Stiffened panel structural optimization on wing skin of “WHALE” aircraft with local and global buckling criteria

M Ikhsan*, H Syamsudin, M G Suada

Aerospace Engineering, Faculty of Mechanical and Aerospace Engineering, Institut Teknologi Bandung, Indonesia

*mr.ikhsanmuhammad@gmail.com

Abstract. The purpose of this research is to obtain an optimum stiffened panel geometry on wing the skin of the amphibian aircraft “WHALE” with local and global buckling criteria. Local buckling prediction was done based on ESDU while global buckling prediction was based on empirical equation which utilizes radius of gyration of the structure, so that the calculation doesn’t require a high computational capability. Genetic Algorithm (GA) multi-objective optimization was performed to determine the optimum type of stiffener and the geometry of the stiffened panel which provides adequate buckling strength yet lightweight structure. A predetermined value of bending moment on the aircraft wing root was selected as a case study which result in a uniaxial compression load on the stiffened panel. The optimization results show that the more the number of stringers, the efficiency of a stiffened panel in resisting buckling will increase until a certain number of stringer. In the selected case study, integral J-stringer was the optimum type of stiffener with skin thickness of 2.45 mm, stiffener and flange thickness of 1.34 mm, stiffener height of 38.6 mm, stiffener pitch of 80 mm and 11.65 mm stiffener flange width. In comparison with the so called initial sizing method, this optimization shows a promising result of 27% weight reduction.

1. Introduction

In aircraft, where weight is always a critical problem, integrally stiffened structural sections have proved particularly effective as a lightweight, high strength construction [1]. In this research, the type of instability discussed is limited to local buckling and global buckling where local buckling is indicated by the skin waving between stringers in a half-wavelength comparable with the stringer pitch, while global buckling (flexural instability or called Euler mode) is indicated by simple strut instability of the skin-stringer construction in a direction normal to the plane of the skin, the half-wavelength is generally equal to the rib or frame spacing.

There are so many considerations in order to obtain a lightweight stiffened panel; therefore, optimization needs to be done. It is important to choose the most appropriate type of stiffener and the geometry of the stiffened panel in order to obtain a lightweight structure. Genetic algorithms are computerized search and optimization algorithms based on the mechanics of natural genetics and natural selection. Genetic algorithms (GAs) are well suited for solving mixed continuous–discrete variables, and discontinuous and nonconvex design spaces problems, and in most cases they can find the global optimum solution with a high probability [2].
Arunkumar, et al. [3] optimized a stiffened panel by using FEM (Finite Element Method). The thickness of a flat plate thickness was changed until buckling factor 1 was reached, then the model was updated by including stiffener on the previous model. The result shows that the more number of stiffeners on a stiffened panel, the weight is reduced while maintaining its strength as shown in figure 1.

![Figure 1. Stiffened panel weight versus number of stringer by Arunkumar.](image)

In this research, optimization was done on a stiffened panel under a uniaxial compression load. Three types of stiffened panels considered are integral blade stringer, integral J-stringer and built-up Zed-stringer. Genetic Algorithm (GA) multi-objective optimization method was used to obtain the optimum geometry with local and global buckling criteria. The use of ESDU and empirical equation in predicting buckling load was mainly aimed at achieving the optimum structure without the needs of high computational capability.

For simplicity, stiffener pitch would be discretely varied 40, 50, 60, ..., 160 mm. While the other parameters (see subchapter 3.1) are varied as a continuous variables. That is, the skin thickness, stringer thickness, stringer height, stringer flange width and height were simultaneously optimized to obtain the optimum stiffened panel. And then variable stress/area would be used as an additional criterion to determine the optimum stiffened panel geometry. Lastly, the obtained geometry would be compared to stiffened panel geometry obtained from initial sizing method in order to see how much weight reduction could be achieved.

2. Local buckling and global buckling

2.1 Local buckling

ESDU (Engineering Science Data Unit) provides reliable validated engineering design data, methods and software to engineers and designers. In this research, ESDU 70003 and ESDU 71014 (reference [5] and [6]) were used to accurately predict local buckling of the designed stiffened panel. Here are the step by step calculation provided by those two items:

a. Input

The input are stiffened panel geometry and material data.

