Finite-Time Mittag–Leffler Synchronization of Neutral-Type Fractional-Order Neural Networks with Leakage Delay and Time-Varying Delays

Călin-Adrian Popa 1,2,† and Eva Kaslik 3,*,†

1 Department of Mathematics, West University of Timișoara, Blvd. V. Pârvan, No. 4, 300223 Timișoara, Romania; calin.popa@cs.upt.ro
2 Department of Computer and Software Engineering, Polytechnic University Timișoara, Blvd. V. Pârvan, No. 2, 300223 Timișoara, Romania
3 Department of Computer Science, West University of Timișoara, Blvd. V. Pârvan, No. 4, 300223 Timișoara, Romania

* Correspondence: eva.kaslik@e-uvt.ro
† These authors contributed equally to this work.

Received: 15 June 2020; Accepted: 9 July 2020; Published: 13 July 2020

Abstract: This paper studies fractional-order neural networks with neutral-type delay, leakage delay, and time-varying delays. A sufficient condition which ensures the finite-time synchronization of these networks based on a state feedback control scheme is deduced using the generalized Gronwall–Bellman inequality. Then, a different state feedback control scheme is employed to realize the finite-time Mittag–Leffler synchronization of these networks by using the fractional-order extension of the Lyapunov direct method for Mittag–Leffler stability. Two numerical examples illustrate the feasibility and the effectiveness of the deduced sufficient criteria.

Keywords: fractional-order neural networks; finite-time synchronization; neutral-type neural networks; leakage delay; Mittag–Leffler function

1. Introduction

Fractional calculus studies the different possibilities of defining real or complex orders for the differentiation and integration operators. Although it has a long history, only recently it has been successfully applied to physics and engineering problems. As such, in the past few years, it became clear for engineers and scientists that some phenomena can be more accurately described by employing the fractional derivative. Fractional differential equations have been proved to better describe many systems in interdisciplinary fields, such as chemistry, biology, physics, mechanics, electromagnetism, heat transfer, acoustics, economy, and finance.

Fractional-order systems have been proven to possess infinite memory. Taking this fact into account, an extremely important improvement would be the introduction of a memory term (represented by a fractional derivative or integral) into a neural network model. Thus, fractional-order artificial neural networks were developed in [1]. Since then, many properties of this type of networks were studied: asymptotic stability and synchronization [2–7], Mittag–Leffler stability and synchronization [8–13], dissipativity [14–16], etc.

The finite-time stability and synchronization properties of fractional-order neural networks were intensely studied over the last few years. Concretely, “finite-time stability analysis of fractional-order complex-valued memristor-based neural networks with time delays” was done in [17]. Finite-time stability criteria for fractional-order delayed neural networks were also established in [18–20]. A more general model, namely fractional-order Cohen–Grossberg BAM neural networks with time delays was researched in [21], from the finite-time stability point of view. Finite-time stability analysis
was undergone in [22], for fractional-order complex-valued memristor-based neural networks with both leakage and time-varying delays. Fractional-order complex-valued neural networks with time delays were discussed in [23], in terms of finite-time stability. Then, finite-time stability criteria for delayed memristor-based fractional-order neural networks were deduced in [24]. A different model, namely the fractional-order fuzzy neural networks with proportional delays, was presented in [25], by giving finite-time stability sufficient criteria for these networks. More recently, finite-time stability results were established for fractional-order complex-valued neural networks with time delay in [26]. Discrete fractional-order complex-valued neural networks with time delays were, on the other hand, the focus of [27], where the existence and finite-time stability for these networks were researched.

Finite-time synchronization was also a topic of interest in the recent past. “Finite-time synchronization of fractional-order memristor-based neural networks with time delays” was studied in [28]. Sufficient criteria that realize “finite-time projective synchronization of memristor-based fractional-order neural networks with delays” were presented in [29]. Fractional-order memristive neural networks with discontinuous activation functions were researched in [30], where criteria for their finite-time synchronization were deduced. Then, both finite-time stability and finite-time synchronization were discussed in [31], for memristor-based fractional-order fuzzy cellular neural networks. Further finite-time synchronization analysis was undergone in [32], where the models were fractional-order memristive competitive neural networks with leakage delay and time-varying delays. Finite-time projective synchronization sufficient criteria were also deduced in [33], for fractional-order complex-valued memristor-based neural networks with delay. More recently, fully complex-valued fractional-order neural networks were the focus of [34], in which finite-time synchronization conditions for these models were given. Lastly, finite-time impulsive synchronization of fractional order memristive BAM neural networks was undergone in [35].

The property of finite-time Mittag–Leffler synchronization was introduced in [36], where sufficient criteria to attain this type of synchronization were given for fractional-order memristive BAM neural networks. Then, novel methods to finite-time Mittag–Leffler synchronization were given in [37], for fractional-order quaternion-valued neural networks. As such, the subject remains largely unexplored in the existing literature.

Fractional-order neural networks with neutral-type delays were also very rarely studied in the available literature. “Delay-independent stability criteria for Riemann–Liouville fractional-order neutral-type neural networks” were established in [38]. Then, the “delay-dependent stability analysis of QUAD vector field fractional-order quaternion-valued memristive uncertain neutral-type leaky integrator echo state neural networks” was done in [39]. No other results for neutral-type fractional-order neural networks exist in the literature, to the best of our knowledge.

On the other hand, fractional-order neural networks with leakage delay were much more in the focus of researchers in the recent years. The “stability analysis of fractional-order complex-valued neural networks with both leakage and discrete delays” was undergone in [40]. New bifurcation results for fractional-order BAM neural networks with leakage delay were given in [41]. As already mentioned, “fractional-order complex-valued memristor-based neural networks with both leakage and time-varying delays” were the focus of [22], where sufficient criteria for finite-time stability were deduced for these networks. Then, also as already mentioned, in [32], fractional-order memristive competitive neural networks with leakage delay and time-varying delays were discussed, from the finite-time synchronization point of view. The “impact of leakage delay on bifurcation in high-order fractional-order BAM neural networks” was researched in [42]. “Novel results on bifurcation for fractional-order complex-valued neural networks with leakage delay” were established in [43]. Lastly, the “dynamic stability of stochastic delayed fractional-order memristor-based fuzzy BAM neural networks with leakage delay” was studied in [44].

Time delays are known to occur in practical implementations of neural networks, which can cause instability or chaotic behavior, because the amplifiers have a finite switching speed. This is the reason why we chose to consider both leakage delay and time-varying delays in our model. On the other
hand, in neutral-type systems, past derivative information has also been observed to influence the present state. The properties of neural reaction processes that occur in the real world can be more accurately described by these systems. The study of these systems is more complicated than that of the usual time-delayed models because of the existence of the neutral-type delay. This type of delay is relevant in many application domains, like automatic control, population dynamics, and vibrating masses attached to an elastic bar. Neutral delay may also appear when implementing neural networks in VLSI circuits. These facts compelled us to also add neutral-type delay to our model. Taking all the above into account, we consider neutral-type fractional-order neural networks with leakage delay and time-varying delays in this paper, and study their finite-time synchronization and finite-time Mittag–Leffler synchronization based, respectively, on two general state feedback control schemes.

