Flavor in Supersymmetry: Anarchy versus Structure

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Abstract

Future high-precision flavor experiments may discover a pattern of deviations from the standard model predictions for flavor-changing neutral current processes. One of the interesting questions that can be answered then will be whether the flavor structure of the new physics is related to that of the standard model or not. We analyze this aspect of flavor physics within a specific framework: supersymmetric models where the soft breaking terms are dominated by gauge-mediation but get non-negligible contributions from gravity-mediation. We compare the possible patterns of non-minimally flavor-violating effects that arise if the gravity-mediated contributions are anarchical vs. the case that they are structured by a Froggatt-Nielsen symmetry. We show that combining information on flavor and CP violation from meson mixing and electric dipole moments is indicative for the flavor structure of gravity-mediation.

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I. INTRODUCTION

The program of high $p_T$ experiments at the Tevatron and at the LHC, and the program of low energy flavor machines, such as present and future B-factories and the LHCb, are complimentary to each other. On one hand, understanding the flavor structure of TeV scale new physics is likely to shed light on the underlying theory at much higher energy scales, perhaps as high as the Planck scale. On the other hand, measuring new flavor parameters may lead to progress in understanding the flavor structure of the standard model itself, namely the smallness and hierarchy that appear in the Yukawa couplings. In this work, we explore this complementarity in a specific new physics framework, and provide a concrete demonstration of how high precision flavor measurements will lead to progress in understanding both the new physics and the standard model flavor puzzles.

The consistency of all measurements of flavor-changing neutral current (FCNC) processes with the standard model predictions requires that the flavor structure of new physics at the TeV scale is highly nontrivial. In particular, it seems likely that this flavor structure should be closely related to the flavor structure of the standard model Yukawa couplings. The most extreme application of this hint from low energy flavor measurements is the assumption of minimal flavor violation (MFV) \[1\]–\[5\]. The MFV principle states that the only source of flavor violation, even in interactions involving new particles, are the Yukawa matrices of the standard model. However, while the flavor constraints suggest that the dominant flavor structure of new physics should be MFV, there is certainly room for sub-dominant contributions that are not MFV. The discovery of such non-MFV physics will be of utmost interest. Thus, the first questions that future flavor measurements should explore, relevant to the new physics flavor puzzle, are the following:

- Are there non-MFV effects in the new physics? At what level do they appear?

If, indeed, non-MFV interactions are established, then finding out their pattern would be of much interest. Indeed, there is a wealth of possible FCNC sectors. In the quark sector alone, there are six different relevant transitions: $s \to d$, $c \to u$, $b \to d$, $b \to s$, $t \to c$ and $t \to u$. Three of these – the $b$ and $c$ decays – can be better measured in the B-factories and in LHCb, while the $t$ decays can be explored at the LHC. Understanding this pattern will allow us to answer yet another question, relevant to the standard model flavor puzzle:
• Is the flavor pattern of the non-MFV new physics related to the standard model flavor pattern or not?

Having these questions in mind, we focus in this work on supersymmetric (SUSY) models with a hybrid gauge- and gravity-mediation of supersymmetry breaking [6]. Gauge-mediation is well motivated since it solves the flavor problem of generic supersymmetric models. It should be kept in mind, however, that, in principle, gravity-mediated contributions are unavoidable. What is called pure gauge-mediation has an implicit assumption that the gravity-mediated contributions are quantitatively negligible. Indeed, this is the case if the source of supersymmetry breaking are $F$-terms with scales that are many orders of magnitude below $(m_Z m_{Pl})$. Pure gauge-mediation, if realized in Nature, will not provide us with additional data to try and understand the standard model flavor puzzle, namely the physics that leads to the structure observed in the Yukawa sector.

In contrast, in the framework called hybrid gauge- and gravity-mediation, there is an $F$-term within a few orders of magnitude below $(m_Z m_{Pl})$. This class of models provides an example of a well-motivated theoretical framework where the dominant flavor structure of the new physics (the soft supersymmetry breaking parameters) is MFV, coming from gauge-mediation, but there are sub-dominant contributions, from gravity-mediation, that are non-MFV and that lead to potentially observable deviations in precision flavor measurements.

In a previous work [7], we assumed that the structure of the non-MFV terms is related to that of the Yukawa couplings. Specifically, we assumed that there is an approximate Froggatt-Nielsen (FN) symmetry [8] that leads to selection rules which, in turn, dictate the structure of both the Yukawa couplings and the soft supersymmetry breaking terms [8, 9, 10]. In this work, in order to further explore the questions formulated above, we take a different path, where the structure of non-MFV terms is not related to the standard model one. Specifically, we assume that the gravity-mediated contributions to the squark mass-squared are anarchical, namely they are all of the same order, with no special features.

Such unrelated structures might arise because the Yukawa couplings come from the superpotential, while the soft mass-squared come from the Kahler potential. The two sectors may have different dynamics, or different selection rules from approximate symmetries. Model building directions are suggested by the framework of Ref. [11] that involves a strongly coupled sector, which is approximately conformally invariant and leads to large anomalous
dimensions of some of the quark fields over a large range of energies. If the CFT sectors are not separable, the relevant Yukawa couplings are suppressed (though not necessarily hierarchical), yet the corresponding soft terms are anarchical.

While we assume the anarchical structure for the quadratic squark masses throughout this work, we investigate three different scenarios for the trilinear scalar couplings (the $A$-terms): anarchical, vanishing, or of a structure similar to the Yukawa couplings. We do so because the $A$-terms, unlike the soft masses, come from the superpotential, as do the Yukawa couplings, and furthermore they transform under the flavor symmetry in precisely the same way as the Yukawa couplings. It could thus well be that their structure is related to the Yukawa sector, while the quadratic terms are not.

Our work here is aimed to answer the question formulated above, of whether a relation between the new physics and the standard model flavor structures exists at all. If the answer will end up being in the affirmative, we will be able to go a step further. Indeed, one can think of various mechanisms that would relate the two sectors, and the final goal would be to distinguish between them and answer questions such as the following:

- Does the flavor structure of the standard model come from an approximate symmetry, or from some dynamical mechanism? If it is a symmetry, is it Abelian or non-Abelian?

It should be interesting to pursue these questions in detail.

Within our framework, we impose the constraints that follow from low energy flavor measurements, and obtain the upper bounds on possible deviations that might still be discovered in the future. In this way, we show how the future flavor measurements may provide answers to the questions posed above, and by that lead to insights concerning the underlying theory of supersymmetry breaking mediation (the new physics flavor puzzle) and the theory of flavor (the standard model flavor puzzle).

The outline of this paper is as follows. In Section I we set our notations for the supersymmetric flavor parameters and review the FCNC and CP constraints. In Section II we present the soft terms at the high and at the weak scale in our framework of hybrid gauge-gravity mediation of supersymmetry breaking. In Section IV we summarize the analysis of Ref. [7] of the implications of FCNCs on models in which the gravity-mediated contributions are subject to an FN mechanism. In this work we extend this study by considering $A$-terms as well. Section V contains the main bulk of our work. In this Section we analyze
the implications of FCNC and CP-violating processes on models where gravity-mediated contributions are anarchical. In Section [VI] we discuss how, in the future, a pattern of deviations from the standard model predictions for FCNCs can shed light on basic flavor puzzles. Further technical details are given in Appendix [A], where we explain how the low energy flavor-violating parameters are related to the high scale soft supersymmetry breaking terms.

II. FCNC AND CP CONSTRAINTS ON SUSY PARAMETERS

Measurements of various low energy processes put strong indirect restrictions on physics beyond the standard model. Here, we briefly review the constraints from flavor-violating and CP-violating processes on the SUSY parameters relevant to our analysis.

Supersymmetric models provide, in general, new sources of flavor violation. These are most commonly analyzed in the basis in which the corresponding (down or up) quark mass matrix and the neutral gaugino vertices are diagonal. In this basis, which we label by a tilde, the squark mass matrices are not necessarily flavor-diagonal, and have the form

\[
\tilde{q}_M^*(\tilde{M}_q^2)_{ij}^{MN} \tilde{q}_N = (\tilde{q}_L^* \tilde{q}_R^*) \begin{pmatrix} (\tilde{M}_q^2)_{ij}^{Ll} & \tilde{A}_q^{Lq} v_q \\ \tilde{A}_q^{Lq} v_q & (\tilde{M}_q^2)_{kl}^{Rl} \end{pmatrix} \begin{pmatrix} \tilde{q}_L^* \\ \tilde{q}_R^* \end{pmatrix}.
\] (2.1)

Here, \(M, N = L, R\) label the chirality, and \(i, j, k, l = 1, 2, 3\) are generation indices. \(\tilde{M}_q^2\) and \(\tilde{M}_q^2\) are the supersymmetry-breaking squark masses-squared. The \(\tilde{A}_q\) parameters enter in the trilinear scalar couplings \(\tilde{A}_q \phi_q q_L q_R\), where \(\phi_q (q = u, d)\) is the \(q\)-type Higgs boson. The latter develop a vacuum expectation value \(v_q = \langle \phi_q \rangle\), with \(v = \sqrt{v_u^2 + v_d^2} \sim 174\) GeV and \(v_u/v_d = \tan \beta\).

