Dynamics of plasma piston composition

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Abstract. The article is devoted to the stratification of plasma formation in the railgun channel. The motion model for a composition of plasma pistons interacting with each other, rails and railgun walls was proposed. It was shown that the maximum impulse that could be transmitted to the throwing body in this system substantially depends on the initial position of the plasma pistons, according to the simulation results. The rational initial system configuration is obtained, at which the maximum value of the momentum transferred to the throwing body is reached. Possible options for obtaining such a configuration and a qualitative model of the stratification of a single plasma formation into several plasma pistons were proposed.

1. Introduction

The development of electromagnetic acceleration technologies is one of the priorities in many areas of science and technology: shock-wave generators [1], high-energy physics [1], space technologies [2], materials science [3], etc. The main obstacle for widespread use is low efficiency (up to 30–40%). The best-known reasons for the loss of efficiency are the speed skin effect [4], emission crisis [1] and interactions of current carriers in the plasma piston with the throwing body and railgun channel walls [5]. With this background, the problem of the "collapse" (stratification) of the plasma piston (PP) into several pistons was overlooked. The loss of efficiency, in this case, is significant due to the complex interaction of the PP between them. Experimental data presented in [4] suggest that the initiation process of plasma formation in the railgun is accompanied by its stratification. By detailed examination of the physics of the process and the data of [4], it can be concluded that the instability of plasma formation can lead to the formation of plasma bunches, which are difficult to consider as a single continuous environment. Depending on the initial and boundary conditions, the number of these formations may vary.

In this article, the interaction of plasma formation elements with each other is presented like an interaction of a set of plasma pistons having different parameters. We constructed a model of the motion of plasma pistons interacting with each other, the throwing body and the railgun walls. A model of the process of plasma formation decay into the PP system due to the occurrence of a stationary wave in the streamer channel between the rails is proposed.

The goal of the work is to determine the current parameters, as well as the initial and boundary conditions that provide the highest average total momentum of the plasma piston system in the railgun channel, ensuring a rational mode of acceleration of the throwing body. An additional goal is to determine the characteristics of a set of plasma pistons according to the known initial and boundary conditions at the initial time.
2. Physics-based model

In the article [6], the movement of a single jumper, which performs the role of the accelerating pan, was considered. In articles [1, 6], the motion of a single PP was considered in accordance with the Artsimovich's model [6] and its interaction with the gaseous environment filling the channel of the railgun.

In the proposed model, we investigated the motion of the plasma pistons system taking into account the electromagnetic interaction between them. The plasma piston system can be specially created due to the railgun design or obtained as a result of wave interaction at the initial stage of acceleration. This issue is discussed in more detail in the corresponding section of this article.

The analysis has shown that in the considered circuits the EMFs of mutual induction are small compared to the voltage on the capacitor at the initial stage of acceleration.

The resolving system of differential equations (1) for calculating the effective currents in the system is composed by the mesh current method:

![Figure 1: Equivalent circuitry.](image)

The parameters of the system for conducting a numerical experiment corresponded to the parameters used in the articles [1, 5]:

| i  | Rᵢ, Ω | Lᵢ, μH | mᵢ, g | C, μF | U₀, kV |
|----|-------|--------|-------|-------|--------|
| 1  | 1.00  | 1.26   | 0.0024 |       |        |
| 2  | 2.00  | 2.36   | 0.0043 | 200   | 32     |
| 3  | 6.00  | 3.46   | 0.0046 |       |        |

*i* – the number of the plasma piston, *Rᵢ* – the internal resistance of the *i*-th PP, *Lᵢ* – the internal inductance of the *i*-th PP, *mᵢ* – the mass of the *i*-th PP, *C* and *U₀* – the supply battery capacity and the initial voltage on the capacitor, respectively.

