MEGABLOCKS: EFFICIENT SPARSE TRAINING WITH MIXTURE-OF-EXPERTS

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ABSTRACT

We present MegaBlocks, a system for efficient Mixture-of-Experts (MoE) training on GPUs. Our system is motivated by the limitations of current frameworks, which restrict the dynamic routing in MoE layers to satisfy the constraints of existing software and hardware. These formulations force a tradeoff between model quality and hardware efficiency, as users must choose between dropping tokens from the computation or wasting computation and memory on padding. To address these limitations, we reformulate MoE computation in terms of block-sparse operations and develop new block-sparse GPU kernels that efficiently handle the dynamism present in MoEs. Our approach never drops tokens and maps efficiently to modern hardware, enabling end-to-end training speedups of up to 40% over MoEs trained with the state-of-the-art Tutel library and 2.4× over DNNs trained with the highly-optimized Megatron-LM framework.

1 INTRODUCTION

Exploiting sparsity in the weights, activations and input data of deep neural networks (DNNs) is an effective technique for reducing the amount of computation that is needed to achieve a given model quality (Han et al., 2015; Gale et al., 2019). The past decade has seen significant progress in algorithms and high-performance software to make sparsity practically useful (Gray et al., 2017; Narang et al., 2017; Kalchbrenner et al., 2018; Elsen et al., 2020; Gale et al., 2020). One area that remains a challenge for sparsity is model training on accelerators. DNNs are most commonly trained on hardware accelerators like GPUs (NVIDIA, 2020) and TPUs (Jouppi et al., 2017), which exploit the regularity of dense computation to deliver high performance. Consequently, fine-grained sparse computation is less efficient on these processors. To enable efficient computation on accelerators, structure can be enforced on the sparse matrices (Narang et al., 2017; Gray et al., 2017; Yao et al., 2019).

An emerging class of models with underlying structured sparsity is Mixture-of-Experts (MoEs) (Shazeer et al., 2017). Each layer in an MoE is a collection of experts, which are themselves small DNNs. As data is passed through the MoE layers, each token is dynamically routed to a subset of the experts for computation. By exploiting this sparse computation, MoEs have reduced training times by as much as 4× for applications in natural language processing and computer vision (Artetxe et al., 2021; Riquelme et al., 2021). These gains have translated to new levels of scale for model training, pushing model sizes past 1 trillion parameters (Artetxe et al., 2021; Du et al., 2021; Fedus et al., 2022).

The challenge in computing MoEs efficiently is handling the dynamic routing and load-imbalanced computation that are fundamental to these architectures. However, existing hardware and software for deep learning make it difficult to meet this challenge. For example, TPUs and their XLA compiler require all tensor shapes to be known statically and often struggle with fine-grained operations like scatters and gathers (Fedus et al., 2022). These constraints make it difficult to implement MoEs directly on TPUs. While GPUs are more flexible, the sparse computation in MoEs does not map cleanly to the software primitives supported in major frameworks and libraries.

State-of-the-art frameworks for MoE training sidestep these challenges by placing rigid constraints on MoE routing. In order to remove the dynamism from the computation, the set of tokens mapped to each expert are trimmed or padded to a user-specified size (Lepikhin et al., 2020; Fedus et al., 2022; Hwang et al., 2022). This procrustean formulation introduces a tradeoff between model quality and hardware efficiency, as users must decide whether to drop tokens or waste computation and memory on padding. This decision is often made through hyperparameter tuning, which increases the complexity of using MoEs.

To address these challenges, we develop an approach for MoE routing and computation based on sparse primitives. Our approach never drops tokens and maps efficiently to modern GPUs, enabling end-to-end training speedups of up to 40% and 2.4× over state-of-the-art frameworks for MoE and DNN training, respectively. We make the following
specific contributions:

- We show how the computation in an MoE layer can be expressed as block-sparse operations to accommodate imbalanced assignment of tokens to experts. We use this formulation to train dropless-MoEs (dMoEs).
- We develop high-performance GPU kernels for block-sparse matrix products that efficiently handle dynamic MoE computation. Our kernels use two techniques, blocked-CR-CSR encoding and transpose indices, to enable efficient matrix products with sparse inputs and outputs in transposed or non-transposed order.

We have implemented these techniques in a system called MegaBlocks, which builds on the state-of-the-art Megatron-LM library for training Transformer models (Shoeybi et al., 2019). We evaluate our system through both microbenchmarks and end-to-end training of Transformer language models.

