Imaging spectroscopy of solar radio burst fine structures

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Solar radio observations provide a unique diagnostic of the outer solar atmosphere. However, the inhomogeneous turbulent corona strongly affects the propagation of the emitted radio waves, so decoupling the intrinsic properties of the emitting source from the effects of radio wave propagation has long been a major challenge in solar physics. Here we report quantitative spatial and frequency characterization of solar radio burst fine structures observed with the Low Frequency Array, an instrument with high-time resolution that also permits imaging at scales much shorter than those corresponding to radio wave propagation in the corona. The observations demonstrate that radio wave propagation effects, and not the properties of the intrinsic emission source, dominate the observed spatial characteristics of radio burst images. These results permit more accurate estimates of source brightness temperatures, and open opportunities for quantitative study of the mechanisms that create the turbulent coronal medium through which the emitted radiation propagates.
During sporadic periods of activity, the Sun produces the largest magnetic energy release events in the solar system: solar flares and coronal mass ejections (CMEs). Flares emit radiation across the electromagnetic spectrum from gamma- and X-rays to radio waves. Solar radio bursts originate from the acceleration of electrons in the relatively tenuous (electron number density \( n \lesssim 10^8 \text{ cm}^{-3} \)) solar corona, a region that, because of its low plasma density, produces very low, and hence undetectable, levels of X-ray and Extreme Ultra-Violet (EUV) emission. The radio bursts produced in such regions are, however, easily observable and thus provide unique diagnostics of electron acceleration and propagation in the outer corona and surrounding heliosphere. They provide information on the impulsive initial evolution of solar eruptions, information that is essential to the overall understanding of such events and hence to developing an effective system of space weather prediction and mitigation.

Most of the brightest solar radio bursts are due to coherent radio plasma emission processes: the injection of non-thermal electrons into the solar corona leads to the generation of Langmuir plasma waves through the electron-electron two-stream instability, and these plasma waves are converted into radio emission at both the plasma (fundamental) \( f_p \sim 9 \times 10^{-3} \sqrt{n(\text{cm}^{-3})} \text{ MHz} \) and second-harmonic \( f_{2p} = 2f_p \) frequencies. Radio bursts produced by electrons moving away from the Sun along open magnetic field lines are known as Type III bursts. The fine frequency structures, so-called Type IIIb bursts, are commonly believed to be caused by density inhomogeneities in the background plasma. The radiation propagates through the solar corona where it is both refracted and scattered by turbulent plasma processes; hence understanding these propagation effects is critical to a correct interpretation of solar radio burst images. Because coherent plasma emission produces radio waves at frequencies close to the local plasma frequency, propagation effects are particularly significant and must therefore be carefully considered in determining both the intrinsic properties of the surrounding plasma (e.g., density, magnetic field, density gradient, and turbulence) and of the emitting high-energy electron beams (e.g., location, energy).

High-time resolution one-dimensional scans or single-frequency images have demonstrated that Type III radio sources expand with time. This could (for such single-frequency observations) be due to either propagation effects or intrinsic variations in the structure of the Type III burst. Further, imaging observations with the Culgoora radioheliograph have also revealed an interesting enigma for the events at the limb: sources of fundamental emission are radially shifted outwards with respect to harmonic emission (and hence are apparently situated at different heights in the solar atmosphere), although the physics of the responsible coherent plasma emission mechanism requires that they are produced cospatially. This is particularly puzzling, since the refraction of radio waves shifts sources radially inwards and, since the fundamental component is refracted more than the second-harmonic component, the fundamental component should appear lower. Although a variety of possible resolutions of this paradox have been presented, a possible resolution that is consistent with the observed increase in source size with time involves radio wave propagation effects, which could shift the observed positions of fundamental radiation upward (radially outward). However, to date there have been no observations that permit the decoupling of propagation effects from intrinsic source variations, and hence there has been no quantitative assessment of the reasons behind this paradox.

