KEPLER-79’S LOW DENSITY PLANETS

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ABSTRACT

Kepler-79 (KOI-152) has four planetary candidates ranging in size from 3.5 to 7 times the size of the Earth, in a compact configuration with orbital periods near a 1:2:4:6 chain of commensurability, from 13.5 to 81.1 days. All four planets exhibit transit timing variations with periods that are consistent with the distance of each planet to resonance with its neighbors. We perform a dynamical analysis of the system based on transit timing measurements over 1282 days of Kepler photometry. Stellar parameters are obtained using a combination of spectral classification and the stellar density constraints provided by light curve analysis and orbital eccentricity solutions from our dynamical study. Our models provide tight bounds on the masses of all four transiting bodies, demonstrating that they are planets and that they orbit the same star. All four of Kepler-79’s transiting planets have low densities given their sizes, which is consistent with other studies of compact multiplanet transiting systems. The largest of the four, Kepler-79 d (KOI-152.01), has the lowest bulk density yet determined among sub-Saturn mass planets.

Key words: planetary systems – stars: individual (KOI-152, Kepler-79) – techniques: photometric

Online-only material: color figures, machine-readable table

1. INTRODUCTION

Within our solar system, the Earth and smaller bodies are primarily mixtures of refractories, rock, and metals. In the outer solar system, the bodies that are too small to retain deep atmospheres contain rock and ices. In the larger planets, including Uranus and Neptune, the light elements H and He dominate the volume. There are no local examples of bodies that are intermediate in size or have a mass between Earth (1 $R_\oplus$, 1 $M_\oplus$) and Uranus/Neptune, both of which are larger than 3.8 $R_\oplus$ and more massive than 14 $M_\oplus$. Mass determinations of transiting exoplanets are beginning to allow the characterization of planets in this size range. As more planetary masses and radii are measured, their bulk densities will provide more constraints on their compositions.

Both accurate mass and radius determinations are required to derive meaningful planetary densities. The ratio of the planetary radius to the stellar radius is a direct measurement from transit light curves, where the fraction of starlight blocked during transit is a simple measure of the projected area of the planet. The uncertainty in the measurement of the size of a transiting planet typically rests on the accuracy to which the stellar radius can be constrained. The star 55 Cancri is unique as the host of a transiting sub-Neptune exoplanet in having a direct measurement of its radius via interferometry (von Braun et al. 2011). The high resolution spectral classification of the atmosphere of Kepler planetary host stars gives a model dependent measurement of the stellar radius with a typical uncertainty of 10%.

The measurement of stellar radius from spectral classification and modeling can be improved upon with additional information. The gold standard for this purpose is where asteroseismic oscillations are detected in the photometric light curve. Among these stars, uncertainties in mass and radius can be reduced to ~1%, although asteroseismic detections in the Kepler data are only available for giant stars and the brightest dwarf stars (Huber et al. 2013).

Another constraint on the stellar density, and hence the star’s radius, can be gleaned from the orbital constraints of exoplanets. The scaled semimajor axis, $a/R_*$, (where $R_*$ is the stellar radius) can be roughly estimated from the transit and ingress durations, but an accurate measurement of $a/R_*$ requires additional information about the orbital eccentricity and alignment ($\omega$), the argument of the pericenter, as measured from the ascending node in the sky-plane toward the Earth. For a measured fractional transit depth $\delta$, transit duration $T$, and either the ingress or egress duration $\tau$,

$$\frac{a}{R_*} = \frac{3^{1/4}}{\pi} \frac{P}{\sqrt{T\tau}} \left( \frac{\sqrt{1-e^2}}{1+e \sin \omega} \right)$$

(Winn 2011). For the purposes of measuring the stellar radius, the information about the orbital eccentricity obtained from dynamical fits, used in Equation (1), can provide an independent constraint on the stellar bulk density following Kepler’s Third Law (Seager & Mollén-Ornelas 2003; Winn 2011):

$$\rho_* \approx \frac{3\pi}{GP^2} \left( \frac{a}{R_*} \right)^3.$$

Here $\rho_*$ is the bulk density of the star, $G$ is the gravitational constant, and $P$ is the orbital period of the transiting planet. In their study of Kepler-11, Lissauer et al. (2013) used dynamical solutions for the orbital eccentricities, alongside high resolution spectra to reduce the uncertainty on the stellar radius to 2%. As a result, the planetary radii were measured with nearly that precision.

Of the transiting exoplanets, the majority of mass determinations to date are the result of radial velocity (RV) spectroscopy, and among these, most are the so-called hot jupiters, planets with substantial envelopes that orbit very close to their host star. The transiting Neptunes and sub-Neptunes with measured RV masses all have short orbital periods, the longest being Kepler-68 b (Gilliland et al. 2013) at 5.4 days.
Kepler-18’s planets have had their masses measured by the combined constraints of RV and transit timing variations (TTVs), with the farthest one orbiting every 14.9 days (Cochran et al. 2011).

TTVs exploit the high degree of accuracy in measuring the transit times of transiting exoplanets, with transit time uncertainties as low as a few minutes in some cases. These probe interplanetary perturbations, and in general are sensitive to the mass ratio of perturbing neighboring planets to the host star (Agol et al. 2005; Holman & Murray 2005). The strongest signals in TTVs occur when planets are near (but not trapped in) mean motion resonances, and the resonant argument cycles with a periodicity that is well sampled over a baseline of transit timing measurements. Near-first-order resonances, the coherence time is long enough for perturbations to build constructively to an easily detectable amplitude. Too far from resonance, the perturbations lose their coherence rapidly, and the TTV amplitude is reduced. Too near resonance, Kepler's 4 yr of observations do not cover a complete cycle. Nevertheless, because TTVs are sensitive to interplanetary perturbations and not the effect of the planet on the star, planetary masses can be determined to far greater orbital periods than RV, as long as enough transit times have been measured to detect timing variations. The super-Neptune Kepler-30 d, with an orbital period of 143 days (Sanchis-Ojeda et al. 2012) has the longest orbital period for an exoplanet with a determined mass represented on the mass–radius diagram. The difference in orbital periods probed by these two separate techniques highlight their respective biases. RV planets orbit close to their star, are hotter, and several, particularly the Earths and super-Earths—including Kepler-10 b (Batalha et al. 2011), CoRoT-7 b (Ferraz-Mello et al. 2011), KOI-94 b (Weiss et al. 2013), Kepler-20 b (Gautier et al. 2012), Kepler-78 b (Howard et al. 2013), and potentially Kepler-18 b (Cochran et al. 2011)—seem to lack deep atmospheres. The TTV mass determinations, such as Kepler-11 (Lissauer et al. 2011, 2013), Kepler-18 (Cochran et al. 2011), Kepler-30 (Sanchis-Ojeda et al. 2012), and Kepler-36 (Carter et al. 2012) are all either compact multiplanet systems with a small period ratio between neighboring planets, or near resonance. Although larger masses certainly improve the chance of detecting a TTV signal, the orbital stability of compact multiplanet systems like Kepler-11 require smaller masses and/or eccentricities. On a practical note, a compact configuration with high multiplicity reduces the risk of intermediate, non-transiting planets confusing the TTV dynamical models. In such dynamically packed systems, there is little room for other massive planets between the transiting planets without making the configuration unstable.

