Correlation between direct dark matter detection and $\text{Br}(B_s \rightarrow \mu\mu)$ with a large phase of $B_s-\bar{B}_s$ mixing

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Abstract

We combine the analyses for flavor changing neutral current processes and dark matter solutions in minimal-type supersymmetric grand unified theory (GUT) models, SO(10) and SU(5), with a large $B_s-\bar{B}_s$ mixing phase and large $\tan\beta$. For large $\tan\beta$, the double penguin diagram dominates the SUSY contribution to the $B_s-\bar{B}_s$ mixing amplitude. Also, the $\text{Br}(B_s \rightarrow \mu\mu)$ constraint becomes important as it grows as $\tan^6\beta$, although it can still be suppressed by large pseudoscalar Higgs mass $m_A$. We investigate the correlation between $B_s \rightarrow \mu\mu$ and the dark matter direct detection cross-section through their dependence on $m_A$. In the minimal-type of SU(5) with type I seesaw, the large mixing in neutrino Dirac couplings results in large lepton flavor violating decay process $\tau \rightarrow \mu\gamma$, which in turn sets upper bound on $m_A$. In the SO(10) case, the large mixing can be chosen to be in the Majorana couplings instead, and the constraint from $\text{Br}(\tau \rightarrow \mu\gamma)$ can be avoided. The heavy Higgs funnel region turns out to be an interesting possibility in both cases and the direct dark matter detection should be possible in the near future in these scenarios.
1 Introduction

Recently, CDF and DØ collaborations have announced the analysis of the flavor-tagged $B_s \to J/\psi \phi$ decay. The decay width difference and the mixing induced CP violating phase, $\phi_s$, have been extracted from their analysis \cite{1}. In the Standard Model (SM), the CP violating phase is predicted to be small, $\phi_s = 2\beta_s \equiv 2 \arg \left( -V_{ts}V_{tb}^*/V_{cs}V_{cb}^* \right) \simeq 0.04$. However, the measured values of the phase are large:

$$\phi_s (\text{CDF}) \in [0.28, 1.29] \ (68\% \ C.L.),$$ (1)

$$\phi_s (\text{DØ}) = 0.57^{+0.30}_{-0.24}\text{(stat)}^{+0.02}_{-0.07}\text{(syst)}.$$ (2)

Combined data analyses including the semileptonic asymmetry in the $B_s$ decay indicate that the CP violating phase deviates about $3\sigma$ from the SM prediction \cite{2}. If this large phase still persists in the upcoming results from Fermilab, it implies the existence of new physics (NP) beyond the SM and that the new physics requires a flavor violation in $b$-$s$ transition as well as a phase in the transition.

Supersymmetry (SUSY) is the most attractive candidate to build NP models. As it is well known, SUSY models have a natural dark matter candidate which is the lightest SUSY particle (LSP). Besides, the gauge hierarchy problem can be solved and a natural aspect of the theory can be developed from the weak scale to the ultra high energy scale. In fact, the gauge coupling constants of the Standard Model gauge symmetries can unify at a high scale using the renormalization group equations (RGEs) involving the particle contents of the minimal SUSY standard model (MSSM). This indicates the existence of grand unified theories (GUTs). The well motivated SUSY GUTs have always been subjects of intense experimental and theoretical investigations. Identifying a GUT model, as currently is, will be a major focus of the upcoming experiments.

The nature of the flavor changing neutral currents (FCNCs) and the CP violating phase is very important to test the existence of new physics beyond the standard model. In SUSY models, the SUSY breaking mass terms for squarks and sleptons must be introduced, and they have sources of FCNCs and CP violation beyond the Kobayashi-Maskawa theory. In general, the soft breaking terms generate too large FCNCs, hence flavor universality is often assumed in squark and slepton mass matrices to avoid large FCNCs in the meson mixings and lepton flavor violations (LFV) \cite{3}. The flavor universality is expected to be realized by the Planck scale physics. However, even if universality is realized at a scale such as the GUT scale or the Planck scale, non-universality in the SUSY breaking sfermion masses is still generated from the evolution of RGEs, and this can lead to a small flavor violating transitions, which could be observed in the ongoing experiments.
In the MSSM with right-handed neutrinos, the induced FCNCs from the RGE effects are not large in the quark sector, while sizable effects can be generated in the lepton sector due to the large neutrino mixing angles [4]. Within GUTs, however, loop effects due to the large neutrino mixings can induce sizable FCNCs also in the quark sector since the GUT scale particles which connect quark and lepton sectors can propagate in the loops [5]. As a result, the patterns of the induced FCNCs highly depend on the unification scenario and the heavy particle contents. Therefore, it is important to investigate FCNC effects to obtain a footprint of the GUT models. If quark-lepton unification is manifested in a GUT model, the flavor violation in $b$-$s$ transition can be responsible for the large atmospheric neutrino mixing [6], and thus, the amount of the flavor violation in $b$-$s$ transition (the second and the third generation mixing), which is related to the $B_s$-$\bar{B}_s$ mixing and its phase, has to be related to the $\tau \to \mu \gamma$ decay [7, 8, 9, 10] for a given particle spectrum. The branching ratio of the $\tau \to \mu \gamma$ decay is being measured at the $B$-factory, and thus, the future results on LFV and from the ongoing measurement of the phase of $B_s$-$\bar{B}_s$ mixing will provide an important information to probe the GUT scale physics.

