A light-front coupled cluster method\textsuperscript{a}

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Abstract

A new method for the nonperturbative solution of quantum field theories is described. The method adapts the exponential-operator technique of the standard many-body coupled-cluster method to the Fock-space eigenvalue problem for light-front Hamiltonians. This leads to an effective eigenvalue problem in the valence Fock sector and a set of nonlinear integral equations for the functions that define the exponential operator. The approach avoids at least some of the difficulties associated with the Fock-space truncation usually used.

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I. INTRODUCTION

Nonperturbative Hamiltonian methods that rely on Fock-space truncation are plagued by uncanceled divergences. The truncation removes contributions from couplings to higher Fock sectors that would otherwise cancel against contributions that are kept. When these contributions separately diverge with respect to an infinite-regulator-scale limit, an uncanceled divergence is born. For example, the Ward identity in QED requires contributions that differ in the number of photons required, as illustrated in Fig. 1; a truncation in photon number will break the Ward identity [1] and introduce an uncanceled divergence.

One approach to the handling of uncanceled divergences is to not take the infinite-regulator limit and instead seek a range of regulator scales over which physical quantities are slowly varying [2]. In this range, one has a compromise between errors due to truncation, which increase as the regulator scale is increased, and errors due to the presence of the regulator, which decrease as the scale is increased. This approach has been used in nonperturbative calculations of the anomalous magnetic moment of the electron regulated by inclusion of Pauli–Villars (PV) fields [2, 3].

An alternative approach is sector-dependent renormalization [4], where the bare parameters are allowed to depend on the Fock sector. This absorbs the uncanceled divergences into redefinitions of the coupling constants. Calculations of the anomalous moment have been done this way [5, 6]. However, the Fock-state wave functions become ill-defined without proper normalization [7], and the bare coupling is driven to zero at finite regulator scales [5]. In fact, the sector-dependent charge renormalization is not ordinary charge renormalization; it occurs in truncated QED without the presence of fermion loops and is the result of wave function renormalization in the broken Ward identity.

To avoid these difficulties we consider a new method, the light–front coupled-cluster (LFCC) method [8], that does not involve Fock-space truncation. It is instead based on the exponential-operator technique of the standard many-body coupled-cluster (CC) method [9] frequently employed in nuclear and chemical physics [10]. We construct an eigenstate of a light-front Hamiltonian as $\sqrt{Z} e^T \langle \phi |$, where $\langle \phi |$ is a valence state with a minimum number of constituents, $T$ is an operator that increases particle number without violating any conservation laws, and $\sqrt{Z}$ is a normalization factor. The truncation that is made is in $T$ but not in the exponentiation, so that all relevant Fock states are retained. This infinite expansion in Fock states necessitates some care in the computation of matrix elements, to avoid doing an infinite sum, but the techniques of the standard CC method can be used, with some extensions, to do the computations.

In the standard CC method $\langle \phi |$ is a product of single-particle states and terms in $T$.

FIG. 1. Graphs that contribute to the Ward identity at lowest order. The dashed vertical lines mark intermediate states. Only the first graph survives a truncation to Fock states with only one photon.
annihilate states in $|\phi\rangle$ and create excited states, to build in correlations. The operator $T$ is then truncated at some number of excitations, usually no more than four. The focus is on finding the solution in the valence sector with a large but fixed number of particles. In contrast, in the LFCC method, the focus is on finding a solution in the infinite Fock space that has the same quantum numbers as the valence state $|\phi\rangle$. The number of particles in the valence state not large, and how this state is to be determined is left unspecified.

Some applications to field theory of the CC method have been previously considered \[11\], for Fock-state expansions in equal-time quantization. The focus is on the non-trivial vacuum structure, which is avoided in light-front quantization, and particle states are then built on the vacuum. There was some success in the analysis of $\phi_{1+1}^4$, particularly of symmetry-breaking effects.

