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Abstract. A class of discrete flavor-symmetry-based models predicts constrained neutrino mass matrix schemes that lead to specific neutrino mass sum-rules (MSR). We show how these theories may constrain the absolute scale of neutrino mass, leading in most of the cases to a lower bound on the neutrinoless double beta decay effective amplitude.

1. Introduction

The discovery of neutrino oscillations implies non-vanishing neutrino masses and mixing providing one of the most solid indications for physics beyond the Standard Model. The fact that neutrinos have very tiny masses, in contrast to charged leptons and quarks, and that two of the mixing angles are large, is among the deepest theoretical puzzles in particle physics. Since neutrinos carry no electric charge, they are expected on general grounds to be Majorana particles [1], leading to the existence of lepton number violating processes. This intriguing possibility will be hopefully confirmed by the observation of neutrinoless double beta decay ($0\nu\beta\beta$) processes [2, 3]. Indeed, upcoming $0\nu\beta\beta$ experiments are expected to improve the sensitivity by up to about one order of magnitude.

The observed pattern of lepton mixing angles suggests the existence of an underlying flavor symmetry of some sort, either an Abelian [4] symmetry or a non-Abelian one [5]. In the former case one typically obtains texture zeros for the mass matrices but is unable to predict mixing angles. In contrast, non-Abelian flavor symmetries are potentially more powerful, allowing also in principle for mixing angle predictions. As an example, several realizations of non-Abelian discrete flavor symmetry schemes lead to an effective neutrino mass matrix which corresponds to a numerical (parameter-free) prediction for lepton mixing. A popular example of such neutrino mass matrix $M^\nu$ is the tri-bimaximal (TBM) [6] type, characterized by

$$M^\nu = M_{TBM} \equiv \begin{pmatrix} x & y & y \\ y & x+z & y-z \\ y & y-z & x+z \end{pmatrix},$$

(1)

which depends only on three complex parameters $x$, $y$ and $z$. In the mass eigentstate basis the three complex parameters $x$, $y$ and $z$ would correspond to three neutrino mass parameters plus two Majorana CP phases [1, 7], as the Dirac phase disappears since $\theta_{13} = 0$. Many such schemes are characterized by a specific (complex) relation among the parameters $x$, $y$ and $z$ (see [8] for a complete list of models that predicts such relationships), leaving only two free complex parameters, further reducing the number of independent model parameters describing the lepton
sector. In the mass basis these correspond to only two independent neutrino mass eigenvalues (the other follows from the existence of a neutrino mass sum rule), plus two Majorana CP phases (as mentioned, the Dirac phase is unphysical).

2. Mass relations

In this section we focus on a general sub-class of mass matrices leading to a numerical (parameter-free) prediction for the lepton mixing matrix, where the following types of mass relations hold:

\[ A) \quad \chi m_2^\nu + \xi m_3^\nu = m_1^\nu, \]
\[ B) \quad \frac{\chi}{m_2^\nu} + \frac{\xi}{m_3^\nu} = \frac{1}{m_1^\nu}, \]
\[ C) \quad \chi \sqrt{m_2^\nu} + \xi \sqrt{m_3^\nu} = \sqrt{m_1^\nu}. \]

Here \( m_i^\nu = m_i^0 \) denote neutrino mass eigenvalues, up to a Majorana phase factor, while \( \chi \) and \( \xi \) are free parameters which specify the model, taken to be positive without loss of generality. For the sake of completeness, we also consider a fourth mass relation,

\[ D) \quad \frac{\chi}{\sqrt{m_2^\nu}} + \frac{\xi}{\sqrt{m_3^\nu}} = \frac{1}{\sqrt{m_1^\nu}}. \]

These kind of mass sum-rule can be obtained in the scenario of inverse seesaw mechanism [9, 10, 11], which arises when introducing a fermion singlet \( S \) with opposite lepton number with respect to the right-handed neutrinos, so that the effective light neutrino mass matrix is \( m_\nu = m_D M T m_D^T \). Assuming \( m_D \) and \( \mu \) to be proportional to the identity matrix and \( M \sim M_{TBM} \), it is straightforward to show that we can obtain the mass sum-rule of type (D).

3. Lower bound for neutrinoless double-\( \beta \) decay

Let us first consider the amplitude for neutrinoless double-\( \beta \) decay within a flavor-generic model. One can plot the effective neutrino mass parameter \( |m_{ee}| \) determining the \( 0\nu\beta\beta \) decay amplitude, as a function of the lightest neutrino mass. As is well-known, by varying the neutrino oscillation parameters \( \Delta m_{atm}^2 \), \( \Delta m_{sol}^2 \), \( \theta_{12} \), \( \theta_{13} \), \( \theta_{23} \) in their allowed ranges one obtains two types of relatively broad bands in the \( (|m_{ee}|, m_{\text{light}}) \) plane corresponding to normal and inverse hierarchy spectra.