## 2 Background: MoE Layers

MoE layers are made up of many experts, which are themselves small neural networks. Each token\(^1\) is dynamically routed to a subset of the experts for computation based on scores computed by a router. The experts are commonly defined to be small multi-layer perceptrons (MLPs). It is typical for tokens to be sent to a small number of experts, often between 1 and 4 (Fedus et al., 2022).

MoE layers are often interleaved with other DNN layers and are most commonly used to replace the feed-forward network (FFN) layers in Transformers (Shazeer et al., 2017; Fedus et al., 2022). This hybrid architecture has demonstrated strong results on both natural language and vision tasks (Du et al., 2021; Riquelme et al., 2021). It is conjectured that these improvements are a result of experts specializing to different parts of the data distribution (Shazeer et al., 2017).

We illustrate an MoE layer in Figure 1 and describe it in detail in the remainder of this section.

### 2.1 Routing

The first stage of an MoE layer is the router, which is responsible for determining the assignment of tokens to experts. In addition to expert assignments, MoE routers also produce probabilities for each assignment that reflect the confidence of the mapping. These weights are encoded as a matrix of scores for each token-expert pair, which are used to linearly combine the top \(k\) expert outputs for each token (see §2.4).

The most common style of MoE routing is the learned router proposed by Shazeer et al. (2017). In this router, the tokens are projected from hidden size elements to num experts scores by multiplying with a weight matrix that is learned jointly with the other model parameters. The scores are normalized with a softmax and the routing decisions are made by greedily selecting the top \(k\) scoring experts for each token.

### 2.2 Permutation

State-of-the-art MoE implementations aim to compute all expert layers in parallel in order to make effective use of the parallelism available on GPUs and TPU (Lepikhin et al., 2020; Fedus et al., 2022; Hwang et al., 2022). The standard primitive used by implementations is batched matrix
The capacity_factor can be thought of as a parameter that reduces the chance of dropping a token. This hyperparameter represents a tradeoff between additional computation and model quality. As such, it is desirable to minimize the amount of load imbalance in the assignment of tokens to experts. The typical mechanism for doing so is auxiliary load balancing losses, which incentivize the router to produce a balanced assignment (Shazeer et al., 2017; Lepikhin et al., 2020; Fedus et al., 2022). These losses additionally help to ensure that all experts see a similar number of tokens during training. This is thought to be important to avoid degenerate states where some experts are assigned zero tokens and stop receiving gradient updates (Zhou et al., 2022).

In addition to enabling batched computation of the expert layers, these constraints allow all tensor shapes to be known statically, which is required by TPUs and XLA.

### 2.3 Computation

Once the data has been permuted, the experts can be computed in parallel. For models where the experts are MLPs, this entails computing each layer for all experts using batched matrix multiplication. For convolutional experts, the layers can be computed with grouped convolutions.

### 2.4 Un-permutation

After the experts are computed, the resulting feature vectors are un-permuted such that their ordering matches that of the input to the layer. The last step in MoE computation is to scale the output tokens by the scores with which they were assigned to their respective experts. When tokens are routed to more than one expert, these weighted results are summed to produce the final layer output for each token.

### 3 Motivation: Token Dropping in MoEs

Despite the use of load balancing losses, prior work has shown that token routing is still highly imbalanced (Hwang et al., 2022). To quantify the effect of token dropping on model quality, we trained MoE language models on The Pile (Gao et al., 2020) with a range of capacity factors. We train Transformer MoEs similar to those used by Fedus et al. (2022), where each model is a Transformer with the FFN layers replaced with 64-expert MoE layers where each expert is a 2-layer MLP matching the original FFN dimensions. We used top-1 routing and based our MoE model dimensionality, which computes a set of matrix products of the same shape (see Figure 3A). However, mapping MoE computation to this primitive is non-trivial. In order to respect the shape constraints of batched matrix multiplication, the experts must be constrained to have weight matrices of the same shape and the number of tokens assigned to each expert must be equal. The latter constraint is particularly problematic because the learned routing algorithm described above provides no guarantees of a load balanced assignment of tokens to experts.

In order to satisfy this constraint, prior work has defined a fixed expert capacity, which is the number of tokens that each expert can be assigned (Lepikhin et al. (2020); Fedus et al. (2022)). If the number of tokens assigned to an expert exceeds its capacity, the extra tokens are dropped. That is to say, they are not passed to any expert for computation and the model relies on a residual connection to reintroduce the dropped tokens’ representation after the MoE layer. If an expert layer is not assigned enough tokens to fill its capacity, its set of tokens is padded to fill the remaining space. Expert capacity is typically specified in terms of a capacity_factor hyperparameter, which is a multiplier on the expected number of tokens that would be assigned to each expert under a perfect uniform distribution:

$$\text{expert\_capacity} = \frac{\text{num\_tokens}}{\text{num\_experts}} \times \text{capacity\_factor}$$

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Compute a set of independent matrix multiplications of
the same size in parallel.

