Quantum generalization of the classical rotating solutions of the O(N) model

D. Vautherin\textsuperscript{1} and T. Matsui\textsuperscript{2}

\textsuperscript{1} Physique Théorique des Particules Elémentaires
Université Pierre et Marie Curie
F-75252, Paris Cedex 05, France

\textsuperscript{2} Yukawa Institute for Theoretical Physics
Kyoto University, Kyoto 606, Japan

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Abstract

Analytic solutions of the mean field evolution equations for an N-component scalar field with O(N) symmetry are presented. These solutions correspond to rotations in isospin space. They represent generalizations of the classical solutions obtained earlier by Anselm and Ryskin. As compared to classical solutions new effects arise because of the coupling between the average value of the field and quantum fluctuations.
1 Introduction

A class of analytic solutions to the classical equations of motion of the non-linear sigma model have been constructed by Anselm and Ryskin[1] who conjectured the possible relevance of such solutions to high energy heavy ion collisions where a large number of pions are produced in the final states and a part of them may be described by a coherent state wave function. Subsequently, Blaizot and Krzywicki[2] presented another type of classical time-dependent solutions which possess a symmetry under the Lorentz-boost in the direction of colliding nuclei and used them to describe the soft pion emission in ultra-relativistic nuclear collisions. Interest in these solutions grew up because of their possible relevance for the observation of a disoriented chiral condensate[3, 4, 5] which may be formed when the chiral symmetry, temporarily restored in hot hadronic matter produced by the nuclear collision, gets broken again by the rapid cooling of the matter by expansion.

These classical oscillating solutions correspond, in the quantum language, to coherent states of quantum fields, rotating in the internal symmetry space of the fields. One may consider them as arising due to the spontaneous breaking of the internal symmetry, similar to nuclear deformation and rotation. In the latter case, it is known that the nuclear rotation affects the nuclear deformation through the coupling of the collective motion of the deformed nuclear mean field to the motions of intrinsic nucleonic degrees of freedom in it[6]. It is the purpose of this note to show that similar phenomenon arises in the case of oscillating coherent quantum fields when their dynamics is treated in the mean field approximation. For this purpose we use the framework of the time-dependent mean field theory of the quantum fields which can be derived from the Gaussian Ansatz to the time-dependent variational wave functional of the functional Schrödinger equation[7, 8]:

$$\Psi [\varphi; t] \equiv \langle \varphi | \Psi(t) \rangle = \mathcal{N} \exp \left( i \langle \pi | \varphi - \bar{\varphi} \rangle \right) \times \exp \left( - \langle \varphi - \bar{\varphi} | \left( \frac{1}{4G} + i\Sigma \right) | \varphi - \bar{\varphi} \rangle \right),$$

where $\varphi(x) = \langle x | \varphi \rangle$ is the coordinate of the quantum field and the $G, \Sigma, \bar{\varphi}, \pi$ define respectively the (time-dependent) real and imaginary part of the kernel of the Gaussian and its average position and momentum. Although this approach appears to sacrifice the covariance with respect to Lorentz
transformation, one can show that the resultant equations of motion can be rewritten in a covariant form and becomes equivalent to the ones obtained from an explicitly covariant formulation [3].

2 Mean Field Equations

For generality we consider an N-component scalar field $\varphi_a(x)$, $a=1, \ldots, N$ with O(N) symmetry, characterized by a bare mass $m_0^2$ and a coupling constant $\lambda$. The index $a$ will be referred to in what follows as flavor or isospin index. In the mean field approximation the evolution of the mean value $\bar{\varphi}$ of the field (often called condensate) [8]

$$\bar{\varphi}_a(x) = \langle \Psi(t)|\varphi_a(x)|\Psi(t) \rangle,$$

is governed by the following equation of motion

$$\left[\Box + m_0^2 + \frac{\lambda}{6}\bar{\varphi}^2 + \frac{\lambda}{6}\text{tr}S(x, x) + \frac{\lambda}{3}S(x, x)\right]\bar{\varphi}(x) = 0. \quad (1)$$

