A Gradient Model for the Spatial Patterns of Cities

Jie Chang, Guofu Yang, Shun Liu, Hanhui Jin, Zhaoping Wu, Ronghua Xu, Yong Min, Kaiwen Zheng, Bin Xu, Weidong Luo, Feng Mao, Ying Ge,* and Kang Hao Cheong*

The dynamics of a city’s structures are determined by the coupling of functional components (such as restaurants) and human population. Yet, there lacks mechanism models to quantify the forces on the spatial distribution of the components. Here, a gradient model is explored to simulate the individual density curves of multiple types of city functional components based on the equilibria of gravitational and repulsive forces along with the urban–rural gradient. The model is concise by relying on four key variables, the attributes of components include net ecosystem service (m) and environmental index (γ); and the attributes of cities include land rent exponent (σ) and population attenuation coefficient (β). The model has been used to simulate the distribution curves of 22 types of components on the urban–rural gradients in 13 cities in two periods. The model reveals a bottom-up mechanism that the patterns of the components in a city are determined by the economic, ecological, and social attributes of both cities and components. Strongly backed by empirical data, the model can predict the distribution curves of many types of components along with the development of cities. This model provides a general tool for analyzing the distribution of multiple objects on the gradients.

1. Introduction

A fascinating event in human activities is the formation, development, expansion, and renewal of cities.[1–3] A city is composed of human beings and multiple types of functional components (or namely facilities), which are enterprises, firms, and other institutions that provide goods and services to people.[4,5] The enclosed industrial components are just defined as organaras (big organs) considering a modern industrial city as a supercell living system, and a city center is a citynucleus.[6] Some types of components, such as banks and restaurants, are concentrated near the citynucleus.[7,8] In contrast, as a city develops, some components, such as manufacturies, tend to move outward from the city center,[8,9] while some emerging industries (such as delivery stores) spring up in the urban area (Figure 1a). The emergence, coexistence, competition, migration, and extinction of the components shape the spatial structure of the city.[9–11] Despite many studies investigating the city structure, some theoretical aspects remain unanswered. For example, how much is the impact of land rent on the distribution of functional components? How the population pattern drives the change of component spatial distribution? How the residents’ ecological preferences drive the change of component distribution? To uncover the evolutionary mechanism of the components, the gradients can deepen the fundamental understanding of the city’s functional spatial structure.
In general, each type of functional component has many individuals in a city.[5,12,13] For example, a type of fast food restaurant can be seen as a group and occupies a niche in a city, a phenomenon akin to a biological population within a community.[14] The present models for the number of components are statistical and they study the response of the local individual density of components—for example, the number of restaurants, schools, hospitals, and banks—to the human population density in a location.[15–17] These models can explain neither the whole city’s structure nor the complex driving forces—apart from the population—behind the distribution of components. A series of spatial economic models, pioneered by von Thünen, can explain the mechanism for the locations of land use types and functional components that are affected by land rent and transport cost gradients.[3,18,19–21] However, these spatial economic models have not involved the impact of people’s ecological preferences. Using concepts from physics, we utilize the relevant theories from geography and try to uncover the mechanism of city structure and its evolution.

1.1. The Spatial Pattern of City Components in Urban–Rural Landscapes

In this paper, a “city” means an urban–rural system, which has been defined as a eukarcty.[6] A great number of various types of functional components separate or overlapping distribute in the city core (citynucleus) and the periphery rural areas (Figure 1a). We investigated 24 types of components and chose four types that implement urban processes that meet the living demands of the local residents. The real-world data of 2 years of the components are investigated and compiled. Results showed that the Kentucky Fried Chicken shops (KFCs) are concentrated near the city nucleus (Figure 1b), the Zhongtong express outlets (ZTOs), which provides domestic parcel service, are concentrated on the outer side of KFCs; the cultivated greenhouses (GHs) are located in suburban areas near the urban fringe, and the dairy farms (DFs) are located in ex-urban areas. Each type of component has \( \approx 10^2 \) to \( \approx 10^3 \) individuals in a city except for dairy farms, which are rare (Table S1, Supporting Information). The individual density of a type of component varied with the distance \( d \) from the city nucleus (Figure 1c). The univariate polynomial regressions show that, for a type of component, the individual density curve has a peak \( P_{\text{max}} \), which is located at a position \( d^* \) along with the urban–rural gradient (Figure 1c; Table S2, Supporting Information). The rank correlation showed that the \( d^* \) of a type of component is related to the supply ability of the target services, which is the economic return to the investor for constructing and operating the components (Figure 1c,d). An exception is that the \( d^* \) of DF is outside GH, although the net service, which is the sum of target and accompanied services, of DF is higher than GH. We find that the higher \( \gamma \) value, which is the ratio of
environmental impacts to target services, pushes DF outside GH (Table S2, Supporting Information), and we denote \( \gamma \) as the “environmental index.” The statistical model\(^{[22]} \) we used can help us acquire the characteristics of component distribution (Table S2, Supporting Information), but cannot uncover the driving factors and mechanisms.

