Relating leptogenesis parameters to light neutrino masses

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We obtain model independent relations among neutrino masses and leptogenesis parameters. We find exact relations that involve the CP asymmetries $\epsilon_{N_\alpha}$, the washout parameters $m_\alpha$ and $\theta_{\alpha\beta}$, and the neutrino masses $m_i$ and $M_\alpha$, as well as powerful inequalities that involve just $\tilde{m}_\alpha$ and $m_i$. We prove that the Yukawa interactions of at least two of the heavy singlet neutrinos are in the strong washout region ($\tilde{m}_\alpha \gg 10^{-3}$ eV).

Introduction. Singlet neutrinos with heavy Majorana masses and with Yukawa couplings to the active neutrinos generate light neutrino masses via the see-saw mechanism and a baryon asymmetry via leptogenesis [1], providing attractive qualitative solutions to these two important puzzles. To be quantitatively successful, the seesaw mechanism should lead to the two observed mass scales,

$$m_s \equiv (\Delta m^2_{sol})^{1/2} \approx 0.009 \text{ eV},$$

$$m_\alpha \equiv (\Delta m^2_{atm})^{1/2} \approx 0.05 \text{ eV},$$

while leptogenesis should lead to the value extracted from observations,

$$Y_{\text{obs}} \equiv n_B - n_{\overline{\nu}} = (8.7 \pm 0.3) \times 10^{-11}.$$  \hspace{1cm} (2)

Unfortunately, because the leptogenesis parameters – the CP asymmetries and the washout factors – directly involve the heavy singlet neutrinos, we cannot realistically hope that they will be measured. In order to make further progress in the investigation of leptogenesis, it is highly desirable to relate the leptogenesis parameters to measurable mass parameters. The purpose of this work is to obtain such relations.

The relations that we obtain involve the washout parameters of all the heavy singlet neutrinos $N_\alpha$. While most leptogenesis studies have focussed on the contributions from the decays of $N_1$, the lightest heavy singlet, it has been realized that, in general, the contributions from the decays of the heavier singlet neutrinos must not be neglected [2, 3, 4]. Indeed, our results reinforce this statement.

Notations. The relevant Lagrangian terms involve, in addition to the $N_\alpha$’s, the light lepton SU(2)-doublets $L_i$ and SU(2)-singlets $E_i$ ($i = e, \mu, \tau$ is a flavor index), and the standard model Higgs $H$:

$$-\mathcal{L} = \frac{1}{2} M_\alpha N_\alpha \bar{N}_\alpha + \lambda_{\alpha i} H N_\alpha L_i + Y_i H^\dagger L_i E_i.$$  \hspace{1cm} (3)

Eq. (3) is written in the mass basis for the singlet neutrinos and for the charged leptons, that is, $M$ and $Y$ are diagonal.

The light neutrino mass matrix is given by

$$m_\nu = v^2 \lambda^T M^{-1} \lambda,$$  \hspace{1cm} (4)

where $v = \langle H \rangle$. Reversing this relation, one can express the Yukawa couplings $\lambda_{\alpha i}$ in terms of the diagonal mass matrix $M$, the matrix $m = \text{diag}(m_1, m_2, m_3)$ (where $m_i^2$ are the eigenvalues of $m_d m_\nu^T$), the lepton mixing matrix $U$ and an orthogonal complex matrix $R$ [5]:

$$\lambda = \frac{1}{v} M^{1/2} R M^{1/2} U^\dagger.$$  \hspace{1cm} (5)

The baryon number generated from the decays of the $N_\alpha$ neutrinos can be written as follows:

$$Y_B = -1.4 \times 10^{-3} \sum_{\alpha, \beta} \epsilon_{N_\alpha} \eta_{\alpha\beta},$$  \hspace{1cm} (6)

where $\epsilon_{N_\alpha}$ is the CP asymmetry generated in $N_\alpha$ decays:

$$\epsilon_{N_\alpha} = \frac{\Gamma(N_\alpha \rightarrow \ell H) - \Gamma(N_\alpha \rightarrow \ell \bar{H})}{\Gamma(N_\alpha \rightarrow \ell H) + \Gamma(N_\alpha \rightarrow \ell \bar{H})},$$  \hspace{1cm} (7)

and $\eta_{\alpha\beta}$ denotes the efficiency factor related to the washout of the asymmetry $\epsilon_{N_\alpha}$ due to $N_\beta$ interactions. (If leptogenesis takes place at $T \lesssim 10^{12}$, flavor indices should be added [2, 3, 4, 5].) It is convenient for our purposes to further define a matrix of dimensionful quantities $\tilde{m}_\alpha$:

$$\tilde{m} = v^2 M^{-1/2} \lambda \lambda^\dagger M^{-1/2}.$$  \hspace{1cm} (8)

