DIMENSIONAL CROSSOVER IN HEAVY FERMIONS

Mucio A. Continentino

Instituto de Física, Universidade Federal Fluminense

Campus da Praia Vermelha, Niterói, 24.210-340, RJ, Brasil

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Recently we have shown that a one-parameter scaling, \( T_{coh} \), describes the physical behavior of several heavy fermions in a region of their phase diagram. In this paper we fully characterize this region, obtaining the uniform susceptibility, the resistivity, and the specific heat. This allows for an explicit evaluation of the Wilson and the Kadowaki-Woods ratios in this regime. These quantities turn out to be independent of the distance \( |\delta| \) to the critical point. The theory of the one-parameter scaling corresponds to a zero dimensional approach. Although spatial correlations are irrelevant in this case, time fluctuations are critically correlated and the generalized hyperscaling relation is satisfied for \( d = 0 \). The crossover from \( d = 0 \) to \( d = 3 \) is smooth. It occurs at a length scale which is inversely related to the stiffness of the lifetime of the spin fluctuations.

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I. INTRODUCTION

Most of the physical properties of heavy fermions can be attributed to the fact that these systems are close to a quantum critical point (QCP) \(^1\). The critical point arises as a competition between Kondo effect and magnetic order induced by RKKY coupling. In the non-critical side of the phase diagram, where the system never orders, a scaling approach reveals the existence of a new characteristic temperature, the coherence temperature \( T_{coh} \propto |\delta|^{\nu z} \), below which the system exhibits Fermi liquid behavior \(^3\). In this equation, \( |\delta| = |J_Q - J_Q^c| \) measures the distance to the \( T = 0 \) critical point and \( \nu \) and \( z \) are respectively the correlation length and dynamic critical exponents. \( J_Q \) is the coupling between the local moments and \( J_Q^c \) its critical value, at which the magnetic instability characterized by the wavevector \( Q \) occurs. At the QCP, i.e., \( |\delta| = 0 \), the system does not cross the coherence line and consequently exhibits non-Fermi liquid behavior down to \( T = 0 \) \(^1\).

Recently we have shown that a one-parameter scaling, the coherence temperature, is able to describe the pressure behavior of several physical quantities for different heavy fermions \(^1\). In this report we show this is due to the flatness of the spectrum of spin fluctuations. We use the spin fluctuation theory of a nearly antiferromagnetic system \(^4\) \(^5\) \(^6\) to fully characterize this one-parameter scaling regime and calculate the specific heat, the uniform susceptibility and the resistivity. This allows for an explicit evaluation of the Wilson ratio and the Kadowaki-Woods ratio \(^7\) between the coefficient of the \( T^2 \) term in the resistivity and that of the linear term of the specific heat.

We make use of the spin fluctuation model since it is a Gaussian theory and for the problem considered here, where the effective dimension \( d_{eff} = d + z > d_c = 4 \) with \( d_c \) the upper critical dimension for the magnetic transition, it gives the correct description of the quantum critical behavior \(^8\) \(^9\).

II. SPECIFIC HEAT

We start from the expression for the specific heat given by the spin-fluctuation theory for a nearly antiferromagnetic electronic system \(^3\). We will use here the notation of Ref. \(^3\),

\[
C/T = \frac{\partial^2}{\partial T^2} \left[ \frac{3}{\pi} \sum q T \int_0^{\infty} \frac{d\lambda}{e^{\lambda T} - 1} \tan^{-1} \left( \frac{\lambda T}{q Q} \right) \right]
\]

(1)

where

\[
\Gamma_q = \Gamma_L(1 - J_Q\chi_L) + \Gamma_L\chi_L A q^2
\]

\( \Gamma_L \) and \( \chi_L \) are local parameters defined through the local dynamical susceptibility \(^6\)

\[
\chi_L(\omega) = \frac{\chi_L}{1 - \omega^2/\Gamma_L}
\]