Expert computation can equivalently be computed using block diagonal matrix
products with equal sized blocks along the diagonal.

We can enable load imbalanced routing and variable sized experts by
expressing expert computation as block sparse matrix multiplication.

Figure 3. Expert Computation in an MoE Layer. Shown with num_expert=3. (A) State-of-the-art MoE implementations use batched
matrix multiplication to compute all experts within a layer in parallel. This introduces the constraints that all experts are assigned the same
number of tokens and that all experts have the same shape. (B) Expert computation can be analogously posed in terms of block diagonal
matrix multiplication with identically sized blocks. (C) In order to relax these constraints, we can construct a block diagonal matrix with
variable sized blocks made up of many smaller blocks. We can compute this matrix efficiently using block-sparse matrix multiplication.

sions on the Transformer-Small model described in Table
1. All models were trained using the tokenization from
GPT2 (Radford et al., 2019) for 10B tokens with sequence
length 1024, the Adam optimizer, and the learning rate and
gradient clipping settings from Shoeybi et al. (2019). We
trained all models on a single A100 GPU with a batch size
of 512 sequences. We trained MoEs with capacity factor 1, 1.5, and 2 as well as the dynamic capacity factor tech-
nique proposed by Tutel (Hwang et al., 2022), where the
capacity factor is set dynamically to the minimum value
that would avoid token dropping. As a baseline, we trained
standard Transformer models across a range of sizes. All
Transformer and MoE models have vocabulary size 51200,
sequence length 1024 and an attention head size of 64. Our
model configurations are summarized in Table 1 and the
results of the experiments are shown in Figure 2.

For these models, we observed that the impact of token
dropping is significant. While the MoE with capacity factor
of 1 achieved a 0.15 reduction in validation loss, the MoE
that avoided dropping tokens provided a reduction of 0.26,
1.73× larger than the gain of the former model and enough
to exceed the quality of Transformer-Medium.

While dropping tokens reduces model quality, increasing
capacity factor comes at the cost of additional computation
and memory. In this example, MoE-layer math operations
increased by over 2× in order to avoid dropping tokens.
Hwang et al. (2022) showed that some MoEs require capac-
ity factors as high as 11 in order to avoid dropping tokens,
and other models where the necessary capacity factor to
avoid dropping tokens spiked unpredictably during training.

In addition to the computational overhead of increasing the
capacity factor, having to tune an additional hyperparameter
can significantly increase the number of models that need to
be trained for a target task. This is particularly cumbersome
for large neural networks, where the cost to train a single
model can run into the hundreds of thousands of dollars
(MosaicML, 2022). Possibly as a result of this, some large
studies on MoEs have declined to explore different capacity
factors at all (Artetxe et al., 2021; Clark et al., 2022).

4 No-Token-Left-Behind with Block Sparsity

This section describes how we formulate MoE layer com-
putation in terms of block-sparse computation in order to
avoid dropping tokens. The motivation for using block-
sparse primitives to express MoE computation is manifold.
First, as we show below, block-sparse matrices are a nat-
ural and flexible way of describing the dynamic and
toal balanced computation in MoEs. Second, block sparsity
maps efficiently to hardware accelerators built around syst-
olic array matrix multipliers like GPUs and TPUs. Because
of the coarse granularity of MoE experts, we can select a
block size for our implementation that is large enough to
enable the computation to realize high fractions of peak
device throughput. Lastly, block-sparse kernels like matrix
multiplication and convolution are general-purpose primi-
tives that are useful across a range of applications (Narang
et al., 2017; Gray et al., 2017; Child et al., 2019; Elsen et al.,
2020). This makes investment in high-performance ker-
nels more practical, as work can be amortized across target
tasks. We could similarly invest in variable sized batched
matrix multiplication kernels, but the utility of this would
be limited to MoE architectures as they are designed today.

In addition to these considerations, the block-sparse formu-
lation of MoEs exposes a new perspective on these algo-
rithms as a form of dynamic, structured, activation sparsity.
This perspective draws parallels to much of the literature on sparse training algorithms and opens up the opportunity to further improve MoEs with insights from this adjacent field.

