The impact of atomic precision measurements in high energy physics

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Abstract. In this talk I discuss the relevance of atomic physics in understanding some important questions about elementary particle physics. A particular attention is devoted to atomic parity violation measurements which seem to suggest new physics beyond the Standard Model. Atomic physics might also be relevant in discovering possible violations of the CPT symmetry.

INTRODUCTION

The aim of this talk is to review some of the atomic precision measurements in atomic physics leading to precious informations in the realm of high-energy physics. The idea of atomic physics bringing light on the high-energy physics world requires some qualification due to the very different scales of energy involved in the two cases. In fact, typically one has a separation of about six or seven order of magnitude between the two scales and one expects the two physics being almost decoupled. In fact, if we look at some observable, $A$, at a scale $\Lambda_1 \ll \Lambda_2$, we expect that the observable can be represented in the form

$$A(\Lambda_1, \Lambda_2) = A(\Lambda_1) + \mathcal{O}\left(\frac{\Lambda_1}{\Lambda_2}\right)^n$$  \hspace{1cm} (1)

In order to be able to derive informations about the physics at the scale $\Lambda_2$, being at the scale $\Lambda_1$, one starts considering a combination of observables corresponding to the corrections coming from the higher scale

$$B = c \left(\frac{\Lambda_1}{\Lambda_2}\right)^n$$  \hspace{1cm} (2)

In order to measure $B$ one needs either the coefficient $c$ being very large in such a way to partially compensate the scale factor, or having an extremely good experimental sensitivity. In this talk I will consider two particular examples of situations

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1) Talk given at the XVII International Conference on Atomic Physics, ICAP 2000, Florence, June 4-9, 2000
where atomic physics can be relevant to high-energy physics, namely Atomic Vi-
olation of Parity (APV) and possible violations of the discrete symmetry CPT,
that is the product of charge-conjugation, parity and time-reversal. In fact, using
heavy atoms like cesium in APV measurements one can get a good enhancement
factor. On the other hand, CPT symmetry can be tested by using the extraor-di-
nary opportunities offered by the atomic traps in order to obtain a very accurate
determination of frequencies.

**ATOMIC PARITY VIOLATION IN ATOMS**

In this Section I will discuss mainly the latest determination of the weak charge
in atomic cesium and some of its implications in models of physics beyond the
Standard Model (SM). The SM has been tested very precisely at machines such
as LEP and SLC, where, working at an energy around the Z mass, one is mainly
testing the property of the Z itself. Therefore, the physics beyond the SM that can
be looked for at these machines is the one giving corrections to the Z-propagator
and/or to the couplings of the Z with fermion-antifermion pairs. Namely, new
massive vector bosons, Z', which mix to the Z, or new particles running in loops
and contributing to the Z self-energy or to vertex corrections. But consider, for
instance, the case of a massive vector boson which does not mix to the Z, and
therefore invisible at LEP (except for tiny radiative corrections). If the Z' is coupled
to fermions, in the low-energy limit it gives rise to an effective four-fermi interaction.
Therefore, low-energy experiments are complementary to the high-energy ones, and
furthermore they are able to measure directly the couplings of the Z to light quarks;
something that at LEP and SLC can be done only in an indirect way. Among the
low-energy experiments a particular role is played by the APV experiments, due to
the precision almost at the level of the one reached at LEP/SLC.

Let us now recall some feature of APV in atoms. First of all, within the SM the
four-fermi parity violating hamiltonian density for nucleons is given by

\[
\mathcal{H}^{PV} = \frac{G_F}{\sqrt{2}} \left[ (\bar{e} \gamma_\mu \gamma_5 e) \sum_{N=p,n} c_{1N} \bar{N} \gamma^\mu N + (\bar{e} \gamma_\mu e) \sum_{N=p,n} c_{2N} \bar{N} \gamma^\mu \gamma_5 N \right]
\]

where

\[
c_{ip} = -2c_{iu} - c_{id}, \quad c_{in} = -c_{iu} - 2c_{id}, \quad i = 1, 2
\]

and

\[
c_{1q} = -8v_e v_q = -(T_3^q - 2s_\theta^2 Q^q), \quad c_{2q} = -8v_e a_q = -T_3^q (1 - 4s_\theta^2), \quad q = u, d
\]

Here \( v_e, v_q, a_e \) and \( a_q \) are the vector and vector-axial couplings of the Z to
the electrons and quarks. For a point-like nucleus with Z protons and N neutrons, the
hamiltonian density, in the non-relativistic limit, is given by
\[ \mathcal{H}_{PV} = \frac{G_F}{4\sqrt{2}m_e} \left[ Q_W(Z, N) \vec{\sigma} \cdot [\vec{p}, \delta^3(\vec{r})]_+ + 2(c_{2p}\vec{S}_p + c_{2n}\vec{S}_n) \cdot [\vec{p}, \delta^3(\vec{r})]_+ \right. \\
\left. - 2i\vec{\sigma} \wedge (c_{2p}\vec{S}_p + c_{2n}\vec{S}_n) \cdot [\vec{p}, \delta^3(\vec{r})]_+ \right] \]

where \( \vec{p} \) is the momentum of the electron, \( \vec{S}_{p(n)} \) the total spin of the protons (neutrons) and \( m_e \) the electron mass. I have also defined the weak charge of the atom as

\[ Q_W(Z, N) = 2 \left[ c_{1p}Z + c_{1n}N \right] \]

Notice that for a heavy atom (large values of \( Z \)) the matrix element of the first term in \( \mathcal{H}_{PV} \) is roughly proportional to \( Z^3 \), one factor coming from \( Q_W \), one from the momentum of the electron and the third one from the wave function evaluated at the origin. This coherence effect was noticed by Bouchiat and Boucharat [1] and it provides, in the case of cesium (\( Z = 55 \)) an enhancement factor of about \( 10^5 \), more or less what is necessary in order to compensate for the decoupling factor from the scales mentioned in the Introduction.

