Alive and well: mimetic gravity and a higher-order extension in light of GW170817

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Abstract

The near-simultaneous multi-messenger detection of the gravitational wave (GW) event GW170817 and its optical counterpart, the short \(\gamma\)-ray burst GRB170817A, implies that deviations of the GW speed from the speed of light are restricted to being of \(\mathcal{O}(10^{-15})\). In this note, we study the implications of this bound for mimetic gravity and confirm that in the original setting of the theory, GWs propagate at the speed of light, hence ensuring agreement with the recent multi-messenger detection. A higher-order extension of the original mimetic theory, appearing in the low-energy limit of projectable Hořava–Lifshitz gravity, is then considered. Performing a Bayesian statistical analysis where we compare the predictions of the higher-order mimetic model for the speed of GWs against the observational bound from GW170817/GRB170817A, we derive constraints on the three free parameters of the theory. Imposing the absence of both ghost instabilities and superluminal propagation of scalar and tensor perturbations, we find very stringent 95\% confidence level upper limits of \(\sim 7 \times 10^{-15}\) and \(\sim 4 \times 10^{-15}\) on the coupling strengths of Lagrangian terms of the form \(\nabla^\mu \nabla^\nu \phi \nabla_\mu \nabla_\nu \phi\) and \((\Box \phi)^2\) respectively, with \(\phi\) the mimetic field. We discuss implications of the obtained bounds for mimetic theories. This work presents the first ever robust comparison of a mimetic theory to observational data.
Keywords: modified gravity, gravitational waves, mimetic gravity

(Some figures may appear in colour only in the online journal)

1. Introduction

The recent joint detection of the gravitational wave (GW) event GW170817 by the LIGO/Virgo collaboration [1], and of its optical counterpart, the short γ-ray burst GRB170817A, by the Fermi gamma-ray burst monitor and the anti-coincidence shield for the INTEGRAL spectrometer [2], has inaugurated the era of multi-messenger astronomy. The near-simultaneous detection of the two signals implies that GWs travel at a speed $c_T$ which is nearly the speed of light [2]. Many exotic theories of gravity feature an effective cosmological medium which spontaneously breaks Lorentz invariance, implying that GWs (the excitations of the medium) no longer travel at the speed of light [3] (see also [4, 5]). As a consequence, several previously theoretically motivated modified gravity theories [6–10] are no longer viable [11–15] (see [4, 16] for important early work, and [17–29]).

A particularly interesting modified gravity theory is represented by mimetic gravity (MimG). Proposed in 2013 by Chamseddine and Mukhanov [30], the theory is related to general relativity via a non-invertible disformal transformation of the metric, involving a mimetic scalar field $\phi$ [35–37]. The non-trivial vacuum solutions of the theory effectively mimic cold dark matter (DM) on cosmological scales, while simple generalizations of the original model can mimic any given cosmological evolution [38] (see [39] for a recent review).

As recently noticed in [13], MimG is a particular case of a degenerate higher order scalar-tensor (DHOST) theory [34, 40] obtained by conformally transforming a Horndeski theory with $c_T = 1$: this implies that $c_T = 1$ also in MimG, ensuring the consistency of the original theory with GW170817. It is unclear, however, whether this conclusion carries over to the many proposed extensions of MimG. Given the interest spurred by MimG in the community, addressing this issue is important and timely.

In this note, it is our aim to study the viability of extended mimetic gravity models in light of GW170817. We focus on a particular model proposed by Cognola et al in [42]. Our choice is motivated by theoretically appealing properties of the model, which can be viewed as the low-energy limit of projectable Hořava–Lifshitz gravity. We show that GW170817 sets stringent constraints on a term of the form $\nabla^\mu \nabla^\nu \phi \nabla_\mu \nabla_\nu \phi$ in the action. Performing a Bayesian statistical analysis where the predictions of the model are compared against observational data from GW170817, we obtain constraints on the three free parameters of the model. This represents the first time that a mimetic theory is robustly constrained against observational data.

This note is then organized as follows. In section 2, we very briefly review mimetic gravity and its extensions, and in section 3 we define the mimetic model we will be considering and study scalar and tensor perturbations within this model, as well as introduce the constraints on tensor perturbations imposed by the GW170817/GRB170817A detection. In section 4, we describe our analysis methodology, before proceeding to section 5 where we discuss our results. Finally, we conclude in section 6.

