Modulational instability and ion-acoustic envelope solitons in four component plasmas

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Modulational instability (MI) of ion-acoustic waves (IAWs) has been theoretically investigated in a plasma system which is composed of inertial warm adiabatic ions, isothermal positrons, and two temperature superthermal electrons. A nonlinear Schrödinger (NLS) equation is derived by using reductive perturbation method that governs the MI of the IAWs. The numerical analysis of the solution of NLS equation shows the existence of both stable (dark envelope solitons exist) and unstable (bright envelope solitons exist) regimes of IAWs. It is observed that the basic features (viz. stability of the wave profile and MI growth rate) of the IAWs are significantly modified by the superthermal parameter ($\kappa$) and related plasma parameters. The results of our present investigation should be useful for understanding different nonlinear phenomena in both space and laboratory plasmas.

I. INTRODUCTION

During the last few decades, the research about electron-positron-ion (e-p-i) plasma has been spectacularly increasing because the observational (Viking Satellite [1] and THEMIS mission [2]) evidence exposed the existence of large amount e-p-i plasma in the space (solar atmospheres [3] [4], pulsar magnetosphere [5] [6], polar regions of neutron stars [7]) and laboratories plasmas [8]. Many authors encounter with wave dynamics [9–17] (such as electron-acoustic waves (EAWs), positron-acoustic waves (PAWs), and IAWs) to understand the physics of collective behaviour in such kind of space and laboratories plasmas.

In case of natural space or in laboratories plasmas (i.e., hot, tenuous, and collisionless) high energy particles may co-exist [18] with isothermal distributed particles and their characteristics are deviated from the eminent Maxwellian distribution. Sometimes this type of particles can be modeled by non-Maxwellian high-energy tail distribution which is known as generalized Lorentzian (kappa) distribution [19–24]. The kappa distribution and its relation to the Maxwellian distribution was first introduced by Vasylinas [19]. This type of distribution may be arisen [25], due to the external forces acting on the natural space plasmas or to wave particle interaction. The Lorentzian or kappa distribution function [20] in three dimensional case can be written in the form

$$F_k(\nu) = \frac{\Gamma(\kappa + 1)}{(\pi \kappa \theta^2)^{3/2} \Gamma(\kappa - 1/2)} \frac{1 + \nu^2}{\kappa^2}^{(\kappa+1)/2}, \quad \kappa \neq 0, \quad \nu > 0,$$

where $\theta = [(2\kappa - 3)/\kappa]V_i$, is the effective thermal speed which depends on the usual thermal velocity $V_i = (k_B T/m)^{1/2}$, $\Gamma$ is the standard gamma function, $T$ is the characteristic kinetic temperature, $V_i$ is the phase velocities of both modes. Masud et al. [40] numerically shown that how exist domains for arbitrary amplitude IA solitons and double layers are determined by cut off conditions.

The investigations of the MI of IAWs both theoretically and experimentally have been increasing day by day due...
to their successful applications in space as well as laboratory plasmas. The NLS equation have been used to understand different nonlinear phenomena such as single pulse and envelope structures respectively, observed in space and laboratory plasmas. Recently, a number of authors investigated the MI and envelope solitons structure in pair and e-p-i plasmas. By using reductive perturbation method, most of them has obtained envelope solitons in unmagnetized electron-ion plasmas. In unmagnetized electron-ion plasmas Ju-Krylov-Bogoliubov-Mitropolsky (KBM) method \cite{54,48} has obtained envelope solitons \cite{12,46,49}. The electrostatic envelope solitons have also been studied by using Krylov-Bogoliubov-Mitropolsky (KBM) method \cite{51}. For inertialess cold electron, we can obtain the expressions for cold electron number densities as

\[ n_c = \left[ 1 - \frac{\phi}{(\kappa - 3/2)} \right]^{-\kappa + 1/2}, \]

where

\[ C_1 = \frac{\kappa - 1/2}{\kappa - 3/2}, \quad C_2 = \frac{1}{2} \left( \frac{\kappa^2 - 1/4}{(\kappa - 3/2)^2} \right), \]

\[ C_3 = \frac{1}{6} \left( \frac{(\kappa - 1/2)(\kappa + 1/2)(\kappa + 3/2)}{(\kappa - 3/2)^3} \right). \]

For inertialess hot electron, we can obtain the expressions for hot electron number densities as

\[ n_h = \left[ 1 - \frac{\mu \phi}{(\kappa - 3/2)} \right]^{-\kappa + 1/2}, \]

where

\[ \alpha = \frac{T_i}{T_h}, \quad \gamma = \frac{n_{hi}}{n_{ei}}, \quad \lambda = \frac{T_e}{T_i}, \quad \sigma = \frac{n_{ei}}{n_{hi}}, \quad \mu = \frac{\mu}{\mu}. \]

The manuscript is organized as follows: The basic governing equations of our considered plasma model is presented in Sec. II. By using reductive perturbation technique, we derive a NLS equation which governs the slow amplitude evolution in space and time is given in Sec. III. The stability analysis is presented, in Sec. IV. The envelope solitons are presented in sec. V. The discussion is provided in Sec. VI.

