Online Algorithms for Dynamic Matching Markets in Power Distribution Systems

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Abstract—In this paper we address the problem of designing online algorithms for dynamic matching markets in distribution systems whose objective is to maximize social welfare while being effective in integrating renewable energy by leveraging load flexibility. With the intuition that the performance of any online algorithm would worsen with increasing randomness, we propose two indicators for measuring the effectiveness of an online algorithm. First one is convergence to optimality (CO) as the randomness goes to zero. The second one focuses on the deviation from optimality measured as a function of the standard deviation, \( \sigma \), of the underlying randomness: renewable generation and customer loads. We take into account the fact that a customer’s value decreases with delay in load servicing.

We present a pair of online matching algorithms for the following generation-consumption conditions: (i) when the mean of renewable generation (\( \mu_r \)) is greater than the mean of the number of customers (\( \mu_c \)) (assumed to be unit demand), and (ii) when the condition (i) is reversed. The online algorithm we present for the first case satisfies CO with a deviation that varies as \( \sim O(\sigma) \). But the same algorithm fails to satisfy CO for the second case. We then present an extension of this algorithm and show that the modified algorithm satisfies CO for the second case with a deviation that varies as \( \sim O(\sigma) \) plus an offset that is \( O(\mu_r - \mu_c) \). Thus, there are two distinct regimes in the set of all possible generation-consumption conditions, with the platform requiring a distinct algorithm for each regime to be effective in terms of the indicators described above.

I. INTRODUCTION

Electric power grid is undergoing a major transformation driven, to a significant extent, by the imperative of decarbonization of the energy system for global warming mitigation. A major approach to achieve this goal is through large scale integration of renewable energy sources (RES). Renewable generation in power grid is usually a mix of utility scale centralized or distributed wind and solar generation. Integration of RES in the management and control of the grid is a significant challenge because PV solar and wind are highly uncertain, inherently variable, and largely uncontrollable. The information and decision complexity of managing the distributed resources renders the conventional top-down centralized approach of dispatching resources impractical.

Market platforms in distribution systems facilitate decentralized management and control and can provide an effective solution for managing such distributed RES. Essentially, such platforms can leverage the flexibility of loads to manage the variability of RES locally. This can allow the grid to be locally self-sufficient and resilient reducing the dependence on large centralized fossil fuel based generators. Managing such platforms is challenging because the future load arrivals and renewable generation are uncertain. Specifically, scheduling is a challenging problem because it depends on the load arrivals and generation over the entire duration of which only partial information is available at any point of time.

The principal objective of this work is to design an online algorithm for a dynamic matching market in distribution systems, whose objective is to maximize social welfare subject to servicing customers’ load reliably, and being effective in integrating renewable energy. A central question is: how do we measure the effectiveness of such an algorithm in utilizing the flexibility in loads to integrate renewable generation? The metric that is typically used to measure the relative performance of an online algorithm is the so-called competitive ratio (CR), i.e., the ratio of the expected social welfare of the algorithm and the expected social welfare of the oracle optimal algorithm. We expect that any online algorithm will fall short of the social welfare that the oracle optimal achieves in proportion to the extent of the randomness in renewable generation and load variability. With this as a working hypothesis, we propose the following measures (defined mathematically later in the paper): (i) convergence to optimality, i.e., convergence of CR to one as the randomness goes to zero, and (ii) deviation from optimality, where the deviation is measured as a function of the standard deviation of the distribution of renewable supply and the number of customers who arrive on the platform.

A. Related Work

Online matching has been extensively studied both in adversarial and stochastic setting [1]–[6]. These works provide algorithms that achieve a lower bound of \( (1 - 1/e) \) for the CR for any distribution or for any scenario. In this sense the algorithms are robust. In [7], the authors studied the online market clearing setting for a general commodity market without service constraints and provided algorithms with robust lower bounds. In contrast to these works, we design an online algorithm and provide a lower bound for its performance that is instance based rather than a single value that applies to all scenarios. This allows us to characterize the variation of an algorithm’s performance with the variation in the randomness.

There is a large body of work related to load flexibility and DR in the energy systems related literature. We refer the
reader to [8]–[20] for a sampling of this literature. Among these, there are a couple of papers that are closer to the setting we consider. Bitar et al. [19] and Nayyar et al. [20], propose a forward market for deadline or duration differentiated service. They show that there exists a competitive equilibrium with a price that is deadline or duration differentiated respectively. These works assume that all the customers arrive at the market at the initial time and that the customers are a continuum. In contrast the setting we consider is an online setting and the customers are not a continuum.

