A Monte Carlo Model for 'Jet Quenching'

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Abstract We have developed the Monte Carlo simulation program \textsc{Jewel 1.0} (Jet Evolution With Energy Loss), which interfaces a perturbative final state parton shower with medium effects occurring in ultra-relativistic heavy ion collisions. This is done by comparing for each jet fragment the probability of further perturbative splitting with the density-dependent probability of scattering with the medium. A simple hadronisation mechanism is included. In the absence of medium effects, we validate \textsc{Jewel} against a set of benchmark jet measurements. For elastic interactions with the medium, we characterise not only the medium-induced modification of the jet, but also the jet-induced modification of the medium. Our main physics result is the observation that collisional and radiative medium modifications lead to characteristic differences in the jet fragmentation pattern, which persist above a soft background cut. We argue that this should allow to disentangle collisional and radiative parton energy loss mechanisms by measuring the $n$-jet fraction or a class of jet shape observables.

1 Introduction

In ultra-relativistic heavy ion collisions, the produced QCD matter reduces significantly the energy of high transverse momentum partons. Experiments at the Relativistic Heavy Ion Collider RHIC give strong support to this picture [1–4]. Most of the experimental evidence comes from studying the leading hadronic fragments of the parent partons via single inclusive hadron spectra and jet-like particle correlations. Since the medium-induced suppression of these spectra persists unattenuated up to the highest transverse momenta accessed experimentally at RHIC, it is likely to play a dominant role also in the much wider transverse momentum range soon to be explored in heavy ion collisions at the CERN Large Hadron Collider LHC [5–7].

On general grounds, the energy lost by the leading fragment in a parton shower must manifest itself in the associated particle yield. So, parton energy loss is expected to result in a medium modification of the \textit{entire} parton fragmentation pattern ('jet quenching') and not only in the suppression of leading hadrons. The corresponding study of medium-modified jets beyond their leading fragments is of great interest for several reasons, in particular:

i) At the LHC a larger fraction of the entire medium-modified jet fragmentation pattern will become accessible above background. ii) Studying the distribution of subleading fragments is likely to discriminate between different microscopic mechanisms conjectured to underly jet quenching, thereby helping to characterise more precisely the properties of matter tested by jet quenching. iii) Modelling the distribution of subleading jet fragments is essential for any operational procedure aiming at disentangling jets from background or characterising the jet-induced modification of the background. Such reasons motivate the development of tools which account dynamically for the interaction between jet and medium, and which model medium-modified jets on the level of multi-particle final states.

The interactions of a partonic jet component with some target (typically, a quark or a gluon) in the medium can be elastic or inelastic. In the elastic case, the projectile parton can lose energy only by transferring recoil momentum to the target ('collisional parton energy loss' [8–16]). In the inelastic case, the dominant source of energy degradation of the partonic projectile is the medium-induced breakup of the projectile via gluon radiation ('radiative parton energy loss' [17–22]), resulting in two or more projectile components of lower energy.

The Monte Carlo (MC) technique provides a powerful tool for the simulation of multi-particle final states. In the absence of medium-effects, the perturbative dynamics of parton fragmentation maps with known accuracy onto a probabilistic iteration of parton splittings, and the Monte Carlo technique is widely used for the simulation of final state parton showers in the vacuum [23–25]. In the presence of medium effects, we expect that a parton shower
can contribute to understanding ‘jet quenching’, in particular for the following reasons:

1. **Baseline for the study of medium-modified multi-particle states.**
   The standard experimental procedure of characterising medium-modifications of jet-like multi-particle final states is to compare to an experimental baseline in which medium effects are absent. A realistic modelling should be able to parallel this experimental procedure.

2. **Exact implementation of energy and momentum conservation.**
   Multi-particle final states are known to be sensitive to trigger biases and they are constrained to share the energy and momentum of a parent parton. For the description of this constrained dynamics, exact energy-momentum conservation is likely to be important. It may be numerically more important than the treatment of interference effects, on which analytical calculations of radiative parton energy loss tend to focus.

3. **Testing a wide range of microscopic mechanisms for the interaction between projectile and target.**
   A Monte Carlo tool should allow the user to change easily between different descriptions of the medium and of the interaction between the medium and the partonic projectile. This is important to constrain the microscopic mechanism underlying parton energy loss and to better identify the specific properties of the medium tested by jet quenching.

4. **Interface with experiment.**
   The modelling of realistic multi-hadron final states is clearly beneficial for comparing theory and data, including detailed detector acceptance, resolution and response.

We note that the application of Monte Carlo techniques for the modelling of medium-effects leaves open conceptual issues in the current description of jet quenching. In particular, medium-induced gluon radiation is expected to arise from an interference phenomenon whose mapping onto a probabilistic description of multi-gluon emission is not known. Also, while the iteration of collisional mechanisms may be probabilistic, its interface with a standard parton shower involves modelling assumptions. In the present work, we have bypassed these fundamental issues in order to arrive at a tool which realises the practically important points listed above.

The working name of this Monte Carlo code is **JEWEL**, standing for Jet Evolution With Energy Loss. In the absence of a medium, the JEWEL parton shower is ordered in virtuality, and strict angular ordering is enforced by phase space constraints. The medium is modeled as a collection of scattering centres whose cross sections and distribution in phase space can, in principle, be chosen freely. To decide whether a parton splits as in the vacuum or whether it interacts with a target parton in the medium, the code compares the mean free path between scattering centres with the mean lifetime of the virtual parton. Within this framework, JEWEL aims at realising the objectives listed above.

One approach currently used to connect parton energy loss calculations to measurable quantities is based on energy loss probabilities (‘quenching weights’ [26,27]), defined by the probabilistic iteration of single medium-induced gluon emissions. Alternatively, the interpretation of medium-induced gluon radiation as a modification of parton splitting functions has motivated attempts to evolve partons via medium-modified DGLAP evolution equations. Both approaches lead to comparable results for leading hadronic fragments [28]. Also, collisional energy loss mechanisms have been used recently to define quenching weights and medium-modified DGLAP evolutions [16]. We note that these approaches are largely limited to leading hadron spectra, or special classes of two-particle correlation functions [29], mainly since they treat subleading fragments with kinematic approximations. Recently, there have been many efforts to go beyond leading hadron spectra within the framework of the models discussed above. This included works on jet broadening [30], the angular dependence of two-particle correlations [31], the effect of directed momentum transfer (a.k.a. flow) on the jet fragmentation [32,33], the modification of jet multiplicity distributions [30,34] and jet hadrochemistry [35]. We expect that Monte Carlo models of jet quenching will greatly advance this field of study, mainly because they can implement the four important features listed above.

In recent years, several Monte Carlo models implementing parton energy loss have been developed already. In particular, the code **HIJING** [37] simulates complete nucleus-nucleus collisions and provides a simplified model for radiative energy loss derived from analytical calculations. There are also codes, which simulate jet quenching without modeling the entire nucleus-nucleus collision, such as **PQM** [38], which is a Monte Carlo implementation of the BDMPS quenching weights, and **PyQuen** [36], which modifies the standard **PYTHIA** 6.2 jet events essentially by reducing the energy of partons in the shower and by adding additional gluons to that shower according to distributions motivated by parton energy loss calculations. There are also two models of medium modified parton showers, namely **Q-PYTHIA** [39] with modified splitting functions including induced gluon radiation in the BDPMPS model and a model where the virtuality of partons in the shower is increased due to interactions with the medium thus stimulating additional gluon emissions [40]. All these approaches go beyond leading hadron spectra.