In order to get a rough idea of the bounds on new physics that can be obtained by a measurement of \( Q_W \) with a given sensitivity, we parametrize the new physics contribution to \( Q_W \) by a four-fermi effective interaction [2]

\[ \mathcal{L}_{NP}^{PV} = \frac{g^2_{NP}}{\Lambda^2} \bar{e} \gamma_\mu \gamma_5 e \sum_{q=u,d} \hbar_{1q} \bar{q} \gamma^\mu q \]

If we assume \( h_{1q} \approx c_{1q} \), for a sensitivity \( \Delta Q_W/Q_W \approx 1\% \) one gets a bound

\[ \Lambda \approx (5 g_{NP}) \, TeV \]

If new physics is strongly interacting (\( g^2_{NP} \approx 4\pi \), then \( \Lambda \approx 17 \, TeV \), whereas in the weakly interacting case (\( g^2_{NP} \approx 4\pi\alpha \)) we get \( \Lambda \approx 1.5 \, TeV \). In any case we see that at 1% level of sensitivity, \( Q_W \) is able to test new physics for scales greater than 1 TeV.

In APV measurements one looks at optical transitions between a pair of states \( |\psi_\pm\rangle \) mixed by \( \mathcal{H}_{PV} \) and a state \( |\psi_0\rangle \) of the same nominal parity as \( |\psi_+\rangle \). The mixing of the two eigenstates of parity is given by

\[ \eta = \frac{\langle \psi_-|H_{PV}|\psi_+\rangle}{\Delta E} \]

where \( \Delta E \) is the splitting between the two levels. If I denote by \( M_1 \) and \( E_1^{PV} \) the amplitudes for the two unperturbed transitions \( |\psi_+\rangle \rightarrow |\psi_0\rangle \) and \( |\psi_-\rangle \rightarrow |\psi_0\rangle \), the transition probability, after the mixing, is given by

\[ W = M_1^2 + |E_1^{PV}|^2 \pm 2 \text{Im}(E_1^{PV}) M_1 \]
The choice of the sign depends on the helicity of the photon which is emitted or absorbed in the transition. In the actual experiment on cesium one measures the \textit{circular dichroism}, that is the asymmetry for the absorption cross-section

\begin{equation}
\delta = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \approx 2 \frac{\text{Im} (E_{1PV})}{M_1} \tag{13}
\end{equation}

Of course, the PV amplitude $E_{1PV}$ is proportional to the mixing parameter $\eta$ and therefore measuring $\delta$ one can get the matrix element of the PV hamiltonian. These ideas have been applied in particular to the transition $6S \rightarrow 7S$ in atomic cesium $^{133}\text{Cs}$ [3–5], but also to other atoms as thallium [6]. The typical value of $\delta$ is $10^{-4} \div 10^{-5}$, but there is a strong background which can be overcome by letting the PV amplitude to interfere with a large electro-induced (Stark) transition. Eventually one extracts from the experiment the matrix element of $H_{PV}$ which is proportional to $Q_W$ times an atomic form factor $\kappa_{PV}$ which must be evaluated theoretically in order to extract the value of the weak charge. Therefore the measurement must be coupled with theoretical calculations of similar accuracy in order to get a precise determination of $Q_W$. In the case of atomic cesium the calculation of $\kappa_{PV}$ was performed independently by two groups [7,8]. This calculation is not an easy task, as one has to use many-body perturbation theory coupled with Hartree-Fock techniques. The theoretical errors are quite difficult to estimate. The authors of Refs. [7,8] did their estimate by looking at the differences between the theoretical and the experimental values of parity conserving quantities as dipole matrix elements and hyperfine splittings for the $6S_{1/2}$, $7S_{1/2}$, $6P_{1/2}$ and $7P_{1/2}$ states. In this way the error $\Delta \kappa_{PV}/\kappa_{PV} \approx 1\%$ was obtained. After the new measurement of the weak charge of the cesium by the Boulder group [5], which improved the accuracy of the previous experiment [4] by more than a factor five, Bennett and Wieman [9] re-examined the theoretical errors on $\kappa_{PV}$. In fact, since the time of the previous estimate there have been a number of new and more precise measurements of the quantities of interest. The result is that now the agreement is much better than before, and as a consequence Bennett and Wieman got the estimate $\Delta \kappa_{PV}/\kappa_{PV} \approx 0.4\%$. It should be noticed that there is a third element which contributes to the extraction of $Q_W$ from the data. This is the Stark mixing-induced electric dipole moment amplitude, $\beta$. The experiments in Refs. [3,4] were using a theoretical determination of $\beta$. In [5] the ratio $M_{hf}/\beta$ has been measured. The off-diagonal magnetic dipole moment induced by the hyperfine interaction is well known empirically and it is possible to extract a precise value for $\beta$. However, in a contribution to this Conference [10], the matrix element $M_{hf}$ has been accurately calculated with the result that the empirical formula for it should be corrected by a factor of 0.24% increasing the discrepancy with the SM (see later). I would like also to comment about some possible neglected contribution in the evaluation of the atomic form factor. It has been pointed out in ref. [11] that there could be a contribution arising from the difference of neutron and proton spatial distributions inside the nucleus. This contribution turns out to be very difficult to estimate,
in fact it is quite model dependent. Most probably it could introduce a further error on \( Q_W(Cs) \) of about 0.3. This would not change the conclusions in a very significant way. Another point has been raised recently in ref. [12]. This author argues that the contribution from the Breit interaction (exchange of a transverse photon between two electrons) could have been underestimated. The Breit interaction contribution to the atomic form factor was estimated in [7] and it was found to be very small. However in ref. [12] it is found that the total effect, taking into account also second and third order contributions, is about twice the first order effect. As a consequence, if "all" the higher order contributions could be shown to be negligible, the experimental measure would reconcile with the SM expectation for \( Q_W(Cs) \). However, see also ref. [13].

