Magnetorotational core collapse of possible GRB progenitors. III. Three-dimensional models

M. Obergaulinger1,2, M.Á. Aloy1
1 Departament d’Astronomia i Astrofísica, Universitat de València, Edifici d’Investigació Jeroni Munyoz, C/ Dr. Moliner, 50, E-46100 Burjassot (València), Spain
2 Institut für Kernphysik, Technische Universität Darmstadt, Schloßgartenstraße 2, 64289 Darmstadt, Germany

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ABSTRACT

We explore the influence of non-axisymmetric modes on the dynamics of the collapsed core of rotating, magnetised high-mass stars in three-dimensional simulations of a rapidly rotating star with an initial mass of $M_{\text{zams}} = 35 M_\odot$ endowed with four different pre-collapse configurations of the magnetic field, ranging from moderate to very strong field strength and including the field predicted by the stellar evolution model. The model with the weakest magnetic field achieves shock revival due to neutrino heating in a gain layer characterised by a large-scale, hydrodynamic $m = 1$ spiral mode. Later on, the growing magnetic field of the proto neutron star launches weak outflows into the early ejecta. Their orientation follows the evolution of the rotational axis of the proto neutron star, which starts to tilt from the original orientation due to the asymmetric accretion flows impinging on its surface. The models with stronger magnetisation generate mildly relativistic, magnetically driven polar outflows propagating over a distance of $10^4$ km within a few $100$ ms. These jets are stabilised against disruptive non-axisymmetric instabilities by their fast propagation and by the shear of their toroidal magnetic field. Within the simulation times of around $1$ s, the explosions reach moderate energies and the growth of the proto neutron star masses ceases at values substantially below the threshold for black hole formation, which, in combination with the high rotational energies, might suggest a possible later proto-magnetar activity.

Key words: Supernovae: general - gamma-ray bursts: general

1 INTRODUCTION

Stars of masses in excess of about $8 M_\odot$ experience the collapse of their cores after the end of their hydrostatic burning phases. In most cases, the collapse and the subsequent phase in which the shock wave launched at the formation of a proto neutron star (PNS) stalls inside the core leads to a core-collapse supernova (CCSN) explosion powered by the standard neutrino mechanism (Bethe & Wilson 1985; Janka 2012). In a small fraction of progenitor stars, however, fast rotation and strong magnetic fields, only of minor importance in the neutrino mechanism, may have a strong impact on the evolution.

A large number of theoretical and numerical studies has been devoted to addressing the conditions for rotation and magnetic fields to affect core collapse, their possible dynamical consequences, and the observational signatures and remnants of such events. The numerical costs of simulations of such a complex system are reflected in a gradual increase of the degree of realism of the models in terms of their resolution and dimensionality as well as their modelling of the nuclear physics and the transport and interactions of neutrinos (see, e.g. Bisnovatyi-Kogan et al. 1976; Müller & Hillebrandt 1979; Symbalisty 1984; Akiyama et al. 2003; Kotake et al. 2004; Thompson et al. 2005; Moiseenko et al. 2006; Obergaulinger et al. 2006a,b; Dessart et al. 2007; Burrows et al. 2007; Sawai et al. 2013; Bugli et al. 2020). The current state of the art is represented by simulations combining three-dimensional (3D), (Newtonian or relativistic) magnetohydrodynamics (MHD) and a leakage scheme (Winterer et al. 2012; Mösta et al. 2014, 2015) or a two-moment transport scheme (Kuroda et al. 2020) for the neutrinos.

The strongest impact of rotation and magnetic fields can be expected if their energies reach, at least locally, equipartition with the (non-rotational) kinetic or even the internal energies (e.g. Meier et al. 1976). Even accounting for their growth due to the compression of the core and, for the magnetic energy, due to mechanisms such as the ensuing differential rotation and instabilities such as convection (e.g. Thompson & Duncan 1993; Raynaud et al. 2020) and the standing-accretion shock instability (SASI, e.g. En Ive et al. 2010; Guilet & Foglizzo 2010) or the magneto-rotational instability (MRI, e.g. Balbus & Hawley 1998; Akiyama et al. 2003; Obergaulinger et al. 2009; Móst et al. 2015; Masada et al. 2015; Rembiasz et al. 2016; Guilet et al. 2015; Rembiasz et al. 2016), this condition corresponds to rotational velocities and magnetic field strengths that can be expected only in an, as yet undetermined, though likely
rather small, fraction of the progenitor stars (e.g. Heger et al. 2005; Woosley & Heger 2006; Aguilerà-Dena et al. 2018).

Stars that meet these conditions may produce explosions powered by the rotational energy magnetically extracted from the PNS rather than by neutrino heating. Such an explosion mechanism may lead to the generation of very violent hypernova explosions (Iwamoto et al. 1998) with much higher energies than the $10^{51}$ erg typical for neutrino-driven standard CCSNe and the production of very fast collimated outflows. The most extreme cases within this spectrum includes gamma-ray bursts (GRBs) driven by the spin down of rapidly rotating PNSs with very strong magnetic fields (proto-magnetars, PMs [Metzger et al. 2011], and superluminous supernovae (SLSNe) [Gal-Yam 2019]. This last possibility, commonly supported in two-dimensional (2d) axisymmetric models (e.g. Ober-gaulinger 2004; Ober-gaulinger et al. 2006a, b; Ober-gaulinger & Aloy 2017), has been questioned based on some 3D simulations in which non-axisymmetric instabilities disrupt jets briefly after their formation (Mosta et al. 2014; Kuroda et al. 2020) but note that other 3d, low-resolution models do not fully support this claim, e.g. Ober-gaulinger & Aloy 2020; Aloy & Ober-gaulinger 2020 hereinafter Paper I and Paper II respectively). Before reaching a radius of around 1000 km, the jets are quenched and turn into wide lobes expanding in a less collimated geometry. Such an evolution might still be consistent with the subsequent generation of GRBs in the collapsar scenario (MacFadyen & Woosley 1999), if ongoing accretion causes the PNS to collapse to a black hole (BH) and sufficient rotational energy permits the formation of an accretion disk (though see Paper II for the possibility of forming a Type-III collapsar without a well developed accretion disc).

In previous studies (Ober-gaulinger & Aloy 2017; Paper I; Paper II) we have investigated the collapse of stars considered potential progenitors of the class of very violent explosions outlined above. In order to cover the large parameter space adequately and achieve long simulation times, we had reduced the computational costs by restricting the majority of our simulations to axisymmetry (only two low-resolution 3D models were included in the previous work to support, to some extent, some of our findings in axial symmetry). We had confirmed the aforementioned evolutionary paths of mostly bipolar explosions driven by a combination of neutrino heating and magneto-rotational stresses, depending on the pre-collapse values of rotational frequency and magnetic field strength. In this article, we extend our work to 3d models. Due to the higher computational effort per model, we select only a small subset of four of the models investigated in axisymmetry, varying the magnetic field of a zero-age main-sequence mass $M_{\text{AMS}} = 35 M_\odot$ (Woosley & Heger 2006) around the predictions made by the stellar evolution calculation for this star.

Specific questions to be addressed here are:

- What is the explosion mechanism, how is it affected by the presence of a strong field, and how do these results differ from the axisymmetric case?
- Are MHD-driven jets destroyed by strong 3d instabilities or do they remain collimated over a long time?
- How does the PNS evolve, does it show non-axisymmetric modes, and do 3D effects alter the likelihood of BH formation?

The paper is organised as follows. We discuss the methodology and initial data of our models in Sect. 2 describe our results in Sect. 3 and present a summary and conclusions from the results in Sect. 4.

2 NUMERICAL METHOD AND SIMULATION SETUP

As a continuation of our previous work, the present study uses the same input physics and numerical method as Ober-gaulinger & Aloy (2017) [Paper I] and Paper II. We refer to these publications for a detailed description of these aspects.

Our 3D grid is formed by $(n_r, n_\theta, n_\phi) = (300, 64, 128)$ numerical zones in the $r$, $\theta$- and $\phi$-directions; to be compared with the 2D grid used in previous papers $(n_r, n_\theta) = (300, 128)$. The radial zones are spaced logarithmically up to an outer radius of $r_{\text{max}} = 5 \times 10^{11}$ cm. The relatively coarse grid is compensated for by the high-resolution methods used in solving the MHD and transport equations with a spatial reconstruction in 5th order (Suresh & Huynh 1997). In energy space, we used $n_e = 10$ logarithmically distributed bins in the interval [3 MeV, 240 MeV].

We selected four of our axisymmetric models for resimulation in 3D (see Tab. 1). They are all based on the stellar model 35OC for a $M_{\text{AMS}} = 35 M_\odot$ star (Woosley & Heger 2006). The spherically symmetric stellar evolution model includes the effects of rotation and magnetic fields according to the prescription for magnetic instabilities, a dynamo, and the redistribution of angular momentum of (Spruit 2002). At the time of collapse, the star has a mass of $M_{\text{PNS}} \approx 28.1 M_\odot$ and a large Fe core of $M_{\text{Fe}} \approx 2.02 M_\odot$. Its centre rotates with an angular velocity of $\Omega_c \approx 2.0 \text{ s}^{-1}$. The model contains a dominantly toroidal magnetic field in radiative zones of the star with a maximum field strength of $B_{\text{max}} \approx 1.2 \times 10^{12}$ G. Convective regions are not magnetised in the pre-SN model 35OC.