where $x=(t, x)$, $\bar{\varphi}(x)$ stands for the N-component vector $\bar{\varphi}_a(x)$, and $\bar{\varphi}^2 = \bar{\varphi}_1^2 + \ldots + \bar{\varphi}_N^2$. Here it is implicit that the first four terms carry the $N \times N$ unit matrix. In the previous equation, $S$ is a $N \times N$ matrix operator in flavor space which is related to the kernel of the Gaussian wave-functional

$$S(x, x) = \langle x|G(t)|x \rangle$$

and the trace runs over the flavor indices. $S(x, y)$ is the Feynman propagator which satisfies

$$\left[\Box_x + m^2(x)\right]S(x, y) = i\delta^4(x - y),$$

with the boundary condition $S(x, y) = S(y, x)$; it is symbolically written by

$$S = \frac{i}{\Box + m^2(x) - i\epsilon} \quad (2)$$

where the $N \times N$ mass matrix is

$$m^2(x) = m_0^2 + \frac{\lambda}{6}\bar{\varphi}^2(x) + \frac{\lambda}{6}\text{tr}S(x, x)$$

$$+ \frac{\lambda}{3}\bar{\varphi}(x) \times \bar{\varphi}(x) + \frac{\lambda}{3}S(x, x). \quad (3)$$
In this expression the symbol $\bar{\varphi}(x) \times \bar{\varphi}(x)$ denotes the $N \times N$ matrix whose $(a, b)$ matrix element is $\bar{\varphi}_a(x)\bar{\varphi}_b(x)$. The previous equations (1) – (3) are non-linear because the motion of the condensate involves the mass matrix $m^2(x)$ which in turn involves the values of the condensate and of the propagator. A partial differential form of the mean field equations can be written down by introducing the so-called mode functions [10]. Note that the first three terms in equation (1) correspond to the classical approximation considered by Anselm and Ryskin. The next two correspond to the contribution of quantum fluctuations whose effect is the object of the present study.

### 3 Rotating Solutions

Let us look for solutions of the previous equations by means of the following Ansatz

$$\bar{\varphi}(x) = U(x)\bar{\varphi}^{(0)} = \exp\{i(q \cdot x)\tau_y\}\bar{\varphi}^{(0)},$$

where $q_{\mu} = (\omega, \mathbf{q})$ and $\tau_y$ is a generator of rotation in the subspace of flavor 1 and 2:

$$\tau_y = \begin{pmatrix} 0 & -i & 0 & \cdots \\ i & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$\bar{\varphi}^{(0)}$ can be interpreted as the mean field in the rotating frame and we assume it to be independent of space and time, pointing in the direction of the first flavor:

$$\bar{\varphi}^{(0)} = \begin{pmatrix} \varphi_0 \\ 0 \\ \vdots \end{pmatrix}$$

(4)

We also introduce the propagator $S^{(0)}$ in the rotating frame by

$$S(x, y) = U(x)S^{(0)}(x, y)U^\dagger(y),$$

with

$$S^{(0)} = \frac{i}{(\partial_{\mu} + iq_{\mu}\tau_y)(\partial^{\mu} + iq^{\mu}\tau_y) + M^2 - i\varepsilon}.$$
We assume that the $N \times N$ matrix $M$ is time and position independent. This implies that $S^{(0)}(x, x)$ has the same property. Indeed

$$S^{(0)}(x, y) = -\int \frac{d^4p}{(2\pi)^4} S^{(0)}(p) e^{ip \cdot (x-y)},$$

with

$$S^{(0)}(p) = \frac{i}{(p_\mu + q_\tau y_\tau)(p^\mu + q^\mu \tau y) - M^2 + i\varepsilon},$$

as can be checked by comparing the action of the two operators on a plane wave state $\exp(ik \cdot y)$.

