A note on two-loop effects in the DMSSM

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Abstract

We investigate the proposed “D-brane alternative” to the MSSM model (DMSSM) which is a type II B string orientifold model with N=1 supersymmetry, three generations and a $SU(3) \times SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ gauge group. An accurate analysis at two-loop level is performed to show that unification constraints predict a “left-right” symmetry breaking scale in the TeV region. The exact value of this scale is the result of the competing effects of the two loop terms against the low energy supersymmetric threshold effects. The model accommodates logarithmic unification of the gauge couplings at an intermediate scale of $10^{12}$ GeV and the necessary conditions to achieve this are addressed.

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1 Introduction

The Minimal Supersymmetric Standard Model (MSSM) is currently the most studied supersymmetric extension of the Standard Model, and it provides a consistent framework for investigating the phenomenological aspects and possible signatures of low energy supersymmetry. The MSSM model may be regarded as the low energy limit of four dimensional heterotic string models where the latter provide a fully unified theory with gravity and may predict upon appropriate compactifications the MSSM gauge group and massless spectrum and amount of supersymmetry. Examples in this respect are provided by the $E_8 \times E_8$ models compactified on a Calabi-Yau manifold \[2\] leading to the $E_6$ gauge group which may then be broken in the presence of Wilson lines \[3\] to a Standard Model-like gauge group. Further, 4D $N=1$ supersymmetry is broken by non-perturbative effects \[4\]. In a compactified string theory standard $SU(5)$-like hypercharge normalisation as in the MSSM is possible without a stage of grand unified group, as this depends on the Kac-Moody level used. For level-1 case an $SU(5)$ relationship emerges, even though the group is just the Standard Model. This is one of the possibilities as other $U(1)_Y$ normalisations are allowed (for related developments see \[5\]). Nevertheless such $SU(5)$-like normalisation together with the massless spectrum and symmetry group of the MSSM enable one to claim circumstantial evidence for (a logarithmic) unification of the gauge couplings within this model or similar ones. Further, the unification is in general stable \[6\] under the inclusion (in addition to the MSSM spectrum) of extra heavy states predicted by the string. Indeed, taking as input the low energy values of $\sin^2 \theta_W(M_Z)$ and $\alpha_3(M_Z)$ and performing a two-loop RG flow to include radiative effects, leads to a value of the unification scale equal to $\approx 2 \times 10^{16}$ GeV. This value is within a factor of 20 \[7\] (see also \[8\]) from the heterotic string prediction obtained after fixing the string tension to the value leading to the correct gravitational strength coupling. Such discrepancy factor may be regarded as a mismatch of the two scales, but may not be so significant given the many orders of magnitude over which we extrapolate the running of the gauge couplings. Further, this discrepancy may be accounted for by string threshold corrections in the presence of Wilson line background \[8\] without going to the strong coupling regime of the string. Therefore the unification of the gauge couplings in MSSM-like models may be regarded as circumstantial support for supersymmetry and a fully embedding of the MSSM into the (weakly coupled) heterotic string theory may be possible.

Alternative possibilities exist in the context of strongly coupled heterotic scenarios (M theory). In this case the string scale \[9\], $M_s \sim g M_P e^{-\phi}$ where $g$ is the gauge coupling, $\phi$ is the dilaton, $M_P$ is the Planck mass. Such a relation allows for a low string scale through the choice of the dilaton v.e.v. $<\phi>$. This has caused much interest for it may bring \[9\] the string scale prediction into better agreement with the aforementioned MSSM unification scale. It was also noticed \[10\] that the mechanism can be applied to lower the string scale even further, perhaps even down to the “TeV region”, to give a low compactification scale \[11\], \[12\] and a low string scale as well.

