Motivated by a recent detection of 511 keV γ-ray line from the center of our Galaxy, we calculate the spectrum of the soft γ-ray background of the redshifted 511 keV photons from cosmological halos. Annihilation of dark matter particles into electron-positron pairs makes a substantial contribution to the γ-ray background. Mass of such dark matter particles must be \( \lesssim 100 \) MeV so that resulting electron-positron pairs are non-relativistic. On the other hand, we show that in order for the annihilation not to exceed the observed background, the dark matter mass needs to be \( \gtrsim 20 \) MeV. We include the contribution from the active galactic nuclei and supernovae. The halo substructures may increase the lower bound to \( \gtrsim 60 \) MeV.

Recently, a narrow 511 keV γ-ray line from the central part of our Galaxy has been detected and mapped by the SPI spectrometer on the INTERnational Gamma-Ray Astrophysics Laboratory (INTEGRAL) satellite\(^1\). This line should be produced by annihilation of non-relativistic electron-positron pairs, and one of the possible origins is the dark matter particles annihilating into electron-positron pairs\(^2\), which explains the measured injection rate of positrons and morphology of the signal extended over the bulge region. Therefore, 511 keV lines from our Galaxy as well as other galaxies may provide a smoking-gun for the existence of light dark matter particles\(^3\). Motivated by this idea, we calculate the spectrum of possible hard x-ray/soft γ-ray particles\(^3\). We include the contribution from the active galactic nuclei and supernovae. The halo substructures may increase the lower bound to \( \gtrsim 60 \) MeV.

\begin{align}
I_\nu &= \frac{c}{4\pi} \int \frac{dz}{H(z)(1+z)^4} \frac{P_\nu((1+z)\nu, z)}{H(z)(1+z)^4},
\end{align}

where \( \nu \) is an observed frequency, \( H(z) \) is the expansion rate at redshift \( z \), and \( P_\nu(\nu, z) \) is the volume emissivity of 511 keV photons in units of energy per unit time, per unit frequency and per unit proper volume:

\begin{align}
P_\nu = \delta \left( (1+z)\nu - \nu_{511} \right) \alpha_{511} h\nu_{511} \langle \sigma v \rangle n_X^2.
\end{align}

Note that an electron-positron pair creates two 511 keV photons, and we assume that the universe is optically thin at \( h\nu \gtrsim 10 \) keV. A parameter \( \alpha_{511} \) determines the fraction producing an electron-positron pair per one dark matter annihilation process. If annihilation occurs predominantly via positronium formation, then \( \alpha_{511} = 1/4 \). While it has been shown that the positronium formation is the dominant process at the center of our Galaxy\(^4\), we assume that annihilation occurs directly, \( \alpha_{511} = 1 \), for other galaxies. Here, \( h\nu_{511} = 511 \) keV and \( \langle \sigma v \rangle \) is the thermally averaged annihilation cross section.

For \( \Omega_X h^2 = 0.116 \) one finds \( \langle \sigma v \rangle = [3.4, 2.4, 2.9] \times 10^{-26} \) cm\(^3\) s\(^{-1}\) (or \([1.1, 0.80, 0.97]\) pb) for \( m_X = [1, 10, 100] \) MeV, respectively (e.g. Eq. (1) in \(10\)). Here, we assume that \( \langle \sigma v \rangle \) is velocity-independent (S-wave annihilation). Boehm et al.\(^2\) argue that the S-wave cross section overpredicts the flux from the Galactic center; however, we argue that it is still consistent with the data for \( m_X \gtrsim 20 \) MeV and \( \rho \propto r^{-0.4} \) (or shallower). This profile explains morphology of the Galactic signal\(^2\), and one then finds that \( \langle \sigma v \rangle \sim 0.8 \) pb\((m_X/20 \) MeV\(^2\)) is allowed at the 2-σ level (see Eq. (9) in \(8\) where the r.h.s. should be multiplied by 37.6/17.3 ≈ 2.2 for \( \rho \propto r^{-0.4} \)). As we show later, constraints from γ-ray background favor this parameter range, and our results do not violate the Galactic constraint.

