Abstract

Inspired by the recent determination of the W-boson mass by the CDF collaboration, we revisit an SO(10) axion model in which a scalar SU(2)_L triplet field with zero hypercharge is known to acquire a non-zero VEV through its mixing with the Standard Model Higgs doublet. The triplet VEV provides a sizable contribution to the W mass, which helps in significantly lowering the 7σ discrepancy between the Standard Model prediction and the higher CDF value for m_W. We show that the relatively light triplet mass (∼ (1−50) TeV) is compatible with gauge coupling unification and observable proton decay. An unbroken Z_2 gauge symmetry, coupled with the presence of two fermionic 10-plets required to resolve the axion domain wall problem, means that both axions and a stable intermediate mass (∼ 10^9 − 10^{10} GeV) fermion are plausible dark matter candidates. We also display the gravitational wave spectrum from the intermediate scale topologically stable cosmic strings predicted by the model.
1 Introduction

In a recent paper [1] largely concerned with the electroweak monopole in Grand Unified Theories (GUTs), it was briefly noted that a specific $SO(10)$ axion model contains a $SU(2)_L$ scalar triplet field with hypercharge $Y = 0$ that acquires a non-zero vacuum expectation value (VEV) through its mixing with the Standard Model (SM) Higgs doublet. It is well-known that this VEV only contributes to the $W$-boson mass, which makes the $SO(10)$ axion model attractive in light of the recent measurement $m_{W} = 80.4335 \pm 0.0094$ GeV [2]. The CDF result is about $7\sigma$ away from the central value estimated within the SM [3–5], and a large number of papers that can explain this deviation has been proposed [6–60].

In order to realize (better) agreement with the higher value for $m_{W}$ determined by CDF, the triplet VEV should make a significant contribution to the $W$ mass while maintaining compatibility with the SM $\rho$ parameter. This requires the triplet mass to be of order $10$ TeV or so, and we show how this is achieved in the $SO(10)$ axion model while preserving the unification of the SM gauge couplings. In addition to the axion, the model also contains an intermediate-mass fermion dark matter (DM) candidate whose stability is guaranteed by a discrete $Z_2$ gauge symmetry. This $Z_2$ symmetry is also responsible for the existence of topologically stable cosmic strings.

The plan of the paper is as follows. In Section 2 we summarize the salient features of the model including the symmetry breaking pattern and the realization of higher $m_{W}$ compared to the SM. Section 3 deals with gauge coupling unification and implications for proton decay. Section 4 discusses the DM candidates consisting of axions and intermediate-mass neutral fermions. The gravitational wave (GW) spectrum from the intermediate scale cosmic strings is discussed in Section 5, and we conclude with a summary in Section 6.

2 The Model

In this section, we briefly outline the salient features of the $SO(10)$ axion model and refer the reader to Refs. [61,62] for additional details. To start with, we first describe the particle content of the setup and then present all the relevant interactions of these particles. We denote the fermion multiplets present in the model as

$$\psi^{(i)}_{10}(1), \; \psi^{(\alpha)}_{10}(-2), \; (i = 1, 2, 3), \; (\alpha = 1, 2),$$

and the scalar multiplets as

$$\phi_{10}(-2), \; \phi_{45}(4), \; \phi_{126}(2), \; \phi_{210}(0).$$

Here, the subscripts refer to the dimension of the representations under $SO(10)$, the Peccei-Quinn (PQ) charges ($Q_{PQ}$) are quoted within parentheses, and $i$ and $\alpha$ are the generation indices for the fermions. With the knowledge of the particle spectrum and symmetries of the model, next, we present all the relevant interactions involving these fields. The Yukawa couplings
are
\[ \psi^{(i)}_{16} \psi^{(j)}_{16} \phi_{10}, \quad \psi^{(j)}_{16} \psi^{(j)}_{126} \phi_{10}, \quad \psi^{(1)}_{10} \psi^{(2)}_{10} \phi_{45}, \] (3)
and the scalar couplings include
\[ \phi_{210}^{\dagger} \phi_{126}^{\dagger} \phi_{45}, \quad \phi_{210}^{\dagger} \phi_{126} \phi_{10} \phi_{45}, \quad \phi_{210} \phi_{126} \phi_{10}. \] (4)

The SM fermions of each family, accompanied by a SM singlet right-handed neutrino, reside in the 16-dimensional representation of \( SO(10) \). In addition, two generations of fermionic 10-plets are included to overcome the axion domain wall problem \([61,62]\). The electroweak sector of the model contains the SM Higgs doublet, which is a linear combination of the two \( SU(2)_L \) doublets from \( \phi_{10} \) and two doublets from \( \phi_{126} \). The three remaining scalar doublets obtain masses of order \( M_{II} \) \([63]\).

At this stage some remarks about the so-called quality problem in axion models are in order. If one makes the rather arbitrary assumption that Planck scale suppressed operators are present in axion models, it then follows that they must not be permitted to spoil the axion resolution of the strong CP problem. We are therefore lead to the conclusion that the coefficients accompanying the potentially dangerous operators, dimension five (e.g., \( \phi_{45}^{4} \phi_{210}^{4} \), \( \phi_{45}^{3} \phi_{210}^{3} \)) and some higher ones in our case, must be adequately suppressed. Clearly, such operators do not arise in the renormalizable \( SO(10) \) framework, and their occurrence in the presence of gravity has not been convincingly demonstrated. Indeed, it has been suggested that wormhole tunneling may give rise to \( U(1)_{\text{PQ}} \) symmetry violating effects with exponentially suppressed coefficients, and they only become important for \( f_a \gtrsim 10^{17} \) GeV \([64-66]\). In our model the axion decay constant \( f_a \) is, of course, orders of magnitude smaller than \( 10^{17} \) GeV and the problem is therefore avoided.

Be that as it may, perhaps a more elegant approach for resolving the axion quality problem is to assume that a suitable discrete gauge symmetry effectively behaves as \( U(1)_{\text{PQ}} \). Discrete gauge symmetries routinely arise from the four dimensional compactification of higher dimensional superstring theories, and the first examples based on this idea have been discussed in Ref. \([67]\).