For the particular form we assumed for $\bar{\phi}$ and $S(x, y)$ the self consistent mass reads

$$m^2(x) = U(x) \left[ m_0^2 + \frac{\lambda}{6} \varphi_0^2 + \frac{\lambda}{6} \text{tr} S^{(0)}(x, x) + \frac{\lambda}{3} \bar{\varphi}(0) \times \bar{\varphi}(0) + \frac{\lambda}{3} S^{(0)}(x, x) \right] U^\dagger(x).$$

Using the formula

$$U(x) \partial_\mu U^\dagger(x) = \partial_\mu - iq_\mu \tau_y,$$

it can be seen that the previous equation implies that our Ansatz actually solves the mean field equations provided the mass matrix $M$ in the propagator $S^{(0)}$ satisfies

$$M^2 = m_0^2 + \frac{\lambda}{6} \varphi_0^2 + \frac{\lambda}{6} \text{tr} S^{(0)}(x, x) + \frac{\lambda}{3} \bar{\varphi}(0) \times \bar{\varphi}(0) + \frac{\lambda}{3} S^{(0)}(x, x).$$

This $N \times N$ matrix equation defines the mass gap $M$. It will be referred to below as gap equation in the rotating frame. To have a closed set, the previous equations must be supplemented by the relation satisfied by the condensate $\bar{\varphi}(0)$ in the rotating frame

$$\left[ -q^2 + m_0^2 + \frac{\lambda}{6} \varphi_0^2 + \frac{\lambda}{6} \text{tr} S^{(0)}(x, x) + \frac{\lambda}{3} S^{(0)}_{11}(x, x) \right] \varphi_0 = 0.$$
where we have use the assumption that the condensate in the rotating frame is position and time independent and takes the form (4). This equation has a trivial solution $\varphi_0 = 0$ having $O(N)$ symmetry when $q = 0$. The interesting solution however (at least when $q = 0$) is expected to have a non-vanishing value of the condensate.

4 Gap Equation in the Rotating Frame

Let us first show that the solution of the gap equation is a diagonal matrix which is furthermore a multiple of the unit matrix in the subspace $a=3,\ldots N$ i.e. $M_{ab}^2 = M_a^2 \delta_{ab}$ with $M_a = \mu$ for $a=3,\ldots N$.

Let us consider the propagator in the subspace of the first two flavors $S^{(0)}(p)$ in the momentum representation. It can be written as the following $2 \times 2$ matrix

$$S^{(0)}(p) = \frac{1}{\Delta} \begin{pmatrix} p^2 + q^2 - M_1^2 & -2ip \cdot q \\ 2ip \cdot q & p^2 + q^2 - M_2^2 \end{pmatrix},$$

where $\Delta$ is the determinant

$$\Delta = (p^2 + q^2 - M_1^2)(p^2 + q^2 - M_2^2) - 4(p \cdot q)^2.$$

In order to build the quantity $S^{(0)}(x,x)$ we have to integrate $S^{(0)}(p)$ over $p$. Since off-diagonal elements are odd functions of momentum, we see that the assumption of a diagonal mass matrix is compatible with the structure of the gap equation.

The equations determining the four quantities characterizing the solution $M_1^2, M_2^2, \mu^2$ and $\varphi_0$ are the following

$$M_a^2 = m_0^2 + \frac{\lambda}{6} \varphi_0^2 + \frac{\lambda}{6} \text{tr} S^{(0)}(x,x)$$

$$+ \frac{\lambda}{3} \left[ S_{aa}^{(0)}(x,x) + \delta_{a1} \varphi_0^2 \right],$$

while the condensate satisfies

$$\left( M_1^2 - \frac{\lambda}{3} \varphi_0^2 - q^2 \right) \varphi_0 = 0.$$
The diagonal elements of $S^{(0)}(x,x)$ have the following expression

$$S^{(0)}_{aa}(x,x) = -\int \frac{d^4 p}{(2\pi)^4} S^{(0)}_{aa}(p),$$

where for the first two flavors $a=1,2$ one has

$$S^{(0)}_{aa}(p) = \frac{i(p^2 + q^2 - M_1^2 - M_2^2 + M_a^2)}{(p^2 + q^2 - M_1^2)(p^2 + q^2 - M_2^2) - 4(p \cdot q)^2},$$

where the masses appearing in this equation are supposed to carry a vanishingly small negative imaginary part. For the flavors $a=3,\ldots N$ one has

$$S^{(0)}_{aa}(p) = \frac{i}{p^2 - \mu^2 + i\epsilon},$$

a quantity which is manifestly flavor independent.

