Edge Spin Current and Spin Polarization in Quantum Hall Regime

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We study the edge spin current and spin polarization and their responses to an external electric field in a two-dimensional electron gas with a Rashba spin-orbit coupling in the quantum Hall regime. The edge state carries a large spin current, which drops sharply away from the boundary. The spin Hall current of the edge states has a resonance at a critical magnetic field, accompanying a spin rotation in the bulk. The spin Hall current is shown to be proportional to the spin polarization, which provides an explicit way to extract the spin current in experiment.

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The edge state and edge charge current in two-dimensional electron gas (2DEG) in a magnetic field have been well studied in the past two decades and have played an important role in understanding the quantum Hall effect [1, 2, 3]. Recent progresses on spintronics stimulate extensive study of spin generation and transport in semiconductors [4]. The relativistic quantum effect of moving electrons in a confining potential in 2DEG induces a spin-orbit coupling to split the energy spectra of electrons, and provides an efficient way to control the electron’s spin. Driven by an external electric field moving electrons may generate a pure spin Hall current via the magnetic impurity or spin-orbit coupling [2]. Very recently the spin accumulation has been observed in both n-type semiconductors and p-type heterojunctions in experiments, which provides substantial evidence of pure spin current [2]. In the presence of a strong magnetic field, the competition between the Rashba spin-orbit coupling and the Zeeman splitting in 2DEG introduces an additional degeneracy and gives rise to somewhat new effects, such as the resonant spin Hall conductance [2] and the anisotropic spin transport [3]. In this Letter we study the edge spin current and spin polarization, and their responses to an external electric field in 2DEG with a Rashba coupling in a magnetic field. The edge effect will cause the anticrossing of Landau levels. The spin Hall conductance is calculated using the edge state approach, and the result can be used to complement the bulk theory for the resonant spin Hall effect. The spin current is shown to be proportional to the spin polarization along the y-direction in a finite magnetic field, which provides an explicit way to extract the spin current in experiments.

We consider a 2DEG with the Rashba coupling in the x-y plane of area $L \times L$ subject to a perpendicular magnetic field $\vec{B} = -B\hat{z}$. The electrons are confined between $-L/2$ and $L/2$ in the y-direction by an infinite potential wall, and its wavefunction is periodic along the z-direction. We choose a Landau gauge $\vec{A} = By\hat{x}$. The Hamiltonian for a single electron with a Rashba coupling is given by

$$H = \vec{\Pi}^2/2m - g_s \mu_B \vec{B} \sigma_z/2 + (\lambda/\hbar)(\Pi_x \sigma_y - \Pi_y \sigma_x),$$

(1)

where the confining potential is implied. $m$, $-e$, and $g_s$ are the electron’s effective mass, charge, and Lande-g factor, respectively. And $\vec{\Pi} = \vec{p} + e\vec{A}/c$ is the kinetic operator, $\mu_B$ is the Bohr magneton, and $\sigma_a$ are the Pauli matrices. The edge state and the edge charge current in the absence of the Rashba coupling have been studied previously [2, 3]. In that case, the eigenstate is given by $\Phi_{n,y_0,0}(y) = e^{i \pi y/2} \phi_{n,y_0}(y) x_{\pm 1/2}$, satisfying the boundary condition $\phi_{n,y_0}(\pm L/2) = 0$, where $\phi_{n,y_0}(y) = (y + y_0 + i\delta_{y_0}'\sqrt{\mathcal{D} y_0}) / \sqrt{\mathcal{D} y_0}$ and $[a_{y_0}, a_{y_0}^\dagger] = \delta_{y_0,y_0}'$. The energy eigenvalues are $E_{n,y_0}(y) = (\nu_{n,y_0} + (1 - g_s)/2) e\omega$, where $\omega = eB/mc$ is the cyclotron frequency. For $|y_0| < L/2$ far away from the edges, the problem is reduced to a simple harmonic oscillator and the eigen value $\nu_{n,y_0} = n$, a non-negative integer. At the two edges $y_0 = \pm L/2$, the boundary condition leads to $\nu_{n,y_0} = 2n + 1$.