1.2. Modeling Overview

The distribution patterns of the functional components are mainly driven by economic factors.\(^{[19,23]} \) The components, which are artificial systems, need to provide enough economic returns (target services) to the investors. The economic constraints for the distribution of components are the land rent and transport costs,\(^{[23]} \) while the ecological constraints are environmental impacts.\(^{[8]} \) In an ideal city with no geographical difference\(^{[19]} \) and only one citynucleus, the human population density monotonically decreases from the urban center to rural areas, within which the components are located and provide services for people (Figure 2a). The 2D pattern of attributes of city and components can be described as a 1D individual density curve along with the urban–rural gradient, \( P(d) \), in a city (Figure 2b,c). It is emphasized that, although the above principle depicts a single-center city, it is also suitable for polycentric cities. The gradient and equilibrium of gravitational and repulsive forces can also be separated from the sub-center of a polycentric city.

The gravitational force (\( F_g \)) along with an urban–rural gradient for a type of component is mainly determined by the spatial pattern of the product’s transport cost. For the life-support components (such as restaurants), the cost of the products of a component is negatively correlated to the density of the local human population (Figure 2d). The transportation cost depends on human population density, road condition, and product transportability.\(^{[19,24]} \) Densely populated areas attract the distribution of functional components\(^{[15,25]} \) due to a small average transportation distance (\( \bar{r} \)) from products to consumers (Figure S9a, Supporting Information). In the citynucleus, the population
is much higher than in urban fringe and rural areas.\(^{19,24}\) For the human population density in locations in response to the distance \(d\) from city nucleus, the exponent function \(Y = ae^{bd}\) is the best fit that supports Clark\(^{26}\) if the range is within the urban areas; while if we include the suburban area, the power law function will be the best-fit regressions (Table S8, Supporting Information),

\[
Pop(d) = Ad^\beta
\]  
(1)

where \(Pop(d)\) is the average human population density of the ring located at a distance \(d\) from the city nucleus. The \(Pop(d)\) values were measured through random quadrat investigation on the ring (Figure 2a). \(A\) is the fitted coefficient. The attenuation coefficient \((\beta)\) is for the human population density on the urban–rural gradient.

Based on the negative correlation between product transportation distance and population density,\(^{13}\) the mean distance \(\bar{r}\) for a certain type of component at \(d\) ring is,

\[
\bar{r}(d) = -\beta \ln(d)
\]  
(2)

where \(\bar{r}(d)\) is the average “last kilometer” distance between the components’ products and the consumers at location \(d\) on the urban–rural gradient, \(d\) and \(\beta\) are from Equation (1).

The transportation cost also depends on product transportability, which varies greatly among the different types of components. For example, we have found that the transport of KFC’s take-out components away from the city center.\(^{19,28}\) Only the components with high environmental impacts are distributed near the city nucleus, while a larger ecosystem dis-service pushes the component outward (Figure S9c, Supporting Information). The “iceberg transport model” to calculate the value loss in the transportation process\(^{27}\) is introduced to calculate the transport cost \((C_T)\) of the products,

\[
C_T(d) = I \left(1 - e^{-\tau I}\right)
\]  
(3)

where \(C_T(d)\) is the transport cost at location \(d\) on the urban–rural gradient, \(I\) is the coefficient for the initial value of a type of product; and \(\tau \in (0,1)\) is the iceberg coefficient, which is the proportion of value lost per unit of distance transported.

