Wave Conditions Variation Effects on Energy Recovery for a Dexa Wave Energy Converter

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Abstract. The aim of this paper is to present the effects of wave amplitude and wave frequency variations on the energy recovered by a DEXA Wave, which is a wave energy converter (WEC) device. The two-body articulated floating WEC consists of two connected rigid bodies, the energy is recovered by taking advantage of the relative movement between the two parts set in motion differently by the wave. The results show that the value of the wave amplitude and wave period have an impact on the recovered energy by the WEC. The present work shown that for this type of WEC, an increase in the amplitude of the wave does not necessarily improve the energy recovered by the WEC also it is shown that the considered WEC could be typical for low wave amplitude areas.

1. Introduction
In recent decades, the attention to marine renewable energy is growing all over the world. More information about these systems can be found in [1-3]. Among the designs for wave energy converter devices, the floating types are of particular interest insofar as they do not require complex and expansive platforms for harvesting wave energy. Among the systems which are in development, AQUABUOY [4] SEAREV [5] and DEXA Wave [6] are constantly improved by optimizing key factors such as power take off devices (PTO), shapes, and other parameters.

Zanuttigh et al. [6] analyzed the power and hydraulic performance of a floating DEXA Wave Energy Converter, to optimize its design for installation in arrays. They indicate that the device length must be tuned based on the local peak wavelength to achieve higher efficiency. Luca and Barbara [7] investigated the interactions between a DEXA Wave device and its mooring system under a different wave condition (regular and irregular). Experiments show that decreasing mooring compliancy enhances energy production. Erfan et al. [8] analyzed the CETO6-project WECs separation in an array with different numbers of devices and arrangements. For the sake of assessing the impact of different wave models, numerical analysis in four real wave scenarios were performed and compared. Their results show a direct relationship between the number of devices and optimal inter-distance among them. Recently, Hong and Xiao [9] investigated the effects of the hydraulic power take-off parameters on power extraction by using the MATLAB/Simulink to simulate system models.

In the present work, a DEXA Wave device is studied. The considered floating WEC is a freely floating wave energy converter system developed by DEXA Wave Energy ApS in 2009 in Denmark [6]. This system consists of two articulated rigid bodies where the energy is recovered by exploiting the relative movement between the two parts set in motion differently by the wave. More recent work on DEXA could be found in [10]. The forces due to the fluid action on the solid parts of the system are
modelled by an Archimedean thrust exerted on the submerged parts of the system and a Morison Force [11], which makes it possible to consider the effects of added mass as well as a drag between the solid and the fluid.

A fourth-order Runge-Kutta method is used to solve the system of five nonlinear differential equations that govern the plane movement of the WEC [12]. The obtained results for the DEXA Wave show the effects of the wave’s amplitude and his angular frequency on the efficiency of the wave energy converter and show that the variation of these parameters significantly impacts the performance of the WEC. Therefore, the dimensioning of the WEC should be adaptive according to the characteristics of the waves to allow a more advanced optimization of the recovered energy.

This paper is structured as follows: the first section is dedicated to the mathematical models of the considered systems. Section two presents the results and their discussion.

2. Mathematical formulation

The DEXA Wave system consists of two rigid bodies $S_1$ and $S_2$ (figure 1). Each part carries a cylinder of center $O_i$ ($i=1$ for cylinder 1 and $i=2$ for cylinder 2) and a floater of center $j$ ($j = A$ for floater 1 and $j = B$ for floater 2), the two bodies are paired by a linking bar. As part of a first approach, the floaters are modeled by cylinders of radius $r_j$ and length $H$.

![Figure 1. Schema 2D of DEXA Wave.](image)

Considering the connections between system components as shown in figure 1, and knowing that only plane WEC movements are allowed, we introduce five degrees of freedom for the mechanical system which are the surge ($x_A$), the heave ($y_A$), and the pitch ($\alpha_1$) for solid 1, the pitch ($\alpha_2$) for solid 2 and the angle ($\alpha_b$) for the linking bar.

Let us consider the non-inertial reference frame $\mathcal{R}(O, x_0, y_0, z_0)$, where $O$ is an arbitrary point taken at the moving free surface of the fluid and $y_0$ is the upward vertical. It should be noted that the mathematical formulation will be established in $\mathcal{R}(O, x_0, y_0, z_0)$. For the reference frame linked to solid 1, the reference frame of center A (center of gravity of floater 1) $\mathcal{R}_A(A, x_A, y_A, z_A)$ is considered where:

$$\overrightarrow{OA} = x_A \hat{x}_0 + y_A \hat{y}_0 \quad (1)$$

In the non-inertial frame $\mathcal{R}(O, x_0, y_0, z_0)$, Newton's second law of motion applied to each part of the system separately [solid 1, solid 2, linking bar] is expressed for solid $i$ ($i = 1$ for solid 1 and $i = 2$ for solid 2) as follows:

$$[\tau_{Di}] = [\tau_{Mi}] + [\tau_{Loi}] + [\tau_{pi}] + [\tau_{Atl}] + [\tau_{pto}] - [\tau_{ei}] \quad (2)$$

where $[\tau_{Di}]$ is the dynamic torsor, $[\tau_{Mi}]$ is the Morison force torsor, which represents the fluid's inertia and viscous forces acting on the system, $[\tau_{Loi}]$ denotes the reactions torsor at the connection between the solid $i$ and the linking bar, $[\tau_{pi}]$ is the gravity force torsor, $[\tau_{Atl}]$ represents the Archimedes thrust torsor, $[\tau_{pto}]$ is the forces torsor for the power take off system of the WEC and $[\tau_{ei}]$ is the inertia force torsor related to the non-inertial character of the considered reference frame. The torsor elements are applied for $S_1$ in $\mathcal{G}_1$ (the gravity center of solid 1), and for $S_2$ in $\mathcal{G}_2$ (the gravity center of solid 2).

For the linking bar, Newton's second law of motion is written as:

$$[\tau_{Db}] = [\tau_{pb}] - [\tau_{LG_1}] - [\tau_{LG_2}] - [\tau_{eb}] \quad (3)$$
where \([\tau_p b], [\tau_p b], [\tau_L b], [\tau_L b]\) and \([\tau_L b] \) represent respectively the dynamic torsor, the gravity force torsor, the reactions torsor between the solid 1 and the linking bar, the reactions torsor between the solid 2 and the linking bar, and the inertia force torsor related to the non-inertial character of the considered reference frame.

By inserting the expressions of the torsors in Eqs. (2) and (3), and after rearrangement, one obtains the following system of five coupled differential equations for the five degrees of freedom \(x_A, y_A, \alpha_b, \alpha_1\) and \(\alpha_2\):

\[
M_T \ddot{x}_A - C_T \sin(\alpha_1) \dot{\alpha}_1 - M_{T2} C_T \sin(\alpha_2) \dot{\alpha}_2 + F_{mA1x} + F_{mA2x} - C_T \cos(\alpha_1) \dot{\alpha}_1^2 + F_{mCy1x} + F_{mCy2x}
\]

\[
- \left(M_{T2} + \frac{m_b}{2}\right) L \cos(\alpha_b) \dot{\alpha}_b^2 = 0
\]

\[
M_T \ddot{y}_A + C_T \cos(\alpha_1) \dot{\alpha}_1 + M_{T2} C_T \cos(\alpha_2) \dot{\alpha}_2 + M_T g + F_{mA1y} + F_{mCy1y} + F_{mA2y} + F_{mCy2y}
\]

\[
- C_T \sin(\alpha_1) \dot{\alpha}_1^2 + \left(M_{T2} + \frac{m_b}{2}\right) L \cos(\alpha_b) \dot{\alpha}_b - M_{T2} C_T \sin(\alpha_2) \dot{\alpha}_2^2 - F_{arA1} - F_{arCy1}
\]

\[
- M_T \omega^2 A_m \cos(\omega t) - F_{arA2} - F_{arCy2} - \left(M_{T2} + \frac{m_b}{2}\right) L \sin(\alpha_b) \dot{\alpha}_b^2 = 0
\]

\[
C_b \sin(\alpha_1) \dot{x}_A - C_b \cos(\alpha_1) \dot{y}_A - C_b \dot{\alpha}_1 + C_b \cos(\alpha_1) \dot{x}_A \dot{\alpha}_1 + C_b \sin(\alpha_1) \dot{y}_A \dot{\alpha}_1 - \beta (\dot{\alpha}_1 - \dot{\alpha}_2)
\]

\[
g M_T (C_1 - l_1) \cos(\alpha_1) + M_{T2} \omega^2 A_m (C_1 - l_1) \cos(\alpha_1) \cos(\omega t) + (C_1 - l_1) \cos(\alpha_1)
\]

\[
\left[ -F_{mA1y} - F_{mCy1y} - F_{arA1} - F_{arCy1} + (C_1 - l_1) \sin(\alpha_1) \right] \left[ F_{mA1x} + F_{mCy1x} \right] = 0
\]

\[
C_{t1} \sin(\alpha_1) \dot{x}_A - C_{t1} \cos(\alpha_1) \dot{y}_A - C_{t1} l_1 \cos(\alpha_1 - \alpha_2) \dot{\alpha}_1 + [C_5 + M_{T2} C_T^2 \dot{\alpha}_2 + g M_{T2} C_T \cos(\alpha_2)]
\]

