Constitutive relations for isotropic materials allowing quasi-linear approximation of the deformation law

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Abstract. Variants of deformation potentials for isotropic materials that do not obey the "single curve" hypothesis are considered. The advantage of the potentials formulated in two normalized stress spaces in comparison with other variants of quasi-linear models of the constitutive relations for isotropic materials is demonstrated. It is shown that the claims of some authors on the originality and universality of the deformation potentials proposed by them are not justified, and their variants of relations are reduced to two forms of writing in normalized spaces.

1. Introduction

Two centuries of comprehensive use of the generalized Hooke's law approved it as an inviolable physical law. However, a wide range of deformation experiments on materials such as cast iron [1 – 5], graphite and graphite plastics [6 – 18], technical and construction ceramics [19], concrete [20 – 24] of most composite materials [25 – 30], confirm the fact that the linear representation of the equations of the relationship between stresses and strains with the establishment of the elastic modulus $E$ and the Poisson's ratio $\nu$ even in a very narrow range of deformation can not be called correct. Much more accurately represented the analytical interpretation of the experimental results to test the uniaxial tension and uniaxial compression in the form of linear functions with the module definition, $E'$ (secant modulus of elasticity) under uniaxial tension and the modulus $E$ under uniaxial compression. At the same time, Poisson's ratios $\nu'$ and $\nu$ (transverse strain coefficients) are calculated. In this key characterizes the properties of an isotropic elastic material, the mechanical characteristics of which depend on the type of stress state. Such a representation in the 60s of the twentieth century was called the multi-modulus theory of elasticity, based on quasi-linear deformation equations.

The indisputable primacy in the transformation of the fundamental law of Hooke in the case of materials that are sensitive to the type of stress state, was assigned to Russian scientists and their colleagues from some republics of the former Soviet Union. Research in this area of materials science and deformation theory, which occurred before the 1960-s, were random and did not attract much interest. In subsequent years, the development of regular research was stimulated by the widespread use of new materials, which obviously caused the emergence of major fundamental results in the formulation of equations of state for materials whose mechanical characteristics depend on the type of stress state. The increased interest in such a physical and mechanical phenomenon, which has arisen in re-
cent years, is caused by the extensive use of polymer and composite materials, the most characteristic feature of the mechanical resistance of which, as many authors note, is a significant deviation of the deformation characteristics from the seemingly unshakable "hypothesis of a single curve" [1–30].

2. Analysis of quasi-linear potentials

A review of most works devoted to the construction of equations of state, taking into account the dependence of the deformation and strength characteristics of isotropic materials to the type of stress state, is available in [31, 32]. In the same works, within the framework of two normalized stress spaces, rather general potential relations between strains and stresses in a quasi-linear form are formulated. And from these relations, as a special case, follows almost all known variants of the physical equations proposed by other authors.

The deformation potential is derived from the first space in the form of:

\[ W = 0.5 [(A + B \cdot \alpha_1)\sigma_1^2 + (A + B \cdot \alpha_2)\sigma_2^2 + (A + B \cdot \alpha_3)\sigma_3^2] + [C + E \cdot \alpha_3 + D \cdot (\alpha_1 + \alpha_2)] \cdot \sigma_1 \cdot \sigma_2 + [C + E \cdot \alpha_1 + D \cdot (\alpha_2 + \alpha_3)] \cdot \sigma_2 \cdot \sigma_3 + [C + E \times \alpha_2 + D \times (\alpha_1 + \alpha_3)] \times \sigma_1 \times \sigma_3, \]

(1)

and from the second – its analog in the following form:

\[ W = (\tilde{b}_1 + \tilde{b}_3 \cdot \xi) \cdot \sigma_2^2 + (\tilde{b}_2 + \tilde{b}_5 \cdot \eta \cdot \cos 3\phi) \cdot \tau^2, \]

