I. INTRODUCTION

Systems with long range interactions abound in nature [1]. Several examples in the current condensed matter arena include Coulomb gases (plasmas) which encompass amongst many others, the omnipresent free electron gas, quantum Hall systems [2], [3], [4], adatoms on metallic surfaces, amphiphilic systems [5], interacting elastic defects (dislocations and disclinations) in solids [6], interactions amongst vortices in fluid mechanics [7], and superconductors [8], crumpled membrane systems [9], wave-particle interactions [10], interactions amongst holes in cuprate superconductors [11], [12], [13], [14], manganates and nickelates [15], [16], some theories of structural glasses [17], [18], [19], [20], [21], and colloidal systems [22], [23]. Needless to say, the list of systems (and works) goes far beyond the little outlined here. Much of the work to date focused on the character of the transitions in these systems and the subtle thermodynamics that is often observed (e.g., the equivalence between different ensembles in many such systems is no longer as obvious, nor always correct, as it is in the “canonical” short range case [24]). Other very interesting aspects of different systems have been addressed in [25].

In the current article, we focus on translationally invariant systems harboring several interactions of different ranges. To avoid many of the subtleties related to pure long range interactions, we will (unless stated otherwise) examine situations wherein screening is present. With these ingredients in place, we will find that the correlation functions of systems with screened long range interactions exhibit, in the exactly solvable large $n$ limit, a peculiar- rather universal- divergence of correlation lengths at high temperatures in many such systems. Notwithstanding this effect, none of the standard correlation functions exhibit any pathology- all correlators decay monotonically with increasing temperatures. In many such systems, there are new emergent modulation lengths governing the size of various domains. We find that these modulation lengths often also adhere to various scaling laws, sharp crossovers and divergences at various temperatures (with no associated thermodynamic transition). We also find that in such systems, correlation lengths generically evolve into modulation lengths (and vice versa) at various temperatures. The behavior of correlation and modulation lengths as a function of temperature will afford us with certain selection rules on the possible underlying microscopic interactions. In their simplest incarnation, for systems hosting two competing interactions, our central results are two fold: (i) In canonical systems harboring competing short and long range interactions, modulated patterns appear whose characteristic modulation length is minimal within the ground state and slowly slowly increases as as temperature is raised. The modulation length $L_D$ associated with these patterns diverges at a crossover temperature $T^*$ above which a uniform phase with multiple correlation lengths appears. The largest correlation length monotonically increases even as $T \to \infty$ although the prefactor associated with these correlation rapidly diminishes. (ii) By sharp contrast, in systems with only finite range interactions, the system exhibits a constant number of modulation and correlation lengths at all temperatures. Furthermore, in these systems, canonically, modulations (if they transpire) span a maximal length scale within the ground state and in the case of two competing interactions, the modulation $L_D$ length decreases as the temperature is raised. Armed with these general characteristics, we may easily discern the viable microscopic interactions (exact or effective) which underlie temperature dependent patterns such as those displayed in Fig. 1. Taken at face value, our results on the modulation lengths would suggest that any two component interaction theories underlying panel A of Fig. 1 may involve a confluence of long and short range interactions whereas those underlying panel B might involve only effective short range interactions. This may be said without knowing, a priori, the detailed microscopic interactions driving these non-uniform patterns. The simple treatment presented below does not account for the curvature of bubbles and the like. These may be easily augmented by inspecting energy functionals (and their associated free energy extrema) of various continuum field morphologies under the the addition of detailed domain wall tension forms- e.g. explicit line integrals along the perimeter where surface tension exists- and the imposition of additional constraints via Lagrange multipliers.
For our purposes, it suffices to note that the two spin correlator,
\[ G(\vec{x}) \equiv \langle S(0)S(\vec{x}) \rangle = k_B T \int \frac{d^d k}{(2\pi)^d} \frac{e^{i\vec{k} \cdot \vec{x}}}{v(k) + \mu}, \]
with \(d\) the spatial dimension. To complete the characterization of the correlation functions at different temperatures, we note that the Lagrange multiplier \(\mu(T)\) is given by the implicit equation \(1 = G(\vec{x} = 0)\) (Eq. (2) fused with translational invariance). We now investigate the general character of the correlation functions given by Eq. (3) for rotationally invariant systems. If the minimum (minima) of \(v(\vec{k})\) occur(s) at momenta \(q\) far from the Brillouin zone boundaries of the cubic lattice then we may set the range of integration in Eq. (3) to be unrestricted. The correlation function is then dominated by the location of the poles (and/or branch cuts) of \([v(k) + \mu]\).

