New path integral representation for Hubbard model: I. Supercoherent state

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Abstract

The Hubbard model is used to study an electronic system. In this paper we present the new path integral representation for Hubbard model. We have constructed the new supercoherent state which appears from a set of eigenfunctions of atomic limit of strongly correlated systems. Exact calculation of nonlinear representation of a supergroup has been carried. This group defines the transformation of atomic base. The general formalism we elaborate for Hubbard model is the one widely used in the gauge field theory of the nonlinear representation of a superconformal group.
1 Introduction

The Hubbard model was originally constructed to describe a metal-insulator transition for spin-dependent fermions in a simple way \cite{1, 2, 3}.

Today this model is still remain the main workspace for investigation of the strong electronic correlation. There exist many approaches to this model for describing many electron system: the band limit approximation for the weak interaction between electrons and the atomic limit for electrons with the strong coulomb repulsion. We start the series of papers in which we are intending to elaborate the new approach to the Hubbard model.

We will present a new path integral approach to the strong interaction regime. The main ingredients for us is the usage of a supercoherent state acting upon an atomic base and the work with effective functional for an electronic system. We will develop the procedure of geometric quantization for strongly correlated systems of electrons. Let us make the brief sketch of the program firstly developed in the series of works of one of the authors \cite{4}. One of the main distinction of this approach to the Hubbard model is that we treat the local supergroup of the local space-time as the main object of our theory. This supergroup generates the transformation of the space-time coordinates which assign the arguments for any function describing this system. This representation of a supergroup in the superspinor space has to contain a Lorents or an SO(4) subgroup equal to the even subalgebra of the Hubbard operators. Odd Hubbard operators produce some superextension of the Lorents group into some supergroup. This dynamic supergroup is given by local dynamic superfields describing the local degrees of freedom of the strongly correlated electron system. We shall introduce the supercoherent state depending on generalized angles equal to the bosonic and fermionic fields of the system. Parametrised by x,y,z,t space-time manifold which determines the arguments of wave function defines us the spinor and superspinor bundle. This superbundle is determined by the supercoherent state.

There are two kinds of the gauge fields in superspinor bundle: one sort of fields is the composite fields equal to the quadratic combination of odd grassmann fields and other sort are nonlinear fields determining the local coordinates frame of the four-dimensional space-time. In general, the local superspinor bundle defines the nonlinear representation of a superconformal group as a maximal group of 4-dimentional space-time of interacting fermionic system.

As the first step of quantization of electronic system with strong coulomb repulsion we perform the reformulation of Hubbard model into the atomic limit formalism. This approach is well known in Mott-Hubbard insulators theory. We want to point out that our approach includes all elements of geometric quantisation \cite{5}: for example possess the algebra of 4-dimensional rotations (Lorents) group, Cartan differencial one-forms which give us the lagrangian of system, the nonlinear representation of underlying supergroup as a ground for the supercoherent state.

In this first paper of this series we will fulfill the exact calculation of the nonlinear representation of those supergroup generators of which appear in operator approach of atomic limit of the Hubbard model. This gives us the possibility to introduce the supercoherent state for the strongly correlated electron system and as a result to obtain the effective action in a future paper.

Let us give the brief description of the calculation method for the finding representation.
of dynamic supergroup in strongly interacting models. Our task is to find in this model such
group structure which could help us to describe the specificity of strong correlation. We take
the following construction as a base:

1) we will collect space coordinates together with the time coordinate and will consider
some curved space-time as a base in which Lorentz subalgebra of superconformal group act
in the spinor base on dynamical fields.

2) full bases of electronic operators gives us the supergroup which is parametrised by 8
dynamical fermionic fields: this fields comprise the conformal spinor.

3) the superspinor representation of a supergroup gives us the supercoherent state de-
scribed by the nonlinear function over odd grassmanian fields. This function characterises
the local properties of the strongly correlated system.

2 Atomic description of Hubbard model

We consider the Hubbard model:

\[ H = -W \sum_{ij\sigma} \alpha_{\sigma,i}^+ \alpha_{\sigma,j} + U \sum_{i,\sigma} n_{\sigma,i} n_{-\sigma,i} + \mu \sum_{\sigma,i} n_{\sigma,i}, \]  

(1)

here \( \alpha_{\sigma,i}^+ \alpha_{\sigma,j} \) - electron creation and annihilation operators. \( n_{\sigma,i} \) - electron density
operator W, U, - band width, one-site electron repulsion and chemical potential.

At first we represent the Hubbard model in atomic bases which determine the atomic
limit. This limit appear as a result of the following procedure. A zero approximation of the
atomic limit is described by one-site repulsion term:

\[ U \sum_{i,\sigma} n_{\sigma,i} n_{-\sigma,i} + \mu \sum_{\sigma,i} n_{\sigma,i}. \]

This hamiltonian can be diagonalized by the following one-site atomic eigenfunction:

\[ |0 >;|+ > = \alpha_{+}^+ |0 >;|> - = \alpha_{-}^+ |0 >;|2 > = \alpha_{+}^+ \alpha_{-}^+ |0 >. \]  

(2)

This bases gives us the fundamental representation of some supergroup in the space of
dimension (2, 2). Point out that the states \(|+ >;|> - = \alpha_{-}^+ \alpha_{+}^+ |0 >;|2 > = \alpha_{+}^+ \alpha_{-}^+ |0 >;|2 > - on even order of the grassmann fields). All operators in this bases will
be the matrix which are determined by commutation and anticommutation relation giving
some superalgebra. Full set of the Hubbard operators have 16 operators part of which

\[ (X^{0+}, X^{0-}, X^{-0}, X^{+0}, X^{2+}, X^{-2}, X^{20}, X^{02}, X^{00} - X^{22}) \]

are the fermionic operators, but other part

\[ (X^{+-}, X^{-+}, X^{++} - X^{--}, X^{02}, X^{20}, X^{00} - X^{22}) \]

are the bosonic operators. \( X^{ij} \) - Hubbard operators contain only one non-zero element equal
1 sitting on site \((i, j)\) in the matrix representation. Point out that this set of operators gives
some bases for some superalgebra.
We have the following representation for creation-annihilating operators in this bases:

\[ \alpha_i^+ = X^{i0} + X^{i2} - \alpha_i^+ = X^{-i0} + X^{-i2} \]  \hspace{1cm} (3)

The Hubbard model in this representation has the form:

\[ H = U \sum_{i,p} X^{ip} - W \sum_{i \alpha \beta} X^{-i \alpha} X^{\beta} \]  \hspace{1cm} (4)

3 Supercoherent state for Hubbard model

In constructing the supercoherent state we use the following interesting observation in interpretation of the set of atomic operators and function for on-site Hubbard repulsion. This observation can be formulated as the following statement: six even Hubbard operators constitute the subalgebra isomorphic with algebra of Lorentz group or algebra of four dimensional rotation group in spinor representation. Complete derivation of this statement will be obtain in subsequent paper.

