The Analysis Research on the Scheme of Moving Functions in the NC Lathe Turning the Free Surface

Qiang Xu 1,a,*, Hai jian 1, b, Jun Wan 1, c, Vernon Baker 2, d

1 Department of Mechanical Engineering, Hefei University. Hefei 230601, China
2 Department of Manufacturing Engineering, University of Texas at Arl. Arlington 76019, USA

*a, *Corresponding author e-mail: xuq@hfuu.edu.cn, bjiangh@hfuu.edu.cn, cwangj@hfuu.edu.cn, dVernon@uta.edu

Abstract. In order to solve the design problem of the scheme of moving functions in the NC lathe turning the free surface, the work piece is cut into n equal parts along the height h, set up the continuous and smooth Centre line of the n equal parts, each equal part is compared of a generatrix and a guide line. Using the generative design principle, the mathematical describing matrixes of the generating line and the cutting point of turning tool are first established. Thus, determine the positional and gesture matrixes of the turning tool. Since the positional and gesture matrixes are achieved through the cascade matrixes, the possible motion schemes of the tandem machine tools for turning free-form surface is decided, a solid foundation is laid for optimizing the design of the CNC lathes and compound machine tools.

Keywords: free surface; generatrix; guide line; turning tool.

1. Introduction
With the development of science and technology, products with complex free surface is widely used in the fields in aviation, spacelift, mould and the other industries, research work on the machining of free form surfaces has become a hot spot in our country, the design of moving functions in machine tools is the first task of the machine tools design. The generation design method can find the all possible scheme of moving functions in machine tools, and many domestic scholars make generation design for motion schemes of machining tools used for machining plane, cylinder, irregular cylinder[1-3], the very little research on the generation design for machining free surface is made, the less research is executed in turning free surface. The article uses the generation design method, the generation design of the machining tools for turning free surface is decided, and all of possible motion schemes are comprehensively determined, a solid foundation is laid for the optimization design of the machine tools structure.
2. The determination of the position and posture matrices of turning tool for turning free surface

Figure 1 is a schematic of turning tool processing free surface. Suppose the free surface is cut into n equal parts along the height, the processing curves that each equal part corresponds to is as shown in Figure 2, the height of each equal part is $\Delta h$.

![Fig.1 The free surface turned by turning tool](image1)

Set central line L go through the centroid of the processing curves that each equal part corresponds to, the turning tool machines the curves (a certain height $h$), it processes the straight line $\Delta L$ along the slope of $\Delta L$ at the same time ($\Delta L$ is smaller in length and is regarded as the straight line approximately), the curve is the generatrix, the guide line is the line segment $\Delta L$ with the slope of $\Delta L$. The more n equal parts is, the smaller $\Delta h$ is, and the higher the accuracy machining free-form surface is.

3. The determination of the mathematical describing matrix of the turning free surface and the turning tool

Figure 2 is the processing curve at the height $h$, OW XW YW ZW is the work piece coordinate system, OW is the centroid and the rotation center of the curve, OS XS YS ZS is the coordinate system at the contact point between machining surface and the turning tool, the XS axis goes through the curvature center of the point s on the machining curve, the describing matrixes of the machining curve of Figure 2 is the transformation matrixes from OWXWYWZW to OSXSYSZS. The origin OW of OWXWYWZW is translated to the origin OS of OSXSYSZS, and OWXWYWZW turns $\theta$ around ZW, the describing matrix of the machining curve is obtained as follows[4]

According the characteristics of the turning tool, the describing matrix of the cutting point c on the turning tool in tool coordinate system $O_pX_pY_pZ_p$ below [5]
4. The determination of the positional and posture matrix of the turning tool

In NC machining, the reasonable angle between the work piece and the turning tool is needed to maintain, namely the correct tool position and posture, so the following is obtained

\[
[\text{W}_P^T] = \begin{bmatrix}
1 & 0 & 0 & x_0 \\
0 & 1 & 0 & y_0 \\
0 & 0 & 1 & z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

The matrix \([\text{W}_P^T]\) describes the relative motion between the turning tool and work piece, namely the positional and posture matrixes are gained as follows

\[
[\text{W}_P^T]^{-1} [\text{P}_m^T] [\text{C}_p^T] = [\text{T}]
\]

As the formula (1) describes the matrix of the generatix at the height \(h\), the formula (3) describes the tool’s positional and pasture matrixes of the generatix at the same level, for ease of calculation, \(z\) takes zero in the describing matrix of the turning tool, so the positional and posture matrixes of the turning tool processing the generatix is given below

\[
[\text{C}_p^T] = \begin{bmatrix}
1 & 0 & 0 & x_0 \\
0 & 1 & 0 & y_0 \\
0 & 0 & 1 & z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