Kepler-79 (with a Kepler magnitude 13.9, located at R.A. = 20^h 02'04.11, decl. = 44°22'53.7") is an excellent candidate for TTV modeling. It was among the first known system with multiple candidates in the Kepler data (Steffen et al. 2010). It has four planetary candidates near-first-order mean motion resonances, with clear evidence of TTVs. Steffen et al. (2012) first noted TTVs at KOI-152 following six quarters of data, although only three candidates were known at the time. These three are near the 1:2:4 resonance, suggesting that multiplanet resonances are reasonably common (Wang et al. 2012). Wu & Lithwick (2013) noted the TTVs at KOI-152, and also found evidence that the inner two planetary candidates have significant eccentricities. This suggests that circular fits to the transit times would be hampered by the mass–eccentricity degeneracy (Lithwick et al. 2012; Wu & Lithwick 2013), casting uncertainty on measured masses. Xie (2013) estimated upper limits on the masses of the inner pair of planets at KOI-152, recognizing that significant eccentricities are likely in the system, and that the third planet may confuse the TTV signal of the second planet. The inner pair orbiting the star have been confirmed as planets, and the star has been renamed Kepler-79. In this paper, our analysis confirms that the remaining two candidates are planets, and throughout this paper we refer to KOI-152.01 and KOI-152.04 as Kepler-79 d and e, respectively.

In Section 2, we introduce our methodology for measuring transit times, and in Section 3, we examine the TTVs, evaluating the applicability of zero-eccentricity analytical solutions for the planetary masses. In Section 4, we describe our numerical models and fits to the TTVs, and present our results for the planetary masses, orbital parameters, and stellar parameters. In Section 5, we consider the potential effects of non-transiting perturbers on our four-planet model. In Section 6 we characterize the planets’ masses, radii, and bulk densities, and in Section 7, we compare our results for Kepler-79’s planets with other known planets of masses less than 30 $M_{\oplus}$ on the mass–radius diagram.

2. MEASUREMENT OF TRANSIT TIMES FROM KEPLER PHOTOMETRIC TIME SERIES

Variations in the brightness of Kepler-79 were monitored with an effective duty cycle exceeding 90% starting at barycentric Julian date (BJD) 2454964.512, with all data returned to Earth at a cadence of 29.426 minutes (long cadence, LC); data were also returned at a cadence of 58.85 s (short cadence, SC) beginning from BJD 2455093.216. Here and throughout we base our timeline for transit data from $T = JD-2,454,900$. Our analysis uses SC data where available, augmented by the LC data set primarily during the epoch prior to $T < 193$ days, for which no SC data were returned to Earth. We obtained these data from the publicly accessible MAST archive at http://archive.stsci.edu/kepler. To measure the transit times from the light curve, we adopt the procedure explained in detail in Appendix 7.1 of Lissauer et al. (2013). The full list of measured transit times and their uncertainties of Table 1 is available in the online journal.

3. ANALYTICS

We begin with the orbital periods based on a linear fit to the observed transit times, summarized in Table 2. We shall solve for the orbital parameters of these planets at $T = 780.0$ days, an epoch chosen to be near the middle of our data set. For each candidate, the first transit time after this chosen epoch, which was calculated from a linear ephemeris to the set of transit times, is at time $T_0$.

This configuration of planetary orbits lies close to a 1:2:4:6 resonance chain of orbital periods, and this system is known to exhibit TTVs (Steffen et al. 2012; Wu & Lithwick 2013; Mazeh et al. 2013; Xie 2013; also see http://exoplanet-science.com/KOI-152.html). Following the convention of Lithwick et al. (2012) and Wu & Lithwick (2013), we can measure the proximity, $\Delta$, of each adjacent pair in this chain to the nearest-first-order ($j : j - 1$) resonance as follows:

$$\Delta_j = \frac{P_j}{P} - j - 1,$$ (3)
Table 1
Measured Transit Times (JD-2,454,900) through Q16 and the Uncertainties for the Four Known Planets of Kepler-79

| Planet                  | Period (days) | $T_0$  |
|-------------------------|---------------|--------|
| Kepler-79 b (KOI-152.03)| 69.62898 ± 0.00778 | 66.61523 ± 0.00716 |
| Kepler-79 c (KOI-152.02)| 83.10956 ± 0.00736 | 94.02236 ± 0.00765 |
| Kepler-79 d (KOI-152.01)| 96.59011 ± 0.00643 | 121.4303 ± 0.00491 |
| Kepler-79 e (KOI-152.04)| 110.07083 ± 0.00829 | 148.83993 ± 0.00631 |
| Kepler-79 f (KOI-152.05)| 123.56791 ± 0.00965 | 176.24028 ± 0.00598 |
| Kepler-79 g (KOI-152.06)| 137.04513 ± 0.00591 | 203.64014 ± 0.00386 |
| Kepler-79 h (KOI-152.07)| 150.52842 ± 0.00664 | 231.04435 ± 0.00390 |
| Kepler-79 i (KOI-152.08)| 177.48572 ± 0.00689 | 258.44548 ± 0.00382 |
| Kepler-79 j (KOI-152.09)| 190.97697 ± 0.01045 | 285.84739 ± 0.00398 |
| Kepler-79 k (KOI-152.10)| 204.47418 ± 0.00637 | 313.25506 ± 0.00421 |
| Kepler-79 l (KOI-152.11)| 217.95003 ± 0.00465 | 340.66602 ± 0.00415 |
| Kepler-79 m (KOI-152.12)| 231.42280 ± 0.00461 | 368.06007 ± 0.00392 |
| Kepler-79 n (KOI-152.13)| 244.91971 ± 0.00468 | 395.47789 ± 0.00363 |
| Kepler-79 o (KOI-152.14)| 258.40553 ± 0.00454 | 422.86349 ± 0.00356 |
| Kepler-79 p (KOI-152.15)| 271.88660 ± 0.00440 | 450.26762 ± 0.00355 |
| Kepler-79 q (KOI-152.16)| 285.37307 ± 0.00671 | 477.67564 ± 0.00354 |
| Kepler-79 r (KOI-152.17)| 298.85861 ± 0.00496 | 505.07279 ± 0.00358 |
| Kepler-79 s (KOI-152.18)| 312.32839 ± 0.00496 | 532.46992 ± 0.00369 |

Table 2
A Linear Fit to Sixteen Quarters of Kepler’s Observed Transit Times for the Planets at Kepler-79 (Formerly KOI-152), Specified as Orbital Periods and the First Calculated Transit Time after $T = 780$ Days

| Planet                  | Period (days) | $T_0$  |
|-------------------------|---------------|--------|
| Kepler-79 b (KOI-152.03)| 13.4845 | 784.3010 |
| Kepler-79 c (KOI-152.02)| 27.4023 | 806.4999 |
| Kepler-79 d (KOI-152.01)| 52.0909 | 821.0171 |
| Kepler-79 e (KOI-152.04)| 81.0631 | 802.1119 |

