Effect of Nyquist noise on the Nyquist dephasing rate in 2d electron systems

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We measure the effect of externally applied broadband Nyquist noise on the intrinsic Nyquist dephasing rate of electrons in a two-dimensional electron gas at low temperatures. Within the measurement error, the phase coherence time is unaffected by the externally applied Nyquist noise, including applied noise temperatures of up to 300 K. The amplitude of the applied Nyquist noise from 100 MHz to 10 GHz is quantitatively determined in the same experiment using a microwave network analyzer.

What is the mechanism of electronic decoherence in disordered 2d conductors? One mechanism is the so-called Nyquist mechanism of electron-electron interactions involving small energy transfer. This mechanism is believed to be equivalent to the interaction of an electron with the fluctuating electromagnetic field (i.e. the Nyquist/Johnson noise) produced by all the other electrons in the system [1], hence the name Nyquist dephasing. If this physical picture is correct, then applying a fluctuating electric field (i.e. Nyquist/Johnson noise) from an external circuit should effect the coherence time measured by weak localization in the same way as the fluctuating electric field produced by the sample itself. Although this effect was first discussed over 20 years ago [2], it has never been tested experimentally [3]. We have performed this experiment, and present the results in this paper.

The central result of this paper is to ask the following question: What is the effect of broadband (100 MHz to 10 GHz) thermal fluctuations in the electric field with noise temperature $T_B$ (amplitude) of up to 300 K on the decoherence rate measured by weak localization on electrons in 2 dimensions? We apply the fluctuating voltage across the terminals of a two dimensional electron gas (see below) by terminating the room temperature end of the coaxial cable shown in the layout in figure 1 with a 50 Ω resistor. From the fluctuation-dissipation theorem, the resistor generates a noise voltage with spectral-density given by

$$V_n^2 = 4k_B T R,$$

(1)

where $T$ is the physical temperature of the external resistor (300 K) and $R$ is the value of the external resistor (50 Ω). The spectrum of these fluctuations is white up to frequencies of order $k_B T$. In the same experiment, we measure the coupling from the sample to the resistor up to 10 GHz with a vector network analyzer. The unique aspect of this experiment is that we quantify the amplitude of the applied fluctuating voltages to the sample terminals over a broad range of frequencies around $\tau_{tr}^{-1}$ using careful microwave engineering. We discuss the microwave circuit models first, then present the amplitude of the applied rf fluctuating voltage based on these circuit models, and then discuss the measured magnetoresistance and inferred phase-coherence times.

The sample studied is a GaAs/AlGaAs modulation-doped heterojunction grown by molecular beam epitaxy. The sample geometry is indicated schematically in figure 1. A hall bar mesa is lithographically defined with four ohmic contacts from diffused Ni/Au/Ge. The sample density and mobility are 1.25 $10^{11}$ $cm^{-2}$ and 600,000 $cm^2/V − s$, respectively, with a corresponding sheet resistance of roughly 80 $Ω$/sq. Two additional capacitive contacts are provided to allow for the application of high frequency signals; these are evaporated Al gates. The gate-2DEG separation is about $5000 \AA$ and the gate area is about $0.25 cm^2$, so that the capacitance value is about 50 pF. At frequencies above roughly 100 MHz, the capacitor does not effect the rf voltage. The d.c. current and voltage leads are several cm long gold wires of diameter 50 μm which act as inductive blocks at frequencies above roughly 100 MHz. The physical temperature of the sample is held at 300 mK for the entire experiment.

In order to determine the amplitude of the applied fluctuating electric field, we consider the effective circuit diagram shown in figure 2. The sample circuit model is that of a capacitively coupled resistor. In reality, there will also be an inductive component at frequencies above $\tau_{tr}^{-1}$, where $\tau_{tr}$ is the transport (momentum) scattering time. (For the sample studied here, this time is about 20 ps.) We studied this circuit model in detail in another publication [4], and found it to be valid up to 10 GHz. In this experiment, we measure the coupling of the sample to the coaxial cable with a microwave vector network analyzer, as in our previous publication: we measure the (frequency dependent) microwave reflection
coefficient defined as
\[
\Gamma(\omega) = \frac{Z_{\text{sample}}(\omega) - 50 \, \Omega}{Z_{\text{sample}}(\omega) + 50 \, \Omega}.
\]  
(2)

This measurement is carried out by inserting the network analyzer at the end of the coax in place of the 50 Ω resistor. We find that the circuit model in figure 2 for the sample describes the coupling to the sample (defined as \(1 - |\Gamma|^2\)) to within 20% over almost the entire frequency range considered, with a sample resistance of 150 Ω and capacitance of 50 pF.

