PRINCIPLES OF A UNIFIED THEORY OF SPACETIME AND PHYSICAL INTERACTIONS

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Abstract

Principles of a new approach (binary geometrophysics) are presented to construct the unified theory of spacetime and the familiar kinds of physical interactions. Physically, the approach is a modified S-matrix theory involving ideas of the multidimensional geometric models of physical interactions of Kaluza-Klein’s type as well as Fokker-Feynman’s action-at-a-distance theory. Mathematically, this is a peculiar binary geometry being described in algebraic terms. In the present approach the binary geometry volume is a prototype of three related notions: the S-matrix, the physical action (Lagrangian) of both strong and electroweak interactions, and the multidimensional metric. A transition from microworld geometrophysics to the conventional physical theory in classical spacetime are characterized.

1 Introduction

In attempts to construct microworld physics for the last half of a century hopes have been pinned successively on a few key ideas and principles. So, in the fifties those were the principles of quantum field theory, in the sixties attempts were made to construct the S-matrix approach and quantum theory axiomatics, in the seventies and early eighties a primary emphasis was placed upon group methods and gauge theory of physical interactions, at the very end of the 20th century hopes were pinned first on supersymmetric theories and then on superstrings and p-branes. At the same time, other (side) concepts: Fokker-Feynman’s action-at-a-distance theory, Penrose’s twistor theory, multidimensional geometrical models of Kaluza-Klein’s type etc. were investigated.

In all these investigations a relation to the coordinate spacetime was formulated in any event. The latter was a priori assumed four-dimensional, either was generalized to superspaces or multidimensional manifolds, or it was considered that for constructing microworld physics needless to proceed from the coordinate spacetime, and the momentum space should be used as the basis for constructing a theory (as was declared by Chew in the S-matrix approach [1]), either it was proposed to start from complex notions of another kind, e.g. from twistors in Penrose’s program [2, 3], then arriving at both quantum theory and the classical spacetime.

In the present paper a new approach is proposed to construct microworld physics, involving a number of ideas and methods of the previous research and allowing one to consider the essence of spacetime and physical interactions from another viewpoint. The theory being put forward is called binary geometrophysics. Its premises have somewhat in common with Penrose’s twistor program, however our starting points are more abstract and richer in their consequences. Binary geometrophysics is closest to the S-matrix theory. The word “binary” reflects two sets of states of the system, i.e. the initial (i) and the final (out) ones, forming the basis of the theory. The theory itself is a peculiar binary geometry describing microworld physics.

As known, the idea of S-matrix approach was put forward by J. Wheeler [4] and W. Heisenberg [5] and was developed into the S-matrix in the sixties in the works by a number of authors (e.g., see [1, 6]). Its basis constituted principles of Lorentz invariance, analyticity, causality etc. Particles and their characteristics were related to poles in the complex plane.
Binary geometrophysics and the S-matrix theory share, apart from the two kinds of states, a possibility of constructing a theory “independently of whether microscopic spatial-temporal continuum does exist or not” [1, p.17]. In binary geometrophysics there is no a priori spacetime for microworld. It arises only in the relations between macro-objects (according to the idea of a macroscopic nature of the classical spacetime [7]). On the very elementary level there becomes invalid the differential and integral calculus alongside spatial-temporal continuum. There remain only algebraic methods.

To construct binary geometrophysics, the mathematical (algebraic) formalism of Yu.I. Kulakov’s binary physical structures [8, 9, 10] has been used. (The latter was previously developed for other purposes.) As in the S-matrix theory, of importance here is a symmetry, however, in our theory there emerges a more general than the Lorentzian symmetry from which one may turn to Lie groups, in particular, to the Lorentz group and internal symmetries being used in gauge theories.

In the present approach the function continuity condition is used only at the first stage to find algebraic laws of structures (relation systems), and then from merely algebraic considerations one may arrive at a definition of particles and a relationship between constants and charges in strong and electroweak interactions. But above all, in the framework of binary geometrophysics, from the unified expressions prototypes of three related notions: the S-matrix, the action (Lagrangian) for strong and electroweak interactions of elementary particles and the multidimensional metric are derived.

The ideas and methods used in the multidimensional geometrical models of physical interactions of Kaluza-Klein’s type theories (e.g., see [13, 14]) and Fokker-Feynman’s action-at-a-distance theory [15–17] prove necessary in constructing binary geometrophysics [11, 12].

It should be emphasized that binary geometrophysics has a relational pattern. Relations between elements, i.e. primary notions of the theory, play the key part in it. Specifically, the microanalogue of reference frames in Relativity [18], called a binary system of complex relations (BSCR), is put in the forefront. If micro-object are denoted by the symbol \( \mu \), and macro-objects by the symbol \( m \), then the starting points of the relational theory should be defined by the symbol \( R_\mu(\mu) \), where \( \mu \) in parentheses means that micro-objects are considered in it, and the subscript \( \mu \) means that this is done with respect to micro-objects as well. In such terms classical mechanics, describing macro-objects with respect to macroinstruments, is denoted as \( R_m(m) \), and quantum mechanics – as \( R_m(\mu) \).

The first two sections present principal ideas and principles of binary geometrophysics at the level \( R_\mu(\mu) \) in the absence of notions of the classical spacetime, and in the fourth section a transition to the conventional theory \( R_m(m) \) in the presence of these notions is discussed.

2 Binary Geometry of the Microworld

Briefly outline the principal notions of BSCR forming the basis of the theory \( R_\mu(\mu) \) and give their physical interpretation.