To conclude this analysis I think that an evaluation of the atomic form factor by taking into account the next order in the many-body perturbative theory is highly desirable in order to settle the question. In any case I find of some interest to assume that the theoretical error is indeed at the level of 0.4\% in order to see which are the possible implications of the APV in high-energy physics.

I can start now to discuss the experimental results on \( Q_W(Cs) \). It is interesting to recall the value obtained in [4] combined with the theoretical determination of \( \kappa_{PV} \) [7,8]

\[
Q_W(Cs) = -71.04 \pm (1.58)_{\text{exp}} \pm (0.88)_{\text{th}}
\]

The total error of these measurement on \( Q_W(Cs) \) is at 2.5\% level of accuracy that, at that time, was comparable with the sensitivity obtained at LEP1. In fact, this determination of \( Q_W(Cs) \) lead to the first indication that technicolor models, in their most simple version obtained from scaling of QCD, could not possibly fit the data. The new experimental result on \( Q_W(Cs) \) [5] combined with the new determination of the theoretical error [9] gives

\[
Q_W(Cs)^{\text{exp}} = -72.06 \pm (0.28)_{\text{exp}} \pm (0.34)_{\text{th}}
\]

A result at 0.6\% level of accuracy. On the theoretical side, \( Q_W \) can be expressed as [14]

\[
Q_W(Cs)^{\text{th}} = -72.72 \pm 0.13 - 102\epsilon_3^{\text{rad}} + \delta_N Q_W
\]

including hadronic-loop uncertainty. I use here the variables \( \epsilon_i \) (i=1,2,3) of ref. [15], which include the radiative corrections, in place of the set of variables \( S, T \) and \( U \) originally introduced in ref. [16]. In the above definition of \( Q_W^{\text{th}}(Cs) \) I have explicitly included only the Standard Model (SM) contribution to the radiative corrections. New physics (that is physics beyond the SM) contributions are represented by the term \( \delta_N Q_W \). Also, I have neglected a correction proportional to \( \epsilon_1^{\text{rad}} \). In fact, as well known [14], due to the particular values of the number of neutrons (\( N = 78 \)) and protons (\( Z = 55 \)) in cesium, the dependence on \( \epsilon_1 \) almost cancels out. For a top mass of 175 GeV and \( m_H = 100(300) \) GeV the value of \( \epsilon_3^{\text{rad}} \) is given by [17]
For \( m_H = 100 \text{ GeV} \), corresponding roughly to the lower experimental bound from direct search at LEP2 [18], one gets

\[
Q_W(Cs)^\text{exp} - Q_W(Cs)^\text{SM} = 1.18 \pm 0.46
\]  

(18)

giving rise to a deviation of about 2.57 \( SD \). Furthermore, for increasing mass of the Higgs the discrepancy increases. Therefore, if we assume as being correct the experimental result, the theoretical evaluation of \( \kappa_{PV} \) and the evaluation of the theoretical errors, we are forced to conclude that the SM is disfavored at 99\% CL.

We can draw another conclusion, that is, that in order to explain the data on \( Q_W(Cs) \) we need new physics not constrained by the LEP and SLC data. In fact, as an example let me consider a type of new physics visible at LEP as, for instance, contributing to the self-energy of the \( Z \), the so called oblique corrections. In such a case one can write \( \delta_N Q_W(\text{oblique}) = -102 \epsilon_{3N} \), and in order to compensate for the discrepancy on \( Q_W(Cs) \) one needs

\[
\epsilon_{3N} = (-11.6 \pm 4.5) \times 10^{-3}
\]  

(19)

whereas from LEP and SLC data one can determine the sum

\[
\epsilon_3^\text{exp} = \epsilon_3^\text{rad} + \epsilon_{3N} = (4.19 \pm 1) \times 10^{-3}
\]  

(20)

Therefore one gets \( \epsilon_{3N} \approx 10^{-3} \), one order of magnitude too small to explain the data on \( Q_W(Cs) \).

I would like also recall the experimental result of APV on Thallium [6]

\[
Q_W(Tl)^\text{exp} = -114.8 \pm 1.2 \text{exp} \pm 3.4 \text{th}
\]  

(21)

This result is not as precise as the one on Cs, and in fact the total error is about 3\%. At this level it is perfectly compatible with the SM prediction

\[
Q_W(Tl)^\text{SM} = -116.7 \pm 0.1
\]  

(22)

A new experiment on cesium is being planned in Paris but the experimental sensitivity is going to be lower than the one obtained in Boulder.

