Soliton Structure of Heavy Baryons

Dong-Pil MIN\textsuperscript{a}, Yongseok OH\textsuperscript{a}, Byung-Yoon PARK\textsuperscript{b}
and
Mannque RHO\textsuperscript{c}

\textsuperscript{a} Department of Physics and Center for Theoretical Physics,
Seoul National University, Seoul 151–742, Korea
\textsuperscript{b} Department of Physics, Chungnam National University, Taejeon 302-764, Korea
\textsuperscript{c} Service de Physique Théorique, C.E. Saclay, 91191 Gif-sur-Yvette, France.

ABSTRACT

Heavy-quark baryons are described as a bound heavy-meson-soliton system in a Lagrangian that combines chiral symmetry and heavy-quark symmetry. We introduce a “Wess-Zumino type” term and show that it dominates the binding of a heavy meson to a soliton. The connection between this model and the Callan-Klebanov model is established to $O(N_c^{-1} \cdot m_\Phi^0)$ where $m_\Phi$ is the mass of the heavy-meson (isospin) doublet $\Phi$ or $\Phi^*$. 
It has recently been shown [1, 2] that the skyrmion description [3] of heavy baryons as one or more heavy pseudoscalar mesons in isospin doublet "wrapped" by an \( SU(2) \) soliton works surprisingly well not only for strange hyperons but also for charmed as well as bottom hyperons provided one takes empirical values for the decay constants and masses of the pseudoscalar mesons. The results for spectra and magnetic moments were found to be remarkably close to the results of quark models which are expected to fare better heavier the quark involved. Analogy to the induced gauge potentials that describe the excitations of diatomic molecules has led to the suggestion [4] that the hyperfine (and fine) splitting for baryons (\( Qqg \)) where \( Q \) represents a heavy quark and \( q \) a light quark of flavor up and down takes the form

\[
\Delta E_{hf} \sim \frac{1}{2I} (\vec{J}_l + c\vec{J}_Q)^2,
\]

where \( I \) is the moment of inertia of the soliton, \( \vec{J}_l \) stands for the angular momentum lodged in the light-quark (soliton) system and \( \vec{J}_Q \) carried by heavy meson denoted generically \( \Phi \) and that in the limit that the heavy-quark mass \( m_Q \) or equivalently the heavy-meson mass containing \( Q \) denoted \( m_\Phi \) goes to infinity, the hyperfine coefficient \( c \) goes to zero and hence the heavy-quark spin \( J_Q \) decouples. Such a limiting behavior would be consistent with the heavy-quark symmetry of Isgur and Wise [5]. Unfortunately up to date, we have been unable to derive (1) from the point of view of induced gauge structure or to show from the Callan-Klebanov (CK) formulation that the coefficient \( c \) indeed has the right asymptotic property.

The purpose of this paper is to examine the heavy-quark limit of the skyrmion description by taking the heavy-meson limit on the effective Lagrangian used in Refs.[1, 2] and comparing with a Lagrangian recently constructed by Wise [6] and Yan et al. [7] to satisfy both the chiral symmetry of light quarks and the Isgur-Wise symmetry of heavy quarks. We show in particular that in the heavy-quark limit, \( c \) vanishes. Our work overlaps closely with – and in part is stimulated by – the recent work of Jenkins, Manohar and Wise (JMW) [8] and Guralnik et al. [9] on the structure of heavy baryons at the fine- and hyperfine-structure level. There is however a distinct difference from Refs.[6]∼[9] in that we start with chiral symmetry in the CK framework and approach the heavy-quark limit from below. In the CK approach, the soliton contributes to the heavy baryon mass a term of \( O(N_c^{-1}) \), the binding energy of the soliton and meson contributes at \( O(N_c^{0}) \) and while the fine and hyperfine splitting occurs at \( O(N_c^{-1}) \), arising from the collective rotation of the soliton and the bound meson, it is formally at \( O(m_\Phi^{-1}) \). Thus the standard \( N_c \)–counting is still valid in the infinite mass limit. In Ref.[9], the hyperfine splitting occurs at \( O(m_\Phi^{-1} \cdot N_c^{-1}) \). We will argue shortly that there is no disagreement on this since in our approach, there is a hidden \( m_\Phi^{-1} \) dependence in the hyperfine coefficient \( c \) of (1).

