Coherent bremsstrahlung in periodically deformed crystals with a complex base

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Abstract

In the present paper we investigate coherent bremsstrahlung of high energy electrons moving in a periodically deformed single crystal with a complex base. The formula for corresponding differential cross-section is derived for an arbitrary deformation field. The conditions are discussed under which the influence of the deformation is important. The case is considered in detail when the electron enters into the crystal at small angles with respect to a crystallographic axis. It is shown that in dependence of the parameters, the presence of the deformation can either enhance or reduce the bremsstrahlung cross-section.

Keywords: Interaction of particles with matter; coherent bremsstrahlung; physical effects of ultrasonics.

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1 Introduction

The high-energy electromagnetic processes in crystals have been under active theoretical investigations for long time (see, for instance, [1]-[5] and references therein). These investigations are of interest not only from the viewpoint of underlying physics but also from the viewpoint of practical applications. When the formation length exceeds the interatomic spacing, the interference effects from all atoms within this length are important and the cross-sections of the electromagnetic processes in crystals can change essentially compared with the corresponding quantities for an isolated atom. From the point of view of controlling the parameters of the high-energy electromagnetic processes in a medium it is of interest to investigate the influence of external fields, such as acoustic waves, temperature gradient, etc., on the corresponding characteristics. The considerations of concrete processes, such as the diffraction radiation, transition radiation, parametric X-radiation, channelling radiation, electron-positron pair creation by high-energy photons, have shown that the external fields can essentially change the angular-frequency characteristics of these processes (see, for example [6]-[12]).

The coherent bremsstrahlung of high-energy electrons moving in a crystal is one of the most effective methods to produce intense beams of highly polarized and monochromatic photons (for recent experiments in this direction see, for instance, [13] and references therein). Such radiation...
has a number of remarkable properties and at present it has found many important applications. This motivates the importance of investigations for various mechanisms of controlling the radiation parameters. As such a mechanism, in [13] we have discussed the influence of hypersonic waves excited in a crystal on the process of the bremsstrahlung of high-energy electrons. In this paper, the case of a simplest crystal with a single atom in the elementary base and the sinusoidal deformation field generated by the hypersound were considered. To have an essential influence of the acoustic wave, high-frequency hypersound is needed. Usually this type of waves are excited by high-frequency electromagnetic fields through the piezoelectric effect in crystals with a complex elementary base. In the present paper we generalize the results of [13,15] for crystals with a complex base and for acoustic waves with an arbitrary profile. The numerical calculations are carried out for the case of the most popular quartz piezocrystal.

The paper is organized as follows. In the next section we derive the general formula for the coherent part of the bremsstrahlung cross-section averaged over the thermal fluctuations and the conditions are specified under which the influence of the deformation field can be considerable. The analysis of the general formula in the case when the electron enters into the crystal at small angles with respect to the crystallographic axes or planes is given in section 3. The results of the numerical calculations for the cross-section as a function of the photon energy and the amplitude of the external excitation are presented. Section 4 summarizes the main results of the paper. Throughout of the paper the system of units $\hbar = c = 1$ is used.

# 2 Influence of external excitations on bremsstrahlung in crystals

We consider the bremsstrahlung of high energy electrons moving in a crystal. Let $(\omega,k)$ be the frequency and the wave vector for the radiated photon and $(E_1,p_1), (E_2,p_2)$ be the energies and momenta for the electrons in the initial and final states respectively. We denote by $d^4\sigma_0/d\omega d^3q = \left| u_{q}(j) \right|^2 \sigma_0 (q)$ the cross-section of the bremsstrahlung on an isolated $j$-th atom as a function of the transfer momenta $q = p_1 - p_2 - k$. In this representation $u_{q}(j)$ is the Fourier transform of the potential for $j$-th atom and the factor $\sigma_0 (q)$ does not depend on the type of the atom. The quantity $u_{q}(j)$ usually is presented in the form $4\pi Z_j e^2 \left[ 1 - F^{(j)} (q) \right]/q^2$, where $Z_j$ and $F^{(j)} (q)$ are the number of electrons in the atom and the atomic form-factor. The differential cross-section for the bremsstrahlung in a crystal by high energy electron can be written in the form (see, for example [1,2])

$$\sigma (q) \equiv \frac{d^4 \sigma}{d\omega d^3q} = \sum_{n,j} u_{q}(j) e^{i\mathbf{q}\mathbf{r}_n} \left| \sigma_0 (q) \right|^2,$$

(1)

where the vector $\mathbf{r}_n(j)$ specifies the positions of the atoms in the crystal and the collective index $n$ numerates elementary cells.

