Gauged supersymmetries in Yang-Mills theory

Matthieu Tissier

Laboratoire de Physique Théorique de la Matière Condensée,
Université Pierre et Marie Curie, 4 Place Jussieu 75252 Paris CEDEX 05, France

Nicolás Wschebor

Instituto de Física, Facultad de Ingeniería, Universidad de la República, J.H. y Reissig 565, 11000 Montevideo, Uruguay

In this paper we show that Yang-Mills theory in the Curci-Ferrari-Delbourgo-Jarvis gauge admits some up to now unknown local linear Ward identities. These identities imply some non-renormalization theorems with practical simplifications for perturbation theory. We show in particular that all renormalization factors can be extracted from two-point functions. The Ward identities are shown to be related to supergauge transformations in the superfield formalism for Yang-Mills theory. The case of non-zero Curci-Ferrari mass is also addressed.

I. INTRODUCTION

In Yang-Mills theories, for almost all calculations aside from lattice simulations of gauge-invariant quantities, one needs to fix the gauge. In order to choose a sort of “optimal” gauge-fixing among a large number of possibilities, one would like to preserve as many properties of the non-fixed gauge theory as possible. In particular, it is convenient to choose a gauge fixing that preserves Lorentz invariance, the global color symmetry group, the renormalizability of the theory, its locality and BRST symmetry [1-3].

As a non trivial subgroup of the gauge symmetry. Of course, one also wants the resulting model to be physically acceptable preserving, in particular, the unitarity. There exist gauge-fixings that, at the perturbative level, satisfy all these requirements. The most popular are the linear covariant gauges, including in particular the Landau gauge. However, such gauge-fixings are ambiguous because of the Gribov copies problem [4, 5, 6].

One manifestation of this problem is that if one tries to construct a nonperturbative version of the BRST symmetry on the lattice, the expectation value of gauge-invariant quantities is an undefined 0/0 expression. This is sometimes called the Neuberger’s 0 problem [7, 8]. These zeroes originate from the compensation in the functional integral of the contributions of pairs of Gribov copies that come with opposite weights. One therefore faces the alternative of either working with a gauge-fixing with a Gribov ambiguity or loose one of the above mentioned properties. In fact, recent works propose a third option, that is to calculate some gauge-invariant quantities without fixing the gauge (see [9] and references therein).

If one chooses the second option, one can, for example:

- Use the axial gauge that explicitly breaks Lorentz invariance but does not have Gribov problem.

- Use the maximal Abelian gauge that breaks the global color symmetry group; in this case, the partial gauge fixing to the maximal abelian subgroup of the gauge group has been proven to avoid the Gribov problem [10].

- Use the absolute Landau gauge by imposing a global extremization condition of a certain functional (see, for example, [11]); however no local action is known to implement this gauge-fixing and the very useful BRST symmetry is also lost. Moreover, no efficient algorithm is known to implement that idea in practice.

In this paper we will follow a more heterodox strategy, which consists in taking the Curci-Ferrari (CF) model [12, 13] that corresponds to the Yang-Mills theory in a particular gauge, supplemented with a mass for gluons and ghosts. This model is not unitary [13, 14, 15] but the presence of the masses lifts the degeneracy of contributions coming from different Gribov copies and therefore regularizes the Neuberger’s zero [16, 17]. If one studies the model directly at zero mass, one has a standard gauge fixing sometimes called Curci-Ferrari-Delbourgo-Jarvis (CFDJ) gauge [12, 13, 18], with all good properties, including unitarity, except that it has a Gribov ambiguity. It is actually possible to have unitarity and regularize the 0/0 expressions by computing physical observables in the massive theory and then taking the limit of vanishing masses.

The mass term can also be seen as a source for the dimension-two composite operator \( \frac{1}{2}(A_\mu^c)^2 + \xi_0 \bar{c}^a c^a \) that attracted a lot of attention recently in relation with nonperturbative effects on the behavior of the correlation functions of ghosts and gluons (see for example [19, 20, 21, 22]).

All these reasons strongly motivate the use of the CF model. However, this model is not widely used in practice, mainly because it seems much more cumbersome than the linear gauges. For instance it has a four-ghosts interaction. In this paper we show that this widespread prejudice should be reconsidered. We will show that beside the large symmetry group of the CF
model, there exist local transformations that induce very simple variations of the action. Therefore, although these transformations are not symmetries of the full action, one can deduce from them useful linear Ward identities. We show that there are actually underlying symmetries associated with these transformations that clearly appear in the superspace formulation of Yang-Mills theory [18, 23, 24, 25]. In this formulation, gluons and ghosts are part of a single supervector in a superspace with 4 bosonic coordinates and 2 anticommuting Grassmann coordinates. We show that the transformations associated with the new Ward identities are, in fact, supergauge transformations. The associated identities allow us to deduce non-renormalization theorems reducing the number of independent renormalization factors from five \[15\] to three. The situation is very close to the gauge-fixed Schwinger model, and interpret in this context the mentioned section VI we review the superspace formulation of Yang-renormalization theorem. In section V we analyze the renormalization properties of the model taking into account the CF mass and deduce another non-renormalization theorem. In section III, we analyze the renormalization properties of the model and its symmetries in the massless case. Finally, we give our conclusions in section VII.

\[ \mathcal{L}_{YM} \] is the Yang-Mills lagrangian:

\[ \mathcal{L}_{YM} = \frac{1}{4} F_{\mu \nu}^a F_{\mu \nu}^a, \]

\[ F_{\mu \nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_0 f^{abc} A_\mu^b A_\nu^c \]

is the bare field strength, \( g_0 \) is the bare gauge coupling, \( A_\mu^a \) is the gauge field, and \( f^{abc} \) denotes the structure constants of the gauge group that are chosen completely antisymmetric. \( \mathcal{L}_{GF} \) is the gauge-fixing term, which includes a ghost sector. It takes the form:

\[ \mathcal{L}_{GF} = \frac{1}{2} \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{1}{2} (D_\mu \bar{c}) \partial_\mu c^a + \frac{\xi_0}{2} h^a h^a + ih^a \partial_\mu A_\mu^a - \xi_0 \frac{g_0^2}{8} (f^{abc} \bar{c}^b c^c)^2. \]

Here, \( c \) and \( \bar{c} \) are ghost and antighosts fields respectively, and \( (D_\mu \varphi)^a = \partial_\mu \varphi^a + g_0 f^{abc} A_\mu^b \varphi^c \) is the covariant derivative for any field \( \varphi \) in the adjoint representation. The main interest of the CFDJ lagrangian \[18\] is that the ghost-antighost exchange symmetry is explicit and that it preserves the linear realization of some continuous symmetries \[18\]. This is not the case if the Lagrange multiplier \( h^a \) is introduced, as often done, in a non-symmetric way:

\[ \mathcal{L}_{GF} = \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{\xi_0}{2} h^a h^a + ih^a \partial_\mu A_\mu^a -\frac{i \xi_0}{2} g_0 f^{abc} \bar{c}^b c^c - \frac{\xi_0 g_0^2}{4} (f^{abc} \bar{c}^b c^c)^2. \]

However, these two versions of the CF model are in fact equivalent: indeed, one obtains \([4]\) by performing the change of variables \( ih^a \rightarrow i \bar{c}^a + \frac{g_0}{2} f^{abc} \bar{c}^b c^c \) in \([3]\).

