Large scale link based latent Dirichlet allocation for web document classification

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Abstract

In this paper we demonstrate the applicability of latent Dirichlet allocation (LDA) for classifying large Web document collections. One of our main results is a novel influence model that gives a fully generative model of the document content taking linkage into account. In our setup, topics propagate along links in such a way that linked documents directly influence the words in the linking document. As another main contribution we develop LDA specific boosting of Gibbs samplers resulting in a significant speedup in our experiments. The inferred LDA model can be applied for classification as dimensionality reduction similarly to latent semantic indexing. In addition, the model yields link weights that can be applied in algorithms to process the Web graph; as an example we deploy LDA link weights in stacked graphical learning. By using Weka’s BayesNet classifier, in terms of the AUC of classification, we achieve 4% improvement over plain LDA with BayesNet and 18% over tf.idf with SVM. Our Gibbs sampling strategies yield about 5-10 times speedup with less than 1% decrease in accuracy in terms of likelihood and AUC of classification.

Keywords: Web document classification, latent Dirichlet allocation, topic distribution

1 Introduction

In this paper we demonstrate the applicability of latent Dirichlet allocation \cite{5}, a computationally challenging but very powerful generative model for large scale Web document classification relying on hyperlinkage in addition to text content. Web content classification is a research area that abounds with opportunities for practical solutions. The performance of most traditional machine learning methods is limited by their disregard for the interconnection structure between web data instances (nodes). At the same time, relational machine learning methods often do not scale to web-sized data sets and, prior to our result, LDA models in general and in particular those that leverage on the link structure were thought to require an unfeasibly large amount of resources on the Web scale.

We apply one of the most successful generative topic models, latent Dirichlet allocation (LDA) developed by Blei, Ng and Jordan \cite{5} for Web site classification. Generative topic models \cite{11,18,5} have a wide range of applications \cite{14,20,3,29,31} in the fields of language processing, text mining and information retrieval, including categorization, keyword extraction, similarity search and statistical language modeling. An LDA model consists of latent topics described by distributions over vocabulary terms, and every term occurrence arises based on the topic distribution corresponding to the document in question. As a starting point of our results, we may use latent topics for dimensionality reduction prior to classification as already suggested but since then less explored in \cite{5}.

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Recently several models extend LDA to exploit links between web documents or scientific papers [9, 13, 12, 21]. In these models the term and topic distributions may be modified along the links. All these models have the drawback that every document is thought of either citing or cited, in other words, the citation graph is bipartite, and influence flows only from cited documents to citing ones.

In this paper we develop the linked LDA model, in which each document can cite to and be cited by others and thus be influenced and influence other documents. Linked LDA is very similar to the copycat model of Dietz, Bickel and Scheffer [12] with the main difference that in our case the citation graph is not restricted to be bipartite. This fact and its consequences are the main advantage of our model, namely, that the citation graph is homogeneous, and so one does not have to take two copies, citing and cited, of every document. In addition we give a flexible model of all possible effects, including cross-topic relations and link selection. As an example, we may model the fact that topics Business and Computer are closer to one another than to Health as well as the distinction between topically related and unrelated links such as links to software to view the content over a Health site. The model may also distinguish between sites with strong, weak or even no influence from its neighbors. The linked LDA model is described in full detail in Section 2.2.

We demonstrate the applicability of linked LDA for text categorization, an application although explicitly mentioned in [15] but, in our best knowledge, justified prior to our work only in special applications [4]. The inferred topic distributions of documents are used as features to classify the documents into categories. In the linked LDA model a weight is inferred for every link. In order to validate the applicability of these edge weights, we show that their usage improves the performance of stacked graphical classification, a meta-learning scheme introduced in [19].

The crux in the scalability of LDA for large corpora lies in the understanding of the collapsed Gibbs sampler for inference. In the first application of the Gibbs sampler to LDA [16] as well as in the fast collapsed Gibbs sampler [23] the unit of sampling or, in other terms, a transition step of the underlying Markov chain, is the redrawing of one sample for a single term occurrence. The storage space and update time of all these counters prohibit sampling for very large corpora. Since however the order of sampling is neutral, we may group occurrences of the same term in one document together. Our main idea is then to re-sample each of these term positions in one step and assign a joint storage for them. We introduce three strategies: for aggregated sampling we store a sample for each position as before but update all of them in one step for a word, for limit sampling we update a topic distribution for each distinct word instead of drawing a sample, while for sparse sampling we randomly skip some of these words. All of these methods result in a significant speedup of about 5-10 times, with less than 1% decrease in accuracy in terms of likelihood and AUC of classification. The largest corpus where we could successfully perform classification using these boostings consisted of 100k documents (that is web sites with a total of 12M pages), and altogether 1.8G term positions.

