We demonstrate VALMOD, our scalable motif discovery algorithm interchangeably. Throughout this paper, we will use the terms radial profiles in astronomy, mass in mass spectroscopy, position in genome sequences, time series distances, top motifs, we call the output, which contains the offsets of the best matches. In Figure 1(c) we plot the Index profile, which contains the offsets of the best matches.

Motif discovery of different lengths. Exact Motif discovery has merely become a single input parameter problem, namely the length of the patterns we want to mine. Unfortunately, this technique comes with an important lack. It does not provide an effective solution for trying several motif length in a range. If one has no cues about an effective fixed length, the simplest solution would be to run the algorithm over all lengths in the range and rank the various motifs discovered, picking eventually the patterns, which contain the desired insight. Clearly, this possibility is not optimal for at least two reasons; the scalability, since finding motif of one fixed length takes $O(n^2)$ time, and also because it does not provide an effective way to compare motifs of different lengths. In this work, we demonstrate the solution to this problem, we recently introduced in [4], to mine Motif discovery of variable lengths. In our contribution we propose VALMOD, the first approach for mining top-k motif pairs of variable length, which is up to orders of magnitude faster/more scalable than the alternatives that have been proposed in the literature.

In order to show the superiority of variable-length motif discovery, consider the following example. In Figure 1 (left) we depict a snippet of an Electrocardiogram (ECG) recording in (a), paired with its Matrix profile, computed with fixed subsequence length: $\ell = 50$ in (b). Note that each value in the Matrix profile corresponds to a subsequence of length $\ell$. Hence, given a data series $D$ of length $|D|$, a Matrix profile records $|D| - \ell + 1$ distances, avoiding trivial matches [4]. In Figure 1(c) we plot the Index profile, which contains the offsets of the best matches.

Looking at the Matrix profile in this example, we note four deep valleys, which suggest the presence of very close matches, namely the motifs. Starting from the Matrix profile, it suffices to follow the dotted lines upwards, in order to detect the motifs, and downwards for finding the position of each subsequence best match. Despite the motifs (heartbeats) are easily detectable to the naked eye, since the snippet is relatively short, the highlighted motifs in Figure 1(a) (red/orange subsequences), just report the second half of a ventricular contraction, giving thus a partial and unsatisfactory result.

In the next section we present the complete details of the VALMOD algorithm.

### 2 VALMOD MOTIF MANAGEMENT

**VALMOD algorithm** As previously introduced, our algorithm, VALMOD (Variable Length Motif Discovery), given a data series
Figure 1: Left) (a) Snippet of ECG recording with highlighted motifs of length 50. (b) Matrix profile computed with subsequence length 50. (c) Index profile, reporting the offsets of the best match. Right) (d) Snippet of ECG recording with highlighted motifs

Figure 2: (a) ECG snippet with distance profile of subsequence D\textsubscript{601,600}. (b) Partial distance profiles computation for length 601.

D, starts by computing the Matrix profile using the smallest subsequence length, namely \(\ell_{min}\), within a specified input range \([\ell_{min}, \ell_{max}]\).

The key idea of our approach is to minimize the work that needs to be done for succeeding subsequence lengths \((\ell_{min} + 1, \ell_{min} + 2, \ldots, \ell_{max})\). To explain the main components and the idea of our algorithm we present a short example in Figure 2.

We start to consider the data series \(D\) in (a) (snippet of ECG recording). To compute the Matrix profile, VALMOD considers all the contiguous subsequences of length \(\ell_{min}\), computing for each one the Distance profile in \(O(|D|)\) time. This latter, contains the z-normalized Euclidean distance between a subsequence and all the other in \(D\). In Figure 2.(a) we report a distance profile for the subsequence \(D_{601,600}\) (the subscript denotes offset=600 and length=600). The minimum distance of each distance profile is a point of the Matrix profile.

