Structure function evolution at next-to-leading order and beyond
Andreas Vogt

Instituut-Lorentz, University of Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands

Results are presented of two studies addressing the scaling violations of deep-inelastic structure functions. Factorization-scheme independent fits to all ep and µp data on $F_2$ are performed at next-to-leading order (NLO), yielding $\alpha_s(M_Z) = 0.114 \pm 0.002_{\text{exp}} \pm 0.004_{\text{th}}$. In order to reduce the theoretical error dominated by the renormalization-scale dependence, the next-higher order (NNLO) needs to be included. For the flavour non-singlet sector, it is shown that available calculations provide sufficient information for this purpose at $x > 10^{-2}$.

1. Introduction

One of the important objectives of studying structure functions in deep-inelastic scattering (DIS) is a precise determination of the QCD scale parameter $\Lambda$ (i.e., the strong coupling $\alpha_s$) from their scaling violations. In this talk we briefly present results of two studies [1, 2] aiming at an improved control and a reduction of the corresponding theoretical uncertainties.

2. Flavour-singlet evolution in NLO [1]

The evolution of structure functions is usually studied in terms of scale-dependent parton densities and coefficient functions. In this case the predictions of perturbative QCD are affected by two unphysical scales: the renormalization scale $\mu_r$ and the mass-factorization scale $\mu_f$. While the former is unavoidable, the latter can be eliminated by recasting the evolution equations in terms of observables [3]. In the flavour-singlet sector, this procedure results in

$$\frac{d}{d \ln Q^2} \left( \frac{F_2}{F_B} \right) = \mathcal{P} \left( \alpha_s(\mu_r), \frac{\mu_r^2}{Q^2} \right) \otimes \left( \frac{F_2}{F_B} \right)$$

with $F_B = dF_2/d\ln Q^2$ or $F_B = F_L$. The kernels $\mathcal{P}$ are combinations of splitting functions and coefficient functions which become prohibitively complicated in Bjorken-x space at NLO. Thus Eqs. (1) are most conveniently treated using modern complex Mellin-moment techniques [4].

We have performed leading-twist NLO fits to the $F_2^p$ data of SLAC, BCDMS, NMC, H1, and ZEUS. Statistical and systematic errors have been added quadratically, the normalization uncertainties have been taken into account separately. The singlet/non-singlet decomposition has been constrained by the $F_2^n/F_2^p$ data of NMC. The initial shapes $F_{2,B}(x, Q_0^2)$ are expressed via standard parametrizations for parton densities at $\mu_f = Q_0$.

Figure 1. The dependence of the fit results for the energy-momentum sum and for $\alpha_s(M_Z)$ on the $Q^2$-cut imposed in addition to $W^2 > 10$ GeV$^2$. 

---

*Work supported by the EC network ‘QCD and Particle Structure’ under contract No. FMRX-CT98-0194.
In order to establish the kinematic region which can be safely used for fits of $\alpha_s$ in the leading-twist NLO framework, the lower $Q^2$-cut applied to the data has been varied between 3 and 30 GeV$^2$. When the normalized momentum sum of the partons defining the $F_{2,B}$ initial distributions is left free, the fits with $Q^2_{\text{cut}} < 10$ GeV$^2$ prefer values significantly different from unity, see Fig. 1. Also shown in this figure is the $Q^2_{\text{cut}}$-dependence of the fitted values for $\alpha_s(M_Z)$, now imposing the momentum sum rule. The results for $Q^2_{\text{cut}} \leq 7$ GeV$^2$ tend to lie above the $Q^2_{\text{cut}} \geq 10$ GeV$^2$ average of $\alpha_s(M_Z) = 0.114$ (dashed line).

In Fig. 2 we display the renormalization scale dependence of the $\alpha_s(M_Z)$ central values for the safe choice $Q^2_{\text{cut}} = 10$ GeV$^2$. The conventional, but somewhat ad hoc, prescription of estimating the theoretical error by the variation due to $0.25 \leq \mu_R^2/Q^2 \leq 4$ results in

$$\alpha_s(M_Z) = 0.114 \pm 0.002 \exp^{+0.006}_{-0.004} \left(\text{scale}\right). \tag{2}$$

Other theoretical uncertainties are considerably smaller and can be neglected at this point. The uncertainty due to possible higher-twist contributions, for instance, can be estimated at about 1\% via the target-mass effects included in the fits.

3. Non-singlet evolution in NNLO \cite{2}

The theoretical error in Eq. (2) clearly calls for NNLO analyses. The necessary contributions to the $\beta$-function \cite{5} and the coefficient functions \cite{6} are known. However, only partial results are available for the 3-loop terms $P^{(2)}(x)$ in the splitting-function expansion ($a_s \equiv \alpha_s/4\pi$)

$$P = a_sP^{(0)} + a_s^2P^{(1)} + a_s^3P^{(2)} + \ldots. \tag{3}$$

For the non-singlet part of $F_2$ considered here (NS$^+$), present information comprises the lowest five even-integer moments \cite{7}, the full $N_f^2$ piece \cite{8}, and the most singular small-$x$ term \cite{9}.

