Influence of the Pauli principle on the dynamics of the Quark-Gluon Plasma

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Abstract

A simple parton cascade model for the ultra-relativistic nucleus-nucleus collisions is proposed to investigate the Pauli blocking at the transition from hadronic to partonic state, i.e. QGP. A boost invariant study of the Pauli blocking is implemented in the Monte Carlo simulation for the first time. It turns out that the higher reaction energy the stronger the partonic Pauli effect is. It amounts to about ten percent at SPS and RHIC energies but nearly reaches twenty percent at LHC energy. Thus the effect is relevant to the formation and evolution of the quark-gluon plasma.

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Pauli blocking plays an important role in nucleus-nucleus collisions at low and intermediate energies because the available phase space volume is quite small [1, 2, 3, 4, 5]. However, in the ultra-relativistic nucleus-nucleus collisions since the available momentum phase space is huge and more than 95% produced particles are bosons one might despise the role of Pauli effect. Even if the available momentum space in ultra-relativistic nucleus-nucleus collision is really huge, the coordinate space, on the other hand, is strongly contracted at the earliest stage of the collision. Although pions, kaons etc. do not suffer with Pauli effect, their precursors, u, d, and s quarks and their antiquarks, do really have all the influences of Pauli blocking if a Quark-Gluon Plasma (QGP) is formed and survives long enough in a limited region of phase space. Furthermore, since Pauli blocking is stronger for u and d quarks than s quark, it might benefit the production of strangeness relatively and the strong constrain of strange suppression could be somewhat overturned. Thus it is worth to study the effect of partonic Pauli blocking on the hadronic final state in ultra-relativistic nucleus-nucleus collisions.

Even though our parton cascade model (see below) is crude in some aspects, the basic assumption is that at each nucleon-nucleon collision the produced particles (mainly pions) are deconfined at least for the time of the reaction, a few fm/c for instance, which is however very short. If a QGP is formed at the maximum compression stage we have then to test for the influence of Pauli principle since all the particles are fermions. In our model the elementary probability of particle production are taken from data [6], thus we expect the...
model to be reasonable in describing the dynamics of particle production. However, the mean field effects and the dynamics of confinement are neglected and we assume that the quarks recombine simply when the density in phase space is small. The later, however, is a rather reasonable assumption since if the density is very high, the quarks are clearly deconfined. As we will show below, our calculations underestimate the available experimental data nearly a factor of 1.5, thus we expect that we are underestimating the Pauli blocking since it is more severe for larger densities (i.e. higher number of produced partons). More refined models which include the dynamics of the QGP plasma must really take the Pauli blocking into account. Simpler models which deal with hadrons only, should estimate the maximum density in phase space reached. If such density is too large then it is necessary to deal with the structure of the hadrons. From an experimental point of view, since the quarks recombine mainly in pions which are bosons, most of the Pauli blocking effect might be hidden. However, some important informations on the phase space density might be obtained from baryon-baryon coincidence measurement, especially neutrals to avoid the distortions of the long range Coulomb force.

To our knowledge the quantum statistic effect in ultra-relativistic heavy ion collisions was really implemented in the study of bosonic enhancement of pion transverse momentum and was also considered formally in the partonic transport equation. It should be emphasized that a great effort was made in after the pioneering study in the field of parton cascade model. However, the partonic Pauli blocking effect was not really incorporated in
FIG. 3: Rapidity (left panel) and transverse momentum (right panel) distributions of Ξ⁻ in Pb+Pb collisions at $\sqrt{s_{NN}}=5500$ GeV.

The parton cascade model for ultra-relativistic heavy ion collision is generally composed of the parton initial state, the parton evolution, the hadronization, and the hadron evolution. However, the models in [12, 13] lack the last two parts at present, for instance. The partonic...
Pauli blocking, of course, plays a role in the former two parts only. As a first step the proposed simple parton cascade model is used for investigating mainly the Pauli blocking effect in the generation of parton initial state from the hadronic distributions at the highest density stage.

There are two ways of creating the parton initial state. In [10, 11, 14] the parton initial state was composed of partons from the mini-jets production in nucleus-nucleus collision and the HIJING multiple mini-jet generator [15] was specified in [10, 11]. The parton initial state in [12, 13] was created via probability distributions first for the spatial and momentum coordinates of nucleons in colliding nuclei and then for the flavor and spatial and momentum coordinates of partons in nucleons. Our parton cascade model follows the former way, however, the JPCIAE multiple mini-jet generator [16] is used instead of HIJING.

