STEREO Observations of Non-linear Plasma Processes in Solar Type III Radio Bursts

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Abstract. Solar type III radio bursts are the fast drifting radio emissions excited by the flare accelerated electron beams in the corona and interplanetary medium. In this paper, we report the observations of several nonlinear processes, obtained by the time domain sampler (TDS) of the WAVES experiment of the STEREO spacecraft in the source region of one of these bursts. These include: (1) a four wave-wave interaction process called the oscillating two stream instability (OTSI) \( L + L \rightarrow L_U + L_D \), where \( L \) is the beam excited Langmuir wave, \( S \) is the ion sound wave, and \( L_U \) and \( L_D \) are the up- and down-shifted Langmuir waves, respectively, and (2) two wave-wave merging processes \( L_U + L_D \rightarrow T_{2fpe} \) and \( L + T_{2fpe} \rightarrow T_{3fpe} \), where \( T_{2fpe} \) and \( T_{3fpe} \) are the electromagnetic waves at the second and third harmonic of the electron plasma frequency, \( f_{pe} \) and \( L \) can be either a beam-excited Langmuir wave, or an up- or down-shifted sideband. The newly developed higher order spectral (HOS) techniques have been used to identify the signatures of these wave-wave interactions in the waveform data. The implication of these findings is that strong Langmuir turbulence processes play key roles in excitation of solar type III radio bursts, especially, in the beam stabilization and conversion of Langmuir waves into electromagnetic waves at the fundamental and higher harmonics of the electron plasma frequency, \( f_{pe} \).

1. Introduction

The most intense radio emissions from the Sun, which are referred to as the type III radio bursts occur as the fast drifting emission bands in the dynamic spectrum. Usually, these bursts are characterized by the fundamental and harmonic bands. Ginzburg and Zhelenyakov [9] were the first to suggest that these bursts are probably excited by a mechanism called the plasma mechanism, which involves: (1) formation of bump-on-tail distributions by the flare accelerated electrons, (2) excitation of Langmuir waves by the bump-on-tail distributions, and (3) conversion of these Langmuir waves into electromagnetic waves at the fundamental and second harmonic of the electron plasma frequency, \( f_{pe} \). This suggestion has been well confirmed by the in situ detection of bump-on-tail distributions of electron beams [19] and Langmuir waves [11, 20, 12, 27, 29, 30] in type III radio bursts. However, answers to some science questions remain unsettled,
which include: (1) How does the electron beam preserve its bump-on-tail over distances of the order of 1 AU and more against the quasi-linear relaxation, and (2) What mechanisms are responsible for conversion of Langmuir waves into radio emissions at the fundamental and higher harmonics of the electron plasma frequency, $f_{pe}$.

The electron beam can maintain its bump-on-tail against the quasi-linear relaxation if the resonance between Langmuir waves and the beam is disrupted well before they back react and relax the beam. The induced scattering off ion clouds, which can pump the Langmuir waves toward lower wave numbers is suggested to accomplish such a task [14]. If the electron temperature $T_e$ is higher than the ion temperature $T_i$, the induced scattering is known to act as the electrostatic decay (ESD) $L \rightarrow L' + S$, where $L$ and $L'$ are the oppositely propagating beam-excited and back-scattered Langmuir waves, respectively, and $S$ is the ion sound wave. The ESD also can facilitate the excitation of second harmonic electromagnetic waves $T_{2f_{pe}}$ through the three wave interaction: $L + L' \rightarrow T_{2f_{pe}}$. In some type III events, evidence for ESD has been reported [20, 12, 28, 29, 31, 13].

According to some authors [25, 26, 10], the intensities of type III burst associated Langmuir waves are expected to well exceed the strong turbulence thresholds, and, therefore, the oscillating two stream instability (OTSI), and, related soliton formation and Langmuir collapse [44, 24] probably are the relevant processes. As far as the stabilization of the electron beam is concerned, the strong turbulence processes can disrupt the resonance between the Langmuir waves and the beam by pumping the Langmuir waves towards higher wave numbers. The OTSI generates an ion sound wave $S$ of frequency and wave number $(f_S, k_S)$, which in turn beats with two beam-driven Langmuir waves of frequency and wave number $(f_{pe}, k_L)$ and excites two oppositely propagating up- and down- shifted side bands with $(f_{pe} + f_S, k_L \pm k_S)$, respectively. Thus, the OTSI can be represented as the four wave interaction

$$L + L \xrightarrow{S} L_U + L_D,$$  \hspace{1cm} (1)

where $L$ is the beam-excited Langmuir wave, and $L_U$ and $L_D$ are the oppositely propagating up- and down-shifted sidebands, respectively. The OTSI also facilitates the excitation of $T_{2f_{pe}}$ through wave-wave merging [25]

$$L_U + L_D \rightarrow T_{2f_{pe}},$$ \hspace{1cm} (2)