With growing interest in the physics of large extra dimensions and low scale string models, alternative, low energy supersymmetric models to the MSSM were suggested \[11\], \[13\], \[14\], \[15\]. The presence of a low string scale may require a significant change of gauge couplings running, if the couplings are still supposed to meet at the string scale. In some cases this may be explained by threshold effects power-like in the scale \[11\], \[16\], due to Kaluza Klein states, although these seem to bring some amount of fine-tuning \[17\]. This may in principle be avoided if the couplings unify not at the string scale but at the first winding mode above it \[16\] \[18\], which may be close to the Planck scale, restoring MSSM-like logarithmic unification. However, consistent model building along these ideas lacks the accuracy and consistency of the situation MSSM vs. heterotic string case and in general, low (string) scale unification (power-like or not) may not be easy achieve \[19\].
One of the possible solutions to deriving low energy supersymmetric models which bear some similarities to the MSSM case, but have a low unification/string scale, was provided by type IIB $Z_N$ orientifold models \cite{13, 15}, with D3 branes placed at $C^3/Z_N$ singularities. These models reproduce the desirable features of a particle theory model: they have N=1 supersymmetry, SM gauge group (at least below some scale), three quark-lepton generations. One characteristic they bring is a non-standard hypercharge normalisation. Examples of this type have been analysed at one loop level \cite{14, 15} and at two loop level \cite{19}. The purpose of this work is to investigate another model of this class \cite{15} the so-called “D-brane alternative” to the MSSM (DMSSM). A one loop analysis \cite{15} has shown that the model presents a logarithmic unification of the couplings at a scale close to $10^{12}$ GeV and may be able to fit the experimental constraints on $\alpha_3(M_Z)$, $\sin^2 \theta_W(M_Z)$ and $\alpha_{em}(M_Z)$. The reason why the couplings may unify at a lower (than in the MSSM) scale is not due to power-like running, but to the different symmetry group and spectrum above some “left-right symmetry” breaking scale $M_R$. We argue that for an accurate investigation one should perform a two loop analysis, given the present accuracy of low energy experimental data. We employ a simple method to perform such a two loop investigation and analyse the constraints this model must respect in order to achieve logarithmic unification. In particular we stress the importance of competing effects of the one-loop low energy supersymmetric thresholds against pure two-loop effects. These effects have strong implications for the existence of a “left-right” symmetry breaking scale $M_R$, whose value and correlation to one-loop supersymmetric thresholds is analysed.

2 Description of the DMSSM

A brief outline of the DMSSM model and its rather distinct features relative to the MSSM case are outlined below.

- The gauge group above the scale $M_R$ is a minimal “left-right” extension of the Standard Model, $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Below the scale $M_R$ the usual $SU(3) \times SU(2)_L \times U(1)_Y$ gauge group applies.

- The charge of the $U(1)_{B-L}$ group as well as that of the hypercharge group $U(1)_Y$ (below $M_R$) have non-standard normalisation, $k_{B-L} = 32/3$ and $k_y = 11/3$.

- The unification scale is lowered from the MSSM case. This is not the result of power-like RG flow of the couplings, but of the non-standard normalisation of the $U(1)$ groups and the enhanced gauge symmetry (and also different spectrum above $M_R$). This provides a specific model with low scale logarithmic unification.

- The representations predicted by this model have the following quantum numbers with respect to the aforementioned gauge group above $M_R$: the Higgs sector: $3 \times (1, 2, 2, 0)$, the quarks sector: $3 \times (3, 2, 1, 1/3) + 3 \times (\bar{3}, 1, 2, -1/3)$, the leptons sector: $3 \times (1, 2, 1, -1) + 3 \times (\bar{1}, 1, 2, 1)$. The model has the nice feature of predicting three generations as a result of three additional complex (compact) dimensions \cite{15}.

