Parallel Photonic Quantum Computation Assisted by Quantum Dots in One-Side Optical Microcavities

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Universal quantum logic gates are important elements for a quantum computer. In contrast to previous constructions on one degree of freedom (DOF) of quantum systems, we investigate the possibility of parallel quantum computations dependent on two DOFs of photon systems. We construct deterministic hyper-controlled-not (hyper-CNOT) gates operating on the spatial-mode and the polarization DOFs of two-photon or one-photon systems by exploring the giant optical circular birefringence induced by quantum-dot spins in one-sided optical microcavities. These hyper-CNOT gates show that the quantum states of two DOFs can be viewed as independent qubits without requiring auxiliary DOFs in theory. This result can reduce the quantum resources by half for quantum applications with large qubit systems, such as the quantum Shor algorithm.
information processing. With these realizations, one half of the quantum resources may be saved. This saving is very important for quantum applications with large qubit systems, such as the quantum Shor algorithm and quantum communications.

**Results**

The spatial-mode and the polarization DOFs of a photon can be manipulated easily using linear optical elements\(^{10,20,31-33}\). Thus, it is convenient to use these DOFs as independent qubits in quantum applications. To show the independence of two DOFs of the polarization DOF and spatial DOF of each photon for any quantum schemes, it is necessary to show that all quantum transformations in SU(2\(^2\)) may be realized on the two DOFs. Note that the CNOT gate on two qubit states and rotation operations on one qubit are universal quantum logic gates\(^{8-10}\) to simulate any quantum transformation on two qubit states and rotation operations on one qubit. Thus, the CNOT gate on a two-qubit system and the CNOT gate on two DOFs of the photon system, i.e., four CNOT gates on a two-photon system (each DOF of one photon is used) implemented. Because of the different roles played by each DOF of the QD-cavity system, we only need to realize the CNOT gate on two DOFs of the photon system, while rotation operations on one DOF are easily implemented. Because of the different rules played by each DOF of the photon in a CNOT gate, there are six different forms of one CNOT gate on two DOFs of the photon system, i.e., four CNOT gates on a two-photon system (each DOF of one photon is used) and two CNOT gates on a one photon system. None of these gates require changing these DOFs during the transformations, nor do they require any auxiliary DOFs. Thus one DOF of one photon may be regarded as the controlling qubit while the other is the target qubit. Note that the CNOT gate on a two-qubit system and the rotation operations on one-qubit system are universal logic gates for general quantum computation. From our six CNOT gates, all quantum global transformations on the same DOF of multiple photon systems can be realized on different DOFs of these photon systems. In this case, each DOF of the photon systems can be viewed as an independent qubit in quantum applications, which means that one half of quantum resources may be saved.

**Quantum-dot-cavity system.** To complete our CNOT operations, the following optical property\(^{38-46}\) and QD-cavity system are used for our schemes. The QD-cavity system used in our proposal is constructed by a singly charged QD [a self-assembled InGaAs QD or a GaAs interface QD] located in the center of a one-sided optical resonant cavity to achieve the maximal light-matter coupling, as shown in Figure 1. According to the Pauli exclusion principle\(^45\), a negatively charged exciton \(X^-\) consisting of two electrons bound to one hole can be optically excited when an excess electron is injected into QD. For the excess electron-spin state \(|0\rangle\) or \(|1\rangle\), a negatively charged exciton \(|\downarrow\downarrow\rangle\) or \(|\downarrow\uparrow\rangle\) with two electron spins antiparallel\(^46\) is created by resonantly absorbing \(|L\rangle\) [left circularly polarized photon] or \(|R\rangle\) [right circularly polarized photon], respectively. Here, \(|\uparrow\rangle\) and \(|\downarrow\rangle\) describe the heavy-hole spin states \(\pm \frac{3}{2}\) and \(\pm \frac{1}{2}\), respectively.

