On the Injection Scale of the Turbulence in the Partially Ionized Very Local Interstellar Medium

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Abstract

The cascade of magnetohydrodynamic (MHD) turbulence is subject to ion–neutral collisional damping and neutral viscous damping in the partially ionized interstellar medium. By examining the damping effects in the warm and partially ionized local interstellar medium, we find that the interstellar turbulence is damped by neutral viscosity at \(\sim 261\) \(\text{au}\) and cannot account for the turbulent magnetic fluctuations detected by Voyager 1 and 2. The MHD turbulence measured by Voyager in the very local interstellar medium (VLISM) should be locally injected in the regime where ions are decoupled from neutrals for its cascade to survive the damping effects. With the imposed ion–neutral decoupling condition and the strong turbulence condition for the observed Kolmogorov magnetic energy spectrum, we find that the turbulence in the VLISM is sub-Alfvénic, and its largest possible injection scale is \(\sim 194\) \(\text{au}\).

Unified Astronomy Thesaurus concepts: Magnetohydrodynamics (1964); Interstellar plasma (851); Heliosphere (711)

1. Introduction

Turbulent magnetic fluctuations following a Kolmogorov spectrum were observed by Voyager 1 and 2 in the outer heliosheath (Burlaga et al. 2018; Lee & Lee 2020; Zhao et al. 2020; Fraternale & Pogorelov 2021; Burlaga et al. 2022). While the source(s) of the turbulence in the very local interstellar medium (VLISM) are under debate (Holzer 1989; Zank 2015; Zank et al. 2019), it is believed to play a crucial role in affecting the transport of energetic particles and cosmic rays (Lazarian & Opher 2009; Krimigis et al. 2013; Stone et al. 2013; López-Barquero et al. 2017; Fraternale et al. 2022), and the structure of the Interstellar Boundary Explorer (IBEX) ribbon (Giacalone & Jokipii 2015; Zirnstein et al. 2020). In particular, modeling by Zirnstein et al. (2020) suggests that the turbulent magnetic fields with scales \(< 100\) \(\text{au}\) are important for producing the ribbon structure similar to IBEX observations, and such turbulence is likely not of pristine interstellar origin, whereas turbulent fluctuations at scales \(\geq 100\) \(\text{au}\) produce features inconsistent with IBEX observations.

Turbulence and turbulent magnetic fields are ubiquitous in the interstellar medium (ISM; e.g., Armstrong et al. 1995; Chepurnov & Lazarian 2010; Gaensler et al. 2011; Xu & Zhang 2017; Lazarian et al. 2018), with the turbulent energy injected by supernova explosions on large length scales (\(\sim 100\) \(\text{pc}\); Breitschwerdt et al. 2017) and cascading down toward smaller scales. In the warm and partially ionized local interstellar medium (LISM; Slavin & Frisch 2008; Frisch et al. 2011), the magnetohydrodynamic (MHD) turbulence is subject to the damping effects due to the frictional collisions between ions and neutrals (i.e., ion–neutral collisional damping) and among neutrals (i.e., neutral viscosity) (Xu & Lazarian 2017). The damping effects cause the cutoff of MHD turbulence cascade when the damping rate exceeds the cascading rate of MHD turbulence.

The linear analysis of MHD waves in a partially ionized medium was performed by Kulsrud & Pearce (1969) and more recently by, e.g., Pudritz (1990), Balsara (1996), Mouschovias et al. (2011), Zaqarashvili et al. (2011), and Soler et al. (2013). Different from linear MHD waves, MHD turbulence is characterized by the nonlinear cascade of turbulent energy, with scale-dependent turbulent anisotropy and a limited timescale of turbulent motions (Goldreich & Sridhar 1995; Lazarian & Vishniac 1999). The ion–neutral collisional (IN) damping of MHD turbulence in a highly ionized medium at a high plasma \(\beta\) was studied by Lithwick & Goldreich (2001). The neutral viscous (NV) damping of MHD turbulence was analyzed by Lazarian et al. (2004), which tends to dominate over the IN damping toward a higher ionization fraction and a higher temperature (Xu & Lazarian 2017). Xu et al. (2015, 2016) performed a general analysis including both damping effects of MHD turbulence in different interstellar phases with varying ionization fractions.

