Numerical modeling of heat transfer at cooling a high-temperature steel cylinder by the flow of a water-air medium with account taken of the gamma - alpha transformation

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Abstract. Based on the mathematical model of the conjugate heat transfer from a steel cylinder to a water-air medium, the results of the calculation of the cooling rate and the mass fraction of the α-phase in supercooled austenite during the γ → α transformation are obtained. To find the mass fraction of the α-phase in austenite during supercooling, a modified Kolmogorov – Johnson – Mel – Avrami equation is used. The calculation results have been obtained during the cooling of the high-temperature metal cylinder by the laminar gas-liquid flow in an annular channel with account taken of vaporization in the liquid. The regulation of the heat and mass transfer balance of the system during the vaporization is based on the energy model of the heat balance and is performed using the effective thermophysical parameters of the cooling medium. The mathematical model is presented in a two-dimensional non-stationary formulation taking into account the symmetry of the cooling medium flow relative to the longitudinal vertical axis of the cylinder. To solve the system of differential equations, the control volume method is used. The parameters of the cooling medium flow are calculated using an algorithm SIMPLE. For the iterative solution of the system of linear algebraic equations, the methods of Gauss-Seidel and under-relaxation are used. The calculations, are carried out using a grid with a condensing profile at the «metal cylinder – liquid» and «liquid - outer metal ring» boundaries on the liquid side and the metal side.

1. Introduction
In [1, 2] a mathematical model of conjugate heat exchange in the heterogeneous system «solid - gas-liquid medium» was offered. The temperature two-dimensional field in a liquid and a solid was determined at the fulfillment of the condition of coupling at the boundary of the media. In addition, near the surface of the high-temperature metal cylinder cooled by the longitudinal water flow, the vaporization was taken into account. A numerical algorithm was used for studying heat transfer at cooling a high-temperature metal blank in the form of a cylinder from structural steel by the gas-liquid flow [1, 3, 4]. It is known, there are various inclusions in water such as air and other gases, which are either dissolved or present in the form of small bubbles in the liquid. The numerical results [1, 2-4]
for conjugate heat transfer at cooling a high-temperature metal cylinder by the flow of water were obtained without considering the presence of air. The natural content of the air fraction $Y_a$ in water is insignificant and in normal conditions it is in the range of 1% - 2.5% [5]. However, in real technological processes of cooling there is a possibility to obtain water-air mixtures with an arbitrary fraction of air while the uniformity of the main flow of a liquid is retained [6]. The objective of the present paper is the simulation of heat transfer at cooling a high-temperature metal cylinder with the flow of air-containing water in a vertical annular channel, as well as [1], for the purpose of revealing the regularities of the influence of the heat transfer parameters on the cooling rate with taking into account the phase $\gamma \rightarrow \alpha$ transformation in the material of the cylinder.

In the present paper, a mathematical model is considered for conjugate heat transfer from the metal cylinder to the flow of the water-air medium in the two-dimensional axisymmetric non-stationary formulation. A metal cylinder with a radius $r_m$ and length $L$ is cooled by the liquid flow moving in the direction of the vertical axis $x$ in the annular space $r_l - r_m$ with the initial velocity $u_0$. The outer radius of the metal wall is $r_{m1}$. Figure 1 shows the calculation diagram for the considered model [1].

![Figure 1. The calculation diagram of the model: 1 – the metal cylinder; 2 – the axis of the cylinder; 3 – the flow of a liquid; 4 – the outer wall.](image)

### 2. Mathematical model

The system of equations describing the flow of the water-air medium taking into account the vaporization in the flow of the liquid is as follows [1]:

\[
\frac{\rho \partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = -\rho g - \frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial u}{\partial r}) + \frac{1}{\rho} \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial r} \right)
\]

(1)

\[
\frac{\rho \partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial r} = -\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v}{\partial r} \right)
\]

(2)

\[
\frac{\partial}{\partial t} (\rho u) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial (pu)}{\partial r} \right) = 0
\]

(3)

\[
\frac{\partial}{\partial t} (\rho c_T u) + \rho c_T u \frac{\partial T}{\partial x} + \rho c_T u \frac{\partial T}{\partial r} = \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) - \dot{m}_Q - \rho g u
\]

(4)

\[
\frac{\rho \partial Y_a}{\partial t} + \rho u \frac{\partial Y_a}{\partial x} + \rho v \frac{\partial Y_a}{\partial r} = 0
\]

(5)

\[
\frac{\rho \partial Y_v}{\partial t} + \rho u \frac{\partial Y_v}{\partial x} + \rho v \frac{\partial Y_v}{\partial r} = \dot{m}_v
\]

(6)
Here, the specific mass velocity of evaporation is \( \dot{m}_v = \frac{(\rho c \Delta T)}{(Q, \Delta t)} \), where

\[
\Delta T = \begin{cases} 
0, & \text{if } T(t + \Delta t) < T_s \\
[T(t + \Delta t) - T_n(t)], & \text{if } T(t + \Delta t) > T_s
\end{cases}
\]

- the liquid overheating ; \( T_n(t) = \max[T(t); T_s] \).