As shown, the gauge group above the scale $M_R$ is enhanced from that of the MSSM. One important consequence is the non-standard normalisation of the charges of the $U(1)$ above and below the scale $M_R$ as discussed below. The initial starting gauge group in the DMSSM contains $U(3) \times U(2)_L \times U(2)_R$ which includes three $U(1)$ gauge groups. Of these $U(1)$’s only one is anomaly-free, with the

\footnote{Such one-loop threshold effects are comparable to two loop effects.}
other two (anomalous) $U(1)$ to become massive and decouple due to a generalised Green-Schwarz mechanism \[14\], \[24\]. As a consequence of the presence of three independent $U(1)$’s coming from the non-Abelian sector (rather than two, as in the MSSM), the hypercharge normalisation will be different since the number of non-Abelian gauge groups controls the normalisation of the anomaly-free $U(1)$ group \[13\] as given by the formula $k_y = 5/3 + 2(N - 2)$ where $N$ is the number of Abelian $U(1)$’s, each coming from a non-Abelian group of the model \[13\]. Thus for a starting non-Abelian gauge group $U(3) \times U(2)$ as in the MSSM we have $N = 2$ and $k_y = 5/3$ emerges. For a starting non-Abelian group $U(3) \times U(2)_L \times U(2)_R$ as in the DMSSM, the same formula gives $k_y = 11/3$ ($N = 3$). For the same model one also has $k_{B-L} = 8/3 + 8(N - 2)$, after using the relations between hypercharge, $SU(2)_R$ and $U(1)_{B-L}$ generators. This gives $k_{B-L} = 32/3$, ($N = 3$) which is different from the standard $SO(10)$ embedding corresponding to $k_{B-L} = 8/3$. This observation has strong implications for the boundary conditions of the running of the associated gauge couplings and the value of the unification scale. Further, these boundary conditions are exactly those derived from embedding the gauge group in a D-brane scheme \[15\] with D-branes placed at the value of the unification scale. Further, these boundary conditions are related to the (number of) non-Abelian gauge groups to start with, is also related to the number of generations (three) and of complex dimensions equally twisted \[13\].

Since the gauge group above the scale $M_R$ is enhanced with states also charged under $SU(2)_R$ in addition to $SU(3) \times SU(2)_L$, the running of the gauge couplings above the scale $M_R$ will be affected significantly. If compared to the MSSM, two-loop effects will be enhanced, and for an equally accurate analysis in the DMSSM, a two loop evaluation of the RG flow is necessary. This is due to the fact that for rather similar matter spectrum, the wavefunction renormalisation of the matter fields (two loop effect for gauge couplings) also receives gauge corrections from the additional gauge bosons of the $SU(2)_R$ group. Such enhancement of the two loop effects is rather generic in models with larger gauge group. Further, a simple one-loop result for the DMSSM shows that

$$\alpha_3^{-1}(M_Z) = -\frac{15}{2\pi} \ln \frac{M_R}{M_Z} + \frac{3}{2} \left(1 - 4\sin^2 \theta_W\right) \alpha_{em}^{-1}(M_Z) - \frac{15}{2\pi} \ln \frac{T_{eff}}{M_Z} + \text{two-loop} \quad (1)$$

where $T_{eff}$ accounting for the low energy supersymmetric threshold effects will be defined later. One important observation of this one loop result in case $T_{eff} \approx M_Z$, is that the value of the scale $M_R$ (parameter of the model) is required to have values relatively close to the electroweak scale $M_Z$ due to the requirement $\alpha_3(M_Z)$ be positive, given the experimental value of $\sin^2 \theta_W(M_Z)|_0 = 0.23114 \pm 0.00016 \ [1]$. These requirements may change upon including two loop terms and it is thus important to know how stable these one loop bounds on $M_R$ are. Further, these bounds are affected by the effects of $T_{eff}$ which are also comparable to two loop effects. This issue is relevant because the value of $M_R$ should not be too large, otherwise one would have to explain why the mass of the two Higgs pairs (whose mass we set for simplicity to $M_R$) present in addition to the MSSM Higgs sector should be large compared to that of the usual Higgs.

