Dynamical holographic QCD with area-law confinement and linear Regge trajectories

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We construct a new solution of five-dimensional gravity coupled to a dilaton which encodes essential features of holographic QCD backgrounds dynamically. In particular, it implements linear confinement, i.e. the area law behavior of the Wilson loop, by means of a dynamically deformed anti-de Sitter metric. The predicted square masses of the light-flavored natural-parity mesons and their excitations lie on linear trajectories of approximately universal slope with respect to both radial and spin quantum numbers and are in satisfactory agreement with experimental data.

PACS numbers: 11.25.Tq, 11.25.Wx, 14.40.-n, 12.40.Yx

Over the past decade a qualitatively new perspective on strong-interaction physics emerged from gauge/string dualities\(^1\) and the underlying holographic principle. These dualities map (i) string theory spectra on asymptotically AdS \(\times X\) spacetimes (i.e. AAdS \(\times X\), where \(X\) is a compact space) into gauge invariant, local operators of the dual field theory, (ii) the fields parameterizing the boundary conditions into sources for the dual operators, and (iii) the string theory partition function (or its low-energy gravity limit) into the generating functional of the field-theory correlators. As a consequence, the notoriously complex strong-coupling regime of large-\(N_c\) gauge theories can be approximated (in low-curvature regions) by weakly coupled and hence analytically treatable classical gravities. The gauge/gravity correspondence thereby supplies new analytical tools for the study of hadronic observables in the non-perturbative regime of the strong force.

Applications of gauge/gravity dualities to “QCD-like” gauge theories either start from specific D-brane setups in ten- (or five-) dimensional supergravity and derive the corresponding gauge theory properties, or try to guess a suitable background and to improve it in bottom-up fashion by comparing the predictions to QCD data. Even the simplest and oldest bottom-up (or “AdS/QCD”) model, the hard wall\(^2\), reproduces a surprising amount of hadron phenomenology\(^3\). The conformal invariance of AdS\(_5\) in the UV reproduces, in particular, the counting rules which govern the scaling behavior of hard QCD scattering amplitudes, while an infrared cutoff on the fifth dimension at the QCD scale \(\Lambda_{\text{QCD}}\) implements the mass gap and discrete hadron spectra.

The hard-wall predictions for the squared masses of light-flavor hadrons depend quadratically on the principal and spin excitation quantum numbers\(^3\), however, in contrast to the theoretically expected and approximately observed linear Regge behavior\(^4\). A straightforward way to correct this shortcoming was suggested in Ref.\(^5\) where the AdS\(_5\) geometry is kept intact while an additional dilaton background field with quadratic dependence on the extra dimension is exclusively responsible for conformal symmetry breaking. This dilaton soft-wall model indeed generates linear Regge trajectories \(m_{n,S}^2 \sim n + S\) for light-flavored mesons of spin \(S\) and radial excitation level \(n\). (Regge behavior can alternatively be encoded via IR deformations of the AdS\(_5\) metric\(^5\).)

However, the dilaton soft wall fails a quintessential test for confining gauge theories: the resulting vacuum expectation value (vev) of the Wilson loop does not exhibit the area-law behavior which a linearly confining static quark-antiquark potential would generate. This is because the Wilson loop vev is determined by the area of the dual string world sheet in the five-dimensional spacetime\(^8\), i.e. it depends exclusively on the background geometry. Since the latter remains exact AdS\(_5\) (and thus conformal) in the soft-wall model, the Wilson loop vev shows a non-confining perimeter law. (The hard wall, on the other hand, also confines magnetic charges instead of screening them\(^5\).) A second, common shortcoming of both hard- and soft-wall backgrounds is that they are not solutions of a dual gravity. Hence their relation to the dynamics of the sought after QCD dual remains obscure and all gauge theory vacuum properties (including confinement, chiral symmetry breaking and condensates) have to be imposed by hand\(^10\).