Finally, as shown in Ref. \([68]\), it is possible that \( U(1)_{\text{PQ}} \) may appear as an accidental symmetry in \( SO(10) \) models supplemented by a suitable continuous gauge symmetry. In this case too the axion quality problem is suitably ameliorated.

For definiteness, we employ a specific symmetry breaking pattern of \( SO(10) \) shown in Eq. (5) which, among other things, also allows a light \( SU(2)_L \) scalar triplet from \( \phi_{45} \) that remains compatible with the unification of the SM gauge couplings. Note that the induced VEV of the
the electroweak mixing angle and $Z$ and $v$ 

The scalar triplet arises from the coupling $\phi_{10}\phi_{10}\phi_{45}$.

\[
SO(10) \times U(1)_{PQ} \xrightarrow{\langle 210(0) \rangle_{M_U}} SU(2)_L \otimes SU(2)_R \otimes SU(4)_C \times U(1)_{PQ} \xrightarrow{\langle 1,1,15 \rangle_{45}} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes \mathbb{Z}_2 \times U(1)_{PQ} \xrightarrow{\langle 1,1,0 \rangle_{45(4)}} SU(3)_C \otimes U(1)_Q \otimes \mathbb{Z}_2.
\]

We employ two-loop renormalization group equations (RGEs) to estimate the GUT scale ($M_U$) and the two gauge symmetry breaking intermediate-scales $M_I$ and $M_{II}$. We find it instructive and useful to distinguish the two latter scales from the axion symmetry breaking scale $f_a$ ($\leq M_{II}$). The remnant anomalous global symmetry after $M_{II}$ is $U(1)_{PQ'}$, which is generated by $Q_{PQ} = 5Q_{PQ} - 3(B - L) + 4T^3_R$, where $T^3_R$ is the diagonal generator of $SU(2)_R$. The $U(1)_{PQ}^0$ symmetry is broken by the VEV of $(1, 1, 15)$ and $(1, 3, 1)$ in $45(4)$ at the scale $f_a$. The fermions from $\psi_{10}$ acquire masses during this symmetry breaking which we assume are all of the same order of magnitude, $m_{DM} = y_{45} \langle \phi_{45} \rangle$. The lightest neutral fermion from the 10-plets along with the axion can account for the observed dark matter relic density of the universe [69] (see Ref. [62] for details).

At this stage, it is important to point out that the above breaking chain allows a light $SU(2)_L$ scalar triplet from $\phi_{45}$ that is compatible with the unification of the gauge couplings. We now shed light on this scalar triplet and describe its role in raising the $W$-boson mass above the SM prediction as suggested by the CDF result. We can write the scalar triplet interaction that arises from the term $\phi_{10}\phi_{10}\phi_{45} + \text{h.c.}$ of Eq. (4) as

\[
-\lambda m_T H^u_i T^i H^d_{10} + \text{h.c.}
\]

Here $T^i \equiv (1, 3, 0)$ is the complex triplet scalar from $\phi_{45}$, and $H^u_{10}(\equiv (1, 2, 1/2)) \oplus H^d_{10}(\equiv (1, 2, -1/2))$ arise from the bi-doublet $(2, 2, 1) \in \phi_{10}$. As a result of electroweak breaking and the presence of this term, a non-zero VEV is induced for the scalar triplet

\[
v_T = \sqrt{2} \lambda v_{10}^u v_{10}^d / m_T,
\]

where $\langle T^3 \rangle = v_T / \sqrt{2}$ and $\langle H^d_{10} \rangle = v_{10}^d / \sqrt{2}$, $\langle H^u_{10} \rangle = v_{10}^u / \sqrt{2}$. The induced triplet VEV modifies the $W$-boson mass such that

\[
m^2_W = \frac{g^2}{4} \left( v_{SM}^2 + 4v_T^2 \right), \quad \text{with } g_{2L}(m_Z) \equiv g,
\]

and $v_{SM} = 246$ GeV is the SM VEV. This means (as also shown in Eq. (11) of Ref. [21]) that the electroweak mixing angle and $Z$-boson mass remain unaltered, and the change in the $\rho$
parameter is solely due to the $W$-mass anomaly. Following Eq. (8), the $\rho$-parameter can be expressed as

$$\rho = 1 + 4\left(\frac{v_T}{v_{SM}}\right)^2.$$  \hspace{1cm} (9)

The experimental value of $\rho$ in this case is $1.00219 \pm 0.00044$ (see Ref. [21]) which, in turn, implies that the central value of the triplet VEV is $v_T = 5.7561$ GeV.

Before closing this section, we would also like to make a few remarks about the topological defects in this model. The $SO(10)$ breaking at $M_U$ to $SU(2)_L \otimes SU(2)_R \otimes SU(4)_C$ yields a topologically stable monopole that subsequently turns into a superheavy monopole carrying a single unit of Dirac magnetic charge as well as some color magnetic charge. This monopole is inflated away within a suitable inflationary setting as shown, for instance, in Refs. [70, 71]. The second breaking yields a stable monopole significantly lighter than $M_U$ that carries two quanta of Dirac charge as well as color charge. Depending on the magnitude of the symmetry breaking scale $M_I$ versus $H_{inf}$, the Hubble parameter during inflation, this monopole with mass $\sim 10M_I$ may be present in our galaxy at an observable level.

As previously mentioned the unbroken $Z_2$ gauge symmetry implies the presence of topologically stable cosmic strings whose mass scale is determined by the second intermediate scale $M_{II}$. We will discuss these strings and their gravitational wave emission in Section 5. Finally, for completeness let us note that the axion strings in this model appear after inflation and form a string-wall system at the QCD phase transition. The strings are superconducting and the loops emit axions and perhaps even the intermediate scale fermion dark matter.

