Development of methods to measure the forces at the rear suspension of a race car during racetrack driving

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Abstract. The forces generated by the tyres of a vehicle are responsible for the maximum achievable performance for a race car. In this work, a geometric matrix method combined with a sensitivity matrix method has been applied to a rear multi-link suspension of a rear-wheel-drive race car to estimate the tyre forces from the measurement of the loads acting on the suspension arms. The geometric matrix method calculates the wheel forces from the equilibrium of the wheel assembly, thus involving all reaction forces exchanged with the suspension arms. The reaction forces have been measured through the application of axial strain gauge bridges on the link arms; however, the lower arm has a complex geometry and exchanges multi-axial forces with the upright, therefore a sensitivity matrix method has been implemented. The strain gauges positions have been identified with FE analyses and after installing the sensors, the calibration of the entire suspension assembly has been performed in a dedicated test rig where known forces at the ideal contact point between the wheel and the ground can be applied. After the calibration and validation in the laboratory, the instrumented suspension has been installed in a race vehicle and multiple racetrack acquisitions have been successfully performed.

1. Introduction

In the race vehicles field, minimizing the lap-time is always a performance target, which means to maximize the lateral and longitudinal accelerations developed in the track. The acceleration limits in cornering and braking are typically set by the performance of the tyres [1]. Measuring the wheel forces during racetrack driving allow to characterize and compare different types of tyres [2], but also to estimate the vertical load transfer and the aerodynamic forces, the friction coefficient of the brakes and the design loads.

In the literature, different methods are proposed to measure the forces acting between the tyre and the ground, which can be distinguished between indirect and direct approaches. Indirect approaches estimate the wheel forces from easily available sensor signals like the vehicle accelerations, the longitudinal velocity and the steering wheel angle. A vehicle dynamics model is required, which is usually implemented in the form of a Kalman Filter [3,4] and the accuracy of force estimation depends on the uncertainties in the parameters of the vehicle model, like inertia properties and aerodynamic coefficients which need to be measured through specific tests. On the other hand, direct approaches, on which this study is focused, estimate the loads by measuring the deformations or the accelerations of the tyre, of the wheel or of the suspension components. Braghin et al. [5] attached three accelerometers in the inner liner of the tyre to estimate the vertical load, while Tuononen [6] estimated the vertical, lateral
and longitudinal forces by measuring the tyre carcass displacements with optical sensors. These approaches require to take into account the effects of the tyre speed and internal pressure, increasing the calibration complexity. Measurement systems independent from the tyre operating conditions are the wheel force transducers [7] which are mounted in place of the original wheel, or as an interface with the hub, and then leading to a variation of the upsprung mass that influence the vehicle dynamic response. To maintain unchanged the vehicle static and dynamic characteristics, instrumentation of the vehicle components is required. Cheli et al. [8] measured the wheel forces and moments by applying three strain gauge bridges to the rim, but for motorsport applications where the frequent tyres change is done by replacing the entire wheel, multiple identical measuring systems would be necessary. To overcome these limitations, in a previous work [9], a double-wishbone front suspension of a rear-wheel-drive race vehicle has been instrumented to measure the tyre forces. The axial loads, which equilibrate the tyre forces, have been measured by applying and calibrating a strain gauge bridge on each arm. The tyre forces have been calculated with a geometric matrix method that has been validated with a laboratory test rig before performing racetrack acquisitions. In this paper, the geometric matrix method combined with the sensitivity matrix method is applied to a multi-link suspension. The lower arm of the suspension has a complex geometry and reacts with multi-axial forces to the upright, which have been estimated through a sensitivity matrix method. Then the tyre forces are calculated from the wheel assembly equilibrium through the geometric matrix method.

2. Methods for measuring the suspension forces

The object of the study is a rear left multi-link suspension of a rear-wheel-drive race vehicle. Figure 1 shows the 3D model of the suspension in the static configuration with the acronyms of each arm and the ISO [10] reference system used in this paper. The static configuration is set by the suspension travel of the car at rest with its weight sustained by the wheels.

