Low–Energy $M1$ and $E3$ excitations in the proton–rich Kr–Zr region

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Abstract

Low-energy intrinsic $K_z^\pi=1^+$, $0^-$, $1^-$, $2^-$, and $3^-$ states in the even-even proton-rich Sr, Kr, and Zr nuclei are investigated using the quasiparticle random phase approximation. In the $Z \simeq N$ nuclei the lowest-lying $1^+$ states are found to carry unusually large $B(M1)$ strength. It is demonstrated that, unlike in the heavier nuclei, the octupole collectivity in the light zirconium region is small and, thus, is not directly correlated with the systematics of the lowest negative parity states.

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I. INTRODUCTION

It has been shown experimentally that shape coexistence, large deformations, the presence of well-deformed intruder orbitals, quenching of pairing correlations, low-lying octupole states, and dramatic shape changes induced by rotation are quite common phenomena in the zirconium region (Z\(\approx\)N\(\approx\)40). The microscopic reason for such a strong variation of collective properties is the low single-particle level density in these medium-mass nuclei. Because of spectacular shape effects, relatively small size, and high collectivity, the nuclei from the A\(\approx\)80 mass region have become favorite testing grounds for various theoretical approaches. Calculations based on the mean-field approach applied to nuclei in the light-Zr region suggest an interpretation of experimental data in terms of well-deformed prolate shapes, weakly-deformed oblate shapes, and spherical (shell-model) configurations [1,2].

There exist a number of mean-field calculations for the light-Zr region [for references, see review [4]]. In most cases calculations give similar equilibrium deformations, but they differ in their predictions for excitation energies of shape-coexisting states. Best examples of the ground-state shape isomerism in nuclei in the light-Zr region are the Ge-Kr isotopes with \(A\approx70\). Calculations suggest the interpretation in terms of two competing configurations: one at an oblate shape, and the other at a prolate shape. Oblate ground states are predicted for Ge- and Se-isotopes and for most Kr-isotopes. For light Sr-isotopes the prolate configuration lies lower in energy. Because of the mutual interaction [of the order of a few hundred keV] the prolate and oblate bands are strongly disturbed in the low-spin region.

The single particle diagram representative of the discussed nuclei is shown in Fig. 1. In the A\(\approx\)80 region both protons and neutrons lie in the same \((p_{1/2}, p_{3/2}, f_{5/2}, g_{9/2})\) shell. For \(T_z=0\) systems, the proton and neutron shell corrections add coherently and, consequently, dramatic shape effects are expected. A beautiful experimental signature of large prolate deformations in the A\(\approx\)80 region, attributed to the large single-particle gaps at Z,N=38 and 40, was observation of very collective rotational bands in neutron-deficient Sr and Zr isotopes [3].

The investigation of the medium-mass N=Z nuclei has been the proprietary niche of groups who made investigations using the Daresbury Recoil Separator. Pioneering works from Daresbury include the spectroscopy of \(^{64}\)Ge, \(^{68}\)Se, \(^{72}\)Kr, \(^{76}\)Sr, \(^{80}\)Zr, and \(^{84}\)Mo [see ref. [4]]. These studies confirmed earlier theoretical predictions of shape transition from strongly oblate shapes in \(^{68}\)Se and \(^{72}\)Kr to strongly prolate shapes in \(^{76}\)Sr, and \(^{80}\)Zr (actually, \(^{76}\)Sr and \(^{80}\)Zr are, according to calculations, very deformed, with the ground state deformation around \(\beta_2=0.4\)). The nucleus \(^{84}\)Mo is the heaviest Z=N system known so far.

Spectroscopy in the light-Zr region will certainly become one of the main arenas of investigations around the proton drip line. The physics of exotic nuclei with \(T_z<0\) is one of the fastest developing subjects in nuclear physics, thanks to exotic (radioactive) ion beam (RIB) facilities currently under construction in Europe, U.S.A., and Japan. In particular, the combination of RIB and the new-generation multidetector arrays should open up many new avenues of exploration [10].

The main motivation of this paper is to make predictions for low-energy collective \(M1\) and \(E3\) excitations around \(^{76}\)Sr. Since the \(M1\) collectivity of low-lying \(1^+\) states increases with deformation (though the energies of those states may increase), it is anticipated that in some well deformed nuclei in the A\(\approx\)80 mass region the strong magnetic dipole strength
should lie low in energy. The existence of collective octupole states in this region is a long-standing question. The low-lying negative-parity states, often interpreted as octupole vibrations, can be of a single-particle character [1]. To shed some light on both issues we performed calculations based on the quasiparticle Random Phase Approximation (RPA). We hope, that those predictions will stimulate experimental investigations of medium-mass nuclei around the N=Z line.

