Reconstruction of multi-balloon copying curved surface based on linear distance field function optimisation

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Abstract: Aiming at the problems of low precision of three-dimensional (3D) curved surface reconstruction and poor surface smoothness of the reconstruction result, a method based on linear distance field function optimisation is proposed for the reconstruction of multi-balloon copying curved surface. First, curvature weighting is conducted for the curved surface point cloud model constructed with the Poisson’s equation (PE), and the boundary value limiting condition of average curvature is used to conduct the smoothness processing of the curved surface; second, due to the dynamic deformation characteristic of multi-balloon copying curved surface, the linear distance field function is used to globally register the non-deformable area and the deformable area, and the distance field gradient is used to estimate the non-deformable area. Moreover, self-adaptive weight optimisation is conducted with the matching confidence level for deformable regions, thus to improve the registration accuracy of 3D point clouds. The simulation results show that the method proposed in this study improves the registration accuracy and surface reconstruction smoothness of standard PE.

1 Introduction

The multi-balloon flexible copying robot is a new research topic in the field of flexible robots presently [1–3]. The copying quality of the copying robot mainly depends on the precision of the fitting of the reconstruction result of the copying curved surface with the objective model. And the reconstruction result of the general curved surface reconstruction [4–7] for the dynamic deformation targets is far from satisfactory.

The surface reconstruction of the dynamic targets is mainly the result of a dynamic matching of three-dimensional (3D) point clouds. At present, there is some research progress at home and abroad. Rostam et al. [8] proposed a 3D curved surface reconstruction method based on the stereo matching algorithm, which improved the edge matching of the 3D point cloud with the self-adaptive weighting and led the filter to reduce the noises of point cloud through the joint weighting. Hitendra et al. [9] proposed a 3D curved surface reconstruction method based on surface stripes, which applied photometric stereo stripe analysis to the estimation of the roughness of outline curves and therefore made the reconstructed curved surface smoother. Carmelo et al. [10] researched the testing of the boundary points of the 3D point cloud, which applied the polygonal function as the curve fitting method to reduce the impact of noise on the curved surface reconstruction. Ojaswa and Nidhi [11] conducted by the smooth of irregular objects, research of the 3D curved surface reconstruction method, and improved the reconstruction precision through the triangular grid interpolation optimisation strategy. Dou et al. [12] performed iterations of the 3D point cloud with the neural network, proposed a deep recurrent neural network algorithm, and constructed a 3D curved surface reconstruction model by the multi-viewpoint point cloud. Reyes et al. [13] proposed, by the dynamic deformable objects, the fuzzy control strategy, and conducted a 3D curved surface reconstruction of the dynamic targets with the constant-scale characteristic-transforming strategy. Florian et al. [14] proposed an approach to 3D curved surface reconstruction by the sparse point cloud, which models the front surface with the statistical form model and conducts the fitting of the curved surface with the mixed Gaussian model. Alma et al. [15] proposed an approach to 3D curved surface reconstruction based on the radial-based RBF neural network and the particle swarm, which reduces the operations of the 3D point cloud through the methods of pattern recognition and environment mapping. Jules et al. [16] proposed an approach to 3D curved surface reconstruction based on the Poisson equation (PE), which constructs the gradient vector field with the implicit zero-level function and closes the scattered point cloud into a closed model. Yukie et al. [17] proposed, through the analysis of the surface grid, to conduct calculations by transforming the surface grids into the equivalent surface, and to improve the robustness of the reconstruction through the randomly selected projection directions. Jung et al. [18] studied the finite-element grid model, conducts the gridding division for the curvature information with the K-means cluster algorithm, and performs the curved surface transformation with the B-spline.

Although the above methods can effectively register and reconstruct the curved surface of the 3D point cloud, there are still problems with them, such as low registration accuracy and poor smoothness of the reconstructed surface and so on. The remaining of this paper is organised as follows: Section 2 constructs a surface reconstruction model based on PE; Section 3 proposes a curvature-weighted surface smoothing method; Section 4 uses a linear distance field function to optimise the registration accuracy of the 3D point cloud; Section 5 conducts simulation tests on the improved algorithm.