\( J_Q \) as before is the \( q \)-dependent exchange coupling between f-moments and \( A \) is the stiffness of the lifetime of the spin fluctuations defined by the small wavevector expansion of the magnetic coupling close to the wavevector \( Q \), i.e., \( J_Q - J_{Q+q} = A q^2 + \cdots \). Then \( \Gamma_q \) can be rewritten as

\[
\Gamma_q = \Gamma_L\chi_L A\xi^{-2}[1 + q^2\xi^2]
\]

where the correlation length \( \xi = (A/|J_Q - J_Q^c|)^{1/2} \) diverges at the critical value of the coupling \( J_Q^c = \chi_L^{-1} \) with the Gaussian critical exponent \( \nu = 1/2 \). Consequently we have for the specific heat

\[
C/T = \frac{\partial^2}{\partial T^2} \left[ \frac{3}{\pi} \sum q T \int_0^{\infty} \frac{d\lambda}{e^{\lambda T} - 1} \tan^{-1} \left( \frac{\lambda T}{\Gamma_L\chi_L A(1 + q^2\xi^2)} \right) \right]
\]

(2)
where the dynamic critical exponent $z = 2$, typical of antiferromagnetic spin fluctuations. The exponential cuts off the contribution for the integral from large values of $A$, consequently for $(T\xi^q)/\Gamma_{L\chi L}A) < < 1$ we can expand the $\tan^{-1}$ for small values of its arguments. The above condition can be written as $T << T_{coh}$, where the coherence temperature

$$k_B T_{coh} = \Gamma_{L\chi L}|J_Q - J_Q^c| \propto |\delta|^{\nu z}$$

is independent of $A$ and $\nu z = 1$. In this regime the system shows Fermi liquid behavior and we get

$$C/T = \frac{\partial^2}{\partial T^2} \left[ \frac{3T^2\xi}{\pi^3} e^\chi - 1 \sum_q \frac{1}{1 + q^2\xi^2} \right]$$

Changing the $\sum_q$ into an integral we find (d=3),

$$C/T = \frac{\partial^2}{\partial T^2} \left[ \frac{\pi^2\xi^{(d-4)}/4V}{\pi^3} \frac{d\lambda}{1 + \frac{1}{2}y^2} \right]$$

which yields

$$C/T = \frac{\pi^2\xi^{(d-4)}/4V}{\Gamma_{L\chi L}A} \frac{1}{1 + \frac{1}{2}y^2}$$

Taking the limit $q_c\xi << 1$ and since $\tan^{-1} y \approx y - y^3/3 + y^5/5 + \cdots$ for small $y$, we get

$$C/T = \frac{\pi^2\xi^{(d-4)}/4V}{\Gamma_{L\chi L}A} \left[ \frac{1}{3} (q_c\xi)^2 - \frac{1}{5} (q_c\xi)^4 + \cdots \right]$$

The first term is independent of $A$ and yields essentially the result of the local interacting model [9], i.e.,

$$C/T = \frac{\pi N k_B^2}{\Gamma_{L\chi L} |J_Q - J_Q^c|}$$

In fact this could have been obtained directly from Eq.3, neglecting the $q$-dependence of $\Gamma_q$ and with $\sum_q \rightarrow N$ [9]. In the equation above the correct units have been restored. Note that the limit $q_c\xi << 1$ may be written as $q_c (A/|J_Q - J_Q^c|)^{1/2} << 1$. This can be satisfied either because the system is far away from the critical point, i.e., $|J_Q - J_Q^c|$ is large, or because $A$ is small. If we write the condition $q_c\xi << 1$ in the form $q_c/\sqrt{|J_Q - J_Q^c|} << 1/\sqrt{\lambda}$, we notice that when $A \rightarrow 0$ this condition becomes valid arbitrarily close to the quantum critical point.

We can rewrite the equation above for the specific heat as $C/T = \pi N k_B^2/T_{coh}$. The large effective masses of heavy fermions are then related to the smallness of $T_{coh}$ consistent with the experimental observations.

### III. SUSCEPTIBILITY AND WILSON RATIO

The zero temperature uniform susceptibility of the nearly antiferromagnetic system in the limit $q_c\xi << 1$ can be directly obtained from the magnetic field ($h$) dependent, $T = 0$, $q$-independent free energy [10], [11], [12].

$$f = \frac{3N}{2\pi} \int_0^{\omega_E} d\omega \tan^{-1} \left[ \frac{\omega + h}{\Gamma_{L\chi L} |J_Q - J_Q^c|} \right]$$

Integrating, differentiating once, twice, taking the value at $h = 0$ and the limit $\omega_E \rightarrow \infty$ we obtain

$$\chi_0 = -\left( \frac{\partial f}{\partial h} \right)_{h=0} = \frac{3N\mu^2}{2\pi\Gamma_{L\chi L} |J_Q - J_Q^c|}$$

or $\chi_0 = 3N\mu^2/2\pi T_{coh}$.