At nonzero temperature one can decompose the radius-vector as $\mathbf{r}_n(j) = \mathbf{r}_n(j)^0 + \mathbf{u}_{tn}(j)$, where $\mathbf{u}_{tn}(j)$ is the displacement of $j$-th atom with respect to the equilibrium position $\mathbf{r}_n(j)^0$ due to the thermal vibrations. After averaging over thermal fluctuations, we can write cross-section as (see for example, [1,2] for the case of a crystal with simple cell)

$$\sigma (q) = \left[ N \sum_{j} \left| u_{q}(j)^0 \right|^2 \left( 1 - e^{-q^2 u_{q}(j)^2} \right) + \left| \mathbf{U}_q \right|^2 \right] \sigma_0 (q),$$

(2)

where $N$ is the number of cells, $u_{q}(j)^2$ is the temperature dependent mean-squared amplitude of
For the deformation field given by Eq. (5), introducing the Fourier transform of the function that determines the positions of the atoms of one of the primitive lattices, and we have introduced the notation

\[ U_q = \sum_{n,j} u_q^{(j)} e^{i q r_{n0}^{(j)}} e^{-\frac{i q^2 u_q^{(j)^2}}{2}}, \]

with \( e^{-\frac{i q^2 u_q^{(j)^2}}{2}} \) being the corresponding Debye-Waller factor. In formula (2) the first term in the square brackets does not depend on the direction of the momentum transfer \( q \) and it determines the contribution of the incoherent effects. The contribution of the coherent effects is presented by the second term which depends on the orientation of the crystal axes with respect to the vector \( q \). By taking into account the formula for the cross-section on an isolated atom, in the range of the transferred momenta \( q \ll m_e \), with \( m_e \) being the mass of electron, the coherent part of the cross-section can be presented in the form

\[ \sigma_c(q) = \frac{e^2}{8\pi^3 E_1} \frac{q^2}{q_i} \left( 1 + \frac{\omega\delta}{m_e^2} - \frac{2\delta}{q_i} + \frac{2\delta^2}{q_i^2} \right) |U_q|^2, \]

where \( q_i \geq \delta \). In formula (4), \( e \) is the electron charge, \( q_i \) and \( q_{\perp} \) are the parallel and perpendicular components of the vector \( q \) with respect to the direction of the electron initial momentum \( p_1 \), \( \delta = 1/\lambda_e \) is the minimum longitudinal momentum transfer, and \( \lambda_e = 2E_1 E_2/ (\omega m_e^2) \) is the formation length for the process of bremsstrahlung. The latter determines the effective longitudinal dimension of the interaction region for the phase coherence of the radiation process.

When external influences are present (for example, in the form of acoustic waves) the radius-vector of an atom in the crystal can be written as \( r_n^{(j)} = r_n^{(j)} + u_n^{(j)} \), where \( r_n^{(j)} \) determines the equilibrium positions of the atom in the situation without the deformation, \( u_n^{(j)} \) is the displacement of the atom caused by the external influence. In this paper we will consider deformations having the periodical structure:

\[ u_n^{(j)} = u_0 f(k_s r_{ne}^{(j)}), \]

where \( u_0 \) and \( k_s \) are the amplitude and the wave vector of the deformations field, \( f(x) \) is an arbitrary function with the period \( 2\pi \), \( \max f(x) = 1 \). In the discussion below we will assume that \( f(x) \in C^\infty (R) \). The time-dependence of \( u_n^{(j)} \) in the case of acoustic waves we can disregard, as for the electron energies we are interested in, the characteristic time for the change of the deformation field is much greater than the time of passage of the particles through the crystal. For the deformation field given by Eq. (5), introducing the Fourier transform of the function \( e^{ixf(t)} \),

\[ F_m(x) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} e^{ixf(t)} e^{imt} dt, \]

the sum over \( n \) in (3) is presented in the form

\[ \sum_n u_q^{(j)} e^{i q r_{n0}^{(j)}} = \sum_{m=-\infty}^{+\infty} F_m(qu_0) \sum_n u_q^{(j)} e^{i q m^* r_{ne}^{(j)}}, \]

where \( q_m = q + m k_s \).