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6 See however [31–34] for important earlier work.
7 See also [37, 41] for related discussions on the relation between MimG and DHOST theories.
8 We use natural units, where the speed of light takes the value 1.
2. Mimetic gravity and its extensions

Mimetic gravity can be obtained by starting from the Einstein–Hilbert (EH) action and reparametrizing the physical metric $g_{\mu\nu}$ in terms of an auxiliary metric $\tilde{g}_{\mu\nu}$ and the mimetic field $\phi$:

$$g_{\mu\nu} = -\tilde{g}_{\mu\nu} \tilde{g}^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi.$$  \hspace{1cm} (1)

When varying the action with respect to $g_{\mu\nu}$, taking into account its dependence on $\tilde{g}_{\mu\nu}$ and $\phi$ through equation (1), the resulting gravitational field equations feature an extra term which, on a flat FLRW background, behaves as a pressureless fluid and hence mimics cold DM [30]. The gradient of the mimetic field is required, for consistency, to satisfy the condition

$$X \equiv g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi / 2 = -1/2,$$

which can be implemented as a constraint at the level of the action through a Lagrange multiplier term $\lambda$ [38]. Various works focusing on cosmological and astrophysical issues within MimG have been conducted in the subsequent literature, for an incomplete list see e.g. [43–78].

The original MimG theory was constructed starting from a ‘seed’ EH action, and various extensions of the theory have been considered by starting from different ‘seed’ actions. Mimetic Horndeski gravity uses Horndeski’s theory, the most general 4-dimensional scalar-tensor theory of gravity with second-order field equations [79], as ‘seed’ theory [80], and has received particular interest recently [42, 81–83].

In the original MimG model, the sound speed $c_s$ (i.e. the speed of propagation of scalar perturbations) is identically 0 [38]. This is problematic if one wants to define quantum fluctuations in the mimetic field. Later it was shown that the problem actually persists also in mimetic Horndeski gravity [82]. In [42], some of us studied an explicit mimetic Horndeski model, from which we then constructed a higher-order mimetic model by explicitly breaking the original Horndeski structure in order to achieve $c_s \neq 0$. In this note, we will be considering this higher-order mimetic model, which for simplicity we refer to as HOMim model.

3. Scalar and tensor perturbations in the HOMim model

The action of the HOMim model is given by [42]:

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[ R(1 + ag_{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi) - \frac{c}{2} (\Box \phi)^2 \right. \left. + \frac{b}{2} (\nabla_\mu \nabla_\nu \phi)^2 - \frac{\lambda}{2} (g_{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + 1) - V(\phi) \right].$$  \hspace{1cm} (2)

Setting $b = c = 4a$ in the action recovers the Horndeski structure of the theory. The equations of motion of the theory can be found in our companion paper [87]. In [42], it was argued that the model can be viewed as the low-energy limit of projectable Hořava–Lifshitz gravity [88], a power-counting renormalizable candidate theory of quantum gravity, hence lending to its theoretical appeal\(^{10}\).

\(^{9}\)Note that the same model was later studied in [84], see also [85, 86] for related studies.

\(^{10}\)The mimetic Horndeski model from which the HOMim action was constructed was inspired by previous work in the context of ‘covariant renormalizable gravity’ models [89–93], wherein power-counting renormalizability is achieved by breaking Lorentz invariance dynamically. In this way, theoretical issues present in Horava–Lifshitz gravity and connected to the explicit breaking of diffeomorphism invariance [94–96] are avoided.
By perturbing a background FLRW line element appropriately, we have derived the sound speed $c_s$ and the gravitational wave speed $c_T$. Here we simply quote the result and refer the reader to [87] for further details on the calculation\textsuperscript{11}:

\begin{equation}
  c_s^2 = \frac{2(b - c)(a - 1)}{(2a - b - 2)(4 - 4a - b + 3c)}, \quad (3)
\end{equation}

\begin{equation}
  c_T^2 = \frac{2(1 - a)}{2(1 - a) + b}. \quad (4)
\end{equation}

From equation (4) it is clear that $b \neq 0$ is a necessary condition for obtaining $c_T \neq 1$. We see that bounds on the GW speed from GW170817 will constrain the parameters $a$ and $b$, whereas further information on the sound speed is necessary in order to put constraints on $c$. Notice also that, when $a = b = c = 0$, we recover $c_s^2 = 0$ and $c_T^2 = 1$, in agreement with expectations from the original MimG model, and in full agreement with the GW170817/GRB170817A detection.

