Constraints on $f(R)$ theories of gravity from GW170817

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Abstract

A novel constraint on $f(R)$ theories of gravity is obtained from the uncertainty in the observed chirp mass associated to the gravitational wave emitted from the binary neutron star merger event GW170817. The $f(R)$ theories possess an additional massive scalar degree of freedom apart from the massless spin-2 modes. The corresponding scalar field contributes an additional attractive, short-ranged “fifth” force affecting the gravitational wave radiation process. We found that chameleon screening is necessary to conform with the observation. We obtained a model independent bound $|f'(R_0) - 1| < 3 \times 10^{-3}$ where the prime denotes the derivative with respect $R$ and $R_0$ is the curvature of our Universe at present. Though our calculation is nonrelativistic, still the bound is stronger/equivalent compared to some earlier other bounds such as from the Cassini mission in the Solar-System, Supernova monopole radiation, the observed CMB spectrum, galaxy cluster density profile, etc. Using the bound obtained, we also constrain the parameter space in the Hu-Sawicki, Starobinsky, and Tsujikawa models as the alternatives to dark energy.

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I. INTRODUCTION

The recent detection of gravitational waves (GW) by the LIGO collaboration [1–5] provides an unprecedented opportunity to test the theories of gravity beyond GR in the extreme stellar environment or strong-field regime per se. Previously, no significant deviation from GR was found in vacuum or in the weak-field regime through several precision tests [6]. Recently, some model independent constraints on deviations from GR have been studied based on various GW generation and propagation mechanisms in the observed GW signals from compact black hole binaries [7, 8]. More recently, constraints on a number of theories beyond GR have been obtained from the constraints on the speed of gravitational waves [9, 10].

There are several unsolved puzzles in GR, such as resolving the singularities (in black holes and the big bang singularity in cosmology), understanding the dark matter and dark energy, etc. which motivate many researchers to pursue modified gravity theories in the classical domain which deviate from GR in ultraviolet and/or infrared energy scales. The simplest and well studied modification is the $f(R)$ theory of gravity which is a generalization of the Einstein-Hilbert action by replacing the Ricci scalar ($R$) by a function $f(R)$ (see [11] and the references therein for a review). Such theories have some important cosmological implications. For example, Starobinsky [12] gave the first successful $f(R) = R + \alpha R^2$ ($\alpha > 0$) model of cosmic inflation, which can account for the early inflationary era with out any inflationary scalar field. The observed cosmic acceleration (at present) can arise in some $f(R)$ theories of gravity with out requiring the cosmological constant and the dark energy i.e. a new exotic form of matter. Initial form of the models proposed for this purpose was $f(R) = R - \alpha / R^n$ ($\alpha > 0$, $n > 0$) [13]. However, this model suffers from various instability problems [14] mainly due to the fact that $f_{,RR} = \partial^2 f / \partial R^2$ is negative in this model. Also, it does not satisfy the local gravity constraints [15]. Later, Hu and Sawicki [16] designed a class of models which avoid the instability problems and do satisfy cosmological and Solar-System constraints under certain limits of parameter space. Other such viable $f(R)$ models were proposed by Starobinsky [17] and Tsujikawa [18]. Modification at the large scale dynamics in these $f(R)$ models leaves several interesting observational signatures such as the modification to the spectra of galaxy clustering, CMB, weak lensing, etc. [19]. For astrophysical and other works in $f(R)$ gravity, see in [20].
An important feature of $f(R)$ gravity is that it carries a massive scalar degree of freedom apart from the usual massless spin-2 tensor modes [21]. It can be shown that $f(R)$ gravity is dynamically equivalent to Einstein gravity minimally coupled to a scalar field in the Einstein frame [11]. The scalar field is associated with a nontrivial potential that depends upon the form of the $f(R)$ model and couples to matter through the trace of the energy-momentum tensor. In the nonrelativistic limit, the scalar field sources a (finite-range) fifth force which is added to the usual Newtonian force. In some $f(R)$ theories, the fifth force can be screened only at the galactic or Solar-System scales through the chameleon mechanism [16, 22, 23]. This mechanism facilitates the above mentioned viable models to conform the local gravity constraints as well as the modified dynamics at the large scale.

Constraints on such $f(R)$ theories were obtained by several authors in Solar-System tests [16], and cosmology [24–30] using various observations such as galaxy cluster profiles [25], cluster abundances [26, 27], CMB [28], redshift-space distortions [29], etc. For astrophysical tests based on the studies of stellar structure, distance measurements, galaxy rotation curves, etc. see Refs. [31–33]. On the other hand, binary systems of compact objects are excellent laboratory to probe the gravity in the strong field regime. Recently, the authors of Ref. [34] obtained the constraints from the study of orbital period decay of quasicircular neutron star-white dwarf (NS-WD) binary systems using the observational data of PSR J0348 +0432 and PSR J1738 + 0333 [35]. In Ref. [36], the authors compute the waveforms of gravitational-waves (GWs) emitted by such inspiral compact binaries such as Neutron star- black hole (NS-BH) and use it to constrain screened modified gravity including the $f(R)$ theories.

