Supersymmetric Three Family SU(5) Grand Unified Models from Type IIA Orientifolds with Intersecting D6-Branes

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Abstract

We construct some \(N = 1\) supersymmetric three-family SU(5) Grand Unified Models from type IIA orientifolds on \(T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)\) with D6-branes intersecting at general angles. These constructions are supersymmetric only for special choices of untwisted moduli. We show that within the above class of constructions there are no supersymmetric three-family models with 3 copies of 10-plets unless there are simultaneously some 15-plets. We systematically analyze the construction of such models and their spectra. The M-theory lifts of these brane constructions become purely geometrical backgrounds: they are singular \(G_2\) manifolds where the Grand Unified gauge symmetries and three families of chiral fermions are localized at codimension 4 and codimension 7 singularities respectively. We also study some preliminary phenomenological features of the models.

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I. INTRODUCTION

Grand unification [1] is an attractive possibility of physics beyond the Standard Model, as it provides a natural explanation for the unification of strong and electro-weak forces at an energy scale of the order of $10^{15-16}$ GeV. Over the years, many Grand Unified Theories (GUTs) have been proposed and their phenomenological features have been thoroughly analyzed. The fact that the GUT scale is remarkably close to the Planck (or string) scale is tantalizing as it suggests that in addition to the elementary particle forces, unification may include also naturally gravity. It is therefore a relevant question as to whether grand unification is realized in string theory, and if so, in what way does the GUT symmetry arise.

It is difficult to address these questions without some concrete models at hand. To make progress, it is important to develop techniques of constructing GUT models (as well as other extensions of the Standard Model) from string theory. With the insights gained from studying some concrete models, we can then examine whether string theory could shed new light on some of the long-standing problems in grand unification. Furthermore, we can contrast these constructions with string models that exhibit the Standard Model gauge symmetry at the string scale to understand what are the advantages and shortcomings of GUTs in the framework of string theory.

These issues surrounding grand unification have been explored extensively in the context of weakly coupled heterotic string [2]. In recent years, however, the emergence of M theory has opened up many new avenues for the construction of consistent string models. In particular, the advent of D-branes has allowed us to construct open string models that are non-perturbative from the dual heterotic string description [3]. The techniques of conformal field theory in describing D-branes and orientifold planes in exactly solvable backgrounds (especially orbifolds) have played a key role in the construction of four-dimensional chiral models with $\mathcal{N} = 1$ supersymmetry. There are two broad ways in which chiral theories can be constructed from D-branes. In the Type II orientifold models of Refs. [4–15], chiral fermions appear on the worldvolume of D-branes when the branes are located at singularities. In this context, an example of a three-family $SU(5)$ GUT model was constructed in [9]. However, in this model there are no Higgs fields either to break the $SU(5)$ gauge symmetry or to give rise to the $SU(2)$ Higgs doublets of the Standard Model. Hence the model is not fully realistic for further phenomenological studies.

Another context in which chiral fermions arise is when D-branes intersect at angles [16]. The spectrum of open strings stretched between the intersecting D-branes contains chiral fermions which are localized at the intersection. This fact was employed in [23–27] (and
subsequently in [28–31]) in the construction of non-supersymmetric brane world models. In particular, numerous examples of three-family Standard-like models as well as GUT models were obtained. However, the dynamics to determine the stability of non-supersymmetric models are not well understood, especially when the string scale is close to the Planck scale (since the non-supersymmetric models are subject to large quantum corrections). Typically, the models are unstable when D-branes are intersecting at angles (since supersymmetry is generically broken). Nevertheless, supersymmetric orientifold models with branes at angles have been constructed [17–19], resulting in the first examples of $\mathcal{N} = 1$ supersymmetric four-dimensional models with the quasi-realistic features of the Standard Model in this context. Subsequently, the phenomenological features of this class of models were explored in [20–22]. In addition to the Standard-like Models, an example of a supersymmetric $SU(5)$ GUT model with four families of quarks and leptons (i.e., a net number of four $10$-plets and four $\bar{5}$-plets) was presented in [18]. The purpose of this paper is to extend this analysis and further explore the possibilities of constructing more realistic supersymmetric $SU(5)$ models in this framework.

Just like the models in [17–19], the supersymmetric orientifold models considered here correspond in the strong coupling limit to compactifications of M theory on certain singular $G_2$ manifolds. As discussed in [19], the D-brane picture provides a simple description of how chiral fermions arise from singularities of $G_2$ compactifications [35–37,17,18]. More recently, there have been some interests in exploring the phenomenological properties (e.g., the problem of doublet-triplet splitting, threshold corrections, and proton decay) of GUT models derived from $G_2$ compactifications [38,39]. It is therefore interesting to explore if the features suggested in [38,39] apply to this class of orientifold models.

The purpose of this paper is few-fold. We shall systematize the techniques of orientifold constructions with intersecting branes to facilitate the search for realistic models. We consider the most general intersecting D6-brane configurations that are compatible with supersymmetry. We then perform a systematic search for three-family SU(5) GUT models within this framework for the case of $T^6/Z_2 \times Z_2$, and show that in this construction there are no three-family models (i.e., models containing three copies of $10$-plets) unless some $15$-plets are present. We therefore relax our criteria by allowing for the appearance of $15$-plets and systematically construct some three-family GUT models. We also briefly explore the

\[^1\text{Recently, a supersymmetric three-family left-right symmetric model based on } T^6/Z_4 \text{ orientifold was constructed [32].}\]
phenomenological features of these constructions.

This paper is organized as follows. In Section 2 we briefly summarize the constraints in constructing supersymmetric orientifold models with branes at angles. We have changed our notation slightly from [17,18] in order to simplify the consistency conditions (tadpole cancellations) and supersymmetry constraints. In Section 3, we classify the brane configurations that preserve supersymmetry. In Section 4, we discuss in detail how to perform a systematic search for the three-family $SU(5)$ models. The spectra of some of these models are tabulated in the appendix. In Section 5 we conclude and briefly discuss some physics implications as well as potential phenomenological difficulties of these models.

II. $Z_2 \times Z_2$ ORIENTIFOLD MODELS WITH BRANES AT ANGLES

The rules to construct supersymmetric type IIA orientifolds on $T^6/(Z_2 \times Z_2)$ with D6-branes at generic angles, and to obtain the spectrum of massless states were discussed in [18]. In this section we recall the essential points of the construction and emphasize some changes in notation which could greatly simplify the systematic search for consistent models.

We start with type IIA theory on $T^6/(Z_2 \times Z_2)$, where the orbifold group generators $\theta$, $\omega$ act on the complexified coordinates on $T^6$ as

$$
\theta : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3)
$$

$$
\omega : (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3). \tag{1}
$$

We assume $T^6$ can be written as a product of three two-tori.

We implement an orientifold projection by $\Omega R$, where $\Omega$ is world-sheet parity, and $R$ acts as

$$
R : (z_1, z_2, z_3) \rightarrow (\bar{z}_1, \bar{z}_2, \bar{z}_3). \tag{2}
$$

There are then four kinds of orientifold 6-planes (O6-planes), associated with the actions of $\Omega R$, $\Omega R\theta$, $\Omega R\omega$, and $\Omega R\theta\omega$, as shown in Figure 1.

The orientifold group acts on the Chan-Paton indices of the branes by the following actions

$$
\gamma_{\theta,a} = \text{diag} \left( i^{1_{N_a/2}}, -i^{1_{N_a/2}} ; -i^{1_{N_a/2}}, i^{1_{N_a/2}} \right)
$$

$$
\gamma_{\omega,a} = \text{diag} \left( \begin{pmatrix} 0 & 1_{N_a/2} \\ -1_{N_a/2} & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 1_{N_a/2} \\ -1_{N_a/2} & 0 \end{pmatrix} \right)
$$
The actions for the orbifold group form a projective representation as explained in [18].

To cancel the RR charge of the O6-planes, we introduce D6-branes wrapped on three-cycles that are products of one-cycles in each of the three two-tori (Figure 2). Let \([a_i], [b_i], i = 1, 2, 3,\) be a canonical basis of homology cycles. We consider \(K\) stacks of \(N_a\) D6-branes, \(a = 1, \ldots, K,\) wrapped on the \(n^i_{a}[a_i] + m^i_{a}[b_i]\) cycle in the \(i^{th}\) two-torus. The complex structure of the tori is arbitrary but it should be consistent with the orientifold projection. The only allowed possibilities then are shown in Figure 2. We can either have a rectangular torus (a) or a tilted torus (b) for which the lattice vectors are \(e'_1 = e_1 + e_2/2, e'_2 = e_2.\) If the above homology basis refers to a rectangular torus, the cycle \([a] + \frac{1}{2}[b]\) is not closed for a rectangular torus (Figure 3 (i)). However, it becomes closed for a tilted torus because the complex structure compensates for the offset (Figure 3 (ii)). Therefore, a generic one-cycle on a rectangular torus takes the form \(n^i_{a}[a_i] + m^i_{a}[b_i],\) where \(n^i_{a}, m^i_{a}\) are integers, while on a tilted torus it takes the form \(n^i_{a}[a'_i] + m^i_{a}[b_i],\) where \(n^i_{a}, m^i_{a}\) are again integers but \([a'_i] = [a_i] + \frac{1}{2}[b_i]\) in terms of the rectangular torus cycles. So the one-cycles on tilted tori can be written as \(n^i_{a}[a_i] + \tilde{m}^i_{a}[b_i],\) where \(\tilde{m}^i_{a} = m^i_{a} + n^i_{a}/2\) is a half integer. It is convenient to describe rectangular and tilted tori cycles in a common notation and to this end we introduce

\[
l^i_{a} \equiv m^i_{a}, \text{ rectangular,} \quad l^i_{\tilde{a}} \equiv 2\tilde{m}^i_{a} = 2m^i_{a} + n^i_{a}, \text{ tilted.} \tag{4}\]

FIG. 1. O6-planes in the orientifold of \(T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2).\)
FIG. 2. The D6-branes wrap one-cycles on each two-torus. The one-cycles make an angle $\theta$ with the $\Omega R$ orientifold plane which lies along the $R_1$-axis on all three two-tori. The tori can be rectangular (a) or tilted (b).

FIG. 3. The cycle $[a] + \frac{1}{2}[b]$, depicted as cycle C, is not closed for an untilted torus (i). However, for a tilted torus (ii) the complex structure makes C a closed cycle $[a']$. 
With this definition we can label a generic one-cycle on either a rectangular or a tilted torus by \((n^i_a, l^i_a)\). Note that for a tilted torus \(l - n\) is necessarily even. In addition, to avoid multiply wrapped branes we require that \(m\) and \(n\) are relatively coprime, for both tilted and untitled tori. Under an \(\Omega R\) reflection a cycle \((n^i_a, l^i_a)\) is mapped to \((n^i_a, -l^i_a)\). So, in order to implement the orientifold projection at the level of the spectrum, for a stack of \(N_a\) D6-branes along cycle \((n^i_a, l^i_a)\) we also need to include their images with wrapping numbers \((n^i_a, -l^i_a)\). For branes on top of the O6-planes we also count branes and their images independently.

As discussed above, the homology three-cycles for stack \(a\) of \(N_a\) D6-branes and its orientifold image \(a'\) are given by

\[
[\Pi_a] = \prod_{i=1}^{3} (n^i_a[a_i] + 2^{-\beta_i}l^i_a[b_i]), \quad [\Pi_a'] = \prod_{i=1}^{3} (n^i_a[a_i] - 2^{-\beta_i}l^i_a[b_i])
\]  
(5)

where \(\beta_i = 0\) if the \(i\)th torus is not tilted and \(\beta_i = 1\) if it is tilted. The homology three-cycles wrapped by the four orientifold planes are

\[
\Omega R : [\Pi_1] = 8[a_1] \times [a_2] \times [a_3], \quad \Omega R\omega : [\Pi_2] = -2^{3-\beta_2-\beta_3}[a_1] \times [b_2] \times [b_3]
\]

\[
\Omega R\theta\omega : [\Pi_3] = -2^{3-\beta_1-\beta_3}[b_1] \times [a_2] \times [b_3], \quad \Omega R\theta : [\Pi_4] = -2^{3-\beta_1-\beta_2}[b_1] \times [b_2] \times [a_3]
\]

and we define \([\Pi_{O6}] = [\Pi_1] + [\Pi_2] + [\Pi_3] + [\Pi_4]\). The intersection numbers of the various homology cycles are easily computed using the fact that the canonical homology one-cycles obey the Grassmann algebra \([a_i][b_j] = -[b_j][a_i] = \delta_{ij}, \quad [a_i][a_j] = [b_i][b_j] = 0\). One finds

\[
I_{ab} = [\Pi_a][\Pi_b] = 2^{-k} \prod_{i=1}^{3} (n^i_a l^i_b - n^i_b l^i_a), \quad I_{ab'} = [\Pi_a][\Pi_{b'}] = -2^{-k} \prod_{i=1}^{3} (n^i_a l^i_b + n^i_b l^i_a)
\]

\[
I_{aa'} = [\Pi_a][\Pi_{a'}] = -2^{3-k} \prod_{i=1}^{3} (n^i_a l^i_{a'}), \quad I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{3-k} (-l^1_a l^2_a l^3_a + l^1_a n^2_a n^3_a + n^1_a l^2_a n^3_a + n^1_a n^2_a l^3_a)
\]

where \(k = \beta_1 + \beta_2 + \beta_3\) is the total number of tilted tori.

As is shown in Figure 1 there are two orientifold planes that wrap around each of the two non-contractible cycles in a rectangular torus. So, if all tori are rectangular there are eight orientifold planes of each type. For a tilted torus, however, one of the two possible positions for the \([b]\)-cycle is lost (fig. 4). So, depending on how many and which tori are tilted, there could be less than eight orientifold planes of the types \(\Omega R\theta, \Omega R\omega, \text{ and } \Omega R\theta\omega\).

Equation 6 gives exactly the homology three-cycle of the four types of orientifold planes \(\text{times}\) their multiplicity for an arbitrary number of tilted tori. Note that this normalization
is different from that used in [18], where the multiplicity of the planes was not included in this definition of the cycles. The new definition allows the tadpole cancellation and supersymmetry conditions to be expressed in a form that is independent of the number of tilted tori. The results, of course, are insensitive to which convention one uses.

