Spin supercurrent in Josephson contacts with noncollinear ferromagnets

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Abstract. We present a theoretical study of the Josephson coupling of two superconductors that are connected through a diffusive contact consisting of noncollinear ferromagnetic domains. The leads are conventional s-wave superconductors with a phase difference of $\phi$. Firstly, we consider a contact with two domains with magnetization vectors misoriented by an angle $\theta$. Using the quantum circuit theory, we found that in addition to the charge supercurrent, which shows a 0–$\pi$ transition relative to the angle $\theta$, a spin supercurrent with a spin polarization normal to the magnetization vectors flows between the domains. While the charge supercurrent is odd in $\phi$ and even in $\theta$, the spin supercurrent is even in $\phi$ and odd in $\theta$. Furthermore, with asymmetric insulating barriers at the interfaces of the junction, the system may experience an antiferromagnetic–ferromagnetic phase transition for $\phi = \pi$. Secondly, we discuss the spin supercurrent in an extended magnetic texture with multiple domain walls. We find the position-dependent spin supercurrent. While the direction of the spin supercurrent is always perpendicular to the plane of the magnetization vectors, the magnitude of the spin supercurrent strongly depends on the phase difference between the superconductors and the number of domain walls. In particular, our results reveal the high sensitivity of spin- and charge-transport in the junction to the number of domain walls in the ferromagnet. We show that superconductivity in coexistence with noncollinear magnetism can be used in a Josephson nanodevice to create a controllable spin supercurrent acting...
as a spin transfer torque on a system. Our results demonstrate the possibility of coupling the superconducting phase to the magnetization dynamics and, hence, constituting a quantum interface, for example between the magnetization and a superconducting qubit.

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1. Introduction

The Josephson effect refers to the coherent transfer of Cooper pairs between two weakly coupled superconductors [1]. In a Josephson contact with a normal metal between the superconductors, the underlying microscopic mechanism is Andreev scattering [2] at the two normal-metal–superconductor interfaces, which converts electron and hole excitations of opposite spin directions into each other by creating a Cooper pair. The resulting dissipationless electrical current is driven by the difference between the phases of the superconducting order parameters across the contact. From the fact that superconductivity is a coherent state of spontaneously broken \( U(1) \)-symmetry, it follows that the Josephson effect is a response of this coherent state to an inhomogeneity over the junction which is produced by the variation of the phase. An analogous non-superconducting effect is predicted to exist in the magnetic tunnel barrier between two ferromagnets with the \( SO(3) \) symmetry-breaking coherent states [3, 4]. In this case, the misorientation angle \( \theta \) of the two magnetization vectors is the driving potential for a dissipationless spin supercurrent, similar to the exchange interaction.

In this paper, we develop the circuit theory of the superconducting spin Josephson effect in an inhomogeneous ferromagnetic (F) contact between two conventional superconductors. We show that when the F contact consists of two domains whose magnetization vectors enclose an angle \( \theta \), in addition to the charge supercurrent, a spin supercurrent will also appear. This spin supercurrent is created by the simultaneous existence of the superconducting states and a noncollinear orientation \( \theta \) of the magnetization vectors. Interestingly, we found that the spin supercurrent and the corresponding spin-transfer torque is directed perpendicular to the plane of the two magnetization vectors, such that it would lead to a precession of the magnetizations around each other. In addition to extensive theoretical [5–8] and experimental [9–13] studies of the spin-transfer torque in F spin-valve and domain structures, there have been studies devoted to the spin-transfer torque in structures with superconducting parts [14–26]. The essential effect is the production of long-ranged spin-triplet superconducting correlations by the interplay between the induced spin-singlet correlations and the noncollinearity of the magnetization profile in F contact [27–33]. Compared to the previous studies, we present a quantum circuit theory calculation that takes the spatial variation of Green’s functions as well as the nonlinearity of
the proximity effect fully into account. By this method, we are specifically able to obtain the inhomogeneity of the spin supercurrent and to define a spin-transfer torque in noncollinear ferromagnetic Josephson contacts.

Several experimental studies on this triplet proximity effect have been carried out \cite{34, 35}. Recently, Khaire et al \cite{36} reported the observation of the long-range supercurrent in Josephson junctions, which is controllable by varying the thickness of one of the ferromagnetic domains. Also, Robinson et al \cite{37, 38} detected the flow of a long-range supercurrent in the ferromagnetic Josephson junction with a magnetic Ho–Co–Ho trilayer and found an enhancement of the critical currents in the antiparallel configuration of the junctions with a trilayer Fe/Cr/Fe barrier.

