Hard hadronic diffraction is not hard

Boris Kopeliovich
Departamento de Física, Universidad Técnica Federico Santa María; Centro Científico-Tecnológico de Valparaíso, Casilla 110-V, Valparaíso, Chile
bzk@mpi-hd.mpg.de

Roman Pasechnik
Department of Astronomy and Theoretical Physics, Lund University, SE-223 62 Lund, Sweden
Roman.Pasechnik@thep.lu.se

Irina Potashnikova
Departamento de Física, Universidad Técnica Federico Santa María; Centro Científico-Tecnológico de Valparaíso, Casilla 110-V, Valparaíso, Chile
irina.potashnikova@usm.cl

Received Day Month Year
Revised Day Month Year

Hadronic diffractive processes characterised by a hard scale (hard diffraction) contain a nontrivial interplay of hard and soft, nonperturbative interactions, which breaks down factorisation of short and long distances. On the contrary to the expectations based on the factorization hypothesis, assuming that hard diffraction is a higher twist, these processes should be classified as a leading twist. We overview various implications of this important observation for diffractive radiation of Abelian (Drell-Yan, gauge bosons, Higgs boson) and non-Abelian (heavy flavors) particles, as well as direct coalescence into the Higgs boson of the non-perturbative intrinsic heavy flavour component of the hadronic wave function.

Keywords: Diffractive Drell-Yan process; diffractive factorisation breaking; Higgsstrahlung; intrinsic heavy flavor.

PACS numbers: 13.87.Ce, 14.65.Dw, 14.80.Bn

1. Introduction

Nowadays, factorisation of hard and soft interactions in Quantum Chromo Dynamics (QCD) in inclusive reactions is one of the most powerful and widely used theoretical tools [1]. While soft long-range interactions are poorly known, they factorise from well-studied short-distance interactions. Typically, the soft part is represented in terms of parton distribution functions (PDFs) whose dependence on the hard factorisation scale is provided by evolution equations with non-perturbative (supposedly, universal) starting distributions parameterised from the data. Making a
plausible (although, not rigorously proven) assumption about universality of the soft interactions, one often constrains those with electro-weak hard probes such as the Deep Inelastic Scattering (DIS) and Drell-Yan (DY) processes and then applies to hard hadronic processes. One may naturally be tempted to extend such a factorization scheme to diffractive reactions with a large rapidity gap. However, such diffractive factorisation turns out to be unavoidably broken and does not hold in practice [2, 3].

Inelastic diffraction in the Good-Walker picture is treated as a shadow of inelastic processes [4, 5, 6]. Consider the incoming plane wave which scatters off the target. If it has differently interacting components, the outgoing wave will necessarily have a different composition, namely, a new (diffractive) state emerges (for more details, see Ref. [2, 7]). Purely soft hadronic diffraction incorporates unknown non-perturbative physics and thus is theoretically very difficult to predict. Instead, diffractive processes involving a hard scale attract particular attention. Although factorisation of interactions at short and long distances still holds in diffractive DIS, the corresponding fracture functions are proven not be universal; they are process-dependent and hence cannot be used for other diffractive processes.

Traditionally, the Pomeron-hadron total cross section is studied by means of diffractive excitations [7]. Being applied to DIS, it enables to constrain the structure function of the Pomeron directly from the data [8]. One way to study diffractive processes is to extrapolate the ideas of QCD factorisation by employing the PDFs in the Pomeron which would, in principle, allow to predict the hard diffractive cross sections in hadronic collisions as is illustrated in Fig. 1. It is well-known, however, that such predictions for hard diffractive observables, for example, on high-\(p_T\) dijet production, strongly fail by about an order of magnitude (see e.g. Refs. [9, 10]). This happens, in particular, due to a strong breakdown of diffractive factorisation due to the unavoidable presence of spectator partons at typically large (hadronic) separations [11].

![Fig. 1. An illustration of the DIS process on hadronic taget (left panel) and on the Pomeron which is often treated as a target (right panel).](image)

Within the quantum-mechanical Good-Walker mechanism of diffraction [4, 5, 6], the off-diagonal amplitude should be treated as a linear combination of diag-
Hard hadronic diffraction is not hard

Onal (elastic) diffractive amplitudes corresponding to different Fock components in the projectile hadronic wave function. In this picture, a result of cancellations between various elastic amplitudes, the diffractive amplitude exhibits a common factor giving rise to the absorptive corrections which, thus, naturally emerge at the amplitude level, and do not require an additional gap survival factors in the cross section. This is especially transparent and clear in the color dipole approach [12, 14] where any diffractive scattering is viewed as a superposition of elastic scatterings of $\bar{q}q$ dipoles of varied sizes arising from a combination of projectile Fock states in the initial hadronic wave function. As was explicitly demonstrated for the first time in Ref. [11], the long-range soft interactions with spectators comes together with hard interactions in the DY reactions associated with a short-distance virtual $\gamma^*$-radiation, and the latter two cannot be consistently separated.