B. Our Contribution

Our primary technical contribution in this paper lies in new theoretical results on the design and performance of online algorithms for dynamic matching markets in distribution systems. Our algorithms do not require centralized control. The key contribution in this paper is a pair of online algorithms that are suitable for two distinct generation-consumption conditions. These conditions are: (i) the mean of renewable generation is greater than the mean of the number of customers (assumed to be unit demand) and (ii) when the inequality in (i) is reversed. Our online algorithm for the the first condition is shown to satisfy convergence to optimality and we also provide a lower bound for deviation from optimality. We also show that this algorithm is ineffective for the second condition. We then propose a modified algorithm for this case and provide results for its convergence and deviation properties.

II. Generation and Consumption Models

The setting described in this paper is motivated by the setting for distribution market platforms such as electric vehicle charging platforms. In such platforms, the electric vehicles function as the flexible loads. The arrival process of the vehicles is stochastic and the vehicles have varying service windows. The price that the vehicle is willing to pay might also decrease with delay depending on the criticality of the electric charge that the vehicle needs. The generation from the local renewable sources are by nature stochastic. The setting we consider in our paper is modeled after this setting.

The matching algorithm for distribution platforms can leverage the local weather forecast, for eg., whether the day will be sunny or cloudy. It can also learn the distribution of the arrival process of the loads and the distribution of renewable generation for different weather conditions by observing these variables over a period of time. The forecast and the learned distributions can be leveraged to improve the timing of the scheduling of the flexible loads. Hence, we assume in our setting that the forecast of the weather and the knowledge of the stochastic process that governs the load arrival process and renewable generation are available to the platform.

A. Supply Model

We consider two sources of supply for the dynamic matching market platform: 1) upstream grid supply, and 2) distributed renewable energy sources (D-RES) in the distribution network. We assume that upstream grid supply, shown by $p_t$, is sufficiently large and that it is priced at $c$ $$/kWh.

The D-RES, such as PV solar and wind generation, are by nature variable and uncertain, and their availability depends on weather, e.g., solar irradiance, wind speed, etc. Let us denote the D-RES generated at time $t$ by $S_t$, which is governed by a discrete-time stochastic process. We assume that the process $S_t$ conditioned on the weather is independent and identically distributed (i.i.d). We denote the mean and standard deviation of $S_t$ conditioned on $w$ by $\mu_{S,t} =: \mathbb{E}[S_t|w]$, $\sigma_{S,t} =: \mathbb{E}[(S_t - \mu_{S,t})^2|w]$, where $S_t$ is bounded by a constant $\bar{S}$. We assume that the weather condition $w$ is available as a forecast.

B. Flexible Demand Model

Let us denote the number of customers who arrive at the platform at time $t$ by an independently and identically distributed stochastic process $n_t$, which is bounded by a constant $\mathbb{N}$. The mean and standard deviation of $n_t$ are respectively denoted by $\mu_{n,t} =: \mathbb{E}[n_t]$, $\sigma_{n,t} =: \mathbb{E}[(n_t - \mu_{n,t})^2]$. Denote the set of customers who arrive at the platform by $\mathcal{K}$. Each customer $k \in \mathcal{K}$ is characterized by three parameters $\{a^k, d^k, b^k\}$, where $a^k$ is the arrival time of the customer, and $d^k$ is the specified deadline time to serve the customer. The parameter $b^k$ is the criticality of customer $k$, which represents the rate at which a customer’s willingness to pay decreases over time. The heterogeneity of customers lie in the differing deadlines and criticality. When customer $k$ arrives in the platform it reports its service deadline $d^k$ and the value $b^k$. This paper assumes that the customers report truthfully on arrival. The utility function of a customer, shown by $\pi^k_t$, represents the customer’s willingness to pay for power, and is defined as follows:

$$\pi^k_t = \begin{cases} c - b^k(t - a^k) & \text{if } a^k \leq t < c/b^k + a^k \\ 0 & c/b^k + a^k \leq t \leq d^k \\ -\infty & \text{if } t > d^k \end{cases}$$

The customers utility function for different values of the criticality parameter $b^k$ is illustrated in Fig. [1] In Fig. [1] customers with a positive criticality parameter would be willing to pay less over time, which communicates their preference to get served at the earliest. In Fig. [1], customer’s willingness to pay is less than or equal to the grid supply price $c$ $$/kWh. This is reasonable considering that the grid supply is available at this price at all times. We assume without loss of generality that the customers are unit demand customers.

