Partial deconfinement: a brief overview

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Received 17 August 2022 / Accepted 20 October 2022 / Published online 9 November 2022
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Abstract The confinement/deconfinement transition in gauge theory plays important roles in physics, including the description of thermal phase transitions in the dual gravitational theory. Partial deconfinement implies an intermediate phase in which color degrees of freedom split into the confined and deconfined sectors. The partially deconfined phase is dual to the small black hole that lies between the large black hole and graviton gas. Better understandings of partial deconfinement may provide us with a clue how gravity emerges from the field theory degrees of freedom. In this article, we briefly review the basic properties of partial deconfinement and discuss applications.

1 Brief overview of partial deconfinement

In large-\(N\) gauge theories, the confined and deconfined phases typically have entropy and energy of order \(N^0\) and \(N^2\), respectively. An immediate question would be: what happens between these two situations, e.g., when the energy is \(N^1\) or \(\epsilon N^2\) with \(\epsilon \ll 1\)? The answer is partial deconfinement \([1–6]\).

Partial deconfinement can be understood as the coexisting phenomenon in the space of the color degrees of freedom, as depicted in Fig. 1. The partially deconfined phase (equivalently, partially confined phase) is distinguished from the completely confined phase (\(M = 0\)) and completely deconfined phase (\(M = N\)), where \(M\) is the “size” of the deconfined sector. In particular at large \(N\), two thermal phase transitions, the Hagedorn transition \([7]\) (\(M = 0\) to \(M > 0\)) and the Gross–Witten–Wadia (GWW) transition \([8, 9]\) (\(M < N\) to \(M = N\)), clarify the distinction among three phases.

A formal description of partial deconfinement can be obtained by the distribution of Polyakov-loop phases \([3, 5]\). Let \(A_t\) be the temporal gauge field \(A_t\) and define the Polyakov loop by

\[
P = \frac{1}{N} \text{Tr} \mathcal{P} \exp \left[ i \int_0^\beta dt A_t \right] = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j} = \int \mathcal{D} \theta \rho^{(P)}(\theta) e^{i\theta},
\]

where \(\beta\) is the circumference of the temporal direction identified with inverse temperature \(\beta = 1/T\), and \(\mathcal{P}\) represents the path ordering. Diagonalizing the matrix before taking trace provides \(N\) phases \(\theta_j\) lie in \([−\pi, \pi)\) called the Polyakov line phase, and in \(N \to \infty\) limit, the summation in the middle expression of (1) can be replaced by the integration of the phase introducing the distribution function \(\rho^{(P)}(\theta)\). For theories with center symmetry, such as pure Yang–Mills theory or maximally supersymmetric Yang–Mills theory, the Polyakov loop is a convenient order parameter for the phase transition associated with the breaking of the center symmetry. Recently, it has been shown \([5]\) that the phase transition can be determined from the distribution function \(\rho^{(P)}(\theta)\) even for theories without center symmetry, such as QCD. In the completely confined phase, the phases distribute uniformly between \(±\pi\), i.e., \(\rho^{(P)}(\theta) = \frac{1}{2\pi}\). In the partially deconfined phase, they distribute non-uniformly and non-zero everywhere. At the GWW transition, a gap opens at \(\theta = \pm \pi\), and the distribution is disjoint at \(\pm \pi\) in the completely deconfined phase. The minimum value of \(\rho^{(P)}(\theta)\) is related to the “size” \(M\) as \(\frac{1}{\pi} \left(1 - \frac{M}{N}\right)\).

Note that the partial deconfinement can also be understood by the analogous notion of Bose–Einstein condensation (BEC) \([5]\). In the quantum system of many-body identical bosons, an enhancement associated with the redundancy brought by the permutation of particles enables us to express that system in the language of gauge theory. In this correspondence, the number of degrees of freedom in the confined sector on gauge theory side can be identified with the number of

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and $N \times N$ matrices $\tau^\alpha$ ($\alpha = 1, \ldots, N^2-1$) represent the SU($N$) generators normalized as $\text{Tr}(\tau^\alpha \tau^\beta) = \delta_{\alpha\beta}$. The canonical commutation relations $[X_{I\alpha}, P_{J\beta}] = i\delta_{IJ}\delta_{\alpha\beta}$ is imposed.

By the creation and annihilation operators $\hat{A}^\dagger_I = \frac{\hat{X}_{I-} + i\hat{P}_I}{\sqrt{2}}$ and $\hat{A}_I = \frac{\hat{X}_{I+} + i\hat{P}_I}{\sqrt{2}}$, the gauge-invariant Fock states can be written by

$$\text{Tr}(\hat{A}^\dagger_I \hat{A}^\dagger_J \hat{A}^\dagger_K \ldots)|0\rangle = \sum_{i,j,k,l,\ldots=1}^N (\hat{A}^\dagger_{i,j} \hat{A}^\dagger_{j,k} \hat{A}^\dagger_{K,l} \ldots)|0\rangle,$$

(4)

by restricting the range of the indices. The difference between (4) and (5) is that the indices right-hand side runs to $M$ instead of $N$ under restricting an SU($M$)-subsector. By combining with that type of state, the energy eigenstate preserving SU($M$) symmetry $|E;\text{SU}(M)\rangle$ can be constructed, which is the state corresponding to the field configuration shown in Fig. 1. Although $|E;\text{SU}(M)\rangle$ is not SU($N$)-invariant state, the SU($N$)-symmetrization

$$|E\rangle_{\text{inv}} \equiv \frac{1}{\sqrt{\text{Vol(SU($N$))}}} \int_{\text{SU($N$)}} dU(|E;\text{SU}(M)\rangle),$$

(6)

provides an SU($N$)-invariant eigenstate. Here $U$ represents the SU($N$) gauge transformations associated with a group element $U$, and the integration is performed over SU($N$) with the Haar measure. By construction, $|E\rangle_{\text{inv}}$ is SU($N$)-invariant. It shows a mapping of the eigenstates from the SU($M$) to SU($N$) theories in a gauge-invariant manner. Such SU($N$)-invariant states are dominant in the partially-deconfined phase [4].