Note that $\tilde{m}$ is a positive matrix and, in particular, $|\tilde{m}_{\alpha\beta}|^2 \leq \tilde{m}_{\alpha\alpha} \tilde{m}_{\beta\beta}$. In terms of the parametrization [5], we have

$$\tilde{m}_{\alpha\beta} = \sum_i m_i R_{\alpha i} R^\dagger_{\beta i}.$$  \hspace{1cm} (9)

In a large part of the parameter space, the washout factors $\eta_{\alpha\beta}$ depend on the mass and the couplings of $N_\alpha$ only via the combination $\tilde{m}_\alpha \equiv \tilde{m}_{\alpha\alpha}$ [3]. For example,
for $M_1 \ll 10^{14}$ GeV and $\bar{m}_\alpha \gg m_s = 2.2 \times 10^{-3}$ eV, we have

$$\eta_{\alpha\alpha} \approx \left( \frac{5.5 \times 10^{-4} \text{ eV}}{\bar{m}_\alpha} \right)^{1.16}. \quad (10)$$

When we talk in this work about the “washout parameters” we refer mainly to the $\bar{m}_\alpha$’s. The off-diagonal terms in $\bar{m}$ do, however, play important roles in leptogenesis. First, the CP asymmetries depend on $\Im m(\bar{m}_{\alpha\beta})$ (see, for example, Eq. (22) below). Second, $|\bar{m}_{\alpha\beta}|$ determines the overlap between the lepton doublet states $\ell_\alpha$ and $\ell_\beta$ to which $N_\alpha$ and $N_\beta$ decay, respectively [4]:

$$|\ell_\alpha\rangle = (\lambda\lambda^\dagger)_{\alpha\alpha}^{-1/2} \sum_\iota \lambda_{\alpha\iota}|\ell_\iota\rangle,$$

$$\cos^2 \theta_{\alpha\beta} \equiv \langle |\ell_\alpha\rangle|\ell_\beta\rangle|^2 = |\bar{m}_{\alpha\beta}|^2/(\bar{m}_\alpha \bar{m}_\beta). \quad (11)$$

For the case of strong hierarchy between the masses and the lifetimes of, say, $N_1$ and $N_2$, and $\bar{m}_1 \gg m_s$, the interactions of $N_1$ first project $\bar{m}_N$ on the directions aligned with or orthogonal to $\ell_1$ and then washout the asymmetry in the $\ell_1$ direction [4]. For this case, we use an approximate expression for the total lepton asymmetry generated in $N_2$ and $N_1$ decays:

$$Y_B \approx -1.4 \times 10^{-3} \left[ \epsilon_{N_5,\eta_{11}} + \epsilon_{N_3,\eta_{22}}(\cos^2 \theta_{12}\eta_{11} + \sin^2 \theta_{12}) \right]. \quad (12)$$

The basic relations. The key point for our results is that $\bar{m}^*\bar{m}$ and $m_\alpha m_\beta^*$ are similar. In particular, the following three relations hold:

$$\det(m_\alpha m_\beta^*) = \det(\bar{m}^*\bar{m}),$$

$$\text{Sym}_2(m_\alpha m_\beta^*) = \text{Sym}_2(\bar{m}^*\bar{m}),$$

$$\text{Tr}(m_\alpha m_\beta^*) = \text{Tr}(\bar{m}^*\bar{m}), \quad (13)$$

where $\text{Sym}_2(A) = \frac{1}{2} \left\{ [\text{Tr}(A)]^2 - \text{Tr}(A^2) \right\}$. These equations can be written as exact relations involving the light neutrino masses $m_\iota$, the washout parameters $\bar{m}_\alpha$, and the off-diagonal terms $\bar{m}_{\alpha\beta}$. The latter can be expressed in terms of the CP asymmetries $\epsilon_{N_\alpha}$ and the projections $\cos^2 \theta_{\alpha\beta}$.

