The origin of dark matter, matter-anti-matter asymmetry, and inflation

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A rapid phase of accelerated expansion in the early universe, known as inflation, dilutes all matter except the vacuum induced quantum fluctuations. These are responsible for seeding the initial perturbations in the baryonic matter, the non-baryonic dark matter and the observed temperature anisotropy in the cosmic microwave background (CMB) radiation. To explain the universe observed today, the end of inflation must also excite a thermal bath filled with baryons, an amount of baryon asymmetry, and dark matter. We review the current understanding of inflation, dark matter, mechanisms for generating matter-anti-matter asymmetry, and the prospects for testing them at ground and space based experiments.

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I. INTRODUCTION

This review aims at building a consistent picture of the early universe where the three pillars of modern cosmology: inflation, baryogenesis and the synthesis of dark matter can be understood in a testable framework of physics beyond the Standard Model (SM).

Inflation (Guth, 1981), which is a rapid phase of accelerated expansion of space, is the leading model that explains the origin of matter: during this phase, primordial density perturbations are also stretched from sub-Hubble to super-Hubble length scales (Mukhanov et al., 1992). A strong support for such an inflationary scenario comes from the precision measurement of these perturbations in the cosmic microwave background (CMB) radiation, e.g. by the Cosmic Background Explorer (COBE) (Smoor et al., 1992) and the Wilkinson Microwave Anisotropy Probe (WMAP) (Komatsu et al., 2011) satellites. However, one of the most serious challenges faced by inflationary models is that only a few of them provide clear predictions for crucial questions regarding the nature of the matter created after inflation and the mode of exiting inflation in a vacuum that can excite the SM degrees of freedom (\(d.o.f\)) (Mazumdar and Rocher, 2011).

From observations we know that the current universe contains 4.6% atoms, 23% non-relativistic, non-luminous dark matter, and the rest in the form of dark energy. While some 13.7 billion years ago it was 37% atoms,
photons and neutrinos, and 63% non-relativistic dark matter \cite{Komatsu+2011}. Therefore, it is mandatory that the inflationary vacuum must excite these SM baryons, and create the right abundance of dark matter. Since the success of Big Bang Nucleosynthesis (BBN) \cite{Iocco+2009} requires an asymmetry between the baryons and anti-baryons of order one part in \(10^{10}\), it is necessary that the baryonic asymmetry must have been created dynamically in the early universe before the BBN \cite{Sakharov1967}.

The prime question is what sort of visible sector beyond the SM would accomplish all these goals – inflation, matter creation, and seed perturbations for the CMB. Beyond the scale of electroweak SM (at energies above \(\geq 100 - 1000\) GeV) there are plethora of candidates, e.g. \cite{Bustamante+2009}. However the low scale supersymmetry (SUSY) provides an excellent platform, which have been built on the success of the electroweak physics \cite{Chung+2005, Haber+Kane1983, Martin1997, Nilles1984}. The minimal supersymmetric extension of the SM, known as MSSM, or its minimal extensions, provides many testable imprints at the collider experiment \cite{Nath+2010}. In particular, the lightest SUSY particle (LSP) can be electrically neutral, and will be an ideal candidate for the weakly interacting massive particle (WIMP) as a dark matter \cite{Ellis+1984, Goldberg1983}, whose abundance can now be calculated from the direct decay of the inflaton, or from the decay products of the inflaton, as shown in Fig. 1.

If such a visible sector, with the known gauge interactions, can also provide us with an inflationary potential capable of matching the current CMB data, then we would be able to identify the origin of the inflaton, its mass and couplings, and the vacuum energy density within a testable theory, such as the MSSM. The inflaton’s gauge invariant couplings would enable us to ascertain the post-inflationary dynamics, and the exact mechanism for particle creation from the inflaton’s coherent oscillations, known as (p)reheating \cite{Allahverdi+2010}. We would be able to precisely determine the largest reheat temperature, \(T_R\), of the post-inflationary universe, during which all the MSSM d.o.f come in chemical and in kinetic equilibrium for the first time ever. Once the relevant d.o.f are created it would be possible to build a coherent picture where we will be able to understand the origin of baryogenesis and the dark matter in a consistent framework as illustrated in Fig. 1.

This review is divided into three parts. In the first part we will discuss the origin of inflation, and how to connect the models of inflation to the current CMB observations. We will keep our discussions general and provide some examples of non-SUSY models of inflation. We will mainly focus on SUSY based models and its generalization to supergravity (SUGRA). We will discuss the epoch of reheating, preheating and thermalization for an MSSM based model of inflation. In the second part of the review, we will focus on baryogenesis. We will state the conditions for generating baryogenesis. We will discuss electroweak baryogenesis, baryogenesis induced by lepton asymmetry, known as leptogenesis, and the MSSM based Afflck-Dine baryogenesis which can create non-topological solitons, known as Q-balls. In the third part, we will consider general properties of dark matter, various mechanisms for creating them, some well motivated candidates, and link the origin of dark matter to the origin of inflation within SUSY. We will briefly discuss the ongoing searches of WIMP as a dark matter candidate.

**II. PARTICLE PHYSICS ORIGIN OF INFLATION**

There are two classes of models of inflation, which have been discussed extensively in the literature, e.g. see reviews \cite{Lyth+Riotto1999, Mazumdar+Rocher2011}. In the first class, the inflaton field belongs to the hidden sector (not charged under the SM gauge group). The direction along which the inflaton field rolls belongs to an absolute gauge singlet, whose couplings to the visible and hidden sectors without any biased – such as the case of gravity which is a true singlet, and a color and flavor blind. In the second class, the inflaton candidate distinctly belongs to the visible sector, where the inflaton is charged under the SM or its minimal extension beyond the SM gauge group. This has many advantages, which we will discuss in some details.

Any inflationary models are required to be tested by the amplitude of the density perturbations for the observed large scale structures \cite{Mukhanov+1992}. Therefore the predictions for the CMB fluctuations are the most important ones to judge the merits of the models, which would contain information about the power spectrum, the tilt in the spectrum, running in the tilt, and tensor to scalar ratio. These observable quantities can be recast in terms of the properties of the poten-

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**FIG. 1:** An illustration of a visible sector model for the early universe. EW stands for the electroweak.
tial which we will discuss below. From the particle origin point of view, one of the successful criteria is to end inflation in the right vacuum - where the SM baryons are excited naturally for a successful baryogenesis before BBN (Mazumdar and Rocher, 2011).

A. Properties of inflation

The inflaton direction which leads to a graceful exit needs to be flat with a non-negligible slope provided by a potential \( V(\phi) \) which dominates the energy density of the universe. A completely flat potential, or a false vacuum with a very tiny tunneling rate to a lower vacuum, would render inflation future eternal, but not past (Borde et al., 2003; Borde and Vilenkin, 1994; Liddle, 1983, 1986; Linde, 1994). A past eternal inflation is possible only if the evolution of the inflaton is approximated as:

\[
H^2 \approx \frac{V(\phi)}{3M_p^2},
\]

\[
3H\dot{\phi} \approx -V'(\phi),
\]

where prime denotes derivative with respect to \( \phi \). There exists slow-roll conditions which constrain the shape of the potential, are give by:

\[
\epsilon(\phi) \equiv \frac{M_p^2}{2} \left( \frac{V''}{V} \right)^2 \ll 1,
\]

\[
|\eta(\phi)| \equiv \frac{M_p^2}{2} \left| \frac{V'''}{V'} \right| \ll 1.
\]

These conditions are necessary but not sufficient for inflation. The slow-roll conditions are violated when \( \epsilon \sim 1 \), and \( \eta \sim 1 \), which marks the end of inflation.

However, there are certain models where this need not be true, for instance in hybrid inflation models (Linde, 1994), where inflation comes to an end via a phase transition, or in oscillatory models of inflation where slow-roll conditions are satisfied only on average (Damour and Mukhanov, 1998; Liddle and Mazumdar, 1998, 1999), or inflation happens in oscillations (Biswas and Mazumdar, 2002), or in fast roll inflation where the slow-roll conditions are never met (Linde, 2001). The K-inflation where only the kinetic term dominates where there is no potential at all (Armendariz-Picon et al., 1999).

One of the salient features of the slow-roll inflation is that there exists a late time attractor behavior, such that the evolution of a scalar field after sufficient e-foldings become independent of the initial conditions (Salopek and Bond, 1990).

The number of e-foldings, \( N_{\text{end}} \), and the end of inflation, \( t_{\text{end}} \), is defined by:

\[
N \equiv \ln \frac{a(t_{\text{end}})}{a(t)} = \int_t^{t_{\text{end}}} H dt \approx \frac{1}{M_p^2} \int_{\phi_{\text{end}}}^\phi \frac{V}{\sqrt{V}} d\phi,
\]

where \( \phi_{\text{end}} \) is defined by \( \epsilon(\phi_{\text{end}}) \sim 1 \), provided inflation comes to an end via a violation of the slow-roll conditions. The number of e-foldings can be related to the Hubble crossing mode \( k = a_k H_k \) by comparing with the present Hubble length \( a_0 H_0 \). The final result is (Liddle and Leach, 2003):

\[
N(k) = 62 - \ln \frac{k}{a_0 H_0} - \ln \frac{10^{16} \text{GeV}}{V_k^{1/4}} + \ln \frac{V_k^{1/4}}{V_{\text{end}}^{1/4}} - \frac{3}{3} \ln \frac{V_{\text{end}}^{1/4}}{\rho_R^{1/4}},
\]

where the subscripts \( \text{end} \) (R) refer to the end of inflation (end of reheating). Today’s Hubble length would correspond to \( N_Q \equiv N(k = a_0 H_0) \) number of e-foldings, whose actual value would depend on the equation of state, i.e. \( \omega = p/\rho \) (\( p \) denotes the pressure, \( \rho \) denotes the energy density), from the end of inflation to radiation and matter dominated epochs. A high scale inflation with a prompt reheating with relativistic species would yield approximately, \( N_Q \approx 50 - 60 \). A significant modification can take place if the scale of inflation is low (Arkani-Hamed et al., 2000; Green and Mazumdar, 2002; Lyth and Steward, 1995, 1996; Mazumdar, 1999, Mazumdar and Perez-Lorenzana, 2001).

B. Density Perturbations

1. Scalar perturbations

Small inhomogeneities in the scalar field, \( \phi(x,t) = \phi(t) + \delta \phi(x,t) \), such that \( \delta \phi \ll \phi \), induce perturbations in the background metric, but the separation between the background metric and a perturbed one is not unique. One needs to choose a gauge. A simple choice would be to fix a gauge where the non-relativistic limit of the full perturbed Einstein equation can be recast as a Poisson equation with a Newtonian gravitational potential, \( \Phi \). The induced metric can be written as, e.g. (Mukhanov et al., 1992):

\[
ds^2 = a^2(x) \left[ (1 + 2\Phi) dx^2 - (1 - 2\Psi) \delta_{ij} dx^i dx^j \right],
\]

Only in the presence of Einstein gravity and when the spatial part of the energy momentum tensor is diagonal, i.e. \( \delta T^i_j = \delta^i_j \), it follows that \( \Phi = \Psi \).

During inflation the massless inflaton (with mass squared: \( m^2 \sim V'' \ll H^2 \)) perturbations, \( \delta \phi \), are stretched outside the Hubble patch. One can track their perturbations from a sub-Hubble to that of a super-Hubble length scales. Right at the time when the wave numbers are crossing the Hubble patch, one finds a solution for \( \delta \phi \) as

\[
\langle |\delta \phi_k|^2 \rangle = \left( H(t_*)^2/2k^3 \right),
\]

where \( t_* \) denotes the instance of Hubble crossing. One can define a power spectrum for the perturbed scalar field

\[
\mathcal{P}_\phi(k) = \frac{k^3}{2\pi^2} \left( \frac{H(t_*)}{2\pi} \right)^2 \left[ \frac{H(t_*)}{2\pi} \right]^2_{k=aH}.
\]
Note that the phase of $\delta \phi_k$ can be arbitrary, and therefore, inflation has generated a Gaussian perturbation. Now, one has to calculate the power spectrum for the metric perturbations. For a critical density universe

$$\delta_k \equiv \frac{\delta \rho}{\rho} \bigg|_k = -\frac{4}{3} \left( \frac{k}{aH} \right)^2 \Phi_k,$$  

(10)

where $\Phi_k(t) \approx (3/5) H(\delta \phi_k/\dot{\phi})|_{k=aH}$. Therefore, one obtains:

$$\delta_k^2 = \frac{4}{9} \rho_k \phi_k = \frac{4}{9} \left( \frac{H}{\dot{\phi}} \right)^2 \left( \frac{H}{2\pi} \right)^2,$$  

(11)

where the right hand side can be evaluated at the time of horizon exit $k = aH$. The temperature anisotropy seen by the observer in the matter dominated epoch is proportional to the Newtonian potential, $\Delta T_k/T = -(1/3)\Phi_k$.

