Energy reconstruction of hadronic showers at the CERN PS and SPS using the Semi-Digital Hadronic Calorimeter

The CALICE Collaboration

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The CALICE Semi-Digital Hadronic CALorimeter (SDHCAL) is the first technological prototype in a family of high-granularity calorimeters developed by the CALICE Collaboration to equip the experiments of future lepton colliders. The SDHCAL is a sampling calorimeter using stainless steel for absorber and Glass Resistive Plate Chambers (GRPC) as a sensitive medium. The GRPC are read out by 1 cm $\times$ 1 cm pickup pads combined to a multi-electronics. The prototype was exposed to hadron beams in both the CERN PS and the SPS beamlines in 2015 allowing the test of the SDHCAL in a large energy range from 3 GeV to 80 GeV. After introducing the method used to select the hadrons of our data and reject the muon and electron contamination, we present the energy reconstruction approach that we apply to the data collected from both beamlines and we discuss the response linearity and the energy resolution of the SDHCAL. The results obtained in the two beamlines confirm the excellent SDHCAL performance observed with the data collected with the same prototype in the SPS beamline in 2012. They also show the stability of the SDHCAL in different beam conditions and different time periods.

**KEYWORDS:** Calorimeter methods; Calorimeters

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1 Introduction

The Semi-Digital Hadronic CALorimeter (SDHCAL) [1] is the first of a series of technological high-granularity prototypes developed by the CALICE collaboration. The SDHCAL is a sampling calorimeter using stainless steel for absorber and Glass Resistive Plate Chambers (GRPC) for its sensitive medium. The SDHCAL is designed to be as compact as possible with its mechanical structure being part of the absorber. The GRPC and the readout electronics are conceived to achieve minimal dead zones [1]. This design renders the SDHCAL optimal for the application of the Particle Flow Algorithm (PFA) techniques [2–4]. Indeed, the high granularity of the SDHCAL would allow a better reconstruction of the hadronic showers and thus a better separation of nearby showers originating from different charged and neutral particles. The better the separation the more efficient the PFA techniques. The latter aim at individually measuring the energy or the momentum of each particle using the most appropriate sub-detector. For this a good separation of the different particles is required. For instance, charged hadron momentum is better measured in a tracker than in the calorimeter. Thus a better separation between neutral and charged hadrons in the hadronic calorimeter will improve the Jet Energy Reconstruction (JER).

The SDHCAL, shown in figure 1, comprises 48 active layers, each of them equipped with a $1 \times 1$ GRPC and an Active Sensor Unit (ASU) of the same size made of a Printed Circuit Board (PCB) hosting on one side (the one in contact with the GRPC) pickup pads of $1 \text{cm} \times 1 \text{cm}$ size each and 144 HARDROC2 ASICs [5] on the other side (figure 2). The GRPC and the ASU are assembled within a cassette made of two stainless steel plates, 2.5 mm thick each. The 48 cassettes are inserted
in a self-supporting mechanical structure made of 49 plates, 15 mm thick each, of the same material as the cassettes, bringing the total absorber thickness to 20 mm per layer. The empty space between two consecutive plates is 13 mm to allow the insertion of one cassette of 11 mm thickness. In total, the SDHCAL represents about 6 interaction lengths \( \lambda_I \). The HARDROC2 ASIC has 64 channels to read out 8\times8 pickup pads of 1 cm\times1 cm each. Each channel has three parallel digital circuits whose parameters can be configured to provide 2-bit encoded information per channel indicating if the charge seen by each pad has passed any of the three different thresholds associated to each digital circuit. Only pads with charge higher than the first threshold fixed at 114 fC are considered. The second and the third thresholds are chosen to be 5 and 15 pC respectively. The use of these thresholds is to allow separating pads that are crossed either by one, a few or many particles within a shower and this information is used to improve on the energy reconstruction of hadronic showers at high energy (\textgreater{} 30 GeV) with respect to the simple binary readout mode. The value of the first threshold is chosen to eliminate the different noise sources. The other two values are selected to allow an optimal energy reconstruction as explained in ref. [6].