For a lattice with a complex cell the coordinates of the atoms can be presented in the form \( r_{ne}^{(j)} = R_n + \rho^{(j)} \), where \( R_n \) determines the positions of the atoms of one of the primitive lattices,
and \( \rho^{(j)} \) are the equilibrium positions for the other atoms inside \( n \)-th elementary cell with respect to \( R_n \). Now the Fourier transform of the effective potential can be written as

\[
U_q = \sum_{m=-\infty}^{+\infty} F_m(q u_0) S(q, q_m) \sum_n e^{i q m R_n},
\]

where the factor \( S(q, q_m) \) determined by structure of the elementary cell is given by the formula

\[
S(q, q_m) = \sum_j u_q^{(j)} e^{-\frac{1}{2} q^2 u_q^{(j)2}} e^{i q m \rho^{(j)}}.
\]

In the case of thick crystals, the sum over the cells in (8) is expressed in terms of a sum over the reciprocal lattice and one finds

\[
U_q = \frac{(2\pi)^3}{\Delta} \sum_{m=-\infty}^{+\infty} F_m(q u_0) S(q, q_m) \sum_g \delta(q - g m),
\]

where \( \Delta \) is the volume of the unit cell, \( g \) is the reciprocal lattice vector, and we have introduced the notation

\[
g_m = g - m k_s.
\]

Note that the function \( U_q \), given by formula (10), is the Fourier transform of the periodic function

\[
U(r) = \frac{1}{\Delta} \sum_{m,g} F_m(g_m u_0) S(g_m, g) e^{-i g_m r}.
\]

Thus, in the presence of the periodic deformation field (5), the process of coherent bremsstrahlung can be considered as a result of the electron motion in the continuous periodic potential given by (12). This field introduces additional momentum to the problem changing the kinematics. By taking into account the delta-function in formula (10), the momentum conservation law is written in the form

\[
p_1 = p_2 + k + g - m k_s,
\]

where \(-m k_s\) is the momentum transfer to the external field.

For thick crystals, by using (10), for the factor in the coherent part of the cross-section we have

\[
|U_q|^2 = \frac{(2\pi)^3}{\Delta} \sum_{m,g} F_m(g_m u_0) S(g_m, g) \delta(q - g_m)
\]

\[
\times \sum_{m'} F^{*}_{m'}(q u_0) S^{*}(q, q_{m'}) \sum_n e^{-i(n-m') k_s R_n}.
\]

Under the assumptions for the function \( f(x) \) given above, by making use the stationary phase method for the integral (5), we can see that for a fixed \( x \) one has \( F_m(x) \sim \mathcal{O} \left( |m|^{-\infty} \right) \) for \( m \to \infty \). Using this property, in the way similar to that used in (11), we can see that in the sum over \( m \) the main contribution comes from the terms for which \( |m k_s u_0| \lesssim |g u_0| \), or \( |m| \lesssim \lambda_s/a \), where \( \lambda_s = 2\pi/k_s \) is the wavelength of the external excitation, and \( a \) is of the order of the lattice constants. Under the condition \( u_0/\lambda_s \ll 1 \), in the summation on the right of formula (14) the
contributions of the terms with $m \neq m'$ are small compared to the diagonal terms $m = m'$ and the sum over $n$ is equal to the number of the cells $N$ in the crystal. As a result we find

$$|U_q|^2 = N \frac{(2\pi)^3}{\Delta} \sum_{m, g} |F_m (g_m u_0)|^2 |S (g_m, g)|^2 \delta (q - g_m).$$ (15)