From now on we drop the subscript $t$ in the moments of the random variables $S_t$ and $n_t$, since they are i.i.d. Also, we denote the combined standard deviation of the number of load arrivals and renewable based generation by
We denote the expectation with respect to all sources of randomness by $E[.]$.

### III. Online Matching Algorithm

In this section, we propose an online algorithm to implement dynamic matching markets in distribution systems. The objective of the proposed online algorithm is to maximize the social welfare of trading energy in the distribution system, subject to serving the customers in the market. Denote the energy allocated to customer $k$ at time $t$ by $q^k_t$, the price that customer pays for $q^k_t$ by $\pi^k_t$ and the unit cost incurred by the platform for providing $q^k_t$ by $c^k_t$. We denote the energy purchased from the grid and energy utilized from the renewable supply at time $t$ by $p_t$ and $s_t$ respectively, where $s_t \leq S_t$. Given these definitions, the social welfare for servicing the customers is defined as the sum of the utility of the customers minus the cost incurred by the market to serve the customers. The social welfare, $W$, is formulated as:

$$W := \sum_{k \in K} (\pi^k_t - c^k_t)q^k_t.$$ 

Thus, the objective of the online algorithm can be formally stated as follows:

$$\max E[W] \text{ s.t. } \sum_k q^k_t = p_t + s_t \forall t. \quad (2)$$

We use $M_o$ to denote an online algorithm for solving the optimization problem (2). The so-called oracle optimal algorithm, denoted by $M_o$, is the solution to the optimization problem assuming it has complete information on the customers’ arrival, their characteristics, and renewable generation. Given this information, the oracle optimal algorithm can compute the optimal matching ahead of time, and achieves the maximum possible social welfare. We use the oracle algorithm as the benchmark for measuring $M_o$’s relative performance, using the metric competitive ratio (CR) defined as follows. Denote the social welfare achieved by the platform’s matching algorithm $M_o$ over the time horizon $T$ by $W[M_o]$ and similarly denote the social welfare achieved by the oracle algorithm by $W[M_o]$. The CR for the matching algorithm $M_o$ is given by:

$$\frac{E[W[M_o]]}{E[W[M_o]]} \text{ (Competitive Ratio (CR))} \quad (3)$$

We propose the following indicators based on the CR for measuring the effectiveness of an algorithm: (i) convergence to optimality (CO) as randomness reduces to zero, (ii) deviation from optimality (DO) measured as a function of combined standard deviation $\sigma$, which are formally defined below.

**Definition 1:** Matching algorithm is said to achieve Convergence to Optimality if the expected welfare $E[W[M_o]]$ converges to $E[W[M_o]]$ (i.e., CR converges to 1) as $\sigma \to 0$.

**Definition 2:** Deviation from Optimality is the function $D(\sigma)$ such that:

$$\frac{E[W[M_o]]}{E[W[M_o]]} \geq 1 - D(\sigma). \quad (4)$$

In particular we are interested in determining an upper bound to $D$ of the form $\sigma^r$. If $D \leq O(\sigma^r)$ then we say $r$ is the convergence rate. The notation $O(.)$ denotes that the term that accompanies the argument as a factor is a constant and does not scale with the problem’s time horizon $T$. We say that the rate of deviation is linear if $r = 1$. We note that convergence is only a necessary property for being effective in managing the uncertainty in generation and loads. Deviation from optimality is a more well rounded measure as it describes the variation in the competitive ratio as the randomness varies.

In the following, we present a pair of online matching algorithms for the following generation-consumption conditions: (i) $\mu_n < \mu_s$ and (ii) $\mu_n \geq \mu_s$. In presenting the results for these scenarios we assume that the weather condition or $w$ does not change. We also assume that the customers are not strategic and $b^k$ is identical across customers, i.e., the customers have identical criticality. The online algorithm we present for the first case, $\mu_n < \mu_s$, satisfies the convergence property and a deviation that varies linearly with $\sigma$. We find that this same algorithm does not satisfy the convergence property for the second case where $\mu_n \geq \mu_s$. Hence, for the second case, we present a slightly modified version of the previous algorithm. We show that this modified algorithm satisfies the convergence property and achieves a deviation that varies linearly with $\sigma$ but with an offset that is $O(\mu_n - \mu_s)$. We note that when $\mu_n - \mu_s$ is very small, the rate of deviation for all practical purposes is linear.