In developing JEWEL, our main focus was to arrive at a code which allows to study in detail the dynamics relating the evolution of the parton shower to the microscopic modeling of the medium interactions. In particular, we are interested in specifying dynamically the physically interesting (but model-dependent) relation between parton energy loss, $p_T$-broadening, recoil momentum, change in jet multiplicity, and other characteristic features of parton energy loss.

This paper describes the physics encoded in JEWEL version 1.0, and demonstrates its use for the study of jet quenching phenomena on the level of multi-hadron final states. The paper is organised as follows: In section 2, we
discuss the Monte Carlo final state parton shower and its interface with a hadronisation mechanism in the absence of medium effects. To arrive at a manageable inclusion of medium-effects on the hadron level, the hadronisation model used by JEWEL is less sophisticated than what is implemented in state of the art Monte Carlo event generators. We discuss these differences and show that JEWEL provides an adequate description of many jet observables, including event shapes, n-jet fractions and intra-jet multiplicity distributions. In section 3 we discuss how scattering in the plasma is included in JEWEL. Most of our discussion will focus on the case of elastic scatterings. For this case, we test in section 3 a well-studied theoretical baseline, namely the in-medium propagation of a parton in the absence of parton splitting. Section 4 addresses the question to what extent the medium-modifications included in JEWEL affect experimentally accessible jet observables in heavy ion collisions. JEWEL, as any newly developed Monte Carlo event generator, is a work in progress. In the conclusions, we summarise what we have learned from the present exploratory study and highlight some directions for future work.

2 The Monte Carlo Model in the absence of medium effects

In this section, we introduce the baseline on top of which medium-effects are included and show that it basically reproduces the observed QCD radiation physics of jets in the vacuum. The evolution variable of a parton shower is not unique. We have decided to use the virtuality $Q^2$. As explained in more detail in section 3, this has the advantage that the evolution variable traces the lifetime $1/Q$ of the virtual states, which facilitates the embedding of the parton shower in the spatiotemporal geometry of a medium. We interface this parton shower with a hadronisation scheme which implements the idea that colour neutralisation occurs locally during hadronisation. However, the scheme invoked here is less sophisticated than the hadronisation prescriptions used in modern event generators, in particular in that it does not require knowledge about the event-specific colour flow in the parton shower. This is a technical simplification, which - in contrast to standard treatments - allows for a straightforward extension of the hadronisation mechanism in the presence of a medium.

2.1 Final state parton shower in the absence of medium effects

2.1.1 Parton evolution

We want to describe the evolution of a parton of initial energy $E$, produced in a hard scattering process. This parton fragments into a multi-parton final state. In the absence of a medium, the JEWEL parton shower is closely related to the mass-ordered shower in the PYTHIA 6.4 event generator [23]. The kinematics of each $a \to b+c$ parton branching is given in terms of the virtuality of the parent parton and the momentum fraction $z$ carried by one of its daughters. The probability that no splitting occurs between an initial and final virtuality $Q_i$ and $Q_f$, respectively, is described by the Sudakov factor

$$S_a(Q_i^2, Q_f^2) = \exp \left[ - \int \frac{dQ^2}{Q^2} \int dz \frac{\alpha_s(z(1-z)Q^2)}{2\pi} \sum b,c \hat{P}_{a\to bc}(z) \right].$$

(1)

Here, $\hat{P}_{a\to bc}(z)$ are the standard LO parton splitting functions for quarks and gluons $(a, b, c \in \{q, g\})$. In order to regularise the integral one has to define an infrared cut-off scale below which splittings are considered to be not resolvable. In practice, we require a minimal virtual mass $Q_0/2$ for the daughters. Given this cut-off, the condition $k_T^2 \approx (1-z)Q^2 > \Lambda^2_{\text{QCD}}$ translates directly into the allowed $z$ range

$$z_{\pm}(Q^2, E) = \frac{1}{2} \left\{ \frac{1}{2} \left( 1 - \frac{Q_0^2 + 4\Lambda^2_{\text{QCD}}}{Q^2} \right) \right\} \frac{1}{1 - \frac{Q^2}{E^2}}.$$ 

(2)

To avoid the Landau pole in equation (1), we increase the nominal cut on $k_T$ by $10\%$. These are the same prescriptions as used in the mass-ordered PYTHIA cascade. With the no-splitting probability equation (1) the probability density $\Sigma_a(Q_i^2, Q_f^2)$ for a splitting to happen at virtuality $Q^2$ is given by

$$\Sigma_a(Q_i^2, Q_f^2) = \frac{dS_a(Q_i^2, Q_f^2)}{d(\ln Q^2)} = S_a(Q_i^2, Q_f^2) \sum_{b,c} W_{a\to bc}(Q^2),$$

(3)

where

$$W_{a\to bc}(Q^2) = \int dz \frac{\alpha_s(z(1-z)Q^2)}{2\pi} \hat{P}_{a\to bc}(z).$$

(4)

is the differential probability for the splitting $a \to b+c$ at $Q^2$, and the Sudakov form factor $S_a(Q_i^2, Q_f^2)$ denotes the probability for evolving from $Q_i^2$ to $Q_f^2$ without splitting.

We determine the virtuality $Q_a^2$ of the parent parton according to the probability density $\Sigma_a(E^2, Q_a^2)$. Consistent with the probability distributions $W_{a\to bc}$, we select the type of parton splitting, which occurred for parton $a$. The momentum fraction $z$ of the splitting is then chosen within the kinematically allowed range $z \in [z_{-}(Q_a^2, E), z_{+}(Q_a^2, E)]$ for the decay of a parton of virtuality $Q_a$, requiring that the virtualities of both partons are larger than the hadronisation scale parameter $Q_0/2$. Subsequently, the virtual masses of the two daughter partons are determined with the help of equation (4), subject to three constraints: The virtualities $Q_b$, $Q_c$ of the daughters are required to be smaller than their energy $z E$ or $(1-z)E$, respectively, and they must be larger than the hadronisation scale $Q_0/2$. In addition, the virtual masses
of the daughters satisfy the constraint $Q_b^2 + Q_c^2 < Q_a^2$. The branching $a \rightarrow b + c$ is finally accepted if the momentum fraction $z$ chosen initially lies within the kinematically allowed range for daughters of ‘mass’ $Q_b$ and $Q_c$, respectively. Otherwise, new values for $Q_b$ and $Q_c$ are chosen, in simulations with angular ordering it may also be necessary to reject the $z$ value and try with a new one. If the branching is accepted, the full four-momentum is reconstructed for both daughters, assuming an azimuthally isotropic decay of the parent. The procedure is then iterated for the daughter partons, based on their assigned virtualities. Parton evolution is stopped for daughter partons which do not split above the scale $Q_0^2$ and these partons are declared to be on mass shell. Angular ordering is enforced by allowing only splittings with decreasing emission angle. The coupling $\alpha_s$ is running at one loop. This shower is essentially the ‘global’ ‘constrained’ evolution which is one of the alternatives of the PYTHIA event generator.

