A dynamic functional model of diode bridge rectifier for unbalanced input voltage conditions

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Abstract
Diode bridge rectifiers are employed in many industrial applications wherein, under practical operating conditions, input voltages can be occasionally or inherently unbalanced. Using the switching function method, this paper proposes a new accurate functional model of the diode bridge rectifier capable of dealing with unbalanced input voltage conditions. To take all degrees of unbalanced conditions into account, this paper introduces uneven commutation intervals phenomenon between diodes occurring under unbalanced voltage conditions and proposes dynamically updating switching functions.

1 | INTRODUCTION
Diode bridge rectifiers (DBRs) are widely used in many industrial applications. The model of DBR is required for studying input current characteristics of variable-speed drives, modelling and evaluation of synchronous machine-rectifier systems, and accurate active current injection for rectifier power factor correction (PFC), etc. Because of the diversity of DBR’s applications, the model of DBR must be able to deal with both balance and unbalance conditions.

Switching functions of diodes are mathematical functions representing switching instants of diodes. Any degree of unbalance in input voltages changes the switching instants. Therefore, switching functions calculated for the balanced condition are not valid for unbalanced voltage conditions. According to the authors’ knowledge, switching functions of DBR have not been developed to take unbalanced conditions into account. This paper extends the switching function model of DBR to unbalanced voltage conditions by continuously updating switching functions consistent with input voltages.

In [1], using the switching function method, a harmonic matrix of DBR has been developed for balanced voltage conditions. Another matrix model of DBR based on the switching function method was proposed in [2]. This model exploits a fast model parameters calculation method. However, it can only deal with balanced voltage conditions. Using the dynamic phasor (DP) method, a switching function model of DBR in $dq$ frame was proposed in [3]. In this model, AC-side inductances were modelled by adding an imaginary resistor to the DC-side circuit, but the resistor was calculated for balanced voltages. In [4, 5], switching instants of DBR were calculated by solving second-order differential equations for various commutation subintervals. The need for solving the equations for several subinterval equivalent circuits increases the complexity of the analyses. In [6], based on circuit equations, an analytical model of DBR was proposed for a slightly unbalanced grid. However, AC source inductances were neglected. The presence of AC source inductances prevents the instantaneous transfer of current between diode-bridge legs leading to commutation intervals between diodes. This paper illustrates that, in unbalanced voltage conditions, durations of commutation intervals is unique for each pair of commutating diodes.

Therefore, variable switching instants and unequal commutation intervals are two consequences of unbalanced voltage conditions. Accordingly, with the aim of extending the switching function method to unbalanced conditions, this paper proposes a new set of switching functions with the ability to dynamically update switching instants and take uneven commutation intervals into account.
2  Switching Functions of DBR Under Balanced Input Voltage Conditions

Using the switching function method, bidirectional relationships between AC and DC variables of DBR (Figure 1) are [7]

\[ v_{dc} = v_a \cdot S_{V,a} + v_b \cdot S_{V,b} + v_c \cdot S_{V,c} \]  \hspace{1cm} (1)

\[
\begin{bmatrix}
    i_a \\
    i_b \\
    i_c \\
\end{bmatrix}
= \begin{bmatrix}
    S_{I,a} \\
    S_{I,b} \\
    S_{I,c} \\
\end{bmatrix} \cdot i_{dc} \hspace{1cm} (2)
\]

where, \( S_{V,i} \) and \( S_{I,i} \) are voltage and current switching functions, respectively, \( v_a, v_b, \) and \( v_c \) are AC-side phase voltages, \( i_a, i_b, \) and \( i_c \) are AC-side phase currents, \( v_{dc} \) and \( i_{dc} \) are DC-side voltage and current, respectively.

Figure 2 shows piecewise switching function waveforms of phase \( a \) under balanced operating conditions. In Figure 2, \( A_a \) and \( B_a \) indicate start and end points of phase \( a \) ON-state switching interval, respectively. As can be seen, during commutation interval where three phases carry current simultaneously, values of voltage switching functions in (1) for commutating phases are 0.5. More details are addressed in [7]. Piecewise switching functions can be approximated using the Fourier series of quasi-square wave functions. In Figure 2, commutation angle \( \mu \) can be calculated as [8]

\[ \mu = \cos^{-1} \left( 1 - \frac{2 \cdot \omega \cdot I_c \cdot i_{dc}}{V_{LL}} \right) \]  \hspace{1cm} (3)

where \( \omega \) is AC-side angular frequency, \( I_c \) is commutation inductance, \( V_{LL} \) is AC-side line-to-line voltage magnitude.

