Ideal with Micro Topological Space

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Abstract. In this paper studied *-\(m_i\)-sets, *-\(m_I\)-sets, *-\(m_I\)-sets and \(*I\)-sets on micro ideal topological space and specific properties of this inspected us also inspected the idea of \(\mathcal{R}_e\) sets and discussed the relationships with examples.

Keyword. microopen set, semi microopen set, ideal \(I\), \(\alpha\)-microopen and pre\(-\)microopen.

Introduction
In 1964 Vaidyanathaswamy [12] study ideal \(I\) of a topological space \((X,\tau)\) If the \(\mathcal{F}(X)\) is family from all subset of \(X\) is given topological space operator \((A)\ast: \mathcal{F}(X) \rightarrow \mathcal{F}(X)\), called local function of \(B\) with connect at \(\tau\) and \(I\) is defined as follows , for \(B \subseteq X\), \(B\ast(I,\tau) = \{x \in X: U \cap B \notin I \text{ for every } U \in \tau\}\).

preliminary

Definition 2.1[13]
Any ideal \(I\) on a topological space \((X,\tau)\) If the \(\mathcal{F}(X)\) is family from all subset of \(X\) is given topological space operator \((A)\ast: \mathcal{F}(X) \rightarrow \mathcal{F}(X)\), called local function of \(B\) with connect at \(\tau\) and \(I\) is defined as follows , for \(B \subseteq X\), \(B\ast(I,\tau) = \{x \in X: U \cap B \notin I \text{ for every } U \in \tau\}\).

Definition 2.2[8]
Let \(U\) be a non empty finite set of objects called the universe and \(R\) be an equivalence relation on \(U\) named as the indiscernibility relation. Then \(U\) is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair \((U,R)\) is said to be the approximation space. Let

\[X \subseteq U.\]

1- The lower approximation of \(X\) with respect to \(R\) is the set of all objects, which can be for certain classified as \(X\) with respect to \(R\) and it is denoted by \(L(R)(X)\). That is, \(L(R)(X) = \{\forall x \in U: R(x) \subseteq X\}\) where \(R(x)\) denotes the equivalence class determined by \(x \in U\).
2- The upper approximation of \( X \) with respect to \( R \) is the set of all objects, which can be possibly classified as \( X \) with respect to \( R \) and it is denoted by \( U_{\delta}(X) \). That is, \( U_{\delta}(X) = \{ x \in \mathbb{R} : R(x) \cap X = \emptyset \} \).

3- The boundary region of \( X \) with respect to \( R \) is the set of all objects, which can be classified neither as \( X \) nor as \( BR(X) \) denotes not-\( X \) with respect to \( R \) and it. That is, \( B_{\delta}(X) = U_{\delta}(X) - L_{\delta}(X) \).

**Definition 2.3** [4]

Let \( U \) be an universe, \( R \) be an equivalence relation on \( U \) and \( \tau_{\delta}(X) = \{ U, \emptyset, L_{\delta}(X), U_{\delta}(X), B_{\delta}(X) \} \) where \( X \subseteq U \) satisfies the following axioms:

- \( U, \emptyset \in \tau_{\delta}(X) \)
- The union of the elements of any sub-collection of \( \tau_{\delta}(X) \) is in \( \tau_{\delta}(X) \)
- The intersection of the elements of any finite sub collection of \( \tau_{\delta}(X) \) is in \( \tau_{\delta}(X) \)

Then \( \tau_{\delta}(X) \) is called the Nano topology on \( U \) with respect to \( X \). The space \( (U, \tau_{\delta}(X)) \) is the Nano topological space. The elements are called Nano open sets.

**Definition 2.4** [2]

\( (U, \tau_{\delta}(X)) \) is a Nano topological space here \( \mu_{\delta}(X) = \cup \{ N \cup (N^0 \cap \mu) : \mu, N \in \tau_{\delta}(X) \} \) and called it Micro topology of \( \tau_{\delta}(X) \) by \( \mu \) where \( \mu, \nu \in \tau_{\delta}(X) \).

**micI\(_C\)**-sets and *-**micI\(_D\)**-sets

**Definition 3.1**

A subset \( A \) in a space \( (U, \tau_R(X), \mu_R(X), I) \) is said \( mic^* \)-closed if \( A^* \subseteq A \). The complement of an \( mic^* \)-closed set is called \( mic^* \)-open.