In this paper, we constrain independently the $f(R)$ gravity (with chameleon mechanism) from the observed GW signals at the LIGO-VIRGO detectors. However, we note that GWs from the binary black hole mergers (BH-BH) [2–4] are insensitive to $f(R)$ gravity as well as other scalar-tensor theories. Black holes in these theories do not have scalar hair [37] and, therefore, are indistinguishable from the black holes in GR. The authors of [38] have studied the possibilities to use the future observations of gravitational radiation from the binary neutron star mergers (BNS) as the probe of $f(R)$ gravity. However, they do not consider the chameleon mechanism. Our study is aimed at a phenomenological insight of the observed GW170817 [5] from a BNS merger, although our approach is fairly modest and we work in the Newtonian limit. In Section II, we discuss the GW radiation from the coalescence of binaries in the presence of additional short-ranged scalar force and use GW170817 to
constrain it. We use this result in Section III to show that Chameleon screening is must in \( f(R) \) gravity. Then, in Section IV, we obtain constraints on general \( f(R) \) theories which accommodate Chameleon mechanism and apply it on the specific dark-energy models such as the Hu-Sawicki, Starobinsky, and Tsujikawa models. Finally, we summarize our results in Section V.

II. COALESCEENCE OF BINARIES AND GW RADIATION FOR THE NEWTONIAN-YUKAWA POTENTIAL

Consider a binary system of two compact objects with masses \( m_1 \) and \( m_2 \) moving around each other. Fig. 1 shows the schematic diagram of the orbital motion in the centre of mass frame. The orbital motion of mass \( m_1 \) and \( m_2 \) are confined in the \( X-Y \) plane.

Let us assume the presence of a short-ranged Yukawa-type modification to the gravitational potential (originated from some scalar field) in addition to the Newtonian term. The

![Schematic diagram of the coalescing binary system in the centre of mass frame.](image.png)
effective Lagrangian for the motion of the system becomes

$$L = \frac{1}{2\mu} \left( \dot{r}^2 + r^2 \dot{\theta}^2 \right) + \frac{Gm_1 m_2}{r} + \frac{\alpha q_1 q_2}{r} e^{-m_\phi r},$$

(1)

where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is the reduced mass, $r = r_1 + r_2$ is relative separation between the compact objects, $\alpha$ is the coupling constant of the scalar interaction, $q_1$ and $q_2$ are the scalar charges, and $m_\phi$ defines the length scale for which the modification in the potential is important. The effect of such an additional term in the gravitational potential will be observed in the Ligo-Virgo detection window of gravitational waves, if $m_\phi^{-1} > \mathcal{O}(10)$ km. When the distance between the binary compact objects such as NS-NS is large ($r \gg m_\phi^{-1}$) the modification in the gravitational interaction can be neglected. However, when they are close enough ($r \lesssim m_\phi^{-1}$) the gravitational “fifth” force is switched on. At very small separation ($r \ll m_\phi^{-1}$) the “fifth” force behaves like a Newtonian gravitational force (or, massless in the Yukawa potential). Therefore, NS mergers are particularly good for constraining the gravitational “fifth” force and its origin in the range $m_\phi^{-1} > \mathcal{O}(1000)$ km (the typical binary separation when the signal enters LIGO-VIRGO window is up to $\mathcal{O}(1000)$ km). Such a short-ranged “fifth” force can also be originated from interaction between charged asymmetric dark matter particles trapped in binary NS (BNS) system, mediated by the massive but ultralight dark photons [39]. However, they used different observation windows in their case.

Let us assume that distance between the two coalescing neutron stars are small enough such that $r \ll m_\phi^{-1}$, when the signal enters LIGO-VIRGO detectors. Then the modified Kepler’s law becomes

$$\omega^2 = \frac{G(m_1 + m_2)}{r^3} (1 + \tilde{\alpha})$$

(2)

where $\tilde{\alpha} = \frac{\alpha q_1 q_2}{Gm_1 m_2}$. As the inspiraling binary radiate gravitational waves, the orbital energy ($E$) of the binary system decreases, where

$$E = -\frac{GM\mu}{2r} (1 + \tilde{\alpha}) = -\frac{1}{2} \mu v^2.$$  

(3)

In the above equation, $M = m_1 + m_2$ and $v = \omega r$.

The luminosity of gravitational waves emitted is related to the quadrupole moment of the binary mass and is given by,

$$L_{GW} = \frac{32G}{5c^5} \mu^2 r^4 \omega^6.$$  

(4)
Using Eq. (2), we get

\[ L_{GW} = \frac{32}{5} \xi^5 \eta^2 \left( \frac{v}{c} \right)^{10} \frac{1}{(1 + \tilde{\alpha})^2}, \tag{5} \]

where \( \eta = \frac{\mu}{M} \) is called the symmetric mass ratio.

