A geometric algorithm for automatic riser determination and shrinkage identification in directionally solidifying castings

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Abstract. A geometric approach for analysis of castings is outlined and contrasted with physics-based simulations. The approach uses a linear-time algorithm to create an implicit geometry representation known as a distance field. Local maxima are extracted from the distance field using heuristic search logic to determine isolated heavy sections and hotspots. Unsound sections are determined by combining hotspot results with regions of constant distance-field value. Possible uses of the algorithm as an augmentation for application of physics-based solvers is explored. The algorithm is compared against a commercially available simulation software package.

1. Introduction
Casting design engineers working with steel are familiar with the Heuver circle method for determining isolated heavy sections in 2-dimensional sections of castings. Heuver spheres are a 3-dimensional extension of Heuver circles which may be applied to fully 3-dimensional geometries. Heuver spheres of locally maximal radius indicate locations which will be last to solidify, and which may be subject to shrinkage porosity if not properly fed. Isolated heavy sections which consist of a continuum of such points, such as flat plates without taper, may be subject to centerline shrink and microporosity [1]. While it is straightforward to determine the location of locally maximal Heuver circles for a 2-dimensional casting section by inspection, it can be challenging to do the same with a fully 3-dimensional model of even modest complexity. Except at inside corners where the sand-fillet effect is significant, Heuver spheres are identical to inscribed spheres, and the latter will be used as an approximation. By using concepts related to inscribed spheres, the approximate locations of maximal Heuver spheres for polyhedral parts may be determined by a linear time algorithm, allowing for rapid insight into and automation of a portion of part rigging design.

The medial axis skeleton of a polyhedron can be thought of as the set of inscribed sphere centers for which the spheres are maximal [2] [3]. The boundary of a polyhedron is composed of 0-, 1-, and 2-dimensional boundary elements: vertices, edges, and faces. Likewise, the medial axis skeleton is composed of medial axis components: junctions, seams, and sheets [4]. Any medial axis component which does not neighbor any boundary elements will be referred to as a central component. As an example, an ell of square cross-section is shown in figure 1a, its medial axis skeleton is shown in figure 1b, and its central components are shown in figure 1c. Note that the central components consist of two linear seams connected to the sharp corners of a quarter-circle-bounded sheet.
Figure 1. (a) Ell with square cross-section; (b) medial axis; (c) central components consisting of two linear seams and a sheet with quarter-circle boundary.

The Euclidean Distance Function (EDF) of a polyhedron is the function that gives, for each point in space, the distance between that point and the closest point on the boundary of the polyhedron. It is possible to demonstrate that the function which assigns the radii value of inscribed spheres to their center points is equivalent to the EDF. The EDF, thusly based on inscribed spheres, increases in value away from the part boundary. Likewise, solidification of a casting occurs inward from the boundary, so the EDF may be thought of as a geometric approximation to a time-to-solidification map. Central components of locally maximal value must be the last to solidify, and indicate isolated heavy sections.

The EDF is also related to the medial axis skeleton. All points on medial axis sheets are equidistant from exactly two distinct boundary elements [5]. If the two boundary elements are parallel, then their shared medial axis sheet must have constant EDF value. Similarly, all points on seams are equidistant from at least three distinct boundary elements [5]. Seams which are equidistant from three or more parallel boundary elements have constant EDF value. Such sheets and seams will be called constant sheets and seams. Because the EDF is a geometric approximation to a time-to-solidification map, it follows that two or more solidification fronts meet uniformly at constant sheets and seams. When such a meeting occurs there is a solidification instability and microporosity is expected in steel alloys [6]. Thus, from a casting perspective, constant seams and sheets may be thought of as geometric approximations to the locations of centerline shrink in steel castings. Determining the location of such sheets and seams is a major goal of our algorithm.

2. Implementation of the Euclidean Distance Function

While the medial axis is a useful approximation of centerline shrink locations, direct determination of an exact representation of the medial axis skeleton is computationally intensive and complicated [5]. Approximations to the medial axis skeleton exist [7], but unfortunately the EDF is also required to determine locations of centerline shrink. The medial axis skeleton may be approximated from the EDF by finding regions of undefined gradient. However, because the EDF is the absolute value of the solution to a non-linear hyperbolic PDE, analytic solutions even in two dimensions are often impossible to determine except in special cases. Discrete EDF approximations have been determined using a tetrahedral mesh discretization with an algorithm of linearithmic time complexity in the number of mesh elements, but the solutions require high-quality meshes which are computationally expensive, and can have significant errors [8]. Additionally, acceptable determination of the medial axis skeleton from the resulting EDF approximation may require iterative mesh refinement near the medial axis skeleton, increasing computational complexity and run-time.
Another approach is to use image analysis-based methods on a 3-dimensional image of a given geometry, whose mesh element equivalent is a volume element, or voxel. An algorithm which has linear time complexity in the number of voxels and exact computation of the discrete EDF exists for 3-dimensional images, and is used in our algorithm [9]. The discrete image-based EDF is called the Euclidean Distance Transform (EDT), and an example of the image-based EDT applied to an ell with square cross-section is shown in figure 2. The 3-dimensional image has been sectioned at half its depth into the page to give a 2-dimensional image showing the largest variation in value.

