Minimal String Unification and Hidden Sector in $Z_8$ Orbifold Models

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Abstract

We study the minimal supersymmetric standard model derived from the $Z_8$ orbifold models and its hidden sectors. We use a target-space duality anomaly cancellation so as to investigate hidden sectors consistent with the MSSM unification. For the allowed hidden sectors, we estimate the running gauge coupling constants making use of threshold corrections due to the higher massive modes. The calculation is important from the viewpoint of gaugino condensations, which is one of the most promising mechanism to break the supersymmetry.
Superstring theories are only candidates for unified theories of all the known interactions. To contact the ‘measurable’ world, we have to derive in a low energy limit the standard model including recent LEP measurements, which show that all gauge coupling constants of the model are unified simultaneously at $M_{\text{GUT}} = 10^{16}\text{GeV}$ within the framework of the minimal supersymmetric standard model (MSSM) [1-4]. Much work has been devoted to obtain the MSSM as string massless spectra. The string theory implies that all coupling are identical at a string scale $M_{\text{string}} = 5.27 \times g_{\text{string}} \times 10^{17}\text{GeV}$ [5, 6], where $g_{\text{string}} \simeq 1/\sqrt{2}$ is the universal string coupling constant. This difference between $M_{\text{GUT}}$ and $M_{\text{string}}$ seems to reject the possibility of a minimal superstring model which has the same matter fields as the MSSM.

However this situation could change by threshold corrections due to higher massive modes. Threshold corrections have been calculated in the case of the orbifold models [5-8]. A target-space duality symmetry [11] plays an important role in the calculation and becomes anomalous by loop effects. This anomaly can be cancelled by the Green-Schwarz mechanism [11] and threshold effects due to towers of massive modes. This anomaly cancellation and the unification of the $SU(3)$, $SU(2)$ and $U(1)_Y$ gauge coupling constants were investigated systematically in ref.[12], which shows that all the $Z_N$ orbifold models except $Z_6$-II and $Z_8$-I have no candidate for the minimal superstring unification. Within the framework of the $Z_8$-I orbifold model, explicit search for the minimal string model was studied in ref.[13].

In addition, effective field theories derived from the superstring theories have another problem on a mechanism of the SUSY-breaking. Some realistic SUSY-breaking is expected to occur in a hidden sector. Ignorance about the hidden sector also makes it difficult to search realistic models. A gaugino condensation mechanism is one of the promising candidates for the realistic SUSY-breaking with a hierarchy [14-23]. The scale of the condensation $M_{\text{COND}}$ and that of the observable SUSY-breaking $M_{\text{SUSY}}$ are related as $M_{\text{SUSY}} \simeq M_{\text{COND}}^3/M_P^2$, where $M_P$ is the Planck scale. In order to lead to the SUSY-breaking at $1\text{TeV}$, the condensation must occur near by $10^{13}\text{GeV}$. Therefore it is important to study coupling constants of the hidden gauge groups around the scale, although we have never understood the condensation mechanism.

In this paper, using the $Z_8$-I orbifold model we investigate all the possible minimal string models which have the $SU(3) \times SU(2) \times U(1)_Y$ gauge group, three generations, two Higgs particles, their superpartners and no extra matter except singlets as the observable sector. Further we study the unification of the gauge coupling constants
of $SU(3)$ and $SU(2)$ among the models. We have an ambiguity on a normalization of the $U(1)_Y$ charges. Hereafter we do not discuss on the $U(1)_Y$ group. Next we investigate the hidden sectors consistent with the above observable sector from the viewpoint of the duality anomaly cancellation and analyze values of their coupling constants at $10^{13.0}\text{GeV}$ through renormalization group equations including threshold corrections.