#### A. Online Algorithm for the Case $\mu_n < \mu_s$

We call the online algorithm we present for this case by $M_1$. This algorithm matches the load for which customers are willing to pay the highest among the currently active customers to the available renewable supply. Any remaining load with an immediate deadline is matched to the grid supply. Theorem 1 below describes the properties of algorithm $M_1$.

**Theorem 1:** When $\mu_n < \mu_s$, the online algorithm $M_1$ satisfies:

$$\frac{E[W[M_o]]}{E[W[M_1]]} \geq 1 - O\left(\sqrt{\sigma_n^2 + \sigma_s^2}\right) \quad (5)$$

From the lower bound it follows that the algorithm satisfies CO with a deviation that varies linearly with $\sigma$.

Proof of Theorem 1 is provided in the Appendix. Let $\delta_{d_m}$ denote the minimum possible service window. From the exact form of the deviation function $D$ for the lower bound derived in the previous theorem (please see Appendix) the following corollary trivially follows.

**Corollary 1:** For the online algorithm in Thm[2] we get,

$$\lim_{\delta_{d_m} \to \infty} \frac{E[W[M_1]]}{E[W[M_o]]} = 1$$

Algorithm $M_1$ is a “greedy” algorithm as it tries to maximize the welfare it can gain at the current time by matching the highest valued customers among those that are active to the renewable supply generated at the current time. We note that the algorithm does not match any of the remaining customers, unless they have an immediate deadline, and they remain as active. This is done with the expectation that the algorithm would be able to find adequate renewable supply in the future, which is the cheaper source of energy supply. In fact, the online algorithm will achieve the optimal welfare if it is able to find renewable supply to service any waiting customer. But that would not be the case for every instance of renewable
generation and customer arrivals. So the social welfare attained by the algorithm can end up deviating from the optimal welfare that the oracle optimal achieves in certain instances.

What we have shown is that the deviation from the oracle optimal is at least $O(\sigma)$. Hence, the rate at which the deviation varies, i.e., $r = 1$. This suggests that $r = 1$ is achievable when $\mu_n < \mu_s$. The lower bound in Theorem 1 also reveals that when customers who arrive at the market have sufficiently large service window (note that $\delta d_m$ is minimum of service window of all the customers to arrive on the platform), the algorithm achieves a social welfare near to the optimal welfare. This is reasonable to expect because under this condition the platform should be able to find renewable supply to service a waiting customer at some point of time in the future.

The results discussed in this section lead to the question: is there an algorithm that achieves a deviation that varies at a faster rate? This is currently an open question. We conjecture that a rate that is strictly greater than linear is not achievable.

B. Online Algorithm for the Case, $\mu_n \geq \mu_s$

We start with a brief argument for why algorithm $M_1$ fails to satisfy CO for this case. Algorithm $M_1$ waits to serve a customer until the renewable generation is available to supply the customer. When $\mu_n > \mu_s$, the total amount of renewable energy generated over a large duration of time would fall short of the number of customers active during this period. Thus, in this case, algorithm $M_1$ would fail, with a high probability, to find renewable supply for certain customers. It is straightforward to show that this probability approaches to one as the randomness goes to zero. Consequently, algorithm $M_1$ would incur a net loss relative to the optimal welfare with probability one as the randomness goes to zero. We present the properties of algorithm $M_1$ for this case formally as a lemma. The proof is similar to the previous theorem.

Lemma 1: When $\mu_n \geq \mu_s$, algorithm $M_1$ fails to satisfy CO and,

$$\frac{E[W|M_o]}{E[W|M_n]} \geq 1 - O(\mu_n - \mu_s) - O\left(\frac{\sigma^2}{\sigma_n^2} + \frac{\sigma^2}{\sigma_s^2}\right) \tag{6}$$

Here we modify algorithm $M_1$, and develop Algorithm $M_2$ for the case when $\mu_n \geq \mu_s$. Algorithm $M_2$ is the following: do the same steps as in $M_1$. In addition, match up to $\mu_n - \mu_s$ of the remaining customers that just arrived to the grid supply, starting from earliest deadline first. This additional commitment on arrival ensures that the algorithm trivially satisfies CO. We present the properties of this algorithm as Theorem 2 below.