To characterise the state of the system by coherent state we input some fields which depend on coordinates \(x, y, z\) and the time \(t\). We have three component dynamic vector of the electrical field

\[ \mathbf{E} = (E^+(x, y, z, t), E^-(x, y, z, t), E^z(x, y, z, t)) \]

three component dynamical vector of the magnetic field

\[ \mathbf{h} = (h^+(x, y, z, t), h^-(x, y, z, t), h^z(x, y, z, t)) \]

and four component dynamical odd grassmann fields

\[ \chi^*_1(x, y, z, t), \chi^*_2(x, y, z, t), \chi^*_3(x, y, z, t), \chi^*_4(x, y, z, t) \]

which are the fermionic fields giving the components of majorana field. All dynamical fields appear in supercoherent state in the following manner:

\[ | G > = e^{\text{exp} \left[ \begin{array}{cccc} E_z & 0 & 0 & E^+ \\ \chi^*_1 & h^- & h^+ & 0 \\ \chi^*_2 & h^- & -h_z & 0 \\ E^- & -\chi^*_3 & \chi^*_4 & -E_z \end{array} \right]} | 0 >, \]  \hspace{1cm} (5)

\[ | G > \] Exponent here act in space of atomic eigenfunction (\(| 0 >, | + >, | - >, | 2 >\)), Function \(| 0 >\) is highest weight vector of supergroup which representation is given by exponential.

4 Evolution operator for electronic systems

The transition amplitude of the evolution operator of the quantum systems is given by the following expression: \(< Z_f | e^{-iH(t_f-t_i)} | Z_i >\). We want to obtain the expression for the effective functional using the states \(| Z >\). Time evolution of the system is given by the following operator:
\[ U(t, t_0) = T_{or} \exp(-i \int_{t_0}^{t} H(\tau) d\tau); \]

if \( t - t_0 = \delta t \) is small, ie \( \delta t \ll 1 \), then

\[ U(t_0 + \delta t, t_0) = 1 - i \int_{t_0}^{t_0+\delta t} H(\tau) d\tau. \]

It is follow from this expression that the symbol for evolutionary operator has the following form:

\[ U(Z, Z^*|t_0 + \delta t, t_0) = \exp(-i \int_{t_0}^{t_0+\delta t} H(Z, Z^*|\tau) d\tau). \]

We devide time interval \([t_0, t]\) by the number \( N \) and obtain \( N \) small intervals for finding the expression for symbol \( U(Z, Z^*|t, t_0) \). Consider the matrix elements of evolution operator \( \exp(-i H(t_f - t_i)) \) between the states \( < Z_f | \) and \( |Z_i \rangle \). Factorising operator \( \exp(-i H(t_f - t_i)) \) by inserting the identity operator \( |H \rangle \) in the exponent we can transform this formula and place the symbol of operator \( H \) in the exponent

\[
\frac{< Z_{k+1} | e^{-i\epsilon H} | Z_k >}{< Z_{k+1} | Z_k >} = \frac{< Z_{k+1} | (1 - i\epsilon H) | Z_k >}{< Z_{k+1} | Z_k >} = e^{-i \frac{< Z_{k+1} | H | Z_k >}{< Z_{k+1} | Z_k >}} + O(\epsilon^2). \tag{6}
\]

As a result we obtain the representation

\[
< Z_f | e^{-i H(t_f - t_i)} | Z_i >= \lim_{N \to \infty} \int \prod_{k=1}^{N} d\mu(Z_k) < Z_{k+1} | Z_k > e^{-it \frac{< Z_{k+1} | H | Z_k >}{< Z_{k+1} | Z_k >}}, \tag{7}
\]

here \( |Z_0 > = |Z_i > ; < Z_{N+1} | = < Z_f |. \) Let define a variation of the following type \( |Z >: |\delta Z_{k+1} > = |Z_{k+1} > - |Z_k >. \) We have:

\[
< Z_f | e^{-i H(t_f - t_i)} | Z_i >= \lim_{N \to \infty} \int \prod_{k=1}^{N} d\mu(Z_k) < Z_k | Z_k > < Z_f | Z_N >
\]

\[
\exp\left(\sum_{k=1}^{N} (\ln(1 - \frac{< Z_k | \delta Z_k >}{< Z_k | Z_k >}) - i\epsilon \frac{< Z_k | H | Z_{k-1} >}{< Z_k | Z_{k-1} >}) \right).
\]

In linear-slice approximation. We take the following expression for the time derivative: Considering the first order in \( \epsilon \) we can take the following expression for the time derivative:
\[ \frac{d|Z\rangle}{dt} = \frac{\delta|Z\rangle}{\epsilon}. \]

In first order in \( \epsilon \), we obtain the final path integral representation of the evolutionary operator in coherent state formalism

\[ < Z_f|e^{-iH(t_f-t_i)}|Z_i> = \int_{|Z(t_i)>=|Z_i>} D(Z, Z^*) e^{-iS[Z,Z^*]}; \tag{8} \]

\[ S[Z, Z^*] = \int_{t_i}^{t_f} dt \int_V dr \left( \frac{< Z(r,t)|i\frac{\partial}{\partial t} - H|Z(r,t)>}{<Z(r,t)|Z(r,t)>} - i[Ln(<Z_f|Z(t_f)>) - Ln(<Z_i|Z(t_i)>)] \right). \]

Measure of integration is given by the following expression:

\[ D[Z, Z^*] = \prod_{t_i<t<t_f} \prod_r d\mu[Z(r,t)^*, Z(r,t)] < Z(r,t)|Z(r,t)>. \]

This form of path integral representation will be the starting point of our consideration.

5 Nonlinear representation of supergroup in Hubbard model

In construction of supercoherent state we have the following supermatrix which we must compute analytically:

\[ U = \exp \begin{pmatrix} E_z & 0 & 0 & E^+ \\ \chi_1 & h_z & h^+ & 0 \\ \chi_2 & h^- & -h_z & 0 \\ -E^- & \chi_3 & \chi_4 & -E_z \end{pmatrix} \]

In this expression we have the fields of different statistics: for example, set of the fields

\( (\chi_1(t,x,y,z), \chi_2(t,x,y,z), \chi_3(t,x,y,z), \chi_4(t,x,y,z)) \)

are the odd grassman valued function of space-time coordinates and describe the fermionic degree of freedom but fields

\( (E_z(t,x,y,z), E^+(t,x,y,z), E^-(t,x,y,z), h_z(t,x,y,z), h^+(t,x,y,z), h^-(t,x,y,z)) \)

describe the electro-magnetic degree of freedom, equal to two three component vectors of the space-time coordinates and are bosonic. In this paper we concentrate in calculating exact representation of exponent of the supermatrix in the coherent state. We take the dynamic electrical, magnetic and grassmann fields which depend on coordinates of 4-dimensional space-time manifold on definite the coordinates and omit \((t, x, y, z)\) coordinates in subsequent formulas.
Our general strategy will be to expand the supermatrix to N-order in fields. Then we can isolate and collect certain series and get some recurrent formular for general term in infinite series. Using this formula we can sum all terms to anytical compact representation. Analytical representation of the supermatrix elements will be the final point of our work. As a starting point we have the following exponencial expression for the representation of super extension of the Lorentz group in the spinor representation.

Expanding this exponent in series we can obtain first and second order in the parameters $b$ and $h$. We see that the polynomial series on the grassmann numbers can be classified in grassmann order $n$. All the supermatrix elements can be represented as a coefficients in grassmann polynomials of order $n$, where $n=0,1,2,3,4$.