The material used is D.T.D 5020 as mentioned in chapter 1, while the geometry are as follows:

- Integral blade stringer: t, ts, h, b (see figure 2)
- Integral J-stringer: t, ts, h, b, d, td (see figure 3)
- Built-up Zed-stringer: t, ts, h, b, d, td (see figure 4)

where

- \( t \) = skin thickness (mm)
- \( ts \) = stiffener thickness (mm).
(1)

\( \frac{b}{t} > 5 ; \frac{h}{ts} > 5 ; \frac{d}{ts} > 5 \) 

(2)

\[
(fb)e = K \times E \times \left( \frac{t}{b} \right)^2
\]

(3)

\[ fb = \eta \times (fb)e \]

2.2 Global buckling

Howe, D [8] stated that overall buckling is determined primarily by rib pitch and the second moment of area of a skin-stringer element, and is effectively given by equation:

\[ gb = K \times E \times \left( \frac{b}{L} \right)^2 \]

where

\( gb = \text{global buckling stress (MPa)} \)

\( K = \text{global buckling coefficient} = (\pi \times k/b)^2 \)
With $\kappa = \text{bending radius of gyration of a skin-stringer element} = \sqrt{\frac{\text{inertia}}{\text{Area}}}$

$E = \text{Young’s modulus}$
$L = \text{ribs pitch}$

3. **Optimization formulation**

3.1 **Design variables**

As stated in section 2.1.a, generally there are 6 design variables $t$, $ts$, $h$, $b$, $d$, and $td$. From manufacturing point of view, it is better to define flange thickness ($td$) the same as stringer thickness ($ts$). So that the design variable $x$ becomes

$$x = \begin{cases} (t, ts, h, b)\top &; \text{Integral Blade stringer} \\ (t, ts, h, b, d)\top &; \text{Integral J - stringer and Built – up Zed – stringer} \end{cases}$$

(5)

3.2 **Objective functions**

The main objectives are adequate strength and lightweight structure. In this research, the first objective is defined such that local buckling stress coincide (or as close as possible) with global buckling stress. As stated in [1] that the best designs (of an integrally stiffened panel) are assumed to be those for which the initial buckling and Euler instability stresses coincide. While the second objective is defined as minimum area of stiffened panel cross-section, since the area clearly represents the structure weight. Those two objectives are formulated as follows:

$$Obj (1) = \left(\frac{fb}{gb} - 1\right)^2$$

(6)

The first objective function, $Obj (1)$, is formulated as a quadratic equation since it has one peak (maximum or minimum). The peak is reached when $fb/gb$ equals to 1 which means that local buckling ($fb$) equals to global buckling ($gb$). It is undesirable that the structure has a good resistance to one of the failure modes but susceptible to another failure mode.

$$Obj (2) = A = A_{\text{skin}} + A_{\text{stiffener}}$$

(7)

$$Obj (2) = w \times t + floor\left(\frac{w}{b} + 1\right) \times (h - 0.5 \times t) \times ts + i \times d \times td$$

(8)

where $i =$

0; integral blade stringer

1; integral J - stringer

2; built - up zed stringer

$w = \text{stiffened panel cross-sectional width}$

In the second objective function, $Obj (2)$, the variables $t$, $ts$, $h$, $b$, $d$ and $td$ refer to section 2.1.a, while “floor” is a rounding down to the smaller integer since the stiffened panel cross-section consists of one skin and several stringers.

3.3 **Optimization constraints**

Based on its origin, the constraints are grouped as follows:

a. Data of $K$ (buckling constant)

   The data of $K$ is available on a certain ratio of $h/b$, $ts/t$, $d/h$, $td/ts$

b. ESDU applicability requirement, as stated in section 2.1.b

c. Data of $\eta$ (plasticity reduction factor)

   The data of $\eta$ is available on a certain ratio of $(fb)e/fn$
d. Structural buckling strength
Local buckling and global buckling needs to be higher than the compression load ($P$).
In detail, the constraints is as stated in table 2 as follows.

| Table 1. Optimization constraints. |
|-----------------------------------|
| Origin                      | Integral Blade stringer | Integral J-stringer | Built-up Zed stringer |
|-------------------------------|--------------------------|---------------------|-----------------------|
| Data of $K$                   |                           |                     |                       |
| $0.05 \leq h/b \leq 1$        | $0.1 \leq h/b \leq 1$    | $0.1 \leq h/b \leq 1$ |
| $0.75 \leq ts/t \leq 3$       | $0.5 \leq ts/t < 2$      | $0.5 \leq ts/t < 2$  |
| ESDU Applicability requirements | $b/t > 5$                 | $b/t > 5$             | $b/t > 5$             |
| Data of $\eta$                |                           |                     |                       |
| $0.72 \leq (fb)e/fn < 2$      | $0.6 \leq (fb)e/fn < 2$  | $0.6 \leq (fb)e/fn < 2$ |
| Equality constrain            | $b = 40, 50, 60, \ldots, 160$ mm |
| Structural strength           | $fb*A \geq P$ and $gb*A \geq P$ |