Specifically, if \(k^s[v(\vec{k}) + \mu]\), with \(s\) an integer, is a polynomial
\[ P(z) = \sum_{m=0}^{M} a_m z^m \]
in \(z = k^2\) then, upon insertion into Eq. (4) we will find that the correlators generally display a net of \(M\) correlation and modulation lengths. At very special temperatures, the Lagrange multiplier \(\mu(T)\) may be such that several poles degenerate into one- thus lowering the number of correlation/modulation lengths at those special temperatures. It is important to emphasize that this multiplicity of roots and thus of correlation/modulation lengths occurs generally for any \(v(k)\) for which \(P(z)\) is a polynomial of degree \(M \geq 2\); multiple length scales appear irrespective of any competing interactions (alternating or uniform signs in \(k^s v(k)\)). What underlies multiple length scales is the existence of terms of different ranges (different powers of \(z\) in the illustration above)- not frustration.

II. CORRELATION FUNCTIONS IN THE LARGE N LIMIT- GENERAL CONSIDERATIONS

All of the results reported in this article were computed within the spherical or large \(n\) limit \([24]\). We focus on translationally invariant systems whose Hamiltonian is given by

\[ H = \frac{1}{2} \sum_{\vec{x}, \vec{y}} V(\vec{x}, \vec{y}) S(\vec{x}) S(\vec{y}). \]  

(1)

Here, the fields \(S(\vec{x})\) may portray classical spins or bosonic fields. The sites \(\vec{x}\) and \(\vec{y}\) lie on a hypercubic lattice of size \(N\) of unit lattice constant. In what follows, \(v(\vec{k})\) and \(S(\vec{k})\) will denote the Fourier transforms of \(V(\vec{x} - \vec{y})\) and \(S(\vec{x})\). For analytic interactions, \(v(\vec{k})\) is a function of \(k^2\) (to avoid branch cuts). The spins satisfy a single global spherical constraint,

\[ \sum_{\vec{x}} \langle S^2(\vec{x}) \rangle = N \]  

(2)

enforced by a Lagrange multiplier \(\mu\) which renders the model quadratic (as both Eqs. (1, 2) are) and thus solvable, see e.g. [25]. In the below we report the results for classical fields; the results for bosonic systems are qualitatively the same with additional Matsubara frequency summations in tow.

III. GENERAL PRELIMINARIES: SHORT AND LONG RANGE INTERACTIONS

A screened Coulomb interactions of screening length \(\lambda\) (i.e. a two spin potential \(V = \frac{1}{8\pi} \frac{1}{|\vec{x} - \vec{y}|} e^{-\lambda|\vec{x} - \vec{y}|}\) in \(d = 3\), and \(V = \frac{1}{16} e^{-\lambda|\vec{x} - \vec{y}|} \ln |\vec{x} - \vec{y}|\) in \(d = 2\) has the continuum Fourier transformed interaction kernel \(v(k) = [k^2 + \lambda^{-2}]^{-1}\). On a hypercubic lattice, the nearest neighbor interactions in real space have the lattice lattice Laplacian

\[ \Delta(\vec{k}) = 2 \sum_{l=1}^{d} (1 - \cos k_l) \]  

(5)

as their Fourier transform. The real lattice Laplacian

\[ \langle \vec{x} | \Delta | \vec{y} \rangle = \begin{cases} 2d & \text{for } |\vec{x} - \vec{y}| = 1 \\ -1 & \text{for } |\vec{x} - \vec{y}| = 1 \end{cases} \]  

(6)

Notice that \(\langle \vec{x} | \Delta^R | \vec{y} \rangle = 0\) for \(|\vec{x} - \vec{y}| > R\), where \(R\) is the spatial range over which the interaction kernel is non-
vanishing. Eq. (6) corresponds to a system is of Range=2,
\[
\langle \vec{x} | \Delta^2 | \vec{y} \rangle = \begin{cases} 
2d(2d + 2) & \text{for } \vec{x} = \vec{y} \\
-4d & \text{for } |\vec{x} - \vec{y}| = 1 \\
2 & \text{for } (\vec{x} - \vec{y}) = (\pm \epsilon_1 \pm \epsilon_2') \text{ where } \ell \neq \ell' \\
1 & \text{for } \pm 2\epsilon \text{ separation.} 
\end{cases} 
\] (7)

In the continuum (small \( k \)) limit, \( \Delta \to z = k^2 \).

