Comment on “On the nature of magnetic stripes in cuprate superconductors,” by H. Jacobsen et al., arXiv:1704.08528v2

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Abstract
Dynamics reduces the orthorhombicity of magnetic stripes in $La_2CuO_{4+y}$. The measured stripe incommensuration can be used to determine the oxygen content of the sample.
With elastic and inelastic neutron-scattering experiments on $La_2CuO_{4+y}$ Jacobsen et al. have provided a valuable contribution to clarify the longstanding puzzle of static vs. dynamic stripes in the pseudogap state of cuprate superconductors. The authors’ main finding is a discrepancy between the incommensuration $\tilde{\delta}_j(\epsilon)$ of dynamic magnetic density waves (MDWs), extrapolated to vanishing energy $\epsilon$, and the value $\tilde{\delta}_j$ from static MDWs,

$$\tilde{\delta}_j(\epsilon \to 0) \equiv \tilde{\delta}_j^0 \neq \tilde{\delta}_j \quad (j = h, k).$$

(1)

Here $h$ and $k$ label the components of unidirectional MDWs, approximately along the $Cu-O$ bonds in the $CuO_2$ plane, with respect to the orthorhombic $a$ and $b$ axes. The values are listed in Table I, reproduced from Ref. 1 (supplementary material). The purpose of this comment is twofold: (1) To show that a modified display of the data, Table II, provides more insight into similarities and differences of dynamic vs. static MDWs and qualitatively suggests a plausible explanation. (2) To determine the level of super-oxygenation $y$ from the measured (average) incommensuration $\delta_{hk}$.

| Neutron scattering | Peak position | MDW mode | $\delta_h$ (r.l.u.) | $\delta_k$ (r.l.u.) |
|--------------------|---------------|----------|---------------------|---------------------|
| Elastic (010)      | static        | 0.1233(3)| 0.0950(5)           |
| Inelastic (010)    | static        | 0.1038(15)| 0.1147(8)           |
| Elastic (100)      | static        | 0.1102(2)| 0.1237(3)           |
| Inelastic (100)    | dynamic       | 0.1093(2)| 0.1173(3)           |

TABLE I: Incommensuration $\tilde{\delta}_j$ and $\tilde{\delta}_j^0$ ($j = h, k$) of static and dynamic magnetic density waves, respectively, in $La_2CuO_{4+y}$, expressed in orthorhombic coordinates. Error bars are denoted in parentheses.

| Scattering (Peak) | Average $\delta_{hk}$ | Deviation $\Delta \delta_h$ | Deviation $\Delta \delta_k$ | Orthorhombicity $\Omega$ |
|-------------------|------------------------|-----------------------------|-----------------------------|--------------------------|
| Elastic (010)     | $\delta_h$ , $\delta_k$ = 0.1102 | - 0.0135                  | + 0.0135                   | 0.12                      |
| Inelastic (010)   | $\delta_h$ , $\delta_k$ = 0.1093 | - 0.0055                   | + 0.0055                   | 0.05                      |
| Elastic (100)     | $\delta_h$ , $\delta_k$ = 0.1092 | + 0.0141                  | - 0.0141                   | 0.13                      |

TABLE II: Incommensuration $\tilde{\delta}_j$ and $\tilde{\delta}_j^0$ of Table I expressed in terms of their average, $\delta_{hk} \equiv (\delta_h + \delta_k)/2$, deviation from the average, $\delta_j = \delta_{hk} + \Delta \delta_j$ ($j = h, k$), and orthorhombicity $\Omega \equiv |\Delta \delta/\delta_{hk}|$. 

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1. Magnetic density waves. The cross components of the incommensurations at the symmetry-related elastic peaks in Table I are found so close to be essentially equal, $\delta_h(100) \simeq \delta_k(010) \simeq 0.0959(5)$, and vice versa, $\delta_k(100) \simeq \delta_h(010) \simeq 0.1235(3)$. In contrast, a clear difference exists between the static and extrapolated dynamic values, $|\delta_h - \tilde{\delta}_h| = 0.07$ and $|\delta_k - \tilde{\delta}_k| = 0.09$, taken at the (100) peak, with the static values bracketing the dynamic ones. This is the surprising result of Ref. 1.