Here we study the $Z_8$-I orbifold models [24, 25]. The 6-dim orbifold is obtained through a division of $\mathbb{R}^6$ by space group elements $(\theta, e_b)$, where $e_b$ are vectors spanning an $SO(9) \times SO(5)$ lattice and $\theta$ is an automorphism of the lattice. The twist $\theta$ has eigenvalues $\exp[2\pi i(1, 2, -3)/8]$ in a complex basis $(X_i, \tilde{X}_i)$ $(i = 1, 2, 3)$. The orbifold models consist of the string on the 4-dim space-time and the orbifold, its right-moving superpartner (RNS string) and a left-moving $E_8 \times E'_8$ gauge part, whose momenta $P^I$ $(I = 1 \sim 16)$ span an $E_8 \times E'_8$ lattice. When we bosonize the RNS part, their momenta span an $SO(10)$ lattice. The twist $\theta$ is embedded into the $SO(10)$ and $E_8 \times E'_8$ lattices in terms of shifts $v^t$ $(t = 1 \sim 5)$ and $V^I$ $(I = 1 \sim 16)$, respectively. The shift $v^t$ is obtained as $v^t = (1, 2, -3, 0, 0)/8$ and the shift $V^I$ should satisfy a condition: $8V^I = 0$ (mod $E_8 \times E'_8$ lattice). All possible shifts $V^I$ are shown explicitly in ref.[26]. Also the vector $e_b$ is embedded into the $E_8 \times E'_8$ lattice by a Wilson line $a^I_{e_b}$, whose characteristics depend on the structure of the orbifold. Refs.[27, 28] show that the $Z_8$-I orbifold has two independent Wilson lines with order two.

Closed strings on the orbifold are classified into untwisted strings and twisted ones. Gauge bosons belong to the untwisted sector and their momenta $P^I$ satisfy $P^IV^I =$integer and $P^Ia^I_{e_b} =$integer. Massless matter fields of the untwisted sector have the momenta $P^I$ satisfying $8P^IV^I =$1, 2, 5 (mod 8) and $P^Ia^I_{e_b} =$integer. The other untwisted states satisfying $P^Ia^I_{e_b} =$integer correspond to antimatter fields. We can find $E_N$ ($N = 6, 7, 8$), $SO(2N)$ ($N \leq 8$) and $SU(N)$ ($N \leq 8$) as the gauge subgroups through the shift $V^I$ and the Wilson lines $a^I_{e_b}$ in the $Z_8$-I orbifold models [29]. Further we study in detail combinations of the shift $V^I$ and the Wilson lines $a^I_{e_b}$ leading to $SU(3) \times SU(2) \times U(1)^5$ as the observable gauge group. We assume that some Higgs mechanism breaks undesirable $U(1)$ symmetries including $U(1)$’s of the hidden sector except $U(1)_Y$. An explicit analysis on all the possible shifts and Wilson lines shows the $Z_8$-I orbifold model with the observable gauge group have the number of the $(3, 2)$ untwisted matter fields associated with the $i$-th plane, $N^i_{(3, 2)}$ as follows,

$$N^i_{(3, 2)} \leq (1, 1, 0),$$  \hspace{1cm} (1)
where the underline represents any permutation of the elements and $N_{(3,2)}^i \leq (a, b, c)$ implies $N_{(3,2)}^1 \leq a$, $N_{(3,2)}^2 \leq b$ and $N_{(3,2)}^3 \leq c$ simultaneously. Similarly we obtain

$$N_{(3,1)}^i \leq (2, 1, 1) \text{ or } (2, 0, 2),$$



$$N_{(1,2)}^i \leq (2, 0, 1), \ (2, 1, 0), \ (1, 2, 1) \text{ or } (2, 0, 2),$$



where $N_{(3,1)}$ and $N_{(1,2)}$ represent the number of the (3, 1) and (1, 2) untwisted matters.

For the twisted sector, we can find matter fields in $\theta^m$-twisted states ($m = 1, 2, 4, 5$). Massless matter fields of the twisted sector satisfy the following condition:

$$h_{KM} + N_{OSC} + c_m - 1 = 0,$$

where $h_{KM}$ is a conformal dimension of the $E_8 \times E_8'$ gauge part, $N_{OSC}$ is a number operator and $c_m$ is obtained as

$$c_m = \frac{1}{2} \sum_{t=1}^3 \left( |mv^t| - \text{Int}(|mv^t|) \right) \left( 1 - |mv^t| + \text{Int}(|mv^t|) \right),$$

where Int($a$) represents an integer part of $a$. A representation $R$ of the group $G$ contributes to the conformal dimension as