Theorem 2: When $\mu_n \geq \mu_s$, the online algorithm (M2) satisfies CO and,

$$\frac{E[W|M_o]}{E[W|M_n]} \geq 1 - O(\mu_n - \mu_s) - O\left(\frac{\sigma^2}{\sigma_n^2} + \frac{\sigma^2}{\sigma_s^2}\right).$$

Proof of Theorem 2 is provided in the Appendix. The main feature of algorithm $M_2$ is that it matches an additional set of customers that just arrived to the grid supply. The additional commitment on arrival ensures that the platform services certain customers earlier for which it could have failed to find renewable supply to service at a later time. This ensures that the algorithm satisfies convergence to optimality. From Theorem 1 it follows that the upper bound to the deviation from optimal welfare varies linearly with $\sigma$ but there is an offset, that is $O(\mu_n - \mu_s)$. We note that the rate of deviation is for all practical purposes linear when $\mu_n - \mu_s$ is very small. We also note that the general rate of deviation of this algorithm is an open question.

IV. CASE STUDIES

We provide an example for each generation condition here. In this example, the number of customers who arrive on the platform at any point of time is not random, and $n_t = 3$. Renewable supply $S_t$ is uniformly distributed over the set \{0, 1, 2, ..., 10\}. From the distribution we get that, $\mu_s = 5$, $\sigma_s = 3.16$, $n = 10$, $\sigma_{m} = 2.45$, and,

$$\text{P}(S_t < n_t) = \text{P}(S_t < 3) = \frac{3}{11}$$

The unit cost of grid supply, $c = 1$, criticality of a customer, $b = 0.5$ and $\delta d_m = 2$. Using expression for the lower bound from the proof of Theorem 1 we get the following lower bound for algorithm $M_1$ in this case:

$$\frac{E[W|M_o]}{E[W|M_n]} \geq 0.76 \tag{7}$$

In the next example we consider the second generation condition. In this example, $n_t = n_m = 6$ for all times. Renewable supply $S$ is uniformly distributed over the set \{0, 1, 2, ..., 10\}. From the distribution it follows that $\mu_s = 5$, $\sigma_s = 3.16$, $n = 10$, and $\sigma_{m} = 4.1$. The unit cost of grid supply, $c = 1$, criticality of a customer, $b = 0.5$ and $\delta d_m = 2$. For the condition described here, numerically we found that,

$$\text{P}(S_t < n_t) \approx 0.5, \text{P}\left\{\sum_{i=1}^{\delta d_m+1} S_t < \sum_{i=1}^{\delta d_m+1} n_t\right\} \approx 0.5$$

Using these numbers in the lower bound for CR derived in Thm. 2 we get the following lower bound for algorithm $M_2$ in this case:

$$\frac{E[W|M_o]}{E[W|M_n]} \geq 1 - \frac{0.5}{4.1} - \frac{3.16 \times 0.5}{4.1} \approx 0.5.$$
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VII. APPENDIX

VIII. PROOF OF THEOREM [1]

In the proofs we use customers in place of loads because a single customer is equivalent to a unit of load. Let \( W_{rs} \) be the welfare generated by the algorithm from matching loads to the renewable supply and \( W_{gs} \) be the welfare generated from matching loads to the GS. The proof entails the following steps.

Step (i): We show that
\[
W[\pi_{M_{\sigma}}] = W_{rs} + W_{gs} \geq W[\pi_{M_0}] + W_{gs}
\]
where \( W_{gs} \) is the amount that the grid pays to the platform. Firstly, the distribution platform cannot increase welfare by changing the matching of a customer who has been matched to renewable supply only if \( (s_d - c) \). Consider all such customers who are matched to the GS. Denote the set of such customers to be \( \Theta_{gs} \). Then using (O-1) it follows that,
\[
W[\pi_{M_{\sigma}}] = W_{rs} + \sum_{\theta \in \Theta_{gs}} (\pi_{\theta} - c) \geq W[\pi_{M_0}] - \sum_{\theta \in \Theta_{gs}} c = W[\pi_{M_0}] + W_{gs}
\]

