IS GRB 050904 A SUPERLONG BURST?

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ABSTRACT

By considering synchrotron radiative processes in the internal shock model and assuming that all internal shocks are nearly equally energetic, we analyze the gamma-ray burst (GRB) emission at different radii corresponding to different observed times. We apply this model to GRB 050904 and find that our analytical results can provide a natural explanation for the multiband observations of GRB 050904. This suggests that the X-ray flare emission and the optical emission of this burst could have originated from internal shocks due to collisions among nearly equally energetic shells ejected from the central engine. Thus, GRB 050904 appears to be a burst with superlong central engine activity.

Subject headings: gamma rays: bursts — hydrodynamics — relativity — shock waves

1. INTRODUCTION

The gamma-ray burst GRB 050904 was an explosive event at a redshift of $z = 6.29 \pm 0.01$, which has been measured through various methods (Kawai et al. 2005; Price et al. 2006; Haislip et al. 2006). After the Swift trigger (Cummings et al. 2005), multiwavelength observations of this high-redshift burst were performed. The $\gamma$-ray isotropic equivalent energy of this burst $E_{\gamma,iso}$ was between $6.6 \times 10^{53}$ and $3.2 \times 10^{54}$ ergs (Cusumano et al. 2006). Boër et al. (2006) observed the optical emission at times of 86–1666 s. The optical afterglow and its spectrum were analyzed quickly (Haislip et al. 2006; Tagliaferri et al. 2005). A break of the optical afterglow light curve was also observed at about 2.6 ± 1.0 days (Tagliaferri et al. 2005). The optical emission appears to be understood in either the late internal shock model or the reverse-forward shock model (Wei et al. 2006).

One of the most remarkable features of this burst is a long-lasting variability of the X-ray emission showing several X-ray flares (XRFs). Watson et al. (2006) and Cusumano et al. (2006) suggested that this variability may be due to a long-lasting activity of the central engine. The XRFs were also found in some other GRBs (Nousek et al. 2006; O’Brien et al. 2006). Burrows et al. (2005), Fan & Wei (2005), Zhang et al. (2006), and Wu et al. (2005) assumed late central engine activities to interpret the XRFs. Plausible origin models of XRFs have recently been proposed, e.g., the magnetic activity of a newborn millisecond pulsar (Dai et al. 2006), fragmentation of a neutron star by a black hole in a compact object binary (Faber et al. 2006), and accretion disk (Perna et al. 2006), and magnetic-barrier-driven modulation of an accretion disk (Proga & Zhang 2006). However, there has not yet been any explicit and detailed investigation on this highly variable light curve. Enlightened by the internal shock model for interpreting the $\gamma$-ray emission (Rees & Mészáros 1994) and prompt optical emission (Mészáros & Rees 1999), we assume that all the highly variable X-ray emission (even at very late times) originates from internal shocks. These shocks occur when many faster shells with equal energy and equal mass catch up with a slower shell with much more kinetic energy and mass. By fitting the peaks of the X-ray and optical light curves, we find that these internal shocks can lead to the observed X-ray and optical emission. We describe an internal-shock model in § 2 and fit the peaks of X-ray and optical light curves of GRB 050904 in § 3. We summarize our results in § 4.

2. MODEL

We assume two cold shells labeled 1 and 4, whose isotropic kinetic energies are $E_1$ and $E_4$, widths are $\Delta_{1,0}$ and $\Delta_{4,0}$ in the cosmological rest frame, and Lorentz factors are $\gamma_1$ and $\gamma_4$ ($\gamma_4 > \gamma_1 \gg 1$), respectively. The shells come from the central engine with time lag $T$ (also in the rest frame). Shell 4 catches up with shell 1 at radius $r_{int} \sim 2\gamma_1^3 cT$, and then two internal shocks occur, that is, a forward shock that propagates into shell 1 and a reverse shock that propagates into shell 4. Four regions are divided by the reverse and forward shocks: unshocked shell 1 (region 1), shocked gas of shell 1 (region 2), shocked gas of shell 4 (region 3), and unshocked shell 4 (region 4), with a contact discontinuity (CD) surface separating region 2 and region 3. Figure 1 gives a sketch of this model. The number densities of region 1 and region 4 at radius $r$ are $n_1 = E_1/(4\pi\gamma_1^2 \Delta_{1,0} m_p c^2)$ and $n_4 = E_4/(4\pi\gamma_4^2 \Delta_{4,0} m_p c^2)$, respectively. To produce plenty of X-rays and $\gamma$-rays, the reverse-shocked materials are required to be relativistic, i.e., $\gamma_{34} \sim (1/\sqrt{2}(\gamma_4/\gamma_1)^{3/2})^{1/2} \sim 1$ ($\gamma_2$ is the Lorentz factor between regions 2 and 3), and the number density ratio is defined as $f = n_4/n_1$.