![Figure 1](https://www.nature.com/scientificreports)

**Spatial-CNOT gate on a two-photon system.** The schematic deterministic CNOT gates on the same DOF of a two-photon system are shown in Figure 2. Here, the circuit shown in Figure 2(a) is designed for the deterministic CNOT gate on the spatial modes of two photons using a one-sided QD-cavity system. The spatial-mode state of the photon \(a\) is the control qubit, while the spatial-mode state of the photon \(b\) is the target qubit. The initial state of the excess electron spin in QD is \(|\downarrow\rangle\) = \(1/\sqrt{2} (|\uparrow\rangle + |\downarrow\rangle)\). The initial state of the photon \(a\) is \(|p\rangle = (|z_2\rangle + |z_1\rangle)\), where \(|z_1\rangle\) and \(|z_2\rangle\) are two spatial modes of the photon \(a\). The detailed circuit is described as follows. First, the photon \(a\) from the spatial mode \(a_1\) passes through the subcircuit \(S1\) [CPBS, \(X\), QD, X, CPBS, sequentially]. From the equation (4), the joint system of the excess electron spin \(e\) and the photon...
Fig. 2 | Schematic hyper-CNOT gate on a two-photon system.
(a) Schematic spatial-CNOT gate operating on the spatial-mode degrees of freedom of a two-photon system simultaneously. CPBS represents a polarizing beam splitter in the circular basis, which transmits |R⟩ and reflects |L⟩. BS represents a 50/50 beam splitter to perform the Hadamard operation on the spatial-mode DOF of a photon. X represents the bit-flip operation X = |R⟩⟨L| + |L⟩⟨R|. W represents the Hadamard operation on the excess electron spin e in QD. (b) Schematic polarization-CNOT gate operating on the polarization degrees of freedom of a two-photon system simultaneously. H represents a half-wave plate (HWP) to perform the Hadamard operation on the polarization DOF of a photon. S1, S2, S3, S4 denote four subcircuits.

\[ \Psi_{ae}^{\prime} = (\gamma_1 |a_1⟩ + \gamma_2 |a_2⟩) \otimes (x_i |R⟩ + x_2 |L⟩) \] (8)

after one W operation being performed on the electron spin e.

Finally, after measuring the excess electron spin e under the basis \{ | + ⟩, | − ⟩ \}, the joint system of the photons a and b collapses into

\[ \Psi_{ab} = (x_i |R⟩ + x_2 |L⟩) \otimes (\beta_1 |R⟩ + \beta_2 |L⟩) \]

by performing the phase-\( Z \) operation \( Z = |a_i⟩⟨a_i| - |a_2⟩⟨a_2| \) on the spatial mode of the photon a for the measurement outcome | + ⟩ of the excess electron spin e. Thus, the deterministic spatial-CNOT gate is completed.

Polarization-CNOT gate on a two-photon system. The schematic deterministic CNOT gate on the polarization DOFs of a two-photon system is shown in Figure 2(b). Here, the polarization state of the photon a is the control qubit, while the polarization state of the photon b is the target qubit. The initial states of the excess electron spin e and the photons a and b are | + ⟩_e, \( |φ⟩_a \) and \( |ψ⟩_b \), respectively, shown in Figure 2(a). In contrast to the step one of the spatial-CNOT gate shown in Figure 2(a), photon a first passes through the subcircuit S3 [CPBS, QD, CPBS, sequentially]. The joint system of the photon a and the electron spin e is changed from \( |φ⟩_{ae} = |φ⟩_a ⊗ | + ⟩_e \) to

\[ \Phi_{ae}^{\prime} = \frac{1}{\sqrt{2}} \left[ | + ⟩_e (\gamma_1 |a_1⟩ + \gamma_2 |a_2⟩) + | - ⟩_e (\gamma_1 |a_1⟩ - \gamma_2 |a_2⟩) \right] \otimes (x_i |R⟩ + x_2 |L⟩) \] (6)

This is the controlled-Z gate \( CZ_{ae} \) on the electron spin e and the spatial-mode state of the photon a [with the electron spin e as the control qubit and the photon a as the target qubit], i.e.,

\[ CZ_{ae} := \left| + ⟩_e (\gamma_1 |a_1⟩ + \gamma_2 |a_2⟩) + | - ⟩_e (\gamma_1 |a_1⟩ - \gamma_2 |a_2⟩) \right] \otimes (x_i |R⟩ + x_2 |L⟩) \] (7)

Then, the photon b with the form \( |ψ⟩_b = (\beta_1 |R⟩ + \beta_2 |L⟩) \otimes (x_i |b1⟩ + x_2 |b2⟩) \) passes a BS (the spatial Hadamard transformation) in the subcircuit S2 and is changed into \( |ψ⟩_{ae} = (\beta_1 |R⟩ + \beta_2 |L⟩) \otimes (δ_1 |b1⟩ + δ_2 |b2⟩) \) with \( δ_1 = (2 + \sqrt{2})/2 \) and \( δ_2 = (2 - \sqrt{2})/2 \). At the same time, the joint system of the electron spin e and the photon a is changed from \( |φ⟩_{ae} \) to

\[ \Phi_{ae}^{\prime} = \frac{1}{\sqrt{2}} \left[ | + ⟩_e (\gamma_1 |a_1⟩ + \gamma_2 |a_2⟩) \otimes (x_i |R⟩ + x_2 |L⟩) \right] \] (9)

