Investigation of $\Xi^0_c$ in a chiral quark model

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Abstract Recently, three new states of $\Xi^0_c$ were observed in the invariant mass spectrum of $\Lambda_c^+ K^-$ by LHCb collaboration. In this work, we use a chiral quark model to investigate these three exited states with the help of Gaussian expansion method both in three-quark structure and in five-quark structure with all possible quantum numbers $IJ^P = \frac{1}{2}(\frac{1}{2})^-$, $\frac{1}{2}(\frac{3}{2})^-$, $\frac{3}{2}(\frac{1}{2})^-$, $\frac{3}{2}(\frac{3}{2})^-$ and $\frac{5}{2}(\frac{5}{2})^-$. The calculations shows that the masses of $2S$ and $1D$ states of $\Xi_c$ are comparable to experimental results; In addition, the resonance states of five-quark configuration are possible candidates of these new states with negative parity by using the real scaling method and their decay width is also given.

1 Introduction

During recent years, many narrow $\Xi_c$ states composed of a charmed quark (c) a strange quark (s) and a light quark (u or d) have been reported by Belle and BaBar Collaborations [1–6]. Very recently, LHCb Collaboration announced that three other $\Xi^0_c$ resonances $\Xi_c(2923)^0, \Xi_c(2938)^0$ and $\Xi_c(2965)^0$ have been observed in the $\Lambda_c^+ K^-$ mass spectrum [7]. The mass and widths of the observed resonances are shown below,

$$\Xi_c(2923)^0 : M = 2923.04 \pm 0.25 \pm 0.20 \pm 0.14\text{MeV}$$
$$\Gamma = 7.1 \pm 0.8 \pm 1.8\text{MeV}$$

$$\Xi_c(2938)^0 : M = 2938.55 \pm 0.21 \pm 0.17 \pm 0.14\text{MeV}$$
$$\Gamma = 10.2 \pm 0.8 \pm 1.1\text{MeV}$$

$$\Xi_c(2965)^0 : M = 2964.88 \pm 0.26 \pm 0.14 \pm 0.14\text{MeV}$$
$$\Gamma = 14.1 \pm 0.9 \pm 1.3\text{MeV}$$

The above results have led to heated discussion while the quantum numbers of these resonances are still unknown. In Ref. [7], LHCb Collaboration pointed out that the $\Xi_c(2930)^0$ [2,3] should be the overlap of state $\Xi_c(2923)^0$ and state $\Xi_c(2938)^0$ while the $\Xi_c(2970)^0$ [4,5,8] may be different from the state $\Xi_c(2965)^0$. In Ref. [9], three newly observed states were studied in the QCD sum rule and light-cone sum rules, and their results suggest that three new states can be well explained as $\Xi^+_c$ baryons with quantum numbers $\frac{1}{2}(\frac{1}{2})^-$ or $\frac{1}{2}(\frac{3}{2})^-$. In Ref. [10], the three $\Xi^0_c$ states and their two body strong decays were evaluated within a chiral quark model and it was found that the resonances $\Xi_c(2923)^0$ and $\Xi_c(2938)^0$ are most likely to be $1P\dagger$ $\Xi'_c$ states with $IJ^P = \frac{1}{2}(\frac{1}{2})^-$ while the $\Xi_c(2965)^0$ should be $1P\dagger$ $\Xi'_c$ states with $IJ^P = \frac{1}{2}(\frac{3}{2})^-$. In Ref. [11], the author performed a $3P_0$ model analysis which contain strong decay behaviors and suggested that the $\Xi_c(2923)^0$ and $\Xi_c(2938)^0$ might be $1P\dagger$ $\Xi'_c$ states and $\Xi_c(2965)^0$ can be $2S\dagger$ $\Xi'_c$ states. In addition, lattice simulations also investigated these heavy flavored baryons [12–14]. Since the quantum numbers of these new states are not determined for the moment, the explanation of them as the excited states of three-quark baryon may be reasonable.

However, the possibility of the five-quark structure of these newly observed states cannot be excluded. From PDG [1], the recognized three $\Xi^0_c$ ground states are $\Xi_c(2471)^0$ with $J^P = (\frac{1}{2})^+$, $\Xi_c(2579)^0$ with $J^P = (\frac{3}{2})^+$ and $\Xi_c(2645)^0$ with $J^P = (\frac{5}{2})^+$. The mass difference between the three newly observed excited states and the ground states ranges from 320 to 494 MeV, which is enough to excite a light quark-antiquark pair from the vacuum to form a pentaquark structure. Thus it is also reasonable for us to use five-quark structure dynamical calculation to analyze the inner structure of the three newly observed states. In Ref. [15], the excited $\Xi^0_c$ states were explained as molecular states of $D\Sigma-DA$. In fact, several theoretical analyses and predictions described the $\Xi^0_c$ as molecular states [16–19].

From the situation we have presented so far, different calculations give different explanations for the three newly observed states’ structures. So it is still controversial for the three states to be described as three-quark structure or as five-quark structure or even as a mixture of them.
Since the quark model was proposed by Gell-Mann and Zweig in 1964, respectively [20,21], it has become the most common approach to study the multiquark system as it evolves. In this work, a constituent quark model (ChQM) is employed to investigate systematically the three-quark and five-quark states corresponding to $\Xi^+_c$. To calculate accurately the results of each possible system, Gaussian expansion method (GEM) [22], an accurate and universal few-body calculation method is used. Within this method, the orbital wave functions of all relative motions of the systems are expanded by gaussians. After considering all possible color (color-singlet and color-octet), spin and flavor configurations, we can completely determine the structures of the system. Finally, with the help of “real scaling method”, we can find the genuine five-quark resonances and their respective decay width.