2 Curved surface reconstruction based on PE

The PE [19] is a common method of 3D curved surface reconstruction, whose essence is to calculate the 3D point cloud by solving the function, extract the equivalent surface of the 3D point cloud, and, based on this, splice the equivalent surface to complete the reconstruction of the 3D curved surface. The flowchart is shown in Fig. 1.

First, the octree topology model is constructed with the target point cloud [20], and the assignment function $F_a$ of every node $a$ of the octree topology model is shown in the following equation:

$$F_a(t) = F_t \left( \frac{t - am}{ad} \right)$$  \hspace{1cm} (1)

The $am$ is the central point of the octree node $a$, and $ad$ is the distance of the octree node $a$.

And then, linear interpolation is conducted with the eight neighbouring nodes, and the gradient function represents the

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approximate estimate of the vector field $G$, shown in the following equation:
\[ G(t) = \sum_{n \in N} \sum_{i=1}^{L(n)} \lambda_{n,i} F_d(t) n \cdot U \] (2)

The $L(n)$ is the eight neighbouring nodes of the node $n$, $\{\lambda_{n,i}\}$ is the linear interpolation weight factor, $S$ is the objective point cloud, and $n \cdot U$ is the normal vector of the vertex.

Then the simplified equation is established as
\[ \sum_{n \in N} \| (\Delta t, F_d) - (\nabla \cdot G, F_d) \|^2 \] (3)

Solving the minimum function $\eta$ of (3) yields the equivalent surface, and then the equivalent surface is spliced to complete the 3D curved surface reconstruction.

Let's take the Rabbit point cloud data set by Stanford University as an example. Simulation experiments with the above model yield the point cloud registration accuracy shown in Fig. 2 and the curved surface reconstruction result shown in Fig. 3.

As shown in Figs. 2 and 3, the precision of the curved surface reconstruction method based on the PE is reduced, and the surface smoothness of the reconstruction result is poorer.

3 Curved surface smoothing based on curvature weighting

To make the reconstructed surface smoother, this paper optimises it with the curvature-weighted approach. The flowchart is shown in Fig. 4.

Assume the open-surface function of the curved point cloud model constructed based on the PE is $\mu(x, y)$, and its continuous condition is as shown in the following equation:
\[ \frac{\partial^2 x}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = \rho \cdot f(x, y) \] (4)

where $\rho$ is a weight factor.

As shown in (4), the curved surface constructed by point $(x, y, \mu(x, y))$ meets the smoothness conditions, and its average curvature is $f(x, y)$.

Assume that the surface of the surface point cloud model constructed based on the PE cannot meet the above conditions. Then set the initial value as the $z$-coordinate of the matrix mesh node, and the $\mu(x, y)$ of every node is iterated until the above conditions are met.

The average curvature iteration equation of the non-smooth curved surface is shown in the following equation:
\[ z_{i,j} = \frac{s_x^2}{2(s_x^2 + s_y^2)} (z_{i+1,j} + z_{i-1,j} - 2z_{i,j}) + \frac{s_y^2}{2(s_x^2 + s_y^2)} (z_{i,j+1} + z_{i,j-1} - 2z_{i,j}) - \frac{s_x s_y}{2(s_x^2 + s_y^2)} f_{i,j} \] (5)

$s_i$ is the step in the direction of $x$, and $s_j$ is the step in the direction of $y$.

Since there is no boundary set in the above iteration process, it is easy to lead to changes in its original form. So, the original point cloud is classified with the minimum absolute value of the average curvature at the extreme points as the boundary value, as shown in the following equation:
\[ \begin{align*}
\frac{\partial \mu(x, y)}{\partial x} &= 0 \\
\frac{\partial \mu(x, y)}{\partial y} &= 0
\end{align*} \] (6)

The region that is larger than the boundary value belongs to the deformation region, and is eliminated out of the iteration process, thus ensuring the correctness of the original form.

Simulation tests are conducted with the above-curved surface smoothing approach, as shown in Figs. 5 and 6.