Here we report imaging spectroscopy observations of fine frequency structures associated with a solar radio burst. These observations with high spatial and temporal resolution demonstrate radio wave propagation effects in the solar corona.
burst is concentrated at frequencies around 20 MHz while the emission in the second burst is concentrated around 40 MHz.)

The Type III burst in this event is rather typical; for example, the peak flux density between 32 and 40 MHz is 100–200 solar flux units (sfu) (1 sfu = 10^{-22} J s^{-1} m^{-2} Hz^{-1}), and it has circular polarizations of ~15% and <5% for the fundamental and harmonic components, respectively. The rapid downward drift of frequency with time is a defining characteristic of solar Type III bursts; it results from the rapidly decreasing ambient density (and hence decreasing plasma frequency away from the Sun) as the emitting electron beam propagates upward through the decreasing density of the solar atmosphere. Since the plasma frequency f \approx n/e^2, it follows that (1/f)df/dt = (1/2n) dn/dt = (1/2)(dln(n)/dr)(dr/dt) = v/L, where L = (dln(n)/dr)^{-1} is the density scale height and v = dr/dt is the vertical component of the velocity of the exciting electron beam. Using the Newkirk density model of the solar corona as a typical model, the characteristic density scale height is L \approx 0.3R_S \approx 2 \times 10^{10} cm at a level in the atmosphere corresponding to plasma frequencies around 32 MHz. Therefore, the observed frequency drift rate df/dt \approx 7 MHz s^{-1} at the f = 32 MHz point in the fundamental frequency burst component corresponds to v = (2L/f)(df/dt) \approx 10^{10} cm s^{-1} \approx c/3, where c is the speed of light. The speed c/3 is a typical speed for the ~30 keV electrons that excite Type III bursts.

The expanded view in Fig. 1 shows that the fundamental component of the burst consists of multiple fine-structured striae; such fine structure is the characteristic signature of Type III bursts. The number of striae increases with decreasing frequency, so that below ~30 MHz, the frequency structure of the burst looks quasi-continuous. These fine frequency structures are believed to be due to small-scale density fluctuations that modulate the resulting radio emission; they have full width at half-maximum (FWHM) durations around 1 s at a given frequency (Fig. 1). The presence of these fine striae in the fundamental component of the burst provides an estimate of the characteristic size of the emitting volume (intrinsic emission source size), which can then (see below) be compared to the source sizes obtained from direct imaging in order to evaluate the effects of radio wave propagation on the observed source size. Specifically, the individual striae (see the zoomed-in dynamic spectrum in Fig. 1) have FWHM frequency widths \Delta f/\sim 0.3 MHz. Although the relationship between \Delta f and the size of the radio emitting source is model dependent, and in particular depends on the angle between the direction of beam propagation and the direction of the density gradient, an order-of-magnitude estimation based on the plasma emission mechanism suggests that the limited frequency range corresponds to a vertical extent \Delta r \approx 2L(\Delta f/f) \approx 4 \times 10^6 cm. We note this would be the size of a density inhomogeneity leading to an enhanced level of Langmuir waves, while the electron beam generating the Langmuir waves is extended over a much larger distance. Such a characteristic size of the fundamental emitting source extends over an angle \theta \approx 0.1 arcmin at the Sun and hence subtends a very small solid angle (\Omega \approx 10^{-2} arcmin^2) on the sky. The harmonic emission is likely to form over a much larger region in physical space, a feature that is also evident from the dynamic spectra—the fundamental component has peak striae, but the harmonic is rather smooth.