In Refs. [9, 11], two of us have studied the correlation between $\text{Br}(\tau \to \mu \gamma)$ and $\phi_s$, the phase in $B_s$-$\bar{B}_s$ mixing, comparing between SU(5) and SO(10) GUT models, and investigated the observational constraints in these models in order to decipher the GUT physics. The flavor violation, originating from the loop corrections via heavy particles, can be characterized by the CKM (Cabibbo-Kobayashi-Maskawa) quark mixing matrix and the MNSP (Maki-Nakagawa-Sakata-Pontecorvo) neutrino mixing matrix, as well as the size of the Yukawa couplings. Since the CKM mixings are small, it is expected that the neutrino mixings dominate the source of FCNCs at low energy. It is important to know whether the large neutrino mixings originate from Dirac-type or Majorana-type neutrino Yukawa couplings. When the large neutrino mixings originate from the Dirac neutrino Yukawa couplings in a particular GUT model, the (squared) right-handed down-type squark mass matrix, $M^2_{\tilde{D}_c}$, as well as the left-handed lepton doublet mass matrix, $M^2_{\tilde{L}}$, can have flavor non-universality. When the large mixings originate from the Majorana Yukawa couplings, the left-handed squark mass matrix, $M^2_{\tilde{Q}}$, can also have flavor non-universality in addition to the other sfermions. In the minimal-type of SU(5) GUT, the large neutrino mixings originate from Dirac neutrino couplings provided that there is no fine-tuning in the seesaw neutrino matrix. On the other hand, in the minimal-type of SO(10) GUT, the large neutrino mixings can originate from Majorana-type couplings. In general, since SU(5) is a subgroup of SO(10), one can construct a model, a non-minimal-type of SU(5) GUT, where the neutrino mixings originate from Majorana-type couplings. Conversely, if we allow fine-tuning in the Yukawa coupling matrices, Dirac neutrino Yukawa couplings can be the source of the large mixings even in the SO(10) model. Actually, there is a little ambiguity to determine...
the minimal SU(5) or SO(10) GUT model because the very minimal versions of the GUT models have problems with phenomenology and a slight modification is needed. (That is why we call our models “minimal-type”.) Here, we call a typical boundary condition as minimal-type SU(5) GUT condition when the off-diagonal elements of $M^2_{Dc}$ and $M^2_{L}$ are correlated due to the Dirac neutrino couplings in the model. Another type of boundary condition where the $M^2_Q$ is correlated to $M^2_{Dc, ar{U}c}$ and $M^2_{L}$ due to the Majorana coupling in SO(10) model is called the minimal-type SO(10) GUT boundary condition. The large phase of $B_s - \bar{B}_s$ mixing in combination with the other flavor violating processes, can tell us which type of boundary condition is preferable.

We analyzed the case of a large tan $\beta$ (which is the ratio of the vacuum expectation values of up- and down-type Higgs fields) in the Ref.\[11\]. In large tan $\beta$ case, the so-called double penguin contribution \[12, 13\] can dominate the SUSY contribution to the $B_s - \bar{B}_s$ mixing amplitude over the box contribution unless the pseudo Higgs field is heavy. When the double penguin contribution is enhanced by a smaller pseudo Higgs field mass, the $B_s \rightarrow \mu \mu$ decay \[14, 13\] is also enhanced close to its experimental bound \[15\]. In other words, if the large phase of $B_s - \bar{B}_s$ mixing originates from the double penguin contribution, the $B_s \rightarrow \mu \mu$ decay should be observed very soon, and therefore it is worthwhile to examine the constraints to see if the large phase is really generated from the double penguin contribution. An important constraint to obtain a large phase of $B_s - \bar{B}_s$ in GUT models comes from the experimental bound of $\tau \rightarrow \mu \gamma$ decay.

Due to the quark-lepton unification, when the flavor violation of $b-s$ transition is large, the $\tau - \mu$ flavor violation is expected to be large as well. This is significant especially in the minimal-type of SU(5) model. As a result, a wide region of the parameter space is excluded. This result is important to distinguish among the solutions of the dark matter relic density as measured by Wilkinson Microwave Anisotropy Probe (WMAP) \[16\] in the context of SUSY dark matter. Since imposing supersymmetric solution to the dark matter content of the universe puts a tremendous constraint on the SUSY model parameter space, it is interesting to see whether the constraints from the flavor violating processes can be satisfied by the dark matter allowed regions. The allowed parameter space, satisfying both of these constraints, can then be probed directly at the large hadron collider (LHC).

In this paper, we assume the lightest neutralino as dark matter in the GUT models and combine the dark matter analyses with the flavor constraint analyses. Consequently, some of the solutions for the neutralino dark matter are disfavored, and the so-called funnel solution (in which the neutralinos annihilate through the heavy Higgs bosons) turns out to be an interesting one for the large phase of $B_s - \bar{B}_s$ mixing and large tan $\beta$ case considered here. In the solution which satisfies all the constraints, the branching ratio of the $B_s \rightarrow \mu \mu$ decay is predicted, and we will show that it is in the range
to be observed soon. In scenarios with large $\tan\beta$, the dark matter direct detection can be correlated to the branching ratio of the $B_s \rightarrow \mu\mu$ decay since both are enhanced by a small pseudoscalar Higgs mass [17]. We investigate this correlation for the case with large phase of $B_s$-$\bar{B}_s$ mixing.