Note that the considered gauge-fixing lagrangian is different from the more standard linear gauge fixing:

\[ \mathcal{L}_{GF} = \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{\xi_0}{2} h^a h^a + ih^a \partial_\mu A_\mu^a. \]

One cannot obtain one from the other by a change of variables in the fields. However, all these gauge fixings coincide in the particular case of the Landau gauge limit \( \xi_0 \rightarrow 0 \). In fact \([1] \) and \([5]\) are identical in this limit.

Let us list the symmetries of the gauge-fixing lagrangian \([3]\):

a) The euclidean symmetries of the spacetime.

b) The global color symmetry.

c) The already mentioned ghost conjugation symmetry: \( c^a \rightarrow \bar{c}^a \), \( \bar{c}^a \rightarrow -c^a \) without modifying the other fields. This symmetry allows one to obtain most of the relations of this paper by conjugating those explicitly considered.

d) The continuous symplectic group \( SP(2, \mathbb{R}) \) \([20]\) with generators \( N, t \) and \( \bar{t} \) defined by:

\[ \begin{align*}
    t A_\mu^a &= 0 & \bar{t} A_\mu^a &= 0 & N A_\mu^a &= 0 \\
    t c^a &= 0 & \bar{t} c^a &= -\bar{c}^a & N c^a &= c^a \\
    t \bar{c}^a &= c^a & \bar{t} \bar{c}^a &= 0 & N \bar{c}^a &= -\bar{c}^a \\
    t h^a &= 0 & \bar{t} h^a &= 0 & N h^a &= -h^a
\end{align*} \]

II. THE ACTION AND ITS SYMMETRIES

In this section, we analyze the CF model with vanishing masses, i.e. the Yang-Mills theory in the CFDJ gauge \([12, 13, 18]\). We will consider here the model in a four dimensional euclidean space without including matter, but most of the results can be generalized to minkowskian space and the inclusion of matter does not modify the main results. The gauge-fixed lagrangian reads

\[ \mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_{GF}. \]
$N$ is associated with the ghost-number conservation. One observes that $A$ and $h$ are singlets while $c$ and $\bar{c}$ form a doublet of this group. Note that $t$ and $\bar{t}$ have ghost number 2 and -2 respectively.

e) The model is also invariant under the nonlinear BRST and anti-BRST symmetries:

$$s A_\mu = (D_\mu c)^a,$$

$$\bar{s} A_\mu = (D_\mu \bar{c})^a,$$

$$sc^a = -\frac{g_0}{2} f^{abc} c^b \bar{c}^c,$$

$$\bar{s}c^a = -\frac{g_0}{2} f^{abc} \bar{c}^b c^c,$$

$$sc^a = i h^a - \frac{g_0}{2} f^{abc} \bar{c}^b c^c,$$

$$\bar{s}c^a = -i h^a - \frac{g_0}{2} f^{abc} c^b \bar{c}^c,$$

$$s i h^a = \frac{g_0}{2} f^{abc} \left( i h^b c^c + \frac{g_0}{4} f^{cde} c^d \bar{c}^e \right),$$

$$\bar{s} i h^a = \frac{g_0}{2} f^{abc} \left( i \bar{c}^b \bar{c}^c - \frac{g_0}{4} f^{cde} \bar{c}^d c^e \right).$$

These symmetries satisfy the standard nilpotency property ($s^2 = \bar{s}^2 = s\bar{s} + \bar{s}s = 0$).

In order to deduce Slavnov-Taylor identities for these symmetries, it is necessary to introduce sources for the variations of the fields under BRST and anti-BRST symmetries. Since the symmetry is nilpotent, it is sufficient to introduce sources for $s\varphi^a$, $\bar{s}\varphi^a$ and $s\bar{\varphi}^a$ for $\varphi^a = A_\mu^a, c^a$ and $\bar{c}^a$.

As usual, the Slavnov-Taylor identity associated to the BRST symmetry is obtained by performing the change of variables in the functional integral $\varphi \rightarrow \varphi + \zeta \varphi$ for all fundamental fields $\varphi$ with a constant grassmanian parameter $\zeta$. One obtains:

$$\int d^4 x \left( \frac{\delta \Gamma}{\delta \varphi^a} + \frac{\delta K_\mu^a}{\delta \varphi^a} - 2 L^\alpha \frac{\delta \Gamma}{\delta M^\alpha} \right) = 0.$$  \hspace{1cm} (12)

A similar equation can be deduced for the anti-BRST symmetry. However, we will not need it here because its information can be obtained by exploiting the ghost conjugation (c). The physical interpretation of (13) is well-known. If one evaluates it for vanishing sources for composite operators, it says that $\Gamma$ is invariant under $A_\mu^a \rightarrow A_\mu^a + \zeta \delta \Gamma / \delta K_\mu^a, c^a \rightarrow c^a - \zeta \delta \Gamma / \delta L^a, c^a \rightarrow c^a - \zeta \delta \Gamma / \delta L^a, c^a \rightarrow c^a - \zeta \delta \Gamma / \delta L^a, c^a \rightarrow c^a - \zeta \delta \Gamma / \delta L^a$, etc. The symmetry transformation itself acquires quantum corrections.

