The critical radiation intensity for direct collapse black hole formation: dependence on the radiation spectral shape

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ABSTRACT

It has been proposed that supermassive black holes (SMBHs) are originated from direct-collapse black holes (DCBHs) that are formed at \( z > 10 \) in the primordial gas in the case where \( \text{H}_2 \) cooling is suppressed by strong external radiation. In this work, we study the critical specific intensity \( J_{\text{crit}} \) required for DCBH formation for various radiation spectral shapes by a series of one-zone calculations of a collapsing primordial-gas cloud. We calculate the critical specific intensity at the Lyman–Werner (LW) bands \( J_{\text{crit}}^{\text{LW,21}} \) (in units of \( 10^{-21} \text{erg s}^{-1} \text{Hz}^{-1} \text{sr}^{-1} \text{cm}^{-2} \)) for realistic spectra of metal-poor galaxies. We find that \( J_{\text{crit}} \) is not sensitive to the age or metallicity for the constant star formation galaxies with \( J_{\text{crit}}^{\text{LW,21}} = 1300–1400 \), while \( J_{\text{crit}} \) decreases as galaxies become older or more metal-enriched for the instantaneous starburst galaxies. However, for the young (the age < 100 Myr) and/or extremely metal poor (\( Z < 5 \times 10^{-4} Z_\odot \)) instantaneous starburst galaxies, such dependence is not strong and \( J_{\text{crit}}^{\text{LW,21}} = 1000–1400 \). We also find that \( J_{\text{crit}} \) is solely determined by the ratio between the \( \text{H}^– \) and \( \text{H}_2 \) photodissociation rate coefficients, \( k_{\text{H}–,\text{pd}}/k_{\text{H}_2,\text{pd}} \), with which we develop a formula to estimate \( J_{\text{crit}} \) for a given spectrum. The higher value of \( J_{\text{crit}} \) for the realistic spectra than those expected in the literature significantly reduces the estimated DCBH number density \( n_{\text{DCBH}} \). By extrapolating the result of Dijkstra, Ferrara & Mesinger, we obtain \( n_{\text{DCBH}} \sim 10^{-9} \text{cMpc}^{-3} \) at \( z = 10 \), which is roughly consistent with the observed number density of high-redshift SMBHs \( n_{\text{SMBH}} \sim 10^{-9} \text{cMpc}^{-3} \) at \( z \sim 6 \), considering large uncertainties in the estimation.

Key words: galaxies: high-redshift – quasars: supermassive black holes – cosmology: theory.

1 INTRODUCTION

Observations reveal that almost all galaxies host supermassive black holes (SMBHs) at their centres (Ferrarese & Merritt 2000; Gebhardt et al. 2000; Gültekin et al. 2009). Although SMBHs play an important role in the cosmic history by their radiative activities via accretion of surrounding gas, their origin has remained one of the most puzzling mysteries in astrophysics. The discovery of SMBHs with inferred black hole mass \( > 10^9 M_\odot \) at \( z > 6 \) (Fan et al. 2001; Mortlock 2012; Venemans et al. 2013) suggests that SMBH seeds are formed very early in the history of the Universe. In order for remnants of first generation (Pop III) stars (\( M_{\text{PopIII}} \sim 100 M_\odot \)) to be seeds of SMBHs, surrounding gas is needed to accrete at the Eddington limited rate for the entire period of accretion from \( \sim 100 \) to \( > 10^7 M_\odot \). However, the Eddington limited accretion is likely to be prevented by radiative feedback (Johnson & Bromm 2007; Alvarez, Wise & Abel 2009; Milosavljević et al. 2009).

A possible solution to this problem is that SMBH seeds are not remnants of Pop III stars but direct-collapse black holes (DCBHs) that are formed by direct collapse of supermassive stars (SMSs) with mass \( > 10^4 M_\odot \) (Bromm & Loeb 2003). SMSs are expected to be formed from primordial-gas clouds in haloes with virial temperature \( T_{\text{vir}} \sim 10^5 K \), in the case where the clouds collapse isothermally with the temperature of gas \( T_{\text{gas}} \sim 8000 K \) via atomic cooling in the absence of \( \text{H}_2 \) molecules due to strong external radiation (Omukai 2001, hereafter OO1).1 In such a case, it has been shown that fragmentation of the gas is suppressed (Bromm & Loeb 2003; Regan, Haehnelt & Viel 2007; Regan & Haehnelt 2009; Inayoshi, Omukai & Tasker 2014) and that the large accretion rate is expected to continue until SMSs (and subsequently DCBHs) are formed (Hosokawa, Omukai & Yorke 2012; Hosokawa et al. 2013).

1 Other physical mechanisms such as shock heating of gas (Inayoshi & Omukai 2012; Visbal, Haiman & Bryan 2014a) are proposed to suppress \( \text{H}_2 \) formation, but we concentrate on the case of strong external radiation in this paper.

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In this paper, we calculate the critical specific intensities of external radiation $F^\text{crit}$ required for DCBH formation. External radiation reduces the abundance of H$_2$ in two ways: one is by direct photodissociation of H$_2$ with the Lyman–Werner (LW) photons (photons with energies 11.2 eV < $h\nu$ < 13.6 eV); the other is by photodissociation of the intermediary H$^+$ of the dominant H$_2$ formation channel with the photons with energies $\gtrsim$0.76 eV. For the fixed spectral shape of radiation, $F^\text{crit}$ is defined as the critical specific intensity $J^\text{crit}(h\nu = 12.4 \text{ eV})$ (in units of $10^{-21}$ erg s$^{-1}$ Hz$^{-1}$ sr$^{-1}$ cm$^{-2}$) at the centre of the LW bands $(11.2 \text{ eV} < h\nu < 13.6 \text{ eV})$. The critical intensity $F^\text{crit}$ has been obtained in various physical conditions, by using one-zone calculations (O01; Omukai, Schneider & Haiman 2008; Inayoshi & Omukai 2011; Wolcott-Green & Haiman 2011) or three-dimensional hydrodynamic simulations (Shang, Bryan & Haiman 2010, hereafter S10; Latif et al. 2014).

