Quasinormal ringing and echoes of black bounces in a cloud of strings

Yi Yang,† Dong Liu,†‡ Zhaoyi Xu,†§ and Zheng-Wen Long†‡

†College of Physics, Guizhou University, Guiyang, 550025, China

In the string theory, the fundamental blocks of nature are not particles but one-dimensional strings. Therefore, a generalization of this idea is to think of it as a cloud of strings. Rodrigues et al. embedded the black bounces spacetime into string cloud, which demonstrates that the existence of string cloud makes the Bardeen black hole singular, while the black bounces spacetime remains regular [Phys. Rev. D 106, 084016 (2022)]. In this work, we study the quasinormal modes of black bounces spacetime surrounded by a cloud of strings to explore what gravitational effects are caused by string cloud. The quasinormal ringing of the regular black hole and traversable wormhole with string cloud are presented. Our results demonstrate that the black bounce spacetime with strings cloud is characterized by gravitational wave echoes as it transitions from regular black holes to wormholes.

I. INTRODUCTION

Recently, LIGO/VIRGO has made significant progress in the observation of gravitational waves [1–5]. In addition, the Event Horizon Telescope has also made a breakthrough in the imaging of black hole shadows [6]. These results validate the predictions of general relativity (GR) about black holes (BH). It also allows physicists to test new physical features beyond GR [7–10]. Gravitational wave spectroscopy plays a crucial role in the examination of new physical features beyond general relativity [11, 12]. For the gravitational wave signal generated by the binary merger, its late stage always decays in the form of the ringdown. It can usually be described using a superposition of complex frequency damping exponents, which are called quasinormal modes (QNMs) [13–15]. The detection of QNMs can serve as a tool to test GR predictions. Therefore, this makes gravitational wave detectors (LIGO/VIRGO and LISA, etc.) expected to detect some new physical features in the future, such as gravitational wave echoes and so on.

Under the framework of general relativity, with the perturbation of black hole spacetime, it must be accompanied by the emergence of quasinormal modes. Because as long as a black hole is perturbed, it responds to the perturbation by emitting gravitational waves, and the evolution of gravitational waves can be divided into three stages [16]: first, a relatively short initial burst of radiation; then a longer damped oscillation, which depends entirely on the parameters of the black hole; and finally the exponentially decays over a longer period of time. Among these three stages, people are generally most concerned about the middle quasinormal ringing stage. The QNMs of black holes have attracted extensive attention [17–36]. Although there are many indirect ways to identify black holes in the universe, gravitational waves emitted by perturbed black holes will carry unique “fingerprints” that allow physicists to directly identify the existence of black holes. In particular, Cardoso proposes that gravitational wave echoes can be used as a new feature of exotic compact objects [37]. Later, when people studied QNM in various spacetime backgrounds, gravitational wave echoes were discovered in the late stage of quasinormal ringing [38–64]. They look forward to using it to identify different compact objects. Therefore, we will study the quasinormal ringing of black bounces in a cloud of strings. We hope to provide some direction for probing black bounces with strings cloud experimentally after obtaining its relevant basic properties through the study of QNMs.

String theory points out that the fundamental blocks of nature are not particles but one-dimensional strings. Therefore, a generalization of this basic idea is to think of it as a cloud of strings. On the other hand, the black hole in general relativity usually has singularities, which forces theoretical physicists to constantly try to avoid the occurrence of singularities. A black hole without singularities is called a regular black hole (RBH). Bardeen was the first theoretical physicist to propose regular black hole [65]. Ayon-Beato et al. interpret it as a black hole solution for the Einstein equations under the presence of nonlinear electrodynamics [66]. Letelier proposed a black hole solution in 1979, which is surrounded by the string cloud [67]. The string cloud is a closed system, therefore its stress-energy tensor is conserved. Sood et al. proposed an RBH surrounded by the string cloud, but the string cloud makes this black hole solution no longer regular [68]. It would be very fascinating if string cloud would not insert singularities in the RBH. Simpson and Visser proposed a type of regular black hole known as black bounces [69]. The difference between this solution and the standard RBH is that it is achieved by modifying the black hole area, and it allows a nonzero radius throat in \( r = 0 \). Many studies have been done on black bounces in-
II. A BRIEF REVIEW OF THE BLACK BOUNCES IN STRINGS CLOUD

To gain black bounces in a cloud of string, Rodrigues et al. [76] considers the following Einstein equations

