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MUON CAPTURE ON DEUTERON AND $^3$He: A PERSONAL REVIEW

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The present status of theoretical and experimental studies of muon capture reactions on light nuclei is reviewed. In particular, the recent results for the two reactions $^2$H($\mu^-$, $\nu_\mu$)nn and $^3$He($\mu^-$, $\nu_\mu$)$^3$H are presented, and the unresolved discrepancies among different measurements and calculations, open problems, and future developments are discussed.

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1. Introduction

When negative muons pass through matter, they can be captured into high-lying atomic orbitals. Then, in a time-scale of the order of $10^{-13}$ s, they cascade down into the 1s orbit, through Auger processes with atomic electrons and the emission of X-rays. At this point, two competing processes occur: one is ordinary decay

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu ,$$

and the other is the (weak) capture by the nucleus

$$\mu^- + A(Z,N) \rightarrow \nu_\mu + A(Z-1,N+1) ,$$

which can take place from any of the two initial hyperfine states, $f = J_i \pm 1/2$ ($J_i$ is the spin of the initial nucleus $A(Z,N)$). Apart from tiny corrections due to bound-state effects, the decay rate is essentially the same as for a free muon. In light nuclei, this is much larger than the rate for capture, which proceeds predominantly through the basic process

$$\mu^- + p \rightarrow n + \nu_\mu ,$$

induced by the exchange of a $W^+$ boson. Its rate is expected to be proportional to the number of protons in the nucleus and to the probability of finding the muon
at the nucleus. Since the semi-leptonic weak nuclear interaction is effectively a contact interaction, this probability scales like the square of the atomic $1s$ wave function evaluated at the origin, proportional to $Z^3$. The capture rate, therefore, scales roughly like $Z^4$. It is only for nuclei with $Z \geq 12$ that the nuclear capture rate becomes comparable with the decay rate. Muonic capture on light nuclei are therefore experimentally challenging processes. However, they are preferred under the theoretical point of view, as the nuclear effects can be easier and more accurately taken into account, and informations on the basic process of Eq. (3) can be better extracted. For instance, muon capture on hydrogen and hydrogen isotopes is, in principle, best suited to obtain informations on the matrix element of the (charge-changing) single-nucleon weak current

$$ j^\mu = \bar{u}_p \left[ F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_{\nu}}{2M_N} - G_A(q^2)\gamma^\mu\gamma^5 - G_{PS}(q^2)\frac{q^\mu\gamma^5}{2M_N} \right] u_n ,$$

but is experimentally the hardest process. Note that in Eq. (4) we have ignored contributions from second-class currents

$$ \sigma^{\mu\nu} = i/2[\gamma^\mu, \gamma^\nu] \quad \text{and} \quad \bar{u}_p = u_p^\dagger \gamma_0 \gamma_5,$$

Of the four form factors of Eq. (4), $F_1(q^2)$ and $F_2(q^2)$ are related to the isovector electromagnetic form factors of the nucleon by the conserved-vector-current (CVC) constraint. They are well known over a wide range of momentum transfers $q^2$ from elastic electron scattering data on the nucleon. The axial form factor $G_A(q^2)$ is also quite well known: its value at vanishing $q^2$, $g_A = 1.2695 \pm 0.0029$, is from neutron $\beta$-decay, while its $q^2$-dependence is parametrized as

$$ G_A(q^2) = g_A/(1 - q^2/\Lambda_A^2)^2 ,$$

with $\Lambda_A = 1$ GeV from an analysis of pion electro-production data and direct measurements of the reaction $p + \nu_\mu \rightarrow n + \mu + \nu$. Note that a considerably larger value $\Lambda_A = 1.35$ GeV is obtained from current analyses of neutrino quasi-elastic scattering data on nuclear targets. However, these analyses are based on rather crude models of nuclear structure (Fermi gas or a local density approximation of the nuclear matter spectral function) and on simplistic treatments of the reaction mechanism. Also, some discrepancies exist on the neutron $\beta$-decay lifetime, as the world average value used here differs by 6.5 standard deviations from the results obtained from gravitationally trapped ultra-cold neutrons. A discussion of this point is however well beyond the subject of the present review.

The induced pseudoscalar form factor $G_{PS}(q^2)$ is the least known of the four form factors of Eq. (4). The MuCap collaboration at Paul Scherrer Institute (PSI) has recently reported a precise measurement of the rate for reaction (3) in the singlet hyperfine state ($f = 0$): $725.0 \pm 13.7 \text{(stat)} \pm 10.7 \text{(syst)}$ sec$^{-1}$. Based on this value, an indirect "experimental"determination of $G_{PS}$ at the momentum transfer $q_0^2 = -0.88 m_\mu^2$ relevant for muon capture on hydrogen has been given.
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$G_{PS}^{\exp}(q_0^2) = 7.3 \pm 1.2$, by using for the remaining form factors the values discussed above and by evaluating electroweak radiative corrections. These are found to be sizable, of the order of $\sim 3\%$. Theoretical predictions for the induced pseudoscalar form factor were derived long ago based on the notion of a partially conserved axial current (PCAC) and pion dominance, and were later refined by evaluating leading-order corrections to the PCAC result with current algebra techniques $^{14}$. More recently, these predictions have been re-derived in chiral perturbation theory ($\chi$PT) $^{15,16}$, finding

$$G_{PS}^{TH}(q^2) = \frac{2m_\mu g_{\pi pm} f_\pi}{m_\pi^2 - q^2} \left( -\frac{1}{3} g_A m_\mu M_N r_A^2 \right), \quad (6)$$

where $g_{\pi pm} = 13.05 \pm 0.20$ is the $\pi NN$ coupling constant, $f_\pi = 92.4 \pm 0.4$ MeV is the pion decay constant, and $r_A = 0.43 \pm 0.03$ fm$^2$ is the axial radius of the nucleus, related to $\Lambda_A$ of Eq. (5) as $\Lambda_A^2 = 12/r_A^2$. For $q_0^2 = -0.88 m_\mu^2$, $G_{PS}^{TH}(q_0^2) = 8.2 \pm 0.2$ $^{15}$. To be noticed that the evaluation of electroweak radiative corrections $^{13}$ for muon capture on hydrogen is crucial for bringing $G_{PS}^{\exp}$ within less than $1\sigma$ of $G_{PS}^{TH}$.

Besides their relevance for extracting informations on single-nucleon weak current form factors, muon captures on light nuclei also provide a testing ground for the theoretical frameworks used to study those reactions of astrophysical interest whose rates cannot be measured experimentally, and for which one has to rely exclusively on theory $^{17}$. In fact, the same nuclear wave functions and, indirectly, the same model for the nuclear interactions from which these are obtained, and the same nuclear weak current can be used to study neutrino reactions in light nuclei $^{18,20,21}$, weak proton captures on proton and $^3\text{He}$ (the so-called $pp$ and $hep$ reactions) $^{19}$, and muon captures on light nuclei.

In the present review, we focus our attention on the following captures:

$$\mu^- + d \rightarrow n + n + \nu_\mu, \quad (7)$$

$$\mu^- + ^3\text{He} \rightarrow ^3\text{H} + \nu_\mu. \quad (8)$$

Muon capture on $^3\text{He}$ can also occur through the two- ($nd$) and three-body ($npn$) breakup channels of $^3\text{H}$. However, the branching ratios of these two processes are 20 % and 10 %, respectively, and experimental and theoretical work on them is quite limited. They will not be discussed here. A comprehensive and detailed description of these reactions, and more in general of the physics of muon capture and the problem of the induced pseudoscalar form factor, can be found in Refs. $^{22,23}$ and $^{24}$.