To calculate the width of the coasting shell, we define the spreading radius $R_{\Delta} \equiv \gamma^2 \Delta_0$ (Mészáros & Rees 1993; Piran 1993). For the internal shocks of our interest, if $r < \min(R_{\Delta,1}, R_{\Delta,4}) = 1.0 \times 10^{15}\gamma_{1,1.5} \Delta_{1,0,12} \text{ cm}$, $R_{\Delta,1} = 1.0 \times 10^{16}\gamma_{1,4.3} \Delta_{4,0,10} \text{ cm}$, the widths of the shells can be considered to be constant, and for $r > \max(R_{\Delta,1}, R_{\Delta,4})$, the widths should be regarded as $r/\gamma_1^2$ and $r/\gamma_4^2$ for the spreading of the shells. The conventional notation $Q_k = Q/10^{44}$ with cgs units is used in this paper, except for special explanations. We now consider these two cases.

2.1. No-Spreading Case

At radius $r < \min(R_{\Delta,1}, R_{\Delta,4})$, for a certain set of parameters $\Delta_{1,0,12} = \Delta_{4,0,10} = E_{1,54} = E_{4,51} = \gamma_1 = \gamma_4 = 1$, one obtains $\gamma_{1,4}^2 f \sim 2.5 \times 10^{-2} E_1^{-1} E_{4,51}^{-1} \Delta_{1,0,12}^{-1} \Delta_{4,0,10}^{-1} < 1$ (where $\gamma_{1,4}$ is the relative Lorentz factor between regions 4 and 1), implying that the forward shock is Newtonian, and $\gamma_{1,4}^2 \gamma_{1,4} f \sim 2.5 \times 10^6 E_{1,54} E_{4,51}^{-1} \Delta_{1,0,12} \Delta_{4,0,10}^{-1} \gamma_4^{-1} \gamma_1^{-1} \gamma_4^{-1} \gamma_1^{-1} > 1$, which shows that the reverse shock is relativistic. Using the shock conditions (Blandford & McKee 1976), and maintaining equality of the pressures and Lorentz factors of the materials at the two sides of the contact...
discontinuity (Sari & Piran 1995), we can obtain the number densities $n_2$ and $n_3$ and energy densities $e_2$ and $e_3$ of the shocked regions in the comoving frame,

$$n_2 \simeq 1.0 \times 10^{10}(1 + z)^2 E_{4,54} \Delta_{-1,15}^{-1} \gamma_1^{-6} \gamma_2^{-2} \text{cm}^{-3},$$

$$n_3 \simeq 9.3 \times 10^6(1 + z)^2 E_{4,54} \Delta_{-1,15}^{-1} \gamma_1^{-5} \gamma_2^{-2} \text{cm}^{-3},$$

$$e_2 = e_3 \simeq 2.2 \times 10^5(1 + z)^2 E_{4,54} \Delta_{-1,15}^{-1} \gamma_1^{-6} \gamma_2^{-2} \text{ergs cm}^{-3}. $$

The Lorentz factor $\gamma = \gamma_3 \simeq 31.6 \gamma_1$. Here $t_0$ is the observer’s time.

Assuming that the reverse shock last from $r_{\text{int}}$ to $r_{\text{int}} + \delta r$, we find $\delta r \simeq 1.0 \times 10^{12} \gamma_1 \gamma_3 \Delta_{-1,15}^{-1} \text{cm}$, which corresponds to a pulse from the beginning to the peak. In the observer’s frame $[dt_0 = (1 + z)dr/(2\gamma c)]$, the time this lasts is

$$\delta t_0 \simeq 0.17(1 + z) \Delta_{-1,15}^{-1} \text{s}. $$

By considering the synchrotron emission at the reverse shock crossing time, which corresponds to the peak emission time, we obtain the characteristic frequencies $\nu_{m,2,p}$ and $\nu_{m,3,p}$, cooling frequencies $\nu_{c,2,p}$ and $\nu_{c,3,p}$, synchrotron self-absorption frequencies $\nu_{a,2,p}$ and $\nu_{a,3,p}$, and the maximum flux densities $f_{\nu,\text{max},2,p}$ and $f_{\nu,\text{max},3,p}$ in the observer’s frame (see Zou et al. 2005 for original formulae of synchrotron emission),

$$\nu_{m,2,p} \simeq 1.3 \times 10^{11} E_{4,54}^{2/3} E_{4,54}^{2/3} \Delta_{4,10}^{5/2} \frac{1}{6 \gamma_1 \gamma_3 \Delta_{-1,15}^{-2} \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Hz}},$$