II. DEFORMATIONS AND PAIRING CORRELATIONS IN THE A~80 MASS REGION

Calculations of equilibrium deformations of A~80 isotopes were previously performed within the Woods-Saxon-Strutinsky model [12]. In this work, new calculations have been carried out using the same single-particle model but the Yukawa-plus-exponential mass formula of ref. [13]. The particle-particle interaction was approximated by the state-independent monopole-pairing Hamiltonian. The pairing energy was computed using the approximate particle number projection in the Lipkin-Nogami version. The pairing strengths and the average pairing energy were taken according to ref. [14]. The calculated equilibrium deformations for selected Kr, Sr, and Zr isotopes are shown in Table I. It is seen that the deformed → spherical shape transition is expected to occur around N~44. Worth noting are very large equilibrium β2 deformations (~0.4) of the lightest Kr, Sr, and Zr isotopes.

In several nuclei around 82Sr highly-deformed and superdeformed bands (β2>0.4) have been predicted to become yrast at high spin [1,2,15,16]. For example, in 82Sr well deformed nearly-prolate bands involving h11/2 neutrons are expected to become yrast at I>32h. Experimentally, a weak ridge-valley structure with a width of ΔEγ≈150 keV has been seen in the Eγ−Eγ correlation map [17]. This ridge corresponds to β2~0.5 for a deformed rigid rotor. However, no discrete band that could be associated with this ridge-valley was identified so far. Theoretically, the superdeformed band in 82Sr is expected [1] to have deformation β2~0.45, see Table I.

The most important interaction, beyond the single-particle deformed mean field, is the short-ranged pairing interaction. This force is often approximated by means of a state-independent monopole pairing interaction. The general feature of the pairing interaction is that the pair correlation energy is anticorrelated with the shell correction. A smaller pairing gap results from a smaller density of single-particle levels around the Fermi level, which are available for pair correlation. For deformed A~80 nuclei the weakest pairing is expected around the deformed gaps at N (or Z)=38–42 [3]. A further reduction of pairing can occur in excited configurations, due to blocking.

In the A~80 mass region are several good examples of very regular, rigid rotational bands. Among them there are negative parity bands in 76Kr and 78Kr built upon the first Iπ=3− state at 2258 keV and 2399 keV, respectively. These bands are among the best normally-deformed rotors, with remarkably large and nearly constant moments of inertia, J(1)≈J(2) [18,19]. Theoretically, those bands are associated with two-quasiparticle excitations built upon the proton [431 3/2]⊗[312 3/2] Nilsson orbitals which happen to occur just below the strongly deformed subshell closure at Z=38. [The proton character of those bands was recently confirmed by the g-factor measurement [20].] Another good example is the [312
3/2 band in $^{77}$Rb \textsuperscript{21} or the $[422\ 5/2]$ band in $^{81}$Y \textsuperscript{22} having unusually large moments of inertia. In all those cases the BCS calculations \textsuperscript{2} suggest the dramatic reduction (or collapse) of the static pairing.

Weak pairing has important consequences for the low-energy electromagnetic transitions. Since the $B(M1)$ values involving the ground state of even-even nuclei are proportional to the BCS factor $(u_{\mu}v_{\nu} - v_{\mu}u_{\nu})^2$, weaker pair correlations enhance the low-lying $M1$ strength. For electric transitions, the related BCS factor is $(u_{\mu}v_{\nu} + v_{\mu}u_{\nu})^2$. On the average, pairing correlations enhance the collectivity of the low-lying $E3$ transitions from/to the ground state in the Sr-Zr region (see Sec. \textsuperscript{[15]})

III. MAGNETIC DIPOLE STATES

The deformation dependence of $1^+$ states is a current subject of both experimental \textsuperscript{23,24} and theoretical \textsuperscript{23,28} studies. The low-energy $B(M1)$ strength (defined as the summed strength over a given energy interval, e.g., 2–4 MeV in the rare-earth nuclei) increases with quadrupole deformation as, roughly, $\beta_2^2$. Recently, it was demonstrated in ref. \textsuperscript{28} that the sum of $B(M1)$ values in the region of $E_x<10$ MeV at heavy superdeformed nuclei around $^{152}$Dy and $^{192}$Hg was several times larger than that at normal deformations. The reason for this enhancement is twofold. Firstly, the proton convection current contribution to $B(M1)$ increases with deformation and at strongly deformed shapes becomes comparable to the spin-flip contribution in the low-energy region. Secondly, as discussed in Sec. \textsuperscript{[14]}, the $B(M1)$ strength increases if the pair correlations are weak, i.e., exactly what is expected at sd shapes \textsuperscript{29}.

Since some of the A~80 nuclei are very well deformed in their ground states, their equilibrium deformations exhibit rapid isotopic and isotonic variations, and their pairing correlations are predicted to be weak due to deformed subshell closures (Table I). Because the Kr, Sr, and Zr isotopes have these characteristics, they are ideally suited for investigations of the low-energy $M1$ strength and its deformation dependence. (The lighter and heavier systems, such as Ge, Se, and Mo, are less deformed and $\gamma$-soft.)