The observations of Langmuir waves by the Ulysses spacecraft have provided some indication that the strong turbulence processes probably occur in type III burst sources [27, 29, 30, 32]. The much superior STEREO high time resolution in situ wave data have provided evidence for various nonlinear processes in type III bursts [6, 21, 15, 16], which also include strong Langmuir turbulence processes [33, 34, 35, 36, 37, 38, 39, 40, 41, 42].

In this paper, we present the observations of a one-dimensional field aligned localized wave packet, obtained by the time domain sampler (TDS) of the WAVES experiment of the STEREO A spacecraft [4] in the source region of a solar type III radio burst. We show that (1) the peak intensity and short scale length of this wave packet satisfy the threshold conditions of OTSI and soliton formation, and (2) its spectrum contains: (a) sidebands, which satisfy the frequency and wave number resonance conditions of OTSI,
and (b) two harmonic peaks, one at $2f_{pe}$ and another one at $3f_{pe}$ excited probably by three wave interactions. We show that the resonance conditions including the phase coherence required by the four- and three wave interactions are easily satisfied. These findings strongly suggest that the strong Langmuir turbulence processes probably play key roles in the beam stabilization as well as conversion of Langmuir waves into the solar type III radio emissions. In section 2, we describe the observations, in section 3, we describe higher order spectral analysis techniques, and in sections 4 and 5, we present the discussion, and the summary and conclusions, respectively.

2. Observations

![Dynamic spectrum of a local type III radio burst and associated Langmuir waves](image)

**Figure 1.** Dynamic spectrum of a local type III radio burst (fast drifting emission from $\approx 5\, MHz$ down to $\approx 26\, kHz$) and associated Langmuir waves (non-drifting emissions in the frequency interval 19-22 kHz).

In Figure 1, we present the dynamic spectrum of a local type III radio burst and its associated in situ waves, observed by the STEREO WAVES experiment [4], which uses three orthogonal monopole antennas with effective lengths of $\sim 1\, m$. The emission drifting fast from $\approx 5\, MHz$, all the way to $\approx 26\, kHz$ is the type III radio burst. Using its fast negative frequency drift, we estimate the velocity of the electron beam responsible for this event as $v_b \approx 0.37c$ ($c$ is the velocity of light). This estimation is based on the assumptions that the mode of emission is the second harmonic and the ambient electron...
Figure 2. The X, Y and Z-electric field components of one of the intense Langmuir wave packets associated with the type III burst in the spacecraft frame of reference.

density $n_e$ is described by the Radio Astronomy Explorer (RAE) model [7]. We identify the non-drifting emissions seen in the 19-22 kHz interval as the Langmuir waves. The Langmuir wave burst, shown by the arrow in this figure is of interest in the present study.

