Semi-Analytical Modelling of Synthetic Static Stiffness for Machining System of Five Axis Machine Tool

Xianyin Duan 1*, Xinyue Chen 2, Fangyu Peng 3 and Guozhang Jiang 1

1 Key Laboratory of Metallurgical Equipment and Control Technology, Wuhan University of Science and Technology, Wuhan, China
2 Hubei Key Laboratory of Mechanical Transmission and Manufacturing Engineering, Wuhan University of Science and Technology, Wuhan, China
3 State Key Laboratory of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan, China

*Corresponding author’s e-mail: dxy@hust.edu.cn

Abstract. A static stiffness model of the machine tool end considering the whole kinematic chain of the machining system is proposed. The whole kinematic chain is divided into the cutter subsystem and the handle-spindle-motion axes subsystem. The two subsystems are equivalent to a variable diameter cantilever and a variable length cantilever. The 3D static stiffness at the end of the machining system could be resulted by both theoretical calculation and experimental measurement. The static stiffness along different directions are analysed based on the force ellipsoid theory.

1. Introduction
Elastic deformation will occur in the kinematic chain of the machine tool machining system under action of cutting force. Usually, the tool subsystem is a weak stiffness link in the whole machining system and attracts attentions of many scholars. However, under many working conditions, such as large-scale multi-axis linkage machine tools with large number of moving axes, ultra-high strength workpiece processing, parts with abnormal posture to process complex curved surface parts, the mechanical deformation of parts other than tool parts has become very important. Therefore, it is necessary to consider the stiffness of the whole transmission link for the multi-axis linkage machining and obtain the comprehensive stiffness of the machining system.

Jacobi matrix model [1-2] was used to establish the stiffness model of machine tool machining process system and could obtain the analytical expression of the stiffness of the process system. It was convenient to establish the mapping relationship between the velocity and force between the joint space and Cartesian space, but this method was only suitable for modeling the stiffness of the process system under simple deformation conditions.

Matrix displacement model [3-5] included the idea of both discrete elements and matrix set, which could reflect the stiffness characteristics of machining systems with different configurations, and it was based on the principle of virtual work, which could explain the relationship between the deformation and the force of each link in the machining system. So it was suitable for establishing the
rigidity model of special machine tools and it was difficult to model the rigidity field of machine tools with arbitrary structure.

The finite element model [6-8] could reflect the stress and deformation of the process system and had high calculation accuracy. But for the multi-axis machining system, because of the large number of node variables, with the increase of the model mesh, the time cost of calculation was huge, and the model needed to be reconstructed after the system deformation. Moreover, the calculation efficiency was difficult to meet the needs. Therefore, this method was mostly used in simple motion system.

Peng [9] established a general stiffness model of multi-axis machining system by Jacobi and point transformation method. But it is necessary to improve in the way of changing the magnitude of the applied force and achieve a more convenient way to change the magnitude of the applied force and have enough security for the machining system in test. This paper divides the machine tool machining system into the cutter subsystem and the handle-spindle-motion axes subsystem. Stiffness model of the cutter is established by accurate theoretical calculation. Stiffness at the end of the machining system is obtained by stiffness calibration experiment. On these basis, stiffness of the handle-spindle-motion axes subsystem is gotten. The details are as follows.

2. Static stiffness equivalent model of multiple spindle tool

In the kinematic chain of multiple spindle machine tool, the tool is located at the end of the whole kinematic chain as the execution link, and the static stiffness of the tool location is determined by the stiffness of each link including each motion axes of machine tool, the machine tool spindle, the cutter handle and the cutter itself. As shown in Figure 1 (a). The static stiffness of the tool could be calculated more accurately by theoretical calculation because of its regular geometry and good isotropy of material. The static stiffness of the other parts is difficult to be calculated by theoretical calculation or its results far from the actual because of the complexity or inaccessibility of geometry, structural characteristics, coordinate relationship and material properties. Therefore, the whole machining system is divided into the cutter subsystem and the “handle-spindle-motion” axes subsystem.

