A NECESSARY AND A SUFFICIENT CONDITION FOR THE EXISTENCE OF THE POSITIVE RADIAL SOLUTIONS TO HESSIAN EQUATIONS AND SYSTEMS WITH WEIGHTS

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Abstract. In this article we consider the existence of positive radial solutions for Hessian equations and systems with weights and we give a necessary condition as well as a sufficient condition for a positive radial solution to be large. The method of proving theorems is essentially based on a successive approximation. Our results complete and improve a recently work published by Zhang and Zhou (Existence of entire positive k-convex radial solutions to Hessian equations and systems with weights, Applied Mathematics Letters, Volume 50, December 2015, Pages 48–55).

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1. Introduction

Let $D^2u$ be the Hessian matrix of a $C^2$ (i.e., a twice continuously differentiable) function $u$ defined over $\mathbb{R}^N$ ($N \geq 3$) and $\lambda(D^2u) = (\lambda_1, ..., \lambda_N)$ the vector of eigenvalues of $D^2u$. For $k = 1, 2, ..., N$ is defined the $k$-Hessian operator as follows

$$S_k(\lambda(D^2u)) = \sum_{1 \leq i_1 < ... < i_k \leq N} \lambda_{i_1} \cdot ... \cdot \lambda_{i_k}$$

i.e., it is the $k^{th}$ elementary symmetric polynomial of the Hessian matrix of $u$. In other words, $S_k(\lambda(D^2u))$ it is the sum of all $k \times k$ principal minors of the Hessian matrix $D^2u$ and so is a second order differential operator, which may also be called the $k$-trace of $D^2u$. Especially, it is easily to see that the $N$-Hessian is the Monge-Ampère operator and that the 1-Hessian is the well known classical Laplace operator. Hence, the $k$-Hessian operators form a discrete collection of partial differential operators which includes both the Laplace and the Monge-Ampère operator.

In this paper we study the existence of radial solutions for the following Hessian equation

$$S^{1/k}_k(\lambda(D^2u)) = p(|x|) h(u) \quad \text{in} \quad \mathbb{R}^N,$$

and system

$$\begin{cases}
S^{1/k}_k(\lambda(D^2u)) = p(|x|) f(u,v) \quad \text{in} \quad \mathbb{R}^N, \\
S^{1/k}_k(\lambda(D^2v)) = q(|x|) g(u,v) \quad \text{in} \quad \mathbb{R}^N,
\end{cases}$$

where $k \in \{1, 2, ..., N\}$, the continuous functions $p, q : [0, \infty) \to (0, \infty)$, $h : [0, \infty) \to [0, \infty)$ and $f, g : [0, \infty) \times [0, \infty) \to [0, \infty)$ satisfy some of the conditions:

(P1) $p, q$ is a spherically symmetric function (i.e. $p(|x|) = p(|x|)$, $q(x) = q(|x|)$);

(P2) $r^{N \frac{N}{N-2} - p^k(r)}$ is nondecreasing for large $r$;

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Theorem 1. Let \( k \in \{1, 2, \ldots, [N/2]\} \) if \( N \) is odd or \( k \in \{1, 2, \ldots, [N/2] - 1\} \) if \( N \) is even. Suppose that (P1), (P2), (C1), (C3) are satisfied. If there exists a positive number \( \varepsilon \) such that

\[
\int_{0}^{\infty} t^{1+\varepsilon+\frac{2(k-1)}{k+1}} (p(t))^{\frac{2k}{k+1}} dt < \infty,
\]

then system (1.1) has a nonnegative nontrivial radial bounded solution \( u \in \Phi^k(\mathbb{R}^N) \).

Theorem 2. If \( p \) satisfy (P1) and \( f \) satisfy (C1), (C3), then the problem (1.1) has a nonnegative nontrivial entire radial solution \( u \in \Phi^k(\mathbb{R}^N) \). Suppose furthermore that (P2) holds. If \( p \) satisfies

\[
\int_{0}^{\infty} \left( \frac{k}{t^{N-k}} \int_{0}^{t} s^{N-1} p^k(s) ds \right)^{1/k} dt = \infty,
\]

then any nonnegative nontrivial radial solution \( u \in \Phi^k(\mathbb{R}^N) \) of (1.1) is large. Conversely, if (1.1) has a nonnegative entire large radial solution \( u \in \Phi^k(\mathbb{R}^N) \), then one or both of the...
to the problem

