Electronic localization in two dimensions

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Electronic localization refers to situations in which electrons are confined in space, and the wave function decays exponentially along any directions. According to the scaling analysis of localization, there can be no metallic state or metal-insulator transition in two dimensions in zero magnetic field and in the absence of electronic interactions. In other words, all electrons are always localized in two dimensions (2D) purely due to multiple scattering by disorders. This has been the prevailing view for the past twenty years. As this phenomenon is due to the wave nature of electrons, it is also called wave localization. Experiments performed in 1980s on various 2D systems tended to support the prediction of localized waveguide transmissions. An excellent review on the current developments in the study of localization behavior and related phenomena in two dimensions was due to Abrahams et al.

In the last several years, however, experimental evidence started to appear, indicating that the widely accepted view might not always have been correct. Unusual metallic behavior has been reported in a number of two dimensional electronic systems. An excellent review on the current developments in the study of localization behavior and related phenomena in two dimensions was due to Abrahams et al.

As noted by Abrahams et al., one of the main observations to be explained is that the unusual metallic behavior is displayed down to the near zero temperature under conditions in which 2D systems are expected to show insulating behavior because of localization due to disorders, according to the theory of localized waveguide transmissions. Various theories have been put forward to explain this unusual observation, ranging from theory of non-Fermi-liquid states, superconducting-insulator transition, scaling theory including electronic interactions, percolation theory, and so on. These theories have been reviewed critically by Abrahams et al. As pointed out by Abrahams et al., however, while each of the theories is capable of explaining one or another part of the set of experimental observations, none of them is able to reconcile all the experimental results. Although it has been now widely accepted by the community that the unusual metallic behavior shown in 2D systems is caused by electronic interactions, which have not been included in the consideration of the previous theory, significant disputes remain. For example, it is pointed out in that the previous view on 2D localization is apparently incomplete and maybe, in the general case, incorrect. Whether all the electronic states in 2D disordered media are localized without electronic interactions therefore still poses an open question. This motivates us to consider further the problem of electronic localization in 2D disordered systems. The question to be addressed here is whether there could be extended or non-localized states in 2D random media and these states are purely caused by interference of multiple scattering waves, i.e. not caused by delocalization effects such as boundary modes, electronic interactions, random magnetic fields, off-diagonal disorders, correlated bands, waveguide transmissions. These effects are known to be able to delocalize or influence the localization of electrons.

We are naturally led to the question of how to discern the contradiction between the experimental observation and the previous assertion that all electronic waves are localized in 2D disordered systems. In order to resolve this conflict, two possible approaches may be adopted. One is to find a theory that can provide a comprehensive picture to all observations. This is not an easy task and is too ambitious at this stage, mainly because of complications involved the actual experiments. Often, various physical effects interplay with each other, making the data interpretation itself very difficult. The second approach may be taken by asking why all electrons have to be localized in two dimensions after all and whether there is some shortcoming in the previous analysis. These questions are also very difficult and delicate. But there may still be some hope that the second approach may be accomplished by looking back at the previous analysis which has led to the conclusion that all waves are necessarily

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localized in 2D. In this approach, it is to verify what are the conditions that warrant the conclusion. If these conditions are not satisfied or they could actually obscure the discernment of the phenomenon of localization, the conclusion from the analysis may not be applicable. An improvement may thus be worthwhile.

Here we try to take the second approach to the problem, in the hope to stimulate further discussions. To proceed, we first repeat and then discuss the theory in \[3\]. We suggest that the previous analysis may not be complete and might be unable to uniquely single out the localization effect, in line with what has been stated in the introduction of \[15\]. Then we propose an improved analysis to address the problem of whether all waves are indeed always localized in 2D systems. We hope to show that after the improvement not only the results from previous analysis can be recovered as a possibility, but also there is the possibility that the localization behavior in 2D actually bears similarities to that in three dimensions (3D). Specifically, there could exist the chance that in two dimensions the transition between localized and non-localized states is possible, by analogy with 3D. Through the discussion, we perceive that the evident conflict between the observations and the previous analysis is due to the difference in the ways that localization is inferred or interpreted.