In the case of the MSSM a similar calculation gives

$$\alpha_3^{-1}(M_Z) = \frac{1}{7} \left(15 \sin^2 \theta_W - 3\right) \alpha_{em}^{-1}(M_Z) + \frac{19}{28\pi} \ln \frac{M_{eff}}{M_Z} + \text{two-loop} \quad (2)$$

with $M_{eff}$ to account for low energy supersymmetric thresholds \[22\] different from the DMSSM case. Comparing equations \[1\], \[2\] we find that the (absolute value of the) variation of $\alpha_3(M_Z)$ with respect to $\sin^2 \theta_W$ has a steeper behaviour for \[1\] than for \[2\] for values of $\sin^2 \theta_W$ close to
In the notation of (4) is that derived from with the gauge group "a" and its normalisation for "a=0" (as well as that of the gauge couplings) corrections. The coefficient model are set to zero, as all complex dimensions are twisted, thus are not expected to bring string spectrum and number of generations. A simple one loop calculation of the coefficients are equal to unity at the tree level (i.e. one loop for the gauge couplings) thus account for two effects. The renormalisation group (RG) evolution above the scale has the following structure

\[ \alpha_a^{-1}(M_R) = -\Delta_a + \alpha_a^{-1}(M_U) + \frac{B_a}{2\pi} \ln \frac{M_U}{M_R} + \frac{3T_a(G)}{2\pi} \ln \left( \frac{\alpha_a(M_U)}{\alpha_a(M_R)} \right)^{1/3} - \sum_\phi \frac{T_a(R_\phi)}{2\pi} \ln Z_\phi(M_U, M_R) \]

where index “a” runs over indices 0,1,2,3 of the U(1)_{B-L}, SU(2)_R, SU(2)_L, SU(3) respectively. The values of the one-loop coefficients is given by \( B_a = \{3/2, 3, 3, -3\}_a \). The rather small value of U(1)_{B-L} beta function in the \( k_{B-L} = 32/3 \) normalisation, will lead (for fixed value of the unified coupling) to a larger value of this coupling (in this normalisation) at scale \( M_R \) than in the SU(2) groups case. The quantities \( \Delta_a \) may account for additional string thresholds, which in our model are set to zero, as all complex dimensions are twisted, thus are not expected to bring string corrections. The coefficient \( T_a(R_\phi) \) accounts for the Dynkin index of the representation associated with the gauge group “a” and its normalisation for “a=0” (as well as that of the gauge couplings) in the notation of (4) is that derived from \( \alpha_a(M_U) = \alpha \) for all indices “a”. The logarithm of the couplings accounts for pure gauge effects (all orders) to the RG flow. The wavefunction coefficients \( Z_\phi \) are equal to unity at the tree level (i.e. one loop for the gauge couplings) thus account for two loop and beyond effects, induced by the mixing matter-gauge or Yukawa effects. In our analysis we only consider their one loop corrected value (two loop for the gauge couplings running) induced by gauge effects of \( SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \). The sum over \( \phi \) runs over the entire matter spectrum and number of generations. A simple one loop calculation of the coefficients \( Z_\phi \) gives

\[ Z_\phi(M_U, M_R) = \prod_{a=0}^3 \left[ \frac{\alpha_a M_U}{\alpha_a M_R} \right]^{-2C_a(\phi)/B_a} \]

3DMSSM: two-loop results

A two-loop analysis of the DMSSM is easier than expected since we need perform only one-loop (wavefunction) calculations. The result is indeed correct in two loop order for the gauge couplings. For details see [23], [24], [25] with application to phenomenology in [1]. Further, the method has the advantage of unambiguously including the threshold effects present at the scale \( M_R \). This is important because we only need the bare value of \( M_R \) for a two loop RG flow of the couplings [3]. We remind there are two Higgs pairs whose masses are set equal to \( M_R \) likely to bring threshold effects [1]. The renormalisation group (RG) evolution above the scale \( M_R \) has the following structure

\[ \frac{d\alpha_3^{-1}(M_Z)}{d(\sin^2\theta_W)} \bigg|_{\text{MSSM}} = \frac{15}{7} \frac{\alpha_{em}(M_Z)}{\sin^2\theta_W} \bigg|_{\text{DMSSM}} = \frac{6}{\alpha_{em}(M_Z)} \]

which means that the prediction for \( \alpha_3(M_Z) \) close to the experimental point will vary faster in the DMSSM than in the MSSM, leading to potentially larger two loop corrections in the DMSSM case.

