Coherent Waveform Consistency Test for LIGO Burst Candidates

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Abstract. The burst search in LIGO relies on the coincident detection of transient signals in multiple interferometers. As only minimal assumptions are made about the event waveform or duration, the analysis pipeline requires loose coincidence in time, frequency and amplitude. Confidence in the resulting events and their waveform consistency is established through a time-domain coherent analysis: the $r$-statistic test. This paper presents a performance study of the $r$-statistic test for triple coincidence events in the second LIGO Science Run (S2), with emphasis on its ability to suppress the background false rate and its efficiency at detecting simulated bursts of different waveforms close to the S2 sensitivity curve.

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1. Introduction

The Laser Interferometer Gravitational-wave Observatory (LIGO) consists of three detectors: H1 and H2, co-located in Hanford, WA and L1, located in Livingston, LA. The simultaneous availability of interferometric data from detectors with similar sensitivity and orientation allows a coherent coincidence analysis to be implemented in the search for bursts of gravitational waves.

In the LIGO burst analysis pipeline[1, 2, 3], candidate events are identified as excesses of power or amplitude in the data stream of each interferometer by a suite of search algorithms, referred to as Event Trigger Generators (ETG): BlockNormal[4], Excess Power[5], TFClusters[6], and WaveBurst[7, 8]. The ETG tuning is tailored to maximize the detection efficiency for a variety of waveforms (narrow-band, broad-band and astrophysically motivated), with a single interferometer false rate of the order of 1 Hz. This relatively large trigger rate is suppressed by the multi-interferometer analysis, which currently only requires that events be coincident in time and in frequency. The coincidence parameters (time window and frequency tolerance) are tuned according to the principle that the coincident detection efficiency should equal the product of efficiencies in the individual interferometers. Coincidence criteria should be loose enough not to further reduce the detection efficiency, within the limitations imposed by the false alarm rate. The coincidence analysis eventually outputs triggers (start time, duration) when excesses of power or amplitude have been detected simultaneously in all interferometers. The first step toward validation of such events is a comparison of the waveforms as they appear in each detector.

This paper describes a test that exploits cross-correlation between pairs of interferometers and combines them into a multi-interferometer correlation confidence. The test is a powerful tool for the suppression of accidental coincidences without reducing the detection efficiency of the pipeline. Its performance has been tested on a 10% portion of data from the LIGO second science run (S2).

2. The r-statistic Cross Correlation Test

2.1. r-statistic

The fundamental building block for the waveform consistency test is the r-statistic, or the linear correlation coefficient of two sequences \( \{x_i\} \) and \( \{y_i\} \):

\[
r = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}}
\]

This quantity only assumes values between -1 (fully anti-correlated sequences), 0 (uncorrelated sequences) and +1 (fully correlated sequences). More generally, if the two sequences are uncorrelated, we expect the r-statistic to follow a normal distribution, with zero mean and \( \sigma = 1/\sqrt{N} \), where \( N \) is the number of data points used to compute \( r \). A coherent component in the two sequences will cause \( r \) to deviate from the normal
Figure 1. Graphical representation of the $r$–statistic as cosine of the angle between vectors in an $N$-dimensional space. (a) The two sequences are uncorrelated: the vectors are orthogonal and $r = 0$. (b) The two sequences are correlated: as the coherent component dominates over the incoherent noise, $r \to 1$.

distribution. If we think of data sequences as vectors in an $N$–dimensional space, the $r$–statistic can be seen as an estimator for the cosine of the angle between the two vectors (see Figure 1). As a normalized statistic, it is not sensitive to the relative amplitude of the two vectors; the advantage is robustness against fluctuations of detector response and noise floor.

The number of points $N$ or, alternatively, the integration window $\tau$, is the most important parameter in the construction of the $r$–statistic. Its optimal value depends in general on the signal. If $\tau$ is too large, the signal is “washed out” in the computation of $r$; if it is too small, statistical considerations on the distribution of $r$ lose validity. Simulation studies show that a set of three integration times (20, 50 and 100 ms) is suitable for most short signals of interest to the LIGO burst search. A more detailed (and computationally intense) scan of integration windows can be implemented in a targeted analysis as it was done for the LIGO externally triggered search described in [9, 10], which is also based on cross-correlation.