\(^1\)The analogy to the diatomic molecule is seen when the interatomic distance \( R \) goes to infinity at which limit the angular momentum of the induced gauge field decouples.
We start with a chiral Lagrangian that contains vector mesons together with the chiral field. One could make a rather general discussion using a hidden gauge symmetric (HGS) Lagrangian of Bando et al. [14, 11], but for our purpose, it suffices to slightly modify and study the model of Ref.[12] which has proven to be phenomenologically successful. In the notation suitable to our purpose, the Lagrangian can be written as the sum of the $SU(2)$ Skyrme Lagrangian, $L_{SU(2)}$, the HGS Lagrangian without (with) the $\omega$ meson coupling, $L_{HGS}^{HGS}$ ($L_{\omega}^{HGS}$), and the “anomalous parity” Lagrangian, $L_{an}$:

$$L_{SU(2)} = \frac{F_\pi^2}{16} \text{Tr} \left[ \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right] + \frac{1}{32e^2} \text{Tr} \left[ \Sigma^\dagger \partial_\mu \Sigma, \Sigma^\dagger \partial_\nu \Sigma \right]^2,$$

$$L_{HGS} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - m_\Phi^2 \Phi^\dagger \Phi - \frac{1}{2} \Phi^\dagger_\mu \Phi^\mu_\nu + \frac{m_\Phi^2}{2} \left[ \Phi^\dagger_\mu + \frac{2i}{F_\pi g_\Phi} \Phi^\dagger_{\mu} A_\mu \right] \Phi^{*\mu} + \frac{2i}{F_\pi g_\Phi} A^\mu \Phi,$$

$$L_{HGS}^{\omega} = \frac{iN_c}{2F_\pi^2} B_\mu \left[ (\Phi^\dagger D^\mu \Phi - (D^\mu \Phi)^\dagger \Phi) - \left( \Phi^{*\dagger} D^\mu \Phi^{*\nu} - (D^\mu \Phi)^{*\nu} \right) \right],$$

$$L_{an} = -\frac{iN_c}{2F_\pi} B_\mu \left( \Phi^\dagger D^\mu \Phi - (D^\mu \Phi)^\dagger \Phi \right) + \delta L_{an},$$

(2)

where

$$D_\mu = \partial_\mu + V_\mu, \quad \Sigma = \xi \cdot \xi,$$

$$\left( \begin{array}{c} V_\mu \\ A_\mu \end{array} \right) = \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi \pm \xi \partial_\mu \xi^\dagger \right),$$

$$B_\mu = \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} \text{Tr} \left\{ \Sigma^\dagger \partial^\nu \Sigma^\dagger \partial^\alpha \Sigma \Sigma^\dagger \partial^\beta \Sigma \right\},$$

$$\Phi^*_{\mu\nu} = \partial_\mu \Phi_{\nu}^* - \partial_\nu \Phi_{\mu}^* + V_\mu \Phi_{\nu}^* - V_\nu \Phi_{\mu}^*,$$

(3)

with $\epsilon_{0123} = +1$. The Lagrangian $L_{an}$ which contains, in addition to the usual Wess-Zumino term [13], intrinsic-parity-odd four-derivative terms involving vector fields requires some explanation to which we will return below. Here, $\Sigma$ is the $SU(2)$ chiral field, $\Phi$ and $\Phi^*$ are, respectively, the pseudoscalar and vector meson doublets of the form $Q\bar{q}$, $F_\pi$ represents the pion decay constant and $g_\Phi$ is the $\Phi^*$ “gauge” coupling to matter fields. The Skyrme parameter $e$ will be specified later. For instance, if we take the kaons to be heavy mesons, $\Phi^\dagger = (K^-, \overline{K}^0)$, $\Phi^*_{\mu\nu} = (K_{\mu}^0, \overline{K}^0_{\mu})$. This Lagrangian is obtained from that of Ref.12 by integrating out the $\omega$ and $\rho$ meson fields and then taking the limit $m_\Phi = m_{\Phi^*} \to \infty$, neglecting the terms that vanish as $m_\Phi^{-1}$ and $m_{\Phi^*}^{-1}$ or faster. For the purpose of comparing with the Isgur-Wise symmetric limit, it is necessary to keep the vector mesons explicitly instead of integrating them out as we did in Ref.2. The reason for this will become clear later on.