While the essence of the current analysis has been very briefly reported in \[20\], here I would like to expand the discussion. For the sake of convenience, some mathematical derivations are necessarily repeated.

\[\beta = \frac{\partial \ln R}{\partial \ln L}. \tag{3}\]

Taking Eqs. (1) and (2) into (3), we obtain the asymptotic behavior

\[\beta \sim \begin{cases} \ln R, & \text{as } R \to \infty \text{ (Localized)} \\ 2 - d, & \text{as } R \to 0 \text{ (Ohmic)} \end{cases} \tag{4}\]

From the asymptotic behavior in Eq. \(4\), we can sketch the universal curves in \(d = 1, 2, 3\) dimensions. The central assumptions here are (1) \(\beta\) is continuous; (2) \(\beta\) is a function of \(R\) and depends on other parameters such as disorders and length scale only through \(R\); and (3) once wave is localized, the increasing sample size would always mean more localization. These assumptions have been discussed in some detail in \[3\].

The generic behavior of \(\beta\) is plotted in Fig. \(1\). It is clear that in the 3D case, the curve crosses the horizontal axis, yielding an unstable fixed point \((B)\). Above this point, the waves become more and more localized as the sample size increases. Below the critical point, the system tends to follow the Ohmic behavior as the sample size is enlarged. This fixed point separates the localized and non-localized states. For the two dimensional case, in the Ohmic regime \(\beta\) approaches zero as \(\ln(R) \to 0\). But the perturbation calculation including the wave interference effect shows that \(\beta\) is always greater than zero. Therefore for both one and two dimensions, the curves do not cross the horizontal axis, and there is thus no fixed point. As the sample size increases, all states move towards the localization regime, as illustrated in Fig. \(1\).

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The above analysis may require a further consideration. The reasons follow. Whether a system has non-localized or only localized states is an intrinsic property of the system, and should not rely on neither the boundary nor the source. As long as the analysis cannot exclude the possibility that the boundary or the source is playing a role, the consequence from the analysis may become questionable. In order to isolate the localization or non-localization effect, therefore, a genuine analysis should not be affected by any possible boundary effects not only in the localization region but also in the non-localization region. Of course, if the system has indeed only localized states, the boundary is not an issue, as the dependence

**FIG. 1:** The scaling function \(\beta\) versus \(\ln R\) from Eq. \(4\)

First we repeat the previous analysis that leads to the view that all electrons are localized in 2D, partially for the sake of convenience on the general reader’s part. According to \[3\], an hypercubic geometry is used for the scaling analysis. In the metallic state, the resistance follows the Ohmic behavior

\[R \sim L^{2-d}, \tag{1}\]

where \(d\) is the dimension. For a localized state, i.e. large \(R\), the resistance grows exponentially

\[R \sim e^{L/L_1}, \tag{2}\]

where \(L_1\) is the localization length which may differ for different dimensions. A scaling function is defined as

\[\beta = \frac{\partial \ln R}{\partial \ln L}. \tag{3}\]

and depends on other parameters such as disorders and length scale only through \(R\); and (3) once wave is localized, the increasing sample size would always mean more localization. These assumptions have been discussed in some detail in \[3\].

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on the boundary is exponentially vanishing. However, the care must be taken for the non-localized regime. It is not difficult to see that straightly speaking, the above scaling theory works without ambiguities for situations when both probing contacts, used to measure the resistance or conductance from which the localization is inferred, are located outside the sample. In this case, the Ohmic behavior given by Eq. (1) is valid under the condition that the current flows uniformly in one direction. Strictly speaking, this is possible only with (a) properly scaled sources and with (b) the presence of confining boundaries. This may be in conflict with the proclamation that whether it is a localization or non-localized state is the intrinsic property of the system and should not rely on a boundary nor a source, and these effects should not come to play a role on the analysis not only in the localization but also the Ohmic regions. Thus, in the strictly term, the above analysis may be more appropriate for studying transport phenomena. It is our opinion that the reduction in the conductance does not necessarily mean that all waves are actually localized. In other words, it is necessary to differentiate the situation that the electrons are prohibited from transmission from the situation that the system actually has only localized states. Certainly, we should not exclude the possibility that the boundary or source effects just mentioned are minimal and can be practically ignored. In fact, this possibility is also supported by the following improved analysis. We also note in reality boundaries are always present in numerical simulations or experiments. In this case, the influence of the boundaries has to be carefully addressed. Furthermore, the earlier scaling analysis has indeed considered the boundaries.