The open string spectrum of these constructions for branes at generic angles was discussed in detail in [18]. We summarize the results in table I. Notice that because of the change of notation from [17,18], the multiplicities of $\mathfrak{w}$ and $\mathfrak{b}$ in the $aa' + a'a$ sector have slightly different expressions.

A. Tadpole conditions

Cancellation of the Ramond-Ramond charge requires that the total homology cycle, weighted by the D6-brane and O6-plane (-4 in D6-brane units) charge, vanishes. That is

$$\sum_a N_a[\Pi_a] + \sum_a N_a[\Pi_{a'}] - 4[\Pi_{O6}] = 0.$$  \hfill (8)

It is useful to introduce the products of wrapping numbers

$$A_a = -n_a^1 n_a^2 n_a^3 \quad B_a = n_a^1 l_a^2 l_a^3 \quad C_a = l_a^1 n_a^2 l_a^3 \quad D_a = l_a^1 l_a^2 n_a^3$$

$$\tilde{A}_a = -l_a^1 l_a^2 l_a^3 \quad \tilde{B}_a = l_a^1 n_a^2 n_a^3 \quad \tilde{C}_a = n_a^1 l_a^2 n_a^3 \quad \tilde{D}_a = n_a^1 n_a^2 l_a^3$$  \hfill (9)

It is then straightforward to rewrite this as the set of equations

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\(^2\text{See Note Added at the end of the paper.}\)
| Sector  | Representation                       |
|---------|---------------------------------------|
| $aa$    | $U(N_a/2)$ vector multiplet            |
|         | 3 Adj. chiral multiplets              |
| $ab + ba$ | $I_{ab}$ ($\blacksquare$, $\blacksquare$) fermions |
| $ab' + b'a$ | $I_{ab'}$ ($\blacksquare$, $\blacksquare$) fermions |
| $aa' + a'a$ | $-\frac{1}{2}(I_{aa'} - \frac{1}{2}I_{a,O6})$ $\blacksquare$ fermions |
|         | $-\frac{1}{2}(I_{aa'} + \frac{1}{2}I_{a,O6})$ $\blacksquare$ fermions |

TABLE I. General spectrum on D6-branes at generic angles (namely, not parallel to any O6-plane in all three tori). The spectrum is valid for both tilted and untilted tori. The models may contain additional non-chiral pieces in the $aa'$ sector and in $ab$, $ab'$ sectors with zero intersection, if the relevant branes overlap. In supersymmetric situations, scalars combine with the fermions given above to form chiral supermultiplets.

$$\sum_a N_a A_a = \sum_a N_a B_a = \sum_a N_a C_a = \sum_a N_a D_a = -16.$$ (10)

At this point we can introduce an arbitrary number of branes wrapping cycles along the orientifold planes, so called ‘filler branes’, which contribute to the tadpole conditions but trivially satisfy the supersymmetry conditions as we shall see below. Table II shows the wrapping numbers of the four O6-planes. Each of the O6-planes has only one of $A, B, C, D$ equal to $-2^k$ and the rest are zero. So, if we consider $N^{(1)}$ branes wrapped along the first orientifold plane, $N^{(2)}$ along the second and so on, the tadpole conditions are modified to

$$-2^k N^{(1)} + \sum_a N_a A_a = -2^k N^{(2)} + \sum_a N_a B_a =$$

$$-2^k N^{(3)} + \sum_a N_a C_a = -2^k N^{(4)} + \sum_a N_a D_a = -16.$$ (11)

Note that the tadpole conditions are symmetric in $A, B, C$ and $D$.

B. Conditions for supersymmetric brane configuration

The condition for preserving $N = 1$, D=4 supersymmetry is that the angle of rotation of any D-brane with respect to the orientifold plane is an element of $SU(3)$, i.e., the matrix
of rotation acting on the complexified compact coordinates has unit determinant. In other words we require that $\theta_1 + \theta_2 + \theta_3 = 0 \mod 2\pi$ where $\theta_i$ is the angle with the R-invariant axis of the $i$th torus as shown in Figure 2. This is equivalent to $\sin(\theta_1 + \theta_2 + \theta_3) = 0$, and $\cos(\theta_1 + \theta_2 + \theta_3) > 0$. We can easily express the angles $\theta_i$ in terms of the one-cycle wrapping numbers on the $i$th torus as

$$
\sin \theta_i = \frac{2^{-\beta_i} l_i R_i^i}{L_i(n^i, l^i)}, \quad \cos \theta_i = \frac{n_i R_i^i}{L_i(n^i, l^i)},
$$

(12)

where $L_i(n^i, l^i) = \sqrt{(2^{-\beta_i} l_i R_2^i)^2 + (n_i R_1^i)^2}$ is the length of the one-cycle wrapping the $i$th torus. Now one can formulate the supersymmetry conditions in terms of the variables introduced in equation (9). We obtain

$$
x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0
$$

$$
A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0
$$

(13)

where $x_A = \lambda$, $x_B = \lambda 2^{\beta_2 + \beta_3}/\chi_2 \chi_3$, $x_C = \lambda 2^{\beta_1 + \beta_3}/\chi_1 \chi_3$, $x_D = \lambda 2^{\beta_1 + \beta_2}/\chi_1 \chi_2$ and $\chi_i = (R_2/R_1)_i$ are the complex structure moduli. The positive parameter $\lambda$ has been introduced to put all the variables $A, B, C, D$ on an equal footing. However, among the $x_i$ only three of them are independent and in the end of the calculation we should express three of them in terms of the fourth. In contrast to the tadpole conditions, supersymmetry constrains each stack of branes individually and can therefore be used to classify all possible brane configurations that preserve supersymmetry. The problem of model building is then enormously simplified since there is only a finite number of building blocks one can possibly combine to construct consistent models.

TABLE II. Wrapping numbers of the four O6-planes.
We have seen that neither the tadpole nor the supersymmetry conditions differentiate among $A$, $B$, $C$ or $D$. Equivalence of $B$, $C$ and $D$ simply follows from the equivalence of the three tori. The equivalence between $A$ and the rest three variables is related to the configuration of the orientifold planes. One can interchange $A$ with one of the other variables by exchanging $l$ with $n$ for one torus and simultaneously replacing $l$ with $-n$ and $n$ with $-l$ for a second torus. Indeed this is precisely the transformation that exchanges the $\Omega R$-plane with one of the other three orientifold planes. Moreover, it is the presence of the orientifold planes that breaks the symmetry between the two axes of the two-tori and hence differentiates $A$, $B$, $C$ and $D$ from $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$. It is therefore clear that the symmetries between these variables will be different for different orbifold groups and can generically be determined from the configuration of orientifold planes.

III. CLASSIFICATION OF SUPERSYMMETRIC BRANE CONFIGURATIONS

Since at most one of $l$ or $n$ can be zero for each torus any brane configuration belongs to one of four possible classes. It can have a zero wrapping number in all three, two, one or none of the tori. We examine each case separately.

I Three zeros

Supersymmetry requires that at least one of $A$, $B$, $C$ or $D$ be non-zero, in fact negative. So either all $l$'s are zero or two $n$'s and an $l$ are zero. The value of the non-zero wrapping numbers is $\pm 1$ for a non-tilted torus and $\pm 2$ for a tilted torus as required to avoid multiply wrapped cycles. Their relative sign is such that the non-vanishing product, $A$, $B$, $C$ or $D$, is negative. We then find exactly four possible brane configurations, namely one of the orientifold planes and its three images under the other three orientifold planes. Therefore, up to $O6$-plane reflections there are just four inequivalent brane configurations, each one parallel to one of the orientifold planes. These are precisely the ‘filler’ branes which, as advertised, do not constrain the moduli and we have already included the effect of an arbitrary number of each of them in the tadpole conditions.

II Two zeros

Each wrapping number enters in two of the products $A$, $B$, $C$ and $D$ and two of $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$. Since not both $l$ and $n$ can be zero for a given torus we necessarily have one and only one of $A$, $B$, $C$ or $D$ and one of $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ or $\tilde{D}$ non-vanishing. But
then, supersymmetry requires that the latter vanishes and hence there is a third zero which contradicts the hypothesis of two zero wrapping numbers. Therefore there are no supersymmetric configurations with two zero wrapping numbers.

III One zero

Here precisely two of $A$, $B$, $C$ and $D$ and two of $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$ are zero. There are six cases depending on which wrapping number we choose to be zero. As an example let $n^1 = 0$ so that $A = B = \tilde{C} = \tilde{D} = 0$. The identity $CD = -\tilde{A}\tilde{B}$ together with the supersymmetry conditions then imply that

$$C < 0, \ D < 0, \ \tilde{A}\tilde{B} < 0, \ x_A/x_B = -\tilde{B}/\tilde{A}.$$  \hspace{1cm} (14)

We refer to this type of supersymmetric brane configuration as ‘type III’.

IV No zeros

Since no wrapping number is zero here we have $A\tilde{A} = B\tilde{B} = C\tilde{C} = D\tilde{D} = \text{constant} \neq 0$. The supersymmetry conditions then require that one and only one of $A$, $B$, $C$ or $D$ is positive and

$$x_A/A + x_B/B + x_C/C + x_D/D = 0.$$  \hspace{1cm} (15)

We refer to this type of supersymmetric configuration as ‘type IV’.

We are now in a position to start looking for consistent models by combining stacks of type III and type IV branes such that the tadpole conditions, as well as the supersymmetry conditions for the moduli are satisfied. Before proceeding however it is instructive to think about the compatibility of different type III and type IV configurations. Consider first two stacks of type IV, one with, say, $A_1 > 0$ and one with $B_2 > 0$. Each of them gives an equation for the moduli which can only have common solution provided

$$A_1B_2 < B_1A_2.$$  \hspace{1cm} (16)

The case $A_1 > 0$, $A_2 > 0$ requires that two of $|B_1/A_1|$, $|C_1/A_1|$, $|D_1/A_1|$ are smaller than their counterparts for the second stack and the third is bigger, or vice versa. Next consider one stack of type IV with $A_1 > 0$ and a second one of type III. These two configurations are always compatible unless the type III configuration has $A_2 = B_2 = 0$ in which case $x_A/x_B = -\tilde{B}_2/\tilde{A}_2$ and a common solution is only possible if

$$|A_1/B_1| < -\tilde{B}_2/\tilde{A}_2.$$  \hspace{1cm} (17)
Finally consider two stacks of type III. These are generically compatible unless they fix the same moduli ratio, say \( x_A/x_B \), to different values.

We could go on and consider general compatibility conditions among three stacks of branes and so on but the number of cases to be considered increases considerably and it seems more economical to examine case by case as it arises. However it is important to realize at this point that generically three stacks of type III and/or type IV configurations completely fix the three moduli and so there is no freedom of adding a fourth stack of branes. We therefore consider only configurations of up to three stacks of type III and/or type IV branes. Nevertheless, one could imagine the possibility of a consistent model with more than three stacks of branes not parallel to the orientifold planes and with subsets of these stacks giving the same equations for the moduli. A special case of this possibility is realized when a single stack of branes is split into two parallel stacks by moving the position of the three-cycle in one or more of the tori. This special case is taken into account by simply treating the number of branes in each stack as an arbitrary parameter to be fixed by the tadpole conditions as we do in the analysis below. However, since all images of a given brane configuration under the \( O_6 \)-planes contribute in exactly the same way in the tadpole and supersymmetry conditions (any \( O_6 \)-plane reflection leaves \( A, B, C \) and \( D \) invariant and changes the sign of \( \tilde{A}, \tilde{B}, \tilde{C} \) and \( \tilde{D} \)), our analysis automatically accounts for this slightly more general case as well, which cannot be obtained by the splitting mechanism described above. To completely exclude the possibility of more than three stacks not parallel to the orientifold planes one needs to show that a fourth stack necessarily gives a moduli equation that cannot be written as a linear combination of those of the first three stacks.

IV. SEARCH FOR SUPERSYMMETRIC THREE-FAMILY SU(5) GUTS

In the standard SU(5) GUT, the quarks and leptons are embedded in the \( 10 \) and \( \overline{5} \) of SU(5). We are therefore interested in models containing a stack with at least five branes which has three copies of antisymmetric matter, i.e. \( n_{\overline{5}} = \pm 3 \). Let us investigate the consequences of this constraint. From Table I and Equation (7) one sees that

\[
\begin{align*}
    n_{\overline{5}} &= 2^{1-k}[(2A - 1)\tilde{A} - \tilde{B} - \tilde{C} - \tilde{D}], \\
    n_{\overline{5}} &= 2^{1-k}[(2A + 1)\tilde{A} + \tilde{B} + \tilde{C} + \tilde{D}]
\end{align*}
\]  

(18)

These are symmetric in \( A, B, C \) and \( D \) due to the identity \( A\tilde{A} = B\tilde{B} = C\tilde{C} = D\tilde{D} \). For a filler brane both these expressions vanish identically and so the U(5) stack of branes must be either type III or type IV. We examine the latter case first.
A. Type IV brane

Without loss of generality we take $A > 0$, $\tilde{A} > 0$. Then,

$$ n_B = 2^{1-k}(2|A| - 1)|\tilde{A}| + |\tilde{B}| + |\tilde{C}| + |\tilde{D}| \geq 2^{3-k}. \quad (19) $$

We immediately conclude that $k = 2$ or $k = 3$.