The SDHCAL was exposed to different kinds of particles at the CERN SPS beamline in 2012 and its performance was studied in the energy range above 10 GeV [6]. To study its performance at lower energies, the SDHCAL was exposed in 2015 to negatively charged pion beams of 3, 4, 5, 6, 7, 8, 9, 10 and 11 GeV at the PS beamline. It was then exposed with the same configurations to positively charged hadrons of 10, 20, 30, 40, 50, 60, 70 and 80 GeV at the SPS beamline. For each energy point about 10000 events were collected.

In this paper, section 2 gives the details of the collected beam data in 2015 as well as the samples of simulated events used for comparison. The pion selection and the muon and electron contamination rejection using the MultiVariate Analysis (MVA) technique known as Boosted Decision Trees (BDT) to separate pion and electron showers is given in section 3. Energy reconstruction of the selected pion events is discussed in section 4. Finally, in section 5 we present the uncertainties related to the energy reconstruction of the collected data.

## 2 Simulation

The simulation model of SDHCAL, based on Geant4.9.6 toolkit package [7], was developed including the interactions of different kinds of particles such as muons, electrons and pions in the SDHCAL prototype. The simulation takes into account the operation conditions of the GRPC. It uses the same values of 0.114, 5 and 15 pC that are used by the SDHCAL readout system for the first, the second and the third threshold respectively.

Among the different Geant4 physics lists that were used to compare the simulation with the beam data collected in 2012, FTF_BIC was found to provide the best agreement [8, 9] for what concerns the distributions of the different variables related to the structure of the showers in the SDHCAL. Since these variables are the most relevant ones in the present analysis, we use this physics list in this work to simulate events with different kinds of particles having the same energies and impinging on the prototype in the same area as those to which the SDHCAL was exposed during the beam test campaigns at the SPS and PS beamlines.

The simulated events are then used to reject contamination by electron and muons of the pion test beam data. They are also used to estimate possible biases that may influence the data energy reconstruction of hadronic showers in terms of linearity and resolution.
3 Pion events selection

The pion samples of both SPS and PS beams are contaminated by two kinds of particles: electrons and muons. The muon contamination is present in all samples. This includes two different types of muons: cosmic muons and beam muons. The latter are generated by pions decaying before arriving at the prototype. Concerning the electron contamination, it is present in the pion beam. It is negligible in pion samples from 6 to 11 GeV in the PS pion beam but still present in the energy range below 6 GeV at the level of a few percent of the beam content. The electron contamination is also present at the level of a few percent in the SPS pion beam, especially in the energy range between 10 and 50 GeV [6]. At the reconstruction level, the muon rejection is rather easy due to their track-like shape that distinguishes them clearly from the hadronic showers in the SDHCAL. The electron rejection is however harder. Electron showers, in particular at low energy, are similar to the pion ones. Although in both PS and SPS, an electron stopper made of a few millimeters thick lead plate was used, this does not allow to completely eliminate the electron contamination. Inspired
by refs. [10, 11], we propose to use the BDT technique to reject the electron background of our pion samples in an improved way with respect to the one used in a previous analysis applied to data collected by SDHCAL iat the SPS beamline [12] in the energy range between 10 and 80 GeV.

3.1 Electron contamination rejection using Boosted Decision Tree (BDT)

As mentioned in the previous section, electron contamination is present in PS and SPS pion samples. Therefore, it is necessary to check the electron contamination and to eliminate it. Thanks to the high granularity of the SDHCAL prototype, we can use the BDT method to exploit the three dimensional shape of both the electromagnetic and the hadronic showers to classify the electron and pion events in our prototype. The TMVA package [13] contains an implementation of this technique. We adopt this package to build our BDT model to reject the electron contamination.