$$h_{KM} = \frac{C(R)}{C(G) + k},$$

where $k$ is a level of a Kac-Moody algebra corresponding to $G$ and $C(R)$ ($C(G)$) is a quadratic Casimir of the $R$ (adjoint) representation, e.g. $C(G) = N$ for $SU(N)$. Here and hereafter we restrict ourselves to the case where $k=1$. For example, a (3,2) representation of $SU(3) \times SU(2)$ has the conformal dimension $h_{KM} = 7/12$. The (3,2) matter fields can be obtained from states oscillated by $\partial X_i$ with $N_{OSC} = 1/8$ in the $\theta$- and $\theta^5$-twisted sectors, as well as non-oscillated states. Similarly matter fields with $N$-dim fundamental representations of $SU(N)$ ($N = 4 \sim 8$) can be obtained from states with $N_{OSC} = 0, 1/8, 2/8$ in the $\theta$- and $\theta^5$-twisted sectors and $N_{OSC} = 0, 2/8$ in the $\theta^2$-twisted sector. However we can not find the above matter fields in the oscillated states of the $\theta^4$-twisted sector. Note that the $\theta$-twisted states with $N_{OSC} = 2/8$ are created by two oscillators $(\partial X_1)^2$ with $N_{OSC} = 1/8$ and an oscillator $\partial X_2$ with $N_{OSC} = 2/8$. For (3,1) and (1,2) representations of $SU(3) \times SU(2)$, oscillators corresponding to $N_{OSC} = 3/8$ in the $\theta$- and $\theta^5$-twisted sectors are allowed in addition to the above range of $N_{OSC}$ for the $N$-dim fundamental representations of $SU(N)$ ($N = 4 \sim 8$).
In general the orbifold models have the duality symmetry. The effective field theories derived from the string models are invariant under the following \(SL(2, \mathbb{Z})\) transformation of a modulus \(T_i\) \((i = 1, 2, 3)\) associated with the \(i\)-th plane:

\[
T_i \rightarrow \frac{a_i T_i - i b_i}{ic_i T_i + d_i},
\]

(6)

with \(a_i, b_i, c_i, d_i \in \mathbb{Z}\) and \(a_i d_i - b_i c_i = 1\). Under the duality the chiral matter fields \(A_\alpha\) transform as follows,

\[
A_\alpha \rightarrow A_\alpha \prod_{i=1}^{3} (i c_i T_i + d_i)^{n^i_\alpha},
\]

(7)

where \(n^i_\alpha\) is called a modular weight \([29, 12, 30]\). The untwisted matter fields associated with the \(i\)-th plane have \(n^i = -\delta^i_0\). The \(\theta^1\), \(\theta^2\), \(\theta^4\)- and \(\theta^5\)-twisted sectors without oscillators have \(n^i = (-7, -6, -3)/8\), \((-6, -4, -6)/8\), \((-4, 0, -4)/8\) and \((-3, -6, -7)/8\), respectively. An oscillator \(\partial X_i\) reduces the corresponding elements of the modular weight by one and the oscillatory \(\partial \tilde{X}_i\) contributes oppositely. For example the matter fields with the \(N\)-dim fundamental representation of \(SU(N)\) \((N = 4 \sim 8)\) can possess the following modular weights:

\[
n^i = (-1, 0, 0), \ (-7, -6, -3)/8, \ (-15, -6, -3)/8, \ (-23, -6, -3)/8,
\]

\[
(-7, -14, -3)/8, \ (-6, -4, -6)/8, \ (-14, -6, -6)/8, \ (-4, 0, -4)/8.
\]

(8)

Similarly we can obtain allowed modular weights for the (3,2), \((\overline{3}, 1)\) and (1,2) matter fields of \(SU(3) \times SU(2)\), taking into account the possible values of \(N_{osc}\).

Loop effects make the duality symmetry anomalous. Its anomaly coefficient for the \(i\)-th plane is obtained as

\[
b^i_a = -C(G_a) + \sum \mathcal{T}(\mathcal{R})(1 + 2n^i_R),
\]

