Radiative Seesaw in Minimal 3-3-1 Model

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Abstract

We study the neutrino sector in a minimal $SU(3)_L \times U(1)_X$ model, in which its mass is generated at one-loop level with the charged lepton mass, and hence there exists a strong correlation between the charged-lepton mass and the neutrino mass. We identify the parameter region of this model to satisfy the current neutrino oscillation data as well as the constraints on lepton flavor violating processes. We also discuss a possibility to explain the muon anomalous magnetic moment.

Keywords: 3-3-1, radiative neutrino masses, lepton flavor violations

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I. INTRODUCTION

The flavor problem can be one of the biggest challenging issues to be resolved, since the standard model (SM) does not have any theoretical sources (especially) why the number of family for each fermion sector (quarks and leptons) is three. One of the reasonable interpretations is to extend the gauge sector $SU(2)_L \times U(1)_Y$ to $SU(3)_L \times U(1)_X$, so called 3-3-1 model, in which the origin of three family is coming from the number of $SU(3)$ color of quarks (that has three) due to the gauge anomaly cancellation [1, 2]. Because of larger gauge group comparing to the SM one, there are several variations of models extending the Higgs sector [3–7] and revisited models to reanalyze with current experimental data [8–24].

On the other hand, explaining the current neutrino oscillation data and dark matter (DM) candidate might be done by physics beyond SM. It is known that in recent radiative seesaw models they can be only explained simultaneously but also correlated each other. It means that neutrinos do not directly interact with the SM Higgs field but with a DM candidate. As a result, minuscule neutrino masses can be naturally realized. This is because a vast literature has recently arisen along thought of this subject [25–36].

In this paper, we combine the 3-3-1 model and radiative seesaw model based on a minimal model in Ref. [5], in which we do not impose any additional discrete symmetry. Then we can generate the neutrino mass at one-loop level. Moreover since our model has a strong correlation between the charged lepton sector and the neutrino sector due to the same origin of these masses, it may be worth analyzing the neutrino oscillation data as well as lepton flavor violating processes and so on.

This paper is organized as follows. In Sec. II, we show our model including Higgs potential. In Sec. III, we analyze lepton sector and show how to correlate the charged-lepton masses and neutrino masses. Then we also show to compute lepton flavor violating processes and muon anomalous magnetic moment. In Sec. IV, we perform parameter scan to identify allowed parameter regions. We conclude in Sec. V.
II. MODEL SETUP

We discuss a possibility of a one-loop induced radiative seesaw model in the context of 3-3-1 model in [5]. The particle contents are shown in Tab. I. We introduce a gauge triplet fermion $L_L = (\nu_L, e_L, e_R^c)$ with $U(1)_X = 0$. For new bosons, we introduce $SU(3)_L$ sextet scalars $S$ with $U(1)_X = 0$, $SU(3)_L$ triplet scalars $(\eta, \rho, \chi)$ with $U(1)_X = (0, 1, -1)$, respectively. The renormalizable Lagrangian for Lepton Yukawa sector, and the scalar potential under these assignments are given by

$$\mathcal{L}_Y = y_{\ell_L} \sum_{i,j,k=1-3} \bar{L}_{Li}(L_L)^i_j \eta_k \epsilon^{ijk} + y_{\ell_L} \text{Tr}[\bar{L}_L S(L_L)^c] + \text{h.c.}$$

(II.1)

$$\mathcal{V} = m^2_\eta |\eta|^2 + m^2_\rho |\rho|^2 + m^2_\chi |\chi|^2 + m^2_S \text{Tr}[|S|^2]$$

$$+ \lambda_1 |\eta|^4 + \lambda_2 |\rho|^4 + \lambda_3 |\chi|^4 + \lambda_4 \text{Tr}[|S|^2] + \lambda_5 \text{Tr}[|S|^4] + \lambda_6 |\eta|^2 |\rho|^2 + \lambda_7 |\eta|^2 |\chi|^2 + \lambda_8 |\chi|^2 |\rho|^2$$