**Preliminaries: Sparse Matrix Product Notation.** In the remainder of this paper we often refer to matrix multiplication where one of the three matrices (the two inputs and one output) is sparse and the others are dense. We borrow the notation from Triton (Tillet et al., 2019) to describe these different operations. Each operation is described with a three character string where each character is either “S” for sparse or “D” for dense. The order of characters is output, followed by the left input followed by the right input. For example, the product of two dense matrices with a sparse output is “SDD”, which is also referred to as sampled dense-dense matrix multiplication (SDDMM). This notation is useful to distinguish operations like DSD and DDS, which are different forms of sparse matrix-dense matrix multiplication (SpMM). Superscript “T” indicates transposition of the input arguments. For example, SDD$^T$ indicates an SDD where the right-hand input matrix is transposed.

### 4.1 Expert Computation With Block Sparsity

The key insight behind our method is shown in Figure 3. Rather than the prevailing approach of computing the experts within an MoE layer using batched matrix multiplication, we could equivalently compute the experts as an SDD where the output sparse matrix has block diagonal structure, as shown in Figure 3B. In this formulation, allowing for a load-imbalanced assignment of tokens to experts is analogous to allowing for the blocks in the block diagonal matrix to have a variable number of rows. To achieve this, we propose to compute each block as many smaller fixed size blocks using block-sparse matrix multiplication, as shown in Figure 3C. To construct multi-layer experts, we can iterate between SDD and DSD operations (see Figure 6).

In this formulation, we could also relax the constraint on the number of columns in each block to build MoE layers with variable sized experts, as is shown in Figure 3C. While this is an interesting direction for future work, we did not explore these configurations as more research is needed to identify how this capability can be used to increase efficiency.

With sufficiently large blocks, block-sparse matrix multiplication is capable of reaching high fractions of peak throughput on modern GPUs (Gray et al., 2017; NVIDIA, 2021). The coarse-grained sparsity in MoEs lends itself to this requirement - in Transformer models using MoE FFN layers, the number of columns in the blocks shown in Figure 3B corresponds to $\text{ffn\_hidden\_size}$, which is commonly between 1024 and 8192 (Vaswani et al., 2017; Radford et al., 2019; Brown et al., 2020). The number of rows in these blocks corresponds to the number of tokens assigned to each expert, which is expected to be equal to the number of tokens divided by the number of experts under a uniform distribution. This can range from a few thousand to tens of thousands of tokens per expert (Lepikhin et al., 2020; Artetxe et al., 2021; Fedus et al., 2022). These coarse-grained blocks are many times larger than the largest tile dimensions used for dense matrix multiplication kernels, which give us the flexibility to select a block size that can match their throughput.

### 5 MegaBlocks: A Framework for Efficient MoE Training

We implemented our techniques in a system called MegaBlocks, which builds on Megatron-LM (Shoeybi et al., 2019) and PyTorch (Paszke et al., 2019). In addition to high-performance dropless-MoE (dMoE) layers, our system supports distributed training of MoEs with both data and expert model parallelism (Fedus et al., 2022).

This section discusses the design of our dMoE implementation, including our block-sparse kernels, and other considerations for building an efficient system. §5.1.1 discusses the limitations of existing block-sparse kernels. §5.1.2 analyzes the effects of the block size on block-sparse product performance. §5.1.3 describes our hybrid blocked-CSR-COO sparse matrix format, which enables efficient matrix products with sparse input and output operands. §5.1.4 introduces transpose indices as a mechanism for efficient iteration over block-sparse matrices in transposed order. Lastly, §5.2 discusses efficient routing and permutation for dMoEs.

**Preliminaries: Matrix Multiplication on GPUs.** Matrix multiplication kernels on GPUs exploit tiling, where the output matrix is broken up into statically sized two-dimensional blocks of values (NVIDIA, 2022c). The computation of...
these tiles can be parallelized, and the individual tiles can be sized to tradeoff arithmetic intensity and parallelism. The group of threads assigned to a tile is called a threadblock.

### 5.1 Efficient Block-Sparse Kernels for MoEs

To train MoEs with block-sparse kernels we need primitives for the forward and backward passes. Consider an MoE FFN layer where each expert is a 2-layer MLP. For this configuration, the forward pass requires an SDD operation followed by a DSD (Figure 6). For the backward pass, we compute SSD$^T$ and DS$^T$D for the second layer data gradient and weight gradient, respectively, followed by DSD$^T$ and DD$^T$S for the first layer data gradient and weight gradient, respectively.