In the presence of the Rashba spin-orbit coupling, $p_x$ remains to be a good quantum number. In the Hilbert subspace of $y_0$ the Hamiltonian can be written as

$$H(y_0) = \hbar \omega [a_{y_0}^\dagger a_{y_0} + (1 - g_s)/(2 + i\eta (a_{y_0} \sigma_+ - a_{y_0}^\dagger \sigma_-))],$$

(2)

where $\eta = \lambda m l_0/\hbar^2$ is the effective Rashba coupling, and $g = g_s m/2mc$. Far away from the two edges the Rashba coupling mixes only two states $\Phi_{n,y_0,0}$ and $\Phi_{n+1,y_0,0}$ so that an analytical solution can be obtained [5, 6, 7]. Near the boundary, exact analytical solutions seem unlikely. We use $\{\Phi_{n,y_0,0}, \Phi_{n+1,y_0,0}\}$, the eigen states of $H(y_0)$ at $\lambda = 0$, as the base functions. The lower energy
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\[ \{E_{n,s}(y_0)\} \]

d and the corresponding eigenvectors can be obtained numerically by truncating sufficiently 

higher Landau level states. In Fig. 1, we show a typical 

eigenvalue spectrum of the lowest twenty Landau levels at 

$y_0 = L/2 - 4b_0$ as a function of the effective Rashba 

coupling $\eta$, for a set of realistic parameters suitable for 

\[ \text{InSb-GaAs/InSb, } \text{GaAs/InSb, } \text{AlAs/InSb, } \text{As/InSb, } \text{AlGaAs/InSb, } \text{InGaAs/InSb, } \text{InGaAlAs/InSb, } \text{InGaP/InSb.} \]

While the quantitative results depend on the precise parameters, the basic physics we address is quite general. Different from the 

bulk state, where the competition between the Rashba coupling and the Zeeman splitting introduces an additional degeneracy [8], there is a level anticrossing [12] near the boundary of a distance of several magnetic 

length as shown in Fig. 1. The edge effect to the level 

anticrossing may be understood as follows. The Rashba coupling mixes the two edge states of opposite spins in the same Landau level, namely $\Phi_{n,y_0,-1}$ and $\Phi_{n,y_0,1}$, while this mixing vanishes for the bulk states.

We now turn to the discussion of the charge and spin currents and the spin polarization near the boundary.

The charge current operator of a single electron, in the 

Hilbert subspace of $y_0$, is given by 

\[ j_c = -e v_x, \]

and $v_x = [x,H]/ih$. The spin-$\tau$ component current operator is 

\[ j_\tau^\alpha = h(\sigma_\alpha v_x + v_x \sigma_\alpha)/4. \]

Let $(j_x, s)_\tau$ be the charge ($e$) or spin ($s$) current carried by an electron in the eigen state $|\tau\rangle$ of $H$ in Eq. (2) with a collective index $\tau = (n, y_0, s)$ and $(S^\alpha)_\tau = (\tau | \sigma_\alpha | \tau)/2$ the average $\alpha$-component spin polarization in that state. Note that the state represented by $y_0$ is extended in the $x$-direction but localized around $y = -y_0$ in the $y$-direction. The calculated expectation values are plotted in Fig. 2 as functions of the position of the edge state described by $y_0 = L/2 - \zeta l_b$ ($\zeta \geq 0$). In comparison with their bulk values, both $(j_x)_\tau$ and $(j_z)_\tau$ are markedly larger. As for the spin polarization, $(S^x)_\tau$ and $(S^y)_\tau$ display interesting features: the spin is mostly polarized along the $z$-axis in the bulk, while mostly polarized along the $y$-axis at the edges. It is interesting to note from Fig. 2 that the large polarization of $S_y$ coincides with the large charge current at the edge. From a mean field point of view, the Rashba coupling $V_R(y_0) = \lambda m_0 (v_x \sigma_y - v_y \sigma_x)/\hbar - 2m \alpha^2 / \hbar^2$ introduces an effective magnetic field along the $y$-direction, which is proportional to the charge current along the $x$-direction. It is most interesting to note that the average of $S_y$ is identical to the spin current $j_y^s$ except for an overall proportionality, $(j_y^s)_\tau = -0.831 (\hbar/m_0 b)(S^y)_\tau$ for the specific parameters in Fig. 2. In fact, there is an exact relation between the two quantities in the present case,

\[ \lambda (j_y^s)_\tau = -(g_s \hbar \mu_B / 2m) B(S^y)_\tau. \]