Taking the view that localization refers to the situation that the envelope of the wave function decays exponentially from some point in space; any other form of envelope would mean non-localized states, and also taking the view that whether it is localized or non-localized is an intrinsic property of the system, we wish to propose an alternative scaling analysis.

We consider uniformly random systems. Consider an infinite system in either 2D or 3D, we compute the effective resistance between any two space points in the system. Then we investigate how the resistance varies as the separation (L) between the two points increase. In this way, the boundary or source effect is unambiguously eliminated. We note that in real experiments, it is the conductance or the resistance across a sample that is often measured for transport properties.

In line with the above discussion, in the limit of small disorders, by neglecting all interference between successive scatterers the resistance \( R \) is assumed to follow the Ohmic behavior which is guided by \( \vec{j} = \sigma \vec{E} \) with \( \sigma \) being the conductivity. An integration leads to

\[
R \sim \begin{cases} 
L, & \text{for 1D} \\
\ln(L/L_0), & \text{for 2D} \\
\frac{1}{x_0} - \frac{L}{x_0}, & \text{for 3D}
\end{cases}
\]  

(5)

where \( L_0 \) refers to the microscopic size. Eq. (6) indicates that the resistance grows logarithmically either with the sample size or the distance between two space points in 2D. Hereafter, \( L \) can stand for either the sample size or the separation between two points in an infinite space.

Eq. (5) is derived by Ohm’s law. As the example, we present the derivations for 2D. Consider an infinite plate, with two probing contacts at \( \vec{r}_1 \) and \( \vec{r}_2 \). One is the source and the other is the drain. The divergence theorem states

\[
\nabla \cdot \vec{j}(\vec{r}) = -4\pi A \delta^{(2)}(\vec{r} - \vec{r}_1) + 4\pi A \delta^{(2)}(\vec{r} - \vec{r}_2).
\]

The solution is

\[
\vec{j}(\vec{r}) = \frac{A}{|\vec{r} - \vec{r}_1|} \vec{e}_1 - \frac{A}{|\vec{r} - \vec{r}_2|} \vec{e}_2,
\]

where

\[
\vec{e}_1 = \frac{\vec{r} - \vec{r}_1}{|\vec{r} - \vec{r}_1|}, \quad \vec{e}_2 = \frac{\vec{r} - \vec{r}_2}{|\vec{r} - \vec{r}_2|}.
\]

The coefficient \( A \) is determined from

\[
\int |\vec{r} - \vec{r}_1| |d\theta \vec{j} \cdot \vec{e}_1| = I,
\]

with \( I \) being the current. This gives

\[
A = \frac{I}{2\pi}. \tag{6}
\]

Now use Ohm’s law

\[
\vec{j} = \sigma \vec{E}, \quad \text{and} \quad \int d\vec{r} \cdot \vec{j} = \sigma \int d\vec{r} \cdot \vec{E}.
\]

We integrate from \( \vec{r}_1 \) to \( \vec{r}_2 \), but in the integration we need to exclude the singular point \( \vec{r}_1 \) and \( \vec{r}_2 \). Assume the line between the two points is the \( x \)-axis. The integration is

\[
A \int_{x_1 + \epsilon}^{x_2 - \epsilon} \frac{dx}{x - x_1} + A \int_{x_1 + \epsilon}^{x_2 - \epsilon} \frac{dx}{x_2 - x} = \sigma V.
\]

Here the voltage

\[
V = \int_{x_1 + \epsilon}^{x_2 - \epsilon} dx E(x).
\]

The integration gives

\[
2.4 \ln \left( \frac{L - \epsilon}{\epsilon} \right) = \sigma V,
\]

where \( L = x_2 - x_1 \). Considering Eq. (6), we have the resistance

\[
\frac{V}{I} = \frac{1}{\pi \sigma} \ln \left( \frac{L}{\epsilon} \right). \quad (L \gg \epsilon).
\]

Therefore in 2D the resistance scales as

\[
R \sim \frac{1}{\pi \sigma} \ln \left( \frac{L}{L_0} \right) \quad (L_0 = \epsilon),
\]

where \( L_0 \) refers to the microscopic size. Eq. (6) indicates that the resistance grows logarithmically either with the sample size or the distance between two space points in 2D. Hereafter, \( L \) can stand for either the sample size or the separation between two points in an infinite space.
which is the result for 2D given in Eq. (5). The result for 3D can be obtained similarly.

