Generation of circular polarization of CMB via polarized Compton scattering

Ali Vahedi, a Jafar Khodagholizadeh, b Rohoollah Mohammadi c,d and Mahdi Sadegh e

aDepartment of Physics, Kharazmi University, Mofatteh Ave, P.O. Box 15614, Tehran, Iran
bFarhangian University, P.O. Box 11876-13311, Tehran, Iran
cIranian National Museum Of Science and Technology (INMOST), P.O. Box 11369-14611, Tehran, Iran
dSchool of Astronomy, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5531, Tehran, Iran
eDepartment of Physics, Payame Noor University (PNU), P.O. Box 19359-3697, Tehran, Iran

E-mail: vahedi@khu.ac.ir, j.gholizadeh@cfu.ac.ir, rmohammadi@ipm.ir, m.sadegh@pnu.ac.ir

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Abstract. The standard scenario of cosmology which includes only Compton scattering predicts a measurable amount for the linear polarization of the cosmic microwave background (CMB) radiation, while through this scenario there is no any sources to generate circular polarization. The circular polarization of CMB has not been observed so far, however it has not been completely excluded from observational evidences, for example, SPIDER group constraints on $l(l+1)C_l^V/(2\pi)$ for $33 < l < 307$ in the range of $141 \mu K^2$ to $255 \mu K^2$ at 150 GHz, providing 95% C.L. On the other hand, over the last few years, many theoretical works have been done to investigate the generation of CMB circular polarization. In this paper, it is revealed that Compton scattering of CMB photons from polarized cosmic electrons with non-zero bulk velocity $v_e \neq 0$ (polarized Compton scattering) can generate circular polarization. The effects of the external magnetic field on large scale, chiral magnetic instability and new physics interactions on the distribution of cosmic electrons can be considered as possible sources of the polarized cosmic electrons. We show that the power spectrum of circular polarization of CMB $C_l^{V(S)}$ generated by polarized Compton scattering in the presence of
scalar perturbation is proportional to the power spectrum of temperature fluctuation $C_{l}^{I(S)}$ and $\delta^2$ where $\delta$ is the fraction of polarized electron number density to the total one. Also it has been discussed that $\delta < 10^{-4}$ is required to find consistency with the reported upper limit of CMB circular polarization.

**Keywords:** CMBR polarisation, CMBR theory, cosmological parameters from CMBR

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1 Introduction

CMB’s temperature anisotropies detected by COBE in 1992 are believed to result from inhomogeneities in matter distribution in the recombination epoch [1]. Because Thomson scattering is an isotropic process, any primordial anisotropies (as opposed to inhomogeneities) should have been smoothed out before decoupling [2]. This certifies the interpretation of the observed anisotropies as a result of density perturbations which can be a source for the formation of galaxies and clusters. The temperature anisotropies discovered by COBE can be taken as evidence that such density inhomogeneities existed in the early universe [2–4]. The gravitational collapse of these primordial density inhomogeneities appears to have formed the large-scale structures of clusters, super-clusters, and galaxies observed today [2].

