TESTING THE METRIC-FIELD EQUATIONS OF GRAVITATION

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ABSTRACT

Physical consequences from gravitation equations based on Poincaré ideas of relativity of space and time in respect of measuring instruments are considered. The most interesting of them are the possibility of the existence of stable supermassive configurations \((10^2 \div 10^{12} M_\odot)\) which can exist in galactic centres, and an explanation of the acceleration of the Universe expansion as a manifestation of the gravitational force properties in the theory under consideration.

Key words: gravitation; compact objects; cosmology; binary pulsar.

1. INTRODUCTION

At the beginning of the 20th century H. Poincaré convincingly showed that there is no sense to assert that one or other geometry of physical space is a true. Only the aggregate ”geometry + measuring instruments” has a physical, verifiable by experience sense. A. Einstein recognised the justice of Poincaré’s reasons. His theory of gravity does not reject Poincaré’s ideas. It only demonstrates relativity of space and time with respect to distribution of matter. Poincaré’s ideas indicate to another property of physical reality - relativity of the geometrical properties with respect to measuring instruments, dependency on their properties. These ideas, apparently, have never been realized in physics and remain until now, more likely, by a subject for discussions of philosophers. The success of General Relativity almost convinced us that physical space-time in the presence of matter is Riemannian and that this fact does not depend on properties of the measuring instruments. The attempts to describe gravity in flat space-time did not obtain recognition and Poincaré’s ideas proved to be almost forgotten for physics.

However, are Poincaré’s ideas really only a thing of such little use as conventionalism, as it is usually believed? A choice of certain properties of measuring instruments is nothing but the choice of some frame of reference, which is just such a physical device by means of which we test properties of space-time. (Of course, we must understand a distinction between a frame of reference as a physical device, and a coordinate system as only a means of parameterisation of events in space-time). For this reason Poincaré’s reasoning about interdependency between properties of space-time and measuring instruments should be understood as the existence of a connection between geometry of space-time and properties of the frame of reference being used. (Verozub, 2001, 2002)

The existence of such a connection can be shown by an analysis of a simple and well-known example. Disregarding the rotation of the Earth we can consider a reference frame, rigidly connected with the Earth surface, as the inertial one (IFR). An observer, who is located in this frame of reference, can, without coming into conflict with experience, describe the motion of a test particle as the one in Minkowski space-time, under the action of a certain force field \(F\). Consider now an observer, who is located in a frame of reference, the reference body of which formed by identical dot masses, free falling in the field \(F\). Such a reference frame can be named the proper frame of reference (PFR) for the given field. Assume, the observer is deprived of the possibility to see the Earth and stars. This observer does not feel the presence of force field \(F\) in any place of the frame. Therefore, if he proceeds from relativity of space-time in Berkeley-Leibnitz-Mach-Poincaré (BLMP) meaning, then from his viewpoint the accelerations of the dot masses, forming the reference body of his frame, in his physical space must be equal to zero. However, instead, he observes a change in the distances between these dot masses in time. How can he explain this fact? Evidently, only reasonable explanation for him is the interpretation of this observed phenomenon as a manifestation of the deviation of geodetic lines in a Riemannian space of nonzero curvature.

Thus, if an observer in the IFR postulates that space-time is flat, then the observer in a proper reference frames of the force field \(F\), who proceeds from relativity of space and time in the BLMP meaning, is forced to consider it as Riemannian.
2. METRIC-FIELD EQUATIONS OF GRAVITATION

We accept according to the Special Relativity that space-time in IFRs is Pseudo-Euclidean. However, we can be argued (Verhózub [2001, 2003]) that if we proceed from the relativivity of space-time in the BLMP sense, then the differential metric form of space-time in PFRs is of the form

\[ ds = -(m μc)^{-1}dS(x, dx). \]

In this equation \( dS = L(x, \dot{x})dt \) and \( L(x, \dot{x}) \) is the Lagrange function describing in an IFR the motion of identical dot masses \( m_\mu \), forming the reference body of the PFR. Thus, the properties of the space-time in a PFR are defined by properties of the used frame in accordance with the Poincaré idea.