2.2. Data Conditioning

The $r$-statistic test is especially effective when all coherent lines and known spectral features are removed from the raw strain data; for this reason, data conditioning plays an important role in the test.

Raw data from each interferometer is band-passed and decimated, in order to suppress the contribution of seismic noise and instrumental artifacts at low and high frequencies and restrict the coherent analysis to the most sensitive frequency band in the LIGO interferometers. For the performance studies reported in this paper, the band of interest was 100-2048 Hz.

Next, the data is effectively whitened by a linear error predictor filter trained on a 10 sec period before the event start time. This filter, described in more detail
in [11, 12], removes predictable content, such as lines and the spectral shape, and emphasizes transients.

2.3. Trigger Scan

The next step consists of partitioning the trigger duration in intervals equal to the integration window, with 50% overlap, as shown in figure 2. For each pair of interferometers \( l, m \) in \{L1, H1, H2\}, a selected value of the integration window \( p \) in \{20ms, 50ms, 100ms\} and each interval \( j \), data is selected from the conditioned time series of two interferometers. One of the two sequences is time-shifted with respect to the other, yielding a distribution of \( r \) coefficients:

\[
\begin{align*}
   r_{plmj}^k &= \frac{\sum_i (x_{plmj}^i - \bar{x}_{plmj})(y_{plmj}^{i+k} - \bar{y}_{plmj})}{\sqrt{\sum_i (x_{plmj}^i - \bar{x}_{plmj})^2} \sqrt{\sum_i (y_{plmj}^{i+k} - \bar{y}_{plmj})^2}},
\end{align*}
\]

where the index \( k \) represents the time lag between the two series, in steps equal to the inverse of the sampling rate, covering the whole \( \pm 10 \, \text{ms} \) range to account for the light travel time between LIGO sites. The quantity \( r_{plmj}^k \) assumes values in \([-1, 1]\) but, as the test is mostly interested in how much the correlation coefficient deviates from 0, only its absolute value \( |r_{plmj}^k| \) is used.

A Kolmogorov-Smirnov test with 5% significance is used to compare the \( \{|r_{plmj}^k|\} \) distribution to the null hypothesis expectation of a normal distribution, with zero mean and \( \sigma = 1/\sqrt{N_p} \), where \( N_p \) is the number of data samples in the \( p \)-th integration window. If the two are inconsistent, the next step is to compute the one-sided significance and the corresponding confidence:

\[
\begin{align*}
   S_{plmj}^k &= \text{erfc} \left( |r_{plmj}^k| \sqrt{\frac{N_p}{2}} \right); \\
   C_{plmj}^k &= -\log_{10}(S_{plmj}^k).
\end{align*}
\]
A correlation confidence is assigned to interval $j$ for interferometers $l, m$ and integration window $p$ as the maximum confidence over all time lags:

$$\gamma_{plmj} = \max_k C_{plmj}^k.$$ (4)

The degeneracy over interferometer pairs is solved through the arithmetical average of confidences from all combinations of $N_{ifo}$ interferometers:

$$\gamma_{pj} = \frac{1}{N_{ifo}(N_{ifo} - 1)} \sum_{i \neq m} \gamma_{plmj}.$$ (5)

Finally, $\Gamma$, the combined correlation confidence for the event, is obtained maximizing over all time intervals and integration windows:

$$\Gamma = \max_p \left[ \max_j \gamma_{pj} \right]$$ (6)

The event passes the waveform consistency test if:

$$\Gamma > \beta,$$ (7)

where $\beta$ is the threshold imposed on the multi-interferometer correlation confidence. For additional details on the method and its implementation, see [12, 13].

3. Triple Coincidence Performance Analysis in S2

The $r$-statistic test performance has been explored, independently of previous portions of the burst analysis pipeline, by adding simulated waveforms to real interferometer noise and then passing 200 ms of data around the simulated peak time through the $r$-statistic test.