The feasibility of the SMBH formation scenario via DCBH can be tested by comparing the estimated DCBH number density $n_{\text{DCBH}}$ with the observed high-redshift SMNB number density $n_{\text{SMNB}} \sim 10^{-3}{\text{Mpc}}^{-3}$ at $z \sim 6$ (Fan et al. 2001; Venemans et al. 2013). Although $F^\text{crit}$ has wide range of varieties depending on physical conditions (O01; Omukai et al. 2008; S10; Inayoshi & Omukai 2011; Wolcott-Green & Haiman 2011, Latif et al. 2014), they are in general much higher than the averaged cosmic LW background in the whole history of the Universe (see, e.g. O’Shea & Norman 2008; Johnson, Dalla Vecchia & Khochfar 2013a). To be concrete, $J^\text{crit}_{\text{LW21}} = O(10) - O(10^5)$, while $J_{\text{nr,LW21}} \lesssim 0.1$. Thus, $J > J^\text{crit}$ is achievable only in the rare situations when a primordial-gas cloud is irradiated by strong radiation from unusually nearby and/or bright galaxies, and the fraction of primordial-gas clouds with $T^\text{vir} \gtrsim 10^2 \text{ K}$ that can form DCBHs $(J > J^\text{crit})$ is very small. In the literature (Dijkstra et al. 2008; Agarwal et al. 2012, hereafter A12; Agarwal et al. 2014; Dijkstra, Ferrara & Mesinger 2014, hereafter D14; Yue et al. 2014), $f (J > J^\text{crit})$ was obtained from semi-analytical calculations to estimate $n_{\text{DCBH}}$. They showed that the clouds with $J > J^\text{crit}$ are distributed at the high-J tail of the probability density, and that even a small change in the value of $J^\text{crit}$ causes significant difference to the predicted value of $n_{\text{DCBH}}$. Thus, precise determination of $J^\text{crit}$ is very important in estimating $n_{\text{DCBH}}$.

It is known that $J^\text{crit}$ strongly depends on the spectral shape of external radiation (O01). While $J^\text{crit}_{\text{LW21}} = O(10)$ for the blackbody spectrum with $T_{\text{rad}} = 10^4 \text{ K}$ (S10), $J^\text{crit}_{\text{LyC11}} = O(1000)$ for that with $T_{\text{rad}} = 10^5 \text{ K}$ (Wolcott-Green, Haiman & Bryan 2011, hereafter WG11). The blackbody spectrum with $T_{\text{rad}} = 10^2$ and $10^4 \text{ K}$ have frequently been used as approximate spectra of Pop II and Pop III galaxies, respectively, in the literature. However, the hardness of realistic spectra ranges between that of the above two blackbody spectra (Leitherer et al. 1999; Schaerer 2003; Inoue 2011), and thus actual values of $J^\text{crit}$ realized in the Universe are not clear yet. In this paper, we study the dependence of $J^\text{crit}$ on spectra and obtain $J^\text{crit}$ for realistic spectra of galaxies calculated by the stellar population synthesis models (Leitherer et al. 1999; Schaerer 2003; Inoue 2011). This paper is organized as follows. In Section 2, we describe our one-zone model used to calculate the evolution of primordial-gas clouds under external radiation. In Section 3.1, we review physical processes proceeding during the evolution of the clouds, showing several results of our one-zone calculations. We determine $J^\text{crit}$ for the blackbody spectra with various temperatures in Section 3.2, and for realistic spectra in Section 3.3. In Section 3.4, we find the key parameter determining the dependence of $J^\text{crit}$ on spectra, and develop a formula to estimate $J^\text{crit}$ based on this parameter. Finally, we present the summary and discussion of this work in Section 4.

### 2 MODEL

#### 2.1 Basics

In this paper, we use a one-zone model, as described in O01, to follow the gravitational collapse of primordial-gas clouds. By neglecting effects due to rotation or magnetic fields for simplicity, the gravitational collapse is expected to proceed like the self-similarity solution (Larson 1969; Penston 1969; Yahil 1983). It has been confirmed that this simplified dynamical evolution actually describes the essential part of the gravitational collapse in three-dimensional hydrodynamic simulations (S10; Latif et al. 2014). The quantities computed in one-zone models correspond to those in the nearly homogeneous central core of the self-similarity solution. The chemical, thermal and radiative processes are solved in detail. In the following, we briefly explain the basics of our one-zone model, which is almost the same as the literature (O01; Omukai et al. 2008; S10), but with updated microphysics.

For the dynamical evolution, we assume that the collapse of clouds proceeds as

$$\frac{dt}{d\tau} = \frac{\rho_B}{(t_{\text{ff}})^{2}},$$

where $t_{\text{ff}} = \sqrt{3\pi/32G}\rho_{\text{vir}}^{-2}$ is the free-fall time, $G$ the gravitational constant, $\rho = \rho_B + \rho_{\text{DM}}$ the total density, $\rho_{\text{B}}$ the baryonic density and $\rho_{\text{DM}}$ the dark matter (DM) density. We assume that the evolution of $\rho_{\text{DM}}$ is described by the spherical top-hat collapse model until $\rho_{\text{DM}}$ reaches the virial density (see, e.g. O01). At that, we keep $\rho_{\text{DM}}$ constant. We assume that the size of central core equals the Jeans length,

$$\lambda_J = \sqrt{\frac{\pi k T_{\text{gas}}}{G\rho_{\text{B}}\mu_{\text{HI}}}},$$

where $\mu_\text{HI}$ is the proton mass, $\mu$ the mean molecular weight and $k$ the Boltzmann constant.

For the chemical evolution in primordial-gas clouds, we solve the chemical network of nine species, $H, H_2, e, H^+, H^-, H_2^+, He, He^+$ and $He^{++}$, with the chemical reaction rates of Glover & Abel (2008). In this work, we do not consider deuterium since the inclusion of it should make no difference to our results. Deuterium becomes important only in the situation when the gas is cooled below a few hundred kelvin by HD cooling (see, e.g. Nagakura & Omukai 2005; McGreer & Bryan 2008; Nakauchi, Inayoshi & Omukai 2014). In the following, we denote the number density of hydrogen nuclei as $n$, that of helium nuclei as $n_n$, and that of species $A$ as $n(A)$. We also denote the abundance of species $A$ normalized by $n$ as $y(A) = n(A)/n$. The chemical evolution is affected by external radiation via photodissociation processes, which we explain in detail in Section 2.2.

The temperature evolution is described by the energy equation

$$\frac{d\rho_B}{d\tau} = -\frac{d\rho_{\text{B}}}{d\tau} \left( \frac{1}{\rho_B} \right) - \frac{\Delta_{\text{net}}}{\rho_B},$$

where $e = p/\rho_B(y_{\text{ad}} - 1)$ is the internal energy per unit mass of baryon, $p = \rho_{\text{B}}k T_{\text{ad}}/\mu_{\text{HI}}$ the pressure and $y_{\text{ad}}$ the adiabatic...
exponent. The net cooling rate per unit volume $\Lambda_{\text{net}}$ is given by $\Lambda_{\text{net}} = \Lambda_{\text{phot}} + \Lambda_{\text{HI}} + \Lambda_{\text{chem}}$, where $\Lambda_{\text{phot}}$, $\Lambda_{\text{HI}}$, and $\Lambda_{\text{chem}}$ are the cooling rates due to radiative cooling by Ly$\alpha$ (Aanninos et al. 1997) and $H_2$ (Glover & Abel (2008) with the local thermodynamic equilibrium (LTE) value by Hollenbach & McKee (1979)), and due to chemical reaction (Shapiro & Kang 1987), respectively.

