Charge fluctuation induced dephasing of exchange coupled spin qubits

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Exchange coupled spin qubits in semiconductor nanostructures are shown to be vulnerable to dephasing caused by charge noise invariably present in the semiconductor environment. This decoherence of exchange gate by environmental charge fluctuations arises from the fundamental Coulombic nature of the Heisenberg coupling, and presents a serious challenge to the scalability of the widely studied exchange gate solid state spin quantum computer architectures. We estimate dephasing times for coupled spin qubits in a wide range (from 1 ns up to > 1µs) depending on the exchange coupling strength and its sensitivity to charge fluctuations.

A central issue in quantum information processing is quantum coherence, i.e. how long a quantum state survives without decay allowing robust quantum computation. The other central issue is scalability, i.e. whether a practical macroscopic quantum computer can be built by suitably scaling up individual microscopic qubits. The perverse dichotomy in quantum computation has been that architectures that can be scaled up fairly easily (e.g. solid state systems) suffer from serious environmental decoherence problems whereas architectures based on isolated atoms and ions, which have excellent coherence (e.g. ion traps), are typically not easily scalable. In this context, the proposed spin quantum computer (QC) architectures in semiconductor nanostructures look particularly promising since electron spin usually has long coherence time as the relativistic nature of spin produces weak direct environmental coupling, and semiconductors, at least as a matter of principle, allow for relatively easy scaling up. Motivated by the pioneering early suggestions, there has been impressive recent experimental advance in the study of spin qubits in gated GaAs quantum dot systems, an architecture widely regarded as one of the most promising solid state QC architectures.

One of the most significant advantages of a spin qubit is its relative isolation from its environment, leading to exceedingly long relaxation and dephasing times in systems such as isolated donor electron and nuclear spins in Si:P and GaAs quantum dots. Specifically, electron spin relaxation due to spin-orbit interaction and coupling to phonons is significantly reduced at low temperatures and for localized spins. The only important decoherence channel left is the nuclear spin hyperfine coupling induced electron spin spectral diffusion, so that the spin decoherence time for an isolated electron could range from tens of microsecond in GaAs quantum dots up to hundreds of millisecond in Si:P donor electrons.

New decoherence channels could open up when qubits are manipulated and/or coupled. An advantage of spin qubits in solids is the availability of exchange interaction, which originates from Coulomb interaction and Pauli exclusion principle. Even though spins are magnetic and magnetic interactions are weak, two-spin operations can actually be very fast (as short as 100 ps), because the underlying exchange interaction is electrostatic, which is strong. However, therein lies a new decoherence channel (compared to single spins) for the spin qubits: when spin coupling is needed and exchange interaction is turned on using external gates, charge fluctuations in the environment (an important source of decoherence for charge qubits in semiconductor and superconducting structures) could lead to gate errors and dephasing.

In this Letter we quantify how charge fluctuations affect exchange gates for spin qubits in a double quantum dot. In particular, we calculate the modification of exchange coupling in the presence of barrier variation and double dot level detuning (both of which could arise from charge fluctuations in the environment), evaluate errors in exchange gates, and calculate dephasing rates for logical qubits encoded in the double dot singlet-triplet states.

Gated quantum dots are defined electrostatically from a two-dimensional electron gas (2DEG). A movement of a trapped charge nearby changes the confining potential mainly in two ways: rise or fall of the barrier between the dots (thus change in tunneling rates), and detuning of the orbital levels in the two dots, as illustrated in Fig. When the central barrier between the double dot rises or falls, the dot potential minima also shift slightly farther or closer from each other.

With this qualitative understanding of how charge fluctuations affect quantum dot confinement, we evaluate the exchange coupling of two electrons in a gate-defined unbiased double quantum dot in the presence of charge fluctuations. We use a two-dimensional quartic potential: $V(x, y) = \frac{1}{2}m\omega^2[(x^2-L^2)^2/L^2+y^2]$. The central barrier height for this potential is $V_B = m\omega^2L^2/2$, directly related to the interdot distance. We use GaAs parameters in this calculation, with $m = 0.067m_0$ where $m_0$ is the bare electron mass. To calculate the exchange splitting for two electrons in this double dot potential, we employ the Heitler-London (HL) approximation, so that the exchange splitting $J$ between the two-electron
singlet and unpolarized triplet states can be expressed in terms of a few easy-to-calculate matrix elements:

