Improving Accuracy of Source Localization Algorithms Using Kalman Filter Estimator.

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Abstract. Passive geolocation of emitters provides benefits to military and civilian surveillance and in the field of aerospace. Passive geolocation can be done by exploiting some of the parameters of the signal transmitted by the source or emitter. In this paper the Time Difference Of Arrival (TDOA) of the signal at different receivers is used to estimate the position of the emitter. This paper aims at presenting the research for the development of an algorithm which can be used in a distributed network of acoustic sensors consisting of an emitter and five stationary receivers. By means of measurement of time difference of arrival of underwater acoustic signal the source position is estimated using three different algorithms. Taylor series, closed form and two stage weighted least square methods are used to estimate the stationary source location by TDOA measurements of received signal and compared on the basis of Cramer Rao Lower Bound (CRLB). Comparison of performance between these methods are done and analysed. Use of a kalman filter is proposed to improve the accuracy of all the three methods and the results are compared and analysed.

1. Introduction
Localization is defined as the process of obtaining the position estimate from multiple passive sensors. Source localization is an important problem in applications like radar, sonar, seismology, radio astronomy and mobile communications.

Position estimate of the source is obtained by exploiting different measurements of the received signal parameters. Signal measurements used are signal power, time, time difference and angle, the methods associated with these measurements are Received Signal Strength (RSS), Angle of Arrival (AOA) and Time of Arrival (TOA) respectively. AOA requires expensive antenna arrays. RSS requires an accurate propagation model and is affected by non-line of sight, shadowing, channel variations etc. TOA requires accurate time synchronization across source and all receivers. Also it requires the knowledge of the initial time (time at which signal is transmitted by the source).

In underwater localisation, a cooperation is to be established with the acoustic source (in the form of clock synchronization or a send–reply communication), making these methods inapplicable for solving the problem of localisation of an unknown underwater source. But Time Difference of Arrival (TDOA) techniques, which require having at least three receivers that allow the localization of the unknown acoustic source in the horizontal plane [4], can be used as a solution to the above problem. In
general, TDOA is a method that calculates the source location from the differences of arrival times measured on transmission paths between the source and fixed receivers [5]. The TDOA-based localization methods generally consist of two steps: the measurement step and the multilateration step. In the measurement step, the differences of acoustic signal arrival times on several receivers are measured. The source location will be on a hyperbola with constant range difference. The range difference is calculated from the measured difference in TOA and the speed of the acoustic signal. With more receivers, more hyperbolic functions will be computed, which ideally intersect in one unique point, thus establishing the source location [9]. Traditional 3-D underwater localization techniques require minimum four noncoplanar receivers to localize the source successfully [10]. Thus in TDOA method prior knowledge of transmit time is not required. Moreover, time synchronization between source and receivers is not required and hence source can be kept completely unknown. Constant TDOA curves between pairs of receivers are hyperbolic in nature and intersection of multiple hyperbolic curves gives the position estimate.

Various algorithms for position estimation using TDOA methods have been proposed so far. Among them there are three algorithms which use entirely different approaches in solving the constant TDOA curves. In this paper Taylor series, closed form and two stage weighted least square methods are implemented and compared on the basis of CRLB. Moreover accuracy of these algorithms are improved using Kalman filter approach [6]-[8].

2. Localisation scenario

Consider a 3D localization scenario that consists of a single stationary source at location \( u = [x, y, z]^T \) where \( x, y, z \) are the position coordinates along x axis, y axis and z axis respectively. Consider \( N \) number of sensors or receivers at locations \( s_i = [x_i, y_i, z_i]^T \) where \( i = 1, 2, ..., N \). Let first sensor be the reference sensor. The TDOA between \( i^{th} \) sensor and first sensor is \( t_i - t_1 \) where \( i = 1, 2, ..., N \). Hence the corresponding range difference of arrival (RDOA) is

\[
r_{i1} = c(t_i - t_1) = ct_i - ct_1 = r_i - r_1
\]

where \( i = 2, 3, ..., N \), \( c \) is the speed of propagation of emitter wave and \( r_i = \|u - s_i\|, i = 1, 2, ..., N \) is the distance between \( i^{th} \) sensor and source. The above scenario is practically prone to noises or errors. First is sensor position error. In this scenario the sensors are fixed and their positions are known so sensor position error is very negligible. Second is the TDOA measurement error. TDOA measurement error is assumed to be Gaussian with zero mean and variance \( \sigma^2 \). With the knowledge of sensor positions and TDOA measurements, estimate of the unknown source position can be found. The three most significant methods prevailing in literature are Taylor series method, Closed form method and Two stage weighted least square (TSWLS) method.

