Atom–atom entanglement generated at early times by two-photon emission

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Abstract
We analyze entanglement generation between a pair of neutral two-level atoms that are initially excited in a common electromagnetic vacuum. The nonlocal correlations that appear due to the interaction with the field can become entanglement when the field state is known. We distinguish two different situations: in the first, the field remains in the vacuum state and in the second, two photons are present in the final state. In both cases, we study the dependence of the entanglement on time and interatomic distance, at ranges related with locality issues.

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As is well known, an atom $B$ cannot become excited by a physical signal coming from another atom $A$ placed at a distance $r$ at a time $t < t_r = r/c$. This does not prevent the emergence of correlations between these two atoms at $t < t_r$. In particular, when the final state of the field (and/or atom $A$) is fixed, atom $B$ can become excited even when both atoms were initially (at $t = 0$) uncorrelated and the time elapsed was smaller than $t_r$. The specification of this final state is highly nonlocal and a finite excitation probability $P_B(t)$ for atom $B$ at times $t$ earlier than $t_r$ does not imply causality violation but is an artifact of that nonlocality. As shown in [1], causality is fully restored after summing over all the accessible final states for atom $A$ and the electromagnetic field. Now only the state of $B$ is accounted for; this is a local property, no wonder that $P_B(t) = 0$ for $t < t_r$ in this case. In the language of quantum information, the system is nonsignaling [2]. It is well known that the mere existence of correlations is not enough in order to send information between the parties: classical communication is always needed to transform entanglement into information.

Due to the existence of these nonlocal correlations, the reduced state of the atoms at time $t$ is a mixed classically correlated state, even for $ct < r$ [3]. The generation of a correlated state from an uncorrelated one only by local interactions could be attributed to the fact that the field vacuum is an entangled state between spacelike separated regions [4–6] or to the finiteness of the Feynman propagator for $ct < r$ [7]. There are at least two ways to obtain entanglement from these correlations: introducing a suitable time-dependent coupling between the field and the atoms (like in [6, 8] with a scalar field and two classical detectors) or fixing the field state [3, 9, 10]. Here, we will show new examples of the latter case: we will start with both atoms excited in the electromagnetic vacuum, and after an interaction time $t$, we will consider the following field states:

(i) no photons are present and
(ii) two photons are detected.

We shall study the dependence of the entanglement on $t$ and $r$, both for $r > c t$ and $r < c t$.

We assume that the wavelengths relevant to the interaction with the atoms, and the separation between them, are much longer than the atomic dimensions. The dipole approximation, appropriate to these conditions, permits the splitting of the system Hamiltonian into two parts $H = H_0 + H_I$ that are separately gauge invariant. The first part is the Hamiltonian in the absence of interactions other than the potentials that keep $A$ and $B$ stable, $H_0 = H_A + H_B + H_{\text{field}}$. The second contains all the interaction of the atoms with the field

$$H_I = -\frac{1}{\epsilon_0} \sum_{n=A,B} \mathbf{d}_n(x_n, t) \mathbf{D}(x_n, t),$$

where $\mathbf{D}$ is the electric displacement field, and $\mathbf{d}_n = \sum_i e \int d^3x_i \langle E | (x_i - x_n) | G \rangle$ is the electric dipole moment of atom $n$, which we will take here as real and of equal magnitude for both atoms ($\mathbf{d} = \mathbf{d}_A = \mathbf{d}_B$), and $|E\rangle$ and $|G\rangle$ are the excited and ground states of the atoms, respectively.

In what follows, we choose a system given initially by

$$|\psi\rangle_0 = |EE\rangle \cdot |0\rangle$$

in which atoms $A$ and $B$...
are in the excited state $|E\rangle$ and the field in the vacuum state $|0\rangle$. The system then evolves under the effect of the interaction during a lapse of time $t$ into a state

$$|\psi\rangle = e^{-i \int_0^t \omega H(t')/\hbar} |\psi_0\rangle,$$

(2)

which, to order $\alpha$, can be given in the interaction picture as

$$|\text{atom}_1, \text{atom}_2, \text{field}\rangle = ((1 + \alpha) |E\rangle E(b |GG\rangle) |0\rangle$$

$$+ (a_B |GE\rangle + a_B |EG\rangle) |1\rangle + (|EE\rangle + |GG\rangle) |2\rangle,$$