Finally, a one-loop MSSM value of \( \alpha_3(M_Z) \approx 0.116 \) differs significantly from its two loop value 0.126(±0.01) (experimental value 0.119±0.002 [1]) obtained using the unification assumption. Thus two loop effects are important and an enhancement of such a difference is expected in the DMSSM. For the reasons outlined above we conclude that a careful analysis of the DMSSM and of its gauge couplings running should include a full two loop RG approach.

4In principle one needs a string mechanism for fixing the moduli vev’s giving the bare mass of these states.

4The index notation as 1* is chosen to distinguish it from that corresponding to U(1)_Y below the scale \( M_R \), see later.
where we used the notation $C_a(\phi) = \{Q_{B-L}^2/k_{B-L}, 3/4, 3/4, 3/4\}_a$ for the quadratic Casimir operator of $U(1)_{B-L}$, $SU(2)_R$, $SU(2)_L$ and $SU(3)$ respectively. Additional effects on coefficients $Z_\phi$ are expected from Yukawa interactions, controlled by the superpotential terms and strongly model dependent in this case. For this reason we do not include them; they may be accounted for by using in (3) the replacement $Z_\phi \rightarrow Z_\phi \times Z_\phi^y$ with $d/dt \ln Z_\phi(M_U, M_R) = \sum_{\nu} A_\nu(\phi)y_\nu(t) = 1/(2\pi) \ln(\text{scale})$ accounting for Yukawa one loop wavefunction renormalisation and coefficients $A_\nu$ depending on the superpotential. This relation may be integrated analytically to give Yukawa correction $Z_\phi^y$ to eq.(3).

The RG flow below the scale $M_R$ is that familiar for the MSSM with the important observation that the hypercharge normalisation which affects one and two loop contributions is that corresponding to 3/11 (and not 3/5 as in the MSSM). The equations have the structure

$$\alpha^{-1}(M_Z) = -\delta_i + \alpha^{-1}(M_R) + \frac{b_i}{2\pi} \ln \frac{M_R}{M_Z} + \frac{3T_i(G)}{2\pi} \ln \left[ \frac{\alpha_a(M_R)}{\alpha_a(M_Z)} \right]^{1/3} - \sum_{\psi} \frac{T_i(R_\psi)}{2\pi} \ln Z_\psi(M_R, M_Z) \tag{6}$$

with index “i” running over 1,2,3 associated with the groups $U(1)_Y$, $SU(2)_L$ and $SU(3)$ respectively. One loop coefficients $b_i = \{3, 1, -3\}$ and the coefficients $Z_\psi$ are similar to those in eq.(3)

$$Z_\psi(M_R, M_Z) = \prod_{i=1}^{3} \left[ \frac{\alpha_a(M_R)}{\alpha_a(M_Z)} \right]^{-2C_i(\psi)/b_i} \tag{7}$$

with $C_i(\psi) = \{Q_{\psi}^2/k_{\psi}, 3/4, 4/3\}_i$. To include the Yukawa effects below the scale $M_R$, an approach similar to that above $M_R$ may be used.

A numerical investigation of the above equations gives the results of Figures 1-3. Provided that unification takes place, the following (conservative) two-loop bounds may be placed on the intermediate scale $M_R$: $M_Z \times \exp(4.05) \leq M_R \leq M_Z \times \exp(4.4)$ and on the unification scale $M_Z \times \exp(22.55) \leq M_U \leq M_Z \times \exp(23.30)$ which give: $5.233 \text{ TeV} \leq M_R \leq 7.427 \text{ TeV}$ and $5.665 \times 10^{11} \text{ GeV} \leq M_U \leq 1.199 \times 10^{12} \text{ GeV}$. We notice a rather strong variation of the predicted two loop value of $M_R$ from its one loop value of about 1 TeV [13] compatible with the same low energy input for $\alpha_3(M_Z)$ and $\sin^2 \theta_W(M_Z)$. This is essentially due to additional radiative effects induced by the larger (than in the MSSM) (non-Abelian) gauge group. These effects are further strengthened by the presence of the logarithm in front of $M_R$ in the RG equations [14,15].