For simplicity, many of the examples which we will employ to illustrate the general premise of the behavior of correlations in systems hosting long and short range interactions, the kernel \( v(k) \) (the Fourier transform of \( V(\vec{x}, \vec{y}) \) of Eq. (1)) will, in the continuum limit, be a simple function of \( k^2 \) or of \( \Delta \) whenever the finite lattice constant is kept.

Simple effects of tension may be emulated via a \( g(\nabla \phi)^2 \) term in the Hamiltonian where \( \phi \) is a constant in a uniform domain. Upon Fourier transforming, such squared gradient terms lead to an effective \( k^2 \) in the \( \phi \) space kernel. Similarly, the effects of curvature notable in many mixtures and membrane systems are often captured by terms involving \( (\nabla^2 h) \) with \( h \) a variable parameterizing the profile; at times the interplay of such curvature terms with others leads, in the aftermath, to a simple short range \( k^4 \) term in the interaction kernel. An excellent review of these issues is addressed in [3].

Screened dipolar interactions and others may be easily emulated by terms such as \( (k^2 + \lambda^2)^{-p} \) with \( p > 0 \).

### IV. MULTIPLE RANGE INTERACTIONS

We now summarize the situation wherein two dominant interactions compete whenever one of the interactions is of infinite range (albeit being screened) while the other is of finite range (i.e. \( V \) strictly vanishes for separations \( |\vec{x} - \vec{y}| > R \)). Such situations arise in many systems [9].

1) The high temperature correlation functions are, in many instances, sum of several exponential pieces; e.g. the correlation function
\[
\langle S(\vec{x})S(\vec{y}) \rangle = \frac{1}{|\vec{x} - \vec{y}|^{d-2}} \left( A_1 e^{-|\vec{x} - \vec{y}|/\xi_1} + A_2 e^{-|\vec{x} - \vec{y}|/\xi_2} + \ldots \right). 
\] (8)

[At least one of the correlation lengths (\( \xi_i \)) diverges.]

2) At low temperatures, translationally frustrated systems with competing interactions on different length scales display modulations (i.e. an oscillatory spatial dependence of the correlation functions), e.g.
\[
\langle S(\vec{x})S(\vec{y}) \rangle \sim \frac{\cos(p|\vec{x} - \vec{y}|)}{|\vec{x} - \vec{y}|^{d-2}} e^{-|\vec{x} - \vec{y}|/\xi} + \ldots 
\] (9)

3) When present in systems with frustrating long-range interactions (e.g. the Coulomb Frustrated Ferromagnet originally introduced to portray frustrated charge separation in the cuprates, [12, 13, 14]) for which, in Eq. (4), \( v(k) = k^2 + Qk^{-2} \), the modulation lengths monotonically increase with increasing temperatures. In the specific case of the Coulomb frustrated ferromagnet, the modulation length \( (2\pi/p) \) monotonically increases with temperatures until it diverges at a disorder line temperature \( T^* \) [13]. We find that the near this temperature (i.e. for \( T = T^* \)), the modulation length scales as \( T^*-T)^{-1/2} \). It should be emphasized that this divergence notwithstanding, the system does not exhibit a phase transition at \( T^* \). The free energy is analytic at this temperature. Nevertheless, the free energy can be made to have a singularity at \( T = T^* \) if the long range interaction is turned off \( (Q = 0) \); this allows us to view the divergence of the modulation length as sparked by a critical point which is “avoided”. [12, 13]

4) In most systems, the sum of the number of correlation and the number of modulation lengths is conserved as a function of temperature apart from special crossover temperatures. In the example of the Coulomb Frustrated Ferromagnet, at temperatures higher than this crossover temperatures \( T > T^* \) two correlation lengths appear as in (1). After diverging at \( T = T^* \) the modulation length turns into a correlation length at higher temperatures. All crossovers may be traced by examining the dynamics of the poles and branch cuts of \( 1/[v(k)+\mu] \) as the temperature (implicit in \( \mu \)) is varied. The merger of two or more poles at special temperatures leads to a temporary annihilation of one (or more length) which is generically restored as the temperature is continuously varied.