\[
J = \frac{2S^2}{1 - S^4} \left[ \psi_L | V - V_L | \psi_L \right] + \left\langle \psi_R | V - V_R | \psi_R \right\rangle \\
\quad + \frac{2S^2}{1 - S^4} (\psi_L | 1 | \psi_R(2)) | \frac{e^2}{r_{12}} | (\psi_L(1) \psi_R(2)) \\
\quad - \frac{2S}{1 - S^4} (\psi_L | V - V_R | \psi_R) + \left\langle \psi_R | V - V_L | \psi_L \right\rangle \\
\quad - \frac{2}{1 - S^4} (\psi_L(1) \psi_R(2)) | \frac{e^2}{r_{12}} | (\psi_L(2) \psi_R(1)).
\]  

(1)

Here \( V_L \) and \( V_R \) are harmonic potential wells that share the same location as the left and right potential minima of given potential \( V \), \( \psi_L \) or \( \psi_R \) is the ground state if only \( V_L \) or \( V_R \) is present, indices 1 and 2 refer to the two electrons, \( S \) is the overlap integral between the single dot orbitals, and \( r_{12} \) is the inter-electron distance. The first and third terms in Eq. (1) refer to single particle contributions to \( J \), the second term originates from the direct repulsion between the two electrons, and the last term is the exchange contribution. The HL approximation works well for higher confinement energies (\( \hbar \omega \gtrsim 3 \text{ meV} \), corresponding to smaller dots) when the on-site Coulomb interaction is sufficiently large, and for larger interdot distances.

In Fig. 2 we plot the exchange splitting and the barrier-height-V\( _B \)-dependence of the exchange for a series of configurations of double dots. For strongly coupled dots exchange splitting up to 1 meV can be obtained, where \( dJ/dV_B \) can be larger than 1, rendering the coupled spin qubits as sensitive to charge noise as a charge qubit. Notice that at the large exchange limit, more sophisticated method is required for reliable evaluation of the exchange splitting [14,15], though the modifications are generally within one order of magnitude so that the qualitative feature of our current results remains. Charge fluctuations could also introduce a small bias \( \Delta V \) between the originally unbiased double dot, which leads to a second-order correction \( |\Delta J| \propto (\Delta V)^2 \). For relatively small charge fluctuations, we neglect this bias effect on exchange since it is a higher-order effect.

In some experimental situations the double dot is strongly biased so that one of the doubly-occupied two-electron singlet states is the ground state [21,22]. In this case the exchange splitting between the two-dot two-electron singlet and triplet states could be dominated by the tunnel coupling between the two- and one-dot singlet states (while the triplet cannot tunnel due to spin blockade [21]), and takes on the value \( J \approx |t|^2/E_b \), where \( t \) is the tunnel coupling between the two singlet states and \( E_b \) their energy difference, dominated by the applied bias potential between the two dots. Charge fluctuations affect both \( t \) and \( E_b \), so that \( dJ = (\frac{dJ}{dE_b})dE_b - (\frac{dJ}{dV_B})dV_B \) (assuming \( dV_B \) and \( dE_b \) represent different components of charge noise), where \( V_B \) is the barrier potential that determines tunnel coupling \( t \). If \( dJ \) is dominated by \( dE_b \), \( dJ = -(J/E_b)dE_b \). For \( J \sim 1\mu\text{eV} \) and \( E_b \sim 100\mu\text{eV} \), \( |dJ/dE_b| \sim 0.01 \), similar to a weakly coupled unbiased situation. (Here \( dJ/dE_b \) as opposed to \( dJ/dV_B \), controls the charge fluctuation effect on the exchange gate.) Our estimates here are consistent with the experimentally measured exchange dependence on bias voltage presented in Ref. [22], where \( dJ/dE_b \) ranges between 0.01 and \( 10^{-4} \), and falls in the same range as our theoretical results in Fig. 2.
Having analyzed how charge fluctuations affect the exchange splitting of a double dot, we now study how spin quantum computing could be influenced. Charge fluctuations can affect coupled spin qubits in two different ways. If exchange interaction is turned on briefly to perform two-qubit operations, a switching event of a nearby charge trap leads to a gate error. If the exchange interaction is constantly on and the singlet and the unpolarized triplet states are the logical qubit states, background charge fluctuation causes pure dephasing between the two states. Below we analyze these two situations separately.