**Definition 3.2**

A subset \( A \) in a space \( (U, \tau_R(X), \mu_R(X), I) \) is called\n
\[ \alpha = mic^* - open \Longleftrightarrow A \subseteq mic - int(mic - cl'(mic - int(A))). \]

**pre-**micI - open if \( A \subseteq mic - int(mic - cl'(mic - int(A))). \)

**Definition 3.3**

Pre- \( micI \) - closure of a subset \( A \) of an ideal micro topological space \( (U, \tau_R(X), \mu_R(X), I) \) denoted by \( micI_pcl(A) \) is defined is the intersection of all Pre- \( micI \) - closed sets of \( U \) containing \( A \).

**Definition 3.4**

A subset \( A \) of a space \( (U, \tau_R(X), \mu_R(X), I) \) is called semi*- **micI** - open if \( A \subseteq mic - cl(mic - int^*(A)). \)

**Definition 3.5**

A subset \( A \) of a space \( (U, \tau_R(X), \mu_R(X), I) \) is called a\n
\[ t^* - micI \] - sets if \( Mic - int(A) = mic - cl'(mic - int(A)). \)

Pre- \( micI \) - regular if \( A \) is Pre- **micI** - closed and \( t^* - micI \) - set.

**Definition 3.6**

A subset \( F \) of a micro topological space \( (U, \tau_R(X), \mu_R(X)) \) is called micro locally closed set if \( F = H \cap Q \) where \( H \) is an \( mic \) - open set and \( Q \) is a micro closed set of \( U \).

**Definition 3.7**

A subset \( A \) of a space \( (U, \tau_R(X), \mu_R(X), I) \) is called\n
(i) \( * - micI_D \) - sets if \( A = H \cap Q \) where \( H \) is an \( mic \) - open set and \( Q \) is a pre- **micI** - closed set of \( U \).

(ii) \( * - micI_V \) - sets if \( A = H \cap Q \) where \( H \) is an \( mic \) - open set and \( Q \) is a **micI** - closed set of \( U \).

(iii) \( * - micI_C \) - sets if \( A = H \cap Q \) where \( H \) is an \( mic \) - open set and \( Q = Mic - cl(Mic - int^*(Q)). \)

**micI_D** - sets if \( A = H \cap Q \) where \( H \) is an \( mic \) - open set and \( Q \) is Pre- **micI** - regular set of \( U \).

**Theorem 3.8**

(i) Each \( micI_D \) set is \( * - micI_D \) - set.

(ii) Each \( * - micI_C \) - set is \( * - micI_V \) - set.
(iii) Each *-micIγ-sets is * - micIβ-set.

Proof:

(i) Let A is micIβ set then A = H ∩ Q where H is an mic *-open set and Q is a pre-micI regular set. Since pre-micI regular set is Pre-micI-open and t∗-micI-set and take the complement of Pre-micI-open impels that Q is a pre-micI - closed set. Hence A is *-micIβ-set.

(ii) Let A is *-micIγ-set then A = H ∩ Q where H is an mic ∗-open set and Q = MIC – cl(MIC – int*(Q)), since MIC – cl(MIC – int*(MIC – cl(Q))) ⊆ MIC – cl(MIC – int*(MIC – cl(Q))) ⊆ Q, then Q is α-micIβ-set. Hence A is *-micIγ-set.

(iii) Let A is *-micIγ-set then A = H ∩ Q where H is an mic *-open set and Q = MIC – cl(MIC – int*(MIC – cl(Q))) ⊆ MIC – cl(MIC – int*(Q)) then Q is pre-micIβ-closed set. Hence A is *-micIγ-set.

Remark 3.9
Converse of the above theorem need not be true from the following example .

Example 3.10
Let U = {a, b, c, d} with U/R = {{b}, {d}, {a, c}}, X = {c, d} and I = {φ, {d}} then
τR(X) = {φ, U, {d}, {a, c, d}, {c, d}}
τR(X) – closed = {φ, U, {a, b, c}, {b}, {a, b}}, μ = {c} Then
μR(X) = {φ, U, {d}, {a, c, d}, {a, c}, {c, d}}
μR(X) – closed = {φ, U, {a, b, c}, {b}, {b, d}, {a, b}, {a, b}}
Then A = {a, b, d} is *-micIγ-set but not *-micIδ-set.