$$+ \lambda_9 |\eta|^2 \text{Tr}[|S|^2] + \lambda_{10} |\rho|^2 \text{Tr}[|S|^2] + \lambda_{11} |\chi|^2 \text{Tr}[|S|^2] + \lambda_{12} |\eta|^2 + \lambda_{13} |\chi|^2 + \lambda_{14} |\rho|^2$$

$$+ (f_1 \sum_{i,j,k=1-3} \epsilon^{ijk} \eta \rho \chi + \text{h.c.}) + (f_2 \rho^T S^\dagger \chi + \text{h.c.}) + (f_3 \eta^T S \eta + \text{h.c.})$$

$$+ (f_4 \sum_{i,j,k=1-3} \epsilon^{ijk} \epsilon^{lmn} S_{il} S_{jm} S_{kn} + \text{h.c.}) + (f_5 (\eta^\dagger \rho)(\eta^\dagger \chi) + \text{h.c.}) + (f_6 \sum_{i,j,k=1-3} \epsilon^{ijk} S_{i6} \rho j \chi k \eta^* + \text{h.c.})$$

$$+ f_7 \text{Tr}[S^\dagger S \eta^* \eta^T] + f_8 \text{Tr}[S^\dagger S \rho^* \rho^T] + f_9 \text{Tr}[S^\dagger S \chi^* \chi^T] + (f_{10} \sum_{i,j,k=1-3} \epsilon^{ijk} \epsilon^{lmn} S_{il} S_{jm} \eta_k \eta_n + \text{h.c.})$$

(II.2)

|                     | Lepton Fields | Scalar Fields |
|---------------------|--------------|---------------|
| $L_L = (\nu_L, e_L, e_R^c)$ | $S$ | $\eta$ | $\rho$ | $\chi$ |
| $SU(3)_L$           | 3            | 6             | 3         | 3         | 3         |
| $U(1)_X$            | 0            | 0             | 1         | -1        |

TABLE I: Contents of lepton and scalar fields and their charge assignment under $SU(3)_L \times U(1)_X$, where the index of the generation are abbreviated.
where \( y_{\ell_1} \) is an anti-symmetric 3 by 3 matrix and \( y_{\ell_2} \) is a symmetric one. Here the scalar fields can be parameterized as

\[
S = \begin{pmatrix}
\sigma_0^0 & h^-_2 & h^+_1 \\
h^-_2 & H^-_1 & \sigma_0^2 \\
h^+_1 & \sigma_0^2 & H^+_2 \\
\end{pmatrix}, \quad
\eta = \begin{pmatrix}
\eta^0 \\
\eta^-_1 \\
\eta^+_2 \\
\end{pmatrix}, \quad
\rho = \begin{pmatrix}
\rho^+ \\
\rho^0 \\
\eta^{++} \\
\end{pmatrix}, \quad
\chi = \begin{pmatrix}
\chi^- \\
\chi^- \\
\chi^0 \\
\end{pmatrix}, \quad
\sigma_1^0 = \frac{\sigma_{1R} + i\sigma_{1I}}{\sqrt{2}},
\]

(II.3)

\[
\sigma_2^0 = \frac{v_\sigma + \sigma_{2R} + i\sigma_{2I}}{\sqrt{2}}, \quad
\eta^0 = \frac{v_\eta + \eta_{R} + i\eta_{I}}{\sqrt{2}}, \quad
\rho^0 = \frac{v_\rho + \rho_{R} + i\rho_{I}}{\sqrt{2}}, \quad
\chi^0 = \frac{v_\chi + \chi_{R} + i\chi_{I}}{\sqrt{2}}.
\]

(II.4)

Here \( v = \sqrt{v_\sigma^2 + v_\eta^2 + v_\rho^2} \simeq 246 \text{ GeV} \) is the vacuum expectation value (vev), and we have assumed that the vev of \( \sigma_0^0 \) is zero, which is unlike the original model discussed in [5]. The scale of \( v_\chi \), which breaks \( SU(3)_L \) symmetry, is assumed to be of \( \mathcal{O}(\text{TeV}) \).