A switching event during an exchange gate inevitably leads to a gate error. Consider a SWAP gate, where an exchange pulse corresponding to \( \int J dt / \hbar = \pi \) leads to a swap of the states of two spins. If instead \( \int J dt / \hbar = \pi + \delta \), the result of the exchange pulse is

\[
\begin{pmatrix}
\alpha_1 \\
\alpha_2
\end{pmatrix}
\begin{pmatrix}
\beta_1 \\
\beta_2
\end{pmatrix}
\xrightarrow{\text{SWAP}}
\begin{pmatrix}
\beta_1 \\
\beta_2
\end{pmatrix}
\begin{pmatrix}
\alpha_1 \\
\alpha_2
\end{pmatrix}
+ \frac{\alpha_1 \beta_2 - \alpha_2 \beta_1}{\sqrt{2}} (1 - e^{i\delta}) |S\rangle
\]

Here \( |S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2} \) is the two-spin singlet state. Thus the two spins would remain entangled after the SWAP operation. This error is linearly proportional to \( \delta \) when \( \delta \ll \pi \), and its prefactor \( \alpha_1 \beta_2 - \alpha_2 \beta_1 \) does not vanish unless the two single spin states are identical.

If we have three spins and intend to swap the state of the first to the third, errors accumulate linearly, with residual entanglement between qubits 1 and 3, and 2 and 3. The total error should grow with the number of spins \( N \) linearly even if errors in each swap are random, since different errors are not directly additive as they represent different unwanted entanglement after the swaps. One can estimate errors in more complicated operations such as a Controlled-NOT gate \( ^{2,22} \), where the feature of linear increase of error with the number of qubits persists.

If two-spin singlet and unpolarized triplet states are used as logical qubit states and the exchange coupling is kept on, background charge fluctuations cause dephasing between the two states. Since charge fluctuation is also a major source of decoherence for double dot charge qubits \( ^{14} \), we can extract the necessary information needed to calculate spin decoherence rate from the measured charge relaxation rates in the GaAs quantum dot system. Similar to what is well established in the decoherence properties of Cooper pair boxes \( ^{12} \), the relaxation rate at the degeneracy point of a double dot single-electron charge qubit (where the double dot ground and first excited states are split by \( 2|t| \) where \( t \) is the tunneling strength between the two dots) is given by a simple expression \( ^{14} \) \( \Gamma_1 = (\pi / 2\hbar^2) S_V(\omega = 2|t| / \hbar) \), where \( S_V(\omega) = (1 / 2\pi) \int_0^{\infty} d\omega e^{i\omega \tau} \langle V(\tau) V(0) \rangle \) is the charge fluctuation (in terms of the bias gate potential fluctuation) correlation in the environment. From the functional dependence of \( S_V(\omega) \) on \( \omega \) (such as \( 1 / \omega \) noise, which has only one parameter and is used here), we could use the knowledge of \( \Gamma_1 \) to determine \( S_V(\omega) \), and then use it to calculate the time(\( \tau \))-dependent phase diffusion \( \Delta \phi_\tau \) for a charge qubit (so that a factor \( \exp(-\Delta \phi) \) appears in the off-diagonal density matrix element of the two-level system formed from the double dot) when it is far away from the degeneracy point, where the effective Hamiltonian becomes \( H = V \sigma_z \).

\[
\Delta \phi_\tau(\tau) = \frac{1}{2\hbar^2} \int_{\omega_0}^{+\infty} d\omega S_V(\omega) \left( \frac{\sin \omega \tau / 2}{\omega / 2} \right)^2.
\]

Here the integral has a low frequency cutoff that is generally taken as the inverse of the measurement time \( ^{13} \).

In the case of two-spin singlet and unpolarized triplet states, the effective two-level Hamiltonian can be written as \( H = J \sigma_z \) (there is no \( \sigma_x \) term here as spin symmetry prevents direct relaxation between the two states), so that the two-spin dephasing is given by \( ^{15} \)

\[
\Delta \phi_s(\tau) = \frac{1}{2\hbar^2} \int_{\omega_0}^{+\infty} d\omega S_J(\omega) \left( \frac{\sin \omega \tau / 2}{\omega / 2} \right)^2
\]

\[
\cong (\frac{dJ}{dV})^2 \Delta \phi_\tau(\tau),
\]

where \( (dJ/dV)^2 = (\partial \Omega / \partial V_B)^2 + (\partial \Omega / \partial E_A)^2 \). Accordingly, two-spin dephasing should be sensitively dependent on the barrier- and bias-dependence of the exchange splitting \( (dJ/dV)^2 \) (Eq. \( ^{14} \) is valid when \( S_{V_B}(\omega) \) for the inter-dot barrier height and \( S_{V=V_B}(\omega) \) for the inter-dot bias are identical (recall that they originate from the same source of charge fluctuations and therefore should have similar behavior). Figure \( ^{4} \) shows the charge qubit phase diffusion. Two-spin phase diffusion can be obtained according to Eq. \( ^{15} \).

