Soft-hard interplay and factorization for baryon production in the target fragmentation region in ep collisions

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Abstract

We discuss baryon production in the nucleon fragmentation region in deep inelastic scattering. The dependence of the nucleon spectra on Bjorken-$x$ is evaluated within the parton model and in QCD, and ways to look for the break-down of the DGLAP approximation via baryon production are suggested. We argue also that the leading neutron production in these small $x$ processes is rather insensitive to the pion parton densities of the nucleon.

1 Introduction

Current HERA experiments, both in the collider and fixed target modes, allow to take a fresh look at the target fragmentation region in a wide range of $x$.

It has been discussed for a long time within the framework of the parton model that at large $x$ properties of the target fragmentation region should depend on $x$, see e.g. Ref. [1]. On the contrary for small $x$ the Yang limiting fragmentation is expected i.e. the baryon differential multiplicities should be similar to those in soft hadron-nucleon interactions. Really within the parton model hadron radiation is a random process where transverse size of initial configuration diffuses to the transverse size of the target and memory on the hard physics in the hadron final states is restricted by the photon fragmentation region which form small part of rapidity space at fixed $Q^2$ when $x \to 0$.

This diffusion is especially enhanced in PQCD as a result of infrared slavery, i.e., from the infrared pole in the invariant charge. As a result of this diffusion, the interaction with the target is always given by non-perturbative QCD, and it is independent of the projectile. At the same time, it is expected that with increase of $x$ the spectrum of baryons should drop faster at large $z$, where $z$ is the Feynman-$x$ of the produced baryon, see Eq. (1) below.

We reevaluate these expectations by taking into account effects of the gluon bremsstrahlung and consider also new opportunities for testing the small $x$ dynamics of the strong interaction by means of studies of long range correlations in rapidity space in deep inelastic
scattering. This is now practical with HERA’s large acceptance detectors. In particular, we will argue that the lack of correlations between hadron production in the current and target fragmentation regions, which is characteristic for the DGLAP evolution equations, may break down if non-linear effects are really important in the evolution. These non-linearities could be observed simply by switching from inclusive measurements to semi-inclusive studies involving the production of hadrons at central rapidities.

2 Parton model expectations

First, let us summarize the parton model expectations. It is convenient to consider the scattering in the Breit frame, where the colliding proton and the virtual photon have four-momenta \( p_N = (P, P, \vec{0}_t) \) and \( q_\mu = (0, -2xP, \vec{0}_t) \), respectively. After the violent collision, the photon removes a parton with momentum \( xP \) and “turns it around”, while the spectator system of partons with momentum \((1-x)P\) fragments into hadrons in the proton fragmentation region.

It is convenient to define the Feynman-\( x \) for the produced baryon,

\[
z = \frac{P_h}{(1-x)P},
\]

as well as the light-cone fraction \( \beta \) of the initial proton momentum carried by this baryon,

\[
\beta = \frac{P_h}{P} = z(1-x).
\]

Correlations between hadrons produced in the soft interactions are short-range in rapidity, i.e., \( |\Delta y| \leq 2 \). Therefore, one expects that in the limit of small \( x \), when the distance in rapidity between the removed parton carrying the light-cone fraction \( x \) and the leading partons, which form the leading hadron exceeds several units correlations should disappear.

Suppose the leading hadron carries a light-cone fraction of the residual system’s momentum of Feynman-\( z \). Then, the hadron yield,

\[
f(z, p_t, x, Q^2) \equiv \frac{1}{\sigma_{tot}(\gamma^*p)} \frac{d\sigma(\gamma^* + p \rightarrow h + X)}{dz \, dp_t^2},
\]
should be independent of $x$, and be the same as for real photon or hadron projectiles. This is just an analog of Yang limiting fragmentation observed in hadron-hadron collisions. In the limit of $z \to 1$, this relation follows from triple Regge analysis \cite{2, 3}. Here, the same triple Reggeon diagrams enter in the inclusive cross section for different projectiles. The only difference arises from the coupling of the Pomeron to the projectile, which cancels out in the ratio to the total cross section for $\gamma^*(h)p$ scattering, and hence it is canceled also in the ratio in Eq. \eqref{3}.\footnote{Discussion of the diffractive case, i.e, $h=p$ and $z \to 1$, requires special treatment, see review in Ref. \cite{4}.}

The violation of limiting fragmentation may occur due to screening effects (multi-Pomeron exchanges), which are naturally much smaller in the case of $\gamma$ or $\gamma^*$ projectiles than for nucleon projectiles. These effects would lead to a certain enhancement of the spectra of the leading particles for the case of projectiles interacting with a smaller cross section, and to a violation of limiting fragmentation at super-high energies due to an increase of the effective cross section of projectile-nucleon interaction. Indication of such a break-down of factorization for proton production in the triple Pomeron limit was observed at the Tevatron collider, see e.g. Ref. \cite{5}.

