BOUNDARY VALUE PROBLEM FOR A MULTIDIMENSIONAL SYSTEM OF EQUATION WITH RiemANN-LOUIVille DERIVATIVES

M.O. Mamchuev

In the paper boundary-value problem for a multidimensional system of partial differential equations with fractional derivatives in Riemann-Liouville sense with constant coefficients is studied in a rectangular domain. The existence and uniqueness theorem for the solution of the boundary value problem is proved. The solution is constructed in explicit form in terms of the Wright function of the matrix argument.

MSC 2010: Primary 33R11; Secondary 35A08, 35A09, 35C05, 35C15, 35E05, 34A08.

Key Words and Phrases: system of partial differential equations, fractional derivatives, boundary value problem, fundamental solution, Wright’s function of the matrix argument.

1 Introduction

Consider the system of equations

$$\sum_{i=1}^{m} A_i D_{0x}^{\alpha_i} u(x) = Bu(x) + f(x), \quad 0 < \alpha_i < 1,$$

in the domain $\Omega = \{x = (x_1, ..., x_m) : 0 < x_i < a_i \leq \infty, i = 1, m\}$, where $f(x) = \|f_1(x), ..., f_n(x)\|$ and $u(x) = \|u_1(x), ..., u_n(x)\|$ are the given and required $n$-dimensional vectors, respectively, $A_i, B$ are given constant square matrices of order $n$, $D_{ay}^{\nu}$ is the operator of fractional integro-differentiation in Riemann-Liouville sense of order $\nu$.

Operator $D_{ay}^{\nu}$ is determined for $\nu < 0$ by the following formula [1, p. 9]:

$$D_{ay}^{\nu} g(y) = \frac{\text{sgn}(y-a)}{\Gamma(-\nu)} \int_a^y \frac{g(s)ds}{|y-s|^\nu+1},$$

and for $\nu \geq 0$ can be determined with the help of following recursive relation

$$D_{ay}^{\nu} g(y) = \text{sgn}(y-a) \frac{d}{dy} D_{ay}^{\nu-1} g(y),$$

$\Gamma(z)$ is a Euler's gamma-function.

Let all the eigenvalues of matrices $A_i, i = 1, m$ are positive. Without loss of generality, we assume that $A_1 = I$ is the identity matrix of order $n$.

We formulate the boundary value problem for the system (1.1).
Problem 1. Find the solution $u(x)$ of the system (1.1) satisfying the following boundary conditions

$$\lim_{x_i \to 0} D^{\alpha_i - 1}_{0x_i} u = \varphi_i(x_{(i)}), \quad x_{(i)} \in \Omega^i, \quad i = 1, m,$$

(1.2)

where $x_{(i)} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_m)$, $\Omega^i = \omega_{a_1} \times \ldots \times \omega_{a_{i-1}} \times \omega_{a_{i+1}} \times \ldots \times \omega_{a_m}$, $\omega_{a_j} = \{ x_j : 0 < x_j < a_j \}$, $\varphi_i(x_{(i)})$ are given $n$-dimensional vectors.

Let us review the works associated with the investigation of the system (1.1) including in the scalar case $n = 1$. In paper [2] for the equation

$$D^{\alpha}_{0x}(u - h_1(y)) + D^{\beta}_{0y}(u - h_2(x)) = f, \quad 0 < \alpha, \beta < 1, \quad x,y \geq 0,$$

(1.3)

the solvability of the boundary value problem is studied in a class of Holder’s continuous functions with the initial conditions $u(0,y) = h_1(y), u(x,0) = h_2(x)$ and the right hand side $f(x,y)$. The fundamental solution of the equation (1.3) was rewritten in the form

$$\psi_{\alpha,\beta}(x,y) = \int_0^\infty \tau^{-\frac{\alpha}{\beta}} \varphi_\alpha \left( x\tau^{-\frac{1}{\beta}} \right) \varphi_\beta \left( y\tau^{-\frac{1}{\beta}} \right) d\tau,$$

(1.4)

where

$$\varphi_\mu(t) = \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!} \sin(\pi \mu k) \Gamma(\mu k + 1) t^{-\mu k - 1}.$$

Using the equality $\Gamma(1 - z)\Gamma(z) \sin(\pi z) = \pi$, the function $\varphi_\mu(t)$ can be represented in the form

$$\varphi_\mu(t) = \frac{1}{t\phi(-\mu, 0; -t^{-\mu})},$$

where $\phi(\rho, \mu; z) = \sum_{k=0}^{\infty} \frac{z^k}{k! (\rho + k\mu)}$ is the Wright’s function [3], [4]. From the last relation and (1.4) we get the following representation

$$\psi_{\alpha,\beta}(x,y) = \frac{1}{x_0} \int_0^\infty \phi(-\alpha, 0; -\tau x^{-\alpha}) \phi(-\beta, 0; -\tau y^{-\beta}) d\tau.$$

Holder’s smoothness of the following equation’s solution

$$D^{\alpha}_{0x}(u - u_0) + c(x,y)u_y(t,x) = f(x,y), \quad x,y > 0,$$

satisfying the boundary conditions $u(0,y) = u_0(y)$ and $u(x,0) = u_1(x)$, was studied in paper [5].

In the papers [6], [7], the theorems of uniqueness and existence of regular solution are proved for a boundary value problem in a rectangular domain for the equation

$$D^{\alpha}_{0x} u(x,y) + \lambda D^{\beta}_{0y} u(x,y) + \mu u(x,y) = f(x,y), \quad 0 < \alpha, \beta < 1, \lambda > 0, \quad x,y > 0.$$

(1.5)
When $\lambda = 1$ the fundamental solution has the form

$$w(x, y) = \frac{1}{xy} \int_0^\infty e^{-\mu \tau} \phi(-\alpha, 0; -\tau x^{-\alpha}) \phi(-\beta, 0; -\tau y^{-\beta}) d\tau,$$

and when $\mu = 0$ it has the form

$$w(x, y) = \frac{x^{\alpha - 1}}{y} e^{\alpha, 0} \left( -\lambda \frac{x^\alpha}{y^\beta} \right),$$

where

$$e^{\mu, \nu}_{\alpha, \beta}(z) = \sum_{k=0}^\infty \frac{z^k}{\Gamma(\mu + \alpha k) \Gamma(\nu - \beta k)}$$

is the Wrihgt type function [7]. In the case $\alpha = 1$, $\mu = 0$, a boundary value problem with negative coefficient $\lambda < 0$ was studied for the equation (1.5).