The observables of interest for muon capture reactions are the capture rates. In reaction (7), the stopped muons can in principle be captured from the two hyperfine states, $f = 1/2$ or $3/2$. However, it is known that capture takes place practically uniquely from the doublet hyperfine state $^{22,23}$. We will therefore consider only the doublet capture rate $\Gamma^D$. In reaction (8), instead, a difference in the capture rates between the hyperfine states is not expected: a hyperfine transition is highly unlikely, due to the energy difference between the hyperfine states. We will therefore...
consider the total capture rate \( \Gamma_0 \). In the next section we will briefly discuss the experimental situation, while the formalism to derive these observables and the most recent theoretical calculations will be presented in Sec. 3. Some concluding remarks are given in Sec. 4.

2. Experimental situation

The first attempt to measure \( \Gamma^D \) was carried out over forty years ago by Wang et al.\(^{25}\). Using a liquid mixed \( \text{H}_2/\text{D}_2 \) target, they obtained \( \Gamma^D = 365 \pm 96 \text{ s}^{-1} \). A few years later, Bertin et al. measured \( \Gamma^D = 445 \pm 60 \text{ s}^{-1} \) using a gas mixed \( \text{H}_2/\text{D}_2 \) target, and assuming a pure doublet mix of \( \mu d \) spin states. However, a subsequent study of hyperfine depopulation in a \( \text{H}_2/\text{D}_2 \) mixture\(^{27}\) has failed to support this assumption, and therefore the Bertin et al. result is considered controversial\(^{23}\). The most recent measurements have been performed in the eighties by Bardin et al.\(^{28}\) and Cargnelli et al.\(^{29}\). They both used pure deuterium, so that the \( \mu d + d \) collision rate is sufficient to fully depopulate the \( f = 3/2 \) hyperfine state. Furthermore, Bardin et al. used a liquid target and the lifetime method, i.e. compared the negative and positive muon lifetime when stopped in deuterium. For positive muons, the lifetime is the inverse of the muon decay rate, while for negative muons the lifetime is the inverse of the sum of the muon decay rate and the muon capture rate. The lifetime difference thus determines the capture rate, assuming, according to the CPT theorem, the positive and negative muon decay rate identical. Cargnelli et al., instead, used a gas target and the neutron method, i.e. directly detected the recoil neutrons, which is obviously quite challenging. These two measurements gave \( \Gamma^D = 470 \pm 29 \text{ s}^{-1} \)\(^{28}\) and \( \Gamma^D = 409 \pm 40 \text{ s}^{-1} \)\(^{29}\). In conclusion, all the measurements available until now, while consistent with each other, are not very precise, since the errors are in the 6\( \div \)10\% range. However, there is hope to have this situation clarified by the MuSun Collaboration\(^{30,24}\), with their on-going experiment at PSI, which should reach a precision of 1.5\%. The gain in experimental precision relies on the fundamental techniques developed for the MuCap experiment\(^{12}\). Muons will be stopped in an active gas target consisting of a cryogenic ionization chamber operated as time projection chamber with ultra-pure deuterium. The muon stopping point will be reconstructed in 3 dimensions, and this will eliminate the otherwise overwhelming background from muon stops in wall materials. The capture rate will then be determined using the lifetime technique.

The experimental situation for muon capture on \( ^3\text{He} \) is much clearer. After a first set of measurements in the early sixties by Falomkin et al.\(^{31}\), Zaimidoroga et al.\(^{32}\), Auerbach et al.\(^{33}\) and Clay et al.\(^{34}\), a very precise determination was performed by Ackerbauer et al.\(^{35}\) in the late nineties. The basic method involves counting the numbers of muon stops and \( ^3\text{He} \) recoils, when a beam of muons is stopped in \( ^3\text{He} \). Ackerbauer et al. used a gas ionization chamber, which allowed a better separation of muon and \( ^3\text{He} \) signals. The measured total capture rate \( \Gamma_0 \), corresponding to a statistical average population of the 4 different hyperfine states,
is $1496 \pm 4$ s$^{-1}$, a value consistent with those of the earlier measurements, but with a factor 10 of improvement in the experimental accuracy.

If the hyperfine structure of the $(\mu,^3\text{He})$ system is taken into account and the direction of the recoiling triton is detected, there are, in addition to the total capture rate, other observables, i.e. angular correlation parameters or so-called recoil asymmetries, which are more sensitive than the capture rate itself to the value of the induced pseudoscalar form factor $G_{PS}(q^2)$. A first attempt to measure the recoil asymmetry has been made by Souder et al. at TRIUMF. They used a $^3\text{He}$ ionization chamber to stop the incoming muons, re-polarize the $(\mu,^3\text{He})$ system and track the triton recoils. They obtained for the vector asymmetry $A_v$ the value of $0.63 \pm 0.09$ (stat.)$^{+0.11}_{-0.14}$ (syst.). This experimental result, which to our knowledge represents the first measurement of this observable, is affected by large systematic uncertainties. Therefore, a comparison between theory and experiment would not be particularly meaningful. Thus, further experimental work is highly recommended.

3. Theoretical calculations

Before discussing the results of the different theoretical calculations for the capture rates of reactions (7) and (8), we present in the following subsection the formalism used in the calculation of the observables under consideration.

3.1. Theoretical formalism

The muon capture on deuteron and $^3\text{He}$ is induced by the weak interaction Hamiltonian $H_W$

$$H_W = \frac{G_V}{\sqrt{2}} \int dx l_\sigma(x) j^\sigma(x) ,$$

where $G_V$ is the Fermi coupling constant, $G_V = 1.14939 \times 10^{-5}$ GeV$^{-2}$ as obtained from an analysis of $0^+ \to 0^+ \beta$-decays, and $l_\sigma$ and $j^\sigma$ are the leptonic and hadronic current densities, respectively. The former is given by

$$l_\sigma(x) = e^{-ik_\sigma \cdot x} \frac{\pi(k_\nu, h_\nu) \gamma_\sigma (1 - \gamma_5)}{\sqrt{2}} \psi_\mu(x, s_\mu) ,$$

where $\psi_\mu(x, s_\mu)$ is the ground-state wave function of the muon in the Coulomb field of the nucleus in the initial state, and $u(k_\nu, h_\nu)$ is the spinor of a muon neutrino with momentum $k_\nu$, energy $E_\nu (=k_\nu)$, and helicity $h_\nu$. While in principle the relativistic solution of the Dirac equation could be used, in practice it suffices to approximate

$$\psi_\mu(x, s_\mu) \simeq \psi_{1s}(x) \chi(s_\mu) \equiv \psi_{1s}(x) u(k_\mu, s_\mu)$$

$$k_\mu \to 0 ,$$

since the muon velocity $v_\mu \simeq Z\alpha \ll 1$ ($\alpha$ is the fine-structure constant and $Z=1$ or 2 for deuteron or $^3\text{He}$, respectively). Here $\psi_{1s}(x)$ is the $1s$ solution of the Schrödinger equation and, since the muon is essentially at rest, it is justified to replace the two-component spin state $\chi(s_\mu)$ with the four-component spinor $u(k_\mu, s_\mu)$ in the limit
This factor is defined as 

