Robust Adaptive Multilevel Control of a Quadrotor

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ABSTRACT This study proposes a new multilevel control of a quadrotor with dynamic uncertainties and time-varying external disturbances. The quadrotor model is partitioned into three subsystems: the vertical position, the horizontal position and the rotational subsystems. First, a double loop integral fast terminal sliding mode control with an adaptive estimator for disturbances’ upper-bounds (ADIFTSMC) is proposed for the altitude subsystem to ensure that the quadrotor reaches the desired height. Secondly, a radial basis function neural network backstepping controller (RBFNNBC) is applied to the horizontal subsystem. Finally, by combining a finite time exact disturbance observer with backstepping nonsingular fast terminal sliding mode control (FDOBNFTSMC), the rotational angles converge to the reference angles in the presence of the time-varying disturbances. Furthermore, a Lyapunov stability analysis is used to prove that the tracking errors converge to a small neighborhood of the origin. Numerical simulations illustrate the feasibility of the compound control structure.

INDEX TERMS Fast terminal SMC, RBFNN, backstepping, disturbance observer, quadrotor.

I. INTRODUCTION The applications of unmanned aerial vehicles (UAV), quadrotors to be specific, have increased in many areas. The main attributes of quadrotors lie in hovering, very fast maneuvers, vertical take-off and landing (VTOL), low cost and small size [1]–[4]. Quadrotors have several applications such as military surveillance, exploration, wild fire surveillance, delivery, aerial photography, rescue missions, agriculture, mapping, e.t.c [5]–[7]. The quadrotor dynamic model is underactuated strongly coupled nonlinear system. Implementing robust control for such systems is challenging and have attracted tremendous interest from the field automatic control. In recent years, researchers have proposed various nonlinear control methods to cope with the varying operating conditions of the quadrotor [8], [9].

The dynamic model of a quadrotor is usually in strict feedback form which is appropriate for designing backstepping control [10]–[13]. In order to eliminate the steady state errors, an integral error term is included in the backstepping design [14], [15]. Nonetheless, these papers did not account for the uncertainties and external disturbances. An adaptive backstepping controller was used to addressed the uncertainties [16]–[18]. However, adaptive control methods are limited to systems with constant or slow time varying disturbances. A sliding mode control (SMC) technique is essential for mitigating all kind of bounded uncertainties and disturbances [19]–[21]. A SMC was combined with adaptive backstepping approach for the altitude and the attitude tracking of an octotor [22]. Furthermore, fuzzy logic system (FLS) and neural networks (NN) based controllers have recently caught the attention of researchers [23]–[28].

The singularity issue that may arise in the control of a quadrotor is avoided by using the unit quaternion representation of the quadrotor attitude [29]. In [30], a robust chattering-free SMC was studied for the quaternion model of a hovering quadrotor. A quaternion-based attitude tracking controller was proposed for a quadrotor subjected to external disturbances. Nevertheless, the quaternion representation of the quadrotor may bring ambiguities. A nonsingular fast terminal (NFTSMC) was suggested in [31], [32] to avoid singularities and achieve a tracking control of the quadrotor with actuator faults. A robust adaptive NFTSMC was designed in [33] for attitude and displacement tracking control of a quadrotor subjected to unknown modelling errors. An adaptive NFTSMC based on dynamic
inversion has been designed for trajectory tracking of a quadrotor [34].

A cascade structure consisting of the orientation and the position dynamics of the quadrotor was proposed in [35]. The orientation part consists of the fully actuated subsystem while the position part consists of the underactuated subsystem, which were managed by independent controllers. In [36], the quadrotor model is not only divided into fully actuated and underactuated subsystems, but also included the propeller subsystem to tackle the actuator faults. In [37], a linear parameter varying controller and feedback linearization controller were designed for the attitude and the position of a quadrotor respectively. In [38], an active disturbance rejection control (ADRC) and backstepping SMC have been constructed for the quadrotor attitude and the position respectively. In [39], a backstepping SMC and an internal model controller have been implemented for the quadrotor rotational and translational subsystems. In [40], an adaptive backstepping controller was implemented for the position subsystem, and an adaptive backstepping FTCSMC was developed for the attitude subsystem. In [41], a regular SMC and backstepping SMC with adaptive fault observer have been presented for quadrotor attitude and position respectively. In [42], a backstepping controller and a chattering-free backstepping SMC have been presented for the quadrotor rotational and translational subsystems respectively. In [43], a backstepping control method was designed for the displacement subsystem, while a SMC strategy was utilised to control the attitude. In [44], a RBFNN based SMC was developed for the position subsystem and a robust integral of the signum error (RISE) was designed for the orientation subsystem. In [45], an integral SMC has been designed for the position subsystem and a backstepping SMC has been developed for the rotation subsystem.