The equation (1.5) with variable coefficients $\lambda \equiv \lambda(x)$ and $\mu \equiv \mu(x)$, when $\alpha = 1$ and $\lambda(x)$ can have a zero of order $m \geq 0$ at the point $x = 0$ was investigated in the papers [8], [9], [10]. It’s fundamental solution

$$w(x, y; t, s) = \exp(\Lambda(x, t)) \phi\left(-\beta, 0; -M(x, t)(y-s)^{-\beta}\right),$$

where $\Lambda(x, t) = \int_x^t \lambda(\xi)d\xi$, $M(x, t) = \int_x^t \mu(\xi)d\xi$, was constructed, and the existence and uniqueness theorems for the solutions of the boundary-value problem in a rectangular domain and the Cauchy problem were proved.

In [11] the unique solvability of the analogue of problem 1 for the equation (1.5) with the Dzhrbashyan-Nersesyan fractional differentiation operators was investigated in the case $n = 1$, $A_i = \lambda_i > 0$ $(i = 1, m)$, $B = \lambda_0$, and the fundamental solution

$$w(x) = \int_0^\infty e^{-\lambda_0 \tau} \prod_{i=1}^m \frac{1}{x_i} \phi\left(-\alpha_i, 0; -\lambda_i \tau x_i^{-\alpha_i}\right) d\tau$$

of this equation was constructed.

For two-dimensional system

$$D_{\partial x}^\alpha u(x, y) + AD_{\partial y}^\beta u(x, y) = Bu(x, y) + f(x, y), \quad (1.6)$$

the problem 1 was solved in explicite form in [12], when $A$ was an identity matrix and in [13], when $A$ was positive defined matrix. In the paper [13] the fundamental solution of the system (1.6) was constructed in the terms of the introduced Wright’s function of the matrix argument.

In present paper we will use similar approach for the solving of problem 1 in multidimensional case.
2 Wright’s function

Wright function \([3, 4]\) is called an entiere function, which is depended from two parameters \(\rho\) and \(\mu\), and represented by the series

\[
\phi(\rho, \mu; z) = \sum_{k=0}^{\infty} \frac{z^k}{k! \Gamma(\rho k + \mu)}, \quad \rho > -1, \quad \mu \in \mathbb{C}.
\]

It is easy to see that

\[
\phi(\rho, \mu; z) \big|_{z=0} = \frac{1}{\Gamma(\mu)}.
\]

The following differentiation formulas hold \([4, 7]\)

\[
\frac{d}{dz} \phi(\rho, \mu; z) = \phi(\rho, \mu + \rho; z), \quad (2.2)
\]

\[
D_0^\nu y^{\mu-1} \phi(\rho, \mu; -\lambda y^\rho) = y^{\rho-\nu-1} \phi(\rho, \rho - \nu; -\lambda y^\rho), \quad (2.3)
\]

\(\lambda > 0, \rho > -1, \mu, \nu \in \mathbb{R}\). From the relations \((2.2)\) and \((2.3)\) follow

\[
\left( \frac{d}{dz} + \lambda D_0^\beta \right) y^{\mu-1} \phi(-\beta, \mu; -\lambda z y^{-\beta}) = 0, \quad \beta < 1, \quad \lambda > 0. \quad (2.4)
\]

For the Wright function following estimates are hold \([7]\)

\[
|y^{\mu-1} \phi(-\beta, \mu; -\tau y^{-\beta})| \leq C \tau^{-\theta} y^{\mu+\beta-\theta-1}, \quad \tau > 0, \quad y > 0, \quad (2.5)
\]

where \(\beta \in (0, 1)\) and \(\theta \geq 0\) if \(\mu \neq 0, -1, -2, ...\), and \(\theta \geq -1\) if \(\mu = 0, -1, -2, ...\)

\[
|\phi(-\beta, \mu; -z)| \leq C \exp \left( -\sigma z^{\frac{1}{1-\beta}} \right), \quad z \geq 0, \quad (2.6)
\]

where \(\beta \in (0, 1)\), \(\varepsilon \in \mathbb{R}\), \(\sigma < (1 - \beta) \beta \gamma^{1-\sigma}\), here and below \(C\) is a pozitive constant.

In paper \([14]\) following relation

\[
\int_0^\infty t^n \phi(-\beta, \mu; -t) dt = \frac{n!}{\Gamma(\mu + (n + 1)\beta)}, \quad n = 0, 1, ... \quad (2.7)
\]

was obtained. In particular for \(n = 0\), we have

\[
\int_0^\infty \phi(-\beta, \mu; -z) dz = \frac{1}{\Gamma(\mu + \beta)}. \quad (2.8)
\]
3 Wright’s function of the matrix argument

1. Let $A$ be a square matrix of order $n$. In view of the function $\phi(\rho, \mu; z)$ is analytic everywhere in $\mathbb{C}$, following series

$$
\phi(\rho, \mu; A) = \sum_{k=0}^{\infty} \frac{A^k}{k! \Gamma(\rho k + \mu)}, \quad \rho > -1, \quad \mu \in \mathbb{C}
$$

is convergence for any matrix $A$ given over the field of complex numbers $\mathbb{C}$, and determine the Wright’s function of the matrix argument.

Let the matrix $H$ leads the matrix $A$ to Jordan normal form $J(\lambda)$, i.e.

$$
A = H J(\lambda) H^{-1},
$$

where $J(\lambda) = \text{diag}[J_1(\lambda_1), \ldots, J_p(\lambda_p)]$ is the quasi-diagonal matrix with the cells of the form

$$
J_k \equiv J_k(\lambda_k) = \begin{pmatrix}
\lambda_k & 1 & \ldots & 0 \\
\lambda_k & \ddots & \ddots & \vdots \\
0 & \ddots & \ddots & \lambda_k \\
0 & \ldots & \lambda_k & 0
\end{pmatrix}, \quad k = 1, \ldots, p,
$$

