The tune of the universe: the role of plasma in tests of strong-field gravity

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Gravitational-wave astronomy, together with precise pulsar timing and long baseline interferometry, is changing our ability to perform tests of fundamental physics with astrophysical observations. Some of these tests are based on electromagnetic probes or electrically charged bodies, and assume an empty universe. However, the cosmos is filled with plasma, a dilute medium which prevents the propagation of low-frequency, small-amplitude electromagnetic waves. We show that the plasma hinders our ability to perform some strong-field gravity tests, in particular: (i) nonlinear plasma effects dramatically quench plasma-driven superradiant instabilities; (ii) the contribution of electromagnetic emission to the inspiral of charged black hole binaries is strongly suppressed; (iii) electromagnetic-driven secondary modes, although present in the spectrum of charged black holes, are excited to negligible amplitude in the gravitational-wave ringdown signal. The last two effects are relevant also in the case of massive fields that propagate in vacuum and can jeopardize tests of modified theories of gravity containing massive degrees of freedom.

I. INTRODUCTION

A. Constraints on the charge of compact objects

Given that EM fields are ubiquitous and play a key role in most of the known universe, it is only natural to ask whether BHs or other compact objects are endowed with electric charge. Astrophysical BHs are considered to be electrically neutral due to a variety of effects, including electron-positron production and neutralization by the surrounding plasma [12–17]. However, mergers occur in violent conditions – possibly including strong magnetic fields – and sufficiently far away that one may question whether all conditions for neutrality are met [18, 19]. In addition, certain dark matter candidates are (weakly) electrically charged and can circumvent the conditions for neutralization [18].

Motivated by the potential of GW astronomy to explore these issues, the coalescence of charged BH in electrovacuum was studied in recent years, both nonlinearly [20–25] and perturbatively in the extreme mass ratio limit [26].

Neglecting environmental effects [16, 17], it was shown that GW observations of the inspiral stage [18, 27] have the potential to provide interesting constraints on the charge of BHs [18, 27]. Likewise, the ringdown stage can in principle be used to study the EM charge of the remnant: gravitational and EM modes couple, and the characteristic GW vibration modes of the system contain also a new, EM-like, family of modes, similar to the case...
of certain modified theories of gravity with extra scalar degrees of freedom nonminimally coupled to gravity \[28-32\]. Thus, it has been argued that the detection of two or more modes can in principle provide constraints on the mass, spin, and charge of the final BH \[18\].

B. Superradiance and the search for new fundamental fields

Another very concrete example of the discovery potential of BHs and GW astronomy concerns new, fundamental ultralight degrees of freedom. These would render spinning BHs unstable, and lead to a transfer of rotational energy to large-scale condensates in their surroundings; these time-varying structures would then emit quasi-monochromatic GWs, a smoking-gun for new physics \[33, 34\] (see \[9\] for a review). The mechanism at work in superradiant instabilities requires two key ingredients: an ergoregion that “forces” the field to be dragged along with the compact object, transferring energy and angular momentum to the field \[9, 35, 36\], and a massive bosonic field. The field mass effectively confines the entire setup, therefore turning an energy-extraction mechanism into an instability mechanism \[9, 37, 38\]. For a tire setup, therefore turning an energy-extraction mechanism into an instability mechanism \[9, 37, 38\]. For a BH of mass \(M\) and angular momentum \(J = \chi M^2\), and a vector field of mass \(h\mu\), such process may be very efficient \[39, 50\]; the timescale for BH spin-down— and for the build-up of a massive vector condensate— is controlled by the parameter

\[
\gamma = M \mu , \tag{1}
\]

and, in the \(\gamma \ll 1\) regime, is of the order \[9, 38, 51\]

\[
\tau \approx \frac{M}{\chi \gamma} \approx \frac{16}{\chi} \left( \frac{M}{1.0 M_\odot} \right) \left( \frac{0.01}{\gamma} \right)^7 \text{yr}. \tag{2}
\]

The instability is suppressed when the object spins down to \(\chi \sim 2\gamma\). Stringent constraints on the existence of new particles can then be imposed via (the lack of) observations of GWs emitted by the bosonic condensate that develops around the BH, “gaps” in the mass-spin plane of BHs \[33, 34\], polarimetry measurements \[52\], etc, although nonlinear photon effects such as pair production or couplings to Standard Model fields can reduce these bounds \[9, 33, 54\]. A complete review of the status of the field is discussed in Ref. \[9\].

