Derived equivalences and Cohen-Macaulay Auslander algebras

Shengyong PAN¹,², Xiaojin ZHANG³

¹ Department of Mathematics, Beijing Jiaotong University, Beijing 100044, China
² Beijing Center for Mathematics and Information Interdisciplinary Sciences, Beijing 100048, China
³ School of Mathematics and Statistics, Nanjing University of Information Science and Technology, Nanjing 210044, China

Abstract Let A and B be Artin R-algebras of finite Cohen-Macaulay type. Then we prove that, if A and B are standard derived equivalent, then their Cohen-Macaulay Auslander algebras are also derived equivalent. And we show that Gorenstein projective conjecture is an invariant under standard derived equivalence between Artin R-algebras.

Keywords Standard derived equivalence, Cohen-Macaulay Auslander algebra, Gorenstein projective conjecture

MSC 18E30, 16G10, 16S10, 18G15

1 Introduction

Derived categories have been one of the important tools in the study of many branches of mathematics: Lie theory, algebraic geometry, representation theory, etc. In the representation theory of algebras, derived equivalences of algebras or rings are one of the central themes. It is well known that endomorphism algebras of tilting complexes are derived equivalent to the original algebra [19]. Derived equivalences have been shown to preserve many algebraic and geometric invariants and provide new connections. For instance, Hochschild homology and cohomology [20], center of algebra [19], and finiteness of finitistic dimension [18] have been shown to be preserved under derived equivalences.

In order to understand derived equivalences, it is an important problem to construct derived equivalences from given ones. In this paper, we focus on standard derived equivalence for Artin algebras of finite Cohen-Macaulay type.

Our main result on Cohen-Macaulay Auslander algebra reads as follows.

Received August 14, 2013; accepted October 27, 2014
Corresponding author: Shengyong PAN, E-mail: shypan@bjtu.edu.cn
Theorem 1 \ Let $A$ and $B$ be Artin $R$-algebras of finite Cohen-Macaulay type. If $A$ and $B$ are standard derived equivalent, then the Cohen-Macaulay Auslander algebras $\Lambda$ and $\Gamma$ of $A$ and $B$ are derived equivalent.

Remark 1 \ (1) In [15], we proved that, if $A$ and $B$ are Gorenstein Artin $R$-algebras and they are derived equivalent, then their Cohen-Macaulay Auslander algebras are derived equivalent. But in this paper, we change derived equivalences to standard derived equivalences, and our result is true for Artin algebras of finite Cohen-Macaulay type. Note that a standard derived equivalence is a derived equivalence, the converse is not sure as now.

(2) If $A$ and $B$ are Artin $R$-projective algebras and they are derived equivalent, then there is a standard derived equivalence between $A$ and $B$ [20]. Then Theorem 1 is true for $A$ and $B$ being Artin $R$-projective algebras of finite Cohen-Macaulay type.

(3) Particularly, in Theorem 1, if $A$ and $B$ are finite-dimensional $k$-algebras of finite Cohen-Macaulay type, where $k$ is a field, then the Cohen-Macaulay Auslander algebras of $A$ and $B$ are derived equivalent. In the case of finite-dimensional algebras, we generalized the result of [15, Theorem 3.11].

As is known, derived equivalences are related homological conjecture. In [16], we proved that derived equivalences preserve the generalized Auslander-Reiten conjecture. Then we have the following theorem.

Theorem 2 \ Let $A$ and $B$ be Artin $R$-algebras. Suppose that $A$ and $B$ are standard derived equivalent. Then $A$ satisfies the Gorenstein projective conjecture if and only if so does $B$.

This paper is organized as follows. In Section 2, we review some basic facts on derived categories and derived equivalences and give some notions. In Section 3, we give the relationship between standard derived equivalences and Gorenstein projective modules. Finally, we prove Theorem 1. In Section 4, we prove Theorem 2.

2 Preliminaries

In this section, we shall recall some definitions and notations on derived categories and derived equivalences.

Let $\mathcal{A}$ be an abelian category. For two morphisms $\alpha: X \to Y$ and $\beta: Y \to Z$, their composition is denoted by $\alpha \beta$. An object $X \in \mathcal{A}$ is called an additive generator for $\mathcal{A}$ if $\text{add}(X) = \mathcal{A}$, where $\text{add}(X)$ is the additive subcategory of $\mathcal{A}$ consisting of all direct summands of finite direct sums of copies of $X$. A complex $X^\bullet = (X^i, d^i_X)$ over $\mathcal{A}$ is a sequence of objects $X^i$ and morphisms $d^i_X$ in $\mathcal{A}$ of the form

$$\cdots \to X^i \xrightarrow{d^i_X} X^{i+1} \xrightarrow{d^{i+1}_X} X^{i+2} \to \cdots$$

such that $d^i_X d^{i+1}_X = 0$ for all $i \in \mathbb{Z}$. If $X^\bullet = (X^i, d^i_X)$ and $Y^\bullet = (Y^i, d^i_Y)$ are two complexes, then a morphism $f^\bullet: X^\bullet \to Y^\bullet$ is a sequence of morphisms
\( f^i: X^i \to Y^i \) of \( \mathcal{A} \) such that \( d_X^i f^{i+1} = f^i d_Y^i \) for all \( i \in \mathbb{Z} \). The map \( f^* \) is called a chain map between \( X^* \) and \( Y^* \). The category of complexes over \( \mathcal{A} \) with chain maps is denoted by \( C(\mathcal{A}) \). The homotopy category of complexes over \( \mathcal{A} \) is denoted by \( K(\mathcal{A}) \) and the derived category of complexes is denoted by \( D(\mathcal{A}) \). It is well known that, for an abelian category \( \mathcal{A} \), the categories \( K(\mathcal{A}) \) and \( D(\mathcal{A}) \) are triangulated categories. For basic results on triangulated categories, we refer the reader to [7] and [14].

