Implementation Euler implicit scheme of unsteady magnetohydrodynamic micropolar fluid pass a sphere affected by mixed convection

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Abstract. This paper considers the effect of mixed convection and magnetic field sphere in the unsteady magnetohydrodynamic micropolar fluid. For solving this problem, we develop dimensional governing equations from mass conservation law, energy conservation law, and Second Newton Law for momentum and angular momentum. We further convert them into non dimensional equations by using non dimensional variables. These non dimensional equations are further transformed into similarity equations using stream functions. Similarity equations are further solved numerically by implementing numerical method with euler implicit scheme. The result shows that the velocity and temperature will increase when parameter magnetic and mixed convection decrease. The higher magnetic parameter leads to higher microrotation of micropolar fluid particle. Whereas microrotation profile decrease with increasing mixed convection parameter.

1. Introduction
Fluid is a substance with the following characteristic: the condition changes continuously when we apply a shear stress [1]. According to compressibility, fluid consist of compressible and incompressible. Liquid is incompressible fluid and gas is kind of compressible fluid. Phase liquid fluid consist of Newtonian and non Newtonian fluid. One kind of Non Newtonian fluid is Micropolar Fluid. Theory of micropolar fluids introduced by Eringen (1964). Micropolar fluid is a non-Newtonian fluid with microstructure that consists of rigid particles randomly oriented to viscous media that have microrotation [2]. The example of micropolar fluids are animal blood, liquid crystal, and polymeric fluids.
In fluid mechanics, there are some studies about magnetohydrodynamic. Magnetohydrodynamic is a study about the flow of electrically conducting fluids influenced by magnetic field. Several studies about the unsteady flow magnetohydrodynamic micropolar fluid have reported velocity, microrotation, and skin friction on porous sphere [3], velocity and microrotation affected by magnetic field [4,5], skin friction and separation time [6]. That previous study using numerical method Keller-Box scheme. This paper using numerical method Euler Implicit scheme for studies about velocity, temperature, and microrotation of the unsteady magnetohydrodynamic micropolar fluid pass a sphere affected by mixed convection.
2. Numerical Methods

We consider the unsteady two-dimensional solid sphere as shown in Figure 1. For an incompressible micropolar fluid, in the presence of magnetohydrodynamic, by neglecting the body force and body couple.

Figure 1. Physical model for micropolar fluids flow past an amagnetic solid sphere. Figure 1(a) represents spherical coordinates of solid sphere. 1(b) Illustration micropolar fluids flow pass a solid sphere.

Micropolar fluid flows from bottom to up, then pass a magnetic solid sphere. When micropolar fluid rubs a solid sphere, it makes a thin layer along to the surface of sphere, that called boundary layer [1]. The governing equations are developed from mass conservation law, energy conservation law, and Second Newton Law for momentum and angular momentum. This research studies the velocity, temperature, and microrotation profile of micropolar fluid pass a solid sphere, so the governing equation for a sphere (3D) are simplified to 2D. The dimensional governing equations consist of continuity equation, momentum equation, angular momentum equation, and energy equation are as follow.

Continuity equation:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  \( (1) \)

Linear momentum equations:
\[
\begin{align*}
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= -\frac{\partial p}{\partial x} - \beta \rho (\bar{T} - T_\infty) g_x + (\mu + k) \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \sigma B_0^2 \frac{\partial N}{\partial y} + k \frac{\partial N}{\partial x} \\
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) &= -\frac{\partial p}{\partial y} - \beta \rho (\bar{T} - T_\infty) g_y + (\mu + k) \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \sigma B_0^2 \frac{\partial v}{\partial y} + k \frac{\partial N}{\partial x}
\end{align*}
\]  \( (2) \)

Angular momentum equation:
\[
\rho \left( \frac{\partial N}{\partial t} + u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) = \gamma \left( \frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} \right) - k \left( 2 \bar{N} + \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right)
\]  \( (3) \)

Energy equation:
\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{c_p}{\rho c_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]  \( (4) \)

With the initial and boundary condition as below
\[
\begin{align*}
t &< 0: \bar{u} = \bar{v} = \bar{N} = 0, \bar{T} = T_\infty, \text{ for any } \bar{x}, \bar{y} \\
t &> 0: \bar{u} = \bar{v} = 0, \bar{N} = -n \frac{\partial u}{\partial y}, \bar{T} = T_w, \text{ at } \bar{y} = 0 \\
\bar{u} &= \bar{u}_e(\bar{x}), \bar{N} = 0, \bar{T} = T_\infty \text{ as } \bar{y} \to \infty
\end{align*}
\]
Where $\rho$ is density of the fluid, $\beta$ is thermal expansion coefficient, $\sigma$ is the electrical conductivity, $k$ is vortex viscosity, $\iota$ is microinertia density, $c$ is thermal conductivity, and $C_p$ is specific heat at constant pressure.

