On Simple Rules for the Social Cost of Carbon

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Abstract
The objective of this paper is to assess the use of simple rules for the social cost of carbon. It is shown that several interrelated objections may apply. The main issues are the following. First, the underlying theoretical models typically assume that the economy finds itself on a balanced growth path, implying that addressing the issue of designing policies for the short run, which play a role in the actual policy debate, are neglected. Second, for some cases the assumptions made regarding the marginal damages of high temperature or of large atmospheric CO$_2$ stocks are shown to be incompatible with other assumptions made. Third, typically the rules follow from an optimal growth model and associate the social cost for a particular year with GDP for that year, but it is not always acknowledged that it should be optimal rather than actual GDP for that year. I also go into the performance of simple rules as compared to first-best.

Keywords Social cost of carbon · Integrated assessment models · Long run growth

1 Introduction
The Social Cost of Carbon (SCC) is defined as the total value of damages over the entire time horizon from emitting a marginal unit of CO$_2$ into the atmosphere. The policy relevance of the SCC lies in the fact that it can be interpreted as the Pigouvian tax to be imposed on CO$_2$ emissions in a competitive economy in order to restore a social optimum.
at least in the absence of other externalities. Recently, there appears to be a tendency in the economics literature to design and propose so-called simple rules for the SCC. These rules advocate a simple, e.g., linear, relationship between the SCC and national income, GDP, based on an analytical Integrated Assessment Model (IAM). Climate change is a complex problem and a simple rule, derived from such a theoretical economic model, may help the policy maker, or scholars from other disciplines, to better understand the essential economic and climate-related features that stand out in finding an optimal carbon tax. Indeed, this is the strategy followed in the simple rules literature: To study a simple model and make assumptions that lead to analytical expressions for the carbon tax rate, followed by numerical exercises showing that the tax rule performs reasonably well, in some sense.

The principal objective of the present paper is to evaluate this approach from an economic theory perspective. Hence, the emphasis will not be on the performance of the rules, although some attention will of course be given to this issue. To illustrate the problems I consider 5 representative articles written by prominent researchers who offer a simple rule: Golosov et al. (2014), Van den Bijgaart et al. (2016), Rezai and Van der Ploeg (2016), Li (2019), and Dietz and Venmans (2019). They all start from a basic IAM that yields optimal paths over time of relevant variables such as GDP and emissions. To derive their rules they make specific assumptions, which are taken in various combinations and modifications from a set including: The absence of direct climate damages in the social welfare functions, a constant elasticity of marginal utility of consumption, constant growth rates of optimal GDP, consumption and man-made capital, and, finally, a specific relationship in the optimum between marginal damages from accumulated atmospheric greenhouse gases and GDP.

The present paper identifies potential inconsistencies regarding these assumptions and the consequences thereof for the applicability and the performance of the simple rules. Although the first two assumptions mentioned, no direct damages and constant elasticity, are instrumental I will not go into them too deeply. The main problem lies in the assumptions of constant growth rates and the relationship between GDP and marginal damages. First, in some models the chosen production structure is incompatible with marginal damages being proportional to GDP, as postulated by several authors. Second, after establishing the SCC as a relatively simple function of GDP in an optimal growth model, some authors relate the SCC to actual rather than optimal GDP. An additional, and more general, issue is that if the economy is on a balanced growth path, inverted-U shaped patterns of e.g., GDP and hence the SCC are ruled out by definition, whereas inverted U-shaped patterns may and do actually occur in many IAMs, generating non-monotonic optimal GDP patterns. Moreover, with the emphasis on long run growth, short term problems, that are presently faced by governments to fight climate change, get less attention then. Finally, the authors make little effort to justify their postulated growth rates as an endogenous outcome of their modelling, in contrast with e.g., Tsur and Zemel (2009) and Nordhaus and Sztorc (2013).

Although I focus on theoretical issues, I will argue in favor of an alternative approach. Economists’ capacity to solve large dynamic optimization problems is strong enough to present to policy makers solution paths over time, rather than just simple rules. This is precisely what the authors do themselves in the process of assessing their rule: They compare the rule to first-best, which necessitates to calculate the first-best. Moreover, sensitivity analyses can be provided and presented to policy makers, who can then make their choices between scenarios. This approach is certainly not novel and has been implemented in practice. (See e.g., U.S. Interagency Working Group, 2010, as well as Newell and Pizer, 2003). I agree that IAMs can look like black boxes and may be difficult to understand. So, models,
like those used to yield simple rules, might be very useful for explanatory purposes, but not primarily for policy purposes. I will address the merits of each of the individual models in due course.

An important feature of climate change is uncertainty. Therefore the sensitivity of the social cost of carbon with respect to uncertainty clearly deserves attention. The literature on this topic is abundant. See e.g., Cai and Lontzek (2019), Gollier (2002, 2004, 2012), Gollier and Weitzman (2010), Lemoine and Rudik (2017): Nordhaus (1994), Newell and Pizer (2003), Weitzman (1998, 2001 and 2014). See also Bremer and Ploeg (2021) and Olijslagers et al. (2021) for a recent contribution. Special attention has been given to the issue of catastrophic climate change. See e.g. Lemoine and Traeger (2014) and Tsur and Zemel (2021), the latter for a recent survey with due attention to their own contributions. Taking uncertainty into account typically leads to more complicated expressions for the social cost of carbon, in spite of the fact that sometimes analytical results are obtained (see e.g. Van der Bremer and Van der Ploeg, 2021). For the purpose of the present paper, namely to scrutinize simple rules, abstracting from uncertainty suffices.

The remainder of the paper is organized as follows. Section 2 describes an IAM that basically comprises the theoretical models underlying the simple rules that I discuss. It also indicates under what conditions a simple rule can be derived. Section 3 describes the specific models as well as the simple rules proposed by the five groups of scholars mentioned above. Moreover, it is shown that the assumptions made to obtain the simple rule are incompatible with each other in some instances. This is what will be called internal inconsistency. Subsequently, Sect. 4 critically assesses these simple rules further. Assumptions underlying the rules that will be discussed are constant growth rates, and particular assumptions regarding marginal damages. Section 5 discusses the performance of the simple rules. It describes the approaches the authors use to show that their rule is performing well. Finally, Sect. 6 concludes.

2 How to Derive a Simple Rule?

2.1 A prototype IAM model

In the present section a slightly modified version of the model developed by Rezai et al. (2014, 2020) is employed, that can be seen as a prototype of the analytical IAMs used to design and to evaluate climate policy. The models yielding the simple rules that I discuss, are in most instances special cases. BGL, CZL, DV, GHKT and RP stand for Van den Bijgaart et al. (2016), Li (2019), Dietz and Venmans (2019), Golosov et al. (2014) and Rezai and Van der Ploeg (2016), respectively.