6  Matrix series for the nonlinear representation of supergroup

First of all point out that the supermatrix in exponent have two submatrix: one is the odd grassmann matrix and the other is the even submatrix containing only the fields of type $E_i(t, x, y, z)$ and $h_i(t, x, y, z)$ type. As a first step we make expansion of the exponent for the even matrix. Expanding in series this exponent we can obtain first, second order and 3,4,5 order in bosonic fields $E_i, i=1,2,3$. For example, the series for $n=0, 1,2$ has the following form:

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
+ \frac{1}{2!}
\begin{pmatrix}
b^2 & 0 & 0 & 0 \\
0 & h^2 & 0 & 0 \\
0 & 0 & h^2 & 0 \\
0 & 0 & 0 & b^2
\end{pmatrix}
+ \frac{1}{3!}
\begin{pmatrix}
b^4 E_z & 0 & 0 & b^2 E^+ \\
0 & h^2 h_z & h^2 h^+ & 0 \\
0 & h^2 h^- & -h^2 h_z & 0 \\
b^2 E^- & 0 & 0 & -b^2 E_z
\end{pmatrix}
+ \frac{1}{4!}
\begin{pmatrix}
b^4 & 0 & 0 & 0 \\
0 & h^4 & 0 & 0 \\
0 & 0 & h^4 & 0 \\
0 & 0 & 0 & b^4
\end{pmatrix}
+ \frac{1}{5!}
\begin{pmatrix}
b^4 E_z & 0 & 0 & b^4 E^+ \\
0 & h^4 h_z & h^4 h^+ & 0 \\
0 & h^4 h^- & -h^4 h_z & 0 \\
b^4 E^- & 0 & 0 & -b^4 E_z
\end{pmatrix}
\]

here we introduce the following abbriviation $b = \sqrt{E_z^2 + E^+ E^-}$, $h = \sqrt{h_z^2 + h^+ h^-}$.

For the odd grassmann number we have following expansion series of exponent. We write here two terms for the grassmann fields $(\chi_1, \chi_2, \chi_3, \chi_4)$ for obtaining coefficients in higher order in $E_i$ and $h_i$. 


bosonic fields:

\[
\begin{pmatrix}
0 & 0 & 0 & 0 \\
\chi_1 & 0 & 0 & 0 \\
\chi_2 & 0 & 0 & 0 \\
0 & -\chi_3 & \chi_4 & 0
\end{pmatrix} + 
\frac{1}{2!} 
\begin{pmatrix}
\chi_1 E_z + \chi_2 h^+ + \chi_1 h_z & -\chi_3 E^+ & \chi_4 E^+ & 0 \\
\chi_2 E_z + \chi_1 h^+ - \chi_2 h_z & 0 & 0 & \chi_1 E^+
\end{pmatrix}
\]

For even order of the grassmann variables we have following series for the composite bosonic fields:

\[
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
x20 & 0 & 0 & 0
\end{pmatrix} + \frac{1}{3!} 
\begin{pmatrix}
E^+ x20 & 0 & 0 & 0 \\
0 & -\chi_1\chi_3 E^+ & \chi_4 E^+ & 0 \\
0 & -\chi_2\chi_3 E^+ & \chi_2\chi_4 E^+ & 0
\end{pmatrix} + 
\frac{1}{4!} 
\begin{pmatrix}
E^+ he + E^+ E_z x20 & 0 & 0 & E^{+2} x20 \\
0 & E^+ he \chi_1 & -E^+ h^+ x20 & 0 \\
x20 b^2 + h^2 x20 + E^+ E_z x20 & 0 & E^+ he & 0
\end{pmatrix}
\]

here we introduce the following abbreviation: 
\[x20 = -\chi_3\chi_1 + \chi_4\chi_2, \ he = -h_z\chi_2\chi_4 + h^- \chi_1\chi_4 - h_z\chi_2\chi_4 - h^+ \chi_2\chi_3, \]
\[heh = -h_z\chi_1\chi_3 + h^- \chi_1\chi_4 - h_z\chi_1\chi_3 - h^+ \chi_2\chi_3\]

The expansion series for third order of the grassmann variable has the following form of first two terms

\[
\frac{1}{4!} 
\begin{pmatrix}
0 & 0 & 0 & 0 \\
-a13\chi_4 E^+ & 0 & 0 & 0 \\
-a13\chi_3 E^+ & 0 & 0 & 0 \\
0 & a33\chi_2 E^+ & a33\chi_1 E^+ & 0
\end{pmatrix} + 
\frac{1}{5!} 
\begin{pmatrix}
0 & -a33\chi_2 E^{+2} & -a33\chi_1 E^{+2} & 0 \\
-a13\chi_4 E^+ E_z & 0 & 0 & -a13\chi_4 E^{+2} \\
-a13\chi_3 E^+ E_z & 0 & 0 & -a13\chi_3 E^{+2} \\
0 & -a33\chi_2 E^+ E_z & -a33\chi_1 E^+ E_z & 0
\end{pmatrix}
\]

here \(a13 = \chi_1\chi_2, \ a33 = \chi_3\chi_4\).

Forth order of grassmann variable has the following series of three terms:

\[
\frac{1}{5!} 
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
2c4hi E^+ & 0 & 0 & 0
\end{pmatrix} + \frac{1}{6!} 
\begin{pmatrix}
2c4hi E^{+2} & 0 & 0 & 0 \\
0 & -c4hi E^{+2} & 0 & 0 \\
0 & 0 & -c4hi E^{+2} & 0 \\
0 & 0 & 0 & 2c4hi E^{+2}
\end{pmatrix}
\]
Having series representation for supergroup for high order in fields we can obtain the general analytical form for the matrix elements. Our task is to obtain exact dependence of the matrix element over component of fields: \(E_z, E^+, E^-; h_z, h^+, h^-\) but not dependence over \(b\) and \(h\).

For example the general form of \(u_{11}\) supermatrix elements in orders higher than 8 as a functions of the dynamic even and odd grassmanian fields are given by following expressions

\[
\begin{align*}
    u_{11} &= a_{11}^0 + E_z b_{11}^0 + E^+(-\chi_1 + \chi_2)(a_{11}^{21} + E_z b_{11}^{21}) + \\
    E^+(-h_z \chi_1 + h^- \chi_2 - h^+ \chi_2)(a_{11}^{22} + E_z b_{11}^{22}) + \\
    \chi_1 \chi_2 \chi_3 (E^+)^2(a_{11}^4 + E_z b_{11}^4) 
\end{align*}
\]

here the coefficients \(a_i^j\) and \(b_i^j\) are some series in \(b\) and \(h\) variables:
\(a_{11}^0, b_{11}^0\) have some infinite series over \(b\) and \(h\) variables.