Referring to Figure 2, Fourier series approximations of voltage and current switching functions are as follows:

\[ A_a = \frac{2}{n \cdot \pi} \cdot \left[ \sin (n \cdot B_a) - \sin (n \cdot A_a) \right] \]  \hspace{1cm} (4)

\[ S_{V,i} (t) = \sum_{n=1}^{\infty} \left[ A_a \cdot \cos \left( n \omega t - \frac{n \cdot \mu}{2} \right) \right] \cdot \cos \left( \frac{n \cdot \mu}{2} \right) \]  \hspace{1cm} (5)

\[ S_{I,i} (t) = \sum_{n=1}^{\infty} \left[ A_a \cdot \cos \left( n \omega t - \frac{n \cdot \mu}{2} \right) \right] \cdot \sin \left( \frac{n \cdot \mu}{2} \right) \]  \hspace{1cm} (6)

where, \( i = a, b, \) and \( c, A_a \) is coefficient of Fourier series.

For phase \( a \), under balanced input voltages, \( A_a = 300 \) and \( B_a = 60 \). Substituting these values in (4), (5), and (6), expressions of voltage and current switching functions of phase \( a \) are
obtained as follows:

\[ A_n = 4 \cdot \frac{n \cdot \pi}{\pi} \cdot \sin \left( \frac{n \cdot \pi}{2} \right) \cdot \cos \left( \frac{n \cdot \pi}{6} \right) \]  

(7)

\[ S_{V,a}(t) = \sum_{n=1}^{\infty} \left[ 4 \cdot \frac{n \cdot \pi}{\pi} \cdot \sin \left( \frac{n \cdot \pi}{2} \right) \cdot \cos \left( \frac{n \cdot \pi}{6} \right) \right] \cdot \cos \left( n \omega t - \frac{n \cdot \mu}{2} \right) \cdot \sin \left( \frac{n \mu}{2} \right) \left( \frac{\mu}{2} \right) \]  

(8)

\[ S_{L,a}(t) = \sum_{n=1}^{\infty} \left[ 4 \cdot \frac{n \cdot \pi}{\pi} \cdot \sin \left( \frac{n \cdot \pi}{2} \right) \cdot \cos \left( \frac{n \cdot \pi}{6} \right) \right] \cdot \cos \left( n \omega t - \frac{n \cdot \mu}{2} \right) \cdot \sin \left( \frac{n \mu}{2} \right) \left( \frac{\mu}{2} \right) \]  

(9)

3.1 SWITCHING FUNCTIONS UNDER UNBALANCED VOLTAGE CONDITIONS

In a three-phase system, unbalanced voltage conditions can occur under the uneven distribution of single-phase loads or faulty conditions. According to [9], the most common fault condition in a three-phase system is the single-phase fault (with 80% probability of occurrence) leading to unequal voltage magnitudes and fundamental phase angle deviation. In addition, in some applications like rotating DBR of the brushless excitation system of wound-rotor synchronous machine in motoring operation mode, input voltages of DBR are inherently unbalanced with unequal voltage magnitudes and phase angle deviation [10]. Therefore, in this paper, the general unbalanced condition is considered where both unequal voltage magnitude and fundamental phase angle deviation exist.

Fourier series coefficients of switching functions in (7), reported frequently in the literature, have been calculated based on intersection points of balanced phase voltages. Therefore, (7) is only valid for balanced voltage conditions.

As can be seen in Figure 2, under balanced conditions, switching functions are even functions. Thus, the Fourier series representation of switching functions has the following features:

\[ \begin{cases} \text{even functions with half-wave symmetry} \\
 b_n = 0 \\
 a_n = 0 \quad \text{if } n \text{ is even} \end{cases} \]

Figure 3 shows an example of the piecewise voltage switching function of phase a under a given unbalanced voltage condition, where input voltages are: \( v_a = 2 \cdot V_m \cdot \cos(\omega t) \), \( v_b = V_m \cdot \cos(\omega t - 2\cdot\pi/3) \), and \( v_c = V_m \cdot \cos(\omega t + 2\cdot\pi/3) \). As can be seen in Figure 2, the voltage switching function is neither even nor odd. Therefore, (7), (8), and (9) are not valid for this unbalanced condition.

