Constraint relation between steerability and concurrence for two-qubit states

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Entanglement and steering are used to describe quantum inseparabilities. Steerable states form a strict subset of entangled states. A natural question arises concerning how much territory steerability occupies entanglement for a general two-qubit entangled state. In this work, we investigate the constraint relation between steerability and concurrence by using two kinds of evolutionary states and randomly generated two-qubit states. By combining the theoretical and numerical proofs, we obtain the upper and lower boundaries of steerability. And the lower boundary can be used as a sufficient criterion for steering detection. Furthermore, we consider a special kind of mixed state transformed by performing an arbitrary unitary operation on Werner-like state, and propose a sufficient steering criterion described by concurrence and purity.

I. INTRODUCTION

In 1935, Schrödinger initially introduced the concept of steering as a generalization of Einstein-Podolsky-Rosen (EPR) paradox [1]. It is supposed that Alice and Bob share an entangled state in two distant area. Alice can steer the particle of Bob into different states by performing measurements on her own particle. Recently, it has been formalized that steering is a sufficient form of quantum inseparabilities [2]. And quantum inseparability is usually measured by entanglement.

In fact, steerable states form a strict subset of entangled states, and this means that not every entangled state is steerable [2]. This steerability is manifested explicitly by violating the different steering inequalities, and it plays an essential role in better understanding the subtle aspects of quantum mechanics. Historically, the violation of steering inequality is usually used as a criterion for whether two-qubit pure states are entangled. However, for a two-qubit mixed state, it is not applicable in this case. Wiseman et al. demonstrated that Werner state with weak entanglement does not violate any steering inequalities [2]. Afterwards, it is experimentally proved that EPR-steering is captured for mixed entangled states that are Bell local [3]. Later on, Costa et al. proposed a measure of steering that is based on the maximal violation of the line steering inequality in the two- and three-measurement scenarios [4]. Recently, Guo et al. [5] presented a steering criterion which is both necessary and sufficient for two-qubit states under arbitrary measurement sets. And a set of complementarity relations between steering or nonlocal advantage of quantum coherence inequalities can be derived and achieved by various criteria [6]. Das et al. [7] proposed a criterion to detect whether a given two-qubit state is steerable. To be specific, for a target state, the new state is constructed. The steering of target state can be detected via detecting entanglement of the new state [7][8].

Now that both entanglement and steering are used to describe quantum inseparabilities. As two vital quantum resources, we are more concerned about how much domain the values of steerability are limited in the values of entanglement. And what states do the boundary states of steerability located in entanglement region represents? In addition, we aim to explore the constrained relation between two quantum resources for arbitrary two-qubit states.

The remainder of this paper is organized as follows. In Sec. II, we review the related concepts of concurrence and steering. In Sec. III, we investigate the relation between concurrence and steerability by using the evolutionary states (Bell-like state under two kinds of decoherence channels) and randomly generated two-qubit states. Furthermore, it is obtained that the upper and lower boundaries of steerability can be expressed by concurrence and purity. In Sec. IV, we investigate the relation between concurrence and steering for a special kind of mixed states (the states are transformed by an arbitrary unitary operation on Werner-like states). In final, we end up our article with a brief conclusion.

II. PRELIMINARIES

Concurrence is usually used as a measure for entanglement of two-qubit states [9][10]. For a two-qubit pure state \(|\psi\rangle\), its concurrence is defined as [11]

$$C (|\psi\rangle) = \left| \langle \psi | \tilde{\psi} \rangle \right|,$$

(1)

where \(\tilde{\psi} = (\sigma_y \otimes \sigma_y) |\psi^*\rangle\). Here \(|\psi^*\rangle\) is the complex conjugate of the pure state \(|\psi\rangle\) and \(\sigma_y\) is the Pauli-y matrice. For a general two-qubit state \(\rho\), its concurrence is defined by the convex-root [12][13]

$$C (\rho) = \min \left\{ q_n C (|\varphi_n\rangle) \right\} \sum_n q_n C (|\varphi_n\rangle).$$

(2)

