Interaction of long-wave perturbations with a shock wave on a wedge and method of mode decomposition of supersonic flow pulsations

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Abstract. The problem of the interaction of long-wave perturbations of an external flow with a shock wave formed in supersonic flow past a wedge was considered in this paper. The formulas for the amplitudes of long-wave disturbances are obtained from the general analytical solution for the perturbation amplitudes. This data are compared with the results of numerical simulation. It is shown that pulsations of different modes can be separated from the total pulsation field of the incoming flow due to the different amplitude of the transformation of the perturbations of various type behind the shock wave. A method for such a separation is proposed and its approbation is carried out in a numerical experiment.

1. Introduction

To create and operate high-speed aircraft, it is necessary to study the interaction of perturbations of the incoming flow with a shockwave arising at the leading edges of bodies in a supersonic flow. Each of the modes of incoming perturbations, namely the entropy, acoustic, and vortex modes, induce a perturbation-superposition of all these three species [1] behind the shock wave in the case of a shock wave isolated from the body. If an shock wave arises in a flow around a rigid body, the situation is much more complicated due to the attachment of the shock wave to the front edge of the body and the reflection and refraction of perturbations between the surface of the body and the shock wave [2]. In this case, each of the types of waves of the oncoming stream can generate pulsations of all perturbation modes behind the shock wave.

Today, the receptivity and evolution of perturbations in supersonic boundary layers are widely studied by means of wind tunnels. For such studies, the accurate knowledge is needed not only about the intensity of pulsations in the core of the supersonic flow of the wind tunnel, but also about the mode composition of these perturbations. This determines the urgency of developing methods and creating devices for isolating pulsations of certain perturbations modes from the total supersonic flow pulsations field. Parametric calculations of the pressure pulsation fields during the interaction of pulsations of an external flow with a shock wave formed in supersonic flow around a wedge of length L were carried out in [3] in a wide range of propagation angles of long-wave (λ>L) perturbations. Received numerical data on the coefficients of transformation of long-wavelength external perturbations on a shock wave have been represented in the form of analytical dependences on the shock wave inclination angle, the perturbation propagation angle, and the Mach number of the incident flow [3]. These dependences can be used to solve the task of mode decomposition of perturbations in supersonic flows. The range of admissible root-mean-square amplitudes of oscillations of vortex, entropy, and acoustic modes in the free stream generated in IT-302M was obtained by using the pressure fluctuations measured on the model surface and the calculated conversion factors [4]. However, the results of [4] are based on the assumptions about the zero angle of propagation of the
modes of external flow disturbances and the absence of a phase connection between them. Moreover, only time-averaged pulsation amplitudes of different modes were obtained.

In this paper, we propose a method of the mode decomposition of perturbations in real time of measurements with an arbitrary distribution of the perturbations over the propagation angles. The magnitude of a certain disturbances mode of the incoming flow is determined from the pressure pulsations on the surfaces of the wedges and the coefficients of conversion of the long-wave length perturbations of various modes to the shock wave taken from analysis of the analytical solution [2]. In this paper, this method of the modal decomposition of perturbations of the oncoming stream is approbated in real time in a numerical experiment.

2. Analytic solution for the case of long-wave perturbations

The general analytical solution for the perturbation amplitudes in a region bounded by a shock wave and a wedge surface in a supersonic flow was obtained in [2] for perturbations of the external flow in the form of plane monochromatic waves. The solution is given in the form of an infinite series of terms containing Bessel functions of the first kind with a set of coefficients, the determination of whose values can be calculate from recurrent relations. Unfortunately, in the case of large values of the ratio of the longitudinal coordinate of the wedge surface to the perturbation wavelength (\(\chi/\lambda \gg 1\)), calculations become extremely sensitive to the number of taken terms of the series. This significantly complicates the analysis of this solution in far-downstream behavior. However, in the case of long-wave perturbations with \(\chi/\lambda \ll 1\), it’s possible to decompose the general solution obtained in this work for the small parameter \(\chi/\lambda\). As a result of this analysis, the equation for pressure perturbations generation behind the shock wave on a wedge in a supersonic flow was obtained:

\[
p = -e^{i\omega t} \frac{\rho}{2c} a_1.
\]

Here \(\bar{p}, \bar{u}, \bar{c}\) are the pressure, velocity and sound speed behind shock wave, \(\gamma\) is the adiabatic index of the gas,

\[
a_i = 2\frac{\bar{c}}{\eta_i} \left( B_1 - (A_2 B_1 - A_1 B_2) \frac{\bar{u}_{\omega}}{\eta_1} R_0^{(2)} + A_1 (A_2 B_1 - A_1 B_2) \frac{\bar{u}_{\omega}}{\eta_1} \right),
\]

