Constraining the range of Yukawa gravity interaction from S2 star orbits II: bounds on graviton mass

A.F. Zakharov, P. Jovanović, D. Borka and V. Borka Jovanović

Abstract. Recently LIGO collaboration discovered gravitational waves [1] predicted 100 years ago by A. Einstein. Moreover, in the key paper reporting about the discovery, the joint LIGO & VIRGO team presented an upper limit on graviton mass such as $m_g < 1.2 \times 10^{-22}\text{eV}$ [1] (see also more details in another LIGO paper [2] dedicated to a data analysis to obtain such a small constraint on a graviton mass). Since the graviton mass limit is so small the authors concluded that their observational data do not show violations of classical general relativity. We consider another opportunity to evaluate a graviton mass from phenomenological consequences of massive gravity and show that an analysis of bright star trajectories could bound graviton mass with a comparable accuracy with accuracies reached with gravitational wave interferometers and expected with forthcoming pulsar timing observations for gravitational wave detection. It gives an opportunity to treat observations of
bright stars near the Galactic Center as a wonderful tool not only for an evaluation specific parameters of the black hole but also to obtain constraints on the fundamental gravity law such as a modifications of Newton gravity law in a weak field approximation. In particular, we obtain bounds on a graviton mass based on a potential reconstruction at the Galactic Center.

**Keywords:** gravity, modified gravity, GR black holes, gravitational waves / theory

**ArXiv ePrint:** 1605.00913
1 Introduction

Recently long-term efforts of theorists and experimentalists have been led to a remarkable discovery of gravitational waves [1]. The result gives also a confirmation of black hole existence in binary systems. The discovery is also an additional brilliant confirmation of general relativity. However, alternative theories of gravity, including massive graviton theories introduced by Fierz and Pauli [3] are a subject of intensive studies and current and future experiments and observations can give constraints on graviton mass. As it was noted by C. Will [4, 5], gravitational wave observations could constrain a graviton mass since in the case of massive graviton a gravitational wave signal is different from a signal of general relativity and analyzing differences for these two curves calculated within these two models (massive and massless ones) one could obtain a graviton mass constraint. Moreover, one could detect a time delay of gravitational wave signal in respect of electromagnetic and neutrino signals in the case if source of gravitational radiation signal is known.

There are many alternative theories of gravity which have been proposed in last years in spite of the evident success of the conventional general relativity (GR) since after 100 years of its development we do not have direct necessities to change GR with its alternative. However, a slow progress in understanding of dark matter (DM) and dark energy (DE) problems stimulates a growth of interest to alternative theories of gravity. Perhaps, there are deviation from Newtonian gravity at the Solar system scales [6, 7] in spite of a model of the thermal origin of the anomaly [8].

A version of a Lorentz invariant massive gravity has been introduced by Fierz and Pauli [3], however, later people found a number of problems with such theories such as existence of ghosts, vDVZ discontinuity [9–11] and some other technical problems [12]. Some of these problems can be overcame [12–21].

Nowadays some of alternative theories do not have the Newtonian limit in a weak gravitational field approximation. Yukawa-like corrections have been obtained, as a general feature, in the framework of $f(R)$ theories of gravity [22]. A set of gravity theories (including so-called massive graviton theories) have a Yukawa limit for a weak gravitational field. We will discuss observational consequences of massive gravity where one can expect a Yukawa type of weak gravitational field limit instead of Newtonian one.

Different experimental and observational ways to bound a graviton mass have been suggested [23–28]. In particular, it was shown that speed of gravity practically coincides with speed of light [29–33], so one could get constraints of a graviton mass. In paper [34] constraints on the range $\lambda$ of Yukawa type force in Solar system has been considered and one can derive the bound $\lambda > 2.8 \times 10^{12}$ km [5] from these constraints assuming a natural
modification of the Newtonian potential \([5, 14]\):

\[
V(r) = -\frac{GM}{(1 + \delta)r} \left[1 + \delta e^{-\frac{r}{\lambda}}\right],
\]

where \(\delta\) is a universal constant. In our previous paper \([35]\) we found constraints on parameters of Yukawa gravity.

