Dilepton Yield in Heavy-Ion Collisions with Bose Enhancement of Decay Widths

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Abstract

The excess of low invariant mass dilepton yield in heavy ion collisions arising from reduction in the rho meson mass at finite temperatures is partially suppressed because of the effect on the width of the rho meson induced by Bose enhancement, essentially due to emission of pions in a medium of the pion gas in the central rapidity region. The sensitivity of the effect on the initial temperature of the hadronic phase is also examined.

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Heavy ion collisions (HIC) provide an unique probe to study the properties of hot and dense nuclear matter temporarily produced when two highly energetic heavy nuclei collide in the laboratory. Various particles are produced directly and via secondary collisions in such high energy HIC which serve as a tool to extract information about the environment leading to their production; among these the dileptons, as well as the photons, seem to be rather promising, as they interact only electromagnetically with the rest of the matter and therefore, suffer minimum final state interaction. With this in mind several experiments have been performed to measure the dilepton yield in HIC \[1, 2\]. This could also provide important signals for the novel state of matter - the Quark Gluon Plasma (QGP) phase, expected to be produced in collisions of two nuclei at ultra-relativistic energies, as per conventional wisdom of Quantum Chromodynamics.

In reality, the contributing factors to this spectra are many and may in fact come from the different stages of the evolution of the hot and dense hadronic matter temporarily produced in the preliminary stage of the reaction. In the hadronic stage major contributions may come, say for example, from \(\pi\pi \rightarrow e^+e^-\), \(\pi\rho \rightarrow \pi e^+e^-\), \(\omega \rightarrow \pi^0 e^+e^-\), among many others as discussed in Ref. \[3\]. In this letter, we demonstrate the effect on low invariant mass dilepton yield in heavy ion collisions arising from the increase in width of vector mesons (decaying into a pion gas in the central rapidity region) due to Bose enhancement (BE) through the induced emission of pions. For the sake of illustration and concreteness we study in particular the role of the thermal nucleon loop in the rho-mediated production of dileptons, more precisely, the dileptons produced from the pion-pion annihilation via Vector Meson Dominance (VMD) \[4\] as shown in Fig. 1. However, the general re-
marks regarding the effect of the Bose enhancement would be applicable to any other model. To this end we also provide a ‘thumb rule’ to estimate the extent of the effect of BE at the peak of the dilepton spectrum.

Actual estimation of the total dilepton spectra requires a thorough understanding of the dynamics of the evolution from the very initial stage of the collision. In fact, many calculations, have already been performed, incorporating various effects like collision broadening, in-medium effects in the form of the effective mass in dense nuclear matter motivated by the novel idea of the partial restoration of chiral symmetry, softening of the pion dispersion relation \[3, 5, 6, 7\]. Still a complete understanding of the data requires further efforts in this direction. With this emerging scenario in the background, we calculate the effect of the nucleon loop at finite temperature together with the role of the Bose enhancement in the final state taking a particular channel into account. It is seen, in this model, that with temperature the \(\rho\)-meson mass goes down while on the other hand the \(\rho\)-meson decay width at finite temperature shows an interesting behaviour as exposed below.

The importance of the \(\pi\pi \rightarrow e^+e^-\) reaction in the context of high energy HIC has already been addressed by many authors \[8, 9\], especially to explain the observed enhancement of the dilepton yield in the low invariant mass region as reported by DLS collaboration and most importantly in recent times in the SPS 200GeV S+Au reaction. Various attempts have been made to account for the data using different models. Li, Ko and Brown explains the data by taking the density dependent masses of the vector mesons \[8\] where, however, Bose-enhancement effects on decay width were not taken into account. Chanfray observes that medium modified pion dispersion relation is responsible for the excess production of the dileptons in the low mass
region [3]. Haglin shows that the other hadronic processes, in particular, 
\[ \pi\rho \rightarrow a_1(1260) \rightarrow \pi e^+ e^- \], has favourable kinematics to populate masses 
between \(2m_\pi\) and \(m_\rho\), although no medium modification for the \(\rho\) meson 
mass or decay width is considered there [10]. In contrast we show that the 
effect of the dropping rho-meson mass is partly offset by the increase in decay 
width of \(\rho\).

