On the total cross section extrapolations to cosmic-ray energies

E. G. S. Luna and M. J. Menon
Instituto de Física “Gleb Wataghin”
Universidade Estadual de Campinas, UNICAMP
13083-970, Campinas, SP, Brazil

We analyze pp total cross section data from accelerator to cosmic-ray energies and show that the experimental information presently available indicate two different scenarios at the highest energies. One of them accommodates the predictions of the majority of models and the other one shows agreement with a multiple diffraction model recently developed and improved. We argue that these results may bring a kind of warning against some possible precipitated attempts to constraint cosmic-ray estimations from analyzes of accelerator data.

PACS numbers: 13.85.-t, 13.85.Lg, 13.85.Tp

Some years ago, a successful description of pp elastic scattering data in the interval $13.8 \leq \sqrt{s} \leq 62.5$ GeV was obtained through a multiple diffraction model \cite{1}, hereafter referred as MDM. Extrapolations of the predicted $\sigma_{\text{tot}}^{pp}$ showed agreement with the highest quoted values of the cosmic-ray experiments. In particular, the model predicts $\sigma_{\text{tot}}^{pp}(\sqrt{s} = 16 \text{ TeV}) = 147 \text{ mb} \ (\text{without error estimated})$ \cite{1}. To our knowledge this is the only phenomenological approach that indicates that the cross section may rise faster with the energy than generally expected from “standard” model predictions \cite{2,3} and some fit results \cite{7}.

Recently, Pérez-Peraza et al. introduced some improvements in the model predictions by determining confident error bands, through a forecasting regression analysis \cite{7}. According to the authors, the extrapolations present so large error bands that the predictions show agreement with all the cosmic-ray results, including those that are incompatible among themselves. Moreover, the authors show that taking account of the pp data in the interval $13.8 \leq \sqrt{s} \leq 62.5$ GeV together with pp data at 546 GeV and 1.8 TeV, the predictions resulting from a new fit of the model free parameters present a very narrow error band and are compatible with the smallest quoted values of the cosmic-ray experiments. Since this seems to be in agreement with “standard” models and fit results \cite{2,3}, the authors conclude that “extrapolations from accelerator data should be used to constrain cosmic-ray estimations” \cite{7}, which was also suggested in \cite{2} and done in practice in \cite{4}.

However some aspects of the approach \cite{7} must be commented. Firstly, reading from the plot in Fig. 2 of this reference we can infer $\sigma_{\text{tot}}^{pp}(\sqrt{s} = 16 \text{ TeV}) \sim 147 \pm 37 \text{ mb}$. Although this is a large error band ($\sim 25\%$) the results still favor (at least qualitatively) the highest quoted values of the cosmic-ray experiments, as shown in the same Figure (this is not what the authors claimed). Secondly, in the context of a model approach, simultaneous analysis of pp and $\bar{p}p$ scattering should take analyticity and crossing into account, what was not done in \cite{7} and also in \cite{5}. At last, the assumption that pp and $\bar{p}p$ interactions are the same even at 546 GeV is a strong ad hoc hypothesis and since pp and $\bar{p}p$ interactions are different in the ISR energy region, one should expect the use of $\sigma_{\text{tot}}^{\bar{p}p}$ and not $\sigma_{\text{tot}}^{pp}$ in this interval.

In order to attempt to bring some insight on this issue, we performed a simple model-independent analysis of the same pp accelerator data used in \cite{1} and \cite{7}, but including cosmic-ray information presently available. As reviewed in some detail in \cite{7}, at cosmic-ray energies ($\sqrt{s} : 6 - 40 \text{ TeV}$) we can identify two distinct set of estimations, one represented by the results of the Fly’s Eye Collaboration \cite{9} together with those by the Akeno Collaboration \cite{10}, and other by the results of Gaisser, Sukhatme, Yodh (GSY) \cite{11} with those by Nikolaev \cite{12}. All these data and estimations are displayed in Fig. 1 (Akeno results were read from the graph of Fig. 4 in Ref. \cite{10}).

The discrepancies between these two sets of cosmic-ray information comes from both, the uncertainties in the determination of the production cross section $\sigma_{\text{prod}}^{p-\text{air}}$ and the uncertainties in the connection between $\sigma_{\text{prod}}^{p-\text{air}}$ and the pp cross section, as discussed in detail by Engel, Gaisser, Lipari and Stanev \cite{13}: all the cosmic-ray estimations are strongly model dependent. One important point is the fact that the results by Nikolaev and GSY have never been directly criticized in the literature.

For these reasons, we shall investigate the behavior of the total cross section by taking into account the discrepancies that characterize the cosmic-ray information. Also, since the base of our discussion is the predictions of the MDM, we shall consider the same set of pp accelerator data used in references \cite{1} and \cite{7}. In order to accomplish that, we select three ensembles of data and experimental information with the following notation:

- Ensemble I: pp accelerator data \cite{1}.
- Ensemble II: pp accelerator data + Akeno + Fly’s Eye \cite{10,11,13}.
- Ensemble III: pp accelerator data + Nikolaev + GSY \cite{12,11,12}. 