In general, the Fourier series of switching functions under unbalanced voltage conditions has following features:

\[ \begin{cases} \text{may be neither even nor odd,} \\
 \text{always has half-wave symmetry} \\
 a_n = b_n = 0, \quad \text{if } n \text{ is even} \end{cases} \]

Therefore, under unbalanced conditions, Fourier series coefficients must be recalculated in accordance with AC-side voltages. Recalculating Fourier series coefficients requires knowledge of intersection points of phase voltages. Next section proposes a DP-based technique for online detecting of intersection points of DBR input phase voltages.

3.1 Intersection points detection of DBR input phase voltages

As illustrated in Figure 4, zero-crossing points of line-to-line voltages correspond to intersection points of phase voltages. The proposed technique exploits this relationship to detect intersection points of DBR input phase voltages. First, line-to-line input voltages are measured. Then, DP forms of line-to-line voltages are calculated. Finally, zero-crossing points of line-to-line voltages (phase angles of line-to-line voltages) are determined from their DP expression and employed for updating the Fourier series of switching functions. The extracting of phase angles of line-to-line voltages from their DP expressions is described in the next section.
3.2 Dynamic phasor modelling of varying frequency waveforms

DPs are time-varying Fourier series coefficients of a quasi-periodic signal. As a result, each DP is defined as the $k^{\text{th}}$ Fourier series coefficient of the original signal. For signal $x(\tau)$ in complex Fourier series form, DPs are defined as follows:

$$\langle X_k \rangle = X_k(t) = \frac{1}{T} \int_{t-T}^{t} x(\tau) \cdot e^{-j\omega \tau} \cdot d\tau$$

(10)

where $k$ is the order of DP. $T$ and $\omega$ are fundamental period and frequency of the signal, respectively.

Using (10), first-order time-varying phasors of line-to-line voltages are

$$\begin{align*}
\langle v_{ab} \rangle & = V_{ab} \cdot e^{j\varphi_{ab}} \\
\langle v_{bc} \rangle & = V_{bc} \cdot e^{j\varphi_{bc}} \\
\langle v_{ca} \rangle & = V_{ca} \cdot e^{j\varphi_{ca}}
\end{align*}$$

(11)

where $V_{ab}$, $V_{bc}$, and $V_{ca}$ are magnitudes of three-phase line-to-line voltages. $\varphi_{ab}$, $\varphi_{bc}$, and $\varphi_{ca}$ are phase angles of line-to-line voltages.

In fact, DPs of line-to-line voltages are complex numbers with modulus and argument parts. Argument parts of DPs are phase angles of line-to-line voltages, which are zero-crossing points of line-to-line voltages (intersection points of DBR input phase voltages).

Exploiting the symmetry of three-phase switching functions, intersection points of phase input voltages of DBR for phase $a$, $b$, and $c$ can be expressed as

$$\begin{align*}
A_a & = \frac{\pi}{2} - \vartheta_{aa} \\
B_a & = \frac{\pi}{2} - \vartheta_{ab} \\
A_b & = B_a \\
B_b & = \frac{\pi}{2} - \vartheta_{bc} \\
A_c & = B_b \\
B_c & = A_a
\end{align*}$$

(12)

Since line-to-line voltages in the DP-domain have a 90-phase shift with respect to their actual waveforms, the term $\pi/2$ is included in the above relationships. As an illustration, consider the case wherein phase voltages are Cosine functions, so line voltages become Sine functions, but in the DP-domain, line voltages are expressed as Cosine. Therefore, adding $\pi/2$ makes the proposed approach generic for all forms of input voltages.

Using (12), under any unbalanced voltage condition, Fourier series of three-phase switching functions can be updated in accordance with input voltages.