In the following we show how both shortcomings can be overcome, by deriving a rather minimal AdS/QCD background which implements the area law, i.e. linear confinement, dynamically. Our strategy will be to adopt an IR-deformed (and hence non-conformal) AdS\(_5\) ansatz family for the metric which is general enough to generate the area law while keeping the fifth dimension non-compact in order to allow for linear Regge trajectories. We will then construct the corresponding dilaton fields and potentials such that their combination solves the five-dimensional Einstein-dilaton equations, and find those solutions which additionally generate linear Regge trajectories in the highly excited meson spectrum.
To implement the above program, we start from the Einstein-Hilbert action of five-dimensional gravity coupled to a dilaton $\Phi$,

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{|g|} \left( -R + \frac{1}{2} g^{MN} \partial_M \Phi \partial_N \Phi - V(\Phi) \right),$$  

(1)

where $\kappa$ is the five-dimensional Newton constant and $V$ is a still general potential for the scalar field. We then search for static solutions of the corresponding field equations in which the metric is restricted to the form

$$g_{MN} = e^{-2A(z)}g_{MN}$$  

(2)

with $\eta = \text{diag}(1, -1, -1, -1, -1)$ and the dilaton field $\Phi(z)$ depends on the radial coordinate only. We write the warp factor as

$$A(z) = \ln z + C(z)$$  

(3)

(our units are such that the AdS$_5$ curvature radius is unity) where the function $C(z)$ describes non-conformal deformations of the AdS$_5$ metric. We further impose the boundary condition $C(0) = 0$ which restricts the geometry to asymptotically AdS$_5$ (AAdS$_5$) spacetimes and thus ensures conformality in the ultraviolet (UV).

Variation of the action (1) leads to the Einstein-dilaton equations

$$6A'^2 - \frac{1}{2} \Phi'^2 + e^{-2A} V(\Phi) = 0,$$  

(4)

$$3A'' - 3A' \Phi' - e^{-2A} V(\Phi) = 0,$$  

(5)

$$\Phi'' - 3A' \Phi' - e^{-2A} \frac{dV}{d\Phi} = 0$$  

(6)

for the background fields $A$ and $\Phi$. This set of coupled differential equations is redundant, i.e. only two of them are independent [6]. To cast those into the form most suitable for our purposes, we add the two Einstein equations to obtain the derivative of the dilaton field as a function of the warp factor,

$$\Phi' = \sqrt{3} \sqrt{A'^2 + A''}$$  

(7)

(the positive sign is chosen for definiteness). Equation (7) determines (up to a constant to be fixed by a boundary condition) the dilaton field which forms, in combination with a given warp factor $A$, a solution of Eqs. (4) – (6). Equation (7) shows, in particular, that a constant dilaton solution requires the warp factor to satisfy $A'' = -A'^2$, i.e. under $C(0) = 0$ exact AdS$_5$ is the only solution. In other words, the conformal-symmetry breaking solutions of interest require a non-constant dilaton and consequently an IR deformation of the AAdS$_5$ geometry. (The lack of the latter in the soft-wall background [5] explains why it cannot solve Eqs. (4) – (6), as realized in Ref. [11].) Another consequence of Eq. (7) is that there remains only one boundary condition to be imposed on the dilaton field.

In order to recover the full information content of the field equations (4) – (6), it remains to find the dilaton potential $V(\Phi(z))$ (whose specific form played no role in the above discussion) for which a given $A$ and the corresponding $\Phi$ determined by Eq. (7) form a solution. This potential can be obtained either from one of the Einstein equations (4), (5) or (up to a constant) from the dilaton equation (6), by substituting Eq. (7). In either case, the result is the relation

$$V(\Phi(z)) = \frac{3e^{2A(z)}}{2} \left[ A''(z) - 3A'^2(z) \right]$$  

(8)

between the dilaton potential, evaluated at the dilaton solution, and the metric. With the help of Eq. (8) and $\Phi' dV/d\Phi = dV/dz$ it can immediately be verified that all field equations (4) – (6) are indeed fulfilled by dilaton fields which satisfy Eq. (7). Hence Eqs. (7) and (8) are equivalent to the two independent Einstein-dilaton equations, and they allow to obtain explicit solutions $\Phi$ and the corresponding potential $V(\Phi(z))$ from a given warp factor $A$ (without requiring a superpotential).

