Signatures of Neutrino Cooling in the SN1987A Scenario

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ABSTRACT

27 years after the detection of the thermal neutrino single signal from SN1987A, which confirmed the core-collapse scenario and the possible formation of a neutron star, there has not been detected yet its compact remnant. Although this neutron star has eluded all observations to date, theoretical and numerical developments have allowed a glimpse of the fate of it. In particular, the hypercritical accretion model proposed by Chevalier (1989) forecasted the accretion of $\sim 0.15 \, M_\odot$ in two hours and after the submergence of the magnetic field in the newborn neutron star as was proposed by Bernal et al. (2013). In this paper, we revisit such model for SN1987A in a numerical framework, focusing on the neutrino cooling effect in the supernova fallback dynamics. For that, we carry out numerical simulations of the accretion of matter onto the surface of the newborn neutron star in order to estimate the physical parameters such as the high-scale of the neutrino-sphere above surface of the neutron star, the emissivity and luminosity of neutrinos in this regime using both an analytical formula and the tabulated values for several neutrino cooling processes (electron-positron annihilation, inverse beta decay, nucleonic bremsstrahlung and plasmons). We use a customized version of the FLASH code to perform the numerical simulations in a 2D spherical mesh. In addition, due to matter effects thermal neutrinos may oscillate resonantly from one flavor to another when they go through the outer layers of the expanding supernova. To investigate these oscillations, we consider an energy of neutrino in the range of 1-30 MeV, the two- and three-neutrino mixing parameters, splitting then this phase of the supernova in four regions: core surface, accretion shock envelope, free fall region and surface of the star. For the magnetized plasma of the first region, we derive the neutrino effective potential up to order $1/M_\odot^4$. For the other regions and based on each density profile we calculate the probabilities of neutrino oscillations at different radii. Finally, in addition of neutrino luminosity, we estimate the number
of events expected as well as the neutrino flavor ratio on Earth, as a signature of this phase.

*Subject headings:* accretion – hydrodynamics – neutrino: cooling – neutrino: oscillations – stars: neutron – supernovae: individual (SN1987A)
1. Introduction

It took almost 400 years so that another supernova, after Kepler’s supernova in 1604, could be observed at naked eye. The neutrino signal from SN1987A detected in terrestrial observatories (Mont Blanc, Kamiokande, IBM and Baksan), suggested the formation of a neutron star in the supernova core (Woosley 1988). So far the search for this holy grail of the type II supernovae paradigm continues. SN1987A was a core-collapse supernova with known progenitor (Sanduleak -69 202) and an estimated distance of \( d \approx 50 \pm 1 \) kpc (Panagia 2005), in the Large Magellanic Cloud. The progenitor was a blue supergiant spectral class B3Iab, with 20 \( M_\odot \), effective temperature of \( 1.6 \times 10^4 \) K, and an estimated size of 43 \( R_\odot \). The total energy in the collapse was \( \approx 10^{53} \) erg, but only 1% of this energy was released in the shock wave to blow the supernova. In the first few seconds, the debris of core-collapse was dense enough that neutrinos could hardly escape. After that, the neutrino optical depth became \( \approx 1 \), thus allowing the gravitational energy to be carried out by the neutrinos created in the formation of the newborn neutron star, with an inferred energy \( \approx 3 \times 10^{53} \) ergs, temperature \( k_B T \approx 4 \) MeV and decay time-scale of the neutrino burst \( \approx 4 \) s. These parameters were consistent with models in which a degenerate iron core collapsed to form a neutron star (Burrows & Lattimer 1986), although there is not yet any evidence of the presence of a pulsar, or even a quiet neutron star inside SN1987A. The possible scenarios to solve this problem range from the delayed collapse of the neutron star into a black hole (Brown & Bethe 1994), a demagnetized neutron star by hyperaccretion of material (Bernal et al. 2013) to the formation of a quark star (Muslimov & Page 1995).

In the core-collapse scenario, the shock wave gets its way out through the outer layers of the progenitor until it encounters a discontinuity in density. At this point, a reverse shock is generated starting a fall-back phase which can induce hypercritical accretion onto the newborn neutron star surface a few hours after the explosion. This scenario is only possible if the progenitor had
Fig. 1.— Core-collapse evolution in SN1987A. (A) The iron core collapse; (B) a proto-neutron star is created as well as a bounce with a shock expanding outwards; (C) a reverse shock is formed at the H/He interface; (D) an accretion shock is established in the neutrino-cooled regime. In this phase the star is split into four regions: core surface, accretion shock envelope, free fall region and surface of the star, each with a radial density, pressure and velocity characteristic profiles. Taken from [Bernal et al.] (2010).
a tenuous H/He envelope surrounding a dense He core, as SN1987A (Smartt 2009). Chevalier (1989) argued in favor of such scenario of late accretion onto the SN1987A core and developed an analytical model for the hypercritical regime. In such a model, the neutrino cooling plays an important role in the formation of a quasi-hydrostatic envelope around the newborn neutron star. Recently, Bernal et al. (2013, 2010) showed that the magnetic field, for SN1987A parameters, does not play an important role in such regime because it is submerged into the crust of the newborn neutron star, therefore we will adopt the idea of a demagnetized newborn neutron star as compact remnant in this supernova. Fig. 1 shows a schematic evolution of a collapsing stellar core and the hypercritical regime as a possible scenario for SN1987A.

Due to the inverse beta decay, electron-positron annihilation ($e^- + e^+ \rightarrow Z \rightarrow \nu_j + \bar{\nu}_j$) and nucleon-nucleon bremsstrahlung ($N + N \rightarrow N + N + \nu_j + \bar{\nu}_j$) for $j = e, \nu, \tau$, thermal neutrinos will be produced at the core and then they will propagate inside the star. The properties of these neutrinos will get modified when they propagate in a magnetized medium and depending on the flavor of the neutrino, it would feel a different effective potential because electron neutrino ($\nu_e$) interacts with electrons via both neutral and charged currents (CC), whereas muon ($\nu_\mu$) and tau ($\nu_\tau$) neutrinos interact only via the neutral current (NC). This would induce a coherent effect in which maximal conversion of $\nu_e$ into $\nu_\mu$ ($\nu_\tau$) takes place even for a small intrinsic mixing angle. The resonant conversion of neutrino from one flavor to another due to the medium effect is well known as the Mikheyev-Smirnov-Wolfenstein effect (Wolfenstein 1978). These neutrino oscillations have been widely studied in the literature in different scenarios (Ruffert & Janka 1999, Goodman et al. 1987, Volkas & Wong 2000, Janka 2012, 2013).

In this work we do a numerical study of hyperaccretion of matter onto the newborn neutron star surface in the SN1987A scenario. We are considering as input the analytical model proposed by Chevalier (1989) and Brown & Weingartner (1994) about the late accretion of matter onto compact objects, few moments after the core-collapse, focusing on the importance of the neutrino cooling processes on the neutron star surface. We employ the AMR FLASH code to carry out
the numerical simulations of the reverse shock, the subsequent formation of a quasi-equilibrium
envelope and the complex dynamics near stellar surface, including several neutrino cooling
processes, a more detailed equation of state, and an additional degree of freedom in the system.
Additionally, we calculate the neutrino oscillations when they travel through the outer layers of
supernova remnant. The paper is arranged as follows. In section 2 we describe the analytical
model as input on the numerical approach focusing on the neutrino cooling processes present in
the system. In section 3 we develop the neutrino oscillations models. In section 4 we discuss the
radial profiles inside the supernova remnant and the neutrino propagation in such regions. Finally,
In section 5 we present and discuss the results in the SN1987A framework.

