MODEL IDENTIFICATION FOR ARMA TIME SERIES THROUGH CONVOLUTIONAL NEURAL NETWORKS

WAI HOH TANG AND ADRIAN RÖLLIN

National University of Singapore

Abstract

In this paper, we use convolutional neural networks to address the problem of model identification for autoregressive moving average time series models. We compare the performance of several neural network architectures, trained on simulated time series, with likelihood based methods, in particular the Akaike and Bayesian information criteria. We find that our neural networks can significantly outperform these likelihood based methods in terms of accuracy and, by orders of magnitude, in terms of speed.

Keywords: autoregressive moving average time series; ARMA; model identification; model selection; convolutional neural networks (ResNet); Akaike information criterion; Bayesian information criterion.

1 INTRODUCTION

The autoregressive moving average (ARMA) time series model is a classical stochastic model that appears in diverse fields from foreign exchange to biomedical science to rainfall prediction. Using ARMA model for time series analysis typically involves three parts: identification of model orders, estimation of model coefficients and forecasting. Identification of ARMA orders is crucial as this has an impact on the subsequent two parts. While the equation describing an ARMA model is simple and easy to interpret, the task of correctly identifying the orders of the AR and MA components to fit a time series is not straightforward. Various methods have been proposed, such as graphical approaches based on autocorrelation function and partial autocorrelation function by Box and Jenkins (1976) and, more commonly, likelihood based methods, such as Akaike information criterion (AIC); see Akaike (1969) and Durbin and Koopman (2001), Bayesian information criterion (BIC); see Akaike (1977), Rissanen (1978) and Schwarz (1978), Hannan-Quinn information criterion (HQC); see Hannan and Quinn (1979), and minimum eigenvalue criterion (MEV); see Liang et al. (1993).

Artificial intelligence methods such as genetic algorithms; see Ong et al. (2005), Pala-niappan (2006), Abo-Hammour et al. (2011) and Calster et al. (2017), and artificial neural networks; see Lee et al. (1991), Jhee et al. (1992), Lee and Jhee (1994), Lee and Oh (1996), Chenoweth et al. (2000) and Al-Qawasmi et al. (2010), are other paradigms in model identification. There are common features amongst the methodologies of these artificial neural networks studies, namely limiting the range of ARMA orders, limiting the length of time series and the need for pre-processing of time series. Firstly, the range of ARMA orders in these neural networks papers is typically small, for example, Chenoweth et al. (2000) evaluated orders up to 2 and Lee and Oh (1996) evaluated orders up to 5. Lee and Oh (1996) explained that most time series in the real world falls within ARMA(5,5) model. Secondly, the length of time series can vary substantially. Chenoweth et al. (2000) used
time series of length 100 and 3,000 while Al-Qawasmi et al. (2010) used length 1,500. Chenoweth et al. (2000) reasoned that the longer length was chosen to examine an upper limit in accuracy of identification because estimation errors were expected to be minimal for time series of such long length while the shorter length was more representative of real economic data. Thirdly, it is notable that raw time series data are not used directly as inputs in their unprocessed form. Instead, statistical properties or features of time series are used as inputs. Al-Qawasmi et al. (2010) used special covariance matrix of MEV criterion as input and Jhee et al. (1992), Lee and Jhee (1994) and Chenoweth et al. (2000) used extended sample autocorrelation function (ESACF) as inputs. Most papers reckoned that there are promises in using neural networks, and identifications are reasonably accurate but additional work is required for improvement.

In recent years, neural networks have received an increased amount of attention, in particular after Krizhevsky et al. (2012) introduced a convolutional neural network (CNN) architecture, called AlexNet, which had won the ImageNet Large Scale Visual Recognition Challenge in 2012 and dramatically improved the state-of-the-art of visual recognition and object detection; see the survey by LeCun et al. (2015). Although the key ideas of the neural network has already been introduced by LeCun et al. (1990), only advancements in computing power and availability of large data sets have enabled such deep CNN architectures to be built and trained efficiently. Many embellishments of the original architecture have been proposed since; particularly important has been the introduction of so-called skip connections by He et al. (2015), which allowed their architecture ResNet to win the ImageNet competition in 2015.

In this paper, we attempt to harness the strength of CNNs — which are ultimately just hierarchical non-linear filters — for the purpose of ARMA time series model selection. At the most fundamental level, likelihood-based methods and, as matter of fact, all other methods, are simply non-linear functions from the data space into some decision space. In our setting, the data space consists of the raw time series, and the decision space the order of autoregressive (AR) and moving average (MA) components predicted by the non-linear function. Our aim will thus be to find a suitable neural network (serving as the non-linear function) that maximises some objective, which, in our case, is the probability of getting the correct order of the AR or MA component. In contrast to earlier artificial neural networks studies, we seek to evaluate a larger range of ARMA orders with length of time series at 1,000, which is shorter but in the same order as studies by Chenoweth et al. (2000) and Al-Qawasmi et al. (2010). In order to keep the computational load manageable, we have restricted ourselves to a maximal order of the each of the AR and MA components of 9, amounting to a total of 100 different combinations of ARMA(p, q) models. This is not a conceptual restriction — higher orders can easily be achieved by increasing architecture sizes at the cost of also increasing computational time, in particular during training. An important difference in our approach is that time series are used directly as inputs to CNNs without prior needs of computing or using any statistical properties of the time series, apart from centering and scaling.

