The pretzelosity distribution function and intrinsic motion of the constituents in nucleon

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Abstract. The pretzelosity distribution function $h_{qT}^+$ is studied in a covariant the quark-parton model which describes the structure of the nucleon in terms of 3D quark intrinsic motion. This relativistic model framework supports the relation between helicity, transversity and pretzelosity observed in other relativistic models without assuming SU(6) spin-flavor symmetry. Numerical results and predictions for SIDIS experiments are presented.

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1. INTRODUCTION

Transverse parton momentum dependent parton distribution (TMDs) and fragmentation functions [1–6] offer the access to novel information on the nucleon structure [7]. TMDs can be accessed in processes like semi-inclusive deep-inelastic lepton nucleon scattering (SIDIS) [8]. Data on such reactions [9–14] provide first insights [18–24]. However, model calculations play an important role for the understanding of the novel functions [25–37].

An important question in this context is whether it is possible to relate unknown TMDs with possibly better known ones. Such relations cannot be exact, since all TMDs are independent. Approximations motivated partly by data were discussed in [38]. The ideal playground to motivate and test any such relations among TMDs are models.

An interesting relation between pretzelosity $h_{qT}^+$, transversity $h_q^T$ and helicity $g_q^T$ was observed in bag model [26]. The name pretzelosity reflects that this function ‘measures’ an appropriately defined deviation of the nucleon from spherical shape which could look like a pretzel [7]. This relation holds also in the spectator model [25], and was subsequently confirmed in the constituent quark model [27] but not in the model of [28].

The purpose of this work is to study pretzelosity, and its possible relations, in the covariant model of the nucleon of Ref. [30]. In this model the intrinsic motion of partons inside the nucleon is described in terms of a covariant momentum distribution. The model was applied to the study of unpolarized and polarized structure functions accessible in DIS $f_1^q(x)$, $g_1^q(x)$ and $g_2^q(x)$ [30, 31] and extended to compute transversity $h_q^T(x)$ [32]. In this work we will generalize the approach to the description of TMDs, focusing on chiral-odd TMDs accessible with transverse nucleon polarization.
2. CHIRAL-ODD TMDS WITH TRANSVERSE POLARIZATION

We focus on chiral-odd TMDS in a nucleon polarized transversely, e.g. in SIDIS, with respect to the hard virtual photon \( q^\mu = (q^0, |\vec{q}|, 0, 0) \). The light-front quark-correlator with the process-dependent Wilson-link \( \mathcal{W} \) [6] where \( z^\pm = (z^0 \pm z^1)/\sqrt{2} \) etc.,

\[
\phi(x, \vec{p}_T, \vec{S}_T)_{ij} = \int \frac{dz^-d^2z_T}{(2\pi)^3} e^{ipz} \langle P, \vec{S}_T | \psi_j(0) \mathcal{W}(0, z, \text{path}) \psi_i(z) | P, \vec{S}_T \rangle |_{z^+ = 0, p^+ = xP^+},
\]

allows to define (3 out of the 4) chiral-odd TMDS in the nucleon as follows

\[
\frac{1}{2} \text{tr} \left[ i\sigma^j \gamma_5 \phi(x, \vec{p}_T, \vec{S}_T) \right] = S_T^j h_1^i + \frac{(p_T^i p_T^k - \frac{1}{4} \vec{p}_T^2 \delta^{jk}) S_T^k}{M_N^2} h_1^i_T + \frac{\epsilon^{ijk} p_T^k}{M_N} h_1^i_T,
\]

where \( \epsilon^{32} = -\epsilon^{23} = 1 \) and zero else. The only structure surviving the \( \vec{p}_T \)-integration in (1) is transversity \( h_1^i(x) \). Nucleon polarizations and Dirac-structures other than that in Eqs. (1, 2) lead to further leading- and subleading-twist TMDS [3, 4, 16].

