The Asymptotic Dynamics of de Sitter Gravity in three Dimensions

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Abstract

We show that the asymptotic dynamics of three-dimensional gravity with positive cosmological constant is described by Euclidean Liouville theory. This provides an explicit example of a correspondence between de Sitter gravity and conformal field theories. In the case at hand, this correspondence is established by formulating Einstein gravity with positive cosmological constant in three dimensions as an SL(2, C) Chern-Simons theory. The de Sitter boundary conditions on the connection are divided into two parts. The first part reduces the CS action to a nonchiral SL(2, C) WZNW model, whereas the second provides the constraints for a further reduction to Liouville theory, which lives on the past boundary of dS\textsubscript{3}.

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1 Introduction

Recently there has been an increasing interest in gravity on de Sitter (dS) spacetimes \([1–18]\). The motivation for this comes partially from recent astrophysical data indicating a positive cosmological constant \([19]\), but apart from phenomenological aspects, it also remains an outstanding challenge to understand the role of de Sitter space in string theory, to clarify the microscopic origin of de Sitter entropy \([20–25]\), and to study in which way the holographic principle \([26, 27]\) is realized in the case of de Sitter gravity. Whereas string theory on anti-de Sitter spaces is known to have a dual description in terms of certain superconformal field theories \([28]\), no such explicit duality was known up to now for dS spacetimes. Based on \([1]\), where the first evidence for a dS/CFT correspondence was given, and on related ideas that appeared in \([2, 4, 29]\), Strominger proposed recently a holographic duality relating quantum gravity on dS \(_D\) to a Euclidean conformal field theory residing on the past boundary \(I^-\) of dS \(_D\) \([9]\). In particular, he considered three-dimensional de Sitter gravity, and showed that it admits an asymptotic symmetry algebra consisting of two copies of Virasoro algebras with central charges \(c = \tilde{c} = 3l/2G\), where \(l\) is the dS\(_3\) curvature radius, and \(G\) denotes Newton’s constant\(^{1}\). This central charge was then rederived by other methods in \([10]\), where also further evidence for a dS/CFT correspondence was given.

The purpose of the present paper is to show that in the case of 2 + 1-dimensional Einstein gravity with positive cosmological constant, the dual CFT in question is Euclidean Liouville field theory \(^2\). This provides an explicit example of a dS/CFT correspondence, and shows how the asymptotic symmetries of dS\(_3\) are realized in the dual conformal field theory. To establish this correspondence, we shall make use of the Chern-Simons formulation \([33, 34]\) of pure gravity in 2 + 1 dimensions, and we will closely follow the analogous work that has been done for anti-de Sitter space \([35–39]\).

The remainder of this paper is organized as follows:

In the next section, we briefly review the Chern-Simons formulation of three-dimensional Einstein gravity with positive cosmological constant, and translate the de Sitter boundary conditions on the metric \([9]\) in terms of the CS connection. In section 3, we show that the first part of these conditions implies the reduction of the Chern-Simons action to a nonchiral SL(2, C) WZNW model. In 4, we make use of the remaining boundary conditions in order to further reduce the WZNW model to Liouville field theory, which lives on the past boundary \(I^-\) of dS\(_3\). The appendix contains our conventions.

2 De Sitter Gravity as Chern-Simons Theory

Pure 2 + 1-dimensional gravity with positive cosmological constant \(\Lambda = 1/l^2\) is described by the action

\(^{1}\)This generalizes the result of Brown and Henneaux \([30]\) to the case of positive cosmological constant.