(2)

where \( \alpha_k = \sigma_k / S \) – main normalized stresses; \( S = \sqrt{\sigma_k \cdot \sigma_k} \) – norm of the first vector space; \( \cos \psi = \sigma / S_0, \sin \psi = \eta = \tau / S_0 \) – normalized stresses of the second space; \( \cos 3j = \sqrt{2} \times \text{det}(S_j) / \tau^3 \) – phase invariant; \( S_0 = \sqrt{\sigma^2 + \tau^2} \) – norm of the second space; \( \sigma = \delta_{ij} \cdot \sigma_{ij} / 3 \) – normal octahedral stress; \( \tau = \sqrt{S_j S_j} / 3 \) – tangent octahedral stress; \( S_j = \sigma_j - \delta_{ij} \cdot \sigma \) – deviator of the stress tensor, \( \delta_{ij} \) – Kronecker symbols; \( \tilde{b}_1 = 1.5 \cdot (A + 2 \cdot C), \tilde{b}_2 = 1.5 \cdot (A - C), \tilde{b}_3 = 1.5 \cdot (B + 4 \cdot D + 2 \cdot E) / \sqrt{3}, \tilde{b}_5 = 4.5 \cdot (B - E) / \sqrt{3}, \tilde{b}_5 = 0.75 \cdot (B - 2 \cdot D + 2 \cdot E) \cdot \sqrt{2} / \sqrt{3} \) – constants, which are determined by the mechanical characteristics of the material.

Two normalized spaces are interconnected through their invariants:

\[ S_0 = S / \sqrt{3}; I_1 = \sqrt{3} \cdot \xi; \quad \text{III}_a = (3 \cdot \xi + 9 \cdot \xi \cdot \eta^2 + 1.5 \cdot \sqrt{2} \cdot \eta \cdot \cos 3\phi) / \sqrt{3}, \]

(3)

where \( I_a = \alpha_k = \alpha_1 + \alpha_2 + \alpha_3, \quad \text{III}_a = \alpha_k \cdot \alpha_k = 1, \quad \text{III}_a = \alpha_k \cdot \alpha_k \cdot \alpha_k \) – invariants of the first space.

Of the proposed potentials derive generalized laws of deformation:

a) volume changes

\[ e = \sigma / (3 \cdot K_0) + \tau / (3 \cdot D_0); \]

(4)

b) changes of form

\[ \varepsilon = \sqrt{1 + 2g^2\omega} \cdot f \cdot \tau \cdot (2 \cdot G_0) + \sigma / (3 \cdot D_0) f; \]

(5)

c) when the phase characteristics

\[ \operatorname{tgo} = 3 \cdot \tilde{b}_5 \cdot \eta \cdot \sin 3\phi / [3 \cdot \eta / (2 \cdot G_0) + \xi / D_0], \]

(6)

where \( K_0 \) – functional «module» of volume deformations; \( G_0 \) - functional «module» shift; \( D_0 \) – functional «module» dilatancy; \( \omega = \phi - \beta \) – the phase difference between the stress and deformation; \( K_0 = 1 / [2(\tilde{b}_1 + \tilde{b}_3 \xi (2 + \eta^2) - \tilde{b}_5 \eta \cos 3\phi)]; \quad D_0 = 1 / (\tilde{b}_5 \eta^3); \quad 2G_0 = 3 / [2\tilde{b}_5 + \xi^2 (2\tilde{b}_5 - \tilde{b}_1) + \tilde{b}_5 \eta (2 + \xi^2) \cos 3\phi]; \quad 3 \cdot e = 0 = \delta_{ij} \cdot e_{ij} \) – volumetric strain; \( \varepsilon = \gamma / 2; \quad \gamma = \sqrt{4 \cdot e_{ij} \cdot e_{ij} / 3} \) – octahedral shear; \( e_{ij} = e_{ij} - \delta_{ij} \cdot 0 / 3 \) – tensor deviator strain.
In [31, 32] it is demonstrated that the obtained quasi-linear equations of state take into account the dilatation properties of materials. In addition, as follows from the equations (5), the medium stresses affect the shape change, but so that if the shear stresses \( \tau = 0 \), the shape change is absent: \( \mathcal{E} = 0 \), that is purely hydrostatic stress does not lead to a shift. The existence of these properties in materials with different resistance has been repeatedly confirmed in all theoretical and experimental studies [1 – 30].