1. Two Sets of Elements

It is postulated that there are two sets of elements. Denote the first set by a symbol \( \mathcal{M} \), and the second one by \( \mathcal{N} \). Elements of the first set are denoted by Latin letters \( (i, j, k, \ldots) \), and the elements of the second one – by Greek letters \( (\alpha, \beta, \gamma, \ldots) \). Between any pair of the elements of different sets a pair relation – some complex number \( u_{\mu\alpha} \) is given (see Fig. 1). The elements of the two sets have the following physical meaning. The elements of the first set \( \mathcal{M} \) characterize initial states of particles, and the elements of the second one \( \mathcal{N} \) – final states. Thus, an idea of the S-matrix approach (more exactly, an elementary chain of any evolution), i.e an initial-to-final transition, proves to be laid in the very fundamental notions of BSCR. The complex relations between elements in the two states are a prototype of both the amplitudes of a transition between states and the notions of momenta, and finally particle coordinates.

2. Law and Fundamental Symmetry

It is postulated that there exists some algebraic law, connecting all possible relations between any
The integers $r$ and $s$ characterizes rank $(r, s)$ of BSCR. The essential point of the theory is a requirement of fundamental symmetry lying in the law (1) to be valid, while substituting the given set of elements by any others in the corresponding sets. The fundamental symmetry and the continuity condition allow functional-differential equations to be derived from them and the form of both pair relations $u_{i\alpha}$ as well as the function $\Phi_{(r,s)}$ itself to be found (see [8, 9, 10]).

$$\Phi_{(r,s)}(u_{i\alpha}, u_{i\beta}, \ldots, u_{k\gamma}) = 0. \quad (1)$$

In binary geometrophysics BSCR of symmetric ranks $(r, r)$ are used, with nondegenerate and degenerate systems of relations being distinguished. For the nondegenerate BSCR the law is written via a determinant of pair relations:

$$\Phi_{(r,r)}(u_{i\alpha}, u_{i\beta}, \ldots) = \begin{vmatrix} u_{i\alpha} & u_{i\beta} & \cdots & u_{i\gamma} \\ u_{k\alpha} & u_{k\beta} & \cdots & u_{k\gamma} \\ \vdots & \vdots & \ddots & \vdots \\ u_{j\alpha} & u_{j\beta} & \cdots & u_{j\gamma} \end{vmatrix} = 0, \quad (2)$$

where the pair relations are represented in the form

$$u_{i\alpha} = \sum_{l=1}^{r-1} i^l \alpha^l. \quad (3)$$

Here $i^1, \ldots, i^{r-1} - (r - 1)$ parameters of the element $i$, and $\alpha^1, \ldots, \alpha^{r-1} - (r - 1)$ parameters of the element $\alpha$.

3. Elementary Basis

The origin of the element parameters should be especially dwelled on. They are analogues of the notions of coordinates in the conventional geometry. To arrive at them, in the law (1) one should separate $r - 1$ elements of the set $\mathcal{M}$ and $s - 1$ elements of the set $\mathcal{N}$ and consider them to be standard. Fig. 1 denotes these elements by letters $m, n, \mu, \nu$. Then this law may be interpreted as a relationship determining a pair relation between two nonstandard elements (say, the elements $i$ and $\alpha$) in terms of their relations to standard elements (see Fig. 1). Relations between the standard elements themselves may be considered given forever. Then the pair relation $u_{i\alpha}$ proves to be characterized by $s - 1$ parameters of the element $i$ (its relations to $s - 1$ standard elements of the set $\mathcal{N}$) and similar
Fundamental Relations

In the BSCR of rank \((r,r)\) an important part is played by fundamental \((r-1)\times(r-1)\)-relations, being nonzero minors of the maximal order \(3(r-1)\) in the determinant of the law (2). For BSCR of any rank \((r,r)\) the fundamental relations are written in terms of a product of two determinants composed of parameters of one sort:

\[
\begin{bmatrix}
\alpha & \beta & \cdots \\
\iota & k & \cdots
\end{bmatrix}
\equiv
\begin{bmatrix}
u_{\alpha\iota} & u_{\alpha\beta} & \cdots \\
u_{k\alpha} & u_{k\beta} & \cdots \\
\cdots & \cdots & \cdots
\end{bmatrix}
= \begin{bmatrix}i^1 & k^1 & \cdots \\
i^2 & k^2 & \cdots \end{bmatrix}
\times
\begin{bmatrix}\alpha^1 & \beta^1 & \cdots \\
\alpha^2 & \beta^2 & \cdots \end{bmatrix}.
\tag{4}
\]

Linear Transformations

Transitions from one elementary basis to another may be shown (see [11]) to be described by linear transformations of element parameters of two sets:

\[i^s = C_r^s i^r; \quad \alpha^s = C_r^{s\ast} \alpha^r,\]

where \(C_r^s, C_r^{s\ast}\) are the coefficients defining a class of binary systems (standard elements) being used. Restrict ourselves to the case of complex conjugate coefficients.

Two-Component and Finsler Spinors

Low-rank BSCR notions are, in fact, used in modern theoretical physics. In particular, the two-component spinor theory [19] naturally arises in the framework of the minimal rank (3,3) BSCR. Really, according to this theory, the elements are characterized by pairs of complex parameters \(i \rightarrow (i^1, i^2), \alpha \rightarrow (\alpha^1, \alpha^2),\) i.e. are vectors of the two-dimensional complex space where the linear transformations (5) are defined. Restrict ourselves to the linear transformations leaving invariant each of the \(2 \times 2\) determinants on the left in (4). Since each of such determinants is an antisymmetric bilinear form:

\[i^1 k^2 - i^2 k^1 = \text{Inv}; \alpha^1 \beta^2 - \alpha^2 \beta^1 = \text{Inv},\]

the definition of two-component spinors becomes evident. For a selected class of transformations (5) the coefficients satisfy the condition:

\[C_r^1 C_r^2 - C_r^2 C_r^1 = 1,\]

i.e. these transformations belong to a six-parameter group \(SL(2, C)\). Such transformations single out the privileged class of standard elements corresponding to inertial frames of reference in General Relativity.