In Eq. (2.1) we omit flavor-diagonal \(F\)- and \(D\)-term contributions present in the minimal supersymmetric standard model (MSSM) since they are not relevant to our analysis. Note that the \(F\)- and \(D\)-term contributions to the chirality-preserving mass terms \((\tilde{M}_q^2)_{ii}\) are suppressed anyway by \(v^2/\tilde{m}_q^2\) with respect to the SUSY-breaking contributions, where \(\tilde{m}_q^2\) is a representative \(q\)-squark mass scale.

In the tilde-basis, flavor violation takes place through squark mass insertions, bringing in factors of

\[
(\tilde{q}_q^2)_{MN} \equiv (\tilde{M}_q^2)_{ij}^{MN}/\tilde{m}_q^2.
\] (2.2)
TABLE I: The phenomenological upper bounds on the chirality-preserving couplings \( (\delta^q_{ij})_A \) and 

\[ \langle \delta^q_{ij} \rangle \]

on \( q = u, d \) and \( A = LL, RR \). The constraints are given for \( m_{\tilde{q}} = 1 \) TeV and 

\[ x \equiv m_{\tilde{g}}^2/m_{\tilde{q}}^2 = 1. \]

We assume that the phases could suppress the imaginary parts by a factor \( \sim 0.3 \). The bound on \( (\delta^d_{23})_{RR} \) is about 3 times weaker than that on \( (\delta^d_{23})_{LL} \) (given in the table). The constraints on \( (\delta^d_{12,13})_A \), \( (\delta^d_{12})_A \) and \( (\delta^d_{23})_A \) are based on, respectively, Refs. [12, 13] and [14].

| \( q \) | \( i,j \) | \( (\delta^q_{ij})_A \) | \( \langle \delta^q_{ij} \rangle \) |
|-----|-----|------|------|
| \( d \) | 12  | 0.03 | 0.002 |
| \( d \) | 13  | 0.2  | 0.07  |
| \( d \) | 23  | 0.6  | 0.2   |
| \( u \) | 12  | 0.1  | 0.008 |

It is useful to define also

\[ \langle \delta^q_{ij} \rangle = \sqrt{\langle \delta^q_{ij} \rangle_{LL} \langle \delta^q_{ij} \rangle_{RR}} . \] (2.3)

The \( \delta^q \) parameters cause flavor and, if complex, CP violation beyond the standard model, and are constrained by indirect measurements.

In Table I we compile the constraints on the chirality-preserving \( \delta^q_{MM} \) parameters obtained in Refs. [12–14]. Wherever relevant, a mild phase suppression in the mixing amplitude is allowed, namely we quote the stronger between the bounds on \( \Re(\delta^q_{ij}) \) and \( 3\Im(\delta^q_{ij}) \). The dependence of these bounds on the average squark mass \( m_{\tilde{q}} \), the ratio \( x \equiv m_{\tilde{g}}^2/m_{\tilde{q}}^2 \) as well as the effect of arbitrary CP violating phases can be found in Ref. [7], and references therein. For the \( D \) system, we use the recent constraints of Ref. [13] incorporating updated CP-violating effects.

For large \( \tan \beta \), additional constraints with respect to those in Table I arise. In particular the effects of neutral Higgs exchange in \( B_s \) and \( B_d \) mixing are important. For instance, for \( \tan \beta = 30 \) and \( x = 1 \) [7, 15]

\[ \langle \delta^d_{13} \rangle < 0.01 \cdot \left( \frac{M_{A^0}}{200 \text{ GeV}} \right), \quad \langle \delta^d_{23} \rangle < 0.04 \cdot \left( \frac{M_{A^0}}{200 \text{ GeV}} \right), \] (2.4)

where \( M_{A^0} \) denotes the pseudoscalar Higgs mass, and the above bounds scale roughly as \( (30/\tan \beta)^2 \). A more detailed discussion including chargino contributions and the impact of rare decays can be found in Ref. [7].

6
TABLE II: The phenomenological upper bounds on the chirality-mixing parameters \((\delta_{ij}^q)_{NM}\), \(N \neq M\) and \(q = u, d\). The constraints are given for \(m_\bar{q} = 1\) TeV and \(x \equiv m_\bar{q}^2/m_\bar{q}^2 = 1\). We assume that the phases could suppress the imaginary parts by a factor \(\sim 0.3\). The constraints on \((\delta_{12,13}^d)_{NM}\), \((\delta_{23}^d)_{NM}\) and \((\delta_{12}^u)_{NM}\), and \((\delta_{12}^u)_{NM}\) are based on, respectively, Refs. [12], [14] and [16, 17]. The bounds are the same for \(\delta_{LR}^q\) and \(\delta_{RL}^q\), except for \((\delta_{12}^d)_{NM}\), where the bound in parentheses refers to \(NM = RL\).

| \(q\) \(ij\) | \((\delta_{ij}^q)_{LR (RL)}\) |
|---|---|
| \(d\) 12 | \(2 \cdot 10^{-4} (0.002)\) |
| \(d\) 13 | 0.08 |
| \(d\) 23 | 0.01 |
| \(d\) 11 | \(4.7 \cdot 10^{-6}\) |
| \(u\) 11 | \(9.3 \cdot 10^{-6}\) |
| \(u\) 12 | 0.02 |

The experimental constraints on the \((\delta_{ij}^q)_{LR}\) parameters in the quark-squark sector are presented in Table II. Very strong constraints apply for the imaginary part of \((\delta_{11}^q)_{LR}\) from electric dipole moments (EDMs). The bounds given here correspond to the experimental upper limit on the EDM of the neutron, \(d_n < 2.9 \cdot 10^{-26} \text{ e cm}\) [17]. For \(x = 4\) and a phase smaller than 0.1, the EDM constraints on \((\delta_{11}^u,d)_{LR}\) are weakened by a factor of \(\sim 6\).

III. HYBRID GAUGE-GRAVITY MODELS

We consider supersymmetric models with gauge-mediated SUSY breaking in the presence of contributions induced by gravity at the Planck scale. While the former follows the flavor structure dictated by the one already present in the standard model, the latter allows, in general, for further intergenerational sfermion flavor mixings.

In Section IIIA we set our initial conditions at the scale of gauge-mediation, \(m_M\), and in Section IIIB give approximate analytical expressions for the flavor and CP-violating \(\delta^q\) parameters defined in Section II. The soft terms at \(m_M\) and at the electroweak scale, \(m_Z\), are linked by the MSSM renormalization group (RG) equations [18]. Details on the RG
evolution (RGE) are given in Appendix A.

A. High scale

In the discussed hybrid setup, the soft terms at the scale of gauge-mediation can be written as

\[
\begin{align*}
M_{Q_L}^2(m_M) &= \tilde{m}^2(1 + r X_{qL}), \\
M_{D_R}^2(m_M) &= \tilde{m}^2(1 + r X_{dR}), \\
M_{U_R}^2(m_M) &= \tilde{m}^2(1 + r X_{uR}).
\end{align*}
\]

Here, \(\tilde{m}\) is the typical scale of the gauge-mediated contribution to the soft terms, which is universal in the limit of neglecting \(\alpha_1, \alpha_2/\alpha_3\) effects, where \(\alpha_i = g_i^2/(4\pi)\), and \(g_1, g_2, g_3\) denote the gauge couplings of the electroweak sector (strong interaction). Above, the coefficient \(r \lesssim 1\) parameterizes the ratio between the gravity-mediated and the gauge-mediated contributions. Gravity-mediation induces also trilinear terms of the form

\[
\begin{align*}
A_{u}(m_M) &= \tilde{m}\sqrt{r} Z_{A_u}, \\
A_{d}(m_M) &= \tilde{m}\sqrt{r} Z_{A_d}.
\end{align*}
\]

B. The \(\delta^u\) parameters

The initial conditions Eqs. (3.1) and (3.2) hold at the high scale \(m_M\), while flavor-changing and CP-violating processes restrict the weak scale parameters \((\delta_{ij}^q)_{NM}\). Thus, the effect of the RG evolution on the soft terms must be evaluated. Furthermore, the \((\delta_{ij}^q)_{NM}\) parameters are read off from the low energy soft terms in the basis in which the quark mass matrices and gluino couplings are diagonal, which differs from the flavor basis. The requisite rotation of the squarks leaves the parametric pattern of the \(X_{ij}\) and \(Z_{ij}\) unchanged.
In Appendix A, we give a detailed derivation, given our framework and various approximations, of the low energy flavor parameters. It leads to the following expressions at the weak scale:

\[
\begin{align*}
(\delta_{12}^u)_{LL} & \sim \frac{1}{r_3} \max \{ r(X_{uL} + Z_A Z_{A_u}^T )_{12}, c_d y_b^2 |V_{ub} V_{cd}^*| \}, \\
(\delta_{12}^d)_{LL} & \sim \frac{1}{r_3} \max \{ r(X_{dL} + Z_A Z_{A_d}^T )_{12}, c_u y_t^2 |V_{ts} V_{td}^*| \}, \\
(\delta_{i3}^u)_{LL} & \sim \frac{1}{r_3} \max \{ r(X_{uL} + Z_A Z_{A_u}^T )_{i3}, c_d y_b^2 |V_{ib} V_{tb}^*| \}, \\
(\delta_{i3}^d)_{LL} & \sim \frac{1}{r_3} \max \{ r(X_{dL} + Z_A Z_{A_d}^T )_{i3}, c_u y_t^2 |V_{tb} V_{ti}^*| \}, \\
(\delta_{ij}^q)_{RR} & \sim \frac{r}{r_3} (X_{qR} + Z_A Z_{A_q}^T )_{ij}, \quad (i \neq j), \\
(\delta_{ij}^u)_{LR} & \sim (Z_{A_u})_{ij} \sqrt{r v} \sin \beta / (r_3 \tilde{m}), \\
(\delta_{ij}^d)_{LR} & \sim (Z_{A_d})_{ij} \sqrt{r v} \cos \beta / (r_3 \tilde{m}),
\end{align*}
\]