The arms ARA, CA, SD, and TA (Anti-Rotation Arm, Camber Arm, Spring&Damper, Toe Arm) are tubular components constrained by means of two spherical joints at their ends and will be in the paper referred to link arms. The lower arm (figure 1b) is a tubular arc-welded structure stiffened by folded plates; it is constrained with four spherical joints: two of them are connected to the vehicle frame, one to the ARA arm and one to the upright.

*Figure 1. (a) Suspension 3D model in static configuration, (b) exploded view with internal reaction forces and outline of the methods applied; the reference system is the ISO reference system.*

The wheel-road forces are assumed to be applied at the ideal contact point, placed in the wheel plane orthogonal to the wheel axle and containing the wheel centre, at a distance equal to the wheel radius in the static configuration. The geometric matrix method described in [9], calculates the tyre forces from the equilibrium of the wheel assembly, which is composed of the following macro-components: wheel,
upright and hub. The measurement of all the reactions between the suspension arms and the upright is required to calculate the tyre forces, which become the sole unknown variables in the equilibrium equation. As shown in figure 1b, each link arm force act along the axis defined by the two respective spherical joints and can be measured by applying and calibrating an axial strain gauge bridge. The lower arm and the upright exchange in the common spherical joint a reaction force with variable direction and intensity depending on the applied tyre forces. This reaction force can be expressed as three orthogonal components \( (F_{LA,x}, F_{LA,y}, F_{LA,z}) \) that can be measured with the sensitivity matrix method by applying three strain gauge bridges to the component.

2.1. The geometric matrix method

The vector sum of all the reactions developed between the upright and the suspension arms must be in equilibrium with the net force from the three components \( (F_{c,x}, F_{c,y}, F_{c,z}) \) that develop between the tyre and the ground.

In equation (1) the method applied to the suspension of figure 1 is shown, where the upright reaction forces are multiplied by the geometric matrices giving the tyre forces. The link arms forces \( (F_{ARA}, F_{CA}, F_{SD}, F_{TA}) \) are multiplied by the respective \( [G_{LINKS}] \) geometric matrix, which contains the direction cosines of the local axis \( x_i \) with respect to the global reference system \( (X,Y,Z) \) of the tyre forces. For the sake of clarity, the local axis \( x_i \) of each link arm is defined by the centres of the two spherical joints.

For the lower arm a local reference system is defined having axes \( (x_{LA}, y_{LA}, z_{LA}) \) coincident with the global reference system when the suspension is in the static configuration. The forces \( (F_{LA,x}, F_{LA,y}, F_{LA,z}) \) exchanged between the upright and the lower arm are expressed in its local reference system, the orientation of which vary when the suspension is compressed or extended from the static configuration. The geometric matrix \( [G_{LA}] \) contains the director cosines of the local reference system of the lower arm with the global reference system, then it projects the forces \( (F_{LA,x}, F_{LA,y}, F_{LA,z}) \) in the global reference system.

\[
\begin{bmatrix}
F_{c,x} \\
F_{c,y} \\
F_{c,z}
\end{bmatrix} =
[G_{LINKS}] 
\begin{bmatrix}
F_{ARA} \\
F_{CA} \\
F_{SD} \\
F_{TA}
\end{bmatrix} 
+ [G_{LA}] 
\begin{bmatrix}
F_{LA,x} \\
F_{LA,y} \\
F_{LA,z}
\end{bmatrix}
\]  

(1)

The \( [G_{LINKS}] \) and \( [G_{LA}] \) are calculated according to equations (2) and (3) and can be numerically computed from the coordinates of the suspension kinematic points. When the suspension is in the static configuration the \( [G_{LA}] \) matrix is the identity matrix from the assumptions of the previous paragraph.

\[
[G_{LINKS}] =
\begin{bmatrix}
\cos(x_{ARA}x) & \cos(x_{ARA}y) & \cos(x_{ARA}z) \\
\cos(y_{ARA}x) & \cos(y_{ARA}y) & \cos(y_{ARA}z) \\
\cos(z_{ARA}x) & \cos(z_{ARA}y) & \cos(z_{ARA}z)
\end{bmatrix}
\]  