These formulae complete the construction of the rotating solutions of the mean field equations for the sigma model.

5 Structure of the Gap Equation

The previous equations involve divergent integrals which need to be regularized e.g. by the introduction of a cutoff in momentum $\Lambda$. For $q=0$ one recovers the usual static mean field equations for the vacuum state. All cutoff dependence may, in principle, be removed by the proper renormalization of the mass and coupling constants. In the following analysis, we instead treat the $O(N)$ model as an effective theory retaining the cutoff dependence explicitly.

In examining the structure of the coupled integral equations (3) – (6) in the presence of non-vanishing $q_\mu$, it is instructive to absorb $q^2$ into the masses by rewriting the effective masses of the first two flavors by $M_1^2 = M_1^2 - q^2$ and $M_2^2 = M_2^2 - q^2$, while keeping other masses unchanged, $M_a^2 = M_a^2$ for $a \geq 3$. Then we find, for the first two flavors,

$$\mathcal{M}_1^2 = m_0^2 - q^2 + \frac{\lambda}{2} \phi_0^2 + \frac{\lambda}{6} \text{tr} S^{(0)}(x,x) + \frac{\lambda}{3} S^{(0)}_{11}(x,x),$$

$$\mathcal{M}_2^2 = m_0^2 - q^2 + \frac{\lambda}{6} \phi_0^2 + \frac{\lambda}{6} \text{tr} S^{(0)}(x,x) + \frac{\lambda}{3} S^{(0)}_{22}(x,x).$$
where

\[
S^{(0)}_{11}(p) = \frac{i(p^2 - M_1^2)}{(p^2 - M_1^2)(p^2 - M_2^2) - 4(p \cdot q)^2},
\]

\[
S^{(0)}_{22}(p) = \frac{i(p^2 - M_1^2)}{(p^2 - M_1^2)(p^2 - M_2^2) - 4(p \cdot q)^2},
\]

while the condensate equation becomes

\[
\left(M_1^2 - \frac{\lambda}{3} \varphi_0^2\right) \varphi_0 = 0.
\]

One can see in this form that a part of the changes of the gap equation caused by non-vanishing \(q\) is to shift the bare mass parameter \(m_0^2 \rightarrow m_0^2 - q^2\). We expect therefore that time-like \(q\) causes an effect equivalent to increase \(|m_0^2|\), hence it leads to an increase of the amplitude of the condensate. This effect may be compared to that of the centrifugal force in ordinary rotation which would cause elongation of a non-rigid rotating body. Note that for time-like \(q\) one can find a frame where the spatial component of \(q\) vanishes and the condensate rotates uniformly with angular frequency \(\omega = \sqrt{q^2}\). On the other hand, for space-like \(q\) there is a frame where the condensate becomes static and oscillates spatially with the wavelength \(\lambda = 2\pi/\sqrt{-q^2}\). In this case, the amplitude of the condensate tends to diminish from its value at \(q = 0\). These effects are purely kinematical and arises also in the classical limit where the amplitude of the condensate is determined by

\[
-q^2 + m_0^2 + \frac{\lambda}{6} \varphi_0^2 = 0 \quad \text{(classical)}.
\]

The remaining effect of non-vanishing \(q\) is to introduce coupling between the first two flavor states. This effect resembles that of the Coriolis force in nuclear rotation which introduces a coupling between single particle states with different orbital angular momenta in a deformed potential. This is a genuine quantum effect which does not show up in the classical approximation.

We now proceed to estimate the significance of these effects by perturbation theory for a small \(q^2\).
6 Perturbative Calculation of $q^2$-dependence

Let us denote by $\delta M_a^2$ the change in $M_a^2$ when going from a square momentum zero to $q^2$. The corresponding change in the diagonal matrix element of $S^{(0)}$ may be expressed as

$$\delta S^{(0)}_{aa}(x,x) = J_a \delta M_a^2 + K_a q^2$$

(9)