In general, scalar dipole radiation also contributes to the total energy loss. However, it vanishes if the scalar-charge to mass ratio of the compact objects are same (i.e. \( \frac{q_1}{m_1} = \frac{q_2}{m_2} \)) \[39, 40\]. This is true also for the scalar field originated from \( f(R) \) gravity, where for the binaries consisting of the same type of compact objects (such as NS-NS mergers) the scalar dipole radiation vanishes \[38\]. Thus \( L_{GW} = -\frac{dE}{dt} \). Using Eqs. (3) and (5) we get

\[ \frac{d}{dt} \left( \frac{v}{c} \right) = \frac{32 \eta}{5} \frac{\xi^3}{GM} \left( \frac{v}{c} \right)^9 \frac{1}{(1 + \tilde{\alpha})^2} \tag{6} \]

The angular frequency \( (\omega_{gw}) \) of the gravitational wave radiation is directly related to the orbital angular frequency \( (\omega) \) of the binary source such that \( \omega_{gw} = 2\omega \). As the orbit decays, the frequency as well as the amplitude of the gravitational wave sweeps upward. This is known as a chirp and such an inspiral wave form is known as chirp wave form. Using \( \pi f_{gw} = \omega = \frac{\nu^3}{GM(1 + \tilde{\alpha})} \) in Eq. (6), we get

\[ \frac{df_{gw}}{dt} = \frac{96}{5} \pi^{8/3} \left( \frac{G \dot{M}_c}{c^3} \right)^{5/3} f_{gw}^{11/3}, \tag{7} \]

where \( f_{gw} \) is the frequency of the emitted gravitational waves. \( \dot{M}_c \) is the modified chirp mass given by

\[ \dot{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} (1 + \tilde{\alpha})^{2/5} = M_c (1 + \tilde{\alpha})^{2/5}. \tag{8} \]

For \( \tilde{\alpha} = 0 \) we get back the standard chirp mass \( (M_c) \). Switching on and off the short-ranged Yukawa potential translates into two different apparent values of chirp mass, \( M_c \) and \( \dot{M}_c \), for early and late stages of the inspiral. For large enough \( \tilde{\alpha} \), it will be difficult to fit the whole gravitational waveform with a single standard template with a unique chirp mass. Then two templates with different masses \( m_E \) and \( m_L \) are required to fit the early wave form and late waveform, respectively. The value of \( \tilde{\alpha} \) can be obtained from the difference between \( m_E \) and \( m_L \). However in the case of GW170817, if the a single chirp mass \( M_c \) fits the whole waveform \( \tilde{\alpha} \) can be constrained from the uncertainty of chirp mass \( \Delta M_c \). For GW170817, the observed chirp mass is \( M_{c,obs} = 1.188_{-0.002}^{+0.004} M_\odot \) \[5\]. From Eq. (8), \( \frac{\Delta M_c}{M_c} = \frac{\dot{M}_c - M_c}{M_c} \approx \frac{2}{3} \tilde{\alpha} \). Since \( \frac{\Delta M_c}{M_c} < \left( \frac{\Delta M_c}{M_c} \right)_{obs} \), \( \tilde{\alpha} < 0.013 \). Using this bound on \( \tilde{\alpha} \), we can constrain the theories of gravity where such a gravitational short-ranged “fifth” force appear.
III. \( f(R) \) THEORIES OF GRAVITY AND ITS NEWTONIAN LIMIT

Scalar-tensor theories of gravity can be possible origin of the additional short-ranged “fifth” force in the Newtonian limit. Massive scalar mode appears in addition to the massless spin-2 graviton modes in such theories \([42]\). This massive scalar mode coupled with matter can generate Yukawa-type potential and, consequently, the “fifth” force at the Newtonian level. We consider metric \( f(R) \) theories of gravity, which falls under this class, in our study. \( f(R) \) theories of gravity are given by the gravitational action in the Jordan frame

\[
S_J = \frac{1}{16\pi G} \int d^4x \sqrt{-\tilde{g}} f(\tilde{R}) + S_M[\tilde{g}, \Psi],
\]

where \( \tilde{g}_{\mu\nu} \) and \( \tilde{R} \) are metric tensor components and Ricci scalar in Jordan frame and \( \Psi \) is the matter field. The field equations are given as

\[
f'(\tilde{R})\tilde{R}_{\mu\nu} - \frac{1}{2} f(\tilde{R}) \tilde{g}_{\mu\nu} - \tilde{\nabla}_\mu \tilde{\nabla}_\nu f'(\tilde{R}) + \tilde{g}_{\mu\nu} \Box f'(\tilde{R}) = 8\pi G \tilde{T}_{\mu\nu}.
\]

(10)

and trace of the above equation is

\[
3\Box f'(\tilde{R}) + f'(\tilde{R}) \tilde{R} - 2f(\tilde{R}) = 8\pi G \tilde{T}.
\]

(11)

In the Einstein frame, \( f(R) \) theory can be written down in the form of a scalar-tensor gravity \([11]\]

\[
S_E = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{2} \partial_{\mu}\phi \partial^{\mu}\phi - V(\phi) \right] + S_M[A^2(\phi)g_{\mu\nu}, \Psi],
\]

(12)

where the Jordan frame metric is related to Einstein frame metric as \( \tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu} \). The conformal factor \( A^2(\phi) \) is directly related to \( f'(\tilde{R}) = \frac{df}{d\tilde{R}} \) as \( A^2 = f'(\tilde{R})^{-1} \). Here, the scalar field \( \phi \) is defined as

\[
\phi = -\sqrt{\frac{3}{16\pi G}} \ln f'(\tilde{R}).
\]