We moreover introduce a new lower bounding distance\([4]\), which lower bounds the true Euclidean distances between longer subsequences in the distance profiles. We initially compute this lower bound from scratch, using as a base the true Euclidean distances computation of subsequences with length 600. For the larger lengths, we update the lower bound, considering only the variation generated by the trailing points in the longer subsequences. This measure enjoys an important property: if we rank the subsequences according to this measure (ascending order), the same rank will be preserved along all the lower bound updates. We want to exploit this property, in order to prune computation. Hence, when the distance profiles are computed (in this example for length=600), we keep in memory the \(p\) Euclidean distances, which have the smallest lower bounding distance (LB); this is done for each distance profile. We show in Figure 2.(b) how the algorithm proceeds for the length 601. Instead of computing from scratch the whole distance profiles, we consider just the elements we stored in the previous step. Here, each distance profile is denoted as partial distance profile. We proceed computing the true Euclidean distances of each partial distance profile, updating the relative LB (this result is depicted in Figure 2.(b)). After this operation, we may have two cases: if in a new computed distance profile the minimum true distance (\(mindist\)) is shorter than the maximum lower bound (\(maxLB\)), we know that no elements, among those not computed, can be smaller than mindist. In this case a partial distance profile becomes a valid distance profile, as in the case of the subsequence \(D_{601,600}\). On the other hand, when \(maxLB\) is smaller than \(mindist\), as in the case of subsequence \(D_{620,601}\), no true minimum distance is found within the distance profile. At the end of this process, we pick the minimum \(maxLB\) of all the non-valid distance profile, which is denoted as \(minLBabs\). Hence, all the \(mindist\) in the valid (partial) distance profiles, smaller than \(minLBabs\) are considered top-\(k\) motif distances. If no \(mindist\) are smaller than \(minLBabs\), we recompute only the distance profiles, which have the \(maxLB\) smaller than the smallest \(mindist\) found, since only those may contain better matches than the already computed ones. We keep extracting in this way, the top-\(k\) motifs of each length, until \(\ell_{max}\).

Experimental Evaluation. To benchmark VALMOD, we use several different datasets in [4], comparing it with two types of algorithms. The first are two state-of-the-art motif discovery algorithms, which receive a single subsequence length as input: QUICKMOTIF [3] and STOMP [1]. In our experiments, they have been adapted to find all the motifs for a given subsequence length range. The
more, whether and how the motif pair changes, helping the user
accept a range of lengths as input, producing the best motif pair
for each length. We report in Figure 3 a sample of the experiments
we conducted (detailed experimental results on several datasets are
reported elsewhere [4]). Here, we show the results of VALMOD ,
which finds motifs in an Electrocardiogram recording (ECG) and in
a data series representing celestial objects (ASTRO) [4]. We couple
the VALMOD results with those of its competitors. In the plots,
we report the total execution time of VALMOD, which includes all the
operations performed by the algorithm (also the VALMAP computa-
tion introduced later), varying motif length ranges (Figure 3 (top))
and the size of the input data series, considering different prefix
snippets (Figure 3 (bottom)). From this experiment, we observe
that VALMOD maintains a good and stable performance across
datasets and parameter settings, quickly producing results, even in
cases where the competitors do not terminate within a reasonable
amount of time.

**Rank Motif Pairs of Variable Lengths.** Since we can discover
motifs of different lengths, we propose a ranking method, suitable
for comparing different-length patterns. We aim to favor longer
and similar sequences in the ranking process of matches that have
different lengths. As a consequence, we factorize the Euclidean
distance by the following quantity: \(\sqrt{1/\ell}\), where \(\ell\) is the length
of the sequences. We call this new distance, *length normalized
distance* [4].

**VALMAP.** While the proposed motif rank weights the subsequences
importance according to the ratio distance-length, we want to know
also, whether and how the motif pairs changes, helping the user
to extract the desired insights at the correct length. To that ex-
tent, we introduce a new meta-data, called Variable Length Matrix
Profile (VALMAP), maintaining the same logic and structure of the
Matrix profile depicted in Figure 1 (top), with the difference
that this new structure carries length normalized distances and it is
coupled with a new vector called *Length profile*, which con-
tains the lengths of the subsequences. More formally, given a data
series \(D\) and a range of subsequence lengths, whose extremes
are denoted by \(\ell_{\text{min}}\) and \(\ell_{\text{max}}\), we define VALMAP as a triple
\((MP^n \in \mathbb{R}^{#D-\ell_{\text{min}}+1}, IP \in \mathbb{N}^{#D-\ell_{\text{min}}+1}, LP \in \mathbb{N}^{#D-\ell_{\text{min}}+1})\),
where \(MP^n\) is the Matrix profile containing length normalized
distances, whereas \(IP\) and \(LP\) are the relative Index and Length
Profile. If we consider just a fixed length, VALMAP will coincide
with the length normalized version of the Matrix profile, with a flat
Length profile. This is basically the structure that VALMOD builds,
considering subsequences of length \(\ell_{\text{min}}\). In the second stage, we
can update VALMAP using the top-\(k\) motif pairs, computed for each
length until \(\ell_{\text{max}}\). We thus consider each \((D_i, D_{i+1}, D_j, D_{j+1})\) in
a top-\(k\) motif pairs, where \(i, j\) are the subsequences offsets, \(\ell_{\text{min}} + 1\)
their lengths and \(d_{i,j}^n\) their length normalized Euclidean distance.
Note that in a motif pair the right subsequence is the one with the
absolute shortest distance to the one at the left. Hence, VALMAP ,
\(MP^n[i]\) is updated with \(d_{i,j}^n\) if \(d_{i,j}^n < MP^n[i]\), which was containing
the distance between \(D_i, D_{i+1}\) and its best match. If this update
place takes, the Index and Length profile are respectively assigned
with \(j\), the offset of the new best match, and \(\ell_{\text{min}} + 1\) the new
length. The update operation takes place for each top-\(k\) motif pair of
any length between \(\ell_{\text{min}}\) and \(\ell_{\text{max}}\). Once the algorithms ends,
VALMAP contains a picture of the motif pairs showing, at which
length the last update takes place. If a motif pair is updated, this
implies that a longer pattern represent a better match and thus it
might reveal either a new event or the same event lasting longer.

