Multiparty Session Programming with Global Protocol Combinators

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Abstract

Multiparty Session Types (MPST) is a typing discipline for communication protocols. It ensures the absence of communication errors and deadlocks for well-typed communicating processes. The state-of-the-art implementations of the MPST theory rely on (1) runtime linearity checks to ensure correct usage of communication channels and (2) external domain-specific languages for specifying and verifying multiparty protocols.

To overcome these limitations, we propose a library for programming with global combinators—a set of functions for writing and verifying multiparty protocols in OCaml. Local behaviours for all processes in a protocol are inferred at once from a global combinator. We formalise global combinators and prove a sound realisability of global combinators—a well-typed global combinator derives a set of local types, by which typed endpoint programs can ensure type and communication safety. Our approach enables fully-static verification and implementation of the whole protocol, from the protocol specification to the process implementations, to happen in the same language.

We compare our implementation to untyped and continuation-passing style implementations, and demonstrate its expressiveness by implementing a plethora of protocols. We show our library can interoperate with existing libraries and services, implementing DNS (Domain Name Service) protocol and the OAuth (Open Authentication) protocol.

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Supplementary Material A source code repository for the accompanying artifact is available at https://github.com/keigoi/ocaml-mpst/

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1 Introduction

Multiparty Session Types. Multiparty Session Types (MPST) [27, 12, 28] is a theoretical framework that stipulates how to write, verify and ensure correct implementations of communication protocols. The methodology of programming with MPST (depicted in Fig. 1(a)) starts from a communication protocol (a global type) which specifies the behaviour of a system of interacting processes. The local behaviour (a local type) for each endpoint process is then algorithmically projected from the protocol. Finally, each endpoint process is implemented in an endpoint host language and type-checked against its respective local type by a session typing system. The guarantee of session types is that a system of well-typed endpoint processes does not go wrong, i.e. it does not exhibit communication errors such as reception errors, orphan messages or deadlocks, and satisfies session fidelity, i.e. the local behaviour of each process follows the global specification.

The theoretical MPST framework ensures desirable safety properties. In practice, session types implementations that enforce these properties statically, i.e. at compile-time, are limited to binary (two party protocols) [50, 45, 37, 47]. Extending binary session types implementations to multiparty interactions, which support static linearity checks (i.e., linear usage of channels), is non-trivial, and poses two implementation challenges.

(C1) How global types can be specified and verified in a general-purpose programming language? Checking compatibility of two communicating processes relies on duality, i.e., when one process performs an action, the other performs a complementary (dual) action. Checking the compatibility of multiple processes is more complicated, and relies on the existence of a well-formed global protocol and the syntax-directed procedure of projection, which derives local types from a global specification. A global protocol is considered well-formed, if local types can be derived via projection. Since global types are far from the types of a “mainstream” programming language, state-of-the-art MPST implementations [29, 42, 54, 10] use external domain-specific protocol description languages and tools (e.g. the Scribble toolchain [57]) to specify global types and to implement the verification procedure of projection. The usage of external tools for protocol description and verification widens the gap between the specification and its implementations and makes it more difficult to locate protocol violations in the program, i.e. the correspondence between an error in the program and the protocol is less apparent.

(C2) How to implement safe multiparty communication over binary channels? The theory of MPST requires processes to communicate over multiparty channels – channels that carry messages between two or more parties; their types stipulate the precise sequencing of the communication between multiple processes. Additionally, multiparty channels have to be used linearly, i.e exactly once. In practice, however, (1) communication channels are binary, i.e a TCP socket for example connects only two parties, and hence its type can describe interactions between two entities only; (2) most languages do not support typing of linear resources. Existing MPST implementations [29, 42, 54, 10] apply two workarounds. To preserve the order of interactions when implementing a multiparty protocol over binary channels, existing works use code generation (e.g. [57]) and generate local types (APIs) for several (nominal) programming languages. Note that although the interactions order is preserved, most of these implementations [29, 42, 10] still require type-casts on the underlying channels, compromising type safety of the host type system. To ensure linear
usage of multiparty channels, runtime checks are inserted to detect if a channel has been used more than once. This is because the type systems of their respective host languages do not provide static linearity checking mechanism.

**Our approach.** This paper presents a library for programming MPST protocols in OCaml that solves the above challenges. Our library, *ocaml-mpst*, allows to specify, verify and implement MPST protocols in a single language, OCaml. Specifically, we address (C1) by developing global combinators, an embedded DSL (EDSL) for writing global types in OCaml. We address (C2) by encoding multiparty channels into channel vectors – a data structure, storing a nested sequence of binary channels. Moreover, *ocaml-mpst* verifies statically the linear usage of communication channels, using OCaml’s strong typing system and supports session delegation. The key device in our approach is the discovery that in a system with variant and record types, checking compatibility of local types coincides with existence of least upper bound w.r.t. subtyping relation. This realisation enables a fully static MPST implementation, i.e., static checking not only on local but also on global types in a general purpose language.

Programming with *ocaml-mpst* (Fig. 1(b)) closely follows the “top-down” methodology of MPST, but differs from the traditional MPST framework in Fig. 1(a). To use our library, a programmer specifies the global protocol with a set of global combinators. The OCaml typechecker verifies correctness of the global protocol and infers local types from global combinators. A developer implements the endpoint processes using our *ocaml-mpst* API. Finally, the OCaml type checker verifies that the API is used according to the inferred type.

The benefits of *ocaml-mpst* are that it is (1) lightweight – it does not depend on any external code-generation mechanism, verification of global protocols is reduced to typability of global combinators; (2) fully-static – our embedding integrates with recent techniques for static checking of binary session types and linearly-typed lists [33, 31], which we adopt to implement multiparty session channels and session delegation; (3) usable – we can auto-detect and correct protocol violations in the program, guided by OCaml programming environments like Merlin [5]; (4) extensible – while most MPST implementations rely on a nominal typing, we embed session types in OCaml’s structural types, and preserve session subtyping [23]; and (5) expressive – we can type strictly more processes than [55] (see § 7).

**Contributions.** Contributions and the outline of the paper are as follows:

§ 2 gives an overview of programming with *ocaml-mpst*, a library in OCaml for specification, verification and implementations of communication protocols.

§ 3 formalises global combinators, presents their typing system, and proves a sound realisability of global combinator, i.e. a set of local types inferred from a global combinator can
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```ocaml
let oAuth = (s -->c) login 00 (c -->a) pwd 00 (a -->s) auth 00 finish (* global protocol*)

let cliThread () =
  let ch = get_ch c oAuth in
  let login(x, ch) = recv ch#role_S in
  let ch = send ch#role_A#pwd "pass" in
  close ch

let srvThread () =
  let ch = get_ch s oAuth in
  let ch = send ch#role_C#login "Hi" in
  let auth(_,ch) = recv ch#role_A in
  close ch

let authThread () =
  let ch = get_ch a oAuth in
  let pwd(code,ch) = recv ch#role_C in
  let ch = send ch#role_S#auth true in
  close ch