In order to describe gravitation in inertial and proper frames of reference we suppose (Thirring, 1961) that in pseudo-Euclidean space-time gravitation can be described as a tensor field \( ψ_{αβ}(x) \) of spin 2, and the Lagrangian, describing the motion of a test particle with the mass \( m_\mu \), is given by the form

\[ L = -m_\mu c^2[g_{αβ}(ψ) \dot{x}^α \dot{x}^β]^{1/2}, \]

where \( \dot{x}^α = dx^α/dt \) and \( g_{αβ} \) is a symmetric tensor whose components are functions of \( ψ_{αβ} \).

If particles move under influence of the force field \( ψ_{αβ}(x) \), then according to (1) the space-time metric differential form in PFRs of this field takes the form

\[ ds^2 = g_{αβ}(ψ) dx^α dx^β. \]

Therefore, the space-time in the PFRs is Riemannian with curvature other than zero. The tensor \( g_{αβ}(ψ) \) is a space-time metric tensor in PFRs.

Any gauge transformation \( ψ_{αβ} → \bar{ψ}_{αβ} \) of the unknown tensor field \( ψ_{αβ} \) induces some mapping \( g_{αβ} → \bar{g}_{αβ} \) of Riemannian space-time which are not related with coordinate transformations. To ensure the invariance of the equations of the motion of test particles with respect to such mappings, correct differential equations for finding tensor field \( g_{αβ} \) must be invariant with respect to geodesic mappings of Riemannian’s space-time. (Verozub, 1991). These transformation play role of gauge transformation in the theory under consideration. The simplest geodesic-invariant generalisation of the vacuum Einstein equations are:

\[ B^γ_{αβγ} - B^δ_{αδβ}B^δ_{νβ} = 0. \]

These equations are vacuum bimetric differential equations for the tensor

\[ B^γ_{αβ} = \Pi^γ_{αβ} - \delta^γ_{αβ}. \]

(Greek indices run from 0 to 3), where

\[ \Pi^γ_{αβ} = \Gamma^γ_{αβ} - (n + 1)^{-1}\left[ \delta^γ_α \Gamma^μ_β + \delta^γ_β \Gamma^μ_α \right]. \]

\( B^γ_{αβ} \) and \( \Pi^γ_{αβ} \) are the Christoffel symbols of the Minkowski space-time, \( \Gamma^γ_{αβ} \) are the Christoffel symbols of the Riemannian space-time, whose fundamental tensor is \( g_{αβ} \). The semicolon denotes the covariant differentiation in Minkowski space-time.

Some additional conditions can be imposed on the tensor \( g_{αβ} \) because of the geodesic invariance of the equations.

In particular, under the conditions \( Q_α = \Gamma^σ_ασ - \Gamma^σ_ασ = 0 \) Eqs. (4) are reduced to Einstein’s vacuum equations.

Therefore, for solving of concrete problems it is sufficient to solve the system of the differential equations:

\[ R_{αβ} = 0; \quad Q_α = 0 \]

in a selected coordinate system of Minkowski space-time by using mathematical methods elaborated for General Relativity. The additional conditions are covariant and do not impose any restrictions on the coordinate system selection.

The spherically-symmetric solution of these equations in Minkowski space-time have no an event horizon and physical singularity in the centre (Verozub, 1991). If the distance from a point attractive mass is many larger than \( r_g \), the physical consequences, resulting from these equations, are very close to General Relativity results. However, they are principally other at short distances from the central mass.

3. SUPERMASSIVE CONFIGURATIONS

It follows from Eqs. (4) that the gravitational force of a point mass \( M \) affecting a free-falling particle of mass \( m \) is given by (Verozub, 1991)

\[ F = -m \left[ \dot{c^2C^I} / 2A + (A^I / 2A - 2C^I / 2C)r^2 \right], \]

where \( A = f^2 / c, C = 1 - r_g / f, f = (r_g^3 + r^3)^{1/3}, r_g = 2GM/c^2, r \) is the radial distance from the centre, \( G \) is the gravitational constant, \( c \) is the speed of light at infinity, the prime denotes derivatives with respect to \( r, \dot{r} = dr/dt \).