$\lambda_1, \ldots, \lambda_p$ are eigenvalue numbers of the matrix $A$, and $J_k(\lambda_k)$ are the square matrices of order $r_k + 1$, $\sum_{k=1}^{p} r_k + p = n$. Then the function $\phi(\rho, \mu; A z)$ can be represented in the form

$$
\phi(\rho, \mu; A z) = H \phi(\rho, \mu; J(\lambda) z) H^{-1},
$$

where

$$
\phi(\rho, \mu; J(\lambda) z) = \text{diag}[\phi(\rho, \mu; J_1(\lambda_1) z), \ldots, \phi(\rho, \mu; J_p(\lambda_p) z)],
$$

$$
\phi(\rho, \mu; J_k(\lambda_k) z) = \begin{pmatrix}
\phi^0_{p, \mu}(\lambda_k z) & \phi^{1}_{p, \mu}(\lambda_k z) & \cdots & \phi^{r_k}_{p, \mu}(\lambda_k z) \\
\phi_{p, \mu}(\lambda_k z) & \phi_{p, \mu}(\lambda_k z) & \cdots & \phi_{p, \mu}(\lambda_k z) \\
0 & \ddots & \ddots & \ddots \\
0 & \cdots & \phi_{p, \mu}(\lambda_k z)
\end{pmatrix},
$$

$$
\phi^{m}_{p, \mu}(\lambda z) = \frac{1}{m!} \frac{\partial^m}{\partial \lambda^m} \phi(\rho, \mu; \lambda z) = \frac{1}{m!} \phi(\rho, \mu + m \lambda z).
$$

2. By using the representation (3.1) and the equality (2.1), we obtain

$$
\phi(\rho, \mu; A z) \big|_{z=0} = \frac{1}{\Gamma(\mu)} I,
$$

where $I$ is a identity matrix of order $n$.

3. Following differentiation formula holds

$$
\frac{d}{dz} \phi(\rho, \mu; A z) = A \phi(\rho, \rho + \mu; A z).
$$
Indeed, by virtue of equality (2.2) we get
\[ \frac{d}{dz} \phi^m_{\rho,\mu}(\lambda z) = \lambda \frac{z^m}{m!} \phi(\rho, \mu + \rho m; \lambda z) + \frac{z^{m-1}}{(m-1)!} \phi(\rho, \mu + \rho m; \lambda z) = \lambda \phi^m_{\rho,\mu}(\lambda z) + \phi^{m-1}_{\rho,\mu}(\lambda z).\]

Whence, in turn, we have
\[ \frac{d}{dz} \phi(\rho, \mu; J(\lambda)z) = J(\lambda) \phi(\rho, \rho + \mu; J(\lambda)z). \quad (3.4) \]

From (3.4), by taking into account following equality
\[ H \frac{d}{dz} \phi(\rho, \mu; J(\lambda)z) H^{-1} = H J(\lambda) H^{-1} H \phi(\rho, \rho + \mu; J(\lambda)z) H^{-1}, \]
we obtain (3.3).

4. Let all of the eigenvalues of the matrix \( A \) are positive. Consider the functon
\[ y^{\mu-1} \phi(-\beta, \mu; -A\tau y^{-\beta}) = H y^{\mu-1} \phi(-\beta, \mu; -J(\lambda)\tau y^{-\beta}) H^{-1}. \]

Denoting
\[ w_m \equiv w_m^{\mu}(\tau, y) = \frac{y^{\mu-1}}{m!} \left( \frac{-\tau}{y^{\beta}} \right)^m \phi(-\beta, \mu - m\beta; -\lambda \tau y^{-\beta}), \quad m = 0, \ldots, r, \]
we can write
\[ y^{\mu-1} \phi(-\beta, \mu; -J_k(\lambda_k)\tau y^{-\beta}) = \begin{vmatrix} w_0 & w_1 & \ldots & w_r \\ w_0 & w_1 & \ldots & w_{r-1} \\ \vdots & \ddots & \ddots & \vdots \\ w_0 & \ldots & \ldots & \end{vmatrix}, \quad k = 1, \ldots, p. \]

In view of (2.3) we get the relation
\[ D_0^{\delta} w_m^{\mu}(\tau, y) = \frac{(-\tau)^m}{m!} y^{\mu-\delta - m\beta - 1} \phi(-\beta, \mu - \beta m; -\lambda \tau y^{-\beta}) = w_m^{\mu-\delta}(\tau, y). \]

From this relation, similarly as we obtained the equality (3.3), we obtain the equality
\[ D_0^{\delta} y^{\mu-1} \phi(-\beta, \mu; -A\tau y^{-\beta}) = y^{\mu-\delta-1} \phi(-\beta, \mu - \delta; -A\tau y^{-\beta}). \quad (3.5) \]

5. It follows from (3.3) and (3.5) that
\[ \left( \frac{\partial}{\partial \tau} + A D_0^{\delta} \right) y^{\mu-1} \phi(-\beta, \mu; -A\tau y^{-\beta}) = 0. \quad (3.6) \]
6. Let all the eigenvalues of the matrix \(A\) be positive, then the following equality holds
\[
\int_0^\infty \phi(-\beta, \mu; -Az) \, dz = \frac{1}{\Gamma(\mu + \beta)} A^{-1}.
\] (3.7)

Indeed, in view of (2.7) we get
\[
\int_0^\infty \frac{(-z)^m}{m!} \phi(-\beta, \mu - \beta m; -\lambda z) \, dz = \frac{1}{\Gamma(\mu + \beta)} (-1)^m \lambda^{m+1}.
\] (3.8)

From (3.8) we obtain
\[
\int_0^\infty \phi(-\beta, \mu; -J_k z) \, dz = \frac{1}{\Gamma(\mu + \beta)} \left| \begin{array}{ccc}
\lambda_1 & -\frac{1}{\lambda_1} & \frac{1}{\lambda_1} & \cdots & \frac{1}{\lambda_1} \\
\lambda_2 & -\frac{1}{\lambda_2} & \frac{1}{\lambda_2} & \cdots & \frac{1}{\lambda_2} \\
& \ddots & \ddots & \ddots & \ddots \\
& & & \lambda_1 & -\frac{1}{\lambda_1} \\
& & & & \lambda_2 & -\frac{1}{\lambda_2}
\end{array} \right| = \frac{1}{\Gamma(\mu + \beta)} J_k^{-1}.
\] (3.9)

From (3.9), (3.1) and equality
\[
J^{-1}(\lambda) = \text{diag} [J_{-1}^{-1}(\lambda_1), \ldots, J_{-1}^{-1}(\lambda_p)]
\]
follows (3.7).

7. We denote by \(|A(x)|_*\) the scalar function that takes at each point \(x\) the largest of the values of the moduli of the elements of the matrix \(A(x) = \|a_{ij}(x)\|\), that is \(|A(x)|_* = \max_{i,j} |a_{ij}(x)|\). Similarly, for the vector \(b(x)\) with components \(b_i(x)\) we denote \(|b(x)|_* = \max_i |b_i(x)|\).