The coefficients $J_a$ and $K_a$ can be obtained by using the series expansion of the propagator \[G(p)\] and integrating out over the momentum $p$. The result is given by

$$J_1 = G'(M_0^2), \quad J_2 = J_3 = \cdots = J_N = G'(\mu^2),$$

$$K_1 = I(M_0^2, \mu^2), \quad K_2 = I(\mu^2, M_0^2),$$

$$K_3 = \cdots = K_N = 0.$$

where we have introduced the following integrals

$$G(m^2) = -\int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\varepsilon}$$

$$I(M_a^2, M_b^2) = -M_b^2 \int \frac{d^4p}{(2\pi)^4} \frac{i}{(p^2 - M_a^2)^2(p^2 - M_b^2)}.$$  

and defined the function $G'(m^2)$ as the derivative of $G(m^2)$ with respect to $m^2$. Here $M_0$ stands for the value of $M_1$ when $q = 0$ while the unperturbed masses of all other flavors ($a = 2, \ldots, N$) are set to $\mu$. They satisfy the gap equations at $q = 0$.

The integral $G(m^2)$ is divergent and requires regularization. Using a 3-dimensional regularization in momentum space with a cutoff $\Lambda$ it is found to be

$$G(m^2) = \frac{1}{8\pi^2} \left[ \Lambda^2 - m^2 \log \left( \frac{2\Lambda}{m} \right) + \frac{m^2}{2} \right].$$

Differentiating with respect to the square mass gives

$$G'(m^2) = -\frac{1}{8\pi^2} \left[ \log \left( \frac{2\Lambda}{m} \right) - 1 \right].$$
On the other hand, the integral $I(M_a^2, M_b^2)$ is finite. Using the standard integral representation of propagators [11], the following closed expression for this integral can be obtained

$$I(M_a^2, M_b^2) = \frac{1}{16\pi^2} \frac{M_b^2}{M_a^2 - M_b^2} \times \left[ 1 - \frac{M_b^2}{M_a^2 - M_b^2} \log \left( \frac{M_a^2}{M_b^2} \right) \right],$$

The linearization of the gap equation leads to the following relations for the mass changes in the first two flavors

$$\delta M_1^2 = \frac{\lambda}{2} \delta \varphi_0^2 + \frac{\lambda}{6} \text{tr} \delta S^{(0)}(x, x) + \frac{\lambda}{3} \left[ J_1 \delta M_1^2 + K_1 q^2 \right],$$

and

$$\delta M_2^2 = \frac{\lambda}{6} \delta \varphi_0^2 + \frac{\lambda}{6} \text{tr} \delta S^{(0)}(x, x) + \frac{\lambda}{3} \left[ J_2 \delta M_2^2 + K_2 q^2 \right].$$

For other flavors one finds

$$\delta \mu^2 = \frac{\lambda}{6} \delta \varphi_0^2 + \frac{\lambda}{6} \text{tr} \delta S^{(0)}(x, x) + \frac{\lambda}{3} J_2 \delta \mu^2.$$

A closed system of linear equations can be constructed by including the linearized form of the evolution equation for the condensate (11)

$$\delta M_1^2 - \frac{\lambda}{3} \delta \varphi_0^2 - q^2 = 0,$$

and the expression for the total change in the trace of the propagator

$$\text{tr} \delta S^{(0)}(x, x) = J_1 \delta M_1^2 + J_2 \left[ \delta M_2^2 + (N - 2) \delta \mu^2 \right] + (K_1 + K_2) q^2.$$

The solution of this linear set gives for the changes in the masses

$$\delta M_1^2 = (c_1 b_2 - b_1 c_2) \frac{q^2}{\Delta},$$

and

$$\delta \mu^2 = (a_1 c_2 - c_1 a_2) \frac{q^2}{\Delta}.$$
The constants appearing in these equations are

\[ a_1 = \frac{\lambda}{3} J_1, \quad a_2 = 1 + \frac{\lambda}{3} J_1, \]
\[ b_1 = 1 - \frac{\lambda}{3} J_2, \quad b_2 = (N + 1) \frac{\lambda}{3} J_2 - 2, \]
\[ c_1 = 1 - \frac{\lambda}{3} K_1, \quad c_2 = 1 - \frac{\lambda}{3} K_1 - \frac{\lambda}{3} K_2 b_1. \]

The quantity \( \Delta \) in the above equations is the determinant of the linear system

\[ \Delta = (N + 2) \left( \frac{\lambda}{3} \right)^2 J_1 J_2 - \lambda J_1 + \frac{\lambda}{3} J_2 - 1. \]

The change in the condensate is given by equation (10).

