Reflection of electromagnetic wave from the boundary of the piezoelectric half-space with cubic symmetry

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Abstract. A large number of works have been devoted to investigation of the influence of the piezoelectric properties of a material on the propagation of elastic waves [1–3]. Herewith, the quasi-static piezoelasticity model was mainly used. In the problem of an electromagnetic wave reflection from an elastic medium with piezoelectric properties, it is necessary to consider hyperbolic equations [4].

1. Governing equations and problem statement

In a rectangular Cartesian coordinate system $(x, y, z)$, a half-space of a material with piezoelectric properties of cubic symmetry occupies the region $(-\infty < x < \infty, 0 \leq y < \infty, -\infty < z < \infty)$.

In [5], a quasi-hyperbolic model was proposed to investigate the wave propagation in piezoelectric media with cubic symmetry. This model is a simplification of an exact system of equations preserving the property of being hyperbolic.

An analogous quasihyperbolic model for piezoelectrics of a hexagonal system was proposed earlier in [6].

According to [2], the equation of propagation of shear electroelastic waves, has the form

\[
\begin{align*}
c_{44} \Delta w + 2\epsilon_{14} \frac{\partial^2 \varphi}{\partial x \partial y} &= \rho \frac{\partial^2 w}{\partial t^2}, \\
\Delta \varphi - 2\epsilon_{14} \frac{\partial^2 w}{\partial x \partial y} &= \varepsilon \mu \frac{\partial^2 \varphi}{\partial t^2},
\end{align*}
\]

(1)

where $w$ is the elastic displacement, $c_{44}$ is the shift module, $\rho$ is the density of the material, $\epsilon_{14}$ is the piezoelectric coefficient in the case of cubic symmetry, $\varphi$ is the electric potential, $\varepsilon$ and $\mu$ are the dielectric and magnetic permeability of the material.

The material equations have the form

\[
\begin{align*}
\sigma_{13} &= c_{44} \frac{\partial w}{\partial x} + \epsilon_{14} \frac{\partial \varphi}{\partial y}, & \sigma_{23} &= c_{44} \frac{\partial w}{\partial y} + \epsilon_{14} \frac{\partial \varphi}{\partial x}, \\
D_1 &= -\varepsilon \frac{\partial \varphi}{\partial x} + \epsilon_{14} \frac{\partial w}{\partial y}, & D_2 &= -\varepsilon \frac{\partial \varphi}{\partial y} + \epsilon_{14} \frac{\partial w}{\partial x},
\end{align*}
\]

(2)
where $\sigma_{13}$ and $\sigma_{23}$ are components of the shifting tension, $D_1$ and $D_2$ are vectors of the electrical field induction. During the wave propagation, there also arises a plane magnetic field perpendicular to the propagation direction, which is determined by the equation

$$
\frac{\partial H_3^{(1)}}{\partial y} = \frac{1}{c} \frac{\partial E_1}{\partial t}, \quad \frac{\partial H_3}{\partial x} = -\frac{1}{c} \frac{\partial E_2}{\partial t}, \quad \frac{\partial E_2}{\partial x} - \frac{\partial E_1}{\partial y} = -\frac{1}{c} \frac{\partial H_3}{\partial t}.
$$

(4)

According to the above equations (1)–(3), an electromagnetic wave incident on the boundary of the piezoelectric half-space $y = 0$ from the region $y < 0$, which is identified with a vacuum, must satisfy the equations

$$
\frac{\partial H_3}{\partial y} = \frac{1}{c} \frac{\partial E_1}{\partial t}, \quad \frac{\partial H_3}{\partial x} = -\frac{1}{c} \frac{\partial E_2}{\partial t}, \quad \frac{\partial E_2}{\partial x} - \frac{\partial E_1}{\partial y} = -\frac{1}{c} \frac{\partial H_3}{\partial t}.
$$

Thus, the incident (and reflected) electromagnetic wave is a solution of equations (4), and the refracted wave is a solution of equations (1).