After this review of these well-known symmetries and their consequences, we now come to the deduction of other Ward identities that are linear and local. The first one is the equation of motion for the Lagrange multiplier $h^a$. It can be obtained in the usual way by performing an infinitesimal space-time dependent shift on the $h$ field $ih^a(x) \rightarrow ih^a(x) + \lambda^a(x)$. This gives:

$$\frac{\delta \Gamma}{\delta h^a} = \xi_0 h^a + i \left[ \partial_\mu A_\mu^a + (D_\mu \alpha_\mu^a) - g_0 f^{abc} (\bar{\beta} c^b + \bar{c}^b \beta^b) \right].$$  \hspace{1cm} (14)
This equation means that terms in the effective action including the $h$ field are not renormalized. Note that the non-symmetric lagrangian \([\xi]\) contains terms that couple the $h$ field tri-linearly which prevents one to derive a simple equation as \((14)\). Such terms do not exist in lagrangians \([30]\) and \([31]\) giving tractable equations of motion for $h$. Another gauge where tractable equations for the (abelian) Lagrange multiplier can be deduced is the Maximal Abelian gauge.

In the case of linear gauge-fixing, as well as in Maximal Abelian gauge \([30, 31]\), another local and linear identity can be deduced from the equation of motion of the anti-Abelian gauge \([30, 31]\), another local and linear identity. As mentioned in the introduction, these terms behave as in abelian gauge theories where gauge-fixing preserves its bare form under the renormalization process. This is very different from Slavnov-Taylor identities, which are non-linear in $\Gamma$ and therefore much harder to handle. The obtention of local, linear Ward identities is a non-trivial result and is the heart of the present manuscript.

The equations \((13), \overline{15}\) and \((17)\) are very simple and have far reaching consequences. However, to our surprise, they have never been addressed before in the CF model. In the next two sections, we discuss the consequences of these relations showing, in particular, that they induce non-trivial non-renormalization theorems for some quantities.

**III. NON-RENNORMALIZATION THEOREM FOR THE COUPLING**

The four new identities derived in the previous section have many consequences on the form of the effective action. As a concrete example, we analyze in this section the implications on the renormalization properties of the model.

The perturbative renormalizability of this model has been proven by considering five renormalization factors \([15]\), including the renormalization of the mass. Recently, however, one of us \([32]\) proved two non-renormalization theorems that reduce the number of renormalization factors from five to three. We now prove in this section and in the following that these non-renormalization theorems are, in fact, a direct consequence of the new identities discussed in the previous section.

We follow the standard procedure (see for example \([32]\)) of considering terms that can diverge by power counting and constraining them iteratively. In a loop expansion, suppose that all divergences have been renormalized at order $n-1$. Divergent terms that appear at order $n$ in the effective action, have couplings with positive or zero dimension. Let us call them $\Delta \Gamma_{div}^{(n)}$, and take an infinitesimal constant $\epsilon$. If one calls $\Gamma_{div}^{(n)} = S + \epsilon \Delta \Gamma_{div}^{(n)}$, then, in four dimensions, the most general form for this functional at order $n$ that satisfies the linear symmetries (a–d), takes the form:

\[
\Gamma_{div}^{(n)}[A, c, \bar{c}, h, K, \bar{K}, L, \bar{L}, M, \alpha, \beta, \bar{\beta}] = \int d^{4}x \left( Z_{L} \left( \bar{L}^{a} L^{a} - \frac{1}{4} M^{a} M^{a} \right) + Z_{K} \bar{K}^{a} K^{a} \right) + \bar{K}^{a} \bar{s} A^{a} + \bar{s} A^{a} K^{a} + \bar{L}^{a} \bar{s} c^{a} + L^{a} s c^{a} + M^{a} (\bar{s} c^{a} + \bar{s} c^{a}) / 2 \right) \Gamma[\bar{A}, A, \bar{c}, c, h, \alpha, \beta, \bar{\beta}] \tag{18}
\]

We introduced the notation $\bar{s}$ and $\bar{s}$ in terms linear in $K, \bar{K}, L, \bar{L}$ and $M$ in analogy to \((11)\). However, for the...
moment, $\check{s}A^{a}_{\mu}$, $\tilde{A}^{a}_{\mu}$, $\check{c}^{a}$, $\tilde{c}^{a}$ and $\check{c}\check{c}^{a}$ denote arbitrary operators depending on $\{A, c, \check{c}, h, \alpha, \beta, \tilde{\beta}\}$, of dimension two, with the same transformations under linear symmetries as the corresponding bare expressions. In order for $\check{s}$ and $\tilde{s}$ to be the symmetries of $\hat{\Gamma}$ discussed just below equation (13), we complement their definitions (again by analogy with (11) and (7)) by

$$\check{s}ih^{a} = \frac{1}{2} \delta \tilde{\Gamma} \quad \text{(19)}$$

$$\tilde{s}ih^{a} = -\frac{1}{2} \delta \check{\Gamma} \quad \text{(20)}$$

$$\check{c}\check{c}^{a} - \tilde{c}^{a} = 2ih^{a}. \quad \text{(21)}$$

For generic operators, one defines $\check{s}$ as

$$\check{s} = \int d^{4}x \left\{ \check{s}\check{A}^{a}_{\mu}(x) \frac{\delta}{\delta \check{A}^{a}_{\mu}(x)} + \check{c}\check{c}^{a}(x) \frac{\delta}{\delta \check{c}\check{c}^{a}(x)} + \tilde{s}\tilde{c}^{a}(x) \frac{\delta}{\delta \tilde{c}^{a}(x)} \right\} \quad \text{(22)}$$

and similarly for $\tilde{s}$. It is now easy to check that, with these definitions, $\check{s}$ and $\tilde{s}$ are symmetries of $\hat{\Gamma}$.

We now want to solve the Slavnov-Taylor equation (13) together with Eqs. (14,15,17). The calculation is lengthy but straightforward. Some details are given in the Appendix. The resolution simplifies if one introduces the variables

$$\check{c}^{a} = c^{a} + Z_{L}^{\beta} \quad \text{(23)}$$

$$\tilde{c}^{a} = c^{a} + Z_{L}^{\beta} \quad \text{(23)}$$

$$\check{A}^{a}_{\mu} = A^{a}_{\mu} - Z_{K}^{\alpha} \quad \text{(23)}$$

$$\tilde{A}^{a}_{\mu} = A^{a}_{\mu} - Z_{K}^{\alpha} \quad \text{(23)}$$

$$\check{h}^{a} = h^{a} + \frac{Z_{L}}{3} \left( \frac{\hat{D}_{\mu} \alpha^{a}_{\mu}}{2} + \frac{\check{g} f^{abc} (\hat{c}^{a} \beta^{b} - \check{c}^{a} \beta^{b})}{2} \right) \quad \text{(24)}$$

