NMR Determination of 2D Electron Spin Polarization at $\nu = 1/2$

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Using a ‘standard’ NMR spin-echo technique we determined the spin polarization $P$ of two-dimensional electrons, confined to GaAs quantum wells, from the hyperfine shift of Ga nuclei in the wells. Concentrating on the temperature ($0.05 \lesssim T \lesssim 10$ K) and magnetic field ($7 \lesssim B \lesssim 17$ T) dependencies of $P$ at Landau level filling factor $\nu = 1/2$, we find that the results are described well by a simple model of non-interacting composite fermions, although some inconsistencies remain when the two-dimensional electron system is tilted in the magnetic field.

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The fractional quantum Hall effect (FQHE), observed in low-disorder two-dimensional electron systems (2DESs) at low temperature $T$ and high magnetic field $B$, is one of the most fascinating problems involving strongly correlated fermions. Recently, considerable attention has been focused on the FQHE ground states near half-integer Landau level (LL) filling factors ($\nu = 1/2, 3/2, \ldots$), where a large body of experimental and theoretical results can be cast in a surprisingly simple picture of non-interacting composite fermions (CFs) with the same charge and spin as electrons. The simplest realization of a CF is at $\nu = 1/2$ where electrons bind two flux quanta of a fictitious Chern-Simons gauge field.

An important issue in the physics of CFs is the spin polarization of the 2DES at half-integer fillings. Before the remarkable success of the CF model, theoretical results, and experimental results, pointed to the possibility of FQHE states with reversed spins at various fillings ($\nu = 2/3, 2/5, \ldots$). In an effort to understand the spin configurations of these states within the CF picture and particularly, the spin polarization of the 2DES close to $\nu = 1/2$, Park and Jain introduced a new parameter, the CF polarization mass $m_p^*$, which is proportional to the ratio of the cyclotron and Coulomb energies. These authors obtained an estimate for $m_p^*$ at $\nu = 1/2$:

$$m_p^*/m_e \approx 0.60 \sqrt{B_\perp},$$  \hspace{1cm} (1)

where $B_\perp$ (in tesla) is the component of $B$ perpendicular to the 2DES plane. The parameter $m_p^*$, combined with a parabolic dispersion law for CFs at $\nu = 1/2$, uniquely determines the spin polarization $P$ at any given $T$ and $B$. Here we report direct measurements of the 2DES spin polarization as a function of $T$ and $B$, and use our data to critically test the applicability of the non-interacting CF model and $m_p^*$. We find that the data are in excellent agreement with predictions of Ref. [1], except when the 2DES is tilted in $B$.

Nuclear magnetic resonance (NMR) is a sensitive technique for the experimental determination of the spin polarization of two-dimensional (2D) electrons. Prior to this work, however, only the use of optically pumped NMR (OPNMR) was reported. The reason is that the number of active nuclei in a typical 2D system is usually too small to generate a useful signal for ‘standard’ NMR techniques. One way to increase the signal is by optical pumping: polarized electrons are excited in the conduction band by illuminating the sample with circularly polarized light and the strong hyperfine coupling ensures the transfer of this polarization to the nuclei. However, this also implies that in OPNMR experiments the electronic system is observed while nuclei are strongly polarized, well beyond their small equilibrium value. We demonstrate here that the standard pulsed NMR technique, applied to the Ga nuclei in GaAs/AlGaAs multiple-quantum well heterostructures does indeed provide measurable signal for $T \lesssim 10$ K. The NMR signal was observed on samples consisting of 200 quantum wells (QWs) using a state-of-the-art laboratory-built pulsed NMR spectrometer. The method we employ avoids any eventual perturbation of the system by optical pumping and the experimental setup is greatly simplified, making possible, e.g., the use of a $^3$He/$^4$He dilution refrigerator.