Table 1. The main parameters of the numerical experiment.
\[
\begin{align*}
-U + I_1 R_1 + L_1 \frac{dI_1}{dt} + \sqrt{L_1 L_2} \frac{dI_2}{dt} \Theta(d - |x_1 - x_2|) + \sqrt{L_1 L_3} \frac{dI_3}{dt} \Theta(d - |x_1 - x_3|) & = 0 \\
L_1 \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt} + \sqrt{L_1 L_2} \frac{dI_3}{dt} \Theta(d - |x_1 - x_2|) + \sqrt{L_1 L_3} \frac{dI_3}{dt} \Theta(d - |x_1 - x_3|) - I_1 R_1 + I_2 R_2 & = 0 \\
L_2 \frac{dI_2}{dt} + L_3 \frac{dI_3}{dt} + \sqrt{L_2 L_3} \frac{dI_3}{dt} \Theta(d - |x_2 - x_3|) + \sqrt{L_1 L_3} \frac{dI_3}{dt} \Theta(d - |x_1 - x_3|) - I_2 R_2 + I_3 R_3 & = 0
\end{align*}
\]

(1)

\(I_i\) — the current flowing through the corresponding plasma piston, \(U\) — the capacitor voltage, \(x_i\) — the coordinate of the \(i\)-th piston, \(d\) — the characteristic size of the plasma piston (streamer channel), \(\Theta(x)\) — Heaviside function.

Another important difference in the motion of the PP system from the motion of a single PP is the interaction between them through the Ampere forces:

\[F_{ij} = \frac{\mu_0 I_i I_j}{4\pi(x_j - x_i)} \]

\(\mu_0\) — the magnetic constant.

A system of differential equations of motion of PP in the railgun channel is obtained from the law of the dynamics of translational motion, taking into account their interaction:

\[
\begin{align*}
\dot{x}_1 & = \frac{1}{m_1} \left( \frac{1}{2} \frac{\partial L_1}{\partial x_1} I_1^2 + \frac{\mu_0 I_1 I_2}{4\pi(x_2 - x_1)} - \frac{\mu_0 I_1 I_3}{4\pi(x_3 - x_1)} \right) \\
\dot{x}_2 & = \frac{1}{m_2} \left( \frac{1}{2} \frac{\partial L_2}{\partial x_2} I_2^2 + \frac{\mu_0 I_1 I_2}{4\pi(x_1 - x_2)} + \frac{\mu_0 I_2 I_3}{4\pi(x_3 - x_2)} \right) \\
\dot{x}_3 & = \frac{1}{m_3} \left( \frac{1}{2} \frac{\partial L_3}{\partial x_3} I_3^2 + \frac{\mu_0 I_1 I_3}{4\pi(x_1 - x_3)} - \frac{\mu_0 I_1 I_3}{4\pi(x_3 - x_1)} \right)
\end{align*}
\]

(2)

At the initial time, the plasma pistons are separated by a distance \(d\) of the order of the width of the gas discharge channel, which determines the size of the piston.

The resulting system of equations (2) does not have an analytical solution; therefore, a numerical solution was applied in the Maple environment based on the Hermitian interpolation method. We performed the calculations on the most probable variants of the initial positions of the PP in this system.

This allowed us to analyze and determine the optimal initial conditions. The parameters of the system for conducting a numerical experiment corresponded to Table 1.

As can be seen from the simulation results (figure 2), the motion of the plasma pistons transforms into pulsations with a fairly consistent pattern of movement of the elements.

In these pulsations, the peak values of speed are 5 times higher than in a system with one PP, which opens up new possibilities for streamlining the process from the point of selecting system parameters with which interaction between the plasma piston and the throwing body will occur in a resonant mode with the transfer of the maximum possible momentum of the plasma piston to the throwing body. Interestingly, the final speed of the fastest PP after the termination of the oscillatory process is 20% lower than the maximum speed in the system without such interaction and 2.5 times higher than the speed of the fastest element at the same instant of time without interaction, which allows varying power and energy conversion efficiency of installation.