4. Initial Sizing
Initial sizing is based on the direct use of loading data (shear force, bending moment, torsion) to evaluate the initial sizes of the main members of the airframe. This chapter consists of a few equations used in the initial sizing method, while the complete version can be seen in reference [8]. Bending moment on the aircraft wing root cause the upper skin of the wing to undergo compression load. And then the initial size was calculated as follows:

$$t_e = \frac{M}{h w \sigma_b}$$

$t_e = 0.65 \ te$ (approximation)
$\te = 0.68 \ te$
$hs = 16 \ ts$
$b = 2 \ hs$ (ranging from 1.5 \ hs to 5 \ hs)$
$d = 0$ for blade stringer, $0.4$ \ hs for Zed-stringer

where $M = \text{bending moment}$
$h = \text{wingbox height}$
$w = \text{stiffened panel width}$
$\sigma_b = \text{allowable stress}$
$\te = \text{effective thickness}$

5. Case Study
This case study is taken from reference [4] in which the structure to be designed is an upper wing skin of WHALE aircraft. It is a 4-seater amphibian aircraft with one engine as seen in figure 5 in the dashed box.
The ultimate bending moment load on its wing root is 95557.5 Nm and its wingbox height and width are 160 mm and 690 mm respectively. The bending moment caused a uniaxial compression load on its upper wing skin [7] as illustrated in figure 6.

![Figure 6. Stiffened panel under uniaxial compression load [9].](image)

The material used is an isotropic aluminium alloy D.T.D 5020 provided by ESDU as stated in table 1.

| Material Characteristic (see ESDU no.76016) | Value |
|---------------------------------------------|-------|
| m                                          | 15    |

### Table 2. Material data.

| Material                        | Aluminium Alloy D.T.D 5020 |
|---------------------------------|----------------------------|
| Young’s Modulus                 | E 73600 MPa                |
| Poisson’s ratio                 | v 0.34                     |
| Stress at Et = 0.5*E            | fn 317 MPa                 |

6. **Validation**

ESDU provides reliable validated engineering design data, methods and software to engineers and designers. In ESDU 70003, one case example on predicting local buckling stress is given. Therefore, validation was done by comparing this research calculation tools with that case example. In which an unflanged stiffened panel with the dimensions of t = 2.032 mm, ts = 3.81 mm, h = 39.34 mm and b = 63.5 mm was to be manufactured from D.T.D. 5020 aluminium alloy. Table 3 shows that the three
variables (K, η and fb) generated by this research calculation tools are close enough with the one given in ESDU. The error is small (less than 3%) then it is concluded that the local buckling calculation is valid.

Table 3. Calculation validation.

| Information               | Variable                  | ESDU (Case example) | Research code (calculation tools) | Error (%) |
|---------------------------|---------------------------|---------------------|-----------------------------------|-----------|
| Buckling constant         | K                         | 4.51                | 4.57                              | 1.24      |
| Plasticity reduction factor | η                        | 0.917               | 0.896                             | 2.28      |
| Local buckling stress     | fb (MPa)                  | 309.6               | 317.3                             | 2.49      |

7. Result and analysis

7.1 Optimization

Figure 7, figure 8 and figure 9 show the overall result of the optimization. In figure 7, local and global buckling of integral J-stringer and built-up zed-stringer are coincide, while the integral blade stringer is not really coincide to each other but close enough. Integral J-stringer and built-up zed-stringer tend to have a higher buckling stress with the stiffener pitch reduction while the integral blade stringer has a relatively constant buckling stress. It is good to have a high buckling stress but it should not be higher than the material yield stress since the load is an ultimate load.

![Figure 7. Buckling stress versus stiffener pitch result.](image)

Figure 7. Buckling stress versus stiffener pitch result.