Instead of using several progenitors, we explored the influence of variations of the magnetic field on the dynamics. A thorough justification of the modifications included in the magnetic field topology and strength can be found in Paper II. In the present work, we use the same rotational profile for all models, namely, the one provided by the stellar evolution model, but initialise our models with four different distributions of the magnetic fields, related to four of the axisymmetric models:

Model O is a 3D version of the eponymous axisymmetric model. It uses the original magnetic field predicted by the stellar evolution model. In axisymmetry, the model explodes in a bipolar way $t_{\text{ph}} \approx 0.18$ s after bounce ($t_{\text{ph}} = t - t_b$, where $t_b \approx 0.4$ s is the bounce time) predominantly driven by magnetic stresses and produces a BH within about 3.2 s.

Model P corresponds to the axisymmetric model 35OC-Rp3 and is defined by an enhancement of the poloidal field component by a factor 3 with respect to the pre-SN progenitor, while the toroidal component remains unchanged. The axisymmetric model produces a magnetically driven jet-like explosion in the first 80 ms after bounce. Accretion onto the PNS is weaker than in model 35OC-R0 and, instead of collapsing to a BH, the model is a possible candidate for a PM-driven GRB.

Model S starts with a very strong magnetic field set up following Suwa et al. (2007) with both toroidal and poloidal components normalized to central values of $10^{15}$ G. The axisymmetric model 35OC-Rs exhibits the strongest explosion of all of the 2D models, setting in without a delay immediately after bounce. Like model 35OC-Rp3, it does not lead to a BH collapse.

Model W employs the same prescription as model S, but reducing the field strength by two orders of magnitude. Its 2D equivalent, model 35OC-Rw, develops a bipolar neutrino-driven explosion aided by the large rotational energy reservoir of the collapsed core at $t_{\text{exp}} = 378$ ms. At its termination, the model developed a very massive PNS close to the threshold of BH formation.
Table 1. List of our models (first column) and their corresponding axisymmetric versions (second column). The third column shows the type of magnetic field: “Or” indicates the magnetic field profile of the original stellar evolution model, $x\rho y$ means that the original poloidal and toroidal fields have been multiplied by factors $x$ and $y$, respectively, and $x\alpha(x,y)$ stands for an artificial dipolar field with maximum poloidal and toroidal field components of $10^x$ and $10^y$ G, respectively. The fourth column gives the final time of the simulation, $t_f$. The column “fate” gives a brief indication of the evolution of the model: $\nu$ means a standard neutrino-driven shock revival, $\nu-\Omega$ one strongly affected by rotation, and MR a magneto-rotational explosion. The last two columns provide proxy values for the explosion mass ($M_{\text{exp}}$) and for the explosion energy $E_{\text{exp}}$ (in units of $10^{43}$ erg) at the end of the simulations. The next five columns present the PNS mass, rotational energy, magnetic energy (both in units of $10^{51}$ erg), average rotational frequency $\Omega = J^{\text{pol}} / I^{\text{pol}}$ and poloidal and toroidal field strength at the end of the computed time.

| name | 2D name | field | $t_f$ [s] | fate | $M_{\text{exp}}$ [$M_\odot$] | $E_{\text{exp}}$ [$10^{51}$ erg] | $M^{\text{pol}}_{\text{sh};\max}$ [$M_\odot$] | $T_{\text{sh};\max}^{\text{pol}}$ [$10^3$ K] | $g_{\text{sh};\max}^{\text{pol}}$ [$10^3$ K] | $\Omega$ [10$^{-4}$ G] | $E^{\text{pol}}_{\text{surf}}$ [$10^{51}$ G] | $B^{\text{tor}}_{\text{surf}}$ [$10^{51}$ G] |
|------|---------|-------|-----------|------|-----------------|-------------------|-----------------|-----------------|-----------------|-------------|-----------------|-----------------|
| W 350C-Rw | $a(10, 10)$ | $\nu-\Omega$ | 1.50 | 0.58 | 1.7 | 1.88 | 11 | 0.056 | 2.3 | 1.6 | 4.2 |
| O 350C-R0 | Or | MR | 0.85 | 0.21 | 0.62 | 2.16 | 15 | 0.24 | 1.3 | 2.1 | 3.8 |
| P 350C-Rp3 | $3p, 1t$ | MR | 0.81 | 0.16 | 0.50 | 2.03 | 18 | 0.23 | 2.6 | 0.73 | 0.76 |
| S 350C-Rs | $a(12, 12)$ | MR | 1.15 | 1.7 | 13 | 1.75 | 3.3 | 0.078 | 1.3 | 1.9 | 1.4 |

Figure 1. Properties of the explosion. The top and bottom panels show the time evolution of the maximum shock radius and the mass and energy of the ejecta, respectively.

We computed the pre-collapse evolution of all models in axisymmetry and mapped to the 3D grid at the time of bounce.

3 RESULTS

3.1 General overview

All our models develop supernova explosions within the first $\sim 0.25$ s after core collapse. Figures 1 summarize selected global quantities of the evolution of our four models: the run-away of the maximum shock radii, $R_{\text{sh};\max}$, the increase of the diagnostic explosion energy, $E_{\text{ej}}$, and the mass, $M_{\text{ej}}$, of the ejecta (Fig 1b). As we shall see in the following, these properties are ordered according to the poloidal field strength in the initial model.

The stronger the poloidal field, the more energetic the explosions are. Similarly, the ejected mass and the shock radius at times sufficiently separated from core bounce (say, $t_{\text{pb}} > 0.35$ s) grow with larger pre-SN poloidal field strengths. In this regard, 2D and 3D models are qualitatively similar (see Paper II).

The two models W and O initiate the shock expansion almost at the same time, after a phase of about 250 ms during which the shock stagnates at around 200 km (Fig 1a). Immediately thereafter, model O exhibits a rise of the shock radius to $R_{\text{sh};\max} \approx 10^4$ km after 850 ms. The ejecta energy and mass (Fig 1b) increase continuously and very rapidly within the first 100 ms after shock revival, then more gradually. The final state of the simulation with $E_{\text{ej}} \approx 5 \times 10^{50}$ erg and $M_{\text{ej}} \approx 0.15 M_\odot$ suggests a moderately violent supernova. The somewhat stronger magnetised model O reaches similar final ejecta masses and energies, while the maximum shock radius is about twice that of model W. Despite the similar global evolution, a detailed comparison of the two models reveals important differences regarding the explosion mechanism and outflow properties (see below).

We find much stronger explosions for the two models with enhanced pre-collapse magnetic fields. Model P explodes after about 100 ms (i.e. at about the same time as its axisymmetric counterpart model 350C-Rp3; Paper I) with a rapidly growing shock radius. By the end of the simulation, i.e. at $t \approx 1.6$ s, the shock achieves a maximum radius of $R_{\text{sh};\max} \approx 3 \times 10^4$ km. Within less than half a second after the onset of the explosion, energy and mass of the ejecta show a fast early rise to $E_{\text{ej}} \approx 1.15 \times 10^{51}$ erg and $M_{\text{ej}} \approx 0.5 M_\odot$, respectively, in contrast to the ongoing growth of both quantities displayed by the axisymmetric version of this model. The two quantities change only little during the final several hundreds of milliseconds of the simulation. As in 2D, the most extremely magnetised model, S, directly explodes without any shock stagnation. Its shock wave expands the fastest ($R_{\text{sh};\max} \approx 5 \times 10^4$ km at $t \approx 1.15$ s) and its final explosion energy ($E_{\text{ej}} \approx 1.3 \times 10^{52}$ erg) and mass ($M_{\text{ej}} \approx 1.7 M_\odot$) are by far the highest of all models and keep increasing when we had to terminate the simulation. These values put model S into the range of potential hypernovae.

Similarly to the variables characterising the shock wave and the ejecta, the properties of the gain layer (Fig 2) as well as the neutrino emission (Fig 3) exhibit an ordering with the initial (poloidal) magnetic field. It should be noted that an analysis of the gain layer is relevant mostly before shock revival, after which its growth parallels that of the expanding shock.

In the case of model O, the mass in the gain layer ($M_{\text{gain}}$) starts to rise already before the begin of the increase of the maximum
Figure 2. Properties of the gain layer. From top to bottom, the panels display the mass contained in the gain layer, the heating efficiency, the ratio between (total and rotational) kinetic energy and the gravitational energy, and the total and poloidal magnetic energy.

Shock radius, while it continues to gradually decrease for a little longer in model $W$. During shock stagnation, models $W$ and $O$ launch their explosions out of a gain layer of $M_{\text{gain}} \gtrsim 0.02 M_\odot$. Model $W$ emits neutrinos at the highest rate reaching $L_{\nu_e} \approx 6.6 \times 10^{52} \text{erg s}^{-1}$ after the end of the neutrino burst. Consequently, its neutrino heating is strongest of all models with a specific heating rate, $\eta_{\text{gain}} = Q_{\nu}/(M_{\text{gain}} c^2)$, where $Q_{\nu}$ is the energy deposition due to neutrinos averaged over the entire volume of the gain layer (see Eq. 22 of Paper I), peaking at $\eta_{\text{gain}} \approx 0.3 \text{ s}^{-1}$ close to the start of the shock runaway. Model $O$ achieves shock revival at the same time and somewhat more violently despite lower neutrino luminosities (about $10\%$ less than $W$) and heating rate ($\eta_{\text{gain}} \lesssim 0.2 \text{ s}^{-1}$) smaller than in model $W$.

