A PLUG WITH INFINITE ORDER AND SOME EXOTIC 4-MANIFOLDS

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Abstract. Every exotic pair in 4-dimension is obtained each other by twisting a cork or plug which are codimension 0 submanifolds embedded in the 4-manifolds. The twist was an involution on the boundary of the submanifold. We define cork (or plug) with order $p \in \mathbb{N} \cup \{\infty\}$ and show there exists a plug with infinite order. Furthermore we show twisting $(P, \varphi^2)$ gives to enlargements of $P$ compact exotic manifolds with boundary.

1. Introduction

1.1. Smooth structures. Let $X$ be a smooth manifold. If a smooth manifold $X'$ is homeomorphic but non-diffeomorphic to $X$, then we say that $X$ and $X'$ are exotic (pair). Any exotic pair gives a different smooth structure on a topological manifold. It is known that if $X$ has at least two smooth structures, then the dimension is greater than 3.

Cork (or plug) is a pair of a submanifold with codimension 0 in a 4-manifold $X$ and an involution on the boundary. They were defined in [A1] and [GS] and Akbulut-Yasui [AY1]. Twisting the cork (or plug) in $X$ by the involution, we can get an exotic pair. Conversely it is known that any simply connected exotic pair can be obtained by the (contractible) cork twisting as proven in [AM, CFHS, M].

Recently many smooth structures have constructed by using cork and plug as in [A2, AY1, AY2, AY3, AY4, GS]. Since the main idea for the existence of cork and plug is due to the failure of the h-cobordism theorem in 4-dimension, naturally the self-diffeomorphism of the boundary of cork and plug is an involution. For example as appeared in [GS] an exotic pair $E(2)\#\overline{CP^2}$ and $\#^3\overline{CP^2}\#^{20}CP^2$ are obtained by a cork twisting (an involution), that $C$ is a contractible 4-manifold having Mazur type and the involution is by a symmetry of the framed link presenting $C$.
There exist infinite smooth structures as a character of 4-dimension. Fintushel-Stern’s Knot surgery, which is defined below, gives rise to mutually non-diffeomorphic manifolds. Let \( T \subset X \) be an embedded torus with trivial normal bundle. Let \( K \) be a knot in \( S^3 \). The knot surgery is defined by

\[
X_K = [X - \nu(T)] \cup_{\phi} [(S^3 - \nu(K)) \times S^1],
\]

where the definition of \( \phi \) is in [FS] and the notation \( \nu \) stands for the tubular neighborhood of the submanifold. The cut and paste notation will be defined in Definition 1.1.

One answer of the question is the result in [A2]. Our motivation is to construct “a cork (or plug)” representing the infinite of exoticity. We will relax the definition of cork and plug to accept infinite order. In the next subsection we will define such a cork and plug and in next section we show the following.

**Theorem 1.1.** There exists a plug \((P, \phi)\) with infinite order. \( P \) is a simply connected, compact, Stein 4-manifold with \( b_2 = 2 \).

The plug \( P \) can be embedded in an elliptic fibration \( X \) over \( D^2 \) with three vanishing cycles, which exactly two of them are parallel. In this case the plug twist \((P, \phi)\) can change \( X \) to knot surgery \( X_K \), where \( K \) is any unknotting number 1. Furthermore \((P, \phi^n)\) gives \( X_{K^n} \), where \( K^n \) is a knot obtained by \( n \) times iteration of the knotting operation from unknot to \( K \).

The square \((P, \phi^2)\) of the plug twist is a non-contractible cork with infinite order. Using this we obtain the following.

**Theorem 1.2.** Let \( \tilde{Y} \) be a 4-manifold presented by the left diagram in Figure 1. There exists a minimal (not having any \((-1)\)-spheres), symplectic, simply-connected 4-manifold \( Y_2 \) with the same homeomorphism type as \( Y = \tilde{Y} \# \mathbb{C}P^2 \). In particular \( Y \) and \( Y_2 \) are an exotic pair.

Let \( \tilde{Z} \) be a 4-manifold presented by the right diagram in Figure 1. There exists a minimal (not having any \((-1)\)-spheres), symplectic, simply-connected 4-manifold \( Z_2 \) with the same homeomorphism type as \( Z = \tilde{Z} \# 2 \mathbb{C}P^2 \). In particular \( Z \) and \( Z_2 \) are an exotic pair.

