Optimal Aggregation of Forest Units to Clusters as “Danchi” under Lower and Upper Size Bounds for Forest Management in Japan

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Abstract: The existing exact optimization techniques for spatially constrained harvest scheduling problems under area restriction are unable to solve spatial aggregation issues that involve the creation of a set of clusters with lower and upper bounds by size. These bounds are necessary to meet one of Japanese local forestry practices for forming “Danchi”, as a size specific cluster. This paper develops a new model to frame aggregation of forest units bounded by size of clusters for “Danchi” over a forest landscape. The proposed model interprets a forest map as a flow channel or network, then utilizes the concept of maximum flow problem to seek an optimal set of size-bounded clusters over a forest landscape. Computational experiments were conducted to demonstrate how size-bounded clusters were formed and allocated using one hypothetical forest map with 200 forest units as well as a real forest map with 462 units from Kochi prefecture in Japan. The results of the experiments showed that the proposed model is efficient to resolve spatial aggregation issues to form “Danchi” with different sets of bounds by size for Japanese local forestry practices.

Keywords: area limits, maximum flow problem, mixed integer programming, spatial forest planning, sequential triangle connection

1. Introduction

In 2012, the Japanese government introduced a new enforceable forest management planning system (MAFF 2012), which aims to improve the efficiency of forest practices through the management of aggregated forest areas or size specific clusters called “Danchi” in Japanese. Aggregating adjacent small forest units into a larger management cluster under lower and upper size requirements (hereafter known as size bounds) has received a great deal of attention as one of the efforts to achieve low cost forest management with governmental subsidy. A group of forest owners in a cluster or “Danchi” can build a large-scale network of forest logging roads for delivery of logs as well as periodic maintenance and a cost-effective thinning process, which could result in a reduction in timber production cost. Creating “Danchi” should meet size bounds for aggregation. Although this system was introduced for creating “Danchi” under the subsidy program, no methodological approach has been developed to assist forest owners or cooperatives on how to aggregate forest units and create a set of “Danchi” clusters to meet size bounds.

Commonly applied aggregation issues require integration of direct and indirect adjacent forest units to create a set of clusters subject to maximum opening size requirements for harvesting in a contiguous fashion. Murray (1999) defined this type of problem as an area restriction model (ARM) as opposed to conventional adjacency problem, defined as unit restriction model (URM). Several studies have addressed ARM-type problems with an exact method (McDill et al. 2002; Goycoolea et al. 2005; Gunn and Richards 2005; Constantino et al. 2008; and St John and Tóth 2015; McDill et al. 2016). There are three main types of exact solution techniques capable of solving spatially constrained harvest scheduling problems with area restrictions: path formulation (McDill et al. 2002), cluster formulation (Goycoolea et al. 2005), and bucket formulation (Constantino et al. 2008). These techniques have been applied with two typical harvest scheduling specifications, Model I and Model II (Johnson and Scheurman 1977). Difference between Model I and Model II formulations lies in the way the decision variables are defined for a harvest activity. Model I uses a static treatment, which considers a possible set of time-different harvest activities over the planning horizon in one decision variable. On the other hand, one decision variable is defined for harvest activity period by period in a dynamic fashion in Model II. Except for the bucket formulation, the other two require a priori enumeration to create a set of feasible clusters. These efforts also have limited ability to address cluster size issues. That is, only the maximum opening size or upper size bound can be considered now. Yoshimoto et al. (2010) proposed an alternative approach to seek a
set of clusters for “Danchi” based on the “hyper unit” approach, where one hyper unit is generated a priori as a candidate cluster for “Danchi” from each forest unit by a predefined rule to meet lower and upper size requirements. Yoshimoto et al. (2017) incorporated this approach into a Model I type spatially constrained harvest scheduling with multiple harvests over static treatments. Because of the predefined rule or a priori enumeration to generate a set of feasible clusters for “Danchi”, their approach is still not exact to solve lower and upper size-bounded clusters for “Danchi”. Recently, Yoshimoto and Asante (2018) developed a model called MF-Model I to solve the maximum opening size issues without a priori enumeration based on Model I formulation. They used the framework of the maximum flow problems (Harris and Ross 1955) to aggregate forest units into feasible clusters, which meet the maximum opening size requirements.