With increasing $x$, the factorization relation in Eq. \eqref{3} is expected to break down. First, the triple Pomeron piece disappears, i.e., the ladder becomes too short in rapidity to build the $\frac{1}{1-z}$ behavior. Next, at $x \geq 0.2$ the main contribution to DIS starts to originate from the scattering off the target’s valence quarks. But the QCD counting rules indicate \cite{1} that, in this limit,

\begin{equation}
 f(z, p_t, x, Q^2) \propto (1 - z) .
\end{equation}

At sufficiently large $x$ (probably $x \geq 0.5$), DIS selects scattering from the minimal Fock space configuration $|3q\rangle$. The analysis of the leading perturbative QCD diagrams, which give the dominant contribution in the $x \to 1$ limit, indicates that the main contribution stems from configurations where two spectator quarks carry large relative momenta, i.e.,

$$\frac{x_1}{x_2} \gg 1 \text{ or } \frac{x_1}{x_2} \ll 1,$$

where $x_1$ and $x_2$ are the light-cone fractions of the spectator quarks.
As a result, the independent fragmentation of two spectator quarks becomes more and more important. Also, for joint fragmentation of both quarks, the relative chance of producing an excited baryon state increases. Hence, in the limit of large $x$, we expect a gradient decrease of $f(z, p_t, x)$ at large $z$ and an increase of the yield of excited baryon states \[1, 6\].

### 3 QCD radiation effects

For inclusive processes, the major modification to the target fragmentation is due to QCD radiation in the initial state. Really, according to the QCD evolution at large $Q^2$, if the $\gamma^*$ interacts with a parton at a given $x$, latter parton originates from a parent parton at a softer resolution scale $Q^2_0$ with $\bar{x} > x$. As soon as we focus on the production of hadrons at large $z$ and small enough $k_t$ (where $k_t^2 \ll Q^2_0$), we can neglect the fusion of gluons (quarks) emitted in the evolution from $(\bar{x}, Q^2_0)$ to $(x, Q^2)$. The reason is that these partons have transverse momenta larger than $\sqrt{Q^2_0}$. Hence, the simple evolution equations are valid, i.e.,

$$\phi^i(x, Q^2, \beta) q_i(x, Q^2) = \int_1^x d\bar{x} \int_{Q^2_0}^{Q^2} d\ln k_t^2 V^{i,j}\left(\frac{\bar{x}}{x}, \frac{Q^2}{k_t^2}\right) \phi^j(\bar{x}, Q^2_0, \beta) q_j(\bar{x}, k_t^2). \quad (5)$$

Here $i$ and $j$ label the parton flavors, $\phi_i(x, Q^2, \beta)$ is the fragmentation function for a residual system with parton $i$ removed, and $V^{i,j}$ is the hard blob leading to the standard kernel of the DGLAP evolution equation for inclusive scattering. The quantity $\beta = \frac{z}{(1-x)}$, which was defined in Eq. (2), is the light-cone fraction of the momentum of the initial nucleon carried by the spectator baryon. Conservation of $\beta$ in the evolution equation reflects the spectator origin of the leading baryon. Within the discussed above approximations $Q^2$ dependence of $\phi^i(x, Q^2)$ in Eq.(3) is due to the change of $x$ of the "parent" quark at $Q^2_0$ scale.

As long as $\bar{x}$ satisfies the condition

$$\bar{x} < x_{diff} \sim 10^{-2}, \quad (6)$$
the factorization expectations of Eq. (3) should hold. However, for fixed $x$, with increase of $Q^2$ the essential $\bar{x}$ increase, and at some $Q^2$ this inequality should be violated. Hence, in PQCD, we expect that for fixed $x$ with increase of $Q^2$ the factorization relation would gradually break down leading to a decrease of $f(z, p_t, x, Q^2)$ at large $z$.

Very recently, ZEUS has released [7] first data on the production of neutrons in the reaction

$$e + p \rightarrow e + n + X$$

in the target fragmentation region for

$$\langle x \rangle \approx 10^{-3} \text{ and } Q^2 \geq 10 \text{ GeV}^2.$$  (8)

The authors did not explicitly check factorization by comparing their data with real photon data on neutron production or with data on scattering of circulating protons off the gas in the vacuum tube. However, qualitatively, the data seem to be consistent with factorization and a weak $(x, Q^2)$ dependence of the neutron multiplicity. Such comparison high precision tests of factorization since many detector parameter uncertainties would cancel out in the relative measurements.