The main contributions of the paper are: (1) the introduction, for the first time in the literature, to the best of our knowledge, of the fractional-order neural networks with neutral-type delay, leakage delay, and time-varying delays; (2) the use of the generalized Gronwall–Bellman inequality to deduce sufficient criteria for the finite-time synchronization of the introduced networks, using a general state feedback control scheme; (3) the use of the fractional-order extension of the Lyapunov direct method for Mittag–Leffler stability in order to deduce sufficient criteria for the finite-time Mittag–Leffler synchronization of the introduced networks, using a different general state feedback control scheme; (4) the possible use of the methods developed in this paper for more general models, with impulsive effects, reaction–diffusion terms, or Markovian jump parameters.

The summary of the rest of the paper is the following. The neutral-type fractional-order neural networks with leakage delay and time-varying delays are introduced in Section 2, together with definitions regarding fractional calculus, definitions of the finite-time synchronization and the finite-time Mittag–Leffler synchronization, one assumption about the activation functions, and four useful lemmas. Then, Section 3 is dedicated to the deduction of the sufficient criteria which ensure finite-time synchronization and finite-time Mittag–Leffler synchronization, respectively, of the introduced model. Section 4 details the two numerical examples given to illustrate the feasibility and the effectiveness of the deduced sufficient criteria. The conclusions of the paper are presented in Section 5.

Notations: \( \mathbb{R} \) denotes the set of real numbers and \( \mathbb{R}^n \) denotes the Euclidean space of dimension \( n \). \( A^T \) represents the transpose of matrix \( A \). \( I_n \) denotes the identity matrix of order \( n \) and \( 0_n \) the empty matrix of order \( n \). That matrix \( A \) is positive definite (negative definite) is denoted by \( A > 0 \) (\( A < 0 \)). The smallest eigenvalue of positive definite matrix \( P \) is \( \lambda_{\text{min}}(P) \). \( \| \cdot \| \) represents the vector Euclidean norm or the matrix Frobenius norm, and \( | \cdot | \) is the element-wise vector norm or the element-wise matrix norm.

2. Preliminaries

First, we will give a few definitions involving fractional calculus.

Definition 1 ([45]). “The fractional integral of order \( \alpha \) for an integrable function \( x : [t_0, \infty) \to \mathbb{R} \) is defined as:

\[
I_{t_0}^{\alpha} x(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} x(s) \, ds,
\]

where \( t \geq t_0, \alpha > 0 \), and \( \Gamma(\cdot) \) is the gamma function, defined by:

\[
\Gamma(\tau) = \int_{0}^{\infty} t^{\tau-1} e^{-t} \, dt,
\]

for \( \text{Re}(\tau) > 0 \), where \( \text{Re}(\cdot) \) represents the real part.”
Definition 2 ([45]). “The fractional Caputo derivative of order $\alpha$ for a function $x \in C^n([t_0, \infty), \mathbb{R})$ is defined by:

$$D_0^\alpha x(t) = \frac{1}{\Gamma(n - \alpha)} \int_{t_0}^{t} x^{(n)}(s) \frac{1}{(t - s)^{\alpha + 1}} ds,$$

where $t > t_0$ and $n$ is a positive integer, with $n - 1 < \alpha < n$. Moreover, when $0 < \alpha < 1$, we have that:

$$D_0^\alpha x(t) = \frac{1}{\Gamma(1 - \alpha)} \int_{t_0}^{t} \frac{\dot{x}(s)}{(t - s)^{\alpha}} ds.$$”

Definition 3 ([45]). “The Mittag–Leffler function is defined by:

$$E_\alpha(z) = \sum_{p=0}^{\infty} \frac{z^p}{\Gamma(pa + 1)},$$

where $\alpha > 0$ and $z \in \mathbb{C}$. When $\alpha = 1$, we have that $E_1(z) = e^z$.”

Now, the neutral-type fractional-order neural networks with leakage delay and time-varying delays will be considered as the master system:

$$D_0^\alpha x_i(t) = -c_i x_i(t - \mu) + \sum_{j=1}^{N} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{N} b_{ij} f_j(x_i(t - \tau(t))) + g_i D_\eta^\varepsilon x_i(t - \eta) + l_i,$$

for $\forall i = 1, \ldots, N$, where $x_i(t) \in \mathbb{R}$ represents the state of the $i$th neuron at time $t$, $c_i \in \mathbb{R}^+$ represents the self-feedback weight of the $i$th neuron, $a_{ij} \in \mathbb{R}$ is the weight without time delay between the $i$th and $j$th neurons, $b_{ij} \in \mathbb{R}$ is the weight with time delay between the $i$th and $j$th neurons, $g_i \in \mathbb{R}$ is the neutral-type weight of the $i$th neuron, $f_j : \mathbb{R} \to \mathbb{R}$ represent the nonlinear activation functions, $\forall j = 1, \ldots, N$, $l_i \in \mathbb{R}$ is the external input for the $i$th neuron, $\mu$ is the leakage delay, $\tau : \mathbb{R}^+ \to \mathbb{R}^+$ are the time-varying delays, and $\eta$ is the neutral-type delay.

In the following, we assume that the time-varying delays $\tau : \mathbb{R}^+ \to \mathbb{R}^+$ are continuously differentiable functions and there exist $\tau > 0$ and $\tau' < 1$, such that $\tau(t) < \tau$, $\tau(t) \leq \tau'$, $\forall t > 0$. Define $\omega = \max\{\mu, \tau, \eta\}$.

The initial conditions of system (1) are given by

$$x_i(t) = \phi_i(t), \ t \in [-\omega, 0],$$

where $\phi_i \in C([-\omega, 0], \mathbb{R})$, for $\forall i = 1, \ldots, N$. The norm of an element $\phi \in C([-\omega, 0], \mathbb{R}^N)$ is defined as $||\phi|| = \sum_{i=1}^{N} \sup_{t \in [-\omega, 0]} |\phi_i(t)|$.

The slave system is given by

$$D_0^\alpha y_i(t) = -c_i y_i(t - \mu) + \sum_{j=1}^{N} a_{ij} f_j(y_j(t)) + \sum_{j=1}^{N} b_{ij} f_j(y_i(t - \tau(t))) + g_i D_\eta^\varepsilon y_i(t - \eta) + l_i - u_i(t),$$

for $\forall i = 1, \ldots, N$, and $y_i(t) \in \mathbb{R}$ represents the state of the $i$th neuron at time $t$, and $u_i(t)$ represents a control input.