In this paper, the dimensionless variables are given by [6]:

$$x = \frac{\xi}{a}, \quad y = Re^{\frac{1}{2}} \frac{h}{a}, \quad t = \frac{u_\infty t}{a}, \quad u = \frac{a}{u_\infty}, \quad r = \frac{r}{a}, \quad v = Re^{\frac{1}{2}} \frac{h}{a}, \quad p = \frac{p}{\rho u_\infty},$$

$$r(x) = \frac{r(x)}{a}, \quad N = Re^{-\frac{1}{2}} \frac{aN}{u_\infty}, \quad T = \frac{\beta - \beta_\infty}{\beta - \beta_\infty}, \quad g_x = g \sin(\frac{\xi}{a}) = -g \sin \tilde{x}, \quad g_y = g \cos(\frac{\xi}{a}) = g \cos \tilde{y}$$

With non dimensional number as follow:

$$M = \frac{a \sigma B_0^2}{\rho u_\infty}, \quad \alpha = \frac{Gr}{Re}, \quad \alpha = \frac{g \rho (T_\infty - T_0) a^3}{\nu v^2}, \quad \Pr = \frac{\nu \rho C_p}{c}, \quad K = \frac{\kappa}{\mu}$$

By substituting the non dimensional variables to dimensional governing equations, we obtain the non dimensional governing equations as follow:

**Continuity equation:**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5)$$

**Linear momentum equations:**

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \alpha \left(1 + \frac{1 + K}{Re} \frac{\partial^2 u}{\partial x^2} + \frac{1 + K}{Re} \frac{\partial^2 u}{\partial y^2} + Mu + K \frac{\partial N}{\partial y}\right)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} - \frac{\alpha}{Re^2} (1 + K \frac{\partial^2 v}{\partial x^2} + \frac{1 + K}{Re^2} \frac{\partial^2 v}{\partial y^2} + Mu + K \frac{\partial N}{\partial x}) \quad (6)$$

**Angular momentum equation:**

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} = \left(1 + \frac{K}{2}\right) \left(\frac{1}{Re} \frac{\partial^2 \rho}{\partial x^2} + \frac{1}{Re} \frac{\partial^2 \rho}{\partial y^2}\right) - K(2N + \frac{\partial u}{\partial y} - \frac{1}{Re} \frac{\partial v}{\partial x}) \quad (7)$$

**Energy equation:**

$$\frac{\partial \epsilon}{\partial t} + u \frac{\partial \epsilon}{\partial x} + v \frac{\partial \epsilon}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \epsilon}{\partial x^2} + \frac{1}{Pr} \frac{\partial^2 \epsilon}{\partial y^2} \quad (8)$$

These non-dimensional equations are transformed using stream function. The governing equations are further transformed into similarity equation using similarity variable for small time:

$$\psi = \int \tilde{\epsilon} u_\infty(x) r(x) f(x, \eta, t) \, dx, \quad \eta = \frac{\tilde{\epsilon} x}{t \tilde{\epsilon}^2}, \quad N = \int \tilde{\epsilon} u_\infty(x) h c, \quad T = \int \tilde{\epsilon} u_\infty(x) r(x) f(x, \eta, t)$$

Using similarity variable above and by substituting $u_\infty = \frac{3}{2} \sin x$ [7], $\frac{\partial f}{\partial \eta} = f', \frac{\partial s}{\partial \eta} = s', \frac{\partial h}{\partial \eta} = h'$, we obtain model on lower stagnation point $x = 0^\circ$:

**Momentum equation:**

$$(1 + K) f''' + Kh' + \frac{\eta^2 f}{2 \eta^2} + Mt(1 - f') + \frac{\alpha}{3} ast + \frac{3}{2} t(1 + ff'' - f')^2 = t \frac{\partial f}{\partial t} \quad (9)$$