**Figure 1.** \( \tilde{Y} \) and \( \tilde{Z} \).

Finally we define a notation on cut and paste of 4-manifolds, used in the paper.
**Definition 1.1** (Cut and Paste). We define a notation of cut and paste. Let $X$ be a 4-manifold and $Z \subset X$ a compact, codimension-0 submanifold in $X$. Let $Y$ be a 4-manifold with the same boundary $M$ as $\partial Z$. We fix identification $M = \partial Z = \partial Y$. Let $\varphi$ be a self-diffeomorphism of $M$. The manifold obtained by identifying $\partial W$ and $\partial Y$ through $\partial Z = M \xrightarrow{\varphi} M = \partial Y$ is presented as follows:

$$[X - Z] \cup_{\varphi} Y.$$  

If $Z = Y$, we use the obvious map as the identification.

**1.2. Cork and Plug with order $p$.** We define notions of cork and plug with order $p$. See [AY1] for the original cork and plug.

**Definition 1.2** (Cork with order $p$). Let $(C, \varphi)$ be a pair of a compact, contractible, Stein 4-manifold $C$ with boundary and a diffeomorphism $\varphi : \partial C \to \partial C$. The pair $(C, \varphi)$ is called a cork with order $p$ if $C$ satisfies the following properties: the order of $\varphi$ is $p \geq 2$, $\varphi$ can extend to a self-homeomorphism of $C$ but $\varphi^q$ ($1 \leq q < p$) cannot be extended to any self-diffeomorphism of $C$. 

In the case of $\varphi^q \neq \text{id}$ for any natural number $q$ we call $(C, \varphi)$ a cork with infinite order.

Let $(C, \varphi)$ be a cork with order $p$ and $X$ a 4-manifold containing $C$. A cork twist of $X$ is the set of manifolds $[X - C] \cup_{\varphi^q} C$ ($1 \leq q \leq p$). If the cork twist of $X$ gives mutually different $p$ smooth structures, the cork $(C, \varphi)$ is called a cork of $X$ with order $p$.

When we treat a non-contractible submanifold as $C$ as appeared in [AY1], then we call it a generalized cork with order $p$.

**Definition 1.3** (Plug with order $p$). Let $(P, \varphi)$ be a pair of a compact Stein 4-manifold $P$ with boundary and a diffeomorphism $\varphi : \partial P \to \partial P$. The pair $(P, \varphi)$ is called a plug with order $p \geq 2$ if $P$ satisfies the following properties: the order of $\varphi$ is $p \geq 2$, $\varphi$ cannot be extended to any self-homeomorphism of $P$, there exists a 4-manifold $X$ containing $P$ such that $[X - P] \cup_{\varphi^q} P$ ($0 \leq q < p$) are $p$ mutually non-diffeomorphic manifolds. 

In the case of $\varphi^q \neq \text{id}$ for any natural number $q$ we call $(P, \varphi)$ a plug with infinite order.

Let $(P, \varphi)$ be a plug with order $p$ and $X$ a 4-manifold containing $P$. A plug twist of $X$ is the set of manifolds $[X - P] \cup_{\varphi^q} P$ ($0 \leq q < p$). If the plug twist of $X$ gives mutually different $p$ smooth structures, the plug $(P, \varphi)$ is called a plug of $X$ with order $p$.

Any cork (or plug) with order 2 means the original cork (or plug).

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2. A plug \((P, \phi)\) with infinite order.

2.1. The diffeomorphism type of \(P\). We define a plug \(P\) mainly used through the paper.

**Definition 2.1.** We define a compact 4-manifold \(P\) to be a manifold admitting the handle decomposition in Figure 2.

\[
\begin{array}{c}
\begin{array}{c}
\includegraphics[width=0.2\textwidth]{figure2}
\end{array}
\end{array}
\]

**Figure 2.** A handle decomposition of \(P\).

On the other hand the manifold \(P\) is obtained as \(V_2 \times S^1\) with three \(-1\)-framed 2-handles, where \(V_2\) is the genus 2 handlebody.

