Wheel drive with integrated differential

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Abstract. The performance indicators of wheeled arable machine-tractor units, which are accelerated on the working gear, depend on the operating modes of the wheels during this period. When the wheel is skidding, soil lumps break down in the contact spot, the soil structure is destroyed. Based on the system analysis of the wheels operation, the method of their improvement is justified by continuous control of the eccentric point of application of the driving torque and external load. As a result of the analysis for the first time, a soil-sparing wheel mover with the properties of a differential, a tangential force regulator and clearance regulator was developed. In the case of an eccentric application of a vertical load and a longitudinal pushing force, one of the satellites of the wheeled planetary gearbox is the leading and bearing one. The purpose of the article is to analyze the factors influencing the automatic adaptation of the wheel drive to changing operating conditions. It is established the relationship between the driving moment and the rolling resistance moment, the moments of inertia of the wheel and the drive gear of the integrated differential.

1. Introduction

Wheeled mobile energy vehicle (WMEV) for agricultural purposes are equipped with inter-axle and inter-wheel differentials. The smoothness of the torque transmission helps to reduce the dynamic effects on the ground and thereby increase its cross-country ability. An in-depth analysis of the propellers of vehicles designed to work on weak soil or snow is presented in [1]. A quantitative assessment of the compaction of the soil by tires as the load on them increases is given in [2].

Since the losses on the elastic slip of the tire are proportional to the square of the supplied torque, modern agricultural tractors are equipped with multi-stage gearboxes or torque converters that allow maintaining a rational value of the driving torque, tangential force, but this leads to an increase in their cost. Increasing the efficiency of the wheel is facilitated by a decrease in the rolling resistance coefficient, the slip coefficient and an increase in the coefficient of use of the coupling mass. These coefficients depend on the normal load on the wheel, the coupling weight; therefore, their management involves ballasting the agricultural tractors [3]. The latter leads to over-compaction of the soil in the depth of the arable layer and the subsurface horizon. The compaction extends to the subsurface layer up to 40 cm, and on moist soils the degree of compaction is greater [4]. The need for further research on reducing soil compaction by WMEV engines is discussed in [5], by chain-track tractor is discussed in [6], by track-wheel-terrain interaction is discussed in [7].

An urgent scientific task is finding ways to control the structural, operational and technological factors that contribute to the effective operation of the tractor wheel without the use of ballast. In order to improve the performance of the wheel drive, it is proposed to apply the gravity of the tractor and the driving torque to the wheel eccentrically. The position of this point should automatically adapt to...
changes in the terrain of the plowed area, field roads. A wheel with a built-in planetary gearbox with a leading bearing satellite meets these requirements [8]. In the literature available to the authors, the issues of the efficiency of wheel propellers with eccentric application of external load and driving torque were not considered.

The purpose of the study is to analyze the operation of the tractor wheel as a system process, to find ways to reduce soil compaction, to assess the influence of the eccentric application of vertical load and longitudinal pushing force on the wheel propulsion, to establish the relationship between the driving moment and the rolling resistance moment.

2. Methods
The operation of the tractor wheel is considered as a system process. Acceleration of a mobile wheeled power vehicle with wheels with built-in differentials is presented as a two-stage process, during which the wheeled planetary gearbox switches to differential mode, smoothly reducing the amount of driving torque on the wheel rim, preventing the wheel from slipping relative to the bearing surface. The equation of motion of a wheel with a built-in differential is obtained based on the provisions of theoretical mechanics.

3. Results

3.1. Wheel operation as a system process
The main components of the wheel operation as a system process, represented in the form of energy and technological subsystems, are the wheel, the support surface, and the loading mode. The operation of the drive wheel is influenced by structural, operational and technological factors. Structural factors include the parameters and properties of the tire, the ratio of the width of the profile to its height, the design of the tire, its elastic properties.

The operating factors include slipping, the ratio of the input torque and the vertical load, rolling on the surface, relying only on the treadmills of the tire or forming a track. Technological factors are the properties of the support surface, which allow the maximum value of the rolling coefficient to be realized with different probabilities.

The parameters of the wheel as an input element of the system include:
- wheel diameter, which can be represented by structural, kinematic, dynamic, free diameters;
- tire design, the ratio between the width and height of the profile; constant or adjustable tire pressure;
- the rigidity of the wheel, the ability to deform taking into account road conditions and the magnitude of external loads, its main shape: round, incompletely circular, elliptical or otherwise.