**k=2** Here $(2|A| - 1)|\tilde{A}| + |\tilde{B}| + |\tilde{C}| + |\tilde{D}| = 6$ and we need to consider the four-partitions of 6. There are just two, $6 = 3 + 1 + 1 + 1 = 2 + 2 + 1 + 1$. We identify the following inequivalent possibilities

(i) $(2|A| - 1)|\tilde{A}| = 3$, $|\tilde{B}| = |\tilde{C}| = |\tilde{D}| = 1$
(ii) $(2|A| - 1)|\tilde{A}| = 1$, $|\tilde{B}| = 3$, $|\tilde{C}| = |\tilde{D}| = 1$
(iii) $(2|A| - 1)|\tilde{A}| = |\tilde{D}| = 1$, $|\tilde{B}| = |\tilde{C}| = 2$
(iv) $(2|A| - 1)|\tilde{A}| = |\tilde{B}| = 2$, $|\tilde{C}| = |\tilde{D}| = 1$

The first three possibilities are trivially excluded since any non-trivial factor must appear simultaneously in two of the products $A$, $B$, $C$ and $D$ and in two of $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$. However, the last possibility passes this elementary test with the solution $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3) = (-1, -2) \times (-1, -1) \times (-1, -1)$ or $(A, B, C, D) = (1, -1, -2, -2)$ and $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) = (2, -2, -1, -1)$. Since for a tilted torus $n - l$ is even we see that the second and third tori are tilted while the first is not. Next we consider the tadpole conditions. To obtain a U(5) gauge group we need a stack of at least 10 coincident branes but we could have more and then separate the additional branes by the splitting mechanism discussed above. Allowing for $N$ extra branes we obtain

$$ \sum_a N_a A_a = 2(-13 - N/2 - 2N^{(1)}), \quad \sum_a N_a B_a = 2(-3 + N/2 + 2N^{(2)}), $$

$$ \sum_a N_a C_a = 2(2 + N + 2N^{(3)}) > 0, \quad \sum_a N_a D_a = 2(2 + N + 2N^{(4)}) > 0. \quad (20) $$

The only way to satisfy the last two conditions is to add at least two extra stacks of type IV, one with $C_1 > 0$ and a second with $D_2 > 0$. As we have seen above this can only be consistent provided $C_1D_2 < D_1C_2$. We have now increased the total number of non-trivial stacks to three and so we assume we cannot obtain a solution by adding more stacks. The last two tadpole conditions then imply $C_1D_2 > D_1C_2$ which contradicts the above inequality. Therefore there is no solution for $k=2$. 

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Here \((2|A| - 1)|\tilde{A}| + |\tilde{B}| + |\tilde{C}| + |\tilde{D}| = 12\) and we need to consider the four-partitions of 12. There are fifteen such partitions but since all tori are tilted only partitions with all terms either even or odd must be kept. There are just five of those, 

\[
12 = 7 + 3 + 1 + 1 = 4 + 4 + 2 + 2 = 3 + 3 + 3 + 3 = 5 + 5 + 1 + 1 = 5 + 3 + 3 + 1.
\]

The first three can easily be excluded since they cannot be realized by the coefficients \(\tilde{A}, \tilde{B}, \tilde{C}\) and \(\tilde{D}\). Each of the last two partitions gives a unique solution as follows

(i) \((A, B, C, D) = (1, -1, -5, -5),\) \((\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) = (5, -5, -1, -1)\)

(ii) \((A, B, C, D) = (3, -3, -1, -1),\) \((\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) = (1, -1, -3, -3)\)

The first case can be shown to be impossible by an argument similar to that used to exclude case (iv) for \(k=2\) above. The key property is that the tadpole conditions again require the addition of two stacks of type IV which cannot satisfy simultaneously the tadpole and supersymmetry conditions.

The second case however is much more interesting. Remarkably enough it is the only possible configuration that has simultaneously 3 antisymmetric \((10)\) and no symmetric \((15)\) multiplets and therefore comes closer to a realistic SU(5) GUT. Unfortunately it turns out, after considerably more effort than in the previous cases, that again there is no solution, at least with up to three non-trivial stacks of branes. The tadpole conditions in this case are

\[
\sum_a n_a A_a = -2(23 + 3N/2 - 4N^{(1)}), \quad \sum_a n_a B_a = 2(7 + 3N/2 + 4N^{(2)}) > 0, \\
\sum_a n_a C_a = -2(3 - N/2 - 4N^{(3)}), \quad \sum_a n_a D_a = -2(3 - N/2 - 4N^{(4)}).
\]

The second condition requires the addition of a second stack of type IV with \(B_1 > 0\). Compatibility with the \(U(5)\) stack then implies \(|B_1| < |A_1|\). Moreover, the last two tadpole conditions require \(N^{(3)} = N^{(4)} = 0\) and \(N = 0, 2, 4\). Inserting then the first two tadpole conditions into the above inequality one obtains \(4 - N^{(1)} - N^{(2)} > 0\). That is either both \(N^{(1)}\) and \(N^{(2)}\) are zero or one of them is zero and the other 2. Considering then the cases \(N = 0, 2, 4\) separately one easily sees that there is no solution with just two non-trivial stacks unless we are willing to accept the limiting case \(|B_1| = |A_1|\) corresponding to setting two of the moduli to zero. A similar analysis can then be repeated for three non-trivial stacks. There are quite a few cases one needs to consider since the third brane is not constrained a priori and can be either type III or type IV. In either case the supersymmetry conditions from all three stacks
become very constraining and one can systematically show that there is no solution, except for limiting cases similar to the one mentioned above. We have therefore shown that there is no three-family U(5) model from type IV configurations. We turn next to the alternative case of type III configurations.

B. Type III brane

Without loss of generality we take $A = B = \bar{C} = \bar{D} = 0$ i.e. $n^1 = 0$. Then

$$n_1 = -n_1 = \pm 2^{1-k}(|\tilde{A}| - |\tilde{B}|)$$

and so we need $|\tilde{A}| - |\tilde{B}| = \pm 3 \times 2^{k-1}$. Hence $k \geq 1$. Note that for type III brane we necessarily have the same number of symmetric and antisymmetric multiplets which means that any model we construct in this category will have three copies of 15-plets. The tadpole conditions are

$$-2^k N^{(1)} + \sum_a N_a A_a = 0, \quad -2^k N^{(2)} + \sum_a N_a B_a = 0,$$

$$-2^k N^{(3)} + \sum_a N_a C_a = (10 + N)|C| - 16, \quad -2^k N^{(4)} + \sum_a N_a D_a = (10 + N)|D| - 16.$$  (23)

If $|C| = |D| = 1$ then $|\tilde{A}| = |\tilde{B}| = 1$ as well which violates the three families condition. Therefore at least one of $|C|$ and $|D|$ is greater than zero. There are two cases

(i) $|C| > 1$ and $|D| > 1$

(ii) $|C| = 1$ and $|D| > 1$

For the first case the tadpole conditions force us to add two stacks of type IV branes, one with $C_1 > 0$ and one with $D_2 > 0$. One easily concludes then that this case is impossible following an argument similar to the ones used to exclude case (iv), k=2 and case (i), k=3 in the previous section. For the second case we first note that $|C| = 1$ implies $|l^1| = 1$ and since $n^1 = 0$ we conclude that the first torus cannot be tilted. Hence, either k=1 or k=2. The tadpole conditions require that we add a second stack of type IV branes with $D_1 > 0$ and we may add a third stack which must necessarily have $C_2 \leq 0$. $C_2 > 0$ is excluded for the same reason as case (i). In any case one finds $|\tilde{A}| = |l^3|$, $|\tilde{B}| = |n^3|$, $|D| = |l^2 n^3|$. The three families condition then implies $|D| = m(m + 3 \times 2^{k-1}) \geq 4$, where $m$ is a positive integer. At this point one can start a systematic search for solutions by first looking at the case of two stacks of non-trivial branes and then considering all possibilities for a third stack.
We have performed such a systematic search and we list all the models found, together with their spectra, in the appendix. We have found all 10 possible solutions for the case of two stacks of non-trivial branes (models I) and all 149 solutions with a third stack of type III (modes II, III and IV). We also list one solution with a third stack of type IV (model V). It is in general much harder to systematically look for solutions of this class mainly because there are more variables to be fixed and the supersymmetry conditions, although more constraining, are harder to implement as a useful criterion to be used in the search and must instead be checked in the end. Finally, we would like to mention that models I differ qualitatively from the rest in that they have one modulus free. All other models completely fix all three moduli.

V. CONCLUSIONS

In this paper, we have explored the possibility of constructing realistic $SU(5)$ Grand Unified string models from Type IIA orientifolds on $T^6/Z_2 \times Z_2$. Due to the strong constraints from supersymmetry and tadpole cancellations, we found that within this construction there are no three-family supersymmetric $SU(5)$ models that are free of $15$-plets. We then relax our criteria by allowing the appearance of $15$-plets and systemically study the three-family supersymmetric $SU(5)$ models constructed in this framework. The models are not fully realistic as there are a number of phenomenological challenges that they have to face. First of all, under $SU(3) \times SU(2) \times U(1)_Y$, the $15$ representation decomposes as follows:

$$SU(5) \supset SU(3) \times SU(2) \times U(1)_Y$$

$$15 = (6,1)(-\frac{2}{3}) + (1,3)(+1) + (3,2)(+\frac{1}{6})$$

(24)

Therefore, the models we consider here contain in addition to the Standard Model particles some exotic chiral matter fields. Furthermore, in the brane construction, the Standard Model particles are not only charged under the $SU(5)$ gauge group but also an additional $U(1)$ which is the center of mass motion of the stack of five D-branes (so the gauge group is actually $U(5)$). In the minimal $SU(5)$ model\(^3\) in which the only Higgs fields are in the $24$ and $5$ representations, the fermion masses come from the Yukawa couplings: $10 \ 10 \ 5_H$ and

\(^3\)In models with Higgs fields transforming in higher dimensional reps. of SU(5) such as $45$ rep., the fermion masses can come from operators other than $10 \ 10 \ 5_H$ and $5 \ 10 \ \overline{5}_H$. However, the perturbative open string sector does not give rise to such higher dimensional representations.
\( \mathbf{5} \mathbf{10} \mathbf{5}_H \) where the subscript \( H \) stands for the Higgs fields. While the operator \( \mathbf{5} \mathbf{10} \mathbf{5}_H \) is allowed by \( U(1) \) charge conservation, the \( U(1) \) charge carried by \( \mathbf{10} \mathbf{10} \mathbf{5}_H \) is non-zero and hence forbidden\(^4\).

One of the motivations for constructing GUT models in the framework of intersecting D-branes is to explore if there are some novel ways of solving some of the long-standing problems in GUTs, e.g., the doublet-triplet splitting problem [40]. The orientifold models considered here when lifted to M theory correspond to \( G_2 \) compactifications. Therefore, it would be interesting to see if the mechanism suggested in [38] can be applied to these models. The basic idea in [38] is that the GUT symmetry can be broken by Wilson lines in such a way that the doublet and the triplet have different discrete quantum numbers. The discrete symmetry forbid a mass term for the doublet whereas the mass term for the triplet is allowed. Therefore, the doublet could remain light even though the triplet receives a GUT scale mass. However, the Wilson lines in the present context are continuous rather than discrete. Only for some special choice of the continuous parameters of the Wilson lines do we obtain the aforementioned discrete quantum numbers.

Although the models we presented are not fully realistic, the techniques that we developed in analyzing the supersymmetric constraints and tadpole cancellations could easily be applied to the search for realistic models in other orientifold constructions (e.g., \( T^6/\Gamma \) where \( \Gamma \) is a discrete symmetry of \( SU(3) \) other than \( \mathbb{Z}_2 \times \mathbb{Z}_2 \)). The fact that we treat the tilted and rectangular tori in a symmetric manner greatly simplifies the search for solutions to the constraints. For simplicity, in the search for Standard-like models in [17–19], we assume that the angles that the D6-branes are rotated with respect to the orientifold plane take the form of \( (\theta_1, \theta_2, 0) \), \( (0, \theta_2, \theta_3) \) or \( (\theta_1, 0, \theta_3) \). It would be interesting to explore other realistic Standard-like models from more general D6-brane configurations. We hope to return to these problems in the future.

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\(^4\)The \( \mathbf{10} \mathbf{10} \mathbf{5}_H \) coupling vanishes for the same reason in the non-supersymmetric \( SU(5) \) model in [29].
Institute for Theoretical Physics for hospitality during the final stage of this work. Research supported in part by DOE grant DOE-FG02-95ER40893 (MC, IP), NATO linkage grant No. 97061 (MC), National Science Foundation Grant No. INT02-03585 (MC, IP, GS), Class of 1965 Endowed Term Chair (MC), and funds from the University of Wisconsin (GS).

_Note Added_

After this article was published we noticed that the sign of the multiplicities of the symmetric and antisymmetric representations in Table 1 should be reversed. That is there are $\frac{1}{2}(I_{aa'} - \frac{1}{2}I_{a,O6})$ fermions and $\frac{1}{2}(I_{aa'} + \frac{1}{2}I_{a,O6})$ fermions in the $aa' + a'a$ sector. As a consequence, the sign of the multiplicities of the symmetric and antisymmetric representations should be reversed for all models in the Appendix, but all models remain valid solutions to the constraints we imposed.
REFERENCES

[1] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974) 438.
H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).
For a recent review, see S. Raby, Phys. Rev. D 66, 010001 (2002) and references therein.

[2] For a review, see Z. Kakushadze, G. Shiu, S. H. Tye and Y. Vtorov-Karevsky, Int. J. Mod. Phys. A 13, 2551 (1998) and references therein.

[3] J. Polchinski and E. Witten, Nucl. Phys. B 460, 525 (1996) [arXiv:hep-th/9510169].

[4] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Ya.S. Stanev, Phys. Lett. B 385 (1996) 96.

[5] M. Berkooz and R.G. Leigh, Nucl. Phys. B 483 (1997) 187.

[6] Z. Kakushadze and G. Shiu, Phys. Rev. D 56 (1997) 3686; Nucl. Phys. B 520 (1998) 75;
Z. Kakushadze, Nucl. Phys. B 512 (1998) 221; Z. Kakushadze, G. Shiu and S. H. Tye, Nucl. Phys. B 533, 25 (1998).

[7] G. Zwart, Nucl. Phys. B 526 (1998) 378; D. O’Driscoll, hep-th/9801114;

[8] G. Shiu and S.-H.H. Tye, Phys. Rev. D58 (1998) 106007.

[9] J. Lykken, E. Poppitz and S. P. Trivedi, Nucl. Phys. B 543, 105 (1999).

[10] G. Aldazabal, A. Font, L.E. Ibáñez and G. Violero, Nucl. Phys. B 536 (1999) 29.

