Drag force in a D-instanton background

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We study the drag force and diffusion coefficient with respect to a moving heavy quark in a D-instanton background, which corresponds to the Yang-Mills theory in the deconfining, high-temperature phase. It is shown that the presence of the D-instanton density tends to increase the drag force and decrease the diffusion coefficient, reverse to the effects of the velocity and the temperature. Moreover, the inclusion of the D-instanton density makes the medium less viscous.

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I. INTRODUCTION

Heavy ion collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) are believed to produce a new state of matter so-called strongly-coupled quark gluon plasma (QGP). It was shown that the life-time of QGP is very short (~5-10 fm/c), hence direct detection of QGP is not possible. Thus, one needs to rely on indirect measurement using suitable probes. Heavy quarks are considered good probes to study the properties of QGP due to their large mass and other unique properties, and there are extensive experimental and theoretical efforts in the field of heavy-flavor probes, for recent reviews on this topic, see e.g. [1–3].

Anti-de-Sitter space/conformal field theory (AdS/CFT), which maps a d dimensional quantum field theory to its dual gravitational theory, living in d + 1 dimensional, has yielded many important insights for studying different aspects of QGP [4–6]. In this approach, the drag force on a moving heavy quark in $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) plasma was first studied in [7, 8]. Therein, the energy loss of the quark is understood as the momentum flow along the string into the horizon. Subsequently, there are many attempts to address the drag force in this direction. For instance, the effect of chemical potential on the drag force is discussed in [9, 10]. The effect of non-commutativity on the drag force is addressed in [11]. The finite coupling corrections corrections on the drag force are analyzed in [12]. The drag force in three charges non-extremal black hole model is studied in [13]. The drag force in AdS/QCD models is investigated in [14–16]. Other related results can be found, for example, in [17–23].

In fact, there is another check of gauge/gravity duality, the correspondence between non-perturbative objects such as instantons. It was shown [24, 25] that the Yang-Mills instantons are identified with the D-instantons of type IIB string theory. The near horizon limit of D-instantons homogeneously distributed over D3-brane at zero temperature has been studied in [24]. The holographic dual of uniformly distributed D-instantons over D3-brane at finite temperature has been analyzed in [27]. It was argued that the features of D3-D(-1) configuration are similar to QCD at finite temperature. For instance, the chiral symmetry breaking exists in the D-instanton background. The dual gauge theory of the background has a confinement property with the linear quark-antiquark potential. Thus, one expects that the results obtained from these theories could provide qualitative insights into analogous questions in QCD. For that reason, many quantities have been studied in the D-instanton background, such as phase transitions [27], light flavor quark [28], jet quenching parameter and heavy quark potential [29].

In this paper, we study the drag force and diffusion coefficient with respect to a moving heavy quark in the D-instanton background. More specifically, we would like to see how the D-instanton density affects the drag force as well as the diffusion coefficient. This is the purpose of the present work.

The organization of this paper is as follows. In the next section, we briefly review the geometry of the D-instanton background at finite temperature. In section 3, we study the effect of the D-instanton density on the drag force. In section 4, we discuss the relaxation time and the diffusion coefficient in this background as well. In the last section, we end up with some discussions.

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II. BACKGROUND GEOMETRY

In this section we briefly review the D-instanton background. The geometry is the one which is a finite temperature extension of D3/D-instanton background given in [28]. The background has an axion field and a five-form field strength which couples to the D-instanton and D3, respectively. In Einstein frame the ten dimensional super-gravity action is found to be [30, 31]

\[
S = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{g} (R - \frac{1}{2}(\partial \Phi)^2 + \frac{1}{2}e^{2\Phi}(\partial \chi)^2 - \frac{1}{6}F_{(5)}^2),
\]

(1)

where \(G_{10}\) is the 10-dimensional gravitational constant. \(R\) denotes the Ricci scalar. \(\Phi\) represents the dilaton. \(\chi\) refers to the axion. \(F_{(5)}\) stands for the field strength associated with Abelian gauge connection.

If one sets \(\chi = -e^{-\Phi} + \chi_0\) in (1), the dilaton term and the axion term can cancel. After that, the solution of (1) can be written as [32]

\[
ds^2 = e^{\frac{\Phi}{2}}\left[\frac{r^4}{R^2}f(r)dt^2 + \frac{r^2}{R^2}d\vec{x}^2 + \frac{1}{f(r)} \frac{R^2}{r^2}d\sigma^2 + R^3d\Omega_5^2\right],
\]

(2)

with

\[
e^{\Phi} = 1 + \frac{q}{r_t^4} \log \frac{1}{f(r)}, \quad f(r) = 1 - \frac{r_t^4}{r^4},
\]

(3)

where \(R\) is the AdS radius. \(\lambda = g_Y^2 N_c = \frac{\alpha'}{\sigma q}\) with \(\lambda\) the ‘t Hooft coupling and \(\alpha'\) the reciprocal of the string tension. \(\vec{x} = x_1, x_2, x_3\) are the boundary coordinates. \(r\) denotes the radial coordinate. The event horizon is located at \(r = r_t\). The boundary is \(r = \infty\). The parameter \(q\) represents the D-instanton density as well as the vacuum expectation value of the gauge field condensate. Moreover, the temperature of the black hole is

\[
T = \frac{r_t}{\pi R^2}.
\]

(4)

Also, note that for \(q = 0\) in (2), the AdS5-Schwarzschild metric is recovered.

