Model hierarchy of upper-convected Maxwell models with regard to simulations of melt-blowing processes

Manuel Wieland1,*, Walter Arne1, Nicole Marheineke2, and Raimund Wegener1
1 Fraunhofer ITWM, Fraunhofer Platz 1, D-67663 Kaiserslautern, Germany
2 Universität Trier, Lehrstuhl Modellierung und Numerik, Universitätsring 15, D-54296 Trier, Germany

In melt-blowing processes, polymeric jets are extruded into turbulent high-speed airflow to produce fibers of micro- and nanoscale. String models supplemented with viscoelastic material laws build a suitable basis for the modeling and simulation of fibrous jets in such processes. We present a model-hierarchy of upper-convected Maxwell models for the fibers and therein incorporate the turbulent velocity fluctuations of the underlying airflow by a reconstruction technique. Within the model hierarchy, simulation results are analyzed with respect to efficiency and accuracy.

1 Introduction

In industrial melt-blowing processes aerodynamic forces are the key player for fiber thinning. Since the direct numerical simulation of the three-dimensional problem is computationally too demanding the fiber dynamics are usually described by one-dimensional equations. To take viscoelastic effects into account an upper-convected Maxwell (UCM) model for the modeling and simulation of fibers in such processes was employed in [1, 2]. This model can be embedded hierarchically into the UCM fiber model of [3], which was derived asymptotically from a three-dimensional instationary boundary value problem. In this paper we link and compare these two models. The turbulent effects originating from the underlying airflow are incorporated into the fiber models by reconstructing the turbulent velocity fluctuations from a k-ε description of the airflow according to [4]. With the help of an academical melt-blowing setup, we present simulation results for the model hierarchy and discuss them with respect to accuracy and efficiency in view of more complex industrial setups.

2 UCM fiber model hierarchy

Asymptotic model Let Ω = {(ζ,t) ∈ R2 | ζ ∈ (-vin t, 0), t ∈ (0, t_{end})} be the space-time fiber domain with t the time, ζ the material parameter, vin = const being the non-dimensional (scalar) inflow velocity at the nozzle, and t_{end} the dimensionless end time. Without loss of generality, we assume vin = 1. According to [3], where a UCM string model has been systematically derived by slender body asymptotics, our viscoelastic fiber jet model in Lagrangian description has the following non-dimensional form

\[
\begin{align*}
\partial_t \mathbf{r} - \mathbf{v} &= 0, \\
\partial_t \mathbf{v} - \frac{1}{e} \mathbf{n} \cdot \partial_e \mathbf{v} &= 0, \\
\partial_t \mathbf{v} - \partial_\zeta \left( \frac{\mathbf{r} - \mathbf{v}}{e} \right) - \frac{1}{Fr^2} e_g - f_{air} &= 0, \\
De \partial_e \sigma + \left( -De (2\sigma + 3p) - \frac{3}{Re} \right) \frac{t}{e} \cdot \partial_e \mathbf{v} + \sigma &= 0, \\
De \partial_e p + \left( De p + \frac{1}{Re} \right) \frac{t}{e} \cdot \partial_e \mathbf{v} + p &= 0,
\end{align*}
\]

where r is the fiber position, v the velocity, and T the temperature. The fiber tangent r is split into the elongation e = ||r|| and the direction t = r/||r|| parameterized with spherical coordinates, i.e., t = (sin θ cos φ, sin θ sin φ, cos θ) with polar and azimuth angle θ ∈ [0, π], φ ∈ [0, 2π), respectively. The corresponding normal n and binormal b are n = (cos θ cos φ, cos θ sin φ, -sin θ) and b = (-sin θ sin φ, sin θ cos φ, 0). This splitting of the tangent is due to an appropriate closing with physical meaningful boundary conditions. The viscoelastic material laws are based on a UCM model for the fiber stress σ and pressure p. The acting outer forces arise from gravity with direction e_g, ||e_g|| = 1, as well as from the surrounding airflow inducing drag forces f_{air}. Moreover, T_a is the aerodynamic temperature field, and d the fiber diameter, which is introduced as d = 2√(4π). The dimensionless numbers are the slenderness parameter ε = d_0/r_0, the Reynolds number Re = ρv_0r_0/μ, the Deborah number De = θ/t_0, the Froude number Fr = v_0/√(g r_0) and the Stanton number St = α/(c_p v_0), where y_0 indicates the reference of any quantity y used for non-dimensionalization. Here, μ describes the dynamic viscosity, θ the relaxation time, ρ the density, and c_p the specific heat capacity of the fiber jet. Moreover, α denotes the heat transfer coefficient and g the gravitational constant.

* Corresponding author: e-mail manuel.wieland@itwm.fraunhofer.de, phone +49 631 31600 4873

This is an open access article under the terms of the Creative Commons Attribution License 4.0, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.