In the framework of rank (3,3) BSCR, the transformations keeping invariant both the antisymmetric forms on the right in (4) and the pair relations \(u_{\alpha\iota}\) form the three-parameter group \(SU(2)\) corresponding to rotations in three-dimensional space. As follows from the foregoing, one may state that four-dimensionality of the classical spacetime and the signature \((+--\)) are due to the rank (3,3) of the first nondegenerate BSCR.

At this level of theory development in the framework of rank (3,3) BSCR we, in fact, arrive at the notions comprising Penrose’s twistor theory. However in binary geometrophysics, relation systems of a higher rank \((r,r)\) are considered. In them the elements are described by \(r-1\)-dimensional vectors. Requiring the corresponding antisymmetric forms in (4) under the transformations (5), we arrive at a nonstandard generalization of the two-component spinors which are naturally called Finsler \((r-1)\)-component spinors [21]. These transformations form a group \(SL(r-1, C)\).

Description of Elementary Particles

In binary geometrophysics the elementary particles are described by a few elements in each of the two sets. In the framework of a simplified model based on the rank (3,3) BSCR, massive leptons (electrons) are described by pairs of elements, and neutrinos – by one element in each of the sets. Let

\[\text{It should be especially noted that this theory does not introduce anything from outside but uses only those notions which naturally arise in the framework of different rank BSCR.}\]

\[\text{It is conventional to use a generalization of two-component spinors based on Clifford’s algebras over the field of real numbers (e.g., see [20]), where the spinors have }2^n\text{ components.}\]
the electron \((e)\) be described by two pairs of elements: \(i, k\) and \(\alpha, \beta\), then it may be characterized in terms close to the conventional ones, i.e. by the four-component column and line:

\[
e = \begin{pmatrix} i^1 \\ i^2 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} i^1 \\ i^2 \\ \beta^2 \\ -\beta^1 \end{pmatrix}; \quad e^\dagger = (\alpha^1, \alpha^2, k_1, k_2), \tag{6}
\]

where the quantities with subscripts denote covariant components of spinors.

On the elements describing the leptons imposed are constraints along the horizontal (in one set of elements) and along the vertical (in two sets). Thus, for free leptons the vertical constraints mean that the parameters of two pairs of elements in the two sets are complex conjugate to one another, following the rules of quantum mechanics. For the electron in (6) this means that \(i^s = \alpha^s, k^s = \beta^s\). The horizontal constraints connect the elements \(i\) and \(k\). They correspond to Dirac’s equations for free particles in a momentum space \([11]\).

8. Momentum Space (Velocity Space)

In the rank (3,3) BSCR, of element parameters – two-component spinors – the four-vectors physically interpreted as velocity (or momentum) components of particles are conventionally constructed. Introducing Dirac’s four-row matrices in the corresponding representation, the electron four-velocity components may be presented as follows:

\[
\begin{align*}
  u^0 &= \frac{1}{2}(\bar{e}\gamma^0 e) = \frac{1}{2}(i^1\alpha^1 + i^2\alpha^2 + k^1\beta^1 + k^2\beta^2); \\
  u^1 &= \frac{1}{2}(\bar{e}\gamma^1 e) = \frac{1}{2}(i^1\alpha^2 + i^2\alpha^1 + k^1\beta^2 + k^2\beta^1); \\
  u^2 &= \frac{1}{2}(\bar{e}\gamma^2 e) = \frac{i}{2}(i^1\alpha^2 - i^2\alpha^1 + k^1\beta^2 - k^2\beta^1); \\
  u^3 &= \frac{1}{2}(\bar{e}\gamma^3 e) = \frac{1}{2}(i^1\alpha^1 - i^2\alpha^2 + k^1\beta^1 - k^2\beta^2),
\end{align*}
\]

where \(\bar{e} = e^\dagger\gamma^0\). Defining the matrix \(\gamma^\beta\), the left and right components of the electron may be conventionally introduced, then the left component of the electron is described by the pair of elements \(i, \alpha\), and the right component is described by the pair \(k, \beta\). It is evident that there is only one component for the neutrino. In the general case, when the particles are described by a triple of elements, a generalization of the above rules is necessary.

The above-mentioned transition from parameters to velocities may be interpreted as a transition from a binary geometry to a unary one by a specific glueing of two pairs of elements of the two BSCR sets into new elements of one set being described by Lobachevsky’s geometry. It is easy to verify that for one particle being characterized by the expression (6) we have

\[
\begin{vmatrix}
  u_{i\alpha} & u_{i\beta} \\
  u_{k\alpha} & u_{k\beta}
\end{vmatrix} \equiv \begin{bmatrix} \alpha & \beta \\ i & k \end{bmatrix} = g_{\mu\nu} u^\mu u^\nu. \tag{8}
\]

For a free particle the four-dimensional velocities defined in such a way possess the well-known property \(u^\mu u_\mu = \text{Const} = 1\).