(3)

To derive the above relation and to see the required condition for the equation, we consider a commutator [13],

\[ [H, \sigma_x] = -i (4m \lambda / \hbar^2) j_y^s - ig_s \hbar \mu_B B \sigma_y. \]

(4)

In the present case, $g_s$ is a good quantum number, and the energy eigenstate $|\tau\rangle$ is localized in the $y$-direction around $-y_0$ and vanishes at the two edges. Because of these properties, the expectation value of this commutator $\langle [H, \sigma_x] \rangle_\tau = \langle \tau | H \sigma_x - \sigma_x H | \tau \rangle = 0$, and hence Eq. ?? is approved. We note that the relation holds in the presence of an electric field $E = Ey$. Because the relation applies to each eigenstate of $H$ in the absence or in the presence of an electric field, the thermodynamic averages of the spin polarization and the spin current are also proportional. However, one has to be cautious to apply the relation to the case with $B = 0$ and $E \neq 0$. At $B = 0$, the states are extended in both $x$- and $y$-directions, and the above derivation for the vanishing integral may not apply. We note that the spin Hall current at $B = 0$ cannot be obtained by taking $B \rightarrow 0$. Inclusion of the Dresselhaus coupling [14] will also give a relevant relation. In the presence of disorder and electron-electron interactions, the spin current and spin polarization vary in space, but such a relation between the spin current and the spin polarization can be shown to hold at any spatial point in the quantum Hall regime. The detailed discussion will be presented elsewhere.

Now we come to discuss the effect of an electric field along the $y$-direction, $V_E = -eEy$. We use the truncation approximation in order to solve the problem and evaluate the energy eigenvalues and eigenstates numerically. We find that the edge charge current responses linearly to the electric field. Here we focus on the non-linear behaviors of the spin currents and spin polarization at or near the resonant magnetic field $B_0$ where the two levels

![Energy Spectra](image)
decreases quickly to zero, but the \( \varphi \) vary non-linearly with the electric field. When the energy gap \( \hbar \omega \) is large, the two levels give the spin along the arrowed line with numbers indicate the order of the energy eigenstates from the lowest to higher ones.

in the bulk are degenerate. In Fig. 3, we plot \((j_s)_x\), \((S^y)_x\), and \((S^z)_x\) in a weak electric field \( E = 0.01 \text{ volt/m} \). Our data show that \((j_s)_x\) is still proportional to \((S^y)_x\) in the presence of the electric field so we plot \((j_s)_x\) and \((S^y)_x\) in the same figure. At the resonant point we notice that the electric field generates a finite spin current in the bulk which is almost equal to the edge spin current at the edge of \( y_0 = -L/2 \). At the edge of \( y_0 = +L/2 \) the current has the same value, but different direction. \((S^z)_x\) decreases quickly to zero, but the \((S^y)_x\) increases to one in unit of \( \hbar/2 \) approximately. Thus the spin rotates from the \( z\) to \( y\)-direction. Near the resonant magnetic field where the energy gap \( E_G \) of the two levels at the bulk is larger than, but comparable with the electric field energy, \( E_G \approx eE \), both the spin current and spin polarization vary non-linearly with the electric field. When the energy gap \( E_G \) is much larger than the electric field energy, \( E_G \gg eE \), the response becomes linear to the field. The rotation of the spins of the two levels in a weak field can be understood in the theory of resonant spin Hall effect. A weak field removes the degeneracy of the two levels of the bulk along the spin \( z\)-direction, and as a result the mixed two levels give the spin along the \( y\)-direction. The rotation of spin from the spin \( z\)-direction to the spin \( y\)-direction should be observed near the resonant point experimentally.