Having shown how to construct such solutions, we can now pick a specific warp factor suitable for our purposes. To this end, we recall that the vacuum expectation value of the Wilson loop in the dual boundary theory is related to the area of the string world sheet in the five-dimensional space-time [8] and thus determined by the geometry alone. Hence we adopt a subfamily of metrics which encodes (besides the conformal UV behavior) the desired area law behavior. The specific conditions given in Ref. [12] have been used in Ref. [9] to show that the leading IR (i.e. $z \to \infty$) behavior of a linearly confining (and magnetically screening) metric of the form (2) in a non-compact fifth dimension is characterized by $C(z) = \lambda^z + \ldots$ where $\lambda \geq 0$ ensures that the conformal AdS$_5$ metric dominates the UV limit (i.e. $C(0) = 0$) and $\lambda \geq 1$ is required for linear confinement.

In order to transparently elaborate on the impact of such confining $C(z)$ on the meson spectrum, we first neglect subleading terms and begin our quantitative discussion with the minimal choice

$$C_\lambda(z) = \lambda^z$$  

(9)

for the confining AAdS$_5$ warp factor. We then obtain the corresponding dilaton solution by integrating Eq. (7) under the UV-conformality preserving boundary condition $\Phi(0) = 0$. The result is

$$\Phi_\lambda(z) = \sqrt{\frac{\lambda}{\lambda + \lambda^2 + \lambda^2 z^2}} \left[ (1 + \lambda) \ln \left( \lambda z^2 + \sqrt{\lambda + \lambda^2 + \lambda^2 z^2} \right) - (1 + \lambda) \ln \left( \sqrt{\lambda + \lambda^2} + z^{\lambda/2} \sqrt{\lambda + \lambda^2 + \lambda^2 z^2} \right) \right]$$  

(10)

which, together with the AAdS$_5$ metric specified in Eq. (9), forms a new analytical solution family of the five-dimensional Einstein-dilaton equations (4) – (6), with
the dilaton potential given by Eq. (8). For \( \lambda = 2 \) it reduces to a solution given in Ref. [9]. (To elevate the soft-wall dilaton field and the AdS5 metric to a solution of the coupled field equations, in contrast, requires an additional tachyon field [13], as suggested in Ref. [11].) Evaluated at the solution \( \Phi_\lambda \), the dilaton potential is
\[
V(\Phi_\lambda(z)) = -\frac{3}{2} e^{2z^\lambda} \left[ 4 + 7\lambda z^\lambda + \lambda^2 z^\lambda \left(3z^\lambda - 1\right) \right].
\]

(11)

Close to the UV and IR limits the dilaton solution behaves as \( \Phi_\lambda(z) \xrightarrow{z \to 0} c_0 z^{\lambda/2} \) and \( \Phi_\lambda(z) \xrightarrow{z \to \infty} c_\infty z^{\lambda} \) where \( c_0 = 2\sqrt{3(\lambda + 1)} \) and \( c_\infty = \sqrt{3} \). In these limits the potential can be easily expressed as a function of \( \Phi \), i.e. \( V(\Phi) \xrightarrow{\Phi \to 0} -6 + 3(\lambda + 1)(\lambda - 8)\Phi^2 / (2c_0^2) \) and \( V(\Phi) \xrightarrow{\Phi \to \infty} -9\lambda^2\Phi^2 \exp(2\Phi/c_\infty) / (2c_\infty^2) \). The exponential divergence for \( \Phi \to \infty \) is both induced and counterbalanced by the exponential behavior of the metric. Indeed, the dilaton-gravity coupling is necessary to stabilize the classical solution since the dilaton potential is not bounded from below. As expected, the boundary condition \( \Phi(0) = 0 \) ensures conformal invariance in the UV where the potential \( V(0) = -6 \) consists of just the AdS5 cosmological constant term.