Introducing the second particle \(e_2\) being described by the parameters: \(j, s, \gamma, \delta\), the scalar product of velocities (momenta) of two particles may be defined as

\[
u_{1\mu}^{i\nu} = \frac{1}{4}(\bar{e}_1\gamma^\mu e_1)(\bar{e}_2\gamma^\mu e_2) = \frac{1}{2} \left( \begin{bmatrix} \alpha\gamma \\ i j \end{bmatrix} + \begin{bmatrix} \alpha\delta \\ i s \end{bmatrix} + \begin{bmatrix} \beta\gamma \\ k j \end{bmatrix} + \begin{bmatrix} \beta\delta \\ k s \end{bmatrix} \right), \tag{9}
\]

where the expression in square brackets denote the fundamental \(2 \times 2\)-rank (3,3) BSCR relations defined in (4).
For interacting particles the conditions of complex conjugation of elements of the two sets are not satisfied any more. Introducing for particles in initial and final states the complex conjugate quantities and constructing from them, using the well-known formulas, four-velocities (momenta), we arrive at the characteristics of particles before and after interaction typical for the S-matrix.

9. Choice of BSCR Rank

In our works (see [11, 12]) properties of physical theories based on the BSCR of ranks (2,2), (3,3), (4,4), (5,5), (6,6) and higher have been analyzed step by step. The analysis has shown that the prototype of the known kinds of physical interactions is constructed in the framework of rank (6,6) BSCR. Therewith individual elementary particles (fermions) should be described by triples of elements of one set \( \mathcal{M} \) and \( \mathcal{N} \). This corresponds to the familiar notions of the structure of baryons consisting of three quarks. In this approach these notions are extended to leptons as well.

10. Base 6 \( \times \) 6-Relations

As a prototype of such related notions as the S-matrix or the action (Lagrangian) for two interacting particles there should be some expression containing two triples, i.e. six elements of one set \( \mathcal{M} \) and six elements of the set \( \mathcal{N} \). Such is the base 6 \( \times \) 6-relation being written in the framework of the rank (6,6) BSCR as follows

\[
\left\{ \alpha \beta \gamma \delta \lambda \rho \right\} \equiv \begin{vmatrix}
0 & 1 & 1 & 1 & 1 & 1 \\
1 & u_{i\alpha} & u_{i\beta} & u_{i\gamma} & u_{i\delta} & u_{i\lambda} & u_{i\rho} \\
1 & u_{k\alpha} & u_{k\beta} & u_{k\gamma} & u_{k\delta} & u_{k\lambda} & u_{k\rho} \\
1 & u_{j\alpha} & u_{j\beta} & u_{j\gamma} & u_{j\delta} & u_{j\lambda} & u_{j\rho} \\
1 & u_{s\alpha} & u_{s\beta} & u_{s\gamma} & u_{s\delta} & u_{s\lambda} & u_{s\rho} \\
1 & u_{l\alpha} & u_{l\beta} & u_{l\gamma} & u_{l\delta} & u_{l\lambda} & u_{l\rho} \\
1 & u_{r\alpha} & u_{r\beta} & u_{r\gamma} & u_{r\delta} & u_{r\lambda} & u_{r\rho}
\end{vmatrix}
\]

\[
\equiv \begin{vmatrix}
i^1 & k^1 & j^1 & s^1 & l^1 & r^1 \\
i^2 & k^2 & j^2 & s^2 & l^2 & r^2 \\
i^3 & k^3 & j^3 & s^3 & l^3 & r^3 \\
i^4 & k^4 & j^4 & s^4 & l^4 & r^4 \\
i^5 & k^5 & j^5 & s^5 & l^5 & r^5 \\
1 & 1 & 1 & 1 & 1 & 1
\end{vmatrix}
\begin{vmatrix}
\alpha^1 & \beta^1 & \gamma^1 & \delta^1 & \lambda^1 & \rho^1 \\
\alpha^2 & \beta^2 & \gamma^2 & \delta^2 & \lambda^2 & \rho^2 \\
\alpha^3 & \beta^3 & \gamma^3 & \delta^3 & \lambda^3 & \rho^3 \\
\alpha^4 & \beta^4 & \gamma^4 & \delta^4 & \lambda^4 & \rho^4 \\
\alpha^5 & \beta^5 & \gamma^5 & \delta^5 & \lambda^5 & \rho^5 \\
1 & 1 & 1 & 1 & 1 & 1
\end{vmatrix}
\]

(10)

where the vertical lines underline the fact that the first particle is described by the elements \( i, k, j, \alpha, \beta, \gamma \), and the second one is described by the elements \( s, l, r, \delta, \lambda, \rho \). This expression is invariant under the transformations of parameters of the group \( SL(5,C) \) and is a specific volume in the binary geometry of rank (6,6).

The physical interpretation of the base 6 \( \times \) 6-relation is illustrated by the diagrams of Fig. 2, where on the left the 12-tail structure of binary geormetrophysics is depicted. Two triples of the lower lines describe initial states of two particles, and two upper triples – their final states. A generalization of Feynman’s type diagram is presented in the middle, and a standard diagram of particle scattering is given on the right.

3 Transition from the Bionary Volume to the S-Matrix Prototype or the Action (Lagrangian)

To pass from the base 6 \( \times \) 6-relation to prototypes of the S-matrix or the action (lagrangian) of two particles, the following set of approaches, procedures and principles should be used.

1. Splitting Procedures

First of all, it is necessary to perform a procedure of splitting (or reduction) corresponding to the 4-\( s \)-splitting procedure into four-dimensional spacetime and additional dimensions in multidimensional
geometrical models of Kaluza-Klein’s type theory. It lies in separating the parameters with indices 1 and 2 called external ones, from three rest parameters with indices 3, 4, 5 called internal ones. From the external parameters four-dimensional momenta (velocities) are constructed both in the framework of the rank (3,3) BSCR according (7), and from the internal ones charges of elementary particles are constructed. In fact, this procedure means splitting the initial rank (6,6) BSCR into two subsystems: a rank (3,3) BSCR (with two parameters) and a rank (4,4) BSCR (with three parameters). As a result of splitting, the initial transformation group \( SL(5, \mathbb{C}) \) is narrowed down up to two subgroups: \( SL(2, \mathbb{C}) \) – for external parameters and \( SL(3, \mathbb{C}) \) (or a narrower one \( SU(3) \)) – for internal parameters.