In this “generic” case there is a lower bound on the neutrinoless double-\( \beta \) decay effective mass parameter \( |m_{ee}| \) only in the case of inverse mass hierarchy: due to the possibility of destructive interference among the light neutrinos from the effect of having opposite CP signs or due to the effect of majorana phases, no lower bound can be established for the case of normal hierarchy [12, 13, 14].

Let us now turn to the case where MSR relations like (A), (B), (C) and (D) hold. As discussed above these can be obtained in flavour models where the neutrino mass matrix only depends on two independent free parameters, so that the resulting mixing angles are fixed, like for example for the tri-bimaximal or bimaximal mixing patterns.

For definiteness here we focus on the case where the rotation in the neutrino sector is of tri-bimaximal form. Corrections from higher dimensional operators and/or from the charged lepton sector can yield \( \theta_{13} \neq 0 \).

Hence we retain the TBM approximation as a useful starting point to obtain our MSR relations. However, when evaluating a lower bound on the effective neutrino mass parameter \( |m_{ee}| \) determining the neutrinoless double-\( \beta \) decay amplitude, we include explicitly the effects
of non-vanishing $\theta_{13}$. We do this by taking the values at 3 $\sigma$ determined in Ref. [15]. Such a lower bound can be obtained from the following procedure.

We first consider that the neutrino masses are complex parameters, where the two Majorana phases are encoded in $m_2^\nu$ and $m_3^\nu$, i.e.

\begin{align*}
m_1^\nu &= m_1^0, \quad \text{(6)} \\
m_2^\nu &= m_2^0 e^{i\alpha}, \quad \text{(7)} \\
m_3^\nu &= m_3^0 e^{i\beta}. \quad \text{(8)}
\end{align*}

As shown in Fig. 1, the neutrino mass sum-rule can then be interpreted geometrically as a triangle in the complex plane, whose area provides a measure of Majorana CP violation \(^1\). Each of the above equations (6), (7) and (8) can be split into two independent equations for the real and imaginary parts.

For simplicity let us start from the idealized case where the neutrino oscillation parameters $\Delta m^2_{\text{atm}}$, $\Delta m^2_{\text{sol}}$, $\theta_{12}$, $\theta_{13}$, $\theta_{23}$ are perfectly well-measured quantities. In this case one can extract the two Majorana phases $\alpha$ and $\beta$ as functions of the base of the triangle, which is determined by $m_1^0$ in case of normal hierarchy (NH) or by $m_3^0$ in case of inverted hierarchy (IH), as well as the parameters $\chi$ and $\xi$ labeling the particular model under consideration.

These relations obtained can then be inserted into the general expression of $|m_{ee}|$:

\begin{align*}
|m_{ee}| = \left| c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13} e^{i\alpha} m_2 + s_{13}^2 e^{i\beta} m_3 \right|. \quad \text{(9)}
\end{align*}

For each ($\chi$, $\xi$) model this effective mass parameter depends on a single parameter, namely the length of the triangle base, which gives a measure of the absolute scale of neutrino mass.

For instance, for case (A) this procedure gives:

\begin{align*}
\xi \cos \alpha m_2^0 + \chi \cos \beta m_3^0 &= m_1^0, \quad \text{(10)} \\
\xi \sin \alpha m_2^0 + \chi \sin \beta m_3^0 &= 0,
\end{align*}

\(^1\) As the area shrinks to zero one obtains the CP-conserving limits corresponding to the four independent choices of CP sign [12, 13].
so that the Majorana CP phases are determined as:

\[
\cos \alpha = \frac{m_1^2 - \chi^2 (\Delta m_{\text{atm}}^2 + m_1^2) + \xi^2 (\Delta m_{\text{sol}}^2 + m_1^2)}{2m_1 \chi \sqrt{\Delta m_{\text{atm}}^2 + m_1^2}} \\
\cos \beta = \frac{m_1^2 + \chi^2 (m_2^2 + \Delta m_{\text{atm}}^2) - \xi^2 (m_1^2 + \Delta m_{\text{sol}}^2)}{2m_1 \chi \sqrt{\Delta m_{\text{atm}}^2 + m_1^2}}.
\]

(11)

The lower bound for the lightest neutrino mass can be obtained from our MSR, using the triangle inequality in the complex plane as suggested in [16], see Fig.(1) for a schematic view.

