Simulation of an experiment on looking for sterile neutrinos at nuclear reactor

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Abstract. The simulation of an experiment on looking for sterile neutrinos at a nuclear reactor at short distances is presented. It has been shown that statistical fluctuations in experimental bins always imitate the oscillatory behavior of the spectrum. An amplitude of the detectable oscillations decreases when statistics growing up in case of oscillations absence, while mass parameter tends to be accidental. When simulating spectra in detectors with oscillations amplitude parameter fluctuates close to simulated value as well as mass parameter.

1 Introduction

The neutrino oscillations hypothesis has been confirmed in a number of experiments with solar [1], atmospheric [2] and reactors [3]−[5] neutrino. The parameters of all three transitions between active types of neutrino have been identified. However, in some experiments [6]−[8] abnormal results have been observed, which do not fit into the scheme of the three types of neutrino. The concept of the sterile neutrino has been introduced in order to describe these phenomena.

Various experiments to search for oscillations in the sterile state are being proposed [9]−[11], some of them have are underway already. Likewise, sterile neutrino presence analysis is being conducted in many ongoing experiments.

A simulation of the registered antineutrino spectrum from a nuclear reactor in a virtual experiment about sterile neutrino searching has been conducted. According to the received model spectra the analysis for oscillations’ searching with given distance values till the reactor has been conducted.

2 Simulation of the observed spectrum of reactor antineutrino

Antineutrino interacts with the hydrogen nuclei by means of the reaction of inverse beta-decay (IBD) in a detector:

\[ \bar{\nu}_e + p \rightarrow n + e^+ \] (1)

The reaction’s energy threshold (1) is 1.806 MeV. The positron energy is linked with the energy of antineutrino as follows:

\[ E_{\nu} = T_e + E_{\text{thr}} + r_n \] (2)
where $E_\nu$ – antineutrino energy, $T_e$ – positron kinetic energy, $E_{thr}$ – reaction threshold (1) and $r_n$ – neutron recoil energy. The neutron energy is minor and in the first approximation it can be neglected.

In the presence of oscillations, the positrons spectrum in the detector should be changed in accordance with the theory of neutrino oscillations. The probability of preserving its flavor for an electronic antineutrino with energy on distances $L$ from the reactor can be identified as:

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta \cdot \sin^2 \left(1.267\frac{\Delta m^2[eV^2] \cdot R[m]}{E_\nu[MeV]}\right)$$ (3)

If there are no oscillations, then the shape of antineutrino spectrum (as well as positrons) does not change, and spectrum’s integral determining the antineutrino’s streaming in the detector, has to satisfy with the $1/R^2$ law. In the presence of oscillations at specific distances, the spectrum’s shape is being modified and the law’s of inverse squares violation should be observed. On very large distances from the source (more than several dozens of oscillations lengths), the spectrum becomes the same as the source again and an integral reduces approximately twice in comparison with oscillations’ absence. The oscillations’ length is determined by value of a parameter and neutrino’s energy:

$$L_{osc} = \frac{E_\nu[MeV]}{1.267\Delta m^2[eV^2] \cdot R[m]}$$ (4)

In the current project the antineutrino spectrum was used, which was obtained within the project [12], and transferred into the positrons’ spectrum by multiplying it with cross section of the reaction (1). The cross section of the reaction (1) is well described by the formula from the project [13]

$$\sigma_{\nu p} = 10^{-43} p_e E_\nu^{-0.07056 + 0.02018 \ln E_\nu - 0.001953 \ln^2 E_\nu}, E_\nu = E_\nu - 1.2933$$ (5)

Some statistics for positron spectrum were simulated at given distances (for example, $N_1 = 10^5$ and $N_2 = \left(\frac{L_2}{L_1}\right)^2$ for another distance). Wherein, whether the spectrum with oscillations was simulated, the change of total number in the spectrum for the given oscillations was considered by multiplying $N_2$ by the ratio of integrals of oscillations’ functions.