let () = List.iter Thread.join [Thread.create cliThread (); Thread.create srvThread (); Thread.create authThread ()]
```

---

**Figure 2** Global protocol and local implementations for OAuth protocol.

2 We use a simplified syntax that support the in-built communication transport of Ocaml. For the full syntax of the library that is parametric on the transport, see the repository.

* § 4 discusses the design and implementation of global combinators.
* § 5 summarises the ocaml-mpst communication library and explains how we utilise advanced features/libraries in Ocaml to enable dynamic/static linearity checking on channels.
* § 6 evaluates ocaml-mpst. We compare ocaml-mpst with several different implementations and demonstrate the expressiveness of ocaml-mpst by showing implementations of MPST examples, as well as a variety of real-world protocols. We demonstrate our library can interoperate with existing libraries and services, namely we implement DNS (Domain Name Service) and the OAuth (Open Authentication) protocols on top of existing libraries. We discuss related work in § 7 and conclude with future work in § 8. Full proofs, omitted definitions and examples can be found in Appendix. Our implementation, ocaml-mpst is available at https://github.com/keigoi/ocaml-mpst including benchmark programs and results.

## 2 Overview of OCaml Programming with Global Combinators

This section gives an overview of multiparty session programming in ocaml-mpst by examples. It starts from declaration of global combinators, followed by endpoint implementations. We also demonstrate how errors can be reported by an OCaml programming environment like Merlin [5]. In the end of this section, we show the syntax of global combinators and the constructs of ocaml-mpst API in Fig. 5. The detailed explanation of the implementations of the constructs is deferred to § 4.

From global combinators to communication programs. We illustrate global combinators starting from a simple authentication protocol (based on OAuth 2.0 [25]). A full version of the protocol is implemented and discussed in § 6. Fig. 2 shows the complete OCaml implementation of the protocol, from the protocol specification (using global combinators) to the endpoint implementations (using ocaml-mpst API).

The protocol consists of three parties, a service `s`, a client `c`, and an authenticator `a`. The interactions between the parties (hereafter also called roles) proceed as follows: (1) the service `s` sends to the client `c` a `login` message containing a greeting (of type `string`); (2)
the client then continues by sending its password (pwd) (of type string) to the authenticator a; and (3) finally the authenticator a notifies s, by sending an auth message (of type bool), whether the client access is authorised.

The global protocol oAuth in Line 1 is specified using two global combinators, --> and finish. The former represents a point-to-point communication between two roles, while the latter signals the end of a protocol. The operator --> is a right-associative function application operator to eliminate parentheses, i.e., \((c --> a) \text{pwd} \exp\) is equivalent to \((c --> a) \text{pwd} (\exp)\), where --> works as a four-ary function which takes roles c and a and label pwd and continuation exp. We assume that login, pwd and auth are predefined by the user as label objects with their payload types of string, string and bool, respectively. Similarly, s, c and a are predefined role objects. We elaborate on how to define these custom labels and roles in § 4.

The execution of the oAuth expression returns a tuple of three channel vectors – one for each role in the global combinator. Each element of the tuple can be extracted using an index, encoded in role objects (c, s, and a). Intuitively, the role object c stores a functional pointer that points to the first element of the tuple, s points to the second, and a to the third element. The types of the extracted channel vectors reflect the local behaviour that each role, specified in the protocol, should implement. Channel vectors are objects that hide the actual bare communication channels shared between every two communicating processes.

Lines 3–21 present the implementations for all three processes specified in the global protocol. We explain the implementation for the client – cliThread (Lines 3–7). Other processes are similarly implemented. Line 4 extracts the channel vector that encapsulates the behaviour of the client, i.e the first element of oAuth. This is done by using the function get_ch (provided by our library) applied to the role object c and the expression oAuth.

Our library provides two main communication primitives, namely send and recv. To statically check communication structures using types, we exploit OCaml’s structural types of objects and polymorphic variants (rather than their nominal counterparts of records and ordinary variants). In Line 5, ch#role_S is an invocation of method role_S on an object ch. The recv primitive waits on a bare channel returned by the method invocation. The returned value is matched against a variant tag indicating the input label `login with the pair of the payload value x and a continuation ch (shadowing the previous usage of ch). Then, on Line 6, two method calls on ch are performed, e.g ch#role_A#pwd, which extract a communication channel for sending a password (pwd) to the authenticator. This channel is passed to the send primitive, along with the payload value "pass". Then, let rebinds the name ch to the continuation returned by send and on Line 7 the channel is closed. Each operation is guided by the host OCaml type system, via channel vector type. For example, the client channel ch extracted in Line 4 has a channel vector type (inferred by OCaml type checker) <role_S: [`login of string * t] inp> which denote reception (suffixed by inp) from server of a login label, then continuing to t, where t is <role_A:pwd:(string,close) out>> denoting sending (out) to authenticator of a pwd label, followed by closing. Note that the type \(<f: t>\) denotes an OCaml object with a field f of type t; \([m \tau] \) is a (polymorphic) variant type having a tag m of type t. Finally, in Lines 25–28 all processes are started in new threads.

On the expressiveness of well-typed global protocols. Fig. 3 shows two global protocols that extend oAuth with new behaviours. In Fig. 3a, the global combinator choice_at specifies a branching behaviour at role s. In the first case (Line 3), the protocol proceeds

---

3 To be precise, the labels are polymorphic on their payload types which are instantiated at the point where they are used.
let oAuth2 () =
  (choice_at s (to_s login_cancel)
   (s, oAuth ()
    (s, (s -->c) cancel @@
     (c -->a) quit @@
     finish)))

let oAuth3 () =
  fix (fun repeat ->
    (choice_at s (to_s oauth2_retry)
     (s, oAuth2 ()
      (s, (s -->c) retry @@
       repeat))
      repeat))
receive actions (Lines 6 and 5) in the client implementation in Fig. 2. Similarly, errors will also be reported if we misspell any of the methods `pwd`, `role_A`, or `role_C`.

Similarly, an error is reported if the global protocol is not safe (which corresponds to an ill-formed MPST protocols [16]) since this may lead to unsafe implementations. Consider Fig. 6 (b), where we modify OAuth2 such that `s` sends a `cancel` message to `a`. This protocol exhibits a race condition: even if all parties adhere to the specified behaviour, `c` can send a `quit` before `s` sends `login`, which will lead to a deadlock on `s`. Our definition of global combinators prevents such ill-formed protocols, and the OCaml compiler will report an error. The actual error message reported in OCaml detects the mismatch between `a` and `c`, indicating violation of the active role property in the MPST literature [16] – the sender must send to the same role.

3 Formalisms and Typing for Global Combinators

This section formalises global combinators and their typing system, along a formal correspondence between a global combinator and channel vectors. The aim of this section is to provide a guidance towards descriptions of the implementations presented in § 4,5.

We first give the syntax of global combinators and channel vectors in § 3.1. We then propose a typing system of global combinators in § 3.2, illustrating that the rules check their
well-formedness. We define derivation of channel vectors from global combinators in § 3.3. The main theorem (Theorem 3.11) states that a well-typed global combinator always derives a channel vector which is typable by a corresponding set of local types, i.e. any well-typed global combinator is soundly realisable by a tuple of well-typed channel vectors.

3.1 Global Combinators and Channel Vector Types

Global combinators denote a communication protocol which describes the whole conversation scenario of a multiparty session.

▷ Definition 3.1 (Global combinators and channel vector types). The syntax of global combinators, written \( g, g', \ldots \), are given as:

\[
g ::= (p \rightarrow q) m : T g | \text{choice} p \{ g_i \}_{i \in I} | \text{fix} x \rightarrow g | x | \text{finish}
\]

where the syntax of payload types \( S, T, \ldots \) (also called channel vector types) is given below:

\[
T, S ::= !T | ?T | T_1 \times \cdots \times T_n | \{ | \langle l_i : T_i \rangle_{i \in I} | [l_i T_i]_{i \in I} | \mu t . T |
\]

The formal syntax of global combinators comes from Scribble [57] and corresponds to the standard global types in MPSTs [43]. We assume a set of participants (\( R = \{ p, q, r, \ldots \} \)), and that of alphabets (\( A = \{ \text{ok, cancel, } \ldots \} \)). Communication combinator \((p \rightarrow q) m : T g\) states that participant \( p \) can send a message of type \( T \) with label \( m \) to participant \( q \) and that the interaction described in \( g \) follows. We require \( p \neq q \) to prevent self-sent messages. We omit the payload type when unit type \( \bullet \), and assume \( T \) is closed, i.e. it does not contain free recursive variables. Choice combinator \( \text{choice} p \{ g_i \}_{i \in I} \) is a branching in a protocol where \( p \) makes a decision (i.e. an output) on which branch the participants will take. Recursion \( \text{fix} x \rightarrow g \) is for recursive protocols, assuming that variables \( (x, x', \ldots) \) are guarded in the standard way, i.e. they only occur under the communication combinator. Termination \( \text{finish} \) represents session termination. We write \( p \in \text{roles}(g) \) (or simply \( p \in g \)) iff, for some \( q \), either \( p \rightarrow q \) or \( q \rightarrow p \) occurs in \( g \).

▷ Example 3.2. The global combinator \( g_{\text{Auth}} \) below specifies a variant of an authentication protocol in Fig. 3 where \( T = \text{string} \) and client sends \( \text{auth} \) to server, then server replies with either \( \text{ok} \) or \( \text{cancel} \).

\[
g_{\text{Auth}} = (c \rightarrow s) \text{auth}: T (\text{choice} s \{ (s \rightarrow c) \text{ok}: T \text{finish}, (s \rightarrow c) \text{cancel}: T \text{finish} \})
\]

Channel vector types abstract behaviours of each participant using standard data structure and channels. We assume labels \( l, l', \ldots \) range over \( R \cup A \). Types \( !T \) and \( ?T \) denote...
output and input channel types, with a value or channel of type $T$ (note that the syntax includes session delegation). $\langle \mathbb{T} \rangle$ is an io-type which is a subtype of both input or output types [53]. $T_1 \times \cdots \times T_n$ is an n-ary tuple type. $\langle l_i : T_i \rangle_{i \in I}$ is a record type where each field $1_i$ has type $T_i$ for $i \in I$. $\langle 1_i : T_i \rangle_{i \in I}$ is a variant type [53] where each $1_i$ is a possible tag (or constructor) of that type and $T_i$ is the argument type of the tag. In both record and variant types, we assume the fields and tags are distinct (i.e. in $\langle l_i : T_i \rangle_{i \in I}$ and $\langle 1_i : T_i \rangle_{i \in I}$, we assume $1_i \neq 1_j$ for all $i \neq j$). The symbol $\bullet$ denotes a unit type. Type $t$ is a variable for recursion. A recursive type takes an equi-recursive viewpoint, i.e. $\mu t. T$ is viewed as $T(\mu t. T \backslash t)$. Recursion variables are guarded and payload types are closed.

Channel vectors: Session types as record and variant types. The execution model of MPST assumes that processes communicate by exchanging messages over input/output (I/O) channels. Each channel has the capability to communicate with multiple other processes. A local session type prescribes the local behaviour for a role in a global protocol by assigning a type to the communication channel utilised by the role. More precisely, a local session type specifies the exact order and payload types for the communication actions performed on each channel (see Fig. 1(a)). In practice, processes communicate on a low-level bi-directional I/O channels (bare channels), which are used for synchronisation of two (but not multiple) processes. Therefore, to implement local session types in practice, a process should utilise multiple bare channels, preserving the order, in which such channels should be used. We encode local session types as channel vector types, which wrap bare channels (represented in our setting by $? T \cdot T \cdot \mathbb{T}$ types) in record and variant types. This is illustrated in the following table, with the corresponding local session types for reference.

| Behaviour          | Channel vector type | Local session type [56] |
|--------------------|---------------------|-------------------------|
| Selection (Output choice) | $\langle q : (m_i : S_i \times T_i)_{i \in I} \rangle$ | $\langle q \nu_{i \in I} m_i (S_i) \cdot T_i \rangle$ |
| Branching (Input choice) | $\langle q : ? (m_i : S_i \times T_i)_{i \in I} \rangle$ | $\langle q \&(i \in I) m_i (S_i) \cdot T_i \rangle$ |
| Recursion          | $\mu t. T$, $t$      | $\mu t. T$, $t$          |
| Closing            | $\bullet$            | $\mu t. T \backslash t$  |

Intuitively, the behaviour of sending a message is represented as a record type, which stores inside its fields a bare output channel and a continuation; the input channel required when receiving a message is stored in a variant type. Type $\langle q : (m_i : S_i \times T_i)_{i \in I} \rangle$ is read as: to send label $m_i$ to $q$, (1) the channel vector should be ’peeled off’ from the nested record by extracting the field $q$ then $m_i$; then (2) it returns a pair $S_i \times T_i$ of an output channel and a continuation. Type $\langle q : ? (m_i : S_i \times T_i)_{i \in I} \rangle$ says that (1) the process extracts the value stored in the field $q$, then reads on the resulting input channel (?) to receive a variant of type $m_i : S_i \times T_i$; then, (2) the tag (constructor) $m_i$ of the received variant indicates the label which $q$ has sent, and the former’s argument $S_i$ is the payload, and the latter $T_i$ is the continuation.

The anti-symmetric structures between output types $\langle q : (m_i : S_i \times T_i)_{i \in I} \rangle$ and input types $\langle q : ? (m_i : S_i \times T_i)_{i \in I} \rangle$ (notice the placements of $!$ and $?$ symbol in these types) come from the fact that an output is an internal choice where output labels are proactively chosen via projection on a record field, while an input is an external choice where input labels are reactively chosen via pattern-matching among variant constructors.

### 3.2 Typing Global Combinators

A key finding of our work is that compatibility of local types can be checked using a type system with record and variant subtyping. Before explaining how each combinator ensures compatibility of types, we give an intuition of well-formed global protocols following [16].
Well-formedness and choice combinator. A well-formed global protocol ensures that a protocol can be correctly and safely realised by a system of endpoint processes. Moreover, a set of processes that follow the prescribed behaviour is deadlock-free. Well-formedness imposes several restrictions on the protocol structure, notably on choices. This is necessary because some protocols, such as OAuth in Fig. 6(b) (§ 2), are unsafe or inconsistent. More precisely, a protocol is well-formed if local types can be generated for all of its roles, i.e. the endpoint projection function [16, Def. 3.1][Def. F.3 in Appendix (§ F)] is defined for all roles. Our encoding allows the well-formedness restrictions to be checked statically, by the OCaml typechecker. Below, we explain the main syntactic restrictions of endpoint projection, which are imposed on choices and checked statically:

R1 (active role) in each branch of a choice, the first interaction is from the same sender role (active role) to the same receiver role (directed output).

R2 (deterministic choice) output labels from an active role are pairwise distinct (i.e., protocols are deterministic).

R3 (mergeable) the behaviour of a role from all branches should be mergeable, which is ensured by the following restrictions:

- M1 two input choices are merged only if (1) their sender roles are the same (directed input), and (2) their continuations are recursively mergeable if labels are the same.

- M2 two output choices can be merged if they are the same.

Intuitively, the conditions in R3 ensure that a process is able to determine unambiguously which branch of the choice has been taken by the active role, otherwise the process should be choice-agnostic, i.e. it should preform the same actions in all branches. Requirement R3 is known in the MPST literature as recursive full merging [16].

Typing system for global combinators. Deriving channel vector types from a global combinator corresponds to the endpoint projection in multiparty session types [28]. Projection of global protocols relies on the notion of merging (R3). As a result of the encoding of local types as channel vectors with record and variants, the merging relation coincides with the least upper bound (join) in the subtyping relation. This key observation allows us to embed well-formed global protocols in OCaml, and check them using the OCaml type system.

Next we give the typing system of global combinators, explaining how each of the typing rules ensures the verification conditions R1-R3. The typing system uses the following subtyping rules.

Definition 3.3. The subtyping relation $\leq$ is coinductively defined by the following rules.

Among those, the rules $[\text{Osub-L}]$ and $[\text{Osub-R}]$ realise equi-recursive view of types. The only non-standard rule is $[\text{Osub-RcdDepth}]$ which does not allow fields to be removed in the super type. This simulates OCaml’s lack of row polymorphism where positive occurrences of objects are not allowed to drop fields. Note that the negative occurrences of objects in OCaml, which we use in process implementations, for example, do have row polymorphism, which correspond to standard record subtyping: $S_i \leq T_i$, $i \in I$. We use standard record subtyping, when typing processes. Since it permits removal of fields, it precisely simulates session subtyping on outputs. Typing rules for processes are left to Appendix § C.6.
The typing rules for global combinators (Fig. 7) are defined by the typing judgement of the form $\Gamma \vdash_R g : T$ where $\Gamma$ is a type environment for recursion variables (definition follows), $R = p_1, \ldots, p_n$ is the sequence of roles which participate in $g$, and $T = T_1 \times \cdots \times T_n$ is a product of channel vector types where each $T_i$ indicates a protocol which the role $p_i$ must obey. We use the product-based encoding to closely model our implementation and to avoid fixing the number of roles $n$ of finish combinator by using variable-length tuples (see Appendix § E).

**Definition 3.4 (Global combinator typing rules).** A typing context $\Gamma$ is defined by the following grammar: $\Gamma ::= \emptyset \mid \Gamma, x:T$. The judgement $\Gamma \vdash_R g : T$ is defined by the rules in Fig. 7. We say $g$ is typable with $R$ if $\Gamma \vdash_R g : T$ for some $\Gamma$ and $T$. If $\Gamma$ is empty, we write $\vdash_R g : T$.

The rule $\text{[Orc-Com]}$ states that $p_i$ has an output type $(p_j; \langle \text{m!} S \times T_i \rangle)$ to $p_j$ with label $\text{m}$, a payload typed by $S$ and continuation typed by $T_i$; a dual input type $(p_j; \langle \text{m?} S \times T_j \rangle)$ from $p_j$ and continuation typed by $T_j$; and the rest of the roles are unchanged.

Rule $\text{[Orc-Sub]}$ is the key to obtain full merging using the subtyping relation, and along with the rule $\text{[Orc-Choice]}$, is a key to ensure the protocol is realisable, and free of communication errors. The rule $\text{[Orc-Choice]}$ requires (1) role $p_a$ to have an output type to the same destination role $q$, which satisfies $R.1$. The output labels $(\{k\}_{k \in K})$ are mutually disjoint at each branch $g_i$, and are merged into a single record, which ensures that the choice is deterministic (R2). All other types stay the same, up to subtyping. Following rule M1 of R3, a non-directed external choices are prohibited. This is ensured by encoding the sender role of an input type as a record field. As the two different destination role labels would result in two record types with no join, following subtyping rule $\text{[Orc-RecDepth]}$, a non-directed external choices are safely reported as a type error. Non-directed internal choices are similarly prohibited (M2). On the other hand, directed external choices are allowed, as stipulated by M1, and ensured by the subtyping relation on variant types $\text{[Orc-Var]}$. For example, the two input types $(q; \langle \text{m?} S_1 \times T_1 \rangle)$ and $(q; \langle \text{m?} S_2 \times T_2 \rangle)$ can be unified as $(q; \langle \text{m?} S \times T \rangle)$ where $S = S_1 \cup S_2$.