2. The Hypercritical Accretion Model and the Neutrino Cooling

2.1. The Analytical Procedure

In the seminal paper about the neutron star fall-back problem, Colgate (1971) showed that
the neutrino cooling on the neutron star surface would result in a low pressure which could drive
matter toward the neutron star surface. Chevalier (1989) calculated, using this mechanism, that the
amount of material deposited on the newborn neutron star surface, for the SN1987A parameters,
was $0.15 \, M_\odot$ on a time-scale of $7 \times 10^3 \, s$. With these parameters, the accretion rate estimated for
SN1987A in the hypercritical regime was $\dot{M} \simeq 350 \, M_\odot \, yr^{-1}$. He argued that if pressure forces
are important in the flow, it has a sonic point at $R_B/4$ for $\gamma = 4/3$, where $R_B$ is the Bondi radius
and $\gamma$ is the adiabatic index. Inside of such Bondi flow the velocity becomes supersonic. The
conclusion was that the inflow towards the neutron star surface must be a supersonic free-fall.
Chevalier (1989) found a nice solution that consistently allows the flow to pass through the
shock and to decelerate towards the surface of the neutron star. Also, if the infall time is less
than the time-scale of the accretion rate change then the flow can be considered as steady-state.
In addition, because the pressure is very high close to the neutron star surface, it is expected
that neutrinos will carry away the gravitational energy. He made several assumptions to find the self-similar solutions: (i) constant accretion rate and spherical accretion in the hypercritical regime, (ii) negligible magnetic field in the flow, (iii) negligible neutron star rotation in this regime, (iv) pair annihilation as the dominant mechanism in the neutrino cooling, and (v) the polytrope approximation to equation of state.

Because of the reverse shock force a significant fraction of the expanding material will fall-back onto the compact object; this material bounces against the surface of the newly born neutron star building a third expansive shock, which tries to break through the free falling material. This expansive shock builds an atmosphere (or envelope) in quasi-hydrostatic equilibrium, with free falling material raining in over it. In that case, the density and velocity profiles of the free falling material are given by

\[ v_0 = \sqrt{\frac{2GM}{r}} \quad \text{and} \quad \rho_0 = \frac{\dot{M}}{4\pi r^2 v_0}, \]  

where \( M \) and \( \dot{M} \) are the mass and the accretion rate, respectively, and the other parameters have the usual meaning. The structure of the atmosphere in quasi-hydrostatic equilibrium is given by

\[ \rho = \rho_s \left( \frac{r_s}{r} \right)^3, \quad p = p_s \left( \frac{r_s}{r} \right)^4, \quad v = v_s \left( \frac{r_s}{r} \right)^{-1}, \]  

where the subscript \( s \) refers to the value at the shock front. The first two values come from imposing hydrostatic equilibrium of a polytropic equation of state, \( p \propto \rho^\gamma \) and the velocity is fixed by mass conservation. Once the shock position, \( r_s \), is known (see below its determination), \( p_s \) and \( \rho_s \) are determined by the strong shock condition and \( v_s \) by mass conservation as

\[ \rho_s = 7\rho_0, \quad p_s = \frac{6}{\gamma} \rho_0 v_0^2, \quad v_s = -\frac{1}{\gamma} v_0, \]  

where \( \rho_0 \) and \( v_0 \) are evaluated as \( r = r_s \). For a given neutron star mass \( M \) and radius \( R \), and
a fixed accretion rate $\dot{M}$, the radial location of the accretion shock, $r_s$, is controlled by energy balance between the accretion power and the integrated neutrino losses, per unit neutron star surface area

$$\frac{GM\dot{M}}{R} = \int_{R}^{\infty} \dot{\epsilon}_\nu(r) dr.$$  

(4)

That is due to the $r_s$ value dependence on the pressure near the neutron star surface, where it is attaining a value that is needed to lose the gravitational energy in neutrinos, then the neutrino emissivity, $\dot{\epsilon}_\nu(r)$, occurs at the scale height very close to the neutron star surface. Chevalier (1989) approached this value by $R/4$ for the pressure profile in an atmosphere in quasi-hydrostatic equilibrium. With this, the energy balance is given by

$$\frac{GM\dot{M}}{R} = 4\pi R^2 \left( \frac{R}{4} \right) \dot{\epsilon}_\nu.$$  

(5)

The high pressure near the neutron star surface ($p_{ns} \simeq 1.86 \times 10^{-12} \dot{M} r_s^{3/2}$ dyn cm$^{-2}$) allows the pair neutrino process to be the dominant mechanism in the neutrino cooling. The neutrino emissivity can be obtained from Dicus (1972) by

$$\dot{\epsilon}_\nu = 1.83 \times 10^{-34} p^{2.25} \text{erg cm}^{-3} \text{s}^{-1}.$$  

(6)

Including the electron–positron contribution to the pressure, $p_{e^-e^+} = 11/4(aT^4/3)$ where $a$ is the radiation constant, the shock radius is given by

$$r_s \simeq 7.7 \times 10^8 \left( \frac{M}{1.4 M_\odot} \right)^{-0.04} \left( \frac{R}{10^6 \text{ cm}} \right)^{1.48} \left( \frac{\dot{M}}{M_\odot \text{ yr}^{-1}} \right)^{-0.37} \text{ cm.}$$  

(7)

This shock radius is an eigenvalue that allows the gravitational energy to be lost for a given value of $\dot{M}$. For SN1987A parameters, $r_s \simeq 8.81 \times 10^7 \text{ cm}$. 
On the other hand, in the region where the emissivity $\dot{\epsilon}_\nu$ is operative, the temperature is in the range $T = (1 - 5)$ MeV. With $T$ in MeV and including the contribution of pairs electron-positron, the energy density of black body is

$$B(T) = 3.77 \times 10^{26} \left( \frac{T}{\text{MeV}} \right)^4 \text{erg cm}^{-3},$$

and the neutrino emissivity as a function of temperature, in the hypercritical regime, is calculated as

$$\dot{\epsilon}_\nu = 0.97 \times 10^{25} \left( \frac{T}{\text{MeV}} \right)^9 \text{ergs s}^{-1} \text{cm}^{-3}.$$  

For the SN1987A parameters, the estimated temperature on the neutron star surface is $T \simeq 4.5 \times 10^{10}$ K ($\simeq 4$ MeV), and the emissivity is given by $\dot{\epsilon}_\nu \simeq 2 \times 10^{30}$ ergs s$^{-1}$ cm$^{-3}$. As the volume of the neutrino-sphere is $V \simeq \pi R^3 = \pi \times 10^{18}$ cm$^3$, the neutrino luminosity is given by $L_\nu \simeq 6 \times 10^{48}$ ergs s$^{-1}$. Note that in this case other processes of neutrino production are neglected due to the high dependence of the pair annihilation process with temperature. In the numerical approach, we include all the relevant processes.

### 2.2. The Numerical Technique

Although the simplified Chevalier model described above estimates the values of parameters and the radial structure of the atmosphere in quasi-hydrostatic equilibrium in the hypercritical regime, it is only an one-dimensional model and requires several assumptions to be developed: polytrope approximation for the equation of state, only one process for the neutrino cooling (pair production) and negligible magnetic field, for instance. Bernal et al. (2013, 2010) showed that the magnetic field, for SN1987A parameters, is buried and submerged under the stellar surface.
by the accreting material in the hypercritical regime, so it does not play an important roll in the
dynamics of quasi-hydrostatic envelope. Because the ram pressure is greater than the magnetic
pressure in this regime, the magnetic field is confined in a small region where the piling up of
matter takes place. In such cases there must be considered numerical simulations of cartesian
two-three-dimensional accretion columns in order to take into account the roll of the magnetic
field. The results show the piling up of matter onto the neutron star surface, as well as the
submergence of the magnetic field in the new crust formed by such material. For such reasons we
do not take into account the magnetic field in the present work. Also, because we are interested
here in calculating several parameters of the neutrino cooling effect on the stellar surface, a few
moments after the material has fallen back. We include more neutrino processes, a more detailed
equation of state and one additional degree of freedom in the simulations.

In the present case, we carry out hydrodynamic numerical simulations of the hypercritical
accretion regime in a two-dimensional spherical mesh, using the Flash code method developed by
[Fryxell et al.](2000). Flash is a Eulerian, parallelized, multi-physics, adaptive mesh code designed
to handle several problems, in the high energy range, found in various astrophysical environments.
In our case, we use a customized version of Flash code, with the piecewise–parabolic method PPM
solver, which solves a whole set of hydrodynamic equations. This solver uses an algorithm which
is a version of higher order Godunov’s scheme. The matter equation of state is an adaptation of
Flash’s Helmholtz package, which includes contributions from the nuclei, electron-positron pairs,
and radiation, as well as the Coulomb correction.