The initial conditions of system (2) are given by

$$y_i(t) = \psi_i(t), \ t \in [-\omega, 0],$$

where $\psi_i \in C([-\omega, 0], \mathbb{R})$, for $\forall i = 1, \ldots, N$.\n

If we denote by \( e_i(t) = y_i(t) - x_i(t) \) for \( \forall i = 1, \ldots, N \), then, based on the expressions of the master system (1) and the slave system (2), the error system has the form

\[
\begin{align*}
D_0^\alpha e_i(t) &= -c_i e_i(t - \mu) + \sum_{j=1}^{N} a_{ij} \mathcal{I}_j(e_j(t)) + \sum_{j=1}^{N} b_{ij} \mathcal{I}_j(e_j(t - \tau(t))) \\
&\quad + g_i D_{-\eta}^\alpha e_i(t - \eta) - u_i(t),
\end{align*}
\]

for \( \forall i = 1, \ldots, N \), where \( \mathcal{I}_j(e_j(t)) = f_j(e_j(t) + x_j(t)) - f_j(x_j(t)), \forall j = 1, \ldots, N \).

The initial conditions of error system (3) are given by

\[
e_i(t) = \sigma_i(t) = \psi_i(t) - \phi_i(t), \quad t \in [-\omega, 0],
\]

where \( \sigma_i \in C([-\omega, 0], \mathbb{R}) \), for \( \forall i = 1, \ldots, N \).

The state feedback control scheme will be used to obtain finite-time synchronization between master system (1) and slave system (2). In this case, the controller is given by

\[
u_i(t) = k_{i1} e_i(t) + k_{i2} e_i(t - \mu) + k_{i3} e_i(t - \tau(t)) + k_{i4} D_{-\eta}^\alpha e_i(t - \eta),
\]

where \( k_{i1}, k_{i2}, k_{i3}, k_{i4} \in \mathbb{R}^+ \) represent the control gains, for \( \forall i = 1, \ldots, N \). System (3) now becomes

\[
\begin{align*}
D_0^\alpha e_i(t) &= -k_{i1} e_i(t) - (c_i + k_{i2}) e_i(t - \mu) - k_{i3} e_i(t - \tau(t)) + \sum_{j=1}^{N} a_{ij} \mathcal{I}_j(e_j(t)) + \sum_{j=1}^{N} b_{ij} \mathcal{I}_j(e_j(t - \tau(t))) \\
&\quad + (g_i - k_{i4}) D_{-\eta}^\alpha e_i(t - \eta),
\end{align*}
\]

for \( \forall i = 1, \ldots, N \).

System (5) can be written in matrix form as

\[
\begin{align*}
\begin{bmatrix} D_0^\alpha e(t) \\ \vdots \end{bmatrix} &= -K_1 e(t) - (C + K_2) e(t - \mu) - K_3 e(t - \tau(t)) + A \begin{bmatrix} \mathcal{I}(e(t)) \\ \vdots \end{bmatrix} + B \begin{bmatrix} \tau(e(t)) \\ \vdots \end{bmatrix} \\
&\quad + (G - K_4) D_{-\eta}^\alpha e(t - \eta),
\end{align*}
\]

Definition 4. The master system (1) is said to be finite-time synchronized with the slave system (2) based on the controller (4), if there exist positive constants \( \{\delta, \varepsilon, T\} \), \( 0 < \delta < \varepsilon \), such that \( ||\sigma|| < \delta \) implies \( ||e(t)|| < \varepsilon, \forall t \in [0, T) \).

We will also use a different state feedback control scheme to realize finite-time Mittag–Leffler synchronization between master system (1) and slave system (2), for which the controller is given by

\[
u_i(t) = k_{i1} e_i(t) + k_{i2} \text{sign}(e_i(t))|e_i(t - \mu)| + k_{i3} \text{sign}(e_i(t))|e_i(t - \tau(t))| + k_{i4} \text{sign}(e_i(t))|D_{-\eta}^\alpha e_i(t - \eta)|,
\]

where \( k_{i1}, k_{i2}, k_{i3}, k_{i4} \in \mathbb{R}^+ \) represent the control gains, for \( \forall i = 1, \ldots, N \). System (3) becomes in this case

\[
\begin{align*}
D_0^\alpha e_i(t) &= -k_{i1} e_i(t) - c_i e_i(t - \mu) - k_{i2} \text{sign}(e_i(t))|e_i(t - \mu)| - k_{i3} \text{sign}(e_i(t))|e_i(t - \tau(t))| \\
&\quad - k_{i4} \text{sign}(e_i(t))|D_{-\eta}^\alpha e_i(t - \eta)| + \sum_{j=1}^{N} a_{ij} \mathcal{I}_j(e_j(t)) + \sum_{j=1}^{N} b_{ij} \mathcal{I}_j(e_j(t - \tau(t))) \\
&\quad + g_i D_{-\eta}^\alpha e_i(t - \eta),
\end{align*}
\]

for \( \forall i = 1, \ldots, N \).
System (9) can be written in matrix form as
\begin{align*}
\frac{d}{dt}x(t) &= -K_1 e(t) - C e(t - \mu) - K_2 \text{sign}(e(t))|e(t - \mu)| - K_3 \text{sign}(e(t))|e(t - \tau(t))| \\
&\quad - K_4 \text{sign}(e(t))|D^\alpha x(t - \eta)| + A\overline{f}(e(t)) + B\overline{f}(e(t - \tau(t))) + GD^\alpha x(t - \eta).
\end{align*}

**Assumption 1.** The following Lipschitz conditions are satisfied by the activation functions defined above.

**Lemma 1** ([45]). "If \(x \in C^n([t_0, \infty), \mathbb{R})\), then
\[
I_{t_0}^\alpha D^\alpha_{t_0} x(t) = x(t) - \sum_{k=0}^{n-1} \frac{(t - t_0)^k}{k!} x^{(k)}(t_0),
\]
where \(t > t_0\) and \(n\) is a positive integer, with \(n - 1 < \alpha < n\). Moreover, when \(0 < \alpha < 1\), we have that:
\[
I_{t_0}^\alpha D^\alpha_{t_0} x(t) = x(t) - x(t_0).
\]