Let \( R \) be a commutative artinian ring, and let \( A \) be an Artin \( R \)-algebra. Denote by \( A\text{-}\text{Mod} \) and \( A\text{-}\text{mod} \) the category of left \( A \)-modules and finitely generated left \( A \)-modules, respectively. The full subcategory of \( A\text{-}\text{Mod} \) and \( A\text{-}\text{mod} \) consisting of projective modules is denoted by \( A\text{-Proj} \) and \( A\text{-proj} \), respectively. Recall that a homomorphism \( f: X \to Y \) of \( A \)-modules is called a radical map provided that for any \( A \)-module \( Z \) and homomorphisms \( g: Y \to Z \) and \( h: Z \to X \), the composition \( hfg \) is not an isomorphism. A complex of \( A \)-modules is called a radical complex if all of its differential maps are radical maps. For more basic properties of radical complexes, we refer to [10]. Let \( K^b(A) \) denote the homotopy category of bounded complexes of \( A \)-modules. Denote by \( D^b(A) \) the bounded derived category of \( A\text{-}\text{mod} \).

The Morita theory on derived equivalences for module categories of rings has been established by Rickard [19]. For more details on derived equivalences, we refer the reader to [19].

**Theorem 3** [19, Theorem 6.4] Let \( A \) and \( B \) be rings. The following conditions are equivalent:

(i) \( D^- (A\text{-}\text{Mod}) \) and \( D^- (B\text{-}\text{Mod}) \) are equivalent as triangulated categories;
(ii) \( D^b(A\text{-}\text{Mod}) \) and \( D^b(B\text{-}\text{Mod}) \) are equivalent as triangulated categories;
(iii) \( K^b(A\text{-}\text{Proj}) \) and \( K^b(B\text{-}\text{Proj}) \) are equivalent as triangulated categories;
(iv) \( K^b(A\text{-}\text{proj}) \) and \( K^b(B\text{-}\text{proj}) \) are equivalent as triangulated categories;
(v) \( B \) is isomorphic to \( \text{End}_{D^b(A)}(T^*) \) for some complex \( T^* \) in \( K^b(A\text{-}\text{proj}) \) satisfying

(a) \( \text{Hom}_{D^b(A)}(T^*, T^*[n]) = 0 \) for all \( n \neq 0 \);
(b) \( \text{add}(T^*) \), the category of direct summands of finite direct sums of copies of \( T^* \), generates \( K^b(A\text{-}\text{proj}) \) as a triangulated category.

**Remark 2** (1) The complex \( T^* \in K^b(A\text{-}\text{proj}) \) in Theorem 3 (v) which satisfies conditions (a) and (b) is called a tilting complex for \( A \). The rings \( A \) and \( B \) are said to be derived equivalent if \( A \) and \( B \) satisfy the conditions of the above theorem.

(2) By [19, Corollary 8.3], two Artin \( R \)-algebras \( A \) and \( B \) are said to be derived equivalent if and only if their derived categories \( D^b(A) \) and \( D^b(B) \) are equivalent as triangulated categories. By Theorem 3, Artin algebras \( A \) and \( B \) are derived equivalent if and only if \( B \) is isomorphic to the endomorphism algebra of a tilting complex \( T^* \). If \( T^* \) is a tilting complex for \( A \), then there is an equivalence \( F: D^b(A) \to D^b(B) \) that sends \( T^* \) to \( B \). On the other hand,
for each derived equivalence $F: \text{D}^b(A) \to \text{D}^b(B)$, there is an associated tilting complex $T^\bullet$ for $A$ such that $F(T^\bullet)$ is isomorphic to $B$ in $\text{D}^b(B)$.

**Definition 1** [20, Definition 3.4] Let $A$ and $B$ be Artin $R$-algebras. A standard derived equivalence between derived categories $\text{D}^{-}(A\text{-Mod})$ and $\text{D}^{-}(B\text{-Mod})$ is an exact functor if it is an equivalence and is isomorphic to $\text{RHom}_A(X^\bullet, -)$ for some object $X^\bullet$ of $\text{D}^b(A \otimes B^{\text{op}}\text{-Mod})$. An object $X^\bullet$ of $\text{D}^b(A \otimes B^{\text{op}}\text{-Mod})$ is called a two-sided tilting complex if it induces such an equivalence.

**Remark 3** It is an open problem whether all derived equivalences are standard derived equivalence [19,20]. Note that, if $A$ and $B$ are Artin $R$-projective algebras and they are derived equivalent, then there is a standard derived equivalence between $A$ and $B$ [20].

### 3 Derived equivalences for Cohen-Macaulay Auslander algebras

In this section, we shall recall the definition of Gorenstein projective modules and then we give the relationship between derived equivalences and the categories of Gorenstein projective modules. Finally, we prove Theorem 1.

Let $A$ be an Artin algebra. If an $A$-module $X$ satisfies $\text{Ext}^i_A(X, A) = 0$ for $i > 0$, then $X$ is said to be a generalized Cohen-Macaulay $A$-module. Denote by $A\mathcal{X}$ the subcategory of generalized Cohen-Macaulay $A$-modules. It is easy to see that if $A$ is a self-injective algebra, then $A\mathcal{X} = A\text{-mod}$. By a $\text{Hom}_A(-, X)$-exact sequence $\cdots \to P^{-1} \overset{d^{-1}}{\to} P^0 \overset{d^0}{\to} P^1 \overset{d^1}{\to} \cdots$, we mean that the sequence $Y^\bullet = (Y^i, d^i)$, where $P^i$ (for each $i$) and $Q$ are projective $A$-modules. Denote by $A\text{-Gproj}$ the subcategory of $A\text{-mod}$ consisting of Gorenstein projective $A$-modules. Note that Gorenstein projective modules are generalized Cohen-Macaulay $A$-modules. But the converse is not true in general. Particularly, if $A$ is a Gorenstein algebra, then by Happel’s result [8], a generalized Cohen-Macaulay module is a Gorenstein projective $A$-module and $A\mathcal{X} = A\text{-Gproj}$. Following [3, Example 8.4(2)], an Artin algebra $A$ is said to be of finite Cohen-Macaulay type provided that there are only finitely many indecomposable finitely generated Gorenstein projective $A$-modules up to isomorphism. It is easy to see that algebras of finite representation type are of finite Cohen-Macaulay type. Suppose that $A$ is of finite Cohen-Macaulay type. In other words, $A\text{-Gproj}$ has an additive generator $M$, that is, $\text{add}(M) = A\text{-Gproj}$. Then $\text{End}_A(M) = \text{Hom}_A(M, M)$ is called the Cohen-Macaulay Auslander algebra of $A$ [3,4].
Definition 2  For an $A$-module $X$, denote by

$$\varepsilon_X : X \to \text{Hom}_A(\text{Hom}_A(X, A), A),$$

$$x \mapsto (f \mapsto (x)f),$$

the usual evaluation map. If $\varepsilon_X$ is an isomorphism, then we call $X$ a reflexive module.