These equalities [as well as the explicit form (9)] can be further used to obtain simple inequalities involving only the washout parameters $\bar{m}_\alpha$ and the light neutrino masses $m_\iota$. In particular, we are able to show that some (and in some cases all) of the $N_\alpha$ interactions are in the strong washout region.

We note that there is no additional information for us in the leptonic mixing angles. The reason is that $\bar{m}$ is independent of the mixing angles. This can be seen by noting that $\lambda\lambda^\dagger$ is independent of $U$, Eq. (5), or directly from Eq. (9).

We apply Eqs. (13) to two cases, differing in the number of singlet neutrinos $N_\alpha$ that are added to the SM. In the “3+2” framework, two such neutrinos are assumed to be relevant to the see-saw mechanism and to leptogenesis, while in the “3+3” framework, there are three. The $3+2$ case is actually a special limit of the $3+3$ framework. When one of the three $\bar{m}_\alpha \to 0$ (that is, $(\lambda\lambda^\dagger)_{\alpha\alpha} \to 0$ and/or $M_\alpha \to \infty$), the corresponding $N_\alpha$ becomes irrelevant to both the see-saw mechanism and leptogenesis, and the model reduces to effectively a $3+2$ model. We checked that all our results in the $3+3$ framework indeed reproduce the $3+2$ results in this limit.

We do not consider $3+m$ models with $m > 3$ because, in general, they are similar to the $3+3$ model. To understand that, note that $\ell_2$ ($\ell_3$) has, in general, a component that is orthogonal to $\ell_1$ ($\ell_1$ and $\ell_2$). Consequently, part of the asymmetries generated by the decays of $N_2$ and $N_3$ is protected against washout [4]. The light flavor space is, however, three dimensional and therefore spanned, in general, by $\ell_1$, $\ell_2$, and $\ell_3$. Consequently, there is no component in $\ell_{\alpha>3}$ that is orthogonal to all three, and the asymmetries $\epsilon_{N_{\alpha>3}}$ are expected to be washed out.

The $3+2$ framework. When we have only two singlet neutrinos ($\alpha = 1, 2$ in Eq. (3)], one of the light mass eigenvalues vanishes and the other two mass eigenvalues are fixed by phenomenology to one of two discrete possibilities, either normal hierarchy (NH) or inverted hierarchy (IH):

$$m_1 = 0, \quad m_2 = m_s, \quad m_3 = m_\alpha \quad \text{(NH)},$$

$$m_1 = 0, \quad m_{2,3} \approx m_\alpha, \quad m_3 - m_2 = m^2_s/(2m_\alpha) \quad \text{(IH)}.$$  

Using Eqs. (13), we have

$$m_2 m_3 = \bar{m}_1 \bar{m}_2 - |\bar{m}_{12}|^2, \quad (15)$$

$$m_2^2 + m_3^2 = \bar{m}^2_1 + \bar{m}^2_2 + 2\Re(\bar{m}_{12}^2). \quad (16)$$

Using Eqs. (15) and (10), we obtain two inequalities:

$$\bar{m}_2 + \bar{m}_1 \geq m_3 + m_2, \quad (17)$$

$$|\bar{m}_2 - \bar{m}_1| \leq m_3 - m_2. \quad (18)$$

We derived various additional inequalities:

$$\bar{m}_{1,2} \geq m_2, \quad (19)$$

$$\bar{m}_1 \bar{m}_2 \geq m_2 m_3, \quad (20)$$

$$m_2/m_3 \leq \bar{m}_1/\bar{m}_2 \leq m_3/m_2. \quad (21)$$

(The inequality $\bar{m}_1 \geq m_2$ was derived in ref. [11].) However, Eqs. (19)-(21) are redundant when the constraints (17) and (18) are imposed.

The combination of the inequalities (17) and (18), together with the known values of $m_2$ and $m_3$ [Eq. (12)], constrains the allowed region in the $\bar{m}_1 - \bar{m}_2$ plane in a very significant way. We plot these constraints in Fig. 1. In particular, we can draw the following conclusions:

1. For NH, both $\bar{m}_\alpha$ are above $m_s$ and at least one of them is above $m_\alpha/2$. The two washout factors are within a factor of $m_\alpha/m_s \sim 6$ of each other.
Within the 3 + 2 case with normal hierarchy. The grey region is forbidden by Eqs. (17) and (18). The black region is derived by scanning the parameter space (fixing $M_1 = 10^{12}$ GeV and $M_2/M_1 = 10$) and requiring that the resulting baryon asymmetry would be within the 3σ range [Eq. (2)].