For the LFCC method, light-front quantization is crucial, not only for the simple vacuum but also because the internal motion of a coupled system can be factored from the external motion. Of course, this separation can be done in the nonrelativistic CC method but not for a relativistic method in equal-time quantization. Obviously, we then use light-cone coordinates \[12, 13\], which we define as $x^+ = t + z$ for time, $x = (x^-, \vec{x}_\perp)$ for space, with $x^- \equiv t - z$ and $\vec{x}_\perp = (x, y)$. The light-cone energy is $p^- = E - p_z$, and the momentum is $p = (p^+, \vec{p}_\perp)$, with $p^+ \equiv E + p_z$ and $\vec{p}_\perp = (p_x, p_y)$. These bring the mass-shell condition $p^2 = m^2$ to the form $p^- = \frac{m^2 + p^2}{p^+}$. The Hamiltonian eigenvalue problem is

$$P^- |\psi\rangle = \frac{M^2 + P^2}{P^+} |\psi\rangle. \quad (1.1)$$

In the next Section, we give a more detailed description of the LFCC method. This is followed in Sec. \[14\] by a formulation of light-front QED in an arbitrary gauge \[15\]. In Sec. \[16\] the two formulations are applied to a calculation of the electron’s anomalous moment. A brief summary is given in Sec. \[17\]. Additional details can be found elsewhere in these proceedings \[14\].

II. LFCC METHOD

We wish to solve the fundamental eigenvalue problem \[1.1\] and compute physical quantities from matrix elements between eigenstates. The starting point is to write an eigenstate as $|\psi\rangle = \sqrt{Z} e^T |\phi\rangle$. The factor $Z$ maintains the chosen normalization, $\langle \psi | \psi \rangle = \delta (P' - P)$. The valence state $|\phi\rangle$ is also normalized: $\langle \phi' | \phi \rangle = \delta (P' - P)$. The operator $T$ contains terms that only increase particle number, such that it conserves $J_z$, light-front momentum $P_+$ charge, and any other conserved quantum number. Because $p^+$ is always positive, $T$ must include annihilation, and, therefore, powers of $T$ include contractions. These contractions will introduce loop corrections.

To solve the eigenvalue problem, we construct an effective Hamiltonian $\overline{P}^- \equiv e^{-T} P^- e^T$ and let $P_\nu$ project onto the valence Fock sector. With these definitions, we arrive at the coupled system

$$P_\nu \overline{P}^- |\phi\rangle = \frac{M^2 + P^2}{P^+} |\phi\rangle, \quad (1 - P_\nu) \overline{P}^- |\phi\rangle = 0. \quad (2.1)$$

The second equation determines the operator $T$, which then fully defines the effective Hamiltonian in the first equation. The two equations are essentially a coupled set of integral
equations for the valence state and the functions that define the terms in $T$. In general, they must be solved by numerical methods.

The expectation value for an operator $\hat{O}$ is given by

$$\langle \hat{O} \rangle = \frac{\langle \phi | e^{T^\dagger} \hat{O} e^T | \phi \rangle}{\langle \phi | e^{T^\dagger} e^T | \phi \rangle}. \quad (2.2)$$

Direct computation requires infinite sums. Following the techniques of the CC method [10], we instead define $\tilde{O} = e^{-T} \hat{O} e^T$ and

$$\langle \tilde{\psi} | = \langle \phi | \frac{e^{T^\dagger} e^T}{\langle \phi | e^{T^\dagger} e^T | \phi \rangle}. \quad (2.3)$$

We then find that $\langle \hat{O} \rangle = \langle \tilde{\psi} | \tilde{O} | \phi \rangle$ and

$$\langle \tilde{\psi}' | = \langle \phi' | \frac{e^{T^\dagger} e^T}{\langle \phi | e^{T^\dagger} e^T | \phi \rangle} | \phi \rangle = \delta(P' - P). \quad (2.4)$$

The effective operator $\overline{O}$ can be computed from its Baker–Hausdorff expansion

$$\overline{O} = \hat{O} + [\hat{O}, T] + \frac{1}{2} [[\hat{O}, T], T] + \cdots. \quad (2.5)$$

The bra $\langle \tilde{\psi} |$ is a left eigenvector of $\overline{P}^-$, as can be seen from the following:

$$\langle \tilde{\psi} | \overline{P}^- = \langle \phi | \frac{e^{T^\dagger} P^- e^T}{\langle \phi | e^{T^\dagger} e^T | \phi \rangle} = \langle \phi | \overline{P}^- \frac{e^{T^\dagger} e^T}{\langle \phi | e^{T^\dagger} e^T | \phi \rangle} = \frac{M^2 + P_+^2}{P} \langle \tilde{\psi} |. \quad (2.6)$$

