Constraints on violation of Lorentz invariance from atmospheric showers initiated by multi-TeV photons

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Received December 1, 2016
Revised April 14, 2017
Accepted May 11, 2017
Published May 25, 2017

Abstract. Parameterizing hypothetical violation of Lorentz invariance at high energies using the framework of effective quantum field theory, we discuss its effect on the formation of atmospheric showers by very-high-energy gamma rays. In the scenario where Lorentz invariance violation leads to a decrease of the photon velocity with energy the formation of the showers is suppressed compared to the Lorentz invariant case. Absence of such suppression in the high-energy part of spectrum of the Crab nebula measured independently by HEGRA and H.E.S.S. collaborations is used to set lower bounds on the energy scale of Lorentz invariance violation. These bounds are competitive with the strongest existing constraints obtained from timing of variable astrophysical sources and the absorption of TeV photons on the extragalactic background light. They will be further improved by the next generation of multi-TeV gamma-ray observatories.

Keywords: gamma ray theory, gamma ray experiments, modified gravity, gamma ray detectors

ArXiv ePrint: 1611.10125
## Contents

1 Introduction 1  
2 Existing constraints on Lorentz violation 2  
3 Effect of Lorentz violation on atmospheric showers 6  
  3.1 Suppression of the Bethe-Heitler process 6  
  3.2 Constraints from observations of the Crab nebula 7  
  3.3 Estimates for future experiments 10  
4 Conclusions 12

1 Introduction

Very-high energy (VHE) gamma-ray astronomy is a rapidly developing branch of astrophysics [1, 2]. Since the first ground-breaking detection of several multi-TeV gamma-ray events from Crab nebula in 1989 [3], it has evolved into a well-established technique for high-quality astronomical observations. More than hundreds of TeV gamma-ray sources have been discovered and studies of their spectra and variability have made valuable contribution to our understanding of the internal processes in these objects [4]. The propagation of VHE photons is affected by the interstellar medium, in particular, the photon background and magnetic fields. Remarkably, it is also sensitive to tiny deviations from Lorentz invariance (LI).

Possible violation of Lorentz invariance (or Lorentz violation (LV) for short) is motivated by some approaches to quantum gravity (see reviews [5–7] and references therein). Several approaches [8–11] predict that the departures from LI, while being tiny at energies accessible in laboratory, grow with energy and become significant at a certain high energy scale $M_{\text{LV}}$. This scale is conventionally assumed to be of the order of Planck mass $M_P = 1.2 \times 10^{19}$ GeV, but can also lie a few orders of magnitude below.\footnote{For example, non-projectable Hořava-Lifshitz gravity [12, 13] favors $M_{\text{LV}}$ in the range $10^{15} \div 10^{18}$ GeV.}

Discussion of LV phenomenology requires specifying the dynamical framework. We adopt a conservative approach describing departures from LI in the language of effective field theory (EFT) [14–20]. This framework implies, in particular, the usual laws of conservation of total energy and momentum in particle interactions. While being well-defined and predictive, this approach has certain limitations. A number of scenarios of LV proposed in the literature [21–25] may not admit an EFT description and thus are left outside the scope of our analysis.

In the EFT framework one postulates existence of a preferred frame, commonly identified with the rest-frame of the CMB. Typical energy attained by a particle in astrophysical phenomena is significantly higher (in CMB frame) than energy ever obtained in the laboratory, making these phenomena a sensitive probe of LV [14, 15]. The most energetic particles detected in cosmic rays are hadrons (protons or nuclei) with energies up to $10^{20}$ eV [26]. Their observation has been used to set very stringent limits on LV for protons [27–30] and nuclei [31]. However, hadrons are not elementary particles and relating these bounds to the fundamental parameters of a given model presents a complicated task. On the other
hand, particles from the sector of quantum electrodynamics (QED) — photons, electrons and positrons — are elementary\(^2\) and constraints on their properties translate directly into the constraints on the underlying theory.

The key consequence of LV is the change in particles’ dispersion relations [19]. This has two potential implications for VHE gamma rays. First, the dependence of the photon propagation velocity on energy induces delays in the arrival time of photons with different energies that can be constrained by timing observations of variable distant sources [35]. Second, it modifies the rates of particle reactions [14, 15, 17, 18, 36, 37]. Several processes, such as photon decay \(\gamma \to e^+e^-\) or photon splitting \(\gamma \to 3\gamma\), kinematically forbidden in LI theory, can become allowed. Cross-sections of other reactions, allowed also in the standard case (pair production on soft photon background or in the Coulomb field of a nucleus in the atmosphere), get modified. This affects the predictions for the gamma-ray spectra of astrophysical sources. The absence of deviations from the predictions of the standard LI theory in the observed spectra establishes bounds on the parameters describing LV.