**II. GOVERNING EQUATIONS**

We consider an unmagnetized plasma system comprising of inertial warm adiabatic ions, isothermal positrons, and two temperature superthermal electrons (hot and cold). At equilibrium, the quasi-neutrality condition can be expressed as \[ n_{i0} + n_{i0} = n_{h0} + n_{i0}, \] where \[ n_{i0}, n_{p0}, n_{h0} \] and \[ n_{i0} \] are the equilibrium number densities of warm adiabatic ion, isothermal positron, and superthermal hot electron and cold electron, respectively. The normalized equations governing the IAWs in our considered plasma system are given by

\[ \frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(niu_i) = 0, \]

\[ \frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x} - 3\alpha n_i \frac{\partial n_i}{\partial x}, \]

\[ \frac{\partial^2 \phi}{\partial x^2} = n_c + \gamma n_h - \sigma n_i - (1 - \sigma + \gamma)n_p. \]

For inertialess cold electron, we can obtain the expressions for cold electron number densities as

\[ \frac{\partial n_c}{\partial t} + \frac{\partial}{\partial x}(n_cu_i) = 0, \]

\[ \frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x} - 3\alpha n_i \frac{\partial n_i}{\partial x}, \]

\[ \frac{\partial^2 \phi}{\partial x^2} = n_c + \gamma n_h - \sigma n_i - (1 - \sigma + \gamma)n_p. \]

\[ n_c = \left[ 1 - \frac{\phi}{(\kappa - 3/2)} \right]^{-\kappa + 1/2}, \]

\[ = 1 + C_1 \phi + C_2 \phi^2 + C_3 \phi^3 + \cdots, \]
where \( v_g \) is the envelope group velocity and \( \epsilon \) (0 < \( \epsilon < 1 \)) is a small (real) parameter. Then we can write a general expression for the dependent variables as:

\[
G(x,t) = G_0 + \sum_{m=1}^{\infty} c^{(m)}(\xi,\tau) \exp(i\Theta),
\]

\[
G_l^{(m)} = [n_{il}^{(m)}, u_{il}^{(m)}, \phi_l^{(m)}]^T, \quad G_l^{(0)} = [1, 0, 0]^T, \tag{10}
\]

where \( \Theta = (k\tau - \omega t) \), simultaneously \( k \) and \( \omega \) are real variables representing the carrier wave number and frequency, respectively. \( G_l^{(m)} \) satisfies the pragmatic condition \( G_l^{(m)} = G_l^{(m)} \), where the asterisk denotes the complex conjugate. The derivative operators in the above equations are treated as follows:

\[
\frac{\partial}{\partial t} \rightarrow -ev_g \frac{\partial}{\partial \xi} + \epsilon^2 \frac{\partial}{\partial \tau} + \frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial x} + \epsilon \frac{\partial}{\partial \xi}. \tag{11}
\]

Substituting equations (9) – (11) into equations (2), (3), and (8) and the first order (\( m = 1 \)) equations with (\( l = 1 \)), gives

\[
-i\omega n_1^{(1)} + iku_1^{(1)} = 0,
\]

\[
-i\omega u_1^{(1)} + ik\phi_1^{(1)} + i\Omega n_1^{(1)} = 0,
\]

\[
\sigma n_1^{(1)} - k^2 \phi_1^{(1)} - \gamma_1 \phi_1^{(1)} = 0,
\tag{12}
\]

where \( \Omega = 3\alpha \). The solution for the first harmonics read as

\[
n_1^{(1)} = \frac{k^2}{\sigma} \phi_1^{(1)}, \quad u_1^{(1)} = \frac{k\omega}{\sigma} \phi_1^{(1)},
\tag{13}
\]

where \( S = \omega^2 - k^2\Omega \). We thus obtain the dispersion relation for IAWs

\[
\omega^2 = \frac{\sigma k^2}{(k^2 + \gamma_1)} + k^2\Omega. \tag{14}
\]