3. Taylor series method

Taylor series estimation method (also Gauss or Gauss Newton interpolation) [1] is an iterative scheme for solution of the simultaneous set of algebraic position equations. It starts with a rough initial guess and improves the guess at each step by calculating the local linear least-sum-squared-error correction. Four sensors are enough for this method and hence there will be three RDOA equations. RDOA equation between \( i^{th} \) sensor and emitter is

\[
f_k(x, y, z, x_i, y_i, z_i) = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} - \sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2} = m_k
\]

where \( m_k \) is the erroneous RDOA measurement \( k = 1, 2, 3 \) and \( i = k + 1 \). If \( x_v, y_v, z_v \) are guesses of the true source position and expand \( f_k \) in Taylor’s series keeping only terms below second order
\[ f_{kv} + a_{k1} \delta_x + a_{k2} \delta_y + a_{k3} \delta_z \approx m_k \quad (3) \]

Where
\[ f_{kv} = f_k(x_v, y_v, z_v, x_k, y_k, z_k) \]
\[ a_{k1} = \frac{\partial f_{kv}}{\partial x}, \quad a_{k2} = \frac{\partial f_{kv}}{\partial y}, \quad a_{k3} = \frac{\partial f_{kv}}{\partial z} \]

Hence,
\[ a_{k1} = \frac{x - x_k}{\sqrt{(x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2}} - \frac{x - x_1}{\sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2}} \quad (4) \]
\[ a_{k2} = \frac{y - y_k}{\sqrt{(x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2}} - \frac{y - y_1}{\sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2}} \quad (5) \]
\[ a_{k3} = \frac{z - z_k}{\sqrt{(x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2}} - \frac{z - z_1}{\sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2}} \quad (6) \]

In matrix form,
\[ A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (7) \]
\[ \delta = \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} \quad (8) \]
\[ Z = \begin{bmatrix} m_1 - f_{1v} \\ m_2 - f_{2v} \\ m_3 - f_{3v} \end{bmatrix} \quad (9) \]

We can rewrite the equations in matrix form as
\[ A\delta \approx Z \quad (10) \]

The \( \delta \) that gives least sum squared error is
\[ \delta = [A^T R^{-1} A]^{-1} A^T R^{-1} Z \quad (11) \]

And thus replace
\[ x_v \leftarrow x_v + \delta_x, \quad y_v \leftarrow y_v + \delta_y, \quad z_v \leftarrow z_v + \delta_z \]

4. Closed form method
This method provides the first 3D solution for the TDOA problem that is genuinely closed form, i.e., that does not depend on range data\[2\]. This method does not need the calculation of range data and does not depend on any other information other than times of arrival. The solution is based on transforming the TDOA equations which are hyperbolic (non-linear) into a set of vector equations.
4.1. Mathematical procedure
Localization scenario consists of a stationary emitter located at \([X_0,Y_0,Z_0]\) and stationary sensors at positions \([X_i,Y_i,Z_i]\) where \(i = 1,2,\ldots,5\). Let emitter emits an electromagnetic pulse at time \(t_0\). The time of arrival of this pulse at the \(i^{th}\) receiver is

\[
t_i = t_0 + \frac{D_i}{c} (12)
\]

where \(D_i\) is the distance between emitter and the \(i^{th}\) receiver and \(c\) is the speed of electromagnetic wave. We will keep one receiver as the reference receiver (usually the first receiver) so that if there are \(M\) receivers there will be \(M - 1\) TDOA equations. Then the time difference in arrival between receivers 1 and 2 is

\[
t_2 - t_1 = \frac{D_2 - D_1}{c} (13)
\]

This is the TDOA equation. TDOA equation in terms of unknown emitter coordinates \(X_0, Y_0, Z_0\) is

\[
\sqrt{(X_2 - X_0)^2 + (Y_2 - Y_0)^2 + (Z_2 - Z_0)^2} - \sqrt{(X_1 - X_0)^2 + (Y_1 - Y_0)^2 + (Z_1 - Z_0)^2}
= c(t_2 - t_1) (14)
\]

where \(X_2, Y_2, Z_2\) and \(X_1, Y_1, Z_1\) are coordinates of second and first receivers respectively. This method requires a minimum of 5 receivers [2]. A LOS scenario is approximated for aerospace and military applications. All the TDOA equations are transformed to a set of vector equations [2] as shown below

\[
a_{11}X_0 + a_{12}Y_0 + a_{13}Z_0 = b_1 (15)
\]

\[
a_{21}X_0 + a_{22}Y_0 + a_{23}Z_0 = b_2 (16)
\]

\[
a_{31}X_0 + a_{32}Y_0 + a_{33}Z_0 = b_3 (17)
\]