To assess the prediction power of the proposed features, we run experiments on a host-level aggregation of the .uk domain, which is publicly available through the Web Spam Challenge [6]. We perform topical classification into one of 11 top-level categories of the Open Directory (http://dmoz.org). Our techniques are evaluated along several alternatives and, in terms of the AUC measure, yield the improvement of 4% over plain LDA and 18% over tf.idf with SVM (here BayesNet is used on linked LDA based features).

The rest of the paper is organized as follows. Section 2 reviews the main concepts of LDA and then introduces our linked LDA model. Section 3 describes the experimental setup and Section 4 the results.

1.1 Related results

The use of latent topics in information retrieval tasks starts with latent semantic indexing (Deerwester, Dumais, Landauer, Furnas, Harshman [11]), a method that represents documents in a low rank approximation of the term space. Probabilistic latent semantic analysis (PLSA,
Hofmann [18]) extends this idea by defining a generative model over the latent topics that yields a term distribution for each document. As the starting point of our results, latent Dirichlet allocation (LDA, Blei, Ng, Jordan [5]) introduces additional metaparameters and sampling from Dirichlet distributions to yield a model with astonishing performance in various tasks.

We compare our linked LDA model to preexisting extensions of PLSA and LDA that jointly model text and link as well as the influence of topics along links. The first such model is PHITS defined by Cohn and Hoffman [9]. The mixed membership model of Erosheva, Fienberg and Lafferty [13] can be thought of as an LDA based version of PHITS. Common to these models is the idea to infer similar topics to documents that are jointly similar in their bag of words and link adjacency vectors. Later, several similar link based LDA models were introduced, including the copycat model, the citation influence model by Dietz, Bickel and Scheffer [12] and the link-PLSA-LDA and pairwise-link-LDA models by Nallapati, Ahmed, Xing and Cohen [21]. These results extend LDA over a bipartition of the corpus into citing and cited documents such that influence flows along links from cited to citing documents. They are shown to outperform earlier methods [12, 21]. The copycat model is very similar to linked LDA, with the only difference that in the former every document \( d \) is duplicated into a citing and a cited copy, the topics in the citing copy are drawn from \( d \)’s topic distribution, while those in the cited copy are drawn from a cited document’s topic distribution. In contrast, in linked LDA, every topic either from \( d' \) or a cited document’s topic distribution. The citation influence model is a finer version of the copycat model, in that there the citing copy’s topics are drawn either from \( d' \) or a cited document’s topic distribution. The link-PLSA-LDA and pairwise-link-LDA models differ from these in that they generate the links. We make comparisons to the link-PLSA-LDA bipartite model in this paper.

While these four models generate topical relation for hyperlinked documents, in a homogeneous corpus one has to duplicate each document and infer two models for them. This is in contrast to the linked LDA model introduced in this paper whose main advantage is that it treats citing and cited documents identically, and no duplication is needed.

As a completely different direction for link based LDA models, we mention the results [32, 33, 26] which give a generative model for the links of a network, with no words at the nodes.

We also compare the performance of our results to general classifiers aided by the Web hyperlinks. Relational learning methods (presented, for instance, in [15]) also consider existing relationships between data instances. The first relational learning method designed for topical web classification was proposed by Chakrabarti, Dom and Indyk [8] and improved by Angelova and Weikum [1]. Several subsequent results [24] and the references therein] confirm that classification performance can be significantly improved by taking into account the labels assigned to neighboring nodes. In our baseline experiments we use the most accurate hypertext classifiers [7] obtained by stacked graphical learning, a meta-learning scheme introduced by Kou and Cohen [19]. In stacked graphical learning, first a base learner is applied to the training data to produce initial predictions. Then the set of features is expanded by adding the predictions of related instances from the first step. Finally, the base learner is re-applied to the expanded feature set, resulting in a stacked model. Performance of stacked graphical learning is evaluated in Section 4.3 both with various graph based edge weights [10] and with those inferred by our linked LDA model.

Another main contribution of this paper is three new efficient inference methods. Upon introducing LDA, Blei, Ng, Jordan [5] proposed a variational algorithm for inference. Later, several other methods were described for inference in LDA, namely collapsed Gibbs sampling [16], expectation propagation, and collapsed variational inference [27]. Besides, [22] suggests methods how Gibbs sampling can be applied in a paralleled environment.

Among the above collapsed Gibbs sampling methods, fastest convergence is achieved by that of Griffiths and Steyvers [16]. To our best knowledge, prior to our result there has been one type of attempt to speed up LDA inference in general, and LDA-based Gibbs sampling in particular. Porteous, Newman, Ihler, Asuncion, Smyth, Welling [28] modify Gibbs sampling such that it gives the same distribution by using search data structure for sample updates, and
call it fast Gibbs sampler. They show significant speedup for large topic numbers.

