Gaugino Mass Nonuniversality and Dark Matter in SUGRA, Strings and D Brane Models

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Abstract

The effects of nonuniversality of gaugino masses on dark matter are examined within supersymmetric grand unification, and in string and D brane models with R parity invariance. In SU(5) unified models nonuniversality in the gaugino sector can be generated via the gauge kinetic energy function which may depend on the 24, 75 and 200 dimensional Higgs representations. We also consider string models which allow for nonuniversality of gaugino masses and D brane models where nonuniversality arises from embeddings of the Standard Model gauge group on five branes and nine branes. It is found that with gaugino mass nonuniversality the range of the LSP mass can be extended much beyond the range allowed in the universal SUGRA case, up to about 600 GeV even without coannihilation effects in some regions of the parameter space. The effects of coannihilation are not considered and inclusion of these effects may further increase the allowed neutralino mass range. Similarly with the inclusion of gaugino mass nonuniversality, the neutralino-proton ($\chi + p$) cross-section can increase by as much as a factor of 10 in some of regions of the parameter space. An analysis of the uncertainties in the quark density content of the nucleon is given and their effects on $\chi + p$ cross-section are discussed. The predictions of our analysis including nonuniversality is compared with the current limits from dark matter detectors and implications for future dark matter searches are discussed.

1 Introduction

In this paper we study the effects of nonuniversality of gaugino masses on dark matter in SUGRA models\textsuperscript{1} and in string and D brane models. Nonuniversal gaugino masses arise quite naturally in supergravity and string unified theories. Thus, in N=1 supergravity the kinetic energy and the mass terms for the gauge fields and the gauginos are given by\textsuperscript{1}

\[ e^{-1} \mathcal{L} = \frac{1}{4} \Re \left[ f_{\alpha\beta} F^\alpha_{\mu\nu} F^{\beta\mu\nu} \right] + \frac{1}{4} i \Im \left[ f_{\alpha\beta} F^\alpha_{\mu\nu} \tilde{F}^{\beta\mu\nu} \right] + \frac{1}{2} \Re \left[ f_{\alpha\beta} \left( -\frac{1}{2} \lambda^\alpha \partial \lambda^\beta \right) \right] \]

\[ -\frac{1}{8} i \Im \left[ f_{\alpha\beta} e^{-1} D_\mu (e^{\bar{\alpha} \gamma^\mu \gamma_5 \lambda^\beta}) \right] + \frac{1}{4} e^{G/2} G^a (G^{-1})^a_b \left( \partial f^*_{\alpha\beta} / \partial z^b \lambda^\alpha \lambda^\beta \right) + \text{(h)} \]

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Here $\lambda^\alpha$ are the gaugino fields, $G = -\ln[\kappa^6 W W^*] - \kappa^2 d$, where $W$ is the superpotential, $d(z, z^*)$ is the Kahler potential where $z^a$ are the complex scalar fields, and $\kappa = (8\pi G_N)^{-\frac{1}{2}} = 0.41 \times 10^{-18}$ GeV$^{-1}$ where $G_N$ is Newton’s constant. The gauge kinetic energy function $f_{\alpha\beta}$ in general has a non-trivial field dependence involving fields which transform as a singlet or a non-singlet irreducible representation of the underlying gauge group. After the spontaneous breaking of the gauge symmetry at the scale of the Standard Model gauge group $SU(2)_L \times U(1) \times SU(3)_C$, one needs to carry out a rescaling of the gauge kinetic energy in the sector of the gauge group that is preserved. This rescaling generates a splitting of the $SU(2)_L \times U(1) \times SU(3)_C$ gauge couplings at the unification scale $M_X$ and one has\[2, 3, 4\]

$$\alpha_i^{-1}(M_X) = \alpha_X^{-1}(M_X) + \sum_r c_{0r} n^r_i$$

(2)

where $n^r_i$ characterize the Higgs vacuum structure of the irreducible representation $r$ and $c_{0r}$ parametrize its relative strength. After rescaling the gaugino mass matrix takes the form

$$m_{\alpha\beta} = \frac{1}{4} e^{G/2} G^a (G^{-1})^b_a (\partial f_{\alpha\gamma} / \partial z^* b) f^{-1}_{\gamma\beta}$$

(3)

Here one finds that the contribution to nonuniversality of the gauginos mass is controlled not only by the nature of the GUT sector but also by the nature of the hidden sector. Because of this the splitting of the gaugino masses at $M_X$ is characterized by a set of parameters $c_r$ different from $c_{0r}$. Thus after the breaking of the unified gauge group we parametrize the gaugino masses at $M_X$ by $\tilde{m}_i(0)$ where\[3, 4\]

$$\tilde{m}_i(0) = m_2 (1 + \sum_r c_r n^r_i)$$

(4)

The effect of $c_{0r}$ on the gauge coupling unification has been discussed in the previous literature\[4\] and we do not discuss it here. In the analysis of this paper we assume unification of gauge couplings at $M_X$ and the refinement of including $c_{0r}$ correction will not have any significant effect on our analysis.

In $SU(5)$ the gauge kinetic energy function $f_{\alpha\beta}$ transforms as the symmetric product of $24 \times 24$ and contains the following representations

$$(24 \times 24)_{symm} = 1 + 24 + 75 + 200$$

(5)

The singlet leads to universality of the gaugino masses while the non-singlet terms will generate nonuniversality. We consider models where we have a linear combination of the singlet and a non-singlet representation, i.e., $1 + 24, 1 + 75$ or $1 + 200$. 

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The quantities \( n^r_i \) for the representations \( 1, 24, 75, 200 \) are listed in Table 1 [3, 4, 6]. Some phenomenological aspects of nonuniversality of gaugino masses have recently been discussed [3, 4, 8]. We focus here on their effects on event rates in the direct detection of dark matter (see Ref. [4] for previous work on the effects of gaugino mass nonuniversality on dark matter).

Techniques for computing the event rate in the scattering of neutralinos off nuclear targets has been discussed by many authors [10]. We follow here the procedures discussed in Ref. [11]. In our analysis we impose the \( b \to s + \gamma \) constraint [12] and the bounds on SUSY particles from the Tevatron and LEP [13]. Specifically we take for the lower limits \( m_{\chi^+} > 94 \text{ GeV}, m_{\chi} > 33 \text{ GeV}, m_{\tilde{\tau}_R} > 71 \text{ GeV}, m_{\tilde{\tau}} > 87 \text{ GeV}, m_{\tilde{g}} > 190 \text{ GeV}, m_h > 113.5 \text{ GeV} [14] \) and for the \( b \to s + \gamma \) branching ratio we take a 2\( \sigma \) range around the current experiment [15], i.e., we take \( 2 \times 10^{-4} < B(b \to s + \gamma) < 4.5 \times 10^{-4} \). The quantity that constrains theory is \( \Omega_\chi h^2 \) where \( \Omega_\chi = \rho_\chi/\rho_c \), where \( \rho_\chi \) is the neutralino relic density and \( \rho_c = 3H_0^2/8\pi G_N \) is the critical matter density, and \( h \) is the value of the Hubble parameter \( H_0 \) in units of 100 km/s/Mpc. The current limit on \( h \) from the Hubble Space Telescope is \( h = 0.71 \pm 0.03 \pm 0.07 [10] \) and recent analyses of \( \Omega_m \) give [17] \( \Omega_m = 0.3 \pm 0.08 \). Assuming \( \Omega_B \simeq 0.05 \), one gets

\[
\Omega_\chi h^2 = 0.126 \pm 0.043
\]  