VHE photons arriving to the Earth are detected through particle showers that they produce in the atmosphere. The depth at which the shower is initiated is determined by the cross section of the first photon-nucleus interaction, the dominant channel being \(e^+e^-\) production in the Coulomb field of the nucleus — the Bethe-Heitler process [38]. As discussed in [37, 39, 40], the cross section of the latter process sensitively depends on LV parameters in the QED sector. In an interesting parameter range the shower formation is suppressed compared to the LI case, leading to the suppression of the detected photon flux. In this paper we emphasize the role of this effect in setting the constraints on LV and derive the bounds following from the absence of suppression in the measured spectrum of the Crab nebula.

The paper is organized as follows. In section 2 we briefly describe the framework for parameterizing deviations from LI in QED and review the existing constraints on the LV parameters focusing on the case of quartic dispersion relations. In section 3 we discuss the effect of LV on the formation of an atmospheric shower by a VHE photon and derive the corresponding constraints on the scale of LV in the photon dispersion relation using the measurements of the Crab nebula spectrum by HEGRA and H.E.S.S. collaborations. We also estimate the reach of the Cherenkov Telescope Array (CTA) and future extensive air shower arrays in improving these bounds. Section 4 is devoted to conclusions.

### 2 Existing constraints on Lorentz violation

The generic effect of LV is the modification of particles’ dispersion relations. Assuming spatial isotropy in the preferred frame, particle energy \(E\) depends only on the absolute value of momentum \(p\) in that frame. At momenta smaller than the LV scale \(M_{LV}\) it can be expanded in powers of \(p\). Focusing on the QED sector and keeping up to quartic terms, one writes the dispersion relations for photons and electrons/positrons:

\[
E_{\gamma}^2 = p_{\gamma}^2 + \frac{\epsilon_{\gamma}p_{\gamma}^4}{M_{LV,\gamma}^2}, \quad E_{e}^2 = m_e^2 + p_e^2(1 + \delta_e) + \frac{\epsilon_e p_e^4}{M_{LV,e}^2},
\]

where \(\epsilon_{\gamma,e}\) can take values \(\pm 1\) and we allowed the scales suppressing the quartic contributions for photons and electrons/positrons to be different in general. Note that, without loss of

\(^2\)This is true in the simplest setup assumed in this paper. In more complicated scenarios [32–34] the QED states, as well as all other particles of the Standard Model, can be composite, which suppresses observable effects of LV.
generality, we have set the quadratic correction to the photon dispersion relation to zero, so that the low-energy velocity of photons is normalized to one; this can be always achieved by an appropriate rescaling of the space- or time-coordinates.

We have not included cubic terms in (2.1). Within the effective field theory framework, such terms would arise from CPT-odd contributions in the Lagrangian \[17–19, 41, 42\]. Phenomenologically, they are strongly constrained with the required suppression scale being well above the Planck mass, see e.g. \[43\]. In what follows we assume that the underlying theory is CPT invariant,\(^3\) so that cubic corrections to the dispersion relations are absent.

Finally, the expressions (2.1) implicitly assume that the dispersion relations are the same for states with different helicities. For photons, this is guaranteed by the CPT symmetry. On the other hand, for the fermionic states CPT invariance only ensures that the dispersion relation of electron with positive (negative) helicity coincides with the dispersion relation of positron with negative (positive) helicity. We take the equality of the dispersion relations of electrons with opposite helicities as an additional simplifying assumption. In principle, it can be ensured by requiring that the QED sector is invariant under parity \[37\], as it happens in the LI case. Our results will not depend on this assumption.

Note that the parameters in the dispersion relations (2.1) can be connected with the coefficients in the Lagrangian of LV QED in the parameterization of \[44, 45\],

\[
\begin{align*}
\delta_e &= -2c_2^{(4)}, \\
\epsilon_e &= -2c_4^{(6)}, \\
\frac{\epsilon_\gamma}{M_{\text{LV},\gamma}} &= -\frac{c_{(1)00}}{\sqrt{\pi}}.
\end{align*}
\]

We now review the constraints on these parameters.

A. Constraints on LV in electrons. The parameter \(\delta_e\) affects the physics at low energies and can be constrained using terrestrial experiments. The analysis of radiation losses by the electron and positron beams at LEP gives \[46\],

\[
|\delta_e| < 2 \times 10^{-15}.
\]

The constraints on \(M_{\text{LV},e}\) come from the observation of the Crab nebula spectrum in the energy range up to 0.1 GeV. The spectrum has two peaks well described by the synchrotron-self-Compton model (see \[47\] for review). This requires presence of electrons with energies up to \(E_{\text{max},e} \sim 10^3\) TeV in the plasma inside the nebula. They produce synchrotron radiation that corresponds to the low-energy hump of the spectrum and rescatter it by the inverse Compton process giving rise to the high-energy peak. Possible LV in electrons would modify the intensity of the synchrotron radiation and hence change the Crab spectrum \[48, 49\]. This leads to the following bound \[49\],

\[
M_{\text{LV},e} > 2 \times 10^{16}\text{ GeV}.
\]

This analysis is insensitive to LV in photons as the energy of the synchrotron radiation (up to 0.1 GeV) is much smaller than the energy of electrons.