We still have an infinite system of equations. To arrive at a finite set, we truncate $T$ at a fixed increase in particle count and truncate the projection $(1 - P_v)$ to a set of Fock sectors above the valence sector that yield enough equations to solve for the functions in $T$. This leaves a finite system of nonlinear equations for the functions in $T$ and in the valence state. The effective Hamiltonian $\overline{P}^- = e^{-T} \overline{P}^- e^T$ has only a finite number of terms in its Baker–Hausdorff expansion, $\overline{P}^- = \overline{P}^- + [\overline{P}^-, T] + \cdots$ and is computed from these commutators of $T$ and $\overline{P}^-$. Similarly, the expansion of an effective operator $\overline{O}$ has only a finite number of terms. The left-hand eigenstate $|\tilde{\psi}\rangle$ is also truncated to a consistent set of Fock sectors, no more than found in $T|\phi\rangle$.

III. QED IN AN ARBITRARY COVARIANT GAUGE

In order to facilitate a check of gauge invariance for physical observables, we quantize light-front QED in an arbitrary covariant gauge [15]. The PV-regulated Lagrangian is [3]

$$\mathcal{L} = \sum_{i=0}^{2} (-1)^i \left[ -\frac{1}{4} F_{i, \mu \nu} F_{i, \mu \nu} + \frac{1}{2} \mu_i A_i^\mu A_i^\mu - \frac{1}{2} \zeta (\partial^\mu A_i^\mu)^2 \right] + \sum_{i=0}^{2} (-1)^i \tilde{\psi}_i (i \gamma^\mu \partial_\mu - m_i) \psi_i - e \tilde{\psi} \gamma^\mu \psi A_\mu, \quad (3.1)$$
where the fermion and photon fields \( \psi_0 \) and \( A_0 \) are the physical fields and indices of 1 and 2 indicate PV fields. The interaction term is built from null combinations

\[
\psi = \sum_{i=0}^{2} \sqrt{\beta_i} \psi_i, \quad A_\mu = \sum_{i=0}^{2} \sqrt{\xi_i} A_{i\mu}.
\] (3.2)

The coupling coefficients satisfy the constraints

\[
\xi_0 = 1, \quad \sum_{i=0}^{2} (-1)^i \xi_i = 0, \quad \beta_0 = 1, \quad \sum_{i=0}^{2} (-1)^i \beta_i = 0,
\] (3.3)

with \( \xi_2 \) and \( \beta_2 \) fixed by chiral symmetry restoration\(^{[15, 16]}\) and zero photon mass\(^{[17]}\).

To quantize, we apply a light-front analog of Stueckelberg quantization\(^{[18]}\). Consider the Lagrangian of a free massive vector field: \( \mathcal{L} = -\frac{i}{4} F^2 + \frac{1}{2} \mu^2 A^2 - \frac{1}{2} \zeta (\partial \cdot A)^2 \). The field equation is \( (\Box + \mu^2) A_\mu - (1 - \zeta) \partial_\mu (\partial \cdot A) = 0 \), and the light-front Hamiltonian density is

\[
\mathcal{H} = \mathcal{H}|_{\xi=0} + \frac{1}{2} (1 - \zeta) (\partial \cdot A) (\partial_\mu A^\mu - 2 \partial_\perp A^+ - 2 \partial_\perp \cdot \vec{A}_{\perp}),
\] (3.4)

with \( \mathcal{H}|_{\xi=0} = \frac{1}{2} \sum_{\mu=0}^{3} e^\mu [ (\partial_\perp A^\mu)^2 + \mu^2 (A^\mu)^2 ] \) and \( e^\mu = (-1, 1, 1, 1) \). The field equation is satisfied by

\[
A_\mu(x) = \int \frac{dk}{4\pi^2 k^+} \left\{ \sum_{\lambda=1}^{3} e^{(\lambda)}(k) \left[ a_\lambda(k)e^{-ik_\perp} + a_\lambda^\dagger(k)e^{ik_\perp} \right] + e^{(0)}(k) \left[ a_0(k)e^{-ik_\perp} + a_0^\dagger(k)e^{ik_\perp} \right] \right\}
\] (3.5)