For particles at rest \( (\dot{r} = 0) \)

\[ F = -\frac{GmM}{r^2} \left[ 1 - \frac{r_g}{(r^3 + r_g^3)^{1/3}} \right]. \]

Fig. 1 shows the force \( F \) affecting particles at rest and particles, free falling from infinity with zero initial speed, as a function of the distance \( \tau = r / r_g \) from the centre. It follows from this figure that the gravitational force affecting a particle at rest tends to zero when distance from the
centre $r \to 0$. It would therefore be interesting to know which masses of the equilibrium configurations can exist if the gravitational force is given by Eq. (10). To answer this question one can proceed from the answer of the hydrostatic equilibrium

$$dp_m/dr = \rho_m g_m, \quad dM_r/dr = 4\pi r^2 \rho_m,$$  \hspace{1cm} (11)

$$p_m = p_m(\rho_m).$$

In these equations $M_r$ is the mass of matter inside the sphere $O_r$ of the radius $r$, $g_m = F/m$ is the force (10) per unit of mass, $r_g = 2GM_r/c^2$ is the Schwarzschild radius of the sphere $O_r$, and $p = p(\rho_m)$ is the equation of state of matter inside the object. We use the equation of state by (Harrison et al., 1965), which is valid for the density range of $(8 \div 10^{13}) \ g \ cm^{-3}$.

For the gravitational force under consideration there are two types of the solutions of Eqs. (11). Besides the solutions describing white dwarfs and neutron stars, there are solutions with very large masses. For example, for the mass of $3 \times 10^6 M_\odot$ the object radius resulting from Eqs. (11) is about $0.4 R_\odot = 0.034 r_g = 3 \times 10^{13} \ cm$ where $r_g$ is the Schwarzschild radius of the object (Verozub, 1996). These configurations are stable (Verozub and Kochetov, 2001a).

4. A SUPERMASSIVE OBJECT IN THE GALAXY CENTRE

The rate of gas accretion onto the supermassive object in the Galaxy centre, associated with radio-source Sgr A*, due to star winds from surrounding young stars is of the order of $10^{-7} \ M_\odot \ yr^{-1}$. Therefore, if the object has a solid surface, it can have an atmosphere, basically hydrogen. Estimates show that in a time of $10 \ Myr$ (an estimated lifetime of surrounding stars) the mass $M_{atm}$ of the gaseous envelope can reach $10 \ M_\odot$. At the temperature $T = 10^7 K$ the atmosphere height $h_a \sim 10^8 \ cm$, and the maximal atmospheric density $\rho_a \sim 10^5 \ g \ cm^{-3}$. Under such a condition, a hydrogen combustion must begin already in our time. Calculations show (Verozub, 2006) that at $T = 10^7 K$ the luminosity of the object due to the nuclear burning on the surface $L = 2.4 \times 10^{33} \ erg \ s^{-1}$ at $\rho_a = 10^2 \ g \ cm^{-3}$ and $L = 5.6 \times 10^{33} \ erg \ s^{-1}$ at $\rho_a = 10^3 \ g \ cm^{-3}$.

Due to gravitational redshift a local frequency (as measured by a local observer) differs from the one for a remote observer by the factor $1 + z_s = 1/\sqrt{1-C}$ where the function $C = C(r)$ is defined in Eqs. (3). The difference between local and observed frequency can be significant for the emission emerging from small distances from the surface. For example, at the above luminosity $L = 5.6 \times 10^{33} \ erg \ s^{-1}$ the local frequency of the maximum of the blackbody emission is equal to $5.7 \times 10^{14} \ Hz$. The observed frequency of the maximum is equal to $2.7 \times 10^{12} \ Hz$. The corresponding specific luminosity is $\sim 2 \times 10^{21} \ erg \ s^{-1} \ Hz^{-1}$. At $L = 10^{36} \ erg \ s^{-1}$ the frequency of the maximum is equal to $2 \times 10^{15} \ Hz$, the observed frequency is equal to $9.7 \times 10^{12} \ Hz$, and the specific luminosity is $\sim 10^{23} \ erg \ s^{-1} \ Hz^{-1}$. These magnitudes are close to observation data (Melia and Falcke, 2001).

A hot layer appears in the atmosphere because of deceleration and stopping of infalling protons. Fig. 2 shows a typical temperature profile in the region of the infalling protons stopping.