In both models, the total and rotational kinetic energies in the
gain layer grow at a similar rate until the onset of the explosion. The magnetic energy, dominated by the toroidal component, reaches up to 10% of the rotational energy in the gain layer of model O. For model W, it is insignificant in the phase leading to shock revival. Its sharp increase thereafter is the result of the ejection of magnetised, hot matter (with entropy per baryon \( \gtrsim 20 \, k_B \)) from above the north polar region of the PNS (see Sect. 7.2).

Compared to models W and O, P shows a gain layer of a higher mass exposed to slightly lower neutrino luminosities. Consequently, the specific heating rates are low. Despite the higher gain mass, Model P begins its post-bounce evolution with kinetic and rotational energies comparable to models W and O. Its magnetic energies, both for the poloidal and the toroidal components, exceed those of model O by a factor of a few (Fig. 4(d)), consistent with the enhanced poloidal field in the initial conditions, and those of W by several orders of magnitude. The corresponding values of gain mass and kinetic and magnetic energies of model S are still higher than in model P, which reflects the lack of a phase of shock stagnation and the fact that the gain layer is almost identical to the expanding post-shock region at all times.

Important properties characterising the PNS show a similar ordering with the initial magnetic fields as can be seen in Fig. 4(a) for a few 100 ms to maximum values of \( M^{\text{pns}} \approx 2.16 \, M_\odot \), \( 2.03 \, M_\odot \), \( 1.88 \, M_\odot \), and \( 1.75 \, M_\odot \) for models W, O, P, and S, respectively. The PNS mass levels off or, for stronger fields, even enters a phase of decrease showing a qualitative agreement with the behaviour displayed by the equivalent axisymmetric models (see Paper II). PNSs of models with weaker fields possess very high rotational energies (\( T^{\text{pns}} \lesssim 2 \times 10^{52} \, \text{erg}; \) Fig. 4(b)), while stronger fields reduce the rotational energy by a factor of about 2. Even in these cases, the average spin frequencies are above \( \Omega \gtrsim 10^3 \, \text{s}^{-1} \). Right after bounce, the ratio of magnetic to rotational energy of the PNS of Model S is \( B^{\text{pns}}/T^{\text{pns}} \lesssim 0.2 \), i.e., about one order of magnitude higher than in models W and S. The strong field causes the rotation to slow down after 500 ms in models W and S, while models O and P show a growing trend for \( T^{\text{pns}} \). The behaviour of the rotational energy of the 3D models is qualitatively similar to their 2D counterparts. However, the values of \( T^{\text{pns}} \) are systematically larger in axisymmetry. The modification of the rotational profile by the magnetic field explains the behaviour of the magnetic energy of the PNS in all models. As a result, the models with weaker initial fields end up with higher final magnetic energies of around \( B^{\text{pns}} \sim (2 \ldots 5) \times 10^{50} \, \text{erg} \) (Fig. 4(c)).

In the following discussion of individual models, we quantify the deformations of surfaces such as the shock wave or the outer boundary of the PNS by expanding their radii as a function of the angular coordinates, \( R(\theta, \phi) \), into the spherical harmonic components (Superscripts \( ^h \) and \( ^pns \) will be used for the shock wave and the PNS surface, respectively). Following Burrows et al. (2012), we define the amplitudes

\[
a_{lm} = \frac{(-1)^{|m|}}{\sqrt{4\pi (2l+1)}} \int d\Omega R(\theta, \phi) Y_l^m(\theta, \phi) \tag{1}
\]

in terms of the spherical harmonics, \( Y_l^m \), which in turn depend on the associated Legendre polynomials, \( P_l^m(\cos \theta) \).

\[
Y_l^m(\theta, \phi) = \begin{cases} 
N_l^m P_l^m(\cos \theta) & m = 0, \\
\sqrt{2N_l^m} P_l^m(\cos \theta) \cos |m|\phi & m \neq 0.
\end{cases} \tag{2}
\]

\[
N_l^m = \frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}. \tag{3}
\]

The lower order coefficients have direct physical meaning: \( a_{0,0} \) is the average radius, the dipole coefficients \( a_{1,m} \) represent the average displacement of the \( R(\theta, \phi) \)-surface from the origin in the three coordinate directions, and the quadrupole amplitude \( a_{2,0} \) can be used to quantify a prolate or oblate deformation of the surface. For an analysis of the latter property, we furthermore determine the radii along the polar axis and the maximum, minimum, and average equatorial radii of the \( R(\theta, \phi) \)-surface.

We will employ several variables commonly used in the analysis of core collapse such as the mass and (total) energy in the gain layer or the ratio between the timescales for advection of gas through the gain layer and for heating by neutrinos. These variables can be defined in different ways. We will use the definitions of the timescales for advection, \( \tau_{\text{adv}} \), neutrino heating, \( \tau_{\text{heat}} \), and the propagation of Alfvén waves through the gain layer, \( \tau_{\text{TAI}} \), put forward in Paper I—see their Eqs. 24, 25, and 28. In these quantities, we can account for their dependence on the angular coordinates by computing radial integrals or averages on radial rays at fixed \((\theta, \phi)\).
3.2 Model W: neutrino-driven explosion

In the roughly 250 ms between core bounce and the onset of the explosion, the strong centrifugal force of the rapidly rotating matter leads to a decidedly oblate shape of the shock wave. While its polar radii shrink, the shock wave continuously expands in the equatorial plane, as shown in Fig. 5. Consequently, the pole-to-equator axis ratio (compare black lines to yellow line and orange band in Fig. 5(a)) decreases to values around 3:2 after t ≅ 200 ms. This tendency is reflected in the negative values of $a_{3/2}^{-1}$ (Fig. 5(c)). The dipole coefficients $a_{3/2}^{-1}$ show quasi-periodic oscillations with an increasing amplitude and a phase shift of $\approx \pi/2$ between each other. Such an evolution is indicative of a growing $m = 1$ spiral mode rotating around the PNS.

This mode is visible in Fig. 5, showing the shape of the shock wave at different times, $t = 0.265, 0.29, 0.34, 0.41$ s. The panels follow the expansion of the shock surface. Their orientation is such that the rotation of the gas and the spiral pattern around the z-axis proceeds in a counterclockwise sense. In the top and bottom rows of panels, the colours of the surface encode the logarithm of the ratio between the advection and heating timescales, $\tau_{adv}/\tau_{heat}$: red and blue shades correspond in the top panels to regions where heating is slower and faster than advection and in the bottom panel to regions with negative and positive total energies in the gain layer, respectively. Already at an early stage of its growth, the spiral mode shows a close correlation to the pattern of the neutrino heating with the pater of fast/slow cooling and increasingly positive total energy (blue shades in the bottom row) developing behind the triple point of the $m = 1$ mode. This geometry with the most favourable conditions for the explosion ($\tau_{adv}/\tau_{heat} > 1$, positive total energy) found near the equator contrasts with the emission geometry of the neutrinos. The luminosities of all flavours are higher at high latitudes than close to the equator; in Fig. 3 compare the green lines (north and south poles) to the red ones (equator). As a consequence, both $\tau_{adv}$ and $\tau_{heat}$ decrease above the poles, with the net effect being a rise of $\tau_{adv}/\tau_{heat}$ (as in axial symmetry). During the entire pre-explosion phase, $\eta_{gain}$ is consistently larger by a factor of at least 4 at $\theta = 0, 180^\circ$ than at $\theta = 90^\circ$. This enhanced efficiency, however, does not translate into a polar explosion (like in axial symmetry) because the poles are also the locations of the fastest accretion through the gain layer, while the advection time increases in the growing $m = 1$ mode near the equator, thus favouring an oblate explosion.

When the shock wave enters the explosion phase (t$_{pb}$ = 0.34 and 0.41 s; Fig. 5(g) and (h)), the hot bubble has grown such as to encompass almost the entire post-shock layer, with the most notable exception being the north and south poles. As the shock continues to expand, its shape partially loses the oblate character and the polar, equatorial, and average radii grow in parallel. The dipole and quadrupole coefficients of the shock surface evolve slowly. They maintain a small, but non-zero, value until the end of the simulation, indicating a continuing moderate asphericity of the ejecta.

The final stages of the simulation (Fig. 7) exhibit an ejecta propagating in the form of several large bubbles. Inside the shock wave, the gas possesses a highly asymmetric distribution of entropy (Fig. 7(a)) and electron fraction (Fig. 7(b)). Two of the bubbles at the highest radii are filled by relatively cool gas ($s < 18$ kB to the right of the plot). Hotter gas with 18 kB < $s < 26$ kB expands inside the shock in the form of blobs of a wide range of sizes. The largest fraction of the ejecta is slightly neutron-rich with $0.4 < Y_e < 0.455$ (in the figure, the gas contained by the green surface, but outside the red one) and 0.455 < $Y_e < 0.51$ (between the green and blue surfaces). In addition, a more neutron-rich structure is expanding at low to intermediate latitudes (inside the red surface). At very late times, a pair of fast outflows with a relatively narrow angle, filled by hot gas consisting of symmetric matter (the blue structure with $s > 26$ kB, $a$, and $Y_e > 0.51$, (b) panel) is launched into the post-shock region from a region near the PNS. The generation of these outflows begins after a rise of the magnetic energy in the PNS and the gain layer. While the ejecta are mostly only very weakly magnetised, the polar outflows are characterised by $B^{-1} \approx 0.1$ (green structures in Fig. 5(c)). Jet and counter-jet are fairly asymmetric and not perfectly aligned along the rotation axis. Instead, their propagation direction varies substantially with time, partly following the motion of the PNS (see below). Thus, the change in the propagation direction is not totally random as in the most standard jittering jets model (JJM Papish & Soker 2011 but see also Sternberg & Soker 2008 for variants of the model that may follow the precession of the PNS). Furthermore, they tend to widen at higher radii. By the time of the end of our simulation, they have not managed to reach the shock surface. Whether they maintain their coherence for longer time and are able to penetrated beyond the more roundish shock wave remains uncertain. Even if that is not the case, they constitute an additional mechanism for transporting energy from the centre of the core to the surrounding regions and heating the ejecta. Also regarding jets as a mean to transport energy away from the PNS but below the shock radius, the computed evolution shares qualitative similarities with the JJM. We define the specific radial energy flux, $f^e := \rho^{-1} \left[(\epsilon_\star + P_\star)\theta - \mathbf{b}_\theta \mathbf{v}^\perp\right]$, where $\epsilon_\star$ and $P_\star$ are the total energy and pressure, including the magnetic contributions, and show it in Fig. 7(d). The blue surface in the northern hemisphere aligned with the stronger one of the two outflows indicates the most efficient magnetohydrodynamic energy transfer emanating from the centre.