The JPCIAE multiple mini-jet generator for ultra-relativistic nucleus-nucleus collision is based on PYTHIA [6] which is a well known event generator for hadron-hadron collision. In the JPCIAE model the radial position of a nucleon in colliding nucleus A (indicating the atomic number of this nucleus as well) is sampled randomly from Woods-Saxon distribution. Each nucleon is given a beam momentum in z direction and zero initial momenta in x and y directions (fermionic motion is neglected). The Lorentz contraction is taken into account after the initialization of nucleons. A least approaching distance for each colliding nucleon pair along their straight line trajectory is calculated together with its collision time under the requirement that the least approaching distance must be less than or equal to \( \sqrt{\sigma_{\text{tot}}/\pi} \). Here \( \sigma_{\text{tot}} \) refers to the total cross section assumed to be equal to 50 mb for \( Au + Au \) collisions at \( \sqrt{s_{\text{nn}}}=130 \) and 200 GeV and 40 and 100 mb for \( Pb + Pb \) collisions at \( \sqrt{s_{\text{nn}}}=17.3 \) and 5500 GeV [17], respectively. The nucleon-nucleon collision with the least collision time is then selected from the initial collision (time) list performing the first collision. This nucleon-nucleon collision is modeled by PYTHIA with string fragmentation switched off, thus the produced particles are quark pairs, diquark pairs and gluons. To investigate the partonic Pauli blocking efficiently the diquark (anti-diquark) is split into quarks (antiquarks) and the gluons into quark pair randomly. The produced partons propagate along straight line trajectory similarly to the nucleons. However, the partonic interaction is neglected for the moment, thus partons do not collide and possibly create new partons, which is a possible reason for underestimating the data. After a nucleon-nucleon collision both the particle list and the collision list are updated and the new collision list is still composed of nucleon-
nucleon collisions only. The next collision is selected from the new collision list and the processes above are repeated until the collision list is empty.

For each parton \( i \) among \( N_{\text{new}} \) partons produced in the current nucleon-nucleon collision a judgment must be performed for its Pauli blocking and its unblocking probability, \( p_{\text{unb}}(i) \), is calculated (see below). If the product of unblocking probabilities of \( N_{\text{new}} \) produced partons

\[
P_{\text{unb}} = \prod_{i=1}^{N_{\text{new}}} p_{\text{unb}}(i)
\]

satisfies

\[
P_{\text{unb}} \leq \xi
\]

where \( \xi \) is a random number, the current nucleon-nucleon collision is blocked and thrown away. Another nucleon-nucleon collision is selected from the collision list and the processes above are repeated.

In the heavy-ion collision at low and intermediate energies the Pauli effect was treated boost uninvariantly [1, 2, 3, 4, 5]. We introduce a method of boost invariant Pauli blocking in Monte Carlo simulation for the first time. To calculate \( p_{\text{unb}}(i) \) we first select the partons with same flavor as parton \( i \) from both the \( N_{\text{new}} \) partons and the particle list formed before current nucleon-nucleon collision. If the total number of partons picked above is denoted by \( N_{\text{pick}} \) we boost all of \( N_{\text{pick}} \) partons to the rest frame of parton \( i \) to be consistent with the method of least approaching distance (this boost is not necessary indeed, we have checked that the discrepancy between results from with and without this boost is nearly 1%). Two three dimensional cubes: one in coordinate space with size \( \Delta_r \) and the other in momentum space with size \( \Delta_p \), are defined at \( i \) (as origin). The product of these two cubes, which is an invariant scalar [18], is assumed to be \( h^3 \), the consequent values of \( \Delta_r \) and \( \Delta_p \) are 1 fm and 1.24 GeV/c, respectively. If the six dimensional cell spanned between partons \( i \) and \( j \) \((j \in N_{\text{pick}})\), \(|x_{ij} \times y_{ij} \times z_{ij} \times px_j \times py_j \times pz_j|\) (invariant scalar), is within \((\Delta_r \times \Delta_p)^3\), i.e. the conditions

\[
|x_{ij}| \leq \Delta_r/2, |y_{ij}| \leq \Delta_r/2, |z_{ij}| \leq \Delta_r/2,
\]

\[
|px_j| \leq \Delta_p/2, |py_j| \leq \Delta_p/2, |pz_j| \leq \Delta_p/2,
\]

are satisfied simultaneously, the occupation number of parton with flavor as parton \( i \), \( N_{\text{occu}}(i) \), is added by one. The \( x_{ij} \) and \( px_j \) in the above equations, for instance, are the