Now, by performing the second Hadamard operation W on the electron spin e, the joint system of the photon a and the photons b collapses into

\[ \Psi_{ab} = (x_i |R⟩ + x_2 |L⟩) \otimes (\beta_1 |R⟩ + \beta_2 |L⟩) \]

by performing the phase-\( Z \) operation \( Z = |a_i⟩⟨a_i| - |a_2⟩⟨a_2| \) on the spatial mode of the photon a for the measurement outcome | + ⟩ of the excess electron spin e. Thus, the deterministic spatial-CNOT gate is completed.
controlled-phase-flip gates combined with certain Hadamard operations, photons \(a\) and photon \(e\) may be changed into the following joint system 

\[
\langle \Phi_3 \rangle_{ab} = \left\{ |\psi\rangle \rightarrow \frac{1}{\sqrt{2}} \left( |\psi\rangle + |\tilde{\psi}\rangle \right) \right\}
\]

by performing one Hadamard operation on the excess electron spin \(e\).

Now, photon \(b\) passes through the subcircuit \(S4\) [CPBS, QD, CPBS, sequentially, i.e., another CZ_{CP} operation on the electron spin \(e\) and photon \(b\)]. The joint system of the excess electron spin \(e\) and the photons \(a\) and \(b\) is changed from \(\langle \Phi_3 \rangle_{ab} \otimes |\psi\rangle_b\) to

\[
\langle \Phi_4 \rangle_{ab} = \left| \begin{array}{l}
|\psi\rangle_a \\
|\tilde{\psi}\rangle_a
\end{array} \right> \left( |\tilde{\psi}\rangle_b + |\psi\rangle_b \right) \\
+ |\psi\rangle_a \left( |\tilde{\psi}\rangle_b - |\psi\rangle_b \right)
\]

\[
\otimes \left( \frac{1}{\sqrt{2}} \left( |\psi\rangle + |\tilde{\psi}\rangle \right) \right) \otimes \left( |\psi\rangle_b + |\tilde{\psi}\rangle_b \right)
\]

This state may be changed to the following joint system

\[
\langle \Phi_5 \rangle_{ab} = \left\{ |\psi\rangle_a |\tilde{\psi}\rangle_b + |\psi\rangle_a |\tilde{\psi}\rangle_b \\
+ |\tilde{\psi}\rangle_a |\psi\rangle_b + |\tilde{\psi}\rangle_a |\tilde{\psi}\rangle_b \\
- \frac{1}{\sqrt{2}} \left( |\psi\rangle + |\tilde{\psi}\rangle \right) \left( |\psi\rangle + |\tilde{\psi}\rangle \right)
\]

\[
\otimes \left( \frac{1}{\sqrt{2}} \left( |\psi\rangle + |\tilde{\psi}\rangle \right) \right) \otimes \left( |\psi\rangle_b + |\tilde{\psi}\rangle_b \right)
\]

by performing a Hadamard transformation \(W\) on the excess electron spin \(e\) and performing two Hadamard transformations \(H\) on photon \(b\).

Thus, after measuring the electron spin \(e\) under the orthogonal basis \(\{ |\psi\rangle, |\tilde{\psi}\rangle \}\), photons \(a\) and \(b\) may be changed into

\[
\langle \Psi_2 \rangle_{ab} = \left| \begin{array}{l}
|\psi\rangle_a |\tilde{\psi}\rangle_b \\
+ |\tilde{\psi}\rangle_a |\psi\rangle_b \\
- |\psi\rangle_a |\tilde{\psi}\rangle_b \\
+ |\tilde{\psi}\rangle_a |\tilde{\psi}\rangle_b \\
\end{array} \right> \\
\otimes \left( \frac{1}{\sqrt{2}} \left( |\psi\rangle + |\tilde{\psi}\rangle \right) \right) \otimes \left( |\psi\rangle_b + |\tilde{\psi}\rangle_b \right)
\]

by performing the phase-flip transformation \(R = |\tilde{\psi}\rangle |\tilde{\psi}\rangle - |\psi\rangle |\psi\rangle\) on the polarization of the photon \(a\) for the measurement outcome \(|\psi\rangle\) of the excess electron spin \(e\). Thus the deterministic polarization-CNOT gate on two photons is completed.