(13)

Then \( A^2(\phi) \) becomes

\[
A(\phi) = e^{\sqrt{\frac{3G}{16\pi}} \phi}
\]

(14)

and the potential \( V(\phi) \) is

\[
V(\phi) = \frac{\tilde{R} f'(\tilde{R}) - f(\tilde{R})}{16\pi G f'(\tilde{R})^2}.
\]

(15)

However, particles follow the geodesics of Jordan frame metric (\( \tilde{g}_{\mu\nu} \)). In the nonrelativistic limit, it turn out to be \([23]\)

\[
\frac{d^2x^i}{dt^2} = -\partial^i \Phi_N - \frac{\beta(\phi)}{M_{pl}} \partial^i \phi,
\]

(16)
where \( \Phi_N \) is Newtonian potential. Thus the “fifth” force is

\[
a_5 = -\frac{\beta(\phi)}{M_{pl}} \partial^i \phi,
\]

where

\[
\beta(\phi) = M_{pl} \frac{d \ln A}{d \phi}.
\]

Note that \( M_{pl}^{-2} = 8\pi G \). For \( f(\tilde{R}) \) theories of gravity, \( \beta(\phi) = 1/\sqrt{6} \) (using Eq. (14)).

From Eq. (12), the equation of motion of scalar field \( \phi \) is

\[
\Box \phi = \frac{dV(\phi)}{d\phi} - \frac{\beta(\phi) \rho}{M_{pl}} T,
\]

where \( T = g_{\mu\nu} T^{\mu\nu} \). \( T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\partial(\sqrt{-g} L_M)}{\partial g_{\mu\nu}} \) is the stress-energy tensor defined in the Einstein frame. However, it is not conserved \( \nabla_\mu T^{\mu\nu} \neq 0 \). The stress-energy tensor defined in the Jordan frame is \( \tilde{T}^{\mu\nu} = \frac{2}{\sqrt{-\tilde{g}}} \frac{\partial(\sqrt{-\tilde{g}} L_M)}{\partial g^{\mu\nu}} \). Actually, the stress-energy tensor in Jordan frame is physically relevant and also conserved, i.e. \( \nabla_\mu \tilde{T}^{\mu\nu} = 0 \). The definitions of stress-energy tensor in Jordan and Einstein frames are related as \( \tilde{T}^{\mu\nu} = A^{-6} T^{\mu\nu} \). In the non-relativistic limit, \( T = -\rho \approx -\tilde{\rho} = \tilde{T} \). Then Eq. (19) becomes

\[
\nabla^2 \phi = \frac{dV(\phi)}{d\phi} + \frac{\beta(\phi) \rho}{M_{pl}} = \frac{dV_{eff}}{d\phi},
\]

where the effective potential

\[
V_{eff} = V(\phi) + \rho \ln A(\phi).
\]

The scalar field \( \phi \) settle down at the minimum of effective potential \( (V_{eff}(\phi)) \) instead of the actual potential \( (V(\phi)) \). The minimum of the effective potential depends upon the density \( \rho \) of matter distribution. Consider a spherical object of mass \( m \) and radius \( r_\odot \) embedded in the medium of background density \( \rho_0 \). This could represent a star inside a galaxy or a galaxy/dark matter halo/cluster embedded in the cosmological background, in which case \( \rho_0 \) is the mean cosmic density. Then the effective potential has minimum at \( \phi_0 = \phi_{min}(\rho_0) \).

Far away from the object \( \phi(r) \rightarrow \phi_0 \). The object of mass \( m \) act as the source of perturbation in the uniform background scalar field \( \phi_0 \), such that \( \phi = \phi_0 + \delta \phi \). Then Eq. (20) becomes

\[
\nabla^2 \delta \phi - m_0^2(\phi_0) \delta \phi = \frac{\beta}{M_{pl}} \delta \rho(r),
\]

where \( m_0^2(\phi_0) = V''_{eff}(\phi_0) \) and \( \delta \rho(r) \) is the mass density profile of the spherical object.

Outside the source, the solution for \( \delta \phi \) looks like

\[
\delta \phi = \frac{\beta}{4\pi M_{pl}} \frac{f(m, r_\odot)}{r} e^{-m_0 r},
\]

where
where the constant \( f(m, r_\odot) \) depends upon the structure of the spherical object. For a point mass (i.e. \( r_\odot = 0 \)), \( f(m, r_\odot) = m \), and assuming \( m_\phi r << 1 \) in Eq. (23), the “fifth force” (Eq. (17)) becomes
\[
a_5 = -\frac{Gm}{3r^2},
\]
and the total gravitational acceleration (Eq. (16)) in the non-relativistic limit becomes
\[
a_r = -\frac{Gm}{r^2} \left(1 + \frac{1}{3}\right).
\]
This is true irrespective of any model of \( f(R) \) gravity. Thus for point mass in \( f(R) \) theories of gravity and at distances \( m_\phi r << 1 \), the nonrelativistic gravitational force deviates largely from the Newtonian force up to a factor of \( 4/3 \); i.e. \( \tilde{\alpha} \approx 0.3 \) in Eq. (8).