Similar to the MSSM case, our results, Figures 1,2,3 are sensitive to the value of $T_{\text{eff}}$, eq.(4) and which has so far been taken equal to $M_Z$. $T_{\text{eff}}$ is a function of the low energy supersymmetric thresholds $\delta_i$ which affect the prediction of the correlation $\alpha_3 - \sin^2 \theta_W$ at $M_Z$ [20]. $T_{\text{eff}}$ is changed from the MSSM case due to the non-standard hypercharge normalisation and different RG flow above the scale $M_R$. It parametrises our lack of detailed knowledge of the low energy supersymmetry spectrum (similar to the MSSM case) as an overall effect on $\alpha_3(M_Z)$. As a result, its value in eq.(4) is a combination of $\delta_i$’s of (3) giving

$$T_{\text{eff}} = M_Z \sum_{L}^{3} \frac{M_L^{\delta_i}}{M_Z^{3/2}} \sum_{Q}^{3} \frac{M_Q^{\delta_i}}{M_Z^{3/2}} \sum_{M}^{3} \frac{M_M^{\delta_i}}{M_Z^{3/2}} \sum_{U}^{3} \frac{M_U^{\delta_i}}{M_Z^{3/2}} \sum_{E}^{3} \frac{M_E^{\delta_i}}{M_Z^{3/2}} \begin{bmatrix} \mu \end{bmatrix}^{\delta_i} \begin{bmatrix} M_2 \end{bmatrix}^{\delta_i} \begin{bmatrix} M_3 \end{bmatrix}^{\delta_i} \begin{bmatrix} M_H \end{bmatrix}^{\delta_i} \approx M_Z \begin{bmatrix} M_L \end{bmatrix}^{\delta_i} \begin{bmatrix} M_2 \end{bmatrix}^{\delta_i} \begin{bmatrix} M_3 \end{bmatrix}^{\delta_i} \tag{8}$$

where the approximation only holds for degenerate squarks and sleptons, hence $T_{\text{eff}}$ increases with their mass and also with that of gauginos.