In order to obtain an improved control performance and disturbance attenuation, a disturbance observer (DO) can be integrated into the quadrotor control design. A nonlinear DO assimilated with SMC was used to control the hovering state of a quadrotor [46]. A DO based attitude tracking controller was developed in [47] to estimate the Coriolis terms and external disturbances. The actuator faults and the uncertainties hampering the performances of the quadrotor have been tackled using the DO based controllers [48]–[50]. Contrary to the traditional DO which guarantees asymptotic stability, the finite time DO (FDO) can attained a finite time stability [50]–[53].

In the light of the foregoing discussion, we present a new robust control scheme for a quadrotor under time-varying disturbances and model uncertainties. The proposed control strategy is divided into into three subcontrollers: ADIFTSMC for altitude control, RBFNNBC for horizontal position control, and FDOBFTSMC for rotational angle control. The main contributions of the proposed method are presented as follows

1) Contrary to [37]–[43], [45] where the quadrotor control design is divided into two subcontrollers, we divided the proposed control scheme into three subcontrollers: the altitude, the horizontal position and the attitude controllers.

2) Unlike the adaptive NFTSMC [33], ADRC [38], backstepping FTCSMC [40], RISE [44], backstepping SMC [45] implemented for the attitude control, in this paper, a backstepping is combined with NFTSMC and FDO (FDONFTSMC) to control the attitude angles. The advantage of this scheme is that the disturbances are exactly estimated and guarantees the robust trajectory following in finite time.

3) In contrast to Adaptive backstepping [40], backstepping [43], RBFNN based SMC [44], integral SMC [45] developed for the position subsystem, a RBFNN based backstepping is devised to stabilised the horizontal position subsystem and produce the desired Euler angles. The RBFNN identify the uncertain functions together with the external disturbances and the time derivative of the virtual control laws.

4) For the first time, an ADIFTSMC is proposed to control the altitude of the quadrotor in the face of environmental disturbances and dynamic uncertainties. This controller is free from singularities and integrates the advantages of integral and fast terminal SMC. Moreover, it requires no knowledge of the disturbances’ upper-bounds.

5) The finite time stability of the multilevel control, ADIFTSMC-RBFNNBC-FDONFTSMC, has been established using the Lyapunov function.

The remainder of this article is arranged as follows. Section II presents the quadrotor dynamic model. Section III displays the design of the proposed control strategy. Section IV demonstrates the simulation and comparison results. We conclude the paper in Section V.

II. QUADROTOR DYNAMIC MODEL

The schematic representation of the quadrotor is depicted in Fig. 1. The quadrotor dynamic model is a six degrees of freedom underactuated system with four inputs. The dynamic model is given by [19]

\[ \begin{align*}
\ddot{x} &= b_1 \dot{x} + V_x + \Delta_x \\
\ddot{y} &= b_2 \dot{y} + V_y + \Delta_y \\
\ddot{z} &= b_3 \dot{z} + V_z + \Delta_z \\
\ddot{\phi} &= b_4 \dot{\phi}^2 + b_5 \dot{\psi} \dot{\phi} + b_6 \dot{\phi} + a_1 u_2 + \Delta_{\phi} \\
\ddot{\theta} &= b_7 \theta^2 + b_8 \dot{\phi} \dot{\theta} + b_9 \dot{\theta} + a_2 u_3 + \Delta_{\theta} \\
\ddot{\psi} &= b_{10} \psi^2 + b_{11} \dot{\phi} \dot{\psi} + a_3 u_4 + \Delta_{\psi}
\end{align*} \]

