Phase Transitions in the Bilayer $\nu = 2/3$ Quantum Hall Effect

N. Kumada∗, D. Terasawa, Y. Shimoda, H. Azuhata, A. Sawada, and Z. F. Ezawa
Department of Physics, Tohoku University, Sendai 980-8578, Japan

K. Muraki, T. Saku, and Y. Hirayama
NTT Basic Research Laboratories, 3-1 Morinosato-Wakamiya, Atsugi, Kanagawa 243-0198, Japan
(Dated: Version: October 29, 2018)

We measured the magnetoresistance of bilayer quantum Hall (QH) effects at the fractional filling factor $\nu = 2/3$ by changing the total electron density and the density difference between two layers. Three different QH states were separated by two types of phase transition: One is the spin transition and the other is the pseudospin transition. In addition, two different hystereses were detected, one of which is specific to bilayer systems. The phase transitions and the hystereses are described well by a composite fermion model extended to a bilayer system.

PACS numbers: 73.43.-f, 73.43.Nq, 73.21.Fg, 71.10.Pm

In bilayer electron systems, interlayer Coulomb and tunnelling interactions provide an additional degree of freedom, which produce rich phenomena with no counterpart in individual two dimensional systems [1]. A good example is the bilayer quantum Hall (QH) state at the filling factor $\nu = 2$. A phase transition has been observed at $\nu = 2$ as revealed by magnetotransport measurements [2, 3], light-scattering [4, 5] and capacitance spectroscopy [6]. In the weak interlayer correlation case, electrons in layers [2, 3] are subjected to the effective magnetic field $\nu B \perp t$ accurately. This state is the compound state with the CF factor

$$\nu \equiv \left( \frac{n_1 - n_2}{n_1} \right)$$

where basic particles are CFs obtained by attaching an electron to the bilayer system [7]. We measured the magnetoresistance of bilayer QH effects at the fractional filling factor $\nu = 2/3$ as revealed by magnetotransport measurements [2, 3], light-scattering [4, 5] and capacitance spectroscopy [6]. In the weak interlayer correlation case, electrons in each layer configure the monolayer $\nu = 1$ QH state separately. This state is the compound state with $\nu = 1 + 1$, which is a spin-polarized state. When the interlayer correlation is enhanced, this compound state transits to a spin-impurelized QH state. In a conventional system, cyclotron energy is very large and the Landau orbital degree of freedom is frozen. Thus, the observed phase transition is due to the spin and layer degree of freedom.

The fractional QH effect (FQHE) is intuitively understood based on the composite fermion (CF) model [8], where basic particles are CFs obtained by attaching an even number of flux quanta to electron. At the filling factor $\nu = 1/2$, the attached flux exactly cancels the applied magnetic field. As the field deviates from $\nu = 1/2$, CFs are subjected to the effective magnetic field $B^*$ and quantized into CF Landau levels. The series of FQHE at $\nu = p/(2p \pm 1)$ are interpreted as integer QH effect (IQHE) of CF at $\nu_{CF} = p$. For instance, the $\nu = 2/3$ FQHE is mapped to the $\nu = 2$ IQHE. However, there exists significant difference because the CF-cyclotron gap is comparable to the Zeeman energy. In the monolayer QH states, indeed, phase transitions have been observed [9], which arise from a competition between the Zeeman and CF-cyclotron energies. Moreover, the hystereses [10, 11, 12, 13, 14, 15] observed around $\nu = 2/3$ and $\nu = 2/5$ in monolayer systems have attracted much attention recently. Thus we expect rich phases to appear in the bilayer $\nu = 2/3$ QH state, where CF-Landau orbital, spin and layer degrees of freedom all come into play.

In this Letter, we study the magnetoresistance at $\nu = 2/3$ in a bilayer system by changing the total electron density $n_t$ and the density difference between two layers $\sigma = (n_f - n_b)/n_t$ and map out the phase diagram in the $n_t-\sigma$ space, where $n_f$ ($n_b$) is the electron density in the front (back) layer. In the phase diagram, three different QH states and a no-QH area were observed. The QH states were elucidated by the activation energy measurements. In addition, two types of hysteresis were detected. One of the hystereses, specific to bilayer systems, is our new finding. These phases are described well by a CF model extended to a bilayer system on the understanding that collective Coulomb interactions renormalize not only the CF cyclotron energy but also the Zeeman and tunnelling energies.