2 LDA models and Gibbs sampling

2.1 Background

In order to prepare the necessary background and notation for our linked LDA model in the next subsection, we shortly describe the Gibbs sampling method for latent Dirichlet allocation [5]. For a detailed elaboration we refer to Heinrich [17]. We have a vocabulary \( V \) consisting of terms, a set \( T \) of \( k \) topics and \( m \) documents of arbitrary length. For every topic \( z \in T \) a distribution \( \phi_z \) on \( V \) is sampled from \( \text{Dir}(\beta) \), where \( \beta \in \mathbb{R}_+^V \) is a positive smoothing parameter. Similarly, for every document \( d \) a distribution \( \vartheta_d \) on \( T \) is sampled from \( \text{Dir}(\alpha) \), where \( \alpha \in \mathbb{R}_+^T \) is a positive smoothing parameter.

The words of the documents are drawn as follows: for every word position of document \( d \) a topic \( z \) is drawn from \( \vartheta_d \), and then a term is drawn from \( \phi_z \) and filled into that position. The notation is summarized in the widely used Bayesian network representation of LDA in Figure 1.

![Figure 1: LDA as a Bayesian network](image)

In this paper we use Gibbs sampling [16] for LDA model inference. Gibbs sampling is a Monte Carlo Markov chain algorithm for sampling from a joint distribution \( p(x) \), \( x \in \mathbb{R}^n \), if all conditional distributions \( p(x_i|x_{-i}) \) are known (\( x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) \)). In LDA the goal is to estimate the distribution \( p(z|w) \) for \( z \in TP, w \in VP \) where \( P \) denotes the set of word positions in the documents, hence Gibbs sampling makes use of the values \( p(z_i = z'|z_{-i}, w) \) for \( i \in P \). In the initialization step a random topic assignment \( z_i, i \in P \) is chosen.

Gibbs sampling for LDA has an efficiently computable closed form as deduced for example in [17]. Before describing the formula, we introduce the usual notation. We let \( d \) be a document and \( w_i \) its word at position \( i \). We also let count \( N_{dz} \) be the number of words in \( d \) with topic assignment \( z \), \( N_{zw} \) be the number of words \( w \) in the whole corpus with topic assignment \( z \), \( N_d \) be the length of document \( d \) and \( N_z \) be the number of all words with topic assignment \( z \). A superscript \( N^{-i} \) denotes that position \( i \) is excluded from the corpus when computing the corresponding count. Now the Gibbs sampling formula becomes

\[
p(z_i = z'|z_{-i}, w) \propto \frac{N_{z_i}^{-i} + \beta(w_i)}{N_{z_i}^{-i} + \sum_w \beta(w)} \frac{N_d^{-i} + \alpha(z')}{N_d^{-i} + \sum_z \alpha(z)}.
\] (1)

After a sufficient number of iterations we stop with the current topic assignment sample \( z \). From \( z \), the variables \( \varphi \) and \( \vartheta \) are estimated as

\[
\varphi_z(w) = \frac{N_{zw} + \beta(w)}{N_z + \sum_{w \in V} \beta(w)}.
\] (2)
and
\[ \vartheta_d(z) = \frac{N_{dz} + \alpha(z)}{N_d + \sum_{z \in T} \alpha(z)}. \quad (3) \]

The likelihood of an inferred LDA model on a set \( P \) of word positions in a collection of held-out documents is
\[
\prod_{i \in P} p(w_i)^{-1/|P|} \quad \text{where} \quad p(w_i) = \sum_{z \in T} \varphi_z(w_i) \vartheta_d(z), \quad (4)
\]
where \( d \) is document of position containing \( i \). Here \( \vartheta \) is a MAP estimate of the topic-distribution of the document, and is usually approximated by unseen inference.

### 2.2 Linked LDA

Next we extend latent Dirichlet allocation to model the effect of a hyperlink between two documents on topic and term distributions. The key idea, summarized as a Bayes net in Figure 2, is to modify the topic distribution of a position on the word plate based on a link from the current document on the document plate. For each position we select either an outlink or the document itself to modify the topic distribution of the original LDA model.

Formally we introduce linked LDA over the notations of the previous subsection for vocabulary \( V \), the \( k \)-element topic set \( T \) and the document set \( D \). Links are represented by a directed graph with inlinks for cited and outlinks for citing documents. Our model also relies on the LDA distributions \( \varphi_z \) and \( \vartheta_d \). We introduce an additional distribution \( \chi_d \) on the set \( S_d = \{d \text{ and its outneighbors}\} \) for every document \( d \), sampled from Dir(\( \gamma_d \)), where \( \gamma_d \) is a positive smoothing vector on \( S_d \).

As also seen in the Bayes net of Figure 2, the words of the documents are drawn as follows. For every word position \( i \) of document \( d \), we
- draw an influencing document \( r \in S_d \) from \( \chi_d \),
- draw a topic \( z \) from \( \vartheta_r \) (instead of \( \vartheta_d \) as in LDA),
- draw a term from \( \varphi_z \) and fill into the position.

