Evolutionary Models of White Dwarfs with Helium Cores

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Abstract. We present seven evolutionary tracks for low-mass white
dwarfs with helium cores, ranging in mass from 0.179 to 0.414 \( M_\odot \). We
generated the pre-white dwarf models from a 1 \( M_\odot \) sequence ex-
tending up to the tip of its red-giant branch by applying high mass-loss
rates at appropriate positions, and we followed their evolution across the
Hertzsprung-Russell diagram and down the cooling path.

We discuss the internal structures and cooling properties of these
new models and compare them with those of recently published models for
low-mass white dwarfs which are based on simplified initial configurations.
We also demonstrate that our new models seem to remove the apparent
discrepancies between the characteristic ages of millisecond pulsars and
the cooling ages of their white dwarf companions.

1. Introduction

The interest in low-mass white dwarfs has increased recently because they ap-
pear frequently as a binary component, especially in several millisecond pulsar
systems. Detailed evolutionary calculations of possible binary scenarios existed
only for isolated cases or limited mass ranges with \( M > 0.2 \ M_\odot \) (Kippenhahn et
al. 1967, 1968, Refsdal & Weigert 1969, Giannone et al. 1970, Iben & Tutukov
1986, Castellani et al. 1994). Only recently also calculations for \( M < 0.2 \ M_\odot \)
have been presented (Alberts et al. 1996, Sarna et al. 1998).

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The need for more extended sets of white dwarf models with helium cores prompted several authors to generate them from ad hoc assumed, simplified starting configurations which appear not to be consistent with evolutionary considerations (Althaus & Benvenuto 1997, Benvenuto & Althaus 1998, Hansen & Phinney 1998). These calculations are based on the implicit assumption that the contraction time to a real white dwarf structure is short compared to the cooling time itself. Since also the size of any unprocessed hydrogen-rich envelope can only be guessed, no definitive statement on the importance of residual hydrogen burning can be made.

Thus, we felt it necessary to provide a grid of evolutionary models for low-mass white dwarfs with structures being as consistent as possible with their expected evolutionary history, which can be used with confidence for interpreting observational data. In our study we aimed at addressing the following questions in a systematic way:

- How large are the masses of the outer, still unburned hydrogen-rich envelopes on top of the helium cores? Since the size of a white dwarf depends critically on the mass content of its unprocessed envelope, this question is closely related to the mass-radius relation of white dwarfs.

- Can the cooling properties of low-mass white dwarfs be reconciled with estimated spin-down ages of millisecond pulsars?

- Are simplified model calculations useful in interpreting observational data?

2. The evolutionary computations

We used an evolutionary code with the following basic input physics (Blöcker 1995): Nuclear burning was accounted for by a nucleosynthesis network including 31 isotopes and 74 reactions up to carbon burning. Radiative opacities were taken from Iglesias et al. (1992), complemented with those of Iglesias & Rogers (1996) and Alexander & Ferguson (1994) for the low temperature range, all for \((Y, Z) = (0.28, 0.02)\). Convection was treated according to the mixing length theory. The mixing length parameter was chosen to \(\alpha = 1.7\), calibrated by computing a solar model. Coulomb corrections of the equation of state have been taken from Slattery et al. (1982).

The outcome of the mass transfer in a close binary system was simulated in the following simple way: the evolution of a 1 \(M_\odot\) model was calculated up to the tip of the red giant branch (RGB), and depending on the desired final mass, high mass loss was switched on at the appropriate positions. When the model started to leave the RGB, mass loss was virtually switched off (cf. Iben & Tutukov 1986, Castellani et al. 1994). More details of our calculations are given in Driebe et al. (1998).

Our method ensures that these red-giant remnants (= pre-white dwarf models) have a structure which is consistent with their previous evolutionary history, with an electron degenerate helium core and a (mainly) unprocessed envelope. The following evolution of the models across the Hertzsprung-Russell diagram and down the cooling path depends only on their actual structure (and mass-loss for larger luminosities), and not on the details of previous heavy mass-loss
episodes during the supposed binary evolution, provided the mass donor regains its thermal equilibrium before it finally shrinks below its Roche lobe.

Figure 1. Hertzsprung-Russell diagram with complete evolutionary tracks of RGB remnants with different masses (from top: 0.414, 0.331, 0.300, 0.259, 0.234, 0.195, 0.179 $\text{M}_\odot$). The long-dashed curve shows the evolutionary track of the 1 $\text{M}_\odot$ model we used for creating the remnants by mass loss. The short-dashed loops outline the very rapid redward excursions of the 0.259 and 0.234 $\text{M}_\odot$ models caused by hydrogen shell flashes.