Thus \(P\) is simply connected and has \(H_2(P) \cong \mathbb{Z}^2\). The fundamental group of the boundary \(\partial P\) is

\[
\pi_1(\partial P) = \langle a, b, c, d, f | [f, b^{-1}], [f, a^{-1}], a[f, c^{-1}], b[f, d^{-1}], f^{-1}YX \rangle,
\]

where we use generators \(a, b, c, d\) and \(f\) in Figure 3. Elements \(X, Y\) are \(X = [b^{-1}, d]\) and \(Y = [a^{-1}, c]\). Therefore we get \(H_1(\partial P) \cong \mathbb{Z}^2\).

2.2. A diffeomorphism \(\phi\) on \(\partial P\). We define a diffeomorphism on \(\partial P\). Moving the diagram of \(\partial P\) in accordance with the process of Figure 4 obtains a diffeomorphism of \(\partial P\) is denoted by \(\phi\).

From the definition of \(\phi\) the images of the generators \(a, b, c, d, f\) are as follows:

\[
(1) \quad (Xb)^{-1}aXb, \ b, \ (Xb)^{-1}a(Xb)a^{-1}c(Xb), \ d(Xd)^{-1}a(Xd), \ f.
\]

**Lemma 2.1.** The gluing map \(\phi\) acts on \(H_1(\partial P)\) and \(H_2(\partial P)\) trivially.

**Proof.** The images in \([1]\) on the abelianization of \(\pi_1(\partial P)\) are trivial. Then the abelianization of \(\phi_*\) is trivial. Thus \(\phi_*\) acts on \(H_2(\partial P)\) trivially through the Poincaré duality.

2.3. \(P\) is a Stein manifold. To show that \(P\) is a plug, \(P\) must be a Stein manifold (admit a Stein structure). Admitting 4-dimensional Stein structure is due to a description by a specific Legendrian surgery diagram on \(#^{\nu} S^2 \times S^1\). The manifold is constructed by attaching 2-handles on \(\sharp^n D^3 \times S^1\) along the framed Legendrian link, where \(\sharp^n\) stands for the boundary sum. The
condition of the framed Legendrian link is that each framing of attaching 2-handles are \( \text{tb}(K) - 1 \), where \( \text{tb}(K) \) is the Thurston-Bennequin invariant of the Legendrian knot \( K \) (see [GS] for the explanation).

**Proposition 2.1.** \( P \) admits a Stein structure.

**Proof.** \( P \) admits handle decomposition as in Figure 5. Each Thurston-Bennequin invariant of the components is 1. Thus all framings of the components satisfy \( \text{tb}(K) - 1 \). Therefore \( P \) admits a Stein structure. \( \square \)
2.4. **Infinite exotic manifolds from \((P, \varphi)\)-twist.** This subsection is essential for the pair \((P, \varphi)\) to be a plug with infinite order. For any embedding \(P \subset W\) the performance \([W - P] \cup_{\varphi} P\) by using \((P, \varphi)\) is called \((P, \varphi)\)-twist.

**Proposition 2.2.** The pair \((P, \varphi)\) produces infinitely many exotic manifolds.

**Proof.** Let \(K_n\) be a twist knot as in Figure 6. Let \(X\) be a manifold as in the left diagram in Figure 7. Performing knot surgery on \(X\), we get the right diagram in Figure 7. Sliding one of the two \(-1\)-framed 2-handles over 0-framed 2-handle in the diagram, and removing the top 0-framed 2-handle, we get the handle diagram of \(P\) in \(X\).

Here we perform the \((P, \varphi)\)-twist \([X_{K_n} - P] \cup_{\varphi} P\). Keeping track of the diffeomorphisms in Figure 4 for \(X_{K_n}\), consequently from Figure 8 we get the following:

\[ [X_{K_n} - P] \cup_{\varphi} P = X_{K_{n+1}}. \]

Namely \((P, \varphi)\)-twist of \(X_{K_n}\) gives \(X_{K_{n+1}}\).

Here we embed \(X\) in \(E(2)\) (the K3-surface) as an elliptic fibration having three vanishing cycles which exactly two of them are parallel.