We will now derive the mesonic excitation spectrum in the background [10, 11] and then study the existence criteria for another classic confinement signature, namely linear square mass trajectories of Regge type for highly non-Abelian DBI action of flavor brane stacks, or from interactions with additional bulk fields. Their neglect is common practice in the current, first generation of AdS/QCD duals. A more detailed discussion of the ensuing limitations can be found e.g. in Refs. [9, 10].

Important qualitative aspects of the meson spectrum arising from Eq. (12) can be understood by studying the UV (i.e. \( z \to 0 \)) and IR (\( z \to \infty \)) limits of the mode potential. We start with the UV behavior. Keeping only the leading terms for small \( z \) and \( \lambda > 1 \),
\[
\mathcal{V}_S(z) = a_0(S) z^{-2} + a_1(S) z^{-2} + a_2(S) z^{-2} + \ldots
\]

(14)

with the spin-dependent coefficients \( a_0(S) = S^2 - 1/4 \) (which originates from the AdS5 part of the warp factor), \( a_1(S) = \sqrt{3(\lambda + 1)}(S - \lambda/4) \) and \( a_2(S) = \sqrt{3S - 2 - 1}(\lambda + 1)S + 5\lambda + 3)/4. For \( \lambda = 4 \) the dilaton field therefore becomes proportional to \( z^2 \) when \( z \to 0 \) (see the discussion below Eq. (11)), as imposed by hand in Ref. [5]. However, while in this case the \( a_2 \) term in Eq. (14) becomes harmonic and generates approximately linear trajectories for the low-lying radial excitations, the mode potential will grow as \( z^6 \) at large \( z \) and cause strong nonlinearities in the trajectories of high-lying radial and spin excitations.

To implement the linear trajectories for highly excited meson states, which semiclassical arguments predict as a consequence of long, unbroken flux tubes in the large-\( N_c \) limit, we now turn to the large-\( z \) behavior of the mode potential. The leading infrared contribution is
\[
\mathcal{V}_S(z) \xrightarrow{z \to \infty} \frac{\lambda^2}{4} \left( 2S \sqrt{3} - 1 \right)^2 z^{2\lambda - 2}.
\]

(15)

Hence for \( \lambda > 1 \) the entire normalizable spectrum is discrete and has a mass gap (in the absence of normalizable zero modes), as previously found for radial glueball excitations [8] and in remarkable agreement with the area-law condition \( \lambda \geq 1 \) [12] for linear confinement. (In contrast, the soft-wall model generates a non-confining perimeter law [8].)

Asymptotically equidistant \( m_{n,S}^2 \) values corresponding to linear trajectories further require \( \lambda = 2 \) in the IR dominant part [13] of the warp factor, in order to generate a harmonic IR potential. The trajectories resulting from the simplest choice \( C(z) = z^2 \) would be of non-universal slope \( (m_{n,S}^2 \propto n(S + c)) \), however, which can be corrected by adopting
\[
C(z) = \frac{1 + \sqrt{\lambda}}{2S + 3 \sqrt{3} - 1} \left( 1 + e^{(1 - 2z)QCD} \right)^2
\]

(16)

as the non-conformal warp factor. The corresponding metric remains close to AdS5 in the UV but deforms rather rapidly for \( z \gtrsim \Lambda_{QCD}^{-1} \) to approach the confining large-\( z \) asymptotics of Eq. (14) with \( \lambda = 2 \). The associated dilaton field and potential, which turn Eqs. [13],
into a solution of the Einstein-dilaton equations \( \Box \), are then obtained by solving Eq. (10) numerically according to the procedure outlined above. This determines the potential (13), and the masses follow by solving the eigenvalue problem (12) numerically. In Fig. (1) the resulting spectrum is compared to experimental data and hard- and soft-wall model results. Note that the state label \( n \geq 1 \) is chosen such that \( n = 1 \) refers to the nodeless ground state of the radial excitation spectrum. A satisfactory description of the meson mass spectrum with nearly universal Regge slopes is indeed achieved without any tuning of adjustable parameters. (The spin dependent factor in Eq. (10) is required by universality. For a physical interpretation see Ref. [1].) A good analytical approximation to the spectrum for \( \Lambda_{QCD} = 0.3 \text{ GeV} \) (in units of GeV)