In Berkeley and Seattle there are plans for isotope ratio measurements. In this case the dependence on the atomic form factor would go away eliminating the theoretical error. However these ratios depend on the variation of the neutron density along the isotope chain. This would introduce errors at least twice as big as the experimental ones [19].

We are now in the position of discussing the implications of eq. (18) on new physics. Assuming that the contribution of new physics, \( \delta_N Q_W \), is such to reproduce the experimental results, we can make use of eqs. (15) and (16) to write [20]
\[ Q_W(Cs)^{\text{exp}} - Q_W(Cs)^{\text{th}}(m_H) = 0.66 + 102\epsilon_3^\text{rad}(m_H) - \delta_N Q_W \pm 0.46 \] (23)

For \( m_H = 100 \text{ GeV} \) at a 95\% CL we find

\[ 0.28 \leq \delta_N Q_W \leq 2.08 \] (24)

Notice that the lower positive bound arises since the SM (corresponding to \( \delta_N Q_W = 0 \)) does not fit the experimental value of \( Q_W(Cs) \) at this CL value. This is quite important since it implies an upper bound on the scale of new physics. For the same reason new physics with a contribution \( \delta_N Q_W < 0 \) is not allowed. Also notice that lower and upper bounds both increase for increasing Higgs mass.

**Contact interactions from compositness.** A typical four-fermi operator in composite models contributing to the PV lagrangian is [21,20]

\[ \pm \frac{g^2}{\Lambda^2} \bar{e} \gamma_\mu \frac{1-\gamma_5}{2} e \bar{q} \gamma^\mu \frac{1-\gamma_5}{2} q \] (25)

The effect of this interaction is to modify the coefficients \( c_{1u,1d} \)

\[ c_{1u,1d} \rightarrow c_{1u,1d} \pm \frac{\sqrt{2\pi}}{G_F \Lambda^2} \] (26)

where, since composite models correspond to strongly interacting new physics, we have assumed \( g^2 = 4\pi \). From

\[ Q_W = -2[(2Z + N)c_{1u} + (Z + 2N)c_{1d}] \] (27)

we see that the negative sign for the operator (25) is excluded. For the positive sign we get the bounds

\[ 12.1 \leq \Lambda(\text{TeV}) \leq 32.9 \] (28)

The typical lower bound from high energy physics is about 3.5 TeV [22].

**Extra-dimension models.** In ref. [23] a minimal extension to higher dimensions of the SM, with extra dimensions compactified, was considered. In this model the fermions live in a 4-dimensional subspace, the wall, whereas the gauge bosons live in the full D-dimensional space, the bulk. In general, there might be two Higgs fields, one living in the bulk, \( \phi_1 \), and the other living on the wall, \( \phi_2 \). The propagation of the gauge fields in the bulk is equivalent to the exchange of an infinite tower of Kaluza-Klein (KK) excitations with increasing mass. For example, for \( D = 5 \), \( M = n/R, n = 1, \cdots, \infty \), with \( R \) the compactification radius. If only the Higgs field \( \phi_2 \) is present, the ordinary gauge bosons do not mix with the KK resonances and it is easy to see that the contribution of these modes to \( Q_W \) is negative [24]. Therefore the model does not fit the data on \( Q_W(Cs) \). For the more general case of both Higgs fields present it has been shown [24] that the LEP/SLC and \( Q_W(Cs) \) experimental data are not compatible among them at 95\% CL.
**FIGURE 1.** The Figure shows the 95% CL regions allowed by $Q_W$ for the $Z'$ models. The solid contour corresponds to $m_H = 100 \text{ GeV}$, and the dashed one to $m_H = 300 \text{ GeV}$.

*Extra Z' models.* The implications of models with an extra neutral vector boson $Z'$ for APV have been considered in the literature for quite a long time [25,24,26]. The $Z'$ has couplings comparable to the ones of the $Z$ in the SM and therefore this is an example of weakly interacting new physics. There is a continuum of such models characterized by an angle $0^0 \leq \theta_6 \leq 90^0$. To any value of $\theta_6$ it corresponds a different model. The 95% CL regions allowed by $Q_W$, in the plane $(\theta_6, M_{Z'})$, for different values of the Higgs mass, are shown in Fig. 1. In deriving these Figures the assumption of zero mixing between $Z$ and $Z'$ has been made. In the Figure are also shown three popular models: $\eta$ ($\theta_6 \approx -52^0$), $\chi$ ($\theta_6 = 0^0$), $\psi$ ($\theta_6 = 90^0$). We see that the $\eta$ and the $\psi$ models are not allowed by the data. The direct search at the Tevatron for a $Z'$ within the $\chi$ model gives a direct lower bound at 95% CL, $M_{Z'} \geq 590 \text{ GeV}$ (a similar bound holds for all these models). Therefore this model is compatible with the data. A recent best fit to all the data (including APV) gives for the $\chi$ model the following results [26], $M_{Z'} = 812^{+339}_{-152} \text{ GeV}$ and a mixing angle compatible with zero, $\theta_M = (-1.12 \pm 0.80) \times 10^{-3}$.

**ATOMIC PHYSICS AND CPT VIOLATION**

The CPT theorem is one of the fundamental results in local relativistic field theories. Therefore the idea of possible violations of this theorem implies that some of the axioms of these theories should be reviewed. Let me recall here the exact statement of the theorem [27]: In a field theory satisfying

1. **Locality**

2. **Lorentz invariance**

3. **Analiticity of the Lorentz group representations in the boost parameters**
the CPT transformation is a symmetry of the theory itself.

