Thermodynamics of heavy quarkonium in magnetic field background

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Abstract

We study the effect of magnetic field on heavy quark-antiquark pair in both Einstein-Maxwell(EM) and Einstein-Maxwell-Dilaton(EMD) model. The interquark distance, free energy, entropy, binding energy and internal energy of the heavy quarkonium are calculated. It is found that the free energy suppresses and the entropy increases quickly with the increase of the magnetic field B. Then, we can conclude that the quarkonium may dissociate easier by increasing the magnetic field B due to the entropic destruction. For large B, the binding energy will vanish at small distance L which indicates the quark-antiquark pair dissociate at smaller distance. The internal energy which consists of free energy and entropy will increase at large separating distance for non-vanishing magnetic field. These conclusions are consistent both in the EM and EMD model. Moreover, we also find that the quarkonium will dissociate easier in the transverse direction than that in the parallel direction in EM model, but the conclusion is opposite in EMD model. It may indicate the dilaton field will deform the space and interact with magnetic field. Finally, we also show that the free energy, entropy and internal energy of a single quark.

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I. INTRODUCTION

It is believed that the so called quark gluon plasma (QGP) has been created in relativistic heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC)[1–4]. Studying the property of the QGP help us to have further understanding of the nature. Heavy quarkonium, which is a bound state of a quark and its antiquark, are among the most sensitive probes used in the experimental study of the QGP and its properties. Depending on the temperature of QGP or size of the quarkonium state, the heavy quark and antiquark may be screened from each other which affects the production rates of heavy quarkonia in heavy-ion collisions[5].

The QGP created in heavy-ion collisions has indeed been found to be strongly coupled at the experimentally accessible temperatures somewhat above the critical temperature[1–4], then the non-perturbative way become valid. The AdS/CFT correspondence[6–8] or the gauge/gravity duality can provide important information to study the strongly coupled systems. The unique advantage of holography leads us to study the various properties of the QGP with gravitational methods. And this holography has given many important insights to study the different nature of strongly coupled matter.

As we know that the strong magnetic fields play essential roles in the noncentral heavy ion collisions[9–22]. Further, this strong magnetic fields may provide important information of the dynamics of quantum chromodynamics (QCD) and give us a better understanding of the QGP. In holographic model, the presence of magnetic field is introduced by adding a $U(1)$ gauge field in the gravitational action. The QCD phase transition in the presence of magnetic field has been studied in Ref.[23–29]. The holographic entanglement entropy with a magnetic field investigated in[30, 31]. The heavy quark diffusion with a magnetic field report in [33]. The energy loss of heavy and light quarks in holographic magnetized background[32]. And the other relative works, see[34–42].

Moreover, many works use AdS/CFT correspondence to investigate the free energy and potential of a heavy quark-antiquark pair in strongly interacting matter[43–46]. In fact, the binding energy can be regarded as the energy difference of free energy of quark-antiquark pair and two free quarks. The difference between free energy and binding energy has been discussed in[47]. Beside free energy, the entropy would be responsible for melting the quarkonium and may be related to the nature of confinement and deconfinement. Lattice QCD stud-
ies show that the additional entropy associated with the presence of static quark-antiquark pair in the QCD plasma[48, 49]. It is found that the entropy will grow with the interquark distance, and give rise to the entropic forces that tend to destroy the quark-antiquark pair[50, 51]. The heavy quark entropy in strong magnetic fields from holographic black hole engineering studied in [52]. Thermal entropy of a quark-antiquark pair above and below deconfinement discussed in [53]. More interesting work, see[54–59].

The rest of the paper is organized as follows: In section II, we will give a brief introduction on the Einstein-Maxwell system and thermodynamic relationship with the magnetic field. In Section III, we calculate the results of quark-antiquark distance, free energy, entropy, binding energy and internal energy with magnetic field background and discuss the effect of magnetic field in both transverse and parallel to magnetic field direction from the Einstein-Maxwell model. Similarly, we can compute all these quantities in the Einstein-Maxwell-Dilaton system in section IV. The free energy, entropy and internal energy of a single quark with finite the magnetic field have been computed in section V. Finally, the conclusion and discussion will be given in Section VI.