Since the number density of dark matter particles, \( n_X \), is usually unknown, we use the mass density, \( \rho_X = n_X m_X \), instead of \( n_X \). After multiplying by \( \nu = \nu_{511}/(1+z) \), one obtains

\begin{align}
\nu I_\nu &= \frac{c}{H(z)(1+z)^4} \frac{h\nu_{511} \langle \sigma v \rangle}{4\pi m_X^2} \langle \rho_X^2 \rangle_z
\approx 0.4179 \text{ keV cm}^{-2} \text{ s}^{-1} \text{ str}^{-1}
\times \frac{C_X (z)(1+z)^2 (\Omega_X h^2)^2}{\sqrt{\Omega_m h^2(1+z)^3 + \Omega_\Lambda h^2}}
\end{align}
the present-day mean matter density. With this definition one obtains

$$M_F = 8.3 \cdot 10^{-2} M_\odot \left( \frac{\Omega_m h^2}{0.13} \right) \left( \frac{\Omega_X h^2}{0.11} \right) \left( \frac{m_X}{1 \text{MeV}} \right)^{-4}.$$  

(5)

The dark matter Jeans’s mass is given by

$$M_J = 38.79 M_\odot \left( \frac{m_X}{\text{MeV}} \right)^{-3} \left( \frac{x_F}{12} \right) \left( \frac{1+z}{3069} \right)^{3/2}.$$  

(6)

for $1 + z > 1 + z_{eq} \approx 3069$, which is a valid expression for cold thermal relics which decouple from radiation at $x_F = m_X/T_F > 3$ ($x_F \approx [11 - 15]$ for $m_X \approx [1 - 100]$ MeV; e.g. see [10]).

Eq. (4) can be cast into the product of the collapse fraction and the mean “halo clumping factor”:

$$C_X(z) = \Delta \cdot F_{\text{coll}}(z) \cdot [C_X^{\text{halo}}],$$  

(7)

where $\Delta$ is the mean halo overdensity of a halo in units of the cosmic mean matter density, the collapse fraction $F_{\text{coll}}(z) \equiv \int dM \frac{dn(M)}{dn(M)}$ is the mass fraction collapsed into cosmological halos at $z$, $[A] \equiv \int dM \frac{dn(M)}{dn(M)} f_{\text{coll}}$, and $C_X^{\text{halo}} \equiv \int d^3r \left( \frac{\rho_X}{\rho_X^{\text{halo}}} \right)^2 / \int d^3r$ is the “halo clumping factor” defined in terms of the halo mean density $\langle \rho_X \rangle_{\text{halo}} \equiv \int d^3r \rho_X / \int d^3r = \Delta \cdot \langle \rho_X \rangle_z$. As illustrated in Fig. (2), the early time $C_X(z)$ is mainly determined by $F_{\text{coll}}(z)$, while the late time $C_X(z)$ by $[C_X^{\text{halo}}]$. As it can be seen from Eq. (4), $C_X(z)$ is very sensitive to the density profile, whose properties are not fully understood (or observed) yet. In order to quantify uncertainties associated with the density profile, therefore, we adopt two different models for density profiles.

**Case A:** The Navarro-Frenk-White (NFW) Profile.

The NFW profile is an empirical fit to radial profiles of dark matter halos in N-body simulations [10]. This profile has a central cusp, $\rho_X \propto r^{-1}$, and is therefore expected to produce large annihilation signals. The NFW profile is given by $\rho_X(r) = \rho_0 (r/r_s)^{-1}(1 + r/r_s)^{-2}$.

The mass $M$ enclosed within the virial radius, $r_{\text{vir}}$, is $M = 4\pi \rho_0 r_{\text{vir}}^3 [\ln(1 + c) - c/(1 + c)]$, and the scale radius, $r_s$, is $r_s = r_{\text{vir}}/c$, where $c$ is called the concentration parameter. With these definitions, one obtains

$$C_X^{\text{halo}}(c = 3) = 7.3, C_X^{\text{halo}}(c = 10) = 50, C_X^{\text{halo}}(c = 20) = 203.$$  

For instance, $C_X^{\text{halo}}(c = 3) = 7.3, C_X^{\text{halo}}(c = 10) = 50, C_X^{\text{halo}}(c = 20) = 203$. For $\Delta$, we use an approximate form

$$\Delta = (18\pi^2 + 82x - 39x^2)/\Omega(z),$$  

where $\Omega(z)$ is the ratio of mean matter density to the critical density at $z$, and $x = \Omega(z) - 1$. This expression is valid for a flat $\Lambda$CDM universe [17, 18]. The concentration of dark matter halos found in N-body simulations has a log-normal distribution with a median value of $c(M, z) = 4(1 + z_c)/(1 + z)^{1/2}.$