In analogy to the conventional charge Josephson effect, the spin Josephson effect has the tendency to remove the inhomogeneity of the order parameter vector of the spin-triplet superconducting state in the F-contact. This is analogous to the spin Josephson effect in contacts between two unconventional triplet superconductors, where the Cooper pair spin current appears in conjunction with the usual charge supercurrent \cite{39–42}.

The spin-dependent circuit theory has already been used in \cite{32} to study the density of states and the Josephson supercurrent in S/F/S heterostructures, which are shown to be dependent on the configuration of the magnetization in F. Here, we further study the spin supercurrent and spin-transfer torque in such Josephson junctions. We demonstrate the dependence of charge and spin supercurrent on the phase difference \( \phi \) and the angle between magnetizations \( \theta \): the spin supercurrent is an even function of \( \phi \) and an odd function of \( \theta \); the charge supercurrent satisfies the inverse relations relative to the \( \varphi \) and \( \theta \). Further, we study the equilibrium configuration of the exchange field vectors as a function of the phase difference and the temperature. We obtain phase diagrams that show the transition between antiferromagnetic and ferromagnetic states in the system. To our best knowledge, this result has not been reported in the literature before.

We also discuss the generalizability of this effect for other ferromagnetic contacts with a more complex inhomogeneity of the direction of the magnetization vector. In particular, when the ferromagnetic contact consists of an in-plane rotating magnetization vector between two homogeneous domains with antiparallel magnetization, we found that the spin supercurrent is highly sensitive to the value of the wave vector. We show that one can tune the spin supercurrent acting as a spin-transfer torque by changing the phase difference between the superconductors or by the variation of the wave vector. Also, we investigate the position dependence of the spin current in \( S_1F_1DWF_2S_2 \), which to the best of our knowledge has not been studied in any other work. We found that the behavior of spin supercurrent relative to the position depends strongly on the phase difference between the superconductors. We extend the quantum circuit theory by describing how spin-transfer torques can be calculated within this method.

2. The model and the basic equation

We describe the basic theory and the model first for a two-domain ferromagnetic contact between two conventional superconductors, as is shown schematically in figure 1(a). The exchange field of one domain \( F_1 \) makes an angle \( \theta \) with that of the other domain \( F_2 \). We restrict our study to the time-independent case and do not consider changes in the magnetic structure in this paper. The generalization to the structure with a continuous magnetization texture in figure 1(b) is straightforward and is described at the end of this section. To proceed with our work, we make use of quantum circuit theory, which is a finite-element technique for
calculating the quasiclassical Green’s functions in diffusive nanostructures [43–48]. In this technique, we represent each F domain and S reservoir by a single node, which is characterized by an energy-dependent $4 \times 4$-matrix Green’s function $\tilde{G}_i$, in Nambu and spin ($\uparrow$, $\downarrow$) spaces [47, 48]. Furthermore, the two nodes in the F domains are assumed to be weakly coupled to each other by means of a tunneling contact. In terms of its spin-space matrix components $\hat{g}$ and $\hat{f}$, the matrix Green’s function is written as

$$\tilde{G} = \begin{pmatrix} \hat{g} & \hat{f} \\ \hat{f}^\dagger & -\hat{g} \end{pmatrix}, \quad \hat{a} = \begin{pmatrix} a_{\uparrow\uparrow} & a_{\uparrow\downarrow} \\ a_{\downarrow\uparrow} & a_{\downarrow\downarrow} \end{pmatrix}, \quad a = f, g.$$  

(1)

We consider the equilibrium condition where a misorientation angle $\theta$ and phase difference $\phi$ may drive Josephson spin and charge currents between two adjacent nodes. These equilibrium Josephson currents can be extracted from the matrix current defined as

$$\tilde{I}_{ij} = (g_{ij}/2)[\tilde{G}_i, \tilde{G}_j],$$  

(2)

where $g_{ij}$ is the tunneling conductance of the contact between two nodes, and $i$ and $j$ denote the connected nodes. This approach works also if we divide the ferromagnetic region into $n$ nodes, as is necessary in the case of figure 1. Then, we have to take $(n - 1)/g_{ij} = (1/g_{F_1F_2} - (1/g_{S_1F_1} + 1/g_{S_2F_2})$, where the conducting part of F domains is discretized into $n$ nodes. For an $S_1F_1F_2S_2$ structure, $n$ is equal to two. The conductance of the tunnel barrier between $S_{1(2)}$ and $F_{1(2)}$ is denoted by $g_{S_{1(2)}F_{1(2)}}$ and, $g_{F_1F_2}$ is the conductance of the whole $F_1F_2$ contact.

