Two-Dimensional Representation and Connectivity Analysis of Nature Fracture in Fractured Reservoir

Huan Zhao, Wei Li, Siqi Li
College of Petroleum Engineering, Northeast Petroleum University, Daqing 163318, China

Abstract. The mechanism of hydraulic fracturing in fractured reservoirs is complex. To understand the complex distribution characteristics of natural fractures and the mechanism of hydraulic fracture propagation in fractured reservoirs, the natural fracture characterization of a two-dimensional model was established based on the fractal method and topology principle. Furthermore, based on this model, the hydraulic fracture propagation dynamic fluid-structure interaction model was built using the global embedded cohesive element method. The influence of natural fracture connectivity on hydraulic fracture morphology was analyzed. The results show that multi-fracture development can effectively reduce the influence of stress anisotropy on the fracture propagation direction and improve the natural fracture connectivity of the reservoir. Moreover, the reservoir connectivity reached 35.87% after fracturing.

Keywords. Natural fracture, Fractal feature, Connectivity, Cohesive element, Fracture morphology

1. Introduction

Hydraulic fracturing technology is an effective simulation technology used for the large-scale production of fractured reservoirs through the communication of natural fractures in the reservoir. In the process of hydraulic fracturing, the description of natural fractures can help in understanding the reservoir further and effectively guide the fracturing design. The geometric distribution of natural fractures is complex and difficult. Fracture network modeling is a necessary method for understanding the fractured reservoirs, while the discrete fracture network (DFN), stochastic continuous network (SCN), and channel network (CN) models have been applied in the geological modeling of fractured reservoirs and achieved desirable results [1]. However, the characteristics of nature fracture distribution in a reservoir cannot be directly measured but rather at the intersection of the borehole or outcrop. The study of multi-scale fracture has shown that the characteristics of fracture distribution, such as fracture length, fracture number, and displacement, follow a power-law function [2-3]. In many cases, the fractal geometry is well suited for describing multi-scale fracture networks. Fractal sizes, which can be measured by a logarithmic graph, can be used to evaluate the geometry characteristics of fracture networks [4]. The effective size range of fractal geometry is limited by the length of discontinuous geological structure systems, such as stratigraphic distribution, fault zone size, reservoir size [5]. The field observations show that only a small part of the fracture network system contributes to the overall flow [6]. Impenetrable isolated fractures that are mapped in the model cannot form a part of the fracture network connectivity. Therefore, fracture connectivity is an important factor in the fracturing effect. Presently, studies [7-8] have been conducted to understand the interaction mechanism of hydraulic and natural fractures and to introduce the cohesion unit method for establishing hydraulic fracturing
models. Rahman et al. [9] established a hydraulic fracture propagation model of fractured reservoirs based on the fluid–solid coupling method and analyzed the effects of natural fracture development direction and length on hydraulic fracture propagation. Taleghani et al. [10] analyzed the influence of the in situ stress difference on the fracture morphology when the hydraulic fracture intersects the natural fracture through the cohesive unit method. Guo et al. [11] simulated the interaction of hydraulic and natural fractures by coupling the stress-seepage field using the cohesive element method. Although hydraulic fracturing technology has been used for the simulation and reconstruction of the fractured reservoirs for decades [12], the hydraulic fracturing propagation model of complex fractured reservoirs has not been thoroughly studied; thus, the mechanism of hydraulic fracture in a fractured reservoir is not clear. Hence, this study aims at simulating the randomly distributed natural fractures in fractured reservoirs by applying fractal and topological methods and establishing the hydraulic fracture propagation model by using the global embedded cohesive element method. Furthermore, the influence of natural fracture connectivity on the fracture morphology was analyzed. Thus, this study lays the foundation for analyzing the impact of fracture connectivity in fractured reservoirs in oil and gas exploration and development.

