Generating primordial fluctuations from modified teleparallel gravity with local Lorentz-symmetry breaking

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Abstract

In the context of modified teleparallel gravity, we study the generation of primordial density fluctuations in a general scalar-torsion theory whose Lagrangian density is an arbitrary function $f(T,\phi)$ of the torsion scalar $T$ and a scalar field $\phi$, plus the kinetic term of this latter. It is well known that generic modifications of teleparallel gravity are not invariant under six-parameter local Lorentz transformations. In order to restore the local Lorentz symmetry, we have incorporated six additional degrees of freedom in the form of Goldstone modes of the symmetry breaking through a Lorentz rotation of the tetrad field. After integrating out all the auxiliary modes, we obtain a second order action for the scalar and tensor propagating modes and their power spectrum generated during inflation. It is found that an explicit mass term emerges in the second order action for curvature perturbation, describing the effects of local Lorentz violation at first-order of slow-roll. We show that only inflationary models with nonminimal coupling functions $f(T,\phi)$ which are non-linear in $T$, including the case of $f(T)$ gravity with minimally coupled scalar field, can generate primordial fluctuations. For a concrete model of inflation, we study power-law inflation by using the latest Planck data.

1. Introduction

Lorentz invariance is considered as one of the most fundamental symmetries in physics, which provides a fundamental support for the principles of general relativity (GR) and the standard model of particle physics [1]. Nevertheless, at a sufficiently high-energy scale (Planck scale), it is expected that these two field theories merge into a single unified and quantum-consistent theory, under a possible breaking of local Lorentz symmetry [2–7]. Furthermore, if primordial density fluctuations were generated during inflation [8–11], they give us an unique opportunity to learn about physics at energy scales that otherwise would not be accessible, since it is believed that inflation occurs near the scale of grand unification, and therefore, it is not too far from scales where quantum gravity is relevant [12–14].

It is well known that modified gravity theories constructed from the so-called teleparallel equivalent of general relativity, or simply, teleparallel gravity (TG) [15–26], break the local Lorentz symmetry [27–28]. This has caused great interest in the study of cosmic inflation and the effects of local Lorentz violation on the inflationary observables from the framework of modified teleparallel gravity theories [29–34]. Particularly, in [31] the authors have investigated power-law and intermediate inflation in $f(T)$ gravity, whereas in [33] it has been studied the slow-roll inflation in a generalized non-minimally coupled scalar-torsion gravity theory with a Galileon-type self-interaction. In Ref. [32], the authors concentrated their efforts in the investigation of the consequences of local Lorentz violation to the generation of primordial density fluctuations. They showed that due to local Lorentz violation no subhorizon scalar-perturbation mode can survive by the time of horizon crossing, and thus these theories are incapable of generating enough primordial density inhomogeneity, even if it brings some de Sitter background solution. Moreover, in [34], where a nonminimal coupling to the vector torsion has been included, the authors have corroborated the aforementioned result of [32], and they have also concluded that for some relation between the coupling functions to torsion scalar and vector torsion, the scalar field can source the linear perturbations.

We study the generation of primordial density fluctuations in a larger class of generalized teleparallel scalar-torsion $f(T,\phi)$ gravity theories with local Lorentz symmetry breaking. We confirm what was obtained in [32] in regards the case of a nonminimal coupling function $f(T,\phi)$ lineal in $T$, but we also show that inflationary models with nonminimal coupling function $f(T,\phi)$ which is non-linear in $T$, can generate primordial fluctuations. The paper is organized as follows. In section 2 we give a concise introduction to TG. In section 3 we develop the framework of generalized $f(T,\phi)$ gravity theory, calculating the background equations for a Friedmann-Robertson-Walker (FRW) metric, and then analysing the de-Sitter limit. In the sections 4 and 5, we investigate the cosmological perturbations using the ADM formalism for the tetrad fields and using the Maldacena’s method of expanding the action...
until second order for the perturbations [35]. In section 6 we apply our results to the particular case of power-law inflation. Finally, in Section 7 we summarize our findings and present our main conclusions and final remarks.

2. Teleparallel Gravity

Teleparallel Gravity (TG) is a gauge theory for the translation group which constitutes an alternative description of gravity based on torsion [23–26]. The dynamical variable of TG is the tetrad field \( e^A_\mu(x^\nu) \), and it connects the spacetime metric \( g_{\mu\nu} \) and the Minkowski tangent space metric \( \eta_{AB} = \text{diag}(-1,1,1,1) \) through the local relation

\[
g_{\mu\nu} = e^A_\mu e^B_\nu \eta_{AB}, \tag{1}
\]

where \( e^A_\mu \) are the tetrad components in a coordinate base and also satisfying the orthogonality conditions \( e^A_\mu e^A_\nu = \delta^\mu_\nu \) and \( e^A_\mu e^B_\nu = \delta^B_\mu \) with \( e^B_\mu \) the inverse components.

The action functional of TG is given by

\[
S = -\frac{M^2_{\text{pl}}}{2} \int d^4x \epsilon T, \tag{2}
\]

being \( T \) the torsion scalar, \( \epsilon = \det (e^A_\mu) = \sqrt{-g} \), and \( M^2_{\text{pl}} = (8\pi G)^{-1} \) the reduced Planck mass. The torsion scalar is defined as

\[
T = S^{\rho\mu\nu} T_{\mu\nu}^{\rho}, \tag{3}
\]

where

\[
T^{\rho\mu\nu} \equiv e^A_\mu [\partial_\rho e^A_\nu - \partial_\nu e^A_\rho + \omega^A_{\beta\mu} e^\beta_\nu - \omega^A_{\beta\nu} e^\beta_\rho], \tag{4}
\]

are the components of torsion tensor, and

\[
S^{\rho\mu\nu} = \frac{1}{2} \left( K^{\rho\mu\nu} + \delta^{\rho\mu} T^{\sigma\nu\theta} - \delta^{\rho\nu} T^{\sigma\mu\theta} - \delta^{\rho\theta} T^{\sigma\mu\nu} \right), \tag{5}
\]

is the so-called super-potential, with

\[
K^{\rho\mu\nu} = \frac{1}{2} \left( T^{\mu\nu\rho} - T^{\nu\rho\mu} - T^{\rho\mu\nu} \right). \tag{6}
\]

the contorsion tensor.