Combining Equations (2) and (3),

\[
F_g(d) = I \left(1 - e^{\beta \ln(d)}\right) = I \left(1 - d^\beta\right)
\]  
(4)

where \(F_g(d)\) is the gravitational force on urban–rural gradient in a city. Equation (4) shows that the attraction is monotonically decreasing from city nucleus to rural area (Figure 2d).

The repulsive force \((F_r)\) along with an urban–rural gradient is monotonically decreasing from city nucleus outward. Generally, the high land rent \((LR)\) repulses some low economic output components away from the city center.\(^{19,28}\) Only the components with high economic output can be located near the city nucleus as they can afford paying the high land rent. We have found that the land rents \((LR)\) in most cities are power law decreasing on the urban–rural gradient,\(^{18}\)

\[
LR(d) = cd^\sigma
\]  
(5)

where \(LR(d)\) is the land rents at location \(d\) on the urban–rural gradient, \(c\) is the land rent coefficient (USD m\(^{-2}\) year\(^{-1}\)), and \(\sigma\) is the land rent attenuation coefficient.

Besides the impact of land rent, a functional component’s economic outputs \((m)\) and people’s preference for environmental impacts also affect the \(F_r\) for a type of component, i.e.,

\[
F_r(d) = LR(d) / (m/\gamma) = cd^\sigma / (m/\gamma)
\]  
(6)

where \(F_r(d)\) is the repulsive force at location \(d\) on the urban–rural gradient, \(\gamma\) is the absolute value, \(m > 0\). Equation (6) means that a larger net ecosystem service enables a component to be distributed near the city nucleus, while a larger ecosystem dis-service pushes the component outward (Figure S9c, Supporting Information). For example, with the social prosperity in recent decades, the components with high environmental impacts are moved far from the city nucleus due to people’s growing preference for better environmental quality.

The minimum of gravitational plus repulsive forces \((F_g + F_r)\), \(E\), is changing with the development of city, and it is also different among different types of components in a city (Figure 2d). The optimum location \((d^*)\) corresponds to \(E\) for a type of component on the urban–rural gradient where the maximum individual density \((P_{max})\) occurs (Figure 2b,c). Each type of component has a \(d^*\) and some of them can overlap due to the fact that they are distributed across different locations within a ring (Figure 2a).

The total amount of a type of component and the \(P_{max}\) in a city is determined by the total demand \((M)\) and the net ecosystem service \((m)\) of the component. We follow the form of Newton’s gravity model and take the product of these two terms \((M \times m)\) as the numerator term. Then the individual density of a type of component along with an urban–rural gradient is

\[
P(d) = \frac{G \times M \times m}{F_g(d) + F_r(d)}
\]  
(7)

where \(P(d)\) is the component density at location \(d\) on the urban–rural gradient and \(G\) is a coefficient to adjust the order of magnitude of the \(M \times m\) multiplier.

When Equations (4) and (6) are substituted into Equation (7), we see that the individual density curve of a type of component on the urban–rural gradient is

\[
P(d) = \frac{G \times M \times m}{I \left(1 - d^\beta\right) + cd^\sigma / (m/\gamma)}
\]  
(8)

The constant of proportionality \((G)\) in Equation (8) will be acquired via the simulation of the real-world data.

1.3. Model Optimization

The input variables were calculated using the above equations and then simulate the individual density curves using Equation (8). The model fitting is based on the Levenberg–Marquardt algorithm, which can provide numerical solutions of nonlinear minimization (local minimum). The parameters and coefficients (Tables S3, S5, S6, and S9, Supporting Information) were input to the nonlinear fitting module (using Origin Pro 2018, OriginLab).
Different cities.

topredict the pattern of components in different periods or different types of components, the gradient model (Equation 9) was used.

Equation 9 is given by:

\[ P(d) = \frac{M \times m}{I (1 - d^{\sigma}) + cd^{\sigma} / \left( \frac{m}{\sigma} \right) + Z} \]

where parameter \( Z \) represents the other factors besides the gravitational and repulsive forces in the model and we dubbed \( Z \) as the “city index” due to it being related to the city attributes revealed by statistics. We did not further simplify the mathematical form of Equation (9) because each parameter has physical significance and corresponds to the mechanism behind the distribution of components.

