Design and Construction of Homodyne Detectors for the Study of Quantum Optical States

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Abstract. Nowadays the fields of Quantum Optics and Quantum Information are experiencing a mayor development worldwide, in particular, in the experimental arena. The reason relies in the high perspectives of application of these fields in areas as computing, security, telecommunications and metrology, among others. A determinant fact for the experimental and technological development is the ability to design and construct electronic devices suitable for the study of light with quantum features, as well as the software which is necessary for the analysis and processing of experimental data. In this work we present a general overview of the theory, practical design and experimental tests of one of such devices called Homodyne Detector addressed to the study and engineering of quantum states of light.

1. Introduction

The development of Quantum Optics and Quantum Information has had remarkable advances in the last years, particularly in the experimental field, enabling new opportunities in research and technological applications [1–9]. The experimental tools and devices addressed to the measurement of quantum properties, which include data analysis and processing, play a fundamental role in this development and in the implementation of diverse applications.

One of the most commonly used detection techniques in the study of optical states in the quantum regime is Quantum Tomography by means of Balanced Homodyne Detection (BHD) in the domain of time. The BHD is a very useful tool in quantum optics for the processing of quantum information in continuous variables [10–13] due to its ability to measure the quadratures of the electromagnetic field of a light mode and to provide basic information for the characterization of quantum states. Some other advantages of the BHD include its relatively easy construction, its economic accessibility, as well as its design versatility.

With the development of quantum state engineering [14] as well as state tomography [2], the efficiency and performance requirements for homodyne detectors are increasing. This demands a more exhaustive analysis for their improvement. The BHD method has been extensively studied from the theoretical point of view [10, 16]. However, the design and construction aspects of the homodyne detector electronics have not been widely discussed mainly because they are addressed to the necessities of each particular application [3, 4, 17, 20]. This
has led to a lack of information on how to make possible improvements in the design of this detector.

There are several reconstruction algorithms that use the data obtained by the homodyne detector to characterize the optical state, as for instance, the Inverse Radon Transform \[12,21\], the Pattern Functions \[12,22\] and the widely used Maximum-Likelihood Reconstruction (MaxLik) which is based on an algorithm of statistical inference \[10,23\].

The main objective of this work is to present a basic discussion of the operation of homodyne detectors and to introduce a first design for a BHD system which is economical and easy to implement. Our proposal includes also an algorithm addressed to perform the analysis of the experimental data given by the detector. To this end, we will use the Radon transform method due to its simplicity and potentially fast algorithmic operation. The materials in this article are organized as follows. In Section 2 we present the theory of a BHD system, while, in Section 3 we discuss its design. Section 4 include some initial tests involving basic optical states as well as the corresponding results of the tomographic information reconstruction. Finally, we summarize our work and present some perspectives in Section 5.

2. Theory behind balanced homodyne detection

In this section, the main theoretical concepts underlying the operation of the BHD are summarized. First, the theoretical picture of homodyne detectors will be presented, followed by its connection with the electromagnetic field quantization. Subsequently, the Wigner distribution will be introduced as a representation of the quantum state and it will be stated how this representation is convenient to interpret the measurements performed by the detector. This is of great importance as it constitutes the basis of optical tomography.

2.1. Homodyne detection

The basic scheme of homodyne detection has in its core a 50/50 optical beam-splitter (BS). In one of the two input ports of the BS a very weak intensity signal -the state under study, which can have classical or quantum features- is sent. Meanwhile, in the second input a phase reference coherent state \[\alpha_{LO}\] is introduced (see Figure 1a) whose intensity is very strong compared to that of the signal port. The subscript LO means Local Oscillator which is a reminiscent term from radio applications.