Step (ii): We show that \( E[W_{gs}] \geq -O(cT \sqrt{\sigma d + \sigma s}) \). By the definition of the algorithm a customer is matched to grid supply only when no renewable supply is left over after matching to the customers who will pay higher in its service window. Thus, a customer who arrives at time \( t \) is not matched to renewable supply by its deadline only if
\[
\sum_{l=t}^{\delta_{d}+t} S_l < \sum_{l=t}^{\delta_{m}+t} n_l
\]
where \( n_k \) is the number of customers who arrive at time \( k \). We define two quantities:
\[
I_t = P \left\{ \sum_{l=t}^{\delta_{d}+t} S_l < \sum_{l=t}^{\delta_{m}+t} n_l \right\}, P_t = P \left\{ \sum_{l=t}^{\delta_{d}+t} S_l < \sum_{l=t}^{\delta_{m}+t} n_l \right\}
\]
This implies that the amount that the grid pays to platform at time \( t' = \delta d_m + t \), is lower bounded by,
\[
W_{gs}(t') \geq c(S_t - n_t) I_t
\]
This implies that,
\[
E[W_{gs}(t')] \geq c E[ (S_t - n_t) I_t] \geq c E[ (S_t - \mu_s) - (n_t - \mu_n) + (\mu_s - \mu_n)] I_t | S_t < n_t | I_t
\]
Since \( (\mu_s - \mu_n) > 0 \) we get that,
\[
E[W_{gs}(t')] \geq c E[ (S_t - \mu_s) - (n_t - \mu_n)] I_t | S_t < n_t | I_t
\]
Using Cauchy Schwartz inequality we get that,
\[
E[W_{gs}(t')] \geq -c (\sqrt{\sigma d^2 + \sigma s^2}) \sqrt{E[S_t < n_t]} \sqrt{P_t}
\]
The probability factor in the above equation can be simplified further:
\[
P_t = P \left\{ \sum_{l=t}^{\delta_{d}+t} S_l - \sum_{k=t}^{\delta_{m}+t} n_l < 0 \right\}
\]
Using Hoeffding’s inequality we get that,

\[ P_t \leq \exp \left\{ -\frac{2(\mu - \mu_n)^2(\delta d_m + 1)}{(\pi + \tilde{S})^2} \right\} = e \quad (10) \]

This implies that,

\[ \mathbb{E}[\tilde{W}_{gs}(t')] \geq -c \left( \sqrt{\sigma_n^2 + \sigma_s^2} \right) e \]

Hence,

\[ \mathbb{E}[\tilde{W}_{gs}] = \sum_{t=1}^{T} \mathbb{E}[\tilde{W}_{gs}(t)] \geq -cT \left( \sqrt{\sigma_n^2 + \sigma_s^2} \right) e \quad (11) \]

This completes Step 2. From the definition of CR it follows that

\[ \mathbb{E}[W[M_\sigma]] = \mathbb{E}[W_{rs}] + \mathbb{E}[W_{gs}] \geq \mathbb{E}[W[M_o]] + \mathbb{E}[\tilde{W}_{gs}] \]

\[ \geq 1 - \frac{\sigma}{\mu_n} \mathbb{E}[W[M_o]] \quad (12) \]

Let us lower bound \( \mathbb{E}[W[M_o]] \):

\[ \mathbb{E}[W[M_o]] \geq \mathbb{E}\left[ \sum_{t=1}^{T} c \min\{n_t, S_t\} \right] = n_{sm} cT \quad (13) \]

Where \( \mathbb{E}\min\{n_t, S_t\} = n_{sm} \). This implies,

\[ \mathbb{E}[W[M_o]] \geq 1 - \frac{cT \left( \sqrt{\sigma_n^2 + \sigma_s^2} \right) e}{n_{sm} cT} = 1 - \frac{\left( \sqrt{\sigma_n^2 + \sigma_s^2} \right) e}{n_{sm}} \quad (14) \]

The property CO follows trivially from the lower bound derived above. ■

**IX. PROOF OF THEOREM 2**

Let \( W_{rs} \) be the welfare generated by the algorithm from matching loads to the renewable supply and \( W_{gs} \) the welfare generated from matching loads to the GS. Similar to the steps in the proof of Theorem 1 we get,

\[ W[M_\sigma] = W_{rs} + \sum_{\theta \in \Theta_{gs}} (\pi_d - c) \geq W[M_o] - \sum_{\theta \in \Theta_{gs}} c \]

\[ = W[M_o] + \tilde{W}_{gs} \quad (15) \]