Surprisingly, the existence of interstellar plasma permeating the universe could provide an outstanding opportunity to test the existence of ergoregions while simultaneously predicting, or explaining, new phenomena. The interaction between EM waves and ions in a plasma changes the dispersion relation and the effective equations of motion of low-frequency photons \[53, 55\]. The dispersion relation of the photon acquires an effective-mass term given by the plasma frequency \[10, 54, 57\],

\[
\omega_p = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}} = 1.8 \times 10^3 \left( \frac{n_e}{10^{-3} \text{cm}^{-3}} \right)^{1/2} \text{rad s}^{-1}, \tag{3}
\]

where \(n_e\) is the electron number density in the plasma, whereas \(m_e\) and \(e\) are the electron mass and charge, respectively, and \(\epsilon_0\) is the vacuum permeability. It is clear from this that the effective mass would be interesting from the point of view of superradiant instabilities of astrophysical systems, since the controlling parameter \(\gamma\) is appreciable \[11\],

\[
\gamma = \omega_p M \approx 0.09 \left( \frac{\omega_p}{1.8 \times 10^3 \text{Hz}} \right) \left( \frac{M}{10 M_\odot} \right), \tag{4}
\]

and therefore the associated instability timescale \[4\] is relatively small.

Superradiant instabilities in the presence of plasmas have therefore been argued to produce important signatures, including distortions in the cosmic background radiation from primordial BHs \[10\], and have been suggested as a possible explanation for Fast Radio Bursts from stellar-size BHs \[11\]. This has motivated further work on the topic, including the impact of a nonhomogeneous plasma profile around a spinning BH, which reduces the instability rate \[58\]. It has been also argued that the spin of neutron stars could be limited via the same plasma-driven mechanism \[59\], a truly tantalizing prospect to explain the systematic reduction of the spin measured in pulsars compared to the mass-shedding limit.

C. The role of plasma: shortcomings of previous analyses

Here, we argue that the aforementioned analyses have severe shortcomings, all related to the fact that they neglected some key ingredient in the photon-plasma interaction.

Constraints on the EM charge have so far neglected the fact that plasma is not transparent to low-frequency waves. The balance arguments used to understand how inspiral proceeds breakdown if EM radiation is not being transported to spatial infinity. Furthermore, previous studies on GW spectroscopy for charged BHs also overlooked environmental effects and how they impact the ringdown stage, in particular they neglected whether the plasma can affect the amplitude of EM-driven quasinormal modes.

Finally, the mechanisms leading to plasma-driven superradiant instabilities neglect backreaction and nonlinear effects. In particular, a small growing electric field will re-arrange the plasma distribution, changing its “effective mass.” In other words, the plasma is assumed to be totally opaque to such low-frequency radiation, but such assumption needs justification, especially for unstable processes in which the EM field initially grows exponentially.

Here, we wish to examine these questions more closely.
II. NONLINEAR EFFECTS MAKE PLASMA TRANSPARENT TO RADIATION

Previous studies on plasma-driven BH superradiant instabilities are based on the assumption of a fixed constant plasma density (see Ref. [58] for an extension to constant, non-homogenous density profiles with a scalar toy model) and, most notably, of weak electric fields. However, relativistic and nonlinear effects cause waves of frequency

$$\omega_p \left(1 + \frac{e^2 E^2}{m_e^2 \omega^2}\right)^{-1/2} < \omega < \omega_p,$$  \hspace{1cm} (5)

to propagate [60, 61]. Here $E$ is the amplitude of the electric field of the wave. Therefore, when the electric field is weak no frequency $\omega < \omega_p$ can propagate in the plasma, in agreement with the linear analysis. However, there is a critical electric field

$$E_{\text{crit}} = \frac{m_e}{e} \sqrt{\omega_p^2 - \omega^2},$$  \hspace{1cm} (6)

above which waves with frequency $\omega$ propagate into the plasma.

When dealing with superradiant instabilities this is a particularly important effect, because the electric field grows exponentially in the initial phase of the instability, before saturating due to nonlinear effects. Indeed, since the dominant superradiant mode has roughly $\omega \sim \omega_p (1 - \frac{1}{2} \gamma^2)$ [10, 39, 40], in the $\gamma \ll 1$ limit the critical electric field that makes the plasma transparent to this mode is

$$E_{\text{crit}}^{\text{SR}} = \frac{m_e}{e} \omega_p \gamma = 0.27 \left(\frac{n_e}{10^{-3}\text{cm}^{-3}}\right)^{1/2} \frac{\gamma}{0.09} \text{V/m}.$$  \hspace{1cm} (7)