\[
- C_{t1} \cos(\alpha_2 - \alpha_b) \dot{\alpha}_b + C_{t1} l_1 \sin(\alpha_1 - \alpha_2) \dot{\alpha}_1^2 - C_{t1} \omega^2 A_m \cos(\alpha_2) \cos(\omega t) - C_{t1} l_1 \sin(\alpha_1 - \alpha_2) \dot{\alpha}_1 \dot{\alpha}_2 + C_e L \sin(\alpha_2 - \alpha_b) \dot{\alpha}_b \dot{\alpha}_2 - C_{t2} \sin(\alpha_2) \left[ F_{mA2x} + F_{mCy2x} \right] - C_{t2} \cos(\alpha_2) \left[ F_{mA2y} + F_{mCy2y} + F_{arA2} + F_{arCy2} \right] = 0
\]

\[
\left[M_{T2} - M_{T1}\right] \sin(\alpha_b) \dot{x}_A - \left[M_{T2} - M_{T1}\right] \cos(\alpha_b) \dot{y}_A - \left[M_{T2} l_1 - M_{T1} C_1\right] \cos(\alpha_1 - \alpha_b) \dot{\alpha}_1
\]

\[
- M_{T2} C_T \cos(\alpha_2 - \alpha_b) \dot{\alpha}_2 + L \left[ \frac{m_b}{6} - M_{T2}\right] \dot{\alpha}_b + \left[M_{T2} l_1 - M_{T1} C_1\right] \sin(\alpha_1 - \alpha_b) \dot{\alpha}_1^2
\]

\[
+ M_{T2} C_T \sin(\alpha_2 - \alpha_b) \dot{\alpha}_2^2 - \sin(\alpha_b) \left[ F_{mA1x} + F_{mCy1x} - F_{mA2x} - F_{mCy2x} \right] - \cos(\alpha_b) \left[ F_{arA1} + F_{arCy1} + F_{mA2y} + F_{mCy2y} - F_{arA2} - F_{arCy2} - F_{mA1y} - F_{mCy1y} \right] - \left[M_{T2} - M_{T1}\right] \cos(\alpha_b) g
\]

\[
+ \omega^2 A_m \left[ M_{T2} - M_{T1}\right] \cos(\alpha_b) \cos(\omega t) = 0
\]

where \(M_T\) is the total mass of the system, \(M_{T1}\) is the mass of the solid \(i\), \(m_b\) is the mass of the linking bar, \(L\) is the length of the linking bar, \(\dot{x}_A\) and \(\dot{y}_A\) are the two accelerations of the floater 1 along \(\overline{Ox}_0\) and \(\overline{Oy}_0\) axis respectively, \(\dot{\alpha}_i\) is the angular acceleration of the solid \(i\), \(\dot{\alpha}_b\) is the angular acceleration of the linking bar and \(g\) represent gravity acceleration. \(F_{mAix}, F_{mAiy}, F_{mCyix}\) and \(F_{mCyiy}\) are given by Morison equation and are written as:

\[
F_{mAix} = \rho_e C_m V_i \dot{x}_i + \frac{1}{2} \rho_e C_d S_i \dot{\dot{x}}_i |\dot{x}_i|, \quad F_{mAiy} = \rho_e C_m V_i \dot{y}_j + \frac{1}{2} \rho_e C_d S_j \dot{\dot{y}}_j |\dot{y}_j| (\text{with} j=A,B) \quad F_{mCyix} = \rho_e C_m V_i \dot{x}_i + \frac{1}{2} \rho_e C_d S_i \dot{\dot{x}}_i |\dot{x}_i|, \quad F_{mCyiy} = \rho_e C_m V_i \dot{y}_j + \frac{1}{2} \rho_e C_d S_j \dot{\dot{y}}_j |\dot{y}_j| \quad \text{where} \quad \rho_e \quad \text{is the fluid density,} \quad C_m \quad \text{is the added mass coefficient,} \quad C_d \quad \text{is defined as drag coefficient,} \quad S_i = r_i H \sin(\frac{\gamma_j}{r_i}) \quad \text{and} \quad S_i = r_1 H \cos(\frac{\gamma_j}{r_1}) \quad \text{are respectively the wetted cross-section area of the floater} \ j \quad \text{and the cylinder} \ i, \ H \quad \text{is the length of the cylinder} \ i. \quad \text{The Archimedes forces} \quad F_{arAix}, F_{arAiy}, F_{arCyix} \quad \text{and} \quad F_{arCyiy} \quad \text{along the} \ \overline{Ox}_0 \quad \text{and} \ \overline{Oy}_0 \ \text{axis respectively, are defined by} \ -\rho_e V_i g, \quad \text{where} \ V_i \quad \text{is the immersed volume. The coefficient} \ \beta \quad \text{is related to the power take-off device,} \ \dot{\alpha}_i \quad \text{is the angular velocity of the cylinder} \ i. \]
The following notations where introduced:

\[ M_{T1} = m_{cy1} + m_{b1} + m_{a1}, \]
\[ M_{T2} = m_{cy2} + m_{b2} + m_{a2}, \]
\[ C_1 = \frac{l_1}{M_{T1}} \left( \frac{m_{b1}}{2} + m_{cy1} \right), \]
\[ C_2 = \frac{l_2}{M_{T2}} \left( \frac{m_{b2}}{2} + m_{cy2} \right), \]
\[ C_3 = m_{a1} \frac{r_1^2}{2} + m_{cy1} \frac{r_1^2}{2} + m_{b1} \frac{l_1^2}{12} + m_{cy1} l_1 (l_1 - C_1) + m_{b1} \frac{l_1^2}{2} (l_1 - C_1), \]
\[ C_4 = l_1 \left( \frac{m_{b1}}{2} + m_{cy1} \right) - C_1 M_{T1}, \]
\[ C_5 = m_{a2} \frac{r_2^2}{2} + m_{cy2} \frac{r_2^2}{2} + m_{b2} \frac{l_2^2}{12} + m_{a2} l_2 (l_2 - C_2) + m_{b2} \frac{l_2^2}{2} (l_2 - C_2), \]
\[ C_6 = C_2 M_{T2} - l_2 \left( \frac{m_{b2}}{2} + m_{cy2} \right), \]
\[ C_7 = C_1 M_{T1} + l_1 M_{T2} + m_{b1} l_1, \]
\[ C_8 = C_4 + M_{T1} (C_1 - l_1), \]
\[ C_9 = C_3 + M_{T1} (C_1 - l_1) C_1, \]
\[ C_{10} = C_6 - M_{T2} C_2, \]
\[ R_1 = \text{radius of cylinder } i \ (i = 1 \text{ for cylinder 1 and } i = 2 \text{ for cylinder 2}), \]
\[ l_i = \text{length of the linking bar between cylinder } i \text{ and floater } i, \]
\[ m_{a1} = \text{mass of the linking bar between cylinder } i \text{ and floater } i, \]
\[ m_{cy1} = \text{mass of the cylinder } i \text{,} \]
\[ m_{cy2} = \text{mass of the cylinder } i \text{.} \]

The system of coupled differential Eqs. (4)-(8) is solved numerically by using 4th order Runge-Kutta method.

### 3. Results and discussion

In order to enhance the energy recovered by the DEXA wave device, the values of the parameters of the WEC such as the damping coefficient \( \beta \) and the power take off device coefficient \( \beta \) and wave amplitude \( A_m \), where \( H = 1m, r_1 = r_2 = 0.6m, R_1 = R_2 = 0.6m, m_{a1} = m_{a2} = m_{cy1} = m_{cy2} = 579kg, l_1 = l_2 = 1.8m, L = 1.3m, m_{b1} = m_{b2} = 1kg, T = 8s, C_m = 23.18 \text{ and } C_d = 321.02. \) It should be noted the device recover the maximum energy when \( A_m = 0.1 \) and \( T = 8s \) this being due to the increase of average differences between the angular velocities \( \dot{\alpha}_1 \) and \( \dot{\alpha}_2 \). Moreover, the recovered energy doesn’t increase as the wave amplitude increase.

It is shown that the device recovered less energy compered to point absorber, which is recovered more than 20J with same condition. This is due to several reasons, including the fact that the cylinder geometry is not optimized as well as the length of the linking bars. which leads to a decrease in the magnitude of square angular velocity differences to \( 10^{-4} \).

**Figure 2.** The recovered energy by DEXA Wave as a function of the PTO coefficient \( \beta \) and wave amplitude \( A_m \).

Figure 3 represents the optimum energy recovered by the device versus the angular frequency of the incoming wave, where \( H = 1m, r_1 = r_2 = R_1 = R_2 = 0.6m, m_{b1} = m_{b2} = 1kg, m_{a1} = m_{a2} = m_{cy1} = m_{cy2} = 579kg, l_1 = l_2 = 1.8m, L = 1.3m, T = 8s, C_m = 23.18 \text{ and } C_d = 321.02. \) This figure shows that the wave’s frequency has an effect on the energy recovery. Moreover, when the length is 0.32m for \( \omega = 0.78 rad/s \) the device recovers a maximum of energy.
Figure 3. The recovered energy as a function of the angular frequency $\omega$.

4. Conclusion
This study has presented a test for determining the best wave environment for the DEXA Wave device in order to enhance the energy recovery. This test is based on varying amplitude and frequency of the incoming waves with optimizing the PTO coefficients. The achieved results show that the amount of energy recovered is higher for particular values of the wave amplitude and wave period. Furthermore, it is shown that the considered WEC could be typical for low wave amplitude areas.

5. References
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