5) In systems with only two finite range interactions (of different scales), the modulation length monotonically decreases with increasing temperatures. This, combined with (3), affords a stringent selection rule on the viable interactions underlyung various experimentally observed modulation lengths. In canonical systems harboring only finite range interactions, the number of correlation lengths and the number of modulation lengths are both independently conserved at all temperatures (much unlike the case for long range interactions where only the sum of the two is conserved).

Although these characteristics are general, it is useful to provide expressions for specific cases. In what follows, we briefly sketch the behavior when two interactions of two different ranges, appear in unison. In section [7], we discuss the case of an infinite range interaction (wherein \( \langle \vec{x} | V | \vec{y} \rangle \neq 0 \) for all \( \vec{x} \neq \vec{y} \) screened Coulomb interaction existing side by side with a short range nearest neighbor exchange interaction. We follow, in section [7] by an investigation of a system hosting several finite range interactions. We the consider, in section [8], with brief remarks concerning the behavior in the case of a system in which long range interactions compete. General remarks concerning the possibility of first order transitions (section [9]) in the modulation length and a rather universal domain length exponent (section [10]) conclude the article.

### V. COEXISTING SHORT AND LONG RANGE INTERACTIONS

We begin with the “Screened Coulomb Ferromagnet”, capturing the competition between a repulsive screened Coulomb interactions and short range attractions. Here, the Fourier
transform of the interaction kernel of Eq. (1) is

\[ v(k) = k^2 + \frac{Q}{k^2 + \lambda^2} \]

(10)

wherein a screened Coulomb interaction competes with a short range ferromagnetic interaction. Not too surprisingly, such an kernel naturally appears in many systems e.g. screened models of frustrated phase separation in the cuprates [11]. Here, at high temperatures \( T > T^* \) wherein a \( T^* \) is given by \( \mu(T^*) = \lambda^2 + 2\sqrt{Q} \), the pair correlator in dimension \( d = 3 \)

\[ G(\vec{x}) = \frac{k_B T}{8\alpha_1\alpha_2 |\vec{x}|^{\alpha_1}} e^{-\alpha |\vec{x}|} \times \{e^{-\alpha |\vec{x}|}(\lambda^2 - \alpha^2) - e^{-\beta |\vec{x}|}(\lambda^2 - \beta^2)\}. \]

(11)

Here,

\[ \alpha^2, \beta^2 = \frac{\lambda^2 + \mu + \sqrt{(\lambda^2 - \mu)^2 - 4Q}}{2}. \]

(12)

For \( T < T^* \),

\[ G(\vec{x}) = \frac{k_B T}{2\pi |\vec{x}|^{\alpha_1}} e^{-\alpha |\vec{x}|} \times \{(\lambda^2 - \alpha^2) \sin \alpha_2 |\vec{x}| + 2\alpha_1 \alpha_2 \cos \alpha_2 |\vec{x}|\}. \]

(13)

where \( \alpha = \alpha_1 + i\alpha_2 = \beta^* \).

Similarly, in \( d = 2 \), for \( T > T^* \),

\[ G(\vec{x}) = \frac{k_B T}{2\pi \beta^2 - \alpha^2} \times \{e^{-\alpha |\vec{x}|}(\lambda^2 - \alpha^2)K_0(\alpha |\vec{x}|) + (\beta^2 - \lambda^2)K_0(\beta |\vec{x}|)\}. \]

(14)

with the Bessel function \( K_0(x) = \int_0^\infty dt \cos(xt) e^{-t} \). Much as in the three dimensional case, the high temperature correlator may be analytically continued to temperatures \( T < T^* \).