where $(\hat{D}_{\mu} \phi)^{a} = Z \partial_{\mu} \phi^{a} + \check{g} f^{abc} \hat{A}^{b}_{\mu} \phi^{c}$. $Z$ and $\check{g}$ are at this level arbitrary constants. In term of these variables, the solution reads:

$$\hat{\Gamma} = \int d^{4}x \left\{ \frac{\check{Z}}{2} \check{F}^{a}_{\mu} \check{F}^{a}_{\mu} + \frac{Z_{L}}{3Y} (\partial_{\mu} \check{A}^{a}_{\mu})^{2} + \frac{1}{2Y} \left( \partial_{\mu} \check{c}^{a} \right)^{2} - \frac{\check{g}^{2}}{8Y} \left( f^{abc} \check{c}^{b} \check{c}^{c} \right)^{2} \right\} \quad \text{(24)}$$

with $\check{F}^{a}_{\mu} = Z (\partial_{\mu} \check{A}^{a}_{\mu} - \partial_{\mu} \check{A}^{a}_{\mu}) + \check{g} f^{abc} \hat{A}^{b}_{\mu} \check{A}^{c}_{\mu}$ and $Y = 1 - Z_{L} / 2$. The action of $\check{s}$ and $\tilde{s}$ on the fields reads:

$$\check{s} \check{A}^{a}_{\mu} = (\hat{D}_{\mu} \check{c})^{a}, \quad \check{s} \tilde{A}^{a}_{\mu} = (\hat{D}_{\mu} \tilde{c})^{a}, \quad \check{s} \check{c}^{a} = \frac{\check{g}}{2} f^{abc} \check{c}^{b} \check{c}^{c},$$

$$\tilde{s} \check{c}^{a} = \frac{\check{g}}{2} f^{abc} \tilde{c}^{b} \check{c}^{c}, \quad \check{s} \tilde{c}^{a} = - \check{g} \frac{\check{g}}{2} \left[ f^{abc} \check{c}^{b} \tilde{c}^{c} \right] - \frac{Z_{L}}{2} \left( i \check{D}_{\mu} \check{h}^{a} + \frac{\check{g}}{2} \frac{f^{abc} (\check{c}^{b} \beta^{c} - \check{c}^{b} \beta^{c})}{2} \right),$$

$$\check{s} \check{c}^{a} = \check{g} \frac{\check{g}}{2} \left[ f^{abc} \check{c}^{b} \check{c}^{c} \right], \quad \check{s} \check{c}^{a} = - \check{g} \frac{\check{g}}{2} \left[ f^{abc} \check{c}^{b} \check{c}^{c} \right]. \quad \text{(25)}$$

Note that equation (23) is written in terms of the bare gauge parameter $\xi_{0}$. The reason being that Eq. (14) ensures that the $h$-sector of the effective action is not renormalized. Actually, Eqs. (14,15,17) impose two other relations:

$$g_{0} = \check{g} \frac{\check{g}}{2} \left[ f^{abc} \check{c}^{b} \check{c}^{c} \right], \quad 1 + Z_{K} = ZY. \quad \text{(26)}$$

These equations are at the core of the non-renormalization theorem (see below).

The action of $\check{s}$ on $h$ can be deduced from Eq. (19). We just give here the expression at vanishing sources for the composite operators:

$$2i Y \check{s}h^{a} = i Y^{2} \check{g} f^{abc} \check{h}_{\mu}^{b} \check{c}^{c} - \frac{\check{g}}{2} \frac{Z_{L}^{2}}{2} \check{g} f^{abc} \partial_{\mu} \check{A}^{b}_{\mu} \check{c}^{c}$$

$$+ \frac{\check{g}^{2}}{4} \left[ f^{abc} \check{h}^{b}_{\mu} \check{c}^{c} \check{c}^{d} \check{c}^{e} - Z_{L} (\hat{D}_{\mu} \check{c})^{a} \right] \quad \text{(27)}$$

An analogous formula can be derived for $\tilde{s}h$.

A straightforward calculation shows that $\check{s}$ and $\tilde{s}$ are nilpotent on-shell, i.e. when one imposes the equations of motion for the fields $h$, $c$ and $\check{c}$. Actually $\check{s}$ and $\tilde{s}$ can be decomposed in a sum of an off-shell nilpotent symmetry that has the form of the bare symmetry (7) up to multiplicative factors and two trivial symmetries with generators:

$$r_{1} c = r_{1} \check{c} = r_{1} \check{A}^{a}_{\mu} = 0$$

$$r_{1} h^{a} = - \frac{f^{abc}}{2} \frac{\delta \check{\Gamma}}{\delta \check{h}^{b}} \check{c}^{c}, \quad \text{(28)}$$
and
\[ r_2 A_\mu = r_2 c = 0 \]
\[ r_2 \bar{c}^a = -\frac{\delta \hat{\Gamma}}{\delta h^a} \]
\[ r_2 i h^a = -\frac{\delta \hat{\Gamma}}{\delta c^a}. \]
(29)

These generators vanish when one imposes the equations of motion. This is consistent with the on-shell nilpotency of \( \hat{s} \) and \( \hat{\bar{s}} \).

Note that there appears in \( \Gamma \) terms that where not present in the bare action described in Section II. There are terms with two powers of the sources or more and also a term proportional to \( (\partial_\mu A_\mu^a)^2 \). In order to make the theory renormalizable, one needs to include such terms in the bare action. Fortunately, it is not necessary to perform again the analysis with this new action. Indeed, the precise form of the bare action is not necessary to deduce Slavnov-Taylor identities. All that is needed is that the bare action satisfies the Slavnov-Taylor identities \[34\]. Therefore, the form of \( \Gamma \) given in Eqs. (18, 24, 25) is stable under renormalization. Let us comment that the term in \( (\partial_\mu A_\mu^a)^2 \) can be eliminated by a shift proportional to \( \partial_\mu A_\mu^a \) of the Lagrange multiplier.