Recall that the relations between the input and output signals of the BS, in its operatorial form:

\[
\begin{align*}
    \hat{a}_1 & = \frac{1}{\sqrt{2}} (\hat{a}_{\text{in}} + \hat{a}_{\text{LO}}) \\
    \hat{a}_2 & = \frac{1}{\sqrt{2}} (\hat{a}_{\text{in}} - \hat{a}_{\text{LO}})
\end{align*}
\]

**Figure 1.** (a) Schematic representation of a Homodyne Detector system: it incorporates a 50/50 beam-splitter (BS) and two photo-diodes \(D_1\) and \(D_2\) in current subtraction configuration. (b) Schematic representation of a beam-splitter: unprimed annihilation operators denote input ports and primed operators denote output ports.
the measurement \( \langle \hat{I} \rangle \) can be simplified to yield

\[
\langle \hat{I} \rangle \propto |\alpha_{LO}| \langle s | \left( \hat{a}_1^\dagger e^{i\theta} + \hat{a}_1 e^{-i\theta} \right) | \psi \rangle_s.
\]

It must be noticed that, due to its intensity, the coherent state is treated classically. Furthermore, by writing the creation and annihilation operators in the form \( \hat{a}_1 = \frac{1}{\sqrt{2\hbar}} (\hat{x}_s + i\hat{p}_s), \hat{a}_1^\dagger = \frac{1}{\sqrt{2\hbar}} (\hat{x}_s - i\hat{p}_s), \)

where \( \hat{x}_s \) and \( \hat{p}_s \) are the position and momentum quadratures of the signal beam and where we have assumed the field consists of oscillators of unitary mass and frequency, it can be seen that the measurement \( \langle \hat{I} \rangle \) can be expressed as

\[
\langle \hat{I} \rangle \propto |\alpha_{LO}| \langle s | \left( \hat{x}_s \cos \theta + \hat{p}_s \sin \theta \right) | \psi \rangle_s,
\]

with \( \theta \) a reference phase fixed by the coherent beam. This is a very important result because the term in parenthesis can be identified, up to a phase factor, with a rotated quadrature of a single mode of the electromagnetic field associated the signal beam \( [12,13,24] \),

\[
\hat{E}_s \propto -[\hat{x}_s \sin \theta + \hat{p}_s \cos \theta].
\]
Moreover, the expected value \[ \langle x \rangle \] is amplified by the local oscillator amplitude, making the quantum properties of the optical signal observable for the detector. These properties can be obtained by performing a series of measurements and then a statistical analysis of the electronic noise signals (shot noise analysis)

\[
\Delta \hat{I}^2 \propto |\alpha_{LO}|^2 \Delta \hat{x}_{\theta s}^2,
\]

produced by the detector when measuring a particular signal state. Here \( \Delta \hat{I}^2 \) is the variance of the difference photo-current while \( \Delta \hat{x}_{\theta s}^2 \) is the variance of the corresponding rotated quadrature

\[
\hat{x}_\theta = \hat{x}_s \cos \theta + \hat{p}_s \sin \theta.
\]

### 2.2. Wigner function and marginal projections

The Wigner function is a quasi-probability distribution function defined in the phase-space which can be considered as an alternative representation of an optical state [12,13,24,25]. It is defined as follows

\[
W(\rho)(x,p) = \frac{1}{2\pi \hbar} \int_{-\infty}^{\infty} \left\langle x + \frac{1}{2} \Delta \right| \hat{\rho}_s \left| x - \frac{1}{2} \Delta \right\rangle e^{-i\frac{p}{\hbar} \Delta} d\Delta,
\]

where \( \hat{\rho}_s \) is the density operator of the signal state we are considering. The Wigner function allows us to obtain statistical information of the rotated quadratures in a particular state by projecting it as [26]

\[
W(x_\theta) = \frac{1}{\cos \theta} \int_{-\infty}^{\infty} W(\rho) \left( \frac{x_\theta - p \sin \theta}{\cos \theta}, p \right) dp.
\]

This expression, called marginal distribution, contains the information about the statistical fluctuations of the quadrature \( \hat{x}_\theta \) and is interpreted as a normalized projection of the Wigner function onto a plane orthogonal to the phase-space defined by the angle \( \theta \). Figure 2 shows the graphical representation of a marginal distribution. Thus, according to [9], marginal distributions correspond to the noise distributions obtained experimentally by means of a homodyne detector at different local oscillator-phase angles. Figure 3 shows the Wigner functions and marginal projections for the vacuum (a), (b) and a coherent state (c), (d).

