Particle dynamics in sheared granular matter

W. Losert1, L. Bocquet2,3, T.C. Lubensky2, and J.P. Gollub1,2
1 Physics Department, Haverford College, Haverford, PA 19041
2 Physics Department, University of Pennsylvania, Philadelphia, PA 19104
3 Laboratoire de Physique de l’E.N.S. de Lyon, U.M.R. C.N.R.S. 5672, 46 Allée d’Italie, 69364 Lyon Cedex, France

The particle dynamics and shear forces of granular matter in a Couette geometry are determined experimentally. The normalized tangential velocity $V(y)$ declines strongly with distance $y$ from the moving wall, independent of the shear rate and of the shear dynamics. Local RMS velocity fluctuations $\delta V(y)$ scale with the local velocity gradient to the power $0.4 \pm 0.05$. These results agree with a locally Newtonian, continuum model, where the granular medium is assumed to behave as a liquid with a local temperature $\delta V^2$ and density dependent viscosity.

An important property of granular matter is partial fluidization in response to shear stresses. Stationary granular matter can sustain normal loads and shear stresses, but if a threshold shear stress is exceeded, part of the material starts to flow with properties that appear to differ from those of a Newtonian fluid. Unlike ordinary fluids, granular materials do not exhibit intrinsic thermal motion. Instead, the granular ‘temperature’, generally defined as the square of RMS velocity fluctuations $\delta V^2$, is created by the flow itself. As a result, the mean flow and RMS fluctuations are related. This fundamental connection has been investigated experimentally, but remains poorly understood.

The flow of sheared granular materials has been investigated in the steady state in several experiments by shearing material in a Couette cell between a stationary outer cylinder and a rotating inner cylinder. All experiments indicate that the mean particle velocity parallel to the shear direction $V(y)$ decreases faster than linearly away from the inner cylinder.

The velocity profile in three dimensions was determined by Mueth et al. Measurements were carried out both in the interior of the material using X-ray and NMR techniques, and on the bottom surface of the Couette cell by optical imaging. These measurements showed that the velocity profile on the bottom surface and in the interior are the same. $V(y)$ varies from nearly Gaussian for kidney shaped particles to roughly exponential for spherical particles. Layering of particles is suggested as the cause of this variation.

In previous studies in a planar geometry, we have found that most of the flow is confined to $5 - 6$ layers of particles close to the shear plane. The mean particle velocities during brief slips of the shearing plate decrease roughly exponentially with distance away from the moving plate, consistent with the findings in the Couette geometry.

The aim of the present experiment is to understand the relationship between mean velocities, RMS fluctuations and shear forces of a steady state shear flow. We have developed a locally Newtonian, continuum model that describes the granular medium as a liquid with nonuniform temperature and density dependent viscosity. The interplay between mean flow and RMS velocity fluctuations can be understood quantitatively in this context, as we demonstrate.

In order to accomplish fluidization independent of shear, we apply an upward airflow at a variable rate through granular matter sheared in a Couette geometry. We measure both the mean particle velocities $V(y)$ and the velocity fluctuations $\delta V(y)$ on the upper surface of the granular material. These should approximate particle motion in the interior based on the previous 3D measurements described above. Measurements of shear forces in air fluidized granular matter, including a discussion of previous related studies of shear forces, will be presented elsewhere.

In the experimental apparatus the granular material (0.75 mm diameter black glass beads) is confined to a 15 mm gap between a stationary outer cylinder and a rotating inner cylinder ($r = 51$ mm), as shown in Fig. 1. The inner cylinder is hollow to reduce its inertia and is coated with a monolayer of randomly packed glass beads. The outer cylinder is made of glass and is coated with a monolayer of randomly packed glass beads up to the height of the top surface, which allows observation of the top layer of grains through a mirror as shown in Fig. 1. To shear the material, the inner cylinder is rotated at a variable rate of $0.001 - 1$ Hz. The lower 38 mm of the inner cylinder is stationary in order to minimize boundary layer effects.

The inner cylinder is connected to a microstepping motor via a flexible tempered steel spring. The spring bending is proportional to the applied shear force. We measure the spring displacement with a capacitive displacement sensor that is rigidly connected to the motor shaft. This spring configuration allows us to measure instantaneous shear forces with excellent dynamic range and precision, and permits both stick-slip dynamics and continuous motion of the inner cylinder, depending on...
parameters. The trajectories of roughly 100 individual particles in the surface layer are determined with a fast CCD camera at 30 – 1000 frames/sec. Particle motion is extracted from sequences of 2000 images with a spatial resolution of < 0.1 pixels. From the particle tracks we determine average particle velocities \( V(y) \) and RMS velocity fluctuations \( \delta V(y) \) as a function of distance \( y \) from the rotating inner cylinder.

