Numerical investigation of the long-term work of arches under material creep conditions

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Abstract. The numerical analysis results of arched constructions are considered taking into account creep of the material. A typical building material substantially exhibiting creep under long-term loading is concrete. As a basic model of concrete creep, a model of viscoelastic material was selected. The model was verified and calibrated on the experimental studies basis of concrete creep and in accordance with the requirements of regulatory documents. Numerical modeling of arches was carried out in the software environment of SIMULIA Abaqus.

1. Introduction

In the process of material creep, it changes, not only according to its physical properties. This changes the structure configuration, which can lead to a fundamental change in its constructive scheme, a substantial change in the stress-strain state and to destruction. Resonant examples of reinforced concrete structures accidents that resulted from a change in the constructive scheme of the structure during a prolonged period of operation, which led to a critical change in its stress-strain state, were the destruction the Koror arched concrete bridge spanning 240.79 meters " at the age" of 18 years in 1996 in Palau [1] and of the reinforced concrete shell covering sports and entertainment complex "Transvaal Park" with the sector shape of the radius of 74.87 meters in 2004, which stood 2 years in Moscow [2].

In the process of structures calculating, it is important to select the optimal verified calculation instrument, assigning an adequate calculation model, including a rheological model of the material, which is well calibrated both in accordance with regulatory requirements and with the results of experimental studies. Such a model as a result will describe the behavior of structures during a long service life under different characteristic conditions of changing external influences.

The numerical studies results of the arch structures work are below. As a rheological model of concrete, the generalized viscoelastic Maxwell model was chosen [3]. The stress relaxation function participating in it is modeled in the SIMULIA Abaqus program environment [4] by the exponential terms of the Prony series [5]. The model was calibrated and verified in accordance with the regulatory...
requirements and on the results of experimental studies [5, 8, 11]. At the same time, the influence on the constructions stressed-deformed state of their fastenings compliance, the relative height of the arches, and the ambient relative humidity was investigated.

2. Influence of the arch structures shape and of their supports compliance on the stress-strain state

To analyze the effect of the shape and conditions of supporting arch structures on their stressed-deformed state, the work of the arches structure was investigated by a span of \( L = 10 \text{[m]} \) with different height \( f_i \) (\( i = 1, 2, 3 \)): \( f_1 = L/4, f_2 = L/6 \) and \( f_3 = L/8 \) (Figure 1).

![Figure 1. The non hinged variable-pitch \((f_i)\) arches and with variable rotary stiffness of the supports \(S_i\).](image)

The axis of the arches was described by a curve:

\[
y(x)=4f_i\left(\frac{x}{L}\right)^2.
\]

(1)

Arches with a constant along the length rectangular cross-section of width 0.30[m] and a height of 0.50[m] are loaded in the keystone section by a vertical concentrated force \( P = 100\text{kN} \). The work of these arches was considered with varying rotational compliance of the plane arch supports. Modeling of different supports rotatable compliance was carried out by assigning the parameter "D44" [4] corresponding to the support rotational stiffness the \( k_i \) [9]. The stiffness of the supports was assigned by the dimensionless parameter \( S_i \) (2), changing its value from 0 (hinged support) to 1000 (rigid fastening):

\[
S_i = \frac{k_i L}{EJ}; \quad (i = 1, 2).
\]

(2)

where \( k_i \) is the rotational stiffness of the \( i \)-th support, \( E \) is the modulus of the arch material elasticity, and \( J \) is the inertia moment of the arch cross section

Modeling of arches creep material taking into account their relative height, compliance of supports, the ambient humidity based on calculation of creep coefficient \( \phi(t,t_0) \), performed in accordance with regulatory requirements [6]. The creep coefficient \( \phi(t,t_0) \) [10, 7] is a generalizing indicator of the rheological response of the material to the change in its properties over time, the change in the geometric parameters of structural elements, as well as characteristics of the environment where the structure is located. The values of the creep coefficient were used in the calibration of the viscoelastic material numerical model in the form of the Prony series, namely, in calculating the dimensionless
relaxation moduli of the material [5]: the shear relaxation modulus \( g_r(t) \) (3) and the bulk relaxation modulus \( k_r(t) \) (4):

\[
g_r(t) = \frac{1}{1 + \phi(t, t_0)}, \tag{3}
\]

\[
k_r(t) = \frac{1}{1 + \phi(t, t_0)}. \tag{4}
\]

The concrete arches are made of class C20/25, having an average compressive strength \( f_{cm} = 33 \text{ [MPa]} \). In addition, the simulation includes the other material characteristics, i.e., Young’s modulus \( E = 30 \cdot 10^9 \text{ [Pa]} \), Poisson’s ratio \( \nu = 0.2 \), density \( \rho = 2200\text{[kg/m}^3]\). Age of concrete when loading arches \( t_0 = 28 \text{ [days]} \). The creep of arches investigated for \( T = 360 \text{ [days]} \) from its loading moment at relative humidity of the surrounding medium \( RH_1 = 50 \text{ [%]} \) (normal) and \( RH_2 = 85\text{[%]} \) (wet). These characteristics determined the time variation of the material creep coefficient (figure 2), as well as the dimensionless parameters of the adopted numerical model of viscoelastic material (Figure 3). The length of a typical finite element in the calculation model was \( h_0 = 187.5\text{[mm]} \).