The range of length scales for the existence of MHD turbulence depends on the coupling state between ions and neutrals and the corresponding damping effects. Irrespective of the origin, the MHD turbulence measured in the VLISM should survive the damping in the partially ionized medium. In this work, we will examine the damping of the interstellar MHD turbulence driven at large scales. More importantly, we will explore the constraint imposed by the damping effects on the locally driven MHD (LMHD) turbulence in the VLISM, as observed by Voyager. In Section 2, we analyze the damping of the interstellar turbulence driven in the strongly coupled regime and the injection of the LMHD turbulence in the decoupled regime with weak damping. In Section 3, we determine the turbulence regime and the largest injection scale of the LMHD turbulence constrained by the ion–neutral decoupling...
In the case when the NV damping dominates over the IN damping, the damping of MHD turbulence occurs in the strongly coupled regime, with the damping scale (Xu & Lazarian 2017; see Appendix B)

$$I_{\text{dam, NV, } \perp} = \left( \frac{\xi_n}{2} \right)^{\frac{3}{2}} \nu_{th}^2 L^{-2} V_L^{-\frac{1}{2}}$$

(2)

where $\nu_{th} = \nu_{th}/(\sigma_n\sigma_{nn})$ is the kinematic viscosity in neutrals, $\nu_{th}$ is the neutral thermal speed, and $\sigma_{nn}$ is the collisional cross section of neutrals. We note that $I_{\text{dam, NV, } \perp}$ is in fact the damping scale of the turbulent kinetic energy spectrum. The magnetic fluctuations in the subviscous range below $I_{\text{dam, NV, } \perp}$ is termed the new regime of MHD turbulence (Lazarian et al. 2004) (see Section 4.1).

We now discuss whether the magnetic turbulence in the VLISM can come from the interstellar turbulence that is injected by supernova explosions. The LISM near the Sun is warm, low density, and partially ionized, with the temperature $T \approx 6300$ K, $n_h \approx 0.2$ cm$^{-3}$, and $n_i \approx 0.07$ cm$^{-3}$ (Slavin & Frisch 2008; Swaczyna et al. 2020). In addition, we adopt $L_{\text{ISM}} \approx 100$ pc, $V_{\text{LISM}} \approx V_N$, and $B \approx 5 \mu G$ as the typical driving conditions of interstellar turbulence and interstellar magnetic field strength (Crutcher et al. 2010). With this high temperature and moderate ionization fraction, we find that

$$I_{\text{dam, IN, } \perp} \approx 7.6 \times 10^{13} \text{ cm} \left( \frac{n_e}{0.07 \text{ cm}^{-3}} \right)^{\frac{3}{2}} \left( \frac{n_h}{0.2 \text{ cm}^{-3}} \right)^{\frac{1}{2}} \left( \frac{L_{\text{ISM}}}{100 \text{ pc}} \right)^{\frac{3}{2}} \left( \frac{B}{5 \mu G} \right)^{\frac{1}{2}}$$

(3)

is much smaller than

$$I_{\text{dam, NV, } \perp} \approx 3.9 \times 10^{15} \text{ cm} \left( \frac{T}{6300 \text{ K}} \right)^{\frac{3}{2}} \left( \frac{n_h}{0.2 \text{ cm}^{-3}} \right)^{\frac{1}{2}} \left( \frac{L_{\text{ISM}}}{100 \text{ pc}} \right)^{\frac{3}{2}} \left( \frac{B}{5 \mu G} \right)^{\frac{1}{2}}$$

(4)

where we assume $n_h + n_i \approx n_h$, and $\sigma_{nn} \approx 10^{-14}$ cm$^2$ (Krsticevic & Schultz 1998; Vranjes & Krstic 2013) is adopted. We note that the gradients of quantities in the outer heliosheath (Zank et al. 2013) cause uncertainties in the above estimates. By assuming the uncertainties $\sigma(n_e) \sim 0.01$ cm$^{-3}$, $\sigma(n_i) \sim 0.01$ cm$^{-3}$, $\sigma(B) \sim 1 \mu G$, $\sigma(T) \sim 1000$ K, and $\sigma(L_{\text{ISM}}) \sim 10$ pc, we find $\sigma(I_{\text{dam, NV, } \perp}) / 2.8 \times 10^{13}$ cm and $\sigma(I_{\text{dam, IN, } \perp}) / 6.4 \times 10^{14}$ cm.