The paper is organized as follow. In Sect. 2, details of a chiral quark model and calculation method are introduced. The results of the three-quark structure are also presented in Sect. 2. In Sect. 3, the numerical results with analysis and discussion of five-body structure are presented. Finally, We give a brief summary of this work in the last section.

### 2 Chiral quark model and wave function

In this paper, the chiral quark model is employed to investigate the states. The chiral quark model has become one of the most common approaches to hadron spectra, hadron-hadron interactions and multiquark states for its successful descriptions [23]. In this model, in addition to one-gluon exchange (OGE), the massive constituent quarks also interact with each other through Goldstone boson exchange. Besides, the color confinement and the scalar octet (the extension of chiral partner $\sigma$ meson) meson exchange are also introduced. More details of this model can be found in Refs. [23,24]. The Hamiltonian of the chiral quark model is given as follows:

$$H = \sum_{i=1}^{n} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{j>i=1}^{n} V_{ij},$$

$$V_{ij} = V_{ij}^{CON} + V_{ij}^{OGE} + V_{ij}^{\gamma} + V_{ij}^{s},$$

$$V_{ij}^{CON} = \lambda_i^c \cdot \lambda_j^c \left[ -a_0 \sum_{1}^{N} \left( -e^{-\mu r_{ij}} + \Delta \right) \right],$$

$$V_{ij}^{OGE} = \frac{1}{d} a_s \lambda_i^c \cdot \lambda_j^c \left[ \frac{1}{r_{ij}} - \frac{\sigma_i \cdot \sigma_j}{6m_i m_j} \right],$$

$$r_0(\mu) = \frac{r_0}{\mu}, \quad a_s = \frac{a_0}{\ln((\mu^2 + \mu_0^2)/\Lambda_0^2)},$$

$$V_{ij}^{\gamma} = v_\gamma(r_{ij}) \sum_{a=1}^{3} \lambda_i^{a} \lambda_j^{a} + v_K(r_{ij}) \sum_{a=1}^{7} \lambda_i^{a} \lambda_j^{a} + v_\eta(r_{ij})$$

$$+ \left[ \cos \theta_p \left( \lambda_i^{0} \lambda_j^{0} \right) - \sin \theta_p \left( \lambda_i^{8} \lambda_j^{8} \right) \right],$$

$$V_{ij}^{s} = \frac{g_s^2}{4\pi} \frac{m_i^2}{12 m_i m_j} A_{ij}^{2} - m_i^2 m_j^2 Y(m_i r_{ij}) - Y(m_j r_{ij}) (\sigma_i \cdot \sigma_j),$$

$$\chi = \pi, K, \eta.$$

The model parameters are listed in Table 1, and the calculated baryon and meson masses are presented in the Table 2 with the experimental values. From the calculation, the theoretical results of baryons which contain one charm quark can fit well with the theoretical results. For meson spectrum, most of the results were close to experimental values except for $\rho$ and $\omega$ mesons.

Since the newly observed states can be three-quark or five-quark states, a comprehensive calculation involving both three-quark and five-quark structure is necessary. The calculation results of all baryons used in this paper (including three $\Sigma_c$ states at the bottom of the table) for the low-lying excited (2S, 1P and 1D) are listed in Table 3. From the results, we can see that the masses of all three 1P $\Sigma^0_c$ states range from 2750 MeV to 2819 MeV, which are close to the masses of two exited $\Sigma^+_c$ states with negative parity: $\Sigma^0_c$(2790) and $\Sigma^0_c$(2815). Similarly, the masses of 2S states $\Sigma^+_c$ are close to the 2.9 GeV. Considering that our calculation result of $\Sigma^+_c$(2645) are lower than the experimental value of 22 MeV, if we make an energy correction, it will fit better with the experimental values. Thus we think one of the three reported states ($\Sigma^0_c$(2923)) can be explained as the 2S states of $\Sigma^+_c$ if their parity is measured to be positive in the future experimen-
ments. In addition, since the masses of 1D states of $\Xi_c$ are within the range, 2938–2992 MeV, it is possible that two of the reported states ($\Xi_c^0(2938)$ and $\Xi_c^0(2965)$) can also be identified as the $1D$ states of $\Xi_c$ if their parity is measured to be positive (Table 3).

Now, we turn to five-quark structures, where the same set of parameters in Table 1 is used. It is worth mentioning that, the components of three-quark will mix with that of five-quark, so it will be necessary to consider a mixture of the two structures in future work.

In the following, the wave functions of the five-quark systems are constructed. The wave function of the system consists of four parts: orbital, spin, flavor and color. The wave function of each part is constructed by two steps, first construct the wave functions of three-quark cluster and quark-antiquark cluster respectively, and then coupling two clusters wave functions to form the complete five-body one.

The first part is orbital wave function. A five-body system have four relative motions so it is written as follows,

$$\psi_{LMJ} = \left[ \left[ \left[ \left[ \phi_{n_1l_1}(\boldsymbol{r}_1) \phi_{n_2l_2}(\boldsymbol{r}_2) \phi_{n_3l_3}(\boldsymbol{r}_3) \phi_{n_4l_4}(\boldsymbol{r}_4) \phi_{n_5l_5}(\boldsymbol{r}_5) \right] \phi_{n_6l_6}(\boldsymbol{r}_6) \right] \phi_{n_7l_7}(\boldsymbol{r}_7) \right] \phi_{n_8l_8}(\boldsymbol{r}_8) \right]_{LMJ},$$

(9)

where the Jacobi coordinates are defined as follows,

$$\rho = \boldsymbol{x}_1 - \boldsymbol{x}_2,$n

(9)

$$\lambda = \left( \frac{m_1 \boldsymbol{x}_1 + m_2 \boldsymbol{x}_2}{m_1 + m_2} \right) - \boldsymbol{x}_3,$$

$$\varphi = \left( \frac{m_3 \boldsymbol{x}_3}{m_3} \right) - \left( \frac{m_4 \boldsymbol{x}_4 + m_5 \boldsymbol{x}_5}{m_4 + m_5} \right).$$