In the other limit where the disorder is very large, the resistance is large. Therefore an exponential localization is expected. The resistance is anticipated to grow as

$$R \sim e^{L/L_1}. \quad (7)$$

While the behavior remains unchanged for 1D, the asymptotic behavior for the scaling function in both 2D and 3D becomes

$$\beta \sim \begin{cases} e^{-\ln(R)}, & \text{for } \ln(R) \to -\infty; \\ \ln(R), & \text{for } \ln(R) \to \infty \end{cases} \quad (8)$$

From these asymptotic behaviors in the two limits, we expect that the localization behavior in 2D and 3D should be similar. That is, in both cases, the scaling function decreases and then increases linearly with \(\ln R\) in the Ohmic and the localization regimes respectively.

Now since we know the asymptotic behaviors in the two limiting cases (Refer to Eq. (5)), the general behavior of the scaling function \(\beta\) may be obtained in the similar way as outlined above or in [3]. Taking the above assumptions for the scaling function \(\beta\) except that it needs not to be monotonic, the scaling function given by Eq. (8) is conceptually plotted in Fig. 2. It is obvious that the 1D situation is a replicate of that shown in Fig. 1. The consequence is that all waves are localized in one dimension for any given amount of disorders.

Seen from Fig. 2, there are two possibilities for two and three dimensions. In the first instance shown in Fig. 2(a), there are two fixed points: \(A\) and \(B\). It is clear that \(A\) and \(B\) are respectively the stable and unstable fixed points. Point \(B\) separates the localized state and the non-localized state. When \(\ln(R)\) is greater than \(B\), the increasing sample size will lead the system to an infinite resistance in the localized regime; thus the electronic waves are localized. When \(\ln(R)\) is initially below point \(B\), increasing the sample size leads to the fixed point \(A\), where the increasing \(L\) will no longer affect the resistance, indicating a stable non-localized state.

The second possibility is shown in Fig. 2(b). There is no fixed point; at most there is only one single unstable fixed point. In this case, all waves in 2D will be localized like in the 1D situation. This is the case previously considered for 2D. Previous results affirming that all electronic waves are localized in 2D fit in this situation. In this sense, the present analysis accommodates the previous analysis. In this case, the boundary or source effect mentioned above must be unimportant.

By expanding \(\beta\) at point \(B\), the critical behavior can be studied. It is easy to see that at this point, the experimentally observed symmetry relating conductivity and resistivity could follow, by analogy with the discussion in [12]. Around \(B\), we have

$$\beta \approx a(\ln R - \ln R_B)^\gamma, \text{ with } a > 0, \quad (9)$$

To ensure that point \(B\) is unstable, \(r\) must be 1. Then we recover the result in [12]

$$R = \begin{cases} R_B e^{(L/L_B)^\alpha}, & \text{for } R > R_B; \\ R_B e^{-(L/L_B)^\alpha}, & \text{for } R < R_B. \end{cases} \quad (10)$$

Clearly if \(a > 1\), the resistance will grow faster than exponentially. The point \(B\) with \(R_B\) may be reached at the limit \(L \ll L_B\). In this limit, \(R/R_B \approx 1 \pm \left(\frac{L}{L_B}\right)^a\) for above and below \(R_B\) respectively.

To explore the behavior around the stable state at point \(A\), we also expand \(\beta\) at this point,

$$\beta \approx -\alpha(\ln R - \ln R_A)^\gamma, \text{ with } \alpha > 0. \quad (11)$$

The fact that point \(A\) is a stable point requires \(\gamma = 1\). By integration, we find that

$$R = \begin{cases} R_A e^{(L/L_A)^{-\alpha}}, & \text{for } R > R_A; \\ R_A e^{-(L/L_A)^{-\alpha}}, & \text{for } R_A > R. \end{cases} \quad (12)$$