where \( V_{ij} \) are CKM elements, \( y_t, y_b \) denotes the top, bottom Yukawa, respectively, \( i = 1, 2 \) and \( j = 1, 2, 3 \). The factor \( r_3 \) captures the effect of RGE corrections to the diagonal elements of the soft squark mass matrices \( (M_2^2)_{ii} \) and is defined in Eq. (A3). Numerically, \( r_3 = \mathcal{O}(1 - 10) \), depending on the initial conditions, the scale of SUSY breaking and hidden sector effects. In minimal models, typically \( r_3 \sim 3 \). The coefficients \( c_u, c_d \) can be of \( \mathcal{O}(1) \) for \( m_M \) near the GUT scale and are all negative. The expressions Eq. (3.3) hold also for \( (\delta_{jj}^q)_{LR} \) up to MSSM \( F \)-term contributions. Throughout this work, the “\( \sim \)” sign implies a similar parametric suppression but with generally different \( \mathcal{O}(1) \) complex coefficients.

### IV. FN SYMMETRY IN THE GRAVITY SECTOR

A mediation mechanism allowing non-MFV contributions to the soft SUSY breaking terms in which flavor-changing terms are nonetheless suppressed was considered in Ref. [19]. In such a setup, the gauge-mediation contributions are dominant, but gravity-mediation contributions are non-negligible. In Ref. [19], the structure of the gravity-mediated contributions was not arbitrary, but rather set by the same approximate horizontal symmetry which explains the smallness of the Yukawa couplings à la Froggatt-Nielsen [8, 10].

In Section IV A we summarize the implications of FCNC constraints in such hybrid FN models as obtained in Ref. [7], updating these to include the more recent constraints in the
system. In Section IV B we investigate the soft breaking $A$-terms in the presence of the flavor symmetry.

A. Flavor breaking in hybrid FN models

We now summarize the results of Ref. [7], in which the gravity-mediated contributions to the soft supersymmetry breaking terms are assumed to be subject to the selection rules of the FN symmetry. Within the simplest FN models, with a single horizontal $U(1)_H$, the parametric structure of the gravity-mediated contributions to the soft terms (3.1) is given by

\[
(X_{q_{L,R}})_{ii} \sim 1, \quad (X_{q_{L}})_{ij} \sim |V_{ij}|, \quad (X_{q_{R}})_{ij} \sim \frac{m_{q_i}/m_{q_j}}{|V_{ij}|} \quad (i < j), \quad q = u, d,
\]

where $m_{q_i}$ denotes the $i$th generation $q$-type quark mass. For the non-MFV contributions, in which there are uncertainties of order one, we use, for example, $V_{13}$ to represent a parametric suppression that is similar to that of $V_{ub}$ or $V_{td}$. For the MFV contributions, we use notations such as $V_{td}$ to denote the actual contributing CKM element.

Imposing the flavor structure of Eq. (4.1) on the expressions (3.3), we obtain the order of magnitude estimates for the $\delta_{ij}^q$ parameters presented in Table III. The $\hat{r}$ parameter is defined as

\[
\hat{r} \equiv \max \{r, y_b^2\}.
\]

Comparing the phenomenological constraints of Table III to the theoretical predictions of Table IV, we obtain upper bounds on $r$ and on $\hat{r}$. The strongest bound on $r$ comes from the $\langle \delta_{12}^d \rangle$ parameter, i.e. from the neutral Kaon system:

\[
r/r_3 \lesssim 0.01 - 0.03.
\]

Here we use $m_{\tilde{q}} = 1$ TeV; for lighter $m_{\tilde{q}}$ the bounds would be stronger by $m_{\tilde{q}}/(1$ TeV). The stronger bound corresponds to $x = 1$ and a phase of order 0.3, while the weaker bound corresponds to $x = 4$ and a phase smaller than 0.1. Since the $\hat{r}$ parameter affects only the $\delta_{i3}^u$ parameters, there is no phenomenological constraint on its size, and it is only bounded by its definition:

\[
r \leq \hat{r} \lesssim 1.
\]
TABLE III: The order of magnitude estimates for \((\delta_{ij}^{d,u})_{LL,RR}\) and \((\delta_{ij}^{d,u})\) in the hybrid gauge-gravity models with FN structure [7]. The numerical estimates are obtained using quark masses at the scale \(m_Z\) [20], and taking \(r_3 = 3\). All results scale as \((3/r_3)\). For \((\delta_{ij}^{d})\) we use \(|V_3| \sim |V_{ts}|\).

| \(q \ ij\) | \((\delta_{ij}^{d})_{LL}\) | \((\delta_{ij}^{d})_{RR}\) | \((\delta_{ij}^{d})\) |
|---|---|---|---|
| \(d 12\) | \((r/r_3)|V_{12}| \sim 0.08r\) | \(\frac{(r/r_3)(m_d/m_s)}{|V_{12}|} \sim 0.08r\) | \((r/r_3)\sqrt{m_d/m_s} \sim 0.08r\) |
| \(d 13\) | \(y_t^2|V_{id}V_{tb}|/r_3 \sim 0.003\) | \(\frac{(r/r_3)(m_d/m_b)}{|V_{13}|} \sim 0.08r\) | \(y_t\sqrt{m_d/m_b}/r_3 \sim 0.01\sqrt{r}\) |
| \(d 23\) | \(y_t^2|V_{is}V_{tb}|/r_3 \sim 0.01\) | \(\frac{(r/r_3)(m_u/m_d)}{|V_{23}|} \sim 0.2r\) | \(y_t\sqrt{m_u/m_d}/r_3 \sim 0.05\sqrt{r}\) |
| \(u 12\) | \((r/r_3)|V_{12}| \sim 0.08r\) | \(\frac{(r/r_3)(m_u/m_c)}{|V_{12}|} \sim 0.003r\) | \((r/r_3)\sqrt{m_u/m_c} \sim 0.02r\) |
| \(u 13\) | \((\hat{r}/r_3)|V_{13}| \sim 0.001\hat{r}\) | \(\frac{(r/r_3)(m_u/m_t)}{|V_{13}|} \sim 0.0006r\) | \(r\hat{r}m_u/m_t/r_3 \sim 0.0009\sqrt{r}\hat{r}\) |
| \(u 23\) | \((\hat{r}/r_3)|V_{23}| \sim 0.01\hat{r}\) | \(\frac{(r/r_3)(m_u/m_t)}{|V_{23}|} \sim 0.03r\) | \(r\hat{r}m_u/m_t/r_3 \sim 0.02\sqrt{r}\hat{r}\) |

TABLE IV: The order of magnitude upper bounds on \((\delta_{ij}^{d,u})_{LL,RR}\) and \((\delta_{ij}^{d,u})\) for \(r/r_3 \lesssim 0.03\) in hybrid FN models [7]. Entries in parentheses are independent of \(r\), therefore representing estimates rather than upper bounds, and scale as \((3/r_3)\). The bounds on \((\delta_{13,23}^{d})\) scale as \(\sqrt{3/r_3}\). The bounds on \((\delta_{12}^{u})_{LL} (\langle \delta_{12}^{u} \rangle)\) correspond to \(\hat{r} \sim 1\) and scale as \((3/r_3) (\sqrt{3/r_3})\); if \(\hat{r} = r\), these bounds are a factor of 10 \(\sqrt{10}\) stronger and do not scale with \(r_3\).

| \(q \ ij\) | \((\delta_{ij}^{d})_{LL}\) | \((\delta_{ij}^{d})_{RR}\) | \((\delta_{ij}^{d})\) |
|---|---|---|---|
| \(d 12\) | 0.007 | 0.007 | 0.007 |
| \(d 13\) | [0.003] | 0.007 | 0.003 |
| \(d 23\) | [0.01] | 0.01 | 0.01 |
| \(u 12\) | 0.007 | 0.0003 | 0.001 |
| \(u 13\) | 0.001 | 0.00005 | 0.0003 |
| \(u 23\) | 0.01 | 0.003 | 0.006 |

For small values of \(\tan \beta\), \(\hat{r} = r\) and Eq. (1) applies to \(\hat{r}\). Inserting \(r/r_3 \lesssim 0.03\) and \(r \leq \hat{r} \leq 1\) into the predictions of Table III, we obtain the upper bounds on the \(\delta_{ij}^{d}\) presented in Table IV.