(10)

$\boldsymbol{x}_i$ is the position of the $i$-th particle. Then we use a set of gaussians to expand the radial part of the orbital wave function which is shown below,

$$\psi_{1m}(\boldsymbol{r}) = \sum_{n=1}^{n_{\text{max}}} c_{nl} \phi_{nlm}(\boldsymbol{r})$$

(11)

$$\phi_{nlm}(\boldsymbol{r}) = N_{nl} r^l e^{-\nu r^2} Y_{lm}(\hat{r})$$

(12)

where $N_{nl}$ is the normalization constant,

$$N_{nl} = \left( \frac{2^{l+2} (2\nu_n)^{l+3/2}}{\sqrt{\pi (2l+1)!!}} \right) \frac{1}{2},$$

(13)

and $c_{nl}$ is the variational parameter, which is determined by the dynamics of the system. The Gaussian size parameters are chosen according to the following geometric progression:

$$\nu_n = \frac{1}{r_n^2}, r_n = r_{\text{min}} a^{n-1}, a = \left( \frac{r_{\text{max}}}{r_{\text{min}}} \right)^{\frac{1}{n_{\text{max}}-1}},$$

(14)

where $n_{\text{max}}$ is the number of Gaussian functions, and $n_{\text{max}}$ is determined by the convergence of the results. In the present calculation, $n_{\text{max}} = 8$.

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**Table 1** Quark model parameters

|           | $m_u=m_d$ (MeV) | 313 |
|-----------|----------------|-----|
| $m_s$ (MeV) | 525 |
| $m_c$ (MeV) | 1800 |

**Goldstone bosons**

|           | $\Lambda_\pi$ (fm$^{-1}$) | 4.20 |
|-----------|--------------------------|------|
| $\Lambda_\rho = \Lambda_\kappa$ (fm$^{-1}$) | 5.20 |
| $m_\pi$ (fm$^{-1}$) | 0.70 |
| $m_\kappa$ (fm$^{-1}$) | 2.51 |
| $m_\eta$ (fm$^{-1}$) | 2.77 |
| $\rho_{ch}/(4\pi)$ | 0.54 |
| $\theta_p(\circ)$ | -15 |

**Confinement**

|           | $a_c$ (MeV) | 160.5 |
|-----------|-------------|-------|
| $\mu_c$ (fm$^{-1}$) | 0.683 |
| $\Delta$ (MeV) | 68.4 |

**Scalar nonet**

|           | $m_\sigma$ (fm$^{-1}$) | 3.42 |
|-----------|------------------------|------|
| $A_\sigma$ (fm$^{-1}$) | 4.20 |
| $\Lambda_\sigma$ (fm$^{-1}$) | 5.20 |
| $m_\sigma$ (fm$^{-1}$) | 4.97 |

**OGE**

|            | $\hat{r}_0$ (MeV fm) | 30.8 |
|------------|----------------------|------|
| $\alpha_{us}$ | 0.552/0.684 |
| $\alpha_{us}$ | 0.650/0.613 |
| $\alpha_{us}$ | 0.633/0.683 |
| $\alpha_{us}$ | 0.650/-- |

**Table 2** The masses of ground-state baryons and mesons (unit: MeV)

|         | $\Lambda$ | 1105 | 1116 |
|---------|-----------|------|------|
| $\Sigma$ | 1201 | 1189 | 1152 |
| $\Sigma^*$ | 1289 | 1385 | 2471 |
| $\Xi_c$ | 2512 | 2297 |
| $\Xi_c'$ | 2587 | 2286 |

|         | $\Sigma_c$ | 2623 | 2645 |
|---------|------------|------|------|
| $\Sigma_c'$ | 2449 | 2455 | 2520 |
| $\Lambda_c$ | 2483 | 2286 |

|         | $\pi$ | 140 | 140 |
|---------|------|-----|-----|
| $\rho$ | 698 | 775 | 782 |
| $\omega$ | 607 | 782 | 542 |
| $\eta$ | 542 | 493 |
| $K$ | 494 | |

|         | $K^*$ | 868 | 892 |
|---------|------|-----|-----|
| $D$ | 1864 | 1864 | 2007 |
| $D^*$ | 2020 | 2007 |
The spin wave functions of 3-quark and 2-quark clusters are written as follows,

\[
\begin{align*}
|B^0_{1/2}⟩ &= \frac{1}{\sqrt{6}} (2αβ - αα - βα), \\
|B^2_{1/2}⟩ &= \frac{1}{\sqrt{2}} (αβα - βαα), \\
|B^1_{-1/2}⟩ &= \frac{1}{\sqrt{6}} (αβ + βα - 2βα), \\
|B^2_{-1/2}⟩ &= \frac{1}{\sqrt{2}} (αβ β - βα), \\
|B^2_{3/2}⟩ &= ααα, \quad |B^3_{1/2}⟩ = \frac{1}{\sqrt{3}} (ααβ + αβα + βα), \\
|B^3_{-1/2}⟩ &= \frac{1}{\sqrt{3}} (αβα + βαβ - βαα), \quad |B^4_{1/2}⟩ = ββ, \\
|M^0_{1,1}⟩ &= αα, \quad |M^0_{1,0}⟩ = \frac{1}{\sqrt{2}} (αβ + βα), \quad |M^0_{1,-1}⟩ = ββ, \\
|M^0_{0,0}⟩ &= \frac{1}{\sqrt{2}} (αβ - βα).
\end{align*}
\]

(15)