**Imaging.** LOFAR imaging observations were made using 24-core Low Band Antenna stations with tied-array beam forming, an observing mode that provides images with sub-second time resolution and unprecedented frequency resolution in order to resolve the individual striae in the Type III burst. The array of 127 tied-array beams cover the sky out to \sim 2R_S with a mosaic beam spacing of \sim 0.1 degrees. We note that tied-array mosaic imaging is different from the traditional method of producing images from interferometric visibilities. The LOFAR core size of D \approx 3.5 km provides an angular resolution \lambda/D \sim 9 arcmin at 32 MHz (wavelength \lambda = 9.4 m) and the dirty beam FWHM area A_{LOFAR} was \sim 110 arcmin^2 at the time of the observation. The flux was calibrated against the Crab nebula both before and after the burst observations; in addition, the Sun-integrated flux was compared with URAN-2 data, which showed agreement within a factor of \sim 2. The temporal modulation of the URAN-2 flux also demonstrated excellent agreement with the observed fine frequency structures, excluding instrumental effects.

The imaging of the radio emission was performed with time resolution \sim 50 ms, during which radio waves propagate a distance of only \sim 1.5 \times 10^6 cm \sim 0.3 arcmin, allowing us to accurately track variations in both the location and the areal extent of the source on sub-second timescales. For each 12 kHz-wide frequency channel we fitted an elliptical Gaussian to the LOFAR images. The ellipse centroid position (which is determined to an accuracy significantly better than the angular resolution of a single beam measurement) and the FWHM area of each source were estimated for all frequency channels during the radio burst (Fig. 2). Figure 3 shows the size and centroid positions (with uncertainties) of both fundamental (F) and harmonic (H) images for a typical stria near 32 MHz; the FWHM areas are A^F \sim 400 arcmin^2 for the fundamental and A^H \sim 600 arcmin^2 for the harmonic. The accuracy of determining the source position and area is variable, depending on the emission.
pronounced during the decay of the burst. This is consistent with the areal expansion of both F and H components being most related to the motion of F and H sources due to frequency drift between 38 and 48 MHz caused by electron transport is shown in Fig. 5. Figure 4 shows the time evolution of the radial centroid positions and areas in the X–Y plane, for both F and H components (Fig. 3). The areal expansion of both F and H components is most pronounced during the decay of the burst. This is consistent with various wave scattering models, although these models predict different motions and growth rates of the source, depending on the assumed emission and scattering anisotropies. Therefore, we focus on times during the decay and estimate the radial velocity in the X-Y plane and areal expansion rate by fitting linear expressions \( r(x, y) = r_0 + (dr/dt)(t - t_0) \), \( A = A_0 + (dA/dt)(t - t_0) \) during the time intervals shown by the shaded regions in Fig. 4. The centroid of the F-emission moves radially outward at an average speed \( dr/dt \approx 1.8 \text{ arcmin s}^{-1} \approx c/4 \), while its area \( A_F \) grows from \( \sim 420 \text{ arcmin}^2 \) to \( \sim 530 \text{ arcmin}^2 \) within \( \sim 0.6 \text{ sec} \), an average areal expansion rate \( dA_F/dt \approx 180 \text{ arcmin}^2 \text{ s}^{-1} \). On the other hand, the centroid of the harmonic component shows negligible radial motion, while its area \( A_H \) grows from \( \sim 600 \text{ arcmin}^2 \) to \( \sim 760 \text{ arcmin}^2 \) over \( \sim 3 \text{ s} \). The average areal expansion rate is \( dA_H/dt \approx 50 \text{ arcmin}^2 \text{ s}^{-1} \), about one-fourth the areal expansion rate for the fundamental component.

We repeated this analysis for 48 well-observed striae in frequency channels between 32 and 38 MHz (Fig. 6). We excluded frequencies below 32 MHz where the striae start to overlap and images above 38 MHz due to low signal-to-noise ratio. Figure 6 shows that for all well-resolved striae, the rate of areal expansion of the fundamental source is \((2–4)\) times greater than the expansion rate for the harmonic source.

Individual striae start at different times within the Type IIIb burst (Fig. 1). Each stria also initially appears at a different time, as shown in the color scale in the insert. Straight-line fits to the positions of each centroid are shown by the arrows. The time elapsed is measured as time after the LOFAR beam, \( A_{\text{LOFAR}} \), is both four orders of magnitude larger than the emission region size \( \Omega \approx 10^{-2} \text{ arcmin}^2 \) as determined above from considerations of the fine frequency width of individual burst striae.