At the boundary of the section $y = 0$, it is necessary to satisfy the boundary conditions [7]

$$
E_1 = E_1^{(1)}, \quad H_3 = H_3^{(1)}, \quad \sigma_{23} = 0.
$$

(5)

Here, the values of the electromagnetic field without any upper index refer to the region $y < 0$, and those with index (1), to the region $y > 0$.

2. General solution constructing

It is known from (4) that the expressions for the incident and reflected waves of the magnetic field have the form

$$
H_{3n} = A \exp[i(\omega t - k_1 x - k_2 y)],
$$

$$
H_{30} = B \exp[i(\omega t - k_1 x + k_2 y)],
$$

(6)

where

$$
k_2 = \sqrt{\frac{\omega^2}{c^2} - k_1^2}.
$$

(7)

The corresponding quantities for the electric field, in particular, for $E_1$, become

$$
E_{in} = -\frac{k_2 c}{\omega} A \exp[i(\omega t - k_1 x - k_2 y)],
$$

$$
E_{in} = \frac{k_2 c}{\omega} B \exp[i(\omega t - k_1 x + k_2 y)].
$$

(8)

The solution of the system of equations (1) is represented in the form

$$
w = C \exp[i(\omega t - k_1 x - k_3 y)],
$$

$$
\varphi = F \exp[i(\omega t - k_1 x - k_3 y)].
$$

(9)

Substituting (9) into (1), we obtain a system of homogeneous algebraic equations for the constants $C$ and $F$:

$$
(s_1 - k_3^2)C - \frac{2\epsilon_{14}}{c_{44}} k_1 k_3 F = 0,
$$

$$
\frac{2\epsilon_{14}}{\epsilon} k_1 k_3 C + (s_2 - k_3^2) F = 0.
$$

(10)
with the notation
\[ s_1 = \frac{\omega^2}{c_1^2} - k_1^2, \quad s_2 = \frac{\omega^2}{c_1^2} - k_2^2, \quad c_1^2 = \frac{c_{44}}{\rho}, \quad c_2^2 = \frac{1}{\varepsilon \mu}. \quad (11) \]

The condition that the determinant of system (10) is zero leads to the equation
\[ k_3^4 - (s_1 + s_2 - \chi k_1^2)k_3^2 + s_1s_2 = 0, \quad \chi = \frac{4e^2}{c_{44} \varepsilon}. \quad (12) \]

Determining the positive roots \( k_{31} \) and \( k_{32} \) from (12), the general solution of (1) with regard to (9) have the form
\[
\begin{align*}
\omega t &+ c_1k_{3i} \sin \chi k_1x, \\
\varphi &+ C_i \cos \chi k_1x, \quad i = 1, 2.
\end{align*}
\]

According to the equations in the system (10), the arbitrary constants \( C_1, C_2 \) and \( F_1, F_2 \) are connected by the relations
\[ C_i = R_i F_i, \quad R_i = \frac{e^{14}}{c_{44}} k_1 k_{3i} (s_1 - k_3^2)^{-1}, \quad i = 1, 2. \quad (14) \]

The components of the electric field are determined from the condition \( E^{(1)} = -\nabla \varphi \). It follows from the equation that
\[
H_3^{(1)} = \frac{i \omega}{Kc_1} \left\{ (\varepsilon k_{31} - e_{14}k_1R_1)F_1 \exp[i(\omega t - k_1x - k_{31}y)] \\
+ (\varepsilon k_{32} - e_{14}k_2R_2)F_2 \exp[i(\omega t - k_1x - k_{32}y)] \right\}. \quad (15)
\]

3. Boundary conditions and particular solution

The requirement that the solutions (6), (8), (13), (15) satisfy the boundary conditions (5) with regard to (9) and
\[ H_3 = H_{3n} + H_{30}, \quad E_1 = E_{1n} + E_{10}, \quad (16) \]
leads to the following system of equations for the unknown constants \( B, F_1, \) and \( F_2 \):
\[
\begin{align*}
A - B &\approx \frac{k_1 \omega}{k_2 c} (F_1 + F_2), \\
A + B &\approx \frac{\omega}{k_1 c} [(\varepsilon k_{31} - e_{14}k_1R_1)F_1 + (\varepsilon k_{32} - e_{14}k_2R_2)F_2], \\
(k_{31}c_{44}R_1 + e_{14}k_1)F_1 + (k_{32}c_{44}R_2 + e_{14}k_1)F_2 &\approx 0. \quad (17)
\end{align*}
\]

The expressions for \( B, F_1, \) and \( F_2 \) determined from (17) and characterizing the amplitudes of the reflected electromagnetic and refracted electroelastic waves turn out to be quite cumbersome. Here these expressions are given in a certain approximation.

