An Elusive $Z'$ Coupled to Beauty

Paul H. Frampton $(a)$, Mark B. Wise $(b)$ and Brian D. Wright $(a)$

$(a)$ University of North Carolina, Chapel Hill, NC 27599-3255

$(b)$ California Institute of Technology, Pasadena, CA 91125

Abstract

By extending the standard gauge group to $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ with $X$ charges carried only by the third family we accommodate the LEP measurement of $R_b$ and predict a potentially measurable discrepancy in $A^{t\bar{t}}_{FB}$ in $e^+e^-$ scattering and that $D^0\bar{D}^0$ mixing may be near its experimental limit. The $Z'$, which explicitly violates the GIM mechanism, can nevertheless be naturally consistent with FCNC constraints. Direct detection of the $Z'$ is possible but challenging.
Although the Standard Model (SM) survived the high precision LEP measurements almost unscathed, there are a few discrepancies which persist, most of them at a low level of statistical significance and hence quite likely to disappear as more data are collected. One outstanding deviation from the SM which is quite large involves the couplings of the beauty (b) quark. In particular, the ratio \( R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})} \) is predicted by the SM to be \( R_b = 0.2156 \pm 0.0003 \) (where the uncertainty comes from \( m_t \) and \( m_H \)) and is measured to be \( R_b = 0.2219 \pm 0.0017 \), about 3% too high and a significant 3.7\( \sigma \) effect (for a recent analysis see Ref. [1]). In this Letter, we shall thus take the \( R_b \) data at face value and construct an extension of the standard model that explains \( R_b \) and has other testable predictions. The two simplest ways to extend the SM while preserving its principal features are to extend the gauge sector or to extend the fermion sector. In the former approach, the simplest possibility is to extend the gauge sector by a U(1) gauge field which mixes with the usual Z boson and generates non-standard couplings to b quarks and perhaps the other quarks and leptons. Such an approach was first discussed in Ref. [4] and in a different context in Ref. [3]. More recently, attempts have been made to explain the \( R_b \) and \( R_c \) discrepancies with an extra U(1) gauge field which couples also to light quarks [6].

The simplest fermion-mixing model to explain the \( R_b \) (and \( R_c \)) data was proposed in Ref. [7]. It is not difficult to find models in which the radiative corrections can accommodate \( R_b \) measurements [8,9]; however, many popular models fail to provide a convenient solution. The Minimal Supersymmetric Standard Model (MSSM) is a notable example of this. Only a small region of parameter space can yield a consistent result, corresponding to a light supersymmetric spectrum, detectable at LEP II [10,11] (see however Ref. [12] for a light gluino alternative). Two-Higgs doublet models also fall into this category [8,13]. For a comprehensive review of the possibilities see Ref. [9] and references therein.

We extend the gauge sector by adopting the choice of gauge group \( SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_X \). Associated with the additional \( U(1)_X \) gauge group is a new quantum number \( X \) which defines the strength of the beauty and top couplings to the one new gauge boson which will be denoted by \( Z' \) for simplicity (although this \( Z' \) will certainly couple differently
than any other $Z'$ in the literature). To proceed with presenting our model we shall first examine the decay of the $Z$ and its relation to the fundamental $Z$-fermion couplings of the effective Lagrangian. The decay of the $Z$ into a fermion-antifermion pair $f \bar{f}$ is given by:

$$\Gamma(Z \rightarrow f \bar{f}) = \frac{\alpha_{em}(M_Z)CM_Z}{6c_W^2s_W^2} \beta((g_L^{f_2} + g_R^{f_2})(1 - x) + 6xg_L^{f}g_R^{f}),$$

(1)

where $c_W = \cos \theta_W$, $g_L^{f} = T_3^f - Q^f \sin^2 \theta_W$, $g_R^{f} = -Q^f \sin^2 \theta_W$, $x = (m_f/M_Z)^2$ and $\beta = \sqrt{1 - 4x}$. The color factor is $C = 3$ for quarks and $C = 1$ for leptons. For the light fermions, it is an adequate approximation to put $x = 0$ and $\beta = 1$ and, using $\sin^2 \theta_W = 0.232$, this gives the familiar values $\Gamma_e = \Gamma_\mu = \Gamma_\tau \simeq 83$ MeV and $\Gamma_\nu_i \simeq 166$ MeV for $i = e, \mu, \tau$ and for the quarks, $\Gamma_u = \Gamma_c \simeq 285$ MeV and $\Gamma_d = \Gamma_s = \Gamma_b \simeq 367$ MeV.