Due to the anisotropic Compton scattering around the recombination epoch, the generation of some relevant linear polarization (about 10%) in CMB radiation is expected [5–8], and these polarization fluctuations should be smaller than temperature fluctuations [10]. The most attractive results of Planck on cosmological parameters including $r$ (scalar-tensor ratio) and linear polarization map are reported in [11, 12]. On the other hand, according to the standard scenario of cosmology (considering Compton scattering as the mean interaction of CMB and cosmic matter), there is no physical mechanism to generate circularly polarized radiation at the last scattering surface. It should be noted that circular polarization measurements can provide valuable information to test the standard cosmological model and the physics beyond the standard model of elementary particles. However, experimental results confirm that one can still have circular polarization contribution in CMB anisotropy. There are relatively few published limits on the CMB circular polarization [13–16]. Also, several papers’ limits on CMB temperature spectrum gathered by Planck satellite, South Pole Telescope, and Atacama Cosmology Telescope were found [17–19]. All the experimental results have reported an upper limit for circular polarization (V-mode) around $\Delta_V/T_{\text{CMB}} < 10^{-4}$. The Milano Polarimeter (MIPOL) installed at the Testa Grigia Observatory obtained the upper limit to the degree of the CMB circular polarization ranging between $5.0 \times 10^{-4}$ and $0.7 \times 10^{-4}$ at angular scales between $8^0$ and $24^0$, far from the $nK$ region [16]. Using the non-zero circular for linear polarization coupling of the HWP (half-wave plate) polarization modulators, SPIDER collaboration provides a constraint on stokes $V$ at 95 and 150 GHz form $33 < l < 301$ [14]. Other future experiments such as CLASS [20] and PIPER [21] will be used to obtain a new constraint on the sensitivity of CMB polarization.
Several reasons can be provided as to the generation of circular polarization. In the case of a renormalizable and gauge-invariant standard model, the extension of photon coupling to an external vector field via Chern-Simons term, which arises as a radiative correction if gravitational torsion couples to fermions, will be the source of the circular polarization of CMB radiation [22]. The linear polarization of the CMB in the presence of a large-scale magnetic field $B$ can be converted to the circular polarization under the formalism of the generalized Faraday rotation (FR) [23–25] known as the Faraday conversion (FC). Furthermore, the V-mode can be produced with the same mechanism [26, 27]. In a background magnetic field or the quantum electrodynamics sector of the standard model extended by Lorentz non-invariant operators as well as non-commutativity, the CMB polarization acquires a small amount of circular polarization [28]. Photon-photon interactions mediated by the neutral hydrogen background, $\gamma + \gamma + \text{atom} \rightarrow \gamma + \gamma + \text{atom}$, through forward scattering [29], photon-neutrino scattering [31], and Euler-Heisenberg effective Lagrangian given in [32, 33] can produce circular polarization. Other interesting mechanisms can also be observed (e.g. photon-graviton interaction and magneto-optic effects) [34–38].

The scattering of a photon from the polarized electron is another mechanism which can be important for generating circular polarization. The description of polarization phenomena for both electromagnetic radiation and elementary particles as a matrix representation of polarization is given [39]. Some studies have investigated phenomena involving electrons and photons with polarization effect considerations such as Compton scattering and bremsstrahlung [40, 41]. They examined the detection and production of circularly polarized gamma radiation by Compton scattering. The production of a polarized electron by the photoionization of the polarized atomic beam has been reported which is a useful source of polarized electrons [42]. A Compton scattering-based polarimeter for measuring the linear polarization of hard X-rays (100–300 keV) from astrophysical sources has been under development [43]. A Monte Carlo method is described for the multi-pole scattering of linearly polarized gamma rays in non-magnetized solid-state targets and then the cross section and Stokes parameters for spin-polarized have been discussed [44]. Also, the investigation of $\gamma$-ray polarizations has led to the insertion of constraints on the Planck scale violation of special relativity [45]. The final electron polarization was calculated for the scattering of the polarized photon by a polarized electron [46].

In this work, it is revealed that Compton scattering of photons from polarized electrons (polarized Compton scattering) can generate circular polarization in contrast to the ordinary Compton scattering [5]. The asymmetry between left- and right-handed number densities of electrons can be obtained from several sources. For example, in beta decays in Neutron stars as clearly shown in [47], nature just accepts left-handed neutrinos; thus, left-handed electrons’ flux participates in beta decay and, their right-handed partner remains. Another case is axions as a dark-matter candidate which can couple to fermions during inflation and produce both helicity states of the electron but in asymmetrical amounts [48]. In the presence of a magnetic field, the electron should fill Landau levels [49]; while the lowest Landau level can be filled only by left-handed electrons, higher levels are filled with both helicity states of electrons. This will cause an asymmetry between left- and right-handed electrons’ distribution which is in the order of $\sim \frac{eB}{p^2}$, where $B$ is the amplitude of magnetic field, and $p$ is the linear momentum of electrons. By reviewing the chiral magnetic instability for electrons with only electromagnetic interaction, the chiral charge density, $n_5$, is of order $\sim 10^{-14} n_e$ where $n_e$ is the number density of electrons [51]. Also, the electromagnetic interaction of the massive
spin-1/2 Dirac particles can flip their helicity [52]. Moreover, from the inflation models with chiral coupling between axial fermion current and the axion, the helicity imbalance would generate asymmetries between left-handed and right-handed photons and fermions (see [30] for a detailed discussion. The above-mentioned mechanisms motivated us to investigate the circular polarization generation of CMB via polarized Compton scattering.

