Max-Min Fairness in IRS-Aided Multi-Cell MISO Systems via Joint Transmit and Reflective Beamforming

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Abstract—This paper investigates an intelligent reflecting surface (IRS)-aided multi-cell multiple-input single-output (MISO) system consisting of several multi-antenna base stations (BSs) each communicating with a single-antenna user, in which an IRS is dedicatedly deployed for assisting the wireless transmission and suppressing the inter-cell interference. Under this setup, we jointly optimize the coordinated transmit beamforming at the BSs and the reflective beamforming at the IRS, for the purpose of maximizing the minimum weighted received signal-to-interference-plus-noise ratio (SINR) at users, subject to the individual maximum transmit power constraints at the BSs and the reflection constraints at the IRS. To solve the difficult non-convex minimum SINR maximization problem, we propose efficient algorithms based on alternating optimization, in which the transmit and reflective beamforming vectors are optimized in an alternating manner. In particular, we use the second-order-cone programming (SOCP) for optimizing the coordinated transmit beamforming, and develop two efficient designs for updating the reflective beamforming based on the techniques of semi-definite relaxation (SDR) and successive convex approximation (SCA), respectively. Numerical results show that the use of IRS leads to significantly higher SINR values than benchmark schemes without IRS or without proper reflective beamforming optimization; while the developed SCA-based solution outperforms the SDR-based one with lower implementation complexity.

Index Terms—Intelligent reflecting surface (IRS), multi-cell systems, multiple-input single-output (MISO), coordinated transmit beamforming, reflective beamforming, optimization.

I. INTRODUCTION

To enable emerging Internet of things (IoT) and artificial intelligence (AI) applications, the fifth-generation (5G)-and-beyond cellular networks need to support massive wireless devices with diverse quality of service (QoS) requirements, such as significantly increased spectrum efficiency, ultra-low transmission latency, and extremely-high communication reliability [1], [2]. Towards this end, small base stations (BSs) are densely deployed to shorten the distances with cellular subscribers [3], and device-to-device (D2D) communications are enabled underlying conventional cellular transmissions to create more spectrum reuse opportunities [4]. However, the emergence of small BSs and D2D communications in 5G-and-beyond cellular networks also introduces severe co-channel interference among different cells and different D2D links, which needs to be carefully dealt with from technical perspectives. In the literature, various approaches have been proposed to mitigate or even utilize the co-channel interference, some examples including coordinated beamforming [5]–[8] and network multiple-input multiple-output (MIMO) [9]–[11].

Recently, intelligent reflecting surface (IRS) has emerged as a promising technology for beyond-5G cellular networks [12], [13], which can also be used to tackle the critical co-channel interference issue in a cost-effective manner. IRS is a passive meta-material panel consisting of a large number of reflecting units, each of which can introduce an independent phase shift on radio-frequency (RF) signals to change the signal transmission environment. By jointly controlling these phase shifts, the IRS can form reflective signal beams, such that the reflected signals can be coherently combined with the directly transmitted signals at intended receivers for enhancing the desirable signal strength, or destructively combined at unintended receivers for suppressing the undesirable interference. As the IRS is a passive device with no dedicated power consumption, it is envisioned as a green and cost-effective solution to enhance the spectrum- and energy-efficiency of future cellular networks [12], [13].

There have been some prior works [14]–[21] investigating the joint transmit and reflective beamforming design in IRS-aided wireless communication systems. The authors in [14], [15] investigated the received signal-to-noise ratio (SNR) maximization problem in a point-to-point IRS-aided multiple-input single-output (MISO) communication system, which is solved by using the techniques of semi-definite relaxation (SDR) [14] and manifold optimization [15], respectively. Furthermore, [16] considered the signal-to-interference-plus-noise ratio (SINR)-constrained power minimization problem in IRS-aided multiuser MISO downlink communication systems, in which alternating optimization is employed to update the transmit and reflective beamforming vectors in an alternating manner, and SDR is employed to optimize the reflective beamforming. In addition, prior works also studied other communication setups aided by the IRS such as IRS-aided orthogonal frequency division multiplexing (OFDM) [17]–[19], non-orthogonal multiple access (NOMA) [20], [21] and simultaneous wireless information and power transfer (SWIPT) systems [22]. Nevertheless, all the above prior works [14]–[22] focused on a single-cell setup. This thus motivates us to use IRSs to facilitate the multi-cell communications in this work.