![Figure 2: Linked LDA as a Bayesian network](image)

Note that for sake of a unified treatment, \( d \) itself can be an influencing document of itself. This is in contrast to the citation influence model of [12], where for every word a Bernoulli draw decides whether in the citing copy the influencing document is \( d \) itself or an outneighbor of it.

We describe the Gibbs sampling inference procedure for linked LDA. Naturally, here \( N_{dz} \) denotes the number of words with topic assignment \( z \) influenced by document \( d \), and similarly for \( N_{zw}, N_d \) and \( N_z \). Note that this document \( d \) is not necessarily the one containing word \( w \), it can be an outneighbor as well. The goal is to estimate the distribution \( p(r, z|w) \) for
The methods modify the original Gibbs sampling procedure [16]. For simplicity, we describe in this section we describe three strategies for faster inference for both LDA and linked LDA.

### 2.3 Fast Gibbs sampling heuristics

In the fast Gibbs sampling heuristics section, we discuss how to improve the efficiency of the sampling process. By modifying the sampling process, we can reduce the computational cost without significantly impacting the quality of the inference. The heuristics are designed to be particularly effective for large datasets, where traditional Gibbs sampling might be prohibitively slow.

Mathematically, the fast Gibbs sampling heuristics can be summarized as follows:

1. **Two-coordinate Sampling**: Contrast to the general Gibbs procedure which re-samples one coordinate at a time, the fast heuristics consider two coordinates simultaneously, namely the topic assignment for the current word and the topic assignment of out-neighbors.

2. **One-coordinate Sampling for Out-neighbors**: Similarly to LDA, after a sufficient number of iterations, we stop with the current topic assignment for the training documents. For an unseen document, the distribution can be estimated exactly as in (3) and (9), once we have a sample from its word topic assignment and word influencing document assignment. Similarly to LDA, unseen inference for linked LDA is the same as doing linked LDA model inference for the whole corpus (train and test corpora), in such a way that the z and r assignments for the training documents are kept throughout.

The likelihood of the inferred linked LDA model on a held-out corpus is calculated analogously as for LDA in Equation (4):

$$
\prod_{i \in P} p(w_i)^{-1/|P|} \text{ where } p(w_i) = \sum_{z \in T} \varphi_z(w_i) \vartheta_r(z) \chi_d(r).
$$

2.3 Fast Gibbs sampling heuristics

In this section we describe three strategies for faster inference for both LDA and linked LDA. The methods modify the original Gibbs sampling procedure [16]. For simplicity, we describe
them for the plain LDA setting. The speedup obtained by these boostings is evaluated in Section 4.1.

All of our methods start by sorting the words of the original documents so that sampling is performed subsequently for the occurrences of the same word. We introduce additional heuristics to compute the new samples for all occurrences of the same word in a document at once.

In **aggregated** Gibbs sampling we calculate the conditional topic distribution \( F \) as in Equation (1) for the first occurrence \( i \) of a word in a given document. Next we draw a topic from \( F \) for *every* position with the same word without recalculating \( F \) and update all counts corresponding to the same word. In this way the number of calculations of the conditional topic distributions is the number of different terms in the document instead of the length of the document, moreover, the space requirement remains unchanged. Thus the speedup is larger if there are more multiple occurrences of the words. This mostly happens for large corpora. This performance can be further improved by maintaining the aggregated topic count vector for terms with big frequency in the document, instead of the storing the topic at each word.

**Limit** Gibbs sampling heavily relies on the bag of words model assumption that the topic of a document remains unchanged by multiplying all term frequencies by a constant. In the limit hence we may maintain the calculated conditional topic distribution \( F \) for the set of all occurrences of a word, without drawing a topic for every single occurrence. Equation (1) can be adapted for this setting by a straightforward redefinition of the \( N \) counts.

It is easy to check that if the topic distributions for all positions are uniform then we get an instable fixed point of limit Gibbs sampling, provided both \( \alpha \) and \( \beta \) are constant. Clearly, with large probability, these fixed points can be avoided by selecting biased initial topic distributions. We never encountered such instable fixed points during our experiments.

Similarly to aggregated Gibbs sampling, depending on the size and term frequency distribution of the documents, limit sampling may result in compressed space usage.

**Sparse** sampling with sparsity parameter \( \ell \) is a lazy version of limit Gibbs sampling where we ignore some of the less frequent terms to achieve faster convergence on the more important ones. On every document we sample \( \text{doclength}/\ell \) times from a multinomial distribution on the distinct terms with replacement, by selecting a term by a probability proportional to its term frequency \( t_{fw} \) in the document. Hence with \( \ell = 1 \) we expect a performance similar to limit Gibbs sampling, while large \( \ell \) results in a speedup of about a factor of \( \ell \), with a trade-off of lower accuracy and slower convergence. The idea of laziness can be naturally built upon aggregated sampling alone, without limit sampling, and we will indeed evaluate this sampling (called **aggregated sparse**) in Table 3 in Section 4.3.