The TDS of the WAVES experiment provides the high time resolution electric field
Figure 3. The parallel and perpendicular electric field components of the Langmuir wave packet with respect to the magnetic field. The unit vectors of magnetic field and solar wind velocity used in the coordinate transformation are $\vec{b} = [-0.60747, -0.79512, 0.005553]$ and $\vec{v} = [0.99167, -0.12865, 0.0060756]$, respectively.
potential data of all three components, which can be converted into the X-, Y- and Z-components of the wave electric field in the spacecraft frame using the transformation matrix given by [2]. TDS has resolved the Langmuir wave bursts seen in Figure 1 into several intense waveforms. In Figure 2, we present the time profiles of $E_X$, $E_Y$ and $E_Z$ components of one of the waveforms, whose peak amplitudes are 27.8 mVm$^{-1}$, 40.4 mVm$^{-1}$ and 7.5 mVm$^{-1}$, respectively. This waveform contains 16384 samples, acquired at the rate of 250,000 samples per second (a time step of 4 μs for a total duration of 65 ms). We transform the $E_X$, $E_Y$ and $E_Z$ components from the spacecraft into a more useful magnetic field ($\vec{B}$)-aligned coordinate system with X-, Y- and Z-axes aligned along $\vec{b}$, $\vec{b} \times \vec{v}$ and $\vec{b} \times (\vec{v} \times \vec{b})$, respectively ($\vec{b}$ and $\vec{v}$ are the unit vectors directed along the magnetic field, and the solar wind velocity, respectively). The relevant unit vectors in this case are $\vec{b} = (-0.60747, -0.79512, 0.005553)$ and $\vec{v} = (0.9917, -0.12865, 0.0060756)$, and the angle between them is $\approx 120^\circ$. In Figure 3, we present the components of this waveform in the B-aligned coordinate system, where, the peak amplitude of the parallel component is $\approx 48.6$ mVm$^{-1}$, and those of the perpendicular components are $\approx 6.7$ mVm$^{-1}$ and $\approx 8.9$ mVm$^{-1}$, respectively. Since, these peak amplitudes indicate that this is a one-dimensional field aligned wave packet, we mainly focus on the parallel component.

In Figure 4, we present the spectrum of the parallel component. The total spectrum from 0 to 65 kHz (Figure 4a) clearly shows peaks at (approximate) 20 kHz, 40 kHz and 60 kHz, where, the base frequency, 20 kHz, corresponds to the local electron plasma frequency, $f_{pe}$, and the 40 kHz corresponds to the second harmonic and 60 kHz corresponds to the third harmonic. Figure 4a also shows that the second harmonic is weaker than the fundamental, and the third harmonic is weaker than the second harmonic, which is typical of the natural signals [43, 16, 38]. The logarithmic spectrum in the frequency interval from 19 to 21 kHz (Figure 4b) also shows an intense peak (L) of the beam excited Langmuir waves at $f_{pe} \approx 20$ kHz, and two side bands, namely a Stokes peak (D) at $\approx 19.8$ kHz and an anti-Stokes peak (U) at $\approx 20.2$ kHz. As seen from this spectral plot, the anti-Stokes peak is weaker than that of the Stokes peak. The linear spectrum from 0 to 1.5 kHz, presented in Figure 4c shows a low frequency enhancement below $\approx 400$ Hz with a peak $\sim 100$ Hz corresponding probably to the ion sound waves.

In Figure 5, we plot the time profile of total electric field of the wave packet $E_t = \sqrt{E_X^2 + E_Y^2 + E_Z^2}$, which clearly shows that this is a localized wave packet with peak $E_t \sim 49$ mVm$^{-1}$ and $\frac{1}{2}$ power duration $\tau_{0.5}$ of 8.46 ms. By assuming that this wave packet is stationary in the solar wind, we can convert this time scale $\tau_{0.5}$ into spatial scale $L_{0.5}$ by interpreting it in terms of Doppler shift in the solar wind

$$L_{0.5} = \tau_{0.5} v_{sw} \cos \theta.$$  

(3)

Here $v_{sw}$ is the solar wind speed and $\theta$ is the angle between the solar wind velocity and the electric field vector, which is nothing but the angle between the ambient magnetic field and the electric field of the wave packet since it is parallel to the magnetic field.

For electron temperature ($T_e$), we assign a typical value of $10^5$ K; the measurements of $T_e$ are not available. Assuming that the intense peak (L) in the spectrum of the parallel component corresponds to Langmuir waves excited at $f_{pe} \approx 20$ kHz (Figure 4b),
Figure 4. (a): The complete spectrum of the parallel component of the wave packet, which shows harmonic peaks at $f_{pe}$, $2f_{pe}$ and $3f_{pe}$ ($f_{pe}$ is the electron plasma frequency), (b) the narrow spectrum around $f \approx f_{pe} \approx 20.0$ kHz, where L, D, and U correspond to the beam excited Langmuir wave, down-shifted sideband at $\approx 19.8$ kHz, and up-shifted sideband at $\approx 20.2$ kHz, respectively, and (c) the low frequency spectrum, which shows the enhancement below 400 Hz corresponding to ion-sound waves.
Figure 5. The time profile of the total electric field $E_t = \sqrt{E_X^2 + E_Y^2 + E_Z^2}$. The $\frac{1}{2}$-power duration of 8.464 ms is equivalent to the spatial scale of $\sim 227\lambda_{De}$.