$$\nu_{m,3,p} \simeq 1.5 \times 10^{17} E_{4,54}^{1/2} \Delta_{4,10}^{5/2} \frac{1}{6 \gamma_1 \gamma_3 \Delta_{-1,15}^{-2} \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Hz}},$$

$$\nu_{c,2,p} = \nu_{c,3,p} \simeq 1.2 \times 10^{12}(1 + z)^{-4} E_{4,54}^{-3/2} \times \Delta_{4,10}^{5/2} \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Hz},$$

$$\nu_{a,2,p} \simeq 3.2 \times 10^{14}(1 + z)^{-5/18} E_{4,54}^{-7/18} \Delta_{4,10}^{7/18} \frac{1}{6 \gamma_1 \gamma_3 \Delta_{-1,15}^{-2} \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Hz}},$$

$$\nu_{a,3,p} \simeq 1.2 \times 10^{14}(1 + z)^{-1/3} E_{4,54}^{-1/3} \Delta_{4,10}^{1/3} \times \gamma_1^{-5/18} \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Hz},$$

$$f_{\nu,\text{max},2,p} \simeq 8.0 \times 10^8 D_{28}^{-2} E_{4,54}^{1/2} \Delta_{-1,15}^{-1/2} \times \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Jy},$$

$$f_{\nu,\text{max},3,p} \simeq 0.47(1 + z)^2 D_{28}^{-2} E_{4,54}^{1/2} \Delta_{-1,15}^{-1/2} \times \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Jy},$$

where $\zeta = 6(p - 2)/(p - 1)$, $p$ is the spectral index of the shock-accelerated electrons, $D$ is the luminosity distance from the burst to the observer (as a function of redshift $z$), and $\epsilon_e$ and $\epsilon_B$ are the fractions of the internal energy density that were carried by electrons and magnetic fields, respectively. In this paper, $p$ is taken to be 2.2, and $\epsilon_e$ and $\epsilon_B$ are equal for both region 2 and region 3. Note that the expressions of $\nu_{a,2,p}$ and $\nu_{a,3,p}$ are only valid in the cases of $\nu_{c,2,p} < \nu_{m,2,p}$ and $\nu_{c,3,p} < \nu_{m,3,p}$.

From the above equations, we see that $\nu_{c,2,p} < \nu_{m,2,p}$ and $\nu_{a,3,p} < \nu_{c,3,p}$, for typical parameters. The flux density in the fast-cooling case at the peak time is then given by

$$f_{\nu,2,p} \simeq 2.4 \times 10^{16} E_{4,54}^{2} \Delta_{4,10}^{2} \times \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Jy},$$

for $\nu < \nu_{c,2,p} < \nu_{a,2,p}$,

$$f_{\nu,2,p} \simeq 7.1 \times 10^{10} E_{4,54}^{2} \Delta_{4,10}^{2} \times \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Jy},$$

for $\nu_{c,2,p} < \nu < \nu_{a,2,p}$,

$$f_{\nu,2,p} \simeq 2.0 \times 10^{-3} E_{4,54}^{2} \Delta_{4,10}^{2} \times \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Jy},$$

for $\nu_{a,2,p} < \nu < \nu_{m,2,p}$,

$$f_{\nu,3,p} \simeq 3.3 \times 10^{2} E_{4,54}^{2} \Delta_{4,10}^{2} \times \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Jy},$$

for $\nu < \nu_{c,3,p} < \nu_{a,3,p}$,

$$f_{\nu,3,p} \simeq 9.6 \times 10^{9} E_{4,54}^{2} \Delta_{4,10}^{2} \times \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Jy},$$

for $\nu_{c,3,p} < \nu < \nu_{a,3,p}$,

$$f_{\nu,3,p} \simeq 1.6 \times 10^{-2} E_{4,54}^{2} \Delta_{4,10}^{2} \times \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Jy},$$

for $\nu_{a,3,p} < \nu < \nu_{m,3,p}$,

$$f_{\nu,3,p} \simeq 5.2 \times 10^{-2} E_{4,54}^{2} \Delta_{4,10}^{2} \times \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Jy},$$

for $\nu < \nu_{a,3,p} < \nu_{m,3,p}$,

$$f_{\nu,3,p} \simeq 5.2 \times 10^{-2} E_{4,54}^{2} \Delta_{4,10}^{2} \times \gamma_1^{-5} \gamma_2^{-2} \times 10^{12} \Delta_{4,10}^{5/2} \text{Jy},$$

for the reverse shock emission. 