We now make contact with the perturbative results and concentrate on the \( A, c, \bar{c} \) sector once the Lagrange multiplier has been eliminated by its equation of motion. The standard parametrization (see for instance \[22\]) of the effective action reads:
\[ \hat{\Gamma} = \int d^4 x \left\{ \frac{1}{2 Z_c} (\partial_\mu c^a \hat{D}_\mu c^a + \hat{D}_\mu \bar{c}^a \partial_\mu c^a) + \frac{1}{2 Z_\xi} (\partial_\mu A_\mu^a)^2 - \frac{Z_\xi \xi_0 g_0^2}{8 Z_g^2 Z_A Z_c} (f_{abc} \bar{c}^b c^c)^2 + \frac{1}{4 Z_A} \hat{F}_a^{\mu \nu} \hat{F}_a^{\mu \nu} \right\}. \]
(30)
with
\[ \hat{D}_\mu c^a = \partial_\mu c^a + \frac{g_0}{Z_g \sqrt{Z_A}} f_{abc} A_\mu^b \bar{c}^c, \]
\[ \hat{D}_\mu \bar{c}^a = \partial_\mu \bar{c}^a + \frac{g_0}{Z_g \sqrt{Z_A}} f_{abc} A_\mu^b c^c, \]
\[ \hat{F}_a^{\mu \nu} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \frac{g_0}{Z_g \sqrt{Z_A}} f_{abc} A_\mu^b A_\nu^c. \]
(31)
Comparison with Eq. (24) – where \( h \) is eliminated by its equations of motion – yields, together with Eq. (26), the following relations:
\[ Z_A = Z^{-1/2} \hat{Z}^{-1} \]
\[ Z_c = Y Z^{-1} \]
\[ Z_\xi = Y \]
\[ Z_g = Y Z^2 \hat{Z}^{1/2}. \]
(32)
One then easily deduces the non-renormalization theorem:
\[ Z_g = Z^{-1/2} \hat{Z}^{-1} Z_\xi^{-2}. \]
(33)

We postpone to section IV the discussion of this equation together with another non-renormalization theorem to be proven in the next section.

### IV. THE MASSIVE CASE

As said in the introduction, Curci and Ferrari proposed a very natural generalization of Yang-Mills theory in this particular gauge \[12, 13\]. One can add a mass term for the ghosts and gluons that preserves BRST-like symmetries:
\[ \mathcal{L}_m = m_0^2 \left( \frac{1}{2} (A_\mu^a)^2 + \xi_0 c^a c^a \right). \]
(34)

The theory remains renormalizable; however, nilpotency of the BRST symmetry is lost and, as a result, the model is no longer unitary \[13, 14, 15\]. The study performed in the previous sections is generalized here to include the mass term \[34\]. We show that the modifications to Eqs. (14, 15, 17) are very simple. The other striking result is that no independent renormalization factor is needed to renormalize the mass term.

Let us start by discussing the symmetry content of the theory in the presence of the mass term. All the linear symmetries (a–d) are preserved. On the contrary the action is not invariant under the original BRST and anti-BRST transformations (7), but are invariant under modified transformations, \( s_m = s + m_0^2 s_1 \) and \( \bar{s}_m = \bar{s} + m_0^2 \bar{s}_1 \), with
\[ \bar{s}_1 c^a = s_1 \bar{c}^a = s_1 A_\mu^a = 0 \]
\[ \bar{s}_1 \bar{c}^a = s_1 c^a = \bar{s}_1 A_\mu^a = 0 \]
\[ s_1 i h^a = c^a \]
\[ \bar{s}_1 i h^a = \bar{c}^a. \]
(35)

As already mentioned, the new BRST and anti-BRST symmetries transformations are no longer nilpotent. Its algebra becomes \[26\]
\[ s_m^2 = m_0^2 t \]
\[ \bar{s}_m^2 = m_0^2 \bar{t} \]
\[ \{s_m, \bar{s}_m\} = -m_0^2 N. \]
(36)

The Curci-Ferrari mass term induces a change in the Slavnov-Taylor equation. The right-hand side of (13) is not zero anymore and must be replaced by a term proportional to \( m_0^2 \):
\[ m_0^2 \int d^4 x \left( i \frac{\delta \Gamma}{\delta h^a c^a} + \alpha_\mu^a \frac{\delta \Gamma}{\delta K_\mu^a} \right) - 2 \beta_\mu^a \frac{\delta \Gamma}{\delta L^a} + 2 \left( \frac{\delta \Gamma}{\delta M^a} \beta_\mu^a \right). \]
(37)
Slight modifications must be introduced to the Ward identities described in Section [1]. The equation (14) is actually not modified because the mass term (41) is independent of $h$. In the second identity, Eq. (15), one must add $\xi_0 m_0^2 \theta^4$ to the left-hand side. Finally, in Eq. (17), one must add $-m_0^2 \partial_{\mu} A_\mu$ to the left-hand side.

One easily checks that the divergent part (15) of the solution of the modified Slavnov-Taylor equation now reads

$$\Gamma^{(n)}_{1,\text{div}} = \Gamma^{(n)}_{\text{div}} + m_0^2 \Gamma^{(n)}_{1,\text{div}}$$

with

$$\Gamma^{(n)}_{1,\text{div}} = \int d^4x \left( \frac{1}{2} (A_\mu^a)^2 + Z \xi_0 \theta^a \bar{\theta}^a \right) \frac{\bar{c}^c c^a}{ZY} - \frac{Z_\xi}{2} (\bar{a}_\mu^a)^2 + 2Z_L \bar{\beta}_\mu^a \beta^c \right).$$

Note that the renormalization of the mass term does not require any new renormalization factor. This leads to another non-renormalization theorem. If one compares the previous equation at zero sources with the standard parametrization of the mass term (22)

$$\int d^4x \frac{m_0^2}{Z_m} \left( A_\mu^a A_\mu^a \frac{1}{2Z_A} + \xi_0 \frac{Z_\xi}{Z_A Z_c} \bar{c}^c c^a \right),$$

by identification of the $A^2$ terms one deduces that

$$Z_m Z_A = ZY.$$  (40)

The $\bar{c} c$ term does not give new information. Using the identifications (32), one obtains another non-renormalization theorem:

$$Z_\xi^2 = Z_m Z_A Z_c.$$  (41)

V. CONSEQUENCES FOR PERTURBATION THEORY

The two non-renormalization theorems presented in previous sections have far reaching consequences for practical perturbative calculations. First of all, they imply that one can calculate as many renormalization factors as in linear gauges. Moreover, all these renormalization factors can be extracted from the 2-point function of gluons alone. In fact, one possible set of independent renormalization factors are $Z_m$ (the renormalization for the composite operator $A_\mu^a A_\mu^a$), $Z_A$ and $Z_\xi$ that can all be extracted from the zero momentum, transverse and longitudinal parts at order $p^2$ of the quoted correlation function. Other choices may even be more convenient in practice since some of these renormalization factors can be extracted from the 2-ghost function that has simpler kinematics. In any case, there is no need to calculate 3-point or higher vertices, contrarily to what is required in linear gauges. The price to pay is very small: there is a 4-ghost vertex, but the required total number of diagrams seems to be always smaller than that in linear gauges. For example, the 1-loop beta function for pure gauge, can be extracted from three diagrams only. So, in what concerns perturbative calculations, once non-renormalization theorems are exploited, CDFJ gauge is as competitive as linear gauges (same number of renormalization factors) and might even be more convenient (all renormalization factors can be extracted from 2-point functions).