The paper is organized as follows: In section 2, we describe the FCNC sources in SUSY GUT models. The two typical boundary conditions in both SU(5) and SO(10) models are considered. In section 3, we describe the SUSY contributions of $B_s$-$\bar{B}_s$ mixing amplitudes, and the constraints from the other FCNC modes. We also discuss the solutions of the WMAP dark matter relic density which can be allowed in this scheme. In section 4, we show our numerical results on both SU(5) and SO(10) GUT models. Section 5 is devoted to conclusion and remarks.

2 FCNC sources in SUSY GUTs

In SUSY theories, the SUSY breaking terms can be sources of flavor violations. In general, it is easy to include flavor violating terms by hand since the SUSY breaking masses with flavor indices are parameters in the MSSM. However, large FCNCs are induced if these parameters are completely general [3]. Therefore, as a minimal assumption of the SUSY breaking, universality of the scalar masses is often considered. This means that all the SUSY breaking (squared) scalar masses are equal to $m_0^2$ and all the scalar trilinear couplings are proportional to the Yukawa couplings (with the coefficients are universal to be $A_0$) at a unification scale.

Even if the universality is assumed, non-universality in the scalar masses at the weak scale is generated by the evolution of the theory from the GUT scale down to the lower scale via the RGEs. As we have mentioned in the introduction, in the MSSM with right-handed neutrinos ($N^c$) the induced FCNCs from the RGE effects are not large in the quark sector while sizable effects can be generated in the lepton sector due to the large neutrino mixings [4]. The sources of FCNCs in this model are the Dirac neutrino couplings. In GUT models, the left-handed lepton doublet ($L$) and the right-handed down-type squarks ($D^c$) are unified in $\bar{5}$, and the Dirac neutrino couplings can be written as $Y_\nu \bar{5} N^c H_5$. As a result, non-universality in the SUSY breaking mass matrix for $D^c$ is generated from the colored-Higgs and right-handed neutrino loop diagrams, and flavor violation in the quark sector can then also be generated from the Dirac neutrino couplings [5-6].

The light neutrino mass matrix can be written as

$$M^{\text{light}}_\nu = f \langle \Delta_L \rangle - Y_\nu M^{-1}_R Y_\nu^T \langle H_u^0 \rangle^2,$$  \hspace{1cm} (3)

where $\Delta_L$ is an SU(2)$_L$ triplet, and $f$ is a Majorana coupling $\frac{1}{2} L L \Delta_L$. The second term is called type I seesaw term [18]. If the type I seesaw term dominates the light neutrino mass,
the Dirac neutrino coupling must have large mixings to explain the large neutrino mixings in the basis where the charge-lepton Yukawa coupling matrix $Y_e$ is diagonal. On the other hand, when the first term (triplet term) dominates (type II seesaw \[19\]), the Majorana coupling must have large mixings. Distinguishing these two cases is very important in order to understand the source of FCNCs in the GUT models.

The triplet contribution in the type II seesaw is natural in the framework of SO(10) GUT models \[20\]. In the SO(10) models, all matter species are unified in the spinor representation 16. Since the right-handed neutrino is also unified to other matter fields, the neutrino Dirac Yukawa coupling does not have large mixings in the minimal-type of SO(10) models, and the proper neutrino masses with large mixings can be generated from the Majorana couplings $\frac{1}{2}fLL\Delta_L$. The $f$ coupling is unified to the $16 16 126$ term which also includes Dirac Yukawa couplings for fermions, and thus the model is predictive \[21\]. If any of the decomposed fields from $T_{26}$ is lighter than the unification scale, the flavor non-universality for squarks and sleptons is generated. It is then possible that the loop corrections generate the flavor violation for the left-handed quark doublet ($Q$), the right-handed up-type quark ($U^c$) and the right-handed charged-lepton ($E^c$), in addition to $D^c$ and $L$.

We parameterize the non-universality in the squark and slepton mass matrices due to the loop corrections as

$$M_F^2 = m_0^2 \left[ 1 - \kappa_F U_F \text{diag}(k_1, k_2, 1) U_F^\dagger \right],$$