#### 5.1.1 Existing Block-Sparse Primitives

We considered two existing libraries for block-sparse matrix multiplication on GPUs: NVIDIA cuSPARSE (NVIDIA, 2022b) and Triton Blocksparse (Tillet et al., 2019). cuSPARSE supports the blocked-ELL sparse matrix format for DSD. However, as of CUDA 11.8 this operation does not support transposition of the sparse matrix input. cuSPARSE also provides no SDD primitive with a blocked-ELL matrix. In addition to these limitations, the blocked-ELL format requires that all rows in the sparse matrix have the same number of nonzeros, which would defeat our goal of supporting load imbalanced matrices. Blocksparse supports SDD, DSD, and DDS as well as all combinations of transposed and non-transposed inputs. However, these primitives assume that the topology of the sparse matrices does not change between invocations. The library API takes a bitmask describing the sparse operand and then pre-computes look-up tables and block groupings to accelerate computation. For our use case, the sparse matrix topology varies across every iteration of training and every MoE layer in the model. In order to use Blocksparse, we would have to pay the cost of these preprocessing steps repeatedly.

Based on this analysis, we opted to write our own block-sparse primitives in order to tailor them to the dynamism of MoE expert computation. We implemented SDD, DSD, and DDS operations targeting NVIDIA GPUs. Our kernels support all combinations of transposed and non-transposed inputs. The remainder of this section details the design and implementation of our kernels.

#### 5.1.2 Selecting Block Size for MoEs

In order to efficiently use modern GPUs, we want to use sparse blocks that have sufficient arithmetic intensity to keep matrix multiplication units busy. Large blocks are also desirable to amortize the cost of storing and operating on sparse matrix metadata, since metadata like column indices only need to be kept for each block of nonzeros.

To select our target block size, we studied the performance of dense matrix multiplication kernels from NVIDIA CUTLASS (NVIDIA, 2022c) with different tile dimensions. We benchmarked mixed-precision (FP16 + FP32 accumulation) matrix multiplication on square matrices with power of two side lengths from 512 to 16384 and every set of tile dimensions supported in CUTLASS. For rectangular tiles, we show only the configurations where the first tile dimension is larger as we found these to slightly outperform the alternative ordering for these problems. We ran all benchmarks on an A100 SXM4 80GB GPU with CUDA 11.5 and CUTLASS 2.5. These benchmarks are shown in Figure 4.

Across these benchmarks, we observed that 128x128 tiles consistently perform on-par or better than other configurations. Anecdotally, we observe that this same configuration is commonly selected by NVIDIA cuBLAS (NVIDIA, 2022a) for the dense Transformer models we studied. Based on this analysis, we opted to use 128x128 block sparsity. While the tile dimensions of a block-sparse matrix multiplication and the block size in the sparse matrix do not need to be equal, we found that for 128x128 blocks the highest performing tile dimensions in our workloads were also 128x128.

To implement our kernels, we extended CUTLASS (NVIDIA, 2022c) to support block-sparse matrices and reused their machinery for high-performance matrix multiplication with different data types and GPU architectures.
We use blocked compressed sparse row (BCSR) as our primary sparse matrix format. BCSR makes it simple to iterate across the nonzeros in a row, which is necessary for operations like DSD and DDS\textsuperscript{T}. Iterating over blocks also has minimal overhead with BCSR, as identifying a block's position in the matrix only requires a single load of its column index. We discuss our approach for efficiently iterating across the nonzeros in a column with this format in §5.1.4.

One challenge with BCSR sparse matrices is efficiently computing SDD operations in parallel. On kernel launch, each threadblock needs to identify the row and column of its output block so that it knows which rows and columns of the input matrices are needed to compute it. Because BCSR only encodes column indices for each block, identifying the row index of a nonzero block requires a search through the row offsets. One solution to this problem is to launch the maximum number of threadblocks that could be needed to compute each row of the output if it were fully dense. On startup, each threadblock can check whether its column offset is out of range for the number of nonzeros in its row and return if there is no work to do. Gale et al. (2020) showed that the overhead introduced by launching extra threadblocks was negligible for moderately sparse matrices (50 - 90% zeros). We experimented with this approach but observed that for MoEs the cost of launching these unused threadblocks was significant, particularly for models with high expert counts where the level of sparsity in the block-sparse matrices is very high.

To efficiently parallelize SDD, we additionally materialize the row indices for each nonzero block so that threadblocks can trivially look up the coordinates of sparse blocks in the output matrix. The storage required for this additional metadata is negligible since we only need to store one index per 16384 nonzero values in a 128x128 block. Even with this additional metadata, we maintain the row-wise ordering of nonzero blocks so the matrix can be operated on as either BCSR or blocked coordinate format (BCOO). We illustrate this hybrid blocked-CSR-COO encoding in Figure 5.