The Wilson ratio is given by

$$\frac{\chi_0/\mu^2}{C/\pi^2 k_B T} = \frac{3}{2} = 1.5$$

which turns out to be a universal number since the dependence on the distance to the critical point, $|J_Q - J_Q^c|$ and on the dimensionless quantity $\Gamma_{L\chi L}$ cancels out. This ratio can increase if we decrease the energy cut-off of the excitations contributing to the specific heat. We emphasize that the above result is valid in the regime $q_c\xi << 1$, that is, if the system satisfies the condition, $q_c/\sqrt{|J_Q - J_Q^c|} << 1/\sqrt{\lambda}$.

### IV. RESISTIVITY AND KADOWAKI-WOODS RATIO

The resistivity due to spin fluctuations in the regime $q_c\xi << 1$ is given by [10], [11], [12].

$$\rho = \rho_0 \frac{1}{T} \int_0^{\omega_E} d\omega \frac{\omega \Im \chi Q(\omega)}{(e^{\beta \omega} - 1)(1 - e^{-\beta \omega})}$$

where

$$\Im \chi Q(\omega) = \chi_0^2 \frac{\omega \xi^2}{1 + (\omega \xi^2)^2}$$

with

$$\chi_0^2 = \frac{1}{|J_Q - J_Q^c|}$$

and

$$\xi^2 = \frac{\chi_0^2}{\Gamma_{L\chi L}}$$

The quantity $\rho_0$ is given by

$$\rho_0 = \frac{J}{W} \frac{m_c}{n_e e^2 \tau_{pe}} (n/n_c)$$
where \( J \) is the coupling constant per unit cell between localized and conduction electrons. \( W, m_e \) and \( n_e \) are the bandwidth, the mass and the number of conduction electrons per unit volume with Fermi momentum \( k_F \), such that \( h \omega_{F}^{-1} = h^2 k_F^2 / 2m_e \). \( n \) is the number of atoms per unit volume.

Using the definitions above, we can rewrite the resistivity as, \( \rho = \rho_0 \Gamma_{LX} R(T) \), where

\[
R(T) = \frac{1}{T} \int_0^\infty \frac{\tilde{\omega}^2}{(\tilde{\omega}/T - 1) (1 - e^{-\tilde{\omega}/T})} \frac{1}{1 + \tilde{\omega}^2} \quad (11)
\]

with \( \tilde{\omega} = \omega \xi_L \) and \( T = T \xi_L \). For \( T << T_{coh} \) we have

\[
R(T << T_{coh}) = \frac{\rho_0 \Gamma_{LX}}{3} \frac{(T_{coh})^2}{(T_{coh})^2} = A_R T^2
\]

where

\[
A_R = \frac{\rho_0 \pi^2 k_B^2}{3 \Gamma_{LX} |J_Q - J_Q'|^2}
\]

The Kadowaki-Woods ratio \( A_R/(C/T)^2 \) is given by

\[
\frac{A_R}{(C/T)^2} = \frac{\rho_0 \Gamma_{LX}}{3(N k_B)^2}
\]

which depends on the local parameters, \( \Gamma_{LX} \), consequently on the nature of the magnetic ion (\( f \) or \( d \), for example), but not on the distance to the critical point, \( |J_Q - J_Q'| \). We get \( A_R/(C/T)^2 \approx 4.8 \times 10^{-9} \rho_0 \Gamma_{LX} (\text{moleK}/m.J)^2 \). Using the \( T = 0 \) value of \( \Gamma_{LX} = 1/2\pi \) and the experimental value for this ratio we can find \( \rho_0 \) and determine microscopic parameters of the system [1].

We point out that in the \( q \)-dependent regime, \( q \xi \geq 1 \), also \( \rho = A_R^M T^2 \) at low temperatures but the coefficient \( A_R^M \propto |J_Q - J_Q'|^{-1/2} \) and consequently does not scale as \( T_{coh}^{-2} \), in disagreement with experiments in heavy fermions [1].

**V. THE NON-FERMI LIQUID REGIME**

As the system gets close to the QCP and \( q \xi \geq 1 \), the system should be described using the full \( q \)-dependent susceptibility. In particular at the quantum critical point, i.e., \( |\delta| = 0 \) but finite temperatures, the neglect of the \( q \)-dependence of \( \chi(q, \omega) \) leads to unphysical behavior as, a constant resistivity and diverging specific heat. The spin fluctuation theory predicts that the Néel line close to the quantum critical point behaves as \( T_N \propto |\delta|^\psi \), where the shift exponent \( \psi = z/(d+z-2) = 2/3 \neq \nu_z = 1 \).

