratio is inversely and directly proportion with failure load and deflection respectively. As the (L/ts) ratio decreases from 37.5 to 25 the failure load increase by about 165% and the deflection decreased by about 42% as average values for the different reinforcement ratios. If the (L/ts) ratio decreases from 25 to 18.75 the failure load increased by about 96% and the deflection decreased by about 15% as average values at the same reinforcement ratio as shown in Fig. (13).
The un-cracked stiffness, cracked stiffness, and absorbed energy values of the studied slabs are shown in the Figs. (14-a), (14-b) and (14-c) respectively. It is noted that, the un-cracked stiffness, increased by average ratio about 246%, 141 when the (L/ts) ratio decreased from 37.5 to 25 and 25 to 18.75 respectively. While the cracked stiffness, increased by average ratio about 400% and 180% when the (L/ts) ratio decreased from 37.5 to 25 and 25 to 18.75 respectively. The absorbed energy increased by about 36% and 55% in the case of the studied slabs with (L/ts) ratio decrease from 37.5 to 25 and 25 to 18.75 respectively.
B. Effect of bar diameter and spacing (Ø & S) with a constant reinforcement ratio on the behavior of one-way slabs

Twelve models of one way slabs with constant 140mm thickness, 3300mm span length and different reinforcement ratios (from 0.75 % to 1.5 %) were analyzed by ANSYS in this study. For the using of different steel bar arrangement (5, 8 and 10/m) with constant reinforcement ratio, it is found that the steel bar arrangement is insignificant when the reinforcement ratio value not exceed than 1.1%, while it has a significant effect when the reinforcement ratio value increased this ratio as shown in Fig. (15). It is noted that the maximum loads capacity for the studied slab decreased with decreasing the total number of steel bar per meter that is may be due to the steel bar diameter in this case bigger than slab thickness divided to eight (i.e. Ø > ts/ 8) as recommended in the Indian code.
a) Un-cracked stiffness  
b) Cracked stiffness  
c) Absorbed energy

Fig. (16): Effect of bar diameter and spacing (Ø & S) with a constant reinforcement ratio on the behavior of one-way slabs
C. Effect of the Curtailment and Decreasing of Steel Reinforcement (RFT) on the Behavior of One-Way Slabs

Four slabs with constant 140mm thickness and 3300mm span length, control model (COS), model with a 50% reduction only in the secondary reinforcement ratio at the out quarters of the secondary span (SWR), model with the curtailment of the main steel only (SWC) and model with curtailment of the main steel and 50% reduction in the secondary reinforcement ratio at the out quarters of the secondary span (SWCR) as shown in Figs. (17-a), (17-b), (17-c) and (17-d) respectively, were analyzed by ANSYS to study the effect of the curtailment and reinforcement reduction on the behavior of the studied one-way slabs.

Note that.

COS : Control one way slab model.
SWR : model with a 50% reduction only in the secondary reinforcement ratio at the out quarters of the secondary span
SWC : Model with curtailment of the main steel only
SWCR: Model with curtailment in the main steel and 50% reduction in the secondary reinforcement ratio at the out quarters of the secondary span

The theoretical un-cracked and cracked stiffness are listed in Table I. From the analysis results, it is noted that the curtailment of the main steel and the 50% reduction in the secondary reinforcement seem to be insignificant and show a slight change in, the un-cracked and crack stiffness, and the absorbed energy values of the studied one way slabs as shown in Figs. (18-a), (18-b) and (18-c) respectively.

Fig. (17): Typical details of models for one way slabs
TABLE 1. Study Results for One-Way Slabs

| Slab No. | Slab Name | $P_{cr}$ kN/m² | $P_{max}$ kN/m² | $\delta_{cr}$ mm | $\delta_{max.}$ mm | $P_{cr} / P_{max}$ | $\text{Un-cracked. Stiff. } P_{cr} / \delta_{cr}$ | $\text{Cracked. Stiff.}$ |
|----------|-----------|----------------|-----------------|-----------------|-----------------|------------------|---------------------------------|-----------------|
| 1        | COS       | 7.875          | 14.44           | 2.28            | 23.00           | 0.545            | 3.454                           | 0.439           |
| 2        | SWR       | 7.875          | 14.375          | 2.28            | 22.60           | 0.548            | 3.543                           | 0.443           |
| 3        | SWC       | 7.875          | 14.375          | 2.28            | 22.57           | 0.548            | 3.542                           | 0.441           |
| 4        | SWCR      | 7.875          | 14.375          | 2.28            | 22.51           | 0.548            | 3.542                           | 0.437           |

Fig. (18): Effect of Curtailment and Decreasing of Steel Reinforcement (RFT) on the Behavior of One-Way Slabs

From the previous analysis of the studied one way slabs, it’s found that, the 50 % reduction in the secondary reinforcement only, the curtailment of the main steel reinforcement only and the 50 % reduction in the secondary reinforcement in additional to the curtailment of the main steel reinforcement caused a reduction values in the steel reinforcement ratios by about 4.2%, 10.4% and 14.6% respectively as indicated in Table II.
TABLE II. Economical Ratio for One-Way Slabs

| Slab No. | Slab Name | μ% | Steel RFT (kg/m²) | Economical ratio % |
|----------|-----------|----|-----------------|-------------------|
|          |           |    | Main           | Sec.       | total |
| 1        | COS       | 0.40 | 3.768          | 0.7536     | 0     | 0     | 0     |
| 2        | SWR       |     | 3.768          | 0.5652     | 0     | 25    | 4.2   |
| 3        | SWC       |     | 3.297          | 0.7536     | 12.5  | 0     | 10.4  |
| 4        | SWCR      |     | 3.297          | 0.5652     | 12.5  | 25    | 14.6  |

D. Effect of Curtailment and Decreasing of Steel Reinforcement (RFT) on the Behavior of Two-Way Slabs

Seven slabs with constant 100mm thickness and 3300mm span length as indicated in table (3) were analyzed to study the effect of the curtailment and the reduction of steel reinforcement (RFT) on the behavior of the studied by ANSYS two-way slabs as shown in Figs. (19-a to 19-g) respectively. It is noted that the curtailment and the steel reinforcement reduction have insignificant effect on the behavior of two way slabs as shown in Table III.