Will considered an opportunity to evaluate a graviton mass analyzing a time delay in electromagnetic waves such as supernova or gamma-ray burst \([5]\), moreover earlier he demonstrated a possibility to constrain a graviton mass from gravitational wave signal alone \([4]\).

Pulsar timing may be used to evaluate a graviton mass \([36]\). In the paper it was concluded that, with 90\% probability, massless gravitons can be distinguished from gravitons heavier than \(3 \times 10^{-22}\) eV (Compton wavelength \(\lambda_g = 4.1 \times 10^{12}\) km), if bi-weekly observation of 60 pulsars is performed for 5 years with a pulsar rms timing accuracy of 100 ns and if 10 year observation of 300 pulsars with 100 ns timing accuracy would probe graviton masses down to \(3 \times 10^{-23}\) eV (\(\lambda_g = 4.1 \times 10^{13}\) km). These conclusions are based on an analysis of cross-correlation functions of gravitational wave background. An idea to use pulsar timing for gravitational wave detection has been proposed many years ago \([37]\). An analysis of the cross-correlation function between pulsar timing residuals of pulsar pairs could give an opportunity to detect gravitational waves \([38, 39]\). If a graviton has a mass it gives an impact on cross-correlation functions \([36]\). However, as a first step people have to discover stochastic GW signal and only after a detailed analysis of cross-correlation it could help to put constraints on a graviton mass.

Here we show that our previous results concerning the constraints on parameters of Yukawa gravity, presented in the paper \([35]\), can be extended in the way that one could also obtain a graviton mass bounds from the observations of trajectories of bright stars near the Galactic Center. As it is shown below our estimate of a graviton mass is slightly greater than the estimate obtained by the LIGO collaboration with the first detection of gravitational waves from the binary black hole system. However, we would like to note that: a) our estimate is consistent with the LIGO one; b) in principle, with analysis of trajectories of bright stars near the Galactic Center one could obtain such a graviton mass estimate before the LIGO report \([1]\) about the discovery of gravitational waves and their estimate of a graviton mass; c) in the future our estimate may be improved with forthcoming observational facilities.

## 2 Graviton mass estimates from S2 star orbit

Two groups of observers are monitoring bright stars (including S2 one) to evaluate gravitational potential at the Galactic Center \([40–48]\). Recently, the astrometric observations of S2 star \([49]\) were used to evaluate parameters of black hole and to test and constrain several models of modified gravity at mpc scales \([50–54]\). The simulations of the S2 star orbit around the supermassive black hole at the Galactic Centre (GC) in Yukawa gravity \([35]\) and their comparisons with the NTT/VLT astrometric observations of S2 star \([49]\) resulted with the constraints on the range of Yukawa interaction \(\lambda\), which showed that \(\lambda\) is most likely on the order of several thousand astronomical units. However, it was not possible to obtain the reliable constrains on the universal constant \(\delta\) because its values \(0 < \delta < 1\) were highly correlated to \(\lambda\), while the values \(\delta > 2\) corresponded to a practically fixed \(\lambda \sim 5000–6000\) AU. Such behavior of \(\lambda\) indicate that it can be used to constrain the lower bound for Compton
wavelength $\lambda_g$ of the graviton, and thus the upper bound for its mass $m_g = h c/\lambda_g$, assuming Yukawa gravitational potential of a form $\propto r^{-1} \exp(-r/\lambda_g)$ [see e.g. 4]. The goal of this paper is to find these constraints using chi-square test of goodness of the S2 star orbit fits by Yukawa gravity potential (1.1). 