The minimization is taken over all possible decompositions \(\rho\) into pure states. An analytic solution of concurrence can be calculated [11]

$$C (\rho) = \max \left\{ 0, 2\sqrt{\lambda_1 (\rho)} - \sum_{n=1}^{4} \sqrt{\lambda_n (\rho)} \right\},$$

(3)

where \(\lambda_n (\rho)\) are the eigenvalues, in decreasing order, of the non-Hermitian matrix \(\rho \hat{\rho}\). Here, the matrix \(\hat{\rho}\) which is the
spin-flipped density matrix of the state $\rho$, has the following form

$$\hat{\rho} = (\sigma_y \otimes \sigma_y)\rho^* (\sigma_y \otimes \sigma_y),$$  \hspace{1cm} (4)

where the matrix $\rho^*$ is the complex conjugate of the state $\rho$.

Besides, steering inequality violation in quantum mechanics clearly illuminates that quantum correlations are quite different from classical correlations. In the case of two-qubit states, Cavalcanti-Jones-Wiseman-Reid (CJWR) inequality [4] is a well-known steering inequality. And it has the important property that an arbitrary two-qubit pure state may violate CJWR inequality if this state is entangled. The violation of CJWR inequality diagnoses whether a two-qubit state is steerable when Alice and Bob are both allowed to measure $N$ observables in their sites. More recently, Costa et al. gave the analytical results of $N = 2, 3$ for any two-qubit states [4]. Setting $N = 3$, the inequality deduced by a finite sum of bilinear expectation values is

$$F_{CJWR} (\rho, r) = \frac{1}{\sqrt{3}} \left| \sum_{i=1}^{3} \langle A_i \otimes B_i \rangle \right| \leq 1, \hspace{1cm} (5)$$

where $A_i = t_i^A \cdot \sigma_i, B_i = t_i^B \cdot \sigma_i$ are used to describe the projection measurements on sides A and B, respectively. Here, $\sigma_i = (\sigma_x, \sigma_y, \sigma_z)$ is a vector made up of Pauli matrices, $r = \{t_1^A, t_2^A, t_3^A, t_1^B, t_2^B, t_3^B\}$ is the set of measurement directions, and $\langle A_i \otimes B_i \rangle = \text{Tr} (\rho A_i \otimes B_i)$ is the expected value of the projection operator $A_i \otimes B_i$ in the state $\rho$. Considering maximally values of $F_{CJWR} (\rho, r)$, Eq. (5) can be rewritten as

$$F (\rho) = \sqrt{t_1^2 (\rho) + t_2^2 (\rho) + t_3^2 (\rho)} \leq 1, \hspace{1cm} (6)$$

where $t_i (\rho) \ (i \in \{1, 2, 3\})$ are the eigenvalues, in decreasing order, of the matrix $t (\rho) = \sqrt{T^T (\rho) T (\rho)}$ with a correlation matrix $T (\rho)$ and the transpose matrix $T^T (\rho)$. The real matrix $T (\rho)$ is formed by the coefficients $\text{Tr} (\rho \sigma_m \otimes \sigma_n)$ ($m, n \in \{x, y, z\}$). Here the quantity $F (\rho)$ donates a three-measurement bilinear maximally expectation values. In order to render steerability $0 \leq S (\rho) \leq 1$, we consider steerability has the following form

$$S (\rho) = \frac{1}{2} \max \{0, F^2 (\rho) - 1\}. \hspace{1cm} (7)$$

**III. CONSTRAINT RELATION BETWEEN STEERABILITY AND CONCURRENCE**

Following these Refs. [13, 14], we can obtain that any pure states $|\varphi\rangle$ satisfy two kinds of complementary equations, i.e.,

$$1 + 2D^2 (|\varphi\rangle) + F^2 (|\varphi\rangle) = 1, \hspace{1cm} (8)$$

$$C^2 (|\varphi\rangle) + D^2 (|\varphi\rangle) = 1, \hspace{1cm} (9)$$

where $D (|\varphi\rangle)$ is first-order coherence of the pure states $|\varphi\rangle$. Combining Eqs. (8) and (9), the quantity $F (|\psi\rangle)$ can be given by