2.1.2 Hadronisation mechanism

In order to have a hadronisation mechanism which is flexible enough to also be useable for the complex parton state in heavy ion collisions, we have developed a modified application of the string hadronisation approach. The hadronisation prescription assumes maximal colour correlation between partons close in momentum space. The code identifies first the parton with the highest energy in the event (alternatively, the highest $p_L$ may be used in hadron collisions.) If this parton is a gluon, it is split into a collinear quark - antiquark pair with the energy sharing given by $P_s = q_1$. The more energetic of the two is then the endpoint of the first string. In the next step it is connected to the closest parton in momentum space (with the only exception that a quark-antiquark pair from a single gluon splitting is not allowed to recombine into a colour singlet). In case the closest parton is a gluon the procedure is iterated until either another (anti)quark is connected or there is no parton in the same hemisphere left. In the latter case a suitable endpoint is generated by adding the required quark or antiquark with momentum in the beam direction to the event. The whole procedure is repeated until all partons are connected in strings. The strings are then hadronised using the Lund string fragmentation routine of PYTHIA 6.4, with default values of hadronisation parameters. We checked that the strings are sufficiently massive for this string fragmentation routine to apply.

This approach is inspired by the fragmentation of jets in hadronic collisions where the additional endpoints can be thought of as being part of the proton remnants, to which the jet is connected by colour flow. The hadrons associated with this additional parton endpoint tend to go along the beam direction so that they are well separated from the jet. As long as the jet structure is analysed in a restricted rapidity range of approximately $|\Delta y| \leq 1$ around the rapidity of the parent parton, the resulting dependence of the model on this endpoint is negligible. There is the possibility to set a maximum invariant mass of neighboring partons in a string, that could be used to tune the routine to data. However, we did not attempt to fine-tune this simplified hadronisation routine.

This hadronisation mechanism has the advantage of being very flexible. It can be applied to any number of jets in an event and, more importantly, it can be applied also to jets in a nuclear environment. In the latter case, it may be desirable to hadronise only some high energy part of the event. This is also possible, since this hadronisation routine can also handle systems that are not globally colour neutral.

For $e^+e^-$ collisions, beam remnants do not exist and the two hemispheres are colour connected and do not hadronise separately. We therefore implemented a variation of the string finding algorithm, which takes into account these additional constraints on colour flow. For the observables discussed in this paper, we did not identify any significant difference between the two hadronisation mechanisms (MC results not shown). To simplify our presentation, we therefore decided to simulate all observables with the same hadronisation routine described above, irrespective of whether they are for $e^+e^-$ or hadronic collisions.

We checked that the hadronic observables are essentially independent of the cut-off scale $Q_0$, at which the perturbative evolution is interfaced with string hadronisation. The default choice is $Q_0 = 1$ GeV.

2.2 Comparison to data

In this subsection, we compare the final hadronic state from JEWEL to data on jets measured in $\sqrt{s} = 200$ GeV $e^+e^-$ collisions at LEP by the ALEPH collaboration [42]. To select the process $e^+e^- \rightarrow q \bar{q} \rightarrow X$ at $\sqrt{s} = 200$ GeV, these ALEPH data were taken with a veto on initial state radiation. They have been compared already [42] very favourably to standard event generators such as PYTHIA.
and HERWIG. The purpose of the present data comparison is to validate the JEWEL parton shower in the absence of medium effects against a set of benchmark data on jets measured at LEP, before studying the extension of the code to medium effects.

To account for a 2-jet event, JEWEL evolves each parent parton separately. For $e^+e^-$ data, the code determines the initial virtuality of one of the two parent quarks according to the probability distribution $\Sigma_i(E^2, Q^2)$, given in equation (3). Then, there are two options: Either, the virtuality of the second parent quark is fixed to be the same, thus conserving energy and momentum at the hard vertex, or we choose it independently according to the same distribution (3). Since the differences observed between these schemes turned out to be negligible, we present here only results for the latter case.

We focus on three classes of tests. First, we test JEWEL on the level of the overall energy flow, which is characterised by event shapes. Then we study an observable class of event shape observables, in particular to oblate-sphericity, sphericity, planarity, aplanarity and total jet broadening. Finally, we turn to a comparison of inclusive charged hadron distributions inside a jet. Hadronisation will be seen to play a more important role for the latter observables.

In figure 1 we compare to simulations of three event shape observables, namely thrust $T$, thrust major $T_{maj}$ and thrust minor $T_{min}$. For these quantities, one sums over the three-momenta $p_i$ of all final state particles. According to the definition of thrust,

$$ T \equiv \max_{n_T} \frac{\sum_i |p_i \cdot n_T|}{\sum_i |p_i|} ,$$

(5)
a 2-jet event is pencil-like if $T = 1$, that is if all particles are aligned parallel or antiparallel to a thrust axis $n_T$. The event is spherical if $T = 1/2$. Once the thrust axis $n_T$ is known, one can determine the direction $n$ orthogonal to $n_T$, along which the momentum flow is maximal. Thrust major is defined as the projection of all particle momenta on this direction $n$,

$$ T_{maj} \equiv \max_{n \cdot n_T} \frac{\sum_i |p_i \cdot n|}{\sum_i |p_i|} .$$

(6)

Thrust minor sums up the components $p_{ix}$ of the final particle momenta $p_i$, which are orthogonal to the plane defined by $n$ and $n_T$,

$$ T_{min} \equiv \frac{\sum_i |p_{ix}|}{\sum_i |p_i|} .$$

(7)

We note that $T$, $T_{maj}$ and $T_{min}$ are perturbatively calculable, infrared-safe quantities and therefore particularly suitable for testing our parton shower implementation. As seen in figure 1 the final state parton shower provides a reasonable description of these jet event shapes over most of the measured range. We recall that JEWEL does not contain a matching of the parton cascade to exact matrix elements, which could improve the QCD modelling of large angle radiation. This may be the reason why the simulation gives fewer events with large $T_{maj}$ and $T_{min}$. Also, we observe small deviations between data and simulation for very small values of $T_{maj}, T_{min} < 0.05$ at the parton level. This region is known [43, 44] to be very sensitive to non-perturbative effects and hence cannot be reliably described by a leading order parton cascade as illustrated by the dashed curve. Consistent with this general statement, we find that for $T_{maj}, T_{min} < 0.05$, the inclusion of the hadronisation prescription leads to some improvement of the data comparison.

We have compared JEWEL to measurements of a wider class of event shape observables, in particular to oblateness, sphericity, planarity, aplanarity and total jet broadening. The comparison to these event shape observables are of similar or better quality than the comparisons shown in figure 1. With these studies, we have established that JEWEL accounts for global features of jet energy flow with an accuracy which is sufficient to characterise (sufficiently large) medium effects on top of it.