Figure 8 shows the area of stiffened panel or in other word the weight of the structure. Figure 8 clearly shows that integral J-stringer is the best stiffener type since it would result in minimum structural weight. In general, the smaller the stiffener pitch, the smaller the structural weight but there is a certain limit to that trend. This result is in good agreement with Arunkumar, et al. [3]. Figure 9 shows the curve of stress/area versus stiffener pitch. The higher the value of stress/area means that the better the structure since the structure would have a higher strength with the same weight or it would have the lower weight with the same strength. Figure 9 also shows that integral J-stringer is the best stiffener in this case study.
In order to choose the best geometry, it is wise to consider the manufacturing point of view. Let’s say that the aircraft wing is not manufactured in one piece to reduce the cost. Then that means the wing would need to be joined to another piece. There would be several hole on it where each hole has its own design rule, that is, the hole should not be close enough to the structural edge. It means that stiffener pitch should not be too small. E. F. Bruhn [10] provided a better understanding about fitting and connection. From the aeroelasticity point of view, namely flutter; the wing skin should have been thicker in order to increase its torsional stiffness. Based on this consideration, stiffened panel with stiffener pitch of 80 mm was chosen as the best geometry. It is concluded that the best stiffened panel in this case study is integral J-stringer with skin thickness of 2.45 mm, stiffener and flange thickness of 1.34 mm, stiffener height of 38.6 mm, stiffener pitch of 80 mm and stiffener flange width of 11.65 mm. In figure 7, 8 and 9 the optimum stiffened panel is marked as red-dashed circle.

For a deeper understanding, let’s see the optimization result of Integral J-stringer stiffened panel in table 4. “Obj 1” and “Obj 2” is as stated in sub chapter 3.2 and variables t, ts, h, b, d, td as stated in section 2.1.a. The closer the stiffener pitch, the smaller the skin thickness. This tendency can be explained by the theory of effective width in which the stringer supports a portion of the skin near the stringer. The closer the stringer pitch, the skin will have more “support” so that the “unsupported” portion will become smaller. Stiffener and flange thickness also has the tendency of becoming smaller, while the stringer height tends to be constant. In relation with equations in chapter 2, we can see that stiffener pitch and skin thickness affect the local buckling the most, while the global buckling is mainly affected by the stringer height since it will increase the stiffened panel inertia.
Table 4. Integral J-stringer optimization result.

| b (mm) | Objective function | Population | num of stringer | Total area (mm^2) | Local buckling (MPa) | Global buckling (MPa) |
|--------|--------------------|------------|-----------------|------------------|----------------------|----------------------|
|        | obj1 | obj2 (mm^2) | t (mm) | ts = td (mm) | h (mm) | b (mm) | d (mm) | A skin | A stringer | Stress (MPa) | Stress (MPa) |
| 160    | 1.4E-08 | 3392 | 4.08 | 2.27 | 38.9 | 160 | 13.9 | 5 | 3816.6 | 575.6 | 197 | 197 |
| 150    | 1.0E-09 | 3147 | 3.75 | 2.27 | 37.8 | 150 | 13.2 | 5 | 2988.6 | 558.1 | 196 | 195 |
| 140    | 4.1E-01 | 3108 | 3.76 | 1.88 | 40.1 | 140 | 16.7 | 5 | 2952.2 | 515.9 | 214 | 214 |
| 130    | 2.3E-14 | 2917 | 3.46 | 1.75 | 39.3 | 130 | 12.7 | 6 | 2389.3 | 527.7 | 206 | 206 |
| 120    | 1.0E-09 | 2780 | 3.38 | 1.67 | 39.0 | 120 | 14.3 | 6 | 2262.5 | 517.3 | 218 | 217 |
| 110    | 9.1E-10 | 2707 | 2.91 | 2.21 | 35.8 | 110 | 11.0 | 7 | 2006.3 | 700.5 | 222 | 221 |
| 100    | 4.8E-10 | 2552 | 2.85 | 1.69 | 38.7 | 100 | 11.9 | 7 | 1969.6 | 582.6 | 237 | 237 |
| 90     | 3.5E-10 | 2402 | 2.68 | 1.37 | 38.5 | 90 | 13.7 | 8 | 1846.1 | 556.2 | 249 | 249 |
| 80     | 9.5E-12 | 2282 | 2.45 | 1.34 | 38.6 | 80 | 11.6 | 9 | 1688.8 | 593.2 | 262 | 262 |
| 70     | 4.0E-10 | 2226 | 2.04 | 1.79 | 34.3 | 70 | 12.2 | 10 | 1411.0 | 814.7 | 268 | 268 |
| 60     | 3.0E-12 | 2099 | 1.90 | 1.42 | 35.1 | 60 | 12.1 | 12 | 1310.9 | 787.9 | 285 | 285 |
| 50     | 2.4E-15 | 2014 | 1.61 | 1.45 | 34.6 | 50 | 10.8 | 14 | 1108.0 | 903.4 | 297 | 297 |
| 40     | 5.9E-11 | 1944 | 1.13 | 1.62 | 32.7 | 40 | 10.6 | 18 | 795.2 | 1248.6 | 292 | 292 |