**Angular momentum:**

$$\left(1 + \frac{K}{2}\right) h'' + \frac{\eta}{2} h' + \frac{h}{2} + \frac{3}{2} t(h h' - h f') = t \frac{\partial h}{\partial t} + K t(2 h + f'') \quad (10)$$

**Energy equation:**

$$s'' + Pr \frac{3}{2} s' + Pr tf \frac{3}{2} \cos x s' = Pr t \left(\frac{\partial s}{\partial t} + \frac{3}{2} \sin x \left(\frac{\partial f}{\partial \eta} \frac{\partial s}{\partial x} - \frac{\partial f}{\partial x} \frac{\partial s}{\partial \eta} - \frac{f}{r} \frac{\partial s}{\partial \eta} \right) \right) \quad (11)$$

With the initial and boundary condition as below:

$t = 0: f = f' = h = s = 0$, for any $x, \eta$

$t > 0: f = f' = 0, h = -nf''$, $s = 1$ at $\eta = 0$

$f' = 1, h = s = 0$ as $\eta \to \infty$
This mathematics models are solved numerically using Euler implicit methods. This models are discretized using finite difference method. The formed equations are transformed to tridiagonal matrix. We further solve tridiagonal matrix problem using Thomas algorithm.

3. Results and discussion

In this research, micropolar fluid flow pass a magnetic solid sphere affected by mixed convection. That problem are solved by using Euler implicit methods. Velocity, temperature of fluids, and microrotation of micropolar particle are shown. Pratomo et. al [5] using Keller-box to solve numerically of problem: flow of micropolar fluids pass a magnetic solid sphere affected by magnetic field. The results of Pratomo et. al [5] are velocity of fluids and microrotation profile of micropolar particle. This present numerical results are compared to Pratomo, et. al [5] using $\alpha = 0, Pr = 1, M = 0, K = 1$ as shown at Fig 2. The comparasion of velocity and microrotation profil are confirmed with Pratomo et. al. So we are sure that this numerical results can be use for another parameters that appropriate with this problem research.

![Figure 2](image1.png)

**Figure 2.** The comparasion of velocity (Fig. 2.a) and microrotation profile (Fig 2.b)

The similarity equations are solved numerically by using Euler implicit scheme. The effect of magnetic parameter (M) at lower stagnation point ($x = 0^o$) shown at Figure 3.

![Figure 3](image2.png)

**Figure 3.** The effect of magnetic parameter (M). Figure 3(a) show velocity profile for various M. Figure 3(b) show temperature for various M and figure 3(c) is microrotation profile for various M.
The Figure 2 shows the effect of magnetic parameter \((M)\). These numerical results have been made at fixed (parameter micropolar \(K = 1, Pr = 1, \alpha = 1, n = 0.5\)). The figure above shows that parameter magnetics increase when the velocity decrease and there is no negative velocity value. This is because of Lorentz force. Temperature for various \(M\) shows that decreasing \(M\) leads to higher temperature under the influence of magnetohydrodynamic and mixed convection at the surface of magnetic solid sphere. Figure 3. shows the microrotation profile in the magnetohydrodynamic micropolar fluids at various magnetic parameters \(M\). Notice that the microrotation profile increases when magnetic parameter \(M\) increases.

![Image](image_url)

**Figure 4.** The effect of mixed convection parameter (\(\alpha\)). Figure 4(a) show velocity profile for various \(\alpha\). Figure 4(b) show temperature for various \(\alpha\) and figure 4(c) is microrotation profile for various \(\alpha\).

The Figure 4 shows velocity, temperature, and microrotation profiles for variations mixed convection parameter \((\alpha = 3.35, \alpha = 4.85, \alpha = 18.6, \alpha = 21.4)\). Velocity increases from zero to one. The higher mixed convection parameter lead to the higher velocity and temperature. Mixed convection parameter with temperature of fluids. The microrotation decreases when mixed convection parameter increases.

4. **Conclusion**

The governing equations are developed from continuity equation, momentum equation, angular momentum equation, and energy equation. The results show that the velocity profile and temperature of micropolar fluid increase when magnetic parameter \(M\) decrease, mixed convection increase and Prandtl number decrease. The higher magnetic and Prandtl Number lead to higher microrotation of micropolar fluid particle. The higher mixed convection parameter leads to the lower microrotation.

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