The ratio of oscillations probability for the chosen distances is written as a separate function, which would be used for fitting the relations of the simulated (“experimental”) spectra:

$$f_{12}(E_{\nu_e}) = \frac{P(L_2)}{P(L_1)} = \frac{1 - \sin^2 2\theta \cdot \sin^2 \left(1.267\frac{\Delta m^2 L_2}{E_\nu}\right)}{1 - \sin^2 2\theta \cdot \sin^2 \left(1.267\frac{\Delta m^2 L_1}{E_\nu}\right)}$$ (6)
The analysis has been conducted by the function minimization method $\chi^2$, which is written as:

$$\chi^2 = \sum_{i} \frac{(R_i - f_{12,i}(\Delta m^2, \sin^2 2\theta))^2}{\sigma_{1i}^2} + \frac{(I_1 - P_1 \cdot (L_1^2))^2}{\sigma_{2i}^2}$$

(7)

Where $R_i$ – a ratio of positron spectra for two distances, $f_{12,i}(\Delta m^2, \sin^2 2\theta)$ – function of oscillation probability ratio (6), $\sigma_{1i}$ – “experimental” uncertainty of bin’s ratios, $\sigma_{2i}$ – experimental uncertainty of “experimental” spectrum integrals ratio, $I_1$ and $I_2$ – integrals of “experimental” spectra, and $P_1$ and $P_2$ – integrals of oscillation functions.

3 Virtual Experiment of Oscillations Searching

3.1 Simulation of spectra without oscillations

In the initial phase couple of spectra were simulated based on the described above procedure without oscillations and they were given distances of 12 and 14 m between detector and reactor core centers. The simulated spectra for statistics $10^5$ events of the spectrum in the closest detector are shown in fig. 1.

Afterwards the analysis of oscillations searching has been conducted by the method of fitting the ratio of spectra expressions (6). Correlation between
Figure 2: Simulated positron spectra ratio without oscillations. Red line – the result of functions ratio fitting (6). The oscillation parameters were found as a result of value minimization, are shown in the figure insertion.

simulated spectra for statistics $10^5$ events and distances 12 m and 14 m are shown in fig. 2.

The simulation of dozen of spectra couples has been conducted for the event statistics $10^5$ in the closest spectrum. Founded oscillation parameters are shown in the table 1. The table is shown the parameter’s value $\Delta m^2$ is grouped around 1 eV$^2$, and the parameter $\sin^2 2\theta$ is approximately equal to the statistical error.

The study of statistics influence on value of oscillation parameters has become the next phase of simulation. Couples of spectra have been found for the event statistics $10^4$, $10^5$ and $10^6$ within the spectrum integral. The oscillations parameters boundaries are shown on Fig. 3, 4 and 5. It was found that statistics always form false oscillations up to two standard deviations. The ranges of oscillation parameters were searched for the value of likelihood function (7), where $\chi^2$ takes on values corresponding $N\sigma$ for the Gaussian distribution. The Gaussian distribution values with two degrees of freedom are shown in the Table 3.

### 3.2 Simulation of Spectra With Oscillations

Verification of necessary statistics for detecting oscillations has become the second phase of the simulation. The spectra in a detector with introducing of
Table 1: Oscillation parameters found from the analysis of simulated spectra couples based on statistics $10^5$ events for the closest detector position.

| $\Delta m^2$, eV$^2$ | $\sin^2 2\theta$ | $\chi^2_{\text{min}}$ |
|----------------------|------------------|-----------------|
| 0.685                | 0.036            | 19.4            |
| 0.885                | 0.005            | 9.7             |
| 0.972                | 0.006            | 12.5            |
| 1.442                | 0.018            | 8.2             |
| 1.266                | 0.030            | 9.5             |
| 0.943                | 0.027            | 8.7             |
| 0.447                | 0.042            | 19.9            |
| 1.478                | 0.011            | 13.0            |
| 0.655                | 0.013            | 8.1             |
| 0.940                | 0.007            | 23.0            |

Table 2: Values of the oscillation parameters $\Delta m^2$ and $\sin^2 2\theta$ at varied statistics in simulated spectra at different statistics without oscillations.

| Events number $10^n$ | $\Delta m^2$, eV$^2$ | $\sin^2 2\theta$ | $\chi^2_{\text{min}}$ |
|-----------------------|----------------------|------------------|-----------------|
| $10^4$                | 0.770                | 0.956            | 3.9             |
| $10^5$                | 1.266                | 0.030            | 9.5             |
| $10^6$                | 1.247                | 0.005            | 21.3            |

Oscillation with parameters $\Delta m^2 = 1$ eV$^2$ and $\sin^2 2\theta = 0.05$ were simulated.

The analysis was conducted according to the method described above. A dozen pairs of spectra were simulated for the same distances as without oscillations for the events’ statistics $10(5)$ at the near spectrum. The results of the analysis are shown in the Table 4. Unlike to the spectrum simulation without oscillations, the parameters pledged into the spectra modeling are certainly found here.

The ratio of simulated spectra in the antineutrino energy scale is shown at fig. 6 (considering oscillations with statistics $10^5$ events).