Then A={a,d} is *-micIδ-set but not it is micIβ-set.

Example 3.11
Let V = {i, j, k, l, m}, V = {{i}, {j, k, l}, {m}}, X = {j, k} ⊆ V, I = {φ, {j, l}, {l}}
Then τR(X) = {φ, V, {j, k, l}} with μ = {i} then μR(X) = {φ, V, {i}, {i, j, k, l}, {j, l, k}}
μR(X) – closed = {φ, V, {j, k, l, m}, {i, m}, {m}}
Then A={m} is *-micIγ-set but it is not *-micIγ-set.

Remark 3.12
From example 3.10and example 3.11 we have the following diagram

![Diagram](image)

Figure 1.

Theorem 3.13
At a subset A from an ideal micro topological space (U, τR(X), μR(X), I) the following properties are equivalent

1- A is *-micIγ-set and semi *-micIβ-open set on U.

2- A = H ∩ mic – cl(mic – int*(A)) at an mic*-open set Q.

Proof:

(i) → (ii) Suppose that A is *-micIγ-set and semi *-micIβ-open set on U. Since A is *-micIγ-set then A = H ∩ Q where H is an mic *-openset and Q is pre-micIβ-closed set of U, we have A ⊆ Q, so
\[ \text{mic} - \text{cl}(\text{mic} - \text{int}^*(A)) \subseteq \text{mic} - \text{cl}(\text{mic} - \text{int}^*(Q)). \]

Since \( Q \) is a pre-micl-closed set of \( U \), then \( \text{mic} - \text{cl}(\text{mic} - \text{int}^*(Q)) \subseteq Q \). Since \( A \) is semi \text{-micl}-open set on \( U \) then \( A \subseteq \text{mic} - \text{cl}(\text{mic} - \text{int}^*(A)) \). Implies that

\[
A = A \cap \text{mic} - \text{cl}(\text{mic} - \text{int}^*(A)) = H \cap Q \cap \text{mic} - \text{cl}(\text{mic} - \text{int}^*(A))
\]

(i) \( \implies \) (ii) Let \( A = H \cap \text{mic} - \text{cl}(\text{mic} - \text{int}^*(A)) \) where \( H \) is \text{mic}*-open set \( \), we have \( A \subseteq \text{mic} - \text{cl}(\text{mic} - \text{int}^*(A)) \). Implies that semi \text{-micl}-open set on \( U \), since \( \text{mic} - \text{cl}(\text{mic} - \text{int}^*(A)) \) be a closed set \( , \) we have \( \text{mic} - \text{cl}(\text{mic} - \text{int}^*(A)) \) is pre-micl-closed set of \( U \). Hence \( A \) is \text{micl}-set of \( U \).

**Theorem 3.14**

At a subset \( A \) from an ideal micro topological space \((U, \tau_R(X), \mu_R(X), I)\) the following properties are equivalent

(i) \( A \) is \text{-micl}_c-set of \( U \).

(ii) \( A \) is \text{-micl}_y-set and semi \text{-micl}-open set of \( U \).

(iii) \( A \) is \text{-micl}_p-set and semi \text{-micl}-open set of \( U \).

**Proof:**

(i) \( \implies \) (ii) Let \( A \) is \text{-micl}_y-set then \( A = H \cap Q \) where \( H \) is an \text{mic}*-open set and \( Q = \text{mic} - \text{cl}(\text{mic} - \text{int}^*(Q)). \) This imply \( A = H \cap Q \)

\[
= H \cap \text{mic} - \text{cl}(\text{mic} - \text{int}^*(Q))
\]

\[
= \text{mic} - \text{int}^*(H) \cap \text{mic} - \text{cl}(\text{mic} - \text{int}^*(Q))
\]

\[
\subseteq \text{mic} - \text{cl}(\text{mic} - \text{int}^*(H)) \cap \text{mic} - \text{cl}(\text{mic} - \text{int}^*(Q))
\]

\[
= \text{mic} - \text{cl}(\text{mic} - \text{int}^*(H)) \supseteq \text{mic} - \text{cl}(\text{mic} - \text{int}^*(Q))
\]

Consequently \( A \subseteq \text{mic} - \text{cl}(\text{mic} - \text{int}^*(A)) \). Hence \( A \) semi \text{-micl}-open set of \( U \), and by theorem 3.8 (ii), \( A \) is \text{-micl}_y-set.

