Differential Game of Pursuit Time Satisfy the Geometric Constraints in $l_2$ Space

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Abstract. In the present article, we present a differential game of pursuit problem with the case of geometric constraint in the Hilbert space $l_2$. The game is given by system of 2-infinite systems of first order ordinary differential equations (ODEs). Geometric constraint are imposed on the control functions of players. The game is began from a given point $z^0$ called the initial position. It is given another point $z^1$ in the space $l_2$. The Pursuer targeting to bring the state of the system from $z^0$ to $z^1$ where an equation to find a guaranteed pursuit time is obtained while that of the Evader action is opposite. The game is assumed to be completed if $z(t) = z^1$ at some time $t$. Moreover, a control problem is studied and then extended to the differential game of pursuit where the strategy for the Pursuer is constructed explicitly.

1. Introduction

Before going into the literature deeply first must acknowledge the present of famous books by famous mathematicians in the theory of differential games these includes the books of [1], [2], [3], [4], [5], they analyzed the real life conflict problems and lays the strong mathematical foundation for the theory of differential games.

Since then many works by various authors with different approaches had been reviewed and published in advancing the area of differential games for an ordinary differential equations. Moreover, the control functions of the players in the differential games problem are usually subjected to either geometric, integral or both constraints. For examples the work of Chernous’ ko [6], [7], [8], [9], [10], [11], [12].

Decomposition method is among the popular method in studying control and the theory differential game problems in systems of differential equations with distributed parameters where the problems can be reduced to the system of infinite system ODEs such as the research work of [7], [8], [9], [10], [11], [12] [13], [14], [15].

In the work of [11] examined a game of pursuit problem for which the first player tries forward the state of the system from given point into another point in some sense while the second player tries to stop this. The system is described by partial differential equations and the control parameters of the game problem are defined to the right hand side of the equation are in additive form. Therefore, decomposition method and some ideas from research paper of
were imposed to get the different forms of sufficient condition. The method imposed to reduce to infinite system of ODEs was based on Fourier method. To solve the problem, the control functions of the players were imposed on various forms of constraints to complete the game for finite time.

Thus, there is a good relationship between the control problems described by PDE and those described by infinite systems of differential equations. This open up the new prospects ideas to study problem of differential game for infinite system of first order ordinary differential equations and can be studied with different method independent of those that studied by PDE. For example, the work of [16], [17], [18], [19], [20], [21], [22], [23].

In the paper [17], the problem was reduced to an infinite system of differential equation described by infinite system of ODEs. The goal of the Pursuer is to bring the system into zeroth state i.e. $z(t) = 0$, while the Evader strives to prevent this. In game, the control functions of the players satisfied the integral constraints with

$$
\|u(\cdot)\|_I \leq \rho, \quad \|v(\cdot)\|_I \leq \sigma.
$$

Also, in the work of [24], studied two-person zero-sum differential game of pursuit-evasion problem described by the following infinite systems in the Hilbert space $l_2$.

$$
\begin{aligned}
\dot{x}_k &= -\alpha_k x_k - \beta_k y_k + u_{1k} - v_{1k}, \quad x_k(0) = x_{k0} \\
\dot{y}_k &= \beta_k x_k - \alpha_k y_k + u_{2k} - v_{2k}, \quad y_k(0) = y_{k0}
\end{aligned}
$$

where, $\alpha_k, \beta_k$ are real numbers, $\alpha_k \geq 0$ with initial state $x^0 = (x_1^0, x_2^0, \ldots) \in l_2$, $y^0 = (y_1^0, y_2^0, \ldots) \in l_2$, $u = (u_{11}, u_{12}, u_{21}, u_{22}, \ldots)$ is control parameters of the Pursuer and $v = (v_{11}, v_{12}, v_{21}, v_{22}, \ldots)$ is that of the Evader in Hilbert space $l_2$ defined by

$$
l_2 = \left\{ \zeta = (\zeta_1, \zeta_2, \ldots) | \sum_{k=1}^{\infty} |\zeta_k|^2 < \infty \right\}, \quad \zeta_k \in \mathbb{R}^2
$$

with inner product and norm as follows

$$
\langle \zeta, \eta \rangle = \sum_{k=1}^{\infty} \zeta_k \eta_k, \quad \zeta, \eta \in l_2, \quad ||\zeta|| = \left( \sum_{k=1}^{\infty} |\zeta_k|^2 \right)^{1/2}.
$$

In the game, Pursuer working to bring the state of the system towards the origin of the space $l_2$, and the Evader actions to stop this. Control functions of the players satisfied integral constraints where an optimal pursuit time was studied and optimal strategies for the players is constructed in an explicit form.