\(^{2}\)Cf. also \([31, 32]\) for the appearance of the Liouville equation and Liouville action in slightly different contexts.
\[ S = \frac{1}{16\pi G} \int d^3x \sqrt{-g}(R - 2\Lambda). \] (2.1)

The equations of motion following from (2.1) admit the de Sitter solution

\[ ds^2 = -\frac{l^2}{\tau^2}d\tau^2 + \tau^2dzd\bar{z}. \] (2.2)

An asymptotically de Sitter geometry is one for which the metric behaves for \( \tau \to \infty \) as

\[ g_{zz} = \frac{\tau^2}{2} + O(1), \]
\[ g_{zz} = O(1), \]
\[ g_{\tau\tau} = -\frac{l^2}{\tau^2} + O(\tau^{-4}), \]
\[ g_{z\tau} = O(\tau^{-3}). \] (2.3)

Past infinity \( \mathcal{I}^- \) is a spacelike cylinder parametrized by the coordinates \( \phi \sim \phi + 2\pi \) and \( y \), where we set \( z = \phi + iy \). We denote this surface by \( \Sigma \).

In what follows, we shall make essential use of the fact that 2 + 1-dimensional de Sitter gravity can be formulated as an SL(2, \( \mathbb{C} \)) Chern-Simons theory, with action

\[ S = \frac{is}{4\pi} \int \text{Tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A) - \frac{is}{4\pi} \int \text{Tr}(\bar{A} \wedge d\bar{A} + \frac{2}{3} \bar{A} \wedge \bar{A} \wedge \bar{A}), \] (2.4)

where, in our conventions (cf. appendix),

\[ s = -\frac{l}{4G}, \] (2.5)

and \( A \) denotes an SL(2, \( \mathbb{C} \)) gauge field. (2.1) and (2.4) are equivalent, if we identify

\[ A = A^a\tau_a = \left( \omega^a + \frac{i}{l}e^a \right) \tau_a, \quad \bar{A} = \bar{A}^a\tau_a = \left( \omega^a - \frac{i}{l}e^a \right) \tau_a, \quad a = 0, 1, 2, \] (2.6)

---

\(^3\)This is related to the planar slicing of dS\(_3\) given in [9] by setting \( \tau = l \exp(-t) \). The past boundary \( \mathcal{I}^- (t \to -\infty) \) corresponds then to \( \tau \to \infty \). The Carter-Penrose diagram can be found in [9].
where \( e^a \) is the dreibein, \( \omega^a = \frac{1}{2} \epsilon^{abc} \omega_{bc} \) the \( \text{SL}(2, \mathbb{R}) \) spin connection, and the \( \tau_a \) are the \( \text{SL}(2, \mathbb{C}) \) generators (cf. appendix).

Modulo total derivatives\(^4\) the action (2.4) can be rewritten as

\[
S = S_{CS}[A] - S_{CS}[ar{A}],
\]

(2.7)

with

\[
S_{CS}[A] = \frac{is}{4\pi} \int d^3x \text{Tr} (\dot{A}_\tau A_\phi - \dot{A}_\phi A_\tau + 2A_y F_{\tau\phi}),
\]

(2.8)

where a dot denotes the derivative with respect to \( y \), and \( F = dA + A \wedge A \).

In terms of the connection \( A \), the boundary conditions (2.3) read

\[
A = \begin{pmatrix}
\frac{d\tau}{2\tau} & \mathcal{O}(1/\tau) \\
\frac{i\tau}{4\pi} d\bar{z} & -\frac{dr}{2\tau}
\end{pmatrix}, \quad \bar{A} = \begin{pmatrix}
-\frac{dr}{2\tau} & -\frac{i\tau}{4\pi} d\bar{z} \\
\mathcal{O}(1/\tau) & \frac{d\tau}{2\tau}
\end{pmatrix}.
\]

(2.9)

Like in the AdS case\(^{[35]}\), we can state two essential points concerning the asymptotic behaviour (2.9) of the connection:

1. The components \( A_\bar{z} \) and \( \bar{A}_z \) are set to zero asymptotically.

2. \( A_\bar{z} = A_1^1 + iA_2^2 \) and \( \bar{A}_z = \bar{A}_1^1 - i\bar{A}_2^2 \) are independent of \( z, \bar{z} \) to leading order in \( \tau \). Also, \( A_0^0 \) and \( \bar{A}_0^0 \) vanish.