![Diagram of IS GRB 050904 A SUPERLONG BURST?](image-url)
2.2. Spreading Case

In the case of $r > \max (R_2, R_3)$, the widths of the shells become $\Delta_1 \approx r/\gamma_1^2$ and $\Delta_4 \approx r/\gamma_4^2$, because of the spreading effect. For the same parameters as in § 2.1, $\gamma_1^2 f_2 \geq 0.5 E_1^{1/3} E_4^{5/3} \gamma_1^{1.5} \gamma_3^{3/4} < 1$, showing that the forward shock can be considered as Newtonian approximately, and $\gamma_1^2 f_2 \approx 2.5 \times 10^4 E_4^{1/3} E_3^{1/4} \gamma_1^{2.5} \gamma_4^2 \gg 1$, implying that the reverse shock is still relativistic.

Using the shock conditions as in § 2.1, we obtain

$$n_2 \approx 1.7 \times 10^8 (1 + z)^2 E_1^{1.5} \gamma_1^{2.5} \gamma_3^{3/2} \mathrm{cm}^{-3},$$

$$n_3 \approx 1.5 \times 10^8 (1 + z)^2 E_4^{1.5} \gamma_1^{1.5} \gamma_4^{3/4} \mathrm{cm}^{-3},$$

$$e_2 = e_3 \approx 3.7 \times 10^8 (1 + z)^3 E_4^{2.5} \gamma_1^{1.5} \gamma_4^{3/2} \mathrm{ergs cm}^{-3},$$

$$\gamma_2 = \gamma_3 \approx 31.6 \gamma_1^{1.5}.$$

The distance for the reverse shock to cross shell 4 is $\Delta_r \approx 6 \times 10^{12} (1 + z)^{-1} \gamma_4^3 \gamma_1^{1.5} \mathrm{cm}$, and the rising time of the pulse in the observer’s frame is $\Delta_\tau \approx 0.2 \gamma_4^3 \gamma_1^{1.5}$.

The corresponding emission values at the peak time are given by

$$\nu_{m, 2, p} \approx 1.6 \times 10^{11} (1 + z)^{1/2} E_2^{1/4} E_4^{1/2} \mathrm{Hz},$$

$$\nu_{m, 3, p} \approx 2.0 \times 10^{17} (1 + z)^2 E_4^{1/4} \mathrm{Hz},$$

$$\nu_{c, 2, p} = \nu_{c, 3, p} \approx 1.5 \times 10^{15} (1 + z)^{-7/2} E_4^{3/2} \mathrm{Hz},$$

$$\nu_{a, 2, p} \approx 6.4 \times 10^{16} (1 + z)^{1/2} E_4^{1/2} \mathrm{Hz},$$

$$\nu_{a, 3, p} \approx 2.0 \times 10^{13} (1 + z)^{1/2} E_4^{5/4} \mathrm{Hz},$$

$$\nu_{f_{\max}, 2, p} \approx 0.033 (1 + z)^{5/2} D_2^{1/2} E_1^{1/4} \mathrm{Hz},$$

$$\nu_{f_{\max}, 3, p} \approx 0.6(1 + z)^{5/2} D_2^{3/2} E_1^{1/2} \mathrm{Hz}.$$

Note that the expressions of $\nu_{m, 2, p}$ and $\nu_{m, 3, p}$ are valid only in the cases of $\nu_{m, 2, p} < \nu_{c, 2, p}$ and $\nu_{m, 3, p} < \nu_{c, 3, p}$, respectively.

In the case of $\nu_{m, 2, p} < \nu_{c, 2, p}, \nu_{m, 2, p} < \nu_{c, 3, p}$, the corresponding flux density of the forward shock emission is

$$f_{\nu_{c, 2, p}} \approx 2.1 \times 10^7 (1 + z) D_2^{2} E_1^{1/4} E_4^{5/4}$$

$$\times e_{c, 0, -0.5} \gamma_1^{2} \gamma_3^{3/2} \mathrm{Jy},$$

$$f_{\nu_{m, 2, p}} \approx 5.2 \times 10^7 (1 + z)^{1/4} D_2^{2} E_1^{1/4}$$

$$\times e_{c, 0, -0.5} \gamma_1^{2} \gamma_3^{1/2} \mathrm{Jy}.$$

As the cooling frequency $\nu_{c, 2, p}$ exceeds the self-absorption frequency $\nu_{a, 2, p}$ at time $t_0 \approx 70 (1 + z) E_4^{1/4} E_3^{1/4} \Delta_1^{1/2} \Delta_4^{1/2} = 10^{15} E_4^{1/4} E_3^{1/4} \Delta_1^{1/2} \Delta_4^{1/2}$ s, the corresponding emission after this time is then

$$f_{\nu_{c, 2, p}} \approx 2.9 \times 10^7 (1 + z) D_2^{2} E_1^{1/4} E_4^{5/4}$$

