On the effects of geographical constraints on task execution in complex networks

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In the present work we investigate the effects of spatial constraints on the efficiency of task execution in systems underlain by geographical complex networks where the probability of connection decreases with the distance between the nodes. The investigation considers several configurations of the parameters defining the network connectivity, and the Barabási-Albert network model is also considered for comparisons. The results show that the effect of connectivity is significant only for shorter tasks, that the locality of connections implied by the spatial constraints reduces efficiency, and that the addition of edges can improve the efficiency of the execution, although with increasing locality of the connections the improvement is small.

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Great part of the current theoretical and applied research in physics relies on fast execution of relatively complex algorithms. Several problems of current interest can only be solved by using parallel or distributed computing systems. A recent trend, namely grid computing [1], often allows more cost-effective solutions to such problems, involving the combined use of several general purpose machines. Such an architecture promotes the natural scaling of the number of computing elements in terms of their availability and importance of specific problems (more elements can be assigned to more important problems). The interconnection of machines participating in grid computing systems involves not only local area networks, but mainly the Internet. The connectivity in grid computing is intrinsically dynamic because of its own nature, i.e. the fact that the availability of machines varies with time. In addition, as the interconnections between such machines are often implemented through the Internet, the connectivity of the latter inherently determines the grid architecture [2]. Although grid computing is inherently more flexible and scalable than traditional parallel and distributed systems the interconnectivity of the processing elements in a grid system, combined with specific properties of those elements, is fundamental for achieving efficiency [3]. Therefore, given a specific problem, one needs to select a suitable interconnectivity which, in the case of Internet-based grid computing, is inherently constrained by the Internet.

Instead of implementing a specific problem and inferring the respective performance, a more reasonable and effective means is to model and simulate the execution of the given algorithms, which is not only cheaper but can be used to provide additional insights about the effect of modifying the interconnection and algorithms implementation. Because of their flexibility for representing almost any discrete structure, complex networks [4, 5, 6, 7] represent a natural resource for modeling distributed computing systems, where the processing elements are denoted by nodes and their respective interconnections are represented by edges. Such a potential is especially relevant in the case of grid computing, which often involves interconnectivities substantially different from the highly regular connections traditionally adopted in parallel computing (e.g., meshes, tori, hypercubes, etc.). As a matter of fact, the underlying Internet is itself a complex network [8].

This work focuses on the problem of executing independent tasks in diversely interconnected grid computing systems underlain by Internet connectivity. A master node partitions the problem to be computed in independent tasks and distribute the tasks among the other nodes. Such an investigation extends and complements a previous work [3], which applied complex networks in order to investigate the effect of diverse interconnectivities, including uniform random and scale free models, on the efficiency of grid computing. In particular, in the current work we consider a new interconnectivity model (growing geometric model, GGM, as well as its extension to include redundant edges, GGM-RE-n), so as to allow the quantification of the effects of geographically oriented connectivity and redundant edges on the performance of grid computing. This is important because geographical constraints are fundamental in the Internet. The Barabási-Albert network model is also considered for comparison purposes.

GGM is a novel complex network model which takes into account the influence of the position of the nodes during network growth. Once the geometry of the underlying space is defined, no additional parameters are required in order to define the GGM evolution. Instead of assuming a pre-specified spatial distribution of all nodes
(as in previous geometrical complex networks models), the GGM model involves the progressive incorporation of new nodes, which are connected to the closest existing nodes. Such a dynamics is expected to emulate, to some accuracy level, the historical evolution of the Internet in developing regions where, starting from an initial point, the Internet is progressively extended through the addition of new points connected to the closest existing outlet. More specifically, the GGM model involves the following steps: (i) The underlying spatial region is defined (a unit two-dimensional square is adopted in the present work); (ii) one of its points is chosen as the initial node; and (iii) a new point is randomly (uniformly, in the present work) chosen within the underlying region and connected to the closest existing node; (iv) the previous step is repeated until the desired number of nodes is reached. An immediate consequence of such a growth dynamics is that the resulting network always contains a single connected component. More precisely, the obtained network is a tree devoid of cycles, self-connections and isolated nodes. In order to allow for the presence of cycles, which are found in the Internet, especially at later developmental stages, we also considered an extension of the GGM model incorporating redundant edges, henceforth called GGM-RE. By redundant it is meant that the additional edges will not change the accessibility to any given node, though they will change other properties such as the node degree, shortest path length and clustering coefficient. Note that the inclusion of these redundant edges will necessarily create cycles in the network. We will express the fact that a model has an n% increase in the number of edges with respect to the GGM model by the abbreviation GGM-RE-n, such that GGM-RE-0 corresponds to the initial model (GGM). Note that, although more flexible schemes would be possible, in the present work the redundant edges are incorporated only after the GGM growth is completed. The redundant edges are established by linking randomly chosen pairs of unconnected nodes. The pairs are chosen with the following procedure: first a node i is randomly selected; a different node j is selected, and is connected with i with probability proportional to \( k_{ij} e^{-\alpha d_{ij}} \), where \( k_{ij} \) is the degree of node j, \( d_{ij} \) is the geographical distance between i and j, and \( \alpha \) is the locality constant (its value determines the importance of geographical distance in the establishment of connections). The process is repeated until the desired number of additional edges is achieved. In the present work, GGM-RE-n networks were generated through the addition of 10% to 100% of redundant edges to the same initial GGM network. In order to provide a reference for comparisons, the Barabási-Albert model [9] (BA) was also considered with the same number of nodes and average degree.