(ii) \( \implies \) (iii) in fact Each \text{-micl}_y-set is \text{-micl}_p-set from theorem 3.8 (iii).

(iii) \( \implies \) (i) Let \( A \) is \text{-micl}_p-set and semi \text{-micl}-open set of \( U \), by theorem 3.13 \( A = H \cap \text{mic} - \text{cl}(\text{mic} - \text{int}^*(A)) \) at an \text{mic}*-open set \( Q \), then

\[
\text{mic} - \text{cl}\left(\text{mic} - \text{int}^*(\text{mic} - \text{cl}(\text{mic} - \text{int}^*(A)))\right) = \text{mic} - \text{cl}(\text{mic} - \text{int}^*(A)).
\]

Hence \( A \) is \text{-micl}_c-set of \( U \).

Definition 3.15

A subset \( A \) from an ideal micro topological space \((U, \tau_R(X), \mu_R(X), I)\) is said to be \text{micl}_p-open if \( G \subseteq \text{micl}_p - \text{int}(A) \) where \( G \subseteq A \) and \( G \) is \text{mic}*-closed set of \( U \) when

\[
\text{micl}_p - \text{int}(A) = A \cap \text{mic} - \text{int}(\text{mic} - \text{cl}(A)).
\]

**Theorem 3.16**

At a subset \( A \) from an ideal micro topological space \((U, \tau_R(X), \mu_R(X), I)\), \( A \) is \text{-micl}_p-closed if and only if \( \text{micl}_p - \text{cl}(A) \subseteq G \) where \( A \subseteq G \) and \( G \) is \text{mic}*-open set on \( U \).

**Proof:**

Let \( A \) is \text{-micl}_p-closed set on \( U \), assume that \( A \subseteq G \) and \( G \) is \text{mic}*-open set on \( U \). Then \( U - A \) is \text{-micl}_p-open and \( U - H \subseteq U - A \) where \( U - H \) is \text{mic}*-closed. Since \( U - A \) is \text{-micl}_p-open then \( U - A \subseteq \text{micl}_p - \text{int}(U - A) \), where

\[
\text{micl}_p - \text{int}(U - A) = U - A \cap \text{mic} - \text{int}(\text{mic} - \text{cl}(U - A))
\]

Since, \( \text{micl}_p - \text{cl}(A) = A \cup \text{mic} - \text{cl}(\text{mic} - \text{int}^*(A)) \)

\[
\text{(U - A)} \cap \text{mic} - \text{int}(\text{mic} - \text{cl}(U - A)) = \text{(U - A)} \cap \text{(U - mic - cl(mic - int^*(A))}
\]

Since, \( \text{micl}_p - \text{cl}(A) = A \cup \text{mic} - \text{cl}(\text{mic} - \text{int}^*(A)) \) then

\[
\text{(U - A)} \cap \text{mic} - \text{int}(\text{mic} - \text{cl}(U - A)) = \text{(U - A) mic - cl(mic - int^*(A))}
\]

\[
= U - \text{micl}_p - \text{cl}(A)
\]
Implies that \( \text{micI}_p - \text{int}(U - A) = U - \text{micI}_p - \text{cl}(A) \). This
\( \text{micI}_p - \text{closed}(A) = U - \text{micI}_p - \text{int}(A) \subseteq H \). Hence \( \text{micI}_p - \text{closed}(A) \subseteq H \).
Converse: is first for part like.

**Theorem 3.17**
Let \( (U, \tau_R(X), \mu_R(X), I) \) is a micro ideal topological space and \( A \subseteq U \). Then \( A \) is * \( - \text{micI}_p \) - set on \( U \) if and only if \( A = B \cap \text{micI}_p \text{cl}(A) \) where \( B \) is \( \text{mic} \) *-open set on \( U \).