II. THE SETUP

The action of the gravity background with back-reaction of magnetic field through the Einstein-Maxwell (EM) system [23, 25, 26] is given as:

\[
S = \int \frac{1}{16\pi G_5} \sqrt{-g} (R - F_{MN}F^{MN} + \frac{12}{L_{\text{AdS}}^2}g_{MN}) d^5x. \tag{1}
\]

where \(R\) is the scalar curvature, \(G_5\) is 5D Newton constant, \(g\) is the determinant of metric \(g_{\mu\nu}\), \(L_{\text{AdS}}\) is the AdS radius and \(F_{MN}\) is the tensor of the U(1) gauge field. The Einstein equation for the EM system could be derived as follows,

\[
E_{MN} - \frac{6}{L_{\text{AdS}}^2}g_{MN} - 2(g^{IJ}F_{M,IJ}F_{N,J} - \frac{1}{4}F_{IJ}F^{IJ}g_{MN}) = 0. \tag{2}
\]

where \(E_{MN} = R_{MN} - \frac{1}{2}Rg_{MN}\). \(E_{MN}, R_{MN}\) and \(R\) are Einstein tensor, the Ricci tensor, and the Ricci scalar, respectively. We take \(L_{\text{AdS}} = 1\) in the following. The ansatz of metric is taken as,

\[
ds^2 = \frac{1}{z^2}(-f(z)dt^2 + \frac{1}{f(z)}dz^2 + h(z)(dx_1^2 + dx_2^2) + q(z) dx_3^2) \tag{3}
\]
A constant magnetic field is in the $x_3$ direction in this metric. For a black hole solution, $f(z = z_h) = 0$ at horizon $z = z_h$ and $q(z)$ together with $h(z)$ are regular function of $z$ in the region $0 < z < z_h$. As discussed in Ref.[26], we will only take the leading expansion as

\[ f(z) = 1 - \frac{z^4}{z_h^4} (1 - \frac{2}{3} B^2 z_h^4 \log \left( \frac{z}{z_h} \right)), \]

\[ q(z) = 1 + \frac{2}{3} B^2 \log(z) z^4, \]

\[ h(z) = 1 - \frac{1}{3} B^2 z^4 \log(z). \] (4)

The Hawking temperature and the magnetic field $B$ is given as

\[ T = \frac{1}{\pi z_h} - \frac{B^2 z_h^3}{6 \pi}. \] (5)

one can take proper values of $z_h$ and $B$ to set the temperature $T$ and magnetic field $B$ in the dual 4D theory. Actually, it is found that the approximate of leading order Eq.(4) is good enough for $T \geq 0.15 GeV$ and $B \leq 0.15 GeV^2$ [26, 32, 60, 61].

The Nambu-Goto action of the worldsheet in the Minkowski metric is given by

\[ S_{NG} = -\frac{1}{2\pi \alpha'} \int d^2 \xi \sqrt{-\det g_{ab}}. \] (6)

where $g_{ab}$ is the induced metric and $\frac{1}{2\pi \alpha'}$ is the string tension and

\[ g_{ab} = g_{MN} \partial_a X^M \partial_b X^N, \quad a, b = 0, 1. \] (7)

where $X^M$ and $g_{MN}$ are the coordinates and the metric of the AdS space.