Two forms of strain potential (1) and (2) have five constants to be determined. It is obvious that experiments on uniaxial tension and uniaxial compression, of which four constants are established, are not enough. The following parameters are determined from these experiments:

\[
A = 0.5 \cdot (1 / E^+ + 1 / E^-); \quad B = 0.5 \cdot (1 / E^+ - 1 / E^-); \\
C = -0.5 \cdot (\nu^+ / E^+ + \nu^- / E^-); \quad D = -0.5 \cdot (\nu^+ / E^+ - \nu^- / E^-). \quad (7)
\]

An additional option was to consider the possibility of finding the fifth constant from the experiments on the net shift [31, 32]. From the processing of the shear test results, the value of the material constant \( G_q \) (shear modulus at pure shear) follows, and from the representation of the potential (2) the following dependence

\[
\tilde{b}_5 = 1.5 \cdot (A - C) = 0.75 / G_q. \quad (8)
\]

On the other hand, from the experiments on uniaxial tension and uniaxial compression potential (2) requires equality:

\[
\tilde{b}_5 = 0.75 \cdot [(1 + \nu^+) / E^+ + (1 + \nu^-) / E^-]. \quad (8*)
\]

Obviously, this confirms that the considered quasilinear relations (1) and (2) describe the states of materials with different resistance, in which the shear modulus \( G_q \) is not an independent mechanical characteristic, but is defined as a medium parameter:

\[
1 / G_q = 1 / G = (1 + \nu^+) / E^+ + (1 + \nu^-) / E^- . \quad (9)
\]

The problem of calculating the fifth constant is solved [31, 32] by processing the phase characteristic, as recommended in the nonlinear theory of elasticity [33]. However, in publications concerning experimental studies on deformation of materials with different resistance, there are only fragmentary, insignificant information about the values \( \tan \omega \). Therefore, [31, 32] a priori variants of determining the fifth constant were proposed, which do not contradict the results of experiments and the fundamental laws of mechanics.

In General, three quite acceptable variants of a priori calculation of the fifth constant are recommended.

Option No. 1. It is assumed that the states of the isotropic material are independent of the stress phase, and this leads to equality \( \tilde{b}_5 = 0 \) (2) and from the experiments on uniaxial tension and compression the relations follow

\[
\tilde{b}_1 = 0.75 \cdot [(1 - 2 \cdot \nu^+) / E^+ + (1 - 2 \cdot \nu^-) / E^-]; \\
\tilde{b}_2 = 0.75 \cdot [(1 + \nu^+) / E^+ + (1 + \nu^-) / E^-]; \quad \tilde{b}_3 = -1.5 \cdot \sqrt{3} \cdot (\nu^+ / E^+ - \nu^- / E^-); \\
\tilde{b}_4 = 0.375 \cdot \sqrt{3} \cdot [(3 + 2 \cdot \nu^+) / E^+ - (3 + 2 \cdot \nu^-) / E^-]. \quad (10)
\]

In this case \( E \), the constant in the form (1) is determined by the formula:

\[
E = -0.25 \cdot [(1 + 2 \cdot \nu^+) / E^+ - (1 + 2 \cdot \nu^-) / E^-]. \quad (11)
\]

Then, in the form of potential (1), five calculated constants are stored, and the form (2) becomes four constant:

\[
W = (\tilde{b}_1 + \tilde{b}_3 \cdot \xi) \cdot \sigma^2 + (\tilde{b}_2 + \tilde{b}_4 \cdot \xi) \cdot \tau^2. \quad (2*)
\]

Option No. 2. Mathematically consistent is the assumption of the smallness of the component in the polynomial decomposition in the construction of the potential (1), in the expression of which the
cofactors are simultaneously three main normalized stresses – quantities of small order in comparison with the norm of vector space (normalized space № 1). At the same time, we note that the simultaneous consideration of three normalized stresses among the elements of the second order of smallness aggravates the smallness of this element. Hence, in form (1) can be taken $E = 0$. Then the potential (1) turns into a four-constant:

$$W = 0.5 \cdot \left( (A + B \cdot \alpha_1) \cdot \sigma_1^2 + (A + B \cdot \alpha_2) \cdot \sigma_2^2 + (A + B \cdot \alpha_3) \cdot \sigma_3^2 \right) + \left[ C + D \cdot (\alpha_1 + \alpha_2) \right] \cdot \sigma_1 \cdot \sigma_2 + \left[ C + D \cdot (\alpha_2 + \alpha_3) \right] \cdot \sigma_2 \cdot \sigma_3$$