Two heterostructures, M242 and M280, each composed of one hundred GaAs QWs, separated by AlGaAs barriers which are Si-doped near their centers, were used in this study. Sample M242 (M280) has 250 Å (300 Å) wide QWs, 1850 Å (2500 Å) thick $A_{0.3}Ga_{0.7}As$ ($A_{0.1}Ga_{0.9}As$) barriers, and density $n = 1.4 \times 10^{11}$ cm$^{-2}$ ($8.5 \times 10^{10}$ cm$^{-2}$). From each heterostructure, we cut two $\approx 26$ mm$^2$ pieces and placed the two pieces together into the radio-frequency coil, so that experiments were done on effectively 200 QWs. For $T \gtrsim 1.5$ K, the NMR signal was recorded as a function of both $B$ and tilt angle.
θ between B and the normal to the plane of the 2DES. For very low-T measurements, the radio-frequency coil was mounted into the mixing chamber of a $^3$He/$^4$He dilution refrigerator, and measurements were performed as a function of T at fixed θ and B.

To distinguish between the contributions of Ga nuclei in QWs and barriers and to eliminate the signal from the substrate, we exploited the difference in their nuclear spin-lattice relaxation rates (1/T1) [14]. The NMR pulse sequence is described in Fig. 1b: the nuclear magnetization was first set to zero by a comb of π/2 pulses. After the magnetization has recovered during time $t_R$, its value was measured by a spin-echo sequence ($\pi/2 - \tau - \pi - \tau - \text{echo}$) [12]. Spectra were obtained by Fourier transforming the echo (Figs. 1b and 1c), and the hyperfine shift $K_S$ of Ga nuclei in QWs is here defined as the frequency shift of the NMR line attributed to the QWs with respect to the barriers’ line. This resonance shift is caused by the hyperfine interaction between nuclei and 2D electrons (dominated by the Fermi contact term) [9,10,12]. Note also that our QWs’ NMR line is split by a small and well defined quadrupole coupling which is clearly resolved at high temperatures (Fig. 1b), confirming the high homogeneity of the 2DES.

Before focusing on the spin polarization at $\nu = 1/2$ we first discuss the results at $\nu = 1/3$. Previous OPNMR experiments [6] revealed a completely spin polarized FQHE ground state at $\nu = 1/3$. The low T ($P = 1$) limit of OPNMR $K_S$ data at $\nu = 1/3$, measured for several samples, was successfully used to determine the intrinsic hyperfine shift of Ga nuclei in the center of each QW [$K_{\text{S,int}} = K_S + 1.1 \times (1 - \exp(-K_S/2.0))$] and to establish the relationship $K_{\text{S,int}} = A_c P n / w$, which defines the hyperfine coupling $A_c = (4.5 \pm 0.2) \times 10^{-13}$ cm$^3$/s (w is the QW width) [10]. Applied to our samples, these expressions yield the reference ‘full polarization’ values $K_{S,\text{int}}^{P=1} \approx 12.7$ kHz for M280 and $K_{S,\text{int}}^{P=1} \approx 25.2$ kHz for M242 [13]. Figure 2 shows that the latter value is consistent with our very low T data. The T-dependence of these data is fitted to $K_{\text{S,int}}^{\text{sat}}(\nu \approx 1/3) \tanh(\Delta_{1/3}/4T)$, yielding $K_{\text{S,int}}^{\text{sat}}(\nu \approx 1/3) = 21 \pm 5$ kHz and $\Delta_{1/3} = 1.7\Delta_Z$, in agreement with the OPNMR result $\Delta_{1/3} = 1.82\Delta_Z$ [10] and theoretical estimates $\Delta_{1/3} \approx 2\Delta_Z$ [14]. Here $\Delta_Z = |g| \mu_B B$ is the Zeeman energy, $\mu_B$ is the Bohr magneton, and $g = -0.44$ is the electron g-factor in bulk GaAs.