\[
m^2_{n,S} \simeq \frac{1}{10} (11n + 9S - 9), \quad (n \geq 1)
\]

which makes the approximate universality of the linear trajectory slopes explicit.

\[
\begin{array}{c}
\text{FIG. 1: (a) Radial excitations of the rho meson in the hard-wall (dashed line), soft-wall (solid line), and dynamical soft-wall (solid line, for } \Lambda_{QCD} = 0.3 \text{ GeV} \text{) backgrounds. (b) Square mass predictions of spin excitations compared to PDG values for } \rho(770), \omega(782), \Phi(1020), \pi_1(1400), \omega(1420), \rho(1450), \rho(1570), \pi_1(1600), \omega(1650), \Phi(1680), \rho(1700), \rho(1900), \rho(2150), f_2(1270), a_2(1320), f_2(1430), f_2(1525), f_2(1565), f_2(1640), a_2(1700), f_2(1810), f_2(1910), f_2(1950), f_2(2150), f_2(2300), f_2(2340), f_2(1670), f_3(1690), f_3(1850), f_4(1900), f_4(2250), a_4(2040), f_4(2050), f_4(2300), a_4(2450) \text{ and } f_4(2510). \\
\end{array}
\]

Since the AdS/CFT dictionary translates the \( z \) dependence of the dilaton into the running of the gauge coupling, the latter could be implemented into Eq. (10) for small \( z \ll \Lambda_{QCD}^{-1} \) according to the perturbative QCD \( \beta \) function. This generates the leading correction \( C_{pert}(z) = -(2 \ln z)^{-1} \) which modifies the UV behavior of the string mode potential (12) as \( V_S(z) \sim 0 \) \( z^{-3/2} \) but vanishes in the IR. Hence asymptotic freedom and the perturbative corrections to it can naturally coexist with confinement at large \( z \).

To summarize, we have found a new solution of the five-dimensional Einstein-dilaton equations which provides an approximate dual background for holographic QCD. The method used in its derivation applies to essentially all AAdS(5) (and hence UV conformal) spacetimes with a Poincaré-invariant boundary. The vacuum properties of the boundary gauge theory, including quark confinement and condensates, are dynamically encoded in this solution without the need for additional background fields. In particular, our background generates a confining area law for the Wilson loop (in contrast to the soft-wall model), and we have outlined how the perturbative running of the gauge coupling could additionally be implemented.

For metrics whose warp factors approach the power law \( z^\lambda \) in the infrared of the non-compact extra dimension, we found the existence condition for the confining area law, \( \lambda \geq 1 \), to essentially coincide with the condition \( \lambda = 1 \) for an entirely discrete meson spectrum and for the existence of a mass gap, as previously encountered in the glueball sector. With \( \lambda = 2 \) our background solution satisfies this condition, and it organizes the square masses of both radial and spin excitations into linear trajectories. It further generates the approximately universal slope of the experimentally observed trajectories (without introducing adjustable parameters beyond the QCD scale) and satisfactorily reproduces the empirical spectrum of the light-flavored natural parity mesons.

WP is grateful to C.A. Ballon Bayona, H. Boschi and G. Pimentel for fruitful discussions. We acknowledge partial support from DAAD, CAPES, FAPESP and CNPq.
For a more detailed discussion of the limitations of AdS/QCD models see e.g. H. Forkel, Phys. Rev. D 78, 025001 (2008); T.D. Cohen, arXiv:0805.4813.

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