Influence of light source and detector’s imperfections on the photon counting statistics

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Abstract. Photon counting provides a valuable tool for measuring quantum properties of light. Unavoidable imperfections of photon detectors lead to discrepancies between the statistics of photons and statistics of photon counts. Here we consider dead-time corrections and influence of intensity fluctuations for simulated Poissonian statistics of photo counts for the pulsed or continuous wave coherent light sources. We analyse dependence between the statistical moments of corrected distribution and the mean photon number, dead-time, and level of intensity fluctuations.

1. Introduction

Coherent states of light are widely used for various quantum optical technologies, e.g. quantum imaging, metrology, and quantum key distribution [1–7], to name a few. High-precision detection of coherent states is a well-known long-standing problem which is a subject of active research over the last few years [8–15].

One of the detection approaches exploits single photon counting which leads to the Poisson distribution of photons. However, inherent imperfections of the detector (e.g. dead time) and laser source (e.g. intensity fluctuations) affect the photon counting distribution and complicates discrimination of coherent states.

In this work, we show how various imperfections of the single-photon detector and light source influence the photon counting statistics and establish the explicit connection between the statistics of photons and statistics of photocounts. We also allow corrections produced by the fluctuations of source intensity known as Mandel’s formula [16, 17]. We calculate first statistical moments (mean value, variance, skewness, etc.) of photocounting distribution modified by the dead time corrections and intensity fluctuation influence and compare with moments of numerically simulated statistics. Our statistical model can be supplemented by such imperfections as a photo detector jitter and a time-distributed laser pulse repetition rate.

These research ideas can be adopted for the development of novel key distribution protocols requiring photon counting resolution.
2. Theory and methods

The cumulative distribution function $F_k$ (CDF) is used for calculation of the probability function $p_k$ for Sub-Poisson distribution:

$$p_k(\lambda, \nu T, \nu \tau) = F_{k-1}(\lambda, \nu T, \nu \tau) - F_k(\lambda, \nu T, \nu \tau),$$

(1)

where $\lambda$, $\nu$, $T$, $\tau$ are mean number of photons, repetition rate of measurement, time window of measurement and dead-time respectively. CDF, this convenient implement for the analysis of photon count distribution [1], was calculated for a time interval with non-dead initial point:

$$F_k(\lambda, \nu T, \nu \tau) = \frac{1}{k!} \gamma(k+1, (\nu T - k \cdot \nu \tau) \cdot \lambda) \cdot \theta(T - k \tau),$$

(2)

where

$$\gamma(k+1, \beta) = \int_0^\beta z^k e^{-z} dz, \quad \theta(T - k \tau)$$

are incomplete gamma function and unit step function respectively. There is a natural generalization of this implement for arbitrary type of coherent light sources, modulations of emission and time window parameters.

Previous investigations used non-dead initial point for time interval [17]. But there can be dead-time at the beginning of the time window in the case of a continuous counting process. We recalculate the probability function with corrections of possible dead initial time moment.

Possible dead initial time interval leads to new modified formula, and with conditions $T = 1/\nu$, $\kappa = \nu \tau$ is given by:

$$\tilde{p}_k(\kappa, \lambda) = p_k(\lambda, 1, \kappa) + \frac{\lambda}{1 + \lambda \kappa} \int_0^\kappa p_k(\lambda, 1 - y, \kappa) dy.$$  

(3)

Another calculated correction is due to the fluctuations of source intensity. Mandel’s formula [16] leads to more complicated equation for time-varying intensity:

$$\tilde{P}_k(\kappa, \xi) = \int_0^\infty \tilde{p}_k(\kappa, \lambda) \cdot p_\xi(\lambda) \, d\lambda,$$

(4)

where $\xi$ is the distribution of time-varying intensity and $p_\xi(\lambda)$ is the probability function of this distribution. Pulsed type of source, jitter of time window or frequency fluctuations cause to corrections which can be included as intensity fluctuations in the Eq.(4). For time-distributed parameter $a_i$ with probability function $p_i(a_i)$ is given by:

$$\tilde{P}_k(\kappa, ...) = \int \tilde{p}_k(\kappa, a_i, ...) \cdot p_i(a_i) \, da_i.$$  

(5)

Full integration in previous equations can be easily calculated to an arbitrary order. There is a series expansion of intensity averaged CDF [17]:

$$\langle F_k(\nu T, \nu \tau) \rangle = \frac{\theta(T - k \tau)}{k!} \sum_{s=0}^\infty \frac{(-1)^s \nu T - k \nu \tau)^{k+1+s}}{s! \cdot k + 1 + s} M_{k+1+s},$$

(6)

where $M_i$ is $i$th statistical moment of distribution $\xi$ of time-varying intensity. This expansion with equation (4) and integral operators manipulation provides fast-computing expression for probability function.

The simulated distributions with dead-time and intensity noise corrections are represented in Fig.1.
3. Results and discussion
We calculated statistical moments of the modified photocounting distribution using by equations (3), (4) and (6) with respect to several varying parameters such as average number of photons, dead-time and level of intensity fluctuations. Results of numerical simulations indicate that the calculated distributions correctly describe photon counting statistics. It shows functional connections of parameters of light and detector with statistical moments of measured distribution, which are easy to compute. Fast and precise recovery of light source properties and dead-time is one of the issues which can be solved by using this functional dependencies.

![Figure 1](image.png)
Figure 1. Left: Poisson distribution of photons. Mean number of photons $\lambda = 1$. Center: sub-Poissonian dead-time-corrected distribution of photon counts. Mean number of photons $\lambda = 1$, dead time $\tau = 0.05 \cdot T$, where $T$ is time window of measurement. Right: distribution of photon counts with dead-time and source-noise corrections. Mean number of photons $\lambda = 1$, dead time $\tau = 0.05 \cdot T$, where $T$ is time window of measurement, ratio between intensity fluctuation and intensity $\delta \lambda / \lambda = 0.2$.

4. Acknowledgement
The work is supported by Russian Foundation for Basic Research grant 20-32-51004.

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