Figure 2. The jet rates as a function of jet resolution scale $y_{cut}$ in $\sqrt{s} = 200$ GeV $e^+e^- \to q\bar{q} \to X$ collisions. Left hand side: Simulation of JEWEL with and without hadronisation for evolution down to $Q_0 = 1$ GeV. Right hand side: Data of the ALEPH collaboration [42] compared to simulations of JEWEL with hadronisation.
In comparison to jet event shapes, there are measurements which are more sensitive to the discrete and stochastic nature of partonic processes underlying the QCD jet fragmentation. One such measurement is the so-called a-jet-fraction. Its definition is based on the Durham clustering algorithm [45]. For each pair of final state particles, one defines a distance

\[ y_{ij} = \min(y_{ij}^2, y_{ji}^2) (1 - \cos \theta_{ij}) / E_{cm}^2. \]  

The pair of particles with smallest \( y_{ij} \) is then replaced by a pseudo-particle, whose energy and momentum are the sums of its daughters. The clustering procedure is repeated until all \( y_{ij} \) exceed a given threshold \( y_{cut} \). The number of clusters separated by a distance larger than \( y_{cut} \) is defined to be the number \( n \) of jets. Thus, as one decreases the resolution scale \( y_{cut} \), one becomes sensitive to finer and finer details of the discrete QCD radiation pattern.

Figure 2 shows simulation results for the \( n \)-jet fraction. We find that for \( \log_{10}(y_{cut}) \gtrsim -3 \), the jet resolution scale is sufficiently coarse such that hadronisation plays a negligible role in the \( n \)-jet-fraction. It is only for smaller scales, that the hadronic late stage of QCD fragmentation affects the number of jets identified by the Durham clustering algorithm. The 3-jet fraction is somewhat too small at large \( \log_{10}(y_{cut}) \) while the 2-jet fraction is correspondingly too large, which is again due to the missing 3-jet matrix element. We find that the \( n \)-jet fractions vary mildly with the size of the strong coupling on the partonic as well as on the hadronic level. Our choice of \( A_{QCD} \) leads to a reasonably good description of the data.

In contrast to the measurements discussed so far, the modelling of single inclusive intra-jet hadron distributions and multi-hadron correlations requires detailed knowledge about the hadronisation mechanism. This is seen in figure 3 where we compare results of our simulation to data of the inclusive distribution \( dN_{ch}/d\xi \). (\( \xi = \ln [E_{jet}/p_{hadron}] \)) of charged hadrons in e+e− → q̄q → X events at \( \sqrt{s} = 200 \) GeV. Data of the ALEPH Collaboration [42] are compared to simulations of JEWEL: i) parton level after parton shower evolved down to \( Q_0 = 1 \) GeV, ii) hadron level after parton shower evolution to \( Q_0 = 1 \) GeV followed by hadronisation.

![Figure 3](image-url)

**Figure 3.** The inclusive distribution \( dN_{ch}/d\xi \). \( \xi = \ln [E_{jet}/p_{hadron}] \) of charged hadrons in e+e− → q̄q → X events at \( \sqrt{s} = 200 \) GeV. Data of the ALEPH Collaboration [42] are compared to simulations of JEWEL: i) parton level after parton shower evolved down to \( Q_0 = 1 \) GeV, ii) hadron level after parton shower evolution to \( Q_0 = 1 \) GeV followed by hadronisation.

Within these uncertainties, which are mainly related to the modelling of hadronisation, we have established that JEWEL provides a reliable baseline for the characterisation of jet quenching phenomena.

### 3 ‘Jet Quenching’ in the Monte Carlo Model

A Monte Carlo model of jet quenching requires a picture of how a final state parton shower is embedded in the space-time geometry of a nuclear collision, and how it interacts with the medium. The question where the parton splittings occur in space is likely to affect the resulting medium modifications strongly, since it determines which partonic components interact for how long with the medium. However, very little is known about the spatiotemporal evolution of parton showers in the vacuum or in the medium. Inverting the standard logic, one may even view the medium as a probe of the spatiotemporal evolution of the jet, since the medium interferes with parton fragmentation on length scales comparable to time-dilated hadronisation times.

This highlights that understanding the medium-modification of the jet relies on understanding how the medium interacts with it, and understanding properties of the medium relies on how they are reflected in the medium-modification of jets. Progress on this mutual dependence requires a dynamical description of both the jet and the medium.

On general grounds, one expects that a parton of virtuality \( Q \) branches on a time scale \( 1/|Q| \) in its rest frame,
which translates to a Lorentz time-dilated length scale in the rest frame of the nuclear matter. Our model of the spatiotemporal distribution of parton branchings within a nuclear environment will be based on this relation between time scales and virtuality. While different parton showers use slightly different evolution variables, we have chosen virtuality (consistent with the Pythia 6.4 mass-ordered shower), since it makes the contact with a spatiotemporal picture relatively easy.

### 3.1 Modelling the interactions with the medium

We regard the medium as a collection of partons acting as scattering centres. For the case of elastic interactions between the jet and the medium, each scattering centre displays to the partonic projectiles an elastic $2 \rightarrow 2$ scattering cross section $d\sigma/dt$. The form of this cross section, as well as the momentum distribution, the mass and the density of scattering centres can be specified freely in JEWEL. For the studies presented in this paper, we focus on a specific model, in which the density and momentum distributions of scattering centres are that of a gas of massive quarks and gluons of temperature $T$. The masses of the scattering centres are fixed to $m_{\text{scatter}} = \mu D(T)/\sqrt{2}$, where $\mu D(T)$ is the thermal Debye mass. It is approximated by $\mu D = 3T$ in the simulation. This model is minimal in the sense that the medium is characterised fully by a single parameter, the temperature $T$.

To specify the spatiotemporal structure of the parton shower, we start from the estimate that the parton shower evolves down to components of virtuality $Q_f$ on a time scale $\sim 1/Q_f$. For a parton of energy $E$ and mass $Q_f$, this will be time-dilated in the rest frame of the medium to $\sim E/Q_f^2$. If the parton originated from the branching of some parton of virtuality $Q_i$, then the parton of virtuality $Q_f$ existed for a duration of approximately

$$
\tau = \frac{E}{Q_f^2} - \frac{E}{Q_i^2}.
$$

For the parent parton, which initialised the parton shower, the lifetime is $\tau = \frac{E}{Q_p^2}$. In the case of a medium of constant density $n$, the probability that no scattering occurs during this time is given by

$$
S_{\text{no scatt}}(\tau) = \exp\left[-\sigma_{\text{elas}} n \tau \beta\right].
$$

Here, $\sigma_{\text{elas}}$ is the total elastic scattering cross section and $\beta$ the projectile velocity. For the evaluation of the total scattering cross section in equation (10) the scattering centres are assumed to be at rest, for the simulation of the scattering process they are assigned a thermal momentum. If the density of scattering centres varies along the trajectory of the parton, one would replace this expression by an integral over the parton trajectory between initial and final times, $\exp\left[-\int_{\tau_f}^{\tau_i} \sigma_{\text{elas}}(\xi) n(\xi) d\xi\right]$. Here, we have included the possibility that the elastic cross section changes during evolution, for instance, since its infrared regulation depends on properties of the medium, or $\alpha_s$ has a temperature dependence.