The properties of the $K^\pi=1^+$ states have been investigated using the RPA Hamiltonian

$$H_{QRPA} = h_{s.p.} + V_{pair} + V_{FF} + V_{\sigma\sigma},$$

where the single-particle Hamiltonian,

$$h_{s.p.} = \sum_i (\epsilon_i - \lambda) c_i^\dagger c_i$$

is an axially deformed Woods-Saxon Hamiltonian of ref. \textsuperscript{30} [see ref. \textsuperscript{31} for parameters],

$$V_{pair} = -\Delta \sum_i (c_i^\dagger c_i^\dagger + c_i c_i)$$

is the monopole-pairing field, $V_{FF}$ is a long-ranged residual interaction (mainly of quadrupole-quadrupole type), and $V_{\sigma\sigma}$ is the spin-spin residual interaction. In eq. (3.1)

$$V_{FF} = -\frac{1}{2} \sum_{T=0,1} \kappa_T F_T^+ F_T,$$

(3.4)
where the isoscalar and isovector fields $F$ are given by

\[ F_{T=0} = F_n + F_p \quad ; \quad F_{T=1} = F_n - \xi F_p \]  \hspace{1cm} (3.5)

and

\[ F_{\tau} = \frac{1}{i\hbar} \left[ h_{s.p.}^{(\tau)}, j_{+}^{(\tau)} \right], \quad \tau = n, p, \]  \hspace{1cm} (3.6)

while the residual spin-spin interaction is written as

\[ V_{\sigma \sigma} = \frac{1}{2} \sum_{T=0,1} \chi_T S_T^+ S_T, \]  \hspace{1cm} (3.7)

where

\[ S_{T=0} = S_n + S_p \quad ; \quad S_{T=1} = S_n - S_p. \]  \hspace{1cm} (3.8)

The strength of $V_{\sigma \sigma}$ is taken \[32\] as $\chi_0 = \chi_1 = 100/A \text{ MeV}$.

The residual interaction $V_{F,F}$ gives rise to isoscalar and isovector shape oscillations. The isoscalar-coupling constant, $\kappa_0$, is determined by the condition \[33\] that the lowest RPA frequency for the isoscalar mode vanishes, since the lowest-lying mode with $K^\pi = 1^+$ is spurious and corresponds to a uniform rotation of the system. The value of $\xi$ in (3.5) is determined by the requirement \[34\] that the spurious component should be absent in the RPA solutions with non-zero frequencies. We have numerically checked that the summed probability of the spurious component, $|S\rangle \propto j_+ |g.s.\rangle$, remaining in the RPA solutions with non-zero frequency is less than $10^{-6}$.

The isovector coupling constant, $\kappa_1$, is taken from the self-consistency condition for the harmonic oscillator model \[35\], $\kappa_1 = -3.5 \kappa_0$. In RPA calculations we take into account all two-quasiparticle configurations with excitation energies less than 26 MeV, and have checked that the configuration space is sufficiently large so as to include all $M1$ strengths.

As a representative example, results of calculations for Sr isotopes are shown in Fig. 2, which shows the excitation energies of the low-lying $K^\pi = 1^+$ states. The values $B(M1; g.s. \rightarrow 1^+)$ (in $\mu_2^2 N$) are indicated. The upper diagram was obtained by using the standard pairing gaps of Table I. According to Sec. I, pairing correlations in the excited states of Sr-Zr are expected to be seriously quenched. Therefore, we performed a second set of calculations with $\Delta_p$ and $\Delta_n$ reduced by 50% with respect to the standard values. As discussed in refs. \[26,28\], reduced pairing leads to increased collectivity of the low-lying $1^+$ states; as seen in Fig. 2 the $B(M1)$ values calculated in the “weak pairing” variant are approximately twice as large as the $M1$ rates obtained in the “standard pairing” variant.

The best candidate for low-lying enhanced $1^+$ states in the A~80 mass region is the N=Z nucleus $^{76}$Sr. Its ground state is very well deformed due to the coherent superposition of proton and neutron shell effects associated with the deformed gap at the particle number 38. In Fig. 3 we show the $B(M1; g.s. \rightarrow 1^+)$ strengths of the calculated $K^\pi = 1^+$ RPA excitation modes in $^{76}$Sr (at the ground-state deformation), as a function of excitation energy. The upper (lower) diagram corresponds to the standard (weak) pairing variant. The $M1$ strength arising from only the proton convection current (i.e., $g_s=0$) and the $M1$
strength from only the spin part (i.e., \( g_l = 0 \)) are also plotted in Fig. 3. In both pairing variants of calculations, there appears only one low-lying \( 1^+ \) state which has unusually strong \( M_1 \) collectivity. In the “weak pairing” variant this state is predicted at 2.2 MeV and the corresponding \( B(M1; g.s. \to 1^-) \) transition is 2.16 \( \mu_N^2 \). The main components of the wave function of the \( 1^+ \) state in \( ^{76}\text{Sr} \) are the \( \pi (g_9/2)^2 \) and \( \nu (g_9/2)^2 \) excitations involving the two Nilsson orbitals \([431 3/2]\) and \([422 5/2]\). The largest components of the low-lying \( 1^+ \) states in \( ^{76}\text{Sr} \) in the energy range of 4–5 MeV are the \([431 3/2] \otimes [431 1/2] \) (spin-flip) and \([301 3/2] \otimes [310 1/2] \) two-quasiparticle excitations. The main contribution to the peak in the \( M_1 \) distribution seen in the energy range of 7–9 MeV in Fig. 3 comes almost exclusively from the spin-flip \( f_{7/2} \to f_{5/2} \) and \( g_9/2 \to g_{7/2} \) transitions.