7.2 Optimization versus initial sizing

Table 5 shows the calculation of initial sizing of a J or Z shaped stiffened panel and the result is on the colored rows. This comparison is done since initial sizing method is an initial guess of the structure size based on strength due to external loading, while the designed structure itself could fail under various failure mode. In this research, two failure modes (local buckling and global buckling) are selected as the design criteria.

Table 5. Initial sizing calculation.

| Wing chord | C | 1350 mm |
| Wingbox width | W | 690 mm |
| Bending moment | M | 9.56E+07 Nmm |
| Wingbox height | H | 160 mm |
| Ribs spacing | L = 0.55.sqrt(C) | 639.04 mm |
| Compression load | P = M/h | 597234 N |
| Young's modulus | E | 73600 MPa |
| Material Function (Machined in DTD 5040) | Ā | 180 Mpa^{0.5} |
| FB | 1.02 - |
| σb (MPa) | 213.7 MPa |
| Te | 4.1 mm |
| Machined Zed stringer (Initial sizing) | t = 0.65 te | 2.6 mm |
| ts = 0.68 te | 2.8 mm |
| hs = 16 ts | 44.1 mm |
| b = 2 hs | 88.1 mm |
| d = 0.4 hs | 17.6 mm |
| Area | 3147 mm^2 |

The optimum result obtained in section 7.1 has cross-sectional area of 2282 mm^2, whereas the initial size method gives an area of 3147 mm^2. So that the optimization is successfully reduced the weight of the structure by 27%.

8. Conclusions and recommendations

8.1 Conclusion

- The smaller the stiffener pitch, the structural weight would be smaller until a certain value.
In the selected case, the best stiffened panel is obtained with integral J-stringer cross-section with skin thickness 2.45 mm, stiffener and flange thickness 1.34 mm, stiffener height 38.6 mm, stiffener pitch 80 mm and stiffener flange width 11.65 mm.

The optimization has successfully reduced the weight of the structure by 27% relative to the method of initial sizing.

8.2 Further work and Recommendations

- Buckling analysis should cover the influence of shear stress.
- The optimization should cover other design criteria such as flutter and wing deflection.

References

[1] Niu, Michael C. Y. 1988: Airframe Structural Design. Hong Kong: Commillit Press, Ltd.
[2] Rao, Singiresu S. 2009. Engineering Optimization: Theory and Practice. Fourth Edition. New Jersey: John Wiley & Sons, Inc.
[3] Arunkumar, K.N. dkk. 2012. Effect of Ribs and Stringer Spacings on the Weight of Aircraft Structure for Aluminum Material. Journal of Applied Sciences, 12: 10061012.
[4] Ikhsan, Muhammad. 2016. Perancangan Struktur Komposit Laminate pada Upper Wing Skin Pesawat “WHALE” dengan Kriteria Tsai-Hill dan Buckling. Tugas Akhir Sarjana, Institut Teknologi Bandung.
[5] ESDU 70003. Local Buckling of Compression Panels with Blade stringer Integral Stiffeners.
[6] ESDU 71014. Local Buckling of Compression Panels with J-stringer Stringers.
[7] Hibbeler, R.C. 2008. Mechanics of Materials, Eight Edition. USA: Pearson Prentice Hall.
[8] Howe, D. 2004. Aircraft Loading and Structural Layout. UK: Professional Engineering Publishing.
[9] Toshimi Taki. 2014. Application of MS-Excel "Solver" to Buckling Analysis of Thin Walled Structures. [ONLINE] Available at: http://www.geocities.jp/toshimi_taki/structure/buckling/buckling.htm. [Accessed 2 July 2018].
[10] Bruhn, E.F. 1973. Analysis and Design of Flight Vehicle Structure. Purdue University