In this way, any diffractive amplitude emerges as a difference between the elastic scatterings of different Fock components in the projectile composite state, in particular, that of hadronic states with and without a hard fluctuation (for example, $qq$ and $qq\gamma^*$ states in the DY case),

$$A_{\text{DY}}^{\text{diff}} \propto \sigma_{\bar{q}q}(\vec{R} + \vec{r}) - \sigma_{\bar{q}q}(\vec{R}) \propto \vec{r} \cdot \vec{R} \sim 1/Q,$$

where $\sigma_{\bar{q}q}(r)$ is the universal dipole-nucleon cross section [12, 14] fitted to DIS data, $\vec{R}$ corresponds to a large separation between different constituent projectile quarks in the incoming hadron while small $r \sim 1/Q \ll R$ is related to the hard radiation process (see e.g. Refs. [15, 16]). Obviously, such a mild hard scale $Q$ dependence, corresponding to the leading-twist behavior, strongly contradicts to the diffractive factorisation DIS-like prediction,

$$A_{\text{DIS}}^{\text{diff}} \propto \sigma_{\bar{q}q}(r) \propto r^2 \sim 1/Q^2,$$

which is apparently a higher-twist effect. While the phenomenological dipole cross section (or partial dipole amplitude) is a universal ingredient naturally accumulating the soft interactions and fitted to the available precision data, the diffractive amplitudes are represented in terms of a linear superposition of elastic dipole scatterings at different transverse separations which is process-dependent and accumulates all the relevant absorptive corrections fully dynamically. Naturally, the gap survival amplitude gets singled out from such a superposition as a common factor dependent on soft parameters of the dipole model and on the diffractive process concerned.

In the forward scattering limit and in the absence of spectator co-movers a single quark cannot radiate an Abelian particle ($\gamma, Z, W^\pm, H$) in a diffractive quark-hadron scattering (with zero net momentum transfer), in variance to diffractive factorisation [17]. Only a dipole can diffractively radiate due to a small fluctuation in its size induced by the hard scattering (c.f. Eq. (1)) so diffraction becomes possible although is strongly suppressed. Such a mechanism opens up new possibilities for universal description of diffractive reactions specific to the dipole approach beyond QCD factorisation [12]. The diffractive factorisation breaking in non-Abelian
radiation is also important although the diffractive gluon radiation off a quark
does not vanish in the forward kinematics due to an extra glue-glue interaction.
The universal dipole mechanism of diffraction has been employed in a number of
diffractive processes so far, and this review aims at a short comprehensive overview
of major implications of the diffractive factorisation breakdown in both Abelian
and non-Abelian diffractive radiation.

2. Color dipole picture

The color dipole formalism \cite{12,13,14} is a phenomenological approach which effec-
tively takes into account the higher-order QCD corrections in dipole-target scatter-
ing in the target rest frame. Color dipoles with definite transverse sizes are treated
as eigenstates of interaction, thus, their scattering off a target is universal and can
be used as a building block for more complicated inclusive and diffractive pro-
cesses. The dipole cross section is the basic ingredient of this approach which is
process-independent and is typically extracted from ample DIS phenomenology. In
the simplest form, without an account for QCD evolution, the dipole cross section
at small Bjorken variable \( x < 0.01 \) is conventionally parameterized in the following
saturated form known as the Golec-Biernat–Wusthoff (GBW) ansatz \cite{18},

\[
\sigma_{\bar{q}q}(\vec{r}, x) = \int d^2b 2\text{Im} f_{\text{el}}(\vec{b}, \vec{r}) = \sigma_0 (1 - \frac{r^2}{R_0^2(x)}), \tag{3}
\]

whose parameters \( \sigma_0 = 23.03 \text{ mb} \) and \( R_0(x) = 0.4 \text{ fm} \times (x/x_0)^{0.144} \), \( x_0 = 0.003 \) are
known from fits to the DIS data, and \( f_{\text{el}}(\vec{b}, \vec{r}) \) is the partial dipole amplitude. Such
a simplified parametrisation (cf. Ref. \cite{19}) provides a reasonable description of all
bulk of inclusive and diffractive DIS data in ep collisions at HERA as well as many
other processes in hadron-hadron and hadron-nucleus collisions such as DY, heavy
quark and prompt photon production etc \cite{14,20,21,22,23,24}.

The dipole cross section (3) levels off at \( r \gg R_0 \), the phenomenon commonly
known as saturation. The second important feature is that it vanishes at small
\( r \to 0 \) as \( \sigma_{\bar{q}q} \propto r^2 \) \cite{12} giving rise to the color transparency property. The latte
reflects the fact that a point-like colorless object does not interact with external
color fields. Finally, the quadratic \( r \)-dependence at small \( r \) is a natural consequence
of gauge invariance and nonabeliance of interactions in QCD.

In soft diffractive processes, the ansatz (3) has to be modified since the Bjorken
variable \( x \) is not a proper variable in this case. Instead, the gluon-target collision
c.m. energy squared \( \hat{s} = x_1 s \) given in terms of the gluon momentum fraction \( x_1 \)
and the \( pp \) c.m. energy \( s \) is a more appropriate variable, while the saturated form
(3) is retained. An analytic form of the \( x \)- and \( \hat{s} \)-dependent parameterisations for
the partial amplitude \( f_{\text{el}}(\vec{b}, \vec{r}) \) can be found e.g. in Refs. \cite{25,26,27}.

3. Diffractive Abelian radiation

Naturally, the observables such as total cross sections are Lorentz invariant. A par-
tonic scattering picture of a given process, however, depends on the reference frame
Hard hadronic diffraction is not hard

\[ G = W^\pm, Z^0 \]

\[ q_k \]

\[ q_f \]

(a)

(b)

Fig. 2. Gauge boson radiation by a quark after (a) and before (b) the interaction with the target color field denoted by a shaded circle.