2. Two-dimensional natural fracture characterization model

A reservoir contains randomly distributed natural fractures of different densities and lengths. The mechanical strength of natural fractures is lower than that of reservoirs. During hydraulic fracturing, hydraulic fractures intersect with natural fractures and form a complex fracture network [13]. Studies conducted on natural fracture data have shown that the fracture length distributions are characterized by log–log plots and the cumulative frequency distribution of the fracture length, which is equal to or greater than the number of fractures [14], follows the following pattern:

$$L(r) = L_c r^{-D_R}$$

(1)

where $L_c$ is a constant feature that describes the fracture density and $D_R$ is a power exponent, which represents the fractal dimension of the fracture length.

In the fracture network, there are $m$ groups of fractures. Each group of fractures has properties such as fracture length and fracture number. Thus, we can obtain

$$r_{a'i} = [(1 - \alpha_i)r_{min}^{-D_{Ri}} + \alpha_i r_{max}^{-D_{Ri}}]^{-1/D_{Ri}} \quad (1 \leq i \leq m)$$

(2)

$$N_i(r_{a'i} \geq r_i) = \left(\frac{r_{max}}{r_i}\right)^{D_{Ri}/2} \quad (1 \leq i \leq m)$$

(3)

where $r_{min}$ and $r_{max}$ are the lower and upper limits of the fracture size in the $i$ group, respectively, $L_i$ is the cumulative length of the fracture in the $i$ group, $N_i$ is the cumulative number of the fractal fracture, and $D_{Ri}$ is the cumulative length of the fractal dimension.

The fracture orientation accuracy and dispersion can be estimated by the root-mean-square error and the Fisher distribution. Priest [15] used the Fisher distribution in modeling the reverse cumulative form, expressed as

$$\theta = \cos^{-1} \left( \frac{\ln[e^{K-F(e^{K-e^{-K}})}]}{K} \right)$$

(4)

where $\theta$ is the direction of the fracture surface measured counterclockwise from the x-axis, $K$ is the Fisher coefficient, and $F$ is the uniform deviation range [0, 1]. Note that the value of $\theta$ varies with the value of $F$.

When $K$ takes the best estimate ($K > 3$), it can be expressed as

$$K = \frac{N - 1}{N - L_R}$$

where $N$ is the number of samples and $L_R$ is the length of the resultant vector. In addition, $K$ is used to measure the accuracy of the average inclination of the direction.

Natural fractures have complex network characteristics when the number of fractures and groups reaches a certain degree, as shown in Figure 1. The parameters of fracture in Figures 1a and 1b, such
as fracture length, fracture number, fracture direction, fracture density, and the number of fracture
groups, are the same, but the connectivity and permeability of the two fracture networks are very
different. Therefore, to accurately represent the fracture network, we must consider the geometric
parameters and connectivity of the fractures.

![Fully connected fracture network.](image1)

(a) Fully connected fracture network.  
(b) Partially connected fracture network.

**Figure 1.** Comparison of connectivity in different fracture networks.

Figure 2a shows two unconnected fractures, while Figures. 2 (b–d) show different nodes of two
connected fractures. The O-, V-, Y-, and X-type node connections are shown in Figure 2a, Figure 2b,
Figure 2c, and Figure 2d, respectively.

![O-, V-, Y-, and X-type node connections.](image2)

(a) O-Type Node  
(b) V-Type Node  
(c) Y-Type Node  
(d) X-Type Node

**Figure 2.** Different types of node connections of two fractures.

Based on this fracture network model, the influences of the fractal dimension, number of groups, and
the dip angle of the fracture network are analyzed. In a two-dimensional plane (2D), a single fracture
is affected by parameters such as fracture position, fracture length, fracture width, and fracture angle.
The fracture group is affected by the initial fracture number and length, the number of fracture groups,
and fracture fractal dimension. The position of the fracture is determined by the Poisson function,
while the fracture length and number are determined by the fractal function. Furthermore, the
connectivity of the network is determined by the node model. The simulation parameters are shown in
Table 1. The distribution of natural fractures developed in different fracture groups is shown in Figure
3.

