Optimal Distribution Model of Shared Parking Space in Large Shopping Malls at Night

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Abstract. With the increase of car ownership year by year, the difficult phenomenon of parking space has attracted more and more attention from the society. Especially in the old residential area, the problem of night parking is becoming more and more serious due to the unplanned parking space in the construction process. Therefore, the efficient parking management strategy at night is becoming the focus of academic research. Aiming at this problem, this paper first analyzes the shared berth allocation problem, and takes the profit maximization of shared parking lots as the goal, and proposes an integer linear programming model for shared parking berths at large shopping malls at night. Finally, according to the characteristics of spare parking berths at the commercial area, we design an experiment based on the profit and parking berth utilization rate of the platform operators. Then, we compare the experimental results with the first come first served distribution plan to verify the effectiveness of the model. The results show that this model not only brings huge benefits to the parking management platform operators, but also makes full use of the spare parking places in the large shopping malls, improves the utilization rate of the parking lots at the commercial area, and makes the parking resources allocated reasonably.

1. Introduction
Shared parking is a new technology to solve the parking problem of urban residents based on the idea of "Internet plus". The scientific and rational shared parking organization scheme can improve the utilization rate of parking resources and obtain better economic and social benefits for the regions with large fluctuation of parking demand at different times. Because of the unplanned parking space in the old community, the problem of night parking is getting worse and worse. The old community is usually located in the city center and is usually adjacent to a large number of large commercial areas. The large shopping malls show obvious unbalance of parking demand day and night, and the spare parking berths in the night becomes more. If not utilized effectively, it will cause a great waste of parking resources. The residents near the mall are the potential users of these spare parking spaces. Both the parking supply and the demand for parking are shared by each other through multi-index games such as parking time, walking distance, parking price and so on. Therefore, it is necessary for parking berth providers to establish accurate berth allocation and parking organization model in order to realize the utilization rate of spare parking berths.

In the past, scholars have carried out a number of researches on parking demand prediction, berthing resource allocation, berth management and so on. Guo [1] adopted the simulation method to...
construct the parking repurchase model, which is in order to earn the maximum benefit of the berthing users, and obtain the optimal use strategy of parking berth under the constraint of parking time. To solve local congestion problem caused by group parking guidance under the condition of limited resources, Manzhen Duan [2] put forward the individual shared parking guidance service strategy, and establish shared parking model for residential area. Yuanyuan Wu [3] analyzed the parking choice behavior of users in the shared situation, and established the model of shared parking choice of hotel with Logit Non-set mathematical model. Kai Chen [4] established Binary Logit model under the survey of commuter parking users, realized the adjustment of the total commuting amount through the rate change and policy support, and combined with examples to explain share time windows adjustment process. Zhiguang Liu [5] divided the users of the parking place into fixed users and of elastic users, change and policy support, and combined with examples to explain share time windows adjustment the temporary parking places. Ming Su [8] aimed at the maximum utility demand and supply benefits, temporary parking space characteristic, and proposed shared parking configuration method based on analysis based on the architectural forms of parking peak time characteristic, fixed parking space and in both residential and commercial areas [9-10]. But most of existing research established the parking parking planning of land, space, resource conservation.

Establish double CES planning model from the substitution elasticity and finally realized the shared reference value and practical guiding significance to make full use of parking garages and alleviating balanced parking demand time, and verify the effect analysis. The results of this paper have theoretical programming model for shared parking places at night according to the characteristics of the not-

the strategy of parking berth optimization in a large business district at night. From the perspective of management scheme from the perspective of berth provider or operator. Therefore, this paper studies as the center. However, they seldom established the parking berth allocation and organizational berth selection model from the angle of demanders, and established the objective function to the user as the center. However, they seldom established the parking berth allocation and organizational management scheme from the perspective of berth provider or operator. Therefore, this paper studies the strategy of parking berth optimization in a large business district at night. From the perspective of the optimization of the integrated management of parking berth, we construct an integer linear programming model for shared parking places at night according to the characteristics of the not-balanced parking demand time, and verify the effect analysis. The results of this paper have theoretical reference value and practical guiding significance to make full use of parking garages and alleviating the night parking demand.