The article consists of mainly two parts: First, finding a suitable CNN architecture and training strategy to solve the task of model selection, and second, to compare the best-performing neural network against two classical likelihood-based methods, namely the Akaike and Bayesian information criteria. The two likelihood-based methods each come in two flavours: step-wise model selection and full search.
## 2 METHODS

### 2.1 Simulation of ARMA\((p, q)\) time series

An ARMA time series model consists of an autoregressive part and a moving average part. We follow the usual convention and denote \(p\) for AR order and \(q\) for MA order. A time series \(X_n\) following an ARMA\((p,q)\) model can then be expressed recursively as

\[
X_n = \varepsilon_n + \sum_{i=1}^{p} \varphi_i X_{n-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{n-j},
\]

where \(\varphi_1, \ldots, \varphi_p\) and \(\theta_1, \ldots, \theta_q\) are fixed real-valued coefficients (typically unknown), and where noise \(\varepsilon_n\) are independent and identically distributed random values. For the distribution of \(\varepsilon_n\), we consider two cases in this paper: a standard normal distribution and a \(t\)-distribution with 2 degrees of freedom, the latter representing a very heavy-tailed distribution. These two types of time series are denoted as TS\(_\text{normal}\) and TS\(_\text{t-dist}\) respectively.

Model identification by AIC and BIC selection criteria in Hyndman and Khandakar (2007) requires time series to be stationary and invertible. Time series used for training of CNN are therefore stationary and invertible as well. Coefficients \(\varphi_i\) and \(\theta_j\) are generated such that all roots of AR polynomial and MA polynomial are larger than 1 (i.e. outside of unit circle) in order to guarantee that the time series defined by (2.1) is stationary and invertible. When we generate a time series for a given order \(p\), we start by assigning all coefficients independent standard normal random values. If the stationarity condition is not met, we pick one of the autoregressive coefficients \(\varphi_1, \ldots, \varphi_p\) uniformly at random and halve its value. If the condition is met, we stop, otherwise we again pick one of the coefficients \(\varphi_1, \ldots, \varphi_p\) uniformly at random, halve it, and repeat this procedure until the condition for stationarity is met.

Likewise, we apply the same approach when generating the \(\theta\) coefficients that fulfill the invertibility condition for a given order \(q\). Invertibility condition is the counterpart to stationarity for the moving average coefficients. It implies that noise can be expressed as weighted sum of current and past observations; that is, we can write

\[
\varepsilon_n = \sum_{i=0}^{\infty} \pi_i X_{n-i}
\]

for some real numbers \(\pi_0, \pi_1, \ldots\). In other words, information of noise (which is typically not observable) is equivalent to the information of current and past observable data.

While, for example, Minerva and Poli (2001), Cigizoglu (2003) and Zhang and Qi (2005) use simulated time series to augment a given dataset in training, we only use simulated data in this paper. Both training and testing data are generated based on the model (2.1). We use a standardized length of 1,000 time steps for each time series, excluding the respective burn-in time\(^1\). An effective training of CNN architectures requires a large number of input data and moreover, in order to avoid over-fitting in computer vision tasks, synthetic training data is commonly generated through manipulation of images by cropping, reflecting, etc. Since all our data is simulated and no-time series is seen twice by any CNN, over-fitting is not expected to be an issue — for the same reason we do also not make use dropout like Ioffe and Szegedy (2015) and He et al. (2015).

\(^1\)Burn-in period in R arima.sim function is computed as \(p + q + \lceil 6 / \log(\text{minroots}) \rceil\) when \(p > 0\), where \(\text{minroots}\) is the smallest absolute value of complex roots of AR polynomial. We modify the last term of burn-in calculation in our code to \(\min(50, 000, \lceil 10 / \log(\text{minroots}) \rceil)\). Although this may result in a slight computational overhead, we want to be sure that stationarity has been reached. When \(p = 0\), the burn-in period is set to \(q\).
2.2 CNN architectures

We consider four different architectures of convolutional neural networks in this article: one ‘vanilla’ CNN without skip connections and three ‘residual’ CNNs with skip connections, but with different arrangements of the convolutional layers, rectifier linear unit layers, batch normalisation layers and skip connections. The three residual architectures are adapted from He et al. (2016), and the detailed arrangement of the various layers is illustrated in Figure 1. Benefit and importance of skip connections are discussed by He et al. (2016). Whereas He et al. (2015, 2016) introduce skip connections from the second layer onwards, we introduce skip connections right from the first layer onwards by ‘fanning out’ the input to multiple features from the first layer.

The size of a CNN architecture is determined by the total number of tunable parameters in the architecture. Architecture size can be varied by changing the hyper-parameters depth (counted as the number of convolutional layers), filter width (kW) and number of features. He et al. (2015) introduced very deep ResNet architectures such as ResNet-50, ResNet-101 and most famously ResNet-152, which won the 2015 ImageNet competition, where the digits represent the number of convolutional layers in the respective networks. Zagoruyko and Komodakis (2016) highlighted the large sizes of these ResNet architectures: 25.6, 44.5 and 60.2 million parameters in these three networks respectively. Furthermore, Zagoruyko and Komodakis (2016) showed that a wide but shallower network can achieve the same accuracy of a very deep but thin network of comparable size and it is several times faster to train the former network. Widening of a network is done by increasing the number of features.

In the same vein, we experiment with a range of hyper-parameters and at the same time evaluate whether a small network can perform ARMA model identification adequately. Moreover, a smaller network can be trained faster and thus enabling more experiments to be conducted. In this paper, depth ranges from 8 to 24 in steps of 4, kW takes values 7, 11 and 15, and the number of features ranges from 8 to 68 in steps of 12. Resulting architecture sizes cover a wide range from 3,502 to 1,541,862 parameters. This allows us to investigate how the performance of model identification may vary when architecture size increases. The overall layout of our CNNs are illustrated in Figure 2. Each CNN architecture is trained for estimating either AR orders or MA orders but not both orders concurrently in this paper. For ease of reference, the architectures are denoted by CNN\textsuperscript{AR} and CNN\textsuperscript{MA} respectively. Each output channel in a CNN represents one order. A mean operation, called average pooling, maps each feature in the last convolutional layer to a corresponding output channel, which can then be interpreted as probabilities using a softmax function. When a time series is fed through the networks, the predicted order is read off directly from the output channel with the highest value, i.e. the highest probability.