For that purpose and in order to obtain the reliable statistical criterion, we had to modify the $\chi^2$ measure of goodness of the fit given in [35] to the following expression:

$$\chi^2 = \sum_{i=1}^{n} \left[ \frac{(x_i^o - x_i^c)^2}{\sigma_{xi} + \sigma_{int}} + \frac{(y_i^o - y_i^c)^2}{\sigma_{yi} + \sigma_{int}} \right],$$

(2.1) where $(x_i^o, y_i^o)$ is the $i$-th observed position, $(x_i^c, y_i^c)$ is the corresponding calculated position, $n$ is the number of observations, $\sigma_{xi}$ and $\sigma_{yi}$ are uncertainties of observed positions, while $\sigma_{int}$ accounts for the ”intrinsic dispersion” of the data. $\sigma_{int}$ is usually introduced whenever the observed uncertainties are not mutually consistent, like it is the case with SN Ia data in cosmology (see e.g. [55, 56] and references therein). In our case introduction of $\sigma_{int}$ was necessary because the astrometric accuracy of the S2 star observations changed for more than order of magnitude during the observational period, improving from around 10 mas at the beginning, up to around 0.3 mas at the end. As it will be shown below, $\sigma_{int}$ will not affect the best fit values for $\lambda$, but it will scale $\chi^2$ so that it could be used as a proper statistics for hypothesis testing.

We then performed the new fits of $n = 70$ positions of S2 star observed by NTT/VLT [49] with its simulated orbits in Yukawa gravity potential (1.1), by varying $\lambda$ between 1500 and 20000 AU and assuming two values of $\delta$: $\delta = 1$ (belonging to the region where $\delta$ and $\lambda$ are correlated) and $\delta = 100$ (belonging to the region where there is no correlation between $\delta$ and $\lambda$). In total 4 parameters were fitted: two components of initial position and two components of initial velocity in orbital plane, and thus the number of degrees of freedom is $\nu = 66$ [35]. Fitting is done by minimization of $\chi^2$ given by (2.1), where $\sigma_{int}$ is estimated from the requirement that the global minimum of reduced $\chi^2$ over the whole range of $\lambda$ is $\chi^2/\nu = 1$ (see figure 1). It is found that $\sigma_{int} = 1.13$ mas satisfies this requirement. The resulting values of $\chi^2$ for $\delta = 1$ and $\delta = 100$ as functions of $\lambda$ are presented in figure 1 by red and blue solid curves, respectively. As it can be seen from this figure, $\chi^2$ asymptotically approaches to the corresponding value of the Keplerian fit $\chi^2_{\text{Kepler}} = 71.34$ (horizontal dashed line in figure 1), for the large values of $\lambda$. Besides, $\chi^2$ has global minimum at $\lambda = 5100 \pm 50$ AU $\approx 7.6 \times 10^{11}$ km in the case of $\delta = 1$, and at $\lambda = 7400 \pm 50$ AU $\approx 1.1 \times 10^{12}$ km in the case of $\delta = 100$. Thus, the obtained results for $\lambda$ are consistent with those from [35].

In the next step, we use the obtained values of $\chi^2$ to test the null hypothesis that $\lambda$ should be at least on the order of $10^3$ AU. For testing this hypothesis we assumed the significance level $\alpha = 0.1$ and calculated the critical value $\chi^2_{\nu,\alpha} = 81.08$ for $\nu = 66$ degrees of freedom (horizontal dash-dotted line in figure 1). As it can be seen from figure 1, for both $\chi^2$ curves there is an exclusion range of $\lambda$ with some upper bound $\lambda_x$ where $\chi^2 > \chi^2_{\nu,\alpha}$, so the cases $\lambda < \lambda_x$ can be excluded with very high probability of $1 - \alpha = 90\%$. In the case of $\delta = 1$ this upper exclusion bound is $\lambda_x = 2900 \pm 50$ AU $\approx 4.3 \times 10^{11}$ km, while in the case of $\delta = 100$ it is $\lambda_x = 4300 \pm 50$ AU $\approx 6.4 \times 10^{11}$ km. Since the null hypothesis can be considered as valid for $\lambda > \lambda_x$, this value can be taken as the lower allowed bound for the graviton Compton wavelength $\lambda_g > \lambda_x$. By comparing this result with those presented in figure 8 from [2], one can see that it is in agreement with Solar System and LIGO constraints on $\lambda_g$.