Different analytical parametrizations may fit all these ensembles with quite good statistical results, as shown, for example, in [15]. In this communication we shall choose the simplest form, which corresponds to the same analytical function used to fit the free parameters, depending on the energy, in the MDM. As explained in [1] this concerns a polynomial of second degree in ln \( s \):

\[
\sigma_{pp}^{tot} = A + B \ln \frac{s}{s_0} + C \left[ \ln \frac{s}{s_0} \right]^2,
\]

(1)

where \( A, B, C \) are free parameters and \( s_0 = 1 \text{ GeV}^2 \).

It should be stressed that in [1] the fits involved only the differential cross section and the \( \rho \) parameter data (input information). The total cross section was predicted from parametrizations of essentially two free parameters depending on the energy, one associated with the hadronic form factor and the other with the average elementary scattering amplitude. Since these parameters enter in the eikonal (in the momentum transfer space), the final result for \( \sigma_{pp}^{tot} \) should not be expected to be the same as that obtained with Eq. (1). In addition, it should be noted that other parametrizations including terms like \((\ln s)^D \text{ and } R_s^{-1/2} \) (\( D \) and \( R \) free parameters) lead to a faster rise of \( \sigma_{pp}^{tot}(s) \) than that obtained through Eq. (1), as shown in [15]. In this sense the above choice clearly constraint the rise of \( \sigma_{pp}^{tot} \) and shall be considered as a lower limit (allowing some connection with the parametrizations of the MDM).

The fits have been performed using the CERN-MINUIT routine [16]. The corresponding values of the free parameters and statistical information about each fit are displayed in Table 1. Errors from the free parameters were propagated to \( \sigma_{pp}^{tot}(s) \) through the usual formulas, taking account of both variances and covariances [17]. The results with the corresponding error bands (± one standard deviation) for ensembles I, II and III are shown in Figs. 2, 3, and 4, respectively.

Table 1. Values of the parameters \( A, B \) and \( C \) in Equation (1) (all in mb) and the \( \chi^2 \) per degree of freedom in each fit to Ensembles I, II and III.

| Ensemble: | I   | II  | III  |
|-----------|-----|-----|------|
| \( A \)   | 47.32 | 45.78 | 49.27 |
| \( B \)   | -3.809 | -3.315 | -4.435 |
| \( C \)   | 0.4042 | 0.3654 | 0.4534 |
| \( \chi^2 \) | 8.654 | 11.75 | 15.04 |
| d.o.f.    | 4   | 11  | 11   |
| \( \chi^2/d.o.f. \) | 2.16 | 1.07 | 1.37 |
From Fig. 2, the results with the ensemble I lie between the two sets of discrepant cosmic-ray estimations. It corresponds to extrapolations from a “pure” accelerator information in the interval investigated.

On the other hand, if one or the other cosmic-ray sets are included the situation changes, as shown in Figs. 3 and 4. Taking into account the error bands (one standard deviation), the two fits become discrepant above $\sim 1$ TeV and they are certainly incompatible above 5 TeV. We understand this result as a statistical evidence for the possibility of two distinct scenarios above $\sqrt{s} \sim 5$ TeV.

In particular, from each ensemble, the interpolated results at $\sqrt{s} = 14$ TeV are $\sigma_{\text{tot}}^{\text{pp}} = 125 \pm 7$ mb (I), $\sigma_{\text{tot}}^{\text{pp}} = 119 \pm 5$ mb (II), and $\sigma_{\text{tot}}^{\text{pp}} = 133 \pm 6$ mb (III). In Ref. [15] four different parametrizations were used to fit Ensembles II and III and the two scenarios are also inferred. For example, taking into account the 4 parametrizations (without error estimations) we can infer from Ensemble II the value $\sigma_{\text{tot}}^{\text{pp}}(14 \text{ TeV}) = 113 \pm 5$ mb and from Ensemble III, $\sigma_{\text{tot}}^{\text{pp}}(14 \text{ TeV}) = 140 \pm 7$ mb. At lower values of energy, namely $\sqrt{s}$ in the region $10 - 100$ GeV, there is no significant distinction between the four parametrizations.

![Fig. 2. Parametrization to Ensemble I: pp accelerator data [14].](image)

![Fig. 3. Parametrization to Ensemble II: pp accelerator data + Akeno + Fly’s Eye [14,9,10].](image)
Ensemble II clearly accommodates the predictions from the majority of models and in general is considered as “the cosmic-ray results”. On the other hand, if the estimations in Ensemble III are the correct ones, the experimental information on $\sigma_{pp}^{tot}$ presently available indicate that the $pp$ cross section rises faster with the energy than generally expected or assumed and, in this case, the rise is well described by the MDM. At this point it should be stressed that, due to its phenomenological basis, this model can not provide a “microscopic” explanation for this phenomena (only, perhaps, some naive geometrical arguments) and this puts limitations in the understanding of a “fast rising”.