2. Analysis of Shared Parking Space

As general business activity is very low at night, malls can provide a certain number of parking places open to the outside world. The operators can collect the shared information windows for all berths and send to management platform. Later, according to the shared time windows, the operators can define unit hour parking price. At the same time, they collect the parking demand at night and assign it to the specific parking space by the linear programming principle of binary integer.

Shared parking time at night can be divided into several discrete intervals, each 30-minute interval. Assuming that K as the sum of all of the intervals, \( k = 1 \) for the first-time interval (e.g. 20: 00-20:30) and \( k = K \) for the last time interval (e.g.7:30-8:00); N as the number of shared parking berths for shopping mall. When parking berths \( n \) are available at intervals \( k \), we define \( S_{nk} \) as 1, otherwise 0. So, the parking supply matrix is \( S_{N \times K} = [S_{nk}] \), \( n = 1,2,\ldots,N; k = 1,2,\ldots,K \); \( t_m, t_{m+1} \in [1,K] \), \( m = 1,2,\ldots,M \). \( M \) is the total number of parking demand, \( t_m \) and \( t_{m+1} \) are the start and end time of the parking demand \( m \) respectively. Therefore, the duration of the parking demand \( m \) is \( t_{m+1} - t_m + 1 \). If the parking demand period \( t \) is included in time interval \( K \), then the parking demand is defined as \( d_{nk} = 1 \), whereas \( d_{nk} = 0 \). The corresponding mathematical expression is:

\[
d_{nk} = \begin{cases} 1, & \text{if } (t_m, t_{m+1} \neq \phi) \\ 0, & \text{if } (t_m, t_{m+1} = \phi) \\ \end{cases}
\]

Therefore, the parking demand matrix is \( D = [d_{nk}], m = 1,2,\ldots,M; k = 1,2,\ldots,K \).

2.1. Parking Berth Allocation
Due to the shared parking time windows of shopping malls at night are different from that of public parking space, the platform operators rent out the spare parking time only. The parking demand using the corresponding shared parking berth must satisfy the time window constraints of the rental period, \( x_{mn} \) represents the relationship between the shared parking berth and the parking demand time period. If the parking demand period \( \left[ t_{ma}, t_{mb} \right] \) is matched with the shared parking time window \( t_n \), then \( x_{mn} = 1 \), whereas \( x_{mn} = 0 \), the corresponding mathematical expression is:

\[
x_{mn} = \begin{cases} 
1, & \left( t_{ma}, t_{mb} \right) \in t_n \\
0, & \left( t_{ma}, t_{mb} \right) \notin t_n 
\end{cases}
\] (2)

Therefore, the parking demand decision matrix is

\[
X = \begin{pmatrix} x_{mn} \end{pmatrix}, \quad n = 1, 2, \ldots, N; \quad m = 1, 2, \ldots, M.
\]

According to the parking demand matrix \( D = [d_{mk}] \) and the demand decision matrix \( X = [x_{mn}] \), the parking occupancy matrix is obtained as follows:

\[
O_{N \times K} = [o_{nk}] = X_{N \times K} \times D_{N \times K}.
\] (3)

Obviously, \( o_{nk} = 1 \) indicates that the parking berth \( n \) is occupied in time interval \( k \), whereas \( o_{nk} = 0 \) indicates that the parking space is empty. The corresponding mathematical expression is:

\[
o_{nk} = \begin{cases} 
1, & d_{mk} \times x_{mn} = 1 \\
0, & d_{mk} \times x_{mn} = 0 
\end{cases}
\] (4)