and experiments on uniaxial tension and uniaxial compression are sufficient to calculate constants under the conditions (7). At the same time, the potential record (2) stores five constants and they are calculated by formulas:

$$\tilde{b}_1 = 0.75 \cdot \left\{ \frac{1}{(1-2 \cdot v^+)} / E^+ + (1-2 \cdot v^-) / E^- \right\}; \quad \tilde{b}_2 = 0.75 \cdot \left\{ \frac{1}{(1+v^+)} / E^+ + (1+v^-) / E^- \right\};$$
$$\tilde{b}_3 = 0.25 \cdot \sqrt{2 \cdot \left\{ \frac{1}{(1-4 \cdot v^+)} / E^+ - (1-4 \cdot v^-) / E^- \right\}}; \quad \tilde{b}_4 = 0.75 \cdot \sqrt{2 \cdot \left\{ \frac{1}{(1+4 \cdot v^+)} / E^+ - (1+4 \cdot v^-) / E^- \right\}}; \quad \tilde{b}_5 = 0.125 \cdot \sqrt{ \left\{ \frac{1}{(1+2 \cdot v^+)} / E^+ - (1+2 \cdot v^-) / E^- \right\}}.$$  

Option No. 3. The fifth constant can be hypothetically obtained if we assume that the forms of the law of volume change of quasilinear material with different resistance under hydrostatic compression and tension coincide with the forms arising from the classical generalized Hooke's law:

$$0^- / 3 = \sigma^- \cdot (1-2 \cdot v^-) / E^-; \quad 0^+ / 3 = \sigma^+ \cdot (1-2 \cdot v^+) / E^+.$$  

A comparison of the laws of volume change (4) and (13) in hydrostatic compression ($\xi = -1$, $\eta = 0$) and tension ($\xi = 1$, $\eta = 0$) gives the following results:

$$E = 0.25 \cdot \sqrt{3} \cdot \left\{ \frac{1}{(1-2 \cdot v^+)} / E^+ - (1-2 \cdot v^-) / E^- \right\} - 0.25 \cdot \left\{ \frac{1}{(1-4 \cdot v^+)} / E^+ - (1-4 \cdot v^-) / E^- \right\};$$
$$\tilde{b}_1 = 0.75 \cdot \left\{ \frac{1}{(1-2 \cdot v^+)} / E^+ + (1-2 \cdot v^-) / E^- \right\}; \quad \tilde{b}_2 = 0.75 \cdot \left\{ \frac{1}{(1+v^+)} / E^+ + (1+v^-) / E^- \right\};$$
$$\tilde{b}_3 = 0.75 \cdot \left\{ \frac{1}{(1-4 \cdot v^+)} / E^+ - (1-4 \cdot v^-) / E^- \right\};$$
$$\tilde{b}_4 = 0.375 \cdot \sqrt{2} \cdot \left\{ \frac{1}{(1-2 \cdot v^+)} / E^+ - (1-2 \cdot v^-) / E^- \right\} - \sqrt{2} \cdot \left\{ \frac{1}{(1+2 \cdot v^+)} / E^+ - (1+2 \cdot v^-) / E^- \right\}.$$  

The third option leads to the preservation in both forms of the deformation potential (1) and (2) of five constants with their unambiguous values arising only from two uniaxial experiments.

In article [34] N.M.Matchenko and I.N.Matchenko tried to formulate more general approaches and forms of construction of quasi-linear four-constant deformation potentials than proposed in [31, 32]. This attempt to build new forms of quasi-linear four constants tion deformation potentials [34] proved only by replacing the invariant signs of the potentials (1) and (2). What is the matter here, explain. The existing quasi-linear models of deformation of isotropic materials are based on the introduction of certain quantitative and qualitative parameters of their stress state. In [34] N.M.Matchenko and I.N.Matchenko write down three groups of such parameters:

a) quantitative parameters of the first group –

$$H_I = \sigma_1 + \sigma_2 + \sigma_3; \quad H_{II} = \sigma_1^2 + \sigma_2^2 + \sigma_3^2;$$

b) quantitative parameters of the second group –

$$I_I = \sigma_1 + \sigma_2 + \sigma_3; \quad I_{II} = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2;$$

c) quantitative parameters of the third group –

$$J_I = \sigma_1 + \sigma_2 + \sigma_3; \quad J_{II} = \sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1.$$  