In the framework of conventional parton model in the center-of-mass frame the dilepton production emerges due the quark-antiquark annihilation into a virtual \( \gamma/Z^0 \) boson. In the target rest frame relevant for the dipole model formalism, however, the same process should be considered as a bremsstrahlung of \( \gamma/Z^0 \) and the corresponding diagrams are illustrated in Figs. 2 (a) and (b), rather than \( q\bar{q} \) annihilation [22, 25]. Indeed, the gauge boson can be radiated off an energetic projectile quark both before and after its scattering off the hadronic target such that these contributions interfere. Thus, at high energies the incoming projectile quark effectively probes the dense gluonic field in the target nucleon and is particularly sensitive to the nonlinear effects in multiple dipole-target scatterings. This is in analogy to the inclusive DIS reaction where a virtual photon in the Bjorken frame is considered as a probe for partonic structure of the hadron, while in the target rest frame it is instead viewed as an interaction of partonic components of the projectile photon.

It is well-known that soft fluctuations with a large sizes \( \langle R^2 \rangle \sim 1/m_q^2 \) \( (m_q \) is a constituent light quark mass) corresponding to the asymmetric dipoles play a leading role in the diffractive DIS process, which is in variance with the inclusive DIS one [24]. Indeed, even though the soft fluctuations are rather rare, they interact with a large cross section which in practice compensates their tiny weights \( \sim m_q^2/Q^2 \). The hard fluctuations corresponding to symmetric small dipoles with \( \langle r^2 \rangle \sim 1/Q^2 \) are much more abundant but their scattering is vanishing as \( 1/Q^2 \). While the inclusive DIS is a semi-hard/semi-soft process with the total cross section \( 1/Q^2 \), the diffractive DIS is solely dominated by soft fluctuations \( \sim 1/m_q^2 Q^2 \) leading to nearly \( Q^2 \)-independence of the SD-to-inclusive ratio \( \sigma_{sd}/\sigma_{inc} \) with the higher-twist behaviour of diffractive DIS.

The inclusive dilepton production process mediated by a virtual photon in the dipole framework in \( pp \) and \( pA \) collisions has been intensively studied in the literature so far (see e.g. Refs. [29, 30, 31, 32]). In particular, it has been understood that the phenomenological dipole model predictions for DY observables are practically indistinguishable from those in the NLO collinear factorisation framework [30] and
provide a good agreement with the recent LHC data \[32\]. In the diffractive channel, the DY and electroweak gauge boson production has been studied within the dipole formalism in Ref. \[33\].

The dipole formula for the inclusive DY cross section is similar to the DIS one \[28, 22\]:

\[
\frac{d\sigma_{\text{DY}}^{\text{inc}}(qp \to \gamma^* X)}{d\alpha dM^2} = \int d^2 \vec{r} |\Psi_{q\gamma^*}(\vec{r},\alpha)|^2 \sigma_{\bar{q}q}(\alpha r, x_2),
\]

in terms of \( q \to q\gamma^* \) distribution function \( \Psi_{q\gamma^*}(\vec{r},\alpha) \) where \( \alpha = p_\gamma^* / (p_+ + p^-) \) is the fractional light-cone momentum of the virtual photon. The inclusive DIS and DY processes are related by means of QCD factorisation, and a similarity between them is a source of the hadron PDF universality.

The single diffractive (SD) DY process is characterized by a relatively small momentum transfer between the colliding protons, i.e. both transverse and fractional momenta are small. One of the protons is then treated as a target, another – as a projectile which emits a virtual photon (or any other Abelian particle) and then hadronises into a hadronic system \( X \) in forward region. Both \( X \) and the radiated photon are separated by a large rapidity large from the target which remains intact, i.e.

\[
p_1 + p_2 \to X + (\text{gap}) + p_2, \quad X \equiv \gamma^*(l^+ l^-) + Y.
\]

At large Feynman \( x_F \to 1 \) of the recoil target proton, the diffractive DY process is described by the triple Regge graphs illustrated in Fig. 3.

In fact, diffractive DY process vanishes in the forward direction \[22\] since the
Fig. 5. The SD-to-inclusive ratio of the DY cross sections vs the dilepton mass squared for c.m. energies $\sqrt{s} = 0.04, 0.5$ and $14$ TeV (left panel) and the diffractive DY and gauge $W^\pm, Z^0$ boson production cross sections as functions of diphoton invariant mass squared $M^2$ at $\sqrt{s} = 14$ TeV (right panel).

graphs (a), (b) and (c) shown in Fig. 4 get canceled, i.e.

$$\frac{d\sigma^{DY\,\text{inc}}_{\text{inc}} (qp \rightarrow q\gamma \ast qp)}{d\alpha \, dM^2 \, d^2 p_T} \bigg|_{p_T=0} = 0.$$  \hspace{1cm} (6)

Indeed, only quark interacts with the target in its both Fock components, $|q\rangle$ and $|q\gamma\ast\rangle$, so they interact with an equal strengths and thus cancel in the forward diffractive amplitude in accordance to the Good-Walker picture. The same behavior holds for any diffractive Abelian radiation, in particular, for diffractive production of $\gamma^*, W^\pm, Z^0$ bosons as well as the Higgs boson.

The diffractive (Ingelman-Schlein) factorisation in the diffractive Abelian radiation with a large rapidity gap is broken. Indeed, large- and small-size fluctuations in the projectile cannot be consistently separated and contribute to the diffractive Abelian radiation on the same footing. This is the source of the leading-twist behavior of the diffractive DY cross section whereas the diffractive DIS is determined by soft fluctuations and thus emerges as a higher-twist process \[11, 16\].