The purely inertial spin connection of TG is

\[
\omega^A_{\beta\mu} = \Lambda^A_D (x) \partial_\beta \Lambda^D_{\mu}(x), \tag{7}
\]

with \( \Lambda^A_D (x) \) a local (point-dependent) Lorentz transformation. For this connection the curvature tensor vanishes identically, whereas that the torsion tensor is non-vanishing [36].

The corresponding spacetime-indexed linear connection is

\[
\Gamma^\rho_{\mu\nu} = e^A_\rho [\partial_\mu e^A_\nu + \omega^A_{B\nu} e^B_\mu], \tag{8}
\]

which is the so-called Weitzenböck connection. It is related to the Levi-Civita connection of GR through

\[
\Gamma^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + K^{\rho\mu\nu}. \tag{9}
\]

Using this latter equation, it can be shown that

\[
T = -R - e^{-1} \partial_\mu (e^{T\nu\mu} \beta), \tag{10}
\]

where \( R \) is the curvature scalar of Levi-Civita connection [36]. This equation show that TG and GR are equivalent theories in the level of field equations.

However, when one modifies gravity from the viewpoint of TG, by introducing a non-minimally coupled matter field, for example a scalar field [37–41], or by adding into the action, non-linear terms in the torsion scalar \( T \), as for example in \( f(T) \) gravity [12–15], it is obtained a new class of modified gravity theories with a rich phenomenology and not equivalent to their corresponding counterpart based on curvature [42].

Below, we are going to study the generation of primordial density fluctuations in generalized teleparallel scalar-torsion gravity theories.

3. Generalized Scalar-torsion gravity

3.1. Field equations and local Lorentz invariance

The relevant action is given by

\[
S = \int d^4x \epsilon [f(T,\phi) + P(\phi)X], \tag{11}
\]

where \( f \) is an arbitrary function of \( \phi \) and \( T \), and also \( X = -\partial_\mu \phi \partial^\mu \phi/2 \). This general action includes non-minimally coupled scalar-torsion gravity models with \( f(T,\phi) \) the coupling function, and \( f(T) \) gravity, plus minimally coupled scalar field. For \( f(T,\phi) = -M^2_{\text{pl}} T/2 - V(\phi) \), we recover TG, with \( V(\phi) \) the scalar potential [12].

Varying the action with respect to the tetrad field \( e^A_\mu \) we find the corresponding field equations

\[
f_{,\mu} G_{\mu\nu} + S_{\mu\rho\nu} \partial_\rho f_{,T} + \frac{1}{4} \delta_{\mu\nu} (f - T f_{,T}) + \frac{P}{4} (\delta_{\mu\nu} X + \partial_\mu \phi \partial_\nu \phi) = 0, \tag{12}
\]

which have been expressed in a general coordinate basis, and \( G^\mu_\nu = e^A_\mu G^A_\nu \) is the Einstein tensor, and the tensor \( G^A_\nu \) is defined as \( G^A_\mu \equiv e^{-1} \partial_\nu (e e^A_\mu S^{\mu\rho\nu} - e^A_\mu T^{\lambda\rho} S^\lambda_{\rho\nu} + e^A_{B\lambda} \omega^B_{\lambda\rho\nu}) + e^A_{B\lambda} \omega^B_{\lambda\rho\nu} + \frac{1}{4} \epsilon^{AB} T \). The equation (12) has an antisymmetric part associated with the tensor \( S_{\mu\nu} \) in the second term. It is an expected result as the action (11) is not local Lorentz invariant [27–28]. To see this explicitly, let us consider the infinitesimal point-dependent Lorentz transformation \( e^A_\mu \equiv e^A_\mu + \xi^A_B e^B_\mu \), with \( \xi^{BA} = -\xi^{AB} \). Under this transformation the variation of the field is

\[
\delta S = \int d^4x \epsilon \partial_\rho f_{,T} S_{\mu\rho\nu} \xi^{\mu\nu}, \tag{13}
\]

where \( \xi^{\mu\nu} \equiv e^A_\mu \xi^{AB} e^B_\nu \). The condition \( \delta S = 0 \) for arbitrary \( \xi^{AB} \) leads us to the following constraint

\[
\partial_{\rho} f_{,T} S_{\mu\rho\nu} = 0. \tag{14}
\]
For TG, $f \sim T$, one has $\partial_T f_T = 0$, and thus the left hand side of this latter equation becomes identically equal to zero, and local Lorentz invariance is restored [36]. For modified teleparallel gravity, $\partial_T f_T \neq 0$, it corresponds to a set of six equations for six additional degrees of freedom, due to violation of local Lorentz symmetry.

3.2. Cosmological background

We impose the standard homogeneous and isotropic background geometry by choosing

$$e^A_{\mu} = \text{diag}(1,a,a,a),$$

which corresponds to a flat Friedmann-Robertson-Walker (FRW) universe with metric

$$ds^2 = -dt^2 + a^2 \delta_{ij}dx^i dx^j,$$

where $a$ is the scale factor which is a function of the cosmic time $t$.

Replacing this tetrad field in the field equations (12), we obtain the background equations

$$f(T, \phi) - P(\phi)X - 2Tf_{,T} = 0, \quad (17)$$

$$f(T, \phi) + P(\phi)X - 2Tf_{,T} - 4Hf_{,T} - 4f_{,T} = 0, \quad (18)$$

$$-P_{,\phi}X - 3P(\phi)H\phi - P(\phi) + f_{,T} = 0, \quad (19)$$

where $H = \dot{a}/a$ is the Hubble rate, and a dot represents derivative with respect to $t$. Also, a comma denotes derivative with respect to $\phi$ or $T$. In these equations, we also have $T = 6H^2$, which has been obtained from Eq. (3).