we estimate the electron density as $n_e \approx 5 \times 10^6$ m$^{-3}$ using the relation $f_{pe}[Hz] = 9n_e^{1/2}$. These values yield Debye length, $\lambda_{De} = 69T_e^{1/2}n_e^{-1/2} \approx 9.8$ m. We estimate the wave number of Langmuir waves as $k_L = \frac{2\pi f_{pe}}{v_b} \approx 1.1 \times 10^{-3}$ m$^{-1}$, and $k_L\lambda_{De} \approx 1.1 \times 10^{-2}$ for $v_b \approx 0.37c$. The STEREO/PLASTIC experiment [8] has measured $v_{sw} \approx 525.7$ kms$^{-1}$. Thus, we can estimate the normalized peak energy density as $W_L = \frac{W_t}{n_eT_e} = \frac{e_0E_t^2}{4n_eT_e} \approx 7.7 \times 10^{-4}$ for $E_t = \sqrt{E_X^2 + E_Y^2 + E_Z^2} = 49$ mVm$^{-1}$. The measured $\tau_{0.5} \approx 8.46$ ms yields $L_{0.5} = \tau_0 v_{sw} \cos \theta \approx 227\lambda_{De}$.

3. Higher Order Spectral Techniques

In a turbulence process, energy transfer occurs between interacting modes, which is rather difficult to identify in the time series data. However, interacting waves obey
energy and momentum conservation laws, which exhibit certain statistical properties of observations. The HOS techniques are used to extract the phase information of the interacting waves. The bi-spectral analysis is used to extract the nonlinear characteristics between three spectral components with frequency sum and difference [17]. The bi-spectrum, which is related to the skewness of the signal can detect the asymmetric non-linearity in three wave interactions, such as the electrostatic decay [13] and harmonic generation [37, 38]. The FFT based bi-spectrum is defined as

$$ B(f_1, f_2) = E[X(f_1)X(f_2)X(f_1 + f_2)^*], $$

where E denotes the expected value, $X(f)$ is the Fourier transform of $x(t)$ and $*$ denotes complex conjugation. This equation indicates that the bi-spectrum is zero unless the phase coherence between three waves at frequencies $f_1$, $f_2$ and $f_1 + f_2$ present in the time series data is non-zero. For example, for spontaneously generated non-interacting modes, the phases would be random and therefore the statistical averaging will be zero. The bi-coherence, which is the normalized bi-spectrum

$$ b^2(f_1, f_2) = \frac{|B(f_1, f_2)|^2}{(E[|X(f_1)X(f_2)X^*(f_1 + f_2)|])^2}, $$

provides the quantitative measure of the phase coherence. A unit value for the bi-coherence indicates perfect coupling, a zero value indicates no coupling, and any value between zero and one indicates partial coupling. Generally, the bi-spectrum is estimated using the periodogram (interval segmentation) method. The approximate variance of the bi-coherence estimator is defined as [17]:

$$ var(b^2) \simeq \frac{M}{2N} [1 - b^2(f_1, f_2)], $$

where $N$ is the number of total data points and $M$ is the number of data records into which the time series is divided. To assess the low bi-coherence levels, one needs long and stationary time series with $N >> 1$.

The wavelet based bi-spectral analysis technique, which can handle both short and non-stationary time series is used to study non-linear processes in space plasmas [5, 13]. The wavelet based bi-spectrum is defined as

$$ B(a_1, a_2) = \int W(a_1, \tau)W(a_2, \tau)W^*(a_3, \tau) d\tau, $$

where $W(a, \tau)$ is the CWT at scale $a$ and time $\tau$, and $a_3^{-1} = a_1^{-1} + a_2^{-2}$ (frequency sum rule). Here the integral is taken over a finite time interval $T$: $\tau_0 \leq \tau \leq \tau_1$. The wavelet bi-spectrum measures the amount of phase coupling between the wavelet components of the time series $x(t)$ with scale lengths $a_1$, $a_2$ and $a_3$, such that the sum rule is satisfied. It can be interpreted as the coupling between wavelets of frequencies, such that $f_3 = f_1 + f_2$, within the frequency resolution, where $f = f_0/a$. The squared wavelet bi-coherence, which is the normalized squared wavelet bi-spectrum is defined as

$$ b^2(a_1, a_2) = \frac{|B(a_1, a_2)|^2}{\int |W(a_1, \tau)W(a_2, \tau)|^2 d\tau \int |W(a_3, \tau)|^2 d\tau}. $$
For digital signals, these integrals are replaced by summations over $N$ points. The bispectrum and bi-coherence are usually computed over a triangle of frequencies defined by the relations:

$$0 < f_1 < \frac{f_N}{N}, f_1 < f_2 < \frac{f_N}{2} - f_1$$

where $f_N$ is the Nyquist frequency.