From the estimate (2.5) follows that
\[
|y^{\mu-1} \phi(-\beta, \mu; -Az)\|_* \leq C \exp \left( -\sigma z \right), \quad z \geq 0,
\] (3.10)

where \(\beta \in (0, 1)\), \(\theta > 0\), \(\mu \neq 0, -1, 2, \ldots\), \(\theta \geq 1\), \(\mu - \beta = 0, -1, 2, \ldots\).

8. From (2.6) and (3.1) follows the estimate
\[
|\phi(-\beta, \mu; -Az)\|_* \leq C \exp \left( -\sigma z \right), \quad z \geq 0,
\] (3.11)

where \(\beta \in (0, 1)\), \(\mu \in \mathbb{R}\), \(\sigma < (1 - \beta) (\lambda \beta^\beta)^{-1}\), \(\lambda = \min_{1 \leq i \leq p} \{\lambda_i\}\), \(\lambda_1, \ldots, \lambda_p\) are eigenvalues of the matrix \(A\).
4 Main results

Consider following function

$$\Phi^\delta_\alpha(x) = \int_0^\infty e^{B\tau} \prod_{i=1}^m h_i^\delta(x_i, \tau) d\tau,$$

where $\alpha = (\alpha_1, ..., \alpha_m)$, $\delta = (\delta_1, ..., \delta_m)$.

From the estimates (3.10), (3.11) and

$$|\exp(B\tau)|_* \leq C e^{\gamma \tau}, \quad \gamma = \max_{1 \leq i \leq q} \{ |\Re \gamma_i| \},$$

(4.1)

where $\gamma_1, ..., \gamma_q$ are eigenvalues of matrix $B$, the convergence of the integral $\Phi^\delta_\alpha(x)$ follows for all $\alpha_i, \delta_i \in \mathbb{R}$, $i = 1, m$.

A regular solution of system (1.1) in domain $\Omega$ is defined as a vector function $u(x)$ satisfying system (1.1) at all points $x \in \Omega$, such that $D^\alpha_\delta u(x) \in C(\Omega)$,

$$\prod_{i=1}^m x_{i-1}^{1-\mu_i} u(x) \in C(\bar{\Omega}),$$

(4.2)

$$\prod_{i=1}^m x_{i-1}^{1-\mu_i} f(x) \in C(\bar{\Omega}),$$

(4.3)

$f(x)$ satisfies the Holder condition. Then there exists a unique regular solution of the problem (1.1), (1.2), which can be represented as

$$u(x) = \int_{\Omega} G(x-t) f(t) dt + \sum_{i=1}^m \int_{\Omega_i} A_i G(x-t^i) \varphi_i(t) dt^i,$$

(4.4)

where $G(x) = \Phi^0_\alpha(x)$, $\Omega_x = \omega_{x_1} \times \omega_{x_2} \times ... \times \omega_{x_m}$, $\Omega_i = \omega_{x_1} \times ... \times \omega_{x_{i-1}} \times \omega_{x_{i+1}} \times ... \times \omega_{x_m}$, $\omega_{x_j} = \{ t_j : 0 < t_j < x_j \}$, $t^i = (t_1, ..., t_{i-1}, 0, t_{i+1}, ..., t_m)$.

5 Auxiliary assertions

Let us prove some assertions that we need in the proof of Theorem 5.1.

5.1 Properties of the function $\Phi^\delta_\alpha(x)$

Lemma 1. The estimate

$$|\Phi^\delta_\alpha(x)|_* \leq C \prod_{i=1}^m x_i^{\delta_i + \alpha_i - 1}, \quad \sum_{i=1}^m \theta_i = 1,$$

(5.1)
holds for all \( x_1 \in [0, x_{10}] \), where \( \theta_i > 0 \), for \( \delta_i \neq 0 \), and \( \theta_i > -1 \), for \( \delta_i = 0 \); and constant \( C \) depends on \( x_{10} \).

**Proof.** In view of (3.10) and (4.1) we get

\[
|\Phi_{\alpha}^\delta(x)| \leq C \prod_{i=2}^m x_i^{\delta_i+\alpha_i\theta_i-1} \int_0^\infty e^{\gamma x_i^{\alpha_i}z^{\theta_i-1}} \phi (-\alpha_1, \delta_1; -\tau x_1^{-\alpha_1}) \, d\tau.
\]

After replacing \( \tau = x_1^{\alpha_1}z \), we obtain

\[
|\Phi_{\alpha}^\delta(x)| \leq C \prod_{i=2}^m x_i^{\delta_i+\alpha_i\theta_i-1} \int_0^\infty e^{\gamma x_1^{\alpha_1}z^{\theta_i-1}} \phi (-\alpha_1, \delta_1; -z) \, dz. \tag{5.2}
\]

We represented the integral on the right-hand side \( \tag{5.2} \) as following sum

\[
J_1(x) + J_2(x) = \int_0^{z_0} + \int_{z_0}^\infty z^{-\theta} e^{\gamma x_1^{\alpha_1}z} \phi (-\alpha_1, \mu; -z) \, dz. \tag{5.3}
\]

In view of the boundedness of the function \( \phi (-\alpha_1, \mu; -z) \) on any finite interval \([0, z_0]\), we obtain that

\[
|J_1(x)| \leq C e^{\gamma x_1^{\alpha_1}z_0} \int_0^{z_0} z^{-\theta} \, dz = C_1 e^{\gamma x_1^{\alpha_1}z_0}. \tag{5.4}
\]

Using the estimate \( \tag{2.0} \), we have

\[
|J_2(x)| \leq C z_0^{-\theta} \int_{z_0}^\infty \exp(\gamma x_1^{\alpha_1}z - \rho_0 z^\varepsilon) \, dz,
\]

where \( \rho_0 \leq \alpha_1/(1-\alpha_1)(1-\alpha_1), \varepsilon = 1/(1-\alpha_1) > 1 \). Note that \( \gamma x_1^{\alpha_1}z - \rho_0 z^\varepsilon \leq -z \) for \( z \geq z_0 = ((\gamma x_1^{\alpha_1}z + 1)/\rho_0)^{(1-\alpha_1)/\alpha_1} > 1 \). Therefore

\[
|J_2(x)| \leq C_2 z_0^{-\theta} e^{-z_0}. \tag{5.5}
\]

From (5.3), (5.4), (5.5) and (5.2) the estimate (5.1) follows, where

\[
C \equiv C(x_{10}) = C_1 e^{\gamma x_1^{\alpha_1}z_0} + C_2 z_0^{-\theta} e^{-z_0}.
\]

Lemma 1 is proved.