Aggregated Gibbs sampling has the required distribution \( p(z|w) \) as its unique stationary distribution. Indeed, it is a random walk over a finite state Markov chain which is irreducible and aperiodic as \( \alpha, \beta > 0 \), implying that it has a unique stationary distribution, which is necessarily the distribution \( p(z|w) \) by construction.

As for limit Gibbs sampling, we rely on the assumption, that in the bag of words model, multiplying all term frequencies in the corpus by a constant the semantic meaning of the documents change only moderately. As limit Gibbs sampling arises as a limit of aggregated Gibbs sampling by tending this constant to infinity, its stationary distribution is very close to what the aggregated version samples, that is, the required \( p(z|w) \). This argument is justified by our measurements in Section 4.

Laziness clearly keeps the above arguments valid, thus aggregated sparse Gibbs sampling has stationary distribution \( p(z|w) \), and sparse sampling the same as limit.

### 3 Experimental setup

#### 3.1 The data set and classifiers

In our experiments we use the 114k node host-level aggregation of the WEBSPAM-UK2007 .uk domain crawl, which consists of 12.5M pages and is publicly available through the Web Spam
Challenge [6]. We perform topical classification into one of the 14 top-level English language categories of the Open Directory (http://dmoz.org) while excluding category “World” containing non-English language documents. If a site contains a page registered in DMOZ with some top category, then we label it with that category. In case of a conflict we choose a random page of the site registered in DMOZ and label its site with its top category. In this way we could derive category label for 32k documents.

We perform the usual data cleansing steps prior to classification. After stemming by TreeTagger\(^1\) and removing stop words by the Onix list\(^2\), we aggregate the words appearing in all HTML pages of the sites to form one document per site. We discard rare terms and keep the 100k most frequent to form our vocabulary. We discard all hosts that become empty, that is those consisting solely of probably only a few rare terms. To reduce unnecessary computational load we also discard all unlabeled hosts with more than 100k remaining word occurrences. Finally we weight directed links between hosts by their multiplicity. For every site we keep only at most 50 outlinks with largest weight.

We call this the big corpus, as it contains 1.8G positions, far more than the usual corpora on which LDA experiments are carried out. Since the big corpus is infeasible for the baseline experiments, we also form a small corpus consisting of the labeled hosts only, to be able to compare our results with the baseline. We use the most frequent 20k terms as vocabulary and keep only the 10 largest weight outlink for each host. Note that even the small corpus is large enough to cause efficiency problems for the baseline classifiers.

We chose 11 out of the 14 categories to apply classification to them, as the other 3 categories were very small in our corpus. In our experiments we perform two-class classification for all of these 11 big categories. We use the machine learning toolkit Weka \(^3\) to apply SVM, C4.5 decision tree and the Bayes net implementation of Weka, called BayesNet. As for graph stacking we used a home made Java code integrated into Weka. We use 10-fold cross validation and measure performance by the average AUC over the 11 classes. Every run ((linked) LDA model build and classification) is repeated 10 times to get variance of the AUC classification performance.

### 3.2 Baseline classifiers

As the simplest baseline we use the tf.idf vectors with SVM for the small corpus, as it took a prohibitively long time to run it on the big corpus. Another baseline is to use the LDA delivered $\theta$ topic distributions as features with the classifier BayesNet.

As recently a large number of relational learning methods were invented, a complete comparison is beyond the scope of this paper. Instead we concentrate on the stacked graphical learning method \(^4\) that reaches best performance for classifying Web spam over this corpus \(^7\).

The general stacked graphical procedure starts with one of the base learners of Subsection 3.1 that classifies each element $v$ positive with weight $p(v)$. Positive and negative instances in the training set have $p(v)$ equal 0 and 1, respectively. These values are used in a classifier stacking step to form new features $f(u)$ based on certain $p(v)$. Stacking can recursively be applied, hence if in one step we consider $p(v)$ for the neighbors of $u$, then in a two-layer stacking we gather information from the distance two neighborhood.

We use cocitation to measure node similarities. The cocitation $\text{coc}(u, v)$ is defined as the number of common inneighbors of $u$ and $v$. This measure turned out most effective for Web spam classification \(^2\). We may use both the input directed graph and its transpose by changing the direction of each link. We will refer to these variants as directed and reversed versions. Notice that reversed cocitation denotes bibliographic coupling (nodes pointed to by both $u$ and $v$). Several other options to measure node similarities and form the neighborhood aggregate features are explored in \(^2\)\(^10\).

