Study of the dynamic processes of the vibratory drum rebound from the compacted surface

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Abstract. Fast dynamic processes of material compaction by vibration rollers represent random dynamic processes, wherein characteristics of the strength of compacted material, humidity, temperature, types of materials and their characteristics change stochastically. However, from the total set of these processes it is possible to single out such processes, which belong to the category of random ones because of their insufficient state of knowledge but they are deterministic. Such processes may include the vibratory drum dynamic vertical movements during the material compaction. In the process of the study the vibratory drum downward and upward movement modes are considered under the action of the dynamic driving force, which while projecting onto a vertical axis turns into a harmonic force acting periodically downwards and upwards and it is a deterministic parameter. Other mass and kinematic characteristics of the vibratory drum are also deterministic ones. On the basis of classic laws of dynamics the vibratory drum bouncing dynamic processes in the final stage of the compaction technological process and accompanying process of the vibratory drum vertical downward movement have been analytically considered. The necessity to solve such problem is justified by the fact that the dynamic process parameter determination by experimental methods is connected with significant physical and material expenses. New knowledge has been obtained that is necessary for the improvement of the vibration roller designs and modes of their operation.

Key-words: vibratory drum, dynamic processes, differential equations.

1. Introduction

At present the studies connected with the control over the vibration roller material compaction modes have been intensified. In paper [1] the authors consider the problem connected with the vibratory drum material compaction process improvement by means of the compaction process parameter systematization using the intelligent control.

Paper [2] is devoted to the reliability and durability improvement of the vibratory drum rolling bearings for the material compaction by improving the design and lubrication modes of the bearings.

In paper [3] the author Yan Tao- ping considers the hydraulic control circuit of the pump-motor drive of the vibratory drum vibration exciter. The conditions of the vibration exciter drive parameter selection have been developed.

In paper [4] S. V. Saveliev has developed the numerical method of the dynamic modeling of the vibration roller soil compaction processes. The modes of the road roller vibratory drum efficient operation have been considered.

In paper [5] the team of authors considers the vibration roller intelligent control problems. A new operation mode intelligent control system has been proposed – GA method for the vibratory drum allowing to improve the control process.
In paper [6] Heqing Li, Qing Tan have proposed the vibratory drum reliability control method by means of the BP artificial neural network. The main attention is paid to the processing time of the obtained information and increasing the vibratory drum reliability. A conducted review of papers relative to the problem of the vibration roller study has shown that the authors consider the topical problems of improving the vibration roller operation modes that confirms high relevance of problems in this area. However, the level of the general theory lags behind the technical level of modern vibration rollers in connection with insufficient research of the vibratory drum interaction processes with the material being compacted. The vibratory drum vertical vibration is very effective with the first roller passages when the compacted base settlement during the passages is equal to \( h_{uc} = 10 \div 20 \text{ mm} \). During subsequent vibration roller passages over the compacted surface the total settlement \( h_{uc} \) is decreased and conditions for the vibratory drum vertical rebound from the compacted surface appear. These phenomena are caused by the fact that the dynamic driving force \( P_d \) of a modern vibration roller exceeds 3-4 times the vibratory drum gravity force. These processes are negative and require a detailed study aimed at further improving the technological compaction processes in the road construction. While studying the dynamic processes the theorems of dynamics are used. The novelty of these studies consists in the consideration of the large mass movement of about 6000 \( \div \) 10000 kg for small periods of time \( t = 0.01 \div 0.001 \text{ s} \) with small amplitudes \( A(z) = 0.0001 \div 0.005 \text{ m} \). At present for the overall effectiveness assessment of such dynamic processes the methods of spectral analysis of vibration acceleration frequencies are used. However, experimental results obtained herein are multifactorial random functions, the operational analysis of which is possible only by means of special computer software. The purpose of this study is to obtain new knowledge relative to mechanical fast processes during the soil compaction by a vertical vibration roller.

2. Problem statement

Figure 1, Figure 2 show a vibratory drum consisting of eccentric weight \( I \) with the mass \( m_1 \) with center of mass at point \( C_1 \), mass \( m_2 \) of the vibratory drum, which is conventionally coincides with the mass \( m_3 \) of the vibratory drum vertical cantledge by gravity from the roller frame [7, 8].

![Figure 1](image1.png)  
**Figure 1.** Vibratory drum on bearing surface.  