![Figure 1. kinematic chain of five axis machine tool and static stiffness equivalent model.](image)

Specifically, the “handle-spindle-motion” axes subsystem in machine tool machining systems is equivalent to a variable length cantilever. As shown in the dotted line in Figure 1 (b), a variable length cantilever is a circular cantilever beam with equal cross-section. The base of the fixed-end machine tool of the beam is the G-point in Figure 1 (b), and the end of the beam is the lowest point of handle.
and tool grasp section. The static stiffness of the A point at the end of the beam is the final static stiffness of the “handle-spindle-motion” axes subsystem. In practice, the static stiffness of the “handle-spindle-motion” axes subsystem is different in different directions. Correspondingly, the cantilever beam is an equivalent beam of variable length, and its length is related to the direction of static stiffness, so it is called “a variable length cantilever”. Therefore, the end of the subsystem has a direction of greater static stiffness, and the corresponding length of the variable length cantilever is smaller, while the direction of smaller static stiffness is corresponded to that of the variable length cantilever.

Similarly, the cutter subsystem is equivalent to a variable cross-section cantilever beam consisting of a tool rod cantilever and a tool tooth. As shown in the solid line of Figure 1 (b), the variable cross-section cantilever beam is composed of two sections of circular cross-section beams in different diameters. The fixed end of the beam is connected to the end of the variable length cantilever, that is, the A point in Figure 1 (b). The end of the beam is the cutter location of the tool (point C in Figure 1 (b)). The break point of two sections of the beam is point B in Figure 1 (b), which represents the dividing point between the cutter bar section and the cutter tooth section of the cutter.

As is shown in Figure 2, calculation idea of the synthetic static stiffness is as follows:

- The machine tool processing system is divided into the cutter subsystem and the handle-spindle-motion axes subsystem. The 3D synthetic static flexibility \( S_{L1} \) at the end of the machine tool processing system is obtained by static stiffness test.

- The tool subsystem \( t_1 \) used in the static stiffness testing experiment is theoretically calculated, to obtain the 3D static flexibility \( S_{t1} \) at the end of the subsystem. The tool subsystem is separated from the machine tool processing system and obtained the 3D static flexibility \( S_{A1} \) of the handle-spindle-motion axes subsystem

\[
S_{A1} = S_{L1} - S_{t1}. \tag{1}
\]

- The end of 3D static flexibility \( S_{t2} \) cutter subsystem \( t_2 \) (corresponding tool geometry, material and grasping length) used in practical machining is calculated theoretically.

- Adding up 3D static flexibility \( S_{t2} \) at the end of tool subsystem \( t_2 \) and 3D static flexibility \( S_{A1} \) of the handle-spindle-motion axes subsystem, the 3D static flexibility \( S_{L2} \) at the end of machine tool system is obtained.

\[
S_{L2} = S_{A1} + S_{t2}. \tag{2}
\]

3. Static stiffness of the “motion-spindle-handle” axes subsystem.

Taking the A point as the origin and the X axis, Y axis and Z axis under the machine tool coordinate system as X axis, Y axis and Z axis, the equivalent beam coordinate system \( (O_A, X_A, Y_A, Z_A) \) could be established. The method to establish the static stiffness model of the “motion-spindle-handle” axes subsystem by calculating the equivalent beam length along the coordinate axis through experimental calibration.

As shown in Figure 1 (b), the tool rotation angles are set to zero. Along the X axis direction of the equivalent beam coordinate system at the end of the machining system apply a force \( f_{x0} \), that is, the cutter location C. The coupling effect of deformation between different directions is not considered. Under the force \( f_{x0} \), the displacement deviation at point C in the direction of the force is \( e_{Cx0} \). The deviation of the cutter tooth in this direction of the force is \( e_{fx} \), and angular deviation \( \theta_{Bx} \) is also generated in the Y axis of the equivalent beam coordinate system. The deviation at point B in the
direction of the force is $e_{Bz}$. By applying the deformation superposition principle and considering the above deviations, we could get the following deformation equation:

$$
e_{Cz0} = \frac{f_{z0}}{3\mu t^2EI}(L_f - r)^2 + \frac{f_{z0}}{3EI}(L_{ex} + L_s)^2$$

$$+ \frac{f_{z0}}{2EI}(2(L_{ex} + L_s)(L_{ex} + L_{sf} - r) - (L_{ex} + L_s)^2)(L_f - r),$$

where, $L_f$ is the length of the cutter teeth. It is assumed that the material and section size of a circular equivalent beam are consistent with the tool material and section size used in the calibration.