$$\times e_{c, 0, -0.5} \gamma_1^{2} \gamma_3^{3/2} \mathrm{Jy}.$$

The flux density of the reverse shock emission in the case of $\nu_{a, 3, p} < \nu_{c, 3, p} < \nu_{m, 3, p}$ is given by

$$f_{\nu_{c, 3, p}} \approx 3.5 \times 10^8 (1 + z)^2 D_2^{2} E_1^{1/4} E_4^{5/4}$$

$$\times e_{c, 0, -0.5} \gamma_1^{2} \gamma_3^{3/2} \mathrm{Jy},$$

$$f_{\nu_{a, 3, p}} \approx 5.3 \times 10^8 (1 + z)^{11/2} D_2^{2} E_1^{1/4} E_4^{5/4}$$

$$\times e_{c, 0, -0.5} \gamma_1^{2} \gamma_3^{3/2} \mathrm{Jy},$$

$$f_{\nu_{f_{\max}, 3, p}} \approx 2.3 \times 10^{12} (1 + z)^{1/2} D_2^{2} E_1^{1/4} E_4^{5/4}$$

$$\times e_{c, 0, -0.5} \gamma_1^{2} \gamma_3^{3/2} \mathrm{Jy}.$$

At time $t_0 \approx 3.4 \times 10^7 (1 + z) E_1^{1/4} D_2^{1/2} E_4^{1/4} \Delta_1^{1/2} \Delta_4^{1/2} = 10^{15} E_1^{1/4} D_2^{1/2} E_4^{1/4} \Delta_1^{1/2} \Delta_4^{1/2}$ s, $\nu_{c, 3, p} = \nu_{m, 3, p}$. After that time, the order of the break frequencies becomes $\nu_{a, 3, p} < \nu_{c, 3, p} < \nu_{m, 3, p}$. In this case, the flux density becomes

$$f_{\nu_{c, 3, p}} \approx 2.3 \times 10^8 (1 + z)^2 D_2^{2} E_1^{1/4} E_4^{5/4}$$

$$\times e_{c, 0, -0.5} \gamma_1^{2} \gamma_3^{3/2} \mathrm{Jy}.$$

$$f_{\nu_{a, 3, p}} \approx 1.0 (1 + z)^{7/3} D_2^{2} E_1^{1/4} E_4^{5/4}$$

$$\times e_{c, 0, -0.5} \gamma_1^{2} \gamma_3^{3/2} \mathrm{Jy}.$$
From equations (27) and (28), we can see that the flux densities are the same for the case in which the observed frequency exceeds the three break frequencies.

We consider the case in which many shells like the foregoing shell 4 collide with shell 1 at different radii \( r_m \), where \( r_m \) indicates the observed time by the relation \( t_o \approx (1+z)r/(2\gamma^2 c) \). Thus, the peaks of emission obey a power-law temporal profile. Under more realistic conditions, shell 1 may be alone, proceeding at the front, and many shells like shell 4 ejected from the central engine continually overtake shell 1 (see Fig. 1). As the energy of shell 1 is assumed to be much greater than that of shell 4, the dynamics of shell 1 after the collision is almost unchanged, and thus the above relations are still applicable. In the popular collapsar model, shell 1 is assumed to be much greater than that of shell 4, the dynamics of shell 1 after the collision is almost unchanged, and thus the above relations are still applicable. In the popular collapsar model, shell 1

\[
 f(\nu, \nu_p) \approx 8.9 \times 10^{-3}(1+z)^{21/20}D_{28}^{-2}E_{45,1}^{21/20} \\
 \times \frac{1/2}{6/5} e^{-(5/2-\nu/11/10)} \nu_{18}^{11/10} \nu_{18}^{11/10}, \\
 \text{for} \ \nu_{\alpha,3,p} < \nu_{\epsilon,3,p} < \nu_{m,3,p} < \nu. \quad (28)
\]

3. GRB 050904

Using the above model, we fit the peaks of the X-ray light curves of GRB 050904 in the observer’s frame. We consider the model parameters \( E_1 = 10^{55} \) ergs, \( E_2 = 1.1 \times 10^{51} \) ergs, \( \gamma_1 = 31.6 \), \( \gamma_4 = 1455 \), \( \Delta_{1,0} = 10^{12} \) cm, \( \Delta_{4,0} = 2.0 \times 10^{16} \) cm, \( r_B = 0.1 \), \( \epsilon_e = 0.316 \), and \( p = 2.2 \) and the cosmology with \( \Omega_m = 0.23 \), \( \Omega_{\Lambda} = 0.73 \), and \( H_0 = 71 \) km s\(^{-1}\) Mpc\(^{-1}\). These parameters lead to a Newtonian forward shock and a relativistic reverse shock. As shown in Figure 2, the solid line with four segments is fitted analytically. The flux is an integral of flux density in a frequency range of \( 4 \times 10^{16} - 2 \times 10^{18} \) Hz, which corresponds to the X-Ray Telescope’s (XRT’s) 0.2–10 keV band. The temporal breaks are due to the crossing of the three break frequencies.