3. THE COVARIANT MODEL OF THE NUCLEON AND TMDS

The starting point for the calculation of the chiral-even functions accessible in DIS, \( f_1^u(x), g_1^u(x), g_2^u(x) \), is the hadronic tensor [30, 31]. In the model it is assumed that DIS can be described as the incoherent sum of the scattering of electrons off non-interacting quarks, whose momentum distributions inside the nucleon are given in terms of the scalar functions: \( G = G^\uparrow + G^\downarrow \) for unpolarized and \( H = G^\uparrow - G^\downarrow \) for polarized quarks.

\( G^\downarrow(pP/M) \) denotes the distribution of quarks of some (not indicated) flavour that are polarized parallel (antiparallel) \( \uparrow(\downarrow) \) to the \( i \)-axis, where \( p \) is the quark momentum and \( M \) the nucleon mass. Though all expressions can be formulated in a manifestly covariant way, it is convenient to work in the nucleon rest-frame, where the \( G^\downarrow \) become functions of \( p^0 = \sqrt{\vec{p}^2 + m^2} \) with \( m \) the quark mass, and the distributions are rotationally symmetric.

The chiral-odd \( h_1^i(x) \) cannot be accessed in DIS through the hadronic tensor. However, for theoretical purposes one may consider the auxiliary process described by the interference of a vector and a scalar current, described on the quark level by \( T_\alpha^q = \epsilon_{\alpha\beta\lambda\nu} p^\beta q^\lambda w^\nu \) where \( w^\nu \) is the quark polarization vector. The nucleon current follows from convoluting \( T_\alpha^q \) with the momentum distribution of polarized quarks \( H(p^0) \) and reads

\[
T_\alpha(x) = \frac{1}{2P^0} \epsilon_{\alpha\beta\lambda\nu} q^\lambda \int \frac{d^3p}{p^0} H(p^0) \delta \left( \frac{p^0-p^1}{M} - x \right) p^\beta w^\nu.
\]

The auxiliary current is related to transversity as

\[
2M T_\alpha(x) \epsilon^{\alpha\beta} = S_T^j h_1^q(x).
\]

Before attempting to extend the approach to TMDS, let us stress that the QCD definition of TMDS includes a Wilson line absent in our model with no gauge boson degrees
of freedom. In such an approach time-reversal (T) odd TMDs, such as the Boer-Mulders function $h_1^\perp$ in (2), vanish [5, 6].

Now we turn to the question how to extend the approach to describe of TMDs, focusing here on chiral-odd ones in a transversely polarized nucleon. For that we observe that the expression for the auxiliary current (3) is of the type: $T_\alpha(x) = \int d^2 p_T T_\alpha(x, \bar{p}_T)$. In the following we explore the consequences of what happens if one does not integrate out transverse momenta in this expression.

With $S^\mu$ denoting the nucleon polarization vector (here $S^\mu = (0, 0, \vec{S}_T)$ with $|\vec{S}_T| = 1$) the most general expression [31] for the covariant quark polarization vector $w^\mu$ reads

$$w^\mu = -\frac{pS}{pP + mM} p^\mu + S^\mu - \frac{M}{pP + mM} p^\mu .$$

From (5) we obtain for the 'unintegrated' auxiliary current contracted with $\epsilon^{\alpha j}$ the result

$$2 M T_\alpha(x, \bar{p}_T) \epsilon^{\alpha j} = \int \frac{dp_T^1}{p_0^1} H(p_0^0) \delta \left( \frac{p_0^1 - p_1^1}{M} - x \right) \left\{ S_T^j (p_0^0 - p_1^1) - p_T^j \vec{S}_T \frac{\vec{p}_T}{p_0^0 + m} \right\} .$$