where \( x, y, z \) stand for the position of quadrotor centre of gravity, \( \phi, \theta, \) and \( \psi \) denote the pitch, roll and yaw angles respectively, \( \Omega \) is the moment of inertia of the rotor, \( I_x, I_y, I_z \) are the moment of inertia of the quadrotor, \( \Delta_i \) \( i = x, y, z, \phi, \theta, \psi \) represent the time-varying disturbances, \( b_1 = -\frac{k_{ax}}{m}, b_2 = -\frac{k_{ay}}{m}, b_3 = -\frac{k_{az}}{I_x}, b_4 = -\frac{K_{ax}}{I_x}, b_5 = \frac{(I_x - I_y)}{I_x}, b_6 = -\frac{J_{ax}}{I_x}, a_1 = \frac{u_1}{I_x}, a_7 = -\frac{k_{aw}}{I_x}, b_8 = \frac{(I_x - I_y)}{I_x}, b_9 = \frac{J_{aw}}{I_x}, a_2 = \frac{u_2}{I_x} \).
the four propellers, \( \omega \) treated in Fig. 2. We designed ADIFTSMC, RBFNNBC, and FDOBFTSC for the quadrotor vertical position, horizontal position and attitude subsystems respectively.

III. CONTROL DESIGN

In this section, a nonlinear robust adaptive controllers are developed to steer the quadrotor states to track the reference trajectories in the premise of external disturbances and model uncertainties. The control block diagram is illustrated in Fig. 2. We designed ADIFTSMC, RBFNNBC, and FDOBFTSC for the quadrotor vertical position, horizontal position and attitude subsystems respectively.

A. ALTITUDE CONTROL

For movement in the vertical direction, the tracking error is defined by \( e_z = z - z_d \). The DIFTS surface is given as [54]

\[
\begin{align*}
S_z &= \dot{e}_z + \lambda e_t \\
\dot{e}_t &= e_z + B_z e_t^2, \quad \text{with } e_z(0) = 0
\end{align*}
\]

(12)

where \( 0 < q < r, \beta_z > 0, \lambda_z > 0 \) are constants.

Assumption 1: The disturbance \( \Delta_z \) is bounded by \( \Delta_z < (a_0 + a_1|e_z| + a_2|e_t|^2) \),

where \( a_i \) \((i = 0,1,2)\) are unknown constants.

Theorem 1: Consider the altitude subsystem (3), the DIFTS surface (12), if the controller is constructed as

\[
\begin{align*}
V_z &= V_{z0} + V_{z1} + V_{z2} \\
V_{z0} &= -\lambda_z (e_z + B_z e_t^2) + \tilde{z}_d \\
V_{z1} &= -b_3 \dot{z} \\
V_{z2} &= - (\tilde{a}_0 + \alpha_1 |e_z| + \alpha_2 |e_t|^2) \text{sgn}(S_z)
\end{align*}
\]

(13)

(14)

(15)

(16)

\( \tilde{a}_i \) \((i = 1,2,3)\) are the estimate of \( a_i \) \((i = 1,2,3)\) and they are updated online by the following adaptation laws

\[
\begin{align*}
\dot{\tilde{a}}_0 &= \gamma_0 [S_z - \tau \tilde{a}_0] \\
\dot{\tilde{a}}_1 &= \gamma_1 [S_z e_z - \tau \tilde{a}_1] \\
\dot{\tilde{a}}_2 &= \gamma_2 [S_z e_t^2 - \tau \tilde{a}_2]
\end{align*}
\]

(17)

in which \( \gamma_i > 0 \) \((i = 0,1,2)\) are constant adaptation gains, then the altitude closed-loop system is ultimately bounded.

Proof 1: Take the positive definite Lyapunov function as

\[
L_z = \frac{S_z^2}{2} + \frac{1}{2\gamma_0} \tilde{\alpha}_0^2 + \frac{1}{2\gamma_1} \tilde{\alpha}_1^2 + \frac{1}{2\gamma_2} \tilde{\alpha}_2^2
\]