As can be seen from Figs. 5 and 6, the curved surface smoothing based on the curvature weighting has a better effect.

4 Precision optimisation based on linear distance field function

In accordance with the problem of reducing the accuracy of the surface reconstruction method based on PE, this paper optimises it...
Suppose that the set of the point cloud is $S$, then the shortest distance from a point $a$ in the point cloud set to the very point is shown by the following equation:

$$D_S(a) = \inf_{p \in S} \| p - a \|$$  \hfill (7)

The $a$ is the midpoint of the point cloud set, and $p$ is the closest point of midpoint $a$.

In order to distinguish that the point is within the boundary $\partial U$, linearisation is conducted with the positive and negative signs added. If the point is within the boundary $\partial U$, the sign is defined as negative, and vice versa, as shown in the following equation:

$$\phi_S(a) = \text{sgn}(\alpha) \inf_{u \in \partial U} \| u - a \|$$  \hfill (8)

Performing the global registry for every sub-block $O_i$ yields the parameter increment $\Delta w_i$ of the non-deformable regions

$$E_i = \sum_{p_j \in O_i} (\phi_N(p_j, w_i) - \phi_M(p_j))^2$$  \hfill (10)

Linearising equation (10) and calculating the distance field gradient $\nabla \phi_N$ with the Sobel operator yields the following equation:

$$E = \sum_{O_i \in M} \sum_{p_j \in O_i} (\phi_N(p_j, w_i) - \phi_M(p_j))^2$$  \hfill (9)

where $\phi_M$ is the distance field of the sampling point, $\phi_N$ is the initial distance field, $p_j$ is the sampling point of the block, $w_i$ is the transform function, and $\beta$ is the deformation parameter.

Global registration is conducted for the 3D point cloud model through the above linear distance field function and is classified into global registration of the non-deformable regions and global registration of the deformable regions in accordance with the characteristics of the multi-balloon copying curved surface.

4.1 Global registration of the non-deformable regions

Divide the entire point cloud set into uniform sub-blocks with every sub-block $O_i$ containing the same number of distance field sampling points. The error of the difference between the distance field sampling point and the initial distance field value is calculated by the following equation:

$$E_i = \sum_{p_j \in O_i} (\phi_N(p_j, w_i + \Delta w_i) - \phi_M(p_j))^2$$  \hfill (10)

If $a \in U$, then $\text{sgn}(\alpha) = -1$; otherwise, $\text{sgn}(\alpha) = 1$. 


\[ \Delta w_i = H_i^{-1} \sum_{p_j} \left[ \nabla \phi_N \frac{\partial \beta}{\partial w_i} \right] \left[ \phi_N (p_j) - \phi_N (\beta(p_j, w_i)) \right] \] (11)

\[ H_i = \sum_{p_j} \left[ \nabla \phi_N \frac{\partial \beta}{\partial w_i} \right] \left[ \nabla \phi_N \frac{\partial \beta}{\partial w_i} \right] \] (12)

Update the transform function \( w_i \) with the non-deformable region parameter increment \( \Delta w_i \) until it converges.

### 4.2 Global registration of the deformable regions

In accordance with the deformable regions of the multi-balloon copying curved surface, define all the sampling points \( p \) in the two non-deformable blocks \( O_i \) and \( O_j \), and perform transform iterations \( \beta_i \) and \( \beta_j \). The difference value after iteration is as shown in the following equation:

\[ E_{ij} = \int_{O_i \cup O_j} \| \beta_i p - \beta_j p \|^2 dp \] (13)

To keep the copied curved surface after iterations smooth, find the weighted sum of the local smoothing items, as shown in the following equation:

\[ E_q = \sum_{i,j} E_{ij} (\beta_i, \beta_j) \] (14)

In view that after the balloon is deformed, the point cloud matching degree of the deformable region's changes. So, to increase the matching precision, introduce the matching confidence level \( E_v \), as shown in the following equation:

\[ E_v = \sum_{i} \gamma_i \int_{O_i} (\beta_i p, \beta_i loc p) dp \] (15)

The \( \gamma_i \) is the self-adaptive weight, whose value can be found through the following equation:

\[ \gamma_i = \frac{e_{max} - e_{loc}}{e_{max}} + \sigma \] (16)

In (15), \( e_{loc} \) is the matching error, \( e_{max} \) is the maximum matching error, and \( \sigma \) is the adjustment factor to avoid that the blocks with large errors are not added into the iteration.

