A surface and subsurface model for the simulation of rainfall infiltration in slopes

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Abstract. Rainfall infiltration is one of the major triggering factors leading to slope failures in geotechnical engineering. Numerical investigation on rainfall infiltration is often based on Richards’ equation, which ignores the surface water effects and simplifies the boundary conditions. In reality, rainfall, infiltration, and surface runoff are interrelated simultaneously. In this paper a new conjunctive one-dimensional surface flow and two-dimensional subsurface flow model for geotechnical slope is developed. The interaction between surface and subsurface flow is the interface infiltration rate, which is obtained by iterations. The results of comparisons between coupled and uncoupled models show that the surface water depth rises up as runoff increases and it tends to a dynamic balance state with a steady surface water depth. Interaction between surface and subsurface flow has remarkable effects on infiltration process. According to the results of coupled model, more rainwaters infiltrate into the slope. Therefore, pore water pressure changes faster and the wetting front moves deeper into the soil. Under initial drier condition, the capacity of infiltration is higher and more rainfall can be absorbed into slope, thus the differences of infiltration rate and pore water pressure between the coupled and uncoupled model are more significant.

1. Introduction
Rainfall is within a part of the hydrological cycle. Generally, when rainwater falls on a slope, part of the amount of rainfall will infiltrate into soil mass, while the others will run off along the slope surface if the rainfall intensity is larger than the infiltration capacity of soil. American Petroleum Institute [1] produces a thorough review on numerous estimation methods of infiltration rate through unsaturated zone. Based on the above research, Ravi and Williams [2] classify those estimation methods into three categories, i.e. (a) empirical models; (b) Green-Ampt models; and (c) Richards equation models. The empirical models only depict cumulative infiltration and infiltration rate changes with time, but can not provide pore water pressure and water content information. Green-Ampt models describe one-dimensional infiltration process by assuming water advancing downward like a piston, but those still can not provide pore water pressure distribution information. Richards equation is the mathematical governing equation of physical flow in saturated-unsaturated soils. After applying proper initial and boundary conditions and solving the partial differential equation, the pore water pressure distribution is obtained.

Since pore water pressure directly contributes to soil shear strength and volumetric change, it is the key issue considered by geotechnical engineers in slope engineering. The mechanism of rain-induced slope failure has been studied by early researchers [3-6]. Neuman [7] successfully introduces finite element method to solve Richards equation. Lam [8] successfully solves some saturated-unsaturated...
seepage problems in geotechnical engineering by finite element method. Fredlund [9-10] presents extended shear strength and volumetric change theories for unsaturated soils, which takes the pore water pressure into consideration. Many numerical studies on infiltration in slopes has presented in references [11-17].

The above numerical rainfall infiltration simulations assume the surface water depth is zero when runoff occurs. But, in fact, the physical processes of rainfall infiltration and surface runoff interact simultaneously. To reflect this interaction, conjunctive surface and subsurface flow model should be adopted. Akan and Yen [18-19] develop a conjunctive one-dimensional surface flow and two-dimensional subsurface flow model. Morita and Yen [20-21] extend it to a conjunctive two-dimensional surface and three-dimensional subsurface flow model. Recently, other researchers [22-26] present new models for solving conjunctive surface and subsurface flow.

In this paper, a new conjunctive one-dimensional surface flow and two-dimensional subsurface flow model is developed. The governing equations of one-dimensional surface water flow on steep slope are modified Saint Venant equations [27] where the effect factor of slope angle is introduced into momentum equation. The governing equation of two-dimensional subsurface flow equation is Richards equation. It is solved by finite element method. The interaction between surface and subsurface flow is the interface infiltration, which is determined by iterations between surface and subsurface flow models. To verify the validity and illustrate the characteristics of conjunctive surface and subsurface flow model, four numerical study cases are performed. A summary of those numerical study cases is presented in Table 1.

| Case No | 1 | 2 | 3 | 4 |
|---------|---|---|---|---|
| SWCC    | See Figure 5 | See Figure 5 | See Figure 5 | See Figure 5 |
| $K_s$ (m/s) | $2.0 \times 10^{-5}$ | $1.23 \times 10^{-5}$ | $1.23 \times 10^{-5}$ | $1.23 \times 10^{-5}$ |
| Dimension | 1D | 1D | 2D | 2D |
| Boundary type | Head | Flux | Head/Flux automatically specified | Head/Flux automatically specified |
| Rainfall intensity (m/s) | $2.5 \times 10^{-6}$ | $2.74 \times 10^{-5}$ | $2.74 \times 10^{-5}$ |
| Initial condition | Constant saturation $S=0.1$ | Negative hydrostatic distribution | Steady state solution with low groundwater table | Steady state solution with high groundwater table |
| Remark | For model verification | For model verification | For model comparison | For model comparison |