Similarly, we can construct the flavor wave function. We have three flavor configurations of the system: \((qqs)(c\bar{q}), (qqc)(s\bar{q})\) and \((qsc)(q\bar{q})\) \((q = u \text{ or } d)\). The flavor wave functions of three-quark and quark-antiquark subclusters can be written as follows,

\[
\begin{align*}
|B^1_{1,1}⟩ &= uuc, \quad |B^1_{1,0}⟩ = \frac{1}{\sqrt{2}} (udc + duc), \quad |B^1_{1,-1}⟩ = ddc, \\
|B^1_{0,0}⟩ &= \frac{1}{\sqrt{2}} (udc - duc), \\
|B^2_{1,1}⟩ &= uus, \quad |B^2_{1,0}⟩ = \frac{1}{\sqrt{2}} (uds + dus), \quad |B^2_{1,-1}⟩ = dds, \\
|B^2_{0,0}⟩ &= \frac{1}{\sqrt{2}} (uds - dus), \\
|B^3_{1,1}⟩ &= \frac{1}{\sqrt{2}} (usc + suc), \quad |B^3_{1,0}⟩ = \frac{1}{\sqrt{2}} (dsc + sdc), \\
|B^3_{0,0}⟩ &= \frac{1}{\sqrt{2}} (usc - suc), \quad |B^3_{0,0}⟩ = \frac{1}{\sqrt{2}} (dsc - sdc), \\
|B^4_{1,1}⟩ &= \frac{1}{\sqrt{2}} (usc), \quad |B^4_{1,0}⟩ = \frac{1}{\sqrt{2}} (dsc), \quad |B^4_{0,0}⟩ = \frac{1}{\sqrt{2}} (usc), \\
|M^1_{1,1}⟩ &= s\bar{d}, \quad |M^1_{1,0}⟩ = -s\bar{u},
\end{align*}
\]

Table 3 The masses of ground-state and low-lying excited state baryons (unit: MeV)

|          | \(\Lambda(1116)\) | \(\Sigma(1189)\) | \(\Sigma^*(1385)\) | \(\Lambda_c(2286)\) | \(\Sigma_c(2455)\) | \(\Sigma^*_c(2520)\) |
|----------|------------------|------------------|------------------|------------------|------------------|------------------|
| 1S       | 1105             | 1201             | 1289             | 2297             | 2449             | 2483             |
| 2S       | 1432             | 1512             | 1559             | 2621             | 2735             | 2753             |
| 1P       | 1356             | 1468             | 1484             | 2560             | 2668             | 2675             |
| 1D       | 1559             | 1660             | 1665             | 2741             | 2838             | 2841             |

\[
\begin{align*}
\Sigma_+(2471) &= 2512, \quad \Sigma'_+(2579) = 2587, \quad \Sigma''_+(2645) = 2623, \\
2S &= 2824, \quad 2S = 2880, \quad 2S = 2900, \\
1P &= 2750, \quad 1P = 2810, \quad 1P = 2819, \\
1D &= 2938, \quad 1D = 2989, \quad 1D = 2992.
\end{align*}
\]
\[ M_{\frac{1}{2}^+} = c\bar{d}, \quad M_{\frac{1}{2}^-} = -c\bar{u}, \]
\[ M_{1^1} = u\bar{d}, \quad M_{1^3} = \frac{1}{\sqrt{2}}(-u\bar{u} + d\bar{d}), \quad M_{1^1} = -d\bar{u}, \]
\[ M_{3^0} = \frac{1}{\sqrt{2}}(-u\bar{u} - d\bar{d}). \]

For the five-quark system under the present investigation, the possible isospin quantum numbers are $\frac{1}{2}$ and $\frac{3}{2}$. The flavor wave functions of five-quark system with isospin $I = \frac{1}{2}$ can be obtained as follows,

\[ \chi_{\frac{1}{2}^+} = \sqrt{\frac{2}{3}}B_{1,1} \left| M_{\frac{1}{2}^+} \right| \left| M_{\frac{1}{2}^-} \right| - \sqrt{\frac{1}{3}}B_{1,0} \left| M_{1^1} \right| \left| M_{1^1} \right|, \]
\[ \chi_{\frac{1}{2}^-} = B_{0,1} \left| M_{\frac{1}{2}^-} \right| \left| M_{\frac{1}{2}^-} \right|, \]
\[ \chi_{1^3} = \sqrt{\frac{2}{3}}B_{1,1} \left| M_{1^3} \right| \left| M_{1^3} \right| - \sqrt{\frac{1}{3}}B_{1,0} \left| M_{1^1} \right| \left| M_{1^1} \right|. \]

The flavor wave functions of five-quark system with isospin $I = \frac{3}{2}$ can be obtained in the following,

\[ \chi_{\frac{3}{2}^+} = \sqrt{\frac{2}{3}}B_{1,1} \left| M_{\frac{3}{2}^+} \right| \left| M_{\frac{1}{2}^-} \right| - \sqrt{\frac{1}{3}}B_{1,0} \left| M_{1^1} \right| \left| M_{1^1} \right|, \]
\[ \chi_{\frac{3}{2}^-} = B_{0,1} \left| M_{\frac{3}{2}^-} \right| \left| M_{\frac{3}{2}^-} \right|, \]
\[ \chi_{1^3} = \sqrt{\frac{2}{3}}B_{1,1} \left| M_{1^3} \right| \left| M_{1^3} \right| - \sqrt{\frac{1}{3}}B_{1,0} \left| M_{1^1} \right| \left| M_{1^1} \right|. \]

The last part is the color wave function. We consider both two kinds of color structures, color-singlet and color-octet. The total color wave functions are written directly:

\[ |\chi^{(1)}\rangle = \frac{1}{\sqrt{18}}(rb - grb + grb - grb + grb)\bar{r}\bar{r} + \bar{g}\bar{g} + \bar{b}\bar{b}). \]
\[ |\chi^{(2)}\rangle = \frac{1}{\sqrt{192}}((rb - grb + grb - grb)(2\bar{r}\bar{r} - \bar{r}\bar{r} - \bar{g}\bar{g}) + (2grb - rb - grb - bgr)(\bar{r}\bar{r} - \bar{g}\bar{g}) \]
\[ = +2(2rgr - rgr - grb)\bar{r}\bar{b} + 2(2rb - rb - grb)\bar{r}\bar{g} - 2(rb - rb - brb)\bar{b}\bar{g} + 2(2rbg - grb - bgb)\bar{g}\bar{g} + 2(gbb + bgb - 2bbg)\bar{b}\bar{b}. \]

Where $\chi^{(1)}$ represents the color wave function of the color singlet structure, $\chi^{(2)}$ and $\chi^{(3)}$ represent the symmetric and antisymmetric structures of the color-octet wave functions respectively, which means their symmetry is between the first and second quark in baryon cluster.