### 5 Simulation testing and analysis

#### 5.1 Registration precision simulation testing

To test the effectiveness of the algorithm, take the Rabbit point cloud dataset as an example. If the original registration data is shown in Fig. 7, the feature points extracted by the iterated partial evaluation (IPE) algorithm in this paper are shown in Fig. 8, and the point cloud registration results are shown in Fig. 9.

Select a different number of characteristic points, compare and analyse the iterative closest point (ICP) algorithm, standard PE and the approach in this paper (IPE). And the results are shown in Figs. 10–13 and Table 1.

As can be seen from the results of Figs. 10–13 and Table 1, with the increase of the number of the characteristic points, the matching precision of the ICP algorithm and the standard PE is becoming increasingly lower, while the approach proposed in this paper can ensure certain matching precision.
5.2 Simulation of the balloon curved surface reconstruction

In accordance with the multi-balloon copying curved surface reconstruction, perform curved surface reconstruction for the balloon in different air-filling states. The matching precision of the balloon state testing ICP algorithm, standard PE and the approach of this paper (IPE) at five different time points are shown in Figs. 14–18.

As can be seen from the simulation results shown in Figs. 11–15, the approach presented in this paper can yield higher registration precision.

Take the balloon at the instant of t5 as an example. The curved surface reconstruction results of the three approaches are shown in Figs. 19–21.

As can be seen from the results in Figs. 19–21, the approach proposed in this paper has a better smoothness of curved surface construction.

6 Conclusions

The research on the copying curved surface reconstruction of the multi-balloon flexible robot helps to simulate the curved surface of

| Iteration times | 100 Feature points | 200 Feature points | 300 Feature points | 400 Feature points |
|-----------------|--------------------|--------------------|--------------------|--------------------|
|                 | ICP                | PE                 | IPE                | ICP                |
| 2               | 0.77 0.76 0.63     | 1.33 1.21 0.69     | 1.42 1.38 1.02     | 1.73 1.41 1.22     |
| 4               | 0.67 0.71 0.39     | 1.22 0.78 0.71     | 1.33 1.25 0.83     | 1.69 1.39 1.18     |
| 6               | 0.58 0.68 0.26     | 0.83 0.73 0.57     | 1.30 1.26 0.75     | 1.27 1.22 1.01     |
| 8               | 0.70 0.53 0.27     | 0.80 0.71 0.56     | 1.17 1.15 0.69     | 1.32 1.36 0.72     |
| 10              | 0.52 0.54 0.26     | 0.85 0.68 0.54     | 0.81 0.77 0.61     | 1.35 1.28 0.68     |
| 12              | 0.59 0.58 0.24     | 0.84 0.69 0.46     | 0.89 0.73 0.63     | 1.33 1.21 0.66     |
| 14              | 0.54 0.49 0.23     | 0.87 0.56 0.45     | 0.83 0.71 0.62     | 1.36 1.25 0.65     |

Table 1 Comparison results of matching accuracy of different feature points

Fig. 14 Comparison of matching precision at instant t1

Fig. 15 Comparison of matching precision at instant t2

Fig. 16 Comparison of matching precision at instant t3

Fig. 17 Comparison of matching precision at instant t4

Fig. 18 Comparison of matching precision at instant t5

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6 Conclusions

The research on the copying curved surface reconstruction of the multi-balloon flexible robot helps to simulate the curved surface of
and deformable region of the 3D surface are optimised, respectively, which improves the registration accuracy and the smoothness of surface reconstruction of standard PE. Although the improved algorithm in this paper has some advantages, its operation time is longer than the standard algorithm, so that it will be optimised and improved in future work.

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