**Proof:**
Let \( A \) is * \( - \text{micI}_p \) - set then \( A = B \cap P \) with any \( \text{mic} \) *-open set \( B \) and a pre- \( \text{micI} \) - closed set \( P \), but
\( A \subseteq \text{micI}_p - \text{closed}(A) \) and also \( A \subseteq \text{micI}_p - \text{closed}(A) \subseteq P \) implies that
\( A = A \cap \text{micI}_p - \text{closed}(A) \)
\( = A \cap P \cap \text{micI}_p - \text{closed}(A) \)
\( = A \cup \text{micI}_p - \text{closed}(A) \)
Conversely : prove that \( \text{micI}_p - \text{closed}(A) \) is a pre- \( \text{micI} \) - closed set but \( \text{micI}_p - \text{closed}(A) \subseteq P \), for a pre- \( \text{micI} \) - closed set containing \( A \). Hence
\( \text{mic} \) \( - \text{cl}(\text{mic} - \text{int}^* (\text{micI}_p - \text{cl}(A) \subseteq \text{mic} \) \( - \text{cl}(\text{mic} - \text{int}^*(P) \subseteq P \).
\( \rightarrow \text{mic} \) \( - \text{cl}(\text{mic} - \text{int}^*(\text{micI}_p - \text{cl}(A) \subseteq A \cap P \)
\( P \) is pre- \( \text{micI} \) - closed set = \( \text{micI}_p - \text{closed}(A) \).

**Theorem 3.18**
Let \( (U, \tau_R(X), \mu_R(X), I) \) is an ideal micro topological space and \( A \subseteq U \), the following properties are equivalent.
\( A \) is pre- \( \text{micI} \) - closed set on \( U \).
\( A \) is * \( - \text{micI}_p \) - set and is * \( - \text{micI}_p \) - closed set on \( U \).

**Proof:**
(i) \( \rightarrow \) (ii) let \( A \) is pre- \( \text{micI} \) - closed set ,then \( A \) is * \( - \text{micI}_p \) - set because any
* \( - \text{micI}_p \) - set= \( H \cap Q \) where \( H \) is an \( \text{mic} \) *-open set and \( Q \) is a pre- \( \text{micI} \) - closed set and \( A \) is * \( - \text{micI}_p \) - closed on \( U \) by Definition 3.15
(ii) \( \rightarrow \) (i) assume that \( A \) is * \( - \text{micI}_p \) - set and is * \( - \text{micI}_p \) - closed on \( U \), since
\( A \subseteq H \) and \( A \) is * \( - \text{micI}_p \) - closed set then by theorem 3.17 \( A = H \cap \text{micI}_p \text{cl}(A) \) where \( H \) is \( \text{mic} \) *-open set on \( U \).
Since \( A \subseteq H \) and \( A \) is * \( - \text{micI}_p \) - closed on \( U \) then \( \text{micI}_p - \text{closed}(A) \subseteq H \), impalas that
\( \text{micI}_p - \text{closed}(A) \subseteq H \cap \text{micI}_p \text{cl}(A) = A \). This \( A = \text{micI}_p \text{closed}(A) \). Hence \( A \) is pre- \( \text{micI} \) - closed set on \( U \).

**Theorem 3.19**
Let \( (U, \tau_R(X), \mu_R(X), I) \) is a micro ideal topological space and \( A \subseteq U \). If \( A \) is * \( - \text{micI}_p \) - set on \( U \) then \( \text{micI}_p - \text{cl}(A) - A \) is pre- \( \text{micI} \) - closed set and \( A \cup U - \text{micI}_p \text{cl}(A) \) is pre- \( \text{micI} \) - open set on \( U \).

**Proof:**
Let \( A \) is * \( - \text{micI}_p \) - set on \( U \) by theorem 3.17 \( A = B \cap \text{micI}_p \text{cl}(A) \) where \( B \) is \( \text{mic} \) *-open set on \( U \).
Impulse that
\( \text{micI}_p - \text{cl}(A) - A = \text{micI}_p - \text{cl}(A) - (B \cap \text{micI}_p \text{cl}(A) = \text{micI}_p - \text{cl}(A) \cap (U - (B \cap \text{micI}_p \text{cl}(A) \)
\( = \text{micI}_p - \text{cl}(A) \cap ("U - B") \cup (U" - \text{micI}_p \text{cl}(A) \)
\( = \text{micI}_p - \text{cl}("A) \cap (U - B") \cup (\text{micI}_p \text{cl}(A) \cap (U - \text{micI}_p \text{cl}(A) \)
\( = \text{micI}_p - \text{cl}("A) \cap (U - B") \cup q = \text{micI}_p - \text{cl}("A) \cap (U - B") \)
This \( \text{micI}_p - \text{cl}("A) - A" = \text{micI}_p - \text{cl}("A) \cap (U - B") \). Hence \( \text{micI}_p - \text{cl}("A) - A" \)
is pre- \( \text{micI} \) - closed set and since \( \text{micI}_p - \text{cl}(A) - A \) is pre- \( \text{micI} \) - closed set on \( U \).Then
\( U - (\text{micI}_p - \text{cl}(A) - A) = U - \text{micI}_p - \text{cl}(A) \cap (U - A) \)
\( = U - (\text{micI}_p - \text{cl}(A) \cap A) \) is pre- \( \text{micI} \) - open set.This
\( U - (\text{micI}_p - \text{cl}(A) - A) = U - (\text{micI}_p - \text{cl}(A) \cup A) \) is pre- \( \text{micI} \) - open set on \( U \).