Remark 4  For any complex $X^\bullet$, we have a functorial homomorphism

$$\varepsilon_X : X^\bullet \to \text{Hom}_A(\text{Hom}_A(X^\bullet, A), A)$$

such that $\varepsilon_{X^\bullet} = \varepsilon_X$.

Lemma 1 [12, Lemma 2.3]  For $X^\bullet \in \text{D}(A\text{-Mod})$ and $Y^\bullet \in \text{D}(A^{\text{op}}\text{-Mod})$, we have a bifunctorial isomorphism

$$\theta_{Y^\bullet,X^\bullet} : \text{Hom}_{\text{D}(A^{\text{op}})}(Y^\bullet, \text{RHom}_A(X^\bullet, A)) \simeq \text{Hom}_{\text{D}(A)}(X^\bullet, \text{RHom}_A(Y^\bullet, A)),$$

where $\text{D}(A^{\text{op}}) = \text{D}(A^{\text{op}}\text{-Mod})$ and $\text{D}(A) = \text{D}(A\text{-Mod})$.

Definition 3  We set

$$\eta_{X^\bullet} = \theta_{\text{RHom}_A(X^\bullet, A), X^\bullet}(\text{id}_{\text{RHom}_A(X^\bullet, A), X^\bullet})$$

for $X^\bullet \in \text{D}(A\text{-Mod})$.

Lemma 2 [12, Lemma 2.5]  For $P^\bullet \in \text{K}(A\text{-Proj})_p$, we have a bifunctorial isomorphism

$$\delta_{P^\bullet} : \text{Hom}_A^*(P^\bullet, A^\bullet, A) \simeq \text{RHom}_A(\text{RHom}_A(P^\bullet, A), A)$$

such that

$$\eta_{P^\bullet} = \delta_{P^\bullet} \circ \varepsilon_{P^\bullet},$$

where $\text{K}(A\text{-Proj})_p$ denotes the full triangulated subcategory of $\text{K}(A\text{-Mod})$ consisting of complexes $X^\bullet$ such that $\text{Hom}_{\text{K}(A\text{-Mod})}(X^\bullet, -)$ vanishes on cyclic complexes.

Definition 4 [12, Definition 2.7]  A complex $X^\bullet \in \text{D}^b(A)$ is said to have finite Gorenstein dimension if $\text{Hom}_{\text{D}^b(A)}(X^\bullet, [-i]) = 0$ vanishes on $A$-proj for $i \gg 0$, and $\eta_{X^\bullet}$ is an isomorphism. In particular, a complex $X^\bullet \in \text{D}^b(A)$ is said to have Gorenstein dimension zero if $\text{Hom}_{\text{D}^b(A)}(X^\bullet, [-i]) = 0$ vanishes on $A$-proj for $i \neq 0$, and $\eta_{X^\bullet}$ is an isomorphism.

Remark 5  (1) Denote by $\text{D}^b(A)_{\text{Gfd}}$ the full subcategory of $\text{D}^b(A)$ consisting of complexes of finite Gorenstein dimension. Then it is a triangulated subcategory.

(2) Gorenstein projective modules are stalk complexes of Gorenstein dimension zero. Then $A$-Gproj is a full subcategory of $\text{D}^b(A)_{\text{Gfd}}$. 


In the literature, Gorenstein projective modules are also called modules of G-dimension zero [1]. Then we have the following lemma.

**Lemma 3** [12, Lemma 2.9] For $X \in A$-mod, the following statements is equivalent:

1. $X \in A$-Gproj;
2. $\text{Ext}^i(X, A) = 0 = \text{Ext}^i(Z^{-2}(\text{Hom}(P_X^\bullet), A), A), \ i \geq 1$,
   where $P_X^\bullet$ is a projective resolution of $X$ and $Z^{-2}$ is the $-2$-th cocycle;
3. $X$ is reflexive and $\text{Ext}^i(X, A) = 0 = \text{Ext}^i(\text{Hom}_A(X, A), A), \ i \geq 1$.

### 3.1 Derived equivalences and Gorenstein projective modules

We briefly recall some basic results on the relationship between derived equivalences and the categories of generalized Cohen-Macaulay modules from [15], which are needed in our proofs.

Suppose that $A$ and $B$ are Artin algebras. Let $F: \text{D}^b(A) \rightarrow \text{D}^b(B)$ be a derived equivalence, and let $P^\bullet$ be the tilting complex associated to $F$. Without loss of generality, we assume that $P^\bullet$ is a radical complex of the form

$$0 \rightarrow P^0 \rightarrow P^1 \rightarrow \cdots \rightarrow P^{n-1} \rightarrow P^n \rightarrow 0,$$

and that $P^0 \neq 0 \neq P^n$.

**Remark 6** As in Section 2, we give the definition of radical complexes. If $P^\bullet$ is the tilting complex associated to $F$, then $P^\bullet$ is in $K^b(A\text{-proj})$. Then $P^\bullet$ is isomorphic in $K^b(A\text{-proj})$ to a radical complex [10,17]. Up to shift, we assume that $P^\bullet$ is of the above form.

We have the following fact.

**Lemma 4** [10, Lemma 2.1] Let $F: \text{D}^b(A) \rightarrow \text{D}^b(B)$ be a derived equivalence between Artin algebras $A$ and $B$. Then there is a tilting complex $\overline{P^\bullet}$ for $B$ associated to the quasi-inverse of $F$ of the form

$$0 \rightarrow \overline{P}^0 \rightarrow \overline{P}^1 \rightarrow \cdots \rightarrow \overline{P}^{n-1} \rightarrow \overline{P}^n \rightarrow 0,$$

with the differentials being radical maps.