Computing forward and backward passes for model training requires sparse matrix transposition. However, iterating over BCSR matrices in transposed order requires searching through each row to identify if the block in the target column is nonzero (Buluğ et al., 2009). We could materialize a transposed version of the sparse matrix explicitly, but this would incur runtime and storage costs as all of the nonzero values in the matrix would need to be copied. To enable efficient iteration over BCSR matrices in transposed order, we construct the metadata for the transposed matrix but do not explicitly transpose the nonzero values. Instead, we construct an array of indices, one for each nonzero block, which are stored in transposed order and contain the offset of each nonzero block in memory. This additional metadata allows efficient iteration through the matrix in transposed order with a layer of indirection, as shown in Figure 5.

This idea is similar to a secondary index in a database, which allows efficient access to entries in a different order than the primary index. Similar to our hybrid Blocked-CSR-COO encoding, this technique relies on the fact that storage and computation is many times cheaper for metadata than it is for nonzero values thanks to our large block sizes.

### 5.2 Efficient Routing and Permutation

As currently implemented, our block-sparse matrix multiplication kernels require the number of tokens assigned to each expert to be a multiple of the block size. In order to respect this constraint, we pad each group of tokens with zeros to the nearest multiple of 128 and fuse this operation into custom permutation kernels. We could remove this constraint by supporting partial blocks at the fringes of the problem similar to how matrix multiplication handles matrices that are not divisible by the tile dimensions. However, the performance impact of this feature would be minimal given we expect the number of tokens assigned to each expert to be thousands or tens of thousands.

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**Figure 6. Pseudo-Code for a dMoE.** The code follows Figure 1 with three changes. First, we construct the sparse matrix topology from Figure 3C from expert assignments (line 12). Second, we pad each expert batch to a multiple of the block size during permutation (line 15, §5.2). Lastly, we compute the experts in parallel by iterating between SDD and DSD operations (lines 22-23, §4.1).
Table 2. MoE Model Configurations. These models correspond to the Transformer configuration of the same size, but with each FFN layer replaced with a 64-expert MoE layer.

| MoE    | num experts | top_k | Weights (M) | GFLOPs |
|--------|-------------|-------|-------------|--------|
| XS     | 64          | 1     | 839         | 316    |
| Small  | 64          | 1     | 3,693       | 879    |
| Medium | 64          | 1     | 13,041      | 2487   |

Table 3. Micro Batch Sizes Used for Model Training. We used the largest micro_batch_size that fit in memory for all experiments.

| Model         | micro_batch_size |
|---------------|------------------|
| Megatron-LM   |                  |
| Transformer-XS| 64               |
| Transformer-Small | 32           |
| Transformer-Medium | 16           |
| Transformer-Large | 16           |
| Transformer-XL | 8               |
| MegaBlocks    |                  |
| dMoE-XS       | 64               |
| dMoE-Small    | 32               |
| dMoE-Medium   | 8                |
| Tutel         |                  |
| dMoE-XS       | 32               |
| dMoE-Small    | 8                |
| dMoE-Medium   | 1                |

Once the expert assignments have been computed by the router, we create the metadata for the block-sparse matrix using a custom CUDA kernel. We additionally construct the transposed metadata at this time to amortize the cost over the multiple block-sparse matrix multiplications that use it across forward and backward computation.

6 Experiments

This section analyzes the performance of our system compared to state-of-the-art libraries, Microsoft Tutel (Hwang et al., 2022) and NVIDIA Megatron-LM (Shoeybi et al., 2019), for training Transformer MoEs and standard Transformers respectively. In order to ensure fair comparisons, we extended Megatron-LM to additionally support MoE training using Tutel’s MoE layer. All experiments were conducted on NVIDIA A100 SXM4 80GB GPUs with CUDA 11.5, CUTLASS 2.5 and used mixed-precision training (Micikevicius et al., 2018) as implemented in Megatron-LM.

6.1 MoE Training Without Dropping Tokens

To assess the efficiency of our technique for avoiding token dropping, we compared to the dMoE method proposed by Hwang et al. (2022) where the capacity factor is set dynamically to the minimum value that avoids token dropping.