Besides tracking the perturbations in the longitudinal gauge with the help of Newtonian potential, there exists another useful gauge known as the comoving gauge. By definition, this choice of gauge requires a comoving hypersurface on which the energy flux vanishes, and the relevant perturbation amplitude is known as the comoving curvature perturbation, $\zeta_k$ (Lukash, 1980; Mukhanov et al., 1992). For the super-Hubble modes, $k \to 0$, the comoving curvature perturbation, $\zeta_k$ is a conserved quantity, and it is proportional to the Newtonian potential, $\zeta_k = -(5/3)\Phi_k$. Therefore, $\delta_k$ can also be expressed in terms of curvature perturbations (Liddle and Lyth, 1993, 2000)

$$\delta_k = 2 \left( \frac{k}{aH} \right)^2 \zeta_k,$$  

(12)

and the corresponding power spectrum $\delta_k^2 = (4/25)P_\zeta(k) = (4/25)(H/\dot{\phi})^2(H/2\pi)^2$. With the help of the slow-roll equation $3H\dot{\phi} = -V'$, and the critical density formula $3H^2M_p^2 = V$, one obtains

$$\delta_k^2 \approx \frac{1}{75\pi^2 M_p^6 V^2} = \frac{1}{150\pi^2 M_p^4 V^2},$$

(13)

$$P_\zeta(k) = \frac{1}{24\pi^2 M_p^2 V},$$

where we have used the slow-roll parameter $\epsilon \equiv (M_p^2/2)(V'/V)^2$. The COBE satellite measured the CMB anisotropy and fixes the normalization of $P_\zeta(k)$ on very large scales. If we assume that the primordial spectrum can be approximated by a power law (ignoring the gravitational waves and the $k-$dependence of the power $n_s$) (Komatsu et al., 2009)

$$P_\zeta(k) \approx (2.445 \pm 0.096) \times 10^{-9} \left( \frac{k}{k_0} \right)^{n_s-1},$$  

(14)

where $n_s$ is called the spectral index (or spectral tilt), the reference scale is: $k_0 = 7.5a_0H_0 \sim 0.002$ Mpc$^{-1}$, and the error bar on the normalization is given at 1$\sigma$, and

$$n_s(k_0) = 0.960 \pm 0.13$$  

(15)

It is important to stress that these central values and error bars vary significantly when other parameters are introduced to fit the data, in part because of degeneracies between parameters (in particular $n_s$ with $\Omega_b h^2$, the optical depth $\tau$, its running, the tensor-to-scalar ratio, $r$, and the fraction of cosmic strings). The spectral index $n(k)$ is defined as

$$n(k) - 1 = \frac{d\ln P_\zeta}{d\ln k}.$$  

(16)

This definition is equivalent to the power law behavior if $n(k)$ is close to a constant quantity over a range of $k$ of interest. One particular value of interest is $n_s = n(k_0)$. If $n_s = 1$, the spectrum is flat and known as Harrison-Zeldovich spectrum (Harrison, 1970; Zeldovich, 1970). For $n_s \neq 1$, the spectrum is tilted, and $n_s > 1$ ($n_s < 1$) is known as a blue (red) spectrum. In the slow-roll approximation, this tilt can be expressed in terms of the slow-roll parameters and at first order:

$$n_s - 1 = -6\epsilon + 2\eta + O(\epsilon^2, \eta^2, \epsilon\eta, \xi^2),$$  

(17)

where

$$\xi^2 \equiv M_p^2 V'(d^3V/d\phi^3)/V^2,$$  

$$\sigma^3 \equiv M_p^2 V''(d^4V/d\phi^4)/V^3.$$  

(18)

The running of these parameters are given by (Salopek and Bond, 1990). Since the slow-roll inflation requires that $\epsilon \ll 1, |\eta| \ll 1$, therefore naturally predicts small variation in the spectral index within $\Delta \ln k \approx 1$ (Kosowsky and Turner, 1995)

$$\frac{dn(k)}{d\ln k} = -16\epsilon + 24\epsilon^2 + 2\xi^2.$$  

(19)

It is possible to extend the calculation of metric perturbation beyond the slow-roll approximations based on a formalism similar to that developed in Refs. (Kolb et al., 1993; Mukhanov, 1983, 1989; Sasaki, 1986).

2. Multi-field perturbations

Inflation can proceed along many flat directions with many light fields. Their perturbations can be tracked conveniently in a comoving gauge, on large scales $\zeta = H\delta \phi/\dot{\phi}$ remains a good conserved quantity, provided each field follow slow-roll condition. The comoving curvature perturbations can be related to the number of e-foldings, $N$, given by (Salopek, 1995; Sasaki and Stewart, 1996)

$$\zeta = \delta \phi = (\partial N/\partial \phi_a)\delta \phi_a,$$  

(20)

where $N$ is measured by a comoving observer while passing from flat hypersurface (which defines $\delta \phi$) to the comoving hypersurface (which determines $\zeta$). The repeated indices are summed over and the subscript $a$ denotes a component of the inflaton (Lyth and Liddle).
If the random fluctuations along $\delta \phi_a$ have an amplitude $(H/2\pi)^2$, one obtains:

$$\delta^2_k = \frac{4}{25} P_\zeta = \frac{V}{15\pi^2 M^2_p} \frac{\partial N}{\partial \phi} \frac{\partial N}{\partial \phi_a}. \quad (21)$$

For a single component $\partial N/\partial \phi = (M_p^{-2} V V')$, and then Eq. (21) reduces to Eq. (19). By using slow-roll equations we can again define the spectral index

$$n - 1 = - \frac{M^2_p V a N_a}{V^2} - \frac{2}{M^2_p N_a N_a} + 2 \frac{M^2_p N_a N_b V_{ab}}{V N_c N_c}, \quad (22)$$

where $V_a \equiv \partial V/\partial \phi_a$, and similarly $N_a \equiv \partial N/\partial \phi_a$. For a single component we recover Eq. (17) from Eq. (22). In the case of multi-fields, one has to distinguish adiabatic from isocurvature perturbations. Present CMB data rules out pure isocurvature perturbations. For ST inflation stochastic gravitational waves are expected to be produced similar to the scalar perturbations during inflation. Therefore it is usually assumed that their contribution to the CMB anisotropy is small. The corresponding spectral index can be expanded in terms of the slow-roll parameters at first order as

$$r \equiv \frac{P_{\text{grav}}}{P_\zeta} = 16\epsilon, \quad n_t = \frac{d \ln P_{\text{grav}}(k)}{d \ln k} \simeq -2\epsilon, . \quad (26)$$

Note that the tensor spectral index is negative. It is expected that PLANCK could detect gravity waves if $r \gtrsim 0.1$, however the spectral index will be hard to measure in forthcoming experiments. The primordial gravity waves can be generated for large field value inflationary models. Using the definition of the number of $e$-folding it is possible to derive the range of $\Delta \phi$ (Hotchkiss et al. 2008; Lyth 1997; Lyth and Liddle, 2009)

$$16\epsilon = r < 0.005 (50/N)^2 (\Delta \phi/M_p). \quad (27)$$

Note that it is possible to get sizable, $r$, for $\Delta \phi \ll M_p$ in assisted inflation (discussed below), and in inflection point inflation discussed in Ref. Ben-Dayan and Brustein, 2010. If the tensor-to-scalar ratio $r$ and/or a running $\alpha_s$ are introduced, the best fit for $n_s$ and error bars (at 1$\sigma$) $n_s = 1.017^{+0.042}_{-0.043}$, $\alpha_s = -0.028 \pm 0.020$ $n_s = 0.970 \pm 0.015$, $r < 0.22$ (at 2$\sigma$), $n_s = 1.089^{+0.070}_{-0.069}$, $r < 0.55$ (at 2$\sigma$), $\alpha_s = -0.053 \pm 0.028$ (Komatsu et al. 2009). These data therefore suggest that a red spectrum is favored ($n_s = 1$ excluded at $2.5\sigma$ from WMAP and at $3.1\sigma$ when other data sets are included) if there is no running.

### 3. Gravitational waves

During inflation stochastic gravitational waves are expected to be produced similar to the scalar perturbations (Allen, 1988; Grishchuk, 1975; Grishchuk and Sidorov, 1989; Sahni, 1990). For reviews on gravitational waves, see (Maggiore, 2000; Mukhanov et al., 1992). The gravitational wave perturbations are described by a line element $ds^2 + d\delta s^2$, where

$$ds^2 = a^2(\tau)(d\tau^2 - dx^i dx_i), \quad d\delta s^2 = -a^2(\tau) h_{ij} dx^i dx^j. \quad (23)$$

The gauge invariant and conformally invariant 3-tensor $h_{ij}$ is symmetric, traceless $\delta^i h_{ij} = 0$, and divergenceless $\nabla_i h_{ij} = 0$ ($\nabla_i$ is a covariant derivative). Massless spin 2 gravitons have two transverse degrees of freedom (d.o.f).

For the Einstein gravity, the gravitational wave equation of motion follows that of a massless Klein Gordon equation (Mukhanov et al., 1992). Especially, for a flat universe

$$\ddot{h}_{ij} + 3H \dot{h}_{ij} + (k^2/a^2) h_{ij} = 0, \quad (24)$$

As any massless field, the gravitational waves also feel the quantum fluctuations in an expanding background. The spectrum mimics that of Eq. (9)

$$P_{\text{grav}}(k) = \frac{2}{M_p^2} \left( \frac{H}{2\pi} \right)^2 \left|_{k=aH} \right. . \quad (25)$$

Note that the spectrum has a Planck mass suppression, which suggests that the amplitude of the gravitational waves is smaller compared to that of the scalar perturbations. Therefore it is usually assumed that their contribution to the CMB anisotropy is small. The corresponding spectral index can be expanded in terms of the slow-roll parameters at first order as

$$r \equiv \frac{P_{\text{grav}}}{P_\zeta} = 16\epsilon, \quad n_t = \frac{d \ln P_{\text{grav}}(k)}{d \ln k} \simeq -2\epsilon, . \quad (26)$$

### C. Generic models of inflation

#### 1. High scale models of inflation

The most general form for the potential of a gauge singlet scalar field $\phi$ contains an infinite number of terms, $V = V_0 + \sum_{\alpha=2}^{\infty} \frac{\lambda_\alpha}{M_p^{2\alpha}} \phi^\alpha$. (28)

The renormalizable terms allows to prevent all terms with $\alpha \geq 4$. By imposing the parity $Z_2$, under which $\phi \to -\phi$, allows to prevent all terms with $\alpha$ odd. Most phenomenological models of inflation proposed initially assume that one or two terms in Eq. (28) dominate over the others, though some do contain an infinite number of terms.

**a. Power-law chaotic inflation:** The simplest inflation model by the number of free parameters is perhaps the chaotic inflation (Linde, 1983) with the potential dominated by only one of the terms in the above series

$$V = \frac{\lambda_\alpha}{M_p^{2\alpha-2}} \phi^\alpha, \quad (29)$$

with $\alpha$ a positive integer. The first two slow-roll parameters are given by

$$\epsilon = \frac{\alpha^2 M_p^2}{2 \phi^2}, \quad \eta = \frac{\alpha(\alpha - 1) M_p^2}{\phi^2}. \quad (30)$$
Inflation ends when $\epsilon = 1$, reached for $\phi_s = \alpha M_p/\sqrt{2}$. The largest cosmological scale becomes super-Hubble when $\phi_Q = \sqrt{2N_Q c M_p}$, which is super Planckian; this is the first challenge for this class of models. The spectral index for the scalar and tensor to scalar ratio read:

$$n_s = 1 - \frac{2 + \alpha}{2N_Q + \alpha/2}, \quad r = \frac{4 \alpha}{N_Q + \alpha/4}.$$  \hspace{1cm} (31)

The amplitude of the density perturbations, if normalized at the COBE scale, yields to extremely small coupling constants: $\lambda_s \ll 1$ (for i.e. $\lambda_4 \approx 3.7 \times 10^{-14}$). The smallness of the coupling, $\lambda_s/M_P^{\alpha-4}$, is often considered as an unnatural fine-tuning. Even when dimension full, for example if $\alpha = 2$, the generation (and the stability) of a mass scale, $\sqrt{2}N_Q M_p \approx 10^{13}$ GeV, is a challenge in theories beyond the SM, as they require unnatural cancellations. These class of models have an interesting behavior for initial conditions with a large phase space distribution where there exists a late attractor trajectory leading to an end of inflation when the slow-roll conditions are violated close to the Planck scale (Brandenberger and Kung, 1990; Kofman et al., 2002; Lindell, 1983, 1985).

Note that the above mentioned monomial potential can be a good approximation to describe in a certain field range for various models of inflation proposed and motivated from particle physics; natural inflation when the inflaton is a pseudo-Goldstone boson (Freese et al., 1990), or the Landau-Ginzburg potential when the inflaton is a Higgs-type field (Bezrukov and Shaposhnikov, 2008). The necessity of super Planckian VEVs represents a challenge to such embedding in particle physics and supergravity (SUGRA).

b. Exponential potential: An exponential potential also belongs to the large field models:

$$V(\phi) = V_0 \exp \left(-\frac{\sqrt{2} \phi}{p M_p}\right).$$ \hspace{1cm} (32)

It would give rise to a power law expansion $a(t) \propto t^p$, so that inflation occurs when $p > 1$. The case $p = 2$ corresponds to the exact de Sitter evolution and a never ending accelerated expansion. Even for $p \neq 2$, violation of slow-roll never takes place, since $\epsilon(\phi) = 1/p$ and inflation has to be ended by a phase transition or gravitational production of particles (Copeland et al., 2001; Lyth and Riotto, 1999).

The confrontation to the CMB data yields: $n_s = 1 - 2/p$ and $r = 16/p$; the model predicts a hight tensor to scalar ratio and it is within the one sigma contour-plot of WMAP (with non-negligible $r$) for $p \in [73 - 133]$.

2. Assisted inflation

Many heavy fields could collectively assist inflation by increasing the effective Hubble friction term for all the individual fields (Liddle et al., 1998). This idea can be illustrated with the help of ‘$m$’ identical scalar fields with an exponential potentials, see Eq. (32), where now $\phi \rightarrow \phi_i$, where $i = 1, 2, \ldots, m$. For a particular solution; when all the scalar fields are equal: $\phi_1 = \phi_2 = \cdots = \phi_m$.

$$H^2 = \frac{1}{3M_p^2} m[V(\phi_1) + \phi_1^2/2],$$ \hspace{1cm} (33)

$$\ddot{\phi}_1 = -3H\dot{\phi}_1 - dV(\phi_1)/d\phi_1.$$ \hspace{1cm} (34)

These can be mapped to the equations of a model with a single scalar field $\phi$ by the redefinitions $\tilde{\phi}_i^2 = m \phi_i^2$; $\tilde{V} = M_\phi V$ ; $\tilde{\rho} = m \rho$, so the expansion rate is $a \propto t^{\tilde{p}}$, provided that $\tilde{p} > 1/3$. The expansion becomes quicker the more scalar fields there are. In particular, potentials with $p < 1$, which for a single field are unable to support inflation, can do so as long as there are enough scalar fields to make $mp > 1$.

In order to calculate the density perturbation produced in multi-scalar field models, we recall the results from Eq. (21). Since $N = -\int H dt$, we have $\sum_i \frac{\partial^2}{\partial \phi_i^2} \phi_i = -H$, we yield: $P_\zeta = (H/2\pi)^2(1/m)(H^2/2\tilde{p})^2$.