![Figure 2. A typical temperature jump as the function of the distance $r$ from the centre inside of the region of protons stopping for the object of the radius $R = 0.04 r_g$. The plots for three value of the magnetic field $B$ are shown.](image)

Because of comptonization of the Planckian radiation of the burning layer on the surface of the central object this radiation can manifest itself at high frequencies. Fig. 3 shows an approximate emergent spectrum of the nuclear radiation at the surface after comptonization obtained for the case when the temperature at the surface is $10^7 K$ and $\rho_a = 3 \times 10^3 \ g \ cm^{-3}$. The results are plotted for several values of the optical thickness $\tau$.

It follows from this figure that the spectrum may give a contribution to the observed Sgr A* radiation (Baganoff et al., 2003; Porquet et al., 2003) in the range $10^{15} \div 10^{20} \ Hz$.

It is necessary to note that the timescale $\delta t$ of a variable...
process is connected with the size $\Delta$ of its region by the relationship $\Delta \leq c \delta t/(1 + z_g)$. For example, the variations in the radiation intensity of $\sim 600$ s on clock of a remote observer occur at the distance $d \leq 1.7R$ from the surface. For this reason high-energy flashes in the Galaxy centre can be interpreted as processes near the surface of the central objects.

Some authors (Robertson and Leiter, 2002) find in spectra of candidates into Black Holes some evidences for the existence of a proper magnetic field of these objects that is incompatible with the existence of the event horizon. In order to show how the magnetic field near the surface of the object in question can manifests itself, we have found a contribution of a magnetic field to the spectrum of the synchrotron radiation. We assume that the magnetic field is of the form $B_{\text{int}} = B_0 (R/r)^3$, where $B_0 = 1.5 \times 10^4$ G. Fig. 4 shows the spectrum of the synchrotron radiation in the band of $\nu = 10^{11} \div 2 \times 10^{15}$ Hz for three cases:

- for the magnetic field $B_{\text{int}}$ of the object, supposing the grid distribution of electrons (Özel, Psaltis and Narayan, 2000) (the dashed line),
- for the sum of $B_{\text{int}}$ and an external equipartition magnetic field $B_{\text{ext}} = \left(M c^2 v_{\text{esc}}^{-2}\right)^{1/2}$ which may exist in the accretion flow (Melia and Falcke, 2001) (the solid line),
- for the magnetic field $B_{\text{int}}$ without non-thermal electrons (the dotted line which at the frequencies $\nu < 10^{14}$ Hz coincides with the dash line), in the presence of the proper magnetic field It follows from the figure that the proper magnetic field can manifest itself for the remote observer at the frequencies larger that $2 \times 10^{12}$ Hz – in IR and X-ray radiation.

Consider another consequence of the gravitational equations under consideration. It is follows from fig. 5 that the gravitational force in flat space-time acting on free moving particles essentially differs from that acting on the particles at rest.

A magnitude which is related with observations in the expanding Universe is a relative velocity of distant star objects with respect to an observer. The radial velocity $v = \frac{dR}{dt}$ of particles on the surface of a selfgravitating expanding homogeneous sphere of a radius $R$ can be obtained from equations of the motion of a test particle (Verozub, 1991):

$$v = c \frac{CF^2}{R^2} \sqrt{1 - \frac{C}{E^2}},$$  \hspace{1cm} (12)$$

where $A$ and $C$ are the functions of the distance $R$ of the object from an observer in which $M = (4/3)\pi \rho R^3$ is the matter mass inside of the sphere of the radius $R$, $\rho$ is the observed matter density, and $r_g = (8/3)\pi c^2 G \rho R^3$ is Schwarzschild’s radius of the matter inside of the sphere. The parameter $E$ is the total energy of a particle divided by $mc^2$. Fig. 5 shows the radial acceleration $\ddot{R} = c^2 v \dot{v}$ of a particle on the surface of the sphere of the radius $R$ in flat space-time. Two conclusions can be made from this figure.

1. At some distance from the observer the relative acceleration changes its sign. If the $R < 2 \times 10^{27} cm$, the radial acceleration of particles is negative. If $R > 2 \times 10^{27} cm$, the acceleration is positive. Hence, for sufficiently large distances the gravitational force affecting particles is repulsive and gives rise to a relative acceleration of particles.

