Screening of qubit from zero-temperature reservoir

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We suggest an application of dynamical Zeno effect to isolate a qubit in the quantum memory unit against decoherence caused by coupling with the reservoir having zero temperature. The method is based on using an auxiliary casing system that mediate the qubit-reservoir interaction and is simultaneously frequently erased to ground state. This screening procedure can be implemented in the cavity QED experiments to store the atomic and photonic qubit states.

I. INTRODUCTION

For efficient quantum information processing, the robust quantum memories that store information encoded in the state superposition are needed. The main obstacle in their realization arises from the difficulty to isolate a quantum mechanical system from the environment that causes decoherence effect. To avoid the decoherence, several strategies have been developed: the quantum error correction schemes [1], feedback implementations [2], dynamical decoupling techniques [3] and the engineering of pointer state methods [1]. In this work, we suggest a mechanism to physically isolate the qubit stored in the memory unit. The basic idea has been inspired by a screening effect in the electromagnetic field theory, where a device can be screened from the disturbing effects of the external electromagnetic fields, if it is closed in the earth-connected metal casing. To implement this idea in the quantum memories, we use a quantum casing system that is frequently erased to the ground state, as is depicted in Fig. 1. As the number of erasing events increases during the time evolution, the system is better isolated from an action of the zero-temperature reservoir. From different point of view, the effect reminiscents dynamical Zeno effect [3], where the system dynamics can mimic the standard Zeno effect [3]. The possible experimental applications could be found, for example, in the cavity QED experiments to screen the atomic and photonic qubit, however, it may be also consider more generally to isolate the system from the reservoir influence.

To illustrate the properties of the screening method, we analyze the standard example of decoherence effect: atomic qubit coupled to the external reservoir on zero temperature, for review [3]. A two-level atom placed in the free-space, initially prepared in the pure state superposition $|\Psi\rangle = \sqrt{1-p}|g\rangle + \exp(-i\psi)\sqrt{p}|e\rangle$ of the excited state $|e\rangle$ and ground state $|g\rangle$ and resonantly interacting with a effectively infinite reservoir of free-space vacuum modes of electromagnetic field exhibits exponential decay of the atomic state. It can be described by the following master equation in Markovian approximation

$$\frac{d}{dt}\rho = \frac{\gamma}{2} \left( 2\sigma\rho\sigma^\dagger - \sigma^\dagger\sigma\rho - \rho\sigma^\dagger\sigma \right),$$

where $\sigma = |g\rangle\langle e| + |e\rangle\langle g|$ are the deexcitation and excitation operators of the atomic system. After the reservoir interaction, the initial state $|\Psi\rangle$ transfers to the reduced density matrix

$$\rho = (1 - P(t))|g\rangle\langle g| + P(t) |e\rangle\langle e| + V(t)(|g\rangle\langle e| \exp(i\psi) + |e\rangle\langle g| \exp(-i\psi)),$$

where $P(t) = P(0)\exp(-\gamma t)$, $P(0) = p$ is probability of the excited state and $V(t) = V(0)\exp(-\gamma t/2)$, $V(0) = \sqrt{p(1-p)}$ is atomic interference term. Thus, the parameters $P(t)$ and $V(t)$ describe the energy and interference evolution in the two-level atom. The fidelity $F = \langle \Psi|\rho|\Psi\rangle$ with initial pure atomic state $|\Psi\rangle$ given by

$$F(t) = (1 - P(0)) (1 - P(t)) + P(0)P(t) + 2V(0)V(t)$$

exponentially vanishes in course of time, according to time-behavior of the depletion coefficients $P(t)$ and $V(t)$. This fidelity vanishing is one from the main obstacles that inhibit the efficiency of state preserving in the qubit memories.

II. SCREENING PROCEDURES

The idea of screening is based on inserting an auxiliary casing system, as is depicted in Fig. 1 and frequent erasing of its state. According to the example discussed in the previous Section, we have placed the atom with Rabi frequency $\Omega$ in the resonant one-mode high-Q cavity with damping constant $\gamma$ [4], as is depicted in Fig. 2. The casing system is the mode of electromagnetic field inside the cavity, initially prepared in the vacuum state. We assume that the off-resonant dissipations described by the damping constant $\Gamma$ can be neglected in comparison with Rabi frequency $\Omega$ and focus on the elimination of decoherence for the resonant coupling. According to screening idea, we in addition divide the evolution to the...
N identical time interval $T = t/N$ by the cavity erasing. In quantum theory, an erasure of a single unknown state $|\psi\rangle$ may be thought of as ideal quantum transmission $|\psi\rangle|0\rangle \rightarrow |0\rangle|\psi\rangle$ with some blank ground state $|0\rangle$ followed by the damping of $|\psi\rangle$ to the environment. The erasing mechanism can be implemented by an additional stream of the two-level atoms also resonantly interact with the cavity field, as is depicted in figure, and initially prepared in the ground state. The interaction is adjusted in such a way, that the photon (emitted by the atom) is quickly extracted in deterministic way from the cavity.