We trained decoder-only Transformer language models on The Pile (Gao et al., 2020) with the same hyperparameters described in §3. For Transformer MoEs, we trained models scaled from our XS, Small, and Medium models with each FFN layer replaced with 64-expert MoE layers using top-1 routing. We also trained standard Transformer models from 46M to 1.3B parameters, equivalent to Transformer-Base (Vaswani et al., 2017) up to GPT3-XL (Brown et al., 2020), as a dense baseline. We trained all models on 8 A100 SXM4 80GB GPUs using 8-way expert model parallelism for MoE layers and data parallelism for all other layers. We use gradient accumulation for all models and train with a batch size of 512 sequences and the largest micro_batch_size that does not run out of memory (Narayanan et al., 2021a). Our model configurations are summarized in Tables 1 and 2. For each model, we report the end-to-end training time and final loss achieved on a validation set in Figure 7.

Compared to the prevalent padding-based approach for avoiding token dropping, our technique for adaptive MoE computation with block sparsity enables end-to-end training speedups of $1.38 \times$, $2.0 \times$ and $4.35 \times$ for MoE-XS, MoE-Small, and MoE-Medium, respectively. In addition to computational overhead, the padding-based approach implemented in Tutel significantly increases the amount of memory required to store activations in the MoE layers. This is particularly problematic because MoEs already require many times more storage for their large weight matrices compared to standard Transformers. For these models, we observed this increase in memory usage reduced the maximum micro_batch_size that Tutel could use by $2 \times$, $4 \times$, and $8 \times$ compared to MegaBlocks for MoE-XS, MoE-Small, and MoE-Medium, respectively. This in turn increases training time because of reduced hardware efficiency. As a result,
we observe that the advantage of MegaBlocks over Tutel grows with model size. The micro\textit{batch size} used for each model configuration are shown in Table 3.

Compared to Transformer models trained with Megatron-LM, dMoEs trained with MegaBlocks reduce the training time required to reach a given validation loss by $1.8 \times - 2.4 \times$. The variation in this comparison is primarily a result of the increased weight memory usage of MoE models, which forced MegaBlocks to use a 2x smaller micro\textit{batch size} for MoE-Medium than the analogous Transformer model. These results highlight the importance of reducing memory usage in MoEs as a direction for future research.

For these Transformer models, we observed that Megatron-LM sustains between 21% and 48% of the 2.5 petaFLOP peak throughput of this 8-GPU system with efficiency increasing with model size. The speedups achieved by MegaBlocks over this state-of-the-art framework demonstrates the efficiency of our system and the efficacy of MoEs.

### 6.2 MoE Training With Token Dropping

We additionally compare our dMoE models to token-dropping MoEs trained with Tutel. In order to find the most efficient configurations, we trained MoE-XS, MoE-Small and MoE-Medium models with capacity factors of $1 \times$, $1.5 \times$, and $2 \times$ for a total of 9 additional models. For these configurations, all token-dropping MoE models were able to use the same micro\textit{batch size} as the analogous dMoE without running out of GPU memory. We report the end-to-end training time and validation loss for these models along with our dMoE and standard Transformer results in Figure 8. Comparing MoEs and dMoEs for the same accuracy is non-trivial because token dropping degrades model quality. For each dMoE, we estimated the runtime of the MoE that would achieve the same validation loss by comparing to the loss-equivalent point on the MoE Pareto frontier.

Even with the most efficient capacity factor for each MoE, dMoEs trained with MegaBlocks reduce the training time required to reach a given validation loss by $1.38 \times$, $1.37 \times$ and $1.18 \times$ for MoE-XS, MoE-Small and MoE-Medium respectively. In addition to significant reductions in end-to-end training time, our system reduces the cost of using MoEs by decreasing the number of hyperparameters that need to be re-tuned for each model and task. These computational savings could in turn be applied to exploring other parameters to further improve model quality.

For MoE-Medium, we observe some loss of efficiency in our implementation due to the relatively small micro\textit{batch size} that could be used while fitting in limited GPU memory. For small batch sizes, smaller tile dimensions (e.g., 64x128 or 64x64) in our block-sparse kernels could improve performance by reducing the amount of wasted computation when the problem dimensions are not divisible by 128. Another direction for increasing efficiency is to reduce the memory usage per device such that larger batch sizes can be used, either through parallelization over more devices or techniques like selective recomputation (Korthikanti et al., 2022).

### 6.3 Block-Sparse Matrix Multiplication Performance

To assess the quality of our block-sparse matrix multiplication kernels, we benchmarked the problem configurations used in training MoE-XS, MoE-Small and MoE-Medium models and compared to cuBLAS batched matrix multiplication. This includes the forward pass, backward weights, and backward data operations for the two layers in each FFN layer. In total, we benchmark 18 problems - 6 problems for each model and task. To allow for comparison with batched matrix multiplication, we benchmarked each problem with a uniform distribution of tokens to experts and the same micro\textit{batch size} listed in Table 3. These benchmarks can be viewed as an ablation assessing the overhead that would be introduced if one were to use our block-sparse kernels to implement a standard, token-dropping MoE. For each problem we averaged throughput over 100 executions. We do not include the time taken to construct the sparse matrix metadata in these benchmarks as these operations amortize over all 6 problems within an FNN layer. The results of these benchmarks are shown in Figure 9.