2. The gravitational force, affecting the particles, tends to zero when $R$ tends to infinity. (The same fact takes place as regards the force acting on particles in the case of a static matter.) The reason of the fact is that at a sufficiently large distance $R$ from the observer the Schwarzschild radius of the matter inside the sphere of the radius $R$ become of the same order as its radius. Approximately at $R \sim 2 r_g$, the gravitational force begin to decrease. The ratio $R/r_g$ tends to zero when $R$ tends

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**Figure 3.** Emergent spectra of nuclear burning after comptonization by a homogeneous hot layer for the optical thickness $\tau = 0.01, 0.03, 0.1, 0.5$. The crosses denote the observation data (Baganoff et al., 2003) (Porquet et al., 2003).

**Figure 4.** The spectrum of the synchrotron radiation

5. ACCELERATION OF THE UNIVERSE EXPANSION AS A CONSEQUENCE OF GRAVITY PROPERTIES
The gravitational redshift is caused by the matter inside of the sphere of the radius $E = 1.2 \times 10^{-29}$ cm$^{-3}$. The matter density is equal to $1.2 	imes 10^{-29}$ g cm$^{-3}$. To infinity, and under the circumstances the gravitational force in the theory under consideration tends to zero. The above ball can be considered as a part of a flat accelerating Universe. To calculate the velocity of a particle at the distance $R$ from the observer there is no necessity to demand a global spherical symmetry of matter outside of the sphere due to the second properties of the gravitational force, because the gravitational influence of very distant matter is neglected small. Therefore a relative velocity of particles at the distance $R$ from the observer is determined by gravitational field of the matter inside of the sphere of the radius $R$. Proceeding from this equation we can find Hubble’s diagram, following basically to the method being used in (Zeldovich & Novikov, 1971). Let $v_0$ be a local frequency in the proper reference frame of a moving source at the distance $R$ from an observer, $v$ be this frequency in a local inertial frame, and $\nu$ be the frequency as measured by the observer in the sphere centre. The redshift $z = (v_0 - v)/v$ is caused by both Doppler-effect and gravitational field. The Doppler-effect is a consequence of a difference between the local frequency of the source in inertial and comoving reference frames, and it is given by $v = v_0[1 - (1 - v/c)(1 + v/c)]$.

The gravitational redshift is caused by the matter inside of the sphere of the radius $R$. The local frequency $v_0$ at the distance $R$ from an observer are related with the observed frequency $\nu$ by equality $\nu = v_0 \sqrt{C}$. Thus, the relationship between frequency $\nu$ as measured by the observer and the proper frequency $v_0$ of the moving source in the gravitational field takes the form

$$v/v_0 = [C(1 - v/c)/(1 + v/c)]^{1/2}. \quad (13)$$

Equation (13) yields the quantity $z = v_0/v - 1$ as a function of $R$. By solving this equation numerically we obtain the dependence $R = R(z)$ of the measured distance $R$ as a function of the redshift. Therefore the distance modulus to a distant star object is given by

$$\mu = 5 \log_{10}[R(z)](z + 1)] - 5 \quad (14)$$

where $R(1 + z)$ is a bolometric distance (in pc) to the object.

If we demand that equation (13) must to give a correct radial velocity of distant star objects in the expanding Universe, it must to lead to the Hubble law at small distances $R$. At this condition the Schwarzschild radius $r_s = (8/3)\pi G \rho R^3$ of the matter inside of the sphere is very small compared with $R$. For this reason $f \approx r$, and $C = 1 - r_g/r$. Therefore, at $E = 1$, and we obtain from (12) that $v = HR$ where $H = \sqrt{(8/3)\pi G \rho}$. If $E \neq 1$ equation (12) does not lead to the Hubble law, since $v$ does not tend to 0 when $R \to 0$. For this reason we set $E = 1$ and look for a value of the density at which a good accordance with observation data can be obtained.

The fig. 6 show the Hubble diagram compared with observations data (Riess et al., 2004). It follows from this figure that the model under consideration does not contradict observation data.