In this paper, with the focus of optimal allocation of forest units into a set of clusters for “Danchi”, a new mathematical programming approach is proposed to assist “Danchi” issues or aggregation issues subject to lower and upper size requirements for clusters. The framework of maximum flow problems is adapted from Yoshimoto and Asante (2018) for the proposed approach, which does not require any predefined rule by a priori enumeration to generate candidate clusters under size bounds for “Danchi”. Since forming the most clusters for “Danchi” over space is still unsolved, and is one of the main issues facing Japanese local forestry practices, my focus in this paper is to concentrate on spatial formation of clusters for “Danchi”.

2. Methods

In this section, a mathematical programming model is developed to create an optimal set of clusters as “Danchi”, which meets lower and upper size bounds on cluster size, within the framework of maximum flow problems, as described in Yoshimoto and Asante (2018). First, let us interpret a forest map as a flow network and describe “Danchi” issues as a maximum flow problem. A forest unit is regarded as a node for the flow source, while the connections between nodes are regarded as an arc, so that arcs are used as connection over common line boundary between two adjacent forest units or nodes for aggregation. In addition, an artificial super node (indexed by “0”) is introduced as a sink from all sources through the flow network. By representing the area of the i-th forest unit by \( L_i \), as a flow into each node or source and out towards the super node or sink, we can formulate aggregation of units to create “Danchi” within the framework of maximum flow problem. Figure 1a shows a hypothetical forest map with 10 units. The circled number is the identity (ID) of the forest units, while the value below the ID shows its area. Based on this map, the flow network can be generated as shown in Figure 1b. For each node, the inflow represents the amount of forest unit area toward the super node. The solid line represents the existent bi-directional arc between nodes, while the dotted line is the one-directional artificial arc toward the super node, “0”.

In order to formulate “Danchi” issues as a maximum flow problem, let us introduce a binary variable for a directional arc corresponding to a flow defined by:

\[
y_{ij} = \begin{cases} 
1 & \text{if a flow takes place from } i\text{-th to } j\text{-th nodes} \\
0 & \text{otherwise}
\end{cases}
\]

and \( y_{i0} \) as a one-way directional arc to the artificial super node. To restrict arc connection for one-way directional flow, we have

\[ y_{ij} + y_{ji} \leq 1, \quad \forall j \in \mathbb{NB}_i, \forall i, \]

where \( \mathbb{NB}_i \) is an index set of forest units adjacent to the i-th forest unit over a common boundary. In order to have one flow out from a node, only one arc is restricted to be connected from the i-th node to one of its adjacent nodes including the super node:

\[ y_{i0} + \sum_{j \in \mathbb{NB}_i} y_{ij} = 1, \quad \forall i. \]

Defining non-negative continuous variables for the amount of flow defined by \( w_{ij} \) from the i-th node to the j-th node, arc connection, \( y_{ij} \), needs to be associated with flow, \( w_{ij} \). That is, any flow means ensured connection and should be less than or equal to upper bound for cluster, \( L_{up} \):

\[ w_{ij} \leq L_{up} \cdot y_{ij}, \quad \forall j \in \mathbb{NB}_i, \forall i. \]
To avoid any leakage of flow from the network, the amount of inflow is equated to the amount of outflow at each node by the following balance equation or flow conservation restrictions:

\[ w_{i0} + \sum_{j \in \mathbb{N}_i} w_{ij} = \sum_{j \in \mathbb{N}_i} w_{ji} + L_i, \quad \forall i, \]

while the total area flow balance is ensured at the super node by:

\[ \sum_{i=1}^{m} w_{i0} = L, \]

where \( L = \sum_{i=1}^{m} L_i \) is the total area with \( m \) as the total number of units. With the above constraints, Eqs. [1] to [5], we know which nodes or forest units are connected by the variables, \( \{y_{ij}\} \), for aggregation.

Upper bound for cluster size requirement, \( L_{up} \), is imposed as a capacity constraint on the arc toward the super node from such nodes with \( L_i < L_{up} \) by:

\[ w_{i0} \leq L_{up} \cdot y_{i0}, \quad \forall i \in \mathbb{C}, \]

where \( \mathbb{C} \) is a set of units whose area size is less than \( L_{up} \). Note that area restriction using size as bounds is only applied to those clusters created. If some forest units are aggregated, this area restriction must be imposed on its corresponding flow from one of the aggregated nodes directly connected to the super node, \( \{w_{i0}\} \). To meet this requirement, another binary variable is introduced to identify aggregation into the \( i \)-th node connecting to the super node by:

\[ s_i = \begin{cases} 1 & \text{if nodes are aggregated into the } i \text{-th node toward the super node} \\ 0 & \text{otherwise} \end{cases} \]