The corresponding upper bound for graviton mass, $m_g = h c/\lambda_x$, according to the fits of the astrometric observations of S2 star by its simulated orbits in Yukawa gravity,
Figure 1. $\chi^2$ (solid lines) as a function of Yukawa range of interaction $\lambda$, i.e. the graviton Compton wavelength $\lambda_g$, obtained from the fits of NTT/VLT observations of S2 star [49] using the gravity potential (1.1) for $\delta = 1$ (red) and $\delta = 100$ (blue). For comparison, the value of the Keplerian fit $\chi^2_{\text{Kepler}} = 71.34$ is also denoted by the horizontal dashed line. The critical value for $\nu = 66$ degrees of freedom and $\alpha = 0.1$ significance level, $\chi^2_{\nu,\alpha} = 81.08$, is presented by the horizontal dash-dotted line, and the upper bounds $\lambda_x$ of the corresponding exclusion regions for $\lambda_g$ by the vertical dotted lines. The values $\lambda_g < \lambda_x$ can be excluded with 90% probability.

is $m_g = 2.9 \times 10^{-21}$ eV in the case of $\delta = 1$ and $m_g = 1.9 \times 10^{-21}$ eV in the case of $\delta = 100$. These constraints are consistent with those obtained from a gravitation wave signal GW150914 recently detected by LIGO [2], and significantly exceeds $1.2 \times 10^{-22}$ eV which represents 90% probability limit for distinguishing massless gravitons (predicted by General relativity) from massive gravitons (predicted by modified gravity theories with Yukawa type of gravitational potential) using pulsar timing experiments [see e.g. 36].

3 Conclusions

In this paper we consider phenomenological consequences of massive gravity and show that an analysis of bright star trajectories could bound the graviton mass. Using simulations of the S2 star orbit around the supermassive black hole at the Galactic Center in Yukawa gravity [35] and their comparisons with the NTT/VLT astrometric observations of S2 star [49] we get the constraints on the range of Yukawa interaction which showed that $\lambda > 4.3 \times 10^{11}$ km. Taking this value as the lower bound for the graviton Compton wavelength, we found that the corresponding most likely upper bound for graviton mass is $m_g < 2.9 \times 10^{-21}$ eV. This result is consistent with the constraints obtained from a gravitation wave signal GW150914 recently detected by LIGO, and significantly exceeds 90% probability limit for distinguishing massless gravitons from massive gravitons using pulsar timing experiments.

Planned observations of trajectories of bright stars near the Galactic Center with GRAVITY [57], E-ELT [58] and TMT [59] may improve the discussed estimates of graviton masses.
Acknowledgments

A.F.Z. thanks a senior scientist fellowship of Chinese Academy of Sciences for a partial support and prof. K. Lee (KIAA PKU) for useful discussion of an opportunity to constrain a graviton mass with observational data from pulsar timing. A.F.Z. thanks also NSF (HRD-0833184) and NASA (NNX09AV07A) at NASA CADRE and NSF CREST Centers (NCCU, Durham, NC, U.S.A.) for a partial support. P.J., D.B. and V.B.J. wish to acknowledge the support by the Ministry of Education, Science and Technological Development of the Republic of Serbia through the project 176003 “Gravitation and the large scale structure of the Universe”.

References

[1] Virgo, LIGO Scientific collaboration, B.P. Abbott et al., Observation of Gravitational Waves from a Binary Black Hole Merger, *Phys. Rev. Lett.* 116 (2016) 061102 [arXiv:1602.03837] [nSPIRE].

[2] Virgo, LIGO Scientific collaboration, B.P. Abbott et al., Tests of general relativity with GW150914, arXiv:1602.03841 [nSPIRE].

[3] M. Fierz and W. Pauli, On relativistic wave equations for particles of arbitrary spin in an electromagnetic field, *Proc. Roy. Soc. Lond.* A 173 (1939) 211 [nSPIRE].

[4] C.M. Will, Bounding the mass of the graviton using gravitational wave observations of inspiralling compact binaries, *Phys. Rev. D* 57 (1998) 2061 [gr-qc/9709011] [nSPIRE].

[5] C.M. Will, The Confrontation between General Relativity and Experiment, *Living Rev. Rel.* 17 (2014) 4 [arXiv:1403.7377] [nSPIRE].