Fig. 1 illustrates the result of our calculations in the Hertzsprung-Russell diagram, encompassing remnant masses from well below 0.2 up to above 0.4 $\text{M}_\odot$. The two sequences between 0.2 and 0.3 $\text{M}_\odot$ experienced typical thermal instabilities of thin burning shells when the CNO cycle shuts off (cf. Kippenhahn et al. 1968, Iben & Tutukov 1986, Castellani et al. 1994). The latter authors find CNO flashes for masses above 0.3 $\text{M}_\odot$ only for Pop. II compositions.

3. Structures and cooling properties

We found a steep correlation between the remnant masses and the sizes of their hydrogen-rich envelopes, ranging from $5 \cdot 10^{-2}$ $\text{M}_\odot$ for our 0.179 $\text{M}_\odot$ model, down to $2 \cdot 10^{-3}$ $\text{M}_\odot$ for 0.414 $\text{M}_\odot$. Our envelope masses agree, for the mass range in common, $M > 0.3$ $\text{M}_\odot$, with those given in Castellani et al. (1994). For $M < 0.2$ $\text{M}_\odot$ they agree in mass and helium enrichment with those of the Sarna
et al. (1998) models. It should be emphasized that these evolutionary envelope masses are larger, for a given remnant mass, than those adopted recently by Benvenuto & Althaus (1998) and Hansen & Phinney (1998).

![Log g - Log T_{eff} Diagram](image-url)

**Figure 2.** $\log g - \log T_{eff}$ diagram with evolutionary tracks of white dwarfs with different masses (from top) of 0.179, 0.195, 0.234, 0.259, 0.300, 0.331, 0.414 M$_{\odot}$ with helium cores, and of 0.524, 0.605, 0.696, 0.836, 0.940 M$_{\odot}$ with carbon-oxygen cores. Isochrones are given between 0.3 and 10 Gyr, and the position of the PSR J1012+5307 companion is also indicated (van Kerkwijk et al. 1996), together with the 6 Gyr isochrone (dotted).

Hydrogen burning continues to increase the helium core at the expense of the envelope, and the pp cycle takes over on the cooling branch and completely determines the cooling rate for the lower-mass models (Webbink 1975, Castellani et al. 1994). The continued burning at the base of the envelope reduces its mass considerably along the cooling path, leading to a complicated dependence of the white dwarf’s size on total mass and effective temperature. Using our evolutionary mass-radius relationships instead of the ones available in the literature, larger white dwarf masses would follow for given surface gravities and effective temperatures (see Driebe et al. 1998 for more details).

At the hot end of the cooling branch the hydrogen luminosity can exceed the gravothermal contribution to the white dwarf’s energy budget by up to a factor of 100. Even at effective temperatures as low as 5,000 K the pp cycle still dominates in the models with $M < 0.2$ M$_{\odot}$. Since the evolution along the cooling sequence is slowed down accordingly, the isochrones for helium dwarfs...
differ in shape from those for CO dwarfs: they are shifted and turned over to the left (Fig. 3).

The position of the PSR J1012+5307 white-dwarf companion is met by our 0.195 $M_\odot$ track for a cooling age of 6 Gyr, which agrees well with the spin-down age estimate of the pulsar, 7 Gyr. It should be mentioned that this result is in close agreement with that of Sarna et al. (1998) who made a rather detailed modeling of this particular system (see also Alberts et al. 1996).

4. Comparison with non-evolutionary models

![Graph](image)

Figure 3. $\log g - \log T_{\text{eff}}$ diagram with a 0.195 $M_\odot$ evolutionary and non-evolutionary white-dwarf track as computed by us with identical chemical structures, and with three non-evolutionary tracks taken from Althaus & Benvenuto (1997). Selected evolutionary ages are marked, and the position of the PSR J1012+5307 companion as well.

It is very instructive to compare the behaviour of our evolutionary models with that of non-evolutionary ones, as is done in Fig. 3 (see also Blöcker et al. 1997). Despite the completely different initial structures of the two sets of non-evolutionary models shown in the figure, and somewhat different physical assumptions as well, the cooling properties are identical, and the models predict an age of only 0.4 Gyr for the PSR J1012+5307 companion, in variance with the pulsar’s spin-down age. Note also that the structure of our non-evolutionary
model does not approach that of the evolutionary model before an effective
temperature of about 5 000 K is reached.

From a thorough comparison between evolutionary and non-evolutionary
models (cf. Blöcker et al. 1997) we can make the following safe conclusions:

- Evolutionary white dwarf models are more compact than non-evolutionary
  models of the same mass and chemical structure.
- Envelope masses are inversely correlated with the white-dwarf mass.
- At lower masses, hydrogen burning via the pp cycle controls the pace of
  cooling.
- The thermo-mechanical structures of low-mass non-evolutionary models
do not converge with those of evolutionary models within a reasonable
time.

Given these facts, the use of non-evolutionary helium white-dwarf models to
interpret observational data appears not to be advisable.

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