Finally, the total wave function of the five-quark system is written as:

\[ \Psi_{JM} = A \left[ |\psi_{LS}\rangle \right] _{JM} \chi_{JM} \chi_{JM} \chi_{JM}, \]

where the $A$ is the asymmetry operator of the system which guarantees the antisymmetry of the total wave functions when identical particles exchange.

The last step, we solve the following Schrödinger equation to obtain eigen-energy of the system.

\[ H\Psi_{JM} = E\Psi_{JM}, \]

with the help of the Rayleigh-Ritz variational principle, the final result can be easily obtained if we just consider the situation five-quark systems are all in ground states. It is worthwhile to mention that if the orbital angular momenta of the system is not zero, it is necessary to use the infinitesimally shifted Gaussian method [22].

### 3 Results and discussions

In this section, we will present calculation of all low-lying states of the $(uuc)(c\bar{q}), (uuc)(s\bar{q})$ and $(uuu)(c\bar{q})$ pentaquark system with all possible quantum numbers $IJ^P = \frac{1}{2}^+, \frac{1}{2}^+, \frac{1}{2}^-, \frac{1}{2}^-, \frac{1}{2}^-$ and $\frac{3}{2}^-$ by ChQM. All the orbital angular momenta of the system are set to zero, so the corresponding parity is negative. To see if there exists genuine resonance states, we employ the real-scaling method [25–28] to make a check. In this method, the Gaussian size parameters $r_n$ for the basis functions between baryon and meson clusters for the color-singlet channels are scaled by multiplying a factor $\alpha$: $r_n \rightarrow \alpha r_n$. As a result, the continuum state will fall off towards its threshold, and the resonant state would act with the scattering states and emerges as avoid-crossing structures with the variation of $\alpha$. A schematic diagram of a resonance is shown in Fig. 1. The top line...
is a scattering state, and it would fall to the corresponding threshold with increasing $\alpha$. However, the down line, resonance state, would interact with the scattering state, which could result in a avoid-crossing structure. If this behavior is repeated periodically, the avoid-crossing point may be a stable resonance [29].

The calculation results that we consider important are listed in Tables 4 and 5. In each table, columns 2–5 represent the wave functions for the three degrees of freedom and their physical channels of five-quark system. In column 6, eigen-energy of the each channel is shown and the theoretical thresholds with increasing $\alpha$ is along with it in column 7. Column 8 gives the binding energy, which is the difference between the eigen-energy and the theoretical thresholds. Finally, we will give the experimental thresholds (the sum of the masses of the corresponding baryon and meson in theory) is along with it in column 7. Column 8 gives the binding energy, which is the difference between the eigen-energy and the theoretical thresholds. Finally, we will give the experimental thresholds (the sum of the experimental masses of the corresponding baryon and meson) along with corrected energy (the sum of experimental thresholds and the binding energy, $E' = E_B + E^{theo}_T$) in last two columns. We hope that this method can reduce the calculation error caused by the model parameters in five-quark calculation partly.

Since the energy of the three newly observed states ranges from 2.9 to 3.0 GeV, for the systems with $I = \frac{3}{2}$, there is hardly any bound state (even if there is a bound state, the energy is much higher than 3.0 GeV). Also for the system with $IJ^P = \frac{1}{2}^-$, the energy of all channels are all around 3.4 GeV. So we focused our analysis on systems with $IJ^P = \frac{1}{2}^-$ and $IJ^P = \frac{1}{2}^-$. We analyze the results of these systems in detail in the following:

(a) $IJ^P = \frac{1}{2}^-$: The single channel calculations show that there are weak attractions for the $\Sigma D$ channel which is consistent with conclusions that there are attractions between $D$ meson and $\Sigma$ baryon in Refs. [16,18,19]. After coupling to respective hidden-color channels, our results didn’t change. Then, we make full-channel coupling and the calculations show that no bound state can be formed. However, resonances can be formed due to the fact that there exist attractions in $\Sigma D$ channel. Hence, real-scaling method is needed in this system.

(b) $IJ^P = \frac{3}{2}^-$: We have the Similar results with the case of $IJ^P = \frac{1}{2}^-$. There are weak attractions in most single channels of $(uus)(c\bar{q})$ systems ($\Sigma D^*, \Sigma^* D$ and $\Sigma^* D^*$). In addition, after coupling to their respective hidden-color channels, the attractions increase by a few MeV, which is the typical range of binding energy of two-hadron molecules. There may exist good resonances in this system though no bound state can be formed after multi-channel coupling. So it is necessary to do a test of resonances by using real-scaling method.

(c) For $IJ^P = \frac{5}{2}^-$, we have the Similar results with the case of $IJ^P = \frac{1}{2}^-$. However, resonances can be formed due to the presence of attractions in $\Sigma D$ channel. Hence, real-scaling method is needed in this system.