2.3 Training of a CNN architecture

Training data are generated on the fly during training. Each training batch comprises 100 time series of length 1,000: one time series for each of the 100 possible combinations of $p$ and $q$ orders from 0 to 9. For each time series, the coefficients are chosen at random as described in Section 2.1, independently of all other time series, and independently of other batches. The noise component, $\varepsilon$, is drawn from a standard normal distribution. Due to memory constrains, these 100 time series are further divided into two mini-batches randomly, and each mini-batch is used for one iteration of a forward and back-propagation step. Since all data is simulated and no two time series ever used twice on a particular neural network, it is not meaningful to use the concept of ‘epochs’ in our setting.

As loss function, we use the cross-entropy criterion, and as gradient descent optimiz-
ers, we use Adam and Nesterov’s Accelerated Gradient (NAG). The Adam optimizer is a popular method in computer vision deep learning. The first moment coefficient $\beta_1$ and second moment coefficient $\beta_2$ of the Adam optimizer are set at recommended default values of 0.9 and 0.999 respectively; see Kingma and Ba (2015). Momentum parameter $\mu$ of NAG is tested at two different values, 0.95 (see Sutskever et al. (2013)) and 0.75. We keep track of mean error, which is the average of loss function during training. Learning rate starts from 0.1 and is halved if there is no reduction in mean error after 6 consecutive 100 batches. Training stops when learning rate is lower than 0.0001.

We make use of GPUs for faster training of the neural networks. Forward and backward propagations can be done with matrix multiplications, which are computed efficiently by GPU. We use NVIDIA(R) GeForce(R) GTX 1080 GPUs for training in this paper.

2.4 Selection of a ResNet architecture

When we repeat the training of an architecture for a fixed set of hyper-parameters, the outcome will naturally vary because the weights and biases of the architecture are initialized randomly and the simulated time series are completely different. In order to ascertain it is not by random chance that one ResNet architecture performs better than the other two, these architectures are ‘pitched’ against each other. To do so, we firstly identify the best performing neural network of each ResNet architecture in Section 2.3 and obtain its corresponding set of hyper-parameters. With these three sets of hyper-parameters, we then consider the combinations of all ResNet architectures and these hyper-parameters. We conduct 100 repeated training for all the nine resultant CNNs. In other words, each architecture will be trained with 3 different sets of hyper-parameters, 100 repetitions for each set of hyper-parameters. Through the box plots of this experiment, we choose the ResNet architecture in Figure 1 that works best.

2.5 Progressive retraining of CNN architectures

After determining the best architecture and corresponding hyper-parameters, the performance of this selected CNN can be improved by a series of progressive retraining. To illustrate this method, assume we are retraining a CNN$^\text{AR}$. We perform a series of training where we vary the values of MA orders by increasing the orders gradually, thus the term progressive. The $q$ values are fixed throughout in one training and increase in a subsequent training. For instance, we start by restricting $q = 0$, retrain the network, then repeat the retraining at $q = 1$ and so on until we reach $q = 9$. Subsequently, we use a range of $q = \{0, 1, 2, \ldots, k\}$, where $k$ is increased from 1 to 9 progressively. In a way, the network adapts to the increasing variation in the time series due to increasing MA orders. Vice versa, we will control the AR orders in the same fashion when we retrain CNN$^\text{MA}$. This retraining approach is analogous to the idea of transfer learning in computer vision. We are able to move from one local minima to a lower minima with this process.

\footnote{Before we change $q$ from one value to another, we impose a stopping condition and perform recursive rounds of training so that we are indeed improving the performance of the network. Every time a CNN$^\text{AR}$ is fully trained, the resulting mean error is compared with the mean error of previous round of training. If the new mean error is lower, we earmark the newly trained CNN as the selected CNN, otherwise it is discarded. We then reset learning rate to a higher value of 0.5 in hope of the optimizer finding a lower minima and perform another round of training. These steps are repeated over and over until there is no further reduction in mean error for 6 consecutive rounds. We then proceed with a different $q$ value.}
2.6 Validation of test suites

We generate two test suites for each type of time series where the white noise comes from different underlying distributions, as mentioned in Section 2.1. The test suites are validated by AIC and BIC selection criteria and our trained CNN architectures. In our training process, a CNN architecture is trained with only one type of time series but it will be tested against both types of time series during validation. This will examine the robustness of CNN architectures against variation in underlying white noise of time series. Each test suite contains 10,000 time series (i.e. 100 batches of 100 time series, one time series for each of the 100 possible combinations of \( p \) and \( q \) orders from 0 to 9 in each batch).

Accuracy is the first metric for performance of identifications. We examine the percentage of correct identifications and mean square error (mse), which is the average of square of difference between classified orders and actual orders. A lower mse means that classified orders are more concentrated around actual values. These two measurements are usually positively correlated. Furthermore, confidence intervals of the percentage of correct identifications are estimated by assuming a binomial distribution. Computational time is the second metric.

Since there is a separate CNN architecture for identification of AR and MA orders respectively, the architectures are eventually assembled together to evaluate each test suite. There are two configurations of such assembly. The first configuration is done by feeding the test suite to CNN\(^\text{AR}\) and CNN\(^\text{MA}\) independently. The layout is in Figure 3 and this configuration is denoted as CNN (Separate). The second configuration makes use of an ensemble of intermediate CNN\(^\text{MA}\) that are produced during the progressive retraining in Section 2.5. The layout in Figure 4 illustrates the mechanism where an appropriate CNN\(^\text{MA}\) is chosen accordingly, conditional on the output of CNN\(^\text{AR}\). This configuration is denoted as CNN (Joint).