The matrix current obeys the following law of current conservation in matrix form:

$$\tilde{I}_{oo} + \tilde{I}_{ss} + \tilde{I}_{ij} = 0.$$  

(3)

Here $\tilde{I}_{oo} = -G_Q(\omega/\delta_i)[\tilde{f}_3, \tilde{G}_i]$, with $\delta_i$ being the electronic level spacing of the node, is the matrix of the leakage current which takes into account the dephasing of electrons and holes due to their finite dwell time in node $i$, and $\tilde{I}_{ss} = i(G_Q/\delta_i)[(h_3 \hat{\sigma}_3 + h_2 \hat{\sigma}_2 + h_1 \hat{t}_3 \hat{\sigma}_3), \tilde{G}_i]$ is the corresponding matrix current representing the leakage caused by the spin splitting due to an exchange field $\tilde{h}_i$ ($G_Q = e^2/(2\pi \hbar)$ is the quantum of the conductance). The third term represents

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**Figure 1.** (a) The $S_1F_1F_2S_2$ junction, where each ferromagnet is represented by a node. $\theta$ is the angle between the magnetization directions of $F_1$ and $F_2$. (b) The $S_1F_1DWF_2S_2$ junction with the Neel domain wall (DW); $\alpha$ is the angle that the local exchange field in the domain wall makes with the $y$-direction.
the matrix currents from the neighboring nodes \(i-1, i+1\). From equation (3), we can find the spin-torque, e.g. in the \(z\)-direction as \(\tau_{zi} = I_{zi,i+1} + I_{zi,i-1}\).

Equation (3) is given for all nodes and is supplemented by boundary conditions, which are the values of \(\hat{G}\) in the \(S\) reservoirs. We neglect the inverse proximity effect in the reservoirs and set the matrix Green’s function in \(S_1\) normal to the plane of the magnetization vectors. Further, we found a transition of the supercurrent with a polarization directed along the \(z\)-axis. The spin supercurrent is given by

\[
I = \frac{Q L_F}{2\pi},
\]

where \(Q\) denotes the charge supercurrent and the component of the spin supercurrent vector \(\tilde{I}\). The temperature dependence of the amplitude of the order parameter is well approximated by \(|\Delta| = 1.76 T_c \tanh(1.74 \sqrt{T_c/T - 1})\). We note that the matrix Green’s function satisfies the normalization condition \(\hat{G}^2 = \hat{1}\).

We have solved these equations numerically by an iteration method. In our calculation, we start by choosing a trial form of the matrix Green’s functions of the nodes, for a given \(\phi\), and iteratively refine the initial values until the Green’s functions are calculated in each of two nodes with the desired accuracy. Note that in general for any phase difference \(\phi\), the resulting Green’s functions vary from one node to another, simulating the spatial variation along the \(F\) contact (see [49]). From the resulting Green’s functions and equation (2), we find the matrix currents. Then, we calculate the charge supercurrent \(I\) and the components of the spin supercurrent vector \(\tilde{I}\) from the relations

\[
I_{ij} = \text{tr} \hat{\sigma}_3 \tilde{I}_{ij}, \quad I_{\tau z_{i,j}} = \text{tr} \hat{\tau}_3 \tilde{\tau}_3 \tilde{I}_{ij}, \quad I_{\tau x_{(y)_{i,j}}} = \text{tr} \hat{\sigma}_{1(2)} \tilde{I}_{ij},
\]

in which \(\text{tr} \ldots = (\pi T/2e) \sum_\omega \text{Tr} \ldots\), with \(\text{Tr}\) denoting the trace in Nambu–spin spaces. In the next steps, we change to the next Matsubara frequency and use the results from the previous one \(\omega_{m-1}\) as the initial guess. We find the respective contribution to the spectral currents and continue to higher frequencies until the required precision of the summation over \(m\) is achieved.

In the following, we scale the length of the system, \(L\), in units of the diffusive superconducting coherence length \(\xi\), and use the dimensionless parameters of \(h/T_c\) and \(t = T/T_c\) as measures of the amplitude of the exchange field \(h\) and the temperature \(T\). To describe a continuous domain wall, we use as parameter the wave vector \(Q\) associated with one full winding of the magnetization by \(2\pi\). Hence, the total number of windings of the magnetization is given by \(Q L_F/2\pi\), where \(L_F\) is the length of the inhomogeneous region (see figure 1). The currents are expressed in units of \(I_0\), the amplitude of the critical Josephson charge current at \(h = 0\) and \(t = 0\).