$$F (|\varphi\rangle) = \sqrt{1 + 2C^2 (|\varphi\rangle)}. \hspace{1cm} (10)$$

Therefore, for the pure state $|\varphi\rangle$, the relation between concurrence $C (|\varphi\rangle)$ and steerability $S (|\varphi\rangle)$ is

$$S (|\varphi\rangle) = C (|\varphi\rangle). \hspace{1cm} (11)$$

It shows that steerability is equivalent to concurrence in a pure state system in Eq. (11). However, for a general two-qubit mixed state $\rho$, the relation between concurrence $C (\rho)$ and steerability $S (\rho)$ is intricate. It is well known that steerable states form a strict subset of entangled states. In other words, steerable states must be entangled states, but entangled states are not necessarily steerable states. For more clearly investigating the relation between steering and concurrence, we consider the cases for the evolutionary states of Bell-like state going through the amplitude damping (AD) and phase damping (PD) channels, respectively. And we obtain that steerability can be expressed via concurrence and purity. For any two-qubit states, we give out a constraint inequality relation between two quantum resources and verify it by using lots of randomly generated two-qubit states.

**A. Evolutionary state corresponding to the AD channel**

We consider the output state $\rho_{BAD}$ which is formed by the particle (A or B) of Bell-like state $|\varphi_B\rangle$ going through the AD channel. And the state $\rho_{BAD}$ has the following concise form

$$\rho_{BAD} = \sum_{i=0}^{1} K_i |\varphi_B\rangle \langle \varphi_B | K_i^\dagger, \hspace{1cm} (12)$$

where $K_0 = |0\rangle \langle 0| + \sqrt{1-\eta} |1\rangle \langle 1|$ and $K_1 = \sqrt{\eta} |0\rangle \langle 1|$ are the Kraus operators of AD channel. From Eq. (4), one obtain that the non-Hermitian matrix $\rho_{BAD} \hat{\rho}_{BAD}$ is a matrix of rank 1, i.e.,

$$R (\rho_{BAD} \hat{\rho}_{BAD}) = 1. \hspace{1cm} (13)$$

Obviously, concurrence $C (\rho_{BAD})$ and first-order coherence $D (\rho_{BAD})$ satisfy the following relation [13]

$$C^2 (\rho_{BAD}) + D^2 (\rho_{BAD}) = \text{Tr} \left( \rho_{BAD}^2 \right). \hspace{1cm} (14)$$

By some calculations, concurrence of the state $\rho_{BAD}$ can be obtained

$$C (\rho_{BAD}) = \sqrt{\lambda_1 (\rho_{BAD})} = \sqrt{1-\eta} C (|\varphi_B\rangle). \hspace{1cm} (15)$$

Following the Ref. [14], the state $\rho_{BAD}$ satisfies the complementary equation, i.e.,

$$1 + 2D^2 (\rho_{BAD}) + F^2 (\rho_{BAD}) = \text{Tr} \left( \rho_{BAD}^2 \right). \hspace{1cm} (16)$$
Combining Eqs. (14) and (16), the quantity $F (\rho_{BAD})$ for state $\rho_{BAD}$ can be expressed as

$$F (\rho_{BAD}) = \sqrt{2C^2 (\rho_{BAD}) + 2Tr (\rho_{BAD}^2)} - 1.$$  \hfill (17)

Therefore, steerability $S (\rho_{BAD})$ can be expressed in terms of concurrence $C (\rho_{BAD})$ and purity $Tr (\rho_{BAD}^2)$, i.e.,

$$S (\rho_{BAD}) = \sqrt{\max \{0, Q^2 (\rho_{BAD}) − 1\}},$$  \hfill (18)

where $Q (\rho_{BAD}) = \sqrt{C^2 (\rho_{BAD}) + Tr (\rho_{BAD}^2)}$. It shows that when concurrence and purity meet the condition $C^2 (\rho_{BAD}) + Tr (\rho_{BAD}^2) > 1$, steerability of the state $\rho_{BAD}$ can be detected.