Then, the Nambu-Goto action can be rewritten as

\[ S_{NG} = -\frac{\mathcal{T}}{2\pi \alpha'} \int_{-L/2}^{L/2} dx \sqrt{g_1(z) \frac{dz^2}{dx^2} + g_2(z)}. \] (8)

$\mathcal{T}$ is the timelike worldlines of length of Wegner-Wilson loop. We can find the magnetic field is $x_3$ direction. So, we parametrize them for parallel direction with $\xi^0 = t, \xi^1 = x_3, g_{1\text{par}}^z(z)$ and $g_{2\text{par}}^z(z)$ are

\[ g_{1\text{par}}^z(z) = \frac{1}{z^4}, \] (9)

\[ g_{2\text{par}}^z(z) = \frac{1}{z^4} f(z) q(z). \] (10)

And for transverse direction with $\xi^0 = t, \xi^1 = x_1, g_{1\text{tra}}^z(z), g_{2\text{tra}}^z(z)$ are...
\[ g_1^{\text{tra}}(z) = \frac{1}{z^4}, \]
\[ g_2^{\text{tra}}(z) = \frac{1}{z^4} f(z) h(z). \] \tag{11}

The separating distances of \( Q\bar{Q} \) pair is
\[ L = 2 \int_0^{L/2} dx = 2 \int_0^{z_0} dx \frac{dz}{dz} = 2 \int_0^{z_0} \left[ \frac{g_2(z)}{g_1(z)} - 1 \right]^{-1/2} dz. \] \tag{12}

From the relation of free energy with Wegner-Wilson loop, the renormalized free energy of \( Q\bar{Q} \) pair is written as
\[ \frac{\pi F_{Q\bar{Q}}}{\sqrt{\lambda}} = \int_0^{z_0} dz \left( \sqrt{\frac{g_2(z)g_1(z)}{g_2(z) - g_2(z_0)}} - \sqrt{g_2(z \to 0)} \right) - \int_{z_0}^{z_h} \sqrt{g_2(z \to 0)} dz. \] \tag{13}

We set the constant \( \sqrt{\lambda} = \frac{1}{\alpha'} \) to one for convenience. The entropy of \( Q\bar{Q} \) pair is given as
\[ S_{Q\bar{Q}} = -\frac{\partial F_{Q\bar{Q}}}{\partial T} = -\frac{\partial F_{Q\bar{Q}}}{\partial z_h} \frac{\partial z_h}{\partial T}. \] \tag{14}

where \( T \) is the temperature of the QGP. And the binding energy is defined as \( E_{Q\bar{Q}} = F_{Q\bar{Q}} - 2F_Q \)\[^{[47]}\], equivalently,
\[ \frac{\pi E_{Q\bar{Q}}}{\sqrt{\lambda}} = \int_0^{z_0} dz \left( \sqrt{\frac{g_2(z)g_1(z)}{g_2(z) - g_2(z_0)}} - \sqrt{g_2(z \to 0)} \right) - \int_{z_0}^{z_h} \sqrt{g_2(z \to 0)} dz. \] \tag{15}

The binding energy \( E_{Q\bar{Q}} \) vanishes when the free energy of the interacting quark pair equals the free energy of non-interacting heavy quarks pair. The free energy of a single quark can be calculated by (13), but let upper limit of integral is \( z_h \). Namely,
\[ \frac{F_Q}{\sqrt{\lambda}} = \frac{1}{\pi} \left( \int_0^{z_h} dz \left( \sqrt{g_1(z)} - \frac{1}{z^2} \right) - \frac{1}{z_h} \right). \] \tag{16}

According to the Ref.\[^{[62]}\], the internal energy of the quark-antiquark pair can be calculated by \( U_{Q\bar{Q}} = F_{Q\bar{Q}} + TS_{Q\bar{Q}} \). If we consider the total energy of the system, we should add \( MB \) term, where \( M \) represents the magnetization which is associated to \( B \). But we only consider the energy of quark-antiquark pair in our paper.
III. THERMODYNAMIC QUANTITIES OF HEAVY QUARKONIUM IN EINSTEIN-MAXWELL (EM) SYSTEM WITH MAGNETIC FIELD

Fig. 1. (a) The dependences of interquark distance $L$ of $Q\bar{Q}$ pair on $z_0$ in the situation magnetic field. Black line is $B = 0\text{GeV}^2$, blue line is $B = 0.1\text{GeV}^2$ and red line is $B = 0.15\text{GeV}^2$ for vanishing chemical potential. The solid line is for transverse direction and dashed line is for parallel direction. (b) is a partly enlarged view of (a).