For the interstellar turbulence driven in the Local Interstellar Cloud, the turbulent velocity of a few km s$^{-1}$ is similar to that from the cascade of supernova-driven turbulence at $\sim 2$ pc (Spangler et al. 2011). It might be a part of the global cascade of the interstellar turbulence. Therefore, it is also subject to the damping effects in the partially ionized LISM and damped at the damping scale similar to that of the supernova-driven interstellar turbulence.

The above calculations show that the damping of the MHD turbulence in the LISM from the interstellar origin is dominated by NV in the strongly coupled regime. The corresponding damping scale is $\sim 3.9 \times 10^{15}$ cm, i.e., 261 au. This means that the turbulence observed by Voyager 1 and 2 is unlikely due to the pristine interstellar origin.
2.2. **LMHD Turbulence in the Decoupled Regime**

As discussed above, for the MHD turbulence injected in the strongly coupled regime, its cascade is cut off either by the IN or NV damping. When ions are decoupled from neutrals, however, the driven MHD turbulence is no longer subject to the NV damping, and the IN damping becomes constantly weak (Xu et al. 2016). The MHD turbulence injected in ions that are decoupled from neutrals, i.e., LMHD turbulence in the decoupled regime, is not cut off due to the damping effects arising in a partially ionized medium.

Based on the in situ measurements of turbulent magnetic energy spectrum by Voyager 1 in the VLISM (Lee & Lee 2020), we have the ratio of the turbulent component to the average magnetic field strength \( \delta B_{\text{obs}} / B \approx 0.06 \) measured at \( l_{\text{obs}} \approx 3 \times 10^{16} \text{ cm} \) (corresponding to the smallest wavenumber of the measured spectrum and assumed to be in the inertial range of turbulence). By assuming that the magnetic fluctuations are mainly induced by Alfvénic turbulence (Cho & Lazarian 2002; Lee & Lee 2020; Hu et al. 2022), the local turbulent velocity can be estimated as

\[
v_{\text{obs}} = \frac{\delta B_{\text{obs}}}{B} V_{\text{Al}} = 2.5 \text{ km s}^{-1} \left( \frac{\delta B_{\text{obs}} / B}{0.06} \right) \left( \frac{n_e}{0.07 \text{ cm}^{-3}} \right)^{-\frac{1}{2}} \left( \frac{B}{5 \mu \text{G}} \right).
\]

By assuming \( \sigma(\delta B_{\text{obs}}) \approx 0.1 \mu \text{G} \) and the uncertainties of other parameters (see Section 2.1), we find \( \sigma(v_{\text{obs}}) \approx 0.85 \text{ km s}^{-1} \).

The large uncertainties in our calculations are mainly caused by the large uncertainties in magnetic field strength measurements (Burlaga et al. 2018). Such a large turbulence level at the measured length scale cannot be accounted for by the interstellar turbulence, which is cut off at a larger length scale \( l_{\text{dam, NV, } \perp} \) (Equation (4)). So the measured turbulence is likely to be driven in the VLISM. Given \( v_{\text{obs}} / l_{\text{obs}} (\approx 8.3 \times 10^{-10} \text{ s}^{-1}) > \nu_{\text{in}} (\approx 1.8 \times 10^{-10} \text{ s}^{-1}) \), the LMHD turbulence measured by Voyager 1 should be injected in the decoupled regime.

3. **Constraint on the Injection Scale of the LMHD Turbulence in the VLISM**

We now discuss the constraint on the injection scale \( L \) of the LMHD turbulence. The three cases we consider are (a) super-Alfvénic turbulence with isotropic injection scale, (b) sub-Alfvénic turbulence with isotropic injection scale, and (c) sub-Alfvénic turbulence with anisotropic injection scale. In Figure 1, we illustrate the scalings of super- and sub-Alfvénic turbulence (see also Appendix A) for isotropic and anisotropic injection.

The condition for the LMHD turbulence to arise in ions alone is

\[
\Gamma_L > \nu_{\text{in}}, \tag{6}
\]

where \( \Gamma_L \) is the driving rate of the turbulence, i.e., the cascading rate of turbulence at \( L \), and \( \nu_{\text{in}} = \gamma_d \rho_i / \mu \) is the ion-neutral collisional frequency.