For convenience, we define here two quantities that will be used to characterize a burst signal. $h_{rss}$ is the square root of the total burst energy, in unit of strain/$\sqrt{\text{Hz}}$:

$$h_{rss} = \sqrt{\int_0^\infty |h(t)|^2 \, dt} = \sqrt{\int_{-\infty}^{\infty} |\tilde{h}(f)|^2 \, df}.$$ (8)

This quantity can be compared directly to the sensitivity curves, as is also reflected in this definition of signal-to-noise ratio:

$$\text{SNR} = \sqrt{2 \int_0^\infty \frac{|\tilde{h}(f)|^2}{S_h(f)} \, df},$$ (9)

where $S_h(f)$ is the single-sided detector noise. In particular, for narrow-band bursts with central frequency $f_0$, this “excess-power” definition of SNR becomes the ratio of $h_{rss}$ to the detector sensitivity at frequency $f_0$: $\text{SNR} \approx h_{rss}/\sqrt{S_h(f_0)}$. In the following, $S_h(f)$ will be the single-sided reference noise for the S2 run.

The LIGO burst search has adopted sine-gaussians as standard narrow-band waveforms to test the search algorithms:

$$h(t) = h_{peak} \sin (2\pi f_0 (t - t_0)) e^{-(t-t_0)^2/\tau^2}; \quad h_{rss} = h_{peak} \sqrt{Q/4\pi f_0},$$ (10)
where \( Q = \sqrt{2\pi f_0} \) is the number of cycles folded under the gaussian envelope. As instances of broad-band, limited-duration bursts, one can also consider Gaussians of the form:

\[
h(t) = h_{\text{peak}} e^{-(t-t_0)^2/\tau^2}; \quad h_{\text{rss}} = h_{\text{peak}} \sqrt{\pi \tau/2}.
\]  

(11)

Table 1 reports the resulting sensitivity of the triple-coincidence \( r \)-statistic analysis, quoted as the \( h_{\text{rss}} \) value that is detected with 50% efficiency for three possible values of the \( \beta \) threshold. These values are affected by a \( \sim 10\% \) systematic error, due to calibration, and a \( \sim 10\% \) statistical error.

| waveform | \( \beta = 3 \) | \( \beta = 4 \) | \( \beta = 5 \) |
|---------|----------------|----------------|----------------|
| SG Q=9 f_0 = 153 Hz | 5.5 \times 10^{-21} | 6.7 \times 10^{-21} | 8.1 \times 10^{-21} |
| SG Q=9 f_0 = 235 Hz | 2.6 \times 10^{-21} | 3.2 \times 10^{-21} | 3.9 \times 10^{-21} |
| SG Q=9 f_0 = 361 Hz | 3.3 \times 10^{-21} | 3.8 \times 10^{-21} | 4.6 \times 10^{-21} |
| SG Q=9 f_0 = 554 Hz | 4.5 \times 10^{-21} | 5.4 \times 10^{-21} | 6.3 \times 10^{-21} |
| SG Q=9 f_0 = 850 Hz | 8.7 \times 10^{-21} | 1.0 \times 10^{-20} | 1.2 \times 10^{-20} |
| SG Q=9 f_0 = 1304 Hz | 1.8 \times 10^{-20} | 2.1 \times 10^{-20} | 2.2 \times 10^{-20} |
| GA \( \tau = 1.0 \) ms | 6.2 \times 10^{-21} | 7.3 \times 10^{-21} | 8.3 \times 10^{-21} |

Note that the \( \beta = 3(4, 5) \) threshold shown here has been selected from first principles, as it can be tracked back to a \( 10^{-3}(10^{-4}, 10^{-5}) \) false probability in the correlation between two interferometers on a single interval of duration equal to an integration window. In order to get a more complete picture of the test efficiency for different values of the \( \beta \) threshold, Figures 5 and 6 show, for the 235 Hz sine gaussian and the 1.0 ms gaussian pulses, the detection probability versus false probability for signals between SNR=1 and SNR=10 relative to the least sensitive interferometer (H2).
The false probability, or the probability that an accidental event passes the \( r \)-statistic test, was obtained from a sample of \( 1.7 \times 10^5 \) 200 ms events randomly selected in the S2 playground. As one can see in Figure 7, this statistic is sufficient to estimate the false probability only up to \( \beta = 3 \); a fit to an exponential decay was used to extrapolate the false rate for the ROC curves to \( \beta > 3 \). The choice of \( \beta \) will ultimately be set by requirements on the false alarm rate in the full burst pipeline.