Even in the absence of such mechanisms, the assumption that low-frequency EM waves do not propagate in the plasma breaks down when the plasma becomes hot and relativistic, i.e., when the collisional velocity $\sim c$. Since the change in the momentum of one electron over a time $\Delta t$ is $\Delta P = F \Delta t$, we get the critical electric field for this to happen $E_{\text{rel}} = m_e c / (e \Delta t)$ with $\Delta t$ the mean collision time between electrons and ions in the plasma. For relativistic electrons, $\Delta t = \ell_e / c$, with $\ell_e = n_e^{-1/3}$ being the mean separation. Therefore, neglecting nonlinear effects the critical value of the electric field above which plasma confinement breaks down reads

$$E_{\text{rel}}^{\text{SR}} = \frac{m_e n_e^{1/3}}{e} = 5 \times 10^6 \left(\frac{n_e}{10^{-3}\text{cm}^{-3}}\right)^{1/3} \text{V/m}.$$  \hspace{1cm} (8)

When the electric field grows to the above values, the plasma becomes transparent, a burst ensues, and the process starts anew. One could estimate how much energy is removed from the rotating object in each cycle. The density of energy is $\sim \epsilon_0 E^2$, and extended over a spatial distance $L \sim 5M/\gamma^2$, set by the size of the bosonic cloud [9]. Thus, the total energy in the condensate is

$$U = \epsilon_0 (E_{\text{SR}}^{\text{SR}})^2 L^3,$$  \hspace{1cm} (9)

for $E_{\text{SR}} = E_{\text{SR}}^{\text{SR}}$ or $E_{\text{SR}}^{\text{SR}} = E_{\text{rel}}^{\text{SR}}$. On the other hand, the rotational energy of a compact object is $K \sim M R^2 \Omega^2$, with $R \sim 2M$ and $\Omega \gtrsim \gamma^2$. Using Eq. (7), one finds

$$\frac{U}{K} \lesssim \frac{125 M^2 m_e n_e}{4 \gamma^6}$$  \hspace{1cm} (10)

$$\approx 9 \times 10^{-39} \left(\frac{M}{10M_\odot}\right)^2 \left(\frac{n_e}{10^{-3}\text{cm}^{-3}}\right)^2 \left(\frac{0.09}{\gamma}\right)^6.$$  \hspace{1cm} (11)

We note that, even in the absence of the above bound, the threshold (Eq. [3]) at which electrons become relativistic (thus emitting bremsstrahlung radiation and breaking all the assumptions leading to the plasma cutoff frequency) yields

$$\frac{U}{K} \lesssim \frac{125 \epsilon_0 M^2 m_e^2 n_e^{2/3}}{4 \gamma^8 e^2}$$  \hspace{1cm} (11)

$$\approx 3 \times 10^{-24} \left(\frac{M}{10M_\odot}\right)^2 \left(\frac{n_e}{10^{-3}\text{cm}^{-3}}\right)^{2/3} \left(\frac{0.09}{\gamma}\right)^8.$$  \hspace{1cm} (12)

Therefore, the above simple analysis shows that relativistic and nonlinear effects hamper dramatically the tapping of rotational energy from a spinning BH. In practice, immediately after the instability occurs the electric field in the BH surroundings becomes large enough as to render the plasma transparent to low-frequency photons, making the trapping (and, in turn, the whole instability) inefficient.

III. PLASMAS AND MERGING CHARGED BHS

As previously mentioned, several mechanisms contribute to the neutralization of BHs. For example, charged BHs are quickly discharged by Hawking radiation or by pair-production. In addition, a small amount of external plasma with total mass $M_{\text{plasma}} \sim 10^{-10} M$ is sufficient to discharge a BH on a short timescale $\tau \sim 10^{-11}$ yr [13]. Neutralization from surrounding plasma would normally screen charge on a Debye lengthscale: the charge of a BH is effectively screened on the spatial scale (exceptions to this rule may occur with stars, but not with objects without atmospheres [62])

$$\lambda_D = \sqrt{\frac{e k_B T}{n_e e^2}} = 7 \left(\frac{T}{10^9 \text{K}}\right)^{1/2} \left(\frac{10^{-3}\text{cm}^{-3}}{n_e}\right)^{1/2} \text{m}.$$  \hspace{1cm} (12)

Colder or denser media yield even tighter atmospheres. Thus, for all practical purposes BHs surrounded by plasmas are neutral.