3.3 Uneven commutation intervals under unbalanced voltage conditions

This section introduces uneven commutation intervals phenomenon happening under unbalanced voltage conditions and proposes equations for calculating and taking uneven commutation intervals into account. Under unbalanced voltage conditions, since magnitudes of phase voltages may be unequal, the commutation angle must be calculated for each combination of two commutating phases as follows:

$$\begin{align*}
\mu_{ab} & = \cos^{-1} \left( 1 - \frac{2 \cdot \omega \cdot L_c \cdot i_{dc}}{V_{ab}} \right) \\
\mu_{bc} & = \cos^{-1} \left( 1 - \frac{2 \cdot \omega \cdot L_c \cdot i_{dc}}{V_{bc}} \right) \\
\mu_{ca} & = \cos^{-1} \left( 1 - \frac{2 \cdot \omega \cdot L_c \cdot i_{dc}}{V_{ca}} \right)
\end{align*}$$

(13)

where $\mu_{ij}$ ($i,j = a, b,$ and $c$) is the commutation angle between phase $i$ and $j$.

3.4 Updating switching functions

Taking unequal commutation angles into account, for phase $a$, Fourier series coefficients of the voltage switching function $\langle A_n \rangle$ and $B_n$ are calculated as

$$\begin{align*}
A_n & = \frac{1}{n \cdot \pi} \cdot [\sin(n \cdot B_a) - \sin(n \cdot A_a) \\
+ \sin(n \cdot (B_b + \mu_{ab})) - \sin(n \cdot (A_a + \mu_{ca}))]
\end{align*}$$

(14)
\[ B_n = \frac{1}{n \cdot \pi} \cdot \left[ \cos(n \cdot A_i) - \cos(n \cdot B_i) \right. \\
+ \cos(n \cdot (A_i + \mu_{ab})) - \cos(n \cdot (B_i + \mu_{ab})) \right] \]  

(15)

Fourier series coefficients of the current switching function \((A_{n,i} \text{ and } B_{n,i})\) of phase \(a\) are calculated as

\[
A_n = \frac{2}{n \cdot \pi} \left[ \sin \left( n \cdot (B_n + \mu_{ab}) \right) \cdot \sin \left( \frac{n \cdot \mu_{ab}}{2} \right) \\
- \sin \left( n \cdot (A_n + \mu_{ab}) \right) \cdot \sin \left( \frac{n \cdot \mu_{ab}}{2} \right) \right] 
\]

(16)

\[
B_n = \frac{2}{n \cdot \pi} \left[ \cos \left( n \cdot (A_n + \mu_{ab}) \right) \cdot \sin \left( \frac{n \cdot \mu_{ab}}{2} \right) \\
- \cos \left( n \cdot (B_n + \mu_{ab}) \right) \cdot \sin \left( \frac{n \cdot \mu_{ab}}{2} \right) \right] 
\]

(17)

As commutation angles are already included in Fourier series coefficients (in (14) to (17)), Fourier series of voltage and current switching functions can be expressed as

\[
S_{V_i}(t) = \sum_{n=1}^{\infty} \left[ A_n \cdot \cos(n \omega t) + B_n \cdot \sin(n \omega t) \right] 
\]

(18)

\[
S_{I_i}(t) = \sum_{n=1}^{\infty} \left[ A_n \cdot \cos(n \omega t) + B_n \cdot \sin(n \omega t) \right] 
\]

(19)

For phase \(b\), Fourier series coefficients of voltage and current switching functions can be obtained from (14), (15), (16), and (17) by replacing \(A_a, B_a, \mu_{ab}\) and \(\mu_{ca}\) with \(A_b, B_b, \mu_{ba}\), and \(\mu_{ab}\), respectively. For phase \(c\), in (14), (15), (16), and (17), \(A_a, B_a, \mu_{ab}\) and \(\mu_{ca}\) must be replaced by \(A_c, B_c, \mu_{ca}\) and \(\mu_{bc}\), respectively.

