Quarkonium dissociation in the presence of a small momentum space anisotropy

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Abstract

We consider the dissociation of heavy quarkonium in a medium close to thermal equilibrium but with a small momentum space anisotropy. Dissociation is defined to take place when the width of the ground state equals its binding energy. We show that if the anisotropic medium is obtained isentropically from the equilibrium one, then to first order in the anisotropy parameter the dissociation temperature remains unchanged. If, in contrast, the non-equilibrium system has a smaller entropy density than the equilibrium one, then the dissociation temperature increases with respect to the isotropic case, by up to $\sim 10\%$ for modest anisotropies.
The physical picture of quarkonium dissociation within a deconfined medium [1] has undergone a slight refinement within the last couple of years. The heavy quark and antiquark are bound together by almost static (off-shell) gluons, and the issue boils down to how the gluon self-energy looks at high temperatures. It turns out that the gluon self-energy has both a real and an imaginary part. The real part is known as Debye screening, while the imaginary part is referred to as Landau damping. Traditionally, it was thought that quarkonium dissociates when Debye screening becomes so strong that the corresponding Schrödinger equation supports no more bound state solutions. The new suggestion is that quarkonium effectively dissociates already at a lower temperature [2, 3], when the binding energy is non-zero but overtaken by the Landau-damping induced thermal width [2]. (Finite width phenomena have also been discussed in a phenomenological approach [4].)

More quantitatively, let us denote by $M$ the heavy quark pole mass, by $T$ the temperature, by $r$ the quarkonium Bohr-type radius, and by $g$ the QCD coupling constant ($\alpha_s = g^2/4\pi$). Then, as is familiar from the hydrogen atom, $r \sim 1/g^2 M$. If quarkonium dissociates through Debye screening, the dissociation temperature is parametrically determined by $r m_D \sim 1$, where $m_D \sim gT$ is the Debye mass, leading to $T \sim gM$. However, the imaginary part of the real-time static potential [2] overtakes the binding energy already at smaller temperature, with $m_D r < 1$; more precisely, in the range $g^2 M < T < gM$ [3]. By formulating the problem in an effective theory framework, this estimate was refined to $T \sim g^{4/3} M$ in ref. [5], and further to $T \sim g^{4/3} \ln^{-1/3} (1/g) M$ in ref. [6]. The last estimate was independently reproduced with another approach in ref. [7].

To summarise, the Debye-screened picture of quarkonium dissociation seems, in retrospective, overly conservative. To rephrase this more polemically, one can argue that Debye screening is not the dominant mechanism responsible for dissociating the quarkonium state in a thermal environment [7].

All of the considerations above refer to the theoretically transparent situation that the medium is in thermal equilibrium. For phenomenological applications it may be interesting to study non-equilibrium environments as well. For instance, in ref. [8] the case was considered that the quarkonium system has a non-zero velocity with respect to the medium. Here, in contrast, we inspect a system in which quarkonium is at rest but the “hard” partonic degrees of freedom have distribution functions in momentum space which contain an anisotropy, or a preferred direction. This situation could emerge as a result of Bjorken expansion at the early stages of a central heavy ion collision, and it has therefore been extensively studied in the literature recently, for instance in the context of plasma instabilities ([9] and references therein) as well as for observables such as heavy quark energy loss [10], heavy quark momentum broadening [11], photon production [12], dilepton production [13], jet quenching [14, 15], and, most relevantly for us, the heavy quark potential [16] and solutions of the Schrödinger equation [17].
2. Strictly speaking, the suggestion to give the hard modes an anisotropic momentum
distribution does not lead to a self-consistent non-equilibrium “ground state”: rather, the
system has “tachyonic” modes, meaning that one is trying to compute around a wrong
extremum. However, as we will see, for our observable these problems are absent in the
limit of a small anisotropy, and we restrict to this case in the following.

For a general momentum distribution, the Hard Thermal Loop gluon self-energy obtains
the form \[ \Pi_{\mu \nu}^{(R)}(K) = g^2 \int \frac{d^3p}{(2\pi)^3} v^\mu \partial^\alpha f(p) \left( \delta^\nu_\alpha - \frac{v^\nu k_\alpha}{v \cdot K + i0^+} \right), \] (1)
where Minkowskian conventions are assumed, \( v \equiv (1, p/p) \), and the subscript \( R \) indicates
that the self-energy appears in the inverse of the retarded propagator. Following ref. [19],
the hard mode momentum distribution function is assumed to be of the form
\[ f(p) \equiv f_{\text{iso}}(\sqrt{p^2 + \xi p_z^2}). \] (2)
Here \( \xi \in (-1, \infty) \) is a real parameter, and \( p_z \) the momentum component in the beam direction;
the values corresponding to the Bjorken expansion induced anisotropy are \( \xi > 0 \).