We must first select the biggest side of the triangle; calling them \(x_i\), \(x_j\) and \(x_k\), we can obtain a lower limit for the lightest neutrino mass from the triangle inequality

\[|x_i| \leq |x_j| + |x_k|\]

so that the Majorana CP phases are determined as:

\[
\cos \alpha = \frac{m_1^2 - \chi^2 (\Delta m_{\text{atm}}^2 + m_1^2) + \xi^2 (\Delta m_{\text{sol}}^2 + m_1^2)}{2m_1 \chi \sqrt{\Delta m_{\text{atm}}^2 + m_1^2}} \\
\cos \beta = \frac{m_1^2 + \chi^2 (m_2^2 + \Delta m_{\text{atm}}^2) - \xi^2 (m_1^2 + \Delta m_{\text{sol}}^2)}{2m_1 \chi \sqrt{\Delta m_{\text{atm}}^2 + m_1^2}}.
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We must first select the biggest side of the triangle; calling them \(x_i\), \(x_j\) and \(x_k\), then the triangle inequality \(|x_i| \leq |x_j| + |x_k|\) must be fulfilled, where \(|x_i| \equiv \text{Max}(|x_1|, |x_2|, |x_3|)\) and \(i \neq j \neq k\). In case (A) and assuming NH for the neutrino mass spectrum, we always have \((\chi m_2^0, \xi m_3^0) > m_1^0\) and the largest side of the triangle can be either \(\chi m_2^0\) or \(\xi m_3^0\); so we must consider separately these two cases. After rewriting two masses in terms of the two squared mass differences, we can obtain a lower limit for the lightest neutrino mass from the triangle inequality \(|x_i| \leq |x_j| + |x_k|\). For the other cases we follow the same procedure. The lower bound on the lightest neutrino mass obtained in this way is then used to estimate the lower bound for \(|m_{ee}|\) from the general expression in eq. (9). Notice that, although we focus here on TBM schemes, some of the MSR considered in our analysis can also be derived using bimaximal mixing as a starting point.

**Classification**

| \(\chi, \xi\) | A—NHA | IH | Ref. | B—NH | B—IH | Ref. | C—NHC | C—IH | Ref. | D—NHD | D—IH | Ref. |
|----------------|--------|-----|------|-------|-------|------|-------|-------|------|-------|-------|------|
| 1,1             | 0.010  | 0.044     | †   | 0.008  | 0.036 | †   | 0.006  | 0.029 | -    | 0.005  | 0.008  | -    |
| 1,2             | *      | 0.046 | -    | 0.008  | 0.027 | -    | *      | 0.014 | -    | 0.004  | 0.026 | †    |
| 1,3             | *      | 0.011 | -    | 0.030  | 0.005 | -    | *      | 0.014 | -    | 0.018  | 0.025 | -    |
| 2,1             | 0.006  | *     | †    | 0.006  | 0.007 | †    | 0.000  | *     | †    | *      | 0.007 | -    |
| 2,2             | 0.019  | 0.026 | -    | 0.023  | 0.008 | -    | 0.017  | *     | -    | 0.003  | 0.015 | -    |
| 2,3             | *      | 0.044 | †    | 0.007  | 0.008 | -    | *      | 0.031 | -    | 0.005  | 0.026 | -    |
| 3,1             | 0.004  | *     | -    | 0.004  | 0.008 | -    | *      | *     | -    | *      |      | -    |
| 3,2             | 0.011  | *     | -    | 0.004  | 0.021 | -    | 0.000  | *     | -    | *      | 0.007 | -    |
| 3,3             | 0.023  | 0.061 | -    | 0.029  | 0.031 | -    | 0.011  | 0.019 | -    | 0.018  | 0.016 | -    |

**Table 1.** Minimal values for the effective 0νββ decay mass parameter \(|m_{ee}|\), in eV, see text for details.

The results obtained from the procedure discussed above are summarized in Tab.1, where we report the lower limits of \(|m_{ee}|\) corresponding to different integer choices of \((\chi, \xi)\) between 1 and 3 and for each of the four MSR considered in eqs. (2)-(5), for both normal and inverted hierarchies. The cases already discussed in the literature are indicated by a †, see [8] for a complete list of references. The entries denoted with the symbol (*) represent situations that do not satisfy the inequality for any value of the lightest neutrino mass \(m_{\text{light}}\). Cases marked by a (-) correspond to models which, as far as we can tell, have not been considered. Some comments

2 Notice that there are three inequalities of the type \(|x_i| \leq |x_j| + |x_k|\) obtained by permuting the three indices \(i, j\) and \(k\), but only one of these constrains the lightest neutrino mass.
are in order. First let us consider the effect of a possible non-zero effect of $\theta_{13}$. In Fig 2 we show the prediction for $|m_{ee}|$ as function of $m_{\text{light}}$ obtained from the MSR $\frac{1}{\sqrt{m_2}} + \frac{2}{\sqrt{m_3}} = \frac{1}{\sqrt{m_1}}$. The red band corresponds to the normal hierarchy case and the green band the inverse hierarchy case, varying the values of $\theta_{13}$, $\theta_{23}$ and $\theta_{12}$ within their $3\sigma$ C.L. interval. By looking at Fig. 2

one sees that, indeed, the $0\nu\beta\beta$ lower bound is sensitive to the value of $\theta_{13}$.