2. For IH, both $\tilde{m}_\alpha$ are above $m_\alpha$, while the difference between them is very small, $\leq m_\alpha^2/(2m_\alpha)$.

3. In either case, both $N_2$ interactions and $N_1$ interactions are in the strong washout regime.

As concerns the two CP asymmetries, $\epsilon_{N_{1,2}}$, they can be written as

$$
\epsilon_{N_\alpha} = f_\alpha(x_{12}) \frac{M_\alpha \text{Im}(\tilde{m}_\alpha^2)}{v^2 \tilde{m}_\alpha}.
$$

The functions $f_\alpha(x_{12})$ can be found in the literature [12]. Instead of the inequalities (17) and (18), one can combine Eqs. (19), (20), and (22) to obtain an exact relation between the neutrino masses, the washout parameters and the CP asymmetries:

$$
4(\tilde{m}_1\tilde{m}_2 - m_2m_3)^2 - (\tilde{m}_1^2 + \tilde{m}_2^2 - m_2^2 - m_3^2)^2 = 4\frac{v^4\tilde{m}_1^2\epsilon_{N_1}^2}{M_1^2[f_1(x_{12})]^2} = 4\frac{v^4\tilde{m}_2^2\epsilon_{N_2}^2}{M_2^2[f_2(x_{12})]^2}.
$$

The 3 + 2 framework with strong $M_2/M_1$ hierarchy. While our main focus here is on model independent relations, we can gain some further understanding by assuming mass hierarchy between the two singlet neutrinos. (It also explains features of the black region in Fig. 1 which corresponds to $M_2/M_1 = 10$.) In the hierarchical case ($x_{12} \ll 1$), we have

$$
f_1(x_{12}) = -3/(16\pi),
$$

$$
f_2(x_{12}) = -x_{12}^2|\ln(x_{12}) + 1|/(4\pi),
$$

$$
\epsilon_{N_2}/\epsilon_{N_1} = -\frac{4\tilde{m}_1}{3m_2}x_{12}\left(\ln \frac{1}{x_{12}} - 1\right).
$$

Using Eqs. (6) and (10), we can give a rough estimate of the ratio between the respective contributions to $Y_R$:

$$
\frac{\epsilon_{N_2}/\tilde{m}_2}{\epsilon_{N_1}/\tilde{m}_1} \sim \frac{\tilde{m}_1^2 M_1}{\tilde{m}_2^2 M_2}\left(\ln \frac{M_2}{M_1} - 1\right).
$$

We would like to emphasize the following points:

1. For a mild hierarchy between $M_1$ and $M_2$, $N_2$-leptogenesis must not be neglected. (In this case, $\eta_{12}$ and $\eta_{21}$ have to be taken into account.)

2. In the NH case, only for a very strong hierarchy, $M_2/M_1 \gg 10^2$, it is guaranteed that $N_1$ leptogenesis dominates. For IH, the contribution from $\epsilon_{N_1}$ is always larger.

3. Given that $\epsilon_{N_2}$ and $\epsilon_{N_1}$ have opposite signs, partial (and potentially significant) cancellation between the two contributions is quite possible.

In the $3 + 3$ framework, when the heavy neutrino masses are strongly hierarchical, $\epsilon_{N_1}$ is subject to an upper bound [13, 14]:

$$
|\epsilon_{N_1}| \leq \frac{3}{16\pi} \frac{M_1(m_3 - m_2)}{v^2},
$$

We first note that an even stronger bound applies to $|\epsilon_{N_2}|$:

$$
|\epsilon_{N_2}| \leq \frac{4}{3} \frac{M_1}{M_2} \left(\ln \frac{M_2}{M_1} - 1\right) \epsilon_{DI}.
$$

Second, we note that $m_3 - m_2$ is fixed to either $m_a - m_s$ (NH) or $m_a^2/(2m_a)$ (IH) [15], and so we can be more specific in Eq. (26):

$$
\epsilon_{DI} = \begin{cases} M_1/(2.5 \times 10^{16} \text{ GeV}) & \text{NH} \\ M_1/(10^{18} \text{ GeV}) & \text{IH} \end{cases}
$$

