On the importance of the beam reference line

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Abstract. The development of a new beam reference line to improve bend-twist response modelling is presented. The beam reference line is derived as the locus of the cross-section elastic energy centres under pure bending moment. The performance of the new reference line is assessed via experimental measurements performed on a Siemens Gamesa wind turbine blade. Bend-twist modelling capability is assessed by comparing the location of the zero-twist rotation line and zero-twist-rate lines.

1. Introduction
A known method to reduce wind turbine loads is to introduce couplings in blade deformation modes, an approach commonly referred to as aeroelastic tailoring of blades. One way to achieve such coupling is through the introduction of sweep in the blade. The significant performance effects of improperly modelling this behaviour puts focus on the accuracy of aeroelastic model.

In most commercial aeroelastic models, the blade structure is modelled using a series of straight beam elements connecting nodes that are positioned on a beam reference line. The beam element stiffness matrix is calculated by integrating a series of stiffness matrices which describe the stiffness of cross-sections at different radial positions along the blade and which are perpendicular to the predefined beam reference line. This approach implies a “beam direction” (prismatic-direction) along the reference line, with bending moment axes lying in the cross-section plane resulting primarily in normal stress on the cross-section, and torsion moment around the reference line resulting in in-plane-normal shear stress. While the displacement degrees of freedom can be expressed in any reference frame, the choice of beam reference line determines the decomposition of the total internal moment-vector at each node into bending and torsion components.

For large wind turbine blades geometric slenderness, anisotropic materials, and dominating magnitude of flapwise and edgewise bending moments result in highly anisotropic cross-section stiffness properties and associated deformations, while the relatively small torsional moment and associated twist directly impact angle of attack and loads. Consequently, appropriate choice of the beam reference line is very important.

1.1. Beam reference line choice
It is common practice when modelling wing-like structures with a beam model to define the beam reference line on which the nodes are placed based on cross-section geometrical properties such as the half-chord location [1,2] or cross-section stiffness properties such as the shear centre or the tension centre (also called the elastic axis) [3]. Other reference lines, such as the flexural axis, are commonly used to interpret the flap-twist response of wing-like structures, independent of the structural model.
[4,5]. Their dependence on the spanwise load distribution has been recognized, and methods to identify the response for general loads have been developed, evaluating the wing’s structural response under loads at different spanwise and chordwise positions [5].

The present study deals primarily with the importance of appropriate selection of the beam reference line for flap-twist response modelling of wind turbine blades with geometrical sweep, including curvature and significant, spanwise tailored trailing edge reinforcement. Methods and concepts related to the flexural axis identification are subsequently used to demonstrate the improved fidelity of the model’s compared to experimental response.

The use of the tension centres to define the beam reference line for example is intuitively plausible for a structure where bending and extensional stiffness are governed by the same structural components like in the case of aircraft wings. It is worth pointing that in the case of prismatic beams, modern beam theories are insensitive to the choice of beam reference line [6]. Consider, however, the beam shown in figure 1. At its section A, the beam consists of an I-beam providing normal stiffness, enclosed in a cylindrical shell providing shear stiffness. At Section B, a point stiffness is added at the shell on the (symmetry-axis) x, offsetting that sections tension centre. Loaded with a bending moment M_x, the structure, due to symmetry, will only experience a bending-deformation around x. For a uniform prismatic beam of either Section A or B, this response may be obtained using above defined elastic axis as beam reference line. For the structure with the section-transition, however, using the tension centre locus as beam reference line will result in a torsion moment component and, in general, associated erroneous twist deformation not only around the beam reference line but also around global z-axis. Furthermore, the resulting beam reference line is not smooth, which causes issues with beam spanwise nodal positioning. A similar result could occur if geometrical properties such as the half-chord were used to define the beam reference line.

A wind turbine blade contains significant, sometimes discrete spanwise structure variations, for example web-start or -end, and trailing edge and spar car reinforcement starts and -drops. Consequently, the cross-section elastic centres and shear centres vary significantly, and its locus does not in general represent the direction of the main bending-carrying structure. Under such conditions, the elastic axis is not an appropriate beam reference line.