Using the average, $\delta_{hk} \equiv (\delta_h + \delta_k)/2$, and the deviation from the average, $\Delta \delta_j$ ($j = h, k$), the same data are displayed in Table II, to be viewed as $\delta_j = \delta_{hk} + \Delta \delta_j$. It now becomes obvious that the static and dynamic averages are essentially equal, $\overline{\delta_{hk}} \simeq \overline{\tilde{\delta}_{hk}} \simeq 0.1095$, suggesting a commonality of static and dynamic MDWs. The average $\delta_{hk}$ would be the tetragonal approximation of the orthorhombic incommensurations. The deviations from $\delta_{hk}$ mark their orthorhombicity, $\Omega \equiv |\Delta \delta/\delta_{hk}|$, being considerably larger for static MDWs than for dynamic ones, $\overline{\Omega}/\overline{\Omega^0} \simeq 2.5$.

Why have the dynamic incommensurations $\tilde{\delta}_{j}^0$ less orthorhombicity? A static MDW can be regarded a (magnetic) superlattice of lattice constants $D_j = 1/\delta_j$. A dynamic MDW, in contrast, can be considered a standing wave of oscillating magnetic dipoles with wavelength components $\tilde{\lambda}_{j}^0 = 1/\tilde{\delta}_{j}^0$. Generally, the motional aspect of dynamics promotes isotropy (here, in the CuO$_2$ plane). The oscillations of the dynamic MDWs then tend to tetragonalize their orthorhombic wavelengths $\tilde{\lambda}_{j}^0$, preventing a relaxation of the magnetic dipoles to a (static) superlattice of larger orthorhombicity. This notion is corroborated by still less orthorhombicity of dynamic MDWs with higher energy $\epsilon$—albeit to a much lesser degree, $\Delta \tilde{\Omega}/\Delta \epsilon = -0.007$/meV (based on Fig. 3 of Ref. 1).

2. Oxygen content. A frequent problem with oxygen-enriched cuprates is uncertainty about the exact level of super-oxygenation $y$. In many cases samples are characterized by the superconducting transition temperature $T_c$ instead of the value of $y$. The sample used in Ref. 1 has $T_c \simeq 40$ K, similar to the sample used by Lee et al. A thermogravitimetric analysis of the latter sample gave an estimate of oxygen enrichment $y = 0.12 \pm 0.01$. The main quantity of interest is, of course, the hole doping level $p$ (per Cu atom in the CuO$_2$ plane) caused by super-oxygenation $y$. No such problem occurs in the much-studied companion lanthanum cuprates $La_{2-x}Ae_xCuO_4$ ($Ae = Sr, Ba$) that are hole-doped through infravalent cation doping $x$ with a hole doping level $p = x$. In the latter materials MDWs
and charge-density waves (CDWs) appear together, called “stripes.” Their incommensuration, \( \delta(x) \propto \sqrt{x - x_0^N} \), depends on the cation doping \( x \), diminished by the Néel point \( x_0^N = 0.02 \) (collapse of 3D antiferromagnetism at \( T = 0 \)).

It is tempting to extend the incommensuration formula from the cation-doped to the oxygen-enriched \( La_2CuO_4 \) compounds. Assuming that each enriching oxygen atom in \( La_2CuO_{4+y} \) gives rise to two doped holes, \( p = 2y \), the formula for MDWs, expressed in terms of oxygen enrichment and orthorhombic coordinates (but in tetragonal approximation) becomes,

\[
\delta_{hk}(y) = \frac{1}{4}\sqrt{2y - x_0^N}.
\]

Solving for \( y = 8\delta_{hk}^2 + x_0^N/2 \) and using the average incommensuration \( \delta_{hk} = 0.1095 \) r.l.u. gives \( y = 0.105 \pm 0.005 \), comparable with the estimate by Lee et al., \( y = 0.12 \pm 0.01 \).

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