To construct a prototype of the action, the base \( 6 \times 6 \)-relation should be represented in the form reduced to four-dimensional spacetime when parameters with indices 1 and 2 are singled out, and the final expression has the Lorentz-invariant (\( SL(2, \mathbb{C}) \)-invariant) form. This may be performed by expansion of the determinants on the right in (10) with respect first two lines. Multiplying them, we arrive at a set of 225 expressions of the form

\[
\{ \alpha \beta \gamma \delta \lambda \rho \} = \sum_{i s}^{225} \left[ \alpha \delta \right] \left( \beta \gamma \lambda \rho \right),
\]

where the square brackets denote the fundamental \( 2 \times 2 \)-relations constructed from parameters with indices 1 and 2, and the parentheses denote combinations of internal parameters of the form

\[
\begin{pmatrix} \beta \gamma \lambda \rho \\ k j l r \end{pmatrix} = \begin{vmatrix} k^3 & j^3 \\ k^4 & j^4 \\ k^5 & j^5 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} l^3 & r^3 \\ l^4 & r^4 \\ l^5 & r^5 \\ 1 & 1 \end{vmatrix} \times \begin{vmatrix} \beta^3 & \gamma^3 & \lambda^3 & \rho^3 \\ \beta^4 & \gamma^4 & \lambda^4 & \rho^4 \\ \beta^5 & \gamma^5 & \lambda^5 & \rho^5 \\ 1 & 1 & 1 & 1 \end{vmatrix}.
\]

2. Classification and Physical Interpretation of Terms of the Base \( 6 \times 6 \)-Relation

The set of 225 terms of the form (11) should be divided into 9 subsets depicted as subblocks of a
contains terms of the form \(\{\alpha\beta\gamma\delta\lambda\rho\}\), where individual terms are denoted by points or asterisks. The latter mark terms of utmost importance in this theory (with a special order of terms).

The subblocks differ in the character of the fundamental \(2 \times 2\)-relations. The middle \(9 \times 9\)-subblock contains terms of the form \(\{\cdot\cdot\cdot\}\), where the vertical line divided the parameters characterizing two particles. Such terms describe vector-vector interactions of two particles. In the gauge theory they correspond to the interactions via intermediate vector bosons (gluons, photons, Z- or W-bosons).

The left lower or right upper \(3 \times 3\)-subblocks \(M[(2),(2)]\) contain terms of the form \(\{\}\), where the horizontal line divides the parameters characterizing two particles. Such terms describe scalar interactions of two particles. They correspond to Higgs’ scalar bosons. These subblocks will be called mass ones.

The diagonal left upper \(M(4,0)\) and right lower \(M(0,4)\) subblocks contain external parameters of only one of the particles. They correspond to the action (Lagrangian) of “free” particles. Such submatrices will be called free for the first and second particles.

The rest four extreme \(3 \times 9\) \(-\) \(9 \times 3\)-subblocks contain three parameters of one particle and one parameter of another particle. There are criteria allowing contributions of such terms to be excluded.

### 3. Unified Principle of Describing Baryons and Leptons

The theory being presented here permits interactions (processes involving) baryons, massive leptons and neutrinos to be described in the same way. All particles mentioned are described by triples of elements in each of the two sets (states). The baryons are generally characterized by all three two-component columns of the external parameters containing nonzero components. Thus, e.g., in the set \(\mathcal{M}\) the baryon \((b)\) is characterized by a rectangular \(3 \times 5\)-matrix of the parameters of its constituents:

\[
\begin{pmatrix}
   i^1 & j^1 \\
   i^2 & j^2 \\
   i^3 & j^3 \\
   i^4 & j^4 \\
   i^5 & j^5 \\
\end{pmatrix}
\equiv
\begin{pmatrix}
   c^1_{(1)} & c^1_{(2)} & c^1_{(3)} \\
   c^2_{(1)} & c^2_{(2)} & c^2_{(3)} \\
   c^3_{(1)} & c^3_{(2)} & c^3_{(3)} \\
\end{pmatrix},
\]

where individual terms are denoted by points or asterisks. The latter mark terms of utmost importance in this theory (with a special order of terms).
where two upper lines correspond to external two-component spinors, and the three rest – to additional parameters (three-component Finsler spinors) describing internal degrees of freedom (charges) of an interacting particle.

For massive leptons (electrons (e)) the matrix has the same form (14), however, one of the upper two-component columns consists of zero external parameters. Let it be the third column: \( j^1 = j^2 = 0 \). For neutrinos (\( \nu \)), being also described by \( \times 5 \)- (14), two two-component columns of the external parameters are zero. Let it be the first two columns: \( i^1 = i^2 = 0 \), \( k^1 = k^2 = 0 \). For the leptons defined in such a way the number nonzero terms in the base \( 6 \times 6 \)-relation (13) reduces considerably. It should be noted that the number zero columns of the external parameters is an invariant property of particles with respect to distinguished parameter transformation groups. The particle definitions by external parameters given here are in agreement with those presented in paragraph 7 of the previous section.