The quantitative parameters of the stress state (15) – (17) introduced in this way are nothing but another interpretation of the values used in [31, 32], and namely, for the first group the introduced parameters [34] are equivalent to the following [31, 32]:
\[ H_I = 3\sigma = \delta_{ij}\sigma_{ij}; \quad H_{II} = S^2 = \sigma_k \cdot \sigma_k = 3 \cdot S_0^2; \]  
\[ (15^*) \]
for the second group these parameters [34] coincide with invariants [31, 32]:
\[ I_I = 3 \cdot \sigma = \delta_{ij}\cdot \sigma_{ij}; \quad I_{II} = 9 \cdot \tau^2; \]  
\[ (16^*) \]
for the third group the recorded parameters [34] are replaced by expressions [31, 32]:
\[ J_I = \delta_{ij}\cdot \sigma_{ij}; \quad J_{II} = 3 \cdot (S_0^2 - 1.5 \cdot \tau^2). \]  
\[ (17^*) \]

The qualitative characteristic of the stress state for all these groups in [34] is proposed to use one parameter presented in the form of:
\[ \chi = (\sigma_1 + \sigma_2 + \sigma_3)/\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2} = 3 \cdot \sigma / S = 3 \cdot \sigma / (\sqrt{3} \cdot S_0), \]  
\[ (18) \]
and this is nothing but a combination of invariants considered in [31, 32]:
\[ \chi = 3 \cdot \sigma / S = 3 \cdot \sigma / (\sqrt{3} \cdot S_0). \]  
\[ (18^*) \]

Thus, when using the considered H.M. Matchenko and I.N. Matchenko's parameters (15) – (18) allowed them to [34] record three forms of quasi-linear potential:
\[ W = (A_h + a_h \cdot \chi) \cdot H_I^2 + (B_h + b_h \cdot \chi) \cdot H_{II}; \]  
\[ (19) \]
\[ W = (A_i + a_i \cdot \chi) \cdot I_I^2 + (B_i + b_i \cdot \chi) \cdot I_{II}; \]  
\[ (20) \]
\[ W = (A_j + a_j \cdot \chi) \cdot J_I^2 + (B_j + b_j \cdot \chi) \cdot J_{II}. \]  
\[ (21) \]

It is easy to notice that all these forms of potential (19) – (21) taking into account the equations (15*) – (18*) are unambiguously transformed and completely coincide with the representation (2*). Therefore, the expressions (19) - (21) have no independent meaning, but represent only a different form of writing. At the same time, in [31, 32] it was found that the five-constant potential has a higher accuracy when compared with experimental deformation diagrams of most materials with different resistance under complex stress conditions.

In another work [35], A.V. Berezin argues that he obtained a more general quasilinear potential than formulated in [31, 32] and this potential is as follows:
\[ \Phi = \Phi_1 \cdot [\sigma_i \cdot \phi_1(u, \zeta)] + \Phi_2 \cdot [\sigma_0 \cdot \phi_2(u, \zeta)] + \Phi_3 \cdot [S_3^{1/3} \cdot \phi_3(u, \zeta)], \]  
\[ (22) \]
where \( \sigma_i \) - the stress intensity; \( \sigma_0 \) - hydrostatic stress; \( u = \sigma_0 / \sigma_i \) - function, taking into account the type of stress state (disadvantages of this qualitative parameter discussed in [32]); \( \phi_1(u, \zeta), \phi_2(u, \zeta), \phi_3(u, \zeta) \) – functions to be experimentally determined; \( S_3 \) – the third invariant of the stress deviator tensor.

At the same time, the author himself [35] without specifying the functions \( \phi_1(u, \zeta), \phi_2(u, \zeta), \phi_3(u, \zeta) \), argues that «the greatest difficulty is the definition of functions \( \phi_i(u, \zeta) \) (i = 1, 2, 3). However, this was done in [31] under some assumptions». These assumptions are presented here above (see expressions (7) – (14)).

3. Summary
From the given analysis of the presented potential dependences, it can be concluded that at present the most common quasi-linear constitutive relations for isotropic materials suitable for problem-free use in solving applied scientific and engineering tasks are two forms of deformation potential (1) and (2).

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