To conclude, let us make three final remarks. First, these two renormalization factors have been found previously by one of us. However the proof of these non-renormalization theorems presented in [32] requires extensive use of equations of motions because it is formulated without the introduction of the Lagrange multiplier field $h$. The physical content of these identities is therefore hidden. Here, these relations are shown to be consequences of the new Ward identities. Second, one can check that the 3-loop renormalization factors (40) satisfy the two non-renormalization theorems (33, 41). Actually, it was observed in [35] that the 3-loops renormalization factors satisfy the identity (41) without giving a general proof. Finally, for the Landau gauge ($\xi = 0$), $Z_\xi = 1$ and one recovers the well-known non-renormalization theorem for the coupling constant (22) as well as the more recent one for the mass (30).

VI. SUPERSPACE INTERPRETATION

A. Flat superspace

It has been shown in the 80’s that reinterpreting the theory in a superspace enables one to give a geometric meaning to the symmetries of the model, in particular to BRST and anti-BRST symmetries [18, 23, 24, 25]. We review here the superfield formalism and subsequently reinterpret the new Ward identities described in the previous section in this context.

In the following, we consider a $4 + 2$ dimensional superspace, with 4 standard bosonic coordinates, noted $x^\mu$, and 2 Grassmanian – anticommuting – coordinates $\theta$ and $\overline{\theta}$: $\theta^2 = \overline{\theta}^2 = \theta \overline{\theta} + \overline{\theta} \theta = 0$. The (super)fields are now functions of $x^\mu$, $\theta$ and $\overline{\theta}$. In the following, capital indices vary on the 4 bosonic directions $\mu$ and on the 2 Grassmanian directions: for instance $x^M = (x^\mu, \theta, \overline{\theta})$. Because of the Grassmanian character of $\theta$ and $\overline{\theta}$, the Taylor-expansion in powers of these variables gives a finite number of terms:

$$f(x^\mu, \theta, \overline{\theta}) = f_{00}(x^\mu) + \theta f_{10}(x^\mu) + \overline{\theta} f_{01}(x^\mu) + \overline{\theta} \theta f_{11}(x^\mu),$$

with $f_{ij}(x^\mu) = \frac{\partial^i f_{j0}}{i!} \big|_{00}$, $f_{ij}(x^\mu) = \frac{\partial^i \partial_{\mu} f_{j0}}{i!} \big|_{00}$. Observe that the derivatives with respect to $\theta$ and $\overline{\theta}$ are nilpotent, just as BRST and anti-BRST symmetries. It is actually possible to make this analogy stronger, if one writes a 4-dimensional field $\phi$ and its BRST/anti-BRST variations
as a $4+2$ dimensional superfield $\Phi$

$$\Phi(x^M) = \phi(x^\mu) + \bar{\theta} s \phi(x^\mu) - \theta \bar{s} \phi(x^\mu) + \bar{\theta} \theta s \bar{s} \phi(x^\mu),$$

(43)

where now it is clear that $s$ and $\bar{s}$ act on the superfield as $\partial_{\mu}$ and $-\partial_{\mu}$ respectively.

Note moreover that the vectorial superfields, like the gauge field, have $4+2$ components $A^\mu$, $A^\bar{\nu}$ and $\bar{A}^\nu$, which have ghost numbers 0, 1 and -1 respectively. One can therefore merge the 4-dimensional gauge field, the ghost, anti-gauge and all BRST/anti-BRST variations of these fields in a unique vectorial superfield:

$$A^\mu(x^M) = A^\mu + \bar{\theta} s A^\mu - \theta s \bar{s} A^\mu + \bar{\theta} \theta s A^\mu$$

$$A^\bar{\nu}(x^M) = c + \bar{\theta} s c - \theta \bar{s} c + \bar{\theta} \theta s c$$

$$\bar{A}^\nu(x^M) = \bar{c} + \bar{\theta} s \bar{c} - \theta \bar{s} \bar{c} + \bar{\theta} \theta s \bar{c},$$

(44)

where we have omitted the color index and the bosonic space variable. The BRST/anti-BRST symmetries can therefore be interpreted as the invariance under translation in the grassmanian directions. It is important to understand at this level that the components $\theta$, $\bar{\theta}$ and $\delta_{\mu}$ of the fields $A^\mu$, $A^\bar{\nu}$ and $\bar{A}^\nu$ are not independent of the $\theta = \bar{\theta} = 0$ part of the fields. Indeed, these are explicit functions of $A^\mu$, $c$, $\bar{c}$ and $h$, as given in Eqs. (43). Consequently, the superfield is constrained and cannot be used as it stands in a functional integral. These constraints are sometimes called “transversality conditions” [18, 23, 24, 25, 37].

The symmetries $t$ and $\bar{t}$ and $N$ given in Eq. (6) also have a simple geometric interpretation in superspace: they correspond to the invariance under “rotations” in the grassmanian directions.

The Lagrangian is easily recast in terms of superspace and superfield. One finds for instance [18]

$$L_{GF} = - \int d\theta d\bar{\theta} \frac{1}{2} A^M g_{MN} A^N,$$

(45)

with $g$ a metric in the superspace, defined as

$$g_{MN} = \begin{cases} \\
\delta_{\mu\nu} & \text{if } M = \mu, \ N = \nu \\
-\xi_{\theta\bar{\theta}}/2 & \text{if } M = \theta, \ N = \bar{\theta} \\
\xi_{\theta\bar{\theta}}/2 & \text{if } M = \bar{\theta}, \ N = \theta \\
0 & \text{otherwise.} 
\end{cases}$$

(46)

The gauge-fixing term appears formally as a mass term in the theory. Observe that $\xi_{\theta\bar{\theta}}$ appears as a different normalization of the bosonic and fermionic coordinates that can be reabsorbed by a change of variables, in the same way as the speed of light can be eliminated in Minkowskian space.