In this analysis we make a somewhat liberal choice for the error corridor on \( \Omega_\chi h^2 \), i.e., we choose \( 0.02 \leq \Omega_\chi h^2 \leq 0.3 \). The choice of a more restricted corridor does not affect the general conclusions arrived at in this analysis. In the theoretical computation of the relic density we use

\[
\Omega_\chi h^2 \approx 2.48 \times 10^{-11} \left( \frac{T_\chi}{T_\gamma} \right)^3 \left( \frac{T_\gamma}{2.73} \right)^3 \frac{N_f^{1/2}}{J(x_f)}
\]

\[
J(x_f) = \int_0^{x_f} dx \langle \sigma v \rangle (x) GeV^{-2}
\]

where \( \left( \frac{T_\chi}{T_\gamma} \right)^3 \) is the reheating factor, \( N_f \) is the number of degrees of freedom at the freeze-out temperature \( T_f \) and \( x_f = kT_f/m_\tilde{\chi} \). In determining \( J(x_f) \) we use the method developed in Ref. [18]. The role of \( J \) in the context of nonuniversalsities will be elucidated in Sec. 5. A number of effects on neutralino dark matter have already been studied. These include the effects of nonuniversality of the scalar masses at the unification scale [13, 20], effects of variations of the WIMP velocity [21], [22, 23], effects of rotation of the galaxy [24], effects of CP violation with EDM
constraints\textsuperscript{[25]}, and effects of coannihilation\textsuperscript{[20]}. The focus of this analysis is on the effects of nonuniversality of the gaugino masses on dark matter. In the present analysis we do not include the effects of coannihilation. These effects become important when the NLSP mass \(m_i\) lies close to the LSP mass, i.e., \(\Delta_i = (m_i/m_\chi - 1) < 0.1\). We have identified several regions of the parameter space where coannihilations involving \(\tilde{\tau}_1, \tilde{e}_R\), the light chargino \(\chi^+_1\), or the next to the lightest neutralino \(\chi^0_2\) occur. However, as we stated above we do not consider coannihilation in this paper and thus eliminate such regions of the parameter space by imposing the constraint \(\Delta_i > 0.1\). An analysis in this region requires a separate treatment and will be reported elsewhere\textsuperscript{[27]}. Recent analyses\textsuperscript{[28, 29]} have pointed to the uncertainties in the quark masses and in the quark content of the nucleon that enter in analyses of dark matter. We give in this paper an independent analysis of the errors in the quark densities and compute their effects on dark matter.

The outline of the rest of the paper is as follows: In Sec.2 we discuss the basic formulae used to compute the neutralino-proton cross-section. An analysis of errors in the quark densities that enter in the scalar \(\sigma_{\chi p}^{(scalar)}\) cross-section is also given. In Sec.3 we first give an analysis of \(\sigma_{\chi p}^{(scalar)}\) for the universal SUGRA case and analyze the effect of errors on the quark densities of the proton on it. We then discuss three nonuniversal scenarios where we consider admixtures of the singlet with the 24 plet, the 75 plet and the 200 plet representations for the gauge kinetic energy function. In Sec.4 we extend the analysis of \(\sigma_{\chi p}^{(scalar)}\) to the case of the O-II string model and a brane model based on 9 branes and 5\(i\) branes. In Sec.5 we discuss the origin of the enlargement of the allowed LSP domain consistent with the relic density constraints due to the presence of nonuniversalities. Conclusions are given in Sec.6. In Appendix A we give an analytic solution of the sparticle masses using the one loop renormalization group equations including the effect of gaugino mass nonuniversalities. Using results of Appendix A we compute in Appendix B the effects of nonuniversalities on the \(\mu\) parameter. The analytic results of Appendices A and B provide a deeper understanding of the gaugino-mass nonuniversality effects discussed in Secs. 3, 4 and 5.

## 2 Neutralino-proton cross-section

For heavy target nuclei such as germanium the neutralino-nucleus scattering cross-section is dominated by the scalar part of the neutralino-quark interaction and it is the quantity \(\sigma_{\chi p}^{(scalar)}\) on which constraints have been exhibited in the recent
experimental works\cite{30,31}. For this reason we focus in this paper on the analysis of $\sigma_{\chi p}(scalar)$. The basic interaction governing the $\chi - p$ scattering is the effective four-fermi interaction given by\cite{32}

$$L_{eff} = \bar{\chi} \gamma_\mu \gamma_5 \chi \gamma^\mu (A P_L + B P_R) q + C \bar{\chi} \chi m_q \bar{q} q + D \bar{\chi} \gamma_5 \chi m_q \bar{q} \gamma_5 q + E \bar{\chi} i \gamma_5 \chi m_q \bar{q} q$$

$$+ F \bar{\chi} \chi m_q \bar{q} i \gamma_5 q$$

where the interaction relevant to our analysis is parametrized by $C$. The $\chi - p$ cross-section arising from scalar interactions is given by

$$\sigma_{\chi p}(scalar) = \frac{4\mu_r^2}{\pi} \left( \sum_{i=u,d,s} f_i^p C_i + \frac{2}{27} (1 - \sum_{i=u,d,s} f_i^p) \sum_{a=c,b,t} C_a \right)^2$$

Here $\mu_r$ is the reduced mass, $f_i^p$ (i=u,d,s quarks) are defined by

$$m_p f_i^p = \langle p | m_q \bar{q}_i q_i | p \rangle$$

and $C$ is given by

$$C = C_{h^0} + C_{H^0} + C_{\tilde{f}}$$

where $C_{h^0}, C_{H^0}$ are the contributions from the s-channel $h^0$ and $H^0$ exchanges and $C_{\tilde{f}}$ is the contribution from the t-channel sfermion exchange. They are given by\cite{32}

$$C_{h^0}(u,d) = -(+) \frac{g^2}{4 M_W M_{h^0}^2} \frac{\cos \alpha (\sin \alpha)}{\sin \beta (\cos \beta)} \text{Re} \sigma$$

$$C_{H^0}(u,d) = \frac{g^2}{4 M_W M_{H^0}^2} \frac{\sin \alpha (\cos \alpha)}{\sin \beta (\cos \beta)} \text{Re} \rho$$

$$C_{\tilde{f}}(u,d) = - \frac{1}{4 m_q} \frac{1}{M_{\tilde{q_1}}^2 - M_{\chi}^2} \text{Re}[C_{qL} C_{qR}^*] - \frac{1}{4 m_q} \frac{1}{M_{\tilde{q_2}}^2 - M_{\chi}^2} \text{Re}[C_{qL'} C_{qR'}^*]$$

Here (u,d) refer to the quark flavor, $\alpha$ is the Higgs mixing angle, and $C_{qL}, C_{qL'}$ etc. are as defined in Ref.\cite{32}, and $\sigma$ and $\rho$ are defined by

$$\sigma = X_{40}^* (X_{20}^* - \tan \theta_W X_{10}^*) \cos \alpha + X_{30}^* (X_{20}^* - \tan \theta_W X_{10}^*) \sin \alpha$$

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\[ \rho = -X_{40}^*(X_{20}^* - \tan \theta_W X_{10}^*) \sin \alpha + X_{30}^*(X_{20}^* - \tan \theta_W X_{10}^*) \cos \alpha \]  