Table 3
Orbital Period Ratios in the Kepler-79 (KOI-152) System, and Their Proximity to First-order ($j : j - 1$, Third Column) and Second-order (Fourth Column) Resonances

| Pair       | Period Ratio | $\Delta_1$ | $\Delta_2$ | Expected TTV Period (days) |
|------------|--------------|------------|------------|---------------------------|
| b.c        | 2.032        | 0.016      | ...        | 852.8                     |
| c.d        | 1.901        | -0.050     | ...        | 525.9                     |
| d.e        | 1.557        | 0.037      | ...        | 721.4                     |
| c.e        | 2.959        | -0.014     | ...        | (1942.0)                  |

Notes. The final column denotes expected TTV periodicities for each pair of potential interactions, with the periodicities near resonances that are not first order, and are thus likely to produce weak perturbations in parentheses.

where $P$ and $P'$ are the orbital periods of the inner and outer planets, respectively. The expected TTV period in this case is:

$$P_{\text{TTV}} = \left| \frac{j}{P'} - \frac{j - 1}{P} \right|^{-1}.$$  (4)

We seek a similar measure for proximity to second-order resonances ($\Delta_2$), where the expected TTV period replaces $j - 1$ with $j - 2$ in Equations 3 and 4. Table 3 highlights the proximity of each pair to first- or second-order resonances. Each pair is close to a first-order resonance (either 2:1 or 3:2). More distant pairings are close to high-order (weaker) resonances, the lowest of which is the near 3:1 resonance between c and e.

Note that for comparable TTV amplitudes, and assuming the TTVs are linear, the TTVs on “c” and “d” are likely a superposition of the two periodicities caused by their immediate inner and outer neighbors. Assuming there are no unseen perturbers, we seek to assess a model for the observed transit times to each transiting planet as the sum of the perturbations of its nearest neighbors. In Figure 1, we show the TTVs for each transiting planet, and solve for sinusoidal fits to the TTVs. The TTV periods are fixed at their expected values based on the orbital periods, and the best-fit amplitudes and phases are solved with a Markov Chain Monte Carlo (MCMC) algorithm. The solutions to the amplitudes and phases are in Table 4. The uncertainties were measured by recording all extrema in TTV phase and the amplitude for models within one reduced $\chi^2$ unit from the best-fit solution. The uncertainties were largely symmetric, so we quote the average of positive and negative uncertainties for simplicity. Xie (2013) noted that the apparent anti-correlated TTVs for the inner pair of Kepler-79 are likely affected by Kepler-79 d, which is confirmed in the observed TTVs for Kepler-79 c with 16 quarters of Kepler data.

For each pair of neighboring planets orbiting Kepler-79, we calculate the time, $T_\phi$, that the TTVs in our sinusoidal fits transition from positive to negative for inner planets, and where there is a transition from negative to positive for outer planets of the same pair, such that exactly anti-correlated TTVs from pairwise interactions would have the same $T_\phi$. Planets on very nearly circular orbits should have perfectly anti-correlated sinusoidal TTVs. For most alignments, the eccentric orbits lead to a detectable phase shift with respect to anti-correlated TTVs. To look for evidence of orbital eccentricity, we compare the phases of planet pairs, to see if there is a detectable sinusoidal TTV fit between the fitted sinusoids ($T_\phi$, with its uncertainties) and the time that conjunctions cross the line of sight ($T_{\text{conj}}$). Lithwick...
et al. (2012) define the phases “φ_{in}” or “φ’_{in}” of TTVs for inner and outer planets of a pair respectively, with respect to when planetary conjunctions pass the line of sight. This is determined by calculating when the longitude of conjunctions,

$$\lambda^j = j\lambda' - (j-1)\lambda = 0.$$  \hspace{1cm} (5)

Here, \(\lambda\) and \(\lambda'\) are the mean longitudes of the inner and outer planets, respectively (Agol et al. 2005; Lithwick et al. 2012).

The times \(T_{\phi0}\) in the fourth column of Table 4 are consistent with the TTVs being anti-correlated—the \(T_{\phi}\) values for neighboring pairs agree at the 1\(\sigma\) level. Thus in the relative phases of the sinusoids of neighboring planets, there is no significant evidence of orbital eccentricity. However, the difference between the times \(T_{\phi}\) and the date that the longitude of the conjunctions is closest to the line of sight \((T_{\text{conj}})^0\), determined by comparing \(\phi_{\text{in}}\) and \(\phi'_{\text{in}} + 180°\), does show evidence of orbital eccentricity. There are significant phase shifts in “b” and the TTVs induced in “c” by “b.” Wu & Lithwick (2013) noted these among the inner pair as evidence of significant free eccentricity. We also find a significant phase shift in the TTVs induced in “d” by “c.” Hence, at a minimum, either “c” must have significant eccentricity or “b” and “d” must have significant eccentricity. However, there is no strong evidence for free eccentricity in the interaction between “d” and “e.” A detectable phase shift implies that free eccentricities are present. When free eccentricities are present, they typically increase the effect of each perturbation causing the TTVs. However, the inverse of \(\Delta\) determines the coherence time for perturbations to cause sinusoidal TTVs (Lithwick et al. 2012).

Before we relax the assumption of circular orbits, we can calculate a quick estimate of the planetary masses. Nominal estimates of the masses of these planets can be made following the solutions of Lithwick et al. (2012) and Wu & Lithwick (2013), assuming that the orbits are circular. For an interacting pair of planets of mass \(m\) and \(m'\), orbiting a star of mass \(M_\star\), near a \((j : j - 1)\) resonance at periods \(P\) and \(P'\), respectively

$$m = M_\star \left| \frac{V\Delta}{P g} \right| \pi j \hspace{1cm} (6)$$

$$m' = M_\star \left| \frac{V\Delta}{P f} \right| \pi^{2/3} j^{-1/3}, \hspace{1cm} (7)$$

where \(f(\Delta)\) and \(g(\Delta)\) are numerical coefficients of the disturbing function near resonance, calculated in Lithwick et al. (2012). Here \(V\) and \(V'\) are the TTV amplitudes. Note that with our longer baseline, the amplitude of the TTVs for the inner two candidates are slightly less than the measured values of Wu & Lithwick (2013), and agree within 1\(\sigma\) uncertainties. For planets “c” and “d”, we calculate two estimates of the mass. In general, these estimates are upper limits, as significant free eccentricities are more likely to increase TTVs than to decrease them. We tabulate these nominal masses in Table 5. Note that for “c,” the nominal masses estimated by the TTVs induced on its inner and outer neighbors are inconsistent. This implies that either the TTVs are caused by unseen perturbers, or that the circular orbits are a poor fit to the data. Due to the degeneracy between mass and eccentricity in TTVs, the expectation of eccentricity casts doubt upon planetary mass estimates made under an assumption of circular orbits.

TTVs with significant non-sinuousoidal components contain information that is difficult to probe analytically. Hence we perform numerical fits to the transit times to solve for masses and the osculating orbital parameters of the planets at the epoch \(T = 780\) days.