4 Break-down of factorization in semi-inclusive processes as a test of the DGLAP approximation at small $x$

In the kinematics of sufficiently small $x$ in DIS, where the QCD evolution equation should be violated, a contribution of multi-Pomeron exchanges may appear significant if the rapid increase of the parton distribution with decreasing $x$ would not be stopped through the diffusion of small configurations to the soft scale. Indeed, if we write $\sigma_{tot}$ as a sum of diffractive (rapidity gap) events and the inelastic contribution,

$$\sigma_{tot} = \sigma_{inel} + \sigma_{diff} ,$$  (9)
and assume that only double Pomeron exchanges are important, we obtain, using AGK cutting rules \[8\], that single multiplicity and double multiplicity events have the cross sections

\[
\begin{align*}
\sigma_{\text{single-mult}} &= \sigma_{\text{tot}} - 3 \sigma_{\text{diff}}, \\
\sigma_{\text{double-mult}} &= 2 \sigma_{\text{diff}},
\end{align*}
\]

respectively. Hence,

\[
\frac{\sigma_{\text{double-mult}}}{\sigma_{\text{single-mult}}} = \frac{2 \sigma_{\text{diff}}}{\sigma_{\text{tot}} - 3 \sigma_{\text{diff}}} = 0.55 - 1.0,
\]

where we take the diffraction fraction to be 15–20\%. For diagrams where two Pomerons are cut, the spectrum of leading nucleons (with \( z \geq 0.5 \)) should be reduced substantially similar to the reduction of the spectrum of leading nucleons in the \( p + A \to N + X \) reaction when the incoming nucleon interacts with two nucleons in the target.

Substantial fluctuations in the differential multiplicities of particles produced in the central rapidity range were observed at HERA for the soft \( \gamma p \) scattering \[9\]. AGK cutting rules have predicted such fluctuations. Models which include multi-Pomeron exchanges can quantitatively describe the observed multiplicity fluctuations, for a recent summary see Ref. \[10\]. Thus, it is quite likely that multi-Pomeron exchanges contribute significantly to the large multiplicity tail of these distributions.

Hence, we suggest to measure the multiplicity of the leading neutrons as a function of the multiplicity of the hadrons produced at central rapidities. This quantity can be defined as:

\[
g(z, k_t, x, Q^2, N_h(y_1, y_2)) \equiv \frac{1}{N(x, Q^2, N_h(y_1, y_2))} \frac{dN(x, Q^2, z, k_t, N_h(y_1, y_2))}{dz d^2k_t},
\]

where \( N_h(y_1, y_2) \) is the multiplicity of hadrons produced in the rapidity interval \( y_1 \leq y \leq y_2 \), and \( N(x, Q^2, N_h(y_1, y_2)) \) is the number of DIS events with multiplicity \( N_h(y_1, y_2) \). Screening effects would lead to a decrease of \( g(z, k_t, N_h(y_1, y_2)) \) for large \( z \) if one selects events with particle multiplicity in the central rapidity range satisfying the inequality

\[
N_h(y_1, y_2) \geq 2 \langle N_h(y_1, y_2) \rangle.
\]
Obviously, no such correlations are expected in the framework of the DGLAP evolution equations.

5 Factorization and the possibility of extracting the pion structure function from the $e + p \rightarrow e + n + X$ reaction

It was suggested in a number of papers, see e.g. Refs. [11, 12, 13], that the HERA collider data on the process of Eq. (7) could be used to measure the pion structure function at small $x$. Approximate factorization characteristic of soft QCD processes makes this extremely difficult (see however discussion in the end of the paragraph) since, in this limit, scattering off the Pomeron dominates. Scattering of any small $x$ partons will lead to essentially the same spectrum of leading neutrons as does hadron-proton scattering. Hence, the spectrum of leading neutrons would be fitted well by a parameterization corresponding to the sum of $PR_1R_2$ triple Reggeon terms, where $R_i = \pi, \rho, ...$—reggeons, see e.g. Ref. [14]. Note that in the high-energy limit, which we discuss, the contribution of $R_1R_2R_3$ terms is negligibl2. At the same time, there is no simple way to distinguish, in the $(x, Q^2)$ limit of Eq. (8) which was studied by ZEUS, the scattering of the virtual photon off pions belonging to the nucleon’s $|\pi N\rangle$ and $|\pi \Delta\rangle$ components from scattering off other components in the nucleon’s wave function.

To estimate the contribution of the scattering off the pion field to the proton’s structure function $F_2(x, Q^2)$, we use our analysis [16] of the nucleon’s antiquark distributions at $x \geq 0.15$. The latter provides lower limits on the slopes of the $\pi NN$ and $\pi N\Delta$ form

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Note that the $\rho$-Reggeon has no simple connection to the $\rho$-meson exchange used in models of the low-energy NN interaction, see e.g. Ref. [14], since the effective spin of the $\rho$-Reggeon is $\approx 1/2$ for small $t$ and not 1. The same is true for pion exchange away from $t \sim 0$. For example, for $-t \sim 0.2 - 0.3$ GeV$^2$, which gives the dominant contribution in pion models of the nucleon’s antiquark sea [16], the pion’s effective spin is $\alpha_\pi(t) = (t - m_\pi^2) + \alpha't \approx -(0.2 - 0.3)$.