Observe that we can rewrite (2.1) as follows:

Proof of the Theorem 1. Assume that (1.3) holds. We prove the existence of a positive number $\varepsilon$ such that

$$
\int_0^\infty t^{1+\varepsilon+\frac{2(k-1)}{k+1}} (p(t))^{\frac{2k}{k+1}} dt = \infty
$$

for every $\varepsilon > 0$;

Remark 5. (see, for example, [12], [16]) If $u : \mathbb{R}^N \to \mathbb{R}$ is radially symmetric then a calculation show

$$
S_k \left( \lambda (D^2 u(r)) \right) = r^{1-N} C_{N-1} \left( \left[ \frac{r^{N-k}}{k} \right]^{1/k} u'(r) \right)^k,
$$

where the prime denotes differentiation with respect to $r = |x|$ and $C_{N-1} = (N-1)!/[(k-1)!(N-k)!]$.

2. Proofs of the main results

In this section we give the proofs of Theorems 1 - 4. The main references for proving Theorems [11, 12] is the work of Lair [13] and Delanoë [5] see also Afrouzi-Shokooh [1].

Proof of the Theorem 1. Assume that (1.3) holds. We prove the existence of $w \in \Phi^k (\mathbb{R}^N)$ to the problem

$$
S_k^{1/k} (\lambda (D^2 w(|x|))) = p(|x|) h (w(|x|)) \text{ in } \mathbb{R}^N.
$$

Observe that we can rewrite (2.1) as follows:

$$
\left[ \frac{r^{N-k}}{k} \left( w'(r) \right)^k \right]' = \frac{r^{N-1}}{C_{N-1}} p^k (r) h^k (w(r)), \ r = |x|.
$$
Then radial solutions of (2.1) are any solution \( w \) of the integral equation

\[
 w (r) = 1 + \int_0^r \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}} p^k (s) h^k (w (s)) \, ds \right)^{1/k} \, dt.
\]

To establish a solution to this problem, we use successive approximation. Define sequence \( \{ w^m \}_{m \geq 1} \) on \([0, \infty)\) by

\[
\begin{align*}
  w^0 &= 1, \quad r \geq 0, \\
  w^m (r) &= 1 + \int_0^r \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}} p^k (s) h^k (w^{m-1} (s)) \, ds \right)^{1/k} \, dt.
\end{align*}
\]

We remark that, for all \( w \).

Moreover, proceeding by induction we conclude \( \{ w^m \}_{m \geq 1} \) are non-decreasing sequence on \([0, \infty)\). We note that \( \{ w^m \}_{m \geq 1} \) satisfies

\[
\left\{ \frac{r^{N-k}}{k} \left[ (w^m (r))' \right]^k \right\}' = \frac{r^{N-1} - 1}{C_{N-1}} p^k (r) h^k (w^{m-1} (r)).
\]

By the monotonicity of \( \{ w^m \}_{m \geq 1} \) we have the inequalities

\[
(2.2) \quad \left\{ \frac{r^{N-k}}{k} \left[ (w^m (r))' \right]^k \right\}' = \frac{r^{N-1} - 1}{C_{N-1}} p^k (r) h^k (w^{m-1} (r)) \leq \frac{r^{N-1} - 1}{C_{N-1}} p^k (r) h^k (w^m (r)).
\]

Choose \( R > 0 \) so that \( r^{N+k-2} p^k (r) \) are non-decreasing for \( r \geq R \). We are now ready to show that \( w^m (R) \) and \( (w^m (R))' \), both of which are nonnegative, are bounded above independent of \( m \). To do this, let

\[
\phi^R = \max\{ p^k (r) : 0 \leq r \leq R \}.
\]

Using this and the fact that \( (w^m)' \geq 0 \), we note that (2.2) yields

\[
\begin{align*}
  r^{N-k} \left[ (w^m (r))' \right]^{k-1} (w^m (r))'' &\leq \frac{N-k}{k} r^{N-k-1} \left[ (w^m (r))' \right]^k + r^{N-k} \left[ (w^m (r))' \right]^{k-1} (w^m (r))'' \\
  &\leq \phi^R \frac{r^{N-1} - 1}{C_{N-1}} h^k (w^m (r)),
\end{align*}
\]

and moreover

\[
\begin{align*}
  r^{N-k} \left[ (w^m (r))' \right]^{k-1} (w^m (r))'' &\leq \phi^R \frac{r^{N-k} - 1}{C_{N-1}} h^k (w^m (r)) \leq R^{k-1} \phi^R \frac{r^{N-k}}{C_{N-1}} h^k (w^m (r)),
\end{align*}
\]

from which we have

\[
\begin{align*}
  \left[ (w^m (r))' \right]^{k-1} (w^m (r))'' &\leq R^{k-1} \phi^R \frac{1}{C_{N-1}} h^k (w^m (r)).
\end{align*}
\]