The rest of the rules are standard. Rule $\text{[Orc-fix]}$ is for recursion; it assigns the recursion variable $x$ a sequence of distinct fresh type variables in the continuation which is later looked up by $\text{[Orc-Choice]}$. In $\text{fix}(t, T)$, we assign a unit type if the role does not contribute to the recursion (i.e., $T = t'$ for any $t'$), or forms a recursive type $\mu_t.T$ otherwise.

**Example 3.5 (Typing a global combinator).** We show that the global combinator $g_{\text{Auth}} = \langle c \to s \rangle \text{auth} \left(\text{choices} \{(s \to c) \text{ ok finish}, (s \to c) \text{ cancel finish}\} \right)$ has the following type under $s,c$:

$\langle c; \langle \text{auth} \times T \times (c; \langle \text{ok!} T \times \bullet, \text{cancel!} T \times \bullet \rangle) \rangle\rangle \times \langle \text{auth} \times T \times (c; \langle \text{ok?} T \times \bullet, \text{cancel?} T \times \bullet \rangle) \rangle$
First, see that \( g_1 = ((s \rightarrow c) \text{ok} \text{ finish}) \) has a typing derivation as follows (note that we omit the payload type \( T \) in global combinators):

\[
\Gamma \vdash \text{finish} : \bullet \times \bullet \\
\Gamma \vdash s \rightarrow c \vdash \text{ok} \vdash \text{finish} : (t : (\text{ok} : T \times \bullet)) \times (s : ?(\text{ok} : T \times \bullet))
\]

For \( g_2 = ((s \rightarrow c) \text{cancel} \text{ finish}) \) we have similar derivation. Then, type of role \( c \) (the second of the tuple) is adjusted by \([\text{o}c\text{-Sub}]\), \( (s : ?(\text{ok} : T \times \bullet)) \leq (s : ?(\text{ok} : T \times \bullet, \text{cancel} : T \times \bullet)) \) and \( (s : ?(\text{cancel} : T \times \bullet)) \leq (s : ?(\text{ok} : T \times \bullet, \text{cancel} : T \times \bullet)) \), thus we have:

\[
\Gamma \vdash s \rightarrow c \vdash g_1 : (c : (\text{ok} : T \times \bullet)) \times (s : ?(\text{ok} : T \times \bullet, \text{cancel} : T \times \bullet)) \\
\Gamma \vdash s \rightarrow c \vdash g_2 : (c : (\text{cancel} : T \times \bullet)) \times (s : ?(\text{ok} : T \times \bullet, \text{cancel} : T \times \bullet))
\]

Then, by \([\text{o}c\text{-Choos}]\), we have the following derivation:

\[
\Gamma \vdash s \rightarrow c \vdash g_1 : (c : (\text{ok} : T \times \bullet)) \times (s : ?(\text{ok} : T \times \bullet, \text{cancel} : T \times \bullet)) \\
\Gamma \vdash s \rightarrow c \vdash g_2 : (c : (\text{cancel} : T \times \bullet)) \times (s : ?(\text{ok} : T \times \bullet, \text{cancel} : T \times \bullet)) \\
\Gamma \vdash s \rightarrow c \vdash \text{choice} s \{ g_1, g_2 \} : (c : (\text{ok} : T \times \bullet, \text{cancel} : T \times \bullet)) \times (s : ?(\text{ok} : T \times \bullet, \text{cancel} : T \times \bullet))
\]

Note that, in the above premises, the first element of the tuple specifying the behaviour of choosing role \( s \), namely \( (c : (\text{ok} : T \times \bullet)) \) and \( (c : (\text{cancel} : T \times \bullet)) \), are disjointly combined into \( (c : (\text{ok} : T \times \bullet, \text{cancel} : T \times \bullet)) \) in the conclusion. Then, by applying \([\text{o}c\text{-Coma}]\) again, we get the type for \( g_{\text{Auth}} \) presented above.

### 3.3 Evaluating Global Combinators to Channel Vectors

Channel vectors are data structures which are created from a global combinator at initialisation, and used for sending/receiving values from/to participants. Channel vectors implement multiparty session programming as nested binary i/o-typed channels.

**Definition 3.6 (Channel vectors).** Channel vectors \((c, c', \ldots)\) and wrappers \((h, h', \ldots)\) are defined as:

\[
c, c' ::= v, \ldots \mid s, s', \ldots \mid (c_1, \ldots, c_n) \mid [\lambda = c] \mid (l_i = c_{i})_{i \in I} \mid \mu x. c \mid [s_i @ h_i]_{i \in I}
\]

\[
h, h' ::= [\lambda] \mid [\lambda = h] \mid (c_1, \ldots, c_n) \mid [\lambda = c_1, \ldots, l_k = h, \ldots, l_n = c_n] \mid 1 := p \mid m
\]

Channel vectors \( c \) are either base values \( v \) or runtime values generated from global combinators which include \text{names} (simply-typed binary channels) \( s, s', \ldots \), \text{tuples} \((c_1, \ldots, c_n)\), \text{variants} \([\lambda = c]\), \text{records} \((\lambda = c_{i})_{i \in I}\), and \text{recursive values} \(\mu x. c\) where \( x \) is a bound variable.

We introduce an extra runtime value, \text{wrapped names} \([s_i @ h_i]_{i \in I}\), inspired by Concurrent ML’s \text{wrap} and \text{choose} functions [52], which are a sequence \([\ldots]_{i \in I}\) of pairs of input name \( s_i \) and a \text{wrapper} \( h_i \). A wrapper \( h \) contains a single hole \([\_]\). An input on wrapped names \([s_i @ h_i]_{i \in I}\) is \text{multiplexed} over the set of names \( \{s_i\}_{i \in I} \). When a sender outputs value \( c' \) on name \( s_j \) \((j \in I)\), the corresponding input waiting on \([s_i @ h_i]_{i \in I}\) yields a value \( h_j [c'] \) where the construct \( h[c] \) denotes a value obtained by replacing the hole \([\_]\) in \( h \) with \( c \) (i.e. applying function \( h \) to \( c \)). We write \([l_i = (s_j,c_j)]_{i \in I}\) for \([s_i @ l_i = ([\ldots])_{i \in I}]_{i \in I}\).

**Definition 3.7 (Typing rules for channel vectors).** Fig. 8 gives the typing rules for channel vectors and wrappers. The typing judgement for (1) channel vectors has the form \( \Gamma \vdash c : T \); (2) wrappers has the form \( \Gamma \vdash h : H \) where the type for wrappers is defined as \( H := T[S] \); We assume that all types in \( \Gamma \) are closed.

The rules for channel vectors are standard where the subtyping relation in rule \([\text{o}c\text{-Sub}]\) is defined at Definition 3.3. For wrappers, rule \([\text{o}c\text{-Warp}])\) types wrapped names where the payload type \( S' \) of input channel \( s \) is the same as the hole’s type, and all wrappers have the same result type \( T \). Rule \([\text{o}c\text{-Warp}])\) checks type of a channel vector \( c = h[x] \) and replaces \( x \) with the hole \([\_]\).
where the smallest to define evaluations of global combinators. The generated channels are interconnected to each other and the created channel.

Definition 3.8 (Operations). (1) The unfolding unfold\(^n\)(c) of a recursive value is defined by the smallest \(n\) such that unfold\(^n\)(c) = unfold\(^{n+1}\)(c), and unfold(c) is defined as:

\[
\text{unfold}(\mu x.c) = c\{\mu x.c/x\} \quad \text{unfold}(c) = c \quad \text{otherwise}
\]

where \(f^{n+1}(x) = f(f^n(x))\) for \(n \geq 2\) and \(f^1(x) = f(x)\). (2) \(c\#1\) denotes the record projection, which projects on field 1 of record value \(c\), defined as: \((1_i=c_i)_{i \in I} \#\#_k =\) unfold\(^*\)(ck), where \# is left-associative, i.e. \(c\#1\#\#\#_1 = (\cdots(\#1\#\#\#)\#\#\#\#\#)\). (3) The \(i\)-th projection on a tuple, \(a(i)\) is defined as \((c_1, \ldots, c_n)(i) = c_i\) for \(1 \leq i \leq n\). (4) fix\((x, x') = 0\); otherwise fix\((x, c) = \mu x.c\).

Definition 3.9 (Evaluation of a global combinator). Given \(R\) and fresh \(s\), the evaluation \(\llbracket g \rrbracket^s_R\) of global combinator \(g\) is defined in Fig. 9. We write \(\llbracket g \rrbracket^s\) if \(R = \text{roles}(g)\).

The evaluation for communication \((p_j \rightarrow p_k) \equiv S\) \(g\) connects between \(p_j\) and \(p_k\) by the name \(s(p_j, p_k, m, i)\) by wrapping \(j\)-th and \(k\)-th channel vector with an output and an input structure, respectively. The name \(s(p_j, p_k, m, i)\) is indexed by two role names \(p_j, p_k\), label \(m\) and an index \(i\) so that (1) it is only shared between two roles \(p_j\) and \(p_k\), (2) communication only occurs when it tries to communicate a specific label \(m\), and (3) both the sender and the receiver agree on the payload type. Here, the index \(i\) is used to distinguish between
Multiparty Session Programming with Global Protocol Combinators

names generated from the same label \( a' \) but different payload type \( m:T \) and \( m:T' \), ensuring consistent typing of generated channel vectors. The choice combinator \( \text{choice}_{p_a}(g_x)_{x \in I} \) extracts the output channel vector (i.e. the nested records of the form \( q=(q_i=c_k)_{k \in K_i} \)) at \( p_a \) from each branch \( g_x \), and merges them into a single output. Channel vectors for the other roles are merged by \( c_1 \sqcup c_2 \) where merging for the outputs is an intersection of branchings from \( c_1 \) and \( c_2 \), while merging of the inputs is their union. We explain merging by example (Example 3.10) and leave the full definition in Fig. 18 in § A.1.

For the recursion combinator, function \( \text{fix}(x_i \cdot c_i) \) forms a recursive value for repetitive session, or voids it as \( \varnothing \) if it does not contain any names.

\[ [\mathcal{g}]_s^{\text{Auth}} = [(c \to s) \mathcal{a}(\text{choice}s \{ (s \to c) \mathcal{a}\text{ok finish}, (s \to c) \mathcal{a}\text{cancel finish} \})]^s \]

Here, we have

\[ \begin{align*}
\mathcal{g}_L &= \langle (s = \mathcal{a}\text{ok}(s_1,0)), (c = \mathcal{a}\text{ok}(s_1,0)), \rangle, \\
\mathcal{g}_R &= \langle (s = \mathcal{a}\text{cancel}(s_2,0)), (c = \mathcal{a}\text{cancel}(s_2,0)), \rangle,
\end{align*} \]

concatenating

\[ \begin{align*}
\text{unfold}^s(\mathcal{g}_L)^s(2) &= \langle s = \mathcal{a}\text{ok}(c_{L2}), c_{L2} = (s_1,0), \rangle, \\
\text{unfold}^s(\mathcal{g}_R)^s(2) &= \langle s = \mathcal{a}\text{cancel}(c_{R2}), c_{R2} = (s_2,0), \rangle
\end{align*} \]

\( = \langle c = \mathcal{a}\text{cancel}(s_3, (c = \mathcal{a}\text{ok}(c_{R2}, c_{L2})) \rangle \}

The following main theorem states that if a global combinator is typable, the generated channel vectors are well-typed under the corresponding local types.

\[ \text{Theorem 3.11 (Realisability of global combinators). If } \vdash \mathcal{g} : T, \text{ then } [\mathcal{g}]^s_R = c \text{ is defined and } \{ s_i : S_i \}_{s_i \in \mathcal{I}(\mathcal{O})} \vdash c : T \text{ for some } \{ \mathcal{S}_i \}. \]

This property offers the type soundness and communication safety for \texttt{ocaml-mpst} endpoint programs: a statically well-typed \texttt{ocaml-mpst} program will satisfy subject reduction theorem and never performs a non-compliant I/O action w.r.t. the underlying binary channels. We leave the formal definition of \texttt{ocaml-mpst} endpoint programs, operational semantics, typing system, and the subject reduction theorem in § C.

4 Implementing Global Combinators

We give a brief overview on the type manipulation techniques that enable type checking of global combinators in native OCaml. § 4.1 gives a high-level intuition of our approach, § 4.2 illustrates evaluation of global combinators to channel vectors in pseudo OCaml code, and § 4.3 presents the typing of global combinators in OCaml. Furthermore, in Appendix § E, we develop variable-length tuples using state-of-art functional programming techniques, e.g., GADT and polymorphic variants, to improve usability of \texttt{ocaml-mpst}.

4.1 Typing Global Combinators in OCaml: A Summary

In Fig. 10 we illustrate the type signature of each global combinator, which is a transliteration of the typing rules (Fig. 7) into OCaml. In the figure, OCaml type \( (t_1 \cdot \ldots \cdot t_n) \) corresponds to a \( n \)-tuple of channel vector types \( t_1 \times \cdots \times t_n \). The implementation makes use of variable-length tuples to represent tuples of channel vectors, and therefore the developer
Global Combinator , Type

\begin{align*}
\text{finish} & \quad (\text{close} \cdots \text{close}) \\
(t_i \rightarrow v_j) \circ g & \quad \text{Given } g : (t_i \cdots \text{close}) , \\
& \quad \text{Return } (t_i \cdots \text{close}) , \text{for each } i.
\end{align*}

\begin{align*}
\text{choice_at } r & \quad (r \text{, } g) \\
(r_i , g_1) & \quad \text{Given } 1 \leq a \leq n , \\
& \quad \text{Return } (t_i \cdots \text{close} \cdots t_{i+a}) \text{, and } \text{for each } i.
\end{align*}

\begin{align*}
\text{fix } (\text{fun } x \rightarrow g) & \quad \text{Given } g : (t_i \cdots \text{close}) \text{ under assumption that } x : (t_i \cdots \text{close}) , \\
& \quad \text{Return } (t_i \cdots \text{close}) , \text{for each } i.
\end{align*}

\begin{align*}
\text{closed_at } r & \quad g \\
(r_i , g) & \quad \text{Given } g : (t_i \cdots \text{close} \cdots t_{i+a}) \text{ and } 1 \leq a \leq n , \\
& \quad \text{Return } (t_i \cdots \text{close} \cdots t_{i+a}) \text{, for each } i.
\end{align*}

Figure 10 Type of Global Combinators in OCaml

Channel vector types in OCaml.

The OCaml syntax of channel vector types is given on the right. The difference with its formal counterparts is minimal. In particular, records are implemented using OCaml object types, and record fields correspond to object methods, i.e. role q is a method. In type \([\langle \text{close} \rangle = \text{unit}]_\text{close} \rightarrow g\), the symbol \(\rightarrow\) marks an open polymorphic variant type which can have more tags. The types \(\text{inp}\) and \(\text{out}\) stand for an input and output types with a payload type \(v_i\) and a continuation \(t_i\). Recursive channel vector types are implemented using OCaml equi-recursive types.

On branching and compatibility checking. As we explained in § 3.2, branching is the key to ensure the protocol is realisable, and free of communication errors. To ensure that the choice is deterministic, it must be verified that the set of labels in each branch are disjoint. Since OCaml objects do not support concatenation (combining of multiple methods e.g., [64, 26]), and cannot automatically verify that the set of labels (encoded as object methods) are disjoint, the user has to manually write a disjoint merge function \(\text{mrg}\) that concatenates two objects with different methods into one (see § E.5 for examples). This part can be completely automated by PPX syntactic extension in OCaml. On compatibility checking of non-choosing roles, external choice \(\langle \text{close} \rangle = \text{unit}\) and \(\langle \text{close} \rangle = \text{unit}\), the types can be recursively merged by OCaml type inference to \(\langle \text{close} \rangle = \text{unit}\) thanks to the row polymorphism on polymorphic variant types \((\rightarrow)\), while non-directed external choices and other incompatible combination of types (e.g., input and output, input and closing, and output and closing) are statically excluded.

On unguarded recursion. The encoding of recursion \(\text{fix } (\text{fun } x \rightarrow g)\) has two caveats w.r.t the typing system: (1) OCaml does not check if a recursion is guarded, thus for example \(\text{fix } (\text{fun } x \rightarrow x)\) is allowed. We cannot use OCaml value recursion, because global
combinators generate channels at run-time. (2) Even if a loop is guarded, Hindley-Milner
type inference may introduce arbitrary local type at some roles. For example, consider the
global protocol \( \text{fix } \text{fun } x \rightarrow (r_a \rightarrow r_b) \text{mag } x \) which specifies an infinite loop for roles
\( \notin \{r_a, r_b\} \), and does not specify any behaviour for any other roles. To prevent undefined
behaviour, the typing rule marks the types of the roles that are not used as closed \( \text{tfix(t, T)} \).
Unfortunately, in type inference, we do not have such control, and the above protocol will
introduce a polymorphic type \( \forall r_i \) for role \( r_i \notin \{r_a, r_b\} \), which can be instantiated by any
local type.

**Fail-fast policy.** We regard the above intricacies on recursion as a *fact of life* in any
programming language, and provide a few workarounds. For (1), we adopt a “fail-fast” policy:
Our library throws an exception if there is an unguarded occurrence of a recursion variable.
This check is performed when evaluating a global combinator before any communication
is started. As for (2), we require the programmer to adhere to a coding convention when
specifying an infinite protocol. They have to insert additional combinator \( \text{closed_at } r_a g \),
which consistently instantiates type variable \( \forall r_i \) with \( \text{close} \), leaving other roles intact. If the
programmer forgets this insertion, fail-fast approach applies, and our library throws a runtime
exception before the protocol has started. In addition, self-sent messages \( (r \rightarrow r) \text{mag} \) for
any \( r \) are reported as an error at runtime.

### 4.2 Implementing Global Combinator Evaluation

Following § 3.3, in Fig. 11, we illustrate the implementation of the global combinators,
by assuming that method names and variant tags are *first class* in this pseudo-OCaml.
Communication combinator \( \rightarrow \) is presented in Fig. 11 (a) where the communication
combinator \( (r_i \rightarrow r_j) \text{mg } g \) yields two reciprocal channel vectors of type \( \forall r_i \forall r_j \forall t \forall v :<m:\text{out}>r_j:<m:\text{out}>r_i:<m:\text{out}>v\forall t_j\forall v\forall t_j\forall v\forall t_j\forall v\forall t_j\). Line 6–9 specifies that the channel
is a polymorphic type \( \forall t \) for role \( r_i \notin \{r_a, r_b\} \), which can be instantiated by any
local type.

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which consistently instantiates type variable \( \forall r_i \) with \( \text{close} \), leaving other roles intact. If the
programmer forgets this insertion, fail-fast approach applies, and our library throws a runtime
exception before the protocol has started. In addition, self-sent messages \( (r \rightarrow r) \text{mag} \) for
any \( r \) are reported as an error at runtime.

The implementation starts by extracting the continuations (the channel vectors) at each
role (Line 3). Line 4 creates a fresh new channel \( s \) of a polymorphic type \( \forall v \text{ channel} \) shared
among two roles, which is a source of type safety regarding *payload* types. Line 6 creates
an output channel vector. We use a shorthand \( <m = e \text{ end} > \) to represent an OCaml object
*object method* \( m = e \) *end*. Thus, it is bound to \( c_r \), by nesting the pair \( (s, c_r) \) inside two
objects, one with a method role, and another with a method label, forming type \( <r_j :<m : (v, t_r) \text{ out}> > \) and \( <r_i :<m : (\forall v \forall t_j) \text{ inp} > > \).

Similarly, Line 8 creates an input channel vector \( c_r \), by wrapping channel \( s \) in a polymorphic variant using \( \text{Event.wrap} \) from Concurrent ML and nesting it in an object type,
forming type \( <r_i :<m : (\forall v \forall t_r) \text{ inp} > > \). This wrapping relates tag \( m \) and continuation \( t_j \)
to the input side, enabling external choice when merged. Finally, the newly updated tuple of
channel vectors is returned (Line 10).

Fig. 11 (b) illustrates the choice combinator \( \text{choice_at} \). Line 6–9 specifies that the channel
vectors at non-choosing roles are *merged*, using a *merge* function. Intuitively, *merge* does
a type-case analysis on the type of channel vectors, as follows: (1) for an input channel vector,
it makes an *external choice* among (wrapped) input channels, using the \( \text{Event.choose} \)
function from Concurrent ML; (2) for an output channel vector, the bare channel is *unified*
label-wise, in the sense that an output on the unified channel can be observed on both input
sides, which is achieved by having channel type around a reference cell; and (3) handling of
channel vector of type \( \text{close} \) is trivial.

**First-class methods.** Method names \( r_i, r_j \) and \( m \) and the variant tag \( m \) occurring in
\( (r_i \rightarrow r_j) \text{mg } g \) are assumed in § 4.1 to be first-class values. Since such behaviour is not
readily available in vanilla OCaml, we simulate it by introducing the type \( \text{method_} \) (Line 2 in
Implementation of first-class methods and labels

Fig. 12, which creates values that behave like method objects. The type is a record with a constructor function `make_obj` and a destructor function `call_obj` (see example in Lines 3–6).

We use that idea to implement labels and roles as object methods. The encoding of local types stipulates that labels are object methods (in case of internal choice) and as variant tags (in case of external choice). Hence, the `label` type (Line 9 in Fig. 12), is defined as a pair of a first-class method, i.e using `method_`, and a variant constructor function. While object and variant constructor functions are needed to compose a channel vector in `-->`, object destructor functions are used in `merge` in `choice_at`, to extract bare channels inside an object. Variant destructors are not needed, as they are destructed via pattern-matching and merging is done by `Event.choose` of Concurrent ML. Roles are defined similarly to labels. See example in Line 15 (the full definition of role type is available in § E.2).

### 4.3 Typing Global Combinators via Polymorphic Lenses

This section shows one of our main implementation techniques – the use of polymorphic lenses [19, 48] for index-based updates on tuple types. This is essential to the implementation of the typing of Fig. 10 in OCaml. To demonstrate our technique, we sketch the type of the branching combinator, in a simplified form. The types of all combinators, incorporating first-class methods and variable-length tuples, can be found in § E.4. The branching combinator demonstrates our key observation that merging of local types can be implemented using row polymorphism in OCaml, which simulates the least upper bound on channel vector types.

