Load Shedding Model and Algorithm Based on Minimum Power Outage Lose

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Abstract. This paper combines the power outage loss caused by load shedding with the optimization problem of load shedding during power shortage, and proposes a load shedding model that seeks the minimum power outage loss under the system's security and economic constraints. The model is a kind of optimal power flow. In order to reduce the complexity of solving the model, this paper adopts an improved linear programming algorithm to solve the model. With the help of Jacobian matrix, the objective function and constraints of the system are linearized and the linear optimal solution in the iterative space of control variables can be found. Through multiple iterations the scheme with the minimum power outage loss is obtained that satisfies various constraints while the total power generation cost of the system is minimized. The correctness and superiority of the model and algorithm are verified in the IEEE14-bus system.

1. Introduction
When the power system has generator downtime, line break and other faults, the power grid will encounter emergency situations such as bus voltage is too low or branch power is too high. At this situation, the grid can be restored by cutting off part of the load. But in order to maximize the protection of user electricity supply, load shedding should be minimized under various constraints of the power system.

Based on saddle point bifurcation theory and power flow Jacobian matrix singular value decomposition theory, the worst load growth direction is calculated, and an optimal load shedding model is given. The optimal point of this scheme is to use the smallest load shedding capacity to realize the maximum load margin increase [1]. When the power system fails, the optimal load shedding plan to ensure the stable operation of the system with the minimum cost of load shedding, which effectively improves the load shedding effect and efficiency [2]. [3] presented an emergency load control method, which takes into account the severity of line overheating. The method takes the heating value of the line as a constraint, considers the output cost of the generator, and calculates the optimal load reduction with the purpose of minimizing the economic compensation after the power outage. From the perspective of system transients, the optimal load control strategy is obtained by...
maintaining system transient stability as a constraint condition [4]-[6]. [7] proposed a hierarchical optimization load shedding strategy for the grid. Different load shedding plans are determined according to the characteristics of different voltage levels of power grids, which is conducive to the online application of the project. Under the conditions of system safety and stability, a load shedding scheme with the minimum power outage loss is established, and efficient calculation is realized. However, the economic problem of system and the branch power constraint is not considered [8].

In this paper, the concept of power outage loss is applied to the load shedding optimization problem, and the system power generation cost is considered. Under the various security constraints of the power grid, a load shedding plan is constructed, and then an efficient iterative linear programming is used to solve it. Finally, the load shedding scheme with the minimum loss of power outage is obtained and the total power generation cost of the system is minimized.

2. Power outage loss function

Power outage loss is economic or social loss to users due to power shortage [9], and it is a function of power outage capacity or power outage ratio. During power shortage, the unimportant load is usually cut first to ensure that the overall power outage loss is minimized.

In this paper, the power outage cost is used to measure the loss caused by the power outage to various loads. According to the importance of the load, the bus load is divided into class-I load, class-II load, class-III load and flexible load. Deriving the function relationship between power outage loss and power outage ratio, assuming that the proportion and power outage cost of various loads of typical load bus are as shown in Table 1.

| Load type     | Proportion | Power outage cost ($/MW) |
|---------------|------------|--------------------------|
| class-I load  | 30%        | 500                      |
| class-II load | 40%        | 300                      |
| class-III load| 20%        | 200                      |
| flexible load | 10%        | 100                      |

Set $P_{Li}$ and $\alpha_i$ ($0 \leq \alpha_i \leq 1$) as the active load and power outage ratio of bus $i$ respectively. When it is necessary to cut the load of the bus in an emergency, cutting off the load according to the principle of power outage cost from low to high. The power outage loss of the bus is the sum of the product of the power outage capacity and the power outage cost. The function between the bus outage loss $F_i(\alpha_i)$ and the outage ratio $\alpha_i$ obtained above is a piecewise linear function. The functional relationship between $F_i(\alpha_i)$ and $\alpha_i$ can be obtained by quadratic fitting of the piecewise function.

$$F_i(\alpha_i) = (210.86\alpha_i^2 + 107.97\alpha_i)P_{Li}$$ (1)

The power outage loss function should have the following properties:

1. The function zero-crossing point, there is no loss when there is no power outage.
2. The function is an increasing function. The greater outage ratio, the greater the loss.
3. The function is a concave function. The greater outage ratio, the greater power loss per unit capacity.

The above power outage loss function is obtained using typical load bus data as an example. In application, it should be calculated based on the actual data of each load bus to obtain a more accurate power outage loss function for the load bus.