\[ R_{\mu} \]

where in this case

\[ q \]

with the leptonic momentum transfer \( q \) and final states. For muon capture on deuteron,

\[ \psi' \]

where \( \psi' \) evaluated at the origin, and

\[ l_\sigma(h_\nu, s_\mu) \equiv \pi(k_\nu, h_\nu) \gamma_\sigma (1 - \gamma_5) u(k_\mu, s_\mu) \tag{14} \]

and the Fourier transform of the nuclear weak current has been introduced as

\[ j^\sigma(q) = \int dx e^{iq\times x} j^\sigma(x) \equiv \langle \rho(q), j(q) \rangle \tag{15} \]

with the leptonic momentum transfer \( q \) defined as \( q = k_\mu - k_\nu \simeq -k_\nu \). The function \( \psi_{1s}(x) \) has been factored out from the matrix element of \( j^\sigma(q) \) between the initial and final states. For muon capture on deuteron, \( \psi_{1s}^{av} \) is approximated as

\[ |\psi_{1s}^{av}|^2 = |\psi_{1s}(0)|^2 = \frac{(\alpha \mu_{ud})^3}{\pi} \tag{16} \]

where \( \psi_{1s}(0) \) denotes the Bohr wave function for a point charge \( e \) evaluated at the origin, and \( \mu_{ud} \) is the reduced mass of the \((\mu, d)\) system. For muon capture on \({^3}\text{He}\), \( \psi_{1s}^{av} \) is approximated as

\[ |\psi_{1s}^{av}|^2 = R \frac{(2 \alpha \mu_{\mu^3\text{He}})^3}{\pi} \tag{17} \]

where in this case \( \mu_{\mu^3\text{He}} \) is the reduced mass of the \((\mu, {^3}\text{He})\) system, and the factor \( R \) approximately accounts for the finite extent of the nuclear charge distribution \( \psi_{1s}^{av} \). This factor is defined as

\[ R = \frac{|\psi_{1s}^{av}|^2}{|\psi_{1s}(0)|^2} \tag{18} \]

with

\[ \psi_{1s}^{av} = \frac{\int dxe^{iq\times x}\psi_{1s}(x)\rho(x)}{\int dxe^{iq\times x}\rho(x)} \tag{19} \]
where \( \rho(x) \) is the \(^3\)He charge density. It has been calculated explicitly in Ref.\(^{39}\) by using the charge densities corresponding to two realistic Hamiltonian models, the AV18/UIX and N3LO/N2LO (see below), and has been found for both models to be within a percent of 0.98, the value obtained from the experimental charge density and commonly adopted in the literature\(^{37}\).

In the case of muon capture on deuteron, the final state wave function is expanded in partial waves as

\[
\Psi_{p,s_1,s_2}(nn) = 4\pi \sum_{S} \left( \frac{1}{2} s_1, \frac{1}{2} s_2 \right) |SS_z\rangle \sum_{LJJ_z} i^L Y^*_{LLz}(\hat{p}) \langle SS_z, LLz |JJ_z \rangle \Psi_{nn}^{LSJJ_z}(p),
\]

(20)

where \( \Psi_{nn}^{LSJJ_z}(p) \) is the \( nn \) wave function. The calculation is typically restricted to \( J \leq 2 \) and \( L \leq 3 \), since it has been proven that higher order partial waves give negligible contributions\(^{39}\). Therefore, in spectroscopic notation, only the \(^1S_0\), \(^3P_0\), \(^3P_1\), \(^3P_2\), \(^3P_3\), and \(^1D_2\) partial waves are considered.

Now, standard techniques\(^{20,37}\) are used to carry out the multipole expansion of the weak charge, \( \rho(q) \), and current, \( j(q) \), operators. For muon capture on deuteron, we find

\[
\langle \Psi_{nn}^{LSJJ_z}(p)|\rho(q)|\Psi_{d}(s_d) \rangle = \sqrt{4\pi} \sum_{\Lambda \geq 0} \sqrt{2\Lambda + 1} i^\Lambda \frac{\langle 1s_d, \Lambda 0 |JJ_z \rangle}{\sqrt{2J + 1}} C^{LSJ}_\Lambda(q),
\]

(21)

\[
\langle \Psi_{nn}^{LSJJ_z}(p)|j_z(q)|\Psi_{d}(s_d) \rangle = -\sqrt{4\pi} \sum_{\Lambda \geq 0} \sqrt{2\Lambda + 1} i^\Lambda \frac{\langle 1s_d, \Lambda 0 |JJ_z \rangle}{\sqrt{2J + 1}} L^{LSJ}_\Lambda(q),
\]

(22)

\[
\langle \Psi_{nn}^{LSJJ_z}(p)|j_\lambda(q)|\Psi_{d}(s_d) \rangle = 2\sqrt{2\Lambda + 1} i^\Lambda \frac{\langle 1s_d, \Lambda - \lambda |JJ_z \rangle}{\sqrt{2J + 1}} \left[ -\lambda M^{LSJ}_\Lambda(q) + E^{LSJ}_\Lambda(q) \right],
\]

(23)

where \( \lambda = \pm 1 \), and \( C^{LSJ}_\Lambda(q) \), \( L^{LSJ}_\Lambda(q) \), \( E^{LSJ}_\Lambda(q) \) and \( M^{LSJ}_\Lambda(q) \) denote the reduced matrix elements (RME’s) of the Coulomb \((C)\), longitudinal \((L)\), transverse electric \((E)\) and transverse magnetic \((M)\) multipole operators, as defined in Ref.\(^{20}\). Since the weak charge/current operators have scalar/polar-vector \((V)\) and pseudo-scalar/axial-vector \((A)\) components, each multipole consists of the sum of \( V \) and \( A \) terms, having opposite parity under space inversion\(^{20}\). The contributing multipoles for the \( S-, P-, \) and \( D- \) channels mentioned above in muon capture on deuteron are given in Table\(^{11}\) where the superscripts \( LSJ \) have been dropped. In the case of muon capture on \(^3\)He, explicit expressions for the multipole operators are given by\(^{20}\)

\[
\langle \Psi_3\pi(Hs_3)|\rho(q)|\Psi_3\pi(He)(s_3) \rangle = \sqrt{2\pi} \sum_{l=0,1} \sqrt{2l + 1} i^l d^l_{m,0}(-\theta) \left( \frac{1}{2} s_3, l m, \frac{1}{2} s_3 \right) C_l(q),
\]

(24)
\[
\langle \Psi_3^H(s'_3)|j_z(q)|\Psi_3^He(s_3) \rangle = -\sqrt{2\pi} \sum_{l=0,1} \sqrt{2l+1} i^l d^d_{m,0}(\theta) \langle \frac{1}{2} s_3, l m | \frac{1}{2} s'_3 \rangle L_l(q) ,
\]
(25)

\[
\langle \Psi_3^H(s'_3)|j_\lambda(q)|\Psi_3^He(s_3) \rangle = \sqrt{3\pi} i d^d_{m,-\lambda}(\theta) \langle \frac{1}{2} s_3, 1 m | \frac{1}{2} s'_3 \rangle [\lambda M_1(q) + E_1(q)] ,
\]
(26)

where \( m = s'_3 - s_3 \), and the \( d^d_{m,m'} \) are rotation matrices in the standard notation of Ref. \[41\]. Applying parity and angular momentum selection rules, it has been shown \[40\] that the only contributing RME’s are \( C_0(V) \), \( C_1(A) \), \( L_0(V) \), \( L_1(A) \), \( E_1(A) \), and \( M_1(V) \).