The contribution to the \( B(M1) \) strength coming from the unique-parity high-\( j \) excitations, such as \((h_{11/2})^2 \) or \((g_9/2)^2 \), has a simple shell model interpretation (in terms of a single-\( j \) shell) and cannot be viewed as coming from a collective “scissors” mode [see discussion in ref. \([32]\)]. The synthetic orbital scissors state is defined as

\[
| R \rangle = \mathcal{N}^{-1} \left( l_+^{(n)} - \alpha l_+^{(p)} \right) | g.s. \rangle, \tag{3.9}
\]

where \( \mathcal{N} \) is a normalization factor and the parameter \( \alpha \) is determined by the requirement that the mode (3.9) is orthogonal to the spurious reorientation mode \([28,36,37]\), i.e.,

\[
\alpha = \frac{\langle g.s. | j_+^{(n)} l_+^{(n)} | g.s. \rangle}{\langle g.s. | j_+^{(p)} l_+^{(p)} | g.s. \rangle}. \tag{3.10}
\]

The calculations show that for the lowest \( 1^+ \) state in \( ^{76}\text{Sr} \) the overlap between its RPA wave function and the state (3.9) is only about 12\%. Consequently, although this state is predicted to carry an unprecedented \( M1 \) strength, it cannot be given a geometric interpretation of the “scissors” mode. The \( K^\pi=1^+ \) isovector giant quadrupole resonance in \( ^{76}\text{Sr} \) lying at \( E_{\text{ex}} \sim 32 \) MeV carries a significant \( M_1 \) strength (\( \sim 4 \mu_N^2 \)) and contains a major component of the “scissors mode” (around 50\%).

Figures 4 and 5 show the calculated \( 1^+ \) states in Kr and Zr isotopes, respectively. As seen in Figs. 2, 4, and 5 when moving away from \( ^{76}\text{Sr} \), the low-energy \( M1 \) strength becomes more fragmented. Good prospects where to find large \( M1 \) strength at low energies are the well-deformed prolate nuclei \( ^{78}\text{Sr} \) (where the \( 1^+ \) state is built mainly from the \( \pi (431 3/2) \otimes [422 5/2] \) and \( \nu (431 3/2) \otimes [431 7/2] \) two-quasiparticle excitations), \( ^{80}\text{Sr}, ^{80}\text{Zr} \) (\( \pi (422 5/2) \otimes [413 7/2] \) and \( \nu (422 5/2) \otimes [413 7/2] \)), \( ^{82}\text{Zr} \), and \( ^{74}\text{Kr} \). The most promising oblate-shape candidate is the \( N=Z \) nucleus \( ^{72}\text{Kr} \). Similar to \( ^{76}\text{Sr} \), the \( 1^+ \) state in \( ^{72}\text{Kr} \) has a \((g_9/2)^2 \) character. However, in this case the main contribution comes from the high-\( \Omega \) substates, i.e., \( \pi (413 7/2) \otimes [404 9/2] \) and \( \nu (413 7/2) \otimes [404 9/2] \).

As discussed in Sec. \([\text{I}]\), the best prospects for superdeformation in the \( A \sim 80 \) region are in the nuclei around \( ^{82}\text{Sr} \). The calculations performed for superdeformed configuration of \( ^{82}\text{Sr} \) predict two states (around 3 MeV and 4 MeV) that carry a large \( M1 \) strength (see Fig. 2). They can be associated with the \( \pi (431 3/2) \otimes [422 5/2] \), \( \nu (422 5/2) \otimes [431 7/2] \) and \( \nu (541 3/2) \otimes [550 1/2] \) two-quasiparticle excitations.
IV. OCTUPOLE CORRELATIONS

In the light zirconium region octupole correlations can be associated with the $g_{9/2}$ and $p_{3/2}$ subshells. Because of their rather large energy separation and a small number of coupling matrix elements, no pronounced octupole instability is expected. In addition, the small number of active subshells makes the octupole effect more sensitive to quadrupole distortion than in heavier nuclei around $^{146}$Ba or $^{222}$Th [38].