We start the calculation at the turnaround time, when the motion of the gas and DM turns from expansion to collapse. We assume that the turnaround time is at $z = 16$, and that initial values for physical quantities are given by $n = 4.5 \times 10^{-3}$ cm$^{-3}$, $T_{\text{gas}} = 21$ K, the ionizing degree $y(e) = 3.7 \times 10^{-4}$ and the $H_2$ fraction $y(H_2) = 2 \times 10^{-6}$, reflecting the condition of the universe at $z = 16$ (Omukai et al. 2008). It has been confirmed that the results are almost independent of the initial conditions as long as realistic values are chosen (Omukai et al. 2008).

2.2 The effects of external radiation on the $H_2$ abundance

In this section, we briefly review the key processes determining the $H_2$ abundance under the influence of external radiation (for more detailed review, see e.g. 001). As explained in the introduction, primordial-gas clouds collapse via atomic cooling in the case where strong external radiation suppresses $H_2$ cooling. In the following, we review two $H_2$ formation and dissociation channels and three photodissociation processes.

Let us start with reviewing the $H_2$ formation channel via intermediary $H^+$, which is the dominant $H_2$ formation channel in most cases. This channel begins with the $H^+$ formation reaction,

$$H + e \rightarrow H^+ + \gamma,$$

which is followed by the $H_2$ formation reaction,

$$H^+ + H \rightarrow H_2 + e.$$

We denote the reaction rate coefficients for equations (4) and (5) as $k^{(1)}_{\text{form}}$ [cm$^3$ s$^{-1}$] and $k^{(2)}_{\text{form}}$ [cm$^3$ s$^{-1}$], respectively. In the above chain, not all $H^+$ molecules formed via equation (4) are used for $H_2$ formation but some of them go back to $H$ by the $H^+$ photodissociation reaction,

$$H^+ + \gamma \rightarrow H + \gamma.$$

Here, the photodissociation rate coefficient, denoted as $k_{H^+}\text{pd}$ [s$^{-1}$], is proportional to the number density of photons of external radiation. The rates of competing reactions given by equations (5) and (6) determine the branching ratio of formed $H_2$ to be used for $H_2$ formation. Since the reactions of equations (5) and (6) proceed much faster than that of equation (4), the formation rate of $H_2$ per unit volume per unit time can be written as $k^{(\text{eff})}_{\text{form}} n(H) n(e)$, where the effective $H_2$ formation rate coefficient $k^{(\text{eff})}_{\text{form}}$ [cm$^3$ s$^{-1}$] is given by

$$k^{(\text{eff})}_{\text{form}} \equiv k^{(1)}_{\text{form}} \left[ \frac{k^{(2)}_{\text{form}} n(H)}{k^{(2)}_{\text{form}} n(H) + k_{H^+}\text{pd}} \right].$$

Next, we would like to review another $H_2$ formation channel via intermediary $H_2^0$, which is less effective than the $H_2$ formation channel via $H^+$ in most cases. This channel begins with the $H_2^0$ formation reaction,

$$H + H^+ \rightarrow H_2^0 + \gamma,$$

which is followed by the $H_2$ formation reaction,

$$H_2^0 + H \rightarrow H_2 + H^+.$$

This channel can be regarded as an analogue of the $H_2$ formation channel via $H^+$. In this case, however, the reaction chain begins with collision of $H$ with $H^+$ instead of $e$. In a similar way to the $H_2$ formation channel via $H^+$, not all the $H_2^0$ molecules formed via equation (8) are used for $H_2$ formation due to the $H_2^+$ photodissociation reaction,

$$H_2^+ + \gamma \rightarrow H + H^+.$$

where the photodissociation rate coefficient $k_{H^+}\text{pd}$ [s$^{-1}$] is proportional to the density of photons of external radiation. Here, again, the rates of competing reactions given by equations (9) and (10) determine the branching ratio of formed $H_2^+$ to be used for $H_2$ formation. In principle, the $H_2$ formation channel via $H_2^+$ can overwhelm that via $H^+$ by suppressing only latter by $H^+$ photodissociation. However, it is unlikely to be realized in our calculations since the strength of $H_2^+$ and that of $H^+$ photodissociation are closely related, as explained in the last part of this section.

There are two important $H_2$ dissociation channels. When $n$ is small, the dominant channel is the $H_2$ photodissociation reaction,

$$H_2 + e \rightarrow 2H,$$

where the $H_2$ photodissociation rate coefficient $k_{H_2}\text{pd}$ [s$^{-1}$] is proportional to the density of photons of external radiation. On the other hand, when $n$ is large, the dominant channel is the collisional dissociation reaction,

$$H_2 + H \rightarrow 3H,$$

where we denote the collisional dissociation rate coefficient as $k_{H_2}\text{cd}$ [cm$^3$ s$^{-1}$]. The main $H_2$ dissociation channel changes at $n \sim 10^3$ cm$^{-3}$ for the case where external radiation with realistic spectra is strong enough to form DCBH.

In the following, we review the three photodissociation processes due to external radiation: $H_2$, $H^+$ and $H_2^+$ photodissociation.

First, let us review $H_2$ photodissociation given by equation (11). $H_2$ photodissociation is one of the key processes in our calculations because it suppresses the $H_2$ abundance by directly dissociating $H_2$ molecules. As in the literature (e.g. Omukai et al. 2008), we estimate the photodissociation rate coefficient $k_{H_2}\text{pd}$ as

$$k_{H_2}\text{pd} \approx k_{H_2}\text{pd} J_{\text{LW}},$$

where $k_{H_2}\text{pd}$ = $1.4 \times 10^6$ (in cgs unit) and $J_{\text{LW}} = J(h\nu = 12.4\text{ eV})$ is the specific intensity of external radiation at the centre of LW bands (11.2 eV $< h\nu < 13.6\text{ eV}$). Although, strictly speaking, $k_{H_2}\text{pd}$ depends on the spectral shape of external radiation, we use the above value throughout this paper because the spectral shape dependence of $k_{H_2}\text{pd}$ is not strong, owing to the narrow frequency range of the LW bands (see Fig. 1). Note that even for very soft spectra, e.g. the blackbody spectrum with $T_{\text{rad}} = 10^4$ K, with non-negligible frequency dependence within the LW bands, $k_{H_2}\text{pd}$, defined with the intensity at the centre of LW bands, does not change significantly: in Draine & Bertoldi (1996), it was shown that $k_{H_2}\text{pd} = 1.4 \times 10^6$ (in cgs unit) obtained for the radiation corresponding to the blackbody spectrum with $T_{\text{rad}} = 2.9 \times 10^4$ K changes to $k_{H_2}\text{pd} = 1.1 \times 10^6$ for that with $T_{\text{rad}} = 1.3 \times 10^4$ K. Thus, we expect that the error due to estimating $k_{H_2}\text{pd}$ by equation (13) does not make large difference to our results.