The equations of motion derived from the action in equation (2) are of at most fourth order, as shown in the companion paper [87]. In particular the Klein–Gordon equation, equation (5) in [87], is of fourth order. However, by imposing the constraint coming from the Lagrange multiplier $\lambda$ (equation (6) in [87]) and by choosing a FLRW metric, we obtain equations of motion of second order in the Hubble rate $H$, see equations (9)–(11) in [87]. Hence, at the background level there do not appear to be ghosts. At the perturbative level, the situation is more delicate, since as soon as the sound speed is non-zero there is an extra propagating scalar mode, for a total of three degrees of freedom. We have discussed this issue in more detail in [87], where we have computed the quadratic action of the scalar and tensor perturbations, and found that the theory is free of ghosts only when choosing $a < 1$ and $c > 0$. Heretofore, we shall impose these conditions in order to ensure the theoretical consistency of the model. In addition to these conditions ensuring the absence of ghosts, stability arguments impose the conditions $0 \leq c_s^2 \leq 1$ and $0 \leq c_T^2 \leq 1$. The upper limit of 1 on $c_s^2$ and $c_T^2$ enforces the absence of superluminal propagation of scalar and tensor modes. It has been shown that superluminality in a theory of gravity is incompatible with an UV-complete theory whose S-matrix satisfies canonical analyticity constraints [97, 98].

The recent near-simultaneous detection of GW170817 [1] and its optical counterpart GRB170817A [2], has placed very stringent constraints on $\delta c_T$, the fractional deviation of the GW speed from the speed of light. Following [13], we will consider the following bound:

\begin{equation}
  |\delta c_T| < 5 \times 10^{-16}. \quad (5)
\end{equation}

The bound in equation (5) provides a very stringent constraint on deviations of $c_T$ from the speed of light (note that the region $c_T < 1$ was already previously constrained by non-observation of gravi-Čerenkov radiation from cosmic rays, although the bound is not competitive with the GW170817/GRB170817A one [99]). Let us imagine for a moment fixing the requirement $c_T \equiv 1$, which is trivially satisfied in the baseline MimG model (setting $a = b = 0$ in equation (4)). In the context of the HOMim mimetic model, from equation (4) it follows that $c_T \equiv 1$ instead imposes the very stringent constraint $b = 0$. This implies that a term of the form $\nabla^\mu \nabla^\nu \phi \nabla_\mu \nabla_\nu \phi$ is prohibited from appearing in the HOMim action. In the remaining part of the work, we will entertain the possibility of a tiny violation of the constraint $c_T \equiv 1$, in

\textsuperscript{11} Notice that $c_s$ and $c_T$ do not depend on the scalar field derivative $\dot{\phi}$, unlike what happens in Horndeski gravity, because in mimetic gravity the Lagrange multiplier constraint fixes $\dot{\phi} = 1$ on a FLRW background.
accordance with the the bounds on \( c_T \) provided by equation (5), and explore the implications of this bound on the parameters of the HOMim model.

4. Analysis methodology

We perform a standard Bayesian statistical analysis to constrain the three parameters of the extended mimetic model \((a, b, \text{ and } c)\) in light of the near-simultaneous GW170817/GRB170817A detection. The constraint on \( \delta c_T \) of equation (5) can only be used to provide bounds on \( a \) and \( b \) (equation (4)). The first part of our analysis is therefore concerned with determining the joint and marginalized posterior probability distributions of \( a \) and \( b \), in light of observational data \( d \) given the constraint on \( \delta c_T \) of equation (5).

We begin by considering the parameters \( \theta = (a, b) \). To proceed, we need to construct the likelihood \( L(\theta) \), consisting of the probability of observing the data \( d \) given a choice of model parameters \( \theta \): \( L(\theta) = P(d|\theta) \). Following equation (5), we model the likelihood as an univariate Gaussian in \( \delta c_T \), centered around \( \delta c_T = 0 \):

\[
L(\theta) \equiv L(a, b) = \exp \left\{ -\frac{[\delta c_T(a, b)]^2}{2\sigma_{\delta c_T}} \right\},
\]

where \( \delta c_T(a, b) \), following equation (4), is given by:

\[
\delta c_T(a, b) = \sqrt{\frac{2(1-a)}{2-2a+b}} - 1.
\]

Finally, in equation (6), \( \sigma_{\delta c_T} \) denotes the uncertainty on \( \delta c_T \), which we estimate as \( \sigma_{\delta c_T} = 5 \times 10^{-16} \) following equation (5).