The tri-spectral analysis technique can yield the higher order coherence among four spectral components and modulation sideband. A cumulant based tri-spectral method has been applied to synthetic [18], and in situ wave data [33, 34, 36, 37, 38]. The tri-spectrum is defined in terms of the complex Fourier components of the signal $X_1$, $X_2$, $X_3$ and $X_4$ at frequencies $f_1$, $f_2$, $f_3$ and $f_4$ as [18]

$$T(1, 2, 3) = E[X_1X_2X_3^*X_4^*] - G(1, 2, 3, 4),$$

where $G(1, 2, 3, 4) = E[X_1X_2]E[X_3^*X_4^*] + E[X_1X_3^*]E[X_2X_4^*] + E[X_1X_4^*]E[X_2X_3^*]$ and $f_4 = f_1 + f_2 - f_3$, and $E[]$ is the expectation operator. The tri-coherence, which is the normalized tri-spectrum eliminates the dependence of tri-spectrum on the amplitude of the signals. Most importantly, it quantifies the phase coherence amongst the spectral components of the wave packet. The squared tri-coherence is expressed as [18]:

$$t^2(1, 2, 3) = \frac{|T(1, 2, 3)|^2}{(E[|X_1X_2X_3^*X_4^*|] + |G(1, 2, 3, 4)|)^2}.$$  

A unit value of the tri-coherence indicates perfect coupling, a zero value indicates no coupling, and a value between zero and one indicates partial coupling. The tri-coherence quantifies the fraction of the total product of powers at the frequency quartet, $(f_1, f_2, f_3, f_1 + f_2 - f_3)$, that is owing to cubically phase-coupled modes.

The tri-spectrum and tri-coherence are estimated using the method of periodograms, which involves (1) the division of the data record into M segments, (2) application of an appropriate window to each segment to reduce leakage, (3) computation of the tri-spectrum and tri-coherence for each segment by using the DFT, and (4) taking the average of the tri-coherence across segments in order to reduce the variance of the estimator. The tri-spectrum estimator is symmetric with respect to permutations of its arguments $f_1$, $f_2$ and $f_3$. The principal domain for the interaction of the type $f_1 + f_2 = f_3 + f_4$ is determined as [18] $0 \leq f_1 \leq f_N$, $0 \leq f_2 \leq f_1$, $0 \leq f_3 \leq f_2$, and $f_3 \leq f_1 + f_2 - f_3 \leq f_N$, where $f_N$ is the Nyquist frequency.

### 4. Discussion

The dispersion relations of the relevant Langmuir and ion sound waves are

$$\omega_L = \omega_{pe}(1 + \frac{3}{2}k_L^2\lambda_{De}^2),$$

and

$$\omega_S = k_S c_S,$$
respectively, where $\omega_{pe} = 2\pi f_{pe}$ and $c_s$ is the ion sound speed. If

$$\frac{W_L}{n_e T_e} > (k_L \lambda_{De})^2,$$

(14)

the weak turbulence approximation is not valid, and the strong turbulence processes, such as the OTSI, soliton formation and spatial collapse become important [44]. In such a case, there won’t be one to one correspondence between $\omega_L$ and $k$, since $\frac{W_L}{n_e T_e}$ determines the bandwidth. Since the Langmuir oscillations get trapped in the self-generated density cavities, the turbulence becomes highly inhomogeneous consisting of localized collapsing soliton-caviton pairs. The energy transfer in this case occurs in the direction of higher values of $k$.