The neutrino energy losses are dominated by the $e^- - e^+$ annihilation process which involves the
formation of a neutrino-antineutrino pair when an electron-positron pair is annihilated near the
stellar surface ($e^- + e^+ \rightarrow \nu + \bar{\nu}$). However, we also include other relevant neutrino processes
present in such regime: (i) the photo-neutrino process, in which the outgoing photon in a Compton
scattering is replaced by a neutrino-antineutrino pair ($\gamma + e^{\pm} \rightarrow e^{\pm} + \nu + \bar{\nu}$); (ii) the plasmon
decay process, in which a photon propagating within an electron gas (plasmon) is spontaneously transformed in a neutrino-antineutrino pair \( \gamma \rightarrow \nu + \bar{\nu} \); (iii) and the Bremsstrahlung process, in which the photon of the standard process is replaced by a neutrino-antineutrino pair, either due to electron-nucleon interactions \( e^\pm + N \rightarrow e^\pm + N + \nu + \bar{\nu} \) or nucleon-nucleon interactions \( N + N \rightarrow N + N + \nu + \bar{\nu} \). All these processes, which are implemented in a customized module in the code, are described in Itoh et al. (1996).

We used a 2D spherical mesh \((r, \theta)\) to perform the numerical simulations. The radial component \( r \) lies in the range \( 10^6 \text{cm} \leq r \leq 3 \times 10^6 \text{cm} \), and the angle \( \theta \) in the range \( \pi/4 \leq \theta \leq 3\pi/4 \), i.e., we simulated only a quarter of the total domain, which has \( 2048 \times 2048 \) effective zones.

As boundary conditions, we impose mass inflow along the top edge of the computational domain, and periodic conditions along the sides. At the bottom, on the neutron star surface, we use a custom boundary condition that enforces hydrostatic equilibrium. We assume the accreting matter to be non-magnetized. We are interested in following the evolution of the system from the instant when the material in free fall bounces against the neutron star surface. Bernal et al. (2010, 2013) showed that the radial profiles for density, pressure and velocity described in equation 2 are achieved when the system evolves a long time. They found, besides the aforementioned profiles predicted by the analytical model, a submergence of the neutron star magnetic field in the new crust formed by the material piled on the stellar surface. In this work we are not interested in following the shock evolution and the consequent formation of a quasi-hydrostatic equilibrium envelope, but we focus on the dynamics very close to the neutron star surface where the neutrino cooling takes place. The boundary condition on the top edge is adaptable, i.e., when the shock leaves the computational domain, the injection of mass changes from free fall to Chevalier mode. We start the simulation with the free fall profiles described by equation 1 with an initial temperature of \( T = 10^7 \text{K} \). The code found the correct pressure profile a after few steps of simulation. The time step in the Flash code is adaptive and depends on local conditions.
Fig. 2.— Radial profiles of density, pressure, velocity, and temperature from simulations of cartesian accretion column, for highly hypercritical accretion rates, compared with the SN1987A parameters (labeled 1). Note the agreement with the analytical profiles and the piling up of matter near to the neutron star surface (not taken into account in the analytical model). Taken from (Bernal et al. 2013).
Typically, the time resolution of the simulations is $dt \simeq 10^{-7}$ s. It is important to note that once the atmosphere in quasi-hydrostatic equilibrium is formed, it will expand, or shrink, so that the physical conditions at its base allow neutrinos to carry away all the energy injected by the accretion. Once emitted, neutrinos will act as an energy sink provided the material is optically thin to them. Under the present conditions of density and temperature at the base of the flow, the fluid consists mainly of free neutrons, protons and electrons. The main sources of neutrino opacity are then coherent scattering of neutrons and protons and pair annihilation. For example, the corresponding cross section for coherent scattering is $\sigma_N = (1/4)\sigma_0[E_\nu/(m_e c^2)]^2$, where $\sigma_0 = 1.76 \times 10^{-44}$ cm$^2$. As these are thermal neutrinos, their energy is $E_\nu \sim k_B T$, and with temperatures $T \lesssim 10^{11}$ K $\sim$ 10 MeV as we will find (see section 2), we have $\sigma_N \lesssim 7 \times 10^{-42}$ cm$^2$. The maximum densities reached at the bottom of the envelope will be below $10^{11}$ g cm$^{-3}$, and in such conditions, the neutrino mean free-path is $l_\nu = (n_N \sigma_N)^{-1} \gtrsim 2.5 \times 10^6$ cm, which is safely larger than the depth of the dense envelope, of the order of a few km. Above this dense region the envelope density decreases rapidly and the whole envelope is transparent to neutrinos. Then, we ignore neutrino absorption and heating. Also, if we ignore convection effects, the time scale required for the quasi-stationary solution to set in is a few sound crossing times, $t_{\text{cross}} \simeq r_s/c_s$. For a shock radius of $r_s \simeq 50$ km and sound velocity $c_s \simeq c/10$, this is $t_{\text{cross}} \simeq 1 - 2$ ms. The simulations presented in Bernal et al. (2013, 2010) ran for hundreds of ms, so this is established quite rapidly. In the present case, our simulations ran for various ms, time enough to analyze the initial transient and the neutrino cooling effect over the stellar surface. Because the neutrino cooling depends on temperature, being more or less constant near the neutron star surface, the energy loss is also more or less constant in the hypercritical regime where such emissivity is operative. The time scale for convection is of course much longer, and will depend on the equilibrium between infall and cooling at the base of the envelope.

Fig. 2 shows the radial profiles for density, pressure, velocity and temperature for several accretion rates, as were obtained by Bernal et al. (2013). The accretion rate labeled (1) correspond
Fig. 3.— Color maps of the evolution of density (up), total energy (right), neutrino emissivity (down) and pressure (left) for the SN1987A parameters: (A) initial transient, $t = 0.1$ ms, (B) shock evolving in the domain, $t = 0.5$ ms, (C) shock leaves the domain and the transient has vanished, $t = 5$ ms, (D) quasi-hydrostatic equilibrium envelope is formed, $t = 10$ ms.
Fig. 4.— Left: Domain of validity of the neutrino cooling processes in the density-temperature space. We used the tabulated values performed by Itoh et al. (1996) for several neutrino processes. Right: neutrino luminosity integrated in the whole computational domain. The analytical formula from Dicus (1972) and the tabulated numerical values from Itoh et al. (1996) are compared.
to the SN1987A accretion rate. The accretion rates labeled (10) and (100) corresponds to accretion rates one and two order of magnitude greater than the SN1987A accretion rate. The piling up of matter takes place at a scale height where the magnetic field is confined. This is a new phenomenon no described by the analytical model. This scale height is also very close to the size of the neutrino-sphere in the accretion column. Fig. 3 shows color maps of density (up), total energy (right), emissivity (down) and pressure (left) for the SN1987A parameters. In (A) we show the initial transient, with free-falling material bouncing on the stellar surface and building an expansive shock, which makes its way through the material falling onto the remnant. Note the high pressure and energy very close to the neutron star surface where the neutrino cooling take place \( t = 0.1 \text{ ms} \). In (B) the shock evolution is evident and a rich morphology is observed in the system. Hydrodynamic instabilities are observed inside the envelope while the shock evolves in the computational domain \( t = 0.5 \text{ ms} \). In (C) the shock has left the computational domain while at the base of the envelope the neutrino cooling is very effective creating an energy sink which allows the material to be deposited on the surface slowly. At this stage the system begins to relax \( t = 5 \text{ ms} \). In (D) the system has nearly reached a quasi-hydrostatic equilibrium. Due to the additional degree of freedom, the flow passes freely through the lateral boundaries, preventing a full equilibrium state from reaching. Nevertheless, it can be observed that the height scale where neutrino-sphere is operating the emissivity is more efficient.

