Directed and Elliptic Flow in $^{112}\text{Sn} + ^{112}\text{Sn}$ Collisions below 100 MeV/nucleon

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The directed and elliptic flow in collisions of $^{112}\text{Sn} + ^{112}\text{Sn}$ at energies from 35 to 90 MeV/nucleon were studied in an isospin-dependent quantum molecule dynamics model (IQMD). With increasing incident energy, the directed flow rises from the negative flow to the positive flow. Its magnitude depends on the nuclear equation of state (EOS). However, the elliptic flow shows decrease with increasing incident energy and its magnitude is not very sensible to EOS. Systematic studies of the impact parameter dependence and cluster mass dependence were also performed. The study of directed flow at intermediate energies thus provides a means to extract the information on the nuclear equation of state.

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Heavy ion collisions (HIC) provide a possibility to study the properties of nuclear matter in conditions vastly different from that in normal nuclei, such as high density and excitation as well as large difference in the proton and neutron numbers. Such knowledge is not only of interest in itself but also useful in understanding astrophysics phenomena such as the evolution of the early universe. One observable that has been extensively used for studying such information from heavy ion collisions is the collective flow of various particles [1]. The prediction of collective flow in heavy ion collisions by the hydrodynamics model [2] has yielded a powerful tools for the investigation of excited nuclear matter. The main goals are to determine the nuclear equation of state (EOS) and the in-medium nucleon-nucleon cross section [3, 4, 5, 6, 7].

Recently, the isospin dependence of collective flow has also become a very interesting subject of theoretical and experimental studies [8, 9]. One knows that nuclear collective flow is a kind of collective phenomenon found in intermediate and high energy HIC, and the studies of the dependence of collective flow on beam energy, mass number, and impact parameter have revealed much interesting physics about the properties and origin of collective flow. In past years, either directed flow or the elliptic flow was studied separately in some papers, but the combined researches are still very few. In this work, an endeavor will be made along this direction.

The isospin dependence of collective flow has been studied by Li et al. [10] and Zheng et al. [11] in term of an isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) model in which the initial proton and neutron densities were calculated from the nonlinear relativistic mean-field (RMF) theory while the isospin dependence enters the model by using the experimental N-N cross sections and the isospin dependent nuclear mean field. However, the IBUU model cannot describe the fragment flow physically since it is a one-body transport model and does not contain many-body correlation.

In the QMD model the nucleons are represented by Gaussian-shaped density distribution. They are initialized in a sphere with a radius $R=1.12A^{1/3}$, according to the liquid drop model. Each nucleon is supposed to occupy a volume of $h^3$, so that the phase space is uniformly filled. In order to explain some experimental results, we have improved the original version of the QMD model [12] to include explicitly isospin degrees of freedom and get an isospin-dependent QMD model (called IQMD model) which includes isospin-dependent Coulomb potential, symmetry potential, N-N cross sections, and Pauli blocking. In addition, in initialization of projectile and target nuclei, we sample neutrons and protons in phase space separately. Using the IQMD model, the directed and elliptic flow in reaction of $^{112}\text{Sn}+^{112}\text{Sn}$ system in the low-intermediate energy domain has been studied. Meanwhile, the roles of the impact parameter and the nuclear equation of state (EOS) have been also studied in this paper. One of the important advantages of the IQMD model is that it can explicitly represent the many body state of the system and thus contains correlation effects to all orders. Therefore, the IQMD model provides important information about both the collision dynamics and the fragmentation process. Even though the BUU model can describe one body observable very successfully, it fails to describe the formation of clusters. In this paper, we construct clusters in terms of the so-called coalescence model, in which particles with relative momentum smaller than $P_0$ and relative distances smaller than $R_0$ are considered to belong to one cluster. We adopted the parameter $R_0 = 2.4$ fm and $P_0 = 200$ MeV/c. In addition, in order to get rid of nonphysical clusters, only the clusters with reasonable proton number Z and neutron number N are selected. Taking the beam direction along the z-axis and the reaction plane on the x-y plane, the elliptic flow is then determined from the average difference between the square of the x and y components of particle transverse momentum [13, 14, 15, 16].

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FIG. 1: Excitation function of the directed flow with the soft or stiff EOS in different impact parameter zone.

\[ V_2 = \frac{\langle P_x^2 - P_y^2 \rangle}{\langle P_x^2 + P_y^2 \rangle} \]  

(1)

It corresponds to the second Fourier coefficient in the transverse momentum distribution and describes the eccentricity of an ellipse-like distribution. In the intermediate energy domain, \( V_2 > 0 \) indicates the in-plane enhancement of particle emission, i.e., a rotation-like behavior, while \( V_2 < 0 \) characterizes the squeeze-out effect perpendicular to the reaction plane, and \( V_2 = 0 \) means an isotropic distribution in the transverse plane. Usually, \( V_2 \) is extracted from the mid-rapidity region. The directed flow relates with the slope of in-plane transverse momentum on the mid-rapidity in the C.M. system, its flow parameter \( F \) at mid-rapidity is defined by

\[ F = d \frac{\langle P_x \rangle}{A} / dy \]  

(2)