Figure 3 shows the temporal evolution of the centroid location, and the areal extent, of the fundamental and harmonic sources (both observed at a frequency of 32.5 MHz, so that the H-radiation is produced in a region with a density one-fourth that of the region emitting the fundamental—and a few seconds later, when the emission at the fundamental frequency has drifted downward to 16.25 MHz). The centroid of the fundamental frequency radiation moves in a direction roughly parallel to the local solar radius (i.e., north-west in the plane-of-image; see full-disk image in Fig. 3), whereas the centroid of the source of harmonic radiation moves in a roughly transverse direction. The motion of F and H sources due to frequency drift between 38 and 32 MHz caused by electron transport is shown in Fig. 5. Figure 4 shows the time evolution of the radial centroid positions and areas in the X–Y plane, for both F and H components (Fig. 3). The areal expansion of both F and H components is most pronounced during the decay of the burst. This is consistent with the areal expansion of the fundamental source being \( \sim 760 \text{ arcmin}^2 \) over \( \sim 3 \text{ s} \). The average areal expansion rate is \( dA_F/dt \approx 180 \text{ arcmin}^2 \text{ s}^{-1} \), about one-fourth the areal expansion rate for the fundamental component.

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us to exclude refraction effects in the Earth’s ionosphere as an explanation for the observed motion of the fundamental stria component. Further, the very similar expansion rates inferred from observations in 235 different frequency channels, at 48 different striae spread over 2 s (larger than ~1 s duration of a stria) allow us to infer with a high degree of confidence that observed regions of fundamental radiation expand faster than regions of harmonic radiation. As we argue next, this result is not supported by any reasonable variation in the intrinsic source sizes in existing models2, but is consistent with propagation-scattering effects.

An intrinsic variation of source size with time at a given frequency requires that the emitting source grows in time as larger and larger iso-density surfaces start to emit17. However, in order to produce the observed striae, which are very narrow in frequency and large in imaged sources, such a model would require two essential features. Firstly, the emitting region is distributed over a thin but large (and changing) volume, all at the same plasma frequency and thus density; any density inhomogeneities would have to be always parallel to iso-frequency surfaces, and secondly, the positions of the stria, which originate at different locations, have nearly identical centroid motions. Moreover, in such a scenario, the expansion rate is related to the structure of the iso-density surface and it is challenging to explain why the expansion has a similar rate at all frequencies, or equivalently why all the iso-density surfaces, which are spread over a height range ~0.2R⊙, expand at nearly identical rates. Finally, such a model does not explain why the centroids of the fundamental and harmonic components behave differently. Therefore, we reach the rather inescapable conclusion that the emitting sources most probably have sizes comparable to the inhomogeneity scale; they are randomly located within the corona and are responsible for the individual striae; and that the observed extent of the radio burst is primarily determined not by the size of the emitting region but rather by wave propagation effects in the surrounding atmosphere.

**Radio wave propagation.** A simple model6,11 for multiple scattering off density inhomogeneities does indeed produce a (linear) increase in the area of a source (in solid angle units) with time (see Equation 64 from11): \( \Delta A(f) = \Delta A(\mu^2) \mu^2 \), \( \mu^2 \) is frequency dependent, so the rate of areal increase for radiation near the plasma frequency will always be larger than that for radiation at the harmonic frequency, qualitatively consistent with the observations in Fig. 6. As the waves propagate away from the source into regions of lower density (and so lower plasma frequency), the (local) plasma frequency \( f_p \) becomes progressively smaller than the wave frequency \( f \). The refractive index approaches unity for both fundamental and harmonic radiation, and the expansion rate \( \propto \frac{f_p^2}{\mu^4} \) is greatly reduced. The location of the radio source is ~7 arcmin from the solar disk center, so the short temporal extent (average FWHM \( \Delta t \approx 1.1 \) s) of the striae constrains the radio wave broadening along the line of sight direction to be less than \( c \Delta t \approx 3 \times 10^{10} \) cm \( \approx 8 \) arcmin. Since the perpendicular to line of sight size (~20 arcmin on the plane of the sky) is larger than the line of sight 8 arcmin size, the combined effect of scattering and emission directivity5 is likely to be anisotropic, with the dominant effect being perpendicular to the line-of-sight direction. While both the fundamental and harmonic regions expand, the fundamental source is also radially moving, and this suggests a rather small
The source sizes are very similar to the average source sizes characteristics of the density turbulence spectrum. Indeed, the harmonic emission source is ~1.4 times larger than the intrinsic emission source size ~0.1 arcmin.