## 3 Unification Solutions

We aim to obtain unification solutions compatible with the electroweak observables (see Table 1) in terms of the unified gauge coupling ($g_U$), intermediate scales ($M_I$ and $M_{II}$), and unification scale ($M_U$) for different choices of $m_{DM}$ and triplet scalar mass ($m_T$). We minimize the $\chi^2$

| Z-boson mass, $m_Z$ | 91.1876(21) GeV |
|---------------------|-----------------|
| Strong fine structure constant, $\alpha_{3C}$ | 0.1179(10) |
| Fermi coupling constant, $G_F$ | $1.1663787(6) \times 10^{-5}$ GeV$^{-2}$ |
| Electroweak mixing angle, $\sin^2 \theta_W$ | 0.23121(4) |

Table 1: Electroweak observables at $m_Z$ [3].

defined at $m_Z$ and given by

$$\chi^2 = \sum_{i=1}^{3} \frac{(g_i^2 - g_{i,exp})^2}{g_{i,exp}^2},$$  \hspace{1cm} (10)

where $g_i$ ($i = Y, 2L, 3C$) are the SM gauge couplings at $m_Z$ obtained through the RGEs starting from the unified gauge coupling at the unification scale and $g_{i,exp}$ are their experimental values.
We compute the \( \beta \)-coefficients as outlined in Refs. [72–74]. The one- and two-loop \( \beta \)-coefficients governing the renormalization group evolution of the gauge couplings at different stages are given in Table 2.

| \( G_{2_L} \) | \( G_{2_R} \) | \( U(1) \) | \( PQ \) |
|-----------------|-----------------|-----------------|-----------------|
| \( 2 \) | \( 4 \) | \( (32/3) \) | \( (3/5) \) |
| \( 2 \) | \( 51 \) | \( 105/2 \) | \( 249/2 \) |
| \( 1 \) | \( 525/2 \) | \( 3551/6 \) | \( 2 \) |

\[
\begin{align*}
\left( \begin{array}{ccc}
4 & 51 & 249 \\
32 & 884 & 1245 \\
3 & 2 & 2 \\
\end{array} \right), & \quad \left( \begin{array}{ccc}
2 & 3 & 4 \\
51 & 2 & 2 \\
525 & 6 & 2 \\
\end{array} \right) \\
\left( \begin{array}{ccc}
108 & 884 & 1245 \\
105 & 2 & 2 \\
3551 & 6 & 2 \\
\end{array} \right) \\
\left( \begin{array}{ccc}
108 & 51 & 525 \\
105 & 2 & 2 \\
3551 & 6 & 2 \\
\end{array} \right)
\end{align*}
\]

Table 2: One- and two-loop beta coefficients for the renormalization group evolution of the gauge couplings at different stages of gauge symmetry.

Figure 1: Renormalization group evolution of the gauge couplings for a unification solution with \( m_T = 10 \) TeV, \( m_{DM} = 10^{10} \) GeV, \( M_{II} = 5.0 \times 10^{10} \) GeV, \( M_I = 2.15 \times 10^{13} \) GeV, \( M_U = 3.8 \times 10^{16} \) GeV, and \( g_U = 0.624 \).

We show the RGE running of the gauge couplings in Fig. 1 for a unification solution with \( m_T = 10 \) TeV, \( m_{DM} = 10^{10} \) GeV, \( M_{II} = 5.0 \times 10^{10} \) GeV, \( M_I = 2.15 \times 10^{13} \) GeV, \( M_U = 3.8 \times 10^{16} \) GeV, and \( g_U = 0.624 \).

Next, in Table 3 we show the unification solutions for two typical values \( 10^9 \) GeV and \( 10^{10} \) GeV of \( m_{DM} \) with \( m_T = \{1, 5, 10, 50\} \) TeV for each value of \( m_{DM} \). In Fig. 2, we have plotted the unification scale \( (M_U) \) and first intermediate scale \( (M_I) \) as functions of the second intermediate
Figure 2: Variation of unification scale \((M_U)\) and first intermediate breaking scale \((M_I)\) with the second intermediate breaking scale \((M_{II})\) for different choices of the triplet scalar mass \((m_T)\) and dark matter mass \((m_{DM})\). The horizontal dot-dashed lines at \(\log_{10} (M_{II}/\text{GeV})\) equal to 15.7 and 15.9 are the lower bound on \(M_U\) from the Super-Kamiokande experiment and the sensitivity of the proposed Hyper-Kamiokande experiment respectively. The horizontal dot-dashed line at \(\log_{10} (M_I/\text{GeV}) = 13.3\) is the lower bound from the MACRO experiment within the inflationary scenario driven by the Coleman-Weinberg potential of a real GUT singlet [71].
The non-observation of proton decay in the Super-Kamiokande (Super-K) experiment has pushed the partial lifetime bound for the decay channel $p \rightarrow e^+ \pi^0$ to be above $2.4 \times 10^{34}$ yrs \cite{75}, which constrains the unification scale $M_U \gtrsim 5.3 \times 10^{15}$ GeV. On the other hand, the Hyper-Kamiokade (Hyper-K) experiment has $3\sigma$ discovery potential to probe the channel $p \rightarrow e^+ \pi^0$ with partial lifetime $10^{35}$ yrs \cite{76} which corresponds to $M_U \simeq 7.5 \times 10^{15}$ GeV. We have indicated the Super-K lower limit and the Hyper-K sensitivity in Fig. 2. There are unification solutions that are compatible with the Super-K bound and a part of them will be probed by the Hyper-K experiment as can be seen in Fig. 2. The monopoles produced during the symmetry breaking at the scale $M_I$ should be partially inflated to satisfy the lower bound on the monopole flux $2.8 \times 10^{-16}$ cm$^{-2}$s$^{-1}$sr$^{-1}$ \cite{77}. In the inflationary scenario driven by the Coleman-Weinberg potential of a real GUT singlet, the monopoles undergo a sufficient number of $e$-foldings to comply with the MACRO bound for $M_I \gtrsim 2 \times 10^{13}$ GeV \cite{71} which is compatible with a good part of the unification solutions as shown in Fig. 2.