The simulations consider that the initial problem has been partitioned into \( M \) independent tasks, which are distributed by the master processing element amongst the slaves (i.e. each of the other processing elements represented by the \( N-1 \) remaining nodes, where \( N \) is the number of nodes in the network), which execute the tasks in parallel. All tasks take the same amount of time \( L \) to conclude. The master processing element does not execute tasks itself, but coordinates their distribution and collects the results. It is assumed that the master can perform unimpeded by delays or bottlenecks. The slave processing elements process each task independently, i.e. without the need to communicate with other slaves. Because the number of tasks is not necessarily equal to an integer multiple of the number of slave processors, idle processing elements may be found at any time during the overall execution. By similar reasoning, some slaves may never receive a task in case the number of slaves is larger than the number of tasks. The simulations assume a communication cost proportional to the total number of links between the master and each of the slaves, with the same communication times for all links; the time taken for the communication through one link is used as the unity of time for the simulation. As no additional costs (e.g., due to routing or congestion) are considered, the performance of the overall execution is directly affected by the network topology. For instance, the shortest the distance between the nodes, the better the performance. At the same time, the selection of the node used as master can significantly influence the overall performance. In case a poorly connected node is chosen for that role, higher communication costs will be implied. In order to average over such effects, the simulation algorithm considers each of the nodes as master for each generated network. The parallel execution time \( T_P \) is defined as the time taken from the dispatch of the first task to a slave until the arrival of the result from the last task in the master node. The parallel speedup \( S \) is the ratio between the time that would be taken for the sequential execution \( (ML \text{ in the considered application model}) \) and the parallel execution time, \( S = ML/T_P \). The performance of each simulation is quantified in terms of its efficiency \( E \), defined as the speedup divided by the number of processing elements, \( E = S/N \). The values reported are averages \( \langle E \rangle \) of the efficiency over 30 networks for each model and parameter values.

Networks of \( N = 1000 \) nodes were generated according to the above described models; all results are averages of 30 realizations. Figure 1 shows the effect of the size of the tasks \( L \) on the efficiency, for fixed \( M = 10000 \). It is clear that, for sufficiently large tasks, the network topology is not an important factor in the efficiency, with significant differences only for \( L \) smaller than 100. Network topology influences the processing efficiency through communication costs; if the computation time of a task is of the order of the time taken to send it to the client and than back to the master, communication cost becomes an important factor; for large \( L \) (about two orders of magnitude larger than the communication latency), the communi-
cation cost, and therefore the network topology, has only small influence in the efficiency. The figure also shows the effect (specially for smaller values of $L$) of increased connectivity: additional edges shifts the efficiency curve up. But the effect on efficiency of adding new edges is decreasing; this can be better seen in the inset, where average efficiency versus additional edges is plotted for $L = 50$. Comparing the GGM-RE model without additional edges with the BA model with $m = 1$ we see that the efficiencies are almost indistinguishable. This result agrees with previous results [3], which found that the efficiency is strongly influenced by the average degree and the fraction of nodes in the largest connected component. In the present case, both networks have the same average degree and are totally connected. The same is not true for GGM-RE with 100% additional edges and BA with $m = 2$ (again, both with the same average degree), where the efficiency of the BA model is clearly superior. This is due to the emphasis on connecting spatially close nodes in the geographical model: this reduces the amount of shortcuts between different regions of the network, increasing average distances between nodes. This effect is more pronounced as the locality constant is increased, as can be seen in the inset: networks with large locality constants almost do not profit from new edges. The effect of geographical locality in the construction of the network can also be seen in Figure 2 which shows the efficiency as a function of task size for different values of the locality constant. An higher locality constant is associated with a lower efficiency, as the number of connections among geographically distant regions tend to decrease, increasing the average distances between nodes and therefore the communication times. The inset in Figure 2 plots efficiency against the locality constant, and shows that small locality constants have little effect, though for values greater than 5 the efficiencies decrease (note that the range of efficiency values spanned is small).

The results presented in this letter show that spatial restrictions, which influence the development of connectivity in complex networks, are a limiting factor for the efficiency distributed task execution in such networks. Because of the lower probability of creation of shortcuts among distant sets of nodes, the efficiency of information transfer in the network is reduced. With the aim of achieving efficient execution of tasks in complex networks it is therefore important to promote the formation of long-range connections during the network construction process.

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