Our sample was grown by molecular beam epitaxy on a (100)-oriented GaAs substrate. It consists of two GaAs quantum wells of 200 Å width separated by a 31-Å-thick barrier of Al0.33Ga0.67As [16]. The tunnelling energy gap is $\Delta_{SAS} = 10.9$K. The electron density in each layer is controlled by adjusting the front and back gate voltages. The low temperature mobility is $2 \times 10^6$cm$^2$/Vs with $n_t = 2 \times 10^{11}$cm$^{-2}$. Measurements were performed with the sample mounted on a goniometer with a superconducting stepper motor [17] in a dilution refrigerator. Standard low-frequency ac lock-in techniques were used. All magnetoresistance data were taken at 50 mK with the magnetic field sweep rate 0.06 T/min and the current 20 nA.

In Fig. 1, we show how the $\nu = 2/3$ state evolves as the total electron density $n_t$ changes in the perpendicular field ($B_{tot} = B_\perp$), where $B_{tot}$ is the total magnetic field. Data are given for two limiting cases, i.e. the balanced density point ($\sigma = 0$) and the monolayer point ($\sigma = 1$). At both points, the magnetoresistance $R_{xx}$ becomes zero at higher density, where QH states are well developed. At $\sigma = 0$, the minimum of $R_{xx}$ gets weaker with decreasing $n_t$, and collapses at $n_t = 0.8 \times 10^{11}$cm$^{-2}$, followed by a reappearance of the $\nu = 2/3$ minimum at $n_t = 0.6 \times 10^{11}$cm$^{-2}$ [18, 19]. At $\sigma = 1$, on the other
FIG. 1: Magnetoresistances at the balanced density point \( \sigma = 0 \) and at the monolayer point \( \sigma = 1 \) for several total densities. They are plotted as a function of \( 1/\nu = eB_\perp/hn_t \). Traces are vertically offset by 5 kΩ for each curve. Hysteresis between the upward (solid trace) and downward (dashed trace) field sweeps is seen at \( \sigma = 1 \).

hand, we observed a totally different behavior: A significant difference in \( R_{xx} \) between upward (solid trace) and downward (dashed trace) magnetic-field sweeps was observed for a wide range of magnetic field covering the entire \( \nu = 2/3 \) region. We call it type-G hysteresis (G stands for the ground state, as we explain later).

We repeated similar measurements for \( 0 \leq \sigma \leq 1 \) in the range of \( 0.5 \leq n_t \leq 1.8 \times 10^{11} \text{ cm}^{-2} \) and made a phase diagram (Fig. 2) [20]. In the phase diagram, four QH (black) areas labeled I, II, III and IV are clearly recognized. Area I is completely separated by the no-QH (white) area, while area IV is connected with areas II and III by the type-G hysteresis (hatched) region. On the other hand, the low density phase at \( \sigma = 0 \) (area II) evolves continuously into the high density phase at \( \sigma = 1 \) (area III). In these two areas (II and III), we observed a new type of hysteresis (small white squares in the black region), which we name type-E hysteresis (E stands for excitation levels). We discuss this hysteresis later in Fig. 5.

We explain these four QH areas based on the CF model extended to a bilayer system. CF energy levels are split by the Zeeman \( \Delta_Z \), pseudo-Zeeman \( \Delta_{ba} \) and CF-cyclotron \( \Delta_{cy} \) energies (Fig. 3). We use the pseudospin language to deal with the layer degree of freedom. The pseudo-Zeeman energy is the energy gap between the bonding (b) and antibonding (a) states [22], \( \Delta_{ba} = \Delta_{SAS} \sqrt{1/(1-\sigma^2)} \), which is equal to \( \Delta_{SAS} \) at \( \sigma = 0 \) and increases with applying a bias voltage. The origin of the CF-cyclotron gap is a Coulomb interaction [21], and we set \( \Delta_{cy} = C(\sigma)e^2/4\pi\epsilon l_B^2 \propto \sqrt{B_\perp} \), where \( C(\sigma) \) is a dimensionless coefficient, \( \epsilon \) is the dielectric constant, \( l_B^2 \) is the magnetic length in the effective magnetic field \( B^* = |B_\perp - B_1/2| \) and \( B_1/2 \) is the magnetic field at \( \nu = 1/2 \). We label each CF energy level as \((N_{CF}, ps, s)\)