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\(^1\)http://www.ims.uni-stuttgart.de/projekte/corplex/TreeTagger/
\(^2\)http://www.lextek.com/manuals/onix/stopwords1.html
3.3 LDA inference

In our LDA inference the following parameter settings are used. The number of topics is chosen to be \( k = 30 \). The Dirichlet parameter vector \( \beta \) is constant \( 200/|V| \), and \( \alpha \) is constant \( 50/k \). For linked LDA we also consider directed links between the documents. For a document \( d \), the smoothing parameter vector \( \gamma_d \) was chosen in such a way that

\[
\gamma_d(c) \propto w(d \rightarrow c) \quad \text{for all } c \in S_d, c \neq d \text{ and}
\]

\[
\gamma_d(d) \propto 1 + \sum_{c \in S_d, c \neq d} w(d \rightarrow c)
\]

such that \( \sum_{c \in S_d} \gamma_d(c) = |d|/p \), where \( |d| \) is the number of word positions in \( d \) (the document length), and \( w(d \rightarrow c) \) denotes the multiplicity of the \( d \rightarrow c \) link in the corpus. As a quick parameter sweep, we tried three values \( p = 1, 4, 10 \) with plain Gibbs sampler and BayesNet classifier. The accuracy was 0.835, 0.852 and 0.854 resp., so we chose \( p = 10 \) in the subsequent experiments.

We take the \( \vartheta \) topic distribution vectors as features for the documents and use the classifiers of Subsection 3.1. For Gibbs sampling in LDA and linked LDA we apply the baseline as well as the aggregated, limit and sparse heuristics. Altogether this results in eight different classes of experiments.

As another independent experiment we also measure the quality of the inferred edge weight function \( \chi \). This weight can be used in the stacked graphical classification procedure of Section 3.2 over the \( \vartheta \) topic distribution feature and the tf.idf baseline classifiers, for both the link graph and its reversed version.

The same experiments are performed with the link-PLSA-LDA model \cite{21}, using the C-code provided to us by Ramesh Nallapati. We also compared the running time of our Gibbs sampling strategies with the fast Gibbs sampling method of \cite{23}, using the C-code referred to therein\cite{3}.

For results, see Subsection 4.1.

We developed an own C++-code for LDA and linked LDA containing plain Gibbs sampling and the three Gibbs sampling boostings proposed in this paper. This code is publicly available\cite{4} together with the used DMOZ labels of the .uk sites. The computations were run on Linux machines with 50GB RAM and multicore 1.8GHz AMD Opteron processors with 1MB cache.

4 Results

4.1 Speedup with the Gibbs sampling strategies

Applying aggregated, limit and sparse Gibbs samplings results in an astonishing speedup, see Tables 1\textsuperscript{2} and 2\textsuperscript{2}. Experiments were carried out on the small corpus, and the models were run with \( k = 30 \) topics.

Observe that the speedup of aggregated Gibbs sampling is striking, and this does not at all come at the expense of lower accuracy. Indeed, results of Subsections 4.2 and 4.3 show less than 1% decrease in terms of likelihood and AUC value after 50 iterations when using aggregated Gibbs sampling. The advantage of limit sampling over aggregated sampling (no need to draw from the distribution) turns out to be negligible. For sparse Gibbs sampling the speedup is a straightforward consequence of the fact that we skip many terms during sampling, and thus the running time is approximately a linear function of \( 1/\ell \) where \( \ell \) is the sparsity parameter. The constant term in this linear function is apparently approximately 75sec for LDA and linked LDA, certainly, this is the time needed to iterate through the 16G memory. As for the choice \( \ell = 10 \), note that the running time of one iteration for LDA is only 9% of the one with plain Gibbs sampling, and still, the accuracy measured in AUC is only 2% worse as seen in Table 4.

\footnote{http://www.ics.uci.edu/~iporteou/fastlda/}

\footnote{http://www.ilab.sztaki.hu/~ibiro/linkedLDA/}

\footnote{http://www.ics.uci.edu/~iporteou/fastlda/}

\footnote{http://www.ilab.sztaki.hu/~ibiro/linkedLDA/}
We point out that though we used the same C-code than [23], our measurement on fast Gibbs sampling demonstrates a somewhat poorer performance than the results presented in [23], even if one takes into account that fast Gibbs sampling is proven to have better performance with a large number of topics ($k \approx 1000$) (with $k = 100$ topics we experienced 2100 sec on average). On the other hand, fast Gibbs sampler gets faster and faster for the consecutive iterations: in our experiments with $k = 30$ topics we measured 2656 sec for the first iteration (much worse than for plain Gibbs) and 757 sec for the 50th. Thus the calculated average would be better with more iterations – to which, however, there is no real need by the observations in Subsection 4.2.

As the number of iterations with variational inference is usually chosen to be around 50, the same as for Gibbs sampling, we feel the above running times for Gibbs sampling and link-PLSA-LDA with its variational inference are comparable.