All the meanings of the model parameters are shown in Table 1. After validating the simulation of the distribution of multiple types of components, the gradient model (Equation 9) was used to predict the patterns of components in different periods or different cities.

| Parameters | Symbol |
|------------|--------|
| Input variable | |
| Total demand for specific goods and services of cities | \( M \) |
| Net ecosystem services of the components | \( m \) |
| Iceberg attenuation index in transportation of products | \( \tau \) |
| Coefficient of statistical function of population distribution | \( \beta \) |
| Distance from city nucleus | \( d \) |
| Coefficient of statistical function of land rent distribution | \( c \) |
| Land rent attenuation coefficient | \( \sigma \) |
| Ratio of ecosystem dis-services to target services | \( \gamma \) |
| Simulated or adjusted parameter | |
| Adjustment constant for \( M \times m \) | \( G \) |
| Coefficient in transport cost function | \( \iota \) |
| City index | \( Z \) |

Corporation) to estimate the coefficients (\( G, \iota \)) by fitting the real-world data of the four types of functional components in 13 cities. The fitting accuracy of the model parameters is evaluated and improved in iteration, and this process continues until the accuracy of the model parameters can no longer be improved. Unfortunately, the simulated distribution individual density curves of KFC, ZTO, and GH on the urban–rural gradient in most cities were not good fits to the data, and particularly, the kurtosis did not conform to the real-world data. This means that Equation (8) overlooks some factors.

After re-evaluation, a new parameter, \( Z \), was introduced into the denominator of the model. Finally, we finish the construction of a gradient model for functional components along with the urban–rural gradients:

\[ P(d) = G \frac{M \times m}{I (1 - d^{\sigma}) + cd^{\sigma} / \left( \frac{m}{\sigma} \right) + Z} \]

where parameter \( Z \) represents the other factors besides the gravitational and repulsive forces in the model and we dubbed \( Z \) as the “city index” due to it being related to the city attributes revealed by statistics. We did not further simplify the mathematical form of Equation (9) because each parameter has physical significance and corresponds to the mechanism behind the distribution of components.

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2. Results

2.1. Model Simulation and Validation

Using the model (Equation 9), we run the fitting module again. The simulated individual density curves coincide with the real-world data in two periods (Figure 3a–d; Table S3, Supporting Information). Results show that the fittings are significant (the minimum \( R^2 > 0.72 \) of the fitting curves). The individual density curves and the relationship between the gravitational and repulsive forces support our hypothesis in Figure 2. The simulation for the individual density curves of the components shows that the repulsive forces to the four components are ordered KFC < ZTO < GH < DF, and so are the gravitational forces (Figure 3e–h). The minimum points (\( F \)) of repulsive plus gravitational forces correspond to the \( d^* \) (Table S2, Supporting Information). For example, along with the development of Shanghai City, the curve of land price and population along with the urban–rural gradients tend to be gentle (absolute values of \( \sigma \) and \( \beta \) decrease), resulting in the \( P_{\text{max}} \) of KFC, ZTO, GH, and DF decreasing by 20%, 29%, 13%, and 21%, respectively, and all the \( d^* \) moving outward (Figure 3a–d). The reason is that with the development of the city, both gravitational and repulsive forces for the four components increase, and the equilibrium changes (Figure 3e–h). All the above model behaviors support our hypothesis in Figure 2, which is that the equilibrium of forces determines the \( d^* \), and the city’s attributes determine \( P_{\text{max}} \). In addition, the \( d^* \) of GH is located at the edge of the built-up area, that reflects the shape and size of the urban area (Figure 3c). This means that the driving force of the components quantified by the gradient model can be used to study the evolution of the scale of cities.

The four types of components simulated by the gradient model in 13 cities showed that all the individual density curves of the components fitted the real-world data significantly except KFC in Shaoxing City and ZTO in Jiaxing City, where there are insufficient individuals for simulation (Figure 3–4; Figure S1, Supporting Information). The individual density curves of KFC, ZTO, GH, and DF were simulated well by gradient (\( P < 0.05 \)), with average \( R^2 \) of 0.88, 0.81, 0.77, and 0.87, respectively. More importantly, the gradient model can simulate the individual density curves of multiple components (based on corresponding coefficients and parameters) in a city at the same time, and simulate their coexistence at any location within cities (Figure 4; Figure S2, Supporting Information). The gradient model clearly shows the mechanism of gravitational-repulsive force shapes the specific niche of components on the urban–rural gradient (Figure S3, Supporting Information).