(2)

\[
[G_{LA}] =
\begin{bmatrix}
\cos(x_{LA}x) & \cos(y_{LA}x) & \cos(z_{LA}x) \\
\cos(x_{LA}y) & \cos(y_{LA}y) & \cos(z_{LA}y) \\
\cos(x_{LA}z) & \cos(y_{LA}z) & \cos(z_{LA}z)
\end{bmatrix}
\]  

(3)

2.2. The sensitivity matrix method

The forces on the suspension arms, required for the geometric matrix method, are estimated through the application of strain gauges to each component. The link arms are subjected to pure axial load along their direction, then an axial full-bridge configuration is suitable to measure the exchanged forces at the upright interface. Equation (4) shows the relation, for a generic link arm \( i \), between the strain gauge
channel CH\(_i\) and the respective F\(_i\) axial force. The sensitivity s\(_i\) is constant and it is calibrated through a uniaxial load testing machine, as described in [9].

\[ F_i = \frac{CH_i}{s_i} \] (4)

The lower arm exchanges multi-axial forces: with the vehicle frame, with the upright in the common joint, with the ARA and ARB link arms along their respective directions. To replicate the suspension constraints, as graphically shown in figure 2a, zero displacements and free rotations (UX=UY=UZ=0) are applied at the two centres of the chassis joints, while the displacement along the direction of the ARA arm is set to be null (UX\(_{ARA}=0\)). The forces (F\(_{LA,x}\), F\(_{LA,y}\), F\(_{LA,z}\)) and the ARB link force are then external forces acting on the lower arm. A sensitivity matrix method (equation (5)) has been applied to the lower arm through the realization of three strain gauge channels (CH\(_{LA,x}\), CH\(_{LA,y}\), CH\(_{LA,z}\)), designed to minimize cross-sensitivity. A strain gauge channel has been realized on the ARB link to measure the force F\(_{ARB}\), the sensitivity of the CH\(_{LA}\) channels to the force F\(_{ARB}\) has been considered by adding a row and a column to the matrix, according to equation (5).

\[
\begin{bmatrix}
CH_{LA,x} \\
CH_{LA,y} \\
CH_{LA,z} \\
CH_{ARB}
\end{bmatrix}
= [S_{LA}]
\begin{bmatrix}
F_{LA,x} \\
F_{LA,y} \\
F_{LA,z} \\
F_{ARB}
\end{bmatrix}
= \begin{bmatrix}
s_{x,x} & s_{x,y} & s_{x,z} & s_{x,ARB} \\
s_{y,x} & s_{y,y} & s_{y,z} & s_{y,ARB} \\
s_{z,x} & s_{z,y} & s_{z,z} & s_{z,ARB} \\
0 & 0 & 0 & s_{ARB}
\end{bmatrix}
\begin{bmatrix}
F_{LA,x} \\
F_{LA,y} \\
F_{LA,z} \\
F_{ARB}
\end{bmatrix}
\] (5)

After calibration, the forces (F\(_{LA,x}\), F\(_{LA,y}\), F\(_{LA,z}\), F\(_{ARB}\)) can be calculated from the inverse of the sensitivity matrix multiplied by the measured signals (CH\(_{LA,x}\), CH\(_{LA,y}\), CH\(_{LA,z}\), CH\(_{ARB}\)).

3. FEA evaluation of the lower arm sensitivity matrix

Finite element simulations have been performed with the software ANSYS\textsuperscript{®} to determine the strain gauges positions. The tubular components and the plates have been discretized with 8-node quadrilateral shell elements, while for the forks and the mountings of the spherical joints 10-node tetrahedral elements have been used; the meshing element size has been set to 2.5mm. The different parts, that in the real component are welded together, have been coupled through beam elements with a circular solid cross-section of 1.25mm and with the same material properties of the lower arm. The arm has been constrained according to paragraph 2.3 and figure 2a, and the forces (F\(_{LA,x}\), F\(_{LA,y}\), F\(_{LA,z}\), F\(_{ARB}\)) have been each one individually applied to evaluate the component surfaces subjected to the higher strains.