In addition, since for NH(IH) $\tilde{m}_1 \geq m_a(m_a)$, we have $\eta_{11} \leq 0.04(0.006)$. The upper bounds on $\epsilon_{N_1}$ and on $\eta_{11}$ give lower bounds on $M_1$,

$$
M_1 \gtrsim \begin{cases} 3.6 \times 10^{10} \text{ GeV} & \text{NH} \\ 1.3 \times 10^{13} \text{ GeV} & \text{IH} \end{cases}
$$

Alternatively, one can write upper bounds on $\tilde{m}_1$:

$$
\tilde{m}_1 \lesssim \begin{cases} m_s \times [M_1/(3.6 \times 10^{10} \text{ GeV})]^{0.86} & \text{NH} \\ m_a \times [M_1/(1.3 \times 10^{13} \text{ GeV})]^{0.86} & \text{IH} \end{cases}
$$

The 3 + 3 framework. Within the 3 + 3 framework, we can distinguish three different types of light neutrino spectra, Normal hierarchy (NH), inverted hierarchy (IH), and quasi degeneracy (QD):

$$
m_1 \ll m_s, \ m_2 \sim m_3, \ m_3 \approx m_a \ (\text{NH});
m_1 \ll m_a, \ m_2,3 \approx m_s, \ m_3 - m_2 = \frac{m_s^2}{2m_a} \ (\text{IH});
m_1 \sim \tilde{m} \gg m_a, \ m_3 - m_2 = \frac{m_a^2}{2\tilde{m}}, \ m_2 - m_1 = \frac{m_a^2}{2\tilde{m}} \ (\text{QD}).
$$
Using Eq. (9), we obtain lower bounds on the washout parameters (the first relation was derived in Ref. [16]):

\[ \tilde{m}_a \geq m_1, \]
\[ \tilde{m}_1 + \tilde{m}_2 + \tilde{m}_3 \geq m_1 + m_2 + m_3. \]  

Evaluating Eqs. (13), we obtain

\[ m_1^2 + m_2^2 + m_3^2 = \tilde{m}_1^2 + m_2^2 + m_3^2 + 2\Re(\tilde{m}_{12}^2 + \tilde{m}_{13}^2 + \tilde{m}_{23}^2), \]  
\[ m_1^2 m_2^2 + m_1^2 m_3^2 + m_2^2 m_3^2 = (\tilde{m}_{11}^2 - |\tilde{m}_{12}|^2)^2 + (\tilde{m}_{22}^2 - |\tilde{m}_{12}|^2)^2 + (\tilde{m}_{23}^2 - |\tilde{m}_{13}|^2)^2 + 2\Re(\tilde{m}_{11}^2 - \tilde{m}_{12}^2 - \tilde{m}_{23}^2 - \tilde{m}_{13}^2)^2 \]  
\[ m_1^2 m_2^2 m_3^2 = \tilde{m}_{11}^2 \tilde{m}_{22}^2 \tilde{m}_{33}^2 - \tilde{m}_{11}^2 |\tilde{m}_{12}|^2 - \tilde{m}_{22}^2 |\tilde{m}_{12}|^2 - \tilde{m}_{23}^2 |\tilde{m}_{12}|^2 - 2\Re(\tilde{m}_{11}^2 \tilde{m}_{22} \tilde{m}_{33} \tilde{m}_{12}) \]
\[ \leq \tilde{m}_1 \tilde{m}_2 \tilde{m}_3. \]  

Using the fact that \( \tilde{m} \) is hermitian and positive, and the general property that for any positive definite matrix \( A \) one has \( \text{Tr}[(A^\dagger)^{-1/2}] \geq \text{Tr}[(A A^\dagger)^{-1/2}] \), we obtain

\[ \tilde{m}_1 \tilde{m}_2 + \tilde{m}_2 \tilde{m}_3 + \tilde{m}_3 \tilde{m}_1 - (m_1 m_2 m_3 + m_2 m_3 m_1 - m_3 m_1 m_2) \]
\[ \geq |\tilde{m}_{12}|^2 + |\tilde{m}_{23}|^2 + |\tilde{m}_{13}|^2. \]  