Let introduce following abbriviation for coefficients which appear in the suprmatrix elements:
\(a_{11}^{21} = f2EE; \quad b_{11}^{21} = f1EE; \quad a_{11}^{22} = f1EE; \quad b_{11}^{22} = fEE; \quad a_{11}^4 = DEDEf1; \quad b_{11}^4 = DEDEf\)

Another matrix element equal to

\[
\begin{align*}
    u_{21} &= (a_{21}^0 + E_z b_{21}^0) \chi_1 + (h_z \chi_1 + h^+ \chi_2)(a_{21}^1 + E_z b_{21}^1) - \\
    E^+ \chi_1 \chi_2 \chi_3 (a_{21}^3 + E_z b_{21}^3)
\end{align*}
\]

and we have some function for the coefficients:
\(a_{21}^0 = f4; \quad a_{21}^0 = f4; \quad b_{21}^0 = f3; \quad a_{21}^1 = f3; \quad b_{21}^1 = f2; \quad a_{21}^3 = f1EE; \quad b_{21}^3 = fEE\)

For \(u_{31}\) we have

\[
\begin{align*}
    u_{31} &= (a_{31}^0 + E_z b_{31}^0) \chi_2 + (-h_z \chi_2 + h^- \chi_1)(a_{31}^1 + E_z b_{31}^1) - \\
    E^+ \chi_1 \chi_2 \chi_3 (a_{31}^3 + E_z b_{31}^3)
\end{align*}
\]

Expanding this series on 12 order in even and odd parameters we can obtain the analytical representation for the supermatrix elements.
and for coefficients:

\[ a_{31}^0 = f4; \quad b_{31}^0 = f3; \quad a_{31}^1 = -f3; \quad b_{31}^1 = f2 \]

Last element of first column is

\[ u_{41} = E^{-b_{11}^0} + (-\chi_3\chi_1 + \chi_4\chi_2)(a_{41}^{21} + E^+ E^{-b_{41}^{21}}) + \]

\[ (-h_z\chi_3\chi_1 - h^+\chi_3\chi_2 - h_z\chi_4\chi_2)(a_{41}^{22} + E^+ E^{-b_{41}^{22}}) + \]

\[ 2\chi_1\chi_2\chi_3\chi_4 E^+(a_{41}^4 + E^+ E^{-b_{41}^4}) \]

For coefficients in this expression we have:

\[ b_{11}^0 = \sinh(b)/b; \quad a_{41}^{21} = f3; \quad b_{41}^{21} = f1EE; \]

\[ a_{41}^{22} = f2; \quad b_{41}^{22} = fEE; \quad a_{41}^4 = fEE; \quad b_{41}^4 = DEDEf \]

For second column we have

\[ u_{12} = -E^+a_{12}^0 + E^+(h_z\chi_3 - h^-\chi_4)a_{12}^1 - \]

\[(E^+)^2\chi_2\chi_3\chi_4 a_{12}^3\]

For coefficients \( a_i^j \) and \( b_i^j \):

\[ a_{12}^0 = f3; \quad a_{12}^1 = -f2; \quad a_{12}^3 = fEE \]

For \( u_{22} \) we have:

\[ u_{22} = a_{22}^0 + h_z b_{22}^0 - E^+\chi_1\chi_3 a_{22}^2 + E^+ h^- h^+ h^- (-\chi_3\chi_1 + \chi_4\chi_2) a_{22}^{21} \]

\[ E^+ (-h_z\chi_1\chi_3 - h^-\chi_1\chi_4 - h^+\chi_2\chi_3 - h_z\chi_1\chi_3) (a_{22}^{22} + h_z b_{22}^{22}) - \]

\[ \chi_1\chi_2\chi_3\chi_4(E^+)^2 (a_{22}^4 + h_z b_{22}^4) \]

For coefficients:

\[ a_{22}^2 = -f2; \quad a_{22}^{21} = fhh; \quad a_{22}^{22} = f1hh; \quad b_{22}^{22} = fhh; \quad a_{22}^4 = f3; \quad b_{22}^4 = f2 \]

For \( u_{32} \) we have:

\[ u_{32} = h^- b_{32}^0 - E^+\chi_2\chi_3 a_{32}^{21} + E^+ h^- (-\chi_3\chi_1 + \chi_4\chi_2) (a_{32}^{22} + h_z b_{32}^{22}) \]

\[ E^+ h^- (-h_z\chi_2\chi_4 + h^-\chi_1\chi_4 - h^+\chi_2\chi_3 - h_z\chi_2\chi_4) a_{32}^{22} - \]

\[ \chi_1\chi_2\chi_3\chi_4(E^+)^2 h^- a_{32}^4 \]

For coefficients:

\[ a_{32}^{21} = f2; \quad a_{32}^{22} = f1hh; \quad b_{32}^{22} = fhh; \quad a_{32}^{22} = fhh; \quad a_{32}^4 = DEDEh f2 \]
For $u_{42}$ we have:

$$u_{42} = (a_{42}^0 + E_z b_{42}^0) \chi_3 + (h_z \chi_3 - h^- \chi_4)(a_{42}^1 + E_z b_{42}^1) + E^+ \chi_2 \chi_3 \chi_4(a_{42}^4 + E_z b_{42}^4)$$

For coefficients:

$$a_{42}^0 = f4; \quad b_{42}^0 = f3; \quad a_{42}^1 = f3; \quad b_{42}^1 = f2; \quad a_{42}^4 = f1EE; \quad b_{42}^4 = fEE$$

For $u_{13}$ we have:

$$u_{13} = E^+ a_{13}^0 - E^+(h_z \chi_4 + h^- \chi_3) a_{13}^1 - (E^+)^2 \chi_1 \chi_3 \chi_4 a_{13}^3$$

For coefficients:

$$a_{13}^0 = f3; \quad a_{13}^1 = f2; \quad a_{13}^2 = fEE; \quad a_{13}^3 = fEE$$

For $u_{32}$ we have:

$$u_{32} = h^+ b_{22}^0 + E^+ \chi_1 \chi_4 a_{23}^{21} + E^+ h^+ (-h_z \chi_3 + h^- \chi_4)(a_{23}^{22} + h_z \chi_3) a_{23}^2$$

$$E^+ h^+ (-h_z \chi_1 \chi_3 + h^- \chi_1 \chi_4 - h^+ \chi_2 \chi_3 - h_z \chi_1 \chi_3) a_{23}^2 - \chi_1 \chi_2 \chi_3 \chi_4 (E^+) h^+ a_{23}^4$$

For coefficients:

$$a_{23}^{21} = f2; \quad a_{23}^{22} = f1h; \quad b_{23}^{22} = fhh; \quad a_{23}^2 = fhh; \quad a_{23}^4 = DEDhf2$$

For $u_{33}$ we have:

$$u_{33} = a_{22}^0 - h_z b_{22}^0 + E^+ \chi_2 \chi_4 a_{33}^{21} + E^+ h^- h^+ (-h_z \chi_1 + h^- \chi_2) a_{33}^2$$

$$E^+ (-h_z \chi_2 \chi_4 + h^- \chi_1 \chi_4 - h^+ \chi_2 \chi_3 - h_z \chi_2 \chi_4)(a_{33}^{22} - h_z b_{33}^{22})$$

$$\chi_1 \chi_2 \chi_3 \chi_4 (E^+)^2 (a_{33}^4 - h_z b_{33}^4)$$

For coefficients:

$$a_{33}^{21} = f2; \quad a_{33}^{22} = fhh; \quad a_{33}^2 = Dfhh; \quad b_{33}^{22} = fhh; \quad a_{33}^4 = DEDhf3; \quad b_{33}^4 = DEDhf2$$

For $u_{43}$ we have:

$$u_{43} = (a_{43}^0 + E_z b_{43}^0) \chi_4 + (h_z \chi_4 + h^+ \chi_3)(a_{43}^1 + E_z b_{43}^1) - E^+ \chi_1 \chi_3 \chi_4 (a_{43}^3 + E_z b_{43}^3)$$
For coefficients:
\begin{align*}
a_{43}^0 &= f4; & b_{43}^0 &= f3; & a_{43}^1 &= f3; & b_{43}^1 &= f2; & a_{43}^3 &= f1EE; & b_{43}^3 &= -fEE
\end{align*}