We trivialize the tubular neighborhood \(E(2) \ni \nu(T) = D^2 \times T^2\) of the general fiber \(T\), where the vanishing cycles of \(E(2)\) generate homology classes of \(\{pt\} \times T\) and the direction \(\partial D^2 \times \{pt\}\) corresponds to a section of \(E(2)\).
This section is mapped to the longitude of $K_n$ by the gluing map of knot surgery.

The obvious map $i : \partial X_{K_n} \rightarrow \partial (X_{K_n} - P) \cup \varphi P = \partial X_{K_{n+1}}$ preserves the two directions corresponding to the vanishing cycles (see Figure 11). Furthermore $i$ maps the direction of the section in $\partial X_{K_n}$ (namely longitude of $K_n$) to the direction of the section in $\partial X_{K_{n+1}}$ (see Figure 12 where the framings of unlabeled links are 0). Thus in this case the replacement of $P$ by $\varphi$ of $E(2)$ gives $E(2)_{K_{n+1}}$.

In general for any 4-manifold $E$ containing $X$ the replacement of $P$ by $\varphi$ gives some knot surgery $E_K$. $K$ is determined depending on $P \hookrightarrow X$.

By the Seiberg-Witten formula on knot surgery in [FS]

$$SW_{E(2)_{K_n}} = nt - 2n - 1 + nt^{-1},$$

thus $E(2)_{K_n}$ are infinitely many mutually non-diffeomorphic manifolds by $(P, \varphi)$-twist. As a result $(P, \varphi)$-twist can give rise to infinite exotic pairs. $\square$

Namely the map $\varphi$ has infinite order.

2.5. **Extendability as a homeomorphism.** Here we show that $\varphi$ cannot be extended to a homeomorphism of $P$.

**Proposition 2.3.** Let $n$ be any integer. The diffeomorphism $\varphi^n : \partial P \rightarrow \partial P$

\[
\begin{cases}
\text{cannot be extended to } P \rightarrow P \text{ as any homeomorphism} & \text{ } n \text{ odd} \\
\text{can be extended to } P \rightarrow P \text{ as a homeomorphism} & \text{ } n \text{ even}.
\end{cases}
\]

In particular $P \cup_{id} \mathbb{T}$ and $P \cup_{\varphi} \mathbb{T}$ are diffeomorphic to $\#^2 S^2 \times S^2$ and $\#^2(\mathbb{CP}^2 \# \overline{\mathbb{CP}^2})$ respectively.
Figure 9. The two longitudes preserves by the operation.

Proof. We take the double \( D(P) := P \cup_{id} \overline{P} \) by using the identity map. This manifold is diffeomorphic to \( \#^2 S^2 \times S^2 \) due to Figure 10. On the other hand the manifold \( D(P, \varphi) := P \cup_{\varphi} \overline{P} \) glued by use of \( \varphi \) is diffeomorphic to \( \#^2(\mathbb{C}P^2 \# \mathbb{C}P^2) \) as in Figure 11. Since \( \#^2 S^2 \times S^2 \) and \( \#^2(\mathbb{C}P^2 \# \mathbb{C}P^2) \) are not homeomorphic, then \( \varphi \) cannot be extended to any homeomorphism of \( P \).

In general \( D(P, \varphi^n) = P \cup_{\varphi^n} \overline{P} \) is diffeomorphic to the left in Figure 12. Replacing \(-1\)-framed 2-handle at the bottom in this picture with 0-framed 2-handle, we get a manifold \( D_n \) with the same intersection form as \( D(P, \varphi^n) \).

From Freedman’s result \( D(P, \varphi^n) \) and \( D_n \) are homeomorphic. The diagram of \( D_n \) in Figure 12 can be simplified by iterating the local process in Figure 13. As in Figure 14 \( D_n \) is diffeomorphic to

\[
\begin{cases} 
\#^2(\mathbb{C}P^2 \# \mathbb{C}P^2) & \text{n odd} \\
\#^2(S^2 \times S^2) & \text{n even.}
\end{cases}
\]

Hence \( D_{\varphi^n}(P) \) is homeomorphic to \( \#^2 S^2 \times S^2 \) or \( \#^2(\mathbb{C}P^2 \# \mathbb{C}P^2) \) respectively namely it is spin (or non-spin) if \( n \equiv 0(2) \) (or \( n \equiv 1(2) \)).