From the simulations, the estimated radius where the neutrino loss is effective (including all the relevant processes) is \( r \simeq 3.2 \times 10^5 \text{ cm} \simeq \left(1/3\right)R \). The mean value of emissivity in such region is \( \dot{\epsilon}_\nu \simeq 2.2 \times 10^{30} \text{ ergs s}^{-1} \text{ cm}^{-3} \) (very similar to the one analytically estimated). In this case, the volume of the neutrino-sphere is \( V \simeq \left(4/3\right)\pi R^3 \simeq \left(4/3\right)\pi \times 10^{18} \text{ cm}^3 \), thus the neutrino luminosity is given by \( L_\nu \simeq 8 \times 10^{48} \text{ ergs s}^{-1} \). The good agreement of these results with the estimated values from the analytical model is surprising. We show in Fig. 4 in addition to the parameter space where different neutrino processes are effective, the neutrino luminosity integrated in the whole computational domain, for the SN1987A parameters. Note that after an
initial transient, the neutrino luminosity oscillates about a fixed value. Two approaches are shown, the analytical method from [Dicus(1972)] and the tabulated method from [Itoh et al. (1996)]. The luminosity values are in perfect agreement with those calculated analytically above. We can infer that the small difference between the analytical and numerical values calculated by integrating the whole computational domain, is due to others neutrino production processes not taken into account in the analytical model. We confirm that this nice analytical model accounts for many important physical processes in the hypercritical regime. However, numerical simulations allow us to go beyond and analyze other relevant physical processes that are lost in purely analytical models, such as the magnetic field submergence, the piling up of matter onto the neutron star surface and the rich hydrodynamic morphology and instabilities that only numerical simulations can reproduce.

On the other hand, when such neutrinos are produced in the neutrino-sphere, they can have a very complex behavior. In the following sections, we calculate the neutrino effective potential and the resonant oscillations of these neutrinos since they are produced in the neutrino-sphere until they go through the upper layers of the supernova progenitor and reach the Earth.

3. Neutrino Oscillations

The properties of these neutrinos are modified when they propagate in such magnetized and thermal medium and depending on the flavor of the neutrino, it would feel a different effective potential because electron neutrino ($\nu_e$) interacts with electrons via both neutral and charged currents (CC), whereas muon ($\nu_\mu$) and tau ($\nu_\tau$) neutrinos interact only via the neutral current (NC), as previous by said. This would induce a coherent effect in which maximal conversion of $\nu_e$ into $\nu_\mu$ ($\nu_\tau$) takes place even for a small intrinsic mixing angle. The resonant conversion of neutrino from one flavor to another due to the medium effect is well known as the Mikheyev-Smirnov-Wolfenstein effect (Wolfenstein 1978). These neutrino oscillations have been
widely studied in the literature in different scenarios (Volkas & Wong 2000; Goodman et al. 1987; Ruffert & Janka 1999; Sahu & D’Olivo 2005; Sahu et al. 2009a,b; Fraija 2014b; Osorio Oliveros et al. 2013; Fraija 2014a; Fraija & Moreno Mendez 2014; Fraija & Moreno M´endez 2014b,a).

3.1. Two-Neutrino Mixing

In this subsection, we will consider the neutrino oscillation process $\nu_e \leftrightarrow \nu_\mu,\tau$. The evolution equation for the propagation of neutrinos in the above medium is given by

$$
i \begin{pmatrix} \nu_e \\ \bar{\nu}_\mu \end{pmatrix} = \begin{pmatrix} V_{ee} - \Delta \cos 2\theta & \frac{\Delta}{2} \sin 2\theta \\ \frac{\Delta}{2} \sin 2\theta & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \bar{\nu}_\mu \end{pmatrix}$$

(10)

where $\Delta = \delta m^2 / 2E_\nu$, $V_{ee} = \sqrt{2}G_F N_e$ is the effective potential, $E_\nu$ is the neutrino energy, and $\theta$ is the neutrino mixing angle. For anti-neutrinos one has to replace $N_e$ by $-N_e$. The conversion probability for a given time $t$ is

$$P_{\nu_e \rightarrow \nu_\mu(\nu_\tau)}(t) = \Delta \sin^2 \theta \frac{\omega^2}{\omega^2} \sin^2 \left( \frac{\omega t}{2} \right),$$

(11)

with

$$\omega = \sqrt{(V_{eff} - \Delta \cos 2\theta)^2 + \Delta^2 \sin^2 2\theta}.$$  

(12)

The oscillation length for the neutrino is given by

$$L_{osc} = \frac{L_v}{\sqrt{\cos^2 2\theta (1 - \frac{V_{eff}}{\Delta \cos 2\theta})^2 + \sin^2 2\theta}},$$

(13)

where $L_v = 2\pi / \Delta$ is the vacuum oscillation length. If the density of the medium is such that the condition $\sqrt{2}G_F N_e = \Delta \cos 2\theta$ is satisfied, then, the resonance condition and resonance length can be written as

$$V_{eff,R} = \Delta \cos 2\theta,$$

(14)
and
\[ L_{\text{res}} = \frac{L_v}{\sin 2\theta}, \quad (15) \]
respectively. Combining eqs (15) and (14) we can obtain the resonance density as a function of resonance length
\[
\rho_R = \begin{cases}
\frac{3.69 \times 10^2}{E_{\nu, MeV}} \left[ 1 - \frac{E_{\nu, MeV}^2}{E_{\nu, MeV}^2 - l_r} \right]^{2 \sin^2 \theta} \frac{\text{gr/cm}^3}{1/2} & \text{solar parameters,} \\
\frac{1.39 \times 10^4}{E_{\nu, MeV}} \left[ 1 - \frac{E_{\nu, MeV}^2}{E_{\nu, MeV}^2 - l_r} \right]^{2 \sin^2 \theta} \frac{\text{gr/cm}^3}{1/2} & \text{atmospheric parameters,} \\
\frac{3.29 \times 10^6}{E_{\nu, MeV}} \left[ 1 - \frac{E_{\nu, MeV}^2}{E_{\nu, MeV}^2 - l_r} \right]^{2 \sin^2 \theta} \frac{\text{gr/cm}^3}{1/2} & \text{acceleration parameters,}
\end{cases}
\]

We will be using the best fit values of two-neutrino mixing (solar, atmospheric and accelerator neutrino experiments) as follows. From solar neutrinos parameters are \( \delta m^2 = (5.6^{+1.9}_{-1.4}) \times 10^{-5} \text{eV}^2 \) and \( \tan^2 \theta = 0.427^{+0.033}_{-0.029} \) [Aharmim & et al. 2011], from atmospheric parameters are \( \delta m^2 = (2.1^{+0.9}_{-0.4}) \times 10^{-3} \text{eV}^2 \) and \( \sin^2 2\theta = 1.0^{+0.09}_{-0.07} \) [Abe & et al. 2011] and from accelerator parameters \( \delta m^2 = (7.9^{+0.6}_{-0.3}) \times 10^{-5} \text{eV}^2 \) and \( \tan^2 \theta = 0.4^{+0.10}_{-0.07} \) [Araki & et al. 2005; Shirai & KamLAND Collaboration 2007; the KamLAND Collaboration & Mitsui 2011].

### 3.2. Three-neutrino Mixing

To determine the neutrino oscillation probabilities we have to solve the evolution equation of the neutrino system in the matter. In a three-flavor framework, this equation is given by
\[
i \frac{d\vec{\nu}}{dt} = H\vec{\nu}, \quad (16)
\]
and the state vector in the flavor basis is defined as
\[
\vec{\nu} \equiv (\nu_e, \nu_\mu, \nu_\tau)^T. \quad (17)
\]
The effective Hamiltonian is

\[ H = U \cdot H_0^d \cdot U^\dagger + \text{diag}(V_{\text{eff}}, 0, 0), \]  

(18)

with

\[ H_0^d = \frac{1}{2E_\nu} \text{diag}(-\Delta m_{21}^2, 0, \Delta m_{32}^2). \]  

(19)

with the same potential \( V_{\text{eff}} \) given for two-neutrino mixing subsection and \( U \) the three neutrino mixing matrix given by Gonzalez-Garcia & Nir (2003); Akhmedov et al. (2004); Gonzalez-Garcia & Maltoni (2008); Gonzalez-Garcia (2011)

\[ U = \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12} & -s_{23}c_{12} - s_{13}s_{12}c_{23} & c_{23}c_{13} \end{pmatrix} \]  

(20)

where \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \cos \theta_{ij} \) and we have taken the Dirac phase \( \delta = 0 \). The survival and conversion probabilities for electron, muon and tau are given in Sahu et al. (2009b,a); Fraija (2014b). The oscillation length for the neutrino is given by

\[ L_{\text{osc}} = \frac{L_\nu}{\sqrt{\cos^2 2\theta_{13}(1 - \frac{2E_\nu V_e^2}{\Delta m_{42}^2 \cos 2\theta_{13}})^2 + \sin^2 2\theta_{13}}}, \]  