```
(* the definition of the type method_*)
type ('obj, 'mt) method_ = {make_obj: 'mt -> 'obj; call_obj: 'obj -> 'mt}
(* example usage of method_ *)
val login_method : (Login : 'mt>, 'mt) method_ (* the type of login_method *)
let login_method =
  {make_obj=(fun v -> object method login = v end); call_obj=(fun obj -> obj#login)}

(* the definition of the type label*)
type ('obj, 'ot, 'var, 'vt) label = {obj: ('obj, 'ot) method_; var: 'vt -> 'var}
(* example usage of label *)
val login : ((Login : 'mt>, 'mt, [> 'login of 'vt], 'vt) label
let login = {obj=login_method; var=(fun v -> 'login(v))}

(* example usage of role: *)
let s = {index=Zero; label=(make_obj=(fun v -> object method role_S=v end); call_obj=(fun o -> c#role_S))}
```

---

**Figure 11** Implementation of communication combinator and (a) branching combinator (b)

**Figure 12** Implementation of first-class methods and labels
We have implemented the type-case analysis for merge mentioned in § 4.2 via a wrapper called mergeable around each channel vector, which bundles a channel vector and its merging strategy.
these mechanisms, by comparing their API usages in Fig. 14 and types in Fig. 13, where the dynamic version stays on the left while the static one is on the right.

**Dynamic Linearity Checking.** Dynamic checking, where linearity violations are detected at runtime, is proposed by [62] and [29], and later adopted by [47, 54]. In ocaml-mpst, dynamic linearity checking is implemented by wrapping the input and output channels, with a boolean flag that is set to true once the channel has been used. If linearity is violated, i.e., a channel is accessed after the linearity flag has been set to true, then an exception `InvalidEndpoint` will be raised. Note that our library correctly handles output channels between several alternatives being used only once; for example, from a channel vector `c` of type `<r: <ok: (string,close) out; cancel: (string,close) out>>`, the user can extract two channels `c#r#ok` and `c#r#cancel` where an output must take place on either of the two bare channels, but not both. In addition, our library wraps each bare channel with a fresh linearity flag on each method invocation, since in recursive protocols, a bare channel is often reused, as the formalism (§ 3) implies.

**Static Linearity Checking with Monads and Lenses.** The static checking is built on top of linocaml [31]; a library implementation of linear types in OCaml which combines the usage of parameterised monads [2] and polymorphic lenses (see § 4.3), to enable static type-checking on the linear usage of channels. In particular, we reuse several techniques from [31, 33]. A parameterised monad, which we model by the type `((pre, post, v) monad)`, denotes a computation of type `v` with a pre- and a post-condition, and they are utilised to track the creation and consumption of resources at the type level. A well-known restriction of parameterised monads in the context of session types, is that they support communication on a single channel only, and hence are incapable of expressing session delegation and/or interleaving of multiple session channels. To overcome this limitation, the slot monad proposed in [31, 33] extends the parameterised monad to denote multiple linear resources in the pre- and post-conditions. The resources are represented as a sequence, and each element is modified using polymorphic lenses [48].

We incorporate the above-mentioned techniques of linocaml so that, instead of having a single channel vector in the pre and post conditions, we can have a sequence of channel vectors, and we use lenses to focus on a channel vector at a particular slot. If we do not require delegation or interleaving, then the length of the sequence is one and the monadic operations always update the first element of the sequence. In particular, as in [33], if a channel is delegated i.e., sent through another channel, that slot (index) of the sequence is updated to `unit`, marking it as consumed.

The ocaml-mpst API, for static linearity checking, is given in Fig. 14(b), where `s_i` and `s_j` in delegation, denote lenses pointing at `i`-th and `j`-th slot in the monad. The binary channels in the channel vector, used within the monadic primitives `send` and `receive`, are of the types given in Fig. 13(b). Functions `send` and `receive` both take (1) a lens `s_i` pointing to a channel vector; and (2) a selector function which extracts, from the channel vector at index `s_i`, a channel `((v data, t_1) out` for output and `a inp` for input. Type `data` denotes unrestricted (non-linear) payload types, whose values are matched against ordinary variables. The result of the monadic primitives is returned as a value of either type `'t lin` for output or `'a lin` for input, which is matched by `match%lin` or `let%lin`, ensuring the channels (and payloads, in case of delegation) are used linearly. A `lin` type must be matched against `lens-pattern` prefixed by `#`. Note that, linocaml overrides the `let` syntax and `#` pattern, in the way that `let%lin #s_i=exp` updates the index `s_i`, in the sequence of channel vectors, with the value returned from `exp`.

To realise session delegation, we have implemented a separate monadic primitive, `deleg_send`. 

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Dynamic | Static (monadic)
---|---
let s = send s #role_q m v in e | let %lin #s_i = send s_i (fun x -> x #role_q_m) v in e
let s = send s #role_q m s' in e | let %lin #s_i = deleg_send s_i (fun x -> x #role_q_m) s_j in e
match receive s #role_p with | match %lin receive s_i (fun x -> #role_p) with
| m_1 (x, s) -> e_1 | | m_1 (x, s_i) -> e_1
| m_2 (s', s) -> e_2 | | m_2 (s_j, s_i) -> e_2 (delegation)
close s | close s

Figure 14 OCaml API for MPST with Dynamic (a) and Static (b) linearity checks

\[ s_i \ (fun \ x \to \ x #p_1) \ s_j , \text{ presented in Fig. 14(b).} \] The primitive extracts the channel vector at position \( s_i \) and then updates the channel vector at position \( s_j \). As a result, the slot for \( s_j \) is returned and used in further communication, the slot \( s_i \) is updated to \texttt{unit}. An example program that uses \texttt{ocaml-mpst} static API is given in Fig. 4(b).

6 Evaluation

We evaluate our framework in terms of run-time performance (§ 6.1) and applications (§ 6.2, § 6.3). We compare the performance of \texttt{ocaml-mpst} with programs written in a continuation-passing-style (following the encoding presented in [60]) and untyped implementations (Bare-OCaml) that utilise popular communication libraries. In summary, \texttt{ocaml-mpst} has negligible overhead in comparison with \texttt{unsafe} implementations (Bare-OCaml), and CPS-style implementations. We demonstrate the applicability of \texttt{ocaml-mpst} by implementing a lot of use cases. In § 6.3, we show the implementation of the OAuth protocol, which is the first application of session types over \texttt{http}.

6.1 Performance

The runtime overhead of \texttt{ocaml-mpst} stems from the implementation of channel vectors, more specifically: (1) extracting a channel from an OCaml object when performing a communication action, and (2) either (2.1) dynamic linearity checks or (2.2) more closures introduced by the usage of a slot monad for static checking.

Our library is parameterised on the underlying communication transport. We evaluate its performance in case of synchronous, asynchronous and distributed transports. Specifically, we use the following communication libraries:

(1) \texttt{ev}: OCaml’s standard \texttt{Event} channels which implements channels shared among POSIX-threads;

(2) \texttt{lwt}: Streams between lightweight-threads [63], which are more efficient for I/O-intensive application in general, and broadly-accepted by the OCaml communities, and

(3) \texttt{ipc}: UNIX pipes distributed over UNIX processes.

Note that \texttt{ev} is synchronous, while the other two are asynchronous. Also, due to current OCaml limitation, POSIX-threads in a process cannot run simultaneously in parallel, which particularly affects the overall performance of (1). As OCaml garbage collector is not a concurrent GC, only a single OCaml thread is allowed to manipulate the heap, which in general limits the overall performance of multi-threaded programs written in OCaml. For (3), we generate a single pipe for each pair of processes, and maintain a mapping between a local channel and its respective dedicated UNIX pipe. In addition, we also implement an optimised variant of \texttt{ocaml-mpst} in the case of \texttt{lwt}, denoted as \texttt{lwt-single} in Fig. 15; it reuses a single stream among different payload types, instead of using different channels for types. In particular, we cast a payload to its required payload type utilising \texttt{Obj.magic}, as proposed...
and examined by [46, 32]. Our benchmarks are generalisable because each microbenchmark exhibits the worst-case scenario for its potential source of overhead.

We compare implementations, written using (1) ocaml-mpst static API, (2) ocaml-mpst dynamic API, (3) a Bare-OCaml implementation using untyped channels as provided by the corresponding transport library, and (4) a CPS implementation, following the encoding in [54]. We have implemented the encoding manually such that a channel is created at each communication step, and passed as a continuation. Fig. 15 reports the results on three microbenchmarks.

**Setup.** We use the native ocamlopt compiler of OCaml 4.08.0 with Flambda optimiser\(^5\). Our machine configurations are Intel Core i7-7700K CPU (4.20GHz, 4 cores), Ubuntu 17.10, Linux 4.13.0-46-generic, 16GB. We use Core_bench\(^6\), a popular benchmark framework in OCaml, which uses its built-in linear regression for estimating the reported costs. We repeat each microbenchmark for 10 seconds of quota where Core_bench takes hundreds of samples, each consists of up to 246705 runs of the targeted OCaml function, we obtain the average of execution time with fairly narrow 95% confidence interval.

**Ping-pong** benchmark measures the execution time for completing a recursive protocol between two roles, which are repeatedly exchanging request-response messages of increasing size (measured in 16 bit integers). The example is communication intensive and exhibits no other cost apart from the (de)serialisation of values that happens in the ipc case, hence it demonstrates the pure overhead of channel extraction, dynamic checks and parameterised monads. In the case of a shared memory transports (ev and lwt), we report the results of a payload of one integer since the size of the message does not affect the running time.

The slowdown of ocaml-mpst is negligible (approx. 5% for Dynamic vs Bare-OCaml, and 13% for Static vs Bare-OCaml) when using either ev, Fig. 15 (a1), or ipc, Fig. 15(a2).

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\(^5\) [https://caml.inria.fr/pub/docs/manual-ocaml/flambda.html](https://caml.inria.fr/pub/docs/manual-ocaml/flambda.html)

\(^6\) [https://blog.janestreet.com/core_bench-micro-benchmarking-for-ocaml/](https://blog.janestreet.com/core_bench-micro-benchmarking-for-ocaml/)
Multiparty Session Programming with Global Protocol Combinators

as a transport, since the overhead cost is overshadowed by latency. The shared memory case using \texttt{lwt}, Fig. 15(a3), represents the worse case scenario for \texttt{ocaml-mpst} since it measures the pure overhead of the implementation of many interactions purely done on memory with minimal latency. The slowdown in the static version is expected [33] and reflects the cost of monadic closures, as the current implementation does not optimise them away. The linearity monad is implemented via a state monad [31], which incurs considerable overhead. The OCaml Flambda optimiser could remove more closures if we annotate the program with inlining specifications. The slowdown (although negligible) in comparison with CPS is surprising since we pre-generate all channels up-front, while the CPS-style implementation creates a channel at each interaction step. Our observation is that the compiler is optimised for handling large amounts of immutable values, while OCaml objects (utilised by the channel vector abstraction) are less efficient than normal records and variants.

Fig. 15 (c) reports on the memory consumption (in terms of words in the major and minor heap) for executing the protocol. Channel vectors with dynamic checking have approximately the same memory footprint as Bare-OCaml, and significantly less footprint when compared with a CPS implementation.

\textbf{n-Ping} is a protocol of increasing size, \texttt{nping} global combinator forming repeated composition of the communication combinators defined by $g_i = (a \rightarrow b) \text{ ping } g \odot (b \rightarrow a) \text{ pong } g \odot g_{i-1}, g_0 = t$ and \texttt{nping} = \textit{fix} (fun t \rightarrow g_n), where $n$ corresponds to the number of ping and pong states. In contrast to Ping-Pong, this example generates a large number of channels and large channel vector objects, evaluating how well \texttt{ocaml-mpst} scales w.r.t the size of the channel vector structure. We show the results for transports \texttt{lwt} and \texttt{lwt-single} in Fig. 15 (b). The static version of \texttt{lwt-single} has a constant overhead from Bare-OCaml. Although the static checking implementation is in general slower, the relative overhead, in comparison with dynamic checking, decreases as the protocol length increases.

\textbf{Chameleons} protocol specifies that $n$ roles ("chameleons") connect to a central broker, who picks pairs and sends them their respective reference, so they can interact peer-to-peer. The example tests delegation (central broker sends a reference) and creation of many concurrent sessions (peer-to-peer interaction of chameleons). The results reported in Fig. 15 (d) show that the implementation of delegation with static linearity checking scales as well as its dynamic counterpart. The cost of linearity (monadic closures) is less than the cost of dynamic checks for many concurrent sessions over \texttt{lwt} transport.

6.2 Use Cases

We demonstrate the expressiveness and applicability of \texttt{ocaml-mpst} by specifying and implementing protocols for a range of applications, listed in Fig. 16. We draw the examples from three categories of benchmarks: (1) \textbf{session benchmarks} (examples 1-9), which are gathered from the session types literature; (2) \textbf{concurrent algorithms} from the Savina benchmark suit [34] (examples 10-13); and (3) \textbf{application protocols} (examples 14-16), which focus on well-established protocols that demonstrate interoperability between \texttt{ocaml-mpst} implemented programs and existing client/servers. For each use case we report on Lines of Code (LoC) of global combinators and the compilation time (\textit{CT} reported in milliseconds). We also report if the example requires full-merge [15] (FM) – a well-formedness condition on global protocols that is not supported in [54], but supported in \texttt{ocaml-mpst}.

Examples 1-9 are gathered from the official Scribble test suite\textsuperscript{7} [59], and we have converted

\textsuperscript{7} https://github.com/scribble/scribble-java
Scribble protocols to global protocol combinators. Examples 10-13 are concurrent algorithms and are parametric on the number of roles (n). To realise the scatter-gather pattern required in the examples, we have added two new constructs, scatter and gather, which correspond to a subset of the parameterised role extension for MPST protocols [10].

To test the applicability of ocaml-mpst to real-world protocols we have specified, using global combinators, a core subset of three Internet protocols (examples 14-16), namely the Simple Mail Transfer Protocol (SMTP), the Domain Network System (DNS) protocol and the OAuth protocol. Using the ocaml-mpst APIs, it was straightforward to implement compliant clients in OCaml that interoperate with popular servers. In particular, we have implemented an SMTP client that inter-operates with the Microsoft exchange server and sends an e-mail, an OAuth authorisation service that connects to a Facebook server and authenticates a client, and a DNS client and a server, which are implemented on top of a popular DNS library in OCaml (ocaml-dns). Note that DNS has sessions, as the DNS protocol has an ID field to discriminate sessions; and a request forwarding in the DNS protocol involves more than two participants (i.e. servers).

### 6.3 Session Types over HTTP: Implementing OAuth

In this section, we discuss more details about ocaml-mpst implementation of OAuth\(^8\), which is an Internet standard for authentication. OAuth is commonly used as a way for Internet users to grant websites or applications access to their information on other websites but without giving them the passwords by providing a specific authorisation flow. Fig. 17 shows the specification of the global combinator, along with an implementation for the authorisation server. We have specified a subset of the protocol, which includes establishing a secure connection and conducting the main authentication transaction. Using OAuth as an example, we also discuss practically motivated extensions, explicit connection handling akin to the one in [30], to the core global combinators. We present that a common pattern when HTTP is used as an underlying transport.

**Extension for handling stateless protocols.** The protocol has a very similar structure to the oauth protocol, presented in § 2. However, the original OAuth protocol is realised over a RESTful API, which means that every session interaction is either an HTTP request or an HTTP response. To handle HTTP connections, we have implemented a thin wrapper around an HTTP library, Cohttp\(^9\), and we make HTTP actions explicit in the protocol by proposing two new global combinators, `connection establishing` combinator (`→`) and `disconnection` combinator (`→`)
The global combinator \( fb\_oauth \) is given in Fig. 17 (a). As before, the protocol consists of three parties, a service \( s \), a client \( c \), and an authorisation server \( a \). First, \( c \) connects to \( s \) via a relative path "/start_oauth" (Line 2). Then \( s \) redirects \( c \) to \( a \) using HTTP redirect code \_302 (Line 3). As a result the client sees a login form at "/login_form" (Lines 4-5), where they enter their credentials (Line 6). Based on the validity of the credentials received by \( c \), \( a \) sends \_200\_success (Line 8) or \_200\_fail. If the credentials are valid, \( c \) proceeds and connects to \( s \) on path "/callback" (Line 9), requesting to get access to a secure page. The service \( s \) then retrieves an access token from \( a \) on URL "/access_token" (Lines 10-11), and navigates the client to an authorised page, finishing the session (Lines 12-13). If the credentials are not valid, the client reports the failure to \( s \) (Lines 15-16), and the session ends (Line 17).

The server role of \( fb\_oauth \) is faithfully implemented in Lines 18-35 which provides an OAuth application utilising Facebook’s authentication service. Line 18 starts a thread which listens on a port 8080 for connections. Essentially it starts a web service at an absolute URL "mpst-oauth" (i.e. relative URLs like "/callback" are mapped to "https://.../mpst-oauth/callback"). The recursive function \( facebook\_oauth\_consumer \) starting from Line 19 is the main event loop for \( s \). Line 20 extracts a channel vector from the global combinator \( fb\_oauth \), of which type is propagated to the rest of the code. Then it generates a session id via a random number generator (Random.int () ) (Line 21), and waits for an HTTP request from a client on \( fb\_acceptor \) (Line 22). When a client connects, the connection is bound to the variable \( conn \) associated with the pre-generated session id. Note that the channel vector expects a connection since no connection has been set for the client yet. Here, the connection is supplied to the channel vector via function application (ch \( conn \)). On Line 24,
expression (fb_redirect_url sid "\callback") prepares a redirect URL to an authentication page of a Facebook Provider (https://www.facebook.com/dialog/oauth) After sending back (HTTP Response) the redirect url to the client with _302 label (Line 25), the connection is implicitly closed by the library. Note that we do not need to supply a connection to the channel vector on Line 25; because a connection already exists, we have already received an HTTP request from the user and Line 25 simply performs HTTP response. The next lines proceed as expected following the protocol, with the only subtlety that we thread the connection object in subsequent send/receive calls.

The full source code of the benchmark protocols and applications and the raw data are available from the project repository.

7 Related Work

We summarise the most closely related works on session-based languages or multiparty protocol implementations. See [59] for recent surveys on theory and implementations.

The work most closely related to ours is [54], which implements multiparty session interactions over binary channels in Scala built on an encoding of a multiparty session calculus to the π-calculus. The encoding relies on linear decomposition of channels, which is defined in terms of partial projection. Partial projection is restrictive, and rules out many protocols presented in this paper. For example, it gives an undefined behaviour for role c and s for protocols OAuth2 and OAuth3 in Fig. 3. Programs in [54] have to be written in a continuation passing style where a fresh channel is created at each communication step.

In addition, the ordering of communications across separate channels is not preserved in the implementation, e.g. sending a login and receiving a password in the protocol OAuth is decomposed to two separate elements which are not causally related. This problem is mitigated by providing an external protocol description language, Scribble [57], and its API generation tool, that links each protocol state using a call-chaining API [29]. The linear usage of channels is checked at runtime.

An alternative way to realise multiparty session communications over binary channels is using an orchestrator – an intermediary process that forwards the communication between interacting parties. The work [7] suggests addition of a medium process to relay the communication and recover the ordering of communication actions, while the work [8] adds annotations that permit processes to communicate directly without centralised control, resembling a proxy process on each side. Both of the above works are purely theoretical.

Among multiparty session types implementations, several works exploit the equivalence between local session types and communicating automata to generate session types APIs for mainstream programming languages (e.g., Java [29, 36], Go [10], F# [54]). Each state from state automata is implemented as a class, or in the case of [36], as a type state. To ensure safety, state automata have to be derived from the same global specification. All of the works in this category use the Scribble toolchain to generate the state classes from a global specification. Unlike our framework, a local type is not inferred automatically and the subtyping relation is limited since typing is nominal and is constrained by the fixed subclassing relation between the classes that represent the states. All of these implementations also detect linearity violations at runtime, and offer no static alternative.

In the setting of binary session types, [33] propose an OCaml library, which uses a slot monad to manipulate binary session channels. Our encoding of global combinators to simply-typed binary channels enable the reuse of the techniques presented in [33], e.g. for delegations and enforcement of linearity of channels.
FuSe [47] is another library for session programming in OCaml. It supports a runtime mechanism for linearity violations, as well as a monadic API for a single session without delegation. The implementation of FuSe is based on the encoding of binary session-typed process into the linear $\pi$-calculus, proposed by [13]. The work [55] also implements this encoding in Scala, and the work [54] extends the encoding and implementations to the multiparty session types (as discussed in the first paragraph).

Several Haskell-based works [50, 45, 37] exploit its richer typing system to statically enforce linearity with various expressiveness/usability trade-offs based on their session types embedding strategy. These works depend on type-level features in Haskell, and are not directly applicable to OCaml. A detailed overview of the different trade-off between these implementations in functional languages is given in Orchard and Yoshida’s chapter in [59]. Based on logically-inspired representation of session types, embedding higher-order binary session processes using contextual monads is studied in [61]. This work is purely theoretical.

Outside the area of session-based programming languages, various works study protocol-aware verification. Brady et al. [6] describe a discipline of protocol-aware programming in Idris, in which adherence of an implementation to a protocol is ensured by the host language dependent type system. Similarly, [58] proposes a programming logic, implemented in the theorem prover Coq, for reasoning on protocol states. A more lightweight verification approach is developed in [1] for a set of protocol combinators, capturing patterns for distributed communication. However, the verification is done only at runtime. The work [9] presents a global language for describing choreographies and a global execution model where the program is written in a global language, and then automatically projected using code generation to executable processes (in the style of BPMN). All of the above works either develop a new language or are built upon powerful dependently-typed host languages (Coq, Idris). Our aim is to utilise the MPST framework for specification and verification of distributed protocols, proposing a type-level treatment of protocols which relies solely on existing language features.

### 8 Conclusion and Future Work

In this work, we present a library for programming multiparty protocols in OCaml, which ensures safe multiparty communication over binary I/O channels. The key ingredient of our work is the notion of global combinators – a term-level representation of global types, that automatically derive channel vectors – a data structure of nested binary channels. We present two APIs for programming with channel vectors, a monadic API that enables static verification of linearity of channel usage, and one that checks channel usage at runtime. OCaml is intensively used for system programming among several groups and companies in both industry and academia [41, 3, 38, 39, 40, 18, 11, 51]. We plan to apply ocaml-mpst to such real-world applications.

We formalise a type-checking algorithm for global protocols, and a sound derivation of channel vectors, which, we believe, are applicable beyond OCaml. In particular, TypeScript is a promising candidate as it is equipped with a structural type system akin to the one presented in our paper.

To our best knowledge, this is the first work to enable MPST protocols to be written, verified, and implemented in a single (general-purpose) programming language and the first implementation framework of statically verified MPST programs. By combining protocol-based specifications, static linearity checks and structural typing, we allow one to implement communication programs that are extensible and type safe by design.
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Multiparty Session Programming with Global Protocol Combinators

Gabriel Radanne, Jérôme Vouillon, and Vincent Balat. Eliom: A core ML language for tierless web programming. In Programming Languages and Systems - 14th Asian Symposium, APLAS 2016, Hanoi, Vietnam, November 21-23, 2016, Proceedings, pages 377–397, 2016. URL: http://dx.doi.org/10.1007/978-3-319-47958-3_20.

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In this section, we review more examples of global combinators. Merging encounters the pair again, forming an appropriate loop and ensuring termination. For given channel vectors to each recursion binder on each side. It adds a mapping between the recursion variable and the pair it on top of the channel vector being generated, then it continues merging by expanding the continuations are merged. For recursions, it generates a fresh recursion variable and bind \( p \in \chi \) to \( x \). Otherwise, it generates the continuations that are the same set of output labels at any branches), and for each field it puts the name from left hand side (left/right does not matter since both names are identical if the global combinator is well-typed) and the continuation is obtained by merging the ones from both hand sides. Merging for input \( \mu x. c1 \perp x \) is more permissive, as it keeps variant tags which do not exist in the other hand side as-is. For the overlapping tags, names from the left hand side is taken as well and the continuations are merged. For recursions, it generates a fresh recursion variable and bind it on top of the channel vector being generated, then it continues merging by expanding the recursion binder on each side. It adds a mapping between the recursion variable and the pair of given channel vectors to \( \chi \), so that the corresponding recursive variable is returned if the merging encounters the pair again, forming an appropriate loop and ensuring termination.

**Appendices**

### A Auxiliary Definitions

#### A.1 Merging of Channel Vectors

On merging \( \perp \chi \), an extra bookkeeping \( \chi \) is introduced to ensure termination. Note that, according to our typing rule for \texttt{choice}, both hand sides must have the same type. Merging for output \( \langle p = \langle m_i = \langle s_i, c_i \rangle \rangle \rangle \) requires both branches to have an intersection. It generates a record only with the overlapping fields (which means we have the same set of output labels at any branches), and for each field it puts the name from left hand side (left/right does not matter since both names are identical if the global combinator is well-typed) and the continuation is obtained by merging the ones from both hand sides. Merging for input \( \langle p = \langle m_i = \langle s_i, c_i \rangle \rangle \rangle \) is more permissive, as it keeps variant tags which do not exist in the other hand side as-is. For the overlapping tags, names from the left hand side is taken as well and the continuations are merged. For recursions, it generates a fresh recursion variable and bind it on top of the channel vector being generated, then it continues merging by expanding the recursion binder on each side. It adds a mapping between the recursion variable and the pair of given channel vectors to \( \chi \), so that the corresponding recursive variable is returned if the merging encounters the pair again, forming an appropriate loop and ensuring termination.

### B More Examples on Global Combinators

In this section, we review more examples of global combinators.

**Example B.1** (Global combinator evaluation). Let \( s_1 = s\{c, s, ok, 0\} \), \( s_1 = s\{c, s, cancel, 0\} \), \( s_3 = s\{c, s, auth, 0\} \), \( s_4 = s\{s, a, ok, 1\} \), and \( s_5 = s\{s, a, cancel, 2\} \). Then:

```plaintext
\begin{align*}
\langle p = \langle m_i = \langle s_i, c_i \rangle \rangle \rangle \perp \chi \quad \langle p = \langle m_j = \langle s_j, c_j \rangle \rangle \rangle \perp \chi = \langle p = \langle m_k = \langle s_k, c_k \rangle \rangle \perp \chi \rangle \quad \text{where } s_k = s_{1k} = s_{2k} \text{ for all } k \in I \cap J
\end{align*}
```
\[ \text{choices}\{(s \to c)\text{ ok finish}, (s \to c)\text{ cancel finish}\} \]
\[ \{\langle s,\text{ok}\rangle = S_1(0),\text{cancel}\rangle = S_2(0)\} \]
\[ \{\langle c,\text{ok}\rangle = S_1(0),\text{cancel}\rangle = S_2(0)\} \]

\[ \text{choices}\{(s \to a)\text{ ok finish}, (s \to a)\text{ cancel finish}\} \]
\[ \{\langle s,\text{ok}\rangle = S_1(0),\text{cancel}\rangle = S_2(0)\} \]
\[ \{\langle c,\text{ok}\rangle = S_1(0),\text{cancel}\rangle = S_2(0)\} \]

This example shows how channel vectors for roles not participating in a choice (here \(a\)) are merged:
\[ \text{choices}\{(s \to a)\text{ ok finish}, (s \to a)\text{ cancel finish}\} \]
\[ \{\langle s,\text{ok}\rangle = S_1(0),\text{cancel}\rangle = S_2(0)\} \]
\[ \{\langle c,\text{ok}\rangle = S_1(0),\text{cancel}\rangle = S_2(0)\} \]

The following example illustrates channel vectors and the usage of unfold\(^*\)(). for the syntax of processes we use MIo, defined in § C

**Example B.2.** Let
\[ e_{\text{calc}} = \text{fix } x \to \text{choice} \{\langle c \to s\rangle\text{ loop } x, \langle c \to s\rangle\text{ stop } ((s \to c)\text{ stop finish})\} \]
\[ e_c = \text{let } x_1 = \text{send } x_0\#\#\text{ loop } () \text{ in let } x_2 = \text{send } x_2\#\#\text{ loop } () \text{ in let } x_3 = \text{send } x_2\#\#\text{ stop } () \text{ in let } x_4 = \text{send } x_2\#\#\text{ step } () \text{ in } \]
\[ e_a = \text{letrec } X(x) = x_0\text{ in } X(x') \]
\[ e_{a0} = \text{match } x\#\text{ with } \langle\text{loop}(\_\to x_1)\rangle \text{ stop}(\_\to x_2)\rangle \text{ let } x_3 = \text{send } x_2\#\#\text{ stop } () \text{ in } \]
\[ e_{a1} = \text{let } x_0, x_0' = e_{a0}\text{ in } (e_c\{c/x_0\}) \]

Then, \( e'_{\text{calc}} \mapsto e'_{\text{calc}} = (x_{s1}, x_{s2}, x_{s'}) \{e_c\{c/x_0\}\} \mid \text{letrec } X(x) = e_{a0}\text{ in } (e_c\{c/x_0'\}) \}

\[ e_c = \mu x_c.\langle s,\text{loop}(s_1.x_c),\text{stop}(s_2,\langle s,\text{stop}(s'_0,())\rangle)\rangle \]
\[ e_s = \mu x_s.\langle s,\text{loop}(s_1.x_s),\text{stop}(s_2,\langle s,\text{stop}(s'_0,())\rangle)\rangle \]
\[ \{\langle s = s_1,\text{loop}(\_\to x_s),\text{stop}(\_\to x_{s_2},\langle s,\text{stop}(\_\to x_{s_2'},())\rangle)\rangle \} \]

See that
\[ \text{unfold}^*(e_c\#\#\text{ loop}) = \langle x_{s1}, c_c \rangle \]
\[ \text{unfold}^*(e_c\#\#\text{ stop}) = \langle x_{s2}, s = \text{stop}(s'_0,()) \rangle \]
\[ \text{unfold}^*(e_c\#\text{ c}) = \text{loop}(x_{s1}, c_c), \text{stop}(x_{s2}, c = \text{stop}(s'_0,()) \rangle \]
\[ = \langle x_{s1}, \text{loop}(\_\to x_{c_c}), s_{s2} = \text{stop}(\_\to x_{s_2'},()) \rangle \]

and each time client sends a label, server makes an external choice between \(s_{1}\) and \(s_{2}\), and they reduce as follows: \( e'_{\text{calc}} \mapsto e'_{\text{calc}} = (x_{s1}, x_{s2}, x_{s'}) \mid \text{letrec } X(x) = e_{a0}\text{ in } e'_{\text{calc}} \}

The types are further elaborated by subtyping with I/O types [49] which is defined in Definition 3.3.

**Example B.3 (Merging via subtyping).** The following typing involves merging where the behaviour of two or more channel vector types in the branches are mixed into one, as

\[ \langle a, b, c \rangle\text{ choices}\{(s \to a)\text{ ok finish}, (s \to a)\text{ cancel finish}\};\]
\[ T_s \times T_c \times T_a \text{ where } T_s = \langle c;\text{ok}\rangle + (\text{a};\text{ok}\times\text{a};\text{ok}\times\text{a}),\text{cancel}\rangle + (\text{a};\text{cancel}\times\text{a}
\]
\[ T_c = \langle c;\text{ok}\rangle + (\text{a};\text{ok}\times\text{a}),\text{cancel}\rangle + (\text{a};\text{cancel}\times\text{a}) \text{ at role } a \text{ respectively, where each of them receives label ok and cancel from } s. \text{ By subtyping, they are amalgamated into a common super type } a;\text{[ok}\_\times\text{a},\text{cancel}\_\times\text{a}] \text{ which can now receive both labels. This is underpinned by the subtyping relation as } (a;\text{[ok}\_\times\text{a}) \subseteq (a;\text{[cancel}\_\times\text{a}) \text{ which is justified by [Orc-Sun].} \]
Example B.4 (Recursion). The following typing derivation is valid under $\mathbb{R} = s, c, a$:

\[
\frac{\vdash_{\mathbb{R}} x : t_4 \times t_4 \times t_4 \times t_4}{\vdash_{\mathbb{R}} x : t_4 \times t_4} \vdash_{\mathbb{R}} x : t_4 \times t_4 \times t_4 \times t_4
\]

\[
\vdash_{\mathbb{R}} x : t_4 \times t_4 \times t_4 \times t_4 \vdash_{\mathbb{R}} x : \langle s, \text{ok}, t_4 \rangle \times t_4 \times \langle s, \text{ok}, t_4 \rangle
\]

\[
\vdash_{\mathbb{R}} x : t_4 \times t_4 \times t_4 \times t_4 \vdash_{\mathbb{R}} (s \rightarrow c) \text{ok} (s \rightarrow a) \text{ok} x : \langle c, \text{ok}, \langle s, \text{ok}, t_4 \rangle \times t_4 \rangle \times \langle s, \text{ok}, t_4 \rangle
\]

\[
\vdash_{\mathbb{R}} x : t_4 \times t_4 \times t_4 \times t_4 \vdash_{\mathbb{R}} x : \langle s, \text{ok}, t_4 \rangle \times t_4 \times \langle s, \text{ok}, t_4 \rangle
\]

Example B.5 (Loops and the finished session). The following example shows the usage of the function $\text{tfix}(\cdot)$ in the rule [recursion] comes from the corresponding case in the End Point Projection in MPST [56]. It declares the termination of the session for a role in a loop in which the role in question never participate in.

\[
\vdash_{\text{p.q.r}} (\text{fix} x \rightarrow (\text{p} \rightarrow \text{r}) \text{ok} x) : \mu t \langle \text{p}; \text{r}; \text{ok}!; t_p \rangle \times \mu t x \times (\text{ok} \times t_q)
\]

where the channel vector type for $q$ is the finished session $\bullet$ because $\mu t (t_q, t_q) = \bullet$.

C MiO: A minimal ocaml-mpst calculus

We introduces a minimal functional calculus, MiO and its typing systems. The calculus distils the main features required for embedding session types in OCaml, notably equi-recursive types, record and variant types, structural subtyping, and simply-typed I/O channels. We prove the type soundness for MiO (Theorem C.8).

C.1 MiO: Syntax and Dynamic Semantics

This section introduces the syntax and operational semantics of MiO.

C.1.1 MiO Program

We introduce the syntax of MiO program, which is expression by the programmer.

Definition C.1 (MiO program). The program (or expression) of MiO is defined as:

\[
e ::=
\]

(let \(x_1, \ldots, x_n = g \text{ in } e\)) (initiation)

(let \(x = \text{send } y \# q \# m \text{ in } e\)) (send)

(let \(x = \text{recv } y \# q \text{ in } e\)) (receive)

(v ::= \(x, y, z, \ldots\)) (values)

match \(x \text{ with } \{m_i(x, y_i) \rightarrow e_i\}_{i \in I}\) (pattern match)

\(\bullet \mid e \mid e'\) (unit, par)

letrec \(D \text{ in } e\) (recursion)

\(D ::= X(\bar{x}) = e\) (declaration)

We assume mutually disjoint sets of variables \((x, y, \ldots)\), and function variables \((X, X', \ldots)\). In \(\text{let } x = \ldots \text{ in } e\), variable \(x\) in \(e\) is bound. Similarly, \(\text{match } x \text{ with } \{m_i(x, y_i) \rightarrow e_i\}_{i \in I}\) and \(\text{letrec } X(x_1, \ldots, x_n) = e \text{ in } \ldots\), variables \(x_i\) and \(y_i\) in \(e_i\) \((i \in I)\) and \(x_1, \ldots, x_n\) in \(e\) are bound, respectively. \(\text{letrec } X(\bar{x}) = e\) in \(e_1\) in \(e_2\) binds \(X\) in both \(e_1\) and \(e_2\). \(fv(e)\) / \(fn(e)\) denote the set of free variables/labels (introduced later) in \(e\). \(fv(e)\) is the set of free function variables in \(e\), and \(dfv(D)\) is the set of declared function variables in \(D\) (i.e. \(dfv(X(\bar{x}) = e) = \{X\}\)). (\(\bullet\) denotes unit value and _ stands for unused binding variables.

Program includes initiation which generates a series of interconnected channels from a global combinator \(g\), each of which corresponds to a role occurring in \(g\). This expression corresponds to Line 1 and Line 8 in Figure 2. Output expression \(\text{let } x = \text{send } y \# q \# m \text{ in } e\) sends label \(m\) with payload \(v\) via channel \(y\) to role \(q\), then binds the continuation to \(x\), and proceeds to \(e\). Input expression \(\text{let } x = \text{recv } y \# q \text{ in } e\) receives on \(y\) from \(q\) then binds the received value to \(x\), and proceeds to \(e\). The received value will have the form \([m = (v_1, v_2)]\) where
\(m\) and \(v_1\) are the label and payload sent from \(q\), and \(v_2\) is a continuation. The received value is decomposed by \textit{pattern matching} expression \textbf{match} \(x\) with \(\{m(x, y) \triangleright v_i\}_{i \in I}\) which matches against patterns \(\{m_i = (x_i, y_i)\}_{i \in I}\), and if \(m = m_k\), it continues to \(v_k\) after simultaneously substituting \(x_k\) and \(y_k\) with \(v_1\) and \(v_2\), respectively. \textbf{Recursive function definition} \textbf{letrec} \(X(x_1, ..., x_n) = e_1\) \textbf{in} \(e_2\) defines a recursive function \(X\) with parameters \(x_1, ..., x_n\) and body \(e_1\) which is local to \(e_2\). A unit value \(\ast\) represents an inactive thread. \textbf{Parallel} \(e_1 | e_2\) represents two threads running concurrently. \(X(\ast)\) is the \textit{function application}.

We use the following shorthand for expressions with \(z\) fresh:
\[
\textbf{match} \textbf{recv} x_0 \# q\textbf{ in} \{m(x, y) \triangleright v_i\}_{i \in I} \quad \textbf{let} \ z = \textbf{recv} x_0 \# q\textbf{ in} \{m(x, y) \triangleright v_i\}_{i \in I}
\]
\[
\textbf{let} \ m(x, y) = \textbf{recv} x_0 \# q \textbf{ in} e \quad \textbf{let} \ z = \textbf{recv} x_0 \# q \textbf{ in} e
\]

\textbf{Example C.2}. The following expressions implement the protocol in Example 3.2:
\[
e_{\text{Auth}} = \textbf{let} x, x' = \textbf{Auth} \textbf{ in} \{e_c | e_a\}
\]

where, with \(\text{fv}(e_c) = \{x\}\), \(\text{fv}(e_a) = \{x'\}\), and \(\text{fv}(e_{\text{Auth}}) = \{\}\), and \(e_c\) and \(e_a\) are following:
\[
e_c = \textbf{let} x_1 = \textbf{send} x \# s \# \text{auth} \# \text{"password"} \textbf{ in} \{\text{match} \text{recv} x_1 \# s \textbf{ with} \{\text{ok}(\_x, 2) \triangleright \ast; \text{cancel}(\_x, 3) \triangleright \ast\}\}
\]
\[
e_a = \textbf{let} \text{auth}(\_x) = \text{recv} x' \# c \textbf{ in let} x_2 = \textbf{send} x_1 \# c \# \text{"ok"} \# \text{in} \ast
\]

\textbf{Example C.3}. Let
\[
e'_{\text{Cal}} = \text{fix}\ x \rightarrow \text{choice}\ c\ \{\text{let} x_1 = \text{send} x_0 \# \text{# loop}\ (\text{let} x_2 = \text{send} x_1 \# \text{# loop}\ \text{in let} x_2 = \text{send} x_2 \# \text{# stop}\ \text{in}\ \text{let stop}(\_x) = \text{recv} x_2 \# s\textbf{ in} \ast)
\]
\[
e_c = \text{let} x_1 = \text{send} x_0 \# \text{# loop}\ (\text{let} x_2 = \text{send} x_1 \# \text{# loop}\ \text{in let} x_2 = \text{send} x_2 \# \text{# stop}\ \text{in}\ \text{let stop}(\_x) = \text{recv} x_2 \# s\textbf{ in} \ast
\]
\[
e_a = \text{let} \text{recv} x_0 \# c\textbf{ with} \{\text{loop}(\_x)\# c\# X(x_1); \text{stop}(\_x)\# x_2}\textbf{ let} x_3 = \text{send} x_2 \# c\# \text{# stop}\ \text{in}\ \ast
\]
\[
e_{\text{Cal}} = \text{let} x_0, x_0' = \text{fix}\ x_0\textbf{ in} (e_c, e_a)
\]

Then, \(e'_{\text{Cal}} \rightarrow e'_{\text{Cal}}' = (\{\mu s_1, s_2, s'\\}(e_c, x_0) \mid \text{letrec} X(x) = e_{s_0}\textbf{ in} (e_{s_0}, e_{s_0})\})\) where
\[
e_c = \mu x_c. \{s = \text{loop}(s_1, x_c); \text{stop}(s_2, s = \text{stop}(s', O))\}
\]
\[
e_a = \mu x_a. \{s = \text{loop}(s_1, x_a); \text{stop}(s_2, s = \text{stop}(s', O))\}
\]
\[
e_{\text{Cal}} = \mu x_{\text{Cal}}. \{s = \text{loop}(s_1, x_{\text{Cal}}); \text{stop}(s_2, s = \text{stop}(s', O))\}
\]

See that
\[
\text{unfold}^{*}(e_c \# \# \text{# loop}) = (s_1, x_c)
\]
\[
\text{unfold}^{*}(e_c \# \# \text{# stop}) = (s_2, s = \text{stop}(s', O))
\]
\[
\text{unfold}^{*}(e_a \# c) = \text{loop}(s_1, x_c); \text{stop}(s_2, c = \text{stop}(s', O))
\]

and each time client sends a label, server makes an external choice between \(s_1\) and \(s_2\), and they reduce as follows: \(e'_{\text{Cal}} \rightarrow (\mu s_1, s_2, s')\ast \mid \text{letrec} X(x) = e_{s_0}\textbf{ in} \ast \equiv \ast\).

\textbf{C.1.2 Dynamic Semantics of MiO}

We introduce a reduction semantics of expressions, which is a standard MPST \(\pi\)-calculus, with extra handling on channel vectors.

\textbf{Definition C.4}. The reduction relation \(\rightarrow\) of the expressions is defined by the rules in Fig. 20. The syntax of MiO in Definition C.1 is extended to the \textit{runtime syntax} as follows:
\[
e ::= \textbf{let} x = \textbf{send} c \# q \# m \ast e \mid \textbf{let} x = \textbf{recv} c \# q \ast e \mid \textbf{match} c \textbf{ with} \{m_i(x_i, y_i) \triangleright e_i\}_{i \in I} \mid X(\ast) \mid (\nu s) e
\]

A \textit{reduction context} \(E\) is defined by the following grammar:
\[
E ::= E \mid e \mid (\nu s) E \mid \textbf{letrec} X(\ast) = e \textbf{ in} E \mid [\ ]
\]
\[ e \mid e' \equiv e \mid e \mid e'' \equiv e \mid (e' \mid e'') e \mid \bullet \equiv e \quad (\nu s) \bullet \equiv \bullet \]

\[(\nu s)(\nu s')e \equiv (\nu s')(\nu s)e \quad (\nu s)(e \mid e') \equiv e \mid (\nu s)e' \quad \text{if } s \notin \text{fn}(e)\]

\[
\text{letrec } D \text{ in } \bullet \equiv \bullet \quad \text{letrec } D \text{ in } (\nu s)e \equiv (\nu s)(\text{letrec } D \text{ in } e) \quad \text{if } s \notin \text{fn}(D)
\]

\[
\text{letrec } D \text{ in } (e \mid e') \equiv (\text{letrec } D \text{ in } e) \mid e' \quad \text{if } \text{dfv}(D) \cap \text{ffv}(e') = \emptyset
\]

\[
\text{letrec } D \text{ in } (\text{letrec } D' \text{ in } e) \equiv \text{letrec } D' \text{ in } (\text{letrec } D \text{ in } e)
\]

\[
\text{if } (\text{dfv}(D) \cup \text{ffv}(D)) \cap \text{dfv}(D') = \emptyset \quad (\text{dfv}(D') \cup \text{ffv}(D')) \cap \text{dfv}(D) = \emptyset