The intensity in the LW bands is self-shielded by $H_2$ molecules when the $H_2$ column density of the central core $N_{H_2}$ becomes large. We take this effect into account in our one-zone model by multiplying the intensity in the LW bands by a self-shielding factor $f_{\text{sh}}$. It seems that there is no complete agreement on the form of $f_{\text{sh}}$ yet (see WG11 and Richings, Schaye & Oppenheimer 2014, hereafter R14), although, in principle, it should be determined uniquely by studying the effective amount of self-shielding with level-by-level radiative
Finally, let us comment on $\mathrm{H}_2^+$ photodissociation given by equation (10). The $\mathrm{H}_2^+$ channel of the $\mathrm{H}_2$ formation may become the dominant channel when $\mathrm{H}^-$ photodissociation suppresses the $\mathrm{H}^-$ channel, which is dominant without any photodissociation. In the following, we check which channel is dominant, in the case where the $\mathrm{H}_2^+$ channel, as well as the $\mathrm{H}^-$ channel, is suppressed by photodissociation. We use the cross-section $\sigma_{\mathrm{H}_2^+}^\mathrm{pd}(\nu, T_{\mathrm{gas}})$ given by Stancil (1994, $T_{\mathrm{gas}} > 2000$ K) and Mihajlov et al. (2007, $T_{\mathrm{gas}} < 2000$ K). The cross-section $\sigma_{\mathrm{H}_2^+}^\mathrm{pd}$ depends on $T_{\mathrm{gas}}$ because $\mathrm{H}_2^+$ is easier to be dissociated from excited states, which are assumed to be populated according to the LTE distribution with $T_{\mathrm{gas}}$. In the case where $T_{\mathrm{gas}} \sim 8000$ K, the frequency range contributing to $\mathrm{H}_2^+$ photodissociation is wider than that contributing to $\mathrm{H}^-$ photodissociation, while $\sigma_{\mathrm{H}_2^+}$ is smaller than $\sigma_{\mathrm{H}^-}$ by an order of magnitude at $\nu > 0.76$ eV, as shown in Fig. 1, and thus the frequency-integrated $\mathrm{H}_2^+$ photodissociation rate coefficient $k_{\mathrm{H}_2^+}^\mathrm{pd}$, defined in a similar way to equation (17), becomes also large in the case $k_{\mathrm{H}_2^-}^\mathrm{pd}$ is large. Therefore, the $\mathrm{H}_2^+$ channel is always subdominant even if the photodissociation processes are considered.

3 RESULTS

3.1 The evolution of primordial-gas clouds under external radiation

In this section, we review physical processes proceeding during the gravitational collapse of primordial-gas clouds under external radiation, showing several results of our one-zone calculations. Following the argument in O01, we see how the evolutionary trajectories bifurcate to the atomic and $\mathrm{H}_2$ cooling tracks depending on the strength and spectral shape of external radiation. We make calculations of collapsing clouds irradiated by the blackbody spectra with $T_{\mathrm{rad}} = 10^4$ and $10^5$ K. We specify the strength of radiation using $J_{\mathrm{LW}, 21}$, in units of $10^{-21}$ erg s$^{-1}$ Hz$^{-1}$ sr$^{-1}$ cm$^{-2}$.

The results are shown in Fig. 2, where it can be clearly seen that the evolutionary trajectories bifurcate to one of two types of tracks: the atomic and $\mathrm{H}_2$ cooling tracks. In the case where the atomic cooling track is chosen, the clouds evolve almost isothermally with $T_{\mathrm{gas}} \sim 8000$ K by atomic cooling. On the other hand, in the case where the $\mathrm{H}_2$ cooling track is chosen, the evolutionary trajectories rapidly merge to the $\mathrm{H}_2$ cooling track when $\mathrm{H}_2$ cooling becomes effective and the clouds cool down to $T_{\mathrm{gas}} \lesssim 1000$ K. By increasing the cross-section, the trajectories get closer to the atomic cooling track, and finally merge to the atomic cooling track at $J_{\mathrm{LW}, 21} = 100$ and 10 000 for $T_{\mathrm{rad}} = 10^4$ and $10^5$ K, respectively. The atomic cooling track is chosen in the case where $\mathrm{H}_2$ molecules needed for $\mathrm{H}_2$ cooling are suppressed by the strong external radiation.

The conditions of the gas when $n$ is close to the critical density $n_{\mathrm{cr}}$ are crucial in determining which track is finally chosen (O01). Here, $n_{\mathrm{cr}}$ is defined as the density above which the population of the internal states of $\mathrm{H}_2$ is determined by the LTE distribution due to sufficient collisional excitation. For both vibrational and rotational excitations of $\mathrm{H}_2$ molecules, $n_{\mathrm{cr}}$ is about $10^5$ cm$^{-3}$ for $T_{\mathrm{gas}} \sim 8000$ K. The former and latter excitations are closely related to the collisional dissociation and cooling processes, respectively. When $n > n_{\mathrm{cr}}$, $\mathrm{H}_2$ molecules are easily dissociated via collisional dissociation and, in addition, cooling rate per $\mathrm{H}_2$ molecule saturates and $\mathrm{H}_2$ cooling becomes less effective than compressional heating. On the other hand, the $\mathrm{H}_2$ formation channel given by equations (4) and (5) becomes effective as $n$ increases. Thus, although $\mathrm{H}_2$ becomes easier to be formed as $n$ increases until $n_{\mathrm{cr}}$, once a trajectory passes through $n_{\mathrm{cr}}$,
Equation (19) helps us to physically understand the $T_{\text{rad}}$ and $J_{\text{H}_2}$ dependence of the evolution. To begin with, we explain why the clouds under the radiation with the same $J_{\text{H}_2,21} = 100$ evolve along the atomic cooling track in the case $T_{\text{rad}} = 10^4$ K but along the H$_2$ cooling track in the case $T_{\text{rad}} = 10^5$ K. In the $T_{\text{rad}} = 10^4$ K case, we obtain $k_{\text{H}_2} \approx 5 \times 10^{-6}$ s$^{-1}$ with equation (17) and $k_{\text{form}} n_{\text{cr}} \sim 1 \times 10^{-6}$ s$^{-1}$ with $n_{\text{cr}} \sim 10^3$ cm$^{-3}$ and $k_{\text{form}} = 1.3 \times 10^{-7}$ cm$^3$ s$^{-1}$ (Glover & Abel 2008). Thus, the second square bracket of equation (19) is significantly smaller than 1, meaning H$^+$ photodissociation plays a role in suppressing H$_2$ formation. On the other hand, in the $T_{\text{rad}} = 10^5$ K case, $k_{\text{H}_2} \approx 1 \times 10^{-9}$ and the second square bracket of equation (19) is almost 1, meaning the effect of H$^+$ photodissociation is negligible. Therefore, the two clouds with different $T_{\text{rad}}$ evolve along different tracks although the strength of H$_2$ photodissociation is the same due to the same $J_{\text{H}_2}$.