The maximal possible effects in the neutral \(B_d\), \(B_s\) and \(D\) systems are thus as follows...
(for $r_3 = 3$):

\begin{align*}
B_d : \left| \frac{M_{12}^{\text{susy}}}{M_{12}^{\text{exp}}} \right| & \lesssim 0.002, \\
B_s : \left| \frac{M_{12}^{\text{susy}}}{M_{12}^{\text{exp}}} \right| & \lesssim 0.005, \\
D : \left| \frac{M_{12}^{\text{susy}}}{M_{12}^{\text{exp}}} \right| & \lesssim 0.03.
\end{align*}

(4.5)

The sensitivity in the $D$ system is slightly modified in comparison to Ref. [7] due to the use of the updated analysis of Ref. [13], see also Table I.

The mixing amplitudes of the $B_{d,s}$ mesons can be significantly enhanced for low $M_{A^0}$ and large $\tan \beta$. By comparing the phenomenological constraints of Eq. (2.4) to the predictions of Table III one finds (for $r_3 = 3$, $\tan \beta = 30$ and $M_{A^0} = 200$ GeV):

\begin{align*}
B_d : \left| \frac{M_{12}^{\text{susy}}}{M_{12}^{\text{exp}}} \right| & \lesssim 0.10, \\
B_s : \left| \frac{M_{12}^{\text{susy}}}{M_{12}^{\text{exp}}} \right| & \lesssim 0.13.
\end{align*}

(4.6)

The effects on these systems are maximized when the RGE suppression is minimal. Further details, such as a variant of FN models with holomorphic zeros, where the gravity-mediated contribution to the $D^0 - \bar{D}^0$ mixing amplitude can be of $\mathcal{O}(1)$, can be found in Ref. [7].

B. $A$-terms in hybrid FN models

Going beyond Ref. [7], we investigate here the trilinear $A$-terms in hybrid models with a FN flavor symmetry. In such scenarios, the $A$-terms follow the same parametric suppression as the corresponding Yukawa matrices $Y$. At the high, messenger scale:

\begin{equation}
(A^{u,d})_{ij}(m_M) \sim \sqrt{r} \tilde{m} \nu^{u,d}_{ij}. \tag{4.7}
\end{equation}

Since the $A$-terms are only similar in texture to the Yukawa matrices, but not proportional to them, rotating to the mass basis leaves the $A$-terms undiagonalized ($q = u, d$)

\begin{equation}
(Z_{A_q})_{ij} \sim Y^q_{ij} \sim V_{ij}m_q/v_q. \tag{4.8}
\end{equation}

The resulting chirality-mixing $\delta^q_{LR}$ parameters, see Eq. (3.3), are less important for flavor physics than the chirality-preserving $\delta^q_{LL, RR}$ parameters. If CP-violating, the $\delta^q_{LR}$ induce a neutron EDM allowed by Eq. (4.3):

\begin{equation}
|d_n^{\text{susy}}/d_n^{\text{exp}}| \lesssim 0.02 \ (0.002), \tag{4.9}
\end{equation}

where the first value corresponds to $x = 1$ and a phase suppression in $(\delta^0_{11})_{LR}$ of $\sim 0.3$, and the value in parentheses is obtained for $x = 4$ and a phase suppression of $\sim 0.1$. 

12
V. ANARCHY IN THE GRAVITY SECTOR

Thus far, we have considered flavor-changing processes within supersymmetric models with hybrid gauge-gravity mediation, in which the structure of the gravity contributions is dictated by the FN mechanism. However, the gravity sector need not obey such selection rules and may, for example, be of anarchical character. By anarchy we mean structure-less gravity contributions, such that all terms of the (hermitian) matrices in Eq. (3.1) obey

\[(X_{qA})_{ij} \sim O(1),\]  \hspace{1cm} (5.1)

and carry, in general, order one CP-violating phases. In particular we do not consider accidental suppressions in the magnitude of individual matrix elements. We now study which measurements can reveal the existence of such anarchical models.

Assuming anarchical structure for the squark masses-squared, one can still consider various structures for the trilinear scalar couplings. The effect of non-vanishing \(A\)-terms is two-fold: First, the RG evolution of the soft terms is modified, and second, chirality-mixing processes may get direct contributions from these terms. We explore three different scenarios for the \(A\)-terms:

1. Section V A: vanishing \(A\)-terms;
2. Section V B: anarchical \(A\)-terms;
3. Section V C: Yukawa-like textured \(A\)-terms.

Before we start a detailed discussion, a comment regarding the MFV terms is in order. In the current context of an anarchical texture in the \(X_{qA}\) matrices Eq. (5.1), non-MFV effects are non-negligible in the \(\delta_{LL}^i\) parameters provided \(r \gtrsim y_t^2 \lambda^5 \sim 3 \cdot 10^{-4}\), where \(\lambda \sim |V_{12}| \sim 0.2\), as can be seen from Eq. (3.3). This is a weaker condition than in the analogous FN case, where interesting, gravity-dominated effects require \(r \gtrsim y_t^2 \lambda^4 \sim 2 \cdot 10^{-3}\). This in turns implies that for the gravity-mediated contributions to have observable consequences, the messenger scale can in principle be lower in the anarchical setup than in the framework with a FN flavor structure.
A. Vanishing $A$-terms

We consider anarchical models with vanishing $A$-terms at the messenger scale:

$$A^{u,d}(m_M) = 0.$$  \hfill (5.2)

Hence, our starting point is Eq. (3.3) with

$$(X_{qa})_{ij} = O(1), \quad Z_{A_q} = 0.$$  \hfill (5.3)

We obtain the following flavor parameters, at the $m_Z$-scale ($i = 1,2$):

$$(\delta_{u12}^u)_{LL} \sim \frac{r}{r_3}, \quad (\delta_{d12}^d)_{LL} \sim \frac{r}{r_3},$$  \hfill (5.4)

$$(\delta_{u3}^u)_{LL} \sim \frac{\hat{r}_i}{r_3}, \quad (\delta_{d3}^d)_{LL} \sim \frac{\hat{r}_i}{r_3},$$

and ($i \neq j$, $j = 1,2,3$):

$$(\delta_{ij}^q)_{RR} \sim \frac{r}{r_3},$$  \hfill (5.5)

where

$$\hat{r}_i \equiv \max\{r, y_b^2 V_{ub} V_{ub}^*\}, \quad \hat{r}_i^u \equiv \max\{r, y_b^2 V_{ub}^* V_{ub}\}.$$  \hfill (5.6)

The resulting order of magnitude estimates for the chirality-preserving $\delta_{ij}^q$ parameters are presented in Table [V]. The chirality-mixing $(\delta_{ij}^q)_{LR}$ parameters play no role in constraining SUSY flavor in this scenario.

By comparing the phenomenological constraints of Table [II] to the order of magnitude predictions of Table [V], we obtain an upper bound on $r$ and find the maximal possible effects in this scenario. The strongest bound on $r$ comes from the neutral Kaon system constraint on $(\delta_{12}^d)$:

$$r/r_3 \lesssim 0.002 - 0.006.$$  \hfill (5.7)

The range corresponds to the same assumptions that enter Eq. (4.3).

The $\hat{r}_i$ ($\hat{r}_i^u$) parameters affect only the $\delta_{i3}^u$ ($\delta_{i3}^d$) parameters, and they are bounded only by their definition:

$$\hat{r}_1 = \max\{r, y_b^2 V_{ub} V_{ub}^*\} \sim \max\{r, 0.004 y_b^2\},$$

$$\hat{r}_2 = \max\{r, y_b^2 V_{cb} V_{cb}^*\} \sim \max\{r, 0.04 y_b^2\},$$  \hfill (5.8)
TABLE V: The order of magnitude estimates for \((\delta_{ij}^{d,u})_A\), \(A = LL, RR\) and \((\delta_{ij}^{d,u})\) in the models defined by Eq. (5.3). The numerical estimates are obtained using quark masses at the scale \(m_Z\) \[^20\] and for \(r_3 = 3\). All numerical estimates scale as \((3/r_3)\).

| \(q\ ij\) | \((\delta_{ij}^q)_{LL}\) | \((\delta_{ij}^q)_{RR}\) | \((\delta_{ij}^q)\) |
|---------|-----------------|-----------------|-------|
| \(d 12\) | \(r/r_3 \sim 0.3r\) | \(r/r_3 \sim 0.3r\) | \(r/r_3 \sim 0.3r\) |
| \(d 13\) | \(\hat{r}_{1}^{u}/r_3 \sim 0.3\hat{r}_{1}^{u}\) | \(r/r_3 \sim 0.3r\) | \(\sqrt{r_3^{1r}/r_3} \sim 0.3\sqrt{r_3^{1r}}\) |
| \(d 23\) | \(\hat{r}_{2}^{u}/r_3 \sim 0.3\hat{r}_{2}^{u}\) | \(r/r_3 \sim 0.3r\) | \(\sqrt{r_3^{2r}/r_3} \sim 0.3\sqrt{r_3^{2r}}\) |
| \(u 12\) | \(r/r_3 \sim 0.3r\) | \(r/r_3 \sim 0.3r\) | \(r/r_3 \sim 0.3r\) |
| \(u 13\) | \(\hat{r}_{1}^{u}/r_3 \sim 0.3\hat{r}_{1}^{u}\) | \(r/r_3 \sim 0.3r\) | \(\sqrt{r_3^{1r}/r_3} \sim 0.3\sqrt{r_3^{1r}}\) |
| \(u 23\) | \(\hat{r}_{2}^{u}/r_3 \sim 0.3\hat{r}_{2}^{u}\) | \(r/r_3 \sim 0.3r\) | \(\sqrt{r_3^{2r}/r_3} \sim 0.3\sqrt{r_3^{2r}}\) |