\section{Axion and Intermediate Scale Fermion Dark Matter}

In this section we investigate the scenario of axion and the lightest neutral component of the 10-plet as DM candidates in the model such that

$$\Omega_{\text{Total}}h^2 = \Omega_a h^2 + \Omega_{10} h^2, \quad (11)$$

where $\Omega_a h^2$ and $\Omega_{10} h^2$ denote the axion and fermion relic densities respectively. It is interesting to point out that in this model axions can be produced by two different mechanisms, namely (a) the misalignment mechanism \cite{78–81} and (b) the decay of axionic strings \cite{81, 82}. The relic axion abundance produced by the misalignment mechanism is expressed as \cite{81}

$$\Omega^{\text{mis}}_a h^2 \simeq 0.236 \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{7/6} \langle \theta^2 f(\theta) \rangle, \quad (12)$$

| $m_{DM}$ (GeV) | $m_T$ (TeV) | $\log_{10} \left( \frac{M_U}{\text{GeV}} \right)$ | $\log_{10} \left( \frac{M_I}{\text{GeV}} \right)$ | $\log_{10} \left( \frac{M_{II}}{\text{GeV}} \right)$ | $g_U$ |
|----------------|-------------|--------------------------------|--------------------------------|--------------------------------|-------|
| 1              | 1           | 17.75,15.65                    | 12.57,13.75                    | 9.0,12.3                      | 0.679,0.605 |
| 5              | 1           | 17.68,15.63                    | 12.71,13.88                    | 9.0,12.2                      | 0.671,0.600 |
| 10             | 1           | 17.65,15.66                    | 12.77,13.91                    | 9.0,12.1                      | 0.668,0.600 |
| 50             | 1           | 17.58,15.63                    | 12.91,14.04                    | 9.0,12.0                      | 0.661,0.596 |
| 10             | 1           | 17.14,15.65                    | 12.86,13.74                    | 10.0,12.3                     | 0.649,0.600 |
| 5              | 1           | 17.07,15.62                    | 13.01,13.87                    | 10.0,12.2                     | 0.642,0.596 |
| 10             | 1           | 17.03,15.65                    | 13.07,13.90                    | 10.0,12.1                     | 0.639,0.596 |
| 50             | 1           | 16.96,15.63                    | 13.23,14.03                    | 10.0,12.0                     | 0.633,0.592 |

Table 3: Unification solutions for the unification scale $M_U$, intermediate scales $M_I$ and $M_{II}$, and unified coupling $g_U$ for different choices of $m_{DM}$ and $m_T$. 
where \( \theta \) denotes the misalignment angle that lies in the interval \([-\pi, \pi]\) [83]. The function \( f(\theta) \) contains the anharmonicity of the axion potential, and \( \langle \theta^2 f(\theta) \rangle \) evaluated in the interval \([-\pi, \pi]\) turns out to be around 8.77 [81]. As previously discussed in Section 2, the decay of \( U(1)_{\text{PQ}} \) strings also contributes significantly in the production of axions and hence cannot be ignored. This contribution to the relic density can be expressed as [81]

\[
\Omega_{\text{str}} h^2 \approx 0.34 \left( \frac{f_a}{10^{12} \text{GeV}} \right)^{7/6}.
\]

The total axion relic density is thus given by

\[
\Omega_a h^2 = \Omega_{\text{mis}} h^2 + \Omega_{\text{str}} h^2 \approx 2.41 \left( \frac{f_a}{10^{12} \text{GeV}} \right)^{7/6}.
\]

In Fig. 3, we show how \( \Omega_a h^2 \) varies with \( f_a \), with the black dashed line denoting the Planck limit [69] on the relic DM abundance. With \( f_a \approx 8 \times 10^{10} \text{ GeV} \), the axion saturates the observed DM relic density.

![Figure 3: Variation of the axion relic density with the axion decay constant \( f_a \). The black dashed line corresponds to \( \Omega_{\text{DM}} h^2 = 0.12 \).](image)

For \( f_a \) smaller than \( 8 \times 10^{10} \text{ GeV} \), the contribution from the fermionic DM component should be taken into account. We do not aim to discuss the production mechanism of the fermion DM but provide, instead, a rough analytical estimate of its abundance \( (Y_{\text{DM}} = n_{\text{DM}}/s) \). The relic density of the fermion DM can be expressed as

\[
\Omega_{\text{Total}} h^2 - \Omega_a h^2 = \frac{m_{\text{DM}} Y_{\text{DM}} s_0}{\rho_c},
\]

where \( s_0 \approx 2890 \text{ cm}^{-3} \) is the present entropy density and \( \rho_c \approx 1.05 \times 10^{-5} \text{ GeV cm}^{-3} \) is the present day critical density. Using Eq. (15), we find that

\[
Y_{\text{DM}} \approx 4.36 \times 10^{-10} (\Omega_{\text{Total}} h^2 - \Omega_a h^2) \left( \frac{\text{GeV}}{m_{\text{DM}}} \right).
\]
In Fig. 4, we show the variation of the asymptotic yield of the fermion DM with its mass for three different values of the axion decay constant $f_a$. The solid red line that corresponding to $f_a = 10^{10}$ GeV suggests that around 91% of the total relic density of the DM is composed of intermediate mass scale fermions, with the remaining 9% coming from axions. As expected, making $f_a$ larger increases the axion contribution to the total DM relic density (see Fig. 3), and the corresponding contribution from the fermion DM has to be reduced. This can be seen from the red dashed ($f_a = 5 \times 10^{10}$ GeV) and red dot-dashed ($f_a = 7 \times 10^{10}$ GeV) lines in Fig. 4.

### 5 Gravitational Waves from Cosmic String Loops

The spontaneous symmetry breaking at $M_{II}$ generates local cosmic strings which are topologically stable. The dimensionless tension of the strings is given by

$$G\mu = \frac{1}{8} \left( \frac{M_{II}}{m_{Pl}} \right)^2,$$

where $G$ is Newton’s gravitational constant and $m_{Pl}$ is the reduced Planck mass. The strings intercommute and form loops that decay by emitting GWs. We estimate the gravitational wave spectra following the burst method described in Refs. [84–86]. To this end, we need the loop distribution function $n(l, t)$ (the number density of loops per unit loop length $l$) at the time of GW emission. This is given in the different cosmic epochs in Refs. [87, 88] (also see the supplemental material of Ref. [89]).