with \( \mu_\lambda = \mu \) for \( \lambda = 1, 2, 3 \), but \( \mu_0 = \bar{\mu} \equiv \mu/\sqrt{\zeta} \), \( \vec{k} = \vec{k}_\perp \), and \( \vec{k}^- = (k_\perp + \bar{\mu}^2)/k^+ \). The polarization vectors

\[
e^{(1,2)}(k) = (0, 2 \hat{e}_{1,2} \cdot \vec{k}_\perp/k^+, \hat{e}_{1,2}), \quad e^{(3)}(k) = \frac{1}{\mu} \left( (k_\perp^2 - \mu^2)/k^+, k^+, \vec{k}_\perp \right), \quad e^{(0)}(k) = \vec{k}/\mu,
\] (3.6)

satisfy \( k \cdot e^{(\lambda)} = 0 \) and \( e^{(\lambda)} \cdot e^{(\lambda')} = -\delta_{\lambda\lambda'} \) for \( \lambda, \lambda' = 1, 2, 3 \). The first term in \( A_\mu \) satisfies \( (\Box + \mu^2) A_\mu = 0 \) and \( \partial_\perp \cdot A = 0 \). The \( \lambda = 0 \) term violates each, but the field equation is satisfied. The nonzero commutators are \( [a_\lambda(k), a_{\lambda'}(k')] = e^\lambda \delta_{\lambda\lambda'} (\vec{k} - \vec{k}') \). The resulting light-front Hamiltonian is

\[
\mathcal{P}^- = \int d^3x \mathcal{H}|_{x^+ = 0} = \int \frac{dk}{4\pi^2} \sum_{\lambda} e^{\lambda \hat{k}^2 / k^+ + \mu^2} a_{\lambda}^\dagger(k) a_\lambda(k).
\] (3.7)

The nondynamical components of the fermion fields satisfy the constraints \( (i = 0, 1, 2) \)

\[
i(-1)^i \partial_\perp \psi_i - e A_\perp \sqrt{\beta_i} \sum_j \psi_j = (i \gamma^0 \gamma^\perp) \left[ (-1)^i \partial_\perp \psi_i + i e A_\perp \sqrt{\beta_i} \sum_j \psi_j \right] - (-1)^i m_i \gamma^0 \psi_i^+. \] (3.8)

When multiplied by \( (-1)^i \sqrt{\beta_i} \) and summed over \( i \), this becomes

\[
i \partial_\perp \psi_\perp = (i \gamma^0 \gamma^\perp) \partial_\perp \psi_\perp - \gamma^0 \sum_i m_i \psi_i^+, \] (3.9)
which is the same constraint as for a free fermion field, in any gauge.

Thus, the light-front QED Hamiltonian can be constructed in any covariant gauge, to obtain
\[
P^- = \sum_{i,s} \int dp \frac{m_i^2 + p_i^2}{p^+} (-1)^j b_{i,s}^\dagger(p) b_{i,s}(p) + \sum_{i,s} \int dk \frac{\mu_i^2 + k_i^2}{k^+} (-1)^j \epsilon_i a_i(k) a_s(k),
\]