In this equation, $E$ is the elastic modulus of the tool material. $I$ is the inertia moment of the tool rod part. $\mu t$ is the effective diameter coefficient of the tool tooth part, and $\mu t = D_e/D$ (the effective diameter of the tool tooth part). $L_r$ is the overhanging length of the tool rod part. $L_{sf}$ is the total overhanging length of the tool, and $L_{sf}' = L_e + L_f$.

From Eq. (3) could get

$$L_{ex} = \frac{\sqrt{9L_f^4 - \frac{6}{\mu t^4}L_f^3 - \frac{4}{\mu s^4}L_f^2 + \frac{18EI}{K_{Cz0}} + \frac{12EI}{K_{Cz0}} - 3L_f^2}}{3L_f' + 2},$$

where, $K_{Cz0}$ is the measured static stiffness of the end of the machine tool machining system, which is obtained by the calibration experiment, and $K_{Cz0} = f_{z0}/e_{Cz0}$. $L_f'$ is an intermediate variable, which is obtained by $L_f' = L_f - r$.

Similarly, $L_{ey}$ could be gotten as

$$L_{ey} = \frac{\sqrt{9L_f^4 - \frac{6}{\mu t^4}L_f^3 - \frac{4}{\mu s^4}L_f^2 + \frac{18EI}{K_{Cy0}} + \frac{12EI}{K_{Cy0}} - 3L_f^2}}{3L_f' + 2},$$

where, $K_{Cy0}$ is the measured value of static stiffness at the end of machine tool processing system, which could obtain from calibration experiments, and $K_{Cy0} = f_{y0}/e_{Cy0}$.

Along the X axis of the equivalent beam coordinate system, the added force of facilities is $(f_z)$. Under this force the deviation of the C point at the end of the machine tool processing system in the direction of the force is $e_{Cz0}$, the deviation of cutter teeth, cutter rod cantilever and equivalent beam under force $(f_z)$ are $e_{Bz}$, $e_{Bz}$ and $e_{Az}$ respectively. By applying the deformation superposition principle and considering the above deviations, we could get the following deformation equation

$$e_{Cz0} = \frac{f_{z0}L_f}{\mu t^2EA_s} + \frac{f_{z0}L_{ex}}{EA_s} + \frac{f_{z0}L_{ex}}{EA_s},$$

where, $A_s$ is the cross-section area of the cutter bar part.

From Eq. (6), we could get

$$L_{ez} = \frac{EA_s}{K_{Cz0}} - \frac{L_f}{\mu t^2} - L_s,$$

where, $K_{Cz0}$ is the measured value of static stiffness at the end of machine tool processing system, which could obtain from calibration experiments, and $K_{Cz0} = f_{z0}/e_{Cz0}$.

Thus the “variable length equivalent beam” model is established. As mentioned above, the material and section size of the beam in the model are taken as the material and section size of the tool in the calibration experiment. Then the static stiffness at the end of the “motion-spindle-handle” axes subsystem is obtained by applying the deformation equation of circular cantilever beam as
\[ K_A = \left( \frac{3EI}{L_{ex}^3} \right) \left( \frac{3EI}{L_{ey}^3} \right) \left( \frac{EA_t}{L_z} \right). \] (8)

4. Static stiffness of cutter subsystem

As shown in Figure 1 (b), because of the symmetry of the cutter along the section direction, by using the deformation superposition principle, considering the force and deformation of the AB section of the cutter bar, the force and deformation of the BC section in the cutter tooth and the angle deformation \( \theta_B \) of point B at the boundary between the cutter bar and the cutter tooth. The static stiffness model of tool subsystem in each coordinate axis along its own coordinate system is established

\[ K_i = \begin{cases} \frac{6EI}{3L_z^2(L_{sf} - r) - L_z^3 + 2L_z^2/\mu_s^4 + 3(2L_z(L_{sf} - r) - L_z^2)(L_f - r)} \end{cases} \] (9)