We need to explain a bit what $L_{an}$ is in the context of the heavy-meson limit that we are interested in. The first term is what one obtains from the topological Wess-Zumino term written down by Witten [13] when expanded à la Callan-Klebanov. This is intrinsically tied
to anomalies in effective theory. Later, as the heavy quark mass increases, this term will disappear\footnote{M.A. Nowak and I. Zahed, private communication. This point is discussed further in\cite{14}.} However the second term, which is intrinsic-parity odd as the Wess-Zumino term is and involves the vectors $P^*$'s, needs not vanish in the heavy-quark limit. We expect them to modify the constants of the main term responsible for the binding of the mesons $\Phi$ and $\Phi^*$ to a soliton. As we know from the work of\cite{10,11}, they are fixed at low energy by low-energy theorems. They can be fixed in the heavy-quark limit by the heavy-quark symmetry.

To see what remains of the Lagrangian (2) in the heavy-quark limit, we make the meson-field redefinition as in Ref.[7], (taking $m = m_{\Phi} = m_{\Phi^*}$)

\[
\begin{align*}
\Phi^*_{\mu}^{\dagger} &= e^{-imv \cdot x} P^*_{\mu} / \sqrt{m}, \\
\Phi^\dagger &= e^{-imv \cdot x} P / \sqrt{m},
\end{align*}
\]

so that the fields $P^*_{\mu}$ and $P$ are independent of the meson mass and obtain

\[
\begin{align*}
\mathcal{L}_{\Phi}^{\text{HGS}} &= -iP v \cdot \vec{D} P_{\mu}^{\dagger} \vec{B} + iP_{\mu}^{\dagger} P \vec{v} \cdot \vec{D} P_{\mu} + i\sqrt{2} \left( P A_{\mu}^{\dagger} P_{\mu} + P_{\mu}^{\dagger} A P_{\mu}^{\dagger} \right), \\
\mathcal{L}_{\omega}^{\text{HGS}} &= \frac{N_c}{2 F_\pi^2} B_{\mu} \left( P v_{\mu} P_{\mu}^{\dagger} - P_{\mu}^{\dagger} v_{\mu} P_{\mu} \right) \\
\mathcal{L}_{\text{an}} &= 2 \frac{N_c}{F_\pi^2} B_{\mu} P v_{\mu} P_{\mu}^{\dagger} + \delta \mathcal{L}_{\text{an}}
\end{align*}
\]

where

\[
\begin{align*}
(D_{\mu} P)_{\dagger} &= (\partial_{\mu} + V_{\mu}) P_{\dagger}, \\
A \vec{D} B_{\dagger} &= A(\vec{D} B)_{\dagger} - (\vec{D} A) B_{\dagger}.
\end{align*}
\]

