Free Convection in a Triangular Cavity Filled with Hybrid-Nanofluid along with Sinusoidal Heat

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Abstract—A numerical study on convective heat transfer of hybrid nanofluid packed in a right angled triangular cavity heated by a sinusoidal temperature maintained from lower side and subjected to a constant magnetic field have been studied in this work. The hypotenuse side of the triangular cavity has been kept in uniform cool temperature while the remaining side is insulated. The governing equations of the problem have been discretized numerically with help of finite element method. A fixed Prandtl number Pr=6.2 has been used for the numerical solution. Several values of Rayleigh number Ra=10^6-10^10, and Hartmann number Ha=0-100 which are the non-dimensional governing parameters have been examined. The volume fraction φ=0.01, 0.05 and 0.1 and the heat generation parameter Q = 1 have been taken for this work. Calculate and the graph of Nusselt number corresponding to different parameters have been presented. The results show that Nusselt number has been decreasing function of nanoparticles Rayleigh number and also it is a decreasing function of Hartmann number. Obtained results has been compared with previously obtained data by other authors.

Index Terms—FEM Model, Free Convection, Hybrid-Nanofluid, MHD Effects.

I. INTRODUCTION

In recent years, convective flow in a cavity with numerous thermal boundary conditions have created an eagerness to several researchers due to its vast applications in industry and environment. Nonuniform temperature distribution keep on the active wall due to shading or other effects in the fields, such as solar energy collection and cooling of electronic components. Therefore, the study on convection heat transfer in cavity with sinusoidal heated wall(s) is important in such situations. For efficient improvement of many thermal systems like micro electronic devices, heat exchangers, refrigerators, solar collectors, cooling of electronic components, nuclear reactors, transformers, and engine/vehicle etc by adopting heat transfer enhancement, one of the most resourceful strategy is using nonofluid/hybrid nanofluid. Nikbakhti and Rahimi [1] studied the double-diffusive natural convection with different heating sections and different parameters in a rectangular cavity with partially heated walls filled with air. Kwak and Hyun [2] studied the natural convection in a cavity with time-varying temperature on the sidewall. For improving heat transfer performance, geometrical arrangement may have the fruitful effect. Saidi et al. [3] presented numerical and experimental results of flow and heat transfer from a sinusoidal cavity. Sarris et al. [4] investigated the natural convection numerically in a rectangular enclosure with a sinusoidal temperature profile on the top wall. There was a clear effect related with Rayleigh number. When the increasing of Rayleigh number, the circulation patterns and their centers are move toward the upper wall corners. The term “nanofluid” was first used by Choi [5] refer to a fluid in which nanoparticles are suspended. Khanaf et al. [6] studied two-dimensional problem of buoyancy-driven heat transfer in a cavity. A nanofluid is a fluid comprehending nanoparticles (less than 100 nm-diameters) dispersed in a regular base liquid (water, ethylene glycol, or oil, etc.). Ho et al. [7] studied natural convection by using three sizes of vertically square cavity filled by water-Al₂O₃ nanofluids. Bilgen and Yedder [8] analyzed the natural convection with sinusoidal temperature profiles in a wall in a rectangular enclosure. They showed that when the heated section is in the lower half of the enclosure in this case the heat transfer is higher. Deng and Chang [9] studied numerically the natural convection with two spatially varying sinusoidal temperature distributions on the left and right sidewalls in a rectangular enclosure. They found that the nonuniform temperature distribution increases heat transfer as compared to the case of uniform wall temperature. Sheikholeslami et al. [10] studied natural convection heat transfer in a cavity with sinusoidal wall filled with CuO-water nanofluid in presence of magnetic field. B. Ghasemi, S.M. Aminossadati [11] studied natural convection of nanoparticles in a triangular enclosure using Brownian motion. Hybrid nanofluid can be found by appending more than one type of nanoparticles with base fluid. Hybrid nanoparticles mixture procedure has been reviewed widely by Sarkar et al. [12] in details. Botha et al. [13] did an experiment of hybrid nanofluid based on silver-silica-oil. Chamikha et al. [14] analyzed natural convection in a semi-circular cavity for unsteady conjugate filled with a hybrid water-based suspension of Al₂O₃ and Cu nanoparticles. Rashad et al [15] studied natural convection in a triangular cavity filled with Cu-Al₂O₃/water hybrid nanofluid with MHD effect. They showed, when the natural convection is very small then results shows significant effect of increasing volume fraction of hybrid nanofluid. The objective of this paper is an effort to contribute in starting the groundwork of this research field. Therefore, this present study focused on triangular cavity heated sinusoidal from below with the presence of magnetic field and internal heat generation with a constant heat flux.