Multiply this by \( (w^m (r))' \) we obtain

\[
(2.3) \quad \left\{ \left[ (w^m (r))' \right]^{k+1} \right\}' \leq \frac{(k+1) R^{k-1} \phi^R}{C_{N-1}^{k-1}} h^k (w^m (r)) (w^m (r))'.
\]

Integrate (2.3) from 0 to \( r \) to get

\[
(2.4) \quad \left[ (w^m (r))' \right]^{k+1} \leq \frac{(k+1) R^{k-1} \phi^R}{C_{N-1}^{k-1}} \int_0^r h^k (w^m (s)) (w^m (s))' \, ds = \frac{(k+1) R^{k-1} \phi^R}{C_{N-1}^{k-1}} \int_1^r h^k (s) \, ds
\]

\[
\]
for $0 \leq r \leq R$, which yields
\[
\int_1^{w^m(R)} \left[ \int_1^t h^k (s) \, ds \right]^{-1/(k+1)} \, dt \leq \left( \frac{k}{k-1} \right) ^{k+1} \frac{1}{C^{k-1}} \cdot R^{k+1}.
\]

It follows from the above relation and by the assumption (C2) that $w^m_1(R)$ is bounded above independent of $m$. Using this fact in (2.4) shows that the same is true of $(w^m(R))'$. Thus, the sequences $w^m(R)$ and $(w^m(R))'$ are bounded above independent of $m$.

Finally, we show that the non-decreasing sequences $w^m$ is bounded for all $r \geq 0$ and all $m$. Multiplying the equation (2.2) by $r^{N+\frac{N}{k+1}+2} (w^m(r))'$, we get
\[
(2.5) \quad \left\{ r^{\frac{N}{k+1}} \left( w^m(r) \right) \right\}^{k+1} = \frac{k+1}{C^{k-1}} \cdot p^k (r) h^k (w^m(r)) r^{N+\frac{N}{k+1}+2} (w^m(r))'.
\]

Integrating from $R$ to $r$ gives
\[
\left[ r^{\frac{N}{k+1}} \left( w^m(r) \right) \right]^{k+1} = \left[ R^{\frac{N}{k+1}+2} (w^m(R)) \right]^{k+1} + \frac{k+1}{C^{k-1}} \int_R^r p^k (s) h^k (w^m(s)) s^{N+\frac{N}{k+1}+2} (w^m(s))' \, ds,
\]
for $r \geq R$. Noting that, by the monotonicity of $s^{N+\frac{N}{k+1}+2} p^k (s)$ for $r \geq s \geq R$, we get
\[
\left[ r^{\frac{N}{k+1}} \left( w^m(r) \right) \right]^{k+1} \leq C + \frac{k+1}{C^{k-1}} r^{N+\frac{N}{k+1}+2} p^k (r) H \left( w^m(r) \right)
\]
where $C = \left[ R^{\frac{N}{k+1}+2} (w^m(R)) \right]^{k+1}$, which yields
\[
r^{\frac{N}{k+1}} \left( w^m(r) \right)' \leq C \cdot r^{1-\frac{N}{k+1}} + \left( \frac{k+1}{C^{k-1}} \right) \cdot \frac{1}{k+1} r^{N-\frac{N}{k+1}+2} \cdot p^k (r) \cdot H^{\frac{1}{k+1}} \left( w^m(r) \right)
\]
or, equivalently
\[
(w^m(r))' \leq C \cdot r^{1-\frac{N}{k+1}} + \left( \frac{k+1}{C^{k-1}} \right) \cdot \frac{1}{k+1} r^{1-\frac{N}{k+1}} \cdot p^k (r) \cdot H^{\frac{1}{k+1}} \left( w^m(r) \right)
\]
and hence
\[
(2.6) \quad \frac{d}{dr} \int_{w^m(r)}^R H (t)^{-1/(k+1)} \, dt \leq C \cdot r^{1-\frac{N}{k+1}} H^{\frac{1}{k+1}} \left( w^m(r) \right) + \left( \frac{k+1}{C^{k-1}} \right) \cdot \frac{1}{k+1} \cdot \left( r^{k-1} p^k (r) \right) H^{\frac{1}{k+1}} \left( w^m(r) \right).
\]