Carrying out a change of variables to \( \bar{p} \equiv \sqrt{p^2 + \xi p_z^2} \), and denoting \( p_z \equiv p \cos \theta_p \), the
momentum integration may be written as
\[ \int \frac{d^3p}{(2\pi)^3} F(v, \sqrt{p^2 + \xi p_z^2}) = \int \frac{d^3\bar{p}}{(2\pi)^3} \frac{F(v, \bar{p})}{(1 + \xi \cos^2 \theta_p)^{1/2}}, \] (3)
where \( v \) is a unit vector, with \( v_z = \cos \theta_p \); the angular variables determining the direction of
\( v \) remain unchanged in the substitution; and \( F \) is an arbitrary function.

For the real-time static potential, we need the component \( \Pi_{00}^{(R)} \) of the self-energy, near the
static limit \( |k_0| \ll k \). Expanding to first order in \( \xi \) and \( k_0 \) yields
\[ \Pi_{00}^{(R)}(K) \approx m_D^2 \left\{ -1 + \xi \left[ \frac{1}{6} - \frac{1}{2} \cos(2\theta_k) \right] - \frac{i\pi}{2} \frac{k_0}{k} \left[ 1 + \xi \cos(2\theta_k) \right] \right\}, \] (4)
where \( \theta_k \) is the angle between \( k, z \) and we denoted
\[ m_D^2 \equiv -g^2 \int \frac{d^3\bar{p}}{(2\pi)^3} \frac{df_{\text{iso}}(\bar{p})}{d\bar{p}}. \] (5)

3. For the real and imaginary parts of the real-time static potential, we need the gluon
propagator near the static limit. More precisely, an explicit computation of the static potential \( \text{à la ref. [2]} \) can be rephrased by noting that, if the static limit exists (it does at \( O(\xi) \)
but not at \( O(\xi^2) \)), it is the time-ordered gluon propagator that matters [20], as would be
expected in the naive real-time formalism:
\[ \lim_{t \to \infty} V_{>}(t, r) = g^2 C_F \int \frac{d^3k}{(2\pi)^3} \frac{e^{ik \cdot r} + e^{-ik \cdot r} - 2}{2} i\Delta_{\perp}^{(0)}(0, k), \] (6)
where \( C_F \equiv (N_c^2 - 1)/2N_c \) and \( N_c = 3 \). The time-ordered propagator can in turn be written as
\[
 i\Delta^{00}_T = \Delta^{00}_R + 2in_B(k^0) \text{Im} \Delta^{00}_R \approx \Delta^{00}_R + \frac{2iT}{k^0} \text{Im} \Delta^{00}_R ,
\]
where \( \Delta^{00}_R \) is the 00-component of the retarded propagator, and \( n_B \) is the Bose-Einstein distribution function. We have here assumed that, unlike the hard modes, the soft gluons are already in thermal equilibrium at a temperature \( T \). It can be verified that at the required order in \( k^0 \) and \( \xi \), the self-energies \( \Pi^{00}_R, \Pi^{0i}_R, \Pi^{ij}_R \) do not contribute to \( \Delta^{00}_R \). Making use of eq. \( (11) \) and expanding to first order in \( \xi \), we thus obtain that in the static limit
\[
 i\Delta^{00}_T(0, k) = -\frac{1}{k^2 + m_D^2} + \frac{\xi m_D^2}{6(k^2 + m_D^2)^2} \left[ 3\cos(2\theta_k) - 1 \right] + \frac{i\pi m_D^2 T}{k(k^2 + m_D^2)^2} + \frac{i\xi m_D^2 T}{3k^2 - m_D^2} \left[ 3k^2 \cos(2\theta_k) + m_D^2 \right] .
\]
Inserting eq. \( (11) \) into eq. \( (10) \), and noting that \( \cos \theta_k = \cos \theta_r \cos \theta_{kr} + \sin \theta_r \sin \theta_{kr} \cos \phi_{kr} \), where \( \theta_i \) is the angle between \( r, z \) and \( \theta_{kr}, \phi_{kr} \) are the angular variables between \( k, r \), we can integrate over \( \theta_{kr}, \phi_{kr} \). The potential becomes
\[
 \lim_{t \to -\infty} V_{\rightarrow}(t, r) = -\frac{g^2 C_F m_D}{4\pi} \left\{ \frac{e^{-\hat{r}}}{\hat{r}} + 1 + \xi \left[ \frac{e^{-\hat{r}} - 1}{6} + (1 - 3\cos^2 \theta_r)\rho(\hat{r}) \right] \right\} - i\frac{g^2 C_F T}{4\pi} \left\{ \phi(\hat{r}) - \xi \left[ \frac{1}{3} \phi_1(\hat{r}) + \frac{4}{15} (1 - 3\cos^2 \theta_r)\phi_2(\hat{r}) \right] \right\} ,
\]
where \( \hat{r} \equiv m_D r \). Furthermore, we have defined
\[
 \rho(\hat{r}) \equiv e^{-\hat{r}} \left( \frac{1}{6} + \frac{1}{2\hat{r}} + \frac{1}{\hat{r}^2} \right) + \frac{e^{-\hat{r}} - 1}{\hat{r}^3} ,
\]
\[
 \phi(\hat{r}) \equiv 2 \int_0^\infty \frac{d\hat{k}}{(\hat{k}^2 + 1)^2} \left[ 1 - \frac{\sin(\hat{kr})}{\hat{k}\hat{r}} \right] ,
\]
\[
 \phi_1(\hat{r}) \equiv 2 \int_0^\infty \frac{d\hat{k}}{(\hat{k}^2 + 1)^2} \left[ 1 - \frac{\sin(\hat{kr})}{\hat{k}\hat{r}} \right] ,
\]
\[
 \phi_2(\hat{r}) \equiv 5 \int_0^\infty \frac{d\hat{k}}{(\hat{k}^2 + 1)^3} \left[ \frac{3\sin(\hat{kr})}{(\hat{k}\hat{r})^3} - \frac{3\cos(\hat{kr})}{(\hat{k}\hat{r})^2} - \frac{\sin(\hat{kr})}{\hat{k}\hat{r}} \right] .
\]
The real part of the potential agrees with the result of ref. \cite{16}. The functions appearing in the imaginary part are finite for all \( \hat{r} \); \( \phi_1 \) and \( \phi_2 \) vanish both at small and large distances.