5. The motion analysis for machining the generatix and the guide line

5.1. The motion analysis for turning the generatix

The motion functions of the machine tools are synthesized by the motor function units generating the generatix and the motion functional units generating the lead line. For the tandem machine tools, the motion cascade matrixes corresponding to each motion unit are as follows[6]

\[
[\text{T}_u^T] = \begin{bmatrix}
\text{M}_1 & \text{M}_2 & \cdots & \text{M}_n
\end{bmatrix}
\]

The positional and posture matrixes are realized through the motion cascade matrixes as follows

\[
[\text{W}_p^T] = \begin{bmatrix}
\text{C}_0 & -\text{S}_0 & 0 & -\text{x}_0\text{C}_0 + y_0\text{S}_0 + z_0\text{C}_0 \\
\text{S}_0 & \text{C}_0 & 0 & -y_0\text{C}_0 + x_0\text{S}_0 + z_0\text{S}_0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

In formula (6), \(\text{M}_i\) represents the matrix corresponding to a motion function, for the machine tools with Cartesian coordinate system, there are 6 basic motion functions: the linear motions along the coordinates \(X, Y, Z\), and the rotation round the three coordinates.

By analyzing the characteristics of \([\text{T}_u^T]\), set the corresponding motion units, namely the rotation around \(Z\) and the liner motions along \(X, Y\), the formula (7) is solved by analytic method, the set unit is the function of the machined surface parameters, it shows that the motion units are necessary, otherwise the set units are redundant, namely
In formula (8), $X$, $Y$ are the linear motions, $V$ is the rotational motion around $Z$, $b_1$, $a_1$, $a_3$, $b_2$ are constants, the following equation is gotten from the formulas (5) (7) (8):

$$
\begin{bmatrix}
CV - SV 0 & a_1 \\
SV CV 0 & b_1 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
100X \\
0 & 10b_2 \\
0 & 010Y \\
0 & 0010
\end{bmatrix}
\begin{bmatrix}
100a_3 \\
0 & 010Y \\
0 & 0010 \\
0 & 0001
\end{bmatrix}
$$

In formula (8), $X$, $Y$ are the linear motions, $V$ is the rotational motion around $Z$, $b_1$, $a_1$, $a_3$, $b_2$ are constants, the following equation is gotten from the formulas (5) (7) (8):

$$
\begin{align*}
\Theta_2 &= V \\
-x_0C\Theta_2 + y_0S\Theta_2 + r_zC\Theta_2 &= (a_2 + X)CV - (b_2 + Y)SV + a_1 \\
-y_0C\Theta_2 + x_0S\Theta_2 + r_zC\Theta_2 &= (a_2 + X)SV + (b_2 + Y)CV + b_1
\end{align*}
$$

X, Y, V are the motion variables, there exists the unique solution because of three equations, it shows that the set motion units have no redundancy, if the rotation is taken as main movements, the motion schemes for turning the generatrix is gotten as follows:

$\begin{array}{c}
W/X, Y \cdot V_c/T
\end{array}$

If the $Z$-axis of the work piece is in the same direction as the $X$-axis or the $Y$-axis of the machine tools, the two other motion schemes for turning the generatrix is obtained as follows:

$\begin{array}{c}
W/Z, Y \cdot V_A/T \\
W/X, Z \cdot V_B/T
\end{array}$

5.2. The motion analysis for turning the guide line

For the guide line, while the ZW-axis (the line segment $\Delta L$) of the work piece is essentially in the same direction as the $Z$-axis of the turning-lathe, as shown in Figure 3. First the ZW-axis of the

![Fig.3 The machining guide line](image)
Work piece must be made to be in the same direction as the Z-axis, the machine tool needs the rotational motion Vc around the Z-axis, the motion VA around the X-axis or the motion VB around the Y-axis, meanwhile, the turning tool is needed to make linear movement along the guide line (the Z-axis of the machine tools), therefore the motion schemes for machining the guide line is as follows[8]

\[ W/V_A, V_C, Z/T \text{ or } W/V_B, V_C, Z/T \]  

(12)

Similarly, while the Z-axis (the line segment \( \Delta L \)) of the work piece is essentially in the same direction as the X-axis or the Y-axis of the machine tools, the four other motion schemes for machining the guideline are obtained below

\[ W/V_A, V_B, X/T \text{ or } W/V_A, V_C, X/T \]  

(13)

\[ W/V_B, V_C, Y/T \text{ or } W/V_B, V_A, Y/T \]  

(14)

### 6. The generation of the CNC lathe structure

The expression above represents the required motion for machining the generatix and the guide line, if the center line of the work piece is in the basic same direction as the Z-axis of the machine tools (the included angle between the center line and the Z-axis is an acute angle), the motions for turning the generatix and the guide line are synthesized, the four other motions for turning the surface of the center line segment \( \Delta L \) are obtained below

\[ W/X, Y, V_A, Z \cdot V_C/T \text{ or } W/X, V_B, Z, Y \cdot V_C/T \]  