The systematics of the lowest $3^{-}$ excitations in the Zr-region is shown in Fig. 6. It is seen that $E_{3^-}$ tends to decrease when approaching the nucleus $^{76}$Sr. On the other hand, the shell correction calculations [11,39,40] predict octupole softness only in the transitional isotopes of Zn-Se with $N \leq 36$. Is the presence of low-lying negative-parity state always a good fingerprint of octupole collectivity? The answer to this question is negative. There are many nuclei that possess relatively high-lying negative parity excitations but still are considered as good examples of systems with strong octupole correlations. In fact, the systematics of experimental $B(E3)$ values in the light-Zr region [11,41] indicates that no correlation can be found between the behavior of the lowest negative-parity states shown in Fig. 6 and the $B(E3; g.s. \rightarrow 3^-)$ strength.

According to the energy systematics presented in Fig. 6, the lowest negative-parity states are observed in strongly deformed nuclei with particle number (N or Z) close to 38. For example, in the nucleus $^{76}$Kr two negative-parity rotational bands built upon the $(3^-)$ (2258 keV) and $(2^-)$ (2227 keV) bandheads are known. However, the coexisting prolate and oblate minima in this nucleus are predicted [11] to be fairly rigid with respect to the reflection-asymmetric distortion. In ref. [12], based on energy systematics, it has been argued that some negative parity bands in well-deformed nuclei from the $A \sim 80$ mass region can be interpreted as collective (aligned) octupole bands. However, it is not the excitation energy of the negative parity band itself that determines the collective character of the underlying intrinsic configuration. In $\pi = -$ bands pairing correlations are usually reduced due to blocking and there is also significant Coriolis mixing. Consequently, these bands have usually larger moments of inertia than ground bands and, in some cases, can become yrast at high spins. In our opinion, the observed lowering of negative-parity states around the particle number 38 does not necessarily indicate strong octupole correlations as suggested in ref. [12] but rather has a non-collective origin, see below.

In order to clarify the issue of octupole collectivity around $Z=38$, $N=38$ we performed the RPA calculations with the Hamiltonian

$$
H_{QRPA} = h_{s.p.} + V_{pair} - \frac{1}{2} \sum_{K} \chi_{3K}^{T=0} Q''_{3K} Q''_{3K} - \frac{1}{2} \sum_{K} \chi_{3K}^{T=1} (\tau_3 Q_{3K})'' (\tau_3 Q_{3K})'' + \frac{1}{2} \sum_{K} \chi_{1K}^{T=1} (\tau_3 D_{1K})'' (\tau_3 D_{1K})''.
$$

(4.1)

where $h_{s.p.}$ is a single-particle Nilsson Hamiltonian, $V_{pair}$ is given by (3.3), and $Q''_{3K} = (r^3 Y_{3K})''$ [$D_{1K}'' = (r Y_{1K})''$] are the doubly-stretched octupole (dipole) operators [13]. A large configuration space composed of 7 major shells (for both protons and neutrons) was used when solving the coupled RPA equations. The octupole isoscalar coupling strengths, $\chi_{3K}^{T=0}$, were determined by the self-consistency condition for the harmonic oscillator model [33,13],
The strength of the isovector octupole mode was taken from ref. [44]
\[ \chi_{3K}^{T=0} = \frac{4\pi}{7} M\omega_0^2 \left\{ \langle (r^4)'' \rangle_0 + \frac{2}{7} (4 - K^2) \langle (r^4 P_2)'' \rangle_0 \right\} + \frac{1}{84} \left[ K^2 (7K^2 - 67) + 72 \right] \langle (r^4 P_4)'' \rangle_0 \left\}^{-1} . \] (4.2)

The strength of the isovector octupole mode was taken from ref. [44]
\[ \chi_{3K}^{T=1} = -0.5 \chi_{3K}^{T=0} , \] (4.3)

while for the isovector dipole mode we used the value [35,43],
\[ \chi_{1K}^{T=1} = \frac{\pi V_1}{\langle (r^2)'' \rangle M\omega_0^2} \] (4.4)
with \( V_1=140\text{MeV} \). A similar model has been used recently [45,46] to discuss octupole excitations built upon superdeformed shapes.