4. Exchange Character of Physical Interactions

If one write the base \( 6 \times 6 \)-relation for two particles with the same internal parameters, then it vanishes due to antisymmetry of determinant columns. A nonzero result arises in using the exchange mechanism of physical interactions. It is based on the postulate that, according to values of the external parameters, the particles may be in two kinds of states: in the \( U \)-state (“normal”) or in one of the \( X \)-states (“excited”). The interaction process consists in an exchange of states between particles.

The “normal” or \( U \)-state is characterized by a nonzero \( 3 \times 3 \)-determinant of the internal parameters. The analysis of possible simplest kinds of \( X \)-states (taking account of the principle of correspondence to the conventional theory) shows that the \( X \)-states may be of two kinds. One type (\( X_C \)) is also characterized by a nonzero determinant of three columns of the external parameters, and for another type (\( X_N \)) such a determinant is zero.

4a) \( X_C \)-states (charged)

The simplest variant of determining \( X_C \) states leading to nonzero base \( 6 \times 6 \)-relations lies in changing the sign of of the three columns of the internal parameters of the \( U \)-state. In all there are three such possibilities (channels) which, according to the column number with changed sign, are called \( X_X \)-, \( X_Y \)- and \( X_Z \)-states. It can be shown that for these \( X_C \)-states only three pairs of terms in the \( M(2,2) \)-matrix prove to be nonzero. They are located in its \( 3 \times 3 \)-subblocks similarly to nonzero elements in six nondiagonal Gell-Mann’s matrices \( \lambda_n \). Such terms correspond to interactions via charged vector bosons in the conventional theory.

4b) \( X_N \)-states (neutral)

In \( X_N \)-states a pair or all three columns (three-dimensional vectors) of the internal parameters are collinear. In the simplest case on may assume that all three vectors (of the column) \( \vec{c}'(s) \), where \( s = 1,2,3 \), are collinear, i.e. representable as \( \vec{c}'(s) = C'_s \vec{c}' \), where \( \vec{c}' \) is a three vector, and \( C'_s \) are three coefficients. The coefficients \( C'_s \) in diagonal summands of the submatrix \( M(2,2) \) may be shown to enter only as differences, i.e. only two combinations of them are independent. One may choose the combinations as follows:

\[
C' = \frac{1}{2}(C'_1 + C'_2 - 2C'_3); \quad \tilde{C}' = -\frac{1}{2}(C'_1 - C'_2).
\]

Two particular cases: \( C' \neq 0; \quad \tilde{C}' = 0; \quad \tilde{C}' = 0; \quad C' \neq 0 \) define two channels: an \( - \)-channel with the corresponding \( X_{X_4} \)-state and \( + \) a channel with the corresponding \( X_{X_2} \)-state, which should be compared to two channels of interactions via neutral vector bosons in the conventional theory. The two combinations of the coefficients in (15) correspond to two Gell-Mann’s diagonal matrices: \( \lambda_3 \) and \( \lambda_8 \) in the conventional representation.

The above considerations characterize, so far only qualitatively, an essence of five channels (three charged and two neutral) in the known types of physical interactions. Details will be presented in the next article.
5. Intermediate Vector Boson Treatment

In the given approach, corresponding to (Fokker-Planck’s) action-at-a-distance concept, there are not intermediate carriers of interactions (vector bosons). To them correspond the above channels of interactions (types of \(X\)-states of elementary particles). For strong interactions eight gluons correspond to these channels: three pairs of charged gluons corresponding to the \(X_{X^-}\), \(X_{Y^-}\) and \(X_{Z^-}\)-states, and two neutral - -gluons corresponding to the states \(X_A\) and \(X_B\). Similarly “intermediate” vector bosons “carrying” electroweak interactions.

6. “Matreshka” Principle

In the conventional gauge field theory three types of spaces are, in fact, used to describe interactions: 1) an external one corresponding to the classical four-dimensional spacetime with the Lorentz transformation group (\(SL(2,C)\) group), 2) an internal space of electroweak interactions in which there takes place the group \(SU(2) \times U(1)\) and 3) an internal (chromatic) space of strong interactions with the group \(SU(3)\). The theory is being constructed as a composition of these spaces, which is called the “bricks” principle.

Binary geometrophysics assumes a more economical approach of electroweak interactions being considered as truncated strong interactions, which is achieved by fixing one of the lines of the internal parameters in (14) (let it will be a line of the parameters with index 3), when the group \(SL(3,C)\) (or \(SU(3)\)) of admissible transformations of the internal parameters is narrowed down up to the subgroup \(SL(2,C)\) (or \(SU(2)\)) of the internal parameters with indices 4 and 5. It turns out that in such a truncated theory there remain valid considerations presented above of two types of interactions – analogues of those via charged and neutral vector bosons, with the difference that for one particle generation only one channel (of three \(X_{C}\)) survives, which corresponds to interactions via one pair of charged vector bosons.

In such a theory, electromagnetic interactions may be shown to correspond to strong ones via neutral A-gluons, weak interactions via neutral vector Z-bosons to strong ones via B-gluons, and electroweak interactions via charged vector \(W^{\pm}\)-bosons to strong ones via one of the pairs of charged vector gluons (say, \(X^{\pm}\)-gluons).

Such a principle of actual embedding of one type of physical interaction to another may be called the “matreshka” principle, an alternative to the conventional “bricks” principle.

7. Physical Interaction Channel Symmetry Principle

The above procedures and principles allow prototypes of the action (Lagrangian) of strong and electroweak interaction of baryons in terms of quarks (their constituents) as well as electroweak interaction of leptons (massive ones and neutrinos) between one another and with baryons (quarks) to be written down in the same way. These prototypes are constructed as combinations of products of the four-velocities (of the external parameters) of the particle (quark) components with the corresponding coefficients of the internal parameters having a physical meaning of charges in strong and electroweak interactions. However, therewith there remain uncertain quantities and values of the independent constants and charges in the corresponding interactions. This gap is eliminated by using the physical interaction channel symmetry.