(18)

where \( \tilde{a}_i = a_i - \hat{a}_i \) \((i = 0,1,2)\). Differentiating \( L_z \) with respect to time yields

\[
\begin{align*}
\dot{L}_z &= S_z \dot{S}_z + \frac{1}{\gamma_0} \tilde{\alpha}_0 \dot{\tilde{a}}_0 + \frac{1}{\gamma_1} \tilde{\alpha}_1 \dot{\tilde{a}}_1 + \frac{1}{\gamma_2} \tilde{\alpha}_2 \dot{\tilde{a}}_2 \\
&= S_z (b_3 \dot{z} + V_z + \Delta_z - \tilde{z}_d + \lambda_z (e_z + B_z e_t^2)) \\
&\quad + \frac{1}{\gamma_0} \tilde{\alpha}_0 \dot{\tilde{a}}_0 + \frac{1}{\gamma_1} \tilde{\alpha}_1 \dot{\tilde{a}}_1 + \frac{1}{\gamma_2} \tilde{\alpha}_2 \dot{\tilde{a}}_2
\end{align*}
\]

(19)

Substituting (13)-(16) into (19) gives

\[
\begin{align*}
\dot{L}_z &\leq [\Delta_z - (a_0 + a_1 |e_z| + a_2 |e_t|^2)] |S_z| \\
&\quad + (\tilde{a}_0 + \hat{a}_0 |e_z| + \hat{a}_2 |e_t|^2) |S_z| \\
&\quad + \frac{\tilde{\alpha}_0 (\tilde{a}_0 - \tilde{a}_0) + \tilde{\alpha}_1 (\tilde{a}_1 - \tilde{a}_1) + \tilde{\alpha}_2 (\tilde{a}_2 - \tilde{a}_2)}{\gamma_i} \leq [\Delta_z - (a_0 + a_1 |e_z| + a_2 |e_t|^2)] |S_z| \\
&\quad + \tilde{\alpha}_0 |S_z| - \frac{\tilde{\alpha}_0}{\gamma_0} + \tilde{\alpha}_1 |S_z| - \frac{\tilde{\alpha}_1}{\gamma_1} \\
&\quad + \tilde{\alpha}_2 |S_z| - \frac{\tilde{\alpha}_2}{\gamma_2}
\end{align*}
\]

(20)

Using the adaptive laws (17) and considering Assumption 1, one has

\[
\dot{L}_z \leq -\epsilon |S_z| + \tau \tilde{a}_0 \hat{a}_0 + \tau \tilde{a}_1 \hat{a}_1 + \tau \tilde{a}_2 \hat{a}_2
\]

(21)

where \( \epsilon = (a_0 + a_1 |e_z| + a_2 |e_t|^2) - \Delta_z > 0 \).
The horizontal subsystem can be rewritten as
\[
\dot{\chi} = f(\chi) + V + \Delta
\]  
(26)
where \( \chi = [x\; y T] \), \( f(\chi) = [b_1 \dot{x}\ b_2 \dot{y}]^T, V = [V_x V_y]^T \), \( \Delta = [\Delta_x \; \Delta_y]^T \). Define the tracking error variable \( e_\chi = \chi - \dot{\chi}_d \). The Lyapunov function is selected as \( L_1 = \frac{1}{2} e_\chi^T \dot{e}_\chi \). The derivative of \( L_1 \) with respect to time is thus
\[
\dot{L}_1 = e_\chi^T \dot{\chi} - \dot{\chi}_d
\]  
(27)

The virtual controller is designed as \( \alpha_\chi = \dot{\chi}_d - \Lambda_\chi e_\chi \), with \( \Lambda_\chi = \Lambda_\chi^T > 0 \) is a constant diagonal matrix. The error between \( \dot{\chi} \) and \( \alpha_\chi \) is computed as \( \epsilon_\chi = \dot{\chi} - \alpha_\chi \). Substituting \( \dot{\chi}_d = \alpha_\chi + \Lambda_\chi e_\chi \) into (27) yields
\[
\dot{L}_1 = e_\chi^T (\dot{\chi} - \alpha_\chi - \Lambda_\chi e_\chi) = e_\chi^T (\dot{\chi} - \alpha_\chi) - e_\chi^T \Lambda_\chi e_\chi
\]  
(28)
The time derivative of \( \epsilon_\chi \) yields
\[
\dot{\epsilon}_\chi = \dot{\chi} - \alpha_\chi = f(\chi) + V + \Delta - \dot{\chi}_d
\]  
(29)