**Remark.** From the equalities (3.3) and (3.4), it is easy to see that formula

\[
D_{\nu, \alpha}^{\delta_i} \Phi_{\alpha}^\delta(x) = \Phi_{\alpha}^{(\delta_1, \ldots, \delta_i, \ldots, \delta_m)}(x) \tag{5.6}
\]

is valid for \( \nu = 0 \) and \( \nu \in \mathbb{N} \). In other cases it is required that the function \( \Phi_{\alpha}^\delta(x) \) has an integrable singularity for \( x_i = 0 \). As follows from the estimate (5.1), for this is sufficient \( \delta_i + \alpha_i > 0 \).
Lemma 2. Let \( A_iB = BA_i, \delta_i + \alpha_i > 0 \ (i = 1, m) \), then the equality
\[
\left( \sum_{i=1}^{m} A_iD_{0x_i}^{\alpha_i} - B \right) \Phi_\alpha^\delta(x) = \prod_{i=1}^{m} \frac{x_i^{-\delta_i-1}}{\Gamma(\delta_i)} I,
\]
holds for all \( x \in \Omega \), where \( I \) is identity matrix.

Proof. By virtue of (3.5) we obtain the expression
\[
D_{0x_k}^{\alpha_k} \Phi_\alpha^\delta(x) = \int_0^\infty e^{B\tau} D_{0x_k}^{\alpha_k} h_k^{\delta_k}(x_k, \tau) \prod_{i=1, i \neq k}^{m} h_i^{\delta_i}(x_i, \tau) d\tau, \quad k = 1, ..., m
\]
for the derivatives with respect to \( x_k \). We transform the previous formula for \( k = 1 \).

Using the equation (3.6), by the formula of integration by parts, and the relations (3.2) and (3.11), we obtain
\[
D_{0x_1}^{\alpha_1} \Phi_\alpha^\delta(x) = -\int_0^\infty e^{B\tau} \left[ \frac{\partial}{\partial \tau} h_1^{\delta_1}(x_1, \tau) \right] \prod_{i=2}^{m} h_i^{\delta_i}(x_i, \tau) d\tau =
\]
\[
= -e^{B\tau} \prod_{i=1}^{m} h_i^{\delta_i}(x_i, \tau) \bigg|_{\tau=\infty}^{\tau=0} + \int_0^\infty h_1^{\delta_1}(x_1, \tau) \frac{\partial}{\partial \tau} \left[ e^{B\tau} \prod_{i=1}^{m} h_i^{\delta_i}(x_i, \tau) \right] d\tau =
\]
\[
= \prod_{i=1}^{m} \frac{x_i^{\delta_i-1}}{\Gamma(\delta_i)} I + B \Phi_\alpha^\delta(x) + \int_0^\infty h_1^{\delta_1}(x_1, \tau) e^{B\tau} \frac{\partial}{\partial \tau} \prod_{i=2}^{m} h_i^{\delta_i}(x_i, \tau) d\tau =
\]
\[
= \prod_{i=1}^{m} \frac{x_i^{\delta_i-1}}{\Gamma(\delta_i)} I + B \Phi_\alpha^\delta(x) + \sum_{k=2}^{m} \int_0^\infty e^{B\tau} \left[ \frac{\partial}{\partial \tau} h_k^{\delta_k}(x_k, \tau) \right] \prod_{i=1, i \neq k}^{m} h_i^{\delta_i}(x_i, \tau) d\tau. \quad (5.9)
\]
From the relations (5.8) and (5.9) follows that
\[
\left( \sum_{k=1}^{m} A_kD_{0x_k}^{\alpha_k} - B \right) \Phi_\alpha^\delta(x) = \prod_{i=1}^{m} \frac{x_i^{\delta_i-1}}{\Gamma(\delta_i)} I +
\]
\[
+ \sum_{k=2}^{m} \int_0^\infty e^{B\tau} \prod_{i=1, i \neq k}^{m} h_i^{\delta_i}(x_i, \tau) \left[ \frac{\partial}{\partial \tau} h_k^{\delta_k}(x_k, \tau) + A_kD_{0x_k}^{\alpha_k} \right] h_k^{\delta_k}(x_k, \tau) d\tau.
\]
From this, taking into account (3.6), we obtain (5.7). Lemma 2 is proved.

5.2 Representation of the solution

Lemma 3. Let \( A_iB = BA_i \), then any regular in the domain \( \Omega \) solution \( u(x) \) of the problem (1.1) - (1.2) can be represented as (4.4).
Proof. Let the matrix $V(x)$ be a solution of equation

$$\sum_{i=1}^{m} D_{0x_i}^{\alpha_i} V(x)A_i = V(x)B + I,$$

(satisfying conditions

$$V(x^i) = 0, \quad i = 1, \ldots, n,$$

where $I$ is the identity matrix of order $n$. In view of Lemma 2 the function

$$V(x) = \int_{\Omega_x} G(t) dt = \Phi^{(1, \ldots, 1)}(x),$$

is a solution of equation

$$\sum_{i=1}^{m} A_i D_{0x_i}^{\alpha_i} V(x) = BV(x) + I.$$

Hence, taking into account the fact that the matrix $\Phi^{(\delta)}(x)$ commutes with the matrices $A_i$ and $B$, we obtain that $V(x)$ is a solution of the equation (5.10).

It follows from (5.1) that the estimate

$$|V(x)|_* \leq C \prod_{i=1}^{m} a_i^{\alpha_i \theta_i}, \quad \theta_i > 0,$$

from which, in turn, follows (5.11). That is, $V(x)$ is the solution of the problem (5.10), (5.11).