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa^2 T_{\mu\nu} = \kappa^2 T_{\mu\nu}^M + \kappa^2 T_{\mu\nu}^{CS}, \quad (1) \]

where

\[ T_{\mu\nu}^M = T_{\mu\nu}^{SV} + T_{\mu\nu}^{NMC}, \quad (2) \]

where \( T_{\mu\nu}^{SV} \) denotes the stress-energy tensor related to the Simpson-Visser spacetime, and the information about the non-minimum coupling between the string cloud and the Simpson-Visser spacetime is included in the stress-energy tensor \( T_{\mu\nu}^{NMC} \). Furthermore, \( T_{\mu\nu}^{CS} \) in Eq. (1) represents the stress-energy tensor of the string cloud, which can be written as

\[ T_{\mu\nu}^{CS} = \frac{\rho \sum_{\alpha} \Sigma_{\alpha} \Sigma_{\alpha}}{8\pi \sqrt{-\gamma}}, \quad (3) \]

where \( \rho \) represents the density of the string cloud. \( T_{\mu\nu}^{CS} \) must satisfy the following conservation laws

\[ \nabla_{\mu} T_{\nu}^{CS} = \nabla_{\mu} \left( \frac{\rho \sum_{\alpha} \Sigma_{\alpha} \Sigma_{\alpha}}{8\pi \sqrt{-\gamma}} \right) \]

\[ = \nabla_{\mu} (\rho \Sigma_{\alpha} \Sigma_{\alpha}) \frac{\Sigma_{\alpha} \Sigma_{\alpha}}{8\pi \sqrt{-\gamma}} + \rho \Sigma_{\alpha} \Sigma_{\alpha} \nabla_{\mu} \left( \frac{\Sigma_{\alpha} \Sigma_{\alpha}}{8\pi \sqrt{-\gamma}} \right) = 0. \quad (4) \]

By solving the above Einstein field equations, Rodrigues et al. obtain the following black bounces in string cloud [76]

\[ ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - R^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (5) \]

where

\[ f(r) = 1 - L - \frac{2m}{\sqrt{a^2 + r^2}}, \quad R = \sqrt{a^2 + r^2}. \quad (6) \]

If \( a = 0 \), this spacetime can be reduced to the Letelier spacetime, and this spacetime can be reduced to the Simpson-Visser spacetime when \( L = 0 \). If \( L = 1 \), this spacetime will have no event horizon, so the range of \( a \) is \( 0 < a < 1 \). In addition, the value of the parameter \( a \) has a critical value

\[ a_c = \frac{2M}{\sqrt{1 - 2L + L^2}}. \quad (7) \]

The black bounce in string cloud will correspond to a different spacetime for different \( a \): 1) regular black hole with string cloud for \( 0 < a < a_c \); 2) one-way wormhole with string cloud for \( a = a_c \); 3) traversable wormhole with string cloud for \( a > a_c \).

III. MASTER WAVE EQUATION

The covariant equations of scalar field perturbation and electromagnetic field perturbation can be written as

\[ \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Psi) = 0, \quad (8) \]

\[ \frac{1}{\sqrt{-g}} \partial_\mu (F_{\mu\nu} g^{\mu\nu} g^\sigma \sqrt{-g}) = 0, \quad (9) \]

where \( F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} \). In order to achieve separation of variables, for the scalar field perturbation we adopt the following ansatz

\[ \Psi(t, r, \theta, \phi) = \sum_{l,m} \psi(t, r) Y_{lm}(\theta, \phi) / R(r), \quad (10) \]

where \( R(r) \) is the function of radial coordinate \( r \) and the parameter \( a \). After separating the variables, we can simplify equations (1) and (2) to the following form

\[ \frac{d^2 \psi}{dt^2} - \frac{d^2 \psi}{dr^2} + V(r) \psi = 0, \quad (11) \]

where tortoise coordinate \( r_* \) can be defined by

\[ dr_* = \frac{1}{f(r)} dr = \frac{1}{1 - L - \frac{2M}{\sqrt{a^2 + r^2}}} dr. \quad (12) \]

Moreover, the effective potentials can be written as
where \( s = 0 \) represents scalar field perturbation and \( s = 1 \) represents electromagnetic field perturbation.