It turns out that for obtaining the familiar relations between charges it suffices to know, first, the fact of an existence of the above channels, second, the character of the presence of charges in the corresponding interactions (multiplicativity in charges in the interactions via charged vector bosons) and, third, the conditions of total symmetry of the above channels for all quarks in strong interactions or for left components of particles (quarks or leptons) in electroweak interactions. Therewith strong interactions are unambiguously shown to be characterized by only one constant corresponding to the constant \(g_0\) in chromodynamics, and electroweak ones – by two constants corresponding to the familiar charges \(g_1\) and \(g_2\) in Weinberg-Salam’s model.

In so doing it is natural that one relationship for two interaction constants is written down, which permits Weinberg’s angle to be calculated theoretically.

8. Description of Elementary Particle Generations
The approach proposed (in the framework of binary geometrophysics) allows one to resolve (or opens a new avenue of attack on) a number of problems of the modern theory of physical interactions. For example, there opens a possibility of theoretical justification of the existence of just three generations of elementary particles. Simplifying the situation, one may argue that the availability of three generations of elementary particles (quarks and leptons) is closely related to the existence of three channels of strong interactions of quarks via charged vector bosons. As mentioned here, in electroweak interactions of the particles of the same generation an analogue of only one pair of charged gluons in the form of W-bosons; in interactions of the particles of two other generations there appear analogues of two other pairs of charged gluons. For leptons, in particular, the definitions of three generations are also related to arrangement of zero columns of the external parameters.

It should be particularly emphasized that the approach proposed corresponds basically to the conventional gauge models of physical interactions (e.g., see [22, 23]). In its framework notions of antiquarks and antileptons are easily defined, besides introduced are mesons as well as particles being described by pairs of elements in each of two sets of the rank (6,6) BSCR, with, to fit the conventional approach, one of these elements corresponding to the definition of a quark, and another – of an antiquark. A number of other problems will be elucidated in a succession of articles.

4 Architectonics of Binary Geometrophysics

The aforesaid relates only to the fundamentals of binary geometrophysics. The simplest act of evolution – a transition of two interacting particles from one state to another has been discussed, with elementary particles being as elementary bases for the latter. This always meant three specific subsets of elements comprising the first and second interacting particles and elementary basis.

An important problem of binary geometrophysics program lies in constructing a transition from an algebraic theory of the level $R_\mu(\mu)$ to the conventional spatial-temporal and other attendant notions of modern physics. The most essential points here are as follows.

First of all, to the three specific subsets of elements mentioned the fourth subset, formed by all other elements that did not enter in these three subsets, should be added. The first three of these subsets may be simple (individual particles) or rather complex up to macro-objects, whereas the added fourth subset is extremely complex by definition.

To turn to the conventional physical theories, it is necessary to take the steps describable as the following chain:

$$R_\mu(\mu) \rightarrow R_\mu(\mu) \rightarrow R_\mu^M(\mu) \rightarrow R_\mu^M(\mu) \rightarrow R_\mu^M(m).$$

This route has been described in our book [12] on the basis of a simplified model in the framework of the rank (4,4) BSCR. In such an approach the base 6 $\times$ 6-relations (or 4 $\times$ 4-relations in the simplified model) generate an interval squared of Kaluza-Klein’s multidimensional metric:

$$(\text{Base } 6 \times 6 - \text{relation}) \rightarrow d\Sigma^2 = G_{AB}dx^Adx^B,$$

where $G_{AB}$ are the multidimensional metric tensor components, and indices $A$ and $B$ run the values: 0, 1, 2, 3, 4, 5, · · ·. Therewith part of terms with the indices $A = \mu$ and $B = \nu$, taking the values: 0, 1, 2, 3, is due to an angular diagonal subblock of the base 6 $\times$ 6-relation $M(4,0)$, the terms with mixed indices (one index is four-dimensional, and another index with the values 5, 6, · · ·) are due to the middle submatrix $M(2,2)$. An especial role plays diagonal components of the multidimensional metric with indices $A, B > 3$.

The prototypes of coordinate shifts are formally introduced in terms of four-dimensional momenta $p^\mu = mdx^\mu/d\Sigma$, where $d\Sigma = dx^4$ is the prototype of the multidimensional interval. The square-law characteristic of the multidimensional metric prototype is due to the square-law of particle momenta in a number of terms of the base 6 $\times$ 6-relations. The additional momentum components and shift prototypes are obtained from the internal parameters of elements.
1. **The first step**, in fact, has been discussed in the previous section of the article. It consisted in passing from the general notions of the theory of relations between abstract elements to prototypes of the action (Lagrangian) of interacting particles and other conventional physical notions. This step may be denoted as a transition $R_\mu^1(\mu) \rightarrow R'_\mu(\mu)$, where $R'_\mu(\mu)$ means the theory obtained from rank $(6,6)$ BSCR, as a result of applying the procedures and principles of the previous section.

At this stage being described in algebraic terms there do not exist most of the habitual notions of modern physics. First of all, there is no a notion of the classical spacetime as well as many attendant macronotions: there are no metric, causality, world lines; the notions of wave functions, particles and fields – interaction carriers are absent; there are no field propagators and singular functions troubling physicists very much in the twentieth century.

There are no many other notions, which makes the conventional field theory unthinkable. Finally, gravitational interactions are absent at this level.