The first two conditions say that one is dealing with a local relativistic field theory, whereas the third one is satisfied in any finite-dimensional representation of the Lorentz group. It is interesting to notice that unitary representations fail to be analytic and as a consequence the CPT theorem can be violated in this case. The first example of this situation dates back to Majorana [28] when he formulated a first order wave equation without negative-energy solutions. He was able to do that by making use of a unitary infinite-dimensional representation of the Lorentz group. Since this theory does not contain antiparticles the CPT symmetry is broken. However, the quarks and leptons described by the SM belong to finite-dimensional representation of the Lorentz group and therefore this does not seem a possible way to break the theorem. It seems also very hard to give up locality, since it guarantees the microcausality of the theory. Therefore, the only sensible way to avoid the consequences of the CPT theorem in a local field theory seems to break Lorentz invariance. A situation of this type could arise at a more fundamental level as in string theory, where it is possible that Lorentz invariance is spontaneously broken around the Planck mass, $M_P$ [29]. One can take into account these effects by writing down a local effective lagrangian with Lorentz and CPT breaking terms. These terms can be written as an expansion in derivatives over the Planck mass. For instance, considering a single fermion, the violating term can be written as

$$\mathcal{L}_v = \sum_n \frac{g_n}{M_P^T} \bar{\psi} \Gamma (i \partial)^n \psi$$

(29)

I have used a somewhat symbolic notation where $\Gamma$ stays for a generic combination of Dirac matrices and $T$ is a constant tensor and I take the mass dimensions of $g_n$ as $[g_n] = 1$. Furthermore I will assume the same internal symmetries as in the SM, that is $SU(3) \otimes SU(2) \otimes U(1)$ [30,31]. Since the breaking terms should vanish in the limit $M_P \to \infty$ also for $n = 0$, I will require

$$g_0 = c_0 \frac{m^2}{M_P}$$

(30)

where $m$ is some low-energy mass scale parameter. We see that the relevant terms are the ones with $n = 0$ and $n = 1$, and therefore the resulting theory preserves the renormalizability property.

Let me now consider a single fermion interacting with the electromagnetic field. One adds to the standard QED lagrangian the following two terms

$$\mathcal{L}_v^{(n=0)} = \bar{\psi} \left[ -a_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu - \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu} \right] \psi$$

(31)

and

$$\mathcal{L}_v^{(n=1)} = \bar{\psi} \left[ ic_{\mu\nu} \gamma^\mu D^\nu + id_{\mu\nu} \gamma_5 \gamma^\mu D^\nu \right] \psi$$

(32)
where \( D_\mu = \partial_\mu - iqA_\mu \), with \( q \) the electric charge of the fermion. There are other possible terms with \( n = 1 \), but they are not compatible with the symmetries of the SM and therefore they should be suppressed. The following orders of magnitude are expected

\[
a_\mu, b_\mu, H_{\mu\nu} \approx O\left(\frac{m^2}{M_P}\right), \quad c_{\mu\nu}, d_{\mu\nu} \approx O\left(\frac{m}{M_P}\right)
\]

The terms in \( L_v^{(n=0,1)} \) violate Lorentz invariance, since all the tensors in eq. (33) are constant ones. However only the terms proportional to \( a_\mu \) and \( b_\mu \) violate CPT symmetry since \( \gamma_\mu, \gamma_\mu\gamma_5 \) and \( D_\mu \) are CPT odd, whereas the other covariant terms are CPT even. Therefore, in the following I will take into consideration only \( L_v^{(n=0)} \). Notice also that when dealing with a single fermion the term in \( a_\mu \) does not have physical meaning since we can write \( a_\mu = \partial_\mu (a \cdot x) \), showing that \( a_\mu \) is a trivial gauge background field. Of course, the situation changes when dealing with different fermions having different \( a_\mu \)'s. From eq. (33) we expect that the order of magnitude of the CPT and Lorentz breaking terms is given by \( m/M_P \approx 10^{-22} \div 10^{-17} \) for \( m = m_e \div v \), where \( m_e \) is the electron mass and \( v \approx 250 \text{ GeV} \) is the electroweak symmetry breaking scale. Lorentz and CPT breaking terms could appear also in the photon part of the total lagrangian. This instance is discussed thoroughly in the second paper of ref. [30], but I will not consider it in this talk.

Here I want to illustrate some atomic physics experiment about CPT violation. But before doing that let me just give a list of other existing or planned experiments about the violation of this fundamental symmetry

- \( K - \bar{K} \) mass difference. This experiment gives the best high-energy result [22]

\[
\left| \frac{m_K - m_{\bar{K}}}{m_K} \right| \lesssim 10^{-18}
\]

- Experiments on neutral meson oscillations to be done at meson factories [32].
- Experiments on muons [33].
- Experiments with spin-polarized solids [34].
- Experiments from clock-comparison [35].

CPT violation may have also some relevance for baryogenesis and this subject has been discussed in ref. [36].