**Example of VALMAP Expressiveness.**

In order to show the expressiveness of VALMAP , we ran VALMOD
on the ECG data snippet previously considered, showing the VALMAP
structure in Figure 1 (right). We use the following input parameter:
\(\ell_{\text{min}} = 50\) and \(\ell_{\text{max}} = 400\). We note that VALMAP reports the
motif with the shortest length normalized distance of length 56,
which is the same partial event detected by the Matrix profile in the
fixed length case, at the top of the picture.

If we look at the Length profile in Figure 1 (f), we observe that,
at an earlier time than the discovered motifs pair, a sequence of
contiguous updates took place, as we reported. The subsequences
concerned have distances almost as short as the one of the best
motifs in VALMAP , thus, remaining longer and possibly valid
matches.

In Figure 1 (d) we depict and highlight the motif pair of length 400.
Immediately, we can note that, the subsequences in red, which com-
pose this motif, are a better representation of a recurrent heartbeat.
In fact, the two typical components (Artia and Ventricles contract)
correctly detected.

**3 SYSTEM DESCRIPTION**

We now describe the architecture of our system, depicted also in
Figure 4. The input is represented by a data series of interest. As
a starting point, the user has the possibility to inspect the data
and also setting the desired parameter (lengths range \([\ell_{\text{min}}, \ell_{\text{max}}]\)).
Afterwards, she can run the VALMOD algorithm, which is a part
of the system back-end we implemented in C. Once terminated,
VALMOD outputs the VALMAP meta-data. This latter is thus sent to
the front-end, implemented in Python. Here, the user can interact
with the system analyzing the showcased elements, such as:

- the checkpoints of the VALMAP, namely all the updates oc-
curred from the length \(\ell_{\text{min}}\) till the desired length, selected
with a dedicated slider.

![Figure 3: (top) Time over motif length ranges (default \(\ell_{\text{min}}=1024\), data series length - 0.5M. (bottom) Time over series length (default length range - 100).](image-url)
all the top-k motifs of variable length, which VALMAP reports.
- expand a selected motif pair to the relative Motif Set, containing all the similar subsequences of the pair in the data.

In Figure 5 we show a screen-shot of the VALMAP analysis in our demonstration.

### 4 DEMONSTRATION

We now present the scenarios proposed to the audience. **Need for Variable Length Motifs.** We will showcase variable length motif discovery using VALMOD on different real datasets [4], including ECG and ASTRO, as well as datasets coming from the domains of *Entomology* and *Seismology*. In these two particular cases, the user can understand the importance of using variable length motif detection (with the support of VALMAP), in order to identify patterns of interesting behavior exhibiting themselves as sequences of different lengths.

**Traditional Motif discovery VS VALMOD.** In this scenario, we will challenge the user to find the motifs without having any knowledge of their lengths, just by inspecting the data themselves. When this takes place, the user can experience the VALMOD support in finding motif pairs that can be of variable length, understanding the quantity and quality of the insights that are not achievable with a simple raw data visual analysis.

**VALMOD VS Competitors.** In this scenario, the user can compare VALMOD to alternative approaches used for motif discovery. Specifically the audience will note the performance improvement, concerning fixed and variable length motif discovery, and the increased expressiveness provided by VALMAP.

### 5 CONCLUSIONS

In this work, we present VALMOD, a system that can efficiently find data series motif of variable length. As opposed to the other approaches, our framework provides a new meta data-series (VALMAP), which ranks motif pairs of variable length, using a new length normalized distance. Our system provides enriched insights, which help to detect not only the correct resolution (length) of an interesting event, but also the occurrences of repeated patterns with different meanings, which are typical in numerous domains.

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