[11] Z. Kakushadze, Phys. Lett. B 434 (1998) 269; Phys. Rev. D 58 (1998) 101901; Nucl. Phys. B 535 (1998) 311.

[12] M. Cvetič, M. Plümiacher and J. Wang, JHEP 0004 (2000) 004.

[13] M. Klein, R. Rabadán, JHEP 0010 (2000) 049.

[14] M. Cvetič, A. M. Uranga and J. Wang, Nucl. Phys. B 595, 63 (2001).

[15] G. Aldazabal, L. E. Ibanez, F. Quevedo and A. M. Uranga, hep-th/0005067.

[16] M. Berkooz, M. R. Douglas and R. G. Leigh, Nucl. Phys. B 480 (1996) 265.

[17] M. Cvetic, G. Shiu and A. M. Uranga, Phys. Rev. Lett. 87, 201801 (2001).

[18] M. Cvetič, G. Shiu and A. M. Uranga, Nucl. Phys. B 615, 3 (2001).
[19] M. Cvetić, G. Shiu and A. M. Uranga, hep-th/0111179.

[20] M. Cvetić, P. Langacker and G. Shiu, Phys. Rev. D 66, 066004 (2002) [arXiv:hep-ph/0205252].

[21] M. Cvetić, P. Langacker and G. Shiu, Nucl. Phys. B 642, 139 (2002) [arXiv:hep-th/0206115].

[22] M. Cvetić, P. Langacker and J. Wang, “Dynamical Supersymmetry Breaking in the Three Family Standard-like orientifold model”, in preparation.

[23] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, JHEP 0010 (2000) 006.

[24] G. Aldazabal, S. Franco, L. E. Ibáñez, R. Rabadán and A. M. Uranga, Journal of Mathematical Physics, vol. 42, number 7, p. 3103, hep-th/0011073; JHEP 0102 (2001) 047.

[25] R. Blumenhagen, B. Körs and D. Lüst, JHEP 0102 (2001) 030.

[26] L. E. Ibáñez, F. Marchesano and R. Rabadán, hep-th/0105155.

[27] C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, Phys. Lett. B 489 (2000) 223.

[28] S. Förste, G. Honecker and R. Schreyer, Nucl. Phys. B 593 (2001) 127; JHEP 0106 (2001) 004.

[29] R. Blumenhagen, B. Körs and D. Lüst, T. Ott, Nucl. Phys. B616 (2001) 3.

[30] D. Bailin, G. V. Kraniotis, and A. Love, Phys. Lett. B 530, 202 (2002); Phys. Lett. B 547, 43 (2002); hep-th/0210219; hep-th/0212112.

[31] C. Kokorelis, JHEP 0209, 029 (2002); JHEP 0208, 036 (2002); hep-th/0207234; JHEP 0211, 027 (2002); hep-th/0210200.

[32] R. Blumenhagen, L. Görlich and T. Ott, arXiv:hep-th/0211059.

[33] B. S. Acharya, hep-th/0011089.

[34] M. Atiyah, J. Maldacena and C. Vafa, hep-th/0011256.

[35] M. Atiyah and E. Witten, hep-th/0107177.

[36] E. Witten, hep-th/0108165.

[37] B. Acharya and E. Witten, hep-th/0109152.
[38] E. Witten, arXiv:hep-ph/0201018.

[39] T. Friedmann and E. Witten, arXiv:hep-th/0211269.

[40] For a review of the doublet-triplet splitting problem, see, e.g., L. Randall and C. Csaki, arXiv:hep-ph/9508208.
## Appendix

In the following, we tabulate all the models we have found together with their spectra. $a, b, c$ denote the stacks of D-branes not parallel to the orientifold planes, giving $U(N_a/2)$ gauge group, while $a', b', c'$ denote their $\Omega R$ image. 1,2,3,4 denote filler branes respectively along the $\Omega R$, $\Omega R\omega$, $\Omega R\theta\omega$ and $\Omega R\theta$ orientifold plane, resulting in a $USp(N^{(i)})$ gauge group. $N$ is the number of branes in each stack. The third column shows the wrapping number of the various branes. Although we do not specify in each model how many and which tori are tilted, this can be seen most easily from the wrapping numbers of the O6-planes. (2,0) or (0,2) signify a tilted torus (see Table II). For the very few cases where there are no filler branes (e.g. model II.1.1) we remind the reader that at least one torus is tilted for all these models and for a tilted torus $n$ and $l$ are either both odd or both even. The intersection numbers between the various stacks are given in the remaining columns to the right. For example, the intersection number $I_{ac}$ between stacks $a$ and $c$ is found in row $a$ column $c$. For convenience we also list the relation among the moduli imposed by the supersymmetry conditions, as well as the gauge group for each model. The numbering of the models reflects the order in which they were found in a systematic search and, to the extent this was possible, how closely the various models are related to one another. In particular, models I have two stacks of branes not parallel to the orientifold planes, models II, III and IV have a third stack of type III branes and model V has a third stack of type IV branes. In some cases (e.g. for models III.4.1-56) we have parametrized a number of closely related models (56 in this case) with one or more non-negative integers which must satisfy definite constraints, e.g. $I_{c/c}^2 = 24 - N^{(i)}$ in this example. These constraints are also shown in the tables.

| model I.1 | $U(5) \times U(1) \times USp(6)$ |
|-----------|---------------------------------|
| stack $N$ $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $b'$ | 1 |
| $a$ 10 $(0, -1) \times (1, 4) \times (1, 1)$ | 3 | -3 | 0 | 0 | 4 |
| $b$ 2 $(-2, 3) \times (-1, 4) \times (1, 1)$ | 27 | 69 | - | - | -12 |
| 1 6 $(1, 0) \times (1, 0) \times (2, 0)$ | $x_B = 4x_A$, $x_A + x_C/3 = x_D/12$ |

| model I.2 | $U(5) \times U(1) \times USp(4) \times USp(2) \times USp(6)$ |
|-----------|----------------------------------------------------------|
| stack $N$ $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $b'$ | 1 | 2 | 4 |
| $a$ 10 $(0, -1) \times (1, 1) \times (4, 1)$ | -3 | 3 | -12 | 1 | -4 | 0 |
| $b$ 2 $(-2, 3) \times (-1, 3) \times (2, 1)$ | 47 | 97 | - | -9 | -6 | 2 |
| 1 4 $(1, 0) \times (2, 0) \times (1, 0)$ | $x_A = 4x_B$, $7x_B + 2x_C = x_D/3$ |
| 2 2 $(1, 0) \times (0, 2) \times (0, -1)$ |
| 4 6 $(0, 1) \times (0, -2) \times (1, 0)$ |

| model I.3 | $U(5) \times U(1) \times USp(2) \times USp(2)$ |
|-----------|---------------------------------|
| stack $N$ $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $b'$ | 1 | 2 |
| $a$ 10 $(0, -1) \times (1, 7) \times (1, 1)$ | 3 | -3 | 10 | 8 | 7 | -1 |
| $b$ 2 $(-2, 3) \times (-1, 3) \times (3, 1)$ | 37 | 71 | - | -9 | -9 |
| 1 2 $(1, 0) \times (2, 0) \times (2, 0)$ | $x_B = 7x_A$, $4x_A + x_C = x_D/9$ |
| 2 2 $(1, 0) \times (0, 2) \times (0, -2)$ |
model I.4 \[ U(5) \times U(1) \times USp(2) \times USp(24) \]

| stack | \( N \) | \( (n^1, t^1) \times (n^2, t^2) \times (n^3, t^3) \) | \( \sigma \) | \( b \) | \( b' \) | 1 | 4 |
|-------|-------|----------------------------------|-------|-------|-------|---|---|
| \( a \) | 10 | \((0, -1) \times (1, 4) \times (1, 1)\) | 3 | -3 | 16 | 0 | 4 | 0 |
| \( b \) | 2 | \((-2, 3) \times (-1, 4) \times (3, 1)\) | 101 | 187 | -12 | 2 |
| 1 | 2 | \((1, 0) \times (1, 0) \times (2, 0)\) | \( x_B = 4x_A, \ 2x_A + x_C = x_D/16 \) |
| 4 | 24 | \((0, 1) \times (0, -1) \times (2, 0)\) |

model I.5 \[ U(5) \times U(1) \times U(2) \times USp(2) \times USp(2) \]

| stack | \( N \) | \( (n^1, t^1) \times (n^2, t^2) \times (n^3, t^3) \) | \( \sigma \) | \( b \) | \( b' \) | 1 | 2 |
|-------|-------|----------------------------------|-------|-------|-------|---|---|
| \( a \) | 10+2 | \((0, -1) \times (1, 4) \times (1, 1)\) | 3 | -3 | 8 | 0 | 4 | 0 |
| \( b \) | 4 | \((-1, 1) \times (-1, 3) \times (3, 1)\) | 4 | 42 | -4 | -1 |
| 1 | 2 | \((1, 0) \times (1, 0) \times (2, 0)\) | \( x_B = 4x_A, \ 5x_A + 3x_C = x_D/3 \) |
| 2 | 2 | \((1, 0) \times (0, 1) \times (0, -2)\) |

model I.6 \[ U(5) \times U(1) \times U(2) \times USp(24) \times USp(8) \]

| stack | \( N \) | \( (n^1, t^1) \times (n^2, t^2) \times (n^3, t^3) \) | \( \sigma \) | \( b \) | \( b' \) | 1 | 4 |
|-------|-------|----------------------------------|-------|-------|-------|---|---|
| \( a \) | 10+2 | \((0, -1) \times (1, 4) \times (1, 1)\) | 3 | -3 | 8 | 0 | 4 | 0 |
| \( b \) | 4 | \((-1, 1) \times (-1, 3) \times (3, 1)\) | 6 | 42 | -4 | -1 |
| 1 | 2 | \((1, 0) \times (1, 0) \times (2, 0)\) | \( x_B = 4x_A, \ 4x_A + 3x_C = x_D/4 \) |
| 4 | 8 | \((0, 1) \times (0, -1) \times (2, 0)\) |

model I.7 \[ U(5) \times U(1) \times U(1) \times USp(4) \]

| stack | \( N \) | \( (n^1, t^1) \times (n^2, t^2) \times (n^3, t^3) \) | \( \sigma \) | \( b \) | \( b' \) | 1 |
|-------|-------|----------------------------------|-------|-------|-------|---|
| \( a \) | 10+2 | \((0, -1) \times (1, 1) \times (4, 1)\) | -3 | 3 | -10 | -18 | 1 |
| \( b \) | 2 | \((-1, 1) \times (-1, -4) \times (2, 1)\) | 38 | 90 | -1 | -8 |
| 1 | 4 | \((1, 0) \times (2, 0) \times (1, 0)\) | \( x_A = 4x_B, \ 9x_B + 4x_C = x_D/2 \) |

model I.8 \[ U(5) \times U(1) \times U(1) \times USp(4) \]

| stack | \( N \) | \( (n^1, t^1) \times (n^2, t^2) \times (n^3, t^3) \) | \( \sigma \) | \( b \) | \( b' \) | 1 |
|-------|-------|----------------------------------|-------|-------|-------|---|
| \( a \) | 10+2 | \((0, -1) \times (1, 4) \times (1, 1)\) | 3 | -3 | 8 | 0 | 4 |
| \( b \) | 2 | \((-1, 1) \times (-1, 3) \times (3, 2)\) | 38 | 90 | -1 | -8 |
| 1 | 4 | \((1, 0) \times (1, 0) \times (2, 0)\) | \( x_B = 4x_A, \ 9x_B + 4x_C = x_D/2 \) |

model I.9 \[ U(5) \times U(1) \times U(1) \times USp(2) \times USp(8) \]

| stack | \( N \) | \( (n^1, t^1) \times (n^2, t^2) \times (n^3, t^3) \) | \( \sigma \) | \( b \) | \( b' \) | 1 | 4 |
|-------|-------|----------------------------------|-------|-------|-------|---|---|
| \( a \) | 10+2 | \((0, -1) \times (1, 1) \times (4, 1)\) | -3 | 3 | -5 | -21 | 1 | 0 |
| \( b \) | 2 | \((-1, 1) \times (-1, -4) \times (4, 2)\) | 60 | 132 | -1 | -8 | 2 |
| 1 | 2 | \((1, 0) \times (2, 0) \times (1, 0)\) | \( x_A = 4x_B, \ 19x_B + 12x_C = x_D \) |
| 4 | 8 | \((0, 1) \times (0, -2) \times (1, 0)\) |

model I.10 \[ U(5) \times U(1) \times U(1) \times USp(16) \]

| stack | \( N \) | \( (n^1, t^1) \times (n^2, t^2) \times (n^3, t^3) \) | \( \sigma \) | \( b \) | \( b' \) | 1 |
|-------|-------|----------------------------------|-------|-------|-------|---|
| \( a \) | 10+2 | \((0, -1) \times (1, 1) \times (4, 1)\) | -3 | 3 | 0 | -24 | 0 |
| \( b \) | 2 | \((-1, 1) \times (-1, 3) \times (3, 1)\) | 82 | 174 | -2 | 2 |
| 4 | 16 | \((0, 1) \times (0, -2) \times (1, 0)\) | \( x_A = 4x_B, \ 5x_B + 4x_C = x_D/4 \) |
| model II.1.1 | $U(5) \times U(1) \times U(1)$ |
|---|---|
| stack | $N$ $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | b $c$ $b'$ $c'$ |
| a | 10 $(0, -1) \times (1, 4) \times (1, 1)$ | 3 -3 0 24 0 0 |
| b | 2 $(-1, 3) \times (-1, 4) \times (1, 1)$ | 6 42 -0 -96 |
| c | 2 $(-1, 0) \times (-1, 4) \times (7, 1)$ | -27 -27 - - - |