\( \Delta T \) value can be either positive or negative. In the first case, the amount of heat energy required for the « liquid-vapor» phase transition is taken into account. In the second case, the heat effect produced by the ‘vapor-liquid’ condensation is taken into consideration provided that there is vapor in the medium, i.e. \( Y_v > 0 \). Thus, the energy heat-mass transfer balance of the system is regulated.

The energy equation for the metal cylinder and the outer metal ring is

\[
\rho_m c_m \frac{\partial T_m}{\partial t} = \frac{\partial}{\partial x} \left( \lambda_m \frac{\partial T_m}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \rho_m \lambda_m \frac{\partial T_m}{\partial r} \right) + q_v
\]  

(7)

The effective thermal-physical parameters of the gas-liquid medium are determined as follows [7]:

\[
\rho = \sum \rho_i Y_i, \quad c = \sum c_i \rho_i Y_i / \rho, \quad \lambda = 1/2 \left[ \sum Y_i \lambda_i + \left( \sum Y_i \lambda_i \right)^{-1} \right], \quad \mu = 1/2 \left[ \sum Y_i \mu_i + \left( \sum Y_i \mu_i \right)^{-1} \right]
\]

\[\sum Y_i = 1.\]  

In the equations, the following symbols are used: \( c \) – specific heat capacity, \( J/(kg \cdot grad) \); \( \rho \) - density, \( kg/m^3 \); \( \lambda \) - heat conduction, \( W/(m \cdot grad) \); \( \mu \) – dynamic viscosity, \( Pa \cdot s \); \( T \) – temperature, \( C \); \( t \) – time, \( s \); \( r \) – radial coordinate, \( m \); \( x \) - longitudinal coordinate, \( m \); \( L \) – computational domain length, \( m \); \( u \) – velocity component along \( x \), \( m/s \); \( v \) – velocity component along \( r \), \( m/s \); \( Y \) – volume concentration, \( Q \) – specific heat of vaporization, \( J/kg \). Indices: \( m \) – metal, \( l \)–liquid, \( v \) – vapor, \( 0 \) – initial value, \( s \) – saturation parameters.

The heat effect of the \( \alpha \to \gamma \) transformation is calculated on the basis of the methods described in [8, 9].

The mass fraction, \( X \), of the \( \alpha \) phase in the austenite in the process of supercooling can be found using a modified Kolmogorov–Johnson–Mehl–Avrami equation analyzed in [10 - 12] which allows to predict the kinetics of the formation of new phases and structural components both at isothermal holding and continuous cooling; the equation has the form [8]:

\[
X = 1 - e^{(-b \cdot \tau^n)}
\]  

(8)

here \( b = -0.0097 \frac{V_m}{T_m} + 14.521 ; \ n = 0.0187 \frac{V_m}{T_m} - 9.293 ; \ \tau = \left( T_m^s - T_m^i \right) / \left( T_m^s - T_m^i \right) \)

The values of the temperatures \( T_m^s \) of the beginning of the \( \gamma \to \alpha \) transformation are determined according to the method described in [9] using the following relation:

\[
T_m^s = k V_m + m V_m^n + T_m^i
\]  

(9)

Here \( V_m \) - cooling rate, \( k,m,z \) - reduced coefficients, \( T_m^i \) - the value of the temperature for the isothermal \( \gamma \to \alpha \) transformation with account taken of the chemical composition of steel.

The temperature \( T_m^f \) of the end of the \( \gamma \to \alpha \) transformation is determined according to the method described in [13] using the following relation:

\[
T_m^f = k V_m - m V_m^n + T_m^i
\]  

(10)

The local rate of cooling in the metal cylinder volume is determined as follows:

\[
V_m = \frac{T_m(t + \Delta t) - T_m(t)}{\Delta t}
\]  

(11)

Depending on the fraction of the formed \( \alpha \) phase, the specific heat of the \( \gamma \to \alpha \) transformation can be found from the following relation [13]:
\[ q_v = q_{v0} \frac{\Delta X}{\Delta t}, \]  
(12)

where \( \Delta X \) - increment of the mass fraction of the \( \alpha \) phase in the supercooled austenite and \( q_{v0} \) - heat generation due to the \( \gamma \rightarrow \alpha \) transformation of pure iron.

Thermophysical parameters of the metal cylinder are determined according to [14] and the value of the specific heat of the metal is calculated according to the formula:
\[
c_m = c_\alpha X + c_\gamma (1 - X),
\]  
(13)

where \( c_\alpha, c_\gamma \) - specific heat of the \( \alpha \) and \( \gamma \) phase.