\( \text{micI}_C \) - set and \( \mathcal{R} - \text{micI} \) - open sets

**Definition 4.1**
A subset $B$ of a micro ideal topological space $(U, \tau_R(X), \mu_R(X), I)$ as said to be $\mathcal{R} - \text{micl} - \text{open}$ set if $B = \text{mic} - \text{int}(\text{mic} - \text{cl}^*(B))$, and $B = \text{mic} - \text{cl}(\text{mic} - \text{int}^*(B))$ as $\mathcal{R} - \text{micl} - \text{closed}$ set.

**Theorem 4.2**

From a micro ideal topological space $(U, \tau_R(X), \mu_R(X), I)$ and a subset $B$ of $U$, then the following properties are equivalent.

1. $B$ is a $\mathcal{R} - \text{micl} - \text{closed}$ set on $U$.
2. $B$ is semi* - micl-open and micl - closed set.

**Proof:**

(i) Suppose that $B$ is a $\mathcal{R} - \text{micl} - \text{closed}$ set then $B = \text{mic} - \text{cl}(\text{mic} - \text{int}^*(B))$.

Imply that $B$ is semi* - micl-open and micl - closed set on $U$.

(ii) Let $B$ is semi* - micl-open and micl - closed set on $U$ then $B \subseteq \text{mic} - \text{cl}(\text{mic} - \text{int}^*(B))$, since $B$ is micl - closed set then $\text{mic} - \text{cl}(\text{mic} - \text{int}^*(B)) \subseteq \text{mic} - \text{cl}(B) = B \subseteq \text{mic} - \text{cl}(\text{mic} - \text{int}^*(B))$.

This $B = \text{mic} - \text{cl}(\text{mic} - \text{int}^*(B))$. Hence $B$ is $\mathcal{R} - \text{micl} - \text{closed}$ set on $U$.

**Theorem 4.3**

From a micro ideal topological space $(U, \tau_R(X), \mu_R(X), I)$ and a subset $B$ of $U$, then the following properties are equivalent.

1. $B$ is a $\mathcal{R} - \text{micl} - \text{closed}$ set on $U$.
2. $\exists$ a mic* - open set $H$ such that $B = H \cap \text{mic}$ - closed set.

**Proof:**

(i) Suppose that $B$ is a $\mathcal{R} - \text{micl} - \text{closed}$ set then $B = \text{mic} - \text{cl}(\text{mic} - \text{int}^*(H))$.

Us pick $H = \text{mic} - \text{int}^*(B)$ implies that $H$ is mic* - open set and $B = \text{mic} - \text{cl}(H)$.

(ii) Let open set $H$ such that $\exists$ a mic* - open set $H$ such that $B = \text{mic} - \text{cl}(H)$.

Since $H = \text{mic} - \text{int}^*(H)$ then $\text{mic} - \text{cl}(H) = \text{mic} - \text{cl}(\text{mic} - \text{int}^*(H))$, implies that

$$\text{mic} - \text{cl}(\text{mic} - \text{int}^*(H)) = \text{mic} - \text{cl}(\text{mic} - \text{int}^*(\text{mic} - \text{cl}(H)))$$

$$= \text{mic} - \text{cl}(\text{mic} - \text{int}^*(H))$$

This $\text{mic} - \text{cl}(\text{mic} - \text{int}^*(H))$. Hence $B$ is a $\mathcal{R} - \text{micl} - \text{closed}$ set on $U$.