$x_B = 4x_A = 4x_C$

| model II.1.2 | $U(5) \times U(1) \times U(1) \times USp(2)$ |
|---|---|
| stack | $N$ $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | b $c$ $b'$ $c'$ |
| a | 10 $(0, -1) \times (1, 4) \times (1, 1)$ | 3 -3 0 16 0 0 4 |
| b | 2 $(-1, 3) \times (-1, 4) \times (1, 1)$ | 6 42 -0 -72 -12 |
| c | 2 $(-1, 0) \times (-1, 4) \times (5, 1)$ | -19 -19 - - - 0 |
| 1 | 2 $(1, 0) \times (1, 0) \times (2, 0)$ | $3x_B = 12x_A = 8x_C$ |

| model II.1.3 | $U(5) \times U(1) \times U(1) \times USp(4)$ |
|---|---|
| stack | $N$ $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | b $c$ $b'$ $c'$ |
| a | 10 $(0, -1) \times (1, 4) \times (1, 1)$ | 3 -3 0 8 0 0 4 |
| b | 2 $(-1, 3) \times (-1, 4) \times (1, 1)$ | 6 42 -0 -48 -12 |
| c | 2 $(-1, 0) \times (-1, 4) \times (3, 1)$ | -11 -11 - - - 0 |
| 1 | 4 $(1, 0) \times (1, 0) \times (2, 0)$ | $3x_B = 12x_A = 4x_C$ |

| model II.1.4 | $U(5) \times U(1) \times U(1) \times USp(2)$ |
|---|---|
| stack | $N$ $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | b $c$ $b'$ $c'$ |
| a | 10 $(0, -1) \times (1, 4) \times (1, 1)$ | 3 -3 0 18 0 8 -1 |
| b | 2 $(-1, 3) \times (-1, 4) \times (1, 1)$ | 6 42 -18 -72 -3 |
| c | 2 $(-1, 0) \times (-1, 2) \times (7, 1)$ | -13 -13 - - - 0 |
| 2 | 2 $(1, 0) \times (0, 1) \times (0, -2)$ | $7x_B = 28x_A = 20x_C$ |

| model II.1.5 | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(2)$ |
|---|---|
| stack | $N$ $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | b $c$ $b'$ $c'$ | 1 2 |
| a | 10 $(0, -1) \times (1, 4) \times (1, 1)$ | 3 -3 0 12 0 6 4 -1 |
| b | 2 $(-1, 3) \times (-1, 4) \times (1, 1)$ | 6 42 -12 -54 -3 |
| c | 2 $(-1, 0) \times (-1, 2) \times (5, 1)$ | -9 -9 - - - 0 0 |
| 1 | 2 $(1, 0) \times (1, 0) \times (2, 0)$ | $x_B = 4x_A = x_C$ |
| 2 | 2 $(1, 0) \times (0, 1) \times (0, -2)$ |

| model II.1.6 | $U(5) \times U(1) \times U(1) \times USp(4) \times USp(2)$ |
|---|---|
| stack | $N$ $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | b $c$ $b'$ $c'$ | 1 2 |
| a | 10 $(0, -1) \times (1, 4) \times (1, 1)$ | 3 -3 0 6 0 4 4 -1 |
| b | 2 $(-1, 3) \times (-1, 4) \times (1, 1)$ | 6 42 -6 -12 -36 -3 |
| c | 2 $(-1, 0) \times (-1, 2) \times (3, 1)$ | -5 -5 - - - 0 0 |
| 1 | 4 $(1, 0) \times (1, 0) \times (2, 0)$ | $3x_B = 12x_A = x_C$ |
| 2 | 2 $(1, 0) \times (0, 1) \times (0, -2)$ |
| model II.2.1 | $U(5) \times U(1) \times U(1) \times USp(6)$ |
|------------|----------------------------------|
| stack | $N (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $c$ | $b'$ | $c'$ | $d$ |
| $a$ | 10 | $(0, -1) \times (1, 4) \times (1, 1)$ | 3 | -3 | 0 | 18 | -2 | 8 | 0 |
| $b$ | 2 | $(-1, 3) \times (-1, 6) \times (1, 1)$ | 10 | 62 | -36 | -96 | 1 |
| $c$ | 2 | $(-1, 0) \times (-1, 2) \times (7, 1)$ | -13 | 13 | - | - | - | - | 1 |
| $d$ | 6 | $(0, 1) \times (0, -1) \times (2, 0)$ | $15x_B = 60x_A = 16x_C$ |

| model II.2.2 | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(6)$ |
|------------|----------------------------------|
| stack | $N (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $c$ | $b'$ | $c'$ | $d$ |
| $a$ | 10 | $(0, -1) \times (1, 4) \times (1, 1)$ | 3 | -3 | 0 | 12 | 6 | 4 | 0 |
| $b$ | 2 | $(-1, 3) \times (-1, 6) \times (1, 1)$ | 10 | 62 | -24 | -72 | -18 | 1 |
| $c$ | 2 | $(-1, 0) \times (-1, 2) \times (5, 1)$ | -9 | 9 | - | - | - | 0 | -1 |
| $d$ | 6 | $(0, 1) \times (0, -1) \times (2, 0)$ | $15x_B = 60x_A = 8x_C$ |

| model II.3.1 | $U(5) \times U(1) \times U(1) \times USp(6)$ |
|------------|----------------------------------|
| stack | $N (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $c$ | $b'$ | $c'$ | $d$ |
| $a$ | 10 | $(0, -1) \times (1, 1) \times (4, 1)$ | -3 | 3 | -4 | 6 | -6 | -20 | 0 |
| $b$ | 2 | $(-1, 3) \times (-1, 3) \times (2, 1)$ | 16 | 56 | -12 | -96 | 1 |
| $c$ | 2 | $(-1, 0) \times (-1, 5) \times (6, 1)$ | -29 | 29 | - | - | - | 1 |
| $d$ | 6 | $(0, 1) \times (0, -2) \times (1, 0)$ | $x_A = 4x_B = 16x_C$ |

| model II.3.2 | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(6)$ |
|------------|----------------------------------|
| stack | $N (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $c$ | $b'$ | $c'$ | $d$ |
| $a$ | 10 | $(0, -1) \times (1, 1) \times (4, 1)$ | -3 | 3 | -4 | 6 | -10 | -4 | 0 |
| $b$ | 2 | $(-1, 3) \times (-1, 3) \times (2, 1)$ | 16 | 56 | -6 | -72 | -6 | 1 |
| $c$ | 2 | $(-1, 0) \times (-1, 3) \times (6, 1)$ | -17 | 17 | - | - | - | 0 | 1 |
| $d$ | 6 | $(0, 1) \times (0, -2) \times (1, 0)$ | $x_A = 4x_B = 8x_C$ |

| model II.3.3 | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(6)$ |
|------------|----------------------------------|
| stack | $N (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $c$ | $b'$ | $c'$ | $d$ |
| $a$ | 10 | $(0, -1) \times (1, 1) \times (4, 1)$ | -3 | 3 | -4 | 6 | -16 | 1 | 0 |
| $b$ | 2 | $(-1, 3) \times (-1, 3) \times (2, 1)$ | 16 | 56 | -6 | -72 | -9 | 1 |
| $c$ | 2 | $(-1, 0) \times (-1, 5) \times (4, 1)$ | -19 | 19 | - | - | - | 0 | 1 |
| $d$ | 6 | $(0, 1) \times (0, -2) \times (1, 0)$ | $3x_A = 12x_B = 28x_C$ |

| model II.3.4 | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(6)$ |
|------------|----------------------------------|
| stack | $N (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $c$ | $b'$ | $c'$ | $d$ |
| $a$ | 10 | $(0, -1) \times (1, 1) \times (4, 1)$ | -3 | 3 | -4 | 6 | -16 | 1 | 0 |
| $b$ | 2 | $(-1, 3) \times (-1, 3) \times (2, 1)$ | 16 | 56 | -6 | -72 | -9 | 1 |
| $c$ | 2 | $(-1, 0) \times (-1, 5) \times (4, 1)$ | -19 | 19 | - | - | - | 0 | 1 |
| $d$ | 6 | $(0, 1) \times (0, -2) \times (1, 0)$ | $3x_A = 12x_B = 28x_C$ |
### model II.3.4

| stack | \( N (n^1, n^2) \times (n^2, n^3) \times (n^3, n^4) \) | \( b \) | \( c \) | \( b' \) | \( c' \) | \( 1 \) | \( 2 \) | \( 3 \) | \( 4 \) |
|-------|-------------------------------------------------|----------|----------|----------|----------|------|------|------|------|
| \( a \) | \( (0, -1) \times (1, 1) \times (4, 1) \) | -3 | -3 | -6 | -6 | -8 | 1 | -4 | 0 |
| \( b \) | \( (-1, 3) \times (-1, 3) \times (2, 1) \) | 16 | 56 | 0 | -54 | -9 | -6 | 1 |
| \( c \) | \( (-1, 0) \times (-1, 3) \times (4, 1) \) | -11 | 11 | - | - | - | 0 | 0 | -1 |

\[ x_A = 4x_B = 4x_C \]

### model II.3.5

| stack | \( N (n^1, n^2) \times (n^2, n^3) \times (n^3, n^4) \) | \( b \) | \( c \) | \( b' \) | \( c' \) | \( 1 \) | \( 2 \) | \( 3 \) | \( 4 \) |
|-------|-------------------------------------------------|----------|----------|----------|----------|------|------|------|------|
| \( a \) | \( (0, -1) \times (1, 1) \times (4, 1) \) | -3 | -3 | -6 | -6 | -12 | 1 | 0 |
| \( b \) | \( (-1, 3) \times (-1, 3) \times (2, 1) \) | 16 | 56 | 0 | -48 | -9 | -1 |
| \( c \) | \( (-1, 0) \times (-1, 5) \times (2, 1) \) | -9 | 9 | - | - | 0 | 0 | 1 |

\[ 3x_A = 12x_B = 8x_C \]

### model II.4.1

| stack | \( N (n^1, n^2) \times (n^2, n^3) \times (n^3, n^4) \) | \( b \) | \( c \) | \( b' \) | \( c' \) | \( 1 \) | \( 2 \) | \( 3 \) | \( 4 \) |
|-------|-------------------------------------------------|----------|----------|----------|----------|------|------|------|------|
| \( a \) | \( (0, -1) \times (1, 1) \times (4, 1) \) | -3 | -12 | -10 | -8 | 1 | 0 |
| \( b \) | \( (-1, 3) \times (-1, 5) \times (2, 1) \) | 30 | 90 | -12 | -96 | 1 |
| \( c \) | \( (-1, 0) \times (-1, 3) \times (6, 1) \) | -17 | 17 | - | - | - | 1 |

\[ 33x_A = 132x_B = 16x_C \]

### model II.4.2

| stack | \( N (n^1, n^2) \times (n^2, n^3) \times (n^3, n^4) \) | \( b \) | \( c \) | \( b' \) | \( c' \) | \( 1 \) | \( 2 \) | \( 3 \) | \( 4 \) |
|-------|-------------------------------------------------|----------|----------|----------|----------|------|------|------|------|
| \( a \) | \( (0, -1) \times (1, 1) \times (4, 1) \) | -3 | -6 | -12 | -8 | 1 | 0 |
| \( b \) | \( (-1, 3) \times (-1, 5) \times (2, 1) \) | 30 | 90 | -12 | -72 | -15 | 1 |
| \( c \) | \( (-1, 0) \times (-1, 3) \times (4, 1) \) | -11 | 11 | - | - | - | 0 | 1 |

\[ 33x_A = 132x_B = 4x_C \]

### model II.5

| stack | \( N (n^1, n^2) \times (n^2, n^3) \times (n^3, n^4) \) | \( b \) | \( c \) | \( b' \) | \( c' \) | \( 1 \) | \( 2 \) |
|-------|-------------------------------------------------|----------|----------|----------|----------|------|------|
| \( a \) | \( (0, -1) \times (1, 7) \times (1, 1) \) | 3 | -3 | 5 | 12 | 4 | 3 |
| \( b \) | \( (-1, 3) \times (-1, 3) \times (3, 1) \) | 14 | 40 | 3 | -48 |
| \( c \) | \( (-1, 0) \times (-1, 5) \times (5, 1) \) | -12 | 12 | - | - | - |