Due to the internal transverse motion of the projectile valence quarks inside the incoming proton, which corresponds to finite large transverse separations between them, the forward photon radiation does not vanish \[11, 16\]. These large distances are controlled by a nonperturbative (hadron) scale $\vec{R}$, such that the diffractive amplitude has the Good-Walker structure \[1\], such that the ratio of the cross sections reads

$$\frac{\sigma^{DY\,\text{sd}}}{\sigma^{DY\,\text{incl}}} \propto \left[ \sigma_{\bar{q}q}(\vec{R} + \vec{r}, x_2) - \sigma_{\bar{q}q}(\vec{R}, x_2) \right]^2 \propto \exp\left(-2R_0^2/R_0^2(x_2)\right).$$  \hspace{1cm} (7)

Thus, the soft part of the diffractive DY interaction is not enhanced such that the process is semi-hard/semi-soft similarly to the inclusive DIS one. The emergent
linear dependence on the hard scale \( r \sim 1/M \ll R_0(x_2) \) indicates that even at a hard scale the diffractive Abelian radiation is very sensitive to the hadron scale, thus, diffractive factorisation does not hold and should not be imposed in practice [30]. The observation that factorisation in diffractive DY process fails due to the presence of spectator partons in the Pomeron has been first made in Refs. [37, 38]. Recently, in Refs. [11, 16, 33] it was explicitly shown that factorisation in diffractive Abelian radiation is therefore even more broken due to presence of spectator partons in the colliding hadrons. Besides, the saturated shape of the dipole cross section (3), therefore, leads to several unusual features of diffractive DY cross section. Namely, the fractional diffractive DY cross section is steeply falling with energy, but rises with the scale as is demonstrated in Fig. 5.

In general, diffractive radiation of any Abelian particle is given by the same graphs as the diffractive DY process, with an appropriate use of couplings and spin structure [33]. In Fig. 5 (right panel) we show the single diffractive cross sections for \( Z^0, \gamma^* \) and \( W^\pm \) bosons production as functions of the dilepton mass squared, \( d\sigma_{\text{sd}}/dM^2 \). The SD-to-inclusive ratios of the DY cross sections for diffractive \( Z^0 \) and \( W^\pm \) production are shown in Fig. 6 in comparison with the CDF data [35].

4. Gap survival effect at the amplitude level

A suppression factor in diffractive cross sections which parameterises the unitarity corrections is known as the survival probability. The latter significantly reduces the diffractive observables in hadronic collisions and there is no process-independent way to compute it consistently beyond the Regge theory. The soft survival probability emerges due to the long-range interactions of (soft) spectator partons in the projectile hadron wave function which are not present in the case of diffractive DIS. Hence, the transverse motion of spectators is the basic source of diffractive

![Fig. 6. The SD-to-inclusive ratio vs dilepton invariant mass squared in comparison with the CDF data [35].]
factorisation breaking in hadronic collisions compared to diffraction in $ep$ collisions.

\[ \sigma_{qq}(\vec{r}_1 - \vec{r}_2) \]
\[ \vec{r}_1 - \vec{r}_2 + \alpha \vec{r} \]

Fig. 7. Diffractive photon radiation in the dipole-target scattering as an elementary ingredient of diffractive DY process in hadronic collisions.

The forward diffractive DY process in hadronic collisions is dominated by a large-size dipole elastic scattering off a given potential as illustrated in Fig. 7. Let $\vec{r}_{1,2}$ are the positions of projectile quarks in the transverse plane of the hadronic wave function such that the dipole size is $|\vec{r}_1 - \vec{r}_2| \sim R_{\text{had}}$. As was mentioned above, a quark emitting a virtual photon shifts its position by a value determined by the hard scale $\delta = \alpha \vec{r} \sim 1/M \ll R_{\text{had}}$ such that the dipole size is changed. As was discussed above, the interference between the two graphs in Fig. 7 leads to a non-vanishing contribution to the diffractive DY cross section. The dipole partial amplitude can then be written in the eikonal form as

\[ \text{Im} f_{\text{el}}(\vec{b}, \vec{r}_1 - \vec{r}_2) = 1 - \exp \left( i \mathcal{V}(\vec{r}_1) - i \mathcal{V}(\vec{r}_2) \right), \quad \mathcal{V}(\vec{b}) = - \int_{-\infty}^{\infty} dz V(\vec{b}, z), \quad (8) \]

where the scattering potential is denoted as $V(\vec{b}, z)$. At high energies, this amplitude is nearly imaginary such that the diffractive DY amplitude off a dipole is provided by

\[ \text{Im} f_{\text{el}}(\vec{b}, \vec{r}_1 - \vec{r}_2 + \alpha \vec{r}) - \text{Im} f_{\text{el}}(\vec{b}, \vec{r}_1 - \vec{r}_2) \simeq \exp \left( i \mathcal{V}(\vec{r}_1) - i \mathcal{V}(\vec{r}_2) \right) \exp \left( i \alpha \vec{r} \cdot \nabla \mathcal{V}(\vec{r}_1) \right), \]

where the first exponential factor represents the gap survival amplitude which accounts for all absorptive corrections, provided that the universal dipole cross section is fitted to the data and accounts for both hard and soft interactions. The survival amplitude indeed vanishes in the black disk asymptotics as required. While conventionally the gap survival factor is incorporated directly into the diffractive cross sections and is thus treated probabilistically, in the color dipole framework the corresponding effects are accounted automatically and treated more naturally quantum-mechanically.
5. Diffractive heavy flavor production

Understanding of both inclusive and diffractive hadroproduction of heavy quarks at large Feynman $x_F \to 1$ is a longstanding controversial problem. Indeed, QCD factorisation predicts vanishing $\bar{Q}Q$ production cross sections at large $x_F$ due to a steeply decreasing gluon density in the forward kinematics which contradicts to the end-point behavior predicted by the Regge asymptotics (see e.g. Ref. [3] and references therein). A similar contradiction arises for the DY reaction at large $x_F$ which was seen from the data [40]. Both examples apparently indicate that the conventional QCD factorisation does not hold, at least, at large Feynman $x_F$ [41].