© 2019 The Authors Proceedings in Applied Mathematics & Mechanics published by Wiley-VCH Verlag GmbH & Co. KGaA Weinheim

https://doi.org/10.1002/pamm.201900018

© 2019 The Authors Proceedings in Applied Mathematics & Mechanics published by Wiley-VCH Verlag GmbH & Co. KGaA Weinheim

1 of 2
Assuming $De \neq 0$, $\sin \vartheta \neq 0$, the system (1) can uniquely be written as quasi-linear system of first order [3], i.e., $\partial_\vartheta^T \mathbf{y} + M(\mathbf{y}) \cdot \partial_\vartheta \mathbf{y} + \mathbf{m}(\mathbf{y}) = 0$ with $\mathbf{y}$ the vector of unknowns. The eigenvalue structure of the system matrix $M$ under the assumption of a pure hyperbolic behavior in combination with the physical setup yields the following initial and boundary conditions [5]:

**Initial-boundary conditions at the nozzle ($t \geq 0$):**

$\mathbf{r}(-t, t) = \mathbf{r}_{in}$, $\vartheta(-t, t) = \vartheta_{in}$, $\varphi(-t, t) = \varphi_{in}$,

$v(-t, t) = e_y$, $T(-t, t) = T_{in}$, $p(-t, t) = p_{in}$,

**Initial conditions ($t = 0$):**

$e(0, 0) = 1$, $\sigma(0, 0) = \sigma_{in}$, $p(0, 0) = p_{in}$,

**Boundary conditions at the nozzle ($t > 0$):**

if $\sqrt{\sigma_{int}} < 1$:

$\sigma(-t, t) = \sigma_{in}$, $p(-t, t) = p_{in}$,

if $\sqrt{3/(ReDe)} + \sigma_{in} + 3p_{in} < 1$:

$e(-t, t) = 1$,

**Boundary conditions at the fiber end ($t > 0$):**

$e(0, t) = 1$, $\sigma(0, t) = 0$, $p(0, t) = 0$.

To close the system (1) a model for the aerodynamic drag force $f_{air}$ is employed. The drag function depends on the direction of the fiber tangent $t$ and the relative velocity between fiber and airflow $v - v_\star$, where the airflow velocity $v_\star = v_\star + v'_\star$ consists of a deterministic part $v_\star$ and a stochastic part $v'_\star$. We assume the deterministic airflow velocity $v_\star$ to be known from a $k_\star-\epsilon_\star$ simulation. Furthermore, from the $k_\star-\epsilon_\star$ turbulence description, we reconstruct the local velocity fluctuations as homogeneous, isotropic Gaussian random fields. The global turbulent velocity $v'_\star$ is then obtained by superposition and yields lift forces on the fiber by being plugged into the air drag function. For details see [4, 5].

**Pressure-free model** Under the assumption of a positive strain rate $v = \partial_\vartheta e/e = t/e \cdot \partial_\vartheta v > 0$ the absolute pressure $p$ is at least one order of magnitude smaller than the stress $\sigma$, i.e., $|p| \leq 0.1 \sigma$, if the relation $v \geq 0.35/(\ell De)$ holds [3]. This means that the pressure is negligibly small for high strain rates. Introducing the splitting $\sigma = m - p$ in the asymptotic model system (1) with new stress variable $m$ and setting $p = 0$ leads to a pressure-free UCM fiber model. This resulting pressure-free model is instantaneously employed in [1, 2] for the modeling of melt-blowing processes. The boundary conditions change accordingly.

### 3 Academical setup

For the solution of the fiber models we employ the numerical scheme described in [5]. The spatial discretization is realized with the help of a finite volume scheme with a Lax-Friedrichs flux approximation. An implicit Euler scheme is used for the temporal discretization. We compare the asymptotic and the pressure-free fiber model by the help of an academical setup where we choose all airflow fields being constant in space and time. The deterministic air speed is $v_\star = 10 e_y$, the turbulent kinetic energy $k_\star = 1$, and the viscous dissipation rate $\epsilon_\star = 1$ with reference values $v_{x,0} = v_0 = 10$ m/s, $k_{x,0} = 2.05 \cdot 10^7$ m$^2$/s$^2$, and $\epsilon_{x,0} = 1.27 \cdot 10^7$ m$^2$/s$^3$ used for non-dimensionalization. We set $\varepsilon = 2.9 \cdot 10^{-3}$, $Re = 8.5 \cdot 10^3$, $De = 2.5 \cdot 10^4$, $Fr = 9.2$, and $St = 5.5 \cdot 10^{-3}$ and fix the (dimensionless) end time $t_{end} = 0.2$ as well as $\sigma_{in} = p_{in} = 0$. For the computation we choose the mesh sizes $\Delta \zeta = 0.1/t_{end}$. Although the strain rate $v$ is not exclusively positive in this academical setup, the pressure in the asymptotic model (1) is orders of magnitude smaller than the stress $\sigma$ shortly away from the nozzle, see Fig. 1. Comparing the solutions at time $t_{end}$ the relative $L^2$-error in all variables $|\varphi|$ (except pressure $p$) between the asymptotic and pressure-free model is $5.14 \cdot 10^{-4}$. The computation times are 6.2 h for the first and 5.4 h for the latter model. Consequently, the simplified model might be preferred for efficiency reasons for industrial melt-blowing setups with even higher strain rates.

**References**

1. S. Sinha-Ray, A. L. Yarin, and B. Pourdeyhimi, J. Appl. Phys. 108(3), 034912 (2010).
2. A. L. Yarin, S. Sinha-Ray, and B. Pourdeyhimi, J. Appl. Phys. 108(3), 034913 (2010).
3. N. Marheineke, B. Liljegren-Sailer, M. Lorenz, and R. Wegener, Math. Mod. Meth. Appl. Sci. 26(3), 569–600 (2016).
4. F. Hübsch, N. Marheineke, K. Ritter, and R. Wegener, J. Stat. Phys. 150(6), 1115–1137 (2013).
5. M. Wieland, W. Arne, N. Marheineke, and R. Wegener, arXiv:1902.01811 (2019).