**Case (a).** If the driven turbulence is super-Alfvénic, i.e., \( M_A = V_L / V_{\text{Al}} > 1 \), where \( V_{\text{Al}} = B / \sqrt{4 \pi \rho_i} \) is the Alfvén speed in ions, \( V_L \) is related to \( v_L \) by (Lazarian & Vishniac 1999)

\[
v_L = V_L \left( \frac{L}{L} \right)^{\frac{1}{2}}, \tag{7}
\]

where \( v_L \) is the local turbulent velocity measured at length scale \( l < L \). With the cascade of turbulent energy, the turbulent velocity becomes equal to \( V_{\text{Al}} \) at the Alfvénic scale \( L_{\text{Al}} = L M_{\text{Al}}^{-3} \) (Lazarian 2006). Below \( L_{\text{Al}} \), the effect of magnetic fields on turbulence becomes important, resulting in turbulence anisotropy (see Figure 1). Therefore, \( l \) in the above equation should be replaced by \( l_L \) when \( l \) is smaller than \( l_{\text{Al}} \), where \( l_L \) is the length scale measured perpendicular to the local magnetic field (Cho & Vishniac 2000). By using \( \Gamma_L = V_L / L \) and the scaling relation in Equation (7), the condition Equation (6) in this case becomes

\[
L < (\nu_{\text{in}} v_L^{-1} l_{\perp})^{-\frac{1}{2}}, \quad l_{\perp} < l < L,
\]

\[
L < (\nu_{\text{in}} v_L^{-1} l_{\perp})^{-\frac{1}{2}}, \quad l < l_{\perp}. \tag{8}
\]

The above expressions provide constraints on the maximum \( L \) of the LMHD turbulence in the decoupled regime if it is super-Alfvénic.

The in situ measurements show that the LMHD turbulence has \( v_L \approx v_{\text{obs}} \) at \( l_{\perp} \approx l_{\text{obs}} \) (Section 2.2). We consider the largest observed scale \( l_{\text{obs}} \) as the perpendicular scale because the trajectory of Voyager 1 is nearly perpendicular to the background magnetic field (Izmodenov & Alexashov 2020; Fraternale & Pogorelov 2021; Dialynas et al. 2022). \( v_L \) is
smaller than $V_{Ai}$ (see Equation (5)) and related to $V_{Ai}$ by
\[ v_l = V_{Ai} \left( \frac{l_l}{L} \right)^{\frac{1}{3}}, \]

yielding
\[ l_A = L \left( \frac{v_l}{V_{Ai}} \right)^{-3} = 1.4 \times 10^{18} \text{ cm} \left( \frac{n_e}{0.07 \text{ cm}^{-3}} \right)^{\frac{1}{2}} \left( \frac{v_l}{2.5 \text{ km s}^{-1}} \right)^{-3} \times \left( \frac{l_l}{3.0 \times 10^{14} \text{ cm}} \right) \left( \frac{B}{5 \mu G} \right)^{\frac{1}{3}}, \]

with the estimated uncertainty $\sigma(l_A) \sim 1.7 \times 10^{18}$ cm. The condition in Equation (8) for super-Alfvénic turbulence can be written explicitly as
\[ L < 2.9 \times 10^{15} \text{ cm} \left( \frac{n_e}{0.2 \text{ cm}^{-3}} \right)^{\frac{1}{2}} \left( \frac{v_l}{2.5 \text{ km s}^{-1}} \right)^{\frac{3}{2}} \times \left( \frac{l_l}{3.0 \times 10^{14} \text{ cm}} \right)^{\frac{1}{2}}, \]

with the estimated uncertainty $\sigma(L) \sim 1.5 \times 10^{15}$ cm. The super-Alfvénic condition $M_A > 1$ requires $L > l_A$, which is not satisfied by the above values. We conclude that the LMHD turbulence cannot be super-Alfvénic.