By virtue of the integration by parts formula and (5.11), we obtain

$$\int_{\Omega_x} V(x - t) \sum_{i=1}^{m} A_i D_{0x_i}^{\alpha_i} u(t) dt = -\sum_{i=1}^{m} \int_{\Omega_x \setminus \Omega_x^i} \left[ \frac{\partial}{\partial t_i} V(x - t) \right] A_i D_{0x_i}^{\alpha_i - 1} u(t) dt -$$

$$-\sum_{i=1}^{m} \int_{\Omega_x \setminus \Omega_x^i} V(x - t) \Big|_{t_i = \varepsilon_i} A_i D_{0x_i}^{\alpha_i - 1} u(t) dt, \quad i = 1, \ldots, m,$$

where $\Omega_x = \omega_{\varepsilon_1} \times \ldots \times \omega_{\varepsilon_m}$, $\Omega_x^i = \omega_{\varepsilon_1} \times \ldots \times \omega_{\varepsilon_{i-1}} \times \omega_{\varepsilon_{i+1}} \times \ldots \times \omega_{\varepsilon_m}$, $\omega_{\varepsilon_j} = \{ t_j : 0 < t_j < \varepsilon_j \}$, $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$. Passing in the last equality to the limit as $\varepsilon_i \to 0$, taking into account (1.1), (1.2), (5.10) and equality [1, p. 34]

$$\int_0^x v_1(t) D_{0t}^\nu v_2(t) dt = \int_0^x [D_{xt}^\nu v_1(t)] v_2(t) dt, \quad \nu < 0,$$

we obtain

$$\int_{\Omega_x} u(t) dt = \int_{\Omega_x} V(x - t) f(t) dt + \sum_{i=1}^{m} \int_{\Omega_x^i} V(x - t^i) A_i \varphi_i(t) dt(i).$$
Differentiating the equality (5.13) over all $x_i$ and taken into account (5.11), we obtain

$$u(x) = \int_{\Omega_x} V_{x_1...x_m} (x - t)f(t)dt + \sum_{i=1}^{m} \int_{\Omega_x} V_{x_1...x_m} (x - t^i)A_i\varphi_i(t(i))dt(i). \quad (5.14)$$

From (5.14), and the equality $V_{x_1...x_m}(x) = G(x)$, we get (4.4). Lemma 3 is proved.

5.3 Properties of the fundamental solution

**Lemma 4.** The following estimates hold

$$|G(x)|_s \leq C \prod_{i=1}^{m} x_i^{\alpha_i \theta_i - 1}, \quad \sum_{i=1}^{m} \theta_i = 1, \quad \theta_i > -1; \quad (5.15)$$

$$|D_{0x_s}^{\alpha_s} G(x)|_s \leq C x_s^{1 - \alpha_s} \prod_{i=1}^{m} x_i^{\alpha_i \theta_i - 1}, \quad \sum_{i=1}^{m} \theta_i = 1, \quad \theta_i > \left\{ \begin{array}{ll} 0, & i = s, \\ -1, & i \neq s; \end{array} \right. \quad (5.16)$$

$$|D_{0x_s}^{\alpha_s} G(x)|_s \leq C x_s^{-\alpha_s} \prod_{i=1}^{m} x_i^{\alpha_i \theta_i - 1}, \quad \sum_{i=1}^{m} \theta_i = 1, \quad \theta_i > -1, \quad (5.17)$$

where $C$ is the positive constant.

**Lemma 5.** The equality

$$\sum_{i=1}^{m} D_{0x_s}^{\alpha_i} G(x) = BG(x).$$

holds for all $x \in \Omega$.

Lemmas 4 and 5 follow from Lemmas 1 and 2 and the formula (5.6).

**Lemma 6.** Let the conditions of Theorem 1 be fulfilled, then the function $u(x)$ defined by the equality (4.4) is a solution of the equation (1.1), such that $D_{0x}^{\alpha_i} u \in C(\Omega)$.

**Proof.** Denote

$$u_f(x) = \int_{\Omega_x} G(x - t)f(t)dt, \quad u^i(x) = A_i \int_{\Omega_x} G(x - t^i)\varphi_i(t(i))dt(i), \quad i = 1, ..., m.$$  

From (5.17) we obtain

$$|D_{x,x_s}^{\alpha_s} G(x - t^i)|_s < C x_s^{\alpha_i \theta_i - 1}(x_s - t_s)^{-\alpha_s} \prod_{j=1}^{m} (x_j - t_j)^{\alpha_j \theta_j - 1}, \quad s \neq i, \quad (5.18)$$

$$|D_{0x_s}^{\alpha_s} G(x - t^i)|_s < C x_s^{\alpha_s \theta_s - \alpha_s - 1} \prod_{j=1}^{m} (x_j - t_j)^{\alpha_j \theta_j - 1}, \quad (5.19)$$

12
where \( \theta_i > \begin{cases} 1, i = s, \\ -1, i \neq s, \end{cases} \sum_{i=1}^{n} \theta_i = 1. \)

By virtue of formula [15, c. 99]

\[
D_{\nu} v_1(x-t)v_2(t) dt = \int_{0}^{x} v_1(x-t)D_{\nu} v_2(t) dt + v_1(x) \lim_{x \to 0} D_{\nu}^{-1} v_2(t), \quad 0 < \nu < 1,
\]

and estimates (5.18) and (5.19), the inclusions

\[
D_{\alpha} s \sum_{j=1}^{m} A_j D_{\alpha} s u_j(x) = B u_i(x), \quad i = 1, ..., m
\]

are valid.

Consider following integrals

\[
J(x, t_{(k)}) = \int_{0}^{x} [D_{x,t_{(k)}}^{\alpha} G(x-t)] f(t) dt,
\]

\[
J_{\varepsilon}(x, t_{(k)}) = \int_{0}^{x} [D_{x,t_{(k)}}^{\alpha} G(x-t)] f(t) dt.
\]

It is obvious that \( \lim_{\varepsilon \to 0} J_{\varepsilon}(x, t_{(k)}) = J(x, t_{(k)}) \). By (5.17), and the fact that \( f(x) \) satisfies the Holder condition, we obtain

\[
|D_{x,t_{(k)}}^{\alpha} G(x-t)[f(t_{(k)}, t_{(k)}) - f(x, t_{(k)})]| \leq C(x_k - t_k)^{q + \alpha_k(\theta_k - 1) - 1} \prod_{j=1}^{n} t_j^{\alpha_j} (x_j - t_j)^{\theta_j - 1},
\]

here \( \theta_k > 1 - \frac{1}{\alpha_k}, \sum_{j=1}^{n} \theta_j = 1. \) Hence it is easy to see that the integral on the right-hand side of

\[
\frac{\partial}{\partial x_k} J(x, t_{(k)}) = \int_{0}^{x} \frac{\partial}{\partial x_k} D_{x,t_{(k)}}^{\alpha} G(x-t) [f(t_{(k)}), f(x, t_{(k)})] dt_k -
\]
\[-D_{x_1 t_k}^{\alpha_k-1}G(x-t)|_{t_k=0}^{t_k=x_1} f(x_k, t(k)) + [D_{x_2 x_1-\epsilon}^{\alpha_k-1}G(x-t)]f(x_k - \epsilon, t(k))\]

converges uniformly on the set \(\Omega \times \Omega_x^k\) for all \(q \in (0, 1]\). Therefore

\[
\lim_{\epsilon \to 0} \frac{\partial}{\partial x_k} J_\epsilon(x, t(k)) = \frac{\partial}{\partial x_k} J(x, t(k)) = [D_{x_1 t_k}^{\alpha_k-1}G(x-t)]f(x_k, t(k)) + \int_0^{x_k} [D_{x_2 t_k}^{\alpha_k} G(x-t)]f(t_k, t(k)) - f(x_k, t(k)) dt_k.
\]