2. Conjunctive surface-subsurface flow model

2.1. Theory of surface flow model

The governing equations for the gradually varied unsteady shallow water flow are commonly known as the Saint Venant equations [28]. In one-dimensional condition, Yen [27] extended the Saint Venant equations to accommodate the rapid varied unsteady flow in steep slope condition as shown in Equations (1) and (2).

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = r - f$$  \hspace{1cm} (1)
\[
\frac{\partial (uh)}{\partial t} + \frac{\partial (u^2h)}{\partial x} + gh \cos \theta \frac{\partial h}{\partial x} = gh(S_o - S_f)
\]

(2)

A sketch of variables definition of the modified Saint Venant equations is presented in Figure 1. \(S_o = \sin \theta\) is the bottom slope; and \(S_f\) is the friction slope. The main difference between modified and original Saint Venant equations is the pressure force term in the momentum equation. When the slope angle is close to zero, factor \(\cos \theta\) tends to 1.0 and modified Saint Venant equations return to its original form.

![Figure 1. Description of variables in Saint Venant equations](image)

The friction slope \(S_f\) is derived according to the Darcy-Weisbach [28] formula

\[
S_f = f_d \frac{u^2}{8gh}
\]

(3)

where \(f_d\) is the frictional resistance coefficient, which is calculated from an approximated form of the Moody diagram [29]. Under laminar, transitional, and turbulent flows conditions, \(f_d\) is calculated as follows respectively:

\[
f_d = \frac{C_L}{R_e} \quad \text{For laminar flow}
\]

(4)

\[
f_d = \frac{0.223}{R_e^{0.25}} \quad \text{For transition flow}
\]

(5)

\[
f_d = \left[2 \log \left(\frac{2h}{k}\right) + 1.74\right]^{-2} \quad \text{For turbulent flow}
\]

(6)

where \(R_e\) is the Reynolds number, which is the ratio of inertia force to viscous force \(\left(R_e = \frac{uh}{\nu}\right)\) with \(\nu\) being the kinematic viscosity; \(k\) is a length measure of surface roughness; \(C_L\) is a coefficient which is a function of rainfall intensity. If the effect of infiltration on the resistance is assumed negligible, \(C_L\) is equal to 24 [27-28].

2.2. Numerical scheme for surface flow model

The traditional Preissmann four point implicit finite difference scheme is used to solve Equations (1) and (2), which has the advantage of producing unconditionally stable solutions. Newton iteration technique is used to deal with the non-linear problem. Since the flow regime may become supercritical instead of subcritical [28], the boundary conditions for the Saint Venant equations will be changed...
from one upstream and one downstream to two upstream boundary conditions automatically. A schematic flow chart of surface flow is illustrated in Figure 2.

2.3. Theory of subsurface flow model
Two-dimensional Richards equation and the flux boundary condition without consideration of source term are presented as follows

\[
m_w \gamma_w \frac{\partial H_w}{\partial t} - \frac{\partial}{\partial x} \left( k_x \frac{\partial H_w}{\partial x} \right) - \frac{\partial}{\partial y} \left( k_y \frac{\partial H_w}{\partial y} \right) = 0 \quad \text{on } \Omega
\]

\[
(k \cdot \nabla H_w) \cdot n - \overline{q} = 0 \quad \text{on } \Gamma
\]

where \( H_w \) is total head; \( k_x \) and \( k_y \) are hydraulic conductivity in the x-direction and y-direction respectively; \( m_w = \partial \theta_w / \partial u_w \), which is the slope of soil-water characteristic curve, where \( \theta_w \) is volumetric water content and \( u_w \) is pore water pressure; \( \gamma_w \) is the unit weight of water; \( t \) is the time; \( k \) is hydraulic conductivity tensor; \( \nabla H_w \) is the gradient of total head at boundary surface; \( n \) is the unit normal direction vector; and \( \overline{q} \) is the boundary flux.