The first segment represents the emission from the relativistic reverse shock with a constant shell width, which satisfies equation (13) with a temporal index of 1/2. At a time of \( \sim 42 \) s, the corresponding radius is \( R(42, s) \approx 2\gamma^2 ct_0/(1+z) \approx 3.5 \times 10^{14} \) cm, while the spreading radii of region 1 and region 4 are \( R_{1,1} \approx \gamma_1^2 \Delta_{1,0} \approx 10^{15} \) cm and \( R_{4,4} \approx 4.2 \times 10^{14} \) cm, respectively. The three radii are approximately equal, and thus \( R(42, s) \) can be taken as the common spreading radius of regions 1 and 4.

After the first break time, regions 1 and 4 should be spreading, and the flux density is described by the third expression of equation (27). The temporal index is \(-1/4\). Up to \( \sim 300 \) s, the frequency \( \nu_{m,3,p} \approx 2.6 \times 10^{17} \) Hz decreases to the observed frequencies of XRT, and then the order of the break frequencies becomes \( \nu_{\alpha,3,p} < \nu_{\epsilon,3,p} < \nu_{m,3,p} < \nu \) or \( \nu_{\alpha,3,p} < \nu_{m,3,p} < \nu_{\epsilon,3,p} < \nu \). The flux density is given by the last expression of equation (27), and the temporal index of the flux is \(-23/20\). After \( \sim 3 \times 10^4 \) s, the order is \( \nu_{\alpha,3,p} < \nu_{m,3,p} < \nu \) and the temporal index becomes \(-12/5\).

Observationally, it was reported that the photon indices are \( \sim -1.3 \) and \( \sim -1.9 \) (whose corresponding spectral indices are \(-0.3 \) and \(-0.9 \) respectively) at early and late times, respectively (Cummings et al. 2005; Watson et al. 2006). The observer’s time at which the spectral index changed is about 350 s. This time can be naturally understood as the crossing time of \( \nu_{m,3,p} \) and \( \nu \). Before this crossing time, \( \nu_e < \nu < \nu_{m,3,p} \) and the spectral index is \(-1/2\), and after this crossing time, \( \nu_e < \nu < \nu_{m,3,p} \) and the index is \(-1.1\). Figure 3 shows the spectral index evolution and a sketch for the evolution of the spectral energy distribution. These spectral indices are generally slightly smaller than the observed ones, because our model only gives the spectral indices at the peak times, but the observations were performed in relatively broad ranges of time. At the beginning of each pair of reverse and forward shocks, the cooling frequency \( \nu_c \) is very large (Zou et al. 2005), because this frequency is proportional to \( t_c^{-2} \), where \( t_c \) is the comoving time of the shocks. Thus, \( \nu_c \) can be larger than \( \nu \) during the initial stage of the shocks. In the case of \( \nu < (\nu_{m,3,p}, \nu_e) \), the
The solid line indicates the total peak flux density, and the others are fitted peaks that are greater than the observed data. As the fitted X-ray peak line in Figure 2. The indices are \( \frac{1}{3} \) and \( \frac{1}{2} \) for the lower frequency emission than for the X-ray band. Furthermore, as the number density of the forward-shocked region is much larger than that of the reverse shocks (see eqs. [14] and [15]), the contribution to the J-band emission exceeds that of the reverse shocks. To show the contribution of the forward shocks, Figure 4 duplicates Figure 2, but includes the forward shocks. It is reasonable to see fitted peaks that are greater than the observed data. As the fitted lines stand for the peak fluxes of the emission of internal shocks, these fluxes should be greater than the time-averaged observed data, especially for the optical emission, which lasts a relatively longer time for the observation. If there are not reverse-forward shocks at a certain time, the observed flux density should be much less than the expected peak one. Therefore, the fitted solid line is basically consistent with the observed data.

For the highest (fourth) J-band point (449–589 s) and the X-ray peak nearly at the same time, we suggest that they could arise from the same internal shocks, and the X-ray peak could occur only at a time slightly earlier than \( \approx 440 \) s (we circled them in Fig. 4 for emphasis). There is a time lag (\( \approx 0–140 \) s) between the X-ray emission and the J-band emission. This may be caused by a spectral time lag (Norris et al. 2000), the different refractions of photons in different energy bands during the propagation from the source to the Earth, or a quantum gravity effect (which is also due to different refractions in a vacuum for different photons; Amelino-Camelia et al. 1998). There has also been an example with a long time lag, GRB 980425 (Norris et al. 2000), which shows the probability of a time lag of tens of seconds.