Next, we explain why the clouds under the radiation with same $T_{\text{rad}} = 10^5$ K evolve along the atomic cooling track in the case $J_{\text{H}_2,21} \leq 1000$ but along the H$_2$ cooling track in the case $J_{\text{H}_2,21} = 10000$. Even in the case $J_{\text{H}_2,21} = 10000$, the second square bracket of equation (19) is almost 1 and the effect of H$^+$ photodissociation is negligible. In the case $J_{\text{H}_2,21} = 10000$, however, H$_2$ photodissociation is very strong and the first square bracket of equation (19) becomes very small. Therefore, the cloud under the radiation with $T_{\text{rad}} = 10^5$ K and $J_{\text{H}_2,21} = 10000$ evolutions along the atomic cooling track by suppressing the H$_2$ abundance with strong H$_2$ photodissociation without any help of H$^+$ photodissociation.

### 3.2 $F^{\text{crit}}$ for blackbody spectra

In this section, we present $F^{\text{crit}}$ for the blackbody spectra with temperatures $7000 \lesssim T_{\text{rad}} \lesssim 200000$ K, to understand the dependence of $F^{\text{crit}}$ on the hardness of the spectrum of external radiation.$^3$ We calculate $F^{\text{crit}}$ with different forms of self-shielding factors because there is some disagreement on the form of self-shielding factor, as mentioned in Section 2.2. We also make comparison of our results with the literature (S10 and WG11). In practice, $F^{\text{crit}}$ is calculated by the bisection method by examining whether $T_{\text{gas}}$ is larger or smaller than 4000 K at $n = 10^3$ cm$^{-3}$. In this section, we use both $J_{\text{H}_2}$ and $J_{\text{H}_2,21}$ to specify the strength of radiation, in order to make it easier to compare our result with the literature ($J_{\text{H}_2,21}$ was used in, e.g. O01, S10, WG11). Other than this section, however, we preferentially use $J_{\text{H}_2}$ because the specific intensity at the LW bands is more directly related to the physics we are interested in.

The results for our fiducial model, in which the self-shielding factor of WG11 is used, are shown as the red lines in Fig. 3. As $T_{\text{rad}}$ increases, $F^{\text{crit}}$ becomes larger: $F^{\text{crit}}_{21,\text{LW}}$ becomes 2 at $T_{\text{rad}} = 10^4$ K and $F^{\text{crit}}_{21,\text{LW}} = 1200$ at $T_{\text{rad}} = 2 \times 10^5$ K. However, the $T_{\text{rad}}$ dependence of $F^{\text{crit}}_{21,\text{LW}}$ becomes very weak for the case $T_{\text{rad}} \gtrsim 3 \times 10^5$ K, for which $F^{\text{crit}}_{21,\text{LW}} \sim 1400$ and is almost constant.

The $T_{\text{rad}}$ dependence of $F^{\text{crit}}$ can be understood with equation (19) in a similar manner to that in Section 3.1. As $T_{\text{rad}}$ decreases, $F^{\text{crit}}$ becomes smaller because H$^+$ photodissociation suppress H$_2$ formation more effectively [the second square bracket of equation (19) becomes smaller]. For high $T_{\text{rad}}$ ($T_{\text{rad}} \gtrsim 3 \times 10^5$ K), the dependence of $F^{\text{crit}}_{21,\text{LW}}$ on $T_{\text{rad}}$ is weak, because the effect of H$^+$ photodissociation

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$^3$ Similar dependence was studied by Wolcott-Green & Haiman (2012), who obtained the $T_{\text{rad}}$ dependence of the critical specific intensity required for minihaloes with virial temperature $T_{\text{vir}} = 10^4$ K to form stars, instead of that of $F^{\text{crit}}$ for DCBH formation studied here.
is negligibly small [the second square bracket of equation (19) is almost unity].

### 3.2.1 Influence of using different self-shielding factors

In the following, we discuss the influence of using different forms of self-shielding factors $f_{\text{sh}}$. The form of $f_{\text{sh}}$ derived in Draine & Bertoldi (1996, hereafter DB96) has been widely used in the literature (e.g. O01; S10). However, WG11 modified it to reproduce the results of their radiative transfer calculations with three-dimensional hydrodynamic simulations for gas with $T_{\text{gas}} \sim 8000$ K. Recently, R14 proposed another form of $f_{\text{sh}}$, arguing that the form derived in WG11 underestimates the strength of self-shielding compared to their radiative transfer calculations with CLOUDY (Ferland et al. 1998).

In this section, we introduce the three different forms of $f_{\text{sh}}$. First, the form of $f_{\text{sh}}$ derived in WG11 is given by equation (14). Secondly, the form derived in DB96 is given by

$$f_{\text{sh}}(N_{H_2}) = \min \left[ 1, \left( \frac{N_{H_2}}{10^4 \text{ cm}^{-2}} \right)^{-3/4} \right].$$  \hspace{1cm} (20)

Thirdly, the form derived in R14 is given by

$$f_{\text{sh}}(N_{H_2}, T_{\text{gas}}) = \frac{1 - \alpha_{H_2}(T_{\text{gas}})}{(1 + x')^{0.5}} \exp \left[ -5 \times 10^{-7} (1 + x') \right]$$

$$+ \frac{\alpha_{H_2}(T_{\text{gas}})}{(1 + x')^{0.5}} \exp \left[ -8.5 \times 10^{-4} (1 + x')^{0.5} \right],$$  \hspace{1cm} (21)

where

$$x' = \frac{N_{H_2}}{N_{\text{crit}}(T_{\text{gas}})}.$$

### 3.3 $J_{\text{crit}}$ for realistic spectra

In this section, we present $J_{\text{crit}}$ for realistic spectra considering various models of source galaxies and intergalactic medium (IGM) radiative transfer. It is needed to obtain $J_{\text{crit}}$ for each realistic spectrum, because realistic spectra in general do not look like the blackbody spectra and cannot be parametrized with a single parameter like $T_{\text{rad}}$. To obtain the spectra of galaxies, we adopt the spectral model by Inoue (2011), who added nebular lines and continua to the stellar population synthesis models of Schaerer (2003) for $Z/Z_\odot = 0$ (Pop III) and $Z/Z_\odot = 5 \times 10^{-4}$, and of starburst99 (Leitherer et al. 1999) for $Z/Z_\odot = 0.02$ and 0.2 by a metallicity-dependent way.