For \( u_{14} \) we have:
\[ u_{14} = E^+b_{11}^0 + (E^+)^2(-\chi_3\chi_1 + \chi_4\chi_2)a_{14}^{21} + \chi_1\chi_2\chi_3\chi_4(E^+)a_{14}^4 \]

For coefficients:
\begin{align*}
a_{14}^{21} &= f1EE; & a_{14}^4 &= DEDEf
\end{align*}

For \( u_{24} \) we have:
\[ u_{24} = E^+b_{24}^0\chi_1 + (h_z\chi_1 + h^+\chi_2)E^+b_{24}^1 - (E^+)\chi_1\chi_2\chi_4a_{24}^{22} \]

For coefficients:
\begin{align*}
a_{24}^0 &= f2; & b_{24}^0 &= f3; & b_{24}^1 &= f2; & a_{24}^{22} &= fEE
\end{align*}

For \( u_{34} \) we have:
\[ u_{34} = E^+b_{34}^0\chi_2 + (-h_z\chi_2 + h^-\chi_1)E^+b_{34}^1 - (E^+)\chi_1\chi_2\chi_3a_{34}^{22} \]

For coefficients:
\begin{align*}
b_{34}^0 &= f3; & b_{34}^1 &= f2; & a_{34}^{22} &= fEE
\end{align*}

For \( u_{44} \) we have:
\[ u_{44} = a_{44}^0 - E_zb_{11}^0 + E^+(-\chi_3\chi_1 + \chi_4\chi_2)(a_{44}^{21} + E_zb_{44}^{21}) + E^+(-h_z\chi_3\chi_1 + h^-\chi_4\chi_1 - h^+\chi_3\chi_2 - h_z\chi_4\chi_2)(a_{44}^{22} - E_zb_{44}^{22}) + 2\chi_1\chi_2\chi_3\chi_4(E^+)a_{44}^{22}(a_{44}^4 - E_zb_{44}^4) \]

For coefficients:
\begin{align*}
a_{44}^{21} &= -f2EE; & b_{44}^{21} &= f1EE; & a_{44}^{22} &= f1EE; & b_{44}^{22} &= fEE;
a_{44}^4 &= DEDEf1; & b_{44}^4 &= DEDEf
\end{align*}

We see latter that many series in our list are equivalent to each other. After such selection between similar ones we have only some different series.
8 Analytical representation for series

Collecting the terms in series expansion for a and b coefficients to 12 order we obtain for example the following representation for:

\[ f_2 = \frac{1}{3!} + \frac{b^2 + h^2}{5!} + \frac{b^4 + h^4}{7!} + \frac{b^6 + h^6}{9!} + \frac{b^8 + h^8}{11!} + \frac{b^{10} + h^{10}}{13!} \]

Let us show how the summation of this series can be performed. We can make the summation of subpart of hole series:

\[ f = \frac{1}{5!} + \frac{b^2 + h^2}{7!} + \frac{b^4 + h^4}{9!} + \frac{b^6 + h^6}{11!} + \frac{b^8 + h^8}{13!} + \frac{b^{10} + h^{10}}{15!} \]

\[ f = \frac{1}{11!} + \frac{b^2 + h^2}{13!} + \frac{b^4 + h^4}{15!} + \frac{b^6 + h^6}{17!} + \frac{b^8 + h^8}{19!} + \frac{b^{10} + h^{10}}{21!} \]

It is seen that the first series equal to

\[ \cosh(b) = 1 + \frac{b^2}{2!} + \frac{b^4}{4!} + \frac{b^6}{6!} + \frac{b^8}{8!} + \frac{b^{10}}{10!} + \ldots \]

and the second series equal to

\[ \sinh(b)/b = 1 + \frac{b^2}{3!} + \frac{b^4}{5!} + \frac{b^6}{7!} + \frac{b^8}{9!} + \frac{b^{10}}{11!} + \frac{b^{12}}{13!} + \ldots \]

For sum of two series we have the following expression \( \cosh(b) + E_z \sinh(b)/b \)

The main series for us is the following expansion:

\[ f = \frac{1}{5!} + \frac{b^2 + h^2}{7!} + \frac{b^4 + h^4}{9!} + \frac{b^6 + h^6}{11!} + \frac{b^8 + h^8}{13!} + \frac{b^{10} + h^{10}}{15!} \]

\[ f = \frac{1}{11!} + \frac{b^2 + h^2}{13!} + \frac{b^4 + h^4}{15!} + \frac{b^6 + h^6}{17!} + \frac{b^8 + h^8}{19!} + \frac{b^{10} + h^{10}}{21!} \]

Let us show how the summation of this series can be performed. We can make the summation of subpart of hole series:
\begin{align*}
&\frac{b^5}{5!} + \frac{b^7}{7!} + \frac{b^9}{9!} + \frac{b^{11}}{11!} + \frac{b^{13}}{13!} + \ldots = \sinh(b) - b - \frac{b^3}{3!} \\
&h^2\left(\frac{b^7}{7!} + \frac{b^9}{9!} + \frac{b^{11}}{11!} + \frac{b^{13}}{13!} + \ldots\right) = h^2\left(\sinh(b) - b - \frac{b^3}{3!} - \frac{b^5}{5!}\right) \\
&h^4\left(\frac{b^9}{9!} + \frac{b^{11}}{11!} + \frac{b^{13}}{13!} + \ldots\right) = h^4\left(\sinh(b) - b - \frac{b^3}{3!} - \frac{b^5}{5!} - \frac{b^7}{7!}\right)
\end{align*}

Having this series representation we can rewrite expression for \( f \)

\[
\frac{1}{5!} + \frac{b^2}{7!} + \frac{b^4 + b^2 b^2 + b^4}{9!} + \frac{b^6 + h^2 b^4 + h^4 b^2 + h^6}{11!} + \frac{b^8 + h^2 b^6 + h^4 b^4 + h^6 b^2 + h^8}{13!} + \\
\frac{b^{10} + h^2 b^8 + h^4 b^6 + h^6 b^4 + h^8 b^2 + h^{10}}{15!} + \\
\frac{b^{12} + h^2 b^{10} + h^4 b^8 + h^6 b^6 + h^8 b^4 + h^{10} b^2 + h^{12}}{17!} + \ldots = \\
\frac{1}{b^5}(\sinh(b) - b - \frac{b^3}{3!}) + \frac{1}{b^7} h^2(\sinh(b) - b - \frac{b^3}{3!} - \frac{b^5}{5!}) + \frac{1}{b^9} h^4(\sinh(b) - b - \frac{b^3}{3!} - \frac{b^5}{5!} - \frac{b^7}{7!}) + \ldots = \\
\sinh(b)/b^5(1 + \frac{h^2}{b^2} + \frac{h^4}{b^4} + \ldots) + (-b - \frac{b^3}{3!})/b^5(1 + \frac{h^2}{b^2} + \frac{h^4}{b^4} + \ldots) + \\
(-\frac{b^5}{5!})\frac{h^2}{b^2}(1 + \frac{h^2}{b^2} + \frac{h^4}{b^4} + \ldots) + (-\frac{b^7}{7!})\frac{h^4}{b^4}(1 + \frac{h^2}{b^2} + \frac{h^4}{b^4} + \ldots) + \ldots
\]