To analyse the de-Sitter limit [17], we use the values $\phi = \phi_*$ and $H = H_*(t)$ in the background equations, so we obtain $\dot{H}_* = 0$ and $f_{,T}(T_*, \phi_*) = 0$. After that, we study the perturbations of this de-Sitter limit, using $\phi = \phi_* + \delta\phi(t)$ and $H = H_* + \delta H(t)$. Then, we expand equation [17] to first order and we find $\delta H = 0$. Next, expanding equation [19] to first order, we find the equation for $\delta(\phi(t))$,

$$\delta\phi + 3H_\delta \phi + f_{,\phi\phi}(T_*, \phi_*) P(\phi_*) \delta \phi = 0,$$

whose solution is given by,

$$\delta\phi(t) = C_1 e^{\mu_+ t} + C_2 e^{\mu_- t}, \quad (21)$$

where $C_1$ and $C_2$ are integration constants, and

$$\mu_{\pm} = -\frac{3H_*}{2} \left[ 1 \pm \sqrt{1 + \frac{4f_{,\phi\phi}(T_*, \phi_*)}{9H_*^2P(\phi_*)}} \right].$$

Therefore, the perturbation $\delta \phi$ is stable only for

$$f_{,\phi\phi}(T_*, \phi_*) P(\phi_*) < 0. \quad (22)$$

For non-phantom scalar fields, it is required $P(\phi_*) > 0$, and then the above constraint becomes $f_{,\phi\phi}(T_*, \phi_*) < 0$. For $f(T)$ gravity, without dynamical scalar field, one has $f_{,\phi\phi}(T_*, \phi_*) = 0$, and then the eigenvalues are $\mu_- = 0$ and $\mu_+ = -3H_*$. Therefore, in this latter particular case, the de-Sitter background is always (marginally) stable [44].

In order to realize the slow-roll approximation into the present scenario, from Eqs. (17) and (18) we obtain

$$\epsilon = \delta P_X + \delta f_{,T}, \quad (24)$$

where we have introduced the slow-roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \delta P_X = -\frac{P(\phi)X}{2H^2f_{,T}}, \quad \delta f_{,T} = \frac{\dot{f}_{,T}}{f_{,T}H} \quad (25)$$

Also, it is useful to split the parameter $\delta f_{,T}$ as

$$\delta f_{,T} = \delta f_{,H} + \delta f_{,X}, \quad (26)$$

where we define

$$\delta f_{,H} = \frac{\dot{f}_{,TT} T'}{f_{,T} H}, \quad \delta f_{,X} = \frac{f_{,T\phi} \dot{\phi}}{H f_{,T}}. \quad (27)$$

Thus, from the relations (24) and (25), it is easy to obtain

$$\delta f_{,H} = -\frac{2\mu}{1 + 2\mu} (\delta P_X + \delta f_{,X}), \quad (28)$$

$$\delta f_{,T} = \frac{1}{1 + 2\mu} (\delta f_{,X} - 2\mu \delta P_X) \quad (29)$$

and then

$$\epsilon = \frac{1}{1 + 2\mu} (\delta P_X + \delta f_{,X}), \quad (30)$$

where $\mu = T f_{,TT}/f_{,T}$ is the so-called deviation parameter, which is a constant during inflation [45].

The time dependence of the slow-roll parameters is calculated as

$$\frac{\delta P_X}{H \delta P_X} = \dot{\epsilon} + 2G_{\phi} + 2\epsilon - \delta f_{,T}, \quad (31)$$

$$\frac{\delta f_{,T}}{H \delta f_{,T}} = \delta f_{,TT} + \delta P_X + (\delta f_{,\phi} - \delta f_{,TT}) \frac{\delta f_{,X}}{\delta f_{,T}}, \quad (32)$$

$$\frac{\delta f_{,X}}{H \delta f_{,X}} = \delta f_{,\phi} + \delta \phi + \delta P_X, \quad (33)$$

where it has also been defined the slow-roll parameters

$$\delta P = \frac{\dot{P}}{H P}, \quad \delta \phi = \frac{\dot{\phi}}{H \phi}, \quad \delta f_{,TT} = \frac{\dot{f}_{,TT}}{H f_{,TT}}, \quad \delta f_{,\phi} = \frac{\dot{f}_{,\phi}}{H f_{,\phi}}. \quad (34)$$

During slow-roll inflation $\delta P_X \sim \delta f_{,T} \sim \delta f_{,X} \sim \mathcal{O}(\epsilon^2)$, and similarly for the other parameters.
4. Scalar Perturbations

4.1. Second order action

In order to study primordial density fluctuations, we start from the Arnowitt-Deser-Misner (ADM) decomposition of the tetrad field \( 30 \)
\[
e^0_\mu = (N, 0), \quad e^a_\mu = (N^a, h^a_i), \quad e^0_\mu = (1/N, -N^i/N), \quad e^a_\mu = (0, h^a_i),
\]
where \( N^i = h^i_a N^a, \) with \( h^a_i h^b_j = \delta^a_j, \) being \( h^a_i \) the induced tetrad field.

Using the uniform field gauge, \( \delta \phi = 0, \) a convenient ansatz for the fields is
\[
N = 1 + \alpha, \quad N^a = a^{-1} e^{-R} \delta_i^a \partial^j \psi, \quad h^i_j = a e^{R} \delta_j^a \delta_i^a,
\]
which gives the corresponding perturbed metric \( 49 \)
\[
ds^2 = -\left[ (1 + \alpha)^2 - a^{-2} e^{-2R} (\partial \psi)^2 \right] dt^2 + 2\alpha \psi dt dx^i + a^2 e^{2R} \delta_{ij} dx^i dx^j.
\]
The additional degrees of freedom due to local Lorentz violation can be incorporated in the form of Goldstone modes of the symmetry breaking, by performing a Lorentz rotation of the tetrad field \( 5 \). So, under the transformation
\[
\Lambda^A B = (e^A)^B C \delta^A + x^A B + \frac{1}{2} x^A C x^B + O(\chi^3),
\]
and keeping fixed the zero spin connection for the cosmological background, the full tetrad field is written as
\[
e^A_\mu = (e^A)^B C e^B_\mu, = e^A_\mu + x^A B e^B_\mu + \frac{1}{2} x^A C x^B e^B_\mu + O(\chi^3).
\]
The matrix \( \chi_{AB} = -\chi_{BA} \) is parametrized as
\[
\chi^0_B = (0, \chi_b), \quad \chi^a_B = (\chi^a_B, B^a_{ hi}),
\]
where \( \chi^a = \eta^{ab} \chi_b \) and \( B_{ab} = -B_{ba}. \) It is defined the spatial vector \( \chi^i = h^i_a \chi_a = \partial_i \beta + \chi_i^{(T)}, \) and the spatial antisymmetric tensor \( B_{ij} = \delta_{ij}^{(T)} B_{hi} B_{ab} = -B_{ji} = -\epsilon_{ijk} B^k. \)