It is worth noting that, because we use the doubly-stretched \( Q_{3K}'' \) in \( Q_{3K}'' \) interactions, there is no simple correlation between the number of two-quasiparticle configurations contributing to an excited state and the corresponding \( B(E3) \) value. That is, an excitation which looks fairly collective in terms of the RPA amplitudes (i.e., appreciable size of backward-going amplitudes), it still can have a very small \( B(E3) \) value. Indeed, the ordinary octupole strengths \( |\langle n|Q_{3K}|0\rangle|^2 \) are quite different from the doubly-stretched octupole strengths \( |\langle n|Q_{3K}'|0\rangle|^2 \) in well-deformed nuclei. For example, in case of the prolate superdeformed harmonic oscillator potential (\( \omega_\perp = 2\omega_3 \)), ratios of the energy-weighted sum rule values \( S_{3K} \) (for \( Q_{3K} \) operators) and \( S_{3K}'' \) (for \( Q_{3K}' \) operators) are given by [47]
\[ S_{3K} : S_{3K}'' = \begin{cases} 50 : 11, & \text{for } K = 0, \\ 13 : 4, & \text{for } K = 1, \\ 1 : 1, & \text{for } K = 2, \\ 1 : 4, & \text{for } K = 3, \end{cases} \] (4.5)
while in the oblate superdeformed case (\( \omega_3 = 2\omega_\perp \)),
\[ S_{3K} : S_{3K}'' = \begin{cases} 5 : 8, & \text{for } K = 0, \\ 17 : 26, & \text{for } K = 1, \\ 1 : 1, & \text{for } K = 2, \\ 4 : 1, & \text{for } K = 3. \end{cases} \] (4.6)

Therefore, in the well-deformed prolate (oblate) configurations, \( B(E3) \) values overestimate (underestimate) the collectivity (in the sense of the RPA with doubly-stretched interaction) for the \( K^π=0^- \) and \( 1^- \) states, while they underestimate (overestimate) the “doubly-stretched” octupole collectivity of the \( K^π=3^- \) states.

The results of calculations for the Sr isotopes are shown in Fig. 7, which displays the predicted excitation energies of intrinsic \( K^π=0^- \), \( 1^- \), \( 2^- \), and \( 3^- \) states and the corresponding \( B(E3) \) values (in s.p.u.). The forward RPA amplitudes for the \( 0^- \), \( 1^- \), \( 2^- \), and \( 3^- \) states built upon prolate configurations in \( ^{76,78,80,82}\text{Sr} \), are plotted in Figs. 8-11, respectively. In none of the nuclei considered, the low-lying negative-parity excitations can be considered as highly-collective states.
In the N=Z nucleus $^{76}$Sr the lowest negative-parity excitations with $K=1$ and 2 can be considered as weakly collective. The $K=1$ octupole phonon has a large component of the two-quasiparticle $[312\,3/2] \otimes [422\,5/2]$ neutron configuration, see Fig. 9. The $K^\pi=2^-$ mode is less collective but it lies lower in energy. As seen in Fig. 10, the main contribution to its wave function comes from the $[310\,1/2] \otimes [422\,5/2]$ proton and neutron excitations. The lowest $K^\pi=0^-$ excitation is mainly built upon the $[310\,1/2] \otimes [431\,3/2]$ excitations. The $K^\pi=3^-$ state is predicted to be a non-collective $[310\,1/2] \otimes [422\,5/2]$ state, see Fig. 11. Of course, all those intrinsic states are expected to be mixed through the Coriolis interaction \[48\]. In the “weaker pairing” variant of the calculations, the $B(E3; g.s. \rightarrow 1^-)$ rate is reduced by a factor of $\sim 3$. This is because the “particle-particle” and “hole-hole” components such as $[301\,3/2] \otimes [422\,5/2]$ or $[310\,1/2] \otimes [431\,3/2]$ have much less effect. A similar quenching is calculated for the $0^-$ state, which becomes a pure particle-hole excitation if pairing is reduced. On the other hand, the characteristics of the $2^-$ state are only weakly influenced by pairing.

The lowest $K^\pi=0^-$ excitations in prolate configurations of $^{78,80,82}$Sr carry a rather weak collectivity. Like in $^{76}$Sr, in the “weak pairing” variant those states become almost pure particle-hole excitations. A similar situation is predicted for the $K^\pi=1^-$ and $3^-$ states. The $K^\pi=2^-$ modes are found to be slightly more collective compared to other modes with $K=0$, 1, and 3. They are expected to appear at about $E_{ex}=2.7$ MeV and they carry $E3$ strength around 6 s.p.u. On the other hand, if pairing is reduced those states become less collective.

The most collective octupole excitations in the oblate configuration of $^{82}$Sr are the $K^\pi=1^-$ and $2^-$ states [$E_{ex}\sim 2.7$ MeV, $B(E3)\sim 7$ s.p.u.]. The calculations also predict a low-lying weakly-collective $K^\pi=1^-$ excitation in the superdeformed configuration of $^{82}$Sr ($E_{ex}\sim 2.3$ MeV, $B(E3)\sim 10$ s.p.u.).

Figures 12 and 13 display calculated low-lying negative parity states built upon the oblate and prolate configurations in the Kr isotopes, respectively. On the average, negative parity states in Kr’s are slightly more collective than those in Sr’s. The $K^\pi=0^-$ prolate excitations are almost pure two-quasiparticle states. The $K^\pi=1^-$ states and the $K^\pi=2^-$ oblate states resemble octupole vibrations; they have $E_{ex}\sim 2.5$ MeV, $B(E3)\sim 7$ s.p.u. The most collective octupole state in the Kr isotopes is the $K^\pi=3^-$ excitation ($E_{ex}\sim 3.2$ MeV, $B(E3)\sim 10$ s.p.u.) in $^{72}$Kr built upon the oblate minimum. However, when pairing is reduced this state becomes almost a pure particle-hole excitation.