By composing of the embedding functor $A$-Gproj $\hookrightarrow \text{D}^b(A)_{\text{IGd}}$ with the localization functor $\text{D}^b(A)_{\text{IGd}} \rightarrow \text{D}^b(A)_{\text{IGd}}/K^b(A\text{-proj})$, we obtain a natural functor

$L' : A$-Gproj $\rightarrow \text{D}^b(A)_{\text{IGd}}/K^b(A\text{-proj})$.

The following theorem is well known [12] and announced by L. L. Avramov. For the convenience, we give the proof.
**Theorem 4** The quotient category

\[ D^b(A)_{fGd}/K^b(A\text{-proj}) \]

is equivalent as a triangulated category to \( A\text{-Gproj} \).

**Remark 7** This theorem is true not only for Artin algebras, but also for left coherent rings [12]. Note that \( A\text{-Gproj} \) is a triangulated category with shift functor \( \Omega^{-1} \).

**Proof of Theorem 4** Denote

\[ L: A\text{-Gproj} \to D^b(A)_{fGd}/K^b(A\text{-proj}). \]

The proof will be divided into three steps.

**Step 1** \( L \) is an exact functor.

Consider a distinguished triangle

\[ X \to Y \to Z \to \Omega^{-1}(X) \]

in \( A\text{-Gproj} \). This distinguished triangle is obtained from the following pushout diagram:

\[
\begin{array}{c}
0 \to X \to I(X) \to \Omega^{-1}(X) \to 0 \\
0 \to Y \to Z \to \Omega^{-1}(X) \to 0
\end{array}
\]

From exact sequences

\[ 0 \to X \to I(X) \to \Omega^{-1}(X) \to 0, \quad 0 \to Y \to Z \to \Omega^{-1}(X) \to 0, \]

we have distinguished triangles

\[ X \to I(X) \to \Omega^{-1}(X) \to X[1], \quad Y \to Z \to \Omega^{-1}(X) \to Y[1] \]

in \( D^b(A)_{fGd} \).

Since \( I(X) \) is projective, which is a projective-injective object in \( A\text{-Gproj} \), we have \( L'(I(X)) = 0 \) in \( D^b(A)_{fGd}/K^b(A\text{-proj}) \). Therefore, we deduce that

\[ L(X) \to L(Y) \to L(Z) \to L(X)[1] \]

is a distinguished triangle in \( D^b(A)_{fGd}/K^b(A\text{-proj}) \). Consequently, \( L \) is an exact functor.

**Step 2** \( L \) is a fully faithful functor.

First, \( L' \) is a full functor. We see that the map

\[ \text{Hom}_A(X, Y) \to \text{Hom}_{D^b(A)_{fGd}/K^b(A\text{-proj})}(X, Y) \]
sending \( f: X \to Y \) to \( f/\text{id} \) is surjective, where \( f/\text{id} \) is \( X = X \xrightarrow{f/\text{id}} Y \). Take a left roof \( X \overset{s}{\to} Z^\bullet \overset{\Delta}{\to} Y \) in \( \text{Hom}_{\text{D}^b(A)_{\text{fGd}}/\text{K}^b(A\text{-proj})}(X,Y) \), where \( Z^\bullet \in \text{D}^b(A)_{\text{fGd}} \). It follows that there is a distinguished triangle

\[
Z^\bullet \xrightarrow{s} X \to M(s) \to Z^\bullet[1]
\]

in \( \text{D}^b(A)_{\text{fGd}} \) such that the mapping cone \( M(s) \) is in \( \text{K}^b(A\text{-proj}) \). Applying the cohomological functor \( \text{Hom}_{\text{D}^b(A)_{\text{fGd}}/\text{K}^b(A\text{-proj})}(-,Y) \) to the distinguished triangle \( Z^\bullet \xrightarrow{s} X \to M(s) \to Z^\bullet[1] \), we have an exact sequence

\[
\cdots \to \text{Hom}_{\text{D}^b(A)_{\text{fGd}}/\text{K}^b(A\text{-proj})}(M(s),Y) \to \text{Hom}_{\text{D}^b(A)_{\text{fGd}}/\text{K}^b(A\text{-proj})}(X,Y) \xrightarrow{\langle s,Y \rangle} \text{Hom}_{\text{D}^b(A)_{\text{fGd}}/\text{K}^b(A\text{-proj})}(Z^\bullet,Y) \to \text{Hom}_{\text{D}^b(A)_{\text{fGd}}/\text{K}^b(A\text{-proj})}(M(s)[-1],Y) \to \cdots.
\]

Furthermore,

\[
\text{Hom}_{\text{D}^b(A)_{\text{fGd}}/\text{K}^b(A\text{-proj})}(M(s)[-1],Y) = 0
\]

since \( M(s)[-1] \in \text{K}^b(A\text{-proj}) \). Then \( g = sf \). Therefore, \( f/\text{id} \) and \( g/s \) are equivalent in \( \text{D}^b(A)_{\text{fGd}}/\text{K}^b(A\text{-proj}) \). This implies that \( L \) is a fully functor.

We claim that if \( X \) is an object of \( A\text{-Gproj} \), then \( L(X) = 0 \) if and only if \( X = 0 \). Indeed, no non-projective object in \( A\text{-Gproj} \) is isomorphic to an object of \( \text{K}^b(A\text{-proj}) \).

Assume that \( \alpha: X \to Y \) is in \( A\text{-Gproj} \) such that \( L(\alpha) = 0 \). There is a distinguished triangle

\[
X \overset{\alpha}{\to} Y \overset{x}{\to} Z \to \Omega^{-1}_F(X)
\]

in \( A\text{-Gproj} \). Since \( L \) is an exact functor, we see that

\[
L(X) \xrightarrow{L(\alpha)} L(Y) \to L(Z) \to L(X)[1]
\]

is a distinguished triangle in \( \text{D}^b(A)_{\text{fGd}}/\text{K}^b(A\text{-proj}) \). Since \( L(\alpha) = 0 \), we conclude that

\[
L(Z) \simeq (L(Y) \oplus L(X)[1]).
\]

Then we have a distinguished triangle

\[
Y \overset{\delta}{\to} Y \to V \to \Omega^{-1}(Y)
\]

in \( \mathcal{A}/\mathcal{P}(F) \). Thus, we get a distinguished triangle

\[
L(Y) \xrightarrow{id} L(Y) \to L(V) \to L(Y)[1]
\]
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in $D^b(A)_{I_Gd}/K^b(A\text{-proj})$. Consequently, $L(V) = 0$. Therefore, we have $V = 0$ by our claim. It follows that $xy$ is an isomorphism and $x$ is split monomorphism. Then $\alpha = 0$ and $L$ is a faithful functor.