In the eventuality that the near-horizon region is plasma-depleted and that neutralization does not occur, then two BHs can indeed be orbiting each other and emit EM radiation. However, also in this case the plasma surrounding the binary strongly hampers EM emission, as discussed in the next section.
A. The inspiral phase

The low-frequency, nearly Newtonian stage in the life of a binary of two compact objects provides stringent constraints on the gravity theory and possible new interactions [4, 63]. The underlying mechanism is as follows. Compact objects such as neutron stars or BHs on tight orbits dissipate energy mostly through GW emission. Mass loss via winds, tidal acceleration, or friction with the environment are all negligible in comparison. In these circumstances, the energy loss can be computed using numerical methods or a post-Newtonian expansion.

It is found that GW emission quickly circularizes the orbit [61]. It is thus customary to assume that the two orbiting objects are on quasi-circular orbits. To lowest order in the orbital velocity, the energy loss is given by the quadrupole formula

\[
\frac{dE_{\text{grav}}}{dt} = -\frac{32 m_1^2 m_2^2 (m_1 + m_2)}{5 r_0^5},
\]

where \( r_0 \) is the orbital radius and \( m_i \) is the mass of the \( i \)-th body. On the other hand, energy loss to radiation triggers an evolution of the binary parameters. In particular, this can be obtained using a quasi-adiabatic approximation to evolve the otherwise constants of motion such as the binary’s energy and angular momentum. Such evolution can be determined using the expression for the orbital energy

\[
E_{\text{orb}} = -\frac{m_1 m_2}{2 r_0},
\]

equating \( dE_{\text{orb}}/dt \) to \( dE_{\text{grav}}/dt \) and promoting the orbital radius \( r_0 \) to be a function of time.

New physics will impact both the emission of radiation, changing Eq. (13), and the orbital relation (14). For example, if the objects carry electric charge there is emission of mostly dipolar EM waves in addition to (mostly quadrupolar) GW emission. When the charge-to-mass ratio is small, dipolar radiation is of the order [18, 27]

\[
\frac{dE_{\text{elec}}}{dt} = -\frac{2}{3} \left( \frac{q_1}{m_1} - \frac{q_2}{m_2} \right)^2 \frac{m_1^2 m_2^2}{r_0^5},
\]

Thus, EM emission dominates at large separations and causes a distinct evolution of the orbital phase as time progresses. These effects can be used to impose stringent constraints on the charge of the inspiralling objects [18, 25, 27].

Unfortunately, the Larmor result (15) is valid in electrovacuum, but not in the presence of a plasma. It can be immediately recognized that the plasma frequency is much larger than the orbital frequency of astrophysical BHs or neutron stars. Thus, the assumption that the generated waves are able to travel freely and contribute to energy loss is incorrect. In fact, the entire calculation underlying (15), in particular the imposition of Sommerfeld conditions at infinity, is not justified.

Is it possible that, just like the superradiance clouds in the previous section, photons are still able to tunnel through via nonlinear effects? We now show that this is not possible. Take a pointlike mass \( M \), carrying charge \( Q \), in motion with acceleration \( a \). The electric field in the wave zone is of order [65]

\[
E = \frac{Q}{4\pi e_0 a} \frac{a}{r^2}.
\]

When the motion is circular and dictated by the inverse-square law, the acceleration is directly related to the inverse of the distance between the two bodies, \( a \). Thus, at small \( r \) the electric field is large and possibly larger than the critical field for transparency which, to be conservative, can be assumed to be the largest value among those discussed in the previous section, i.e. Eq. (6), \( E_{\text{crit}} \sim m_e c \omega / e \). Assume that the bodies are weakly charged, so that the acceleration is mostly provided by gravity (the calculation generalizes trivially). Taking into account the typical plasma frequency (3), one concludes that during the inspiral of astrophysical BHs, \( \omega < \omega_p \). Thus, one finds the distance \( R \) below which the electric field of the radiation is larger than \( E_{\text{crit}} \),

\[
\left( \frac{R}{a} \right)^2 \approx \delta^2 \left( \frac{M}{a} \right)^4 \frac{1}{\alpha^2 \omega_p^2}.
\]

where \( \delta < 1 \) is the BH charge-to-mass ratio. This quantity is always smaller than unity for astrophysical objects. In other words, in the radiation zone, the electric field is always sub-critical. This means that the plasma is not transparent to this radiation.