When the flow from such a node to the super node becomes greater than its area, \( L_i \), we know that aggregation has taken place on that node. Thus, the following constraint is introduced to associate \( s_i \) with the amount of flow to the super node, \( w_{i0} \):

\[ L_i \cdot s_i < w_{i0} \leq L_i + L \cdot s_i, \quad \forall i. \]

Eq. [7] implies that when \( w_{i0} \) becomes greater than its area, \( L_i \), aggregation takes place in the \( i \)-th node with \( s_i = 1 \). With the use of \( s_i \), the following constraint is imposed to limit cluster size by a lower bound, \( L_{lo} \):

\[ L_{lo} \cdot s_i \leq w_{i0}, \quad \forall i. \]

Only when \( s_i = 1 \) and thus \( y_{i0} = 1 \) for aggregation with Eq. [6], we have \( L_{lo} \leq w_{i0} \leq L_{up} \), so that size bounds become effective for a cluster generated based on the \( i \)-th node.
Given the above set of constraints from Eqs.\[1\] to \[8\], we can seek a set of feasible clusters for “Danchi” over the forest landscape. In this paper, the objective function is assumed to demand more aggregation for “Danchi” and to penalize individual units that do not aggregate by:

\[ Z = \max\left(\sum_{i=1}^{m} s_i - \sum_{i=1}^{m} y_{i0}\right). \]

It is of interest to investigate if the objective value becomes 0, by which we know that no individual forest unit remains unaggregated. In Yoshimoto et al. (2010) and Yoshimoto et al. (2017), because of a priori enumeration, there are always some individual forest units that may not be aggregated for “Danchi”. Note that the objective can be arranged in many ways on forest managers’ preference. In this paper, the main purpose is to show the mathematical programming formulation to form patterns for “Danchi”, so that the above simple objective was used.

As a result, the proposed model here is expressed by:

\[ Z = \max\left(\sum_{i=1}^{m} s_i - \sum_{i=1}^{m} y_{i0}\right) \]

subject to

\[ y_{ij} + y_{ji} \leq 1, \quad \forall j \in \mathbb{NB}_i, \forall i \]
\[ y_{i0} + \sum_{j \in \mathbb{NB}_i} y_{ij} = 1, \quad \forall i \]
\[ w_{ij} \leq L_{up} \cdot y_{ij}, \quad \forall j \in \mathbb{NB}_i, \forall i \]
\[ w_{i0} + \sum_{j \in \mathbb{NB}_i} w_{ij} = \sum_{j \in \mathbb{NB}_i} w_{ji} + L_i, \quad \forall i \]
\[ \sum_{i=1}^{m} w_{i0} = \bar{L} \]
\[ w_{i0} \leq L_{up} \cdot y_{i0}, \quad \forall i \in \mathbb{C} \]
\[ L_i \cdot s_i < w_{i0} \leq L_i + \bar{L} \cdot s_i, \quad \forall i \]
\[ L_{i0} \cdot s_i \leq w_{i0}, \quad \forall i \]
\[ s_i \leq y_{i0}, \quad \forall i \]
\[ \{\{y_{ij}\}, \{s_i\}\} \in \{0, 1\}, \{w_{ij}\} \geq 0 \]

Note that Eq.\[10\] was added to accelerate computational performance based on personal experiments to improve the computational burden, though it is obvious for any feasible solution to meet Eq.\[10\]. Also it is noteworthy that if all units have area size less than $L_{i0}$, Eqs.\[7\] and \[8\] can be combined to $L_{i0} \cdot s_i \leq w_{i0} \leq L_i + \bar{L} \cdot s_i$.

3. Data

Computational demonstration was conducted with one hypothetical forest map as well as one real forest map. The hypothetical map was created by a Voronoi diagram with Carson’s Voronoi polygons function\footnote{https://carsonfarmer.com/2009/09/voronoi-polygons-with-r/, (Accessed 1 June 2019)} in “R” statistical software. Details of the procedure can be found in Yoshimoto and Asante (2018). The other map is a map from Sagawa Village of Kochi prefecture, Japan. Figure 2 shows a hypothetical forest map with 200 unit (called EX200 hereafter) and the histogram of forest unit area, while Figure 3 shows for Sagawa map with 462 units (called SG462 hereafter). As can be seen in Figure 3b, the area distribution of SG462 is heavily skewed toward the small around 1ha or less as compared to that of EX200 in Figure 2b. The total area of EX200 is 1000ha with 5ha as
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an average size, while SG462 has 472.42ha with 1.02ha as an average size ranging from 0.03ha to
9.56ha. SG462 can be seen as common forest landscape in Japan, where there are many small-scale
forests. Table 1 shows the summary of both forest maps*2.