We have not written out the term $\delta \mathcal{L}_{\text{an}}$ since while the coefficients are known phenomenologically in the light-quark sector, they are not known in the regime we are concerned with. We expect that it will include terms of the form

\[
\frac{iN_c}{F_\pi^2} e^{\mu\nu\alpha\beta} v_{\mu} \left( a P A_{\nu} A_{\alpha} P_{\beta}^{\dagger} - b P_{\beta}^{\dagger} A_{\alpha} A_{\nu} P_{\beta} \right)
\]

with $a$ and $b$ unknown constants\footnote{For light-quark baryons, low-energy theorems fix it to $a = b = 1/3$. For heavy-quark baryons, the Isgur-Wise symmetry requires that $a = b$ but the overall constant is not known. See later.}. Note that since in SMNR\cite{12} we started with an apparently $SU(3)$ symmetric Lagrangian (apart from the meson mass term) with the flavor $Q$ put on the same footing as the light quarks, Eq. (3) results from the $\omega$-meson coupling terms and hence the constant $N_c/F_\pi^2$ is fixed. In the heavy-quark limit, the heavy pseudoscalar decouples from the Wess-Zumino term, so the first term of Eq.(7) which comes from the primordial Wess-Zumino term will disappear. However, the second term will remain to modify effectively the coefficient of Eq.(6) which came from the $\omega$-meson coupling with the
heavy mesons $P$ and $P^*$. That such a term must be present can be seen by bosonizing light and heavy quarks from QCD \[15\].

If the $P$ and $P^*$ are degenerate, there are additional terms in an HGS Lagrangian that can contribute at the same order in the normal as well as anomalous parts of the Lagrangian. One such term comes from the intrinsic-parity odd term accompanying the Wess-Zumino term in the HGS Lagrangian \[11\] that survives in the heavy-meson mass limit

$$c_4 L^{(4)} = -c_4 2 i g_0^2 \epsilon^{\mu \nu \alpha \beta} v_\mu P^\nu A_\alpha P^{\mu \dagger},$$

with $c_4$ the coefficient of $L^{(4)}$ \[11\]. This is effectively a four-derivative term that belongs to the same intrinsic-parity class as $\delta L_{\text{an}}$ discussed above. The coefficient $c_4$ is fixed in the light-quark sector to $c_4 = i N_c / 16 \pi^2$ from the decay $\omega \to \rho \pi$ \[11\]. We will determine later its value in the heavy-quark sector.

Our Lagrangian \((5)\)–\((10)\) can now be compared with the one used by JMW \[8\]

$$L_{\text{JMW}} = -i \text{Tr} [P \cdot \partial H_a + i \text{Tr} \bar{H} H_b v^\mu (V_\mu)_{ba} + i g \text{Tr} \bar{H} H_b \gamma^\mu \gamma_5 (A_\mu)_{ba} + \cdots],$$

with the heavy meson field $H_a$ (where $a$ labels the light-quark flavor) defined as

$$H = \left(1 + \frac{\gamma^5}{2}\right) \left[P^\mu \gamma^\mu - P^5 \gamma^5\right].$$

Substituting Eq.\((12)\) into \((11)\) and taking the trace over the gamma matrices, we have \[7\]

$$L_{\text{JMW}} = -i PV \cdot \bar{D} P^\dagger + i PV \cdot \bar{D} P^\mu P_{\mu \dagger} + 2ig \left\{P^\mu A_\mu P^\dagger + P A^\mu P^\mu P_{\mu \dagger}\right\} + 2g \epsilon^{\mu \nu \alpha \beta} v_\lambda P_\mu A_\nu P_{\alpha \dagger}.$$  

\((13)\)

Ignoring for the moment the term \((6)\) which we will take up shortly, we see that the SMNR Lagrangian \((3)\) with \((10)\) is identical to the JMW Lagrangian \((13)\) if we identify $g = 1 / \sqrt{2} g_0$ and $c_4 g_0^2 = ig$. JMW did not take into account the term \((6)\) since it involves higher derivatives. This procedure is perhaps justified in the meson sector since $B_\mu$ involves three derivatives and hence is suppressed by the factor $p^3 / \Lambda^3$ where $\Lambda$ is the chiral scale of order $m_\rho$ and $p$ is of order $m_\pi$. However in the nontrivial topological sector $B_\mu$ is the baryon current and is of $O(1)$. Furthermore in the Callan-Klebanov scheme, the terms \((3)\) and \((7)\) play the key role in binding the heavy pseudoscalar doublet to the soliton. We therefore propose to add to the JMW Lagrangian \((11)\) the $L^{\text{HGS}}_{\omega}$, Eq.\((8)\), preserving chiral and Isgur-Wise symmetries

$$L^{\text{HGS}}_{\omega} = \alpha B_\mu j_\mu,$$

$$j_\mu = \text{Tr} \left(\bar{H} \gamma^\mu H\right).$$

\((14)\)