The appropriate generalized scaling form of the free energy for this case is given by \( f \propto |\delta(T)|^{2-\alpha} F_c[t] \) with \( t = T/|\delta(T)|^{2-\alpha} \) and \( \delta(T) = \delta(T = 0) - u T^{1/\nu} \) where \( u \) is a constant [2]. The singularities along the Néel line, \( |\delta(T)| = 0 \), are described by tilde exponents \( \tilde{\alpha}, \tilde{\nu}, \) etc., different from those associated with the zero temperature fixed point (the non-tilde exponents) [3].

The scaling function \( F \) is not \( \delta(T = 0) \) constant and \( F_c[t \to \infty] \propto t^x \) with \( x = (\tilde{\alpha} - \alpha)/\nu_z \) such that close to the critical Néel line we obtain the correct asymptotic behavior, \( f \propto A(T)|\delta(T)|^{2-\tilde{\alpha}} \), where the amplitude \( A(T) = T^{2/\nu_z} \). For the specific heat we find

\[
C/T \approx u^{2-\tilde{\alpha} T}(2-\tilde{\alpha}(\nu_z - \tilde{\nu})) T^{-\tilde{\nu}} \quad (12)
\]

Assuming thermal Gaussian exponents, essentially \( \tilde{\alpha} = 1/2 \), we get, \( C/T \propto u^{3/2} T^{5/4} \) for \( \psi = 2/3, \nu = 1/2 \) and \( z = 2 \), instead of \( C/T \propto T^{3/2} \) for the case of extended scaling \( \nu_z = \psi \). The staggered susceptibility \( \chi(\delta = 0, T) \propto T^{-\tilde{\gamma}/\nu} = T^{3/2} \) since \( \gamma = \tilde{\gamma} = 1 \) [3]. For the correlation length we get \( \xi^{-2} \propto u T^{2+\tilde{\gamma}/\nu} \) since \( \nu = \tilde{\nu} \). This should be compared to Eq. 3.11 of Ref. 1. Notice that the thermal and \( T = 0 \) Gaussian critical exponents are the same, except for the exponent \( \alpha \) of the free energy \( \alpha = 2 - \tilde{\nu}d = 1/2 \), \( \alpha = 2 - \nu(d+z) = -1/2 \) and this is the reason our prediction for the specific heat at \( |\delta| = 0 \) is different from the spin fluctuation result [3].

Notice that while the Gaussian theory yields the correct exponents for the zero temperature transition, this is not the case for the finite temperature Neel transitions and the expression above for \( C/T \) may be used with non-Gaussian thermal exponents, for example, with those of the 3d Heisenberg model.

**VI. CONCLUSION**

We have calculated the Wilson and Kadowaki-Woods ratio for a model of nearly antiferromagnetic systems in the regime \( q \xi \leq 1 \). These quantities turn out to be constants, i.e., independent of the distance \( |\delta| \) to the critical point. The spin fluctuation theory in the regime \( q \xi \leq 1 \) corresponds to a local interacting model and yields a one-parameter scaling since \( C/T \propto T_{coh}^{-\chi_0} \propto T_{coh}^{-1} \), \( A_R \propto T_{coh} \) with \( T_{coh} \propto \xi^{-2} \). The local interacting model becomes valid arbitrarily close to the QCP as the stiffness \( A \) of the lifetime of the spin fluctuations vanishes. It can be regarded as a zero dimensional theory since, in spite that spatial correlations are irrelevant in this regime, time fluctuations are critically correlated and the quantum hyperscaling relation \( 2 - \alpha = \nu(d+z) \) is satisfied for \( d = 0 \) [1]. This is a direct consequence of the quantum character of the transition. As the system gets closer to the critical point and \( q \xi \geq 1 \), the full \( q \)-dependence of the dynamic susceptibility must be taken into account. The crossover from \( d = 0 \) to three dimensional behavior is smooth. It occurs at a length scale which is inversely related to the stiffness \( A \). It is possible that there is an intervening region dominated by two-dimensional fluctuations [12] before the system finally settles in three dimensional criticality.
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