The models of galaxies and IGM radiative transfer explored in this paper are summarized in Table 1. Since we are interested in the early universe, we focus on metal-poor galaxies with $Z/Z_\odot = 0$ (Pop III), $5 \times 10^{-4}$, 0.02 and 0.2, where the solar metallicity $Z_\odot = 0.02$. We assume a Salpeter-type initial mass function (IMF) with the stellar mass range 1–100 M_\odot. We consider two types of star formation: instantaneous star formation (IS) and constant star formation (CS).

**Table 1.** Galaxy/IGM models explored.

| IMF                                      | Salpeter IMF with 1–100 M_\odot (fixed) |
|-------------------------------------------|------------------------------------------|
| Metallicity ($Z/Z_\odot$)                | 0 (Pop III), $5 \times 10^{-4}$, 0.02, 0.2 |
| SF type: age                             | 1, 10 Myr, 100 Myr–1 Gyr$^a$            |
| Escape fraction ($f_{\text{esc}}$)       | 0, 0.5                                   |
| IGM absorption (Lyα)                     | Complete/no absorption                   |
| IGM absorption (Lyβ)                     | Complete absorption (fixed)              |

$^a$IS galaxies with age between 100 Myr and 1 Gyr are studied with the step width of 0.1 on a logarithmic scale.
The critical LW intensity $\alpha_{\text{crit}}$, for realistic spectra of the IS galaxies with $Z = 0$, $5 \times 10^{-4}$, 0.02 and $0.2Z_\odot$. We assume complete Ly$\alpha$ absorption and $f_{\text{esc}} = 0$. The horizontal axis is the time since the burst. $J_{\text{LW},21}^{\text{esc}} \sim 1400$ at 1 Myr since the burst irrespective of metallicity. In the case of the CS galaxies, $J_{\text{LW},21}^{\text{esc}} = 1300–1400$, irrespective of the metallicity and the duration of SF.

As examples, we show the spectra of the $Z = 0$ (Pop III) and $Z/Z_\odot = 0.2$ galaxies in Fig. 4. The spectra are roughly flat in the frequency range $h\nu \lesssim 10$ eV due to the superposition of the stellar emission with various effective temperatures. The number of the LW photons from the old IS galaxies is exponentially suppressed, because the high-temperature stars that contribute to producing the LW photons no longer emit radiation in such galaxies due to their short lifetimes.

The results of $F^{\text{crit}}$ for realistic spectra are summarized as follows. While $J_{\text{LW},21}^{\text{esc}}$ depends on $Z$ and the duration of SF, $F^{\text{crit}}$ has a wide range of values for the IS galaxies depending on the models, as shown in Fig. 5. We plot only the cases of complete Ly$\alpha$ absorption and $f_{\text{esc}} = 0$ in Fig. 5, because the effects of changing Ly$\alpha$ absorption and $f_{\text{esc}}$ make at most 5 per cent difference to the values of $F^{\text{crit}}$, although the Ly$\alpha$ line and the nebular emission contribute to $H^-$ photodissociation and slightly reduce the value of $F^{\text{crit}}$. The critical intensity $F_{\text{crit}}$ decreases as the IS galaxies become older or more metal-enriched, although the dependence is weak for the young or extremely metal poor galaxies. For the young galaxies with the time since burst less than 100 Myr, $J_{\text{LW},21}^{\text{esc}} = 1000–1400$, while for the extremely metal poor and not very old galaxies ($Z \lesssim 5 \times 10^{-4} Z_\odot$ and the time since burst is less than 500 Myr), $J_{\text{LW},21}^{\text{esc}} \approx 1400$ and is almost constant.

### 3.4 The reason for the dependence of $F^{\text{crit}}$ on spectra

In this section, we explain the reason for the dependence of $F^{\text{crit}}$ on the spectral shape of external radiation, as seen in Sections 3.2 and 3.3, by pointing out the key parameter determining $F^{\text{crit}}$. We then develop a method to estimate $F^{\text{crit}}$ for a given spectrum without calculating the evolution of the clouds.

#### 3.4.1 The key parameter determining $F^{\text{crit}}$

In this section, we propose a hypothesis that the ratio between the $H^-$ and $H_2$ photodissociation rates, $k_{\text{H}^-}/k_{\text{H}_2}$, is the key parameter determining the dependence of $F^{\text{crit}}$ on spectra, and prove its
validity in the following. We come up with this hypothesis because, as explained in Section 3.1, in the cases strong \( \text{H}^- \) photodissociation suppresses \( \text{H}_2 \) formation, smaller \( J_{\text{LW}} \) (and hence weaker \( \text{H}^- \) photodissociation) is needed to suppress \( \text{H}_2 \) cooling. In this section, the quantity written as \( k_{\text{LW}, 1} \) is not the true value realized in clouds during the evolution but that defined by equation (13) without considering the effect of self-shielding. Here, we are interested in the quantity directly related to external radiation.

To demonstrate that \( k_{\text{LW}, 1} \) is the only parameter determining \( F^{\text{crit}} \), we obtain \( k_{\text{LW}, 1} \) for the realistic and blackbody spectra studied in this paper and plot them with \( J^{\text{crit}} \) in Fig. 6. It is clear from Fig. 6 that there is a one-to-one correspondence between \( k_{\text{LW}, 1} \) and \( F^{\text{crit}} \). In other words, \( F^{\text{crit}} \) is solely determined by \( k_{\text{LW}, 1} \). We plot only the results for the complete Ly\( \alpha \) absorption and \( f_{\text{esc}} = 0 \) cases in Fig. 6, because the effects of changing Ly\( \gamma \) absorption and \( f_{\text{esc}} \) make little difference. The \( k_{\text{LW}, 1} \) dependence of \( J^{\text{crit}} \) can be understood with equation (19), in the same manner as the \( T_{\text{rad}} \) dependence (see the last part of Section 3.2).

Note that \( k_{\text{LW}, 1} \) can be regarded as a proxy for the hardness of the spectrum. The relation between \( T_{\text{rad}} \) and \( k_{\text{LW}, 1} \) is given in Table 2. The ratio \( k_{\text{LW}, 1} \) increases as \( T_{\text{rad}} \) decreases, corresponding to the fact that \( \text{H}^- \) photodissociation becomes more effective as the spectrum becomes soft. The dependence of \( k_{\text{LW}, 1} \) on \( T_{\text{rad}} \) becomes weak for high \( T_{\text{rad}} \) (\( \geq 5 \times 10^4 \) K). This is because for such high \( T_{\text{rad}} \), the spectrum obeys the Rayleigh–Jeans law (\( J(\nu) \propto \nu^2 \)) in the frequency range contributing to \( \text{H}^- \) and \( \text{H}_2 \) photodissociation (0.76 eV < \( \nu \) < 13.6 eV).