3.1.1 BDT input variables

Based on the difference in topology between electromagnetic and hadronic showers, we choose eight variables as inputs of the BDT model to help discriminate pion against electron events. Hereafter, a description of each of these variables is given:

- *First layer of the shower (Begin)*: the SDHCAL layers being perpendicular to the beam, to define the layer in which the shower starts, we look for the first layer which contains at least 4 fired pads. To eliminate fake shower starts due to accidental noise or a local high multiplicity, the following 3 layers after the first layer are also required to have more than 4 fired pads (hits) for each of them. If no layer fulfils this, a value of $-10$ is assigned to the variable. Since each layer of the SDHCAL represents about 1.2 radiation lengths ($X_0$), electromagnetic showers start developing in the first layers. For pions, their interaction probability density is given by $1 - \exp\left(-\frac{l}{\lambda_i}\right)$ where $l$ is the length of the pion trajectory in the calorimeter medium before interacting. Figure 3 shows the distribution of the first layer index of the shower in the SDHCAL prototype for pions and electrons as obtained from the simulation.

![Graph showing the distribution of the first layer index of pion and electron showers in the PS energy range of 1-12 GeV and the SPS energy range of 10-80 GeV.](image)

**Figure 3.** Distribution of the first layer index of pion and electron showers in the PS energy range of 1-12 GeV (left) and the SPS energy range of 10-80 GeV (right) of pions and electrons as given by the simulation. The red line corresponds to pions and the black one to electrons.

\footnote{The number of hit due to noise in one SDHCAL event is very low and was estimated to be lower than 2.3 per event in [6].}
• **Number of track segments in the shower (nTrack):** applying the Hough Transform (HT) method to single out the tracks in each event as described in ref. [14], we obtain the number of track segments in the pion, electron events. A HT-based segment candidate is considered as a track segment if there are more than 6 aligned hits with not more than one layer separating two consecutive hits. Due to their pure electromagnetic interaction and their content made essentially of photons, electrons and positrons, showers produced by electrons feature almost no track segment while most of the hadronic showers have at least one. The distribution of nTrack can be seen in figure 4.

![Figure 4](image1.png)

**Figure 4.** Distribution of the number of track segments in pion and electron showers in the PS energy range of 1-12 GeV (left) and the SPS energy of 10-80 GeV (right) as given by the simulation. The red line corresponds to pions and the black one to electrons.

• **Number of clusters of the shower (nCluster):** all hits in a given layer are clustered using a nearest-neighboring algorithm described in ref. [1]. It consists in merging in each GRPC plate the hits sharing a common edge. This variable defines the number of clusters of the shower and its distribution is shown in figure 5. Indeed, the compactness of the electromagnetic shower leads to a reduced number of clusters in an electron shower with respect to that of a pion shower of the same energy.

![Figure 5](image2.png)

**Figure 5.** Distribution of the number of clusters in pion and electron showers in the PS energy range of 1-12 GeV (left) and the SPS range of 10-80 GeV (right) as given by the simulation. The red line corresponds to pions and the black one to electrons.
• **Ratio of the number of shower layers over the total number of fired layers (nInteractingLayers/nLayers):** this is the ratio between the number of the shower layers defined as those in which the Root Mean Square (RMS) of the hits’ position in the $x$-$y$ plane exceeds 5 cm in both $x$ and $y$ directions and the total number of layers with at least one hit. This variable allows, as can be seen in figure 6, a good separation between pions and electrons at low energy.

![Figure 6](image)

**Figure 6.** Distribution of the ratio of number of shower layers over the total number of fired layers in pion and electron showers in the PS energy range of 1-12 GeV (left) and the SPS energy range of 10-80 GeV (right) as given by the simulation. The red line corresponds to pions and the black one to electrons.

• **The average number of hits per fired layer (nHit/nLayer):** this is the ratio between the total number of fired pads over the number of layers with at least one fired pad. The distribution of this variable is shown in figure 7.