To summarize, the use of EM dipolar losses [Eq. (15)] to constrain the charge of astrophysical objects is not justified. Careful understanding of the role of plasma is necessary, but not available at this point. Indeed, the problem of a two orbiting charged particles in a plasma shares some similarities with that of two particles in a perfectly-reflecting box. In the latter case energy cannot be radiated to infinity and stationary solutions where the orbit does not shrink exist (see, e.g., Ref. [66] for the case of a binary in asymptotically anti de-Sitter spacetime). However, the case of the plasma is much more complex since: i) the reflection of the radiation is frequency dependent; and ii) as discussed before plasma is opaque only at the linear level, in reality we expect both the radiation and the binary motion to affect the plasma dynamics and profile around the binary, potentially modifying the propagation/absorption of EM waves.

B. The ringdown stage

The existence of couplings between two fields introduces mixing of modes in the ringdown. In the context of collisions of two charged BHs, gravitational and EM perturbations are coupled to each other. This leads to mode mixing in the ringdown, which is described by
gravitational-led and EM-led modes. The former (resp., latter) are those that correspond to the standard gravitational (resp., EM) modes of a Reissner-Nordstrom BH in the neutral limit ($Q \to 0$).

We illustrate this effect in Appendix A where we overview the collision of two electrically charged BHs within a perturbative approach. The effect is more pronounced when the coupling between the two sectors is large, which in the case of electrically charged BHs is related to the product of their charges. To take plasma physics into account demands a more careful analysis, as one should consider the perturbations induced by the matter surrounding each BH and possibly the binary as a whole. We are currently unable to deal with this problem in full generality. However, we are interested in understanding one particular and crucial aspect of plasma physics, which is the introduction of an effective mass term.

1. Toy model

We wish to study how does spacetime react when couplings to (possibly effective) massive fields are considered. Following the discussions presented in the previous section, this is precisely what happens when plasma is present in the surroundings of BHs. Our prototype model, which presents the main desired features, is dynamical Chern-Simons theory with a self-interacting potential \[ \beta \]. In this theory the axial gravitational perturbations couple to a (pseudo)scalar field. The action of the theory is

\[
S = \kappa \int d^4x \sqrt{-g} R + \frac{\alpha}{4} \int d^4x \sqrt{-g} \vartheta * R R
\]

where $\vartheta$ is the scalar field coupled to gravity, and

\[ * R R = \frac{1}{2} R_{abcd} \varepsilon^{bcde} R^{cd} \varepsilon_{ef} \]

is the Pontryagin term. The equation of motion can be obtained by varying the action with respect to the metric $g_{ab}$ and the scalar field $\vartheta$, namely:

\[
G_{ab} + \frac{\alpha}{\kappa} C_{ab} - \frac{1}{2\kappa} T^a_{ab} = \frac{1}{2\kappa} T_{ab},
\]

\[
\Box \vartheta - \frac{dV}{d\vartheta} = -\frac{\alpha}{4\beta} * R R,
\]

where $G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R$ is the Einstein tensor, \[ T^a_{ab} = \beta \left[ \vartheta_{,a} \vartheta_{,b} - \frac{1}{2} g_{ab} \Box \vartheta - g_{ab} V(\vartheta) \right] \], $\Box = g^{ab} \nabla_a \nabla_b$ is the D’Alembertian operator, and

\[ C^{ab} = \vartheta_{,c} \varepsilon^{cdef} (\nabla_e R^{df})_{,d} + \vartheta_{,cd} * R^{(ab)cd}, \]

where $\vartheta_{,a} = \nabla_a \vartheta$, $\vartheta_{ab} = \nabla_a \nabla_b \vartheta$, and $* R^{abef} = \frac{1}{2} \varepsilon^{abef} R^{ef}$. In the geometric units adopted so far, $\kappa = 1/16\pi$.

For concreteness, we consider the simplest potential for a massive scalar field,

\[ V(\vartheta) = \frac{\mu^2}{2} \vartheta^2, \]

where $\mu \hbar$ is the mass of the scalar.

We consider a spherically symmetric background, which is described by the Schwarzschild solution also in this theory. Gravitational perturbations on this background can be expanded in a basis of (scalar, vector, and tensor) spherical harmonics. The scalar field can be expanded in scalar spherical harmonics as follows

\[ \vartheta = \frac{\Theta}{r} Y^{lm} e^{-i\omega t}. \]