(21)

where \( L_\nu = 4\pi E_\nu/\delta m_{32}^2 \) is the vacuum oscillation length. From the resonance condition, \( \sqrt{2}G_F N_e = \Delta \cos 2\theta_{13} \), the resonance length and density are related as

\[ \rho_R = \frac{1.9 \times 10^4}{E_{\nu, \text{MeV}}} \left[ 1 - E_{\nu, \text{MeV}}^2 \left( \frac{8.2 \times 10^4 \text{ cm}}{l_r} \right)^2 \right]^{1/2} \text{ gr/cm}^3. \]  

(22)

On the other hand, combining solar, atmospheric, reactor and accelerator parameters, the best fit values of the three neutrino mixing are for \( \sin^2 \theta_{13} < 0.053 : \Delta m_{21}^2 = (7.41^{+0.21}_{-0.19}) \times 10^{-5} \text{ eV}^2; \tan^2 \theta_{12} = 0.446^{+0.030}_{-0.029} \) and for \( \sin^2 \theta_{13} < 0.053 : \Delta m_{21}^2 = (7.41^{+0.21}_{-0.19}) \times 10^{-5} \text{ eV}^2; \tan^2 \theta_{12} = 0.446^{+0.030}_{-0.029} \) (Aharmim & et al. 2011).
3.3. Neutrino Oscillation from Source to Earth

Between the surface of the star and the Earth the flavor ratio $\phi_{\nu_e}^0 : \phi_{\nu_\mu}^0 : \phi_{\nu_\tau}^0$ is affected by the full three description flavor mixing, which is calculated as follow. The probability for a neutrino to oscillate from a flavor estate $\alpha$ to a flavor state $\beta$ in a time starting from the emission of neutrino at star $t = 0$, is given as

$$P_{\nu_\alpha \rightarrow \nu_\beta} = |<\nu_\beta(t) | \nu_\alpha(t = 0)>| = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left( \frac{\delta m^2_{ij} L}{4 E_\nu} \right).$$

(23)

With the three-mixing parameters of neutrino oscillation, we can write the mixing matrix as

$$U = \begin{pmatrix}
0.816669 & 0.544650 & 0.190809 \\
-0.504583 & 0.513419 & 0.3694115 \\
0.280085 & -0.663141 & 0.694115
\end{pmatrix}$$

(24)

Averaging the sin term in the probability to $\sim 0.5$ for larger distances $L$ ([Learned & Pakvasa 1995]), the probability matrix for a neutrino flavor vector of $(\nu_e, \nu_\mu, \nu_\tau)_{source}$ changing to a flavor vector $(\nu_e, \nu_\mu, \nu_\tau)_{Earth}$ is given as

$$\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau_{Earth}
\end{pmatrix} = \begin{pmatrix}
0.534143 & 0.265544 & 0.200313 \\
0.265544 & 0.366436 & 0.368020 \\
0.200313 & 0.368020 & 0.431667
\end{pmatrix} \begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau_{source}
\end{pmatrix}$$

(25)

for distances longer than the Solar System.

4. Density Profiles

When neutrinos propagate inside a star they have to go through different stratified regions. In order to obtain the density profiles, we have selected four important regions inside the star. The surface of the core, the accretion shock region, the free fall region and the surface of the
star. Analytical and numerical models of density distribution in a pre-supernova have shown a decreasing dependence on radius $\rho \propto r^{-n}$, with $n = 3/2$ (3) for convective (radiative) envelopes (Woosley et al. 1993; Shigeyama & Nomoto 1990; Arnett 1991).

4.1. Core Surface

On the surface of the core, we have a magnetized and thermal plasma (i.e. the neutrinosphere). In this region ($1.0 \times 10^6 \text{cm} \leq r_1 \leq 1.33 \times 10^6 \text{cm}$), the magnetic field and temperature are $B = 5 \times 10^{12} \text{G}$ and $T = 4.31 \text{MeV}$. The neutrino effective potential for $B \leq B_c = m^2/e \simeq 4.14 \times 10^{13} \text{G}$ is given by (Fraija 2014b)

$$V_{\text{eff}} = \frac{\sqrt{2} G_F m^2_e B}{\pi^2 B_c} \left[ \sum_{l=0}^{\infty} (-1)^l \sinh \alpha_l \left\{ \frac{m^2_e}{m^2_W} \left( 1 + 4 \frac{E^2}{m^2_e} \right) K_1(\sigma_l) \right. \right.$$

$$\left. + \sum_{n=1}^{\infty} \lambda_n \left( 2 + \frac{m^2_e}{m^2_W} \left( 3 - 2 \frac{B}{B_c} + 4 \frac{E^2}{m^2_e} \right) \right) \times K_1(\sigma_l \lambda_n) \right\} - 4 \frac{m^2_e}{m^2_W} \frac{E_e}{m_e} \sum_{l=0}^{\infty} (-1)^l \cosh \alpha_l \left\{ \frac{3}{4} K_0(\sigma_l) + \sum_{n=1}^{\infty} \lambda^2_n K_0(\sigma_l \lambda_n) \right\} \right].$$

where $K_i$ is the modified Bessel function of integral order $i$ and the parameters as function of magnetic field ($B$), temperature ($T$) and chemical potential ($\mu$) are $\lambda^2_n = 1 + 2n B/B_c$, $\alpha_l = (l + 1) \mu/T$ and $\sigma_l = (l + 1) m_e/T$.

As shown in Fig. 5, the effective potential is an increasing function of magnetic field which has a value of $7.4 \times 10^{-10} \text{eV}$ for $B = 5 \times 10^{12} \text{G}$. In this figure, we can observe that the effective potential is positive, therefore due to the positivity of the effective potential ($V_{\text{eff}} > 0$) neutrinos can oscillate resonantly. By considering two: solar (left-hand figure above), atmospheric (right-hand figure above) and accelerator (left-hand figure below), and three-neutrino (right-hand figure below) mixing, we have analyzed the resonance condition which is plotted in Fig. 6. In this figure, one can see that temperature is a decreasing function of chemical potential independently of neutrino energy. In these plots we can see that temperature is a decreasing function of chemical potential and for the values of temperature and chemical potential in the range $0.8 \text{MeV} \leq T \leq 5 \text{MeV}$ and $10^{-1} \text{eV} \leq \mu \leq 10^3 \text{eV}$, respectively, neutrinos oscillate...
Fig. 5.— The effective potential ($V_{\text{eff}}$) as a function of magnetic field (B) is plotted for 4.31 MeV.
Fig. 6.— Plot of temperature \((T)\) as a function of chemical potential \((\mu)\) for which the resonance condition is satisfied. We have used the best fit parameters of the two-flavor: solar (left-hand figure above), atmospheric (right-hand figure above) and accelerator (left-hand figure below), and three-flavor (right-hand figure below) neutrino oscillation, the value of magnetic field of \(B = 5 \times 10^{12}\) G and taken into account four different neutrino energies: \(E_{\nu} = 1\) MeV (green thin-solid line) (red dashed line), \(E_{\nu} = 5\) MeV (blue dashed line), \(E_{\nu} = 20\) MeV (black dot-dashed line) and \(E_{\nu} = 30\) MeV (bred dotted line)).
resonantly. Also can be seen from these plots that the chemical potential achieves the largest value as accelerator parameters are considered and the smallest one as solar parameters are taking into account. In addition as shown in fig 7 for the three-neutrino mixing we study the survival and conversion probabilities for the active-active ($\nu_e,\mu,\tau \leftrightarrow \nu_e,\mu,\tau$) neutrino oscillations as a function of distance (left figure) and energy (right figure). From these plot one can see that independently of flavor and energy the probabilities go from 0 to unity.

Fig. 7.— We plot the oscillation probability as a function of distance (left figure) and energy (right figure) when neutrinos are propagating through region 1.

conversion probabilities for the active-active ($\nu_e,\mu,\tau \leftrightarrow \nu_e,\mu,\tau$) neutrino oscillations as a function of distance (left figure) and energy (right figure). From these plot one can see that independently of flavor and energy the probabilities go from 0 to unity.