With the periodic boundary condition along the \( x\)-direction the velocity operator in the Hilbert subspace of \( y_0 \) can be written as \( v_x = (\ell_0^2/\hbar)(\partial H/\partial y_0) \) and thus the spin current operator \( j_s^z = \ell_0^2/2(\partial H/\partial y_0\sigma_z + \sigma_z\partial H/\partial y_0)/2 \). Following the works by Halperin and MacDonald and Streda, the total spin Hall current in the filled Landau level \((n, s)\) can be expressed as

\[
(j_s^z)_{n,s} = \int_{-L/2}^{+L/2} \frac{dy_0}{4\pi} \frac{\partial E_{n,s}(y_0)}{\partial y_0} \langle \tau | \sigma_z | \tau \rangle - \sum_{n', s'} \int_{-L/2}^{+L/2} \frac{dy_0}{8\pi} \left( E_{n,s}(y_0) - E_{n', s'}(y_0) \right) \times \\
\left( \langle \tau | \partial_{y_0} | \tau' \rangle \langle \tau' | \sigma_z | \tau \rangle - \langle \tau | \sigma_z | \tau' \rangle \langle \tau' | \partial_{y_0} | \tau \rangle \right)
\]

where \( \tau' = (n', y_0, s') \). Without the spin-orbit coupling the energy eigenstate satisfies \( \langle \tau | \sigma_z | \tau' \rangle = s\delta_{n,n'}\delta_{s,s'} \) so that \((j_s^z)_{n,s} = -seV/4\pi\), which is only determined by the voltage difference at the two edges, \( E_{n, s}(L/2) - E_{n, s}(-L/2) = -eV\), and the inclusion of impurities and Coulomb interactions in the Hamiltonian does not affect this result as for the charge Hall current. The spin Hall conductance displays a series of plateau in the quantum Hall regime, \( G_s = (1 - (-1)^n)e/8\pi \) corresponding to the quantum Hall conductance, \( G_c = ne^2/h \). In the presence of spin-orbit coupling the states with different spins will be mixed together, and the spin gradually deviates from the \( z\) to \( y\)-direction and the spin Hall conductance varies with the effective Rashba coupling or magnetic field through tuning the energy gap between the two states especially near the Fermi level. We calculate
the total spin Hall conductance numerically as a function of $1/B$ for a fixed chemical potential assuming that the voltage drops only near the edges and the bulk state does not contribute to the total spin Hall current. Both charge and spin Hall conductances are plotted in Fig. 4. As expected the Hall conductance is quantized, the spin Hall conductance has the order of $e/4\pi$ if odd number of Landau levels is occupied, and is of order of $10^{-3} – 10^{-4}e/4\pi$ if even number of Landau levels is occupied. The spin Hall conductance is a function of the effective spin-orbit coupling, which varies with the magnetic field. The resonant peak appears only when the two degenerate bulk Landau levels crosses over a special value of chemical potential with decreasing the magnetic field. In the inset of Fig. 4 we extract the spin Hall conductance for a fixed density of charge carriers, $n_c = 1.9 \times 10^{12}$ cm$^{-2}$.

The spin and charge edge currents will become dominant when the bulk states are localized due to the impurities and the charge Hall conductance is quantized. In a realistic sample of a finite size both charge and spin bulk Hall current densities will not be suppressed completely, but decays with the size of the sample if the impurities are taken into account. The total charge and spin Hall currents consist of both the edge and bulk current. The ratio of the edge and bulk Hall currents or the distribution of the electric field are determined by the disorder, Coulomb interaction, and the rigidity of the confining potential wall as studied by several authors. The quantum Hall effect can be understood from both the bulk and edge state point of view. In this Letter we have presented a picture of edge states for spin Hall conductance for a 2DEG with a Rashba coupling. The theory is consistent with and complementary to the bulk theory for resonant spin Hall conductance, especially in quantum Hall regime. In this system the motion of electrons will induce an effective magnetic field via the Rashba coupling, and generate spin polarization. Since the spin current is proportional to the spin polarization along the $y$-direction this relation will provide an explicit way to extract spin current from the distribution of spin polarization.

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