Substituting the expression (15) into the formula (14) and integrating over the vector $q$, for the cross-section one obtains

$$d\sigma = \int \sigma (q) d\mathbf{q} = N_0 (d\sigma_n + d\sigma_c),$$ (16)

with $d\sigma_n$ and $d\sigma_c$ being the incoherent and coherent parts of the cross-section per atom, $N_0$ is the number of atoms in the crystal. The coherent part of the cross-section is determined by the formula

$$\frac{d\sigma_c}{d\omega} = \frac{e^2 N}{N_0 E_1^2 \Delta} \sum_{m, g} \frac{g_m^2}{g_m} \left[ 1 + \frac{\omega^2}{2E_1 E_2} - 2 \frac{\delta}{g_m} \left( 1 - \frac{\delta}{g_m} \right) \right] |F_m (g_m u_0)|^2 |S (g_m, g)|^2,$$ (17)

where the vector $g_m$ is defined by relation (11) and the summation is carried out under the condition $g_m \geq \delta$. For monoatomic single crystals and for sinusoidal deformation fields, $f (z) = \sin (z + \varphi_0)$, the formula (17) is reduced to the result obtained in [14]. Note that for this type of deformation one has the Fourier-transform

$$F_m (z) = e^{im\varphi_0} J_m (z)$$ (18)

with $J_m (z)$ being the Bessel function. The formula (17) differs from the corresponding expression for the bremsstrahlung in undeformed crystals by the replacement $g \rightarrow g_m$, and by an additional summation over $m$ with the weights $|F_m (g_m u_0)|^2$. This corresponds to the presence of an additional one-dimensional superlattice with the period $\lambda_s$ and with the reciprocal lattice vector $mk_s$, $m = 0, \pm 1, \pm 2, \ldots$. In the presence of the deformation field the number of possibilities to satisfy the condition $g_m \geq \delta$ in the summation of formula (17) increases due to the term $mk_s$ in the expression for $g_m$. As we will see below, this leads to the appearance of additional peaks in the spectral distribution of the radiated photons. As the main contribution into the coherent part of the cross-section comes from the terms with $g_m \sim \delta$, the influence of the deformation field may be considerable if $|mk_s| \gtrsim \delta$. Combining this with the previous estimate that the main contribution into the series over $m$ comes from the terms $|m| \lesssim \lambda_s / a$, we find the following condition: $u_0 / \lambda_s \gtrsim a / (4\pi^2 c)$. At high energies one has $a/l_c \ll 1$ and this condition can be consistent with the condition $u_0 / \lambda_s \ll 1$.

3 Limiting cases and numerical results

In this section we consider the general formula (17) for the cross-section of the coherent part of the bremsstrahlung in special cases when the coherence effects are important. We will assume that the crystal lattice is orthogonal. The reciprocal lattice vector components are given by $g_i = 2\pi n_i / a_i$, $n_i = 0, \pm 1, \pm 2, \ldots$, where $a_i$, $i = 1, 2, 3$, are the lattice constants along the corresponding directions. Let $\theta$ be the angle between the initial electron momentum $p_i$ and the crystallographic $z$-axis. For the parallel component of the vector $g_m$ we have the expression

$$g_m = g_m \cos \theta + (g_{my} \cos \alpha + g_{mx} \sin \alpha) \sin \theta,$$ (19)
where $\alpha$ is the angle between the projection of the vector $\mathbf{p}_1$ on the plane $(x, y)$ and $y$-axis.

If the electron moves in an unoriented crystal, in formula (17) the summation over $g$ can be replaced by the integration and the cross-section for the bremsstrahlung coincides with that in an amorphous medium. Coherent effects appear when the electron enters into the crystal at small angles $\theta$. In this case the main contribution to the cross-section give the terms with $g_z = 0$, and from formula (17) one finds

$$
\frac{d\sigma_c}{d\omega} \approx \frac{e^2 N}{E_1^2 N_0} \sum_{m, g_y} \frac{g^2_{m \perp}}{g^2_{m \parallel}} \left[ 1 + \frac{\omega^2}{2E_1 E_2} - \frac{\delta}{g_{m \parallel}} \left( 1 - \frac{\delta}{g_{m \parallel}} \right) \right] |F_m (g_m u_0)|^2 |S (g_m, g)|^2, \tag{20}
$$

where the summation goes under the condition $g_{m \parallel} \geq \delta$ with

$$
g_{m \parallel} \approx -mk_{s \parallel} + (g_y \cos \alpha + g_x \sin \alpha) \theta. \tag{21}
$$

Note that in the arguments of the functions $F_m$ and $S$ we can put $g_m \approx (g_x, g_y, 0)$.