\[ 9x_B = 63x_A = 14x_C \]
| Model          | Stack | \(N(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) | \(a\)   | \(b\)   | \(c\)   | \(b'\)  | \(c'\)  |
|---------------|-------|--------------------------------------------------|--------|--------|--------|--------|--------|
| Model II.6    | stack | \(U(5) \times U(1) \times U(1) \times USp(24)\) | \(a\)  | \(b\)  | \(c\)  | \(b'\) | \(c'\) | 4      |
|               | 1     | \((0, -1) \times (1, 1) \times (1, 1)\)          | 3      | -3     | 8      | 16     | 0      | 0      |
|               | 2     | \((-1, 3) \times (-1, 4) \times (3, 1)\)         | 40     | 104    | -0     | -96    | 1      |
|               | 3     | \((-1, 0) \times (-1, 4) \times (5, 1)\)         | -19    | 19     | -1     | -1     | -1     |
|               | 4     | \((0, 1) \times (0, -1) \times (2, 0)\)          | 3\(x_B = 12x_A = 2x_C\) |
| Model II.7.1  | stack | \(U(5) \times U(1) \times U(1) \times USp(4)\)   | \(a\)  | \(b\)  | \(c\)  | \(b'\) | \(c'\) | 4      |
|               | 1     | \((0, -1) \times (1, 1) \times (4, 1)\)          | -3     | 3      | -6     | 4      | -12    | 0      |
|               | 2     | \((-1, 2) \times (-1, 5) \times (2, 1)\)         | 17     | 63     | -8     | -64    | 1      |
|               | 3     | \((-1, 0) \times (-1, 3) \times (6, 1)\)         | -17    | 17     | -1     | -1     | -1     |
|               | 4     | \((0, 1) \times (0, -2) \times (1, 0)\)          | 11\(x_A = 44x_B = 8x_C\) |
| Model II.7.2  | stack | \(U(5) \times U(1) \times U(1) \times USp(2) \times USp(4)\) | \(a\)  | \(b\)  | \(c\)  | \(b'\) | \(c'\) | 4      |
|               | 1     | \((0, -1) \times (1, 1) \times (4, 1)\)          | -3     | 3      | -6     | 0      | -12    | -8     | 1      | 0      |
|               | 2     | \((-1, 2) \times (-1, 5) \times (2, 1)\)         | 17     | 63     | -8     | -64    | 1      |
|               | 3     | \((-1, 0) \times (-1, 3) \times (4, 1)\)         | -11    | 11     | -1     | -1     | -1     |
|               | 4     | \((1, 0) \times (2, 0) \times (1, 0)\)           | 11\(x_A = 44x_B = 2x_C\) |
| Model II.1    | stack | \(U(5) \times U(1) \times U(1) \times USp(2) \times USp(2)\) | \(a\)  | \(b\)  | \(c\)  | \(b'\) | \(c'\) | 4      |
|               | 1     | \((0, -1) \times (1, 1) \times (4, 1)\)          | -3     | 3      | -6     | 1      | -12    | 0      | -4     | 0      |
|               | 2     | \((-2, 3) \times (-1, 3) \times (2, 1)\)         | 47     | 97     | -5     | -2     | -6     | 2      |
|               | 3     | \((-1, 1) \times (-1, 1) \times (1, 0)\)         | 0      | 0      | -1     | -1     | 0      |
|               | 4     | \((1, 0) \times (0, 2) \times (0, -1)\)          | \(x_A = 4x_B = 4x_C\) |
| Model II.2    | stack | \(U(5) \times U(1) \times U(2) \times USp(2)\)   | \(a\)  | \(b\)  | \(c\)  | \(b'\) | \(c'\) | 2      |
|               | 1     | \((0, -1) \times (1, 1) \times (4, 1)\)          | -3     | 3      | -6     | 1      | -12    | 0      | -4     | 0      |
|               | 2     | \((-2, 3) \times (-1, 3) \times (2, 1)\)         | 47     | 97     | -5     | -2     | -6     | 2      |
|               | 3     | \((-1, 1) \times (-1, 3) \times (1, 0)\)         | -5     | 5      | -1     | -1     | -1     |
|               | 4     | \((1, 0) \times (0, 2) \times (0, -1)\)          | \(x_A = 4x_B = 24x_C\) |
| Model II.3    | stack | \(U(5) \times U(1) \times U(2) \times USp(2)\)   | \(a\)  | \(b\)  | \(c\)  | \(b'\) | \(c'\) | 2      |
|               | 1     | \((0, -1) \times (1, 1) \times (4, 1)\)          | -3     | 3      | -6     | 1      | -12    | 0      | -4     | 0      |
|               | 2     | \((-2, 3) \times (-1, 3) \times (2, 1)\)         | 47     | 97     | -12    | 0      | -6     |
|               | 3     | \((-1, 1) \times (-1, 3) \times (1, 0)\)         | -1     | 1      | -1     | -1     | -3     |
|               | 4     | \((1, 0) \times (0, 2) \times (0, -1)\)          | 3\(x_A = 12x_B = 8x_C\) |
| model III.1.4 | $U(5) \times U(1) \times U(2) \times USp(2) \times USp(4)$ |
|----------------|--------------------------------------------------|
| stack | $N \times (n_1, l_1) \times (n_2, l_2) \times (n_3, l_3)$ | $\begin{array}{cccccc} a & b & c & b' & c' & 1 \\ 10 & 0 & -1 & (1,1) \times (4,1) & -3 & 3 -8 2 -12 -10 -14 0 1 \\ 2 & (-2,3) \times (-1,3) \times (2,1) & 47 & 97 & -8 & - & -8 & 6 \\ 4 & (2,1) \times (-1,1) \times (-0,0) & -1 & 1 & - & - & -1 & 0 \\ 2 & (1,0) \times (0,2) \times (0,1) & x_A = 4x_B = 8x_C \\ 4 & (0,1) \times (0,2) \times (1,0) & \end{array}$ |

| model III.1.5 | $U(5) \times U(1) \times U(2) \times USp(2) \times USp(2)$ |
|----------------|--------------------------------------------------|
| stack | $N \times (n_1, l_1) \times (n_2, l_2) \times (n_3, l_3)$ | $\begin{array}{cccccc} a & b & c & b' & c' & 1 \\ 10 & 0 & -1 & (1,1) \times (4,1) & -3 & 3 -8 2 -12 -11 -1 0 1 \\ 2 & (-2,3) \times (-1,3) \times (2,1) & 47 & 97 & 0 & -3 & -9 & -6 \\ 4 & (1,1) \times (-1,1) \times (-0,0) & 2 & 2 & - & - & - & 0 1 \\ 1 & (1,0) \times (2,0) \times (1,0) & x_A = 4x_B = 12x_C \\ 2 & (1,0) \times (0,2) \times (0,1) & \end{array}$ |

| model III.1.6 | $U(5) \times U(1) \times U(2) \times USp(2) \times USp(2)$ |
|----------------|--------------------------------------------------|
| stack | $N \times (n_1, l_1) \times (n_2, l_2) \times (n_3, l_3)$ | $\begin{array}{cccccc} a & b & c & b' & c' & 1 \\ 10 & 0 & -1 & (1,1) \times (4,1) & -3 & 3 -8 1 -12 -10 1 -1 \\ 2 & (-2,3) \times (-1,3) \times (2,1) & 47 & 97 & -9 & - & -9 & -6 \\ 4 & (1,3) \times (-1,1) \times (-0,0) & 2 & -2 & - & - & - & 0 3 \\ 1 & (1,0) \times (2,0) \times (1,0) & 3x_A = 12x_B = 4x_C \\ 2 & (1,0) \times (0,2) \times (0,1) & \end{array}$ |

| model III.1.7 | $U(5) \times U(1) \times U(2) \times USp(2) \times USp(2)$ |
|----------------|--------------------------------------------------|
| stack | $N \times (n_1, l_1) \times (n_2, l_2) \times (n_3, l_3)$ | $\begin{array}{cccccc} a & b & c & b' & c' & 1 \\ 10 & 0 & -1 & (1,1) \times (4,1) & -3 & 3 -8 1 -12 -10 0 1 -1 \\ 2 & (-2,3) \times (-1,3) \times (2,1) & 47 & 97 & -7 & - & 2 & -9 & -6 2 \\ 4 & (1,2) \times (-1,1) \times (-0,0) & -1 & 1 & - & - & - & 0 2 0 \\ 1 & (1,0) \times (2,0) \times (1,0) & x_A = 4x_B = 2x_C \\ 2 & (1,0) \times (0,2) \times (0,1) & \end{array}$ |

| model III.1.8 | $U(5) \times U(1) \times U(2) \times USp(2) \times USp(2)$ |
|----------------|--------------------------------------------------|
| stack | $N \times (n_1, l_1) \times (n_2, l_2) \times (n_3, l_3)$ | $\begin{array}{cccccc} a & b & c & b' & c' & 1 \\ 10 & 0 & -1 & (1,1) \times (4,1) & -3 & 3 -8 1 -12 -10 0 1 -4 0 \\ 2 & (-2,3) \times (-1,3) \times (2,1) & 47 & 97 & -5 & - & -2 & -9 & -6 2 \\ 4 & (1,1) \times (-1,1) \times (-0,0) & 0 & 0 & - & - & - & 0 1 0 \\ 1 & (1,0) \times (2,0) \times (1,0) & x_A = 4x_B = 4x_C \\ 2 & (1,0) \times (0,2) \times (0,1) & \end{array}$ |
### model III.2.2

| stack | $N(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $c$ | $b', c'$ | $x_B = 4x_A = 3x_C$ |
|-------|------------------------------------------------|-----|-----|-----|---------|----------------|
| $a$   | 10 $(-2, 3) \times (-1, 2) \times (3, 1)$ | 47  | 97  | -9  | -9      |                |
| $b$   | 2 $(-2, 3) \times (-1, 2) \times (3, 1)$ | 47  | 97  | -9  | -9      |                |
| $c$   | 2 $(1, 3) \times (1, 0) \times (0, -2)$  | 4   | -4  | -   | -       |                |
| 2     | $4 \times (0, 1) \times (0, -2)$ |    |     |     |         |                |

### model III.2.3

| stack | $N(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $c$ | $b', c'$ | $2x_B = 8x_A = x_C$ |
|-------|------------------------------------------------|-----|-----|-----|---------|----------------|
| $a$   | 10 $(-2, 3) \times (-1, 2) \times (3, 1)$ | 47  | 97  | -9  | -9      |                |
| $b$   | 2 $(-2, 3) \times (-1, 2) \times (3, 1)$ | 47  | 97  | -9  | -9      |                |
| $c$   | 2 $(1, 3) \times (1, 0) \times (0, -2)$  | 4   | -4  | -   | -       |                |
| 2     | $4 \times (0, 1) \times (0, -2)$ |    |     |     |         |                |
| 4     | $2 \times (0, 1) \times (0, -2)$ |    |     |     |         |                |

### model III.2.4

| stack | $N(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $c$ | $b', c'$ | $x_B = 4x_A = 2x_C$ |
|-------|------------------------------------------------|-----|-----|-----|---------|----------------|
| $a$   | 10 $(-2, 3) \times (-1, 2) \times (3, 1)$ | 47  | 97  | -9  | -9      |                |
| $b$   | 2 $(-2, 3) \times (-1, 2) \times (3, 1)$ | 47  | 97  | -9  | -9      |                |
| $c$   | 2 $(1, 3) \times (1, 0) \times (0, -2)$  | 4   | -4  | -   | -       |                |
| 2     | $4 \times (0, 1) \times (0, -2)$ |    |     |     |         |                |
| 4     | $2 \times (0, 1) \times (0, -2)$ |    |     |     |         |                |

### model III.2.5

| stack | $N(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $c$ | $b', c'$ | $x_B = 4x_A = x_C$ |
|-------|------------------------------------------------|-----|-----|-----|---------|----------------|
| $a$   | 10 $(-2, 3) \times (-1, 2) \times (3, 1)$ | 47  | 97  | -9  | -9      |                |
| $b$   | 2 $(-2, 3) \times (-1, 2) \times (3, 1)$ | 47  | 97  | -9  | -9      |                |
| $c$   | 2 $(1, 3) \times (1, 0) \times (0, -2)$  | 4   | -4  | -   | -       |                |
| 2     | $4 \times (0, 1) \times (0, -2)$ |    |     |     |         |                |
| 4     | $2 \times (0, 1) \times (0, -2)$ |    |     |     |         |                |

### model III.3.1

| stack | $N(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $c$ | $b', c'$ | $3x_A = 12x_B = 40x_C$ |
|-------|------------------------------------------------|-----|-----|-----|---------|----------------|
| $a$   | 10 $(-2, 3) \times (-1, 2) \times (3, 1)$ | 47  | 97  | -9  | -9      |                |
| $b$   | 2 $(-2, 3) \times (-1, 2) \times (3, 1)$ | 47  | 97  | -9  | -9      |                |
| $c$   | 2 $(1, 3) \times (1, 0) \times (0, -2)$  | 4   | -4  | -   | -       |                |
| 2     | $4 \times (0, 1) \times (0, -2)$ |    |     |     |         |                |
| 4     | $2 \times (0, 1) \times (0, -2)$ |    |     |     |         |                |

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| Stack | \( n^1, l^1 \) | \( n^2, l^2 \) | \( n^3, l^3 \) | \( a \) | \( b \) | \( c \) | \( b' \) | \( c' \) | \( 2 \) |
|------|----------------|----------------|----------------|-----|-----|-----|-----|-----|-----|
| \( U(5) \times U(1) \times U(1) \times USp(2) \) | \( a \) | \( 10 \) | \( (0, -1) \times (1, 1) \times (4, 1) \) | -3 | 3 | -4 | 2 | -14 | -4 | -4 |
|       | \( b \) | \( 2 \) | \( (-2, 3) \times (-1, 3) \times (3, 1) \) | 74 | 142 | -48 | -9 |
|       | \( c \) | \( 2 \) | \( (2, 5) \times (-1, 3) \times (-1, 0) \) | -1 | 1 | - | - | - | 5 |
|       | \( 2 \) | \( 2 \) | \( (1, 0) \times (0, 2) \times (0, -1) \) | 5x_A = 20x_B = 24x_C |

| Stack | \( n^1, l^1 \) | \( n^2, l^2 \) | \( n^3, l^3 \) | \( a \) | \( b \) | \( c \) | \( b' \) | \( c' \) | \( 2 \) |
|------|----------------|----------------|----------------|-----|-----|-----|-----|-----|-----|
| \( U(5) \times U(1) \times U(1) \times USp(2) \) | \( a \) | \( 10 \) | \( (0, -1) \times (1, 1) \times (4, 1) \) | -3 | 3 | -4 | 16 | -14 | -14 | -4 |
|       | \( b \) | \( 2 \) | \( (-2, 3) \times (-1, 3) \times (3, 1) \) | 74 | 142 | -48 | -36 | -9 |
|       | \( c \) | \( 2 \) | \( (2, 1) \times (-1, 15) \times (-1, 0) \) | -29 | 29 | - | - | - | 1 |
|       | \( 2 \) | \( 2 \) | \( (1, 0) \times (0, 2) \times (0, -1) \) | \( x_A = 4x_B = 120x_C \) |

| Stack | \( n^1, l^1 \) | \( n^2, l^2 \) | \( n^3, l^3 \) | \( a \) | \( b \) | \( c \) | \( b' \) | \( c' \) | \( 2 \) |
|------|----------------|----------------|----------------|-----|-----|-----|-----|-----|-----|
| \( U(5) \times U(1) \times U(1) \times USp(2) \) | \( a \) | \( 10 \) | \( (0, -1) \times (1, 1) \times (4, 1) \) | -3 | 3 | -4 | 2 | -14 | 0 | -4 |
|       | \( b \) | \( 2 \) | \( (-2, 3) \times (-1, 3) \times (3, 1) \) | 74 | 142 | -36 | -48 | -9 |
|       | \( c \) | \( 2 \) | \( (2, 15) \times (-1, 1) \times (-1, 0) \) | 13 | -13 | - | - | - | 15 |
|       | \( 2 \) | \( 2 \) | \( (1, 0) \times (0, 2) \times (0, -1) \) | \( 15x_A = 60x_B = 4x_C \) |