In this paper, we consider an IRS-aided multi-cell MISO system, where an IRS is dedicatedly deployed at the cell boundary to assist the wireless transmission from BSs to users and suppress their inter-cell interference. We assume that there

This work was supported in part by the National Key R&D Program of China (No. 2018YFB1800800), the Natural Science Foundation of China (Nos. 61871137 and 11671419), the Guangdong Province Key Area R&D Program (No. 2018B030338001), and the Guangdong Province Basic Research Program (Natural Science) (No. 2016ZKZX0285). J. Xu is the corresponding author.
is one multi-antenna BS serving one single-antenna user in each cell. Our objective is to jointly optimize the coordinated transmit beamforming at the multiple BSs and the reflective beamforming at the IRS, to maximize the minimum weighted received SINR at users, subject to the individual maximum transmit power constraints at BSs, and the reflection constraints at the IRS. However, the formulated minimum SINR maximization problem is highly non-convex due to the coupling between the transmit and reflective beamforming vectors. To solve this difficult problem, we propose efficient algorithms based on alternating optimization, in which the transmit and reflective beamforming vectors are optimized in an alternating manner. In particular, under any given reflective beamforming, we obtain the optimal coordinated transmit beamforming via second-order cone programming (SOCP); while under any given coordinated transmit beamforming, we develop two efficient designs to update the reflective beamforming by using the techniques of SDR and successive convex approximation (SCA), respectively. It is observed that the performance of the SDR-based solution generally depends on the randomization procedure, while the SCA-based solution can always converge towards a stationary point. Numerical results show that the use of IRS leads to significant performance gains over benchmark SDR-based solution generally depends on the randomization (SCA), respectively. It is observed that the performance of the techniques of SDR and successive convex approximation.

Let $s_i$ denote the transmitted signal by each BS $i$ and $w_i \in \mathbb{C}^{M \times 1}$ the corresponding transmit beamforming vector. We assume that $s_i$’s are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) random variables with zero mean and unit variance, i.e., $s_i \sim \mathcal{CN}(0,1)$. The transmitted signal by each BS $i$ is thus given by $x_i = w_is_i, \forall i \in K$. Suppose that each BS has a maximum power budget denoted by $P_i$. Then we have $E[|x_i|^2] = \|w_i\|^2 \leq P_i, \forall i \in K$, where $E(\cdot)$ denotes the stochastic expectation.

As for the reflection at the IRS, let $\theta_n \in [0, 2\pi)$ and $\beta_n \in [0,1]$ denote the phase shift and the reflection amplitude imposed by the $n$-th reflecting unit on the incident signal, respectively. Accordingly, let $\Theta = \text{diag}(\beta_1e^{j\theta_1}, \ldots, \beta_Ne^{j\theta_N})$ represent the reflection coefficient matrix at the IRS, where $j \triangleq \sqrt{-1}$, and $\text{diag}(a_1, \ldots, a_N)$ denotes a diagonal matrix with its diagonal elements being $a_1, \ldots, a_N$. Furthermore, let $v = [\beta_1e^{j\theta_1}, \ldots, \beta_Ne^{j\theta_N}]H$ denote the reflective beamforming vector, where each element $\beta_n$ denoted by $\beta_n$, must satisfy $|\beta_n| \leq 1, \forall n \in N$. Here, the superscript $H$ denotes the conjugate transpose of a vector or matrix. As a consequence, we have the combined reflective channel from BS $k$ to user $i$ as $f_k^H\Theta g_k = v^H\Phi_k$, where $\Phi_k = \text{diag}(f_k^H)G_k$. Notice that this transformation separates the reflective beamforming vector $v$ from the reflective channels, which will significantly facilitate our derivation later. By combining the directly transmitted and reflected signals, the received signal at user $i$ is accordingly expressed as

$$y_i = (v^H\Phi_{i,k} + h_{i,k}^H)w_is_i + \sum_{k \neq i,k \in K} (v^H\Phi_{i,k} + h_{i,k}^H)w_k s_k + n_i,$$