2.2. Prediction and Application Ability of the Gradient Model

In order to validate the robustness and the predictive power of the gradient model, we analyze the relationship between the coefficients of the model and the attributes of the case cities and components. Regression analysis shows that the Z of the components that are located in urban areas are related to urban central land price (\( c \)), while the Z of components that are located in rural areas are most closely related to urban areas (Figure S4, Supporting Information). However, \( I \) is not related to any attributes. We then try to find if there are indirect relationships, and then the aggregate force of the model, the gravitational and repulsive forces are derived.

\[ F' (d^*) = \left[ I (1 - d^*c\sigma) + cd^* \frac{m}{\sigma} \right] = 0 \]
Figure 3. Simulated individual density curves of the components and the driven forces in Shanghai City in two periods. a–d) Simulated individual density curves and real-world data of the four types of components along with the urban-rural gradient: a) Kentucky Fried Chicken shops; b) ZTO express outlets; c) cultivated greenhouses, red dotted line indicates the location of the urban fringe; d) dairy farms; e–h) the repulsive force (solid line) and the reverse-gravitational force (dashed line) of the components correspond to a–d, respectively.

where the symbols are the same as in Equation (9). When the sum of gravitational and repulsive forces is the minimum, the individual density of components reaches the peak value, and the derivative of the sum of repulsive and gravitational forces is 0. Therefore, $I$ can be calculated via

$$I = \frac{cf\sigma}{\tau\beta m} d^{\alpha-\beta}$$

(11)

The $G$ value of the same type of component can be obtained by the geometric mean of the same component in multiple cities,

$$G = \left( \prod_{i=1}^{N} G_i \right)^{1/N}$$

(12)

where $G_i$ is the constant of proportionality in city $i$, the value of which is obtained by model fitting, and $n$ is the number of the case cities.

The regression analysis showed that the $d^*$ of a type of component was significantly related to the urban population of the 13 cities (Figure S4, Supporting Information). Then $I$ can be derived from $d^*$, which is determined by the gravitational and repulsive force. In order to study the effects of city development on component distribution, the attributes of the latest period in Hangzhou and Ningbo were used to predict the distribution of KFC, ZTO, and GH over time. The predictions matched the three components’ distribution curves well ($R^2 > 0.53$, Figure S5, Supporting Information).

The magnitudes of the average $G$ of KFC and ZTO located in urban areas are both $10^{-12}$, while the magnitudes of the average $G$ of GH and DF located outside the city are $10^{-6}$ and $10^{-8}$, respectively. The difference in the $G$ values of components inside and outside the city is of 4 to 6 orders of magnitude (Table S4, Supporting Information).

We predict the individual density curves of the four types of components in the other two cities (Wuhan and Nanjing City) according to the relationship between the model parameters and the attributes in 13 cities (Figure S4, Supporting Information). The predicted curves match well ($P < 0.05$) with the real-world data of the distribution of components, respectively (Figure S6, Supporting Information). The predictions validate the universality of the model among different cities (Tables S5 and S6, Supporting Information). Furthermore, the gradient model could be extended to simulate many types of components co-existing in an urban-rural system (Figure S7, Supporting Information).

3. Discussion

John von Neumann used to say, “with four parameters I can fit an elephant”.(29) It means that excessive arbitrary parameters made model lose its significance. Fortunately, our gradient model has only one arbitrary parameter $G$, and the other two parameters $I$ and $Z$, can be calculated from real-world data (Figure S4, Supporting Information). The parameters used in the gradient model are easy to observe and calculate, and the simulation results can be verified by filed investigations. It means that our gradient model is reliable and robust, for it stands on the solid ground.
of physics. Former models mainly consider the two-dimensions mainly identify “islands” or “lowlands” in urban area for some parameters.[9] Their data and analysis can easily be transformed, and apply this 1D gradient model, and quantify the relevant variables of spatial distribution of components, including the economic revenue, population distribution, and ecological preference. For example, this 1D gradient model can provide a quantitative demonstration to Christaller’s central hypothesis.[30] Urban planners can use this model to accurately predict, how cities will develop over time, by considering the changes of population, land rent, and ecological consciousness.

