Motion control of the spacecraft with a low thrust engine during flight to near-Earth asteroids

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Abstract. This work aims to simulate the flight of a spacecraft with an electric propulsion system to the near-Earth asteroid 2011ES₄. A control law was formed that ensures the minimum fuel consumption when the engine is running without shutdowns. The simulation of the flight of the spacecraft to the point of the asteroid's orbit closest to the Earth is carried out. An additional study was also carried out, which simulates the flight of a spacecraft with a liquid-propellant engine. A comparison of the flight simulation results using two different propulsion systems was made. A comparison of the mass flow rate, the relative velocity between the spacecraft and the asteroid, and the flight times using these propulsion systems is made.

1. Introduction
Much attention has recently been paid to the study of near-Earth asteroids. This is related to issues of asteroid hazard [1] and the study of their physical parameters. For a more detailed study of asteroids, it will be necessary to send spacecraft to them. You can choose from a variety of propulsion systems to fly, ranging from traditional liquid-propellant engines to various types of electric propulsion systems.

In this paper, we simulate the flight to the near-Earth asteroid 2011ES₄ when it approaches the Earth and carry out a comparative analysis of the flight using a liquid-propellant engine (LPE) and an electric propulsion engine (EPE). The aim is to compare the results of modeling and determine the appropriateness of using a particular propulsion system, and the effectiveness of the selected criterion. Relative speed at the point of meeting of the spacecraft with the asteroid and fuel consumption for the flight are taken as optimality criteria.

2. Description of the asteroid
Near-Earth asteroids are asteroids whose orbits lie close to or intersect the Earth's orbit. One of these asteroids is the asteroid 2011ES₄. It belongs to the Apollo group, its diameter is 25 m. September 01-03, 2020, and this asteroid will approach the Earth at a minimum distance of 120,000 km. On September 1, its speed relative to the Earth will be 8.16 km/s. This moment is convenient for launching a spacecraft to it. The trajectory of the asteroid when flying near the Earth is shown in Figure 1. This trajectory was determined using the JPL Horizons system.
For the mission to the asteroid, it is planned to use a spacecraft equipped with an electric propulsion system. The goal of the mission is to meet the asteroid at the lowest possible relative speed at the meeting point.

3. Description of the problem and mathematical model
Let us assume that the spacecraft at the initial moment of time moves in a near-earth circular orbit of height $H$. The characteristics of the spacecraft and the propulsion system are also considered known. At the moment of the beginning of the maneuver, the engine is switched on, and the spacecraft begins its flight to the asteroid in accordance with the selected control law. It is necessary to choose such a control program and such a launch time that will ensure the minimum fuel consumption and at the same time ensure the minimum relative speed between the asteroid and the spacecraft at the meeting point.

3.1. Control law
The spacecraft fuel consumption is determined by the following ratio [2]:

$$\dot{m}_r = \frac{P}{I},$$

where $P$ – engine thrust;
$I$ – specific impulse.

The spacecraft motion will be described by the following system of equations:

$$\begin{cases}
\ddot{r} = \vec{f}_G + \vec{f}_s + \vec{f}_M + \ddot{a} \\
\dot{\dot{m}}_r = \frac{P}{I}
\end{cases},$$

where $\vec{f}_G$ – acceleration due to the Earth's gravity;
$\vec{f}_s$ – acceleration due to the sun's gravity;
$\vec{f}_M$ – acceleration due to the moon's gravity;
\(\tilde{a}\) – engine acceleration.

In this work, it is assumed that the engine runs without shutdown after the start of the maneuver, only the thrust direction changes. In this case, the acceleration from the engine will be determined by the following relationship:

\[
\tilde{a} = \frac{P}{m_0 - m_f(t)} \tilde{e} = \frac{P}{I t} \tilde{e}.
\]

Here \(\tilde{e} = (e_x, e_y, e_z)^T\) – a unit vector showing the direction of acceleration from the motor. The minimum fuel consumption is considered as an optimality criterion:

\[
\tilde{e} = \arg\min_t m_f(t, \tilde{e}) .
\]

We will investigate the dates of approach to the asteroid in order to ensure the lowest relative speed between the spacecraft and the asteroid.