(16)

where \( X_{n0} \) are the components of the LSP

\[ \chi = X_{10}^* \tilde{B} + X_{20}^* \tilde{W}_3 + X_{30}^* \tilde{H}_1 + X_{40}^* \tilde{H}_2 \]  

(17)

We discuss now the amount of uncertainty connected with the determination of \( f_i^p \). The quantities that are used as inputs are \( \sigma_{\pi N} \), \( x \), and \( \xi \) defined by

\[ < p | 2^{-1} (m_u + m_d) (\bar{u}u + \bar{d}d) | p > = \sigma_{\pi N}, \]  

(18)

\[ x = \frac{\sigma_0}{\sigma_{\pi N}} = \frac{< p | \bar{u}u + \bar{d}d - 2\bar{s}s | p >}{< p | \bar{u}u + \bar{d}d | p >}, \]  

(19)

and

\[ \xi = \frac{< p | \bar{u}u - \bar{d}d | p >}{< p | \bar{u}u + \bar{d}d | p >}. \]  

(20)

We can determine \( f_i^p \) in terms of these and find

\[ f_u^p = \frac{m_u}{m_u + m_d} (1 + \xi) \frac{\sigma_{\pi N}}{m_p} \]  

\[ f_d^p = \frac{m_d}{m_u + m_d} (1 - \xi) \frac{\sigma_{\pi N}}{m_p} \]  

\[ f_s^p = \frac{m_s}{m_u + m_d} (1 - x) \frac{\sigma_{\pi N}}{m_p} \]  

(21)

A similar analysis holds for the neutralino-neutron scattering and one can determine \( f_i^n \) in terms of \( \xi \), \( x \), \( \sigma_{\pi N} \) as follows

\[ f_u^n = \frac{m_u}{m_u + m_d} (1 - \xi) \frac{\sigma_{\pi N}}{m_p} \]  

\[ f_d^n = \frac{m_d}{m_u + m_d} (1 + \xi) \frac{\sigma_{\pi N}}{m_p} \]  

\[ f_s^n = \frac{m_s}{m_u + m_d} (1 - x) \frac{\sigma_{\pi N}}{m_p} \]  

(22)

We note in passing that from Eqs.21 and 22 one has the relation

\[ f_u^p f_d^p = f_u^n f_d^n \]  

(23)

which holds independent of the details of the input parameters. We discuss now the numerical evaluation of \( f_i^p \) and \( f_i^n \). The various determinations of \( \sigma_{\pi N} \), \( \sigma_0 \) and \( x \) using analyses of Ref.\[33, 34, 35, 36, 37, 28\] are summarized in Table2. For \( \sigma_{\pi N} \)
one finds an average value of $48 \pm 9$ MeV, and for $\sigma_0$ an average value of $35.5 \pm 6$ MeV. These give $x = 0.74 \pm 0.25$. Further, there are two independent lattice gauge determinations\cite{36,37} of $y = 1 - x$ which we list in Table 2. In recording the result of $x$ for Ref.\cite{37} we have reduced the $y$ value by 35\% as discussed in Ref.\cite{36}. The average of these lattice gauge calculations gives $x = 0.61 \pm 0.08$. Taking the average yet again of this $x$ and of $\bar{\sigma}_0/\pi \sigma_{N}$ we get the average $\bar{x}$ listed in Table 2. In addition to the above we need to determine the symmetry breaking parameter $\xi$. Here as in the work of Ref.\cite{29} we use the analysis of Ref.\cite{38} on baryon mass splittings to obtain

$$\xi = \frac{(\Xi^- + \Xi^0 - \Sigma^+ - \Sigma^-) x}{\Xi^- + \Xi^0 + \Sigma^+ + \Sigma^- - 2m_p - 2m_n}$$

(24)

where $x$ is as defined by Eq.(19). Numerically one finds $\xi = 0.196x$ and on using Table 2 we find

$$\xi = 0.132 \pm 0.035$$

(25)

In addition to the above one needs the ratios of the quark masses for which we use\cite{34}

$$\frac{m_u}{m_d} = 0.553 \pm 0.043, \quad \frac{m_s}{m_d} = 18.9 \pm 0.8$$

(26)

On using Eqs.(21),(25),(26), and Table 2 we find

$$f_u^p = 0.021 \pm 0.004$$
$$f_d^p = 0.029 \pm 0.006$$
$$f_s^p = 0.21 \pm 0.12$$

(27)

Similarly for $f_i^n$ we find

$$f_u^n = 0.016 \pm 0.003$$
$$f_d^n = 0.037 \pm 0.007$$
$$f_s^n = 0.21 \pm 0.12$$

(28)

For the more general case of neutralino-Nucleus ($\chi - N$) scattering one has

$$\sigma_{\chi-N}(scalar) = \frac{4m_{\chi}^2}{\pi} (Z \sum_{i=u,d,s} f_i^p C_i + \frac{2}{27} Z (1 - \sum_{i=u,d,s} f_i^p) \sum_a C_a$$

$$+(A-Z) \sum_{i=u,d,s} f_i^n C_i + \frac{2}{27} (A-Z) (1 - \sum_{i=u,d,s} f_i^n) \sum_a C_a)^2$$

(29)

Using Eqs.21 and 22 we write the above in the form
\[ \sigma_{\chi-N}(\text{scalar}) = \frac{4m_r^2A}{\pi} \left( f \frac{m_u C_u + m_d C_d}{m_u + m_d} + \hat{f} \xi \Delta \frac{m_u C_u - m_d C_d}{m_u + m_d} + f C_s \right) \]

\[ + \frac{2}{27} (1 - f - \hat{f} - \hat{f} \xi \Delta \frac{m_u - m_d}{m_u + m_d} \sum_{a=e,b,t} C_a)^2 \]  

(30)

where \( \Delta = (2Z - A)/A \), \( f = f_s \), \( \hat{f} = \sigma_{\pi N}/m_p \) and numerically on using Table 2 we have

\[ \hat{f} = 0.05 \pm 0.01 \]  

(31)

We note that while the \( \chi-p \) and \( \chi-n \) cross-sections depend on \( \xi \), the \( \xi \) dependent term has a cancellation in (\( \chi - N \)) cross-section because of the \( (2Z - A) \) factor. Further, if the target nucleus has \( A = 2Z \), i.e., \( \Delta = 0 \), then the \( \xi \) dependent term will drop out of the \( \chi - N \) cross-section. Because of the above it is experimentally better to plot the \( \chi - N \) cross-section rather than the \( \chi - p \) cross-section as is currently the practice[30, 31].

3 Dark matter in GUT models with gaugino mass nonuniversality

As discussed at the beginning of this section we consider here models where the nonuniversalities arise from admixtures of the singlet with the 24 plet, the 75 plet and the 200 plet representations. However, we begin first by exhibiting the result for the universal SUGRA case. The soft SUSY breaking sector of the theory, under the assumption that SUSY breaking is communicated from the hidden to the visible sector by gravitational interactions, is parametrized in this case by the universal scalar mass \( m_0 \), the universal gaugino mass \( m_{\tilde{g}} \), the universal trilinear coupling \( A_0 \) all taken at the GUT scale, and tan \( \beta = < H_2 > / < H_1 > \) where \( H_2 \) gives mass to the up quark and \( H_1 \) gives mass to the down quark. Throughout this analysis we assume that the Higgs mixing parameter \( \mu \) (which appears in the superpotential as \( \mu H_1 H_2 \)) is determined via the electro-weak symmetry breaking constraint. The range of the parameters are limited by a naturalness constraint. We mean this to imply that \( m_0, m_{\tilde{g}} \leq 1 \text{ TeV} \), where \( m_{\tilde{g}} \) is the gluino mass, tan \( \beta \leq 25 \), and \( A_0 \), or equivalently \( A_t \), the value of \( A_0 \) at the electro-weak scale in the top channel, is limited by the electro-weak symmetry breaking constraint. For the analysis here we choose \( \mu > 0 \) while for the other \( \mu \) sign the allowed parameter space for dark
matter is strongly limited due to the $b \rightarrow s + \gamma$ constraint\[12\]. The results for $\mu < 0$ look qualitatively different in that the cross-sections are significantly smaller.