The potential of the IQMD model used in the present study includes an asymmetry term in the nuclear mean-field potential. The nuclear mean-field potential is parameterized as

\[ U(\rho, \tau_z) = a(\rho/\rho_0) + b(\rho/\rho_0)^c + c\tau_z(\rho_p - \rho_n)/\rho_0 \]  

(3)

In the above, \( \rho_0 \) is the normal nuclear density; \( \rho, \rho_n, \) and \( \rho_p \) are nucleon, neutron, and proton densities, respectively; and \( \tau_z \) equals 1 for proton and \( \tau_z = -1 \) for neutron and \( c \) is chosen to be 32 MeV. The parameter \( a \) and \( b \) was chosen to have a stiff equation of state with a compressibility of \( K = 380 \text{ MeV} \) and a soft equation of state with a compressibility of \( K = 220 \text{ MeV} \) as done in [12]. The directed and elliptic flows in reaction \( ^{112}\text{Sn} + ^{112}\text{Sn} \) are calculated at different incident energies. For each impact parameter, a calculation of 200 events is performed. In the present calculations, it is found that the directed and elliptic flow have been saturated by the end of 120 fm/c, therefore the results actually correspond to the statistical average value of 1000 ”events” which come from the sum of five time intervals from 120 to 200 fm/c in each event.

Fig. 1 shows the directed flow for \( ^{112}\text{Sn} + ^{112}\text{Sn} \) collisions at different incident energies. Up-triangle symbols correspond to soft potential and down-triangle symbols to stiff potential. From Fig. 1, with increasing incident energy, the directed flow increases. The balance energy, i.e., the energy of disappearance of directed flow, for soft potential is larger than that of stiff potential. From Fig. 1 (a), when the impact parameter is 3.0~4.5fm, the balance energy with soft potential is near 85MeV and that with stiff potential is near 70MeV. From Fig. 1 (b), when the impact parameter is 0~1.5fm, the balance energy with soft potential is near 70 MeV and that with stiff
potential is near 60 MeV. Therefore, the balance energy depends strongly on the nuclear equation of state as well as the impact parameter. In this context, we can try to extract the information on the nuclear equation of state from the excitation function of the directed flow.

Fig. 3 shows the impact parameter dependence of the directed flow (a) and the elliptic flow (b). The four impact parameter regions 0∼1.5fm, 1.5∼3.0fm, 3.0∼4.5fm and 4.5∼6.0fm and incident energy 50MeV/nucleon are adopted in the calculation. The directed flows are all negative while the elliptic flows are all positive at different impact parameter. The directed and elliptic flows depend strongly on impact parameter. With the impact parameter increases, the directed flow increases in negative direction and reaches the maximum in semi-peripheral collision and decreases in negative direction in large impact parameter. While, the elliptic flow always increases in positive direction. These are consistent to the experimental observation and other theoretical works [18,19,20,21,22,23].

When the impact parameter is smaller, the elliptic flow is smaller, seen from Fig. 2(b), so a large impact parameter of 3.0∼4.5fm is adopted in the Fig. 3 to present the excitation function of elliptic flow. Fig. 3 shows the excitation function of elliptic flow for $^{112}\text{Sn}+^{112}\text{Sn}$ collisions.

All the values of the elliptic flow are positive and decrease with the incident energy, indicating the rotational behavior becomes weak with increasing the incident energy, but the squeeze-out never reveal below 90MeV/u even though the directed flow shows the change from the negative to positive. When the incident energy is smaller, the elliptic flow has large difference between the case of soft potential and the case of stiff potential due to the fact that nucleon-nucleon scattering effects at low energies are not strong enough and the nuclear mean-field potential is dominated. But, there will be only a little difference of the elliptic flow between the soft potential and the stiff potential with the incident energy increases. Overall speaking, the elliptic flow is not very sensible to the nuclear equation of state in the studied reaction system.

As said above, one of advantages of QMD is its fragment formation, we show the directed and elliptic flow for some cluster combinations in FIG. 4. The soft potential and the incident energy 50MeV/u are adopted in the FIG. 4. In order to accumulate enough statistics, we define three bins of different fragments region according to the charge, the solid circles are the flow of neutrons and protons, the up-triangles are the flow of fragments with $Z=2$∼4 and the down-triangles are the flow of fragments with $Z\geq 5$. It is obvious to see that the absolute values of the directed flow and the elliptic flow enhance with the charge (or mass) increases, i.e. the heavier fragment flow is stronger than the lighter one.
In summary, the IQMD model has been used to study the directed and elliptic flows in the collisions of $^{112}\text{Sn} + ^{112}\text{Sn}$ at different impact parameters and different fragment charges from 35 to 90 MeV/nucleon. It is found that a transition in directed flow from the negative to positive flow is revealed as the incident energy increases. A strong dependence on the nuclear EOS is seen in the directed flow at different incident energies and at different impact parameters. The elliptic flow decreases with increasing the incident energy and increases with increasing the impact parameter. Meanwhile, the elliptic flow and the directed flow is observed to be stronger as the fragment charge (mass) increases. In comparison with the directed flow, the elliptic flow is not sensitive to the EOS. By this study, we found that there exists different sensitivity to EOS for different kind of flow. Hence, it will be helpful to choose more sensitive flow probe to EOS before the comparison with the data is made.

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