In all cases considered so far, the sensitivity of the \( r \)-statistic is comparable to or better than that of the ETGs (see for instance [4, 7]), whose 50% detection efficiency in triple coincidence typically is in the SNR=5-10 range. This means the \( r \)-statistic test with \( \beta = 3, 4 \) has a very small effect on the detection efficiency of the burst analysis pipeline.

It is worth emphasizing that these false alarm probabilities have been computed by applying the \( r \)-statistic test to random times in the S2 dataset. However, when the \( r \)-statistic is fully integrated in the burst pipeline it acts on triggers that are pre-selected by the ETGs and their coincidence analysis. Such events share a minimal set of properties in all three interferometers: at the very least, they are simultaneous excesses of power in overlapping frequency bands. It is reasonable to expect a larger portion of these events to survive the \( r \)-statistic test than what is shown above for random events, at fixed \( \beta \). Nevertheless, a preliminary analysis of ETG background coincident triggers in S2 indicate the \( r \)-statistic test can effectively suppress the accidental rate by 2-4 orders of magnitude, depending on the value of \( \beta \), at negligible cost for the detection efficiency.

### 4. Conclusion

The LIGO burst S1 analysis [1] exclusively relied on event trigger generators and time/frequency coincidences. The search in the second science run (S2) includes a new module of coherent analysis: the \( r \)-statistic waveform consistency test. By thresholding on \( \Gamma \), the correlation confidence of coincident events, the test can effectively suppress the burst false alarm rate by 2-4 orders of magnitude. Tests of the method, using simulated signals on top of real S2 noise, yield 50% triple coincidence detection efficiency for narrow-band and broad-band bursts at SNR=3-5 relative to the least sensitive detector.

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Figure 3. Q=9 sine-gaussian signal with $f_0 = 235$ Hz.
(a) Detection efficiency of the $r$-statistic test as function of the amplitude of the simulated signal. The three curves correspond to three values of the $\beta$ threshold.
(b) S2 reference sensitivity curves for the three interferometers. The star’s horizontal position is the central frequency of the sine-gaussian (235 Hz); its vertical position is the $h_{rss}$ with 50% survival probability if $\beta = 3$ is used. This point corresponds to $h_{rss} = 2.6 \times 10^{-21}/\sqrt{\text{Hz}}$ and SNR=9 for L1, SNR=4.5 for H1 and SNR=3.3 for H2. Sensitivity curves and $h_{rss}$ have units of strain$/\sqrt{\text{Hz}}$ (scale to the left). The dashed curve represents the single-sided spectrum for the corresponding waveform, in units of strain/Hz (scale to the right).

Figure 4. As Figure 3, for Gaussian pulses with $\tau = 1.0$ ms.
Note that for broad band signals, the characteristic frequency, the frequency that maximizes the SNR integrand $|\tilde{h}(f)|^2/S_h(f)$, is different for the three interferometers: $f_{\text{char}} = 162$ Hz for L1, $f_{\text{char}} = 223$ Hz for H1 and $f_{\text{char}} = 251$ Hz for H2. The 50% detection probability, with $\beta = 3$ is at $h_{rss} = 6.2 \times 10^{-21}/\sqrt{\text{Hz}}$ and SNR=13 for L1, SNR=6.3 for H1, SNR=4.5 for H2.
Figure 5. Sine-Gaussian Q=9 $f_0 = 235$ Hz: Receiver Operating Characteristics, or detection probability vs false rate curves, parametrized with the $\beta$ threshold. Each curve corresponds to a different signal amplitude: the top legend quotes the $h_{rss}$, while the bottom legend shows the corresponding SNR in the three interferometers.

Figure 6. Gaussian $\tau = 1.0$ ms: Receiver Operating Characteristics, or detection probability vs false rate curves, parametrized with the $\beta$ threshold. Each curve corresponds to a different signal amplitude: the top legend quotes the $h_{rss}$, while the bottom legend shows the corresponding SNR in the three interferometers.
Figure 7. False rate versus $\beta$ from a sample of $1.7 \times 10^5$ events randomly selected in the S2 playground. The dashed line is a fit with an exponential decay, applied to data points with $\beta > 1.5$. The fit has been used to extrapolate the false rate for $\beta > 3$ in the construction of the ROCs in Figures 5 and 6.