3. Results and discussions

From the numerical calculations, we obtained the spin supercurrent with a polarization directed normal to the plane of the magnetization vectors. Further, we found a transition of the favorable configuration of the domain, from antiparallel to parallel, as the exchange field of
Figure 2. (a) Plot of spin supercurrent versus $\theta$ for different values of $\phi$ when $L/\xi = 1.0$, $h/T_c = 5.0$, $t = 0.5$. (Inset) $\theta$ dependence of charge supercurrent for the same system. (b) Plot of $I$ versus $\phi$ for different values of $\theta$ for the previous system. (Inset) $\phi$ dependence of spin supercurrent for the same system. (c) Critical spin supercurrent versus $\phi$ for different temperatures, which shows the appearance of $0-\pi$ transition relative to the $\phi$. (Inset) Critical charge supercurrent versus $\phi$ for the same system. (d) Light dashed, dotted and dashed-dotted lines are, respectively, the maximum spin supercurrent as $\phi$ varies between 0 and $\pi$ versus $\theta$ when $t = 0.1$, 0.5 and 0.9. Dark dashed, dotted and dashed-dotted lines are, respectively, the spin supercurrent for the $\phi$ value that maximizes the charge supercurrent for different temperatures in the same situation.

the asymmetric domains increases. Also, we showed that in a system with a more complex configuration of direction of the magnetization, the profile and penetration depth of the spin supercurrent are highly dependent on the number of rotations that the magnetization vector has undergone across the domain wall.

3.1. $S_1F_1F_2S_2$ junction

Using the method described in section 2, we have calculated the spin and charge supercurrents for the two domains of the F contact of figure 1(a) when the exchange field vectors are taken to be in the $x-y$-plane. We found that the spin supercurrent has a polarization that is aligned along the $z$-axis, namely perpendicular to the plane of the exchange fields of $F_1$ and $F_2$. Our results on the dependence of the spin $I_z$ and charge $I$ supercurrents on the misorientation angle $\theta$, the phase difference $\phi$ and the temperature $t$ are shown in figure 2. We found that, in general, the
spin supercurrent obeys the symmetry relations $I_z(\varphi) = I_z(-\varphi)$ and $I_z(\theta) = -I_z(-\theta)$, which are analogues of the relations $I(\varphi) = -I(-\varphi)$ and $I(\theta) = I(-\theta)$ for the charge supercurrent (see figures 2(a) and (b)). This behavior suggests that one can change the direction of the spin supercurrent, which is proportional to the induced spin-transfer torque [7], by changing the phase difference between two superconductors when $\theta$ is fixed. We note that a nonzero spin supercurrent is provided by a noncollinear orientation of the exchange field vectors and the existence of the superconductivity ($|\Delta| \neq 0$), even for $\varphi = 0$.

We define the critical spin supercurrent, $I_{zcr}(\varphi)$, as the maximum of the absolute value of the spin supercurrent as a function of $\varphi$ for a given $\varphi$, similar to the definition of the charge critical supercurrent. We may also use a distinct definition, which we denote by $I_{z\text{max}}(\theta)$, as the absolute value of the spin supercurrent for a value of $\varphi$ that maximizes the charge supercurrent as a function of $\varphi$, for a given $\theta$. Figure 2(c) shows the behavior of $I_{z\text{cr}}$ as a function of $\varphi$ for different temperatures. At a given temperature $t$, $I_{z\text{cr}}$ shows a change of sign at a phase difference that depends on $t$. This change of sign may be recognized as the signature of a transition between 0 and $\pi$ spin Josephson couplings, in analogy to the charge 0–$\pi$ transition in F Josephson junctions [50–59]. The corresponding critical charge current shows a 0–$\pi$ transition with varying $\theta$. Note that both $I_{z\text{cr}}(\varphi)$ and $I_{z\text{cr}}(\theta)$ have a nonzero value at the transition point at low temperatures, as the signature of nonzero second harmonic in the current–phase and the current–angle relations [60–62]. In figure 2(d), we have also plotted $I_{z\text{max}}(\theta)$, which shows a change of sign at $\theta = \pi$ for all temperatures [19].

We have also studied the dependence of the spin supercurrent on the absolute value of the exchange field. The results are shown in figure 3, in which $I_{z\text{cr}}$ is plotted as a function of $h/T_c$ for different $\varphi$ and $\theta$. These results show that the sign and amplitude of the spin supercurrent can also be modulated by varying $h/T_c$, which can be used for further tuning the corresponding spin-transfer torque. We note that for strong ferromagnets with $h \gg \Delta$, the spin supercurrent vanishes. This is due to the suppression of the amplitude of the Andreev reflection at $S_1F_1$ and $S_2F_2$ interfaces in this limit, which suppresses the proximity effect.