where $F = Q, U^c, D^c, L, E^c$. The quantity $\kappa_F$ denotes the amount of the off-diagonal elements and it depends on the sfermion species. The unitary matrices $U_F$ is equal to the neutrino mixing matrix in a limit. We note that the unitary matrices $U_F$ should be defined in the basis where charged-lepton Yukawa matrix, and down-type quark Yukawa matrix are diagonal in order to calculate the flavor violating processes such as $\tau \rightarrow \mu \gamma$, and $B_s - \bar{B}_s$ mixing. In the minimal SU(5) GUT where only $H_5$ and $\bar{H}_5$ couple to the fermions by renormalizable terms, $U_{D^c}$ is exactly same as $U_L$ and has large mixing angles, while $U_{Q,U^c,E^c}$ have small mixings relating the CKM mixings. In general, fermion mass matrices come from the sum of the Yukawa terms, and the equality of $U_{D^c}$ and $U_L$ can be completely broken when there are cancellations among the minimal Yukawa term and additional Yukawa terms. Here, we consider a model where the (nearly) equality between $U_{D^c}$ and $U_L$ (especially for 23 mixing angle of them) is maintained as a “minimal-type” assumption. The assumption is natural if there is a dominant Yukawa contribution and corrections to fit realistic masses and mixings are small. In the minimal-type of SO(10) model, all $U_F$ can have large mixings responsible for the neutrino mixings. The detail physical interpretation of this parameterization is given in \[11, 22\]. When the Dirac neutrino Yukawa coupling $Y_\nu$ or the Majorana coupling $f$ is hierarchical, we obtain $k_1, k_2 \ll 1$.
and then the 23 element of the sfermion mass matrix is $-\frac{1}{2} \kappa_0^2 \sin 2\theta_{23} e^{i\alpha}$. The magnitude of the FCNC between 2nd and 3rd generations is controlled by $\kappa \sin 2\theta_{23}$, where $\theta_{23}$ is the mixing angle in the unitary matrix. The phase parameter $\alpha$ also originates from the unitary matrix, and it will be the origin of a phase of the FCNC contribution.

It is interesting that the flavor violation pattern in the lepton sector and the quark sector can depend on the SO(10) symmetry breaking vacua. Actually, in order to forbid a rapid proton decay, the quark flavor violation should be larger than the lepton flavor violation among the symmetry breaking vacua [23]. Namely, it is expected that $\kappa_Q$, $\kappa_{Uc}$, and $\kappa_{Dc}$ are much larger than $\kappa_L$ and $\kappa_{Ec}$. For example, if only the Higgs fields $(8, 2, \pm 1/2)$ are light compared to the breaking scale (which is the most suitable case), one obtains $\kappa_Q = \kappa_{Uc} = \kappa_{Dc}$, and only quark flavor violation is generated, while the lepton flavor violation is not generated. On the other hand, when the flavor violation is generated from the minimal-type of SU(5) vacua with type I seesaw, the quantities $\kappa$’s have relations as $\kappa_L \sim \kappa_{Dc}$, and $\kappa_Q, \kappa_{Uc}, \kappa_{Ec} \sim 0$, effectively. Actually, when we take the threshold effect into account, it is expected that $\kappa_L$ is always larger than $\kappa_{Dc}$ since the right-handed Majorana mass scale is less than the scale of colored Higgs mass. Therefore, the existence of $b$-$s$ transition indicated by the experimental results in Fermilab predicts the sizable lepton flavor violation in the minimal-type of SU(5) model. Thus, if the results of large $B_s$-$\bar{B}_s$ phase is really an evidence of NP, the GUT models are restricted severely [8, 9, 10]. Therefore, investigating the quark and lepton flavor violation is very important to decipher the GUT symmetry breaking especially when the $B_s$-$\bar{B}_s$ phase is large [9].

3 $B_s$-$\bar{B}_s$ mixing and direct dark matter detection

Let us briefly see the phase of $B_s$-$\bar{B}_s$ mixing. We use the model-independent parameterization of the NP contribution:

$$C_{B_s} e^{2i\phi_{B_s}} = \frac{M_{12}^{\text{full}}}{M_{12}^{\text{SM}}}$$

(5)

where ‘full’ means the SM plus NP contribution, $M_{12}^{\text{full}} = M_{12}^{\text{SM}} + M_{12}^{\text{NP}}$. The NP contribution can be parameterized by two real parameters $C_{B_s}$ and $\phi_{B_s}$. The time dependent CP asymmetry $(S = \sin \phi_s)$ in $B_s \rightarrow J/\psi\phi$ is dictated by the argument of $M_{12}^{\text{full}}$: $\phi_s = -\arg M_{12}^{\text{full}}$, and thus $\phi_s = 2(\beta_s - \phi_{B_s})$. It is important to note that large SUSY contribution is still allowed even though the mass difference of $B_s$-$\bar{B}_s$ [24] is kept fairly consistent with the SM prediction. This is because the mass difference, $\Delta M_{B_s}$, can be just twice the absolute value of $M_{12}^{\text{full}}$. The consistency of the mass difference between the SM prediction and the experimental measurement just means $C_{B_s} \sim 1$, and a large $\phi_{B_s}$ is still allowed. For example, when $C_{B_s} \simeq 1$, the phase $\phi_{B_s}$ is related as $2 \sin \phi_{B_s} \simeq A_{s}^{\text{NP}}/A_{s}^{\text{SM}}$, where $A_{s}^{\text{NP,SM}} = |M_{12}^{\text{NP,SM}}|$. The argument of $M_{12}^{\text{NP}}$, being free in GUT
models, is due to the phase in off-diagonal elements in SUSY breaking mass matrix (in the basis where $Y_d$ is a real diagonal matrix), and one can choose an appropriate value for the new phase in the NP contribution. Therefore, the experimental data constrains $A_{s}^{\text{NP}}/A_{s}^{\text{SM}}$, and therefore, $\kappa \sin 2\theta_{23}$ is constrained for a given SUSY particle spectrum when the phase of $B_{s}\bar{B}_{s}$ is large.