Case (b). If the driven turbulence is sub-Alfvénic, i.e., $M_A = V_L/V_{Ai} < 1$, the MHD turbulence is weak with weak interactions between counterpropagating Alfvén wave packets (Galtier et al. 2000) over scales $[L_r, l_{\text{tran}}]$, where $l_{\text{tran}} = LM_A^3$ is the perpendicular transition scale from weak to strong MHD turbulence (Lazarian & Vishniac 1999; see Figure 1). For weak turbulence, there is no parallel cascade. Its cascade to smaller perpendicular scales strengthens until the cascade becomes strong (Goldreich & Sridhar 1997). The scaling of weak turbulence follows $v_l = V_L(l_l/L)^{1/2}$ (Lazarian & Vishniac 1999), while in the strong turbulence regime, the local turbulent velocity follows the scaling (Lazarian & Vishniac 1999)
\[ v_l = V_L \left( \frac{l_l}{L} \right)^{1/2} M_A^{\frac{1}{4}}. \]

For the weak turbulence at $L$, there is
\[ \Gamma_L = \frac{V_L}{L} M_A. \]

By combining Equations (12) and (13), the condition in Equation (6) becomes
\[ L < \nu_{in}^{-2} v_l^{3/2} l_{\text{tran}}^{-1} V_{Ai}^{-1}. \]

The spectral indices of magnetic fluctuations in weak and strong MHD turbulence are $-2$ and $-5/3$, respectively. With the Kolmogorov slope ($-5/3$) found for the measured magnetic energy spectrum (Burlaga et al. 2018; Lee & Lee 2020), we consider that the locally measured turbulence is in the strong MHD turbulence regime. For the driven sub-Alfvénic turbulence, the condition in Equation (14) gives
\[ L < 3.6 \times 10^{14} \text{ cm} \left( \frac{n_e}{0.07 \text{ cm}^{-3}} \right)^{\frac{1}{2}} \left( \frac{n_H}{0.2 \text{ cm}^{-3}} \right)^{-2} \times \left( \frac{v_l}{2.5 \text{ km s}^{-1}} \right)^{\frac{3}{2}} \left( \frac{l_l}{3.0 \times 10^{14} \text{ cm}} \right)^{-\frac{1}{2}} \left( \frac{B}{5 \mu G} \right)^{-1}, \]

with $\sigma(L) \sim 3.8 \times 10^{14}$ cm. By using Equations (12) and (13), the condition in Equation (6) can also be written as
\[ V_L < \left( \nu_{in}^{-1} v_l^{3} l_{\text{tran}}^{-1} \right)^{\frac{1}{2}} \left( \frac{n_H}{0.07 \text{ cm}^{-3}} \right)^{\frac{1}{2}} \left( \frac{v_l}{2.5 \text{ km s}^{-1}} \right)^{\frac{3}{2}} \times \left( \frac{l_l}{3.0 \times 10^{14} \text{ cm}} \right)^{-\frac{1}{2}} \left( \frac{B}{5 \mu G} \right)^{-1}, \]

with $\sigma(V_L) \sim 2.7$ km s$^{-1}$. The corresponding $M_A$ is
\[ M_A = \frac{V_L}{V_{Ai}} < 0.13 \left( \frac{n_e}{0.07 \text{ cm}^{-3}} \right)^{\frac{1}{2}} \left( \frac{n_H}{0.2 \text{ cm}^{-3}} \right)^{-1} \times \left( \frac{v_l}{2.5 \text{ km s}^{-1}} \right)^{\frac{3}{2}} \left( \frac{l_l}{3.0 \times 10^{14} \text{ cm}} \right)^{-\frac{1}{2}} \left( \frac{B}{5 \mu G} \right)^{-1}, \]

with $\sigma(M_A) \sim 0.072$. Given such a small $M_A$ value, the implied $l_{\text{tran}} = LM_A^{\frac{1}{2}}$ is $\lesssim 6.1 \times 10^{12}$ cm. This means that the observed turbulence would be in the weak MHD turbulence regime with $l_{\text{obs}} > l_{\text{tran}}$. This is inconsistent with the observed Kolmogorov spectrum for strong MHD turbulence.