**Theorem 4.4**

From a micro ideal topological space $(U, \tau_R(X), \mu_R(X), I)$ and a subset $B$ of $U$, then $B$ is semi* - micl-open if $B = H \cap P$ where $H$ is a $\mathcal{R} - \text{micl} - \text{closed}$ set and $\text{mic} - \text{int}(P)$ is a mic - dense set.

**Proof:**

Assume that $B = H \cap P$ where $H$ is a $\mathcal{R} - \text{micl} - \text{closed}$ set and $\text{mic} - \text{int}(P)$ is a mic* - dense set. Then $\exists$ a mic* - open set $G$ such that $H = \text{mic} - \text{cl}(G)$.

Us pick

$$F = G \cap \text{mic} - \text{int}(P)$$

Imply that $F$ is mic* - open and $F \subseteq B$.

Furthermore,

$$\text{mic} - \text{cl}(F) = \text{mic} - \text{cl}(G \cap \text{mic} - \text{int}(P))$$

and $\text{mic} - \text{cl}(G \cap \text{mic} - \text{int}(P)) \subseteq \text{mic} - \text{cl}(G)$.

Since $\text{mic} - \text{int}(P)$ is a mic* - dense set. Then $G = G \cap \text{mic} - \text{cl}^*(\text{mic} - \text{int}(P))$

$$\subseteq \text{mic} - \text{cl}^*(G \cap \text{mic} - \text{int}(P))$$

$$\subseteq \text{mic} - \text{cl}(G \cap \text{mic} - \text{int}(P))$$

Imply that $\text{mic} - \text{cl}(G) \subseteq \text{mic} - \text{cl}(G \cap \text{mic} - \text{int}(P))$. Moreover $\text{mic} - \text{cl}(F) = \text{mic} - \text{cl}(G \cap \text{mic} - \text{int}(P))$.

$$\subseteq \text{mic} - \text{cl}(G) = H \subseteq \text{mic} - \text{cl}(G \cap \text{mic} - \text{int}(P)) = \text{mic} - \text{cl}(F)$$. This $F \subseteq B \subseteq H = \text{mic} - \text{cl}(F)$.

Hence then $B$ is semi* - micl-open set.

**Definition 4.5**

The semi* - micl closure of a subset $A$ of a micro ideal topological space $(U, \tau_R(X), \mu_R(X), I)$
Denoted by $S_{\text{mic}}^{*} \text{cl}(A)$ defined by the $\cap$ of all semi$^{*} - \text{micl}$-closed sets of $U$ containing $A$.

**Definition 4.6**

If $(U, \tau(X), \mu_R(X), I)$ is a micro ideal topological space and $A \subseteq U$, $A$ is called

(i) Generalized semi$^{*} - \text{micl}$-closed ($gS_{\text{micl}}^{*}\text{closed}$) on $(U, \tau(X), \mu_R(X), I)$ if $S_{\text{micl}}^{*} \text{cl}(A) \subseteq F$.

Where $A \subseteq F$ and $F$ is mic-open set on $(U, \tau(X), \mu_R(X), I)$.

(ii) Generalized semi$^{*} - \text{micl}$-open ($gS_{\text{micl}}^{*}\text{open}$) on $(U, \tau(X), \mu_R(X), I)$ if $U - A$ is $gS_{\text{micl}}^{*}\text{closed}$ set on $(U, \tau(X), \mu_R(X), I)$.

**Theorem 4.7**

Of a subset $A$ for a micro ideal topological space $(U, \tau(X), \mu_R(X), I)$, $A$ is $gS_{\text{micl}}^{*}\text{open}$ if and only if $K \subseteq S_{\text{micl}}^{*}\text{int}(A)$ where $K \subseteq A$ and $K$ is a mic$^{-}$ closed set on $(U, \tau(X), \mu_R(X), I)$, whenever $S_{\text{micl}}^{*}\text{int}(A) = A \cap \muic - \text{cl}(\text{mic} - \text{int}^{*}(A))$.