The results of $IJ^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ with help of real-scaling method are shown in Figs. 2 and 3. In these four figures, we have marked the threshold with lines (red online) and the physical components are also tagged. The mass for $2S$ states of $\Xi^c$, $\Xi^c_2$ and $\Xi^c_5$ are given in Table III. For the possible resonances, we mark the energy with blue line. Since we focus on the energy around 3.0 GeV, we set the energy range between 2650 MeV and 3080 MeV. From the figures, we can see that all thresholds appear as horizontal lines. In addition, we can find genuine resonances whose energy are stable with the increase of $\alpha$. For $IJ^P = \frac{1}{2}^-$ system, we got two resonances that energy are 2975 MeV and 3029 MeV around 3.0 GeV. In the same way, we also find one resonance in $IJ^P = \frac{3}{2}^-\Sigma D^*$ : 3048 MeV. Because of too many channels involved, we did a simple energy correction to these resonances and the result is 2954 MeV (the main component is $\Sigma D$ with color-singlet structure), 3017 MeV (the main component is $\Sigma D$ with color-octet structure) and 3020 MeV (the main component is $\Sigma D^*$ with color-singlet structure). Considering the deviations of quark model calculations and the energy of the resonances are very close to the newly observed states, these two resonances are good candidates of the newly reported states.

We also do a further calculation about partial decay width of these two resonances. The decay width for three resonances to possible open channels is obtained by the following formula:

$$\Gamma = 4V(\alpha)\frac{\sqrt{k_r k_c}}{|k_r - k_c|}, \ (24)$$
Table 4 The eigen-energy of channels with $IJ^P = \frac{1}{2}^+$. "cc1" denotes the color-singlet channels coupling and "cc2" means full channels coupling (unit: MeV)

| Index | $\psi_i$ | $\psi_{\sigma_j}$ | $\psi^{c_k}$ | Physical channel | $E$ | $E^{\text{theo}}_{i_{th}}$ | $E_B$ | $E^{\text{Exp}}_{i_{th}}$ | $E'$ |
|-------|----------|-------------------|--------------|-----------------|----|----------------|-----|----------------|----|
| 1     | $i = 1$  | $j = 1$           | $k = 1$      | $\Sigma_i \bar{K}$ | 2944 | 2944           | 0   | 2949           | 2949|
| 2     | $i = 1$  | $j = 1, 2$        | $k = 1, 2, 3$|                 | 2944 |                |     |                |     |
| 3     | $i = 1$  | $j = 3$           | $k = 1$      | $\Sigma_i \bar{K}^*$ | 3317 | 3317           | 0   | 3347           | 3347|
| 4     | $i = 1$  | $j = 3, 4$        | $k = 1, 2, 3$|                 | 3317 |                |     |                |     |
| 5     | $i = 1$  | $j = 5$           | $k = 1$      | $\Sigma_i^* \bar{K}^*$ | 3351 | 3351           | 0   | 3412           | 3412|
| 6     | $i = 1$  | $j = 5$           | $k = 1, 2, 3$|                 | 3351 |                |     |                |     |
| 7     | $i = 2$  | $j = 2$           | $k = 1$      | $\Lambda_i \bar{K}$ | 2791 | 2791           | 0   | 2780           | 2780|
| 8     | $i = 2$  | $j = 1, 2$        | $k = 1, 2, 3$|                 | 2791 |                |     |                |     |
| 9     | $i = 2$  | $j = 2$           | $k = 1$      | $\Lambda_i \bar{K}^*$ | 3165 | 3165           | 0   | 3178           | 3178|
| 10    | $i = 2$  | $j = 1, 2$        | $k = 1, 2, 3$|                 | 3165 |                |     |                |     |
| 11    | $i = 3$  | $j = 1$           | $k = 1$      | $\Sigma D$ | 3062 | 3065 − 3       | 3053 | 3050          |     |
| 12    | $i = 3$  | $j = 1, 2$        | $k = 1, 2, 3$|                 | 3062 | − 3            |     |                |     |
| 13    | $i = 3$  | $j = 3$           | $k = 1$      | $\Sigma D^*$ | 3221 | 3221           | 0   | 3196           | 3196|
| 14    | $i = 3$  | $j = 3, 4$        | $k = 1, 2, 3$|                 | 3221 |                |     |                |     |
| 15    | $i = 3$  | $j = 5$           | $k = 1$      | $\Sigma^* D^*$ | 3309 | 3309           | 0   | 3389           | 3389|
| 16    | $i = 3$  | $j = 5$           | $k = 1, 2, 3$|                 | 3309 |                |     |                |     |
| 17    | $i = 4$  | $j = 2$           | $k = 1$      | $\Lambda D$ | 2969 | 2969           | 0   | 2980           | 2980|
| 18    | $i = 4$  | $j = 1, 2$        | $k = 1, 2, 3$|                 | 2969 |                |     |                |     |
| 19    | $i = 4$  | $j = 2$           | $k = 1$      | $\Lambda D^*$ | 3125 | 3125           | 0   | 3123           | 3123|
| 20    | $i = 4$  | $j = 1, 2$        | $k = 1, 2, 3$|                 | 3125 |                |     |                |     |
| 21    | $i = 5$  | $j = 1$           | $k = 1$      | $\Xi_i \pi$ | 2652 | 2652           | 0   | 2611           | 2611|
| 22    | $i = 5$  | $j = 1, 2$        | $k = 1, 2, 3$|                 | 2652 |                |     |                |     |
| 23    | $i = 6$  | $j = 2$           | $k = 1$      | $\Xi_i' \pi$ | 2727 | 2727           | 0   | 2719           | 2719|
| 24    | $i = 6$  | $j = 1, 2$        | $k = 1, 2, 3$|                 | 2727 |                |     |                |     |
| 25    | $i = 5$  | $j = 3$           | $k = 1$      | $\Xi_i \rho$ | 3210 | 3210           | 0   | 3245           | 3245|
| 26    | $i = 5$  | $j = 3, 4$        | $k = 1, 2, 3$|                 | 3210 |                |     |                |     |
| 27    | $i = 6$  | $j = 4$           | $k = 1$      | $\Xi_i' \rho$ | 3285 | 3285           | 0   | 3354           | 3354|
| 28    | $i = 6$  | $j = 3, 4$        | $k = 1, 2, 3$|                 | 3285 |                |     |                |     |
| 29    | $i = 7$  | $j = 1$           | $k = 1$      | $\Xi_i \eta$ | 3054 | 3054           | 0   | 3019           | 3019|
| 30    | $i = 7$  | $j = 1, 2$        | $k = 1, 2, 3$|                 | 3054 |                |     |                |     |
| 31    | $i = 8$  | $j = 2$           | $k = 1$      | $\Xi_i' \eta$ | 3129 | 3129           | 0   | 3127           | 3127|
| 32    | $i = 8$  | $j = 1, 2$        | $k = 1, 2, 3$|                 | 3129 |                |     |                |     |
| 33    | $i = 7$  | $j = 3$           | $k = 1$      | $\Xi_i \omega$ | 3119 | 3119           | 0   | 3258           | 3258|
| 34    | $i = 7$  | $j = 3, 4$        | $k = 1, 2, 3$|                 | 3119 |                |     |                |     |
| 35    | $i = 8$  | $j = 4$           | $k = 1$      | $\Xi_i' \omega$ | 3194 | 3194           | 0   | 3361           | 3361|
| 36    | $i = 8$  | $j = 3, 4$        | $k = 1, 2, 3$|                 | 3194 |                |     |                |     |
| 37    | $i = 5$  | $j = 5$           | $k = 1$      | $\Xi_i^* \rho$ | 3321 | 3321           | 0   | 3423           | 3423|
| 38    | $i = 5$  | $j = 5$           | $k = 1, 2, 3$|                 | 3321 |                |     |                |     |
| 39    | $i = 7$  | $j = 5$           | $k = 1$      | $\Xi_i^* \omega$ | 3230 | 3230           | 0   | 3427           | 3427|
| 40    | $i = 7$  | $j = 5$           | $k = 1, 2, 3$|                 | 3230 |                |     |                |     |