(9)

where \(a\) represents a suffix for a gauge group and \(\mathcal{T}(\mathcal{R})\) is an index given as \(\mathcal{T}(\mathcal{R}) = C(\mathcal{R})\dim(\mathcal{R})/\dim(G)\), e.g., \(\mathcal{T}(\mathcal{R}) = 1/2\) for the \(N\)-dim fundamental representation of \(SU(N)\). This anomaly can be cancelled by two ways. One is the Green-Schwarz mechanism, which is independent of the gauge groups. The other is due to the threshold effects. Since only the \(N = 2\) supermultiplets contribute to these effects, the threshold corrections depend on the modulus whose plane is unrotated under some \(\theta^m\) twist. Thus for the \(\mathbb{Z}_8\)-I orbifold the corrections depend on \(T_2\). Further for the first and third planes the duality anomaly should be cancelled only by the Green-Schwarz
mechanism. Therefore we obtain necessary conditions for the anomaly cancellation as
\[ b_{SU(3)}^i = b_{SU(2)}^i = b_a^i, \quad (i = 1, 3), \]  
(10)
where \( b_a^i \) implies an anomaly coefficient of some hidden sector. We assign all the possible modular weights to the three (3,2), six (3,1) and five (1,2) matter fields of \( SU(3) \times SU(2) \) using eqs.(1) and (2), and analyze the anomaly cancellation condition for the \( SU(3) \) and \( SU(2) \) parts of eq.(10). We can find 2946 combinations of \( (b_{SU(3)}^i, b_{SU(2)}^i) \) satisfying eq. (10), up to the values of \( b_{SU(3)}^2 \) and \( b_{SU(2)}^2 \).

Now we consider running gauge coupling constants including the threshold corrections. The one-loop coupling constants \( g_a(\mu) \) at a scale \( \mu \) are obtained as
\[ \frac{1}{g_a^2(\mu)} = \frac{1}{g_{\text{string}}^2} + \frac{b_a}{16\pi^2} \log \frac{M_{\text{string}}^2}{\mu^2} - \frac{1}{16\pi^2}(b_a^2 - \delta_{GS}^2)\log[(T_2 + \overline{T}_2)|\eta(T_2)|^4], \]  
(11)
where \( \eta(T) \) is the Dedekind function, \( \delta_{GS}^2 \) is a gauge group independent GS coefficient and \( b_a \) are \( N = 1 \beta \)-function coefficients, i.e., \( b_{SU(3)} = -3 \) and \( b_{SU(2)} = 1 \). From this renormalization group flow, we can derive a unified scale \( M_{a-b} \) of two couplings, \( g_a \) and \( g_b \) through the following relation:
\[ \log \frac{M_{a-b}}{M_{\text{string}}} = \frac{b_b^2 - b_a^2}{2(b_a - b_b)}\log[(T_2 + \overline{T}_2)|\eta(T_2)|^4]. \]  
(12)
Here we discuss the unification of the \( SU(3) \) and \( SU(2) \) gauge coupling constants. They should be unified at \( M_{\text{GUT}} \) as shown in the measurements. Note that \( [(T_2 + \overline{T}_2)|\eta(T_2)|^4] \) is always less than one. To derive \( M_{\text{GUT}} < M_{\text{string}} \) from eq.(12), the anomaly coefficients must satisfy the following condition:
\[ b_{SU(3)}^2 > b_{SU(2)}^2. \]  
(13)
We obtain the range of \( \Delta b^2 = b_{SU(3)}^2 - b_{SU(2)}^2 \) for the assignments of the MSSM matter fields allowed by eqs.(10) and (13) as \( 1/4 \leq \Delta b^2 \leq 17/2 \). We use \( M_{\text{GUT}} = 10^{16.0}\text{GeV} \) and \( M_{\text{string}} = 5.27/\sqrt{2} \times 10^{17.0}\text{GeV} \) so as to get a minimum value \( ReT_2 = 5.5 \) in the minimal string unification derived from the \( Z_8 \)-I orbifold models.

Now we investigate hidden sectors consistent with the observable sector discussed above, i.e., the possible assignments for the observable MSSM unification satisfying eqs.(10) and (13). At first we consider the hidden gauge subgroups which do not have matter fields with non-trivial representations. In the case the anomaly coefficients \( b_a^i \) are determined by \( C(G_a) \), i.e., \( b_a^i = -C(G_a) \), where \( C(E_8) = 30 \), \( C(E_7) = 18 \),
$C(E_6) = 12$ and $C(SO(2N)) = 2N - 2$. The Casimir $C(G)$ for all the possible subgroups are listed in the first column of Table 1. For these values of $b_a^n$ we study whether the hidden sectors satisfies the condition (10) with the observable MSSM unification. The results are shown in the second column of Table 1. In this case the groups with $C(G) \geq 14$ are ruled out as the hidden sector of the MSSM. Then we obtain the range of $\Delta b^2$ for the allowed combinations of the observable and hidden sectors, and the ranges are found in the third column of Table 1. The column also shows the corresponding values of $\Re T_2$. It seems natural that the value of $\Re T_2$ is of order one. That requires larger values of $\Delta b^2$. Hereafter we discuss the case where $\Delta b^2 > 3$. Actually the value $\Delta b^2 = 3$ corresponds to $\Re T_2 \sim 12$.