TABLE VI: The order of magnitude upper bounds on \((\delta_{ij}^{d,u})_{LL,RR}\) and \((\delta_{ij}^{d,u})\) corresponding to \(r/r_3 \lesssim 0.006\) and the bounds of Eqs. (5.8, 5.9). Entries with an \(r_3\) dependence are indicated so.

| \(q\ ij\) | \((\delta_{ij}^q)_{LL}\) | \((\delta_{ij}^q)_{RR}\) | \((\delta_{ij}^q)\) |
|---------|-----------------|-----------------|-------|
| \(d 12\) | 0.006 | 0.006 | 0.006 |
| \(d 13\) | max\{0.006, 0.003(3/r_3)\} | 0.006 | max\{0.006, 0.004\sqrt{3/r_3}\} |
| \(d 23\) | max\{0.006, 0.01(3/r_3)\} | 0.006 | max\{0.006, 0.009\sqrt{3/r_3}\} |
| \(u 12\) | 0.006 | 0.006 | 0.006 |
| \(u 13\) | max\{0.006, 0.001y_b^2(3/r_3)\} | 0.006 | max\{0.006, 0.003y_b\sqrt{3/r_3}\} |
| \(u 23\) | max\{0.006, 0.001y_b^2(3/r_3)\} | 0.006 | max\{0.006, 0.009y_b\sqrt{3/r_3}\} |

and

\[
\hat{r}_{1}^{u} = \max\{r, y_t^2V_{td}^*V_{tb}\} \sim \max\{r, 0.009\},
\hat{r}_{2}^{u} = \max\{r, y_t^2V_{ts}^*V_{td}\} \sim \max\{r, 0.04\}. \tag{5.9}
\]

Inserting the bounds (5.7), (5.8) and (5.9) into the predictions of Table V, we obtain the upper bounds on the \(\delta_{ij}^q\) given in Table VI.

We learn that the maximal possible effects in the neutral \(B_d\), \(B_s\) and \(D\) systems are as
follows (for $r_3 = 3$):

$$B_d : |M_{12}^{\text{susy}} / M_{12}^{\text{exp}}| \lesssim 0.007,$$

$$B_s : |M_{12}^{\text{susy}} / M_{12}^{\text{exp}}| \lesssim 0.002,$$

$$D : |M_{12}^{\text{susy}} / M_{12}^{\text{exp}}| \lesssim 0.6. \tag{5.10}$$

Note that these systems do not depend on $\hat{r}_1$, and so are independent of $y_b$.

We emphasize the following points:

- The bound in the $D$ system comes from $\langle \delta_{12}^u \rangle$ and is $r_3$ independent.

- For $r_3 = \mathcal{O}(1 - 10)$, the bound in the $B_s$ system comes from $\langle \delta_{23}^d \rangle$. For $r_3 = \mathcal{O}(1 - 7)$ it scales with $3/r_3$; for $r_3 > 7$ it does not scale with $r_3$.

- For $r_3 = \mathcal{O}(1 - 10)$, the bound in the $B_d$ system comes from $\langle \delta_{13}^d \rangle$. For $r_3 = \mathcal{O}(1 - 1.5)$ it scales with $3/r_3$; for $r_3 > 1.5$ it does not scale with $r_3$.

As mentioned in Section II, for large $\tan \beta$ and low $M_{A^0}$, the $B_{d,s}$ mixing amplitudes can be significantly enhanced. Indeed, comparing the constraints of Eq. (2.4) to Table V, we obtain for $r_3 = 3$ (and $\tan \beta = 30$, $M_{A^0} = 200$ GeV):

$$B_d : |M_{12}^{\text{susy}} / M_{12}^{\text{exp}}| \lesssim 0.36,$$

$$B_s : |M_{12}^{\text{susy}} / M_{12}^{\text{exp}}| \lesssim 0.05. \tag{5.11}$$

The $r_3$ dependence of upper bounds on the supersymmetric contributions to $B_d$ and $B_s$ mixings is shown in Fig. [1].

**B. Anarchical $A$-terms**

We consider scenarios with anarchical $A$-terms at the messenger scale:

$$(A_{u,d})_{ij}(m_M) \sim \sqrt{r} \tilde{m}, \tag{5.12}$$

so that

$$(X_{q_A})_{ij} = \mathcal{O}(1), \quad (Z_{A_q})_{ij} = \mathcal{O}(1). \tag{5.13}$$

We insert the anarchical structure (5.13) into the expressions of the $\delta^q$ parameters in Eq. (3.3). We then compare these predictions to the bounds given in the Tables II and II. Note that these bounds are obtained for $m_{\tilde{q}} = 1$ TeV. Recalling that parametrically $m_{\tilde{q}}^2 \sim r_3 \bar{m}^2$, we use $\bar{m} \sim 1$ TeV/\sqrt{r_3}.
FIG. 1: Maximum reach in $B_d$ (solid) and $B_s$ (dashed) mixing, $|M_{12}^{\text{asy}}/M_{12}^{\text{exp}}|$, as a function of the RGE-factor $r_3$, for hybrid anarchy models with vanishing soft trilinear couplings as in Eq. (5.2). The two upper curves correspond to $\tan \beta = 30$ and $M_{A^0} = 200$ GeV, while the two lower ones correspond to low $\tan \beta$.

The anarchical $A$-terms induce $O(1)$ changes in the $X_{qA}$ matrices in the chirality preserving LL and RR blocks of the squark mass-squared matrices. Hence, the order of magnitude estimates for the chirality-preserving $(\delta_{qij}^{it})_{LL,RR}$ parameters given in Eqs. (5.4) and (5.5) remain standing. The new ingredients in this analysis with full anarchy are the chirality-mixing parameters $\delta_{LR}^{i}$. We find that the strongest bound on $r/r_3$ comes from the EDM constraints on $(\delta_{11}^{u,d})_{LR}$:

$$ r/r_3 < \begin{cases} 
(7.3 \cdot 10^{-10} - 2.8 \cdot 10^{-8}) \frac{1 + \tan^2 \beta}{(Z_{A^0})_{11}^2} & \text{down sector,} \\
(2.9 \cdot 10^{-9} - 1.1 \cdot 10^{-7}) \frac{1 + \tan^2 \beta}{\tan^2 \beta (Z_{A^0})_{11}^2} & \text{up sector.} 
\end{cases} \quad (5.14) $$

The stronger bounds correspond to $x = 1$ and phases $\lesssim 0.3$. The weaker bounds correspond to $x = 4$ and phases $\lesssim 0.1$. For $\tan \beta \gtrsim 2(\gtrsim 2)$ the down (up) sector represents the stronger bound in Eq. (5.14). The $(\delta_{11}^{u,d})_{LR}$-related bound is largely insensitive to the value of $\tan \beta$.

We also give the maximal values of the chirality-changing $\delta_{LR}^{i}$ parameters in full anarchy using Eq. (5.14) and $m_{\tilde{q}} = 1$ TeV (for $i \neq j$):

$$ (\delta_{ij}^{u})_{LR} \lesssim 6 \cdot 10^{-5}, $$
$$ (\delta_{ij}^{d})_{LR} \lesssim 6 \cdot 10^{-5}/\tan \beta. \quad (5.15) $$
TABLE VII: The order of magnitude estimates for \((\delta_{ij}^{d,u})_{LL,RR}\) and \((\delta_{ij}^{d,u})\) in the models defined by Eq. (5.13). The numerical estimates are obtained using quark masses at the scale \(m_Z\) and for \(r_3 = 3\) and \(\tan \beta = 3\). All numerical estimates scale as \((3/r_3)\).

| \(q \ ij\) | \((\delta_{ij}^{d})_{LL}\) | \((\delta_{ij}^{u})_{RR}\) | \((\delta_{ij}^{d})\) |
|---|---|---|---|
| \(d \ 12\) | \(y_t^2|V_{ts}V_{td}^*|/r_3 \sim 10^{-4}\) | \(r/r_3 \sim 0.3r/\sqrt{r}\) | \(0.006\) |
| \(d \ 13\) | \(y_t^2|V_{tb}V_{td}^*|/r_3 \sim 0.003\) | \(r/r_3 \sim 0.3r/\sqrt{r}\) | \(0.006\) |
| \(d \ 23\) | \(y_t^2|V_{tb}V_{ts}^*|/r_3 \sim 0.01\) | \(r/r_3 \sim 0.3r/\sqrt{r}\) | \(0.007\) |
| \(u \ 12\) | \(\tilde{r}/r_3 \sim 0.3\tilde{r}/r_3 \sim 0.3\sqrt{\tilde{r}}\) | \(\tilde{r}/r_3 \sim 0.3\sqrt{\tilde{r}}\) | \(0.004\) |
| \(u \ 13\) | \(y_b^2|V_{ub}V_{tb}^*|/r_3 \sim 4 \cdot 10^{-6}\) | \(r/r_3 \sim 0.3r/\sqrt{r}\) | \(0.004\) |
| \(u \ 23\) | \(y_b^2|V_{cb}V_{tb}^*|/r_3 \sim 4 \cdot 10^{-5}\) | \(r/r_3 \sim 0.3r/\sqrt{r}\) | \(0.004\) |

The constraints in Eq. (5.14) imply that, for reasonable values of the RG-factor \(r_3 = O(1 - 10)\), most of the \(\delta_{LL}^{d}\) parameters are dominated by MFV effects:

\[
\begin{align*}
(\delta_{12}^{d})_{LL} & \sim \frac{c_u y_t^2 |V_{ts}V_{td}^*|}{r_3}, \\
(\delta_{i3}^{u})_{LL} & \sim \frac{c_d y_b^2 |V_{ub}V_{tb}^*|}{r_3}, \\
(\delta_{i3}^{d})_{LL} & \sim \frac{c_u y_b^2 |V_{ub}V_{tb}^*|}{r_3}, \\
\end{align*}
\]

(5.16)

where \(i \neq 3\), whereas

\[
(\delta_{12}^{u})_{LL} \sim \frac{\tilde{r}}{r_3},
\]

(5.17)

with

\[
\tilde{r} \equiv \max\{r, c_d y_b^2 |V_{ub}V_{cb}^*|\} \sim \max\{r, 2 \cdot 10^{-4} y_b^2\},
\]

(5.18)

can be either MFV or gravity dominated. The \(\delta_{RR}^{d}\) parameters in full anarchy are still non-MFV dominated, with the RR-mixing being subject to some subtleties discussed below.