Note that this last expression only contains one of the scalar fields, chosen arbitrarily to be $\phi_1$. The estimation for the spectral tilt is given by: $n - 1 = -2/mp$, which matches that produced by a single scalar field with $\tilde{p} = mp$. The more scalar fields there are, the closer to scale-invariance is the spectrum that they produce. The above calculation can be repeated for arbitrary slopes, $p_i$. In which case the spectral tilt would have been given by $n = 1 - 2/p$, where $p = \sum p_i$. The above scenario has been generalized to study arbitrary exponential potentials with couplings, $V = \sum_i z_i \exp(\sum_i \alpha_i \phi_j)$ (Copeland et al., 1999; Green and Lidsey, 2000).

a. Assisted chaotic inflation: Multi-scalar fields of chaotic type has interesting properties (Jokinen and Mazumdar, 2004):

$$V \sim \sum_i f(\phi_i^n/M_p^{n-4})$$ \hspace{1cm} (35)

(for $n \geq 4$). The chaotic inflation can now be driven at VEVs, $\phi_i \ll M_p$, below the Planck scale (Kanti and Olive, 1999). The effective slow-roll parameters are given by: $\epsilon_{eff} = \epsilon/m \ll 1$ and $|\eta_{eff}| = |\eta|/m \ll 1$, where $\epsilon, \eta$ are the slow-roll parameters for the individual fields. Inflation can now occur for field VEVs (Jokinen and Mazumdar, 2004):

$$\frac{\Delta \phi}{M_p} \sim \left(\frac{600}{m}\right) \left(\frac{N_Q}{60}\right) \left(\frac{\epsilon_{eff}}{2}\right)^{1/2} \ll 1,$$ \hspace{1cm} (36)

where $N_Q$ is the number of e-foldings. Obviously, all the properties of chaotic inflation can be preserved at VEVs $\ll M_p$, including the prediction for the tensor to scalar ration for the stochastic gravity waves, i.e. $r = 16\epsilon_{eff}$. For $\epsilon_{eff} \sim 0.01$ and $m \sim 100$, it is possible to realize a sub-Planckian inflation, the spectral tilt close to the flatness: $n_s - 1 = -6\epsilon_{eff} + 2\eta_{eff}$, and large tensor to scalar ratio, i.e. $r \sim 0.16$. 
b. N-flation: Amongst various realizations of assisted inflation, N-flation is perhaps the most interesting one. The idea is to have $N \sim 300 (M_P/f) \sim 10^4$ number of axions, where $f$ is the axion decay constant, of order $f \sim 0.1 M_P^{-1}$ drive inflation simultaneously with a leading order potential (Dimopoulos et al., 2005):

$$V = V_0 + \sum_i \Lambda_i^4 \cos(\phi_i/f_i) + ... \tag{37}$$

where $\phi_i$ are axion fields correspond to the partners of Kähler moduli. The ellipses contain higher order contributions. In a certain Type-IIB compactification, it is assumed that all the moduli are heavy and thus stabilized by prior dynamics, including that of the volume modulus. Only the axes of $T_i = \phi_i/f_i + i M_P^2 R_i^2$ are light (Dimopoulos et al., 2005). The assumption of decoupling the dynamics of Kähler moduli from the axions is still a debatable issue, see (Kallosh, 2008). After rearranging the potential for the axions, and expanding them around their minima for a canonical choice of the kinetic terms, the Lagrangian simplifies to the lowest order in expansion:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi_i \partial^\mu \phi_j - \frac{1}{2} \sum_i m_i^2 \phi_i^2 + \cdots. \tag{38}$$

The exact calculation of $m_i$ is hard, assuming all of the mass terms to be the same $m_i \sim 10^{13}$ GeV, and $N \gg (M_P/f)^2$, then it is possible to match the current observations with a tilt in the spectrum, $n \sim 0.97$, and large tensor to scalar ratio: $r \sim 8/N_Q \sim 0.13$ for $N_Q \approx 60$. There are also realizations of assisted inflation via branes (Becker et al., 2003; Cline and Stoica, 2005; Mazumdar et al., 2001).

3. Hybrid inflation

The end of inflation can happen via a waterfall triggered by a Higgs (not necessarily the SM Higgs) field coupled to the inflaton, first discussed in (Copeland et al., 1994; Lindesay, 1991, 1994). The model is based on the potential given by (Linde, 1991, 1994)

$$V(\phi, \psi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \left( \psi^2 - M^2 \right)^2 + \frac{\lambda'}{2} \phi^2 \psi^2, \tag{39}$$

where $\phi$ is the inflaton and $\psi$ is the Higgs-type field. $\lambda$ and $\lambda'$ are two positive coupling constants, $m$ and $M$ are two mass parameters. It is the most general form (omitting a quartic term $\lambda'' \phi^4$) of renormalizable potential satisfying the symmetries: $\psi \leftrightarrow -\psi$ and $\phi \leftrightarrow -\phi$. Inflation takes place along the $\psi = 0$ valley and ends with a tachyonic instability for the Higgs-type field. The critical point of instability occurs at:

$$\phi_c = M \sqrt{\lambda/\lambda'} . \tag{40}$$

The system then evolves toward its true minimum at $V = 0$, $\langle \phi \rangle = 0$, and $\langle \psi \rangle = \pm M$.

The inflationary valley, for $\langle \psi \rangle = 0$, where the last 50–60 e-foldings of inflation is assumed to take. This raises the issue of initial conditions for $\langle \phi, \psi \rangle$ fields and the fine tuning required to initiate inflation (Clesse and Rocher, 2008; Lazarides and Vlachos, 1997; Meinedos and Liddle, 2000; Panagiotakopoulos and Tetradis, 1999; Tetradis, 1998). In Ref. (Clesse and Rocher, 2008) it was found that when the initial VEV of the inflaton, $\phi \ll M_P$, a subdominant but non-negligible part of the initial conditions for the phase space leads to a successful inflation, i.e. around less than 15% depending on the model parameters. Initial conditions with super-Planckian VEVs for $\phi \gg M_P$ automatically leads to a successful inflation similarly to chaotic inflation. In the inflationary valley, $\langle \psi \rangle = 0$, the effective potential is given by:

$$V_{eff}(\phi) \simeq \frac{\lambda M^4}{4} + \frac{1}{2} m^2 \phi^2, \tag{41}$$

The model predicts a blue tilt in the spectrum, i.e. $n_s > 1$, in the small field regime, $\phi_Q < M_P$, which is slightly disfavored by the current data.

Two variations of the hybrid inflation idea were proposed assuming that the term $\phi^2$ is negligible. The two-field scalar potentials are of the form:

$$V_{pq}(\phi, \psi) = M^4 \left[ 1 - \left( \frac{\phi}{\phi} \right)^p \right]^2 + \lambda \phi^2 \psi^q. \tag{42}$$

They share the common feature of having an inflationary trajectory during which $\langle \psi \rangle$ is varying and not vanishing. For $(p, q) = (1, 2)$, the model is known as Mutated hybrid (Stewart, 1995b), and $(p, q) = (4, 6)$ corresponds to Smooth hybrid inflation (Lazarides and Panagiotakopoulos, 1995). The latter involves non-renormalizable terms of order $M_P^{-2}$ to keep the potential bounded from below. The potential is valid in the large field limit $\phi \gg \phi_s$, since in the small field limit, the potential is not bounded from below and should be completed. For mutated, the model predicts a red spectral index and negligible tensor to scalar ratio, $n_s - 1 \approx -3/(8N_Q) \approx 0.97$, and $r \approx 3m/(2\Lambda N_Q^{3/2}) \ll 3/(8N_Q^2) \sim 10^{-4}$, if we assume $N_Q \approx 60$. For smooth, the end of slow-roll inflation happens by a violation of the conditions; $\epsilon, \eta \ll 1$, since no waterfall transition takes place. This allows the predictions for the spectral index to be $n_s - 1 \approx -5/(3N_Q) \approx 0.97$ (Lazarides and Panagiotakopoulos, 1995), and the ratio for tensor to scalar is found to be negligible.

4. Inflection point inflation

One of the challenges for inflation is to realize inflation at low scales, preferably below $M_P$, with the right tilt and the amplitude of the power spectrum. Inflection point inflation admits a large amount of flexibility in the field space – similar to the analogy of a ball rolling on an elastic surface following the least action principle. With
the help of two independent parameters, $A$ and $B$, it is possible to obtain a large range of tilt in the spectrum, while keeping the amplitude of the perturbations intact. Let us consider a simple realization of such a potential:

$$V(\phi) = A\phi^2 - C\phi^3 + B\phi^4,$$

(43)

where $C = f(A, B)$ in order to obtain a point of inflection suitable for inflation. The VEV at which inflation occurs is intimately related to the two independent parameters and can happen at wide ranging scales below $M_P$, and for wide ranging values of $(A, B)$.

Here we will generalize this potential to any generic potential $V$ which can be written in the following form (here $t$ denotes differentiation with respect to $\phi$) ([Enqvist et al. 2010a; Hotchkiss et al. 2011]):

$$V = V_0 + a(\phi - \phi_0) + \frac{b}{2}(\phi - \phi_0)^2 + \frac{c}{6}(\phi - \phi_0)^3 + \cdots (44)$$

where $V_0 \equiv V(\phi_0)$, $a \equiv V'(\phi_0)$, $b \equiv V''(\phi_0)$, $c \equiv V'''(\phi_0)$, which is the Taylor expansion, truncated at $n = 3$, around a reference point $\phi_0$, which we choose to be the point of inflection where $V''(\phi_0) = 0$, or saddle point where $V'''(\phi_0) = V'(\phi_0) = 0$. The higher order terms in Eq. (44) can be neglected during inflation, provided that

$$|V_{0''''}| \gg \left| \frac{d^n V}{d\phi^n}(\phi_0) \right| |\phi_c - \phi_0|^{m-3}, \quad m \geq 4,$$

(45)

where $\phi_c$ corresponds to the field value at the end of inflation. Assuming that the slow-roll parameters are small in the vicinity of the inflection point $\phi_0$, and that the velocity $\dot{\phi}$ is negligible, the potential energy $V_0$ gives rise to a period of inflation.

Inflation ends at the point $\phi_e$ where $|\eta| \sim 1$. By solving the equation of motion, the number of e-folds of inflation during the slow-roll motion of the inflaton from $\phi$ to $\phi_e$, where $\phi_0 - (\phi_0 - \phi_e) < \phi < \phi_0 + (\phi_0 - \phi_e)$, is found to be ([Enqvist et al. 2010a])

$$N = \frac{V_0}{M_P^2} \sqrt{\frac{2}{ac}} \left[ F_0(\phi_e) - F_0(\phi) \right],$$

$$F_0(z) = \text{arccot} \left( \sqrt{\frac{c}{2a}} (z - \phi_0) \right).$$

(46)

It useful to define the parameters $X = \frac{aM_P}{\sqrt{2V_0}}$ and $Y = \sqrt{2NMPX}$. Note that $X$ is the square root of the slow-roll parameter $\epsilon$ at the point of inflection. The slow-roll parameters can then be recast in the following form:

$$\epsilon = \frac{2Y^2}{e^2M_P^2N^2} \left( \frac{Y}{S} \right)^4,$$

(47)

$$\eta = \frac{2}{N} \frac{Y}{S} \left( \sqrt{1 - X \cos Y - \sqrt{X} \sin Y} \right),$$

(48)

$$\xi^2 = \frac{2}{N^2} \left( \frac{Y}{S} \right)^2,$$

(49)

where $S = \sqrt{1 - X \sin Y + \sqrt{X} \cos Y}$. One can solve Eqs. (47-49), for $X$, $Y$ and $N$ in terms of the slow-roll parameters. The equations are non-linear and in general cannot be solved analytically. However, since $\epsilon \ll |\eta|, \xi$, one can find a closed form solution provided that $V_{01/4} \leq 10^{16}$ GeV and $X \leq \sqrt{\epsilon} \ll 1$ ([Allahverdi et al. 2007c; Bueno Sanchez et al. 2007; Enqvist et al. 2010a; Hotchkiss et al. 2011]):

$$P_{\zeta}^{1/2} = \frac{1}{2\sqrt{24\pi^{3/2}e\epsilon^{1/2}M_P^2}} \frac{V_0^{1/2}}{\pi \sqrt{6M_P^2X}} \sin^2 Y, \quad (50)$$

$$n_s = 1 + 2\eta - 6\epsilon = 1 - \frac{4}{NQ} Y \cot Y, \quad (51)$$

$$\alpha = -\frac{4}{N_Q^2} \left( \frac{Y}{\sin Y} \right)^2.$$  

(52)

One can derive the properties of a saddle point inflation provided $Y/\sin Y \to 1$, and $Y \cot Y \to 1$. The model favors the current observations by matching the COBE normalization and the spectral tilt ranging from $n_s \in [0.93, 1.0]$. For instance, the lowest value corresponds to the saddle point inflation for $N_Q \sim 60$.

D. Supersymmetric models

One of the most compelling virtues of SUSY is that it can protect the quadratically divergent contributions to the scalar mass, which arise in one-loop computation from the fermion contribution and quartic self interaction of the scalar field. Such corrections generically spoil the flatness of the inflaton potential. The quadratic divergence is independent of the mass of the scalar field and cancel, exactly if $\lambda_s = \lambda_f^2$, where $\lambda_f$ is the fermion Yukawa and $\lambda_s$ is the quartic scalar coupling. However this procedure fails at 2-loops and one requires fine tuning of the couplings order by order in perturbation theory. In the case of the SM Higgs, a precision of roughly one part in $10^{17}$ is required in couplings to maintain the Higgs potential, often known as the hierarchy problem or the naturalness problem. The electroweak symmetry is still broken by the Higgs mechanism, but the quadratic divergences in the scalar sector are absent. In the SUSY limit the fermion and scalar masses are degenerate, but the SUSY has to be broken softly at the TeV scale in such a way that it does not spoil the solution to the hierarchy problem, see [Chung et al. 2003; Haber and Kane, 1983; Martin, 1997; Nilles, 1984].