**Hybrid spatial-polarization CNOT gate on a two-photon system.**

The schematic deterministic hybrid spatial-polarization CNOT gate on a two-photon system is shown in Figure 3(a). Here, the spatial state of photon \(a\) is the control qubit, while the polarization state of the photon \(b\) is the target qubit. All initial states are identical to ones for the spatial-CNOT gate on the two-photon system shown in Figure 2. The new circuit is derived from both subcircuits in Figure 2. Firstly, the subcircuit \(S1\) shown in Figure 2(a) is used to complete the controlled-phase-flip gate described in equation (7) on the electron spin \(e\) and photon \(a\), following the joint system \(\langle \Phi_3 \rangle_{ab}\) shown in equation (12). Then, using the subcircuit \(S4\) shown in Figure 2(b) for the excess electron spin \(e\) and photon \(b\), the total system is changed from \(\langle \Phi_3 \rangle_{ab} |\psi\rangle_b\) to

\[
\langle \Phi_5 \rangle_{ab} = \left\{ |\psi\rangle_a |\tilde{\psi}\rangle_b + |\tilde{\psi}\rangle_a |\psi\rangle_b \\
+ |\tilde{\psi}\rangle_a |\tilde{\psi}\rangle_b \\
\end{array} \right> \\
\otimes \left( \frac{1}{\sqrt{2}} \left( |\psi\rangle + |\tilde{\psi}\rangle \right) \right) \otimes \left( |\psi\rangle_b + |\tilde{\psi}\rangle_b \right)
\]

by performing the phase-flip operation \(Z\) on the spatial DOFs of the photon \(a\) for the measurement outcome \(|\psi\rangle\) of the excess electron spin \(e\). Thus, the deterministic hybrid spatial-polarization CNOT gate on a two-photon system is completed.

**Hybrid polarization-spatial CNOT gate on a two-photon system.**

The schematic deterministic hybrid polarization-spatial CNOT gate on a two-photon system is shown in Figure 3(b). Here, the polarization state of photon \(a\) is the control qubit while the spatial state of photon \(b\) is the target qubit. The new circuit is derived from both subcircuits in Figure 2. First, the subcircuit \(S3\) shown in Figure 2(b) is used to complete the controlled-phase-flip gate shown in equation (13) on the excess electron spin \(e\) and photon \(a\), following the joint system \(\langle \Phi_3 \rangle_{ab}\) shown in equation (12). Then, using the subcircuit \(S2\) shown in Figure 2(a) for the excess electron spin \(e\) and photon \(b\), the total system is changed from \(\langle \Phi_3 \rangle_{ab} |\psi\rangle_b\) to

\[
\langle \Phi_6 \rangle_{ab} = \left\{ |\psi\rangle_a |\tilde{\psi}\rangle_b + |\tilde{\psi}\rangle_a |\psi\rangle_b \\
+ |\tilde{\psi}\rangle_a |\tilde{\psi}\rangle_b \\
\end{array} \right> \\
\otimes \left( \frac{1}{\sqrt{2}} \left( |\psi\rangle + |\tilde{\psi}\rangle \right) \right) \otimes \left( |\psi\rangle_b + |\tilde{\psi}\rangle_b \right)
\]

by performing the phase-flip operation \(R\) on the polarization of photon \(a\) for the measurement outcome \(|\psi\rangle\) of the excess electron spin \(e\). Thus the deterministic polarization-CNOT gate on two photons is completed.
spin $e$. So, the deterministic hybrid polarization-spatial CNOT gate on two photons is completed.

**Hybrid spatial-polarization CNOT gate on a one-photon system.**

The schematic deterministic hybrid spatial-polarization CNOT gate on a one-photon system is shown in Figure 3(c). Here, the spatial state of photon $a$ is the control qubit while the spatial state of photon $b$ is the target qubit. The circuit is similar to the one on a two-photon system, shown in Figure 3(a). The difference is that the two subcircuits S1 and S4 are implemented on only one photon. In detail, after passing the subcircuit S1 shown in Figure 2(a), the joint system of the electron spin $e$ and photon $a$ is $|\Phi_{1a}\rangle_{ac}$, as shown in equation (6). Then, using the subcircuit S4 shown in Figure 2(b), the joint system of the excess electron spin $e$ and photon $a$ is changed into

$$
|\Phi_{2a}\rangle_{ae} = |\uparrow\rangle_{e}|\gamma_{1}\rangle_{a}|(x_{1}|R\rangle + x_{2}|L\rangle)
+ |\downarrow\rangle_{e}|\gamma_{1}\rangle_{a}|(x_{1}|R\rangle + x_{2}|L\rangle)
+ |\uparrow\rangle_{e}|\gamma_{2}\rangle_{a}|(x_{2}|R\rangle + x_{1}|L\rangle)
+ |\downarrow\rangle_{e}|\gamma_{2}\rangle_{a}|(x_{2}|R\rangle + x_{1}|L\rangle).
$$