From the latter, taking into account

\[
\text{In view of } (5.12), \text{ we have }
\]

\[
\text{From this estimate follows } (5.25).
\]

We get

\[
\sum_{k=1}^m A_k D_{0 x_k}^{\alpha_k} u_f(x) = \frac{\partial}{\partial x_k} \int_{\Omega_x} [D_{x_1 t_k}^{\alpha_k-1}G(x-t)]f(t)dt = \frac{\partial}{\partial x_k} J(x, t(k)) dt(k) \in C(\Omega).
\]

(5.23)

In view of (6.12), we have

\[
\int_{\Omega_x} \left[ \sum_{k=1}^m A_k D_{0 x_k}^{\alpha_k} - B \right] u_f(t)dt = \int_{\Omega_x} \left[ \sum_{k=1}^m A_k D_{x_1 t_k}^{\alpha_k} - B \right] V(x-t)f(t)dt = \int_{\Omega_x} f(t)dt.
\]

Which implies that

\[
\sum_{k=1}^m A_k D_{0 x_k}^{\alpha_k} u_f(x) = Bu_f(x) = f(x).
\]

(5.24)

The validity of the lemma 6 follows from (5.20) - (5.24). Lemma 6 is proved.

**Lemma 7.** Let the function \(\varphi_j(x_{(j)})\) satisfies the condition (4.2), then the following relations

\[
\lim_{x_s \to 0} D_{0 x_s}^{\alpha_s-1} \int_{\Omega_x^s} G(x-t^s)\varphi_j(t_{(j)})dt_{(j)} = 0, \quad s \neq j, \quad x_{(s)} \in \Omega^s \setminus \Omega_x^s, \quad (5.25)
\]

\[
\lim_{x_s \to 0} D_{0 x_s}^{\alpha_s-1} \int_{\Omega_x^s} G(x-t^s)\varphi_s(t_{(s)})dt_{(s)} = \varphi_s(x_{(s)}), \quad x_{(s)} \in \Omega^s \setminus \Omega_x^s \quad (5.26)
\]

hold, and the limits are uniform on any closed subset of the domain \(\Omega^s\).

**Proof.** In view of (5.10) we have the estimate

\[
|D_{0 x_s}^{\alpha_s-1} u^s(x)| \leq C x_s^{\alpha_x+\mu_x-\alpha_s}, \quad s \neq j, \quad (5.27)
\]

for \(x_{(s)} \in \Omega^s \setminus \Omega_x^s\). From this estimate follows (5.25).
Consider the integral

\[ D_{0x_s}^{\alpha_s-1}u^s(x) = A_s D_{0x_s}^{\alpha_s-1} \int_{\Omega_s} G(x - t^s) \varphi_s(t(s)) dt(s) = \]

\[ = A_s \left( \int_{\Omega_s^x} + \int_{\Omega_s^x \setminus \Omega_s^z} \right) D_{0x_s}^{\alpha_s-1} G(x_s, t(s)) \varphi_s(x(s) - t(s)) dt(s), \quad (5.28) \]

where \( G(x_s, t(s)) = G(t)|_{t_s = x_s}, \Omega_s^x = \omega_{x_s} \times ... \times \omega_{x_{i-1} \times \omega_{x_{i+1}} \times ... \times \omega_{x_n}}, \omega_{\varepsilon_j} = \{t_j : 0 < t_j < \varepsilon_j\}, \varepsilon = (\varepsilon_1, ..., \varepsilon_n). \) The limit of the second integral for \( x_s \to 0 \) is zero, due to the estimate

\[ |D_{0x_s}^{\alpha_s-1}G(x_s, t(s))| \leq C x_s^{\alpha_s \theta}, \quad 0 < \theta \leq 1, \quad x_s \in \Omega_s \setminus \Omega_s^z, \]

and the boundedness of the integral \( \int_{\Omega_s^x \setminus \Omega_s^z} \varphi_s(x_s - t(s)) dt(s). \) We denote the first integral \( I_1(x), \) then

\[ I_1(x) = A_s \int_{\Omega_s^x} D_{0x_s}^{\alpha_s-1} G(x_s, t(s)) \left[ \varphi_s(x(s) - t(s)) - \varphi_s(x(s)) \right] dt(s) + \]

\[ \quad + A_s \int_{\Omega_s^x \setminus \Omega_s^z} D_{0x_s}^{\alpha_s-1} G(x_s, t(s)) dt(s) \varphi_s(x(s)) = \]

\[ = A_s \int_{\Omega_s^x} \left[ \int_0^\infty e^{B_\tau} h_{1-x_s}^{1-\alpha_s}(x_s, \tau) \prod_{i=1}^m h_0^0(t_i, \tau) d\tau \right] \left[ \varphi_s(x(s) - t(s)) - \varphi_s(x(s)) \right] dt(s) + \]

\[ \quad + A_s \int_0^\infty e^{B_\tau} h_{1-x_s}^{1-\alpha_s}(x_s, \tau) d\tau \int_{\Omega_s^x \setminus \Omega_s^z} \prod_{i=1}^m h_0^0(t_i, \tau) dt(s) \varphi_s(x(s)). \quad (5.29) \]

Using the fact that by virtue of (3.35)

\[ \int_0^{\varepsilon_i} \frac{1}{t_i} \phi \left(-\alpha_i, 0; -A_i \tau t_i^{-\alpha_i} \right) dt_i = \phi \left(-\alpha_i, 1; -A_i \tau \varepsilon_i^{-\alpha_i} \right), \]

and then replacing the integration variable, we transform the integral

\[ \int_0^\infty e^{B_\tau} h_{1-x_s}^{1-\alpha_s}(x_s, \tau) d\tau \int_{\Omega_s^x \setminus \Omega_s^z} \prod_{i=1}^m h_0^0(t_i, \tau) dt(s) = \int_0^\infty e^{B_\tau} h_{1-x_s}^{1-\alpha_s}(x_s, \tau) \prod_{i=1}^m h_0^0(\varepsilon_i, \tau) d\tau = \]