![Figure 2](image2.png)  
**Figure 2.** Start of vibratory drum bouncing process.

The mass \( (m_2 + m_3) \) is considered as a mass applied at the center of mass of the vibratory drum at point \( C_2 \), in this case the point \( C \) is the center of mass of the mechanical system. We use the principle of motion of center of mass. In this case during the eccentric weight rotation with the mass \( m_1 \) the center of mass of the system \( C \) maintains the state of rest in space [9].
Figure 1, Figure 2 show the acting external forces $m_1g$, $(m_2 + m_3)g$ and reaction force $R_z$ of the vibratory drum on the bearing surface. Figure 1 and Figure 2 do not show the d’Alembert inertia force of the eccentric weight, because it is the vibration exciter internal force, which is balanced by the reaction in the support $C_2$ of the eccentric weight. The principle of motion of center of mass allows to write down the differential equations of the vibratory drum bouncing and dropping on the compacted surface for the considered mechanical system.

3. Theory

The principle of motion of center of mass is formulated as follows. The center of mass of the mechanical system is the point where the mass of the entire system is concentrated and where all forces acting on the mechanical system are applied [9].

The principle of motion of center of mass relative to the vertical axis $z$ (see Figure 1, Figure 2) is written in the form of the basic equation of Newton dynamics [10, 11]

$$\sum k_zc F_z = \sum m_k \frac{dz_c}{dt^2}. \quad (1)$$

For the solution of equation (1), we preliminarily determine the coordinate of the center of mass $z_c$ (see Figure 2)

$$z_c = \frac{\sum m_k z_k}{\sum m_k} = \frac{m_1 z_1 + (m_2 + m_3) z_2}{m_1 + m_2 + m_3}. \quad (2)$$

The coordinate $z_1$ can be written as the time function.

$$z_1 = r_1 \sin pt, \quad z_2 = 0, \quad (3)$$

where $r_1$ – is the eccentricity of the unbalanced mass of the vibration exciter eccentric weight; $p$ is the vibration exciter eccentric weight rotation frequency.

The second derivative of expression (2) taking into account (3) has the following form

$$\ddot{z}_c = -\frac{m_1 r_1 p^2}{m_1 + m_2 + m_3} \sin pt. \quad (4)$$

Inserting (4) into equation (1) we obtain

$$-m_1 r_1 p^2 \sin pt = R_z - (m_1 + m_2 + m_3)g. \quad (5)$$

From equation (5) it is possible to obtain the modulus of the vibration exciter eccentric weight radial driving force

$$P_d = m_1 r_1 p^2. \quad (6)$$

From equation (5) we can determine the vertical support reaction of the vibratory drum

$$R_z = (m_1 + m_2 + m_3)g - P_d \sin pt. \quad (7)$$

4. Results discussion

For one period $T$ of the eccentric weight rotation for multiple angles $\varphi = pt$ the Table shows the values of the vibratory drum reaction $R_z$ (see Figure 1, Figure 2).

The calculation of the vibratory drum vertical upward movement by the analytical method is of great importance, because the experimental determination of this parameter is difficult in connection with the large number of random factors affecting this process. Information about the vibratory drum raising value above the support surface is necessary to study the effect of the vibratory drum dropping process on the vibratory drum material compaction process.

In Figure 1 the vibratory drum eccentric weight is in the initial position of the vibratory drum raising process. However, during a certain rotation angle $\varphi = \varphi_0$ (see Figure 2) the vibratory drum will not take off from the bearing surface, as the eccentric weight lifting force upwards $P_d < P_d \sin pt$ is insufficient for overcoming the vibratory drum gravity force.
In subsequent studies Figure 3 and Figure 4 show the d’Alembert inertia force $P_d$ as an active radial acting force [10, 11] being the main factor of the vibratory drum support reaction $R_z$ changing (see the table).