The threshold for the supersonic collapse is [44]

$$\frac{W_L}{n_e T_e} \geq \frac{m_e}{m_i},$$

(15)

where $m_e$ and $m_i$ are the electron and ion masses, respectively. This condition signifies that the compression due to ponder-motive force and self focusing overwhelms the wave packet spreading due to dispersion. The criterion for the formation of the collapsing wave packet is [23]

$$\frac{W_L}{n_e T_e} \geq (\Delta k \lambda_{De})^2,$$

(16)

where $\Delta k = \frac{2\pi}{L}$ is the wave number characteristic of the envelope. In the present case, the conditions (14-16) are easily satisfied, since $\frac{W_L}{n_e T_e} \approx 7.7 \times 10^{-4}$ and, $(k_L \lambda_{De})^2$ is $\approx 1.2 \times 10^{-4}$ for $k = k_L \sim 1.1 \times 10^{-3}$ m$^{-1}$, $\frac{m_e}{m_i} \approx 5.5 \times 10^{-4}$ and $(\Delta k \lambda_{De})^2 \approx 7.7 \times 10^{-4}$ estimated for the spatial scale $L$ of $\approx 227\lambda_{De}$.

4.1. Four Wave Interactions

The OTSI is a four wave interaction process, in which two Langmuir waves with frequencies and wave numbers ($f_L$, $k_L$) serve as pump waves, and the up- and down-shifted Langmuir sidebands with ($f_U$, $k_U$) and ($f_D$, $k_D$) serve as the daughter waves. The non-linear coupling between these four modes occurs through a purely growing ion sound mode with ($f_S$, $k_S$). The relevant frequency, wave number and phase matching conditions are

$$2f_L = f_D + f_U$$

(17)

$$2k_L = k_D + k_U$$

(18)

$$2\phi_L = \phi_D + \phi_U,$$

(19)

where, the subscripts L, D and U correspond to the beam-excited Langmuir wave, down- and up-shifted sidebands, respectively.

The spectrum of the parallel component (Figure 4b) shows a strong Langmuir wave peak with up- and down-shifted sidebands, together with low frequency enhancement (Figure 4c). We can verify, whether these observed spectral peaks correspond to the
beam-excited Langmuir waves, and to the OTSI excited sidebands and low frequency waves. Here, we assume that \( k \approx k_L \). As far as the frequency matching condition \( 2f_L = f_D + f_U \) is concerned, it is easily satisfied, since the frequency shifts \( \Delta f \) of the down- and up-shifted side bands are symmetric with respect to the Langmuir wave pump, being \( \approx 300 \) Hz and \( \approx 300 \) Hz, respectively. Moreover, the frequency differences \( \Delta f = |f_L - f_{U,D}| \) are also in good agreement with the frequency of the ion sound waves of < 400 Hz. The matching condition \( k_{U,D} = k_L \pm k_S \) is also reasonably well satisfied, for the wave numbers of the ion sound waves \( k_S \approx 0.04 \), estimated using the expression for the Doppler shift, \( k_S = \frac{2\pi f_S}{\nu_{sw}} \) and for \( f_S = 300 \) Hz and \( \nu_{sw} = 527 \) kms\(^{-1} \), which yields \( |k_{U,D}| \approx |k_S| \), since \( k_L \approx 1.1 \times 10^{-2} \) is three to four times less than \( k_S \approx 0.04 \). Thus, the frequency and wave number matching conditions of OTSI are easily satisfied.

As far as the phase coherence between the Langmuir pump waves and side bands is concerned, we have to calculate the tri-coherence spectrum as a function of three frequencies. Using equation (11), we have computed the tri-coherence spectrum of the parallel component in the frequency ranges [20 kHz \( \leq f_k \leq 20.75 \) kHz, 20 kHz \( \leq F_1 \leq 20.25 \) kHz, 19.75 kHz \( \leq F_p \leq 20.25 \) kHz], for \( N = 1000 \) (0.004s) and \( M = 16 \) with a Hamming window, where \( N \) is the segment length, and \( M \) is the number of segments. We display the computed three dimensional tri-coherence spectrum in Figure 6. As seen from this Figure, the 3-D tri-coherence spectrum shows a peak at \( (\approx 20.25, \approx 20, \approx 19.75) \) kHz with a statistically significant peak value of \( t^2 \approx 0.33 \). This tri-coherence spectral peak is indicative of the phase relation \( 2\phi_L = \phi_D + \phi_U \).