The total capture rate for the two reactions under consideration is then defined as

\[
d\Gamma = 2\pi \delta(\Delta E) |T_W|^2 \times \text{phase space}
\]
(27)

where \( \delta(\Delta E) \) is the energy-conserving \( \delta \)-function, and the phase space is \( dp d\kappa / (2\pi)^6 \) for reaction \[17\] and just \( d\kappa / (2\pi)^3 \) for reaction \[8\]. The following notation has been introduced: (i) for muon capture on deuteron

\[
|T_W|^2 = \frac{1}{2f+1} \sum_{s_1s_2s_\nu f_{s}} |T_W(f,f_2;s_1,s_2,s_\nu)|^2 ,
\]
(28)

and the initial hyperfine state has been fixed to be \( f = 1/2 \); (ii) for muon capture on \( ^3\text{He} \)

\[
|T_W|^2 = \sum_{s'_3s_\nu f_{f_2}} P(f,f_2) |T_W(f,f_2;s'_3,s_\nu)|^2 ,
\]
(29)

where \( P(f,f_2) \) is the probability of finding the \( (\mu,^3\text{He}) \) system in the total-spin state \( f,f_2 \) and \( P(f,f_2) = 1/4 \) when the same probability to the different hyperfine states is assigned.

| Partial wave | Contributing multipoles |
|--------------|-------------------------|
| \(^1S_0\)     | \( C_0(A) \), \( L_1(A) \), \( E_1(A) \), \( M_1(V) \) |
| \(^3P_0\)     | \( C_1(V) \), \( L_1(V) \), \( E_1(V) \), \( M_1(A) \) |
| \(^3P_1\)     | \( C_0(A) \), \( L_0(A) \), \( C_1(V) \), \( L_1(V) \), \( E_1(V) \), \( M_1(A) \), \( C_2(A) \), \( L_2(A) \), \( E_2(A) \), \( M_2(V) \) |
| \(^3P_2-3F_2\) | \( C_1(V) \), \( L_1(V) \), \( E_1(V) \), \( M_1(A) \), \( C_2(A) \), \( L_2(A) \), \( E_2(A) \), \( M_2(V) \), \( C_3(V) \), \( L_3(V) \), \( E_3(V) \), \( M_3(A) \) |
| \(^1D_2\)     | \( C_1(A) \), \( L_1(A) \), \( E_1(A) \), \( M_1(V) \), \( C_2(V) \), \( L_2(V) \), \( E_2(V) \), \( M_2(A) \), \( C_3(A) \), \( L_3(A) \), \( E_3(A) \), \( M_3(V) \) |

Table 1. Contributing multipoles in muon capture on deuteron, for all the \( nn \) partial waves with \( J \leq 2 \) and \( L \leq 3 \). The spectroscopic notation is used. See text for further explanations.
After carrying out the spin sums, the total rate and recoil asymmetry for muon capture on $^3$He are

$$\Gamma_0 = G_V^2 E_\nu^2 \left(1 - \frac{E_\nu}{m_3^{3\text{He}}} \right) |\psi_{1s}^{av}|^2$$

$$\left[ |C_0(V) - L_0(V)|^2 + |C_1(A) - L_1(A)|^2 + |M_1(V) - E_1(A)|^2 \right],$$

(30)

with $E_\nu$ given by

$$E_\nu = \frac{(m_\mu + m_{3\text{He}})^2 - m_{3\text{He}}^2}{2(m_\mu + m_{3\text{He}})},$$

(31)

and

$$A_v = 1 + 2 \text{Im} \left[ \frac{(C_0(V) - L_0(V))(C_1(A) - L_1(A))^*}{|C_0(V) - L_0(V)|^2 + |C_1(A) - L_1(A)|^2 + |M_1(V) - E_1(A)|^2} \right] - |M_1(V) - E_1(A)|^2.$$  

(32)

In the case of muon capture on deuteron, the differential rate reads

$$\frac{d\Gamma_D}{dp} = E_\nu \left[1 - \frac{E_\nu}{m_\mu + m_d} \right] |\psi_{1s}^{av}|^2 \frac{P^2 dp}{8\pi^2 |T_W|^2},$$

(33)

where

$$E_\nu = \frac{(m_\mu + m_d)^2 - 4m_n^2 - 4p^2}{2(m_\mu + m_d)}.$$  

(34)

In Eqs. (30–34), $m_\mu$, $m_n$, $m_d$, $m_{3\text{H}}$, $m_{3\text{He}}$ are the muon, neutron, deuteron, $^3\text{H}$ and $^3\text{He}$ masses. The integration over $\hat{p}$ in Eq. (33) is performed numerically using Gauss-Legendre points. A limited number of them, of the order of 10, is necessary to achieve convergence to better than 1 part in $10^3$. In order to calculate the total capture rate $\Gamma^D$, the differential capture rate is plotted versus $p$, and numerically integrated. Usually, about 30 points in $p$ are enough for this integration in each partial wave.  

3.2. Results

Theoretical work on reactions (7) and (8) is just as extensive as the experimental one (see Sec. 2). A list of publications, updated to the late nineties, is given in Table 4.1 of Ref. 22, in Ref. 23 and Ref. 24. Here, we limit our considerations to the calculations performed since the year 2000. The starting point will be our studies of Refs. 39 and 40, for both muon capture reactions under consideration, as, to our knowledge, are the most recent ones published. These results will be compared with the calculations of Ando et al. 43 and Ricci et al. 44 for reaction (7), and Gazit 45 for reaction (8). We will comment also on our early study of reaction (8) 40, and on the results of Ho et al. 46 and Chen et al. 47. The most recent studies (available only as preprint) of Refs. 48 and 49 will be also briefly discussed.

The theoretical results of Refs. 39, 43 and 44 for the capture rate $\Gamma^D$ of reaction (7) from the initial doublet hyperfine state and of Refs. 39, 40 and 45 for the total capture rate $\Gamma_0$ of reaction (8) are summarized in Tables 2 and 3 respectively.
Table 2. Summary of the theoretical results for the doublet capture rate $\Gamma^D$ (in s$^{-1}$) of muon capture on deuteron. Only the calculations after the year 2000 are considered. The results obtained with the $^1S_0$ nn final state are also shown. There where possible, the theoretical uncertainty is also indicated.

| Ref.            | $\Gamma^D$ | $\Gamma^D(^1S_0)$ |
|-----------------|------------|-------------------|
| Ando et al.     | 386        | 254 ± 1           |
| Ricci et al.    | 423 ± 7    | 261 ± 7           |
| Marcucci et al. | 392 ± 2.3  | 248.6 ± 2.7       |

Let us review our work of Ref. 39. The first ingredient for any theoretical study of the reactions under consideration is the realistic Hamiltonian model used to describe the initial and final $A = 2$ and 3 nuclear wave functions entering in Eqs. (12) and (13). Two representative two-nucleon interaction models have been used, the phenomenological Argonne $v_{18}$ (AV18) 50 and the potential derived within chiral effective field theory ($\chi$EFT) up to next-to-next-to-next-to leading order (N3LO) by Entem and Machleidt 51. These two models both reproduce the deuteron observables and the large two-nucleon scattering database with a $\chi^2$/datum $\simeq 1$. Given the significant differences in their derivation and structure, they are believed to be a representative subset of the accurate two-nucleon interaction models available in the literature. To accurately describe the $A = 3$ nuclear systems, it is well known that the two-nucleon potentials need to be augmented by three-nucleon interactions. The Urbana IX (UIX) 52 model has been used in conjunction with the AV18, and the chiral three-nucleon interaction, derived up to next-to-next-to-leading order (N2LO) in Ref. 53, has been used together with the N3LO. The hyperspherical-harmonics (HH) method has been used to solve the $A$-body bound and scattering problem, also in the context of $A = 2$ systems, for which of course wave functions could have been obtained by direct solution of the Schrödinger equation. The HH method for $A \geq 3$ has been reviewed in considerable detail in a series of recent publications 54,55,56.