In the radiation dominated universe, we have

$$n_r(l, t) = \frac{0.18}{t^{3/2} (l + \Gamma G \mu)^{5/2}} \Theta(0.1t - l),$$
where $\Gamma \simeq 50$. In the matter dominated universe there are two contributions. For the loops that are remnants from the radiation era

$$n_{rm}(l, t) = \frac{0.18t_{eq}^{1/2}}{t^2(l + \Gamma G\mu t)^2} \Theta(0.18t_{eq} - l - \Gamma G\mu t - t_{eq}) ,$$

(19)

where $t_{eq}$ is the equidensity time, and for the loops that are produced during the matter dominated era

$$n_m(l, t) = \frac{0.27 - 0.45(l/t)^{0.31}}{t^2(l + \Gamma G\mu t)^2} \Theta(0.18t - l)\Theta(l + \Gamma G\mu t - t_{eq} - 0.18t_{eq}) .$$

(20)

Assuming cusp domination, the waveform at frequency $f$ and redshift $z$ is given by

$$h(f, l, z) = g_{1c} \frac{G\mu l^{2/3}}{(1 + z)^{1/3}r(z)} f^{-4/3},$$

(21)

where $g_{1c} \simeq 0.85$ [89] and $r$ is the proper distance

$$r(z) = \int_0^z \frac{dz'}{H(z')},$$

(22)

with $H$ being the Hubble parameter. For the burst rate per unit space-time volume we have

$$\frac{d^2R}{dzdl} = N_c H_0^{-3}\phi_V(z) \frac{2n(l, t(z))}{l(1 + z)} \left( \frac{\theta_m(f, l, z)}{2} \right)^2 \Theta(1 - \theta_m),$$

(23)

where $H_0$ is the present value of the Hubble parameter,

$$\theta_m(f, l, z) = \left[ \frac{\sqrt{3}}{4} (1 + z) f l \right]^{-1/3}$$

(24)

is the beam opening angle, and

$$\phi_V(z) = \frac{4\pi H_0^3 r^2}{(1 + z)^3 H(z)}.$$ 

(25)

We have taken $N_c = 2.13$ as in Ref. [85].

The GW background is given by

$$\Omega_{GW}(f) = \frac{4\pi^2}{3H_0^2} f^3 \int_{z_*}^{t_F} dz \int dl h^2(f, l, z) \frac{d^2R}{dzdl} ;$$

(26)

where $t_F$ is the time when loop formation starts and the lower limit $z_*$ in the integral in Eq. (26) leaves out the infrequent bursts from the stochastic background [85] so that

$$\int_0^z dz \int dl \frac{d^2R}{dzdl} = f.$$ 

(27)

We have taken the integration limit on $l$ to be from 0 to $2t$ ($3t$) for the radiation (matter) domination. The various Heaviside $\Theta$ functions will anyway control the upper and lower integration limits during numerical evaluations.
Figure 5: Gravitational wave spectra from cosmic strings generated during the symmetry breaking at $\log_{10}(M_{II}/\text{GeV}) = [9.5, 12.3]$. The sensitivity curves [90, 91] for PPTA [92] and various proposed experiments, namely, SKA [93, 94], CE [95], ET [96], LISA [97, 98], DECIGO [99], BBO [100, 101], HLVK [102], etc. are shown on the plot.

The gravitational wave spectra for the breaking scales $M_{II} \in [10^{9.5}, 10^{12.3}]$ GeV are shown in Fig. 5. They satisfy the present PPTA bound [92] and can be probed in various proposed experiments, including SKA [93, 94], CE [95], ET [96], LISA [97, 98], DECIGO [99] and BBO [100,101]. We have assumed, without loss of any generality, that the network of the string loops is present in the horizon from a very early time $t_F = 10^{-25}$ sec. In fact, in an inflationary universe driven by the Coleman-Weinberg potential of a real GUT-singlet [103, 104], inflation ends at a cosmic time $8.3 \times 10^{-37}$ sec and the phase transitions occur during inflation only if the corresponding symmetry breaking scales $\gtrsim 10^{13}$ GeV [105]. Therefore, the strings in the present case are produced after the end of inflation during the inflaton oscillation [71]. Needless to say, the new radiation temperature dominates over the Hawking temperature from the inflaton oscillations soon after inflation. Consequently, the Ginzburg criterion [106] for a phase transition is governed by the radiation temperature which approaches the reheat temperature (see Ref. [71]) at the reheat time $t_r \simeq 2.3 \times 10^{-25}$ sec. The smaller loops formed during the inflaton oscillation era do not contribute to the gravitational wave background within the frequency range of nHz to kHz. Therefore, we can safely take $t_F = 10^{-25}$ sec to compute the GW spectra.

6 Summary

We have discussed how the recent measurement of $m_W$ by CDF can be readily incorporated into a well-motivated axion model based on $SO(10)$ grand unification. No ad-hoc additional symmetries are imposed, in line with the spirit of the Standard Model. The axion symmetry breaking
scalar field contains an $SU(2)_L$ triplet component that acquires a non-zero VEV through its mixing with the SM doublet. We show how the unification of the SM gauge couplings is preserved with an appropriate symmetry-breaking pattern of $SO(10)$. The proton lifetime is estimated to lie within the reach of future experiments. The model contains two 10-plets of fermions that are introduced to resolve the axion domain wall problem. An unbroken $Z_2$ gauge symmetry from $SO(10)$ ensures the presence of a stable intermediate-mass fermion from these 10-plets which, in addition to the axion, is a plausible dark matter candidate. The $Z_2$ symmetry also yields topologically stable intermediate scale cosmic strings whose gravitational wave spectrum we have also provided.

Acknowledgements

This work is supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.) under the “First Call for H.F.R.I. Research Projects to support Faculty Members and Researchers and the procurement of high-cost research equipment grant” (Project Number: 2251). R.R. also acknowledges the National Research Foundation of Korea (NRF) grant funded by the Korean government (NRF-2020R1C1C1012452).

References

[1] G. Lazarides and Q. Shafi, *Electroweak monopoles and magnetic dumbbells in grand unified theories*, Phys. Rev. D 103 (2021) 095021 [2102.07124].

[2] CDF collaboration, *High-precision measurement of the W boson mass with the CDF II detector*, Science 376 (2022) 170.

[3] Particle Data Group collaboration, *Review of Particle Physics*, PTEP 2020 (2020) 083C01.

[4] C.-T. Lu, L. Wu, Y. Wu and B. Zhu, *Electroweak Precision Fit and New Physics in light of W Boson Mass*, 2204.03796.