**Figure 1.** Sixteen quarters of transit timing variations for Kepler-79 (colored points) using primarily short cadence data, supplemented by long cadence data where short cadence was unavailable. The TTVs are the difference between the observed transit times and a calculated linear ephemeris (O–C). The solid curves are best-fit sinusoids (“b” and “c”), or the sum of two sinusoids (“c” and “d”: shown as dashed curves). The sinusoidal fits solve for amplitude and phase, while the periods remain fixed at the expected TTV period, given in Table 3. (A color version of this figure is available in the online journal.)
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O transit times (mutual inclinations between planetary orbits) in all of our free parameters, and have assumed co-planarity (i.e., negligible to our chosen epoch. We also keep all planetary masses as phas is specified by the midpoint of the first transit subsequent and phase of each of the planets are free parameters. The has ninth-order errors. In all of our models, the orbital period dynamical models. To integrate planetary motions, we adopt fits to this data set. Our free parameters were orbital parameters. Assuming Circular Orbits the Planets of Kepler-79, Relative to the Host Star Table 4 The Superposition of TTVs at Kepler-79, with Their Expected TTV Periods (Second Column), Their Best-fit Amplitudes (Third Column), and Times that the TTVs Transition Across Zero (Fourth Column), as Depicted Graphically in Figure 1, Shown Here with Measured Uncertainties.

| Candidate | TTV Period (days) | TTV Ampl. (minutes) | $T_\phi$ (days) | $T_{\text{conj}}$ | TTV Phase |
|-----------|-------------------|---------------------|----------------|----------------|------------|
| b         | 852.8             | 6.24 ± 1.35         | 195.4 ± 30.4   | 256            | $\phi_{\text{ttv}} = -26 ± 13^\circ$ |
| c         | 852.8             | 14.24 ± 1.59        | 161.9 ± 14.3   | 256            | $\phi_{\text{ttv}} = 140 ± 6^\circ$  |
|           | 525.9             | 5.67 ± 1.48         | 562.9±23.0     | 574            | $\phi_{\text{ttv}} = -8 ± 16^\circ$  |
| d         | 525.9             | 6.56 ± 0.96         | 539.8 ± 13.1   | 574            | $\phi_{\text{ttv}} = 157 ± 9^\circ$  |
|           | 721.4             | 8.31 ± 0.52         | 614.9 ± 14.2   | 605            | $\phi_{\text{ttv}} = 5 ± 7^\circ$    |
| e         | 721.4             | 19.06 ± 3.81        | 625.0 ± 24.1   | 605            | $\phi_{\text{ttv}} = 190 ± 12^\circ$ |

Notes. The fifth column lists the times that conjunctions in mean longitude pass the line of sight, and the final column measures the TTV phase in degrees, where for an inner planet with no TTV phase shift, $\phi_{\text{ttv}} = 0^\circ$, and for an outer planet with no phase shift, $\phi_{\text{ttv}} = 180^\circ$. All times are measured in days unless otherwise indicated.

Table 5 Nominal Mass Estimates Assuming Circular Orbits for the Planets of Kepler-79, Relative to the Host Star

| Planet | $m_p$ ($10^{-6} M_\ast$) |
|--------|--------------------------|
| b      | 98.7 ± 11.0              |
| c      | 22.3 ± 4.8               |
|        | 44.5 ± 6.5               |
| d      | 27.3 ± 7.1               |
|        | 25.5 ± 5.1               |
| e      | 19.1 ± 1.2               |

Notes. For “c” and “d,” perturbations on two neighboring planets permit two estimates of masses, at the amplitudes calculated in Table 4. These are likely upper limits on planetary masses. Here, the inconsistency for the mass of “c” may be due to significant orbital eccentricities. Our preferred estimates for the planetary masses are given in Section 4.

4. DYNAMICAL MODELS OF KEPLER-79 WITH FOUR PLANETS

Figure 2 shows the TTVs for the four known candidates and a dynamical fit to this data set. Our free parameters were orbital period, $T_0$ the time of the first transit after epoch ($T = 780$ days), the eccentricity vector components $e \cos \omega$ and $e \sin \omega$ (where $\omega$ is measured from the sky-plane and reaches 90° if the transit is at the pericenter), and planetary mass. The dynamical models measure planetary mass as a fraction of the stellar mass. However, following Lissauer et al. (2013), an accurate constraint on $\rho_*$, and hence the stellar mass, is one of the benefits of dynamical models. To integrate planetary motions, we adopt the eighth-order Runge–Kutta Dormand–Prince method, which has ninth-order errors. In all of our models, the orbital period and phase of each of the planets are free parameters. The phase is specified by the midpoint of the first transit subsequent to our chosen epoch. We also keep all planetary masses as free parameters, and have assumed co-planarity (i.e., negligible mutual inclinations between planetary orbits) in all of our dynamical models. We make no attempt to model transit durations or impact parameters in our dynamical simulations.

Our integrations produce an ephemeris of simulated transit times, $S$, and we compare these simulated times to the observed transit times ($O$). We employ the Levenberg–Marquardt algorithm to search for a local minimum in $\chi^2$. The algorithm evaluates the local slope and curvature of the $\chi^2$ surface. Once it obtains a minimum, the curvature of the surface is used to populate the covariance matrix and evaluate local uncertainties. Other parameters are allowed to float when determining the limits on an individual parameter’s error bars. Assuming that the $\chi^2$ surface is parabolic in the vicinity of its local minimum, its contours are concentric ellipses centered at the best-fit value. The orientations of these ellipses depend on correlations between parameters. The errors that we adopt account for the increase in uncertainty in some dimensions due to such correlations.

Our best-fit model for the nominal 190 transit times (shown as TTVs in Figure 1) of the four known planetary candidates leaves six data points that are outliers beyond 3σ (where $O - S/\sigma > 3$) (of which there should be ≈0.4, i.e., likely zero or one, if the errors were Gaussian), and 20 points that are between 2σ and 3σ (of which there should be ≈8 if the errors were Gaussian). Clearly the uncertainty in some of the transit times is either under-estimated or the four-planet model is wrong. The outliers may be due to errors in some of the measured transit times that are not incorporated into timing uncertainty estimates; sources of such errors could be stellar activity (including flares and starspots), instrumental effects, and so on.

To assess our dynamical model with $\chi^2$ minimization requires a method for dealing with these outliers. Lissauer et al. (2013), in their TTV analysis of Kepler-11, compared three independent methods of measuring transit times from light curves to filter out outlying or anomalous transit times. They discarded points that did not have overlapping error bars for transit times with at least one of the other two transit time measurements. Here, for Kepler-79, we seek a method to self-filter a single measured set of transit times, noting that the distribution of measured times has far too many 3σ outliers, as well as too many 2σ outliers. Hence, we compare the best-fit dynamical models against the combined “raw” set of short and LC transit times with best-fit models where outliers beyond 3σ or 2σ, respectively, are removed, and a data set of SC only transit times with outliers beyond 3σ removed. We thus conduct our dynamical fits against four sets of transit times.

For each set of transit times, we find a best-fit solution and then evaluate which outliers, if any, are beyond our 2σ or 3σ threshold. These points are removed and another best-fit solution is found with dynamical models. Iterations continue until there are no more outliers at the best-fit solution.