[6] J.D. Anderson, P.A. Laing, E.L. Lau, A.S. Liu, M.M. Nieto and S.G. Turyshev, Indication, from Pioneer 10/11, Galileo and Ulysses data, of an apparent anomalous, weak, long range acceleration, *Phys. Rev. Lett.* 81 (1998) 2858 [gr-qc/9808081] [nSPIRE].

[7] J.D. Anderson, J.K. Campbell, J.E. Ekelund, J. Ellis and J.F. Jordan, Anomalous Orbital-Energy Changes Observed during Spacecraft Flybys of Earth, *Phys. Rev. Lett.* 100 (2008) 091102 [nSPIRE].

[8] S.G. Turyshev, V.T. Toth, G. Kinsella, S.-C. Lee, S.M. Lok and J. Ellis, Support for the thermal origin of the Pioneer anomaly, *Phys. Rev. Lett.* 108 (2012) 241101 [arXiv:1204.2507] [nSPIRE].

[9] V.I. Zakharov, Linearized gravitation theory and the graviton mass, *JETP Lett.* 12 (1970) 312 [nSPIRE].

[10] H. van Dam and M.J.G. Veltman, Massive and massless Yang-Mills and gravitational fields, *Nucl. Phys.* B 22 (1970) 397 [nSPIRE].

[11] Y. Iwasaki, Consistency condition for propagators, *Phys. Rev. D* 2 (1970) 2255 [nSPIRE].

[12] V.A. Rubakov and P.G. Tinyakov, Infrared-modified gravities and massive gravitons, *Phys. Usp.* 51 (2008) 759 [arXiv:0802.4379] [nSPIRE].

[13] A.I. Vainshtein, To the problem of nonvanishing gravitation mass, *Phys. Lett.* B 39 (1972) 393 [nSPIRE].

[14] M. Visser, Mass for the graviton, *Gen. Rel. Grav.* 30 (1998) 1717 [gr-qc/9706051] [nSPIRE].

[15] G.R. Dvali, G. Gabadadze and M. Porrati, Metastable gravitons and infinite volume extra dimensions, *Phys. Lett.* B 484 (2000) 112 [hep-th/0002190] [nSPIRE].
[16] I.I. Kogan, S. Mouslopoulos, A. Papazoglou and G.G. Ross, \textit{Multi-brane worlds and modification of gravity at large scales}, \textit{Nucl. Phys. B} \textbf{595} (2001) 225 [\texttt{hep-th/0006030}] [SPIRE].

[17] C. Deffayet, G.R. Dvali, G. Gabadadze and A.I. Vainshtein, \textit{Nonperturbative continuity in graviton mass versus perturbative discontinuity}, \textit{Phys. Rev. D} \textbf{65} (2002) 044026 [\texttt{hep-th/0106001}] [SPIRE].

[18] L.S. Finn and P.J. Sutton, \textit{Bounding the mass of the graviton using binary pulsar observations}, \textit{Phys. Rev. D} \textbf{65} (2002) 044022 [\texttt{gr-qc/0109049}] [SPIRE].

[19] L.S. Finn and P.J. Sutton, \textit{Bounding the mass of the graviton using binary pulsar observations}, \textit{Phys. Rev. D} \textbf{65} (2002) 044022 [\texttt{gr-qc/0109049}] [SPIRE].

[20] M. Maggiore, \textit{Gravitational Waves}, vol. VI, Oxford University Press, Oxford U.K. (2008).

[21] K. Hinterbichler, \textit{Theoretical Aspects of Massive Gravity}, \textit{Rev. Mod. Phys.} \textbf{84} (2012) 671 [arXiv:1105.3735] [SPIRE].

[22] S. Capozziello, A. Stabile and A. Troisi, \textit{A General solution in the Newtonian limit of f(R)-gravity}, \textit{Mod. Phys. Lett. A} \textbf{24} (2009) 659 [arXiv:0901.0448] [SPIRE].

[23] A.S. Goldhaber and M.M. Nieto, \textit{Mass of the graviton}, \textit{Phys. Rev. D} \textbf{9} (1974) 1119 [SPIRE].

[24] P.G.O. Freund, A. Maneshwari and E. Schonberg, \textit{Finite-Range Gravitation}, \textit{Astrophys. J.} \textbf{157} (1969) 857.

