Hire the Experts: Combinatorial Auction Based Scheme for Experts Selection in E-Healthcare

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Abstract. During the last decade, scheduling the healthcare services (such as staffs and OTs) inside the hospitals have assumed a central role in healthcare. Recently, some works are addressed in the direction of hiring the expert consultants (mainly doctors) for the critical healthcare scenarios from outside of the medical unit, in both strategic and non-strategic settings under monetary and non-monetary perspectives. In this paper, we have tried to investigate the experts hiring problem with multiple patients and multiple experts; where each patient reports a preferred set of experts which is private information along with their private cost for consultancy. To the best of our knowledge, this is the first step in the direction of modelling the experts hiring problem in the combinatorial domain. In this paper, the combinatorial auction based scheme is proposed for hiring experts from outside of the hospitals to have expertise by the preferred doctors set to the patients.

1 Introduction

The scarcity of well qualified healthcare experts such as physicians, surgeons, nurses etc, during the critical healthcare scenario inside the hospitals (or medical units) has been highlighted as one of the biggest obstacles in achieving the high quality medical services. In our future references healthcare experts will be used as experts. Also medical units, hospitals, and organizations will be used interchangeably. As the experts are scarce, the immediate questions that comes in the mind is: how to efficiently and effectively schedule the experts and services of the medical units so as to meet their demand up to some extent? Answering to the above raised question, the substantial number of works have been done in literature to schedule the healthcare services mainly physicians [17], nurses [1], and Operation theatres (OTs) [8][4] inside the hospitals. Today with the advancement in technologies, more precisely advancement in medical technologies, the experts can provide their expertise to the patients admitted miles away from the experts resident with their physical or virtual presence. It is to be noted that, the virtual presence of the experts can be achieved by utilizing several communication medias say video conferencing, smartphone etc. In recent years, several works have been initiated in the direction
of hiring expertise from outside of the in-house hospitals for the patients, under both monetary \cite{15} \cite{12} and non-monetary environment \cite{11} \cite{13}.

In this paper, we have tried to investigate the more general setup in the expert hiring problem in healthcare domain with multiple patients and multiple experts. In this set-up, each patient reveals a preferred set of experts and the private cost for consultancy that is meant as the consultation fees of the experts, if allocated. Our goal is to allocate the requested set of experts to the patients in a conflict-free way with the constraint of maximizing the total cost for consultancy, in literature it is termed as social welfare.

The remainder of the paper is structured as follows. Section 2 elucidates the preliminary concepts about scheduling of healthcare services inside the hospitals and scheduling of doctors outside of the hospitals. Section 3 describes our proposed model. The proposed mechanisms is illustrated in section 4. The paper is concluded with the possible future directions in section 5.

2 Literature review

The prior art on scheduling the hospital resources such as operation theatres (OTs), physicians, nurses etc.) can be classified into two broad categories— one addressing the scenario of scheduling the hospital resources inside the in-house hospitals, with the other addressing scheduling of experts outside the in-house hospitals. Our paper can be classified more in the second category. While there are many fundamental questions that lie at the core of this area, our work finds relevance to only few of them such as a) Which experts are to be hired? b) How to fulfil the multiple demand of the patients for the experts in a conflict-free way? c) What is to be paid to the hired experts?

A vast majority of the literature in scheduling the hospital resources is dedicated to the problem of scheduling the hospital resources inside the in-house hospitals. A wide variety of scheduling techniques have been proposed to scheduling OTs \cite{6} \cite{8} \cite{4} \cite{2} \cite{5} \cite{7}, internal staffs (such as nurses \cite{1}, physicians \cite{17} etc.) inside the hospital.

The study of hiring experts, as considered here, was initiated by \cite{15} in non-strategic setting and by \cite{12} in strategic setting. Talking about the works done in non-strategic setting, in \cite{15} a doctor is providing his expertise to a patient admitted to distant hospital by its virtual presence by utilizing the communication media such as video conferencing. In \cite{16} the context of the patients (such as age, sex, medical report etc.) and the context of doctors (such as expertise area) are utilized to recommend the doctors from outside the admitted hospitals to the patients. Now moving to the setting where the patients and doctors (collectively called agents) are strategic in nature. In \cite{12} the set of doctors are hired from outside of the hospital for a patient admitted to a hospital. To allocate the doctors to a patient an auction based framework is utilized. Extending the work done in \cite{12}, in \cite{14} the more realistic situation of hiring the doctors from outside the hospitals is considered where the patient is constrained by budget. One interesting situation is addressed in \cite{11} that tackles
the situation of hiring of socially motivated doctors who are intended to provide free of cost services. In [11] the mechanisms are developed to allocate a doctor to a patient. Following [11], in [13] an effort has been made to model the ECs hiring problem as a two sided preference market in healthcare domain.