**Step 3** The functor $L$ is dense.

Let $X^\bullet \in D^b(A)_{I_Gd}/K^b(A\text{-proj})$. Then $X^\bullet$ is in $D^b(A)_{I_Gd}$. Take a projective resolution $P^\bullet_X$ of $X^\bullet$ with $P^\bullet_X \in K^{-b}(A\text{-proj})$. Without loss of generality, we assume that $P^\bullet_X$ has the following form:

\[ \cdots \rightarrow P^{-r+2}_X \rightarrow P^{-r+1}_X \rightarrow P^{-r}_X \xrightarrow{d^{-r}} \cdots \rightarrow P^0_X \rightarrow 0 \rightarrow \cdots \]

such that $H^i(P^\bullet_X) = 0$ for $i < -r$. Then we deduce that

\[ \cdots \rightarrow P^{-r+1}_X \rightarrow \text{Im}(d^{-r-1}) \rightarrow 0 \rightarrow \cdots \]

is an acyclic complex. Therefore, the complex $P^\bullet_X$ is isomorphic in $D^b(A)_{I_Gd}$ to a complex of the form

\[ 0 \rightarrow P^{-r}_X / \text{Im}(d^{-r-1}_X) \xrightarrow{d^{-r+1}} P^{-r+1}_X \rightarrow \cdots \rightarrow P^0_X \rightarrow 0 \rightarrow \cdots \]

We conclude that

\[ P^{-r}_X / \text{Im}(d^{-r-1}_X)[r] \simeq P^\bullet_X \simeq X^\bullet \]

in $D^b(A)_{I_Gd}/K^b(A\text{-proj})$. So $P^{-r}_X / \text{Im}(d^{-r-1}_X)$ is a Gorenstein projective $A$-module [12, Proposition 2.10]. Then there is an exact sequence

\[ 0 \rightarrow P^{-r}_X / \text{Im}(d^{-r-1}_X) \rightarrow Q^0 \rightarrow Q^1 \rightarrow \cdots \rightarrow Q^{r-1} \rightarrow M \rightarrow 0 \]

such that

\[ M \simeq \Omega^{-r}(P^{-r}_X / \text{Im}(d^{-r-1}_X)), \]

where $Q^i$ are projective for $0 \leq i \leq r - 1$. Then

\[ P^{-r}_X / \text{Im}(d^{-r-1}_X)[r] \simeq M \]

in $D^b(A)_{I_Gd}/K^b(A\text{-proj})$. Therefore, $L(M) \simeq X^\bullet$. This shows that $L$ is dense.

\[ \square \]

**Lemma 5** [12, Theorem 4.2] Let $\Omega^\bullet$ be a two-sided tilting complex in $D^b(B \otimes A^{op}\text{-Mod})$. Then the standard derived equivalence

\[ F = \Omega^\bullet \otimes^L_A : D^b(A) \rightarrow D^b(B) \]

induced an equivalence

\[ D^b(A)_{I_Gd} \simeq D^b(B)_{I_Gd}. \]

The following lemma is very useful in our proof. Then we see that standard derived equivalences preserve Gorenstein projective modules.
Lemma 6 Let $F: \mathsf{D}^b(\mathcal{A}) \rightarrow \mathsf{D}^b(\mathcal{B})$ be a standard derived equivalence between Artin algebras $\mathcal{A}$ and $\mathcal{B}$, and let $G$ be the quasi-inverse of $F$. Suppose that $P^\bullet$ and $\mathcal{P}$ are the tilting complexes associated to $F$ and $G$, respectively. Then

(i) for $X \in \mathsf{A-proj}$, the complex $F(X)$ is isomorphic in $\mathsf{D}^b(\mathcal{B})$ to a radical complex $\mathcal{P}^0_X$ of the form

$$0 \rightarrow \mathcal{P}^0_X \rightarrow \mathcal{P}^1_X \rightarrow \cdots \rightarrow \mathcal{P}^{n-1}_X \rightarrow \mathcal{P}^n_X \rightarrow 0$$

with $\mathcal{P}^0_X \in \mathsf{B-proj}$ and $\mathcal{P}^i_X$ projective $\mathcal{B}$-modules for $1 \leq i \leq n$;

(ii) for $Y \in \mathsf{B-proj}$, the complex $G(Y)$ is isomorphic in $\mathsf{D}^b(\mathcal{A})$ to a radical complex $\mathcal{P}_Y^\bullet$ of the form

$$0 \rightarrow \mathcal{P}_Y^{-n} \rightarrow \mathcal{P}_Y^{-n+1} \rightarrow \cdots \rightarrow \mathcal{P}_Y^{-1} \rightarrow \mathcal{P}_Y^0 \rightarrow 0$$

with $\mathcal{P}_Y^{-n} \in \mathsf{A-proj}$ and $\mathcal{P}_Y^i$ projective $\mathcal{A}$-modules for $-n + 1 \leq i \leq 0$.

Proof We only show the first case. The proof of (ii) is similar to that of (i).