Figure 2 shows the time dependences of the kinematic characteristics of the plasma piston system, inherent in the rational configuration for transferring the total momentum. We obtained this data from numerical experiment. The average total momentum transmitted to the throwing body with this initial configuration exceeds the average momentum in all considered configurations by more than 50%. The
momentum values are given in table 2, and figure 3 shows a bar chart of values that clearly demonstrates the optimality of the chosen option. Here, the PP with the lowest mass is the furthest one from the beginning of the railgun (x₁), which allows the PP system to maintain a consistent motion for a time equal to the time constant of the circuit. Simultaneously, the amplitude of oscillations both in speed and along the coordinate of PP1 increases resonantly, which allows, while reducing the current flowing through the system, to transfer most of the energy to one of the heavier pistons, contributing to its exit from the pulsation’s volume.

Table 2: Results of numerical experiments.

| Experiment number | The initial position of PP #1 (x₁, m) | The initial position of PP #2 (x₂, m) | The initial position of PP #3 (x₃, m) | Average total momentum (p, kg·m·s⁻¹) |
|-------------------|--------------------------------------|--------------------------------------|--------------------------------------|-------------------------------------|
| 1                 | 0.0                                  | 0.1                                  | 0.2                                  | 0.049                               |
| 2                 | 0.0                                  | 0.2                                  | 0.1                                  | 0.140                               |
| 3                 | 0.1                                  | 0.0                                  | 0.2                                  | 0.099                               |
| 4                 | 0.1                                  | 0.2                                  | 0.0                                  | 0.087                               |
| 5                 | 0.2                                  | 0.0                                  | 0.1                                  | 0.196                               |
| 6                 | 0.2                                  | 0.1                                  | 0.0                                  | 0.232                               |

Figure 2: Kinematic characteristics of the plasma piston system in numerical experiment. a) Coordinate versus time for a plasma piston system; b) Speed versus time for a plasma piston system.
Figure 3: Bar chart of average total momentums transmitted to the throwing body with different initial configurations.

The proposed model was generalized on systems with large number of PPs, but the results of these numerical experiments are not the subject of this article.

3. Possible reason for plasma formation’s stratification

Since the motion of the PP in the channel of a rail accelerator is of the pulsating nature, at the initial moment of time the formation of standing waves in the streamer channel is possible both between conducting rails and between side walls.

The cyclic frequency (ω) is the lowest common multiple (hereinafter LCM) for the Langmuir frequencies of each of the plasma components. Consequently, there are areas of separated particles, which, due to the difference in specific charge, have the different amplitude of oscillations (figure 4).

Figure 4: Scheme of the stationary wave in the railgun channel.

In the numerical experiment, multicomponent plasma was used, consisting of the chemical elements most frequently found in fuels: hydrogen, oxygen, and carbon. The data of Langmuir frequencies are presented in Table 3.
Table 3: Langmuir frequencies of $H^+$, $O_2^-$, and $C^{4+}$ ions.

| Frequency | Hz     |
|-----------|--------|
| $\omega_{H}$ | $0.03\times10^{6}$ |
| $\omega_{O}$ | $0.14\times10^{6}$ |
| $\omega_{C}$ | $0.04\times10^{6}$ |
| LCM         | $0.84\times10^{6}$ |

The Fourier transform of the total current in the system, obtained using the MATHEMATICA package and presented in figure 5, shows that the maximum amplitude is observed in the same range of values as the LCM of the stationary wave frequency. This confirms the hypothesis of the occurrence of a standing wave at the initial moment of time in the streamer channel of a plasma formation between the accelerator rails and side walls.

Figure 5: Fourier transform of the current in the system.

Due to the physical process of separation of plasma formation into heavy ions and lighter current carriers, the latter are located at the edges of heavier ions due to the greater amplitude of oscillations. Therefore, the main component link for the system is the three PP: in the middle there are heavy ions, along the edges there are light current carriers. It was the process of separation of ions into heavy and light ones that allowed focusing attention in a numerical experiment on a three-piston configuration.

4. Conclusion

The process of plasma stratification causes sufficient variation of system effectiveness as shown by simulation. Nevertheless, the suggested model requires the detailed elaboration of the plasma pistons’ diffusion issue. It is going to be investigated in further articles.

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