3. Minimum load shedding model and algorithm

3.1. Minimum load shedding model

For a power grid with $n_b$ buses and $n_l$ branches, setting bus $n_b$ as a balanced bus. The load on the buses adopts constant power model, and the power outage loss is used to measure the impact of load
shedding on the load. It is assumed that the power factor remains unchanged before and after load shedding. In order to ensure that the total power generation cost of the system is better after the load shedding, the cost needs to be added to the objective function. At this time, the objective function of the minimum load shedding problem is derived:

\[
\text{Min} \quad W = \sum_{i=1}^{n_b} F(\alpha_i) + \sum_{i=1}^{n_g} E(G_{i,G})
\]  

(2)

Where \( F(\alpha_i) \) and \( E(P_{i,G}) \) are the power outage loss function and the power generation cost function of bus \( i \) respectively. The power outage loss caused by the shedding of the load is much greater than the power generation cost corresponding to this part of the load. Therefore, the load shedding plan with the minimum power outage loss can be obtained by using this objective function for optimization, while the total power generation cost is minimized.

Constraint conditions of the minimum load shedding model are as follows:

\[
\begin{align*}
\Delta P_i &= G_{i,G} (T) \cos \theta_i + B_{i,G} (T) \sin \theta_i) = 0 \\
\Delta Q_i &= G_{i,G} (T) \sin \theta_i - B_{i,G} (T) \cos \theta_i = 0 \\
|P_i| &\leq P_{i,\text{max}} \quad \text{for} \quad i = 1, 2, \ldots, n_b \\
0 &\leq \alpha_i \leq 1 \quad \text{for} \quad i = 1, 2, \ldots, n_b \\
V_{i,\text{min}} &\leq V_i \leq V_{i,\text{max}} \quad \text{for} \quad i = 1, 2, \ldots, n_b \\
P_{i,G} &\leq P_{i,G} \leq P_{i,G,\text{max}} \quad \text{for} \quad i = 1, 2, \ldots, n_b \\
Q_{i,G} &\leq Q_{i,G} \leq Q_{i,G,\text{max}} \quad \text{for} \quad i = 1, 2, \ldots, n_b
\end{align*}
\]  

(3)

Equation (3) is the constraint of bus power balance. Formula (4) is the branch power constraint. The branch includes transmission lines and transformer branches. Considering that most reactive power in the power grid is compensated locally and the reactive power flow on the transmission network is small, therefore, the branch active power limit is used to replace the branch power limit to simplify the calculation. Formula (5) is the ratio constraint of power outage at load buses. Formula (6) are bus voltage amplitude constraint. Formulas (7) and (8) are active and reactive power output constraint of generators.

3.2. Minimum load shedding algorithm

The minimum load shedding model is essentially an optimal power flow model. The solution of the optimal power flow is an optimization process. The optimal objective function value is found by adjusting the control variables of the system [10]. In this model, the control variables of the system are the bus power outage ratio, generator output power and its voltage, and the state variables are branch active power, the amplitude and phase angle of bus voltage. This paper uses iterative linear programming method to solve the model. First, linearize the nonlinear model to obtain the linear objective function and linear constraint conditions at the initial operating point. Then perform linear programming calculations in the constraint space of each variable. Modify each variable according to the planning results to obtain a new operating point. Repeat the above steps until the objective function reaches a stable optimal solution. The specific process is as follows:

Before linear programming, a set of initial values \( P_i^0, \alpha_i^0, V_i^0, P_{i,G}^0, Q_{i,G}^0 \) need to be given, which satisfy all constraints of the system. Then linearize the objective function at the initial values to obtain the linear objective function:

\[
\min \sum_{i=1}^{n_b} \left[ \frac{dF(\alpha_i)}{d\alpha_i} \Delta \alpha_i + \frac{dE(P_{i,G})}{dP_{i,G}} \Delta P_{i,G} \right]
\]  

(9)
Then linearize the equation constraint (3) with the aid of the system Jacobian matrix:

$$
\begin{bmatrix}
\Delta \theta_i \\
\Delta V_i \\
\Delta P_i \\
\Delta Q_i
\end{bmatrix}
= \begin{bmatrix}
\Delta \alpha_i & 0 & 0 & 0 \\
0 & \Delta \alpha_i & 0 & 0 \\
\Delta \alpha_i P_L & \Delta \alpha_i Q_L & \alpha_i & 0 \\
\Delta \alpha_i P_Q & \Delta \alpha_i Q_Q & 0 & \alpha_i
\end{bmatrix}
\begin{bmatrix}
\Delta \theta_i \\
\Delta V_i \\
\Delta P_i \\
\Delta Q_i
\end{bmatrix}
$$

(10)

Where $\Delta \alpha_i$, $\Delta P_i$, and $\Delta Q_i$ are the change of the control variable during this iteration, $\Delta P_i$ and $\Delta Q_i$ are the unbalanced power of the bus caused by the change of the control variable, $J|_{\delta_i, \gamma_i, \omega_i}$ is the conventional Jacobian matrix at the current operating point, $\Delta \theta_i$ and $\Delta V_i$ are the change of the state variable caused by the change of the control variable.