4. In order to obtain a theoretically consistent result in the weak-coupling regime, it is important to systematically account for all the effects to a given order, emerging from the various momentum and frequency scales of the problem. This can be achieved by employing
effective field theory methods [5, 21] (for a review, see ref. [6]). The upshot is that \( \hat{r} \) is formally a small parameter, \( \hat{r} < 1 \) [3], and can be expanded in.

Expanding in \( \hat{r} \), the term proportional to \( \xi \) of the real part of the potential is seen to be of \( O(\hat{r}^2 m_D^2) \), i.e. suppressed by two powers of \( \hat{r} \) with respect to the \( \xi \)-independent term. Therefore it can be omitted in the weak-coupling limit.

In contrast, in the imaginary part (the second line of eq. (9)) the corrections from \( \xi \) are of the same order as the leading term:

\[
\begin{align*}
\phi(\hat{r}) &\approx \frac{\hat{r}^2}{3} \left( \frac{1}{\hat{r}} - \gamma_E + \frac{4}{3} \right), \\
\phi_1(\hat{r}) &\approx \frac{\hat{r}^2}{3} \left( \frac{1}{\hat{r}} - \gamma_E + \frac{5}{6} \right), \\
\phi_2(\hat{r}) &\approx \frac{\hat{r}^2}{3} \left( \frac{1}{\hat{r}} - \gamma_E + \frac{47}{60} \right).
\end{align*}
\]

In particular, at leading logarithmic order, all three functions behave identically.

5. Before we inspect more precisely the consequences of eq. (9), we need to discuss the value of the parameter \( m_D \) appearing in it, defined by eq. (5).

Often the function \( f_{\text{iso}} \) in eq. (5) is taken to be a sum of Bose-Einstein and Fermi-Dirac distribution functions, with a “temperature” \( T \) appearing as a parameter (see, e.g., eq. (1) of ref. [22]). It is clear, however, that since \( f_{\text{iso}} \) was part of a specification of a non-equilibrium state (cf. eq. (2)), a precise physical meaning can only be given to \( T \) through further arguments (cf. ref. [23] and references therein). We have already specified our “late-time” setting above (strongly interacting soft modes thermalized at a physical temperature \( T \), weakly interacting hard modes still anisotropic), so we need to rethink the meaning of the parameter appearing in \( f_{\text{iso}} \) for this case. We denote this parameter by \( T' \) in the following.