Ref. \[49\] performs the analysis under the assumption \(\delta_e = 0\). However, relaxing this assumption is not expected to significantly change the constraint (2.4). Further, it is instructive to estimate the bound on \(\delta_e\) that can be obtained if the analysis is performed allowing for

\(^3\)We do not consider in this paper loop corrections to the dispersion relations that can induce a logarithmic running of the coefficients in (2.1) with momentum.

\(^4\)While the CPT symmetry follows from LI, the converse is not true: a theory can be CPT invariant and Lorentz violating at the same time.
its non-zero values. The relevant quantity for the synchrotron radiation is the deviation of the group velocity of electrons from unity. Comparing the contributions to the group velocity from the quadratic and quartic terms in the electron dispersion relation, we find that the bound (2.4) can be translated into (cf. [50]),

$$\delta_e \lesssim 3\left(E_{\text{max},e}/M_{\text{LV},e}\right)^2 \sim 10^{-20}.$$  \hspace{1cm}  (2.5)

Of course, this is only a crude estimate and a careful analysis taking into account the dynamical processes in the Crab nebula is required to set rigorous bounds on $\delta_e$. The fact that (2.5) is more than five orders of magnitude stronger than the best laboratory constraint (2.3) makes such analysis promising. However, it is beyond the scope of the present paper.

We are going to see that the constraints on LV in the photon dispersion relation that can be obtained from the current data are significantly weaker than for electrons. Therefore we will neglect LV in electrons from now on.

**B. Photon time of flight from distant sources.** A quartic correction in the dependence of photon energy on momentum implies the dependence of photon phase and group velocities on energy. Depending on the sign of the correction, high-energy photons from fast flares in distant sources would arrive earlier or later than low-energy ones. The time of flight analysis has been performed for active galactic nuclei (AGN) [51], gamma-ray bursts (GRB) [52] and pulsars [53]. Absence of statistically significant time-lags between photons with different energies yields,

$$M_{\text{LV},\gamma} > 6.4 \times 10^{10} \text{ GeV}, \quad \text{AGN [51]},$$

$$M_{\text{LV},\gamma} > 1.3 \times 10^{11} \text{ GeV} \quad \text{GRB [52].}$$  \hspace{1cm}  (2.6, 2.7)

The bound from pulsars is significantly weaker.

The time of flight bounds have the advantage of directly constraining the photon dispersion relation, independently of the effects of LV on the interactions. However, they are somewhat sensitive to the model of the source flare that contributes the largest uncertainty in the analysis. Stronger bounds on $M_{\text{LV},\gamma}$ are obtained by considering the physical processes affecting the propagation and detection of VHE photons. The relevant processes differ depending on whether $\epsilon_\gamma$ is positive or negative. With some abuse of language, we will refer to these cases as “superluminal” and “subluminal” respectively.

**C. Photon decay to $e^+e^-$ pair.** In the superluminal case ($\epsilon_\gamma = +1$) a high-energy photons can decay into $e^+e^-$ pairs in the vacuum. This process occurs only if the photon energy exceeds a certain threshold that can be found as follows. The quartic contribution to the dispersion relation can be thought of as an effective momentum-dependent “photon mass”,

$$m_{\gamma,\text{eff}}^2(p_\gamma) \equiv E_\gamma^2 - p_\gamma^2 = \frac{p_\gamma^4}{M_{\text{LV},\gamma}^2}.$$  \hspace{1cm}  (2.8)

It characterizes the amount of energy that can be transferred from the photon to the decay products. The process $\gamma \rightarrow e^+e^-$ becomes allowed once $m_{\gamma,\text{eff}}$ exceeds $5^2 m_e$. The pair is created with approximately equal momenta — half of the initial photon momentum. Above the threshold the decay is very rapid$^6$ and leads to a sharp cutoff in photon spectrum of

$^5$Recall that we neglect LV in electrons.

$^6$When $m_{\gamma,\text{eff}} \gg 2m_e$ the decay width is given by $\Gamma_{\gamma \rightarrow e^+e^-} = (\alpha p_\gamma^5)/(3M_{\text{LV},\gamma}^2)$, where $\alpha$ is the fine structure constant [37].
all astrophysical sources: no high-energy photons can reach the Earth from astronomical distances [17, 18]. Thus, an observation of gamma rays of astrophysical origin with an energy $E_\gamma$ gives the bound,

$$M_{\text{LV,}\gamma} > \frac{E_{\gamma}^2}{2m_e}.$$  

(2.9)

The recent analysis [54] using the highest-energy photons observed from the Crab nebula sets the constraint,

$$M_{\text{LV,}\gamma} > 2.8 \times 10^{12} \text{ GeV} \quad (\epsilon_\gamma = +1).$$  

(2.10)

Even if the photon decay into $e^+e^-$ is kinematically forbidden, the flux from astrophysical sources can be depleted by photon splitting $\gamma \to n\gamma$. This process is kinematically allowed whenever the photon dispersion relation is superluminal. Splitting into 3 photons $\gamma \to 3\gamma$ was analyzed in [55] for the case of cubic corrections to the photon dispersion relation and the width of this process was found to strongly depend on energy and the LV scale. Thus, observations of multi-TeV photons of astrophysical origin put restrictive bounds on the latter. However, a study for the case of quartic dispersion relation is missing in the literature. We leave the derivation of the corresponding bounds for future.