We alert the reader that two correlation lengths appear in the ground state. In the presence of screening, the pole trajectories are slightly skewed yet for \( Q > \lambda^4 \), \( \alpha \) tends to the ground state modulation wavenumber \( \sqrt{\sqrt{Q} - \lambda^2} \). If the screening is sufficiently large, i.e. if the screening length is shorter than the natural period favored by a balance between the unscreened Coulomb interaction and the nearest neighbor attraction \( \lambda > Q^{1/4} \), then the correlation functions never exhibit oscillations. In such instances, the poles continuously stay on the imaginary axis and, at low temperatures, one pair of poles veers towards \( k = 0 \) reflecting the uniform ground state of the heavily screened system. In Figs. 23, the evolution of the pole locations in traced at different temperatures \( T > T^* \), \( T < T^* \) leading to a “disorder line” like transition.

An analytical thermodynamic crossover does occur at \( T = T^* \). A large calculation of the free energy via equi-partition reveals that the internal energy per particle

\[ \frac{U}{N} = \frac{1}{2}(k_B T - \mu), \]

(15)

To ascertain a crossover in \( U \) and that in other thermodynamic functions, the forms of \( \mu \) both above and below \( T^* \) may be easily gleaned from the spherical normalization condition to find that the real valued functional form of \( \mu(T) \) changes [27].

We note, in passing, that the system orders at \( T = T_c \) given by

\[ \frac{1}{k_B T_c} = \int \frac{d^d k}{(2\pi)^d} \frac{1}{v(k) - v(\vec{q})}. \]

(16)

Here, for \( Q > \lambda^4 \), the modulus of the minimizing (ground state) wavenumber \( (|\vec{q}|) \) is given by

\[ q = \frac{2\pi}{L_D^D} = \sqrt{\sqrt{Q} - \lambda^2}, \]

(17)

with \( L_D^D \) the ground state modulation length. Associated with this wavenumber is the kernel \( v(\vec{q}) = 2\sqrt{Q} - \lambda^2 \) to be inserted in Eq. (16) for an evaluation of the critical temperature \( T_c \). Similarly, the ground state wavenumber \( \vec{q} = 0 \) whenever \( Q < \lambda^4 \). Needless to say, whenever \( Q > \lambda^4 \) and modulations transpire for temperatures \( T < T^* \), the critical temperature at which the chemical potential of Eq. (3),

\[ \mu(T_c) = \lambda^2 - 2\sqrt{Q}, \]

is lower than the crossover temperature \( T^* \) (given by \( \mu(T^*) = \lambda^2 + 2\sqrt{Q} \)) at which modulations first start to appear. The Yukawa Ferromagnet is found to have \( T_c(Q > \lambda^3) > 0 \) in \( d > 4 \) and in any dimension \( T_c(Q > \lambda^3) = 0 \). For small finite \( n \) a first order Brazovskii transition may replace the continuous transition occurring at
$T_c$, within the large $n$ limit [28]. Depending on parameter values such an equilibrium transition may or may not transpire before a possible glass transition may occur [19].

VI. MULTIPLE SHORT RANGE INTERACTIONS

Our central thesis is that in many models in which short range interactions compete with one another, the modulation length always varies with temperature. This is in contrast to in systems with infinite range interactions wherein the modulation length increases as temperature is raised.

We now illustrate this general premise by a few examples:

(i) With the conventions of the Fourier transformed kernel of Eq. (1) and Eqs. (3, 5), the Fourier transformed interaction kernel

$$v(k) = (\Delta^3 - \Delta_0^3)^2$$  \hspace{1cm} (18)

corresponds to a finite range interaction linking sites which are, at most, six lattice units apart. In what follows we employ the shorthand $c \equiv (\mu - \mu_{\text{min}})$ with $\mu_{\text{min}} = \Delta_0^3$. The poles, \{\Delta_0^{\pm} \}_{i=1}^3, of the correlator $G(k) = k_B T [c + v(k)]^{-1}$ are given by

$$\Delta_0^3 \pm i \sqrt{c}^{1/3} \exp (2\pi \alpha/3).$$  \hspace{1cm} (19)

