The Orbital of Satellite Transfer with Inclination Change Using a New Techniques

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Abstract
The process of satellite transition without perturbations effect were studded in this work. The initial elliptical orbit (e= 0.01, a= 6745.45 km, i= 0°) was transfer to geostationary orbit through the transfer elliptical orbit with inclination variation between the initial and final orbit Δi= 0˚,1,2 to 180˚. A program was designed to calculate the velocity needed to transition (ΔVhohman), the velocity needed to rotation the plane of orbit (ΔVi), ΔVtotal, mass of fuel needed and time of transition we assume that the time of rotation equal one period. Hohmann transfers method between coaxial elliptical orbits to Satellite transmission was used. It is the process of transition the satellite from an elliptical initial orbit to a circular final orbit through an elliptical transition orbit some assumption were used in our techniques of transition orbit. The results indicated that the best method for the transfer process that is transfer the satellite from the perigee of the initial elliptical orbit and then perform the orbit rotation process of the final upper orbit. The inclination angle can be reach is 60 degree where at this Δi low velocity or low energy needed and the rotation can happen during one period. The amount of fuel needed to transition of the satellite can be achieved.

Keywords: orbital dynamic, orbital transfer, Hohmann, parking orbit.

1. Introduction
The conducts of satellite on orbit maneuvers is to correct the orbit shape, size, or location of the satellite [1]. The Satellite orbits are classified as many types according to inclination and altitude as well as the mission for which the satellite was launched [2]. The orbital maneuvers is transfer a spacecraft from one orbit to another orbit. Orbital changes can be gets many ways and steps, such as the transfer from a low-earth parking orbit to an interplanetary orbit. Changing orbits requires firing of onboard rocket engines. The impulsive maneuvers will be concerned only in
which the rockets fire in relatively short bursts to produce the required velocity change \( \Delta v \) [3]. The Hohmann transfer method required \( \Delta v \) is less than the other transition methods while The bi-elliptic transfer method requires \( \Delta t \) longer than Hohmann transfer [4]. The transfer is happen through one period or less which is no longer than 10 hours. The transition Maybe from elliptical orbit to circular or from circular to circular or from circular to elliptical, has a eccentricity \( e \). The transition between the two orbits having the same inclination \( i \) from equatorial. The transition may be from perigee or from the apogee of the initial orbit, and the transition may be in one or several stages. In this project we will study the transition from low elliptical orbit with eccentricity \( e=0.01 \) to Geostationary orbit As well as variation the inclination of the final orbit between \( 0-180 \). For the two cases, the rotation before the transition and the rotation after the transition, as well as the study of the two cases of transition from perigee and apogee. To search for the ideal possible transition and rotation [5].

1.1. Artificial satellite orbits:

a. Low Earth Orbits (LEO): These orbits are close to the surface of the earth, where the height ranges less than 1500 km, and has a circular or semi-circular shape \( e < 0.1 \), the satellites have a high velocity reach to 27359 km / hr or 7.6 km/sec. The time of period is in the range 90-225 min, therefore these satellites can be monitored from the earth station at time no more than 10 min. these type of orbit exposure to atmospheric drag which decrease exponentially with altitude and increase with area/mass of satellite, as well as the effect of other perturbations [6,7].

b. Mid Earth Orbits (MEO): The perigee height of these orbits ranges from (2000-35000 km) and their period (4-8 hr). In such a high orbit, the satellite can be monitored for two hours or more from the earth station. An example of these orbits (GPS satellite) in this orbit no drag effect [7].

c. High Earth Orbits (HEO): These orbits are more than (35000 km) from the Earth and its period is about a day or a little more orbit and is characterized by the longevity of the sun and the moon An example of these orbits Geosynchronous Earth Orbit GEO (circular orbit \( r = 42164 \) km) [7].

1.2. Types of transition

Any of the orbit may be classified depending on the inclination angle to three types (equatorial, inclined, polar) orbit.

a. Hohmann transfer method.

The Hohmann transfer method is an elliptical orbit that contacts both inner and outer orbits, the transfer orbit from perigee or apogee of the initial orbit to the final orbit is direct in half period. The Hohmann transfer methods have different efficiency for two-impulse maneuver and depend on the height of the initial orbit [8].

b. Hohmann transfers between coaxial elliptical orbits method.

This method contain two-impulsive maneuvers happen between elliptical orbits is called as a coaxial elliptical orbits, the transition occur in the original orbit to target orbit in two cases either from perigee or from apogee show in figure(1), to find which of two-transfer required low energy must calculated the individual total change of velocity requirement for the transfer The perigee and the apogee that determined, where the orbital parameters of the orbits
rA = 6858 km, \ rA' = 7818 km, \ rB = 10218 km, \ rB' = 8298 km [9].