The matter fields for $N = 1$ SUSY are chiral superfields $\Phi = \phi + \sqrt{2}\theta \psi + \theta \theta F$, which describe a scalar $\phi$, a fermion $\psi$, and a scalar auxiliary field $F$. The SUSY scalar potential $V$ is the sum of the $F$- and $D$-terms:

$$V = \sum_i |F_i|^2 + \frac{1}{2} \sum_a g^2_a D^a D^a,$$

$$F_i \equiv \frac{\partial W}{\partial \phi_i}, \quad D^a = \phi^i T^a \phi,$$

(53)
where $W$ is the superpotential, and $\phi_i$ transforms under a gauge group $G$ with the generators of the Lie algebra given by $T^a$. Note that all the kinetic energy terms are included in the $D$-terms. For inflation, the effects of supergravity (SUGRA) becomes important. At tree level, $N = 1$ SUGRA potential is given by the sum of $F$ and $D$-terms, see [Nilles, 1984]

$$
V = e^{\frac{\kappa(\phi_i^*)}{M_P^2}} \left[\frac{(K^{-1})_{ij} F_i F_j - \frac{3}{2}|W|^2}{M_P^2}\right] + \frac{g^2}{2} \text{Re} f_{ab}^{-1} \hat{D}^a \hat{D}^b,
$$

(54)

$$
F^i = W^i + K^i \frac{W}{M_P^2}, \quad \hat{D}^a = -K^a(T^a)^{ij} \phi_j + \xi^a.
$$

(55)

where we have added the Fayet-Iliopoulos contribution $\xi^a$ to the $D$-term, and $D_a = D^a/g_a$, where $g_a$ is gauge coupling. Here $K(\phi_i, \phi^*)$ is the Kahler potential, which is a function of the fields $\phi_i$, and $K_{ij} = \partial K/\partial \phi_i \partial \phi_j$. In the simplest case, at tree-level $K = \phi_i \phi^* \phi_i$ (and $K_{ij} = (K^{-1})_{ij} = \delta_{ij}$). In general the Kahler potential can be expanded as: $K = \phi_i \phi^* \phi_i + (k_{ij} \phi_i \phi_j + c.c.)/M_P^2 + (k_{ij} \phi_i \phi_j + \phi_i \phi^* \phi_j + c.c.)/M_P^2 + \cdots$.

The kinetic terms for the scalars take the form:

$$
\frac{\partial^2 K}{\partial \phi_i \partial \phi_j} D_{\mu} \phi_i D^\mu \phi_j.
$$

(56)

The real part of the gauge kinetic function matrix is given by $\text{Re} f_{ab}$. In general, $f_{ab} = \delta_{ab}(1/g_a^2 + f_a^2 \phi_i / M_P + \cdots)$. The gauginos masses are typically given by $m_{\chi} = \text{Re} f_{(a)} (F_1)/2M_P$. For a universal gaugino masses, $f_a$ are the same for all the three gauge groups of MSSM. In the simplest case, it is just a constant, $f_{ab} = \delta_{ab}/g_a^2$, and the kinetic terms for the gauge potentials, $\lambda_{\mu}$, are given by:

$$
\frac{1}{4} \text{Re} f_{ab} F^a_{\mu\nu} F^a_{\mu\nu}.
$$

(57)

SUGRA will be broken if one or more of the $F_i$ obtain a VEV. The gravitino, spin $\pm 3/2$ component of the graviton, then absorb the Goldstino component to become massive. Requiring classically $\langle V \rangle = 0$, as a constraint to obtain the zero cosmological constant, one obtains

$$
m_{3/2}^2 = \frac{(K_{ij} F_i F_j)}{3M_P^2} e^{(K)/M_P^2} |(W)|^2/M_P^4.
$$

(58)

1. F-term inflation

The most well-known model of SUSY inflation driven by $F$-terms is of the hybrid type and based on the superpotential [Copeland et al., 1994; Dvali et al., 1994; Linde and Riotto, 1997]

$$
W = \kappa S(\Phi \bar{\Phi} - M^2).
$$

(59)

where, $S$ is an absolute gauge singlet, while $\Phi$ and $\bar{\Phi}$ are two distinct superfields belonging to complex conjugate representation, and $\kappa$ is an arbitrary constant fixed by the CMB observations. It is desirable to obtain an effective singlet $S$ superfield arising from a higher gauge theory such as GUT [Langacker, 1981]. However, to our knowledge it has not been possible to implement this idea, see the discussion in [Mazumdar and Rocher, 2011]. Typically $S$ would have other (self)couplings which would effectively ruin the flatness required for hybrid inflation.

This form of potential is protected from additional destabilizing contributions with higher power of $S$, if $S$, $\Phi$ and $\bar{\Phi}$ carrying respectively the charges $+2$, $\alpha$ and $-\alpha$ under R-parity. Then $W$ carries a charge $+2$ so that the action $S = \int d^4 \theta W + \cdots$ is invariant.

The tree level scalar potential derived from Eq. (59) reads

$$
V_{\text{tree}}(S, \phi, \bar{\phi}) = \kappa^2|S|^2 - \bar{\phi} \phi + \kappa^2|\phi|^2 + |\bar{\phi}|^2,
$$

(60)

where we have denoted by $S, \phi, \bar{\phi}$ the scalar components of $S, \Phi, \bar{\Phi}$. Note the similarity between Eq. (39) and Eq. (60), where $m = 0$, and both $\lambda$ and $\lambda'$ are replaced by only $\kappa^2$. We will also assume $\phi^* = \bar{\phi}$ along this direction, and the kinetic terms for the superfields are minimal, i.e. with a k"ahler potential: $K = |S|^2 + |\bar{\phi}|^2 + |\Phi|^2$.

Let us define two effective real scalar fields canonically normalized, $\sigma \equiv \sqrt{2} \text{Re}(S)$, and $\psi \equiv 2 \text{Re}(\Phi) = 2 \text{Re}(\bar{\Phi})$, the overall potential can then be recast as:

$$
V_{\text{tree}}(\sigma, \psi) = \kappa^2\left(2M^2 - \frac{\psi^2}{4}\right) + \frac{\kappa^2}{4} \sigma^2 \psi^2.
$$

(61)

The global minimum of the potential is located at $S = 0$, $\phi^* = M^2$. At large VEVs, $S > S_c \equiv M$, the potential also possesses a local valley of minima (at $\langle \psi \rangle = 0$) in which the field $\sigma$, now rolls on with $V_{\text{tree}} = V_0 \equiv \kappa^2 M^4$. This non-vanishing value of the potential both sustain the inflationary dynamics and induces a SUSY breaking.

This induces a splitting in the mass of the fermionic and bosonic components of the superfields $\Phi$ and $\bar{\Phi}$, with $m_\Phi^2(S) = \kappa^2|S|^2 \pm \kappa^2 M^2$ and $m_{\bar{\Phi}}^2 = \kappa^2|S|^2$. Note that this description is valid only as long as $S$ is sufficiently slow-rolling such that $\kappa^2|S|^2|\Phi|^2$ can be considered as a mass term. Therefore radiative corrections do not exactly cancel out [Dvali et al., 1994; Lazarides, 2000], and provide a one-loop potential which can be calculated by using the Coleman-Weinberg formula [Coleman and Weinberg, 1973],

$$
V_{1-\text{loop}}(\phi) = V_{\text{inf}}(\phi) + \Delta V
$$

$$
\Delta V = \frac{1}{64 \pi^2} \sum_i (-)^{F_i} M_i(\phi)^4 \ln \frac{M_i(\phi)^2}{\Lambda(\phi)^2},
$$

(62)

where $V_{\text{inf}}$ is now the renormalized potential, $\Lambda(\phi)$ is the renormalization mass scale. The sum extends over all helicity states $i$, $F_i$ is the fermion number, and $M(\phi)$ is the mass of the $i$-th state. One obtains:

$$
V_{1-\text{loop}}(S) = \frac{\kappa^4 N M_A^4}{32 \pi^2} \left[2 \ln \frac{s^2 \kappa^2}{\Lambda^2} + (z + 1)^2 \ln(1 + z^{-1}) + (z - 1)^2 \ln(1 - z^{-1})\right],
$$

(63)
where \( z = |S|^2/M \equiv x^2 \), \( \Lambda \) represents a non-physical energy scale of renormalization and \( \mathcal{N} \) denotes the dimensionality. Note that the perturbative approach of Coleman and Weinberg breaks down when close to the inflection point at \( z \approx 1 \). For small coupling \( \kappa \), the slow-roll conditions (for \( \eta \)) are violated infinitely close to the critical point, \( z = 1 \), which ends inflation.

The normalization to COBE allows to fix the scale \( M \) as a function of \( \kappa \). If the breaking of \( G \) does not produce cosmic strings, the contribution to the quadrupole anisotropy simply comes from the inflationary contribution (see Eq. (13)) and the observed value can be obtained even with a coupling \( \kappa \) close to unity (Dvali et al. 1994). Small values of \( \kappa \) can render the scale of inflation very low, as low as the TeV scale (Bastero-Gil et al., 2003; Bastero-Gil and King, 1998; Randall et al., 1996; Randall and Thomas, 1995).

However it has been shown that the formation of cosmic strings at the end of \( F \)-term inflation is highly probable when the model is embedded in SUSY GUTs (Jeannerot et al. 2003). In this case, the normalization to COBE receives two contributions, one from inflation and other from cosmic strings (Jeannerot, 1997; Rocher and Sakellariadou, 2005a), which affects the relation \( M(\kappa) \) at large \( \kappa \), and imposes new stringent bounds on \( M \lesssim 2 \times 10^{15} \text{GeV} \), and (Jeannerot and Postma, 2005; Rocher and Sakellariadou, 2005b)

\[
\kappa \lesssim 7 \times 10^{-7}(126/N_Q),
\]

by demanding that the cosmic strings came at best contribute less than \( \lesssim 10\% \) of isocurvature fluctuations (Bevis et al., 2008). Once \( M \) is fixed, the spectral index \( n_s \) can be computed as the range is found to be: \( n_s \in [0.98,1] \) whether cosmic strings form or not (Jeannerot and Postma, 2005; Senoguz and Shaﬁ, 2003), and by including the soft-SUSY breaking terms within minimal kinetic terms in the Kähler potential, the spectral index can be brought down to \( 0.928 \leq n_s \leq 1.008 \) (Rehman et al. 2004).

2. SUGRA corrections and solutions

For inflaton VEVs below the Planck scale, the SUGRA effects can become important and may ruin the ﬂatness of the potential. The \( N = 1 \) SUGRA potential is now given by Eq. (54), where the \( F \)-terms containing an additional exponential factor. Various cross terms between the Kähler and the superpotential leads to the soft breaking mass term for the light scalar fields (Bertolami and Ross, 1987; Copeland et al., 1994; Dine et al., 1984; 1995b; 1996b; Linde and Riotto, 1997)

\[
m_{\text{SUGRA}}^2 \sim m_{\text{susy}}^2 + \frac{V}{3M_P^2} \sim \mathcal{O}(1)H^2,
\]

where \( m_{\text{susy}} \sim \mathcal{O}(100) \text{GeV} \) contains soft-SUSY breaking mass term for the low scale SUSY breaking scenarios. Once the inflaton gets a mass \( \sim H \), the contribution to the second slow-roll parameter \( \eta \) becomes order unity and the slow roll inflation ends, i.e. \( \eta \equiv \frac{M_P^2V''}{V} \sim m_{\text{SUGRA}}^2H^2 \sim \mathcal{O}(1) \). This is known as the SUGRA-\( \eta \) problem.

When there are more than one chiral superfields, as in the \( F \)-term hybrid model, it can be possible to cancel the dominant \( \mathcal{O}(1)H \) correction to the inflaton mass by choosing an appropriate Kähler term (Copeland et al., 1994; Steward, 1995a). For non-minimal Kähler potentials, such as

\[
K = |S|^2 + |\Phi|^2 + \Phi^4 + \kappa S|S|^4/M_P^2 + \ldots,
\]

the kinetic terms \( K^{ij} \partial_i \Phi_j \partial_i \Phi_i \) are non-minimal because \( K^{ij} \neq \delta^{ij} \). One obtains: \( \langle \partial_S \cdot K \rangle^{-1} \sim 1 - 4\kappa_S|S|^2/M_P^2 + \ldots \) One again obtains a problematic contribution to the inflaton mass, i.e. \( \kappa_S \times \mathcal{O}(1)H \). Several mechanisms have been proposed to tackle this \( \eta \)-problem. One can impose, \( \kappa_S \sim 10^{-4} \), which is sufficient to keep the slow roll inflation safe, but without much physical motivation. For a generic inflationary model it is not possible to compute \( \kappa_S \) at all.

- a. Shift and Heisenberg symmetry: Safe non-minimal Kähler potentials have also been proposed (Antusch et al., 2009; Bastero-Gil and King, 1999; Brax and Martin, 2005; Pallas, 2009) making use of the shift symmetry.

- b. Inflection point inflation: For any smooth potential, it is possible to drive inflation near the saddle point, \( V' = 0, V'' = 0, V''' \neq 0 \), or near the point of inflection, \( V' \neq 0, V'' = 0, V''' \neq 0 \). These are special points where the effective mass term of the inflaton vanishes and the potential does not suffer through SUGRA-\( \eta \) problem (Allahverdi et al., 2007a, 2006; Mazumdar et al., 2011). In the saddle point case
the potential can be made so flat that inflation can be driven eternally (Allahverdi et al. 2006, 2007d).

From the low-energy perspective, the most generic and dangerous SUGRA corrections to the inflaton potential (with minimal and non-minimal Kähler potentials for \( \phi \)) would have a large vacuum energy contribution. To complicate further, one may even assume that the flatness of \( \phi \) is lifted by non-renormalizable contribution to the potential (Mazumdar et al. 2011):

\[
V(\phi) = V_c + c_H H^2 \frac{n}{2} |\phi|^2 - \frac{a_H H}{n M_P^2} \phi^n \left( \frac{\phi^{2(n-1)}}{M_P^{2(n-3)}} \right),
\]

where \( V_c = 3H^2M_P^2 \). As mentioned above the interesting observation is that, in fact, there always exists a range of field values, \( \Delta \phi \), for which a full potential admits a point of inflection with all known sources of corrections taken into account. Now, all the uncertainties in the corrections to the Kähler potential can be absorbed in the full potential, such that the flat region admits a slow roll inflation with \( \Delta \phi \gg \phi_0 \) (Mazumdar et al. 2011). The condition for this inflection point is \( \alpha_H \approx 8(n-1)c_H \), where we characterize the fine-tuning by \( \beta \) defined as:

\[
\frac{\alpha_H}{8(n-1)c_H} = 1 - \frac{(n-2)^2}{4} \beta^2.
\]

When \( |\beta| \) is small, a point of inflection \( \phi_0 \) exists such that \( V''(\phi_0) = 0 \), with

\[
\phi_0 = \left( \sqrt{\frac{c_H}{2(n-1)} H M_P^{n-3}} \right)^{\frac{1}{n-1}}.
\]

We can Taylor expand the potential about \( \phi_0 \) as discussed in section II.C.3 and analyze the CMB constraints as shown in Fig. 2.