**Table.** Dependence of the vibratory drum support reaction $R_z$ on the angle of the eccentric weight rotation $\phi$

| Process phase                                      | Angle $\phi = \phi_0$ | Reaction $R_z$ |
|----------------------------------------------------|------------------------|----------------|
| Start of the vibratory drum raising                | 0                      | $R_z = (m_1 + m_2 + m_3)g$ |
| Middle of the time period of the vibratory drum raising process | 0.5$\pi$               | $R_{z_{\text{min}}} = (m_1 + m_2 + m_3)g - P_d$ |
| Termination of the vibratory drum raising process and start of the vibratory drum lowering process (start of the material compaction process) | $\pi$                  | $R_z = (m_1 + m_2 + m_3)g$ |
| Middle of the time period of the vibratory drum downward movement (material compaction) | $3\pi/2$               | $R_{z_{\text{max}}} = (m_1 + m_2 + m_3)g + P_d$ |
| Termination of the vibratory drum downward movement (end of the material compaction process) | $2\pi$                 | $R_z = (m_1 + m_2 + m_3)g$ |

In other words the d’Alembert inertia force has been transferred to the category of active forces (Figure 3, Figure 4), that allows to use the basic equation of Newton dynamics.

In Figure 3 the vertical driving force $P_z = P_d\sin\phi_0$ overcame the vibratory drum gravity forces, that’s why so the reaction $R_z = 0$.

For Figure 3 we can write the equation of equilibrium of forces

$$P_d\sin\phi_0 = (m_1 + m_2 + m_3)g,$$

where $\phi_0$ – the delay angle of the vibratory drum taking off process from the compacted surface.

The delay angle of the vibratory drum taking of process from the bearing surface can be determined by the formula from equation (8)

$$\phi_0 = \arcsin \frac{(m_1 + m_2 + m_3)g}{P_d}.$$

The obtained value of the angle $\phi_0$ allows to determine the delay time of the vibratory drum raising process start above the bearing surface

![Figure 3. Start of the vibratory drum upward movement process above the compacted surface.](image)

![Figure 4. Start of the vibratory drum downward movement onto the compacted surface.](image)
\[ \tau = \frac{\varphi_0}{\pi}, \quad (10) \]

With the eccentric weight uniform rotation \( p = \text{const} \) the delay time is proportional to the angle \( \varphi_0 \) according to equation (10).

The equation of the mechanical system mass center dynamics according to Newton relative to the vertical axis \( z \) for the vibratory drum (see Figure 3) can be written in the following form [11]

\[ (m_1 + m_2 + m_3) \ddot{z} = P_d \sin(pt - \varphi_0) - (m_1 + m_2 + m_3)g. \quad (11) \]

For the solution of differential equation (11) the harmonic function on the right-hand side of the equation is replaced by the average effective constant value for the semi-period of the eccentric weight rotation [11]

\[ P_d \sin pt = 0.6366 P_d, \quad 0 \leq t \leq 0.5T. \quad (12) \]

The delay angle \( \varphi_0 \) as a function of time can be taken into account by introducing the delay time \( (t - \tau) \), where \( \tau \) is the delay time.

Equation (11) of the vibratory drum upward movement as a result of replacement according to equation (11) becomes a differential equation with the constant right-hand side

\[ (m_1 + m_2 + m_3) \ddot{z} = 0.6366 P_d - (m_1 + m_2 + m_3)g. \quad (13) \]

Differential equation (13) is the equation of the upward movement of the mechanical system mass center which is brought to the following form

\[ \ddot{z} = f_z. \quad (14) \]

The right-hand side of equation (14) has the following form

\[ f_z = \frac{0.6366 P_d - (m_1 + m_2 + m_3)g}{m_1 + m_2 + m_3}. \quad (15) \]

Differential equation (14) has the analytical solution

\[ \ddot{z} = f_z (t - \tau) + C_1. \quad (16) \]

\[ z = f_z \frac{(t - \tau)^2}{2} + C_1 (t - \tau) + C_2. \quad (17) \]

For the determination of the integration constants \( C_1, C_2 \) we use the initial conditions: at \( t=0 \) \( z = z_0 = 0; \quad \ddot{z} = \dot{z}_0 = r_c p \), where \( r_c \) is the radius of the mechanical system mass center rotation (see Figure 1)

\[ r_c = \frac{m_1}{m_1 + m_2 + m_3} r_1. \quad (18) \]

The integration constants have the following values

\[ C_1 = r_c p; \quad C_2 = 0. \quad (19) \]

The equation of the vibratory drum upward movement has the following form

\[ z_n = f_z \frac{(t - \tau)^2}{2} + r_c p(t - \tau), \quad 0 \leq t \leq 0.5T. \quad (20) \]

5. Consideration of the results

For the performance of numerical experiments we use the data of the vibration roller DM-614: \( (m_1 + m_2 + m_3)=6000 \text{ kg}; \quad m_1 = 70 \text{ kg}; \quad \text{frequency } f = 30 \text{ Hz}; \quad p = f2\pi = 188.495 \text{ rad/s}; \quad P_d = 215 \text{ kN}; \quad \varphi_0 = 18^\circ; \quad \tau \]
\begin{align*}
T &= 0.0333 \text{ s};
r_c &= 0.001008 \text{ m};

\text{vibratory drum bouncing height } z_n &= 0.0046 \text{ m (according to equation (20))}.
\end{align*}

After the fulfillment of the vibratory drum bouncing operation the dynamic process of the vibratory drum lowering (dropping) starts.