\]

\[\text{let } x_1, \ldots, x_n = \text{in}(e_1 | \cdots | e_n) \rightarrow (\nu s)(e_1[c_1/x_1] | \cdots | e_n[c_n/x_n])\]

\[\text{let } x = \text{send } c_p \# q \# m \# c' \in e_1 \quad \text{let } y = \text{recv } c_q \# p \in e_2 \rightarrow e_1[c_1/x] | e_2[c_2/y]\]

\[
\text{match with } \{ m_k(x_1, y_1) \in I \} \rightarrow e_k[c_1/k] \{ c_2/y \} \quad \text{match with } \{ e_k[c_1/k] \{ c_2/y \} \}
\]

\[\text{letrec } X(\bar{x}) = e_1 \in (X(\bar{x}) | e_2) \rightarrow \text{letrec } X(\bar{x}) = e_1 \in (e_1[c_1/\bar{x}] | e_2) \quad \text{if } e \rightarrow e' \]

\[\text{letrec } X(\bar{x}) = e_1 \in (X(\bar{x}) | e_2) \rightarrow \text{letrec } X(\bar{x}) = e_1 \in (e_1[c_1/\bar{x}] | e_2) \quad \text{if } e \rightarrow e' \]

**Restriction** $(\nu s)e$ denotes session channel $s$ binding all free channels in the form of $s(p, p, m, i)$ which are generated by $[\llbracket g \rrbracket]$. The structural congruence $\equiv$ (adapted from [56]) is inductively defined by the rules in Figure 19.

The reduction rules of MiQO are defined in Figure 20. Rule **[Ored-Int]** generates a tuple of channel vectors $(c_1, \ldots, c_n)$ with fresh name $s$ from a global combinator $[\llbracket g \rrbracket]$ and then substitutes them to variables $x_i$ and continue to $e$. We assume that $x_i$ freely occurs in $e_j$ only, but not in $e_j$ where $i \neq j$. The names introduced by channel vectors are bound by restriction by $s$. In rule **[Ored-Comm]** the sender and receiver interact via two interconnected channel vectors $c_p$ and $c_q$ at role $p$ and $q$, respectively. They have the form $(\text{send } c_p \# q \# m \# c')$ and $(\text{recv } c_q \# p \in e_2)$ which communicates label $m_k$ and payload $c'$ from $p$ to $q$. On sender’s side, record projection $c_p \# q \# m \in s_k(c_1)$ where $s_k$ takes a form of $s_k[p, q, m, i]$. On the receiver’s side, evaluation of $c_q \# p \in e_2$ yields wrapped names $[m_k(\{c'_1/c_1\})]_{i \in I}$ where each $c'_k$ takes a form of $s'_k[p, q, m, j]$. The communication happens if they both are generated from the same global combinator and interconnected via the same name $s = s'$ and the same index $i' = j'$.

After communication, the sender binds $c_1$ to $x$ and continues to $e_1$. The receiver receives the variant value $c_2 = h_k[e'] = [m_k[\{x_i/c_i\}][c']] = [m_k[c', c'_k]]$ which contains both received payload $c'$ and continuation $c'_k$, and binds it to $y$ and continues to $e_2$, and the variant value is matched in the subsequent reductions.

Rule **[Ored-Match]** matches the variant values of the form $[l_k=(c_1, c_2)]$ yielded by recv against patterns $[m_k=(x_1, y_1)]_{i \in I}$, and if $k \in I$, it binds $c_1$ and $c_2$ to $x_k$ and $y_k$ respectively, and reduces to $e_k$.

The rest of the rules are standard from [56]. Rule **[Ored-Rec]** instantiates a recursive call to its body $e$; Rule **[Ored-E]** defines a reduction up to the structural congruence defined in Figure 19. Rule **[Ored-Ctx]** is a contextual rule.

**Example C.5** (Reduction). Recall Examples 3.2, C.2 and 3.10. We have:
\[
\begin{align*}
\text{[Orc-Init] } \text{rules}(g) &= \{ p_1, \ldots, p_n \} \quad \vdash p_1, \ldots, p_n \ g : T_1 \times \cdots \times T_n \quad \Theta \cdot \Gamma, x_i : T_i \vdash e_i \quad \forall i \in \{ 1, \ldots, n \} \\
\text{[Orc-\Gamma]} \quad \Gamma \vdash c : q : (m \mapsto T \times T') \quad \Gamma \vdash c' : T \quad \Theta \cdot \Gamma, x : T' \vdash e \\
\text{[Orc-\Theta]} \quad \Theta \cdot \Gamma \vdash e_1 \quad \Theta \cdot \Gamma \vdash e_2 \\
\Theta \cdot \Gamma \vdash \text{let } x_1, \ldots, x_n = g \text{ in } e_1 \quad \text{[Orc-rece]} \quad \Theta \cdot \Gamma \vdash e \quad \Theta \cdot \Gamma \vdash c \cdot \Theta \cdot \Gamma \vdash c' \cdot \Theta \cdot \Gamma \vdash e \\
\Theta \cdot \Gamma \vdash \text{let } x = \text{recv } e \# q \# m \text{ in } e \\
\Theta \cdot \Gamma \vdash \text{match } c \text{ with } \{ m(y_i, x_i) \mapsto e_i \} \in I \\
\Theta \cdot \Gamma \vdash \text{letrec } X(x_1 : T_1, \ldots, x_n : T_n) = e_1 \in e_2 \\
\Theta \cdot \Gamma \vdash X(x_1, \ldots, x_n) \quad \Theta \cdot \Gamma \vdash (\nu s)e \\
\text{\textbf{Figure 21} The Typing Rules for Expressions} \quad \Theta \cdot \Gamma \vdash e
\end{align*}
\]

let \( x, x' = g_{\text{Auth}} \text{ in } (e_c | e_a) \rightarrow (\nu s)(e_c(e_i, x_2) | e_a(x_3, x_4)) \\
= (\nu s)(\text{let } c_2 = \text{send } x \# s \# \text{auth in } \cdots | \text{let } e_c = \text{recv } x' \# c \text{ in } \cdots \\
\text{They interact on } s_3, \text{ since } c_3 = (s = \text{auth} = (s_3, c_\theta)) \text{ and } e_3 = (c = \text{auth} = (s_3, c_\theta)) \\
\text{ where } c_\theta = (s = \text{ok} = (s_1, s_2), \text{cancel} = (s_3, s_4)) \text{ and } e_\theta = (c = \text{ok} = (s_1, s_2), \text{cancel} = (s_3, s_4)) \\
\rightarrow (\nu s)\left(\text{match } e_c \# s \# \text{with } (\text{ok} \mapsto x_2) \mapsto \bullet \text{; cancel} \mapsto x_3 \mapsto \bullet \right) \text{ | let } x_2 = \text{send } c_\theta \# e_\theta \text{ in } \bullet \\
\text{(Here, the sender selects } \text{ok}, \text{ interacting on } s_1 \text{ and evolving to:)}
\rightarrow (\nu s)\left(\text{match } e_\theta \text{ with } (\text{ok} \mapsto x_2) \mapsto \bullet \text{; cancel} \mapsto x_3 \mapsto \bullet \right) \mapsto (\nu s)(\bullet \mapsto \bullet) \equiv \bullet
\]

\section*{C.2 Static Semantics and Properties of MiO}

This section summarises the typing systems of MiO; then proves type soundness of MiO. Typing MiO is divided into three judgements (channel vectors, wrappers and expressions).

\begin{definition}[Typing rules] Figure 8 and Figure 21 give the typing rules. We extend the syntax of typing contexts \( \Gamma \) from Definition 3.4 as \( \Gamma ::= \| \Gamma, s : T \) and introduce context for recursive functions \( \Theta \) as: \( \Theta ::= \emptyset | \Theta, X : T_1, \ldots, T_n \). Here, \( X : T_1, \ldots, T_n \) states that the parameter type of an \( n \)-ary function \( X \). The typing judgement for (1) channel vectors has the form \( \Gamma \vdash c : T \); (2) wrappers has the form \( \Gamma \vdash h : H \) where the type for wrappers is defined as \( H ::= T[S] \); and (3) expressions has a form \( \Theta \cdot \Gamma \vdash e \). We assume that all types in \( \Gamma \) and \( \Theta \) are closed.

The rules for channel vectors are standard where the subtyping relation in rule [Orc-Sub] is defined at Definition 3.3 in Section 3.2.

For wrappers, rule [Orc-WrapIns] types wrapped names where the payload type \( S' \) of input channel \( s \) is the same as the hole’s type, and all wrappers have the same result type \( T \). Rule [Orc-WrapApp] checks type of a channel vector \( c = b[x] \) and replaces \( x \) with the hole \([\cdot]\).

For expressions, rule [Orc-Init] types the initialisation with a typed global combinator. Rule [Orc-\Theta] types the output expression which sends a label \( m \) and a payload \( c' \) with as a nested record at \( c \). Rule [Orc-rece] is the dual rule for the input expression. Rule [Orc-\Theta] hides all indexed \( s \) by \( s \). Other rules are standard from [56].

\begin{example}[Typing expression] Recall that \( e_{\text{Auth}} = \text{let } x, x' = g_{\text{Auth}} \text{ in } (e_c | e_a) \) from Example C.2. Typing of \( e_c \) has the following derivation:
\end{example}


\[
\begin{align*}
\Gamma', \tau: \{ok_{\tau} \times \cdot, cancel_{\tau} \times \cdot\}, x_2: \_T \vdash \cdot \quad & \quad \Gamma', \tau: \{ok_{\tau} \times \cdot, cancel_{\tau} \times \cdot\}, x_3: \_T \vdash \cdot \\
\Gamma', \tau: \{ok_{\tau} \times \cdot, cancel_{\tau} \times \cdot\} \vdash \text{match : \{ok_{\tau} \times x_2\} }& \cdot \\
\Gamma', x_1: \{ok_{\tau} \times \cdot, cancel_{\tau} \times \cdot\} & \vdash \text{let : recv_{x_1 \#s} in match : \{ok_{\tau} \times x_2\} }& \cdot
\end{align*}
\]

where \( \Gamma = \Gamma' \times \{\text{auth}_{\tau} \times x\} \), \( \Gamma' = \Gamma, x_1: \{\text{ok}_{\tau} \times \cdot, cancel_{\tau} \times \cdot\} \), and \( e' = \text{match}_{\tau} \times x_{1 \#s} \) with \( \{\text{ok}_{\tau} \times x_2\} \cdot \text{cancel}_{\tau} \times x_3\) which is expanded to \( \text{let : recv}_{x_{1 \#s}} \) construct.

Similarly, \( e' \) can be typed as follows:

\[
\begin{align*}
(c \langle \text{ok}_{\tau} \times \cdot, cancel_{\tau} \times \cdot\rangle & \leq (c \langle \text{ok}_{\tau} \times \cdot, cancel_{\tau} \times \cdot\rangle) \\
\Gamma'_{x_1}, x_2: (c \langle \text{ok}_{\tau} \times \cdot, cancel_{\tau} \times \cdot\rangle), x_3: \_T \vdash \cdot \\
\Gamma', x_1: (c \langle \text{ok}_{\tau} \times \cdot, cancel_{\tau} \times \cdot\rangle) & \vdash \text{match : \{auth}_{\tau} \times \cdot} & \cdot \\
\Gamma, x_1: (c \langle \text{ok}_{\tau} \times \cdot, cancel_{\tau} \times \cdot\rangle) & \vdash \text{let : send}_{x_{1 \#s}} \in \cdot
\end{align*}
\]

where \( \Gamma = x: \{\text{auth}_{\tau} \times (c \langle \text{ok}_{\tau} \times \cdot, cancel_{\tau} \times \cdot\rangle)\}, \Gamma' = \Gamma, x_1: (c \langle \text{ok}_{\tau} \times \cdot, cancel_{\tau} \times \cdot\rangle)\) and

\( \Gamma'_{x_1} = \Gamma'_{x_1} : (c \langle \text{ok}_{\tau} \times \cdot, cancel_{\tau} \times \cdot\rangle) \).

See that the output is typed via subtyping. Then, we have:

\[
\begin{align*}
\vdash_{\text{Auth}} T_{x_{1 \#s}} & \vdash_{\text{Auth}} T_{x_{1 \#s}}, x_3: \_T \vdash \text{let : send}_{x_{1 \#s}} \in \cdot
\end{align*}
\]

\( \vdash_{\text{Auth}} T_{x_{1 \#s}} = \vdash_{\text{Auth}} T_{x_{1 \#s}}, x_3: \_T \vdash \text{let : send}_{x_{1 \#s}} \in \cdot \)

\[ \vdash_{\text{Auth}} T_{x_{1 \#s}} = \vdash_{\text{Auth}} T_{x_{1 \#s}}, x_3: \_T \vdash \text{let : send}_{x_{1 \#s}} \in \cdot \]

**Theorem C.8** (Subject reduction). If \( \Theta \cdot \Gamma \vdash e \) and \( e \rightarrow e' \), then \( \Theta \cdot \Gamma \vdash e' \).

### D Proofs for Basic Properties of MiO

#### D.1 Substitution Lemma and other lemmas

**Lemma D.1** (Substitution lemma). Followings hold:

1. (a) If \( \Gamma, \Gamma': c : T \) and \( \Gamma \vdash e\{c'/x\} : T' \), then \( \Gamma \vdash e : T'[T_0] \) and \( \Gamma \vdash e' : T' \), and \( \Gamma \vdash e : T'[T_0] \).
2. If \( \Gamma \vdash h : [T'] \) and \( \Gamma \vdash e \in c : T \), then \( \Gamma \vdash h[e] : T' \).
3. If \( \Theta \cdot \Gamma, \Gamma : c \in T \) and \( \Theta \vdash e \in c : T' \), then \( \Theta \vdash e : T'[T_0] \).

**Proof.** Follows.

1. We proceed by mutual induction on the derivation trees of \( \Gamma \vdash c : T \) and \( \Gamma \vdash h : T[T_0] \).

**Case** (Otc-O). \( e = (\cdot) \). Trivial.

**Case** (Otc-?). \( e = y \). If \( x = y \), we have \( y\{c'/x\} = c' \), and by rule (Otc-?), we have \( T = T' \).

By assumption, we get \( \Gamma \vdash e' : T' \). If \( x \neq y \), since \( y\{c'/x\} = y \), it trivially holds.

**Case** (Otc-s). \( e = s \). We have \( s\{c'/x\} = s \) and it trivially holds.

**Case** (Otc-Tup). \( e = (c_1, \ldots, c_n) \). For each \( i \in \{1, \ldots, n\} \), exists \( T_i \) such that \( T = T_i \times \ldots \times T_n \) and \( \Gamma, x : T' \vdash (c_1, \ldots, c_n) : T_1 \times \ldots \times T_n \), and we have \( \Gamma, x : T' \vdash (c_1, \ldots, c_n) : T_1 \times \ldots \times T_n \). By induction hypothesis, we have \( \Gamma \vdash c_i\{c'/x\} : T_i \) \( (i \in \{1, \ldots, n\}) \). Then, by applying (Otc-Tup), we get \( \Gamma \vdash (c_1, \ldots, c_n)\{c'/x\} : T_1 \times \ldots \times T_n \).

**Case** (Otc-Rec) and (Otc-Variant). \( e = (l_i\{c_i\})_{i \in I} \) and \( e' = (l\{c'\}) \). Similar.

**Case** (Otc-WrapSp). \( e = [s_i@h_i]_{i \in I} \). From rule (Otc-WrapSp), \( T = T'' \) for some \( T'' \), and for each \( i \in I \), there exists \( T_i \) such that \( \Gamma, x : T' \vdash s_i : T_i \) and \( \Gamma, x : T' \vdash h_i : T''[T_0] \).

By induction hypothesis, we have \( \Gamma \vdash s_i\{c'/x\} : T_i \) and \( \Gamma \vdash h_i\{c'/x\} : T''[T_i] \) for each \( i \in I \), and by applying (Otc-WrapSp), it follows \( \Gamma \vdash [s_i@h_i]_{i \in I}\{c'/x\} : T'' \).

**Case** (Otc-Suc). We have \( S \) such that \( S \subseteq T \) and \( \Gamma, x : T' \vdash c : S \). By induction hypothesis, \( \Gamma \vdash c\{c'/x\} : S \). Again, by applying (Otc-Suc), we get \( \Gamma \vdash c\{c'/x\} : T' \).

For (b), we have \( \Gamma \vdash h : T[T_0] \) and the only rule is (Otc-Wrapper). By the rule, we have c,
y such that $y \notin \text{fn}(h)$, $c = h[y]$ and $\Gamma, y : T_0 \vdash c : T$. Note that, by Barendregt convention, we can assume $x \neq y$ and $y \notin \text{fn}(c')$. By induction hypothesis, $\Gamma, y : T_0 \vdash c'[x/y] : T$, Furthermore, we see $c'[x/y] = h[c'[x/y]]$ and, $y \notin \text{fn}(h[c'[x/y]])$ (since $y \notin \text{fn}(h))$. By $\text{[Oto-Wrapper]}$, $\Gamma \vdash h[c'[x/y]] : T[T_0]$.

2. From the derivation of $\Gamma \vdash h : T[T']$, for some $x$ and $c'$ we have $c' = h[x]$ such that $\Gamma, x : T' \vdash c' : T$ By (1), we have $\Gamma \vdash c'[x/y] : T$ and since $h[c] = c'[x/y]$, we get $\Gamma \vdash h[c] : T$.

3. By induction on the derivation of $\Theta \cdot \Gamma \vdash e$.

\textbf{Case} $\text{[or-s] Trivial.}$

\textbf{Case} $\text{[or-letrec]}$ We have $e = \text{letrec } X(x_1:T_1, \ldots, x_n:T_n) = e_1 \text{ in } e_2$ and we assume $x \notin \{x_i\}_{i \in 1..n}$. By induction hypothesis, $\Theta, X,T_1, \ldots, T_n \cdot \Gamma, x_1 : T_1, \ldots, x_n : T_n \vdash e_1 [\text{c'/x}]$ and $\Theta, X,T_1, \ldots, T_n \cdot \Gamma \vdash e_2 [\text{c'/x}]$, and by applying $\text{[or-letrec]}$, we get $\Theta \cdot \Gamma \vdash (\text{letrec } X(x_1:T_1, \ldots, x_n:T_n) = e_1 \text{ in } e_2)[\text{c'/x}]$.

\textbf{Case} $\text{[or-x]}$ We have $e = X(e_1, \ldots, e_n)$ and $\Gamma \vdash e_i : T'$ for $i \in \{1, \ldots, n\}$. By (1), we have $\Gamma \vdash e_1 [\text{c'/x}] : T'$ and By $\text{[or-x]}$, it follows that $\Theta \cdot \Gamma \vdash (X(e_1, \ldots, e_n))[\text{c'/x}]$.