The model captures the essence of IAMs. It includes a climate module and describes the accumulation of capital, in a closed economy. The labor force $L$ equals population and grows at an exogenously given and constant growth rate $\mu$. Initial labor is set equal to unity without loss of generality. Instantaneous welfare $W$ depends positively on per capita consumption $C/L$ and negatively on temperature $T$. Some authors introduce the notion of an effective $CO_2$ stock that causes damages. From a mathematical perspective this approach is identical to relating damages to temperature.
E. Instantaneous utility is discounted at a constant rate of pure time preference $\rho$. The time argument is omitted when there is no danger of confusion. Total welfare

$$\int_0^\infty e^{-\rho t} LW(C/L, T) dt,$$

is to be maximized. The economy’s GDP is

$$GDP = Y(K, L, F, R, S, T, t) = \dot{K} + \mu K + C,$$ (1)

where GDP has the following components,

$$Y(K, L, F, R, S, T, t) = Z(K, L, F + R, t)$$

$$- D(T) Z(K, L, F + R, t) \dot{Z}(K_0, L_0, F(0))$$

$$+ R(0), 0)^{1-\varepsilon} - G(S) F - bR.$$ (2)

Here $K$ denotes man-made capital, $F$ is the input from a non-renewable resource (fossil fuel), and $R$ is the use of a renewable resource that is perfectly substitutable for the non-renewable one. The argument $t$ allows for exogenous technological progress. Temperature has a negative impact on production, due to climate change: $D'(T) \geq 0$, brought about by the accumulation of greenhouse gases. The unit extraction cost of fossil fuel from the stock $S$ is represented by $G(S)$ where $G$ is a decreasing function. Renewables are produced with a linear technology, requiring an amount $b$ of output per unit of production of the renewable resource. GDP equals gross production

$$Z(K, L, F + R, t)$$

minus the sum of losses due to climate change, given by $D(T) Z(K, L, F + R, t) \dot{Z}(K_0, L_0, F(0) + R(0), 0)^{1-\varepsilon}$, extraction cost of fossil fuel $G(S) F$ and the cost of producing renewables $bR$. The parameter $\varepsilon$ is introduced to make a distinction between additive ($\varepsilon = 0$) and multiplicative ($\varepsilon = 1$) damages and to allow for intermediate cases.\(^2\) Emissions of $CO_2$ are proportional to fossil fuel use. Part $q_L$ of emissions stays in the atmosphere forever. The accumulated stock from these emissions thus follows from

$$\dot{E}_1 = q_L F.$$ (3)

The transient stock of $CO_2$ follows from

$$\dot{E}_2 = (1 - q_L) \varphi_0 F - \varphi \dot{E}_2.$$ (4)

\(^2\) The formula $D(T) Z(K, L, F + R, t) \dot{Z}(K_0, L_0, F(0) + R(0), 0)^{1-\varepsilon}$ appears in Rezai et al. (2014). It is argued there that on the one hand higher temperatures lead to respiratory and other diseases inducing lower levels of health, productivity and aggregate output. Climate change also destroys the production potential of agriculture, the more so the higher is the temperature. This justifies the multiplicative formulation of damages. On the other hand, climate change destroys habitats and is detrimental to biodiversity, which can be represented as damages not proportional to GDP. Intermediate cases have $0 < \varepsilon < 1$. It was pointed out by a referee that in formula (2) with $\varepsilon = 0$, damages are proportional to initial gross production, which is an endogenous entity in case energy input is to be chosen optimally. For the case of additive damages ($\varepsilon = 0$) it would therefore be better to replace initial gross production by an exogenous parameter.
with $\varphi_0$ a scale parameter and $\varphi$ a decay factor. Total accumulated emissions are $E = E_1 + E_2$. Temperature follows from

$$\dot{T} = \frac{1}{\varphi T}(E - T) = \frac{1}{\varphi T}(E_1 + E_2 - T).$$

(5)

The fossil fuel stock is depleted by extraction. Moreover, the stock should stay non-negative:

$$\dot{S} = -F, \quad S(t) \geq 0.$$  

(6)

All initial stocks are given and denoted as: $K_0$, $L_0$, $S_0$, $T_0$, $E_{10}$, $E_{20}$, $E_0$.

The optimal path of the economy is derived from maximizing social welfare subject to the conditions described above, including the obvious non-negativity conditions. Omitting the arguments of $Y$ and setting $L_0 = 1$, one can write the Hamiltonian of the problem as

$$\Lambda = e^{(\pi - \rho)t} W(e^{-\pi t}C, T) + \lambda[-F] + v_1[\varphi_L F] + v_2[\varphi_0 (1 - \varphi_L) F - \varphi E_2]$$

$$+ \kappa[Y - C - \mu K] + v[\frac{1}{\varphi T}(E_1 + E_2 - T)].$$

Here $\lambda$, $v_1$, $v_2$, $\kappa$ and $\nu$ are the co-states or shadow prices corresponding with the resource stock, the permanent $CO_2$ stock, the transient $CO_2$ stock, capital and temperature, respectively. According to the Pontryagin maximum principle the necessary conditions for optimality entail the maximization of the Hamiltonian with respect to the instruments $C$, $F$ and $R$. Moreover, the shadow price $p$ of a state variable $X$ satisfies $-\dot{p} = \partial H/\partial X$. This yields as necessary conditions (with for each necessary condition the relevant policy and state variables between parentheses):

$$e^{\pi t} W_C(e^{-\pi t}C, T) = \kappa, \quad (C),$$

(7)

$$Y_F = \frac{\lambda}{\kappa} + \frac{-v_1 \varphi_L - v_2 \varphi_0 (1 - \varphi_L)}{\kappa} \quad \text{if} \quad F > 0, \quad (F),$$

(8)

$$Y_R = 0 \quad \text{if} \quad R > 0, \quad (R),$$

(9)

$$-\dot{\lambda} = -\kappa Y_S, \quad (S),$$

(10)

$$-\dot{\nu} = e^{(\pi - \rho)t} W_2(C/L, T) + \kappa Y_T - \nu \frac{1}{\varphi T}, \quad (T),$$

(11)

$$-\frac{\dot{\kappa}}{\kappa} = Y_K - \mu, \quad (K),$$

(12)

$^3$ This equation captures the discrete-time version used by RP where $T_t - T_{t-1} = (E_t - T_{t-1})/\varphi T$ to allow for a delay between temperature and accumulated $CO_2$. With $\varphi_T = 1$ it holds that $E_t = T_t$, and adjustment is immediate. This corresponds with $\varphi_T = 0$ in this continuous time version.
The interpretation of these conditions is straightforward. Equation (8), for example, says that in an optimum where fossil fuel is used, i.e., \( F > 0 \), the marginal product of fossil fuel, net of extraction cost \( Y_F \), equals the sum of the Hotelling rent \( \lambda / \kappa \), and the cost of the externality caused by the use of fossil fuel. The latter is the social cost of carbon SCC: The loss of welfare due to a marginal increase of emissions. Hence

\[
SCC = -\frac{\varphi_L v_1 + \varphi_0 (1 - \varphi_L) v_2}{\kappa}.
\]

This is also the tax rate \( \tau \) to be imposed in order to induce perfectly competitive firms to use the optimal fossil fuel input.