While the polar (Zerilli) sector is the same as in General Relativity and reduced to a single, second-order, radial differential equation \[ \theta(r) \leftrightarrow \theta(r) \], the axial (Regge-Wheeler) sector is coupled to scalar perturbation and reduces to the system \[ \beta \] (we omit for simplicity the indices $l$ and $m$ in the variables $\Psi$ and $\Theta$),

\[
\begin{align*}
\frac{d^2}{dr^2} \Psi + (\omega^2 - V_1) \Psi &= V_2 \Theta, \\
\frac{d^2}{dr^2} \Theta + (\omega^2 - V_2) \Theta &= V_1 \Psi,
\end{align*}
\]

where we have defined $\beta$,

\[
\begin{align*}
V_{11} &= f(r) \left[ l(l+1) - \frac{6M}{r^2} \right], \\
V_{12} &= f(r) \frac{96\pi M}{r^3}, \\
V_{21} &= f(r) \frac{6M(l+2)!}{r^3 \beta(l-2)!}, \\
V_{22} &= f(r) \left[ \frac{l(l+1)}{r^2} \left( 1 + \frac{576\pi M^2}{\beta r^6} \right) + \frac{2M}{r^3} + \mu^2 \right]
\end{align*}
\]

with $f(r) = 1 - 2M/r$, and $r_* = r + 2M \ln(r/2M - 1)$ being the tortoise coordinate.

From the above equations, one immediately sees that the scalar perturbations source the gravitational ones with a relative coupling $\propto \Theta/\Psi$, whereas the gravitational perturbations source the scalar ones with a relative coupling $\propto \Psi/(\beta \Theta)$. Therefore, when $\beta$ is small (large), the scalar field is strongly (weakly) sourced by the gravitational perturbation, while the latter depends on $\beta$ only indirectly through the value of $\Theta$.

2. Results

We have studied Eqs. (25)-(26) with two different methods. In particular, we used established techniques

\[
\begin{align*}
\frac{d^2}{dr^2} \Psi + (\omega^2 - V_1) \Psi &= V_2 \Theta, \\
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\end{align*}
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where $\vartheta_{,a} = \nabla_a \vartheta$, $\vartheta_{ab} = \nabla_a \nabla_b \vartheta$, and $* R^{abef} = \frac{1}{2} \varepsilon^{abef} R^{ef}$. In the geometric units adopted so far, $\kappa = 1/16\pi$.

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2. Results

We have studied Eqs. (25)-(26) with two different methods. In particular, we used established techniques
to look for the characteristic modes (quasinormal modes) of such a system in the frequency domain \[72, 73\]. These frequencies tell us how fluctuations decay at very late times. However, such a knowledge is insufficient, if not accompanied by the relative amplitudes of such fields. To understand this aspect, we also performed evolutions of the corresponding 1+1 partial differential equations: following Refs. \[25, 74\], we use the light-cone coordinates, \( u = t - r_a \) and \( v = t + r_a \). Then, the field equations can be written in matricial form as 
\[
\Phi = \begin{pmatrix} \Psi \\ \Theta \end{pmatrix}, \quad V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}.
\]

We consider three different initial data \( ID_{2, \Psi, \Theta} \) of the form
\[
\Phi(0, v) = \begin{pmatrix} \epsilon_\Psi e^{-(v-v_c)^2/2\sigma} \\ \epsilon_\Theta e^{-(v-v_c)^2/2\sigma} \end{pmatrix}, \quad \Phi(u, 0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix},
\]
with \( (\epsilon_\Psi, \epsilon_\Theta) = (1, 1), (1, 0), (0, 1) \) for \( ID_2, ID_\Psi, ID_\Theta \), respectively. In other words, \( ID_2 \) corresponds to both fields initially perturbed, whereas \( ID_\Psi \) and \( ID_\Theta \) correspond to only \( \Psi \) or only \( \Theta \) initially perturbed, respectively. We focus on width \( \sigma/M = 1 \) and a Gaussian located at \( v_c/M = 10 \). The ranges of \( u \) and \( v \) are both \( (0, 400M) \), and we extract the data at \( r_a = 50M \).

For \( \beta = 100 \), a frequency-domain analysis of the quadrupole modes with \( l = 2 \) and mass coupling \( M\mu = 0.1 \) indicates the presence of various characteristic modes. One is located at \( M\omega \sim 0.309 - 0.092i \) and can be identified with a gravitational-led mode. In the decoupled \( \beta \to \infty \) limit, this is lowest and dominant gravitational quasinormal mode \[72, 73\]. A second dominant mode is located at \( M\omega \sim 0.506 - 0.094i \) which in the massless decoupled limit is the lowest quasinormal frequency of a minimally coupled scalar field. A mass term gives rise to new phenomena, in particular the appearance of quasibound, Hydrogen-like states \[9, 35, 74\]. For \( \mu = 0.1 \) we find \( M\omega \sim 0.0999 - 2 \times 10^{-17}i \). In general this mode is characterized by \( \text{Re}(\omega) \sim \mu \). For \( \beta = 1 \) the modes’ frequencies change. The lowest gravitational-driven mode is at \( 0.29 - 0.098i \), while the scalar-driven goes up substantially to \( \omega \sim 1.4 - 0.14i \). The quasi-bound state remains at \( \omega \sim \mu \), with the imaginary part increasing \[74\].