cc1 2652 0
cc2 2652

...
Table 6 The eigen-energy of channels with $\mathcal{J}P = \frac{1}{2}^\pm$ “cc1” denotes the color-singlet channels coupling and “cc2” means full channels coupling (unit: MeV)

| Index | $\psi_{i}^c$ | $\psi_{i}^{\sigma_i}$ | $\psi_{i}^{\pi_i}$ | Physical channel | $E$ | $E_{i}^{theo}$ | $E_{i}^{th}$ | $E_{iB}$ | $E_{i}^{Exp}$ | $E_{i}^{Exp}$ |
|-------|-------------|-----------------|-----------------|-----------------|-----|-------------|-------------|---------|------------|------------|
| 1     | $i = 1$     | $j = 6$         | $k = 1$         | $\Sigma_1^− \bar{K}^+$ | 3317 | 3317        | 0           | 3344    | 3344       |
| 2     | $i = 1$     | $j = 6, 7$      | $k = 1, 2, 3$  | $\Sigma_1^+ \bar{K}$  | 2978 | 2978        | 0           | 3014    | 3014       |
| 3     | $i = 1$     | $j = 8$         | $k = 1$         | $\Sigma_2^− \bar{K}$  | 3550 | 3550        | 0           | 3412    | 3412       |
| 4     | $i = 1$     | $j = 8$         | $k = 1, 2, 3$  | $\Sigma_2^+ \bar{K}^*$ | 3550 | 3550        | 0           | 3412    | 3412       |
| 5     | $i = 1$     | $j = 9$         | $k = 1$         | $\Lambda_1 \bar{K}^+$  | 3165 | 3165        | 0           | 3178    | 3178       |
| 6     | $i = 1$     | $j = 9$         | $k = 1, 2, 3$  | $\Lambda_2 \bar{K}^*$  | 3165 | 3165        | 0           | 3178    | 3178       |
| 7     | $i = 2$     | $j = 7$         | $k = 1$         | $\Lambda_1 \bar{K}^+$  | 3165 | 3165        | 0           | 3178    | 3178       |
| 8     | $i = 2$     | $j = 6, 7$      | $k = 1, 2, 3$  | $\Lambda_2 \bar{K}^*$  | 3165 | 3165        | 0           | 3178    | 3178       |
| 9     | $i = 3$     | $j = 6$         | $k = 1$         | $\Sigma D^*$            | 3217 | 3221        | -4          | 3196    | 3192       |
| 10    | $i = 3$     | $j = 6, 7$      | $k = 1, 2, 3$  | $\Sigma^+ D$            | 3215 | 3215        | -6          | 3190    | 3190       |
| 11    | $i = 3$     | $j = 8$         | $k = 1$         | $\Sigma^+ D$            | 3152 | 3152        | -1          | 3246    | 3245       |
| 12    | $i = 3$     | $j = 8$         | $k = 1, 2, 3$  | $\Sigma^+ D$            | 3151 | 3151        | -2          | 3244    | 3244       |
| 13    | $i = 3$     | $j = 9$         | $k = 1$         | $\Sigma^+ D$            | 3305 | 3309        | -4          | 3389    | 3385       |
| 14    | $i = 3$     | $j = 9$         | $k = 1, 2, 3$  | $\Sigma^+ D$            | 3304 | 3304        | -5          | 3384    | 3384       |
| 15    | $i = 4$     | $j = 7$         | $k = 1$         | $\Lambda D^*$            | 3125 | 3125        | 0           | 3123    | 3123       |
| 16    | $i = 4$     | $j = 7$         | $k = 1, 2, 3$  | $\Lambda D^*$            | 3125 | 3125        | 0           | 3123    | 3123       |
| 17    | $i = 5$     | $j = 6$         | $k = 1$         | $\Xi^− \rho$            | 3210 | 3210        | 0           | 3245    | 3245       |
| 18    | $i = 5$     | $j = 6, 7$      | $k = 1, 2, 3$  | $\Xi^− \rho$            | 3210 | 3210        | 0           | 3245    | 3245       |
| 19    | $i = 5$     | $j = 8$         | $k = 1$         | $\Xi^− \pi$             | 2763 | 2763        | 0           | 2785    | 2785       |
| 20    | $i = 5$     | $j = 8$         | $k = 1, 2, 3$  | $\Xi^− \pi$             | 2763 | 2763        | 0           | 2785    | 2785       |
| 21    | $i = 5$     | $j = 9$         | $k = 1$         | $\Xi^− \rho$            | 3321 | 3321        | 0           | 3423    | 3423       |
| 22    | $i = 5$     | $j = 9$         | $k = 1, 2, 3$  | $\Xi^− \rho$            | 3321 | 3321        | 0           | 3423    | 3423       |
| 23    | $i = 6$     | $j = 7$         | $k = 1$         | $\Xi^− \rho$            | 3285 | 3285        | 0           | 3354    | 3354       |
| 24    | $i = 6$     | $j = 7$         | $k = 1, 2, 3$  | $\Xi^− \rho$            | 3285 | 3285        | 0           | 3354    | 3354       |
| 25    | $i = 7$     | $j = 6$         | $k = 1$         | $\Xi^+ \omega$          | 3119 | 3119        | 0           | 3259    | 3259       |
| 26    | $i = 7$     | $j = 6, 7$      | $k = 1, 2, 3$  | $\Xi^+ \omega$          | 3119 | 3119        | 0           | 3259    | 3259       |
| 27    | $i = 7$     | $j = 8$         | $k = 1$         | $\Xi^+ \eta$            | 3165 | 3165        | 0           | 3193    | 3193       |
| 28    | $i = 7$     | $j = 8$         | $k = 1, 2, 3$  | $\Xi^+ \eta$            | 3165 | 3165        | 0           | 3193    | 3193       |
| 29    | $i = 7$     | $j = 9$         | $k = 1$         | $\Xi^+ \omega$          | 3230 | 3230        | 0           | 3427    | 3427       |
| 30    | $i = 7$     | $j = 9$         | $k = 1, 2, 3$  | $\Xi^+ \omega$          | 3230 | 3230        | 0           | 3427    | 3427       |
| 31    | $i = 8$     | $j = 7$         | $k = 1$         | $\Xi^+ \omega$          | 3194 | 3194        | 0           | 3361    | 3361       |
| 32    | $i = 8$     | $j = 6, 7$      | $k = 1, 2, 3$  | $\Xi^+ \omega$          | 3194 | 3194        | 0           | 3361    | 3361       |