4 CASE STUDY AND MODEL VALIDATION UNDER AN UNBALANCED VOLTAGE CONDITION

To analyze the performance and accuracy of the proposed DBR model, computer simulations are carried out under unbalanced and fault operation conditions by using a detailed model of DBR in Matlab/Simulink. In the simulation, input voltages of the rectifier are

\[
\begin{align*}
  v_a &= 10 \cdot \cos \left( \omega t + \frac{\pi}{6} \right) \\
  v_b &= 18 \cdot \cos \left( \omega t - \frac{2 \cdot \pi}{3} \right), \quad \omega = 400 \cdot 2 \cdot \pi \\
  v_c &= 15 \cdot \cos \left( \omega t + \frac{\pi}{2} \right)
\end{align*}
\]

(20)
To verify inclusiveness of the proposed model, a severe unbalanced voltage condition with unequal voltage magnitudes and phase angle deviations is considered. At $t = 0.2$ s open-circuit fault occurs in phase $b$. In the simulation, $L_c$ is 20 μH and DC load consists of 1 Ω resistor in series with 8 mH inductor. Waveforms of voltage switching functions of all three phases and current switching function of phase $a$ are shown in Figure 5(a). Twenty-five Fourier series components have been included in the Fourier series approximation of switching functions. As can be seen, commutation intervals are not equal. For instance, the commutation interval between phase $b$ and phase $c$ is shorter than the commutation interval between phase $a$ and phase $c$. One cycle delay which arises from updating of intersection points of phase voltages can be seen in Figure 5(a) to (d). The reason is that after the fault occurrence, intersections between phases changed and consequently the switching functions must be updated. Since intersection angles are calculated from line voltages in DP-domain, and in fact, DP is one period average of a signal, one period delay is imposed. The open-circuit fault occurs at $t = 0.2$ s, and at $t = 0.2025$ s (one time period of input voltages) switching functions were updated.

Figure 5(b) and (c) show rectifier output voltage and current, simulated by the detailed model and the proposed model. The estimated current of phase $a$ in comparison with detailed model is shown in Figure 5(d). From Figure 5, it can be seen that results obtained by the proposed model accurately trace waveforms obtained from the detailed model.

5 | CONCLUSION

Unlike the balanced voltage condition where uncontrolled switching instants of DBR diodes are fixed, under unbalanced voltage conditions, switching instants change with respect to input voltages. Moreover, if magnitudes of unbalanced input voltages are unequal, durations of commutation intervals between diodes will be non-identical and differ from their values under balanced voltage conditions. Taking these two differences between balanced and unbalanced voltage conditions into account, this paper proposed DBR switching functions updating continuously in accordance with input voltages. Therefore, proposed switching functions can deal with unbalanced input voltages. The proposed method can be applied to other DBR systems with higher pulse numbers.

REFERENCES

1. Sun, Y., et al.: Frequency-domain harmonic matrix model for three-phase diode-bridge rectifier. IET Gener. Transm. Distrib. 10(7), 1605–1614 (2016)
2. Zhai, H., et al.: Fast calculation method for rectifier matrix model and its application in optimised control of SAPF for network-wide harmonic suppression. IET Gener. Transm. Distrib. 12(8), 1897–1905 (2018)
3. Yang, T., et al.: Fast functional modelling of diode-bridge rectifier using dynamic phasors. IET Power Electron. 8(6), 947–956 (2015)
4. Mayordomo, J.G., et al.: An analytical procedure for calculating harmonics of three-phase uncontrolled rectifiers under nonideal conditions. IEEE Trans. Power Delivery 30(1), 144–152 (2015)
5. Mayordomo, J.G., et al.: A detailed procedure for harmonic analysis of three-phase diode rectifiers under discontinuous conduction mode and nonideal conditions. IEEE Trans. Power Delivery 33(2), 741–751 (2018)
6. Fang, Z., et al.: Performance analysis and capacitor design of three-phase uncontrolled rectifier in slightly unbalanced grid. IET Power Electron. 8(9), 1429–1439 (2015)
7. Pejovic, P., Three-Phase Diode Rectifiers with Low Harmonics-, Springer, US. Power Electronics and Power Systems, 1 , 318pp. (2007) https://www.springer.com/gp/book/9780387293103
8. Ren, Z., et al.: Performance of homopolar inductor alternator with diode-bridge rectifier and capacitive load. IEEE Trans. Ind. Electron. 60(11), 4891–4902 (2013)
9. Bollen, M.H., Voltage sags characterization, In:Understanding Power Quality Problems: Voltage Sags and Interruptions, , pp. 139–251 IEEE, Piscataway, NJ (2000)
10. Deriszadeh, A., et al.: Excitation procedure for brushless wound-rotor synchronous starter generator with seamless transitions. IET Power Electron. 12(11), 2873–2883 (2019)

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