For the completeness of our validation, we train another two separate CNN\(^\text{AR}\) and CNN\(^\text{MA}\) with TS\(_t\)-dist, i.e. drawing the noise, \( \varepsilon \), in Section 2.3 from a heavy-tailed distribution now. These two CNNs will also be tested against the two test suites in the same CNN (Separate) and CNN (Joint) configurations. Instead of repeating the process of selecting ResNet architecture and set of hyper-parameters, we utilize the same ResNet architecture and corresponding hyper-parameters that we have obtained in the steps earlier when we are working with TS\(_t\)normal.

Validation by AIC and BIC selection criteria is done by auto.arima function in R. It is done in both stepwise and non-stepwise settings. We shall call the non-stepwise setting as full evaluation for ease of comparison. Stepwise algorithm is faster because fewer combinations of \( p \) and \( q \) are considered. A full evaluation produces better accuracy naturally because all combinations of \( p \) and \( q \) are considered, but this approach takes much longer. Parameters of auto.arima function are set such that unnecessary computations are avoided\(^3\), which will maximize processing time without affecting identification accuracy.

We perform the validations on different machines concurrently because of the long processing time in R. AIC and BIC tests are performed on Intel(R) Xeon(R) CPU E7-4850 v2 @ 2.30GHz and Intel(R) Xeon(R) CPU E7-4870 @ 2.40GHz computers while CNN architectures are tested on a Intel(R) Xeon(R) CPU E5-2620 v4 @ 2.10GHz computer. In order to compare the processing time of these different methods, an exact computational

\(^3\) Setting of some parameters of auto.arima function. Order of first-differencing is set to 0, seasonal is set to FALSE, stationary is set to TRUE and allowmean is set to FALSE since the inputs are stationary ARMA\((p,q)\) time series.
task was timed on all three machines. The performance on Intel(R) Xeon(R) CPU E5-2620 v4 @ 2.10GHz computer is used as the denominator to standardize all processing time.

3 RESULTS

3.1 Training: Selection of CNN architectures and hyper-parameters

We observe from Figure 5 to 10 that ResNet architectures outperform ‘vanilla’ CNN architecture. ReLU before activation ResNet architecture produces the best CNNAR and CNNMA when training are conducted with NAG optimizer, momentum of 0.75. Furthermore, progressive retraining methodology also improves accuracy of identifications. Mean error of CNNAR decreases from 1.7104 in Figure 6 to 1.5490. Mean error of CNNMA decreases from 1.8486 in Figure 9 to 1.6203.

Difference in arrangement of layers in ResNet architectures does have a significant effect. We shortlist the hyper-parameters of the best performing ReLU before activation, Original and Full pre-activation architectures from Figure 6 and 9. These hyper-parameters are tabulated in Table 1 and 2 respectively. The box plots of 100 repetitive training of each combination of ResNet architecture and these hyper-parameters in Figure 11 and 12 show clearly that ‘ReLU before activation’ architecture outperforms the other two ResNet architectures regardless of hyper-parameters for both identification of AR and MA orders. Moreover, we observe from Figure 13 that introduction of skip connections from first convolutional layer instead of second convolutional layer improves accuracy of identifications.

In general, it is seen in our linear regression analysis that mean error decreases as architecture size increases or in other words, accuracy of identifications improves when architecture size increases. This is observed in Figure 14 and 15, where training of ‘ReLU before activation’ for the entire range of architecture size from 3,502 to 1,541,862 is repeated 5 times. Table 3 and 4 summarize the negative correlation between mean error and log of architecture size. The results tell us that architecture size is an important component. Moreover, for a given architecture size, it seems depth and kW hyper-parameters are more significant than number of features. As the hyper-parameters are intertwined given an architecture size, increasing one hyper parameter will require reduction in one or both of the other two hyper-parameters. Therefore when architecture size is a limitation, the results suggest that focus should be placed on depth and kW.

The role of NAG optimizer in the training of CNNMA is an interesting finding from our training. While Adam is a popular optimizer in the field of computer vision, it does not train CNNMA properly however. We find that the predicted MA orders tend to degenerate to a few orders regardless of CNN architectures trained with Adam optimizer, thus explaining why the mean errors of these architectures remain relatively unchanged between 2.20 to 2.35 in Figure 8 even when architecture size increases. On the other hand, we observe a decreasing mean error trend for architectures trained with NAG optimizer in Figure 9 and 10. CNN architectures perform comparably well in identification of AR orders when they are trained with either Adam or NAG optimizers.

\footnote{A code to compute the first 800 digits of \(\pi\) for 200,000 iterations. It is based on the code from https://crypto.stanford.edu/pbc/notes/pi/code.html.}
3.2 Validation: Comparison of CNN architectures against AIC and BIC selection criteria

The results of first performance benchmarking with a set of 10,000 TS\textsubscript{normal} are in Table 5. As expected, full evaluations by AIC and BIC selection criteria perform better than stepwise evaluations in terms of accuracy of identifications of AR and MA orders. The improvement in accuracy when performing full evaluations comes at a significant increase in computational costs however. Processing time increases from 3 hours in a stepwise BIC evaluation to 79 hours in a full BIC evaluation and from 8 hours in a stepwise AIC evaluation to 165 hours in full AIC evaluation. While the improvement from a stepwise to full BIC evaluation is significant, it is only marginal in the case of AIC selection criteria. Accuracies of classification of AR and MA orders by a full BIC evaluation are at 35.23% and 35.58% respectively vis-à-vis corresponding 23.84% and 26.71% by full AIC evaluation. Our observation of BIC selection criteria outperforming AIC selection criteria is consistent with Hannan (1980) where BIC is also found to be a better criterion than AIC.