Figure 4 shows the initial state of the mechanical system for downward movement from the height of 
\( z_n = 0.0046 \text{ m} \). The vibratory drum dropping phase makes the angle \( \varphi = 0.5\pi \). The vibratory drum dropping is a forced movement, because it is performed under the action of the vibratory drum gravity force and eccentric weight driving force directed downwards. The harmonic sinusoidal function \( P_d \sin pt \) at a quarter-period \( t = 0.25T \) can be replaced by the equivalent average acting constant force \( P_z \) directed downwards.

\begin{equation}
\quad P_z = P_d \sin pt = 0.3183P_d. \tag{21}
\end{equation}

The equation of the forced downward movement of the vibratory drum mass center for Figure 4 has the following form

\begin{equation}
(m_1 + m_2 + m_3)\ddot{z} = 0.3183P_d + (m_1 + m_2 + m_3)g. \tag{22}
\end{equation}

The analytical solution of differential equation (22) has the following form

\begin{equation}
z = f_z t^2/2 + r_c pt, \quad 0 \leq t \leq 0.25T. \tag{23}
\end{equation}

where \( f_z \) − is the acceleration of the vibratory drum vertical dropping, which is determined by the formula

\begin{equation}
f_z = \frac{0.3183P_d}{m_1 + m_2 + m_3} + g. \tag{24}
\end{equation}

Thus, the differential equation has been obtained of the vibratory drum downward movement after bouncing over the bearing surface.

In equation (23) the height of the vibratory drum dropping \( z = z_n \) is known, that’s why it’s possible to determine the time \( t \) of the vibratory drum dropping from the height \( z_n \)

\begin{equation}
t_n = -\frac{r_c P}{f_z} \pm \left[ \frac{r_c P}{f_z} \right]^2 + \frac{2z_n}{f_z}. \tag{25}
\end{equation}

Equation (25) allows to determine the time of the vibratory drum forced downward movement from the raised position on the bearing surface.

For the vibration roller the time of the vibratory drum dropping according to equation (25) is equal to \( t = 0.01111 \text{ s} \).

With a uniform rotation of the eccentric weight with an angular velocity \( p \) the quarter-period of rotation \( t = 0.25T \) corresponds to the time \( t = 0.008325 \text{ s} \), however, the vibratory drum dropping time appeared to be longer and was equal to \( t_n = 0.01111 \text{ s} \). This means that at the moment of the vibratory drum contact with the bearing surface at point \( K \) the eccentric weight is able to turn at an angle \( \varphi_n > 0.5\pi \) (Figure 5).

Figure 5 shows the moment of the vibratory drum dropping process termination. The point \( A \) determines the eccentric weight position at the beginning of the vibratory drum dropping with the driving force horizontal position \( P_d \) of the eccentric weight. The eccentric weight rotation angle when dropping from the height \( z_n \) is determined by the formula

\begin{equation}
\varphi_n = pt_n 180/\pi = 119.9^\circ. \tag{26}
\end{equation}
Figure 5. The end of the vibratory drum lowering process on the compacted surface at the point K: OB – the direction of the driving force $P_d$ at the moment of the vibratory drum dropping process termination.

6. Conclusion
The developed methods allowed to obtain new knowledge relative to the vibratory drum operation modes, differential equations describing the vibratory drum bouncing and dropping on the compacted surface were obtained. The considered modes are negative phenomena for the compaction process. In the technical literature many authors note the vibro-impact nature of such vibratory drum operation modes, which are observed experimentally.
The disadvantage of the vibratory drum bouncing and dropping mode consists in the complete exclusion of the vibration exciter driving force from the material and soil compaction process during the time of the vibratory drum upward and downward movements. That’s why the optimization of the vibratory drum operation modes should be based on the study of contact interactions of the vibratory drum with compacted material without a rebound and bounces on the bearing surface.

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