### 2.3. Quantum state tomography

As mentioned before, a homodyne detector is capable of measuring the marginal distributions of a signal state at different phase angles. If our objective is to obtain all the possible statistical
information about the state, the best option to represent this information is to reconstruct the corresponding Wigner function from a set of marginal distributions. This process is called Quantum Tomography and there are some well known methods to perform it [10–12].

In this work we will use the so called Radon Transform tomography due to its easy implementation and relatively fast operation. This transform was first introduced by Johann Radon in 1917 [27] and has become a commonly used mathematical tool particularly in medical physics and image processing (see, e.g. [28]). The Radon transform is obtained by inverting expression (12) by means of Fourier type transformations [26],

\[ W_\rho(x, p) = \frac{1}{(2\pi)^2\hbar} \int_{-\infty}^{\infty} \int_{-\pi/2}^{\pi/2} \int_{-\infty}^{\infty} |t'| W(x_\theta) e^{it'(x_\theta - x \cos \theta - p \sin \theta)} dx_\theta d\theta dt'. \] (13)

where \( t' \) is interpreted as a cutoff frequency parameter that allows us to manipulate the frequency contribution in the reconstruction process [12,26].

In an experimental situation we will collect discrete data from the homodyne detector system, i.e., the experimental marginal distributions will correspond to normalized histograms instead of continuous probability distributions. Therefore, a discretized version of the Radon transform is needed. Different methods of discretization can be found in the literature [29–32]. Our proposal, addressed to the reconstruction of the Wigner function from the marginal distributions, reads

\[ W_\rho(x', p') = \frac{1}{(2\pi)^2\hbar} \sum_{j=-N_l}^{N_l} \sum_{l=0}^{\pi} \sum_{k=-N_x}^{N_x} |t_j| W(x_k) \times \cos \left[ t_j \left( x_k - x' \cos \theta_l - p' \sin \theta_l \right) \right] \Delta x_k \Delta \theta_l \Delta t, \] (14)

where \( W(x_k) \) are the discrete marginal distributions obtained from the homodyne detector, \( \theta_l \) and \( x_k \) are the discrete phase angle and position for a certain marginal projection data point, \( \Delta \theta_l, \Delta x_k \) their corresponding resolution steps, \( \Delta t \) is the cutoff step value, the summation limits \( N_l \) and \( N_x \) depend on the number of data points taken for each parameter, and \( x' \) and \( p' \) are the discretized position and momentum variables, respectively. All these parameters are set according to the resolution requirements chosen experimentally. Equation (14) is the base for the tomographical applications of the homodyne detection measurement device, which will allow us to reconstruct the Wigner function of the signal state measured by the detector.

3. Homodyne detector design and implementation

Nowadays there are two main forms of measuring experimentally the quantum properties of light. The most common one is through the use of avalanche photo-diodes (APD), which are capable of detecting light quanta up to an efficiency of 70% [33]. These detectors work in a
Geiger mode, and, in consequence, require high resolution counting electronics which turns the detection system expensive.

The second way of measuring the quantum properties of light is by means of homodyne detectors. The main advantage of these devices is their ability to resolve states in the Fock basis (including states with \( n > 1 \)) and other non-classical states (squeezed states or cat states) \([10][12][13]\). Their efficiency can be comparable to the APD one, and can be built with common electronic components easily available on the market. This makes its implementation more economically accessible compared with an APD.