4.2. Accretion shock region

In this region, $(1.33 \times 10^6 \text{cm} \leq r_2 \leq r_s)$ with $r_s$ given by eq. (7), the density profile can be obtained from the Chevalier model (eqs. 1, 2 and 3) and written as

$$\rho_2(r_2) = 7.70 \times 10^2 \left( \frac{r_2}{r_s} \right)^{-3} \text{g cm}^{-3}$$  

where $r_2$ is the quasi-hydrostatic envelope radius.
4.3. Free fall region

The density of material in free fall, \( r_s \leq r_3 \leq r_h \) where \( r_h = 6.3 \times 10^{10} \) cm, is obtained from eqs. 1 and written as

\[
\rho_3(r_3) = 5.74 \times 10^{-2} \left( \frac{r_3}{r_h} \right)^{-3/2} \text{ g cm}^{-3}
\]  \hspace{1cm} (27)

4.4. Surface of the star

In particular, for the presupernova star of SN1987A (blue supergiant) a density profile was done by Chevalier [Chevalier & Soker 1989]. The analytic form of the density distribution in the outer radiative layer of the star has a polytropic structure, \( \rho = \rho_1 \left( \frac{R_\star}{r} - 1 \right)^n \), where \( R_\star \simeq 3 \times 10^{12} \) cm, \( \rho_1 \simeq 3 \times 10^{-5} \) g cm\(^{-3}\). The corresponding polytropic index for a radiative envelope with constant Thomson opacity is \( n = 3 \). Chevalier [Chevalier & Soker 1989] studied several models for the BSG presupernova of SN1987A, and all models are normalized to give the same density \( \rho = 2 \left( \frac{10^{11}}{10^{10.8}} - 1 \right)^3 = 0.4 \) g cm\(^{-3}\). Then, the final density profile of the outer layer is given by a power-law fit with an effective polytropic index \( n = 17/7 \),

\[
\rho_4(r_4) = 3.4 \times 10^{-5} \text{ g cm}^{-3} \times \begin{cases} 
\left( \frac{R_\star}{r} \right)^{17/7}; & r_h < r < r_b \\
\left( \frac{R_\star}{r_b} \right)^{17/7} \times \left( \frac{r - R_\star}{r_b - R_\star} \right)^5; & r > r_b.
\end{cases}
\]  \hspace{1cm} (28)

where \( r_b = 10^{12} \) cm. Associating the number of electron per nucleon \( Y_e = 0.5 \), we obtained the number density of electrons as \( N_e = N_a \rho(r)Y_e = 1.2 \times 10^{23} \) cm\(^{-3}\) corresponding to a polytropic hydrogen envelope.

Considering the density profiles (eqs. 26, 27 and 28), we present a description of two- and three-flavor neutrino oscillations. Based on these density profiles we calculate the effective potential, the resonance condition and, the resonance length and density. From the resonance
Fig. 8.— We plot the oscillation probability as a function of distance (left figure) and energy (right figure) when neutrinos are propagating through region 2.

Fig. 9.— We plot the oscillation probability as a function of distance (left figure) and energy (right figure) when neutrinos are propagating through region 3.
Fig. 10.— We plot the oscillation probability as a function of distance (left figure) and energy (right figure) when neutrinos are propagating through region 4.

condition, we obtain the resonance density ($\rho_R$) as a function of resonance length ($l_R$) for two (eq. 16) and three flavors (eq. 22). We put together the plots of the density profiles as a function of distance with the resonance conditions (resonance density as a function of resonance length). They are shown in Fig 4.4. For two flavors, we have taken into account solar (left-hand figure above), atmospheric (right-hand figure above) and accelerator (left-hand figure below) parameters of neutrino experiments. Using solar parameters, the resonance length is in the range $\sim (6.1 \times 10^6 - 2.3 \times 10^8)$ cm and resonance density in $\sim (1 - 10^4)$ g/cm$^3$. For atmospheric parameters, the resonance length is less than $4.8 \times 10^6$ cm and the resonance density in $\sim (10^2 - 10^{4.0})$ g/cm$^3$. Using accelerator parameters, the resonance length is less than $4.3 \times 10^5$ cm and the resonance density lies in the range $\sim (10^5 - 10^{6.8})$ g/cm$^3$ and for three flavors, the range of resonance length is less than $\sim 4.2 \times 10^6$ cm and resonance density is $\sim (10^{3.2} - 10^{4.7})$ g/cm$^3$.

In addition as shown in figs. 8, 9 and 10 for the three-neutrino mixing we study the survival and conversion probabilities for the active-active ($\nu_{e,\mu,\tau} \leftrightarrow \nu_{e,\mu,\tau}$) neutrino oscillations as a function of distance (left figure) and energy (right figure). From these plot one can see that that the electron neutrino almost does not oscillate to any other flavor $P_{ee} \simeq 1$, $P_{e\mu} \simeq 0$ and $P_{e\tau} \simeq 0$ and is almost independent of the energy of the neutrinos and the distance, also that the muon and tau neutrinos...
Fig. 11.— Density profiles ($\rho_2$, $\rho_3$ and $\rho_4$) given by eqs. (26), (27) and (28), respectively are plotted. Also from the resonance condition, we plot the resonance density as a function of resonance length for High-energy neutrinos. We have used the best fit parameters of the two-flavor: solar (left-hand figure above), atmospheric (right-hand figure above) and accelerator (left-hand figure below), and three-flavor (right-hand figure below) neutrino oscillation.
oscillate among themselves with equal probability and the oscillation depends on the neutrino energy and distance.

5. Discussion and Conclusions

We have continued the study of hypercritical accretion onto newborn neutron stars, in particular with the aim of studying the effect of the neutrino cooling in the formation of an envelope in quasi-hydrostatic equilibrium, for the SN1987A parameters. This is particularly relevant in the context of making the neutron star eventually invisible as a pulsar following the supernova explosion, if the magnetic field is submerged by the accretion in the hypercritical regime. Extending our exploration of parameter space to spherical symmetry spanning a significant fraction of the stellar surface in the simulations, we have paid special attention to the size of the neutrino-sphere where the emissivity of neutrinos is effective and calculated the neutrino luminosity, in the first instant when the formation of the quasi-steady atmosphere takes place after the reverse shock has reached the hard surface of the star, and how it affects an initial transient in the system.

We made comparisons of our numerical results with analytical model of Chevalier and found an excellent agreement between the neutrino luminosity and height scale values obtained with the Flash Code and those estimated with analytical approximations. The additional neutrino processes involved in the numerical approach slightly increase the neutrino luminosity value, and the height scale. But essentially, the analytical model is an excellent approximation to describe the hypercritical system. Nevertheless, the numerical approach is necessary in order to include more physical ingredients (magnetic field, more realistic equation of state and more neutrino processes) and analyze other interesting phenomena such as the submergence of the magnetic field on the stellar surface, as well as the dynamics of the system with higher degrees of freedom.
We have studied the active-active neutrino process in the supernova framework. We have divided the path of moving neutrinos into four regions. For the first region, we have derived the neutrino effective potential up to order $1/M_W^4$ as a function of chemical potential ($\mu$), temperature ($T$), neutrino energy ($E_\nu$) and magnetic field.

On the other hand, for this phase we calculate the numbers of events that could have been detected in SKII. Then the number of events expected is

$$N_{ev} = VN_A \rho_N \int_t \int_{E'} \sigma_{cc}^{\bar{\nu}_e p} \frac{dN}{dE} dE dT$$

(29)

where $V$ is the effective volume of water, $N_A = 6.022 \times 10^{23} \text{ g}^{-1}$ is the Avogadro’s number, $\rho_N = 2/18 \text{ g cm}^{-3}$ is the nucleons density in water, $\sigma_{cc}^{\bar{\nu}_e p} \simeq 9 \times 10^{-44} E_{\bar{\nu}_e}^2 / \text{MeV}^2$ is the cross section, $dT$ is the detection time of neutrinos and $dN/dE$ is the neutrino spectrum. Taking into account the relationship between the neutrino luminosity $L$ and flux $F$,

$$L = 4\pi D_z^2 F < E > = 4\pi D_z^2 E^2 dN/dE$$

and approximation of the time-integrate average energy $< E_{\bar{\nu}_e } > = 15 \text{ MeV}$ and time, then

$$N_{ev} \simeq \frac{t}{< E_{\bar{\nu}_e } >} VN \rho_N \sigma_{cc}^{\bar{\nu}_e p} E_{\bar{\nu}_e} >^2 \frac{dN}{dE}$$

$$\simeq \frac{4\pi D_z^2}{< E_{\bar{\nu}_e } >} VN \rho_N \sigma_{cc}^{\bar{\nu}_e p} L_{\bar{\nu}_e}. \quad (30)$$