(15)

In the same way, while the center line (the line segment) of the work piece is in the basic same direction as the X-axis or Y-axis, the four other motions for turning the surface of the center line segment \( \Delta L \) are gotten as follows

\[ W/V_B, X, Z, Y \cdot V_A/T \text{ or } W/V_C, X, Z, Y \cdot V_A/T \]  

(16)

\[ W/V_C, Y, X, Z \cdot V_B/T \text{ or } W/V_A, Y, X, Z \cdot V_B/T \]  

(17)

So, get the six different motion schemes as shown in Tab.1

### Tab.1 The different motion schemes for CNC turning free surface

| the direction of the workpiece | the schemes of the machine tool |
|-------------------------------|--------------------------------|
| The center line (the line segment \( \Delta L \)) is in the same direction as the Z-axis of the machine tools | \( W/X, Y, V_A, Z \cdot V_C/T \) |
|                               | \( W/X, V_B, Z, Y \cdot V_C/T \) |
| The center line (the line segment \( \Delta L \)) is in the same direction as the X-axis of the machine tools | \( W/V_B, X, Z, Y \cdot V_A/T \) |
|                               | \( W/V_C, X, Z, Y \cdot V_A/T \) |
| The center line (the line segment \( \Delta L \)) is in the same direction as the Y-axis of the machine tools | \( W/V_C, Y, X, Z \cdot V_B/T \) |
|                               | \( W/V_A, Y, X, Z \cdot V_B/T \) |

If two types of vertical and horizontal machine tools are adopted, the structural forms of the machining tools whose Y-axis or X-axis is in the same direction as the center line (the line segment \( \Delta L \)) of the work piece is basically same, one of them can be omitted.

### 7. Conclusion

With the intruction above, the following conclutions are given

1) The free surface is cut into n equal parts along the height h, set the continuous and smooth curve L, the curve L goes through the centroid of the cross section, the generatix is the curve of the cross section, it is supposed that generatix is continuous and smooth, and is assumed that the turning tool machines the generatix without inference, the guide line is the line segment \( \Delta L \) of the curve L, the turning tool machines each equal part, namely the curved surface body is machined, the thinner the equal part is, the higher the accuracy machining free-form surface is.

2) First determine the describing matrices of the generatix of equal part, and the describing matrixes of the cutting point C on the turing tool, in order to ensure the resonable angle between the workpiece and the turning tool, the position and posture matrixes for machining the generatix are obtained, for the
tendem machine tools, the positional and pasture matrixes of the turning tool are realized through the motion cascade matrices, if the set unit isn’t the function of the machined surface parameters, it shows that the motion unit is redundant.

3) In the same way, the motion schemes for machining the guide line are determined, synthesizes the motions for turning the generatrix and the guide line and the postion of a workpiece on a machine tool, the possible motion schemes of the machine tool are solved.

4) While the motion schemes of the machine tool are determined, the impacts of the velocity of the machine tool are not considered, nor do the impacts of the machining surface on the motion of the machine tools.

Acknowledgements
Natural Science Foundation of Education of Anhui Province (No.KJ2019A0844), Teaching Quality Project of Anhui Province (2018sjjd030) and Science Foundation for Talents of Hefei University (15RC10) support this work.

References
[1] XIAO Ya-ping, Kan Hong-lin. The kinematic function plan of machine tools based on analytic method [J]. Journal of Anhui Polytechnic University, Vol. 28(2013) No.4, p. 44-46.
[2] Zhang Guang-peng, Shi Wenhao, Huang Yumei, et al. Generative design of structure configurations for NC lathes [J]. China Mechanical Engineering, 2003, 14(21): 1805-1808.
[3] Liao De-gang, Cai Yue-hua, Che Xiao-yi. Research on the Technology of Superposition Forming for irregular Cylinder NC Turning and Milling[J]. Journal of Hunan University of Arts and Science, 2005, 17(2): 49-54.
[4] Zhang Guang-peng, Shi Wenhao, Huang Yumei, et al. Generative design of structure configurations for NC lathes [J]. China Mechanical Engineering, 28(2013) No.4, p. 44-46.
[5] Zhidong Huang. Research on motion generation and machining area for machining complex surface [D]. Changchun: Jilin University, 2014, p. 13-29.
[6] Zhao De-hong, Shao Meng, Zhao Min. Design of Turn-milling Combined NC Machine Tool Based on the Movement Function Generative[J]. Stone, 2012(12): 19-24.
[7] Huang Yu-mei. The innovation design of whole scheme of NC machine tools [J]. World Manufacturing Engineering & Market, (2001) No. 1, p. 41-44.
[8] Zhao De-Hong, Sun Jing, Wu Yu-hou. The design and analysis of saw-milling CNC for special shaped stone [J]. Modular Machine Tool & Automatic Manufacturing Technique, (2014) No. 2, p. 102-105.