2 CMB interaction with polarized electrons

To describe an ensemble of a photon-like CMB radiation, one can start with the density matrix:

$$\hat{\rho} = \frac{1}{\text{tr}(\hat{\rho})} \int \frac{d^3k}{(2\pi)^3} \rho_{ij}(k) D_{ij}(k)$$ (2.1)

where $D_{ij}(k) \equiv a_i^\dagger (k) a_j(k)$ and $\rho_{ij}$ are the photon number operator and the general density matrix component in the space of polarization states, respectively, and $k$ indicates the momentum of photons. $I$, $Q$, $U$ and $V$ are Stokes parameters related to $\rho_{ij}(k)$ as follows:

$$\hat{\rho} = \frac{1}{2} \begin{pmatrix} I + Q & U - iV \\ U + iV & I - Q \end{pmatrix}$$ (2.2)

The time evolution of $\rho_{ij}(k)$ as well as Stokes parameters is given [5]:

$$(2\pi)^3 \delta^3(0) 2k^0 \frac{d}{dt} \rho_{ij}(k) = i[[H^0_1(t), D^0_{ij}(k)]] - \frac{1}{2} \int dt ([[H^0_1(t), H^0_1(0), D^0_{ij}(k)]]$$ (2.3)

where $k^0 = |k|$ and $H^0_1(t)$ is the first order of the interacting Hamiltonian. The first term on the right hand side of eq. (2.3) is a forward scattering term, and the second one is a higher order collision term. Using standard calculations of quantum electrodynamics (QED), the interacting Hamiltonian for electron-photon scattering ($\gamma(p) + e(q) \rightarrow \gamma(p') + e(q')$) is given

$$H^0_1(t) = \int d\mathbf{q} d\mathbf{q}' d\mathbf{p} d\mathbf{p}' (2\pi)^3 \delta^3(\mathbf{q}' + \mathbf{p}' - \mathbf{q} - \mathbf{p}) \exp \left[ i t (q^0 + p^0) - q^0 - p^0 \right]$$

$$\times \left[ \hat{b}_{r'}(\mathbf{q}') a_{s'}^\dagger (\mathbf{p}') \mathcal{M}(q'r', p_s, q'r' s') a_s(\mathbf{p}) b_r(\mathbf{q}) \right],$$ (2.4)

where $a_s, a_s^\dagger$ and $b_r, b_r^\dagger$ are annihilation and creation operators of the quantized photon and electron fields, respectively. $\mathcal{M}$ is the Compton scattering amplitude

$$\mathcal{M}(q'r', p_s, q'r' s') = -ie^2 U_{r'}(\mathbf{q}')$$

$$\left[ \frac{\ell_{s'} (p') (q + \mathbf{q} + m) \ell_{s} (p)}{2 q_p} - \frac{\ell_{s} (p) (q' + \mathbf{q} + m) \ell_{s'} (p')}{2 p_{q'}} \right] U_r(\mathbf{q}),$$ (2.5)

where $r, r'$ and $s, s'$ indices run over electron and photon spin states. Note that phase space elements are defined:

$$d\mathbf{q} = \frac{d^3 \mathbf{q}}{(2\pi)^3 q^0}, \quad d\mathbf{p} = \frac{d^3 \mathbf{p}}{(2\pi)^3 p^0}.$$ (2.6)
2.1 Forward scattering terms

The usual assumption of forward scattering is that the fields begin as a free field and end as another free field in which the interactions are isolated from one another. According to this assumption, it will be proved that (as done by [5]):

\[
\bar{U}_r(q)\hat{\epsilon}_{s} (q + m)\hat{\epsilon}_{s'} U_r(q) = \bar{U}_r(q)(2q \cdot \epsilon_s - \hat{\epsilon}_s \cdot q + m\hat{\epsilon}_s)\hat{\epsilon}_{s'} U_r(q) \\
= (2q \cdot \epsilon_s)\bar{U}_r(q)\hat{\epsilon}_{s'} U_r(q) \\
= \frac{2}{m}(q \cdot \epsilon_s)(q \cdot \epsilon_{s'}) \\
= \bar{U}_r(q)\hat{\epsilon}_{s'} (q + m)\hat{\epsilon}_{s} U_r(q) \\
\]

These two terms (in eq. (2.5)) cancel out each other. Thus, the forward scattering of ordinary Compton scattering does not have any contribution to the generation of the circular polarization of CMBs for electrons being unpolarized. However, this term in the presence of neutrino scattering off photons has a non-zero contribution [53]. The treating polarized electrons are the same as neutrinos or any other particle species governed by the Boltzmann equations. With the evaluation of the forward scattering term for Compton scattering, this term will be zero independently of whether the electrons are polarized or unpolarized (for more details, see [5]).