Replacing the values for a water volume of $V = 2.14 \times 10^9 \text{ cm}^3$ (2.14 kton) and neutrino luminosity $L_{\nu} \simeq 8.0/6.0 \times 10^{48} \text{ erg/s}$, we have that the number of neutrinos expected would have been 1.49 which is in the detection limit and in comparison to the initial burst 18.7 is small. Otherwise, if we take into account the volume in SK-III (31.9 kton) we would expect a neutrino number of 22.3 in the hypercritical phase. The amount of these neutrinos would confirm such phase in SN1987A or other SN with the similar characteristics.
On the other hand, we calculate the flavor ratio on Earth four four neutrino energies ($E_\nu = 5$ MeV, 10 MeV, 15 MeV and 20 MeV), as shown in table [1]. From this table we can see a small deviation of the standard flavor 1:1:1 for neutrinos. In this calculation we take into account that for neutrino cooling processes: electron-positron annihilation, inverse beta decay, nucleonic bremsstrahlung and plasmons, only inverse beta decay is the one in producing electron neutrino. It is important to say that our calculations were done for neutrinos instead of anti-neutrinos.

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| $E_{\nu}$ (MeV) | $\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau}$ On the Plasma (1.3×10^6 cm) | $\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau}$ On the Star (3×10^{12} cm) | $\phi_{\nu_e}^0 : \phi_{\nu_\mu}^0 : \phi_{\nu_\tau}^0$ On Earth |
|---------------|--------------------------------------------------------------------------------|--------------------------------------------------------------------------------|----------------------------------|
| 5             | 0.914 : 1.042750 : 1.042                                                        | 0.924 : 1.037 : 1.037                                                          | 0.977 : 1.007 : 1.015            |
| 10            | 1.016 : 0.991800 : 0.991                                                        | 1.013 : 0.993 : 0.993                                                          | 1.004 : 0.998 : 0.997            |
| 15            | 1.106 : 0.946922 : 0.946                                                        | 1.085 : 0.957 : 0.957                                                          | 1.025 : 0.991 : 0.983            |
| 20            | 1.145 : 0.927213 : 0.927                                                        | 1.124 : 0.937 : 0.937                                                          | 1.037 : 0.987 : 0.975            |

Table 1: The flavor ratio on the surface of the plasma, star and Earth for four neutrino energies ($E_{\nu} = 5$ MeV, 10 MeV, 15 MeV and 20 MeV for SN1987A).
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A. Neutrino Effective Potential

We use the finite temperature field theory formalism to study the effect of heat bath on the propagation of elementary particles. The effect of magnetic field is taken into account through Schwinger’s propertime method \cite{Schwinger1951}. The effective potential of a particle...
is calculated from the real part of its self-energy diagram. The neutrino field equation in a magnetized medium is,

$$[k - \Sigma(k)]\Psi_L = 0,$$  \hspace{1cm} (A1)

where the neutrino self-energy operator $\Sigma(k)$ is a Lorentz scalar which depends on the characterized parameters of the medium, as for instance, chemical potential, particle density, temperature, magnetic field, etc. For our purpose, $\Sigma(k)$ can be formed by,

$$\Sigma(k) = R\left(a_\parallel k_\parallel + a_\perp k_\perp + bu + cb\right)L,$$  \hspace{1cm} (A2)

where $k_\mu = (k^0, k^3)$, $k_\perp^\mu = (k^1, k^2)$ and $u^\mu$ stands for the 4-velocity of the center-of-mass of the medium given by $u^\mu = (1, 0)$. The projection operators are conventionally defined as $R = \frac{1}{2}(1 + \gamma_5)$ and $L = \frac{1}{2}(1 - \gamma_5)$. The effect of magnetic field enters through the 4-vector $b^\mu$ which is given by $b^\mu = (0, \hat{b})$. The background classical magnetic field vector is along the $z$-axis and consequently $b^\mu = (0, 0, 0, 1)$. So using the four vectors $u^\mu$ and $b^\mu$ we can express

$$k_\parallel = k_0 u - k_3 b,$$  \hspace{1cm} (A3)

and the self-energy can be expressed in terms of three independent four-vectors $k_\perp^\mu$, $u^\mu$ and $b^\mu$. Therefore we can write ($\Sigma = R\tilde{\Sigma}L$)

$$\tilde{\Sigma} = a_\perp k_\perp + bu + cb.$$  \hspace{1cm} (A4)

The neutrino self-energy in a magnetic background can be found from Eq. (A1),

$$\det[k - \Sigma(k)] = 0.$$ \hspace{1cm} (A5)

Using the Dirac algebra, the dispersion relation, $V_{eff} = k_0 - |k|$, as a function of Lorentz scalars can be written as,

$$V_{eff} = b - c \cos \phi - a_\perp |k| \sin^2 \phi,$$  \hspace{1cm} (A6)

where $\phi$ is the angle between the neutrino momentum and the magnetic field vector. Now the Lorentz scalars $a$, $b$ and $c$ which are functions of neutrino energy, momentum and magnetic field
can be calculated from the neutrino self-energy due to CC and NC interactions of neutrino with the background particles.

**B. One-loop neutrino self-energy**

Let us consider one-loop corrections to the neutrino self-energy in the presence of a magnetic field. The one-loop neutrino self-energy comes from three pieces (Erdas et al. 1998, Sahu et al. 2009a,b, Fraija 2014b), one coming from the $W$-exchange diagram which we will call $\Sigma_W(k)$ (Fig. 1(a)), one from the $Z$-exchange diagram which will be denoted by $\Sigma_Z(k)$ (Fig. 1(b)) and one from the tadpole diagram which we will designate by $\Sigma_t(k)$ (Fig. 1(c)). The total neutrino self-energy in a magnetized medium then becomes:

$$\Sigma(k) = \Sigma_W(k) + \Sigma_Z(k) + \Sigma_t(k).$$  \hspace{1cm} (B1)

The $W$-exchange diagram to the one-loop self-energy is

$$- i\Sigma_W(k) = R \left[ \int \frac{d^4 p}{(2\pi)^4} \left( \frac{-ig}{\sqrt{2}} \gamma_\mu iS_\ell(p) \left( \frac{-ig}{\sqrt{2}} \gamma_\nu W^{\mu\nu}(q) \right) \right) \right] L,$$  \hspace{1cm} (B2)

where $g^2 = 4\sqrt{2}G_F M_W^2$ is the weak coupling constant, $W^{\mu\nu}$ depicts the W-boson propagator which in the eB $\ll M_W^2$ limit and in unitary gauge is given by (Erdas et al. 1998, Sahu et al. 2009a,b),

$$W^{\mu\nu}(q) = \frac{g^{\mu\nu}}{M_W^2} \left( 1 + \frac{q^2}{M_W^2} \right) - \frac{q^\mu q^\nu}{M_W^4} + \frac{3ie}{2M_W^4} F^{\mu\nu}$$  \hspace{1cm} (B3)

here $M_W$ is the W-boson mass, $g^{\mu\nu}$ is the metric tensor and $F^{\mu\nu}$ is the electromagnetic field tensor. $S_\ell(p)$ stands for the charged lepton propagator which can be separated in two charged propagators: one in presence of an uniform background magnetic field ($S_\ell^0(p)$) and the other in a magnetized medium ($S_\ell^\beta(p)$). It can be written as,

$$S_\ell(p) = S_\ell^0(p) + S_\ell^\beta(p).$$  \hspace{1cm} (B4)
Assuming that the z-axis points in the direction of the magnetic field $B$, we can express the charged lepton propagator in presence of an uniform background magnetic field as,

$$iS_0^\ell(p) = \int_0^\infty e^{\phi(p,s)}G(p,s)\,ds,$$  \hspace{1cm} (B5)

where the functions $\phi(p,s)$ and $G(p,s)$ are give by,

$$\phi(p,s) = is(p_0^2 - m_\ell^2) - is[p_3^2 + \frac{\tan z}{z}p_\perp^2],$$

$$G(p,s) = \sec^2 z \left[A + iB\gamma_5 + m_\ell(\cos^2 z - i\Sigma^3 \sin z \cos z)\right],$$  \hspace{1cm} (B6)

where $m_\ell$ is the mass of the charged lepton, $p_\parallel^2 = p_0^2 - p_3^2, p_\perp^2 = p_1^2 + p_2^2$ are the projections of the momentum on the magnetic field direction and $z = eBs$, being $e$ the magnitude of the electron charge. Additionally, the covariant vectors are given as follows, $A_\mu = p_\mu - \sin^2 z(p \cdot u\gamma_\mu - p \cdot b \gamma_\mu), B_\mu = \sin z \cos z(p \cdot u\gamma_\mu - p \cdot b \gamma_\mu)$, and $\Sigma^3 = \gamma_5 \gamma_4 b\gamma_4 u$.