For the differential elastic scattering cross sections, we choose

$$
\frac{d\sigma}{dt} = \frac{\pi \alpha_s^2 C_R}{s^2 + u^2} \left|\frac{C_R}{t}\right|^2 \begin{array}{c}
\text{regularised} \\
\end{array}.
$$

This is the leading $t$-channel exchange term for quark-quark ($C_R = 4/9$), quark-gluon ($C_R = 1$) and gluon-gluon ($C_R = 9/4$) scattering, respectively. The regularisation of equation (11) and the running of the coupling will be specified below. Instead of equation (11), one could include the full LO $2 \rightarrow 2$ parton cross sections [55]. These contain, for instance, also terms singular in the Mandelstam variable $u$. For a scattering in the medium, this corresponds to processes in which almost all the projectile energy is transferred to the scattering partner. Such processes may be viewed not as an energy degradation of the leading parton but as an exchange of the roles of projectile parton and target parton. For the sake of simplicity, the present study does not aim at including and characterising such more detailed features of elastic interactions between projectile and target.

In the Monte Carlo model studied here, we know the virtuality of the partons, which emerge from parton splittings or from elastic scatterings: After a splitting, the virtuality is determined via equation (3), after a scattering, it remains unchanged. This prescription is consistent with the dominance of small angle $t$-channel scatterings, which do not open phase space that could be used to reduce the fast parton’s virtuality. Given this virtuality, we determine the lifetime $\tau$ of the parton according to equation (9). With the probability $S_{\text{no scatt}}(\tau)$, this parton will not scatter, and the code simulates the next splitting as in the vacuum case. With the probability $[1 - S_{\text{no scatt}}(\tau)]$, the parton scatters at some time $\tau' < \tau$. In this case, the parton exchanges momentum with a scattering centre according to the differential cross section equation (11). Then, the code continues to propagate the scattered projectile parton, by checking whether there will be further parton scatterings before splitting. In this procedure, angular ordering is enforced for consecutive scatterings, but it is reset at each scattering, consistent with the assumption that scattering destroys the interference pattern of parton radiation. On-shell partons are allowed to scatter until they leave the medium. It is thus assumed that partons cannot hadronise inside the medium. The code has an option to propagate also the recoiling target partons with subsequent scatterings and scatterings. To simplify the presentation and discussion, we do not explore this option in the present work.

The above prescription for embedding a parton shower in a nuclear environment includes inevitably model-dependent assumptions. In this exploratory work, we do not embark on their systematic study, but we would like to recall some of the major sources of such model dependencies:

First, as mentioned already, the spatiotemporal structure of the parton shower is based on assumptions, which are difficult to constrain. For instance, even if the lifetime
\( \tau \) of a virtual state is given parametrically by \( (8) \), one may distribute this lifetime with an exponential decay law with half time \( (9) \), or one may try to improve the lifetime estimate e.g. by accounting more accurately for the energy of the daughter partons or by replacing it with other assumptions.

Second, the choice of the cross section \( (11) \) invokes the assumption that the interaction between the jet and the medium can be treated perturbatively. This assumption is problematic, since data at RHIC provide some evidence of anomalously strong coupling. In addition, there are quantitative uncertainties arising from the regularisation of the cross section \( (11) \).

Third, the model introduced here does not yet include a mechanism of radiative energy loss. Since this may be the main source of energy degradation, we have included an option to enhance the vacuum splitting functions by a factor \( (1 + f_{\text{med}}) \).

\[
\hat{P}_{a \rightarrow b c}(z) \rightarrow (1 + f_{\text{med}}) \hat{P}_{a \rightarrow b c}(z),
\]

as long as the splitting occurs in the medium. This prescription has been argued to mimic characteristics of radiative energy loss \([34]\). In a probabilistically iterated parton shower, a more satisfactory treatment of radiative energy loss may be to include partonic inelastic \( 2 \rightarrow 3 \) scattering cross sections on the same footing as the elastic \( 2 \rightarrow 2 \) cross sections included here. We did not implement this option in the present study, since its discussion involves significant additional conceptual and technical issues, which - in our view - deserve a separate study.

Fourth, there is the question of how to interface a medium-modified parton shower with a hadronisation model. In the present study, the final partonic state is interfaced with the hadronisation model discussed in section 2 after all interactions with the medium. So, the present version of JEWEL is based on the assumption that the hadronisation of a jet fragment is unaffected by medium effects, but this assumption could be modified in future studies to explore for instance changes in jet hadrochemistry \([35]\).

The present paper does not aim at fully studying or resolving any of these issues. But we expect that JEWEL provides a tool for their systematic exploration in subsequent works.

### 3.2 A baseline of medium-effects: collisional energy loss without parton branching

The case of a single, high energy parton losing energy via multiple elastic interactions in a spatially extended target has been studied extensively in the literature \([8–16]\). This problem can be studied in JEWEL by switching off the option of parton splitting, and specifying the (model-dependent) density of scattering centres and elastic scattering cross sections. The MC then simulates the propagation of an on-shell parton (massless or massive) which undergoes scattering in a medium. The scattering probability is in this case determined only by the path length inside the medium, and the density and cross section of scattering centres. Here, we only consider the collisional energy loss of massless partons and we leave the case of heavy flavours to a dedicated study.

As discussed in section 3.1, we define the density, mass and momentum distribution of scattering centres in terms of a thermal distribution. The same temperature is the model parameter entering the regularisation of elastic scattering cross sections. To make contact with the previous studies, we have chosen two different regularisation schemes for the elastic scattering cross section, namely case I

\[
\sigma_{\text{elas}}^{\text{case I}} = \int_{0}^{t_{\text{max}}} d|t| \frac{\pi \alpha_{s}(|t| + \mu_{B}^{2})}{s^{2}} C_{R} \frac{s^{2} + (s - |t|)^{2}}{|t|^{2} + \mu_{B}^{2}},
\]

which is the default, and case II

\[
\sigma_{\text{elas}}^{\text{case II}} = \int_{\mu_{B}^{2}}^{t_{\text{max}}} d|t| \frac{\pi \alpha_{s}(|t|)}{s^{2}} C_{R} \frac{s^{2} + (s - |t|)^{2}}{|t|^{2}}.
\]

The kinematic boundary is \( |t_{\text{max}}| = 3\sqrt{2}ET \). The total cross sections \( (13) \) and \( (14) \) differ only by their regularisation. In figure 4 we have calculated the resulting average energy loss for an in-medium path length of \( L = 1 \) fm in a medium of temperature \( T \). In general, the average collisional energy loss increases with increasing temperature and projectile energy. Quantitatively, the models I and II show differences of approximately a factor 2 in \( dE/dx \) for a 10 GeV parton. The differences decrease slowly with increasing projectile energy. Cross section II leads to a larger energy loss, as may be expected since there is minimum momentum transfer.