In the MSSM with flavor universality, the chargino box diagram dominates the SUSY contribution to $M_{12}(B_{s})$. In the general parameter space of the soft SUSY breaking terms, the gluino box diagram can dominate the SUSY contribution for a lower $\tan \beta$ (i.e. $\tan \beta \lesssim 30$). The gluino box contribution is enhanced if both left- and right-handed down-type squark mass matrices have off-diagonal elements [7], and therefore, it is expected that the SUSY contribution to the $B_{s}\bar{B}_{s}$ mixing amplitude is large for the SO(10) model with type II seesaw, compared to the minimal-type of SU(5) model [9].

The box diagram does not depend on $\tan \beta$ (ratio of the vacuum expectation values of two Higgs fields) explicitly, whereas, the flavor changing Higgs interaction (through so-called Higgs penguin diagram) directly depend on $\tan \beta$, and the double Higgs penguin contribution to the $B_{s}\bar{B}_{s}$ mixing can become more important than the box diagram when $\tan \beta$ is large and there is an off-diagonal element in the right-handed down-type squark mass matrix [12, 13]. We note that the off-diagonal elements in the left-handed squark mass matrix is less important in order to generate a sizable double penguin contribution. This is because the chargino loop can generate the left-handed Higgs penguin contribution. Therefore, even in the minimal-type of SU(5) model, the double penguin contribution can be sizable when $\tan \beta$ is large. When the off-diagonal elements of left-handed squark mass matrix are generated, the left-handed flavor changing contribution to different processes (e.g., $b \rightarrow s\gamma$) can be modified.

The $B_{s} \rightarrow \mu\mu$ decay can be generated by a single Higgs penguin diagram [14, 13]. The decay amplitude is proportional to the muon Yukawa coupling, and thus the amplitude is proportional to $\tan^3 \beta$. Therefore, the branching ratio is proportional to $\tan^6 \beta$. Since it can be generated by a single left-handed penguin diagram, this decay occurs even in the universal SUSY breaking models like the mSUGRA (minimal supergravity) [25]. The current bound of the branching ratio is $\text{Br}(B_{s} \rightarrow \mu\mu) < 4.7 \times 10^{-8}$ [26]. When $\tan \beta$ is large, this bound gives an important constraint to the parameter space [8, 15, 17, 27]. In other words, one would expect that the $B_{s} \rightarrow \mu\mu$ decay will be observed very soon.

When the lepton flavor violation is correlated to the flavor violation in the right-handed down-type squark as in the minimal-type of SU(5) model, the $\tau \rightarrow \mu\gamma$ decay will give us the most important constraint to obtain the large $B_{s}\bar{B}_{s}$ phase [9, 10]. Furthermore, the squark masses are raised much more compared to the slepton masses due to the gaugino loop contribution since the gluino is heavier compared to the Bino and the Wino at low energy, and
thus the lepton flavor violation will be more sizable compared to the quark flavor violation. The current experimental bound of the branching ratio of $\tau \rightarrow \mu \gamma$ is \cite{28}

$$\text{Br}(\tau \rightarrow \mu \gamma) < 4.5 \times 10^{-8}. \quad (6)$$

In order to allow for a large phase in the $B_s$-$\bar{B}_s$ mixing in the minimal-type of SU(5) model, a large flavor-universal scalar mass (often called $m_0$) at the cutoff scale is preferable. The reasons are as follows. The gaugino loop effects are flavor invisible and they enhance the diagonal elements of the scalar mass matrices while keeping the off-diagonal elements unchanged. If the flavor universal scalar masses at the cutoff scale become larger, both Br($\tau \rightarrow \mu \gamma$) and $\phi_{B_s}$ are suppressed. However, Br($\tau \rightarrow \mu \gamma$) is much more suppressed compared to $\phi_{B_s}$ for a given $\kappa \sin 2\theta_{23}$ because the low energy slepton masses are sensitive to $m_0$ while the squark masses are not so sensitive due to the gluino loop contribution to their masses.

When $\tan \beta$ is large, the $\tau \rightarrow \mu \gamma$ constraint is relaxed for a large $B_s$-$\bar{B}_s$ phase, because the double-penguin contribution to the $B_s$-$\bar{B}_s$ mixing is proportional to $\tan^4 \beta$ while the $\tau \rightarrow \mu \gamma$ is proportional to $\tan^2 \beta$. However, the $B_s \rightarrow \mu \mu$ constraint becomes important in this case since it is proportional to $\tan^6 \beta$. As a result, the branching ratio of $B_s \rightarrow \mu \mu$ decay will have a lower bound in a given large $\tan \beta$ and SUSY spectrum when the phase of $B_s$-$\bar{B}_s$ mixing is large. This is because as follows: The double penguin contribution to the $B_s$-$\bar{B}_s$ mixing and the amplitude of $B_s \rightarrow \mu \mu$ are inversely proportional to $m_A^2$, where $m_A$ is a CP odd Higgs mass. For a given $\tan \beta$ and a large phase of $B_s$-$\bar{B}_s$ mixing, a larger $\kappa$ value is needed if $m_A$ is supposed to be larger. Then, the parameter space is excluded by $\tau \rightarrow \mu \gamma$ constraint, due to the approximate relation $\text{Br}(\tau \rightarrow \mu \gamma) \propto \kappa^2$. Therefore, $m_A$ has an upper bound, and $B_s \rightarrow \mu \mu$ have a lower bound.