One assumption that is commonly made, and in which I will follow the authors, is that instantaneous welfare does not directly depend on the stock of atmospheric \( CO_2 \), as might be the case due to for example increased health problems and the loss of biodiversity (see e.g., the latest report by the Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services (2019)). With a more general instantaneous welfare function the SCC includes this additional externality. Van der Ploeg and Withagen (1990), Tahvonen and Kuuluvainen (1991), Michel and Rotillon (1995), Hoel and Sterner (2007) and Zhu et al. (2019) take this into account. Of the studies discussed below only GHKT go briefly into the issue and argue that their rule may fundamentally change. Another assumption made by some authors is a constant elasticity of marginal utility. In spite of the fact that there is no general agreement on the size of the elasticity (see Drupp et al., 2018 and Groom and Maddison, 2019) and that it may differ across regions (see Anthoff and Emmerling, 2019) it is relatively innocuous and I will not comment on its use any further.

### 2.2 Solving the Model

Define \( x(t) = \frac{v_1(t)}{\kappa(t)} \), \( y(t) = \frac{v_2(t)}{\kappa(t)} \) and \( z(t) = \frac{v(t)}{\kappa(t)} \). Hence, \( x(t) \), \( y(t) \) and \( z(t) \) can be interpreted as the value in terms of final commodities of an increase of the permanent \( CO_2 \) stock, the transient \( CO_2 \) stock and temperature. Then \( SCC(t) = -\varphi_L x(t) - \varphi_0 (1 - \varphi_L) y(t) \). Also

\[
\dot{x} = -\frac{1}{\varphi_T} z - \frac{\dot{k}}{\kappa} x,
\]

\[
\dot{y} = -\frac{1}{\varphi_T} z + (\varphi - \frac{\dot{k}}{\kappa}) y,
\]

\[
\dot{z} = -Y_T + (\frac{1}{\varphi_T} - \frac{\dot{k}}{\kappa}) z.
\]

By \( g_x(t) \), \( g_y(t) \) and \( g_z(t) \) I denote the respective growth rates, whereby the argument \( t \) indicates that the growth rates are not a priori constant. So, \( \dot{x}(t) = g_x(t) x(t) \), \( \dot{y}(t) = g_y(t) y(t) \), \( \dot{z}(t) = g_z(t) z(t) \).
and $z(t) = g_z(t) z(t)$. The differential Eqs. (16), (17), (18) yield the following expression for the social cost of carbon:

$$SCC(t) = \frac{\psi_L}{\kappa + g_z(t)} + \frac{\psi_0(1 - \psi_L)}{\kappa + g_y(t) - \varphi} \frac{Y_T(t)}{1 - \psi_T(\frac{\kappa}{\kappa} + g_z(t))},$$

(19)

with the impact of temperature on GDP given by

$$Y_T(t) = -D'(T(t)) Z(K(t), L(t), F(t)) \varphi_0(1 - \varphi_L) Z(0)^{1-\epsilon},$$

(20)

so that, in shorthand,

$$SCC(t) = \frac{\psi_L}{\kappa(t) + g_z(t)} + \frac{\psi_0(1 - \psi_L)}{\kappa(t) + g_y(t) - \varphi} \varphi_T(\kappa(t) + g_z(t)) - 1.$$

Equation (21) will play a crucial role in the discussion of the literature, to which I proceed in due course, after offering a preview here. The expression is a necessary condition for optimality. It does not look simple at all, but it would be a simple rule if it could be written as $SCC(t) = f(Y(t))$, with $f$ linear or exponential. To arrive at such a rule it is instrumental to relate marginal damages from higher temperatures, $D'(T(t)) Z(T)^{\varphi_0(1 - \varphi_L)} Z(0)^{1-\epsilon}$, to GDP in a simple way. One option used in the literature is to set $\varphi_0 = 1$ and $1 - D(T) = e^{-\beta T}$ (with $\beta$ constant). Moreover, it would be helpful if expressions such as $\kappa(t) + g_z(t)$ could be reduced to constants. This recipe works well if the elasticity of marginal utility is constant, as is assumed, and the growth rate of consumption in the optimum is a constant, say $g$, because then the Ramsey rule applies: $r(t) = Y_K - \mu = -\kappa(t) = \rho + \eta(g - \pi)$, saying that the discount rate $r$ is the sum of the pure rate of time preference and the product of the elasticity of marginal utility and per capita consumption growth. Hence, the two assumptions of isoelastic utility and a constant growth rate make the discount rate a constant, provided the pure rate of time preference is constant as well. It goes without saying that this is helpful in designing a simple $SCC$, where discounting is required. To end this Section, I introduce two definitions.

**Definition 1** A rule for the social cost of carbon is called simple if it is expressed as $SCC(t) = f(Y(t))$.

**Definition 2** A simple rule is internally consistent if applying the rule leads to the first-best in the model on which it is based.

Hence, definition 1 does not allow for rules that have multiple variables, be it economic or non-economic, as arguments. Definition 2 does not allow for rules that are based on assumptions that are incompatible with each other.

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4 More on the application of the Ramsey Rule in Integrated Assessment Models can be found in Tol (2009), who surveys 211 estimates, and in Drupp et al. (2018), who have develop a survey among experts on the constituent parts of the Ramsey Rule. See also Ramsey (1928) and Kolstad (2014) on the rate of pure time preference. Weitzman (2001), Pindyck (2016), and Arrow et al. (2012, 2013) write on the role of experts, Groom and Maddison (2019) on the elasticity of marginal utility. For real world applications see Werkgroep Discontovoet (2015) and Centraal Planbureau (2015) for the Netherlands, Cropper (2012) for the US, and the Committee for an Official Shadow Price of Carbon (2018) for France.
3 Simple Rules in the Literature

In this Section I discuss five rules for the social cost of carbon. Thereby I will address the consistency issue mentioned above. The notation is unified as much as possible in terms of the model introduced in the previous Section. I will first consider two articles that do not assume constant growth rates from the start, GHTK and CZL. The next triple of articles starts with RP who provide a more general model than the subsequent two articles, followed by BGL and DV. In spite of the focus on critically assessing theoretical issues, the contributions certainly have important merits, that will be mentioned after each article’s discussion.

3.1 Golosov, Hassler, Krusell and Tsyvinski (GHKT)

The model actually studied by GHKT is richer than the version I discuss here. It has more sectors than I describe, I assume away renewables \((R \equiv 0)\) and allow for one type of fossil fuel only, that is costless to extract \((G(S) = 0)\), and some of the parameters that I take as constant are allowed to vary over time in the GHKT paper. They also use a more general climate module than the one introduced in my Sect. 2 above, but that latter module serves as an example given by GHKT. In spite of these simplifications my version of the model illustrates the essence of my concerns. But I do follow the “general” GHKT model in that it has log utility, a constant savings rate \((\sigma)\) (in their model following from log utility, Cobb–Douglas final output and full depreciation, in discrete time), constant population \((\pi = 0)\), and in that it employs a specific damage function:

\[
Y_T = -\vartheta Y.
\]

Without loss of generality temperature can be identified with accumulated greenhouse gases: \(T = E_1 + E_2\), without delay.

