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Quantum Origin of the Early Inflationary Universe

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Abstract

We give a detailed presentation of a recently proposed mechanism of generating the energy scale of inflation by loop effects in quantum cosmology. We discuss the quantum origin of the early inflationary Universe from the no-boundary and tunneling quantum states and present a universal effective action algorithm for the distribution function of chaotic inflationary cosmologies in both of these states. The energy scale of inflation is calculated by finding a sharp probability peak in this distribution function for a tunneling model driven by the inflaton field with large negative constant \( \xi \) of non-minimal interaction. The sub-Planckian parameters of this peak (the mean value of the corresponding Hubble constant \( H \approx 10^{-5}m_p \)), its quantum width \( \Delta H / H \approx 10^{-5} \) and the number of inflationary e-foldings \( N \geq 60 \) are found to be in good correspondence with the observational status of inflation theory, provided the coupling constants of the theory are constrained by a condition which is likely to be enforced by the (quasi) supersymmetric nature of the sub-Planckian particle physics model.

1. Introduction

In this paper we give a detailed account of a recently proposed mechanism for generating the energy scale of the chaotic inflationary Universe by the loop part of the effective action in quantum cosmology [¹].

Quantum cosmology became a theory of the quantum origin of inflationary Universe in early eighties due to the synthesis of the cosmological inflation [²] with the idea of the quantum state [³, ⁴, ⁵], generating the initial conditions for inflationary scenario. One of the main problems of this theory was the formulation of such a quantum state that could describe a very early quantum Universe, its evolution leading to the modern observable large-scale structure. The inflation paradigm is very attractive because
at least heuristically it allows one to avoid applications of quantum gravity since the inflationary epoch has to take place at the energy scale or a characteristic value of the Hubble constant $H = \dot{a}/a \sim 10^{-5} m_P$ much below the Planck one $m_P = G^{1/2}$. The predictions of the inflation theory essentially depend on this energy scale which must be chosen to provide a sufficient number of e-foldings $N$ in the expansion law of a scale factor $a(t)$ during the inflationary epoch, $N = \int_{t_i}^{t_f} dt H \geq 60$, and also generate the necessary level of density perturbations. This quantity, however, is a free parameter in the inflation theory, and, to the best of our knowledge, there are no convincing principles that could fix it without invoking the ideas of quantum gravity and cosmology. These ideas imply that a quantum state of the Universe in the semiclassical regime gives rise to an ensemble of inflationary universes with different values of $H$, approximately evolving at later times according to classical equations of motion. This quantum state allows one to calculate the distribution function of this ensemble and interpret its probability maximum at certain value of $H$ (if any) as generating the quantum scale of inflation.

The implementation of this idea in the tree-level approximation of quantum cosmology [3, 4, 5, 6] has a controversial status and, in our opinion, is not satisfactory. The corresponding distribution functions are extremely flat [7, 8] for large values of $H$ and unnormalizable at $H \to \infty$. This violates the validity of the semiclassical expansion underlying the inflation theory, since the contribution of over-Planckian energy scales is not suppressed to zero, and special assumptions are necessary to establish a Planckian "boundary" [9] to protect semiclassical inflation physics from the nonperturbative realm of quantum gravity. Apart from this difficulty, the possible local maxima of the distribution function for the tree-level quantum states are either generating insufficient amount of inflation violating the above bound [10], or generate too high level of quantum inhomogeneities and require unnaturally strong fine tuning (see Sect.2.1).

The key to the solution of these problems, not resorting to the conjectures on a hypothetical over-Planckian phase of the theory, may consist in the semiclassical $\hbar$-expansion and the search for mechanisms that could justify this expansion. Despite the perturbative nonrenormalizability of quantum gravity, this approach makes sense in problems with quantum states peaked at sub-Planckian energies. In particular, it would work in quantum cosmology with the no-boundary [3, 4] or tunnelling [5] wavefunctions, provided they suppress the contribution of Planckian energies and generate the probability peaks at the lower (preferably GUT) scale compatible with microwave background observations. As shown in authors’ papers [11, 12, 1, 13], the loop effects can drastically change the predictions of the tree-level theory and really allow one to reach this goal. Moreover, as it was briefly announced in [1], one can get a sharp probability peak in the distribution function of inflationary models with characteristic
parameters of GUT and, in this way, provide a numerically sound link between quantum cosmology, inflation theory and the particle physics of the early universe. Thus, the purpose of this paper is to give a detailed presentation of this work.

The organization of the paper is as follows. Sec. 2 presents quantum cosmology as a theory of the quantum origin of the chaotic inflationary Universe. It gives a brief account of the quantum gravitational tunnelling underlying the no-boundary and tunnelling wavefunctions of the Universe, discusses the model with nonminimally coupled inflaton field and presents a special algorithm for the one-loop distribution function of (quasi)-DeSitter models. Sects. 3 - 6 contain detailed calculations of various perturbative contributions to this distribution function. We work within the double perturbation theory: loop expansion in $\hbar$ up to the one-loop order and the expansion of the slow roll approximation up to the subleading order in the corresponding smallness parameter $m_P^2/|\xi|\varphi^2 \ll 1$, where $\varphi$ is a value of the inflaton scalar field and $\xi = -|\xi|$ is a big negative constant of its nonminimal coupling with curvature. Important difference from our previous work [1], where the distribution function was calculated only in the leading order of the slow roll expansion, is that now we find it in a subleading order. This leaves the main conclusions of [1] qualitatively the same and thus proves the stability of the leading order, although gives rise to certain quantitative corrections. Sects. 5 and 6 contain perturbative calculations of the one-loop effective action for generic set of fields of various spins, contributing to the distribution function. In Sect. 7 we present a final answer for this function and find a corresponding probability peak that can be interpreted as generating the energy scale of inflation. In Sect. 8 this result is used for the derivation of the selection criterion for viable particle physics models in the early Universe with nonminimal inflaton scalar field, apparently suggesting their (quasi)supersymmetric nature. This conclusion is based on the observation that the energy scale of inflation is suppressed relative to the Planck scale by the same small factor $\sim 10^{-5}$ that determines a recently observed magnitude of the microwave background radiation anisotropy [14, 15], provided that the two special combinations of coupling constants of the system satisfy certain restrictions. Sect. 9 contains concluding remarks.

2. Quantum cosmology – the theory of quantum origin of the early inflationary Universe

2.1. The no-boundary and tunneling quantum states

It is widely recognized now that the inflationary scenario is one of the most promising pictures of the early Universe [2]. It can be described by the DeSitter or quasi-
DeSitter spacetime generated by an effective cosmological constant $\Lambda$, which in its turn is being generated by other slowly varying fields. Thus, in the model of chaotic inflation with the scalar inflaton field $\phi$, minimally coupled to the metric tensor $G_{\mu\nu}$

$$L(G_{\mu\nu}, \phi) = G^{1/2} \left\{ \frac{m_p^2}{16\pi} R(G_{\mu\nu}) - \frac{1}{2} (\nabla \phi)^2 - U(\phi) \right\},$$

the effective cosmological constant is generated by the potential of the inflaton field $U(\phi)$. For this system rolling down from the potential barrier $U(\phi)$ (which is supposed to be monotonically growing with $\phi$) there exists the so-called slow roll approximation, when the non-stationarity of $\phi$ is much less than the rate of change of the cosmological scale factor $a$ measured by the Hubble constant $H = \dot{a}/a$. In this approximation equations of motion take the form

$$\dot{\phi} \simeq -\frac{1}{3H} \frac{\partial U}{\partial \phi} \ll H \phi, \quad (2.2)$$

$$H = H(\phi) \simeq \sqrt{\frac{8\pi U(\phi)}{3m_P^2}}, \quad (2.3)$$

so that an effective cosmological constant $\Lambda = 3H^2$ is determined by the inflaton field potential. This potential is approximately constant during the inflationary stage due to the slow change of $\phi$ and only at the end of this stage decreases close to zero when the effective cosmological constant "decays" into inflaton oscillations, their energy being spent for the reheating of the Universe and its transition to radiation-dominated and then matter-dominated stages. The duration of the inflation stage usually measured by the number of e-foldings between the beginning $t_I$ and the end $t_F$ of inflation

$$N = \int_{t_I}^{t_F} dt \ H \quad (2.4)$$

determines the coefficient of inflationary expansion of the model $\exp N$ and depends on the initial value of the inflaton field $\phi_I$. This dependence can be approximately obtained by changing the integration variable here to $\phi$ and integrating from $\phi_I$ to zero. This leads to a fundamental bound on the choice of inflationary model and initial value of inflaton $N \geq 60$ [2]

$$N(\phi_I) \simeq \frac{4\pi}{m_P^2} \int_{\phi_I}^{\phi} d\phi \frac{H(\phi)}{[\partial H(\phi)/\partial \phi]} \geq 60. \quad (2.5)$$

The role of quantum cosmology consists in the formulation of the quantum initial data for such a picture in the form of a particular quantum state of the Universe – the wavefunction $\Psi(q)$ usually defined on superspace of 3-metric coefficients and all
matter fields. The implementation of this idea was proposed in the pioneering works of Hartle, Hawking and Vilenkin [3, 4, 5], who suggested that such initial data (and the wavefunction $\Psi(q)$) correspond to the quantum gravitational tunneling that can semiclassically be described by the transition with changing spacetime signature.

In the context of spatially closed cosmology the Lorentzian DeSitter spacetime can be regarded as a result of quantum tunneling from the classically forbidden state described by Euclidean DeSitter geometry. A simple picture of the tunneling geometry demonstrating such a mechanism is presented on Fig.1. The DeSitter solution of the Einstein equations with the cosmological constant $\Lambda = 3H^2$

$$\begin{align*}
    ds^2_L &= -dt^2 + a^2_L(t) c_{ab} dx^a dx^b, \\
    a_L(t) &= \frac{1}{H} \cosh (Ht)
\end{align*} \tag{2.6}$$

describes the expansion of a spacelike spherical hypersurface with a metric of a 3-sphere $a^2(t) c_{ab}$ with the radius (scale factor) $a_L(t)$. Its Euclidean counterpart with the DeSitter positive signature metric

$$\begin{align*}
    ds^2 &= d\tau^2 + a^2(\tau) c_{ab} dx^a dx^b, \\
    a(\tau) &= \frac{1}{H} \sin (H\tau),
\end{align*} \tag{2.7}$$

represents a geometry of a 4-dimensional sphere of radius $R = 1/H$ with 3-dimensional sections (3-spheres) parameterized by a latitude angle $\theta = H\tau$. These two metrics are related by the analytic continuation into a complex plane of the Euclidean time $\tau$ [16, 17]

$$\tau = \pi/2H + it, \quad a_L(t) = a(\pi/2H + it). \tag{2.10}$$

This analytic continuation can be interpreted as a quantum nucleation of the Lorentzian spacetime from the Euclidean one and shown on Fig.1 as a matching of the two manifolds (2.6) - (2.9) across the equatorial section of the 4-sphere $\tau = \pi/2H$ ($t = 0$) - the bounce surface $\Sigma_B$.

Two known quantum states which semiclassically implement this mechanism, are represented by the no-boundary wave function of Hartle and Hawking [3, 4] and the tunneling wave function [6] known for historical reasons as a wavefunction of Vilenkin who pioneered the idea of quantum gravitational tunnelling in [5]. In the approximation of the two-dimensional minisuperspace consisting of the scale factor $a$ and inflaton scalar field $\phi$

$$q^i = (a, \phi), \tag{2.11}$$
these wavefunctions $\Psi_{NB}(a, \phi)$ and $\Psi_T(a, \phi)$ satisfy the minisuperspace version of the Wheeler-DeWitt equation and semiclassically represent its two linear independent solutions

$$\Psi_{NB}(a, \phi) \sim e^{-I(a, \phi)}, \quad \Psi_T(a, \phi) \sim e^{+I(a, \phi)}, \quad (2.12)$$

where the Euclidean Hamilton-Jacobi function $I(a, \phi)$ of the model is calculated at a particular family of solutions of classical Euclidean equations of motion subject to special boundary conditions of Hartle and Hawking at $a = 0$ (the "initial" point of the extremal) and boundary conditions $(a, \phi)$ at the end point of the extremal -- an argument of the wavefunction. At $a = 0$ the derivative of the scalar field with respect to the Euclidean time $\tau$ should be zero while $da/d\tau = 1$ ($\tau$ measures the proper distance), which is equivalent to the requirement of regularity of a 4-metric in the neighbourhood of the pole of a 4-sphere $(2.8) - (2.9)$ at $\tau = 0$. In the leading order of the slow-roll approximation, when the inflaton field is constant, such a solution coincides with the exact round 4-metric $(2.8) - (2.9)$ with the Hubble constant $(2.3)$, and its Hamilton-Jacobi function equals

$$I(a, \phi) = -\frac{\pi m_p^2}{2H^2} \left[ 1 - (1 - H^2(\phi)a^2)^{3/2} \right], \quad H^2(\phi) = \frac{8\pi U(\phi)}{3m_p^2}. \quad (2.13)$$

When the point $(a, \phi)$ belongs to the region of minisuperspace below the curve (see Fig. 2)

$$a = \frac{1}{H(\phi)}, \quad (2.14)$$

the universe exists in the classically forbidden (underbarrier) state described by this Euclidean spacetime. Euclidean extremals, beginning at $a = 0$, have a caustic $^1 (2.14)$ and cannot penetrate into the region $a > 1/H(\phi)$ with real Euclidean time. However, they can be continued into this region by the analytic continuation into the complex time $(2.10)$ which generates the Lorentzian (imaginary) part of the Euclidean function $I(a, \phi)$

$$I(a, \phi) = I(\phi) \pm iS(a, \phi), \quad a > 1/H(\phi), \quad (2.15)$$

$$S(a, \phi) = -\frac{\pi m_p^2}{2H^2} \left( H^2(\phi)a^2 - 1 \right)^{3/2}. \quad (2.16)$$

Here $I(\phi)$ is a Euclidean action of the theory with the Lagrangian $(2.1)$ calculated on the gravitational half-instanton - the hemisphere $(2.8) - (2.9)$ ($0 \leq \tau \leq \pi/2H$)

$$I(\phi) = -\frac{3m_p^4}{16U(\phi)}. \quad (2.17)$$

$^1$In the lowest order of the slow-roll approximation with constant $\phi$ the problem is actually one-dimensional and Eq. $(2.14)$ represents a set of turning points, however beyond this approximation this curve should, in fact, be replaced by the envelope of the family of Euclidean trajectories.
This action determines the amplitude of wavefunctions (2.12) in a classically-allowed (Lorentzian) domain

\[
\Psi_{NB}(a, \phi) \sim e^{-I(\phi) \cos (S(a, \phi) + \pi/4)},
\]

\[
\Psi_T(a, \phi) \sim e^{+I(\phi) + iS(a, \phi)}, \quad a > 1/H(\phi),
\]

which is interpreted in the tree-level approximation as a distribution function for the one-parameter ensemble of Lorentzian inflationary universes characterized by the Hamilton-Jacobi function (2.16). The parameter enumerating the members of this ensemble is a value of the inflaton field \( \phi \) or the corresponding Hubble constant \( H = H(\phi) \) and scalar curvature of the DeSitter space. Its quantum distributions for the no-boundary \( \rho_{NB}(\phi) \) [3] and tunnelling \( \rho_T(\phi) \) [6] quantum states read

\[
\rho_{NB}(\phi) \sim e^{-2I(\phi)}, \quad \rho_T(\phi) \sim e^{+2I(\phi)},
\]

The difference between these two wave functions and their quantum distributions consists in the different boundary conditions in superspace: while the tunneling state \( \Psi_T(a, \phi) \) at \( a > 1/H(\phi) \) contains only the outgoing wave and describes an expanding universe, the no-boundary wave function \( \Psi_{NB}(a, \phi) \) in the Lorentzian regime represents the superposition of states of expanding and contracting cosmologies corresponding to the components of (2.18) of positive and negative frequencies with respect to the minisuperspace coordinate \( a \). The tunneling wave function is defined by the above mentioned outgoing wave conditions in the Lorentzian region of superspace and an additional condition of the \( \phi \)-independence of \( \Psi_T(a, \phi) \) at \( a \to 0 \) [8, 18]. For the no-boundary wavefunction there exists a more fundamental and model-independent prescription in the form of the path integral over regular Euclidean 4-geometries [3, 4], which in the tree-level approximation is dominated by the expression (2.18) – a contribution of the saddle point of this integral – the Euclidean-Lorentzian extremal (2.6) - (2.9).