Let me now consider atomic physics experiments for testing CPT using atomic traps. Several of these experiments have been performed by confining single particles or antiparticles in a Penning trap for a long time. These experiments have a very high precision, of order \( 10^{-9} \) or better, whereas the precision in experiments about mesons (see eq. (34)) is much lower, of order \( 10^{-3} \). I recall here the comparison of the electron and positron gyromagnetic ratios, \( g_\mp \), obtained measuring their cyclotron and anomaly frequencies (see later), which gives the figure of merit [37]
\[\left| \frac{g_- - g_+}{g_{av}} \right| \lesssim 2 \times 10^{-12} \quad (35)\]

Measuring the proton and antiproton cyclotron frequencies, one can get their charge-to-mass ratios. \( r_{p,\bar{p}} \) \[38\]

\[\left| \frac{r_p - r_{\bar{p}}}{r_{av}} \right| \lesssim 9 \times 10^{-11} \quad (36)\]

Analogously, from the charge-to-mass ratio for electron and positron \[39\]

\[\left| \frac{r_{e^-} - r_{e^+}}{r_{av}} \right| \lesssim 1.3 \times 10^{-7} \quad (37)\]

As we see the relevant figures of merit are much bigger than the one for the mass difference \( K - \bar{K} \), although, as noticed, these measurements are about six order of magnitude more sensitivity than the one leading to (34). In ref. \[40\] it has been argued that these figures of merit could not be the relevant ones in testing CPT breaking. In fact, within the approach presented here, at the lowest order in the CPT violating parameters, one has \( g_- = g_+ \), and similarly the charge-to-mass ratios do not depend on these parameters \[40\]. To review this point, let me start by the Dirac equation for an electron or a proton including the breaking terms contained in \( L_v^{(n=0)} \) (of course, the breaking parameters may depend on the type of particle one is considering).

\[
\left( i \gamma^\mu D_\mu - m - b_5 \gamma^\mu - \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu} \right) \psi = 0
\]

\[38\]

In a Penning trap the radial confinement is obtained through a strong axial magnetic field, whereas the axial confinement is obtained by a quadrupole electric field. The main corrections due to the CPT and Lorentz breaking parameters are obtained by taking \( A_\mu \) as the four-potential for a constant magnetic field. Then, to obtain the energy shifts generated by the breaking parameters one makes use of the relativistic Landau levels wave functions and the expressions containing the full QED corrections for the unperturbed levels \[40,41\]. However, the underlying physics can be understood quite simply recalling the expression for the non-relativistic Landau levels

\[ E_{n,\sigma} = \left( n + \frac{1}{2} + \frac{g}{2} \right) \frac{Be}{m}, \quad \sigma = \pm \frac{1}{2} \quad (39)\]

The cyclotron and anomalous frequencies are obtained comparing two Landau levels with different quantum number \( n \) and with the same and opposite spin configurations respectively

\[
\omega_c = E_{1,-1/2} - E_{0,-1/2} = \frac{Be}{m}
\]

\[
\omega_a = E_{0,+1/2} - E_{1,-1/2} = \frac{g - 2 \frac{Be}{m}}{2}
\]

\[40\]
The relevant CPT and Lorentz breaking corrections to the energy levels are given by [41]

\[ \delta E_{n,\pm 1/2}^e = \mp b_3 \pm H_{12}, \quad \delta E_{n,\pm 1/2}^e = \mp b_3 \mp H_{12} \quad (41) \]

where we have taken the third axis along the magnetic field of the trap. The frequencies for the antiparticles that we need according to the CPT theorem are the ones with inverted spin, therefore

\[ \omega_{e^-} = \omega_{e^+} = \omega_c, \quad \text{To conclude this } \omega_a^e = \omega_a \mp 2b_3 + 2H_{12} \quad (42) \]

We get

\[ \Delta \omega_c \equiv \omega_{e^-} - \omega_{e^+} = 0 \quad \Delta \omega_a \equiv \omega_{a^-} - \omega_{a^+} = -4b_3 \quad (43) \]

We recall that these equations hold only at the first order in the breaking parameters and also that the usual relation \((g-2)/2 = \omega_a/\omega_c\) does not hold here since, as noted before, the gyromagnetic ratios do not change at the lowest order.

Since the observables that are measured in a Penning trap are the anomalous and cyclotron frequencies, it seems natural to introduce figures of merit related to these observables. A such figure of merit for CPT violation is [40]

\[ r_{\omega_a}^e = \frac{|E_{n,\sigma}^e - E_{n,-\sigma}^e|}{E_{n,\sigma}^e} = \frac{|\delta E_{n,\sigma}^e - \delta E_{n,-\sigma}^e|}{E_{n,\sigma}^e} \quad (44) \]

where \(E = E + \delta E\). For a weak magnetic field one gets

\[ r_{\omega_a}^e = \frac{2m}{2m} = 2 \frac{|b_3|}{m} \quad (45) \]

A new analysis of the 1987 experiment by Dehmelt et al. [37] has been done recently in ref. [42] obtaining the following bound

\[ r_{\omega_a}^e \lesssim 1.2 \times 10^{-21} \quad (46) \]

However, the vector \(b_\mu\) is absolutely constant and as such it rotates with a diurnal period of 23 h and 56 m, when seen in the laboratory frame which is fixed with respect to the earth. This effect might have given rise to non favorable situations during the observation, and therefore the bound has been a bit relaxed [42]

\[ r_{\omega_a}^e \lesssim 3 \times 10^{-21} \div 2 \times 10^{-20} \quad (47) \]