Figure (1): Hohmann transfers method between coaxial elliptical orbits [9]

c. Bi-Elliptic Transfer method.

Bi-elliptic transfer is the dotted ellipse located inside the upper orbit but outside the lower orbit, in which touching the both orbits. Bi-elliptic transfer required two-transfer orbit and three-impulse burn. Bi-elliptic transfer applied two-coaxial semi ellipses in which extend beyond outer orbit. This type of transfer require long time and high mass of fuel born [9].

2. Theory:

Any two objects in the universe have a gravitational force according to Newton's law and move orbitally according to the Kepler’s law, the satellites orbits are not keplerian orbit because of the effect of perturbations on the orbital elements.

Hohmann transfers method between coaxial elliptical orbits to Satellite transmission was used in this research. It is the process of transition the satellite from a elliptical orbit with (e = 0.01) to a circular orbit through an elliptical transition orbit.

The basic equation of satellite motion on the elliptical orbit without perturbation is show as following equation [10,13].

\[
\ddot{r} = - \frac{\mu}{r^3} \dot{r}
\]  

\[\mu = \frac{G M_e}{r^2}\] where \(G\) is the gravitational constant and \(M_e\) is the Earth mass, \(\dot{r}\) is the position vector for the satellite, \(r\) is the distance between the earth center and satellite at time \((t)\)

\[\ddot{r} = \frac{d^2r}{dt^2}\] is the satellite acceleration at time \((t)\) in unit \(km/s^2\).

The perturbed forces are less than 0.01% of the central force. These perturbation forces are displace the satellite path from the Keparian orbit after many cycles. The satellite have additional accelerations by the perturbation forces, which can be combined into a resulting perturbing acceleration vector \(\ddot{r}_p\). The extended equations of motion are written [10,11,13,14]:

\[
\ddot{r} = - \frac{\mu}{r^3} \dot{r} + \ddot{r}_p
\]  

Perturbing forces are in particular responsible for [10, 13]:

\[\ddot{r}_p = \ddot{r}_E + \ddot{r}_s + \ddot{r}_M + \ddot{r}_{sp} + \ddot{r}_A\]
Where \( \hat{r}_E \) is the non-spherically and inhomogeneous mass distribution within Earth (central body), \( \hat{r}_S \) and \( \hat{r}_M \) is the sun and moon acceleration attraction on the satellite , \( \hat{r}_{sp} \) and \( \hat{r}_A \) is the Accelerations due to direct and Earth reflected solar radiation pressure.

These perturbations were not used in this work because the transition orbit is less than one orbit therefore the perturbations are very small and can be neglected [4,6].

The solution of the equation (1) is get the satellite distance [3]:

\[
r = \frac{h^2}{\mu} \frac{1}{1 + e \cos(f)}
\]  

(4)

\( f \): True anomaly angle \((0,360^\circ)\), \( h \): angular momentum per unit mass.

From equation (4) the perigee and apogee distance \( r_a \), \( r_p \) are calculated at \( f = 0.180 \) degree.

\[
r_a = a (1+e) \quad , \quad r_p = a (1-e)
\]  

(5)

Where \( a \) is the semi-major axis of the elliptical orbit.

The Eccentricity of the orbit can be calculated as the following [3,12]:

\[
e = \frac{r_a - r_p}{r_a + r_p}
\]  

(6)

This equation is used to calculate the angular momentum [3].

\[
h = \sqrt{2\mu} \sqrt{\frac{r_a r_p}{r_a + r_p}} \quad \text{or} \quad h = \sqrt{\mu \cdot a \cdot (1 - e^2)}
\]  

(7)

This equation is used to calculate the velocity for elliptical orbit [3].

\[
v^2 = \mu \left( \frac{2}{r} - \frac{1}{a} \right)
\]  

(8)

The semi major axis \( a \) calculated from equation (5) [12].

\[
a = \frac{r_p}{1 - e}
\]  

(9)

For circular orbit \( r = a \) the velocity at equation (8) become: [3].

\[
v^2 = \frac{\mu}{r}
\]  

(10)

The velocity required to move the satellite from the initial orbit to the final orbit through the transition orbit can be calculated from the following equation \( \Delta v \) is the variation of the velocity required for transfer.