The parameters of the support surface as an input element of the system are: deformable or non-deformable surface; volume crumpling coefficient; shear modulus; coefficient of friction; rheological parameters of support surface.

The parameters of the loading mode that reflect the features of the interaction of the wheel and the support surface include: the kinematic mode of operation, the central or eccentric application of the driving torque, the longitudinal pushing force and the vertical load.

The system process ‘wheel operation’ is also characterized by output elements, their parameters and indicators. The estimated performance parameters at the output of the ‘wheel - bearing surface’ system include cross-country ability; ground clearance; transport capacity; relative value of the road obstacle to be overcome, etc. Indicators of the system at the output are the tangential force of the wheel; the driving moment formed by the wheel; the moment of resistance to the movement of the wheel; the efficiency of the wheel. The effectiveness of the feedback of the output indicators of the wheel with its energy and technological subsystems depends, first of all, on its purpose, the point of application of the driving or braking torque and external loads, the moment of rolling resistance. A
promising method of increasing the efficiency of its use is the combination of the functions of a wheel mover and a tillage working body.

The structural methods of controlling the value of the driving torque of the wheel include changing the parameters of the wheel along the perimeter of the rim: the diameter, width and height of the tire profile, the use of incompletely circular wheels.

Operational methods for controlling the output indicators of the wheel operation process include: tire pressure control, control of the size and distribution of the coupling mass, the use of removable ground hooks, the choice of a rational transmission ratio of the WMEV transmission, including the use of an integrated wheel differential.

Structural methods for controlling the output performance of the wheel include controlling the point of application of the external load and the driving or braking torque to the wheel. In ‘classic’ wheels, the point of their application is usually the center of the wheel. In the wheels proposed by the authors with a built-in differential, the gravity of the transported load and the longitudinal pushing force are applied eccentrically with an automatic change in the coordinates of the point of application, and the gravity of the wheel is applied to its center. This is the essence of the proposed concept of improving the wheel drive. It is implemented by turning the satellite of the planetary gearbox into a bearing driving element, the position of which automatically changes during operation, thereby providing the ability to control the rolling coefficient of the wheel with changing properties of the support surface.

3.2. Working process of a wheel with a built-in differential

The force of gravity $Q$ and the longitudinal pushing force $P_T$ are perceived by the gear 2 mounted on the drive bearing shaft 3 of the gearbox-differential 1, which is supported by the bearings 4 of the centering disk 5 (figure 1).

![Figure 1. Schematic diagram of a wheel with a built-in differential: force of gravity ($Q$), longitudinal pushing force ($P_T$), applications of the longitudinal pushing force ($H_1, H_2, H_3$), the ordinates ($L_2$), force of gravity ($L_3$), radius ($r_1$) of the driving gear, radius ($r$) of the centering disk](image-url)
The working process is based on the automatic transition of the planetary gearbox to the differential mode, depending on the moment of resistance experienced by the wheel. The epicyclical gear (wheel rim) remains stationary as long as the rolling resistance of the wheel exceeds the torque coming from the engine through the transmission. Otherwise, the gear 2 begins to rotate along the stationary epicyclical gear and move the point of application of the vertical load and of the horizontal longitudinal pushing force 3 (figure 1) by the value of \( L \) and in the ordinate by the value of \( H \). Thus, the turning torque \( M_1' \) and the lever torque \( M_1'' \) are formed. The wheel rim begins to move when the summaries torque \( M_2 \) (equation (1)) of the turning torque \( M_1' \) the lever torque \( M_1'' \) and the driving moment from the engine \( M_e \) enlarged by transmission and wheel gearbox exceeds the rolling resistance moment of the wheel \( M_f2 \) (figure 2):

\[
M_\Sigma = M_e k_M i_g l_g + M_1'' + M_1' > M_{f2}
\]  

(1)

In (1) \( k_M \) is the engine's torque adaptability coefficient. At the first stage of acceleration the epicyclical gear does not rotate. The gear ratio of the planetary gear is given by \( i_g = \frac{Z_2}{Z_1} \) where \( Z_1 \) and \( Z_2 \) are the number of teeth of the gear 1 and the epicyclical gear 2 (figure 2).