where $n_i$ denotes the additive white Gaussian noise (AWGN) at the receiver of user $i$ with zero mean and variance $\sigma_i^2$, i.e., $n_i \sim \mathcal{CN}(0, \sigma_i^2), \forall i \in K$. By treating the interference as noise, the received SINR at user $i$ is given by

$$\gamma_i(v, \{w_i\}) = \frac{\|v^H\Phi_{i,k} + h_{i,k}^H\|w_i\|^2}{\sum_{k \neq i,k \in K} \|v^H\Phi_{i,k} + h_{i,k}^H\|w_k\|^2 + \sigma_i^2}.$$

Our objective is to maximize the users’ communication performance in a fair manner. As a result, we consider the max-min fairness problem with the objective of maximizing the minimum weighted SINR of all users, by jointly optimizing the transmit beamforming $\{w_i\}$ at the BSs and the reflective beamforming $v$ at the IRS, subject to the individual transmit power constraints at BSs and the reflection constraints at the IRS. Let $\alpha_i > 0$ denote a weight parameter for user $i \in K$ characterizing the fairness among the $K$ users, where a larger value of $\alpha_i$ indicates that user $i$ has a higher priority in transmission. Therefore, the minimum SINR maximization problem is formulated as

$$\begin{align*}
\text{(P1)}: \max_{v, \{w_i\}} \min_{i \in K} \gamma_i(v, \{w_i\}) \quad \text{subject to} \quad & \|w_i\|^2 \leq P_i, \forall i \in K \quad \text{(4)} \\
& \|n_i\| \leq 1, \forall n \in N. \quad \text{(5)}
\end{align*}$$

To facilitate the derivation, we first introduce an auxiliary variable $t$ and reformulate problem (P1) as the following
In particular, suppose that the optimal solution of \{efficient approaches for updating beamforming vectors\}. In Sections III and IV, we present the effective or combined channel from BS \(k\) iteration manner. For notational convenience, suppose that at each optimization-based algorithms are implemented in an iterative manner. Accordingly, the coordinate transmit beamforming \(\{w_i\}_i\) under given transmit beamforming \(\{w_i\}_i\) and the reflective beamforming vector \(\{w_i\}_i\) are optimized in an alternating manner, with the other being fixed. In particular, the alternating-optimization-based algorithms are implemented in an iterative manner. For notational convenience, we define \(a_{i,k} = \Phi_{i,k}^H v + h_{i,k}\) as the effective or combined channel from BS \(k\) in \(K\) to user \(i\) in \(K\). Accordingly, the coordinate transmit beamforming optimization problem becomes

\[
\text{(P2)} : \max_{\{w_i\}_i} t
\]

\[
\text{s.t. } \sum_{k \neq i, k \in K} |a_{i,k}^H w_i|^2 + \sigma_i^2 \geq \alpha_i t, \forall i \in K, \quad (7) \tag{4}
\]

It is observed that problem (P2) is not still a convex optimization problem. To tackle this issue, we introduce the following feasibility problem (P2.1), which is obtained based on problem (P2) by fixing \(t\).

\[
\text{(P2.1)}: \text{find } \{w_i\}_i
\]

\[
\text{s.t. } (4), (9), \text{ and } (10)
\]

In particular, suppose that the optimal solution of \(t\) to problem (P2) is given by \(t^*\). It is thus clear that if problem (P2.1) is feasible under any given \(t\), then we have \(t \leq t^*\); while if (P2.1) is infeasible, then it follows that \(t > t^*\). Therefore, problem (P2) can be equivalently solved by checking the feasibility of problem (P2.1) under any given \(t > 0\), together with a bisection search over \(t > 0\).