Though, black holes are classically point masses, they do not have scalar charges in \( f(R) \) theories [36, 37] and, hence, the “fifth” force is absent there (i.e. \( \tilde{\alpha} = 0 \)). Such BH-BH mergers are same as in GR and can not be used to constrain \( f(R) \) gravity. Therefore, in our case, we consider the Neutron stars which have finite size. The above mentioned large contribution from the “fifth” force can be suppressed in some \( f(R) \) theories through the Chameleon screening.

### A. Chameleon screening and thin shell effect

For the models of \( f(R) \) theories of gravity which admit Chameleon screening mechanism (see [23] for review), gravitational “fifth” force is suppressed at small scale such as Solar-systems, while strong modification in gravity appears at the cosmological scales. In such models, the form of \( V(\phi) \) becomes such that the effective mass of the scalar field \( m_\phi \) becomes heavier in high density (\( \rho \)) region and lighter in the low density region. The first example of such a model was that of Hu and Sawicki [16]. Other notable examples are Starobinsky [17] and Tsujikawa [18] dark energy models. Chameleon screening is also applicable to finite size compact objects such as Neutron stars. Therefore, using BNS mergers, we can constrain such \( f(R) \) theories of gravity.

In such theories, the field can reach a minimum of the effective potential \( (V'_{\text{eff}}(\phi_s) = 0) \) also at the centre of the spherical object (neutron star) and remain there (\( \phi = \phi_s \)) up to some radius \( r_s \), at which it enters in the second regime and begins to roll towards its asymptotic value (\( \phi_0 \)). Therefore, there is no “fifth” force interior to \( r_s \) called as the screening radius.

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Then Eq. (22) becomes
\[ \nabla^2 \delta \phi = \begin{cases} \ \frac{\beta}{M_{\text{pl}}^2} \delta \rho(r), & r_s \leq r \ll m_{\phi_0}^{-1}, \\ 0, & r < r_s, \end{cases} \tag{26} \]

(a) For \( r \leq r_s \), \( \phi = \phi_s \) and for \( r \to \infty \), \( \phi \to \phi_0 \).

(b) Thin shell effect. Ref. [23].

Ref. [31]

FIG. 2: Chameleon screening.

After integrating Eq. (26) we get
\[ \frac{d\phi}{dr} = \frac{\beta (m(r) - m(r_s))}{4\pi M_{\text{pl}} r^2}, \tag{27} \]

outside the screening radius, where \( m(r) = \int_0^r 4\pi r'^2 \delta \rho(r')dr' \). Then the “fifth” force (Eq. (17)), outside the screening radius, becomes
\[ a_5 = -\frac{G m(r)}{3r^2} \left( 1 - \frac{m(r_s)}{m(r)} \right) = \frac{a_N}{3} \left( 1 - \frac{m(r_s)}{m(r)} \right). \tag{28} \]

If \( r_s \ll r_\odot \), the “fifth” force is of the order of the Newtonian gravitational force \( (a_5/a_N \approx 1/3) \) and hence, the object is said to be unscreened. On the other hand, for the screened object, \( r_s \approx r_\odot \) and \( a_5/a_N \ll 1 \). In this case, the fifth-force only receives contributions from the mass in a thin shell outside the screening radius (see Fig. 2(b)). This phenomena is called as the thin-shell effect [23]. Assuming \( \phi_s \approx 0 \) and after integration of Eq. (26) the field profile can be written in terms of the Newtonian potential \( \Phi_N \) as [31]
\[ \phi(r) \approx \begin{cases} 2\beta M_{\text{pl}} \left[ \Phi_N(r) - \Phi_N(r_s) + r_s^2 \Phi'_N(r_s) \left( \frac{1}{r} - \frac{1}{r_s} \right) \right], & r \geq r_s, \\ 0, & r < r_s, \end{cases} \tag{29} \]
The screening distance $r_s$ is related to the background field $\phi_0$ through the following equation

$$\chi_0 \equiv \frac{\phi_0}{2\beta_0 M_{pl}} = -\Phi_N(r_s) - r_s \Phi_N'(r_s).$$

(30)

From Eq. (13), we note that $\phi_0$ depends on the model of $f(R)$ gravity as

$$|f'(\tilde{R}_0) - 1| = \sqrt{\frac{2}{3} \frac{\phi_0}{M_{pl}}},$$

(31)

where ‘tilde’ denote the Jordan frame. Thus the information about the screening distance from the observations can be used to constrain different $f(R)$ theories.