Figure 2. Moving the driving carrier satellite from position I to position II: the tangential force \( (F_x) \), the resultant of the vertical reaction \( (Rz) \), the torque \( (M_1) \) coming from the engine to the driving gear through the transmission, the torque \( (M_2) \) coming from the engine to the wheel through the transmission, the moment of resistance to rolling of the wheel \( (M_{f2}) \), inertia moment of the wheel \( (M_{j2}) \), in its relative rotational motion, angular velocity \( (\omega_2) \) of the wheel, the height \( (H) \) of the tire profile, the speed \( (v) \) of the translational movement of the wheel, the radius \( (r_1) \) described by the center of the driving gear, the current coordinate \( (z_i) \), of the point of application of longitudinal force, radius \( (r_{20}) \) of the driving gear, kinematic radius \( (r_{20}) \) of the wheel.
3.3. Acceleration of the WMEV with a torque transformer on the driving wheels

Acceleration of the WMEV from a standstill with a torque transformer on the driving wheels takes place in two stages. At the first stage the center of mass of the WMEV occurs within the radius of the wheel (figure 1). The energy of the supplied driving torque is spent on the rotation of the driving satellite along the epicyclical gear, which is part of the wheel rim, to ‘lift’ the masses. There is an accumulation of the potential energy of the system. The moment of resistance to the wheel is greater than the driving moment supplied to the epicyclical gear.

The second stage of acceleration begins when the summary torque exceeds the moment of resistance to rolling the wheel. The wheel begins to roll relative to the support surface.

When the WMEV moves, these stages alternate. Their relative duration depends on the characteristics of the WMEV engine, the wheel drive with a built-in torque transformer, and their ability to adapt to changing operating conditions. The change in the coordinates of the center of the drive shaft of the gear (ordinates-within \( H_1...H_3 \) and abscissa-within \( 0...L_2 \)) indicates the self-adaptation of the wheel mover to road conditions and the possibility of automatic changes in the clearance of the WMEV (figure 1).

3.4. Conclusion of the equations of wheel motion

In order to identify the factors that affect the operation of the proposed wheel, to establish the relationship between the driving torque and the rolling resistance moment, the moments of inertia of the drive gear \( J_1 \) and the wheel \( J_2 \), we will make an equation of the wheel movement with an eccentric supply of the driving torque and the external load (figure 2).

At the beginning of acceleration, the wheel differential operates in the planetary gear mode with a gear ratio of \( i_g \). The equation (2) of motion is represented as:

\[
J_1 \frac{d\omega_1}{dt} + J_2 \frac{d\omega_2}{dt} = M e_i T^i_{i_g} - F_x z_i - F_z x_i + M_{f2}
\]

The torque \( M_2 \) coming from the engine to the wheel through the transmission gear ratio \( i_T \) and the planetary gearbox (figure 2) can be expressed as equation (3):

\[
M_2 = M e_i k_M i_T i_g
\]

where \( k_M \) is the engine's torque adaptability coefficient. The inertia moment of the wheel \( M_{j2} \) in its relative rotational motion is expressed as equation (4):

\[
J_2 \frac{d\omega_2}{dt} = M_{f2}
\]

The direction of action is opposite to the vector of angular velocity \( \omega_2 \) and angular acceleration \( \varepsilon_2 \) of the wheel, and \( \omega_2 = \omega_2 / i_g \).

The inertia moment of the driving gear in its relative rotational motion is expressed as equation (5):

\[
J_1 \frac{d\omega_1}{dt} = M_{j1}
\]

The direction of its action is opposite to the vector of angular velocity \( \omega_1 \) and angular acceleration \( \varepsilon_1 \) of the gear, coincides with the vector of angular velocity \( \omega_2 \) and angular acceleration \( \varepsilon_2 \) of the wheel.

The value of the lever torque of the longitudinal force relative to the stationary epicyclical gear (figure 2) is given by the expression (6):
\[ M''_1 = F_x r_1 \]  

(6)

where \( F_x \) is the tangential force, \( r_1 \) is the radius of the driving satellite 3 (figure 1).

Lever moment relative to the instantaneous center of rotation of the wheel in its uniform rotation can be expressed as equation (7)-(8):

\[ M''_1 = F_x z_i \]  

(7)

\[ z_i = r_{20} \left( 1 - \frac{r'_1}{r_{20}} \cos \phi \right) \]  

(8)

where \( z_i \) is the current coordinate of the point of application of longitudinal force (figure 2); \( r_{20} \) – kinematic radius of the wheel; \( r'_1 \) – the radius described by the center of the driving gear.