Therefore, to solve problem (P2), we only need to solve problem (P2.1) under any fixed \(t > 0\), by using SOCP as follows [24]. Towards this end, we notice that the SINR constraints in (7) can be reformulated as

\[
(1 + \frac{1}{\alpha_i t}) |a_{i,k}^H w_i|^2 \geq \sum_{k \in K} |a_{i,k}^H w_k|^2 + \sigma_i^2, \forall i \in K. \quad (8)
\]

Based on (8), it is evident that if \(\{w_i\}_i\) is a feasible solution to problem (P2.1), then any phase rotation of \(\{w_i\}_i\) will still be feasible. Without loss of optimality, we choose the solution of \(\{w_i\}_i\) such that \(a_{i,k}^H w_k\) becomes a non-negative value for any user \(k \in K\). As a result, we have the following constraints:

\[
a_{i,k}^H w_k \geq 0, \forall i \in K, \quad (9)
\]

where \(a_{i,k}^H w_k\) has a non-negative real part and a zero imaginary part, i.e., \(\text{Re}(a_{i,k}^H w_k) \geq 0\) and \(\text{Im}(a_{i,k}^H w_k) = 0\), with \(\text{Re}(x)\) and \(\text{Im}(x)\) denoting the real and imaginary parts of a complex number \(x\). Accordingly, (8) can be further re-expressed as

\[
\sqrt{1 + \frac{1}{\alpha_i t}} |a_{i,k}^H w_k| \geq \sqrt{\frac{\|A_i^H e_i\|^2}{\alpha_i t}, \forall i \in K. \quad (10)
\]

where \(A \in \mathbb{C}^{K \times K}\) denotes a matrix with the element in its \(i\)-th row and \(j\)-th column being \(a_{i,j}^H w_j\), \(e_i \in \mathbb{C}^{K+1}\) denotes a vector with the \(i\)-th element being one and others being zero, and \(\|\cdot\|_2\) denotes the Euclidean norm of a vector. Therefore, problem (P2.1) is reformulated as the following equivalent form:

\[
\text{(P2.2)}: \text{find } \{w_i\}_i
\]

\[
\text{s.t. } (4), (9), \text{ and } (10)
\]

Problem (P2.1) is an SOCP problem that can be optimally solved by standard convex optimization solvers such as CVX [26]. Therefore, the optimal coordinate transmit beamforming solution to problem (P2) is finally obtained.

III. COORDINATED TRANSMIT BEAMFORMING OPTIMIZATION

In this section, we optimize the coordinate transmit beamforming \(\{w_i\}_i\) under any given reflective beamforming \(v\). For notational convenience, we define \(a_{i,k} = \Phi_{i,k}^H v + h_{i,k}\) as the effective or combined channel from BS \(k\) in \(K\) to user \(i\) in \(K\). Accordingly, the coordinate transmit beamforming optimization problem becomes

\[
\text{(P2)} : \max_{\{w_i\}_i} t
\]

\[
\text{s.t. } \sum_{k \neq i, k \in K} |a_{i,k}^H w_k|^2 + \sigma_i^2 \geq \alpha_i t, \forall i \in K, \quad (7) \tag{4}
\]

Notice that problem (P2.1) or (P1) is difficult to be optimally solved due to the coupling between the transmit beamforming \(\{w_i\}_i\) and the reflective beamforming \(v\) at the SINR terms. In the following, we propose two solutions by application of SDR in reflective beamforming optimization problem. In the following, we propose two solutions by application of SDR in reflective beamforming optimization problem. In the following, we propose two solutions by application of SDR in reflective beamforming optimization problem.

In this section, we optimize the reflective beamforming \(v\) under given transmit beamforming \(\{w_i\}_i\). For notational convenience, we define \(c_{i,k} = \Phi_{i,k}^H v + h_{i,k}\) and \(d_{i,k} = h_{i,k}^H w_i, \forall i, k \in K\). Then, we have

\[
|v^H \Phi_{i,k} + h_{i,k}^H w_k|^2 = v^H C_{i,k} v + 2Re\{v^H u_{i,k}\} + |d_{i,k}|^2, \quad (11)
\]

where \(C_{i,k} = c_{i,k} d_{i,k}^H\) and \(u_{i,k} = c_{i,k} d_{i,k}^H\). Accordingly, the reflective beamforming optimization problem is given by

\[
\text{(P3)} : \max_{v} t
\]

\[
\text{s.t. } \sum_{k \neq i, k \in K} |v^H C_{i,k} v + 2Re\{v^H u_{i,k}\} + |d_{i,k}|^2 \geq \alpha_i t, \forall i \in K, \quad (12) \tag{5}
\]

Notice that problem (P3) is also a non-convex optimization problem. In the following, we propose two solutions by leveraging the SDR and SCA techniques, respectively.