B. Evolutionary state corresponding to the PD channel

We consider the output state $\rho_{BPD}$ which is formed by the particle (A or B) of Bell-like state $|\varphi_B\rangle$ going through the PD channel. And the state $\rho_{BPD}$ has the following concise form

$$\rho_{BPD} = \sum_{i=0}^{1} K_i |\varphi_B\rangle \langle \varphi_B| K_i^\dagger,$$  \hfill (19)

where $K_0 = |0\rangle \langle 0| + \sqrt{1 - \eta} |1\rangle \langle 1|$ and $K_1 = \sqrt{\eta} |1\rangle \langle 1|$ are the Kraus operators of PD channel. Based on Eq. (4), we can obtain that the non-Hermitian matrix $\rho_{BPD} \rho_{BPD}^\dagger$ is a matrix of rank 2, i.e.,

$$R (\rho_{BPD} \rho_{BPD}^\dagger) = 2.$$  \hfill (20)

It reveals that concurrence $C (\rho_{BPD})$ and first-order coherence $D (\rho_{BPD})$ satisfy the following relation \cite{13}

$$C^2 (\rho_{BPD}) + D^2 (\rho_{BPD}) \leq Tr (\rho_{BPD}^2).$$  \hfill (21)

For calculating concurrence of the state $\rho_{BPD}$, we adopt Eq. (3) to obtain its concurrence result, i.e.,

$$C (\rho_{BPD}) = \left| \sqrt{\lambda_1 (\rho_{BPD})} − \sqrt{\lambda_2 (\rho_{BPD})} \right|$$

$$= \sqrt{1 - \eta} C (|\varphi_B\rangle).$$  \hfill (22)

And the purity of state $\rho_{BPD}$ can be given by

$$Tr (\rho_{BPD}^2) = 1 - \frac{\eta}{2} C^2 (|\varphi_B\rangle)$$

$$= 1 - \frac{C^2 (|\varphi_B\rangle) - C^2 (\rho_{BPD})}{2}.$$  \hfill (23)

Besides, the correlation matrix $T(\rho_{BPD})$ can be written as

$$T(\rho_{BPD}) = \begin{pmatrix}
\sqrt{1 - \eta} C (|\varphi_B\rangle) & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}$$

$$= \begin{pmatrix}
C (\rho_{BPD}) & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}.$$  \hfill (24)

One obtain the quantity $F (\rho_{BPD})$ of state $\rho_{BPD}$ can be expressed as

$$F (\rho_{BPD}) = \sqrt{1 + 2C^2 (\rho_{BPD})}.$$  \hfill (25)

Therefore, the relation between steerability $S (\rho_{BPD})$ and concurrence $C (\rho_{BPD})$ has the following form

$$S (\rho_{BPD}) = C (\rho_{BPD}).$$  \hfill (26)

It is apparent that steerability is equivalent to concurrence for the states which is formed by particle (A or B) of Bell-like state going through the PD channel. According to Eq. (23), steerability of the state $\rho_{BPD}$ can also be expressed in terms of its purity, i.e., $S (\rho_{BPD}) = \sqrt{C^2 (|\varphi_B\rangle) + 2Tr (\rho_{BPD}^2) - 2}$.

C. Randomly generated two-qubit states

In Sec. III A and B, we discuss two types of states $\rho_{BAD}$ and $\rho_{BPD}$, respectively. And the states $\rho_{BAD}$ and $\rho_{BPD}$ show the special relation between concurrence and steerability in Eqs. (18) and (26), respectively. For any two-qubit state, what relation could we obtain about concurrence and steerability?