The weak current consists of polar- and axial-vector components, derived within two different frameworks, the “Standard Nuclear Physics Approach” (SNPA) and $\chi$EFT. The first one goes beyond the impulse approximation, by including meson-
exchange currents (MEC’s) and terms arising from the excitation of Δ-isobar degrees of freedom. The second approach includes two-body contributions derived in heavy-baryon chiral perturbation theory (HBχPT) within a systematic expansion, up to N3LO \[21,57\]. Since the transition operator matrix elements are calculated using phenomenological wave functions, this second approach is a “hybrid” χEFT approach (χEFT*). Here we briefly review the main characteristics of the weak current operator, both within SNPA and χEFT*. We consider only the contributions beyond the one-body term, as the one-body operators can be easily obtained performing a non-relativistic reduction of the single-nucleon weak current of Eq. 4, retaining corrections up to order \( (q^2/M_N^2)^3 \) \[20\].

The polar (scalar) weak current (charge) operator is related to the isovector part of the electromagnetic current (charge) via the CVC hypothesis. In SNPA, no free parameters are present in the model for the electromagnetic operator, which is able to reproduce the trinucleon magnetic moments to better than 1% \[59\] as well as a large variety of electromagnetic observables \[58,59,60\]. In the case of χEFT*, no two-body contributions to the scalar charge operator are present at N3LO, while the vector current is decomposed into four terms \[57\]: the soft one-pion exchange \((1\pi)\) term, vertex corrections to the one-pion exchange \((1\pi C)\), the two-pion exchange \((2\pi)\), and a contact-term contribution. Their explicit expressions can be found in Ref. \[57\]. All the \(1\pi, 1\pi C\) and \(2\pi\) contributions contain low-energy constants (LEC’s) estimated using resonance saturation arguments, and Yukawa functions obtained by performing the Fourier transform from momentum- to coordinate-space with a Gaussian regulator characterized by a cutoff \(\Lambda\). This cutoff determines the momentum scale below which these χEFT currents are expected to be valid, i.e. \(\Lambda=500\div800\) MeV \[21\]. The contact-term electromagnetic contribution is given as sum of two terms, isoscalar and isovector, each one with a LEC in front \((g_{4S} \text{ and } g_{4V})\), fixed to reproduce the experimental values of \(A=3\) magnetic moments. The resulting LEC’s are given in Table V of Ref. \[39\] and listed again in Table 4 for completeness. The uncertainties on \(g_{4S}\) and \(g_{4V}\) are not due to the experimental errors on the triton and \(^3\text{He}\) magnetic moments, which are in fact negligible, rather to numerics.

| \(\Lambda\) (MeV) | \(g_{4S}\) | \(g_{4V}\) | \(d_R\) |
|-------------------|------------|------------|--------|
| 500               | 0.69±0.01  | 2.065±0.006| 0.97±0.07 |
| \text{AV18/UIX}   | 0.55±0.01  | 0.793±0.006| 1.75±0.08 |
| 800               | 0.25±0.02  | -1.07±0.01 | 3.89±0.10 |
| \text{N3LO/N2LO}  | 0.11±0.01  | 3.124±0.006| 1.00±0.09 |
The two-body axial current operators in SNPA as used in Ref. [39] as well as in the studies of the \( pp \) and \( hep \) reactions [19,20], can be divided in two classes: the operators of the first class are derived from \( \pi \)- and \( \rho \)-meson exchanges and the \( \rho \pi \)-transition mechanism. These mesonic operators give rather small contributions [39]. The operators in the second class are those that give the largest two-body contributions, and are due to \( \Delta \)-isobar excitation [19,20]. In particular, in the dominant \( N \)-to-\( \Delta \)-transition axial current, the \( N \)-to-\( \Delta \) axial coupling constant (\( g_A^* \)) is retained as a parameter and is determined by fitting the experimental Gamow-Teller matrix element of tritium \( \beta \)-decay (\( GT^{\text{EXP}} \)). Also the pseudoscalar term in the \( N \)-to-\( \Delta \)-transition axial current is retained. It is important to note that the value of \( g_A^* \) depends on how the \( \Delta \)-isobar degrees of freedom are treated. In the muon capture studies presented here, the two-body \( \Delta \)-excitation axial operator is derived in the static \( \Delta \) approximation, using first-order perturbation theory. This approach is considerably simpler than that adopted in Ref. [20] where the \( \Delta \) degrees of freedom were treated non-perturbatively, within the so-called transition-correlation operator approach, by retaining them explicitly in the nuclear wave functions [61]. The results for \( g_A^* \) obtained within the two schemes differ by more than a factor of 2 [20], but the results for the observables calculated consistently within the two different approaches are typically within 1 % of each other. To be noticed that the presented SNPA two-nucleon weak current is not the only model available in the literature. In fact, in Ref. [44] two-body MEC’s are derived from the hard pion chiral Lagrangians of the \( N\Delta\pi\rho\omega a_1 \) system, and are not constrained to reproduce any experimental observable, like \( GT^{\text{EXP}} \). This is typically responsible for large model-dependence in the results, as some of the coupling constants and cutoff parameters entering the axial current are poorly known.

The two-body axial current operator in \( \chi \text{EFT} \) consists of two contributions: a one-pion exchange term and a two-nucleon contact-term. The explicit expressions for these terms can be found in Ref. [21]. While the coupling constants which appear in the one-pion exchange term are fixed by \( \pi N \) data, the LEC which determines the strength of the contact-term \( (d_R) \) has been fixed by reproducing \( GT^{\text{EXP}} \). The values of \( d_R \) for \( \Lambda =500\div800 \) MeV are given in Table 4 [39]. The experimental error on \( GT^{\text{EXP}} \) is primarily responsible for the uncertainty in \( d_R \).

Our results of Ref. [39] for reaction (7) are compared in Table 2 with those of two previous calculations, performed in SNPA [44] and \( \chi \text{EFT}^* \) [43]. The first one uses the Nijmegen I and Nijmegen 93 [62] Hamiltonian models to obtain the nuclear wave functions, and MEC’s derived from the Lagrangians of the \( N\Delta\pi\rho\omega a_1 \) system. The second calculation uses the AV18 [50] potential to derive the wave functions, and the same \( \chi \text{EFT} \) weak current model presented above, constrained to reproduce \( GT^{\text{EXP}} \) in tritium \( \beta \)-decay. However, the \( 1\pi C, 2\pi \) and contact-term contributions to the weak vector current are not included. Furthermore, only the \( S \)-wave contribution in the \( nn \) final scattering state (the \( ^1S_0 \) state) is retained, and higher partial-wave contributions are estimated based on Ref. [63]. By inspection of Table we can conclude that: (i) our calculated \( \Gamma^D \) values are in good agreement with the results of Ando...
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et al. \cite{13}, and the small existing differences has been traced back to the inclusion in the weak vector current of the $1\pi C$, $2\pi$ and contact-term contributions \cite{19}.