Therefore, there are a scalar mode \( \beta, \) a transverse vector mode \( \chi_i^{(T)} \) and a (pseudo) vector mode \( B. \) \( 52, 50. \)

Following \( 33, 35 \), the next step is to expand the action \( 11 \) up to second order to obtain
\[
S^{(2)} = \int dt d^3 x \left[ \frac{2}{a^2} (w_1 \dot{\bar{R}} - w_1 H \alpha) \dot{\beta}^2 \psi + 6w_1 H \alpha \dot{\bar{R}} - 2w_2 \alpha \dot{\beta} \cdot \psi - 2w_3 \alpha \dot{\beta} \dot{\psi} + 2w_4 \beta \dot{\beta} \dot{\psi} + 2w_5 \beta \dot{\psi}^2 + 2w_6 \beta \dot{\psi} \right],
\]
where we have defined the functions
\[
w_1 = -2(f_T + 2f_{TT}),
\]
\[
w_2 = -2f_T,
\]
\[
w_3 = P(\psi) X + T f_T + 2T^2 f_{TT},
\]
\[
w_4 = -2(f_T + T f_{TT}),
\]
\[
w_5 = 4f_T,
\]
\[
w_6 = \frac{4}{3} T f_{TT}.
\]
From action \( 42 \), it can be seen that the scalar modes \( \alpha, \psi \) and \( \beta \) are auxiliary fields and do not propagate. Varying this action with respect to \( \dot{\beta}^2 \psi \) leads us to
\[
w_1 \ddot{R} - w_1 H \alpha + \frac{w_6}{a^2} \partial^2 \beta - w_6 \partial^2 \beta = 0,
\]
whereas variation with respect to \( \dot{\beta}^2 \dot{\beta} \) gives
\[
-4w_2 \dot{\bar{R}} + 4w_4 H \alpha + w_5 \ddot{\bar{R}} - 2 \frac{w_6}{a^2} \dot{\beta}^2 \psi + 2w_5 \partial^2 \beta = 0,
\]
and for \( \alpha \) we have
\[
-2 \frac{w_1}{a^2} H \partial^2 \psi + 6w_1 H \ddot{\bar{R}} - 2 \frac{w_2}{a^2} \partial^2 \bar{R} + 2w_3 \alpha + 4w_4 H \partial^2 \beta = 0.
\]
Solving the above three equations for \( \alpha, \dot{\beta}^2 \psi \) and \( \dot{\beta}^2 \dot{\beta}, \) and after substituting these results in Eq. \( 42, \) the second order action for curvature fluctuation can be written as
\[
S^{(2)} = \int dt d^3 x a^3 Q_s \left[ \dot{\bar{R}}^2 - \frac{2}{a^2} (\partial \bar{R})^2 - m^2 \bar{R}^2 \right],
\]
where
\[
Q_s = \frac{3w_1 H^2 + w_3}{H^2} = P X = \frac{P X}{H^2},
\]
\[
e_s^2 = 1,
\]
\[
m^2 = \frac{w_2}{w_2} \left( 3H + \frac{Q_s}{Q_s} - 2 \frac{w_2}{w_2} + \frac{w_2}{w_2} + w_1 w_2 w_6 Q_s \right).
\]
The first and second term in action \( 47 \) are the usual terms appearing in the quadratic action of perturbations, while the third term is a new explicit mass term, that represents the effects of local Lorentz-symmetry breaking. The origin of this mass term is the Lorentz violating coupling term \( f(T, \psi) \) in action \( 11. \) The emergence of this propagating massive scalar mode could be related to an alternative gravitational Higgs mechanism \( 3, 5. \)

For any theory to be physically viable, it must be free of ghosts and Laplacian instabilities by requiring \( Q_s > 0 \) and \( e_s^2 > 0. \) These two conditions are satisfied by equations \( 48 \) (for \( P > 0 \)) and \( 49. \) Moreover, in the presence of an explicit mass term, there is an additional condition that is the non-occurrence of tachyonic instability \( 51. \) There are two situations in which the tachyonic instability can be avoided. The first possibility is that the mass squared
must be positive, \( m^2 > 0 \), and, the second one, if we have \( m^2 < 0 \), then it is required that \( |m^2| \lesssim H^2 \) \cite{51,52}.

In terms of slow roll parameters we can write
\[
Q_s = w_2 \delta_{P,X} ,
\]
and it is also useful to define
\[
\eta = \frac{Q_s}{H Q_s} = \delta_p + 2 \delta_\phi + 2 \epsilon .
\]

Similarly, the mass term can be written as
\[
\eta_R = \frac{m^2}{3H^2} = \delta_{f,T} \left[ 1 + \left( 1 + \frac{\delta_{fX}}{\delta_{PX}} \right) \delta_{fH} \right] ,
\]
For \( f(T,\phi) \) non-linear in \( T \), and either \( \delta_{fX} = 0 \) or \( \delta_{fX} \neq 0 \), one has that \( \eta_R \sim O(\epsilon) \) is non-zero (and finite). Furthermore, tachyonic instability is avoided as long as \( |\eta_R| \lesssim 1 \). In the absence of coupling between \( T \) and \( \phi \), one has \( \delta_{fX} = 0 \), and thus \( \delta_{fT} = \delta_{fH} \). So, from Eq. \( \S2 \), we find \( \eta_R = 2 \delta_{fH} \sim O(\epsilon) \). This is the explicit mass term arising in \( f(T) \) gravity, plus scalar field. For \( T \), \( f \sim T \), one has \( \delta_{fH} = 0 \), and then \( \eta_R = 0 \), which is an expected result since \( T \) is local Lorentz invariant \cite{39}.