2.4. Numerical scheme for subsurface flow model
The Galerkin method is used to solve the finite element equation (7) and (8). Set \( N \) as the interpolating function vector and \( H \) as the total head vector of the element. Let

\[
C = \int_{\Omega} N^T m_w \gamma_w d\Omega
\]

\[
K = \int_{\Omega} \begin{bmatrix} \frac{\partial N}{\partial x}^T k_x \frac{\partial N}{\partial x} & + & \frac{\partial N}{\partial y}^T k_y \frac{\partial N}{\partial y} \end{bmatrix} d\Omega
\]
Then, the finite element equation can be expressed symbolically as follows:

$$Q = \int \nabla \boldsymbol{N} \cdot \mathbf{q} \, d\Gamma$$

(11)

Using the backward difference method to discretize the time term leads to the following equation:

$$\left[ \frac{C}{\Delta t} + K \right] \mathbf{H}_{n+1} = \left[ \frac{C}{\Delta t} \right] \mathbf{H}_n + \mathbf{Q}_{n+1}$$

(13)

Equation (13) is the matrix form of finite element equation usually used to solve the saturated-unsaturated seepage problem. Based on the subsurface model presented above, a FORTRAN module is written to solve two-dimensional saturated-unsaturated subsurface flow. Lumped mass matrix method is adopted for mass matrix $C$ to enhance the stability of solution. A schematic flow chart of subsurface flow is illustrated in Figure 3.
2.5. Judgment of runoff generation
A trial and error procedure is used to judge runoff generation on slope surface. First assume all the rainfall supplies at that time interval could infiltrate into soil under its own gravity and matric suction gradients. Then calculate the subsurface flow. If positive water pressure occurs somewhere, it means that extra pressure is needed to force the given rainfall infiltrate into slope, which is against the previous assumption of no extra forces. Thus, runoff will generate at that position. In contrast, Morita and Yen [20-21] use the infiltrability of soil to judge runoff generation, but it is hard to be estimated.

2.6. Calculate infiltration and runoff
Infiltration is calculated after solving subsurface flow. It is a backward procedure of solving total head or pore water pressure distribution under a given boundary condition. The right hand side term $Q_{n+1}$ in Equation (13) denotes the equivalent nodal flux. When the total head vector at n+1 time is obtained, it can be calculated as follows:

$$Q_{n+1} = KH_{n+1} + \frac{C}{\Delta t} (H_{n+1} - H_n)$$  \hspace{1cm} (14)

Since the nodal flux is obtained, the unit flux can be easily estimated by dividing the length of its edge. Thus, infiltration and runoff can be obtained.

2.7. Conjunctive surface and subsurface flow model flow chart
The main flow chart of conjunctive surface and subsurface flow model is shown in Figure 4. First, prepared data and parameters are read and a series of pre-processes for conjunctive model analysis are done. Then, it enters into the analysis loop and conducts simulation one time step by one time step. In each time step, firstly run subsurface flow model to check if runoff occurs on slope. Then, estimate surface water depth as trial value and switch flux boundary to head boundary to calculate subsurface flow again. When the convergence is obtained, calculate the infiltration and runoff respectively. Then use the runoff as input to conduct the surface flow modeling and calculate surface water depth along the slope. Finally, compare the trial and calculated surface water depths. If the differences of those depths tend to an acceptable convergence, it will end the current time step, output corresponding results, and return to next analysis step, or else, it will update the trial surface water depth and modify the corresponding head boundary to iterate again.

3. Model verification

3.1. One-dimensional infiltration under head boundary
In case 1, the height of soil column is 4 m. At the top of soil column, the total head boundary is applied with the value of 4 m, which means the pressure head is zero. The saturated and residual volumetric water contents of soil are 0.2 and 0.02 respectively. The porosity is 0.2; saturated conductivity is $2.0 \times 10^{-5}$ m/s. Initial saturation of whole soil column is set to be 0.1. The soil-water characteristic curve and hydraulic conductivity function are illustrated in Figure 5. In numerical simulation, finite element mesh space in vertical direction $dz$ is equal to 0.1 m and the time step $dt$ is equal to 6 seconds. Infiltration simulation in 3 hours duration is performed.

The comparison of saturation profiles among conjunctive model, analytical solution [30], and SEEP/W at 1, 2, and 3 hours respectively are presented in Figure 6. Figure 6 shows that the degree of saturation profiles of conjunctive model are overlapped with that of analytical solution and SEEP/W. The relative errors among those three models are within 0.2%. In this case, the soil column is 4 times deeper and saturated conductivity is about 3 times larger than that of Morita and Yen [20]. But both the differences between numerical and analytical solution are rather small.