IV. CONSTRAINTS ON $f(R)$ THEORIES FROM GW170817

From Eqs. (16) and (28), the total gravitational acceleration outside a neutron star of mass $m$ becomes

$$a_r = -\frac{Gm}{r^2} \left[ 1 + \frac{1}{3} \left( 1 - \frac{m(r_s)}{m} \right) \right].$$

(32)

Using this result for a BNS system of masses $m_1$ and $m_2$, the effective gravitational potential energy of the binary system (in the nonrelativistic limit)

$$V_{grav} = -\frac{1}{2} \sum_{i,j=1,2} Gm_im_j \frac{r_{ij}}{r_{ij}} \left[ 1 + \frac{1}{3} \left( 1 - \frac{m(r_{s,j})}{m_j} \right) \right].$$

(33)

Note that $r_{12} = r_{21} = r$. Then the effective force acting on the reduced mass $\mu$ becomes

$$F_r = -\mu \frac{\partial}{\partial r} V_{grav}(r) = -\frac{Gm_1m_2}{r^2} \left[ 1 + \frac{1}{3} \left( 1 - \frac{1}{2} \left( \frac{m(r_{s,1})}{m_1} + \frac{m(r_{s,2})}{m_2} \right) \right) \right].$$

(34)

Therefore, $\tilde{\alpha}$ in Eq. (8) becomes

$$\tilde{\alpha} = \frac{1}{3} \left[ 1 - \frac{1}{2} \left( \frac{m(r_{s,1})}{m_1} + \frac{m(r_{s,2})}{m_2} \right) \right].$$

(35)

For neutron stars, we assume that the mass density inside the star is almost constant. Therefore, $m(r_s)/m = r_s^3/r_\odot^3$. Further, we assume that the neutron stars for GW170817 are almost similar (i.e. $m_1 \approx m_2$ and $r_{\odot,1} \approx r_{\odot,2} = r_\odot$) and hence, $r_{s,1} \approx r_{s,2} = r_s$. Then

$$\tilde{\alpha} \approx \frac{1}{3} \left( 1 - \frac{r_s^3}{r_\odot^3} \right).$$

(36)

Since $\tilde{\alpha} < 0.013$ from the observations of GW170817, we get $r_s > 0.987r_\odot$ using Eq. (36). The typical neutron star radius is $r_\odot \sim 15$ km. This result reveals that neutron stars
are different from the main sequence stars where a substantial part of the interior can be unscreened such that \( r_s \approx 0.3 \, r_\odot \) [31].

Next we note that the background field \( \phi_0 \) is same for both the neutron stars, i.e.

\[
\chi_0 = \frac{\phi_0}{2 \beta M_{pl}} = -\Phi_N(r_{s,1}) - r_{s,1}\Phi_N'(r_{s,1}) = -\Phi_N(r_{s,2}) - r_{s,2}\Phi_N'(r_{s,2})
\approx \Phi_N(r_s) - r_s\Phi_N'(r_s). \tag{37}
\]

We assume the Newtonian potential for each of the neutron star of masses \( m_1 \approx m_2 = m, \)

\[
\Phi_N(r) \approx \frac{Gm}{2r_\odot^3} (r^2 - 3r_\odot^2). \tag{38}
\]

Then using Eq. (37), we get

\[
\chi_0 \approx \frac{3Gm}{2c^2r_\odot} \left( 1 - \frac{r_s^2}{r_\odot^2} \right) \tag{39}
\]

where we divided r.h.s. by \( c^2 \) to get the match the dimension and get the correct number.

The total mass of BNS merger (GW170817) is \( M = m_1 + m_2 = 2.74^{+0.04}_{-0.01} \, M_\odot \) (\( M_\odot \) is the mass of the Sun.). Hence, we assume \( m \approx 1.37 \, M_\odot \). Then, \( \chi_0 < 5 \times 10^{-3} \). Using the estimated \( \chi_0 \) in Eq. (31) we get

\[
|f'(\tilde{R}_0) - 1| < 3 \times 10^{-3} \tag{40}
\]

Note that this is still an model independent result, provided the model allows the Chameleon screening. This result is consistent with above mentioned difference between the neutron stars and the main sequence stars. For the Sun, an example of a main sequence star, we get

\[
|f'(\tilde{R}_0) - 1| \approx 2 \times 10^{-6} \text{ using } r_s \approx 0.3r_\odot, \quad m = M_\odot = 2 \times 10^{30} \text{ kg(Solar mass), and } r_\odot \approx 7 \times 10^8 \text{ m (Solar radius)}. \]

Also, we note that above analysis and the result is correct when \( m_\phi^{-1} > 1000 \text{ km} \) as mentioned in the Sec. II. This corresponds to the Compton wavelength \( \lambda_c > 1000 \text{ km} \) and an energy scale \( E_\phi < 1.2 \times 10^{-12} eV \). Here, we emphasize on the fact that the energy scale mentioned here is not related to the bound on the graviton mass [41] which was used in [43, 44]. In \( f(R) \) gravity, graviton is massless as the spin-2 modes are massless and the mass of the scalar mode \( (m_\phi) \) signifies only the range of the scalar force and dispersion in the associated scalar wave [42]. Using Eqs. (13), (15), and (21) we get

\[
V_{eff}'(\phi) = \frac{\beta M_{pl}(\tilde{R}f'(\tilde{R}) - 2f(\tilde{R}))}{f'(\tilde{R})^2} + \frac{\beta \rho}{M_{pl}}, \tag{41}
\]

\[
m_\phi^2 = V_{eff}''(\phi) = \frac{1}{3} \left[ \frac{\tilde{R}}{f'(\tilde{R})} + \frac{1}{f''(\tilde{R})} - \frac{4f(\tilde{R})}{f'(\tilde{R})^2} \right]. \tag{42}
\]
At background scalar field ($\phi_0$), $V'_{\text{eff}}(\phi_0) = 0$, which leads to

$$m^2_{\phi}(\phi_0) = \frac{1}{3} \left[ \frac{1}{f''(\tilde{R}_0)} - \frac{\tilde{R}_0}{f'(\tilde{R}_0)} - 16\pi G \rho_0 \right].$$