Fig. 1 shows the dependences of interquark distance $L$ of $Q\bar{Q}$ pair on $z_0$ for different $B$. One can find that the interquark distance $L$ distance increase with $z_0$ and once reaches the maximum value, then it is down to the zero. It may indicate that the $Q\bar{Q}$ pair is in the deconfinement phase. This is because that, in the deconfinement phase, there is a maximum of the separating distance $L$, which is the screening distance $L_c$. If continuing to increase the $z_0$, $U$-shape strings become unstable, then it is down to zero. The unstable branch is not physically favored, we only focus our discussion on $L \leq L_c$ in this paper. Moreover, we can see that larger magnetic field will lead to easier dissolution of the heavy quarkonium. And the heavy quarkonium dissociate easier in the transverse magnetic direction than that in the parallel magnetic direction.
FIG. 2. The free energy of quarkonium on $L$ at different magnetic field. Black line is $B = 0\text{GeV}^2$, blue line is $B = 0.1\text{GeV}^2$ and red line is $B = 0.15\text{GeV}^2$ for vanishing chemical potential. The solid line is for transverse direction and dashed line is for parallel direction. (b) is a partly enlarged view of (a).

The dependence of free energy of heavy $Q\bar{Q}$ pair on the interquark distance $L$ for different $B$ is plotted in Fig. 2. It is shown that the free energy is only a Coulomb potential which lacks a linear potential at large $L$. When adding the dilaton, free energy will have a linear potential at large $L$ which is presented in the next section. And one can find that the free energy is suppressed at larger magnetic field $B$. Further transverse magnetic field will suppress more than the parallel magnetic field.

FIG. 3. The entropy of quarkonium on $L$ at different magnetic field. Black line is $B = 0\text{GeV}^2$, blue line is $B = 0.1\text{GeV}^2$ and red line is $B = 0.15\text{GeV}^2$ for vanishing chemical potential. The chemical potential is 0 and the temperature is $T = 0.2\text{GeV}$. (b) Corresponding entropic force as a function of $L$. The solid line is for transverse direction and dashed line is for parallel direction.

Then, we show the dependence of entropy of heavy $Q\bar{Q}$ pair on the interquark distance
$L$ at different magnetic field $B$ in Fig. 3(a). The entropy increases quickly with the increase of the magnetic field $B$. The increase of entropy will naturally leads to large entropy force $F_e = T \partial S / \partial L$ as shown in Fig. 3 (b). We can see that the entropic force will go to infinite when approaching the the screening distance. Thus, we can conclude that the production rate of quarkonium will suppress with the increase of the magnetic field $B$. As discussed in Ref.[50], the large entropy force is considered as a important reason of driving the dissociation process.

![Graph](image)

FIG. 4. The binding energy of quarkonium on $L$ at different magnetic field. Black line is $B = 0 $GeV$^2$, blue line is $B = 0.1 $GeV$^2$ and red line is $B = 0.15 $GeV$^2$ for vanishing chemical potential. The solid line is for transverse direction and dashed line is for parallel direction. (b) is a partly enlarged view of (a).

We compute the dependence of binding energy of heavy $Q\bar{Q}$ pair on the interquark distance $L$ at different magnetic field $B$ in Fig. 4. The binding energy increases with the increasing of the magnetic field $B$. At a certain distance $L_c$, the free energy of the bound $Q\bar{Q}$ pair equals the free energy of an unbound $Q\bar{Q}$ pair, while for larger distances the free energy of an unbound pair is smaller than that of a bound pair. However, it dose not imply the $Q\bar{Q}$ pair dissociates at this length scale. More discussions about binding energy can be found in Ref.[47]. When increasing $B$, the binding energy will reach to zero at smaller distance. It may indicate that the binding quarks becomes weaker at large magnetic field, especially in the transverse magnetic field.
FIG. 5. The internal energy of quarkonium on $L$ at different magnetic field. Black line is $B = 0\text{GeV}^2$, blue line is $B = 0.1\text{GeV}^2$ and red line is $B = 0.15\text{GeV}^2$ for vanishing chemical potential. The solid line is for transverse direction and dashed line is for parallel direction. (b) is a partly enlarged view of (a).