| Stack | \( n^1, l^1 \) | \( n^2, l^2 \) | \( n^3, l^3 \) | \( a \) | \( b \) | \( c \) | \( b' \) | \( c' \) | \( 2 \) |
|------|----------------|----------------|----------------|-----|-----|-----|-----|-----|-----|
| \( U(5) \times U(1) \times U(1) \times USp(2) \times USp(14) \) | \( a \) | \( 10 \) | \( (0, -1) \times (1, 1) \times (4, 1) \) | 3 | -3 | -4 | 2 | -14 | 0 | -10 |
|       | \( b \) | \( 2 \) | \( (-2, 3) \times (-1, 3) \times (3, 1) \) | 74 | 142 | -8 | -8 | -9 | 2 |
|       | \( c \) | \( 2 \) | \( (2, 1) \times (-1, 1) \times (-1, 0) \) | -1 | 1 | - | - | - | 10 |
|       | \( 2 \) | \( 2 \) | \( (1, 0) \times (0, 2) \times (0, -1) \) | \( x_A = 4x_B = 8x_C \) |
|       | \( 4 \) | \( 14 \) | \( (0, 1) \times (0, -2) \times (1, 0) \) |

| Stack | \( n^1, l^1 \) | \( n^2, l^2 \) | \( n^3, l^3 \) | \( a \) | \( b \) | \( c \) | \( b' \) | \( c' \) | \( 2 \) |
|------|----------------|----------------|----------------|-----|-----|-----|-----|-----|-----|
| \( U(5) \times U(1) \times U(1) \times USp(2) \times USp(6) \) | \( a \) | \( 10 \) | \( (0, -1) \times (1, 1) \times (4, 1) \) | 3 | -3 | -4 | -4 | -2 | -4 | 0 |
|       | \( b \) | \( 2 \) | \( (-2, 3) \times (-1, 3) \times (3, 1) \) | 74 | 142 | -0 | -9 | 2 |
|       | \( c \) | \( 2 \) | \( (2, 1) \times (-1, 1) \times (-1, 0) \) | -3 | 3 | - | - | - | 3 | 0 |
|       | \( 2 \) | \( 2 \) | \( (1, 0) \times (0, 2) \times (0, -1) \) | \( x_A = 4x_B = 8x_C \) |
|       | \( 4 \) | \( 6 \) | \( (0, 1) \times (0, -2) \times (1, 0) \) |
| model III.7 | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(12)$ |
|---|---|
| stack | $N \ (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $c$ | $b'$ | $c'$ | $n$ |
| $a$ | $(0, -1) \times (1, 1) \times (4, 1)$ | 10 | 3 | -3 | -4 | -14 | 4 | -10 | 0 | 0 | 0 | 0 |
| $b$ | $(-2, 3) \times (-1, 3) \times (3, 1)$ | 2 | 74 | 142 | -12 | -8 | -16 | 9 | 2 |
| $c$ | $(2, 1) \times (-1, 3) \times (-1, 0)$ | 2 | -5 | 5 | -5 | -1 | -1 | 1 | 0 |
| $x_A = 4x_B = 24x_C$ |

| model III.8 | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(12)$ |
|---|---|
| stack | $N \ (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $c$ | $b'$ | $c'$ | $n$ |
| $a$ | $(0, -1) \times (1, 1) \times (4, 1)$ | 10 | 3 | -3 | -4 | -2 | -4 | 0 |
| $b$ | $(-2, 3) \times (-1, 3) \times (3, 1)$ | 2 | 74 | 142 | -12 | -8 | -16 | 9 | 2 |
| $c$ | $(2, 1) \times (-1, 1) \times (-1, 0)$ | 2 | 1 | 1 | -1 | -3 | 0 |
| $x_A = 12x_B = 8x_C$ |

| model III.9 | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(10)$ |
|---|---|
| stack | $N \ (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $c$ | $b'$ | $c'$ | $n$ |
| $a$ | $(0, -1) \times (1, 1) \times (4, 1)$ | 10 | 3 | -3 | -4 | -6 | -14 | 4 |
| $b$ | $(-2, 3) \times (-1, 3) \times (3, 1)$ | 2 | 74 | 142 | -8 | -16 | -9 | 2 |
| $c$ | $(2, 1) \times (-1, 5) \times (-1, 0)$ | 2 | -9 | 9 | -3 | -1 | 1 |
| $x_A = 4x_B = 40x_C$ |

| model III.10 | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(10)$ |
|---|---|
| stack | $N \ (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $c$ | $b'$ | $c'$ | $n$ |
| $a$ | $(0, -1) \times (1, 1) \times (4, 1)$ | 10 | 3 | -3 | -4 | 2 | -14 | 0 | -4 | 0 |
| $b$ | $(-2, 3) \times (-1, 3) \times (3, 1)$ | 2 | 74 | 142 | -16 | 8 | -9 | 2 |
| $c$ | $(2, 5) \times (-1, 1) \times (-1, 0)$ | 3 | -3 | -3 | - | -5 | 0 |
| $5x_A = 20x_B = 40x_C$ |

| model III.11 | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(8)$ |
|---|---|
| stack | $N \ (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $c$ | $b'$ | $c'$ | $n$ |
| $a$ | $(0, -1) \times (1, 1) \times (4, 1)$ | 10 | 3 | -3 | -4 | 8 | -14 | 6 | -4 | 0 |
| $b$ | $(-2, 3) \times (-1, 3) \times (3, 1)$ | 2 | 74 | 142 | -16 | -8 | -9 | 2 |
| $c$ | $(2, 1) \times (-1, 7) \times (-1, 0)$ | 2 | -13 | 13 | -3 | -3 | 1 |
| $x_A = 4x_B = 56x_C$ |

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| Model | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(8)$ | Stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $n$ | $b$ | $c$ | $b'$ | $c'$ | $2$ | $4$ |
|-------|------------------------------------------------|-------|-----|-----------------|-----|-----|-----|-----|-----|-----|-----|
| **Model III.3.12** | | $a$ | 10 | $(0, -1) \times (1, 1) \times (4, 1)$ | 3 | -3 | -4 | 2 | -14 | 0 | -4 | 0 |
| | | $b$ | 2 | $(-2, 3) \times (-1, 3) \times (3, 1)$ | 74 | 142 | -20 | -16 | -9 | 2 |
| | | $c$ | 2 | $(2, 7) \times (-1, 1) \times (-1, 0)$ | 5 | -5 | - | - | 7 | 0 |
| | | 2 | $2 \times (0, 2) \times (0, -1)$ | \(7x_A = 28x_B = 8x_C\) |
| | | 4 | $(0, 1) \times (0, -2) \times (1, 0)$ |
| **Model III.3.13** | | $a$ | 10 | $(0, -1) \times (1, 1) \times (4, 1)$ | 3 | -3 | -4 | 10 | -14 | -8 | -4 | 0 |
| | | $b$ | 2 | $(-2, 3) \times (-1, 3) \times (3, 1)$ | 74 | 142 | -24 | -24 | -9 | 2 |
| | | $c$ | 2 | $(2, 1) \times (-1, 9) \times (-1, 0)$ | -17 | 17 | - | - | 1 | 0 |
| | | 2 | $2 \times (0, 2) \times (0, -1)$ | x_A = 4x_B = 72x_C |
| | | 4 | $(0, 1) \times (0, -2) \times (1, 0)$ |
| **Model III.3.14** | | $a$ | 10 | $(0, -1) \times (1, 1) \times (4, 1)$ | 3 | -3 | -4 | 2 | -14 | 0 | -4 | 0 |
| | | $b$ | 2 | $(-2, 3) \times (-1, 3) \times (3, 1)$ | 74 | 142 | -32 | -28 | -9 | 2 |
| | | $c$ | 2 | $(2, 9) \times (-1, 1) \times (-1, 0)$ | 7 | -7 | - | - | 9 | 0 |
| | | 2 | $2 \times (0, 2) \times (0, -1)$ | 9x_A = 36x_B = 8x_C |
| | | 4 | $(0, 1) \times (0, -2) \times (1, 0)$ |
| **Model III.3.15** | | $a$ | 10 | $(0, -1) \times (1, 1) \times (4, 1)$ | 3 | -3 | -4 | 12 | -14 | -10 | -4 | 0 |
| | | $b$ | 2 | $(-2, 3) \times (-1, 3) \times (3, 1)$ | 74 | 142 | -32 | -28 | -9 | 2 |
| | | $c$ | 2 | $(2, 1) \times (-1, 11) \times (-1, 0)$ | -21 | 21 | - | - | - | 1 | 0 |
| | | 2 | $2 \times (0, 2) \times (0, -1)$ | x_A = 4x_B = 88x_C |
| | | 4 | $(0, 1) \times (0, -2) \times (1, 0)$ |
| **Model III.3.16** | | $a$ | 10 | $(0, -1) \times (1, 1) \times (4, 1)$ | 3 | -3 | -4 | 2 | -14 | 0 | -4 | 0 |
| | | $b$ | 2 | $(-2, 3) \times (-1, 3) \times (3, 1)$ | 74 | 142 | -28 | -32 | -9 | 2 |
| | | $c$ | 2 | $(2, 11) \times (-1, 1) \times (-1, 0)$ | 9 | -9 | - | - | 11 | 0 |
| | | 2 | $2 \times (0, 2) \times (0, -1)$ | 11x_A = 44x_B = 8x_C |
| | | 4 | $(0, 1) \times (0, -2) \times (1, 0)$ |
| Model | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(2)$ |
|-------|-------------------------------------------------|
| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $n' | n' | b | c | b' | c' | 2 | 4 |
| a     | 10  | $(0, -1) \times (1, 1) \times (4, 1)$       | 3  | -3 | -4 | 14 | -14 | -12 | -4 | 0 |
| b     | 2   | $(-2, 3) \times (-1, 3) \times (3, 1)$      | 74 | 142 | -40 | -32 | -9 | 2 |
| c     | 2   | $(2, 1) \times (-1, 13) \times (-1, 0)$     | -25 | 25 | - | - | - | 1 | 0 |
| 2     | 2   | $(1, 0) \times (0, 2) \times (0, -1)$       | $x_A = 4x_B = 104x_C$ |
| 4     | 2   | $(0, 1) \times (0, -2) \times (1, 0)$       | $x_A = 4x_B = 104x_C$ |

| Model | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(2)$ |
|-------|-------------------------------------------------|
| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $n' | n' | b | c | b' | c' | 2 | 4 |
| a     | 10  | $(0, -1) \times (1, 1) \times (4, 1)$       | 3  | -3 | -4 | 2 | -14 | 0 | -4 | 0 |
| b     | 2   | $(-2, 3) \times (-1, 3) \times (3, 1)$      | 74 | 142 | -32 | -40 | -9 | 2 |
| c     | 2   | $(2, 13) \times (-1, 1) \times (-1, 0)$     | 11 | -11 | - | - | - | 13 | 0 |
| 2     | 2   | $(1, 0) \times (0, 2) \times (0, -1)$       | $13x_A = 52x_B = 8x_C$ |
| 4     | 2   | $(0, 1) \times (0, -2) \times (1, 0)$       | $13x_A = 52x_B = 8x_C$ |

| Model | $U(5) \times U(1) \times U(1) \times USp(N^{(4)})$ |
|-------|-------------------------------------------------|
| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $n' | n' | b | c | c' | 4 |
| a     | 10  | $(0, -1) \times (1, 1) \times (1, 1)$       | 3  | -3 | 16 | $4 + l^2_c$ | 0 | $4 - l^2_c$ | 0 |
| b     | 2   | $(-2, 3) \times (-1, 4) \times (3, 1)$      | 101 | 187 | - | $36 - 2N^{(4)}$ | 0 | $36 - 2N^{(4)}$ | 2 |
| c     | 2   | $(1, l^2_c) \times (-1, l^2_c) \times (-2, 0)$ | $2l^2_c - 2l^2_c$ | - | - | - | - | 0 |
| 4     | $N^{(4)}$ | $(0, 1) \times (0, -1) \times (2, 0)$ | $x_B = 4x_A = l^2_c x_C/l^1_c$, $l^1_c l^2_c = 24 - N^{(4)}$ |

| Model | $U(5) \times U(2) \times U(1) \times USp(N^{(4)})$ |
|-------|-------------------------------------------------|
| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $n' | n' | b | c | c' | 4 |
| a     | 10  | $(0, -1) \times (1, 1) \times (1, 1)$       | 3  | -3 | 8 | $4 + l^2_c$ | 0 | $4 - l^2_c$ | 0 |
| b     | 4   | $(-1, 4) \times (-1, 3) \times (3, 1)$      | 6  | 42 | - | $4 - N^{(4)}$ | - | $4 - N^{(4)}$ | 1 |
| c     | 2   | $(1, l^2_c) \times (-1, l^2_c) \times (-2, 0)$ | $2l^2_c - 2l^2_c$ | - | - | - | - | 0 |
| 4     | $N^{(4)}$ | $(0, 1) \times (0, -1) \times (2, 0)$ | $x_B = 4x_A = l^2_c x_C/l^1_c$, $l^1_c l^2_c = 8 - N^{(4)}$ |

| Model | $U(5) \times U(2) \times U(2) \times USp(2)$ |
|-------|-------------------------------------------------|
| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $n' | n' | b | c | b' | c' | 2 |
| a     | 10  | $(0, -1) \times (1, 1) \times (4, 1)$       | -3 | 3 | -2 | 1 | -7 | 0 | -4 |
| b     | 4   | $(-1, 1) \times (-1, 3) \times (3, 1)$      | 4  | 32 | - | -2 | 0 | -3 |
| c     | 4   | $(1, 1) \times (-1, 1) \times (-1, 0)$      | 0  | 0 | - | - | - | 1 |
| 2     | 2   | $(1, 0) \times (0, 2) \times (0, -1)$       | $x_A = 4x_B = 4x_C$ |
### model III.6.2  
\[ U(5) \times U(2) \times U(1) \times USp(2) \]

| stack | \(N\) | \((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) | \(n\) | \(n\) | \(l\) | \(b\) | \(b'\) | \(c\) | \(c'\) | \(2\) |
|--------|-------|---------------------------------|------|------|------|-----|------|-----|------|-----|
| \(a\)  | 10    | \((0, -1) \times (1, 4) \times (1, 1)\) | 3    | -3   | 5    | 2   | 3    | -1  |
| \(b\)  | 4     | \((-1, 1) \times (-1, 3) \times (3, 1)\) | 4    | 32   | -4   | -3  |
| \(c\)  | 2     | \((1, 1) \times (-1, 1) \times (-2, 0)\) | 0    | 0    | -    | -   | -    | 2   |
| 2      | 2     | \((1, 0) \times (0, 1) \times (0, -2)\) | \(x_B = 4x_A = x_C\) |       |      |      |      |