![Figure 2](image-url)

**Figure 2.** (a) EDT of an ell of square cross-section sectioned at half depth. Black pixels are nearest the boundary, white pixels are farthest. (b) Close-up showing constant sheet at the bend, connecting to two constant-valued seams extending into each leg, shown in white. Compare with the central components in figure 1c.

Our algorithm determines regions of constant EDT value which can be presented directly as a geometric approximation of the Niyama criterion. To determine the Niyama criterion values, and for other comparisons, a commercial simulation software MAGMA was used [10]. Using the commercial software simulation package, Niyama values were determined in a flat plate with a cylindrical feeder attached to one end [10]. Plots cut halfway through the depth of the plate are shown in figure 3. Figure 3a shows the Niyama values between 0 and 5.6 in gray with larger values in black. Figure 3b shows the central finite difference gradient magnitude of the EDT, with values near zero shown in gray and other values shown in black. Figure 3c shows the EDT with lighter values further from the plate boundary. The plate has dimensions of 19 by 5.5 by 1 inch, and the feeder has height-to-diameter ratio of 1.5 with diameter of 3 inches. The feeder’s circular cross-section is tangent to one end of the plate and centered along the middle dimension. The setup of the simulation is identical to that used by Carlson and Beckermann in demonstrating that dimensionless Niyama criterion is a predictor of microporosity [11].

It is worth noting that the gray portion of the Niyama criterion in figure 3a does not match the central sheet of the gradient magnitude in figure 3b due to edge and corner effects of the physical temperature gradient. In both the physical simulation and the geometric approximation the plate shows an expansive central region where some gradient is sufficiently close to zero. In the case of the Niyama criterion the temperature gradient is close zero, and with the geometric approximation the EDT gradient magnitude is close zero. Because the Niyama criterion may be used as a predictor of microporosity [11], the EDT may be a useful geometric approximation for predicting microporosity.
3. Heuristic determination of hotspots and shrinkage

Determination of locally maximal central junctions from an EDT is relatively straightforward using a linear array search. For polyhedrons with only axially-oriented boundary elements, such as the ell of square cross section in figure 2a, constant seams and sheets are also straightforward to determine, as the central finite difference gradient magnitude values are less than unity if the corresponding voxels contain part of the medial axis skeleton. For general polyhedra it is not required that constant seams or sheets be linear or planar, nor axially oriented. Additionally, imaging discretization makes identification of exact medial axis skeleton geometry difficult and leads to error in identifying constant sheets and seams, and thus regions of centerline shrink. A non-polyhedral example geometry, a rectangular cross-section annulus, showing curved seams and sheets is shown in figure 4.

Figure 3. (a) Niyama criterion value darkened between 0 and 2.8 in a 19 by 5.5 by 1 inch deep plate, sectioned at half depth. Plate has a 3 inch diameter, 4.5 inch deep cylindrical feeder tangent to the bottom edge centered along the width [10]. (b) Gradient magnitude of EDT of the same plate. Gray pixels have gradient value much larger than unity. (c) EDT of the same plate. Black pixels are nearest the boundary, white furthest.

Figure 4. (a) Cylindrical annulus; (b) Medial axis with non-planar central sheet darkened.
To mitigate the problems caused by discretization and oblique or curved medial axis components, heuristic searches may be used to prune a list of voxels identified as potential locally maximal components. The current implementation creates a list of voxels whose EDT values are no lower than their neighbors. The list is sorted in order of decreasing EDT value. For each, a connected basin of EDT values is created around the first element of the list down to some threshold. If the basin of a list voxel overlaps an already existing basin, it is not counted as a maximum. All surviving list voxels are declared local maxima. Extremely low EDT valued voxels are trimmed as well to remove spurious values. Once the local maxima have been located, isolated heavy sections have been determined, and with it a geometric approximation for feeder placement.

Feeder sizes may be determined by application of steel alloy feeding rules to the isolated heavy sections determined via EDT [12]. Essentially, a heuristic akin to a game is played by each of the local maxima. Starting from the largest EDT value, at each step, the local maxima voxels are assigned a feeding distance based on feeding rules from literature [12]. If a neighbor has a greater EDT value, or if it has already been assimilated, or if it is a part boundary voxel, then it may not be assimilated. If a neighboring voxel has approximately equal EDT value, it may be assimilated, but the allowed feeding distance is reduced by the distance between the voxels. If instead the neighboring voxel has lower EDT value, it may be assimilated and the allowed feeding distance is recomputed based on the new EDT value. The heuristic game must converge because either feeding fronts will have insufficient remaining feeding distance to allow further progress, or they will contact each other, or they will contact the part boundary.