(i) For $X \in \mathsf{A-proj}$, by [10, Lemma 3.1], we see that the complex $F(X)$ is isomorphic in $\mathsf{D}^b(\mathcal{B})$ to a complex $\mathcal{P}^0_X$ of the form

$$0 \rightarrow \mathcal{P}^0_X \rightarrow \mathcal{P}^1_X \rightarrow \cdots \rightarrow \mathcal{P}^{n-1}_X \rightarrow \mathcal{P}^n_X \rightarrow 0,$$

with $\mathcal{P}^i_X$ projective $\mathcal{B}$-modules for $i > 0$. We need to show that $\mathcal{P}^i_X$ is a Gorenstein $\mathcal{B}$-module. Then there exists a distinguished triangle

$$\mathcal{P}^0_X \xrightarrow{i} \mathcal{P}_X^i \xrightarrow{j} \mathcal{P}^0_X \rightarrow \mathcal{P}^0_X[1]$$

in $\mathsf{K}^b(\mathcal{B})$, where $\mathcal{P}^0_X$ denotes the complex $\tau_{\geq 1}(\mathcal{P}^0_X)$. For each $i \in \mathbb{Z}$, applying the functor $\hom_{\mathsf{D}^b(\mathcal{B})}(-, \mathcal{B}[i])$ to the above distinguished triangle, we get an exact sequence

$$\cdots \rightarrow \hom_{\mathsf{D}^b(\mathcal{B})}^i(\mathcal{P}^0_X[1], \mathcal{B}[i]) \rightarrow \hom_{\mathsf{D}^b(\mathcal{B})}^i(\mathcal{P}^0_X, \mathcal{B}[i]) \rightarrow \hom_{\mathsf{D}^b(\mathcal{B})}^i(\mathcal{P}_X^i, \mathcal{B}[i]) \rightarrow \hom_{\mathsf{D}^b(\mathcal{B})}^{i+1}(\mathcal{P}_X^i, \mathcal{B}[i]) \rightarrow \cdots .$$

On the other hand,

$$\hom_{\mathsf{D}^b(\mathcal{B})}^i(\mathcal{P}_X^i, \mathcal{B}[i]) \simeq \hom_{\mathsf{K}^b(\mathcal{B})}^i(\mathcal{P}^0_X, \mathcal{B}[i]) = 0, \quad i \geq 0.$$

By [18, Lemma 2.1] and $\hom^i_{\mathcal{A}}(X, \mathcal{A}) = 0$ for $i \geq 1$, $\hom_{\mathsf{D}^b(\mathcal{A})}(X, \mathcal{A}[i]) = 0$. Thus,

$$\hom_{\mathsf{D}^b(\mathcal{B})}(\mathcal{P}_X^i, \mathcal{B}[i]) \simeq \hom_{\mathsf{D}^b(\mathcal{A})}(G(\mathcal{P}_X^i), G(\mathcal{B})[i]) \simeq \hom_{\mathsf{D}^b(\mathcal{A})}(X, \mathcal{A}[i]) = 0$$

for all $i \geq 1$. Consequently, we get

$$\hom_{\mathsf{D}^b(\mathcal{B})}(\mathcal{P}^0_X, \mathcal{B}[i]) = 0, \quad \forall i \geq 1,$$
by the above exact sequence. Therefore,
\[ Ext^i_B(\mathcal{P}^0_X, B) \simeq \text{Hom}_{D^b(B)}(\mathcal{P}^0_X, B[i]) = 0, \quad i \geq 1. \]
In the following, we show that \( \eta_{\mathcal{P}^0_X} \) is an isomorphism. Since \( \text{RHom}_B(-, B) \) is a right derived contravariant functor, there is a commutative diagram

\[
\begin{array}{ccc}
\mathcal{P}^0_X & \to & T^0_X \\
\downarrow \eta_{\mathcal{P}^0_X} & & \downarrow \eta_{T^0_X} \\
\text{R}(\text{R}(\mathcal{P}^0_X, B), B) & \to & \text{R}(\text{R}(\mathcal{P}^0_X, B), B)
\end{array}
\]

where
\[ \text{R}(\text{R}(\mathcal{P}^0_X, B), B) = \text{RHom}_B(\text{RHom}_B(\mathcal{P}^0_X, B), B). \]
To see that the diagram is commutative, we have the following argument. Take a minimal projective resolution \( Q^\bullet_{\mathcal{P}^0_X} \) of \( \mathcal{P}^0_X \). Then \( T^0_X \) is isomorphic to the complex \( \sigma_{\geq 0}(Q^\bullet_X) \).

Then there exists a distinguished triangle
\[ T^0_X \to Q^\bullet_X \to Q^\bullet_{\mathcal{P}^0_X} \to T^0_X[1] \]
in \( D^b(B) \), and a commutative diagram

\[
\begin{array}{ccc}
\mathcal{P}^0_X & \overset{i}{\to} & T^0_X \\
\downarrow \eta_{\mathcal{P}^0_X} & & \downarrow \eta_{T^0_X} \\
\text{R}(\text{R}(\mathcal{P}^0_X, B), B) & \overset{j}{\to} & \text{R}(\text{R}(\mathcal{P}^0_X, B), B)
\end{array}
\]

where
\[ \text{\textbullet}(\text{\textbullet}(\mathcal{P}^0_X, B), B) = \text{Hom}_B(\text{\textbullet}(\mathcal{P}^0_X, B), B). \]
By Lemma 5, \( T^0_X \) and \( \mathcal{P}^0_X \) are in \( D^b(B)_{\text{Gal}} \). Then \( \eta_{\mathcal{P}^0_X} \) and \( \eta_{T^0_X} \) are isomorphisms. Therefore, it follows from the axiom of triangulated category that \( \eta_{Q^\bullet_{\mathcal{P}^0_X}} \) is an isomorphism. Since
\[ \text{RHom}_B(\text{RHom}_B(\mathcal{P}^0_X, B), B) \simeq \text{Hom}_B(\text{\textbullet}(Q^\bullet_{\mathcal{P}^0_X}, B), B), \]
$\eta_{P^0_X}$ is an isomorphism. Thus,

$$P^0_X \simeq \text{RHom}_B(\text{RHom}_B(P^0_X, B), B).$$

Consequently, we get

$$H^i(\text{RHom}_B(\text{RHom}_B(P^0_X, B), B)) = 0, \quad i \neq 0.$$

It follows that

$$P^0_X \simeq \text{Hom}_B(\text{Hom}_B(P^0_X, B), B),$$

$$\text{Ext}^i_B(\text{Hom}_B(P^0_X, B), B) = 0 = \text{Ext}^i_B(P^0_X, B), \quad i \geq 1.$$

By Lemma 3 (3), $P^0_X$ is a Gorenstein projective $B$-module.

The following theorem is well known, which is proved by Beligiannis [3] or Kato [12], but we give the explicit formula by the standard derived equivalence $F: \text{D}^b(A) \to \text{D}^b(B)$.