The source term can also be written as [18]

$$L_{sources} = \int d\theta d\bar{\theta} A^M g_{MN} \mathcal{J}^N,$$

(47)

with

$$\mathcal{J}^\mu = \alpha^\mu - \bar{\theta} K^\mu - K^\mu \theta + \bar{\theta} \theta J^\mu$$

$$\frac{\xi_{\theta\bar{\theta}}}{2} \mathcal{J}^\theta = \beta + \theta M - i R \theta + \bar{\theta} \bar{\theta} L + \bar{\theta} \theta X$$

$$\frac{\xi_{\theta\bar{\theta}}}{2} \mathcal{J}^\bar{\nu} = \bar{\beta} - \bar{\theta} M + i R \bar{\theta} - \theta L + \bar{\theta} \theta X.$$  

(48)

The Yang-Mills term does not have such a nice superspace expression. One can however write it as [18]

$$L_{YM} = \int d\theta d\bar{\theta} \frac{1}{4} \bar{\theta} \theta (\mathcal{F}_{\mu\nu})^2,$$

(49)

with

$$\mathcal{F}_{\mu\nu} = \partial_{\mu} A^a_{\nu} - \partial_{\nu} A^a_{\mu} + g f^{abc} A^b_{\mu} A^c_{\nu}.$$  

(50)

After this review of the superspace formalism, let us now come to the interpretation of the supergauge symmetries described in Section II. The infinitesimal gauge transformations can actually be written in the very concise form:

$$\delta A^a_{\mu} = \partial_{\mu} \Lambda^a + g f^{abc} A^b_{\mu} \Lambda^c,$$

(51)

where $\Lambda$ is an arbitrary function of $x^M$. This transformation has exactly the same form as a standard gauge transformation in Yang-Mills theory. To make contact with the expressions of Section II, we just need to write the Taylor expansion of $\Lambda$ in powers of $\theta$ and $\bar{\theta}$:

$$\Lambda(x^M) = \lambda(x^\mu) + \bar{\theta} \eta(x^\mu) + \bar{\eta}(x^\mu) \theta + \bar{\theta} \theta \lambda(x^\mu).$$

(52)

This transformation not only gives the right gauge transformation of the physical fields $A$, $c$, $\bar{c}$ and $h$, but also give consistent gauge variations for their BRST and anti-BRST variations. Moreover, the Ward identities have a very natural interpretation. Indeed, the Yang-Mills part of the action (52) is manifestly invariant under the supergauge transformation. The gauge-fixing term (53) breaks this symmetry. However its variation under (52) is linear in the field,

$$\delta L_{GF} = - \int d\theta d\bar{\theta} A^a A^M \partial_{\mu} \Lambda^a,$$

(53)

and one can therefore deal with it in the corresponding Ward identity.

B. Curved superspace

If the superspace formulation of the massless CFDJ model has been known for quite some time, the corresponding formulation for the massive CF model has never been addressed before. This is the aim of this subsection. The important observation in this respect is that the BRST and anti-BRST transformations $s_m$ and $\bar{s}_m$ (that were associated with translations in the grassmanian sector in the massless case) do not anticommute.
Their anticommutator is indeed proportional to \( m_0 \) times a “rotation” in the Grassmanian coordinates. This is very similar to what happens when one studies the commutation relations of the rotations of the sphere in the limit of infinite radius, where the sphere approaches a plane. At leading order in the curvature, two rotations can be interpreted as translations (that commute) and the third corresponds to the rotation of the plane. We therefore expect that the theory in the presence of a mass term is associated with a superfield theory in a curved superspace, with curvature proportional to \( m_0 \).

The calculations in a curved superspace require the introduction of a formalism similar to the one of General Relativity. Actually all the standard formulas in a curved space have their superspace equivalent that differ by some signs. We followed the formalism and conventions of \([38, 59]\), except that we work with left-derivatives. In particular, we consider the supercovariant derivative of a supervector \( \mathcal{V}^N \):

\[
\mathcal{D}_M \mathcal{V}^N = \partial_M \mathcal{V}^N + \Gamma^N_{MP} \mathcal{V}^P
\]

with Christoffel symbols

\[
\Gamma^G_{AB} = \frac{(-1)^{bc}}{2} \left( (-1)^{ab+b} \partial_B g_{AD} + (-1)^{b} \partial_A g_{BD} - (-1)^{d(a+b)+d} \partial_D g_{AB} \right) g^{DC}.
\]

Here and below, the lowercase letters are 1 if the associated uppercase is fermionic and 0 otherwise. The covariant derivative \( \mathcal{D}_M \) should not be confused with the derivative \( D_M \) associated with the gauge group, which we used up to now.

As in standard Riemann geometry, superspace symmetries are described by the Killing vectors that satisfy the equation

\[
\mathcal{D}_M \mathcal{X}_N + (-1)^{mn} \mathcal{D}_N \mathcal{X}_M = 0.
\]

Taking the Lie bracket of two Killing vectors \( \mathcal{X} \) and \( \mathcal{Y} \),

\[
[\mathcal{X}, \mathcal{Y}]^M = \mathcal{X}^P \partial_P \mathcal{Y}^M - \mathcal{Y}^P \partial_P \mathcal{X}^M
\]

gives another Killing vector. The corresponding algebra is the Lie algebra of the isometry group of the superspace. Moreover, the Killing vectors generate the infinitesimal field transformations under isometries again by the Lie bracket:

\[
\mathcal{A}^M \rightarrow \mathcal{A}^M + \epsilon [\mathcal{X}, \mathcal{A}]^M.
\]

In the following we consider the metric

\[
g_{MN} = \begin{cases} 
\delta_{\mu\nu} & \text{if } M = \mu, \ N = \nu \\
-\frac{\epsilon_2}{2} (1 + m_0^2 \theta \bar{\theta}) & \text{if } M = \theta, \ N = \bar{\theta} \\
\frac{\epsilon_2}{2} (1 + m_0^2 \theta \bar{\theta}) & \text{if } M = \bar{\theta}, \ N = \theta \\
0 & \text{otherwise}
\end{cases}
\]

Observe first that it identifies with \([40]\) in the limit \( m_0 \rightarrow 0 \). Moreover, it is compatible with Poincaré and symplectic symmetry groups but does not respect the translation invariance in Grassmanian coordinates. From Eq. \([58]\) one can deduce that the non-zero Christoffel symbols are:

\[
\Gamma^{\theta}_{\theta\theta} = -\Gamma^{\bar{\theta}}_{\theta\theta} = -m_0^2 \theta \\
\Gamma^{\bar{\theta}}_{\theta\theta} = -\Gamma^{\theta}_{\theta\theta} = -m_0^2 \bar{\theta}.
\]

Using the expression for the scalar curvature of ref. \([38]\), one finds that the superspace has a finite and homogeneous scalar curvature \( R = -12m_0^2 / \epsilon \).