If the transition is from a circular orbit to a circular orbit, we can use the equation (10). If the transition from an elliptical orbit to an elliptical orbit is used equation (8), this process requires transition time. The transition time when \( \mu = 398602.4415 \) and \( a \) in km for half period in sec can be calculated from the following equation (11).
The mass of satellite is not constant through transition orbit because mass burn. This can be written as the following [3]:

\[ \frac{\Delta m}{m} = 1 - e^{-\frac{\Delta v}{Isp g_o}} \]  

(12)

Where:

\( \Delta m \) is the consume mass for propellant.
\( g_o \) is the gravity standard of acceleration.
\( Isp \) is the impulsive specific of the propellants.

3. The program algorithm:

1- input information

\[ \pi = 3.1415926, \mu = 398602.4415 \text{ m}^3\text{sec}^{-2}, m = 1000 \text{ kg} \]
\[ Iisp = 300 \text{ sec}, g_o = 9.807 \text{ N/kg}, Re = 6378.165 \text{ km}, r_p1 = 6678 \text{ km}, e_1 = 0.01, r_c = 42164 \text{ km} \]

*1 is the initial orbit. **2 is the transition orbit. ***c or 3 is the final orbit

2- Calculation the semi major axis for initial orbit by the formula

\[ a_1 = \frac{r_p1}{1 - e_1} \]

And distance from apogee for initial orbit by the formula

\[ a_1 = \frac{r_a1}{1 + e_1} \]

3- Use \( \Delta \text{inclination} \) by the (inclination for final orbit – inclination for initial orbit) from 0˚ to 180˚.

4- Calculation velocity from perigee for initial orbit

\[ v_p1 = \sqrt{\mu \left( \frac{2}{r_a1} - \frac{1}{a_1} \right)} \]

5- Calculation of the eccentricity and angular momentum and semi major axis for the transition orbit

\[ 2 = \frac{(r_c - r_a1)/(r_c + r_a1)}{h_2} = \sqrt{(r_a1 \mu) (1 + e_2)} \]
\[ a_2 = (r_a1 + r_c)/2 \]

6- Calculation of the velocity at perigee and apogee for the transition orbit, and calculation the transition velocity required \( \Delta V_2 \) (first impulse velocity)

\[ v_p2 = \sqrt{\mu \left( \frac{2}{r_a1} - \frac{1}{a_2} \right)}; \]
\[ v_a2 = \sqrt{\mu \left( \frac{2}{r_c} - \frac{1}{a_2} \right)}; \]
\[ \Delta v_2 = v_p2 - v_p1. \]

7- Calculation of the velocity for the final orbit

\[ \Delta V_3 = \sqrt{\frac{\mu}{r_c}} \] and the second impulse velocity

8- Calculation of the impulse transition velocity \( \Delta V_{hohman} \) and rotational velocity required \( \Delta Vi \) and total velocity required \( \Delta V_i \) and \( T_{tran} \) and \( \Delta m/m \).

\[ \Delta V_{hohman} = (\Delta V_3 + \Delta V_2), \]
\[ \Delta Vi = 2(v_3 \text{ or } v_p2) \sin \left( \frac{\Delta i}{2} \right), \] use \( v_p2 \) at rotation before transition and use \( v_3 \) at transition before rotation.
ΔVt = (ΔVhohman + ΔVi).

9- Ttran = 0.5*(2*π)/(μ) * (a215)/60 in min.

Trot can be calculated by assume that the rotation happen through one period of the initial or final orbit that the rotation happen in it. ΔTtotal = Ttran+Trot

10- using eq 12 to calculate Δm/m where Δv in meter.

4. Results and discussion:

Transition and rotation techniques from orbit with high 300 km and e= 0.01 to the geostationary orbit, the program was designed by us to get the output date (ΔVhohman, ΔVi, ΔVtotal, Δm/m, Ttran) to all Δi = 0 to 180, that mean we have 180 tables at Δi= 0,1,2,……,180 that like the following table(1) at Δi=30.

Table-1: The results date for orbit transition with Δinclination 30.
4.1. Technique with case No. 1: Transfer and then rotate the orbit from apogee.

The variation of the required velocity to make a rotation of the orbit increasing with the required change in orbit inclination, but when it reaches a specific inclination, the velocity begins to less increase gradually as show in figure (2). At $\Delta i$ less than 60 degree the low velocity needed to rotate the orbit can be achieved. At $\Delta i$ more than 150 degree to 180 degree; the low velocity needed to rotate the orbit can be stable 6.2 km/sec approximately.