Consider now the entropy density of the system. Making use of the substitution in eq. (3), it is given by an integral of the type

\[
s_{\text{non-eq}} = \int \frac{d^3p}{(2\pi)^3} \mathcal{F}(\vec{p}) \approx \int \frac{d^3\vec{p}}{(2\pi)^3} \mathcal{F}(\vec{p}) \left( 1 - \frac{3}{2} \xi \cos^2 \theta_p \right),
\]

where we expanded to leading order in \( \xi \). Carrying out the angular integral, and assuming a massless system without chemical potentials, this yields

\[
s_{\text{non-eq}} = \left( 1 - \frac{\xi}{2} \right) c T'^3,
\]

with a certain constant \( c \). Similarly, the energy density becomes \( e_{\text{non-eq}} = \frac{3}{4} (1 - 2\xi/3) c T'^4 \).

We could now envisage two cases. If we define the non-equilibrium state by \( T' \equiv T \), then we observe that, for \( \xi > 0 \), it has less entropy density than the corresponding equilibrium
state, with \( s_{\text{eq}} = cT^3 \). On the other hand, if we impose the physical condition that the non-equilibrium state be related “isentropically” to the equilibrium one, then we are left to conclude that the parameter \( T' \) should be chosen as

\[
T' = \left( 1 + \frac{\xi}{6} \right) T .
\] (19)

The same outcome results if we define the temperature through \( T^{-1} = \partial s_{\text{non-eq}} / \partial e_{\text{non-eq}} \).

6. In order to estimate the temperature at which quarkonium dissociates, we finally carry out a parametric computation according to the discussion above. As we will see the effects of the anisotropy parameter \( \xi \) can be fully accounted for without being concerned about various numerical factors. Thus, we estimate the magnitude of the binding energy by the real part of the static potential, expanded to leading order in \( \hat{r} \) and evaluated at the distance scale of the Bohr radius:

\[
\text{Re}[V>] \sim - \frac{g^2C_F}{4\pi r} \left[ 1 + O\left( \frac{1}{\ln r} \right) \right] \bigg|_{r \sim 1/g^2 M}.
\] (20)

In the imaginary part, in contrast, corrections involving \( \xi \) are of order unity. Considering the \( s \)-wave ground state, the term proportional to \( 1 - 3\cos^2 \theta r \) in eq. (9) averages to zero at first order in perturbation theory (corrections will be of order \( \xi^2 \)), so to leading-logarithmic order

\[
\text{Im}[V>] \sim - \frac{g^2C_F}{4\pi r} \left[ 1 + O\left( \frac{1}{\ln r} \right) \right] \times \left( 1 - \frac{\xi}{3} \right) .
\] (21)

If we now assume the temperature \( T' \) parameterizing the non-equilibrium system to be isentropically obtained from the equilibrium case, leading to eq. (19), then the Debye mass parameter defined in eq. (5) evaluates to

\[
m_D^2 \sim g^2T'^2 \approx g^2T^2 \left( 1 + \frac{\xi}{3} \right) .
\] (22)

Thus \( \xi \)-dependence cancels on the right-hand side of eq. (21) to \( \mathcal{O}(\xi) \); at leading-logarithmic order the dissociation temperature remains at the value

\[
T_{\text{melt}} \sim g^\frac{4}{3} \left( \ln \frac{1}{g} \right)^{-\frac{1}{3}} M ,
\] (23)

obtained in refs. [6, 7].

If, in contrast, we assume \( T' = T \), then \( m_D^2 \sim g^2T^2 \). Temperature appears in eq. (21) cubically, meaning that equality with the real part of eq. (20) is obtained for

\[
T_{\text{melt}} \sim g^\frac{4}{3} \left( \ln \frac{1}{g} \right)^{-\frac{1}{3}} M \times \left( 1 + \frac{\xi}{9} \right) .
\] (24)
For $\xi \sim 1$, up to which range our small-$\xi$ approach might be assumed qualitatively reasonable, we thus obtain a 10% increase in the dissociation temperature.

7. To conclude, while the precise definition of the heavy quarkonium dissociation temperature is ambiguous, requiring a convention on the shape of the corresponding smoothly evolving spectral function (for results within the weak-coupling expansion, see ref. [24]), it appears that for the class of non-equilibrium states introduced in ref. [19], the relative change caused by a momentum space anisotropy can be estimated analytically at leading-logarithmic order. On the other hand, the definition of the non-equilibrium state itself contains a hidden ambiguity, in that a temperature-like parameter is introduced which requires further justification. We have related this parameter to the entropy density of the system, and thus arrived at two physically distinct results, eqs. (23) and (24). In general, it is expected that non-equilibrium states have less entropy than the equilibrium one, under which conditions eq. (24) could be a better estimate than eq. (23), indicating an increase of the dissociation temperature; nevertheless, reasonable arguments could also be given in favour of eq. (19), leading to eq. (23). In any case, our results suggest that quarkonium dissociation is primarily a probe of the entropy density of the system, i.e. of the number of hard modes, rather than of Debye screening, i.e. of soft collective phenomena. At the same time, given the ambiguities appearing, it seemed to us that the analytic leading-logarithmic order-of-magnitude estimate is about as far as one needs to go for small anisotropies; for larger ones, numerical simulations would be needed.