[5] J. de Blas, M. Pierini, L. Reina and L. Silvestrini, *Impact of the recent measurements of the top-quark and W-boson masses on electroweak precision fits*, 2204.04204.

[6] Y.-Z. Fan, T.-P. Tang, Y.-L.S. Tsai and L. Wu, *Inert Higgs Dark Matter for New CDF W-boson Mass and Detection Prospects*, 2204.03693.

[7] X. Liu, S.-Y. Guo, B. Zhu and Y. Li, *Unifying gravitational waves with W boson, FIMP dark matter, and Majorana Seesaw mechanism*, 2204.04834.

[8] P. Athron, M. Bach, D.H.J. Jacob, W. Kotlarski, D. Stöckinger and A. Voigt, *Precise calculation of the W boson pole mass beyond the Standard Model with FlexibleSUSY*, 2204.05285.
[9] H. Song, W. Su and M. Zhang, *Electroweak Phase Transition in 2HDM under Higgs, Z-pole, and W precision measurements*, 2204.05085.

[10] K. Cheung, W.-Y. Keung and P.-Y. Tseng, *Iso-doublet Vector Leptoquark solution to the Muon g − 2, R_{K,K^*}, R_{D,D^*}, and W-mass Anomalies*, 2204.05942.

[11] M. Endo and S. Mishima, *New physics interpretation of W-boson mass anomaly*, 2204.05965.

[12] X.-F. Han, F. Wang, L. Wang, J.M. Yang and Y. Zhang, *A joint explanation of W-mass and muon g-2 in 2HDM*, 2204.06505.

[13] Y.H. Ahn, S.K. Kang and R. Ramos, *Implications of New CDF-II W Boson Mass on Two Higgs Doublet Model*, 2204.06485.

[14] P. Fileviez Perez, H.H. Patel and A.D. Plascencia, *On the W-mass and New Higgs Bosons*, 2204.07144.

[15] J. Kawamura, S. Okawa and Y. Omura, *W boson mass and muon g − 2 in a lepton portal dark matter model*, 2204.07022.

[16] S. Kanemura and K. Yagyu, *Implication of the W boson mass anomaly at CDF II in the Higgs triplet model with a mass difference*, 2204.07511.

[17] K.I. Nagao, T. Nomura and H. Okada, *A model explaining the new CDF II W boson mass linking to muon g − 2 and dark matter*, 2204.07411.

[18] P. Mondal, *Enhancement of the W boson mass in the Georgi-Machacek model*, 2204.07844.

[19] K.-Y. Zhang and W.-Z. Feng, *Explaining W boson mass anomaly and dark matter with a U(1) dark sector*, 2204.08067.

[20] L.M. Carpenter, T. Murphy and M.J. Smylie, *Changing patterns in electroweak precision with new color-charged states: Oblique corrections and the W boson mass*, 2204.08546.

[21] O. Popov and R. Srivastava, *The Triplet Dirac Seesaw in the View of the Recent CDF-II W Mass Anomaly*, 2204.08568.

[22] G. Arcadi and A. Djouadi, *The 2HD+a model for a combined explanation of the possible excesses in the CDF M_W measurement and (g – 2)_µ with Dark Matter*, 2204.08406.

[23] T.A. Chowdhury, J. Heeck, S. Saad and A. Thapa, *W boson mass shift and muon magnetic moment in the Zee model*, 2204.08390.

[24] D. Borah, S. Mahapatra, D. Nanda and N. Sahu, *Type II Dirac Seesaw with Observable ∆N_{eff} in the light of W-mass Anomaly*, 2204.08266.
[25] M. Du, Z. Liu and P. Nath, *CDF W mass anomaly from a dark sector with a Stueckelberg-Higgs portal*, 2204.09024.

[26] K. Ghorbani and P. Ghorbani, *W-Boson Mass Anomaly from Scale Invariant 2HDM*, 2204.09001.

[27] P. Asadi, C. Cesarotti, K. Fraser, S. Homiller and A. Parikh, *Oblique Lessons from the W Mass Measurement at CDF II*, 2204.05283.

[28] A. Bhaskar, A.A. Madathil, T. Mandal and S. Mitra, *Combined explanation of W-mass, muon g − 2, R_{K(∗)} and R_{D(∗)} anomalies in a singlet-triplet scalar leptoquark model*, 2204.09031.

[29] K.S. Babu, S. Jana and V.P. K., *Correlating W-Boson Mass Shift with Muon g − 2 in the 2HDM*, 2204.05303.

[30] A. Ghoshal, N. Okada, S. Okada, D. Raut, Q. Shafi and A. Thapa, *Type III seesaw with R-parity violation in light of m_W (CDF)*, 2204.07138.

[31] F. Arias-Aragón, E. Fernández-Martínez, M. González-López and L. Merlo, *Dynamical Minimal Flavour Violating Inverse Seesaw*, 2204.04672.

[32] K. Sakurai, F. Takahashi and W. Yin, *Singlet extensions and W boson mass in the light of the CDF II result*, 2204.04770.

[33] J. Gu, Z. Liu, T. Ma and J. Shu, *Speculations on the W-Mass Measurement at CDF*, 2204.05296.

[34] A. Batra, S. K. A., S. Mandal and R. Srivastava, *W boson mass in Singlet-Triplet Scotogenic dark matter model*, 2204.09376.

[35] S. Baek, *Implications of CDF W-mass and (g − 2)_μ on U(1)_{Lμ−Lτ} model*, 2204.09585.

[36] D. Borah, S. Mahapatra and N. Sahu, *Singlet-Doublet Fermion Origin of Dark Matter, Neutrino Mass and W-Mass Anomaly*, 2204.09671.

[37] J. Heeck, *W-boson mass in the triplet seesaw model*, 2204.10274.

[38] A. Addazi, A. Marciano, A.P. Morais, R. Pasechnik and H. Yang, *CDF II W-mass anomaly faces first-order electroweak phase transition*, 2204.10315.

[39] Y. Cheng, X.-G. He, F. Huang, J. Sun and Z.-P. Xing, *Dark photon kinetic mixing effects for CDF W mass excess*, 2204.10156.