The distribution functions \( \rho_{NB}(\phi) \) and \( \rho_T(\phi) \) describe the opposite results of the most probable underbarrier tunneling: to the minimum and maximum of the inflaton potential \( U(\phi) \geq 0 \) correspondingly (though in the latter case a minimum \( U(\phi) = 0 \) does not belong, strictly speaking, to the domain of applicability of the slow-roll approximation).

Equations given above apply to the model (2.1), however they can also be used in the theory with non-minimally coupled scalar inflaton \( \varphi \)

\[
L(g_{\mu\nu}, \varphi) = g^{1/2} \left\{ \frac{m^2}{16\pi} R(g_{\mu\nu}) - \frac{1}{2} \xi \varphi^2 R(g_{\mu\nu}) - \frac{1}{2} \nabla^2 \varphi^2 - \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4} \varphi^4 \right\},
\]

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provided \( L(G_{\mu\nu}, \phi) \) above is viewed as the Einstein frame of the Lagrangian \( L(g_{\mu\nu}, \varphi) \) with the fields \( (G_{\mu\nu}, \phi) = ((1 + 8\pi |\varphi^2/m_P^2|)g_{\mu\nu}, \phi(\varphi)) \) related to \((g_{\mu\nu}, \varphi)\) by the known conformal transformation of the metric and the reparametrization of the scalar field \([19, 20, 21]\). For a negative nonminimal coupling constant \( \xi = -\xi \) this model easily generates the chaotic inflation scenario \([22]\) with the following inflaton potential in the Einstein frame parameterization

\[
U(\phi) \bigg|_{\phi = \bar{\phi}(\varphi)} = \frac{m^2 \varphi^2/2 + \lambda \varphi^4/4}{\left(1 + 8\pi |\xi| \varphi^2/m_P^2\right)^2},
\]

including the case of symmetry breaking at scale \( v \) with \( m^2 = -\lambda v^2 < 0 \) in the Higgs potential \( \lambda(\varphi^2 - v^2)^2/4 \). At large \( \varphi \) the potential (2.22) approaches a constant and depending on the parameter

\[
\delta = -\frac{8\pi |\xi| m^2}{\lambda m_P^2} = \frac{8\pi |\xi| v^2}{m_P^2},
\]

has two types of behaviour at the intermediate values of the inflaton field. For \( \delta > -1 \) it does not have local maxima and generates the slow-roll decrease of the scalar field from its initial value \( \varphi_I \) leading to a standard scenario with a finite inflationary stage and approximate e-folding number

\[
N(\varphi_I) = \left(\frac{\varphi_I}{m_P}\right)^2 \frac{\pi (|\xi| + 1/6)}{1 + \delta}.
\]

For \( \delta < -1 \) it has a local maximum at \( \varphi = m/\sqrt{\lambda|1 + \delta|} \), including the case of zero \( \lambda \) when \( \varphi = m_P/\sqrt{8\pi |\xi|} \), and due to a negative slope of the potential leads to the inflation with infinite duration for all models with the scalar field growing from its initial value \( \varphi_I > \varphi \) to infinity.

The tree-level distribution functions (2.20) for such a potential do not suppress the over-Planckian scales and are unnormalizable at large \( \varphi \), \( \int^\infty d\varphi \rho_{NB,T}(\varphi) = \infty \), thus invalidating a semiclassical expansion. Only for \( \lambda = 0 \) the normalizability takes place in the tunnelling case with \( |\xi| \neq 0, \rho_T(\varphi) \sim \exp[-4\pi^2 |\xi|^2 \varphi^2/m^2], \varphi \to \infty \), but this fine tuning is too strong and can hardly survive renormalization of \( \lambda \) by quantum effects.

Only for a tunneling case with \( \delta < -1 \) the distribution \( \rho_T(\phi) \) has a local peak at the maximum of the potential (2.22) \( \varphi \), which could serve as a source of the energy scale of inflation at reasonable sub-Planckian value of the Hubble constant. However, this peak requires a large positive mass of the inflaton field \( m^2 > \lambda m_P^2/(8\pi |\xi|) \), which is too large for reasonable values of \( \xi = -2 \times 10^4, \lambda = 0.05 \) \([19]\). Another (and maybe more serious) difficulty with this inflation scenario starting from the maximum of inflation potential is that according to (2.2) it begins with zero \( \dot{\varphi} \sim 0 \) and generates infinitely large
quantum inhomogeneities (inverse proportional to $\dot{\phi}$ [2]) which are incompatible with the observable large scale structure of the Universe. All this makes questionable the attempts to arrange the quantum origin of our Universe at the tree-level theory and serves as a motivation for considering the loop effects.

2.2. One-loop distribution function of the inflationary cosmologies

Note that the calculation of the tree-level distribution does not require the knowledge of the correct probabilistic inner product of cosmological wave functions. It is enough to calculate and square an amplitude of the wave function, which due to the peculiarities of the model is a function on the section of the two-dimensional minisuperspace, transversal to the coordinate $a$, usually playing the role of time. Therefore the obtained distribution function is defined on the physical subspace of a correct dimensionality – one-dimensional space of spatially homogeneous inflaton field. Beyond the tree-level approximation the situation changes: one needs the knowledge of the wave function with the preexponential factor in the needed approximation, knowledge of the correct inner product and the extension beyond the minisuperspace approximation, because the distribution function contains now a non-trivial contribution from integration over inhomogeneous quantum fields frozen in the tree-level approximation. At the one-loop level which we shall study here it is enough to consider these fields in the linear approximation. Setting of the problem in the model of chaotic inflationary Universe consists in the minisuperspace model with the scale factor $a$ and spatially homogeneous scalar inflaton $\varphi$ and with inhomogeneous fields of all possible spins treated as perturbations on this background. Together they form a superspace of variables

$$q = (a, \varphi, \varphi(x), \psi(x), A_a(x), \psi_a(x), h_{ab}(x), \ldots).$$  \tag{2.25}

On this superspace we shall have to calculate the no-boundary and tunnelling wavefunctions $\Psi_{NB,T}(q)$ and then, by using a proper physical inner product, calculate the distribution function of the collective variable $\varphi$. To make the latter step and even to give a rigorous definition of this distribution, it is better first to make a quantum reduction to the wave function of physical ADM variables $\xi$, which simultaneously disentangles time $t$ (initially parametrized among the superspace variables (2.25) [23])

$$q \rightarrow (\xi, t), \quad \Psi(q) \rightarrow \Psi(\xi, t).$$  \tag{2.26}

Strictly speaking this reduction is not consistent (globally on phase space of the theory), and a complete understanding and the interpretation of the cosmological wavefunction might be reached only in the conceptually more advanced framework (third
quantization of gravity theory, refined algebraic quantization [24, 25], etc.). Although this framework still does not have a status of a well-established physical theory, there exists a good correspondence principle of this framework with the quantization in reduced phase space for systems with a wide class of special (positive-frequency) semiclassical quantum states. For these states the conserved current of the Wheeler-DeWitt equations perturbatively coincides with the inner product of the ADM quantization and thus can be used for the construction of the probability distribution (for a perturbative equivalence of the ADM and Dirac-Wheeler-DeWitt quantization of gravity for such physical states see [26, 27, 12]). The tunneling wavefunction belongs to such a class of states, while the no-boundary one does not and should be supplied with additional (third quantization) principles to be interpreted in terms of the probability distribution of the above type.

It is plausible to make this reduction separately in the minisuperspace sector of the full superspace \( (a, \varphi) \) and its sector of inhomogeneous modes. We choose an inflaton field \( \varphi \) as a physical variable whose distribution function we will calculate, while the solution of classical equations of motion (2.7) with \( H = H(\varphi) \) will be considered as a gauge

\[
\lambda^a(a, \varphi, t) = a - \frac{1}{H(\varphi)} \cosh(H(\varphi)t) = 0. \tag{2.27}
\]

It simultaneously plays the role of the parameterization of minisuperspace coordinates in terms of the physical variable \(^2\). The ADM reduction for linearized inhomogeneous modes of fields boils down to the choice of their transverse \((T)\) and transverse-traceless components \((TT)\), so that the full set of physical variables reads

\[
\xi^A = (\varphi, f), \quad f = (\varphi(x), \psi(x), A^T_a(x), \psi^T_a(x), h^{TT}_{ab}(x), \ldots). \tag{2.28}
\]

At the quantum level the ADM reduction can be easily carried out by the method described in [28, 12, 27] for the tunneling state (2.19). However it stumbles upon the problem of positive and negative frequency components for the Hartle-Hawking wavefunction (2.18) and in the gauge (2.27) encounters the analogue of the Gribov copies problem, corresponding to these components. As discussed in [29], these copies are an artifact of using inappropriate gauge, whose surface intersects twice the classical extremal (2.7) of one and the same Universe before and after its bounce against the minimal value of a cosmological radius \( a = 1/H(\varphi) \). This implies a dubious interpretation of (2.18) as a superposition of two simultaneously existing states of expanding and contracting Universe. This problem can be resolved at the fundamental level by the

\(^2\)This gauge is very convenient because it approximately corresponds to the choice of the proper time with the lapse function \( N^\pm = 1 \) [12].
transition to quantization in conformal superspace in the framework of the York gauge [29], but this framework is not yet developed to be a workable technique. However, in the present model in the semiclassical approximation it is enough merely to consider the quantum ADM reduction for a separate positive frequency (or negative frequency) component of (2.18).

Thus, semiclassically for both cosmological states the quantum ADM reduction boils down to obtaining the corresponding wave function of physical variables $\Psi(\xi, t) = \Psi(\varphi, f | t)$. Then, the distribution function of $\varphi$ should be regarded as a diagonal element of the density matrix of this pure state $\langle \Psi | \Psi \rangle$. It can be obtained from $|\Psi\rangle = \Psi(\phi, f | t)$ by averaging over the rest of the modes of physical fields $f$

$$\rho(\psi | t) = \int df \Psi^*(\phi, f | t) \Psi(\phi, f | t),$$

(2.29)

and does not reduce to a simple squaring of the wave function.

The calculation of the one-loop no-boundary and tunnelling wavefunctions perturbatively in inhomogeneous modes $f$ on the Friedmann-Robertson-Walker background was carried out in many papers [30, 31, 17, 32, 18, 11, 13, 1]. It can be based on the path integration over the fields regular on the Euclidean spacetime with metric (2.8) - (2.9) or by using the known one-loop approximation for the general solution of the Wheeler-DeWitt equations [28, 12, 27]. Then both wavefunctions turn out to be Gaussian in the variables $f$ - their Euclidean DeSitter invariant vacuum [17]. Therefore the integration over $f$ in (2.29) is trivial and leads to the fundamental algorithm which is valid for both no-boundary [11, 12, 13, 33, 1] and tunneling [34] quantum states

$$\rho_{NB,T}(\varphi | t) \approx \frac{\Delta_{\varphi}^{1/2} \varphi}{|v_{\varphi}(t)|^2} e^{\mp 2 I(\varphi) - \Gamma_{1-loop}(\varphi)},$$

(2.30)

It involves the doubled Euclidean action on the hemisphere with the metric (2.8) - (2.9), the linearized mode of the homogeneous inflaton field $v_{\varphi}(t)$ - the basis function of the wave equation on the Lorentzian DeSitter background (2.6) - (2.7), which can be obtained from the \textit{regular} Euclidean linearized mode $u_{\varphi} = u_{\varphi}(\tau)$

$$\frac{\delta^2 I[\xi]}{\delta \xi \delta \xi} u_{\varphi} = 0$$

(2.31)

by the analytic continuation (2.10)

$$\frac{\delta^2 S[\xi]}{\delta \xi \delta \xi} v_{\varphi} = 0,$$

(2.32)

$$v_{\varphi}(t) = [u_{\varphi}(\pi/2H + it)]^*.$$  

(2.33)

Here $I[\xi]$ and $S[\xi]$ are the Euclidean and Lorentzian action functional reduced to the ADM physical variables, so that $\delta^2 I[\xi]/\delta \xi \delta \xi$ and $\delta^2 S[\xi]/\delta \xi \delta \xi$ are the operators of
the physical inverse propagators. The mode $v_\psi(t)$ is normalized in (2.30) to unity with respect to the Wronskian inner product in the space of solutions of this wave equation

$$\Delta_\psi = i v_\psi'(t) \hat{W} v_\psi(t),$$

(2.34)

where $\hat{W} = \hat{W} - \hat{W}$ is a corresponding Wronskian operator linear in time derivative. The one-loop contribution is the same for both quantum states and is determined by the Euclidean effective action of all physical fields $\xi(x)$

$$\Gamma_{1\text{-loop}}(\varphi) = \frac{1}{2} \text{Tr} \ln \left. \frac{\delta^2 I[\xi]}{\delta \xi \delta \xi} \right|_{\text{DS}}.$$  (2.35)

This effective action is calculated on the DeSitter instanton - 4-dimensional sphere of the radius $1/H(\varphi)$ - and, therefore, is a function of $\varphi$ - the argument of the distribution function. Such a closed Euclidean manifold is obtained by the doubling of the half-instanton [13] - two hemispheres match each other along the equatorial hypersurface $\Sigma_B$ (on which a quantum transition with the change of signature takes place). Graphical illustration of the procedure of calculating the distribution function is given on Fig. 3. The wave function and its conjugate, participating in the scalar product (2.29), can be represented by two Euclidean-Lorentzian manifolds. When calculating the inner product, due to implicit unitarity of the theory, the contributions of Lorentzian regions cancel each other and the result boils down to the Euclidean effective action calculated on a closed instanton obtained by gluing the two hemispheres of the above type [13, 33].