Order of magnitude estimates for the various \(\delta_{ij}^{d}\) are given in Table VII.

The situation regarding the RR flavor mixing is driven by three factors: the extremely strong constraint on \(r/r_3\), the fact that the latter stems from LR-mixing, which is only suppressed by \(\sqrt{r/r_3}\), and the absence of significant MFV terms in the RR-mixing as opposed to the LL one.
Concerning the MFV contributions in the RR sector, as implemented in Eq. (3.3), flavor-changing MFV effects in \((\tilde{M}_{qR}^2)_{ij}\) are absent at one-loop. However, they are generated at two loops through the RGE \([18]\). The largest such effect is the contribution to \((\tilde{M}_{dR}^2)_{23}\) and is proportional to \(y_{sk}y_{bt}^3V^*_tb\sim 2\cdot10^{-7}\tan^2\beta\). Estimating very roughly the loop suppression as \(1/(16\pi^2)^2\times \text{logs} \lesssim 10^{-3}\), the non-MFV gravity effects dominate in \((\delta_{23})_{RR}\) provided Eq. (5.14) is saturated. We stress that in either case, the resulting \(\delta_{RR}\) value is very small and irrelevant for flavor phenomenology. In this sense it is not of interest whether such a small value is non-MFV or MFV. The latter can happen if the neutron EDM bound tightens, or for very large values of \(\tan\beta\).

Additionally, due to the smallness of the single insertion \(\delta_{RR}\) parameters, of order \(10^{-7}\), the question arises as to whether products of multiple mass insertions, effectively yielding similar RR squark mixing \((\delta^{u,d}_{ij})_{RR}\), can lead to comparable or larger effects. These receive contributions from \(F\)-terms induced by the supersymmetric Higgs mass term \(\mu\) in the MSSM superpotential. For mixing involving the third generation we estimate \((i = 1, 2)\):

\[
\begin{align*}
(\delta_{i3}^u)_{RR} &= (\delta_{i3}^u)_{RL}(\delta_{33}^u)_{LR} \sim \sqrt{\frac{r_m v \mu}{r_3 m^2_\tilde{q}}} \frac{1}{\sqrt{1 + \tan^2 \beta}} \lesssim \frac{1 \cdot 10^{-5}}{\tan \beta} \left(\frac{\mu}{1 \text{ TeV}}\right), \\
(\delta_{i3}^d)_{RR} &= (\delta_{i3}^d)_{RL}(\delta_{33}^d)_{LR} \sim \sqrt{\frac{r_m v \mu}{r_3 m^2_\tilde{q}}} \frac{\tan \beta}{\sqrt{1 + \tan^2 \beta}} \lesssim 2 \cdot 10^{-7} \left(\frac{\mu}{1 \text{ TeV}}\right),
\end{align*}
\]

where in the inequality we apply the bound Eq. (5.14) for \(\tan \beta \gtrsim 2\) and use \(m_\tilde{q} = 1\) TeV. For \(\mu \sim m_\tilde{q}\) we then find that the maximal reach of \((\delta_{i3}^u,d)_{RR}\) is larger than the corresponding maximal reach of \((\delta_{i3}^u,d)_{RR} \sim r/r_3\). For all other \(\delta^q\) parameters, the single mass insertion is the dominant one.

Combining the constraint Eq. (5.14) to the order of magnitude estimates of Table VII, we obtain the upper bounds on the \(\delta_{ij}^q\) presented in Table VII. Note that in the up sector, entries with an \(r_3\) dependence are MFV, while non-\(r_3\) entries are non-MFV. As the \((\delta_{i3}^u,d)_{RR}\) and the \((\delta_{i3}^u,d)\) parameters will not affect the maximal possible effects in the neutral meson systems, we use the single mass insertions in both Tables VII and VIII.

We learn that the maximal possible effects in the neutral \(B_d\), \(B_s\) and \(D\) systems are (for \(r_3 = 3\) and, for the \(D\) system, \(\tan \beta = 3\)):

\[
\begin{align*}
B_d : |M_{12}^{\text{susy}}/M_{12}^{\text{exp}}| &\lesssim 2 \cdot 10^{-4}, \\
B_s : |M_{12}^{\text{susy}}/M_{12}^{\text{exp}}| &\lesssim 5 \cdot 10^{-4}, \\
D : |M_{12}^{\text{susy}}/M_{12}^{\text{exp}}| &\lesssim 3 \cdot 10^{-10}.
\end{align*}
\]
TABLE VIII: The order of magnitude upper bounds on \((\delta_{d,u}^{q})_{LL,RR}\) and \((\langle \delta_{d,u}^{q} \rangle)_{LL,RR}\) for \(r/r_3 \lesssim 1.2 \cdot 10^{-7}\), obeying the bound of Eq. (5.14) for \(\tan \beta = 3\). Entries in parentheses are independent of \(r\), therefore representing estimates rather than upper bounds.

| \(q \, ij\) | \(\langle \delta_{q}^{ll} \rangle\) | \(\langle \delta_{q}^{rr} \rangle\) | \(\langle \delta_{q}^{q} \rangle\) |
|-------------|------------------|------------------|------------------|
| \(d \, 12\) | \(10^{-4}(3/r_3)\) | 1.2 \cdot 10^{-7} | 4 \cdot 10^{-6} \sqrt{3/r_3}\) |
| \(d \, 13\) | [0.003(3/r_3)] | 1.2 \cdot 10^{-7} | 2 \cdot 10^{-5} \sqrt{3/r_3}\) |
| \(d \, 23\) | [0.01(3/r_3)] | 1.2 \cdot 10^{-7} | 4 \cdot 10^{-5} \sqrt{3/r_3}\) |
| \(u \, 12\) | 10^{-7}\max\{4.8/r_3, 1.2\} | 1.2 \cdot 10^{-7} | 10^{-7}\max\{1.4 \sqrt{3/r_3}, 1.2\} |
| \(u \, 13\) | [4 \cdot 10^{-6}(3/r_3)] | 1.2 \cdot 10^{-7} | 7 \cdot 10^{-7} \sqrt{3/r_3}\) |
| \(u \, 23\) | [4 \cdot 10^{-5}(3/r_3)] | 1.2 \cdot 10^{-7} | 2 \cdot 10^{-6} \sqrt{3/r_3}\) |

The largest possible contributions to the \(B_{d,s}\) mixing amplitudes come from the MFV contributions to the \((\delta_{d}^{q})_{LL}\)'s. Therefore, the effect is not enhanced for large \(\tan \beta\).

We conclude that, for anarchical gravity-mediated contributions to the \(A\)-terms, the effects on FCNC processes are negligibly small. In contrast, any improvements in the neutron EDM measurements may either further strengthen the constraints on this framework or discover its effects.

C. Yukawa-like \(A\)-terms

Here we explore the implications of \(A\)-terms of Yukawa-like texture as in the models with a FN symmetry discussed in Section IV A, specifically, Eq. (4.7), and

\[
(X_{qA})_{ij} = O(1), \quad (Z_{Aq})_{ij} \sim Y_{ij}^{q} \sim V_{ij}m_{qj}/v_{q},
\]

As concerns the \(\delta_{LL,RR}^{q}\) parameters, the effect of such \(A\)-terms can be described as \(O(r)\) changes in the RGE coefficients \(c_u, c_d, c_uR, c_dR\) defined in Eq. (A2); see Eqs. (A6) and (A7). This leads, in turn, to at most \(O(1)\) changes in the \(X_{qA}\) matrices. Therefore, the estimates for the \((\delta_{ij}^{q})_{LL,RR}\) parameters in this scenario vary by at most \(O(1)\) from the estimates obtained in Section IV A for \(Z_{Aq} = 0\) at the high scale.