Consequently, the cavity field decouples from the atomic system and restores the vacuum state in the cavity. To achieve it, the Rabi frequency $\Omega \pi$ of the atoms consisting the erasing mechanism must be adjusted $\Omega \gg \Omega$. Experimentally, the erasing mechanism can be implemented analogically to micromaser operation [3], where the condition $\Omega \gg \Omega$ can be achieved situating the atom $\Lambda$ near the intensity minimum of standing cavity mode, where the coupling constant is much smaller than near the maximum. On the other hand, a stream of atoms consisting the erasing mechanism is focused into the maximum of standing cavity mode.

Thus, the evolution of screened two-level atom consists $N$ same sequential parts separated by the erasing events, in which the atom repeatedly interacts with the damped cavity in the vacuum state. The evolution between erasing events can be described in Markovian approximation by the master equation for atom-field state

$$\frac{d}{dt} \rho = -\frac{i}{\hbar} [H_t, \rho] + \frac{\gamma}{2} \left( 2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a \right),$$

where $H_t = i\hbar \Omega (a^\dagger \sigma - \sigma^\dagger a)$ is interaction Hamiltonian of the resonant atom-field coupling. Assuming the atom initially in the state $|\Psi\rangle$ and the cavity mode in the vacuum state, the master equation can be analytically solved and two regions of evolution are obtained: for the overdamped case $4\Omega/\gamma < 1$, the solution exhibits the hyperbolic character, which changes for the under-damped case $4\Omega/\gamma > 1$ in the oscillatory behavior. After tracing over the cavity field and $N$-times repeated evolution with erasing, we obtain the reduced density matrix of the two-level atom having the following coefficients for $\Omega \neq \gamma/4$

$$P_N(t) = P(0) \exp \left( -\frac{\gamma t}{2} \right) \left( \frac{\gamma}{4\Lambda} \sinh 2\Lambda \frac{t}{N} + \left( \frac{\Omega^2}{2\Lambda^2} \right)^N \right),$$

$$V_N(t) = V(0) \exp \left( -\frac{\gamma t}{2} \right) \left( \cosh \frac{\Lambda}{4} \sinh \Lambda \frac{t}{N} \right)^N,$$

where $\Lambda = \sqrt{(\gamma/4)^2 - \Omega^2}$ and for $\Omega = \gamma/4$,

$$P_N(t) = P(0) \exp \left( -\frac{\gamma t}{2} \right) \left( 1 + \frac{\gamma}{16N^2} t^2 \right)^N,$$

$$V_N(t) = V(0) \exp \left( -\frac{\gamma t}{2} \right) \left[ 1 + \frac{\gamma}{4N} t + \frac{\gamma^2}{16N^2} t^2 \right]^N.$$

Now we arrive to the main result of the paper: as can be proved, for every value of the constants $\Omega$ and $\gamma$, the energy depletion term $P_N(t)$ and interference term $V_N(t)$ tend to initial values $P(0)$ and $V(0)$ as the number $N$ of erasing events increases and consequently, the fidelity

$$F_N(t) = (1 - P(0))(1 - P_N(t)) + P(0)P_N(t) + 2V(0)V_N(t)$$

approaches unity in the limit

$$\lim_{N \to \infty} F_N(t) = 1,$$

as is depicted in Fig. 3. Thus the qubit is isolated from the external influence of zero-temperature. This effect can be employed as screening of quantum system from the reservoir decoherence. The nature of this behavior is dynamical Zeno effect [3], which is achieved by non-exponential character of the decoherence induced by the interaction of the atom with the cavity mode. As the coupling constant $\Omega$ increases, the non-exponential offset, depicted in Fig. 3, is shorter and we need implement the erasing mechanism more quickly. However, number $N$ of erasing events during the same time interval, depicted in Fig. 4, increases only sub-exponentially. Note that the decoherence is the most progressive for unbalanced superpositions $|\Psi\rangle$ with $P \to 1$; for this reason, an unbalanced superposition has been analyzed in Fig. 3 and Fig. 4.

Now we consider different application of the screening for the qubit state of field mode $|\Psi\rangle = a|0\rangle + b|1\rangle$, where $|0\rangle$ and $|1\rangle$ are the vacuum and single photon states, confined in the high-Q cavity. Simultaneously, we will discuss an influence of the screening on the internal qubit evolution.