Such constraints in the minimal-type of SU(5) have an impact on the neutralino dark matter which satisfies the recent WMAP result of relic density. The dark matter constraint is mostly satisfied in the minimal supergravity (mSUGRA) models by the following three scenarios.

1. The stau-neutralino coannihilation region.

2. The (nearly) Higgsino region (i.e. lightest neutralino has a large Higgsino component).

3. The funnel region (i.e. the neutralinos annihilate through heavy Higgs bosons pole).

At first, the stau-neutralino coannihilation region is not favored because the lighter stau is relatively light (almost degenerate with neutralino) in the region. When the stau is light, the $\tau \rightarrow \mu \gamma$ constraint will be severe and a large phase of $B_s$-$\bar{B}_s$ mixing is excluded. Secondly, the small Higgsino mass is not very favored either because it also enhances the $\tau \rightarrow \mu \gamma$ amplitude.
unless the sleptons are very heavy. Such heavy sleptons do not explain the muon $g - 2$ anomaly \[29\]. The third solution (funnel region) is interesting in the current scheme to have a large phase of $B_s - \bar{B}_s$ mixing. In the funnel region, the lightest neutralino mass $M_{\chi_1^0}$ is twice the mass of the heavy Higgs bosons ($2M_{\chi_1^0} \simeq m_A$). In the models that we consider, the masses of heavier CP even Higgs boson and the CP odd Higgs boson are nearly degenerate. As we have mentioned, $m_A$ has an upper bound for a given parameter space, and as result, the mass of the lightest neutralino is bounded from above. In the funnel region, $m_A$ is almost fixed for a given gaugino mass. In the double penguin contribution of $B_s - \bar{B}_s$ amplitude and the $B_s \rightarrow \mu \mu$, $m_A$ is a dominant parameter, and thus the branching ratio of $B_s \rightarrow \mu \mu$ can be predicted.

The Higgs mass $m_A$ is also important for the spin-independent scattering cross-section since it gets the dominant contributions from the Higgs exchange diagrams. In much of the parameter space, the t-channel Higgs exchanges ($h, H$) dominate the proton-neutralino cross-section $\sigma_{\chi_1^0-p}$. The spin-independent scattering cross-section can be written as \[30\]:

$$\sigma_{\chi_1^0-p} \simeq \frac{4}{\pi} m_p^4 |(A^0 f_u/m_u + A^c f_c/m_c + A^f f_i/m_i) + (A^d f_d/m_d + A^s f_s/m_s + A^b f_b/m_b)|^2,$$

(7)

where, $f_q \equiv \langle p | m_q \bar{q} q | p \rangle / m_p$, and $f_u \simeq 0.027$; $f_d \simeq 0.039$; $f_s \simeq 0.36$; $f_c = f_b = f_t \simeq 0.043$ \[31\]. The down- and up-type quark Higgs amplitudes are

$$A^{d,s,b} = \frac{g_2^2 m_{d,s,b}}{4M_W} \left( -\sin \alpha \frac{F_h}{\cos \beta m_h^2} + \cos \alpha \frac{F_H}{\cos \beta m_H^2} \right),$$

(8)

$$A^{u,c,t} = \frac{g_2^2 m_{u,c,t}}{4M_W} \left( \cos \alpha \frac{F_h}{\sin \beta m_h^2} + \sin \alpha \frac{F_H}{\sin \beta m_H^2} \right),$$

(9)

where $g_2$ is the $SU(2)$ gauge coupling, the Higgs mixing angle $\alpha$ is usually small $\sim 0.1$ and

$$F_h = (N_{12} - N_{11} \tan \theta_W)(N_{14} \cos \alpha + N_{13} \sin \alpha),$$

(10)

$$F_H = (N_{12} - N_{11} \tan \theta_W)(N_{14} \sin \alpha - N_{13} \cos \alpha).$$

(11)

Here $N_{ii}$ are the mixing amplitudes for the lightest neutralino $\chi_1^0$ among the Bino ($\tilde{B}$), Wino ($\tilde{W}$) and the two Higgsinos ($\tilde{H}_1, \tilde{H}_2$):

$$\chi_1^0 = N_{11} \tilde{B} + N_{12} \tilde{W} + N_{13} \tilde{H}_1 + N_{14} \tilde{H}_2.$$

(12)

We see that the down-type quark Higgs amplitude, Eq.(8), has larger contribution to the cross-section rather than the up-type ones $A^{u,c,t}$ due to larger $f_u$. It is also apparent that the cross-section increases for smaller value of the heavy Higgs mass, $m_H$, which scales with $m_A$ and smaller values of $\mu$ which increases $N_{13}$.

Therefore, $m_A$ is also important for the proton-neutralino elastic scattering cross-section, and this is particularly the case for models with non-universal Higgs masses \[32\] \[33\] \[34\]. Thus,
there is a correlation between the direct detection of the Milky Way dark matter and $\text{Br}(B_s \rightarrow \mu \mu)$ \cite{17}. Furthermore, the dark matter cross-sections are also affected by its particle contents, i.e., whether the lightest neutralino is gaugino or Higgsino dominated.