(ii) The calculated $\Gamma^D$ value of Ricci et al. \cite{14} differs from the other results by 7\%-10\%.

In order to investigate the origin of the discrepancies between our results \cite{39} and those of Ando et al. \cite{13} on one side, and the results of Ricci et al. \cite{14} on the other, we have repeated \cite{12} the calculation of $\Gamma^D$ (and $\Gamma_0$), including the so-called “potential currents”, i.e. those operators arising when PCAC is implemented at the two-body level. It was argued in fact by Ricci et al. \cite{14} that “omitting the potential current causes an enhancement of the doublet transition rate $\Lambda_1/2$ [i.e. $\Gamma^D$] by $\simeq 1\%$”. These currents were first constructed in Ref. 64, and we have recently reviewed them in Ref. 42, where their explicit expression can be found. The calculation has been performed within $\chi$EFT*, using the AV18 (AV18/UIX for $A = 3$) Hamiltonian model. Again the LEC which determines the strength of the axial current contact-term has been fixed by reproducing GT$_{\text{EXP}}$. The results for $\Gamma^D$ (and $\Gamma_0$) are $393.2 \pm 0.8$ s$^{-1}$ ($1488 \pm 9$ s$^{-1}$), with $\Gamma^D(1S_0)=250.1 \pm 0.8$ s$^{-1}$, in perfect agreement with our previous results of Ref. 39. From this we can conclude that the potential currents proposed by Ricci et al. \cite{14} give negligible contributions to the rate $\Gamma^D$ (and $\Gamma_0$), and the discrepancy between the theoretical calculations is still a puzzling problem. From a historical point of view, it should be noticed that such a discrepancy between different theoretical results for $\Gamma^D$ already existed in the early nineties \cite{63,66,65}. In fact, Adam and Truhl'ík \cite{66} found $\Gamma^D = 416 \pm 7$ s$^{-1}$, while Tatara et al. \cite{63} and Doi et al. \cite{65} found $\Gamma^D = 399$ s$^{-1}$ and 402 s$^{-1}$, respectively.

Finally, we should also mention that a calculation of $\Gamma^D$ has been performed within pionless EFT by Chen et al. \cite{47}. The objective of this work, however, is not to predict $\Gamma^D$, but rather to find the relation between the two-nucleon axial current matrix element entering the muon capture rate on deuteron and the $pp$ weak capture. Within this approach, therefore, a precise experimental determination of $\Gamma^D$ will put a stringent constraint on this matrix element, and consequently on the $pp$ weak capture rate.

Using $A = 3$ nuclear wave functions derived from the AV18/UIX or N3LO/N2LO Hamiltonian models, and the same SNPA or $\chi$EFT* weak charge and current operators presented above, we have studied also the total capture rate $\Gamma_0$ for reaction \cite{8,39}. The results are shown in Table 8 and are compared with other theoretical works of the last ten years \cite{40,45}. Our calculation of Ref. 40 represents the first attempt to study muon capture on $^3$He in a way that is consistent with the approach adopted for the weak proton capture reactions $pp$ and $hep$ \cite{19,20}. The nuclear wave functions were obtained, within the HH method, from the AV18/UIX Hamiltonian model, and the nuclear weak current was derived within the SNPA, as presented above. The theoretical uncertainty reported in Table 8 for $\Gamma_0$ results from the adopted fitting procedure and experimental error on GT$_{\text{EXP}}$. Note that a calculation based on the older Argonne $v_{14}$ (AV14) \cite{67} two-nucleon and Tucson-Melbourne (TM) \cite{68} Hamiltonian model yielded a $\Gamma_0$ of $1486 \pm 8$ s$^{-1}$, suggesting a
weak model-dependence. In fact, we have demonstrated \(^{40}\) that \(\Gamma_0\) roughly scales as the triton binding energy. Therefore, any meaningful comparison between results obtained using different Hamiltonian models requires the inclusion of three-nucleon forces. This is the reason why we have not considered in Table 3 the results of Ho et al. \(^{46}\), obtained, within the SNPA, without the inclusion of MEC’s and, most important, three-nucleon interaction. In Ref. \(^{40}\) we provide also the only available recent theoretical prediction for the recoil asymmetry \(A_v\), found to be \(0.5350 \pm 0.0014\) with the AV18/UIX, in agreement with the experimental result of Ref. \(^{36}\), \(0.63 \pm 0.09\) (stat.)\(^{11}\), \(-0.14\) (syst.). The results for \(A_v\) are very little model-dependent, but very sensitive to \(G_{PS}(q^2)\): \(A_v\) would vary by roughly 20\%, if \(G_{PS}(q^2)\) would be 50\% larger than the PCAC value (see Fig. 1 of Ref. \(^{40}\)). The corresponding variation for \(\Gamma_0\) would be of the order of 5\%.

The first study of reaction (8) within \(\chi\)EFT* approach has been performed by Gazit \(^{45}\). The nuclear wave functions have been obtained with the Effective Interaction HH method \(^{69}\), and the \(\chi\)EFT weak current presented above. However, as in Ref. \(^{43}\) no \(1\pi C\), \(2\pi\) and contact-term contributions to the weak vector current are retained. The theoretical uncertainty reported in Table 3 has two main sources: the experimental uncertainty on the triton half-life, and the calculation of electroweak radiative corrections \(^{13}\). Few comments are here in order: (i) electroweak radiative corrections were not included in our studies of Refs. \(^{39}\) and \(^{40}\). Were to be included, the central value for \(\Gamma_0\) would become 1493 s\(^{-1}\) for both calculations, in nice agreement with the result of Gazit \(^{45}\). (ii) The comparison between our study of Ref. \(^{40}\) and that of Gazit \(^{45}\) suggests that the SNPA and \(\chi\)EFT* results nicely agree, when the MEC’s are constrained to reproduce GT\(^{\text{EXP}}\). We have verified this observation \(^{39}\) for both reaction (7) and (8). However, we have shown that \(1\pi C, 2\pi\), and contact terms in the mesonic \(\chi\)EFT vector current are important in order to achieve such an agreement. If they were to be neglected, \(\Gamma_0\) would be 1453 s\(^{-1}\) \(^{39}\).

Finally, we recall the studies of the early nineties by Congleton and Fearing \(^{70}\) and Congleton and Truhoř\(^{71}\). In the latter work, the nuclear wave functions were obtained from the AV14/TM Hamiltonian model and the nuclear weak current retained contributions similar to those of Ref. \(^{44}\). The value obtained for the total capture rate \(\Gamma_0\) was \(1502 \pm 32\) s\(^{-1}\), the uncertainty due to poor knowledge of coupling constants and cutoff parameters.