In Fig. 2 we present the T-dependence of the 2DES spin polarization at $\nu = 1/2$ at different B for our two samples. The right axes give the measured $K_{\text{S,int}}$, while the deduced spin polarization, defined as $P(T) = K_{\text{S,int}}(T)/K_{\text{S,\text{int}}^{P=1}}$, is indicated on the left axes. Concentrating on the $\theta = 0^\circ$ data (filled circles), we note that $K_{\text{S,int}}$ for the high density 2DES (M242) at $B = 11.4$ T reaches the full polarization value as $T \to 0$, implying that the ground state of the 2DES is fully spin polarized at $\nu = 1/2$. On the other hand, M280 data at $B = 7.1$ T reveal that the low-T $K_{\text{S,int}}$ saturates at $9.5 \pm 1$ kHz, below the expected $K_{\text{S,\text{int}}^{P=1}} \approx 12.7$ kHz for this sample. The ground state in M280 therefore appears to be only partially spin polarized at $\theta = 0^\circ$.

In the remainder of the paper we will discuss how these conclusions, as well as the $T$-dependencies reported in Fig. 2, compare to the non-interacting CF model of Ref. 6. However, without referring to any model, we can already infer useful information from the data presented so far by considering the Zeeman energy normalized to the Coulomb energy ($\Delta_C$) for the two samples. Here $\Delta_C = e^2/\epsilon_l B$, $\epsilon \approx 13$ is the static dielectric constant of GaAs, and $l_B = \sqrt{\hbar/eB}$ is the magnetic length. The $\Delta_Z/\Delta_C$ ratio is 0.019 for M242 at $B = 11.4$ T and 0.016 for M280 at $B = 7.1$ T, implying that the 2DES becomes fully spin polarized for $\Delta_Z/\Delta_C$ above a critical value which lies between 0.016 and 0.019. This conclusion is consistent with magneto-optics data [3] which yielded a critical value of 0.018 for the full spin polarization at $\nu = 1/2$.

We now attempt to understand the $T$-dependence of $P$ based on a simple model of non-interacting CFs. We assume that, consistent with previous work [1], CFs have a $g$-factor roughly the same as electrons and consider parabolic bands occupied by $n$ CFs with mass $m^*_n$.

Hence, the density of states $D(E)$ (for spin-up and spin-down CFs) is $D(\pm E) = D(\pm \Delta_Z/2)$, where $D = m^*_n/(2\pi\hbar^2)$ and $\theta$ is the step function. Making use of Fermi-Dirac distribution we find

$$P = \frac{D}{n} \left[ \Delta_Z - 2k_B T \cdot \tanh^{-1} \left( 1 + \exp\left(\frac{\theta \nu \mu_B B}{\sinh^2\left(\frac{\theta \nu \mu_B B}{2\hbar}\right)}\right)^{-\frac{1}{2}} \right) \right].$$

(2)

Depending on the strength of the magnetic field, this model predicts either partially or completely polarized Fermi sea of CFs in the $T \to 0$ limit, i.e., $P(T = 0) = \min\{D\Delta_Z/n, 1\}$. Note that according to Eq. 2, $P$ at a given $T$ and $B$ depends only on $m^*_n$. Taking $m^*_n$ as a fitting parameter, in Fig. 3 we show the best fits of Eq. 2 to the $\theta = 0^\circ$ data by solid curves. These curves indeed provide a reasonable description of the data. Moreover, the deduced $m^*_n$ values (2.2$m_e$ at $B = 11.4$ T and 1.7$m_e$ at $B = 7.1$ T) are found to be in excellent agreement with the polarization mass predicted by Eq. 2 together with the simple model leading to Eq. 2 give a very good account of the $\theta = 0^\circ$ data without any adjustable parameters.