FIG. 2: Phase diagram illustrated for \( \nu = 2/3 \). The horizontal axis is the normalized density difference between two layers \( \sigma \) and the vertical axis is the total electron density \( n_t \). \( R_{xx} \) minimum for QH state was found in the black area. QH effect was not observed in the white area. Type-G hysteresis was observed in the hatched area as in Fig. 1. The small white square in black region indicates occurrence of different type of hysteresis, where a QH state is developed and the hysteresis is found at fields slightly lower than that at \( \nu = 2/3 \) (Fig. 5).

FIG. 3: Low-lying energy levels of CF are depicted schematically as a function of the magnetic field. At \( \nu = 2/3 \), the lowest two levels are filled up (thick curves), and three QH states are expected to appear.
balanced ($\sigma \neq 0$), the filling factor in the front and back layers deviate from 1/3, and the compound state collapses. Thus, this state is stable only at $\sigma = 0$. In area IV, where $n_t$ is small and $\sigma$ is large, the spin-unpolarized and pseudospin-polarized (SU-PP) state is realized. When $n_t$ is increased from area IV to III, the SU-PP state transits to the spin-polarized and pseudospin-polarized (SP-PP) state. Even at $\sigma = 0$, $\Delta_{ba}$ has a finite value $\Delta_{SAS}$, which exceeds the CF-cyclotron gap at low density. Thus, the QH state in area II is the SP-PP state. Accordingly, the SP-PP state persists continuously from area II to III.

To elucidate these bilayer QH states more in detail, we determine the activation energy $\Delta$ by tilting the sample in the magnetic field with keeping $B_{\perp}$ fixed, where $\Delta$ is determined from the temperature dependence of the magnetoresistance $[23]$: $R_{xx} \propto \exp(-\Delta/2T)$. Figure 4 shows the activation energy in the four QH areas as a function of the total magnetic field $B_{tot} = (B_{\parallel}^2 + B_{\perp}^2)^{1/2}$, where $B_{\parallel}$ is the in-plane magnetic field. As $B_{\parallel}$ is increased, $\Delta_Z = g^* \mu_B B_{tot}$ increases and $\Delta_{SAS} \propto \exp\{-B_{dd}/(B_{\perp}B_{xx})\}$ decreases $[24]$, where $g^*$ is the gyromagnetic ratio, $\mu_B$ is the Bohr magneton and $d$ is the layer separation. In area I and III, $\Delta$ initially increases, which indicates that the excitation gap is the Zeeman energy. In area II, when $B_{tot}$ is increased only by 0.1 T, the activation energy becomes zero. Although we increased $B_{tot}$ up to 13 T, a QH state did not reappear. The excitation gap of this state is $\Delta_{ba}$, because $B_{\parallel}$ decreases $\Delta_{SAS}$. In area IV, the activation energy first decreases and shows a pronounced transition at 5 T. This behavior is interpreted as the phase transition from the SU-PP to SP-PP states due to the increased $B_{\parallel}$ $[25, 26]$. This SP-PP state is the same as the one in area III.

We analyze the phase transition points based on the noninteracting CF model (Fig.3). We start with the phase transition point between the SP-PP (area II and III) and SP-PU (area I) states at the balanced point ($\sigma = 0$). It occurs due to the crossing of the levels (1,b,$\uparrow$) and (0,a,$\uparrow$) at

$$\Delta_{ba} = C(\sigma)e^2/4\pi\epsilon l_B^2.$$  

The transition occurs at $n_t = 0.8 \times 10^{11}$ cm$^{-2}$ in our data (Fig.3). As mentioned above, the SP-PU state made of (0,b,$\uparrow$) and (0,a,$\uparrow$) does not exist in the unbalanced configuration ($\sigma \neq 0$) and the no-QH area develops between the areas I and III as in the white area in Fig.3. Then, eq. (1) determines the phase boundary between the SP-PP state and the no-QH area. The transition point shifts to higher $n_t$ as $\sigma$ is increased, $n_t \propto 1/(C^2(\sigma)(1 - \sigma^2))$.