Inequality (2.6) combined with
\[
\frac{1}{\sqrt{2}} \sqrt{2 \cdot (s^{k-1} p^k (s))^{k+1}} = \frac{1}{\sqrt{2}} \sqrt{2 \cdot s^{k+1} \cdot (s^{k-1} p^k (s))^{k+1}} = \frac{1}{\sqrt{2}} \left[ s^{1+\varepsilon} \left( s^{k-1} p^k (s) \right)^{k+1} + s^{1-\varepsilon} \right]
\]
gives
\[
\int_{w^m(r)}^R H (t)^{-1/(k+1)} \, dt \leq C \cdot \frac{1}{\sqrt{2}} \int_R^r t^{1-\frac{N}{k+1}} H^{\frac{1}{k+1}} \left( w^m(t) \right) \, dt
\]
\[
+ \frac{1}{\sqrt{2}} \left( \frac{k+1}{C^{k-1}} \right) \cdot \frac{1}{k+1} \left[ \int_R^r t^{1+\varepsilon+\frac{2(k-1)}{k+1}} (p(t))^{\frac{2k}{k+1}} \, dt + \int_R^r t^{-1-\varepsilon} \, dt \right]
\]
\[
\leq C \cdot \frac{1}{\sqrt{2}} \int_R^r t^{1-\frac{N}{k+1}} \left( w^m(R) \right) H^{\frac{1}{k+1}} \, dt
\]
\[
+ \frac{1}{\sqrt{2}} \left( \frac{k+1}{C^{k-1}} \right) \cdot \frac{1}{k+1} \left[ \int_R^r t^{1+\varepsilon+\frac{2(k-1)}{k+1}} (p(t))^{\frac{2k}{k+1}} \, dt + \frac{1}{R} \right].
\]

The above relation is needed in proving the bounded of the function $\{w^m\}^m \geq 1$ in the following. Indeed, since for each $\varepsilon > 0$ the right side of this inequality is bounded independent of $m$
(note that \( w^m(t) \geq 1 \), so is the left side and hence, in light of (C2), the sequence \( \{w^m\}_{m \geq 1} \) is a bounded sequence and so \( \{w^m\}_{m \geq 1} \) are bounded. Thus \( \{w^m\}_{m \geq 1} \to w \) as \( m \to \infty \) and the limit functions \( w \) are positive entire bounded solutions of equation (2.1).

Proof of the Theorem 2. We know that for any \( a_1 > 0 \) a solution of

\[
v(r) = a_1 + \int_0^r \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} p^k(s) h^k(v(s)) \, ds \right)^{1/k} \, dt,\]

exists, at least, small \( r \). Since \( v' \geq 0 \), the only way that the solution can become singular at \( R \) is for \( v(r) \to \infty \) as \( r \to \infty \). Thus, we can show that, for each \( R > 0 \), there exists \( C_R > 0 \) so that \( v(R) \leq C_R \), we have existence. To this end, let \( M_R = \max \{p(r) \mid 0 \leq r \leq R\} \) and consider the equation

\[
w(r) = a_2 + M_R \int_0^r \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} p^k(s) h^k(v(s)) \, ds \right)^{1/k} \, dt
\]

where \( a_2 > a_1 \). We next observe that the solution to this equation exists for all \( r \geq 0 \) and of course, it is a solution to \( S^{1/k}_k(\lambda(D^2w(r))) = M_R h(w) \) on \( \mathbb{R}^N \) which is treated in [12, (Theorem 1.1, p. 177)]. We now show that \( v(r) \leq w(r) \) for all \( 0 \leq r \leq R \) and hence we conclude the proof of existence. Clearly \( v(0) < w(0) \) so that \( v(r) < w(r) \) for at least all \( r \) near zero. Let

\[ r_0 = \sup \{r \mid v(s) < w(s) \text{ for all } s \in [0, r]\} \, . \]