\textbf{Case} $\text{[or-recv]}$. We have $\Theta \cdot \Gamma \vdash y = \text{recv } c'[\text{#q in } e]$. By assumption and (1), we have $\Gamma \vdash c'[\text{c'/x}] : \langle q_{\mu \text{-T}}[\text{T}^\alpha_i], T_i \rangle_{i \in I}$ and by assumption and induction hypothesis, we have $\Theta \cdot \Gamma, \mu, y : \{m_\mu, T_i \times T_i^\alpha_i \}_{i \in I} \vdash c'[\text{c'/x}]$. By applying $\text{[or-recv]}$, we get $\Theta \cdot \Gamma \vdash \{\text{let } y = \text{recv } c'[\text{#q in } e] \} [\text{c'/x}]$.

\textbf{Case} $\text{[or-match]}$. We have $e = \text{match } c' \text{ with } \{m_\mu(x_i, y_i) \rightarrow e_i \}_{i \in I}$. By induction and (1), we have $\Gamma \vdash c'[\text{c'/x}] : \langle \mu \{m_\mu \times \text{T}^\alpha_i \}_{i \in I} \rangle$ and $\Gamma \vdash e_1 [\text{c'/x}] : T$. Furthermore, by assumption and induction hypothesis, for each $i \in I$, we have $\Theta \cdot \Gamma, y : T_i \vdash x_i : T_i^\alpha_i \vdash c'[\text{c'/x}]$. By applying $\text{[or-match]}$, we get $\Theta \cdot \Gamma \vdash \{\text{match } c' \text{ with } \{m(x_i, y_i) \rightarrow e_i \}_{i \in I} \} [\text{c'/x}]$.

\textbf{Case} $\text{[or-\#]}$. We have $e = \text{let } x = \text{send } c'[\text{#\#c in } e]$. By assumption and (1), $\Gamma \vdash c'[\text{c'/x}] : \langle \text{q}_{\mu \text{-T}}[\text{T}^\alpha j], T_j \rangle$ and $\Gamma \vdash e_1 [\text{c'/x}] : T$ hold. By assumption and induction hypothesis, $\Theta \cdot \Gamma, y : T' \vdash c'[\text{c'/x}]$. By applying $\text{[or-\#]}$, we get $\Theta \cdot \Gamma \vdash \{\text{let } y = \text{send } c'[\text{#\#c in } e] \} [\text{c'/x}]$.

\textbf{Case} $\text{[or-if]}$. We have $e = c_1 | e_2$. By induction hypothesis, we get $\Theta \cdot \Gamma \vdash c_1 [\text{c'/x}]$ for $i \in \{1, 2\}$. By applying $\text{[or-if]}$, we get $\Theta \cdot \Gamma \vdash (e_1 | e_2)[\text{c'/x}]$.

\textbf{Case} $\text{[or-\#]}$. We have $e = \text{let } x_1, \ldots, x_n = \text{g in } (e_1 | \cdots | e_n)$. By induction hypothesis, we get $\Theta \cdot \Gamma, x_i : T_i \vdash c_1 [\text{c'/x}]$ for each $i \in \{1, \ldots, n\}$ (note that $x \notin \{x_i\}_{i \in \{1, \ldots, n\}}$).

By applying $\text{[or-\#]}$, we get $\Theta \cdot \Gamma \vdash \{\text{let } x_1, \ldots, x_n = \text{g in } (e_1 | \cdots | e_n) \} [\text{c'/x}]$.

\textbf{Case} $\text{[or-\#]}$. We assume $s \notin \text{fn}(c)$. By induction hypothesis, $\Theta \cdot \Gamma, s : T \vdash e[\text{c'/x}]$. By applying $\text{[or-\#]}$, we get $\Theta \cdot \Gamma \vdash \{\text{\langle s \#T\rangle e} \} [\text{c'/x}]$.

\[ \text{Lemma D.2 (Inversion).} \]

\text{Followings hold:}

1. If $\Theta \cdot \Gamma \vdash \text{let } x = \text{send } c' \text{ in } e$ and $d = (s_j, c_j)$ then, $\Gamma \vdash c_j : T$ and $\Theta \cdot \Gamma, x : T \vdash e$, $\Gamma = \Gamma', s_j : T'^{}$, where $j \in I$, and $\Gamma' \vdash c' : T'^{}$ where $T'^{} \leq T'$.

2. If $\Theta \cdot \Gamma \vdash \text{let } x = \text{recv } c' \text{ in } e$ and $d = [s_i : h_i]_{i \in I}$ then, $\Gamma = \Gamma', [s_i : S_i]_{i \in I}$, $\Gamma \vdash h_i : T[T_i]$ and $S_i \subseteq T_i$ for all $i \in I$ and $\Theta \cdot \Gamma, x : T \vdash e$.

3. If $\Theta \cdot \Gamma \vdash \text{match } c \text{ with } \{m(x_i, y_i) \rightarrow e_i \}_{i \in I}$, $c = \langle m_j = (c_j, j) \rangle$ and $j \in I$, then for all $i \in I$, $\Theta \cdot \Gamma, x_i : T_i, y_i : T_i' \vdash e_i$, $\Gamma \vdash c_j : S_j$, $S_j \subseteq T_j$ and $S_j' \subseteq T_j'$. $\\}$

4. If $\Theta \cdot \Gamma \vdash \text{letrec } X(x) = e_1 \text{ in } e_2$, then $\Theta, X,T_1, \ldots, T_n \cdot \Gamma, x_1 : T_1, \ldots, x_n : T_n \vdash e_1$, and $\Theta, X,T_1, \ldots, T_n \cdot \Gamma \vdash e_2$.

5. If $\Theta \cdot \Gamma \vdash e_1 | e_2$, then $\Theta \cdot \Gamma \vdash e_1$ and $\Theta \cdot \Gamma \vdash e_2$.

6. If $\Theta \cdot \Gamma \vdash X(e_1, \ldots, e_n)$, then $\Theta = \Theta', X(T_1, \ldots, T_n, \forall i \in 1..n, \Gamma \vdash c_i : S_i$ and $S_i \subseteq T_i$.

7. If $\Theta \cdot \Gamma \vdash \text{let } x_1, \ldots, x_n = \text{g in } (e_1 | \cdots | e_n)$, then roles(g) = \{p_1, \ldots, p_n\}, $\forall \mu \text{ g: T}_1 \times \cdots \times T_n$ and $\Theta \cdot \Gamma, x_i : T_i \vdash e_i$.

8. If $\Theta \cdot \Gamma \vdash \{\text{\langle s \#T\rangle e} \}$ then $\Theta \cdot \Gamma, s : T \vdash e$.\]
Proof. Standard.

Lemma D.3 (Type preservation for \( \equiv \)). If \( \Theta \cdot \Gamma \vdash e \) and \( e \equiv e' \), then \( \Theta \cdot \Gamma \vdash e' \).

Proof. Standard.

The following lemma relates term-level and type-level projection.

D.2 Type safety for global combinators

Definition D.4. \( \Gamma \) is basic on \( c \), written Basic(\( \Gamma, c \)), if, for all \( x \in \text{fv}(c) \) there is some \( t_x \) such that \( \Gamma \vdash x : t_x \), and \( t_x \neq t_y \) for any \( x, y \in \text{fv}(c) \) s.t. \( x \neq y \).

Lemma D.5. If \( \Gamma \vdash c_i : T \) (\( i \in \{1, 2\} \)) and followings hold:
1. Basic(\( \Gamma, c_i \)) for \( i \in \{1, 2\} \).
2. If \( z \mapsto (c_1, c_2) \in \chi \), then \( \Gamma \vdash c_i : T \) (\( i \in \{1, 2\} \)) and \( \Gamma \vdash z : T \).
Then, \( c_1 \sqcup \chi \subseteq c_2 \) is defined and \( \Gamma \vdash c_1 \sqcup \chi \subseteq c_2 : T \) holds.

Proof. We proceed by the induction on the number of calls of \( c_1 \sqcup \chi \subseteq c_2 \). This induction terminates since the size of the set of pairs \((c_1, c_2)\) accumulated in \( \chi \) is bounded. The interesting cases are ones that involve recursion.

Case \( c_1 = \mu x . c'_1 \). (1) If \( z \mapsto (\mu x . c'_1, c_2) \in \chi \), by the definition, we have \( c_1 \sqcup \chi \subseteq c_2 \). Furthermore, by assumption, we have \( \Gamma \vdash z : T \). (2) If \( z \mapsto (\mu x . c'_1, c_2) \notin \chi \), by inversion lemma, we have \( \Gamma \vdash \mu x . c' : \mu t . T' \) for some \( t, T' \) where \( \mu t . T' \leq T \), and by substitution lemma, we have \( \Gamma \vdash c'_1 \{\mu x . c'/x\} : T' \{t' / t\} \). Furthermore, since \( T' \{t' / t\} \leq T \), we have \( \Gamma \vdash c'_1 \{\mu x . c'/x\} : T \). By induction hypothesis, we have some \( c' = c'_1 \{\mu x . c'/x\} \sqcup \chi \sqcup \cdot \sqcup (\mu x . c'_1, c_2) \) defined, and \( \Gamma, z : T \vdash c' : T \). (Here, beware that both Basic(\( \Gamma, z : T, c'_1 \{\mu x . c'/x\} \)) and Basic(\( \Gamma, z : T, c_2 \)) hold since \( z \) is fresh, i.e. \( z \notin (\text{fv}(c'_1) \cup \text{fv}(c_2)) \)). Then, by [Orc-m], we have \( \Gamma \vdash \mu x . c' : T \).

Case \( c_1 = x \). Since Basic(\( \Gamma, x \)), we have \( \Gamma \vdash x : t_x \), and by inversion lemma, \( T = t_x \). Furthermore, since only possible rule to derive \( \Gamma \vdash c_2 : t_x \) is [Orc-x], and from Basic(\( \Gamma, c_2 \)), we have \( c_2 = x \). Hence, by the definition of \( \sqcup \chi \), we have \( c_1 \sqcup \chi \subseteq c_2 \).

Lemma D.6. If \( \Gamma \vdash c_i : T \) and Basic(\( \Gamma, c_i \)) for all \( i \in I \), then \( \bigcup_{i \in I} c_i \) is defined and \( \Gamma \vdash \bigcup_{i \in I} c_i : T \).

Proof. Straightforward by induction.

Proposition D.7. If \( \vdash_R g : T \), then \( T \) is closed.

Proof. By induction on \( g \).

Lemma D.8. If \( \Gamma \vdash p_1, \ldots, p_n g : T_1 \times \cdots \times T_n \) then \([g]^\Gamma_R = c\) is defined and \( \Gamma' = c : T_1 \times \cdots \times T_n \) where \( \Gamma' = \Gamma, \{s_i : S_i\}_{s_i \in \text{fv}(c)} \) for some \( \{S_i\} \).

Proof. We proceed by induction on the structure of \( g \).

Case \( g = \langle p_j \mapsto p_k \rangle \). By inversion, \( \Gamma \vdash_R g : T_1 \times \cdots \times T_n \) holds. By induction hypothesis, we get \( \Gamma' = \Gamma, \{s_i : S_i\} \) for some \( \{S_i\} \). Let \( \Gamma'' = \Gamma', s : T \) where \( s = \langle p_j, p_k, \cdot \rangle \). For each \( p_i \in \{p_1, \ldots, p_n\} \), we have \( \Gamma' \vdash [g]^\Gamma_R(i) : T_i \) and see that by [Orc-s], \( \Gamma'' \vdash s : T \). Then, by applying typing rules repeatedly, we have: \( \Gamma'' \vdash \langle p_k = \langle m = \langle s, [g]^\Gamma_R(j) \rangle \rangle \rangle : T'_j \) and \( \Gamma'' \vdash \langle p_j = \langle m = \langle s, [g]^\Gamma_R(k) \rangle \rangle \rangle : T'_j \) where \( T'_j = \langle p_k : (m : T_j \times T_j) \rangle \).
and $T_k' = \langle p_j: \rho[m_T \times T_k] \rangle$. Then, by using $\text{[Orc-Top]}$, we have

$\Gamma^{\prime} \vdash g:: T_1 \times \ldots \times T_k' \times \ldots \times T_n$. 

Case $g = \text{choice}_{p_a} \{ g_i \}_{i \in I}$. By inversion, for all $i \in I$ we have $\Gamma \vdash g_i:: T_1 \times \ldots \times T_n$. Then, applying induction hypothesis, we have $\Gamma_i' \vdash [g_i]_R^\ast: T_1 \times \ldots \times \langle p_a: \rho[m_k \times T_k \times T_k' \times \ldots \times T_n] \rangle \ldots \times T_n$ where $\Gamma_i' = \Gamma, \{ s_{ij}: S_{ij} \}$ for some $\{ s_{ij}: S_{ij} \}$. By taking $\Gamma' = \Gamma \cup \bigcup_{i \in I} \{ s_{ij}: S_{ij} \}$ and $c_{ij} = [g_i]_R^\ast(j)$, we have $\Gamma' \vdash c_{ij}: T_j$ and $\Gamma' \vdash \bigcup_{i \in I} c_{ij}: T_j$ for each $j \in \{ 1, \ldots, n \} \setminus \{ a \}$, and $c_{ia} = \langle \rho\hat{a} = (s_{ik}, c_{ik}') \rangle_{k \in K_i}$ then, by applying $\text{[Orc-Record]}$ for $\langle \rho\hat{a} = (s_{ik}, c_{ik}') \rangle_{k \in K} (K = \bigcup_{i \in I} K_i)$ and by using $\text{[Orc-Top]}$, we have the desired typing. Other cases are trivial or similar.

$\blacktriangleleft$

**Theorem 3.11** (Realisability of global combinators). If $\vdash_R g:: T$, then $[g]_R^\ast \vdash c:: T$ for some $\{ \hat{s}_i \}$.

**Proof.** A special case of the above lemma. $\blacktriangleleft$

### D.3 Proof of Subject Reduction

**Theorem C.8** (Subject reduction). If $\Theta \cdot \Gamma \vdash e$ and $e \longrightarrow e'$, then $\Theta \cdot \Gamma \vdash e'$.

**Proof.** Induction on derivation of $e \longrightarrow e'$.

**Case** $\text{[Ored-Comd]}$. $e = \text{let } x = \text{send } c_p \# q \# m_k \ c' \text{ in } e_1 \mid \text{let } y = \text{recv } c_p \# p \text{ in } e_2$, $e' = e_1 \{ c/y \} \mid e_2 \{ h_{ij}/c/y \}$ where $j \in I$, $c_p \# q \# m_k = (s_{ij}, c)$ and $c_{p} \# p = [s_i @ h_{ij}]_{i \in I}$. By applying inversion lemma for $\mid$, $\text{send}$ and $\text{recv}$, we have

$\Theta \cdot \Gamma', s_j: T' \vdash \text{let } x = \text{send } d_1 c' \text{ in } e_1$,

$\Theta \cdot \Gamma', s_j: T' \vdash \text{let } y = \text{recv } d_2 c' \text{ in } e_2$, and $T_i' = T''$.

$\Gamma \vdash c:: T$ and $\Theta \cdot \Gamma, x: T \vdash e_1$,

$\Gamma \vdash c': T''$ and $T'' \leq T'$

For all $i \in I$, $\Gamma \vdash h_i: T''[T_i]$ and $\Theta, \Gamma, y: T'' \vdash e_2$.

By applying substitution lemma on $e_1$, we get $\Theta \cdot \Gamma \vdash e_1 \{ c/y \}$. Next, by applying $\text{[Orc-Sub]}$ to $\Gamma \vdash c': T''$, we have $\Gamma \vdash c': T'' = T'_i$ and by applying substitution lemma on $h_j$, we get $\Gamma \vdash h_j \{ c'/y \}$ and by substitution lemma on $e_2$, we get $\Theta \cdot \Gamma \vdash e_2 \{ h_{ij}/c'/y \}$. Then, from $\text{[ord]}$ we get $\Theta \cdot \Gamma \vdash e_1 \{ c/y \} \mid e_2 \{ h_{ij}/c'/y \}$.

**Case** $\text{[Ored-Match]}$. $e = \text{match } \text{with } [m_i(x_i, y_i) \rightarrow e_i]_{i \in I}$, $e' = e_j \{ c_1/x_j \} \{ c_2/y_j \}$, and $c = [n_i = (c_1, c_2)]$ where $j \in I$. By inversion lemma for $\text{match}$, we have

$\Gamma \vdash c_j: T_j$, $\Gamma \vdash c_j': T'_j$,

and for all $i \in I$, $\Theta \cdot \Gamma, x_i: T_i, y_i: T_i' \vdash e_i$.

By applying substitution lemma on $c_j$ twice, we get $\Theta \cdot \Gamma \vdash e_j \{ c_1/x_j \} \{ c_2/y_j \}$.

**Case** $\text{[Ored-Rec]}$. $e = \text{letrec } X(\bar{x}) = e_1 \text{ in } (X(\bar{c}) \mid e_2)$ and $e' = \text{letrec } X(\bar{x}) = e_1 \text{ in } (e_1 \{ \bar{c}/\bar{x} \} \mid e_2)$.

By inversion of $\text{letrec}$ and $\mid$, we have

$\Theta, X: T_1, \ldots, T_n \vdash x_1: T_1, \ldots, x_n: T_n \vdash e_1$,

$\forall i \in 1, n, \Gamma \vdash c_i: S_i$ and $S_i \leq T_i$, and

$\Theta, X: T_1, \ldots, T_n \vdash e_2$.

By rule $\text{[Orc-Sun]}$, we have $\Gamma \vdash c_1: T_1$ for all $i \in \{ 1, \ldots, n \}$. By applying substitution lemma on $e_1$ repeatedly, we get $\Theta, X: T_1, \ldots, T_n \vdash e_1 \{ \bar{c}/\bar{x} \}$. Finally, by rule $\text{[ord]}$ and $\text{[ord-letrec]}$, we get $\Theta \vdash \text{letrec } X(\bar{x}) = e_1 \text{ in } (e_1 \{ \bar{c}/\bar{x} \} \mid e_2)$.

**Case** $\text{[Ored-Intr]}$. $e = \text{let } x_1, \ldots, x_n = \text{gin } (e_1 \mid \ldots \mid e_n)$, $e' = (\nu \bar{s})(e_1 \{ c/x_1 \} \mid \ldots \mid e_n \{ c/x_n \})$.

From the premise of the rule, we have:

$[g]_{p_1 \ldots p_n} = (c_1, \ldots, c_n)$. 

$\blacktriangleleft$
Figure 22 Implementation of first-class methods and labels

\[ \bigcup_{i \in \{1, n\}} \text{fn}(e_i) = \{s\}, \text{which shares base name s} \]

\[ \{s\} \cap \bigcup_{i \in \{1, n\}} \text{fn}(e_i) = \emptyset. \]

By inversion, we have
\[ \Theta \vdash \Gamma, x_i: T_i \vdash e_i. \]

From Theorem 3.11, for \( \{s_j\}_{j \in J} = \{s\} \) we have \( \{s_j: T_j\}_{j \in J} \vdash e_i: T_i \) for all \( i \in \{1, \ldots, n\} \). By weakening, for all \( i \in \{1, \ldots, n\} \), we have
\[ \Gamma, \{s_j: T_j\}_{j \in J} \vdash c_i: T_i \]

and
\[ \Theta \vdash \Gamma, \{s_j: T_j\}_{j \in J}, x_i: T_i \vdash e_i. \]

By substitution lemma, we get \( \Theta \vdash \Gamma, \{s_j: T_j\}_{j \in J} \vdash e_i(c_i/x_i) \). By applying \( [\text{O} \Gamma c_i] \) and \( [\text{O} \Gamma e_i] \), we finally get \( \Theta \vdash \Gamma \vdash (\nu s)(e_1/c_1) \ldots e_n/c_n) \).