III. LEPTON SECTOR

A. Charged lepton sector

The charged lepton masses can be generated by the terms \( y_{\ell_1} \sum_{i,j,k=1-3} (\bar{L}_i)(L_L)_{ij} \eta_k^* \epsilon^{ijk} \) and \( y_{\ell_2} \text{Tr}[\bar{L}_L S(L_L)]^c \) after the electroweak symmetry breaking, as can be seen in Eq. (II.1). The resulting mass matrix can be written and diagonalized by

\[
(\bar{e}_L)_{a}(m_{\ell})_{ab}(e_R)_{b} \equiv (\bar{e}_L)_{a} \left[ \frac{(y_{\ell_1})v_\eta}{\sqrt{2}} + \frac{(y_{\ell_2})v_\sigma}{\sqrt{2}} \right]_{ab} (e_R)_{b} = (\bar{e}_L)_{a}(V_{eL})_{ai}(m_{\ell}^{\text{diag}})_{i}(V_{eR})_{ib}(e_R)_{b},
\]

(III.1)

\[
\sum_{k=1}^{3} \left[ \frac{(y_{\ell_1})v_\eta}{\sqrt{2}} + \frac{(y_{\ell_2})v_\sigma}{\sqrt{2}} \right]_{ak} \left[ \frac{(y_{\ell_1})v_\eta}{\sqrt{2}} + \frac{(y_{\ell_2})v_\sigma}{\sqrt{2}} \right]_{bk}^{*} = (V_{eL})_{ai}|m_{\ell}^{\text{diag}}|_{i}^{2}(V_{eL})_{ib},
\]

(III.2)

where \( y_{\ell_1} \) is the ant-symmetric matrix and \( y_{\ell_2} \) is the symmetric matrix. \( |m_{\ell}^{\text{diag}}| = (|m_{e}|, |m_{\mu}|, |m_{\tau}|) = (0.5 \text{ MeV}, 105.6 \text{ MeV}, 1777 \text{ MeV}) \). Each of \( V_{eL} \) and \( V_{eR} \) is the unitary matrix to diagonalize the charged leptons.
FIG. 1: An example of one-loop diagrams contributing to neutrino mass.

B. Neutrino sector

Through the following Yukawa interactions,

\[
\mathcal{L} \supset y_{\ell_1} (\bar{\nu}_L e_L^c - \bar{e}_L \nu_L^c) \eta_2 + y_{\ell_2} (-\bar{\nu}_L e_R + e_R^c \nu_L^c) \eta_1^+ \\
+ y_{\ell_3} (\bar{\nu}_L e_L^c + \bar{e}_L \nu_L^c) h_2 + y_{\ell_4} (\bar{\nu}_L e_R + e_R^c \nu_L^c) h_1^+,
\]

(III.3)

the active neutrino mass matrix $m_\nu$ is induced at one-loop level (see Fig. 1 for an example diagram), which is given by

\[
-(M_{\ell}^{th})_{ab} \approx \frac{y_{\ell_1} (m_\ell^+ + m_\ell^c) y_{\ell_1}^T}{(4\pi)^2} \left[ \ln \left( \frac{m_{\eta_2}^2}{m_{\eta_2}^+} \right) \right] \\
- \frac{y_{\ell_2} (m_\ell^+ + m_\ell^c) y_{\ell_2}^T}{(4\pi)^2} \left[ \ln \left( \frac{m_{h_2}^2}{m_{h_2}^+} \right) \right] \\
- \frac{y_{\ell_1} m_\ell^c y_{\ell_2}^T + y_{\ell_2} m_\ell^c y_{\ell_1}^T}{2(4\pi)^2} \left[ \ln \left( \frac{m_{\eta_1 h_1}^2}{m_{\eta_1 h_1}^+} \right) \right] \\
- \frac{y_{\ell_2} m_\ell^c y_{\ell_1}^T + y_{\ell_1} m_\ell^c y_{\ell_2}^T}{2(4\pi)^2} \left[ \ln \left( \frac{m_{\eta_1 h_2}^2}{m_{\eta_1 h_2}^+} \right) \right].
\]