A. SDR-based Solution to Problem (P3)

In this subsection, we use the well-established SDR technique to solve problem (P3). This is motivated by the wide application of SDR in reflective beamforming optimization (see, e.g., [16]). Towards this end, we first define

\[
\bar{v} = [v^H \Phi_{i,k} + h_{i,k}^H w_k, v^H C_{i,k} v + 2Re\{v^H u_{i,k}\} + |d_{i,k}|^2],
\]

where

\[
R_{i,k} = \begin{bmatrix} C_{i,k} & u_{i,k}^T \end{bmatrix} \quad \text{and} \quad \bar{v} = [v^H \Phi_{i,k} + h_{i,k}^H w_k, v^H C_{i,k} v + 2Re\{v^H u_{i,k}\} + |d_{i,k}|^2].
\]

Accordingly, problem (P3) is re-expressed as

\[
\text{(P3.1)} : \max_{\bar{v}} t
\]

\[
\text{s.t. } \sum_{k \neq i, k \in K} |\bar{v}^H R_{i,k} \bar{v} + |d_{i,k}|^2 \geq \alpha_i t, \forall i \in K, \quad (13)
\]

\[
|\bar{v}_n| \leq 1, |\bar{v}_{n+1}| = 1, \forall n \in \mathcal{N}. \quad (14)
\]

Furthermore, we define \(V = \bar{v}^H \bar{v}\) with \(V\) being positive semi-definite (i.e., \(V \succeq 0\) and rank\((V) \leq 1\). Then problem (P3.1) or (P3) is further reformulated as the following equivalent form:
solution to problem (P3.1), denoted by \( \bar{V} \) of a complex number \( \bar{W} \) together with a bisection search over \( V_n \geq 0 \) and \( \text{rank}(V) \leq 1 \).

where \( V_{m,n} \) denotes the element in the \( m \)-th row and \( n \)-th column of the matrix \( V \), and \( \text{Tr}(A) \) denotes the trace of matrix \( A \). However, problem (P3.2) is still challenging to be optimally solved due to the non-convex rank-one constraint in (18). Motivated by the idea of SDR, we relax this constraint, and obtain a relaxed version of (P3.2) as:

\[
(P3.3) : \max_{\tilde{V}} t \\
\text{s.t.} \quad \text{Tr}(R_{i,k}^\dagger V) + |d_{i,k}|^2 \geq \alpha_t, \forall i \in K \tag{15}
\]

Notice that problem (P3.4) is a convex semi-definite program (SDP) and thus can be solved optimally by using CVX [26]. As a result, we have obtained the optimal solution to problem (P3.3), denoted by \( \tilde{V}^* \).

Now, it remains to reconstruct the solution to problem (P3.2) or equivalently (P3.1)/(P3) based on \( \tilde{V}^* \) and \( t^* \). In particular, if \( \text{rank}(\tilde{V}^*) \leq 1 \), then \( \tilde{V}^* \) and \( t^* \) are also the optimal solution to problem (P3.2). In this case, we have \( V^* = \tilde{V}^* \bar{v} \bar{v}^H \), where \( \bar{v} \) becomes the optimal solution to problem (P3.1). However, if \( \text{rank}(\tilde{V}^*) > 1 \), then the following Gaussian randomization procedure [25] needs to be further adopted to produce a high-quality rank-one solution to problem (P3.2) and (P3.1).

Specifically, suppose that the eigenvalue decomposition of \( V^* \) is \( V^* = U \Sigma U^H \). Then, we set \( \tilde{v} = U \Sigma^{1/2} r \), where \( r \) corresponds to a CSGC random vector with zero mean and covariance matrix \( I \), i.e., \( r \sim CN(0, I) \). Accordingly, we construct a feasible solution \( \bar{v}^* \) to problem (P3.1) as \( \bar{v}_n = e^{j\arg(\tilde{v}_n/\tilde{v}_{\text{max}})} \), where \( \tilde{v}_n \) and \( \tilde{v}_{\text{max}} \) denote the \( n \)-th element of vector \( \tilde{v} \) and \( v \), respectively, and \( \arg(x) \) denotes the phase of a complex number \( x \). To guarantee the performance, the randomization process needs to be implemented multiple times and the best solution among them is selected as the obtained solution to problem (P3.1), denoted by \( \bar{v}^* \). In this case, the obtained solution to problem (P3.2) is \( \bar{v}^* \bar{v}^H \). Based on the solution of \( \bar{v}^* \) to problem (P3.1), we can accordingly obtain the solution of (P3) as \( \bar{v}^* \). Therefore, the SDR-based algorithm for solving problem (P3) is complete.