\((15)\)

\(\#4\) This is rather close to the quark-model prediction $g \simeq 0.75$ \[8\] and also to the phenomenological value $g \approx 0.6$ extracted from the CLEO collaboration data \[16\].
with $\alpha$ that as noted above cannot be determined by symmetries alone. Such a term arises in an approximate bosonization of QCD, through the coupling of $H$ to the $\omega$ meson \[^{[15]}\] so there is no reason to ignore it. Although this has nothing to do, at least in the heavy-quark limit, with the *bona-fide* Wess-Zumino term, we will refer, *for convenience*, to it as “Wess-Zumino type” term to suggest that in the limit that $m_Q$ goes to zero, it would have the same form as – and perhaps be linked to – the *topological* Wess-Zumino term. In \[^{[15]}\], $j^\mu$ is the $U(1)$ current of the Lagrangian $L_{\Phi^0}$ corresponding to the heavy-quark flavor which is conserved in our case. Although as mentioned above, the coupling constant $\alpha$ cannot be determined by chiral and Isgur-Wise symmetries alone, we will analyze the structure of heavy baryons in units of $-N_c/2F_\pi^2$, *i.e.*, the coefficient of $L_{\mu 1}$ in Eq. (15). We will normalize the meson field as

$$\int d^3r j^0 = -2 \int d^3r (PP^\dagger + P_i^s P_i^{s\dagger}) = -1$$ (16)

and work in the rest frame of the heavy meson, $v_\mu = (1, 0, 0, 0)$. Note that $P_0^s = 0$ due to $v \cdot P^s = 0$.

The Lagrangian correct to order $O(m_0^0 N_c^0)$ is given by

$$L_B = -M_{sol} - m_\Phi + \int d^3r (L_P + L_W),$$

$$-L_P = 2gi \left\{ P^{s\dagger} A^i P^\dagger + P A^i P^{s\dagger} - i\epsilon^{0ijk} P^{s\dagger} A^j P^{s\dagger} \right\}$$

$$-L_W = 2\alpha B_0 \left( PP^\dagger + P_i^s P_i^{s\dagger} \right).$$ (17)

One can readily see that $L_P$ and $L_W$ are invariant with respect to the global rotation $S \in SU(2)_V$ in the light flavor space (*i.e.*, the isospin space) provided that the fields transform

$$P(x) = \phi(x) S^\dagger,$$

$$P_i^s(x) = \phi_i^s(x) S^\dagger,$$

$$\xi(x) = S \xi_0(\vec{x}) S^\dagger,$$ (18)

with $x = (t, \vec{x})$ and $\xi_0(\vec{x}) = \exp(i\vec{r} \cdot \vec{F}(r)/2)$ in the hedgehog configuration. We should stress that in order for $L_B$ to be invariant, the transformation (18) is *required*. The standard procedure for collective quantization is to elevate $S$ to a dynamical variable by endowing it with the time dependence $S(t) = a_0(t) + \vec{a}(t) \cdot \vec{F}$. Note that as defined, the fields $\phi(x)$ and $\phi_i^s(x)$ are fields living in the rotating frame \[^{\#2}\].