[40] A. Crivellin, M. Kirk, T. Kitahara and F. Mescia, *Correlating t → cZ to the W Mass and B Physics with Vector-Like Quarks*, 2204.05962.

[41] S. Lee, K. Cheung, J. Kim, C.-T. Lu and J. Song, *Status of the two-Higgs-doublet model in light of the CDF m_W measurement*, 2204.10338.
[42] A. Batra, S.K. A, S. Mandal, H. Prajapati and R. Srivastava, CDF-II W Boson Mass Anomaly in the Canonical Scotogenic Neutrino-Dark Matter Model, 2204.11945.

[43] R. Benbrik, M. Boukidi and B. Manaut, W-mass and 96 GeV excess in type-III 2HDM, 2204.11755.

[44] C. Cai, D. Qiu, Y.-L. Tang, Z.-H. Yu and H.-H. Zhang, Corrections to electroweak precision observables from mixings of an exotic vector boson in light of the CDF W-mass anomaly, 2204.11570.

[45] Q. Zhou and X.-F. Han, The CDF W-mass, muon g-2, and dark matter in a U(1)_{L_\mu - L_\tau} model with vector-like leptons, 2204.13027.

[46] C.-R. Zhu, M.-Y. Cui, Z.-Q. Xia, Z.-H. Yu, X. Huang, Q. Yuan et al., GeV antiproton/gamma-ray excesses and the W-boson mass anomaly: three faces of \(~60 \sim 70\) GeV dark matter particle?, 2204.03767.

[47] J.-W. Wang, X.-J. Bi, P.-F. Yin and Z.-H. Yu, Electroweak dark matter model accounting for the CDF W-mass anomaly, 2205.00783.

[48] R. Dcruz and A. Thapa, W boson mass, dark matter and (g – 2)_{\mu} in ScotoZee neutrino mass model, 2205.02217.

[49] X.-Q. Li, Z.-J. Xie, Y.-D. Yang and X.-B. Yuan, Correlating the CDF W-boson mass shift with the b \rightarrow s\ell^+\ell^- anomalies, 2205.02205.

[50] S.-P. He, A leptoquark and vector-like quark extended model for the simultaneous explanation of the W boson mass and muon g – 2 anomalies, 2205.02088.

[51] J. Kim, S. Lee, P. Sanyal and J. Song, CDF m_{W} and the muon g – 2 through the Higgs-phobic light pseudoscalar in type-X two-Higgs-doublet model, 2205.01701.

[52] J.L. Evans, T.T. Yanagida and N. Yokozaki, W boson mass anomaly and grand unification, 2205.03877.

[53] T.A. Chowdhury and S. Saad, Leptoquark-vectorlike quark model for m_{W} (CDF), (g – 2)_{\mu}, R_{K^{(*)}} anomalies and neutrino mass, 2205.03917.

[54] S.-S. Kim, H.M. Lee, A. Menkara and K. Yamashita, The SU(2)_{D} lepton portals for muon g – 2, W boson mass and dark matter, 2205.04016.

[55] P. Athron, A. Fowlie, C.-T. Lu, L. Wu, Y. Wu and B. Zhu, The W boson Mass and Muon g – 2: Hadronic Uncertainties or New Physics?, 2204.03996.

[56] L. Di Luzio, R. Gröber and P. Paradisi, Higgs physics confronts the M_{W} anomaly, 2204.05284.

[57] J.J. Heckman, Extra W-Boson Mass from a D3-Brane, 2204.05302.
[58] T.-K. Chen, C.-W. Chiang and K. Yagyu, *Explanation of the W mass shift at CDF II in the Georgi-Machacek Model*, 2204.12898.

[59] J. Kim, *Compatibility of muon g − 2, W mass anomaly in type-X 2HDM*, 2205.01437.

[60] B. Barman, A. Das and S. Sengupta, *New W-Boson mass in the light of doubly warped braneworld model*, 2205.01699.

[61] R. Holman, G. Lazarides and Q. Shafi, *Axions and the Dark Matter of the Universe*, Phys. Rev. D 27 (1983) 995.

[62] G. Lazarides and Q. Shafi, *Axion Model with Intermediate Scale Fermionic Dark Matter*, Phys. Lett. B 807 (2020) 135603 [2004.11560].

[63] K.S. Babu and S. Khan, *Minimal nonsupersymmetric SO(10) model: Gauge coupling unification, proton decay, and fermion masses*, Phys. Rev. D 92 (2015) 075018 [1507.06712].

[64] S.B. Giddings and A. Strominger, *Axion Induced Topology Change in Quantum Gravity and String Theory*, Nucl. Phys. B 306 (1988) 890.

[65] K.-M. Lee, *Wormholes and Goldstone Bosons*, Phys. Rev. Lett. 61 (1988) 263.

[66] J. Alvey and M. Escudero, *The axion quality problem: global symmetry breaking and wormholes*, JHEP 01 (2021) 032 [2009.03917].

[67] G. Lazarides, C. Panagiotakopoulos and Q. Shafi, *Phenomenology and Cosmology With Superstrings*, Phys. Rev. Lett. 56 (1986) 432.

[68] L. Di Luzio, *Accidental SO(10) axion from gauged flavour*, JHEP 11 (2020) 074 [2008.09119].

[69] PLANCK collaboration, *Planck 2018 results. VI. Cosmological parameters*, Astron. Astrophys. 641 (2020) A6 [1807.06209].

[70] V.N. Şenoğuz and Q. Shafi, *Primordial monopoles, proton decay, gravity waves and GUT inflation*, Phys. Lett. B 752 (2016) 169 [1510.04442].

[71] J. Chakrabortty, G. Lazarides, R. Maji and Q. Shafi, *Primordial Monopoles and Strings, Inflation, and Gravity Waves*, JHEP 02 (2021) 114 [2011.01838].

[72] D.R.T. Jones, *The Two Loop β Function for a G1 × G2 Gauge Theory*, Phys. Rev. D 25 (1982) 581.