However, it should be remembered that several cosmic-ray experiments point out to the possibility of new phenomena in $pp$ collisions at $\sqrt{s} \sim 5$ TeV [18] and this has never been investigated by accelerators. Moreover, recently, more confident analyzes stress the particular features of Centauro events, when compared with normal events observed in emulsion chamber experiments [19]. Since the bulk of the cosmic-ray showers concerns $pp$ and not $\bar{p}p$ interactions, these “exotic” events may represent an additional contribution to the $pp$ channel, resulting in a faster increase of the $\sigma_{tot}$, as claimed in [20].

Certainly, these puzzles shall be resolved by the CERN Large Hadron Collider (LHC) and even the BNL Relativistic Heavy Ion Collider (RHIC). But before this, we understand that the results presented here may serve as a kind of warning against possible precipitated attempts to constraint cosmic-ray estimations from analyzes of accelerator data.

Since in this communication our central goal was to discuss the predictions of the MDM, the same set of $pp$ accelerator data was used. An extend analysis with all experimental data and estimations available, including $\bar{p}p$ scattering, is in course.

We are grateful to Fapesp for financial support and to R.F. Ávila for discussions. M.J.M. is also thankful to N.N. Nikolaev for discussions and CNPq for financial support.

---

FIG. 4. Parametrization to Ensemble III: $pp$ accelerator data + Nikolaev + GSY [4][1][1].

---

[1] A. F. Martini and M. J. Menon, Phys. Rev. D 56, 4338 (1997).
[2] A. Donnachie and P. V. Landshoff, Z. Phys. C 2, 55 (1979); Phys. Lett. B 387, 637 (1996).
[3] P. Desgrolard, M. Giffon, and E. Predazzi, Z. Phys. C 63, 241 (1994).
[4] C. Bourrely, J. Soffer, and T. T. Wu, Nucl. Phys. B247, 15 (1984); Z. Phys. C 37, 369 (1988).
[5] M. M. Block, F. Halzen, and T. Stanev, Phys. Rev. D 62, 077501 (2000); Phys. Rev. Lett. 83, 4926 (1999).
[6] C. Augier et al. (UA4/2 Collaboration), Phys. Lett. B 315, 503 (1993).
[7] J. Pérez-Peraza et al., e-Print Archive: hep-ph/0011167 (2000).
[8] J. Velasco et al., in 26th International Cosmic Ray Conference, 1999, Vol. 1, p. 198; e-Print Archive: hep-ph/9910484
[9] Fly's Eye Collaboration (R. M. Baltrusaitis et al.), Phys Rev. Lett. 52, 1380 (1984).
[10] Akeno Collaboration (H. Honda et al.), Phys. Rev. Lett. 70, 525 (1993).
[11] T.K. Gaisser, U.P. Sukhatme, and G.B. Yodh, Phys. Rev. D 36, 1350 (1987).
[12] N.N. Nikolaev, Phys. Rev. D 48, R1904 (1993).
[13] R. Engel, T. K. Gaisser, P. Lipari, and T. Stanev, Phys. Rev. D 58, 014019 (1998).
[14] Accelerator data on $\sigma_{\text{pp}}^{\text{tot}}$ (the numbers between the squared brackets indicate the center-of-mass energy in GeV): A.S. Carrol et al., Phys. Lett. B 61, 303 (1976) [13.8]; A.S. Carrol et al., Phys. Lett. B 80, 423 (1979) [19.4]; U. Amaldi and K. R. Schubert, Nucl. Phys. B 166, 301 (1980) [23.5, 30.7, 44.7, 52.8, 62.5].
[15] R. F. Ávila, E. G. S. Luna, and M. J. Menon, hep-ph/0105065, submitted to Braz. J. Phys.
[16] F. James, MINUIT - Function Minimization and Error Analysis - Reference Manual, Version 94.1, CERN - D506 (1994).
[17] P. R. Bevington and D. K. Robinson, data reduction and error analysis for the physical sciences, (Mcgraw-Hill, New York, 1992).
[18] Chacaltaya and Pamir Collaboration, Nucl. Phys. B370, 365 (1992).
[19] S. L. C. Barroso et al., Nucl. Phys. B (Proc. Supp.) 75A, 150 (1999); S. L. C. Barroso, Doctoral Thesis, UNICAMP, Brazil, 2000.
[20] M. J. Menon, in International Workshop On Hadron Physics 98, edited by E. Ferreira, F. F. S. Cruz and S. S. Avancini (World Scientific, Singapore, 1999) p. 333; e-Print Archive: hep-ph/9810508