In order for the algorithm to work normally, the phase angle of the balance bus needs to be set as a constant:

$$
\Delta \theta_{b0} = 0
$$

(11)

For branch active power, load bus outage ratio, bus voltage amplitude, generator output should meet the following constraints:

$$
|P_i^0 + \Delta P_i| \leq P_i^{\text{max}}
$$

(12)

$$
0 \leq \alpha_i^0 + \Delta \alpha_i \leq 1
$$

(13)

$$
V_i^{\text{min}} \leq V_i^0 + \Delta V_i \leq V_i^{\text{max}}
$$

(14)

$$
P_{\text{Gi}}^{\text{min}} \leq P_{\text{Gi}}^0 + \Delta P_{\text{Gi}} \leq P_{\text{Gi}}^{\text{max}}
$$

(15)

$$
Q_{\text{Gi}}^{\text{min}} \leq Q_{\text{Gi}}^0 + \Delta Q_{\text{Gi}} \leq Q_{\text{Gi}}^{\text{max}}
$$

(16)

In Equation (12) $\Delta P_i$ is the change of branch active power and its calculation formula is:

$$
\Delta P_i = \frac{\partial P_i}{\partial \theta_i} \Delta \theta_i + \frac{\partial P_i}{\partial V_i} \Delta V_i + \frac{\partial P_i}{\partial \delta_i} \Delta \delta_i + \frac{\partial P_i}{\partial \gamma_i} \Delta \gamma_i
$$

(17)

In equation (17) $\partial P_i / \partial \theta_i$, $\partial P_i / \partial V_i$, $\partial P_i / \partial \delta_i$, $\partial P_i / \partial \gamma_i$ can be obtained indirectly by equation (18).

$$
P_i = -G_i V_i^2 + V_i V_i (G_i \cos \theta_i + B_i \sin \theta_i)
$$

(18)

In order to make the linearization more accurate, it is necessary to limit the range of control variables:

$$
-s_1 \leq \Delta \alpha_i \leq s_1
$$

(19)

$$
-s_2 \leq \Delta P_{\text{Gi}} \leq s_2
$$

(20)

$$
-s_3 \leq \Delta Q_{\text{Gi}} \leq s_3
$$

(21)

Where $s_1$, $s_2$, and $s_3$ are the linearization step length of load bus outage ratio, generator active and reactive power output respectively, which are determined by the approximate degree of linearization.

Perform linear programming calculations under the above constraints, and then modify the system variables according to the planning results to obtain new operating points. Repeat the above steps until the objective function reaches a stable optimal solution, the criterion of stable optimal solution is:

$$
|W_{k+1} - W_k| \leq \varepsilon
$$

(22)

Where $k$ is the number of iteration cycles, $\varepsilon$ is iterative convergence error tolerance.

4. Case analysis

Through case analysis of IEEE-14 bus system to illustrate the effectiveness of the proposed model and algorithm. The structure of the system is shown in Figure 1, including 5 machines, 11 loads, and 20 branches. The upper limit of the active power transmission of the branch is shown in Figure 1, and the system load and power generation data are shown in Table 2.

When the power system is operating normally, the generators share the loads according to the proportion of its own maximum power generation to the total power generation of the system, where $P_{\text{G1}} = 104.34$ MW, $P_{\text{G2}} = P_{\text{G3}} = P_{\text{G6}} = P_{\text{G8}} = 44.56$ MW, the power generation cost is 28.06$/MW, and the
system meets all safety constraints. On this basis, set the transmission line L6-11 has a disconnection fault and the power flow is transferred, which causes the active power of the branch L6-13 and L9-10 to exceed the limit, where \( P_{L6-13} = 25.78 \text{MW} \), \( P_{L9-10} = 32.43 \text{MW} \). Need to carry out emergency load shedding control on the system.

\[
\begin{align*}
P_{L6-13} & = 25.78 \text{MW} \\
P_{L9-10} & = 32.43 \text{MW}
\end{align*}
\]

![Figure 1. The structure diagram of IEEE-14 bus system.](image)

Table 2. System load and power generation data.