The Hamiltonian reads

\[
\Lambda = e^{-\rho t} \ln C + \lambda [-F] + v_1 [\varphi_L F] + v_2 [\varphi_0 (1 - \varphi_L) F - \varphi E_2] + \kappa [Y - C - \mu K].
\]

The relevant necessary conditions are:

\[
e^{-\rho t} \frac{1}{C} = \kappa,
\]

\[
Y_T = \frac{\dot{\lambda}}{\kappa} + \frac{-v_1 \varphi_L - v_2 \varphi_0 (1 - \varphi_L)}{\kappa} = \frac{\dot{\lambda}}{\kappa} + SCC,
\]

\[
-v_1 = \kappa Y_T = -\kappa \vartheta Y,
\]

\[
-v_2 = \kappa Y_T - v_2 \varphi = -\kappa \vartheta Y - v_2 \varphi.
\]

Hence,

\[
-v_1 = \kappa Y_T = -\frac{\vartheta}{1 - \sigma} e^{-\rho t}, -v_2 = -\frac{\vartheta}{1 - \sigma} e^{-\rho t} + v_2 \varphi,
\]

Implying

\[
SCC(t) = \vartheta Y(t) \{ \frac{1}{\rho} \varphi_L + \frac{1}{\rho + \varphi} \varphi_0 (1 - \varphi_L) \}.
\]
Expression (22) is equivalent to (12) in GHKT (o.c., page 54). This is clearly a simple rule. It is also internally consistent, in the sense that no contradictory assumptions have been made. It establishes a relationship in the optimal economy. However, the authors argue that “... no knowledge about future technology, productivity or labor supply is needed to calculate the marginal externality cost of emissions per GDP unit” (o.c., page 54). Indeed, the ratio $\frac{SCC(t)}{Y(t)}$ does not depend on the variables mentioned. And it is true that to calculate the $SCC$ at any instant of time one only needs the value of GDP at that same instant of time. But, GDP needed in the rule is optimal GDP and it is well–known from optimal control theory, and stressed by a.o. Smulders (2012), that through the necessary conditions optimal GDP does depend on future technology etc. To determine the optimal GDP, and therefore the optimal SCC, knowledge of the entire future is therefore indispensable, even if, as assumed by GHKT, the primitives of the model are such that in an optimum the savings rate is constant. Implicitly GHKT acknowledge this by arguing that the input of oil should be optimal and should obey the resource constraint (o.c., page 54). I conclude:

The GHKT rule (22) establishes a simple relationship between optimal SCC and optimal GDP. It is a simple and consistent rule. A fruitful application of the rule requires full knowledge of the optimum in the underlying model.

In spite of these shortcomings the GHKT model is widely cited, because it is a great achievement to find an analytical expression for the SCC in a rather general model (including uncertainty). In Sect. 5 we will have to say more about the numerical performance of the GHKT model.

3.2 Chuan-Zhong Li (CZL)

CZL’s theoretical model (o.c. Section 2) reduces to a special case of the model of Sect. 2. It does not include renewables ($R \equiv 0$, extraction cost of fossil fuel is zero ($G(S) \equiv 0$), the climate module has $E_1 \equiv \varphi_L = \varphi = 0$, so that the temperature increase is written as $\dot{T} = \varphi_0 F$, population is constant ($\pi = 0$). If $\varepsilon = 1$ and $1 - D(T) = e^{-\beta T}$ (with $\beta$ constant), as assumed by CZL, then GDP can be written as $Y(K, F, T, t) = e^{-\beta T} Z(K, F, t)$. So, damages are purely multiplicative. Note that this implies $Y_T = -\beta Y$.

The Hamiltonian reads,

$$\Lambda = e^{-\rho t} U(C) + \kappa [e^{-\beta T} Z(K, F, t) - C - \mu K] + \nu \varphi_0 F - \lambda F,$$

where the co-state variables correspond to the same state variables as in the previous section. The necessary conditions include $-\dot{\varphi} = -\kappa \varphi Y$ and $-\dot{\kappa} = Y_K - \mu$. Also $SCC = -\frac{\dot{\varphi_0} \nu}{\kappa}$. Define $z = \frac{\dot{\kappa}}{\kappa}$. Then $\dot{z} = \frac{\dot{\kappa}}{\kappa} - \frac{\dot{\varphi}}{\varphi} = \beta Y + rx$, with $r = Y_K - \mu$. It is shown in Appendix 1 that under some regularity conditions concerning convergence

$$SCC(t) = \varphi_0 \beta Y(t) \int_0^\infty e^{-\int_0^t (r(s) - g(s))ds} ds,$$

(23)

---

5 GHKT work in discrete time. This explains the difference between the expressions for the SCC in their article and my formula (22).

6 An important topic in climate change economics, namely the transition from fossil to renewables, is therefore not addressed in this model. In a calibrated model later on in the CZL paper it does play a role.
where $\dot{Y}(t)/Y(t) = g(t)$. Hence, the social cost of carbon is proportional to $GDP$. This does not yet make the rule a simple rule according to Definition 1 of the previous Section, because also other variables, namely the future interest rate $r(t)$ and the growth rate $g(t)$, play a role. Consistency in terms of Definition 2 does not pose a problem, like in GHKT. But to apply this rule fruitfully in the design of climate policy knowledge is needed of $GDP$, its growth rate $g(t)$ over the entire future, as well as all current and future interest rates $r(t)$. Moreover, one needs to know not their actual (or expected) values but their optimal values. The latter follow from the optimization of a full model. However, after specifying utility and production functions and calibration CZL employs actual current world GDP which is unlikely to be optimal. I will come back to this later. I conclude.

The CZL rule (23) establishes a simple relationship between optimal SCC and optimal GDP. But it is not a simple rule, since it includes the interest rate as well as the GDP growth rate. The rule is internally consistent. A fruitful application of the rule requires full knowledge of the optimum in the underlying model.

CZL generalizes GHTK in several respects, however at the cost of a more complicated expression for the SCC. CZL allows for general isoelastic utility (instead of log utility) and allows for depreciation of capital (in a continuous-time setting). The numerical applications will be discussed in due course.

### 3.3 Rezai and Van der Ploeg (RP)

The model of Sect. 2, with $D(T)Z(K, L, F + R, t)^{\varepsilon}Z(K_0, L_0, F(0) + R(0), 0)^{1-\varepsilon}$ is also the model used by RP (see footnote 2). Their rule reads, in my notation,

$$SCC(t) = \left[ \frac{\varphi_L}{\rho + \eta(g - \pi) - \varepsilon g} + \frac{\varphi_0 (1 - \varphi_L)}{\varphi + \rho + \eta(g - \pi) - \varepsilon g} \right] \frac{\partial Y(t)^{\varphi} Y(0)^{1-\varphi}}{1 + \varphi T (\rho + \eta(g - \pi) - \varepsilon g)},$$

(o.c. page 497 formula 2), where $\vartheta$ is a constant and all denominators are assumed positive. Recall that the formula derived from optimization in the full-fledged model reads:

$$SCC(t) = \left[ \frac{\varphi_L}{-\frac{\kappa(t)}{\kappa(t)} - g_x(t)} + \frac{\varphi_0 (1 - \varphi_L)}{\varphi - \frac{\kappa(t)}{\kappa(t)} - g_y(t)} \right] D'(T(t))Z(t)^{\varphi} Z(0)^{1-\varphi}.$$  

(25)

Equivalence of the two expressions hinges upon three assumptions, made in RP’s Result 4 (o.c. page 503). One is that the elasticity of marginal utility $\eta$ is constant. As said above, I will not dwell on this. The two other assumptions are.