![Table 1. Simulation parameters of the different fracture groups.](image3)

| Group Number | Fracture numbers | The largest length of fracture (m) | Fractal dimensions | Fracture angle (°) |
|--------------|------------------|------------------------------------|-------------------|-------------------|
| 1 group      | 40               | 100                                | 1.2               | 0                 |
| 2 groups     | 80               | 100                                | 1.2               | 90                |
| 3 groups     | 120              | 100                                | 1.2               | 45                |

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3. The hydraulic fracture propagation model

To simulate hydraulic fracture and natural fracture propagation, the hydraulic fracture propagation model was established through the coupling of four physical processes [12]: the fracture propagation, fluid flow within the fractures, rock deformation in the vicinity of fractures, and fluid leak-off into the porous medium. The basic theory of pore elasticity exhibited an explicit coupling between the dilation of the matrix skeleton and the pressure of the diffusing pore fluid. By definition, the total stress can be represented as the summation of the effective stress and pore pressure and is given as

$$\sigma_{ij} = \sigma'_{ij} + \alpha p \delta_{ij}$$

where $\sigma_{ij}$ is the total stress (MPa), $\sigma'_{ij}$ is the effective stress (MPa), $p$ is the pore pressure, $\alpha$ is the Biot constant, and $\delta_{ij}$ is the Kronecker delta.

For the linear elasticity, the constitutive relationship can be expressed by the effective stress and the strain as follows:

$$d\varepsilon = D e : d\sigma'$$

The numerical simulation of the hydraulic fracture propagation is performed using the cohesive element method. In this model, a predefined path, which is composed of cohesive elements that meet the requirements of the traction-separation law, is embedded in the rock to represent the hydraulic and natural fractures [16]. The cohesive element approach can predict the fluid pressure of the fracture initiation and propagation process of any joint geometry.

The quadratic interaction function involving the nominal stress ratios is unity when the damage is about to occur. The criterion is as follow:

$$\left(\frac{l_n}{l_n^0}\right)^2 + \left(\frac{l_s}{l_s^0}\right)^2 + \left(\frac{l_t}{l_t^0}\right)^2 = 1$$

where $l_n^0$, $l_s^0$, and $l_t^0$ are the peak values of the nominal stress when the deformation is normal to the interface, in the first shear direction, and in the second shear direction, respectively.

4. The analysis of fracture propagation morphology

4.1 The fracture propagation model

Many joints are developed in the fractured reservoir. Combined with the fractal characteristics of natural fractures, the 2D traces of randomly distributed natural fractures were drawn using the fracture network model. The hydraulic fracture propagation models with different groups of nature fractures
were established by the method of global embedded cohesive elements. The perforation position was taken as the center of the model. The rock properties and reservoir characteristics are shown in Table 2.

Table 2. The Parameters of the hydraulic fracturing propagation model in a fractured reservoir.

| Parameters                     | Value  | Parameters                     | Value |
|--------------------------------|--------|--------------------------------|-------|
| Depth (m)                      | 1648   | Rock tensile strength (MPa)    | 4.8   |
| Young's modulus (GPa)          | 22     | Rock density (kg/m$^3$)        | 2300  |
| Poisson's ratio                | 0.3    | Injection rate (m$^3$/min)     | 6     |
| Formation pressure (MPa)       | 17     | Fracturing fluid viscosity     | 100   |
| Maximum horizontal principal stress | 30   | Perforation length (m)         | 0.2   |
| Minimum horizontal principal stress | 27   | Fractal dimension              | 1.2   |

The hydraulic fracture propagation model of the fractured reservoir with one, two, and three groups of fractures were built. The rock model uses the pore fluid grid (4-node bilinear displacement and pore pressure, CPE4P), the natural fracture uses the cohesive element grid (6-node displacement and pore pressure cohesive element, COH2D4P), and the grid is 0.5 m in size. Figure 4 shows the stress distribution and fracture morphology of the fractured reservoirs with different groups of fractures.

![Figure 4](image.png)

Figure 4 shows that the fractures initiate and propagate from multiple directions and form many fracture branches in fractured reservoirs. The propagation direction is not completely controlled by the in situ stress distribution. As the different groups of natural fractures increase, their respective connectivity increase as well and the fracture morphology becomes more complex.