In order to verify that it does correspond to the CF model we calculated the most general Killing vector, obtaining:

\[
\mathcal{X}^\mu = a^\mu + R^{\mu\nu} x^\nu, \\
\mathcal{X}^\theta = \alpha (1 + m_0^2 \theta \bar{\theta}) + \theta \beta - \delta \bar{\theta}, \\
\mathcal{X}^{\bar{\theta}} = \bar{\alpha} (1 + m_0^2 \theta \bar{\theta}) + \bar{\theta} \beta + \delta \theta.
\]

The part proportional to \( a^\mu \) corresponds to translations and the one proportional to \( R^{\mu\nu} x^\nu \) to rotations in bosonic coordinates. The parts proportional to \( \beta \) and \( \bar{\beta} \) correspond to the symmetries \( t \) and \( \bar{t} \) respectively, while the part proportional to \( \delta \) corresponds to the ghost number. Finally, the parts proportional to \( \bar{\alpha} \) and \( \alpha \) correspond to BRST and anti-BRST symmetries respectively (observe that they become translations when \( m_0 \rightarrow 0 \)). By a straightforward calculation one can verify that the Lie bracket of the Killing vectors generate the Lie algebra of symmetries of the CF model as described in section \([IV]\). It is also an easy task to verify that the Killing vectors generate the right fields transformations for the fields \( A, c, \bar{c} \) and \( h \) as defined in section \([IV]\). Finally, one can verify that the only renormalizable lagrangian compatible with the symmetries of the curved superspace is that of the CF model.

\[ \text{VII. CONCLUSION} \]

In the present paper, we have shown that the CFDJ gauge fixing of Yang-Mills theory verifies four non trivial local and linear Ward identities. This result has many consequences. First, it allows the deduction of two non-renormalization theorems, that reduces the number of independent renormalization factors from five to three. Consequently, in perturbation theory, one has to calculate as many renormalization factors as in linear gauges. Moreover, as discussed in section \([IV]\) all these renormalization factors can be extracted from the 2-point functions alone. We expect that this simplifies considerably the perturbative calculations in Yang-Mills theory.

Another important result of the present paper is that the obtained Ward identities can be interpreted in the superfield formalism for Yang-Mills theory as consequences of supergauge transformations. The generalization to the theory with a CF mass term is simple and it is shown to
be equivalent in the superfield formalism to a curvature of the superspace in the grassmannian coordinates. Up to now, however, the superfields are constrained by the so-called “transversality condition”. As a consequence, one cannot use them as they stand in a functional integral. Let us stress that the existence of this supergauge symmetry reinforces our conviction that the superfield formalism is of prime importance in this field. This pushes one to look for the description of the model in terms of unconstrained superfields. This work is currently in progress.

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APPENDIX A: SOLVING SLAVNOV-TAYLOR IDENTITY

In this appendix we give some details of the derivation of equations (22) and (24). We first substitute the expression (18) into the Slavnov-Taylor identity (13) and analyze the terms quadratic in the sources $K, L, \bar{K}, \bar{L}$ and $M$. One easily finds that $\hat{s}A$ and $\hat{s}c$ do not depend arbitrarily on $c, A, \beta$ and $\alpha$, but only through $\bar{c}^a$ and $\bar{A}_\mu^a$ (see Eq. (23)). If one now study the terms linear in $K, L, \bar{K}, \bar{L}$ and $M$, one finds four independent constraints. The two relations
\begin{equation}
\hat{s}^2 A_\mu^a = 0,
\end{equation}
\begin{equation}
\hat{s}^2 c^a = 0,
\end{equation}
give the nilpotency in a particular sector. One also finds
\begin{equation}
\hat{s}\hat{s} A_\mu^a(x) = -Z_K \frac{\delta \hat{\Gamma}}{\delta A_\mu^a(x)} - \frac{\delta \hat{\Gamma}}{\delta \alpha_\mu^a(x)} - \frac{Z_L}{2} \left( \hat{D}_\mu \frac{\delta \hat{\Gamma}}{\delta h} \right)^a.
\end{equation}
Adding to this equations its conjugate one deduces:
\begin{equation}
\{\hat{s}, \hat{s}\} A_\mu^a = 0,
\end{equation}
which again expresses the nilpotency in another sector. The fourth relation reads
\begin{equation}
\hat{s}\hat{s} c^a = -Z_L \frac{\delta \hat{\Gamma}}{\delta c^a} + \frac{\delta \hat{\Gamma}}{\delta \beta^a} - \frac{Z_L}{2} \frac{\delta \hat{\Gamma}}{\delta h} f^{abc} \bar{c}^c.
\end{equation}

Now, the most general operators of dimension two, respecting Lorentz invariance, global color invariance, the symmetry (19), ghost number conservation, the definition (15) and nilpotency (A1) are written in (25).

Equations (A2,A3) take a simpler form if one introduces the variable $h^a$ defined in Eq. (20). Taking as independent variables $A_\mu, c, \bar{c}, h, \alpha, \beta$ and $\beta$ one deduces
\begin{equation}
\hat{s}\hat{s} c^a = \frac{\delta \hat{\Gamma}}{\delta \beta^a},
\end{equation}
\begin{equation}
\hat{s}\hat{s} A_\mu^a = -\frac{\delta \hat{\Gamma}}{\delta \alpha_\mu^a}.
\end{equation}
Note that at this level we have explicit expressions for the $\hat{s}$ and $\hat{s}$ variations of fields $A, c$ and $\bar{c}$ but not $h$. However, the left hand sides of equations (A5) can be computed without knowledge of the variations of $h$. Therefore, we have an explicit expression for the derivatives of $\hat{\Gamma}$ with respect to $\alpha$ and $\beta$ (and by conjugation of $\beta$). One can then integrate these trivial differential equations and obtain the dependence of $\hat{\Gamma}$ on these variables. As a result, we only need to find the part of $\hat{\Gamma}$ that does not depend on the sources. The dependence on $h$ (and then on $\bar{h}$) is trivially deduced from (14). The remaining part is obtained by imposing the invariance of $\hat{\Gamma}$ under $\hat{s}$. One finally obtains the result (24).

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