[25] A.A. Logunov and M.A. Mestvirishvili, \textit{Relativistic theory of gravitation}, \textit{Theor. Math. Phys.} \textbf{61} (1984) 1170.

[26] S. S. Gershtein, A. A. Logunov and M. A. Mestvirishvili, \textit{Gravitational field self-limitation and its role in the Universe}, \textit{Phys. Usp.} \textbf{49} (2006) 1179.

[27] S.S. Gershtein, A.A. Logunov and M.A. Mestvirishvili, \textit{General relativity and the Schwarzschild singularity}, \textit{Phys. Part. Nucl.} \textbf{39} (2008) 1 [SPIRE].

[28] A.S. Goldhaber and M.M. Nieto, \textit{Photon and Graviton Mass Limits}, \textit{Rev. Mod. Phys.} \textbf{82} (2010) 939 [arXiv:0809.1003] [SPIRE].

[29] S.M. Kopeikin, \textit{Testing relativistic effect of propagation of gravity by very long baseline interferometry}, \textit{Astrophys. J.} \textbf{598} (2003) 704 [astro-ph/0302294] [SPIRE].

[30] E.B. Fomalont and S.M. Kopeikin, \textit{The measurement of the light deflection from jupiter: experimental results}, \textit{Astrophys. J.} \textbf{598} (2003) 704 [astro-ph/0302294] [SPIRE].

[31] S.M. Kopeikin, \textit{Speed of gravity in general relativity and theoretical interpretation of the Jovian deflection experiment}, \textit{Class. Quant. Grav.} \textbf{21} (2004) 3251 [gr-qc/0310059] [SPIRE].

[32] C. Lammerzahl, A. Macias and H. Mueller, \textit{Lorentz invariance violation and charge (non-)conservation: A General theoretical frame for extensions of the Maxwell equations}, \textit{Phys. Rev. D} \textbf{71} (2005) 025007 [gr-qc/0501048] [SPIRE].

[33] S.M. Kopeikin, \textit{Comments on the paper by S. Samuel on the speed of gravity and the jupiter/quasar measurement}, \textit{Int. J. Mod. Phys. D} \textbf{15} (2006) 273 [gr-qc/0501001] [SPIRE].

[34] C. Talmadge, J.P. Berthias, R.W. Hellings and E.M. Standish, \textit{Model Independent Constraints on Possible Modifications of Newtonian Gravity}, \textit{Phys. Rev. Lett.} \textbf{61} (1988) 1159 [SPIRE].

[35] D. Borka, P. Jovanović, V.B. Jovanović and A.F. Zakharov, \textit{Constraining the range of Yukawa gravity interaction from S2 star orbits}, \textit{JCAP} \textbf{11} (2013) 050 [arXiv:1311.1404] [SPIRE].

[36] K. Lee, F.A. Jenet, R.H. Price, N. Wex and M. Krämer, \textit{Detecting massive gravitons using pulsar timing arrays}, \textit{Astrophys. J.} \textbf{722} (2010) 1589 [arXiv:1008.2561] [SPIRE].

[37] M.V. Sazhin, \textit{Opportunities for detecting ultralong gravitational waves}, \textit{Sov. Astron.} \textbf{22} (1978) 36.
[38] F.A. Jenet, G.B. Hobbs, K.J. Lee and R.N. Manchester, Detecting the stochastic gravitational wave background using pulsar timing, *Astrophys. J.* 625 (2005) L123 [astro-ph/0504458] [SPIRE].

[39] K.J. Lee, F.A. Jenet and R.H. Price, Pulsar Timing as a Probe of Non-Einsteinian Polarizations of Gravitational Waves, *Astrophys. J.* 685 (2008) 1304.

[40] A. Ghez, M. Morris, E.E. Becklin, T. Kremenek and A. Tanner, The Accelerations of stars orbiting the Milky Way’s central black hole, *Nature* 407 (2000) 349 [astro-ph/0009339] [SPIRE].

[41] G.F. Rubilar and A. Eckart, Periastron shifts of stellar orbits near the Galactic Center, *Astron. Astrophys.* 374 (2001) 95.