7 Discussion

In the above result all quantum effects are contained in the parameters \( J_a \) and \( K_a \) which represent the changes in the vacuum fluctuation caused by non-vanishing \( q \). The classical relation (8) can be obtained from this result by setting \( J_a = K_a = 0 \). In this limit one finds

\[ \frac{\lambda}{3} \delta \varphi^2 = 2q^2 \quad \text{(classical)}. \] (11)

To obtain an estimate of the quantum fluctuations we have chosen \( M_0 \) to be of the order of the sigma mass \( M_\sigma \) i.e. about 500 MeV and set \( N = 4 \). Assuming the value of the condensate \( \varphi_0 \) to be the pion decay constant \( f_\pi = 93 \text{ MeV} \), the condensate equation at \( q = 0 \) gives a coupling constant \( \lambda = 86.7 \). For a momentum cutoff \( \Lambda = 1 \text{ GeV} \), the gaequation gives \( \mu = 224 \text{ MeV} \). Note that \( \mu \) is a variational parameter characterizing the significance of the quantum fluctuation of pion fields; it should not be taken as the pion mass. For the previous values of the masses one has \( J_1 = -0.00489, J_2 = -0.0141, K_1 = 0.00948 \) and \( K_2 = 0.00800 \). Although the coupling constant is large compared to unity the constants \( K_1 \) and \( K_2 \) are sufficiently small to have \( c_1 \) and \( c_2 \) very close to unity. Actual values are

\[ c_1 = 0.973, \quad c_2 = 0.812, \]
which means that the most important explicit $q^2$ dependence is the one arising in the equation for the condensate. The above values give the following expressions for the variations of the masses with momentum

$$\delta M_1^2 = 8.12q^2 \quad \text{and} \quad \delta \mu^2 = 1.48q^2.$$  

The change in the condensate is found to satisfy

$$\frac{\lambda}{3} \delta \varphi_0^2 = 7.15q^2. \quad (12)$$

An increase of the angular velocity (time-like $q^2$) thus produces an increase of the chiral radius, both in the classical approximation and in the mean-field picture, in agreement with the qualitative argument given above. The change is more rapid in the quantum case. Similarly for a space-like $q^2$ a vanishing value of the condensate is reached more rapidly in the quantum case. This is expected because quantum fluctuations smear out the effective potential and make symmetry breaking more difficult to reach. In the quantum case a transition occurs when

$$-q^2 = q_c^2 \simeq M_0^2/7. \quad (13)$$

At this point the mass of the first flavor vanishes which implies that a solution with a non zero condensate can no longer be obtained beyond this point. The corresponding classical excitation energy density of the meson field is

$$-q_c^2 f_\pi^2/2 \simeq (123 \text{ MeV})^4.$$  

It is interesting to compare this critical momentum for space-like condensate to the corresponding value in the classical limit obtained from (8) for $\varphi_0 = 0$:

$$q_c^2 = M_0^2/2 \quad \text{(classical)}$$

where we have used the relation $M_0^2 = -2m_0^2$ in the tree (classical) approximation. The coupling to the quantum fluctuation thus works to suppress the appearance of static condensate with longer wavelengths. This implies that the appearance of the static chiral condensate is more sensitive to the size of the system than the classical treatment predicts. This would have important implications for the dynamics of chiral condensate in high energy nuclear collisions.
It is worthwhile noting that the analogy with nuclear deformation is limited by the fact that in the nuclear case deformation disappears in the classical limit contrary to symmetry breaking in the $O(N)$ model. Evolutions with angular velocity are however similar.

In conclusion we have shown that, as compared to analyses based on classical equations, new effects arise in a quantum framework as a result of the coupling between the evolutions of mean values and fluctuations. Our result indicates that the coupling to the quantum fluctuation suppresses static condensate with spatial oscillation while the condensate oscillating in time is amplified by the quantum effects.

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