The shear dynamics without airflow are found to be very similar to the dynamics of a plate sliding across a granular layer \[9\]. At low shear rates the motion of the inner cylinder is intermittent with short, rapid slips, and long periods of sticking. At sufficiently high shear rates (or with a stiff spring), steady motion of the inner cylinder is observed.

Airflow has a strong effect on the shear forces and shear dynamics. Without airflow, stick-slip motion is observed with a mean shear force of 95 N/m². At small airflow, the shear force is reduced, but stick-slip motion persists. At high airflow rates (with a mean air speed of \( \sim 1.6 \) m/s through the granular material), shear forces are reduced by a factor of 4 to \( \sim 20 \) N/m² and steady sliding motion is observed. The experiments described below were carried out at large airflow just below the threshold of the 'bubbling' instability. For 0.75 mm diameter glass particles no motion on the upper surface is observed at this airflow in the absence of shear. The mean shear stress does not change with shear rate over more than two orders of magnitude in the shear rate. The shear forces will be discussed in detail elsewhere \[8\]. Here we note that airflow strongly reduces the shear strength of the material and changes the dynamics from stick-slip motion to steady sliding.

The average velocity of particles at the surface of the shear cell is shown in Fig. 2 as a function of distance \( y \) from the rotating inner cylinder, for inner wall velocities \( U \) ranging from 0.004 to 0.4 Hz. As found previously, the velocity profile decays strongly to zero over a few particle diameters \( d \). The normalized velocity profile is independent of the imposed shear rate over at least two orders of magnitude in shear rate. We also find that without airflow (at \( U = 0.01 \) Hz, solid triangles), the inner cylinder moves with stick-slip dynamics rather than steady sliding, but the velocity profile remains unchanged.

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as a boundary condition at the wall. $T_0$ is created by the flow in the first layers close to the wall, so that eq. (1) is assumed to apply only for distance $y$ larger than a cutoff distance $y_w$ from the wall: $y_w$ quantifies the width of the boundary layer in which temperature is created by the motion of the inner cylinder. The fit to experimental data yields $y_w = 2.8d$. The effective cell width $H$ is thus $H_0 - y_w$, where $H_0$ is the wall to wall distance. As shown in Fig. 3 this hydrodynamic model shows good agreement with the experimental data.

Within the framework of this phenomenological model, the velocity profile can be constructed from the constant shear-rate condition, $\sigma_{xy} = \eta \dot{\gamma} = \text{const}$, with $\dot{\gamma} = dV_y(y)/dy$ the local shear rate and $\eta$ the viscosity. The viscosity is expected to scale with the collision frequency, $\eta = \eta_0 P/(\rho_c d^2 T^{1/2})$ (where $d$ is the bead diameter, $\eta_0$ a dimensionless number and $\rho \sim \rho_c$ has been assumed), so that the local shear rate obeys $T^{1/2} = \eta_0 P/(\rho_c d^2 \sigma_{xy}) \dot{\gamma}$. This local relationship can be compared to the experimental data by plotting the local RMS velocity as a function of the local velocity gradient $\dot{\gamma}$. As shown in Fig. 4, a remarkable scaling relationship is found between these two local quantities,

$$\delta V_{\text{RMS}} \equiv T^{1/2} \sim \dot{\gamma}^\alpha$$

with an exponent $\alpha \approx 0.4$. This scaling behaviour is not in agreement with the simple scaling of the viscosity with the collision frequency only, for which an exponent of 1 would have been measured.

The actual scaling can be understood if we assume that the dimensionless coefficient $\eta_0$ contains an additional contribution that diverges algebraically, when the density reaches random-close packing, $\rho_c$, i.e. $\eta_0 = \eta_0(1 - \rho/\rho_c)^{-\beta}$. The density can be related to the granular temperature through the constant isotropic pressure $P$, which can be formally written as $P = \rho f(\rho) T$. The dimensionless function $f(\rho)$ is expected to diverge like $(1 - \rho/\rho_c)^{-1}$ at random close packing [1], so that at constant pressure $P$, in a region where $\rho \sim \rho_c$, one has $\rho_c - \rho \sim T$. Note that this relation is consistent with Reynold’s remark that the density has to decrease in order to allow flow, which occurs for non-zero granular temperature. Together with the constant shear rate condition, these relations imply a nonlinear algebraic scaling of $\delta V(y)$ with the shear rate, with an exponent $\alpha = (2\beta + 1)^{-1}$. With the experimentally determined exponent $\alpha = 0.4$, we obtain the overall divergence of the viscosity with density as $\eta \sim (\rho_c - \rho)^{1.75}$.