### Table 2. The relation between \( T_{\text{rad}} \) and \( k_{\text{LW}, 1} \)

| \( T_{\text{rad}} \) (K) | \( 8 \times 10^3 \) | \( 1 \times 10^4 \) | \( 2 \times 10^4 \) | \( 3 \times 10^4 \) | \( 5 \times 10^4 \) | \( 1 \times 10^5 \) | \( 2 \times 10^5 \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( k_{\text{LW}, 1} \) | \( 8.7 \times 10^3 \) | \( 4.6 \times 10^4 \) | \( 2.1 \times 10^5 \) | \( 4.6 \times 10^5 \) | \( 1.7 \times 10^6 \) | \( 1.0 \times 10^8 \) | \( 8.1 \times 10^9 \) |

By obtaining the relation between \( k_{\text{LW}, 1} \) and \( F^{\text{crit}} \) in advance, \( F^{\text{crit}} \) for a given spectrum can be estimated from this relation without calculating the evolution of the clouds. However, there remains one uncertainty. In order to determine the evolution of the clouds, the \( \text{H}^+ \) photodissociation rate should be specified in addition to \( J_{\text{LW}} \) (which determines \( k_{\text{LW}, 1} \) by equation (13)) and \( k_{\text{LW}, 1} \). In the following, we assume \( k_{\text{LW}, 1} \) is 0.1, motivated by the fact that \( k_{\text{LW}, 1} \) is 0.1 when \( T_{\text{gas}} \approx 8000 \) K during the evolution of the clouds under the external radiation with the realistic and thermal spectra. This assumption is further justified by the fact that the detailed value of \( k_{\text{LW}, 1} \) is not important. We have also made calculations for the cases with \( k_{\text{LW}, 1} \) times larger or smaller than the true values, but have found only negligible difference in the results. The relation between \( F^{\text{crit}} \) and \( k_{\text{LW}, 1} \) obtained with the above assumption is shown as the red line in Fig. 6. For the realistic and blackbody spectra studied in this paper, this relation almost perfectly reproduces the \( F^{\text{crit}} \) from the information of \( k_{\text{LW}, 1} \).

For later convenience, we present a fitting formula to the relation given above,

\[
J^{\text{crit}}_{\text{LW}, 21} = \begin{cases} 1400 \\ 1400 \times 10^{(a_1 x + a_2 x^2)} \\ x \leq 0 \\ x > 0 \end{cases}
\]

where

\[
x = \log_{10} \left( \frac{k_{\text{LW}, 1}}{k_{\text{H}_2, 1}} \right) - 2
\]

and

\[
a_1 = -0.19, \quad a_2 = -0.12.
\]

We find that, for the realistic and blackbody spectra studied in this paper, this fitting formula reproduces the \( F^{\text{crit}} \) from \( k_{\text{LW}, 1} \) with at most 10 per cent error in the range \( 1 < k_{\text{LW}, 1} < 10^5 \). With this fitting formula, \( F^{\text{crit}} \) can be easily estimated from \( k_{\text{LW}, 1} \) for a given spectrum.

### 3.4.2 A method to estimate \( J^{\text{crit}} \) for a given spectrum

Here, we propose a simple and easy method to estimate \( F^{\text{crit}} \) for a given spectrum. As mentioned in Section 3.4.1, the relation between \( k_{\text{LW}, 1} \) and \( F^{\text{crit}} \) can be used to estimate \( F^{\text{crit}} \). However, to use this relation, the frequency integral in equation (17) is needed to be evaluated in obtaining \( k_{\text{LW}, 1} \). The information of the spectral shape is contained in \( k_{\text{LW}, 1} \) in equation (18), which can be parametrized with \( \alpha_{\text{LW}, 1} \) as

\[
k_{\text{LW}, 1} = \alpha_{\text{LW}, 1} \kappa_{\text{LW}, 1}^{(0)}
\]

where \( \kappa_{\text{LW}, 1}^{(0)} = 1.1 \times 10^{11} \) (in cgs unit) is defined as \( \kappa_{\text{LW}, 1} \) for the flat spectrum (\( J(\nu) = \text{const.} \)). The values of \( \alpha_{\text{LW}, 1} \) for the power-law spectra \( J(\nu) \propto \nu^s \) are given in Table 3. In the following, we avoid the numerical integration of equation (17) by approximating \( \kappa_{\text{LW}, 1} \)
with $k_{H-\,pd}^{(0)}$. The realistic spectra are roughly flat in the frequency range $h \nu \lesssim 10$ eV, as mentioned in Section 3.3, and thus the error due to this approximation can be estimated with the spread of $\alpha_{k_{H-\,pd}}$ around a flat spectrum and is expected to be small. By using this approximation with equations (13) and (18), we can estimate $k_{H-\,pd}/k_{H_2,\,pd}$ from $J_{2eV}/J_{LW}$ simply as

$$k_{H-\,pd}/k_{H_2,\,pd} \approx 79 J_{2eV}/J_{LW}.$$ (30)

In the light of this relation, we modify the fitting formula given by equations (26), (27) and (28) by redefining $x$ in equation (27) as

$$x = \log_{10} (79 J_{2eV}/J_{LW}) - 2.$$ (31)

To check the validity of the formula given by equations (26), (28) and (31), we obtain $J_{2eV}/J_{LW}$ for the realistic and blackbody spectra studied in this paper and compare them with $F^{\text{crit}}$ estimated from $J_{2eV}/J_{LW}$ with this formula. The result is that the formula reproduces $F^{\text{crit}}$ with at most 30 per cent error for both the realistic and blackbody spectra. Although the error is larger than the formula given in Section 3.4.1, it is still practically negligible considering an order-of-magnitude scatter of $F^{\text{crit}}$ due to the diversity in the three-dimensional structure of the clouds (S10; Latif et al. 2014).

4 SUMMARY AND DISCUSSION

By using the one-zone model described in Section 2, we have calculated the critical intensity of external radiation needed for primordial-gas clouds in haloes with $T_{\text{II}} \gtrsim 10^4$ K to form DCBHs by suppressing $H_2$ cooling. By performing series of calculations for various types of external radiation, we have examined the dependence of $F^{\text{crit}}$ on the spectral shape of external radiation.