(9)
where the collapse redshift, $z_c$, is implicitly given by a relation $M_\ast(z_c) = 10^{-2}M$. ($M_\ast(z)$ is the non-linear mass at $z$.) As lower mass objects collapse at higher $z_c$, the concentration decreases as $M$. (The lower mass objects have steeper profiles.) Following [19], we take into account a log-normal distribution of $c$ with the dispersion of $\sigma(\ln(c)) = 0.2$. While we use Eq. (9) for all the range of $M$ and $z$ in our analysis, we should keep in mind that this fitting formula is valid only for a limited range of $M$ and $z$ covered by N-body simulations.

Fig. 1 shows the predicted soft $\gamma$-ray background for the NFW profile. The predicted signal, $\nu I_\nu$, is roughly proportional to $\nu^{1.3}$, which simply reflects the fact that the universe becomes more clumpy at lower $z$ as $C(z) \propto (1+z)^{-1.8}$ (recall that $h\nu = 511$ keV/(1 + $z$)), as seen in Fig. 2. One should, however, keep in mind that we have simply extrapolated the fitting formula for $c(M, z)$ [Eq. (9)] to the regime where simulations are no longer valid. For the $m_X = 1$ MeV case, for example, the free-streaming mass gives the maximum concentration of $c \sim 70$ at $z = 0$. Since we don't fully understand halo profiles at such low mass or high concentration, we also apply an arbitrary, hypothetical upper limit to $c$ and investigate sensitivity of our results to the change in concentration parameters. This toy model will substantially reduce the contribution from small mass halos with $c(M, z) > c_{\text{max}}$. As $c(M, z) \propto (1+z)^{-1}$, the signal at lower $z$ (higher $h\nu$) is suppressed. As a result, $\nu I_\nu$ becomes almost flat for $\nu > \nu_\ast$, where $\nu_\ast$ corresponds to a redshift, $z_\ast$, after which the clumping factor stops evolving fast and evolves only slowly as $(1+z)^{-1/2}$ or even slower (Fig. 1 [2]). Sensitivity of the predicted spectrum to the concentration parameter model is demonstrated more in Fig. 3. For the canonical concentration parameter model given by Eq. (9) without any upper limit, the smallest mass halos always dominate; on the other hand, once the upper limit on $c$ is imposed, the largest contribution comes from $M \approx M_\ast(z)$, effectively removing contribution from the lower mass halos as the lower mass halos hit $c_{\text{max}}$ earlier than the higher mass halos.

Case B: The Truncated Isothermal Sphere (TIS) Profile. The TIS [20] is an analytical model of the post collapse equilibrium structure of halos resulting from the collapse of a top-hat density perturbation. The TIS is the minimum energy solution to the Lane-Emden equation, matching energy of the TIS to the energy of the top-hat perturbation. The TIS density profile has a soft core and thus provides a much smaller annihilation signal than the NFW profile. Note that the TIS fits the observed rotation curves of dwarf spheroidals and low surface brightness galaxies fairly well, as opposed to the
signal by a factor of calculations, however, show that they overestimated the tion can account for the observed intensity. Our recent Compton-thin AGNs with 

\[ \gamma = 3 \] (they assumed \( \gamma = 10 \)) are probably the most dominant contribution supernovae contribution\[24\], as well as the observed soft \( \gamma \)-ray background\[6\].

The uncertainty includes statistical uncertainty as well as potential systematic uncertainties associated with instrumental calibrations. It has been shown that the background signals measured by various experiments show a large scatter (e.g.,\[25\]), and it may be possible that the measured background is uncertain up to 30%\[22\].

One may relax this constraint in various ways. If the AGN spectrum has a smaller energy cut-off, \( E_{\text{cut}} \approx 100 \) keV, the AGNs do not contribute to the \( \gamma \)-ray background for \( h\nu \gtrsim 100 \) keV, and \( m_X \gtrsim 12 \) MeV becomes acceptable (Fig. 4). Bigger uncertainties come from the halo model. If we adopt the TIS or the NFW with \( \gamma = 3 \), we obtain \( m_X \gtrsim 5 \) and 1.7 MeV for with and without the AGNs contribution, respectively (see Fig. 3).

