Even and odd combinations of nonlinear coherent states

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Abstract. In this work we present some statistical properties of even and odd combinations of nonlinear coherent states associated with two nonlinear potentials; one supporting a finite number of bound states and the other supporting an infinite number of bound states, within the framework of an f-deformed algebra. We calculate their normalized variance and the temporal evolution of their dispersion relations using nonlinear coherent states defined as (a) eigensates of the deformed annihilation operator and (b) those states created by the application of a deformed displacement operator upon the ground state of the oscillator.

1. Introduction

It is well known that the coherent states of the harmonic oscillator are the ones closest to a classical state, when acted on by harmonic interactions these states remain localized around the corresponding classical trajectory and do not change their functional form with time [1], their phase space distribution is well localized in both the position and momentum variables. The coherent states were first studied by Shrödinger in 1926, who referred to them as states of minimum uncertainty product [2]. Almost forty years later, Glauber [3] showed that these states can be obtained starting from any one of three mathematical definitions: i) as the eigenstates of the annihilation operator \( \hat{a}|\alpha\rangle = \alpha|\alpha\rangle \) with \( \alpha \) a complex number, ii) as those sates obtained by the application of the displacement operator \( \hat{D}(\alpha) = \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a}) \) and iii) as the quantum states with a minimum uncertainty product \( \langle (\Delta x)^2 \rangle \langle (\Delta p)^2 \rangle = 1/4 \) where \( x \) and \( p \) are the momentum and position operators. It is also known that, when two coherent states are superposed the resultant states can display nonclassical properties such as squeezing and antibunching effects [4]. In this work we focus our attention in the statistics of the light measured by the normalized variance of the number operator.

In general, a superposition of two coherent states with different complex amplitudes \( \alpha_1 \) and \( \alpha_2 \) can be written as:

\[
|\Psi\rangle = N(e^{i\phi_1}|\alpha_1\rangle + e^{i\phi_2}|\alpha_2\rangle),
\]

where \( N \) is a normalization factor. In particular, here we concentrate our attention on two superpositions with equal amplitudes and phases differing by \( \pi \). These are:
Using the explicit expression for the harmonic oscillator (or field) coherent state in terms of the number states \( |n \rangle \), we get

\[
|+\rangle = N_+ \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{\sqrt{(2n)!}} |2n\rangle, \\
\]

\[
|-\rangle = N_- \sum_{n=0}^{\infty} \frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}} |2n+1\rangle. \\
\]

The state \( |+\rangle \) is called an even coherent state since such a superposition involves only even number states. On the other hand, the state \(|-\rangle\) is an odd coherent state.

For a large enough displacement \( \alpha \), the states \(|\pm\rangle\) can be interpreted as the quantum superposition of two macroscopically distinguishable states, that is, a Shrödinger-cat-like-state [5]. A well-studied example is the case of a two level ion trapped in an external harmonic field. It turns out that the stationary states of the center of mass motion (in the harmonic trap) of a laser-driven ion are even or odd coherent states [6, 7]. Another method suggested for the preparation of cat states involves the coupling of an optical coherent field with a Kerr nonlinear medium [8]. These states are often referred to as optical cats.

Because of this progress in the generation of coherent states and their combinations, we are interested in extending the notion of even and odd coherent states to potentials other than the harmonic oscillator [9]. In particular, we will consider even and odd combinations of nonlinear coherent states (NLCS) for two nonlinear systems. These are the modified and the trigonometric Pöschl-Teller potentials, the former supports a finite number of bound states and the latter an infinite number of bound states. Using the framework of f-deformed algebra, we construct nonlinear coherent states pertinent to Pöschl-Teller like potentials as: (a) eigensates of a deformed annihilation operator (AOCS) and (b) those states created by the application of a deformed displacement operator upon the ground state of the oscillator (DOCS). Here we generate even and odd combinations of these states, characterize their quantum statistical properties and evaluate their dispersion relations as a function of time. These properties are contrasted with those of the even and odd combinations of harmonic oscillator coherent states (CS).