By alternately implementing the SDR-based solution to (P3) and the SOCP-based solution to (P2), we can obtain an efficient solution to the original problem (P1). We refer to this algorithm as alternating optimization with SDR. In summary, the algorithm of alternating optimization with SDR is presented as Algorithm 1.

### Algorithm 1: Alternating optimization with SDR

1: Initialize: \( l = 0 \), \( \bar{w}^{(0)} \) and accuracy threshold \( \epsilon > 0 \).
2: Repeat:
3: \( l = l + 1 \);
4: Under given \( \bar{w}^{(l-1)} \), solve problem (P2) to obtain \( \{w_i^t\} \), and set \( w_i^{(l)} = w_i^t, \forall i \in K \);
5: Under given \( \{w_i^{(l)}\} \), solve problem (P3) to obtain \( \bar{v}^* \), and set \( w_i^{(l)} = \bar{v}^* \);
6: Until the increase of the objective function in (P1) is smaller than \( \epsilon \).

### B. SCA-based Design for Updating Reflective Beamforming

To overcome the above drawbacks of the SDR-based solution, in this subsection, we propose an efficient design for updating the reflective beamforming vector \( v \) by applying the SCA technique. Recall that the update of \( v \) in problem (P3) is implemented iteratively in the alternating-optimization-based algorithm for solving the original problem (P1). Therefore, instead of directly solving (P3), in the SCA-based design we aim to find an updated \( v \) to increase the users’ minimum SINR. Towards this end, we consider a particular iteration \( l \geq 1 \), the local point of \( v \) as \( v^{(l-1)} \), which corresponds to the obtained \( v \) in the previous iteration. Under given \( \{w_i^{(l)}\} \) together with \( v^{(l-1)} \), we denote the achieved minimum SINR at users as \( \min_{i \in K} \gamma_i(v^{(l-1)}, \{w_i^{(l)}\}) \). In the following, we explain how to update \( v \) to increase the minimum SINR at users based on SCA.

For notational convenience, we first define an auxiliary function for user \( i \in K \) as:

\[
F_i(v, \{w_i^{(l)}\}, t) = \alpha_t \left[ \sum_{k \neq i,k \in K} (v^H C_{i,k} v + 2Re\{v^H u_{i,k}\} + |d_{i,k}|^2) + \sigma_i^2 \right] - (v^H C_{i,i} v + 2Re\{v^H u_{i,i}\} + |d_{i,i}|^2),
\]

where \( C_{i,k} \), \( u_{i,k} \), and \( d_{i,k} \), \( i, k \in K \) are defined at the beginning of Section IV. Note that after the update of \( \{w_i\} \) at each iteration \( l \), it must hold that \( \min_{i \in K} F_i(v^{(l-1)}, \{w_i^{(l)}\}) = 0 \). Accordingly, we update the reflective beamforming vector \( v \) at the IRS by solving the following problem:

\[
(P4) : \min_{\bar{t}} \max_{\bar{i} \in K} F_i(v, \{w_i^{(l)}\}, \bar{t}) \tag{21}
\]

s.t. (5).

As \( v^{(l-1)} \) is a feasible solution to problem (P4) leading to an objective value of zero, the optimal solution to problem...
(P4) should be non-positive. Suppose that the obtained solution to (P4) as \( v^i \). If \( \min_{i \in K} F(v^i, u^i, t^i) < 0 \), then it can be easily shown that \( \min_{i \in K} \gamma_i(v^i, u^i, t^i) > \min_{i \in K} \eta_i(v^{(i-1)}, u^{(i-1)}), \) i.e., the minimum SINR is increased. Therefore, we focus on solving problem (P4) next.