### 4.2 Likelihood and convergence of LDA inference

Figures 3-5 show the convergence of the likelihood and the AUC for BayesNet, for some combinations of LDA and linked LDA models run with various Gibbs samplers. The plots range over 50 iterations, and we have stopped inference after every 2 iterations and calculated the AUC of a BayesNet classification over the $\varphi$ features, and the likelihood (as described in Subsections 2.1 and 2.2). The experiments are run on the small corpus, the number of topics was $k = 30$ for all models, and the parameters were as described in Subsection 3.3.

The high pairwise correlation of the otherwise quite different accuracy measures, likelihood and AUC values in Figure 3 is very interesting. This is certainly due to the fact that after the topic assignment stabilizes, there is only negligible variance in the $\varphi$ features. This behavior indicates that the widely accepted method of stopping LDA iterations right after the likelihood has stabilized can be used even if the inferred variables ($\varphi$ in our case) are later input to other classification methods.

The usual choice for the number of Gibbs sampling iterations for LDA is 500-1000. Thus it is worth emphasizing that in our experiments after only 20-30 iterations both likelihood and accuracy stabilizes. This is in accordance with the similar experiments of [16, 28], which found that for plain LDA, likelihood stabilizes after 50-100 iterations, over various corpora. As a consequence, we chose 50 as the number of iterations for the next experiments.

Figure 4 demonstrates that the linked LDA model with plain, aggregated and limit samplers over-perform plain LDA by about 1% in likelihood. This gap increases to about 4% after applying classifiers to the inferred $\varphi$ topic distributions, see Table 4. The aggregated and limit

| Gibbs sampler | LDA | linked LDA |
|---------------|-----|------------|
| plain         | 1000| 1303       |
| aggregated    | 193 | 1006       |
| limit         | 190 | 970        |
| sparse ($\ell = 2$) | 135 | 402       |
| sparse ($\ell = 5$) | 105 | 241       |
| sparse ($\ell = 10$) | 91  | 171       |
| sparse ($\ell = 20$) | 84  | 129       |
| sparse ($\ell = 50$) | 80  | 107       |

Table 1: Average CPU times for one iteration of the Gibbs sampler (in secs)

| model / sampler                  | time  |
|----------------------------------|-------|
| LDA / fast Gibbs [23]           | 949   |
| link-PLSA-LDA / var. inf. [21]  | 19,826|

Table 2: Average CPU times for one iteration for baseline models and samplers (in secs)
Figure 3: Correlation of the likelihood (the lower the better) and the AUC for BayesNet (the higher the better) for three choice of model / sampler combinations.

Figure 4: Convergence of the likelihood for various models and samplers.

Gibbs boostings result in negligible deterioration in the likelihood, though they give 5 times speedup.

The observation that much fewer iterations are enough for Gibbs sampling, combined with our Gibbs boosting methods bids fair that LDA may become a computationally highly efficient latent topic model in the future.

4.3 Comparison with the baseline

We run supervised web document classification as described in Subsection 3.2. The results can be seen in Tables 3-5, the evaluation metric is AUC, averaged over the 11 big categories. See Table 3 for AUC values. The big corpus was too large for the baseline methods to terminate, so comparison with them in the small corpus can be seen in Table 4.

The tables clearly indicate that applying aggregated, limit and sparse Gibbs sampling with sparsity ℓ at most 10 has only a minor negative effect of about 2% on the classification accuracy, albeit they give significant speedup by Table 1. Linked LDA slightly outperforms the LDA-based categorization for all classifiers, by about 4%. This gap is biggest for BayesNet.

Table 5 indicates that the χ link weights delivered by the linked LDA model captures influence very well, as it improves 2% over tf.idf, 4% over LDA and 3% over linked LDA with the cocitation graph, and 3% over link-PLSA-LDA with its own χ weights. This clearly indicates
Figure 5: Convergence of the likelihood for the sparse sampler for LDA with various sparsity parameters

| model / sampler         | BayesNet | SVM  | C4.5 |
|-------------------------|----------|------|------|
| LDA                     | 0.821    | 0.710| 0.762|
| LDA / aggr.             | 0.820    | 0.684| 0.756|
| LDA / limit             | 0.810    | 0.695| 0.739|
| LDA / sparse (ℓ = 10)   | 0.788    | 0.669| 0.719|
| linked LDA              | **0.854**| **0.723**| **0.765**|
| linked LDA / aggr.      | 0.848    | 0.711| 0.754|
| linked LDA / a. sparse (ℓ = 10) | 0.837    | 0.701| 0.733|

Table 3: Big corpus. Classification accuracy measured in AUC for LDA and linked LDA under various Gibbs sampling heuristics. Sparse’ at linked LDA refers to the lazy version of the aggregated Gibbs sampler.

that the χ link weights provided by the linked LDA model are good approximation of the topical similarity along links. Reversing the graph influences behaves in a quite unpredictable way, though rev-cocit is somewhat better than cocit, furthermore, reversion worsens the AUC measure if the weights come from linked or link-PLSA-LDA χ values.