To account for multiple local minima in the $\chi^2$ surface, we used multiple choices of initial conditions, and recorded all solutions with $\chi^2$ values within one reduced $\chi^2$ unit of the best-fit model. The outputs of each local minimum were used
Figure 2. Observed and simulated transit timing variations for the planetary candidates orbiting Kepler-79, using the combined data set of short and long cadence transit times. The panels on the left side compare $O - C$ (colored data) and model times, $S - C$ (black points). The right-hand side plots the residuals with the dynamical model subtracted from the observed transit timing variations. Note the vertical scales for the residuals (right panels) differ from those of the TTVs.

(A color version of this figure is available in the online journal.)
that are within one reduced chi-square unit of the best known fit for each data set of transit times were included. The nominal uncertainties for model fits $\sigma_{\text{norm}}$ are reduced for all minima apart from the one with the lowest chi-square, with uncertainties reduced according to the formula: $\sigma_e = \sigma_{\text{norm}} \sqrt{1 - (\Delta \chi^2 / (\text{dof}))}$. This assumes that the local minimum in $\chi^2$ is on a parabolic surface, and therefore extends error bars to reach where $(\Delta \chi^2 / (\text{dof})) \approx 1$. The results in Figure 3 show that a wide range of eccentricities can satisfy the data for Kepler-79 d, and that the individual best-fit solutions are indeed moderately sensitive at the ~1σ level to the manner in which outlying transit time measurements are handled.

Nevertheless, with our exploration of multiple local minima in the $\chi^2$ surface, our uncertainties are augmented to ensure that our best-fit models cover the solutions and uncertainties of all four sets of transit times. We note that within each data set, the uncertainties with differing data sets are all consistent at the 1σ level. In Figure 4, we show the 1σ uncertainties for the masses and the components of the eccentricity vectors for all four planets. For our total uncertainty for each parameter, we adopt the union of all uncertainties for each parameter as shown in Figure 4, and the median of the four best-fit solutions for each parameter as our nominal solution. The results for all the fitted parameters are in Table 7.

To test the solutions for long-term orbital stability, we simulated the trajectories of the best-fit solutions using a symplectic integrator (Rauch & Hamilton 2012) for 1 Gyr and found the system to be stable. (We ignored mass loss, tidal evolution, and general relativity.) Increasing masses to their 1σ limits resulted in a system that appeared unstable after 300 Myr. However, by increasing the eccentricities to their 1σ maxima and leaving the masses at their nominal values, the system lasted just 70,000 yr before orbits appeared unstable. These tests indicate that dynamically, Kepler-79’s planets are packed close to the stability limit.

The perturbations of each planet orbiting Kepler-79 can be deconvolved as a linear sum of pairwise TTVs to a high degree of accuracy. Figure 5 highlights the contribution of each planetary candidate to the TTVs of the other planets in the system, including the non-sinusoidal components to the TTV signal that are captured by dynamical fits. The remarkable fit of the pairwise perturbations in adding up to match the net TTVs is indicative of the linear nature of Kepler-79’s

| Planet | $\chi^2_{\text{raw}}$ | $\chi^2_2$ | $\chi^2_3$ | $\chi^2_{3\sigma}$ |
|--------|-----------------|----------|----------|------------------|
| b      | 173.27          | 107.51   | 63.79    | 102.12           |
| c      | 88.95           | 56.25    | 32.49    | 51.96            |
| d      | 30.12           | 30.61    | 21.73    | 22.73            |
| e      | 43.52           | 14.74    | 9.79     | 13.44            |
| Total  | 335.87          | 209.11   | 127.80   | 190.24           |

As initial conditions for all other sets of transit times in the search for alternative local minima in the $\chi^2$ surface. The search over different initial conditions continued until at least 25 local minima within one reduced $\chi^2$ unit of the best known fit for each of the four sets of transit times were found.

Table 6 summarizes the goodness of fit of the best-fit model for each set of transit times. Including all measured transit times that lead to $\chi^2 / (\text{dof}) > 1$, whereas, not surprisingly, using the highest threshold of acceptance for the transit times $(O - S)/\sigma < 2$, causes $\chi^2 / (\text{dof}) < 1$. We compare these values to the expected reduction in $\chi^2$ when 2σ or 3σ outliers are removed from a Gaussian distribution. In this case, the removal of 3σ outliers has a small effect on $\chi^2$, whereas with our data, the removal of these outliers significantly reduced $\chi^2$.

We note here that Kepler-79 d contributes the least to the $\chi^2$ when all measured transit times are included, and appears to be the least sensitive to the removal of outlying transit times.

Figure 3 shows the range of masses and eccentricity vectors that are within one reduced $\chi^2$ unit of the best known minimum for each data set of transit times. All model fits within one reduced $\chi^2$ unit of the best known fit for each data set of transit times were included. The nominal uncertainties for model fits $\sigma_{\text{norm}}$ are reduced for all minima apart from the one with the lowest $\chi^2$, with uncertainties reduced according to the formula: $\sigma_e = \sigma_{\text{norm}} \sqrt{1 - (\Delta \chi^2 / (\text{dof}))}$. This assumes that the local minimum in $\chi^2$ is on a parabolic surface, and therefore extends error bars to reach where $(\Delta \chi^2 / (\text{dof})) \approx 1$. The results in Figure 3 show that a wide range of eccentricities can satisfy the data for Kepler-79 d, and that the individual best-fit solutions are indeed moderately sensitive at the ~1σ level to the manner in which outlying transit time measurements are handled.

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Figure 3. Results of fitting best-fit mass and the components of the eccentricity vector for Kepler-79 d using four sets of transit times. Each point represents a local minimum in $\chi^2$ with uncertainties estimated locally from the covariance matrix. For each set of transit times, local minima were included if, compared to the lowest known $\chi^2$ for that data set, $(\Delta \chi^2 / (\text{dof})) < 1$, and their error bars were reduced such that with uncertainties, $(\Delta \chi^2 / (\text{dof})) = 1$.

(A color version of this figure is available in the online journal.)
TTVs over the *Kepler* observational baseline. The orbits of *Kepler*-79’s planets are close enough to resonance for the coherence period, the length of time over which perturbations act constructively, to be much longer than the synodic period. However, the orbits are not so close to resonance that the TTVs reach a high amplitude—the TTVs are always a tiny fraction of an orbital period. We also note that the near-second-order resonance TTVs on *Kepler*-79 e induced by “e” have a predicted amplitude of just one minute and the TTVs of *Kepler*-79 e induced by “c,” have an amplitude of just 2 minutes, even though for these orbital periods near 3:1, |Δ3| = 0.014 is lower than each of the first-order nearness-to-resonance values (Δ1). The expected period of this component of the TTVs is much longer than the baseline of observations and the cycle is incomplete.