Now, let us consider the case of \( f(T,\phi) \) a linear function in \( T \), and \( \delta_{fX} \neq 0 \). This is precisely the non-minimally coupled scalar-torsion theory of Ref. \cite{52}. For this model one has \( \delta_{fH} = 0 \), \( \delta_{fT} = \delta_{fX} \), and then \( |\eta_R| = \infty \). The physical meaning of this is that there are non-zero-momentum solutions for the scalaron, as in this case one would have \( \partial^2 \mathcal{R} = 0 \), from action \( \S2 \), and then, spoiling the generation of primordial density fluctuations. This latter result is consistent with what was obtained in \cite{52}.

### 4.2. Mukhanov-Sasaki equation

It is introduced the canonically-normalized Mukhanov variable
\[
v = z \mathcal{R} ,
\]
where we have also defined
\[
z^2 = 2a^2 Q_s .
\]

Making the change to conformal time \( d\tau = dt/a \), and using the above variables, action \( \S2 \) can be written as
\[
S^{(2)} = \frac{1}{2} \int d\tau d^3 x \left[ (v')^2 - c_s^2 (\partial \nu)^2 - M^2 v^2 \right] ,
\]
where it has been defined the effective mass term as
\[
M^2 = a^2 m^2 - \frac{z''}{\sqrt{2}} ,
\]
where \( m^2 = 3H^2 \eta_R \), with \( \eta_R \) given by \( \S3 \), and \( z''/z \) is the usual effective mass term coming from the interaction between \( \mathcal{R} \) and the cosmological background.

Varying the action \( \S4 \) and using the Fourier expansion
\[
v(\tau, x) = \int \frac{d^3 k}{(2\pi)^3} v_k(\tau) e^{i k \cdot x} ,
\]
it is straightforward to obtain
\[
v'_k + (k^2 + M^2) v_k = 0 .
\]
Furthermore, this equation can be arranged in the way
\[
v'_k + \left[ k^2 - \frac{1}{4} \left( \nu^2 - \frac{1}{4} \right) \right] v_k = 0 ,
\]
where we have defined
\[
\nu = \nu - \eta_R = \frac{3}{2} + \epsilon + \frac{1}{2} \eta - \eta_R .
\]
For \( \nu \) constant and real, the exact solution to \( \S5 \) is
\[
v_k(\tau) = \sqrt{-\nu} \left[ C_1 H^{(1)}_\nu (-k \tau) + C_2 H^{(2)}_\nu (-k \tau) \right] ,
\]
where \( H^{(1)}_\nu \) and \( H^{(2)}_\nu \) are the Hankel’s functions of first and second kind, respectively \cite{53}. By imposing the Bunch-Davies vacuum, such that the solution matches plane-wave solution \( v_k(\tau) = e^{-i k x}/\sqrt{2K} \), at the ultraviolet regime \( k \gg aH \) \( (k \tau \ll 1) \), and using the relations
\[
\lim_{k \tau \rightarrow -\infty} H^{(1,2)}_\nu (-k \tau) = \sqrt{\frac{2}{\pi \sqrt{k}}} e^{\mp i k x} \frac{\Gamma(\nu)}{\Gamma(\frac{1}{2})} \left( \frac{\nu}{k} \right) ,
\]
we find \( c_1 = \sqrt{\frac{\nu}{\pi}} \frac{\Gamma(\nu)}{\Gamma(\frac{1}{2})} \) and \( c_2 = 0 \). Therefore, the exact solution to \( \S6 \) becomes
\[
v_k(\tau) = \sqrt{-\nu} \left[ e^{\mp i k x} (\nu + \frac{1}{2}) \right] H^{(1)}_\nu (-k \tau) .
\]

On super-horizon scales \( k \ll aH \) \( (k \tau \rightarrow 0) \), and using
\[
\lim_{k \tau \rightarrow 0} H^{(1)}_\nu (-k \tau) = \frac{\sqrt{2}}{\pi e^{-i \frac{1}{2}} 2^{-\frac{1}{2} - \frac{1}{2} \nu} \Gamma(\frac{1}{2})} (k \tau)^{-\frac{3}{2}} ,
\]
one finds
\[
v_k(\tau) = e^{i k x} \left[ 2^{-\frac{1}{2} - \frac{1}{2} \nu} \Gamma(\frac{1}{2}) \right] \frac{1}{\sqrt{2k}} (k \tau)^{-\frac{3}{2}} .
\]

Now, taking into account that \( \tau = (-1/aH)(1 + \epsilon) \) (at first-order) and \( \mathcal{R}_k = z^{-1} v_k = (H/k)(k/aH)(2Q_s)^{-1/2} v_k \), we write
\[
|\mathcal{R}_k| \approx \frac{H}{2 \sqrt{k^3 Q_s}} \left( \frac{k}{aH} \right)^{\frac{3}{2}} ,
\]
where \( H_k \) and \( Q_{sk} \) are the values of \( H \) and \( Q_s \) at \( k = aH \).

\begin{equation}
\S5
\end{equation}
The scalar power spectrum of curvature perturbation is calculated as
\[ P_s(k) = \frac{k^3}{2\pi^2} |\mathcal{R}_k(\tau)|^2, \]
\[ \sim \frac{H_k^2}{8\pi^2 Q_{kk}} \left[ 1 + 2\eta_R \ln \left( \frac{k}{aH} \right) \right]. \tag{68} \]

Given that \( \eta_R \sim O(\epsilon) \), the consequence of local Lorentz violation is a slight logarithmic time-dependence of the curvature perturbation and its power spectrum at super-horizon scales. Thus, as a satisfactory approximation, we can evaluate this latter at the horizon crossing [53].