In the further discussion two qualitatively different cases should be distinguished. The first one corresponds to the situation where the electron moves far from the crystallographic planes (angles $\alpha$ and $\pi/2 - \alpha$ are not small). In this case, the summation over the reciprocal lattice vector components $g_x$ and $g_y$ in (20) can be replaced by the integration in accordance with $\sum_{g_x, g_y} \rightarrow (a_1 a_2/4\pi^2) \int dg_x g_y$. For the corresponding cross-section this leads to the result

$$
\frac{d\sigma_c}{d\omega} \approx \frac{e^2 N}{4\pi^2 E_1^2 a_3 N_0} \sum_{m, g_y} \int dg_x dg_y \frac{g^2_{m \perp}}{g^2_{m \parallel}} |F_m (g_m u_0)|^2
$$

$$
\times |S (g_m, g)|^2 \left[ 1 + \frac{\omega^2}{2E_1 E_2} - \frac{\delta}{g_{m \parallel}} \left( 1 - \frac{\delta}{g_{m \parallel}} \right) \right], \tag{22}
$$

where the integration range is determined by the condition $g_{m \parallel} \geq \delta$ with $g_{m \parallel}$ given by (21).

In the second case the electron enters into the crystal at small angles $\theta$ with respect to the crystallographic axis $z$ and near the crystallographic planes $(y, z)$ (the angle $\alpha$ is small). In this case, in dependence of the electron energy, two subcases should be considered separately. Under the condition $\delta \sim 2\pi\theta/a_2$, for the longitudinal component in formula (20) one has

$$
g_{m \parallel} \approx -mk_{s \parallel} + \theta g_y \geq \delta, \tag{23}
$$

and the summation over $g_x$ can be replaced by the integration, $\sum_{g_x} \rightarrow (a_1/2\pi) \int dg_x$, with the result

$$
\frac{d\sigma_c}{d\omega} \approx \frac{e^2 N}{2\pi E_1^2 a_3 N_0} \sum_{m, g_y} \int dg_x g^2_{m \perp} \frac{g^2_{m \parallel}}{g^2_{m \parallel}} |F_m (g_m u_0)|^2
$$

$$
\times |S (g_m, g)|^2 \left[ 1 + \frac{\omega^2}{2E_1 E_2} - \frac{\delta}{g_{m \parallel}} \left( 1 - \frac{\delta}{g_{m \parallel}} \right) \right]. \tag{24}
$$

This formula is further simplified in the case when the amplitude of the deformation field, $u_0$, is perpendicular to the crystallographic $x$-axis. In this case, in the argument of the function $F_m$ in (21) one has $g_m u_0 \approx g_y u_{0y}$ and this function does not depend on the integration variable. As a result, for the cross-section we obtain the formula

$$
\frac{d\sigma_c}{d\omega} \approx \frac{e^2 N}{2\pi E_1^2 a_3 N_0} \sum_{m, g_y} \left[ 1 + \frac{\omega^2}{2E_1 E_2} - \frac{\delta}{g_{m \parallel}} \left( 1 - \frac{\delta}{g_{m \parallel}} \right) \right]
$$

$$
\times |F_m (g_m u_0)|^2 \int dg_x g^2_{m \perp} |S (g_m, g)|^2, \tag{25}
$$
Fourier transforms of the atomic potentials we take the Molière parametrization: the cell including 6 atoms of silicon and 12 atoms of oxygen (elementary cell of Shrauf). For the of bremsstrahlung photons. To deal with an orthogonal lattice, as an elementary cell we choose results show that by adjusting the orientation of the crystal relative to the incident electron values of parameters in the case of SiO$_2$ and, as before, the summation goes under the condition $g_{m||} \geq \delta$.