### 3.2. Motion equations and boundary conditions

Equations of motion in a geocentric inertial reference frame:

\[
\begin{align*}
\dot{x} &= V_x \\
\dot{y} &= V_y \\
\dot{z} &= V_z \\
V_x &= f_{G_x} + f_{S_x} + f_{M_x} + a_z \\
V_y &= f_{G_y} + f_{S_y} + f_{M_y} + a_y \\
V_z &= f_{G_z} + f_{S_z} + f_{M_z} + a_z
\end{align*}
\]

In this case, the unit acceleration vector is found as follows:

\[
\tilde{e} = \begin{pmatrix}
\cos \alpha \\
\cos \beta \\
\sqrt{1 - \cos^2(\alpha) - \cos^2(\beta)}
\end{pmatrix}
\]

In order to find the optimal control over the angles and, we use the Pontryagin maximum principle [3]. Let’s compose the Hamiltonian:

\[
H = \psi_x V_x + \psi_y V_y + \psi_z V_z + \left( f_{G_x} + f_{S_x} + f_{M_x} + \cos \alpha \right) \psi_x V_x + \left( f_{G_y} + f_{S_y} + f_{M_y} + \cos \beta \right) \psi_y V_y + \\
\left( f_{G_z} + f_{S_z} + f_{M_z} + \sqrt{1 - \cos^2(\alpha) - \cos^2(\beta)} \right) \psi_z V_z.
\]

To determine the angles \(\alpha\) and \(\beta\) at which the maximum of the Hamiltonian is ensured, we find the following derivatives:

\[
\begin{align*}
\frac{\partial H}{\partial \alpha} = 0 &\rightarrow \psi_x V_x \cos \alpha \sin \alpha - \psi_x V_x \sin \alpha = 0 \rightarrow \alpha_{opt} = \arccos \left( \frac{\psi_x V_x}{\sqrt{1 - \cos^2(\alpha) - \cos^2(\beta)}} \right) \\
\frac{\partial H}{\partial \beta} = 0 &\rightarrow \psi_y V_y \cos \beta \sin \beta - \psi_y V_y \sin \beta = 0 \rightarrow \beta_{opt} = \arccos \left( \frac{\psi_y V_y}{\sqrt{1 - \cos^2(\alpha) - \cos^2(\beta)}} \right)
\end{align*}
\]

Let us compose the system of canonical equations:
We have obtained a system of 12 equations containing the equations for the change in six phase coordinates, the mass of the spacecraft, and six conjugate variables that describe the optimal flight to the asteroid.

We now turn to the boundary conditions of the boundary value problem. If the launch point in the initial orbit is determined, then we know the orbit and position of the spacecraft at the initial moment of time and we fully know the vector of phase variables. At the same time, the initial conditions for the conjugate variables will be unknown.

At the final moment of time, we know the necessary coordinates, which coincide with the coordinates of the asteroid, and the spacecraft speed is considered to be non-fixed. Based on the transversality conditions, the final conditions for the conjugate variables in coordinates will be non-fixed, and the conjugate variables in velocity at the final moment of time will be equal to zero. As a result, the final conditions will take the following form:

As a result, we have a six parametric boundary value problem consisting of 12 equations, six initial, and six final conditions. Having solved this boundary value problem, we will obtain the optimal trajectory on which the minimum spacecraft fuel consumption and the minimum relative speed at the point of meeting with the asteroid will be ensured.

To solve the boundary value problem, a modified Newton's method was used with the initial conditions obtained from solving the problem of a plane interorbital transition in the framework of the two-body problem.
3.3. Calculation method
When calculating the thrust control of the engine and the trajectory of the spacecraft, the following method was used:
1) Simulation of the flight with known parameters of the reference orbit and characteristics of the spacecraft is carried out. The coordinates of the meeting point and the time of arrival of the asteroid at this point are also known;
2) Upon completion of the simulation, the flight time and the relative speed at the meeting point are determined;
3) Based on the flight time, the start date is determined;
4) The total fuel consumption for the flight is calculated and the relative mass fuel consumption is determined.

3.4. Additional research
Also, as part of the work, an additional study was carried out, which considers a flight to an asteroid using a liquid propellant engine. In this study, a flight with a liquid-propellant engine from the same reference orbit as with an ERE is analyzed, the fuel consumption and relative speed are estimated depending on the launch point, and a comparison is made by these criteria with an EPE. The flight was calculated within the framework of the same motion model and in an impulse setting. The results of this study will be presented in paragraph 4.

4. Research results
We will consider a coplanar flight from the reference orbit with the parameters given in Table 1. The inclination and longitude of the ascending node of the reference orbit correspond to the inclination and longitude of the ascending node of the asteroid’s orbit in the geocentric inertial coordinate system.

| Parameter                        | Value |
|----------------------------------|-------|
| Orbit height $H$, km            | 3000  |
| Eccentricity $e$                | 0     |
| Inclination $i$, deg            | 62    |
| Longitude of the ascending node $\Omega$, deg | 336   |

The spacecraft mass $m$ for research with electric propulsion and rocket engine will be the same and equal to 100 kg. For the study, the parameters of the EPE and LPE were taken, corresponding to real engines. The parameters of the selected motors are shown in Table 2.

| Parameter              | Value |
|------------------------|-------|
| Engine type            | EPE   | LPE  |
| Engine name            | PlaS-40 | -  |
| Thrust $P$, N          | 0.042 | 500  |
| Specific impulse $I$, m/s | 17500 | 3100 |
| Engine mass, kg        | 4      | 1,2  |
Due to the long duration of the flight, it is necessary to take into account a large number of external factors that can affect the spacecraft movement. From external disturbances, we will take into account the noncentrality of the Earth's gravitational field (second and fourth zonal harmonics), the attraction of the Sun, and the attraction of the Moon. These perturbations will be taken into account both during flight with an electric propulsion engine and in an additional study for a flight with an electric propulsion engine.