III. DRAG FORCE

In this section, we study the behavior of the drag force for the background metric (2). It is known that when a heavy quark moves in the plasma, it feels a drag force and consequently loses energy. On the other hand, the energy loss can be described in a dual trailing string picture [7, 8]: a heavy quark moving on the boundary, but with a string tail into the AdS bulk. According to this scenario, the dissipation of the heavy quark can be depicted by the drag force, which is conjectured to be related to a string tail in the fifth dimension.

The drag force is associated with the damping rate \(\mu\) (or friction coefficient), defined by Langevin equation,

\[
\frac{dp}{dt} = -\mu p + f_1,
\]

(5)

subject to a driving force \(f_1\). And for \(dp/dt = 0\), the driving force is equivalent to a drag force \(f\).

Now we discuss a heavy quark moving in one direction, e.g., \(x_1\) direction. The coordinates are parameterized as

\[
t = \tau, \quad x_1 = vt + \xi(r), \quad x_2 = 0, \quad x_3 = 0, \quad r = \sigma.
\]

(6)

The string dynamic is captured by the Nambu-Goto action,

\[
S = \frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-g},
\]

(7)

where \(g\) is the determinant of the induced metric with

\[
g_{\alpha\beta} = g_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta},
\]

(8)

where \(g_{\mu\nu}\) and \(X^\mu\) are the brane metric and the target space coordinates, respectively.
Substituting (6) into (2), the induced metric reads
\[ g_{tt} = -e^{2\Phi} \frac{r^2 f(r)}{R^2}, \quad g_{xx} = e^{2\Phi} \frac{r^2}{R^2}, \quad g_{rr} = e^{2\Phi} \frac{R^2}{r^2 f(r)}, \]
given this, the Lagrangian density is found to be
\[ L = \sqrt{-g_{rr}g_{tt}g_{xx} - g_{rr}g_{xx}v^2 - g_{xx}g_{tt}\xi'^2} = \sqrt{e^{\Phi}[1 - \frac{v^2}{f(r)} + \frac{r^4 f(r)}{R^4} \xi'^2]}, \]
with \( \xi' = d\xi/d\sigma \). As the action does not depend on \( \xi \) explicitly, the momentum is a constant,
\[ \Pi_\xi = \frac{\partial L}{\partial \xi'} = \xi' - \frac{e^{\Phi}r^4 f(r)/R^4}{\sqrt{1 - \frac{v^2}{f(r)} + \frac{r^4 f(r)}{R^4} \xi'^2}} = \text{constant}, \]
which leads to
\[ \xi'^2 = \frac{\Pi_\xi^2[1 - \frac{v^2}{f(r)}]}{r^4 f(r)/R^4 - \Pi_\xi^2}. \]
note that in the right hand side of the above equation, the denominator and numerator are both positive for large \( r \) and negative for small \( r \) near the horizon. On the other hand, \( \xi'^2 \) should be everywhere positive. With these conditions, one finds that the denominator and numerator change sign at the same point. For the numerator, the critical point \( r_c \) satisfies
\[ f(r_c) = v^2, \]
which leads to
\[ r_c = \frac{r_t}{(1 - v^2)^{1/4}}. \]
For the denominator, it also changes sign at \( r_c \), which yields
\[ \Pi_\xi = \sqrt{e^{\Phi(r_c)}r^2_c \sqrt{f(r_c)}} = \sqrt{1 + \frac{q}{r_t^2} \log \frac{1}{v^2} \sqrt{1 - v^2} \frac{r_t^2}{R^2}}. \]
On the other hand, the current density for momentum \( p_1 \) is
\[ \pi_x^r = -\frac{1}{2}\alpha'^{1/2} \frac{g_{tt}g_{xx}}{g}, \]
and the drag force is
\[ f = \frac{dp_1}{dt} = \sqrt{g\pi_x^r}, \]
results in
\[ f = -\frac{1}{2}\alpha'^{1/2} \sqrt{1 + \frac{q}{r_t^2} \log \frac{1}{v^2} \sqrt{1 - v^2} \frac{r_t^2}{R^2}}, \]
where the minus sign implies that the drag force is against the movement.
By using the relations
\[ \lambda = g_{YM}^2 N_c = \frac{R^4}{\alpha'^2}, \quad T = r_t/(\pi R^2), \]
the drag force in the D-instanton background is obtained as
\[ f = -\sqrt{1 + \frac{q}{\pi^4 R^6 T^4} \log \frac{1}{v^2} \frac{\pi T^2 \sqrt{\lambda}}{2} \sqrt{1 - v^2}}. \]
Also, the rate of energy loss in D-instanton background is found to be
\[
\frac{dE}{dt} = \vec{f} \cdot \vec{v} = -\sqrt{1 + \frac{q}{\pi^4 R^8 T^4 \log \frac{1}{v^2}}} \frac{\pi T^2 \sqrt{g_{YM}^2 N}}{2} \frac{v^2}{\sqrt{1 - v^2}}.
\] (21)