Theoretical study of \(\pi^+\pi^- \rightarrow l^+l^-\) dates back to early eighties. Domokos 
and Goldman were among the first to compute the rate for this channel 
using relativistic kinetic theory in which free space pion electromagnetic form 
factor was used and no thermal effects on the \(\rho\)-meson were considered [11]. Later, Pisarski in the context of chiral symmetry restoring phase transition 
included some finite temperature effects on the \(\rho\)-meson [12]. The VMD 
effects, on the other hand, have been used extensively at finite temperature 
by Gale and Kapusta, in which, the sensitivity of the dilepton yield on the 
\(\pi - \rho\) dynamics in presence of a hot pion gas, has been investigated and they 
concluded that medium corrections are rather modest up to \(T=150\) MeV [13].

In the present paper, the thermal nucleon loop brings in a modification of the 
rho dominated pion form factor in the medium which is juxtaposed with the 
effect of the thermal distribution of the pions. Unlike the situation in ref. [13] 
( concentrated on \(\rho\pi\) dynamics ) where the thermal effects through the pion 
loop had little effect, here the nucleon loop at finite temperature is found 
to affect the dilepton yield substantially essentially because the nucleon in 
contrast to the pion suffers a considerable mass reduction. In-medium cross-
sections are estimated considering both the effect of the thermal distribution 
on the phase space and also its influence on the rho-decay widths in the form 
of the Bose enhancement [14] due to induced emission of pions in a medium of
the pion gas. Here a gas of pions are converted into dileptons via rho meson by the VMD where necessary temperature dependent effects are included. It is observed that the interaction of the rho meson with the thermal bath causes an enhancement of the dilepton yield in the low invariant mass region because of the lowering of the $\rho$- meson mass at finite temperature while the Bose enhancement, because of the thermal distribution of pions, causes a reduction by increasing the rho decay widths in a hot pion gas. Still, an overall excess yield is observed taking both the factors into account together with the shift of the peak position towards the low invariant mass region. To incorporate the effect of the nucleon thermal loop, we take recourse to the usual imaginary time formalism of finite temperature field theory.

The Lagrangian describing $\rho - N$ interaction is taken to be

$$L_{int} = g_{\rho NN} [\bar{N} \gamma_\mu \tau^a N - \frac{\kappa}{2M} \bar{N} \sigma_{\mu\nu} \tau^a N \partial^\nu] \rho^\mu$$

(1)

$\tau^a$ are the isospin Pauli matrices and $N$ is the nucleon field. The coupling constant and the strength of the ‘magnetic interaction’ is determined either from the fitting of the nucleon-nucleon interaction data ($g_{\rho NN} = 2.63, \kappa = 6.1$) or from the VMD of nucleon form factors ($g_{\rho NN} = 2.71, \kappa = 3.7$).

The form of the polarization tensor for the rho meson is given by

$$\Pi_{\mu\nu}(q^2) = \frac{2g_{\rho NN}^2}{\beta} \sum_n \int \frac{d^3k}{(2\pi)^3} Tr[G(\vec{k} + \vec{q}, \omega_n)\Gamma_\mu G(\vec{k}, \omega_n)\tilde{\Gamma}_\nu]$$

(2)

where $\beta = 1/kT$, $\Gamma_\mu, \tilde{\Gamma}_\nu$ are the appropriate vertex factors obtained from the Lagrangian. The sum is carried out over the Matsubara frequencies [17], viz. $\omega_n = i(2n + 1)\pi/\beta$. This is analogous to what Blaizot discusses in the context of photon-self energy for vacuum polarization of the QGP plasma [13]. Here, of course we have an additional contribution coming from the tensorial form.
of the $\rho-N$ interaction. In the last equation $G(\vec{k}, \omega_n)$ is the thermal nucleon propagator.

$$G(\vec{k}, \omega_n) = \int_0^\beta d\tau e^{i\omega_n \tau} G(\vec{k}, \tau)$$

(3)

$$G(\vec{k}, \tau > 0) = \Lambda_+(k)\gamma_0(1 - n_-(k))e^{-\epsilon_k \tau} + \Lambda_-(k)\gamma_0 n_+(k)e^{\epsilon_k \tau}$$

(4)

$$G(\vec{k}, \tau < 0) = -\Lambda_+(k)\gamma_0 n_-(k)e^{-\epsilon_k \tau} - \Lambda_-(k)\gamma_0(1 - n_+(k))e^{\epsilon_k \tau}$$

(5)

Here $n_{\pm}(k) = 1/(e^{\beta \epsilon_k} \pm 1)$ and $\Lambda_{\pm}(k) = \frac{1}{2\epsilon_k} (\epsilon_k \pm (\alpha \cdot k + m\gamma_0))$, the latter is the projection operator as discussed in Ref. [15] and $\epsilon_k = \sqrt{k^2 + m^*}$, where the asterisk reminds us that here, instead of the free nucleon mass, the medium modified mass is to be used.