As evidenced by the radii in Fig. 5(b), the PNS is gradually contracting. Around t$_{pb}$ ≅ 0.5 s, this contraction is interrupted by an expansion in the equatorial region. This phenomenon will be explored below. The shape of the PNS is rather aspherical. At early times, we see oscillations similar to the $m = 1$ shock modes in the decomposition of the PNS surface in spherical harmonics (Fig. 5(a)), albeit at lower amplitude and higher frequency.

As a result of a relatively limited time of mass accretion ending after t$_{pb}$ > 0.45 s, the model develops a massive, rapidly rotating and strongly magnetised PNS with values at the end of the computed time of $M_{\text{pns}} \approx 2.16 M_\odot$, $\tau_{\text{pns}} \approx 2.16 \times 10^{52}$ erg and $B_{\text{pns}} \approx 5.3 \times 10^{50}$ erg. While the rotational energy grows parallel to the mass accretion, the magnetic energy experiences its strongest increase after the mass and rotational energy have achieved their maximum values. Its growth between t$_{pb}$ = 0.5 s and t$_{pb}$ = 0.7 s is exponential, though with an e-folding time of about 200 ms much slower than typical timescales of the PNS such as the rotational period or the crossing times of the flow speeds in its interior.

This very low growth rate is, however, in rough agreement with a different dynamical mode present in the PNS during the same interval of time. We show the evolution of the shape of the PNS and its magnetic field in Fig. 8. At t$_{pb} = 0.4$ s (Fig. 8(a)), the PNS has the form of a moderately flattened ellipsoid with a pole-to-equator axis ratio around $8:5$ rotating about the $z$-axis. The magnetic field is dominated by field lines circling the axis. The asymmetry of the surrounding layers, modulated by the strong $m = 1$ mode, translates into a strong asymmetry of the downflows feeding the PNS and a partial tilting of its rotational axis. At t$_{pb} = 0.6$ s, the outer layers rotate about an axis forming an angle of $\approx 30^\circ$ w.r.t. the $z$-axis, whereas the inner regions ($\rho > 10^{12}$ g cm$^{-3}$) initially maintain the
Figure 5. Evolution of shock (left) and PNS (right panel) radii of model W. In the top panels, the radii measured along the polar directions and the maximum radii as well as the average over all longitudes in the equatorial plane and over the entire surface are shown by lines as indicated in the legend, and the range of radii in the equatorial plane is represented by the orange band. The bottom panels show the evolution of the three normalised multipole amplitudes of the shock/PNS surface, $a_{lm,1}^{sh/PNS}/a_{0,0}^{sh/PNS}$ ($m = 0, \pm 1$), and of one of the normalised quadrupole amplitudes, $a_{2,0}^{sh/PNS}/a_{0,0}^{sh/PNS}$.

Figure 6. Shock surface of model W at, from left to right, $t = 0.265, 0.29, 0.34, 0.41$ s. The shock is represented by a surface whose colours display quantities characterising the explosion conditions: in the top and bottom panels, the ratio of heating and advection time scales (blue/red: heating faster/slower than advection), and the specific total energy (blue/red: unbound/bound matter), respectively.

original axis of rotation (Fig. 5b). Following this transition, the axis of the magnetic field loops near the PNS surface is also tilted. The tilt slowly increases from $t_{pb} \approx 0.45$ s to $t_{pb} \approx 0.65$ s.

In order to quantify more precisely the evolution of the PNS tilting angle, $\theta_{\text{tilt}}^{\text{PNS}}$, we express it in terms of the major axes of the PNS.

To this end, we approximate the PNS as well as the shock surface by the ellipsoid determined by the principal axes of the $3 \times 3$-tensor formed from the quadrupole spherical harmonic decomposition (i.e. using Eq. 1 with $l = 2$ and $m = 0, 1, 2$; see Jackson 1962 § 4.1) of this approximate surface. In both cases, the result is a pair of axes...
originally in the equatorial plane and one axis along the z-axis. The direction of these axes will trace the changing orientation of the PNS or shock. These changes are shown in the time evolution of the latitude of the axes in Fig. 7. During the first tens of milliseconds, the PNS and the shock are close to spherical and the angles vary in a random manner. When the PNS starts to tilt and the shock develops its large-scale deformations, the polar PNS axes drifts from its original orientation, parallel to the z-axis, to latitudes of $\theta = -50^\circ$. The other two main axes experience a similar change. Temporarily, they return to their original orientations ($t_{pb} \approx 0.8$ s). The increase of the deviation by the end of the simulation suggests that the PNS might continue to oscillate in a similar way during the subsequent evolution. We note that the shock surface changes its orientation by a similar amount.

The tilt manifests itself in the growth of the quadrupole coefficients, of which $a_{20}^{pns}$ is shown in Fig. 5(d). At times, the rather violent changes of the PNS surface also show up in the large and quick fluctuations of the dipole coefficient $a_{10}^{pns}$. The tilt is also responsible for the interruption of the contraction of the PNS (cf. Fig. 5(b)). With decreasing mass accretion rate, $a_{00}^{pns}$ saturates and later on slowly decreases. At $t_{pb} = 0.8$ s, the PNS shows a dichotomy between a roughly spherical bulge with $\rho \gtrsim 10^{12}$ g cm$^{-3}$ and a flat, disk-like component whose rotational axis is mostly aligned with the original one (Fig. 5(c)). The magnetic field couples the entire PNS and leads to an exchange of the magnetic axes of inner and outer layers such that they are oblique to both each other and to the z-axis. Most likely, this configuration is not the final state and the factors at work up to that point, external ones such as accretion and, possibly later, fall-back or the expulsion of gas from the PNS surface as well as internal ones such as the magnetic coupling will continue to modify the structure and the magnetic field.

### 3.3 Model 0: stronger field

Though happening at a similar time as in model $W$ (cf. the evolution of the shock radii in Fig. 10(a)), the shock revival in model 0 is the result of a rather different evolution and explosion mechanism. The shape of the shock wave before the explosion (times $t_{ph} = 0.24$ s, $0.29$ s, and $t_{ph} = 0.34$ s in Fig. 11) does not exhibit a similarly strong $m = 1$ modification as in the case of the weakest initial field. While not completely suppressed, the spiral mode grow to about half the amplitude as in model $W$, measured in terms of the normalized dipole coefficients $a_{11}^{pns}$ (Fig. 12(c)); note also that these coefficients show quasi-periodic oscillations similar to model $W$, albeit with a lower frequency. As a consequence, the shock is less oblate and the quadrupole coefficient grows to $a_{20}^{sh} \approx 0.08 a_{00}^{sh}$. Most importantly, neutrino heating does not, by itself, achieve conditions favourable for the explosion. The heating timescale usually is longer than the advection timescale at low latitudes (top panels of Fig. 12 and these regions do not reach positive total energies in the gain layer (middle panels). The magnetic field, on the other hand, is dynamically relevant. Although averaged over the gain layer, its energy amounts to only about 5% of the kinetic energy, it is strong enough to cause the partial suppression of the $m = 1$ mode. Near the polar axis, the field is sufficiently strong for Alfvén waves to propagate faster through the gain layer than gas is advected towards the PNS as we show in the blue regions of the shock wave in the bottom panels.