Where

\[
a_{11} = \frac{2}{(t_2 - t_1)} \left( \frac{X_2}{a_2^2} \frac{X_1}{a_1^2} \right) - \frac{2}{(t_3 - t_1)} \left( \frac{X_3}{a_3^2} \frac{X_1}{a_1^2} \right) (18)
\]

\[
a_{12} = \frac{2}{(t_2 - t_1)} \left( \frac{Y_2}{a_2^2} \frac{Y_1}{a_1^2} \right) - \frac{2}{(t_3 - t_1)} \left( \frac{Y_3}{a_3^2} \frac{Y_1}{a_1^2} \right) (19)
\]

\[
a_{13} = \frac{2}{(t_2 - t_1)} \left( \frac{Z_2}{a_2^2} \frac{Z_1}{a_1^2} \right) - \frac{2}{(t_3 - t_1)} \left( \frac{Z_3}{a_3^2} \frac{Z_1}{a_1^2} \right) (20)
\]

\[
b_1 = \frac{1}{(t_2 - t_1)} \left( \frac{X_2^2 + Y_2^2 + Z_2^2}{a_2^2} \right) - \frac{1}{(t_3 - t_1)} \left( \frac{X_1^2 + Y_1^2 + Z_1^2}{a_1^2} \right) + c^2(t_3 - t_2) (21)
\]

Here \(a_e\) are the coefficients showing path delay in multipath environments. Here in LOS Scenario its value is always 1. Rewriting the equations in matrix form we get,

\[
A = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix} (22)
\]
\[ B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \]  

(23)

Therefore the position estimate vector \( X = [x, y, z] \) is

\[ X = (A^T A)^{-1} A^T B \]  

(24)

5. Two stage weighted least square method

This method consists of two stages. In stage 1 an estimate of the target position is calculated. And in the second stage a correction to the previous result is made. This method is a closed form method and it does not require an initial guess of target location [3].

5.1 Mathematical procedure

Let \( \varphi_1 \) be the composite unknown vector to be estimated,

\[ \varphi_1 = [u^T, d_1]^T = (G_1^T W_1 G_1)^{-1} G_1^T W_1 h_1 \]  

(25)

The unknown composite vector has four unknowns i.e; three position coordinates of the emitter and \( d_1 \) the distance between source and the reference sensor that is the first sensor. To solve for four unknowns we need four TDOA equations which requires four sensor pairs and hence the number of sensors required is five in this method. Let

\[ G_1 = \begin{bmatrix} -(s_2 - s_1)^T & r_{21} \\ -(s_3 - s_1)^T & r_{31} \\ -(s_4 - s_1)^T & r_{41} \\ -(s_5 - s_1)^T & r_{51} \end{bmatrix} \]  

(26)

where \( t_{i1} = t_{i1} c \) is the range difference corresponding to \( t_{i1} \), \( t_{i1} \) is the TDOA between sensors \( i \) and sensor 1, \( i = 2, 3, \ldots, 5 \).

\[ h_1 = 0.5 \begin{bmatrix} r_{21}^2 + s_1^T s_1 - s_2^T s_2 \\ r_{31}^2 + s_1^T s_1 - s_3^T s_3 \\ r_{41}^2 + s_1^T s_1 - s_4^T s_4 \\ r_{51}^2 + s_1^T s_1 - s_5^T s_5 \end{bmatrix} \]  

(27)

The weighting matrix is the inverse of covariance of TDOA measurement noise \( Q \)

\[ W_1 = Q^{-1} \]  

(28)

There are two possible errors in this scenario. One is the error in the measurement of TDOA and the other is the sensor position error. Weighting matrix accounts for the total noise or error in the scenario. Since our sensors are fixed, sensors position errors are negligible and can be neglected. So the error to be considered here is the TDOA measurement here. It is assumed to be zero mean Gaussian distributed with variance \( \sigma^2 \) and covariance matrix \( Q \).

This forms the stage 1 of the TSWLS method. Stage 2 updates the stage-1 localization result \( u \) via first finding an estimate of its estimation error and then subtracting it from \( u \).