The above simple scheme holds only in the lowest order of the slow-roll approximation, when the inflaton scalar field is constant on the solution of classical (Euclidean and Lorentzian) equations of motion and generates the effective Hubble and cosmological constants invariable in time. This is the case of the so-called real tunnelling geometry [35]. In the chaotic inflation model, however, the inflaton field is varying with time which makes the spacetime geometry deviating from the exact DeSitter one, and correspondingly the analytic continuation from the Euclidean regime to the Lorentzian one makes the extremal complex. The general case of complex gravitational tunneling was considered in much detail in [13] and for this particular model in [36]. The modification due to the imaginary part of the complex extremal looks as follows. The complex minisuperspace extremal $Q(z) = (a(z), \varphi(z))$ in the complex plane of the Euclidean time $z = \tau + it$ should satisfy the following boundary-value problem

$$\frac{\delta I(Q)}{\delta Q(z)} = 0, \quad \frac{d\varphi(z)}{dz} \bigg|_{z=0} = 0, \quad a(z) \sim z + O(z^2), \quad z \to 0,$$

(2.36)

$$Q(z_+) = q_+ = (a_+, \varphi_+)$$

(2.37)
with the Hartle-Hawking no-boundary conditions at $z = 0$ and real boundary data $(a_+, \varphi_+)$ at the boundary of the spacetime ball $z = z_+$. The Euclidean action $I(a_+, \varphi_+) = I[Q(z)]$ calculated on the solution of this problem is in general complex but cannot be decomposed as before in the purely real contribution of the Euclidean part of the full manifold and imaginary contribution of its real Lorentzian section. Thus, the algorithm (2.30) still holds, but with $I(\varphi)$ replaced by the real part of the complex action

$$I(\varphi) \rightarrow \Re I[Q(z)] = \Re I(a_+, \varphi_+) \quad (2.38)$$

and with all the other quantities calculated on the background of this extremal [13]. The final real point of the extremal $(a_+, \varphi_+)$ should be a subject of the ADM reduction which identifies $\varphi_+$ with the physical variable and expresses $a_+$ as a function of the physical time $t_+$ and $\varphi_+$ in some gauge, like (2.27). In what follows we shall find such a complex extremal in the first subleading order of the slow-roll approximation and calculate the corresponding distribution function. The model we consider will be a chaotic inflationary cosmology with the inflaton field non-minimally coupled to curvature with a large negative coupling constant $\xi = -|\xi|$. As we shall see, a small parameter of the slow roll expansion in this model turns out to be inverse proportional to $|\xi|$, $m_P^2/|\xi|\varphi^2 \ll 1$. This choice is justified by the fact that this model with $|\xi| \simeq 2 \times 10^4$ is regarded as a good candidate for the inflationary scenario compatible with the observational status of inflation theory [19].

3. Non-minimal inflaton scalar field: tree-level approximation

3.1. Perturbation theory for Euclidean classical solutions

Let us find the classical solution of the problem (2.36) in the model (2.21) with the nonminimally coupled inflaton field. We shall begin solving this problem on the Euclidean segment of complex time $z = \tau$. The Euclidean action of this model in the minisuperspace of homogeneous scale factor $a$, inflaton $\varphi$ and lapse $N$ looks like

$$I[N, a, \varphi] = -2\pi^2 \int_{\tau_-}^{\tau_+} d\tau N a^3 \left\{ \frac{1}{2} \left( \frac{3m_P^2}{4\pi} - 6\xi\varphi^2 \right) \left( \frac{1}{a^2} - \frac{\dot{a}^2}{a^2} \right) - \frac{1}{2} \ddot{\varphi}^2 - 6\xi\varphi\frac{\dot{a}}{a} - \frac{1}{2} m^2 \varphi^2 - \frac{1}{4} \lambda\varphi^4 \right\}. \quad (3.1)$$

where the dot denotes the differentiation with respect to proper Euclidean time $d/Nd\tau$. 

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The first order variational derivatives of this action with respect to $\varphi$ and $N$ give the Euclidean equations of motion

$$I_{\varphi} \equiv -A \left\{ \dot{\varphi} + 3\frac{\dot{a}}{a} \dot{\varphi} - \left[ m^2 + \lambda \varphi^2 + 6\xi \left( \frac{1}{a^2} - \frac{\dot{a}^2}{a^2} - \frac{\ddot{a}}{a} \right) \right] \varphi \right\} = 0, \quad (3.2)$$

$$I_{,N} \equiv -\frac{A}{N} \left\{ \frac{1}{2} \left( \frac{3m_P^2}{4\pi} - 6\xi \varphi^2 \right) \left( \frac{1}{a^2} - \frac{\dot{a}^2}{a^2} \right) + \frac{1}{2} \dot{\varphi}^2 + 6\xi \varphi \dot{\varphi} \frac{\dot{a}}{a} \right. \left. \right. - \frac{1}{2} m^2 \varphi^2 - \frac{1}{4} \lambda \varphi^4 \right\} = 0, \quad (3.3)$$

where the normalization factor $A = 2\pi^2 N a^3$. Eq. (3.2) is a dynamical equation of motion for the scalar field $\varphi$, while Eq. (3.3) is a Lagrangian version of the Hamiltonian constraint. There is no need to write down the variation of the action with respect to the scale factor $a$ because this equation is a consequence of (3.2) - (3.3).

We now develop the perturbation theory for the above equations starting with the DeSitter solution as a lowest order approximation of a constant scalar field $\varphi$. This solution in the cosmic time gauge $N = 1$ has the form

$$\varphi^{(0)} = \varphi_0 = \text{const}, \quad a^{(0)} = \frac{1}{H(\varphi_0)} \sin[H(\varphi_0) \tau], \quad (3.4)$$

where the bracketed superscript denotes the order of perturbation theory and $\varphi_0 = \varphi(0)$ is the initial value of the inflaton scalar field at $\tau = 0$ and $a = 0$. Substituting it into Eq. (3.3) we obtain $H(\varphi)$ as a following function of the scalar field:

$$H^2(\varphi) = \frac{m^2 \varphi^2 + \lambda \varphi^4 / 2}{3m_P^2 / 4\pi - 6\xi \varphi^2} = \frac{\lambda \varphi^2}{12|\xi|} \left[ 1 - \frac{m_P^2 (1 + 2\delta)}{8\pi |\xi| \varphi^2} + O \left( \frac{m_P^4}{|\xi|^2 \varphi^4} \right) \right]. \quad (3.5)$$

On the solution (3.4) the equation (3.3) is satisfied exactly, while the equation (3.2) holds only approximately

$$I_{,\varphi}^{(0)} = A_0 M_0^2 \varphi_0, \quad (3.6)$$

$$M_0^2 \equiv M^2(\varphi_0) = \left[ m^2 + \lambda \varphi^2 + 12\xi H^2(\varphi) \right]|_{\varphi = \varphi_0} \quad (3.7)$$

with the effective mass in the dynamical equation for inflaton field $M^2(\varphi)$ which is small in view of (3.5) (remember that $\xi = -|\xi| < 0$) in the limit of large $|\xi|$ and $\varphi^2$

$$M^2(\varphi) = \lambda m_P^2 \frac{1 + \delta}{8\pi |\xi|} + O \left( \frac{m_P^4}{|\xi|^2 \varphi^4} \right), \quad (3.8)$$

provided the parameter $\delta$ defined by Eq.(2.23) is bounded (in the chaotic inflation model with nonminimal inflaton the constant $|\xi|$ is usually chosen of the order of magnitude of ratio of the Planck scale to the GUT scale $v = -m^2 / \lambda$, so that $\delta \ll 1$).
This property of cancellation of the leading in φ contributions to (3.7) underlies the slow-roll approximation in this model of the nonminimal and nonlinear inflaton field. This approximation works for large values of the inflaton, its inverse playing the role of smallness parameter. As we shall now see, big negative ξ further improves this expansion which actually takes place in powers of \( m_P^2 / |\xi| \phi^2 \).

To find the first subleading order of this expansion, we have to expand the equations of motion (3.2)-(3.3) up to the first order in perturbations

\[
I_{,N,\phi} \phi + I_{,N, a} \delta a = -I_{,N} \equiv 0, \tag{3.9}
\]

\[
I_{,\phi,\phi} \delta \phi + I_{,\phi, a} \delta a = -I_{,\phi}^{(0)}, \tag{3.10}
\]

and solve this linear system for \( \delta \phi \) and \( \delta a \). Here we use an obvious notations for the second order variational derivatives of the action, which are the differential operators evaluated at the lowest-order solution (3.4). With the choice of a new (angular) variable on the DeSitter sphere, replacing the Euclidean time,

\[
\theta = H(\phi_0) \tau, \tag{3.11}
\]

these operators take the form

\[
I_{,\phi,\phi} = -AH^2 \left\{ \frac{d^2}{d\theta^2} + 3 \cot \theta \frac{d}{d\theta} - \left( \frac{m^2}{H^2} - 24\xi \right) \right\}, \tag{3.12}
\]

\[
I_{,\phi, a} = -AH^2 \frac{6\xi \phi_0}{a} \left\{ \frac{d^2}{d\theta^2} + 2 \cot \theta \frac{d}{d\theta} + 3 \right\} + \frac{3}{a} I_{,\phi}, \tag{3.13}
\]

\[
I_{,N,\phi} = AH^2 \left\{ 6\xi \phi_0 \left( 1 - \cot \theta \frac{d}{d\theta} \right) + \frac{m^2 \phi_0 + \lambda \phi_0^3}{H^2} \right\}, \tag{3.14}
\]

\[
I_{,N, a} = \frac{AH^2}{a} \left( 3m_P^2 / 4\pi - 6\xi \phi_0^2 \right) \left( 1 + \cot \theta \frac{d}{d\theta} \right), \tag{3.15}
\]

with the normalization factor \( A \), Hubble constant \( H \) and scale factor \( a \) taken in the lowest-order approximation.

For large \( \phi_0 \) and in view of (3.5) the last two equations can be simplified

\[
\frac{1}{AH^2} I_{,N,\phi} \delta \phi = -6\xi \phi_0 \frac{\cos^2 \theta}{\sin \theta} \frac{d}{d\theta} \frac{\delta \phi}{\cos \theta} + O(m_P / \phi_0), \tag{3.16}
\]

\[
\frac{1}{AH^2} I_{,N, a} \delta a = -6\xi \phi_0^2 H \frac{\cos^2 \theta}{\sin^2 \theta} \frac{d}{d\theta} \frac{\delta a}{\cos \theta} \left( 1 + O(m_P^2 / \phi_0^2) \right), \tag{3.17}
\]

and used in the linearized constraint equation (3.9) to give

\[
\frac{d}{d\theta} \frac{\delta a}{\cos \theta} = -\frac{\sin \theta}{\phi_0 H_0} \frac{d}{d\theta} \frac{\delta \phi}{\cos \theta} + O(m_P^5 / \phi_0^5), \tag{3.18}
\]
This equation in its turn can be used in the dynamical linearized equation (3.10). The latter in view of (3.12)-(3.13) can be rewritten in the form

\[
\left\{ \frac{d^2}{d\theta^2} + 3 \cot \theta \frac{d}{d\theta} + 24 \xi \right\} \delta \varphi + 6 \xi \varphi_0 H \left( \frac{d}{d\theta} + 3 \cot \theta - \tan \theta \right) \frac{\cos \theta \frac{d}{d\theta} \delta a}{\sin \theta \frac{d}{d\theta} \cos \theta} = \frac{M_0^2 \varphi_0}{H^2} + O(m_P^3/\varphi_0^3),
\]

which allows one to exclude \( \delta a \) on account of (3.18) and thus arrive at the closed equation for \( \delta \varphi \)

\[
\left\{ \frac{d^2}{d\theta^2} + 3 \cot \theta \frac{d}{d\theta} \right\} \delta \varphi = \frac{1}{1 - 6 \xi} \frac{M_0^2 \varphi_0}{H^2} + O(m_P^3/\varphi_0^3).
\]

Its solution satisfying the Hartle-Hawking boundary conditions [3, 4] \((d/d\theta)\delta \varphi(0) = 0, \delta \varphi(0) = 0\), reads (we use (3.5) and (3.8))

\[
\delta \varphi = -\frac{m_P^2}{\pi \varphi_0} \frac{1 + \delta}{1 - 6 \xi} \left( \ln \cos \frac{\theta}{2} - \frac{1}{4} \tan^2 \frac{\theta}{2} \right) + O(m_P^3/\varphi_0^3).
\]

The corresponding perturbation of the scale factor \( \delta a \) can then be obtained by integrating the equation (3.18) with zero initial condition at \( \theta = 0 \)

\[
\delta a = \frac{m_P^2}{\pi \varphi_0^3 H(\varphi_0)} \frac{1 + \delta}{1 - 6 \xi} \cos \theta \left( \tan \theta \ln \cos \frac{\theta}{2} - \frac{1}{4} \tan^2 \frac{\theta}{2} \tan \theta - \frac{\theta}{4} + \frac{1}{2} \tan \frac{\theta}{2} - \int_0^\theta d\theta' \ln \cos \frac{\theta'}{2} \right) + O(m_P^5/\varphi_0^5).
\]

The results of this section can be summarized in the one-parameter family of Euclidean extremals enumerated by the initial value of the inflaton field \( \varphi_0 \)

\[
\varphi(\tau, \varphi_0) = \varphi_0 + \delta \varphi(\theta, \varphi_0), \quad \theta \equiv H(\varphi_0) \tau
\]

(3.23)

\[
a(\tau, \varphi_0) = \frac{1}{H(\varphi_0)} \sin \theta + \delta a(\theta, \varphi_0), \quad \theta \equiv H(\varphi_0) \tau
\]

(3.24)

with perturbations \( \delta \varphi = O(m_P/\varphi_0) \) and \( \delta a = O(m_P^3/\varphi_0^3) \) given above. These perturbations are small for large \( \varphi_0 \). The corresponding smallness parameter is \( m_P^2/\varphi_0^2 \) (one should remember that for a bounded range of \( \theta \) and in the leading order \( \varphi \sim \varphi_0, a \sim \varphi_0^{-1} \)). A direct inspection of these perturbations also shows that for large negative constant of nonminimal coupling \( |\xi| \gg 1 \) the actual smallness parameter is \( m_P^2/|\xi|\varphi_0^2 \), and the final conclusions of this paper will be obtained for \( m_P^2/|\xi|\varphi_0^2 \ll 1 \).
3.2. **Superspace caustic and complex extremals**

Once we have a perturbative solution parametrized by the initial value of the scalar field $\varphi_0$, we can now perturbatively solve the boundary value problem (2.37) which takes the form of two equations

\[
\varphi_0 + \delta \varphi(\theta_+, \varphi_0) = \varphi_+,
\]

\[
\frac{1}{H(\varphi_0)} \sin \theta_+ + \delta a(\theta_+, \varphi_0) = a_+,
\]

for $(\varphi_0, \theta_+)$ in terms of $(\varphi_+, a_+)$. Like (3.23)-(3.24) this solution can be obtained in a subleading approximation in the form

\[
\varphi_0 = \varphi_0^{(0)} + \varphi_0^{(1)}, \quad \theta_+ = \theta_+^{(0)} + \theta_+^{(1)},
\]

where the superscript in brackets denotes the order of perturbation theory.