The second-order when (\( m = 2 \)) reduced equations with (\( l = 1 \)) are

\[
n_1^{(2)} = \frac{k^2}{S} \phi_1^{(2)}, \quad v_1^{(2)} = \frac{i(v_g k - \omega)}{S} \phi_1^{(1)} - \frac{S}{k^2} \phi_1^{(1)},
\]

\[
u_1^{(2)} = \frac{S}{k\sigma} \phi_1^{(2)} - \frac{(S + \sigma k^2)}{k\sigma^2} \phi_1^{(1)} \tag{15}
\]

with the compatibility condition

\[
v_g = \frac{\partial \omega}{\partial k} = \frac{\sigma \omega^2 - S^2}{k\sigma^2}.
\tag{16}
\]

The amplitude of the second-order harmonics are found to be proportional to \( |\phi_1^{(1)}|^2 \)

\[
n_2^{(2)} = C_4 |\phi_1^{(1)}|^2, \quad n_0^{(2)} = C_7 |\phi_1^{(1)}|^2,
\]

\[
u_2^{(2)} = C_5 |\phi_1^{(1)}|^2, \quad n_0^{(2)} = C_8 |\phi_1^{(1)}|^2,
\]

\[
\phi_2^{(2)} = C_6 |\phi_1^{(1)}|^2, \quad \phi_0^{(2)} = C_9 |\phi_1^{(1)}|^2, \tag{17}
\]

where

\[
C_4 = \frac{\Omega k^6 + 3\omega^2 k^4 + 2C_6 S^2 k^2}{2 S^3},
\]

\[
C_5 = \frac{\omega C_4 S^2 - \omega k^4}{k S^2},
\]

\[
C_6 = \frac{\sigma(\Omega k^6 + 3\omega^2 k^4) - 2\gamma_2 S^3}{2 S^3(4k^2 + \gamma_1) - 2\sigma S^2 k^2},
\]

\[
C_7 = \frac{2\omega v_g k^3 + \Omega k^4 + \omega^2 k^2 + C_6 S^2}{S^2(v_g^2 - \Omega)},
\]

\[
C_8 = \frac{v_g C_7 S^2 - 2\omega k^3}{S^2},
\]

\[
C_9 = \frac{2\sigma \omega v_g k^3 + \Omega \sigma k^4 + \sigma k^2 \omega^2 - 2\gamma_2 S^2(v_g^2 - \Omega)}{\gamma_1 S^2(v_g^2 - \Omega) - \sigma S^2}.
\]

Finally, the third harmonic modes (\( m = 3 \)) and (\( l = 1 \)) and with the help of equations (13) – (17), give a system of equations, which can be reduced to the following NLS equation:

\[
i \frac{\partial \phi}{\partial t} + P \frac{\partial^2 \phi}{\partial \xi^2} + Q|\phi|^2 \phi = 0, \tag{18}
\]

where \( \phi = \phi_1^{(1)} \) for simplicity. The dispersion coefficient \( P \) is

\[
P = \frac{v_g^2 \Omega^2 k^5 - 3v_g k \omega^4 + 4\Omega^2 k^2 \omega^3 + 2v_g \Omega \omega^2 k^3 - 4\Omega \omega^2 k^4}{2\sigma \omega^2 k^2},
\]

and the nonlinear coefficient \( Q \) is

\[
Q = \frac{S}{2 \sigma \omega k^2} \left[ - \frac{(\sigma \omega^2 k^2 + \sigma \Omega k^4)(C_4 + C_7)}{S} + 2S \gamma_2 (C_6 + C_9) + 3S \gamma_3 - 2\sigma \omega k^3 (C_5 + C_8) \right].
\]

IV. STABILITY ANALYSIS

The evolution of IAWs is governed by the equation (18), essentially depends on the coefficients product \( PQ \). Let us consider the harmonic modulated amplitude solution \( \Phi = \Phi_0 \exp(iQ|\Phi_0|^2 t) \). Following the standard stability analysis, one may perturb the amplitude by setting \( \tilde{\Phi} = \Phi_0 + \epsilon \epsilon_{1,0} \exp[i(k_{MI} \xi - \omega_{MI} t)] + c.c \) (the perturbation wave number \( k_{MI} \) and the frequency \( \omega_{MI} \) should be distinguished from their carrier wave homolog quantities, denoted by \( k \) and \( \omega \)). Hence, the nonlinear dispersion relation for the amplitude modulation is

\[
\omega_{MI}^2 = \frac{P^2 k_{MI}^2}{2} \left( k_{MI}^2 - 2 \frac{Q}{P} |\Phi_0|^2 \right). \tag{19}
\]

Clearly, if \( PQ < 0 \), \( \omega_{MI} \) is always real for all values of \( k_{MI} \), hence in this region the IAWs is stable in the presence of small perturbation. On the other hand, when
FIG. 1: (a) Showing the variation of $PQ$ against $k$ for different values of $\kappa$. (b) Plot of $P/Q$ against $k$ for different values of $\kappa$. All the figures are generated by using these values, $\alpha = 0.11, \gamma = 0.85, \lambda = 0.1, \sigma = 0.3, \mu = 0.11,$ and $\kappa = 3$.