**Assumption 1** There exists a constant $g$ such that $\dot{C}(t)/C(t) = \dot{Y}(t)/Y(t) = g$ for all $t \geq 0$.

**Assumption 2** Marginal damages are proportional to $GDP$.

Together with Assumption 1, Assumption 2 should, according to the authors in the proof of their Result 4 (o.c. 518), be interpreted as

$$Y_{\varphi}(t) = -D'(T(t))Z(t)^{\varphi} Z(0)^{1-\varphi} = -\vartheta Y(t)^{\varphi} Y(0)^{1-\varphi},$$

(26)
With these assumptions the two expressions are easily seen to be identical, because then also $\frac{k}{z} = \rho + \eta(g - \pi)$ and we must have $g_z(t) = g_y(t) = g_e(t) = \varepsilon g$, from (16), (17), (18) with $Y(t) = \theta Y(t) - Y(0)^{1-\varepsilon} = \theta e^{gt} Y(0)$ inserted.

The rule developed in RP is a simple rule, according to my Definition 1. But several caveats apply. First, by proportionality is usually meant that one variable, e.g. $Y(t)$, can be written as a constant fraction of another one, e.g., $Y(t)$. If $Z$ has the same growth rate as $Y$ then proportionality implies a constant temperature. If the growth rates differ, but are still constants, then temperature is monotonically increasing or decreasing. It is unclear what possibility needs to be considered. Second, recall that $GDP$ is, in shorthand,

$$Y(t) = Z(t) - D(T)Z(t)e^{Z(0)} - G(S(t))F(t) - bR(t).$$

Hence, (26) requires

$$D'(T(t))Z(t)e^{Z(0)} = \theta \{Z(0) - D(T_0)Z(0) - G(S_0)F(0) - bR(0)\} e^{gt}. \quad (26')$$

It is hard to see what assumptions regarding the primitives of the model need to be made for expression (26') to hold. To illustrate the difficulty, consider the case where $\varepsilon = 1$ and $1 - D(T) = e^{-BT}$. This specification of damages is used by GHKT and it also seems to be assumed in the discussion of the parameter $\theta$ by RP (denoted by $\chi$ in RP on o.c. page 497). The correct expression for the SCC in (25) would include $D'(T)Z(t)Z(0)^{1-\varepsilon} = \theta e^{-BT} Z(t)$. The expression used by RP has $\theta Y(t) = \theta e^{-BT} Z(t) - \theta (G(S(t))F(t) - bR(t))$. Hence, there is a problem because $GDP$ and gross production are not the same. As a second example, consider the case where $\varepsilon = 1$ again and where gross production has the same growth rate $g$ as $GDP$. Then (26') says that temperature is constant. If the economy is in the carbon-free era temperature is decreasing, but the $SCC$ is not that relevant in this period of time: The $SCC$ is then only needed to make fossil fuel too expensive to use. So, suppose fossil fuel is being used. Then the fossil fuel input is constant, in order to maintain a constant temperature. But then proportionality is achieved only for a special extraction technology, namely where per unit extraction cost also rise at a rate $g$. So, the simple rule does not satisfy the consistency definition. Finally, like in the two previous rules, one needs to know the full optimum in order to apply the rule. I conclude.

The RP rule (24) establishes a simple relationship between optimal SCC and optimal GDP. It is a simple rule. The assumption of proportionality between GDP and marginal damages is not justified in many plausible cases so that the rule is internally inconsistent. A fruitful application of the rule requires full knowledge of the optimum in the underlying model.

RP (o.c. page 495) claim that they generalize GHKT. Indeed, expression (24) for the social cost of carbon closely resembles the one derived by GHKT (their Eq. (12) o.c. page 54) if $\eta = 1$, $\pi = 0$, $\varepsilon = 1$, $\beta = g$, and $\varphi_T = 0$, because then

$$SCC(t) = \left[ \frac{\varphi_L}{\rho} + \frac{\varphi_0(1 - \varphi_L)}{\varphi + \rho} \right] \theta Y(t).$$

However, RP arrive at their rule assuming a constant growth rate for GDP. In GHKT the growth rate of GDP can be anything. In that sense the simple rule designed by GHKT is more general. Still, RP are more general in the sense that GHKT use a specific damage function.
3.4 Van den Bijgaart, Gerlagh, Liski (BGL)

The theoretical model studied by BGL is, to a large degree, a simple version of the model used in RP. They assume $E \equiv E_2$, $\phi_L = 0$ and $\phi_0 = 1$. Hence, $\dot{E} = F - \varphi E$. Moreover, renewables do not play a part ($R \equiv 0$), extraction of fossil fuel is costless ($G(S) \equiv 0$), and fossil fuel is abundant ($S_0 = \infty$). Labor input grows at a rate $\pi$. Temperature $T$ affects production and GDP in the following way:
\[
Y(K, F, T, t) = Z(K, F, t) - \alpha T^\psi Z(K, F, t)^\psi (L\bar{y})^{1-\psi},
\]
with $\bar{y}$ a constant, defined as “the reference per capita income at which one-degree temperature rise leads to relative damages $\alpha$.” (o.c. page 79). Finally, the motion of temperature over time is described by $\bar{T} = \gamma(V(E) - T)$.

The Hamiltonian reads:
\[
\Lambda = e^{(\pi - \rho t)}U(C/L) + \kappa[Y - C - \mu K] + \nu[F - \varphi E] + \lambda[\gamma(V(E) - T)].
\]
The necessary conditions include:
\[
e^{-\rho t}U'(C/L) = \kappa,
\]
\[
Y_F = -\frac{\nu}{\kappa}, \text{ if } F > 0,
\]
\[
-\dot{\lambda} = \kappa Y_T - \gamma \lambda.
\]

The social cost of carbon is $SCC = -\nu/\kappa$. Define $x = \nu/\kappa$, $y = \lambda/\kappa$. Then we get the following tuple of differential equations:
\[
\dot{x} = (\varphi - \frac{\kappa}{\kappa})x - \gamma V'(E)y,
\]
\[
\dot{y} = -Y_T + (\gamma - \frac{\kappa}{\kappa})y.
\]

In order to derive the SCC a number of additional assumptions is made. These include constant elasticity of marginal utility and Assumption 1, i.e., constant growth rates of consumption and GDP (equal growth rates of these two variables is equivalent to the BGL assumption of a constant savings rate, o.c. page 79). Another assumption is that temperature $T$ is constant, implying that also accumulated CO$_2$ and fossil fuel use are constant: $V(E_0) = T_0$. Hence we write $F = F(T_0)$ and $E = E(T_0)$. Finally, marginal damages, $Y_T = -\alpha \psi T^{\psi - 1}Z(K, L, F, t)^\psi (L\bar{y})^{1-\psi}$, are supposed to grow at a constant rate $g\epsilon + (1 - \epsilon)\pi$ (o.c. page 79 Eq. (22)). With these assumptions the two differential equations reduce to:
\[
\dot{x} = (\varphi + \rho + \eta(g - \pi))x - \gamma V'(E(T_0))y,
\]
\[
\dot{y} = \alpha \psi T_0^{\psi - 1}Z(K_0, F(T_0), 0)^\psi (\bar{y})^{1-\psi} e^{(g\epsilon + (1-\epsilon)\pi)y} + (\gamma + \rho + \eta(g - \pi))y.
\]