Next we study the running gauge coupling constants of the hidden sector allowed at the previous stage. Making use of eq.(12) we can easily get a scale where the couplings constants of the hidden and the observable $SU(2)$ (or $SU(3)$) gauge groups unify. Further we use eq.(11) so as to derive the hidden gauge coupling constant at $10^4\text{GeV}$ as

$$
\alpha_a^{-1}(t) = \alpha_a^{-1} - \frac{b_a}{2\pi} \log \frac{10^4}{M_{\text{GUT}}} + \frac{1}{2\pi} \left\{ b_a - 1 + \frac{4(b_a^2 - b_{SU(2)}^2)}{\Delta b^2} \right\} \log \frac{M_{\text{string}}}{M_{\text{GUT}}},
$$

where $\alpha_a = \frac{g_a^2}{4\pi}$ and $\alpha_{\text{GUT}}$ is obtained by the unified coupling constant $g_{\text{GUT}}$ of $SU(3)$ and $SU(2)$ as $\alpha_{\text{GUT}} = \frac{g_{\text{GUT}}^2}{4\pi}$. Now we calculate $\alpha_a(t = 13.0)$ of the hidden sector which is consistent with the minimal string unification from the viewpoint of the duality anomaly cancellation. We use $\alpha_{\text{GUT}}^{-1} = 25.7$ at $M_{\text{GUT}} = 10^{16.0}\text{GeV}$ and $M_{\text{string}} = 5.27/\sqrt{2} \times 10^{17.0}\text{GeV}$ to estimate $\alpha_a^{-1}$ at $10^{13.0}\text{GeV}$. The results are found in Table 1, where the fourth column shows the allowed ranges of $\alpha_a^{-1}(t = 13.0)$. If the value of $\alpha_a$ blows up at a higher energy than $10^{13}\text{GeV}$, the fifth column of Table 1 shows the ranges of the scales $M_{\text{blow}}$ where $\alpha_a^{-1} = 0$. The gaugino condensation of $SU(3)$ or $SU(4)$ without non-trivial matter fields might happen around $10^{13}\text{GeV}$, while the larger gauge groups might lead to the condensation at a higher scale than $10^{13}\text{GeV}$.

Now we study the hidden sector of the gauge groups which have the matter fields with the non-trivial representations. Here we restrict ourselves to the $SU(N)'$ gauge groups. First of all, we consider the hidden sectors which has the gauge subgroup $SU(2)'$ and one matter field with the doublet representation. We assign all the possible modular weights to the matter field and calculate the values of $b_{SU(2)}^a$, satisfying eq.(10). In a way similar to the above discussion we estimate the range of the $\alpha_{SU(2)'}^{-1}$
We obtain $14.9 \leq \alpha_{SU(2)'}^{-1} \leq 21.7$ for the hidden $SU(2)'$ coupling constants consistent with the MSSM as the observable sector. If we consider the $SU(2)'$ gauge group with two doublets, we derive from the consistency with the MSSM the values of $\alpha_{SU(2)'}^{-1}$ as $12.5 \leq \alpha_{SU(2)'}^{-1} \leq 32.2$. When we discuss the $SU(N)'$ ($N > 2$) group with matter fields of non-trivial representations, we have to take into account a gauge anomaly. The cancellation of the anomaly requires that there appear the same number of the $N$-dim fundamental representations as their conjugate ones. For example we study the hidden sector which has the $SU(3)'$ gauge group and a pair of $3$ and $\overline{3}$ matter fields. We obtain $6.7 \leq \alpha_{SU(3)'}^{-1} \leq 26.5$ at $10^{13.0}$GeV for gauge coupling constant of the hidden sector consistent with the observable MSSM unification. Similarly we estimate other types of the hidden sectors consistent with the observable MSSM unification using eq.(8). These analyses for the $SU(N)'$ ($N = 4, 5, 6$) gauge subgroups and four or less pairs of matter fields with $N$-dim fundamental and its conjugate representations are found in Table 2. If the gauge coupling constants blow up in some assignments of the modular weights to the matter fields, the table shows the maximum blow-up scales $M_{\text{blow}}$ (GeV) in parentheses instead of the minimum values of $\alpha^{-1}$. We need several matter fields with non-trivial representations unless the coupling constants of $SU(7)'$ and $SU(8)'$ blow up at higher scale than $10^{13}$GeV. The $SU(7)'$ and $SU(8)'$ groups require at least the four and six pairs of $N$-dim fundamental and its conjugate representations, respectively. Actually we have a maximum of $\alpha_{SU(7)'}^{-1}(t = 13.0)$ equal to 1.4 in the hidden sector with $SU(7)'$ and the above four pairs, while the model shows a maximum blow-up scale $M_{\text{blow}}$ is equal to $10^{15.2}$GeV.