In the case of proton and antiproton there is no experiment at the moment. Assuming an experimental sensitivity analogous to the electron positron case (meaning \(\delta \omega_a \approx 2 \text{ Hz}\)) one gets [41]
The last case I consider is the spectroscopy of free or magnetically trapped hydrogen (\(H\)) and antihydrogen (\(\bar{H}\)). This is interesting since the two-photon \(1S-2S\) transition has been measured with a precision of \(3\times10^{-14}\) in a cold atomic beam of \(H\) and with a precision of \(10^{-12}\) in trapped \(H\) [44]. However for the free case the dependence of the \(1S-2S\) transition on the CPT and Lorentz breaking parameters is suppressed by a factor \(\alpha^2/8\pi\), since the \(1S\) and \(2S\) levels shift by the same amount at the leading order in the breaking [45]. Consider now the spectroscopy of \(H\) and \(\bar{H}\) in a magnetic field \(B\). In the basis \(|m_J,m_I\rangle\) the four \(1S\) and \(2S\) hyperfine Zeeman levels are, for \(n=1,2\)

\[
|b_n\rangle = |-1/2,-1/2\rangle, \quad |d_n\rangle = |1/2,1/2\rangle \\
|a_n\rangle = \cos \theta_n |-1/2,1/2\rangle - \sin \theta_n |1/2,-1/2\rangle \\
|c_n\rangle = \sin \theta_n |-1/2,1/2\rangle + \cos \theta_n |1/2,-1/2\rangle
\]

with \(\tan 2\theta_n = (51 \text{ mT})/n^3B\). Transitions of the type \(|c_1\rangle \rightarrow |c_2\rangle\) have leading-order sensitivity to Lorentz and CPT violation, but they are field-dependent. As a consequence there is a problem connected with the broadening of the lines due to trapping field inhomogeneities.

Consider now hyperfine transitions in the ground state. Again there is the problem of the Zeeman broadening. However one can try to eliminate the frequency dependence on \(B\) (at lowest order) by choosing a field independent transition point [45]. For \(B \approx 0.65T\) the state \(|c_1\rangle\) is highly polarized (\(|1/2,-1/2\rangle\)). Then the effect on the transition \(|c_1\rangle \rightarrow |d_1\rangle\) of the CPT and Lorentz violating parameters is \(\delta\omega_{c\rightarrow d} = 2(\mp b_3^p + H_{12}^p)\). Therefore by putting \(\Delta\omega_{c\rightarrow d} = \omega_{c\rightarrow d}^H - \omega_{c\rightarrow d}^{\bar{H}}\) the corresponding figure of merit can be defined as

\[
r_{c\rightarrow d}^H = \frac{|\Delta\omega_{c\rightarrow d}|}{m_H} = 4 \frac{|b_3^p|}{m_H} \quad (50)
\]

Attaining a resolution of \(1 \text{ mHz}\), one would get [45]

\[
r_{c\rightarrow d}^H \lesssim 5 \times 10^{-27} \quad (51)
\]

**CONCLUSIONS**

In this talk I have reviewed some important consequences of atomic physics measurements in the domain of high-energy physics. In particular APV in cesium could be the first real indication of new physics beyond the SM. The atomic physics tests of the CPT symmetry are already at a spectacular level of sensitivity, and the future experiments on \(H\) and \(\bar{H}\) could give bounds well below the one expected from string theory.
REFERENCES