**Lemma 2** ([46]). "Suppose \(\alpha > 0\), \(a(t)\) is a nonnegative function locally integrable on \(0 \leq t < T\) (for some \(T \leq +\infty\)) and \(g(t)\) is a nonnegative, nondecreasing continuous function defined on \(0 \leq t < T\), \(g(t) \leq M\) (\(M\) is a constant), and suppose \(u(t)\) is nonnegative and locally integrable on \(0 \leq t < T\) with
\[
u(t) \leq a(t) + g(t) \int_0^t (t - s)^{a-1} u(s) ds, 0 \leq t < T.
\]
Then
\[
u(t) \leq a(t) + \int_0^t \left[ \sum_{n=1}^{\infty} \frac{(g(t)\Gamma(\alpha))^n}{\Gamma(n\alpha)} (t - s)^{n\alpha-1} a(s) \right] ds, 0 \leq t < T.
\]
Moreover, if \(a(t)\) is a nondecreasing function on \(0 \leq t < T\), then
\[
u(t) \leq a(t)E_a(g(t)\Gamma(\alpha)I^\alpha), 0 \leq t < T.
\]

**Lemma 3** ([47]). "If \(x \in \mathcal{C}^1([t_0, \infty), \mathbb{R}^N)\), then
\[
\frac{1}{2} D^\alpha_{t_0} x^T(t)x(t)) \leq x^T(t)D^\alpha_{t_0} x(t)), \forall t \geq t_0,
\]
where \(0 < \alpha < 1\)."

**Lemma 4** ([48]). "Let \(V \in C([t_0, \infty), \mathbb{R})\) which satisfies
\[
D^\alpha_{t_0} V(t) \leq -\lambda V(t), \forall t \geq t_0,
\]
where \(0 < \alpha < 1\) and \(\lambda > 0\). Then
\[
V(t) \leq V(t_0)E_a(-\lambda I^\alpha), \forall t \geq t_0.
\]
3. Main Results

In the following, we will assume that $0 < \alpha < 1$.

First, we give a sufficient condition that ensures the finite-time synchronization of master system (1) and slave system (2), based on the controller (4).

**Theorem 1.** If Assumption 1 holds, then master system (1) and slave system (2) are finite-time synchronized based on the controller (4) if $\|G - K_4\| < 1$ and there exist positive constants $\{\delta, \epsilon, T\}$ such that the following inequality holds:

$$
\frac{1 + \|G - K_4\|}{1 - \|G - K_4\|} Fa \left( \frac{1}{1 - \|G - K_4\|} (\|K_1\| + \|A\| L) + \|C + K_2\| + \|K_3\| + \|B\| L) t^\alpha \right) < \frac{\epsilon}{\delta}, \forall t \in [0, T).
$$

**Proof.** Integrating relation (6), we get that

$$
\mathcal{I}_0^t D_{-t}^\alpha e(t) = -K_1 \mathcal{I}_0^t e(t) - (C + K_2) \mathcal{I}_0^t e(t - \mu) - K_3 \mathcal{I}_0^t e(t - \tau(t)) + AI_0^t f(e(t)) + BI_0^t \dot{f}(e(t - \tau(t)))
$$

$$
+ (G - K_4) \mathcal{I}_0^t D_{-t}^\alpha e(t - \eta),
$$

or, by using Lemma 1, that

$$
e(t) - e(0) = -K_1 \frac{1}{\Gamma(\alpha)} \int_0^t (t - s)^{\alpha - 1} e(s) ds - (C + K_2) \frac{1}{\Gamma(\alpha)} \int_0^t (t - s)^{\alpha - 1} e(s - \mu) ds
$$

$$
- K_3 \frac{1}{\Gamma(\alpha)} \int_0^t (t - s)^{\alpha - 1} e(s - \tau(s)) ds + A \frac{1}{\Gamma(\alpha)} \int_0^t (t - s)^{\alpha - 1} f(e(s)) ds
$$

$$
+ B \frac{1}{\Gamma(\alpha)} \int_0^t (t - s)^{\alpha - 1} \dot{f}(e(s - \tau(s))) ds + (G - K_4) \mathcal{I}_0^t D_{-t}^\alpha e(t - \eta).
$$

We have that

$$
\mathcal{I}_0^t D_{-t}^\alpha e(t - \eta) = \frac{1}{\Gamma(\alpha)} \int_0^t \left( \frac{1}{\Gamma(1 - \alpha)} \int_{t - \eta}^t \frac{\dot{e}(u)}{(t - u)^\alpha} du \right) ds
$$

$$
= \frac{1}{\Gamma(\alpha)} \int_0^t (t - s)^{\alpha - 1} \left( \frac{1}{\Gamma(1 - \alpha)} \int_{t - \eta}^s \frac{\dot{e}(u)}{(s - u)^\alpha} du \right) ds
$$

$$
= \frac{1}{\Gamma(\alpha) \Gamma(1 - \alpha)} \int_0^t (t - s)^{\alpha - 1} \int_{t - \eta}^s \frac{\dot{e}(u)}{(s - u)^\alpha} du ds
$$

$$
= \frac{1}{\Gamma(\alpha) \Gamma(1 - \alpha)} \int_{t - \eta}^t \dot{e}(u) \int_{t - \eta}^s (s - u)^{\alpha - 1} du ds.
$$

With the change of variable $s - \eta - u = v$, we get

$$
\mathcal{I}_0^t D_{-t}^\alpha e(t - \eta) = \frac{1}{\Gamma(\alpha) \Gamma(1 - \alpha)} \int_{t - \eta}^t \dot{e}(u) \int_{u}^{t - \eta - u} \frac{(t - \eta - u - v)^{\alpha - 1}}{v^\alpha} dv du.
$$

Then, with the change of variable $v = (t - \eta - u) w$, we get

$$
\mathcal{I}_0^t D_{-t}^\alpha e(t - \eta) = \frac{1}{\Gamma(\alpha) \Gamma(1 - \alpha)} \int_{t - \eta}^t \dot{e}(u) \int_{0}^{t - \eta - u} \frac{(t - \eta - u - v)^{\alpha - 1}}{v^\alpha} dv du
$$

$$
= \frac{1}{\Gamma(\alpha) \Gamma(1 - \alpha)} \int_{t - \eta}^t \dot{e}(u) \int_{0}^{t - \eta} \frac{(t - \eta - u - w)^{\alpha - 1}}{(t - \eta - u - w)^\alpha} (t - \eta - u) dw du
$$

$$
= \frac{1}{\Gamma(\alpha) \Gamma(1 - \alpha)} \int_{t - \eta}^t \dot{e}(u) \int_{1}^{1 - \alpha} (1 - w)^{\alpha - 1} w^{1 - \alpha - 1} dw du
$$

$$
= \frac{1}{\Gamma(\alpha) \Gamma(1 - \alpha)} \int_{t - \eta}^t \dot{e}(u) B(1 - \alpha, \alpha) du,
$$

where $B(u, v)$ is the beta function.
where $B(x,y)$ denotes the Euler beta function. Therefore:

$$I_0^\eta D_0^\eta e(t-\eta) = \int_{\eta}^{t-\eta} \dot{e}(u) du = e(t-\eta) - e(-\eta).$$

By taking the norm of relation (12), and also taking into account the above relation, we obtain that

$$\|e(t)\| \leq \|e(0)\| + (|G - K_4| + |e(-\eta)|) \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} |e(s)| ds$$

$$+ |C + K_2| \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} |e(s-\mu)| ds$$

$$+ |K_3| \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} |f(e(s))| ds$$

$$+ |A| \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} |f(e(s))| ds$$

$$+ |B| \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} |f(e(s-x(s)))| ds$$

$$+ |G - K_4| \|e(t-\eta)\|$$

$$\leq (1 + |G - K_4|) |\sigma| + (|K_1| + |A| ||L|| + |C + K_2| + |K_3| + |B| ||L||) \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} v(s) ds,$$

where, for the last inequality, we used Assumption 1.