According to (0.8) Proposition (iii) in Boyer’s paper [B], \( \varphi^n \) can be extended to a homeomorphism of \( P \) if \( n \) is even, and \( \varphi^n \) cannot be extended to any homeomorphism of \( P \) if \( n \) is odd. \( \square \)
As a corollary we get the following corollary.

**Corollary 2.1.** For non-zero integer \( m \), \((P, \varphi^{2m})\) are infinite generalized corks with infinite order.

**Proof.** Assertion in Proposition 2.3 means \( \varphi^{2m} \) can be extended as a homeomorphism of \( P \). Therefore \((P, \varphi^{2m})\) are infinite many corks with infinite order.

**Proof of Theorem 1.1.**
Definition 2.1, Proposition 2.1, 2.2, and 2.3 mean Theorem 1.1.

From the proof of Proposition 2.2, the plug twist by \((P, \varphi)\) means “crossing changing operation” of smooth structures through knot surgery. On the other hand any twist knot \( K_n \) is an unknotting number 1 knot and there exist two embeddings \( \iota_1 : P \hookrightarrow E(2) \) and \( \iota_2 : P \hookrightarrow E(2) \) such that the plug
twists give rise to $E(2)_{K_n}$ and $E(2)_{K_{n+1}}$ as the diagram below.

\[
\begin{array}{c}
E(2) \xrightarrow{(t_1, P, \varphi)} E(2)_{K_n} \\
\downarrow \quad \downarrow (P, \varphi) \\
E(2)_{K_{n+1}}
\end{array}
\]
This means \( \iota_1 \) and \( \iota_2 \) are different embeddings of \( P \) in \( E(2) \).

We raise the several questions.

**Question 2.1.** Are there exist any plug \((Q, \psi)\) with infinite order which \( \psi^n \) for any integer \( n \) cannot be extended to a homeomorphism of \( P \)?

Or if \((Q, \psi)\) is a plug with infinite order, then is \((Q, \psi^2)\) a generalized cork with infinite order?

**Question 2.2.** Are there exist any plug or cork \((Q, \psi)\) with finite order \( p \) (\( 3 \leq p < \infty \))?

3. **Some exotic manifolds.**

We consider enlargements of \( P \) attaching some \(-1\)-framed 2-handles.

3.1. **A manifold exotic to \( Y \).** Attaching \(-1\)-framed 2-handle over one meridian of the two 0-framed 2-handles of \( P \) as in Figure 15 we define the resulting manifold to be \( Y \). Thus \( Y := \tilde{Y} \# \mathbb{CP}^2 \), where \( \tilde{Y} \) is the 4-manifold attached along a satellite link as in right picture in Figure 15.

\( \tilde{Y} \) is a simply connected 4-manifold with \( b_2 = 2 \) and the boundary \( \partial Y \) is a 3-manifold with \( H_1 \cong \mathbb{Z} \).

Let \( W \) be a 4-manifold and \( T \subset W \) an embedded torus with the trivial neighborhood. Let \( L = K_1 \cup \cdots \cup K_n \) an \( n \)-component link. For \( n \) copies \((W_i, T_i)\) of \((W, T)\) we define link operation \( W_L \) as follows:

\[
W_L = \left( (S^3 - \nu(L)) \times S^1 \right) \cup \varphi \left( \bigcup_{i=1}^{n} [W_i - \nu(T_i)] \right),
\]

where the \( n \) attaching maps are the common \( \varphi \) and the definition is in [FS].

In this section we show the following.

**Theorem 3.1.** \( Y \) admits at least two smooth structures \{\( Y, Y_2 \)\}. \( Y_2 \) admits symplectic structure and minimal.
Proof. Twisting \( \tilde{P} \) using the generalized cork \((P, \varphi^2)\), we get Figure 17 (called \( Y_2 \) here). The manifold \( Y_2 \) can be embedded in a link operation \( E(1)_{L_2} \), where \( E(1) \) is the elliptic fibration with 12 nodal singularities over \( S^2 \) and contains a general fiber as an embedded torus with trivial normal bundle. \( L_n \) is the \((2, 2n)\)-torus link. Since the boundaries are homeomorphic by an easy handle calculation and the cork can be extended to \( P \) as a homeomorphism, the two manifolds \( Y \) and \( Y_2 \) are homeomorphic.