Eqs. (33) and (36) can be combined to give

\[ \tilde{m}_1^2 + \tilde{m}_2^2 + \tilde{m}_3^2 - 2(\tilde{m}_1 \tilde{m}_2 + \tilde{m}_2 \tilde{m}_3 + \tilde{m}_3 \tilde{m}_1) \leq m_1^2 + m_2^2 + m_3^2 - 2(m_1 m_2 + m_2 m_3 + m_3 m_1). \]  

We now use the above equations and inequalities to obtain lower bounds on the \( \tilde{m}_a \)'s. We denote by \( \tilde{m}_{\alpha}, \tilde{m}_b, \tilde{m}_c \) the smallest, intermediate and largest \( \tilde{m}_a \), respectively. The left hand side of the inequality (37) is minimized for \( \tilde{m}_b = \tilde{m}_a = 0 \). It is maximized (for \( \tilde{m}_c \geq 2(\tilde{m}_b + \tilde{m}_a) \)) by maximal \( \tilde{m}_c \), which, according to Eq. (32), is given by \( \sum_i m_i - (\tilde{m}_b + \tilde{m}_a) \). We can then write an inequality that depends on the sum of the two smallest \( \tilde{m}_a \):

\[ 3(\tilde{m}_a + \tilde{m}_b)^2 - 4 \sum_i m_i (\tilde{m}_a + \tilde{m}_b) + 4 \sum_{i < j} m_i m_j \leq 0, \]  

This inequality leads to an interesting lower bound,

\[ \tilde{m}_a + \tilde{m}_b \geq \frac{2}{3} \sum_i m_i - \frac{2}{3} \left( \sum_i m_i^2 - \sum_{i < j} m_i m_j \right)^{1/2}. \]  

This bound has interesting implications for models of \( N_2 \) leptogenesis that are based on \( \tilde{m}_1 \) in the weak washout region [18, 19, 20], where it gives \( \tilde{m}_2 \geq m_2 \). (A qualitative statement in this regard was made in ref. [19].) Eqs. (31), (32) and (39) lead to the following lower bounds:

- Normal hierarchy:
  \[ \tilde{m}_c \geq \frac{m_a}{3} \left( 1 + \frac{m_s}{m_a} \right), \quad \tilde{m}_b \geq \frac{m_s}{2} \left( 1 - \frac{m_s}{4m_a} \right), \quad \tilde{m}_a \geq m_1. \]  

- Inverted hierarchy:
  \[ \tilde{m}_c \geq \frac{2m_a}{3}, \quad \tilde{m}_b \geq \frac{m_a}{2}, \quad \tilde{m}_a \geq m_1. \]  

- Quasi degeneracy:
  \[ \tilde{m}_\alpha \geq \tilde{m}. \]  

We conclude that, for hierarchical (quasi-degenerate) light neutrino masses, at least two (all three) of the \( \tilde{m}_\alpha \) are in the strong washout region.

**Conclusions.** We investigated the relations between leptogenesis parameters and light neutrino masses. In particular, we derived exact relations between elements of the \( \tilde{m} \) matrix [defined in Eq. (5)], relevant to leptogenesis, and the light neutrino masses. The diagonal elements, \( \tilde{m}_\alpha \), determine the \( \Delta L = 1 \) washout effects. As concerns the off-diagonal ones, \( \Im(m_{\alpha \beta}) \) determine the size of the CP asymmetries, while \( |m_{\alpha \beta}| \) is related to projections (in heavy flavor space) of the asymmetries generated by heavy singlet neutrinos due to interactions of lighter singlets.

The resulting equations lead to interesting exact relations, such as Eq. (23), between the washout parameters, CP asymmetries and neutrino masses. The various relations lead to simple inequalities between the washout parameters \( \tilde{m}_\alpha \) and the light neutrino masses \( m_i \), see Eqs. (17)–(21) for the 3 + 2 framework and (31)–(39) for the 3 + 3 framework.

For light neutrino masses with normal hierarchy, we find the following results:

- In the 3 + 2 framework, both \( N_1 \) and \( N_2 \) interactions are in the strong washout region, with both \( \tilde{m}_a \geq 0.009 \) eV and at least one \( \geq 0.025 \) eV.

- In the 3 + 3 framework, at least two \( N_\alpha \)'s have interactions in the strong washout region, with \( \tilde{m}_a \geq 0.005 \) eV and at least one \( \geq 0.02 \) eV.

The lower bounds are stronger for inverted hierarchy, and even more so in the 3+3 framework with quasi-degenerate light neutrinos.

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