\[^{\#5}\]It should be noted that the quantization followed in Refs. \[^{[8, 9]}\] differs from our procedure. In Refs. \[^{[8, 9]}\], the heavy meson fields are not rotated, so are not defined in the soliton rotating frame. Instead they are defined in their rest frame. There is of course nothing wrong in their procedure and it explains why they need not resort to the isospin-spin transmutation. The heavy mesons there are not *behaving* like heavy quarks as in our approach. It would be interesting, however, to understand why the sign change of the coupling constant $g$ or of the radial shape function $F(r)$ affects the binding in Refs. \[^{[8, 9]}\].
From the equations of motion for $\phi(x)$ and $\phi^*(x)$ gotten from the Lagrangian valid at $O(m_\Phi^0 \cdot N_c^0)$, one can readily arrive at the feature that the probabilities of the pseudoscalar and vector mesons are peaked at the center of the soliton

$$|\phi(x)|^2 \propto \delta^3(\vec{x}),$$
$$|\phi^*_i(x)|^2 \propto \delta^3(\vec{x}).$$

That this must be so can be understood as follows. In the large $N_c$ and large $m_\Phi$ (or equivalently large $m_\Phi^*$) limit, the soliton and the meson will be on top of each other and hence when the soliton is fixed at the origin, the wavefunction of both $P$ and $P^*$ in the inertial frame of the rotating soliton must be of the delta function type. Given these solutions, it is now a simple matter to calculate the energy shift coming from $-(\mathcal{L}_P + \mathcal{L}_W)$ of (17)

$$E_I = -\int (\mathcal{L}_P + \mathcal{L}_W) d^3r$$
$$= -\frac{1}{2\pi^2} \alpha \{F'(0)\}^3. \quad (19)$$

An important point to note here is that the contribution of $\mathcal{L}_P$ is zero \[^7\]. In fact, this $\mathcal{L}_P$ term is proportional to the scalar product of the isospin vector and the spin vector of the light degrees of freedom of $H$ field \[^6\], i.e., $\vec{I}_H \cdot \vec{S}_{iH}$. Contrary to Ref.\[9\], however, in our formalism, to $O(N_c^0)$ the light degrees of freedom in $H$ field do not possess isospin and spin quantum numbers. The field $H$ gets its quantum numbers only after collective rotation. Therefore in the rotating frame their “expectation values” must be zero. This is guaranteed in our calculation at $O(N_c^0)$ by the exact cancellation among the three terms in $\mathcal{L}_P$. This shows the difference between the quantization procedure adopted here and that adopted by JMW \[^8, 9\].

As suggested above, we take $g = 1/\sqrt{2} \simeq 0.7$, $F'(0) \approx -0.89$ GeV from the literature and the experimental value of $F_\pi = 186$ MeV and $e = 4.75$, with which we find the $\alpha$ value in the b-quark sector should be \[^#6\]

$$\alpha \approx -\frac{1}{2.8} \left( \frac{N_c}{2F_\pi^2} \right) \quad (20)$$

to reproduce $M_{\Lambda_b} - M_N = 4.65$ GeV which is the predicted value of the quark model. This corresponds to the fine splitting of

$$E_I \approx -0.55 \text{ GeV}. \quad (21)$$

Next we consider the effects of $O(m_\Phi^0 \cdot N_c^{-1})$ term in the Lagrangian. For this we define

$$S^\dagger \dot{S} = i\vec{r} \cdot \vec{\Omega} \quad (22)$$

\[^#6\]The $\alpha$ so obtained is numerically not very different from the value if we take $\alpha = -N_c/(2F_\pi^2)$ with $F_D = 1.84F_\pi$ and $F_B = 1.67F_\pi$. These values are consistent with those employed in Ref.\[2\].
and write the corresponding Lagrangian to $O(m_\Phi^0 \cdot N_c^{-1})$

$$L_{(-1)} = \int d^3r L_{(-1)} = 2I\Omega^2 - 2\Omega \cdot \vec{Q}, \quad (23)$$

where

$$\vec{Q} = -\int d^3r \left( \phi \vec{n}(\vec{r}) \phi^\dagger + \phi_i^\dagger \vec{n}(\vec{r}) \phi_i^{\ast\dagger} \right),$$

$$\vec{n} = \frac{1}{2} \left( \xi_0^\dagger \vec{x}_0 + \xi_0 \vec{x}_0^\dagger \right)$$