Invoking the WLS minimization, the estimate of the stage-1 localization error is given by

\[ \varphi_2 = (G_2^T W_2 G_2)^{-1} G_2^T W_2 h_2 \]  

(29)
The weighting matrix $W_2$ is defined as

$$W_2 = (B \text{cov}(\varphi_1) B^T)^{-1} \quad (30)$$

where the covariance matrix of the stage-1 output is

$$\text{cov}(\varphi_1) = (G_1^T W_1 G_1)^{-1} \quad (31)$$

And the other matrices are defined as

$$G_2 = \begin{bmatrix} I_{3 \times 3} \\ -\rho_{U,s_1}^T \end{bmatrix} \quad (32)$$

Where $\rho_{U,s_1} = (u - s_1)/\|u - s_1\|$ and

$$h_2 = \begin{bmatrix} 0_{3 \times 1} \\ d_1 \end{bmatrix} \quad (33)$$

$$B = \begin{bmatrix} -I_{3 \times 3} & 0_{3 \times 1} \\ 0_{2 \times 1} & 1 \end{bmatrix} \quad (34)$$

Thus $u = u - \varphi_2 \quad (35)$

6. Kalman filter

Kalman filter algorithm uses a prediction followed by a correction. It is sometimes called predictor-corrector or prediction-update filter. Starting from some initial state estimate $X_0$ and initial state error covariance matrix $P_0$, the predictor-corrector format is applied recursively at each time step.

6.1 Prediction

First the state vector is predicted from the state dynamics equation or process equation,

$$X_{k|k-1} = AX_{k-1} + w_{k-1} \quad (36)$$

Where $X_{k-1}$ is the previous estimated state vector, $w$ is predicted state noise matrix and $A$ is the system matrix. Then state error covariance matrix is predicted using,

$$P_{k|k-1} = AP_{k-1}A^T + Q_{k-1} \quad (37)$$

Where $Q$ is the process noise covariance matrix.

6.2 Correction

Let the measurement equation be

$$Y_k = H X_{k|k-1} + z_k \quad (38)$$

Where $Y$ is the filter output state vector, $H$ is the observation matrix and $z$ is the measurement noise vector. Then we calculate the Kalman gain matrix

$$K_k = (P_{k|k-1} H)/(HP_{k|k-1} H^T + R) \quad (39)$$
Where $R$ is the measurement noise covariance matrix. Then the current state vector is obtained by correcting the predicted state vector as,

$$X_k = X_{k|k-1} + K_k[Y_k - HX_{k|k-1}] \quad (40)$$

Where $Y$ is the measurement of the output and $HX_{k|k-1}$ is the predicted output. The term $Y_k - HX_{k|k-1}$ is called innovation.

Then the current state error covariance matrix is

$$P_k = (I - K_k H)P_{k|k-1} \quad (41)$$

In Kalman filter algorithm, we start with initial state vector $X_0$ which is the initial position estimate obtained from the source localization algorithms namely, Taylor series method, Closed from method and Two stage weighted least square method and its corresponding error covariance matrix called initial state covariance matrix $P_0$. Initially $P_0 = Q$.

7. Performance analysis

The estimator with the minimum variance or uncertainty is the best estimator. The lowest possible variance achievable by an estimator is called Cramer Rao Lower Bound (CRLB). The estimator whose variance or Mean square error lies close to CRLB is regarded as the best estimator. Lowest variance benchmark is the sum of diagonal elements of the below CRLB matrix

$$CRLB = \begin{bmatrix} X & Y \end{bmatrix}^{-1} \quad (42)$$

where

$$X = \left( \frac{\partial r}{\partial u} \right)^T Q_r^{-1} \frac{\partial r}{\partial u} \quad (43)$$

$$Y = \left( \frac{\partial r}{\partial s} \right)^T Q_s^{-1} \frac{\partial r}{\partial s} \quad (44)$$

$$Z = \left( \frac{\partial r}{\partial s} \right)^T Q_s^{-1} \frac{\partial r}{\partial s} + Q_s^{-1} \quad (45)$$

where

$$\frac{\partial r}{\partial u} = \begin{bmatrix} (u - s_2) & (u - s_1) & \cdots & (u - s_M) & (u - s_1) \\ \frac{r_2}{r_1} & \frac{r_1}{r_1} & \cdots & \frac{r_1}{r_1} & \frac{r_1}{r_1} \end{bmatrix} \quad (46)$$