The turning torque can be expressed as equation (9):

\[ M'_1 = F_2 x_i \]  

(9)

The current value \( x_i \) of the abscissa of the center of the drive gear was defined as equation (10):

\[ x_i = r' \sin \phi \]  

(10)

It changes from zero to \( r'_1 \), and then decreases to zero when the angle changes \( \phi = 0 \ldots 180^0 \). The direction and magnitude of the angular velocity of the center of the drive gear are variable. They depend on the ratio of the rolling resistance of the wheel to the driving torque. The driving torque, in turn, depends on the characteristics of the WMEV engine (3), the current values of the lever torque (6) and the turning torque (9). The moment of resistance to rolling the wheel at the initial moment can be expressed as equation (11):

\[ M_{f_2} = R_x a_2 \]  

(11)

In equation (11) \( R_x \) is the resultant of the vertical reaction of the reference surface; \( a_2 \) - distance of the application of the resultant vertical reaction of the support surface, calculated from the vertical diameter of the wheel in the direction of speed (in figure 2 \( R_x \) and \( a_2 \) are not shown; their action is replaced by the moment of resistance to rolling of the wheel \( M_{f_2} \)).

The value of the moment \( M_{f_2} \) depends on the amount of deformation of the tire and the ground under the oncoming sector of the wheel, the coordinates of the center of pressure, which the vertical reaction of the wheel and the longitudinal component of the tangent reaction of the road is applied. When the abscissa \( x_i \) exceeds a certain value, the planetary gearbox switches to the differential mode with the gear ratio \( i_d \). The rotation of the epicyclical gear begins. The second stage of acceleration of the WMEV begins.

The equation of the wheel motion at this stage can be expressed as equation (12):

\[ J_{\Sigma 1} \frac{d\omega_1}{dt} + J_{\Sigma 2} \frac{d\omega_2}{dt} = M_{x_1} i_d \omega_1 - F_x z_1 - F_2 x_1 + M_{f_2} \]  

(12)

The gear ratio of the differential is given by the expression: \( i_d = (Z_2/Z_1)-1 \).

The total moment of inertia of the wheel \( J_{\Sigma 2} \) when turning relative to the instantaneous center of rotation (ICR) at the time of the beginning of the movement of the epicyclical gear is expressed as equation (13):
\[
J_{\Sigma 2} = \frac{3}{2} m_2 r_2^2
\]  

(13)

The moment of inertia of the drive gear (1) (figure 2) with the applied force of gravity \(Q\) (figure 1) or \(m_G\) (in figure 2 is not shown) relative to the ICR, taking into account its distance from the dividing circle can be expressed as equation (14):

\[
J_{\Sigma 1} = \frac{3}{2} (m^2 + 1) m_G r_1^2
\]  

(14)

The values of the required torque are determined from equations (2) and (12).

At the first stage of overclocking the required torque is given by expression (15):

\[
M_{e1} = \frac{1}{i_1 i_g} \left( J_1 \frac{d\omega_1}{dt} + J_2 \frac{d\omega_2}{dt} + F_x z_1 + F_z x_1 + M_{f2} \right)
\]  

(15)

At the second stage of acceleration the required torque can be expressed as equation (16):

\[
M_{e2} = \frac{1}{i_1 i_d} \left( J_{\Sigma 2} \frac{d\omega_2}{dt} + J_{\Sigma 1} \frac{d\omega_1}{dt} + F_x z_1 + F_z x_1 + M_{f2d} \right)
\]  

(16)

Since in equations (15) and (16) there are the following inequalities \(M_{f2} > M_{f2d}\) and \(i_g > i_d\), then \(M_{e2} > M_{e1}\). Thus, the value of the required drive torque of the WMEV engine increases smoothly.