Every run (LDA model build and classification with and without graph stacking) is repeated 10 times to get variance of the AUC measure. Somewhat interestingly, this was at most 0.015 throughout, so we decided not to quote them individually.

5 Conclusion and future work

In this paper we introduced the linked LDA model which integrates the flow of influence along links into LDA in such a way that each document can be citing and cited at the same time. By our strategies to boost Gibbs sampling we were able to apply our model to supervised web document classification as a feature generation and dimensionality reduction method. In our experiments linked LDA outperformed LDA and other link based LDA models by about 4% in AUC. One of our Gibbs sampler heuristics produced 10-fold speedup with negligible deterioration in convergence, likelihood and classification accuracy. Over our data set of Web hosts, these boostings outperform the fast Gibbs sampler of [23] in speed to a great extent. We also note that our samplers use ideas orthogonal to fast Gibbs sampling [23] and the paralleled sampling of [22], and so these methods can be used in combination. It would be interesting to explore other domains than LDA where our Gibbs sampling strategies can be applied. Limit Gibbs sampling makes it possible to have arbitrary non-negative real numbers as word counts in
Table 4: Small corpus. Classification accuracy measured in AUC for LDA and linked LDA under various Gibbs sampling heuristics as well as the baseline methods.

| model / sampler          | BayesNet | SVM   | C4.5  |
|--------------------------|----------|-------|-------|
| LDA                      | 0.817    | 0.705 | 0.767 |
| LDA / aggr.              | 0.813    | 0.691 | 0.750 |
| LDA / limit              | 0.808    | 0.662 | 0.720 |
| LDA / sparse (ℓ = 2)     | 0.805    | 0.654 | 0.719 |
| LDA / sparse (ℓ = 5)     | 0.799    | 0.671 | 0.713 |
| LDA / sparse (ℓ = 10)    | 0.791    | 0.667 | 0.711 |
| LDA / sparse (ℓ = 20)    | 0.764    | 0.649 | 0.689 |
| LDA / sparse (ℓ = 50)    | 0.735    | 0.624 | 0.670 |
| linked LDA               | 0.850    | 0.709 | **0.777** |
| linked LDA / aggr        | 0.849    | 0.696 | 0.771 |
| linked LDA / limit       | 0.845    | 0.688 | 0.761 |
| linked LDA / sparse (ℓ = 2) | 0.840 | 0.683 | 0.758 |
| linked LDA / sparse (ℓ = 5) | 0.836 | 0.679 | 0.753 |
| linked LDA / sparse (ℓ = 10) | 0.827 | 0.673 | 0.751 |
| linked LDA / sparse (ℓ = 20) | 0.799 | 0.656 | 0.726 |
| linked LDA / sparse (ℓ = 50) | 0.768 | 0.630 | 0.705 |
| link-PLSA-LDA/var. inf.  | 0.827    | 0.687 | 0.754 |
| tf.idf                   | 0.569    | **0.720** | 0.565 |

a document, instead of the usual tf counts. To this end, we plan to measure whether accuracy of LDA is improved if the tf counts are replaced with the pivoted tf.idf counts of [25]. As a further research we will investigate possible application of the linked LDA model to other domains, like web spam filtering.

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| model + graph          | BNet  | C4.5   |
|------------------------|-------|--------|
| LDA + cocit            | 0.830 | 0.762  |
| LDA + rev-cocit        | 0.831 | 0.770  |
| linked LDA + $\chi$    | 0.863 | 0.785  |
| linked LDA + rev-$\chi$| 0.857 | 0.780  |
| linked LDA + cocit     | 0.838 | 0.765  |
| linked LDA + rev-cocit | 0.839 | 0.752  |
| link-PLSA-LDA + own-$\chi$ | 0.839 | 0.755 |
| link-PLSA-LDA + own-rev-$\chi$ | 0.837 | 0.755 |
| link-PLSA-LDA + cocit  | 0.835 | 0.754  |
| link-PLSA-LDA + rev-cocit | 0.840 | 0.763  |
| tf.idf + cocit         | 0.597 | 0.589  |
| tf.idf + rev-cocit     | 0.595 | 0.593  |
| tf.idf + linked LDA-$\chi$ | 0.611 | 0.601 |
| tf.idf + linked LDA-rev-$\chi$ | 0.609 | 0.598 |
| tf.idf + link-PLSA-LDA-$\chi$ | 0.604 | 0.597 |
| tf.idf + link-PLSA-LDA-rev-$\chi$ | 0.606 | 0.596 |

Table 5: Small corpus, classification with graph stacking. The base learner may be one of tf.idf, LDA, linked LDA (both without Gibbs sampling heuristics) and link-PLSA-LDA. Edge weights may arise by cocitation or the inferred $\chi$ of the linked LDA and link-PLSA-LDA models. These weights may be obtained over the reversed graph, which is indicated by rev.

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