(a) Forest map with 200 units

(b) Histogram of forest unit area

Figure 2. A hypothetical forest map with 200 units, EX200.

(a) Forest map with 462 units

(b) Histogram of forest unit area

Figure 3. A real forest map with 462 units, SG462.

Table 1. Summary of area of forest units for two forest maps.

| Map  | Minimum | 1st Quartile | Median  | Mean  | 3rd Quartile | Maximum | Area (ha) |
|------|---------|--------------|---------|-------|--------------|---------|-----------|
| EX200| 0.19    | 2.73         | 4.40    | 5.00  | 6.49         | 18.17   | 1000.00   |
| SG462| 0.03    | 0.26         | 0.58    | 1.02  | 1.08         | 9.56    | 472.42    |

*2GIS files are available from https://www.formath.jp/resource/, (Accessed 1 June 2019)
4. Results

Computational experiments were conducted by iMac Pro with 10 Cores (Intel Xeon W Processor 3GHz) and 128GB memory under Mac OS Mojave. The solver for optimization was IBM ILOG CPLEX Ver.12.6.1 under its interactive mode. That is, an input file generator was constructed first for the solver, then the solver was used to seek an optimal solution by a shell script. In order to investigate how size-bounded clusters or “Danchi” are formed over forest landscape, three sets of size bounds, \((L_{lo}, L_{up})\), were used, changing from 20 to 40ha for \(L_{lo}\) and from 30 to 50ha for \(L_{up}\) with 10ha difference, respectively. All instances used here took less than 10 second of CPU time to reach an optimal solution (see Table 2).

Table 2. Computational performance for optimality.

| Map   | Binary | Const | \((L_{lo}, L_{up})\) | Obj | CPU |
|-------|--------|-------|----------------------|-----|-----|
| EX200 | 1504   | 2858  | (20, 30)             | 0   | 3.07|
|       |        |       | (30, 40)             | 0   | 2.80|
|       |        |       | (40, 50)             | 0   | 2.72|
| SG462 | 3328   | 6380  | (20, 30)             | 0   | 1.63|
|       |        |       | (30, 40)             | 0   | 1.91|
|       |        |       | (40, 50)             | 0   | 3.62|

1: # of binary variables, 2: # of linear constraints, 3: Objective value, 4: CPU time (sec).

The resultant “Danchi” patterns for EX200 are shown in Figure 4. Summary of clusters in all cases for EX200 is provided in Table 3. For all cases, no individual unit remained unaggregated over the landscape with the objective value of 0 in Eq.[9]. With \((L_{lo}, L_{up}) = (20, 30)\), 39 clusters were created for “Danchi” with an average size of 25.64ha, ranging from 20.20 to 30.00 ha. The minimum number of units in a cluster was 2 and the maximum was 9 as shown in Table 4. When the bounds increased to \((L_{lo}, L_{up}) = (30, 40)\), the number of clusters was reduced to 29 with an average size of 34.481ha. The minimum and maximum number of units in a cluster was 3 and 11, respectively. Further increase in the bounds to \((L_{lo}, L_{up}) = (40, 50)\), resulted in 22 clusters for “Danchi” with 45.45 ha as an average size. The minimum and maximum number of units also increased to 4 and 14. Note that grey polygons in Figures 4a to 4c, are those clusters with small boundary to aggregate units.

Table 3. Summary of forest clusters for a hypothetical forest map with 200 units, EX200.

| \((L_{lo}, L_{up})\) | Clusters | Minimum | 1st Quantile | Median | Mean | 3rd Quantile | Maximum |
|---------------------|----------|---------|--------------|--------|------|--------------|---------|
| (20, 30)            | 39       | 20.20   | 23.66        | 26.28  | 25.64| 27.63        | 30.00   |
| (30, 40)            | 29       | 30.12   | 31.01        | 34.30  | 34.48| 37.81        | 39.72   |
| (40, 50)            | 22       | 40.33   | 43.84        | 46.63  | 45.45| 47.63        | 48.89   |

Table 4. Summary of the number of units in clusters as “Danchi” for EX200.