It is also interesting to study the equilibrium configuration of the exchange field vectors as a function of the phase difference and temperature. The equilibrium angle can be obtained by minimizing the free energy, $F$, of the contact as a function of $\theta$. We have calculated the $\theta$ dependence of $F$ by integrating the spin supercurrent over $\theta$ and the charge supercurrent over $\phi$:

$$F(\varphi, \theta) = \int_0^\varphi I(\varphi', 0) \, d\varphi' + \int_0^\theta I_z(\varphi, \theta') \, d\theta'. \quad (7)$$

Our calculation shows that the exchange field vectors favor either parallel ($\theta = 0$) or antiparallel ($\theta = \pi$) configurations, depending on $\varphi$, $t$ and $h/T_c$. The behavior of this superconductivity-induced exchange coupling differs for the two cases of a contact with symmetric barriers with $g_{S_1F_1} = g_{S_2F_2}$ and an asymmetric contact with very different $g_{S_1F_1}$ and $g_{S_2F_2}$. For a symmetric system, we found that the coupling is antiferromagnetic ($\theta = \pi$) for $\varphi = 0$, but becomes ferromagnetic ($\theta = 0$) for $\varphi = \pi$. This behavior is found to hold irrespective of the values of $L/\xi$, $h/T_c$.

However, for an asymmetric system it is possible to change the coupling from ferromagnetic to antiferromagnetic and vice versa by varying $L/\xi$, $h/T_c$ or $t$, for the phase difference $\varphi = \pi$. In figure 4, we have shown the ferromagnetic–antiferromagnetic coupling phase diagram of the system in the plane of $h/T_c$ and $t$, when $\varphi = \pi$ and for different values.
of $L/\xi$. This phase diagram is similar to the 0–$\pi$ Josephson couplings phase diagram of a homogeneous F contact between two superconductors; see [49]. As we show in the inset of figure 4, the minimum of $F$ as a function of $\theta$ shifts from $\theta = \pi$ for low values of $h/T_c$ to $\theta = 0$ at higher values of $h/T_c$, when $L/\xi = 1$ and $t = 0.4$. The temperature-induced transition between the ferromagnetic and antiferromagnetic phases is also possible, but only over a finite interval $\Delta h$ of the amplitude of the exchange field of the F domains. This width of the temperature-induced transition increases with decreasing $L/\xi$. For $\varphi = 0$, the coupling between the exchange fields of the two domains is found to be always antiferromagnetic in an asymmetric structure, which is very similar to the symmetric case.

3.2. $S_1 F_1 \text{DWF}_2 S_2$ junction with the Neel domain wall

The superconducting spin Josephson effect described above may take place in F contacts with a more complex profile of the exchange field vector. An interesting case is a finite-width F domain wall between two domains, where the exchange field has a continuous spatial rotation between two homogeneous F domains. Here, we present the results of our calculation of the spin supercurrent and the spin-transfer torque for a Neel domain wall junction, which is shown

Figure 3. (a) $I_z$ versus $h/T_c$ for different values of $\theta$ when $\varphi = \pi/2$, $L/\xi = 1.0$ and $t = 0.1$. (b) $I_z$ versus $h/T_c$ for the same system but for different $\varphi$ when $\theta = \pi/2$. Panels (a) and (b) are logarithmic plots that show the 0–$\pi$ transition relative to the exchange field.

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Figure 4. Phase diagram of the transition of the minimum of free energy from $\theta = \pi$ to $\theta = 0$ for asymmetric systems with $g_{S_1 F_1} = 0.1 g_{S_2 F_2}$ and different lengths. The inset shows the $F - \theta$ plane of the three-dimensional energy plot when $L/\xi = 1$ and $t = 0.4$. Here, $F_0$ is the free energy of the system when the magnetization is collinear.

schematically in figure 1(b). Note that the spin-transfer torque acting on the local magnetization on node $i$ is obtained from $\tau_{zi} = I_{zi,i+1} + I_{zi,i-1}$, as we have shown earlier. We model the local angle of the exchange field vector with respect to the $y$-axis to vary as $\alpha(x) = (QL_F)(x/L_F)$.