2. Superposition of nonlinear coherent states for the modified Pöschl-Teller potential

The modified Pöschl Teller (MPT) potential relies among the most studied nonlinear systems. This potential supports a finite number of bound states and it is pertinent for the description of vibrational excitations of molecular bending modes [10, 11]. The MPT potential is given by [12]

\[
V(x) = U_0 \tanh^2(ax), \\
\]

where \( U_0 \) is the depth of the well, \( a \) the range of the potential and \( x \) the relative distance from the equilibrium position. The solutions of the Schrödinger equation associated to this potential and its eigenvalues are [13]

\[
\psi_n(\zeta) = N_n \left(1 - \zeta^2\right)^{s/2} F(-n, \epsilon + s + 1; \epsilon + 1, (1 - \zeta)/2) \\
E_n = U_0 - \frac{\hbar^2 a^2}{8m} \left(-2n + 1 + \sqrt{1 + \frac{8mU_0}{\hbar^2 a^2}}\right)^2, \\
\]

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where $N'_n$ is a normalization constant, $\zeta = \tanh(ax)$, $m$ is the reduced mass of the molecule, $s$ is related with the depth of the well so that $s(s + 1) = 2mU_0/(\hbar^2a^2)$, $\epsilon = \sqrt{-2m(E - U_0)/\hbar a}$ and $F(a, b; c, z)$ stands for the Hypergeometric function [14]. If we write the eigenvalues in terms of the parameter $s$ we obtain

$$E_n = U_0 - \frac{\hbar^2a^2}{2m} (s - n)^2 = \frac{\hbar^2a^2}{2m} \left( s + 2sn - n^2 \right). \quad (8)$$

The number of bound states is determined by the dissociation limit $\epsilon = s - n = 0$. For integer $s$ the state associated to null energy is not normalizable. In this case the last bound state corresponds to $n_{\text{max}} = s - 1$ [11].

Let us now introduce deformed operators $\hat{A}$, $\hat{A}^\dagger$ defined by [15]

$$\hat{A} = \hat{a}f(\hat{n}), \quad \hat{A}^\dagger = f(\hat{n})\hat{a}^\dagger, \quad \hat{n} = \hat{a}^\dagger\hat{a} \quad (9)$$

and consider a quantum f-oscillator with the Hamiltonian

$$\hat{H}_D = \frac{\hbar\Omega}{2} \left( \hat{A}\hat{A}^\dagger + \hat{A}^\dagger\hat{A} \right) = \frac{\hbar\Omega}{2} \left( (\hat{n} + 1)f^2(\hat{n} + 1) + \hat{n}f^2(\hat{n}) \right). \quad (10)$$

By choosing a deformation function as

$$f^2(\hat{n}) = \frac{\hbar a^2}{2m\Omega} \left( 2s + 1 - \hat{n} \right), \quad (11)$$

the deformed Hamiltonian (10) becomes

$$\hat{H}_D = \frac{\hbar^2a^2}{2m} \left( -\hat{n}^2 + 2s\hat{n} + s \right) = \hbar\omega \left( \hat{n} + \frac{1}{2} - \frac{\hat{n}^2}{2s} \right), \quad (12)$$

where $\omega = \frac{\hbar a^2 s}{m}$. It is to be noted that the spectrum of Eq. (12) is identical to that of Eq. (8). The harmonic limit is recovered by taking $s \to \infty$, $a \to 0$ and $sa^2 \to m\Omega/\hbar$.

The nonlinear coherent states of an f-deformed algebra can be constructed as eigenstates of the deformed annihilation operator [15, 16],

$$\hat{A}\vert\alpha, f\rangle = \alpha\vert\alpha, f\rangle, \quad (13)$$

where $\alpha$ is a complex number. The states $\vert\alpha, f\rangle$ have the number state expantion

$$\vert\alpha, f\rangle = N_f \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!f(n)!}} \vert n \rangle, \quad (14)$$

with $N_f$ a normalization constant and $f(n)! = f(0)f(1)\cdots f(n)$. However, since we are considering a system which supports only $s - 1$ bound states, Eq. (14) becomes

$$\vert\alpha, f\rangle \approx N_f \sum_{n=0}^{s-1} \frac{\alpha^n}{\sqrt{n!f(n)!}} \vert n \rangle, \quad (15)$$

and these states are not exact eigenstates of the deformed annihilation operator because the summation is finite.
Even and odd combinations of these nonlinear coherent states are obtained as [17]

\[ |\pm\rangle_A = N_\pm (|\alpha, f\rangle \pm |-\alpha, f\rangle), \]
\[ = N_\pm N_f \sum_{n=0}^{s-1} \frac{(1 \pm (-1)^n)\alpha^n}{\sqrt{n!f(n)!}}|n\rangle, \]
\[ N_{2\pm} = \frac{1}{2} \left( 1 \pm N_f^2 \sum_{n=0}^{s-1} \frac{(-1)^n|\alpha|^{2n}}{n!f(n)!^2} \right)^{-1}. \]