| \((L_{lo}, L_{up})\) | Minimum | 1st Quantile | Median | Mean | 3rd Quantile | Maximum |
|---------------------|---------|--------------|--------|------|--------------|---------|
| (20, 30)            | 2       | 4.0          | 5.0    | 5.1  | 6.5          | 9       |
| (30, 40)            | 3       | 5.0          | 7.0    | 6.9  | 8.0          | 11      |
| (40, 50)            | 4       | 7.3          | 9.5    | 9.1  | 11.0         | 14      |

With the same sets of size bounds, a forest map of SG462 for Sagawa Village resulted in 18, 14 and 10 clusters for “Danchi” under \((L_{lo}, L_{up})\) equal to (20, 30), (30, 40) and (40, 50), respectively (see Table 5). Although the model with a priori enumeration used in Yoshimoto et al. (2010) left individual units unaggregated for “Danchi”, the proposed model completely allocated all units into clusters as can be seen in Figure 5. The set of the minimum and maximum number of units in a cluster became (6, 46), (11, 60), and (16,73) for \((L_{lo}, L_{up})\) equal to (20, 30), (30, 40) and (40, 50), respectively (see Table 6). Due to the large number of small-scale forest units, the maximum number of units in “Danchi” was much larger than that in the case of EX200.
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(a) \((L_{lo}, L_{up}) = (20, 30)\)
(b) \((L_{lo}, L_{up}) = (30, 40)\)
(c) \((L_{lo}, L_{up}) = (40, 50)\)

Figure 4. An optimal set of clusters for “Danchi” with 200 units, EX200.
Note: Grey area is an aggregated unit with small connected boundary.
Value in a polygon indicates area size.

Table 5. Summary of forest clusters for Sagawa forest map with 462 units, SG462.

| \((L_{lo}, L_{up})\) | Clusters | Minimum | 1st Quantile | Median | Mean | 3rd Quantile | Maximum |
|---------------------|----------|---------|--------------|--------|------|--------------|---------|
| (20, 30)            | 18       | 21.69   | 24.16        | 26.62  | 26.25| 29.09        | 30.00   |
| (30, 40)            | 14       | 30.02   | 30.94        | 31.66  | 33.74| 38.29        | 39.89   |
| (40, 50)            | 10       | 40.47   | 47.15        | 48.66  | 47.24| 49.54        | 49.98   |

Table 6. Summary of the number of units in clusters as “Danchi” for SG462.

| \((L_{lo}, L_{up})\) | Minimum | 1st Quantile | Median | Mean | 3rd Quantile | Maximum |
|---------------------|---------|--------------|--------|------|--------------|---------|
| (20, 30)            | 6       | 15.0         | 24.0   | 25.7 | 37.8         | 46      |
| (30, 40)            | 11      | 18.5         | 31.5   | 33.0 | 45.8         | 60      |
| (40, 50)            | 16      | 29.5         | 52.5   | 46.2 | 55.8         | 73      |

5. Conclusions

In this paper, a new model was proposed to frame aggregation of forest units bounded by the size of clusters for “Danchi” over the forest landscape. The target optimization problem was subject to area restriction constraints by lower and upper bounds on size of clusters for “Danchi” over the forest landscape. It is noteworthy that the proposed model does not require any predefined rule to create possible clusters or preparation of candidate clusters by a priori enumeration. The benefit of the proposed modeling approach stems from the use of maximum flow problems to specify unit aggregation for a cluster, where area of each forest unit was regarded as a flow into each unit. Adjacent connection over a common boundary was regarded as arc to deliver a flow between adjacent units. By connecting all units to an artificial super node, the proposed model finds an optimal set of clusters as well as individual forest units not aggregated over forest landscape. With two sets of forest maps used here, the computational experiments showed that all instances were solved within 10 second of CPU time for optimality.

Japanese government passed a new law on forest management in 2019, which seeks sustainable
forest resource utilization (MAFF 2019). The main point of the law is to allow local government to take responsibility for forest planning from forest owners, so that it becomes possible to separate the management scheme from ownership. In such a case, proposing an efficient forest plan plays a critical role in sustaining forest management. Currently, an emerging issue in local forestry practices is to resolve a problem of small-scale forestry practices by forming a certain sized cluster as “Danchi” so that it becomes possible to apply large scale and long-term forest planning efficiently. The proposed model is promising as it can resolve “Danchi” issues by employing mathematical programming approach. In the demonstrative examples conducted here, only lower and upper size bounds were assumed on aggregation. However, in a practice, there may be some units that may be unfordable to aggregate, or there may already exist some clusters for “Danchi”. Even in such a case, the proposed model can seek the best set of clusters given those units set aside.

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