(3)

where

$$\begin{align*}
\alpha &= \frac{1}{2} \langle 0 | T(S_A^{+}S_A^{-} + S_B^{+}S_B^{-}) |0\rangle, \\
b &= \langle 0 | T(S_B^{+}S_A^{-}) |0\rangle, \\
a_A &= \langle 1 | S_A^{-} | 0\rangle, \\
a_B &= \langle 1 | S_B^{-} | 0\rangle, \\
f &= \frac{1}{2} \langle 2 | T(S_A^{+}S_A^{-} + S_B^{+}S_B^{-}) |0\rangle, \\
g &= \langle 2 | T(S_B^{+}S_A^{-}) |0\rangle,
\end{align*}$$

(4)

where $S = -\frac{i}{\hbar} \int_0^t \omega H(t') dE(x, t')$, with $T$ being the time ordering operator, $n$, $|n\rangle$, $0$, $1$, $2$, stands for the state of $n$ photons with definite momenta and polarizations, i.e. $|1\rangle = |k, e_1\rangle$, etc. The superscript signs in $S^{\pm}$, to be defined in (5) below, are associated with the energy difference between the initial and final atomic states of each transition. Among all the terms that contribute to the final state (3), only $b$ corresponds to interaction between both atoms. This would change at higher orders in $\alpha$. Here, $a$ describes intra-atomic radiative corrections, $a_A$ and $a_B$ describe single-photon emission by one atom and $g$ by both the atoms, whereas $f$ corresponds to two-photon emission by a single atom. In quantum optics, virtual terms such as $b$ and $f$, which do not conserve energy and appear only at very short times, are usually neglected by the introduction of a rotating wave approximation (RWA). But here we are interested in the short time behavior, and therefore all the terms must be included, as in [1, 11, 12]. Actually, only when all these virtual effects are considered, can it be said properly that the probability of excitation of atom $B$ is completely independent of atom $A$ when $r > ct$ [11, 12].

Finally, in the dipole approximation, the actions $hS^{\pm}$ in (4) reduce to

$$S^{\pm} = -\frac{i}{\hbar} \int_0^t dE(x, t'),$$

(5)

where $\Omega = \omega_E - \omega_G$ is the transition frequency, and we are neglecting atomic recoil. Equation (5) depends on the atomic properties $\Omega$ and $d$ and on the interaction time $t$. In our calculations, we will take $(\Omega d/\hbar c) = 5 \times 10^{-3}$, which is of the same order as the $1s \rightarrow 2p$ transition in the hydrogen atom, consider $\Omega t > 1$ and analyze the cases $(r/ct) \simeq 1$ near the time $t = t_c$, where one atom could begin to receive signals from the other.

Given a definite field state $|n\rangle$, the pair of atoms is in a projected pure two-qubit state as shown in (3). We will denote these states by $|A, B, n\rangle$, $\rho_{AB}^{(n)} = |A, B, n\rangle \langle A, B, n|$, and will compute the concurrence $C^{(n)}$ [13]:

$$C^{(n)} = \max_0, \sqrt{\lambda_i - \sum_{j \neq i} \sqrt{\lambda_j} },$$

(6)

where $\lambda_i$ is the largest of the eigenvalues $\lambda_j (j = 1, \ldots, 4)$ of

$$\begin{pmatrix}
(\sigma_j^{\alpha} \otimes \sigma_j^{\beta}) \rho^{(n)}_{AB} (\sigma_j^{\alpha} \otimes \sigma_j^{\beta})
\end{pmatrix}.$$

We begin with the case $n = 0$, where the field is in the vacuum state and, following (3), the atoms are in the projected pure state $|AB\rangle = ((1 + \alpha) |EE\rangle + b |GG\rangle)/c_0$, where $c_0 = \sqrt{1 + a^{2} + b^{2}}$ is the normalization, giving a concurrence

$$C^{(0)} = 2|b||1 + a|/c_0^2.$$

(7)

The computation of $a$ and $b$ can be performed following the lines given in [3], where they were computed for the case of an initial atomic state $|E\rangle$. We will consider that the dipoles are parallel along the $z$-axis, while the atoms remain along the $y$-axis. Under these conditions, using the dimensionless variables $x = r/ct$ and $z = \Omega r/c$,

$$\begin{align*}
a &= \frac{4i}{3\pi} \sum_{x} \left( \ln \left| 1 - \frac{\max z}{x} \right| + 2i\pi \right), \\
b &= \frac{\alpha d_x d_y}{\pi \varepsilon_0^2} \left( -\nabla^2 \delta_{ij} + \nabla_i \nabla_j \right) I_x,
\end{align*}$$

(8)

with $K = \alpha |d|^2/\varepsilon_0^2 r^2$ and $I_x = I_x + I_y$, where

$$I_\pm = \frac{-ie^{i\pi/2}}{2\pi} \left[ \pm 2 \cos \left( \frac{z}{x} \right) e^{i\pi/2} Ei \left( \mp iz \right) + e^{-i\pi/2} \right]$$

$$\times Ei \left( iz \left( 1 \pm \frac{1}{x} \right) \right) e^{-i\pi/2} \left( 1 \pm \frac{1}{x} \right),$$

(9)

for $x > 1$ with the additional term $-2\pi e^{i\pi/2} (1/(1-x))$ for $x < 1$. We use the conventions and tables of [14].

