Magnetohydrodynamic Simulations of Accretion Disks around a Weakly Magnetized Neutron Star in Strong Gravity

Y. Kato\textsuperscript{1}, M. R. Hayashi\textsuperscript{2}, S. Miyaji\textsuperscript{1}, and R. Matsumoto\textsuperscript{1}

\textsuperscript{1}Graduate School of Science and Technology, Chiba University, 1-33 Yayoi-cho, Inage-ku, Chiba 263-8522, Japan
\textsuperscript{2}National Astronomical Observatory, Mitaka, Tokyo 181, Japan

ABSTRACT

We carried out two dimensional high-resolution magnetohydrodynamic (MHD) simulations of an accretion disk around a weakly magnetized neutron star. General relativistic effects are taken into account by using pseudo-Newtonian potential of Paczynski, B., P. J. Witt (1980). When magnetic loops connect the neutron star and the accretion disk, the twist injection from the disk or from the rotating neutron star triggers expansion of the loops. Since the expanding magnetic loops prevent inflow toward the magnetic poles of the neutron star, disk matter accumulates on the boundary between the magnetosphere and the disk. Magnetic reconnection taking place in the loops creates a channel along which the disk matter can accrete and unloads the magnetosphere. This process produces quasi-periodic variation of the accretion flow in the innermost region of the disk. We found two kinds of oscillations. One is the magnetospheric oscillation regulated by magnetic reconnection. The other is the radial disk oscillation. The typical frequency of the oscillations is 100 Hz to 2 kHz. Furthermore, we predict that QPO sources inevitably accompany X-ray flares by magnetic reconnection and bipolar outflows of hot X-ray emitting plasma similar to the optical jets in protostars.

INTRODUCTION

Shortly after the quasi-periodic oscillations of low mass X-ray binaries were found, various models have been proposed (e.g., Lamb, F. K., N. Shibazaki, M., M. A. Alpar, and J. Shaham, 1985; Miller, M. C., F. K. Lamb, and D. Psaltis, 1998). Although the disk-magnetosphere interaction is believed to be essential in such models, its detailed physical mechanisms have not been worked out yet.

Nonlinear simulations of the interaction between the dipole magnetic field of a star and its surrounding disk were carried out by Hayashi, M. R., K. Shibata, and R. Matsumoto (1996). By assuming Newtonian gravity and neglecting the stellar rotation, they showed that the magnetic interaction can explain the X-ray flares and outflows observed in protostars. When the dipole magnetic field of the star is twisted by the rotation of the disk, the magnetic loops connecting the star and the disk expand. Magnetic reconnection taking place in the expanding loops produces X-ray flares and hot plasma outflows. We extend this model to the neutron star. The dynamics around the marginally stable radius in the strong gravity of the neutron star is mimicked by using pseudo-Newtonian potential of the form: $\Psi_{PN} = -GM/(R-r_g)$ where $r_g(=2GM/c^2)$ is the Schwarzschild radius and $R(=\sqrt{r^2+z^2})$ the distance from the center of the neutron star. In this paper, we primarily concern with the magnetic interaction around the boundary between the magnetosphere and the accretion disk surrounding the neutron star.

SIMULATION MODELS

We numerically solved axisymmetric MHD equations in a cylindrical coordinate by using a 2-D MHD code based on a modified Lax-Wendroff scheme with artificial viscosity. Assuming that a neutron star is an aligned rotator, the dipole magnetic moment of the neutron star is...
### Table 1. Model Parameters

| Parameter                        | Model-A | Model-B |
|----------------------------------|---------|---------|
| NS mass ($M_\odot$)              | 1.4     |         |
| NS radius ($r_g$)                | 2.0     |         |
| Corotation radius ($r_h$)        | 13      | $\infty$|
| Plasma $\beta$                   | 10      | 100     |
| Magnetic Reynolds Number         | $R_m = 1000$ |       |
| Magnetic Diffusivity             | $\eta = 1/R_m$ |       |
| Center of torus ($r_g$)          | 13      |         |
| Computational area ($r_g$)       | $30 \times 30$ |     |
| Number of Grids                  | $500 \times 500$ |     |

Fig. 1. Initial State of Numerical Simulation

aligned with stellar rotation axis. We put a cold sub-Keplerian disk around the star (see Figure 1). We adopt polytropic equation of state $P = K \rho^{5/3}$ for the disk. The radius of the density maximum of the disk is $13 \, r_g$. The inner edge of the disk initially locates at $11 \, r_g$. Outside the disk, we assumed non-rotating, spherical, and isothermal hot halo. The radius of the neutron star is assumed to be smaller than the marginally stable radius. We impose the absorbing boundary condition on the surface of a neutron star at $R = 2.0 \, r_g$. The strength of the dipole magnetic moment is parameterized by plasma $\beta$ defined as the ratio of gas pressure to magnetic pressure at $(r, z) = (13 \, r_g, 0)$. The unit of length is Schwartzschild radius, and the velocity is normalized by the speed of light [$r_g = c$ (the speed of light) = 1]. We divided the computational area into $500 \times 500$ grids and solved axisymmetric resistive MHD equations by applying a 2-D resistive MHD code which is originally developed by Hayashi, M. R., K. Shibata, and R. Matsumoto (1996). Resistivity is assumed to be uniform.

### SIMULATION RESULTS

We carried out simulations of the following two models. Model-A is for a rotating neutron star and model-B is for a non-rotating neutron star. These model parameters are summarized in table 1. The reason why we choose these two models is that the angular momentum transport between the neutron star and the accretion disk intrinsically depends on the differential rotation between them.

