We introduce preferential behavior into the study on statistical mechanics of money circulation. The computer simulation results show that the preferential behavior can lead to power laws on distributions over both holding time and amount of money held by agents. However, some constraints are needed in generation mechanism to ensure the robustness of power-law distributions.

Keywords: money circulation; power law; preferential behavior; econophysics

1. Introduction

Empirical studies on many social issues show that numerous statistical distributions follow power laws, such as stock-price fluctuations, the probability distribution of the population of cities, the degree distributions in many networks and distribution of income or wealth. In particular, the wealth distribution which is an old topic has recently been renewed by a small band of econophysicists. They proposed multi-agent interaction models to explore the mechanism of wealth distribution, in which wealth is simply represented by money. However, to our knowledge, the simulation results of these works show that the steady states of money have Gibbs or Gamma distributions, leaving the power-law phenomena of wealth distribution unexplained. It has been shown that preferential behavior assumption plays an essential role in generating power-law distributions in some cases, such as in networks and in money dynamics. The soul of preferential assumption is to break the equality among agents, some of which are entitled to more power in getting corresponding entity. In this paper, we investigate how the individual preferential behavior generates power law phenomena in money circulation process. The statistical distributions involved in monetary circulation are composed of two aspects. One is over the time interval the money stays in agents' hands which is named as holding time, the other is over the amount of money held by agents.
2. Power Law Distribution over Holding Time

The model in this section is an extension of ideal gas-like model\textsuperscript{7}, where each agent is identified as a gas molecule and each trading as one elastic (two-body) collision. In this model, the economic system is assumed to be closed, thus the total amount of money $M$ before and after transaction is conserved and the number of agents $N$ remains constant. Since the scale and initial distribution of money have no effect on the final results, most of our simulations were carried out with $N = 250$ and $M = 25,000$ and the amount of money held by each agent was set to be $M/N$ at the beginning. The money is possessed by agents individually and agents can exchange money with each other. In each round, an arbitrary pair of agents $i$ and $j$ gets engaged in a trade among which agent $i$ is randomly picked to be the “payer” and the other one $j$ becomes the “receiver”. The amount of money transferred is determined by the following equation

$$\Delta m = \varepsilon (m_i + m_j)/2,$$

where $\varepsilon$ distributes randomly and uniformly within an interval $[0, 1]$.

In the original ideal gas-like models, the money paid out is chosen with equal probability, which has been discussed in Ref. 10. In the extension model, the preferential behavior is introduced by imposing the unequal probability. In a given round, agent $i$ with money $m_i$ in hand is the payer, the probability of money $k$ among $m_i$ to be transferred is given by:

$$p(k) = \frac{l_k + 1}{\sum_{n=1}^{m_i} (l_n + 1)},$$

where $l_n$ is the times that money $n$ has participated in trade since the beginning of simulation. Here, we express the probability with exchange times plus 1 instead of exchange times in case that denominator be zero at the beginning of simulations.

In simulations, the interval between the first two exchanges for one unit of money to participate in is recorded as its holding time after most of money ($\geq 99.9\%$ in our simulations) had been exchanged at least one time. Then, after most of money ($\geq 95\%$) are recorded, the sampling of the holding times of money in this system is completed.

Each of the results shown in Figure 1 is an average of 500 simulations. It can be found that the holding time distribution does not follow Gibbs-Boltzmann distribution any more after the introduction of preferential behavior. There is an initial growth of $P(t)$ from $t = 0$, which quickly saturates and then a long range of power-law decay in $P(t)$ for large $t$ value is observed. This decay, when fitted to power law $P(t) \propto t^{-\nu}$, gives $\nu = -3.67 \pm 0.05$. We also examined the holding time distributions in several periods long enough after the system had achieved stationary state, and we found the power distribution is remarkably robust, e.g., the holding time distribution is stationary even after $t = 500,000$ while it has been observed after $t = 1,000$. 
3. Power Law Distribution over Money Held

Previous studies on money distribution are all carried out within the framework of ideal gas-like models. We introduced the preferential behavior into such kinds of model by assuming that the agent with more money has larger probability to win or to be chosen to participate in the trade. However slightly the preferential propensity is set, we get the same final result: one agent achieves all of the money. This indicates that preferential behavior is not enough to produce robust power-law distributions in such case. Thus, in this section, the effects of preferential behavior on money distribution will be analyzed within a new framework.