Fig. 8. Typical contributions to inclusive production of a heavy quark pair in a quark-proton collision.

A detailed analysis of various contributions into the diffractive $Q\bar{Q}$ production from both diffractive gluon and quark excitations has been performed in Ref. [39]. For example, in the case of diffractive quark excitation $q+g \to (\bar{Q}Q)+q$ the dynamics of inclusive heavy flavor production is characterised by five distinct topologies which can be classified as: (i) bremsstrahlung (like in DY), and (ii) production mechanisms as illustrated by the Feynman graphs in Fig. 8 such that the total amplitude

$$A_{\text{diff}}^{Q\bar{Q}} = A_{\text{BR}} + A_{\text{PR}}.$$  \hspace{1cm} (9)

Each of these two contributions is gauge invariant and can be described in terms of three-body dipole cross sections, $\sigma_{gqg}$ and $\sigma_{gQQ}$, respectively, which strongly motivates such a separation. Similar graphs and classification hold for the diffractive gluon excitation $g+g \to (Q\bar{Q})+g$ as well. The amplitudes for each of the two mechanisms are expressed via the amplitudes $A_i$ corresponding to the graph numbering in Fig. 8. As was elaborated in Ref. [39] such a grouping can be performed for both transversely and longitudinally polarised intermediate gluons. The bremsstrahlung and production components have the following form,

$$A_{\text{BR}} = A_1 + A_2 + \frac{Q^2}{M^2 + Q^2} A_3;$$ \hspace{1cm} (10)\hspace{1cm} \\
$$A_{\text{PR}} = \frac{M^2}{M^2 + Q^2} A_3 + A_4 + A_5,$$ \hspace{1cm} (11)

where $Q^2 = (p_i - p_j)^2$ in terms of the initial $p_i$ and final $p_j$ projectile quark momenta, and $M$ is the invariant mass of the $Q\bar{Q}$ pair.
For diffractive production one has to provide a colorless two-gluon exchange. In analogy to the leading-twist DIS diffraction at large photon virtualities $\gamma^* \rightarrow Q\bar{Q}g$, the BR and PR contributions are dependent on two characteristic length scales: the small separation between the $\bar{Q}$ and $Q$, $s \sim 1/m_Q$, and a typically large separation between $q$ and $Q\bar{Q}$, $\rho \sim 1/m_q$. In analogy to diffractive DY, the diffractive excitation of a quark thus turns out to be a higher twist effect as is depicted in Fig. 9 (left). The leading twist contributions to diffractive $Q\bar{Q}$ production come from both sources: when both exchanged gluons couple to the valence quark which gives rise to the $Q\bar{Q}$ pair, and when one of the gluons is coupled to another spectator quark not participating in the hard scattering as is shown in Fig. 9 (right) [39] (for more details, see Ref. [3]). So the interaction with spectators again plays an important role as one of the source for the diffractive factorisation breaking.

The $Q\bar{Q}$ production amplitudes in diffractive quark scattering off a proton target are related to the effective dipole cross sections $\Sigma_{1,2}$ for colorless $gqq$ and $gQ\bar{Q}$.
Boris Kopeliovich, Roman Pasechnik, Irina Potashnikova

systems as

\[ A_{BR} \propto \Phi_{BR}(\vec{p}, \vec{s}) \Sigma_1(\vec{p}, \vec{s}) \sim \langle s^2 \rangle \sim \frac{1}{m_Q^2}, \]

\[ A_{PR} \propto \Phi_{PR}(\vec{p}, \vec{s}) \Sigma_2(\vec{p}, \vec{s}) \sim \vec{s} \cdot \vec{p} \sim \frac{1}{m_q m_Q}, \]

where \( \Phi_{BR/PR} \) are complicated distribution amplitudes for the \( q + g \rightarrow (Q\bar{Q}) + q \) subprocess. The bremsstrahlung contribution is of a higher twist effect and is therefore suppressed while for diffractive Abelian radiation it is equal to zero. In opposite, the production contribution is of the leading twist and is thus much larger than the bremsstrahlung term in analogy to the diffractive DY reaction. This is again due to the presence of spectators at large distances from the \( Q\bar{Q} \) pair despite of non-Abelian nature of the process which is a rather non-trivial fact. The non-Abelian interactions, however, introduce extra important leading-twist terms into the “production” mechanism, which are independent of the structure of the hadronic wave function, in addition to those from the spectators’ interactions.

The leading-twist behavior \( 1/m_Q^2 \) of the diffractive cross section is confirmed by E690 [42] and CDF [43] data as demonstrated in Fig. 10 (left panel), where the corresponding cross sections for charm, beauty and top quarks, \( p + p \rightarrow Q\bar{Q}X + p \), are shown as functions of c.m.s. \( pp \) energy. Besides, on the right panel we show differential cross section in \( x_1 \)-variable, \( d\sigma/dx_1 \), for diffractive charm quark production at two different energies \( \sqrt{s} = 0.5 \) and 14 TeV.