Using Bayes’ theorem, we construct the joint posterior distribution of \( a \) and \( b \) as the product of the likelihood (equation (6)) and the prior probability distributions we assign to \( a \) and \( b \). The choice of prior is dictated by a combination of theoretical and phenomenological considerations. Following our previous discussion, we first of all impose the requirement of subluminality of tensor perturbations: \( c_T^2(a, b) \leq 1 \).

In the action equation (2), the term multiplying the Riemann tensor is \( 1 + ag^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi = 1 + 2aX = 1 - a \). As this term controls the strength of the effective Newton constant, we must impose its non-negativity, which implies \( a < 1 \). Notice that, as discussed in section 3, requiring the absence of ghosts anyway led to the condition \( a < 1 \). In addition, guided by perturbativity arguments, we expect \( |a| \lesssim O(1) \), as in general it could be problematic to embed a coupling constant \( |a| \gg O(1) \) in the context of a UV-complete theory of gravity. Guided by these considerations, we choose for simplicity to impose a top-hat (flat) prior on \( a \) within the range \([-1, 1] \). We assess \textit{a posteriori} that our results are only mildly affected by other choices of flat prior as long as the upper and lower limits are \( \sim O(1) \) in modulo.

Concerning \( b \), we already know that this parameter is required to be \( \ll O(1) \), in order for the bound in equation (5) to be satisfied. Moreover, we see from equation (4) that for \( b < 0 \), one would obtain \( c_T^2 > 1 \), which violates the subluminality requirement. Based on these arguments, we impose a top-hat prior on \( b \) within the range \([0, 1] \). In conclusion, the joint posterior distribution of \( a \) and \( b \) we sample from is given by:

\[
P(a, b|d) = \exp \left\{ -\frac{[\delta c_T(a, b)]^2}{2\sigma_{\delta c_T}^2} \right\} \Theta(c_T^2) \Theta(1 - c_T^2) \times \Theta(1 + a) \Theta(1 - a) \Theta(b) \Theta(1 - b),
\]
where $\Theta(x)$ denotes the Heaviside step function.

In the second part of the analysis we include the parameter $c$ as well, which requires additional information to be taken into account beyond the constraint on $\delta c_T$ of equation (5). Since $c$ enters in the expression for the sound speed $c_s$ (equation (3)), we additionally impose the subluminality of scalar perturbations\textsuperscript{12}. In addition, as discussed in section 3, requiring the absence of ghosts leads to the condition $c > 0$. Therefore, guided by considerations on the absence of ghosts and perturbativity as per our previous discussion, we impose a top-hat prior on $c$ within the range $[0, 1]$. We will later anyway see that data require $c \ll O(1)$. In this case, the joint posterior distribution of $a$, $b$, and $c$, given the data $d$, is given by:

$$P(a, b, c|d) = \exp \left\{ -\frac{[\delta c_T(a,b)]^2}{2\sigma^2_{\delta c_T}} \right\} \times \Theta(1 + a)\Theta(1 - a)\Theta(b)\Theta(1 - b)\Theta(c)\Theta(1 - c) \times \Theta(c^2_T - 1)\Theta(1 - c^2_T)\Theta(c^2_T - 1 - c^2_T).$$

(9)

To sample the posterior distributions of equations (8) and (9), we make use of Markov chain Monte Carlo (MCMC) methods, by implementing the Metropolis–Hastings algorithm.

We use the cosmological MCMC sampler Montepython \cite{101}, configured to act as a generic sampler. We monitor the convergence of the MCMC chains using the Gelman and Rubin parameter $R - 1$, which we require to be < 0.01 in order for the chains to be considered converged.

5. Results

We first sample the joint $a$-$b$ posterior distribution given by equation (8). We show the results in the triangular plot of figure 1, whose diagonal contains the marginalized probability distributions of the two parameters.

We find $a < 0.55$ at 95\% confidence level (C.L.), while the marginalized posterior of $b$ is, as expected, peaked at $b = 0$ and falls rapidly as $b$ increases, indicating $b \lesssim 5.11 \times 10^{-15}$ at 95\% C.L.. The reason for these very tight bounds is readily found by inspecting equation (4). As $b$ moves away from 0 (at fixed $a$), $c^2_T$ rapidly moves away from 1, and hence the probability density of the given point in $(a, b)$ parameter space decreases.