The effective nucleon mass is calculated self-consistently by taking only the tadpole diagram arising out of the sigma meson exchange. Here also imaginary time formalism is invoked [17] and one subtracts out the divergent part as only the shift in the nucleon mass is relevant.

The covariant rate of the dilepton production for a static thermal system is given by the expression

$$\frac{dN}{d^4x} = \int d^3\vec{p}_1 d^3\vec{p}_2 f(E_1) f(E_2) |\mathcal{M}|^2 (2\pi)^4 \delta(l_+ + l_- - p_1 - p_2) d^3\vec{l}_+ d^3\vec{l}_-$$

(6)

where $d^3\vec{p}_i = \frac{d^3p_i}{2E_i(2\pi)^3}$ and $f(E_1), f(E_2)$ are the pion distribution functions. First, we perform the integration over lepton momenta and then the integration over the initial momenta are done taking into account the fact that, here, the collision is not necessarily collinear.

The observed dilepton spectra originating from an expanding hadronic system (in our case a pionic gas) is obtained by convoluting the static (fixed
temperature) emission rate with the expansion dynamics. Relativistic hydrodynamics is a convenient tool to describe the space time evolution of such hadronic systems [19]. In this work we will use the Bjorken model [20] of boost invariant longitudinal expansion, according to which the cooling of the system is governed by $T(\tau) = \left(\tau_i/\tau\right)^{1/3}T_i$, where $T_i$ is the initial temperature at the initial proper time $\tau_i$. Within the framework of this model the invariant mass distribution of dileptons at mid-rapidity ($y=0$) is given by,

$$
\frac{dN}{dM^2 dy} = \frac{3}{\pi R_A^2} \left(\frac{c}{4a_k} \frac{dN}{dy}\right)^2 \frac{1}{2(2\pi)^3} \int p_T dp_T \frac{dT}{T^2} M^2 \tilde{\sigma}_i(M) K_0(M_T/T) \tag{7}
$$

where $R_A$ is the radius of the projectile nucleus, $c \approx 3.6$, $a_k = \pi^2/30$, $dN/dy$ is the pion multiplicity, $K_0$ is the zeroth order modified Bessel function of second kind. $\tilde{\sigma}_i(M)$ is the in-medium cross section, and also it is to be noted that in eq.(6) the matrix element also suffers medium modification because of the thermal interaction as represented by the blob in Fig. 1. The in medium decay width is also determined in a similar fashion by taking the Bose enhancement factor due to the thermal distributions of the pions and, in fact, $\Gamma_{\rho \to \pi\pi}$ appears in the denominator of the matrix element $\mathcal{M}$ together with the effective nucleon mass. In none of these cases, however, the vertex correction is taken into account.

In Fig. 2, the variation of the effective mass of the nucleon and the $\rho$ meson is presented to show the consistency of our calculation with that of others, say for example, Ref. [18]. One observes that with temperature(T) the $\rho$ meson mass decreases appreciably which results in a shift of the total dilepton yield towards the low invariant mass region.

The variation of the in-medium decay width ($\Gamma_\rho$) of the rho meson as a function of T which shows an interesting behaviour is depicted in Fig. 3. Actually the decay width, at finite temperature, is expected to show a
reduction as the phase space available is less because of the reduction of the 
\( \rho \) meson mass in the thermal bath, while, on the other hand the presence 
of the pions would cause an enhancement of the \( \rho \to \pi\pi \) decay as evident 
from the Fig. 3. The rapid fall of the \( \rho \)-mass with increasing temperature 
and the consequent reduction in \( \rho \)-width is dramatically offset by the Bose 
enhancement so that up to a temperature of 160 MeV, the width remains in 
the region \( \sim 155 \) to 161 MeV. This particular feature of the \( \rho \) decay width 
has not been remarked upon earlier. The \( \rho\pi\pi \) coupling constant, is estimated 
from the decay width of the rho meson in free space and is given by 
\[ g^2_{\rho\pi\pi}/4\pi \sim 2.9. \] 
We also note from Fig. 2 that the nucleon effective mass, while showing 
a reduction at finite \( T \), is still large compared to the rho meson mass and 
therefore the nucleon-antinucleon channel would still be closed and have no 
effect on the decay width.