In figure 2b-d contour plots of maximum and minimum principal elastic strain are shown and the areas subjected to the higher elastic strains for each load case are highlighted with white dashed lines. The strain gauges of each channel have been positioned according to the directions of the principal elastic strains in the respective load case and in order to minimize cross-sensitivity. Figure 3 shows the strain gauges configurations described below:

- CH\(_{LA,x}\): two adjacent uniaxial strain gauges aligned with the axis of the tubular component, half-bridge configuration (strain gauges on opposite Wheatstone bridge branches);
- CH\(_{LA,y}\): two adjacent XY rosettes with one grid aligned to the axis of the tubular component, full-bridge configuration;
- CH\(_{LA,z}\): two adjacent XY rosettes with grids at an angle of 45° to the axis of the tubular component, full-bridge configuration.
Figure 2. FE model of the lower arm subjected to each force component at the kinematic common point with the upright.

Figure 3. Strain gauges positioning on the lower arm.

The sensitivity matrix has been evaluated from the FE model: each coefficient of equation (6) has been calculated by summing the deformation of each strain gauge grid according to the Wheatstone bridge equation and dividing by the applied force.

\[
\begin{bmatrix}
CH_{LAX} \\
CH_{LAY} \\
CH_{LAz} \\
CH_{ARB}
\end{bmatrix}_{[\mu e]} = \begin{bmatrix}
-0.056 & +0.036 & +0.010 & -0.017 \\
-0.043 & +0.063 & -0.016 & -0.028 \\
-0.024 & -0.043 & +0.163 & -0.051 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
F_{LAX} \\
F_{LAY} \\
F_{LAz} \\
F_{ARB}
\end{bmatrix}_{[N]} = \begin{bmatrix}
\mu e \\
\mu e \\
\mu e \\
\mu e
\end{bmatrix}
\]
4. Test rig calibration of the lower arm

The suspension has been instrumented according to the previous paragraphs, 8 strain gauge channels were realized, one for each link arm (ARA, CA, SD, TA, ARB) and three for the lower arm. The link arms have been individually calibrated to measure the axial force with a uniaxial testing machine, while for the lower arm calibration a test rig was re-designed (figure 4) from the one used for the front suspension [9].

4.1. Test rig design

The test rig is shown in figure 4, where the chassis constraints have been replicated on a frame plate constrained to a fixed support, then each suspension arm has been connected at its respective fork and assembled at the upright. The suspension spring/damper has been replaced by a dummy tubular component to set a constant suspension travel and to deal with a load-independent geometric matrix.

The length of the rigid dummy can be changed, and the chassis plate is designed with multiple holes to switch between three different suspension travel configurations. The three suspension positions considered are the static, the half-extended and the half-compressed suspension configuration. A plate was mounted in the place of the wheel and it carries the forks that allow the forces (F_{cx}, F_{cy}, F_{cz}) application by four servo-hydraulic actuators. The forks are positioned in order to ensure that the axes of the actuators intersect at the ideal wheel-ground contact point when no load is applied. The two MTS servo-hydraulic actuators in the y direction have a load capacity of ±15kN, while the two ITALSIGMA actuators along the x and z direction have a load capacity of ±10kN; they are controlled through an MTSFlextest FT 60 controller.

To reproduce the forces in a forward acceleration, a positive F_{cx} force is applied by the respective actuator and the M_y moment around the hub axis is delivered to the driveshaft which is constrained at its ends with two tripods. For the reproduction of the braking events, a pneumatic manual actuator operates on the pump rod through a leverage mechanism (figure 4). The pressurized caliper blocks the disk and a negative F_{cx} braking force can be applied by the actuator: the M_y moment around the hub axis is delivered mostly to the upright, while a minor component is carried by the driveshaft simulating the engine brake torque.

Lastly, the ARB force can be manually applied and adjusted through a tie rod that connects its link to a fixed support.