We note that in calculations based on finite temperature field theory, the medium specifies a preferred Lorentz
frame and the effect of collisional energy loss is in general not fully described by an elastic scattering cross section of the form $d\sigma/dt$. This complicates analytical comparisons of our model with recent studies of collisional energy loss [8–16]. On the other hand, to the best of our knowledge, no characteristic medium-induced deviations from the form $d\sigma/dt$ have been explored so far, and a comparison of our calculations for $dE/dx$ on the numerical level seem meaningful. We find that the temperature dependence of $dE/dx$ shown in figure 4 is consistent with the dependences reported previously. The differences between case I and case II are representative of the typical factor 2 uncertainties between different model studies, though some recent studies lead to slightly larger values of $dE/dx$ than those shown in figure 4 see e.g. [14]. Here, we do not enter the current discussion of whether an improved understanding of the scale dependence of elastic interactions may allow to narrow these uncertainties. Rather, we take equation (13) as an example of an elastic cross section whose strength and $t$-dependence may be changed in future simulations of JEWEL. We finally note that in a Monte Carlo simulation, the average energy loss does not grow linearly with the in-medium path length. This is because energy and momentum are conserved exactly so that subsequent collisions occur at decreasing centre of mass energies, which introduces a mild non-linearity.

For single inclusive spectra, which fall steeply in transverse momentum, the dependence of parton energy loss is not determined by the average energy loss but by the least energy loss which a significant fraction of the projectiles suffer. For this reason, it has become the standard in studies of collisional energy loss to study $dN/d\Delta E$ of partons suffering an energy loss $\Delta E$. Our result for this distribution is shown in figure 5 and is seen to peak sharply for negligible energy loss. This reflects the fact that elastic interactions are dominated by small angle scattering, for which the longitudinal component of the momentum transfer between projectile and target becomes negligible in the high energy limit.

We note that the temperature dependence of collisional parton energy loss models shows several generic features. Qualitatively, these may be understood by observing that the strength of the average collisional energy loss is essentially governed by the product of the density of scattering centers times the cross section, which each of these scattering centers displays. The density rises proportional to $T^3$ with increasing temperature. The cross section, however, is dominated by the lowest typical momentum transfer, which is governed by the Debye screening mass, so parametrically $\sigma \propto 1/m_D^2 \propto 1/T^2$. For increasing temperature, the strong increase in density is largely balanced by a reduction of the effective cross section per scattering center, and the net effect is an average energy loss $\Delta E$ which shows a mild, approximately linear increase with $T$, as seen in Fig. 4. For the same reason, increasing the Debye screening mass by increasing $T$, one re-

---

**Figure 5.** Distribution of energy loss $\Delta E$ for different parameter choices (top panels) and the probability for an energy loss smaller than $\Delta E$ after passage of a medium of length $L$ (bottom panels).
This quantity is plotted in the lower panel of figure 5. Upon proper normalisation, the distribution $dN/d\Delta E$ can be turned into a probability distribution for a high energy quark to lose less than a specific amount of energy $\Delta E$. This quantity is plotted in the lower panel of figure 5. One sees that for an in-medium path-length of $L = 1$ fm, almost 65% of all projectile partons do not lose any energy in our models, even if the temperature is taken to be 500 MeV. For 3 fm in-medium path length, there are still typically 25% of all projectiles which emerge unscathed. This indicates that elastic interactions alone are unlikely to account for a large fraction of the suppression of single inclusive hadron spectra.

3.3 Characterising the Recoiling Medium

The energy lost by a jet is redistributed amongst the target components. If one could characterise the amount of energy recoiling in the medium, this would be an unambiguous determination of collisional effects. (We note that also $2 \rightarrow 3$ and $2 \rightarrow n$ processes can contribute to such recoil effects.) More generally, characterising the recoil of the medium may provide a means to disentangle different mechanisms of parton energy loss. In this section, we...
characterise the momentum distribution of target partons, recoiling against a projectile parton which undergoes elastic $2 \rightarrow 2$ processes in a thermal medium.

In figure 6 the momentum and angular distribution of recoiling scattering centres is compared to the undisturbed thermal medium. We investigate the cases, that i) an on-shell projectile is propagated without splitting (upper panel of figure 6) or that ii) the splitting of the projectile is included (lower panel of figure 6). The results are very similar for both cases, indicating that the recoil is only weakly sensitive to the projectile energy.

The scattering centres in the medium show initially an exponentially falling $p_\perp$-spectrum and an isotropic momentum distribution, characteristic for a thermal distribution. After interaction with the projectile, the $p_\perp$-spectrum of the recoil follows a power law at intermediate and high $p_\perp$. The high-$p_\perp$ tail is more pronounced at higher temperatures. Using the kinematics of $2 \rightarrow 2$ scattering, one estimates

$$E_\text{out} = \frac{|t| + 2E_\text{in}^2}{2E_\text{in}} \Rightarrow \frac{d\sigma}{dE_\text{out}} = 2E_\text{in}\frac{d\sigma}{d|t|},$$

where $E_\text{in}$ and $E_\text{out}$ are the energies of the incoming and outgoing scattering centre, respectively, and the average over the angle between the initial partons was taken. We note that despite this power law, the yield of recoil particles lies mainly at relatively soft momentum, $p_\perp < 1 - 2$ GeV.

The recoil moves predominantly in the jet direction with a maximum at $\Delta\phi \sim 0.8$ nearly independent of the temperature. The results shown here are for cross section I; with option II the result is qualitatively the same. There is a moderate change in the shape of the distribution, but the position of the maximum remains unchanged.

Most of the features of the angular correlation can be understood from a simplified analytical model where the scattering centre is assumed to be at rest and the cross section is approximated by $d\sigma/d|t| \propto |t|^{-2}$ (very similar results are obtained with $d\sigma/d|t| \propto (|t| + \mu_0^2)^{-2}$). In this approximation $\cos(\Delta\phi)$ can only be positive. Given the shape of the distribution the position of the maximum can be approximated by the expectation value

$$\langle \cos(\Delta\phi) \rangle \simeq \left( \sqrt{3} - 1 \right) + \frac{m_\text{scatt}}{E_p} \left( 2\sqrt{3} - 3 \right),$$

which is practically constant for large projectile energies $E_p$. It has, in particular, only a weak temperature dependence through the mass $m_\text{scatt}$ of the scattering centre. However, the position of the maximum is determined by the regularisation of the cross section. We therefore investigate the dependence on the infrared regulator by treating it as an independent variable. After expansion around $\mu_0^2 = 2m_\text{scatt}$ one gets for the shift of the maximum position

$$\Delta(\cos(\Delta\phi)) \simeq \left[ \sqrt{3} - 1 - \frac{1}{\sqrt{3}} \right] \frac{\mu_0^2 - 2m_\text{scatt}^2}{2m_\text{scatt}^2}.$$  \hspace{1cm} (17)

This finding is in agreement with the results from the full Monte Carlo simulation.