By utilising the approximation property of RBFNN, we have
\[
f(\chi) + \Delta - \dot{\chi}_d = W^T \xi(\chi, h_i, c_i) + \varphi
\]  
(30)
where \( W = [W_x^T \; W_y^T]^T \) is the optimal weight vector, \( \xi = \text{diag}(\xi_x, \xi_y) \) is the basis function matrix with \( h_i \) as the width, \( c_i \) is the center, \( \varphi = [\varphi_x, \varphi_y]^T \) is the RBFNN approximation error vector satisfying \( \|\varphi\| \leq \varphi^* \), with \( \varphi^* \) being a positive constant. The second Lyapunov function is define as follows
\[
L_x = \frac{1}{2} e_\chi^T e_\chi + \frac{1}{2} e_\chi^T \dot{\chi}_d + \frac{1}{2} \dot{\tilde{W}}^T \gamma^{-1} \dot{\tilde{W}}
\]  
(31)
where \( \gamma \) is a constant diagonal matrix, \( \tilde{W} = W - \hat{W} \) is estimation error. The time-derivative of \( L_x \) is
\[
\dot{L}_x = e_\chi^T \dot{e}_\chi + e_\chi^T \dot{\chi}_d + \dot{\tilde{W}}^T \gamma^{-1} \dot{\tilde{W}}
\]  
(32)
\[
= -e_\chi^T \Lambda e_\chi + \gamma^T (e_\chi + W^T \xi + \varphi + V) - \dot{\tilde{W}}^T \gamma^{-1} \dot{\tilde{W}}
\]

**Theorem 2:** Considering the horizontal subsystem (26), if the controller (33) and RBFNN weight update rule (34) are designed, the signals in (31) are ultimately bounded.

**Proof 2:** Substituting (33) and (34) into (32), one gets
\[
\dot{L}_x = -e_\chi^T \Lambda e_\chi - e_\chi^T \dot{\varphi} + \tau W^T \dot{\tilde{W}}
\]  
(35)
The following inequalities exist
\[
e_\chi^T \dot{\varphi} \leq \frac{\|e_\chi\|^2}{2} + \frac{\|\varphi\|^2}{2}
\]
\[ \mathbf{W}^T \mathbf{W} = \mathbf{W}^T (\mathbf{W} - \mathbf{W}) \leq \frac{\| \mathbf{W} \|^2}{2} - \frac{\| \mathbf{W} \|^2}{2} \]

\[ \dot{L}_x \leq -\| \Lambda \| e_x^2 - \left( \frac{\| K \|}{2} - \frac{1}{2} \right) \| e_x \|^2 - \tau \frac{\| \mathbf{W} \|^2}{2} + \frac{\| \mathbf{W} \|^2}{2} + \frac{\| \varphi \|^2}{2} \]

\[ \leq -a_x L_x + b_x \]  \hspace{1cm} (36)

where \( a_x = \min(2\| \Lambda \|, 2\left( \frac{\| K \|}{2} - \frac{1}{2} \right), \tau) \), \( b_x = \frac{\| \mathbf{W} \|^2}{2} + \frac{\| \varphi \|^2}{2} \). By integrating (36) and considering the definition of the Lyapunov function (31), one can have

\[ \frac{\| \xi_x \|^2}{2} \leq (L_x(0) - \frac{b_x}{a_x}) \Rightarrow \| \xi_x \| \leq \sqrt{2(L_x(0) - \frac{b_x}{a_x})} \]  \hspace{1cm} (37)

Therefore, the error signals \( e_x, \chi_x \) and \( \mathbf{W} \) are bounded in the compact set \( \vartheta_x \equiv \{ \xi_x : \| \xi_x \| \leq \epsilon_x \} \), with \( \epsilon_x \equiv \sqrt{2(L_x(0) - b_x/a_x)} \).