In the above example, the meaning of the “splitting” of color degrees of freedom is clear: Only the deconfined sector is excited. This is analogous to the system of $N$ non-interacting indistinguishable bosons, in which some particles are excited while the rest falls into the one-particle ground state, i.e., Bose–Einstein condensation (BEC). In fact, BEC is the same phenomenon as confinement, when the system of $N$ indistinguishable bosons is regarded as a gauge theory with $S_N$ gauge symmetry [4].

There is a superselection associated with the large-$N$ limit that allows us to use $|E;\text{SU}(M)\rangle$ instead of $|E\rangle_{\text{inv}}$. We can interpret this as a “spontaneous breaking” of SU($N$) gauge symmetry [4].
Fig. 2 Phase structure of the type IIB superstring theory on $\text{AdS}_5 \times S^5$. Same phase structure is expected to be realized for the $\mathcal{N} = 4$ SYM on $S^3$ at finite temperature (see e.g., Ref. [16]).

group. Therefore, by mimicking how BEC is generalized to interacting theories, we can generalize partial deconfinement to finite coupling [5, 6].

Reference [11] provided an explicit demonstration at strong coupling by studying a matrix model. The separation in Fig. 1 was confirmed as a property of lattice configurations with a certain gauge-fixing condition.

3 Partial deconfinement and holography

The confinement/deconfinement phase transition is closely related to the geometric transition in the dual gravitational system via the gauge/gravity duality. The $\text{AdS}_5/\text{CFT}_4$ correspondence [15] is a well-established example of the duality which gives the relation between 4d $\mathcal{N} = 4$ super Yang-Mills theory on $S^3$ and type IIB superstring theory on $\text{AdS}_5 \times S^5$.

The phase structure had been discussed from both sides in the duality [16, 17] (see also Fig. 2); In the duality, the thermal-AdS phase (i.e., graviton gas phase) and the large black hole phase, which appear as thermodynamically stable phases, are known as dual to the confined and deconfined phase on the CFT side. These two phases are minima of free energy and correspond to the completely confined and completely deconfined phases.

At the intermediate energy, the “small” black hole and the Hagedorn string also exist as thermodynamical states. Although these intermediate phases are not favored in the canonical ensemble, they are stable physical states in the microcanonical ensemble [16]. The original motivation for considering partial deconfinement was to describe the small black hole in terms of the dual gauge theory [1–3]. The small black hole can be described approximately as the 10d Schwarzschild solution.\(^2\) The energy of such a phase scales as $E \sim N^2 T^{-7}$ where $N^2$ can be translated into the Newton constant via the AdS/CFT dictionary, and therefore has the negative specific heat. It has been a puzzle to identify and express such objects in the dual gauge theory, and after that, partial deconfinement in gauge theory was proposed as the dual of the small black hole and Hagedorn string.

The supersymmetric matrix models (the BFSS and BMN matrix models [18, 19]) have been studied intensively in the context of gauge/gravity duality [20] as well. These models are expected to describe both string theory and M-theory [18, 20, 21]. Of particular interest is how the ‘transition’ between string theory and M-theory takes place. The deconfined phase has been studied intensively, and the comparison with the dual type IIA black zero-brane solution was conducted very precisely (see the pioneering works [22–33] and Ref. [34] for the latest result). Recently, the signal of the confined phase in those matrix models at low temperatures was reported [35]. The existence of the confined phase and the first-order confinement/deconfinement transition can be natural consequences of dual M-theory description. In this scenario, the partially deconfined phase corresponds to the 11d Schwarzschild black hole [36–38].

Moreover, the simple picture adopted above enables us to encode the geometry into color degrees of freedom [6, 39]; Very long strings (black hole and Hagedorn string) are dual to the deconfined blocks. In terms of operators, long traces express them. Short strings (e.g., gravitons) are short traces corresponding to tiny deconfined blocks. Reference [40] has recently proposed that the black hole evaporation can be visualized in terms of colors, and the Page curve can be derived straightforwardly.

4 Conclusion and discussion

The confinement/deconfinement transition is one of the nontrivial features in gauge theory at finite temperature and provides plenty of physical consequences. This review focused on partial deconfinement as a key concept and its applications to quantum gravity via holographic duality.

Another class of important applications is the standard model of particle physics and beyond-standard-model physics. In the context of the standard model, one possible scenario is that the ‘cross-over’ region of
QCD is partially deconfined with higher-order (i.e., third order) phase transitions. It would be interesting if there are experimental signals. The study of the strongly coupled lattice gauge theory shows that flux tubes are formed in the confined sector [41] and the chiral symmetry breaks at the GWW point where the confinement sets in [42]. Although the notion of the GWW transition requires infinite $N$, chiral symmetry breaking is a well-defined concept even at finite $N$. Therefore, we might be able to generalize partial deconfinement to finite $N$ using chiral symmetry [42]. In the context of beyond-standard-model physics, the nature of thermal phase transition may be useful to test the models based on cosmological observations.

Data Availability No data associated in the manuscript.

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