First, it is taken into account that the electromechanical coupling coefficient \( x \) is sufficiently small and hence, instead of (9), the following approximate expressions can be obtained:
\[ \tilde{k}_{3i} \approx \left[ s_1 \left( 1 - \frac{\chi k_1^2}{s_1 - s_2} \right) \right]^{1/2}, \quad (i = 1, 2). \quad (18) \]
Also using the inequality \( c_t \ll c_1 \), for constants \( B \) and \( F_2 \), we have

\[
B = \frac{p_1 - p_2}{p_1 + p_2} A, \quad F_2 = -i \frac{2}{\omega (p_1 + p_2)} A,
\]

where

\[
\begin{align*}
p_1 &= \frac{\varepsilon}{2k_1 c_1} [k_{32} (1 - 0.5 \chi \eta) - \chi k_{31} \gamma (1 + \eta)], \quad \eta = \frac{k_1^2 c_1^2}{\omega^2}, \\
p_2 &= \frac{k_1}{k_2 c} (1 - \gamma), \quad \gamma = \frac{2 - \eta}{2 \eta (1 + 0.5 \chi \eta) (1 + \chi k_1^2 s^2)}.
\end{align*}
\]

According to (14) and the last equation in (17), the remaining unknown constants are determined by the formulas

\[
C_i = R_i F_i, \quad F_1 = -\chi \gamma F_2.
\]

4. Results discussion

It is shown in section 3 that a flexible wave arises when an electro-magnetic wave is incident on a tension-free surface. However, a flexible wave can also arise during the incidence of an electro-magnetic wave on a consolidated boundary [8].

Assume that, instead of condition (5), we have the following condition on the half-space boundary:

\[
E_1 = E_1^1, \quad H_3 = H_3^1, \quad \omega = 0, \quad \text{when } y = 0.
\]

Substituting (6), (8), (13), (15) into the boundary condition (22), we obtain a chain of equations of the form (170 for the required constants, where the third equation must be replaced by the following one:

\[
R_1 F_1 + R_2 F_2 = 0.
\]

Using (23) results in the first two equations of (17) of the form:

\[
\begin{align*}
A - B &= \frac{ik_1 \omega}{k_2 c} \left( 1 - \frac{R_2}{R_1} \right) F_2, \\
A + B &= \frac{\omega \varepsilon}{k_1 c_1} \left( k_{32} - k_{31} \frac{R_2}{R_1} \right) F_2.
\end{align*}
\]

From here, the desired reflection coefficient \( B \) and one of the refraction coefficients \( F_2 \) can be determined as

\[
B = \frac{q_1 - q_2}{q_1 + q_2} A, \quad F_2 = -i \frac{2}{\omega (q_1 + q_2)} A,
\]

where

\[
q_1 = \frac{\varepsilon}{k_1 c_1} (k_{32} - k_{31}) \frac{R_2}{R_1}, \quad q_2 = \frac{k_1}{k_2 c} \left( 1 - \frac{R_2}{R_1} \right).
\]

The reflection coefficient \( F_1 \) is determined from (23), while \( c_1 \) and \( c \) are determined from (14).

Conclusion

The electromagnetic wave incident on a half-space of a piezoelectric material with cubic symmetry excites an elastic wave. The amplitude of the elastic wave depends on the angle of incidence of the electromagnetic wave and the electromechanical coupling factor. No elastic wave is excited in normal incidence of an electromagnetic wave \( (k_1 = 0) \).
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