Quantum effects amplification due to energy dissipation in parametrically driven qubit-cavity systems

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Abstract. A qubit system coupled to a resonator may be treated as a quantum metamaterial. Quantum nature of such a system results in large variety of non-trivial effects. In particular, we study a parametrically driven system of qubits coupled to a single-mode resonator and show that the energy dissipation in the resonator can enhance population of the excited state of the qubits and generate the entanglement between qubits. The energy dissipation in the qubit subsystem can enhance the number of photons generated from vacuum. We demonstrate that these effects survive in the steady state as well. The optimal parameters for these two cases of dissipation-induced quantum effects amplification are shown to be results of fine balance of different processes in this hybrid system.

1. Introduction
A key feature of a quantum metamaterial is a coherent quantum dynamics. One of the possible quantum object which can be used as an element of such a metamaterial is a superconducting qubit [1]. It has prominent perspectives from experimental point of view due to its high degree of flexibility and tunability [2], and especially due to the possibility of dynamical changing of the coupling strength [3, 4, 5]. Thus, the investigation of parametrically driven systems is important for fundamental science and applications.

2. Hamiltonian and basic equations
We study a hybrid system of qubits coupled to a single-mode quantum resonator. The Hamiltonian of the system reads as

\[
\mathcal{H}(t) = \omega a^\dagger a + \sum_j \epsilon_j \sigma_j^+ \sigma_j^- + \sum_j g(t)(\sigma_j^+ + \sigma_j^-)(a^\dagger + a),
\]  

(1)
where $a^\dagger$ and $a$ are photon creation and annihilation operators, $\sigma^+_j$, $\sigma^-_j$ are Pauli operators acting in the subspace of $j$-th qubit degrees of freedom, and $g(t)$ is dynamically tunable coupling constant. The last term of the Hamiltonian is related to the interaction of qubits and photons in the resonator. It contains two contributions. The first one is rotating wave term, $V_1 = \sum_j g(t)(\sigma^+_j a + \sigma^-_j a^\dagger)$). The second one is counter-rotating wave term, $V_2 = \sum_j g(t)(\sigma^+_j a^\dagger + \sigma^-_j a)$. $V_2$ term can be essential for correct description of the system’s dynamics when the coupling constant is periodically modulated [6, 7, 8].

To take into account the decoherence we solve numerically the Lindblad equation

$$\partial_t \rho(t) - \Gamma[\rho(t)] = -i[H(t), \rho(t)],$$

where $\rho(t)$ is time-dependent full density matrix of the systems. We assume that the bath is Markovian so that the decoherence depends on the rates of energy dissipation in the resonator $\kappa$, in the qubit $\gamma$, and on the pure qubit dephasing rate $\gamma_\varphi$:

$$\Gamma[\rho] = \kappa \left( a_\rho a^\dagger - \frac{1}{2} \{a^\dagger a, \rho\} \right) + \gamma \left( \sigma^-_\rho \sigma^+_\rho - \frac{1}{2} \{\sigma^+_\rho \sigma^-_\rho, \rho\} \right) + \gamma_\varphi (\sigma^z_\rho \rho \sigma^z_\rho - \rho).$$

3. The effect of energy relaxation on qubit entanglement

For the simplicity we study in this section two identical qubits coupled to the resonator. We assume that initially qubits and resonator are in their ground states, $|\downarrow\downarrow, 0\rangle$. The effect of coupling constant modulation is strongest when parametric resonance takes place, i.e. the modulation frequency is twice the resonator frequency [9, 10]. To further simplify the description we separate fast and slow degrees of freedom and perform time averaging over the fast ones. This procedure eliminates fast oscillations with the frequency of the photon field which have small amplitude. Only oscillations with much smaller frequencies of the order of the Rabi frequency survive. In the limit $\{g(t)\} \ll \omega$, the system’s dynamics is controlled by two Fourier components [9, 10], $\langle g(t) \rangle_t \equiv p$ and $\langle g(t) \exp(-2i\omega t) \rangle_t \equiv q$. The only energy scale of the problem is defined by the coupling strength $g_0$ and we represent $p$ and $q$ as $p = g_0\theta$, $q = g_0(1 - \theta)$, where $0 \leq \theta \leq 1$. Parameters $p$ and $q$ control the coupling strength in $V_1$ and $V_2$ channels respectively.