2.2. The parking berth allocation of the same shared time window at night

Under special circumstances, all the parking spaces in a large mall at night can be shared at the same period \( \left( t_{mc}, t_{md} \right) \). In this case, only the acceptance of the user's parking request is required. If the request is accepted, then \( x_{mn} = 1 \), otherwise \( x_{mn} = 0 \). The corresponding mathematical expression is:

\[
x_{mn} = \begin{cases} 
1, & r \in \left( t_{mc}, t_{md} \right) \\
0, & r \notin \left( t_{mc}, t_{md} \right)
\end{cases}
\] (5)

3. Shared parking berth allocation model

3.1. The berth allocation model for different shared time windows

Assuming that \( g_s \) and \( g_b \) represent the price of parking space at night in a large shopping mall and the price of parking space for the operator respectively, the total revenue of the parking management platform is \( G = G_s - G_b - G_j \).

Among them,

\[
G_s = g_s \cdot \sum_{n=1}^{N} \sum_{k=1}^{K} o_{nk}
\] (6)

\[
G_b = g_b \cdot \sum_{n=1}^{N_b} K
\] (7)

And \( G_j \) represents the penalty for the platform operator's refusal to parking request.

Therefore, the shared berth allocation problem of large shopping malls at night can be expressed by integer linear programming:

\[
\max G = \max \{ G_s - G_b - G_j \}
\] (8)

\[
= g_s \cdot \sum_{n=1}^{N} \sum_{k=1}^{K} o_{nk} - g_b \cdot \sum_{n=1}^{N_b} K - \alpha \cdot \sum_{m=1}^{M} \sum_{n=1}^{N} x_{mn}
\]

Among them,

\[
o_{nk} \leq s_{nk}, \quad n = 1, 2, \ldots, N; \quad k = 1, 2, \ldots, K
\] (9)

Equation (6) represents the revenue generated by shared parking berths; Equation (7) represents the total expenditure of the management platforms (or operators) to purchase parking berths; Equation (8)
represents the net income of the management platforms, namely the profit; Equation (9) indicates that the shared parking berth does not exceed the spare parking berth supply of the mall; Among them, $\sum_{m=1}^{M}\sum_{n=1}^{N}x_{mn}$ represents all unaccepted parking demands and $\mu$ is a penalty factor for refusing a parking request.

### 3.2. The berth allocation model for the same shared time window

The allocation of the same shared time window is compared to the berth allocation at different time windows. It does not need to match the accepted parking request to the parking berth of the corresponding time window. The model is as follows:

$$\text{max } G = \max(G_s - G_b - G_j)$$  \hspace{1cm} (10)

$$= g_s \cdot \sum_{n=1}^{N} \sum_{k=1}^{K} x_{mk}d_k - g_b \cdot N_b \cdot K \cdot \alpha \cdot (M - \sum_{m=1}^{M} x_m)$$  \hspace{1cm} (11)

$$\sum_{m=1}^{M} x_{mk}d_k \leq N, k=1,2,\ldots K$$

The objective function value (10) depends on the total parking supply and the final acceptance of the parking request; Equation (11) indicates that the accepted parking demand cannot exceed the parking supply at each interval.

### 4. Numerical analysis

This section takes a business center as an example to analyze the distribution results of shared berth allocation. Supposing that the time interval of the shared berth is 0.5h, and the time period starts from 7 p.m. to 7 a.m. the next day, namely $K=24$; At the same time, assuming that the total number of the parking berths operators can buy is $N=80$, parking fee is $g_s = 1.5$ (rmb/h), each berth purchase cost is $g_b = 0.5$ (rmb/h), and impact factor is $\alpha=2$. If the parking time is more than 6 h, the parking fee and purchase cost no longer increase. In addition, assuming that during the entire modeling period, the user's arrival rules obey the poisson distribution, and the parking time follows the negative exponential distribution. The average parking time is $T=11$ (h), and the number of night parking requirements varies from 0 to 500.