Let us discuss results. First, for \( q = 0 \) in (20), the drag force of \( \mathcal{N} = 4 \) SYM theory [7, 8] can be reproduced, as expected. Furthermore, to study the effect of D-instanton density on the drag force, we plot \( f/f_{SYM} \) for various temperatures and velocities in fig.1. The left is plotted for a low temperature \( (r_t = 1) \) while the right is for higher one \( (r_t = 2) \). From the figures, one can see that the drag force in D-instanton background is larger that of \( \mathcal{N} = 4 \) SYM theory. Also, increasing \( q \) leads to increasing \( f \), opposite to the effects of \( v \) and \( T \) (or \( r_t \)). Therefore, one concludes that the D-instanton density has the effect of increasing the drag force. Interestingly, it was shown [29] that the presence of the D-instanton density enhances the jet quenching parameter, which means that regarding the energy loss, the effects of D-instanton density on the drag force and jet quenching parameter are consistent.

IV. DIFFUSION COEFFICIENT

The diffusion coefficient, a fundamental parameter of plasma at RHIC and LHC for heavy quarks, can be derived from the drag force. In this section, we follow the argument in [7, 8] to study the effect of the D-instanton density on the diffusion coefficient.

To begin with, we recall the results of \( \mathcal{N} = 4 \) SYM theory in [7, 8] as follows. The drag force is
\[
 f_{SYM} = -\frac{\pi T^2 \sqrt{\lambda}}{2} \frac{v}{\sqrt{1 - v^2}},
\] (22)

and the relaxation time is
\[
 t_{SYM} = \frac{2m}{\pi T^2 \sqrt{\lambda}},
\] (23)

where \( m \) is the mass of the heavy quark.

The diffusion coefficient is given by
\[
 D_{SYM} = \frac{T}{m} t_{SYM} = \frac{2}{\pi T \sqrt{\lambda}}.
\] (24)

Likewise, one can derive the relaxation time and diffusion coefficient with the effect of the D-instanton density from (20). The relaxation time is
\[
 t = \frac{1}{\sqrt{1 + \frac{q}{\pi^4 R^8 T^4 \log \frac{1}{v^2}}} \frac{2m}{\pi T^2 \sqrt{\lambda}}},
\] (25)
The diffusion coefficient is
\[ D = \frac{1}{\sqrt{1 + \frac{q}{\pi R^8 T^4} \log \frac{v}{\pi T \sqrt{\lambda}}} \pi T \sqrt{\lambda}}. \]  
\( \text{(26)} \)

To illustrate the effect of the D-instanton density on the diffusion coefficient, we plot \( D/D_{SYM} \) at two fixed temperature \( (r_h = 1) \) for two different velocities in fig.2. One finds that the D-instanton density decreases the diffusion coefficient, while the velocity and temperature have opposite effect. Also, the effect of the D-instanton density on the relaxation time is similar to that on the diffusion coefficient.

On the other hand, one would like to discuss the mass of the heavy quark. It was shown that \[ t > \frac{1}{T}. \]  
\( \text{(27)} \)

In terms of \( t = \frac{q}{\pi T} D \), one finds
\[ m > \sqrt{1 + \frac{q}{\pi R^8 T^4} \log \frac{1}{v^2}} \frac{2}{\pi T \sqrt{\lambda}}, \]  
\( \text{(28)} \)
which implies that the presence of the D-instanton density increases the lower limit of the mass of the heavy quark.

\[ \text{V. CONCLUSION} \]

Drag force and diffusion coefficient are two important quantities that can be related to the energy loss in dissipative processes in medium. In this paper, we studied these two quantities in an AdS configuration generated by a dilaton field, corresponding to the D-instanton density contributions. It is shown that the presence of the D-instanton density tends to increase the drag force and decrease the diffusion coefficient, reverse to the effects of the velocity and the temperature. Interestingly, it was argued that the D-instanton density enhances the jet quenching parameter. Thus, one concludes that the effects of D-instanton density on the drag force and the jet quenching parameter are consistent.

Moreover, one can analyze the effects of D-instanton density on the viscosity. It is known that a stronger force and associated smaller diffusion coefficient imply a more strongly coupled medium, closer to an ideal liquid. Since the D-instanton density has the effect of increasing the drag force, one argues that the presence of the D-instanton density tends to decrease the viscosity of QGP.

\[ \text{VI. ACKNOWLEDGMENTS} \]

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