The growth rates of both $x$ and $y$ are $g\epsilon + (1 - \epsilon)\pi$ and the social cost of carbon equals
\[
SCC(t) = \frac{\gamma \alpha \psi T_0^{\psi - 1}Z(K_0, F(T_0), 0)^\psi (\bar{y})^{1-\psi} e^{(g\epsilon + (1-\epsilon)\pi)y}}{(\varphi + \rho + (\eta - \epsilon)(g - \pi) - \pi)(\gamma + \rho + (\eta - \epsilon)(g - \pi) - \pi)}.
\]

This is formula (24) in BGL, with $\psi T_0^{\psi - 1} = 1.3e^{\psi}/m$, a constant. The assumption regarding marginal damages is also necessary, given the other assumptions.

\footnote{Note that this formulation circumvents the problem discussed in footnote 6.}
The resulting SCC is a simple rule again. But several remarks are in order. First, two crucial assumptions made are incompatible. On the one hand it is assumed that GDP, \( Y(t) = Z(t) - \alpha T^\omega Z(t)^\gamma (L(t)\bar{y})^{1-\varepsilon} \), grows at a rate \( g \). On the other hand, marginal damages \( Y_T(t) = -\alpha \gamma T^\omega T^\gamma Z(t)^\gamma (L(t)\bar{y})^{1-\varepsilon} \) grow at a rate \( g + (1 - \varepsilon)\gamma \), since temperature is assumed to be constant. Hence, there is an inconsistency unless \( \varepsilon = 1 \). However, \( \varepsilon \neq 1 \) is presented as one on the main innovations of the paper. Second, BGL argue that \( Z(K_0, F(T_0), 0) \) is actual current income (o.c. page 80), but, as in RP, income \( Y \neq Z \). Hence, the rule does not meet the requirements to be consistent. Moreover, \( Z(K_0, L_0, F(T_0), 0) \) is not current production but optimal production. Hence, the same problem arises as in the articles discussed earlier. I conclude.

The BGL rule (26) establishes a relationship between optimal SCC and optimal GDP. It is a simple rule. The rule is not internally consistent. The assumptions regarding the growth rates of GDP and marginal damages, jointly with a constant temperature, are only justified if GDP and gross production are proportional. A fruitful application of the rule requires full knowledge of the optimum in the underlying model.

The merit of the work by BGL lies in the comparison with other IAMs to which I will return in Sect. 5

3.5 Dietz and Venmans, DV

The aim of the study by Dietz and Venmans (2019) is not only to derive a closed-form solution for the SCC but also, in particular, to show that many IAMs greatly overestimate the delay between carbon emissions and temperature increase. For the purpose of the present paper the focus is on the former part of their work, while acknowledging the merits of looking at the implications of the latter issue, namely the delay. DV assume a constant elasticity of marginal utility, a constant growth rate of consumption (o.c. page 113) and a constant savings rate. GDP is specified as \( Y(K, L, T, F, t) = Q(K, e^{\omega t} L) e^{-\delta F^2/2} e^{\xi F - 2F^2/2} \) with \( Q \) linearly homogeneous and where the parameter \( \omega \) is the rate of Harrod-neutral technical progress. Hence, renewables do not play a role, fossil fuel is costless to extract and \( Y_T = -\delta TY \). They also postulate \( \dot{E} = F \) and \( \dot{T} = \gamma (\zeta E - T) \). Fossil fuel reserves are non-exhaustible. Finally, it is assumed that \( \dot{E}/E = \dot{T}/T = \delta \), a constant. This poses a first problem, of which the authors seem to be aware, but they only mention it without considering its implications (see o.c. page 114 and footnote 7). If \( \delta > 0 \) then \( F(t) = \delta L(t) \to \infty \) as \( t \to \infty \) so that \( -\gamma T^2(t)/2 + (\xi F(t) - \xi F^2(t)/2) \to -\infty \) as \( t \to \infty \) and the marginal product of energy becomes negative. This cannot be optimal, because it unnecessarily hampers GDP in the long run. Hence, to avoid inconsistencies, it must be the case that \( \delta = 0 \) so that the atmospheric \( CO_2 \) stock is constant and emissions as well as fossil fuel use are zero. In Appendix 2 it is shown that the following simple rule for the social cost of carbon results:

\[
SCC(t) = \frac{\gamma \zeta \theta T_0 Y(0)e^{\gamma t}}{(\rho + \eta(g - \pi) - g)(\gamma + \rho + \eta(g - \pi) - g)}.
\] (28)

This is a simple rule indeed. But with \( \delta = 0 \) the theoretical model mathematically boils down to the Ramsey-Koopmans-Cass model yielding, as is well-known, a long run constant growth rate of capital equal to the sum of the labor growth rate and the rate of technical progress. \( g = \omega + \pi \). Moreover, the SCC only serves to keep firms from using fossil fuel. Hence, the rule only gives a lower bound on the carbon tax, with the purpose to ban fossil fuel from the outset.
Therefore, if initial temperature is ‘too’ low, the assumption of a constant temperature is not warranted, at least not in an optimum. I conclude.

The DV rule (27) establishes a simple relationship between optimal SCC and optimal GDP. It is a simple and internally consistent rule if the economy is carbon-free from the start. A fruitful application of the rule requires full knowledge of the optimum in the underlying model.  

4 Constant and Exogenous Growth Rates

In the present Section I comment on the constant growth rates assumption made in RP, BGL and DV. I will argue that assuming constant growth rates requires specific initial conditions in growth models. Moreover, many plausible IAMs may have negative growth rates in the long run.

The assumption of positive constant growth rates of GDP, consumption and the SCC is ad hoc. A growth rate should be endogenous, explained by e.g., population growth, technical progress, accumulation of human capital, and innovation in energy technologies. The arguments in favor of constant exogenous growth rates are not explicitly given in the theoretical models that we have considered. Moreover, exogeneity suggests that growth rates are not affected by climate policies. However, climate change policy is a non-marginal phenomenon and thus affects growth. Burke et al. (2015) show that the GDP growth rate is affected by climate change. Moreover, Dietz and Hepburn (2013) provide an overview of the literature, showing that the economics profession has been aware for long of the consequences of large projects for cost–benefit analysis (see also Dasgupta et al. 1972 and Stiglitz 1974). Dietz and Hepburn present a convincing argument why valuing the reduction of global carbon emissions as if it were a marginal project, may lead to serious errors. Also the study by Drupp et al. (2018), a survey among “experts”, shows a large variation in the projected per capita growth rates.