1. Bouchiat M.A. and Bouchiat C.C., *Phys. Lett.* **B48**, 111 (1974); *J. Phys.* **35**, 899 (1974).
2. Ramsey-Musolf M.J., *physics/0001250*, (2000).
3. Bouchiat M.A., Guena J., Hunter L. and Pottier L., *Phys. Lett.* **B117**, 358 (1982).
4. Noecker M.C., Masterson B.P. and Wieman C.E., *Phys. Rev. Lett.* **61**, 310 (1988).
5. Wood C.S., Bennett S.C., Cho D., Masterson B.P., Roberts J.L., Tanner C.E. and Wieman C.E., *Science* **275**, 1759 (1999).
6. Vetter P.A., Meekhof D.M., Majumder P.K., Lamoreaux S.K. and Fortson E.N., *Phys. Rev. Lett.* **71**, 3442 (1993); Edwards N.H., Phipp S.J., Baird E.G., Nakayama S., *Phys. Rev. Lett.* **74**, 2654 (1995).
7. Bouchiat M.A., Flambaum V.V., Silvestrov P. and Sushkov O., *Phys. Lett.* **A141**, 147 (1989).
8. Blundell S.A., Johnson W.R. and Sapirstein J., *Phys. Rev. Lett.* **65**, 1411 (1990).
9. Bennett S.C. and Wieman C.E., *Phys. Rev. Lett.* **82**, 2484 (1999).
10. Bouchiat M.A., Flambaum V.V. and Ginges J.S.M., contribution A11 to this meeting, *Conference Abstracts*, eds. F. Fusi and F. Cervelli. See also Bouchiat M.A. and Flambaum V.V., *physics/0005038*, (2000).
11. Pollock S.J. and Welliver M.C., *Phys. Lett.* **B464**, 177 (1999).
12. Derevianko A. *physics/0001046*, (2000); Derevianko A., *hep-ph/0005274*, (2000).
13. Kozlov M.G., Porsev S.G. and Tupitsyn I.I., *physics/0004076*, (2000) and contribution A15 to this meeting, *Conference Abstracts*, eds. F. Fusi and F. Cervelli.
14. Marciano W.J. and Rosner J.L., *Phys. Rev. Lett.* **65**, 2963 (1990); Altarelli G., Lectures given at the *Les Houches Summer School: Particles In The Nineties*, 30 Jun - 26 Jul 1991, Les Houches, France.
15. Altarelli G., Barbieri R. and Jadach S., *Nucl. Phys.* **B369**, 3 (1992); Altarelli G., Barbieri R. and Caravaglios F., *Nucl. Phys.* **B405**, 3 (1993); *ibidem Phys. Lett.* **B349**, 145 (1995).
16. Peskin M.E. and Takeuchi T., *Phys. Rev. Lett.* **65**, 964 (1990); *ibidem Phys. Rev.* **D46**, 381 (1991).
17. Altarelli G., Barbieri R. and Caravaglios F., *Int. J. Mod. Phys.* **A13**, 1031 (1998), and updating of these results as communicated to me by Dr. Caravaglios.
18. The actual experimental lower bound on the Higgs mass is 107.9 GeV at 95% CL, see: ALEPH, DELPHI, L3 and Opal Collaborations. The LEP Working group for Higgs boson searches, CERN-EP-2000-055, April 2000.
19. Pollock S.J., Fortson E.N. and Wilets L., *Phys. Rev.* **C46**, 2587 (1992); Chen B.Q. and Vogel P., *Phys. Rev.* **C48**, 1392 (1993).
20. Casalbuoni R., De Curtis S., Dominici D. and Gatto R., *Phys. Lett.* **B460**, 135 (1999).
21. Langacker P., *Phys. Lett.* **B256**, 277, (1991).
22. Particle Data Group, Caso C. *et al.*, *Eur. Phys. J.* **C3**, 1 (1998).
23. Pomarol A. and Quiros M., *Phys. Lett.* **B438**, 255 (1998); Delgado A., Pomarol A. and Quiros M., *Phys. Rev.* **D60**, 95008 (1999); Masip M. and Pomarol A. *Phys. Rev. D60*, 96005 (1999).
24. Casalbuoni R., De Curtis S., Dominici D. and Gatto R., *Phys. Lett.* B462, 48 (1999).
25. Amaldi U. *et al.*, *Phys. Rev.* D36, 1385 (1987); Marciano W.J. and Rosner J.L., *Phys. Rev. Lett.* 65, 2963 (1990); Altarelli G. *et al.* *Phys. Lett.* B261, 146 (1991); Mahantappa K.T. and Mohapatra P.K., *Phys. Rev.* D43 3093 (1991); Rosner J.L., *Phys. Rev.* D61, 016006 (2000). In these papers it is also possible to find a complete list of references to the Z' models.
26. Erler J. and Langacker P., *Phys. Rev. Lett.* 84, 212 (2000).
27. Streater R.F. and Wightman A.S., *PCT, spin and statistics and all that*, edited by W.A. Benjamin, Inc., New York, Amsterdam, 1964.
28. Majorana, E., *Il Nuovo Cimento* 9, 335 (1932). This paper is in italian and it was translated in english by Fradkin E.S., *AJP* 34, 314 (1966).
29. Kostelecký V.A. and Potting R., *Nucl. Phys.* B359, 545, (1991); *ibidem, Phys. Lett.* B381, 389, (1996); Kostelecký V.A. and Samuel S., *Phys. Rev. Lett.* 63, 224, (1989); *ibidem, Phys. Rev. Lett.* 66, 1811, (1991); *ibidem, Phys. Rev.* D39, 683, (1989); *ibidem, Phys. Rev.* D40, 1886, (1989).
30. Colladay D. and Kostelecký V.A., *Phys. Rev.* D55, 6760, (1997); *ibidem, D58*, 116002, (1998).
31. For a recent review of the results on this subject, see Kostelecký V.A., *hep-ph/0005280*, (2000).
32. Kostelecký V.A., *Phys. Rev. Lett.* 80, 1818, (1998).
33. Bluhm R., Kostelecký V.A. and Lane C.D., *Phys. Rev. Lett.* 84, 1098, (2000).
34. Bluhm R. and Kostelecký V.A., *Phys. Rev. Lett.* 84, 1381, (2000).
35. Kostelecký V.A. and Lane C.D., *Phys. Rev.* D60, 116010, (1999).
36. Bertolami O. *et al.*, *Phys. Lett.* B395, 178 (1997).
37. Van Dyck R.S. Jr., Schwinberg P.B. and Dehmelt H.G., *Phys. Rev. Lett.* 59, 26 (1987); *ibidem Phys. Rev.* D34, 722 (1986). For a review of the principles of the Penning trap, see: Brown L.S. and Gabrielse G. *Rev. Mod. Phys.* 58, 233 (1986).
38. Gabrielse G. *et al.*, *Phys. Rev. Lett.* 82, 3198 (1999).
39. Schwinberg P.B., Dyck R.S. Jr and Dehmelt H.G., *Phys. Lett.* A81, 119 (1981).
40. Bluhm R., Kostelecký V.A. and Russell N., *Phys. Rev. Lett.* 79, 1432 (1997).
41. Bluhm R., Kostelecký V.A. and Russell N., *Phys. Rev.* D57, 3932 (1998).
42. Dehmelt H., Mittleman R., Van Dyck R.S. Jr and Schwinberg P., *Phys. Rev. Lett.* 83, 4694 (1999).
43. Udem T. *et al.*, *Phys. Rev. Lett.* 79, 2646 (1997).
44. Cesar C.L. *et al.*, *Phys. Rev. Lett.* 77, 255 (1996).
45. Bluhm R., Kostelecký V.A. and Russell N., *Phys. Rev. Lett.* 82, 2254 (1999).