It is seen that the series of the type \( 1 + \frac{h^2}{b^2} + \frac{h^4}{b^4} + \ldots \) describe geometric series and gives the following result:

\[
1 + \frac{h^2}{b^2} + \frac{h^4}{b^4} + \ldots = \frac{1}{1 - h^2/b^2} = \frac{b^2}{b^2 - h^2}
\]

If we insert this result we obtain the representation for:

\[
\frac{1}{5!} + \frac{b^2}{7!} + \frac{b^4 + b^2 b^2 + b^4}{9!} + \frac{b^6 + h^2 b^4 + h^4 b^2 + h^6}{11!} + \frac{b^8 + h^2 b^6 + h^4 b^4 + h^6 b^2 + h^8}{13!} + \\
\frac{b^{10} + h^2 b^8 + h^4 b^6 + h^6 b^4 + h^8 b^2 + h^{10}}{15!} + \ldots = \frac{\sinh(b)}{b^5} + (-b - \frac{b^3}{3!})/b^5 b^2 - h^2 + (\frac{b^5}{5!})\frac{h^2}{b^2} b^2 - h^2 + (\frac{b^7}{7!})\frac{h^4}{b^4} b^2 - h^2 + \ldots
\]
\[
\begin{align*}
\frac{\sinh(b)}{b^2 - h^2} &= \frac{1}{b^2} - \frac{1}{b^4} + \left(- \frac{1}{3!} - \frac{h^2}{5!} - \frac{h^4}{7!} - \ldots \right) \frac{1}{b^2 - h^2} \\
\frac{\sinh(b)}{b^4} &= -\frac{1}{b^2} - \frac{1}{b^6} + \left(- \sinh(h)/h^2 + \frac{1}{h^2} \right) \frac{1}{b^2 - h^2} \\
\frac{\sinh(b)}{b^6} - \frac{\sinh(h)}{h^3} + \frac{1}{b^2 h^2} + \frac{1}{b^2h^2} \\
\end{align*}
\]

Let us consider two series: one is \( f \) and second is \( f_1 \). If we multiply \( f \) by coefficient \( a^5 \) and make following substitution \( b - > ba, h - > ha \) we obtain the following series

\[
\frac{a^5}{5!} + \frac{1}{7!} + \frac{a^7 b^2 + h^2}{9!} + \frac{a^9 b^4 + h^2 b^2 + h^4}{11!} + \frac{a^{11} b^6 + h^2 b^4 + h^4 b^2 + h^6}{13!} + \frac{a^{13} b^8 + h^2 b^6 + h^4 b^4 + h^6 b^2 + h^8}{15!}
\]

\[
\frac{a^{15} b^{10} + h^2 b^8 + h^4 b^6 + h^6 b^4 + h^8 b^2 + h^{10}}{17!} + \ldots
\]

It is obvious that if we take derivative we can reduce factorial in our series for example:

\[
f_1 = \frac{\partial (a^5 f)}{\partial a}, \quad f_2 = \frac{\partial f_1}{\partial a}, \quad f_3 = \frac{\partial f_2}{\partial a}, \quad f_4 = \frac{\partial f_3}{\partial a}
\]

here we must insert \( b - > ab, h - > ah \) and \( a \) put to 1 after differentiation. Taking derivatives we obtain the analytical expression for \( f_1 \):

\[
f_1 = \frac{\cosh(b) - \cosh(h)}{b^2 - h^2}, \quad f_2 = \frac{\sinh(b)}{b^2} - \frac{\sinh(h)}{h^2}, \quad f_3 = \frac{\cosh(b) - \cosh(h)}{b^2 - h^2}, \quad f_4 = \frac{b \sinh(b) - h \sinh(h)}{b^2 - h^2}
\]

Series for \( f_i \) have the following forms:

\[
f_1 = \frac{1}{4!} + \frac{b^2 + h^2}{6!} + \frac{b^4 + h^2 b^2 + h^4}{8!} + \frac{b^6 + h^2 b^4 + h^4 b^2 + h^6}{10!} + \frac{b^8 + h^2 b^6 + h^4 b^4 + h^6 b^2 + h^8}{12!} + \frac{b^{10} + h^2 b^8 + h^4 b^6 + h^6 b^4 + h^8 b^2 + h^{10}}{14!} + \frac{b^{12} + h^2 b^{10} + h^4 b^8 + h^6 b^6 + h^8 b^4 + h^{10} b^2 + h^{12}}{16!} + \frac{b^{14} + h^2 b^{12} + h^4 b^{10} + h^6 b^8 + h^8 b^6 + h^{10} b^4 + h^{12} b^2 + h^{14}}{17!} + \ldots
\]

\[
f_2 = \frac{1}{3!} + \frac{b^2 + h^2}{5!} + \frac{b^4 + h^2 b^2 + h^4}{7!} + \frac{b^6 + h^2 b^4 + h^4 b^2 + h^6}{9!} + \frac{b^8 + h^2 b^6 + h^4 b^4 + h^6 b^2 + h^8}{11!} + \ldots
\]
obtain the series for $fEE$. For series $fhh$

$$f_3 = \frac{1}{2!} + \frac{b^2 + h^2}{4!} + \frac{b^4 + h^2 b^2 + h^4}{6!} + \frac{b^6 + h^2 b^4 + h^4 b^2 + h^6}{8!} + \frac{b^8 + h^2 b^6 + h^4 b^4 + h^6 b^2 + h^8}{10!} + \ldots$$

$$f_4 = 1 + \frac{b^2 + h^2}{2!} + \frac{b^4 + h^2 b^2 + h^4}{4!} + \frac{b^6 + h^2 b^4 + h^4 b^2 + h^6}{6!} + \frac{b^8 + h^2 b^6 + h^4 b^4 + h^6 b^2 + h^8}{8!} + \ldots$$

If we take the series for $f$ and multiply it by $b^2$ and take following derivative $\frac{\partial f(h^2)}{\partial (b^2)}$ we obtain the series for $fEE$. For series $fhh$ we must make multiplication of $f$ on $h^2$ and make derivative on $h^2$ :

$$fhh = \frac{\partial (h^2 f)}{\partial (h^2)}$$

$$fhh = \frac{1}{5!} + \frac{2b^2 + h^2}{7!} + \frac{3b^4 + 2h^2 b^2 + h^4}{9!} + \frac{4b^6 + 3h^2 b^4 + 2h^4 b^2 + h^6}{11!} + \frac{5b^8 + 4h^2 b^6 + 3h^4 b^4 + 2h^6 b^2 + h^8}{13!} + \frac{6b^{10} + 5h^2 b^8 + 4h^4 b^6 + 3h^6 b^4 + 2h^8 b^2 + h^{10}}{15!}$$
To evaluate \( f_{1EE}, f_{2EE}, f_{1hh} \) we multiply \( fEE \) by \( a^5 \) and make substitution \( b - \rightarrow ba, h - \rightarrow ha \). After calculation we fix \( a = 1 \).