In Fig.1 we plot the maximum and the minimum of $\sigma_{\chi p}(scalar)$ as a function of $m_\chi$ where the parameters are allowed to vary over the naturalness range discussed above. The analysis is done for three sets of $f_p^p$ values corresponding to the corridor given by Eq.(27). They correspond to (I) $f_p^u = 0.025$, $f_p^d = 0.035$, $f_p^s = 0.33$, (II) $f_p^u = 0.021$, $f_p^d = 0.029$, $f_p^s = 0.21$, (III) $f_p^u = 0.017$, $f_p^d = 0.023$, $f_p^s = 0.09$. From Fig.1 we see that the different sets can lead to a variation in $\sigma_{\chi p}(scalar)$ up to a factor of about 5. For the rest of the analysis in this paper we use set (II).

Next we consider the $1 + 24$ model. We find that in this case $\sigma_{\chi p}(scalar)$ typically decreases for positive values of $c_{24}$ and increases with negative values of $c_{24}$. This behavior arises primarily from the dependence of the gaugino-Higgsino components $X_{n0}$ of the LSP on the gaugino mass nonuniversality. Thus $X_{n0}$ are sensitive to the gaugino nonuniversality through their dependence on $\tilde{m}_1$, $\tilde{m}_2$ and $\mu$. In Fig.2 we exhibit the dependence of $X_{n0}$ on $C_{24}$ for some typical input values. The quantity $\sigma_{\chi p}(scalar)$ depends on the direct product of the gaugino and Higgsino components of $\chi$. Specifically, $\sigma_{\chi p}(scalar)$ vanishes if $\chi$ is a pure Bino. From Fig.2 we see that negative values of $c_{24}$ increase the Higgsino components and hence increase the neutralino-quark scattering and lead to an enhancement of $\sigma_{\chi p}(scalar)$ while the opposite situation is realized for positive values of $c_{24}$. This is what is found in Fig.3 where we give a plot of the maximum and the minimum of $\sigma_{\chi p}(scalar)$ for the cases $c_{24} = -0.08$, $c_{24} = 0$ and $c_{24} = 0.1$ when the soft SUSY breaking parameters are varied over the assumed naturalness range as in Fig.1. The analysis shows that for $c_{24} = -0.1$ an enhancement of $\sigma_{\chi p}(scalar)$ by as much as a factor of 5 can occur as a result of the gaugino mass nonuniversality and the allowed range of $m_\chi$ is also increased in this case beyond the range allowed in the universal SUGRA case.

In Fig.4 we give an analysis of the maximum and the minimum of $\sigma_{\chi p}(scalar)$ for the $1 + 75$ case for three different values of $c_{75}$, i.e., $c_{75} = -0.06$, $c_{75} = 0$ and $c_{75} = 0.04$. As for the $1 + 24$ case, $\sigma_{\chi p}(scalar)$ typically increases for negative values of $c_{75}$ and decreases for positive values of $c_{75}$. Again this can be understood by analysing the gaugino-Higgsino components of the LSP as a function of $c_{75}$. Thus here as in the $1 + 24$ case one finds that the Higgsino components of the LSP increase as $c_{75}$ decreases and decrease as $c_{75}$ increases. Because of this there is an enhancement of $\sigma_{\chi p}(scalar)$ for $c_{75} < 0$. A comparison of $c_{75} = 0$ and $c_{75} = -0.06$
cases in Fig. 4 shows that an enhancement of $\sigma_{\chi_p}(\text{scalar})$ up to a factor of 5 or more occurs in this case. As in the case of $1 + 24$ here also one finds that the allowed range of $m_\chi$ consistent with the constraints is extended beyond the values allowed in the universal SUGRA case.

In Fig. 5 we give an analysis of the maximum and the minimum of the $\sigma_{\chi_p}(\text{scalar})$ for the $1 + 200$ case when $c_{200} = -0.08$, $c_{200} = 0$ and $c_{200} = 0.1$. In this case the dependence of $\sigma_{\chi_p}(\text{scalar})$ on $c_{200}$ is opposite to that one has in the previous cases, i.e., the $1 + 24$ case and the $1 + 75$ case. Here it is for positive values of $c_{200}$ that $\sigma_{\chi_p}(\text{scalar})$ increases and it is for negative values of $c_{200}$ that $\sigma_{\chi_p}(\text{scalar})$ decreases. The origin of this reversal lies in $n_i^r$ (see Table 1) and can be understood from the dependence of $X_{n0}$ on $c_{200}$. The above implies that the Bino component of $\chi$ decreases and the Higgsino components increase for $c_{200} > 0$ while the opposite situation occurs when $c_{200} < 0$. This dependence implies that $\sigma_{\chi_p}(\text{scalar})$ should increase for $c_{200} > 0$ and decrease for $c_{200} < 0$ which is what is observed in Fig. 5. In this case one finds that $\sigma_{\chi_p}(\text{scalar})$ can increase up to a factor of 10 or more because of nonuniversality. The analysis also shows that as in the case of $1 + 24$ and $1 + 75$ the allowed range of the LSP mass is extended beyond what is allowed in the universal SUGRA case.

One can gain a deeper understanding of the dependence of $X_{n0}$ on $c_r$ and hence a deeper understanding of the dependence of $\sigma_{\chi_p}(\text{scalar})$ on $c_r$ from studying the dependence of the Higgs mixing parameter $\mu$ on $c_r$. To appreciate why $\mu$ is such an important parameter in this discussion it is useful to look at large $\mu$, i.e., the case $\mu^2 >> M_Z^2$. In this limit one finds that the LSP eigenvector is given by

\[
X_{10} \simeq 1 - \left(\frac{M_Z^2}{2\mu^2}\right) \sin^2 \theta_W, \quad X_{20} \simeq -\left(\frac{M_Z^2}{2}(\tilde{m}_2 - \tilde{m}_1)\mu\right) \sin 2\theta_W \sin 2\beta, \\
X_{30} \simeq \left(M_Z/\mu\right) \sin \theta_W \sin \beta, \quad X_{40} \simeq -\left(M_Z/\mu\right) \sin \theta_W \cos \beta, \quad (32)
\]

Eq.32 shows that in the large $\mu$ limit $\chi$ is mostly a Bino and the corrections to the pure Bino limit are proportional to $\left(M_Z^2/\mu^2\right)$ while the Higgsino components are proportional to $\left(M_Z/\mu\right)$. As already pointed out the $\sigma_{\chi_p}(\text{scalar})$ depends on the interference of the gaugino and Higgsino components of the LSP, i.e., $X_{i0} \times X_{a0}$ ($i=1,2; \alpha = 3,4$). Clearly then as $|\mu|$ increases we go more deeply into the pure Bino region reducing $\sigma_{\chi_p}(\text{scalar})$. Likewise as $|\mu|$ decreases $\chi$ develops larger Higgsino components $X_{a0}(\alpha = 3,4)$, even though it is still dominantly a Bino, $\sigma_{\chi_p}(\text{scalar})$ decreases. Thus to gain an insight on the effect of $c_r$ on $X_{n0}$ and hence on the effect of $c_r$ on $\sigma_{\chi_p}(\text{scalar})$ we need to understand how $c_r$ affects $\mu$. We address this topic below.
In SUGRA models $\mu^2$ is determined via the breaking of the electro-weak symmetry and thus depends on the gaugino mass nonuniversality through the Higgs mass parameters (see Eqs.39 and 43 in Appendix A and Eq.50 in Appendix B).