Finally, the results for the Zr isotopes are shown in Fig. 14. The lowest negative-parity excitations in $^{80}$Zr and $^{82}$Zr (prolate configuration) have a two-quasiparticle character. The $K^\pi=0^-$, $1^-$, and $2^-$ modes in the oblate minimum of $^{82}$Zr are weakly collective, with $B(E3)\sim 5$–9 s.p.u. Interestingly, the $B(E3)$ rates for these states do not depend strongly on pairing. This is because their dominant two-quasiparticle components are the particle-like $(g_9/2)_{1/2,3/2,5/2}$ orbitals and the hole-like negative-parity $p_{3/2} \oplus f_{5/2}$ levels with $\Omega=1/2$ and $3/2$.

V. CONCLUSIONS

In the light zirconium region there are many excellent candidates for the low-lying $1^+$ states with unusually large $B(M1; 0^+ \rightarrow 1^+)$ rates, around 1–2 $\mu_N^2$. The best prospects
are the Z=N nuclei, such as $^{76}$Sr (prolate), $^{80}$Zr (prolate), and $^{72}$Kr (oblate), where protons and neutrons contribute equally strongly to the $M1$ collectivity. Interestingly, the unusually strong low-energy $M1$ strength in those nuclei has a simple interpretation in terms of $(g_{9/2})^2$ excitations, i.e., it does not result from a simplistic scissors mode. Also, it does not resemble the strong $M1$ transitions known in the light Z=N nuclei [49], mainly of the spin-flip origin.

In $^{76}$Sr and neighboring nuclei, the $1^+$ excitations are predicted to appear just above the $\pi=-$ intrinsic states. Generally, the $K^\pi=0^-, 1^-, 2^-$, and $3^-$ bandheads are calculated to be very weakly collective in well-deformed proton-rich Kr, Sr, and Zr nuclei (except maybe $^{72}$Kr). Namely, the low-lying negative-parity states have a dominant two-quasiparticle character when they are built on an intrinsic state with a large quadrupole deformation. There is no clear correlation between the excitation energy of the $3^-$ state and the magnitude of the $B(E3)\uparrow$ value in the nuclei from the proton-rich Sr-Zr region.

The results of our calculations are quite sensitive to the strength of pairing interaction. In general, the weaker the pairing correlations, the more (less) collective are the $M1$ ($E3$) excitations. There exists some indirect experimental evidence supported by calculations, see Sec. II, that pairing is seriously reduced in some excited states of well-deformed nuclei from the A~80 mass region. We hope that future measurements of excited states in the well-deformed nuclei around $^{76}$Sr, especially their lifetimes, will shed new light on the collectivity of $M1$ and $E3$ states and, indirectly, on the magnitude of pairing correlations in this mass region.

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Figure captions

**Figure 1:** Neutron single-particle levels in $^{78}$Kr as functions of the quadrupole deformation $\beta_2$ ($\beta_4=0$). The Nilsson states are labelled by means of the asymptotic quantum numbers, $[Nn_z\Lambda \Omega]$.

**Figure 2:** Predicted excitation energies of low-lying $1^+$ states of prolate configurations in $^{76,78,80,82}$Sr, oblate minimum in $^{82}$Sr $[82(o)]$, and the superdeformed configuration in $^{82}$Sr $[82(SD)]$. The numbers indicate the $B(M1; \text{g.s.} \rightarrow 1^+)$ values (in $\mu_2 N$) for transitions greater than 0.5 $\mu_2 N$. Only states with $B(M1; \text{g.s.} \rightarrow 1^+)>0.1 \mu_2 N$ are shown (solid lines: $B(M1)>0.3 \mu_2 N$, dashed lines: $B(M1)<0.3 \mu_2 N$). The upper portion shows the results obtained with standard pairing, $\Delta_{std}$, see Table I. The results obtained with pairing reduced by 50% are displayed in the lower portion.

**Figure 3:** $B(M1; \text{g.s.}, K^\pi=0^+ \rightarrow K^\pi = 1^+)$ values for $^{76}$Sr calculated in RPA as a function of the excitation energies of $1^+$ states. The summed values per 1 MeV energy bin are plotted as a histogram (solid lines). For reference, the $B(M1)$ values associated with spin part only ($g_l=0$, dotted line) or orbital part only ($g_s=0$, dashed line) are also shown. The $g$-factors used are $g_l=g_l^{\text{free}}$ and $g_s=(0.85)g_s^{\text{free}}$. The upper (lower) diagram represents the “standard pairing” (“weak pairing”) variant of the calculations.