\(A_i, B_i\) are functions containing the parameters of the unperturbed flow from [2], \(\varphi\) is the shock wave angle of inclination, \(\alpha\) is the angle of inclination of the wedge,

\[
\eta_1 = A_1 \tan (\varphi - \alpha) \left( u_{\omega} - \bar{u}_{\omega} \right) + A_2 \bar{u}_{\omega},
\]

\[
\eta_2 = \bar{u}_{\omega} \left( A_2 B_1 - A_1 B_2 \right) \frac{\bar{u}_{\omega}}{\eta_1} - \left( A_3 B_1 - A_1 B_3 \right).
\]

Functions \(u_{\omega}, \bar{u}_{\omega}\) are unperturbed normal to shock components of the flow velocity ahead and behind the shock wave, and the tangential velocity component, respectively. Values are normalized in the same way as in [2].

Functions

\[
R_0^{(1)} = u_{\omega},
\]

\[
R_0^{(2)} = (A_1 + A_2) u_{\omega} + A_2 \bar{u}_{\omega} \rho_{\omega} + \frac{p_{\omega}}{2\gamma u_{\omega} M_{\omega}},
\]

\[
R_0^{(3)} = \left[ \frac{1}{\rho} (B_1 - 1) + \frac{\bar{u}_{\omega}}{u_{\omega}} \right] u_{\omega} + \left[ \frac{u_{\omega}}{\rho} (B_1 - 1) - \frac{1}{(\gamma - 1) u_{\omega} M_{\omega}} \right] \rho_{\omega} + \frac{p_{\omega}}{(\gamma - 1) u_{\omega} M_{\omega}^2},
\]

linearly depend on the amplitudes of normal and tangential to shock wave velocities \(u_{\omega}\) and \(\bar{u}_{\omega}\), amplitudes of density \(\rho_{\omega}\) and pressure \(p_{\omega}\) perturbations. \(\bar{\rho}\) is the density of the unperturbed flow. The analytical dependencies obtained from the general solution allow to determine the transformation coefficients of the acoustic \(g_a\), entropy \(g_e\), and vortex \(g_v\) mode in the pressure pulsation on the wedge surface in the following form:
\[ g_a = A^* + B^* \cos \left( \frac{\pi}{2} - \varphi^* - \theta \right); \\
\quad g_e = C^*; \\
\quad g_\varphi = \gamma B^* \sin \left( \frac{\pi}{2} - \varphi^* - \theta \right). \]  

Here \( A^*, B^*, C^*, \varphi^* \) are functions of the unperturbed flow parameters. It should be noted that the transformation coefficients obtained from the results of numerical simulation [3] are consistent with the general solution obtained for the case of long-wave perturbations.

3. Numerical simulation of supersonic flow past a wedge

Numerical simulation of disturbed supersonic flow around a wedge was carried out using the ANSYS Fluent software package with the addition of built-in UDF modules for introducing perturbations of various modes. The system of two-dimensional Navier-Stokes equations for viscous and inviscid cases was solved. An explicit scheme of second-order accuracy with respect to space with Roe-FDS by the method of splitting convective flows and the explicit Runge-Kutta method with respect to time was used. The procedure of the numerical method is described in more detail in [5,6].

The length of wedge generatrix was \( L = 0.1 \) m. The computational domain was a rectangle with the part of its lower side consist the wedge. The left (input) boundary was located some distance upstream from the leading edge of the wedge, the height of the computational domain was chosen from the condition that the bow shock wave did not interact with the upper boundary of the computational region. The right boundary is shifted from the rear end of the model, so that the flow in the output section is supersonic. On the left and upper bounds of the computational domain, the parameters of the supersonic incoming flow \( p_\infty, M_\infty, T_\infty \) were set, and the downwash condition was used at the right output boundary. Non-slip conditions and constant temperature of \( T_\infty = 300K \) was set to calculate the viscous flow on the surface of the wedge. The angle of the wedge solution was taken into account by the corresponding direction of the velocity vector of the incident flow \( u_\infty \) at an angle \( \alpha \) to the wedge. The computational area was covered with a rectangular grid.

After calculation of the main unperturbed flow, the flow with pressure, velocity and temperature perturbations was calculated. For this, a superposition of the main flow (quantities with the indices \("\infty\"\)) and plane monochromatic waves of the corresponding mode was set at the input boundary of the computational domain: 
\[ p = \tilde{p}_\infty + p_\infty; \\
T = \tilde{T}_\infty + T_\infty; \\
u = \tilde{u}_\infty + u_\infty \] 

similar to [5]. The amplitude of the disturbances normalized to the pressure in the unperturbed flow, \( A_0 = 0.03 \). The angle of propagation of external disturbances had a starting point from the direction of the velocity vector of the running flow.