Let $v(t) = \sup_{t-\omega \leq s \leq t} |e(s)|$, $\forall t > 0$, then $\|e(t)\| \leq v(t), \|e(t-\mu)\| \leq v(t), \|e(t-x(s))\| \leq v(t), \|e(t-\eta)\| \leq v(t), \forall t > 0$. Also, $v(0) = |\sigma|$. We now have that

$$\|e(t)\| \leq (1 + |G - K_4|) |\sigma| + |G - K_4| v(t)$$

$$+ (|K_1| + |A| ||L|| + |C + K_2| + |K_3| + |B| ||L||) \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} v(s) ds,$$

which yields

$$v(t) \leq (1 + |G - K_4|) |\sigma| + |G - K_4| v(t)$$

$$+ (|K_1| + |A| ||L|| + |C + K_2| + |K_3| + |B| ||L||) \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} v(s) ds,$$

which is equivalent with

$$v(t) \leq \frac{1 + |G - K_4|}{1 - |G - K_4|} |\sigma| + \frac{1}{1 - |G - K_4|}$$

$$\times (|K_1| + |A| ||L|| + |C + K_2| + |K_3| + |B| ||L||) \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} v(s) ds. \ (12)$$

If we denote

$$a(t) = \frac{1 + |G - K_4|}{1 - |G - K_4|},$$

$$g(t) = \frac{1}{1 - |G - K_4|} (|K_1| + |A| ||L|| + |C + K_2| + |K_3| + |B| ||L||) \frac{1}{\Gamma(\alpha)},$$
then inequality (12) becomes:

\[ v(t) \leq a(t) + g(t) \int_0^t (t-s)^{a-1} v(s) \, ds. \]

Now, applying Lemma 2, we have

\[
||e(t)|| \leq v(t) \\
\leq a(t) E_\alpha \left( \frac{1}{1 - ||G - K_4||} \left( ||K_1|| + ||A||L|| + ||C + K_2|| + ||K_3|| + ||B||L|| \right) t^\alpha \right) \\
< \delta \cdot \frac{\epsilon}{\delta} = \epsilon, \quad \forall t \in [0, T),
\]

where, for the last inequality, we used condition (10). This completes the proof of the theorem.

**Remark 1.** The condition in Theorem 1 only needs the computation of the norm of 8 matrices, which is easily done using the default function in MATLAB. Also, the verification of the inequality only needs computing the Mittag–Leffler function for the biggest argument, because it is a strictly increasing function.

We now give a sufficient condition that ensures the finite-time Mittag–Leffler synchronization of master system (1) and slave system (2), based on the controller (7).

**Theorem 2.** If Assumption 1 holds, then master system (1) and slave system (2) are finite-time Mittag–Leffler synchronized based on the controller (7) if

\[
K_1 - ||A||L > 0, \quad K_2 - C > 0, \quad K_3 - ||B||L > 0, \quad K_4 - ||G|| > 0,
\]

and there exist positive constants \( \{\delta, \epsilon, T\} \) such that

\[
E_\alpha(-\lambda t^\alpha) < \frac{\epsilon^2}{\delta^2}, \quad \forall t \in [0, T),
\]

where \( \lambda = 2\lambda_{\min}(K_1 - ||A||L). \)

**Proof.** The following Lyapunov function will be considered:

\[
V(t) = \frac{1}{2} e^T(t)e(t).
\]

Taking into account Lemma 3, and computing the fractional-order derivative of \( V(t) \) along the trajectories of system (9), it follows that
\begin{align*}
D_0^\alpha V(t) &= D_0^\alpha \left( \frac{1}{2} e^T(t) e(t) \right) \\
&\leq e^T(t) D_0^\alpha e(t) \\
&= e^T(t) \left[ -K_1 e(t) - C e(t) - K_2 \text{sign}(e(t)) |e(t) - \mu| - K_3 \text{sign}(e(t)) |e(t - \tau(t))| \right. \\
&\quad \left. - K_4 \text{sign}(e(t)) D_{\alpha,y} e(t - \eta) \right] + A^2 e(t) + B^2 e(t - \tau(t)) + G D_{\alpha,y} e(t - \eta) \\
&= -e^T(t) K_1 e(t) - e^T(t) C e(t) - e^T(t) K_2 |e(t) - \mu| - e^T(t) K_3 |e(t - \tau(t))| \\
&\quad - e^T(t) K_4 D_{\alpha,y} e(t - \eta) + e^T(t) A^2 e(t) + e^T(t) B^2 e(t - \tau(t)) + e^T(t) G D_{\alpha,y} e(t - \eta) \\
&\leq \left| e^T(t) \right| K_1 \left| e(t) \right| + \left| e^T(t) \right| (C - K_2) \left| e(t - \mu) \right| - e^T(t) K_3 \left| e(t - \tau(t)) \right| \\
&\quad - e^T(t) K_4 D_{\alpha,y} e(t - \eta) + e^T(t) \left| (A L) e(t) \right| + e^T(t) \left| (B L) e(t - \tau(t)) \right| \\
&\quad + e^T(t) \left| (G) \right| D_{\alpha,y} e(t - \eta) \\
&= -e^T(t) \left| (K_1 - |A L|) e(t) \right| - e^T(t) \left| (K_2 - C) e(t - \mu) \right| \\
&\quad - e^T(t) \left| (K_3 - |B L| e(t - \tau(t)) \right| - e^T(t) \left| (K_4 - |G|) D_{\alpha,y} e(t - \eta) \right|,
\end{align*}

where, for the last inequality, we used Assumption 1.