![Fig. 11. Typical Feynman graphs for the diffractive Higgsstrahlung process off a heavy quark which involve interactions of spectator partons.](image)

6. Diffractive Higgs production

6.1. Higgsstrahlung

Consider single diffractive Higgs boson production in hadron-hadron collisions. The Higgs boson decouples from light quarks, in particular, due to a smallness of the corresponding Yukawa coupling so the Higgsstrahlung by light hadrons is vanishingly small. Although a light projectile quark does not radiate the Higgs boson directly,
Hard hadronic diffraction is not hard

it can do it via production of heavy flavors. Similarly to the diffractive \( Q\bar{Q} \) production considered above, the diffractive Higgsstrahlung process off a heavy quark is dominated by the diagrams involving interactions of spectators at large transverse separations as illustrated in Fig. 11. Therefore, the Higgsstrahlung mechanism is closely related to the non-Abelian mechanism for diffractive heavy quark production discussed in the previous section. In a sense, it is also similar to diffractive \( DY, Z^0 \) and \( W^\pm \) production since in all these cases the radiated particle does not participate in the interaction with the target although \( gg \rightarrow Q\bar{Q} + H \) subprocess is rather involved and more complicated Fock states containing heavy flavors need to be resolved by the exchanged gluons.

![Graph](image)

Fig. 12. The differential cross section of single diffractive Higgs boson production in association with a heavy quark (\( b\bar{b} \) and \( t\bar{t} \)) pair vs Higgs boson rapidity (left panel) and the SD-to-inclusive ratio for the Higgsstrahlung process as a function of the \( QQH \) invariant mass (right panel) (see more details in Ref. [44]).

The rapidity-dependent cross section of diffractive Higgs boson production off \( t\bar{t} \) and \( b\bar{b} \) at the LHC energy \( \sqrt{s} = 14 \text{ TeV} \) is plotted in Fig. 12 (left panel). At Higgs mid-rapidities, the top and bottom contributions are comparable to each other, whereas top quark provides a wider rapidity distribution and dominates at large Higgs boson transverse momentum \( \pT \). The total cross section is rather small and below 1 fb. In Fig. 12 (right panel) we present the SD-to-inclusive ratio of the corresponding Higgsstrahlung cross sections for different c.m. energies \( \sqrt{s} = 0.5, 7, 14 \text{ TeV} \) and for two values of the Higgs boson rapidities \( Y = 0 \) and 3 as functions of \( QQH \) invariant mass. This ratio is in overall agreement with the corresponding data for diffractive beauty production \[43\].

As expected from above discussion, the diffractive factorisation in diffractive Higgsstrahlung is broken by transverse motion of spectator valence quarks in the projectile hadron leading to a growth of the SD-to-inclusive ratio with the hard scale, \( M \), and its decrease with \( \sqrt{s} \). Such a behavior is opposite to the one predicted by diffractive factorisation and is in full analogy with the diffractive Abelian radiation.
6.2. Direct heavy flavour fusion

The Higgs boson can also be diffractively produced due to fusion of the intrinsic heavy flavours (IQ) in light hadrons, $\bar{Q}Q \rightarrow H$, as is depicted in Fig. 13, left.

![Diagram](image)

Fig. 13. The two-gluon exchange diagram for the Higgs exclusive production via coalescence of intrinsic heavy quarks, $\bar{Q}Q \rightarrow H$ (left panel), and the cross section of diffractive exclusive Higgs production off different intrinsic flavors as a function of the Higgs boson mass [45] (right panel).

Such exclusive Higgs production process, $pp \rightarrow Hpp$ was analysed in Refs. [45, 46]. The diffractive cross section has the form,

$$d\sigma(pp \rightarrow ppH) = \frac{1}{(1-x_2)16\pi^2} |A(x_2, \vec{p}_1, \vec{p}_2)|^2,$$

(12)

where the diffractive amplitude in Born approximation reads,

$$A(x_2, \vec{p}_1, \vec{p}_2) = \frac{8}{3\sqrt{2}} \int \frac{d^2Q}{q^2} \frac{d^2k}{k^2} \alpha_s(q^2)\alpha_s(k^2) \delta(\vec{q} + \vec{p}_2 + \vec{k}) \delta(\vec{k} - \vec{p}_1 - \vec{Q})$$

$$\times \int d^2\tau |\Phi_p(\tau)|^2 \left[ e^{i(\vec{k}+\vec{q}) \cdot \vec{\tau}/2} - e^{i(\vec{q}+\vec{k}) \cdot \vec{\tau}/2} \right] \int d^2R d^2\rho H^1(\vec{r}) e^{iq \cdot \vec{r}/2}$$

$$\times (1-e^{-i\vec{q} \cdot \vec{r}}) \Psi_p(\vec{\rho}) e^{i\vec{k} \cdot \vec{\rho}/2} \left(1-e^{-i\vec{k} \cdot \vec{\rho}} \right) \Psi_p(\vec{\rho}, \vec{r}, \vec{\rho}, z) e^{i\vec{Q} \cdot \vec{R}}. $$

(13)

Here $(1-x_1)(1-x_2) = M_H^2/s$. $\Psi_p(\vec{R}, \vec{r}, \vec{\rho}, \vec{z})$ is the light-cone wave function of the IQ component of the projectile proton with transverse separations $\vec{R}$ between the $\bar{c}c$ and $3q$ clusters, $\vec{r}$ between the $c$ and $\bar{c}$, $\vec{Q}$ is the relative transverse momentum of the $3q$ and $\bar{c}c$ clusters in the projectile and $\vec{\rho}$ is the transverse separation of the quark and diquark which couple to the final-state proton $p_2$. The density $|\Phi_p(\tau)|^2$ is the wave function of the target proton which we also treat as a color dipole quark-diquark with transverse separation $\tau$. (The extension to three quarks is straightforward [12]). The fraction of the projectile proton light-cone momentum carried by the $\bar{c}c$, $z \approx 1 - x_1$. This wave function is normalized as,

$$\int_0^1 dz \int d^2R d^2\rho d^2p_1 |\Psi_p(\vec{R}, \vec{r}, \vec{\rho}, z)|^2 = P_{IQ},$$

(14)
where $P_{IQ}$ is the weight of the IC component of the proton, which is suppressed as $1/m_Q^2$ \[47\], and is assumed to be $P_{IC} \sim 1\%$. The amplitudes $H(\vec{r})$ and $\Phi_p(\vec{p})$ denote the wave functions of the produced Higgs and the outgoing proton, respectively, in accordance with Fig. 13 left.