We can understand the effect of nonuniversality on $\mu$ analytically by expanding $\mu$ for the nonuniversal case around the universal value using $c_r$ as an expansion parameter

$$\tilde{\mu}^2 = \mu_0^2 + \sum_r \frac{\partial \mu^2}{\partial c_r} c_r + O(c_r^2)$$ (33)

where $\mu_0$ is the value of $\mu$ for the universal case. Using the analysis of Appendix B the pattern of breaking in the Higgs structure of $24,75,200$ shows that

$$\frac{\partial \mu^2_{24}}{\partial c_{24}} > 0, \quad \frac{\partial \mu^2_{75}}{\partial c_{75}} > 0, \quad \frac{\partial \mu^2_{200}}{\partial c_{200}} < 0$$ (34)

Thus in the neighborhood of $c_r = 0$ a negative value of $c_{24}$ gives a smaller value of $|\mu|$ leading to larger Higgsino components in $\chi$ and hence a larger $\sigma_{\chi p}(\text{scalar})$ as is observed in the numerical analysis. A similar situation holds for the nonuniversality effects from $c_{75}$. However, for the nonuniversality effects from $c_{200}$ an opposite situation holds because of the opposite sign of the derivative term as given by Eq.34. More generally one finds the same behavior in a larger $c_r$ domain, i.e., $\mu^2_{24} < \mu_0^2$ for $c_{24} < 0$, $\mu^2_{75} < \mu_0^2$ for $c_{75} < 0$, and $\mu^2_{200} < \mu_0^2$ for $c_{200} > 0$ and observations similar to those valid for small $c_r$ also apply here. These results imply that gaugino nonuniversality which makes $\mu$ small produces a deviation of the LSP from the approximate Bino limit in a direction which leads to a larger value of $\sigma_{\chi p}(\text{scalar})$.

Nonuniversality of the gauginos masses also has implications for naturalness. One convenient definition of naturalness is via the fine tuning parameter $\Phi$ introduced in Ref.[39] defined by $\Phi = \frac{1}{4} + \frac{\mu^2}{M_Z^2}$. Using this definition one can easily compare values of fine tuning for the universal and nonuniversal cases. One finds $\Phi_{24} < \Phi_0$ ($c_{24} < 0$), $\Phi_{75} < \Phi_0$ ($c_{75} < 0$), $\Phi_{200} < \Phi_0$ ($c_{200} > 0$), where $\Phi_{24}$ is $\Phi$ for the case $1 + 24$ etc, and $\Phi_0$ is $\Phi$ for the universal case. Since the correction to $\mu^2$ is negative for the case of larger Higgsino components one finds that the deviation from the approximate Bino limit is in the direction of a smaller value of $\Phi$ and towards the direction of greater naturalness relative to the universal case. Thus a smaller $\mu$ leads to a larger $\sigma_{\chi p}$ and a larger detection rate and also makes the model more natural by making $\Phi$ small. In this sense the more natural the SUSY model the larger is the detection rate.
4 Dark matter in string/brane models

One of the main hurdles in the analysis of SUSY phenomenology based on string models is that there is as yet not a full understanding of the breaking of supersymmetry here. However, there do exist efficient ways to parametrize SUSY breaking and one such parametrization is

\[ F^S = \sqrt{3}m_3^2(S + S^*) \sin \theta e^{-i\gamma_S}, \quad F^i = \sqrt{3}m_3^2(T + T^*) \cos \theta \Theta_i e^{-i\gamma_i} \tag{35} \]

where \( F^S \) is the dilation VEV, \( F^i \) are the moduli VEVs, \( \theta \) (\( \Theta_i \)) parametrizes the Goldstino direction in the S (\( T_i \)) field space and \( \gamma_S (\gamma_i) \) is the phase. The \( \Theta_i \) obey the constraint \( \Theta_1^2 + \Theta_2^2 + \Theta_3^2 = 1 \). We begin with an example of the O-II string model with the soft SUSY breaking sector parametrized by

\[ \tilde{m}_i = \sqrt{3}m_3^2(\sin \theta e^{-i\alpha_S} - \gamma_i \epsilon \cos \theta e^{-i\alpha_T}) \]

\[ m_0^2 = \epsilon'(-\delta_{GS})m_3^2, \quad A_0 = -\sqrt{3}m_3^2 \sin \theta e^{-i\alpha_S} \tag{36} \]

Here \( \gamma_1 = -\frac{33}{5} + \delta_{GS}, \gamma_2 = -1 + \delta_{GS}, \gamma_3 = 3 + \delta_{GS} \) where \( \delta_{GS} \) is the Greene-Schwarz parameter which is fixed by the constraint of anomaly cancellation in a given orbifold model. Further, as in the GUT analyses we treat \( \mu \) as an independent parameter. The phenomenology of this model has been discussed in Ref. [41] and the EDM constraints in Refs. [42, 43]. However, in the analysis below we do not impose the accelerator constraints [41] and as in the analysis for GUT models we do not assume CP violation and thus set the CP phases to zero.

The structure of the soft SUSY parameters for the model discussed above shows that the nonuniversality of the gaugino masses is controlled by several parameters in this case: \( \delta_{GS}, \theta \) and \( \epsilon \) which play a role similar to the role played by the parameters \( c_r \) in the case of the GUT models. The presence of several parameters leads to many different possibilities for generating gaugino mass nonuniversality. In addition, there is a new feature in this string model, not present in GUT models, in that the universal scalar (mass)\(^2\) at the unification scale, i.e. \( m_0^2 \), is dependent on \( \delta_{GS} \) which therefore correlates the universal scalar mass with the gaugino mass nonuniversality. An analysis of the maximum and the minimum curves for \( \sigma_{\chi p}(scalar) \) as a function of \( m_\chi \) is given in Fig.6 when the parameters in the model are varied over a range of allowed values. A comparison with Fig.1 shows
that $\sigma_{\chi p}(\text{scalar})$ can be larger than for the universal SUGRA case by a factor of as much as 10. One also finds that the range of the neutralino mass extends to about 575 GeV significantly beyond what one finds in the universal SUGRA case even without inclusion of the coannihilation effects.