E Implementation: Omitted Type Signatures and Explanations

This section gives the OCaml type signatures and implementations of the main implementation building blocks using sophisticated functional programming techniques based on GADT and polymorphic variants. Namely, first-class methods and labels are explained in § E.1, roles and variable-length tuples in § sec:vartup, input and output channels in § E.3, and global combinators in § E.4.

E.1 First-Class Methods and Labels

As we show in § 4.1, the definition of roles and labels use methods of an object. To enable this encoding, we introduce first-class methods – the type method_ defined on Line 2 in Fig. 22. The type is a record with a constructor function make_obj and a destructor function call_obj. An example usage of the type method_ is given on Line 6 by defining the type login_method.

In make_obj, the expression (object method login=v end) creates an object that consists of a method login with no parameter, returning (v: 'mt). Field call_obj simply implements a method invocation (obj#login).

Our encoding of local types requires label names to be encoded as an object method (in case of internal choice) and as a variant tag (in case of external choice). Hence, the label type, Line 9, is defined as a pair of a first-class method and a variant constructor function. As in § 4.2, while object and variant constructor functions are needed to compose a channel vector in (-->), object destructor functions are used in merge in choice_at, to extract bare channels inside an object. Variant destructors are not needed, as they are destructed via...
We declare variable-length tuple type \((\text{t\_tup})\) as a Generalised Abstract Data Type [20], as follows:

\[
\text{type _ tup = Nil: nil tup } | \text{Cons : 'hd * 'tl tup } \rightarrow \text{['cons of 'hd * 'tl] tup}
\]

The type \(\text{t\_tup}\) consists of two constructors \(\text{Nil}\) and \(\text{Cons}\) which construct tuples \((c_1, c_2, \ldots, c_n)\) as a cons-list \((\text{Cons}(c_1, \text{Cons}(c_2, \ldots, \text{Cons}(c_n, \text{Nil})))\)). The element types can be heterogeneous; in type \((\text{t\_tup})\) the argument \(t\) denotes tuple type \(t_1 \times \cdots \times t_n\) by the nested sequence of polymorphic variant types as \((\text{['cons of } t_1 \times \cdots \times t_n\text{] tup})\). Here, the auxiliary type \(\text{nil}\) is defined by an infinite sequence of \(\text{unit}\) types defined in the second line, \((\text{['cons of } \text{unit} \times 'a}\text{) as 'a})\) where outer \('a\) binds the whole \(\text{nil}\) type, forming an equi-recursive type which essentially states that the the rest of roles have a closed session \(\text{unit}\). Thus, \(\text{finish}\) combinator is defined as \(\text{let finish : nil tup = Nil which has an infinite sequence of } \text{units}\) on types, denoting a terminating protocol for any number of roles.

Then, taking inspiration from [31, § 3.2.4], we define the type-level index type on this variable-length tuple as polymorphic lenses (see § 4.3), again using GADTs. The index type \(\text{idx}\) has two constructors \(\text{Zero}\) and \(\text{Succ}\), making an index in a tuple via Peano numbers. The constructor \(\text{Zero}\) says that the lens refers to the 0-th element i.e. the head of a cons, while \(\text{Succ}\) takes a lens and constructs a new lens which refers to a position deeper by one. By applying \(\text{Succ}\) repeatedly, elements at arbitrary depths can be referred. We store the lens for each channel vector inside the role object. For example, the roles \(s\) and \(c\) from the \(\text{OAuth}\) protocol in § 2 are implemented as the records \(\text{let c = \{index = Zero, ...\}}\) and \(\text{let s = \{index = Succ(Zero), ...\}}\) respectively.

\[
\text{type _ tup \_ idx =}
\]

\[
\text{Zero : (['cons of 't * 'tl], 't, [\text{cons of 'u * 'tl}], 'u) idx}
\]

\[
\text{Succ : (['tl1, 't, 'tl2, 'u) idx \rightarrow (['cons of \text{'}hd * 'tl1], 't, [\text{cons of 'hd * 'tl2}], 'u) idx}
\]

\[
\text{val tup \_ get : 'ts tup \rightarrow ('ts, 't, 'us, 'u) idx \rightarrow 't}
\]

\[
\text{val tup \_ put : 'ts tup \rightarrow ('ts, 't, 'us, 'u) idx \rightarrow 'u \rightarrow 'us tup}
\]

**Roles.** By pairing first-class methods and indices, we develop the role type, defined in Fig. 23. The role type is a record with two fields, \(\text{role\_index}\) denotes the index of the role within the global combinator sequence, while \(\text{role\_label}\) is a first-class encoding of the role label as a method in an object. The full declaration of the role \(s\) is given on Line 6.
E.3 Input and output types

To represent communication channels, we use the OCaml module Event, which provides a synchronous inter-thread communications over channels. For each communication action we generate a fresh channel and wrap it in a channel vector structure. The output ⟨m: (v*t) out⟩ is an object with a method m proactively called by the sender’s side choosing label m, of which return type (v*t) out is just a pair of channel and continuation. type (v, t) out = 'v Event.channel * 't (* abstract *) where 'v Event.channel is a standard synchronous channel type of value 'v in OCaml. Note that this pair structure is abstract i.e., hidden outside the module, to prevent abusing of the continuation 't before sending on 'v channel. The output on 'v channel does not transmit any labels, but they are implicitly passed. The transmission of the label m implicitly happens, when output labels are proactively chosen by calling a method m. The input [">m of v*tl] inp makes an external choice as an idiomatic pattern-matching on variants, enabling a case analysis on continuations based on labels. This is done by Event.wrap function, which originates from Concurrent ML [52]. The wrap function works as a map on received values; thus, by wrapping v channel with a function v -> [">m of v*tl], we obtain an input of type [">m of v*tl] inp.

E.4 Global Combinators

This section gives the types for all global combinators.

Communication combinator is a 4-ary combinator. Its type signature has many type variables which are resolved by unification, as we already observed in § 4.3. The types signature is given below, which realises the typing rule [|G|-Comm|] in Fig. 7 using lenses in role type and first-class methods in label type:

\[
\begin{align*}
\text{val ( --> ) : (}'g1, 'ti, 'g2, 'ui, ('ri as 'uj), 'var inp\text{) role }&\Rightarrow (\text{ sending role type }) * \\
('g0, 'tj, 'g1, 'uj, ('rj as 'ui), 'obj) role &\Rightarrow (\text{ receiving role type }) * \\
('obj, (v, 'ti) out, 'var, (v * 'tj) label &\Rightarrow (\text{ the type of the label }) * \\
'g0 tup &\Rightarrow (\text{ the type of the initial tuple of channel vectors }) * \\
'g2 tup &\Rightarrow (\text{ the type of the resulting tuple }) *
\end{align*}
\]

In the expression (m -->j: m g), the continuation g holds the tuple type ('g0 tup). By index-based update via role types, the tuple type ('g1 tup) is updated to ('g2 tup) such that rj’s channel vector in ('g1 tup) is updated to <role_rj: <m: (v, ti) out>>, while that of ri becomes <role_rj: [">m of (v * 'tj] inp>. Assuming that the indices of ni and nj are i and j respectively, 'g0 is updated first to 'g1 by changing its j-th element 'tj to 'uj. Then, it is further updated to 'g2 by changing i-th element 'ti to 'ui. Furthermore, the part ('ri as 'uj) which equates 'uj and 'ri determines the form of 'uj (at role ri) being <role_rj: 'var inp>. Type 'var has the form of a variant type [">m of 'v * 'tj] which results in a faithful encoding of a receiving type, since it is a part of variant constructor function (specified by the parameters of type label). By a similar argument, type 'ui equated to 'rj has the form <role_rj: <m: (v, ti) out>>, which describes the session at ri.

Loops via lazy evaluation The signature of the loop combinator fix is given below.

\[
\begin{align*}
\text{val fix : (}'t tup &\Rightarrow 't tup\text{) } &\Rightarrow 't tup \\
\text{let fix = let rec body = lazy (f (RecVar body)) in Lazy.force body}
\end{align*}
\]

Function fix takes a function f and returns a fixpoint of it (x = f x) by utilising lazy evaluation and a value recursion which makes a cyclic data structure. We extend the tuple types for global combinators, i.e. 't tup type, with a new constructor RecVar which discriminates recursion variables from other constructors. Lazy.force tries to expand unguarded recursion
variables which occurs right under the fixpoint combinator. This enables the “fail-fast” policy, explained in § 4.1. For example, an unguarded loop like \((\text{fix } (\text{fun } t \rightarrow t))\) fails with \texttt{UnguardedLoop} exception.

**Branching combinator: Merging and object concatenation** In a similar way, from \cite{ORC} the type of the binary branching combinator \((\text{choice@}_r mrg (r_u, g_l) (r_u, g_r))\) is implemented as follows:

\[
\begin{align*}
\text{val choice@}_t & : (\text{'g0, unit, 'g, 'tlr, 'ra, _}) \rightarrow ('tlr, 'tl, 'tr) \text{ disj} \\
& ((\text{'gl, 'tl, 'g0, unit, 'ra, _}) \rightarrow ('gl \text{ tup}) \rightarrow ('gr, 'tr, 'g0, unit, 'ra, _) \rightarrow ('gr \text{ tup}) \rightarrow ('g \text{ tup})}
\end{align*}
\]

The lens part is same as in § 4.3. Additionally, the role-label part ‘\(\text{ra}\)’ ensures that the three roles are same. Types ‘\(\text{'tl}\)’ and ‘\(\text{'tr}\)’ are output type of form \(\text{<role}_q:\ <m_r: (v_i, t_i) \text{ out}>_{i \in I}\) \(\text{and} \text{<role}_q:\ <m_r': (v_j', t_j') \text{ out}>_{j \in J}\) where \(q\) is the destination role and \(\{m_i\} \text{ and } \{m_j'\}\) are the set of output labels which should be disjoint from each other. The following type \((l_r, l, r) \text{ disj}\) denotes a constraint that type \(l_r\) is the type concatenated from mutually-disjoint \(l\) and \(r\):

\[
\text{type ('lr, 'l, 'r) disj =} \\
\{\text{disj_merge: 'l }\rightarrow 'r \rightarrow 'lr; \text{disj_split_L: 'lr }\rightarrow 'l; \text{disj_split_R: 'lr }\rightarrow 'r\}
\]

**E.5 Example of Concatenating Two Disjoint Objects**

The functions \(\text{disj_merge}\) concatenates two disjoint objects ‘\(l\)’ and ‘\(r\)’ into one, while \(\text{disj_split\{L,R\}}\) splits an object ‘\(lr\)’ to ‘\(l\)’ and ‘\(r\)’, respectively. Both are used in the definition of a branching operator. This constraint must manually be supplied by programmers. For example, the following \(\text{left_or_right}\) states a concatenation of type ‘\(<\text{left}: 'tl\)’ and ‘<\text{right}: 'tr>’ into ‘\(<\text{left}: 'tl; \text{right}: 'tr>\)’:

\[
\begin{align*}
\text{val left_or_right & : (}<\text{left}: 'l; \text{right}: 'r>, <\text{left}: 'l>, <\text{right}: 'r>) \text{ disj} \\
& \text{let left_or_right } = \\
& \{\text{disj_merge=fun l r }\rightarrow \text{object method left=l#left method right=r#right end}; \\
& \text{disj_split_L=(fun obj }\rightarrow \text{obj#left); \text{disj_split_R=(fun obj }\rightarrow \text{obj#right})}
\end{align*}
\]

**F Multiparty Session Types and Processes**

This section quickly outlines the multiparty session types \cite{12, 56}. For the syntax of types, we follow \cite{4} which is the most widely used syntax in the literature. A **global type**, written \(G, G', \ldots\), describes the whole conversation scenario of a multiparty session as a type signature, and a **local type**, written by \(S, S', \ldots\). Let \(\mathcal{P}\) be a set of participants fixed throughout the section: \(\mathcal{P} = \{p, q, r, \ldots\}\), and \(\mathcal{A}\) is a set of alphabets.

**Definition F.1 (Global types).** The syntax of a **global type** \(G\) is:

\[
G ::= p \rightarrow q; \{m_i(S_i).G_i\}_{i \in I} \ | \ \mu t.G \ | \ t \ | \ \text{end} \quad \text{with } p \neq q, \ I \neq \emptyset, \ \text{and } \forall i \in I : \text{fv}(S_i) = \emptyset
\]

We write \(p \in \text{roles}(G)\) (or simply \(p \in G\)) iff, for some \(q\), either \(p \rightarrow q\) or \(q \rightarrow p\) occurs in \(G\).

**Definition F.2 (Local types).** The syntax of **local types** is:

\[
S.T ::= p k_{i \in I} m_i(S_i).S'_i \ | \ p \oplus_{i \in I} m_i(S_i).S'_i \ | \ \text{end} \ | \ \mu t.S \ | \ t \quad \text{with } I \neq \emptyset, \ \text{and } m_i \text{ pairwise distinct}
\]

We require types to be closed, and recursion variables to be guarded.

The relation between global and local types is formalised by **projection** \cite{4, 27}.

**Definition F.3 (projection).** The **projection of** \(G\) **onto** \(p\) (written \(G|_p\)) is defined as:
We say that $G$ is well-formed if for all $p \in \mathcal{P}$, $G|p$ is defined.

Below we define the multiparty session subtyping relation, following [24][17]. Intuitively, a type $S$ is smaller than $S'$ when $S$ is “less demanding” than $S'$, i.e., when $S$ imposes to support less external choices and allows to perform more internal choices. Session subtyping is used in the type system to augment its flexibility.

> **Definition F.4** (Session subtyping). The subtyping relation $\leq$ is coinductively defined as:

\[
\begin{align*}
\forall i \in I & \quad S_i \leq T_i \quad S_i' \leq T_i' & \text{[Sub-k]} \\
& \frac{p \wedge_{i \in I} m_i(S_i) . S_i' \leq p \wedge_{i \in I} m_i(T_i) . T_i'}{S[p.S/t] \leq T} & \text{[Sub-end]} \\
& \frac{S \leq T \mu.T/t}{\mu t.S \leq T} & \text{[Sub-µL]} \\
& \frac{S \leq \mu t.T}{\mu t.S \leq T} & \text{[Sub-µR]}
\end{align*}
\]

For projection of branchings, we appeal to a merge operator along the lines of [14], written $G \sqcap S'$, ensuring that if the locally observable behaviour of the local type is dependent on the chosen branch then it is identifiable via a unique choice/branching label. The merging operation $\sqcap$ is used in the type system to augment its flexibility.

For convenience, we use the “channel-oriented” order of [21, 54] for our subtyping relation. For a comparison with “process-oriented” subtyping of [17], see [22].