2. **The second step (chain)** consists in taking account of all particle of the surrounding world. In modern literature this is called Mach’s principle [24]. This step may denoted as a transition $R'_\mu(\mu) \rightarrow R^M_\mu(\mu)$, where to the two above factors we added one more, i.e. the external world, designated by the subscript $M$. The basic idea of this step lies in a transition from one base $r \times r$-relation to the sum of such base relations which necessarily contain a fixed number each of elements of an isolated (first) particle in each of two BSCR sets $\mathcal{M}$, $\mathcal{N}$. Symbolically, we represent this transition by the formula:

$$\left\{ \alpha \beta \gamma | \delta \lambda \rho \right\} \rightarrow \Sigma(1, 1') = \sum_{s,l,r} \left\{ \alpha \beta \gamma | \delta \lambda \rho \right\}$$

where

$$\left\{ \alpha \beta \gamma | \delta \lambda \rho \right\} = \left\{ i k j | s l r \right\}$$

where the prime means the base $6 \times 6$-relations interpreted according to the previous section. Here the summing is performed over all elements of two BSCR sets $\mathcal{M}$ and $\mathcal{N}$, except for elements determining the first particle, i.e. over all particles of the surrounding world.

In this case we are dealing with a set of second particles (but not one particle) each of which contributes to the value of the prototype of multidimensional interval of the first particle. It may be singled out by considering separately contributions of the base $6 \times 6$-relations containing elements of the second particle.

In the framework of this step (stage of theory development) one needs to perform the procedure of $4+1+\cdots$-splitting corresponding to reducibility (splitting) of the rank $(6,6)$ BSCR into two subsystems of ranks $(3,3)$ and $(4,4)$. In Kaluza-Klein’s multidimensional theory context this procedure corresponds to employment of the monadic, dyadic, $\cdots$, $s$-dic methods of splitting [14, 18] of the $n$-dimensional manifold into the four-dimensional spacetime and additional dimensions orthogonal to it.

It should be especially emphasized that **only after application of the splitting procedure one may speak about appearance of gravitational interaction**. In the expression (16) the contributions of other particles are due to electroweak interactions. After the splitting procedure there arise prototypes of the components of the four-dimensional curved metric being defined by a sum of Minkowski space and quadratic contributions of mixed components of the metric $G_{\mu a}$, where $a = 4, 5, \cdots$, which corresponds to the form of Kaluza-Klein’s metric [14]. Therewith the resulting components of the metric for additional dimensions, physically interpretable as prototypes of the electromagnetic and other “intermediate vector fields” field strengths, contain sums of linear contributions of surrounding particles.

Thus, **both electromagnetic and gravitational interactions are determined by the same contributions, but they are summing up linearly for an electromagnetic interaction, and quadratically for a gravitational one.** Remind that the square of a sum of terms is inequal to a sum of squares of the same terms. This means that a linear sum of contributions of electromagnetic interaction may be zero, whereas the sum of squares of the same contributions will be always nonzero, i.e. gravitation interaction will exist even in the absence of an observable electromagnetic field. This conclusion determines a new glance to the nature of gravity as well as the essence of Einstein’s General Relativity. In particular,
this determines another approach to many fundamental problem of the modern theory of gravity.

After the first stage and performing the splitting procedure in the theory there arise large sums of the terms corresponding to the surrounding world. In Wheeler-Feynman’s absorber theory [16], in the framework of Fokker-Planck’s at-a-distance theory, such sums are, in fact, approximated by integrating over matter uniformly distributed in the whole space.

3. **The third step** consists in a transition from the elementary bases $\mu$ to rather complex systems of base elements and in the limit to a macroinstrument $m$: $R^M_m(\mu) \rightarrow R^M_m(\mu)$, which, in fact, corresponds to quantum mechanics. Symbolically, this step may be described by the formula:

$$\Sigma(1,1') \rightarrow \sum_{\text{basis}} \left( \sum_{\text{World}} \delta_{\lambda \rho} \{ \alpha \beta \gamma | \delta \lambda \rho \}^{i'} \right) \rightarrow d\Sigma^2(1,1').$$

(18)

Here there arise one more summing over the elementary bases comprising a macroinstrument. The sums are complicated considerably, however, at this stage the new sums are approximated by integrating over the momenta of transfer from some objects to others. These sums correspond to Fourier integrals in the conventional representations of vector field potentials.

Only after performing the second stage there arise a possibility of speaking about the wave functions of spinor particles and about boson fields – interaction carriers. At this stage there arise notions of retarded and advanced interactions (potentials). The latter are eliminated according to Feynman-Wheeler’s procedure of allowing for world’s absolute absorber. At this stage there arise quantum mechanics and quantum field theory.

4. **The fourth step** consists in a transition from consideration of microparticles to description of micro-objects: $R^M_m(\mu) \rightarrow R^M_m(\mu)$, which means a transition to classical physics. The latter is realized by summing (averaging) over all particles comprising macro-objects (a isolated particle). Strictly speaking, only after this stage the classical spacetime is said to be constructed from primary relations. It is here that there arise notions of distances and time intervals meant in the previous paragraph, when speaking of the wave functions and boson interaction field-carriers.

5 Conclusion

In the present article the main principles forming the basis for a new approach to constructing the unified theory of spacetime and physical interactions have been indicated, only a brief characteristic of the most essential results obtained in the framework of binary geometrophysics are given. A thorough presentation will be given in a run of articles devoted to separate above-mentioned problems.

The author thanks the participants of the Russian Gravitational Society Seminar at Physics Faculty, Moscow State University, as well as the participants of the Theoretical Physics Laboratory Seminar at Joint Institute for Nuclear Research in Dubna for detailed discussions and valuable comments.

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