On these problems, we observe that our block-sparse kernels are able to realize 98.6% of the throughput of cuBLAS with a standard deviation of 4%. The maximum relative throughput was 104% and the minimum was 91%. Overall, our kernels slightly outperformed cuBLAS on half of the...
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Figure 9. Block-Sparse Matrix Multiplication Throughput Compared to cuBLAS Batched Matrix Multiplication. Benchmarked for the problem configurations used in training MoE-XS, MoE-Small and MoE-Medium models. For these problems, our block-sparse matrix multiplication kernels realize 98.6% of the throughput achieved by cuBLAS on average with a standard deviation of 4% and a maximum and minimum relative throughput of 104% and 91% respectively.

Figure 9 illustrates the throughput comparison between our block-sparse matrix multiplication kernels and cuBLAS batched matrix multiplication. The throughput is relative to cuBLAS and shows the performance of our kernels for different problem configurations used in training MoE-XS, MoE-Small, and MoE-Medium models. Our kernels realize an average throughput of 98.6% with a standard deviation of 4%, and the throughput ranges from 104% to 91%

problems and slightly underperformed on the other half.

While benchmarking CUTLASS, we observed that altering the order in which tiles of the output matrix are computed can change the throughput of the operation by as much as 10% due to L2 caching effects. We believe that most of the performance discrepancy in these results can be attributed to the re-ordering of computation that occurs with block-sparse matrices, although further investigation is needed.

One case where we note additional overhead is in the DSTD operations used to compute weight gradients. Because we use a secondary index to iterate over the sparse operand in transposed order the access patterns when iterating through this matrix exhibit little spatial locality which in turn reduces the throughput of the overall operation. While this is an interesting problem for further study, the overall impact on model performance is minimal because of the limited opportunity for improvement (<10%) combined with the relatively small amount of end-to-end runtime that these two operations represent.

7 RELATED WORK

MoE Routing. Improved routing algorithms for MoEs is an active area of research. BASE layers formulate MoE routing as a linear assignment problem trying to maximize the aggregate token-expert affinities under the constraint of a perfectly balanced assignment (Lewis et al., 2021). This method guarantees no tokens are dropped by re-routing tokens to different experts as needed. Clark et al. (2022) found that BASE layers can incur significant runtime overhead and proposed an approximate version using the Sinkhorn algorithm. Because their approximation is no longer guaranteed to avoid token dropping, Clark et al. (2022) use a capacity factor of 2 for all experiments. Other techniques have been proposed to statically decide tokens to expert mappings ahead of time based on hash functions (Roller et al., 2021). However, Clark et al. (2022) observed that this approach did not perform as well as the other routing algorithms they studied. More recently, Zhou et al. (2022) proposed to reverse the routing problem such that each expert selects its top-k scoring tokens. While this guarantees a load balanced assignment of tokens to experts, this method still suffers from token dropping because the same token can be selected by multiple experts. We expect that improved routing algorithms complement our method for efficient and flexible expert computation. Exploring how these methods could be combined is an interesting direction for future research.

High-Performance MoEs. To scale MoE training, Tutel implements optimized distributed communication primitives for MoEs and techniques for hiding the communication costs of expert model parallelism (Hwang et al., 2022). He et al. (2022) proposed FasterMoE, a system for distributed training of MoEs based on efficient communication strategies and changes to the MoE routing algorithm to avoid network congestion. Our implementation could additionally benefit from these techniques, particularly for large-scale distributed training.

Sparse Kernels. Sparse matrix formats that allow for efficient transposed access are well studied (Buluç et al., 2009; Smith & Karypis, 2015; Li et al., 2018). Exploring how these formats can be adapted to large block sparsity on modern GPUs is an interesting direction for future research.

8 CONCLUSION

We introduced MegaBlocks, a system for efficient MoE training on GPUs. Our system is based on a reformulation of MoEs in terms of block-sparse operations and new, block-sparse GPU kernels that efficiently handle the dynamism present in MoEs. Our approach never drops tokens and maps efficiently to modern hardware accelerators, enabling end-to-end training speedups of up to 40% over MoEs trained with the state-of-the-art Tutel library and 2.4× over DNNs trained with the highly-optimized Megatron-LM framework.
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