The computed tri-coherence spectrum can also be displayed as 2-D cross-sections at different frequencies. We present such cross sections in Figure 7. The cross-section at \( F_p = f_D = 19.75 \) kHz (top panel) shows the tri-coherence spectral peak at \( (\approx 20, \approx 20, \approx 19.75) \) kHz with \( t^2 \approx 0.33 \); this value agrees with that of 3-D spectrum. This peak tri-coherence again quantifies the phase relation \( 2\phi_L = \phi_D + \phi_U \), where \( \phi_L, \phi_D \) and \( \phi_U \) are the phases of the beam-excited Langmuir wave (\( \approx 20 \) kHz), Stokes (\( \approx 19.75 \) kHz) and anti-Stokes (\( \approx 20.25 \) kHz) modes, respectively, i.e., the frequency quartet in this case is \( (\approx 20, \approx 20, \approx 19.75, \approx 20.25) \) kHz. The cross-section at \( F_p = f_L = 20.25 \) kHz (second panel) shows a relatively weaker spectral feature, with peak value of \( \approx 0.30 \) at \( \approx 20.25 \) kHz, \( \approx 20.5 \) kHz, \( \approx 20.5 \) kHz). In the bottom panel, we present the cross-section at \( F_p = f_U = 20.75 \) kHz, which also shows a much weaker peak of \( t^2 \approx 0.1 \) at \( \approx 20.75, \approx 21 \) kHz. These tri-coherence spectral peaks reflect the four wave interactions involving the sidebands themselves. However, the tri-coherence spectral peak \( t^2 \approx 0.33 \) (top panel) is more significant in the statistical sense, which indicates that the four wave interaction \( 2f_L \rightarrow f_D + f_U \) is the dominant one. Thus, the significant tri-coherences corresponding to \( 2\phi_L = \phi_D + \phi_U \) in the tri-coherence spectrum at Stokes, anti-Stokes and beam-excited Langmuir wave frequencies provide evidence for the OTSI type of four-wave interaction.

4.2. Wave-Wave Merging

As seen in Figure 4a, the spectrum of the parallel component contains harmonics. One can interpret these spectral peaks at \( 2f_{pe} \) and \( 3f_{pe} \) in terms of second and third harmonic electromagnetic waves excited by wave-wave interactions, as discussed by several authors
Figure 6. The three dimensional tri-coherence spectrum $t^2(F_k, F_l, F_p)$ of the $E_{||}$ component of Figure 3a. The peak tri-coherence of $\approx 0.33$ at $(\approx 20.25, \approx 20, \approx 19.75, \approx 20)$ kHz quantifies the phase matching condition $2\phi_L = \phi_D + \phi_U$, where $L$, $D$ and $U$ correspond to beam-excited, Stokes and anti-Stokes modes, respectively (see for example, [45]). This can be possible only if the waves corresponding to these spectral peaks satisfy the following matching rules

$$L_1 + L_2 \rightarrow T_{2f_{pe}},$$

(20)

and

$$L + T_{2f_{pe}} \rightarrow T_{3f_{pe}}.$$  

(21)

As far as the frequency and wave number resonance conditions are concerned, they are easily satisfied for these two type of three wave interactions (see, for example, [38]).
Figure 7. The cross-sections of the tri-coherence spectrum of the $E_{\parallel}$ component (Figure 3a). The top, middle and bottom panels show the cross-sections at $F_p \approx F_D \approx 19.75$ kHz, $F_p \approx f_L \sim 20.25$ kHz, and $F_p \approx f_U \approx 20.75$ kHz, respectively (the subscripts L, D and U correspond to beam-excited, Stokes and anti-Stokes modes, respectively.)
Figure 8. The wavelet based bicoherence spectrum of the $E_{||}$-component. The bicoherence spectral peaks at (20, 20) kHz and (40, 20) kHz correspond to the three wave interactions $L_U + L_D \rightarrow T_{2f_{pe}}$ and $L + T_{2f_{pe}} \rightarrow T_{3f_{pe}}$, respectively. Here, $L_U$ and $L_D$ correspond to up- and down-shifted side bands excited by the OTSI, and $T_{2f_{pe}}$ and $T_{3f_{pe}}$ correspond to the second and third harmonic electromagnetic waves, respectively.