It is seen that expression for \( f_{1EE}, f_{2EE}, f_{1hh} \) are the following: \( f_{1EE} = (\frac{\partial fEE}{\partial b})_{a=1} \)

\[
f_{1EE} = \frac{1}{4!} + \frac{2b^2 + h^2}{6!} + \frac{3b^4 + 2b^2h^2 + h^4}{8!} + \frac{4b^6 + 3h^2b^4 + 2h^4b^2 + h^6}{10!} + \frac{5b^8 + 4h^2b^6 + 3h^4b^4 + 2h^6b^2 + h^8}{12!} + \frac{6b^{10} + 5h^2b^8 + 4h^4b^6 + 3h^6b^4 + 2h^8b^2 + h^{10}}{14!} + \ldots\ldots
\]

Repeating all operation we can obtain for \( f_{2EE} = (\frac{\partial^2 fEE}{\partial^2 a})_{a=1} \)

\[
f_{2EE} = \frac{1}{3!} + \frac{2b^2 + h^2}{5!} + \frac{3b^4 + 2b^2h^2 + h^4}{7!} + \frac{4b^6 + 3h^2b^4 + 2h^4b^2 + h^6}{9!} + \frac{5b^8 + 4h^2b^6 + 3h^4b^4 + 2h^6b^2 + h^8}{11!} + \frac{6b^{10} + 5h^2b^8 + 4h^4b^6 + 3h^6b^4 + 2h^8b^2 + h^{10}}{13!}
\]

and for \( f_{1hh} = (\frac{\partial^2 fhh}{\partial a})_{a=1} \)

\[
f_{1hh} = \frac{1}{4!} + \frac{2h^2 + b^2}{6!} + \frac{3h^4 + 2b^2h^2 + b^4}{8!} + \frac{4h^6 + 3b^4h^2 + 2b^4h^2 + b^6}{10!} + \frac{5h^8 + 4b^2h^6 + 3b^4h^4 + 2b^4h^2 + b^6}{12!} + \frac{6h^{10} + 5b^2h^8 + 4b^4h^6 + 3b^6h^4 + 2b^8h^2 + b^{10}}{14!} + \ldots\ldots
\]

Series for \( DEDE \) equal to derivative of \( fEE \) on \( b^2 \). It is seen if we compare series for \( fEE \) and \( DEDE \): \( DEDE = \frac{\partial fEE}{\partial (b^2)} \)

\[
DEDE = \frac{2}{7!} + \frac{6b^2 + 2h^2}{9!} + \frac{12b^4 + 6h^2b^2 + 2h^4}{11!} + \frac{20b^6 + 12b^2h^4 + 6h^4b^2 + 2h^6}{13!} + \ldots\ldots
\]
\[ \frac{30b^8 + 20h^2b^6 + 12h^4b^4 + 6h^6b^2 + 2h^8}{15!} + \ldots \]

We can obtain the following representation for \( DEDEf_1 \): \( DEDEf_1 = \frac{\partial f_{hh}}{\partial (b^2)} \)

\[ DEDEf_1 = 2 \frac{6b^2 + 2h^2}{8!} + \frac{12b^4 + 6h^2b^2 + 2h^4}{10!} + \frac{20b^6 + 12h^2b^4 + 6h^4b^2 + 2h^6}{12!} + \frac{30b^8 + 20h^2b^6 + 12h^4b^4 + 6h^6b^2 + 2h^8}{14!} + \ldots \]

Comparing series for \( DEDh f_2 \) and series for \( f_2 \) we can obtain \( DEDh f_2 = \frac{\partial^2 f_2}{\partial (b^2) \partial (h^2)} \)

\[ DEDh f_2 = \frac{1}{7!} + \frac{2b^2 + 2h^2}{9!} + \frac{3b^4 + 4h^2b^2 + 3h^4}{11!} + \frac{4b^6 + 6h^2b^4 + 6h^4b^2 + 4h^6}{13!} + \frac{5b^8 + 8h^2b^6 + 9h^4b^4 + 8h^6b^2 + 5h^8}{15!} + \frac{6b^{10} + 10h^2b^8 + 12h^4b^6 + 12h^6b^4 + 10h^8b^2 + 6h^{10}}{17!} + \ldots \]

and for \( DEDh f_3 \) we can obtain the following representation: \( DEDh f_3 = \frac{\partial^2 f_3}{\partial (b^2) \partial (h^2)} \)

\[ DEDh f_3 = \frac{1}{6!} + \frac{2b^2 + 2h^2}{8!} + \frac{3b^4 + 4h^2b^2 + 3h^4}{10!} + \frac{4b^6 + 6h^2b^4 + 6h^4b^2 + 4h^6}{12!} + \frac{5b^8 + 8h^2b^6 + 9h^4b^4 + 8h^6b^2 + 5h^8}{14!} + \frac{6b^{10} + 10h^2b^8 + 12h^4b^6 + 12h^6b^4 + 10h^8b^2 + 6h^{10}}{16!} + \ldots \]

Summing our calculation let’s write all analytical formula for the function in the matrix elements:

\[ f_{EE} = \frac{2 \sinh(h)b^3 + h(b^2 - h^2) \cosh(b)b + (h^3 - 3b^2h) \sinh(b)}{2b^3h(b^2 - h^2)^2}; \]

\[ f_{1EE} = \frac{-2b \cosh(b) + 2b \cosh(h) + (b^2 - h^2) \sinh(b)}{2b(b^2 - h^2)^2}; \]

\[ f_{2EE} = \frac{b(b^2 - h^2) \cosh(b) - (b^2 + h^2) \sinh(b) + 2bh \sinh(h)}{2b(b^2 - h^2)^2}; \]

\[ f_2 = \frac{h \sinh(b) - b \sinh(h)}{b^3h - bh^3}; \quad f_3 = \frac{\cosh(b) - \cosh(h)}{b^2 - h^2}; \]

\[ f_4 = \frac{b \sinh(b) - h \sinh(h)}{b^2 - h^2}; \quad f = \frac{\sinh(b) - \sinh(h)}{b^2 - h^2}; \]
9 Analytical representation for supercoherent state

Having nonlinear representation of underlying symmetry group of Hubbard model we can construct the supercoherent state as the action of the supergroup on some fix vector, for example \( |0 \rangle \) - the height weight vector of the fundamental representation.

\[
| G \rangle = \exp \begin{pmatrix}
E_z & 0 & 0 & E^+ \\
\chi_1 & h_z & h^+ & 0 \\
\chi_2 & h^- & -h_z & 0 \\
E^- & \chi_3 & \chi_4 & -E_z
\end{pmatrix} | 0 \rangle
\]

here \( |0 \rangle \) is the eigenfunction of the Hubbard repulsion, describing the state with no electron.