In section 3, following\(^{[33,40]}\), we show that the first condition implies a reduction of the Chern-Simons action to a nonchiral WZNW model. The second condition then provides the constraints for a further reduction from the WZNW model to Liouville theory.

### 3 Reduction to a WZNW Model

When \( A_\bar{z} \) and \( \bar{A}_z \) are required to vanish on the boundary \( \mathcal{I}^- = \Sigma \), the action (2.7) is not extremal on-shell. Instead, \( \delta S \) equals the surface term \( \delta \left[ -\frac{s}{4\pi} \int_{\Sigma} d\phi d\tau \text{Tr} (A_\phi^2 + \bar{A}_\phi^2) \right] \).

In order to cancel this, we must add a boundary term to the action (2.7), leading to the improved action\(^5\).

\(^4\)Boundary terms will be considered below.

\(^5\)Surface terms that arise at \( y_1 \) and \( y_2 \) will be discussed in section 4. Besides, de Sitter space has two spacelike conformal boundaries \( \mathcal{I}^- \) and \( \mathcal{I}^+ \), so that in principle additional boundary terms at \( \mathcal{I}^+ \) have to be taken into account. However, since we are only interested in the asymptotic dynamics of the gravitational field near \( \mathcal{I}^- \), we will ignore such surface terms. Some discussion of the problems that arise due to the presence of a second boundary can be found in\(^{[6]}\), where it was argued that in spite of the fact that de Sitter space has two boundaries, the dual description of de Sitter gravity is provided by a single CFT.
\[ S = S_{CS}[A] + \frac{s}{4\pi} \int_{\Sigma} d\phi dy \text{Tr}[(A_\phi)^2] - S_{CS}[\bar{A}] + \frac{s}{4\pi} \int_{\Sigma} d\phi dy \text{Tr}[(\bar{A}_\phi)^2]. \quad (3.1) \]

We recognize that \( A_y \) and \( \bar{A}_y \) are Lagrange multipliers that implement the Gauss law constraints \( F_{\tau\phi} = \bar{F}_{\tau\phi} = 0 \). These are easily solved by

\[
A_\mu = G_1^{-1} \partial_\mu G_1, \quad \bar{A}_\mu = G_2^{-1} \partial_\mu G_2, \quad (3.2)
\]

where \( G_{1,2} \) have the asymptotic behaviour

\[
G_1 \rightarrow g_1(z, \bar{z}) \left( \begin{array}{cc} \sqrt{\frac{\tau}{l_0}} & 0 \\ 0 & \sqrt{\frac{\tau}{l_0}} \end{array} \right), \quad G_2 \rightarrow g_2(z, \bar{z}) \left( \begin{array}{cc} \sqrt{\frac{1}{\tau}} & 0 \\ 0 & \sqrt{\frac{1}{\tau}} \end{array} \right), \quad (3.3)
\]

and \( g_{1,2}(z, \bar{z}) \) are arbitrary elements of \( \text{SL}(2, \mathbb{C}) \). Strictly speaking, the Gauss law constraints imply \((3.2)\) only for \( \mu = \tau, \phi \), whereas \((3.2)\) for \( \mu = y \) is a gauge choice. \((3.3)\) implies the boundary condition \((2.9)\) for \( A_\tau \) and \( \bar{A}_\tau \), whereas for the tangential components one obtains

\[
A_j = \begin{pmatrix} -\frac{i}{2} a_j^0 & \frac{i}{2} a_j^- \\ \frac{i}{2} a_j^+ & \frac{i}{2} a_j^0 \end{pmatrix}, \quad \bar{A}_j = \begin{pmatrix} -\frac{i}{2} \tilde{a}_j^0 & \frac{i}{2} \tilde{a}_j^- \\ \frac{i}{2} \tilde{a}_j^+ & \frac{i}{2} \tilde{a}_j^0 \end{pmatrix}, \quad (3.4)
\]

where \( a_j \equiv g_1^{-1} \partial_j g_1, \tilde{a}_j \equiv g_2^{-1} \partial_j g_2 \), and \( j = z, \bar{z} \).