As we discussed before, we use $U_{Q\bar{Q}} = F_{Q\bar{Q}} + TS_{Q\bar{Q}}$ to define the internal energy of $Q\bar{Q}$. The dependence of internal energy of heavy $Q\bar{Q}$ pair on the interquark distance $L$ of $Q\bar{Q}$ pair at different magnetic field $B$ is shown in Fig. 5. For small distances $L$, the behavior of the internal energy is slightly suppressed. However, for larger distances $L$, the internal energy will increase with the increase magnetic field. This difference may be due to the contribution of entropy at large interquark distance while $F$ is dominant at small interquark distance.

IV. THERMODYNAMIC QUANTITIES OF HEAVY QUARKONIUM IN EMD MODEL WITH MAGNETIC FIELD

In this section, we will consider the effect of dilaton filed. A conformal theory often can not describe the real QCD. The dilaton introduced in the action achieved many successes in QCD phase transition, meson spectrum and other aspects. In our paper, at least, a action without the dilaton can not get a proper behavior of single quark as we will discuss in the next section. Thus, as a compare, we also show the results of EMD model.

A five dimensional EMD model with Maxwell fields[63],

$$S = \int -\frac{1}{16\pi G_5}\sqrt{-g}(R - \frac{f_1(\phi)}{4}F_{(1)MN}^2 - \frac{f_2(\phi)}{4}F_{(2)MN}^2 - \frac{1}{2}\partial_M\phi\partial^M\phi - V(\phi))d^5x.$$  

(17)
Where $F_{(1)MN}$ and $F_{(2)MN}$ are the field strength tensors, $\phi$ is the dilaton field. $f_1(\phi)$, $f_2(\phi)$ are the gauge kinetic functions. The solution of metric in string frame is

$$ds^2 = \frac{e^{2A_s(z)}}{z^2} \left[ (-g(z)dt^2 + \frac{1}{g(z)}dz^2 + dy_1^2 + e^{B^2 z^2}(dy_2^2 + dy_3^2)) \right].$$

(18)

Magnetic field is in $y_1$ direction and $A_s = A(z) + \sqrt{\tfrac{1}{6}} \phi(z)$ with

$$A(z) = -az^2,
K_3 = -\frac{1}{2c} \int_0^{z_h} d\xi \xi^3 e^{-B^2 \xi^2 - 3A(\xi) - c \xi^2},
g(z) = 1 + \int_0^z d\xi \xi^3 e^{-B^2 \xi^2 - 3A(\xi)} (K_3 + \frac{\mu^2}{2c e^{cz_h^2}}),
\phi(z) = (9a - B^2) \log(\sqrt{6a^2 - B^4} \sqrt{6a^2z^2 + 9a^2 - B^4z^2 - B^2 + 6a^2z - B^4z})
+ z \sqrt{6a^2z^2 + 9a - B^2(B^2z^2 + 1)} - \frac{(9a - B^2) \log(\sqrt{9a - B^2} \sqrt{6a^2 - B^4})}{\sqrt{6a^2 - B^4}}.$$  

(19)  
(20)  
(21) 
(22) 
(23)

We take $a = 0.15 GeV^2$ and $c = 1.16 GeV^2$. $\tilde{\mu}$ is related to the chemical potential, we only consider the vanishing chemical potential. The Hawking temperature is given as

$$T = -\frac{z_h e^{A(z_h) - B^2 z_h^2}}{4\pi} (K_3 + \frac{\tilde{\mu}^2}{2c e^{cz_h^2}}).$$

(24)