**Theorem 5** Let $F: \text{D}^b(A) \to \text{D}^b(B)$ be a standard derived equivalence. Then there is an equivalence $F: \text{A-Gproj} \to \text{B-Gproj}$ sending $X$ to $P^0_X$, such that the diagram

$$\begin{array}{ccc}
\text{A-Gproj} & \xrightarrow{\simeq} & \text{D}^b(A)_{\text{Igd}}/K^b(\text{A-proj}) \\
F \downarrow & & \downarrow F \\
\text{B-Gproj} & \xrightarrow{\simeq} & \text{D}^b(B)_{\text{Igd}}/K^b(\text{B-proj})
\end{array}$$

is commutative up to natural isomorphism.

**Proof** This theorem is a straightforward consequence of Lemma 6.

**Remark 8** If $A$ is a Gorenstein algebra, then $\text{D}^b(A)_{\text{Igd}} = \text{D}^b(A)$ [12]. Then the canonical functor $\text{A-Gproj} \to \text{D}^b(A)/K^b(\text{A-proj})$ is an equivalence. This result is due to Buchweitz [5, Theorem 4.4.1] and independently to Happel [8, Theorem 4.6].

### 3.2 Proof of Theorem 1

Keep notations as before. We shall give the proof of Theorem 1.

Let $A$ be an Artin algebra, and let $X$ be in $_A\mathcal{X}$, which is not a projective $A$-module. Set

$$U = A \oplus X, \quad \Lambda = \text{End}_A(U), \quad V = B \oplus F(X), \quad \Gamma = \text{End}_B(V).$$

Let $T^\bullet$ be the complex $P^0_/\oplus P^0_X$. Then $T^\bullet$ is in $K^b(\text{add}_B V)$.

**Lemma 7** [15, Lemma 3.7] We have the following statements:

1. $\text{Hom}_{K^b(\text{add}_B V)}(T^\bullet, T^\bullet[i]) = 0$ for $i \neq 0$;
(2) \( \text{add} \mathcal{T}^\bullet \) generates \( \text{K}^b(\text{add}_B V) \) as a triangulated category.

Recall that \( \mathcal{T}^\bullet \) is the complex \( \mathcal{P}^\bullet \oplus \mathcal{P}_X^\bullet \), which is of the form

\[
0 \to \mathcal{T}^0 \to \mathcal{T}^1 \to \cdots \to \mathcal{T}^n \to 0.
\]

As before, the complex \( \text{Hom}_B(V, \mathcal{T}^\bullet) \) is of the form

\[
0 \to \text{Hom}_B(V, \mathcal{T}^0) \to \cdots \to \text{Hom}_B(V, \mathcal{T}^n) \to 0,
\]

and the differential of \( \text{Hom}_B(V, \mathcal{T}^\bullet) \) is \( \text{Hom}_B(V, d^\bullet) \), where \( d^\bullet \) is the differential map of \( \mathcal{T}^\bullet \) and \( \text{Hom}_B(V, \mathcal{T}^\bullet) \) is a projective \( \text{End}_B(V) \)-module. Then we have the following lemma which shows that the complex \( \text{Hom}_B(V, \mathcal{T}^\bullet) \) is a tilting complex for \( \text{End}_B(V) \).

**Lemma 8** [15, Proposition 3.8] The complex \( \text{Hom}_B(V, \mathcal{T}^\bullet) \) is a tilting complex for \( \text{End}_B(V) \) with endomorphism algebra isomorphic to \( \text{End}_A(U) \).

We now have all the ingredients to complete the proof of Theorem 1.

**Proof of Theorem 1** Let \( F : \text{D}^b(A) \to \text{D}^b(B) \) be a standard derived equivalence. By Theorem 5, there is an equivalence between \( A\text{-Gproj} \) and \( B\text{-Gproj} \). Therefore, \( A \) is of finite Cohen-Macaulay type if and only if so is \( B \).

If \( X \) is an additive generator of \( A\text{-Gproj} \), then \( X = \oplus_{0 \leq i \leq n} X_i \), where each \( X_i \) is an indecomposable non-projective Gorenstein projective \( A \)-module. Set \( \Lambda = \text{End}_A(A \oplus X) \). Then \( \Lambda \) is the Cohen-Macaulay Auslander algebra of \( A \).

By Theorem 5, it follows that \( Y_i = \mathcal{E}(X_i) \) is an indecomposable non-projective Gorenstein projective \( B \)-module. Set \( Y = \oplus_{0 \leq i \leq n} Y_i \). Then \( \Gamma = \text{End}_B(B \oplus Y) \) is the Cohen-Macaulay Auslander algebra of \( B \). Let \( N \) be the \( B \)-module \( (B \oplus Y) \) and let \( \mathcal{T}^\bullet \) be the complex \( F(A \oplus X) \). Thus, we construct a tilting complex \( \text{Hom}(N, \mathcal{T}^\bullet) \). The result follows from Lemma 8 and Theorem 3. \( \square \)

**Corollary 1** Let \( A \) and \( B \) be finite-dimensional \( k \)-algebras of finite Cohen-Macaulay type. If \( A \) and \( B \) are derived equivalent, then the Cohen-Macaulay Auslander algebras \( \Lambda \) and \( \Gamma \) of \( A \) and \( B \) are also derived equivalent.

As a corollary of Theorem 1, we re-obtain the following result of [15] for finite-dimensional Gorenstein \( k \)-algebras, where \( k \) is a field.

**Corollary 2** [15, Theorem 3.11] Let \( A \) and \( B \) be finite-dimensional Gorenstein \( k \)-algebras of finite Cohen-Macaulay type. If \( A \) and \( B \) are derived equivalent, then the Cohen-Macaulay Auslander algebras \( \Lambda \) and \( \Gamma \) of \( A \) and \( B \) are also derived equivalent.

We end this section with the following question.