[42] R. Schödel et al., A Star in a 15.2 year orbit around the supermassive black hole at the center of the Milky Way, *Nature* 419 (2002) 694 [SPIRE].

[43] A.M. Ghez et al., The first measurement of spectral lines in a short-period star bound to the galaxy’s central black hole: A paradox of youth, *Astrophys. J.* 586 (2003) L127 [astro-ph/0302299] [SPIRE].

[44] A.M. Ghez et al., Variable infrared emission from the supermassive black hole at the center of the Milky Way, *Astrophys. J.* 601 (2004) L159 [astro-ph/0309076] [SPIRE].

[45] A.M. Ghez et al., Measuring Distance and Properties of the Milky Way’s Central Supermassive Black Hole with Stellar Orbits, *Astrophys. J.* 689 (2008) 1044 [arXiv:0808.2870] [SPIRE].

[46] S. Gillessen, F. Eisenhauer, S. Trippe, T. Alexander, R. Genzel, F. Martins et al., Monitoring stellar orbits around the Massive Black Hole in the Galactic Center, *Astrophys. J.* 692 (2009) 1075 [arXiv:0810.4674] [SPIRE].

[47] S. Gillessen et al., A gas cloud on its way towards the supermassive black hole in the Galactic Centre, *Nature* 481 (2012) 51 [arXiv:1112.3264] [SPIRE].

[48] L. Meyer et al., The Shortest Known Period Star Orbiting our Galaxy’s Supermassive Black Hole, *Science* 338 (2012) 84 [arXiv:1210.1294] [SPIRE].

[49] S. Gillessen et al., The orbit of the star S2 around SgrA* from VLT and Keck data, *Astrophys. J.* 707 (2009) L114 [arXiv:0910.3069] [SPIRE].

[50] A.F. Zakharov, A.A. Nucita, F. De Paolis and G. Ingrosso, Apostron Shift Constraints on Dark Matter Distribution at the Galactic Center, *Phys. Rev. D* 76 (2007) 062001 [arXiv:0707.4423] [SPIRE].

[51] A.F. Zakharov, S. Capozziello, F. De Paolis, G. Ingrosso and A.A. Nucita, The Role of Dark Matter and Dark Energy in Cosmological Models: Theoretical Overview, *Space Sci. Rev.* 48 (2009) 301.

[52] D. Borka, P. Jovanovic, V.B. Jovanovic and A.F. Zakharov, Constraints on $R^n$ gravity from precession of orbits of S2-like stars, *Phys. Rev. D* 85 (2012) 124004 [arXiv:1206.0851] [SPIRE].

[53] A. F. Zakharov, Possible Alternatives to the Supermassive Black Hole at the Galactic Center, *J. Astrophys. Astron.* 36 (2015) 539.

[54] A.F. Zakharov, Is there an ordinary supermassive black hole at the Galactic Center?, *Gravitation, Astrophysics, and Cosmology* (2016) 176, proceedings of the Twelfth Asia-Pacific International Conference on Gravitation, Astrophysics, and Cosmology, Moscow, Russia, 28 Jun–5 July 2015.

[55] SNLS collaboration, P. Astier et al., The Supernova legacy survey: Measurement of $\omega(m)$, $\omega(\lambda)$ and $W$ from the first year data set, *Astron. Astrophys.* 447 (2006) 31 [astro-ph/0510447] [SPIRE].
[56] A.F. Zakharov and V.N. Pervushin, *Conformal cosmological model and SNe Ia data*, *Phys. Atom. Nucl.* 75 (2012) 1418.

[57] N. Blind et al., *GRAVITY: the VLTI 4-beam combiner for narrow-angle astrometry and interferometric imaging*, arXiv:1503.07303.

[58] A. Ardeberg et al., *An Expanded View of the Universe Science with the European Extremely Large Telescope*, M. Lyubenova and M. Kissler-Patig eds., ESO, Munich Germany (2009).

[59] W. Skidmore et al., *Thirty Meter Telescope Detailed Science Case: 2015*, TMT.PSC.TEC.07.007.REL02, TMT Observatory Corporation (2015).