We further show that our model is consistent with the observed data in some other aspects. First, the Lorentz factor of region 1 should not change significantly during the period of X-ray emission. Because faster ejected shells (region 4) have much less kinetic energy than the front slower one (region 1) and the forward shocks are Newtonian, the Lorentz factor and the particle number of region 1 do not increase significantly when it is overtaken by faster shells. On the other hand, we estimate the deceleration radius \( r_d \equiv \left[ 3E_0 / \left( 4 \pi \gamma^2 m_p c^2 \right) \right] ^{1/3} \), where \( E_0 \) is the isotropic equivalent kinetic energy and \( m \) is the medium density. Letting the gamma-ray isotropic equivalent energy be \( E_{\gamma,\text{iso}} = 2.5 \times 10^{50} \) ergs, which is consistent with the observation range (Cusumano et al. 2006), and assuming the radiative efficiency \( \eta = 0.2 \) and the medium density \( n = 3.16 \text{ cm}^{-3} \), we obtain \( E_0 = E_{\gamma,\text{iso}} (1-\eta)/\eta \approx 1 \times 10^{55} \) ergs. This energy is consistent with the parameters assumed above. Then, the deceleration radius is \( r_d \approx 8.0 \times 10^{15} \) cm, which corresponds to the observed time \( t_0 \approx 9.7 \times 10^4 \) s. This deceleration time is larger than the X-ray duration time, indicating that region 1 is not decelerated by the ambient medium during this period.

Second, the Lorentz factor at the jet-break time (Tagliaferri et al. 2005) is about \( 1/\theta_j \approx 17.5 \) (\( \theta_j \approx 0.057 \)). It should be greater at an earlier time. This is in agreement with the parameter we used (\( \gamma_1 = 31.6 \)).

Third, we note the photon indices at the second break of the fitted X-ray peak line in Figure 2. The indices are \( \approx 1–2.2 \) during the time period of \( (149 \text{ s}, 223 \text{ s}) \) and \( \approx 1.5 \pm 0.3 \) during the time period of \( (223 \text{ s}, 303 \text{ s}) \) in the Burst Alert Telescope (BAT) data and \( \approx 1.13 \pm 0.07 \) during the time period of \( (169 \text{ s}, 209 \text{ s}) \), \( \approx 1.31 \pm 0.06 \) during the time period of \( (209 \text{ s}, 269 \text{ s}) \), and \( \approx 1.34 \pm 0.06 \) during the time period of \( (269 \text{ s}, 371 \text{ s}) \) in the XRT data [Table 1 of Cusumano et al. (2006), where the time is divided by \( 1+z \) to give the local observer’s frame]. It can be seen that the photon indices of the BAT data are greater than those of the XRT data. In our model, the frequency \( \nu_m \) is crossing the observed frequency during this period. For the frequency \( \nu < \nu_m < \nu_m \) the flux densities are scaled as \( f_\nu \propto \nu^{-1/2} \) and \( f_\nu \propto \nu^{-11/10} \), respectively. Therefore, the spectrum is harder for the lower frequency emission than for the higher frequency emission, which is consistent with the observed data.

Fourth, the mean intrinsic column density is \( N_{\text{HI}} = (2.30 \pm 0.50) \times 10^{22} \text{ cm}^{-2} \), as measured by XRT (Cusumano et al. 2006). This high density cannot be explained by the ambient medium with an assumed number density of \( 3 \text{ cm}^{-3} \). Because the radiation originates from the reverse-forward shocks and should pass through shell 1, we find that the column density of shell 1 is \( N_{\text{HI}} = E_1 / \left( \gamma_1^2 4 \pi r_m^2 m_p c^2 \right) \), which is \( 1.67 \times 10^{23} \text{ cm}^{-2} \) at radius \( r = 10^8 \text{ cm} \) and \( 1.67 \times 10^{24} \text{ cm}^{-2} \) at radius \( r = 10^9 \text{ cm} \). These values are basically consistent with the measured mean column density. Therefore, the materials in shell 1 provide an explanation for the high column density measured by XRT.

Finally, there are several low peaks in Figure 2 at about \( 10^3 \) s. They may be caused by nonuniform Lorentz factors or energies of the ejected rapid shells.

In short, the X-ray flares are understood as being due to the internal shocks. These shocks are produced by the continually ejected rapid shells and the faster ones, such as with the slowest shell at different radii. These collisions then lead to reverse-forward shocks. The shocks produce the X-ray and lower energy emission by synchrotron radiation. We obtained a “normal” set of parameters to fit the peak fluxes of XRFs of GRB 050904 and

![Figure 4](image-url)

**Fig. 4.**—Same as Fig. 2, except including the contribution of the forward shocks. The solid line indicates the total peak flux density, and the others are fitted peaks that are greater than the observed data.
found that our model is well consistent with the observations including the early optical data.