Since the Seiberg-Witten invariant of \( E(1)_{L_2} \) is computed as the multi-valuable Alexander polynomial of \( L_2 \), it is
\[
t_1 t_2 + t_1^{-1} t_2^{-1},
\]
where each \( t_i \) is the Poincaré dual \( PD([T_i]) \) of a general fiber \( T_i \) of the two \( E(1) \). Namely the basic classes of \( E(1)_{L_2} \) are \( \{ \pm PD([T_1] + [T_2]) \} \). If \( Y \) and \( Y_2 \) are diffeomorphic, then there exists an embedding \( Y \hookrightarrow E(1)_{L_2} \). This means \( E(1)_{L_2} = Y' \# \mathbb{CP}^2 \). The Seiberg-Witten basic classes \( B_{E(1)_{L_2}} \) of \( E(1)_{L_2} \) have to be of form \( B_{E(1)_{L_2}} = \{ K \pm PD([E_1])|K \in B_{Y'} \} \), where \( E_1 \) are the exceptional sphere. Thus \( [E_1] = \pm([T_1] + [T_2]) \) holds. From the square \(-1 = [E_1]^2 = ([T_1] + [T_2])^2 = 0 \) this is contradiction. Therefore \( Y \) and \( Y_2 \) are non-diffeomorphic and \( Y_2 \) does not have any \((-1)\)-sphere. Thus \( Y \) and \( Y_2 \) are exotic manifolds.

Furthermore \( L_2 \) is a fibered link, thus \( E(1)_{L_2} \) is a symplectic manifold. In particular \( Y_2 \) is a minimal symplectic manifold. □

Applying the same argument to the case of the generalized cork twist \((P, \varphi^{2n})\) of \( Y \), we obtain \( Y_{2n} \hookrightarrow E(1)_{L_{2n}} \).

\[
SW_{E(1)_{L_{2n}}} = \Delta_{L_{2n}}(t_1, t_2) = (t_1 t_2)^{2n-1} + (t_1 t_2)^{2n-3} + \cdots + (t_1 t_2)^{-2n+1}
\]
As a result \( Y \) and \( Y_{2n} \) are non-diffeomorphic. Since each \( Y_{2n} \) does not contain \((-1)\)-sphere, \( Y_{2n} \) is a minimal symplectic 4-manifold. However whether these manifolds \( Y_{2n} (n \geq 1) \) are mutually non-diffeomorphic or not is not known.
3.2. A manifold exotic to $Z$. Next we define $Z = P \cup h_1 \cup h_2 = \tilde{Z} \# \mathbb{CP}^2$, where the attaching circles of $h_1$ and $h_2$ are the meridians of 0-framed 2-handles in Figure 15. The framings of $h_1$ and $h_2$ are $-1$. $\tilde{Z}$ is a simply connected manifold obtained by attaching two 2-handles over the same link as $\tilde{Z}$ with framing $(1, 1)$. $Z$ has $b_2 = 4$ and $b_2^+ = 2$. The boundary $\partial Z$ is a homology sphere (a torus sum of two copies of the trefoil complement).

In the same way we assert the following.

**Theorem 3.2.** $Z$ admits at least two smooth structures $Z, Z_2$. $Z_2$ admits symplectic structure and minimal.

**Proof.** The manifold $Z_2$ which is obtained by the generalized cork twist $(P, \varphi^2)$ can be embedded in $E(1)L_2$. Therefore by Freedman’s result, $Z$ and $Z_2$ are homeomorphic. In particular they have the same boundary.

If $Z_2$ is diffeomorphic to $Z$, the same argument of the Seiberg-Witten invariant as the previous one leads to contradiction. Therefore $Z_2$ is non-diffeomorphic to $Z$. □

**Proof of Theorem 1.2** Theorem 3.1 and Theorem 3.2 imply Theorem 1.2. □

**Conjecture 3.1.** $\{Y_{2m}\}$ and $\{Z_{2m}\}$ are infinitely many mutually exotic manifolds to $Y$ and $Z$ respectively.

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