(22)

Thus, after measuring the excess electron spin $e$ under the basis $|\uparrow\rangle, |\downarrow\rangle$, photon $a$ may be changed into

$$
|\Psi_{5a}\rangle = |\gamma_{1}\rangle_{a}|(x_{1}|R\rangle + x_{2}|L\rangle)
+ |\gamma_{2}\rangle_{a}|(x_{2}|R\rangle + x_{1}|L\rangle)
$$

(23)

by performing the phase-flip operation $Z$ on the spatial modes of photon $a$ for the measurement outcome $|\uparrow\rangle$ of the excess electron spin $e$. So, the deterministic hybrid spatial-polarization CNOT gate is completed on a one-photon system.

**Hybrid polarization-spatial CNOT gate on a one-photon system.**

The schematic deterministic hybrid polarization-spatial CNOT gate on a one-photon system is shown in Figure 3(d). Here, the polarization state of photon $a$ is the control qubit while the spatial state of photon $b$ is the target qubit. The circuit is similar to the one on a two-photon system, shown in Figure 3(b). The difference is that the two subcircuits S3 and S2 are implemented on only one photon. In detail, after passing the subcircuit S3 shown in Figure 2(b), the joint system of the electron spin $e$ and photon $a$ is $|\Phi_{3a}\rangle_{ae}$ shown in equation (12). Then, using the subcircuit S2 shown in Figure 2(a), $|\Phi_{3a}\rangle_{ae}$ is changed into

$$
|\Phi_{4a}\rangle_{ae} = |\uparrow\rangle_{e}|x_{1}|R\rangle|\gamma_{1}\rangle_{a}| + |\uparrow\rangle_{e}|x_{2}|L\rangle|\gamma_{2}\rangle_{a}|
+ |\downarrow\rangle_{e}|x_{2}|R\rangle|\gamma_{2}\rangle_{a}| + |\downarrow\rangle_{e}|x_{1}|L\rangle|\gamma_{1}\rangle_{a}|
$$

(24)

Thus, after measuring the electron spin $e$ under the basis $|\uparrow\rangle, |\downarrow\rangle$, photon $a$ may be changed into

$$
|\Psi_{6a}\rangle = |x_{1}|R\rangle|\gamma_{1}\rangle_{a}| + |x_{2}|L\rangle|\gamma_{2}\rangle_{a}|
+ |x_{2}|R\rangle|\gamma_{2}\rangle_{a}| + |x_{1}|L\rangle|\gamma_{1}\rangle_{a}|
$$

(25)

by performing the phase-flip operation $R$ on the polarization modes of photon $a$ for the measurement outcome $|\uparrow\rangle$ of the excess electron spin $e$. So, the deterministic hybrid polarization-spatial CNOT gate is completed on one photon.

**Discussion**

Experimentally, the transition rules in equation (5) may fail due to decoherence and dephasing. The fidelities of our hyper-CNOT gates are reduced. The spin-dependent transition rule is imperfect and decreases the fidelities by a few percent if heavy-light hole mixing is considered. Fortunately, the hole mixing can be reduced by improving the shape, size, and type of QDs.\(^{46}\) Although electron spin decoherence can also decrease the fidelities of the hyper-CNOT gates, however, this effect may be reduced by extending the electron coherence time to $\mu s$ using spin echo techniques.\(^{47}\) The spin superposition states $|+\rangle$ and $|\rangle$ are generated using nanosecond electron spin resonance microwave pulses or picosecond optical pulses, of which the preparation time ($ps$) is significantly shorter than the spin coherence time.\(^{48}\) Then, the Hadamard operation for transforming electron spin states $|\rangle$ and $|\rangle$ to $|+\rangle$ and $|\rangle$ can be achieved.