The polar regions are, furthermore, subject to the most intense neutrino radiation. The PNS has an even more aspherical shape than that of model $W$. From Fig. 10(b), we can extract a drop of the pole-to-equator axis about to 1 : 2 and a growth of the quadrupole coefficient to $a_{20}^{pns} \approx -0.14 a_{00}^{pns}$ until $t_{pb} = 0.35$ s. The larger equatorial radius reduces the the part of the neutrino luminosity powered by the accretion rather than thermal cooling of the PNS emitted into low latitudes as well as the total luminosity. Close to the pole, on the other hand, the neutrino fluxes are as high as in model $W$. Consequently, a favourable ratio of both the heating and the Alfvén timescales to the advection timescales develop there (Fig. 11(top and bottom panels). These conditions stabilize the shock at the pole at radii $R_{sh} \approx 200$ km in contrast to the gradual receding it undergoes in model $W$. Ultimately, they combine to launch a polar, rather than equatorial, runaway of the shock wave. The explosion starts with a peanut-shaped shock wave that is at first expanding faster into the southern hemisphere ($a_{10}^{sh} < 0$), but soon thereafter evolves into a largely symmetric pair of prolate outflows with the quadrupole coefficient approaching $a_{20}^{sh} \approx 0.1 a_{00}^{sh}$. We present the structure of the ejecta of model 0 at a late stage in the simulation ($t_{ph} \approx 0.8$ s) in Fig. 12. Their bipolar morphology offers a stark contrast to model $W$ (Fig. 9). The shock wave (red surface in Fig. 12(a) showing the entropy) possesses a mostly axisymmetric shape with an equatorial radius of $R_{sh}^{eqtl} = 4.5 \times 10^3$ km and a polar elongation of $R_{sh}^{pol} = 8.9 \times 10^3$ km. In the largest part of the volume of the post-shock region, the gas has only a small positive or negative radial velocity. The PNS is fed by an oblate accretion flow at low latitudes with peak velocities exceeding $10^4$ cm s$^{-1}$. It extends to a distance of about 2000 km. In the innermost 1000 km, the downflow deviates strongly from axisymmetry. In this region, matter falls onto the PNS in the form of a spiral mode that coexists with pockets of outwards moving matter. The downdrafts decelerate considerably at $r \approx 300$ km, where the local rotational energy reaches half of the local gravitational energy and, hence, matter is...
Figure 8. Structure of the PNS and its magnetic field in model \( \tilde{W} \) at times \( t_{pb} \approx 0.4, 0.6, 0.8 \) s (left to right). The orange surface, cut open at \( y = 0 \), represents the specific entropy on a iso-density surface of \( \rho = 10^{10} \) g cm\(^{-3} \) and the colours on the field lines display the magnetic field strength.

Figure 9. Time evolution of the orientation of the PNS and the shock surfaces of model \( \tilde{W} \). Red and blue lines show the latitudinal direction of the three major axes of the PNS and the shock, respectively.

nearly entirely supported by centrifugal forces. The gas with positive radial velocity in the equatorial region does not manage to overcome the ram pressure of the downflows. Instead, the core ejects gas in fast \( v_r \gtrsim 4 \times 10^9 \) cm s\(^{-1} \), blue surfaces in Fig. 12) bipolar outflows along the rotational axis. The magnetisation of the jets is moderate, reaching \( \beta^{-1} \approx 0.1 \) along its beam and \( \beta^{-1} \approx 0.01 \) in the cocoon (blue and green surfaces in in Fig. 12(c)), while the magnetic field is insignificant in the downflows.

These jets maintain their stability and coherence over a distance of several 1000 km. Within the first 1000 km of their propagation, they expand laterally to a diameter of 500 km. Thereafter, the lateral spreading slows down and the outflows resemble collimated jets. Though non-axisymmetric modes modify their geometry, they do not suffice to quench the outflow in a similar manner as observed by Møsta et al. (2014). Since this stability is a common feature of all our magneto-rotational outflows, we defer a discussion to a separate section comparing models \( O, P, \) and \( S \).

The outflows are more neutron-rich than in model \( \tilde{W} \) (Fig. 12(b)). The beam of the jet contains relatively hot, almost symmetric \( Y_e \approx 0.5 \) matter and is surrounded by cooler gas with an electron fraction as low as \( Y_e \approx 0.32 \). The composition of the outflows shows a higher north-south asymmetry than the velocity. The southern outflow contains a large shroud of material with \( Y_e < 0.4 \) enclosing the beam (dark blue surface) as well as a cloud with \( Y_e \approx 0.35 \) (green) expanding non-axisymmetrically at the edge of the ejecta.

The PNS evolves in an even more oblate manner than in model \( \tilde{W} \) (Fig. 10(b) and (d)). The polar radii (black lines, Fig. 10(b)) continuously contract to \( R_{\text{PNS,pol}} \approx 23 \) km at the end of the simulation, while the equatorial radii saturates at \( R_{\text{PNS,eqtr}} \approx 50 \) km (blue lines). The high degree of flattening is reflected in the drift of the quadrupole coefficient to \( a_{2,0}^{\text{PNS}} \approx -0.12a_{2,0}^{\text{pol}} \). We do not observe a gyration of the PNS axis similar to the case of model \( \tilde{W} \). As a consequence, the magnetic field retains a more ordered geometry. Inside of and around the PNS, it mainly consists of an \( m = 1 \) mode that spirals out from its centre at \( r = 0 \) through the entire equatorial plane of the PNS into the aforementioned partially centrifugally supported region up to \( r \approx 300 \) km (Fig. 13 where we represent the inner \( \approx 160 \) km). Inside this flux tube, \( \beta^{-1} \) can reach values of \( \beta^{-1} \gtrsim 0.1 \). This structure can account for a significant modification of the rotational profile. Around the PNS surface, its Maxwell stress component \( M^\phi = \beta \phi r \) corresponds to a relative local rate of change of the angular momentum of \( \tau_j^{-1} := J^{-1}d_JJ \sim J^{-1}r^{-2}\partial_r (r - r^2M^\phi) \gtrsim 10^{-1} \). This rate decreases by one order of magnitude towards the centre of the PNS, but remains comparable to the secular timescales of the evolution of the PNS. Similarly to many of our axisymmetric models, the outward transport of angular momentum increases the centrifugal support in the outer layers of the PNS (Paper II). Along the rotational axis, the field lines form a helical structure that connects the PNS with the polar jets.

3.4 Models P and S: MHD explosions

Models P and S produce explosions setting in with hardly any shock stagnation (at \( \Delta t_{\exp} \approx 100 \) ms, for model P) and promptly after bounce (for model S). As a result, there is no time for the development of an \( m = 1 \) mode in the shock before the onset of the explosion (Fig. 14 note the difference with models \( \tilde{W} \) and 0 in Figs. 15 and 10 respectively). Shock revival starts from a virtually spherical shape (note that all lines in the Fig. 14(a) cluster around the same values and the comparably low magnitude of \( a_{2,0}^{\text{pol}} \) between \( t_{pb} \approx 0.05 \) s and \( t_{pb} \approx 0.1 \) s in Fig. 14(c)). Thereafter, the asymmetry rises quickly as a prolate explosion sets in, first in the northern, and later also in the southern hemisphere. At \( t_{pb} \approx 0.35 \) s, the northern and southern shock radii start to agree and expand at the same velocity. In the equatorial region, the shock expands slower, leading...
Figure 10. Same as Fig. 5 but for model 0.

Figure 11. Shock surface of model 0 at, from left to right, $t_{pb} = 0.24, 0.29, 0.34, 0.41$ s. The top and middle rows show the same quantities as in Fig. 6, i.e., the advection-to-heating timescale ratio and the specific total energy, respectively. The bottom row adds the ratio between the advection and the Alfvén timescales, $\tau_{adv}/\tau_{Alf}$.
to a relatively constant pole-to-equator axis ratio of 2 : 1 and a very high quadrupole coefficient maintaining a value of $a_{2,0}^\text{sh} \approx 0.09 a_{2,0}^\text{sh}$ for a long time. The shock expansion proceeds faster in model S and without any intermediate state of a more or less spherical shape. The shock radii show a minor north-south asymmetry (cf. black lines in Fig.15(a)) for the polar shock radii and the moderate value of $a_{2,0}^\text{sh}$ in Fig.15(c)). The ejecta show the most extreme morphology of all models with an axis ratio around 4 : 1 and a quadrupole coefficient in the range $0.12 \leq a_{2,0}^\text{sh} / a_{2,0}^\text{sh} \leq 0.25$.

In both models, neutrino heating contributes very little to the explosion. Immediately after bounce, the PNS does not become highly oblate. Consequently, the neutrino emission is relatively isotropic (Fig.15(c), (d)) and we do not find an enhancement of the neutrino heating in the polar regions in the same way as in model 0. The ratios between advection and heating timescales, presented in the top rows of panels of Figs.16 and 17, do not favour shock revival at any location. In model P, we only find an increase of $\tau_{\text{adv}} / \tau_{\text{heat}}$ beyond unity once the advection timescale rises after the shock expands along the rotational axis. In model S, $\tau_{\text{adv}} / \tau_{\text{heat}}$ remains below unity until well after the explosion has started. Then the shock expansion leads to an increase of $\tau_{\text{adv}}$ and, consistently, an increase of $\tau_{\text{adv}} / \tau_{\text{heat}}$, in qualitative agreement with the evolution of $\tau_{\text{adv}} / \tau_{\text{heat}}$ displayed by the corresponding axisymmetric model.

350C-Rs (Paper I). The strong magnetic fields lead to short Alfvén timescales and, thus, $\tau_{\text{adv}} / \tau_{\text{Alf}} > 1$ at high latitudes (in qualitative similarity to the respective axisymmetric models), indicating a magnetically driven launch of the shock (bottom rows of panels). They cause the gas in these regions to become gravitationally unbound as we show in the blue regions in the middle rows of panels.

The outflows of both models, visualized in Fig.18 and Fig.19, present a typical jet-like morphology (as in axial symmetry; Paper I) with a narrow beam with maximum velocities up to $v_{\text{max}} \approx c / 2$ and similar values of the normalised energy fluxes along the rotational axis. They are surrounded by a cocoon moving at much smaller radial speeds. Model P shows a relatively wide cocoon morphology similar to model 0 and a slightly curved beam. Model S features a narrower cocoon and a beam of a conical shape, widening as it propagates outwards. Similarly to model 0, model P hosts a large, very anisotropic downflow near the equator transporting matter from a radius of $r \approx 10000\, \text{km}$ to under 1000 km. Like in model 0, the downflow is interspersed with matter with positive radial velocities. However, unlike in that model, an $m = 1$ mode is less prominent and restricted to a smaller region around the PNS. In model S, we find only small clumps of gas falling towards the PNS rather than a large-scale downflows. Both the equatorial expansion of the bow shock driven by the jets and the injection of part of the downflowing mass into the polar outflows act as a feedback mechanism on the mass accretion rate onto the PNS. Due to this feedback, the mass accretion rate is reduced and, consistently, models P and S display smaller PNS masses than models 0 and 0 (Fig.4).