(43)

From Eq. (40), we can safely use $f'(\tilde{R}_0) \approx 1$ in Eq. (43). We can also assume $\tilde{R}_0 \approx 8\pi G \rho_0$. Then Eq. (43) becomes

$$m^2_{\phi}(\phi_0) \approx \frac{1}{3f''(\tilde{R}_0)} - 8\pi G \rho_0.$$  

(44)

Considering the cosmological background and using the above said assumption on mass of the scalar field ($m_{\phi^{-1}} > 1000$ km), we get

$$f''(\tilde{R}_0) > 3 \times 10^{12} m^2.$$  

(45)

Using the bound on $f'(\tilde{R}_0)$ (40) and assumption on $f''(\tilde{R}_0)$ (45), we next constrain Hu-Sawicki, Starobinsky, and Tsujikawa dark energy models.

**A. Hu-Sawicki model**

The Hu-Sawicki dark-energy model is given by

$$f(\tilde{R}) = \tilde{R} - \frac{\mu \tilde{R}_0 (\tilde{R}/\tilde{R}_0)^{2n}}{b (\tilde{R}/\tilde{R}_0)^{2n} + 1},$$

(46)

where $n \geq 1$, $\mu, b > 0$ for the stability of the model [16]. Note that $n$, $\mu$, and $b$ are dimensionless quantities.

At present, the Universe is mostly dominated by dark energy. So, we work in the constant curvature (de Sitter) cosmological background. Then, from Eq. (11), we get

$$f'(\tilde{R}_0) \tilde{R}_0 - 2f(\tilde{R}_0) \approx 0,$$

(47)

where $\tilde{R}_0 = 4\Lambda$ and $\Lambda$ is the cosmological constant. Using the Hu-Sawicki model (Eq. (56)) in Eq. (47), we get [43]

$$b_{\pm} = -1 + \mu \pm \sqrt{\mu(\mu - 2n)}.$$  

(48)

Note that $\mu > 2n$. From Eq. (31), we have

$$|f'(\tilde{R}_0) - 1| = \frac{2n\mu}{(1 + b_{\pm})^2} < 3 \times 10^{-3}. $$

(49)
Assuming \( n/\mu \ll 1 \), the above inequality cannot be satisfied for \( b_- \). Therefore the allowed root is \( b_+ \). Then we obtain
\[
\frac{n}{\mu} < 6 \times 10^{-3}. \tag{50}
\]

On the other hand we have
\[
\begin{aligned}
f''(\tilde{R}_0) &= \frac{1}{\tilde{R}_0} \left[ \frac{4n^2\mu + 2n\mu}{(b_+ + 1)^2} - \frac{8n^2\mu}{(b_+ + 1)^3} \right] \\
&\approx \left( \frac{n^2 + n/2}{\mu \tilde{R}_0} \right), \quad n/\mu \ll 1. \tag{51}
\end{aligned}
\]

Then using Eq. (45) and \( \Lambda \approx 1.11 \times 10^{-52} \text{ m}^{-2} \), we obtain
\[
\left( \frac{n^2 + n/2}{\mu} \right) > 1.3 \times 10^{-39}. \tag{52}
\]

Thus, from Eqs. (50) and (53), we get for \( n = 1 \), \( 10^{39} > \mu > 167 \), and, for \( n = 2 \), \( 4 \times 10^{39} > \mu > 333 \).

Relating the galactic density (\( \rho_{gal} = 10^{-24} \text{ g cm}^{-3} \) for the Milky Way) to the cosmological density we find
\[
|f'(\tilde{R}_{gal}) - 1| \approx \left( \frac{8\pi\rho_{gal}G}{4c^2\Lambda} \right)^{-2n-1} |f'(\tilde{R}_0) - 1|, \tag{54}
\]

where we used \( b_+ \approx 2\mu \gg 1 \), \( \tilde{R}_{gal} > \tilde{R}_0 \), \( \tilde{R}_{gal} \approx 8\pi\rho_{gal}G/c^2 \), and \( \tilde{R}_0 \approx 4\Lambda \) (\( \Lambda = 1.1 \times 10^{-52} \text{ m}^{-2} \)). Using \( n = 1 \) and Eq. (31) we get at the galactic scale,
\[
|f'(\tilde{R}_{gal}) - 1| < 4 \times 10^{-17}. \tag{55}
\]

Above bound on \( f'(\tilde{R}_{gal}) \) is stronger than the bound from Cassini test where \( |f'(\tilde{R}_{gal}) - 1| < 5 \times 10^{-11} \) [16].