In ideal conditions, one may neglect the cavity side leakage, and the reflection coefficients are $|r_{0}| \approx 1$ and $|r_{0}| \approx 1$. The corresponding fidelities of our six hyper-CNOT gates are nearly 100%. Unfortunately, it is impossible to neglect side leakage from the cavity in the experiment.\(^{27-31,44-51}\) The general fidelity is defined by $F = \langle \phi | \phi \rangle$, where $| \phi \rangle$ and $| \phi \rangle$ are the final states of an ideal condition and a real situation with side leakage, respectively. In the resonant condition with $\omega_{e} = \omega_{c}$, if the cavity side leakage is considered, the optical selection rules for a QD-cavity system given by the equation (4) become

$$
|R| |\uparrow\rangle \rightarrow r_{0}|R| |\uparrow\rangle \rightarrow r_{0}|R| \rightarrow \rightarrow r_{0}|L| |\uparrow\rangle
$$

(26)

Due to the exchangeability of the polarization DOF and the spatial DOF of one photon with respect to random initial photon, the fidelities of the hybrid spatial-polarization CNOT and the hybrid polarization-spatial CNOT on a two-photon are same to those of the spatial-CNOT and the polarization CNOT on a two-photon, respectively. However, for the spatial-CNOT and the polarization CNOT, we cannot directly get the same fidelities from the exchangeability of polarization DOF and spatial DOF because the exchanging of two DOFs at one time may cause confusing implementation due to the different circuits of these hyper CNOT gates, shown in Figure 2. Moreover, from Figure 3 for the one-photon system, one cannot change the DOFs during the implementation process because the hyper-CNOT is not realized in a one-shot manner. Thus, we need to compute the fidelities of the spatial-CNOT and polarization-CNOT on a two-photon system and the hybrid spatial-polarization CNOT and the hybrid polarization-spatial CNOT on a one-photon system.

In detail, by replacing the optical selection rules in equation (5) with the ones in equation (26), using the same procedures as in the Results section, one can obtain the final states after the spatial CNOT gate on a two-photon system, polarization-CNOT gate on a two-photon system, hybrid spatial-polarization CNOT on a one-photon system, or hybrid spatial-polarization CNOT on a one-photon system. Therefore, the fidelities $F$ of these four hyper-CNOT gates are

$$
F_{2}^{2SSD} = \left( \frac{\delta_{1}^{2}(\psi_{1}^{2} + 0.5\psi_{2}^{2}(r + r_{0})) + \delta_{2}^{2}(\psi_{1}^{2} + 0.5\psi_{2}^{2}(r + r_{0}))}{\delta_{1}^{2}(\psi_{1}^{2} + 0.5\psi_{2}^{2}(r + r_{0})) + \delta_{2}^{2}(\psi_{1}^{2} + 0.25\psi_{2}^{2}(r + r_{0}))} \right)^{2},
$$

(27)

$$
F_{2}^{2SSD} = \left( \frac{\beta_{1}^{2}(\psi_{1}^{2} + 0.5\psi_{2}^{2}(r + r_{0})) + \beta_{2}^{2}(\psi_{2}^{2} + 0.5\psi_{2}^{2}(r + r_{0}))}{\beta_{1}^{2}(\psi_{1}^{2} + 0.5\psi_{2}^{2}(r + r_{0})) + \beta_{2}^{2}(\psi_{1}^{2} + 0.25\psi_{2}^{2}(r + r_{0}))} \right)^{2},
$$

(28)

$$
F_{2}^{2PS} = \left( \frac{\gamma_{1}^{2}(\psi_{1}^{2} + 0.5\psi_{2}^{2}) + 0.5\gamma_{2}^{2}(r + r_{0})(\psi_{1}^{2} + \psi_{2}^{2})}{\gamma_{1}^{2}(\psi_{1}^{2} + 0.25\psi_{2}^{2}(r + r_{0})) + \gamma_{2}^{2}(\psi_{1}^{2} + 0.25\psi_{2}^{2}(r + r_{0}))} \right)^{2},
$$

(29)

$$
F_{2}^{2PS} = \left( \frac{\gamma_{1}^{2}(\psi_{1}^{2} + 0.5\psi_{2}^{2}) + 0.5\gamma_{2}^{2}(r + r_{0})(\psi_{1}^{2} + \psi_{2}^{2})}{\gamma_{1}^{2}(\psi_{1}^{2} + 0.25\psi_{2}^{2}(r + r_{0})) + \gamma_{2}^{2}(\psi_{1}^{2} + 0.25\psi_{2}^{2}(r + r_{0}))} \right)^{2},
$$