where $V(\omega)$ is the minimum energy difference, while $k_c$ and $k_r$ stand for the slopes of scattering state and resonance state at the avoid-crossing point respectively. More details can be found in Ref. [30]. The results of decay width are shown in Table 6.

We can see that for $\Sigma_c(2975)$, $\Sigma_c(3029)$ and $\Sigma_c(3048)$ states, the decay width is 8.2 MeV, 10.1 MeV and 12.9 MeV respectively. After considering the statistic uncertainty and systematic uncertainty of experimental results, we think resonance $\Sigma_c(2975)$ can be identified as $\Sigma_c(2923)^0$ state while resonance $\Sigma_c(3029)$ as $\Sigma_c(2938)^0$ and resonance $\Sigma_c(3048)$ as $\Sigma_c(2965)^0$ if future data from the experiment suggest the parity of three reported $\Sigma_c^0$ states is negative.
4 Summary

In this paper, we investigate the three-quark $usc$ states and five-quark $uscu \bar{u}$ or $us cud \bar{d}$ states in the framework of the chiral constituent quark model with the help of Gaussian expansion method. For the result of three-quark system, we find that both first radial excitation state ($2S$) of $\Xi^c_-(2645)$ and $1D$ states of $\Xi_c^0$ have the energy around 2.9–3.0 GeV, which are possible candidates of the newly observed excited states of $\Xi_0^c$ reported by LHCb Collaboration. Particularly, $\Xi_c(2923)^0$ could be the $2S$ state of three-quark configuration while $\Xi_c(2938)^0$ and $\Xi_c(2965)^0$ could be $1D$ states if the parity of these states is positive. It is worth mentioning that our preliminary calculation indicated the energy of $1P$ states of $\Xi_c^0$ are around 2.8 GeV, which are good candidates of $\Xi_c^0(2790)$ and $\Xi_c^*(2815)$ from PDG with consistent negative-parity. For the five-quark systems, we investigate all the possible systems with negative-parity, especially systems with $IJ^P = \frac{1}{2}^-$ and $IJ^P = \frac{3}{2}^-$ and the calculation shows that there is no bound state for these systems. However, the resonances are possible. We use real-scaling method to find the genuine resonances and also study their decay width. Three resonance states $\Xi_c(2975)$, $\Xi_c(3029)$ and $\Xi_c(3048)$ with energy around 2.9 GeV to 3.0 GeV have been found in the $IJ^P = \frac{1}{2}^-$ and $IJ^P = \frac{3}{2}^-$ systems. These three resonance states are possible candidates of the newly reported excited states of $\Xi_c$ by LHCb Collaboration.

However, we cannot jump to conclusions with calculations of three-quark system and five-quark system in this paper because experimental and theoretical studies of these three newly observed states is not enough. In addition, a mixed system of three-quark baryon and pentaquark cannot be ignored. So, the study of $\Xi_c$ in the framework of the unquenched quark model, including the higher Fock components with more future experimental and theoretical data is our future work.

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