Although deviations of $c_T$ from 1 are mostly controlled by $b$, the parameter $a$ does nonetheless play a role. In fact, from the orientation of the joint $a$-$b$ posterior distribution (lower left panel in figure 1), we see that the two parameters exhibit a mild negative correlation (also referred to as parameter degeneracy). That is, it is possible to increase/decrease one parameter and correspondingly decrease/increase the other, and still maintain consistency with the GW170817 bound on $c_T$. This observation can be rigorously shown by Taylor expanding $\delta c_T$ in the limit of small $b/(2 - 2a)$:

$$\delta c_T = \left(1 + \frac{b}{2 - 2a}\right)^{-\frac{1}{2}} - 1 \xrightarrow{a \to 0} - \frac{b}{4(1 - a)}. \quad (10)$$

\textsuperscript{12} There exist upper limits on the sound speed of DM from observations of the CMB and large-scale structure, which suggest $c^2_s \lesssim 10^{-10.7}$ \cite{100}. However, these bounds are not entirely model-independent: the analysis should be re-performed for the HOMim model, by solving the relevant Einstein–Boltzmann equations. Moreover, the propagating scalar mode in the HOMim model does not exclusively mimic DM, but a combination of DM and dark energy (see \cite{87}). In order to be conservative, we have decided to not impose these upper limits on $c_s$. 
From equation (10), we see that the more \( b \) increases, the more \( c_T \) deviates from 1. Moreover, the smaller \( a \) is, the larger the term \( 4(1-a) \) in equation (10) is, implying that it is consequently possible to ‘tolerate’ larger values of \( b \) and still be consistent with the deviation of \( c_T \) from 1 allowed by GW170817/GRB170817A. This explains the mild negative correlation between \( a \) and \( b \). From our MCMC chains we estimate the correlation coefficient between the two parameters to be \( \approx -0.40 \).

Next, we sample the joint \( a \)-\( b \)-\( c \) posterior probability distribution given by equation (9). We show the results in the triangular plot of figure 2. Quoting all 95% C.L. upper bounds, we find \( a < 0.27 \), \( b < 7.18 \times 10^{-15} \), and \( c < 4.68 \times 10^{-15} \). In order to explain the results we find, it is useful to consider the expression for the sound speed squared, equation (3), and the combinations of the three parameters \( a \), \( b \), and \( c \) necessary to keep this quantity positive. It is then quite easy to see that, in the limit where \( b \ll a \) and \( c > 0 \), it is possible to keep \( c_s^2 \geq 0 \) by requiring that \( c \) be smaller than \( b \), i.e. \( c \lesssim b \sim O(10^{-15}) \), while also having \( 1-a \gg O(10^{-15}) \) (i.e. \( a \) is sufficiently far from 1). In this limit, the sound speed is approximately given by:

\[
c_s^2(a, b, c) \approx \frac{(b - c)}{4 - 4a - b + 3c}.
\]

Although there appears to be a mild peak in the posterior distribution of \( b \), we find that the distribution is consistent with \( b = 0 \) at \( \sim 2 \sigma \). Therefore, as is standard practice in the field, we only quote an upper limit for \( b \) instead of a ‘detection’ of non-zero \( b \). Recall also that we had chosen the upper and lower limits for our priors based on perturbativity considerations. We have checked that our results, and in particular the upper limits on \( b \) on \( c \), are only very marginally affected (within the same order of magnitude) by other choices for the upper and lower limits of the prior which still are \( O(1) \) in modulo. We therefore consider our results relatively robust against the choice of prior.
where the condition $c \lesssim b \sim O(10^{-15})$ now ensures that the numerator of equation (11) is positive, while the condition $(1-a) \gg O(10^{-15})$ ensures that there are no ‘accidental cancellations’ between the terms $4(1-a)$ and $3c-b$ in the denominator which might otherwise make it negative, i.e. that the denominator is approximately given by $4(a-1)$ and hence is always positive since $a<1$. This discussion explains why the upper limit on $c$ is approximately of the same order of the upper limit on $b$, i.e. of order $10^{-15}$, since $c$ is required to be positive (to avoid ghosts) but smaller than $b$ (to have $c^2_s \geq 0$).