Problem (P4) is still non-convex as the objective function is non-convex with respect to \( v \). To address this issue, we apply the SCA technique to approximate the second convex term in the right-hand-side of (20) by its first-order Taylor expansion.

Note that a convex function is lower bounded by its first-order Taylor expansion at any given point. Thus, at the local point of \( v^{(i-1)} \), we have

\[
F_i(v, \{u_i^l\}, t^i) \leq \alpha_i t^i \sum_{k \neq i, k \in K} (v^l C_{i,k} v + 2Re\{v^l u_{i,k}\} + |d_{i,k}|^2) + \sigma_i^2
\]

By introducing an auxiliary variable \( z \) and replacing \( F_i(v, \{u_i^l\}, t^i) \) by \( F_{z,i} \), (P4) is approximated as the following problem:

\[
(P4.1): \min \ z
\]

\[
s.t. \ F_{z,i}(v, \{u_i^l\}, t, t^i) \leq z, \forall i \in K, \quad (23)
\]

Problem (P4.1) is a convex problem that can be solved optimally by CVX [26]. Suppose that the optimal solution to problem (P4.1) is denoted as \( v^{**} \) and \( t^{**} \). By substituting \( v^{**} \) into \( F_i(v, \{u_i^l\}, t) \), it is evident that as any feasible solution problem (P4.1) is also feasible for problem (P4), the obtained value of (P4.1) by \( v^{**} \) is smaller than that of (P4). Therefore, \( v^{**} \) leads to an increased SINR value. Therefore, we can directly update \( v \) as \( v^{**} \), i.e., \( v^i = v^{**} \). In summary, the alternating optimization with SCA is presented as Algorithm 2.

Algorithm 2: Alternating optimization with SCA

1: Initialize: \( l = 0, v^{(0)} \) and accuracy threshold \( \epsilon > 0 \).
2: Repeat:
3: \( l = l + 1 \);
4: Under given \( v^{(l-1)} \), solve problem (P2) to obtain \( \{u_i^l\} \) and \( t^l \), and set \( w_i^l = u_i^l, \forall i \in K \), and \( t^{(l)} = t^l \);
5: Under given \( \{u_i^l\}, t^l, \) and \( v^{(l-1)} \), solve problem (P4.1) to obtain \( v^{**} \), and set \( v^{(l)} = v^{**} \);
6: Until the increase of the objective function in (P1) is smaller than \( \epsilon \).

It is worth noting that for the algorithm of alternating optimization with SCA, it is ensured that after each update of \( v \) by SCA, the minimum SINR among all users is always non-decreasing. Therefore, the objective value of (P1) is ensured to be non-decreasing at each iteration. As a result, this algorithm will converge towards a stationary solution to problem (P1).

V. NUMERICAL RESULTS

In this section, we provide numerical results to validate the performance of the proposed alternating-optimization-based algorithms in the IRS-aided multi-cell MISO system. In the simulation, there are \( K = 3 \) BSs located at \((100, 0, 0), (100, 0, 0)\), and \((0, 100, 0)\), respectively, each of which is equipped with \( M = 2 \) antennas. We consider a scenario with symmetrically distributed users unless otherwise stated, where the three users are located at \((-d_{user}, 0), (d_{user}, 0)\) and \((0, d_{user})\), with \( d_{user} = 5 \) m. An IRS with \( N = 20 \) reflecting units is deployed at \((0, -d_{IRS}),\) with \( d_{IRS} = 10 \) m. Furthermore, we set the maximum transmit power at all BSs to be identical, i.e., \( P_i = P_{max}, \forall i \in K \), and we are interested in the minimum SINR at users by setting \( \alpha_i = 1, \forall i \in K \). In addition, we consider the distance-dependent path loss model as

\[
P_L = C_0 \left( \frac{d}{d_0} \right)^{-\alpha},
\]

where \( C_0 = -30 \) dB denotes the path loss at the reference distance of \( d_0 = 1 \) m, \( \alpha \) denotes the path loss exponent, \( d \) denotes the distance between the transmitter and receiver. For the BS-user, BS-IRS and IRS-user links, we set the path-loss exponents \( \alpha \) to be 3.6, 2, and 2.5, respectively. Furthermore, we consider line-of-sight (LOS) channels from BSs to the IRS, and Rayleigh fading for the BS-user and IRS-user links. The noise power at each user \( i \) is set as \( \sigma_i^2 = -80 \) dBm, \( \forall i \in K \).

