Emergence of long wavelength pion oscillations following a rapid QCD phase transition

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ABSTRACT

To model the dynamics of the chiral order parameter in a far from equilibrium QCD phase transition, we consider quenching in the O(4) linear sigma model. We summarize arguments and numerical evidence which show that in the period immediately following the quench arbitrarily long wavelength modes of the pion field are amplified. This results in large regions of coherent pion oscillations, and could lead to dramatic phenomenological consequences in ultra-relativistic heavy ion collisions.

My talk was a description of work done with Frank Wilczek [1]. In these proceedings, I sketch our central results, emphasizing several points that were raised in discussions at the conference. The interested reader should, however, consult Ref. [1] and our earlier work [2] for a more detailed exposition.

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1. Misalignment of the chiral condensate

Among the most interesting speculations regarding ultra-high energy hadronic or heavy nucleus collisions is the idea that regions of misaligned vacuum might occur [3]. Misaligned regions are places where the four-component field $\phi^a \equiv (\sigma, \vec{\pi})$, that in the ground state takes the value $(v, 0)$ is instead partially aligned in the $\vec{\pi}$ directions. Because of the explicit chiral symmetry breaking (i.e. because the pion is not massless), in such a region $\phi$ would oscillate about the $\sigma$ direction. If they were produced, misaligned vacuum regions would relax by coherent pion emission — they would produce clusters of pions bunched in rapidity with highly non-Gaussian charge distributions. In each such cluster, the ratio

$$R \equiv \frac{n_{\pi^0}}{n_{\pi^0} + n_{\pi^+} + n_{\pi^-}}$$  \hspace{1cm} (1.1)$$

is fixed. Among different clusters, $R$ varies and is distributed according to

$$\mathcal{P}(R) = \frac{1}{2} R^{-1/2}.$$  \hspace{1cm} (1.2)$$

As an example of (1.2), we note that the probability that the neutral pion fraction $R$ is less than .01 is .1! This is a graphic illustration of how different (1.2) is from what one would expect if individual pions were emitted with no isospin correlations many pions. We have proposed [1] a concrete mechanism by which such phenomena may arise in heavy ion collisions for which the plasma is far from thermal equilibrium.
Emergence of long wavelength pion oscillations following a quench

In studying the behaviour of the plasma in the central rapidity region of a heavy ion collision at RHIC energies or higher, it seems reasonable to assume that after a time of order 1 fm a hot plasma is formed in which the chiral order parameter is disordered and in which the baryon number density is low enough that it can be neglected. Our goal is to study the behaviour of the long wavelength modes of the chiral order parameter as this plasma loses energy and $\sigma$ develops an expectation value.

In previous work (Ref. [2], references therein, and Wilczek’s talk at this conference) we considered the equilibrium phase structure of QCD. We argued that QCD with two massless quark flavours probably undergoes a second order transition. At first sight, this might seem ideal for the development of large regions of misaligned vacuum, since the long wavelength critical fluctuations characteristic of a second order transition are such regions. Unfortunately, the effect of the light quark masses spoil this possibility [2]. While in lattice simulations it is in principle possible to reduce the light quark masses below their physical values and get arbitrarily close to the second order critical point, in heavy ion experiments we must live with a pion which has a mass comparable to the transition temperature. Near $T_c$, the correlation length in the pion channel is shorter than $T_c^{-1}$ [2], and as a result the misaligned regions almost certainly do not contain sufficient energy to radiate large numbers of pions.

Here, we consider an idealization which is in some ways opposite to that of thermal equilibrium, that is the occurrence of a sudden quench from high to low temperatures, in which the $(\sigma, \vec{\pi})$ fields are suddenly removed from contact with a high temperature heat bath and subsequently evolve mechanically according to zero temperature equations of motion. In a real heavy ion collision, the phase transition proceeds by a process in between an equilibrium phase transition in which the temperature decreases arbitrarily slowly and a quench in which thermal
fluctuation ceases instantaneously. Our goal is not to quantitatively model a realistic heavy ion collision as a quench. Rather, in studying the dynamics of the long wavelength modes of the chiral order parameter in a quench, our hope is that the qualitative behaviour in this model is representative of the physics which occurs in real processes in which the QCD plasma cools rapidly and is far from thermal equilibrium.