We have performed numerical calculations for the bremsstrahlung cross-section for various values of parameters in the case of SiO$_2$ single crystal at zero temperature. The corresponding results show that by adjusting the orientation of the crystal relative to the incident electron momentum and the parameters of the external influence it is possible to enhance the number of bremsstrahlung photons. To deal with an orthogonal lattice, as an elementary cell we choose the cell including 6 atoms of silicon and 12 atoms of oxygen (elementary cell of Shrauf). For the Fourier transforms of the atomic potentials we take the Moliere parametrization:

$$u_q^{(j)} = \sum_{i=1}^{3} \frac{4\pi Z_j e^2 \alpha_i}{q^2 + (\chi_i/R_j)^2},$$

with the values of the parameters $\alpha_i = \{0.1, 0.55, 0.35\}$, $\chi_i = \{6.0, 1.2, 0.3\}$, and with $R_j$ being the screening radius of the $j$-th atom in the elementary cell. The calculations are carried out for the sinusoidal transversal acoustic wave of the S-type (for the corresponding parameters see, for instance, Ref. [17]). For this wave the vector of the amplitude of the displacement is directed along the $X$-direction of the quartz single crystal, $u_0 = (u_0, 0, 0)$, and the velocity is equal $4.687 \times 10^5$ cm/sec. The vector determining the direction of the hypersonic propagation lies in the $YZ$-plane and forms the angle $0.295 \text{ rad}$ with the $Z$-axis. As the $z$-axis we choose the axis $Z$ of the quartz crystal. The corresponding function $F(x)$ is determined by formula [18]. The numerical calculations show that, in dependence of the values for the parameters $E_1$, $\theta$, $\alpha$, $u_0$, $\lambda_s$, the external excitation can either enhance or reduce the cross-section of the bremsstrahlung process.

As an illustration of the enhancement for the coherent bremsstrahlung cross-section, in the left panel of figure [1] we have plotted the quantity $10^{-6} (m_2^2 \omega/e^6) \frac{d\sigma_c}{d\omega}$, evaluated by formula (25), as a function of the ratio $\omega/E_1$, for $u_0 = 0$ (dashed curve, deformation is absent) and for $2\pi u_0/a_2 = 0.55$ (full curve) in the case $\theta = 0.00024$ and for the electron of energy $E_1 = 70$ MeV moving in SiO$_2$ single crystal. The corresponding deformation field is generated by the transversal acoustic wave of the S-type with the frequency $\nu_s = 5$ GHz. In the right panel of figure [1] the cross-section $10^{-6} (m_2^2 \omega/e^6) \frac{d\sigma_c}{d\omega}$, evaluated by formula (25), is presented as a function of the relative amplitude of the deformation, parameter $2\pi u_0/a_2$, for the energy of photon corresponding to $\omega/E_1 = 0.0002$. The values of the other parameters are the same as those for the left panel. In the left panel of figure [2] we have presented the cross-section $10^{-4} (m_2^2 \omega/e^6) \frac{d\sigma_c}{d\omega}$, evaluated by formula (26), as a function of the ratio $\omega/E_1$ for $u_0 = 0$ (dashed curve) and $2\pi u_0/a_1 = 0.25$ (full curve) in the case $\psi = 0.00092$. The values of the other parameters are the same as those for figure [1]. In the right panel of figure [2] we have plotted the cross section $10^{-4} (m_2^2 \omega/e^6) \frac{d\sigma_c}{d\omega}$, evaluated by formula (26), as a function of the parameter $2\pi u_0/a_1$ for $\omega/E_1 = 0.0001$ with the same values of the other parameters as those for figure [1].
As we see from the presented examples, the presence of the deformation field leads to the appearance of additional peaks in the spectral distribution of the radiated photons for frequencies lower than the frequency of the first peak in the situation where the deformation is absent. As we have already mentioned before, this is related to that in the presence of the deformation field the number of possibilities to satisfy the condition $g_{m\parallel} \geq \delta$ in the summation in formula (17) increases due to the presence of the additional term $mk_{s\parallel}$ in the expression for $g_{m\parallel}$.