Finally, the scale-dependence of the scalar power spectrum is
\[ n_s - 1 \equiv \frac{d\ln P_s(k)}{d\ln k} \bigg|_{k=aH} = -2\epsilon - \eta + 2\eta_R. \tag{69} \]
This carries out the effects of local Lorentz violation on the scalar power spectrum through the term \( 2\eta_R \), at first-order in slow-roll approximation.

5. Tensor perturbations

From the ADM decomposition for the tetrad field presented in Eqs. [35] and [36], and using the uniform field gauge, \( \delta \phi = 0 \), we take [39]
\[ N = 1, \quad N^a = 0, \quad h^a_i = a(\delta^a_i + \frac{1}{2}\gamma^a_i). \tag{70} \]
Then the induced 3–metric is
\[ g_{ij} = \eta_{ab}h^a_ih^b_j = a^2 \left[ \delta_{ij} + h_{ij} + \frac{1}{4}\gamma_{ij}\gamma^b_j \right], \tag{71} \]
where we have defined
\[ h_{ij} = \frac{1}{2} \eta_{ab} \left( \delta^b_i \gamma_j^b + \delta^b_j \gamma_i^b \right) = \frac{1}{2} \left( \gamma_{ij} + \gamma_{ji} \right), \tag{72} \]
and \( \gamma^a_i = \gamma^{ij}\gamma^a_j \). Given that the \( \gamma^2 \) term has contribution only in cubic calculations of the Lagrangian [35], we keep only until the second term \( h_{ij} \) in the induced metric. Also, the tensor \( \gamma_{ij} \) can be split in the form \( \gamma_{ij} = \gamma_{(ij)} + \gamma_{[ij]} \). The symmetric part \( h_{ij} = \gamma_{(ij)} \) fulfills the transverse and traceless conditions, \( \partial^\lambda h_{ij} = h^i_j = 0 \), to be gauge invariant [12]. On the other hand, the antisymmetric part matches the gauge degrees of freedom in the local Lorentz invariant theory, and then we identify \( B_{ij} \) with \( \gamma_{[ij]} \).

Then, using the tetrad formalism we find the second-order action for the tensor modes, \( h_{ij} = h_+\epsilon^+_i + h_\times\epsilon^\times_i \), in the way
\[ S_T = \sum_\lambda \int dt d^3x a^3Q_T \left[ h^2_\lambda - \frac{c^2}{a^2} \left( \partial h_\lambda \right)^2 \right], \tag{73} \]
where two polarization states are given by \( \lambda = +, \times \). We have also defined
\[ Q_T = -\frac{1}{2} f_T, \tag{74} \]
and the squared tensor propagation speed is
\[ c_T^2 = 1. \tag{75} \]
The non-ghost condition is satisfied only for \( f_T < 0 \). Besides the usual transverse massless graviton modes, propagating at speed of light, there are no additional propagating modes in the quadratic action [73], which is consistent with local Lorentz invariance of tensor perturbations [34].

The power spectrum for tensor perturbations becomes
\[ P_T = \frac{H_k^2}{2\pi^2 Q_{Tk}}, \tag{76} \]
with \( H_k \) and \( Q_{Tk} \) the values of \( H \) and \( Q_T \) at \( k = aH \). Thus, the spectral index is
\[ n_T \equiv \frac{d\ln P_T}{d\ln k} \bigg|_{k=aH} = -2\epsilon - \delta f_T. \tag{77} \]
Tensor-to-scalar ratio, evaluated at the horizon crossing, is given by
\[ r = \frac{P_T}{P_s} \simeq 16\delta f_X = 16 \left( \epsilon - \delta f_T \right). \tag{78} \]
Using the Eqs. (77) and (78), we obtain the consistency relation
\[ r = 8 \left( -n_T - 3\delta f_T \right). \tag{79} \]
This is agreement with the standard inflation limit where \( r = -8n_T \). The quantity \( \delta f_T \) appears as a small correction to the value of standard inflation.

6. Application to a concrete model of inflation
We consider the ansatz
\[ f(T, \phi) = -\frac{M^2}{2} T - G(T)F(\phi) - V(\phi), \tag{80} \]
with \( G(T) = T^a \), \( F(\phi) = \xi \phi^c \), and \( V(\phi) = \lambda \phi^d \), where \( s, c, d, \xi, \lambda \) are positive constants.

Under the slow-roll approximation, \( \dot{\phi}^2/2 \ll V \) and \( |\dot{\phi}| \ll H|\dot{\phi}| \) [12], the backgrounds equations (17) and (19) give
\[ T \simeq \left[ \frac{\lambda}{\xi (2s - 1)} \right]^{\frac{1}{2}} \phi^\frac{d-c}{d}, \tag{81} \]
where \( \phi \) becomes
\[ \phi^2 \frac{d-c}{2} + \frac{d-c}{d} \sim 2\lambda \left[ \frac{\xi (2s - 1)}{\lambda} \right]^{\frac{1}{2}} \left( \frac{c}{2s - 1} + d \right) N, \tag{82} \]
and we have introduced the e-folds number \( N \) [12]. Here, we have also applied the high energy limit \( G_{TT}/M^2_{pl} \gg 1 \), for \( N \gg 1 \), with \( \mu \equiv TG_{TT}/G_{TT} = s - 1 \).
Thus, from Eq. (53), we obtain
\[ \eta_R = -\frac{2\lambda}{s} \left[ \frac{\xi (2s - 1)}{\lambda} \right]^{\frac{1}{2}} \left( \frac{c}{2s - 1} + d \right) \times \left[ \frac{c(3s - 2)}{2s - 1} + 2d(s - 1) \right]^{\frac{d-c}{d}}. \tag{83} \]
For $s = 1$ there is a divergence. So, it is required $s > 1$.