We write expression for \( |G\rangle \) in the following form:

\[
| G \rangle = \begin{pmatrix}
g_0^0 + g_0^1 \chi_i \chi_j + g_0^2 \chi_i \chi_2 \chi_3 \chi_4 \\
g_1^1 \chi_1 + g_2^2 \chi_2 + g_1^2 \chi_1 \chi_2 \chi_4 \\
g_1 \chi_1 + g_2 \chi_2 + g_1^2 \chi_1 \chi_2 \chi_3 \\
g_0^0 + g_2^2 \chi_i \chi_j + g_1^2 \chi_1 \chi_2 \chi_3 \chi_4
\end{pmatrix}
\]

here coefficients \( g \) are equal to \( g_0^0 = \cosh(b) + E_z \sinh(b)/b \);

\[
g_0^3 = -E^+(a_{11}^2 + E_z b_{11}^2) - E^+ h_z (a_{11}^2 + E_z b_{11}^2) = -E^+ (\frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(b)}{b^2 - h^2}))) + (E_z + h_z) \frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(h)}{b^2 - h^2})) + h_z E_z \frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(h)}{b^2 - h^2}));
\]

\[
g_0^{02} = E^+(a_{11}^2 + E_z b_{11}^2) - E^+ h_z (a_{11}^2 + E_z b_{11}^2) = E^+ (\frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(h)}{b^2 - h^2}))) + (E_z - h_z) \frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(h)}{b^2 - h^2})) - h_z E_z \frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(h)}{b^2 - h^2}));
\]

\[
g_0^{01} = E^+ h^- (a_{11}^2 + E_z b_{11}^2) = E^+ h^- (\frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(h)}{b^2 - h^2}))) + E_z \frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(h)}{b^2 - h^2}));
\]

\[
g_0^{02} = -E^- h^+ (a_{11}^2 + E_z b_{11}^2) = E^- h^+ (\frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(h)}{b^2 - h^2}))) + E_z \frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(h)}{b^2 - h^2}));
\]

\[
\geq
\]
\[
g_4^0 = (E^+)^2(a_{11}^4 + E_z b_{11}^4) = (E^+)^2(\frac{\partial^2}{\partial^2(b^2)}(b^2(\frac{\sinh(b) - \sinh(h)}{b^2 - h^2}))) + E_z \frac{\partial^2}{\partial^2(b^2)}(b^2(\frac{\sinh(b) - \sinh(h)}{b^2 - h^2})));
\]

\[
g_1^+ = (a_{21}^0 + E_z b_{21}^0) + h_z(a_{21}^0 + E_z b_{21}^0) = \frac{b \sinh(b) - h \sinh(h)}{b^2 - h^2} + \]

\[
(E_z + h_z)\frac{\cosh(b) - \cosh(h)}{b^2 - h^2} + E_z h_z \frac{\sinh(b)}{b^2 - h^2} - \frac{\sinh(h)}{h};
\]

\[
g_2^+ = h^+(a_{21}^0 + E_z b_{21}^0) = h^+ \left(\frac{\cosh(b) - \cosh(h)}{b^2 - h^2} + E_z \frac{\sinh(b)}{b^2 - h^2} - \frac{\sinh(h)}{h}\right);
\]

\[
g_{12}^+ = E^+(a_{21}^3 + E_z b_{21}^3) = E^+ \left(\frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(b) - \sinh(h)}{b^2 - h^2}))) + E_z \frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(b) - \sinh(h)}{b^2 - h^2})));\]

\[
g_1^- = h^-(a_{31}^1 + E_z b_{31}^1) = h^- \left(\frac{\cosh(b) - \cosh(h)}{b^2 - h^2} + E_z \frac{\sinh(b)}{b^2 - h^2} - \frac{\sinh(h)}{h}\right);
\]

\[
g_2^- = (a_{31}^0 + E_z b_{31}^0) - h_z(a_{31}^0 + E_z b_{31}^0) = \frac{b \sinh(b) - h \sinh(h)}{b^2 - h^2} + \]

\[
(E_z - h_z)\frac{\cosh(b) - \cosh(h)}{b^2 - h^2} - E_z h_z \frac{\sinh(b)}{b^2 - h^2} - \frac{\sinh(h)}{h};
\]

\[
g_{123}^- = -E^+(a_{31}^3 + E_z b_{31}^3) = -E^+ \left(\frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(b) - \sinh(h)}{b^2 - h^2}))) + E_z \frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(b) - \sinh(h)}{b^2 - h^2})));\]

\[
g_0^0 = E^+ \sinh(b)/b;
\]

\[
g_2^1 = -(a_{41}^2 + E^+ E^- b_{41}^2) - h_z(a_{41}^2 + E^+ E^- b_{41}^2) = -h_z(\frac{\sinh(b) - \sinh(h)}{b^2 - h^2}) - \frac{\cosh(b) - \cosh(h)}{b^2 - h^2} - \]

\[
E^+ E^- \left(\frac{\partial}{\partial(b^2)}(b^2(\frac{\cosh(b) - \cosh(h)}{b^2 - h^2}))) + h_z \frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(b) - \sinh(h)}{b^2 - h^2})));\]

\[
g_{32}^2 = a_{41}^2 + E^+ E^- b_{41}^2 - h_z(a_{41}^2 + E^+ E^- b_{41}^2) = -h_z(\frac{\sinh(b) - \sinh(h)}{b^2 - h^2}) + \frac{\cosh(b) - \cosh(h)}{b^2 - h^2} + \\

\[
E^+ E^- \left(\frac{\partial}{\partial(b^2)}(b^2(\frac{\cosh(b) - \cosh(h)}{b^2 - h^2}))) - h_z \frac{\partial}{\partial(b^2)}(b^2(\frac{\sinh(b) - \sinh(h)}{b^2 - h^2})));\]

\]
\[ g_{41}^2 = h^-(a_{41}^{22} + E^+ E^- b_{41}^{22}) = h^- \left( \frac{b}{b^2 - h^2} - \frac{\sinh(b)}{h} \right) + E^+ h^- \frac{\partial}{\partial(b^2)} \left( b^2 \left( \frac{b}{b^2 - h^2} - \frac{\sinh(b)}{h} \right) \right); \]

\[ g_{32}^2 = -h^+(a_{41}^{22} + E^+ E^- b_{41}^{22}) = -h^+ \left( \frac{\sinh(b)}{b^2 - h^2} \right) - E^+ h^+ \frac{\partial}{\partial(b^2)} \left( b^2 \left( \frac{b}{b^2 - h^2} - \frac{\sinh(b)}{h} \right) \right); \]

\[ g_4^2 = 2E^+ (a_{41}^4 + E^+ E^- b_{41}^4) = 2E^+ \left( \frac{\partial}{\partial(b^2)} b^2 \left( \frac{\sinh(b)}{b^2 - h^2} - \frac{\sinh(h)}{h^2} \right) \right) + E^+ E^- \frac{\partial^2}{\partial^2(b^2)} b^2 \left( \frac{\sinh(b)}{b^2 - h^2} - \frac{\sinh(h)}{h^2} \right); \]

10 Conclusion

We have calculated the exact representation of the supergroup as well as the supercoherent state in the Hubbard model. These constructions naturally appear in the strongly correlated electronic systems in the case of introducing atomic base in limit of large on-site Hubbard repulsion. The dynamical supergroup which operates in a local superbundle determined by any on-site eigenfunction gives us the wave function in the form of a superspinor. This superspinor describes a local supercoordinates frame in the curved supermanifold. The operator spinor part acting in tangent and cotangent bundles of this supermanifold in supergroup can be reformulated in the terms of the atomic Hubbard operators. Next step lies in the calculation of effective functional of Hubbard model.

References

[1] J. Hubbard, Phys. Rev. Lett. 3, 77 (1959)

[2] E. Fradkin, Field Theories of Condensed Matter Systems, Addison - Wesley, Redwood City (1991)

[3] P. Fulde, Electron Correlations in Molecules and Solids, Springer-Verlag, Berlin (1993)

[4] V.M.Zharkov, Teoret. and Math.Fiz., 60:3, (1984),404-412;

77:1, (1988),1077-1084;

86:2, (1991),181-188.

[5] N.M.J. Woodhouse, Geometric Quantization. Clarendon Press.(1991)