On the other hand, the charged lepton propagator in a magnetized medium is given by,

$$S^\beta_\ell(p) = i\eta_F(p \cdot u)\int_0^\infty e^{\phi(p,s)}G(p,s)\,ds,$$  \hspace{1cm} (B7)

where $\eta_F(p \cdot u)$ contains the distribution functions of the particles in the medium which are given by:

$$\eta_F(p \cdot u) = \frac{\theta(p \cdot u)}{e^{\beta(p \cdot u - \mu_\ell)} + 1} + \frac{\theta(-p \cdot u)}{e^{-\beta(p \cdot u - \mu_\ell)} + 1},$$  \hspace{1cm} (B8)

where $\beta$ and $\mu_\ell$ are the inverse of the medium temperature and the chemical potential of the charged lepton.

The Z-exchange diagram to the one-loop self-energy is

$$-i\Sigma_Z(k) = R \left[\int \frac{d^4 p}{(2\pi)^4} \left(\frac{-ig}{\sqrt{2} \cos \theta_W}\right) \gamma_\mu iS_{\nu}(p) \left(\frac{-ig}{\sqrt{2} \cos \theta_W}\right) \gamma_\nu Z^{\mu\nu}(q)\right] L,$$  \hspace{1cm} (B9)

$\theta_W$ is the Weinberg angle, $Z^{\mu\nu}(q)$ is the Z-boson propagator in vaccum, $S_{\nu}$ is the neutrino propagator in a thermal bath of neutrinos.

The Tadpole diagram to the one-loop self-energy is
\[ i\Sigma_t(k) = R \left[ \left( \frac{g}{2 \cos \theta_W} \right)^2 \gamma_\mu iZ^{\mu\nu}(0) \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left[ \gamma_\nu (C_V + C_A \gamma_5) iS_\ell(p) \right] \right] L, \]  

(B10)

where the quantities \( C_V \) and \( C_A \) are the vector and axial-vector coupling constants which come in the neutral-current interaction of electrons, protons \((p)\), neutrons \((n)\) and neutrinos with the \( Z \) boson. Their forms are as follows,

\[ C_V = \begin{cases} 
-\frac{1}{2} + 2 \sin^2 \theta_W & e \\
\frac{1}{2} & \nu \\
\frac{1}{2} - 2 \sin^2 \theta_W & p \\
-\frac{1}{2} & n 
\end{cases} \]  

(B11)

and

\[ C_A = \begin{cases} 
-\frac{1}{2} & \nu, p \\
\frac{1}{2} & e, n 
\end{cases} \]  

(B12)

By evaluating eq. (B2) explicitly we obtain

\[ \text{Re} \Sigma_W(k) = R \left[ a_{W_L} k_\perp + b_W k + c_W b \right] L \]  

(B13)

where the Lorentz scalars are given by

\[ a_{W_L} = -\frac{\sqrt{2}G_F}{M_W^2} \left[ \left\{ E_{\nu_e}(N_e - \bar{N}_e) + k_3(N^0_e - \bar{N}^0_e) \right\} \\
+ \frac{eB}{2\pi^2} \int_0^\infty dp_3 \sum_{n=0}^\infty (2 - \delta_{n,0}) \left( \frac{m^2_{\nu_e}}{E_n} - \frac{H}{E_n} \right) (f_{e,n} + \bar{f}_{e,n}) \right]. \]  

(B14)

\[ b_W = \sqrt{2}G_F \left[ \left( 1 + \frac{3 m^2_{\nu_e}}{2 M_W^2} + \frac{E_{\nu_e}^2}{M_W^2} \right) (N_e - \bar{N}_e) + \left( \frac{eB}{M_W^2} + \frac{E_{\nu_e} k_3}{M_W^2} \right) (N^0_e - \bar{N}^0_e) \\
- \frac{eB}{2\pi^2 M_W^2} \int_0^\infty dp_3 \sum_{n=0}^\infty (2 - \delta_{n,0}) \left\{ 2k_3 E_n \delta_{n,0} + 2E_{\nu_e} \left( E_n - \frac{m^2_{\nu_e}}{2E_n} \right) \right\} (f_{e,n} + \bar{f}_{e,n}) \right]. \]  

(B15)
and

\[ e_W = \sqrt{2} G_F \left[ \left( 1 + \frac{m_e^2}{2 M_W^2} - \frac{k_3^2}{M_W^2} \right) (N_e^0 - \bar{N}_e^0) + \left( \frac{e B}{M_W^2} - \frac{E_{\nu_e} k_3}{M_W^2} \right) (N_e - \bar{N}_e) \right. \]

\[-\frac{e B}{2\pi^2 M_W^2} \int_0^\infty dp \sum_{n=0}^\infty (2 - \delta_{n,0}) \left\{ 2 E_{\nu_e} \left( E_n - \frac{m_e^2}{2 E_n} \right) \delta_{n,0} + 2 k_3 \left( E_n - \frac{3 m_e^2}{2 E_n} - \frac{H}{E_n} \right) \right\} (f_{e,n} + \bar{f}_{e,n}) \bigg]. \]  

(B16)

where the electron distribution function is given by

\[ f(E_{e,n}, \mu) = \frac{1}{e^{E_{e,n} - \mu} + 1}. \]  

(B17)

and \( \bar{f}_{e,n}(\mu, T) = f_{e,n}(-\mu, T) \). We can also express the Eq. (B9), for Z-exchange, as

\[ Re \Sigma_Z(k) = R(a_Z k + b_Z \mu) L \]  

(B18)

and explicit evaluation gives

\[ a_Z = \sqrt{2} G_F \left[ \frac{E_{\nu_e}}{M_Z^2} (N_{\nu_e} - \bar{N}_{\nu_e}) + \frac{2}{3} \frac{1}{M_Z^2} \left( \langle E_{\nu_e} \rangle N_{\nu_e} + \langle \bar{E}_{\nu_e} \rangle \bar{N}_{\nu_e} \right) \right] \]  

(B19)

and

\[ b_Z = \sqrt{2} G_F \left[ (N_{\nu_e} - \bar{N}_{\nu_e}) - \frac{8 E_{\nu}}{3 M_Z^2} \left( \langle E_{\nu_e} \rangle N_{\nu_e} + \langle \bar{E}_{\nu_e} \rangle \bar{N}_{\nu_e} \right) \right] \]  

(B20)

where the four vector \( k \) can be decomposed in the four vectors \(\hat{k}\) and \(\hat{b}\) in accordance with Eq. (A3).

From the tadpole diagram, Eq. (B10), we obtain

\[ Re \Sigma_t(k) = \sqrt{2} G_F R \left\{ C_{V_e} (N_e - \bar{N}_e) + C_{V_p} (N_p - \bar{N}_p) + C_{V_{\mu}} (N_{\mu} - \bar{N}_{\mu}) + (N_{\nu_e} - \bar{N}_{\nu_e}) \right. \]

\[ + (N_{\nu_{\mu}} - \bar{N}_{\nu_{\mu}}) + (N_{\nu_{\tau}} - \bar{N}_{\nu_{\tau}}) \right\} \left( t - C_{A_e} (N_e^0 - \bar{N}_e^0) \right) b \bigg] L. \]  

(B21)

For anti-neutrinos we must change \((N_x - \bar{N}_x)\) by \(- (N_x - \bar{N}_x)\). Following Fraija (2014b) for \( B \leq B_c \), solving the integral-terms in the Lorentz scalars eqs. (B14), (B15) and (B16), then the neutrino effective potential \( A_6 \) for \( \phi = 0 \) can be expressed as shown in eq. 26.