In the complex plane, the poles \{\Delta_0^{\pm}\} lie on the vertices of a hexagon. There exist, at least, two poles with different values of $|\text{Im}\{k_i\}|$ leading to, at least, two different correlation lengths, $\xi_i = |\text{Im}\{k_i\}|^{-1}$ at all temperatures (and apart from special degenerate usually to three correlation lengths). The existence of multiple correlation lengths \{$\xi_i$\} are the rule in systems of multiple range interactions- including unfrustrated systems with no competing interactions. Their presence (as well as the existence of multiple modulation lengths) enriches the scope of possible scaling functions, allowing us to construct a greater multitude of scaling functions $\{\xi_i\}, \{L_{\alpha D_i}\}$, with $\Gamma$ a set of external parameters, than those present in standard critical systems where only one length (a single correlation length) sets the scale. For a finite ranged interaction kernel $V$ which is a general polynomial of the lattice Laplacian $\Delta$, the system possesses several modulation lengths and several correlation lengths at all temperatures; barring few exceptions - their net number is conserved as temperature is varied. Generically, for a finite ranged interaction which is a polynomial in the Lattice Laplacian $\Delta$, the system possesses a fixed number of correlation lengths and a fixed number of modulation lengths; there is no sharp analogue of $T_c$ wherein modulation lengths turn into correlation lengths. Such multiple correlation lengths often present for such kernels $v(k)$ which are functions of the lattice Laplacian $\Delta$ are accompanied by a $T_c$ discontinuity (an "avoided critical point" [12, 13]) in sufficiently high dimension when appropriate competition is present to allow real roots $0 < \{\Delta_i\} < 4d$ when $\mu = \mu_{\text{min}}$. All cross-over temperatures $[T_{i=1,2,...,p}]$ (including low dimensional systems which possess no critical behavior for zero frustration), at which correlation lengths disappear and turn into modulation lengths, tend continuously to the avoided critical temperature (or its analytic continuation for low dimensions - in high dimensions such an “avoided critical temperature” [12, 13] becomes critical for zero frustration). These crossover temperatures $[T_{i=1,2,...,p}]$ are more dramatic for non-analytic functions of $\Delta$ such as the frustrated screened Coulomb interaction investigated in Section VI, wherein the domain length diverges. (ii) We next consider a very prototypical interaction kernel appearing, amongst others, in amphiphile problems and crumpled elastic membrane systems, see e.g. [5]. For the short-range (Teubner-Strey) correlator

$$G^{-1}(k) = a_2 + c_1 k^2 + c_2 k^4,$$  \hspace{1cm} (20)

it is a simple matter to show that

$$G(x) \sim \frac{\sin kx}{kx} \exp[-x/\xi],$$  \hspace{1cm} (21)
where
\[ \kappa = \sqrt{\frac{1}{2} \sqrt{\frac{a_2}{c_2} - \frac{c_1}{4c_2}}} \]
\[ \xi^{-1} = \sqrt{\frac{1}{2} \sqrt{\frac{a_2}{c_2} + \frac{c_1}{4c_2}}}. \] (22)

In amphiphilic systems \( a_2 \) and \( c_{1,2} \) are functions of amphiphile concentration (as well as temperature).

The above two examples reaffirm out claim that in many simple thermodynamical models, short range interactions, the modulation length (if it exists) typically increases as the temperature is lowered. The converse typically occurs in systems hosting competing infinite range and finite range interactions.

When, in the notation of the Fourier transformed kernel of Eq. (1), \( v(\vec{k}) = k^4 \),
\[ G(\vec{x}) = \frac{1}{4 \pi x \sqrt{d}} \exp \left[ -x/\xi \right] \sin \kappa x. \] (23)

Thus within the spherical model of this range two system with a sole overall ferromagnetic interaction (\( v(\vec{k}) = k^4 \)) will display thermally induced oscillations. At \( T = 0 \) the (ferromagnetic) ground state is unmodulated.

VII. MULTIPLE LONG RANGE INTERACTIONS

A different behavior is seen when two long range interactions compete (e.g., \( v(k) = Ak^{-2} + Bk^{-4} \) with \( A < 0 \) and \( B > 0 \)). In such instances, modulations are present at all temperatures. Moreover, by sharp contrast to the competing long-range and short range interactions investigated earlier, within the large \( n \) limit, the modulations become more and more acute with a length which tend to zero in the high temperature limit.