![Figure 7](image)

**Figure 7.** Distribution of the average number of hits per fired layer in pion and electron showers in the PS energy range of 1–12 GeV (left) and the SPS range of 10–80 GeV (right) as given by the simulation. The red line corresponds to pions and the black one to electrons.

• **Shower density (Density):** this is the average number of the neighbouring hits located in the $3 \times 3$ pads around one of the hits (including the hit itself) for the hits in all layers in the given event. Figure 8 shows clearly that electromagnetic showers are more compact than the hadronic ones as expected.
Figure 8. Distribution of the density of pion and electron showers in the PS energy range of 1–12 GeV (left) and the SPS range of 10–80 GeV (right) as given by the simulation. The red line corresponds to pions and the black one to electrons.

- **Shower radius (meanRadius):** this is the RMS of hits distance with respect to the event axis. To estimate the event axis, the average positions of the hits in each of the ten first fired layers of an event are used to fit a straight line. The straight line is then used as the event axis. The electromagnetic shower being more compact than the hadronic shower, its radius is expected to be smaller as can be seen in figure 9.

Figure 9. Distribution of the meanRadius of pion and electron showers in the PS energy range of 1–12 GeV (left) and the SPS range of 10–80 GeV (right) as given by the simulation. The red line corresponds to pions and the black one to electrons.

- **Ratio of the number of third-threshold hits over the total number of hits (nHit3/nHit):** the three thresholds indicate the amount of charge collected in each pickup pad. The third one is set to single out the pads with high collected charge that may be induced by the passage of many particles in the cell associated to the pickup pad. The nHit3 is the number of third-threshold hits in one event. The ratio of nHit3 to the total number of hits helps to distinguish electromagnetic-like events and separate them from hadronic-like ones since the relative number of hits with the third threshold is higher in the former than in the latter due to the difference of their compactness. The distribution of this ratio can be seen in figure 10.
Figure 10. Distribution of the ratio of number of third threshold hits over the total number of hits in pion and electron showers in the PS energy range of 1–12 GeV (left) and the SPS range of 10–80 GeV (right) as given by the simulation. The red line corresponds to pions and the black one to electrons.

Table 1. The chosen BDT hyperparameters.

| Option                                      | Setting |
|---------------------------------------------|---------|
| Ntrees (Number of trees in the forest)      | 1000    |
| nCuts (Number of steps during node cut optimisation) | 20      |
| MaxDepth (Max depth of the decision tree allowed) | 4       |

3.1.2 Training and testing details of the BDT

For the training and testing process, 200000 pion and 200000 electron simulated events are used to form a training set (66.7%) and a testing one (33.3%). Another independent 400000 events including pions and electrons are used as a validation set. The same number of events is simulated for each energy point. The hyperparameters resulting from the BDT optimisation procedure such as maxDepth\(^2\) are described in table 1. After feeding the eight topological variables to the BDT model using the training and testing sets, the performance of our model is shown in figure 11. It clearly shows the strong separation power between pions and electrons. At the same time, the BDT response of the validation sets has very good agreement with training ones. This confirms that our model is performing very well and is not subject to overfitting. After applying the muon rejection cuts to be explained in section 3.2 to our collected data, we apply our BDT models on the selected events. Figure 12 shows the BDT output of 6 GeV (left plot) and 11 GeV (right plot) pion runs which are supposed to be free of electron contamination. The performance of the pion event selection matches the one obtained with the simulated pion events quite well as shown in this figure, thus confirming that our model is reliable. Figure 13 shows the result of 3 GeV and 5 GeV pion beam runs which, in principle, may contain electron contamination. From this figure, one can see that most of the events are located in the region associated to pions and a good agreement is observed between the data and the simulation even in the region of overlap between the pions and the electrons. Therefore, the electron contamination in pion runs in the 3–5 GeV energy range is rather small even

\(^2\)It controls the maximum depth of the tree that will be created. It can also be described as the length of the longest path from the tree root to a leaf. The root node is considered to have a depth of 0.
if it is more important than that of the PS higher energy runs. By requiring the BDT response to be larger than 0.0, we reject almost all of the electrons (> 99%) keeping rather pure pion events.