3. D-term inflation

In Refs. (Binétruy and Dvali, 1996; Halvach, 1996; Stewart, 1995a), it was noticed that a perfectly flat inflaton potentials can be constructed using a constant contribution coming from the D-term. In addition, the SUGRA-\( \eta \) problem arising in F-term models does not appear for D-terms driven inflation because the D-sector of the potential does not receive exponential contributions from non-minimal SUGRA. The model however requires the presence of a Fayet-Iliopoulos (FI) term \( \xi \), and therefore a \( U(1)_\xi \) symmetry which generates it. For a Kähler potential \( K(\Phi_m, \Phi_n) \), the D-terms

\[
D^a = -g_a [D_a = \phi_i (T_a)^i] J^j + \xi_a
\]

(where \( K^m = \partial K/\partial \Phi_m \)) give rise to a scalar potential:

\[
V(\phi, \phi^*) = \frac{1}{2} |\text{Re} \phi|^{-1} \sum D^a D_a + F-\text{terms}
\]

where \( g_a \) and \( T^a \) are respectively the gauge coupling constants and the generators of each factors of the symmetry of the action, 'a' running over all factors of the symmetry, and \( f(\phi) \) is the gauge kinetic function. If this symmetry contains a factor \( U(1)_\xi \), the most general action then allows for the presence of a constant contribution \( \xi \).

The simplest realization of D-term inflation reproduces the hybrid potential with three chiral superfields, \( S, \phi_+, \) and \( \phi_- \) with non-anomalous \( U(1)_\xi \) (an abelian theory is said to be anomalous if the trace of the generator is non-vanishing \( \sum q_n \neq 0 \)) charges \( q_n = 0, +1, -1 \) (Binétruy and Dvali, 1996; Halvach, 1996). The superpotential can be written as

\[
W^D = \lambda_S \phi_+ \phi_-. \tag{71}
\]

In what follows, we assume the minimal structure for \( f(\Phi_i) \) (i.e., \( f(\Phi_i) = 1 \)) and take the minimal Kähler potential, i.e. \( K = |\phi_-|^2 + |\phi_+|^2 + |S|^2 \). Then the scalar potential reads

\[
V_{D-\text{SUGRA}} = \lambda^2 \exp \left( \frac{\phi_-^2 + \phi_+^2 + |S|^2}{M_P^2} \right)
\]

\[
\left[ |\phi_+ \phi_-|^2 \left( 1 + \frac{|S|^4}{M_P^2} \right) + |\phi_+ S|^2 \left( 1 + \frac{|\phi_-|^4}{M_P^2} \right) \right] +
\]

\[
+ \frac{g_\xi^2}{2} (|\phi_+|^2 - |\phi_-|^2 + \xi)^2,
\]

where \( g_\xi \) is the gauge coupling of \( U(1)_\xi \). The global minimum of the potential is obtained for \( \langle S \rangle = 0 \) and \( \langle \Phi_i \rangle = \sqrt{\xi} \), which is SUSY preserving but induces the breaking of \( U(1)_\xi \). For \( S > S_{\text{inst}} \equiv g_\xi \sqrt{\xi} \) the potential is minimized for \( |\phi_+| = |\phi_-| = 0 \) and therefore, at the tree level, the potential exhibits a flat inflationary valley, with vacuum energy \( V_0 = g_\xi^2 \sqrt{\xi}^2 / 2 \).

The radiative corrections depend on the splitting between the effective masses of the components of the superfields \( \Phi_+ \) and \( \Phi_- \), because of the transient D-term SUSY breaking. The radiative cor-
rections are given by the Coleman-Weinberg expression (Coleman and Weinberg, 1973) and the full potential inside the inflationary valley reads

\[ V_{\text{eff}}^{\text{D-SUGRA}} = \frac{g_2^2 \xi^2}{2} \left( 1 + \frac{g_2^2}{16\pi^2} \frac{2 \ln \frac{\Lambda^2}{\Lambda^2}}{\Lambda^2} \exp \left( \frac{|S|^2}{M_p^2} \right) + \left( z + 1 \right)^2 \ln(1+z^{-1}) + (z-1)^2 \ln(1-z^{-1}) \right) , \]

with \( z = (\Lambda^2 |S|^2 / g_2^2 \xi) e^{S^2 / M_p^2} \). Inflation ends when the slow-roll conditions break down, that is for \( z_{\text{end}} \sim 1 \), and the predictions for the inflationary parameters are very similar to the previous discussion on \( F \)-term inflation.

### E. MSSM flat direction inflation

So far we have discussed inflationary models where the inflaton sector belongs to the hidden sector (not charged under the SM gauge group), such models will have at least one SM gauge singlet component, whose couplings to other fields and mass are chosen just to match the CMB observations. These models are simple but lack proper embedding within MSSM or its minimal extensions.

In order to construct a predictable hidden sector model of inflation, one must know all the inflaton couplings to the hidden and visible matter. One such unique model has been constructed within type IIB string theory, where it was found that all the inflaton energy is transferred to exciting the \( \tilde{X} \)'s, and the universe could be prematurely dominated by the hidden sector dark matter. Such obstacles do not arise if the last phase of inflation occurs within MSSM.

1. Introducing MSSM and its flat directions

In addition to the usual quark and lepton superfields, MSSM has two Higgs fields, \( H_u \) and \( H_d \). Two Higgses are needed because \( H^\dagger \) is forbidden in the superpotential. The potential for the MSSM is given by, see (Chung et al., 2003; Haber and K粉碎, 1985; Martin, 1997; Nilles, 1984)

\[ W_{\text{MSSM}} = \lambda_u Q H_u u + \lambda_d Q H_d d + \lambda_3 LH u e + \mu H_u H_d , \]

where \( H_u, H_d, Q, L, u, d, e \) in Eq. (74) are chiral superfields, and the dimensionless Yukawa couplings \( \lambda_u, \lambda_d, \lambda_3 \) are 3 \times 3 matrices in the family space. We have suppressed the gauge and family indices. The \( H_u, H_d, Q, L \) fields are \( SU(2) \) doublets, while \( u, d, e \) are \( SU(2) \) singlets. The last term is the \( \mu \) term, which is a SUSY version of the SM Higgs boson mass. Terms proportional to \( H_u^2 H_u \) or \( H_d^2 H_d \) are forbidden in the superpotential, since \( W_{\text{MSSM}} \) must be analytic in the chiral fields. \( H_u \) and \( H_d \) are required not only because they give masses to all the quarks and leptons, but also for the cancellation of gauge anomalies. The Yukawa matrices determine the masses and CKM mixing angles of the ordinary quarks and leptons through the neutral components of \( H_u = (H_u^+, H_u^0) \) and \( H_d = (H_d^0 H_d^{-}) \). Since the top quark, bottom quark and tau lepton are the heaviest fermions in the SM, we assume that only the third family, \((3,3)\) element of the matrices \( \lambda_u, \lambda_d, \lambda_3 \) are important.

The \( \mu \) term provides masses to the Higgsinos

\[ \mathcal{L} \supset - \mu (\tilde{H}_u^+ \tilde{H}_d^- - \tilde{H}_u^0 \tilde{H}_d^0) + \text{c.c.} , \]

and contributes to the Higgs (mass)\(^2\) terms in the scalar potential through

\[ - \mathcal{L} \supset V \supset |\mu|^2 (|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2) . \]

Note that Eq. (70) is positive definite. Therefore, it cannot lead to electroweak symmetry breaking without including SUSY breaking (mass)\(^2\) soft terms for the Higgs fields, which can be negative. Hence, \( |\mu|^2 \) should almost cancel the negative soft (mass)\(^2\) term in order to allow for a Higgs VEV of order \( \sim 174 \) GeV. That the two different sources of masses should be precisely of same order is a puzzle for which many solutions has been suggested (Casas and Munoz, 1993; Giudice and Masiero, 1988; Kim and Nilles, 1984).

Within MSSM one can construct gauge invariant \( D \)- and \( F \)-flat directions, for the list of MSSM flat directions see (Dine et al., 1996a; Gherghetta et al., 1996). A flat direction can be represented by a composite gauge invariant operator, \( X_m \), formed from the product of \( k \) chiral superfields \( \Phi_i \) making up the flat direction: \( X_m = \Phi_1 \Phi_2 \cdots \Phi_m \). The scalar component of the superfield \( X_m \) is related to the order parameter \( \phi \) through \( X_m = e^{i \phi} m \) (Dine et al., 1996b).

An example of a \( D \)-and \( F \)-flat direction is provided by (Dine and Kusenko, 2004; Enqvist and Mazumdar, 2003)

\[ H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix} , \quad L = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix} , \]

where \( \phi \) is a complex field parameterizing the flat direction, or the order parameter, or the AD field. All the other fields are set to zero. In terms of the composite gauge invariant operators, we would write \( X_m = LH_u (m = 2) \). Note that a flat direction necessarily carries a global \( U(1) \) quantum number, which corresponds to an invariance of the effective Lagrangian for the order parameter \( \phi \) under phase rotation \( \phi \rightarrow e^{i \theta} \phi \). In the MSSM the global \( U(1) \) symmetry is \( B - L \). For example, the \( L \)-direction has \( B - L = -1 \).

From Eq. (80) one clearly obtains \( F^{H_u} = \lambda_u Q u + \mu H_u = F^{H_d}_L = \lambda_d H_d e \equiv 0 \) for all \( \phi \). However there exists a non-zero F-component given by \( F^{H_d}_L = \mu H_u \). Since \( \mu \) can not be much larger than the electroweak scale
$M_W \sim O(1)$ TeV, this contribution is of the same order as the soft SUSY breaking masses, which are going to lift the degeneracy. Therefore, following (Dine et al. 1996b), one may nevertheless consider LH$\phi$ to correspond to a F-flat direction. The relevant D-terms read

$$D^a_{SU(2)} = H^a_\tau \tau_3 H_u + L^a \tau_3 L = \frac{1}{2} |\phi|^2 - \frac{1}{2} |\phi|^2 \equiv 0.$$  \hspace{1cm} (78)

Therefore the LH$\phi$ direction is also D-flat.

2. Gauge invariant inflatons of MSSM

A simple observation was first made in (Allahverdi et al. 2007c, 2006, 2007d), where the inflaton properties are directly related to the soft SUSY breaking mass term and the A-term of the MSSM. Within MSSM, it is possible to lift the flatness of the gauge invariant combinations of squarks and sleptons away from the point of enhanced gauge symmetry by the F-term, while maintaining the D-flatness.

**a. Squarks and sleptons driven inflation:** Let us consider a non-renormalizable superpotential term (Dine and Kusenko 2004; Enqvist and Mazumdar, 2003):

$$W_{non} = \sum_{n>3} \frac{\lambda_n}{n} \frac{\Phi^n}{M_P^n},$$ \hspace{1cm} (79)

Where $\Phi = \phi \exp[i\theta]$, while $\theta$ is the phase term is a gauge invariant superfield which contains the flat direction. Within MSSM (with conserved R-parity) all the flat directions are lifted by the non-renormalizable operators with $4 \leq n \leq 9$ (Gherghetta et al. 1996). Two distinct directions are: udd and LLc, up to an overall phase factor they are parameterized by:

$$u^a_i = \frac{1}{\sqrt{3}} \phi, \quad d^a_j = \frac{1}{\sqrt{3}} \phi, \quad d^a_k = \frac{1}{\sqrt{3}} \phi. \hspace{1cm} (80)$$

$$L^a_i = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad L^a_j = \frac{1}{\sqrt{3}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad c_k = \frac{1}{\sqrt{3}} \phi. \hspace{1cm} (81)$$

where $1 \leq a, \beta, \gamma \leq 3$ ($a \neq \beta \neq \gamma$) are color indices, and $1 \leq i, j, k \leq 3$ ($j \neq k$) denote the quark families for udd, and $1 \leq a, b, c \leq 2$ ($a \neq b$) are the weak isospin indices and $1 \leq i, j, k \leq 3$ ($i \neq j \neq k$) denote the lepton families for LLc. Both these directions are lifted by $n = 6$ non-renormalizable operators (Gherghetta et al. 1996),

$$W_6 \supset \frac{1}{M_P^6} (LLe)(LLe), \quad W_6 \supset \frac{1}{M_P^6} (udd)(udd).$$ \hspace{1cm} (82)

Rest of the directions within MSSM are lifted by hybrid operators of type, $W \sim (1/M_P^{-3} \Psi \Phi^{n-1})$, which does not lead to cosmologically flat potential viable for slow-roll inflation (Allahverdi et al. 2008, 2006, 2007d).

The scalar potential along these directions includes softly broken SUSY mass term for $\phi$ and an $A$-term gives rise to a specific potential (Allahverdi et al. 2008, 2006, 2007d)

$$V(\phi) = \frac{1}{2} m_\phi^2 |\phi|^2 - \frac{\lambda_\phi^6}{6 M_P^6} + \frac{\lambda_2 |\phi|^4}{M_P^4},$$ \hspace{1cm} (83)

The $A$-term is a positive quantity with dimension of mass. Note that the first and third terms in Eq. (83) are positive definite, while the $A$-term leads to a negative contribution along the directions whenever cos($n\theta + \theta_A) < 0$. The above potential is similar to Eq. (44). It is possible to find a point of inflection, $\phi_0$, provided that $A^2/40m_\phi^2 \equiv 1 + 4\alpha^2$, where $\alpha \ll 1$, and at the lowest orders in $O(\alpha^2)$, we obtain:

$$V(\phi_0) = \frac{4}{15} m_\phi^2 \phi_0^2 + \cdots, \quad V'(\phi_0) = 4a^2 m_\phi^2 \phi_0 + \cdots, \quad V''(\phi_0) = 0, \quad V'''(\phi_0) = 32a^2 \phi_0^2 + \cdots.$$

$$\phi_0 = \left( m_\phi M_P^3/\lambda \sqrt{10} \right)^{1/4} + O(\alpha^2).$$ \hspace{1cm} (84)

In the case of gravity-mediated SUSY breaking scenarios, $m_\phi \sim A \sim m_{3/2} \sim (100 \text{ GeV} - 1 \text{ TeV})$. Therefore the condition $A^2 \sim 40m_\phi^2$ can indeed be satisfied. Inflation occurs within an interval $|\phi - \phi_0| \sim \phi_0^3/60 M_P^2 < M_P$, in the vicinity of the point of inflection, $\phi_0 \sim O(10^{14} \text{ GeV})$. Within which the slow-roll parameters, $\epsilon, \eta \ll 1$. The Hubble expansion rate during inflation is given by

$$H_{\text{MSSM}} \simeq \frac{1}{\sqrt{45}} \frac{m_\phi \phi_0}{M_P} \sim (100 \text{ MeV} - 1 \text{ GeV}).$$ \hspace{1cm} (85)

The amplitude of density perturbations $\delta_H$ (see Eqs. (13), (50), (51)) and the scalar spectral index $n_s$ are given by (Allahverdi et al. 2006, 2007d). (Allahverdi and Mazumdar, 2006a, Bueno Sanchez et al. 2007):

$$\delta_H = \frac{8}{\sqrt{5\pi}} \frac{m_\phi M_P}{\phi_0} \frac{1}{\Delta^2} \sin^2[N_Q \Delta^2]$$ \hspace{1cm} (86)

$$n_s = 1 - 4 \sqrt{\Delta^2} \cot[N_Q \Delta^2],$$ \hspace{1cm} (87)

where $2 \times 10^{-6} \leq \Delta^2 \equiv 900a^2 N_Q^2 (M_P/\phi_0)^4 \leq 5.2 \times 10^{-6}$, and $N_Q \sim 50$. Running in the tilt is very small. In this case the universe thermalizes in to MSSM radiation instantaneously in less than one Hubble time after the end of inflation (Allahverdi et al. 2011b), see the discussion in Sect. 11.F.3.