### C. ATTITUDE CONTROL

The attitude subsystem is as follows

\[
\begin{align*}
\dot{\eta} &= \Omega \\
\dot{\Omega} &= f(\Omega) + gu + \Delta \eta
\end{align*}
\]  \hspace{1cm} (38)

where \( \eta = [\phi, \theta, \psi]^T \) is the orientation of the quadrotor, \( \Omega = [\dot{\phi}, \dot{\theta}, \dot{\psi}]^T \), \( \Delta \eta = [\Delta \phi, \Delta \theta, \Delta \psi]^T \), \( u = [u_1 u_2 u_3]^T \)

\[ f = \begin{bmatrix} b_9 \dot{\phi}^2 + b_5 \psi \dot{\theta} + b_3 \dot{\psi} \\ b_7 \dot{\theta}^2 + b_8 \phi \dot{\psi} + b_6 \dot{\phi} \\ b_{10} \dot{\psi}^2 + b_{11} \phi \dot{\theta} \end{bmatrix} \quad g = \begin{bmatrix} \frac{1}{I_x} & 0 & 0 \\ 0 & \frac{1}{I_y} & 0 \\ 0 & 0 & \frac{1}{I_z} \end{bmatrix} \]

A finite time disturbance observer [55] can be designed for (38) as follows

\[
\begin{align*}
\dot{\mu}_0 &= \zeta_0 + f(\Omega) + gu \\
\zeta_0 &= h_1 R^{1/3} \text{sgn}^{2/3}(\mu_0 - \Omega) + \mu_1 \\
\dot{\mu}_1 &= \zeta_1 \\
\zeta_1 &= h_2 R^{1/2} \text{sgn}^{1/2}(\mu_1 - \zeta_0) + \mu_2 \\
\dot{\mu}_2 &= h_3 \text{Rsgn}(\mu_2 - \zeta_1)
\end{align*}
\]  \hspace{1cm} (39)

where \( \mu_0, \mu_1 \) and \( \mu_2 \) are the estimate of \( \Omega, \Delta \eta \) and \( \dot{\Delta} \eta \), respectively, \( \text{sgn}^p(\chi) = |\chi|^p \text{sgn}(\chi) \). Subsequently, \( \Delta \eta \) can be exactly estimated in finite time.

Define the observation errors as

\[ \pi_1 = \mu_0 - \Omega, \quad \pi_2 = \mu_1 - \Delta \eta, \quad \pi_3 = \mu_2 - \dot{\Delta} \eta \]  \hspace{1cm} (40)

The observation error dynamics is thus

\[
\begin{align*}
\dot{\pi}_1 &= \zeta_0 + f(\Omega) + gu - f(\Omega) - gu - \Delta \eta \\
&= -h_1 R^{1/3} \text{sgn}^{2/3}(\pi_1 + \pi_2) \\
\dot{\pi}_2 &= -h_2 R^{1/2} \text{sgn}^{1/2}(\mu_1 - \zeta_0) + \mu_2 - \dot{\Delta} \eta \\
&= -h_2 R^{1/2} \text{sgn}^{1/2}(\pi_2 - \pi_1 + \pi_3) \\
\dot{\pi}_3 &= -h_3 \text{Rsgn}(\pi_3 - \zeta_1) - \Delta \eta \\
&= -h_3 \text{Rsgn}(\pi_3 - \pi_2 - \dot{\Delta} \eta)
\end{align*}
\]  \hspace{1cm} (41)

From [55], the observation errors will converge to zero in finite time. Then, \( \mu_0 \equiv \Omega, \quad \mu_1 \equiv \Delta \eta, \quad \mu_2 \equiv \dot{\Delta} \eta \). The attitude tracking errors are as follows

\[ \begin{align*}
\dot{e}_\eta &= \eta - \eta_d \\
\dot{\epsilon}_\eta &= \Omega - \dot{\eta}_d
\end{align*} \]  \hspace{1cm} (42)

where \( e_\eta = [\phi_\theta, \theta_\eta, \psi_\eta]^T \), \( \eta_d = [\phi_d, \theta_d, \psi_d]^T \) is the vector of desired attitude angles. The NFTSMC surface is expressed as [56]

\[ S = e_\eta + q \text{sgn}(e_\eta) + \Delta \text{sgn}^r(\dot{\epsilon}_\Omega) \]  \hspace{1cm} (43)

where \( S = [S_\phi, S_\theta, S_\psi]^T \),

\[ \text{sgn}^r(e_\eta) = \begin{bmatrix} |e_\phi| \text{sgn}(e_\phi), |e_\theta| \text{sgn}(e_\theta), |e_\psi| \text{sgn}(e_\psi) \end{bmatrix}^T \]

\[ \text{sgn}^r(\dot{\epsilon}_\eta) = \begin{bmatrix} |e_\phi| \text{sgn}(e_\phi), |e_\theta| \text{sgn}(e_\theta), |e_\psi| \text{sgn}(e_\psi) \end{bmatrix}^T \]

\[ q > 0, \quad \Lambda > 0 \] are design parameters, \( q > 2 \) and \( q > r \).