D. Modification of pair production on background photons. Standard LI physics predicts that a VHE photon interacts with extragalactic background light (EBL) producing an $e^+e^-$ pair, $\gamma\gamma_b \to e^+e^-$, where $\gamma$ is the VHE photon and $\gamma_b$ is a photon from the background. The mean free path of a photon with energy of several $\sim 100$ TeV is $\sim 1$ Mpc [56], which leads to an attenuation of VHE photon flux from extragalactic sources. For sources within the Milky Way this process is irrelevant.

Subluminal LV in photons ($\epsilon_\gamma = -1$) shifts the threshold of pair production upward [57–61]. This leads to higher predictions for the VHE photon flux from extragalactic sources than in the LI case. Non-detection of large fluxes constrains LV. Ref. [62] uses the data on the Mrk 501 flare in 1997 [63, 64] to establish a bound on the cubic correction to the photon dispersion relation. Translating it into the bound on the quartic term one obtains,

$$M_{\text{LV,}\gamma} \gtrsim 3 \times 10^{11} \text{ GeV} \quad (\epsilon_\gamma = -1).$$  

(2.11)

Recent analysis of the VHE part of the spectrum of Mrk 501 during the 2014 flare leads to a stronger limit [65]:

$$M_{\text{LV,}\gamma} > 7.5 \times 10^{11} \text{ GeV} \quad (\epsilon_\gamma = -1)$$  

(2.12)

at 95% confidence level (CL). It is worth noting that these bounds rely on the assumption that the observed cutoff in the Mrk 501 spectrum is not intrinsic to the source, but is fully accounted for by absorption on EBL. Besides, they require modeling of the EBL spectrum. While the understanding of EBL has significantly improved over the last decade (see [62, 66] and references therein), some uncertainties still remain [67, 68].

In refs. [69–71] it was suggested that a very strong constraint,

$$M_{\text{LV,}\gamma} \gtrsim 1.2 \times 10^{22} \text{ GeV} \quad (\epsilon_\gamma = -1),$$  

(2.13)

The width of splitting into 2 photons $\gamma \to \gamma\gamma$ is generally expected to be more suppressed by additional powers of the LV scale as in the limit of LI QED the matrix element with an odd number of external photon legs identically vanishes (this is the statement of the Furry theorem), see a discussion in [6].
can be obtained from non-observation of a photon component in ultra-high-energy (UHE) cosmic rays (energies $\gtrsim 10^{19}$ eV). In the LI case photons with such energies get absorbed through pair production on the cosmic microwave background (CMB), whereas LV at a scale below $(2.13)$ would suppress this process and UHE photons would reach the Earth. Clearly, this argument requires UHE photons to be produced in the Universe in the first place. As such, it essentially relies on the assumption that the dominant component of UHE cosmic rays are protons that give rise to UHE photons through a cascade starting with a pion production on CMB — the GZK process [72, 73]. At the moment it is not clear whether this assumption actually holds [74].

3 Effect of Lorentz violation on atmospheric showers

The bounds on $M_{LV,\gamma}$ reviewed in the previous section have been derived under the assumption of standard interaction of high-energy photons with the Earth’s atmosphere. We now discuss the validity of this assumption and obtain new constraints by considering the effect of LV on photon-induced atmospheric showers. We follow the approach of [40] which we adapt here to the case of multi-TeV energies.

3.1 Suppression of the Bethe-Heitler process

A primary photon interacts with the atmosphere mainly through the Bethe-Heitler process — pair production in the Coulomb field of an atomic nucleus in the air. The standard LI result for the cross section of this process reads [38],

$$\sigma_{BH} = \frac{28Z^2\alpha^3}{9m_e^2}\left(\log\frac{183}{Z^{1/3}} - \frac{1}{42}\right) ,$$

(3.1)

where $\alpha$ is the fine structure constant and $Z$ is the charge of the nucleus; for scattering on nitrogen ($Z = 7$) this gives $\sigma_{BH} \approx 0.51$ b. The depth of the first interaction $X_0$ is a random variable obeying exponential distribution with the mean value $\langle X_0 \rangle = m_{at}/\sigma_{BH} \simeq 57$ g cm$^{-2}$, where $m_{at}$ is the average mass of the atoms of the air (typically, nitrogen). The first interaction leads to the development of an electromagnetic cascade with the number of particles in the cascade reaching its maximum at the depth $X_{max}$. The length of the shower development $\Delta X \equiv X_{max} - X_0$ follows the Gaussian statistics. The mean value $\langle \Delta X \rangle$ depends logarithmically on the primary photon energy and varies between 200 g cm$^{-2}$ and 250 g cm$^{-2}$ in the relevant energy range (from 100 GeV to 100 TeV). Within this range the dispersion $\Sigma_{\Delta X} \approx 50$ g cm$^{-2}$ is approximately constant [75].