The above discussion also suggests that we can expect a strong positive correlation between $b$ and $c$ (since increasing $c$ requires increasing $b$ in order to keep the numerator of equation (11) positive). In fact, we find a correlation coefficient of 0.66 between $b$ and $c$, which is stronger than the already strong correlation we previously found between $a$ and $b$. On the other hand, we find a weaker correlation between $a$ and $c$, with correlation coefficient of $-0.28$, which is induced by the mutual correlations of these two parameters with $b$. The negative correlation between $a$ and $c$, and the positive one between $b$ and $c$, explain why introducing the parameter $c$ has respectively tightened and loosened the upper limits we previously derived on $a$ and $b$. 

Figure 2. As in figure 1, with the addition of the parameter $c$ and the further imposition of subluminality of scalar perturbations.
Note

when only considering these two parameters (recall that the upper limit shifted from 0.55 to 0.27 for $a$, and from $5.11 \times 10^{-15}$ to $7.18 \times 10^{-15}$ for $b$). We plot a heatmap of the correlation coefficients between the 3 parameters in figure 3, where it is clear that the strongest correlation is that between $b$ and $c$, for reasons already discussed previously.

6. Conclusions

In this note, we have examined the status of mimetic gravity in light of the recent near-simultaneous detection of the GW event GW170817 [1] and its optical counterpart, the short γ-ray burst GRB170817A [2]. The original theory [30] is in perfect agreement with this multi-messenger detection, as the speed of GWs $c_T$ is therein identically equal to the speed of light: $c_T = 1$.

We have then considered a theoretically motivated extended higher-order mimetic model (the HOMim model, equation (2)), which appears in the low-energy limit of projectable Hořava–Lifshitz gravity [42]. Entertaining the possibility of a tiny violation of the $c_T \equiv 1$ constraint, in agreement with experimental constraints from GW170817/GRB170817A (equation (4)) [2], we have performed a Bayesian statistical analysis to derive observational constraints on the three free parameters of the HOMim model. In particular, we have found that $b$ and $c$, the coefficients of the terms $\nabla^\mu \nabla^\nu \phi \nabla_\mu \nabla_\nu \phi$ and $(\Box \phi)^2$ in the action respectively (with $\phi$ the mimetic field), are subject to the very stringent constraints $0 \leq b < 7.18 \times 10^{-15}$ and $0 \leq c < 4.68 \times 10^{-15}$ at 95% confidence level. In light of these very tight limits it is tempting to conclude that, to avoid incurring in fine-tuning and naturalness issues, both parameters

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Figure 3. Heatmap of the correlation matrix between the 3 parameters ($a$, $b$, and $c$) we are examining. We visually see that the strongest correlation is that between $b$ and $c$, resulting from the necessity of avoiding ghost instabilities (which requires $c > 0$) while needing $c_5^2 \geq 0$ (which requires $c < b$), as discussed in the text.
should in fact be $0^{14}$. In this case, the gravitational wave speed is identically equivalent to the speed of light, while the sound speed squared is $1/4(1 - a)$ and always positive as long as $a < 1$, which is required to avoid ghost instabilities. We stress that in our paper it is the first time that robust observational constraints are placed on a mimetic theory.

We conclude mentioning theoretical issues pertaining the instability of the theory. It has recently been argued that mimetic gravity and its higher-order extensions suffer from gradient and ghost instability issues, which undermine the theoretical consistency of the theory $^{[106–115]}$. Although in this work we have imposed specific conditions on the parameters to avoid ghost instabilities, we notice that more generally there exist paths to curing these instabilities, for instance by considering direct couplings between higher derivatives of the mimetic field and curvature $^{[110–112, 114, 115]}$. One would in general expect such couplings to lead to deviations of $c_T$ from the speed of light, rendering the theory at odds with observational constraints, and warranting a new analysis along the lines of the one conducted here. Other future research directions could include constraining mimetic gravity from the study of primordial gravitational waves, i.e. from measurements of the cosmic microwave background BB spectrum, along the lines of the study performed in $^{[116]}$ for the case of $f(T)$ gravity. We defer these studies to future work.

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See instead $^{[87]}$ for a brief discussion concerning another possibility, namely that the parameters $b$ and $c$ might still be small but non-zero, and might arise as quantum corrections $^{[103–105]}$. Although in principle imposing $c_T^2 \geq 0$ and $c_L^2 \geq 0$ as we did should at least alleviate the gradient instability issues.
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