**Figure 11.** Distribution of the BDT output of training and validation set using the simulated electron (black) and pion (red) events from 1 GeV to 80 GeV. The solid line is from training set while the dashed one is from validation set.

**Figure 12.** The BDT output of 6 GeV (left) and 11 GeV (right) beam runs after muon rejection. The solid line is from pion beams and dashed one is from training set.

### 3.2 Muon contamination rejection

The main contamination in our beam data is that of muons, including beam muons and cosmic ones. To eliminate these two kinds of muons, we use the information based on the different behaviours of muons and pions in the SDHCAL prototype. Basically, muons cross the prototype and only leave a straight track in the prototype like the one shown in figure 14. The mean of hits distance (described by the variable meanRadius hereafter) of muon hits with respect to the global event axis is thus very often less than 1.5 cm (≈ 1.5 pads) as shown in figure 15.

To eliminate most of the muon contamination, we require that the meanRadius is greater than 2 cm. To further reduce the muon contamination, including the so-called radiative muons that...
Figure 13. The BDT output of 3 GeV (left) and 5 GeV (right) beam runs after muon rejection. The solid line is from pion beams and dashed one is from training set.

Figure 14. Event display of one 6 GeV simulated muon with the green, blue and red colour indicating the first, second and third threshold hits respectively. The third threshold is often absent in muon tracks because of the small amount of charge produced by muons in the GRPC.

produce a few hit clusters around the muon track, we require the ratio of the number of shower layers to the total number of layers with at least one hit to be more than half.

To check the rejection power of the muon cuts, we apply it to dedicated muon runs. Figure 16 shows the distribution of the number of hits before and after muon rejection for 120 GeV muon runs. It clearly shows the rejection power of this selection which is higher than 99.0%.

The result of the selection including the muon rejection and electron cut (BDT response > 0.0) is shown in figure 17 and figure 18 for PS and SPS beam data runs respectively.

4 Energy reconstruction

The rejection of electrons present in the pion data sample using the BDT but also that of the muons allows us to have pure pion sample as explained in the previous section. The selected pion events of
Figure 15. Distribution of the meanRadius of 1 GeV to 12 GeV muons as given by the simulation (green). MeanRadius of electrons (black) and that of pions (red) of the same energies are also shown.

Figure 16. Distribution of the number of hits for 20 GeV muon run before (solid line) and after (dashed line) muon cut.

Figure 17. The number of hits for 3, 7 and 11 GeV pion beam runs before (blue) and after (red) muon selection.
Figure 18. The number of hits for 20, 40 and 60 GeV pion beam runs before (blue) and after (red) muon selection.

the PS beam energy from 3 GeV to 11 GeV and those in the range of 10-80 GeV of the SPS, can then be used to reconstruct energy. Based on the information of the number of hits belonging to first threshold (nHit1), second threshold (nHit2) and third threshold (nHit3), the hadronic shower energy can be reconstructed as described in ref. [6] using the following formula:

\[ E_{\text{reco}} = \alpha \times n\text{Hit}1 + \beta \times n\text{Hit}2 + \gamma \times n\text{Hit}3 \]  

(4.1)

where \( \alpha, \beta \) and \( \gamma \) are weight factors which are parametrised as second order polynomials of the total number of hits \( n\text{Hit} = n\text{Hit}1 + n\text{Hit}2 + n\text{Hit}3 \):

\[
\begin{align*}
\alpha &= \alpha_1 + \alpha_2 \times n\text{Hit} + \alpha_3 \times n\text{Hit}^2 \\
\beta &= \beta_1 + \beta_2 \times n\text{Hit} + \beta_3 \times n\text{Hit}^2 \\
\gamma &= \gamma_1 + \gamma_2 \times n\text{Hit} + \gamma_3 \times n\text{Hit}^2
\end{align*}
\]  