The lowest order approximation

\[
\varphi_0^{(0)} = \varphi_+,
\]

\[
\theta_+^{(0)} = \arcsin [H(\varphi_+) a_+]
\]

shows that only the points $(\varphi_+, a_+)$ lying below the curve

\[
a = \frac{1}{H(\varphi)}
\]

can be reached by Euclidean trajectories (with the no-boundary initial conditions) in real time. This curve consists of the points of maximal expansion of the Euclidean model at $\theta = \pi/2$ and can be regarded as a caustic of the family of solutions in the lowest order of the slow roll expansion. The points of two-dimensional minisuperspace above this curve can be reached only in complex Euclidean time because

\[
\theta_+^{(0)} = \frac{\pi}{2} + iH(\varphi_+) t_+,
\]

\[
H t_+ = \text{arcosh} \left[ H(\varphi_+) a_+ \right],
\]

where $t_+$ can be interpreted as the Lorentzian time in the DeSitter space nucleating at $t_+ = 0$ from the Euclidean DeSitter hemisphere parameterized by the angular coordinate $\theta = H(\varphi_+) \tau$, $0 \leq \theta \leq \pi/2$. Thus in this case the combined Euclidean-Lorentzian evolution takes place on a contour in a complex plane of time, consisting of the Euclidean $0 \leq \tau \leq \pi/2H(\varphi_+)$ and Lorentzian $\tau = \pi/2H(\varphi_+) + i t$, $0 \leq t \leq t_+$ segments.

In the lowest order of the slow roll approximation the point of Euclidean-Lorentzian transition coincides with the point of maximal expansion of $a$ on the Euclidean trajectory where the both velocities $\dot{\varphi}$ and $\dot{a}$ vanish. This situation corresponds to the
so-called real tunnelling [35] when the analytically matched Euclidean and Lorentzian extremals both have real values of the configuration space coordinates. Beyond the leading order of the slow roll approximation this property does not hold.

The first subleading approximation for \((\varphi_0, \theta_+\) can be obtained by making in Eqs. (3.25)-(3.26) the first-order iteration in \(\delta \varphi\) and \(\delta a\)

\[
\varphi_0^{(1)} = -\delta \varphi(\theta_+^{(0)}, \varphi_+),
\]

\[
\theta_+^{(1)} = \frac{1}{\cos \theta_+^{(0)}} \left( H(\varphi_+) \delta a(\theta_+^{(0)}, \varphi_+) + \sin \theta_+^{(0)} \delta \varphi(\theta_+^{(0)}, \varphi_+)/\varphi_+ \right) = \frac{m^2_\pi}{\pi \varphi_0^2} \frac{1 + \delta}{1 - 6\xi} \left( \frac{\theta_+^{(0)}}{4} - \frac{1}{2} \tan \frac{\theta_+^{(0)}}{2} + \int_0^{\varphi_0} d\theta \ln \cos \frac{\theta}{2} \right) + O(m^4_\pi / \varphi_+^4). \quad (3.34)
\]

In this approximation the point of maximal expansion (extremum of (3.26)) at \(\theta_m\) differs from \(\pi/2\)

\[
\theta_m = \frac{\pi}{2} + \varepsilon_m, \quad (3.35)
\]

\[
\varepsilon_m = \frac{m^2_\pi}{\pi \varphi_0^2} \frac{1 + \delta}{1 - 6\xi} \left( -\frac{3}{2} + \frac{\pi}{8} - \frac{\pi}{2} \ln 2 + G \right), \quad (3.36)
\]

where \(G\) is the Catalan constant \((G = \int_0^{\pi/4} d\theta \ln \cot \theta = 0.915\ldots)\). The set of these points, however, does not form the caustic curve: in contrast with the lowest order of the slow roll approximation, in which the dynamics is actually one-dimensional, \(\dot{\varphi}_0 = 0\), now the both minisuperspace coordinates depend on time and the equation that determines the envelope of the \(\varphi_0\)-parameter family of extremals (3.23)-(3.24) is given by the following requirement. At the envelope curve the vector tangential to the extremal \((\dot{\varphi}, \dot{a})\) and the vector transversal to their one parameter family \((\partial \varphi/\partial \varphi_0, \partial a/\partial \varphi_0)\) are collinear, i.e.

\[
\dot{\varphi} \frac{\partial a}{\partial \varphi_0} - \dot{a} \frac{\partial \varphi}{\partial \varphi_0} = 0. \quad (3.37)
\]

From this equation it follows that the extremal starting at \(\tau = 0\) with the initial scalar field \(\varphi_0\) hits the caustic at

\[
\theta_+ (\varphi_0) = \frac{\pi}{2} + \varepsilon_+ (\varphi_0), \quad (3.38)
\]

\[
\varepsilon_+ (\varphi_0) = \frac{m^2_\pi}{\pi \varphi_0^2} \frac{1 + \delta}{1 - 6\xi} \left( -\frac{1}{2} + \frac{\pi}{8} - \frac{\pi}{2} \ln 2 + G \right). \quad (3.39)
\]

The caustic curve when parametrized by the value of \(\varphi_0\) can be obtained by substituting \(\theta_+ (\varphi_0)\) in (3.23) and (3.24). Then, with \(\varphi_0\) excluded in terms of \(\varphi\) from the second of the resulting equations, the first of them becomes the equation of the caustic as a
graph of $a$ against $\varphi$. Simple calculations show that the first order corrections cancel out and the caustic equation remains the same as the leading-order one (3.30) $a = 1/H(\varphi) + O(m_p^4/\varphi^5)$. The meaning of this curve implied by the equation (3.37) is that its points belonging to a particular extremal $a = a(\tau, \varphi_0)$. $\varphi = \varphi(\tau, \varphi_0)$ are no longer in one-to-one correspondence with the value of the Euclidean time $\tau$ and initial value of the scalar field $\varphi_0$ enumerating the extremals

$$\frac{\partial (\varphi, a)}{\partial (\tau, \varphi_0)} = 0. \quad (3.40)$$

One cannot go beyond this curve in real Euclidean time: the continuation in real $\theta > \theta_0(\varphi_0)$ results in the trajectory bouncing back to the Euclidean domain under the caustic curve $a < 1/H(\varphi) + O(m_p^4/\varphi^5)$. Similarly to the lowest order approximation the points beyond this curve can be reached only in complex time. Now, however, we have two important differences. Firstly, the both velocities $(\dot{a}, \dot{\varphi})$ at the nucleation point (as anywhere else on every extremal) are nonvanishing simultaneously which results in the complex valuedness of the extremal ending at the point $(\varphi_+, a_+)$ beyond the caustic (the discussion of matching conditions at the Euclidean-Lorentzian transition for this case of complex tunnelling can be found in the previous authors’ work [13]). And, secondly, the point of nucleation (the corresponding value of the Euclidean time) is a nontrivial function of the end point of the extremal. To show this take $a_+ > 1/H(\varphi_+)$ and calculate the final value of the complex Euclidean time $\theta_+$ given by Eqs. (3.27), (3.31), (3.32), (3.34). It has a real part

$$\text{Re} \theta_+ \equiv \theta_N(\varphi_+, t_+) = \frac{\pi}{2} + \varepsilon_N(\varphi_+, t_+), \quad (3.41)$$

$$\varepsilon_N(\varphi_+, t_+) = \frac{m_p^2}{\pi \varphi_+^2} \left( \frac{1}{1 - 6 \xi} \left( -\frac{1}{2 \cosh[\frac{1}{2} \ln 2 + G + \frac{1}{2} H(\varphi_+) \int_0^{t_+} dt \arctan \sinh[\frac{1}{2} H(\varphi_+) t]} \right) \right), \quad (3.42)$$

which can be identified with the nucleation point provided we choose the contour of complex “angular” time $\theta$ joining the points $\theta = 0$ and $\theta_+$ with the union of two segments $0 \leq \theta \leq \theta_N$ and $\theta = \theta_N + i H(\varphi_+) t$, $0 \leq t \leq t_+$. In contrast with the case of real tunnelling this is just a convention, because on both segments the fields are complex and analyticity does not give any preference to this particular choice. Comparison of $\varepsilon_m, \varepsilon_c$ and $\varepsilon_N$ shows that all these deviations from the lowest order nucleation point $\theta = \pi/2$ are different: $\varepsilon_m < \varepsilon_c$ and $\varepsilon_N = \varepsilon_c$ only at $t_+ = 0$, that is when the end point of the extremal lies on the caustic, while for positive Lorentzian time $\varepsilon_N > \varepsilon_c$.

\footnote{Note that our perturbation theory for $\theta_N$ and other Lorentzian quantities in its present form is...}
Once we have a solution for equations (2.36)-(2.37) we can calculate the corresponding Hamilton-Jacobi function and, in particular, its real part \( \Re I(a_+, \varphi_+). \) When calculated in the lowest order approximation on the solution (3.4) the doubled real part of the action obviously coincides with the Euclidean action on the full four-dimensional sphere of the radius \( 1/H(\varphi_+) \) and constant scalar field \( \varphi = \varphi_+. \) When expanded in powers of \( m_P^2/\varphi_+^2 \) it equals

\[
2 \Re I(a_+, \varphi_+) = I_0 + \frac{I_1}{\varphi_+^2} + O \left( \frac{m_P^4}{\varphi_+^4} \right),
\]

\[
I_0 = -\frac{96\pi^2|\xi|^2}{\lambda},
\]

\[
I_1 = -\frac{24\pi m_P^2|\xi(1+\delta)}{\lambda}.
\]

Since we want to have this quantity in the first subleading approximation in \( m_P^2/\varphi_+^2, \) a priori we have to include corrections to this result linear in \( \delta \varphi \) and \( \delta a. \) By direct calculations these corrections can be shown to vanish due to certain intrinsic cancellations. The mechanism of these cancellations follows from the parametrized nature of the gravity theory and looks as follows. Using the notations of (2.36)-(2.37) (with complex \( \theta \) replacing \( z \)) we can write down the total action on the extremal subject to boundary conditions \( q_+ \) at \( \theta_+ \) as an integral over \( \theta \) of the corresponding Lagrangian

\[
I_{\theta_+}[Q(\theta)] = \int_{\theta_0}^{\theta_+} d\theta L(Q, \dot{Q})
\]

with a subscript indicating the upper limit of integration \( \theta_+ \) over the complex angular “time” which is a solution of the boundary condition (2.37). We have a solution of classical equations of motion as a perturbation expansion \( Q(\theta, \varphi_0) = Q^{(0)}(\theta, \varphi_0) + \delta Q(\theta, \varphi_0) \) and in two subsequent orders of this perturbation theory the boundary conditions have the form

\[
Q^{(0)}(\theta_+, \varphi_0^{(0)}) = q_+, \quad (3.47)
\]

\[
Q^{(0)}(\theta_+^{(0)} + \theta_+^{(1)}, \varphi_0^{(0)} + \varphi_0^{(1)}) + \delta Q(\theta_+^{(0)}, \varphi_0^{(0)}) = q_+, \quad (3.48)
\]

whence it follows that

\[
\left[ \frac{\partial Q^{(0)}}{\partial \varphi_0} \varphi_0^{(1)} + \dot{Q}^{(0)} \theta_+^{(1)} + \delta Q \right]_{\theta_+} = 0, \quad (3.49)
\]

valid only for small values of the Lorentzian time \( t, \) \( H(\varphi_+) t \sim 1, \) to be valid in the slow roll limit of big \( H(\varphi_+). \) In the Euclidean context one of the motivations for introducing the coordinate \( \theta = H \tau \) instead of \( \tau \) was the fact the range of \( \theta \) below the caustic is bounded by \( \pi/2 \) which is different from the Lorentzian domain where the hyperbolic “angle” \( H t \) is unbounded from above.
where the dot denotes the derivative with respect to $\theta$. Now, the total action (3.46) calculated in a first subleading order of perturbation theory reads

$$I_{\epsilon_+^{(0)}+\epsilon_+^{(1)}}[Q^{(0)}(\theta, \varphi_0^{(0)} + \varphi_0^{(1)} + \delta Q)]$$

$$= I_{\epsilon_+^{(0)}}[Q^{(0)}(\varphi_0^{(0)}, \theta)] + I_{\epsilon_+^{(1)}} + L \theta_+^{(1)} + \left[ \frac{\partial L}{\partial Q} \left( \frac{\partial Q^{(0)}}{\partial \varphi_0^{(0)}} \varphi^{(1)}_0 + \delta Q \right) \right]$$

(3.50)

(here the first order variation of the functional argument of the action reduces to the standard surface term at $\theta_+$, because this variation is being calculated at the solution of the classical equations). Then from (3.49) the total first order correction induced by $\delta Q$ reduces to

$$\left( L - \frac{\partial L}{\partial Q} \right) \theta_+^{(1)} = 0$$

(3.51)

and vanishes because the coefficient of $\theta_+^{(1)}$ here in a reparametrization invariant theory boils down to a Hamiltonian constraint identically satisfied for a classical background. Thus the final expression for the real part of the classical action reduces in the subleading approximation to (3.43)-(3.45).