2.2 Damping terms

The contribution of the damping term (usual cross section) of the Compton scattering for the generation of CMB polarization has been studied in numerous studies [See, for example, [5] and its references]. First, we review the result presented in [5] for the case of Compton scattering of photons from unpolarized electrons. The Boltzmann equation is:

\[
2k^0\hat{\rho}_{ij}(k) = \frac{1}{4} \int d\mathbf{q}^\prime dp^\prime (2\pi)^4 \delta^4(q^\prime + p - q - k)M(q^\prime, ps_1, qr, ks_1)M^\dagger(qr, ks^\prime_2, q^\prime r^\prime, ps_2) \\
\times \left[n_e(x, q^\prime)\delta_{s_2s_1^\prime} \delta_{js_1^\prime} \hat{\rho}_{ijs_1}(k) + \delta_{j^\prime s_2} \delta_{ijs_1}(k)\right] - 2n_e(x, q^\prime)\delta_{s_1s_1^\prime} \delta_{ijs_1}(k) - 2n_e(x, q^\prime)\delta_{s_1s_1^\prime} \delta_{ijs_2}(k) \right \]

where \(n_e(x, q)\) is the electron distribution function. The distribution function of the cosmic electrons, which is known as a thermal Maxwell-Boltzmann distribution [5], is:

\[
n_e(x, q) = n_e(x) \left(\frac{2\pi}{mT_e}\right)^{3/2} \exp \left[-\frac{(q - m\mathbf{v}(x))^2}{2mT_e}\right] \\
\]

where \(n_e(x)\), \(m\), \(T_e\) and \(\mathbf{v}(x) = v_e(x)\hat{\mathbf{v}}\) are electron number density, electron mass, electron temperature and electron bulk velocity, respectively. Let us also write the following useful integrals:

\[
\int \frac{d^3q}{(2\pi)^3} n_e(x, q) = n_e(x), \\
\int \frac{d^3q}{(2\pi)^3} q n_e(x, q) = mn(x)n_e(x).
\]
Now, with this furnishing, we go through eq. (2.8). First, we can simplify Compton scattering amplitude, eq. (2.5), as follows:

\[
\mathcal{M}(q'r', ks_1, qr, ps'_1) = -ie^2 \bar{U}_{r'}(q') \left[ 2q \cdot k \right] \left[ 2q \cdot p \right] \frac{\epsilon}{2} \frac{\gamma_m}{2m} \left( \frac{1}{2} + \gamma_S \right) \left( s_1 \right)\bar{U}_r(q) \left( s_1' \right) \frac{\gamma_m}{2m} \left( \frac{1}{2} + \gamma_S \right) \left( s_1 \right)\bar{U}_r(q) \left( s_1' \right) \frac{\gamma_m}{2m} \left( \frac{1}{2} + \gamma_S \right) \left( s_1 \right)\bar{U}_r(q) \left( s_1' \right)
\]

Then, the squared Compton amplitude in the abbreviated form is:

\[
\mathcal{M}(q'r', ps'_1, qr, ks_1)\mathcal{M}(qr, ks'_2, q'r', ps_2) = e^4 \sum \left\{ \bar{U}_{r'}(q')T(s_1, s'_1)U_r(q)\bar{U}_r(q)T(s_2, s'_2)U_{r'}(q') \right\},
\]

where

\[
T(s_1, s'_1) = \frac{f_{s'_1}(p)}{2q \cdot k} \left[ 2q \cdot \epsilon_{s_1}(k) - f_{s_1}(k) \right] - \frac{f_{s_1}(k)}{2q \cdot p} \left[ 2q \cdot \epsilon_{s'_1}(p) + f_{s'_1}(p) \right]
\]

\[
\bar{T}(s_2, s'_2) = \frac{1}{2q \cdot k} \left[ 2q \cdot \epsilon_{s_2}(k) - f_{s_2}(k) \right] s_2(p) - \frac{1}{2q \cdot p} \left[ 2q \cdot \epsilon_{s'_2}(p) + f_{s'_2}(p) \right] s'_2(k)
\]