4 Conclusion

The simulation of an experiment on searching for sterile neutrinos at nuclear reactor was done. The pairs of positron spectra with definite statistics with and without oscillations were simulated by Monte-Carlo method. Oscillations were accepted with the parameters of $\Delta m^2 = 1$ eV$^2$ and $\sin^2 2\theta = 0.05$. In current work the distances of 12 and 14 m were chosen to spectra, which are close to the distances used in the DANSSE experiment [11]. In regards to that, the
Table 3: Deviation from the minimum value of $\chi^2_{\text{min}}$ for the Gaussian distribution with two degrees of freedom.

| $N\sigma$ | $\Delta\chi^2$ | $P,\%$ |
|-----------|----------------|--------|
| 1.0       | 2.30           | 68.27  |
| 1.64      | 4.61           | 90.0   |
| 2.0       | 6.18           | 95.45  |
| 3.0       | 11.83          | 99.73  |
| 4.0       | 19.35          | 99.9937|
| 5.0       | 28.44          | 99.999943|

Table 4: Oscillation parameters found from the analysis of simulated pairs of spectra based on statistics $10^5$ events for the closest detector position with oscillations $\Delta m^2 = 1 \text{eV}^2$ and $\sin^22\theta = 0.05$.

| $\Delta m^2$, $\text{eV}^2$ | $\sin^22\theta$ | $\chi^2_{\text{min}}$ |
|-----------------------------|------------------|------------------------|
| 1.045                       | 0.059            | 18.2                   |
| 1.004                       | 0.058            | 10.9                   |
| 0.994                       | 0.049            | 12.0                   |
| 0.991                       | 0.046            | 10.2                   |
| 1.011                       | 0.045            | 10.7                   |
| 0.997                       | 0.069            | 9.4                    |
| 0.977                       | 0.061            | 17.9                   |
| 1.050                       | 0.043            | 10.5                   |
| 0.982                       | 0.048            | 8.0                    |
| 1.005                       | 0.051            | 20.9                   |

Statistics on further distance were simulated with consideration of the distance and chosen oscillation parameters.

The analysis of the spectra simulations with oscillations has shown that with minor statistics ($\sim 10^4$ of events in each spectrum) the oscillations are detected with 95% C.L. The statistics do not allow determining pledged oscillations on the higher level of credibility. At the events' statistics $10^5$ the credibility level is rising till the value corresponding to the four standard deviations, but is not reaching 5$\sigma$. However, it is getting possible to reach 5$\sigma$ value if the events statistics is above $10^6$.

Without oscillations and at any statistics the rapture of oscillation parameters areas had been occurred at variations from the minimal value $\chi^2_{\text{min}}$ at 2.5-3$\sigma$. Moreover, founded oscillation parameters have demonstrated the following: with increasing statistics the parameter $\sin^22\theta$ has decreased to approximately statistical error, and the parameter $\Delta m^2$ has shown random values in the range from 0.1 up to 10 $\text{eV}^2$. 
At this stage, the systematic error was not taken into consideration.

In the real experiment the systematic error to be added to the statistical error thus the statistics should be increased in order to identify the oscillations effect on the credibility level of 5σ.

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Figure 4: The same as on the fig. 3 for the events statistics $10^5$ events within spectrum for the closest distance.

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Figure 5: The same as on the fig. 3 for the events statistics $10^6$ events within spectrum for the closest distance.

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Table 5: Values of the oscillation parameters in simulated spectra at different statistics with used parameters $\Delta m^2 = 1\,\text{eV}^2$ and $\sin^2 2\theta = 0.05$.

| Events number | $\Delta m^2$, eV$^2$ | $\sin^2 2\theta$ | $\chi^2_{\text{min}}$ |
|---------------|----------------------|------------------|---------------------|
| $10^4$        | 0.879                | 0.070            | 5.7                 |
| $10^5$        | 1.011                | 0.045            | 10.7                |
| $10^6$        | 1.036                | 0.048            | 20.6                |

Figure 6: Simulated positron spectra ratio with chosen oscillations. Red line – the result of functions ratio fitting (6). The oscillation parameters were founded as a result of value minimization, are shown in the figure insertion.
Figure 7: Oscillation parameters boundaries for statistics $10^4$ events within spectrum for the closest distance. The shown areas: red line $-1\sigma$, green line $-2\sigma$ and blue line $-3\sigma$. Oscillation parameters are $\Delta m^2 = 1 \text{ eV}^2$ and $\sin^2 2\theta = 0.05$. 
Figure 8: The same as on the fig. 7 for the events statistics $10^5$ events within spectrum for the closest distance. Violet line – 4σ, light blue line – 5σ
Figure 9: The same as on the fig. 7 for the events statistics $10^6$ events within spectrum for the closest distance. Number of standard deviation is shown for each curve in insertion.