**Figure 4:** Similar to Fig. 2 (standard pairing) but for the Kr isotopes.

**Figure 5:** Similar to Fig. 2 (standard pairing) but for the Zr isotopes.

**Figure 6:** The lowest $3^-$ energy level (in keV), observed experimentally for doubly even nuclei from the light zirconium region. The dashed lines represent the lowest contours, at 2.2 and 2.3 MeV.

**Figure 7:** Predicted excitation energies of low-lying intrinsic $K^\pi=0^-, 1^-, 2^-, 3^-$ states in $^{76,78,80,82}$Sr. The numbers indicate the $B(E3; \text{g.s.} \rightarrow K^-)$ values in s.p.u. $[1 \text{ s.p.u.}=0.41610^{-6}\text{A}^2\text{e}^2\text{b}^3$, cf. ref. [3]]. They are shown for the states with $B(E3)>1\text{s.p.u.}$ Other states represent non-collective $\pi=-$ excitations. The solid lines correspond to states with $B(E3)>3\text{s.p.u.}$ while the dashed lines correspond to states with $B(E3)<3\text{s.p.u.}$ The results were obtained with standard pairing, $\Delta_{std}$, see Table I.

**Figure 8:** Absolute values of forward RPA amplitudes of the lowest $K^\pi=0^-$ states built upon prolate minima in the Sr isotopes versus the quasiparticle configuration (numbered according to their excitation energies) for neutrons (solid lines) and protons (dashed lines). All amplitudes whose absolute values greater than $5\times10^{-2}$ are indicated. (Note that due to the time-reversal symmetry each amplitude contributes to the intrinsic wave function twice.) The results were obtained with standard pairing, $\Delta_{std}$, see Table I.

**Figure 9:** Similar to Fig. 8 but for the lowest $K^\pi=1^-$ states in the Sr isotopes.

**Figure 10:** Similar to Fig. 9 but for the lowest $K^\pi=2^-$ states in the Sr isotopes.
**Figure 11:** Similar to Fig. 9 but for the lowest $K^{π}=3^{-}$ states in the Sr isotopes.

**Figure 12:** Similar to Fig. 7 but for the lowest $π=−$ states in oblate configurations in the $^{72,74,76}$Kr isotopes.

**Figure 13:** Similar to Fig. 7 but for the lowest $π=−$ states in prolate configurations in the $^{72,74,76}$Kr isotopes.

**Figure 14:** Similar to Fig. 7 but for the lowest $π=−$ states in the $^{80,82}$Zr isotopes.
TABLE I. Calculated equilibrium shape deformations $\beta_2$ and $\beta_4$, and proton and neutron pairing gaps, $\Delta_p$ and $\Delta_n$ (in MeV), at selected oblate and prolate configurations of Kr, Sr and Zr isotopes. According to calculations, the oblate $I=0$ minima lie lower in energy than the prolate $I=0$ minima in $^{72,74,78}$Kr, $^{82}$Sr, and $^{82}$Zr. For $^{82}$Sr the calculations were also performed at superdeformed configuration with $\beta_2=0.45$.

| Nucleus | Z | N  | $\beta_2$ | $\beta_4$ | $\Delta_p$ | $\Delta_n$ | $\beta_2$ | $\beta_4$ | $\Delta_p$ | $\Delta_n$ |
|---------|---|----|---------|---------|---------|---------|---------|---------|---------|---------|
| Oblate  |    |    |         |         |         |         |         |         |         |         |
| 36      | 36 | 36 | -0.31   | -0.010  | 1.34    | 1.23    | 0.35    | 0.016   | 1.40    | 1.31    |
| 38      | 38 | 38 | -0.30   | -0.016  | 1.26    | 1.46    | 0.37    | 0.0     | 1.31    | 1.12    |
| 40      | 40 | 40 | -0.25   | -0.036  | 1.32    | 1.54    | 0.36    | -0.016  | 1.24    | 1.25    |
| 42      | 42 | 42 | -0.24   | -0.050  | 1.28    | 1.48    | 0.32    | -0.023  | 1.18    | 1.46    |
| 44      | 44 | 44 | -0.23   | -0.050  | 1.24    | 1.46    |         |         |         |         |
| Prolate |    |    |         |         |         |         |         |         |         |         |
| 38      | 38 | 38 |         |         |         |         |         | 0.39    | -0.016  | 1.14    | 0.99    |
| 40      | 40 | 40 |         |         |         |         |         | 0.39    | -0.029  | 1.01    | 1.04    |
| 42      | 42 | 42 |         |         |         |         |         | 0.37    | -0.030  | 0.93    | 1.34    |
| 44      | 44 | 44 | -0.22   | -0.065  | 1.35    | 1.37    | 0.28    | -0.020  | 1.15    | 1.48    |
| 44      | 44 | 44 |         |         |         |         |         | 0.45    | 0.0     | 0.83    | 1.45    |
| 40      | 40 | 40 | -0.22   | -0.078  | 1.39    | 1.31    | 0.39    | -0.038  | 0.96    | 1.26    |