As pointed out in [37, 39], LV changes the cross section of the Bethe-Heitler process. Qualitatively this can be understood as follows. The electron mass in the expression (3.1) characterizes the momentum transfer between the photon and nucleus required to produce the $e^+e^-$ pair. In the LV case the momentum transfer is shifted due to the presence of the effective photon mass (2.8). Thus, up to a factor of order one, the modified Bethe-Heitler cross section can be estimated as (3.1) with the replacement

$$m_e^2 \mapsto |m_e^2 - m_{\gamma,eff}^2(p_{\gamma})/4| ,$$

(3.2)

This modification is not relevant for superluminal photons as the cross section essentially remains close to its value in the LI theory as long as $0 < m_{\gamma,eff}^2(p_{\gamma}) < 4m_e^2$, i.e. as long as the photon decay is forbidden; for higher values of $m_{\gamma,eff}^2$ photon decay provides the dominant
signature of LV. However, for subluminal photons the modification of the Bethe-Heitler cross section can be important. If
\[ m_{\gamma,\text{eff}}^2(p_{\gamma}) < 0, \quad |m_{\gamma,\text{eff}}^2(p_{\gamma})| \gg 4m_e^2 \]
the cross section gets strongly suppressed. These qualitative arguments are supported by an explicit calculation in LV QED. Under the conditions (3.3) the modified cross section reads [37],
\[ \sigma_{\text{BH}}^{\text{LV}} = \frac{16Z^2\alpha^3}{3|m_{\gamma,\text{eff}}^2(p_{\gamma})|} \log \frac{1}{\alpha Z^{1/3}} \log \frac{|m_{\gamma,\text{eff}}^2(p_{\gamma})|}{2m_e^2}. \]  

The suppression factor
\[ \frac{\sigma_{\text{BH}}^{\text{LV}}}{\sigma_{\text{BH}}} \approx \frac{12m_e^2M_{\text{LV},\gamma}^2}{7E_\gamma^4} \cdot \log \frac{E_\gamma^4}{2m_e^2M_{\text{LV},\gamma}^2} \]  
quickly decreases with energy.

Smaller cross section delays the formation of the electromagnetic cascade which is now initiated deeper in the atmosphere. Correspondingly, the depth of the maximal shower development also increases. If it exceeds certain limiting value \( X_{\text{lim}} \) which depends on the experimental setup, the event cannot be recognized as a photon. This implies a fast drop in the number of registered photons above certain energy which is determined by the LV scale \( M_{\text{LV},\gamma} \). Note that this effect is opposite to the other consequence of LV discussed in the previous section, namely, inefficient absorption of multi-TeV photons on EBL which leads to the increase of the photon flux from extragalactic sources. Therefore, in analyzing the constraints on LV it is important to make sure that these two effects do not compensate each other.

### 3.2 Constraints from observations of the Crab nebula

Absence of evidence for the suppression of the shower formation in the observational data can be used to derive constraints on \( M_{\text{LV},\gamma} \). The most energetic photon events have been detected from the Crab nebula. The spectra were measured independently by the HEGRA experiment up to \( E_\gamma \sim 75\,\text{TeV} \) [76] and by H.E.S.S. up to \( E_\gamma \sim 40\,\text{TeV} \) [77]; they are shown in figure 1. Both are well described by a power law

\[ \left( \frac{d\Phi}{dE} \right)_{\text{obs}} \propto E^{-n}, \quad n = \begin{cases} 2.62 \pm 0.02 & \text{HEGRA [76]} \\ 2.7 \pm 0.1 & \text{H.E.S.S. [77]} \end{cases} \]  

without any significant evidence for a cutoff. As the Crab nebula is a galactic source, there is no significant absorption on EBL.