VIII. FIRST ORDER TRANSITIONS IN THE MODULATION LENGTH

In the examples furnished above and several of our general maxims in the introduction, we focused on systems in which only two interactions of different ranges exist in a single system. In these systems, we found within the large \( N \) limit that the modulation lengths were always monotonic in temperature. Needless to say, this need not be the case yet within the large \( n \) limit this generally requires the existence of interactions spanning more ranges. The ground state modulation lengths (the reciprocals of Fourier modes \( \{ \hat{q}_i \} \) minimizing the interaction kernel) need not be continuous as a function of the various parameters: a “first order transition” in the value of the ground state modulation lengths can occur. Such a possibility is quite obvious and need not be expanded upon in depth. Consider, for instance, the Range=3 interaction kernel
\[ v(\vec{k}) = a[\Delta + \epsilon] + \frac{1}{2} b[\Delta + \epsilon]^2 + \frac{1}{3} c[\Delta + \epsilon]^3, \] (24)
with \( 0 < \epsilon \ll 1 \) and \( c > 0 \). If \( a > 0 \) and \( b < 0 \), then there are three minima, i.e., \( |\Delta + \epsilon| = 0 \) and \( |\Delta + \epsilon| = \pm m_+^2 \), where \( m_+^2 = \frac{1}{2c_a \pm \sqrt{b^2 - 4ac}} \), the locus of points in the \( ab \) plane where the three minima are equal is determined by \( v(\vec{k}) = 0 \), which leads to \( m_+^2 = -\frac{b}{a} \). Thus, \( b = -4\sqrt{ac}/3 \) is a line of “first order transitions”, in which the minimizing \( |\Delta + \epsilon| \) (and thus the minimizing wavenumbers) changes discontinuously by an amount \( \Delta m = (-\frac{4a}{b})^{1/2} = \frac{3a}{c}^{1/2} \).

IX. A UNIVERSAL DOMAIN LENGTH EXPONENT

Invoking the normalization condition \( G(\vec{x} = 0) = 1 \) in Eq. (4), we find that given the competing screened Coulomb and short range attraction of Eq. (10) and more general kernels \( v(\vec{k}) \) in which infinite-range interactions augment finite ones, in any problem in which the crossover temperature \( T^* \) is finite, the modulation length
\[ L_D \sim (T^* - T)^{-\nu_L}, \] (25)
with the (large \( n \)) domain length exponent \( \nu_L = 1/2 \) in any dimension \( d \).

X. CONCLUSIONS

In conclusion, our major finding is a general evolution of modulation and correlation lengths as a function of temperature in different classes of systems, those harboring infinite range (including screened) interactions and those having only short range interactions. These “selection rules” impose constraints on candidate theories describing a system in which the empirical behavior of modulation lengths is known. We further elucidated on the peculiar thermal evolution of the multiple correlation lengths in many systems having long range interactions, the largest of which may increase monotonically in temperature (even as \( T \to \infty \)).

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[27] For instance, in three dimensions, with $\Lambda = \frac{2\pi}{a}$ an ultraviolet cutoff with $a$ the lattice unit length, at high temperatures, $T > T^*$, this leads to the following implicit equation for $\mu(T)$ in the case of the screened Coulomb ferromagnet,

$$\frac{1}{T} = \frac{\Lambda}{2\pi^2} + \frac{\sqrt{2}}{4\pi^2}$$

$$\times \left( \frac{\lambda^2 \mu - \mu^2 + \mu p - 2Q}{\sqrt{\lambda^2 + \mu + p}} \tan^{-1} \left( \frac{\Lambda \sqrt{2}}{\sqrt{\lambda^2 + \mu + p}} \right) \right)$$

Here, we employed the shorthand $p \equiv \sqrt{(\mu - \lambda^2)^2 - 4Q}$. This parameter $p$ vanishes at the crossover temperature $T^*$ at which a divergent modulation length makes an appearance, $p(T = T^*) = 0$. At low temperatures, $T < T^*$, $p$ becomes imaginary and an analytical crossover occurs to another real functional form.

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