**Discussion**

We have reported imaging spectroscopy observations of fine frequency structures in solar Type III radio bursts. The high frequency-time resolution of these observations have allowed us to image the radio waves as they emerge from the solar atmosphere.

The observed sources have linear extents, corrected for the finite size of the LOFAR beam, of $\sqrt{(A^F - A_{LOFAR})} \simeq (17 - 22)$ arcmin near the peak of the fundamental component (Fig. 4). The source sizes are very similar to the average source sizes $\simeq 20$ arcmin inferred for Type III bursts at 43 MHz and are much larger than the intrinsic emission source sizes ~0.1 arcmin deduced from the appearance of fine temporal substructures within the burst. The simulations of LOFAR response (Methods section) show the small source required by plasma emission cannot explain the LOFAR observations. We have also found that the areal extent of the fundamental component grows more rapidly than the harmonic component, consistent with a model involving scattering off density inhomogeneities. The measured expansion rates (Fig. 6), as well as the sub-second fundamental source motion, provide valuable information on the fibrous structure of the corona and on the (currently poorly known) characteristics of the density turbulence spectrum.

Because the intrinsic source size is so much smaller than the apparent source sizes, the brightness temperature of the source must be similarly ($A^F / \Omega \sim 10^7$) larger than that obtained using the apparent source areas. A flux of 100 solar flux units at the fundamental frequency corresponds to a brightness temperature $\sim 10^{14}$ K, which is larger than what is typically assumed and interestingly close to the maximum brightness temperatures observed for type III solar radio bursts.

These results also resolve a long-standing problem in solar radio astronomy—why the fundamental and harmonic sources, which result from the same physical process, do not generally appear to coincide spatially (Figs. 2 and 3). Because propagation effects result in a large increase in source size with time, the apparent source locations of the F and H sources are controlled primarily by these propagation-scattering effects, rather than by properties of the underlying emitting sources. We encourage further tests of this conclusion through analysis of radio data in different frequency bands.

The differential rates of propagation of F and H-radiation can easily lead, within a modest time, to a systematic displacement of the apparent source centroids, which Fig. 3 shows to be in the radial direction.

These observations also allow new testing and improvement of radio wave propagation models in turbulent coronal plasma, tests that have hitherto not been available for the solar corona. In particular, since the areal expansion coefficient is proportional to the size of the density fluctuations, this provides a diagnostic of the latter quantity and opens up previously unavailable opportunities for further study of various manifestations of solar activity, such as flares, coronal mass ejections and formation of the solar wind.

**Methods**

**Polarization.** The Sun-integrated flux densities observed by LOFAR and URAN-2 described above have been compared in Fig. 7. Polarization measured by URAN-2 is given for the same frequency.