**Question** Let \( A \) and \( B \) be Artin \( R \)-algebras of finite Cohen-Macaulay type. If \( A \) and \( B \) are derived equivalent, then are the Cohen-Macaulay Auslander algebras of \( A \) and \( B \) also derived equivalent?
4 Derived equivalences and Gorenstein projective conjecture

As is known, the vanishing of Ext groups and functors play a crucial role in the study of algebras, in general, of rings and their modules. Auslander conjectured that every Artin algebra satisfies a certain condition on vanishing of cohomology of finitely generated modules. Auslander’s conjecture is related to the finitistic dimension conjecture and Nakayama conjecture for finite-dimensional algebras [9]. In [11], Jørgensen and Şega proposed a counterexample which fails to Auslander’s condition. Furthermore, Auslander and Reiten conjectured [2] that a finitely generated module \( M \) over an Artin algebra \( A \) is projective if

\[
\text{Ext}^i_A(M, M) = 0 = \text{Ext}^i_A(M, A), \quad i \geq 1.
\]

This conjecture is still open now. Christensen and Holm [6] studied the rings that satisfy Auslander’s condition on vanishing of cohomology and proved that the Auslander-Reiten conjecture is true for the rings which satisfy Auslander’s condition.

As a special case of Auslander-Reiten conjecture, Luo and Huang [13] proposed the following Gorenstein projective conjecture. Let \( A \) be an Artin algebra, and let \( M \) be a Gorenstein projective module. Then \( M \) is projective if and only \( \text{Ext}^i_A(M, M) = 0 \) for \( i \geq 1 \).

We refer the reader to [21] and references therein for some new advances on the conjecture.

We have shown in [15] that a derived equivalence \( F \) between \( A \) and \( B \) induces an additive functor \( F \) between the stable categories \( A^\mathcal{D} \to B^\mathcal{D} \), where \( A^\mathcal{D} \) is the stable category of \( A^\mathcal{D} \). The following lemma is useful in our proof.

**Lemma 9** [17, Lemma 3.8] For \( X \in A^\mathcal{D} \), we have the following results.

1. For each positive integer \( k \), there is an isomorphism
   \[
   \beta_k : \text{Hom}_{D^b(A)}(X, X[k]) \to \text{Hom}_{D^b(B)}(F(X), F(X)[k]).
   \]
   Here, we denote the image of \( g \) under \( \beta_k \) by \( \beta_k(g) \).

2. For positive integers \( k \) and \( l \), the \( \beta_k \) and \( \beta_l \) satisfy the following rule:
   \[
   \beta_k(g)(\beta_l(h)[k]) = \beta_{k+l}(g(h[k]))
   \]
   for all \( g \in \text{Hom}_{D^b(A)}(X, X[k]) \) and \( h \in \text{Hom}_{D^b(A)}(X, X[l]) \).

Motivated by Lemma 9, we consider the relationship between the Auslander-Reiten conjecture and derived equivalences. In [16], we proved that derived equivalences preserve the generalized Auslander-Reiten conjecture. We use the method in [16] to prove derived equivalences preserve the Auslander-Reiten conjectures [17].

**Theorem 6** Let \( A \) and \( B \) be Artin \( R \)-algebras. Suppose that \( A \) and \( B \) are standard derived equivalent. Then \( A \) satisfies the Gorenstein projective conjecture if and only if so does \( B \).
Proof. We assume that the Gorenstein projective conjecture is true for \( B \). If \( X \) is a Gorenstein projective \( A \)-module which satisfies \( \text{Ext}^i_A(X, X) = 0 \) for \( i \geq 1 \), then it follows from Lemma 6 that \( \overline{P}_X^0 \) is a Gorenstein projective \( B \)-module. By Lemma 9, it is easy to see that, for \( i \geq 1 \),

\[
\text{Ext}_B^i(\overline{P}_X^0, \overline{P}_X^0) \cong \text{Hom}_{\text{D}^b(B)}(\overline{P}_X^0, \overline{P}_X^0[i]) \\
\cong \text{Hom}_{\text{D}^b(A)}(X, X[i]) \\
\cong \text{Ext}_A^i(X, X) = 0.
\]

So, by our assumption, we see that the \( B \)-module \( \overline{P}_X^0 \) is projective. Therefore, \( X \cong G(\overline{P}_X^0) \in \text{K}^b(\text{A-proj}) \). Then it is easy to see that \( \text{proj.dim}(X) < +\infty \), where \( \text{proj.dim}(X) \) is the projective dimension of \( X \). Let

\[ 0 \rightarrow P_X^{-m} \rightarrow \cdots \rightarrow P_X^0 \rightarrow X \rightarrow 0 \]

be a minimal projective resolution of \( X \), where \( P_X^i \) is projective \( A \)-module for each \( -m \leq i \leq 0 \). Applying the functor \( \text{Hom}_A(-, X) \) to the above sequence, it follows that \( \text{Hom}_A(X, A) \) is a projective \( A^{\text{op}} \)-module and \( X \) is reflexive, since \( \text{Ext}_A^i(X, A) = 0 \) for \( i \geq 1 \). Then \( X \) is a projective \( A \)-module. Similarly, we can prove that the converse is also true. \( \square \)

In the following, we give some applications of Theorem 6. We recall the following theorem from [21] without a proof. For details of the proof, we refer to [21, Theorems 3.2, 4.6].

**Theorem 7** Let \( A \) be an Artin algebra. If \( A \) is a CM-finite Artin algebra, then \( A \) satisfies the Gorenstein projective conjecture.

By Theorems 6 and 7, we immediately have the following result.

**Corollary 3** Let \( A \) be an Artin algebra in Theorem 7. If \( B \) is an Artin algebra standard derived equivalent to \( A \), then \( B \) satisfies the Gorenstein projective conjecture.

We end this paper with the following question.

**Question** Let \( A \) and \( B \) be Artin algebras. Suppose that \( A \) and \( B \) are derived equivalent. Then does \( A \) satisfy the Gorenstein projective conjecture if and only if so does \( B \)?

**Acknowledgements** S. Y. Pan was supported by the National Natural Science Foundation of China (Grant No. 11201022), the Fundamental Research Funds for the Central Universities (2013JBM096, 2013RC027), and the Scientific Research Foundation for the Returned Overseas Chinese Scholars, Ministry of Education. This revision of the first draft was done when S. Y. Pan was a postdoctor of Bishop’s University, he would like to thank Professor Thomas Brüstle for his warm hospitality. X. J. Zhang was supported by National Natural Science Foundation of China (Grant No. 11101217).
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