15
\[
\int_0^\infty \phi(-\alpha_s, 1 - \alpha_s; -A\epsilon z) F(x_s, z) dz.
\] (5.30)

where 
\[
F(x_s, z) = e^{Bx_s t} \prod_{i \neq s} \phi(-\alpha_i, 1; -A_i \epsilon^{-\alpha_i} x_s^\alpha z). 
\]

It follows from the estimate (2.5) that there exists a uniform limit for all 
\(z \in [0, z_0]\) \(z_0 < \infty\)
\[
\lim_{x_s \to 0} F(x_s, z) = I,
\] (5.31)

and, that \(|F(x_s, z)| \leq \exp(\gamma x_s z)\), for each finite \(x_s \leq x_{s0}\). It follows from the latter that the integral (5.30) converges uniformly in all \(x_s \in [0, x_{s0}]\). Passing to the limit in the integral (5.30) as \(x_s \to 0\), by taking into account (5.31) and the formula (3.7), we get
\[
\lim_{x_s \to 0} \int_0^\infty e^{Bx_s t} \int_0^\infty \prod_{i \neq s} h_i^0(t_i, \tau) dt_i = 
\int_0^\infty \phi(-\alpha_s, 1 - \alpha_s; -A_s z) dz = A_s^{-1}.
\] (5.32)

The function \(\varphi_s(t_{(s)})\) is continuous on \([x - \epsilon, x]\), therefore 
\[
\omega(\epsilon) = \sup |\varphi_s(x_{(s)} - t_{(s)}) - \varphi_s(x_{(s)})| \to 0
\]
for \(\epsilon \to 0\). Because of the arbitrariness of the choice of \(\epsilon\) and (5.32), for \(x_s \to 0\) the first term in (5.29) tends to zero, and the second to \(\varphi_s(x_{(s)})\). Thus 
\[
\lim_{x_s \to 0} I_1(x) = \varphi_s(x_{(s)}).
\]

From the latter together with (5.28) it follows (5.26). Lemma 7 is proved.

6 Proof of Theorem 1

Proof. Taking into account that \(f^*(x) = \prod_{i=1}^m x_i^{1-\mu_i} f(x) \in C(\Omega)\) and using (4.3) and (5.15) we get 
\[
|u_f(x)|, \leq C \prod_{i=1}^m x_i^{\mu_i + \alpha_i \theta_i - 1}, \quad \theta_i > 0, \quad \sum_{i=1}^m \theta_i = 1.
\] (6.1)

It follow from (6.1) that 
\[
\lim_{x_s \to 0} D_{0x_s}^{\alpha_s - 1} u_f(x) = 0.
\]

16
From Lemma 7 and the last relation, the fulfillment of the boundary conditions (1.2) follows. From the estimate (6.1) it also follows that \( \prod_{i=1}^{n} x_i^{1-\mu_i} u_f \in C(\Omega) \).

Using (4.2) and (5.15) give

\[
|u_k(x)| \leq C x_k^{\alpha_k} x_1^{1-\mu_k} \prod_{i=1 \atop i \neq k}^{m} x_i^{\alpha_i+\theta_i-1}.
\]

From the last inequality it follows that \( \prod_{i=1}^{m} x_i^{1-\mu_i} (u - u_f) \in C(\Omega) \). The foregoing, together with Lemma 6 proves the existence of a regular solution of the problem (1.1) – (1.2). The uniqueness of the solution of the problem follows from Lemma 5. Theorem 1 is proved.

References

[1] Nakhushev A. M. Fractional calculus and its applications, Moscow: Fizmatlit, 2003. (In Russian).

[2] Clement Ph., Gripenberg G., Londen S-O. Schauder estimates for equations with fractional derivatives, Trans. of the Amer. Math. Soc., 352:5, (2000), 2239–2260.

[3] Wright E. M. On the coefficients of power series having exponential singularities, J. London Math. Soc., 8:29, (1933), 71–79.

[4] Wright E. M. The asymptotic expansion of the generalized Bessel function, Proc. London Math. Soc. Ser. II, 38, 257–270.

[5] Clement Ph., Gripenberg G., Londen S-O. Holder regularity for a linear fractional evolution equation, Progr. Nonlinear Differ. Equat. and Their Appl., 35, (1999), 62–82.

[6] Pskhu A. V. Solution of a boundary value problem for a fractional partial differential equation, Differential Equation, 39:8, (2003), 1092-1099. (In Russian).

[7] Pskhu A. V. Fractional partial differential equations, Moscow: Nauka, 2005. (In Russian).

[8] Mamchuev M. O. A boundary value problem for a first-order equation with a partial derivative of a fractional order with variable coefficients, Reports of Circassian International Academy of Sciences. 11:1, (2009), 32–35. (In Russian).

[9] Mamchuev M. O. Cauchy problem in non-local statement for first order equation with partial derivatives of fractional order with variable coefficients, Reports of Circassian International Academy of Sciences. 11:2, (2009), 21–24. (In Russian).
[10] Mamchuev M. O. Boundary value problems for equations and systems with the partial derivatives of fractional order, Nalchik: Publishing house KBSC of RAS, 2013. (In Russian).

[11] Pskhu A. V. Boundary value problem for a multidimensional fractional partial differential equation, Differential Equation, 47:3, (2011), 385-395. (In Russian).

[12] Mamchuev M. O. Boundary value problem for a system of fractional partial differential equations, Differential Equations, 44:12, (2008), 1737–1749.

[13] Mamchuev M. O. Boundary value problem for a linear system of equations with the partial derivatives of fractional order, Chelyabinsk Physical and Mathematical Journal, 2:3, (2017), 295–311. (In Russian).

[14] Gorenflo R., Luchko Yu., Mainardi F. Analytical properties and applications of the Wright function, Fractional Calculus and Applied Analysis, 2:4, (1999), 383–414.

[15] Podlubny I. Fractional differential equations, New-York: Acad. press, 1999.

Institute of Applied Mathematics and Automation of KBSC RAS
"Shortanov" Str., 89 A, Nal’chik – 360000, RUSSIA
e-mail: mamchuev@rambler.ru