The economics literature provides many examples of models of economic growth where along a social welfare optimum GDP asymptotically approaches a path with constant growth, or, under specific initial conditions, finds itself on balanced growth from the outset. Oftentimes convergence is monotonic, like in the seminal Ramsey (1928) model or its extensions with population growth and Harrod-neutral technical progress. But there are also models from resource economics with exhaustible fossil fuel, where optimal consumption and GDP are inverted-U shaped. See Dasgupta and Heal (1974), Stiglitz (1974), Withagen (1990). Early analytical papers on capital accumulation and pollution include Forster (1973) and Keeler et al. (1971). More recently, and inspired by climate change,

\[ \xi Y(t) \leq SCC(t) = \frac{\gamma \xi \theta T_0 Y(t)}{(\rho + \eta (g - \pi) - g)(\gamma + \rho + \eta (g - \pi) - g)} . \]

Two minor remarks are in order. With a constant temperature their Proposition 2 is not correct, because then fossil fuel use is zero. Their Corollary 2 also holds for the more general specification of damages, as long as temperature is constant.

Studies where the SCC and the growth rate are endogenously derived from an Integrated Assessment Model are e.g., Tsur and Zemel (2009) and Nordhaus and Sztorc (2013), already mentioned.

Whether or not there is a phase with increasing consumption depends on the level of the pure rate of time preference and the initial resource stock (see e.g., Benchekroun and Withagen, 2011).
the focus is on the transition from fossil fuel to renewables. A recent analytical model that closely resembles the model of Sect. 2 was developed by Van der Ploeg and Withagen (2014). They find that in the long run the economy will (asymptotically) converge to the carbon-free steady state. If the economy is originally abundant in fossil fuel, consumption will initially rise, overshooting the steady state carbon-free consumption rate, and will eventually decrease towards the steady state. Simulations show that the carbon tax may be inverted-U shaped, depending on the question whether the economy is still developing or mature, in terms of capital. CZL does not impose constant growth rates in his theoretical model, and in a calibrated model he confirms the optimality of a $\text{SCC}$ that is initially rising and eventually decreasing. Also Okullo et al. (2020) should be mentioned, who reconsider GHKT and find that in an optimum the $\text{SCC}$ is inverted-U shaped as well. I will come back to the implications for social welfare in the next Section.

The assumption of steady growth may also cause a difficulty with the initial conditions. As an example consider RP with $\varepsilon = 1$. If the steady state prevails from the outset then.

$$Y_{K}(0) = Z_{K}(K_{0}, 1, F(0)) (1 - D(T_{0})) = \rho + \eta(g - \pi) + \mu.$$ 

Also

$$Y_{T}(0) = -D'(T_{0})Z(K_{0}, F(0)) = -\theta Y(K_{0}, F(0))
= -\theta[(1 - D(T_{0}))Z(K_{0}, 1, F(0)) - G(S_{0}) F(0)]$$

So, with a given initial temperature, to be on a balanced growth path from the outset, initial fossil fuel input and initial capital should take specific values. A similar issue presents itself in the BGL model, with a constant temperature. Maintain the assumption of $\varepsilon = 1$, for the sake of exposition. Then, with a constant growth rate $g$ of consumption it holds that

$$Y_{K}(K_{0}, 1, F(T_{0}), T_{0}, 0) = \rho + \eta(g - \pi) + \mu.$$ 

$$Y_{F}(K_{0}, F(T_{0}), T_{0}, 0) = \text{SCC}(0) = \frac{\gamma \alpha \gamma T_{0}^{w-1}Z(K_{0}, 1, F(T_{0}), 0)}{(\phi + \rho + (\eta - 1)(g - \pi) - \pi)(\gamma + \rho + (\eta - 1)(g - \pi) - \pi)}.$$ 

These are two equations with the initial capital stock and the initial temperature as variables. Hence, under the balanced growth assumption there are a specific temperature and a specific capital stock that yield balanced growth. If the actual temperature and the actual initial capital stock deviate, the $\text{SCC}$ does not inform the policy maker of the optimal tax. Finally, also in DV specific initial values are needed.

5 Performance of the Simple Rules

One question the authors of simple rules tend to ask themselves is how well their rule performs. Obviously, the answer depends on the entity to which the simple rule is compared and on the criterion that is used to evaluate differences. In this Section we briefly discuss and compare the ways the papers dealt with in this article address the performance as well as the robustness of the rules designed. We keep the same order as in the previous section.

A preliminary remark is in order. Different studies come up with different (long run) growth rates of the $\text{SCC}$. Let the growth rate of $\text{GDP}$ (be it actual or optimal) be equal to $g(t)$. Then the growth rates of the $\text{SCC}$ in GHKT and CZL are left unspecified; in DV it is $g$, in RP it is $\varepsilon g$, and in BGL it is $\varepsilon(g + \pi) - \pi$, where $g$ is constant and $\varepsilon$ is the
constant elasticity of damages with respect to gross production: \( D(E(t)) Z(t)^\varepsilon Z(0)^{1-\varepsilon} \). For \( \varepsilon = 1 \) they are all equal. But for \( \varepsilon \neq 1 \) divergence occurs. The question arises what topology is needed to make these growth rates the same in the corresponding metric, even if there exists agreement on the value of \( g \). Indeed, although for small differences in \( \varepsilon \) growth rates of GDP are close, levels may differ substantially in the long run. Hence, if one would want to answer the question which simple rule is “the best”, the answer would hinge on the assumed growth rates.

The procedure chosen in evaluating the simple rules consists in considering a somewhat more general model than the theoretical ones on which the simple rule is based, to calculate the optimum, and then see in how well the simple rule SCC performs to achieve the optimum. Also several robustness checks are done. CZL argues that his rule comes close to the first-best carbon tax in DICE2016. Also BGL extensively “test” their formula on DICE. In addition they acknowledge potential differences and identify their causes. RP claim that their rule is very close to first-best, in a calibrated extension of their original model. DV do not compare their rule to others, but they do mention similarities between the growth rates of the SCC and other studies. Finally, GHKT advocate their rule, without referring to other IAMs, apart from mentioning that by a proper choice of parameters, their rule can reproduce the SCC advocated by Nordhaus (2008) and Stern (2007).

Since the main contribution of this paper concerns the theoretical aspects of the studies I only briefly mention the work by Okullo et al. (2020), who investigate the robustness of the SCC developed by GHKT. GHKT perform several numerical exercises with an extension of the small model that we discussed in Sect. 3, but maintain the assumptions of log preferences, Cobb–Douglas production function (and full depreciation in a period of 10 years). They carefully select parameter values from various studies. They calculate the optimum as well as the outcomes under laissez-faire. GHTK argue (o.c. page 82) that “the optimal tax computation relying on our closed form is likely robust to a number of extensions”, for example by considering values of the coefficient of relative risk aversion (see also Barlage (2014)). Okullo et al. (2020) cast some doubt on this statement at least with respect to the modelling of preferences and production. With general isoelastic utility and CES production they consider a GHKT-like model that allows for a transition from fossil fuel to renewables. They perform two types of optimization. First, they calculate the overall first-best. Second, they calculate a constrained optimum, where the government is restricted to imposing a carbon tax that is linear in GDP, as advocated in GHKT, say \( SCC(t) = \alpha Y(t) \), where the coefficient is constant, but can be chosen optimally. Then they find that first-best tax and the simple tax rule do not deviate too much with low substitutability between renewables and fossil but this is no longer so when renewables and fossil fuel are close substitutes. A similar comparison holds for the welfare analysis where welfare is expressed in so-called balanced growth equivalents. With high substitutability the rule performs as well as with no tax at all, i.e., business as usual. If the constant \( \alpha \) is a priori set equal to the constant used by GHKT, the outcome is even worse, of course. These results also seem to run counter to the findings of RP, who assume perfect substitutability, isoelastic utility and CES production and claim that their rule “yields only negligible welfare losses compared to the true welfare optimum...” (RP, p.493), because RP also use a linear rule.