The problem of a flight with an electric propulsion engine will be solved using the control law derived earlier. With such a control law, the engine thrust will be directed along the speed vector, and the engine will operate without shutdown. With this control law, the spacecraft will spin up in a spiral. The flight trajectory with this control law and the above initial data is shown in Figure 2.

![Figure 2. Flight trajectory with the EPE.](image)

According to JPL Horizons, the asteroid will arrive at the found meeting point on September 1st at 16:00. The flight to the nearest point of the asteroid's orbit was 15.5 days. Based on this, we can conclude that the maneuver of the flight with the electric propulsion engine must be started on August 17 at 04:00 to meet the asteroid at this point. The spacecraft reaches the meeting point at a speed of 1.88 km/s, the asteroid has a speed of 8.16 km/s. Consequently, the relative speed between the spacecraft and the asteroid will be 6.28 km/s. Despite the long duration of the flight, the acting disturbances did not have a strong effect on the trajectory of the spacecraft, and it entered the orbit of the asteroid.

**4.1. Additional research results**

As part of an additional study, a flight to an asteroid using an LPE is considered. The problem of a flight with a liquid-propellant engine will be considered in an impulse setting. The flight will be operated
using a single-pulse scheme. As a result, the flight path will be a Homan ellipse (Figure 3). The impulse value required for the flight will be determined as follows:

\[ \Delta V = \frac{P}{m} t_p, \]

where \( t_p \) – firing duration.

In the case of a flight from a liquid-propellant engine, the starting point of the maneuver in the reference orbit will vary. By changing the starting point, the spacecraft will go to different points of the asteroid's orbit. As a consequence, the flight time and the impulse value required for the flight will change. The results of this study are shown further in Figures 4-6.
Figure 4. Dependence of the relative velocity on the characteristic speed.

Figure 4 shows the dependence of the relative velocity between the spacecraft and the asteroid depending on the magnitude of the momentum required for the flight and the launch angle. As can be seen from the graph, during a single-pulse flight, when reaching the points of the asteroid's orbit other than the nearest one, which corresponds to a launch angle of 98°, the flight time increases significantly. The value of the required impulse also increases, but the relative velocity between the spacecraft and the asteroid decreases insignificantly. Therefore, it is optimal for a transfer from a liquid-propellant engine to reach the nearest point of the orbit. When the spacecraft and the asteroid meet at this point x, the relative speed will be 7.46 km/s.

Figure 5. Dependence of the flight time on the launch angle.
Figure 5 shows the time dependence of the spacecraft flight time to the asteroid’s orbit. As can be seen from the graph, at the beginning of the maneuver at angles from 80 to 150 degrees, the flight time is approximately the same and is equal to one day. When trying to reach the distant points of the asteroid’s orbit, the flight time begins to grow at a high speed.

Figure 6 shows the dependence of the characteristic speed on the starting angle.

Figure 6. The dependence of the characteristic speed on the starting angle.

4.2. Comparison of the results
Below, in Table 3, the results of modeling the flight to the nearest point of the asteroid's orbit using EPE and LPE are given. It provides data on flight times, engine operation and total fuel consumption.

|        | Firing duration | Flow rate, kg | Total flow, kg | The ratio of the initial mass to the mass of fuel consumed | Relative speed, km/s |
|--------|-----------------|---------------|---------------|----------------------------------------------------------|---------------------|
| EPE    | 15.5 days       | 2.4E-06       | 3.21          | 31.1                                                     | 6.28                |
| LPE    | 464 s           | 0.161         | 74.8          | 1.33                                                     | 7.46                |

As can be seen from this table, the flight of a spacecraft with an electric propulsion engine in terms of relative flow rate significantly exceeds the flight performed by a spacecraft with a liquid propellant engine. Despite the significantly longer flight time, external disturbances did not because deviations of the spacecraft transfer orbit; therefore, the use of an EPE is preferable.

5. Conclusion
Within the framework of this work, a study was carried out of a spacecraft transfer with an electric propulsion engine to the orbit of the near-earth asteroid 2011ES₄. A control law was derived to ensure minimum fuel consumption when the engine is running without shutdowns. An additional study was also carried out, in which the flight of a spacecraft with an LPE to the same asteroid was considered. As a result of the work, it turned out that the use of an electric propulsion engine is more appropriate.
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References
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