We have performed a systematic study of the constraints imposed on $P_{\text{NS}}^{(2)+}(x)$ by these results. Four approximations spanning the current uncertainty range are shown in Fig. 3, together with their convolutions with a typical input shape.

![Figure 2](image1.png)

$\alpha_s(M_Z)$

- $F_2/F_L$
- $F_2/dF_2$

$Q^2_{\text{cut}} = 10$ GeV$^2$

$\mu_R^2/Q^2$

Figure 2. The dependence of the optimal values for $\alpha_s(M_Z)$ on the renormalization scale $\mu_r$.

![Figure 3](image2.png)

$P_{\text{NS}}^{(2)+}(x<1)$

$N_f = 0$ approx.

$P_{\text{NS}}^{(2)+}$

$x(P_{\text{NS}}^{(2)+} \otimes f)$

$xf = x^{0.5}(1-x)^3, \ N_f = 0$

A
B
C
D

Figure 3. Representative approximate results for the flavour-number independent part of the 3-loop non-singlet $\overline{\text{MS}}$ splitting function.
\[
\frac{d \ln F_2^{NS}}{d \ln Q^2} = x^{0.5}(1-x)^3, \quad \alpha_s = 0.2
\]

Figure 4. The first three steps in the expansion of the scaling violations of the non-singlet component of \(F_2\) for typical input parameters.

\(P^{(2)+}_{NS}(x)\) is well determined at \(x \geq 0.15\), with a total spread of about 15% at \(x \approx 0.3\). At (non-asymptotically) small \(x\) its behaviour is rather unconstrained despite the known leading \(x \to 0\) contribution. As the splitting functions enter scaling violations always via convolutions

\[
(P \otimes f)(x) = \int_{x}^{1} dy/y \ P(x/y) f(y) \tag{4}
\]

with smooth initial distributions \(f(x)\), the residual uncertainties are much reduced for observables over the full \(x\)-range. In the present case they prove to be negligible at \(x > 0.02\).

The net effect of the NNLO correction is finally illustrated in Fig. 4, where the scale-derivative of \(F_2^{NS}\) is shown for \(\mu_r = Q\) and \(N_f = 4\), using an \(\alpha_s\)-value typical for the fixed-target region. The inclusion of this correction into fits is expected to lead to a slightly lower central value for \(\alpha_s\) and a considerably reduced theoretical uncertainty.

4. Summary and outlook

We have analyzed present \(ep/\mu p\) \(F_2\)-data in a factorization-scheme independent framework \[1\]. We find that \(Q^2, W^2 > 10\ \text{GeV}^2\) is a safe region for leading-twist NLO fits of \(\alpha_s\). Our central value is close to that of the standard pre-HERA analysis in \[10\], but lower than the recent result of \[11\] using a lower \(Q^2\)-cut of 2 GeV\(^2\). The irreducible renormalization-scale uncertainty turns out to be larger than expected from \[10\].

We have derived approximate \(x\)-space expressions for the 3-loop non-singlet splitting functions \(P^{(2)}_{NS}\), including error estimates \[3\]. This approach is complementary to, but more flexible than, the integer-moment procedures pursued in \[12\]. The remaining uncertainties of \(P^{(2)}_{NS}\) are small for the evolution at \(x > 10^{-2}\), thus allowing for detailed NNLO analyses in this region. An extension to the singlet case is in preparation.

REFERENCES

1. J. Blümlein and A. Vogt, to appear.
2. W.L. van Neerven and A. Vogt, to appear.
3. W. Furmanski and R. Petronzio, Z. Phys. C11 (1982) 293; S. Catani, Z. Phys. C75 (1997) 665; and references therein.
4. M. Glück, E. Reya, and A. Vogt, Z. Phys. C48 (1990) 471; Ch. Berger et al., Z. Phys. C70 (1996) 77; J. Blümlein and A. Vogt, Phys. Rev. D58 (1998) 014020.
5. O.V. Tarasov, A.A. Vladimirov, and A.Yu. Zharkov, Phys. Lett. B93 (1980) 429; S.A. Larin and J.A.M. Vermaseren, Phys. Lett. B303 (1993) 334.
6. E.B. Zijlstra and W.L. van Neerven, Phys. Lett. B272 (1991), 127; ibid. B273 (1991) 476.
7. S.A. Larin, T. van Ritbergen, and J.A.M. Vermaseren, Nucl. Phys. B427 (1994) 41; S.A. Larin et al., Nucl. Phys. B492 (1997) 338.
8. J.A. Gracey, Phys. Lett. B322 (1994) 141.
9. J. Blümlein and A. Vogt, Phys. Lett. B370 (1996) 149.
10. A. Milsztajn and M. Virchaux, Phys. Lett. B274 (1992) 221.
11. A.D. Martin et al., Eur. Phys. J. C4 (1998) 463.
12. A.L. Kataev, G. Parente, A.V. Sidorov, hep-ph/9904332 (these proceedings), hep-ph/9905310, and references therein.
13. J. Santiago and F.J. Yndurain, hep-ph/9904344.