Theorem 1. For a general two-qubit state $\rho$, as long as the sum of concurrence square and purity is greater than 1, then the quantum state $\rho$ exists steerability. And it can be described by the following inequality

$$S (\rho) \geq \max \{0, C^2 (\rho) + Tr (\rho^2) - 1\}.$$  \hfill (27)

Proof of theorem 1. The Ref. \cite{14} gives a complementarity equation of first-order coherence and correlation for a general state $\rho$.

$$1 + D^2 (\rho_A) + D^2 (\rho_B) + F^2 (\rho) = \frac{4}{Tr (\rho^2)}.$$  \hfill (28)

And the Ref. \cite{13} gives a complementarity relation of first-order coherence and concurrence for a general state $\rho$, i.e.,

$$\frac{D^2 (\rho_A) + D^2 (\rho_B) + C^2 (\rho)}{2} \leq \frac{1}{Tr (\rho^2)}.$$  \hfill (29)

Combining Eqs. (28) and (29), we can require

$$F^2 (\rho) \geq 2 \left[ Tr (\rho^2) + C^2 (\rho) \right] - 1.$$  \hfill (30)

Therefore, based on Eqs. (7) and (30), one derive Eq. (27). By using lots of randomly generated two-qubit states, we visually display the lower boundary of steerability about certain concurrence and purity in Fig.1.

Theorem 2. For a general two-qubit state $\rho$, the steerability is bounded by a quantity related to concurrence and purity. And theorem 2 can be described by the following inequality

$$S (\rho) \leq \min \left\{ C (\rho), \sqrt{\max \{0, 2Tr (\rho^2) - 1\}} \right\}.$$  \hfill (31)

Proof of theorem 2. According to Eq. (28), we obtain the relation between the quantity $F (\rho)$ and purity $Tr (\rho^2)$, i.e.,

$$F^2 (\rho) \leq 4Tr (\rho^2) - 1.$$  \hfill (32)
IV. STEERABILITY AND CONCURRENCE FOR A KIND OF MIXED STATE

At the front, we have proposed the inequality relation between concurrence and steerability for a general two-qubit state. And we have also investigated the upper and lower upper boundary states. Next, we will study concurrence and steerability for the mixed state $\rho_{WU}$ which is transformed by performing an arbitrary unitary operation $U$ on Werner-like state $\rho_W = p|\varphi_B\rangle\langle\varphi_B| + (1-p)\frac{I_{16}}{4}$, where $|\varphi_B\rangle$ is a Bell-like state. And the purity of states $\rho_{WU}$ can be given by $\text{Tr} (\rho_{WU}^2) = \text{Tr} (\rho_W^2) = \frac{1 + 3p^2}{4}$. And the mixed state $\rho_{WU}$ has the following form

$$\rho_{WU} = U\rho_W U^\dagger = p|\varphi\rangle\langle\varphi| + (1-p)\frac{I_{16}}{4}, \quad (33)$$

where $|\varphi\rangle = U|\varphi_B\rangle$ is a pure state. For the state $\rho_{WU}$, we obtain two properties about concurrence $C(\rho_{WU})$ and steerability $S(\rho_{WU})$.

**Property 1.** Steerability $S(\rho_{WU})$ of the state $\rho_{WU}$ is related to steerability $S(|\varphi\rangle)$ of the pure state $|\varphi\rangle$. And the correlation can be expressed as

$$S(\rho_{WU}) = \frac{1}{2} \max\{0, p^2 [1 + 2S^2 (|\varphi\rangle)] - 1\}. \quad (34)$$

**Proof of property 1.** The correlation function $T_{ij} (\rho_{WU})$ corresponding to the state $\rho_{WU}$ can be reduced as

$$T_{ij} (\rho_{WU}) = \text{Tr} (\rho_{WU} \sigma_i \otimes \sigma_j)$$

$$= p\text{Tr} (|\varphi\rangle\langle\varphi| \sigma_i \otimes \sigma_j) + \frac{1-p}{4}\text{Tr} (\sigma_i \otimes \sigma_j)$$

$$= pT_{ij} (|\varphi\rangle) + \frac{1-p}{4}\text{Tr} (\sigma_j) \text{Tr} (\sigma_j)$$

$$= pT_{ij} (|\varphi\rangle). \quad (35)$$

Thus, the value $F (\rho_{WU})$ of state $\rho_{WU}$ is closely related to the value $F (|\varphi\rangle)$ of the pure state $|\varphi\rangle$

$$F (\rho_{WU}) = pF (|\varphi\rangle). \quad (36)$$

Combining Eqs. (7), (10) and (11), we can obtain Eq. (34).