II. PROBLEM FORMULATION

The schematic diagram of the studied configuration is
described in Fig 1. In this study, the steady two dimensional natural convection flow in a right angle triangular cavity of vertical length \(H\) and horizontal length \(H\) has been considered in the presence of internal heat generation and uniform external magnetic field has been applied in the horizontal direction normal to the vertical wall.

![Image](http://dx.doi.org/10.24018/ejers.2019.4.12.1613)

**Fig. 1. Schematic diagram of the present geometry and coordinate system.**

The enclosure has been filled with hybrid Al\(_2\)O\(_3\)-Cu/water nanofluid. The inclined wall of the enclosure has been cooled at a constant temperature \(T_c\). A sinusoidal heat source has been located at the bottom wall. It has been also assumed that both the fluid and hybrid nanoparticles are in thermal equilibrium and there is no slip between them. The hybrid nanofluid used in the work has been considered as laminar and incompressible. It has also been assumed that the gravitational acceleration acts to the vertical downward surface. Fluid and hybrid nanofluid Properties has been given in Table 1, assumed constant except for the density variation, which is maintained on Boussinesq approximation.

| TABLE I: THERMOPHYSICAL PROPERTIES OF WATER, ALUMINA AND COPPER [15] |
|-----------------|-----------------|-----------------|
| Property        | Water           | Alumina (Al\(_2\)O\(_3\)) | Copper (Cu) |
| \(c_p\) (J/kg\(^\circ\)K\(^-1\)) | 4179            | 765             | 385         |
| \(\rho\) (kg/m\(^3\))     | 997.1           | 3970            | 8933        |
| \(k\) (W/m\(^\circ\)K)     | 0.613           | 40              | 401         |
| \(\alpha\) (K\(^-1\))      | 21 \times 10\(^{-5}\) | 85 \times 10\(^{-5}\) | 1.67 \times 10\(^{-5}\) |
| \(\sigma\) (\(\mu\) Sm\(^{-1}\)) | .05             | 1 \times 10\(^{10}\) | 5.96 \times 10\(^{7}\) |

**III. MATHEMATICAL FORMULATION**

By considering the problem described above the equations for the conservation of mass, momentum and energy in Cartesian coordinate system for hybrid nanofluid [15] for fluid and solid are given below:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}
\]

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\rho_f}{\rho_{hnf}} \left( \frac{\partial^2 U}{\partial^2 X} + \frac{\partial^2 U}{\partial^2 Y} \right) \tag{2}
\]

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\rho_f}{\rho_{hnf}} \left( \frac{\partial^2 V}{\partial^2 X} + \frac{\partial^2 V}{\partial^2 Y} \right) \tag{3}
\]

\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{hnf}}{\alpha_f} \left( \frac{\partial^2 \theta}{\partial^2 X} + \frac{\partial^2 \theta}{\partial^2 Y} \right) + \frac{\rho C_p f}{\rho_{hnf} C_p} Q \tag{4}
\]

where \(P_r = \frac{\nu f}{\alpha_f} \cdot R_a = \frac{\nu f}{\alpha_f} \cdot k_{hnf} f \cdot \alpha_f = B_0 H \sqrt{\frac{\nu f}{\rho_{hnf}}} \) are the Prandtl number, Rayleigh number and the magnetic Hartmann number respectively and \(Q\) is the dimensionless heat generation coefficient.

The equation (1) to (4) are made dimensionless by using the following relations

\[
X = \frac{xH}{H}, \quad Y = \frac{yH}{H}, \quad U = \frac{uH}{\alpha_f}, \quad V = \frac{vH}{\alpha_f}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad \frac{P}{\rho_{hnf} C_p} = \frac{\nu f}{\alpha_f} \cdot k_{hnf} f \cdot \alpha_f = \frac{B_0 H}{\sqrt{\frac{\nu f}{\rho_{hnf}}}} \frac{\nu f}{\alpha_f} \quad (5)
\]

The boundary condition for the problem is given below:

On the left wall: \(U = V = 0, \quad \frac{\partial \theta}{\partial X} = 0\), on the inclined wall:

\(U = V = 0, \quad \theta = 0\) and on the bottom wall: \(U = V = 0, \quad \theta = \sin(\alpha X)\).