Inserting \((3.2)\) into the action \((3.1)\) yields a sum of two chiral WZNW models,

\[
S = S^R_{WZNW}[g_1] - S^L_{WZNW}[g_2], \quad (3.5)
\]

where

\[
S^R_{WZNW}[g_1] = \frac{s}{2\pi} \int_{\Sigma} d\phi dy \text{Tr}[g_1^{-1} \partial z g_1 g_1^{-1} \partial \phi g_1] - \frac{i s}{12\pi} \int \text{Tr}(G_1^{-1} dG_1)^3, \n
S^L_{WZNW}[g_2] = -\frac{s}{2\pi} \int_{\Sigma} d\phi dy \text{Tr}[g_2^{-1} \partial z g_2 g_2^{-1} \partial \phi g_2] - \frac{i s}{12\pi} \int \text{Tr}(G_2^{-1} dG_2)^3. \quad (3.6)
\]

These first order actions describe respectively a holomorphic group element \( g_1(z) \) and an antiholomorphic group element \( g_2(\bar{z}) \). One has thus \( a_\bar{z} = \tilde{a}_z = 0 \) on-shell, so that the first part of the boundary conditions is indeed satisfied.

\[^6\text{We did not consider possible holonomies, that appear as additional zero modes in } (3.2).\]
The sum (3.3) of right- and left chiral actions is equivalent to the nonchiral WZNW action with dynamical variable $g = g_1^{-1}g_2$. To see this equivalence, we rewrite the action (3.3) in terms of the new variables $g$ and $\Pi \equiv -g_2^{-1}\partial_\phi g_1 g_2^{-1} - g_2^{-1}\partial_\phi g_2$. This leads to

$$S = \frac{s}{2\pi} \int d\phi dy \, \text{Tr} \left[ \frac{i}{2} \Pi g^{-1}\partial_\phi g + \frac{1}{4} \Pi^2 + \frac{1}{4} g^{-1}\partial_\phi gg^{-1}\partial_\phi g \right] + \frac{is}{12\pi} \int \text{Tr} (G^{-1}dG)^3, \quad (3.7)$$

with $G = G_1^{-1}G_2$. (3.7) is exactly the nonchiral WZNW model in first order formalism. Eliminating the auxiliary variable $\Pi$ by its equation of motion, one gets finally

$$S = \frac{s}{2\pi} \int d\phi dy \, \text{Tr} \left[ g^{-1}\partial_z gg^{-1}\partial_Z g \right] + \frac{is}{12\pi} \int \text{Tr} (G^{-1}dG)^3, \quad (3.8)$$

which is the standard WZNW action.

### 4 Further Reduction to Liouville Theory

Up to now, we have implemented only part 1 of the boundary conditions on the Chern-Simons connection. We must still incorporate the second part, which, in terms of the Kac-Moody currents, read

$$J^-_z \equiv (g^{-1}\partial_z g)^- = -2i, \quad \tilde{J}^+_z \equiv (\partial_z gg^{-1})^+ = -2i, \quad (4.1)$$

and

$$J^0_z = \tilde{J}^0_z = 0. \quad (4.2)$$

The constraints (1.1) are first class among themselves, and therefore generate a gauge symmetry, while the conditions (1.2) can be viewed as gauge conditions for the symmetry generated by (1.1) [37]. If one implements (1.1) and (1.2) by means of Lagrange multipliers, one gets the (gauged-fixed version of the) action for the gauged WZNW model, in which one has gauged the subgroup of $\text{SL}(2, \mathbb{C})$ generated by the first class constraints [37]. It is well known [11–14] that this model is equivalent to Liouville theory. To see this equivalence at the level of the action, we parametrize $g \in \text{SL}(2, \mathbb{C})$ according to the Gauss decomposition