The sensitivity analysis of the gradient model shows that the distributions of multiple types of components are more sensitive to four input parameters, m, γ, σ, and r than other parameters. These four parameters largely determine the gravity and repulsion forces (Table S7, Supporting Information). It means that the attributes of components together with the attributes of cities determine the functional spatial structure of cities. The attributes of components include net ecosystem service (m) and environmental index (γ), and the attributes of cities include land rent exponent (σ) and population attenuation coefficient (β). Many researchers find that the land rent and freight rate affect the distribution of components,[21] but few models quantify the affects. This 1D gradient model quantifies the influence of driving factors and find out the key factors. For example, the ecosystem service intensity m of greenhouses is much smaller than that of KFCs, so it cannot afford the high land rent, therefore, they are repulsed by the city center. Meanwhile, the fresh-keeping distance r of greenhouse products is longer than that of KFCs, so they are less attracted by the city center. The equilibrium of the two forces mainly drives those greenhouses located outside the urban areas and far from other functional components.

The gradient model uncovers the social-economic and ecological mechanism of development of city’s functional structure. The technological innovations of the components increase the productivity m, while decreasing the environmental impact γ of the components. The improvements of the net benefit (combined m and γ) of the components raise the gravitation force to the components, and then uplift the individual density peak (Pmax) of the individuals and pull the Pmax to move inward (dℓ decreases).
on urban-rural gradients. (Figure S8, Supporting Information). These results support the view that the enterprises (city functional components or organaras) have to keep up with technological innovations continuously.\textsuperscript{[12,31]} Furthermore, the individual density curve response of the environmental index $γ$ reveals that innovations must also reduce the environmental impact of components in order to adapt to the changes in human preference, which tends to be stricter on environment quality with increasing social prosperity.\textsuperscript{[12,31]} The super-cell city model has analyzed the spatial relationships between organaras and citynucleus.\textsuperscript{[6]} The gradient model further quantifies the important impacts of the production capacity, ecological characteristics, and technological progress of organaras on the city’s functional spatial structure.

The land rent reflects the result of competition among components for a scarce land resource.\textsuperscript{[37]} The development of cities flattens the spatial patterns of land rent and human population.\textsuperscript{[18]} As the land rent curves become gentler, the $P_{\text{max}}$ of the four types of components decrease, but only the $d^*$ of GH and DF located outside the urban area moves outward (Table S7, Supporting Information). In other words, the growth of population in the peripheries of the city is caused by the urbanization (Figure S9b, Supporting Information) that will stimulate the riphreries of the city is caused by the urbanization (Figure S9b, Supporting Information). The gradient model further quantifies the important impacts of the production capacity, ecological characteristics, and technological progress of organaras on the city’s functional spatial structure.

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Although each case city has its unique spatial structure and development trend, the gradient model in this study has been proven to be general enough to analyze the spatial distribution of city functional components. In fact, the key input parameters are the spatial distribution function of human population and land price, as well as the component production capacity. Due to its generalizability, our model has the potential to be applied in other cities around the world. We have found that the trends of land rent curve and population density curve along with the urban–rural gradient of some cities in the United States, Japan, South Korea, Australia, and Europe are similar to the patterns of the Chinese cities, revealing the monotonic attenuation trend from city center outward.\textsuperscript{[18,35]} On the other hand, recent studies have shown that the affluent population mainly occupies the position of the urban center, even in megacities such as New York and London\textsuperscript{[36]} that are similar to the case of Chinese cities in this study. Although some rich people tend to live in the outer suburbs, these small populations cannot change the spatial pattern of high accessibility of urban centers. Therefore, once the input parameters are obtained, the model can then be used to describe the distribution pattern of functional components in other cities worldwide. Our studies can also be carried out across regions and countries for comparing the similarities and differences of diverse cities and countries, and this motivates future work.

This model integrates the various factors and their interactions into a 1D urban–rural gradient and finds quantified rules. This static model describes the interaction results of various factors as the eco-economic system in relative equilibrium state. Such a mechanism model, instead of the statistic model,\textsuperscript{[7]} reveals the general law of the development, expansion, and renewal of the cities. This mathematical gradient model deepens the understanding of city structure based on a conceptual supercell model.\textsuperscript{[6]} Based on the historical distributions of the functional components and related parameters, the model can be used to predict the development of the city morphology and structure. The model can also help to explore the limitations of the current city in structure and function, and to find out the crucial optimization points to promote sustainable development in the context of global urbanization. It can be further developed and inspire interdisciplinary studies across fields such as ecology, economics, urban planning and management, and engineering.