To bring the effect of Bose enhancement into bold relief, we discuss the 
effect of the nucleon thermal loop on the dilepton spectra with and without 
the Bose enhancement factor included for the \( \rho \to \pi\pi \) channel. In our ex-
ample we have taken \( R_A = 4.6 \) fm, \( dN/dy = 225 \), \( T_i = 185 \) MeV, \( T_F = 130 \) 
MeV, the results are plotted in fig.4, where shift of the peak position towards 
a low invariant mass is observed together with an enhancement of the total 
yield. It is to be noted that inclusion of the Bose enhancement suppresses 
the yield substantially by increasing the decay width as expected from Fig. 
3. Quantitatively, a suppression \( \sim 30\% \) of the total dilepton yield is observed 
when the effect of BE on decay width is considered compared to the results 
without BE effects everything else remaining the same. However, still an 
excess production of the dileptons at finite temperature is expected once we 
include the effect of the thermal loop. Our findings without the effect of BE
on decay width are consistant with what Li, Ko and Brown observes taking into account the matter induced modification of the $\rho$ meson mass. It may be noted that upto a temperature ($T$) $\sim 100 - 150$ MeV, the BE is dominant as evident from Fig. 3 beyond which the mass modification takes over causing an over all reduction of the decay width indicating the sensitivity of the total dilepton yield on the initial temperature of the hadronic phase. To bring this effect into clear focus we also present the spectra for $T_i = 165$ MeV, keeping all the other parameters same as before. The results are displayed in fig.5.

The Bose enhancement factor for the rho width due to induced emission of pions is clearly given by $(1+e^{-\beta m_\rho/2})^2$ as each pion carries the energy $m_\rho/2$. Thus the square modulus of the vector meson dominated pion form factor at the $\rho$-peak ($m^2_\rho/\Gamma^2$) receives a reduction factor of $(1+e^{-\beta m_\rho/2})^4$ and accordingly this would reflect itself on the lepton yield from this channel. However, since the basic process has to be integrated over the space-time evolution and the concomittant thermal history incorporated in the above reduction factor, an effective temperature ($T_{eff}$) needs to be inserted. In order to determine $T_{eff}$, the peak in the invariant mass of the total yield is found and in the given model the temperature at which the vector meson mass achieves the value $m_{peak}$ gives us an estimate of $T_{eff}$. Thus the effective reduction factor may be estimated to be $(1+e^{-m_\rho/2kT_{eff}})^{-4}$. It is easily checked that this roughly agrees with our detailed determination of this value. It should be noted that in a case where the vector meson (say $\omega$), dominantly decays into three pions the suppression due to Bose enhancement will be the sixth power of the relevant analogous factor.

In conclusion, we observe that the thermal nucleon loop and the thermal distribution of the pion gas affect the dilepton yield in the low invariant mass
region quite appreciably and should therefore be included in the dynamical
calculation to move towards a more complete understanding of the dilepton
production in high energy heavy ion collision. The main result of the present
analysis is that the effect of BE on the widths of vector mesons with domi-
nant pionic decay channels can have substantial influence on the dilepton
yield depending on the initial temperature. Studies are in progress to inves-
tigate these effects for other hadronic channels which may contribute to the
production of leptons.

We gratefully acknowledge useful discussions with J.P. Blaizot (SACLAY)
K. Haglin (MSU), R. Bhalerao (TIFR), D. K. Srivastava (VECC) and K.
Mukherjee (SINP). One of the author (BS) acknowledges stimulating discus-
sions with C. M. Ko, G. E. Brown and M. Prakash.
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Figure captions

1. Feynman diagram for dilepton production via Vector Meson Dominance from pion-pion annihilation where the blob represents the interaction of the $\rho$ meson with the thermal bath.

2. Effective masses are depicted as a function of temperature both for nucleon and rho meson taking the effect of thermal loops into account.

3. The variation of the decay width with temperature for three different cases are shown. The value of the free decay width is 151 MeV. Solid, dashed and dotted curves represent decay width using the free $\rho$ mass with BE, medium modified $\rho$ width with and without BE respectively.

4. Effect of the thermal bath on the dilepton yield for $T_i = 185\, MeV$ and $T_F = 130\, MeV$. Solid, dashed, and dotted represent contribution with free (without medium effect), medium-modified without BE factor and medium-modified with BE factor respectively.

5. Same as Fig.4 for $T_i = 165\, MeV$. 

\[ \pi^+ + \pi^- \rightarrow \rho^+ + \gamma \rightarrow \pi^+ + e^- + \nu_e \]