To characterize the host star, we used the light curve alongside spectral classification. For fitting the light-curve, we adopt the analytic model of Mandel & Agol (2002) for a planet transiting a stellar surface described by a quadratic limb-darkening law, and we adopt the limb-darkening parameters of Claret & Bloemen (2011). We modeled the orbits of each planet as non-interacting Keplerian orbits, and fit the light curve for best-fit parameters of the mean stellar density, ρ*, the photometric zero point for each planet, the center of transit time for the first observed transit $T_0$, the orbital period $P$, the impact parameter $b$, the scaled planetary radius $R_p/R_*$, and the components of the eccentricity vector, $e \cos \omega$ and $e \sin \omega$. To account for the TTVs, the light curve model cadence was contracted and expanded based on a linear interpolation of measured transit times for each planet to match the observed transit times.

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**Figure 4.** Best-fit solutions for planetary masses and the eccentricity vector components $e \sin \omega$ and $e \cos \omega$ using four different methods of accounting for outliers among the measured transit times. The orbital periods are displayed with a stagger to allow for comparison between the solutions. Here, the dynamical fit against raw transit times is marked by the gray points, the combined data set of short and long cadence transit times with 3σ outliers removed is in blue, the same set with 2σ outliers removed is in red, and the short cadence data only with 3σ outliers removed is in green. The black squares mark the median of the four solutions, with error bars showing the range of all uncertainties.

(A color version of this figure is available in the online journal.)

**Table 7**

| Planet | Period (days) | $T_0$ (JD-2,545,900) | $e \cos \omega$ | $e \sin \omega$ | $10^6 M_p/M_*$ |
|--------|--------------|----------------------|------------------|-----------------|-----------------|
| b      | 13.4845±0.0002 | 784.307±0.002 | $-0.015^{+0.006}_{-0.011}$ | $-0.003^{+0.005}_{-0.005}$ | 28.0±19.0 |
| c      | 27.4029±0.0006 | 806.475±0.004 | $-0.020^{+0.013}_{-0.016}$ | $-0.022^{+0.004}_{-0.022}$ | 15.1±6.0 |
| d      | 52.0902±0.0009 | 821.011±0.002 | $0.014^{+0.009}_{-0.015}$ | $0.020^{+0.045}_{-0.019}$ | 15.3±4.2 |
| e      | 81.0659±0.0015 | 802.126±0.003 | $0.012^{+0.032}_{-0.013}$ | $0.002^{+0.013}_{-0.019}$ | 10.7±3.0 |

**Notes.** The parameters we measure include the orbital periods (second column), time of first transit after epoch (third column), $e \cos \omega$ (fourth column), $e \sin \omega$ (fifth column), and planetary mass relative to the mass of the star (sixth column).
To calculate the posterior distributions of model parameters, we used an MCMC routine described in Section 4.1 of Rowe et al. (2014). We used the determination of $e \cos \omega$ and $e \sin \omega$ from dynamical modeling of the TTVs as constraints characterized by a Gaussian distribution. We generated $4 \times 1,000,000$ Markov Chains (after burn-in) and calculated the median value for each model parameter and its 1σ uncertainty interval, which we list in Table 8.

The light curve model gives a geometrical measurement of $\rho_*$, which we combined with the spectroscopic determination of $T_{\text{eff}}$ and [Fe/H]. We matched these to stellar evolution models (Demarque et al. 2004), to estimate the stellar mass and radius. Our uncertainties follow from the posterior distributions of our MCMC analysis. Our results are in Table 9.

### 5. NON-TRANSITING PERTURBERS?

Near strong mean motion resonances, pairwise interactions cause a TTV frequency at the circulating argument of the nearest resonance. The period of this signal increases closer to the resonance. Thus in Table 3, the highest values in the final column mark the orbit pairs closest to resonance. For the known candidates in this system, all pairwise TTVs due to first-order resonance have a complete cycle of observed transit times, hence the well-constrained model fit. It follows from the good fits that we have found, to the transit time data at the expected TTV periods, that these candidates are very likely to orbit the same star.

Figure 6 displays the periodicities detected in the TTVs. For this figure we have constructed periodograms of the TTVs $(O-C)$, as well as the residuals $(O-S)$. For each periodogram the frequency was limited at the Nyquist frequency (half the sample rate as the maximum frequency, corresponding to twice the orbit period). The minimum frequency corresponds to the maximum period that is sampled over the observational baseline. For this limit, we have chosen twice the observational baseline, which would be a lower limit on any incomplete TTV period, or an upper limit on its frequency, under the assumption

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**Figure 5.** Pairwise contributions to TTVs from each planet (simulated transit times minus a calculated linear ephemeris to the transit times) in the best-fit solution to the data set of transit times with 3σ outliers removed for “b” (top), “c,” “d,” and “e” (bottom). In each panel, contributions from “b” are in blue, “c” in green, “d” in orange, and “e” in red. The contributions are summed and displayed in gray, although these curves are largely obscured by the black curves in most of the panels, which denote the best-fit solution, which was in close agreement with the sum of linear pairwise TTVs. (A color version of this figure is available in the online journal.)
Figure 6. Periodograms of the observed TTVs of the planets and residuals of best-fit models for each of the four data sets of transit times. Here, the normalized power measured in parts per million (ppm), and the periodograms have been normalized by summing the power over 10,000 equally spaced frequency channels between an assumed half-wave over the TTV baseline and the Nyquist frequency. The graphs highlight the dominant frequencies in the best-fit model (left) and the residuals (right). In each graph, the dark gray curves mark the best fit to the raw transit times, the purple curves mark the best fit to the combined SC and LC data set with 3σ outliers removed, the blue curves mark the best fit to the combined data set with all 2σ outliers removed, and the brown curves mark the best fit to SC-only times with 3σ outliers removed.

(A color version of this figure is available in the online journal.)
that a quadratic TTV signal could represent a half-cycle or a longer period. In these plots, the absolute power in each peak need not correspond between models, because the combined data sets and the short, cadence-only data set have different baselines and therefore different frequency domains. However, the relative height of the peaks within each data set and their locations in frequency are of interest. In each case the peak in the periodogram corresponds to the expected TTV signal from the last column in Table 3. However, the peaks are broad enough such that for the planets with two neighbors, “c” and “d,” there are not two clear peaks in the periodogram. For “c” the single broad peak encompasses the expected TTV signal at 853 days (0.0136 μHz) and 526 days (0.0220 μHz). For “d” the shape of the broad curve hints at two peaks where we would expect them to be near 526 days (0.0220 μHz) and 721 days (0.0161 μHz), respectively, although they cannot be resolved.