In this case, \( \tilde{W}_{gs} \) can be divided into two parts. One part corresponds to the payment made by the grid when the customer is matched on arrival to the GS, \( \tilde{W}_{gs1} \). The other part corresponds to the payment made by the grid for the customers who are matched to GS later than their arrival time, \( \tilde{W}_{gs2} \). It follows that

\[ \mathbb{E}[W[M_\sigma]] \geq \mathbb{E}[W[M_o]] + \mathbb{E}[\tilde{W}_{gs1}(t)] + \mathbb{E}[\tilde{W}_{gs2}(t)] \]

Note that a customer is matched on arrival at \( t \) when \( S_t < n_t \). And up to \( \mu_n - \mu_s \) are matched. This implies,

\[ \mathbb{E}[\tilde{W}_{gs1}(t)] \geq -c\mathbb{E}[\sigma_n - \sigma_s] \mathbb{P}\{S_t < n_t\} \]

That is,

\[ \mathbb{E}[\tilde{W}_{gs1}(t)] \geq -c(\mu_n - \mu_s) \mathbb{P}\{S_t < n_t\} \]

The lower bound for \( \mathbb{E}[\tilde{W}_{gs2}(t)] \): If a customer is matched later than its arrival time \( t \) to the GS then it should be that \( S_t < n_t - (\mu_n - \mu_s) \). And only up to \( \tilde{n}_t = \tilde{n}_t - (\mu_n - \mu_s) \) number of customers of the customers who arrive at \( t \) can be matched later than \( t \) to the GS. In addition, the customer is matched to GS at a time later than its arrival time only at its deadline. Thus, if the customer does get matched to the GS later than its arrival time then it is necessary that the cumulative sum of the renewable supply generated from its arrival time up to its deadline is insufficient to service the customers who arrive at the platform during this period i.e. \( \sum_{t=1}^{\delta_d + t} S_t < \sum_{t=1}^{\delta_d + t} n_t \). Thus, it follows that

\[ \mathbb{E}[\tilde{W}_{gs2}(t)] \geq c \mathbb{E}\left[ \left( \tilde{S}_t - \tilde{n}_t \right) \mathbb{I}\{\tilde{S}_t < \tilde{n}_t\} I_t \right], \]

where \( \tilde{S}_t = S_t - \mu_s \) and \( \tilde{n}_t = n_t - \mu_n \). Then, from Cauchy Schwartz inequality we get that,

\[ \mathbb{E}[\tilde{W}_{gs2}(t)] \geq -c \left( \sqrt{\sigma_n^2 + \sigma_s^2} \right) \sqrt{\mathbb{P}\{\tilde{S}_t < \tilde{n}_t\} \sqrt{P_t}} \quad (16) \]

Combining the expression for the lower bound of \( \mathbb{E}[\tilde{W}_{gs1}] \) and \( \mathbb{E}[\tilde{W}_{gs2}] \) we get,

\[ \mathbb{E}[W[M_\sigma]] \geq \mathbb{E}[W[M_o]] - c \sum_{t=1}^{T} (\mu_n - \mu_s) \mathbb{P}\{S_t < n_t\} \]

\[ - c \sum_{t=1}^{T} \left( \sqrt{\sigma_n^2 + \sigma_s^2} \right) \sqrt{P_t} \quad (17) \]

Following steps similar to the steps in the proof of Theorem 1 we get that,

\[ \frac{\mathbb{E}[W[M_\sigma]]}{\mathbb{E}[W[M_o]]} \geq 1 - \tilde{c}_1 (\mu_n - \mu_s) - \tilde{c}_2 \left( \sqrt{\sigma_n^2 + \sigma_s^2} \right), \]

where,

\[ \tilde{c}_1 = \frac{\mathbb{P}\{S_t < n_t\}}{n_{sm}}, \tilde{c}_2 = \frac{\sqrt{\mathbb{P}\{\tilde{S}_t < \tilde{n}_t\} \sqrt{P_t}}}{n_{sm}}. \]

CO follows trivially from the definition of the algorithm. ■