In the presence of LV the measured flux gets reduced,
\[ \left( \frac{d\Phi}{dE} \right)_{\text{LV}} = P_{\text{reg}}(E_\gamma) \cdot \left( \frac{d\Phi}{dE} \right)_{\text{LI}}, \]  
where \( P_{\text{reg}}(E_\gamma) \) is the probability to actually register a photon with energy \( E_\gamma \). The latter is equal to the probability that \( X_{\text{max}} \) of the shower induced by the photon does not exceed \( X_{\text{lim}} \). To find \( P_{\text{reg}} \) we assume that LV affects only the cross section of the first interaction and does not modify the subsequent development of the shower. This is justified as the secondary
The registration probability starts deviating significantly from unity at the energies where \( \langle X_0 \rangle_{LV} \) becomes comparable to \( X_{\text{lim}} \). In the limit \( \langle X_0 \rangle_{LV} \gg X_{\text{lim}} \) it tends to

\[
P_{\text{reg}}(E_{\gamma}) \simeq \frac{X_{\text{lim}} - \langle \Delta X \rangle}{\langle X_0 \rangle_{LV}},
\]

which reflects the fact that for large \( \langle X_0 \rangle_{LV} \) the probability to form a shower is uniformly distributed over the depth of the atmosphere.

The effect of LV on the prediction for the Crab spectrum is illustrated in figure 1. We take the primary spectrum to be power-law with the spectral index fixed by the data points at energies below 20 TeV. One clearly sees a break in the highest-energy tail of the spectrum predicted by the model due to the suppression of shower formation. We now analyze the HEGRA and H.E.S.S. datasets separately and obtain the constraints on \( M_{LV,\gamma} \) from the excess of the number of actually observed events over that predicted by the LV model.

A. HEGRA data.  

HEGRA experiment has collected 385 hours of data of the Crab photon spectrum in the multi-TeV range during the period from 1997 to 2002 [76]. The obtained spectrum shows power-law dependence till the last energy bin\(^8\) centered at \( E_{\text{max}} = 75 \text{ TeV} \) (see figure 1, left panel). Numbers of events in the direction of the source \( N_{\text{on}} \) together with the numbers of events in a region of the sky away from the source \( N_{\text{off}} \) characterizing the background are listed for each energy bin in table 3 of ref. [76]. The method of gamma-hadron separation used in the HEGRA analysis does not include cuts on \( X_{\text{max}} \); conservatively, we

\(^8\)A slight steepening may be seen at the end of the spectrum, but its significance is less than 2 \( \sigma \).
take the depth of the atmosphere at the HEGRA location (approximately 1000 g cm$^{-2}$ for showers from the zenith angle $\sim 45^\circ$) as the limiting shower depth $X_{\text{lim}}^{\text{max}}$.

Data in the highest energy bin have the strongest power in constraining LV. We apply the likelihood ratio method [78, 79] to these data to test the one parameter family of LV hypotheses parameterized by $M_{\text{LV},\gamma}$. The observed values ($N_{\text{on}} = 36, N_{\text{off}} = 104$) are assumed to be random realizations of Poisson distributions with the average values

$$\langle N_{\text{on}} \rangle = \langle N_s \rangle + \langle N_b \rangle, \quad \langle N_{\text{off}} \rangle = \alpha^{-1} \langle N_b \rangle,$$

where $\langle N_s \rangle, \langle N_b \rangle$ are the expectation values of the signal and background respectively, and $\alpha = 0.2$ is the ratio of the on- and off-exposures [76]. The expectation value of the signal in the presence of LV is given by,

$$\langle N_s \rangle^{\text{LV}} = P_{\text{reg}}(E_{\text{max}}) \langle N_s \rangle^{\text{LI}},$$

where $\langle N_s \rangle^{\text{LI}}$ is the expectation value of the signal in the standard LI theory; it is obtained by extrapolating the flux from energies below 20 TeV with a power-law. The expectation value of the background $\langle N_b \rangle$ is unknown and is marginalized over. The likelihood is calculated as the probability to have the observed realization ($N_{\text{on}}, N_{\text{off}}$) for a given value of $M_{\text{LV},\gamma}$, normalized to the maximal value of the probability over all possible choices of $M_{\text{LV},\gamma}$. It is known that the logarithm of the likelihood, multiplied by $(-2)$, obeys the $\chi^2$ distribution.

The resulting likelihood profile is shown in figure 2, left panel. From it one reads the constraint

$$M_{\text{LV},\gamma} > 2.1 \times 10^{11} \text{ GeV} \quad (\epsilon_\gamma = -1) \quad \text{at 95\% CL.}$$

In the effective field theory parameterization of [44] this translates into a one-sided bound on the coefficient $c_{(I)00}^{(6)}$,

$$c_{(I)00}^{(6)} < 4 \times 10^{-23} \text{ GeV}^{-2} \quad \text{at 95\% CL.}$$

The data exhibit a slight preference for $M_{\text{LV},\gamma} \approx 6 \times 10^{11} \text{ GeV}$, but it is not statistically significant. It is due to the fact that the observed flux in the last bin lies below the best power-law fit.

Let us comment on the sensitivity of the bound (3.13) to the assumptions about the intrinsic spectrum of the Crab nebula. If instead of a pure power-law, we use a model with a cutoff in the intrinsic spectrum, the bound on LV scale will become stronger. Indeed, the slight steepening of the observed spectrum in the last bins will be accounted for by the intrinsic cutoff, leaving no room for an additional suppression due to LV. On the other hand, if the model for the intrinsic spectrum includes hardening at high energies, the bound on $M_{\text{LV},\gamma}$ will get weaker. However, this scenario is disfavored according to the present theoretical understanding of the VHE photon emission in the Crab nebula [80]. Thus, the bound (3.13) can be considered as conservative.