For $\phi_0 \sim 10^{14} \text{ GeV}$, there is an apparent fine-tuning in the parameters $A/m_\phi = a \sim 10^{-10}$, which may look unpleasant. However note that this fine tuning between the two MSSM parameters in the ratio is energy dependent and valid only at the scale of inflation at $10^{14} \text{ GeV}$, but at the TeV scale where the soft masses would be measured at the collider there is no apparent fine tuning in the parameters (Allahverdi et al. 2010c). As shown in Sect. 11.D.2.d see Fig. 2 the SUGRA corrections will ameliorate the tuning down to $\beta \equiv$
\[\alpha \sim 10^{-2}\], virtually addressing any fine tuning required for the success of MSSM inflation. It was shown in Refs. \cite{Allahverdi:2008}, that the\footnote{The inflection point for the MSSM inflaton is an attractor solution, provided there exists a phase of inflation for the MSSM inflaton is an attractor along this direction, after the minimization along this direction, is found to be \cite{Allahverdi:2007}.} inflection point for the MSSM inflaton is an attractor solution, provided there exists a phase of inflation prior of that of the MSSM with \(N \geq 10^{10}\) e-foldings. Such large e-foldings can be generated within string theory landscape \cite{Allahverdi:2007}, or within MSSM \cite{Allahverdi:2008}.

b. Renormalizable superpotential: The left handed neutrinos can be of Dirac type with an appropriate Yukawa coupling. The simplest way to obtain this would be to augment the SM symmetry by, \(SU(3)C \times SU(2)L \times U(1)_Y \times U(1)_{B-L}\), where \(U(1)_{B-L}\) is gauged. The relevant superpotential term is

\[W = h N H_u L.\] (88)

Here \(N, L\) and \(H_u\) are superfields containing the RH neutrinos, left-handed (LH) leptons and the Higgs which gives mass to the up-type quarks, respectively. Note that the \(N H_u L\) monomial represents a \(D\)-flat direction under the \(U(1)_{B-L}\), as well as the SM gauge group.

The value of \(h\) needs to be small, i.e. \(h \leq 10^{-12}\), in order to explain the light neutrino mass, \(\sim \mathcal{O}(0.1\ eV)\) corresponding to the atmospheric neutrino oscillations detected by Super-Kamiokande experiment. The potential along this direction, after the minimization along the angular direction, is found to be \cite{Allahverdi:2007}.

\[V(|\phi|) = \frac{m_\phi^2}{2} |\phi|^2 + \frac{k^2}{12} |\phi|^4 - \frac{A h}{6\sqrt{3}} |\phi|^3.\] (89)

For \(A \approx 4m_\phi\), there exists an inflection point for which \(V'(\phi_0) \neq 0, V''(\phi_0) = 0\), where inflation takes place

\[\phi_0 = \sqrt{3} \frac{m_\phi^2}{h} = 6 \times 10^{12} m_\phi \left(\frac{0.05 \text{eV}}{m_\nu}\right),\]

\[V(\phi_0) = \frac{m_\phi^4}{4h^2} = 3 \times 10^{24} \left(\frac{m_\phi}{m_\nu}\right)^2 \left(\frac{0.05 \text{eV}}{m_\nu}\right)^2.\] (90)

The amplitude of density perturbations follows from Eqs. (89, 91).

\[\delta_H \approx \frac{1}{5\pi} \frac{H_{inf}^2}{\nu} \approx 3.5 \times 10^{-27} \left(\frac{m_\nu}{0.05 \text{eV}}\right)^2 \left(\frac{M_p}{m_\phi}\right) N^2.\] (91)

Here \(m_\phi\) denotes the loop-corrected value of the inflaton mass at the scale \(\phi_0\) in Eqs. (90, 91). The spectral tilt as usual has a range of values \(0.90 \leq n_s \leq 1.0\) \cite{Allahverdi:2007}.

c. MSSM Higgses as inflaton: The MSSM Higgses are another fine example of a visible sector inflaton provided some restrictions are met \cite{Chatterjee:2011}. The required superpotential is given by

\[W = \mu H_u H_d + \sum_k \frac{\lambda_k}{k} \frac{(H_u H_d)^k}{M_p^{2k-3}},\] (92)

This is the \(\mu\)-term which were considered an ideal candidate to generate the density perturbations \cite{Enqvist:2004}, but now they can also provide the required vacuum energy to inflate the universe \cite{Chatterjee:2011}. The scalar potential along the \(H_u H_d\) D-flat direction is given by

\[V(\varphi, \theta) = \frac{1}{2} m^2(\varphi)^2 + (-1)^{(k-1)/2} 2 \lambda_k \mu \cos((2k - 2)\theta) \varphi^{2k} + 2 \lambda_k \varphi^{2(2k-1)},\] (93)

where \(\varphi = \sqrt{2}\varphi \cos \theta\), and \(H_u = (1/\sqrt{2})(\phi, 0)^T\), \(H_d = (1/\sqrt{2})^{-1}(0, \phi)^T\), and

\[m^2(\theta) = \frac{1}{2} (m_{H_u}^2 + m_{H_d}^2 + 2\mu^2 - 2B\mu \cos 2\theta),\]

\[\lambda_k = \frac{\lambda_k}{2(2k-1)M_p^{2k-3}}.\] (95)

For simplicity, we may assume \(\mu\) and \(B\) to be real. This choice is compatible with the experimental constraints, mainly from the Electron Dipole Moment measurements \cite{Pospelov:2005}. The inflection point can be obtained for \(\theta = 0, \pm \pi/2\), for simplicity let us consider the case for \(\theta = 0\), and when the following condition is satisfied, \(m_\phi^2 = k^2 \mu^2/(2k - 1) + \lambda^2\), where \(\lambda\) is the tuning required to maintain the flatness of the potential. Although, this tuning could be harsh at the inflationary scale, \(\varphi \sim 10^{14}\) GeV, but the ratio evolves to \(m_\phi^2/\mu^2 \sim \mathcal{O}(1)\) at the electroweak scale by virtue of running of the renormalization group equations \cite{Chatterjee:2011}. The amplitude
of the CMB perturbations can be obtained for $\lambda_2 \sim 10^{-8}$ and $\lambda_3 \sim 0(1)$, it is possible to obtain a similar plot like Fig. 4 for Higgs mass $m(\theta = 0) \sim 100 - 250$ GeV, which yields the spectral tilt in the range $0.93 \leq n_s \leq 0.98$.

F. Preheating, reheating, thermalization

Reheating at the end of inflation is an important aspect of inflationary cosmology. Without reheating the universe would be empty of matter, for a review see [Alihverdi et al., 2010a]. Reheating occurs through coupling of the inflaton field $\phi$, to the SM matter. Such couplings must be present at least via gravitational interactions. In particular, if the inflaton is a SM gauge singlet, the relevant couplings to SM are: $\nabla^2 \phi \delta H(q) q R$, $\nabla^2 \phi F_{\mu \nu} F^{\mu \nu}$, $g^2 \phi^2 H H$, where $M$ is the scale below which all these effective operators are valid, $\lambda$, $g \sim 0(1)$, $H$ is the SM Higgs doublet, and $q_L, q_R$ are the left and the right handed SM fermions (Alihverdi and Mazumdar, 2007b).

Similar couplings would arise if $\phi$ is replaced by right handed sneutrinos, axions, moduli, or any other hidden sector field. Being a SM singlet, $\phi$ can as well couple to other hidden sectors, moduli, axions, etc. Since the hidden sectors are largely unknown, it becomes a challenge for a singlet inflaton to decay solely into the SM d.o.f (Cicoli and Mazumdar, 2010a).

After the end of inflation, the inflaton starts coherent oscillations around its minimum. The frequency of oscillations are determined by the frequency of oscillations, $\omega \sim m_{\text{eff}} \geq H_{\text{inf}}$. During this epoch the inflaton can decay perturbatively (Albrecht et al., 1982; Dolgov and Kirillova, 1990; Kolb and Turner, 1988; Turner, 1983). Averaging over many oscillations results in a pressureless equation of state where $p = \langle \hat{\phi}^2 / 2 - V(\phi) \rangle$ vanishes, so that the energy density starts evolving like a matter domination (in a quadratic potential) with $\rho_\phi = \rho_i (a_i / a)^3$ (subscript $i$ denotes the quantities right after the end of inflation). For $\lambda \phi^4$ potential the coherent oscillations yield an effective equation of state similar to that of a radiation epoch. If $\Gamma_\phi$ represents the total decay width of the inflaton to pairs of fermions. This releases the energy into the thermal bath of relativistic particles when $H(a) = \sqrt{(1/3M_P^2)\rho_i (a_i / a)^3} / 2 \approx \Gamma_\phi$. The energy density of the thermal bath is determined by the reheat temperature $T_R$, given by:

$$T_R = \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{\Gamma_\phi M_P} = 0.3 \left( \frac{200}{g_*} \right)^{1/4} \sqrt{\Gamma_\phi M_P},$$

where $g_*$ denotes the effective relativistic d.o.f in the plasma. However the inflaton decay products need to thermalize, which requires acquiring kinetic and chemical equilibrium.

1. Non-perturbative particle creation

If the inflaton coupling to the matter field is large, a completely new channel of reheating opens up due to the coherent nature of the inflaton field, proposed by (Kofman et al., 1994; 1997; Shtanov et al., 1995; Traschen and Brandenberger, 1990), known as preheating. Let us first consider a simple toy model with interaction Lagrangian

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}g^2 \chi^2 \phi^2,$$

where $\chi$ is another scalar field, in a realistic set-up $\chi$ could take the role of the SM Higgs. We can neglect the effect of expansion provided that the time period of preheating is small compared to the Hubble expansion time $H^{-1}$, this is reasonable in many cases.

The quantum theory of $\chi$ particle production in the external classical inflaton background begins by expanding the quantum field $\hat{\chi}$ into creation and annihilation operators $\hat{a}_k$ and $\hat{a}_k^\dagger$ as:

$$\hat{\chi}(t, \mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left( \chi_k^*(t) \hat{a}_k e^{i k \mathbf{x}} + \chi_k(t) \hat{a}_k^\dagger e^{-i k \mathbf{x}} \right),$$

where $k$ is the momentum. Since the equation of motion for $\chi$ is linear it can be studied simply mode by mode in Fourier space. The mode functions then satisfy:

$$\chi_k + \left( k^2 + m_\chi^2 + g^2 \Phi^2 \sin^2(2m_d t) \right) \chi_k = 0,$$

where $\Phi$ is the amplitude of oscillation in $\phi$. This is the Mathieu equation which is written in the form

$$\chi_k'' + (A_k - 2q \cos 2z) \chi_k = 0,$$

where the dimensionless time variable is $z = m_d t$ and a prime now denotes the derivative with respect to $z$. Comparing the coefficients, we find

$$A_k = \frac{k^2 + m_\chi^2}{m_d^2} + 2q \quad q = \frac{g^2 \Phi^2}{4m_d^2},$$

The growth of the mode function corresponds to particle production (Birrell and Davies, 1982). It is well known that the above Mathieu equation Eq. 100 has instabilities for certain ranges of $k$:

$$\chi_k \propto \exp(\mu_k z),$$

where $\mu_k$ is called the Floquet exponent. For small values of $q$, i.e. $q \ll 1$, resonance occurs in a narrow instability band about $k = m_d$, known as a “narrow resonance” band (Traschen and Brandenberger, 1990). The resonance is much more efficient if $q \gg 1$ (Kofman et al., 1994; 1997). In this case, resonance occurs in broad bands, i.e. the bands include all long wavelength modes $k \to 0$, known as broad resonance. This can be understood by studying the condition for particle production in the WKB approximation for the evolution of $\chi$. 

field which is violated. In the WKB approximation: 
\[ \chi_k \propto e^{\pm i \int \omega_k dt} \]
which is valid as long as the adiabaticity condition
\[ \frac{d\omega_k^2}{dt} \leq 2\omega_k^3 \]  \hspace{1cm} (103)
is satisfied. In the above, the effective frequency \( \omega_k \) is given by
\[ \omega_k = \sqrt{k^2 + m_{\chi}^2 + g^2 \Phi(t)^2 \sin^2(m_{\phi} t)} \]  \hspace{1cm} (104)
By inserting the effective frequency Eq. (104) into the condition Eq. (103) and following some algebra, the adiabaticity condition is violated for momenta
\[ k^2 \leq \frac{2}{3\sqrt{3}} gm_{\phi} \Phi - m_{\chi}^2. \]  \hspace{1cm} (105)
For modes with these values of \( k \), the adiabaticity condition breaks down in each oscillation period when \( \phi \) is close to zero. The particle number does not increase smoothly, but rather in “bursts” \cite{Kofman1994, Kofman1997}.

The above analysis of neglecting the expansion of the universe is self-consistent. However, as discussed in detail in \cite{Kofman1997}, the expansion of space can be included. The equation of motion for \( \chi \) becomes
\[ \ddot{\chi}_k + 3H \dot{\chi}_k + \left( \frac{k^2}{a^2} + m_{\chi}^2 + g^2 \Phi(t)^2 \sin^2(m_{\phi} t) \right) \chi_k = 0. \]  \hspace{1cm} (106)
The adiabaticity condition is now violated for momenta satisfying:
\[ \frac{k^2}{a^2} \leq \frac{2}{3\sqrt{3}} gm_{\phi} \Phi - m_{\chi}^2. \]  \hspace{1cm} (107)
Note that the expansion of space makes broad resonance more effective since more \( k \) modes are red-shifted into the instability band as time proceeds. The detailed analysis yields the same expression for the resonance band except for the exact value of the numerical coefficient of the first term on the r.h.s.. Broad parametric resonance ends when \( q \leq 1/4 \).