(III.4)

Note here that the mass insertion approximation has been used, that is, $(M_+)^2 = (M_+)^2_{\text{diag}} + (M_+)^2_{\text{off-diag}}$, with $(M_+)^2_{\text{diag}} \gg (M_+)^2_{\text{off-diag}} \equiv \delta m_{f_i f_j}^{+2}$, $(f_i, f_j = \eta_i, h_j)$, where $M_+$ is the singly charged boson mass matrix, and

\[
\delta m_{\eta_2}^{+2} = \sqrt{2} f_3 v_\eta + (f_5 v_\rho v_\chi)/2, \quad \delta m_{h_2}^{+2} = 3\sqrt{2} f_4 v_\sigma, \\
\delta m_{\eta_1 h_1}^{+2} = (4f_{10} + f_7)v_\eta v_\eta/(2\sqrt{2}), \quad \delta m_{\eta_2 h_1}^{+2} = f_3 v_\eta + (f_6 v_\rho v_\chi)/(2\sqrt{2}), \\
\delta m_{\eta_1 h_2}^{+2} = f_3 v_\eta + (f_6 v_\rho v_\chi)/(2\sqrt{2}), \quad \delta m_{\eta_2 h_2}^{+2} = (4f_{10} + f_7)v_\sigma v_\eta/(2\sqrt{2}).
\]

(III.5)
Neutrino mass matrix can be diagonalized by a unitary matrix $U_\nu$ as $m_\nu = U_\nu m_\nu^{\text{diag}} U_\nu^T$, and hence the MNS matrix $U_{\text{MNS}}$ is defined as

$$U_{\text{MNS}} = V_{eL} U_\nu. \tag{III.6}$$

### C. Lepton Flavor Violations

Here we discuss the lepton flavor violating processes in our model, which can be induced through the mixings among singly charged bosons in Eq. (III.3). The branching ratio of the lepton flavor violating decays are given by

$$\text{BR}(\ell_i \rightarrow \ell_j \gamma) = \frac{12\pi^2}{m_i^2 G_F^2} \left[ |A_{Lji}|^2 + |A_{Rji}|^2 \right], \tag{III.7}$$

where

$$A_{Lji} = m_j \left[ G_{L}^{(ii)}(\eta_2, h_2) + G_{L}^{(iii+)}(\eta_2, h_2) \right] + m_i \left[ G_{L}^{(i)}(\eta_1, h_1) + G_{L}^{(iii+)}(\eta_1, h_1) \right], \tag{III.8}$$

$$A_{Rji} = m_j \left[ G_{R}^{(i)}(\eta_1, h_1) + G_{R}^{(iii+)}(\eta_1, h_1) \right] + m_i \left[ G_{R}^{(ii)}(\eta_2, h_2) + G_{R}^{(iii+)}(\eta_2, h_2) \right], \tag{III.9}$$

$$G_{L(R)}^{(i)}(\eta_1, h_1) \approx \frac{\delta m_{ij}^2}{12(4\pi)^2 m_{\eta_1}^2 m_{h_1}^2}, \tag{III.10}$$

$$G_{L(R)}^{(ii)}(\eta_2, h_2) \approx \frac{\delta m_{ij}^2}{12(4\pi)^2 m_{\eta_2}^2 m_{h_2}^2}, \tag{III.11}$$

$$G_{L(R)}^{(iii+)}(\eta_1, h_1) \approx G_{L(R)}^{(iii+)}(\eta_2, h_2) \approx \frac{1}{12(4\pi)^2} \left[ \frac{(y_{\ell_a} y_{\ell_b})'}{m_{\eta_2}^2} + \frac{(y_{\ell_a} y_{\ell_b})'}{m_{h_2}^2} \right], \tag{III.12}$$