At the measured Higgs mass value $125$ GeV the intrinsic bottom and top provide comparable contributions as can be seen in Fig. 13 right. Comparing the Higgsstrahlung cross section off the produced heavy quarks, i.e. $gg \rightarrow Q\bar{Q}H$, and that off the intrinsic component one concludes that the intrinsic contribution to the diffractive Higgs boson production can be relevant at forward Higgs boson rapidities $y_H > 3.5$ \[44\].

7. Summary

In this short review, we discussed the most important implications of the diffractive factorisation breaking in hard diffractive hadronic collisions. Indeed, forward diffractive Abelian radiation such as radiation of direct photons, Drell-Yan dileptons, and gauge $Z^0$ and $W^\pm$ bosons by a projectile parton is forbidden. Nevertheless, a finite-size hadron can diffractively radiate in the forward direction due to soft interactions of its spectators with the target nucleon. Such a feature of the diffractive Abelian radiation breaks factorisation between soft and hard interactions resulting in a leading-twist behavior, i.e. to the $1/M^2$ scaling of the corresponding cross section with the boson mass $M$.

The non-Abelian forward diffractive radiation of heavy flavors is permitted even for an isolated parton. However, interaction with spectators provides the dominant contribution to the diffractive cross section. It comes from the interplay between large and small distances similar to that in the diffractive Drell-Yan process. In particular, this leads to unusual properties of the SD-to-inclusive ratios such as growth with the hard scale $M$ and decrease with c.m. energy $\sqrt{s}$. The latter behavior holds, in fact, for various Abelian and non-Abelian radiation processes in diffractive hadronic collisions, in variance with predictions of diffractive factorisation. The experimental data confirm the leading-twist behavior of the observables.

The diffractive Higgsstrahlung off heavy flavors is possible as a double-step process when a heavy quark pair is produced first, $g \rightarrow Q\bar{Q}$, and then the Higgs boson is radiated off $Q$ or $\bar{Q}$. Such a mechanism of diffractive Higgs boson production in association with a heavy quark pair at the LHC is highly suppressed ($\lesssim 1$ fb) but is not negligible compared to the conventional loop-induced central exclusive Higgs boson production. Another important contribution to the diffractively produced Higgs boson which dominates over the Higgsstrahlung off the produced heavy quarks comes from coalescence of intrinsic heavy quarks in the proton. For $M_H = 125$ GeV and forward rapidities $y_H > 3.5$ dominance of intrinsic bottom and top is expected.

Due to a decisive impact of soft spectator interactions and the universal interplay between the soft and hard interactions, the same unconventional hard scale
and energy dependences of the SD-to-inclusive ratio has been observed in all typical diffractive reactions. This strongly motivates further deeper phenomenological studies of diffractive factorisation breaking effects at various energies providing an access to soft dynamics of partons in a nucleon.

Acknowledgments

This study was partially supported by Fondecyt (Chile) grants No. 1130543, 1130549, as well as by CONICYT grant PIA ACT1406 (Chile). R. P. was partially supported by Swedish Research Council Grant No. 2013-4287.