In principle, it is also possible to excite the fermions non-perturbatively, in spite of the fact that the occupation number of any fixed state cannot be greater than one (because of the Pauli exclusion principle) \cite{Baacke1998, Giudice1999a, Greene1999, Giudice1999b, Peloso2000, Giudice1999c, Kallosh2000a, Kofman1999, Kofman1999b, Maroto2000, Nilles2001a, Nilles2001b}.

a. Tachyonic preheating: It is possible that effective frequency of certain mode can be negative. For example in a symmetry breaking potential: 
\[ V(\phi) = \frac{1}{2} A (\phi^2 - \eta^2)^2 \]
for small field values, the effective mass of the fluctuations of \( \phi \) is negative and hence a “tachyonic” resonance will occur, as studied in \cite{Felder2001}. For small field values, the equation for the fluctuations \( \phi_k \) of \( \phi \) is
\[ \ddot{\phi}_k + (k^2 - m_{\phi}^2) \phi_k = 0. \]  \hspace{1cm} (108)
The modes with \( k < m \) grow with an exponent which approaches \( \mu_k = 1 \) in the limit \( k \to 0 \). Given initial vacuum amplitudes for the modes \( \phi_k \) at the intial time \( t = 0 \) of the resonance, the field dispersion at a later time \( t \) will be given by
\[ \langle \delta\phi^2 \rangle = \int_0^m \frac{k dk}{4\pi} e^{2i\sqrt{m_{\phi}^2 - k^2}}. \]  \hspace{1cm} (109)
The growth of the fluctuations modes terminates once the dispersion becomes comparable to the symmetry breaking scale.

Tachyonic preheating also occurs in hybrid inflation models, see Eq. (69). In this case, it is the fluctuations of \( \psi \) which have tachyonic form and which grow exponentially \cite{Felder2001}\cite{Kallosh2001}. Preheating in hybrid inflation was first studied in \cite{Garcia-Bellido1998} using the tools of broad parametric resonance.

b. End of preheating: In the above analysis we have neglected the back-reaction of the produced \( \chi \) and \( \phi \) particles on the dynamics. The presence of \( \chi \) particles changes the effective mass of the inflaton oscillations. This back-reaction effect is negligible as long as the change \( \Delta m_{\phi}^2 \) in the square mass of the inflaton is smaller than \( m_{\phi}^2 \). In the Hartree approximation, the change in the inflaton mass due to \( \chi \) particles is given by \( \Delta m_{\phi}^2 = g^2 (\chi^2) \) \cite{Kofman1997}. Another important condition is that the energy in the \( \chi \) particles should be sub-dominant. Therefore, \( \rho_{\chi} \approx \langle (\nabla \chi)^2 \rangle = k^2 (\chi^2) \ll m_{\phi}^2 (\phi^2) \). It was found that \( \rho_{\chi} \) is smaller than the potential energy of the inflaton field at the time \( t_1 \) as long as the value \( q \) at the time \( t_1 \) is larger than 1, i.e. \( q(t_1) > 1 \). This is roughly speaking the same as the condition for the effectiveness of broad resonance \cite{Kofman1997}.

2. Thermalization

Neither the perturbative decay of the inflaton nor the preheating mechanism produce a thermal spectrum of decay products. In a full thermal equilibrium the energy density \( \rho \) and the number density \( n \) of relativistic particles scale as: \( \rho \sim T^4 \) and \( n \sim T^3 \), where \( T \) is the temperature of the thermal bath. Thus, in full equilibrium the average particle energy is given by: \( \langle E \rangle_{eq} = (\rho/n) \), which obeys the scaling, \( \langle E \rangle_{eq} \sim \rho^{1/4} \sim T \).

a. Perturbative reheating and thermalization: If the inflaton decays perturbatively, then right after the inflaton has decayed completely, the energy density of the universe is given by: \( \rho \approx 3 (\Gamma_{\phi} M_{Pl})^2 \), \( \langle E \rangle \approx m_{\phi} \gg \rho^{1/4}. \) From conservation of energy, the number density of decayed particles is: \( n \approx (\rho/m_{\phi}) \ll \rho^{3/4}. \) Hence, perturbative decay results in a dilute plasma that contains a small number of very energetic particles. A local thermal equilibrium requires re-distribution of the energy among

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different particles, kinetic equilibrium, as well as increasing the total number of particles, chemical equilibrium. Therefore both number-conserving and number-violating reactions must be involved.

The most important processes for kinetic equilibration are $2 \rightarrow 2$ scatterings with gauge boson exchange in the $t$-channel. Due to an infrared singularity, these scatterings are very efficient even in a dilute plasma (Allahverdi and Mazumdar, 2006; Davidson and Sarkar, 2000). Chemical equilibrium is achieved by changing the number of particles in the reheating plasma. In order to reach full equilibrium the total number of particles must increase by a factor of $n_{eq}/n$, where $n \approx \rho/m$ and the equilibrium value is: $n_{eq} \sim \rho^{3/4}$. This can be a very large number, i.e. $n_{eq}/n \sim \mathcal{O}(10^3)$. It was recognized in (Allahverdi and Drees, 2002; Davidson and Sarkar, 2000), see also (Allahverdi, 2000; Jaikumar and Mazumdar, 2004) that the most relevant processes are $2 \rightarrow 3$ scatterings with gauge-boson exchange in the $t$-channel. When these scatterings become efficient, the number of particles increases very rapidly, and full thermal equilibrium is established shortly after that (Engvist and Sirkka, 1993).

b. Non-perturbative preheating and thermalization: In this case the occupation numbers of the excited quanta are typically very high after the initial stages of preheating. Once the occupation numbers of the resonant modes become sufficiently large, re-scattering of the fluctuations begins (Khlebnikov and Tkachev, 1996; 1997a; b; Micha and Tkachev, 2003; 2004) which terminates the phase of exponential growth of the occupation numbers. The evolution of the field fluctuations evolves to a regime of turbulent scaling which is characterized by the spectrum $n(k) \sim k^{-3/2}$ (Micha and Tkachev, 2003; 2004), which is non-thermal (for a thermal distribution we would expect $n(k) \sim k^{-1}$). The phase of turbulence ends once most of the energy has been drained from the inflaton field. At this time quantum processes take over and lead to the thermalization of the spectrum.

3. Calculation of $T_R$ within MSSM

In the case of MSSM inflation, the inflaton couplings to MSSM d.o.f are known (Allahverdi et al., 2011b). It is therefore possible to track the thermal history of the universe from the end of inflation. When the MSSM inflaton passes through minimum, i.e. $\phi = 0$, the entire gauge symmetry gets restored and all the d.o.f associated with the MSSM gauge group become massless, which is known as the point of enhanced gauge symmetry.

These are the massless modes which couple to the inflaton directly, for instance the d.o.f corresponding to $SU(2)_W \times U(1)_Y$, or that of $SU(3)_c \times U(1)_Y$. At VEVs away from the minimum, the same modes become heavy and therefore it is kinematically unfavorable to excite them. The actual process of excitation depends on how strongly the adiabatic condition for the time dependent vacuum is violated for the inflaton zero mode.

a. Couplings for LLe inflaton: Let us illustrate this with $L_1 L_2 e_4$ flat direction as an inflaton. The inflaton non-zero VEV completely breaks the $SU(2)_W \times U(1)_Y$ symmetry. This results in four massive real scalars, whose masses are obtained from the $D$-terms (Allahverdi et al., 2011b)

$$V \supset \frac{1}{12} g_W^2 \phi^2 (\chi_1^2 + \chi_2^2 + \chi_3^2) + \frac{1}{4} g_\psi^2 \phi^2 \chi_4^2.$$  (110)

Here $g_W$, $g_\psi$ are the $SU(2)_W$ and $U(1)_Y$ gauge couplings respectively, and $\phi$ denote the inflaton, see Eq. (51). The $\chi$’s are Goldstone bosons from breakdown of $SU(2)_W \times U(1)_Y$. They are eaten by the Higgs mechanism and give rise to longitudinal components of the electroweak gauge fields. In the unitary gauge, they are completely removed from the spectrum. The $\chi$ particles decay to squarks, the Higgs particles, and the $L_3$, $e_1$, $e_2$ sleptons with the decay rates given by: $\Gamma_{\chi_1} = \Gamma_{\chi_3} = \Gamma_{\chi} = 3 g_{W}^{2} \phi / 8 \pi \sqrt{6}$, $\Gamma_{\chi_4} = (9 g_{W}^{2} \phi / 16 \pi \sqrt{2})$. Note that the decay rate is proportional to the VEV of the inflaton, which sets the mass of $\chi$ fields. Couplings of the inflaton to the gauge fields are obtained from the flat direction kinetic terms (Allahverdi et al., 2011b)

$$\mathcal{L} \supset \frac{g_{W}^{2}}{12} \phi^{2} (2 W^{+\mu} W^{-\mu} + W_{3}^{\mu} W_{3}^{\mu}) + \frac{g_{\psi}^{2}}{4} \phi^{2} B^{\mu} B_{\mu},$$  (111)

where $W^{+} = (W_{1} - i W_{2}) / \sqrt{2}$, $W^{-} = (W_{1} + i W_{2}) / \sqrt{2}$, $W_{i\mu}$ and $B_{\mu}$ are the $SU(2)_W$ and $U(1)_Y$ gauge fields respectively. The gauge fields decay to $(s)quarks$, Higgs and Higgsino particles, and $L_3$, $e_1$, $e_2$ sleptons with the total decay widths: $\Gamma_{W^{+}} = \Gamma_{W^{-}} = \Gamma_{W_3} = (3 g_{W}^{2} \phi / 8 \pi \sqrt{6})$, $\Gamma_{B} = (9 g_{W}^{2} \phi / 16 \pi \sqrt{2})$. Couplings of the inflaton to fermions can also be found in a similar way (Allahverdi et al., 2011b).

b. Instant preheating and thermalization: The fields that are coupled to the inflaton acquire a VEV-dependent mass that varies in time due to the inflaton oscillations. For illustration, we first focus on the $\chi_1$ scalar, which are produced every time the inflaton goes through zero. The Fourier eigenmodes of $\chi_1$ have the corresponding energy

$$\omega_k = \sqrt{k^2 + m_{\chi}^2 + g_{W}^{2} \phi(t)^2 / 6}$$  (112)

where $m_{\chi}$ is the bare mass of the $\chi$ field. The growth of the occupation number of mode $k$ can be computed exactly for the first zero-crossing, $n_{k,x} = \text{exp} \left[-\pi \sqrt{6}(k^2 + m_{\chi}^2)/(g_{W} \phi_0)\right]$, where the inflaton near the zero-crossing is given by $\phi_0 = (2V(\bar{\phi}))^{1/2}$, from the conservation of energy, where $\bar{\phi}$ is the amplitude of the inflaton oscillations, $\bar{\phi} \approx \phi_0 / \sqrt{3} \sim 10^{13}$ GeV, where $\phi_0$ is the inflection point for inflation, Eq. (51). Note that after a few oscillations, $\phi_0 \approx m_{\phi} \phi$, since the expansion...
rate during the inflaton oscillations is negligible by virtue of $m_{\phi} \sim 100 \text{ GeV}$ and $H(t) \lesssim H_{inf} \sim 1 \text{ GeV}$. The total number density of particles thus produced follows:

$$n_{\chi_1} = \int_0^{\infty} \frac{q^2 k}{(2\pi)^3} n_k \chi = \frac{m_{\phi}^2 q^{3/4}}{2\sqrt{2\pi}} \exp \left( -\frac{\pi m_{\phi}^2}{2m_{\phi}^2\sqrt{q}} \right).$$

(113)

where $q \equiv (g_{\phi}^2 \phi_0^2/24m_{\phi}^4) \gg 1$. This expression corresponds to the asymptotic value and assumes there is no perturbative decay of the produced $\chi$ particles. However, immediately after adiabaticity is restored, $\tau > \tau_c = \sqrt{2} q^{-1/4}$, $\chi_1$ particles decay into lighter particles (i.e. those particles that have no gauge coupling to the inflaton). In the case of $L_1L_2e_3$ inflaton these are the (s)quarks, $H_u$ Higgsinos, and $L_3$, $e_1$, $e_2$ (s)leptons.

Thus the fraction that is transferred from the inflaton to $\chi_1$'s, and through their prompt decay into relativistic squarks, at every inflaton zero-crossing, can be computed analytically, and they are given by,

$$\frac{n_{reco}}{n_\phi} \sim 0.0067 g_{W}^2 - \frac{n_{\gamma}}{2m_{\phi}^2\sqrt{g_{W}}}. \quad \text{(114)}$$

The total number of d.o.f coupled to the $L_1L_2e_3$ inflaton is 32 (4 from scalars, $4 \times 3 = 12$ from gauge fields, and $4 \times 4 = 16$ from fermions). Therefore the fraction of the inflaton energy density that is transferred to relativistic squarks and squarks, see Eq. (113), for $q_{\gamma} \approx q_{\gamma} \approx 0.6$, has to be multiplied by $(1+3+4) = 8$ [Allahverdi et al. 2011b]:

$$\frac{\rho_{reco}}{\rho_\phi} \sim 10.6\% \quad \text{(per zero \text{ --} crossing).} \quad \text{(115)}$$

Note that this fraction is independent from the amplitude of oscillations. The draining the inflaton energy is quite efficient, nearly 10% of the inflaton energy density gets transferred to the relativistic species – but not all the SM d.o.f are in thermal equilibrium after one oscillation. It takes near about 120 oscillations to reach the full chemical and kinetic equilibrium via processes requiring $2 \leftrightarrow 2$ and $2 \leftrightarrow 3$ interactions. However due to the hierarchy between $H_{inf} \sim 10^{-3} m_{\phi}$, this happens within a single Hubble time after the end of inflation. One can estimate the final reheat temperature [Allahverdi et al. 2011b]

$$T_R = (30/\pi^2 g_*)^{1/4} \rho_0^{1/4} \approx 2 \times 10^8 \text{ GeV}, \quad \text{(116)}$$

where $g_* = 228.75$ and $\rho_0 = (4/15)m_{\phi}^2\phi_0^2$, see Eq. (S4).