FIG. 2: The scaling function \(\beta\) vs \(\ln(R)\) for 1, 2, and 3 dimensions in the new scaling.
with $L_A$ is a measure of how fast $R$ approaches $R_A$ and it may depends on various parameters such as disorders. In the three dimensional case, when $\alpha = 1$ and $R_A \sim \frac{1}{\sigma L_A}$, the state at point $A$ is an Ohmic state, in accordance with Eq. 6. In this case, as $L$ increases, $R$ will converge to the Ohmic result $R_A$. For the two dimensions, point $A$ refers to a stable state, and when $\ln(R)$ is initially below $B$ the system will converge to $A$. The state at point $A$, however, is not of the Ohmic nature. In fact, if the system were Ohmic, Eq. 6 would imply a logarithm growth in the resistance with $L$. Therefore, we are led to the conclusion that although there can be non-localized states in 2D, the exact Ohmic state is absent, for an infinite sample or when the separation between two measuring points is exceedingly large. Due to possible slow convergence of $R$, however, the resistance may behave nearly like Ohmic for a wide range of $L$. Furthermore, in 2D, the existence of the stable state at $A$ would imply that the added small disorder tends to slow down the increase of resistance near the stable fixed point and the resistance will be eventually saturated at $A$, as $L$ goes to infinity, tending to comply with the result shown on P. 203 in Ref. [22]. This is a state that the classical estimate of the conductance approaches a minimum conductance due to the wave nature of electrons, expected from the Landauer formula for the conductance. The non-increasing resistance state may correspond to the unusual metallic behavior discussed in the literature. Whether this state can be observed depends on samples and experimental conditions. Nonetheless, the possibly important finding reported here is that a stable non-localized state is possible in two dimensions, not excluding the other possibility that two dimensional random media only support localized states as indicated by Fig. 2(b).

Now we discuss further the above results. First, we pointed out that the previous scaling analysis has been questioned in the literature in the past, and more recently by a number of authors who demonstrated some results in one dimensional random media that are in contrary to the prediction of the previous analysis. And a novel approach to look into the problem of localization has been suggested. In addition to the recent experiments, there were also other experiments which cannot be accounted for by the previous analysis. As pointed out in 26, two exceptions predicted by the scaling theory to occur at the center of a Landau level in the quantum Hall effect, and the Anderson transition which occurs in zero magnetic field if there is a significant spin-orbit interaction.

However, it should also be pointed out that, the previous scaling analysis was supported by a number of simulations, for instance, by a simulation of the 2D Anderson model with diagonal disorders. And there were also many experiments that affirm the previous analysis not only in the electronic systems (e.g. 4, 5, 6, 32) but in phonic systems 33. The fact that the previous analysis was supported by some research while not by some others may thus imply that the previous analysis of the 2D localization is incomplete and is likely model-dependent, as suggested in 13, 24. As aforementioned, the present analysis accommodates the prediction of the previous analysis (Referring to Fig. 2(b)) and therefore may also accommodate these theoretical and experimental supports.

Next, we may point out some recent evidence that may support the present analysis for possible non-localized states (Referring to Fig. 2(a)). First, our recent exact numerical results indeed indicate that there is indeed a need to differentiate the situation that the waves are prohibited from transmission across a sample from the situation that the system actually has only localized states. There, the acoustic propagation and scattering in water containing many parallel air-filled cylinders is studied in an exact manner. Two situations are compared: (1) wave propagating through the array of cylinders, imitating common experimental setups, as summarized in 25, 26, and the scenario of the previous scaling analysis 6, and (2) wave transmitted from a source located inside the ensemble. It was shown that waves can be blocked from propagation by disorders in the first scenario, but such an inhibition does not necessarily lead to actual wave localization in the medium. Note that the electronic system is more complicated because effects such as the Coulomb interaction can make data interpretation difficult. In this sense, classical systems are advantageous in studying localization effects.

Furthermore, the present scaling analysis has been pointed out (Y. Tarasov, private communication) to be absolutely in line with recent findings obtained through direct calculation of the conductance with the use of the Kubo formula 15, 35. In these references, it was shown that the localization behavior in both 2D and 3D can be similar.

Finally, we note that a suggestion of two fixed points similar to what was shown in Fig. 2(a) has earlier discussed in a general description of scaling theories of localization by Janssen 36. There, however, no concrete physical quantity was used. We stress that the above inclusion of published research is far from completeness. There is a great amount of excellent works which can be referred to in the recent monograph 18.

In summary, we have attempted to present an analysis of electronic localization in random media. The study suggests that the localization behavior is similar in both two and three dimensions. The transition between localized and non-localized states is possible in both dimensions. A new state is predicted as a possible non-localized state for 2D disordered systems. It must be stressed that these findings do not exclude the possibility that all waves are localized in two dimensional random media. As shown above, the present analysis also supports the possibility that all waves are localized in two dimensional random media, a profound principle which has guided significantly previous investigations of localization since its inception.
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