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factors. Note that the lower limit corresponds to the maximal possible contribution of the pion field. It was demonstrated in Ref. [16] that, at small $x$ and especially with increasing $Q^2$, the pion contribution should be a relatively small fraction of the sea’s parton density. It is straightforward to extend this analysis to the even smaller $x$ of the ZEUS experiment [7]. We find that, though in this model the contribution of scattering off the pion field may exceed 50% for $Q^2 \leq 4$ GeV$^2$, it cannot exceed 30 – 35% of $F_2(x, Q^2)$ for the kinematics given by Eq. (8), see Fig. 1. There, we show results from fits to two experiments which measured the pion’s parton distributions via Drell-Yan scattering. The fit to the NA24 data [17] leads to a value of the ratio $F_{2\pi}(x, Q^2)/F_2(x, Q^2)$ close to 1 for $x \sim 10^{-3}$ and $Q^2 = 10$ GeV$^2$, while the NA10 fit [18] corresponds to a ratio of about 0.3. So, the results of the calculation based on the NA24 fit can be considered an upper limit for the pion contribution to $F_2(x, Q^2)$.

Besides, we want to stress that since the average relative distances in the $|\pi N(\Delta)\rangle$ configuration are small ($\leq 1$ fm) [16], screening effects, which we neglected in our model calculation of the pion contribution to $F_2$, would further reduce this contribution.

The main contribution to $F_2(x, Q^2)$ in the small $x$ and $Q^2 \geq 10$ GeV$^2$ range comes thus from the scattering off the gluon field, which, at low $Q^2$ resolution, originates predominantly from gluon emission from the valence quarks as well as non-perturbative gluons, and hence does not belong to the pions. However, based on the factorization argument, we expect that these configurations give a regular, unsuppressed contribution to the neutron spectrum.

Within models which explicitly include pion degrees of freedom to the nucleon’s wave function, scattering off a number of components, $|\pi^+ n\rangle$, $|\pi^0 p\rangle$, $|\pi^- \Delta^{++}\rangle$, $|\pi^0 \Delta^+\rangle$ and $|\pi^+ \Delta^0\rangle$, is important, see Fig. 2. Only about 50% of the final states result in the production of neutrons. Hence, the total multiplicity of neutrons in DIS due to the pion mechanism cannot exceed 15%, though for typical inelastic (non-diffractive) events one expects that the baryon multiplicities of protons and neutrons are about equal and close to 50% (where we neglected the small correction due to the production of strange baryons).
This expectation is consistent with bubble chamber neutrino data, which were analyzed, for instance, in Ref. [19].

To really measure the pion structure function, one would have to extrapolate to the pion pole, which requires measurements for $-t \sim m^2_\pi$. Since

$$-t = \frac{m^2_N (1-x) (1-x-z)^2}{z} + \frac{k^2_t (1-x)}{z}, \quad (15)$$

only the region of $z \geq 0.9$ and $k_t \leq m_\pi$ could be used for such measurements. The current angular and momentum resolution of the ZEUS detector [20] is approaching requirements necessary for such studies. However counting rates in this region would be quite low.

Note, also, that even interpolation to the pion pole for small $t$ would not be simple due to screening effects. These effects lead to a non-zero value for the cross section at $t = 0$, though in a naive pion exchange model this cross section would be proportional to $t$. This situation is analogous to the one encountered in low-energy charge exchange reactions where it causes substantial problems for an accurate determination of the $g_{\pi NN}$ coupling constant, for a recent discussion see Ref. [21].

To summarize, a study of leading baryon production in deep inelastic scattering can shed new light on the interplay of soft fragmentation dynamics and the properties of the nucleon’s parton wave function. It can furthermore provide an effective probe of the deviations from the DGLAP picture of small $x$ deep inelastic scattering.

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FIGURE CAPTIONS.

Fig.1. The relative contribution of scattering off the virtual pion cloud to the proton’s structure function $F_2(x, Q^2)$. Results are shown for various $Q^2$ and for the NA24 [17] and NA10 [18] pion structure functions. For further details see Ref. [16].

Fig.2. The relative contributions of the various virtual $p \rightarrow B\pi$ processes to $F_2^\pi(x, Q^2)$, the contribution to the proton’s structure function from scattering off its virtual pion cloud. Results are shown for $Q^2 = 10 \text{ GeV}^2$ and for the NA24 [17] pion structure functions.
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$Q^2 = 10 \text{ GeV}^2$

NA24