Both outflows are highly magnetised at their base. In the case of model P, $B^{-1} \approx 20$ over the polar caps of the PNS where the gas is accelerated. It drops, however, with radius. The magnetisation is strongest at the outer edge of the beam, where a sheath-like region with $B^{-1}$ larger than unity extends up to $z \approx 20000\, \text{km}$. Beyond that point, this condition can be fulfilled in clumps propagating outwards in the jet. This geometry is similar in model S, though in a modified manner owing to its more coherent magnetic field. Near the jet base, the magnetisation exceeds $B^{-1} \gtrsim 200$, and the continuous sheath of super-equipartition magnetic fields ($B^{-1} > 1$) dissolves into smaller clumps at radii $r > 10000\, \text{km}$.

The jets contain very hot ejecta with entropies around $s \sim 200\, \text{k}\text{g}$ in the beam of model P. The jet of model S is very inhomogeneous with $s \gtrsim 100\, \text{k}\text{g}$ in large parts of it, even reaching values $s \gtrsim 500\, \text{k}\text{g}$ occasionally. Model S ejects predominantly matter with $Y_e \approx 0.5$ (although close to the flanks of the jets matter with $0.4 < Y_e < 0.45$ is also launched; blue and green shades in Fig.19b)), whereas model P produces a significant amount of neutron-rich ejecta with an electron fraction down to $Y_e \approx 0.33$. Like in model 0, this component is distributed in several large bub-
bles at mid latitudes around the beam of the jets where, typically, \( Y_e \approx 0.5 \).

The PNSs of the two models is prolate, albeit less than in model O at late times due to their lower rotational energies (Fig. 14 (b), (d)) and Fig. 15 (b), (d)). The polar radii continuously contract to \( R_{\text{PNS, pol}} \approx 25 \text{km} \) over the course of the simulation. This process is accompanied by shrinking equatorial radii. The PNSs possess a moderate pole-to-equator axis ratio of up to \( 1 : 1.6 \) (P) and \( 1 : 2.5 \) (S), but decreasing towards the end of the simulation. The quadrupole coefficient achieves peak values around...
Figure 16. Same as Fig.11 but for model P at, from left to right, \( t_{pb} = 0.11, 0.15, 0.19, 0.23 \) s.

\[ a_{sh}^{\text{P}}, 0 \approx -0.06 a_{sh}^{\text{S}}, 0 \] (P) and \( a_{sh}^{\text{P}}, 0 \approx -0.13 a_{sh}^{\text{S}}, 0 \) (S). Both measures of the asymmetry decrease during the last few hundred ms of the simulation (more pronouncedly in model S) as the contraction slows down along the rotational axis while continuing at a higher rate in the equatorial region. We note that the rotational axes of the PNSs remain aligned with the original axis and do not change in a similar way as in model W.

Both PNSs are strongly magnetized with maximum field strengths of up to \( B_{\text{max}} \leq 10^{16} \) G. The PNS is threaded by field lines wound up around the rotation axis with a much stronger toroidal than poloidal component (see Fig.20). The poloidal field is most notable close to the axis and in the regions above the polar caps of the PNS from which the jets are launched. Both the dominance of the toroidal over the poloidal magnetic field and the reinforcement of the poloidal component close to the axis are in qualitative agreement with the axisymmetric results of Paper II. From the polar caps, a helical field extends in the surrounding gas. In both models, the magnetic field makes a strong contribution to the transport of angular momentum, in particular via the helical components in the polar regions.

### 3.5 Jet stability

As described above, three of our models develop magnetically driven jets. While they differ in important properties such as the time of explosion, the propagation speed, or the magnetization, a common feature is their stability. Unlike the MHD jets found in the models of Mösta et al. (2014) and Kuroda et al. (2020), once generated our jets propagate outwards at high speeds without being disrupted by non-axisymmetric instabilities. While we could not follow our models until the jets have entered the outer layers of the stars or until break-out from the stellar surface, it stands to reason that our models would produce much more asymmetric and polar explosions than the ones found by these authors, for which the jets are quenched after a comparably short distance and a more roundish explosion ensues. The following subsection is dedicated to an inquiry into this difference.

We first point out several of the many physical and numerical differences between our simulations and the others. Among them, the most important may be the following:

- numerical grid: our simulations were performed on spherical grids, whereas the other authors used Cartesian coordinates and an adaptive mesh refinement;
- as a consequence, it is difficult to compare the grid resolutions;
- the simulations of both Mösta et al. (2014) and Kuroda et al. (2020) were run in full general relativity rather than using an approximate GR potential in special relativistic simulations as in our case (however the instabilities found in the jets by other authors happen in a range of radii where the GR effects are small);
- the three works employ three different approaches to neutrino transport: Mösta et al. (2014) used a leakage scheme, whereas Kuroda et al. (2020) and we performed the simulations with a two-moment scheme, though with differences to ours at the level of both neutrino-matter interactions and energy-coupling terms depending on velocity and gravity;
- all studies started from different pre-collapse models, in terms
Figure 17. Same as Fig. 11 but for model S at, from left to right, $t_{pb} = 0.01, 0.03, 0.05, 0.07$ s.

Figure 18. Same as Fig. 7 but for model P at $t_{pb} = 1.45$ s.

Figure 19. Same as Fig. 7 but for model S at $t_{pb} = 1.05$ s.
of the progenitor masses (25 $M_\odot$, 20 $M_\odot$, and 35 $M_\odot$) for Mösta et al. (2014), Kuroda et al. (2020), and this work, respectively) and of the initial rotational profiles (artificially imposed, parametrized profiles for the other two works and rotational profiles taken from the stellar evolution model here) and magnetic fields (artificially added fields or ones based on the progenitor models).

While strictly speaking the shorter simulation times of the other studies limit a comparison to the first phases after bounce, we will extend our analysis to the later phases during which the jets propagate to many 1000 km. Following Mösta et al. (2014), Kuroda et al. (2020), we compute, after mapping to Cartesian coordinates $x$, $y$, $z$, the position of the barycentre of the magnetic pressure as a function of vertical coordinate, $z$, and time, $t$:

\[
\xi_b = \left( \int_{\mathcal{A}(z)} dA \xi b^2 \right) \left( \int_{\mathcal{A}(z)} dA b^2 \right)^{-1},
\]

where $\xi$ stands for $x$ and $y$ and the integration surface $\mathcal{A}(z)$ limits the analysis to a region around the axis defined by the relation $\varpi < \max(|z|/5, 50 \text{ km})$. We further set $\sigma_b = \sqrt{\xi_b^2 + \xi_b^2}$. Mösta et al. (2014) found an exponential growth of the displacement of the barycentre from its initial position on the rotational axis at heights below 100 km already during the first 20 ms. Kuroda et al. (2020) found an increase of $\sigma_b$ in a similar range of times and positions. This amplification was attributed to kink-mode instabilities growing in the outflow on timescales of the order of 1 ms and vertical lengthscales of few km.

We first summarize important results regarding non-axisymmetric instabilities of the outflows in our simulations. We show the structure of the magnetic field of these three models at late times in Fig. 21. In a striking difference to the aforementioned studies, the jets launched by models 0, P, and S close to the PNS propagate over large distances without being disrupted by strong instabilities. In particular for the strongest fields of P and S, the jet beam coincides with a column of helical field roughly aligned with the rotational axis. The magnitude of deviations of these columns from the axis is anti-correlated with the field strength with model O and S showing the largest and smallest displacements, respectively.

As mentioned above, the analyses of Mösta et al. (2014) and Kuroda et al. (2020) concentrate on the immediate post-bounce phase ($t \leq 30 \text{ ms}$) and low radii ($r \leq 74 \text{ km}$), i.e. a regime in which the outflow has not yet fully developed. Hence, they are dealing with the instabilities of the magnetic field inside the PNS affecting the evolution after the outflows have left the PNS. We will start our look at the dynamics in a similar regime. For a quantitative evaluation, we point to Fig. 22 displaying the time evolution of $\sigma_b$ for all three models with jets during the first 0.1 s and at various heights up to $z \approx 156 \text{ km}$. During this period, the displacement of the barycentre grows on timescales of few ms to values of $\sigma_b \gtrsim 1 \text{ km}$.

The rapid increase sets in after the shock has passed a given location, i.e. almost immediately after bounce for all lines shown in Fig. 22 except for the yellow ones ($z \approx 156 \text{ km}$) for which the growth is delayed as the shock takes several tens of ms to reach this height. We note that the red and yellow lines, respectively corresponding to $z \approx 95 \text{ km}$ and $156 \text{ km}$, are already beyond the analysis heights of Mösta et al. (2014) and Kuroda et al. (2020) and, most importantly, outside the PNS. For the former analysis heights, the growth of the barycentre deviation happens on scales of $\sim (30 - 50) \text{ ms}$, not on scales of $\sim 1 \text{ ms}$ as stated by Mösta et al.