At one-loop level the effect of increasing $T_{\text{eff}}$ is to increase $\alpha_3(M_Z)$, see eq.(4). This behaviour is different from the MSSM case where the opposite effect is manifest (see eq.(2) with $T_{\text{eff}} \rightarrow M_{\text{eff}}$).
The curves mark the limiting values (conservative estimates) of $M_R$ and $M_U$ for which one may still simultaneously fit the experimental constraints on $\alpha_3(M_Z)$ and $\sin^2\theta_W(M_Z)$. The difference is in essence due to different RG flow/gauge structure above the scale $M_R$. The increasing effect on $\alpha_3(M_Z)$ due to (increasing) $T_{eff}$ may be compensated for by decreasing $M_R$ (see Figures 1,4) and also $M_U$ (see Figure 2). This may also be seen in one loop order from eqs. (1), (8) which show that for fixed low energy input, the combination $M_R \times T_{eff}$ must stay constant in this approximation. This is an important effect that we would like to stress. We mentioned that pure one loop effects predict ($\alpha_3(M_Z)$, $\sin^2\theta_W$ fixed) a value for $M_R$ of order 1 TeV which is further increased by two loop effects to values of order 5 – 7 TeV. This may be a concern since one must explain why the mass of the additional Higgs sector (also of mass equal to $M_R$) is so high compared to the electroweak scale. However, low energy supersymmetric thresholds ($T_{eff}$) can reduce significantly the predicted value of $M_R$ for fixed low energy input, see Figure 4. For example sleptons and gaugino masses with (lower bounds on) masses in the region of 300 (200) GeV increase the value of $T_{eff}$ (initially set to $M_Z$) by a factor of $\approx 7.90(3.91)$ which thus reduces $M_R$ (Figure 4) in the region 450 (1000) GeV. For a general picture of this situation see also Figures 5 and 6 where the dependence of $\alpha_3$ and $\sin^2\theta_W$ is showed as a parametric plot for different values of $T_{eff}$ and $M_R$. Therefore, the presence of a low energy supersymmetric spectrum ensures that there is a left-right symmetry breaking scale not far above $M_Z$. This avoids the difficulty and need for a rather large $M_R$ of Figures 1,2,3 where $T_{eff} = M_Z$. In addition, (lower) bounds on $M_R$ may also exist to avoid FCNC problems rather generic in “left-right” symmetric models with $M_R$ in the region of 1 TeV. This issue was addressed in [27] and it would be useful to have a detailed investigation in the DMSSM model. The (lower) bounds on $M_R$ would in turn be related to $T_{eff}$ (see Figure 4) to provide upper bounds for it and thus for (the combination of) the low energy supersymmetric spectrum (via eq. (8)).
Figure 3. Two-loop (continuous line) correlation for \( \ln M_R/M_Z = 4.2 \) \((M_R \approx 6\text{TeV})\). The one-loop case (dashed line) corresponding to the same value of \( M_R \) cannot fit the low energy values of \( \alpha_3(M_Z) \), \( \sin^2 \theta_W(M_Z) \) unless \( M_R \) is reduced by a factor of \( \approx 4 \), in the region of 1.5 TeV. This shows a significant change of the one-loop from the two-loop prediction for \( M_R \) with \( \alpha_3(M_Z) \), \( \sin^2 \theta_W(M_Z) \) fixed to the experimental values.

Figure 4. Two-loop plot of \( T_{\text{eff}} \) versus \( M_R \) for \( \alpha_3(M_Z) \) and \( \sin^2 \theta_W(M_Z) \) fixed to their (central) experimental values.

4 Conclusions and outlook

We presented a simple method to compute two-loop effects in a model with different symmetry groups above/below the scale \( M_R \) and to account for the threshold effect at this scale. The model is successful in achieving a low scale of unification with logarithmic running only for the gauge couplings. The low scale of unification is due to the enhanced gauge symmetry and non-standard hypercharge \( U(1)_Y \) and \( U(1)_{B-L} \) normalisations. We showed that the two loop prediction for the “left-right” symmetry scale is the result of the competing effects between pure two loop terms and one loop supersymmetric thresholds. The latter thus ensure that the value of \( M_R \) may be kept rather small, of order of 1 TeV or even less. Further quantitative analysis of this model is however required to explain how the breaking of the “left-right” symmetry is induced. Finally, a comparative analysis to the MSSM case of the correlation \( (\alpha_3(M_Z), \sin^2 \theta_W(M_Z)) \) (as in [21]) could provide an insight into the relative amount of fine tuning of the high scale one needs perform to keep such correlation stable against high scale physics. Such analysis would help understand which model, DMSSM or MSSM is more predictive/less fine-tuned.

Acknowledgements

The author would like to thank Fernando Quevedo and Louis Ibáñez for helpful discussions on this work. The work was supported by the University of Bonn under the European Commission RTN programme HPRN-CT-2000-00131.
Figure 5. Parametric plot of $\alpha_3(M_Z)$ versus $M_R$ ($\sin^2 \theta_W$ fixed to its experimental range), computed for different values of the low energy supersymmetric threshold $T_{\text{eff}} = kM_Z$ with $k$ increasing from right to left by unity, as shown in the figure. The dashed lines mark the experimental limits.

Figure 6. Parametric plot for $\sin^2 \theta_W$ versus $M_R$ ($\alpha_3(M_Z)$ fixed to 0.119), calculated for different values of the low energy supersymmetric threshold $T_{\text{eff}} = kM_Z$ with $k$ increasing downwards, as shown in the figure. The dashed lines mark the experimental limits.

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