Using the previously obtained temperature profile, Eq. (5) can be integrated (with no-slip boundary conditions at both walls) to give the velocity profile. The corresponding result is plotted in Fig. 4 as a dashed line, indicating good agreement with the experimental profile. The calculation also yields a relation between the boundary temperature $T_0$ and the shear rate $U$, as $T_0 \sim U^\alpha$ in agreement with the experimental data (not shown).

In a phenomenological description, it is tempting to consider these RMS velocity fluctuations as an effective granular temperature and consider a local hydrodynamic model as a starting point, as has already been proposed in the literature (see e.g. [11]). Within such a picture the granular temperature obeys a transport equation characterizing the balance between “heat flow” and dissipation due to the inelastic collisions:

$$\partial_y \lambda \partial_y T - \epsilon T = 0. \quad (1)$$

(Note that a local heating term has been omitted, which can be shown to be valid far from the moving boundary.) This local equation introduces two transport coefficients, the thermal conductivity $\lambda$ and the energy loss rate $\epsilon$. Both $\lambda$ and $\epsilon$ are proportional to the collision frequency. In the high density limit we consider, $\rho \sim \rho_c$ with $\rho_c$ the density at random close packing, $\lambda$ and $\epsilon$ are proportional to $P/T^{1/2}$, where $P$ is the isotropic pressure [1]. Stress conservation in the $y$ direction implies that $P$ is constant over the cell. From eq. (1), one can obtain the temperature profile as

$$T^{1/2}(y) = T_0^{1/2} \frac{\cosh((H - y)/\delta)}{\cosh(H/\delta)} \quad (2)$$

where $\delta$, defined as $\delta^2 = 2\lambda/\epsilon$, is proportional to the bead diameter $d$, and is independent of position and temperature. A vanishing heat flux has been assumed at the outer stationary boundary in agreement with additional experimental measurements for wider shear regions, while the temperature at the moving boundary, $T_0$, is introduced.
FIG. 4. Connection between the local RMS velocity fluctuations and local shear rate (same symbols as for Fig. 3). Local fluctuations are found to increase approximately as a power law of the local velocity gradient, with a power of 0.4 (dashed line).

Finally, we measure granular flow when the gap between the two cylinders is only 5 particle diameters wide. In this case, the inner cylinder jams and can only be moved at high airflow rates and high rotation rates. For the narrow gap we find that $V(y)$ decreases linearly with $y$ and that $\delta V(y)$ is uniform across the gap at shear rates of 0.1 rev./sec and 0.02 rev./sec. This result is consistent with our model: A linear velocity profile is expected for a constant granular temperature. A constant granular temperature in turn is expected for cell widths smaller than the previously defined decay length $\delta$.

The success of the present local hydrodynamic model in describing the experimental data (even for small driving velocities) calls for a deeper understanding. The basic ingredient of the present model is the algebraically diverging viscosity close to random close packing. In a granular material, the ability to flow relies mainly on a decrease in local density. Such a decrease can be caused by the onset of RMS velocity fluctuations. Lowering the density allows some particles to move, creating “fluidized regions”, while other regions remain in clusters at rest. The flow properties can be thus formulated in terms of local fluidization probability, or cluster lifetime. In a granular material, these quantities might be estimated by means of a free volume estimate, along the lines proposed by Edwards [12] for granular matter and earlier by Cohen and Turnbull [13] in the context of transport in glassy systems. These models predict a very strong divergence of the viscosity close to random close packing, in qualitative agreement with the present results. More precisely, our data are consistent with an algebraic divergence, which is also expected in supercooled liquids when the density moves off the critical density [14].

The model quantitatively describes all aspects of the experimental results: (i) The RMS velocity fluctuations increase with the local shear rate to a given power. (ii) The velocity decreases strongly away from the moving wall. (iii) The normalized velocity profile is independent of shear rate and shear dynamics.

More generally, our experimental results indicate that there may be a useful analogy between the dynamics of granular materials and the behavior of supercooled liquids close to the glass transition. Such an analogy was proposed in a recent paper by Liu and Nagel [15]. Our results support this conjecture and indicate the quantitative relationship: The flow properties reported here are quantitatively predicted from a locally Newtonian, continuum model, provided that the local temperature is identified with $(\delta V)^2$ and that the viscosity is assumed to diverge as the density approaches a critical value. The specific properties of individual particles only enter through the few adjustable parameters of the model. Once determined in a specific geometry, the flow properties in any geometry can be predicted. Further experiments are underway to verify this predictive power of the model.

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