$$= \cos F(r)\vec{\tau} - (\cos F(r) - 1)\vec{\tau}(\vec{\tau} \cdot \hat{r}), \quad (24)$$

and $I$ is the moment of inertia of the $SU(2)$ soliton determined from the properties of the $N$ and $\Delta$. As suggested by JMW [8], because of the $\delta$-function structure of the meson wavefunctions and a parity-flip at the origin, it is more convenient to transform the heavy-meson fields to

$$\phi \rightarrow \phi' = \phi \xi_0,$$

$$\phi_i^\ast \rightarrow \phi_i^{\ast'} = \phi_i^\ast \xi_0,$$

$$\vec{n} \rightarrow \xi_0 \vec{n} \xi_0^\dagger. \quad (25)$$

Note that the binding energy is not affected by this transformation. With the primed fields, $\vec{Q}$ is of the form

$$\vec{Q} = -\frac{1}{2} \int d^3r \left\{ \phi' \left( \Sigma^\dagger \vec{\tau} \Sigma + \vec{\tau} \right) \phi'^\dagger + \phi_i^{\ast'} \left( \Sigma^\dagger \vec{\tau} \Sigma + \vec{\tau} \right) \phi_i^{\ast\dagger} \right\}. \quad (26)$$

Now since in the soliton rotating frame, the “isospin” of the meson is transmuted to spin, we can identify

$$\vec{Q} = c\vec{J}_Q, \quad (27)$$

namely, proportional to the angular momentum lodged in the meson which is 1/2. Canonical quantization of (23) leads to an $O(m_\Phi^0 \cdot N_c^{-1})$ splitting $^7$ in energy given by (1) [12],

$$\Delta E_{hf} = 2I\Omega^2$$

$$= \frac{1}{2I} \left( cJ(J + 1) + (1 - c)J_\ell(J_\ell + 1) + c(c - 1)J_Q(J_Q + 1) \right), \quad (28)$$

where $J_\ell$ is the spin lodged in the rotor as discussed in Ref. [12]. The total spin $\vec{J}$ of the system is

$$\vec{J} = \vec{J}_\ell + \vec{J}_Q. \quad (29)$$

By an explicit calculation [17], we find that with (26)

$$c = 0, \quad (30)$$

$^7$Modulo a hidden $m_\Phi^{-1}$ dependence in $c$ explained below.
where we used the normalization of (16) which is invariant under the transformation (25). The way we arrive at this result is quite intriguing and highly nontrivial. The first term of (26) coming from the $P$ mesons gets cancelled exactly by the second coming from the $P^*$ mesons. If the $P$ and $P^*$ were not degenerate the cancellation would not occur. This suggests the following scenario. For not too large $m_\Phi$, say, $m_K$, $c$ can be substantial, of $O(1)$, since the $K^*$ is rather high-lying compared with the $K$. As $m_\Phi$ becomes large, the $P^*$ comes near the $P$, thus decreasing $c$ such that in the heavy-quark limit, we get $c = 0$. Thus our formula found in Refs. [1, 2] has now the correct Isgur-Wise limit.

Given that $c = 0$ in the Isgur-Wise limit, we have the splitting

$$\Delta E_{hf} = \frac{1}{2J} J(J+1).$$  \hspace{1cm} (31)$$

This $\Delta E_{hf}$ predicts that there is an effective “fine” splitting of right sign and magnitude between $\Lambda$ and the degenerate $\Sigma$ and $\Sigma^*$. The predicted mass spectrum (denoting the mass by the particle symbol) for b-quark baryons, with $\Lambda - N = 4.65$ GeV to fix $\alpha$, is

$$\Sigma_b - N = \Sigma^*_b - N = 4.84 \text{ GeV.}$$ \hspace{1cm} (32)