As concerns the \(\delta_{LR}^{q}\) parameters, the parametric suppression of \(\tilde{A}_{ij}^{q}\) can be extracted from
$A_{ij}^q$ at the high scale $m_M$. This statement can be straightforwardly understood in the up sector, regardless of the structure of the $A$-terms, and in the down sector, for vanishing or anarchical initial $A$-terms. It is a little more subtle for the down sector in the case of Yukawa-like textured initial $A$-terms, but still holds true due the fact that the various RGE contributions in the last two lines of Eq. (A8) are at most comparable to the direct term proportional to $A_{ij}^d$. For instance, in the $(\delta_{LR}^d)_{11}$ term, which is relevant for the EDM constraints, $Y_{11}^d$ and $V_{td}^* Y_{31}^d$ are comparable. Thus, the expressions of (3.3) for the $\delta^q$ parameters hold with $Z_{A_q}$ of the structure (5.21).

We conclude that the strongest constraint on $r/r_3$ in this scenario comes from $\langle \delta_{12}^d \rangle$, as was the case for vanishing $A$-terms. Thus the bound of Eq. (5.7) holds, and the estimates and constraints that apply in the case of Yukawa-like $A$-terms are the same as those that apply in the case of vanishing $A$-terms.

An exception to this is the prediction for the neutron EDM, which is induced by the chirality-mixing $\delta_{LR}^d$ parameters in models with texture (5.21):

$$|d_{n}^{\text{susy}}/d_{n}^{\text{exp}}| \lesssim 0.01 (0.001).$$

Here, the first value corresponds to $x = 1$ and a phase suppression in $(\delta_{11}^d)_{LR}$ of $\sim 0.3$, and the value in parentheses is obtained for $x = 4$ and a phase suppression of $\sim 0.1$.

**D. Flavor constraints on the messenger scale**

The flavor and CP bounds on $r/r_3$, Eq. (4.3) or (5.7) or (5.14), imply upper bounds on the scale of gauge-mediation $m_M$ or, put differently, a minimal separation between the scales of gauge- and gravity-mediation. In minimal models [4], when the highest $F$-term contributes to both gauge and gravity mediation,

$$r \sim \left( \frac{m_M}{m_{p1}} \right)^2 \left( \frac{4\pi}{\alpha_3(m_M)} \right)^2 \frac{3}{8} \frac{1}{N_M},$$

where $m_{p1}$ is the Planck scale and $N_M$ denotes the number of messengers. The dependence of $r_3$ on $m_M$ is only logarithmic, see [4] for details. Fig. 2 presents the FCNC and CP constraints within the different flavor scenarios.

The difference between a FN-model (dotted lines) and models with anarchy but vanishing or Yukawa-like $A$-terms (dashed lines) is small, with the maximal messenger scales being
FIG. 2: The three solid blue curves give $r/r_3$ as a function of the messenger scale for $N_M = 1$ (upper), $N_M = 3$ (middle) and $N_M = 10$ (lower). The three pairs of horizontal lines give upper bounds on $r/r_3$ and correspond to the following flavor scenarios for the gravity-mediated contributions: (i) Full anarchy (solid, pink) – Eq. (5.14) with $\tan \beta = 3$; (ii) Anarchy with vanishing or Yukawa-like $A$-terms (dashed, green) – Eq. (5.7); (iii) FN structure (dotted, black) – Eq. (4.3). For a given number of messengers and a given flavor scenario, FCNC and CP constraints give an upper bound on the messenger scale, which can be read from the crossing point of the corresponding blue and horizontal curves.

related by $(m_M^{\text{max}})_{A \sim 0,Y} \sim (m_d/m_s)^{1/4} (m_M^{\text{max}})_{FN}$. In both cases,

$$m_M/m_{\text{Pl}} \lesssim 10^{-3} \quad (\text{FN, or anarchy with } A \sim 0,Y).$$

(5.24)

In the fully anarchical case, the EDM constraints yield a stronger bound:

$$m_M/m_{\text{Pl}} \lesssim 10^{-5} \quad (\text{full anarchy}).$$

(5.25)

VI. CONCLUSIONS

Measurements in low energy experiments of flavor-changing and of CP-violating processes will complement direct searches for new physics. Our goal in this work has been to understand concrete ways in which such complementarity will take effect. In order to do that, we chose to work in a specific framework, which is supersymmetry where the soft breaking terms receive dominant contributions from gauge-mediation and sub-dominant contributions from
gravity-mediation. Within this framework, we investigated three main possible scenarios, concerning the flavor structure of the gravity-mediated soft breaking terms:

1. The flavor structure of all such terms is dictated by a Froggatt-Nielsen symmetry, which is also responsible for the smallness and hierarchy in the Yukawa terms.

2. The flavor structure of the squark mass-squared matrix is anarchical, while the $A$-terms are subject to the FN selection rules.

3. The flavor structure of all soft supersymmetry breaking terms is anarchical.

The first scenario has been studied in a previous paper [7], while the latter two are explored in this work. Additional scenarios that we investigated (one where holomorphic zeros play a role and another where the $A$-terms vanish at the Planck scale) provide further variations, but we will explain our main conclusions on the basis of the three main ones.

Within this framework, we are able to answer the following questions, which will be relevant whatever type of new physics will be discovered:

- Which processes are most likely to show deviations from the standard model?
- At what level can these effects appear?
- Can we tell whether the flavor pattern of the gravity-mediated contributions is related to the standard model flavor pattern or not?

Table IX should be helpful in clarifying our main conclusions.

We learn that any improvement in the upper bound on CP violation in neutral $D$ mixing or on the neutron EDM will have an impact on our framework. It will make the upper bound on the ratio of gravity-to-gauge mediation stronger (in, respectively, the An-FN and An-An scenarios) or, equivalently, it will strengthen the upper bound on the messenger scale of gauge-mediation. Conversely, if non-MFV effects are observed, their size will provide a lower bound on the size of gravity-mediated contributions. In models of a single scale supersymmetry breaking, such a bound can be translated into a lower bound on this scale.

We can further make the following general statements, some of which are valid well beyond the specific framework of new physics that we have studied:
TABLE IX: The maximal size of possible effects in the mixing of $B_d - \bar{B}_d$, $B_s - \bar{B}_s$ and $D^0 - \bar{D}^0$ for low $\tan \beta$ [Eqs. (4.5), (5.10), (5.20)], normalized to the experimental value, and in the neutron EDM $d_n$ [Eqs. (4.9), (5.22)], normalized to the experimental upper bound. FN (An) means that the structure of the corresponding soft terms defined in Eqs. (3.1) and (3.2) is dictated by a Froggatt-Nielsen symmetry (is anarchical).

|   | $B_d$ | $B_s$ | $D^0$ | $d_n$      |
|---|-------|-------|-------|-----------|
| FN FN | 0.002 | 0.005 | 0.03  | 0.02      |
| An FN | 0.007 | 0.002 | $O(1)$| 0.01      |
| An An | $2 \cdot 10^{-4}$ | $5 \cdot 10^{-4}$ | $O(10^{-10})$ | 1         |

- If the flavor and CP structure of the new physics is anarchical, it is quite possible that its effects will be discovered in CP-violating flavor-diagonal observables rather than in flavor-changing measurements.

- If new flavor effects of similar relative sizes are discovered in both third generation ($b$ or $t$) decays and second generation ($c$ decays), then the new physics is likely to have a flavor structure. Furthermore, this structure may well be related to the standard model one.

- A situation where non-MFV effects related to third generation physics are larger (in their relative size) than those related to charm physics requires that the flavor structure of the new physics is not related to the standard model one.

It is amusing to note that since the strong suppression of gravity-mediated flavor within full anarchy makes the latter almost MFV-like, it allows for the possibility of a long-lived light stop with macroscopic decay length given a suitable particle spectrum [21].

We conclude that the search for new physics in high-precision flavor experiments and in EDM measurements will be very informative about the underlying, very high scale new physics. It will further allow us to test whether mechanisms such as a Froggatt-Nielsen symmetry dictate all flavor structures or not.
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Appendix A: The high scale-low scale connection

In supersymmetric models with hybrid gauge- and gravity-mediation of supersymmetry breaking, the form of the soft terms at the scale of gauge-mediation $m_M$ is given in Eqs. (3.1) and (3.2). In this Appendix, we consider the RGE effects and provide approximate analytical expressions for the soft terms at the weak scale, $m_Z$.

1. Weak scale

The weak-scale squark mass-squared matrices $\tilde{M}_{q_A}^2$ have the following form:

\[
\begin{align*}
\tilde{M}_{D_L}^2 &= M_{Q_L}^2 + D_{D_L} \mathbf{1} + m_D m_D^\dagger, \\
\tilde{M}_{U_L}^2 &= M_{Q_L}^2 + D_{U_L} \mathbf{1} + m_U m_U^\dagger, \\
\tilde{M}_{D_R}^2 &= M_{D_R}^2 + D_{D_R} \mathbf{1} + m_D^\dagger m_D, \\
\tilde{M}_{U_R}^2 &= M_{U_R}^2 + D_{U_R} \mathbf{1} + m_U^\dagger m_U, \\
\end{align*}
\]

where $m_{U,D}$ are the up and down quark mass matrices in the flavor basis, $D_{q_A}$ are the $D$-term contributions, and all quantities should be evaluated at the electroweak scale $\mu \sim m_Z$.

The initial conditions Eqs. (3.1) and (3.2) hold at the scale of gauge-mediation $m_M$, while flavor-changing processes restrict the weak scale parameters $(\delta_{ij}^q)_{NM}$ $(N, M = L, R)$, and so the effect of RG evolution on the soft terms must be taken into account.