Finally, our work is related to and build upon the concept of combinatorial auction, studied in the field of algorithmic mechanism design [3][10]. Our setting is different from the above addressed set-ups due to the reason that there are multiple patients and multiple experts alongside with the constraint that the patients request for the doctors set is combinatorial in nature. By combinatorial nature, we mean that the patients can place their interest on the combination of experts, rather than individual expert. We have designed a combinatorial auction based scheme motivated by [4] for the discussed set-up.

3 System model and problem formulation

Let \( \mathcal{P} = \{p_1, p_2, \ldots, p_n\} \) be the set of all patients and \( \mathcal{S} = \{s_1, s_2, \ldots, s_m\} \) the set of all experts. If not specified explicitly, \( n \) and \( m \) denote the total number of patients and the total number of experts respectively. For the time being, we assume that the expertise level (or quality) of all the experts are similar. Each patient’s valuation is the cost he/she (henceforth he) is willing to pay for the consultancy received from the preferred set of experts. The valuation vector of the patient set is given as \( v = \{v_1, v_2, \ldots, v_n\} \); where \( v_i \in \mathbb{R}^+ \) is the valuation of patient \( p_i \) if he gets the specified set of experts otherwise 0.

Each of the patients place their private valuation in sealed bid manner. It is to be noted that due to strategic nature of the patients, they can mis-report their respective private values. So, it is convenient to represent the bid value reported by patient \( p_i \) for receiving the consultancy as \( v'_i \). \( v'_i = v_i \) describes the fact that any patient \( p_i \) is not deviating from its true valuation. On the other hand, the true demand vector of all the available patients is given as \( \mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_n\} \); where \( \mathcal{D}_i \in \mathcal{D} \) is the preferred doctor set of the patient \( p_i \). It is convenient to represent the reported demand of experts by any patient \( p_i \) as \( \mathcal{D}'_i \). \( \mathcal{D}'_i = \mathcal{D}_i \) ensure the fact that the demand of expert set reported by patient \( i \) is true. The two tuples, in combination, will be termed as the type of the patients. The type profile of all the patients is given as \( \pi = \{\pi_1, \pi_2, \ldots, \pi_n\} \): where \( \pi_i = < \mathcal{D}_i, v_i > \). \( \pi'_i = \pi_i \) ensure the fact that the type profile reported by patient \( i \) is true. The type of all the patients except \( i \) can be represented as \( \pi_{-i} = \{\pi_1, \pi_2, \ldots, \pi_{i-1}, \pi_{i+1}, \ldots, \pi_n\} \). Let \( \mathcal{A} \) be the set of all possible allocation of \( m \) experts to \( n \) patients. More formally, a mechanism \( \mathcal{M} = (f, \tilde{p}_i) \) consists of an allocation mechanism \( f : \pi^n \rightarrow \mathcal{A} \) and the payment functions \( \tilde{p}_i : \pi^n \rightarrow \mathbb{R} \) for \( i = 1, \ldots, n \). The mechanism \( \mathcal{M} \) is said to be computationally efficient if \( f \) and \( \tilde{p}_i \) can be determined in poly-time. As the patients are individually rational, they will try to maximize their utility. The utility function for \( i^{th} \) patient is \( u_i \) and is given as \( u_i : \mathcal{A} \times \pi_i \rightarrow \mathbb{R} \). The utility of any patient can be defined as the true valuation minus the payment that is to be paid by him if the preferred


4 Truthful Greedy Mechanism for Hiring Experts

In this section, we propose an approximately efficient and truthful greedy mechanism motivated by [3][9] for the experts hiring problem. The proposed mechanism consists of an allocation rule and payment rule, those when tide up leads to an incentive compatible mechanism.

**Allocation rule** In the Allocation rule, firstly each of the patients are placed at the suitable position in the decreasing ordering of the patients based on the criteria \( \frac{v_i}{\sqrt{|D_i|}} \) such that \( \frac{v_1}{\sqrt{|D_1|}} \geq \frac{v_2}{\sqrt{|D_2|}} \geq \ldots \geq \frac{v_n}{\sqrt{|D_n|}} \). Once sorting is done, the next step is, in each iteration the allocation mechanism greedily selects a patient \( p_i \) that satisfies the condition that the set of preferred experts by \( p_i \), say \( D_i \) and the set of preferred experts by the patients in partially determined winning set \( Q \) i.e. \( \cup_{k \in Q} D_k \) shares no common experts i.e. \( D_i \cap (\cup_{k \in Q} D_k) = \phi \). Finally, the allocation rule terminates once all the patients in set \( P \) is accessed.

**Payment rule** Once the winning set of patients i.e. \( Q \) is determined, next goal is to decide the amount they will pay against the consultancy by the preferred experts. In the payment rule, for determining the payment of each patient \( p_i \in Q \) first the smallest index \( j \) in the sorted ordering is determined such that \( D_i \cap D_j \neq \phi \), and also for all index \( \ell < j \) except index \( i \), \( D_\ell \cap D_j = \phi \). Once the index \( j \) is determined the payment of any patient \( p_i \in Q \) is given as \( \frac{1}{\sqrt{|D_i|/|D_j|}} \). In the
similar fashion, the payment of the remaining patients in the winning set $Q$ will be determined.