with $(y_{\ell_a} y_{\ell_b})' \equiv V_{eR} y_{\ell_a} y_{\ell_b} V_{eR}^\dagger$, and $(y_{\ell_a} y_{\ell_b})' \equiv V_{eL} y_{\ell_a} y_{\ell_b} V_{eL}^\dagger$, with $(a, b) = 1, 2$. The upper indices of $G$, respectively, represent a difference among the singly charged bosons exchanging processes. Therefore, $(i)$ is the $\eta_1^+$ and $h_1^+$ mixing process, $(ii)$ is the $\eta_2^+$ and $h_2^+$ mixing one, $(iii)$ is the $\eta_1^+$ no mixing one, $(iv)$ is the $\eta_2^+$ no mixing one, $(v)$ is the $h_1^+$ no mixing one, and $(vi)$ is the $h_2^+$ no mixing one.

### D. Muon Anomalous Magnetic Moment

The muon anomalous magnetic moment, so-called the muon $g - 2$, has been measured at Brookhaven National Laboratory. The current average of the experimental results is given
\[ a_{\mu}^{\text{exp}} = 11659208.0(6.3) \times 10^{-10}. \]  

(III.13)

It has been known that there is a discrepancy from the SM prediction by 3.2σ \[38\] to 4.1σ \[39\]:

\[ \Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (29.0 \pm 9.0 \text{ to } 33.5 \pm 8.2) \times 10^{-10}. \]  

(III.14)

The contribution by the singly charged bosons is evaluated as

\[ \Delta a_{\mu} = -\frac{m_{\mu}^2}{2} \left[ G_{L}^{(i)}(\eta_1, h_1) + G_{R}^{(i)}(\eta_1, h_1) + G_{L}^{(ii)}(\eta_2, h_2) + G_{R}^{(ii)}(\eta_2, h_2) 
+ 2G_{L}^{(i\mu+v)}(\eta_1, h_1) + 2G_{L}^{(i\mu+v)}(\eta_2, h_2) \right]_{i=j=\mu}. \]  

(III.15)

IV. NUMERICAL ANALYSIS

For simplicity, we assume the normal hierarchy for the neutrino mass spectrum with one zero mass eigenvalue, and parameterize the neutrino mass matrix as

\[ \mathcal{M}_{\nu}^{\text{exp}} = U_{\text{PMNS}} \text{diag}(0, m_{\nu_2}, m_{\nu_3}) U_{\text{PMNS}}^{T}, \]  

(IV.1)

with the (real) standard form of the PMNS matrix,

\[ U_{\text{PMNS}} \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \]  

(IV.2)

with \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \cos \theta_{ij} \). Then, one finds a relation between \( \mathcal{M}_{\nu}^{\text{th}} \) and \( \mathcal{M}_{\nu}^{\text{exp}} \) given by

\[ \mathcal{M}_{\nu}^{\text{exp}} = V_{eL}^{\dagger} \mathcal{M}_{\nu}^{\text{th}} V_{eL}, \]  

(IV.3)

where we also define \( V_{eL(R)} \) as the standard parametrization as analogy to the MNS matrix,

\[ V_{eL(R)} \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{eL(R)23} & s_{eL(R)23} \\ 0 & -s_{eL(R)23} & c_{eL(R)23} \end{bmatrix} \begin{bmatrix} c_{eL(R)13} & 0 & s_{eL(R)13} \\ 0 & 1 & 0 \\ -s_{eL(R)13} & 0 & c_{eL(R)13} \end{bmatrix} \begin{bmatrix} c_{eL(R)12} & s_{eL(R)12} & 0 \\ -s_{eL(R)12} & c_{eL(R)12} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \]  

(IV.4)
with \( s_{eL(R)ij} = \sin \theta_{eL(R)ij} \) and \( c_{eL(R)ij} = \cos \theta_{eL(R)ij} \).