In Section 3.2, we have seen how $F^{\text{crit}}$ changes depending on the temperature of the blackbody spectra within $7000 < T_{\text{rad}} < 200 000$ K. In Section 3.3, we have determined $F^{\text{crit}}$ for the realistic spectra of the metal-poor galaxies, by taking the data from the stellar population synthesis models. We have found that $F^{\text{crit}}$ is not sensitive to the age or metallicity for the CS galaxies with $J_{LW,\,21}^{\text{crit}} \approx 1300–1400$, while $F^{\text{crit}}$ decreases as galaxies become older or more metal-enriched for the IS galaxies. However, such dependence for the IS galaxies is weak for the young or extremely metal poor galaxies: $J_{LW,\,21}^{\text{crit}} \approx 1000–1400$ for the young (the age less than 100 Myr) galaxies and $J_{LW,\,21}^{\text{crit}} \approx 1400$ for the extremely metal poor ($Z < 5 \times 10^{-4} Z_\odot$) and not very old (the age less than 500 Myr) galaxies. It should be noted that the above values of $F^{\text{crit}}$ are obtained with $f_{\text{sh}}$ of WG11, and that those obtained with $f_{\text{sh}}$ of R14 are about two times larger than the above values, as shown in Section 3.2.1. It is important to precisely determine the form of $f_{\text{sh}}$, but it is beyond the scope of this work.

We have also found that the dependence of $F^{\text{crit}}$ on the spectral shape is totally attributable to a single parameter $k_{H-\,pd}/k_{H_2,\,pd}$ in Section 3.4.1. By using the one-to-one correspondence between $k_{H-\,pd}/k_{H_2,\,pd}$ and $F^{\text{crit}}$ and the approximate relation between $k_{H-\,pd}$ and $J_{2eV}$, we have proposed a formula given by equations (26), (28) and (31) to estimate $F^{\text{crit}}$. With this formula, $J_{2eV}$ is reproduced with at most 30 per cent error from the information of $J_{2eV}/J_{LW}$ for the realistic and blackbody spectra studied in this paper.

Let us discuss the implication of our results. In the following, we adopt $J_{LW,\,21}^{\text{crit}} = 1400$ as the fiducial value, because it is the typical value for young and metal-poor galaxies commonly present in the supposed DCBH formation era at $z \gtrsim 10$. It has been known that the value of $F^{\text{crit}}$ is much higher than the averaged cosmic LW background $J_{bg,\,LW,\,21} \lesssim 0.1$ in the whole history of the Universe (see, e.g., O’Shea & Norman 2008; Johnson et al. 2013a), and thus the clouds need to be irradiated by unusually nearby and/or strong sources to achieve $J_{LW} > J_{LW,\,21}^{\text{crit}}$. By performing semi-analytical computations with $N$-body simulations (A12) and Monte Carlo simulations (D14), A12 and D14 estimated the DCBH number density $n_{\text{DCBH}}$ with assumption that DCBHs are formed in all the atomic cooling haloes with $J_{LW} > J_{LW,\,21}^{\text{crit}}$. However, they may have overestimated $n_{\text{DCBH}}$ due to the smaller values of $F^{\text{crit}}$ used in their estimation ($J_{LW,\,21}^{\text{crit}} = 30$ and 300 are used in A12 and D14, respectively).

We re-estimate $n_{\text{DCBH}}$ with our $J_{LW,\,21}^{\text{crit}} = 1400$ by extrapolating the results of A12 and D14. The estimate of $n_{\text{DCBH}}$ changes from $n_{\text{DCBH}} \sim 10^{-3}$ to $10^{-10}$ cMpc$^{-3}$ at $z = 10$ according to fig. Cl of D14, and from $n_{\text{DCBH}} \sim 10^{-4}$ to $10^{-6}$ cMpc$^{-3}$ at $z = 12$ according to fig. 7 of A12. Although the values of $n_{\text{DCBH}}$ estimated according to D14 and A12 do not match each other, both decrease by two or three orders of magnitude. The observed high-redshift SMBH number density is $n_{\text{SMBH}} \sim 10^{-9}$ cMpc$^{-3}$ at $z \sim 6$ (Fan et al. 2001; Venemans et al. 2013), which is in the same order as $n_{\text{DCBH}}$ estimated according to D14. In order to test the scenario of SMBH formation via DCBH by comparing predicted $n_{\text{DCBH}}$ and observed $n_{\text{SMBH}}$, it is crucial to more precisely estimate $n_{\text{DCBH}}$ in the light of our $J_{LW,\,21}^{\text{crit}} = 1400$. We would like to note that the present-day SMBH number density inferred from observed luminosity function of active galactic nuclei is $n_{\text{SMBH}} \sim 10^{-4}$ cMpc$^{-3}$ (Shankar, Weinberg & Miralda-Escudé 2009; Johnson et al. 2013b), which is several orders of magnitude higher than our estimate of $n_{\text{DCBH}}$, and it is thus unlikely that all of the SMBHs are originated from DCBHs.

In order to achieve such strong external radiation as $J_{LW,\,21}^{\text{crit}} \gtrsim 1400$, source galaxies need to be very close to the DCBH-forming haloes. In such cases, we expect that dynamical interactions between the sources and clouds cannot be overlooked, and thus the above extrapolation of the results of A12 and D14 may be no longer correct. One possibility is that large fraction of the pairs merge to single larger haloes due to the gravitational interactions before forming DCBHs, as suggested by cosmological simulations (Chon et al., in preparation). Another is that radiation from one of pairs of haloes to another realizes $J_{LW,\,21} \gtrsim 1000$ during the synchronized evolution of the pairs (Visbal, Haiman & Bryan 2014b). In any case, it is necessary to understand how strong external radiation $J_{LW,\,21} \gtrsim 1400$ is realized, by studying further the effects of interactions between primordial-gas clouds and radiation sources.

In this paper, we have studied the dependence of $F^{\text{crit}}$ on spectra of external radiation. However, $F^{\text{crit}}$ also depends on other physical conditions of the clouds and their environment. Inayoshi & Omukai (2011) found that $F^{\text{crit}}$ increases in the presence of cosmic ray and/or X-ray, although it is not clear yet how much cosmic ray and/or X-ray are emitted from the same galaxy as the source of radiation. Omukai et al. (2008) found that the conditions of metallicity allowed them to form DCBHs, although it is not yet clear how $F^{\text{crit}}$ changes in the case where the metallicity is very small but not exactly zero.D14 and Agarwal et al. (2014) phenomenologically took into account the effect of metal enrichment from the same galaxy as the source of radiation in their simulations. S10 and Latif et al. (2014) found that $F^{\text{crit}}$ has an order-of-magnitude scatter due to three-dimensional structures of the clouds, such as shocks and turbulence. R14 argued that self-shielding of the LW photons is suppressed by the turbulence in the clouds due to the Doppler broadening of lines. On the other hand, the LW photons irradiated on the clouds may be reduced by the Lyman series absorption of neutral hydrogen in the IGM.

In future studies, it is important to determine the probability distribution of $F^{\text{crit}}$, considering various physical conditions of clouds and their environment. By comparing $n_{\text{DCBH}}$ predicted with such
probability distribution and observed $n_{\text{SMBH}}$, a high-precision test of the SMBH formation scenario via DCBH becomes possible.

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