The initial conditions: \( u = u_0 \), \( v = 0 \), \( T_i = T_{i0}, T_m = T_{m0}, T_{ml} = T_{l0}, Y_a = Y_{a0} \)

The boundary conditions:
\[
x = 0: 0 < r < r_m, \quad \frac{\partial T}{\partial x} = 0; \\
r_m < r < r_l, \quad T = T_{i0}, u = u_0, v = 0, Y_v = 0, Y_a = Y_{a0}, \frac{\partial p}{\partial x} = 0 \\
x = L: 0 < r < r_m, \quad \frac{\partial T}{\partial x} = 0; \\
r_m < r < r_{ml}, \quad \frac{\partial T}{\partial x} = 0 \\
r_m < r < r_l, \quad \frac{\partial T}{\partial x} = 0, \frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial x} = 0, \frac{\partial Y_v}{\partial x} = 0, \frac{\partial Y_a}{\partial x} = 0, p = 0 \\
0 < x < L: r = 0 \quad \frac{\partial T}{\partial r} = 0 \\
r = r_m, -\lambda_m \frac{\partial T_m}{\partial r} = -\lambda_l \frac{\partial T_l}{\partial r}, T_m = T_l, u = 0, v = 0, \frac{\partial Y_v}{\partial r} = 0, \frac{\partial Y_a}{\partial r} = 0 \\
r = r_l, -\lambda_m \frac{\partial T_{ml}}{\partial r} = -\lambda_l \frac{\partial T_l}{\partial r}, T_{ml} = T_l, u = 0, v = 0, \frac{\partial Y_v}{\partial r} = 0, \frac{\partial Y_a}{\partial r} = 0 \\
r = r_{ml}: \quad \frac{\partial T}{\partial r} = 0.
\]

3. Solution procedures
The system of differential equations (1) - (7) is solved by the control volume method. The flow field parameters (1) - (3) are calculated with the help of an algorithm SIMPLE [15] which is used for modeling flows of a liquid with heat-mass transfer. The differential equations are reduced to the system of linear algebraic equations and solved iteratively using the Gauss-Seidel method in combination with the under-relaxation factor. Equations (8) - (13) are solved at each calculation step with respect to time.

4. Numerical calculation results
For the investigation, let us assume that the material of the cylinder is steel 40. The values of the temperatures of the beginning and end of the \( \gamma \rightarrow \alpha \) transformation are calculated according the dependences (9) and (10):
\[
T_m^s = -0.07V_m - 63V_m^{0.167} + 782; \quad T_m^f = -2.3041V_m - 65.386V_m^{0.194} + 695.
\]
As the preliminary analysis has shown the values of the temperatures \( T_m^s \) and \( T_m^f \) are decreasing with increasing rate of cooling. The above relations have physical meaning at the rate of cooling in the
range of $V_m \approx (0-200) ^\circ C/s$. In this connection, let us assume that at the exceedance of $V_m \approx 200 ^\circ C/s$ the process of the $\gamma \rightarrow \alpha$ transformation takes place at $V_m = 200 ^\circ C/s$. The value of $q_v$ is taken equal to 136.93 MJ/m$^3$ [12]. For the calculation, let us take the following geometrical dimensions (see Fig.1): $r_m = 0.015$ m, $r_i = 0.025$ m, $r_{ml} = 0.03$ m, $L = 0.1$ m. The initial temperature $T_{m0} = 850 ^\circ C$.

The outer metal wall is from steel 12N18N9T. The cooling medium is water, $T_{l0} = 20 ^\circ C$, $Y_a = 0.025$, the water flow velocity $u_0 = 0.1$ m/s, as well as [1].

Figure 2 presents the results of the numerical calculations of the radial cooling rate in the centre of the metal cylinder. The results show that the rate of the cooling of the metal cylinder decreases with time, and the radially directed profile has a sloping character.

Figure 3 shows the values of the mass fraction $X$ of the $\alpha$ phase in the supercooled austenite along the length of the being cooled cylinder.

It can be seen that with increasing cooling rate there is the irregular $\gamma \rightarrow \alpha$ transformation throughout the volume of the steel cylinder. The non-linearity of the processes is due to the irregular heat exchange between the cylinder and the ambient. It can be seen that at the threefold increase in the time of cooling, the mass fraction $X$ of the volume of the $\alpha$ phase increases on the average by a factor.
of two. The calculations show that the value $q_v$ of the specific heat of the $\gamma \rightarrow \alpha$ transformation insignificantly influences the variation of the system heat balance. When $q_v$ is taken into account, in energy equation (7) the calculated value of the temperature in the calculated volume of the steel cylinder increases on the average by $(2 - 2.5)\%$ for the temperature range of the $\gamma \rightarrow \alpha$ transformation.

5. Conclusions
Thus, the presented mathematical model of the conjugate heat transfer at cooling the high-temperature metal cylinder by the flow of water containing air in the vertical circular channel can be used for finding the regularities of the influence of the heat transfer parameters on the cooling rate when the phase $\gamma \rightarrow \alpha$ transformation in the material of the cylinder is taken into account. The possibility is shown for finding the mass fraction $X$ of the $\alpha$ phase in austenite at supercooling depending on the space variation of the rate of the cooling of steel. The developed model can find various engineering applications associated with the process of cooling high-temperature metal bodies with account taken of the $\gamma \rightarrow \alpha$ transformation.

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