Now, taking into account conditions (13), inequality (16) becomes:

\[ D_0^\alpha V(t) \leq -|e^T(t)| (K_1 - |A L|) |e(t)| \]

\[ \leq -\lambda V(t), \]

where \( \lambda = 2 \lambda_{\text{min}}(K_1 - |A L|) > 0 \). By Lemma 4, we have that

\[ V(t) \leq V(0) E_a(-\lambda t^\alpha), \]

or, equivalently,

\[ \frac{1}{2} ||e(t)||^2 \leq \frac{1}{2} ||e(0)||^2 E_a(-\lambda t^\alpha) \]

\[ \leq \frac{1}{2} ||\sigma||^2 E_a(-\lambda t^\alpha), \]

which further leads to

\[ ||e(t)|| \leq ||\sigma|| (E_a(-\lambda t^\alpha))^\frac{1}{2} < \frac{\epsilon}{\delta} \]

where, for the last inequality, we used condition (14). This completes the proof of the theorem. \( \square \)

**Remark 2.** The condition in Theorem 2 only needs to verify that 4 matrices are positive definite. Again, the verification of the inequality only needs computing the Mittag-Leffler function for the biggest argument, because it is a strictly increasing function.

### 4. Numerical Examples

Two numerical examples shall be given in this section to illustrate the feasibility and the effectiveness of the sufficient criteria deduced above.
Example 1. The two-neuron neutral-type fractional-order neural network having leakage delay and time-varying delays will be the master system:

\[
D_0^\alpha x(t) = -Cx(t - \mu) + \sum_{j=1}^{N} Af(x(t)) + \sum_{j=1}^{N} Bf(x(t - \tau(t))) + GD_{-\eta}^{\alpha} x(t - \eta) + I,
\]

(16)

the fractional-order neural network with two neurons will be the slave system:

\[
D_0^\alpha y(t) = -Cy(t - \mu) + \sum_{j=1}^{N} Af(y(t)) + \sum_{j=1}^{N} Bf(y(t - \tau(t))) + GD_{-\eta}^{\alpha} y(t - \eta) + I - u(t),
\]

(17)

and the controller will be:

\[
u(t) = K_1e(t) + K_2e(t - \mu) + K_3e(t - \tau(t)) + K_4D_{-\eta}^{\alpha}e(t - \eta),
\]

(18)

where \(e(t) = y(t) - x(t)\), and \(\alpha = 0.5\),

\[C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad A = \begin{bmatrix} 0.2 & -0.2 \\ -0.2 & 0.2 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 & 0.1 \\ 0.1 & -0.1 \end{bmatrix}, \quad G = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.5 \end{bmatrix}, \]

\[f(x) = \frac{1}{1 + e^{-x}}, \quad x \in \mathbb{R},\]

from which we deduce that \(L = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}\), and so Assumption 1 is fulfilled. The leakage delay is \(\mu = 0.04\), the time-varying delays are \(\tau(t) = 0.1|\cos t|\), and the neutral-type delay is \(\eta = 0.05\), from where we get that \(\tau = \tau' = 0.1\) and \(\omega = \max\{\mu, \tau, \eta\} = 0.1\).

The values of \(K_1, K_2, K_3, K_4\) are designed as:

\[
K_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad K_3 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad K_4 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix}.
\]

With these values we get that \(|G - K_4| = 0.6 < 1\), and, if we take \(\delta = 0.1\) and \(\varepsilon = 100\), then condition (10) holds for \(T = 2.1411\), which means that the conditions of Theorem 1 are fulfilled. Thus, we obtain that master system (16) is finite-time synchronized with slave system (17), based on controller (18).

The state trajectories and the phase trajectories of the errors \(e_1\) and \(e_2\) are depicted in Figures 1–3, for 8 initial values.

Example 2. Now, the master system will be the fractional-order neural network having two neurons:

\[
D_0^\alpha x(t) = -Cx(t - \mu) + \sum_{j=1}^{N} Af(x(t)) + \sum_{j=1}^{N} Bf(x(t - \tau(t))) + GD_{-\eta}^{\alpha} x(t - \eta) + I,
\]

(19)

the slave system will be the fractional-order neural network having two neurons:

\[
D_0^\alpha y(t) = -Cy(t - \mu) + \sum_{j=1}^{N} Af(y(t)) + \sum_{j=1}^{N} Bf(y(t - \tau(t))) + GD_{-\eta}^{\alpha} y(t - \eta) + I - u(t),
\]

(20)
and the controller will be:

\[ u(t) = K_1e(t) + K_2\text{sign}(e(t))|e(t - \mu)| + K_3\text{sign}(e(t))|e(t - \tau(t))| + K_4\text{sign}(e(t))|D^\alpha_\eta e(t - \eta)|, \]  

(21)

where \( e(t) = y(t) - x(t) \), and \( \alpha = 0.75 \),

\[
C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad A = \begin{bmatrix} 0.2 & -0.2 \\ -0.2 & 0.2 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 & 0.1 \\ 0.1 & -0.1 \end{bmatrix}, \quad G = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad f(x) = \frac{1}{1 + e^{-x}}, \quad x \in \mathbb{R},
\]

from which we get that \( L = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix} \), which means that Assumption 1 is fulfilled. If the leakage delay is \( \mu = 0.07 \), the time-varying delays are \( \tau(t) = 0.1|\sin t| \), and the neutral-type delay is \( \eta = 0.06 \), then \( \tau = \tau' = 0.1 \) and \( \omega = \max\{\mu, \tau, \eta\} = 0.1 \).

The values of \( K_1, K_2, K_3, K_4 \) are designed as:

\[
K_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad K_3 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad K_4 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}.
\]

With these values we get that \( K_1 - |A|L > 0, K_2 - C > 0, K_3 - |B|L > 0, K_4 - |G| > 0, \) and \( \lambda = 2\lambda_{\text{min}}(K_1 - |A|L) = 0.2 > 0 \), so conditions (13) hold. Condition (14) is satisfied for any \( \varepsilon > \delta \) and any \( T > 0 \), because \( E_\alpha(-\lambda\theta^\alpha) \leq 1, \forall t \geq 0 \). By applying Theorem 2, we get that master system (19) is finite-time Mittag–Leffler synchronized with slave system (20), based on controller (21).

The state trajectories and the phase trajectories of the errors \( e_1 \) and \( e_2 \) are depicted in Figures 4–6, for 8 initial values.
Figure 2. State trajectories of the error $e_2$ in Example 1, for 8 initial values (depicted with different colors). Best viewed in color.

Figure 3. Phase trajectories of the errors $e_1$ and $e_2$ in Example 1, for 8 initial values (depicted with different colors). Best viewed in color.

Figure 4. State trajectories of the error $e_1$ in Example 2, for 8 initial values (depicted with different colors). Best viewed in color.
Figure 5. State trajectories of the error $e_2$ in Example 2, for 8 initial values (depicted with different colors). Best viewed in color.

Figure 6. Phase trajectories of the errors $e_1$ and $e_2$ in Example 2, for 8 initial values (depicted with different colors). Best viewed in color.