3.2. One-dimensional infiltration under head boundary
In case 2, the height of soil column is 4 m. A kind of homogeneous fine sand [17] is used with the $K_s$ is $1.23 \times 10^{-5}$ m/s (44.3mm/hr) and the porosity is 0.347. Its soil-water characteristic curve and hydraulic
conductivity function are shown in Figure 7. The residual volumetric water content is about 0.184. The rainfall duration is 12 hours. The initial pore water pressure distribution in soil column is in negative hydrostatic distribution. The rainfall is applied at the top surface and the bottom of soil column is closed and impermeable.

![Soil-water characteristic curve and hydraulic conductivity function](image)

**Figure 5.** Soil-water characteristic curve and hydraulic conductivity function used in analytical model

![Saturation profile comparison](image)

**Figure 6.** Comparisons of saturation profile among conjunctive model, analytical solution, and SEEP/W under head boundary

The calculated results of conjunctive model and SEEP/W are illustrated in Figure 8. At 4 hours rainfall, the wetting front affects to about 0.7 m in depth. The maximum difference of pore water pressure at wetting front between conjunctive model and SEEP/W is about 3 kPa. At the end of 8 and 12 hours rainfall, the wetting front advances to about 1.1 m and 1.6 m in depth respectively and the maximum difference of pore water pressure at wetting front between two models is still about 3 kPa. While, in
In the nearly saturated zone, the pore water pressure changes mildly against that on wetting front and the pore water pressure result of conjunctive model is highly consistent with that of SEEP/W. The pore water pressure of nearly saturated zone is converged to a value of about -7 kPa, which is consistent with the pore water pressure value in hydraulic conductivity function with respect to conductivity of $2.5 \times 10^{-6}$ m/s. The difference of pore water pressure distribution between conjunctive model and SEEP/W is small and stable within 12 hours rainfall duration. It shows that the result of conjunctive model is valid and reliable in solving flux boundary problem.

As a result, the subsurface model in conjunctive surface and subsurface flow model is verified under both head boundary and flux boundary. The results show that it is robust in solving highly non-linear saturated-unsaturated flow.

![Figure 7. Soil-water characteristic curve and hydraulic conductivity function of fine sand](image)

![Figure 8. Comparisons of pore water pressure distribution between conjunctive model and SEEP/W under flux boundary](image)
4. NUMERICAL SIMULATION

4.1. Study cases
In case 3, the initial groundwater table is low and the slope soil is dry, while in case 4 the initial groundwater table is high and the slope soil is relatively wet. The soil properties are the same as that of case 2. A 4 hours uniform rainfall with intensity of $2.74 \times 10^{-3}$ m/s (98.6 mm/hour) is adopted. The numerical time step is about 6 seconds. Total simulation time is 48 hours. The total rainfall amount is close to a high daily rainfall 394 mm that is presented in Ng and Shi’s study [11], but the hourly rainfall intensity is several times larger.

4.2. Geometry and meshes
The geometry and finite element mesh for subsurface flow analysis are presented in Figure 9. The length of the slope is 50 m; the slope angle is 30°; and the depth of calculation zone is 15 m. The whole subsurface flow domain is divided into 2500 quadrilateral elements with 2626 nodes.

4.3. Initial and Boundary conditions
Initial groundwater table and pore water pressure are estimated by running steady state flow under proper boundary conditions. A small rainfall intensity with the value of $1.0 \times 10^{-6}$ m/s (0.0864 mm/day) is applied on the slope surface. A river is assumed at the toe of slope with the elevation of 12.9 m in case 3 and 18.0 m in case 4. The positions of two initial groundwater tables are illustrated in Figure 9.

As for surface flow, the initial condition is assumed that the slope surface is dry. For subsurface flow, the slope surface is a flux boundary subjected to rainfall. When the surface is saturated, the flux boundary will switch to head boundary. The proportion of right side boundary above the river level is specified as potential seepage surface and the exit point will be checked during iterations. Besides, the other boundaries are impermeable. For surface flow, the surface water depth at upstream is equal to the base flow water depth, i.e. 0.15 mm. If surface flow is subcritical, the
downstream water depth is specified to be equal to normal depth. If surface flow is supercritical, the upstream inflow velocity is specified according to base flow.