(4.2)

The nine parameters \( \alpha_i=1,2,3, \beta_j=1,2,3 \) and \( \gamma_k=1,2,3 \) are obtained, as described in ref. [6], from a part of the data samples of a few energy points by minimising the following \( \chi^2 \) expression:

\[
\chi^2 = \sum_{i=1}^{N} \frac{(E_{\text{beam}}^i - E_{\text{reco}}^i)^2}{\sigma_i^2}
\]  

(4.3)

where the \( E_{\text{beam}}^i \) denotes the beam energy and the \( E_{\text{reco}}^i \) is the reconstructed energy. \( N \) is the number of total events and \( \sigma_i = \sqrt{E_{\text{beam}}^i} \) where the choice of \( \sigma = \sqrt{E_{\text{beam}}} \) is motivated by the fact that the expected energy resolution is approximately given by the stochastic term: \( \sigma_{E_{\text{beam}}} = \frac{\sigma}{\sqrt{E_{\text{beam}}}} \).

Since the PS raw 10 GeV sample is almost free of electron contamination, it is therefore expected to be less impacted by the BDT-based selection. On the contrary, the SPS raw 10 GeV sample electron contamination is relatively higher and could thus be impacted by the BDT-based selection. To check however that this selection only eliminates the electrons without changing the pion sample characteristics, the reconstructed energy of the PS 10 GeV sample without BDT selection is compared to that of the SPS one after applying the BDT selection. Figure 19 shows the normalised reconstructed energy distribution of these two samples as well as their relative difference. The rather good agreement between the two distributions confirms the absence of bias of the BDT selection and its efficiency in rejecting the electron contamination.
4.1 Energy resolution and linearity

The two purified samples; the one of 3–11 GeV and the one of 10–80 GeV collected at the PS and the SPS beamline respectively, are then used to reconstruct the pion energy in the SDHCAL following the method described in ref. [6].

The reconstructed energy distributions of 3, 7 and 11 GeV pion data samples collected at PS are shown in figure 20.

![Reconstructed energy distributions for 3 (left), 7 (middle) and 11 GeV (right) pion data samples collected at the PS. The distributions are fitted with a double sided Crystal Ball function. The variance of the Gaussian part of the Crystal Ball function is used to estimate the resolution of the reconstructed energy.](image)

The reconstructed energy distributions of 20, 40 and 60 GeV pion data samples collected at SPS are shown in figure 21.

The reconstructed energy distribution of each energy point from 3 to 80 GeV is fitted using a double-sided Crystal Ball function. This function takes into account that the reconstructed energy distribution is not a pure gaussian one and thus adds tails of polynomial shape to better determine the range where the distribution is rather gaussian.

![Reconstructed energy distributions of 20, 40 and 60 GeV pion data samples collected at SPS.](image)
Figure 21. Reconstructed energy distributions for 20 (left), 40 (middle) and 60 GeV (right) pion data samples collected at SPS. The distributions are fitted with a double-sided Crystal Ball function. The variance of the Gaussian part of the Crystal Ball function is used to estimate the resolution of the reconstructed energy.

Figure 22. Mean reconstructed energy of pion showers as a function of the beam energy as well as relative deviation of the pion mean reconstructed energy with respect to the beam energy (left) and resolution of the reconstructed hadron energy as a function of the beam energy (right). Both statistical and systematic uncertainties are included in the error bars. Dashed line on the left plot indicates the ideal linearity response of the calorimeter.

The mean value and standard deviation of the Gaussian part of the latter are taken as the reconstructed energy and its resolution respectively. In figure 22, the energy linearity (left) and resolution results (right) are shown using both PS and SPS data.