Next, we discuss the transition point between the SP-PP (area II and III) and SU-PP (area IV) states, where the Zeeman $\Delta_Z$ and CF-cyclotron $\Delta_{cy}$ gaps become equal, $g^* \mu_B B_{\perp} = C(\sigma)e^2/4\pi\epsilon l_B^2$. 

The transition occurs at $n_t = 0.95 \times 10^{11}$ cm$^{-2}$ in the monolayer limit ($\sigma = 1$) in our data (Fig.3). When $\sigma$ decreases, $C(\sigma)$ decreases because electrons tend to extend over both of the layers and the Coulomb energy decreases. It follows from eq. (2) that $B_{\parallel} \propto C(\sigma)^2$. Hence, the total density at the transition point decreases as $n_t = veB_{\perp}/h \propto C(\sigma)^2$ $[27]$. This explains why the SP-PP state is realized at higher density near the monolayer point (area III) but at lower density around the balanced point (area II).

In this way, the phase diagram for $\nu = 2/3$ is interpreted qualitatively by the noninteracting CF model. However, some refinement is needed quantitatively. We focus on the coefficient $C(\sigma)$ of the CF cyclotron gap. From eq. (2) we obtain $C(1) = 0.026$ at $\sigma = 1$. We have argued that it decreases as $\sigma$ decreases, and hence $C(0)$ must be much smaller than 0.026, as is suggested in our phase diagram (Fig.3) $[27]$. However, from eq. (1) we obtain $C(0) = 0.17$ at $\sigma = 0$, which is almost ten times bigger than the one implied by eq.(1). We have reached this inconsistency by assuming that collective Coulomb interactions renormalize only the CF cyclotron energy as in the noninteracting CF theory of the monolayer FQHE. This simple picture fails in the bilayer FQHE. The above inconsistency would be resolved by the understanding that collective Coulomb interactions renormalize also the Zeeman and tunnelling energies of CFs.

Finally, we discuss the two types of the hysteresis. Type-G hysteresis was observed between the SU-PP and SP-PP states. This type of hysteresis has already been
observed in various monolayer systems, and argued to be caused by domain morphology in the ground state [12, 13, 14]. The ground state consists of the spin-polarized and spin-unpolarized domains at the crossover point of the two states. The hysteresis would result from a conspiracy between electronic and nuclear polarization [10, 11, 13].

Type-E hysteresis is distinguished from type-G one by the fact that the ground state does not change. Thus, type-E hysteresis is not due to the domain morphology of the ground state. We notice that type-E hysteresis appears not exactly at $\nu = 2/3$ but only on the lower field side of $\nu = 2/3$ (Fig. 5). We speculate the origin of the hysteresis as follows. At the lower field side of $\nu = 2/3$, many quasiparticles are created in excitation levels of $\nu = 2/3$. The SP-PP state in areas II and III have different excitation levels $(0,a,\uparrow)$ and $(0,b,\downarrow)$, as shown in the insets of Fig. 4. These upper CF levels are almost degenerate with one another in the small white square area in the phase diagram (Fig. 3). We would conclude that type-E hysteresis occurs due to the domain morphology involving different spin excitation levels. As far as we are aware of, type-E hysteresis has not been observed before.

In summary, we made the phase diagram for $\nu = 2/3$ by changing the total electron density and the density difference between two layers. In the phase diagram, three different spin/pseudospin QH states were observed. The SP-PP state is realized at lower density around the balanced point, while at higher density near the monolayer point. A no-QH area develops between the SP-PU and SP-PP states. Moreover, two types of hysteresis were observed. One is due to the crossover of different spin ground states, and the other is associated with crossing of different spin excitation levels. We have interpreted the phase diagram by a CF model extended to a bilayer system. These phases are caused by the spin, layer and composite fermion Landau orbital degrees of freedom.

The research was supported in part by Grant-in-Aids for the Scientific Research from the Ministry of Education, Science, Sports and Culture (Nos. 10203201, 11304019, 08159), the Mitsubishi Foundation and the Asahi glass Foundation, CREST-JST and NEDO "NTDP-98" projects.