First, Fig. 1 shows the convergence behaviour of the two alternating-optimization-based algorithms, where \( P_{max} = 35 \) dBm. It is observed that the alternating optimization with SCA leads to monotonically increasing SINR values over iterations, thus converging towards a stationary solution; while the alternating optimization with SDR results in fluctuated SINR values due to the randomization process. Furthermore, it is also observed that running on a computer with E5-2667v4 CPU and 32G memory, the average run time of the alternating optimization with SDR is 895.1628 seconds, while that of the alternating optimization with SCA is 432.7232 seconds. This shows the advantage of SCA again in terms of the implementation complexity.

Next, we evaluate the performance of the proposed two alternating-optimization-based algorithms, as compared with the following two benchmark schemes.

Benchmark scheme with random reflective phases: We set the phase shift \( \theta_n \) of each unit \( n \) at the IRS as a random value uniformly distributed in \([0, 2\pi]\), and set \( \beta = 1 \). Under such given reflective beamforming, we solve problem (P2) to obtain the corresponding coordinated transmit beamforming.

Benchmark scheme without IRS: Without IRS deployed, we only need to optimize the coordinated transmit beamforming by solving problem (P2), in which \( \{a_{i,k}\} \) is replaced as \( \{h_{i,k}\} \).

Fig. 2 shows the minimum SINR at users versus the maximum transmit power \( P_{max} \) at each BS, in which the results are averaged over 100 random channel realizations. First, it is observed that the two proposed alternating-optimization-based algorithms considerably outperform the two benchmark schemes, and the performance gains become more significant when \( P_{max} \) gets large. This shows the benefit of IRS in both signal enhancement and interference suppression, especially when the interference (or transmit power) becomes strong. Next, the alternating optimization with SCA is observed to lead to higher minimum SINR values than that with SDR. This is consistent with the observation in Fig. 1. Furthermore, the benchmark scheme with random reflective phases is observed to have a similar performance as that without IRS. This shows that the benefit of IRS can only be gained via proper reflective beamforming optimization.

To further reveal the practical performance, Fig. 3 shows the minimum SINR at users versus \( P_{max} \) in another scenario with the three users uniformly distributed within a triangle.
In this paper, we investigated the IRS-aided multi-cell MISO system, with the objective of maximizing the minimum weighted SINR at all users by jointly optimizing the coordinated transmit beamforming at BSs and reflective beamforming at the IRS, subject to individual power constraints at BSs. We proposed efficient alternating-optimization-based algorithms to update the transmit and reflective beamforming vectors in an alternating manner. In particular, we used the SOCP to optimize the transmit beamforming, and proposed two designs based on SDR and SCA for updating the reflective beamforming. Numerical results demonstrated that the dedicatedly deployed IRS considerably improves the performance of the multi-cell MISO system by not only enhancing the received signal strength but also suppressing the inter-cell interference, especially for cell-edge users. It was also shown that the SCA-based design is an efficient algorithm for optimizing the reflective beamforming at the IRS with guaranteed convergence, which outperforms the conventionally adopted SDR-based reflective beamforming optimization.

VI. CONCLUSION

In this paper, we investigated the IRS-aided multi-cell MISO system, with the objective of maximizing the minimum weighted SINR at all users by jointly optimizing the coordinated transmit beamforming at BSs and reflective beamforming at the IRS, subject to individual power constraints at BSs. We proposed efficient alternating-optimization-based algorithms to update the transmit and reflective beamforming vectors in an alternating manner. In particular, we used the SOCP to optimize the transmit beamforming, and proposed two designs based on SDR and SCA for updating the reflective beamforming. Numerical results demonstrated that the dedicatedly deployed IRS considerably improves the performance of the multi-cell MISO system by not only enhancing the received signal strength but also suppressing the inter-cell interference, especially for cell-edge users. It was also shown that the SCA-based design is an efficient algorithm for optimizing the reflective beamforming at the IRS with guaranteed convergence, which outperforms the conventionally adopted SDR-based reflective beamforming optimization.

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