At last we discuss the case where gauginos of two gauge subgroups condensate. Refs.\cite{19, 23} show that two or more condensations lead to more realistic SUSY-breaking than the unique condensation. Here we consider the hidden sectors which have the $SU(4)'$ (or $SU(3)'$) gauge group without non-trivial representation matter and the $SU(N)'$ gauge group with one or two pairs of $N$-dim fundamental and its conjugate representations, and we investigate their consistency with the MSSM unification. It is obvious that there never appear simultaneously different gauge groups without nontrivial matter. For example we consider the hidden sector which has the $SU(4)'$ gauge group without nontrivial matter and $SU(4)'$ gauge group with a pair of 4 and $\overline{4}$ matter fields. Explicit analysis shows that the above combination is allowed only at $b_{SU(4)'}^2 = 3$ as well as $b_{SU(4)'}^i = 4$ ($i = 1, 3$) for the latter $SU(4)'$. Further in a way similar to the above we obtain the range of $\alpha_{SU(4)'}^{-1}$ for the latter $SU(4)'$ as $6.0 < \alpha_{SU(4)'}^{-1} < 11.5$ at $10^{13.0}$GeV. Of course the $SU(4)'$ gauge subgroup without
matter gives the same values of $\alpha^{-1}$ as in Table 1. The results for the allowed hidden sectors are shown in Table 3, where the second and third rows show $\alpha^{-1}(t = 13.0)$ of the hidden sectors consistent with $SU(3)'$ and $SU(4)'$ hidden subgroups which have no nontrivial matter. In this case, the given values of $b_{SU(N)'}^2$ are restricted tightly and the corresponding values are written in the second and third rows. The condition due to the duality anomaly cancellation forbids the other combinations of the $SU(4)'$ (or $SU(3)'$) group without matter and the $SU(N)'$ ($N \leq 8$) group with the two or less pairs of matter fields which have the $N$-dim fundamental and its conjugate representations. For the allowed combinations, the ranges of $\alpha^{-1}$ become narrow compared with those in Table 2. Therefore imposition of the two or more gaugino condensations avails to constrain the hidden sector.

To sum up, we have studied all the possible MSSM’s from the $Z_8$-I orbifold models and their hidden sectors. The orbifold models rule out the gauge groups with Casimir $C(G) \geq 14$ and no nontrivial matter field as the hidden sector of the observable MSSM. Further we have estimated the gauge coupling constants of the hidden sectors using the threshold corrections. It is interesting to apply the above analyses to the case of the $Z_6$-II orbifold model, which is another promising model for the MSSM among the $Z_N$ orbifold models. Extension to $Z_N \times Z_M$ orbifold models is more complicated, since three planes of the orbifolds are unrotated under some twist. However it is important to discuss the orbifold models from the viewpoint of the above approach.
Note added

After almost completion of this work, the author received by S. Stieberger a paper [31], which show threshold corrections are modified in the case where orbifolds are not decomposed into three 2-dim orbifolds, although they do not discuss the $Z_8$-I orbifold models. Even if we need a modification on the threshold corrections, the above discussions do not change except replacement of the value of $ReT_2$ leading to $M_{\text{GUT}}$.