The couplings $g_{L,R}^{f}$ are modified when the $Z$ mixes with a $Z'$. The effective Lagrangian for the $Z$ and $Z'$ coupling to fermions is

$$L_{eff} = g_Z Z^\mu \bar{f} \gamma_\mu (g_L^f P_L + g_R^f P_R)f + g_X Z'^\mu \bar{f} \gamma_\mu (X_L^f P_L + X_R^f P_R)f,$$

(2)

where $g_Z = g_2/c_W = 0.739$, and $P_{R,L} = (1 \pm \gamma_5)/2$. This $Z'$ does not mix with the photon and the electric charge still given by $Q = T_3 + Y/2$, where $Y$ is the hypercharge and $T_3$ the third component of weak isospin. The mass eigenstates are mixtures of these states with a mixing angle according to $\hat{Z} = Z \cos \alpha - Z' \sin \alpha$ and $\hat{Z}' = Z' \cos \alpha + Z \sin \alpha$. If the mass matrix is given by

$$
\begin{pmatrix}
Z \\ Z'
\end{pmatrix}
= 
M^2 \begin{pmatrix}
\delta M^2 \\ M^2
\end{pmatrix}
\begin{pmatrix}
Z \\ Z'
\end{pmatrix},
$$

(3)

then the mixing angle is given by

$$\tan \alpha = \frac{\delta M^2}{M^2 - M_Z^2} = \frac{\delta M^2}{M^2 - M_Z^2},$$

(4)

where the hats denote mass eigenvalues. Because of the level of agreement between the SM and leptonic $Z$ decays at LEP, $\cos^2 \alpha$ must be near unity. In the presence of the $Z'$, we see from Eq. (3) that the $Z$ couplings are modified according to:
\[ \delta g_L^f = -\frac{g_X}{g_Z} X_L^f \tan \alpha , \quad \delta g_R^f = -\frac{g_X}{g_Z} X_R^f \tan \alpha , \]  

(5)

where we have factored out a \( \cos \alpha \) factor common to all the mass eigenstate \( \hat{Z} \) couplings.

The change \( \delta R_b \) is given at lowest order in the mixing by

\[ \delta R_b = R_b - R_b^{(0)} = 2 R_b^{(0)} (1 - R_b^{(0)}) \left( \frac{g_L^{b(0)} \delta g_L^b + g_R^{b(0)} \delta g_R^b}{(g_L^{b(0)})^2 + (g_R^{b(0)})^2} \right) , \]  

(6)

where the superscript 0 denotes SM quantities and \( g_L^{b(0)} = -0.423 \) and \( g_R^{b(0)} = 0.077 \).

Requiring \( R_b \) to be within one standard deviation of the experimental value means that

\[ 0.0080 > \delta R_b > 0.0046 . \]

Depending on the U(1) charges of the \( t \) and \( b \) quarks we consider adding a second \( (\phi', X_{\phi'} = +1) \) and possibly third \( (\phi'', X_{\phi''} = -1) \) Higgs doublet to the SM doublet \( (\phi, X_{\phi} = 0) \).

First consider the case of only two Higgs doublets. Here \( \phi' \) couples to both \( b \) and \( t \) and so \( X_{\phi'} = X_L^b - X_R^b = -X_L^t + X_R^t \). Then we can write \( \delta M^2 = -X_{\phi'} g_X g_Z |\langle \phi' \rangle|^2 \) and using Eq. (4) we see that \( X_{\phi'} \tan \alpha < 0 \). If only \( b_L \) or \( b_R \) has nonzero \( X \) charge then \( X_{\phi'} = X_L^b \) or \( X_{\phi'} = -X_R^b \) respectively and because of the signs of \( g_L^{b(0)} \) and \( g_R^{b(0)} \) in Eq. (6), \( R_b \) would always be decreased. We must therefore consider both \( X_{L,R}^b \) nonzero. Then we can write (6) numerically as

\[ \delta R_b = g_X \tan \alpha (1.05 X_{\phi'} + 0.86 X_R^b) , \]  

(7)

so \( -X_R^b/X_{\phi'} \gtrsim 1.2 \) in order to get a positive effect. To see that this is inconsistent, we must use another constraint: the measured \( Z \)-pole forward-backward asymmetry in \( e^+ e^- \to \bar{b}b \), \( A_{FB}^{(0,b)} \). To leading order it is given by

\[ \delta A_{FB}^{(0,b)} = A_{FB}^{(0,b)} - A_{FB}^{(0,b)(SM)} = A_{FB}^{(0,b)(SM)} \frac{4(g_L^{b(0)})^2 (g_R^{b(0)})^2}{(g_L^{b(0)})^4 - (g_R^{b(0)})^4} \left( \frac{\delta g_L^b}{g_L^{b(0)}} - \frac{\delta g_R^b}{g_R^{b(0)}} \right) . \]  