We have obtained the position-dependent spin supercurrent, which is perpendicular to the plane of the magnetization and flows in the homogeneously and inhomogeneously magnetized parts of the system. To the best of our knowledge, there have been no previous studies investigating the position dependence of the spin current and spin-transfer torque. Figures 5(a) and (b) show the behavior of the spin-transfer torque versus position for a system with $L_F/L = 1/3$. We can see that the system has interesting behavior depending strongly on the value of $QL_F$. As is evident from figure 5(a), the spin-transfer torque penetrating the homogeneous ferromagnets becomes of negligible constant value. This shows that the spin supercurrent in $F_1$ and $F_2$ has a linear position dependence when $\phi = 0$. In particular, this is true when the value of the spin supercurrent is comparable to the one in the nonhomogeneous parts.

Also figure 5(b) shows that the penetrating spin-transfer torque in the homogeneous parts is nearly zero for $QL_F = \pi, 3\pi$ and is much smaller than the one in the domain wall region for $QL_F = 2\pi, 4\pi$ when $\phi = \pi$. In addition, we note that the spin-transfer torque always has a symmetric position dependence around $x = L/2$, whereas the spin supercurrent always shows asymmetric behavior. Our calculations demonstrate further that the behavior of the spin current and spin-transfer torque versus position for a system with $QL_F = n\pi$ and $\phi = \pi$ is similar to that of a system with $QL_F = (n + 1)\pi$ and $\phi = 0$; see figure 5. This observation of a symmetry between the magnetic winding number and the superconducting phase is at present not fully understood, but will be the subject of future research.
Finally, we have also studied Josephson systems without the homogeneous ferromagnetic parts $F_1$ and $F_2$, which correspond to $L_F = L$. In this way, we would like to check how the spin currents in the homogeneous part, which are appreciable in size but give rise to a negligible spin-transfer torque, influence the spin-torque on the magnetization texture. We show the corresponding dependence of the spin supercurrent on the phase difference and position in figure 6 for a wall with rotation angle $\pi$. The first thing to note is that we observe a qualitatively similar phase dependence as in the case of two homogeneous ferromagnets. In particular, the $I_z(\phi)$ relation is always symmetric. Also, for a specific position, the sign of the spin-transfer torque can be changed by varying the phase difference between the superconductors. In addition, for a simple Neel domain wall, when $QL_F = \pi$, we see that the behavior of the spin-transfer torque can be approximately described by the relation $\cos[(x/L)(QL_F) + \pi/2]$ for $\phi = 0$, and turns into $\cos[3(x/L)(QL_F) + \pi/2]$ for $\phi = \pi$. These findings have the same symmetry, as we mentioned in the previous discussion of a system with a homogeneous ferromagnet attached. Also, this result shows the strong dependence of the spin-transfer torque on the phase difference between the superconductors, which can be interpreted as a direct interplay between the induced spin-singlet correlations of the superconductors and the inhomogeneity of the magnetization in the domain wall. In fact, this interplay leads to the
generation of triplet correlations in the contact region, whose inhomogeneity drives the spin supercurrent and therefore the spin-transfer torque.

Figures 7(a) and (b) show the spin-transfer torque as a function of the phase difference, $\phi$, and the wave vector, $QL_F$ (which is more or less given by the number of $180^\circ$ domain walls). In figures 7(a) and (b), the behavior for different positions $x/L = 1/2$ and $x = L/6$ is shown. For the position closer to the superconductor, $x/L = 1/6$, the spin-transfer torque goes to zero much faster for larger wave vectors than in the middle of the ferromagnet at $x = L/2$. Hence, we see that the suppression of the spin-transfer torque strongly depends on the position in the domain wall. Also, figures 7(a) and (b) show that the spin-transfer torque oscillations versus wave vector depend strongly on the phase difference and the position. While they start for small $QL_F$ in an exactly opposite fashion for both positions, the oscillations in the middle of the domain wall are well behaved also for large $QL_F$, whereas the behavior is more complex in the ferromagnetic part close to the superconductor. In that limit, no well-defined oscillation period can be identified. Furthermore, while the direction of the spin supercurrent is fixed, we find that the sign and magnitude of the spin supercurrent and spin-transfer torque can be modulated by changing the wave vector or the phase difference. We have found that superconductivity in coexistence with noncollinear magnetism can be used in a Josephson nanodevice to create the tunable spin supercurrent that acts as a spin-transfer torque on the junctions’ magnetization.