$$g = \begin{pmatrix} 1 & X \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \exp(\frac{i}{2} \Phi) & 0 \\ 0 & \exp(-\frac{i}{2} \Phi) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ Y & 1 \end{pmatrix}, \quad (4.3)$$
where $X, Y, \Phi \in \mathbb{C}$. It will be shown below that actually $\Phi$ is real. With (4.3), the action (4.8) reads

$$S = \frac{s}{2\pi} \int d\phi dy \left[ \frac{1}{2} \partial_z \Phi \partial_{\bar{z}} \Phi + 2 \partial_{\bar{z}} X \partial_z Y \exp(-\Phi) \right]. \quad (4.4)$$

In terms of the ”momenta” $\Pi_X = \partial\mathcal{L}/\partial\dot{X}$ and $\Pi_Y = \partial\mathcal{L}/\partial\dot{Y}$ conjugate to $X, Y$, one obtains for the constraints (4.1)

$$\Pi_X = \frac{is}{2\pi} \partial_{\bar{z}} Y \exp(-\Phi) = \frac{s}{2\pi}, \quad \Pi_Y = -\frac{is}{2\pi} \partial_z X \exp(-\Phi) = -\frac{s}{2\pi}. \quad (4.5)$$

In order to implement (4.5), we have to go from (4.4) to the reduced action (Routhian function),

$$S \to S - \int d\phi dy \left[ \dot{X} \Pi_X + \dot{Y} \Pi_Y \right], \quad (4.6)$$

i.e. we have to perform a partial Legendre transformation with respect to $\dot{X}, \dot{Y}$. This is equivalent to the procedure used in [35], which consists in adding a boundary term to (4.4), to get an improved action

$$S_{\text{impr}} = S + \frac{is}{2\pi} \int d\phi \left[ Y \partial_{\bar{z}} X - X \partial_z Y \right] \exp(-\Phi) \bigg|_{t_1}^{t_2}. \quad (4.7)$$

One can then insert (4.5) into (4.6) or (4.7), to obtain finally

$$S = \frac{s}{2\pi} \int d\phi dy \left[ \frac{1}{2} \partial_z \Phi \partial_{\bar{z}} \Phi + 2 \exp(\Phi) \right], \quad (4.8)$$

which is the action of Euclidean Liouville field theory. We have thus shown that the asymptotic dynamics of three-dimensional de Sitter gravity is described by Liouville field theory. This provides an explicit example of the dS/CFT correspondence proposed by Strominger [9]. The two sets of Virasoro generators of Liouville theory are related to the residual Kac-Moody symmetries preserving the constraints (4.1) [35, 43], given by

$$g(z, \bar{z}) \rightarrow \Omega(z) g(z, \bar{z}) \tilde{\Omega}^{-1}(\bar{z}), \quad (4.9)$$

with

$$\Omega(z) = \pm \left( \begin{array}{cc} 1 & f(z) \\ 0 & 1 \end{array} \right), \quad \tilde{\Omega}(\bar{z}) = \pm \left( \begin{array}{cc} 1 & 0 \\ \bar{f}(\bar{z}) & 1 \end{array} \right), \quad (4.10)$$
where $f(z)$ ($\tilde{f}(\tilde{z})$) is an arbitrary holomorphic (antiholomorphic) function.

We still have to show that the Liouville mode $\Phi$ is real. This is not evident from the Gauss decomposition (4.3), as this in general requires a complex $\Phi$ for the group $SL(2, \mathbb{C})$. We did however not yet implement all the constraints. As our $SL(2, \mathbb{C})$ generators satisfy $\tau^a = \sigma \tau_a \sigma$ (cf. appendix), with $\sigma$ given in (A.5), the identification (2.6) leads to the pseudoreality condition

$$A^\dagger = \sigma A \sigma,$$  \hspace{1cm} (4.11)