**Theorem 3:** Consider the attitude subsystem (38) with control law (44)-(47) for reaching and staying on the surface (43), and supposed that the parameters of the FDO (39) are properly selected so that the disturbances can be exactly estimated, then the attitude signals track the desired angles in finite time.

\[ u = u_e + u_d + u_r \]  \hspace{1cm} (44)

\[ u_e = g^{-1}[-(f - \dot{\eta}_d) - |\dot{e}_\eta|^{2-r} I + a q |e_\eta|^{q-1} \text{sgn}(\dot{e}_\eta) - |\dot{\epsilon}_\eta|^{1-r} e_\eta - \mu_1] \Lambda r \]  \hspace{1cm} (45)

\[ u_d = g^{-1} \mu_1 \]  \hspace{1cm} (46)

\[ u_r = g^{-1}[-K S - \gamma \text{sgn}^r(\dot{\epsilon}_\Omega)] \]  \hspace{1cm} (47)

where \( K = K^T > 0 \) and \( \gamma = \gamma^T > 0 \) are constant matrices, \( 0 < r < q < 1 \), \( I \) is identity matrix, \( |S|^q = |S_\phi|^q, |S_\theta|^q, |S_\psi|^q \), \( \mu_1 \) is the estimate of \( \Delta \eta \) computed by the FDO, \( |e_\phi| = \text{diag} \{ |e_\phi|, |e_\theta|, |e_\psi| \} \), \( |\dot{\epsilon}_\eta| = \text{diag} \{ |\dot{\epsilon}_\phi|, |\dot{\epsilon}_\theta|, |\dot{\epsilon}_\psi| \} \). The virtual control input is designed as

\[ \dot{\Omega} = -Ce_\eta + S + \dot{\eta}_d \]  \hspace{1cm} (48)

where \( C = C^T > 0 \) is a constant diagonal matrix.

**Proof 3:** Consider a positive Lyapunov function as follows:

\[ L_\eta = \frac{e_\eta^T e_\eta}{2} + \frac{S^T S}{2} \]  \hspace{1cm} (49)

By differentiating \( L_\eta \) with respect to time, we have

\[ \dot{L}_\eta = e_\eta^T \dot{e}_\eta + S^T \dot{S} \]

\[ = e_\eta^T (\Omega - \dot{\eta}_d) + S^T [\dot{e}_\eta + q \text{sgn}(e_\eta)|^{q-1} \dot{e}_\eta + \Lambda \text{sgn}^r(\dot{\epsilon}_\eta)|^{q-1} \dot{\epsilon}_\eta] \]  \hspace{1cm} (50)
Using the virtual controller (48), one has

\[
\dot{L}_\eta = -e^T_\eta C e_\eta + S^T \left[ e_\eta + \dot{e}_\eta + \varrho q |e_\eta|^{q-1} \dot{e}_\eta + \Delta r |\dot{e}_\eta|^{r-1} \left[ f(\Omega) + gu + \Delta_\eta - \tilde{u}_d \right] \right]
\]

(51)

Substituting the overall control law (44)-(47) into (51), we have

\[
\dot{L}_\eta = -e^T_\eta C e_\eta + S^T \left[ e_\eta + \dot{e}_\eta + \varrho q |e_\eta|^{q-1} \dot{e}_\eta + \frac{3}{q} q |\dot{e}_\eta|^q - \frac{1}{q-1} \frac{1}{S} \frac{S}{e_\eta} \frac{S}{r}^{q/r} \text{sgn}(S) \right]
\]

(52)

Note that the FDO accurately estimate \( \Delta_\eta \) in finite time, e.g., \( \mu_1 \equiv \Delta_\eta \), then

\[
\dot{L}_\eta \leq -\|C\| \|e_\eta\|^2 - \|\varrho q |\dot{e}_\eta|^{q-1} \| \|K\| S^2 + \|S\|^{r/q+1} \|Y\| \|
\]

(53)