We show in figure 1 the concurrence $C^{(0)}$ (7) for $x$ near 1 for given values of $z$. As in the case where $|E\rangle$ is the initial atomic state, $C^{(0)}$ jumps at $x = 1$ and has different behaviors on both sides.
In figure 2 the concurrence is sketched as a function of $z$ for given values of $\Omega t = z/x$. The tiny values of the concurrence for the region $z > \Omega t$ (which corresponds to $x > 1$) diminish as $t$ grows and will eventually vanish, since $b$ is a non-RWA term.

The case $n = 1$ analyzed in [9] shows atom–atom correlations due to the uncertainty of the photon source. Here, we will focus on the two-photon case. The final atomic state $|AB \rangle = (f \langle EG | + g \langle GE |)/c_2$, with $c_2 = \sqrt{\left| f \right|^2 + \left| g \right|^2}$, is in the same subspace as for $n = 0$. The normalization $c_2$ is $O(\alpha)$ like the expectation values $f$, $g$, so that all the coefficients in $\rho^{(2)}$ may be large. Therefore, although the probability of attaining this state is small, the correlations are not. The concurrence is

$$C^{(2)} = 2|f g^*/c_2^2.$$  

We find that

$$f = \theta(t_1 - t_2)(u_A(t_1) u_A^*(t_2) + u_A(t_1) v_A^*(t_2)) + v_B(t_1) u_B^*(t_2) + u_B(t_1)v_B^*(t_2)),$$

$$g = u_B u_A^* + u_A u_B^*$$

with $u_A = (1|S_0^A|0)$ and $v_A = (1|S_0^B|0)$. The primes account for the two single photons, i.e. $|2\rangle = |k\epsilon_x, k'\epsilon_x'\rangle$. The quantities $\left|u_A\right|^2 = \left|u_B\right|^2 = \left|u\right|^2$, $\left|v_A\right|^2 = \left|v_B\right|^2 = \left|v\right|^2$, $l = u_A v_B^* = u_B v_A^*$, $u v^* = u_A v_A^*$, $u_B u_B^* = u_B u_B^*$, $u_A v_B^* = u_A v_B^*$, have been computed in [3].

In figure 3, we show $C^{(2)}$ in front of $x$ for given values of $z$. When $x \to 0$ ($t \to \infty$, i.e. the quantum optics regime), $f$ vanishes and the final atomic state would be the separable state $|G G\rangle$, with zero concurrence. Entanglement is sizeable for $x > 1$, and could be maximized if a particular two-photon state was detected [10].

In figure 4, $C^{(2)}$ is sketched as a function of $z$ for given values of $\Omega t = z/x$. Again, the concurrence for the region $z > \Omega t$ ($x > 1$) diminishes as $t$ grows and will eventually vanish, since it is due to $f$, which is a non-RWA term. Interestingly, as we noted in [9] for the single-photon emission, $x = 1$ is a singular point that divides the spacetime into two different regions. This occurs, as in [9], even if in these cases $t$ is not the propagation time of any physical signal between the atoms. This effect comes from the appearance of effective interaction terms such as $l$, which would be missing if we could discriminate the source of emission of each photon.

As a summary, we have computed the concurrence generated between a pair of neutral two-level atoms by the interaction with the electromagnetic field, in order to study the role of locality in the growing up of quantum correlations. We have considered the initial state with both atoms excited and the field in the vacuum, and two different situations after time $t$: the field remains in the vacuum and two photons have been emitted. In the former case, entanglement is generated by the interaction amplitude $b$. For $r < ct$ photon exchange makes correlations grow during a short time, before vanishing eventually due to the non-RWA nature of $b$. For $r > ct$ although $b$ is nonzero, its amplitude is negligible compared with $r < e t$. The situation is different when two photons are present. There is a relevant probability that both photons come from the same atom, which in our scheme is represented by the non-RWA amplitude $g$. As long as we cannot distinguish the source of emission of each photon, concurrence is sizeable for $r > c t$ and $r < c t$, and will eventually vanish. Although there are no interaction terms between the atoms, the indistinguishability construes effective interaction terms and $r = c t$ plays the role of a frontier between two different spacetime regions.
Acknowledgments

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