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Table 1. hidden sectors without matter

The second column shows whether or not each gauge group is allowed as the hidden sector of the MSSM unification from the viewpoint of the duality anomaly cancellation. + and – imply allowed groups and forbidden ones, respectively.

| $C(G)$ | Anom. canc. | $\Delta b'^2$ ($ReT_2$) | $\alpha^{-1}$ | $M_{\text{blow}}$ (GeV) |
|--------|-------------|-------------------------|----------------|------------------------|
| 2      | +           | $8 \sim 1/4$ (5.8 $\sim$ 115) | 13.5 $\sim$ 17.8 | —                      |
| 3      | +           | $8 \sim 1/4$ (5.8 $\sim$ 115) | 8.1 $\sim$ 12.0 | —                      |
| 4      | +           | $7 \sim 1/4$ (6.4 $\sim$ 115) | 2.0 $\sim$ 6.2  | —                      |
| 5      | +           | $7 \sim 1/4$ (6.4 $\sim$ 115) | —               | $10^{13.0} \sim 10^{13.7}$ |
| 6      | +           | $13/2 \sim 1/4$ (6.7 $\sim$ 115) | —               | $10^{14.8} \sim 10^{14.9}$ |
| 7      | +           | $13/2 \sim 1/4$ (6.7 $\sim$ 115) | —               | $10^{14.4} \sim 10^{14.9}$ |
| 8      | +           | $9/2 \sim 1/4$ (8.9 $\sim$ 115) | —               | $10^{14.9} \sim 10^{15.2}$ |
| 10     | +           | $7/2 \sim 1/4$ (11 $\sim$ 115) | —               | $10^{15.7}$            |
| 12     | +           | $3/2 \sim 1/4$ (21 $\sim$ 115) | —               | —                      |
| 14     | —           | —                        | —               | —                      |
| 18     | —           | —                        | —               | —                      |
| 30     | —           | —                        | —               | —                      |

Table 2. hidden sectors with matter fields

The second row shows $\alpha_{SU(N)}^{-1}(t = 13.0)$ of the corresponding hidden gauge subgroups $SU(N)'$ ($N = 4, 5, 6$) with one pair of $N$-dim fundamental and its conjugate representations. Similarly the third, fourth and fifth rows show $\alpha_{SU(N)}^{-1}(t = 13.0)$ of the subgroups with two, three and four pairs of the above representations.

| Matter | $SU(4)$ | $SU(5)$ | $SU(6)$ |
|--------|---------|---------|---------|
| 1 pair | 3.3 $\sim$ 11.5 | $(10^{13.5})$ $\sim$ 5.8 | $(10^{14.2})$ $\sim$ 0.1 |
| 2 pairs| 3.6 $\sim$ 13.8 | $(10^{13.3})$ $\sim$ 8.1 | $(10^{13.9})$ $\sim$ 2.4 |
| 3 pairs| 3.2 $\sim$ 16.2 | $(10^{13.4})$ $\sim$ 10.5 | $(10^{14.2})$ $\sim$ 4.7 |
| 4 pairs| 3.5 $\sim$ 18.5 | $(10^{13.4})$ $\sim$ 12.8 | $(10^{14.4})$ $\sim$ 7.1 |
Table 3. hidden sectors with two gaugino condensations

In the first row \((N, M)\) represents the gauge group \(SU(N)'\) with \(M\) pairs of \(N\)-dim fundamental and its conjugate representations as the hidden sectors.

| \((N, M)\) | \((4,1)\) | \((5,1)\) | \((4,2)\) | \((5,2)\) | \((6,2)\) |
|------------|----------|----------|----------|----------|----------|
| \(SU(3)\) | 4.6 \(\sim\) 10.1 | — | 7.0 \(\sim\) 12.5 | 0.0 \(\sim\) 5.4 | — |
| \(b_{SU(N)}^2\) | \((-5)\) | — | \((-4)\) | \((-7)\) | — |
| \(SU(4)\) | 6.0 \(\sim\) 11.5 | \((10^{13.2}) \sim 4.4\) | 5.0 \(\sim\) 13.8 | 1.3 \(\sim\) 6.8 | \((10^{13.8}) \sim 0.4\) |
| \(b_{SU(N)}^2\) | \((-3)\) | \((-6)\) | \((-2,-4,-5,-6,-7)\) | \((-5)\) | \((-7)\) |