Guaranteed pursuit time was obtained by [25] for the game of two players with different target described by system of infinite differential equations (1) in the space $l_2$, when the controls functions of the players are subjected to geometric constraints. The target of the first player is to bring the state of the system towards the origin against the actions of the second player tries avoid this. In the game, the strategy of the first player happen to be a Pursuer is constructed in an explicit form.

Likewise in the paper [26] differential game of pursuit time was considered for the system (1) in Hilbert space $l_2$ where integral constraints are imposed on the control functions of the players. The present paper is in similar approach to that of [26] but with different case of constraints, i.e, geometric constraint.
2. Statement of Problem

In the space $l_2$, we study a differential game of two players i.e., Pursuer and the Evader described by the following system of 2-infinite systems of first order ODEs

$$\begin{align*}
\dot{x}_k &= -\alpha_k x_k - \beta_k y_k - u_{1k} + v_{1k}, \quad x_k(0) = x^0_k, \quad k = 1, 2, ..., \\
\dot{y}_k &= \beta_k x_k - \alpha_k y_k - u_{2k} + v_{2k}, \quad y_k(0) = y^0_k,
\end{align*}$$

(2)

with initial state $x^0 = (x^0_1, x^0_2, ...) \in l_2$, $y^0 = (y^0_1, y^0_2, ...) \in l_2$, where $\alpha_k, \beta_k$ are real numbers and $\alpha_k > 0$, $k = 1, 2, ...$, $u = (u_{11}, u_{12}, u_{21}, u_{22}, ...)$ is control parameters of the first player and $v = (v_{11}, v_{12}, v_{21}, v_{22}, ...)$ to be of second one.

Let $x^1 = (x^1_1, x^1_2, ...) \in l_2$, $y^1 = (y^1_1, y^1_2, ...) \in l_2$ be another state, and let $T$ be a sufficiently large positive number.

**Definition 2.1** A function $w(\cdot), w: [0, T] \to l_2$, with measurable coordinates $w_{1k}(t), w_{2k}(t), 0 \leq t \leq T, k = 1, 2, \ldots$, so that $w(\cdot) = (w_{11}(\cdot), w_{21}(\cdot), w_{12}(\cdot), w_{22}(\cdot), ...)$, subject to

$$\sum_{k=1}^{\infty} \left( w_{1k}^2(s) + w_{2k}^2(s) \right) ds \leq \rho_0^2,$$

is called admissible control, where $\rho_0$ is a given positive number. Denote $S(\rho_0)$ to be the set of all admissible controls.

Let $C(0, T; l_2)$ denote the space of continuous functions $z(\cdot) = (z_1(\cdot), z_2(\cdot), ...)$ with values $z(t) \in l_2$, $0 \leq t \leq T$. If $w(\cdot)$ belong to the set $S(\rho_0)$, then the following system of 2-infinite system of first order ODEs

$$\begin{align*}
\dot{x}_k &= -\alpha_k x_k - \beta_k y_k + w_{1k}, \quad x_k(0) = x^0_k, \quad k = 1, 2, ..., \\
\dot{y}_k &= \beta_k x_k - \alpha_k y_k + w_{2k}, \quad y_k(0) = y^0_k,
\end{align*}$$

(3)

where $w = (w_{11}, w_{21}, w_{12}, w_{22}, ...)$ is control parameter for the player, has a unique solution of the form $z(t) = (z_1(t), z_2(t), ...), [0, T]$ in $C(0, T; l_2)$ [27], e.i.,

$$z_k(t) = Q_k(t) \left( z^0_k + \int_0^t Q_k(-s) w_k(s) ds \right), \quad k = 1, 2, \ldots$$

(4)

and $Q_k(t), k = 1, 2, ...$ can be defined as

$$Q_k(t) = \begin{bmatrix}
-e^{-\alpha_k t} \cos(\beta_k t) & -e^{-\alpha_k t} \sin(\beta_k t) \\
e^{-\alpha_k t} \sin(\beta_k t) & e^{-\alpha_k t} \cos(\beta_k t)
\end{bmatrix}, \quad k = 1, 2, \ldots$$

The matrix $Q_k(t)$ possesses the following properties

i. $Q_k(h + t) = Q_k(t)Q_k(h) = Q_k(h)Q_k(t),$

ii. $|Q_k(t)z_k| = |Q_k(t)z_k| = e^{-\alpha_k t}|z_k|$, 

iii. $|Q_k(t)Q^*_k(t)z_k| = |Q^*_k(t)Q_k(t)z_k| = e^{-2\alpha_k t}|z_k|$, 

where $Q^*$ is the transpose of matrix $Q$.