In what follows, the initial setting of the system is the same with the ideal gas-like model, \( N = 250, M = 25,000 \) and each agent has 100 units of money in hand at the beginning. The main novel mechanism introduced here is to assume that every agent pays money out to others in each round, and the amount of money paid out is determined randomly. As to how to dispense the money, there are two modes. One is that the others have equal probability to receive each unit of money; the other one is that the probability \( p_{i,j} \) at which agent \( i \) gets the money from agent \( j \) satisfies

\[
p_{i,j} = \frac{m_i}{\sum_{n=1}^{N} (n \neq j) m_n},
\]

where \( m_i \) is the amount of money held by agent \( i \) before the trade. Please note the constraint \( n \neq j \) in this rule eliminates the possibility for the payer to get back the
money he has paid out. It is obvious that the second mode is with preference, in which the rich have higher probability to get richer.

The simulation results of the two modes are shown in figure 2, both of which are averages of 500 simulations. And they reveal clearly that the preferential behavior does have effects on the probability distribution of money. The stationary distribution of money without preference is a Gamma type. After the preferential behavior is introduced into the model, the power-law distribution is observed, and the fitting to power law gives exponent $\gamma = -1.60 \pm 0.02$. Further measurement performed after $t = 500,000$ shows that the distribution is quite robust.

In the simulations, if we removed the constraint $n \neq j$ in Eq.(3), we found that for $t = 500,000$, more than 80% of the money was acquired by one agent. It can be forecasted that after enough long time, all of money would be held by one agent. This fact means that only preferential behavior without any constraints can not induce power laws.

4. Conclusion

In this paper we studied the effect of preferential behavior on probability distributions of both aspects of the circulation of money. We performed computer simulations to show how the preferential behavior produces power-law distributions over holding time and money held respectively. It is also worth noting that some constraints may be necessary to ensure the robustness of power laws. The conclusion may be valuable to the understanding on the mechanism of power laws.

Acknowledgements

We would like to thank Prof. Zengru Di for encouragement and enlightening communications. This work was supported by the the National Science Foundation of China under Grant No. 70371072.

References

1. P. Bak, K. Chen, J.A. Scheinkman and M. Woodford, Ric. Economichi 47, 3 (1993); M.H.R. Stanley et al. Nature 379, 804 (1996).
2. S.C. Manrubia, D.H. Zanette, Phys. Rev. E. 58, 295 (1998).
3. R. Albert and A.-L. Barabási, Reviews of Modern Physics 74, 47 (2002).
4. V. Pareto, Cours d’Economie Politique, (Librairie Droz, Geneva, 1897).
5. A. Drăgulescu, and V. M. Yakovenko, Physica A 299, 213 (2001); A. Drăgulescu and V. M. Yakovenko, Eur. Phys. J. B 20 585-589 (2001).
6. B. Hayes, Am. Scientist 90, 400 (2002).
7. A. A. Drăgulescu and V. M. Yakovenko, Eur. Phys. J. B 17, 723(2000).
8. A. Chakrabarti and B. K. Chakrabarti, Eur. Phys. J. B 17, 167 (2000); M. Patriarca, A. Chakrabarti and K. Kaski, arXiv: cond-mat/0312167.
9. N. Ding, Y. Wang and Li Zhang, Eur. Phys. J. B 36, 149-153(2003).
10. Y. Wang, N. Ding and L. Zhang, Physica A 324, 665-667(2003).