The current highest sensitivity of direct detection is about $5 \times 10^{-8}$ pb for neutralino mass $\lesssim 100$ GeV \cite{35,36}. This is expected to increase to $2 \times 10^{-9}$ pb soon \cite{37} for neutralino mass $\lesssim 100$ GeV (for the neutralino mass we have considered in this paper the expected limit should be around $5 \times 10^{-9}$ pb). For the models we consider, this constraint can exclude the parameter space with small $m_A$ and/or small $\mu$.

4 Numerical results

In order to illustrate the features described in the previous section, we plot the figures when the NP/SM ratio of the $B_s$-$\bar{B}_s$ amplitude is 0.5, $A_s^\text{NP}/A_s^\text{SM} = 0.5$, and the absolute value of the full amplitude is same as SM amplitude, $C_{B_s} = 1$. Under these choices, one can obtain that $|2\phi_{B_s}|$ is about 0.5 (rad). We consider that the SUSY breaking Higgs squared masses, $m_{H_u}^2$ and $m_{H_d}^2$, are not related to other scalar masses in order to make $m_A$ and $\mu$ free parameters, since these two parameters are important for Higgs penguin contribution and the proton-neutralino cross-section.

Figure 1 is drawn in the case of minimal-type of SU(5) model. We choose the $\kappa$ values for the non-universality to be same for left-handed sleptons and right-handed down-type squarks, for simplicity. To draw, we choose $\tan \beta = 40$, and the universal trilinear scalar coupling at GUT scale is zero ($A_0 = 0$). The unified gaugino mass $m_{1/2}$ is chosen to be 500 GeV and 800 GeV, and the sfermion masses at GUT scale is chosen to be 500 GeV and 1 TeV. These ranges of mass parameters can be probed at the LHC. We plot $\text{Br}(B_s \rightarrow \mu \mu)$ and the proton-neutralino spin independent cross-section ($\sigma_{\chi-p}$) contours using black and green lines respectively. The blue strips are the 2-std region of the WMAP dark matter relic density, assuming that the entire dark matter content is made of neutralino. The gray shaded region is excluded by the experimental bound of $\text{Br}(B_s \rightarrow \mu \mu)$. The red shaded region (bottom-right corner) corresponds to the stau LSP and hence is disallowed by the dark matter requirement. The yellow shaded region is excluded by the experimental bound of $\text{Br}(\tau \rightarrow \mu \gamma)$. As one can see from the figures, the $\tau \rightarrow \mu \gamma$ bound is relaxed for a larger sfermion mass $m_0$. For a larger gaugino mass $m_{1/2}$, the WMAP allowed strips for the funnel region (i.e., the vertical strips for $2M_{\tilde{\chi}_1^0} \sim m_A$) shift to the right. As mentioned in the previous section, the stau-neutralino coannihilation region (close to the red shaded region) and the small Higgsino mass (i.e. small $\mu$) is excluded by the bound of $\text{Br}(\tau \rightarrow \mu \gamma)$. On the other hand, the funnel region is still allowed. When $m_{1/2} = 800$ GeV, the funnel regions (for $\mu < 1$ TeV) shift to the region excluded by $\tau \rightarrow \mu \gamma$. In order to
Figure 1: The $m_A - \mu$ plane for minimal-type of SU(5) with non-universal Higgs masses, for \( \tan \beta = 40, A_0 = 0 \) and \((m_{1/2}, m_0) = (a) (500,500) \text{ GeV}, (b) (500,1000) \text{ GeV}, (c) (800,500) \text{ GeV} \) and (d) (800,1000) \text{ GeV} \) respectively.
Figure 2: The $m_A - \mu$ plane for minimal type SO(10) models with non-universal Higgs masses, for $\tan \beta = 40$, $A_0 = 0$ and $(m_{1/2}, m_0) = (a) (500,500)$ GeV, and (b) (800,500) GeV respectively.

allow these regions, $\mu > 2$ TeV and $\mu > 1.2$ TeV are needed for $m_0 = 500$ GeV and 800 GeV respectively. As a consequence, if $\mu$ is restricted to be less than 1 TeV, $m_{1/2}$ is bounded and then the cross-section for the direct dark matter detection is bounded from below. Note that the neutralino mass, which depends mostly on $m_{1/2}$, in these plots, is large (about 200 GeV for (a) and (b), and 320 GeV for (c) and (d)). At these masses, the current sensitivity of the direct detection experiments (as can be seen in Fig. 4 of [35] and Fig. 4 of [36]) is still lower than what is needed to exclude more of the parameter space. With the expected increase of the sensitivity by an order of magnitude in the near future, this constraint would become more severe.