**Definition 2.2** A function $u(\cdot) \in S(\rho)$, where $\rho$ is a given positive numbers is called an admissible controls of the Pursuer and $v(\cdot) \in S(\sigma)$, where $\sigma$ is given positive numbers be that of the Evader.
Consider the following equation

\[ \theta > t > \theta \]

to complete the game for the time say \( l \).

In this section, we study a differential game of pursuit in 3-infinite system of first order ODEs (2), and construct the strategy of the Pursuer.

2. Differential Game of Pursuit Time

The system of 2-infinite system of first order ODEs can be transform into a system of infinite first order ODEs as

\[ z(t) = (z_1(t), z_2(t), ...), \quad |z_k| = \sqrt{x_k^2 + y_k^2}, \quad z_k(t) = (x_k(t), y_k(t)), \]

\[ z^0 = (z^0_1, z^0_2, ...), \quad z^1 = (z^1_1, z^1_2, ...). \]

The following statement can be proved similar to Lemma 1 in [21].

Definition 2.3 The strategy of Pursuer is defined as a function in the form

\[ u(t, v) = (u_1(t, v), u_2(t, v), ...), \quad u : [0, T] \times l_2 \to l_2 \]

whose components \( u_k = (u_{1k}, u_{2k}) \) has the form

\[ u_k(t, v) = v_k(t) + w_k(t), \quad w_k = (w_{k1}, w_{k2}), \quad v_k = (v_{1k}, v_{2k}), \]

for which the system (2) has a unique solution at \( u(t) = u(t, v) \), where \( v(\cdot) = (v_1(\cdot), v_2(\cdot), ...) \) is any admissible control of the second player i.e, Evader and \( w(\cdot) = (w_1(\cdot), w_2(\cdot), ...) \in S(\rho - \sigma) \) is any function.

The strategy of Pursuer is defined as a function in the form

\[ u(t, v) = (u_1(t, v), u_2(t, v), ...), \quad u : [0, T] \times l_2 \to l_2 \]

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for which the system (2) has a unique solution at \( u(t) = u(t, v) \), where \( v(\cdot) = (v_1(\cdot), v_2(\cdot), ...) \) is any admissible control of the second player i.e, Evader and \( w(\cdot) = (w_1(\cdot), w_2(\cdot), ...) \in S(\rho - \sigma) \) is any function.

The system of 2-infinite system of first order ODEs can be transform into a system of infinite first order ODEs as

\[ z(t) = (z_1(t), z_2(t), ...), \quad |z_k| = \sqrt{x_k^2 + y_k^2}, \quad z_k(t) = (x_k(t), y_k(t)), \]

\[ z^0 = (z^0_1, z^0_2, ...), \quad z^1 = (z^1_1, z^1_2, ...). \]

Definition 2.4 Generally, pursuit can be completed in the game (2) for the time say \( \theta > 0 \) if for any given strategy of Pursuer \( u(t, v) \) and for any given admissible control \( v(t) \) of Evader, the equality \( z(\tau) = z^1 \) at some \( \tau, 0 \leq \tau \leq \theta \) satisfied. The number \( \theta \) is also called a guaranteed pursuit time.

Research Problem 2.5 Find an equation for guaranteed pursuit time in differential game of 2-infinite system of first order ODEs (2), and construct the strategy of the Pursuer.

3. Differential Game of Pursuit Time

In this section, we study a differential game of pursuit in \( l_2 \) space in which the first player tries to complete the game for the time say \( \theta > 0 \) and the second player actions is opposite.

Consider the following equation

\[ E(t) = \sum_{k=1}^{\infty} (2|z_k^0|^2 e^{2\alpha_k t} B_k^2(t) + 2|z_k^1|^2 A_k^2(t)) = (\rho - \sigma)^2, \quad t > 0, \quad z^0, z^1 \in l_2, \]

where \( B_k(t) = \frac{2\alpha_k}{e^{2\alpha_k t} - 1}, \quad A_k(t) = -B_k(-t), \quad t > 0, \quad k = 1, 2, ..., \)

The following statement can be proved similar to Lemma 1 in [21].