5. Conclusions

Two sufficient criteria which ensure the finite-time synchronization and finite-time Mittag–Leffler synchronization of fractional-order neural networks with neutral-type delay, leakage delay, and time-varying delays were given, by making the assumption that the activation functions satisfy the Lipschitz conditions. The generalized Gronwall–Bellman inequality was used to realize the finite-time synchronization of the introduced networks, based on a general state feedback control scheme. Then, the fractional-order extension of the Lyapunov direct method for Mittag–Leffler stability was used to realize the finite-time Mittag–Leffler synchronization of the same networks, based on a different general state feedback control scheme. The feasibility and the effectiveness of the theoretical results was illustrated by providing two numerical examples.

The methods developed in the paper are general, and can be used to obtain sufficient criteria for the finite-time synchronization and the finite-time Mittag–Leffler synchronization of neural network models with impulsive effects, reaction–diffusion terms, or Markovian jump parameters. These developments represent promising future work directions.
Author Contributions: Conceptualization, C.-A.P.; methodology, C.-A.P. and E.K.; software, C.-A.P.; validation, C.-A.P.; formal analysis, C.-A.P. and E.K.; investigation, C.-A.P. and E.K.; resources, C.-A.P.; data curation, E.K.; writing—original draft preparation, C.-A.P.; writing—review and editing, C.-A.P. and E.K.; visualization, C.-A.P.; supervision, C.-A.P. and E.K.; project administration, C.-A.P. and E.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References
1. Kaslik, E.; Sivasundaram, S. Nonlinear dynamics and chaos in fractional-order neural networks. Neural Netw. 2012, 32, 245–256. [CrossRef]
2. Wang, L.; Song, Q.; Liu, Y.; Zhao, Z.; Alsaadi, F.E. Global asymptotic stability of impulsive fractional-order complex-valued neural networks with time delay. Neurocomputing 2017, 243, 49–59. [CrossRef]
3. Chen, J.; Chen, B.; Zeng, Z. Global asymptotic stability and adaptive ultimate Mittag–Leffler synchronization for a fractional-order complex-valued memristive neural networks with delays. IEEE Trans. Syst. Man Cybern. Syst. 2018, 49, 2519–2535. [CrossRef]
4. Hu, T.; Zhang, X.; Zhong, S. Global asymptotic synchronization of nonidentical fractional-order neural networks. Neurocomputing 2018, 313, 39–46. [CrossRef]
5. Chen, J.; Li, C.; Yang, X. Asymptotic stability of delayed fractional-order fuzzy neural networks with impulse effects. J. Frankl. Inst. 2018, 355, 7595–7608. [CrossRef]
6. Chen, L.; Huang, T.; Machado, J.T.; Lopes, A.M.; Chai, Y.; Wu, R. Delay-dependent criterion for asymptotic stability of a class of fractional-order memristive neural networks with time-varying delays. Neural Netw. 2019, 118, 289–299. [CrossRef] [PubMed]
7. Ali, M.S.; Hymavathi, M.; Senan, S.; Shekher, V.; Arik, S. Global asymptotic synchronization of impulsive fractional-order complex-valued memristor-based neural networks with time varying delays. Commun. Nonlinear Sci. Numer. Simul. 2019, 78, 1–21. [CrossRef]
8. Chen, J.; Chen, B.; Zeng, Z. O(\(t^{-\alpha}\))-synchronization and Mittag–Leffler synchronization for the fractional-order memristive neural networks with delays and discontinuous neuron activations. Neural Netw. 2018, 100, 10–24. [CrossRef] [PubMed]
9. Pratap, A.; Raja, R.; Sowmiya, C.; Bagdasar, O.; Cao, J.; Rajchakit, G. Robust generalized Mittag–Leffler synchronization of fractional order neural networks with discontinuous activation and impulses. Neural Netw. 2018, 103, 128–141. [CrossRef]
10. Yang, X.; Li, C.; Song, Q.; Chen, J.; Huang, J. Global Mittag–Leffler stability and synchronization analysis of fractional-order quaternion-valued neural networks with linear threshold neurons. Neural Netw. 2018, 105, 88–103. [CrossRef] [PubMed]
11. Wang, L.F.; Wu, H.; Liu, D.Y.; Boutat, D.; Chen, Y.M. Lur’e Postnikov Lyapunov functional technique to global Mittag–Leffler stability of fractional-order memristive neural networks with piecewise constant argument. Neurocomputing 2018, 302, 23–32. [CrossRef]
12. You, X.; Song, Q.; Zhao, Z. Global Mittag–Leffler stability and synchronization of discrete-time fractional-order complex-valued neural networks with time delay. Neural Netw. 2020, 122, 382–394. [CrossRef] [PubMed]
13. Ali, M.S.; Narayanan, G.; Shekher, V.; Alsaedi, A.; Ahmad, B. Global Mittag–Leffler stability analysis of impulsive fractional-order complex-valued BAM neural networks with time varying delays. Commun. Nonlinear Sci. Numer. Simul. 2020, 83, 1–22. [CrossRef]
14. Ding, Z.; Shen, Y. Global dissipativity of fractional-order neural networks with time delays and discontinuous activations. Neurocomputing 2016, 196, 159–166. [CrossRef]
15. Velmurugan, G.; Rakkiyappan, R.; Venkatarayanan, V.; Cao, J.; Alsaedi, A. Dissipativity and stability analysis of fractional-order complex-valued neural networks with time delay. Neural Netw. 2017, 86, 42–53. [CrossRef] [PubMed]
16. Fan, Y.; Huang, X.; Wang, Z.; Li, Y. Global dissipativity and quasi-synchronization of asynchronous updating fractional-order memristor-based neural networks via interval matrix method. J. Frankl. Inst. 2018, 355, 5998–6025. [CrossRef]
17. Rakkiyappan, R.; Velmurugan, G.; Cao, J. Finite-time stability analysis of fractional-order complex-valued memristor-based neural networks with time delays. *Nonlinear Dyn.* 2014, 78, 2823–2836. [CrossRef]
18. Wu, R.; Lu, Y.; Chen, L. Finite-time stability of fractional delayed neural networks. *Neurocomputing* 2015, 149, 700–707. [CrossRef]
19. Yang, X.; Song, Q.; Liu, Y.; Zhao, Z. Finite-time stability analysis of fractional-order neural networks with delay. *Neurocomputing* 2015, 152, 19–26. [CrossRef]
20. Chen, L.; Liu, C.; Wu, R.; He, Y.; Chai, Y. Finite-time stability criteria for a class of fractional-order neural networks with delay. *Neural Comput. Appl.* 2015, 27, 549–556. [CrossRef]
21. Rajivganthi, C.; Rihan, F.A.; Lakshmanan, S.; Muthukumar, P. Finite-time stability analysis for fractional-order Cohen–Grossberg BAM neural networks with time delays. *Neural Comput. Appl.* 2018, 29, 1309–1320. [CrossRef]
22. Wang, L.; Song, Q.; Liu, Y.; Zhao, Z.; Alsaadi, F.E. Finite-time stability analysis of fractional-order complex-valued memristor-based neural networks with both leakage and time-varying delays. *Neurocomputing* 2017, 245, 86–101. [CrossRef]
23. Ding, X.; Cao, J.; Zhao, X.; Alsaadi, F.E. Finite-time stability of fractional-order complex-valued neural networks with time delays. *Neural Process. Lett.* 2017, 46, 561–580. [CrossRef]
24. Chen, C.; Zhu, S.; Wei, Y.; Yang, C. Finite-time stability of delayed memristor-based fractional-order neural networks. *IEEE Trans. Cybern.* 2018, 50, 1607–1616. [CrossRef] [PubMed]
25. Tyagi, S.; Martha, S. Finite-time stability for a class of fractional-order fuzzy neural networks with proportional delay. *Fuzzy Sets Syst.* 2020, 381, 68–77. [CrossRef]
26. Hu, T.; He, Z.; Zhang, X.; Zhong, S. Finite-time stability for fractional-order complex-valued neural networks with time delay. *Appl. Math. Comput.* 2020, 365, 1–17. [CrossRef]
27. You, X.; Song, Q.; Zhao, Z. Existence and finite-time stability of discrete fractional-order complex-valued neural networks with time delays. *Neural Netw.* 2020, 123, 248–260. [CrossRef]
28. Velmurugan, G.; Rakkiyappan, R.; Cao, J. Finite-time synchronization of fractional-order memristor-based neural networks with time delays. *Neural Netw.* 2020, 1309–1320. [CrossRef]
29. Zheng, M.; Li, L.; Peng, H.; Xiao, J.; Yang, Y.; Zhao, H. Finite-time projective synchronization of memristor-based delay fractional-order neural networks. *Nonlinear Dyn.* 2017, 89, 2641–2655. [CrossRef]
30. Li, X.; Fang, J.; Zhang, W.; Li, H. Finite-time synchronization of fractional-order memristive recurrent neural networks with discontinuous activation functions. *Neurocomputing* 2018, 316, 284–293. [CrossRef]
31. Zheng, M.; Li, L.; Peng, H.; Xiao, J.; Yang, Y.; Zhang, Y.; Zhao, H. Finite-time stability and synchronization of memristor-based fractional-order fuzzy cellular neural networks. *Commun. Nonlinear Sci. Numer. Simul.* 2018, 59, 272–291. [CrossRef]
32. Pratap, A.; Raja, R.; Cao, J.; Rajchakit, G.; Alsaadi, F.E. Further synchronization in finite time analysis for time-varying delayed fractional order memristor competitive neural networks with leakage delay. *Neurocomputing* 2018, 317, 110–126. [CrossRef]
33. Zhang, Y.; Deng, S. Finite-time projective synchronization of fractional-order complex-valued memristor-based neural networks with delay. *Chaos Solitons Fractals* 2019, 128, 176–190. [CrossRef]
34. Zheng, B.; Hu, C.; Yu, J.; Jiang, H. Finite-time synchronization of fully complex-valued neural networks with fractional-order. *Neurocomputing* 2020, 373, 70–80. [CrossRef]
35. Zhang, L.; Yang, Y. Finite time impulsive synchronization of fractional order memristive BAM neural networks. *Neurocomputing* 2020, 384, 213–224. [CrossRef]
36. Xiao, J.; Zhong, S.; Li, Y.; Xu, F. Finite-time Mittag–Leffler synchronization of fractional-order memristive BAM neural networks with time delays. *Neurocomputing* 2017, 219, 431–439. [CrossRef]
37. Xiao, J.; Cao, J.; Cheng, J.; Zhong, S.; Wen, S. Novel methods to finite-time Mittag–Leffler synchronization problem of fractional-order quaternion-valued neural networks. *Inf. Sci.* 2020, 526, 221–244. [CrossRef]
38. Zhang, H.; Ye, R.; Cao, J.; Alsaedi, A. Delay-independent stability of Riemann–Liouville fractional neutral-type delayed neural networks. *Neural Process. Lett.* 2018, 47, 427–442. [CrossRef]
39. Pahnehkolaei, S.M.A.; Alfi, A.; Machado, J.T. Delay-dependent stability analysis of the QUAD vector field fractional order quaternion-valued memristive uncertain neutral type leaky integrator echo state neural networks. *Neural Netw.* 2019, 117, 307–327. [CrossRef]
40. Zhang, L.; Song, Q.; Zhao, Z. Stability analysis of fractional-order complex-valued neural networks with both leakage and discrete delays. *Appl. Math. Comput.* 2017, 298, 296–309. [CrossRef]
41. Huang, C.; Meng, Y.; Cao, J.; Alsaedi, A.; Alsaadi, F.E. New bifurcation results for fractional BAM neural network with leakage delay. *Chaos Solitons Fractals* **2017**, *100*, 31–44. [CrossRef]