The velocity needed to transfer is 3.932710 km/s Added to all values that of velocity cause rotation $\Delta V_i$, which makes behavior $\Delta V_{\text{total}}$ Looks like a behavior $\Delta V_i$ as show in figure (3). The amount of fuel needed to transition $\Delta V_{\text{total}}$ is constant while $(\Delta m/m)$ rotation was various the rotation orbit with $\Delta i$ convexity as increased, when $\Delta i$ reaches more than 50 degree become difficult due to the need for more fuel, which causes an increase in the mass of the satellite and an increase in the cost of the satellite as show in figure (4).

![Graph](image)

(Figure 2): $\Delta V_i$ with $\Delta i$ for Technique 1.
4.2. Technique with case No. 2: rotate and then Transfer the orbit from apogee.

The variation of the required velocity to make a rotation of the orbit increasing with the required change in orbit inclination, it is behavior same case 1, but for case 2 the required values of velocity or energies to transfer and to rotate the orbit were increased to more than as show in figure (5,6) dapple values in case 1. The velocity needed to transfer is 4.011877 km/s Added to all the velocity values that needed rotation, which makes behavior $ΔV_{total}$ Looks like a behavior $ΔV_{i}$ as show in figure (6). The amount of fuel needed to rotate of the satellite, which is added to the satellite’s mass the variation of $Δm/m$ with $Δi$ Its have sharp increased with when $Δi$ reaches more than 20 degree We are facing difficulty in transportation due to the need for more fuel, which causes an increase in the mass of the satellite and an increase in the cost of the satellite, At $Δi$ more than 100 degree; the $Δm/m$ needed to rotate the orbit can be stable at value 1 as show in figure (7).
Figure (5): $\Delta V_i$ with $\Delta i$ for Technique 2.

Figure (6). $\Delta V_{\text{total}}$ with $\Delta i$ for Technique 2.

Figure (7): $\Delta m/m$ (rotation) with $\Delta i$ for Technique 2.

4.3. Technique with case No. 3: Rotate and then Transfer the orbit from perigee.

The variation of the required velocity to make a rotation of the orbit increasing with the required change in orbit inclination, it is behavior same Technique 2, figure (5). But there is very little change in the values as show on figure (8). The velocity needed to transfer is 3.989597 km/s. Added to all the velocity values that cause rotation, which makes behavior $\Delta V_{\text{total}}$ Looks like a behavior $\Delta V_i$ as show in figure (9). The amount of fuel needed to rotation of the satellite, which is added to the satellite’s mass the variation of $(\Delta m/m)$ rotation with $\Delta i$. Its sharp with $\Delta i$ increased when $\Delta i$ reaches more than 19 degree show a difficulty in transportation due to the need for more fuel, which causes an increase in the mass of the satellite and an increase in the cost of the satellite. At $\Delta i$ more than 100 degree; the $\Delta m/m$ (rotation) needed to rotate the orbit can be stable as show in figure (10).
4.4. Technique with case No. 4: Transfer and then the rotate orbit from perigee

The variation of the required velocity to make a rotation of the orbit increasing with the required change in orbit inclination, it is behavior same Technique 1, figure (2). But there is very little change in the values as show on figure (11). The velocity needed to transfer is 3.907747 km/s Added to all the velocity values that cause rotation, which makes behavior $\Delta V_{\text{total}}$ Looks like a behavior $\Delta V_{i}$ as show in figure (12). The amount of fuel needed to rotation of the satellite, which is added to the satellite’s mass the variation of $(\Delta m/m)$ rotation with $\Delta i$ It increases as the inclination increases when reaches more than 50 degree We are facing difficulty in transportation
due to the need for more fuel, which causes an increase in the mass of the satellite and an increase in the cost of the satellite, At $\Delta i$ more than 100 degree; the $\Delta m/m$ (rotation) needed to rotate the orbit can be stable 1 kg approximately as show in figure (13).

Figure (11): $\Delta V_i$ with $\Delta i$ for Technique 4.

Figure (12). $\Delta V$ total with $\Delta i$ for Technique 4.

Figure (13): $\Delta m/m$ (rotation) with $\Delta i$ for Technique 4.
**Conclusion**

In this research, we concluded that the best method for the transfer process is that we transfer the satellite and then perform the orbit rotation process. It’s difficult to rotate the orbit reach $\Delta i$ (60) degree or more. Because in this inclination the low velocity this mean low energy and the low velocity needed to rotate the orbit can be achieved and the amount of fuel needed to rotation of the satellite can be achieved also. But the higher the inclination, the more. The need for a higher velocity increases, which means we need higher energies, and the amount of fuel needed to transition of the satellite cannot be achieved.

**Techniques 1 and 2 are needed high energy to transfer and rotate and they are not proper to used when the rotation angle is great than 20 degree therefore techniques 3 or 4 must be uses.**

*** Technique no 3 is the best case to uses.

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