There is a strong correlation between the scattering angle and the momentum transfer, such that the most energetic partons are closer to the jet axis. If one allows the incoming scattering centre to carry momentum but averages over its direction, then the analytical model discussed above leads to an angular dependence of the outgoing scattering centre with respect to the incoming projectile,

$$\cos(\Delta\phi) = \frac{E_\text{out}(E_p + E_\text{in}) - E_pE_\text{in} - E_\text{in}^2}{p_\perp \sqrt{E_\text{out}^2 - m_\text{scatt}^2}}.$$  \hspace{1cm} (18)

In figure 7 this result is compared to the Monte Carlo simulation. The overall behaviour is similar, but the Monte Carlo result is distributed wider due to the spread in energy and angle of the incoming scattering centres.

At face value, figure 7 indicates that a jet can be accompanied by additional multiplicity which has its maximum separated from the jet axis by a characteristic finite angle $\Delta\phi$, and whose yield is expected to die out quickly with increasing transverse momentum. Similarly, figure 7 indicates that scatterings with large energy transfer lead to a relatively large projectile energy loss of the leading parton, but that the recoil parton is very close to the jet axis so that the energy may stay within the jet cone. Typical momentum transfers, on the other hand, tend to scatter the recoil to a characteristic and relatively large angle. We note, however, that whether the partonic distributions of figure 6 and 7 will or will not change substantially upon
hadronisation may depend on details of the hadronisation mechanism. In particular, if the partons entering the peak of the \( \cos(\Delta \phi) \) distribution are connected by colour strings with the leading jet fragments at \( \cos(\Delta \phi) \sim 1 \), then the hadronic distribution will peak much closer to \( \cos(\Delta \phi) = 1 \) than the partonic one shown in figure [6]. On the other hand, multiple gluon exchanges with the target break the colour flow between hard projectile fragments and target recoils. If such a mechanism is invoked, then the hadronic distribution is likely to follow the partonic one shown in figure [6]. Here, we limit our discussion to the kinematic constraints of the underlying partonic distributions in figure [6] and [7], but we do not explore the model dependence associated with the hadronisation of these distributions.

4 Medium-modifications of jet measurements

In this section, we study to what extent the jet measurements discussed in section 2 are sensitive to the medium effects introduced in section 3. A medium does not only affect jet fragmentation. It also results in a high ‘background’ multiplicity of soft particles. This complicates the characterisation of jets and their medium modifications. To establish which jet measurements are experimentally feasible despite this background, one would ideally like to embed the parton shower simulated by JEWEL into full event simulation of a heavy ion collision. This could be done but it lies beyond the scope of the present paper. In this section, we compare analyses of the full simulated jets with analyses, for which we only include hadrons above a cut \( E_{\text{cut}} = 2 \text{ GeV} \). This comparison may indicate the extent to which different jet medium modifications remain visible above the hadronic background of the heavy ion collision.

For \( e^+e^- \)-collisions, thrust, thrust major and thrust minor are of particular interest for testing QCD radiation physics, since they are infrared safe and thus perturbatively calculable quantities. Applying the definitions [5], [6] and [7] to a single jet or to the jet activity above background, the feature of infra-red safety is lost. But \( T \), \( T_{\text{maj}} \) and \( T_{\text{min}} \) will still provide a characterisation of the global energy flow in a jet. Here, we ask to what extent these quantities may provide a useful characterisation of jet medium modifications. Figure 8 shows the result of simulations of the JEWEL parton shower. If collisional energy loss is included, then the jet is expected to broaden. This is clearly seen in the broadening of \( T \), \( T_{\text{maj}} \) and \( T_{\text{min}} \) in the upper left panel of figure 8. However, the kinematics of elastic \( 2 \rightarrow 2 \) scatterings dictates that the more energetic projectiles are deflected by smaller angles. We checked that the collisional broadening observed in figure 8 is due to recoil. If the recoiling scattering centres are removed from the final state and do not hadronise, the thrust distributions with medium are practically indistinguishable from the vacuum results. The recoiling scattering centres have mostly relatively low momenta so that the medium-induced broadening is much reduced and becomes small if only hadrons of energy above \( E_{\text{cut}} = 2 \text{ GeV} \) are included in the analysis (see lower left panel of figure 8).

In contrast to collisional energy loss, single partonic components of medium-induced additional radiation have a higher probability to carry a significant energy fraction of the initial projectile energy. So, on general grounds, one expects that the medium-induced broadening of the distributions in thrust, thrust major and thrust minor will persist even if soft hadrons of energy \( E_h < E_{\text{cut}} = 2 \text{ GeV} \) are dropped from the analysis. This is clearly seen in figure 8. We note that the value of \( f_{\text{med}} \) entering these simulations is a priori a free model parameter. We estimated, however, that the choice \( f_{\text{med}} = 3 \) lies within the range of parameters consistent with a nuclear modification factor \( R_{\text{PbPb}} \approx 0.2 \) in central lead-lead collisions.

The medium-modification of the \( n \)-jet fraction may provide another tool for disentangling elastic from inelastic jet quenching mechanisms. The potential impact of elastic \( 2 \rightarrow 2 \) processes on the \( n \)-jet fraction is limited to two numerically rather minor effects: First, elastic collisions are a source of angular broadening, which can increase the distance \( y_{ij} \) between partons and this implies that additional jets can become visible on a coarser scale \( y_{\text{cut}} \). However, the higher the energy of the partonic projectile, the smaller is this broadening effect. For this reason, one expects the dependence of the \( n \)-jet fraction on collisional energy loss to be relatively small and mainly visible for relatively small values of \( y_{\text{cut}} \) which are sensitive to small energies. Second, collisional mechanisms may kick target components towards higher transverse momenta. In this way, hard recoils may be counted as jets, thereby increasing the \( n \)-jet fraction. However, from the momentum space distribution of recoil partners studied in figures 6 and 7 we know that this effect will remain limited to very fine jet resolution scales. This is confirmed by the results shown in figure 9 which shows that even after an in-medium path length of \( L = 5 \text{ fm} \) in a medium of temperature \( T = 500 \text{ MeV} \), the \( n \)-jet fraction remains unaffected above \( \log_{10}(y_{\text{cut}}) < -1.5 \) and the mild medium-induced deviations seen at higher jet resolution scale can be attributed largely to soft hadrons with energy \( E_h < E_{\text{cut}} = 2 \text{ GeV} \).

In contrast to elastic interactions, medium-induced radiation results in a distribution of sub-leading jet fragments up to characteristically larger energies. This is clearly seen in the medium-modified \( n \)-jet fractions of figure 8 which shows a marked increase in the average number of jets even for a very coarse resolution scale \( \log_{10}(y_{\text{cut}}) > -1 \). After a background cut at \( E_{\text{cut}} = 2 \text{ GeV} \), this medium-modification remains clearly visible on the logarithmic scale \( \log_{10}(y_{\text{cut}}) \).