When we contrast accuracies of CNN architectures with full BIC evaluation in Table 5, we see that CNN architectures yield better results. The best performing CNN setup, CNN\textsubscript{normal} (Joint), records 41.01% and 40.00% accuracies for identifications of AR and MA orders respectively. Moreover, the time taken is less than an hour. While CNN\textsubscript{normal} (Joint) performs well in identifying individual AR and MA orders, its accuracy in identifying both orders concurrently is 20.47%, which is marginally lower than 20.98% by full BIC evaluation. A CNN architecture that is trained with TS\textsubscript{t-dist}, i.e. a different type of time series, fairs poorly however when evaluating TS\textsubscript{normal} test suite, especially in terms of identification of MA orders. The output of CNN\textsubscript{t-dist} (Joint) is 30.91% and 12.25% for corresponding identifications of AR and MA orders.

Table 6 shows the second performance benchmarking with a set of 10,000 TS\textsubscript{t-dist}. When we examine CNN\textsubscript{t-dist}, i.e. the CNN architecture trained with time series of the same type of white noise, we see that CNN (Joint) setup perform better than CNN (Separate) setup again. Other observations are similar to Table 5 with an exception that identification of MA orders by CNN\textsubscript{t-dist} (Joint) at 35.53% falls short of full BIC evaluation at 38.20%. One notable observation however is the performance of CNN\textsubscript{normal}. Unlike the significant drop in accuracies when there is a mismatch of trained CNN architectures and types of time series in Table 5, CNN\textsubscript{normal} (Joint) is able to evaluate TS\textsubscript{t-dist} at a comparable accuracies of 36.21% and 34.89% for respective identifications of AR and MA orders.

Table 7 to 10 show detailed breakdown of identification of respective AR and MA orders by full BIC evaluation and CNN (Joint). The highest values of each row in these tables are generally located along the diagonals for both approaches, which show that both methods identify well across the whole range of p and q. We notice a pattern where accuracy is higher at lower p or q orders but it drops as the orders increase. Classifications of q of order 7 and 8 by CNN\textsubscript{MA} in Table 10 are not as good as full BIC evaluation, which results in an overall lower accuracy in terms of identification of MA orders in Table 6.

3.3 Validation: Comparison against results by Chenoweth et al. (2000)

Chenoweth et al. (2000) evaluated a smaller range of ARMA models namely up to order 2 with the exclusion of ARMA(0,0), in other words, a total of 8 ARMA models. 800 test time series corresponding to these 8 models were used in their evaluations, which equates to 100 time series for each ARMA model. They obtained a mean correct percentage of 49.38% (with 95% confidence interval [42.34%, 56.36%]) accurate identifications for time series of length 3,000 and 20.38% (with 95% confidence interval [16.44%, 24.31%]) for time
series of length 100. We examine a subset of our results for the same ARMA models to get a ballpark comparison to the results by Chenoweth et al. (2000). The number of time series in the subset of our test suites for this range of ARMA models happen to be 800 time series as well. Our results show that a full BIC evaluation correctly identifies 63.0% (with 95% confidence interval [59.62%, 66.52%]) of both orders while CNN\textsubscript{normal} (Joint) gets both orders correctly at 50.1% (with 95% confidence interval [46.62%, 53.84%]).

4 DISCUSSION

In this paper, CNN architectures can identify ARMA orders as well as, if not better than, likelihood based methods. Furthermore, CNN architectures perform the task at a fraction of the time taken by AIC and BIC methods. Although the time taken by AIC and BIC methods can be shortened by choosing a stepwise approach, it adversely affects accuracy and CNNs are still much faster. It is a common knowledge that a neural network typically does not perform well on a task or input that it has not been trained on or seen before. We test the robustness our CNNs by evaluating test time series that are statistically different from training data. We generate two types of time series of different white noise; one which is coming from a standard normal distribution and another from a $t$-distribution with 2 degrees of freedom, which represents a very heavy-tailed distribution. While performance of CNN architectures does depend on training data used, we notice that CNN\textsubscript{normal} is more robust than CNN\textsubscript{$t$-dist}. Put it differently, CNN\textsubscript{normal} is still able to evaluate TS\textsubscript{$t$-dist} relatively well.

Out of the three ResNet architectures evaluated, we find ‘ReLU before activation’ to be the best candidate for ARMA model identification. This finding is contrary to the result in He et al. (2016) where it is the worst performing ResNet architecture for the task of computer vision. Moreover, contrary to common usage of Adam optimizer in computer vision, we find that NAG optimizer with momentum of 0.75 is key in the training of CNN\textsuperscript{MA} and produces better outcome too for both CNN\textsuperscript{AR} and CNN\textsuperscript{MA}. We observe that accuracy of identifications improves as architecture size increases. Our linear regression analysis seem to suggest a deeper architecture with wider filter width is needed to capture the correlation information of the time series.

Apart from choosing a good architecture, we introduce a few improvements that produce better trained CNNs. Training outcomes of two CNNs, despite having the same architecture and same set of hyper-parameters, can be very different. The outcomes vary because firstly, the weights and biases of each CNN are initialized randomly and secondly, training time series, which are generated on the fly, are different. We describe a progressive retraining strategy that sharpens a trained CNN. Introducing skip connections right from the first convolutional layer is another subtle tweak that improves the performance. When we assemble individual CNN\textsuperscript{AR} and CNN\textsuperscript{MA}, which are trained separately, we find the arrangement in a CNN(Joint) setup works better.

ACKNOWLEDGEMENTS

The authors thanks Chen Ying and Alexandre Thiery for helpful discussions.

REFERENCES

Abo-Hammour, Z. S., Alsmadi, O. M. K., Al-Smadi, A. M., Zaqout, M. I., and Saraireh, M. S. (2011). ARMA model order and parameter estimation using genetic algorithms. Mathematical and Computer Modelling of Dynamic Systems 18(2), 201–221.
Al-Qawasmi, K. E., Al-Snadi, A. M., and Al-Hammami, A. (2010). Artificial neural network-based algorithm for ARMA model order estimation. *Network Digital Technologies 2010*, 184–192.