(30)
Figure 4 | Average fidelities of the present hyper-CNOT gates. (a) The average fidelity of the spatial-CNOT gate on a two-photon system. (b) The average fidelity of the polarization-CNOT gate on a two-photon system. (c) The average fidelity of the hybrid spatial-polarization CNOT gate on a one-photon system. (d) The average fidelity of the hybrid polarization-spatial CNOT gate on a one-photon system. The coupling strength is defined by $\tilde{g} = \alpha_{k}$. The average fidelity is computed as the expectation of random input photons.

where $\alpha_{k}$, $\beta_{k}$, $\gamma_{k}$, $\delta_{k}$ are the coefficients of the initial photons and satisfy $\alpha'_{1} + \alpha'_{2} = \beta'_{1} + \beta'_{2} = \gamma'_{1} + \gamma'_{2} = \delta'_{1} + \delta'_{2} = 1$, and $\alpha'_{1} = \alpha_{1} + \alpha_{2}$, $\alpha'_{2} = \alpha_{1} - \alpha_{2}$, $\beta'_{1} = \beta_{1} + \beta_{2}$, $\beta'_{2} = \beta_{1} - \beta_{2}$, $\gamma'_{1} = \gamma_{1} + \gamma_{2}$, $\gamma'_{2} = \gamma_{1} - \gamma_{2}$, $\delta'_{1} = \delta_{1} + \delta_{2}$, $\delta'_{2} = \delta_{1} - \delta_{2}$. We generally consider the real $r$, $s$, $t$, and real coefficients $\alpha_{k}$, $\beta_{k}$, $\gamma_{k}$, $\delta_{k}$ because these fidelities depend on the coefficients of the initial photons, we present them in Figure 4 as the expectations of the initial photons by evaluating the average fidelity of 10$^5$ random initial photons. Here, $r = \frac{k^2 - 1}{k^2 + 1}$ [from the equation (3)] and $r = \frac{(k' - 1)(1 + \kappa)^2 + 10g^2}{1/(1 + \kappa)^2 + 10g^2}$ [from the equation (2) and $\xi = 0.1\kappa_{a}$] under the resonant condition $\omega_{k} = \omega = \omega_{a}$ and $\kappa' = \kappa_{k}/\kappa_{a}$ and $g' = g/(\kappa_{k} + \kappa)$. Generally, strong coupling strength and the low side leakage and cavity loss rate ($\kappa_{k}/\kappa_{a}$) are all required for high fidelity. Experimentally, the strong coupling strength $g/(\kappa_{k} + \kappa)$ can be raised to 2.4 by improving the sample designs, growth, and fabrication$^{38,49}$. For our hyper-CNOT gates, if $g/(\kappa_{k} + \kappa) = 0.5$ and $\kappa_{k}/\kappa_{a} = 0$, their fidelities are greater than 90%. When the coupling strength $g/(\kappa_{k} + \kappa) = 2.4$ with $\kappa_{k}/\kappa_{a} = 0$, the fidelities are approximately 100%. If the side leakage and cavity loss rate are $\kappa_{k}/\kappa_{a} = 0.3$ for $g/(\kappa_{k} + \kappa) = 2.4$. The fidelities are greater than 95%. The side leakage and cavity loss rate have been reduced to $\kappa_{k}/\kappa_{a} = 0.7$ with $g/(\kappa_{k} + \kappa) = 1^{38,43,44,60}$. Recently, a quantum gate between the spin state of a single trapped atom and the polarization state of an optical photon contained in a faint laser pulse has been experimentally achieved$^{41}$. We believe that their hybrid gate may soon be extended to our general hyper-CNOT gates on the two-photon system or the one-photon system because the main primitive gate of our CNOT gates is the controlled-phase-flip gate shown in equations (7) and (13).