TABLE III. Study Results for Two-Way Slabs

| Slab No | Slab Name | Pcr, kN/m² | Pmax, kN/m² | δcr, mm | δmax, mm | Pcr/Pmax | Un-cr. Stiff. | Cr. Stiff. |
|---------|-----------|------------|-------------|---------|---------|----------|-------------|-----------|
| 1       | CTS       | 7.875      | 12.125      | 2.113   | 9.3     | 0.649    | 3.727      | 2.229     |
| 2       | SWR1      | 7.875      | 12.050      | 2.114   | 13.33   | 0.654    | 3.725      | 2.264     |
| 3       | SWR2      | 7.875      | 12.125      | 2.115   | 13.78   | 0.649    | 3.723      | 2.220     |
| 4       | SWC1      | 7.875      | 11.805      | 2.114   | 8.76    | 0.667    | 3.725      | 2.226     |
| 5       | SWC2      | 7.875      | 12.125      | 2.113   | 12.99   | 0.649    | 3.727      | 2.267     |
| 6       | SWCR1     | 7.875      | 12.050      | 2.114   | 13.19   | 0.654    | 3.726      | 2.223     |
| 7       | SWCR2     | 7.875      | 11.850      | 2.114   | 9.42    | 0.665    | 3.725      | 2.218     |

Note that:
CTS : Control two way slab model.
SWR1: Model with a 50% reduction in the steel reinforcement in quarters of one direction
SWR2: Model with a 50% reduction in the steel reinforcement in quarters of two directions
SWC1: Model with curtailment in one direction
SWC2: Model with curtailment in two directions
SWCR1: Model with curtailment and 50% reduction in the steel reinforcement in one direction
SWCR2: Model with curtailment and 50% reduction in the steel reinforcement in two directions
From the analysis of the studied two way slabs, it’s found that, the variations of the curtailment of steel bars and steel reinforcement reduction ratio have a significant effect on the building costs. The studied case of model with curtailment and 50% reduction in the steel reinforcement in the two directions has the highest reduction in the steel reinforcement ratios by about 31.25% without any effect on the behavior of the studied slabs as indicated in Table. IV.

### TABLE IV. Economical Ratio for Two-Way Slabs

| Slab No | Slab Name | Steel RFT (kg/m²) | Economical ratio % |
|---------|-----------|-------------------|--------------------|
|         |           | main | Sec. | main | Sec. | Total |
| 1       | CTS       | 1.57 | 1.57 | 0    | 0    | 0     |
| 2       | SWR1      | 1.57 | 1.1775 | 0 | 25 | 12.5 |
| 3       | SWR2      | 1.1775 | 1.1775 | 25 | 25 | 25 |
| 4       | SWC1      | 1.57 | 1.37375 | 0 | 12.5 | 6.25 |
| 5       | SWC2      | 1.37375 | 1.37375 | 12.5 | 12.5 | 12.5 |
| 6       | SWCR1     | 1.57 | 1.09735 | 0 | 31.25 | 16.6 |
| 7       | SWCR2     | 1.09735 | 1.09735 | 31.25 | 31.25 | 31.25 |
V. CONCLUSIONS

Several studied cases of the one and two way slabs under uniformly distributed loads were analyzed in this paper to study the effect of the two main factors, span to depth ratio and the variation of reinforcement ratio at different places. From the previous theoretical analysis and the results, it can be recommended that:

1. The minimum thickness for the analyzed simply supported one-way slab is equal to the slab span length divided by 20, i.e. \( L/20 \) for span length 10m and 7m under total loads equal to 3.5 and 7 KN/m² respectively without checking of the deflection.

2. For the studied one-way slabs, about 14% of the slab total reinforcement without any changes in the slabs behavior can be saved by the 50 % reduction in the secondary reinforcement at the out quarters of the secondary span in additional to the curtailment of the main steel.

3. For the studied two-way slabs, about 30% of the slab total reinforcement without any changes in the slabs behavior can be saved by the 50 % reduction in the two reinforcement directions at the out quarters of the spans in additional to 50% curtailment of the steel bars in the two reinforcement directions.

4. The bar arrangement is insignificant when the reinforcement ratio value not exceed than 1.1%, while it has a significant effect when the reinforcement ratio value increased over this ratio, especially for the studied slabs with steel bar diameter bigger than the slab thickness divided by eight (i.e. \( \Phi > ts/8 \)).

5. The un-cracked stiffness, increased by an average ratio about 246%, 141 when the \( (L/ts) \) ratio decreased from 37.5 to 25 and 25 to 18.75 respectively, while the cracked stiffness, increased by an average ratio about 400% and 180%.

6. The absorbed energy increased by about 36% and 55% in the case of the studied slabs when the \( (L/ts) \) ratio decreased from 37.5 to 25 and 25 to 18.75 respectively.

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