$PQ > 0$, the MI would set in as $\omega_{MI}$ becomes imaginary and the envelope is unstable for $k_{MI} < k_c = \sqrt{2Q|\Phi_0|^2/P}$, where $k_c$ is the critical value of the wave number of modulation and $\Phi_0$ is the amplitude of the carrier waves. In the region $PQ > 0$ and $k_{MI} < k_c$, the growth rate ($\Gamma_g$) of MI is given by

$$\Gamma_g = |P| k_{MI}^2 \sqrt{\frac{k_c^2}{k_{MI}^2} - 1}.$$  \hfill (20)

Clearly, the maximum value $\Gamma_{g(max)}$ of $\Gamma_g$ is obtained at $k_{MI} = k_c/\sqrt{2}$ and is given by $\Gamma_{g(max)} = |Q||\Phi_0|^2$.

The coefficients of dispersion term $P$ and nonlinear term $Q$ are dependent on various plasma parameters, such as $\alpha, \beta, \gamma, \sigma, \mu, \lambda$ and $\kappa$. Thus, these parameters may be controlled the stability conditions of the IAWs. Therefore, we have investigated the stability of the profile by depicting the ratio of $P/Q$ versus $k$ for different plasma parameters. When the sign of the ratio $P/Q$ is negative, the modulated envelope pulse is stable, while the sign of the ratio $P/Q$ is positive, the modulated envelope pulse will be unstable against external perturbations. It is clear that both stable and unstable region are obtained from the figures 2 − 4. When $P/Q \rightarrow \pm \infty$, the corresponding value of $k (= k_c)$ is called critical or threshold wave number for the onset of MI. This critical value separates the unstable ($P/Q > 0$) from the stable region ($P/Q < 0$) one.

V. ENVELOPE SOLITONS

If $PQ < 0$, the modulated envelope pulse is stable and in this region dark envelope solitons exist, on the other hand when $PQ > 0$, the modulated envelope pulse which is unstable against external perturbations and lead to formation of bright envelope solitons. A solution of equation (18) may be sought in the form $\Phi = \sqrt{\psi} \exp(i\theta)$, where $\psi$ and $\theta$ are real variables which are determined by substituting into the NLS equation and separating real and imaginary parts. An interested reader is referred to [30, 54–58] for details. The different types of solution thus obtained are clearly summarized in the following paragraphs.
FIG. 3: Showing the variation of $P/Q$ against $k$ for different plasma parameters, (a) For $\lambda$, (b) For $\mu$, and (c) For $\sigma$. (d) Plot of the of MI growth rate ($\Gamma_g$) against $k_{mi}$ for different values of $\kappa$. Along with $k = 1.2$ and $\Phi = 0.06$.

FIG. 4: Plot of the of MI growth rate ($\Gamma_g$) against $k_{mi}$ for different plasma parameter, (a) For $\alpha = 3$, (b) For $\gamma$. Along with $k = 1.2$ and $\Phi = 0.06$.

A. Bright solitons

When $PQ > 0$, we find bright envelope solitons. The general analytical form of bright solitons reads

$$\psi = \psi_0 \text{sech}^2 \left( \frac{\xi - UT}{W} \right),$$

$$\theta = \frac{1}{2P} \left[ U\xi + \left( \Omega_0 - \frac{U^2}{2} \right) \tau \right].$$

(21)

Here, $U$ is the propagation speed (a constant), $W$ is the soliton width, and $\Omega_0$ oscillating frequency for $U = 0$. Figure 6(a) represents the bright envelope solitons.
B. Dark solitons

When $PQ < 0$, we find dark envelope solitons whose general analytical form reads as

$$\psi = \psi_0 \tanh^2 \left( \frac{\xi - U\tau}{W} \right),$$
$$\theta = \frac{1}{2P} \left[ U\xi - \left( \frac{U^2}{2} - 2PQ\psi_0 \right) \tau \right]. \quad (22)$$

Interestingly, in both of the latter two equation, the relation between soliton width $W$ and the constant maximum amplitude $\psi_0$ are related by

$$W = \sqrt{\frac{2|P/Q|}{\psi_0}}. \quad (23)$$

The ratio $P/Q$ determines the soliton width $W$ as $\psi_0 W \sim (P/Q)^{1/2}$. So lower $P/Q$ values suggest narrower solitons and vice versa. Figure 6(b) represents the dark envelope solitons.