References

1. J. C. Collins, D. E. Soper and G. F. Sterman, Adv. Ser. Direct. High Energy Phys. 5, 1 (1989).
2. B. Z. Kopeliovich, I. K. Potashnikova, I. Schmidt, Braz. J. Phys. 37, 473 (2007).
3. R. Pasechnik, B. Kopeliovich and I. Potashnikova, Adv. High Energy Phys. 2015, 701467 (2015).
4. R. J. Glauber, Phys. Rev. 100, 242 (1955).
5. E. Feinberg and I. Ya. Pomeranchuk, Nuovo. Cimento. Suppl. 3 (1956) 652.
6. M. L. Good and W. D. Walker, Phys. Rev. 120 (1960) 1857.
7. A. B. Kaidalov, Phys. Rept. 50 (1979) 157.
8. G. Ingelman, Phys. Lett. B152 256 (1985).
9. A. Brandt et al. [UA8 Collaboration], Phys. Lett. B297, 417 (1992).
10. Bonino et al. [UA8 Collaboration], Phys. Lett. B311, 239 (1988).
11. B. Z. Kopeliovich, I. K. Potashnikova, I. Schmidt, A. V. Tarasov, Phys. Rev. D74, 114024 (2006).
12. B. Z. Kopeliovich, L. I. Lapidus and A. B. Zamolodchikov, JETP Lett. 33, 595-597 (1981) [Pisma Zh. Eksp. Teor. Fiz. 33, 612 (1981)].
13. A. H. Mueller, Nucl. Phys. B 415, 373 (1994).
14. N. N. Nikolaev, B. G. Zakharov, Z. Phys. C 64, 631 (1994).
15. B. Z. Kopeliovich, I. K. Potashnikova, I. Schmidt, A. V. Tarasov, Phys. Rev. D74, 114024 (2006).
16. R. S. Pasechnik and B. Z. Kopeliovich, Eur. Phys. J. C 71, 1827 (2011).
17. B. Z. Kopeliovich, A. Schafer and A. V. Tarasov, Phys. Rev. D62, 054022 (2000).
18. K. J. Golec-Biernat, M. Wusthoff, Phys. Rev. D 59, 014017 (1998).
19. J. Bartels, K. J. Golec-Biernat and H. Kowalski, Phys. Rev. D 66 (2002) 014001.
20. B. Z. Kopeliovich, in Proceedings of the international workshop XXIII on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Austria, 1995, edited by H. Feldmeyer and W. Nörenberg (Gesellschaft Schwerionenforschung, Darmstadt, 1995), p. 385.
21. S. J. Brodsky, A. Hebecker and E. Quack, Phys. Rev. D 55, 2584 (1997).
22. B. Z. Kopeliovich, A. Schafer, and A. V. Tarasov, Phys. Rev. C 59, 1609 (1999).
23. B. Z. Kopeliovich, J. Raufiesen, and A. V. Tarasov, Phys. Lett. B 503, 91 (2001).
24. N. N. Nikolaev, G. Piller and B. G. Zakharov, Z. Phys. A 354, 99 (1996);
    B. Z. Kopeliovich and A. V. Tarasov, Nucl. Phys. A 710, 180 (2002).
25. B. Z. Kopeliovich, H. J. Pirner, A. H. Rezaeian and I. Schmidt, Phys. Rev. D 77, 034011 (2008) [arXiv:0711.3010 [hep-ph]].
26. B. Z. Kopeliovich, I. K. Potashnikova, I. Schmidt and J. Soffer, Phys. Rev. D 78, 014031 (2008) [arXiv:0805.4534 [hep-ph]].
27. B. Z. Kopeliovich, A. H. Rezaeian, I. Schmidt, Phys. Rev. D78, 114009 (2008)
28. B. Z. Kopeliovich, proc. of the workshop Hirschegg 95: Dynamical Properties of Hadrons in Nuclear Matter, Hirschegg January 16-21, 1995, ed. by H. Feldmeyer and W. Nörenberg, Darmstadt, 1995, p. 102 [hep-ph/9609385].
29. B. Z. Kopeliovich, J. Raufeisen, A. V. Tarasov and M. B. Johnson, Phys. Rev. C 67, 014903 (2003).
30. J. Raufeisen, J.-C. Peng and G. C. Nayak, Phys. Rev. D 66, 034024 (2002); M. B. Johnson, B. Z. Kopeliovich, M. J. Leitch, P. L. McGaughey, J. M. Moss, I. K. Potashnikova and I. Schmidt, Phys. Rev. C 75, 035206 (2007); M. B. Johnson, B. Z. Kopeliovich and I. Schmidt, Phys. Rev. C 75, 064905 (2007).
31. M. A. Betemps, M. B. Gay Ducati and M. V. T. Machado, Phys. Rev. D 66, 014018 (2002); M. A. Betemps, M. B. G. Ducati, M. V. T. Machado and J. Raufeisen, Phys. Rev. D 67, 114005 (2003); M. A. Betemps, M. B. Gay Ducati and E. G. de Oliveira, Phys. Rev. D 74, 094010 (2006); M. B. Gay Ducati and E. G. de Oliveira, Phys. Rev. D 81, 054015 (2010); M. B. G. Ducati, M. T. Griep and M. V. T. Machado, Phys. Rev. D 89, no. 3, 034022 (2014).
32. E. Basso, V. P. Goncalves, J. Nemchik, R. Pasechnik and M. Sumbera, Phys. Rev. D 93, no. 3, 034023 (2016).
33. R. Pasechnik, B. Kopeliovich and I. Potashnikova, Phys. Rev. D 86, 114039 (2012).
34. B. Z. Kopeliovich and B. Povh, Z. Phys. A 356, 467 (1997).
35. T. Aaltonen et al. [CDF Collaboration], Phys. Rev. D 82, 112004 (2010).
36. A. Donnachie, P. V. Landshoff, Nucl. Phys. B303, 634 (1988).
37. J. C. Collins, L. Frankfurth, M. Strikman, Phys. Lett. B307, 161-168 (1993) [hep-ph/9212212].
38. J. C. Collins, Phys. Rev. D57, 3051-3056 (1998) [hep-ph/9709499].
39. B. Z. Kopeliovich, I. K. Potashnikova, I. Schmidt and A. V. Tarasov, Phys. Rev. D 76, 034019 (2007).
40. K. Wijesooriya, P. E. Reimer and R. J. Holt, Phys. Rev. C 72, 065203 (2005) [nucl-ex/0509012].
41. B. Z. Kopeliovich, J. Nemchik, I. K. Potashnikova, M. B. Johnson and I. Schmidt, Phys. Rev. C 72, 054606 (2005) [hep-ph/0501260].
42. M. H. L. S. Wang, M. C. Berisso, D. C. Christian, J. Felix, A. Gara, E. Gottschalk, G. Gutierrez and E. P. Hartouni et al., Phys. Rev. Lett. 87, 082002 (2001).
43. T. Affolder et al. [CDF Collaboration], Phys. Rev. Lett. 84, 232 (2000).
44. R. Pasechnik, B. Z. Kopeliovich and I. K. Potashnikova, Phys. Rev. D 92, no. 9, 094014 (2015).
45. S. J. Brodsky, B. Kopeliovich, I. Schmidt and J. Soffer, Phys. Rev. D 73, 113005 (2006).
46. S. J. Brodsky, A. S. Goldhaber, B. Z. Kopeliovich and I. Schmidt, Nucl. Phys. B 807, 334 (2009).
47. M. Franz, M. V. Polyakov and K. Goeke, Phys. Rev. D 62, 074024 (2000).