Case (c). In the above calculations we assume that the injection scale of turbulence is isotropic. In the presence of strong background magnetic fields, the driven sub-Alfvénic turbulence is likely to have anisotropic $L$ (see, e.g., Pogorelov et al. 2017). With a sufficiently small perpendicular component of injection scale $L$, the shear in the direction perpendicular to the magnetic field can cause significant distortions of magnetic field lines within the Alfvén wave period. If the anisotropy is sufficiently large so that the nonlinear interaction between counterpropagating Alfvén wave packets is strong and the critical balance relation (Goldreich & Sridhar 1995) is satisfied at $L$, the entire turbulent cascade would be in the strong MHD turbulence regime (see Figure 1). In this case, we have
\[ v_l = V_L \left( \frac{l_l}{L} \right)^{\frac{1}{4}}, \]

and $\Gamma_L = V_L/l_{\perp}$. The condition in Equation (6) leads to the constraint on the perpendicular injection scale,
\[ l_{\perp} < \left( \nu_{in} v_l^{-1} l_{\perp}^{-1} \right)^{\frac{1}{2}} \approx 2.9 \times 10^{15} \text{ cm} \left( \frac{n_H}{0.2 \text{ cm}^{-3}} \right)^{-\frac{1}{2}} \left( \frac{v_l}{2.5 \text{ km s}^{-1}} \right)^{\frac{1}{2}} \times \left( \frac{l_l}{3.0 \times 10^{14} \text{ cm}} \right)^{-\frac{1}{2}}, \]

with the uncertainty $\sigma(L) \sim 1.5 \times 10^{15}$ cm, and the same constraint on $V_L$ as in Equation (16).

Among the three cases, only case (c), i.e., sub-Alfvénic turbulence with anisotropic injection scale, provides a self-
consistent result. In Figure 2, we present $2\pi / L_{\perp,\text{max}}$ together with the observationally measured magnetic energy spectrum taken from Lee & Lee (2020) for the period from 2012 August to 2019 December and Burlaga et al. (2018) for intervals 2013.3593–2014.6373 and 2015.3987–2016.6759. Here $L_{\perp,\text{max}}$ (Equation (4)) is the largest possible perpendicular injection scale of the LMHD turbulence in the VLISM. We find that $L_{\perp,\text{max}}$ is close to $l_{\text{dam,IN,\perp}}$ of the interstellar turbulence given by Equation (4).

4. Discussion

4.1. New Regime of MHD Turbulence

In the case when the NV damping dominates over the IN damping (see Section 2.1), the turbulent motions are damped at the neutral viscous scale, but the magnetic fluctuations can exist on smaller scales. There is a new regime of MHD turbulence in the subviscous range (Cho et al. 2002; Lazarian et al. 2004; Xu & Lazarian 2016), where the kinetic energy spectrum is steep with the spectral index $-4$ and the magnetic energy spectrum is flat with the spectral index $-1$. We note that such a flat magnetic energy spectrum corresponds to scale-independent magnetic fluctuations. The subviscous magnetic fluctuations are caused by the shear of viscous-scale turbulent eddies in the direction perpendicular to the magnetic field. As the IN damping suppresses magnetic fluctuations, the damping scale of the subviscous magnetic fluctuations is determined by the balance between the eddy-turnover rate at $l_{\text{dam,IN,\perp}}$ and the IN damping rate,

$$\frac{v_{\nu}}{l_{\text{dam,IN,\perp}}} = \omega_{\text{d,IN}},$$

where $v_{\nu} = V_{\text{LISM}}(l_{\text{dam,IN,\perp}}/L_{\text{ISM}})^{1/3}$ is the turbulent velocity at $l_{\text{dam,IN,\perp}}$. The IN damping rate in this case is

$$\omega_{\text{d,IN}} = \frac{\xi_{\perp} k^2 (\delta V_A)^2}{2 \nu_{\text{ni}}},$$

where $\delta V_A = \delta B/\sqrt{4\pi \rho}$, and $k^2 (\delta V_A)^2$ is the magnetic force per unit mass per unit displacement corresponding to the subviscous magnetic fluctuation $\delta B$ perpendicular to the magnetic field. Equation (21) is different from the expression for IN damping rate of Alfvén waves in Kulsrud & Anderson (1992), as over subviscous scales, there are no Alfvén wave motions of magnetic fields. By assuming that the Alfvénic component dominates the turbulent motion, we approximately have $\delta V_A \approx v_{\nu}$. Then we obtain from Equation (20)