4.2 The analysis of the fracture morphology

To further analyze the effects of fracture initiation and morphology in fractured reservoirs, the injection pressures and the number of opened natural fractures in the fractured reservoir with different groups of natural fractures were drawn, as shown in Figure 5 and Figure 6, respectively.
Figure 5 shows that the injection pressure increases with the injection of the fracturing fluid. However, the injection pressure quickly decreases when the fracture pressure adds to the fracture initiation pressure. The injection pressure curve fluctuates because the hydraulic fracture encountered multiple nature fractures. With an increase in the fracture groups, the injection pressure decreases. This shows that the hydraulic fracture is easier to initiate when the nature fractures increase. Figure 6 shows that the number of nature fractures intersecting with hydraulic fractures increases when the groups of nature fractures increase. This is because the distribution of natural fractures in the reservoir becomes complicated when the number of natural fracture groups increases. The existence of natural fractures effectively eliminates the anisotropy of the reservoir.

Figure 7 shows that at the early stage of fracture propagation, the fracture length is similar for the three different groups. The fracture length quickly increases when the hydraulic fracture intersects the natural fractures. The more the groups of the natural fracture, the faster the increase in the fracture length. This is because the fracture length grows quickly when the hydraulic fracture intersects and propagates along the nature fracture. Furthermore, the number of nature fractures opened by the hydraulic fracture increases when the groups of nature fractures increase. Figure 8 shows that at the early stage of fracture propagation, the fracture width is similar for the three different groups. The more the groups of the natural fracture, the smaller the maximum fracture width. In summary, the fracture propagation length and fracture width are related to the nature fracture in which the hydraulic fractures intersect. When one group of natural fractures occurs in the reservoir, the hydraulic fracture is short and wide. Conversely, when more than one group of natural fractures occur in the reservoir, the hydraulic fracture becomes long and narrow.

4.3 The connectivity of the fracture
To analyze the connectivity of natural fractures in the reservoir, the connectivity of natural fracture clusters in the two groups of fractures model was drawn, as shown in Figure 9. Two groups of natural...
fractures are present with a total of 92 fractures. Table 3 shows that the maximum connected and the unconnected ratio of the model are 29.35 and 36.96%, respectively.

Table 3. Fracture network of the fractured reservoir.

| No. | The fracture number of the fracture cluster | The number of fracture cluster | Connected type | The total of fracture number | The proportion |
|-----|-------------------------------------------|-------------------------------|----------------|-----------------------------|----------------|
| 1   | 1 fracture                                 | 34                            | Un-connected   | 34                          | 36.96%         |
| 2   | 2 fractures                                | 4                             | connected      | 8                           | 8.70%          |
| 3   | 3 fractures                                | 2                             | connected      | 6                           | 6.52%          |
| 4   | 6 fractures                                | 1                             | connected      | 6                           | 6.52%          |
| 5   | 11 fractures                               | 1                             | connected      | 11                          | 11.96%         |
| 6   | 27 fractures                               | 1                             | connected      | 27                          | 29.35%         |

In comparison with the fracture cluster distribution and fracture propagation shown in Figures 10 and 12, respectively, we can deduce that the hydraulic fractures communicate with the natural fracture cluster, as shown in Figure 9 (d) and (f). The connectivity of the fracture is 35.87% after the hydraulic fracturing. With the injection of the fracturing fluid, the fracture connectivity is improved. Furthermore, the hydraulic fracturing technology can effectively improve the connectivity of fractured reservoirs.

Figure 10. The connectivity of nature fracture.  
Figure 11. Fracture propagation morphology.
5. Conclusions
(1) A novel model of the nature fracture connectivity was established using the fractal dimension theory and topological principle. The model can accurately represent the random nature fracture in the reservoir.
(2) In consideration of the random natural fracture, the hydraulic fracturing fluid–solid coupling dynamic propagation model was established by combining the cohesive element method and the natural fracture network model. Furthermore, the fracture morphology in the fractured reservoir with different groups of fractures was analyzed.
(3) The influence of the fracture group distribution on fracture connectivity was analyzed. When the number of natural fracture groups increases, the connectivity of natural fractures increases, and the morphology of the hydraulic fracture tends to be long and narrow. When two groups of fractures are present in the reservoir, the fracture connectivity improvement effect of 35.87% was achieved by applying the hydraulic fracture technology.

6. References
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