Model 0 shows the non-magnetically driven $m = 1$ shock deformations also found in the least magnetised model $W$, albeit at a lower amplitude. Disentangling the effects of this non-magnetic instability from similar $m = 1$ modes originated by magnetic effects on the evolution of $\sigma_b$ is difficult during this phase. The increase is similar, though slightly slower, for model P. Model S, which launches a prompt jet-like explosion, presents a very rapid increase of $\sigma_b$ behind the shock front, which nevertheless levels off at values $\sigma_b \sim$ few kilometres (similarly to Models 0 and P). The amplitude as well as the growth times and the location at which the non-axisymmetric modes grow are similar to the simulations of Mösta et al. (2014) and Kuroda et al. (2020).

In contrast to the results of Mösta et al. (2014) and Kuroda et al. (2020), the early development of non-axisymmetric modes does not quench the jets. Indeed, our results qualitatively coincide with the ones of Bromberg & Tchekhovskoy (2016), who find that

\footnote{Here and in the following, results are presented for the northern hemisphere. We note that they equally apply to the southern hemisphere.}
relativistic magnetized jets propagating in collapsar progenitors are relatively immune to global kink modes and, hence, able to maintain their stability well beyond the breakout through the stellar surface. Nevertheless, all jets show moderate deviations from axisymmetry as they propagate through the star (as predicted by the analytic work of [1]). For the evolution over a wider range of times and heights, we refer to Fig. [23] (top panels). The importance of non-axisymmetric modes, expressed in terms of the relative displacement $\eta_{\phi}/|z|$, correlates inversely with the power of the jets:

- In the case of model 0, the jets experience strong corrugation of their shape with $\eta_{\phi}/|z|$ peaks at similar magnitudes, model P is characterised by lower values at the base of the jet at heights between the region where the jet is accelerated outside the PNS at several tens of km and about 200 km as well as near the jet head.
- This tendency of weaker deformations continues to the most energetic jets in model S, for which the deviation only occasionally reaches the levels of the other models.

The estimates for the time and wavelength of the fastest growing kinks in model [24] are based on the approximations obtained for cylindrical jets (e.g. [25]). However, the jets in our models develop a narrow angle conical shape. The growth of kink instabilities in conically expanding jets was investigated by, e.g., [26]. Despite several differences, their setup and ours are sufficiently similar for their results to guide our analysis. Following them, we define the magnetic pitch as (note the extra factor 2$\pi$ with respect to commonly used definitions in the literature, e.g. [27])

$$H = 2\pi\sigma\left|b'/b\right|$$

(5)

(note that $b'$ is the radial component of the magnetic field in spherical coordinates; in our simulations $b' \approx b\beta$) and the Alfvén crossing time as the time taken by an Alfvén wave to orbit the jet axis,

$$\tau_c = \frac{2\pi\sigma c^\phi}{\Lambda \Sigma \gamma_{\text{adv}}},$$

(6)

The second term in the denominator of Eq. (6) accounts for the potential widening of a conical jet, which increases the travel time of an Alfvén wave along the field lines towards higher radii. We note that the estimation of Eq. (6) is non-relativistic, but sufficient for our jets, which are only mildly relativistic and, hence, the Lorentz factor that should multiply the former expression (e.g. [28]) is approximately one and has been dropped for the sake of simplicity. The Alfvén crossing time is finite only for

$$\tau_{\text{c}}^\phi > 2\pi.$$  

(7)

It roughly sets the timescale on which kink modes grow and, thus, $\tau_{\text{kink}} \sim \tau_c$. As noted by [29], the condition in (7) can be regarded as a criterion for instability, which may be modified by additional effects such as the differential rotation of the jet, and the magnetic shear created by it, may further suppress the growth of kink modes. This shear can be quantified in terms of $\alpha = \frac{d\ln b/\tau}{d\ln z}$. According to [30], instability of a mode with vertical wave number $k$ may set in if $\alpha > \frac{1}{2}(m+k\pi)^2 - 1$, with $m = \pm 1$. Irrespective of $\kappa$, $\alpha > -1$ is an absolute minimum for the instability.

We note, furthermore, that heuristic arguments and numerical evidence shows that the full development of kink modes, such that the jet is significantly deformed requires $(5-10)\tau_{\text{kink}}$ (e.g. [31]). If Eq. (7) is fulfilled, kink modes grow on length scales $\lambda_{\text{kink}} > H$. Otherwise, the jet expands too fast for an Alfvén wave to orbit its centre and, thus, for kink modes to grow. Taking into account the vertical propagation of matter in the jet, one may derive the following additional criterion of instability (Bromberg & Tchekhovskoy 2016)

$$\Lambda := \frac{\Sigma \tau_{\text{kink}}}{t_{\text{dyn}}} \geq 1,$$

(8)

where $\Sigma = 5\ldots 10$ is a numerical parameter and $t_{\text{dyn}} = z/v^\gamma$ is the dynamical timescale for the expansion of the jet. In general, this criterion will be more restrictive than the one stated by Eq. (7).

In our collimated outflows neither the velocity inside of the jet is ultrarelativistic, nor the magnetic field dominates the dynamics so that they would be force-free. Hence, the whole jet is causally connected in the direction perpendicular to its axis and the causal restrictions for the growth of kink modes (e.g. [32]) are not relevant in our analysis.

The middle and bottom subpanels of Fig. [23] display the value of $\Lambda$ in regions where it is below unity, i.e., where an instability is possible, and the magnetic shear parameter $\alpha$, respectively. During the first few tens of ms after bounce and at radii inside the PNS, Alfvén crossing times are around $\tau_c \sim 1$ ms and magnetic pitches are in the range of $H \approx O(1)$ km. Furthermore, $\Lambda < 1$ and $\alpha > 0$ are indicative of an instability. These values are consistent with the observed growth we find in this phase of the three models. However,
of propagation, but not to a point where the perturbations would disrupt the jets. In the transition region where the barycentre displacement increases, we find short Alfvén crossing times corresponding to $\tau_b^{-1} > 100 \, \text{s}^{-1}$ and long pitches, $H \sim 300\ldots1000 \, \text{km}$, indicating the possibility of range of growth rates and unstable modes. The data for $A$ and $a$ are rather noisy, but show a tendency towards a growing stability against kink modes as the jet progresses through the star in model P. The magnetic shear confines the unstable region to the immediate vicinity of the $z$-axis offering a possible explanation for the limited growth of kink modes in this model.

Model S shows a similar picture, though more extreme than in P. The jets are subject to very minor deviations from axisymmetry. Potential regions of fast amplification of kink modes can be found at their base with Alfvén crossing times in the range of milliseconds and a wide range of magnetic pitches. The jets are faster than in any of the other models, which, together with magnetic shear expressed in the low values of $a < 0$, with may suppress the instabilities.

To summarize, we find similarities to the works by Mäntylä et al. (2014) and Kuroda et al. (2020) in the growth of $\sigma_b$ during the early phases of the explosion, but also a very different evolution thereafter. Though this phase leads to similar amplitudes of $\sigma_b$, the jets are not quenched, but, in cases of delayed as well as prompt explosions, manage to break out off the inner core to then propagate over a long distance with only a minor to moderate influence of non-axisymmetric modes. We note that our results are at least qualitatively consistent with an analysis of the growth of kink modes following Moll et al. (2008) and Bromberg & Tchekhovskoy (2016). The fact that the nascent jets are able to survive relatively strong deformations is most apparent for model 0, in which they are generated at a relatively late time and in the interior of a stalled shock wave which itself is, even in the absence of any kink modes, dominated by strong deviations from axisymmetry. Although the exact conditions for such modes, created externally or growing in the jet itself, to destroy the highly collimated outflow have to be explored further, we can put forward a tentative explanation. We attribute the strong stability of the jets outside the PNSs mostly to a stabilising profile of the toroidal magnetic field with cylindrical radius with low and, in large regions, negative values of $a$. Further stabilisation may be provided by the fast propagation of the jets, leading to a high ratio between potential kink timescales and dynamic times.

4 SUMMARY AND CONCLUSIONS

We continued our previous investigations of the magnetorotational core collapse of massive stars by performing a series of three-dimensional simulations of possible progenitors of GRBs [Obergaulinger & Aloy (2017), Paper I; Paper II]. As initial model, we chose a star of zero-age main-sequence mass $M_{\text{zams}} = 35 M_\odot$ evolved until the onset of core collapse in spherical symmetry including a model of the magnetic fields and rotation [Woosley & Heger (2006)]. To address the uncertainty of the geometry and the strength of the magnetic field owing to the approximate nature of this model, we computed four versions of the same progenitor star with different field configurations. One of them is based on the stellar-evolution model and possesses a rather strong magnetic field with a strength of up to $b_{\text{max}} \approx 1.3 \times 10^{12} \, \text{G}$ limited to the convectively stable layers of the star (model 0). In another simulation, model P, we explored the effect of an artificial increase of the poloidal component, energetically subdominant in model 0, by a global factor of 3. The remaining models are set up with a large-scale dipolar field geometry normalized to two different central values, viz.

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**Figure 22.** Time evolution of $\sigma_b$ for (from top to bottom) models 0, P, S at various heights in the northern hemisphere as indicated in the legend.
Our simulations were run with our numerical code combining special relativistic MHD with a spectral two-moment neutrino transport and including the relevant reactions between neutrinos and matter.

Our axisymmetric models confirmed the development of highly energetic, strongly bipolar explosions driven by a combination of neutrino heating and magnetic extraction of rotational energy from the core as well as showing paths towards the formation of GRB progenitors driven by proto-magnetars or collapsars. The goal of our present study is to scrutinize these possibilities in full three-dimensional geometry. In this effort, we complement previous work along similar lines done by Scheidegger et al. (2010), Winteler et al. (2012), Mösta et al. (2014, 2018), Kuroda et al. (2020) using a variety of physical approximations and numerical methods. Among issues of the explosion mechanisms, a main question emerging from these studies pertains to the development of non-axisymmetric instabilities perturbing the polar outflows and potentially disrupting them before they manage to break out of the core.