With the value of the density $\rho = 4.5 \cdot 10^{-30}$ g cm$^{-3}$ we obtain that $H = 1.59 \cdot 10^{-18} c^{-1} = 49 km/c^{-1} Mpc$.

6. TESTING BY PSR 1913+16

Our generalisation of the Einstein equations with a matter source are of the form (Verozub and Kochetov, 2000)

$$B_{\alpha\beta\gamma} - B_{\alpha\gamma} B_{\beta\gamma} = k (T_{\alpha\beta} - 1/2 G_{\alpha\beta} T), \quad (15)$$

where $k = 8\pi G/c^4$, $T_{\alpha\beta}$ is the matter energy-momentum tensor, $T = G_{\alpha\beta} T_{\alpha\beta}$, and

$$G_{\alpha\beta} = g_{\alpha\beta} - (n + 1)^{-1} Q_{\alpha} Q_{\beta}, \quad (16)$$

These equations with a matter source were tested (Verozub and Kochetov, 2000) by binary pulsar PSR 1913+16 in the framework of the Blandford - Teukolsky model (Blandford and Teukolsky, 1975). Namely, we take into account the dependence of the phase of the
pulsar radiation on the following three observable parameters which are functions of the unknown masses $m_1$ and $m_2$ of two pulsars: 1) The relative rate of the orbital period $P_b$ increasing: $dP_b/dt = P_b = P_b(m_1, m_2)$, 2) The rate of periastron shift of the pulsar orbit $\dot{\omega} = \omega(m_1, m_2)$, 3) the quantity $\gamma = (t_1 - t_2)/\sin(\tilde{E})$ where $t_1$ is the moment of the radiation of a pulse as measured by a remote observer, $t_2$ is the same moment as measured in the proper frame of reference of the pulsar, $\tilde{E}$ is the eccentric anomaly.

For a system of two gravitationally bound point masses $m_1$ and $m_2$ we have obtained the equation

$$\frac{dP_b}{dt} = \frac{92\pi}{5c^3} (2\pi G)^{5/3} P_b^{8/3} f(e) m_1 m_2 M_p^{-1/3},$$

(17)

where $M_p = m_1 + m_2$,

$$f(e) = (1 - e^2)^{-7/2} (1 + 73/24 e^2 + 37/96 e^4)$$

(18)

and $e$ is the eccentricity. The rate of the periastron shift of the pulsar orbit resulting from our gravitation equations is given by

$$\dot{\omega} = \frac{6\pi GM}{a (1 - e^2) c^2} + \frac{\pi G^2 M^2}{2a^2 (1 - e^2)^2 c^4} f_1(e),$$

(19)

Verozub [1996] where $f_1(e) = (54 + 16e^2 - e^4)$. The second term for the considered system is about 0.01 of the total value of $\dot{\omega}$ and, therefore must be taken into account.

For the value of $\gamma$ we have obtained

$$\gamma = \frac{GP_0 e m_2 (m_1 + 2m_2)}{2\pi ac^2 (m_1 + m_2)^2}.$$  \hspace{1cm} (20)

Fig. [7] shows that in the plane $m_1, m_2$ the curves $P_b = P_b(m_1, m_2)$, $\dot{\omega} = \omega(m_1, m_2)$ and $\gamma = \gamma(m_1, m_2)$ meet in one point. The value of the total mass $M_p$ of the system resulting from the measured value of $P_b$ and $e$ by equation (15) is equal to ($2.82845 \pm 0.003$)$M_\odot$. But then, by using (18) one can find that the pulsar masses are equal to (1.441 $\pm 0.03$)$M_\odot$ and (1.387 $\pm 0.03$)$M_\odot$. These results differ very little from those in General Relativity (Taylor et al., 1992): (1.442 $\pm 0.03$)$M_\odot$ and (1.386 $\pm 0.03$)$M_\odot$, correspondingly. Due to a kinematic effect in our Galaxy Damour and Taylor [1993] a little correction $(-0.017 \pm 0.005) \cdot 10^{-12}$ must be added to the found theoretical value of $P_b = (-2.40249 \pm 0.005) \cdot 10^{-12}$. Taking into account this correction, the ratio of the observed value of $P_b$ to that found theoretically is equal to 1.0023 $\pm 0.0047$.

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