**B. Starobinsky model**

The Starobinsky dark-energy model [17] is given by
\[
f(\tilde{R}) = \tilde{R} + \lambda \left[ \tilde{R}_0 \left( 1 + \frac{\tilde{R}^2}{\tilde{R}_0^2} \right)^{-n} - 1 \right], \tag{56}
\]

where \( n \geq 1, \lambda > 0 \). For this model
\[
|f'(\tilde{R}_0) - 1| = 2^{-n}n\lambda < 3 \times 10^{-3}. \tag{57}
\]
So $\lambda < \frac{3x2^3}{n} \times 10^{-3}$. On the other hand, using Eq. (45), we have

$$f''(\tilde{R}_0) = \frac{n^2 \lambda}{\tilde{R}_0 2^n} > 3 \times 10^{12} m^2.$$  \hspace{1cm} (58)

Then we obtain $\lambda > \frac{2^6}{n} \times 10^{-39}$. For both $n = 1$ and $n = 2$, $2 \times 10^{-39} < \lambda < 6 \times 10^{-3}$.

C. Tsujikawa model

Another such dark energy model is given by [18]

$$f(\tilde{R}) = \tilde{R} - \nu \tilde{R}_0 \tanh \left( \frac{\tilde{R}}{\tilde{R}_0} \right).$$  \hspace{1cm} (59)

For this model

$$|f'(\tilde{R}_0) - 1| = 0.4 \times \nu < 3 \times 10^{-3},$$  \hspace{1cm} (60)

and

$$f''(\tilde{R}_0) = \frac{0.6 \times \nu}{\tilde{R}_0} > 3 \times 10^{12}.$$  \hspace{1cm} (61)

Then we obtain $2.2 \times 10^{-39} < \nu < 7.5 \times 10^{-3}$.

V. CONCLUSIONS

In this paper we constrain $f(R)$ theories of gravity from recently detected gravitational waves at LIGO-VIRGO detectors. In particular, we use the observation of GW170817, the first GW signal from a binary neutron star merger. GWs from BH-BH mergers can not constrain $f(R)$ theories as in these theories black holes do not carry scalar charge.

In $f(R)$ gravity, an extra massive scalar mode appears apart from the massless spin-2 modes. This extra scalar mode affects the GW generation in two ways. One is that an attractive short ranged “fifth” force adds up to to the usual Newtonian gravitational force between two compact objects. The other effect is that the scalar dipole radiation carries away some part of the total mechanical energy of the binary system. However, for the BNS merger, we ignore the scalar dipole radiation. From the uncertainty in the observed chirp mass for GW170817, we obtain bound on the strength of the scalar force ($\tilde{\alpha} < 0.013$). We assumed that the range of the scalar force is greater than the binary separation when the GW signal enters in the LIGO-VIRGO detection window. We noticed that, without any
screening effect, the scalar force can contribute largely ($\alpha = 1/3$). Fortunately, some $f(R)$ models such as the Hu-Sawicki model admit the Chameleon screening which can suppress the effect of the scalar field considerably to conform with the observations. Due to the Chameleon mechanism, the compact objects like stars are self-screened such that only a shell of its interior contributes to the scalar force, which is called as the thin shell effect.

The observation from GW170817 reveals that most part of the interior of the neutron stars are screened ($r_s > 0.987r_\odot$). This results in a model independent bound on $f(R)$ theories of gravity such that $|f'(R_0) - 1| < 3 \times 10^{-3}$ where the $R_0$ is curvature of the cosmological background spacetime at present. Our assumption on the range of scalar force translates into the relation $f''(R_0) > 3 \times 10^{12} m^2$. We applied these two results in the Hu-Sawicki, Starobinsky, and Tsujikawa models to constrain the parameter space.

| Observations                                      | $|f'(R_0) - 1|$ constraints | Ref.     |
|--------------------------------------------------|---------------------------|----------|
| Solar-System bounds (Cassini mission)            | $\lesssim 743^a$          | [16, 23] |
| Supernova monopole radiation                     | $< 10^{-2}$               | [46]     |
| Cluster density profiles (Max-BCG)               | $< 3.5 \times 10^{-3}$    | [25]     |
| CMB spectrum                                     | $< 10^{-3}$               | [28]     |
| **GW170817 (GW from BNS merger)**                | $< 3 \times 10^{-3}$      | **our current work** |
| Cluster abundances                               | $< 1.6 \times 10^{-5}$    | [26, 27] |
| Strong gravitational lensing (SLACS)             | $< 2.5 \times 10^{-6}$    | [45]     |
| Redshift-space distortions                       | $< 2.6 \times 10^{-6}$    | [29]     |
| Distance indicators in dwarf galaxies            | $< 5 \times 10^{-7}$      | [32]     |

$^a$ This is obtained for the Hu-Sawicki model with $n = 1$, when translated from the bound $|f'(R_{gal}) - 1| \lesssim 5 \times 10^{-11}$ at the galactic scale [23].

In the Table I, we compare the constraint on $|f'(R_0) - 1|$ that we obtained with other bounds available in the literature. We note that the bound obtained by us is better than the bounds from Cassini test, Supernova monopole radiation, and also is as good as the bounds from the study of galaxy cluster density profiles and CMB spectrum. Our present work is based on the analysis in the nonrelativistic/Newtonian limit. We intend to study the post-Newtonian phases, in future, which may improve the bound. Also, future observations
of the GWs from other BNS mergers will put tighter constraints on theories of $f(R)$ gravity and other scalar-tensor gravity with Chameleon mechanism.

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