4.4. Results and discussions

In conjunctive model, the upstream surface water depth is fixed as base flow water depth, when runoff discharges from upstream to downstream, the surface water depth increases gradually and the downstream water depth is the highest which is sensitive with time. The comparisons of downstream surface water depth hydrograph within the first 5 hours between two investigated cases are shown in Figure 10. It shows that at about 0.35 hour the downstream point begins to have remarkable surface water under low groundwater table condition. While under high groundwater table condition, the remarkable surface water generates earlier at about 0.22 hour. This is because the slope soil with high groundwater table is wetter than that with low groundwater. And the initially drier soil slope can absorb more water than that of wetter soil slope. So under the same rainfall pattern, the occurrence of surface runoff in initially drier slope will delay a short time. When larger amount of runoff occurs and discharges to downstream, the downstream water depth rises up prominently. From 0.22 to about 1.0 hour, the downstream water depth increases rapidly. During this time, the downstream water depth under high groundwater table condition is larger than that under low groundwater table condition. At about 1.0 hour, the difference of surface water depth tends to quiet small, which shows that the influence of initial condition begins to disappear. At about 1.5 hours, the surface water depth reaches to a limit value of about 5.2 mm, which means that at the coupled processes among rainfall, infiltration, and surface runoff have reached to a relative steady state. At the end of 4 hours, the downstream water depth reaches to about 5.3 mm. After rainfall ceases at 4 hours, the surface water on slope discharges rapidly. The water depth at downstream drops from about 5.3 mm to zero in 7 minutes. In contrast, a similar hydrograph on mild open channel slope is presented by Morita and Yen [21]. The slope length is about 1/5 of our study; uniform rainfall intensity is about 1/2 of our study; and duration is 1/12 of our study. But the downstream surface water depth is about 10 mm which is about twice of those cases.

The results of surface water depth profile under different initial conditions are compared at selected times, which are illustrated in Figure 11. Because the upstream boundary of surface flow is fixed, the surface water depth at distance of zero along slope surface, i.e. upstream point, is equal to 0.15 mm. 4 observing times are selected to illustrate the surface water depth profile, i.e. 0.4, 0.6, 1.0, and 4 hours. Figure 11 shows that at 0.4 hour the surface water is shallow under low groundwater table condition

Figure 10. Downstream surface water depth hydrograph
with the maximum water depth of about 1.4 mm. The surface water depth increases gradually along the slope from upstream to downstream. While, under high groundwater table condition the surface water depth profile rises up to higher position with the maximum water depth of about 3.2 mm. At 0.6 hour, the surface water depth increases quickly under low groundwater table. The maximum water depth reaches to about 3.4 mm, which is 2 mm higher than that at 0.4 hour. While, under high groundwater table the surface water profile increases about 0.8 mm. At this time, the difference of maximum water depth between different initial conditions is about 0.6 mm, which is much smaller than that at 0.4 hour. Then, at 1 hour, the difference of the maximum surface water depth reduces to about 0.15 mm and the difference of surface water profile between different initially conditions becomes to disappear. Finally, when rainfall ceases at 4 hours, the difference of maximum water depth is about 0.01 mm which is too small to distinguish and two profiles are nearly superposed.

Figure 11. Surface water depth profiles at different times

Figure 12. Comparisons of rainfall intensity and infiltration rate in different initial conditions
The comparisons of infiltration rate between different initial conditions are presented in Figure 12. Hillel [31] shows that the infiltration capacity of soil increases as soil becomes dry, thus under initial lower groundwater table condition, the slope soil can fully absorb higher rainfall intensity at beginning. As shown in Figure 12, in the first 0.15 hour, almost all the rainfall water is absorbed into slope soil under initial lower groundwater table condition, while under high groundwater table condition the surface becomes saturated sooner after rainfall applies on slope. Morita and Yen [21] show a similar process. That is, when the infiltration capacity of soil is greater than the rainfall intensity, all rainwater enters into soil. Else, parts of the slope surface become saturated and runoff occurs on it. Then, infiltration rate decreases gradually and tends to the constant value of saturated hydraulic conductivity [31]. At about 1 hour, the infiltration rates under different initial conditions tend to be equal, which is closely consistent with the changes of surface water depth as shown in Figure 11. After about 1.5 hours, the infiltration rate changes slightly and the coupled processes of infiltration and runoff reach to a relative steady state. When rainfall stops, the infiltration continues for about 7 minutes as surface water exists on slope.