$M$ is the number of sensors

$$\frac{\partial r}{\partial s} = \begin{bmatrix} (u - s_1) & \cdots & (u - s_1) \\ \frac{r_1}{r_1} & \cdots & \frac{r_1}{r_1} \\ (u - s_2) & \cdots & 0 \\ \frac{r_2}{r_2} & \cdots & \vdots \\ \vdots & \vdots & \vdots \\ 0 & \cdots & (u - s_M) \frac{r_M}{r_M} \end{bmatrix} \quad (47)$$

8. Simulation

Consider the localization scenario with five stationary sensors and a stationary source at [2000,2500,3000]$^T$. The sensors are positioned at $s_1 = [150,100,50], s_2 = [500,200,400], s_3 = [200,100,300], s_4 = [-300,-200,200], s_5 = [100,100,100]$. Let the unknown source position estimate be $u_0 = [x,y,z]^T$. The source is stationary i.e, velocity $v = [x,y,z]^T = [0]$. In all the three
method we need to get the TDOA measurements and use this data along with the known sensor
positions to find the source position estimate.

The two possible errors in these methods are the error in TDOA measurement and sensor position
error. TDOA measurement error is assumed to be Gaussian, zero mean with covariance matrix $Q_t$.
Sensor position error is also zero mean Gaussian with covariance matrix $Q_s$. Here $Q_t = \sigma_t^2 R$ where $\sigma_t^2$ is the TDOA noise variance and here it is taken to be 0.01. $R$ is the correlation matrix whose
diagonal elements are unity and other elements are 0.5.

$Q_s = \sigma_s^2 J$ where $\sigma_s^2$ is the sensor position noise variance and here it is $10^{-4}$ and $J = diag(0.1,0.1,0.1,0.1,0.1)$. Diagonal elements of $J$ shows the position error at each sensor and is same
for all three sensor coordinates. The localization Mean Square Error (MSE) of each method is
compared with respect to CRLB.

The comparison plot is a function of TDOA noise variance $\sigma_t^2$ and localization MSE where

$$
MSE(x) = \frac{\sum_{l=1}^{L} \| \hat{x}^{(l)} - x \|^2}{L} 
$$

(48)

$\hat{x}$ is the position estimate and $x$ is the actual position and $L$ is number of ensemble runs. Fig.1 shows the source position estimate without Kalman filter. Fig.2 shows the performance analysis of the three
methods in terms of CRLB.

For Kalman filter let the number of iterations is taken as 20 with step size $dt = 1$. The state vector
$X = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T$,

$$
A = \begin{bmatrix}
1 & 0 & 0 & dt & 0 & 0 \\
0 & 1 & 0 & 0 & dt & 0 \\
0 & 0 & 1 & 0 & 0 & dt \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
$$

The observation matrix $H = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}$

The process noise covariance matrix $Q$ and measurement noise covariance matrix $R$ is assumed to
have a variance of 0.1. Fig.3 shows the source position estimate after applying Kalman filter algorithm.
**Figure 1.** Source position estimate without Kalman filter

**Figure 2.** Comparison with CRLB

**Figure 3.** Source position estimate with Kalman filter

**Table 1.** Comparison of position error of three methods and with kalman filter

| Method          | Position estimate error m | Position estimate error with kalman filter m |
|-----------------|---------------------------|---------------------------------------------|
| Taylor series   | 25.09                     | 0.66                                        |
| Closed form     | 183.7                     | 18.9                                        |
| TSWLS           | 29.3                      | 1.5                                         |

Table 1 shows that the error estimates are better for the implementation with kalman filter.

9. Conclusion

From the comparison result we understand that the methods Taylor series Expansion and Two Stage Weighted least square (TSWLS) are equally good. At low noise conditions both methods have a 2-3 dB deviation from CRLB curve. At medium noise conditions the deviation is 3-4 dB and at high noise conditions there is 5 dB deviation.

But the slope is maintained at all noise conditions. So Taylor series method and Two stage weighted least square methods are robust and are closer to CRLB. Closed form method shows a deviation as high as 12-14dB from CRLB at low and medium noise conditions. The slope tend to increase at high noise making it not suitable at high noise conditions. The simulation results indicate that the lowest error possible at high noise condition is achieved by Taylor series method and is approximately 25 m. 25 m error is not a promising number when compared with other localization methods like GPS. Applying Kalman filter, filters out the noise in the position estimate. As per the simulation results the error has decreased to almost less than a metre which is very promising in our future studies.

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