\( x \) component of the coordinate distance between \( i \) and \( j \) and of the momentum of \( j \) in rest frame of \( i \) \((i \in N_{new})\), respectively. Thus the occupation probability of parton with flavor as parton \( i \) reads

\[
p_{occu}(i) = N_{occu}(i)/g
\]

where \( g=6 \) is the spin and color degeneracies of the quark and/or antiquark with a given flavor (identified as u, d, and s). The unblocking probability of parton \( i \) is then

\[
p_{unb}(i) = 1 - p_{occu}(i).
\]

The above partonic system is then hadronized by JETSET \([6]\) after the nucleon-nucleon collision ceased (freeze-out) and the hadronic final state is the consequence. One investigates the partonic Pauli blocking in the generation of the initial parton state and consequently on the hadronic final state by comparing the calculations with partonic Pauli blocking to those without.

In Tab. 1 is given the multiplicity of particles in \( Au + Au \) collisions at \( \sqrt{s_{nn}}=130 \) and 200 GeV (5% most central) and \( Pb + Pb \) collisions at 17.3 GeV (5% most central) and 5500 GeV (10% most central) from calculations with and without Pauli blocking. The decreasing percentage of the particle multiplicity in the calculations with Pauli blocking relative to the ones without Pauli blocking is given in the table as well. In the last row the strangeness multiplicity, the sum of all the strange particles in the table, is shown. One sees in Tab. 1 that the partonic Pauli blocking effect is generally below ten percent at SPS and RHIC energies but reaching nearly twenty percent at LHC energy. The higher reaction energy the stronger Pauli effect because of the increasing number of produced quarks and of the competition between the partonic Pauli blocking, which decreases pion production, and the fragmentation, which benefits low \( p_T \) pions. The partonic Pauli effect is also of benefit to the production of strangeness as seen in \( Pb + Pb \) collision at LHC energy.

The rapidity (left panel) and transverse momentum (right panel) distributions with and without Pauli blocking for \( Pb + Pb \) collisions at \( \sqrt{s_{nn}} = 5500 \) GeV are given, respectively, in Fig. 1, 2, and 3 for \( \pi^+, k^+ \) and \( \Xi^- \), for instance. The discrepancy between the two cases is visible, in rapidity distribution especially.

In summary, a simple parton cascade model for ultra-relativistic nucleus -nucleus collisions is proposed studying the partonic Pauli blocking effect in the parton initial state and consequent hadronic final state. A boost invariant study of the Pauli effect is implemented
in the Monte Carlo simulation for the first time. It turned out that the amount of this Pauli blocking effect is below ten percent at SPS and RHIC energies but reaching nearly twenty percent at LHC energy, i.e. the higher beam energy the stronger the partonic Pauli effect is. Thus the effect is relevant to the formation and evolution of the quark-gluon plasma. However, the results from our model are a kind of under estimation. Since the charge multiplicity in \( Au + Au \) collisions at \( \sqrt{s_{nn}}=200 \) and 130 GeV in the calculations with Pauli blocking is, respectively, 3063 and 2562 (including feed-down correction from \( k^0_s, \Lambda, \) and \( \bar{\Lambda} \)) and the multiplicity of \( \pi^+ \) and \( \pi^- \) in \( Pb + Pb \) collision at \( \sqrt{s_{nn}}=17.3 \) GeV is, respectively, 416.5 and 429.3 in Tab. 1. They are all less than the corresponding experimental data of 4630\( \pm 370 \) [19], 3860\( \pm 300 \) [20], and 619\( \pm 17 \pm 31 \) and 639\( \pm 17 \pm 31 \) [21], respectively. On the other hand, the results of our parton cascade model are not too bad to compare with experimental data, which means that the model is reasonable in its range.

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