![Figure 13. Comparisons of pore water pressure distribution between conjunctive model and SEEP/W at section A-A](image)

The comparisons of pore water pressure distributions in different initial conditions at A-A section is illustrated in Figure 13. Figure 13 shows that under high groundwater table condition, the pore water pressure at slope surface is about -60 kPa, which is 20 kPa greater than that under low groundwater table condition. At 4 hours, the difference of wetting front between conjunctive model and SEEP/W is about 0.4 m under low groundwater table condition, while under high groundwater table condition, that difference is nearly zero. Although the wetting front of coupled model reaches to nearly the same depth under different initial conditions, the infiltration amount is different. Then, at the end of 24 hours, the pore water pressure has redistributed for about 20 hours. The difference of wetting front between conjunctive model and SEEP/W under low groundwater table condition is about 1.6 m, which is about two times of that under high groundwater table condition. Finally, at the end of 48 hours, under low groundwater table condition, the difference of wetting front between conjunctive model and SEEP/W is still large, which is about 2.0 m. While under high groundwater table condition, the wetting front of both conjunctive model and SEEP/W reaches to groundwater table, which causes groundwater table raising 1.2 m and 0.4 m for coupled and uncoupled results respectively. According
to the comparisons, in a word, the difference of pore water pressure distribution between conjunctive model and SEEP/W under initial dry condition is more significant than that under initial wet condition. The comparison of pore water pressure profiles between conjunctive model and SEEP/W at A-A section under low groundwater table condition is shown in Figure 14. It shows that the wetting front of conjunctive model reaches to about 2.4 m in depth from the ground surface after 4 hours rainfall, which is about 0.4 m deeper than that of SEEP/W. The difference reflects that under the positive pressure head of surface water and interaction between infiltration and runoff, the infiltration in each time step is greater than that of uncoupled model SEEP/W. So, during rainfall time, more water infiltrates into slope and the wetting front moves deeper as shown in conjunctive model result. When rainfall stops at 4 hours, in conjunctive model the surface water keeps on affect the infiltration for about 7 minutes. While in SEEP/W, infiltration stops with the cease of rainfall simultaneously. When runoff disappears from the slope surface, pore water pressure begins to redistribute within the slope. The pore water pressure at surface decreases with time. The wetting front continues to move downward. At the end of 12 hours, the wetting front of conjunctive model reaches to about 4.8 m in depth, which is about 1.2 m deeper than that of SEEP/W. In addition, at the end of 24 hours, the wetting front of conjunctive model moves to about 7.6 m in depth, which is about 1.6 m deeper than that of SEEP/W. Finally, at 48 hours, the wetting front is close to groundwater table. As water discharges into groundwater, the difference of pore water pressure distribution between conjunctive model and SEEP/W tends to lessen.

![Figure 14. Comparisons of pore water pressure distribution in different initial conditions at section A-A](image)

5. Conclusions
A new conjunctive one-dimensional surface flow and two-dimensional subsurface flow model is developed to analyze infiltration and runoff simultaneously in steep slope. The governing equations of one-dimensional surface flow are modified Saint Venant equations. The governing equation of two-dimensional subsurface flow is Richards equation. The subsurface model in conjunctive model is verified by head boundary problem and flux boundary problem respectively. Then, the characteristics of conjunctive model are illustrated. Typical surface water depth and infiltration rate changes with rainfall duration are presented. Furthermore, the pore water pressure distributions are compared and
discussed between conjunctive model and SEEP/W, and the effect of initial condition is studied. Based on the above studies, the following conclusions can be drawn:

(1) Using new developed conjunctive surface and subsurface flow model, surface flow on steep slope can be solved; the infiltration rate versus time can be determined; and the actual infiltration and runoff processes can be simulated.

(2) When runoff occurs on slope, the surface water depth can be calculated. As runoff increases, the surface water depth rises up. When infiltration and runoff tend to a relative steady state, the surface water depth also rises to a relative steady level. Under 98.6 mm/hour rainfall intensity, the maximum downstream water depth over a 30° slope after 4 hours duration is about 5.3 mm. When rainfall stops, the surface water discharges rapidly within about 7 minutes.

(3) Surface water depth and the interaction between infiltration and runoff have remarkable effects on rainfall infiltration process. According to the results of conjunctive surface and subsurface flow model, a greater amount of rainwater infiltrates into the slope. Therefore, the pore water pressure changes faster and the wetting front moves deeper into the soil.

(4) Under initial drier condition, the capacity of infiltration is higher and more rainfall can be absorbed into slope. The difference of pore water pressure distribution between conjunctive model and SEEP/W under initial dry condition is more significant than that under initial wet condition.

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