$$l_{\text{dam,IN,\perp,sub-v}} = \frac{\xi_{\perp} n_{\text{ni}}^{-1/2} \nu_{\text{ni}}^{1/2}}{2} \left(\frac{T}{6300 \text{ K}}\right)^{1/2} \left(\frac{n_e}{0.07 \text{ cm}^{-3}}\right)^{-1/2} \left(\frac{n_{\text{H}}}{0.2 \text{ cm}^{-3}}\right),$$

with $\sigma(l_{\text{dam,IN,\perp,sub-v}}) \sim 8.5 \times 10^{13} \text{ cm}$, as the IN damping scale of subviscous magnetic fluctuations (see Figure 2). We see that $l_{\text{dam,IN,\perp,sub-v}}$ is slightly smaller than $l_{\text{dam,IN,\perp}}$ in Equation (4). So the cutoff scale of magnetic fluctuations is close to the cutoff scale of the kinetic energy spectrum of the interstellar turbulence.

For the LMHD turbulence driven in the VLISM, the new regime of MHD turbulence is also expected below the
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Voyager 1 and 2 can be explained by a transmission of fast modes from the inner heliosheath with a conversion to incompressible Alfvén waves. They further assumed that the outer scale of this heliosphere-originated turbulence is around 120 au, and the interstellar turbulence will continue down to scales smaller than those measured by Voyager. Our work, by considering the ion–neutral interactions in the LISM region, suggests that the interstellar turbulence should have a cutoff around 261 au, and a new heliosphere-related turbulence (such as the scenario described by Zank et al. 2019) with an injection scale smaller than \( \sim 2.9 \times 10^{15} \text{ cm} \) (\( \approx 194 \text{ au} \)) is needed to explain the Voyager measurements.

5. Conclusions

The damping effects in the partially ionized LISM determine the range of length scales for the existence of interstellar MHD turbulence and the LMDH turbulence. Due to the high temperature and moderate ionization fraction in the LISM, we find that the dominant damping mechanism of the interstellar MHD turbulence is the NV damping. The NV damping scale of the interstellar turbulence is about 261 au. Below the NV damping scale of turbulent cascade, the new regime of MHD turbulence with constant magnetic fluctuations (Lazaridou et al. 2004) is expected to rise. We find that the subviscous magnetic fluctuations are cut off due to the IN damping at a scale slightly smaller than the NV damping scale. For the LMDH turbulence, when the injection occurs in the regime with ions decoupled from neutrals, the turbulent cascade cannot be cut off by the damping related to partial ionization. Given the turbulent velocity at the largest observed length scale inferred from in situ measurements, after applying the ion–neutral decoupling condition, we find that the LMDH turbulence in the VLISM is sub-Alfvénic with the injected turbulent energy smaller than the magnetic energy. With the trajectory of Voyager 1 approximately perpendicular to the background magnetic field, by assuming anisotropic injection scale of the LMDH turbulence, we further find the upper limit of the perpendicular injection scale \( L_{\text{max}} \sim 2.9 \times 10^{15} \text{ cm} \) \( \approx 194 \text{ au} \), which is close to the NV damping scale of the interstellar turbulence. Our estimated largest outer scale of LMDH turbulence is comparable to the extent of the heliosphere in the upstream direction (Pogorelov et al. 2018) and 1 order of magnitude smaller than the estimate given in Burlaga et al. (2018) and Lee & Lee (2020) by extrapolating the power-law slope to the equipartition between the turbulent fluctuation and the average magnetic field strength. Note that other considerations such as mode conversion (Zank et al. 2019) and IBEX modeling (Zirnstein et al. 2020) could further limit the injection scale down to tens of au.

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Software: MATLAB (MATLAB 2021).
Appendix A
Anisotropy of Strong MHD Turbulence

The theoretically established scale-dependent anisotropy of trans-Alfvénic turbulence (Goldreich & Sridhar 1995) and sub-Alfvénic turbulence (Lazarian & Vishniac 1999) has been tested by MHD turbulence simulations (Cho & Lazarian 2002, 2003) and supported by spacecraft measurements in the solar wind (Horbury et al. 2008; Luo & Wu 2010; Forman et al. 2011). Here we briefly review the anisotropy of strong Alfvénic turbulence.