(8)

Inserting the numerical values, including \( A_{FB}^{(0,b)(SM)} = 0.101 \), we find that

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1 We are here assuming that \( \hat{M}_{Z'} > M_Z \). Models with \( \hat{M}_Z > \hat{M}_{Z'} \) can be constructed but their parameter space is more restricted.
\[
\delta A_{FB}^{(0,b)} = g_X \tan \alpha (0.043 X_{\phi'} + 0.278 X_{R}^b) .
\]

Comparison of the experimental forward-backward asymmetry with the SM prediction allows only a small departure satisfying \(|\delta A_{FB}^{(0,b)}| < 0.003\). Using the lowest consistent value of \(\delta R_b\) then shows that \(A_{FB}^{(0,b)}\) is too big. This excludes all models with only the two scalar doublets \(\phi\) and \(\phi'\).

So we must add a third doublet \(\phi''\) which gives mass to the \(t\) quark, \(\phi'\) still coupling to the \(b\) quark. Thus \(X_{\phi''} = -X_{L}^t + X_{R}^t\) and \(X_{\phi'} = X_{L}^b - X_{R}^b\). In this case we have \(\delta M^2 = -g_X g_Z (X_{\phi'} |\langle \phi' \rangle|^2 + X_{\phi''} |\langle \phi'' \rangle|^2)\) and with opposite signs for \(X_{\phi'}\) and \(X_{\phi''}\) and the natural choice \(|\langle \phi'' \rangle| > |\langle \phi' \rangle|\) we can make \(X_{\phi'} \tan \alpha > 0\). We are thus free to make simple choices for the quark charges. There are two natural choices to consider: (i) \(X_L^b = 1; X_R^b = 0\) and (ii) \(X_L^b = 0; X_R^b = 1\). Of these, (ii) can be shown to be inconsistent with the data, as follows. Equations (7) and (9) give \(\delta R_b = -0.19 g_X \tan \alpha\) and \(\delta A_{FB}^{(0,b)} = 0.24 g_X \tan \alpha\). Requiring \(\delta R_b > 0.0046\), implies \(|\delta A_{FB}^{(0,b)}| > 0.005\) contradicting experiment. This then leaves our preferred model: the charges for the third family - defined more carefully below - are simply \(X_{L}^b = 1\) and \(X_{R}^b = 0\). The model has three Higgs scalar doublets \(\phi, \phi'\) and \(\phi''\) with \(X\) charges 0, +1 and −1 respectively.

Cancellation of chiral anomalies is most economically accomplished by adding two doublets of quarks \((w, w')_L + (w, w')_R\) which are vector-like in weak hypercharge. The doublet \((w, w')_L\) has the opposite \(X\) charge and hypercharge to \((t, b)_L\) while the right-handed doublet has zero \(X\) charge. These acquire mass from a complex weak singlet Higgs scalar. The electric charges of these \textit{weird} quarks are +1/3 and −2/3; they thus give rise to stable fractionally-charged color singlets which may be problematic cosmologically. An alternative anomaly cancellation is to add quark \(SU(2)\) doublets, with \(Y = +1/6\), \((t', b')_L(X = -1) + (t', b')_R(X = 0)\) together with \(SU(2)\) singlet \(Y = -1\) charged leptons \(l_{\ell L}(X = 1) + l_{\ell R}(X = 0)\) and \(l_{\ell L}(X = -1) + l_{\ell R}(X = 0)\).

There is a three-dimensional parameter space for the model spanned by \(\tan \alpha, g_X\) and \(\xi = \hat{M}_Z / \hat{M}_{Z'}\). We consider, for simplicity, only \(\hat{M}_Z < \hat{M}_{Z'}\) and will be able to constrain
these parameters. Using the analysis above we have from the constraint on $R_b$,

$$0.008 \geq g_X \tan \alpha \geq 0.004 , \quad (10)$$

as well as a weaker constraint from the asymmetry: $g_X \tan \alpha < 0.07$. Turning this around using the $\delta R_b$ constraint, gives a prediction for the asymmetry:

$$3 \times 10^{-4} \geq \delta A_{FB}^{(0,b)} \geq 2 \times 10^{-4} . \quad (11)$$