**Radio source position and size.** The imaging has been performed for 48 individual strands in 235 frequency channels and is shown in Fig. 5. The LOFAR observations were made using 24-core Low Band Antenna stations using tied-array beam forming images with sub-second time resolution. The array of 127 tied-array beams, which are the coherent sum of all the station beams, covered the Sun with ~6 arcmin spacing (the data are publicly available via http://lofar.target.rug.nl). Each LOFAR beam measures the flux $F_i$ for a given position on the sky given by $x_i$, $y_i$ in heliocentric coordinates. Figure 2 shows the locations of the beams on the solar disk. The measured flux $F_i$ recorded by LOFAR is a convolution of a LOFAR...
beam point spread function (Fig. 8) and the true source. The positions and the flux values for each beam allow us to reconstruct the images. Iteratively finding the highest value in the array of $F_i$, 1% of the maximum convolved with a point spread function (Fig. 8) has been subtracted from all $F_i$ until the highest value is smaller than 5% of the initial maximum. The resulting cleaned image is shown in Fig. 9. Figure 9 also compares the result of simulated LOFAR images.

In principle, one can apply the cleaning procedure to thousands of images, and then deduce the measured source parameters. However, because we are interested in the characteristic size and source position only, such an extensive calculation effort seems unjustified. Instead, we determine the areal extent and subtract the LOFAR dirty beam area. The side-lobes do not exceed 10%, so the images are weakly affected by cleaning at half-maximum level (the top right (cleaned) image looks similar to the top left (dirty) image at 50% level in Fig. 9). The simulations in Fig. 9 also demonstrate that small data.

To determine the size and the position of the source, we fit an elliptical Gaussian $S(x, y) = S_0 \exp(-x^2/2\sigma_x^2 - y^2/2\sigma_y^2)$, where $x = (x - x_0)\cos(T) - (y - y_0)\sin(T)$ and $y = (x - x_0)\sin(T) + (y - y_0)\cos(T)$, where $T$ is the rotation from the x axis, in the clockwise direction. Minimizing $\chi^2$

$$\chi^2 = \sum_{i=1}^{N} \left( \frac{F_i - S(x_i^0, y_i^0; S_0, x_0, y_0, \sigma_x, \sigma_y, T)}{\delta F} \right)^2,$$

we find the Gaussian parameters for each moment of time and frequency. The inferred parameters are as follows: $S_0$ is the peak amplitude, $x_0, y_0$ are the coordinates of the central source position, $\sigma_x, \sigma_y$ are the rms lengths, $T$ is the rotation of the ellipse from the x axis. The background flux level before the burst for each frequency and beam was taken as the uncertainties on the flux $\delta F$ (typically around 1 sfu). The resulting fitted Gaussian gives the source size (convolved with the LOFAR beam). The half-maximum area of the side-lobes do not exceed 10%, so the images are.

Fig. 7 Total flux and polarization data. a Time profile of the Sun-integrated radio flux at 32 MHz from the LOw Frequency ARray (LOFAR) and Ukrainian Radio interferometer of National Academy of Sciences (URAN-2). b Degree of circular polarization of the radio emission at the same frequency
Similarly, the error on the source half-maximum area $A$ can be estimated

$$\frac{\delta A}{A} \approx 2 \frac{\delta F}{F} \sqrt{\frac{h}{A}}$$

For example, the error on the area becomes $\delta A/A \sim 5\%$ for harmonic near the peak shown in Fig. 4. The above expressions show that the accurate determination of the source positions and the areas becomes available due to high signal to noise ratios $F/\delta F$. We further note that the results do not imply that the source has a Gaussian shape, but show that the multi-beaming measurements can provide estimates of the position better that the resolution of the instrument and the area measurements better than 'dirty' beam half-maximum area.

Data availability. The datasets generated during and/or analyzed during the current study are available in the LOFAR Long Term Archive, http://lofar.target.rug.nl/ and https://sdo.gsfc.nasa.gov/data/ or available from the authors upon request.
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Author contributions

E.P.K. has designed and coordinated the investigation. S.Y., A.A.K. and V.N.M. have contributed to the analysis of data and the creation of the figures. A.G.E., B.A., N.L.S.J., N.H.B., P.S. have contributed to the data analysis and the data interpretation. All authors have contributed to writing the manuscript.

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