The absence of the first-order corrections in $\Re I(a_+, \varphi_+)$ due to $\delta Q$ guarantees the tree level unitarity of the theory – the time independence of the semiclassical wavefunction amplitude. As it follows from (3.43) it depends only on the final value of the scalar field $\varphi_+$, while the contribution of $\delta Q$ could have introduced a nontrivial dependence on $a_+$ (or $t_+$). This property, obvious for real tunnelling, for complex extremals was shown to be a consequence of the Einstein-Hamilton-Jacobi equation for complex semiclassical phase [35, 13].

4. The homogeneous inflaton mode

Here we calculate the contribution of the Lorentzian inflaton mode $v_\varphi(t)$ to the probability distribution $\rho(\varphi, t)$, given by the preexponential factor $\Delta_\varphi^{1/2} / |v_\varphi(t)|$ of (2.30). The algorithm (2.30) was obtained within the ADM reduced phase-space quantization implying that $\varphi$ is a physical degree of freedom while the scale factor $a$ and the lapse function $N$ are determined by the gauge condition (2.27). This gauge condition introduces cosmic time (with unit lapse $N = 1$) [12], therefore the Euclidean and Lorentzian wave equations for $v_\varphi(t)$ (2.31)-(2.32) can be obtained by the same linearization procedure as in Sect.3 (performed in $N = 1$ gauge). After exclusion of $\delta a$ of this section, this equation boils down to the homogeneous version of Eq.(3.20) with $u_\varphi(\tau)$ replacing
\[ \delta \varphi \]

\[
\frac{\delta^2 I[\xi]}{\delta \xi \delta \xi} u_\varphi = \frac{2\pi^2}{H} \left\{ -\frac{d}{d\theta} \sin^3 \theta \frac{d}{d\theta} + O(m_P^2/|\xi| \varphi_0^2) \right\} u_\varphi = 0 \quad (4.1)
\]

(one should remember that the operator in (3.20) enters the quadratic part of the action with the factor \( A H^2 = 2\pi^2 a^3 H^2 = 2\pi^2 \sin^3 \theta / H \). This equation was, however, obtained only in the lowest order approximation while we would need to know \( v_\varphi(t) \) in a subleading order of the slow roll expansion. The remedy is to use the one-parameter family of classical solutions already known in this approximation. The needed linearized mode can be obtained by differentiating this solution (3.24) with respect to \( \varphi_0 \)

\[
u_\varphi(\tau) = \frac{\partial \varphi(\tau, \varphi_0)}{\partial \varphi_0} \quad (4.2)
\]

and then analytically continuing it to the Lorentzian spacetime (2.33). Using (3.21) one has the Euclidean mode in the first subleading approximation

\[
u_\varphi(\tau) = 1 + \frac{m_P^2}{2\pi \varphi_0^2} \frac{1 + \delta}{1 - 6\xi} \left\{ \frac{\theta}{2 + \cos \theta} \tan \frac{\theta}{2} + 2 \ln \cos \frac{\theta}{2} - 1 \right\}, \quad (4.3)
\]

Since it has the form \( 1 + O(m_P^2/|\xi| \varphi_0^2) \) for \( |\xi| \gg 1 \) the modulus of the corresponding Lorentzian mode gives the contribution to the exponential of \( \rho(\varphi, t) \)

\[
\ln \frac{1}{|v_\varphi(t)|} = O(m_P^2/|\xi| \varphi_0^2) \quad (4.4)
\]

which is by one power of \( 1/\xi \) smaller in magnitude than the subleading term \( I_1/\varphi_0^3 \) of the corresponding tree-level contribution (3.43).

Now we have to calculate the Wronskian norm of this mode \( \Delta_\varphi \). From (4.1) it follows that the Wronskian operator in the Euclidean time \( \tau = \theta / H \) (\( \hat{W}^E = \hat{W}^E - \hat{W}^E = i \hat{\rho} \)) equals

\[
\hat{W}^E \equiv W^E = -2\pi^2 \frac{1}{H^2} \sin^3 \theta \frac{d}{d\theta} \quad (4.5)
\]

The time-independent inner product (2.34) can be calculated at the nucleation point \( t = 0 \) where it equals \( 2u_\varphi \hat{W}^E u_\varphi \). Applying the Wronskian operator to (4.3) we get

\[
\Delta_\varphi = \frac{6\pi^2(1 + \delta)m_P^2}{\lambda \varphi_0^2} \left[ 1 + O(m_P^2/|\xi| \varphi_0^2) \right], \quad (4.6)
\]

so that the final contribution of the inflaton mode equals

\[
\ln \frac{\Delta_\varphi^{1/2}}{|v_\varphi(t)|} = \text{const} - 2 \ln \varphi_0 + O(m_P^2/|\xi| \varphi_0^2), \quad (4.7)
\]
5. One-loop effective action on the DeSitter instanton

In this section we begin calculating the contribution of the one-loop effective action (2.35) to the distribution function (2.30). As it was discussed in the end of Sect.2, this quantity should be calculated at the extremal $Q = Q^{(0)} + \delta Q$ which differs from the exact DeSitter background (with constant inflaton) $Q^{(0)}$ by the corrections of the slow roll expansion:

$$\Gamma_{1\text{-loop}} = \frac{1}{2} \text{Tr} \ln \frac{\delta^2 I [\xi]}{\delta \xi \delta \xi} \bigg|_{Q^{(0)} + \delta \xi} = \Gamma^{(0)}_{1\text{-loop}} + \delta \Gamma_{1\text{-loop}}, \quad (5.1)$$

So here we obtain $\Gamma^{(0)}_{1\text{-loop}}$ and in the Sect.6 develop the perturbation theory for $\delta \Gamma_{1\text{-loop}}$.

It is important that in contrast to the 2-dimensional minisuperspace sector $(a, \varphi)$ that was only probed by the tree-level approximation of the theory, the one-loop order involves the contribution of all fields inhabiting the model. Without loss of generality we shall assume that its low-energy (sub-Planckian) sector is given by the inflaton-graviton action (2.21) plus arbitrary set of Higgs scalars $\chi(x)$, vector gauge bosons $A_\mu(x)$ and fermions $\psi(x)$. It can also include gravitino, but we shall mainly focus at this sector of spin 0, 1/2, 1 and spin 2 fields. In the full Lagrangian

$$L(g_{\mu\nu}, \varphi, \chi, A_\mu, \psi) = L(g_{\mu\nu}, \varphi) + g^{1/2} \left( -\frac{1}{2} \sum_\chi (\nabla \chi)^2 - \frac{1}{4} \sum_A F^2_{\mu\nu}(A) - \sum_\psi \bar{\psi} \gamma^\mu \psi \right) + L_{\text{int}}(\varphi, \chi, A_\mu, \psi) \quad (5.2)$$

we single out the interaction of Higgs, vector and spinor fields with the inflaton field $L_{\text{int}}(\varphi, \chi, A_\mu, \psi)$. Its nonderivative part has the form

$$L_{\text{int}} = \sum_\chi \frac{\lambda_\chi}{4} \chi^2 \varphi^2 + \sum_A \frac{1}{2} g_A^2 A_\mu^2 \varphi^2 + \sum_\psi f_\psi \varphi \bar{\psi} \psi + \text{derivative coupling} \quad (5.3)$$

with Higgs $\lambda_\chi$, vector gauge $g_A$ and Yukawa $f_\psi$ coupling constants. In the Lagrangian (5.2) the inflaton field can be regarded as one of the components of one of the Higgs multiplets $\chi$, which has a nonvanishing expectation value in the cosmological quantum state. In its turn the choice of the interaction Lagrangian here is dictated by the renormalizability of the matter field sector of the theory (5.2) and by the requirement of local gauge invariance with respect to arbitrary Yang-Mills group of vector fields $A_\mu$. The terms of derivative coupling in (5.3) should be chosen to guarantee the latter, but their form will not be important for the conclusions of this paper. On the contrary,
the quantum gravitational effects will crucially depend on the nonderivative part of the interaction Lagrangian.

A very important property of the functional (5.1) is that it is calculated on shell, that is on the solution of classical equations, and therefore, as is well known from the theory of gauge fields [37, 38, 13], is independent of the choice of gauge conditions used for its construction (or ADM reduction to physical variables in terms of which it reduces to the functional determinant in the physical sector). This freedom can be used to transform (5.1) identically to the background covariant gauges in which the one-loop action takes the form of the functional determinant of the covariant operator

\[ F = \frac{\delta^2 I_{\text{tot}}[g]}{\delta g \delta g} \]  

acting in the full space of gauge and ghost fields

\[ g = (\varphi(x), \chi(x), \psi(x), A_\mu(x),\psi_\mu(x), g_{\mu\nu}(x), \ldots, C(x), \tilde{C}(x)). \]  

Here \( I_{\text{tot}}[g] \) is a total action defined on this space of fields including the covariant gauge-breaking and ghost terms, \( C \)'s denote all possible gauge and coordinate ghosts. Spatial components of fields in the nonghost sector of (5.5) form the canonical superspace of the theory (2.27). As compared to (2.27) we only added \( \chi(x) \) – the set of all scalar multiplets of the model other than the inflaton field \( \varphi(x) \), introduced above.

The possibility to convert the one-loop action to covariant form is very important, because in this form it admits covariant regularization and renormalization of inevitable ultraviolet divergences and allows one to obtain correct scaling behaviour, quantum anomalies, etc. The full set of gauges for internal gauge symmetries and gravitational diffeomorphisms can be chosen in such a way that the operator (5.4) becomes minimal – diagonal in second-order derivatives forming a covariant \( D' \)Alambertian \( \Box \). Moreover, on the exact DeSitter background with constant inflaton field this operator can as a whole be reduced to the block-diagonal form

\[ F = \text{diag} (-\Box_s + X_s) \]  

with blocks \(-\Box_s + X_s\) belonging to \( O(5) \) irreducible representations of spin \( s = 0, 1/2, 1, 3/2, 2, \ldots\) on the 4-dimensional sphere of the Euclidean DeSitter space. For constant potential terms \( X_s \) of these operators (which is the case of \( Q^{(0)} \)) their spectra are well known [39, 40, 41] and can be used for calculating their functional determinants under a proper covariant regularization. We shall use a \( \zeta \)-functional regularization [42] in which a contribution of every block of (5.6) equals

\[ -\frac{1}{2} \text{Tr} \ln(-\Box_s + X_s) = \frac{1}{2} \zeta'(0) + \frac{1}{2} \zeta(0) \ln \frac{\mu^2}{H^2} \]  

where \( \mu \) is the renormalization scale.
with the generalized \( \zeta \)-functions built of dimensionless eigenvalues of the rescaled operator
\[
\zeta_s(p) = \sum_{\lambda} \lambda^{-p}, \quad H^{-2}(-\Box_s + X_s)\phi_\lambda(x) = \lambda\phi_\lambda(x)
\]
(5.8)
(the scale of the DeSitter instanton is its inverse radius \( H \), so that the differential operator here is effectively defined on a sphere of unit radius with a correspondingly rescaled potential term). As a result the full one-loop action takes the form
\[
-\Gamma^{(0)}_{\text{1-loop}} = \frac{1}{2} \zeta'(0) + \frac{1}{2} S(0) \ln \frac{\mu^2}{H^2}, \\
\zeta'(0) = \sum_s w_s \zeta'_s(0), \quad \zeta(0) = \sum_s w_s \zeta_s(0).
\]
(5.9)
(5.10)
Here the weights \( w_s \) – positive and negative integers – reflect the statistics of the field and also the details of transition from the original local fields (5.5) to the decomposition in irreducible \( O(5) \) representations of (5.6). In the equations above \( \mu^2 \) is a mass parameter reflecting the renormalization ambiguity resulting from the subtraction of logarithmic divergences proportional to \( \zeta(0) \). This quantity plays a very important role because it determines the leading high-energy behaviour of the one-loop action and correspondingly the anomalous scaling behaviour of the distribution function \( \rho \sim H^{-\zeta(0)} \). As it was observed in [11, 1, 13, 34, 29] it can produce a principal quantum cosmological mechanism – to make the distribution function of quasi-DeSitter models normalizable in over-Planckian domain and generate the inflationary probability peak. Therefore we begin with the calculation of this quantity.

5.1. The anomalous scaling \( \zeta(0) \)

Apart from the method of \( O(5) \) irreducible representations and Eqs. (5.8) and (5.10) the total anomalous scaling \( \zeta(0) \) can be obtained by a more universal Schwinger-DeWitt technique [37, 38] as an integral over the spacetime
\[
\zeta(0) = \frac{1}{16\pi^2} \int d^4 x \ g^{1/2} \ a_2(x)
\]
(5.11)
of the second Schwinger-DeWitt coefficient \( a_2(x) \) in the proper time heat kernel expansion for the operator (5.4). This technique easily allows one to derive the mechanism

---

\[4\] This transition involves certain Jacobians which have the form of positive and negative powers of operators in (5.6) [39, 40, 41] leading to sign factors of the above type. The example of this procedure is presented below for gauge vector field.
of suppression of the over-Planckian energy scales due to a big value of the nonminimal coupling constant $|\xi| \gg 1$. Indeed, for large masses of particles the dominant contribution to $a_2(x)$

$$a_2(x) = \frac{1}{2} \left( \sum_x m^4_x + 4 \sum_A m^4_A - 4 \sum_{\psi} m^4_{\psi} \right) + \ldots$$

(5.12)

is quartic in their masses with the sign factor depending on statistics (and weight factors given by the number of the corresponding tensor field components). In the model (5.2)-(5.3) on the background with a big constant field $\varphi_0$ Higgs scalars, vector gauge bosons and fermions acquire by the analogue of the Higgs mechanism the effective masses induced by the interaction Lagrangian

$$m^2_\chi = \frac{\lambda_\chi \varphi^2_0}{2}, \quad m^2_A = g^2_A \varphi^2_0, \quad m^2_\psi = f^2_\psi \varphi^2_0.$$ 

(5.13)

Being integrated over the 4-volume of the instanton $8\pi^2/3H^4$ these masses generate in view of the expression (3.5) for $H(\varphi_0)$ the following dominant contribution to $\zeta(0)$

$$\zeta(0) = Z [1 + O(m^2_\varphi/|\xi| \varphi^2_0)], \quad Z = 6 \frac{\xi^2}{\lambda} A.$$ 

(5.14)

where $A$ is a following fundamental combination of matter fields coupling constants [1]

$$A = \frac{1}{2\lambda} \left( \sum_x \lambda^2_x + 16 \sum_A g^4_A - 16 \sum_{\psi} f^4_\psi \right).$$

(5.15)

Thus for large $|\xi| \gg 1$ and positive $A \sim O(1)$ we have $Z \gg 1$ which is a corner stone of the quantum gravitational mechanism that suppresses the over-Planckian energy scales, $\rho \sim H^{-2} \rightarrow 0$, $H \rightarrow \infty$ and, thus, serves as a justification of the semiclassical expansion.