42. Huang, C.; Cao, J. Impact of leakage delay on bifurcation in high-order fractional BAM neural networks. *Neural Netw.* **2018**, *98*, 223–235. [CrossRef] [PubMed]

43. Yuan, J.; Zhao, L.; Huang, C.; Xiao, M. Novel results on bifurcation for a fractional-order complex-valued neural network with leakage delay. *Phys. A Stat. Mech. Its Appl.* **2019**, *514*, 868–883. [CrossRef]

44. Ali, M.S.; Narayanan, G.; Shekher, V.; Alsulami, H.; Saeed, T. Dynamic stability analysis of stochastic fractional-order memristor fuzzy BAM neural networks with delay and leakage terms. *Appl. Math. Comput.* **2019**, *369*, 1–23. [CrossRef]

45. Podlubny, I. *Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications*; Academic Press: Cambridge, MA, USA, 1998.

46. Ye, H.; Gao, J.; Ding, Y. A generalized Gronwall inequality and its application to a fractional differential equation. *J. Math. Anal. Appl.* **2007**, *328*, 1075–1081. [CrossRef]

47. Aguila-Camacho, N.; Duarte-Mermoud, M.A.; Gallegos, J.A. Lyapunov functions for fractional order systems. *Commun. Nonlinear Sci. Numer. Simul.* **2014**, *19*, 2951–2957. [CrossRef]

48. Li, Y.; Chen, Y.; Podlubny, I. Stability of fractional-order nonlinear dynamic systems: Lyapunov direct method and generalized Mittag–Leffler stability. *Comput. Math. Appl.* **2010**, *59*, 1810–1821. [CrossRef]