Interplay of Rashba and Dresselhaus spin splittings in 2D weak localization

M.M. Glazov and L.E. Golub
A.F. Ioffe Physico-Technical Institute, Russian Academy of Sciences, 194021 St. Petersburg, Russia

The effects of structural (Rashba) and bulk (Dresselhaus) spin-orbit interaction terms on the low-field magnetoresistance are investigated in high-quality two dimensional systems. The weak localization theory accounting for both of these terms valid in the whole range of magnetic fields is proposed. The suppression of antilocalization correction as Rashba and Dresselhaus terms strengths approach each other is demonstrated. The effect of cubic in the wavevector spin-splitting term is analyzed.

INTRODUCTION

Spin properties of carriers in semiconductor heterostructures attract now great attention due to spintronics proposals. From the point of view of fundamental physics as well as of possible applications the control of spin-orbit splitting of electronic energy spectrum in two-dimensional systems is one of the most important aspects. There are two contributions to the spin-orbit splitting in two-dimensional semiconductor structures: the Rashba and the Dresselhaus terms caused by structure and bulk inversion asymmetry respectively. Their magnitudes can be measured in various optical and transport experiments. In this work we investigate the effect of these spin splittings on low-field magnetoresistance caused by weak localization.

As the nature of Dresselhaus and Rashba terms is different they possess different symmetry and they are not additive in various kinetic phenomena. The weak antilocalization theory developed in Ref.[1] was limited to extremely weak magnetic fields where the electron motion can be considered as diffusive and to small values of spin-splittings. The theory valid in the range of the magnetic fields was developed in the Ref.[2] for the case of only one term being relevant. However, the experiment evidences the comparable magnitudes of both contributions, see e.g. Ref.[3]. An attempt to account simultaneously both Dresselhaus and Rashba terms was carried out in Ref.[3] but their theory was limited by rather high magnetic fields.

Here we present a general theory which allows to compute quantum conductivity corrections in the whole range of classically weak magnetic fields and for arbitrary values of Dresselhaus and Rashba spin-orbit splittings. We demonstrate that, for equal magnitudes of these contributions, the weak-localization conductivity correction exactly equals to that in the absence of spin-orbit interaction. In such a case even small cubic in wavevector spin-splitting modifies strongly of magnetoresistance leading to antilocalization minimum.

THEORY

In what follows we concentrate on zinc-blende lattice-based quantum wells grown along [001] direction. The spin-orbit interaction is described by the following Hamiltonian

\[ H_{SO}(k) = \hbar \sigma \cdot [\Omega_R(k) + \Omega_D(k)], \]

where \( k \) is the electron wave vector, \( \sigma \) is the vector of Pauli matrices. \( \Omega_{R(D)} \) characterizes the spin precession frequency due to Rashba (Dresselhaus) term which explicitly read: \( \Omega_D(k) = \Omega_D(\cos \chi, -\sin \chi) \) and \( \Omega_R(k) = \Omega_R(\sin \chi, -\cos \chi) \). Here \( \chi \) is the angle between \( k \) and [100] axis and \( \Omega_D, \Omega_R \) are the coupling strengths.

In accordance with Eq. (1) the electron motion is accompanied with spin rotation yielding the appearance of spin-dependent phase in the Green’s function [2]

\[ G^{R,A}(r, r') = G_0^{R,A}(r) \exp \left[ i \varphi(r, r') - i \sigma \cdot \omega(R) \right], \]

where \( R = r - r' \), \( G_0^{R,A} \) are retarded (R) and advanced (A) Green’s function for electron’s propagation in the short-range random potential without magnetic field \( B = 0 \) and spin splitting \( \Omega = 0 \). \( \varphi(r, r') = (x + x') (y' - y) / 2 l_B^2 \) with \( l_B = \sqrt{\hbar / eB} \) being the magnetic length and vector \( \omega(R) = \Omega_D(k_F l_B^{-1} R) + \Omega_D(k_F l_B^{-1} R) \), \( k_F \) is the Fermi wavevector, \( l \) is the mean free path.

Following the general theory of quantum-conductivity corrections [4] we solve the equation for the interference magnitude Cooperon \( C(r, r') \)

\[ C(r, r') = \frac{\hbar^3}{m \tau} \Phi(r, r') + \int dr_1 \Phi(r, r_1) C(r_1, r'), \]

where \( m \) is electron effective mass, \( \tau \) is the scattering time, \( \Phi_{\alpha\gamma,\beta\delta}(r, r') = \hbar^3 / m \tau G_{\alpha\gamma}^{R}(r, r') G_{\beta\delta}^{A}(r, r') \), expanding the quantities entering in Eq. (3) in the series over the wavefunctions of spinless particle with charge \( 2e \). In the general case where both Rashba and Dresselhaus terms are present in the spin-orbit Hamiltonian, the isotropy of the energy spectrum in the system is removed and all the ‘Landau levels’ of \( 2e \) particle become intermixed as opposed to the case of one term being relevant. In the latter situation the total angular momentum
FIG. 1: Magnetoconductivity computed for the constant Dresselhaus term $\Omega_D \tau = 1$ and different values of Rashba term $\Omega_R \tau$. The phase-breaking time $\tau_o = 100\tau$. The inset shows the minimum position vs. $\Omega_R/\Omega_D$.

is conserved allowing to separate full Cooperon matrix into finite blocks [2]. Finally, as Cooperon is known the computation of conductivity reduces to the simple linear-algebra problem [5].