Note that, in the Compton scattering of unpolarized electrons, there are averaging assumptions on the final and initial helicity states of electrons in eq. (2.13), allowing us to use the ordinary completeness relation \( \sum_r U_r(q)\bar{U}_r(q) = \frac{\gamma_m}{m} \) for both ingoing and outgoing electrons. Nevertheless, here we consider small polarization for ingoing electrons. In this case, the completeness relation of Dirac spinors is modified as [54, 55]

\[
U_r(q)\bar{U}_r(q) = \left[ \frac{\gamma_m + 1}{2m} \right] \left( \frac{1}{2} + \gamma_S \right) \left( s_1 \right)\bar{U}_r(q) \left( s_1' \right)
\]

where \( S_r \) helicity operator with \( r = L, R \) is defined as:

\[
S_R(q) = \left( \frac{q}{m}, \frac{E}{m}, \frac{q}{m} \right), \quad S_L(q) = -S_R(q).
\]

Let us consider a small fraction \( \delta L \) of left-handed polarization for ingoing cosmic electrons while we do not apply any constraint on the outgoing electrons due to their interaction with CMB photons. A reason for this asymmetry between left-handed and right-handed electrons is the cosmic magnetic field, which enforces the lowest Landau level fill just by left-handed electrons (See [50, 61]). The chiral asymmetry is also discussed in the inflation models which is generated from chiral coupling between the axion and the fermion axial current (See [30]). Hence, we have

\[
\mathcal{M}(q'r', ps'_1, qr, ks_1)\mathcal{M}(qr, ks'_2, q'r', ps_2) = e^4 Tr \left\{ \frac{(g' + m)}{2m} T(s_1, s'_1) \left[ \frac{(g + m)}{2m} \right] \left[ \frac{1}{2} + \gamma_S \right] \left( s_1 \right)\bar{U}_r(q) \left( s_1' \right) \left[ \frac{1}{2} + \gamma_S \right] \left( s_1 \right)\bar{U}_r(q) \left( s_1' \right) \left[ \frac{1}{2} + \gamma_S \right] \left( s_1 \right)\bar{U}_r(q) \left( s_1' \right) \left[ \frac{1}{2} + \gamma_S \right] \left( s_1 \right)\bar{U}_r(q) \left( s_1' \right) \left[ \frac{1}{2} + \gamma_S \right] \left( s_1 \right)\bar{U}_r(q) \left( s_1' \right) \left[ \frac{1}{2} + \gamma_S \right] \left( s_1 \right)\bar{U}_r(q) \left( s_1' \right)
\]
It should be noted that, in the above equation, \( q \) and \( q' \) are ingoing and outgoing electrons’ momentum, respectively. One can rewrite eq. (2.18) as the following:

\[
\mathcal{M}(q', p_{s1}', q, k_{s1})\mathcal{M}(q, k_{s2}, q', p_{s2}) = e^4 2 \text{Tr} \left\{ \frac{q' + m}{2m} T(s_1, s_1') \frac{q + m}{2m} T(s_2, s_2') \right\} 
\]

\[+ \frac{e^4}{2} \text{Tr} \left\{ \frac{q' + m}{2m} T(s_1, s_1') \frac{q + m}{2m} (\gamma_5 S_L(q)) T(s_2, s_2') \right\}, \tag{2.19}\]

where the first term is the amplitude of Compton scattering of unpolarized electrons investigated in the standard scenario, whereas the second term indicates the contribution of Compton scattering of polarized electrons represented as \(|\mathcal{M}|^2\). With straightforward calculations (applicable Mathematica package [57, 58]) and keeping the dominated contribution, we have:

\[
|\mathcal{M}|^2 \approx \frac{e^4}{4(q \cdot k)^2} q \cdot \epsilon'_{s_2}(k) \left( k \cdot \epsilon'_{s_1}(p) \hat{q} \cdot \epsilon_{s_1}(k) \times \epsilon_{s_2}(p) + p \cdot \epsilon_{s_1}(k) \hat{q} \cdot \epsilon'_{s_1}(p) \times \epsilon_{s_2}(p) \right) 
\]

\[+ q \cdot \epsilon_{s_2}(p) \left( p \cdot \epsilon_{s_1}(k) \hat{q} \cdot \epsilon'_{s_2}(k) \times \epsilon_{s_1}(p) + \hat{q} \cdot \epsilon_{s_1}(k) \epsilon'_{s_2}(k) \cdot p \times \epsilon_{s_2}(p) \right) 
\]