We discuss next dark matter for a class of D brane models. Over the recent past there has been considerable interest in the study of Type IIB orientifolds and their compactifications\[^{14, 13, 10}\]. We consider here models with compactifications on a six torus of the type $T^6 = T^2 \times T^2 \times T^2$. In models of this type one has a set of 9 branes, $7_i$ ($i=1,2,3$) branes, $5_i$ branes and 3 branes. This set is further constrained by the requirement of $N=1$ supersymmetry which requires that one has either 9 branes and $5_i$ branes, or $7_i$ branes and 3 branes. Model building on branes allows an additional flexibility in that one can associate different parts of the Standard Model gauge group with different branes. In Ref.\[^{12, 10}\] a brane model using two five branes $5_1$ and $5_2$ was investigated while in Ref.\[^{13}\] models using 9 brane and $5_i$ brane were investigated. We pursue here the implications of the latter possibility\[^{13}\]. In one of the models of Ref.\[^{13}\] the Standard Model gauge group is associated with the branes in the following way: the $SU(3)_C \times U(1)_Y$ is associated with the 9 brane while $SU(2)_L$ is associated with the $5_1$ brane. Further, it is assumed that the $SU(2)_R$ singlets are associated with the 9 brane while the $SU(2)_L$ doublets are associated with the intersection of 9 brane and $5_1$ brane. The soft SUSY breaking sector of this $9-5_1$ brane model is then given as follows: The gaugino masses $\tilde{m}_i$ ($i=1,2,3$) corresponding to the gauge group $SU(3)_C$, $SU(2)_L$ and $U(1)$ are parametrized by\[^{13}\]

$$\tilde{m}_1 = \sqrt{3} m_3 \sin \theta e^{-i \gamma_3} = \tilde{m}_3 = -A_0, \quad \tilde{m}_2 = \sqrt{3} m_3 \cos \theta \Theta_1 e^{-i \gamma_1}$$

while the $SU(2)_L$ singlets are parametrized by $m_9$ and $SU(2)_L$ doublets are parametrized by $m_{95_1}$ where\[^{13}\]

$$m_9^2 = m_{95_1}^2 = m_3^2 (1 - 3 \cos^2 \theta \Theta_1^2), \quad m_{95_1}^2 = m_3^2 \left(1 - \frac{3}{2} \cos^2 \theta (1 - \Theta_1^2)\right).$$

Here the $\theta$ and $\Theta_1$ are the directions of the Goldstino in the dilaton and the moduli VEV space as discussed earlier. To avoid generating tachyons in the theory one needs to impose the constraint $\cos^2 \theta \Theta_1^2 < 1$. A second version of this model was also discussed in Ref.\[^{13}\]. Here one associates $SU(3)_C \times U(1)_Y$ with the $5_1$ brane while the $SU(2)_L$ with the 9 brane, and assumes that the $SU(2)_R$ singlets were associated with the $5_1$ brane while the $SU(2)_L$ doublets are associated with the
intersection of 9 brane and 5$_1$ brane as before. The soft SUSY breaking sector of this model can be gotten from the model discussed above by the interchange $\cos \theta \Theta_1 \leftrightarrow \sin \theta$. In the analysis of this paper we focus on the version of the model given by Eqs.37 and 38. As usual we assume that the parameter $\mu$ is free and we determine it via the constraint of the radiative breaking of the electro-weak symmetry. Again as in other cases we have considered we set the CP phases to zero. Eqs.37 and 38 show that one has nonuniversality at the unification scale both in the scalar sector as well as in the gaugino sector. The limit of universal scalar mass corresponds to $\Theta_1 = 1/\sqrt{3}$. Our focus in this paper is on the nonuniversality in the gaugino sector, and so for the numerical analysis we set $\Theta_1 = 1/\sqrt{3}$. In this case the numerical analysis shows that the allowed neutralino mass range extends up to about 650 GeV.

5 Gaugino mass nonuniversality and LSP mass range

In SUGRA models with universal boundary conditions at the GUT scale, the allowed range of the LSP typically does not exceed 200 GeV and with imposition of additional constraints it is often significantly less. As discussed in Secs.3 and 4 in the presence of gaugino mass nonuniversality one finds that the allowed LSP range is increased. For the $1 + 24$ case the allowed LSP range extends to about 220 GeV for the case $c_{24} = -0.1$. For the $1 + 75$ case one finds that $c_{75} = -0.06$ gives an LSP range which extends to 250 GeV, while for the $1 + 200$ case with $c_{200} = 0.1$ the allowed LSP range extends to 375 GeV. A similar situation holds for the string/brane models. Here one finds that the allowed LSP range can extend up to 600 GeV. These extended regions arise even without inclusion of coannihilation effects which is known to extend the allowed regions also up to about 700 GeV[26]. In the regions of the parameters space considered the effects of coannihilations would in fact be negligible since we are considering only those configurations for which $\Delta_i > 0.1$. The specific mechanism by which the neutralino mass range is extended is also different than in the case of coannihilation. Thus for the case of coannihilation the increase in the allowed LSP range occurs as a consequence of a coupled channel effect while in the case of nonuniversalities the extension of the allowed region of the LSP arises due to a significant increase in the value of $J$ for certain ranges of nonuniversalities. Thus one may expand $J$ as the sum over the
final state channels in the $\chi\chi$ annihilation so that $J = J(f\bar{f}) + J(WW) + J(ZZ) + J(Zh^0) + J(ZH^0) + etc$. The region of the nonuniversality parameter space which leads to an enhancement of $\sigma_{\chi p}$ can also lead to an enhancement of the cross-sections for the $\chi\chi$ annihilation into the final states $W^+W^-, ZZ, Zh, ZH$ etc and hence to an increase in $J$ which leads to a decrease in the relic density down to permissible limits consistent with constraints. Thus regions of the parameter space which would otherwise be excluded are now included when the gaugino mass nonuniversalities are included.

6 Conclusion

In this paper we have analyzed the effects of the nonuniversality of the gaugino masses on neutralino dark matter in SUGRA, string and D brane models under the constraint of R parity invariance. It is found that nonuniversality effects can enhance the $\chi - p$ cross-section for scalar interactions by as much as a factor of 10. We also carried out an analysis of the uncertainties in the numerical determination of $f_i^p$ (i=u,d,s) and find that with the current state of uncertainties the $\chi - p$ cross-section cannot be pinned down to better than a factor of about 5. Our analysis of the gaugino mass nonuniversality also exhibits another important phenomena, i.e., that the allowed range of the neutralino mass can be extended up to about 600 GeV even without inclusion of the coannihilation effects. The effects of coannihilation were not considered and inclusion of these effects may further increase the allowed neutralino mass range. The extended LSP mass range should be of interest in the experimental searches for dark matter in the current dark matter detectors [30, 31, 47] and in the design of new dark matter detectors which are at the planning stage [48].