In conclusion, we have investigated the possibility of parallel quantum computation based on two DOFs of photon systems, without using auxiliary spatial modes or polarization modes. We have constructed six deterministic hyper-controlled-not (hyper-CNOT) gates operating on the spatial-mode and the polarization DOFs of a two-photon system or a one-photon system. Compared with the hyper-CNOT gates on the same DOF of a two-photon system$^{28,33,35}$, our gates may be performed on different DOFs of two photons or one photon. Our schemes have also used fewer CPBS, which may be difficult in experiment. In contrast to the hyper-CNOT gates on the photon and stationary electron spins in quantum dots$^{32,36}$, our CNOT gates are ultimately realized on the photon system, and the excess electron spin is an auxiliary resource. Because the side leakage and cavity loss may be difficult to control or reduce for the electron-spin qubit and photonic qubits in the double-sided QD-cavity system, our gates are easier to implement experimentally than the hyper-parallel photonic quantum computation gates$^{33}$ using a double-sided QD-cavity system. Furthermore, their hyper-CNOT gates are only performed on a two-photon system, while our gates have also been implemented on a one-photon system. Even if the different DOFs may be easily changed, the operation of exchanging different DOFs is not convenient for a one-photon system because the hyper-CNOT is not realized in a one-shot manner. In detail, from the schematic circuits shown in Figure 3(c) and (d), one controlled-phase-flip gate is firstly performed on the auxiliary electron spin and one DOF, and then another controlled-phase-flip gate is performed on the auxiliary electron spin and the other DOF. Thus, one cannot exchange the DOFs after the controlled-phase-flip gate, and can only exchange the DOFs before the CNOT gate. This fact is important for the parallel quantum realization of large-scale quantum schemes such as the quantum Shor algorithm or the quantum search algorithm. Therefore, our results are distinct from all previous quantum logic gates on different photons$^{32,37}$. Our theoretical results show that two DOFs of photon systems can be used as independent qubits in quantum information processing. With these realizations, one half of the quantum resources may be saved. Of course, the hyper-CNOT gates may be affected by the cavity leakage, and spin coherence in quantum dots or the exciton coherence in experiment. With the recent experiments of QD-cavity system$^{39,39}$ and the quantum gate between a flying optical photon and a single trapped atom$^{51}$, our results are expected to be applicable for creating photon-photonic entangled states from separable input states, large-scale quantum computation, or quantum communication.
Methods
Measurement of the excess electron spin $e$ in QD. The excess electron spin $e$ is measured using an auxiliary photon $|\psi\rangle_e = \frac{1}{\sqrt{2}} (|R\rangle + |L\rangle) |e\rangle$. Let the right circular polarization $|R\rangle$ of photon $e$ interact with the QD-cavity system (photon $e$ passes a CPBS to splitter the circular polarizations $|R\rangle$ and $|L\rangle$, and then the $|R\rangle$ passes through the QD and combines with $|L\rangle$ of photon $e$ using another CPBS). Then, the final state of the complicated system becomes

$$
\frac{1}{\sqrt{2}} (|R\rangle + |L\rangle) |e\rangle |1\rangle, \quad \frac{1}{\sqrt{2}} (|R\rangle - |L\rangle) |e\rangle |1\rangle.
$$

Thus, the electron spin $e$ can be determined by measuring the photon in the orthogonal basis $\{\frac{1}{\sqrt{2}} (|R\rangle + |L\rangle)\}$. The electron spin is $|1\rangle$ for the measurement outcome $\frac{1}{\sqrt{2}} (|R\rangle + |L\rangle)$ or $\frac{1}{\sqrt{2}} (|R\rangle - |L\rangle)$, respectively.

Single-sided QD-cavity system in quantum information processing based on two DOFs. The spatial-mode and the polarization DOFs may convert into each other if only one DOF is used for encoding information. There are many schemes on the polarization logic gates using the spatial-mode DOF as the assistant [3,23,24,32,25]. Our hyper-CNOT gates show that the spatial-mode and the polarization DOFs of photonic states can be used independent qubits without auxiliary spatial modes. This is simpler than the one using double-sided QD-cavity system [4]. If two DOFs are independently used for encoding different information, their conversions may cause confusions for quantum information processing. For example, two DOFs of the photon system are used as the encoding qubit and the register qubit respectively, one cannot convert them during the Shor decomposition. Moreover, if the spatial-mode DOFs of some photon systems and the polarization DOFs of other photon systems are used as the same type of qubits in application, such as the register qubits in the Shor algorithm, great attentions should be paid because the circuits of six hyper-CNOT gates are different and cannot exchanged in experiment. Of course, the two DOFs of a photon may be used as a four-dimensional state, and the single-sided QD-cavity system can also be used to implement the quantum information process with this normalization of two DOFs.
51. Reiserer, A., Kalb, N., Rempe, G. & Ritter, S. A quantum gate between a flying optical photon and a single trapped atom. *Nature* **508**, 237–240 (2014).

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**Author contributions**

L.M.X. proposed the theoretical method and wrote the main manuscript text. L.M.X. and W.X. reviewed the manuscript.

**Additional information**

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