Only very recently, the first steps to study muon capture reactions in a consistent \(\chi\)EFT framework have been done \(^{48,49}\). Although the results are not yet published, the main ingredients of a \(\chi\)EFT calculation are outlined. In particular, in Ref. \(^{48}\) we have used the N3LO and N3LO/N2LO interaction models and the \(\chi\)EFT weak current operator presented above. Furthermore, the LEC \(d_R\) determining the strength of the axial current contact-term, and the LEC \(c_D\), entering the contact-term three-nucleon interaction at N2LO, have been related, as suggested in
Ref. 72 and 73, as

$$d_R = \frac{M_N}{\Lambda^* g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}, \quad (35)$$

where $c_3$ and $c_4$ are the LEC’s of the $\pi N$ Lagrangian, already part of the chiral two-nucleon potential at NLO, and $\Lambda^* = 700$ MeV is the chiral-symmetry-breaking scale. Then, the calculation is implemented in the following steps: (i) all the LEC’s present in the interaction and in the current are set consistently, and the same cutoff regulator, i.e. $\exp[-(q/\Lambda)^4]$, is used both in the current and in the N2LO three-nucleon interaction. (ii) The $^3$H and $^3$He ground state wave functions are calculated, within the HH method, using the N3LO/N2LO Hamiltonian model, for two values of $\Lambda = 500$ and 600 MeV 51, 74, and the set of values $\{c_D, c_E\}$ are determined, for which the $A = 3$ experimental binding energies (BE’s) are reproduced. A wide range of $c_D$ values has been spanned, and in correspondence to each $c_D$ in this range, $c_E$ has been fixed in order to reproduce either BE($^3$H) or BE($^3$He). (iii) For each set of $\{c_D, c_E\}$, the triton and $^3$He wave functions are calculated and, using the $\chi$EFT axial weak current discussed above, the Gamow-Teller matrix element of tritium $\beta$-decay (GT$_{TH}$) is determined. This allows to determine the range of $c_D$ values for which GT$_{TH} = \text{GT}^{\text{EXP}}$ within the experimental error. A corresponding range for $c_E$ is given from the previous step. (iv) For the minimum and maximum values of $\{c_D; c_E\}$ in the selected range, the LEC’s $g_{4S}$ and $g_{4V}$ entering the two-nucleon contact terms of the electromagnetic current, and therefore the weak vector current, are determined by reproducing the $A = 3$ magnetic moments. At this point, the potential and current models are fully constrained, and the results for $\Gamma^D$ and $\Gamma_0$ are $\chi$EFT predictions. They are found to be $\Gamma^D = 399 \pm 3$ s$^{-1}$ ($\Gamma^D(1S_0^+) = 255 \pm 1$ s$^{-1}$) and $\Gamma_0 = 1494 \pm 21$ s$^{-1}$, including electroweak radiative corrections 13. These results are in good agreement with the ones of the other calculations mentioned above, except for that of Ricci et al. 44, for which the discrepancy remains of the order of 4÷9 %. On the other hand, in a similar calculation, Adam et al. 49 have found $\Gamma^D$ in the range 401.2 s$^{-1}$÷436.6 s$^{-1}$, depending on the $\chi$EFT two-nucleon potential used.

We conclude remarking that a comparison between the calculated and measured rates for muon capture on $^3$He makes it possible to put a constraint on the induced pseudoscalar form factor $G_{PS}(q^2)$ at $q^2_0 = -0.954 m_N^2$, relevant for this reaction. A similar comparison could be done for the muon capture on deuteron. However, being the available experimental data so uncertain, such a comparison would be less significant. Within the $\chi$EFT approach, we have varied $G_{PS}(q^2_0)$ to match the theoretical upper (lower) value with the experimental lower (upper) value for $\Gamma_0$ 15. This has allowed to obtain for $G_{PS}(q^2_0) = 8.2 \pm 0.7$, in very good agreement with the $\chi$PT prediction of Eq. (6), which gives $G_{PS}^{\chi\text{PT}}(q^2_0) = 7.99 \pm 0.20$ 15.
4. Conclusions

Muon capture reactions on light nuclei, in particular deuteron and $^3$He, have demonstrated to be an interesting, fruitful and controversial field of research, both experimentally and theoretically. The work on this subject has been extensive, and the last few years have seen even a growth of interest and research. At this point, the experimental situation can be summarized as follows: (i) the total rate for muon capture on $^3$He, reaction (8), is very well determined, with an accuracy of 0.3 %, hard to be reached by any present theoretical calculation. (ii) The angular correlation parameters, or so-called recoil asymmetries, are poorly known. Only the vector asymmetry $A_v$ has been measured, but the experimental error is still very large, of the order of $\sim 30 \%$. (iii) The rate for muon capture on deuteron, reaction (7), from the doublet hyperfine state is also poorly known, with experimental values which agree among each other, but have uncertainties of $6\div10 \%$. However, the ongoing experiment performed by the MuSun Collaboration at PSI will clarify the situation and determine the rate with a precision of 1.5 %.

The theoretical situation is evolving very fast, and a large effort has been put in the past few years to reduce as much as possible the theoretical uncertainty on the calculated observables. In particular, it has been shown that the model-dependence, relative to the adopted models for the nuclear interaction and weak currents, can be strongly reduced by fitting the unknown parameters of the nuclear currents to some significant observables, as the tritium half-life and the $A=3$ magnetic moments. A crucial role in these calculations is played by the numerical techniques used to calculate the few-body wave functions, with the considered accurate (and highly complex) Hamiltonian models. Without such a fundamental ingredient, all the calculations mentioned above would be affected by a much larger uncertainty. However, significant discrepancies remain between the available theoretical calculations of the rate for muon capture on deuteron, reaction (7), and their origin is still to be understood. Finally, the first steps toward a $\chi$EFT, and ultimately QCD-based, prediction have been made.

Considering the two main motivations to study muon capture on light nuclei, i.e. (i) to provide significant tests for the theoretical frameworks used in the study of reactions of astrophysical interest not accessible experimentally, and (ii) to extract the value for the induced pseudoscalar form factor $G_{PS}(q^2)$ and ultimately validate the $\chi$PT predictions, further investigations are highly recommended. In particular, there are very few studies on the muon capture reactions on $^3$He in the two- and three-body breakup channels, both theoretically and experimentally. Furthermore, an accurate measurement for the angular correlation parameters of muon capture on $^3$He could put even a more stringent constraint on $G_{PS}(q^2)$.