One also finds that, as expected on general grounds, all inverse hierarchy schemes corresponding to various choices of $(\chi, \xi)$ within sum-rules A-D have a lower bound for the parameter $|m_{ee}|$. However, the numerical value obtained depends on the MSR scheme, signalling that not all values within the corresponding band in Fig. 2 are covered.

On the other hand, even though normal hierarchy models do not lead to a lower bound on the $0\nu\beta\beta$ amplitude due to the possibility of destructive interference amongst the light neutrinos, one finds that the possibility of full cancellation is precluded for all schemes in the table, except for the (2,1) case considered in Ref. [17] and the (3,2) scheme, both of which correspond to MSR of type (C). All other NH MSR schemes considered here imply a minimum value for the $0\nu\beta\beta$ decay amplitude 3.

In particular, the maximal value we have found for the lower bound on $|m_{ee}|$ is $|m_{ee}| = 0.061$ eV, obtained in correspondence with the set of values $(\chi, \xi) = (3, 3)$ for the case (A) in IH. Such a value for $|m_{ee}|$ lies within the sensitivity of upcoming experiments; hence it would be interesting, from the model building point of view, to find from first principles a flavor-symmetry-based model predicting such a mass relation.

Mass sum rules are classified in four different categories, some have already been considered in the literature. For each case, we have first extracted the allowed numerical values of $|m_{ee}|$, for both mass orderings of the neutrino mass eigenstates and we have then given the behaviour

$3$ Of course some of the bounds are phenomenologically less interesting since they fall outside realistic sensitivities of coming experiments.
of $|m_{ee}|$ as a function of the lightest neutrino mass. Although our MSR schemes were obtained within the TBM anzatz, we have computed the possible values of $|m_{ee}|$ considering all the neutrino parameters (including a non-vanishing $\theta_{13}$) within their $3\sigma$ allowed ranges. In most MSR schemes one finds a lower bound for the $0\nu\beta\beta$ amplitude, even for NH spectra. We find that the most favourable case (large lower bound) corresponds to a sum-rule of type (A) obtained in correspondence of the set of values $(\chi, \xi) = (3, 3), |m_{ee}| = 0.061 \text{ eV}.$

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References
[1] Schechter J and Valle J. W. F. 1980 Phys. Rev. D22, 2227
[2] Schechter J and Valle J W F 1982 Phys. Rev.D D25, 2951
[3] Duerr M, Lindner M and Merle A 2011 J. High Energy Phys. JHEP 1106(2011)091 (Preprint hep-ph/1105.0901)
[4] Froggatt C D and Nielsen H B 1979 Nucl. Phys. B147 277
[5] Ishimori H et al. 2010 Prog. Theor. Phys. Suppl. 183 1 (Preprint hep-ph/1003.3552)
[6] Harrison P F, Perkins D H and Scott D H 2002 Phys. Lett. B530 167
[7] Rodejohann W and Valle J 2011 Phys. Rev. D 84 073011 (Preprint hep-ph/1108.3484)
[8] Dorame L, Meloni D, Morisi S, Peinado E and Valle J W F 2012 Nucl. Phys. B 861 259 (Preprint hep-ph/1111.5614)
[9] Mohapatra R N and Valle J W F 1986 Phys. Rev. D34 1642
[10] Gonzalez-Garcia M C and Valle J W F 1989 Phys. Lett. B216 360
[11] Dorame L, Morisi S, Peinado E, Valle J W F and Rojas A D 2012 Phys. Rev. D 86 056001 (Preprint hep-ph/1203.0155)
[12] Wolfenstein L 1981 Phys. Lett. B107 77
[13] Schechter J and Valle J W F 1981 Phys. Rev. D24 1883; erratum ibid 1982 D25 283
[14] Valle J W F 1983 Phys. Rev. D27 1672
[15] Schwetz T, Tortola M and Valle J W F 2011 New J. Phys. 13 063004
Schwetz T, Tortola M and Valle J W F 2011 New J. Phys. 13 109401
[16] Barry J and Rodejohann W 2011 Nucl. Phys. B842 33
[17] Hirsch M, Morisi S and Valle J W F 2008 Phys. Rev. D78 093007