### model III.7.1-5  
\[ U(5) \times U(1) \times U(1) \times USp(2) \times USp(N(4)) \]

| stack | \(N\) | \((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) | \(n\) | \(n\) | \(l\) | \(b\) | \(b'\) | \(c\) | \(c'\) | \(2\) |
|--------|-------|---------------------------------|------|------|------|-----|------|-----|------|-----|
| \(a\)  | 10    | \((0, -1) \times (1, 1) \times (4, 1)\) | -3   | 3    | -2   | (1 + \(l_2^2\))/2 | -15 | (1 - \(l_2^2\))/2 | 0   |
| \(b\)  | 2     | \((-1, 1) \times (-1, 3) \times (7, 2)\) | 52   | 116  | -    | 2 - \(N(4)\) | -    | 2 - \(N(4)\) | 7   |
| \(c\)  | 2     | \((1, l_2^1) \times (-1, l_2^3) \times (-1, 0)\) | \(l_2^1\) | -    | \(l_2^1\) | -    | -    | \(l_2^3\) | 0   |
| 2      | 2     | \((1, 0) \times (0, 2) \times (0, -1)\) | \(x_A = 4x_B = 4l_2^1x_C/l_2^1\) |       |
| 4      | \(N(4)\) | \((0, 1) \times (0, -2) \times (1, 0)\) | \(l_2^1l_2^3 = 5 - N(4)\) |       |      |      |      |

### model III.8.1-6  
\[ U(5) \times U(1) \times U(2) \times USp(N(4)) \]

| stack | \(N\) | \((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) | \(n\) | \(n\) | \(l\) | \(b\) | \(b'\) | \(c\) | \(c'\) | \(4\) |
|--------|-------|---------------------------------|------|------|------|-----|------|-----|------|-----|
| \(a\)  | 10    | \((0, -1) \times (1, 1) \times (4, 1)\) | -3   | 3    | -5   | (1 + \(l_2^2\))/2 | -21 | (1 - \(l_2^2\))/2 | 0   |
| \(b\)  | 2     | \((-1, 1) \times (-2, 8) \times (3, 1)\) | 60   | 132  | -    | \(-N(4)/2\) | -    | \(-N(4)/2\) | 2   |
| \(c\)  | 4     | \((1, l_2^1) \times (-1, l_2^3) \times (-1, 0)\) | \(l_2^1\) | -    | \(l_2^1\) | -    | -    | -    | 0   |
| 4      | \(N(4)\) | \((0, 1) \times (0, -2) \times (1, 0)\) | \(x_A = 4x_B = 4l_2^1x_C/l_2^1\) |       |
| 4      | \(N(4)\) | \((0, 1) \times (0, -2) \times (1, 0)\) | \(l_2^1l_2^3 = 4 - N(4)/2\) |       |      |      |      |

### model III.9.1  
\[ U(5) \times U(1) \times U(1) \]

| stack | \(N\) | \((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) | \(n\) | \(n\) | \(l\) | \(b\) | \(b'\) | \(c\) | \(c'\) | \(\) |
|--------|-------|---------------------------------|------|------|------|-----|------|-----|------|-----|
| \(a\)  | 10    | \((0, -1) \times (1, 1) \times (4, 1)\) | -3   | 3    | -5   | 3   | -21  | -3  |
| \(b\)  | 2     | \((-1, 1) \times (-2, 8) \times (3, 1)\) | 60   | 132  | -    | -    | -    | 0   |
| \(c\)  | 2     | \((1, 1) \times (-2, 8) \times (-1, 0)\) | -6   | 6    | -    | -    | -    | -   |
| \(a\)  | 10    | \((0, -1) \times (1, 1) \times (4, 1)\) | -3   | 3    | -5   | 3   | -21  | -1  |
| \(b\)  | 2     | \((-1, 1) \times (-2, 8) \times (3, 1)\) | 60   | 132  | -    | -    | -    | 0   |
| \(c\)  | 2     | \((1, 1) \times (-2, 8) \times (-1, 0)\) | -2   | 2    | -    | -    | -    | 0   |
| 4      | 4     | \((0, 1) \times (0, -2) \times (1, 0)\) | \(x_A = 4x_B = 16x_C\) |       |      |      |      |
| model IV.1.1 | \(U(5) \times U(1) \times U(1) \times USp(4)\) |
| --- | --- |
| stack | \(N \quad (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) |
| \(a\) | 10 | \((0, -1) \times (1, 4) \times (1, 1)\) | 3 | -3 | 8 | 8 | 0 | 12 | -1 |
| \(b\) | 2 | \((-1, 1) \times (-1, 4) \times (3, 1)\) | 6 | 42 | -4 | -48 | -3 |
| \(c\) | 2 | \((-1, 2) \times (-1, 0) \times (5, 1)\) | -9 | 9 | - | - | -10 |
| \(2\) | 4 | \((1, 0) \times (0, 1) \times (0, -2)\) | 10 | \(x_B = 40x_A = x_D\) |

| model IV.1.2 | \(U(5) \times U(1) \times U(1) \times USp(2) \times USp(4)\) |
| --- | --- |
| stack | \(N \quad (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) |
| \(a\) | 10 | \((0, -1) \times (1, 4) \times (1, 1)\) | 3 | -3 | 8 | 4 | 0 | 8 | 4 | -1 |
| \(b\) | 2 | \((-1, 1) \times (-1, 4) \times (3, 1)\) | 6 | 42 | 0 | -36 | -4 | -3 |
| \(c\) | 2 | \((-1, 2) \times (-1, 0) \times (5, 1)\) | -5 | 5 | - | - | - | 0 | -6 |
| \(1\) | 2 | \((1, 0) \times (1, 0) \times (2, 0)\) | 6 | \(x_B = 24x_A = x_D\) |
| \(2\) | 4 | \((1, 0) \times (0, 1) \times (0, -2)\) |

| model IV.2.1 | \(U(5) \times U(1) \times U(1) \times USp(2) \times USp(6)\) |
| --- | --- |
| stack | \(N \quad (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) |
| \(a\) | 10 | \((0, -1) \times (1, 4) \times (1, 1)\) | 3 | -3 | 10 | -4 | 12 | -1 | 0 |
| \(b\) | 2 | \((-1, 1) \times (-1, 0) \times (3, 1)\) | 10 | 62 | 0 | -72 | -3 | 1 |
| \(c\) | 2 | \((-1, 2) \times (-1, 0) \times (5, 1)\) | -9 | 9 | - | - | -10 | 1 |
| \(2\) | 2 | \((1, 0) \times (0, 1) \times (0, -2)\) | 10 | \(x_B = 40x_A = x_D\) |
| \(4\) | 6 | \((0, 1) \times (0, -1) \times (2, 0)\) |

| model IV.2.2 | \(U(5) \times U(1) \times U(1) \times USp(2) \times USp(6)\) |
| --- | --- |
| stack | \(N \quad (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) |
| \(a\) | 10 | \((0, -1) \times (1, 4) \times (1, 1)\) | 3 | -3 | 10 | -4 | 8 | 4 | -10 |
| \(b\) | 2 | \((-1, 1) \times (-1, 0) \times (3, 1)\) | 10 | 62 | 0 | -54 | -3 | 1 |
| \(c\) | 2 | \((-1, 2) \times (-1, 0) \times (5, 1)\) | -5 | 5 | - | - | - | 0 | -6 | 1 |
| \(1\) | 2 | \((1, 0) \times (1, 0) \times (2, 0)\) | 6 | \(x_B = 24x_A = x_D\) |
| \(2\) | 2 | \((1, 0) \times (0, 1) \times (0, -2)\) |
| \(4\) | 6 | \((0, 1) \times (0, -1) \times (2, 0)\) |

| model IV.3.1 | \(U(5) \times U(1) \times U(1) \times USp(12)\) |
| --- | --- |
| stack | \(N \quad (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) |
| \(a\) | 10 | \((0, -1) \times (1, 4) \times (1, 1)\) | 3 | -3 | 16 | 8 | -16 | 12 | 0 |
| \(b\) | 2 | \((-1, 1) \times (-1, 8) \times (3, 1)\) | 22 | 122 | -12 | -144 | 1 |
| \(c\) | 2 | \((-1, 2) \times (-1, 0) \times (5, 1)\) | -9 | 9 | - | - | - | 1 |
| \(4\) | 12 | \((0, 1) \times (0, -1) \times (2, 0)\) | 10 | \(x_B = 40x_A = x_D\) |
| stack | $N$ | $n^1, l^1$ | $n^2, l^2$ | $n^3, l^3$ | $n$ | $b$ | $b'$ | $c'$ | 4  |
|-------|-----|-------------|-------------|-------------|-----|-----|-----|-----|----|

### model IV.3.2

#### $U(5) \times U(1) \times U(1) \times USp(2) \times USp(12)$

| $a$ | 10 | $(0, -1) \times (1, 4) \times (1, 1)$ | 3 | -3 | 16 | 8 | -16 | 12 | 4 | 0 |
| $b$ | 2  | $(-1, 1) \times (-1, 6) \times (3, 1)$ | 22 | 122 | -8 | 0 | -108 | -12 | 1 |
| $c$ | 2  | $(-1, 2) \times (-1, 0) \times (5, 1)$ | -5 | 5  | - | - | - | 0 | 1 |
| 1   | 2  | $(1, 0) \times (1, 0) \times (2, 0)$ |   | 6$x_B = 24x_A = x_D$ |
| 4   | 12 | $(0, 1) \times (0, -1) \times (2, 0)$ |   |   |

### model IV.1

#### $U(5) \times U(1) \times U(1) \times USp(28)$

| $a$ | 10 | $(0, -1) \times (1, 4) \times (1, 1)$ | 3 | -3 | 24 | 4 | -12 | 8 | 0 |
| $b$ | 2  | $(-1, 1) \times (-1, 8) \times (5, 1)$ | 28 | 132 | -8 | - | -96 | 1 |
| $c$ | 2  | $(-1, 2) \times (-1, 0) \times (3, 1)$ | -5 | 5  | - | - | - | -1 |
| 4   | 28 | $(0, 1) \times (0, -1) \times (2, 0)$ | 6$x_B = 24x_A = x_D$ |

### model IV.3.3

#### $U(5) \times U(1) \times U(1) \times USp(2) \times USp(18)$

| $a$ | 10 | $(0, -1) \times (1, 4) \times (1, 1)$ | 3 | -3 | 20 | 4 | -6 | 8 | -10 |
| $b$ | 2  | $(-1, 1) \times (-1, 6) \times (5, 1)$ | 20 | 100 | -6 | - | -72 | -51 |
| $c$ | 2  | $(-1, 2) \times (-1, 0) \times (3, 1)$ | -9 | 9  | - | - | -61 |
| 2   | 2  | $(1, 0) \times (0, 1) \times (0, -2)$ | 6$x_B = 24x_A = x_D$ |
| 4   | 18 | $(0, 1) \times (0, -1) \times (2, 0)$ |   |   |

### model IV.3.3

#### $U(5) \times U(1) \times U(1) \times USp(2) \times USp(8)$

| $a$ | 10 | $(0, -1) \times (1, 4) \times (1, 1)$ | 3 | -3 | 16 | 4 | 0 | 8 | -10 |
| $b$ | 2  | $(-1, 1) \times (-1, 6) \times (5, 1)$ | 12 | 68 | -4 | - | -48 | -51 |
| $c$ | 2  | $(-1, 2) \times (-1, 0) \times (3, 1)$ | -9 | 9  | - | - | -61 |
| 2   | 2  | $(1, 0) \times (0, 1) \times (0, -2)$ | 6$x_B = 24x_A = x_D$ |
| 4   | 8  | $(0, 1) \times (0, -1) \times (2, 0)$ |   |   |

### model IV.5.1

#### $U(5) \times U(2) \times U(1) \times U(1) \times USp(8)$

| $a$ | 10+2 | $(0, -1) \times (1, 4) \times (1, 1)$ | 3 | -3 | 12 | 8 | -8 | 12 | 0 |
| $b$ | 2  | $(-1, 1) \times (-1, 8) \times (3, 1)$ | 14 | 82 | -0 | - | -64 | 1 |
| $c$ | 2  | $(-1, 1) \times (-1, 0) \times (5, 1)$ | -4 | 4  | - | - | -1 |
| 4   | 8  | $(0, 1) \times (0, -1) \times (2, 0)$ | 5$x_B = 20x_A = x_D$ |

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### Model IV.5.2

| Stack | $N$ | $U(5) \times U(2) \times U(1) \times U(1) \times USp(2) \times USp(2)$ |
|-------|-----|---------------------------------------------------------------------|
| $a$   | 10+2 | $0, -1 \times (1, 4) \times (1, 1)$ |
| $b$   | 2    | $-1, 1 \times (-1, 6) \times (3, 1)$ |
| $c$   | 2    | $-1, 1 \times (-1, 0) \times (5, 1)$ |
| 2     | 2    | $(1, 0) \times (0, 1) \times (0, -2)$ |
| 4     | 2    | $(0, 1) \times (0, -1) \times (2, 0)$ |

### Model V

| Stack | $N$ | $U(5) \times U(1) \times U(1) \times USp(2) \times USp(2)$ |
|-------|-----|----------------------------------------------------------------|
| $a$   | 10  | $(0, -1) \times (1, 1) \times (4, 1)$ |
| $b$   | 2   | $(-1, 2) \times (-1, 1) \times (8, 1)$ |
| $c$   | 2   | $(1, -1) \times (-1, 7) \times (2, -1)$ |
| 1     | 2   | $(1, 0) \times (2, 0) \times (1, 0)$ |
| 4     | 2   | $(0, 1) \times (0, -2) \times (1, 0)$ |