The effective properties of Hybrid nanofluid (Al\(_2\)O\(_3\)-Cu/water) is defined as follows:

density: \(\rho_{hnf} = (1-\phi_f) \rho_1 + \phi_2 \rho_2 + \phi_3 \rho_3\);

thermal diffusivity: \(\alpha_{hnf} = \frac{\kappa_{hnf}}{\rho_{hnf} C_p} \)

heat capacitance:

\(\rho C_p f_{hnf} = (1-\phi_f) [\rho C_p f_1 + \phi_2 \rho C_p f_2 + \phi_3 \rho C_p f_3] \)

thermal expansion coefficient:

\((\beta C)_{hnf} = (1-\phi_f) \beta f_1 + \phi_2 \beta f_2 + \phi_3 \beta f_3 \)

viscosity based on Brinkman model [16]:

\(\mu_{hnf} = \frac{\mu_1}{(1-\phi_2 - \phi_3) 2.5} \)

thermal conductivity, according to the Maxwell-Garnetts model [17]:

\(\mu_{hnf} = k_1 \left( \frac{2 \phi_2 k_2 + \phi_3 k_3}{\phi_1} + 2k_1 + 2(\phi_2 k_2 + \phi_3 k_3) - 2\phi_1 k_1 \right) \times \left( \frac{2 \phi_2 k_2 + \phi_3 k_3}{\phi_1} + 2k_1 - 2(\phi_2 k_2 + \phi_3 k_3) + \phi_1 k_1 \right)^{-1} \)

Effective electrical conductivity according to the Maxwell-Garnetts model [17]:

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The effects of various solid volume fraction ($\phi$) of nanoparticles, the Rayleigh number (Ra) and the Hartmann number (Ha) in the cavity are discussed. The results of this study are presented in terms of stream lines and isotherms contour figure together with average Nusselt numbers. The volume fraction $\phi_1$ of the hybrid nanofluid contains of equal amounts of the two different $\phi_2$ (Al$_2$O$_3$) and $\phi_3$ (Cu) nanoparticles. Fig 3a presents the streamlines performance with $\phi_1$ and there is a tiny impact easily observed from the Fig 3a. When nanoparticle’s viscosity increases then there build up in stream line also increases. Natural convection increases as the thermal conductivity increases, shown in Fig 3b.

There was a good agreement shown in Fig 2, between present results and the numerical results of Rashad [16] for both the streamlines and isotherms inside the cavity. These validations boost the confidence that the procedure applied in this work is appropriate and help to stated objectives of the current investigation.

V. RESULTS AND DISCUSSIONS
Fig 4 and Fig 5 are showing that average Nusselt number is decreasing for the increasing of volume fraction ($\phi$) and the Hartmann number (Ha) for different Rayleigh number (Ra) except for $Ra = 10^4$. When $Ha = 0$, vortex inside the cavity flow spontaneously but with the increases of Ha, vortex become restrict movement close to the heating particles due to the magnetic effect as shown in Fig 6a. In the case of isotherm, with the increases of Ha, it distributes uniformly with fewer gradients, shown in Fig 6b. Hybrid nanofluid move energetically and covered all most a full cavity with increases of Rayleigh number as shown in Fig 7a. With the increases of Ra, isotherms bifurcate as shown in Fig 7b.
VI. CONCLUSION

Natural convection of Hybrid nanofluid filled in a triangular cavity in presence of MHD effect horizontally to the cavity with internal heat generation flow and sinusoidal heat from bottom surface has been investigated numerically. Varying the parameters $Ra$, $Ha$ and $\phi$ have headed to the following conclusions:

i. The effectiveness of hybrid nanofluid can be observed when the natural convection is small.

ii. Hartmann number considerably restrict the Nusselt number when the Rayleigh number is higher.

| Greek Symbol | Subscript | Description |
|--------------|-----------|-------------|
| $C_p$ | | specific heat ($J \cdot kg^{-1} \cdot K^{-1}$) |
| $Pr$ | | Prandtl number ($\frac{\nu}{\alpha}$) |
| $g$ | | gravitational acceleration ($ms^{-2}$) |
| $Ha$ | | Hartmann number |
| $Ra$ | | Rayleigh number $\frac{g\beta\Delta H}{\nu^2}$ |
| $k$ | | fluid conductivity ($Wm^{-1}K^{-1}$) |
| $T_s$ | | heat source temperature (K) |
| $H$ | | cavity side height (m) |
| $T_i$ | | heat sink temperature (K) |
| $Q$ | | heat generation ($W$) |
| $Nu$ | | average Nusselt number $(U,V)$ |
| $p$ | | dimensionless pressure $(kgm^{-3}s^{-2})$ |
| $P$ | | non-dimensional pressure $(\frac{p + \rho g v L^2}{\rho u^2})$ |
| | | dimensionless coordinates $(X,Y)$ |
| $\sigma$ | | electrical conductivity $(Sm^{-1})$ |
| $\phi$ | | Nanoparticles volume fraction |
| $MHD$ | | magneto-hydrodynamic |
| $FEM$ | | Finite element method |

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