The cascading rate of Alfvénic turbulence in strong MHD turbulence regime is

$$\tau_{\text{cas}}^{-1} = v_l L_c^{-1} = V_{\text{st}} L_{\text{st}}^{-\frac{4}{3}} L_l^{-\frac{2}{3}}, \quad (A1)$$

where $V_{\text{st}}$ is the turbulent velocity at the outer scale $L_{\text{st}}$ of strong MHD turbulence. More specifically, there is

$$V_{\text{st}} = V_A, \quad L_{\text{st}} = l_A = LM_A^3, \quad (A2)$$

for super-Alfvénic turbulence with $M_A > 1$, and

$$V_{\text{st}} = V_L M_A, \quad L_{\text{st}} = l_{\text{tran}} = LM_A^2, \quad (A3)$$

for sub-Alfvénic turbulence with $M_A < 1$. We note that $V_A$ should be replaced by $V_{Ax}$ when the turbulence is injected in the decoupled regime.

By combining the expression of $\tau_{\text{cas}}^{-1}$ with the critical balance relation

$$\tau_{\text{cas}}^{-1} = \frac{V_A}{l_{\text{||}}}, \quad (A4)$$

we can obtain the anisotropic scaling relation of strong Alfvénic turbulence,

$$l_{\text{||}} = \frac{V_A}{V_{\text{st}}} L_{\text{st}}^{-\frac{2}{3}} L_l^{-\frac{2}{3}}. \quad (A5)$$

It shows that smaller-scale turbulent eddies are more elongated along the local magnetic field.

Appendix B
Damping Scales of MHD Turbulence Cascade in a Partially Ionized Medium

Under the consideration of both IN and NV damping effects in a partially ionized medium, Xu et al. (2015), Xu et al. (2016), and Xu & Lazarian (2017) derived the general expression of the damping rate of Alfvénic turbulence. Here we briefly review its approximate form in different coupling regimes.

In the weakly coupled regime, there is only the IN damping, with the damping rate

$$\omega_d = \frac{\nu_n}{2}. \quad (B1)$$

For the MHD turbulence injected in the decoupled regime, as the cascading rate is always larger than $\nu_n$, and thus larger than $\omega_d$, the cascade is not cut off by the damping in a partially ionized medium.

In the strongly coupled regime, the damping rate can be approximately written as

$$\omega_d = \frac{\xi_n}{2} \left( k^2 \nu_n + \frac{k^2 (\delta V_A^2)}{\nu_{ni}} \right) = \frac{\xi_n}{2} \left( k^2 \nu_n + \frac{k^2 l_{\text{st}}^2 V_A^2}{\nu_{ni}} \right), \quad (B2)$$

where we assume that the magnetic fluctuations are mainly induced by Alfvénic turbulence and apply the critical balance relation for strong MHD turbulence, $k \delta V_A = k \nu_A = k \nu_{\text{st}}$.

Here $V_A$ is the turbulent velocity at wavenumber $k$. The first and second terms of $\omega_d$ correspond to NV and IN damping, respectively.

For the MHD turbulence injected in the strongly coupled regime, when the damping rate exceeds the cascading rate, MHD turbulence cascade is damped. We consider that the damping scale is in the strong MHD turbulence regime. By comparing Equation (B2) with Equation (A1), we find the damping scale

$$l_{\text{dam,NV,\perp}} = \left( \frac{\xi_n}{2} \right) V^2 \frac{L_{\text{st}}^2}{l_{\text{st}}} V_A^{-\frac{2}{3}} \quad (B3)$$

when the NV damping dominates over the IN damping, where we assume $k \sim k_{\perp}$, and

$$l_{\text{dam,IN,\parallel}} = \left( \frac{2 \nu_{ni}}{\xi_n} \right) L_{\text{st}}^{-\frac{1}{3}} l_{\text{st}}^{-\frac{1}{3}} V_A^{-\frac{2}{3}} \quad (B4)$$

in the opposite case. If the MHD turbulence is damped due to neutral viscosity in the strongly coupled regime, there is the so-called new regime of MHD turbulence on scales below the NV damping scale (Lazarian et al. 2004). If the damping of MHD turbulence is caused by neutral–ion decoupling, hydrodynamic turbulent cascade can happen in neutrals that are decoupled from ions on scales smaller than the IN damping scale (Burkhart et al. 2015; Xu et al. 2015).

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