Akaike, H. (1969). Fitting autoregressive models for predictions. *Annals of the Institute of Statistical Mathematics* **21**(1), 243–247.

Akaike, H. (1977). On entropy maximization principle. *Applications of Statistics*, 27–41.

Box, G. E. P. and Jenkins, G. M. (1976). *Time series analysis: forecasting and control*. Holden-Day.

Calster, T. V., Baesens, B., and Lemahieu, W. (2017). ProfARIMA: A profit-driven order identification algorithm for ARIMA models in sales forecasting. *Applied Soft Computing* **60**, 775–785.

Chenoweth, T., Hubata, R. and St. Louis, R. D. (2000). Automatic ARMA identification using neural networks and the extended sample autocorrelation function: a reevaluation. *Decision Support Systems* **29**, 21–30.

Cigizoglu, H. K. (2003). Incorporation of ARMA models into flow forecasting by artificial neural networks. *Environmetrics* **14**, 417–427.

Durbin, J. and Koopman, S. J. (2007). *Automatic time series forecasting: the forecast package for R*. Journal of Statistical Software **27**(3).

Hyndman, R. J. and Khandakar, Y. (2007). Automatic time series forecasting: the forecast package for R. *Journal of Statistical Software* **27**(3).

Ioffe, S. and Szegedy, C. (2015). Batch normalization: accelerating deep network training by reducing internal covariate shift. *Proceedings of the 32nd International Conference on Machine Learning*, 448–456.

Jhee, W. C., Lee, K. C., and Lee, J. K. (1992). A neural network approach for the identification of the Box-Jenkins model. *Network: Computation in Neural Systems* **3**, 323–339.

Kingma, D. P. and Ba, J. L. (2015). Adam: a method for stochastic optimization. arXiv preprint, arXiv:1412.6980.

Krizhevsky, A., Sutskever, I., and Hinton, G. E. (2012). ImageNet classification with deep convolutional neural networks. *Advances in Neural Information Processing Systems* 25.

LeCun, Y., Bengio, Y., and Hinton, G. (2015). Deep learning. *Nature* **521**, 436–444.

LeCun, Y., Boser, B., Denker, J. S., Henderson, D., Howard, R. E., Hubbard, W., and Jackel, L. D. (1990). Handwritten digit recognition with a back-propagation network. *Advances in Neural Information Processing Systems* 2, 396–404.

Lee, J. K. and Jhee, W. C. (1994). A two-stage neural network approach for ARMA model identification with ESACF. *Decision Support Systems* **11**, 461–479.

Lee, K. C. and Oh, S. B. (1996). An intelligent approach to time series identification by a neural network-driven decision tree classifier. *Decision Support Systems* **17**, 183–197.

Lee, K. C., Yang, J. S. and Park, S. J. (1991). Neural network-based time series modeling: ARMA model identification via ESACF approach. *IEEE International Joint Conference on Neural Networks*, 232–236.

Liang, G., Wilkes, D. M. and Cadzow, J. A. (1993). ARMA model order estimation based on the eigenvalues of the covariance matrix. *IEEE Transactions on Signal Processing* **41**(10), 3003–3009.

Minerva, T. and Poli. I. (2001). Building ARMA models with genetic algorithms. *Lecture Notes in Computer Science* **2037**, 335–342.

Ong, C.-S., Huang, J.-J., and Tseng, G.-H. (2005). Model identification of ARIMA family using genetic algorithms. *Applied Mathematics and Computation* **164**, 885–912.
Palaniappan, R. (2006). Towards optimal model order selection for autoregressive spectral analysis of mental tasks using genetic algorithm. *International Journal of Computer Science and Network Security* **6**(1A), 153–162.

Rissanen, J. (1978). Modeling by shortest data description. *Automatica* **14**, 465–471.

Schwarz, G. (1978). Estimating the dimension of a model. *The Annals of Statistics* **6**(2), 461–464.

Sutskever, I., Martens, J., Dahl, G., and Hinton, G. (2013). On the importance of initialization and momentum in deep learning. *Proceedings of 30th International Conference on Machine Learning* **28**, 1139–1147.

Zagoruyko, S. and Komodakis, N. (2016). Wide residual networks. arXiv preprint, arXiv:1605.07146.

Zhang, G. P. and Qi, M. (2005). Neural network forecasting for seasonal and trend time series. *European Journal of Operational Research* **160**, 501–514.
Figure 1: Different architectures tested. The ResNet blocks are based on He et al. (2016) paper.

Figure 2: A general layout of convolutional neural network architecture to illustrate filtering of information from time series input and in between convolutional layers. The rectangle blocks represent stacks of convolutional, batch normalization and/or rectifier linear unit layers as illustrated in Figure 1. Intermediate blocks are represented in dotted lines. Mean operator, i.e. average pooling, is applied to each of last 10 blocks to generate an output list. AR or MA order is identified from the index of largest value in this output list.
Figure 3: CNN (Separate). CNN\textsuperscript{AR} and CNN\textsuperscript{MA} evaluate time series independently.

Figure 4: CNN (Joint). A corresponding CNN\textsuperscript{MA} is chosen accordingly based on identification of AR order by preceding CNN\textsuperscript{AR}.
Figure 5: Mean errors of $\text{CNN}^{\text{AR}}$ architectures trained with Adam optimizer.

Figure 6: Mean errors of $\text{CNN}^{\text{AR}}$ architectures trained with NAG optimizer; momentum of 0.75.
Figure 7: Mean errors of CNN\textsuperscript{AR} architectures trained with NAG optimizer; momentum of 0.95.