4. Conclusion

We have studied the Josephson effect in a diffusive contact consisting of two ferromagnetic domains with noncollinear magnetizations which connects two conventional superconductors. Using quantum circuit theory, we have shown that the spin supercurrent will flow through the contact, due to the generation of inhomogeneous spin-triplet superconducting correlations. The polarization of the spin supercurrent is directed normal to the plane of two magnetization vectors in a way that the resulting spin-transfer torque intends to align the magnetization of the
two domains. The spin and charge current–phase–angle relations obey the specific symmetry relations versus the phase difference $\phi$ and the misorientation angle $\theta$. Whereas, the charge supercurrent satisfies the odd–even relationship on the $\phi$ and $\theta$, the spin supercurrent is an even–odd function of $\phi$ and $\theta$. From these relations, we have predicted a transition between 0 and $\pi$ Josephson coupling by varying the misorientation angle $\theta$. We found a transition of the favorable configuration of the domain, from antiparallel to parallel, as the exchange field of the domains increases. This transition occurs for asymmetric systems with $\phi = \pi$. Also, the domains in the symmetric systems settle in a parallel configuration when $\phi = \pi$.

We have further discussed the generation of the spin supercurrent in magnetic contacts with a more complex configuration of the direction of the magnetization vector. For a domain wall between two domains with antiparallel magnetizations, we have shown that the profile and penetration depth of the spin supercurrent are highly dependent on the number of rotations that the magnetization vector has undergone across the domain wall. In particular, we show that while the direction of the spin supercurrent is always perpendicular to the plane of the magnetization vectors, the sign and magnitude of the spin supercurrent strongly depend on the phase difference between the superconductors and the value of the wave vector. We present a Josephson nanodevice that can be used to create a controllable spin supercurrent and spin-transfer torque.

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Figure 7. (a) Spin-transfer torque of $I_z$ versus $QL_F$ and $\phi$ for the system with $x/L = 1/6, L/\xi = 1.0, L_F/L = 1.0, h/T_c = 5.0$ and $t = 0.1$. (b) The same as (a) but for $x/L = 1/2$. 
References