As pointed out before, the quantum information of the state measured by the HD will be in the statistics of the shot noise produced by the photo-currents. Thus, in order to analyze this information we need to amplify the shot noise produced by the photo-diodes over any other electronic noises (e. g. thermal or radio noises). This can be achieved by using a trans-impedance amplifier design, which is capable of amplifying very small currents up to gains of several thousands depending on the requirements \([34]\) and whose design presents a low susceptibility to external noises. In our particular case we decide to use a Texas Instruments operational amplifier OPA847 with an amplification factor of \(1k\Omega\). This will give the HD a theoretical operational bandwidth of more than 250MHz, well beyond the planned operational frequency of 100MHz. Additionally, the choice of the bandwidth of the detector will be addressed to implement applications with a 87MHz Ti:Sa pulsed laser. Nevertheless, the bandwidth can be modified by changing the amplification factor. Also, for the sake of noise reduction, the electronic design for the HD has high frequency considerations like highly filtered power supply stages, surface mounted electronics (SMD 1206 size) and microstrip transmission line considerations for the printed circuit board (PCB) design \([35][37]\). As for the photo-diodes, we choose a pair of Hamamatsu S5972 silicon photo-diodes with a bandwidth of 500MHz and quantum efficiency of 90\%. These photodiodes will operate in a reverse-bias mode with ±12V supply.

All previous considerations led us to the electronic design and its printed circuit board implementation showed in Figure 4. The implementation was performed with common chemical toner transfer methods.

### 4. Operational tests

The first operation tests for the HD system were performed by considering the vacuum state. In order to do this, it is necessary to construct an interferometric optical system that allows us to balance (or equalize) the amount of light reaching the photo-diodes. Once this is achieved, the input port of the HD beam-splitter where the signal is sent will be blocked so that only the local oscillator, which consist in a 500mW CW He-Ne laser vertically polarized, reaches

**Figure 4.** Homodyne Detector implementation. (a) Simplified circuit design. (b) PCB implementation.
the photo-diodes. To change the phase of the local oscillator for measuring different marginals we made use of a Babinet-Soleil compensator directly in its path (see Figure 5a). This optical system is the Balanced Homodyne Detection for Tomography. The noise signals produced by the HD at different phase angles were analyzed in a 500MHz oscilloscope and a statistical analysis (histograms of events) for voltages was collected for each phase (Figure 5b). The histograms were normalized in order to be fed to our reconstruction algorithm (14) as marginals (Figure 5c). The main characteristic of these marginals is their independence of the phase, which is in agreement with the theoretical predictions shown in Figure 3b.

Once we have compiled the required set of marginal distributions into a matrix arrangement, the Radon transform algorithm took an average of 98s to compute the corresponding Wigner function. The reconstructed Wigner function is shown in Figure 5d. The main feature observed in the reconstructed function is the Gaussian distribution centered at the origin of the phase-space, which is predicted by the theory (Figure 3a).

A second test consisted in sending a coherent state as the input signal of the BS. For this purpose, with the help of a half wave plate (HWP) and a polarized beam-splitter (PBS), 10% of the light intensity of the local oscillator was diverted to the input port of the HD-BS (Figure 6a). In this case, due to the interference between the local oscillator and the coherent signal, the phase shift produced by the Babinet compensator introduces an imbalance of intensity between the HD photo-diodes resulting in an oscillation of the center of the marginals (Figure 6b). The corresponding reconstructed function is presented in Figure 6c. The marginal distributions and the reconstructed Wigner function show the displaced Gaussian distribution characteristic of the coherent state (Figure 3c and 3d).

Figure 5. Experimental scheme for Balanced Homodyne Detection Tomography. (a) Experimental setup. (b) Oscilloscope analysis. (c) Experimental marginals. (d) Reconstructed Wigner function.
5. Conclusions

We experimentally implemented a tomographic system based on a HD. It was tested for vacuum states as well as for coherent states. The reconstructed Wigner functions show the main features of the states under study. Also, the implemented reconstruction algorithm is capable of processing the experimental data in reasonable times (<2min). Based on the present outcomes, the next steps in our project include more operation, characterization and calibration tests which are still necessary in order to improve the performance of the detector. Also, it is necessary to properly quantify the reconstruction quality. This can be done by using a correlation function between experimental and theoretical Wigner functions. A frequency response analysis of our HD is required in order to fully characterize this device. A shot noise level study must be performed in order to know the amplification capabilities of our detector respect to other similar ones. Finally, a tomographic measurement with our detector on non classical states of light produced with non-linear crystals must be performed in order to state the quantum capabilities of the detector. Results on the matter are in progress.

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