B. H.E.S.S. data. The second data sample that we consider in this paper is the measurement of the Crab spectrum resulting from 4.4 hours of data taking by the H.E.S.S. observatory during the flare in March 2013 [77]. The data extend till $E_{\text{max}} \sim 40 \text{ TeV}$, see
Figure 2. Dependence of the likelihood on the scale of Lorentz violation in photons $M_{LV, \gamma}$ obtained using the Crab spectrum measurements by HEGRA (left) and H.E.S.S. (right). The values of $M_{LV, \gamma}$ to the left of the vertical line are excluded at 95% CL.

The number of on- and off-events in the last bin are $(N_{on} = 4, N_{off} = 1)$. The gamma-hadron separation technique implemented in H.E.S.S. uses a multivariate analysis method which includes, in particular, cuts on $X_{\text{max}}$ [82, 83]. Conservatively, we take $X_{\text{max}}^{\text{lim}} = 600 \text{ g cm}^{-2}$: deeper showers certainly would not be recognized as photon events.

We use the same approach as in the case of HEGRA dataset to determine the likelihood of the LV hypothesis. The ratio between the on- and off-exposures for the last bin is taken as $\alpha = 0.095$ [84]. The resulting likelihood curve is shown in the right panel of figure 2. It implies the bound,

$$M_{LV, \gamma} > 1.3 \times 10^{11} \text{ GeV} \quad (\epsilon_\gamma = -1) \quad \text{at 95% CL}, \quad (3.14a)$$

or

$$c_{(6)}^{(I)00} < 10^{-22} \text{ GeV}^{-2} \quad \text{at 95% CL} \quad (3.14b)$$

in the notations of [44]. This constraint is weaker than (3.13), which is a consequence of the lower statistics. A full statistical analysis of the H.E.S.S. data on the Crab flare including determination of $X_{\text{max}}$ for individual events has potential to improve the bound (3.14). Such analysis would require an access to the raw experimental data and is beyond the scope of the present work.

The constraints (3.13), (3.14) obtained in this subsection are of the same order, but somewhat weaker than the bounds from the absorption on EBL (2.11), (2.12). Still, they are important to validate the latter bounds which rely on an implicit assumption that LV does not modify shower formation for photons with energies below $\sim 20 \text{ TeV}$. Our study implies that this assumption is indeed correct.

### 3.3 Estimates for future experiments

It is interesting to analyze how the bounds on LV can be improved by future observations. Cherenkov Telescope Array (CTA) [85] will be able to measure the photon flux from the

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Footnote: The H.E.S.S. data on the Crab spectrum in the quiet state [81], despite being collected over a longer observation time (22.9 hours), terminate at a lower energy $E_{\text{max}} \sim 30 \text{ TeV}$ and thus are less suitable for constraining LV.
Crab nebula at energy $\sim 100\text{ TeV}$ upon 50 hours of data taking for any realistic model of the Crab emission spectrum. This forecast assumes the quality requirements of no less than 10 signal events in each energy bin and the statistical significance of non-zero flux detection at least $5\sigma$ [86]. To estimate the CTA sensitivity to LV we take several sample values of $(N_{\text{on}}, N_{\text{off}})$ that satisfy these requirements (for $\alpha = 0.2$), see table 1. Next, we assume that the expectation value of the signal predicted by the LI model $\langle N_s \rangle_{\text{LI}}$ coincides with the best-fit value following from the measurements. Finally, we calculate the likelihood dependence on $M_{\text{LV},\gamma}$ following the approach described in the previous subsection; we assume that the central energy in the bin is 100 TeV and use $X_{\text{lim}}^{\text{max}} = 600 \, \text{g cm}^{-2}$. The resulting 95% CL bounds are listed in the table 1. Conservatively, we conclude that a $5\sigma$ detection of 100 TeV photon flux by CTA will allow to constrain

$$M_{\text{LV},\gamma} > 1.7 \times 10^{12} \text{ GeV} \quad (\epsilon_\gamma = -1).$$

(3.15)

This is almost an order of magnitude stronger than the limit (3.13a) from HEGRA data and exceeds the best current limit (2.12) by a factor of 2.5. Similar results can be obtained in the case of a 100 TeV photon flux detection by the HAWC experiment [87, 88].