My conclusion is that more research on performance is needed. I leave this for further research since the core of the present paper is theoretical in nature. Nevertheless, any comparison requires the calculation of the ‘real’ first-best, so that one wonders what one loses by imposing the ‘really’ optimal SCC.
6 Conclusion

The objective of the simple rule literature is to be praised: Presenting a tax rule that is relatively easy to understand for policy makers. However, it has been demonstrated in the present paper that one should be careful in proposing simple rules and in believing in such rules. They are derived from theoretical models that are sometimes based on conflicting assumptions. Hence, the problem is not so much that assumptions are made, but that some assumptions cannot hold jointly. This brings to mind a quote from Solow (1956, p. 65): “All theory depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive”. Solow continues: “A "crucial" assumption is one on which the conclusions do depend sensitively, and it is important that crucial assumptions be reasonably realistic. When the results of a theory seem to flow specifically from a special crucial assumption, then if the assumption is dubious, the results are suspect”. In the case at hand assumptions on constant growth rates are difficult to reconcile with the specifications of the technology, including climate damages. And constant GDP growth is difficult to reconcile with constant growth of marginal damages, as in RP. One could argue that the difference is not large, but, without convincing evidence, this is hard to accept when it comes to climate change, which requires action in the short run. The assumption of constant growth rates neglects potentially important short run phenomena, including transition paths, as was seen above. These findings are not only relevant from a purely academic or theory perspective, although these perspectives constitute the focus of the present paper. At least in many EU countries policies are being developed to meet the Paris agreement, which sets targets also for the immediate future, say 2030. In the debates in the policy arena the issue is not so much what needs to be done in the long run but what should be done now. In particular, policy makers want to know the carbon tax for the coming years. The European Commission (2019, 2021) has proposed a Green Deal that should lead to the EU being carbon-free in 2050. Within the EU there is much opposition to this medium run objective, by Poland for example, not only because it would affect their sovereignty, but also because these countries are afraid they cannot meet the targets in the short run without giving up growth. Hence, in practice there is nowadays serious concern that the path to the (new) steady state may be painful. It should therefore be addressed carefully, at least attention should be paid to reconciling the objective with respect to growth and those with respect to climate change.

One may ask whether it wouldn’t be better for the design of policy to confront policy makers with a set of scenarios, describing in some detail the accumulation of atmospheric $CO_2$, paths of capital accumulation, exhaustion of fossil fuel, consumption paths, consistent with each other, and let the policy maker make a choice? This procedure is by no means new. It is followed by inter alia the US Interagency Working Group (2010), which by itself does not reveal its preference for one scenario over another. Technically, the design and presentation of scenarios is relatively easy, given the enormous power we nowadays have in performing numerical calculations. One objection could be that preferences of policy makers should be revealed ex ante, not ex post. However, in this respect I would like to quote from Nobel Prize Winner Tjalling Koopmans: “Ignoring realities in adopting ‘principles’ may lead one to search for a nonexistent optimum or to adopt an optimum that is open to unanticipated objections” (Koopmans, 1965). Indeed, if the policy maker does not understand what the ingredients of the model are, then it is
Appendix 1

Derivation of the CZL rule (22)

The differential equation for $z$ in Sect. 3.2 reads: $\dot{z} = \theta Y + rz$. With $z = uw$ it follows from the\linebreak $\dot{w} = \theta Y(0)e^{\int_0^t (g(t) - \tau(t))dt}$ so that $w(t) = \theta Y(0) \int_0^t e^{\int_0^s (g(s) - \tau(s))ds} ds + w(0)$ and\linebreak $u(t)w(t) = e^{\int_0^t (g(t) - \tau(t))dt} [\theta Y(0) \int_0^t e^{\int_0^s (g(s) - \tau(s))ds} ds + w(0)]$. Provided that $\lim_{t \to \infty} e^{\int_0^t (g(t) - \tau(t))dt} = \infty$ and under the assumption that the SCC is bounded from above, it follows that:\linebreak $\lim_{t \to \infty} [\theta Y(0) \int_0^t e^{\int_0^s (g(s) - \tau(s))ds} ds + w(0)] = 0$ so that $\theta Y(0) \int_0^\infty e^{\int_0^s (g(s) - \tau(s))ds} ds = 0$.

This implies $SCC(t) = \phi_L \theta Y(t) \int_0^\infty e^{-\int_0^s (g(s) - \tau(s))ds} ds$ Q.E.D.

Appendix 2

Derivation of the DV rule (26)

In the notation of Sect. 2 the economic model analyzed by DV reads as follows.

Max $\int_0^\infty e^{-\rho t} L(U(C/L)) dt$,

subject to.

$L/L = \pi$, $L(0) = L_0 = 1$,

$Y = C + K + \mu K$, $K(0) = K_0$,

$\dot{E} = F$, $E(0) = E_0$,

$\dot{T} = \gamma(\zeta E - T)$, $T(0) = T_0$,

$Y(K, L, F, T, t) = Q(K, L)e^{\mu t} e^{-\beta T^2 / 2} e^{F - \zeta E^2 / 2}$.

The Hamiltonian of the problem reads.

$\Lambda = e^{(\pi - \rho)t} U(C/L) + \nu F + \kappa [Y - C - \mu K] + \lambda [\gamma(\zeta E - T)]$.

Necessary conditions for an optimum are

$e^{-\rho t} (C/L)^{-\eta} = \kappa$,

$\kappa Y_F = -\nu$,

$-\dot{\lambda} = -\lambda \gamma + \kappa Y_T$. 

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\[
\dot{k} = \kappa [Y_K - \mu],
\]
\[
\dot{\nu} = \lambda \gamma \zeta.
\]
With \( z = \nu / \kappa \) and \( y = \lambda / \kappa \), we have
\[
\dot{z} = -\epsilon \zeta y - \frac{\dot{k}}{\kappa} z
\]
\[
\dot{y} = (\epsilon - \frac{\dot{k}}{\kappa}) y - Y_T
\]
Moreover, \( Y_T = -\theta TY \). With \( \dot{E}/E = \dot{T}/T = 0 \), and \( g = \dot{Y}/Y = \dot{C}/C = \dot{K}/K \) is follows that \( Y_T = -\theta T_0 Y(0)e^{\theta t} \). Then it is easily seen that
\[
SCC = \frac{\gamma \zeta \theta T(0) Y(0)e^{\theta t}}{(\rho + \eta(g - \pi) - g)(\gamma + \rho + \eta(g - \pi) - g)}.
\]

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