Note added

Recently a paper appeared [25] where the same problem is considered as here. The results deviate slightly from ours because the physics setting is different: unlike in our eq. (7), even the soft gluons are assumed to have a non-thermal distribution function, which leads to an additional term in the gluon propagator (the 2nd line of eq. (14) in ref. [25]). We note, however, that if we re-express within our setting the soft gluon $T'$ in terms of $T'$ from eq. (19), and identify $T'$ with the temperature parameter of ref. [25], then curiously our eq. (21) does reproduce eq. (58) of ref. [25].

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References

[1] T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416.

[2] M. Laine, O. Philipsen, P. Romatschke and M. Tassler, JHEP 03 (2007) 054 [hep-ph/0611300].

[3] Y. Burnier, M. Laine and M. Vepsäläinen, JHEP 01 (2008) 043 [0711.1743].

[4] D. Cabrera and R. Rapp, Phys. Rev. D 76 (2007) 114506 [hep-ph/0611134].

[5] M.A. Escobedo and J. Soto, 0804.0691.

[6] M. Laine, Nucl. Phys. A 820 (2009) 25C [0810.1112].

[7] F. Dominguez and B. Wu, Nucl. Phys. A 818 (2009) 246 [0811.1058].

[8] H. Liu, K. Rajagopal and U.A. Wiedemann, Phys. Rev. Lett. 98 (2007) 182301 [hep-ph/0607062].

[9] A. Rebhan, P. Romatschke and M. Strickland, JHEP 09 (2005) 041 [hep-ph/0505261]; D. Bödeker and K. Rummukainen, JHEP 07 (2007) 022 [0705.0180]; P. Arnold and G.D. Moore, Phys. Rev. D 76 (2007) 045009 [0706.0490]; J. Berges, D. Gelfand, S. Scheffler and D. Sexty, 0812.3859.

[10] P. Romatschke and M. Strickland, Phys. Rev. D 71 (2005) 125008 [hep-ph/0408275].

[11] P. Romatschke, Phys. Rev. C 75 (2007) 014901 [hep-ph/0607327].

[12] B. Schenke and M. Strickland, Phys. Rev. D 76 (2007) 025023 [hep-ph/0611332].

[13] M. Martinez and M. Strickland, Phys. Rev. Lett. 100 (2008) 102301 [0709.3576]; Phys. Rev. C 78 (2008) 034917 [0805.4552]; 0808.3969.

[14] A. Dumitru, Y. Nara, B. Schenke and M. Strickland, Phys. Rev. C 78 (2008) 024909 [0710.1223].

[15] R. Baier and Y. Mehtar-Tani, Phys. Rev. C 78 (2008) 064906 [0806.0954].

[16] A. Dumitru, Y. Guo and M. Strickland, Phys. Lett. B 662 (2008) 37 [0711.4722].

[17] A. Dumitru, Y. Guo, A. Mócsy and M. Strickland, 0901.1998 [hep-ph].

[18] S. Mrówczyński and M.H. Thoma, Phys. Rev. D 62 (2000) 036011 [hep-ph/0001164].

[19] P. Romatschke and M. Strickland, Phys. Rev. D 68 (2003) 036004 [hep-ph/0304092].
[20] A. Beraudo, J.P. Blaizot and C. Ratti, Nucl. Phys. A 806 (2008) 312 [0712.4394].

[21] N. Brambilla, J. Ghiglieri, A. Vairo and P. Petreczky, Phys. Rev. D 78 (2008) 014017 [0804.0993].

[22] M.E. Carrington and A. Rebhan, Phys. Rev. D 79 (2009) 025018 [0810.4799].

[23] A. Rebhan, M. Strickland and M. Attems, Phys. Rev. D 78 (2008) 045023 [0802.1714].

[24] Y. Burnier, M. Laine and M. Vepsäläinen, JHEP 02 (2009) 008 [0812.2105].

[25] A. Dumitru, Y. Guo and M. Strickland, 0903.4703.