### IV. Quasinormal Modes of Black Bounces in a Cloud of Strings

In this section, we plan to numerically solve the wave equation for black bounces in a cloud of strings to obtain the time-domain profiles (TDF). We use the light cone coordinates

\[
\begin{align*}
  u &= t - r_*, \\
  v &= t + r_* ,
\end{align*}
\]

Then Eq. (11) can be written as

\[
\frac{\partial^2}{\partial u \partial v} \psi (u,v) + \frac{1}{4} V(r) \psi (u,v) = 0 .
\]

We adopt the discretization scheme suggested by Gundlach and Price et al. \cite{77, 78}

\[
\psi_N = \psi_E + \psi_W - \psi_S - \Delta u \Delta v V \left( \frac{\psi_W + \psi_E}{8} \right) + \mathcal{O} (\Delta^4) .
\]

where \( S = (u,v), W = (u + \Delta u, v), E = (u, v + \Delta v), N = (u + \Delta u, v + \Delta v) \). Moreover, we use the Gaussian initial pulse \cite{79, 80, 81} for two null surface, i.e.

\[
\begin{align*}
  &\psi (u = u_0, v) = \exp \left[ \frac{-(v-v_0)^2}{2\sigma^2} \right] , \\
  &\psi (u, v = v_0) = 0 .
\end{align*}
\]

In our work, we adopt \( \sigma = 3, v_0 = 10 \).

For the frequency domain, we use the WKB method to calculate the QNM frequencies. Schutz and Will first used the WKB method to calculate the quasi-normal scale of black holes in 1985 \cite{82}, and later they extended it to the third-order WKB method with higher accuracy \cite{83}, and Konoplya extended it to the sixth-order on their basis \cite{84, 85}. When using the Pade approximation \cite{86, 87}, it can even be generalized to the more accurate thirteenth order. The higher-order WKB methods take the form \cite{88}

\[
\begin{align*}
  \omega^2 &= V_0 + A_2 (K^2) + A_4 (K^2) + A_6 (K^2) + \cdots \\
  &- i K \sqrt{-2V_2} \left( 1 + A_3 (K^2) + A_5 (K^2) + A_7 (K^2) \ldots \right) ,
\end{align*}
\]

where \( K \) denotes half-integer values. The correction term \( A_6 (K^2) \) depends on the derivative of the effective potential at its maximum value.

In Table I, we give the fundamental QNM frequencies \( (l = 1, n = 0) \) of scalar field perturbations for black bounces in a cloud of strings with \( M = 0.5 \). One can see that when \( L \) is fixed and \( a \) is changed, both the real and imaginary parts of the QNM frequencies decrease with the increase of \( a \), which implies that its actual oscillation frequencies decrease with the increase of \( a \), while a decrease in its damping rate means that its decay time becomes longer as \( a \) increases. When \( a \) is fixed, both the real and imaginary parts of the QNM frequency are also decreased, which indicates that \( L \) and \( a \) have similar contributions to the QNM frequencies for the scalar field perturbation to black bounces in a cloud of strings. In Table II, we give the fundamental QNM frequencies \( (l = 1, n = 0) \) of electromagnetic field perturbations for black bounces in a cloud of strings with \( M = 0.5 \). Unlike the case of scalar field perturbations, when \( L \) is fixed and \( a \) is changed, the real part of the QNM frequency increases as the parameter \( a \) increases and the imaginary part decreases as the parameter \( a \) increases. These results show that its true oscillation frequency increases with the increase of \( a \), and its decay time increases with the increase of \( a \). When \( a \) is fixed and \( L \) is changed, the results show that the contribution of \( L \) is similar to that of the scalar field.

In Fig. 1, we present the effective potential of the scalar field perturbation for different \( a \) with \( M = 0.5, l = 1, L = 0.1 \) and for different \( L \) with \( M = 0.5, l = 1, a = 0.1 \) as the function of the tortoise coordinate \( r_* \). From Fig. 1, we can clearly see that the effective potential is a single peak, which indicates that the black bounce in a cloud of strings at this time is a black hole space-time. The results show that the effective potential is very sensitive to changes in \( L \), but not particularly sensitive to changes in \( a \).