![Figure 1: The coherent part of the cross-section evaluated by formula (25), as a function of $\omega/E_1$ (left panel) for the electron energy $E_1 = 70$ MeV and $\theta = 0.00024$ in the cases $u_0 = 0$ (dashed curve), $2\pi u_0/a_2 = 0.55$ (full curve), and as a function of $2\pi u_0/a_2$ (right panel) for the photon energy $\omega/E_1 = 0.0002$. The deformation field is generated by the acoustic wave of the $S$-type with the frequency $\nu_s = 5$ GHz.]

4 Conclusion

In the present paper we have investigated the influence of the deformation field of an arbitrary profile on the coherent bremsstrahlung of high energy electrons in oriented single crystals with complex base. The latter can be induced, for example, by acoustic waves. The corresponding results show that the deformation field can serve as a possible mechanism for the control of the angular-frequency characteristics of the radiated photons. The calculations have been done within the framework of the first Born approximation in the crystal potential. In the presence of periodic deformation field, the process of coherent bremsstrahlung can be considered as a result of the electron motion in the continuous periodic potential given by formula (12). The coherent part of the cross-section, averaged on thermal fluctuations of atoms, is given by formula (17) where the factor $|F_m(g_{m\parallel} u_0)|^2$ is determined by the function describing the displacement of the atoms due to the deformation field, and the factor $|S(g_{m\parallel}, g)|^2$ is determined by the structure of the crystal cell. In addition to the summation over the reciprocal lattice vector of the crystal, formula (17) contains a summation over the reciprocal lattice vector $mk_{s\parallel}$ of the one-dimensional super-lattice induced by the deformation field. The influence of the deformation field on the cross-section can be remarkable under the condition $4\pi^2 u_0/a \gtrsim \lambda_s/l_c$. The role of coherence effects in the bremsstrahlung cross-section is essential when the electron enters into the crystal at
small angles with respect to the crystallographic axes. In this case the main contribution into the coherent part of the cross-section comes from the crystallographic planes, parallel to the chosen axis (axis $z$ in our consideration). The behavior of the cross-section as a function of the photon energy essentially depends on the angle $\alpha$ between the projection of the electron momentum on the plane $(x, y)$ and $y$-axis. If the electron moves far from the corresponding crystallographic planes, the summation over the perpendicular components of the reciprocal lattice vector can be replaced by the integration and the coherent part of the bremsstrahlung cross-section is given by formula (22). When the electron enters into the crystal near crystallographic planes, two cases have to be distinguished. For the first one $\theta \sim a_2/2\pi l_c$, the summation over $g_x$ can be replaced by the integration and one obtains formula (24). This formula is simplified to the form (25) in the case when the amplitude of the deformation field is perpendicular to the crystallographic $x$-axis. In the second case one has $\psi = \alpha \theta \sim a_1/2\pi l_c$, and the main contribution into the cross-section comes from the crystallographic planes parallel to the incidence plane. The corresponding formula for the cross-section takes the form (26). The numerical calculations for the cross-section are carried for the SiO$_2$ single crystal with the Moliere parametrization of the screening atomic potentials and for the deformation field generated by the transversal acoustic wave of the $S$-type with frequencies 5 GHz, and for the energy of an electron 70 MeV. Results of the numerical calculations are presented in figures 1 and 2. The values of the parameters $\theta$, $\psi$, $u_0$ are chosen in the way to have an enhancement of the cross-section for the bremsstrahlung. As it is seen from the given figures, the presence of an ultrasonic wave leads to the appearance of new peaks in the cross-section of a coherent bremsstrahlung. This is related to that in the presence of the ultrasonic waves the number of possibilities to satisfy the condition $g_{m\parallel} \geq \delta$ increases. These new peaks are relatively strong in the range of the ratio $\omega/E_1$ from zero up to the first peak of the cross-section in the case when the ultrasonic wave is absent.
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