It is important that this mechanism is entirely generated in the renormalizable matter-field sector of the theory consisting of the multiplets of the standard model, because the graviton-inflaton sector (2.21) yields the contribution independent of $\xi$ [43, 21] as well as the spin-3/2 gravitino field [44] (considered to be noninteracting with inflaton)

$$\zeta(0)_{\text{graviton+inflaton}} = -\frac{171}{10}, \quad \zeta(0)_{\text{gravitino}} = \frac{589}{180}.$$ 

(5.16)

This property suggests that this mechanism is robust against high energy modifications of the fundamental theory designed to solve the problems of nonrenormalizability in
perturbative quantum gravity.

Our purpose now is to go beyond the lowest order approximation (5.14) and find subleading corrections. For this we would need the subleading term in the expression for the Hubble constant (3.5) and also use exact expressions for \( \zeta(0) \) of massive scalar, vector and spinor fields on the Euclidean DeSitter background. For a scalar field with the mass \( m_\chi \) and the constant \( \xi_\chi \) of nonminimal interaction (which is different from \( \xi \equiv \xi_\varphi \)) this expression reads [39, 40, 41]

\[
\zeta_\chi(0) = \frac{29}{90} - 4 \xi_\chi + 12 \xi_\chi^2 - \frac{1}{3} \frac{m^2_\chi}{H^2} + \frac{1}{12} \frac{m^4_\chi}{H^4}.
\]

Using (3.5) and (5.13) in this equation we have

\[
\zeta_\chi(0) = \frac{3\lambda^2}{\lambda^2} \xi^2 + \frac{3\lambda^2}{4\pi \lambda^2} \frac{(1 + 2\delta) |\xi|m_\varphi^2}{\varphi_0^2} - \frac{2\lambda}{\lambda} |\xi| + O(m_\varphi^2/\varphi_0^2).
\]

The structure of this expression

\[
O(|\xi|^2) + O(|\xi|m_\varphi^2/\varphi_0^2) + O(|\xi|) + O(m_\varphi^2/\varphi_0^2)
\]

demonstrates the nature of the perturbation theory that we shall use in what follows. It has two smallness parameters: \( m_\varphi^2/|\xi|\varphi_0^2 \ll 1 \) - the parameter of the slow roll expansion and \( 1/|\xi| \ll 1 \) - the parameter of this particular model with nonminimal inflaton. Below we shall see that \( |\xi| \simeq 2 \times 10^4 \) and the probability maximum in the distribution in question will be for \( m_\varphi/\varphi_0 \simeq 0.03 \). This means that for the most important range of values of the inflaton field

\[
\frac{m_\varphi^2}{|\xi|\varphi_0^2} \gg \frac{1}{|\xi|}
\]

and in the subleading approximation the last two terms in (5.18) and (5.19) can be discarded. In what follows we shall invariably follow this rule. Thus we have

\[
\zeta_\chi(0) = 3\xi^2 \frac{\lambda^2}{\lambda^2} \left[ 1 + \frac{1}{4\pi} \frac{m_\varphi^2}{|\xi|\varphi_0^2} + O(1/|\xi|) \right].
\]

Similar calculations for vector and Dirac spinor fields with masses from (5.13) give the result (taking into account their statistics)

\[
\zeta_A(0) = 48 \xi^2 \frac{g_A^4}{\lambda^2} \left[ 1 + \frac{1}{4\pi} \frac{m_\varphi^2}{|\xi|\varphi_0^2} + O(1/|\xi|) \right],
\]

\[
\zeta_\psi(0) = -48 \xi^2 \frac{f^2}{\lambda^2} \left[ 1 + \frac{1}{4\pi} \frac{m_\varphi^2}{|\xi|\varphi_0^2} + O(1/|\xi|) \right].
\]

\(^5\)Another advantage of this mechanism related to big \( |\xi| \) is that it allows one to disregard the known and thus far unresolved problem of discrepancies between the renormalization in covariant and unitary gauges observed on topologically non-trivial curved spaces for fields of spins \( s \geq 1 \) [45, 46, 47]. These discrepancies are independent of the nonminimal coupling constant and, thus, negligible for \( |\xi| \gg 1 \).
Obviously, the contribution of Majorana or Weyl spinor fields is one half of the expression (5.23).

Thus, the total anomalous scaling of the theory on the exact DeSitter background reads

\[ \zeta(0) = 6 \frac{\xi^2}{\lambda} A \left[ 1 + \frac{1 + 2\delta}{4\pi} \frac{m_\psi^2}{\xi|\varphi_0^2|} + O(1/|\xi|) \right]. \]  

(5.24)

It should be emphasized again that the graviton-inflaton sector does not contribute to this expression. We have seen that dependence on \( \xi \) in \( \zeta(0) \) above arises due to the terms \( m_\chi^4/H^4 \) and \( m_\psi^2/H^2 \). Indeed, in these terms \( H \) depends on \( \xi \). However, while the effective masses \( m_\chi, m_A, m_\psi \) of non-inflaton scalar, vector and spinor fields do not depend on \( \xi \), the effective masses of the graviton and inflaton are \( \xi \)-dependent (see eq.(3.8) for the effective inflaton mass) and this dependence cancels the dependence of \( H \) on \( \xi \). One can show also that the subleading terms in \( \zeta(0)_{\text{gravity+inflaton}} \) will be at most \( O(|\xi|) \) in the terminology of (5.19) and thus will be discarded in what follows. From calculations below it will be clear that the same is true for \( \zeta'(0) \) too. Thus in the subleading approximation the one-loop part of the probability distribution will be contributed by the non-inflaton scalar, vector and spinor fields.

5.2. Calculating \( \zeta'(0) \)

The calculation of \( \zeta'(0) \) requires the knowledge of the finite part of the effective action unrelated to its ultraviolet divergences associated with \( \zeta(0) \). We shall calculate it by the technique of generalized \( \zeta \)-functions developed in [40, 41] for fields in the basis of irreducible \( O(5) \) representations of different spins \( s \) (5, 6). In [41] these calculations for \( \zeta \)-functions (5.8) were put in a unified framework in the form of a special function defined for \( \Re p > 2 \) and analytically continued to \( p = 0 \)

\[ \zeta_s(p) = \frac{1}{3}(2s + 1)F\left(p, 2s + 1, (s + 1/2)^2, b_s\right), \]  

(5.25)

\[ F(p, k, a, b) \equiv \sum_{\nu=\pm k+1}^{\infty} \frac{\nu(\nu^2 - a)}{(\nu^2 - b)^p}, \quad \Re p > 2. \]  

(5.26)

Here the parameters \( b_s \) are related to potential terms of operators in (5.6) according to [41]:

\[ b_0 = \frac{9}{4} - \frac{X_0}{H^2}, \quad b_1 = \frac{13}{4} - \frac{X_1}{H^2}, \quad b_{1/2} = -\frac{X_{1/2}}{H^2}. \]  

(5.27)

Particular values of the function (5.26) equal [41]

\[ F(0, k, a, b) = \frac{1}{4}b(b - 2a) + \frac{1}{24}a(3k^2 + 6k + 2) - \frac{1}{64}\frac{k^2(k + 2)^2}{H^2} + \frac{1}{120}, \]  

(5.28)
\[ F(1, k, a, b) = \frac{1}{2} b - \frac{1}{12} - \frac{1}{8} k(k + 2) - \frac{1}{2}(b - a) \Psi \left( \frac{k}{2} + 1 \pm \sqrt{b} \right), \quad (5.29) \]

\[ \Psi(x \pm y) \equiv \psi(x + y) + \psi(x - y), \quad (5.30) \]

where \( \psi(x) \) is a logarithmic derivative of Euler’s \( \Gamma \) function. Using the corollary of (5.26)

\[ \frac{d}{db} F'(0, k, a, b) = F(1, k, a, b), \quad F' = \frac{dF}{dp}, \quad (5.31) \]

we can find \( F'(0) \) and hence \( \zeta'(0) \) by integrating (5.31)

\[ F'(0, k, a, b) = \frac{1}{4} b^2 - \frac{1}{12} b - \frac{1}{8} b k(k + 2) \]

\[ - \frac{1}{2} \int_0^b d\xi (z - a) \Psi \left( \frac{k}{2} + 1 \pm \sqrt{z} \right) + C, \quad (5.32) \]

where the constant \( C \) is given by derivatives of Hurwitz functions

\[ C = 2\zeta'_H(-3, 1 + k/2) - 2a\zeta'_H(-1, 1 + k/2), \quad \zeta_H(x, y) = \sum_{n=0}^{\infty} \frac{1}{(n + y)^x}. \]

The expression (5.32) is rather complicated and generally can be obtained only numerically. However, we need only its dependence on \( \varphi_0 \) contained in \( b_s \) and in the approximation retaining only the first two terms of the generic expansion (5.19). Since the potential terms of (5.6) are basically given by effective masses induced according to (5.13), \( X_s \sim m_s^2 = O(\varphi_0^2) \), the corresponding parameters \( b_s \) (5.27) have the structure

\[ b_s = b_s^{(0)} + \frac{b_s^{(1)}}{\varphi_0^2} + O(1/|\xi|), \quad b_s^{(0)} \simeq \frac{X_s}{H^2} = O(|\xi|). \quad (5.33) \]

Using (5.28)-(5.29) and expanding (5.25) and (5.32) in \( b_s^{(1)}/\varphi_0^2 \) one obtains

\[ \zeta_s(0) = \zeta_s(0)|_{b_s^{(0)}} + \frac{2s + 1}{6} \frac{b_s^{(0)}}{\varphi_0^2} + O(m_p^2/\varphi_0^2), \quad (5.34) \]

\[ \zeta'_s(0) = \zeta'_s(0)|_{b_s^{(0)}} + \frac{2s + 1}{3} \left\{ \frac{b_s^{(0)}}{2} - \frac{1}{2} \left[ b_s^{(0)} - (s + 1/2)^2 \right] \Psi \left( s + \frac{3}{2} \pm i\sqrt{b_s^{(0)}} \right) \right\} \]

\[ + O \left( \frac{m_p^2}{\varphi_0^2} \right). \quad (5.35) \]

For the calculation of the full effective action (5.7) we would also need \( \ln(\mu^2/H^2) \). It is worth transforming this quantity separately for every irreducible \( s \)-component

\[ \ln \frac{\mu^2}{H^2} = \ln \frac{\mu^2}{\varphi_0^2} + \ln \varphi_0^2 + \ln(-b_s^{(0)}) + \frac{b_s^{(1)}}{b_s^{(0)} \varphi_0^2} + O \left( \frac{1}{|\xi|} \right). \quad (5.36) \]
Then, disregarding the $\varphi_0$-independent part and using in $\Psi\left(s + 3/2 \pm i(-b_s^{(0)})^{1/2}\right)$ the asymptotic expansion for $\psi(z)$ at large $z$ [48], $\psi(z) \sim \ln z - 1/2z + \ldots$, we see that the logarithmic in $b_s^{(0)}$ term of (5.36) gets cancelled by the logarithmic term of $\Psi\left(s + 3/2 \pm i(-b_s^{(0)})^{1/2}\right)$ and the final answer for a partial one-loop action reads

$$
\zeta_s'(0) + \zeta_s(0) \ln \frac{\mu^2}{H^2} = \text{const} + \zeta_s(0) \ln \frac{\mu^2}{\varphi_0^2} + \frac{2s + 1}{3} b_s^{(1)} b_s^{(0)} \left( \frac{3}{4} + \frac{1}{2} \ln \frac{\varphi_0^2}{X_s} \right) + O\left( \frac{m_p^2}{\varphi_0^2} \right). \tag{5.37}
$$

Now we can go over to the calculation of the total one-loop action for the model (5.2). For a Higgs scalar field $\chi$ the contribution reduces to the above equations (5.34) and (5.37) with $s = 0$ and $X_0 = m^2$ (see eq.(5.13) for the effective mass of the Higgs field), so that in view of the expansion for $H$ (3.5)

$$
b_s^{(0)} = -6 |\xi|^\frac{\lambda_s^2}{\lambda}, \quad b_s^{(1)} = -\frac{3\lambda_s (1 + 2\delta)}{4\pi\lambda} m^2_p. \tag{5.38}
$$

For a gauge vector field the situation is more complicated because its one-loop action in the most convenient gauge $\nabla^\mu A_\mu = 0$ (leading to minimal operator with diagonal derivatives) equals [41]

$$
\frac{1}{2} \text{Tr} \ln(-\Box - R^\mu + m^2_A) - \text{Tr} \ln(-\Box) = \frac{1}{2} \text{Tr} \ln(-\Box_0 + m^2 + 3H^2)
$$

$$
+ \frac{1}{2} \text{Tr} \ln(-\Box_0 + m^2_A) - \text{Tr} \ln(-\Box_0), \tag{5.39}
$$

where the subtracted term is a contribution of ghosts and the other terms represent the decomposition of the vector functional determinants into $O(5)$ irreducible components (see footnote after eq.(5.10)). Similar procedure holds for a spinor field operator which technically must be squared to reduce calculations to that of the $\Box^{1/2} + \ldots$ [41]. Finally we have for vector and spinor fields the coefficients

$$
b_s^{(0)} = -12 |\xi|^\frac{f_s^2}{\lambda}, \quad b_s^{(1)} = -\frac{3f_s^2 (1 + 2\delta)}{2\pi\lambda} m^2_p, \tag{5.40}
$$

$$
b_A^{(0)} = -12 |\xi|^\frac{g_A^2}{\lambda}, \quad b_A^{(1)} = -\frac{3g_A^2 (1 + 2\delta)}{2\pi\lambda} m^2_p. \tag{5.41}
$$

and the corresponding $X_A$, $X_\psi$ different from the squared masses (5.13) by terms that go beyond our approximation. Their use in (5.34) and (5.37) allows one to reproduce the expression (5.24) for the total $\zeta(0)$ obtained above by the Schwinger-DeWitt method and get the final algorithm for the total one-loop effective action on the De-Sitter background

$$
-\Gamma_{1\text{-loop}}^{(0)} = \text{const} - 3 \frac{\xi^2}{\lambda} A \left[ 1 + \frac{1 + 2\delta}{4\pi} \frac{m^2_p}{|\xi|^2 \varphi_0^2} \right] \ln \frac{\varphi_0^2}{\mu^2}.$$

30
\[ \frac{3|\xi|(1 + 2\delta)m_p^2}{4\pi\lambda\varphi_0^2} \left( \frac{3}{2} A + B \right) + O\left( \frac{m_p^2}{\varphi_0^2} \right), \]  

(5.42)

where the coefficient \( A \) is defined by Eq. (5.15) and \( B \) is a following new combination of coupling constants

\[ B = -\frac{1}{2\lambda} \left( \sum_\chi \lambda_\chi^2 \ln \frac{\lambda_\chi}{2} + 16 \sum_A g_A^4 \ln g_A^2 - 16 \sum_\psi f_\psi^4 \ln f_\psi^2 \right). \]  

(5.43)

6. Perturbation theory for the one-loop effective action

Due to the presence of slow roll corrections \( \delta Q \) the effective action (5.1) acquires the contribution which in the first order of the perturbation theory equals

\[ \delta \Gamma_{1\text{-loop}} = \frac{1}{2} \text{Tr} \left[ \delta F G \right] \bigg|_{Q=0}, \]  

(6.1)

\[ FG(x, x') = \delta(x, x') \]  

(6.2)

where \( G = G(x, x') \) is the Green’s function of the operator \( F \) (eq.(5.4)) on a four-sphere and \( \delta F \) is the variation of this operator induced by \( \delta Q = (\delta a, \delta \varphi) \).