**Model-A : Fast-Rotator**

Model-A includes the stellar rotation. The rotation period is parameterized by the corotation radius $r_c$. The corotation radius is taken to be $r_c = 13 \, r_g$, which corresponds to 3 milliseconds rotation period of the neutron star. Figure 2 shows the evolution of density and angular momentum distribution. Outside $r_c$, propeller action drives outflows. The magnetic field lines deform its configurations into a shape similar to that of helmet-streamer in the solar corona. Inside $r_c$, the disk material accretes because the magnetic field removes angular momentum from the disk. The deformed magnetic field lines around the disk-magnetosphere boundary is similar to those of Ghosh-Lamb model (Ghosh P., F. K. Lamb 1978).

**Model-B : Slow-Rotator**

We neglect the stellar rotation by assuming that corotation radius is far from the neutron star. Figure 3 shows the evolution of density and angular momentum distribution. A consequence of neglecting the stellar rotation is that the magnetic field efficiently removes the angular momentum from the disk and deposits it to the neutron star. Since the surface layers of the disk most efficiently lose their angular momentum,
Fig. 2. Time evolution of density distribution (upper panels) and angular momentum $L = \rho rv_\phi$ distribution (lower panels) for Model-A. The solid curves indicate the magnetic field lines. The arrows show the velocity vectors. Inside $r_c = 13r_g$, the magnetic torque removes angular momentum from the disk. Outside $r_c$, the propeller action drives outflows.

they accrete in dynamical time-scale. When the strength of magnetic field is not strong enough to keep the Alfvén radius outside the marginally stable radius ($r_{ms}$), the inner edge of the disk reaches $r_{ms}$. During this stage, we found the following quasi-periodic oscillations: (1) The magnetic loops, which connect the neutron star and the disk, are twisted by the rotation of the disk and expand. (2) The expanding loops prevent further accretion toward the neutron star and the magnetosphere is loaded with the accreted material. (3) When magnetic twist exceeds critical angle, magnetic reconnection is triggered in the current sheet formed inside the loops. Magnetic reconnection channels the accretion flow toward the neutron star, and the magnetosphere is unloaded. (4) This process repeats.

Timing Analysis
We analyzed the time variation of accretion rate falling into the neutron star. The left panel of Figure 4 shows the time variation of the equatorial accretion rate at marginally stable radius for Model-B. The right panel shows the power spectral density of the time variation. Typical frequencies are between 100 Hz and 2 kHz. The arrows in the left panel indicate the visible magnetic reconnection events.

Our numerical results indicate that until the inner edge of the disk reaches the marginally stable radius, the magnetospheric oscillation which accompanies magnetic reconnection produces 100 - 1000 Hz QPOs. Once the inner edge reaches the last stable orbit, magnetic reconnection ceases. In the latter stage, disk oscillation around $3r_g$ to $4r_g$ produces kHz QPOs (e.g., Matsumoto, R., S. Kato, and F. Honma 1989).

SUMMARY
Figure 5 summarizes the numerical results. Inside the corotation radius, magnetic torque removes angular momentum from the disk and accretion proceeds. The magnetic loops connecting the neutron star and the disk expand and prevent the plasma from further infall. When the magnetic loops are twisted more than critical angle, magnetic reconnection takes place inside the loops and channels the accretion flow toward the
neutron star. After the magnetosphere is unloaded via accretion, the process repeats (see Goodson, A. P. & R. M. Winglee 1999 for similar mechanism in protostars). In addition, this mechanism excites the radial disk oscillation. As a result, accretion rate changes quasi-periodically. The typical frequency of this QPO is the Keplerian rotation period at the inner edge of the disk, which corresponds to 100 - 1000 Hz when the inner edge is located between $3 r_g$ and $20 r_g$. When the disk inner edge reaches the last stable orbit, the oscillation frequency of the disk saturates around 1 - 2 kHz. This scenario can explain both the positive correlation of X-ray count rate and QPO frequency and kHz QPOs observed by RXTE.

By extending our simulations to 3-D, we will be able to reproduce the two peaks of QPOs conventionally explained by beat-frequency interpretation. Furthermore, we predict that QPO sources inevitably accompany X-ray flares by magnetic reconnection and bipolar outflows of hot X-ray emitting plasma similar to the optical jets in protostars.

ACKNOWLEDGMENTS
Numerical computations were carried out on VPP300/16R and VX/4R at the Astronomical Data Analysis Center of the National Astronomical Observatory, Japan. This work is supported by Japan Science and Technology Corporation (ACT-JST).

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Fig. 4. (a) Time variation of accretion rate for model-B. Simulation time $t_{\text{sim}}$ is converted to real time by $t = 1.4 \times 10^{-5} (M/1.4M_\odot)t_{\text{sim}}$ sec. Arrows indicate visible magnetic reconnection events. (b) Power Spectral Density (PSD) of accretion rate at $(r, z)=(3r_g, 0)$.

Fig. 5. Mass accretion scenario with disk-magnetosphere interaction: CR and MSR stand for the corotation radius and marginally stable radius, respectively. (a) Inside the corotation radius, the disk material loses angular momentum and accretes. (b) The accreted matter accumulates around the boundary between the magnetosphere and the disk. The magnetic loops connecting the neutron star and the disk are twisted by the rotating plasma. (c) When the magnetic twist exceeds critical angle, magnetic reconnection channels the accretion flow onto the magnetic poles of the neutron star and the magnetosphere is unloaded via accretion. The process (a) - (c) repeats. (d) When the magnetosphere is collapsed, the radial disk oscillation modulates the accretion flow.