This will be detectable if the experimental accuracy can be increased by a factor of at least 3 to 5. The quantity $\tan \alpha$ can be further restricted by perturbativity and by custodial SU(2). An upper limit $g_X(M_Z) < \sqrt{4\pi} = 3.54$, combined with the $\delta R_b$ constraint dictates that

$$\tan \alpha > 0.001 . \quad (12)$$

The accuracy of custodial SU(2) symmetry (the $\rho$ parameter) in the presence of multiple $Z$’s can be expressed in terms of $\rho_i = M_W^2/(\hat{M}_Z c_W^2)$ [14]. With just two $Z$’s we have the relationship

$$\tan^2 \alpha = \frac{\bar{\rho}_1 - 1}{\xi - 2 + \bar{\rho}_1} , \quad (13)$$

where $\bar{\rho}_i = \rho_i/\hat{\rho}$ with $\hat{\rho} = 1 + \rho_i$ which takes into account the top quark radiative corrections. Rewriting Eq. (1) in terms of the Fermi constant $G_F$, we find that all the decay rates are multiplied by a factor of $\bar{\rho}_{eff} = \bar{\rho}_1 \cos^2 \alpha$ compared to the SM. Using the the global fit allowing new physics in $R_b$ from Ref. [1] we have $\bar{\rho}_{eff} = 1.0002 \pm 0.0013 \pm 0.0018$ and Eq. (13) gives, for $\alpha \ll 1, \xi \ll 1$,

$$\tan \alpha < 0.045 \frac{\xi}{\sqrt{1 - 2\xi^2}} . \quad (14)$$

Since we have the lower bound on $\tan \alpha$ from Eq. (12), we deduce that $\xi > 0.028$ implying that $\hat{M}_{Z'} < 3.3$ TeV. It is very interesting that the present model produces such an upper limit on the new physics because it implies its testability in the next generation of accelerators.
Because we have assigned $X$-charge asymmetrically to the three families, there is inevitably a violation of GIM suppression \[15\] of the Flavor-Changing Neutral Currents (FCNC). In fact, study of FCNC sharpens the definition of our model. When we assigned $X_{t,b} = 1$, there was an inherent ambiguity of basis for the left-handed doublet $(t, b)_L$ because in general a unitary transformation is needed to relate this doublet to the mass eigenstates. The two most predictive limiting cases, out of an infinite range, are where (i) $t$ (ii) $b$ in $(t, b)_L$ is a mass eigenstate. If $t$ is a mass eigenstate, then the empirical \[2\] value $\Delta m_B = (3.4 \pm 0.4) \times 10^{-13}$GeV imposes an upper limit on the product $(g_X \xi)$ too small, to be consistent with the necessary increase $\delta R_b$. On the other hand, if $b$ is a mass eigenstate the $Z'$-exchange contribution to $\Delta m_B$ vanishes as do the (less constraining) FCNC effects like $\Delta m_K$, $b \to s \gamma$, $b \to s \ell \ell$.

The model with $b$ a mass eigenstate can be made natural by imposing the discrete symmetry $b_R \to -b_R$, $\phi' \to -\phi'$. This symmetry is spontaneously broken at the weak scale but because it suffers from a QCD anomaly there is no domain wall problem \[16\]. With the discrete symmetry the Yukawa couplings of the neutral components of the Higgs doublets are

$$\mathcal{L} = g_t \bar{t}_L t_R \phi^{(0)*} + g_b \bar{b}_L b_R \phi^{(0)*} + g_{ij}^{(u)} \bar{u}_{iL} u_{jR} \phi^{(0)*} + g_{ij}^{(d)} \bar{d}_{iL} d_{jR} \phi^{(0)*} + g_{i3}^{(u)} \bar{u}_{iL} t_R \phi^{(0)*} + h.c.,$$  \[15\]

where $\{i, j\} \in \{1, 2\}$ (the exotic fermions do not have Yukawa couplings to the ordinary ones). The weak eigenstate quark fields are related to primed mass eigenstate fields by

$$u_L = U_{L}^{\dagger} u'_L \quad d_L = T_L^\dagger d'_L$$

$$u_R = U_{R}^{\dagger} u'_R \quad d_R = T_R^\dagger d'_R$$  \[16\]

where (for $T_L$ and $T_R$) $T_{33} = 1$ and $T_{3i} = T_{i3} = 0$. The Kobayashi–Maskawa matrix that occurs in the charged $W$ boson couplings, $(g_2/\sqrt{2}) \bar{u}_{aL} \gamma_\mu V_{a\beta} d_{\beta L} W^\mu$ for $a, \beta \in \{1, 2, 3\}$, is