In Fig. 2, we show the The time-domain profiles of the scalar field perturbation (left panel) for different \( a \) with \( M = 0.5, l = 1, L = 0.1 \), and the time-domain profiles of the electromagnetic field perturbation (right panel) for different \( a \) with \( M = 0.5, l = 1, L = 0.1 \). In Fig. 2, the blue solid line represents \( a = 0.1 \), the black solid line represents \( a = 0.6 \), and the red solid line represents \( a = 1.1 \). The corresponding effective potential is given in Fig. 1. One can see that the decay time of quasinormal ringing is the longest when \( a \) is larger, which indicates that its damping rate should be smaller for the larger \( a \). In other words, the imaginary part of its quasinormal modes frequency is smaller. Such a result is a good validation of our results shown in Table I, i.e. the imaginary parts of the quasinormal mode frequencies decrease as \( a \) increases. In Fig.
TABLE II. Fundamental QNM frequencies ($a = 0$)

| $L$ | $a = 0.1$ | $a = 0.2$ | $a = 0.3$ | $a = 0.4$ | $a = 0.5$ |
|-----|------------|------------|------------|------------|------------|
| $L = 0.1$ | 0.496725 - 0.157866i | 0.496685 - 0.157016i | 0.496617 - 0.155581i | 0.496513 - 0.153536i | 0.496354 - 0.150846i |
| $L = 0.3$ | 0.335962 - 0.0952796i | 0.33595 - 0.0949686i | 0.33593 - 0.094447i | 0.335901 - 0.09371i | 0.335857 - 0.0927514i |
| $L = 0.5$ | 0.199901 - 0.0484858i | 0.199901 - 0.0484048i | 0.199897 - 0.0482694i | 0.199892 - 0.0480792i | 0.199884 - 0.0478334i |
| $L = 0.7$ | 0.0915345 - 0.0173945i | 0.0915324 - 0.0173771i | 0.0915342 - 0.0173771i | 0.0915338 - 0.0173752i | 0.0915333 - 0.0173209i |
| $L = 0.9$ | 0.0173483 - 0.0017282i | 0.0173483 - 0.0017282i | 0.0173483 - 0.0017282i | 0.0173483 - 0.0017282i | 0.0173483 - 0.0017282i |

TABLE II. Fundamental QNM frequencies ($l = 1, n = 0$) of electromagnetic field perturbations for black bounces in a cloud of strings with $M = 0.5$.

| $L$ | $a = 0.1$ | $a = 0.2$ | $a = 0.3$ | $a = 0.4$ | $a = 0.5$ |
|-----|------------|------------|------------|------------|------------|
| $L = 0.1$ | 0.428078 - 0.150395i | 0.4285 - 0.149661i | 0.429193 - 0.148411i | 0.430141 - 0.146608i | 0.431316 - 0.144197i |
| $L = 0.3$ | 0.299214 - 0.0917233i | 0.299349 - 0.0914366i | 0.299574 - 0.0909544i | 0.299883 - 0.0902697i | 0.300273 - 0.0893731i |
| $L = 0.5$ | 0.184002 - 0.0471784i | 0.184031 - 0.0471006i | 0.18408 - 0.0469704i | 0.184147 - 0.0467872i | 0.184233 - 0.0465499i |
| $L = 0.7$ | 0.0870852 - 0.0171203i | 0.087088 - 0.01711i | 0.0870928 - 0.0170928i | 0.0870994 - 0.0170686i | 0.0871079 - 0.0170375i |
| $L = 0.9$ | 0.0170619 - 0.00191716i | 0.0170619 - 0.00191716i | 0.0170619 - 0.00191716i | 0.017062 - 0.00191652i | 0.0170621 - 0.00191613i |

FIG. 1. The effective potential of the scalar field perturbation for different $a$ (left panel) with $M = 0.5, l = 1, L = 0.1$ and for different $L$ (right panel) with $M = 0.5, l = 1, a = 0.1$. 
FIG. 2. The time-domain profiles of the scalar field perturbation (left panel) for different $a$ with $M = 0.5, l = 1, L = 0.1$. The time-domain profiles of the electromagnetic field perturbation (right panel) for different $a$ with $M = 0.5, l = 1, L = 0.1$.

FIG. 3. The time-domain profiles of the scalar field perturbation (left panel) for different $L$ with $M = 0.5, l = 1, a = 0.1$. The time-domain profiles of the electromagnetic field perturbation (right panel) for different $L$ with $M = 0.5, l = 1, a = 0.1$.