Figure 2 is drawn for the case of SO(10) boundary conditions in the same way as for figure 1. We choose the kappa values to be same for both left- and right-handed squarks. As is mentioned, depending on the SO(10) breaking vacua, the flavor non-universality in the slepton mass matrices can be reduced, and we choose them to be zero to escape from the $\tau \rightarrow \mu \gamma$ bound. With this choice, then, there is no upper bound for $m_A$. Therefore, if $\text{Br}(B_s \rightarrow \mu \mu)$ turns out to be small ($\lesssim 10^{-8}$) and $\tan \beta$ large, we should adopt the SO(10) model. As we have mentioned, in the SU(5) case, when we take both the dark matter relic density and $\tau \rightarrow \mu \gamma$ bound into account, $\mu$ needs to be larger for a larger $m_{1/2}$, and consequently the direct detection cross-section would be too small to be probed at the ongoing XENON 100 experiment, as can be seen from figures 1(c,d). In the SO(10) case, on the other hand, the direct detection cross-section
Figure 3: Correlation between $\text{Br}(B_s \rightarrow \mu \mu)$ and $\sigma_{\chi^- p}$ for the minimal-type (a) SU(5) and (b) SO(10) cases, both for $\tan \beta = 40$, $A_0 = 0$, $m_{1/2} = 500$ GeV and $m_0 = 500$ GeV.

even for larger values of $m_{1/2}$ (e.g., figure 2(b)) can still be large and could be probed very soon. This is, in part, because the stau coannihilation and the Higgsino dark matter solutions are also allowed along with the funnel region. If we find evidence for these solutions from the LHC, then the SO(10) model would be preferred.

We comment that the boundary condition for the left-handed squark mass and right-handed up-type squark masses are different from the previous SU(5) boundary condition. Therefore, the RGE running for the SUSY breaking Higgs masses are different, and thus the stau LSP regions (red/pink shaded regions) for the SO(10) figures are different from the SU(5) ones.

In Figure 3, we show the correlation between the $\text{Br}(B_s \rightarrow \mu \mu)$ and the proton-neutralino cross-section for SU(5) and SO(10) boundary conditions. The data points are picked up from the figure 1 and 2 for $m_0 = m_{1/2} = 500$ GeV, and are sliced for fixed values of the Higgsino mass $\mu$. The small circles represents the solution for the WMAP relic density. Since $m_A$ is almost determined for the funnel region of the WMAP solution and the $\mu$ dependence of $\text{Br}(B_s \rightarrow \mu \mu)$ for a given phase of $B_s-\bar{B}_s$ mixing is not large, $\text{Br}(B_s \rightarrow \mu \mu)$ is predictable as one can see from the figures. The proton-neutralino cross-section, on the other hands, depends on the Higgsino mass hence varies with $\mu$. The current experimental sensitivity is still low, but if one of the solutions is correct, the direct detection of the dark matter should be possible in the near future. Note, however, that if the neutralino contributes only a part of the dark matter content (i.e., regions between two circles where the neutralino relic density is lower than the WMAP), the neutralino direct detection rate would be scaled down. In this case, we will need even higher sensitivity for the direct detection to exclude the parameter space regions.
5 Conclusion

We have investigated the GUT models when the $B_s$-$\bar{B}_s$ mixing phase can become really large as indicated in the Fermilab experiments. We considered two cases: one is the minimal-type of SU(5) model with type I seesaw. The other is the minimal-type of SO(10) model with type II seesaw. The difference between the two boundary conditions is whether there exists a sizable off-diagonal element in the left-handed squark mass matrix. We emphasize that the sources of FCNC in the GUT models will be restricted if the large phase of $B_s$-$\bar{B}_s$ mixing persists in the upcoming result from Fermilab.

In the case of a large $\tan\beta$, the double penguin contribution dominates the SUSY contribution to the $B_s$-$\bar{B}_s$ mixing when the pseudo Higgs mass is not too heavy. Especially in the minimal-type of SU(5) model, the pseudo Higgs mass should be low enough to satisfy the experimental constraint from $\tau \to \mu\gamma$ decay, and because of this, the branching ratio of $B_s \to \mu\mu$ is sizable and can be detected very soon. The $\tau \to \mu\gamma$ constraint also restricts the Higgsino mass $\mu$ and the slepton masses, and it may exclude some solutions of the relic density of the neutralino dark matter. In fact, the stau-neutralino coannihilation and the Higgsino dark matter solutions are not favored for the large phase of $B_s$-$\bar{B}_s$ mixing in the minimal-type of SU(5) model. On the other hand, the funnel solution (in which the neutralinos annihilate through the heavy Higgs bosons pole) is favored in this case. For the funnel solution, the pseudo Higgs mass is almost determined, and the branching ratio of the $B_s \to \mu\mu$ decay is more predictive and can be observed soon for the values of soft masses which can be probed at the LHC. The direct detection cross-section depends on the heavy Higgs mass and the Higgsino mass, and correlated to the $B_s \to \mu\mu$ in the case of a large phase of the $B_s$-$\bar{B}_s$ mixing. The dark matter-nuclear cross-section could be in the range to be detected very soon in the upcoming experiments in both SU(5) and SO(10) models.

In this paper, we have concentrated on the importance of the 2nd and 3rd generation FCNC effects such as $\text{Br}(\tau \to \mu\gamma)$ and $\phi_{B_s}$ correlation in GUT models, since they can be correlated directly by the 23 mixing. The constraints from $\text{Br}(\mu \to e\gamma)$ decay, $K$-$\bar{K}$ and $B_d$-$\bar{B}_d$ mixings may also be important, but these effects depend on the details of the flavor structure which can have a freedom of cancellation. We refer to the Ref.\cite{22} for an analysis of flavor violation including the first generation.

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