4.2. Test procedure

A realistic combination of wheel-ground forces has been applied to the suspension, reproducing a racetrack load history in the test rig. The load history, provided by Michelotto S.a.s., is derived from a vehicle simulation on the flipped circuit MotorLand Aragón (Spain). In the flipped circuit the originally
left turns become right turns and vice versa, while preserving the length and the radius of curvature, in this way the loads on the left suspension are maximized since it runs more frequently on the outer side. The testing procedure is described below, while, in the following figures 5-7, the forces and signals of the rear suspension are plotted against vertical axes that have been normalized by a constant and common force value.

a) Application of the simulated forces \((F_{cx}, F_{cy}, F_{cz})\) with the test rig: the loads of the actuators have been acquired by the MTS controller and are shown with continuous lines in figure 5. For the 8 strain gauge channels an IMC CRONOS PL-2 acquisition unit has been used, the three of them applied to the lower arm are plotted in figure 6. The sampling frequency has been set equal to 500Hz.

b) Calculation of the lower arm \((F_{LA,x}, F_{LA,y}, F_{LA,z})\) applied forces from the equation (1) for each timestep of the acquired signals: the link arms \((F_{ARA}, F_{CA}, F_{SD}, F_{TA})\) forces are multiplied by the matrix \([G_{LINKS}]\) and subtracted from the \((F_{cx}, F_{cy}, F_{cz})\) forces, the resultant vector then pre-multiplied by the inverse of the matrix \([G_{LA}]\). The resultant forces are named \((F_{LA,x}, F_{LA,y}, F_{LA,z})\), the first of them is plotted in figure 7 with a continuous line.

c) Evaluation of the sensitivity matrix: by multiple linear regression of the \((F_{LA,x}, F_{LA,y}, F_{LA,z}, F_{ARB})\) forces against the measured lower arm signals \((CH_{LA,x}, CH_{LA,y}, CH_{LA,z}, CH_{ARB})\) the sensitivity matrix coefficients are evaluated. Naming \((F_{LA,x,s}, F_{LA,y,s}, F_{LA,z,s})\) the forces calculated from the inverse experimental sensitivity matrix multiplied by the signals \((CH_{LA,x}, CH_{LA,y}, CH_{LA,z}, CH_{ARB})\), the objective of the multiple linear regression is to minimize the sum of the squared errors between \((F_{LA,x,s}, F_{LA,y,s}, F_{LA,z,s}, F_{ARB})\) and \((F_{LA,x}, F_{LA,y}, F_{LA,z}, F_{ARB})\).

d) Calculation of ideal wheel-ground contact point forces \((F_{cx}, F_{cy}, F_{cz})\) from the instrumented suspension according to equation (1): using as inputs the link arms measured axial forces \((F_{ARA}, F_{CA}, F_{SD}, F_{TA})\) and the lower arm forces \((F_{LA,x,s}, F_{LA,y,s}, F_{LA,z,s})\) calculated from the experimental sensitivity matrix. The instrumented suspension forces \((F_{cx}, F_{cy}, F_{cz})\) are plotted with dashed lines in figure 5 to compare with the forces applied by the actuators, displayed in continuous lines.

![Figure 5](image-url) Simulated load history at the ideal wheel-ground contact point of flipped MotorLand Aragón racetrack, comparison between applied forces by the actuators (continuous lines) and measured forces by the instrumented suspension (dashed lines).
Figure 6: Lower arm $CH_{LA}$ measured signals during application of figure 5 racetrack load history at the ideal wheel-ground contact point. The signals have been normalized by a constant force value, as done in figure 5 and figure 7.

Figure 7: Lower arm $F_{LA,X}$ force during application of figure 5 racetrack load history at the ideal wheel-ground contact point.

4.3. Test rig results

The flipped Aragón circuit simulation consists of three right turns where the instrumented suspension is on the outer side, two left turns, four braking events and two straightaways. The load history has been applied with the test rig in the three suspension travel configurations, considering the three different geometric matrices. In order to minimize the errors on the forces estimated by the instrumented suspension, two different sensitivity matrices have been evaluated.