**Figure 1.** Decoherence-free time evolution of quantum concurrence.

**Figure 2.** Time evolution of quantum concurrence at $\kappa = 0.2g_0$, $\gamma = 0$.

We start from the case of zero decoherence. Fig. 1 demonstrates the time evolution of quantum concurrence $C$, which we calculate with use of a standard procedure [11]. The time evolution of $C$ reveal oscillations with the Rabi frequency. Relatively large maximum value of
\( C \) results from the resonance between excitation energies of qubits and the resonator frequency. As one can expect, non-zero value of the relaxation in the resonator blurs the picture as it is shown in Fig. 2. The effect of non-zero qubit relaxation is also quite expected — the concurrence oscillations become smaller with time and finally concurrence disappears. Surprisingly, turning on both relaxations simultaneously can stabilize concurrence, so that it remains non-zero in steady state even for non-zero qubit relaxation. This effect takes place only when \( \theta > 1/2 \), as it is illustrated in Fig. 3, 4.

![Figure 3](image1.png) **Figure 3.** Time evolution of quantum concurrence at \( \kappa = 0.1g_0, \gamma = 0.2g_0 \).

![Figure 4](image2.png) **Figure 4.** Time evolution of quantum concurrence at \( \kappa = g_0, \gamma = 0.2g_0 \).

Steady state analysis shows that there is an optimal value of resonator relaxation rate \( \kappa \) as it is illustrated in Fig. 5. A non-zero \( \kappa \) allows to create a kind of a circle consisting of three processes. At first, the system is excited from the lowest energy state \(|↓↓, 0\rangle\) to the state \(|↑↓, 1\rangle + |↓↑, 1\rangle\) via \( V_2 \) process. Then states \(|↑↑, 0\rangle\) and \(|↓↓, 2\rangle\) are produced by \( V_1 \) process and \(|↑↑, 0\rangle\) state contributes to \( C \). Non-zero \( \kappa \) leads to the decay of \(|↓↓, 2\rangle\) state to the initial state \(|↓↓, 0\rangle\) and thus closes the cycle. This explains positive role of \( \kappa \) for the quantum concurrence stabilization. For complete analysis the other processes should be taken into account as well. In particular, the decay of \(|↑↓, 1\rangle + |↓↑, 1\rangle\) state may suppress the occupation of \(|↑↑, 0\rangle\) state via \( V_1 \) process. Thus, quantum concurrence appears as a result of a fine balance between different processes.

As seen in Fig. 6, in general the behavior of the mutual information \( I \) is close to the one of \( C \). Significant difference exists in the region \( \theta \leq 1/2 \), where \( C = 0 \), but \( I > 0 \). It means that in this case the qubits are correlated but not entangled.

In steady state zero is the optimal qubit relaxation rate for \( C \) due to the energy dissipation destroys quantum effects in the same channel. As a result, the less qubit relaxation rate the larger the steady state quantum concurrence.

4. The effect of energy relaxation on occupation numbers
For the simplicity in this section we study single qubit coupled to the resonator. It is parametrically excited from its ground state by the counter-rotating term of the Hamiltonian. Results for qubit occupation number in the steady state, \( n_q \), are provided in Fig. 7. It demonstrates a non-trivial behavior — the largest effect of parametric excitation achieved for some optimal value of \( \kappa \). This feature can be treated as a consequence of fine balance of counter-rotating processes and energy relaxation in resonator. The influence of \( \gamma \) is quite predictable — it suppress the population of qubit’s excited state.
The mean photon number in the steady state, $n_{\text{ph}}$, shows tricky behavior. The maximum value of $n_{\text{ph}}$ is achieved at $\theta = 0.5$ and $\gamma = 0$. This regime indicates that the smaller the energy dissipation, the stronger the quantum effects. In addition, there is another local maximum of $n_{\text{ph}}$ at $\theta = 1$ and at finite $\gamma$. These two maxima are connected by a kind of an arc with locally enhanced $n_{\text{ph}}$. As $\kappa$ grows and approaches $g_0$, this local maximum become a global one. Thus, the enhancement of photon generation from vacuum due to the energy dissipation in the qubit subsystem starts to dominate at non-zero threshold of resonator relaxation $\sim g_0$. As one may expect, the increase of $\kappa$ lead to suppression of mean photon number.

We also analyze the influence of pure dephasing $\gamma_{\varphi}$ on our results. Generally, it suppresses the amplification of quantum effects, but this suppression is negligible as long as condition $\gamma_{\varphi} \lesssim \gamma$ is valid.

5. Conclusion
We studied theoretically the dynamics of parametrically driven system of qubits coupled to a single-mode resonator. We demonstrated that the energy dissipation does not always lead to the suppression of quantum effects. Instead, energy dissipation in one subsystem can greatly
increase quantum effects in the other subsystem. Thus, non-zero value of the resonator decay rate can stabilize quantum concurrence and quantum mutual information of two qubits and allows them to survive even in the steady state despite of non-zero qubit relaxation rate. Non-zero qubit relaxation rate, in turn, leads to the amplification of photon generation from vacuum. This effect is more pronounced when coupling constant modulation is strong.

Our results may be useful for further research and development of quantum metamaterials.

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