Such a low concentration parameter or filtering of low mass halos may be possible if dark matter has some finite self-interacting cross section\[26, 27\]. In addition to the average shape of \( \rho_X \), the clumpy substructure within halos would contribute to the signal significantly\[11, 10\]. While the abundance and properties of substructure are uncertain, the analysis in\[1\] shows that the substructures can enhance the clumping factor by more than a factor of 10. As the intensity is proportional to \( m_X^2 \), the contribution from the substructures can increase the lower limit on \( m_X \) by more than a factor of 3, giving \( m_X \gtrsim 60 \) MeV for the NFW profile.

In summary, we have calculated the soft \( \gamma \)-ray background of the redshifted 511 keV photons from the dark

\[ \rho = 1799\Omega_m(z) \left[ \frac{21.38}{9.08^2 + (r/r_0)^2} - \frac{19.81}{14.62^2 + (r/r_0)^2} \right]^2, \]

\[ r_t = \frac{0.187}{1 + z} \left( \frac{M}{10^{12} M_\odot} \right) (\Omega_m h^2)^{-1/3} \text{ Mpc}, \]

and \( r_0 \) is the core radius given by \( r_0 = r_t/29.4 \). We find that the predicted signal for the TIS model is very similar to that for the NFW with \( \gamma = 3 \), which is consistent with the TIS profile being much flatter (i.e., much less clumpy) than the NFW with the canonical concentration. This can be more easily explained by Eq. 7. At \( z \gtrsim 1 \), \( \Delta \approx 178 \) and \( C_X(z) \approx 178 \times 7.3 	imes F_{\text{coll}} \) for the NFW with \( m_X \approx 3 \), while the TIS has \( \Delta \approx 130 \), \( C_{\text{halo}} \approx 11.5 \), and so \( C_X(z) \approx 130 \times 11.5 \times F_{\text{coll}} \).

\[ AGNs \text{ and Supernovae. There are other sources for the soft } \gamma \text{-ray background. The Active Galactic Nuclei (AGNs) are probably the most dominant contribution up to } \sim 100 \text{ keV, and it may continue to dominate up to } \sim 500 \text{ keV}\[21, 22\], depending on a cut-off energy scale of the AGN spectrum, } E_{\text{cut}}, \text{ which is a free parameter. The other candidate source is the Type Ia Supernovae (SNIa).}\[23\] have calculated a spectrum of the soft \( \gamma \)-ray background from SNIa, finding that the SNIa contribution can account for the observed intensity. Our recent calculations, however, show that they overestimated the signal by a factor of \( \sim 10 \) (they assumed \( \sim 10 \) times larger supernova rate than observed). The SNIa contribution is thus negligible at the energy scale of our interest\[24\].

Figure 4 compares the dark matter annihilation, the Compton-thin AGNs with \( E_{\text{cut}} = 500 \) keV\[23\], and the supernovae contribution\[24\], as well as the observed soft \( \gamma \)-ray background\[6\]. We find that an acceptable range for the dark matter mass is \( 20 \lesssim m_X \lesssim 100 \) MeV for the NFW profile. (Note that the upper limit comes from requiring that electron-positron pairs be non-relativistic.)
matter particles annihilating into electron-positron pairs in cosmological halos. Our fiducial model based on the current N-body simulations, compared to the observed γ-ray spectrum, places limits on the dark matter mass, $20 \lesssim m_X \lesssim 100$ MeV. (If the cross section is allowed to be free, we find $(σv)/pb(MeV/m_X)^2 \lesssim 2.1 \times 10^{-3}$, which is consistent with that from the Galactic center $^3$ for $ρ \propto r^{-0.4}$ or shallower profile). Recently, Beacom, Bell and Bertone $^2$ have shown that the internal bremsstrahlung of electrons and positrons contributes to the γ-ray background at $> 1$ MeV. Using COMPTEL and EGRET data, they have obtained $m_X \lesssim 20$ MeV. Putting all the constraints together, $m_X \sim 20$ MeV may be favored. However, in order to have a more robust constraint, further understanding of cosmological non-linear structures is required. Obtaining a more reliable model for AGNs is equally important. Uncertainty in the cut-off energy, $E_{cut}$, is the dominant uncertainty in the predicted soft γ-ray background signal at $hν \gtrsim 100$ keV. If the cut-off energy is smaller than $\sim 100$ keV, the soft γ-ray background signal may serve as evidence for the existence of dark matter particles with $m_X \sim \mathcal{O}(10)$ MeV.

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