By varying the input parameters in the following ranges,

\[
1 \text{ GeV} \leq (v_\eta, v_\sigma) \leq 100 \text{ GeV}, \quad 0.1 \text{ GeV}^2 \leq \delta m_{\eta^+}^2 \leq 10 \text{ GeV}^2, \quad 0.1 \text{ GeV}^2 \leq \delta m_{\eta^+ h_1}^2 \leq 10 \text{ GeV}^2, \nonumber
\]
\[
100 \text{ GeV} \leq m_{\eta^+} \leq 1000 \text{ GeV}, \quad -1 \leq (s_{eL(R)ij}, c_{eL(R)ij}) \leq 1, \tag{IV.5}
\]

we perform parameter scan to reproduce the neutrino oscillation data at 95% confidence level [40],

\[
0.2911 \leq s_{12}^2 \leq 0.3161, \quad 0.5262 \leq s_{23}^2 \leq 0.5485, \quad 0.0223 \leq s_{13}^2 \leq 0.0246, \nonumber
\]
\[
0.00857 \text{ eV} \leq m_{\nu_2} \leq 0.00878 \text{ eV}, \quad 0.0496 \text{ eV} \leq m_{\nu_3} \leq 0.0507 \text{ eV}. \tag{IV.6}
\]

In the parameters scan, we also impose the current experimental bounds on the lepton flavor violating processes [41, 42]:

\[
\text{BR}(\mu \to e\gamma) < 5.7 \times 10^{-13}, \quad \text{BR}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}, \quad \text{BR}(\tau \to e\gamma) < 3.3 \times 10^{-8}. \tag{IV.7}
\]

We show some representative figures of our results which simultaneously satisfy the neutrino oscillation data and the constraints from the LFV processes. Here we have examined \(10^7\) sampling points to search for our allowed parameters. Fig. 2 shows the allowed points in terms of \(s_{eL12}\) and \(s_{eL23}\) to simultaneously satisfy the neutrino oscillation data and the LFV constraints, except the red points which predict too large LFV rates. We see that that LFV constraints are not so stringent. Fig. 3 shows the allowed points to satisfy the current LFV bounds in terms of \(m_{\eta^+}\), along with the future reach in Mu2e experiments [43] at around \(\text{BR}(\mu \to e\gamma) = (2.5 - 6) \times 10^{-17}\) (horizontal lines). For the allowed point, we have also calculated the contribution to the muon \(g - 2\) from charged bosons. Fig. 4 shows the predictions in terms of \(m_{\eta^+}\). We have found that the contribution is negative and cannot reconcile the discrepancy between the experimental result and the SM prediction. See [9] for the contribution from extra gauged bosons, which is found to be positive.

V. CONCLUSIONS

We have proposed a radiative seesaw model with a \(SU(3)_C \times SU(3)_L \times U(1)_X\) gauge symmetry, in which the neutrino mass is induced through one-loop radiative corrections
FIG. 2: Parameter scan in terms of $s_{eL12}$ and $s_{eL23}$ to satisfy the LFV constraints and neutrino oscillation data, where the red points do not satisfy the LFV constraints.

FIG. 3: Allowed points to satisfy the $\mu \to e\gamma$ constraint in terms of $m_{\eta_1^\pm}$, where the horizontal lines denote the future reach by Mu2e experiments at around $\text{BR}(\mu \to e\gamma) = (2.5 - 6) \times 10^{-17}$.

with the charged lepton mass. As a result, there is a strong correlation between the charged lepton and neutrino masses, and it is nontrivial if the current neutrino oscillation data are reproduced. In the model, the LFV processes are also induced via one-loop quantum corrections. We have performed general parameter scan, and found that a large portion of parameter space can simultaneously satisfy the current neutrino oscillation data and the constraints on the LFV processes. The parameter region we have found can be partly tested by future Mu2e experiments. We have also calculated contributions to the muon anomalous magnetic moment from charged scalar particles in our model, and found that the contributions are not significant.
FIG. 4: Muon anomalous magnetic moment in terms of $m_{\eta_1}$ to satisfy the LFV constraints and neutrino oscillation data, where the red points do not satisfy the LFV constraints. One can see that all the absolute values of data is less than $10^{-10}$. Since the observed deviation from the SM prediction is positive and of $\mathcal{O}(10^{-9})$, the other contributions such as extra gauged bosons could be expected.

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