### 4. Experimental Section

#### Selection of Case Cities and Functional Components

The criteria for choosing case cities were as follows: (1) the data for the population and spatial pattern of the population of the city is available; (2) the data for the spatial pattern of the land price of the city is available; (3) a city is self-sustainable with complete processes and function. According to the criteria, 15 cities were selected in China: Shanghai, Beijing, Tianjin, Suzhou, Wuhan, Hangzhou, Nanjing, Wuji, Ningbo, Nantong, Hefei, Changzhou, Shaoxing, Jiaxing, and Zhenjiang City, and several time periods in some cities (Table S1, Supporting Information). The criteria for choosing the type of component were as follows: (1) the function of the components is to provide services to people locally, so their amount is related to the population of the city; (2) each component has a number of individuals that form an individual density distribution curve along with the urban–rural gradient; (3) different types of components have distinguishable distribution curves along with the urban–rural gradient: one type is concentrated near the citynucleus, one type outside the center, one type is near the urban fringe, and one is in the ex-urban area. According to the criteria, the following were chosen: (1) a type of fast food restaurant, Kentucky Fried Chicken shops, most of which are near the citynucleus; (2) a type of express delivery outlets, ZTO express outlets, most of which are outside the citynucleus; (3) a primary biological production component, cultivated greenhouses, which are mainly outside but near the suburban area; (4) a type of secondary biological production component, dairy farms, that are mainly located in ex-urban areas (Figure S10, Supporting Information).

#### Data Source and Ecosystem Services Assessment

The data of city attributes and functional components were obtained by investigation or from public database, for details see Supporting Information. Through the surveying and mapping of the land price of cities, it was found that there were still a large number of single-center cities. Of course, the research on multi-center cities may be disturbed, and needs further study. According to the Millennium Ecosystem Assessment, ecosystem services included provisioning services, regulating services, cultural services, and supporting services.\textsuperscript{[37]} In this study, the ecosystem services (goods and services) were provided by artificial ecosystems (components) were divided into target services and accompanied services separately (Figure S11, Supporting Information).\textsuperscript{[11]} The calculations of the ecosystem services of the components; for details, see Supporting Information.

The target services of a type of component were determined by the investment goal of the artificial ecosystem. This means that they can be
the provisioning, regulating, or cultural services in Millennium Ecosystem Assessment. The target service of KFC, GH, and DF was the provisioning of food, which is equivalent to some of the provisioning services of natural ecosystems, while the target service of ZTO is the regulating service of distributing goods to people. The accompanied services were equivalent to the externalities (positive or negative) in economics. They could be categorized into provisioning, regulating, and cultural services. Regulating services were further divided into positive (services) and negative (dis-services) in this paper, following the guidelines in Liu et al. The net service (NES, m in model) is the sum of the ecosystem services (target service + positive regulating services + cultural services) and disservices (environmental impacts).

\[
NES = \sum_{i=1}^{n} ES_i \tag{13}
\]

where ES, (USD m⁻² year⁻¹) is the value of ecosystem service \(i\), and \(n\) is the number of ecosystem services considered in this study.

Environmental index (\(\gamma\) in the model) is calculated by the ratio of disservices (EDS) to target goods and services (TGS) of a type of component,

\[
\gamma = \frac{EDS}{TGS} \tag{14}
\]

Statistics: The statistical functions for the attributes of cities and components with distance from the citynucleus used linear and nonlinear regressions (Excel 2019, Microsoft Cooperation). Linear and nonlinear regressions were used to study the relationship between \(Z\) and the city attributes, and the best-adjusted \(R^2\) was used to select the regression form.

The nonlinear fitting module in Origin 2018 Pro (OriginLab Corporation) was used to simulate the spatial distribution of functional components. As the initial values of nonlinear fitting parameters may affect the simulation results, the appropriate initial values were carefully cycled through. At the beginning of the model iterative process, the default value of 1 was used as the initial value for \(G\) and \(I\). Meanwhile, \(Z\) was fixed as 0, because \(Z\) only affects the \(P_{max}\) without the peak position. After the model iteration is over at this stage, the condition on \(Z\) is relaxed and continued with the iterative process to obtain the final fitting parameters.

Supporting Information
Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest
The authors declare no conflict of interest.

Data Availability Statement
Research data are not shared.

Keywords
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