We can find the magnetic field is $y_1$ direction. In this case, the $g_{1\text{par}}(z)$, $g_{2\text{par}}(z)$ for parallel direction are

$$g_{1\text{par}}(z) = \frac{e^{A_s}}{z^4},
g_{2\text{par}}(z) = \frac{e^{A_s} g(z)}{z^4}.$$  

(25)  
(26)

And for transverse direction, $g_{1\text{tra}}(z)$, $g_{2\text{tra}}(z)$ are

$$g_{1\text{tra}}(z) = \frac{e^{4A_s}}{z^4},
g_{2\text{tra}}(z) = \frac{e^{4A_s} g(z)}{z^4} e^{B^2 z^2}.$$  

(27)

Similarly, one can use Eq.(12) to Eq.(15) in the last section to get separating distances, free energy, entropy and binding energy.
FIG. 6. The dependences of interquark distance $L$ of $Q\bar{Q}$ pair as a function of $z_0$ for different magnetic fields. Black line is $B = 0$, blue line is $B = 0.15$ and red line is $B = 0.3$ for vanishing chemical potential. The solid line is for transverse direction and dashed line is for parallel direction. (b) is a partly enlarged view of (a).

Fig. 6 shows the dependence of interquark distance $L$ of $Q\bar{Q}$ pair on $z_0$ at different $B = 0, 0.15, 0.3\text{GeV}^2$. This picture also tells us that magnetic field will suppress the screening distance which seems similar to the previous case. But it is found that the magnetic field will affect the screening distance stronger with the presence of dilaton. Moreover, in the enlarged picture, we can see that the screening distance is smaller in the parallel magnetic field than that in the transverse magnetic field which is different from the EM model. Thus, we assume that dilaton will reinforce the magnetic effect and affect the spatial distribution of magnetic field.
FIG. 7. The free energy of quarkonium on $L$ for different magnetic fields. Black line is $B = 0 \text{GeV}^2$, blue line is $B = 0.15 \text{GeV}^2$ and red line is $B = 0.3 \text{GeV}^2$ for vanishing chemical potential. The solid line is for transverse direction and dashed line is for parallel direction. (b) is a partly enlarged view of (a).

The dependence of free energy of heavy $Q\bar{Q}$ pair on the interquark distance $L$ at different $B$ is plotted in Fig. 7. It is shown that the free energy is a Coulomb potential at small $L$ and a linear potential at large $L$ after adding the dilaton field. And one can also find that the free energy is suppressed for larger magnetic field $B$. It is expected due to the screening of color charges in the medium[47].

FIG. 8. The entropy of quarkonium on $L$ at different magnetic field. Black line is $B = 0 \text{GeV}^2$, blue line is $B = 0.1 \text{GeV}^2$ and red line is $B = 0.15 \text{GeV}^2$ for vanishing chemical potential. The solid line is for transverse direction and dashed line is for parallel direction. (b) is a partly enlarged view of (a).

We show the dependence of entropy of heavy $Q\bar{Q}$ pair on the interquark distance $L$ at
different magnetic field $B$ in Fig. 8. The entropy also increases with the increase of the magnetic field $B$ in the EMD model. This increase of entropy will lead to large entropy force. But transverse magnetic field leads to larger entropy than parallel magnetic field at small $L$. When approaching to screening distance $L_s$, the effect of parallel magnetic field will become large, which is different from the EM model.

![Graph](image)

**FIG. 9.** The binding energy $E$ of quarkonium on $L$ at different magnetic field. Black line is $B = 0\text{GeV}^2$, blue line is $B = 0.15\text{GeV}^2$ and red line is $B = 0.3\text{GeV}^2$ for vanishing chemical potential. The solid line is for transverse direction and dashed line is for parallel direction. (b) is a partly enlarged view of (a).