Next, we present our study of $P$ in tilted magnetic fields (Figs. 5 and 6). Unfilled circles in Fig. 5 show $\nu = 1/2$ data at $\theta = 40^\circ$ (M242) and $\theta = 61^\circ$ (M280), taken as a function of $T$ at $B = 14.8$ T. The data for both samples have a qualitatively similar behavior: the polarization at
\( \theta \neq 0^\circ \) is larger than at \( \theta = 0^\circ \) for low and intermediate \( T \) while at highest \( T \) the measured polarization falls below the \( \theta = 0^\circ \) values. In Fig. 3 we present data, taken at \( T \lesssim 1.5 \) K, showing the dependence of \( \mathcal{P} \) on \( B \) for both M242 and M280. Here the spin polarization exhibits a monotonic increase with \( B \).

To compare these data with the predictions of the non-interacting CF model, we show in Figs. 3 and 4 plots of \( \mathcal{P}(T, B) \) according to Eq. 3. Note that here there are no adjustable parameters as we used the \( m^*_p \) values obtained from the fits to the \( \theta = 0^\circ \) data. The only parameter in Eq. 3 that depends on the total magnetic field is \( \Delta \). The results (dotted curves in Fig. 3) qualitatively agree between the calculated curves and the data becomes larger at higher \( B \) (larger tilt angles). These observations suggest that the simple CF model leading to Eqs. 1 and 2 are not directly applicable at large \( \theta \). The deformation of the 2DES wavefunction at large \( \theta \) may be partly responsible for this discrepancy, although we cannot rule out other possibilities.

In conclusion, we demonstrated the feasibility of the 'standard' NMR experiments to investigate the spin polarization of the 2DES in the quantum limit. Our results provide experimental support for the simplest and most straightforward model of \( \mathcal{P}(T, B) \) at \( \nu = 1/2 \). This model, based on the assumption of a parabolic dispersion law of CFs with a polarization effective mass \( m^*_p \), describes well the experimental data when the 2DES is subjected only to a perpendicular magnetic field.

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Note added.- As this manuscript was being completed we became aware of similar work \cite{17} investigating the spin polarization at \( \nu = 1/2 \) by OPNMR.

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FIG. 2. $^{71}$Ga intrinsic hyperfine shift ($K_{\text{Int}}$) vs $T$ for M242 at $\theta = 0^\circ$ and $B = 17$ T ($\nu = 0.335$). The solid curve is a fit to the data (see text).

FIG. 3. $P$ (left axes) and $K_{\text{Int}}$ (right axes) vs $T$ at $\nu = 1/2$ for M242 (top panel) and M280 (bottom panel). The filled circles represent $\theta = 0^\circ$ data and the unfilled circles represent $\theta \neq 0^\circ$ data. In both panels the solid curves represent best fits of Eq. 2 to the $\theta = 0^\circ$ data; these fits give $m_p^*/m_e = 2.2 \pm 0.2$ for M242 and $m_p^*/m_e = 1.7 \pm 0.2$ for M280. The dotted curves represent predictions of Eq. 2 for $B = 14.8$ T and using $m_p^* = 2.2m_e$ (M242) and $m_p^* = 1.7m_e$ (M280).

FIG. 4. $P$ vs $B$ at $\nu = 1/2$ and $T \approx 1.5$ K for M242 (o) and M280 (•). The dotted and solid curves represent predictions of Eq. 3. The solid curve was computed with $m_p^* = 2.2m_e$ (M242), and the dotted one with $m_p^* = 1.7m_e$ (M280).
Fig. 1 'NMR Determination of 2D Electron Spin Polarization at $\nu = \frac{1}{2}$'
Melinte/Freytag/Horvatic/Berthier/Levy/Bayot/Shayegan
Fig. 2 'NMR Determination of 2D Electron Spin Polarization at $v = 1/2$'
Melinte/Freytag/Horvatic/Berthier/Levy/Bayot/Shayegan
Fig. 3 'NMR Determination of 2D Electron Spin Polarization at $\nu = 1/2$
Melinte/Freytag/Horvatic/Berthier/Levy/Bayot/Shayegan
Fig. 4 'NMR Determination of 2D Electron Spin Polarization at $\nu = 1/2$'
Melinte/Freytag/Horvatic/Berthier/Levy/Bayot/Shayegan