[73] J. Chakrabortty, R. Maji, S.K. Patra, T. Srivastava and S. Mohanty, *Roadmap of left-right models based on GUTs*, Phys. Rev. D 97 (2018) 095010 [1711.11391].
[74] J. Chakrabortty, R. Maji and S.F. King, Unification, Proton Decay and Topological Defects in non-SUSY GUTs with Thresholds, Phys. Rev. D 99 (2019) 095008 [1901.05867].

[75] Super-Kamiokande collaboration, Search for proton decay via $p \rightarrow e^+\pi^0$ and $p \rightarrow \mu^+\pi^0$ with an enlarged fiducial volume in Super-Kamiokande I-IV, Phys. Rev. D 102 (2020) 112011 [2010.16098].

[76] Hyper-Kamiokande collaboration, Hyper-Kamiokande, in Prospects in Neutrino Physics, 4, 2019 [1904.10206].

[77] MACRO collaboration, Final results of magnetic monopole searches with the MACRO experiment, Eur. Phys. J. C 25 (2002) 511 [hep-ex/0207020].

[78] J. Preskill, M.B. Wise and F. Wilczek, Cosmology of the Invisible Axion, Phys. Lett. B 120 (1983) 127.

[79] L.F. Abbott and P. Sikivie, A Cosmological Bound on the Invisible Axion, Phys. Lett. B 120 (1983) 133.

[80] F.W. Stecker and Q. Shafi, The Evolution of Structure in the Universe From Axions, Phys. Rev. Lett. 50 (1983) 928.

[81] L. Visinelli and P. Gondolo, Dark Matter Axions Revisited, Phys. Rev. D 80 (2009) 035024 [0903.4377].

[82] C. Hagmann, S. Chang and P. Sikivie, Axion radiation from strings, Phys. Rev. D 63 (2001) 125018 [hep-ph/0012361].

[83] K. Dimopoulos, G. Lazarides, D. Lyth and R. Ruiz de Austri, The Peccei-Quinn field as curvaton, JHEP 05 (2003) 057 [hep-ph/0303154].

[84] S. Olmez, V. Mandic and X. Siemens, Gravitational-Wave Stochastic Background from Kinks and Cusps on Cosmic Strings, Phys. Rev. D 81 (2010) 104028 [1004.0890].

[85] Y. Cui, M. Lewicki and D.E. Morrissey, Gravitational Wave Bursts as Harbingers of Cosmic Strings Diluted by Inflation, Phys. Rev. Lett. 125 (2020) 211302 [1912.08832].

[86] P. Auclair et al., Probing the gravitational wave background from cosmic strings with LISA, JCAP 04 (2020) 034 [1909.00819].

[87] J.J. Blanco-Pillado, K.D. Olum and B. Shlaer, The number of cosmic string loops, Phys. Rev. D 89 (2014) 023512 [1309.6637].

[88] J.J. Blanco-Pillado and K.D. Olum, Stochastic gravitational wave background from smoothed cosmic string loops, Phys. Rev. D 96 (2017) 104046 [1709.02693].
[89] LIGO Scientific, Virgo, KAGRA collaboration, *Constraints on Cosmic Strings Using Data from the Third Advanced LIGO–Virgo Observing Run*, Phys. Rev. Lett. 126 (2021) 241102 [2101.12248].

[90] E. Thrane and J.D. Romano, *Sensitivity curves for searches for gravitational-wave backgrounds*, Phys. Rev. D 88 (2013) 124032 [1310.5300].

[91] K. Schmitz, *New Sensitivity Curves for Gravitational-Wave Signals from Cosmological Phase Transitions*, JHEP 01 (2021) 097 [2002.04615].

[92] R. Shannon et al., *Gravitational waves from binary supermassive black holes missing in pulsar observations*, Science 349 (2015) 1522 [1509.07320].

[93] P.E. Dewdney, P.J. Hall, R.T. Schilizzi and T.J.L.W. Lazio, *The square kilometre array*, Proceedings of the IEEE 97 (2009) 1482.

[94] G. Janssen et al., *Gravitational wave astronomy with the SKA*, PoS AASKA14 (2015) 037 [1501.00127].

[95] T. Regimbau, M. Evans, N. Christensen, E. Katsavounidis, B. Sathyaprakash and S. Vitale, *Digging deeper: Observing primordial gravitational waves below the binary-black-hole-produced stochastic background*, Phys. Rev. Lett. 118 (2017) 151105.

[96] G. Mentasti and M. Peloso, *ET sensitivity to the anisotropic Stochastic Gravitational Wave Background*, JCAP 03 (2021) 080 [2010.00486].

[97] N. Bartolo et al., *Science with the space-based interferometer LISA. IV: Probing inflation with gravitational waves*, JCAP 12 (2016) 026 [1610.06481].

[98] P. Amaro-Seoane et al., *Laser interferometer space antenna*, 1702.00786.

[99] S. Sato et al., *The status of DECIGO*, Journal of Physics: Conference Series 840 (2017) 012010.

[100] J. Crowder and N.J. Cornish, *Beyond LISA: Exploring future gravitational wave missions*, Phys. Rev. D 72 (2005) 083005 [gr-qc/0506015].

[101] V. Corbin and N.J. Cornish, *Detecting the cosmic gravitational wave background with the big bang observer*, Class. Quant. Grav. 23 (2006) 2435 [gr-qc/0512039].

[102] KAGRA, LIGO Scientific, Virgo, VIRGO collaboration, *Prospects for observing and localizing gravitational-wave transients with Advanced LIGO, Advanced Virgo and KAGRA*, Living Rev. Rel. 21 (2018) 3 [1304.0670].

[103] Q. Shafi and A. Vilenkin, *Inflation with SU(5)*, Phys. Rev. Lett. 52 (1984) 691.

[104] G. Lazarides and Q. Shafi, *Extended Structures at Intermediate Scales in an Inflationary Cosmology*, Phys. Lett. B 148 (1984) 35.
[105] G. Lazarides, R. Maji and Q. Shafi, *Cosmic strings, inflation, and gravity waves*, *Phys. Rev. D* **104** (2021) 095004 [2104.02016].

[106] V.L. Ginzburg, *Some Remarks on Phase Transitions of the Second Kind and the Microscopic theory of Ferroelectric Materials*, *Soviet Phys. Solid State* **2** (1961) 1824.