We ran our simulations for a comparatively long times of between 0.8 and 1.5 s after bounce, which extends into a relatively long phase after the four models develop an explosion. Depending on the initial magnetic field, several evolutionary paths are possible.

The relatively weakly magnetized model \( W \) produces shock revival within about 250 ms after bounce due to neutrino heating. Before the explosion, the shock wave gradually expands at low latitudes and experiences a strong \( m = 1 \) spiral deformation, which is also visible in the pattern of efficient neutrino heating and of a favourable ratio of the timescales of advection through the gain layer and neutrino heating. This explosion mechanism leads to a moderately oblate geometry of the shock wave as it propagates outwards. Behind it, large bubbles of hot gas expand in a stochastic geometry, in contrast to the polar explosion of the axisymmetric version of the model. At late times, however, when the PNS has acquired a sufficiently strong magnetic field, a very hot outflow of moderate magnetization emerges from a polar region at the PNS surface and starts to catch up with the more spherical shock wave. This outflow is highly variable and changes its direction, sharing qualitatively some of the properties of the jittering jets model (Papish & Soker, 2011). Whether this is a generic feature of mildly magnetised pre-SN cores requires further exploration with a larger grid of models with different masses and rotational properties.

Compared to model \( W \), the stronger fields of \( 0 \) compensate for a less important neutrino heating such that the explosion time is very similar. The magnetic contribution to the explosion mechanism favours a bipolar rather than equatorial or spherical explosion geometry. The model develops a pair of collimated, fast outflows along the rotational axis that reach a radius of \( r = 10^3 \) km within 0.5 s after they have been launched. By the end of the simulations, the diagnostic explosion energies of the two models level off at comparable values around \( E_{\text{ej}} \approx 5 \times 10^{50} \) erg, i.e., the PNSs of both models retain high rotational energies of more than \( 1.5 \times 10^{52} \) erg by the end of the simulations. If subsequently released by magnetic braking on longer timescales, this energy reservoir would be sufficient to power a hypernova-like explosion. The simulations offer an indication of such a possibility in the maximum of \( T_{\text{fres}} \) reached after more than half a second and the subsequent decline.

The two models with the strongest fields, \( P \) and \( S \), explode due to magnetic fields and rotation alone. They show only a short (P) epoch of shock stagnation or none at all (S). Around the axis of the magnetic field, which is identical to the rotational axis, the Alfvén waves pass faster through the gain layer than fluid elements falling towards the PNS. The corresponding strong magnetic field accelerates the gas along the axis. The resulting jets propagate very rapidly, reaching distances of \( r \approx 10^4 \) km within 0.4 s. The explosions energies are in excess of the canonical value of \( 10^{51} \) erg with a value of \( E_{\text{ej}} \approx 2 \times 10^{51} \) erg for model \( P \) and \( S \) exceeding \( E_{\text{ej}} > 10^{52} \) erg without having converged to a final value by the end.
of the simulation. These two models have rotational energies in the PNS significantly smaller than models $W$ and $0$, though still above $10^{52}$ erg in model $P$, and $\sim 4 \times 10^{51}$ erg in model $S$. Hence, also in model $P$, the prospects of a very energetic SN explosion are large.

Mass accretion onto the PNSs ceases in all models within at most 0.7 s after bounce, which is not sufficient to increase their masses beyond the limit for BH formation. The ordering of final masses is the inverse of the magnetic field with values between $M^{\text{pns}}_{\text{final}} \approx 2.16 \times 10^{50}$ for model $W$ and $M^{\text{pns}}_{\text{final}} \approx 1.75 \times 10^{50}$ for model $S$. The strong MHD explosions of models $P$ and $S$ quench mass accretion most effectively and the PNSs start gradually losing mass. All PNSs possess high rotational and magnetic energies. In models $P$ and $W$, the presence of strong fields, in particular their poloidal components, cause the PNSs to rotate slower than in the other two models. Model $S$ experiences a pronounced spin-down in parallel to the mass loss of the PNSs and the prolonged increase of the explosion energy. The work done by the field leads to a decrease of the magnetic energy, too. The less magnetically dominated PNSs of models $0$ and $W$, on the other hand, maintain or even increase the magnetic energies. At the end of the simulations, the surface-averaged fields of the PNS, ranging between $\sim 7 \times 10^{13}$ G for the poloidal and toroidal components of model $P$ to $\sim 4.2 \times 10^{14}$ G for the toroidal component of $W$, are in the range of magnetar fields. The poloidal and toroidal components tend to be of similar magnitude.

The strong rotation flattens the shapes of the PNSs to a strong degree. In model $W$, we observe highly asymmetric downflows impinging on the PNS that slowly tilt the orientation of its rotation axis. This effect results in a complex topology of the magnetic field characterised by loops aligned along different directions. This geometry differs strongly from that of the other models where the combination of poloidal and toroidal components follows roughly the pattern observed in axisymmetric models.

We summarize elements our results have in common with our axisymmetric models and the simulations of other authors and where they differ from them. The times of the explosion and the mechanisms by which they are initiated are similar to the axisymmetric versions of the models. The neutrino-driven explosion of model $W$, on the other hand, with its strong $m = 1$ mode and the predominantly equatorial shock revival differs from the bipolar explosion in 2D, as does the tilting rotational axis of the PNS. The explosion geometry of the other models is much closer to the axisymmetric models. The explosion energies grow to lower values than in axisymmetry and tend to stabilise within the time simulated here. The exception to the latter behaviour is model $S$ with an ongoing rise of the explosion energy, albeit slower than in 2D. Since at the same time the PNS loses mass and rotational energy, this evolution resembles that of the proto-magnetar cases we had found in axisymmetry. Unlike in 2D, where BH formation is a common outcome, several of the models considered here (in particular models $W$ and $0$), mass accretion stops, at least for the moment, in all 3D models before the PNS reaches a mass sufficient for gravitational instability. Longer simulations would be required to check the possibility of the accretion of fallback material during later epochs.

The explosion mechanisms —rotationally modified neutrino-driven and MHD explosions— as well as the explosion energies agree in general with the results of other groups such as the ones of Takiwaki et al. (2016), Summa et al. (2018) for rotating stars without magnetic fields and of Winteler et al. (2012), Mosta et al. (2014), Kuroda et al. (2020) for magneto-rotational core collapse.

The magnetically driven outflows of models $0$, $P$, and $S$ are not subject to strong non-axisymmetric instabilities. The displacement of the barycentre of the magnetic field in the jets can grow exponentially early on, but the growth is limited and does not lead to a strong perturbation or a disruption of the outflows. On the issue of the disruption of the MHD-driven outflows by non-axisymmetric instabilities, our results are more in line with Winteler et al. (2012), who did not observe such a behaviour, than with Mosta et al. (2014) and Kuroda et al. (2020), whose simulations show strong kink modes. However, the disagreement may not be as large as the dichotomy of failed or successful jets might suggest. We find a growth of non-axisymmetric modes at times and locations similar to the cases presented by Mosta et al. (2014), Kuroda et al. (2020), viz. the innermost few tens of km in the immediate post-bounce phase. However, in our models the jets manage to overcome these perturbations and propagate towards larger radii. After this critical phase, they are subject to only minor influence of kink modes. We find indications for a continuous dependence of the importance of the instabilities on the field strength and, thus, the energetics and speed of the jet with the weaker jets showing stronger distortions than stronger ones. This finding seems to be supported by Mosta et al. (2014) who mention a test simulation with a stronger field than the one in their 3D model showing weaker kink modes.

Though a quantitative comparison is made difficult by the different physical settings, the growth of the kink instabilities is compatible with the analysis and the results for jet propagation of Moll et al. (2008) as well as with the results obtained by Bromberg & Tchekhovskoy (2016) for collapsar jets. According to the findings of Moll et al., the typical length scales and the growth times of the kink modes are given by the magnetic pitch of the helical field, larger for stronger radial field, and the time an Alfvén wave requires for one full revolution along the helical structure. The jet can be stabilised if it accelerates, expands laterally, or if differential rotation generates a strong magnetic shear. During later stages of the evolution, when the jet has propagated beyond several 100 km, the conditions are favourable for the growth of very long (hundreds to thousands of km) modes on short times of several ms. However, all the aforementioned inhibiting effects are also present, reducing the impact of the non-axisymmetric modes. Among them, the morphology of the magnetic field seems to play a prominent role. The toroidal field has only a small positive or even negative gradient with cylindrical radius, which, according to Begelman (1998), may account for a stabilisation of the outflows.

We point out some limitations of our work. Besides a higher grid resolution, a wider scope of initial models, in particular a more realistic magnetic field configuration derived self-consistently from multi-dimensional stellar models, would be highly desirable. Despite these limitations, our results strengthen the case for rapidly rotating and strongly magnetized stars as progenitors of energetic, bipolar CCSNe. Furthermore, the final state of our models, containing PNSs with high rotational energy and strong magnetic fields, as well as the spin-down phase of the strongest magnetised model suggest the possibility of a later transformation into a proto-magnetar-driven GRB. Further exploring this option would require much longer simulation times, which is not feasible using the same numerical methods. Aspects that will be addressed in future research are the production of heavy elements in these models and the multi-messenger observables of gravitational waves and neutrinos.

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DATA AVAILABILITY

Data available on request.

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