The states \(|+\rangle_A\) and \(|-\rangle_A\) indicate the even and odd combination, respectively.

Using the deformation function given by Eq. (11) we obtain the following commutation relations

\[ [\hat{A}, \hat{n}] = \hat{A}, \quad [\hat{A}^\dagger, \hat{n}] = -\hat{A}^\dagger, \quad [\hat{A}, \hat{A}^\dagger] = \frac{\hbar a^2}{m\Omega}(s - \hat{n}). \]

The set of operators \(\{\hat{A}, \hat{A}^\dagger, \hat{n}\}\) form a closed Lie algebra. As a consequence, a unitary displacement operator \(D(\zeta)\) can be constructed in terms of the deformed operators as [18]

\[ D(\zeta) = \exp \left[ \frac{2m\Omega}{\hbar a^2} \zeta \hat{A}^\dagger \left( \frac{1}{1 + |\zeta|^2} \right)^{m\Omega g(\hat{n}; a, s)/\hbar a^2} \exp \left[ -\sqrt{\frac{2m\Omega}{\hbar a^2}} \zeta^* \hat{A} \right] \right], \]

where \(\zeta = e^{i\rho_0} \tan \left( \frac{\hbar a^2}{2m\Omega} |\alpha| \right), g(\hat{n}; a, s) = \hbar a^2(s - \hat{n})/m\Omega \) and \(|\alpha|e^{i\rho_0}\). The approximate nonlinear coherent states generated by the action of \(D(\zeta)\) on the ground state of the oscillator are

\[ |\zeta\rangle \approx \frac{1}{(1 + |\zeta|^2)^{s}} \sum_{n=0}^{s-1} \sqrt{\frac{\Gamma(2s + 1)}{n!\Gamma(2s + 1 - n)}} |\zeta^n|n\rangle. \]

Thus, another way to generalize the concept of even and odd coherent states is to take the combinations

\[ |\pm\rangle_D = \frac{N_\pm}{(1 + |\zeta|^2)^s} \sum_{n=0}^{s-1} \sqrt{\frac{\Gamma(2s + 1)}{n!\Gamma(2s + 1 - n)}} (1 \pm (-1)^n) |\zeta^n|n\rangle, \]

with \(N_\pm\) a normalization constant given by:

\[ N_{2\pm} = \frac{1}{2} \left( 1 \pm \frac{1}{(1 + |\zeta|^2)^2} \sum_{n=0}^{s-1} \frac{\Gamma(2s + 1)}{n!\Gamma(2s + 1 - n)} (-1)^n |\zeta|^{2n} \right)^{-1}. \]

3. Superposition of nonlinear coherent states for the trigonometric Pöschl-Teller potential

Let us consider now the trigonometric Pöschl-Teller potential

\[ V(x) = U_0 \tan^2(ax) \]

where \(U_0\) the potential’s strenght and \(a\) its range. Its eigenfunctions and eigenvalues are [12]:

\[ \psi_n(x) = \sqrt{\frac{a(\lambda + n)\Gamma(2\lambda + n)}{\Gamma(n + 1)}} (\cos(ax))^{1/2} P_{n+\lambda-1/2}^{1/2-\lambda}(\sin(ax)), \]
\[ E_n = \frac{\hbar^2 a^2}{2\mu} (n^2 + 2n\lambda + \lambda), \]  \hspace{1cm} (25)

where \( \mu \) is the mass of the particle and the parameter \( \lambda \) is related to the potential strength and range by \( \lambda(\lambda + 1) = 2\mu U_0/\hbar^2 a^2 \).