For GRB 050904, we also found that the contribution of the forward shock emission is insignificant in the X-ray band, but significant in the $J$ band at early times, because the frequency of emission from the Newtonian forward shocks mainly lies in the optical band. However, we should note that for other bursts, forward shocks may also be important for the X-ray emission. If the leading shell (shell 1) is not more energetic than the following faster shells (shells 4), the velocity of shell 1 cannot be regarded as unchanged after some collisions, and the simple model in § 2 is invalid. If the parameters differ, the deceleration radius may be smaller, and the external shock may become important. All these effects would change the profile of the XRF peaks.

In general, we argued that some long bursts should have the same properties: the peaks are nearly constant before the time at which $\nu_m$ is just equal to the observed frequency $\nu$ and decline rapidly with a temporal index of about $-(3p - 2)/4$ after this time. Therefore, the $\gamma$-ray emission of these bursts persists up to the time when $\nu_m = \nu$ and disappears because of the rapid decline. In this case, the $\gamma$-ray emission ceases when the peak energy $E_p \approx h\nu$. There are two other reasons for the disappearance of $\gamma$-rays. One is that the flux of $\gamma$-ray emission decreases below the detectable limits of telescopes when the frequency order is still $\nu_c < \nu < \nu_m$. The other is the turnoff of the central engine when $\nu_m$ is still larger than $\nu$. Only in the latter case can it be said that the $\gamma$-ray duration is the burst duration.

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REFERENCES

Amelino-Camelia, G., Ellis, J., Mavromatos, N. E., Nanopoulos, D. V., & Sarkar, S. 1998, Nature, 393, 763
Blandford, R. D., & McKee, C. F. 1976, Phys. Fluids, 19, 1130
Boer, M., Atteia, J. L., Dameridi, Y., Gendre, B., Klotz, A., & Stratta, G. 2006, ApJ, 638, L71
Burrows, D. N., et al. 2005, Science, 309, 1833
Cummings, J., et al. 2005, GCN Circ., 3910, http://gcn.gsfc.nasa.gov/gcn/gcn3/3910.gcn3
Cusumano, G., et al. 2006, Nature, 440, 164
Dai, Z. G., Wang, X. Y., Wu, X. F., & Zhang, B. 2006, Science, 311, 1127
Faber, J. A., Baumgarte, T. W., Shapiro, S. L., & Taniguchi, K. 2006, ApJ, 641, L93
Fan, Y. Z., & Wei, D. M. 2005, MNRAS, 364, L42
Haislip, J. B., et al. 2006, Nature, 440, 181
Kawai, N., Yamada, T., Kosugi, G., Hattori, T., & Aski, K. 2005, GCN Circ., 3937, http://gcn.gsfc.nasa.gov/gcn/gcn3/3937.gcn3
Mészáros, P., & Rees, M. J. 1993, ApJ, 405, 278
———. 1999, MNRAS, 306, L39
Norris, J. P., Marani, G. F., & Bonnell, J. T. 2000, ApJ, 534, 248
Nousek, J. A., et al. 2006, ApJ, 642, 389
O’Brien, P. T., et al. 2006, ApJ, in press (astro-ph/0601125)
Perri, R., Armitage, P. J., & Zhang, B. 2006, ApJ, 636, L29
Piran, T., Shemi, A., & Narayan, R. 1993, MNRAS, 263, 861
Price, P. A., Cowie, L. L., Minezaki, T., Schmidt, B. P., Songaila, A., & Yoshii, Y. 2006, ApJ, 645, 851
Proga, D., & Zhang, B. 2006, MNRAS, in press (astro-ph/0601272)
Rees, M. J., & Mészáros, P. 1994, ApJ, 430, L93
Sari, R., & Piran, T. 1995, ApJ, 455, L143
Tagliaferri, G., et al. 2005, A&A, 443, L1
Watson, D., et al. 2006, ApJ, 637, L69
Wei, D., Yan, T., & Fan, Y. Z. 2006, ApJ, 636, L69
Wu, X. F., Dai, Z. G., Huang, Y. F., & Lu, T. 2003, MNRAS, 342, 1131
Xu, X. F., Dai, Z. G., Wang, X. Y., Huang, Y. F., Feng, L. L., & Lu, T. 2005, ApJ, submitted (astro-ph/0512555)
Zhang, B., Fan, Y. Z., Dyks, J., Kobayashi, S., Mészáros, P., Burrows, D. N., Nousek, J. A., & Gehrels, N. 2006, ApJ, 642, 354
Zou, Y. C., Wu, X. F., & Dai, Z. G. 2005, MNRAS, 363, 93