which implies $G_{\tau}^{-1} = \sigma G_1^\dagger \tau$, where $\tau$ denotes an arbitrary constant element of $SL(2, \mathbb{C})$. If we choose e. g. $\tau^\dagger = -\tau$, we obtain the relation\footnote{The same holds for the boundary value $g$.}

$$G^\dagger = -\sigma G \sigma,$$  \hspace{1cm} (4.12)

so that $G$ has the form

$$G = \begin{pmatrix} u & w \\ -\bar{w} & v \end{pmatrix},$$  \hspace{1cm} (4.13)

with $u, v \in \mathbb{R}$, $w \in \mathbb{C}$. It is easy to see that (4.13) parametrizes an element of the coset space $SL(2, \mathbb{C})/SU(2)$. The Gauss decomposition of (4.13) is given by (4.3) with $Y = -X$ and $\Phi$ real. We can now impose this final constraint on the action (4.8) by means of a Lagrange multiplier $\lambda$, and then integrate in the path integral over $\lambda$ and over the imaginary part of $\Phi$, which leads to the same Liouville action, but with real $\Phi$. Alternatively, one can implement (4.12) already after the first reduction step, i. e. , one can restrict the group elements appearing in the WZNW action (3.8) to take values in the coset space $SL(2, \mathbb{C})/SU(2)$. To this end, one first translates (4.12) into a constraint for the currents,

$$J_{\dagger}^\dagger = -\sigma \tilde{J}_z \sigma,$$  \hspace{1cm} (4.14)

which is then imposed by means of a Lagrange multiplier in the action (3.8). This amounts to gauging the subgroup $SU(2)$ of $SL(2, \mathbb{C})$, generated by the constraints (1.14), i. e. , one obtains the action of the $SL(2, \mathbb{C})/SU(2)$ gauged WZNW model. The integration constant $\tau$ relating $G_1$ and $G_2$ determines which subgroup is gauged. For example, the choice $\tau^\dagger = \tau$ leads to a gauging of the subgroup $SU(1, 1)$. The classical Liouville solution corresponding to de Sitter space in horospherical coordinates (2.2) is given by
\[ \exp \Phi = \left[ -uz\bar{z} - iwz + i\bar{w}\bar{z} + \frac{1 - \bar{w}w}{u} \right]^{-2}, \quad (4.15) \]

where \( u \in \mathbb{R} \) and \( w \in \mathbb{C} \) are arbitrary constants. By a combined dilation and translation, \( uz \to z + i\bar{w} \), (4.15) can be cast in the elliptic form [45]

\[ \exp \Phi = \frac{u^2}{[1 - z\bar{z}]^2}. \quad (4.16) \]

Possible further extensions of our work would be the consideration of holonomies, as well as the inclusion of the second boundary. In particular, it would be interesting to verify the argumentation of [9], that the holographic dual is a field theory on one boundary, rather than two. This is currently under investigation.

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**A Conventions**

Our SL(2, \( \mathbb{C} \)) generators are

\[ \tau_0 = \frac{1}{2} \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \quad \tau_1 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (A.1) \]

They satisfy

\[ [\tau_a, \tau_b] = \epsilon_{abc} \tau^c, \quad (A.2) \]

with \( \epsilon_{012} = +1 \), and are normalized according to

\[ \text{Tr}(\tau_a \tau_b) = \frac{1}{2} \eta_{ab}, \quad (A.3) \]

where \( (\eta_{ab}) = \text{diag}(-1, 1, 1) \).

Another useful property is

\[ \tau^\dagger_a = \sigma \tau_a \sigma, \quad (A.4) \]
with $\sigma \in \text{SL}(2, \mathbb{C})$ given by

$$
\sigma = \begin{pmatrix}
i & 0 \\
0 & -i
\end{pmatrix}.
$$

We further define

$$
\tau_\pm = \frac{1}{2}(\tau_1 \mp i\tau_2).
$$

Finally, $d\tau \wedge d\phi \wedge dy$ is chosen to have positive orientation.
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