To which extent are these modes excited during the evolution of our initial conditions? The outcome of the time evolutions are summarized in Fig. 1 (for \( \beta = 100 \)) and Fig. 2 (for \( \beta = 1 \)) for different mass couplings \( M\mu \).

Although we show the evolution of both the scalar and the gravitational field, for the purpose of this discussion let us focus on the gravitational sector alone, the first and third row in Figs. 1-2. The first aspect that stands out is that, in general, the new channel – the scalar – has an impact also in the gravitational sector. The evolution of the gravitational field leads to a ringdown stage which is not a simple damped sinusoid, but a superposition of at least two of the modes discussed above. Thus, the scalar quasinormal mode percolates to the gravitational sector due to the coupling. Furthermore, for massive scalar field, the very late time behavior of the gravitational sector is that of a weakly damped sinusoid controlled by the scalar quasi-bound states, ringing at a frequency \( \omega \sim \mu \) \[74\].

However, our results show that

(i) The amplitude of the scalar-driven quasinormal mode in the gravitational sector is smaller at smaller couplings (large \( \beta \)). This is also apparent from the panels in Figs. 1-2.

(ii) More importantly, at small couplings and large masses the gravitational ringdown is universal. To a good approximation it corresponds to the modes of BHs in vacuum GR. This is depicted in the third row, second column of Fig. 1. Increasing the mass term \( M\mu \) delays the appearance of the quasi-bound state dominance, where the field rings at \( \omega \sim \mu \), a clear imprint from the scalar sector in the gravitational waveform. Notice that even at large couplings this feature is present: the larger the mass \( M\mu \) the more pure and scalar-free is the gravitational waveform. As in the rest of this work, this feature arises because the scalar is unable to propagate and therefore the equations decouple in practice.

(iii) The above behavior holds well even when initially there is only a scalar field, such as in the third row, third column of Figs. 1 and 2. The “contamination” of the gravitational wave by the scalar mode is smaller at large mass couplings. In other words, for large \( M\mu \) the amplitude of the induced EM-led, gravitational mode is small and decreases when \( M\mu \) increases.

IV. CONCLUSIONS

Recent years have witnessed new developments in strong-field tests of gravity based on charged binaries \[18, 20, 27\] and on plasma-driven superradiant effects \[10, 11, 58\]. These tests either neglect the effect of plasma surrounding the binaries, or neglect nonlinear plasma-photon interactions, or anyway treat the photon-plasma coupling in a simplistic way. Unfortunately, as shown in this work, EM emission from binary coalescence, secondary EM-driven modes in the ringdown, and plasma-driven instabilities are strongly suppressed when a more rigorous treatment is performed.

We argue that previous constraints and effects should be revised on the light of our results, and urge for a more detailed treatment of the photon-plasma interactions in strongly-gravitating systems, which is unavailable at the moment. Given the variety of scales and the complex physics involved, full numerical simulations using general-relativistic magneto-hydrodynamics might be required.

Although the main focus of this work was on plasma physics, some of our results are also relevant for tests
FIG. 1. Ringdown time evolution in our toy model in which a massive scalar field $\Theta$ is coupled to axial gravitational perturbations $\Psi$. We consider $\beta = 100$ (small coupling) and the initial data is located at $v_0 = 10$. The top two rows refer to $\mu M = 0, 0.1, 0.2$ (zero or small mass term), whereas the bottom two rows correspond to $\mu M = 0.5, 1.1, 1.5$ (moderately large mass term). From left to right: ID$_2$, ID$_\Psi$, ID$_\Theta$. In all cases we set $\alpha = 1$.

of modified gravity [76]. In particular, the suppression of the extra modes in the ringdown and of the dipolar emission in the inspiral should qualitatively apply also to the case of extra fundamental massive fields that propagate in vacuum. While it is well-known that a massive field suppresses the emission of low-frequency waves (see, e.g., Ref. [77–82] for examples in scalar-tensor theory), we also predict that if the field is massive enough its modes
FIG. 2. Same as Fig. 1 but for $\beta = 1$ (stronger coupling). Notice that the high-frequency scalar-led mode is present in the gravitational signal for low-mass couplings $M\mu$ but is absent when $M\mu$ is large.