Lemma 3.1 Let \( z^0, z^1 \in l_2 \), and the series

\[ \sum_{k=1}^{\infty} \alpha_k |z_k^1|^2 \]

be convergent. Then, for any \( t > 0 \), the series \( E(t) \) converges.

We can vividly see that for any \( k = 1, 2, ..., \) both the functions \( B_k^2(t), e^{2\alpha_k t} B_k^2(t) \) and \( A_k^2(t) \) are also decreasing on \( (0, +\infty) \), and approaches infinity as \( t \to 0^+ \), implies that the left hand side of equation (5) is also decreasing on \( (0, +\infty) \), and approaching infinity as \( t \) approaches \( 0^+ \). Again, \( A_k^2(t) \) approaches \( 4\alpha_k^2 \) as \( t \to +\infty \) and \( B_k^2(t), e^{2\alpha_k t} B_k^2(t) \) approaches zero as \( t \to \infty \). Therefore, the left hand side of equation (5) is approaches

\[ 4 \sum_{k=1}^{\infty} \alpha_k |z_k^1|^2, \quad \text{as} \quad t \to +\infty. \]
In view of the above information, we conclude that

$$E(t) > 4 \sum_{k=1}^{\infty} \alpha_k|z_k^1|^2, \ t > 0. \quad (7)$$

Moreover, equation (5) has the only root $t = \theta_1 > 0$ if and only if the following

$$(\rho - \sigma)^2 > 4 \sum_{k=1}^{\infty} \alpha_k|z_k^1|^2; \quad (8)$$

is satisfied and the root is unique.

**Theorem 3.2** If (8) is satisfied and $\rho > \sigma$, then $\theta_1$ is guaranteed pursuit time in the game (2).

Before proven the above theorem, first we study the following lemma of control problem for the single player of the system: such that to find the time $\theta$

$$z(0) = z^0, \ z(\theta) = z^1.$$

As stated above that

$$E(t) = \rho_0^2, \ t > 0, \quad (9)$$

has a unique root $t = \theta$ if and only if the following inequality

$$\rho_0^2 > 8 \sum_{k=1}^{\infty} \alpha_k^2|z_k^0|^2. \quad (10)$$

**Lemma 3.3** Let $z^0, z^1 \in l_2$ and (8) be satisfied. Then there is a control

$$w_k(t) = \begin{cases} Q_k^*(-t) \left[ Q_k(-\theta)z_k^1 - z_k^0 \right] B_k(\theta), & k = 1, 2, ..., \ 0 \leq t \leq \theta \\ 0, & t > \theta \end{cases} \quad (11)$$

bring the state of the 2-infinite system of first order ODEs (3) from one point $z^0$ into another point $z^1$ at the time $\theta$.

**Proof:** A. Prove that (11) is admissible. Using the properties of $Q_k(t)$, the control (11) and the clear inequality $|a - b|^2 \leq 2|a|^2 + 2|b|^2$, we have

$$\sum_{k=1}^{\infty} |w_k(t)|^2 = \sum_{k=1}^{\infty} Q_k^*(-t) \left| Q_k(-\theta)z_k^1 - z_k^0 \right|^2 B_k^2(\theta) \leq \sum_{k=1}^{\infty} e^{2\alpha_k t} \left( 2e^{2\alpha_k \theta}|z_k^1|^2 + 2|z_k^0|^2 \right) B_k^2(\theta) \leq \sum_{k=1}^{\infty} \left( 2|z_k^0|^2e^{2\alpha_k \theta}B_k^2(\theta) + 2|z_k^1|^2A_k^2(\theta) \right) = E(\theta) = \rho_0^2.$$  

Thus, (11) is admissible.
B. Next, is to steer the state of the system from one point into another point i.e., \( z(\theta) = z^1 \). We have,

\[
z_k(\theta) = Q_k(\theta) \left( z_k^0 + \int_0^\theta Q_k(-s) \left( Q_k^*(-s) \left[ Q_k(-\theta) z_k^1 - z_k^0 \right] B_k(\theta) \right) ds \right)
\]

\[
= Q_k(\theta) \left( z_k^0 + (Q_k(-\theta) z_k^1 - z_k^0) B_k(\theta) \int_0^\theta e^{2\alpha s} ds \right)
\]

\[
= Q_k(\theta) z_k^0 + Q_k(\theta) (Q_k(-\theta) z_k^1 - z_k^0) = z_k^1.
\]

Thus, the system \( z(t) \) can be transferred from \( z^0 \) to \( z^1 \) for the time \( \theta \). The proof of Lemma 3.3
is complete.