FIG. 4. The effective potential and echoes of the scalar field perturbation to black bounces in a cloud of strings with $M = 0.5, l = 1, L = 0.1, a = 1.112$. 
In Fig. 5, we present the effective potential and corresponding quasinormal ringing of the scalar field perturbation to black bounces in a cloud of strings with $M = 0.5, l = 1, L = 0.1, a = 1.12$. One can see two peaks in the effective potential. It implies that the black bounce in a cloud of strings has become a wormhole spacetime. Due to the large distance between the two peaks of the effective potential, the two peaks can scatter waves independently. Therefore, the gravitational wave will be reflected by both peaks and will be repetitively reflected in the potential well, and it will also have a small part of the wave passing through the time barrier. This allows observers to see the gravitational wave echoes. As shown in Fig. 4 right panel. From the right panel of Fig. 4, one can see weaker echoes of gravitational waves after the initial quasinormal ringing. Since the potential well at this time is wider, the time for the gravitational wave to

In Fig. 6, we present the effective potential and corresponding quasinormal ringing of the scalar field perturbation to black bounces in a cloud of strings with $M = 0.5, l = 1, L = 0.1, a = 1.13$. We can clearly see the quasinormal ringing after the initial pulse, which represents the unique “fingerprint” of black bounce in a cloud of strings. In addition, we can find that the contribution of $L$ to the QNM frequency for black bounces in a cloud of strings is similar to that of $a$, but the NQM frequencies are more sensitive to $L$ than $a$. As we expected, there is no gravitational wave echoes signal here. In addition, late-time tails are also shown after quasinormal ringing.

For black bounce in a cloud of strings, when the parameter $a$ increases, it can change from a black hole to a wormhole. In Fig. 4, we present the effective potential and corresponding quasinormal ringing of the scalar field perturbation to black bounces in a cloud of strings with $M = 0.5, l = 1, L = 0.1, a = 1.112$. One can see that when $a$ increases to the threshold, there are two peaks in the effective potential. It implies that the black bounce in a cloud of strings at this time has become a wormhole spacetime. Due to the large distance between the two peaks of the effective potential, the two peaks can scatter waves independently. Therefore, the gravitational wave will be reflected by both peaks and will be repetitively reflected in the potential well, and it will also have a small part of the wave passing through the time barrier. This allows observers to see the gravitational wave echoes. As shown in Fig. 4 right panel. From the right panel of Fig. 4, one can see weaker echoes of gravitational waves after the initial quasinormal ringing. Since the potential well at this time is wider, the time for the gravitational wave to
reach another peak from one peak will be relatively long. Therefore, we see a long time interval between the first gravitational wave echo signal and the initial quasinormal ringing. As $a$ increasing, we can see from Fig. 5 and Fig. 6 that the peak value of the effective potential hardly changes, but its potential well width becomes smaller and smaller. This means that the time required for gravitational waves to reach another peak from one peak becomes shorter so that the gravitational wave echo signal appears sooner after the initial quasinormal ringing, and the time interval between gravitational wave echoes is smaller when $a$ is larger.

V. CONCLUSION

In this work, we study the quasinormal ringing for the regular black hole and traversable wormhole with string cloud. The distinctive feature of the black bounces in a cloud of strings is that when parameter $a$ reaches a certain threshold, it can transform from a black hole to a wormhole. For a regular black hole with a cloud of strings, we calculated its quasinormal frequencies under scalar field perturbation and the electromagnetic field perturbation using the WKB method. Our results show that the real oscillation frequency decreases with the increase of the parameter $a$ under the scalar field perturbation, and the decay time of the gravitational wave is longer with the increase of the parameter $a$. However, its real oscillation frequency increases with the increase of the parameter $a$ under the electromagnetic field perturbation, and its effect on the decay time is similar to that of the scalar field. These conclusions are well verified by the results of quasinormal ringing. In addition, from the time-domain profiles, we can find that a regular black hole with a cloud of strings is very sensitive to the change of parameter $L$. For wormholes with a cloud of string, we obtained clear gravitational wave echo signals after initial quasinormal ringing. Furthermore, as the parameter $a$ increases, the time interval between gravitational wave echoes becomes smaller and smaller.

In addition, by comparing with the work of Churilova and Stuchlik [89], we find that the strings cloud has the following effects on the black bounce spacetime:

1) It extends the parameter range of black bounce spacetime keeping as a regular black hole.

2) The presence of the strings cloud depresses the peak of the barrier.

3) It reduces the real oscillation frequency of gravitational waves and reduces the damping rate of gravitational wave signals.

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