The phase diffusion grows as almost a quadratic function of time (actually \( t^2 \ln t \)). For charge relaxation time in the order of 10 ns \( ^{14} \), charge dephasing time for biased qubit is in the order of 1 ns (lower horizontal line in Fig. \( ^{3} \). Two-spin dephasing time sensitively depends on \( dJ/dV \), which in turn varies widely (see Fig. \( ^{2} \) and \( ^{3} \)). For example, if \( dJ/dV = 1 \) (when the double dot is tightly coupled), the two-spin dephasing time is as short as the charge dephasing time. On the other hand, if \( dJ/dV = 0.01 \), which is a reasonable estimate (see Fig. \( ^{2} \), two-spin dephasing time would be in the order of ten times of the charge relaxation time \( T_1 \sim 10 \) ns, leading to \( T_2 \sim 0.1 \mu s \). If \( dJ/dV \) could be reduced to 0.001, for small \( J \), or using schemes that can minimize \( dJ/dV \) through device design, in the spirit of the pseudo-digital design (to combat gate voltage fluctuations) of Ref. \( ^{22} \), in which \( J \) reaches maximum while \( dJ/dV \) is close to zero, the two-spin dephasing time would be \( \sim 1 \mu s \), in the same order as the single spin decoherence time due to...
illustrated by Eq. (4). The lower horizontal line indicates linearly proportional to the charge qubit phase diffusion, as a qubit can be determined from this figure as well since it is with a cutoff frequency of 1 Hz. Dephasing of a two-spin channel for exchange-coupled spin qubits, and further developments in device design and fabrication is needed to reduce the sensitivity of exchange coupling to charge fluctuations. In particular, the linear scaling of charge fluctuation induced spin dephasing of the exchange gate architecture with the number of gates is a rather serious problem that could limit the scalability of exchange-based spin quantum computation. Our finding of the charge fluctuation induced spin qubit dephasing time being in a wide range from ns to μs points to the need to optimize the double dot design in order to desensitize exchange coupling $J$ to the environmental charge noise.

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FIG. 3: Phase diffusion of a biased double dot charge qubit with a cutoff frequency of 1 Hz. Dephasing of a two-spin qubit can be determined from this figure as well since it is linearly proportional to the charge qubit phase diffusion, as illustrated by Eq. (4). The lower horizontal line indicates that $\Delta \phi_c \sim 1$ corresponds to $t \sim T_1/10$, the middle (upper) horizontal line indicates that $\Delta \phi_c \sim 10^4$ ($10^6$) corresponds to $t \sim 10T_1$ ($100T_1$). In other words, if $dJ/dV \sim 0.001$, $\Delta \phi_c \sim 1$ would correspond to a time of $100T_1$ where charge relaxation time $T_1$ is in the order of 10 ns, so that spin dephasing time $\sim 1 \mu$s.

to nuclear spin induced spectral diffusion in GaAs [13]. In general, extremely small values of $dJ/dV$ is needed to essentially eliminate the charge fluctuation induced spin decoherence.

Charge fluctuation induced spin qubit dephasing is obviously not just limited to GaAs quantum dots. Whenever charge degrees of freedom (e.g. electrostatic gates) are used to boost the speed of a quantum computing scheme, dephasing effect from charge noise in the underlying structure could arise. For instance, charge noise could have negative effects on trapped ions when semiconductor micro-traps are used, so that charge fluctuations in the semiconductor environment could lead to decoherence in the ionic states. Similarly, various proposed solid state quantum computing schemes using only exchange gate architectures are potentially susceptible to charge noise decoherence.

In conclusion, although single spin coherence is unaffected by charge noise, background charge fluctuations could lead to significant gate errors and/or decoherence in semiconductor-based electron spin qubits through inter-qubit exchange coupling. Our results show that charge noise could be the most important decoherence channel for exchange-coupled spin qubits, and further development in device design and fabrication is needed to reduce the sensitivity of exchange coupling to charge fluctuations. In particular, the linear scaling of charge fluctuation induced spin dephasing of the exchange gate

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