If we now propose the deformation function
\[ f^2(\hat{n}) = \frac{\hbar a^2}{2\mu\Omega} (\hat{n} + 2\lambda - 1) \]  \hspace{1cm} (26)

the deformed Hamiltonian (10) transforms into
\[ \hat{H}_D = \frac{\hbar^2 a^2}{2\mu} (\hat{n}^2 + 2\lambda\hat{n} + \lambda) = \hbar\omega \left( \hat{n} + \frac{1}{2} + \frac{\hat{n}^2}{2\lambda} \right), \quad \omega = \frac{\hbar a^2 \lambda}{\mu}, \]  \hspace{1cm} (27)

whose spectrum reproduces that of the trigonometric potential. Even and odd combinations of nonlinear coherent states pertinent to the trigonometric Pöschl-Teller potential are obtained using the expressions given in Eqs. 16 for the eigenstates of the annihilation operator and Eqs. 21 for those obtained with the deformed displacement operator. In this case, the number of bound states supported by the potential is infinite and the nonlinear states we construct are exact.

4. Quantum statistical properties
We now examine the statistical properties of the combinations of nonlinear coherent states. Firstly, let us consider the probability to find \( n \) quanta in \(|\pm\rangle_{A,D}\) which is given by
\[ P_n = |\langle n|\pm\rangle_{A,D}|^2. \]  \hspace{1cm} (28)

For the case of the trigonometric Pöschl-Teller potential we get
\[ \langle n|\pm\rangle_A = N_\pm N_f \frac{(1 \pm (-1)^n)\alpha^n}{\sqrt{n!f(n)!}}, \]  \hspace{1cm} (29)
\[ \langle n|\pm\rangle_D = N_\pm (1 - |\zeta|^2)^{\lambda} \sqrt{\Gamma(n + 2\lambda)/n!}\Gamma(2\lambda) \} (1 \pm (-1)^n)\zeta^n. \]  \hspace{1cm} (30)

The occupation number distribution for the even combination of nonlinear coherent states is displayed in Fig. 1 for the harmonic oscillator combination (third column) and the \(|\pm\rangle_{A,D}\) combinations corresponding to AOCS (first column) and DOCS (second column) for an average value of the number operator \( \langle \hat{n} \rangle = 10 \) and 30. Notice that these states possess only even number states. The probably distribution is very similar for all combinations for an average value \( \langle \hat{n} \rangle = 10 \) corresponding to a small value of \( |\alpha| \). On the other hand, notice that for rather large values of the energy, as in the case with \( \langle \hat{n} \rangle = 30 \), the distribution for the even combination of AOCS is narrower than that for DOCS and the harmonic combination takes intermediate values. This is consistent with Figs. 2(a) and 2(b). We found a similar behavior for the odd combinations of nonlinear coherent states.

In figure 2 we show the normalized variance of the number operator \( \hat{n} \) defined as
\[ \frac{\langle (\Delta \hat{n})^2 \rangle}{\langle \hat{n} \rangle} = \frac{\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2}{\langle \hat{n} \rangle}, \]  \hspace{1cm} (31)

as a function of \( |\alpha| \). The average values were taken between even and odd combinations of exact nonlinear coherent states. For the even and odd combinations corresponding to the AOCS, the behavior for the variance is similar to that of the harmonic combination when \( |\alpha| < 1 \). As
Figure 1. Occupation number distribution $P_n(\alpha)$ as a function of $n$ for $\langle \hat{n} \rangle = 10$ in (a), (b), (c) and $\langle \hat{n} \rangle = 30$ in (d), (e), (f). Even combinations of NLCS: AOCS in (a) and (d), DOCS in (b) and (e). The harmonics oscillator case is in (c) and (f). The NLCS correspond to the trigonometric Pöschl-Teller potential.