The expression (6.1) is incomplete unless one fixes uniquely the Green’s function and specifies the functional composition law \( \delta F G \) in the functional trace. One should remember that the kernel \( G(x, x') \) is not a smooth function of its arguments and its irregularity enhances when it is acted upon by two derivatives contained in \( \delta F \). Therefore one has to prescribe the way these derivatives act on both arguments of \( G(x, x') \) and how the coincidence limit of the resulting singular kernel is taken in the functional trace

\[ \text{Tr} \left[ \delta F G \right] = \int d^4x \text{tr} \left[ \delta F (\nabla', \nabla) G(x, x') \right] \bigg|_{x'=x} \]  

(6.3)

(tr denotes the matrix trace operation over tensor indices). The specification of trace operation follows from the procedure of calculating the Gaussian path integral over quantum disturbances which gives rise to the one-loop functional determinants. As was shown in [49] (see also [50] and [33]) the functional determinant of the differential operator generated by the Gaussian path integral is determined by the variational equation \( \delta \ln \text{Det} F = \delta \text{Tr} \ln F = \text{Tr} \delta F G \), where the Green’s function \( G(x, x') \) satisfies the same boundary conditions as the integration variables in the Gaussian integral and the functional composition law \( \delta F G \) implies a symmetric action of spacetime derivatives on both arguments of \( G(x, x') \). In what follows we shall describe in detail
the calculations for the case of the scalar field and then give the result for other higher spin contributions.

For the field $\chi(x)$ with the wave operator

$$F = -g^{\mu\nu} \nabla_\mu \nabla_\nu + m_\chi^2(\varphi_0)$$

the symmetrized variation of this operator (6.4) looks like

$$\delta F (\nabla', \nabla) = \nabla'_\mu \delta g^{\mu\nu}(x) \nabla_\nu + \delta m_\chi^2(\varphi),$$

where $\nabla'_\mu$ and $\nabla_\nu$ are acting on different arguments of $G(x, x')$ in (6.3).6

The Green’s function for a scalar field on a four-sphere is well known5, however we have to regulate a divergent expression (6.3) and will do it by replacing the Green’s function with its operator power $G^{1+p}(x, x') = F^{-1-p} \delta(x, x'), p \rightarrow 0$,

$$G^{1+p}(x, x') = \frac{1}{\Gamma(p+1)} \int_0^\infty ds s^p \exp(-s F) \delta(x, x').$$

The corresponding heat kernel can be constructed by noting that in view of DeSitter invariance both the Green’s function and its heat kernel are functions of the world function $\sigma(x, x')$ — one half of the square of geodetic distance between the points $x$ and $x'$ which can also be expressed in terms of the angle $\theta = \theta(x, x')$ between these points on a sphere of radius $R = 1/H$, $\sigma(x, x') = R^2 \theta^2 / 2$. Similarly, the dedensitized delta-function above can be constructed in terms of this angular variable $y = \cos \theta$

$$\frac{\delta(x, x')}{g^{1/2}(x')} = \frac{1}{4\pi^2 R^4} \frac{d}{dy} \delta(y - 1).$$

The scalar D’Alambertian acting on this function of $\theta$ yields the operator of the Legendre equation after the commutation with the derivative $d/dy$ above

$$\Box \frac{d}{dy} = \frac{d}{dy} \left[ (1 - y^2) \frac{d^2}{dy^2} - 2y \frac{d}{dy} + 2 \right],$$

which suggests to expand the delta function in the series of Legendre polynomials $P_n(y)$ — eigenfunctions of the Legendre operator with eigenvalues $-n(n + 1)$

$$\delta(y - 1) = \sum_{n=0}^\infty \left( n + \frac{1}{2} \right) P_n(y),$$

6Note that $F$ enters the action with the metric dependent factor $g^{1/2} F$, however $g^{1/2}$ is not varied here, because the contribution of this overall factor is cancelled by the contribution of the local measure [51]. For this reason, in particular, the effective action is given by the functional determinants of $F$ instead of those of $g^{1/2} F$. The difference between the corresponding results as well as the contribution of local measure are formally proportional to unregulated $\delta(0)$ which vanishes in dimensional regularization, but given by $\zeta(0)$ in the zeta-functional one. Therefore in the regularization we use here these terms require a careful bookkeeping.
whence it follows that

\[
\exp(-sF)\delta(x, x') = \frac{g^{1/2}(s')}{4\pi^2 R^4} \sum_{n=1}^{\infty} \left( n + \frac{1}{2} \right) e^{-\frac{(n+1/2)^2 - b_0}{2R^2}} dP_n(y) / dy, \tag{6.10}
\]

where \( b_0 = 9/4 - m^2 R^2 \) coincides with the expression given by (5.27) for a spin-0 case. Substituting it into (6.5), integrating over \( s \) and expressing the Legendre polynomials in terms of the hypergeometric function we finally obtain

\[
G^{1+p}(x, x') = \frac{g^{1/2}(x')}{16\pi^2 R^2-2p} \sum_{n=1}^{\infty} \frac{(-1)^n+1 n(n+1)(2n+1)}{[(n+1/2)^2 - b_0]^{1+p}}
\times F \left( 1 - n, n + 2; 2; \frac{1}{2} + \frac{1}{2} \cos \sqrt{2\sigma(x, x')/R^2} \right). \tag{6.11}
\]

This expression allows one to obtain the coincidence limits of the Green’s function and its derivatives arising in (6.3). In view of the well known coincidence limits \( \sigma(x, x) = 0, \nabla_\mu \sigma(x, x')|_{x' = x} = 0, \nabla_\mu \nabla_\nu \sigma(x, x')|_{x' = x} = -g_{\mu\nu} \), we have

\[
\nabla_\mu \nabla_\nu G^{1+p}(x, x') = \frac{g_{\mu\nu} g^{1/2}(x)}{64\pi^2 R^2-2p} \sum_{n=1}^{\infty} \frac{n(n+1)(2n+1)(1 - n)(n+2)}{[(n+1/2)^2 - b_0]^{1+p}}. \tag{6.13}
\]

Infinite series here are calculable by the technique of [39, 40, 41]. They have the pole structure in \( p \to 0, A/p + B + O(p) \), and lead to \( \delta \Gamma_{1-loop} \) in the form of the finite part of the variation (6.1) \(^7\):

\[
\delta \Gamma_{1-loop} = \frac{1}{2} (I_1 A_1 + I_2 A_2) \ln \frac{\mu^2}{H^2} + \frac{1}{2} (I_1 B_1 + I_2 B_2), \tag{6.14}
\]

where \( I_1 \) and \( I_2 \) are given by the integrals

\[
I_1 = -\frac{1}{16\pi^2 R^2} \int_{S^4} d^4x g^{1/2} \delta m^2, \tag{6.15}
\]

\[
I_2 = \frac{1}{64\pi^2 R^4} \int_{S^4} d^4x g^{1/2} \delta g_{\mu\nu} g_{\mu\nu}, \tag{6.16}
\]

and \( A_i, B_i, i = 1, 2 \), are the following pole and finite parts of the above two sums

\[
A_1 = b_0 - \frac{1}{4}, \quad B_1 = \frac{1}{12} - \left( b_0 - \frac{1}{4} \right) \Psi \left( \frac{1}{2} \pm \sqrt{b_0} \right) + b_0,
\]

\[
A_2 = \frac{9}{4} A_1 - b_0 \left( b_0 - \frac{1}{4} \right),
\]

\[
B_2 = \frac{9}{4} B_1 + \frac{7}{480} - \frac{1}{12} \left( b_0 - \frac{1}{4} \right) + b_0 \left( b_0 - \frac{1}{4} \right) \Psi \left( \frac{1}{2} \pm \sqrt{b_0} \right) - b_0 \left( \frac{3}{2} b_0 - \frac{1}{4} \right). \tag{6.17}
\]

\(^7\)Note that \( \zeta \)-functional regularization is formally free from divergences – pole terms in \( p \) [42]. These terms are artifacts of the variational equation (6.1) which differs from the variation of finite \( \zeta'(0) \) exactly by the pole term in \( p \). Indeed \( \delta \zeta'(p) = -(1 + p d/dy) \) \( Tr \delta F G^{1+p} \) and for \( Tr \delta F G^{1+p} = A/p + O(1) \), \( p \to 0 \), equals the finite part of (6.1) \( \delta \zeta'(0) = [-Tr \delta F G^{1+p} + a/p]_{p=0} \).
Since $\delta m_\chi^2(\varphi) = \lambda_{\chi} \varphi_0 \delta \varphi$, $\delta g^{\mu\nu} = \text{diag}(0, -2g^z \delta a/a)$, with $\delta \varphi$ and $\delta a$ given by eqs. (3.21) and (3.22), the final result for a scalar field contribution to $\delta \Gamma_{1-\text{loop}}$ reads up to terms $O(m_P^2/\varphi_0^2)$

\[
\delta \Gamma_{1-\text{loop}} = \frac{1}{2} \delta \zeta'(0) - \frac{1}{2} \delta \zeta'(0) \ln \frac{H^2}{\mu^2},
\]

\[
\delta \zeta'(0) = -\frac{9m_P^2}{2} \frac{\xi}{\pi^2 \varphi_0^2} \lambda^2, \quad \delta \zeta'(0) = \frac{9m_P^2}{2} \frac{\xi}{\pi^2 \varphi_0^2} \lambda^2 \kappa \ln \frac{6|\xi| \lambda_{\chi}}{\lambda}, \quad (6.18)
\]

\[
\kappa = \frac{\pi}{96} + \frac{\pi \ln 2}{12} - \frac{1}{72} \approx 0.2. \quad (6.19)
\]

The leading terms of $\delta \zeta(0)$ and $\delta \zeta'(0)$ for vector and spinor fields have a similar form and when composed with the contribution of Higgs multiplets above give rise to the total $\delta \Gamma_{1-\text{loop}}$. Similarly to the unperturbed part (5.42) it expresses as a function of the universal combinations of coupling constants $A$ and $B$ given by Eqs.(5.15) and (5.43)

\[
\delta \Gamma_{1-\text{loop}} = \frac{9m_P^2}{2} \frac{|\xi| (1 + \delta)}{\pi^2 \varphi_0^2} \lambda A \ln \frac{\varphi_0^2}{\mu^2} - \frac{9m_P^2}{2} \frac{|\xi| (1 + \delta) m_P^2}{\pi^2 \varphi_0^2} \lambda B. \quad (6.20)
\]

7. Probability maximum of the distribution function

Combining the tree-level part (3.43)-(3.45) with the contributions of the inflaton mode (4.7) and perturbative contributions of the one-loop effective action (5.42) and (6.20) we finally arrive at the distribution function (2.30) of inflationary cosmologies

\[
\rho_{\lambda,NB}(\varphi_0) = N \exp \frac{3|\xi|}{\lambda} \left[ \frac{m_P^2}{\varphi_0^2} \alpha_{\pm} \left[ \frac{m_P^2}{\varphi_0^2} + \frac{\beta A m_P^2}{\varphi_0^2} \ln \frac{\varphi_0^2}{\mu^2} + O \left( \frac{m_P^2}{|\xi| \varphi_0^2} \right) \right] \right], \quad (7.1)
\]

where $N$ is a field independent normalization factor and $\alpha_{\pm}$ and $\beta$ are the following functions of coupling constants of the model

\[
\alpha_{\pm} = 8\pi (1 + \delta) \pm \frac{1 + 2\delta}{4\pi} \left( \frac{3A + B}{2} \right) \pm 3\kappa B \frac{1 + \delta}{\pi}, \quad (7.2)
\]

\[
\beta = \frac{1 + 2\delta}{4\pi} - 3\kappa \frac{1 + \delta}{\pi}, \quad (7.3)
\]

involving the parameter $\delta$ (2.23) and universal combinations $A$ and $B$. 

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The second term in the exponential of (7.1) confirms the conclusions of [11, 12, 1] that loop corrections can drastically change the predictions of classical theory and suppress the contribution of the over-Planckian energy scales due to big positive anomalous scaling of the theory

$$\rho_{T,NB}(\varphi_0) \sim \varphi_0^{-Z^{-2}}, \quad \varphi_0 \to \infty,$$  

(7.4)

with $Z$ given by eq. (5.14). Extra power $-2$ of $\varphi_0$ here comes from the contribution of the inflaton mode (4.7) neglected in (7.1) within the $O(m_P^2/\varphi_0^2)$ accuracy but important for $\varphi_0 \to \infty$. For positive $Z = 6|\xi|^2A/\lambda$ this asymptotics can be regarded as a justification of a semiclassical expansion.