Considering the inequality \( \|S\|^{r/q+1} \geq \|S\|^2 \), \( \|S\| \rightarrow 0 \) and letting \( a_\eta = \{2\|C\|, 2\|\varrho q |\dot{e}_\eta|^{q-1} \| \|K\| - \|T\| \} \), we get

\[
\dot{L}_\eta \leq -a_\eta e^{-a_\eta t} L_\eta
\]

(54)

By integrating (54), we can have

\[
L_\eta \leq L_\eta(0) e^{-a_\eta t}
\]

(55)

IV. SIMULATION

In this section, a comparison of the simulations is carried out to demonstrate the effectiveness of the proposed control scheme. The efficacy of the proposed ADIFTSMC-RBFNNBC-FDOBNFTSMC has been compared with the controllers in [40] where an adaptive backstepping (AB) and adaptive backstepping FTSMC (ABFTSMC) were designed for the position and the attitude subsystems respectively, and in [45] where integral SMC (ISMC) and Backstepping SMC (BSMC) were utilised for the attitude and the position subsystems respectively.

The quadrotor parameters are obtained from [19]. The initial condition of each of the quadrotor states is 0.001. The time varying external disturbances are modelled as sinusoidal functions \( \Delta_i = 2 \sin(0.8t) \) \((i = x, y, z, \phi, \theta, \psi) \). The control design parameters are given in Table 1. The desired signals are given by

\[
x_d = \begin{cases} 1 & 0 \leq t \leq 4 \\ 0.8 & 4 < t \leq 8 \end{cases}, \quad y_d = \begin{cases} 0.8 & 0 \leq t \leq 8 \\ 0.3 & 8 < t \leq 15 \end{cases}, \quad z_d = \sin(t), \quad \psi_d = \sin(0.8t)
\]

| Parameters | Values |
|------------|--------|
| \( \lambda_x, \beta_x, \gamma, \omega_1, \omega_2 \) | (50, 0.1, 0.05, 0.02, 0.02) |
| \( \Delta_x, \Delta_y, \Delta_z \) | diag(5, 0.05, 0.15) |
| \( h_\psi, c_\psi \) | diag(5, 0.05, 0.15) |
| \( c_x, q_1, r \) | diag(5, 0.05, 0.15) |
| \( h_1, h_2, h_3, L \) | diag(5, 0.05, 0.15) |

TABLE 1. Control design parameters.
The simulation results are presented in Figs. 3-7. From the position responses shown in Fig. 3, it can be inferred that the ADIFTSMC/RBFNNBC provide the highest trajectory tracking precision and the quickest dynamical responses compared to both AB and ISMC. Moreover, the tracking error responses in Fig. 4 show that the ADIFTSMC/RBFNNBC converges to zero in the shortest time.

In Fig. 5, the comparison between the FDOBNFTSMC, BSMC and ABFTSMC has been presented. It can be observed that in the presence of the disturbances, the attitude angles followed the reference angles with greater accuracy under the FDOBNFTSMC. Contrary to BSMC and ABFTSMC, the FDOBNFTSMC can exactly estimate and compensate the time varying disturbances in finite time. The tracking responses of the yaw angle are demonstrated in Fig. 6. This figure illustrates that the tracking errors converge to zero in the shortest time under FDOBNFTSMC. The control inputs are presented in Fig. 7.

V. CONCLUSION

In this article, a novel multilevel control of a quadrotor has been developed to guarantee the robust trajectory following in the presence of model uncertainties, system coupling and environmental disturbances. The nonlinear control method is divided into three subcontrollers. Firstly, an ADIFTSMC is developed for the altitude subsystem to keep the quadrotor flying at the desired height irrespective of the external disturbances. Then, the horizontal position subsystem is controlled with a RBFNNBC to stabilise it and generate the reference Euler angles for the attitude subsystem. The RBFNN approximates the uncertain dynamics, external disturbances and the derivatives of the virtual control inputs. Subsequently, a FDOBNFTSMC has been developed for the attitude subsystem. Numerical simulations have shown the superiority of the proposed multilevel control structure, ADIFTSMC-RBFNNBC-FDOBNFTSMC, over some existing methods. The finite time stability of the overall system is presented based on the Lyapunov theory. In future work, the effectiveness of the proposed scheme will be investigated in real-time control of a quadrotor.

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