IV. APPLICATION OF THE LFCC METHOD

We briefly discuss an application to the calculation of the anomalous magnetic moment of the electron. For simplicity, Fock space is truncated to exclude positrons. The valence state is \( |\phi^\pm(p)\rangle = \sum_i z_i b_{i,s}^\dagger(p) |0\rangle \). The T operator is truncated to be
\[
T = \sum_{ijs\lambda} \int dyd\vec{k}_\perp \frac{dp}{\sqrt{16\pi^3 p^+}} \phi_i^s \lambda \sigma_{ijl} \sigma_s^l \sigma_{ijs} \langle \lambda | y^p, y\vec{p}_\perp + \vec{k}_\perp, \vec{k}_\perp | \phi_s \rangle
\]
After exclusion of terms that annihilate the valence state, the effective Hamiltonian is
\[
\overline{P}^- = \sum_{ijs} \int dp (-1)^j \left[ \delta_{ij} \frac{m_i^2 + p_i^2}{p^+} + I_{ji} \right] b_{j,s}^\dagger(p) b_{i,s}(p)
\]
\[
\quad + \sum_{ijs\lambda} \int dyd\vec{k}_\perp \frac{dp}{\sqrt{16\pi^3 p^+}} \left\{ h_{ijl}^s \lambda (y, \vec{k}_\perp) + \frac{1}{2} V_{ijl}^s \lambda (y, \vec{k}_\perp) \right\}
\]
\[
\quad + \left[ \frac{m_j^2 + k_j^2}{1 - y} + \frac{\mu_j^2 + k_j^2}{y} - \frac{m_j}{y} \right] \phi_{ijl}^s \lambda (y, \vec{k}_\perp)
\]
\[
\quad + \frac{1}{2} \sum_{ij'} \left[ I_{jk'} \frac{1}{1 - y} \phi_{ij'}^s \lambda (y, \vec{k}_\perp) - \sum_j (-1)^{i+j'} \phi_{ij'}^s \lambda (y, \vec{k}_\perp) I_{jk'} \right]
\]
\[
\quad \times a_i^\dagger(y^p, y\vec{p}_\perp + \vec{k}_\perp, \vec{k}_\perp) b_{js}^\dagger((1 - y)p^+,(1 - y)\vec{p}_\perp - \vec{k}_\perp) b_{i,s}(p) + \text{H.c.}
\]
with the self-energy \( I_{ji} \) and the vertex loop correction \( V_{ijl}^s \lambda \) defined in [14].

The projection \( P_{\epsilon} \overline{P}^- |\phi^\pm(p)\rangle = \frac{M_a^2 + p_i^2}{p^+} |\phi(p)\rangle \) of the eigenvalue problem yields
\[
m_i^2 + \sum_j I_{ij} \zeta_{ij} = M_a^2 \zeta_{ai},
\]

Projection onto \( |e\gamma\rangle \), orthogonal to \( |\phi\rangle \), gives
\[
\left[ \frac{M_a^2 - M_b^2 + k^2}{1 - y} - \frac{\mu_a^2 + k^2}{y} \right] C_{abl}^{s \lambda} (y, \vec{k}_\perp) = H_{ab}^{s \lambda} (y, \vec{k}_\perp) + \frac{1}{2} \left[ V_{abl}^{s \lambda} (y, \vec{k}_\perp) - \sum_l \frac{I_{bl}^{s \lambda}}{1 - y} C_{abl}^{s \lambda} (y, \vec{k}_\perp) \right],
\]

(4.4)
with

\[ C_{abl}^{\pm s\lambda}(y, \vec{k}_\perp) \equiv \sum_{ij} (-1)^{i+j} z_a^i z_b^j t_{ij} \pm s_{abl}(y, \vec{k}_\perp). \] (4.5)

Notice that \( C_{abl}^{\pm s\lambda} \), the analog of the two-body wave function, satisfies an equation where the bare masses have been replaced by the physical masses \( M_a \). Thus, the LFCC method provides a natural way for physical masses to enter the calculation instead of having them imposed by a sector-dependent renormalization. Also, although the \( T \) operator has been truncated to a single photon emission, the projected eigenvalue problem retains the self-energy corrections and vertex corrections necessary for the Ward identity to be satisfied.

V. SUMMARY

Details of the application to QED can be found in [14]. The result for the anomalous moment agrees with the Schwinger correction at first order and will sum to all orders in \( \alpha \) many of the higher-order corrections.

The key advantage of the LFCC method is that it avoids Fock-space truncations that can induce uncanceled divergences and other inconsistencies, such as Fock-sector dependence and spectator dependence. The approach is systematically improvable, by the addition of terms to the truncated \( T \) operator.

The complete calculation of the dressed-electron state and its anomalous moment, within the given truncation of \( T \), is in progress. It requires a numerical solution of the right-hand and left-hand eigenvalue problems, as well as numerical quadrature for the matrix element that yields the anomalous moment. Work beyond this can include investigation of the dressed-photon state, extension of the dressed-electron state to include \( e^+e^- \) pairs, and study of muonium, positronium, and symmetry breaking in scalar theories. Of particular interest, of course, is the adaptation of these methods to QCD; the LFCC method is general enough that it can be applied once a suitably regulated light-front Hamiltonian is constructed.

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