Driving condition: a positive $F_{c,x}$ is applied at the ideal wheel-ground contact point and the $M_y$ moment around the hub axis is carried by the driveshaft; the $F_{LA,x}$ force is mainly positive as can be seen from figure 7. The corresponding Lower arm sensitivity matrix is displayed in equation (7):

\[
\begin{pmatrix}
CH_{LA,x} \\
CH_{LA,y} \\
CH_{LA,z} \\
CH_{ARB}
\end{pmatrix}_{[\mu e]} =
\begin{bmatrix}
-0.051 & +0.032 & +0.007 & +0.003 \\
-0.036 & +0.055 & -0.006 & -0.005 \\
-0.032 & -0.039 & +0.129 & -0.033 \\
0 & 0 & 0 & s_{ARB}
\end{bmatrix}
\begin{pmatrix}
F_{LA,x} \\
F_{LA,y} \\
F_{LA,z} \\
F_{ARB}
\end{pmatrix}_{[N]}^\text{driving}
\] (7)
Braking condition: a negative $F_{c,x}$ is applied at the ideal wheel-ground contact point and the $M_y$ moment around the hub axis is delivered mostly to the upright while a minor component is carried by the driveshaft; the $F_{LA,z}$ force is mainly negative as can be seen from figure 7. The corresponding Lower Arm sensitivity matrix is displayed in equation (8):

$$
\begin{bmatrix}
CH_{LA,x} \\
CH_{LA,y} \\
CH_{LA,z} \\
CH_{ARB} \end{bmatrix}
\text{[με]} =
\begin{bmatrix}
-0.050 & +0.033 & +0.009 & +0.001 \\
-0.044 & +0.057 & -0.008 & -0.005 \\
+0.005 & -0.043 & +0.136 & -0.034 \\
0 & 0 & 0 & S_{ARB}^{braking} \end{bmatrix}
\begin{bmatrix}
F_{LA,x} \\
F_{LA,y} \\
F_{LA,z} \\
F_{ARB}\end{bmatrix}\text{[N]}
$$

The differences between the experimental (equations (7)-(8)) and the numerical (equation (6)) sensitivity matrices, ranging from 10% to 20% for the coefficients in the main diagonal, are believed to be caused by the following factors:

- the strain gauges of the $CH_{LA,y}$ channel are located in the proximity of multiple nearby weld beads that have been FE modelled with beam elements to simplify the analysis, but leading to different strain distribution in the contiguous mesh elements with respect to the real component;
- the lower arm is over-constrained along the direction defined by the two spherical joints at the chassis, leading to reaction forces that depend on the stiffness of the supports, not being modelled in the FE model;
- possible misalignments during the application of the strain gauges due to the complex geometry of the lower arm.

These differences underline the requirement to have performed the experimental calibration of the sensitivity matrix of the lower arm.

The differences between the braking and traction matrices have been found principally in the ($CH_{LA,x}$, $CH_{LA,y}$, $CH_{LA,z}$) channels response to $F_{LA,x}$. They are explained considering the $F_{LA,x}$ force sign: when negative, the load is mainly carried by the side of the upright fork towards the ARA arm since it is stiffened by the gusset plate; when positive, a higher contribution of the load is carried by the front side of the upright fork. This leads to different strain distribution when $F_{LA,x}$ forces with the same intensity but different sign are applied.

The experimental sensitivity matrices have been used for the calculation of the lower arm forces ($F_{LA,x}$, $F_{LA,y}$, $F_{LA,z}$) from the signals ($CH_{LA,x}$, $CH_{LA,y}$, $CH_{LA,z}$, $CH_{ARB}$), then the forces ($F_{c,x}$, $F_{c,y}$, $F_{c,z}$) have been evaluated according to point (d) of paragraph 4.2. The forces applied by the actuators are plotted in continuous line in figure 5, while the forces estimated from the instrumented suspension are reported in dashed line. The following table 1 summarizes the achieved mean percentage errors on wheel forces estimation.

| Load direction | Mean error |
|----------------|------------|
| $F_{c,x}$ traction | 5%         |
| $F_{c,x}$ braking  | 7%         |
| $F_{c,y}$ outer side turn | 5%         |
| $F_{c,y}$ inner side turn | 10%        |
| $F_{c,z}$            | < 2%       |

5. Racetrack tests

The instrumented rear suspension has been installed in the car to acquire racetrack loads at the MotorLand Aragón circuit. The car is also equipped with five potentiometers which measure the suspension travel and the steering wheel angle, while an accelerometer acquires the longitudinal
acceleration and the lateral acceleration of the vehicle in its center of mass. Other acquired signals are the vehicle longitudinal speed and the brakes pressures. All the signals are acquired from the vehicle data logger via the CAN bus, at a frequency of 100Hz; the post-processing procedure has been implemented through the MATLAB® software in an automated routine, which is schematically depicted in figure 8.