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Appendix A: Effects of gaugino mass nonuniversality on sparticle masses

In this appendix we give analytic solutions to the one loop renormalization group equations including the effect of gaugino mass nonuniversality. The analytic formulae for these with universal boundary conditions were given in Ref. [19] and for nonuniversality in the scalar mass sector in Ref. [19]. Here we limit ourselves to the gaugino mass nonuniversality. These formulae are found useful in gaining an analytic understanding of the nonuniversality effects. The one loop RG formulae
are given in several papers (see, e.g. Ref. [49]) and we do not reproduce them here. Rather we discuss the solutions under the boundary conditions where \( m_0 \) is the universal scalar mass, \( \tilde{m}_i(0) \) (i=1,2,3) are the gaugino masses for the gauge group sectors \( U(1), SU(2), SU(3) \) and \( A_0 \) is the universal trilinear coupling all taken at the unification scale. The Higgs mass parameters, the trilinear couplings, and the squark and slepton masses at the electro-weak scale are all sensitive to the effect of gaugino mass nonuniversality. The simplest case is that of the mass parameter \( m^2_{H_1} \) for the \( H_1 \) Higgs which couples to the down quark. Here one finds

\[
m^2_{H_1} = m_0^2 + \frac{3}{2} f_2(t) + \frac{3}{10} f_1(t) m_2^2
\]

(39)

where

\[
f_i(t) = \frac{1}{\beta_i} \left( 1 - \frac{1}{(1 + \beta_i t)^2} \right) \left( \frac{\tilde{\alpha}_i(0)}{\bar{\alpha}_G} \right) \left( \frac{\tilde{m}_i(0)}{m_{1/2}} \right)^2
\]

(40)

Here \( t = \ln(M_X^2/Q^2) \), \( \beta_i = (b_i/4\pi)\tilde{\alpha}_i(0) \) where \( b_i = (33/5, 1, -3) \) for \( U(1), SU(2), \) and \( SU(3) \), and \( \tilde{\alpha}_i(0) = \alpha_i/4\pi \). The \( f_i(t) \) contain the nonuniversality effects. The evolution of the mass parameter for the Higgs \( H_2 \) involves the evolution of the trilinear coupling in the stop channel at the electro-weak scale and this coupling is given by

\[
A_t(t) = \frac{A_0}{1 + 6Y_0 F} + m_{1/2} \left( \tilde{H}_2 - \frac{6Y_0 H_3}{1 + 6Y_0 F} \right)
\]

(41)

where

\[
\tilde{H}_2 = \tilde{\alpha}_G \left( \frac{16}{3} \tilde{h}_3 + 3 \tilde{h}_2 + \frac{13}{15} \tilde{h}_1 \right), \quad \tilde{H}_3 = \int_0^t E(t) \tilde{H}_2(t)
\]

\[
\tilde{h}_i = t \frac{\tilde{\alpha}_i(0)}{\bar{\alpha}_G} \left( \frac{\tilde{m}_i(0)}{m_{1/2}} \right)
\]

(42)

where \( Y_0 \) is the top Yukawa coupling at the GUT scale and the functions \( E \) and \( F \) are as defined in Ref. [49]. We note that the first term on the right hand side of Eq.(40) which arises purely from the top Yukawa coupling evolution is unaffected by nonuniversality while the second term in affected through the modification of \( \tilde{h}_i \). For the mass parameter \( m_{H_2} \) for Higgs \( H_2 \) that couples with the top one finds

\[
m^2_{H_2} = \frac{m_2^2 \bar{e}(t) + A_0 m_0 m_{1/2} \tilde{f}(t) + m_0^2 (h(t) - k(t) A_0^2)}{m_{1/2}^2}
\]

(43)

where the functions \( h(t) \) and \( k(t) \) are unaffected by nonuniversality and are as given in Ref. [49] while the functions \( \bar{e} \) and \( \tilde{f} \) are modified due to nonuniversality.
\[\tilde{e}(t) = \frac{3}{2} \left[ \tilde{G}_1 + Y_0 \tilde{G}_2 \right] \frac{(\tilde{H}_2 + 6Y_0 \tilde{H}_4)}{D(t)^2} + \tilde{H}_8 \]  

(44)

and

\[\tilde{f}(t) = -\frac{6Y_0 \tilde{H}_3(t)}{D(t)^2}; \quad D(t) = (1 + 6Y_0 F(t)) \]  

(45)

Here the various tilde functions containing the nonuniversality are defined below

\[\tilde{G}_1(t) = \tilde{F}_2(t) - \frac{1}{3} (\tilde{H}_2)^2\]

\[\tilde{G}_2(t) = 6\tilde{F}_3(t) - \tilde{F}_4(t) - 4\tilde{H}_2(t)\tilde{H}_4(t) + 2F(\tilde{H}_2)^2 - 2\tilde{H}_6(t)\]

\[\tilde{F}_3(t) = F(t)\tilde{F}_2(t) - \int_0^t dt' E(t')\tilde{F}_2(t'), \quad \tilde{F}_4(t) = \int_0^t dt' E(t')\tilde{H}_5(t')\]

\[\tilde{H}_4(t) = F(t)\tilde{H}_2(t) - \tilde{H}_3(t), \quad \tilde{H}_5(t) = \tilde{\alpha}_G(-\frac{16}{3} \tilde{f}_3(t) + 6\tilde{f}_2(t) - 22 \tilde{f}_1)\]

\[\tilde{H}_6(t) = \int_0^t dt' (\tilde{H}_2)^2 E(t'), \quad \tilde{H}_8(t) = \tilde{\alpha}_G(-\frac{8}{3} \tilde{f}_3(t) + \tilde{f}_2(t) - \frac{1}{3} \tilde{f}_1)\]  

(46)

The squark and slepton masses are also affected by nonuniversality. For the up squarks in the first two generations one finds

\[m_{u_{\tilde{u}_L}}^2(t) = m_0^2 + m_{u_R}^2 + \tilde{\alpha}_G[\frac{8}{3} f_3 + 3 \tilde{f}_2 + \frac{1}{30} \tilde{f}_1]m_2^2 + (\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W) M_Z^2 \cos 2\beta\]

\[m_{u_{\tilde{u}_R}}^2(t) = m_0^2 + m_{u_R}^2 + \tilde{\alpha}_G[\frac{8}{3} f_3 + \frac{8}{15} \tilde{f}_1]m_2^2 + \frac{2}{3} \sin^2 \theta_W M_Z^2 \cos 2\beta\]  

(47)

The analysis of the first two generations of the down quarks and of the sleptons is similar. Finally we discuss the third generation squarks. Here one finds

\[m_{b_{\tilde{b}_L}}^2(t) = \frac{1}{2} m_0^2 + m_b^2 + \frac{1}{2} m_{b_R}^2 + \tilde{\alpha}_G[\frac{8}{3} f_3 + \frac{1}{15} \tilde{f}_1]m_2^2 + (-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W) M_Z^2 \cos 2\beta\]

\[m_{b_{\tilde{b}_R}}^2(t) = m_0^2 + m_b^2 + \tilde{\alpha}_G[\frac{8}{3} f_3 + \frac{2}{15} \tilde{f}_1]m_2^2 - \frac{1}{3} \sin^2 \theta_W M_Z^2 \cos 2\beta\]

\[m_{\tilde{t}_{\tilde{L}}(t)}^2 = m_{\tilde{t}}^2 + m_1^2 + (\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W) M_Z^2 \cos 2\beta\]

\[m_{\tilde{t}_{\tilde{R}}(t)}^2 = m_{\tilde{t}}^2 + m_1^2 + \frac{2}{3} \sin^2 \theta_W M_Z^2 \cos 2\beta\]  

(48)

where \(m_{\tilde{t}}^2\) and \(m_{\tilde{Q}}^2\) are defined by

\[m_{\tilde{u}}^2 = \frac{1}{3} m_0^2 + \frac{2}{3} \tilde{f}_A m_{\tilde{u}}^2 - \frac{2}{3} k A_0^2 + \frac{2}{3} h m_0^2 + [\frac{1}{3} \tilde{e} + \tilde{\alpha}_G(\frac{8}{3} f_3 - \tilde{f}_2 + \frac{1}{3} \tilde{f}_1)]\]

\[m_{\tilde{Q}}^2 = \frac{2}{3} m_0^2 + \frac{1}{3} \tilde{f}_A m_{\tilde{Q}}^2 - \frac{1}{3} k A_0^2 + \frac{1}{3} h m_0^2 + [\frac{1}{3} \tilde{e} + \tilde{\alpha}_G(\frac{8}{3} f_3 + \tilde{f}_2 - \frac{1}{15} \tilde{f}_1)]\]  

(49)
All the sparticle mass relations limit to the universal case when we set \( \alpha_i(0) \bar{m}_i(0)/\alpha_G m_{\tilde{\chi}} = 1 \).