The aim of the current study is to define an optimal location of a wind turbine beam reference line to properly capture bending-torsion deformation modes. The study is composed of two parts: a theoretical derivation and an experimental validation. In the theoretical part, a new beam reference line is derived as the locus of the cross-section elastic energy centres under pure bending moment. In the second part, the flap-twist modelling response was validated using experimental measurements using a Siemens Gamesa wind turbine blade subject to varying static loads.
2. Centre of strain energy under bending

Consider a beam cross-section with longitudinal coordinate z and cross-section coordinates x and y shown in figure 2. The cross-section coordinates x and y are assumed to have their origin at the elastic centre and are aligned with the cross-section principal axes. This choice of reference axis ensures that the extension and bending deformation are decoupled, e.g. an extension force along z will not cause a rotation of the cross-section about x and y.

![Cross-section coordinate system](image)

Figure 2. Cross-section coordinate system.

The bending deformation of the cross-section is characterized by the bending curvature about the x and y axis, namely $\kappa_x$ and $\kappa_y$, respectively. Following standard beam theory, the strain energy $dU$ of an infinitesimal cross-section area $dA$ when the structure is undergoing a pure bending deformation about x is given as,

$$dU = \frac{1}{2} E \kappa_x^2 y^2 dA.$$ (1)

A measure of the strain energy distribution about the y axis can be obtained by integrating over the entire cross-section area the strain energy component weighted by its x position,

$$U_d = \int_A x dU = \frac{1}{2} \int_A x E \kappa_x^2 y^2 dA.$$ (2)

The weighted energy $U_d$ can also be represented by weighting the total bending energy calculated using the bending stiffness $E_o I_x$ with a distance $x_{be}$,

$$U_d = \frac{1}{2} \int_A x E \kappa_x^2 y^2 dA = \frac{1}{2} x_{be} \kappa_x^2 E_o I_x.$$ (3)

The distance $x_{be}$ can be interpreted as the location where all the strain energy can be located to create the same weighted energy. Analogous to the definition of the center of gravity, $x_{be}$ is coined the center of bending strain energy. Solving for the bending strain energy centre yields

$$x_{be} = \frac{1}{E_o I_x} \int_A E x y^2 dA.$$ (4)

The y component of the bending strain energy centre can be obtained in a similar fashion by considering a bending deformation about the y axis,

$$y_{be} = \frac{1}{E_o I_y} \int_A E x y^2 dA.$$ (5)

It is seen in the above equation how the bending strain energy center does not depend on the magnitude of the bending load applied but only on the distribution of Young’s modulus with respect to the principal axis.
Applying the bending energy center formulation to the example shown in figure 1 would yield a beam reference line going through the center of both section A and section B. A graphical representation of the energy distribution and bending energy center for a blade cross-section section is shown in figure 3. The chordwise location of the bending energy center is found close to the highly loaded beam caps, whereas the trailing-edge reinforcement sees little load and consequently has little effect on the energy center.

![Figure 3. Strain energy centre of wind turbine cross-section. Arrow size and colour indicate strain-energy density.](image)

3. Validation
In this section, numerical and experimental results are presented to validate the new bending energy-based beam reference line definition. The validation is performed using a Siemens Gamesa 59 m wind turbine blade. The blade has by design a significant amount of sweep in its planform making it ideal to test the new beam reference line for non-straight and non-prismatic structures. The experimental data is compared to an ANSYS 3D shell-element finite element model and a beam finite element model. The ANSYS and beam models are shown in figure 4. The shell model is composed of fifty thousand 4-node Shell181 elements with anisotropic material properties representative of the true layup. The beam model uses a co-rotational formulation with 59 elements based on [7]. In the shell model, the load introduction is performed via a load-distributing element whereas it is introduced as an equivalent force and moment applied at a node in the beam model. Both models are run with non-linear geometrical solvers with non-following loads representative of gravity test-load.