Instead of utilizing the standard one-mode cavity to confine the photon state, we consider three coupled one-mode cavities in the configuration depicted in Fig. 5. We suppose that the mode in the sandwiched cavity is under an internal interaction described by interaction Hamiltonian $H'$. We show that an influence of the screening procedure on this internal interaction is reduced simultaneously as the erasing is more frequent. Without loss of generality, we can consider that one side of the sandwiched cavity has perfect reflectivity and thus, we simplify the discussion to one-side case. This situation can be described in Markovian approximation by the following Heisenberg-Langevin equations

$$\frac{d}{dt} a = -\kappa b - \frac{i}{\hbar} [a, H'], \quad \frac{d}{dt} b = -\gamma b + \kappa a + F(t),$$

where $a$ is annihilation operator of the screened field mode, $b$ is annihilation operator of the auxiliary cavity, $F(t)$ is operator Langevin force of the reservoir, $\kappa$ is coupling constant between the cavities, $\gamma$ is damping constant and $H'$ is interaction Hamiltonian of an internal
erasing events, we can describe the evolution after $N$ temperature reservoir. Assuming finite number of external reservoir influence $H$, corresponding to Hamiltonian $D^\prime$ and $\Xi$ is operator of external reservoir influence

$$\Xi(\Delta t) = b(0)\kappa\Delta t(1 - \gamma \Delta t/2) + \kappa \int_0^{\Delta t} \int_0^{\Delta t} F(t')dt''dt', \quad (10)$$

which exhibits vanishing normal ordered moments

$$\langle \Xi^{(m)}(\Delta t)\Xi^{(n)}(\Delta t)\rangle = 0 \quad (11)$$

for the auxiliary mode in vacuum state and the zero-temperature reservoir. Assuming finite number $N$ of erasing events, we can describe the evolution after $N$ iteration of the relation (13)

$$a(t) \approx \left(1 - \frac{it}{\hbar N}D^\prime - \frac{(\kappa t)^2}{2N^2}\right)^N a(0) - \sum_{n=1}^{\infty} c_n(\Delta t)\Xi_n(\Delta t), \quad (13)$$

where $c_n(\Delta t) = \left(1 - (\kappa t)^2/(2N^2)\right)^n$ are the time dependent coefficients arising by iterative procedure, $\Xi_n(\Delta t)$ are independent external influence satisfying $[H', \Xi_n(\Delta t)] = 0$. The term $(\kappa t)^2/2N^2$ vanishing in the limit of erasing events $N \to \infty$ and the the annihilation operator approaches

$$a(t) \approx \exp\left(-\frac{it}{\hbar}D^\prime\right) a(0) - \sum_{n=1}^{\infty} c_n(\Delta t)\Xi_n(\Delta t). \quad (14)$$

The residual influence of the reservoir is eliminated considering (12). Thus, as the number of erasing events $N$ increases, all the normal ordered moments are protected from the decoherence and consequently, the screened state evolve only under the internal evolution. In this way, we may manipulate with the state confined in the memory unit.

In this paper, we suggest an application of dynamical Zeno effect to quantum screening of system qubit from the vacuum reservoir influence. The screening exhibits independently on the coupling and damping parameters and can be applied in both the over-damped and under-damped cases. In addition, the internal system evolution is simultaneously preserved and we thus can operate with quantum state saved in the memory. Two possible experimental implementations have been pointed up, for the two-level atom and for photonic qubit state stored in the cavities.

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FIG. 1: Screening of the qubit state: $A$ – direct qubit interaction with reservoir with damping constant $\gamma$, $B$ – screening of reservoir interaction by frequently erased casing qubit coupled to the system.

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FIG. 2: Schematic setup of the atomic state screening: $A$ – screened atom, $\gamma$ – cavity damping constant, $\Gamma$ – atomic damping constant, $\Omega$ – Rabi frequency of atom-cavity coupling, $\Omega_c$ – Rabi frequency of erasing interaction.
FIG. 3: The slowing down of fidelity degradation by the screening effect: the atom confined in damped cavity for overdamped (under-damped) behavior. For illustration, we assume the dissipative constant $\gamma$ of the cavity same as for the direct atom-reservoir interaction case.

FIG. 4: Number of the erasing events which is needed to preserve the qubit state ($p = 0.9$) for $\gamma t = \pi$.

FIG. 5: Schematic setup of the field state screening: $\gamma$ – cavity damping constant, $\kappa$ – cavity coupling constant, $\Omega_e$ – Rabi frequency of erasing interaction.
$4\Omega/\gamma=0.9$ $(4\Omega/\gamma=2)$, $p=0.9$

- screening
- $N=10$ ($N=150$)
- (no erasing)
- no erasing
- no screening
$\gamma t = \pi$, $p = 0.9$

$N$

$4\Omega/\gamma$

$F = 0.99$

$F = 0.97$

$F = 0.95$