**Appendix B: nonuniversality effects on \( \mu \)**

Since \( \mu \) is determined via the constraint of electro-weak symmetry breaking it is sensitive to the gaugino mass nonuniversality. Thus one has

\[
\mu^2 = (m_{H_1}^2 - m_{H_2}^2 \tan \beta^2)(\tan \beta^2 - 1)^{-1} - \frac{1}{2} M_Z^2 + \Delta \mu^2 \tag{50}
\]

where \( \Delta \mu^2 \) is the loop correction. From the above one finds

\[
\frac{\partial \mu^2}{\partial c_r} = (t^2 - 1)(m_{\tilde\tau}^2 g'_r - t^2(m_{\tilde\tau}^2 e'_r + A_0 m_0 m_{\tilde\tau} f'_r)) + \frac{\partial \Delta \mu^2}{\partial c_r} \tag{51}
\]

Here \( g'_r = \frac{\partial g}{\partial c_r} \) where \( \tilde{g} = \tilde{\alpha}_G(\frac{2}{3} \tilde{f}_2 + \frac{4}{10} \tilde{f}_1) \), and \( f'_r = \frac{\partial f}{\partial c_r} \). For large \( \tan \beta \) Eq.(51) reduces down to

\[
\frac{\partial \mu^2}{\partial c_r} = -(m_{\tilde\tau}^2 e'_r + A_0 m_0 m_{\tilde\tau} f'_r) + \frac{\partial \Delta \mu^2}{\partial c_r} \tag{52}
\]

These results lead to Eq.(34).

**Tables:**

| SU(5) rep | \( n_1^r \) | \( n_2^r \) | \( n_3^r \) |
|----------|-------|-------|-------|
| 1        | 1     | 1     | 1     |
| 24       | -1    | -3    | 2     |
| 75       | -5    | 3     | 1     |
| 200      | 10    | 2     | 1     |

**Table 2: Uncertainties in \( \sigma_{\pi N}, \sigma_0 \) and \( x \)**

| Ref. | \( \sigma_{\pi N} \) (MeV) | Ref. | \( \sigma_0 \) (MeV) | Ref. | \( \sigma_0/\sigma_{\pi N} \) | \( x \) |
|------|-----------------|------|-----------------|------|-----------------|------|
| [33] | 45 ± 8          | [34] | 35 ± 5          |       | 0.74 ± 0.25     |      |
| [35] | 48 ± 10         | [36] | 36 ± 7          |       | 0.64 ± 0.03     |      |
| [37] | 50 ± 10         | [38] | 35.5 ± 6        |       | 0.57 ± 0.1      |      |
| [28] | 49 ± 8          |       | \( \bar{x} \)   | 0.67 ± 0.18     |      |
| \( \bar{\sigma}_{\pi N} \) | 48 ± 9 |      |                 |      |                  |      |

**Figure captions**

Fig.1: Exhibition of the dependence of \( \sigma_{\chi p}(scalar) \) on the uncertainties in \( f_u^p, f_d^p \) and \( f_s^p \) for the universal case. The plots are for three sets of \( f_i^p \) discussed in Sec.3. The parameter space spanned is as discussed in the text.
Fig.2: Plot of gaugino-Higgsino components $X_{n0}$ ($n=1,2,3,4$) as a function of $c_{24}$ for the input data $m_0 = 51$ GeV, $m_\chi = 70$ GeV, $\tan \beta = 10$, $A_t/m_0 = -7$ where $X_{20} = -|X_{20}|$ and $X_{40} = -|X_{40}|$.

Fig.3: Plot of the maximum and the minimum curves $\sigma_{\chi p}(scalar)$ as a function of $m_\chi$ for the case when one considers admixtures of $1 + 24$ representations with $c_{24} = -0.1$ (dashed), $c_{24} = 0$ (solid) and $c_{24} = 0.08$ (dotted) when the other parameters are varied over the assumed naturalness range as discussed in the text.

Fig.4: Plot of the maximum and the minimum curves $\sigma_{\chi p}(scalar)$ as a function of $m_\chi$ for the case when one considers admixtures of $1 + 75$ representations with $c_{75} = -0.06$ (dashed), $c_{75} = 0$ (solid) and $c_{75} = 0.04$ (dotted) when the other parameters are varied over the assumed naturalness range as discussed in the text.

Fig.5: Plot of the maximum and the minimum curves $\sigma_{\chi p}(scalar)$ as a function of $m_\chi$ for the case when one considers admixtures of $1 + 200$ representations with $c_{200} = 0.1$ (dashed), $c_{200} = 0$ (solid) and $c_{200} = -0.08$ (dotted) when the other parameters are varied over the assumed naturalness range as discussed in the text.

Fig.6: Plot of the maximum and the minimum curves for $\sigma_{\chi p}(scalar)$ as a function of $m_\chi$ for the heterotic string model O-II. The range of parameters consists of $\epsilon, \epsilon'$ in the range 0.0025-0.01, $\theta$ in the range 0.1-1.6, $\delta_{GS}$ in the range -1 to -10, $m_{3/2}$ in the range up to 2 TeV, and values of $\tan \beta$ range up to 25.

Fig.7: Plot of the maximum and the minimum curves for $\sigma_{\chi p}(scalar)$ as a function of $m_\chi$ for the $9 - 5_1$ D brane model. The range of parameters consists of $\theta$ in the range $0.1 - 1.6$, $m_{3/2}$ in the range up to 2 TeV, and values of $\tan \beta$ range up to 25.
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$\sigma_{zp} (\text{scalar}) \text{ (cm}^2\text{)}$

- Set I
- Set II
- Set III

$m_\chi (\text{GeV})$
Gauge-Higgsino components of LSP: $x_{n0}$
\begin{align*}
10^{-45} &\leq \sigma_{zp} (\text{scalar}) (\text{cm}^2) \\
10^{-44} &< \sigma_{zp} (\text{scalar}) (\text{cm}^2) \\
10^{-43} &< \sigma_{zp} (\text{scalar}) (\text{cm}^2) \\
10^{-42} &< \sigma_{zp} (\text{scalar}) (\text{cm}^2) \\
10^{-41} &< \sigma_{zp} (\text{scalar}) (\text{cm}^2) \\
10^{-40} &< \sigma_{zp} (\text{scalar}) (\text{cm}^2)
\end{align*}
$\sigma_{\chi p}$ (scalar) (cm$^2$)

$m_\chi$ (GeV)

DAMA
CDMS
CDMS (future)
GENIUS

$c_{200} = 0$
$c_{200} = 0.1$
$c_{200} = -0.08$
The plot shows the variation of $\sigma_{\chi p}$ (scalar) in cm$^2$ as a function of $m_\chi$ (GeV). The different lines represent various experimental results and simulations:
- DAMA
- GENIUS
- CDMS (future)
- CDMS

The heterotic string model is also indicated on the plot.
$\sigma_{\chi p} (\text{scalar}) \ (\text{cm}^2)$

$m_\chi \ (\text{GeV})$

DAMA
CDMS
CDMS (future)

9-5, D Brane model