![Figure 4. ANSYS 3D shell model and beam model of a flapwise pull test at leading edge with applied boundary conditions](image)
The experimental yoke setup is shown in figure 5. Reflective targets were positioned on the surface of the blade. Three-dimensional displacements of these targets were measured with a total meter station. Similarly located targets were added to the shell and beam models to enable an identical postprocessing to determine flap, edge and twist deflections and allow direct comparison of the results.

![Figure 5. Experimental flap-twist coupling test setup showing pulls at outer spanwise location and two chordwise locations, as well as reflective total meter targets for deformation evaluation.](image)

### 3.1. Static test description and analysis

Proper identification of flap-twist response to general loads requires application of flapwise forces at different spanwise and chordwise positions [7]. At two spanwise locations along the blade, namely at 41.85 m and 55.85 m, flapwise pulls at different chordwise positions were hence performed on the real blade and numerical models to determine the flap-twist response, within experimental practical constraints.

The flap-twist responses from the pulls at different chord locations were combined and visualized using the concepts of zero-twist-rotation line and zero-twist-rate-line. The zero-twist-rotation line characterizes the blade flap-twist response for a given spanwise location of flapwise force. It shows the chordwise location for a pull at a given spanwise location resulting in zero twist rotation. The location of the zero-twist-rotation line depends on the entire blade structure between root and the span position of interest. At load introduction it intersects the flexural axis of the blade. A graphical illustration of the location of the zero-twist-line at 30 m from a pull at 41.85 m is shown in figure 6.

Another quantity of interest studied is the zero-twist-rate line. It characterizes the contribution of each span segment to the blade flap-twist response for a given spanwise position of flapwise force. Moreover, it shows the chordwise location of a pull that would result in a zero-twist-rate over a span segment between spanwise pairs of twist measurements. At the load introduction spanwise position, the zero-twist-rate line intersects the shear-center locus. In contrast to the zero-twist-rotation line, this quantity is a local property of the blade.
3.2. Results

The static bend-twist response obtained using the beam model with the new proposed beam reference line is shown in figure 6 and figure 7 for the inner and outer pull test. The results are compared to experimental data, a 3D shell model, and a beam model with its beam reference line aligned with the tension centre. The bend-twist response is illustrated in figure 7 by the location of the zero-twist-rotation line described in the previous section. It can be observed that the shell model results (red line) have the best agreement with testing data (black line). The results obtained using the beam model using the new beam reference line (green line) are close to the shell model results and more accurate than those from the beam model using tension centre line as beam reference line (dark yellow line). As expected, the difference in response between the two beam reference lines is proportional to the change in element orientation.

The zero-twist-rate lines are plotted in figure 8 for the two test pull cases. Again, improvements in the results using the bending energy-based beam reference line can be observed. The good agreement between all numerical models and the experimental data at the two pull spanwise locations indicates that the shear centre at these positions is properly modelled.

It is interesting to note that the zero-twist-rate and zero-twist rotation lines do not coincide for the inner and outer pulls. This indicates that in addition to shear-twist coupling, bending-twist coupling is also present. In general bending-twist coupling can come from structural orientation either from geometry or material properties.
Figure 7. Zero-twist-rotation lines for pulls at 41.85 m and 55.85 m.

Figure 8. Zero-twist-rate lines for pulls at 41.85 m and 55.85 m.
4. Conclusions

It is proposed to replace the use of the tension centre to define the beam reference line of wind turbine blades with the locus of the cross-section elastic energy centres under pure bending moment. This new definition of the beam reference line enables proper identification of flap-twist coupling, which is particularly sensitive to beam reference line chordwise location and orientation. This sensitivity is particularly present in wind turbine blade modelling given the large flapwise loads and small edgewise displacements compared to blade chord length. Conversely, edge-twist coupling is less sensitive to beam reference line definition as flapwise displacement is large compared to airfoil thickness, which bounds both tension centre and energy-centre under bending.

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