The bi-spectral analysis of the wave packet can provide definite clues whether these processes are actually occurring in the present case or not. If these non-linear processes are responsible for the observed spectral peaks, the corresponding waves should be non-linearly coupled to each other, i.e., one would expect peaks at $(f_{pe}, f_{pe})$ and at $(2f_{pe}, f_{pe})$ in the bi-coherence spectrum; these bi-coherence spectral peaks are indicative of the three wave interactions (20) and (21), respectively.

We have computed the wavelet-based bi-coherence spectrum of the parallel component of the waveform. As seen from Figure 8, this clearly shows two intense peaks, one at (20, 20) kHz and another one at (40, 20) kHz, with peak bi-coherences of $b(20 \text{ kHz}, 20 \text{ kHz}) \approx 0.98$ and $b(40 \text{ kHz}, 20 \text{ kHz}) \approx 0.30$, respectively. These bi-coherence spectral peaks provide unambiguous evidence for the non-linear interactions $(20 + 20 \rightarrow 40) \text{ kHz}$ and $(20 + 40 \rightarrow 60) \text{ kHz}$. Thus, the observed second and third harmonic spectral peaks probably correspond to electromagnetic waves, generated as a result of coalescence of two oppositely propagating Langmuir waves, and coalescence of a Langmuir wave with an electromagnetic wave at $2f_{pe}$, respectively.

As far as the three wave interaction $L_1 + L_2 \rightarrow T_{2f_{pe}}$ is concerned, the sidebands $L_U$ and $L_D$ excited by the OTSI are well suited to play the role of the oppositely propagating Langmuir waves $L_1$ and $L_2$. In the case of the third harmonic $T_{3f_{pe}}$ is concerned, it could be excited either by a four wave interaction $L_1 + L_2 + L_3 \rightarrow T_{3f_{pe}}$ or by a three wave interaction $L + T_{2f_{pe}} \rightarrow T_{3f_{pe}}$, where $L_1$, $L_2$ and $L_3$ are the Langmuir waves with different wave vectors. However, the intense bi-spectral peak at $(2f_{pe}, f_{pe})$ supports only the later process, where the Langmuir wave could be either the beam-excited Langmuir wave, or any one of the side bands.
5. Summary and Conclusions
We have presented the observations of an intense localized one dimensional magnetic field aligned Langmuir waveform obtained by the STEREO/TDS in the source region of a local solar type III radio burst. The theories predict that the bump-on-tail distributions of electron beams propagating along the magnetic field excite one-dimensional field aligned Langmuir wave packets. The observations presented in this study represent one of such wave packets. Our analysis has revealed that the peak intensity and short spatial scale of this wave packet easily satisfy the threshold condition for excitation of OTSI and related strong turbulence processes. Our spectral analysis has shown that its spectrum exhibits the characteristic signatures of OTSI, namely, a resonant peak at $f_{pe}$, Stokes and anti-Stokes peaks at $f_{pe} \pm f_S$, and a low frequency enhancement below $f_S$. We have clearly demonstrated that these spectral components easily satisfy the frequency and wave number resonance conditions of the OTSI type of four wave interaction. In order to verify whether the phases of these spectral components satisfy the resonance condition required by the OTSI, we have computed FFT based tri-coherence spectrum of the wave packet. We have shown that the computed tri-coherence spectrum contains the needed peak, which is indicative of phase coherence amongst the spectral components. The spectrum of the wave packet is also found to contain two harmonic peaks, one at $2f_{pe}$ and another one at $3f_{pe}$ in addition to an intense peak at $f_{pe}$. We have calculated its wavelet based bi-coherence spectrum and found that it contains two bi-coherence spectral peaks, one at $(f_{pe}, f_{pe})$ and another one at $(2f_{pe}, f_{pe})$. These bi-coherence spectral peaks provide clear evidence for two kinds of three wave interactions, namely, $L_U + L_D \rightarrow T_{2f_{pe}}$ and $L + T_{2f_{pe}} \rightarrow T_{3f_{pe}}$, respectively. Those observations are consistent with simulation results (see, for example [1]). The one dimensional wave packet presented in this study with its characteristic sideband spectral structure is in the linear regime of OTSI, and the second harmonic electromagnetic wave is most probably excited as a result of merging of oppositely propagating sidebands excited by the OTSI.

Thus, the observations of the one dimensional wave packets presented in this study confirm that the OTSI and related strong turbulence processes play a significant role in the stabilization of the electron beam as well as excitation of second and third harmonic electromagnetic waves.

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6. References
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