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References
1. A. Czarnecki, G.P. Lapage and W.J. Marciano, Phys. Rev. D 61, 073001 (2000).
2. H. Primakoff, Rev. Mod. Phys. 31, 802 (1959).
3. S. Weinberg, Phys. Rev. 112, 1375 (1958).
4. N. Severijns, M. Beck and O. Naviliat-Cuncic, Rev. Mod. Phys. 78, 991 (2006).
5. J.D. Bjorken and S.D. Drell, Relativistic Quantum Mechanics (McGraw-Hill, New York, 1964).
6. C.E. Hyde-Wright and K. de Jager, Ann. Rev. Nucl. Part. Sci. 54, 217 (2004).
7. Particle Data Group (C. Amsler et al.), Phys. Lett. B 667, 1 (2008).
8. E. Amaldi, S. Fubini and G. Furlan, Electroproduction at Low Energy and Hadron Form Factors, Springer Tracts in Mod. Phys., Vol. 83 (Springer-Verlag, New York, 1979), p. 1.
9. T. Kitagaki et al., Phys. Rev. D 28, 436 (1983).
10. C. Juszczak, J.T. Sobczyk and J. Zmuda, Phys. Rev. C 82, 045502 (2010).
11. A.P. Serebrov et al., Phys. Rev. C 78, 035505 (2008); ibid. 82, 035501 (2010).
12. MuCap Collab. (V.A. Andreev et al.), Phys. Rev. Lett. 99, 032007 (2007).
13. A. Czarnecki, W.J. Marciano and A. Sirlin, Phys. Rev. Lett. 99, 032003 (2007).
14. S.L. Adler and Y. Dothan, Phys. Rev. 151, 1267 (1966).
15. V. Bernard, N. Kaiser and U.-G. Meissner, Phys. Rev. D 50, 6899 (1994); D. Gazit and N. Barnea, Phys. Rev. D 50, 192501 (2007).
16. R. Schiavilla et al., Phys. Rev. C 58, 1263 (1998).
17. L.E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky and S. Rosati, Phys. Rev. Lett. 84, 5059 (2000); L.E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky, S. Rosati and J.F. Beacom, Phys. Rev. C 63, 015801 (2000).
18. T.-S. Park et al., Phys. Rev. C 67, 055206 (2003).
19. D.F. Measday, Phys. Rep. 354, 243 (2001).
20. T. Gorringe and H.W. Fearing, Rev. Mod. Phys. 76, 31 (2004).
21. A. Kammel and K. Kubodera, Ann. Rev. Nucl. Part. Sci. 60, 327 (2010).
22. M. Cargnelli et al., Workshop on fundamental µ physics, Los Alamos, 1986, LA 10714C; Nuclear Weak Process and Nuclear Structure, Yamada Conference XXIII, ed. M. Morita, H. Ejiri, H. Ohtsubo, and T. Sato (World Scientific, Singapore), p. 115 (1989).
23. I.V. Falomkin et al., Phys. Lett. 3, 229 (1963).
24. O.A. Zaimidoroga et al., Phys. Lett. 6, 100 (1963).
25. L.B. Auerbach et al., Phys. Rev. 138, B127 (1965).
26. D.R. Clay, J.W. Keuffel, R.L. Wagner and R.M. Edelstein, Phys. Rev. 140, B587 (1965).
35. P. Ackerbauer et al., *Phys. Lett. B* **417**, 224 (1998).
36. P.A. Souder et al., *Nucl. Instr. and Meth. in Phys. Res. A* **402**, 311 (1998).
37. J.D. Walecka, *Theoretical Nuclear and Subnuclear Physics* (Oxford University Press, New York, 1995).
38. J.C. Hardy et al., *Nucl. Phys. A* **509**, 429 (1990).
39. L.E. Marcucci et al., *Phys. Rev. C* **83**, 014002 (2011); L.E. Marcucci, *Few-Body Syst.* **50**, 383 (2011).
40. L.E. Marcucci, R. Schiavilla, S. Rosati, A. Kievsky and M. Viviani, *Phys. Rev. C* **66**, 054003 (2002).
41. A.R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, 1957).
42. L.E. Marcucci and M. Piarulli, *Few-Body Syst.* **49**, 35 (2011).
43. S. Ando et al., *Phys. Lett. B* **533**, 25 (2002).
44. P. Ricci, E. Truhlík, B. Mosconi and J. Smejkal, *Nucl. Phys. A* **837**, 110 (2010).
45. D. Gazit, *Phys. Lett. B* **666**, 472 (2008).
46. E.C.Y. Ho, H.W. Fearing and W. Schadow, *Phys. Rev. C* **65**, 065501 (2002).
47. J.-W. Chen, T. Inoue, X. Ji, and Y. Li, *Phys. Rev. C* **72**, 061001(R) (2005).
48. L.E. Marcucci et al., *arXiv:1109.5563*
49. J. Adam et al., *arXiv:1110.3183*
50. R.B. Wiringa, V.G.J. Stoks, and R. Schiavilla, *Phys. Rev. C* **51**, 38 (1995).
51. D.R. Entem and R. Machleidt, *Phys. Rev. C* **68**, 041001 (2003).
52. B.S. Pudliner, V.R. Pandharipande, J. Carlson and R.B. Wiringa, *Phys. Rev. Lett.* **74**, 4396 (1995).
53. P. Navrátil, *Few-Body Syst.* **41**, 117 (2007).
54. A. Kievsky et al., *J. Phys. G: Nucl. Part. Phys.* **35**, 063101 (2008).
55. M. Viviani et al., *Few-Body Syst.* **39**, 150 (2006).
56. L.E Marcucci, A. Kievsky, L. Girlanda, S. Rosati, and M. Viviani, *Phys. Rev. C* **80**, 034003 (2009).
57. Y.-H. Song, R. Lazauskas and T.-S. Park, *Phys. Rev. C* **79**, 064002 (2009).
58. J. Carlson and R. Schiavilla, *Rev. Mod. Phys.* **70**, 743 (1998).
59. L.E. Marcucci, M. Viviani, R. Schiavilla, A. Kievsky and S. Rosati, *Phys. Rev. C* **72**, 014001 (2005).
60. L.E. Marcucci, M. Pervin, S.C. Pieper, R. Schiavilla and R.B. Wiringa, *Phys. Rev. C* **78**, 065501 (2008).
61. R. Schiavilla, R.B. Wiringa, V.R. Pandharipande and J. Carlson, *Phys. Rev. C* **45**, 2628 (1992).
62. V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen and J.J. de Swart, *Phys. Rev. C* **49**, 2950 (1994).
63. N. Tatara, Y. Kohyama and K. Kubodera, *Phys. Rev. C* **42**, 1694 (1990).
64. B. Mosconi, P. Ricci and E. Truhlík, *Eur. Phys. J. A* **25**, 283 (2005).
65. M. Doi et al., *Nucl. Phys. A* **511**, 507 (1990); *Prog. Theor. Phys.* **86**, 13 (1991).
66. J. Adam and E. Truhlík, *Nucl. Phys. A* **507**, 675 (1990).
67. R.B. Wiringa, R.A. Smith and T.L. Ainsworth, *Phys. Rev. C* **29**, 1207 (1984).
68. S.A. Coon et al., *Nucl. Phys. A* **317**, 242 (1979).
69. N. Barnea, W. Leidemann and G. Orlandini, *Phys. Rev. C* **61**, 054001 (2000); *Nucl. Phys. A* **693**, 565 (2001); N. Barnea and A. Novoselsky, *Ann. Phys. (N.Y.)* **256**, 192 (1997).
70. J.G. Congleton and H.W. Fearing, *Nucl. Phys. A* **552**, 534 (1992).
71. J.G. Congleton and E. Truhlík, *Phys. Rev. C* **53**, 956 (1996).
72. A. Gardestig and D.R. Phillips, *Phys. Rev. Lett.* **96**, 232301 (2006).
73. D. Gazit, S. Quaglioni and P. Navrátil, *Phys. Rev. Lett.* **103**, 102502 (2009).
74. R. Machleidt and D.R. Entem, *Phys. Rep.* **503**, 1 (2011).
75. R. Skibiński, J. Golak, H. Witala and W. Glöckle *Phys. Rev. C* **59**, 2384 (1999).
76. S.E. Kuhn *et al.*, *Phys. Rev. C* **50**, 1771 (1994).
77. V. M. Bystritsky *et al.*, *Phys. Rev. A* **69**, 012712 (2004).