**Algorithm 1: Truthful Greedy Mechanism for Hiring Experts**

| Input          | $P = \{p_1, p_2, \ldots, p_n\}$ |
|----------------|----------------------------------|
| Input          | $\pi = \{\pi_1, \pi_2, \ldots, \pi_n\}$ |
| Output         | $Q \leftarrow \phi, \hat{p} \leftarrow \phi$ |

/* Allocation mechanism */

1. Sort descent($P$) // Sorting in descending order of $\frac{s_i}{\sqrt{|D_i|}} \forall p_i \in P$

2. for $i = 1$ to $n$ do
3.   if $\hat{D}_i \cap (\cup_{j \in Q} \hat{D}_j) = \phi$ then
4.   \hspace{1em} $Q \leftarrow Q \cup p_i$
5.   end
6. end

/* Payment rule */

7. foreach $p_i \in Q$ do
8.   $j^* \leftarrow \arg\min_j \{\hat{D}_i \cap \hat{D}_j \neq \phi\}; \forall j > i$
9.   if such $j$ exists then
10.      $\hat{p}_i \leftarrow \frac{v_{j^*}}{\sqrt{|D_{j^*}|/|D_i|}}$
11.      $\hat{p} \leftarrow \hat{p} \cup \hat{p}_i$
12.         end
13.    else
14.      $\hat{p}_i \leftarrow 0$
15.      $\hat{p} \leftarrow \hat{p} \cup \hat{p}_i$
16.  end
17. return $Q, \hat{p}$

### 4.1 Analysis of Algorithm 1

**Proposition 1.** The proposed mechanism is incentive compatible [3].

**Theorem 1.** The running time of the proposed mechanism (Algorithm 1) is $O(n^2)$.

*Proof.* The running time can be derived in a straightforward way. The sorting in line 1 will take $O(n \log n)$ time. For each iteration of for loop line 2-6 is bounded above by $O(1)$. As the for loop will iterate for $n$ times, so line 2-6 will take $O(n)$ time. For each iteration of for loop, line 8 is bounded above by $O(n)$ and line 9-16 will take $O(1)$ time. As the for loop will iterate for $n$ times in worst case. So, line 7-16 is bounded above by $O(n^2)$. Thus, the running time of the proposed algorithm is bounded above by $O(n \log n) + O(n) + O(n^2) = O(n^2)$.  

**Theorem 2.** Any patient $p_i \in P$ can’t gain by misreporting his true valuation.
Proof. In the similar line in [3] the more detailed proof in our setting is presented. For any patient $i$ the type information can be mis-reported by misreporting the valuation $\pi'_i = <\tilde{D}_i, v'_i>$. So, this gives rise to two different cases:

**Case 1.** Let us suppose that the $i^{th}$ winning patient deviates and reports a type $\pi'_i \neq \pi_i$ such that the valuation $v'_i > v_i$. As the patient was winning with $v_i$, with $v'_i$ he would continue to win and his utility $u_i(f(v'_i, v_{-i}), v'_i) = u_i(f(v_i, v_{-i}), v_i)$. If instead he reports $v'_i < v_i$. Again two cases can happen. He can still win. If he wins his utility, according to the definition will be $u_i(f(v'_i, v_{-i}), v'_i) = u_i(f(v_i, v_{-i}), v_i)$. If loses his utility will be $u_i(f(v'_i, v_{-i}), v'_i) = 0 < u_i(f(v_i, v_{-i}), v_i)$.

**Case 2.** If the $i^{th}$ patient was losing with $v_i$ let us see whether he would gain by deviation. If he reports $v'_i < v_i$, he would still lose and his utility $u_i(f(v'_i, v_{-i}), v'_i) = 0 = u_i(f(v_i, v_{-i}), v_i)$. If instead he reports $v'_i > v_i$. Two cases can occur. If he still loses his utility $u_i(f(v'_i, v_{-i}), v'_i) = 0 = u_i(f(v_i, v_{-i}), v_i)$, but if he wins, then he had to beat some valuation $v_j$ such that $\frac{v'_i}{\sqrt{D_i}} \geq \frac{v'_j}{\sqrt{D_j}}$. Now as he wins his utility $u_i(f(v'_i, v_{-i}), v'_i) < 0$ in this case. So he would have got a negative utility. Hence no gain is achieved.

This concludes the proof. Considering the case 1 and case 2 above, it can be concluded that any patient $i$ can’t gain by mis-reporting his valuation. □

5 Conclusion and future works

In this paper, we have addressed the more general setup in the healthcare system with multiple patients and multiple experts; where each patients has a privilege to report the required set of experts alongwith their private valuations. We have proposed, a combinatorial auction based scheme for this problem that results in an allocation which is no better than $\sqrt{m}$-approximation of the optimal allocation. Designing the more general mechanism for the set-up consisting of $n$ patients and $m$ experts with experts varying in quality can be thought of as our immediate future work.

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