These are comparable to the predictions of quark potential models [18]

$$(\Lambda_b - N)^{QM} = 4.65 \text{ GeV},$$

$$(\Sigma_b - N)^{QM} = 4.86 \text{ GeV},$$

and to those of bag models [19]

$$(\Lambda_b - N)^{BM} = 4.62 \text{ GeV},$$

$$(\Sigma_b - N)^{BM} = 4.80 \text{ GeV}.$$ $^8$

It is possible, within the scheme described so far, to discuss hyperfine splitting with a nonzero $c$. For a finite heavy-quark mass for which $m_\Phi < m^*_{\Phi}$, the CK model indicates that $c \sim 1/m_\Phi$. This is the hidden $m_\Phi^{-1}$ dependence buried in the hyperfine coefficient $c$ alluded above which we conjecture may have an intricate connection to a Berry potential. For a sufficiently large $m_\Phi$, we may therefore assume $c = a/m_\Phi$. Now using (28), we can write for baryons with one heavy quark $Q$$^9$

$$\Sigma_Q - \Lambda_Q = \frac{1}{I}(1 - c_Q) \simeq 195 \text{ MeV}(1 - c_Q).$$ \hspace{1cm} (33)

With the experimental value $\Sigma_c - \Lambda_c \approx 168 \text{ MeV}$ for the charmed baryons, we get $c_c \simeq 0.14$. This means that with $m_D = 1869 \text{ MeV}$, the constant $a \simeq 262$ MeV. So

$$c_\Phi \simeq 262 \text{ MeV}/m_\Phi.$$ \hspace{1cm} (34)$^9$

$^8$ This splitting with $c_Q = 0$ is equal to that of Ref. [4]. In Ref. [4], the authors predicted that $\Sigma_Q - \Lambda_Q \approx 4\Delta M = 195 \text{ MeV}$ where $\Delta M$ is $M_\Delta - M_N$. They have no $1 - c_Q$ dependence in contrast to our eq. (33).

$^9$ It is amusing to note that this formula works satisfactorily even for the kaon for which one predicts $c_s \simeq 0.53$ to be compared with the empirical value 0.62.
Now for b-quark baryons, using $m_b = 5279$ MeV, we find $c_b \simeq 0.05$ which with (33) predicts
\[ \Sigma_b - \Lambda_b \approx 185\text{MeV}. \] (35)

This agrees well with the quark-model prediction. Furthermore the $\Sigma^* - \Sigma$ splitting comes out correctly also. For instance, it is predicted that
\[ \frac{\Sigma^*_b - \Sigma_b}{\Sigma^*_c - \Sigma_c} \approx \frac{m_D}{m_B} \approx 0.35 \] (36)
to be compared with the quark-model prediction $\sim 0.32$. If one assumes that the heavy mesons $\Phi$ are weakly interacting, then we can put more than one $\Phi$'s in the soliton and obtain the spectra for $\Xi$'s and $\Omega$'s as reported in [1, 2]. The agreement with the quark-model results is surprisingly good as pointed out in those references.

We have shown in this paper that one can interpolate the description of baryon structure *smoothly* from light baryons (chiral symmetry) to heavy baryons (Isgur-Wise symmetry) provided extra terms in chiral Lagrangian are implemented to satisfy the Isgur-Wise symmetry. The results obtained in Refs.[8] and [9] which build heavy-quark skyrmions starting with a Lagrangian that satisfies both chiral symmetry and Isgur-Wise symmetry supplemented by symmetry-breaking terms of $O(1/m_Q)$ are strikingly similar to ours which start from chiral symmetry with “higher derivative” terms suitably added in to approach the heavy-quark symmetry. The reason why the CK calculations of heavy baryons of Refs.[1, 2] with the principal contribution coming from the pseudoscalar $P$ and minor contribution from the vector $P^*$ (thus *apparently* possessing no manifest heavy-quark symmetry) were successful may be that the charm quark and bottom quark masses are not really heavy enough to require heavy quark symmetry *ab initio*. This may be somewhat like the strange quark mass which is heavy enough to be considered “heavy” in the sense of the Callan-Klebanov model and light enough to be considered “light” in the sense of the Yabu-Ando model [20]. An attractive feature of our results is that ours are interpretable in terms of nonabelian Berry potentials.

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