We use the linear sigma model to describe the low energy interactions of pions:

\[ \mathcal{L} = \int \text{d}^4x \left\{ \frac{1}{2} \partial^\mu \phi^\alpha \partial_\mu \phi_\alpha - \frac{\lambda}{4} (\phi^\alpha \phi_\alpha - v^2)^2 + H\sigma \right\}, \quad (2.1) \]

where \( \lambda, v, \) and \( H \propto m_q \) are to be thought of as parameters in the low energy effective theory obtained after integrating out heavy degrees of freedom. We treat (2.1) as it stands as a classical field theory, since the phenomenon we are attempting to describe is basically classical and because as a practical matter it would be prohibitively difficult to do better.

Our numerical simulations of quenching in the linear sigma model are described in more detail in [1]. As initial conditions, we choose \( \phi \) and \( \dot{\phi} \) randomly independently on each site of a cubic lattice. Therefore, the lattice spacing \( a \) represents the correlation length in the disordered initial state. In [1] we made a crude attempt to choose initial distributions for \( \phi \) and \( \dot{\phi} \) appropriate for a quench from an initial temperature of \( T = 1.2T_c \). With initial conditions chosen, we then model the \( T = 0 \) evolution of the system after the quench by evolving the initial configuration using a standard finite difference, staggered leapfrog scheme according to the equations of motion obtained by varying (2.1). After each two time steps, we compute the spatial fourier transform of the configuration and from that obtain the angular averaged power spectrum.

The central result of our simulations is that the power in the long wavelength modes of the pion field grows dramatically. While the initial power spectrum is white and while at late times the system is approaching a configuration in which the energy is partitioned equally among modes, at intermediate times of order several
times $m_\pi^{-1}$ the low momentum pion modes are oscillating with large amplitudes. When we used gaussian initial distributions for $\phi$ and $\dot{\phi}$ with width $v/2$ and $v$ respectively, the power in modes with $k = 0.2a^{-1} \simeq 0.3m_\pi$ is more than 1000 times that in the initial white power spectrum. For initial conditions chosen to model an initial temperature of $1.2T_c$, the amplification is less, but is still of order 100.

The amplification of low momentum modes which we observe in the numerical simulations can be qualitatively understood by approximating $\phi^\alpha \phi_\alpha(\vec{x},t)$ in the equations of motion by its spatial average. After doing the spatial fourier transform, the equation of motion for the pion field becomes

$$\frac{d^2}{dt^2} \vec{\pi}(\vec{k},t) = -\{ -\lambda v^2 + \lambda \langle \phi^2 \rangle(t) + k^2 \} \vec{\pi}(\vec{k},t)$$

where $\langle \phi^2 \rangle(t)$ means simply the spatial average of $\phi^2$. At late times, $\langle \phi^2 \rangle$ becomes time independent and takes its vacuum value, and the quantity in brace brackets in (2.2) becomes simply $m_\pi^2 + k^2$. Immediately after the quench, however, $\langle \phi^2 \rangle$ varies with time, and there are periods when $\langle \phi^2 \rangle < v^2$. A wave vector $k$ mode of the pion field is unstable and grows exponentially whenever $\langle \phi^2 \rangle < v^2 - k^2/\lambda$. As $\langle \phi^2 \rangle$ varies, longer wavelength modes are unstable for more and for longer intervals of time, and, in agreement with the numerical simulations, are amplified relative to shorter wavelength modes.