The scalar power spectrum is written as

$$P_s = \frac{(2s - 1)^2}{96\pi^2}\frac{\lambda^2 - 2^d\phi^{3d(c+2)-2d+2}}{c(d-2)}, \quad (84)$$

whereas $n_s$ and $r$ are given by

$$n_s = 1 - \frac{2\lambda}{(s-1)c} \left[ \frac{\xi (2s - 1)}{\lambda} \right] ^{\frac{1}{2}} \times \left[ \frac{c^2 (s-1)}{2s-1} + \frac{2\xi (s-1)(3ds - d + s)}{2s-1} \right] \phi^{c/d + d - 2}, \quad (85)$$

$$r = \frac{16\lambda (2s - 1)}{s} \left[ \frac{\xi (2s - 1)}{\lambda} \right] ^{\frac{1}{2}} \times \left( \frac{c}{2s-1} + 1 \right)^2 \phi^{c/d + d - 2}. \quad (86)$$

Using Eq. (82) and for $N \gg 1$, $n_s$ and $r$ take the form $n_s = 1 - p/N$ and $r = q/N$ with $p = p(s,c,d)$ and $q = q(s,c,d)$, functions of $s$, $c$, and $d$. The latest cosmological data from Planck satellite [54] fixed the values of $n_s$ and $r$ in the ranges $n_s = 0.9649 \pm 0.0042$ at 68% CL, and $r < 0.07$ at 95% CL, which allows us constraint the parameters of $s$, $c$, and $d$. Additionally, from the observational data for the scalar power spectrum, $P_s = 2.141 \times 10^{-9}$ [54], we can obtain an estimate for $\lambda$ and $\xi$. For $c = 0$, which corresponds to $f(T)$ gravity plus scalar field, and for $N = 50$, it is required $s > 2.318$, and $0.3075 \leq \xi \leq 0.3075$. For example, for $s = 3$, we have $0.4042 \leq \xi < 0.4138$, and then $-0.0064 < \eta_R < -0.0062$. In this case, we also obtain $\lambda/M^{4-d}_p \sim 10^{-7}$ and $\xi M^2_\phi \sim 10^{15}$. These results are consistent with the observational data found for the concave potential, $V_{\phi\phi} < 0$, in Ref. [54]. However, in this case, since $d < 1$, the stability condition of de-Sitter background in Eq. (23) is not satisfied.

For non-minimally coupled scalar-torsion models, with $c > 0$, it is required $c < d$, in order to have $H < 0$. Using the observational data of $n_s$ and $r$, and for $N = 70$, and $d = 1$, with $c = 0.05$, we find the range $1.00214 \leq s \leq 1.00445$, and $-0.00924 \leq \eta_R \leq -0.00504$. On the other hand, for $N = 60$, and $d = 2/3$, with $c = 0.01$, we get $1.00015 \leq s \leq 1.00033$, and $-0.00850 \leq \eta_R \leq -0.00438$. For these values we get $\lambda/M^{4-d}_p \sim 10^{-3}$ and $\xi M^2_\phi \sim 10$. In FIG. 1 we show the curves $r(n_s)$ for several different values of the parameters. In FIG. 2 it is depicted the behaviour of $\eta_R$ as function of $N$. It can be verified that $|\eta_R| = |m^2/(3H^2)| \ll 1$.

7. Conclusions

We have studied the generation of primordial fluctuations in generalized teleparallel scalar-torsion gravity theories whose Lagrangian density is an arbitrary function $f(T, \phi)$ of the torsion scalar $T$ and a scalar field $\phi$, plus the kinetic term of this latter. To develop primordial density perturbations, we started from the Arnowitt-Deser-Misner (ADM) formalism of the tetrad field, and we choose the uniform gauge [13]. The tetrad field has sixteen degrees of freedom and local Lorentz invariance of TG allows us to eliminate six degrees of freedom, yielding the same number of independent components of the metric tensor [39]. However, it is well known that the action for modified teleparallel gravity is no longer a local Lorentz invariant, and thus, the field equations are not completely symmetric [27, 28]. In order to restore the local Lorentz invariance, we have introduced six additional degrees of freedom in the form of Goldstone modes of the symmetry breaking, through a Lorentz rotation of the tetrad field [5]. So, the antisymmetric part of field equations constitutes a set of six equations for six extra modes, that is, a scalar, a transverse 3-vector, and a spatial antisymmetric tensor modes.

Putting all these pieces together, and after integrating out the auxiliary fields, we have calculated the second or-
nder action for the propagating modes. As usual, we have treated the scalar and tensor modes separately since they are not coupled. Vector modes decay rapidly with the cosmic expansion, and thus they can be ignored. Furthermore, we have verified that the corresponding additional tensor modes are completely cancelled out from the second order action for tensor perturbations, remaining only the usual transverse massless graviton modes, propagating at speed of light, and therefore indicating the local Lorentz invariance in the tensor perturbations sector.

In the second order action for curvature perturbation, it is observed the emergence of an explicit mass term, which represent the effects of local Lorentz violation. This explicit mass term is of first-order in slow-roll, and it is always nonzero (and finite), for nonminimal coupling functions $f(T,\phi)$, which necessarily leads us to an alternative Higgs mechanism that has no direct analogue in nonabelian gauge theory. As expected, in the case of TG this explicit mass term is equal to zero, because the local Lorentz invariance. On the other hand, when the nonminimal coupling function $f(T,\phi)$ is linear in $T$, like the action considered in , it becomes divergent, which necessarily leads us to $\theta^2\mathcal{R} = 0$, or equivalently, $\mathcal{R}_k = 0$ for all Fourier mode $k$, and hence, it is immediate to conclude that no subhorizon scalar mode could propagate and survive by the time of horizon crossing. This latter result is consistent with what was obtained in.

Our results indicate that only for modified teleparallel gravity theories with non-linear coupling functions $f(T,\phi)$, including the $f(T)$ gravity, plus scalar field, as a particular example. The arising of this propagating massive scalar mode could be related to an alternative Higgs mechanism that has no direct analogue in nonabelian gauge theory. As expected, in the case of TG this explicit mass term is equal to zero, because the local Lorentz invariance. On the other hand, when the nonminimal coupling function $f(T,\phi)$ is linear in $T$, like the action considered in , it becomes divergent, which necessarily leads us to $\theta^2\mathcal{R} = 0$, or equivalently, $\mathcal{R}_k = 0$ for all Fourier mode $k$, and hence, it is immediate to conclude that no subhorizon scalar mode could propagate and survive by the time of horizon crossing. This latter result is consistent with what was obtained in.