We now consider the external circuit model. In general, since the external resistor is “seen” by the sample through a coaxial cable, the effective impedance denoted in figure 2 is a complicated function of the frequency, length and characteristic impedance of the coax, and the external or “load” resistor, with significant real and imaginary components, even though the load resistor is purely real. Historically, this technical difficulty has prevented quantitative analysis of the external impedance and hence the ability to measure its effect on phase coherence in 2d and 1d systems. However, for the special case where the “load” resistance is equal to the characteristic impedance of the coax (50 Ω in this experiment), it can be shown that the effective impedance is real and frequency independent, and is equal to:
\[
Z_{\text{external}}(\omega) = 50 \, \Omega.
\]  
(3)

In this special case, equation 3 is still strictly correct even if there is loss (attenuation) along the coaxial cable.

If the coax is lossless, then the equivalent circuit in figure 2 can be used to calculate the noise voltage at the terminals of the sample, with the value of \(T_{\text{effective}}\) given by the temperature of the external resistor (300 K). The external circuit acts as a noise source with voltage given by the equation in figure 2 and a source impedance given by \(Z_{\text{external}}\). Taking this into account, and the fact that \(Z_{\text{external}}\) is real and equal to 50 Ω, the amplitude of the voltage fluctuations at the terminals of the sample are:
\[
V_n^2(\omega) = 4k_B T_{\text{eff}} 50 \, \Omega \left(\frac{Z_{\text{sample}}(\omega)}{50 \, \Omega + Z_{\text{sample}}(\omega)}\right)^2 = k_B T_{\text{eff}} 50 \, \Omega \left(1 - |\Gamma(\omega)|^2\right),
\]  
(4)

where we have inserted the definition of \(\Gamma\). Since \(\Gamma\) is measured, we know the amplitude of the fluctuating field at each frequency.

If there is loss in the coax, then the voltage fluctuations generated by the external resistor will get attenuated, while the coax itself will generate some noise. By modeling the loss as uniformly distributed along the length of the coax, and by modeling the temperature profile along the length of the coax as linear, we find that equation 3 is still valid provided that the following expression for \(T_{\text{effective}}\) be used:
\[
T_{\text{effective}}(K) = 300 \frac{4.3}{L} \left(1 - 10^{-L/10}\right).
\]  
(5)

Here \(L\) is the loss of the entire coax, in units of dB. (0 dB is no loss; +∞ dB is infinite loss, i.e. no transmission.) In the limit of a lossless coax, the effective external temperature is 300 K, as expected. A more complicated version of equation 3 is used to calculate the voltage noise for a 77 K “load”, i.e. external resistor.

Finally, we plot in figure 3 the applied fluctuating electric field determined from the effective external temperature calculated from equation 3 and 4, the measured coax loss, and the measured sample to coax coupling efficiency. (The measured coax loss varied from 0 dB at low frequencies to 4 dB at 10 GHz.) The effective external temperature determined from equation 3 as well as the coupling measured with a network analyzer are shown in the insets for reference. We also plot the intrinsic noise generated by the sample itself, which is 0.3 K. In the absence of an external circuit, as discussed above, this is expected to be the dominant source of decoherence for the electrons, due to the so-called Nyquist dephasing mechanism.

We now turn to our measurements of the phase coherence times under the applied fluctuating fields shown in fig. 3. Although sample resistances close to 50 Ω allow for good characterization of the microwave coupling, they make measurement of weak localization difficult because of the small resistance changes that must be resolved. Small probe currents must be used to avoid sample heating. For that reason the measured magnetoresistance
In order to be more quantitative, we perform a least squares fit of the peak to the following functional form 

\[ \delta R \propto e^2 R_s \left[ \psi \left( \frac{1}{2} + \frac{H_{tr}}{H} \right) + \frac{1}{2} \psi \left( \frac{1}{2} + \frac{H_{so}}{H} \right) - \frac{3}{2} \psi \left( \frac{1}{2} + \frac{H_{so} + H_{so}}{H} \right) \right], \tag{6} \]

where \( H \) is the applied magnetic field, \( \psi \) is the digamma function, and \( H_i = h/4eL_i^2 \), where \( i \) represents the scattering mechanism, and \( L_i = \sqrt{D \tau_i} \) the corresponding length. The \( i \)'s correspond to \( tr \)=transport, \( so \)=spin orbit, and \( \phi \)=phase breaking. The elastic mean-free path and \( R_{so} \) are related, so that there are effectively three free parameters in the theory curve. In performing a two-parameter fit (holding \( \tau_{so} \) fixed), we find the fit results of \( \tau_{\phi} \) and \( \tau_{tr} \) to be independent of the spin-orbit scattering time, as long as \( \tau_{so} \) is sufficiently larger than \( \tau_{\phi} \) and \( \tau_{tr} \). This is consistent with the results of Dresselhaus [8], who studied the spin-orbit scattering rates in GaAs 2DEGs in detail. In figure 4, we plot the fitted results for a two-parameter fit, keeping \( H_{so} \) fixed at 0.013 Gauss, the value predicted by the Dresselhaus data for our density. We find a value of 34 ps and 37 ps for \( \tau_{\phi} \) in the presence and absence of the externally applied Nyquist noise, respectively [10]. (We find a value of 13 ps for \( \tau_{tr} \) in both cases, in reasonable agreement with that value of 22 ps calculated from the measured value of \( R_{so} \).) The value of \( \tau_{\phi} \) cannot be said to have changed within the measurement error.