Figure 8: Mean errors of CNN\textsuperscript{MA} architectures trained with Adam optimizer.
Figure 9: Mean errors of CNN$^{MA}$ architectures trained with NAG optimizer; momentum of 0.75.

Figure 10: Mean errors of CNN$^{MA}$ architectures trained with NAG optimizer; momentum of 0.95.
Figure 11: Average mean error of training of 100 CNN^{AR} for each combination of architecture and corresponding hyper-parameters. See Section 2.4.

Figure 12: Average mean error of training of 100 CNN^{MA} for each combination of architecture and corresponding hyper-parameters. See Section 2.4.
Figure 13: An experiment to show that introducing skip connections right from the first layer improves the performance of our CNNs. The word ‘(Original)’ in the diagram denotes the architecture, which has skip connections from second layer onwards, just like those in He et al. (2015) and He et al. (2016). The box plots show the average mean errors of training of 100 CNN\textsuperscript{AR} and CNN\textsuperscript{MA} respectively for each architecture. The hyper-parameter values are arbitrary chosen at depth of 20, kW of 11 and number of features of 56 for CNN\textsuperscript{AR} and depth of 24, kW of 15 and number of features of 68 for CNN\textsuperscript{MA}.

Figure 14: Training of CNN\textsuperscript{AR} architectures is repeated 5 times for all combinations of hyper-parameters. Result of regression analysis is in Table 3.
Figure 15: Training of CNN$^{\text{MA}}$ architectures is repeated 5 times for all combinations of hyper-parameters. Result of regression analysis is in Table 4.
Table 1: Mean errors and hyper-parameters of respective CNN\textsuperscript{AR} trained with TS\textsubscript{normal} and NAG optimizer; momentum of 0.75.

| ResNet architecture | Depth | kW  | Number of Features | Size    | Mean Error | Training time (hours) |
|---------------------|-------|-----|--------------------|---------|------------|-----------------------|
| ReLU before activation | 16    | 15  | 68                 | 985,350 | 1.710      | 1.606                 |
| Original            | 8     | 15  | 68                 | 428,838 | 1.751      | 1.079                 |
| Full pre-activation | 20    | 15  | 44                 | 532,518 | 1.750      | 1.675                 |

Table 2: Mean errors and hyper-parameters of respective CNN\textsuperscript{MA} trained with TS\textsubscript{normal} and NAG optimizer; momentum of 0.75.

| ResNet architecture | Depth | kW  | Number of Features | Size    | Mean Error | Training time (hours) |
|---------------------|-------|-----|--------------------|---------|------------|-----------------------|
| ReLU before activation | 24    | 15  | 44                 | 649,206 | 1.849      | 3.065                 |
| Original            | 12    | 15  | 56                 | 481,518 | 1.967      | 2.056                 |
| Full pre-activation | 20    | 15  | 68                 | 1,263,606 | 1.901     | 3.266                 |

Table 3: Regression analysis of CNN\textsuperscript{AR} architectures. Mean error is regressed against log of architecture size, depth, filter size (kW) and number of features. Architecture depth, filter size (kW) and number of features are set as polynomial of degree 2.

| Coefficients      | Estimate | Std. Error | t-value | p-value |
|-------------------|----------|------------|---------|---------|
| Intercept         | 2.037    | 0.060      | 34.059  | ≪0.001  |
| log(ParamSize)    | -0.022   | 0.005      | -4.310  | ≪0.001  |
| Depth             | -0.081   | 0.049      | -1.659  | 0.098   |
| Depth\textsuperscript{2} | 0.0379  | 0.016      | 2.299   | 0.022   |
| kW                | -0.085   | 0.036      | -2.374  | 0.018   |
| kW\textsuperscript{2} | 0.017   | 0.015      | 1.135   | 0.257   |
| Features          | 0.139    | 0.144      | 0.963   | 0.336   |
| Features\textsuperscript{2} | 0.053   | 0.043      | 1.216   | 0.225   |

Residual standard error 0.014
\(R^2\) 0.826
Adjusted \(R^2\) 0.823
N 450
| Coefficients  | Estimate | Std. Error | t-value | p-value |
|---------------|----------|------------|---------|---------|
| Intercept     | 2.526    | 0.210      | 12.018  | <0.001  |
| log(ParamSize)| -0.036   | 0.018      | -2.000  | 0.046   |
| Depth         | -0.531   | 0.171      | -3.110  | 0.002   |
| Depth²        | 0.205    | 0.058      | 3.542   | <0.001  |
| kW            | -1.031   | 0.126      | -8.173  | <0.001  |
| kW²           | 0.249    | 0.051      | 4.840   | <0.001  |
| Features      | -0.160   | 0.508      | -0.315  | 0.753   |
| Features²     | 0.091    | 0.152      | 0.594   | 0.553   |

Residual standard error 0.050

$R^2$ 0.784

Adjusted $R^2$ 0.781

N 450

Table 4: Regression analysis of CNNMA architectures. Mean error is regressed against log of architecture size, depth, filter size (kW) and number of features. Architecture depth, filter size (kW) and number of features are set as polynomial of degree 2.