The equation for the extremum of the obtained distribution at $\varphi_0 = \varphi_1$ can be represented in the form

$$\pm \frac{\varphi_1^2}{m_P^2} = \frac{\alpha_\pm}{|\xi|A} + \frac{\beta}{|\xi|^2} \left( \ln \frac{\varphi_1^2}{\mu^2} - 1 \right) + O\left( \frac{1}{|\xi|^2} \right),$$

(7.5)

where plus or minus signs correspond to the tunnelling or no-boundary wavefunctions respectively. To analyze the existence of its solution we shall have to prescribe a certain reasonable range of parameters and try solving it by iterations. In the next section we shall briefly discuss the model of nonminimal inflation with big $|\xi| \simeq 2 \times 10^4 \gg 1$ and $\delta = O(1)$ that was used in our previous work [1] as a good candidate for a quantum origin of the early Universe at a sub-Planckian energy scale (around GUT scale). In [1] we only took into account the tree-level part and the leading logarithmic behaviour of the one-loop effective action, which correspond to retaining in (7.1) only the first two terms of the exponential with the parameter $\alpha$ truncated to the first ($A$ and $B$-independent) term of (7.2). In [1] it was shown that the requirement of the minimal admissible duration of the inflationary stage (2.5) imposes upper bound on the combination of coupling constants $A \simeq 1.3$. Here we shall show that the qualitative estimates of [1] remain also true after the inclusion of perturbative corrections obtained above.

The justification of the results of [1] consists in the observation that the second term in the right-hand side of (7.5) can be regarded small and taken by perturbations. Indeed, from the upper bound on $A$ we can assume that both $A$ and $B$ are of the order of magnitude one

$$A = O(1), \quad B = O(1).$$

(7.6)

This follows from the comparison of expressions (5.15) and (5.43) and a natural assumption that the estimate for $B$ follows from that for $A$, unless strong cancellation takes place between different separately big terms of the expression (5.15) for $A$. From
these bounds we see that $\alpha_\pm = 8\pi (1 + \delta) + O(1/10)$, $\beta = O(1/10)$ due to the numerical values of the coefficients in (7.2)-(7.3) and the fact that $\delta = O(1)$. Then it follows that $\beta A/\alpha_\pm \sim A/80\pi (1 + \delta) \sim A/250$ and, therefore, the second term in the right-hand side of (7.5) can indeed be treated perturbatively with a good smallness parameter $A/250$. With the same precision the expression for $\alpha_\pm$ reduces in the leading order to $\alpha = 8\pi (1 + \delta)$. It is needless to say that for smaller values of $A$ and $B$ this approximation works even better, because the leading term in the right-hand side of (7.5) grows and the corrections decrease for $A, B \to 0$.

For $\delta > -1$ (which is the case of a standard classical scenario with a finite duration of inflation, see Sect.2) the leading order solution of (7.5) exists only for the case of a tunneling wavefunction (sign plus) and coincides with the result of [1]. Thus, the parameters of the inflation probability maximum – the mean value of the inflaton field $\varphi_I$ and its quantum dispersion $\Delta \varphi = [-d^2 \ln \rho(\varphi_I)/d\varphi_I^2]^{-1/2}$ are

$$\varphi_I = m_P \sqrt{\frac{8\pi (1 + \delta)}{|\xi| A}}, \quad \Delta \varphi = \frac{\varphi_I}{\sqrt{12 A |\xi|}}.$$  \hspace{1cm} (7.7)

For the no-boundary quantum state of the Universe the peak can be realized only for $\delta < -1$ and, thus corresponds to the classical scenario with endless inflation stage [1].

8. Nonminimal inflation and particle physics of the early Universe

The present state of inflation theory is consistent with observations of the cosmic microwave background radiation anisotropy in the COBE [14] and Relikt [15] satellite experiments. In the chaotic inflationary model with a nonminimal inflaton field (2.21) the spectrum of perturbations compatible with these measurements can be acquired in the range of coupling constants $\lambda/\xi^2 \sim 10^{-10}$ [19, 53] (the experimental bound on the gauge-invariant [54] density perturbation $P_\zeta(k) = N_\zeta^2 (\lambda/\xi^2)/8\pi^2$ in the $k$-th mode "crossing" the horizon at the moment of the $e$-foldings number $N_k$). The main advantage of this model is that it allows one to avoid an unnaturally small value of $\lambda$ in the minimal inflaton model [2] and replace it with the GUT compatible value $\lambda \sim 0.05$, provided $\xi \sim -2 \times 10^4$ is chosen to be related to the ratio of the Planck scale to a typical GUT scale, $|\xi| \sim m_P/v$. For these values of coupling constants the parameter (2.23) is $\delta \sim 8\pi v/m_P \sim 10^{-3}$ (thus easily satisfying a much weaker upper bound $\delta = O(1)$ assumed above). As far as it concerns the Hubble parameter (3.5) and the number of $e$-foldings (2.24) at the inflationary peak (7.7) obtained above, in the leading order in
$|\xi| \gg 1$ they are given by

$$H(\varphi_I) = m_p \frac{\sqrt{\lambda}}{|\xi|} \sqrt{\frac{2\pi(1 + \delta)}{3A^2}}, \quad (8.1)$$

$$N(\varphi_I) = \frac{8\pi^2}{A} \quad (8.2)$$

and satisfy the bound $N(\varphi_I) \geq 60$ with a single restriction on $A$, $A \leq 1.3$. This restriction gives rise to the bounds (7.6) and, therefore, justifies the smallness of the perturbative corrections of the above type, that were neglected in our previous paper [1]. These perturbative corrections contained in the second term of the equation (7.5), unfortunately, violate the conclusion of [1] on the absence of renormalization ambiguity in the energy scale of inflation, but as we see this ambiguity in the choice of $\mu^2$ is strongly suppressed by the smallness of the ratio $\beta A/\alpha$.

On the other hand, the restriction on $A$ justifies a slow-roll approximation, because the corresponding smallness parameter is $\dot{\varphi}/H \varphi \simeq -A/96\pi^2 \sim -10^{-3}$. For the above small value of $\delta \ll 1$ and $A \simeq 1$, the obtained numerical parameters describe extremely sharp inflationary peak at $\varphi_I$ with small width and sub-Planckian Hubble constant

$$\varphi_I \simeq 0.03m_p, \quad \sigma \simeq 10^{-7}m_p, \quad H(\varphi_I) \simeq 10^{-5}m_p, \quad (8.3)$$

which is the most realistic range of the inflationary scenario. The smallness of the width does not, however, lead to its quick quantum spreading: the commutator relations for operators $\varphi$ and $\dot{\varphi}$, $[\varphi, \dot{\varphi}] \simeq i/(12\pi^2|\xi|a^3)$ [19], give rise at the beginning of inflation, $a \simeq H^{-1}$, to a negligible dispersion of $\dot{\varphi}$, $\Delta \dot{\varphi} \simeq H^3/12\pi^2|\xi|\sigma \simeq (\sqrt{\lambda}/|\xi|)|\dot{\varphi}| \ll |\dot{\varphi}|$. It is remarkable that the relative width

$$\frac{\Delta \varphi}{\varphi_I} \sim \frac{\Delta H}{H} \sim 10^{-5} \quad (8.4)$$

corresponds to the observable level of density perturbations, although it is not clear whether this quantum dispersion $\Delta \varphi$ is directly measurable now, because of the stochastic noise of the same order of magnitude generated during the inflation and superimposed upon $\Delta \varphi$.

All these conclusions are universal for a generic low-energy model (5.2) and (apart from the choice of $|\xi|$ and $\lambda$) universally depend on one parameter $A$ of the particle physics model. This quantity should satisfy the lower and upper bounds

$$\frac{\sqrt{\lambda}}{|\xi|} \ll A \leq 1.3 \quad (8.5)$$

in order respectively to render $Z$ positive, thus suppressing over-Planckian energy scales, and provide sufficient amount of inflation ($A$ should not, certainly, be exceedingly close to zero, not to suppress the dominant contribution of large $|\xi|$ in (5.14)).
In [1] the conclusion was made that this bound suggests the quasi-supersymmetric nature of the particle model, because supersymmetry can constrain the values of the Higgs $\lambda$, vector gauge $g_A$ and Yukawa $f_0$ couplings so as to provide a subtle balance between the contributions of bosons and fermions in (5.15) and fit the quantity $A$ into a narrow range (8.5). Now, however, with the inclusion of corrections that go beyond the estimates of [1] we have also to provide the boundedness of $B$ (5.43) which is less obvious to be compatible with the bound (8.5), unless all the terms of (5.15) and (5.43) are separately small due to small values of all coupling constants. In the latter case the supersymmetry is not needed to explain the restrictions (8.5) on the choice of a particle physics model. Still, supersymmetry remains a reasonable conjecture consistent with this selection criterion (8.5) and sounds coherent with conclusions of [55] where supersymmetry was argued in the opposite case of small $|\xi|$.

9. Conclusions

Thus, the same mechanism that suppresses the over-Planckian energy scales also generates a narrow probability peak in the distribution of tunnelling inflationary universes and is likely to suggest the (quasi)supersymmetric nature of their particle content. It seems to be consistent with microwave background observations within the model with a strongly coupled nonminimal inflaton field. A remarkable feature of this result is that it is mainly based on one small parameter — the dimensionless ratio of two major energy scales, the GUT and Planck ones, given by the combination of the coupling constants $\sqrt{\lambda}/|\xi| \approx 10^{-8}$.

Big value of $|\xi|$ is actually responsible for the fact that the one-loop corrections qualitatively change the tree-level behaviour and produce the inflationary peak of the above type. In the absence of powerful nonperturbative methods there is no rigorous proof that the inclusion of multi-loop orders will not destroy this one-loop effect. However, the following qualitative arguments support the conjecture that this will not happen. Point is that the effective gravitational constant in this model is inverse proportional to $m_P^2 + 8\pi |\xi| \varphi^2$ and, thus, large $|\xi|$ might improve the loop expansion [21] by suppressing the contribution of multi-loop orders. On the other hand, the power-law mechanism (7.4) of suppressing the high-energy scales is independent of the renormalization ambiguity (the parameter $\mu^2$). This also gives a hope that the obtained effect is robust against inclusion of multi-loop corrections. All these corrections are weighted by this suppressing one-loop factor and are small at least in some range of $\varphi$ above the probability peak value $\varphi_1$ where all their curvature invariants have GUT values rather than Planckian ones. Therefore, irrespective of what happens at Planckian scales this peak at GUT scale will be separated from the unknown Planckian domain by the re-
region with very small values of the distribution function and, at least heuristically, will
not interfere with nonperturbative quantum gravity or its more fundamental (stringy)
generalization.

Obviously, the large value of $|\xi|$ at sub-Planckian (GUT) scale requires explanation
which might be based on the renormalization group approach (and its extension to
non-renormalizable theories [21]). As shown in [21], quantum gravity with nonminimal
scalar field has an asymptotically free conformally invariant ($\xi = 1/6$) phase at over-
Planckian regime, which is unstable at lower energies. It is plausible to conjecture that
this instability can lead (via composite states of the scalar field) to the inversion of the
sign of running $\xi$ and its growth at the GUT scale, thus making possible the proposed
inflation applications.

> From the viewpoint of the theory of the early universe, the obtained results give a
strong preference to the tunneling quantum state. Debates on advantages of the tun-
neling versus no-boundary wavefunction has a long history [2, 5, 6, 56, 8]. At present,
in the cosmological context the tunnelling proposal seems to be more useful and con-
ceptually clearer than the no-boundary one, because for its interpretation one should
not incorporate vague ideas of the third quantization of gravity which inevitably arise
in the no-boundary case: splitting the Lorentzian wavefunction in positive and nega-
tive frequency parts and separately calculating their probability distributions. On the
other hand, the formulation of the tunneling proposal is not so aesthetically closed, for
it involves imposing outgoing condition after the potential barrier, the unit normal-
ization condition – before the barrier at $a = 0$, the requirement of the normalizability
in variables $f$ [34], etc. And all this in contradistinction with the closed path-integral
formulation of the no-boundary proposal, automatically providing many of the above
properties. On the other hand, outside of the cosmological framework, in particular,
within the scope of the wormhole and black hole physics, the tunneling proposal seems
to be helpless. Moreover, at the overlap of the cosmological framework with the theory
of the virtual black holes it leads to contradictions signifying that the quantum birth
of bigger black holes is more probable than of the small (Planckian) ones [57]. All
these arguments can hardly be conclusive, because it might as well happen that the
difference between the no-boundary and tunnelling wavefunctions should be ascribed
to the open problem of the correct quantization of the conformal mode. Note that
the normalizability criterion for the distribution function and its algorithm (2.30) do
not extend to the low-energy limit $\varphi \to 0$, where a naively computed no-boundary
distribution function blows up to infinity, the slow-roll approximation becomes invalid,
etc. This is a domain related to a highly speculative (but, probably, inevitable) third
quantization of gravity [58], which goes beyond the scope of this paper. Fortunately,
this domain is also separated from the obtained inflationary peak by a vast desert
with practically zero density of the quantum distribution, which apparently justifies our conclusions disregarding the ultra-infrared physics of the Coleman theory of baby universes and cosmological constant [58].

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Figure captions

Fig. 1 Graphical representation of the Lorentzian spacetime $L$ nucleating at the bounce surface $\Sigma_B$ from the Euclidean manifold $E$ of the no-boundary type, having the topology of the four-dimensional ball.

Fig. 2 Two-dimensional minisuperspace of the scale factor $a$ and the inflaton field $\phi$ in the chaotic inflation model. The Euclidean extremal (in the slow roll approximation) starts at $a = 0$ with large initial value $\phi_0$ in the form of a trajectory that is reflected from the caustic $\Sigma_B$, $a \simeq 1/H(\phi)$, and enters the region $\phi \to \infty$, $a \to 0$.

Fig. 3 Graphical representation of calculating the quantum distribution of tunnelling Lorentzian universes: a composition of the combined Euclidean-Lorentzian spacetime $M_- \cup L$ with its orientation reversed and complex conjugated copy $M_+ \cup L^*$ results in the doubled Euclidean manifold $2M$ — the gravitational instanton carrying the Euclidean effective action of the theory. The cancellation of the Lorentzian domains $L$ and $L^*$ reflects unitarity of the theory in the physical spacetime of Lorentzian signature.