There are no peaks in the residuals that stand out significantly from the background or noise. For “d” the highest residual peak is less than one tenth as high as the TTV model. Kepler-79 c also has residual peaks significantly lower than the model, by a factor of at least 5. For these two planets, perturbed by two neighbors, notwithstanding that there are more parameters that can be tuned, there is no evidence for an unseen perturber in the TTV fit. For Kepler-79 “b” and “e,” the residuals in the periodograms are higher compared to the model fit. At “b” the highest residual peak is ~1/4 as high the model peak, although no residual peak stands out significantly above other residual peaks. At “e” the highest residual peak is approximately one-third as high as the model fit, although this residual peak is significantly lower for the data sets with outliers removed than it is for the raw data set of transit times, and lowest for the SC-only data with the outliers beyond 3σ removed. We should certainly expect more residual power at higher frequencies where outliers have been included in the fits, and this is apparent in Figure 6. More importantly, high amplitude TTVs only occur where planets are near resonance, and these would show a signal in the low frequencies of the periodograms. High frequency residuals would correspond to TTVs that are far from resonance and therefore significantly weaker and more likely to be lost in the noise. Hence, we see no evidence that any unseen planets at Kepler-79 contribute significantly to the TTVs of the four known planets. Of course, we cannot rule out the possibility of other planets perturbing the four known planets. Because the configuration is so compact, it appears that only an unseen perturber orbiting interior to Kepler-79 b or beyond Kepler-79 e could have any potential of confusing our four-planet model. We note that Kepler-79 e has the highest fraction of its transits that are outliers. This is not surprising since planet e has the lowest transit depth and a high impact parameter. Its transit times are therefore the most sensitive to unknowns like stellar flares and starspots. Could this, nevertheless, be due to the TTVs of an outer perturber increasing the TTV amplitude over the photometric baseline? While we cannot exclude this possibility, we can consider how this hypothetical planet may affect our solutions. We focus on “e” because it is more likely to have an effect on our most surprising result, the low mass of Kepler-79 d.

If a non-transiting planet had an orbital period near 104 days, its orbital period could be near 2:1 with Kepler-79 d and the 4:3 resonance with Kepler-79 e, and it could cause near-first-order resonance TTVs in Kepler-79 d and e. Because we see no residual peaks in the periodogram for Kepler-79 d, a perturber causing first-order TTVs on “d” appears unlikely. Hence, only if the TTVs induced on “d” by an outer planet had a similar period to the TTVs on “d” caused by “e” or “c,” would our model for d’s TTVs be confused. Any other possible near-first-order resonance with Kepler-79 e would leave only high-order TTVs on Kepler-79 d, which would be an insignificant addition to its TTVs. Our solution to the mass of Kepler-79 d is constrained by the well-fitted TTVs it induces on “c” and “e.” A false measurement of Kepler-79 e’s mass would diminish one, but not both, of these constraints, notwithstanding the freedom for eccentricities to adjust to a different mass for “e.” It appears unlikely that Kepler-79 d, given its position between two transiting planets, would have a substantially different mass if a more distant planet were inducing TTVs in just one of its neighbors, namely, Kepler-79 e. The effect of a fifth planet beyond Kepler-79 e would have an even smaller effect on our solutions for Kepler-79 b and c.

### 6. CHARACTERIZING THE PLANETS

Table 10 shows our measured masses, radii, densities, and incident fluxes for each planet based on our best-fit solutions, with uncertainties extended to account for all four sets of transit times. The planetary bulk densities follow from the constraint in stellar density:

\[
\rho_p = \left( \frac{\rho_*}{\rho_*^1} \right) \left( \frac{M_p}{M_*} \right) \left( \frac{R_p}{R_*} \right)^3.
\]

Each of the terms in parentheses in Equation (8) is an independent source of uncertainty that we add in quadrature. We use this formula because it provides a more appropriate accounting for the uncertainties than the uncertainties in the planetary masses and radii. Due to the tight constraints on stellar and planetary radii, the dominant source of uncertainty here is the planet-to-star mass ratios from our dynamical models. Nevertheless, all four candidates can be characterized as having low bulk density, due to the retention of volatiles. Kepler-79 b has the highest bulk density (albeit with large uncertainties), despite having a similar radius to Kepler-79 c and e. The nominal density of Kepler-79 b is slightly less than a pure ice world of the same mass, suggesting either a substantial mass fraction of water and/or a relatively thin H/He envelope, with heavy elements in the interior. Kepler-79 c and e have similar characteristics to the planets of Kepler-11 d and e (Lissauer et al. 2013), and likely have envelopes that are far more significant by volume than by mass (Lissauer et al. 2013). The bulk density of Kepler-79 d is the lowest for a Kepler planet measured to date, although its composition may be sensitive to the temperature of the gas. Although the temperature of Kepler-79 d is unknown, its place on the mass–radius diagram appears to give its envelope a

| Planet | Mass (M⊕) | Radius (R⊕) | Density (g cm⁻³) | a (AU) | e | Flux (F⊙/1AU) |
|--------|-----------|-------------|-----------------|--------|---|----------------|
| b      | 10.90±7.4 | -6.0        | 1.43±0.97       | 0.117±0.002 | 0.015±0.012 | 162 |
| c      | 5.90±1.9  | -2.3        | 0.62±0.20       | 0.187±0.002 | 0.030±0.027 | 63  |
| d      | 6.00±2.1  | -1.6        | 0.09±0.03       | 0.287±0.004 | 0.025±0.059 | 27  |
| e      | 4.10±1.2  | -1.1        | 0.53±0.15       | 0.386±0.005 | 0.012±0.044 | 15  |

Notes. All uncertainties are 1σ confidence intervals, with planetary masses and radii in Earth units, and density in g cm⁻³. The flux (final column) is scaled to the flux received from the Sun at 1 AU.
total mass (as a fraction of the planetary mass) of around 50% at $T = 500 \text{ K}$, and roughly 10% at $1000 \text{ K}$, (Rogers et al. 2011). The equilibrium temperature of a planet assumes zero albedo and no internal heat source, and depends solely on the temperature of the host star and the orbital distance. For low eccentricities,

$$T_{eq} \approx T_* \sqrt{\frac{R_p}{2 a}}.$$  

(9)

Given its distance from the star, the equilibrium temperature of Kepler-79 d is $634 \pm 112 \text{ K}$. Hence, it appears reasonable that Kepler-79 d has an H/He envelope that contributes significantly more than 10% of the planetary mass, but less than 50% of the mass.

7. KEPLER-79’S PLANETS ON THE MASS–RADIUS DIAGRAM

Here we plot Neptunes and sub-Neptunes on the mass–radius diagram, including all planets with measured radii and masses, with nominal mass determinations up to $30 \, M_{\oplus}$. These include mass determinations by RV spectroscopy as well as TTVs. We adopt the data from published studies that include the most recent stellar spectral classification and mass determinations for: HAT-P-11 b (Kepler-3 b) (Bakos et al. 2010; Southworth 2011), HAT-P-26 b (Hartman et al. 2011), 55 Cancri e (von Braun et al. 2011; Winn et al. 2011; Gillon et al. 2012), GJ 3470 b (Bonfils et al. 2012), GJ 436 b (Ehrenreich et al. 2011; Ballard et al. 2010; von Braun et al. 2012), GJ 1214 b (Charbonneau et al. 2009; Valencia et al. 2013), CoRoT-7 b (Ferraz-Mello et al. 2011; Bruntt et al. 2010), Kepler-4 b, (Borucki et al. 2010), Kepler-10 b (Batalha et al. 2011), Kepler-11 b-f (Lissauer et al. 2013), Kepler-18 b-d (Cochran et al. 2011), Kepler-30 b and d (Sanchis-Ojeda et al. 2012), Kepler-36 b and c (Carter et al. 2012), Kepler-68 b (Gilliland et al. 2013), Kepler-78 b Howard et al. (2013), and KOI-94b (Weiss et al. 2013). We exclude KOI-94 c ($M_p = 15.6^{+5.7}_{-15.6} \, M_{\oplus}$) and KOI-94 e ($M_p = 35^{+18}_{-23} \, M_{\oplus}$) from the mass–radius diagram because their masses are poorly constrained, and in the case of KOI-94 e, its nominal mass is beyond $30 \, M_{\oplus}$. For the solar system, we adopt data given in de Pater & Lissauer (2010).