\[+ \hat{q} \cdot \epsilon'_{s_1}(p) \left( q \cdot \epsilon_{s_2}(p) \hat{q} \cdot k \times \epsilon'_{s_2}(k) - q \cdot \epsilon'_{s_2}(k) \hat{q} \cdot k \times \epsilon_{s_2}(p) \right) 
\]

\[+ \epsilon_{s_1}(k) \cdot \epsilon'_{s_1}(p) \left( q \cdot \epsilon_{s_2}(p) \hat{q} \cdot p \times \epsilon'_{s_2}(k) - q \cdot \epsilon'_{s_2}(k) \hat{q} \cdot p \times \epsilon_{s_2}(p) \right) 
\]

\[+ \epsilon_{s_1}(k) \cdot \epsilon_{s_2}(p) q \cdot \epsilon'_{s_2}(k) \hat{q} \cdot p \times \epsilon_{s_1}(p) + \epsilon'_{s_1}(k) \cdot \epsilon_{s_2}(p) q \cdot \epsilon_{s_2}(p) \hat{q} \cdot k \times \epsilon_{s_1}(k) 
\]

\[\delta_{s_2s'_2} q \cdot \epsilon'_{s_2}(k) \hat{q} \cdot k \times \epsilon_{s_1}(k) - \delta_{s_1s'_2} q \cdot \epsilon_{s_2}(p) \hat{q} \cdot p \times \epsilon_{s'_1}(p) \right\}, \tag{2.20}\]

where \( \hat{q} = q/|q| \); then the Boltzmann equation for \( \rho_{ij}(x, k) \) is given by:

\[
\frac{d}{dt} \rho_{ij}(x, k) = \frac{e^4 \delta_{L}}{2k^0} \int dq dp \frac{m}{E(q + k - p)} (2\pi)^3 \delta(E(q + k - p) + p - E(q) - k) 
\]

\[\times \left( n_e(x, q) \delta_{s'js_1s'_1} (\delta_{i0j} \rho_{s'j}(k) + \delta_{p'j} \rho_{s_1s_1}(k)) \right) 2n_e(x, q') \delta_{i0j} \delta_{p'j} \rho_{s_1s_2}(p) |\mathcal{M}|^2 \hat{p}, \tag{2.21}\]

where we introduce \( \delta_L = n_{eL}/n_e \) and \( \delta_R = n_{eR}/n_e \) as a fraction of polarized electron number density to total density with net left- or right-handed polarizations. By running all indices, ignoring the recoil momentum of final electrons and considering the below equations

\[
\delta(E(q + k - p) + p - E(q) - k) \sim \delta(p - k), \tag{2.22}\]

\[
E(q + Q) \sim m \left[ 1 + \frac{q^2}{m^2} + \frac{q \cdot Q}{m^2} + \ldots \right], \tag{2.23}\]

\[
n_e(q + Q) \sim n_e(q) \left[ 1 - \frac{Q \cdot (q - mv)}{mT_e} + \ldots \right], \tag{2.24}\]

\[\quad \quad -6\]
the time evolution of Stokes parameters would have the following form:

\[ \dot{I}(k) = \tilde{\tau}_{pc} \int \frac{d\Omega}{4\pi} \sum_{s} \left[ f_{IS}(\hat{k}, \hat{p})S(k) + g_{IS}(\hat{k}, \hat{p})S(p) \right], \]

\[ \frac{\dot{Q}}{\dot{V}}(k) = \tilde{\tau}_{pc} \int \frac{d\Omega}{4\pi} \sum_{s} \left[ f_{QS}(\hat{k}, \hat{p})S(k) + g_{QS}(\hat{k}, \hat{p})S(p) \right] \]

\[ \dot{U}(k) = \tilde{\tau}_{pc} \int \frac{d\Omega}{4\pi} \sum_{s} \left[ f_{US}(\hat{k}, \hat{p})S(k) + g_{US}(\hat{k}, \hat{p})S(p) \right] \]

\[ \dot{V}(k) = \tilde{\tau}_{pc} \int \frac{d\Omega}{4\pi} \sum_{s} \left[ f_{VS}(\hat{k}, \hat{p})S(k) + g_{VS}(\hat{k}, \hat{p})S(p) \right], \]

where \( S \in \{I, Q, U, V\} \) and

\[ \tilde{\tau}_{pc} = \frac{3 m v_{c}(x)}{2} \frac{n(x)}{k^3} \sigma_T \delta_L n_{e}(x). \]

All coefficients \( f_{IS}, f_{QS}, f_{US} \) and \( f_{VS} \) can be easily obtained from eqs. (2.2) and (2.20). As we are interested in the calculation of circular polarization (i.e. eq. (2.28)), we disregard the time evolution of \( \dot{I}(k), \dot{Q}(k) \) and \( \dot{U}(k) \) for the rest of calculations. Furthermore, in (2.28), the coefficients of \( f_{VQ}, g_{VQ}, f_{VV} \) and \( g_{VV} \) are not considered because \( Q \) and \( U \) are at least one order of magnitude smaller than \( I \) in the case of CMB radiation.