|                | AIC stepwise | AIC full | BIC stepwise | BIC full | CNN\textsubscript{normal} (Joint) | CNN\textsubscript{normal} (Separate) | CNN\textsubscript{dist} (Joint) | CNN\textsubscript{dist} (Separate) |
|----------------|--------------|----------|--------------|----------|----------------------------------|------------------------------------|-------------------------------|-----------------------------------|
| AR(%)          | 21.87        | 23.84    | 22.79        | 35.23    | 41.01                            | 41.01                              | 30.91                         | 30.91                             |
| AR 95% C.I.    | [21.65, 22.69]| [23.00, 24.76]| [21.96, 23.63]| [34.26, 36.21]| [40.00, 42.02]| [40.00, 42.02]| [29.98, 31.85]| [29.98, 31.85]                  |
| MA(%)          | 23.94        | 26.71    | 23.71        | 35.58    | 39.10                            | 40.00                              | 12.03                         | 12.25                             |
| MA 95% C.I.    | [23.09, 24.79]| [25.83, 27.61]| [22.87, 24.57]| [34.61, 36.57]| [38.10, 40.10]| [39.00, 41.01]| [11.41, 12.67]| [11.62, 12.89]                  |
| MA mse         | 3.115        | 2.854    | 3.429        | 2.626    | 2.240                            | 2.230                              | 4.427                         | 4.347                             |
| Both correct(%)| 10.55        | 11.66    | 11.88        | 20.98    | 17.82                            | 20.47                              | 3.73                          | 3.63                              |
| Both correct 95% C.I.| [9.96, 11.14] | [11.05, 12.29] | [11.26, 12.51] | [20.18, 21.79] | [17.07, 18.57] | [19.68, 21.28] | [3.38, 4.09] | [3.28, 3.98] |
| Time taken (hours) | 7.967 | 164.919  | 3.005        | 78.964   | 0.439                            | 0.454                              | 0.438                         | 0.445                             |

Table 5: Classifications of ARMA($p,q$) orders for 10,000 TS\textsubscript{normal}. Time taken has been standardized.

|                | AIC stepwise | AIC full | BIC stepwise | BIC full | CNN\textsubscript{normal} (Joint) | CNN\textsubscript{normal} (Separate) | CNN\textsubscript{dist} (Joint) | CNN\textsubscript{dist} (Separate) |
|----------------|--------------|----------|--------------|----------|----------------------------------|------------------------------------|-------------------------------|-----------------------------------|
| AR(%)          | 23.43        | 30.48    | 22.44        | 34.71    | 36.21                            | 36.21                              | 46.81                         | 46.81                             |
| AR 95% C.I.    | [22.59, 24.28]| [29.55, 31.41]| [21.62, 23.28]| [33.74, 35.68]| [35.23, 37.19]| [35.23, 37.19]| [45.78, 47.84]| [45.78, 47.84]                  |
| AR mse         | 2.958        | 2.699    | 3.173        | 2.471    | 2.095                            | 2.095                              | 1.699                         | 1.699                             |
| MA(%)          | 25.47        | 34.97    | 22.67        | 38.20    | 33.38                            | 34.89                              | 31.47                         | 35.53                             |
| MA 95% C.I.    | [24.60, 26.34]| [34.00, 35.95]| [21.84, 23.56]| [37.21, 39.29]| [32.43, 34.35]| [31.92, 33.86]| [30.53, 32.41]| [34.56, 36.54]                  |
| MA mse         | 3.080        | 2.576    | 3.396        | 2.498    | 2.712                            | 2.470                              | 2.740                         | 2.554                             |
| Both correct(%)| 13.01        | 18.69    | 12.38        | 22.20    | 13.48                            | 15.91                              | 16.40                         | 20.09                             |
| Both correct 95% C.I.| [12.36, 13.66] | [17.84, 19.37] | [11.75, 13.02] | [21.38, 23.03] | [12.82, 14.14] | [15.20, 16.63] | [15.68, 17.13] | [19.30, 20.88] |
| Time taken (hours) | 6.253 | 77.714   | 2.957        | 54.380   | 0.437                            | 0.451                              | 0.439                         | 0.438                             |

Table 6: Classifications of ARMA($p,q$) orders for 10,000 TS\textsubscript{t-dist}. Time taken has been standardized.
### Table 7: Classification of AR orders by full BIC evaluation and CNN\textsubscript{normal} (Joint) in Table 5. Values are in percentages and normalized along the rows.

| BIC | CNN classified AR order | Actual AR order |
|-----|-------------------------|-----------------|
|     |                         |                 |
| 0   |                         |                 |
| 1   |                         |                 |
| 2   |                         |                 |
| 3   |                         |                 |
| 4   |                         |                 |
| 5   |                         |                 |
| 6   |                         |                 |
| 7   |                         |                 |
| 8   |                         |                 |
| 9   |                         |                 |

### Table 8: Classification of MA orders by full BIC evaluation and CNN\textsubscript{normal} (Joint) in Table 5. Values are in percentages and normalized along the rows.

| BIC | CNN classified MA order | Actual MA order |
|-----|-------------------------|-----------------|
|     |                         |                 |
| 0   |                         |                 |
| 1   |                         |                 |
| 2   |                         |                 |
| 3   |                         |                 |
| 4   |                         |                 |
| 5   |                         |                 |
| 6   |                         |                 |
| 7   |                         |                 |
| 8   |                         |                 |
| 9   |                         |                 |

### Table 9: Classification of AR orders by full BIC evaluation and CNN\textsubscript{t-dist} (Joint) in Table 6. Values are in percentages and normalized along the rows.

| BIC | CNN classified AR order | Actual AR order |
|-----|-------------------------|-----------------|
|     |                         |                 |
| 0   |                         |                 |
| 1   |                         |                 |
| 2   |                         |                 |
| 3   |                         |                 |
| 4   |                         |                 |
| 5   |                         |                 |
| 6   |                         |                 |
| 7   |                         |                 |
| 8   |                         |                 |
| 9   |                         |                 |

### Table 10: Classification of MA orders by full BIC evaluation and CNN\textsubscript{t-dist} (Joint) in Table 6. Values are in percentages and normalized along the rows.

| BIC | CNN classified MA order | Actual MA order |
|-----|-------------------------|-----------------|
|     |                         |                 |
| 0   |                         |                 |
| 1   |                         |                 |
| 2   |                         |                 |
| 3   |                         |                 |
| 4   |                         |                 |
| 5   |                         |                 |
| 6   |                         |                 |
| 7   |                         |                 |
| 8   |                         |                 |
| 9   |                         |                 |