3. Charge separation does not occur

The striking prediction (1.2) for the probability distribution of the neutral pion fraction $R$ naturally leads to the question of whether there are similarly unusual fluctuations in the electric charge itself, i.e. in the ratio of $\pi^+$ to $\pi^-$ mesons. Formulae similar to (1.2) hold for the real fields $\pi^1 = \frac{1}{\sqrt{2}}(\pi^+ + \pi^-)$ and $\pi^2 = \frac{1}{i\sqrt{2}}(\pi^+ - \pi^-)$, but not for $\pi^+$ and $\pi^-$. While the total electric charge must be conserved, there is no conservation law prohibiting the separation of charge into
regions of net positive and negative charge. We must determine whether long wavelength oscillations of the electric charge density grow. The charge operator 
\[ j_0 = \pi^1 \frac{\partial}{\partial t} \pi^2 - \pi^2 \frac{\partial}{\partial t} \pi^1 \]
measures rotary motion in the \( \pi^1 - \pi^2 \) plane. However, the amplification mechanism which operates following a quench kicks \( \vec{\pi} \) radially outward, and does not induce rotary motion. This heuristic argument is borne out in the simulations. Long wavelength oscillations of the electric charge density are not amplified.

Notice that the question of whether charge separation occurs is a dynamical one, and has a straightforward dynamical answer for the quench mechanism of generating regions of misaligned condensate. In earlier work, Kowalski and Taylor [3] imposed isospin symmetry by hand in order to avoid the possibility of charge separation, which they consider physically implausible. The coherent states we reach are not isospin singlets, and we see no reason to impose that condition; nevertheless the intuition of Kowalski and Taylor is vindicated and there is no charge separation.

4. How large are the regions of coherent pion oscillations?

This question is of crucial phenomenological interest. In order to be observable, these regions must evolve into sufficiently many pions. We must ask, therefore, what are the longest wavelength modes of the pion field which get amplified? In our simulations, the answer is unequivocal — the wavelength of modes which are amplified is limited only by the size of the lattice on which the simulation is run. Alas, it is much harder to determine what will happen in a real heavy ion collision. The one thing that can be said with certainty is that the effect is \textit{not} cut off by the inverse pion mass. Modes with \( k < m_\pi \) \textit{are} amplified. This is in marked contrast to the situation obtained in thermal equilibrium, and is why the phenomenon will only be detected if the plasma in a heavy ion collision is far enough from thermal equilibrium that quenching is an appropriate idealization. If \( m_\pi^{-1} \) is not the long wavelength cut-off, what is? The most optimistic (and
perhaps implausible) possibility, which is suggested by a literal interpretation of our simulations, is that coherent oscillations of the pion field in regions as large as the transverse extent of the plasma are possible. (The rapid expansion in the longitudinal direction will damp the growth of modes with \( \vec{k} \) parallel to the beam relative to those with \( \vec{k} \) transverse.) In a real collision, the size of modes which grow could perhaps be limited by the time available before the pions no longer interact and therefore can no longer be described by oscillations of a classical field, or perhaps by the size of regions of the plasma in which the energy density is reasonably homogeneous. At this point, the most that can be said is that long wavelength pion oscillations are amplified after a quench, and that their size is not limited by any microphysical length like \( m_\pi^{-1} \) but is limited dynamically, perhaps only by the system size.

5. Outlook

Although we have made many idealizations and approximations, it seems possible that the essential qualitative feature of the phenomenon we have elucidated — long wavelength pion modes experiencing periods of negative mass\(^2\) and consequent growth following a quench — could occur in real heavy ion collisions. Given the explicit symmetry breaking which gives mass to the pions, one might have expected the dynamics following a quench to be featureless. The mechanism here discussed provides a robust counterexample. We have not come close to modelling a real heavy ion collision. While our treatment can surely be improved, it seems doubtful that quantitative theoretical predictions for heavy ion collisions will be possible. At the end of the day, the question of whether or not long wavelength pion oscillations occur will be answered experimentally. If a heavy ion collision is energetic enough that there is a central rapidity region of high energy density and low baryon number, and if such a region cools rapidly enough that the process can be modelled as a quench, this will be detected by observing clusters of pions of similar rapidity in which the neutral pion fraction \( R \) is fixed. This ratio will be
different in different clusters and will follow a distribution like (1.2). Were such a signature to be observed experimentally, it would be clear evidence for an out of equilibrium transition from a QCD plasma in which the chiral order parameter was initially disordered.

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