**Proof:**

(i) Let $A$ is $gS_{\text{micl}}^{*}\text{open}$ set in $U$ suppose that $K \subseteq A$ and $K$ is a mic$^{-}$ closed set on $(U, \tau(X), \mu_R(X), I)$, implies that $U - A$ is a $gS_{\text{micl}}^{*}\text{closed}$ set and $U - A \subseteq U - K$ where $S_{\text{micl}}^{*}\text{cl}(U - A) = (U - A) \cup \muic - \text{int}(\text{mic} - \text{cl}^{*}(U - A)).$ Since $U - (A \cap \muic - \text{cl}^{*}(U - A)) = (U - A) \cup (U - \muic - \text{cl}(\text{mic} - \text{int}^{*}(A))).$

(ii) Let $A$ is $S_{\text{micl}}^{*}\text{cl}(A)$ is mic$^{-}$ closed set on $U$. Hence $A$ is a mic$^{-}$ closed set.

**Theorem 4.8**

$(U, \tau(X), \mu_R(X), I)$, a micro ideal topological spaces while $A \subseteq U$ the following properties are equivalent:

(i) $A$ is a $\mathcal{R} - \text{micl} - \text{open}$ set on $U$.

(ii) $A$ is a mic$^{-}$ open and $gS_{\text{micl}}^{*}\text{closed}$ set.

**Proof:**

(i) Suppose that $A$ is a $\mathcal{R} - \text{micl} - \text{open}$ set on $U$ then $A = \muic - \text{int}(\text{mic} - \text{cl}^{*}(A)).$ Implies that $A$ is open and $S_{\text{micl}}^{*}\text{cl}(A)$ is mic$^{-}$ closed set this $\mathcal{R} - \text{micl} - \text{open}$ set on $U$. Hence $A$ is a mic$^{-}$ closed set.

(ii) Suppose that $A$ is mic$^{-}$ open and $gS_{\text{micl}}^{*}\text{closed}$ set. Then $A \subseteq \muic - \text{int}(\text{mic} - \text{cl}^{*}(A))$, since $A$ is mic$^{-}$ open and $gS_{\text{micl}}^{*}\text{closed}$ set. Then $S_{\text{micl}}^{*}\text{cl}(A) \subseteq A$, since $S_{\text{micl}}^{*}\text{cl}(A) = A \cup \muic - \text{int}(\text{mic} - \text{cl}^{*}(A))$. Then $S_{\text{micl}}^{*}\text{cl}(A) = A \cup \muic - \text{int}(\text{mic} - \text{cl}^{*}(A)) \subseteq A$ then mic$^{-}$ open (mic$^{-}$ cl$^{*}(A)$) $\subseteq A$ and $A \subseteq \muic - \text{int}(\text{mic} - \text{cl}^{*}(A))$. Hence $A$ is a $\mathcal{R} - \text{micl} - \text{open}$ set on $U$.

**Remark 4.9**

All mic$^{-}$open set and all $\mathcal{R} - \text{micl} - \text{closed}$ set on $U$ is a$^{*}\text{-micl}$ - set on $U$. Conversely of Remark is not true on general shown in the following example.

In example 3.10 $A = \{c\}$ a$^{*}\text{-micl}$ - set but it is not $\mathcal{R} - \text{micl} - \text{closed}$ set and $B = \{a, b, c\}$ is a$^{*}\text{-micl}$ - set but it is not mic$^{-}$open set.

**Definition 4.10**

A subset $B$ of $(U, \tau(X), \mu_R(X), I)$ is called micro locally closed if $B = G \cap F$ where $G$ is mic$^{-}$open set and $F$ is mic$^{-}$closed in $(U, \tau(X), \mu_R(X), I)$.

**Remark 4.11**

All a$^{*}\text{-micl}$ - set is micro locally closed set on $U$. Conversely of Remark is not true on general shown in the following example. In example 2.5 $A = \{b, d\}$ is locally closed set but it is not a$^{*}\text{-micl}$ - set.

**Theorem 4.12**
\((U, \tau^*_g(X), \mu^*_g(X), I)\), a micro ideal topological space, \(A \subseteq U\) and \(B \subseteq U\) if \(A\) be a \(S^*_{micro} -\) open set and \(B\) be a mic-open set then \(A \cap B\) is \(S^*_{micro} -\) open.

**Proof:**

Let \(A\) be \(S^*_{micro} -\) open and \(B\) be a mic-open set, implies that 
\[A \cap B \subseteq \text{mic} - cl(\text{mic} - \text{int}^*(A) \cap B) \subseteq \text{mic} - cl(\text{mic} - \text{int}^*(A) \cap B).\]

Hence \(A \cap B\) is \(S^*_{micro} -\) open.

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