We compute the dependence of binding energy of heavy $Q\bar{Q}$ pair on the interquark distance $L$ at different magnetic field $B$ in Fig. 9. The binding energy increases with the increasing of the magnetic field $B$. For increasing $B$, the binding energy at fixed distance becomes weaker(notice binding energy is negative) with the increase of magnetic field.
FIG. 10. The internal energy $U$ of quarkonium on $L$ at different magnetic field. Black line is $B = 0\text{GeV}^2$, blue line is $B = 0.15\text{GeV}^2$ and red line is $B = 0.3\text{GeV}^2$ for vanishing chemical potential. The solid line is for transverse direction and dashed line is for parallel direction. (b) is a partly enlarged view of (a).

The dependence of internal energy of heavy $Q\bar{Q}$ pair on the interquark distance $L$ of $Q\bar{Q}$ pair at different magnetic field $B$ is shown in Fig. 10. As discussed before, for small distances $L$, the behavior of the internal energy is dominant by free energy. However, for larger distances $L$, the internal energy is dominant by the entropy. Thus, the internal energy is suppressed at small $L$ and increase at large $L$ for large magnetic field. And the behavior of internal energy at large $L$ is similar to entropy.

V. THERMODYNAMIC QUANTITIES OF SINGLE QUARK IN EMD MODEL WITH MAGNETIC FIELD

In the vanishing magnetic field, the free energy, entropy and internal energy of single quark can be calculated easily by considering the U-shape string approaches to the horizon $z_h$ in the conformal theory. As given in Ref.[47]:

$$F_Q = -\frac{\sqrt{\lambda}}{2} T, \quad S_Q = \frac{\sqrt{\lambda}}{2}, \quad U_Q = 0$$

(28)

Comparing with lattice results[64], obviously, we find the conformal theory can’t describe the single quark well. But in EMD model, we can do further calculation.
FIG. 11. Free energy of single quark as a function of $T$. Black line is $B = 0\text{GeV}^2$, blue line is $B = 0.3\text{GeV}^2$ and red line is $B = 0.5\text{GeV}^2$ for vanishing chemical potential.

FIG. 12. Entropy of single quark as a function of $T$. Black line is $B = 0$, blue line is $B = 0.3\text{GeV}^2$ and red line is $B = 0.5\text{GeV}^2$ for vanishing chemical potential.

The free energy of a single quark in this model is shown in Fig. 11. One can find that this figure captures the behavior of $F_Q/T$ in lattice QCD calculation in $B = 0$[64]. The increase of $B$ will lead to the increase of free energy. And the $F_Q/T$ will tend to the conformal case at very large $T$ limit for any given magnetic field. The single quark entropy is shown in Fig. 12. And one can find that the $S_Q$ will increase with the increase of magnetic field. At large $T$ limit, the results will tend to conformal case. The internal energy of a single quark is shown in Fig. 13. It is also shown that $U_Q$ will increase with the increase of the magnetic field and tend to the conformal limit at large $T$. 

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VI. SUMMARY AND CONCLUSIONS

In this paper, we study the free energy, entropy, binding energy, internal energy in the 5-dimensional EM and EMD model with the magnetic field. It is found that the increase of magnetic field will suppress the screening distance, and free energy. The entropy will increase quickly with the presence of magnetic field. The induced entropic force will lead to the strong quarkonium suppression due to the entropic destruction. Then, we can see that the absolute value of binding energy (negative energy) will decrease with the increase of magnetic field, which means the binding energy of quarkonium become weaker. The internal energy is dominant by $F$ at small separating distance and dominant by $TS$ at large separating distance. Thus, we can see the internal energy will suppress at small separating distance and increase at large separating distance. Besides, the effect of magnetic field is not significant at the small separating distance.

By comparing two models, we can see effect of dilaton on the thermodynamic qualities. The transverse and parallel magnetic field will have different effect. Thus, we assume that the dilaton will deform the space and influence the effect of magnetic field at different directions. Since a deformed model can describe the thermodynamic qualities of single quark at vanishing magnetic field which is qualitatively similar to the 2+1 flavor lattice QCD calculation\cite{64}, we show the results of single quark and find magnetic field will enhance the free energy, entropy and internal energy in the EMD model.
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