Next, control problem can be extended to a differential game problem by introducing control function of the second player to the game model, i.e., Pursuer and Evader.

**Proof: A.** To prove the theorem of differential game, first we need to construct the strategy of the first player on \([0, \theta_1] \) as

\[
u_k(t) - w_k(t), \quad \nu_k(t) = Q_k^*(-t) \left[ Q_k(-\theta_1) z_k^1 - z_k^0 \right] B_k(\theta_1), \quad \theta_1, k = 1, 2, ...
\]

\[
0, \quad t > \theta_1.
\]

It is not difficult to verify that the system (2) has the solution

\[
z_k(t) = Q_k(t) \left( z_k^0 - \int_0^t Q_k(-s) u_k(s) ds + \int_0^t Q_k(-s) v_k(s) ds \right).
\]

(13)

**B.** Show that pursuit is completed. Using (13) and strategy (12), we obtain

\[
z_k(\theta_1) = Q_k(\theta_1) \left( z_k^0 + \int_0^{\theta_1} Q_k(-s) v_k(s) ds \right.
\]

\[
- \int_0^{\theta_1} Q_k(-s) (v_k(s) - Q_k^*(-s) \left[ Q_k(-\theta_1) z_k^1 - z_k^0 \right] B_k(\theta_1)) ds \right)
\]

\[
= Q_k(\theta_1) \left( z_k^0 + \left[ Q_k(-\theta_1) z_k^1 - z_k^0 \right] B_k(\theta_1) \int_0^{\theta_1} e^{2\alpha s} ds \right)
\]

\[
= Q_k(\theta_1) z_k^0 + Q_k(\theta_1) \left[ Q_k(-\theta_1) z_k^1 - z_k^0 \right] = z_k^1.
\]

**B.** To complete the prove, we must show that strategy (12) is admissible. Using the Minkowskii inequality and recall that \( v(\cdot) \) belongs to set \( S(\sigma) \), we get
\[
\left( \sum_{k=1}^{\infty} |u_k(t)|^2 \right)^{1/2} = \left( \sum_{k=1}^{\infty} |v_k(t) - Q_k^t(-t) [Q_k(-\theta_1)z_k^1 - z_k^0] B_k(\theta_1)|^2 \right)^{1/2} \\
\leq \left( \sum_{k=1}^{\infty} |v_k(t)|^2 \right)^{1/2} \\
+ \left( \sum_{k=1}^{\infty} |Q_k^t(-t) [Q_k(-\theta_1)z_k^1 - z_k^0] B_k(\theta_1)|^2 \right)^{1/2} \\
\leq \sigma + \left( \sum_{k=1}^{\infty} e^{2\alpha_k t} |Q_k(-\theta_1)z_k^1 - z_k^0|^2 B_k^2(\theta_1) \right)^{1/2}.
\]

Now, we imposed the true inequality \(|a - b|^2 \leq 2|a|^2 + 2|b|^2\) in (14), we have

\[
\left( \sum_{k=1}^{\infty} |u_k(t)|^2 \right)^{1/2} \leq \sigma + \left( \sum_{k=1}^{\infty} e^{2\alpha_k t} \left( 2|z_k^0|^2 + 2|z_k^1|^2 e^{2\alpha_k \theta_1} \right) B_k^2(\theta_1) \right)^{1/2} \\
\leq \sigma + \left( \sum_{k=1}^{\infty} \left( 2e^{2\alpha_k \theta_1} |z_k^0|^2 B_k(\theta_1) + 2|z_k^1|^2 A_k(\theta_1) \right) \right)^{1/2} \\
= \sigma + \rho - \sigma = \rho.
\]

Hence, this complete proof of the theorem.

4. Conclusion
In the present paper, a pursuit differential game problem described by the system of 2-infinite systems of first order ODEs has been studied in a space given \(l_2\). Geometric constraints on the control functions of the players are imposed.

We have solved a control problem of forwarding the state of the 2-infinite systems of 1st-order ODEs (3) from one point \(z^0\) into another given point \(z^1\) for finite time. Also, we have obtained sufficient conditions for the completion of the game, we have given an equation for guaranteed pursuit time and constructed a strategy for the Pursuer to complete the pursuit in the game (2).

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