3 Power spectrum of the circular polarization

We continue the calculation in the presence of the primordial scalar perturbations indicated by \( (S) \) which we expand in the Fourier modes characterized by a wave number \( K \). For each given wave number \( K \), it is useful to select a coordinate system with \( K \parallel \hat{z} \) and \( (\hat{e}_1, \hat{e}_2) = (\hat{e}_x, \hat{e}_\phi) \). The baryon bulk velocity \( v \) at the linear order is irrotational, meaning that it is the gradient of a potential, and thus it is parallel to wave number \( K \) in the Fourier space (see [9]):

\[ |v| = |v| \approx (1 + z)^{-1/2}10^{-3}. \]

The temperature anisotropy \( \Delta_I^{(S)} \) and circular polarization \( \Delta_V^{(S)} \) of the CMB radiation can be expanded in the conformal time \( \eta \) and described by multi-pole moments as follows:

\[ \Delta_{I,V}(\eta, K, \mu) = \sum_{l=0}^{\infty} (2l + 1)(-i)^l \Delta_{I,V}^l(\eta, K)P_l(\mu) \]

where \( \mu = \hat{n} \cdot \hat{k} = \cos \theta \), the \( \theta \) is the angle between the CMB photon direction \( \hat{n} = k/|k| \) and the wave vectors \( K \) and \( P_l(\mu) \) is the Legendre polynomial of rank \( l \). Consequently, we continued with the definition\(^{1}\)

\[ \Delta_I^{(S)}(K, k, \eta) \equiv \left( \frac{4k^2 \partial I_0}{\partial k} \right)^{-1} \Delta_I^{(S)}(K, k, \eta). \]

Here, we should define \( \frac{d}{dt} \) in the left hand side of eq. (2.21) to take into account space-time structure and gravitational effects such as the red-shift. For each plane wave, each scattering

\(^{1}\)This is confusing in the literature, but we should note that the right side of \( \Delta_I^{(S)} \) is dimensionless and we continue with it.
and interaction can be described as the transport through a plane parallel medium [59, 60], and finally, Boltzmann equations in the presence of the primordial scalar perturbations are given as:

\[
\frac{d}{d\eta} \Delta^{(S)}_V + iK \mu \Delta^{(S)}_V = -\dot{\tau}_{e\gamma} \left[ \Delta^{(S)}_V - \frac{3}{2} \mu \Delta^{(S)}_{V1} \right] - i2/3 \dot{\tau}_{pc} \left[ P_2(\mu) \Delta^{(S)}_I - \Delta^{(S)}_{I2} \right] \tag{3.4}
\]

where \( \dot{\tau}_{e\gamma} \equiv \frac{d\tau_{e\gamma}}{d\eta} \) in which \( \tau_{e\gamma} \) is the Compton scattering optical depth, and \( a(\eta) \) is the normalized scale factor.

The values of \( \Delta^{(S)}_I(\eta_0, \hat{n}) \) and \( \Delta^{(S)}_V(\eta_0, \hat{n}) \) at the present time \( \eta_0 \) and direction \( \hat{n} \) can be obtained in the following general form by integrating the Boltzmann equation (3.4), along the line of sight [6, 7] and with summing over all the Fourier modes \( K \):

\[
\Delta^{(S)}_V(\hat{n}) = \int d^3 K \xi(K) \Delta^{(S)}_V(K, \eta_0, \eta),
\tag{3.5}
\]

where \( \xi(K) \) is a random variable used to characterize the initial amplitude of each primordial scalar perturbations mode, and then the values of \( \Delta^{(S)}_V(K, \eta_0, \eta) \) are given as:

\[
\Delta^{(S)}_V(K, \mu, \eta_0) \approx \int_0^{\eta_0} d\eta \dot{\tau}_{e\gamma} e^{ix\mu - \tau_{e\gamma}} \left[ \frac{3}{2} \mu \Delta^{(S)}_{V1} - i \frac{2\tau_{e\gamma}}{3\dot{\tau}_{pc}} \left( P_2(\mu) \Delta^{(S)}_I - \Delta^{(S)}_{I2} \right) \right],
\tag{3.6}
\]

where \( x = K(\eta_0 - \eta) \). The differential optical depth \( \dot{\tau}_{e\gamma}(\eta) \) and total optical depth \( \tau_{e\gamma}(\eta) \) due to Thomson scattering at time \( \eta \) have been defined as follows:

\[
\dot{\tau}_{e\gamma} = a_n e \sigma_T, \quad \tau_{e\gamma}(\eta) = \int_0^\eta \dot{\tau}_{e\gamma}(\eta) d\eta.
\tag{3.7}
\]

The power spectrum \( C^{l(S)}_l \) due to Compton scattering in the presence of scalar perturbation is:

\[
C^{l(S)}_l = \langle \Delta^{(S) l}_I \Delta^{(S) l}_I \rangle.
\tag{3.8}
\]

Therefore, the circular power spectrum of the CMB radiation, \( C^{l_V}_l \), due to Compton scattering would be:

\[
C^{l_V}_l = \langle a_{V1}^* a_{V1} \rangle \approx \frac{1}{2l+1} \int d^3 K f^{(S)}_V(K, \tau) \int |d\Omega|^s \int_0^{\eta_0} d\tau e^{-ix\mu - \tau_{e\gamma}} \left[ \frac{2\tau_{pc}}{3\dot{\tau}_{e\gamma}} \left( P_2(\mu) \Delta^{(S)}_I - \Delta^{(S)}_{I2} \right) \right]^2.
\tag{3.9}
\]

and, with the approximation, the power spectrum of circular polarization for \( l < 2 \) can be estimated as:

\[
C^{l(S)}_l \approx \left( \frac{\dot{\tau}_{PC}}{\tau_{e\gamma}} \right)_{av}^2 C^T_l = 10^8 \delta_L^2 C^{l(S)}_l,
\tag{3.10}
\]

where \( C^{l(S)}_l \) is the power spectrum of temperature fluctuation. In addition, using eq. (3.1), we have:

\[
\left. \frac{\dot{\tau}_{PC}}{\tau_{e\gamma}} \right|_{av} \approx \frac{mv}{k^0} \frac{\delta_L}{z_{lss}} \int_0^{z_{lss}} \frac{dz}{(1+z)^3/2} \approx 10^4 \delta_L,
\tag{3.11}
\]

where \( v \approx 10^{-3} \) is the bulk velocity at present time, \( k^0 = 2.7 \) Kelvin and \( z_{lss} = 1100 \) indicates red-shift at the last scattering surface.

\[\text{JCAP01(2019)052}\]
4 Conclusion

In this work, the assumption related to an asymmetry in the number density of left- and right-handed electrons in the universe motivated the authors to calculate the dominated contribution of this asymmetry for the power spectrum of CMB circular polarization $C_l^{V(S)}$. The quantum Boltzmann equation approach was employed in the present study. In this study, by solving the quantum Boltzmann equation for the density matrix of CMB photons as well as CMB Stokes parameters, it is demonstrated that the damping term of polarized Compton scattering in the presence of scalar perturbation can generate circular polarization in CMB radiation. The generated $C_l^{V(S)}$ is proportional to $C_l^{I(S)}$ and $\delta^2$. Based on our results, to generate circular polarization in CMB radiation, the bulk velocity of cosmic electrons should be none zero $v_e \neq 0$. An interesting point is the converting anisotropy intensity $\Delta_I$ to circular polarization [see also refs. [39, 46]] while the mean mechanism of the most theoretical works which is studied the generation of CMB circular polarization is Faraday conversion (converting linear to circular polarization).

Most observational groups have reported an upper limit around $\Delta_V/T_{CMB} < 10^{-4} \sim \Delta_T/T_{CMB}$ [14, 16] which means that $C_l^{V(S)} \leq C_l^{I(S)}$. If this upper limit is applied, the fraction of polarized electron number density to the total one should be less than $\delta < 10^{-4}$. As shown in eqs. (2.25)–(2.27), the polarized Compton scattering can generate the B-mode polarization in the presence of the scalar perturbation, thereby affecting the value of E-mode polarization and the anisotropy of CMB temperature [61].

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