$$V_{a\gamma} = U_{L \alpha \beta} T_{L \beta \gamma}^{\dagger}$$  \[17\]
implying that $V_{a3} = U_{La3}$ and $V_{aj} = U_{Lai}T_{Lij}$. It follows that the flavor changing $Z'$ boson couplings are

$$L_{FCNC} = g_X Z_\mu^I (\bar{u}_{aL} \gamma^\mu V_{a3} V_{33}^* u_{bL})$$  \hspace{1cm} (18)$$

and that the flavor changing neutral Higgs boson couplings are

$$L_{FCNC} = \left( \frac{m_t}{v''} \right) \left( \phi_0^{(0)\alpha*} - \frac{v''}{v} \phi_{\alpha}^{(0)*} \right) \bar{u}_{aL} V_{a3} U_{R33}^* u_{bR} + h.c. \hspace{1cm} (19)$$

The chief FCNC constraint now comes from the experimental bound $\Delta m_D < 1.3 \times 10^{-13}$ GeV. The $Z'$-exchange contribution gives $\delta(\Delta m_D) \simeq (g_X \xi)^2(7 \times 10^{-6}$ GeV$)Re[V_{13}V_{23}^*]^2(f_D/(0.22$ GeV$))^2$ and hence requires instead only a mild constraint $g_X \xi \lesssim 1$, easily consistent with $\delta R_b$. There is also a contribution to $(\Delta m_D)$ from neutral Higgs exchange but the neutral Higgs masses can be chosen so that this is acceptably small. For example, the $\phi -$ and $\phi'' -$ exchange contribution to $DD$ mixing is sufficiently suppressed (by third-family mixing) to allow Higgs masses $\simeq 250$ GeV.

Fitting the hadronic width of $Z$ in our model gives rise to a decrease in $\alpha_s(M_Z)$ and tends to resolve discrepancies with low-energy determinations. Now let us consider the production of $Z'$ in colliders. In $p\bar{p} \rightarrow Z'X$, the $Z'$ is dominantly produced in association with two $b$ quarks. The cross-section at $\sqrt{s} = 1.8$ TeV falls off rapidly with $M_{Z'}$: for example, putting $g_X = g_Z$, it decreases from 16 pb at $M_{Z'} = 100$ GeV to 1 fb at $M_{Z'} = 450$ GeV. Against the $b\bar{b}$ background from QCD such a signal would be difficult to observe at Fermilab. In particular, $Z'$ production leads to final states with four heavy-flavor jets and one expects competition from QCD jet production to be severe. At an $e^+e^-$ collider, sitting at the $Z'$-pole, there is a possibility for detecting the $Z'$. The coupling to $e^+e^-$ is suppressed by $\tan\alpha$ but still the pole can show up above background. In Fig. 1 we display the cross-section for $e^+e^- \rightarrow b\bar{b}$ as a function of $\sqrt{s}$ for $Z'$ masses (a) 500 GeV, (b) 250 GeV and (c) 150 GeV respectively. The shape of the $Z'$ resonance indicates the importance of $Z-Z'$ interference. The parameters $g_X$ and $\alpha$ have been chosen to produce the most marked effect while still remaining within the limits discussed above.
In summary, we have constructed a model which can account for the measured value of $R_b$. It introduces a $Z'$ coupled almost entirely to the third family and to exotic fermions. The model has at least the esoteric interest that $Z'$ couples with sizeable strength to $b$ and $t$ quarks and can naturally avoid disastrous FCNC without a GIM mechanism. There is a prediction for the forward-backward asymmetry $A_{FB}^{(0,b)}$ and $D\bar{D}^0$ mixing may be near its experimental value. This $Z'$ is particularly elusive because it is so difficult to detect at colliders — with the possible exception of $e^+e^- \rightarrow \bar{b}b$ at the $Z'$ pole.

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FIGURES

FIG. 1. Cross-section for $e^+e^- \rightarrow \bar{b}b$ for $Z'$ masses (a) 500 GeV, (b) 250 GeV and (c) 150 GeV. The model parameters for each case are (a) $g_X = 1.0$, $\tan \alpha = 0.008$, $m_t = 180$ GeV giving $\Gamma_{Z'} = 32$ MeV, (b) $g_X = 0.5$, $\tan \alpha = 0.015$, giving $\Gamma_{Z'} = 2.5$ GeV and (c) $g_X = 0.3$, $\tan \alpha = 0.025$ giving $\Gamma_{Z'} = 570$ MeV.