In a fixed situation, figure 1 shows the relationship between profit and demand. With an increase in the number of parking request, profit $G$ has linear growth from negative (parking incomes are below the purchasing cost), the slope is 17 (approximately equal to the average parking time $T = 11$ times $g_s = 1.5$), and then grows to achieve maximum in a non-linear way. When the number of parking requests reaches to 61, the operator is in a break-even state; When the number of parking requests is up to 105, the distribution model is transformed from linear to non-linear. At this time, some parking requests are accepted selectively, while others are rejected. If the penalty item is not considered, the optimal number of requests to maximize profits $M$ is 265. When $M > 265$, the profit remains at the maximum, which means that subsequent parking requests are rejected (see figure 1).

If the penalty item is considered, the number of parking requests for maximum profit $M$ is equal to 198; Then with the increase of $M$, non-linear reduces, and all additional parking request was refused beyond a certain number of parking demands. Therefore, because of the negative impact of parking request was refused, profit has a linear decline with the increase of $M$.

With the increase of the number of parking requests, the number of times that the operation platform refuses to the parking requests is increasing, and the corresponding penalty is more and more, which is basically linear growth.

By analyzing the utilization rate of parking, it is obvious that the optimal allocation model is based on all the parking demand information, and finally makes a parking space arrangement. When parking request reaches 211, utilization rate of the parking place (utilization rate of the parking time) reaches 1.0 (figure 4).
In order to make the final result comparison, this paper chooses a solution that is based on shared parking in a parking lot which can be rented at different time periods, "first come first served" \cite{1}. "First come first served", namely, the driver makes a demand for parking in the parking management platform, then the platform will immediately receive parking request and assign processing to the request, until each parking demand is processed or there is no spare parking area available for parking needs.

![Figure 1. Relationship between profit and parking request in a fixed situation(\(\alpha=0\))](image)

In order to evaluate the validity of the model proposed in this paper, the model is compared with the profit and parking berth utilization ratio of the "first come, first served" distribution scheme. In the "first come, first served" allocation scheme, when there is a request, the parking platform will immediately accept and assign the corresponding shared parking berth, until 80 available shared parking berths are assigned and no spare berth is left. The results showed that only 96 parking demands are accepted and processed, that is, the subsequent parking demand is rejected. When the penalty item is not considered, the maximum profit will be reached when the parking request is 96. If the penalty item is considered, the profit will drop when the parking request is 96, as shown in figure 2. The parking berth utilization rate is also the maximum when the parking request is 96, but the maximum value is only 0.41, as shown in figure 3.

![Figure 2. The relationship between parking utilization rate and parking request](image)

Through the comparison with the results of “first come, first served” allocation, this paper builds a shared parking allocation model which can make the parking platform operators more profitable. At the same time, parking berth utilization has been greatly improved.

5. Conclusion
Parking berths in large shopping malls have the basic conditions of night sharing, scientific and effective parking organization and berth allocation are the prerequisites for success operation. From the perspective of parking management operating platform, this paper takes full advantage of the characteristics of spare parking berths at night, and then parking berth allocation optimization model is
established. Compared with “first come, first served” allocation, it fully proves that the model proposed by this paper can make the parking platform operators obtain more profits and realize the reasonable allocation of parking space to the users, so that the parking resources can be properly configured. It not only solves the parking problem of the residents near some shopping malls, but also provides some economic benefits to the operators of shopping malls and management platforms. The unbalance between supply and demand of parking resources is the key to the existence of parking problems, but there is also a multi-objective game between both suppliers and operators. Therefore, the walking distance, parking price and the fluctuation of berth quantity should be discussed in detail in the follow-up study.

Figure 3. The relationship between profit and stop request of “first come, first served” allocation

Figure 4. The relationship between parking utilization and parking request of “first come, first served” allocation

Acknowledgement
The research is jointly supported by the Ministry of Housing and Urban-Rural Development (2017-R2-032, 2017-K2-009), the National Natural Science Foundation of China (51508122), and Guangxi Key Research and Development Program (GuiKe AB16380280).

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