When the heuristic game has converged, each isolated heavy section may be assigned a feeder volume from the feeding rules [12]. The feeding rules are a collection of non-linear functions which are dependent on several geometric measurements. Two of the measurements are characteristic length $L$ and characteristic width $W$ of the part, which are interpreted to mean the largest and second-largest dimensions of an equivalent plate, respectively. The third measurement used is the half-thickness, $T$, of the equivalent plate, and can also be interpreted to mean the maximal EDT value of the isolated heavy section [12].

The implementation currently used as of this writing uses an axially-oriented bounding box of the assimilated region to determine $L$ and $W$. Associated part volume is calculated conservatively as the volume of the isolated heavy section bounding box, and $T$ is the EDT value of the original isolated heavy section local maxima voxel or voxels. The feeding rules return a required feeder volume to achieve ideal feeding, which can be used to compute the feeder dimensions depending on desired geometry, e.g. cylindrical, prismatic polyhedral, etc. The calculation for a cylindrical feeder of aspect ratio 1.5 is shown in table 1 for the bearing block shown in figure 5. The top row of figure 5 shows the models in a 3D view as reference, while the bottom row shows a side view of each model. Figure 5a shows the original part segmented into two isolated heavy sections by our algorithm, figure 5b shows the part rigged according to the results of our algorithm as calculated in table 1, and figure 5c shows the part rigged according to a solution by Wlodawer [13]. The feeders have diameter 123 mm and have been placed without consideration for chills, feedpads, gating, or any of the other accoutrements associated with engineered rigging designs. The yield of our method is 59%, an improvement on Wlodawer’s solution which has yield of 51% [13].

In Figure 6, the computation time of the same cube part with the same voxel and mesh densities and spacing are compared between the image based algorithm described here and the commercial simulation software. While the power law scaling is nearly the same, the image based algorithm is approximately 17 times faster than the commercial simulation software over the range of mesh elements used in the experiment [10]. While there are speed benefits, our algorithm does not reproduce or replace the functionality of a physics-based solver. Instead our algorithm provides a means of rapidly assessing manufacturability and estimates the locations of certain defects. Some of the components of our algorithm are parallelizable, and should show additional benefits if implemented as such.
Table 1. Computation of feeder size for part shown in figure 5 determined by image-based algorithm, assuming a cylindrical feeder of aspect ratio 1.5.

| Description                        | Expression | Value   | Units |
|------------------------------------|------------|---------|-------|
| Bounding Box Dimensions            | $L \times W \times H$ | 225 x 160 x 130 | mm   |
| Maximum EDT                        | $T$        | 39.1    | mm   |
| Shape Factor                       | $S = \frac{L + W}{T}$ | 9.84    |       |
| Shape Factor Contribution          | $F = S^{-0.74}$ | 0.184   |       |
| Bounding Box Volume                | $V_B = LWH$ | 4.68 x 10^6 | mm^3 |
| Feeder Volume                      | $V_F = 2.51V_B F$ | 2.16 x 10^6 | mm^3 |
| Feeder Radius                      | $R_F = \sqrt[3]{\frac{V_F}{3\pi}}$ | 61.2 | mm   |
| Feeder Height                      | $H_F = 3R_F$ | 183    | mm   |

Figure 5. (a) Bearing block geometry and (b) segmentation into two isolated heavy sections by heuristic algorithm. Riser sizes and positions are calculated and attached to the original geometry to produce a (c) rigged bearing block geometry. The commercial simulation software is run on the result and (d) porosity is calculated [10]. (e) A rigging solution for the bearing block due to Wlodawer, with (f) porosity from the commercial simulation software [10] [13].
Figure 6. Timing comparison between the image-based algorithm and the commercial simulation software [10]. Error bars are smaller than is reasonably visible on the plot and are not shown.

4. Conclusions
It should be noted the authors do not believe their algorithm should serve as a replacement of physics-based casting simulation software, but instead should be used as a design tool to augment physics-based simulation software and optimization algorithms. It can be used to automate some of the design process already used by casting designers, and to create an initial seed for optimization and further design operations. Because the algorithm is fairly rapid, it is also hoped that further image-based tool development will allow for more rapid turnaround time in customer requirements versus process efficiency design iterations. Additionally, costing and other design metrics will be straightforward to plug in to the algorithm, allowing for rapid decision making among a wide variety of geometries which meet customer requirements, minimizing design time and cost, and allowing greater exploration of solution spaces to further maximize process efficiency.

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