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References

[1] S. Weinberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, John Wiley and Sons, New York, 1972.
[2] M. Gasperini, Inflation and Broken Lorentz Symmetry in the Very Early Universe, Phys. Lett. 163B (1985) 84–86.
[3] V. A. Kostelecky, S. Samuel, Gravitational Phenomenology in Higher Dimensional Theories and Strings, Phys. Rev. D40 (1989) 1886–1903.
[4] D. Colladay, V. A. Kostelecky, CPT violation and the standard model, Phys. Rev. D55 (1997) 6760–6774. arXiv:hep-ph/9703454.
[5] R. Bluhm, S.-H. Fung, V. A. Kostelecky, Diffeomorphism Violation, Massive Modes, and Gravity, Phys. Rev. D77 (2008) 065020. arXiv:0712.4119.
[6] R. Bluhm, Gravity Theories with Background Fields and Spacetime Symmetry Breaking, Symmetry 9 (10) (2017) 230. arXiv:1710.10515.
[7] D. Mattig, Modern tests of Lorentz invariance, Living Rev. Rel. 9 (2005) 5. arXiv:gr-qc/0502097.
[8] A. H. Guth, Inflationary universe: A possible solution to the horizon and flatness problems, Phys. Rev. D 23 (2) (1981) 347.
[9] A. A. Starobinsky, A new type of isotropic cosmological models without singularity, Phys. Lett. B 91 (1) (1980) 99–102.
[10] A. D. Linde, A new inflationary universe scenario: a possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems, Phys. Lett. B 108 (6) (1982) 389–393.
[11] V. F. Mukhanov, H. A. Feldman, R. H. Brandenberger, Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions, Phys. Rept. 215 (1992) 203–343.
[12] V. Mukhanov, Physical foundations of cosmology, Cambridge University Press, 2005.
[13] L. Ackerman, S. M. Carroll, M. B. Wise, Imprints of a Primordial Preferred Direction on the Microwave Background, Phys. Rev. D 75 (2007) 083502, [Erratum: Phys.Rev.D 80, 069901 (2009)]. arXiv:astro-ph/0701357.
[14] D. Baumann, L. McAllister, Inflation and String Theory, Cambridge Monographs on Mathematical Physics, Cambridge University Press, 2015. arXiv:1404.2601.
[15] A. Einstein, Riemannian geometry with maintaining the notion of distant parallelism, Sitz. Preuss. Akad. Wiss. 217.
[16] A. Unzicker, T. Cuse, Translation of einstein’s attempt of a unified field theory with teleparallelism, arXiv:physics/0503046.
[17] A. Einstein, A theory of gravitation, Math. Ann. 102 (1930) 685.
[18] A. Einstein, A theory of gravitation, Sitzungssber. Preuss. Akad. Wiss. Phys. Math. Kl. 401.
[19] C. Pellegrini, J. Pilehanski, A theory of gravitation, Math.-Fys. Skr. Dan. Vid. Selskab 2 (2).
[20] C. Møller, On the crisis in the theory of gravitation and a possible solution, K. Dan. Viden. Selsk. Mat.-Fys. Medd. 39 (13) (1978) 1–31.
[21] K. Hayashi, T. Nakano, Extended translation invariance and associated gauge fields, Progress of Theoretical Physics 38 (2) (1967) 491–507.
[22] K. Hayashi, T. Shirafuji, New general relativity, Phys. Rev. D 19 (12) (1979) 3524.
[23] J. G. Pereira, Teleparallelism: A New Insight Into Gravity, in: A. Ashiktar, V. Petkov (Eds.), Handbook of Spacetime, Springer, 2014, pp. 197–212. arXiv:1302.6983.
[24] V. C. de Andrade, L. C. T. Guilleìn, J. G. Pereira, Gravitational energy momentum density in teleparallel gravity, Phys. Rev. Lett. 84 (2000) 4533–4536. arXiv:gr-qc/0003100.
[25] H. I. Arcos, J. G. Pereira, Torsion gravity: A Reappraisal, Int. J. Mod. Phys. D 13 (2004) 2193–2240. arXiv:gr-qc/0501017.
[26] J. G. Pereira, Y. N. Obukhov, Gauge Structure of Teleparallel Gravity, Universe v.6 (2019) 139. arXiv:1906.06287.
[27] T. P. Sotiriou, B. Li, J. D. Barrow, Generalizations of teleparal-
lel gravity and local Lorentz symmetry, Phys. Rev. D83 (2011) 104030. arXiv:1012.4039

[28] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[29] R. Ferraro, F. Fiorini, Modified teleparallel gravity: Inflation without inflaton, Phys. Rev. D75 (2007) 084031. arXiv:gr-qc/0610067

[30] Y.-P. Wu, C.-Q. Geng, Primordial Fluctuations within Teleparallelism, Phys. Rev. D 86 (2012) 104035. arXiv:1012.4039.

[31] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[32] R. Ferraro, F. Fiorini, Modified teleparallel gravity: Inflation without inflaton, Phys. Rev. D 83 (2011) 104030. arXiv:1012.4039.

[33] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[34] R. Ferraro, F. Fiorini, Modified teleparallel gravity: Inflation without inflaton, Phys. Rev. D 83 (2011) 064035. arXiv:1012.4039.

[35] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[36] R. Ferraro, F. Fiorini, Modified teleparallel gravity: Inflation without inflaton, Phys. Rev. D 83 (2011) 064035. arXiv:1012.4039.

[37] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[38] R. Ferraro, F. Fiorini, Modified teleparallel gravity: Inflation without inflaton, Phys. Rev. D 83 (2011) 064035. arXiv:1012.4039.

[39] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[40] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[41] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[42] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[43] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[44] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[45] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[46] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[47] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[48] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[49] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[50] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[51] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[52] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[53] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041

[54] B. Li, T. P. Sotiriou, J. D. Barrow, f(T) gravity and local Lorentz invariance, Phys. Rev. D 83 (2011) 064035. arXiv:1010.1041