[1] Josephson B D 1962 Phys. Lett. 1 251
[2] Andreev A F 1964 Sov. Phys.—JETP 19 1228
[3] Slonczewski J C 1993 J. Magn. Magn. Mater. 126 374
[4] Nogueira F C and Bennemann K H 2004 Europhys. Lett. 67 620
[5] Berger L 1979 J. Appl. Phys. 49 2156
[6] Berger L 1979 J. Appl. Phys. 50 2137
[7] Brataas A, Bauer G E W and Kelly P 2006 Phys. Rep. 427 157
[8] Ralph D C and Stiles M D 2008 J. Magn. Magn. Mater. 320 1190
[9] Freitas P P and Berger L 1985 J. Appl. Phys. 57 1266
[10] Hung C Y and Berger L 1989 J. Appl. Phys. 63 4276
[11] Kläui M, Vaz C A F, Bland J A C, Wernsdorfer W, Faini G, Cambril E and Heyderman L J 2003 Appl. Phys. Lett. 83 105
[12] Tsoi M, Fontana R E and Parkin S S P 2003 Appl. Phys. Lett. 83 2617
[13] Sankey J C, Cui Y-T, Sun J Z, Slonczewski J C, Buhrman R A and Ralph D C 2008 Nat. Phys. 4 67
[14] Waintal X and Brouwer P W 2002 Phys. Rev. B 65 054407
[15] Löffander T, Champel T, Durst J and Eschrig M 2005 Phys. Rev. Lett. 95 187003
[16] Nussinov Z, Shnirman A, Arovas D P, Balatsky A V and Zhu J-X 2005 Phys. Rev. B 71 214520
[17] Linder J, Yokoyama T and Sudbø A 2008 Phys. Rev. B 79 224504
[18] Grein R, Eschrig M, Metalidis G and Schön G 2009 Phys. Rev. Lett. 102 227005
[19] Alidoust M, Linder J, Rashedi G H, Yokoyama T and Sudbø A 2010 Phys. Rev. B 81 014512
[20] González E M, Folgueras A D, Escudero R, Ferrer J, Guinea F and Vicent J L 2007 New J. Phys. 9 34
[21] Zhao E and Sauls J A 2007 Phys. Rev. Lett. 98 206601
[22] Zhao E and Sauls J A 2008 Phys. Rev. B 78 174511
[23] Giazotto F, Taddei F, Fazio R and Beltram F 2005 Phys. Rev. Lett. 95 066804
[24] Giazotto F, Taddei F, D’Amico P, Fazio R and Beltram F 2007 Phys. Rev. B 76 184518
[25] Giazotto F and Taddei F 2008 Phys. Rev. B 77 132501
[26] Holmquist C, Teber S and Fogelström M 2011 Phys. Rev. B 83 104521
[27] Bergeret F S, Volkov A F and Efetov K B 2001 Phys. Rev. Lett. 86 3140
[28] Bergeret F S, Volkov A F and Efetov K B 2001 Phys. Rev. Lett. 86 4096
[29] Bergeret F S, Volkov A F and Efetov K B 2005 Rev. Mod. Phys. 77 1321
[30] Blanter Y M and Hekking F W J 2004 Phys. Rev. B 69 024525
[31] Crouzy B, Tollis S and Ivanov D A 2007 Phys. Rev. B 76 134502
[32] Braude V and Nazarov Yu V 2007 Phys. Rev. Lett. 98 077003
[33] Braude V and Blanter Ya M 2008 Phys. Rev. Lett. 100 207001
[34] Sosnin I, Cho H, Petrushov V T and Volkov A F 2006 Phys. Rev. Lett. 96 157002
[35] Keizler R S, Goennenwein S T B, Klapwijk T M, Miao G, Xiao G and Gupta A 2006 Nature 439 825
[36] Khaire T S, Khasawneh M A, Pratt W P Jr and Birge N O 2010 Phys. Rev. Lett. 104 137002
[37] Robinson J W A, Witt J D S and Blamire M G 2010 Science 329 5987
[38] Robinson J W A, Halász G B, Buzdin A I and Blamire M G 2010 Phys. Rev. Lett. 104 207001
[39] Brydon P M R, Kastening B, Morr D K and Manske D 2008 Phys. Rev. B 77 104504
[40] Brydon P M R and Manske D 2009 Phys. Rev. Lett. 103 147001
[41] Brydon P M R 2009 Phys. Rev. B 80 224520
[42] Brydon P M R, Inirotakis C and Manske D 2009 New J. Phys. 11 055055
[43] Nazarov Y V 1994 Phys. Rev. Lett. 73 1420
[44] Nazarov Y V 1999 Superlattices Microstruct. 25 1221
[45] Nazarov Y V 2005 Quantum transport and circuit theory Handbook of Theoretical and Computational Nanotechnology ed M Rieth and W Schommers vol 1 (Valencia, CA: American Scientific Publishers)
[46] Cottet A, Huertas-Hernando D, Belzig W and Nazarov Y V 2009 Phys. Rev. B 80 184511
[47] Huertas-Hernando D, Nazarov Y V and Belzig W 2002 Phys. Rev. Lett. 88 047003
[48] Huertas-Hernando D and Nazarov Y V 2005 Eur. Phys. J. B 44 373
[49] Shomali Z, Zareyan M and Belzig W 2008 Phys. Rev. B 78 214518
[50] Bulaevskii L N, Kuzii V V and Sobyanin A A 1977 JETP Lett. 25 290
[51] Buzdin A I, Bulaevskii L N and Panyukov S V 1982 Pis’ma Zh. Eksp. Teor. Fiz. 35 147
  Buzdin A I, Bulaevskii L N and Panyukov S V 1982 JETP Lett. 35 178
[52] Buzdin A I and Kupriyanov M Yu 1991 Pis’ma Zh. Eksp. Teor. Fiz. 53 308
  Buzdin A I and Kupriyanov M Yu 1991 JETP Lett. 53 321
[53] Buzdin A I, Bujicic B and Kupriyanov M Yu 1992 Sov. Phys.—JETP 74 124
[54] Ryazanov V V, Oboznov V A, Rusanov A Y, Veretennikov A V, Golubov A A and Aarts J 2001 Phys. Rev. Lett. 86 2427
[55] Chtcchelkatchev N M, Belzig W, Nazarov Y V and Bruder C 2001 Pis’ma Zh. Eksp. Teor. Fiz. 74 357
  Chtcchelkatchev N M, Belzig W, Nazarov Y V and Bruder C 2001 JETP Lett. 74 323
[56] Kontos T, Aprii M, Lesueur J, Genèt F, Stephanidis B and Boursier R 2002 Phys. Rev. Lett. 89 137007
[57] Guichard W, Aprii M, Bourgeois O, Kontos T, Lesueur J and Gandit P 2003 Phys. Rev. Lett. 90 167001
[58] Buzdin A I 2005 Rev. Mod. Phys. 77 935
[59] Vasenko A S, Golubov A A, Kupriyanov M Y and Weides M 2008 Phys. Rev. B 77 134507
[60] Sellier H, Baraduc C, Lefloch F and Calemczuk R 2003 Phys. Rev. B 68 054531
[61] Mohammadkhani G and Zareyan M 2006 Phys. Rev. B 73 134503
[62] Konschelle F, Cayssol J and Buzdin A I 2008 Phys. Rev. B 78 134505

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