From the data shown in figure 4 we can estimate that \( \tau_{\phi}^{-1} \) changed by no more than 50%. To illustrate this point, we plot the predicted curve for a factor of 1.5 change (increase) in \( \tau_{\phi}^{-1} \); this change is clearly ruled out by the experiment. The same conclusion applies if we perform a three-parameter fit (varying \( \tau_{so} \), \( \tau_{\phi} \), and \( \tau_{tr} \)) or a one parameter fit (varying only \( \tau_{\phi} \) and using estimated values for \( \tau_{tr} \), and \( \tau_{so} \)). Thus, the experimental conclusion is robust and independent of the particular curve-fitting procedure used. We measure the magnetoresistance when the physical temperature of the external resistor is changed from 300 K to 77 K, and find a a similar lack of change in \( \tau_{so} \) with the change in applied noise voltages. To an experimental resolution of 0.1 \( e^2/h \), we also find no change in the B=0 conductance under these changes in externally applied noise.

As other groups have done [3], we have been able to suppress the weak localization peak by applying a con-
stant amplitude, single frequency field at various frequencies between 50 MHz and 20 GHz. However, we were unable to separate the effect of heating from the electric field induced decoherence. It is unclear to us (theoretically) whether the effect of a broadband, fluctuating electric field is equivalent to that of a “comb” of fields with constant amplitude distributed broadly in frequency. Even if this were predicted theoretically, it is still important to test experimentally. The case of an externally applied broadband fluctuating field is in some sense a better check of the theory of the Nyquist dephasing mechanism, since the fields generated by the electrons in the sample are themselves broadband and fluctuating.

For the sake of comparison with other experiments which measure the effect of a single frequency constant amplitude field, we can estimate the (rms) value of the electric field strength in our experiment. For single frequency constant amplitude experiments, the important dimensionless measure of the field strength is given by $\alpha = 2e^2DE^2/\hbar^2(2\pi f)^3$, with D the diffusion constant and E the electric field strength. In our experiments, we calculate that the rms voltage is roughly 10 mV, hence an rms electric field strength of 10 V/m. We estimate that $\alpha \approx 4$. For fixed amplitude and frequency experiments, this is theoretically $\geq 4$ enough amplitude to change $\tau_\phi$ by 100%, if a single frequency constant amplitude with the same field strength were applied.

We now turn to the theoretical interpretation of our results. There have only been a few theoretical calculations of the effect of external circuit noise on the phase-coherence of electrons in 2d conductors [1,2]. Equation 3.3.22 of reference [1] seems to predict that in the presence of external circuit noise with noise temperature $T_0$, and with good circuit coupling as we have in this experiment, the measured $\tau_\phi$ should be comparable to the value that one would measure in the absence of such noise if the physical temperature of the electrons was equal to $T_0$. For the experiment considered here, that would imply that our measured value of $\tau_\phi$ in the presence of 300 K of noise should be essentially zero, fully suppressing the weak localization peak, in contradiction to what we observe. For these reasons, we are perplexed as to why the weak localization peak still exists at all, even in the presence of such a high artificial temperature of the electromagnetic field fluctuations.

We speculate on three possible reasons for this robustness of the phase coherence to the externally applied noise. First, it could be due to the fact that in our sample, the electron motion is ballistic at frequencies above $\tau_\phi^{-1}$, which is comparable to $\tau_\phi^{-1}$ in this experiment. Second, even though the amplitude of the field is up to 300K, its frequency is probably not coupled to the sample all the way up to 300 K/$k_B$, which is many THz. There are, to our knowledge, no theoretical predictions of what happens to $\tau_\phi^{-1}$ if a thermal field is applied only over a certain (albeit broad) range of frequencies, corresponding to the situation in our experiment. Finally, we speculate that perhaps in order to efficiently cause dephasing, not only must the frequency of the applied thermally fluctuating field be comparable to $\tau_\phi^{-1}$, but that the wave vector may also need to be of order $L_\phi^{-1}$. In our experiments, the wave vector of the applied fluctuating field is roughly 1/(1 mm), which is much smaller than $1/L_\phi$, roughly 1/(10 μm). These hypotheses suggest a class of future experiments to determine the effect of fluctuating electric fields on systems with shorter mean free paths (such as thin films), as well as other coupling geometries to allow the coupling of higher-wavevector fluctuating electric fields.

In conclusion, we have measured the effect of externally applied broadband Nyquist noise on the intrinsic Nyquist dephasing rate of electrons in 2 dimensions. Within the experimental accuracy, the phase coherence time is unaffected by the externally applied Nyquist noise, including applied noise temperatures of up to 300 K.

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