The flow around the wedge was considered under the following conditions:
\[ I - \tilde{p}_\infty = 494 Pa, \tilde{T}_\infty = 98 K, M_\infty = 7.8, \text{inviscid case} \]
\[ II - \tilde{p}_\infty = 494 Pa, \tilde{T}_\infty = 98 K, M_\infty = 7.8, Re_{1,\infty} = 4 \times 10^4 m^{-1} \]
\[ III - \tilde{p}_\infty = 4237 Pa, \tilde{T}_\infty = 177 K, M_\infty = 5.56, Re_{1,\infty} = 10.38 \times 10^4 m^{-1} \]

for the wedge angles \( \alpha = 5 \pm 15^\circ \) and wide range of angles of propagation of external disturbances \( \theta = -60 \div 60^\circ \).

Figures 1 and 2 show the results of numerical simulation of the action of a fast acoustic wave with a frequency \( f = 50 kHz \) (or \( L/\lambda = 2.88 \)) on a wedge with \( \alpha = 10^\circ \) in comparison with analytical solution [2]. The parameters of the unperturbed flow for obtaining the analytical solution corresponded to the case I (\( M_\infty = 7.8, \alpha = 10^\circ, L/\lambda = 2.88 \)). Figure 1 shows the fields of instantaneous pressure pulsations, obtained in numerical simulation of viscous flow (Figure 1a) and inviscid flow (Figure 1b) and according to theory [2] (Figure 1c) at \( M_\infty = 7.8 \). Figure 1b shows the field of instantaneous pressure pulsations in numerical simulation with parameters (I), and Figure 1c - with parameters (II). The agreement of the amplitudes of instantaneous pulsations and its phase velocities is evident for all cases.

Figure 2 shows a comparison of the rms pressure pulsations amplitudes on the wedge surface normalized to the amplitude of pressure pulsations in the incoming stream, depending on the parameter \( x/L \). It is seen that the pulsation amplitudes are in good agreement with [2] and the calculated data for an inviscid case over the whole surface of the wedge. In the case of calculation taking into account the viscosity, the amplitude of the pressure pulsations for \( x/L > 0.15 \) is slightly
higher. However, all amplitudes are practically the same up to $x/L=0.15$ which corresponds to the case of long-wave disturbances ($\lambda/L < 0.4$). This allows us to use equations (1) and (2) to calculate the perturbation transformation coefficients in viscous supersonic flows. Moreover, the approximation of long-wave perturbations can be used for the mode decomposition of perturbations in viscous supersonic flows.

**Figure 1.** Fields of instantaneous pressure pulsations: (a) – viscous case II, (b) – inviscid case I, (c) – general solution [2].

**Figure 2.** Normalized rms amplitudes of pressure pulsations on the wedge surface: 1 – viscous case II, 2 – inviscid case I, 3 – general solution [2].

4. Mode decomposition of disturbances
The idea of the method of mode decomposition consists in the possibility of separation of the pulsations of individual modes in the incoming flow by relations (1) and measured pressure pulsations on elements of the special device. This device contains several elements with a sharp leading edge, located at different angles of attack. On the surfaces of these elements pressure pulsations are measured. The difference in the angles of inclination of the surfaces of the device leads also a difference in the magnitude of the conversion coefficients of the different modes of perturbations of the external flow, obtained from (2). Then, according to the relations (1), for various angles of inclination of the device surfaces, a system of equations is computed for calculating the pulsations of the individual modes of the incoming flow.

This method is suitable for long-wave perturbations of the incoming flow under the condition \( \lambda \gg H \), where \( H \) is the largest device size. Potentially this allows to separate pressure \( p_\infty \), velocities \( u_{\infty x} \), \( u_{\infty y} \), \( u_{\infty z} \) and entropy \( s_\infty \) fluctuations, which requires using of measuring sensors on 5 different surfaces of the device. However, the analysis of relations (2) shows that \( B^* \sin \varphi \delta^* \) and \( C^* \) are linearly dependent quantities in the inviscid case, and the solution is determined only by four independent variables. In this case, it is sufficient to use two pairs of wedges or plates symmetric with respect to the direction of the incoming flow and located at the angle of attack for the mode decomposition of the perturbations: \( \alpha^+_1 \), \( \alpha^-_1 \), \( \alpha^+_2 \), \( \alpha^-_2 \) (here \( \alpha_1 \neq \alpha_2 \)). Analysis of the solution for the three-dimensional case from [7] shows that the perturbation transformation coefficients do not depend on the velocity fluctuations in the transverse direction to the plane formed by the velocity vector and the normal to the shock wave. Therefore, one pair of wedges \( \alpha^+_1 \) and \( \alpha^-_1 \) for determination of \( u_{\infty y} \) is located so that the normal to the surface was lay in the \((x, y)\) plane. The second pair of wedges \( \alpha^+_2 \) and \( \alpha^-_2 \) is located so that the normal to the surface was lay in the \((x, z)\) plane. The instantaneous values of pulsations superposition of the oncoming stream are:

\[
p_{\infty} = \frac{\sigma}{c_2} \frac{c_{1}^{*}A_{1}^{*} - c_{2}^{*}A_{2}^{*}}{c_{1}^{*}A_{1}^{*} - c_{2}^{*}A_{2}^{*}} + s_{\infty} = \frac{c_{1}^{*}A_{1}^{*} - c_{2}^{*}A_{2}^{*}}{c_{1}^{*}A_{1}^{*} - c_{2}^{*}A_{2}^{*}} u_{\infty x} + \frac{c_{1}^{*}A_{1}^{*} - c_{2}^{*}A_{2}^{*}}{c_{1}^{*}A_{1}^{*} - c_{2}^{*}A_{2}^{*}} u_{\infty y} \frac{\delta_{1}^{*}}{p_{\infty B}^{*} \cos \varphi_{1}^{*}} \frac{\delta_{2}^{*}}{p_{\infty B}^{*} \cos \varphi_{2}^{*}},
\]

where \( \sigma = \frac{p_{w+} + p_{w-}}{2} \) and \( \delta = \frac{p_{w+} - p_{w-}}{2} \) - half-sum and half-difference of pressure pulsations on symmetric surfaces. To separate pulsations \( u_{\infty x} \) and \( s_{\infty} \) the model addition is required by another measuring device for determination of pulsations which are different from pressure pulsations. Such a device, for example, can be a mass flow pulsation sensor (hot-wire anemometer).

In this paper, the approbation of the method of mode decomposition of perturbations was carried out in numerical experiment. For this, calculations of pressure perturbation were made on the surface of the wedge generatrix at the point \( x_0 = 0.0646m \) with the angle of attack \( \alpha_1 = 5^\circ \) and \( \alpha_2 = 15^\circ \) on time. A double calculation was made for each angle of attack in case of external flow disturbances having a propagation angle \( \theta \) which is different from \( 0^\circ \) using wave action with propagation angle of \( \theta \) and \( -\theta \) to simulate symmetrical wedges. The values of the pressure pulsations at the same time points of the calculation were processed using the relations (3) to obtain the instantaneous values of the disturbance amplitudes of the external flow. Figure 3 shows the data of the computational experiment of the mode decomposition under the conditions of the oncoming stream III (see item 3) and under action of fast acoustic wave at frequency 10 kHz \( (x_0/\lambda = 0.38) \) and with the propagation angle \( \theta = 0 \). Figure 3a shows the calculated pressure pulsations on the wedges surfaces (analog of the experimental values), Figure 3b – result of the restoration of pressure pulsations in the oncoming flow and Figure 3c – result of restoration of the combination of pulsations of the longitudinal component of velocity and entropy according to relations (3).
It is seen that the reconstructed amplitudes of perturbation are in good agreement with the initial ones: deviations in amplitude $p'_\infty$ are less than 2%. Recovered values $u'_\infty$ are also consistent with the initial pulsations in the investigated range of propagation angles $\theta$ to 60°. The next stage of investigation will be the experimental implementation of this method on the basis of measurements of pressure pulsations by special device under the action of controlled perturbations of a supersonic flow of known amplitude and known mode composition.

**Figure 3.** Calculated pressure pulsations on wedges surfaces (a), restored pressure pulsations in the incoming stream (b) and restored complex of pulsations of longitudinal component of velocity and entropy (c): 1 - calculated values under the conditions of the oncoming flow (initial), 2 - reconstructed values.

5. Conclusion
Dependences of the transformation coefficients of long-wave disturbances were obtained from the general solution of the inviscid problem of the interaction of perturbations of an incoming supersonic flow with a shock wave on the wedge.

Numerical simulation of perturbed supersonic flow past a wedge in a viscous and inviscid case was carried out. It is shown that the transformation coefficients for inviscid and viscous cases coincides under the conditions of the action of long-wave disturbances.

The method of real time decomposition pulsations of different modes of the incoming flow from the total pulsation field (mode decomposition) was proposed and its validation in a numerical experiment was performed.

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