Figure 2. Plots of the normalized variance $\langle (\Delta \hat{n})^2 \rangle / \langle \hat{n} \rangle$ versus the size of the coherent states $|\alpha|$ for even (a) and (b) and for odd (c) and (d) combinations of AOCS (dashed curve) and DOCS (dashed-dotted curve) corresponding to the trigonometric Pöschl-Teller potential. The continuous curve is for harmonic oscillator coherent states. $|\alpha|$ increases the states exhibit sub-Poissonian character. This implies that the variance in $\hat{n}$ is smaller than its mean ($\langle (\Delta \hat{n})^2 \rangle < \langle \hat{n} \rangle$). For the even and odd combinations of DOCS, we can see once again that they reproduce the harmonic result for $|\alpha| < 1$. As the number of quanta increases, for $|\alpha| \geq 1.5$, the variance is always super-Poissonian in both combinations ($\langle (\Delta \hat{n})^2 \rangle > \langle \hat{n} \rangle$).

Figure 3 shows the normalized variance (31), as a function of $|\alpha|$, with the averages taken between even and odd combinations of approximate nonlinear coherent states corresponding to the modified Pöschl-Teller potential. Here, we considered a potential supporting $s + 1 = 10$ bound states so that occupation number corresponding to the $|\alpha|$ values belong to the lower part of the spectrum. Notice that, on the one hand the even combination of AOCS is always super-Poissonian for $0 \leq |\alpha| \leq 1.5$. On the other hand, the odd combination of AOCS is always sub-Poissonia when $0 \leq |\alpha| \leq 1$. In both combinations, for $|\alpha| \leq 0.4$, the behavior is similar.
Figure 3. Plots of the normalized variance \(\langle (\Delta \hat{n})^2 \rangle / \langle \hat{n} \rangle\) versus the size of the coherent states \(|\alpha|\) for even (a) and (b) and for odd (c) and (d) combinations of AOCS (dashed curve) and DOCS (dashed-dotted curve) corresponding to the modified Pöschl-Teller potential, The continuous curve is for harmonic oscillator coherent states (CS).

to the harmonic case. The even combination of DOCS exhibits super-Poissonian statistics for \(0 \leq |\alpha| \leq 0.9\), while for interval of \(0.9 < |\alpha| \leq 1.5\) the behavior is sub-Poissonian. Finally, for \(|\alpha| \leq 0.4\) the even and odd combination of DOCS is almost identical to the harmonic combination.

5. Dispersion relations

In order to evaluate the temporal evolution of the dispersion relations between even and odd nonlinear coherent states we define the deformed coordinate and momentum as [11]:

\[
X_D = \sqrt{\frac{\hbar}{2\mu\omega}} \left( \hat{A}F(\hat{n}) + F(\hat{n})\hat{A}^\dagger + \hat{A}^3G(\hat{n}) + G(\hat{n})\hat{A}^3 \right) \quad (32)
\]

\[
P_D = -i\sqrt{\frac{\hbar\mu\omega}{2}} \left( \hat{A}R(\hat{n}) - R(\hat{n})\hat{A}^\dagger + \hat{A}^3S(\hat{n}) - S(\hat{n})\hat{A}^3 \right) \quad (33)
\]

where we have kept up to third order terms in the deformed operators and the functions \(F, G, R\) and \(S\) are given by:

\[
F(n) = \sqrt{\frac{2\mu\omega}{\hbar}} \frac{\langle n - 1 | x | n \rangle}{f(n)\sqrt{n}}, \quad (34)
\]

\[
G(n) = \sqrt{\frac{2\mu\omega}{\hbar}} \frac{\langle n - 3 | x | n \rangle}{f(n)f(n - 1)f(n - 2)\sqrt{n(n - 1)(n - 2)}}, \quad (35)
\]

\[
R(n) = i\sqrt{\frac{2}{\hbar\mu\omega}} \frac{\langle n - 1 | \hat{p} | n \rangle}{f(n)\sqrt{n}}, \quad (36)
\]

\[
S(n) = i\sqrt{\frac{2}{\hbar\mu\omega}} \frac{\langle n - 3 | \hat{p} | n \rangle}{f(n)f(n - 1)f(n - 2)\sqrt{n(n - 1)(n - 2)}}. \quad (37)
\]