The planets in Figure 7 are all compared to theoretical models of pure-water ice, silicate rock, or iron planets. Model curves for planetary radii of planets made from pure ice, rock, or iron follow the results of Fortney et al. (2007). Several features stand out from Figure 7. First, among the sub-Neptunes, TTV planets are systematically larger and hence less dense than RV planets in the same mass range. This is most likely due to the biases of both techniques. For given planetary masses, larger planets have deeper transits yielding more precisely measurable transit times. There is thus a bias in TTV detections to large, low density planets. On the other hand, all low mass planets detected with RV have very short orbital periods, and all the known rocky exoplanets are too hot to retain deep atmospheres. This puts a bias in the RV detections to high density planets, in the same mass range as the low density planets of Kepler-11 and Kepler-79.

The bottom panel of Figure 7 highlights the two orders of magnitude in the range in planetary densities that are observed in the mass range up to $30 \, M_{\oplus}$. The RV density determinations appear more tightly correlated than the TTV determinations, although the uncertainty in the density for the low mass, low density TTV planets is certainly larger. Nominally, Kepler-79 d has the lowest planetary bulk density measured to date, although Kepler-12 b, with a mass of $137 \, M_{\oplus}$, and radius of $19 \, R_{\oplus}$, has a very similar bulk density within uncertainties. Kepler-79 d is only $\sim 5\%$ as massive as Kepler-12 b, and also receives roughly 30 times less insolation than Kepler-12 b (Fortney et al. 2011). It is significantly less dense than other characterized sub-Saturn mass planets.

If we parameterize the ratio of the planetary radius to the radius of a planet of the same mass composed purely of ice, of rock, we have a simple test of whether the planet has retained a substantial gaseous envelope. We note that a similar test has been independently proposed by (Kipping et al. 2013). Planets that are less dense than water ice (above the blue line in Figure 8) are likely to have a volumetrically substantial H/He envelope. Planets that are more dense than pure silicate rock, (below the brown line Figure 8), are less likely to retain a substantial amount of volatiles. Between these two limits, the atmosphere of a planet cannot be determined without more information, since it is unknown in what proportions volatiles are likely to be present in the form of ices and gases. Nevertheless, we surmise from Figure 8 that planets with $R_p < 2 \, R_{\oplus}$ appear unlikely to retain deep atmospheres, and from the bottom panel that planets with $R_p > 4 \, R_{\oplus}$ are very likely to retain deep atmospheres. With detailed composition models, Lopez & Fortney (2013b) have found a transition from rocky planets to planets with significant H/He envelopes around 1.75 $R_{\oplus}$. This transition is consistent
Figure 8. Planetary radii compared to solutions of pure silicate rock, marked as a dashed orange line. The radius of a planet made of pure ice (relative to pure rock) is marked in dashed purple. Mass determinations by RV are in red, and TTV in blue, with the planets of Kepler-79 in green. Planets above the dashed purple line are less dense than water ice and are likely to have deep H/He envelopes. Planets below the dashed orange line are denser than rock, and are unlikely to have a substantial atmosphere. Planets between these limits must contain some volatiles, but the unknown tradeoff between water or a combination of H/He and rock precludes a definitive answer on whether there is an H/He envelope. The dashed lines terminate at the size of a 30 M⊕ planet made of pure rock (2.51 R⊕) or pure ice (3.76 R⊕) following Fortney et al. (2007). For pure ice worlds, we limit the theoretical curve to Rp/Rrock > R⊕. Over the range we include, Rrock/Rp = 1.43 with a deviation of <1%. (A color version of this figure is available in the online journal.)

Figure 9. Planetary radii compared to solutions of pure rock (brown line) as a function of equilibrium temperature (with pure ice worlds on the dark blue line for comparison). Mass determinations by RV are in red, and TTV in blue, with the planets of Kepler-79 in green. Here we include only planets with masses <30 M⊕. A sharp transition to planets with little or no volatiles appears beyond 1050 K. In the cooler regime, no exoplanets that are denser than rock have been characterized.

(A color version of this figure is available in the online journal.)

with the mass–radius findings of Wu & Lithwick (2013) using TTV data; and Weiss et al. (2013) using RV data. Both of these studies find an increase in planetary densities at smaller planetary sizes.

In Figure 9, we show the equilibrium temperature of each characterized planet <30 M⊕ against their radii relative to pure rock or ice. We see an abrupt transition around 1050 K. At higher temperatures, there is no evidence of planets harboring substantial gases, and below this temperature, corresponding to ≈240 F⊕ (where F⊕ is the flux intercepted by the Earth), a wide range of planetary densities is observed. One possible exception is Kepler-4b, which has an equilibrium temperature of 1614 K, Rp/Rrock = 1.61, and Rp/R⊕ = 1.12. Borucki et al. (2010) estimated that Kepler-4 b retains a deep H/He atmosphere of about 4%–6% by mass. Of the "hot" planets below 30 M⊕, Kepler-4 b is alone in requiring a deep atmosphere. Nevertheless, we still see a much wider range in bulk densities among the cool sub-Neptunes.

Lopez & Fortney (2013a) found a more gradual transition to mass loss in their theoretical models of mass loss from photoevaporation. The models of Owen & Wu (2013) also show a more gradual transition in the region surrounding 1050 K. Since the plot in Figure 9 is still very sparsely populated, we await more mass determinations to see if the transition at 240 F⊕ or Teq = 1050 K is truly as abrupt as it appears now.

Kepler-36 stands out as having two planets that are close in orbital periods (13.84 and 16.24 days, respectively, Carter et al. 2012), with remarkably different densities. These planets appear to straddle the transition in equilibrium temperature at ~1050 K, with the outer planet retaining an atmosphere, and the inner one denser than pure silicate rock (Lopez & Fortney 2013a). Kepler-79 b lies close to Kepler-36 c on this plot, with an equilibrium temperature of 1004 ± 24 K, although it is marginally denser. Among the "cool" sub-Neptunes, no exoplanets that are denser than pure silicate rock have yet been discovered.

8. CONCLUSION

Assuming a four-planet model for the TTVs of the four transiting planets of Kepler-79, we have added four sub-Uranus masses to the mass–radius diagram. We confirm the planetary nature of the outer pair of candidates and that they are planets in the same system. The planets of Kepler-79 appear to follow the trend of the planets orbiting Kepler-11, being of low density with significant envelopes. Nevertheless, the planets show remarkable variety in their bulk masses. Kepler-79 b, c, and e all have radii between 3.5 and 4.0 R⊕, and yet their masses range from ~3 to ~13 M⊕, and their bulk densities range from 0.3 to 1.6 g cm⁻³. The largest planet, Kepler-79 d (~7 R⊕), has a remarkably low mass given its size, and most likely has the lowest nominal bulk density yet determined among known planets with sub-Saturn masses.

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