Figure 8: Post-processing procedure for wheel forces calculation from racetrack acquired data. The picture of the car is not representative of the real vehicle.

The strain gauge signals of the suspension are zeroed at the start of every acquisition session with the car lifted from the ground, so the arms are loaded only with the wheel assembly load along the negative \( F_{c,z} \) direction: this is set as the zero vertical force value for the instrumented suspension. When the car is lowered to the ground, the instrumented suspension measures a vertical load equal to the sum of the weight of the wheel assembly and the left rear portion of the sprung mass.

The arms signals are then filtered to reduce the signal noise due to the engine vibrations and to the road roughness: a zero-phase digital 4\(^{th}\) order Butterworth filter with a cut-off frequency of 20Hz has been applied through the MATLAB® \texttt{filtfilt} function. Figure 8 reports the comparison between the unfiltered and the filtered signal for the TA link arm and highlights a halving of the standard deviation.
The calculation of the link arms forces is straightforward with equation (4), however, for the lower arm, the brake pressure signal at the rear callipers ($p_{\text{brake}_R}$) is used to switch between the braking and the driving sensitivity matrices. The wheel forces are then calculated according to equation (1), considering a variable geometric matrix depending on the suspension spring displacement. At each step the geometric matrices are calculated from the signal $l_{SD,RL}$ of the rear left suspension potentiometer and multiplied by the respective arm forces resulting in the wheel forces.

In figure 9 the accelerations and wheel forces acquired during a lap are plotted in the reference system of figure 8. The wheel forces have been normalized by the same force value used in the previous paragraph.

![Figure 9: Acquired accelerations and rear left wheel forces from the instrumented suspension for a lap of the MotorLand Aragón circuit. The wheel forces have been normalized by the same force value used in figure 5.](image)

The maximum lateral load $F_{c,y}$ have been measured in the 3rd turn of the circuit where the instrumented suspension is on the outer side, it is associated with the maximum vertical load $F_{c,z}$ of the lap due to the combined lateral and longitudinal load transfer effect. The maximum traction positive $F_{c,x}$
force has been measured at the exit of the 6th turn where the instrumented suspension is on the outer side and thanks to the limited slip differential the stiffer tyre generates the higher longitudinal load. Lastly, the bigger $F_{cx}$ braking force has been measured before the turn 1 of the circuit.

6. Conclusions

In this work, a geometric matrix method combined with a sensitivity matrix method has been developed and experimentally validated to measure the loads acting on a rear multi-link suspension of a rear-wheel-drive race car. The axial loads on the link arms have been measured through the application of axial strain gauges in a full bridge configuration, while two full-bridge and a half-bridge configurations have been applied on the lower arm to measure the three force components exchanged with the upright. The sensitivity matrix has been firstly evaluated through FE analyses and subsequently calibrated in a test rig dedicated to the entire suspension assembly. The calibration has been performed by applying a simulated load history of the MotorLand Aragón circuit at the ideal contact point between the wheel and the ground. Two different sensitivity matrices have been evaluated depending on the sign of the $F_{cx}$ force applied at the ideal contact point, namely for driving and braking conditions, respectively. The experimental coefficients on the main diagonal differs between 10% and 20% from the numerically evaluated ones. After the calibration of the sensitivity matrix of the lower arm, the wheel forces estimated from the instrumented suspension have been compared with the actual applied ones to calculate the errors achieved in a racetrack acquisition. The errors on the estimated tyre forces are lower than 5% for mid-corner, turn-exit and in the straightaways, while they are lower than 7% for braking and turn-in events. After the laboratory tests for the validation, the instrumented suspension has been installed in the vehicle and multiple racetrack acquisitions have been successfully performed. The instrumented suspension will be used in future works for the characterization of the tyre and brake performance starting from racetrack acquisitions.

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