5. Angle static stiffness at the boundary

As shown in Figure 1 (b), there is an angle factor that determined the static stiffness of point C at the end of the machine tool machining system, which is the angle deviation \( \theta_A \) of point A at the boundary of the “motion-spindle-handle” axes subsystem and the cutter subsystem. The effect of the angle deviation on the static stiffness of point C at the end of the machine tool machining system couldn’t be ignored, and the static stiffness of the angle is given as

\[ K_{\theta, \phi} = \begin{cases} \frac{2EI}{L_{ex}(L_{ex} + L_{sf} - r) - L_{ex}^2(L_{sf} - r)}, & \text{for } 6. \text{ Synthetic static stiffness}

We could obtain the synthetic static stiffness at the end of the machine tool machining system by adding up the static stiffness of the “motion-spindle-handle” axes subsystem, the static stiffness of the cutter subsystem and the angular stiffness at the boundary of the two subsystems. When the cutter tilted forward or sideways, it is necessary to consider the influence of the tool tilt angle on the end of 3D static stiffness, and to unite the two subsystems into the same coordinate system.

7. Conclusions

The multi axis CNC machining system is divided into cutter subsystem and the handle-spindle-motion axes subsystem. The former part establishes the stiffness model by accurate theoretical calculation. The latter part obtains the synthetic stiffness at the end of the machining system by stiffness calibration experiment and uses it to subtract the stiffness of the cutter part to obtain the stiffness of this part. The innovation of the stiffness calculation method in this paper is that the difference of the latter part in the direction of three coordinate axes is considered to obtain more accurate results through actual measurement in each direction. Then the 3D stiffness is transformed into the cutter coordinate system by the rotation coordinate transformation of the kinematic chain and added up the partial stiffness of the cutter subsystem to obtain the synthetic stiffness.
Acknowledgments
This work is supported by National Natural Science Foundation of China under Grant No. 51605346, and China Postdoctoral Science Foundation under Grant No. 2016M602374.

References
[1] Simaan N, Shoham M. (2003) Geometric interpretation of the derivatives of parallel robots’ jacobian matrix with application to stiffness control. Journal of Mechanical Design, 125(1): 33-42.
[2] Zhang D, Xi F, Mechefske C.M, et al. (2004) Analysis of parallel kinematic machine with kinetostatic modelling method. Robotics and Computer-Integrated Manufacturing, 20(2): 151-165.
[3] Dong W, Du Z, Sun L. (2005) Stiffness influence atalases of a novel flexure hinge-based parallel mechanism with large workspace. Intelligent Robots and Systems, (IROS 2005). 2005 IEEE/RSJ International Conference on. IEEE, 856-861.
[4] Lv Y.N, Wang L.P, Guan L.W. (2008) Stiffness analysis and optimization of a hybrid machine tool based on the stiffness matrix. Journal of Tsinghua University (Science and Technology), 48(2): 180-183.
[5] Huang T, Zhao X, Whitehouse D J. (2002) Stiffness estimation of a tripod-based parallel kinematic machine. Robotics and Automation, IEEE Transactions on, 18(1): 50-58.
[6] Carbone G, Lim H, Takanishi A, et al. (2006) Stiffness analysis of biped humanoid robot WABIAN-RIV. Mechanism and machine theory, 41(1): 17-40.
[7] Liu Y, Wang J, Wang L. (2006) Stiffness optimization of a heavy hybrid machine tool named XNZH2430. Journal of Tsinghua University, 46(8): 1418.
[8] Mekaouche A, Chapelle F, Balandraud X. (2015) FEM-Based Generation of Stiffness Maps. Robotics, IEEE Transactions on, 31(1): 217-222.
[9] Peng F, Yan R, Chen W, et al. (2012) Anisotropic force ellipsoid based multi-axis motion optimization of machine tools. Chinese Journal of Mechanical Engineering, 25(5): 960-967.