VI. DISCUSSION

In this work, we have considered an unmagnetized four-component plasma consisting of inertial warm adiabatic ions, isothermal positrons, and two temperature superthermal electrons (hot and cold). By employing the reductive perturbation method, a NLS equation is derived, which governs the evolution of IAWs. We have investigated the existence of both stable and unstable regions for IAWs structures and the associated MI of electrostatic wave packets. The results, we have found from this investigation which can be summarized as follows:

1. The variation of $PQ$ with $k$ for different values of superthermality (via $\kappa$) is depicted in Fig. 1(a). One can recognize that when $P$ and $Q$ are opposite sign ($PQ < 0$), there is a stable region (the IAWs are modulationally stable) whereas $P$ and $Q$ are same sign ($PQ > 0$), there is an unstable region (the IAWs are modulationally unstable). With the increasing of the values of $\kappa$ the unstable region is decreasing. The intersecting point of the $PQ$ curve with the $k$-axis is called critical or threshold wave number ($k_c$).

2. The $k_c$ value is greatly controlled by superthermality (via $\kappa$). It may be noted that the smaller value of $\kappa$ means strong superthermality. With the increase of $\kappa$, the value of $k_c$ is decreased, which is depicted in Fig. 1(b). For a large value of $\kappa = 30$ or 100, the $k_c = 0.71$ remains almost constant. But if $\kappa < 3$, the value of $k_c$ is changed rapidly. So stability of the wave profile is so much sensitive to change with $\kappa$, when $\kappa \leq 3$.

3. The effects of ion temperature (via $\alpha$) on the wave profile is extremely high to change the stability of the electrostatic wave packets. It is observed from Fig. 2(a) that with the increasing of ion temperature the $k_c$ is shifted to the lower value that means excited ions minimize the stability region for IAWs. So ion temperature plays a crucial role for controlling the stability of the IAWs profile.

4. In Fig. 2(b) the variation of $P/Q$ with $k$ has been plotted for different values of hot electron concentration (via $\gamma$). We see that $k_c$ increases with the increasing of hot electron concentration, the critical value is shifted to higher value. That means higher concentration of hot electron provides greater restoring force which extend the stable region.

5. It can be observed from the Fig. 3(a), the stability of the IAWs profile is also governed by the positron temperature (via $\lambda$) of our considered plasma model. If the positron temperature of the
system increases, then the value of $k_c$ also decreases. For small wave number there is dark envelope solitons exists whereas bright envelope solitons exists for large wave number.

6. The effects of the cold electron temperature (via $\mu$) on the stability of IAWs profile is analyzed from Fig. 3(b), which depicts the dependence of ratio $P/Q$ on $k$ for different values of $\mu$. As cold electron temperature increases the $k_c$ value is increased.

7. The dependence of ratio $P/Q$ on $k$ for different values of ion number density (via $\sigma$) is depicted in Fig. 3(c). Ion number density plays an important role to control the stability of the profile. Excess number of ion cause to provide large moment of inertia that may be suppressed the stability region.

8. It is observed from Fig. 3(d) that MI growth rate are significantly effected by the values of superthermality (via $\kappa$). With increasing superthermality, the MI growth rates appear to decrease. The lower values of $\kappa$ (excess superthermality) may be enhanced the MI growth rate.

9. The dependence of the MI on ion temperature (via $\alpha$) is shown in Fig. 4(a). With the increase of ion temperature, the growth rate of the instability increases. From Fig. 4(b), similar behaviour (the maximum value of the growth rate increases, with the increasing of hot electron number density) is also observed (via $\gamma$). So $\alpha$ and $\gamma$ are enhanced the instability. Moreover, the growth rate ($\Gamma_g$) increases with increasing of $k_{MI}$. For a particular value of $k_{MI}$, the growth rate ($\Gamma_g$) is reached its critical value ($\Gamma_g \equiv \Gamma_{gc}$). Hence the growth rate ($\Gamma_g$) sharply decreases with further increases the values of $k_{MI}$.

A large number of observations clearly reveal the existence of high-energy/superthermal electrons in various natural space environment (Saturn’s magnetosphere, magnetotail, auroral zones, the ionosphere, solar wind, strong radiation in the interstellar or interplanetary medium etc.) and laboratories plasmas. We are optimistic that our nonlinear analysis will be helped to understand the nonlinear structures (bright and dark envelope solitons) that may be formed in both space and laboratory plasmas which containing of isothermal positrons, two distinct temperature superthermal electrons (hot and cold), and inertial warm adiabatic ions.

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