We have checked that the \( n \)-jet fraction shows a very weak dependence on the total jet energy in the range \( 75 < E_{\text{jet}} < 125 \text{ GeV} \). However, if this jet energy is not measured exactly, \( E_{\text{jet,true}} = f_{\text{corr}} E_{\text{jet,meas}} \), then the \( n \)-jet fraction is shifted by a constant value \( -\log_{10}(f_{\text{corr}}) \) in \( \log_{10}(y_{\text{cut}}) \). For instance, a deviation of the jet energy reconstructed from a calorimetric jet measurement from the true jet energy by 30% amounts to a shift of \( \log_{10}(1.3^2) \approx 0.23 \). This is comparable to the size of the
medium modifications on the right hand side of figure 9. So, our studies indicate that the n-jet fraction may provide a powerful tool for disentangling radiative from collisional energy loss mechanisms, but a proper assessment of various experimental uncertainties will be clearly needed to explore this tool. An analogous statement applies to the medium-modifications of jet shapes studied in figure 8.

In figure 10, we plot the medium modification of the single inclusive distribution \(dN/d\xi\) in the presence of collisional and radiative medium effects. Similar to the case for the vacuum shower in figure 3, hadronisation is seen to lead to a significant softening of the distribution. If elastic interactions with the medium are included but only the cascade is hadronised, then the total jet multiplicity of projectile partons or hadrons depends only very weakly on the medium, since \(2 \rightarrow 2\) processes do not increase the parton multiplicity. However, the total jet multiplicity may increase significantly if recoil partons are counted towards the jet. This is illustrated in figure 10 which also shows the resulting \(dN/d\xi\) distribution after addition of all recoil partons that go in the same hemisphere as the original parton. Thus the jet-modification of the medium and the medium-modification of the jet are two complementary aspects of the same dynamic phenomenon.

As stated above, the present study of radiative parton energy loss mechanisms is based on a simplified model which does not account for the recoil distribution of target partons. However, figure 10 indicates that radiative mechanisms result in a larger increase of intra-jet multiplicities.

5 Conclusion and Outlook

Monte Carlo models for hadronic and nuclear collisions are at the interface between the theory of QCD and experiment. Depending on the class of measurements and the status of theory, they provide a bridge between QCD and data which invokes a varying degree of model assumptions. In this paper, we have started to discuss the Monte Carlo model JEWEL, developed for a class of measurements sensitive to jet quenching phenomena in ultrarelativistic nuclear collisions. In the absence of medium-effects, we have validated JEWEL in section 2 against a set of benchmark jet measurements. In section 3, we have then introduced medium-modifications with a special focus on the dynamic description of elastic interactions between a fragmenting projectile and the medium. The main results, presented in section 4 show that collisional energy
loss is important and demonstrate the basic physical jet quenching effects for given temperature and path length in the plasma. Moreover, we find that quantities like the $n$-jet fraction and jet event shapes may allow us to distinguish between elastic and inelastic mechanisms underlying medium-induced parton energy loss. As discussed in the introduction, the microscopic mechanism(s) underlying jet quenching are not firmly established and their theoretical description is incomplete. In view of the complexity of this problem in heavy ion collisions, the Monte Carlo simulation method is particularly suitable for a detailed treatment. Amongst the many open problems in the theory and phenomenology of jet quenching, we identify in particular the following points which may be accessible by further developments of JEWEL:

1. **Heavy flavour, multi-hadron correlations and other measurements.**
   JEWEL can be extended easily to include the in-medium propagation of heavy quarks, which has attracted significant interest recently [11–14,56]. Simulations of multi-hadron correlations can be extracted from the present version of JEWEL, but are likely to depend strongly on the hadronisation model, as discussed in the context of figure 3.

2. **Realistic description of collision geometry and expansion.**
   On a schematic level, JEWEL can be interfaced with any simulation of the bulk evolution of heavy ion collisions, by supplementing JEWEL with information about a realistic distribution of partonic trajectories inside the medium, and with information about a realistic time-dependent density of scattering centres along these trajectories. In particular, this includes full 2-dimensional ideal [57] or viscous [58, 59] and full 3-dimensional ideal [60] hydrodynamic simulations of the heavy ion collision, as well as other models for an expanding plasma [15].

3. **Radiative parton energy loss.**
   The version of JEWEL presented here mimics radiative energy loss by enhancing the perturbative splitting functions. We regard the substitution of this *ad hoc* prescription by realistic $2 \rightarrow 3$ inelastic parton scattering cross sections as a feasible and interesting future extension of JEWEL. It would provide for a radiative energy loss mechanism which includes recoil effects, and it could define the incoherent limit of any radiative mechanism.

4. **The separation between weakly coupled and strongly coupled regimes and the problem of hadronisation.**

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**Figure 9.** $n$-jet fractions for a single 100 GeV quark jet after hadronisation in vacuum and with medium effects. Left hand side: Collisional energy loss for $T = 500$ MeV and $L = 5$ fm. Recoil partons are either hadronised together with the cascade ('all') or they are not included in the hadronisation ('cascade'). Right hand side: Medium-induced radiation for $f_{med} = 3$ and $L = 5$ fm. In the top row, all hadrons are included, while in the bottom row, only hadrons with energy above $E_{cut} = 2$ GeV are included.
On theoretical grounds, there is ample motivation to use weak coupling techniques for describing the splitting of highly virtual partons, and their sufficiently hard interactions with a medium. However, for soft interactions with the medium, one has to invoke a nonperturbative mechanism for regularising the infra-red singularities, resulting in uncertainties discussed in section 3. Additional complications arise if one considers the propagation of soft projectile fragments whose momentum is close to that of components of the heat bath. Propagation of these fragments within a perturbative parton cascade is expected to be unreliable since the medium is likely to be strongly coupled and the dynamics of the medium cannot be accounted for by partonic 2 → 2 and 2 → 3 processes only. Another open question is how to hadronise these soft jet components within a soft high multiplicity environment, where novel hadronisation mechanisms such as recombination may become relevant. These processes may become a test laboratory for understanding how a well-defined partonic projectile interacts and to what extent it thermalises within a finite size medium. From a pragmatic point of view, a parton shower may contribute to this issue by identifying the momentum scales at which a perturbative description breaks down.

There are many other open issues for improving our understanding of jet quenching in an interplay between theory and experiment. One of the most central ones may be how to best characterise a jet within a high multiplicity environment such that unambiguous information about its medium modifications can be obtained. Standard calorimetric jet measurements are very difficult to apply to heavy ion collisions, though novel jet algorithms may be better suited [61]. On the other hand, the suppression of single inclusive spectra provides unambiguous evidence that there are strong medium modifications. There is no fundamental reason why a similar unambiguous characterisation should not be possible for other aspects of the entire jet fragmentation pattern. From a pragmatic point of view, the question is then how to identify classes of jet measurements, which are sensitive to medium effects but which remain sufficiently insensitive to operational uncertainties in the jet definition. JEWEL may contribute to this central issue on various levels. By superimposing simulations of JEWEL on top of the simulated background of heavy ion collisions (or developing JEWEL to include this) one can test the sensitivity of different jet observables. On the other hand, by simulating the redistribution of ‘background’ multiplicity due to the propagation of a jet, JEWEL provides a mean to go beyond the simplifying assumption that the medium-modified jet is uncorrelated to the underlying background. These features of JEWEL may contribute to establish to what extent an operational procedure of characterising jet medium modifications is suited to draw model-independent conclusions.

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