The same procedure is applied to the SPS sample only. Similar results as the one obtained with both PS and SPS beamlines are obtained as can be shown in figure 23. More importantly, these SPS results are similar to those obtained in 2012 [6]. This confirms the robustness of the SDHCAL prototype over time.
Figure 23. Mean reconstructed energy for pion showers as a function of the beam energy as well as relative deviation of the pion mean reconstructed energy with respect to the beam energy (left) and resolution of the reconstructed hadron energy as a function of the beam energy (right). Both statistical and systematic uncertainties are included in the error bars. Dashed line on the left plot indicates the ideal linearity response of the calorimeter.

5 Uncertainties estimation

The linearity and energy resolution results presented in section 4.1 include statistical and systematic uncertainties. We present here after the main contributions to the systematic uncertainties:

- For the reconstructed energy of all energy points, a double sided Crystal Ball fit function and a Gaussian fit function are used. In the case of the Gaussian case, a first fit is applied providing a rough estimate of the position of the distribution maximum $E_0$ and its standard deviation $\sigma_0$. A second fit in the range $[E_0 - 1.5\sigma_0, E_0 + 1.5\sigma_0]$ is then applied to determine the mean value and the standard deviation that are used in the comparison with the Crystal Ball function. The difference of fitting results obtained from these two fit functions are considered as the value of systematic uncertainties associated to the fit of the reconstructed energy.

- For the muon rejection, using all energy points data samples of PS, the meanRadius varied by an arbitrary 5% in both directions with respect to the nominal values. The maximum deviation with respect to the nominal value is used as an estimate of the systematic uncertainties due to a residual muon contamination.

- For the electron rejection using the BDT method, the BDT cut value is changed from −0.05 to 0.05 with respect to the nominal values 0.0. The maximum deviations are taken and added to the systematic uncertainties as an estimate of the impact of residual electron contamination.
Although the statistical uncertainties are found to be negligible for almost all the runs with respect to systematic uncertainties, their contributions as well as the systematic uncertainties previously discussed are added quadratically to obtain the final uncertainties. The results are summarised in tables 2 and 3. The uncertainty coming from the difference of fit functions is found to be the main component (90%) of the total systematic uncertainties.

6 Conclusion

The data collected from the exposure of the SDHCAL prototype to pion beams in both PS and SPS covering a large range of energy (3-80 GeV) are analyzed. Rejection of muon and electron contamination is performed. For the latter a BDT-based technique is applied. This technique allows the rejection of the electron contamination without reducing the pion sample compared to the analysis used in ref. [6] where the electron contamination was reduced by requiring the interaction to start showering after ten radiation lengths ($X_0$) leading to a loss of about half of the pion events. Energy of the pions collected in both PS and SPS is then reconstructed following the techniques developed in ref. [6] and compared with those obtained with the SPS data only and with those obtained in 2012 following a standard selection analysis. The results show that good performances including excellent linearity and energy resolution, are obtained over a large dynamic range from 3 to 80 GeV.

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| Energy (GeV) | Beam data of PS and SPS |
|-------------|-------------------------|
| 3           | 0.282 ± 0.085           |
| 4           | 0.281 ± 0.081           |
| 5           | 0.279 ± 0.042           |
| 6           | 0.266 ± 0.030           |
| 7           | 0.251 ± 0.025           |
| 8           | 0.244 ± 0.020           |
| 9           | 0.244 ± 0.001           |
| 10          | 0.236 ± 0.001           |
| 11          | 0.224 ± 0.006           |
| 20          | 0.160 ± 0.007           |
| 30          | 0.130 ± 0.001           |
| 40          | 0.117 ± 0.001           |
| 50          | 0.103 ± 0.002           |
| 60          | 0.092 ± 0.005           |
| 70          | 0.081 ± 0.009           |
| 80          | 0.067 ± 0.001           |

Table 3. List of energy resolution observed and associated uncertainties for beam data in the energy range from 3 to 80 GeV.

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