while present in the spectrum of a BH remnant cannot be sufficiently excited during the merger and are, therefore, undetectable. It is interesting that the relevant parameter for these effects is typically the coupling $\mu M$, which is huge for astrophysical BHs and the typical masses of particles in the standard model[83]. Therefore, if a putative extra fundamental field is massive (approximately with mass $\mu h \gtrsim 10^{-10}$ eV so that $\mu M \gtrsim 1$ for a stellar-mass BH) its effects in the inspiral and ringdown are strongly suppressed.
The metric, therefore, can be written as

\[ g_{ab} = g_{ab}^{(0)} + h_{ab}, \]  

with \( g_{ab}^{(0)} = \text{diag}\{-f, f^{-1}, r^2, r^2 \sin^2 \theta\} \), with \( f = 1 - 2M/r + Q^2/r^2 \), where \( M \) is the mass and \( Q \) the BH charge. The metric perturbation \( h_{ab} \) induced by the falling particle can be decomposed into spherical harmonics. We can study the perturbation in the Regge-Wheeler gauge \[68],

\[ h_{ab} = \begin{pmatrix} e^v H_0 & H_1 & 0 & 0 \\ H_1 & e^{-v} H_2 & 0 & 0 \\ 0 & 0 & r^2 K & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta K \end{pmatrix} Y_{lm}(\theta, \phi), \]  

(A2)

with \( Y_{lm}(\theta, \phi) \) being the standard spherical harmonics. The electromagnetic perturbations can be also expanded in harmonics, being described by the perturbation of the vector potential \( \delta A_a = (\delta A_0, \delta A_1, 0, 0)Y_{lm} \). Finally, one can also decompose the particle stress-energy tensor into tensorial harmonics as \[69]\n
\[ T_{ab} = \begin{pmatrix} A_0 & \frac{i}{\sqrt{2}} A_{(1)} & 0 & 0 \\ \frac{i}{\sqrt{2}} A_{(1)} & A & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} Y_{lm}, \]  

(A3)

where the radial functions \( A_0, A_{(1)}, \) and \( A \) depend on the particle’s motion. By plugging this into Einstein-Maxwell equation and upon linearization, we can find equations describing the perturbations of the metric perturbations and of the vector potential. We direct the reader to Ref. 18 for more details.

By considering plunging processes, we can investigate the waveform emitted during the collision. The waveform presents the characteristic ringdown in which the signal oscillates according to the quasinormal mode frequency of the BH decaying exponentially in time. In Fig. 3 we show the GWs emitted from the collision of a BH with charge \( Q \) with a charged particle with charge \( q = -Q \). The left panel shows the case of relatively small charges (\( Q = -q = 0.1M \)), in which the ringdown contains basically the gravitational-led fundamental quasinormal mode of the central Reissner-Nordström BH. For higher charges (right panel of Fig. 3, in which \( Q = -q = 0.9M \)), the signal is less regular, being contaminated by additional modes. The waveform may be thought as the superposition of two types of modes: the standard gravitational-led one and the EM-led one, which is sufficiently excited in the high-charge (i.e. large-coupling) case and it therefore appears in the GW signal. In fact, by considering an expansion of the form

\[ K(t) \approx \sum_n K_n e^{\i \omega_n t}, \]  

(A4)

one can use the Prony method to find that the expansion is predominantly given by the frequencies of the fundamental gravitational-led (with frequency \( \omega M = 0.414 - 0.088i \)) and EM (\( \omega M = 0.620 - 0.096i \)) modes. The superposition of modes can also be seen by analyzing the power spectrum of the GW flux, as shown in Fig. 4 where the vertical lines mark the real parts gravitational-led and the EM-led modes.
FIG. 3. Ringdown produced by the collision of charged BH with a charged particle (taking only the dominant quadrupole term). The left and right panels correspond to the case of relatively small and large charges, respectively. In the former case the signal is essentially dominated by the gravitational-led mode, whereas for highly charged configurations the signal is contaminated by the additional EM-led mode.

The latter case is what one may expect when the BH is isolated, without the presence of plasma. As discussed in the main text, the main effect of the EM-plasma coupling is to provide an effective mass term for the EM mode, suppressing its excitation in the GW signal.

FIG. 4. Gravitational energy flux as a function of the frequency \( \omega \) for the plunge of a particle with mass \( \mu_p \) and charge \( q \) into a BH with charge \( Q \) (taking only the dominant quadrupole term). The vertical dotted lines mark the real part of the gravitational and EM fundamental modes of the corresponding Reissner-Nordstrom BH.

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