As concerns the soft squark masses, we obtain the following approximate form:

\[
\begin{align*}
M_{Q_L}^2 (m_Z) &\sim \tilde{m}_{Q_L}^2 (r_3 \mathbf{1} + c_u Y_u Y_u^\dagger + c_d Y_d Y_d^\dagger + r X_{uL} + r Z_{A_u} Z_{A_u}^\dagger + r Z_{A_d} Z_{A_d}^\dagger), \\
M_{U_R}^2 (m_Z) &\sim \tilde{m}_{U_R}^2 (r_3 \mathbf{1} + c_u Y_u^\dagger Y_u + r X_{uR} + r Z_{A_u}^\dagger Z_{A_u}), \\
M_{D_R}^2 (m_Z) &\sim \tilde{m}_{D_R}^2 (r_3 \mathbf{1} + c_d Y_d^\dagger Y_d + r X_{dR} + r Z_{A_d}^\dagger Z_{A_d}),
\end{align*}
\]

(A2)
where $Y_u$ and $Y_d$ denote the up and down quark Yukawa matrices in the flavor basis, and the effect of trilinear soft couplings is included. (The RG-coefficient in front of the $A$-terms is of order one.) We note the following points:

1. The factor $r_3$ captures the effect of RGE corrections to the diagonal elements of the soft squark mass matrices $(M_{\tilde{q}_A}^2)_{ii}$ via

$$\tilde{m}_{12}^2(\mu = m_Z) = r_3\tilde{m}_{12}^2(\mu = m_M),$$

with the average diagonal mass-squared defined as

$$\tilde{m}_{ij}^2 \equiv \frac{1}{2} \left( (M_{\tilde{q}_A}^2)_{ii} + (M_{\tilde{q}_A}^2)_{jj} \right).$$

For simplicity this factor is taken to be universal among all squarks, as the dominant contribution to the initial squark soft masses and to their RGE is QCD-induced and, in the limit that we neglect the electroweak gauge couplings, is indeed universal.

2. The coefficients $c_u, c_d, c_{uR}, c_{dR}$ can be of $O(1)$ for $m_M \sim m_{GUT}$ and are all negative. Yukawa-related contributions to the RGE thus lower the weak scale values of the diagonal $(M_{\tilde{q}_A}^2)_{33}$ entries with respect to the high scale ones. Sub-dominant MFV terms with higher powers of the Yukawa couplings are henceforth neglected.

3. We use the various $\tilde{m}_{ij}^2(m_Z)$ to evaluate the denominator of the $(\delta_{ij}^q)_A$, neglecting $D$-terms of $O(m_Z^2/\tilde{m}_{ij}^2)$ and $F$-terms of at most $O(m_t^2/\tilde{m}_{i3}^2)$.

4. Eq. (A2) is given in the flavor basis, while the $\delta^q$ parameters are relevant in the mass basis of the quarks. This rotation of the squarks leaves the parametric pattern of the $X_{ij}$ and $Z_{ij}$ unchanged.

As concerns the $A$-terms, we obtain for small to moderate $\tan \beta$ (so that for the purpose of RGE we employ $Y_d \ll 1$) the following approximate form:

$$A^u(m_Z) \sim M_3(a_u + b_u Y_u^\dagger Y_u)\ Y_u + \tilde{m}\sqrt{r}(d_u + e_u Y_u^\dagger)Z_{A_u},$$

$$A^d(m_Z) \sim \tilde{m}\sqrt{r}(d_d + e_d Y_u^\dagger)Z_{A_d},$$

where $\tilde{m}$ represents the typical scale of the $SU(3)$-related contributions to the gauge-mediated soft masses. The $a, b$ coefficients relate to the MFV part of the trilinear scalar couplings and are irrelevant to our discussion. The dimensionless coefficients $d, e$ can be $O(1)$ and, for our purposes, are taken as such.
2. The $\delta^q$ parameters

We now work in the basis in which the quarks mass matrices and gluino couplings are diagonal. We use $q = U, D$, $i = 1, 2$, $j = 1, 2, 3$, $r \ll r_3$ and neglect the masses of the first and second generation quarks. For the LL block, we find

\[ (\tilde{M}^2_{qL}(m_Z))_{33} \sim \tilde{m}^2_{qL}(r_3 + c_u y_t^2 + c_d y_b^2 + r(X_{qL})_{33} + r(Z_{A_u} Z_{A_u}^\dagger)_{33} + r(Z_{A_d} Z_{A_d}^\dagger)_{33}, \]
\[ (\tilde{M}^2_{qL}(m_Z))_{ii} \sim \tilde{m}^2_{qL}(r_3 + r(X_{qL})_{ii} + c_u y_t^2 |V_{ti}|^2 + r(Z_{A_u} Z_{A_u}^\dagger)_{ii} + r(Z_{A_d} Z_{A_d}^\dagger)_{ii}), \]
\[ (\tilde{M}^2_{U_L}(m_Z))_{12} \sim \tilde{m}^2_{qL}(c_d y_b^2 V_{ub} V_{cb}^* + r(X_{uL})_{12} + r(Z_{A_u} Z_{A_u}^\dagger)_{12} + r(Z_{A_d} Z_{A_d}^\dagger)_{12}), \]
\[ (\tilde{M}^2_{U_L}(m_Z))_{33} \sim \tilde{m}^2_{qL}(c_d y_b^2 V_{ub} V_{cb}^* + r(X_{uL})_{33} + r(Z_{A_u} Z_{A_u}^\dagger)_{33} + r(Z_{A_d} Z_{A_d}^\dagger)_{33}), \]
\[ (\tilde{M}^2_{D_L}(m_Z))_{12} \sim \tilde{m}^2_{qL}(c_u y_t^2 V_{tb} V_{td}^* + r(X_{dL})_{12} + r(Z_{A_u} Z_{A_u}^\dagger)_{12} + r(Z_{A_d} Z_{A_d}^\dagger)_{12}), \]
\[ (\tilde{M}^2_{D_L}(m_Z))_{33} \sim \tilde{m}^2_{qL}(c_u y_t^2 V_{tb} V_{td}^* + r(X_{dL})_{33} + r(Z_{A_u} Z_{A_u}^\dagger)_{33} + r(Z_{A_d} Z_{A_d}^\dagger)_{33}). \]  

For the RR block, we find

\[ (\tilde{M}^2_{U_R}(m_Z))_{33} \sim \tilde{m}^2_{t_R}(r_3 + c_u R y_t^2 + r(X_{uR})_{33} + r(Z_{A_u}^\dagger Z_{A_u})_{33}), \]
\[ (\tilde{M}^2_{D_R}(m_Z))_{33} \sim \tilde{m}^2_{d_R}(r_3 + c_d R y_b^2 + r(X_{dR})_{33} + r(Z_{A_d}^\dagger Z_{A_d})_{33}), \]
\[ (\tilde{M}^2_{qR}(m_Z))_{ii} \sim \tilde{m}^2_{qR}(r_3 + r(X_{qR})_{ii} + r(Z_{A_q} Z_{A_q})_{ii}), \]
\[ (\tilde{M}^2_{qR}(m_Z))_{ij} \sim \tilde{m}^2_{qR}(r(X_{qR})_{ij} + r(Z_{A_q} Z_{A_q})_{ij}), \quad (i \neq j). \]  

For the LR block, we find $(i, j \neq 3)$:

\[ (\tilde{A}^u(m_Z))_{33} \sim M_3(a_u + b_u y_t^2) y_t + \tilde{m} \sqrt{r} (d_u + e_u y_t^2) (Z_{A_u})_{33}, \]
\[ (\tilde{A}^u(m_Z))_{ii} \sim \tilde{m} \sqrt{r} d_u (Z_{A_u})_{ii}, \]
\[ (\tilde{A}^u(m_Z))_{3i} \sim \tilde{m} \sqrt{r} (d_u + e_u y_t^2) (Z_{A_u})_{3i}, \]
\[ (\tilde{A}^d(m_Z))_{33} \sim \tilde{m} \sqrt{r} (d_d + e_d y_b^2) (Z_{A_d})_{33}, \]
\[ (\tilde{A}^d(m_Z))_{3j} \sim \tilde{m} \sqrt{r} (d_d (Z_{A_d})_{3j} + e_d y_b^2 V_{tb} V_{ik} (Z_{A_d})_{kj}), \]
\[ (\tilde{A}^d(m_Z))_{ij} \sim \tilde{m} \sqrt{r} (d_d (Z_{A_d})_{ij} + e_d y_b^2 V_{ti} V_{ik} (Z_{A_d})_{kj}). \]  

(In the last two lines summation over $k = 1, 2, 3$ is implied.) In some specific cases, for example, vanishing initial $A$-terms or anarchical ones, the above expressions simplify.

Eqs. (A6), (A7) and (A8) lead to the expressions for the $m_Z$-scale flavor-changing $\delta^q$ parameters given in Eq. (13).
A final comment here concerns $\delta_{LR}^w$. Here, there are contributions from both gravity-medi-ation (via the initial $A$-terms) and gauge-medi-ation (via the gaugino masses). We ne-glect the gaugino contributions when constraining the low energy $\delta_{LR}^w$, since we are interested in the constraints on the gravity parameters. Constraints from the gaugino contributions are similar to those in gauge-medi-ation and do not concern us here.

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