The tangential force \(F_x = -F_T\) at the first stage is balanced by the circumferential force of the gearing pair ‘drive gear – epicyclical gear’ (figure 2). The forces \(F_z = F_T\) are applied to the center of the leading satellite, their directions are opposite. With uniform movement, the traction force of the wheel can be written as equation (17):

\[
F_T = F_X = \left( M_{e2} - M_{f2d} \right) / r_{20}
\]  

(17)

When WMEV driving in difficult road conditions, short-term modes are possible, in which the value \(F_x = 0\) or its vector changes the sign to the opposite. When \(F_x = 0\) the driving wheel traction force \(F_T\), this mode is called free rolling mode. The supplied energy is spent on overcoming the rolling resistance of the free drive wheel and on its acceleration, and with uniform moment \(M_{e2} = M_{f2d}\). In the second case, the force \(F_x\) becomes pushing, and the mode of operation of the drive wheel is called neutral.

4. Discussion
The slipping of the tire is preceded by a stage of elastic sliding, when there is no shift of soil particles in the contact spot. Slipping or external sliding occurs due to the cut of the soil enclosed between the tread protrusions and the sliding of the tread protrusions on the ground. With an increase in the load on the wheel in the incoming zone of the wheel, as the driving torque on the wheel increases, the soil shifts at some distance from the contact surface. At the same time, the effect of increasing the actual diameter of the wheel due to the thickness of the shifted soil layer (Rankine body) is observed in the contact spot. The transition of the built-in planetary gearbox to differential mode ensures a smooth reduction of the driving torque on the wheel rim; the process will be smooth and does not lead to a sharp shift of the ground. Since the power of elastic slip losses increases according to the quadratic dependence on the value of the realized torque and decreases with increasing kinematic radius, the above contributes to reducing the dynamic effects on the ground and increasing the cross-country ability of the wheeled WMES.
5. Conclusion
As a result of the system analysis of the wheel operation, the concept of improving the wheel by eccentric application of the driving torque and external load is substantiated. A smooth change in the tangential force due to the automatic transition of the built-in planetary gear wheel to differential mode helps to reduce slipping. The time-stretched acceleration of the wheeled arable unit allows equipping the tractor with a lower-power engine, increasing its load factor, and reducing the requirements for the coefficient of adaptability to short-term overloads.

References
[1] Muro T and O’Brien J 2004 *Terramechanics: Land Locomotion Mechanics* (London: Taylor and Francis Group) p 322
[2] Botta G, Rivero D, Tourn M, Bellora Melcon F, Pozzolo O, Nardon G, Balbuena R, Tolon Becerra A, Rosatto H and Stadler S 2008 Soil compaction produced by tractor with radially and cross-ply tires in two tillage regimes. *Soil Till. Res.* **101**(1-2) 44 http://dx.doi.org/10.1016/j.still.2008.06.001
[3] Arvidsson J and Keller T 2007 Soil stress as affect by wheel load and tyre inflation pressure. *Soil. Till. Res.* **96**(1-2) 284 http://dx.doi.org/10.1016/j.still.2007.06.012
[4] Jamali H, Nachimuthu G, Palmer B, Hodgson D, Hundte A, Nunnb C and Braunack M 2021 Soil compaction in a new light: Know the cost of doing nothing – A cotton case study. *Soil. Till. Res.* **213**(4) 105158 https://doi.org/10.1016/j.still.2021.105158
[5] Nguyen V, Matsuo T, Inaba S and Kuomoto T 2008 Experimental analysis of vertical soil reaction and soil stress distribution under off-road tires. *J. Terramechanics.* **45**(1-2) 25 http://dx.doi.org/10.1016/j.jterra.2008.03.005
[6] Ma Z-D and Perkins N 2002 A track-wheel-terrain interaction model for dynamic simulation of tracked vehicles. *Vehicle. Syst. Dyn.* **6** 401 http://dx.doi.org/10.1076/vesd.37.6.401.3522
[7] Mudarisov S, Gainullin I, Gabitov I, Hasanov E and Farhutdinov I 2020 Soil compaction management: reduce soil compaction using a chain-track tractor. *J. Terramechanics* **89** 1 http://dx.doi.org/10.1016/j.jterra.2020.02.002
[8] Ilyin V, Andreev R, Ivanshchikov Yu and Pushkarenko N 2016 Functional role of the turn over device of the driving wheel of MEW in transformation of transmission torque. *Proc. of the Int. Sci. and Practical Conf. Scientific and Educat. Env. as the Basis for the Dev. of the Agro-Industrial Complex and Social Infrastructure of the Village* (October 20-21, Cheboksary: Chuvash State Agricultural Academy) p 766 [In Russian]