If \( r_0 = R \), then we are done. Thus assume that \( r_0 < R \). It follows from assumption \( a_2 > a_1 \) that

\[
v(r_0) = a_1 + \int_0^{r_0} \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} p^k(s) h^k(v(s)) \, ds \right)^{1/k} \, dt
\]

\[
< a_2 + M_R \int_0^{r_0} \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} p^k(s) h^k(v(s)) \, ds \right)^{1/k} \, dt = w(r_0) .
\]

Thus there exists \( \varepsilon > 0 \) so that \( v(r) < w(r) \) for all \( [0, R + \varepsilon) \), contradicting the definition of \( r_0 \). Thus we conclude that \( v < w \) on \([0, R]\) for all \( R > 0 \) and hence \( v \) is a nontrivial entire solution of (1.1). Now let \( u \) be any nonnegative nontrivial entire solution of (1.1) and suppose \( p \) satisfies

\[
\int_0^\infty \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} p^k(s) \, ds \right)^{1/k} \, dt = \infty.
\]

Since \( u \) is nontrivial and non-negative, there exists \( R > 0 \) so that \( u(R) > 0 \). On the other hand since \( u' \geq 0 \), we get \( u(r) \geq u(R) \) for \( r \geq R \) and thus from

\[
u(r) = u(0) + \int_0^r \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} p^k(s) h^k(u(s)) \, ds \right)^{1/k} \, dt,
\]

since \( u \) will satisfy that equation for all \( r \geq 0 \), we get

\[
u(r) = u(0) + \int_0^r \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} p^k(s) h^k(u(s)) \, ds \right)^{1/k} \, dt
\]

\[
\geq u(R) + h(u(R)) \int_R^r \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} p^k(s) \, ds \right)^{1/k} \, dt \to \infty \text{ as } r \to \infty.
\]

Conversely, assume that \( h \) satisfy (C1), (C3) and that \( w \) is a nonnegative entire large solution of (1.1). Note also, that \( w \) satisfies

\[
\left[ \frac{r^{N-k}}{k} (w'(r))^k \right]' = \frac{r^{N-1}}{C_{N-1}^{k-1}} p^k(r) h^k(w(r)) .
\]
Using the monotonicity of \( r^{N+\frac{N}{k}} p(r) \) we can apply similar arguments used in obtaining Theorem 1 to get

\[
(w(r))' \leq C^{\frac{1}{k+1}} r^{1-\frac{N}{k}} + \left( \frac{k+1}{C^{k-1}} \right)^{\frac{1}{k+1}} r^{1-\frac{2}{k+1}} p^{\frac{1}{k+1}} (r) H^{\frac{1}{k+1}} (w(r)),
\]

which we may rewrite as

\[
(2.7)
\]

\[
\int_{w(R)} \left[ H(t) \right]^{-1/(k+1)} \, dt \leq C^{\frac{1}{k+1}} \int_{R}^{r} t^{1-\frac{N}{k}} H^{-\frac{1}{k+1}} (w(t)) \, dt
\]

\[
+ \frac{1}{\sqrt{2}} \left( \int_{R}^{r} t^{1+\frac{1}{k+1}} (p(t))^{\frac{2k}{k+1}} \, dt + \int_{R}^{r} t^{-1-\frac{2}{k+1}} \, dt \right)
\]

\[
\leq C^{\frac{1}{k+1}} \int_{R}^{r} t^{1-\frac{N}{k}} \, dt + \frac{1}{\sqrt{2}} \left( \frac{k+1}{C^{k-1}} \right)^{\frac{1}{k+1}} \int_{R}^{r} t^{1+\frac{1}{k+1}} (p(t))^{\frac{2k}{k+1}} \, dt + \frac{1}{\sqrt{2} R^{\frac{1}{k+1}}}.
\]

where

\[
C_{R} = C^{\frac{1}{k+1}} H^{-\frac{1}{k+1}} (w(R)) + \frac{1}{\sqrt{2}} \frac{1}{R^{\frac{1}{k+1}}} \left( \frac{k+1}{C^{k-1}} \right)^{\frac{1}{k+1}}.
\]

By taking \( r \to \infty \) in (2.7) we obtain (1.5) since \( w \) is large and \( h \) satisfies (C3). These observations completes the proof of the theorem.

**Proof of the Theorem 3 and 4.** In order, to obtain the conclusion, combine the proof of Theorem 1 and 2 with some technical results from 3 and 4.

\[\square\]

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