Degeneracies of the radial mass profile in lens models

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Abstract. We discuss a simple parameter degeneracy in lens models which follows directly from the well known mass sheet degeneracy and prevents determining the radial mass distribution from lensing alone for many systems. For cosmological important systems like 1115+080 it is shown that no exact value of $H_0$ can be determined without assuming a mass index $\beta$.

1. The mass sheet degeneracy

The degeneracies examined in this poster are a consequence of the so called “mass sheet degeneracy”. Falco et al. (1985) and Gorenstein et al. (1988) have shown that a lens model can be transformed to an equivalent model by scaling it with a factor of $(1 - \kappa)$ and adding a constant surface mass density $\kappa$. Constant mass densities as well as external shear do not contribute to the time delay. As $H_0 \Delta t$ depends linearly on the mass density, $H_0$ also scales with $(1 - \kappa)$. This has the effect that only an upper limit for $H_0$ can be determined when additional mass sheets may be present.

When modelling lens systems, no decision between different models can be made if the models are equivalent just for the positions of the observed images. We want to examine the impact of the degeneracy for simple parametric lens models.

2. Perturbed spherical lens models

We use the very simple approach of a spherically symmetric mass distribution plus external shear $\gamma$ to illustrate the degeneracy. For a radial deflection angle of $\alpha(r)$ we can find an equivalent model by scaling $\alpha(r)$ and the shear by $(1 - \kappa)$ and adding the surface mass density which gives a contribution of $\kappa r$ to the radial deflection angle.

Whether this leads to a degeneracy of model parameters, depends on the number of images, the image configuration and the number of parameters used to describe the radial deflection angle $\alpha$.

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A very simple while important case is the spherical power law model with a deflection angle of $\alpha = \alpha_0 r^{\beta - 1}$. It has two parameters, the scale $\alpha_0$ and the mass index $\beta$. Special values of $\beta$ are 0 (point mass), 1 (isothermal sphere) and 2 (constant mass sheet). This simple model may not describe the mass distribution of real galaxies exactly but is quite useful to illustrate the parameter degeneracies which exist for other models as well.

3. Application to Einstein cross systems

In systems like the Einstein cross 2237+0305, the images are located at more or less the same distance (about one Einstein radius) from the centre of the lens. Two spherical models are equivalent to first order if the deflection angle as well as the first derivative is the same for both. We use the Einstein radius as unit of $r$ and an isothermal model as reference. A model with $\alpha = \alpha_0 r^{\beta - 1}$ is equivalent to the isothermal model to first order near $r = 1$ if $\alpha_0 = 1$ and $\beta = 1 + \kappa$. This leads to a very simple scaling of the shear, the time delay and $H_0$:

$$1 - \kappa = 2 - \beta = \frac{\gamma(\beta)}{\gamma(\text{iso})} = \frac{H_0(\beta)}{H_0(\text{iso})}$$

We see that the Hubble constant for the more general $\beta$ model $H_0(\beta)$ can differ significantly from the value determined for the isothermal model $H_0(\text{iso})$. Even more important is the fact that the possible systematic error which is made by assuming isothermal models when the real $\beta$ is different is the same for all systems of this type and does not show as scatter in the results for $H_0$. Comparison of our simple analytical results with numerical models from Wambsganss & Paczyński (1994) for the Einstein cross and Schechter et al. (1997) for the “triple quasar” 1115+080 show that the agreement is excellent for the former and still quite good for the latter. The case of 1115+080 is especially interesting, because it is one of the few systems with a measured time delay used for the determination of $H_0$ (Schechter et al. 1997).

4. Further information

The original much more detailed poster presented at the conference is available from [http://www.hs.uni-hamburg.de/english/persons/wucknitz.html](http://www.hs.uni-hamburg.de/english/persons/wucknitz.html) or on request. An article with a discussion of the degeneracy for different types of lens systems is in preparation.

References

Falco, E.E., Gorenstein, M.V., & Shapiro, I.I. 1985, ApJ, 289, L1
Gorenstein, M.V., Shapiro, I.I., & Falco, E.E. 1988, ApJ, 327, 693
Schechter, P., et al. 1997, ApJ, 475, L85
Wambsganss, J., & Paczyński, B. 1994, AJ, 108, 1156