RESULTS AND DISCUSSION

Figure 1 presents the quantum conductivity correction plotted vs. $B/B_{tr}$ ($B_{tr} = \hbar/2e\ell^2$) for different relation of Rashba and Dresselhaus constants. The value of Dresselhaus term is fixed to be $\Omega_D \tau = 1$ for all curves in the figure while Rashba term took different values $\Omega_R \tau$.

The curve corresponding to $\Omega_R = \Omega_D$ exactly coincides with the result for spinless particles [4]. The magnitude of the quantum correction decreases with the magnetic field as long trajectories (longer than $l_B$) are suppressed in the magnetic field. In this case the spin precession axis is directed along either [110] or [110] for any wavevector $k$ and the spin rotation angle for closed trajectories is zero. From the point of view of the quantum mechanics the single-particle spectrum in this case consists of two independent paraboloids which contribute additively to the magnetoconductivity [6].

With the decrease of the Rashba term, the spin rotation angle for closed loops is no longer equals to zero and the magnetoconductivity becomes non-monotonous. As for the given trajectory the spin rotation angle will be the greater the greater $|\Omega^2_R - \Omega^2_D|\tau^2$, the decrease of the Rashba term (at fixed Dresselhaus one) will manifest itself as an increase of spin-orbit interaction [1]. The minimum of the magnetoconductivity shifts to the smaller fields (see inset to Fig. 1) and its depth decreases.

EFFECT OF $k$-CUBIC TERM

Besides the linear-$k$ terms studied above the spin-orbit interaction Eq. (1) contains also cubic in the wavevector contribution originated from the bulk Dresselhaus term

$$\Omega_3^D (k) = \Omega_3 (\cos 3\chi, \sin 3\chi).$$

(4)

The constant $\Omega_3$ can be of the same order of magnitude as $\Omega_D$ and $\Omega_R$ [1, 3] thus its effect on quantum conductivity corrections may be important. Also, the calculation of the zero-field correction shows that for $\Omega_R/\Omega_D = 1$ where linear in $k$ terms compensate each other the conductivity correction is determined to within the 10% by cubic contribution Eq. (4). Thus, we focus on the case where only $k$-cubic term is present in the effective Hamiltonian and compute magnetoconductivity for the different values of the $\Omega_3 \tau$ and $\Omega_D = \Omega_R = 0$, see Figure 2.

Qualitatively the behavior of the curves in Figure 2 coincides with that found for case of linear in the wavevector spin splitting [2]. In the high field case $B \ll \max \{|\Omega_3\tau^2, 1/B_{tr}\}$ all curves have the same asymptotics. As it can be seen from the figure inset the magnetoconductivity minimum shifts to the higher field range with an increase of $\Omega_3 \tau$. The minimum depth is a non-monotonous function of the spin splitting: for the small spin splitting values the increase of the splitting leads to an increase of minimum depth while for higher splittings the behavior is opposite.
CONCLUSION

We put forward a general theory of quantum conductivity corrections valid in the whole range of classically weak magnetic fields and taking into account the conduction band spin-splitting. Both Rashba and Dresselhaus terms were considered. We have shown that if Rashba and Dresselhaus terms are exactly equal the weak-localization correction is the same as in the spinless case. The crossover between weak localization and weak antilocalization is demonstrated as Rashba term value approaches to that of Dresselhaus term. The effect of cubic in the wavevector part of Dresselhaus term is comprehensively studied. The proposed theory can be used in the analysis of the low-field magnetoresistance to precisely extract the spin-splitting magnitudes from the transport measurements.

This work is financially supported by RFBR, Russian President grant for young scientists and by “Dynasty” Foundation — ICFPM.

[1] F. G. Pikus and G. E. Pikus, Phys. Rev. B 51, 16 928 (1995).
[2] L. E. Golub, Phys. Rev. B 71, 235310 (2005).
[3] J. B. Miller, D. M. Zumbühl, C. M. Marcus, Y. B. Lyanda-Geller, D. Goldhaber-Gordon, K. Campman, and A. C. Gossard, Phys. Rev. Lett. 90, 076807 (2003).
[4] V. M. Gasparyan and A. Yu. Zyuzin, Fiz. Tverd. Tela 27, 1662 (1985) [Sov. Phys. Solid State 27, 999 (1985)].
[5] M. M. Glazov and L. E. Golub, Semiconductors, in press.
[6] S. A. Tarasenko and N. S. Averkiev, Pis’ma ZhETF 75, 669 (2002) [JETP Letters 75, 552 (2002)]; N. S. Averkiev, M. M. Glazov and S. A. Tarasenko, Solid State Commun. 133, 543 (2005).