The above analysis provides a simple criterion to estimate the exclusion power of a given experiment that can be used also at higher photon energies. Under the condition of a $5\sigma$ detection of the photon flux, the values $M_{\text{LV},\gamma}$ can be excluded at 95% CL if they suppress the registration probability of the photon (3.8) by at least a factor of two,

$$P_{\text{reg}}(E_\gamma) \leq \frac{1}{2}.$$

(3.16)

Extensive air shower arrays, such as LHAASO [89], TAIGA (HISCORE) [90] and Carpet-2 [91] are designed to register photons with energies up to (a few)$\times 10^{20}$ TeV. If the Crab spectrum does not have a sharp cutoff up to these energies, they will be able to detect the corresponding flux with high significance. Assuming a $5\sigma$ detection of photons with energies $\sim 400\text{ TeV}$ and using (3.16) as the exclusion criterion, one obtains the lower bound,

$$M_{\text{LV},\gamma} \gtrsim 3 \times 10^{13} \text{ GeV} \quad (\epsilon_\gamma = -1).$$

(3.17)

Clearly, the constraint on LV will get even stronger if photons with yet higher energies are observed. At present we do not know if sources of such photons exist in the universe. One possibility could be photons produced by the interaction of UHE cosmic rays with CMB. These photons would have energies $10^{19} \div 10^{20}$ eV, but their flux is highly uncertain depending on the chemical composition of UHE cosmic rays and the unknown radio background. Nevertheless, with an appropriate reconstruction of $X_{\text{max}}$ for individual events, the bounds on LV can be obtained without any assumptions about the origin of primary photons or

| $N_{\text{on}}$ | $N_{\text{off}}$ | $\langle N_s \rangle_{\text{LI}}$ | 95% CL bound on $M_{\text{LV},\gamma}$ (GeV) |
|----------------|-----------------|-------------------------------|---------------------------------------------|
| 11             | 3               | 10.4                          | $1.72 \times 10^{12}$                       |
| 20             | 18              | 16.4                          | $1.90 \times 10^{12}$                       |
| 30             | 42              | 21.6                          | $1.95 \times 10^{12}$                       |

Table 1. CTA exclusion potential for several realizations of the number of events corresponding to $5\sigma$ detection of the photon flux in the energy bin centered at 100 TeV.
their flux, the only requirement being a detection of a few photon-induced showers. In particular, a handful of 5 photon events with energies $\sim 10^{19}$ eV will be sufficient to set strong trans-Planckian constraint $M_{\text{LV,}\gamma} \gtrsim 4 \times 10^{23}$ GeV [40].

4 Conclusions

Using the EFT description of LV QED we have shown that the cross section of the Bethe-Heitler process responsible for the first interaction of VHE photons with the atmosphere is suppressed compared to the LI theory. This increases the depth of the photon-induced showers which, in turn, leads to suppression of the number of registered VHE photons. Using absence of such suppression in the high-energy part of the Crab spectrum we obtained 95% CL lower bounds on the scale of LV in photons. The bound following from the data collected by the HEGRA experiment (3.13) is stronger than the one obtained using the H.E.S.S. data on the Crab March 2013 flare (3.14), which is due to higher HEGRA statistics. A more detailed statistical analysis involving the characteristics of observed showers (in particular, the values of $X_{\text{max}}$) would plausibly improve the H.E.S.S. bound.

The constraints obtained in this work are a few times weaker than the bounds derived from VHE photon absorption on EBL. Still, they play the role of validating the latter bounds which were obtained under the assumption of the standard shower formation probability.

We have analyzed the potential of future experiments such as CTA and extensive air shower arrays to improve the bounds on LV from shower formation. We have found that, depending on the maximal energy of detected photons, the constraints can be improved by one ($E_{\text{max}} \sim 100$ TeV) or two ($E_{\text{max}} \sim 400$ TeV) orders of magnitude. This is comparable to the bound that can be obtained by CTA using the EBL absorption feature in the spectrum of Mrk 501 under the most favorable assumption of the power-law emission spectrum\textsuperscript{10} [92].

It is worth emphasizing that the bounds derived from the shower formation mostly rely on the physical processes happening in the atmosphere and thus are very robust. The only modeling of the source that enters into our analysis is an assumption that a power-law spectrum sets an upper limit on the primary photon flux. Future observations may require more detailed models of the emission spectrum. However, given that the most plausible source of photons with energies (a few)$\times 100$ TeV is the well understood Crab nebula, this does not appear problematic. Moreover, a proper reconstruction of $X_{\text{max}}$ for individual events allows to get rid of any assumptions about the primary flux, making the bounds completely independent of the source model [40].

Acknowledgments

We thank Arnim Balzer, Dieter Horns and Kornelia Stycz for explanations concerning the H.E.S.S. results. We are grateful to Sergey Troitsky and Ksenia Ptitsyna for helpful discussions. We thank Alan Kostelecky for useful comments on the first version of the paper. The comparison of H.E.S.S. and HEGRA observations with the prediction of LV models was supported by the Russian Science Foundation, grant 14-12-01340. P.S. thanks CERN Theory Department for hospitality. The work of S.S. is supported by the Swiss National Science Foundation.

\textsuperscript{10}In the case of a cutoff in the emission spectrum the EBL absorption bound is weaker.
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