| Bus | \( P_{Li} \)/MW | \( Q_{Li} \)/MW | \( F(\alpha_i) \)/(S/MW·h) | \( P_{Gi}^{\text{max}} \)/MW | \( P_{Gi}^{\text{min}} \)/MW | \( Q_{Gi}^{\text{max}} \)/MW | \( Q_{Gi}^{\text{min}} \)/MW | \( E(P_{Gi}) \)/(S/MW·h) |
|-----|----------------|----------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 1   | 0.0            | 0.0            | /                           | 180 | 20 | 60 | -40 | 0.0055\( P_{Gi}^2 + 22.61 P_{Gi} + 227 \) |
| 2   | 21.7           | 12.7           | \( (216.57 \alpha_i^2 + 73.56 \alpha_i) P_{Li} \)| 80 | 10 | 40 | -20 | 0.0061\( P_{Gi}^2 + 24.01 P_{Gi} + 154 \) |
| 3   | 94.2           | 19.0           | \( (176.30 \alpha_i^2 + 106.72 \alpha_i) P_{Li} \)| 80 | 10 | 40 | -20 | 0.0072\( P_{Gi}^2 + 26.11 P_{Gi} + 121 \) |
| 4   | 47.8           | -3.9           | \( (155.63 \alpha_i^2 + 82.31 \alpha_i) P_{Li} \)| / | / | / | / | / |
| 5   | 7.6            | 1.6            | \( (242.76 \alpha_i^2 + 59.26 \alpha_i) P_{Li} \)| / | / | / | / | / |
| 6   | 11.2           | 7.5            | \( (209.43 \alpha_i^2 + 51.25 \alpha_i) P_{Li} \)| 80 | 10 | 40 | -20 | 0.0073\( P_{Gi}^2 + 24.96 P_{Gi} + 116 \) |
| 7   | 0.0            | 0.0            | /                           | / | / | / | / | / |
| 8   | 0.0            | 0.0            | /                           | 80 | 10 | 40 | -20 | 0.0078\( P_{Gi}^2 + 31.05 P_{Gi} + 103 \) |
| 9   | 29.5           | 16.6           | \( (210.86 \alpha_i^2 + 107.97 \alpha_i) P_{Li} \)| / | / | / | / | / |
| 10  | 15.0           | 2.1            | \( (270.61 \alpha_i^2 + 21.84 \alpha_i) P_{Li} \)| / | / | / | / | / |
| 11  | 17.0           | 3.2            | \( (250.28 \alpha_i^2 + 95.54 \alpha_i) P_{Li} \)| / | / | / | / | / |
| 12  | 6.1            | 1.6            | \( (160.86 \alpha_i^2 + 110.13 \alpha_i) P_{Li} \)| / | / | / | / | / |
| 13  | 13.5           | 5.8            | \( (184.67 \alpha_i^2 + 122.27 \alpha_i) P_{Li} \)| / | / | / | / | / |
| 14  | 14.9           | 5.0            | \( (249.90 \alpha_i^2 + 37.46 \alpha_i) P_{Li} \)| / | / | / | / | / |

Set \( \alpha_0 = 0.5 \), \( s_1 = 0.03 \), \( s_2 = s_3 = 5 \text{MW} \), \( \epsilon_1 = 10^{-4} \). Using the algorithm proposed in this article to calculate, the optimization results are as follows:

By adjusting the generator output and cutting part of the load on bus 10 and bus 11, the power grid can return to safe operation state. The outage ratio of bus 10 is 43.68% and the outage load is 6.55+j0.92MW. The outage ratio of bus 11 is 33.48% and the outage load is 5.69+j1.07MW. At this time, the constraint that reaching the limit is \( P_{L9-10} = 20.00 \text{MW} \). After optimization, the incremental
costs of load shedding at the two buses are 260.0$/$MW and 264.6$/$MW respectively. The reason why the power outage ratio of bus 10 is relatively large is that the less important load accounts for a larger proportion. In addition, the incremental cost of load shedding for bus 10 and bus 11 is close, and bus 10 is slightly larger than bus 11, which meet the power flow characteristics of the power grid. After optimization, $P_{G1}=140.88$MW, $P_{G2}=73.07$MW, $P_{G3}=21.93$MW, $P_{G6}=28.00$MW, $P_{G8}=10.00$MW, and the power generation cost is 27.01$/$MW. The more economical generator produces more power. The costs of the system have decreased compared to the generation scheme before optimization. Scheme with the minimum power outage loss has obtained that satisfies various constraints while the total power generation costs of the system is minimized.

5. Conclusions
This paper proposes a load shedding model and algorithm in an emergency situation. This model adjusts the generator output and reduces the bus load under the constraints of the bus voltage, branch power and generator output power to restore the system to normal operation, thereby minimizing the loss of power outage and the total power generation cost of the system. An improved linear programming algorithm is used to solve the model. The algorithm linearizes the objective function and constraint conditions continuously, thereby transforming the nonlinear problem into multiple linear programming problems. After multiple iterations, the stable optimal solution can be found. The algorithm reduces the computational complexity and improves computational efficiency. The analysis of the calculation results of the case also shows the effectiveness of the model and algorithm.

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