The investigation results of the arches relative height influence and compliance of their supports on the change in the values of deflections in the arch keystone section and on the bending moments magnitude in the supports are below.

![Figure 2](image-url). The change in the creep coefficient \( \phi(t, t_0) \) in time \( t \) for two values of the relative humidity of the environment \( RH \).

### 3. Change in the stress-strain state of arch structures under conditions of the material creep

For the numerical analysis of the material creep effect on the behavior of an arch structure in time the work of a shallow arch was investigated. Its axis is described by a curve of the second order

\[
y(x) = -\frac{4f}{L^2} x^2 + \frac{4f}{L} x. \tag{5}
\]

The span of the arch is \( L = 20\text{[m]} \), its height in the central keystone section is \( 2.0\text{[m]} \), supports are hinged, the rectangular cross-section constant in length has a height of \( 0.80\text{[m]} \) and a width of \( 0.30\text{[m]} \). The arch is loaded in the keystone with a concentrated force \( P = 500\text{[kN]} \). Concrete C25/30 is used in the manufacture of the arch. Its modulus of elasticity is \( E = 31\cdot10^9\text{[Pa]} \), Poisson’s ratio \( \nu = 0.2 \), average compressive strength \( f_{cm} = 33\text{[MPa]} \), density \( \rho = 2400\text{[kg/m}^3]\). Ambient humidity \( RH = 85\text{[%]} \), age of material loading \( t_0 = 28\text{[days]} \), arch deformation study period \( t = 516\text{[days]} \). As in
the previous section, the change in the creep coefficient in time was determined according to [6]. Simulation of concrete creep in time was carried out using a numerical model of viscoelastic material with its two key dimensionless parameters: the modulus of shear relaxation $g(t)$ (3) and the bulk relaxation modulus $k(t)$ (4). The length of the calculation model element is $h_0 = 218.2$[mm].

![Graph](image1)

**Figure 3.** Change in the dimensionless relaxation module $k(t)$ and $g(t)$ in time $t$ for two cases of ambient relative humidity $RH$.

Numerical calculation of the arch was carried out according to a deformable scheme with the considered period of its operation (516 days). It was divided into 3 time intervals (figure 4). At the boundaries of the time intervals, due to the accumulated creep displacement taken into account, the real arch configuration (Figure 5) and the dimensionless modules of the material viscoelastic model $g(t)$ and $k(t)$ were corrected (Figure 4).

![Graph](image2)

**Figure 4.** Change in the dimensionless modules $k(t)$ and $g(t)$ in time $t$. 
Figure 5. Change the shape of the arch in three time intervals of its life cycle.

4. Results and conclusions
Numerical study of non-hinged arches with a change behavior in their relative height, compliance of supports at different environment relative humidity and under conditions of the material creep showed a clear effect of these factors on their stress-strain state. Figure 6 shows the change in the deflection of the hinged arch ($S_i = 0$) in its keystone section in time, depending on the height of the arch $f_i$ ($f_i = L/4$ (elevated arch), $f_2 = L/6$ (average) and $f_3 = L/8$ (shallow sloping)) and on the relative humidity of the environment $RH$. So at normal ambient humidity $RH_1 = 50\%$ a decrease in the hinge supported ($S_i=0$) arch height from $f_2 = L/6$ to $f_3 = L/8$ by age 1 year will increase the deflections in its keystone section by 45%. And with a rigid arch support ($S_i = 1000$), this difference is already 70%.

Figure 6. Change in the deflection value $u$ of the articulated arch ($S_i = 0$) in time $t$, depending on the relative height $f_i$ and the relative humidity of the environment $RH$.

Figure 7 illustrates the effect of the supports compliance ($S_i$), as well as of the environment humidity $RH$, on the nature of the deflection development in the arch in its keystone section under creep conditions.

Figure 8 shows the development in time $t$ of the bending moment $M$ in the flexible compliant support ($S_i = 10$) of arches of different relative elevation.
Figure 7. The compliance effect of the $S_i$ supports and the relative humidity of the environment $RH$ on the arch deflection ($f_1 = L/4$) under material creep conditions.

Figure 8. The growth of the bending moment $M$ in time $t$ in the compliant support ($S_i = 10$) of the arches of different elevation $f_i$ under creep conditions of the material.

In the process of shallow arches and shells calculating in time, it is essential to take into account the changes in their configuration due to accumulated creep strains. Figure 9 compares the results of shallow arch (its characteristics are described in Section 3) calculating under conditions of material creep without taking into account the change in its calculation scheme over time and taking into account the change in the arch configuration due to accumulated creep strains. In this case, the arch contour revision was carried out only in three stages of its 16-month life cycle. It can be seen the calculation of a shallow arch by a deformable scheme made it possible to reveal in 500 days after its loading an almost twofold increase in deflections in comparison with the traditional calculation results.
Figure 9. Changing the deflection value of the shallow arch in the keystone section \( u \) in time \( t \) during the normal calculation (the bottom line) and as a result of the calculation according to the deformable scheme (upper lines).

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