By comparing to (2) we read off the following results:

$$h_1^q(x, p_T) = \int \frac{dp_T^1}{p_0^1} H(p_0^0) \delta \left( \frac{p_0^1 - p_1^1}{M} - x \right) \left[ p_0^0 - p_1^0 - \frac{\vec{p}_T^2}{2(p_0^0 + m)} \right] ,$$

$$h_{1T}^q(x, p_T) = \int \frac{dp_T^1}{p_0^1} H(p_0^0) \delta \left( \frac{p_0^1 - p_1^1}{M} - x \right) \left[ -\frac{M^2}{p_0^0 + m} \right] ,$$

and $h_1^\perp q(x, p_T) = 0$. Several comments are in order. First, in our approach the vanishing of the T-odd $h_1^\perp q$ is consistent. Second, integrating in Eq. (7) over $\vec{p}_T$ yields the model expression for $h_1^q(x) \equiv \delta q(x)$ from [32]. Third, $h_{1T}^q \neq 0$ implies non-sphericity in the nucleon in the sense of [7] inspite of a spherically symmetric $H(p_0)$. Forth, adding $h_1^q(x)$ and $h_{1T}^q(x) = \int d^2 p_T \frac{\vec{p}_T^2}{2M^2} h_{1T}^q(x, p_T)$ yields the model expression for $g_1^q(x) \equiv \Delta q(x)$ derived in [31], i.e. we recover the remarkable relation [26]:

$$g_1^q(x) - h_1^q(x) = h_{1T}^{(1)q}(x) .$$

This relation is satisfied in several [25–27] though not all [28] quark models. Remarkably, it follows in our approach without assuming SU(6) spin-flavour symmetry of the nucleon wave function as was done in [25–27]. This is an important observation: SU(6) is not a necessary condition for the relation (9) to be satisfied in a quark model. What is a necessary condition is the absence of gluon degrees of freedom [29].

Finally, we remark that in the chiral limit $m \to 0$ it is possible to relate the transverse moment of pretzelosity to the twist-3 parton distribution function $g_T^q(x)$ [31] as follows

$$h_{1T}^{(1)q}(x) + g_T^q(x) = \mathcal{O} \left( \frac{m}{M} \right) .$$

Since in the model the WW-relation holds, $g_T^q(x) = \int_1^1 dy g_T^q(y) / y + \mathcal{O} \left( \frac{m}{M} \right)$ [31], this offers a possibility to estimate pretzelosity numerically in the model framework.
4. RESULTS AND PHENOMENOLOGY

We estimate $h_{1T}^{(1)q}(x)$ in our approach using (10) and the WW-approximation for $g_T^q(x)$ with $g_T^q(x)$ at a scale of 2.5 GeV$^2$ from [39]. We obtain the results shown in Fig. 1a.

The azimuthal SSA from transversely polarized targets, $A_{UT}^{sin(3\phi_s-\phi)}$, allows to access pretzelosity in SIDIS due to the Collins effect [2], see [3, 26] for details. We use the information on $H_1$ from [20–22]. Fig. 1b shows that the model results for the SSA are compatible with preliminary COMPASS deuteron target data [14]. Fig. 1c shows estimates for the SSA in the kinematics of the CLAS 12 GeV beam experiment. The error projections from [40] included in the plot demonstrate that CLAS will be able to measure effects of pretzelosity of the size predicted by the model.

5. CONCLUSIONS

A generalization of the covariant model [30–32] to the description of TMDs was suggested, and applied to compute the pretzelosity distribution function $h_{1T}^{(1)q}$. In particular, it was shown that the relation between helicity, transversity and pretzelosity [26] is satisfied in this model — remarkably, without assuming SU(6) symmetry.

A numerical estimate of $h_{1T}^{(1)q}$ was presented, and used to compute $A_{UT}^{sin(3\phi_s-\phi)}$, the leading-twist SSA in SIDIS due to Collins effect and pretzelosity. The model results are compatible with the preliminary deuteron target data from COMPASS [14]. Predictions of this observable in the kinematics of the CLAS experiment with upgraded 12 GeV beam suggest that information on pretzelosity is accessible at Jefferson Lab [40].
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