**Property 2.** Concurrence $C(\rho_{WU})$ of the state $\rho_{WU}$ is related to concurrence $C(|\varphi\rangle)$ of the pure state $|\varphi\rangle$. And this relation can be expressed as

$$C(\rho_{WU}) = \max\{0, pC(|\varphi\rangle) - \frac{1-p}{2}\}. \quad (37)$$

**Proof of property 2.** The non-Hermitian matrix $\rho_{WU} \hat{\rho}_{WU}$ can be given by

$$\rho_{WU} \hat{\rho}_{WU} = H + \frac{(1-p)^2}{16} I \otimes I, \quad (38)$$

where $H = p^2 \langle \varphi | \tilde{\varphi} \rangle |\varphi\rangle \langle \varphi| + \frac{p(1-p)}{4} (|\varphi\rangle \langle \varphi| + |\tilde{\varphi}\rangle \langle \tilde{\varphi}|)$. Here, since its eigenvalue-equation are too complicated, we do not directly calculate the eigenvalues of non-Hermitian matrix $\rho_{WU} \hat{\rho}_{WU}$. According to some properties of matrix-rank,
the states \( \rho_{\text{WU}} \) can be expressed as

\[
C (\rho_{\text{WU}}) = \max \left\{ 0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4} \right\} \\
= \max \left\{ 0, \sqrt{\lambda_1 + \lambda_2 - 2\sqrt{\lambda_1\lambda_2}} - \frac{1-p}{2} \right\} \\
= \max \left\{ 0, pC (|\varphi\rangle) - \frac{1-p}{2} \right\} .
\] (42)

Obviously, when the pure state \(|\varphi\rangle\) is a Bell state, both concurrence and steerability of the states \(\rho_{\text{WU}}\) reach maximum. Eq. (34) shows that CJWR inequality can be violated at the case of \(p > 1 \frac{1}{2\sqrt{\text{Tr}(\rho_{\text{WU}}^2)}}\) for the state \(\rho_{\text{WU}}\). Combining Eqs. (11), (34) and (37), steerability can be obtained by concurrence and purity, i.e.,

\[
S (\rho_{\text{WU}}) = \sqrt{\max \left\{ 0, x (\rho_{\text{WU}}) + Q^2 (\rho_{\text{WU}}) - 1 \right\}},
\] (43)

where \(x (\rho_{\text{WU}}) = \frac{1+2C(\rho_{\text{WU}})}{2} \left( 1 - \sqrt{\frac{\text{Tr}(\rho_{\text{WU}}^2)}{3}} \right) \) and \(Q (\rho_{\text{WU}}) = \sqrt{C^2 (\rho_{\text{WU}}) + \text{Tr}(\rho_{\text{WU}}^2)}\). Combining Theorem 2, we can obtain a sufficient criterion for detection steering by using concurrence and purity for the state \(\rho_{\text{WU}}\) (as shown in Fig. 3).

V. CONCLUSION

In this paper, we have investigated the constraint relation between steerability and concurrence for two kinds of evolutionary states and lots of randomly generated two-qubit states. The result shows that the upper and lower boundaries of steerability for any two-qubit state can be exactly expressed based on certain concurrence and purity. Specifically, the lower boundary can be used as a sufficient criterion for steering detection. In other words, a general two-qubit state must be steerable if the sum of purity and concurrence’s square is greater than one. And the upper boundary reveals that steerable states form a strict subset of entangled states. Furthermore, we consider a special kind of mixed state transformed by performing an arbitrary unitary operation on Werner-like state. It can be obtained that the special mixed states concurrence and steerability are related to ones of a pure state transformed by the unitary operation performed on Bell-like state, respectively. And we present and demonstrate a sufficient criterion, which provides an effective theoretic basis to seek steerable states from entangled states.

ACKNOWLEDGEMENTS

This work was supported by the National Science Foundation of China under Grant Nos. 11575001 and 61601002, Anhui Provincial Natural Science Foundation (Grant No. 1508085QF139) and Natural Science Foundation of Education Department of Anhui Province (Grant No. KJ2016SD49).
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