Here, the matrix elements \(\langle n - \beta | x | n \rangle\) and \(\langle n - \beta | \hat{p} | n \rangle\) are evaluated by numerical integration using the eigenfunctions of the corresponding Schrödinger equation.
The temporal evolution of an operator \( \hat{O} \) is calculated taking the expectation values between even and odd coherent states \( |\pm; t\rangle = U_D(t)|\pm\rangle \), where \( U_D(t) = e^{-i\hat{H}_D t/\hbar} \) is the time evolution operator and \( \hat{H}_D \) is the deformed Hamiltonian associated with the potential under consideration. These expressions allow us to obtain the temporal evolution of the dispersion in the coordinate and the momentum through

\[
(\pm; t| (\Delta \hat{O})^2 |\pm; t\rangle = (\pm; t| \hat{O}^2 |\pm; t\rangle - (\pm; t| \hat{O} |\pm; t\rangle)^2, \tag{38}
\]

where we take \( \hat{O} = X_D \) or \( P_D \).

Figure 4. Temporal evolution of the average value of the dispersion \( \langle (\Delta X_D)^2 \rangle \) and of the uncertainty product \( \langle (\Delta X_D)^2 \rangle \langle (\Delta P_D)^2 \rangle \) for even (a),(c) and odd (b),(d) combinations of DOCS corresponding to the modified Pöschl-Teller potential. The continuous curve is for even (left panel) and odd (right panel) combinations of CS. The parameters \( \langle \hat{n} \rangle = 2, s = 10, a = \sqrt{2/9}, \) and \( \hbar = \mu = \Omega = 1 \) are taken.

Figure 5. Temporal evolution of the average value of the dispersion \( \langle (\Delta X_D)^2 \rangle \) and of the uncertainty product \( \langle (\Delta X_D)^2 \rangle \langle (\Delta P_D)^2 \rangle \) for even (a),(c) and odd (b),(d) combinations of DOCS corresponding to the trigonometric Pöschl-Teller potential. The continuous curve is for even (left panel) and odd (right panel) combinations of CS. The parameters \( \langle \hat{n} \rangle = 2, \lambda = 21/2, a = \sqrt{2/21}, \) and \( \hbar = \mu = \Omega = 1 \) are taken.
Briefly, let us describe the time evolution of the dispersion, \(\langle (\Delta X_D)^2 \rangle\), and of the uncertainty product, \(\langle (\Delta X_D)^2 \rangle \langle (\Delta P_D)^2 \rangle\), with the averages taken between even and odd combinations of DOCS associated with the modified (Figure 4) and trigonometric (Figure 5) Pöschl-Teller potentials. For the sake of comparison the even and odd combinations of CS are also calculated. In both combinations of DOCS, notice that the amplitude of the dispersion is smaller than that of the harmonic combinations. We can see that though for the even combinations of DOCS the dispersion in the coordinate exhibits squeezing \(\langle (\Delta X_D)^2 \rangle < 1/2\) with a certain periodicity the dispersion for the odd combinations do not show such behavior. Also, modulations of \(\langle (\Delta X_D)^2 \rangle\) appear for the DOCS, in contrast to the results for the harmonic combinations where the amplitude remains constant. Finally, neither combination is a minimum uncertainty state since the product \(\langle (\Delta X_D)^2 \rangle \langle (\Delta P_D)^2 \rangle\), is always larger than the standard value of 1/4. We found an almost identical behavior for the corresponding time evolution of the even and odd combinations of AOCS.

6. Conclusions
In this work we have presented two alternative generalizations for the construction of non linear coherent states (NLCS) pertinent to potentials of a Pöschl-Teller type. In the first place we considered a potential supporting a finite number of bound states, in this case the NLCS obtained are only approximate. We made even and odd combinations of NLCS defined as eigenstates of the deformed annihilation operator (AOCS) and as those obtained by means of the application of a deformed displacement operator on the ground state of the oscillator (DOCS). We found that their conduct is similar for small values of the parameter \(|\alpha|\) and differences begin to be noticeable for values of this parameter where the average value of the number operator is such that states near the dissociation begin to be non negligible. In the second case we considered a potential supporting an infinite number of bound states, here the NLCS obtained are exact though the AOCS and the DOCS are different. We calculated the temporal evolution of the dispersions and the results obtained with the AOCS are identical to those obtained with the DOCS, however their statistical properties are different as can be seen from the variance of the number operator where in one case the distribution is super-poissonian and in the other case it is sub-poissonian. A measurement of these properties would allow us to specify which one of the two generalizations employed is adequate.

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