III. MATTER-ANTI-MATTER ASYMMETRY

If (p)reheating can provide a thermal bath where all the SM quarks and leptons are excited, it is then an important question to ask – why the present day galaxies and intergalactic medium is primarily made up of baryons rather than anti-baryons?

The baryon abundance in the universe is denoted by $\Omega_b \equiv \rho_b/\rho_c$, which defines the fractional baryon density $\rho_b$ with respect to the critical energy density of the universe: $\rho_c = 1.88 h^2 \times 10^{-26} \text{ g cm}^{-3}$. The observational uncertainties in the present value of the Hubble constant; $H_0 = 100 h \text{ km} \cdot \text{s}^{-1} \text{ Mpc}^{-1}$, are encoded in $h = 0.73$ [Kessler et al. 2004]. It is useful to write in terms of the baryon and photon number densities

$$\eta \equiv \frac{n_b - n_{\gamma}}{n_\gamma} = 2.68 \times 10^{-8} \Omega_b h^2, \quad \text{(117)}$$

where $n_b$ is the baryon number density and $n_\gamma$ is for anti-baryons. The photon number density is given by $n_\gamma \approx (2\zeta(3)/\pi^2) T^3$. The best present estimation of the baryon density comes from BBN, which is based on SM physics with 3 neutrino species [Cyburt et al. 2008; Fields and Sarkar 2006]

$$0.019 \leq \Omega_b h^2 \leq 0.024 \quad (95\% CL), \quad \text{(118)}$$

$$5.1 \times 10^{-10} \leq \eta \leq 6.5 \times 10^{-10} \quad (95\% CL). \quad \text{(119)}$$

The observational data on $D$ and $^4\text{He}$ are consistent with each other and the expectations from the BBN analysis, but both prefer slightly higher value compared to the $^7\text{Li}$ abundance $\text{Li}/H_{abs} = (1.7 \pm 0.06 \pm 0.44) \times 10^{-10}$, which is smaller than $D$ and $^4\text{He}$ by at least $\sim 4.2\sigma$. The $^7\text{Li}$ abundance is primarily measured in the stellar systems such as globular clusters.

From the acoustic peaks of the CMB the baryon fraction can be deduced. The WMAP data imply $\Omega_b h^2 = 0.02273 \pm 0.00062$ or $\eta = 6.23 \pm 0.17 \times 10^{-10}$ [Komatsu et al. 2011]. The WMAP data relies on priors and the choice of number of parameters, it is possible to yield baryon abundance as low as $\Omega_b h^2 = 0.0175 \pm 0.0007$ [Hunt and Sarkar 2010]. In spite of systematic uncertainties the WMAP data is consistent with that of theoretical predictions from BBN. The major unresolved problem is the $\text{Li}$ abundance, stellar $\text{Li}/H_{abs}$ measurements are inconsistent with both WMAP and BBN data, and this could be an useful probe of new physics at BBN, see for a review [Jedamzik and Pospelov 2009].

Often in the literature the baryon asymmetry is given in relation to the entropy density $s = 1.8g_*n_\gamma$, where $g_*$ measures the effective number of relativistic species which itself a function of temperature. At the present time $g_* \approx 3.36$, while during BBN $g_* \approx 10.11$, rising up to 106.75 at $T \gg 100 \text{ GeV}$. In the presence of supersymmetry at $T \gg 100 \text{ GeV}$, the number of effective relativistic species are nearly doubled to 228.75. The baryon asymmetry defined as the difference of baryon and anti-baryon number densities relative to the entropy density, is bounded by

$$58.4(7.2) \times 10^{-11} \leq \frac{n_b - n_{\gamma}}{s} \leq 9.2(9.2) \times 10^{-11}, \quad \text{(120)}$$

at (95\% CL), where the numbers are CMB [Komatsu et al. 2011], and BBN...
bounds (Cyburt et al., 2008; Fields and Sarkar, 2006), respectively.

If at the beginning $\eta = 0$, then the origin of this small number can not be understood in a CPT invariant universe by a mere thermal decoupling of nucleons and anti-nucleons at $T \sim 20$ MeV. The resulting asymmetry would be too small by at least nine orders of magnitude, see (Kolb and Turner, 1988). Therefore it is important to seek mechanisms for generating baryon asymmetry, for reviews see (Dine and Kusenko, 2003; Riotto, 1998; Rubakov and Shaposhnikov, 1996).

A. Requirements for baryogenesis

As pointed out first by Sakharov (Sakharov, 1967), baryogenesis requires three ingredients: (1) baryon number non-conservation, (2) $C$ and $CP$ violation, and (3) out-of-equilibrium condition. All these conditions are believed to be met in the very early universe.

c. Baryon number non-conservation: In the SM, baryon number $B$ is violated by non-perturbative instanton processes (t Hooft, 1976a,b). Due to chiral anomalies both baryon number $J_B^\mu$ and lepton number $J_L^\mu$ currents are not conserved (Adler, 1969; Bell and Jackiw, 1969). However the anomalous divergences come with an equal amplitude and an opposite sign. Therefore $B - L$ remains conserved, while $B + L$ may change via processes which interpolate between the multiple non-Abelian vacua of SU(2). The probability for the $B + L$ violating transition is however exponentially suppressed (t Hooft, 1976a,b), but at finite temperatures when $T \gg M_W$, baryon violating transitions are in fact copious (Manton, 1983).

The $B$ violation also leads to proton decay in GUTs (Langacker, 1985). The dimension 6 operator $(\Lambda QQQL)/\Lambda$ generates observable proton decay unless $\Lambda \geq 10^{15}$ GeV. In the MSSM the bound is $\Lambda \geq 10^{26}$ GeV because the decay can take place via a dimension 5 operator. In the MSSM superpotential there are also terms which can lead to $\Delta L = 1$ and $\Delta B = 1$. Similarly there are other processes such as neutron-anti-neutron oscillations in SM and in SUSY theories which lead to $\Delta B = 2$ and $\Delta B = 1$ transitions. These operators are constrained by the measurements of the proton lifetime, which yield the bound $\tau_p \geq 10^{33}$ years (Nakamura et al., 2010).

d. $C$ and $CP$ violation: The maximum $C$ violation occurs in weak interactions while neutral Kaon is an example of $CP$ violation in the quark sector which has a relative strength $\sim 10^{-3}$ (Nakamura et al., 2010). $CP$ violation is also expected to be found in the neutrino sector. Beyond the SM there are many sources for $CP$ violation. An example is the axion proposed for solving the strong $CP$ problem in QCD (Peccei and Quinn, 1977a,b).

e. Departure from thermal equilibrium: If $B$-violating processes are in thermal equilibrium, the inverse processes will wash out the pre-existing asymmetry ($\Delta n_B$) (Weinberg, 1979). This is a consequence of $S$-matrix unitarity and $CPT$-theorem. However there are several ways of obtaining an out-of-equilibrium process in the early universe. Departure from a thermal equilibrium cannot be achieved by mere particle physics considerations but is coupled to the dynamical evolution of the universe.

1. Out-of-equilibrium decay or scattering: The condition for out-of-equilibrium decay or scattering is that the rate of interaction must be smaller than the expansion rate of the universe, i.e. $\Gamma < H$. The universe in a thermal equilibrium can not produce any asymmetry, rather it tries to equilibrate any pre-existing asymmetry.

2. Phase transitions: They are ubiquitous in the early universe. The transition could be of first, or of second (or of still higher) order. First order transitions proceed by barrier penetration and subsequent bubble nucleation resulting in a temporary departure from equilibrium. The QCD and possibly electroweak phase transitions are examples of first order phase transitions. The nature and details of QCD phase transition is still an open debate (Karsch et al., 2001; Rajagopal and Wilczek, 1993). Second order phase transitions have no barrier between the symmetric and the broken phase. They are continuous and equilibrium is maintained throughout the transition.

3. Non-adiabatic motion of a scalar field: Any complex scalar field carries $C$ and $CP$, but the symmetries can be broken by terms in the Lagrangian. This can lead to a non-trivial trajectory of a complex scalar field in the phase space. If a coherent scalar field is trapped in a local minimum of the potential and if the shape of the potential changes to become a maximum, then the field may not have enough time to readjust with the potential and may experience completely non-adiabatic motion. This is similar to a second order phase transition but it is the non-adiabatic classical motion which prevails over the quantum fluctuations, and therefore, departure from equilibrium can be achieved. If the field condensate carries a global charge such as the baryon number, the motion can charge up the condensate. This is the basis for the Affleck-Dine baryogenesis (Affleck and Dine, 1983).

B. Sphalerons

At finite temperatures $B + L$ violation in the SM can be large due to sphaleron transitions between degenerate gauge vacua with different Chern-Simons numbers (Klinkhamer and Manton, 1984; Manton, 1983). Thermal scattering produces sphalerons which in effect decay in $B + L$ non-conserving ways below $10^{12}$ GeV (Bochkarev and Shaposhnikov, 1987), and thus can exponentially wash away $B + L$ asymmetry. The three im-
important ingredients which play important role are following.

f. Chiral anomalies: In the SM there is classical con-
servation of the baryon and lepton number currents $J_B^a$ and $J_L^a$, but because of chiral anomaly (at the quantum level) the currents are not conserved (Adler, 1969; Bell and Jackiw, 1969). Instead ('t Hooft, 1976b),

$$\frac{\partial}{\partial \tau} J_B^a = -\frac{\alpha_2}{8\pi} N_g W_4^{\mu
u} W_{4\mu
u} + \frac{\alpha_1}{16\pi} N_g F^{\alpha\beta} F_{\alpha\beta},$$

$$\frac{\partial}{\partial \tau} J_L^a = -\frac{\alpha_2}{8\pi} N_g W_4^{\mu
u} W_{4\mu
u} + \frac{\alpha_1}{16\pi} N_g F^{\alpha\beta} F_{\alpha\beta}.$$ (121)

where $N_g$ is the number of generations, $\alpha_2$ and $\alpha_1$ ($W_{4\mu
u}$ and $F_{\mu\nu}$) are respectively the $SU(2)$ and $U(1)$ gauge couplings (field strengths). Note that at the quantum level $B + L \neq 0$ is violated, but $B - L = 0$ is still conserved.

g. Gauge theory vacua: in the $SU(2)$ gauge group, the vacua are classified by their homotopy class ($\Omega_n(t)$), characterized by the winding number $n$ which labels the so called $\theta$-vacua (Moore, 1985). A gauge invariant quantity is the difference in the winding number (Chern-Simons number)

$$N_{\text{CS}} = n_+ - n_- = -\frac{\alpha_2}{8\pi} \int d^4 x W_4^{\mu\nu} W_{4\mu\nu}. \quad (122)$$

In the electroweak sector the field density $W\bar{W}$ is related to the divergence of $B + L$ current. Therefore a change in $B + L$ reflects a change in the vacuum configuration determined by the difference in winding number

$$\Delta(B + L) = \frac{\alpha_2}{4\pi} N_g \int d^4 x W_4^{\mu\nu} W_{4\mu\nu} = -2N_g N_{\text{CS}}. \quad (123)$$

For three generations of SM leptons and quarks the minimal violation is $\Delta(B + L) = 6$. Note that the proton decay $p \rightarrow e^+\pi^0$ requires $\Delta(B + L) = 2$, so that despite $B$-violation, proton decay is completely forbidden in the SM. The probability amplitude for tunneling from an $n$ vacuum at $t \rightarrow -\infty$ to an $n + N_{\text{CS}}$ vacuum at $t \rightarrow +\infty$ can be estimated by the WKB method ('t Hooft, 1976a)

$$P(N_{\text{CS}})_{B+L} \sim \exp \left(-\frac{4\pi N_{\text{CS}}}{\alpha_2(M_Z)}\right) \sim 10^{-162N_{\text{CS}}}.$$ (124)

The baryon number violation rate is negligible at zero temperature, but as argued at finite temperatures the situation is completely different (Kuzmin et al., 1985; Manton, 1983).

h. Thermal tunneling: below the critical temperature of the electroweak phase transition, the sphaleron rate is exponentially suppressed (Carson et al., 1990):$

$$\Gamma \sim 2.8 \times 10^5 \kappa T^4 \left(\frac{\alpha_2}{4\pi}\right)^4 \left(\frac{E_{\text{sph}}(T)}{B(\lambda'/g)}\right)^7 e^{-E_{\text{sph}}/T}. \quad (125)$$

where $\kappa$ is the functional determinant which can take the values $10^{-4} \leq \kappa \leq 10^{-1}$ (Dine et al., 1992). Above the critical temperature the rate is however unsuppressed. Since the Chern-Simons number changes at most by $\Delta N_{\text{CS}} \sim 1$, one can estimate from Eq. (122) that $\Delta N_{\text{CS}} \sim g_{2sph}^2 W_i^2 \sim 1 \rightarrow W_i \sim (1/g_{2sph})$. Therefore a typical energy of the sphaleron configuration is given by $E_{\text{sph}} \sim l_{3sph}^3 (\partial W)^2 \sim (1/g_{2sph}^2)$.

At temperatures greater than the critical temperature there is no Boltzmann suppression, so that the thermal energy $\propto T \geq E_{\text{sph}}$. This determines the size of the sphaleron: $l_{sph} \geq 1/g_{2sph}^2 T$. Based on this coherence length scale one can estimate the baryon number violation per volume $\sim l_{sph}^3$, and per unit time $\sim l_{sph}$. On dimensional grounds the transition probability would then be given by

$$\Gamma_{\text{sph}} \sim (1/l_{sph}^3 t) \sim \kappa(\alpha_2 T)^4.$$ (126)

where $\kappa$ is a constant which incorporates various uncertainties. However, the process is inherently non-perturbative, and it has been argued that damping of the magnetic field in a plasma suppresses the sphaleron rate by an extra power of $\alpha_2$ (Arnold et al., 1997), with the consequence that $\Gamma_{\text{sph}} \sim \alpha_2^2 T^4$. Lattice simulations with hard thermal loops also give $\Gamma_{\text{sph}} \sim \mathcal{O}(10\alpha_2^3 T^4)$ (Moore, 1999).

d. Washing out $B + L$: Assuming that in the early universe the SM $d.o.f$ are in equilibrium, the transitions $\Delta N_{\text{CS}} = +1$ and $\Delta N_{\text{CS}} = -1$ are equally probable. The ratio of rates for the two transitions is given by $\Gamma_{\text{sph}}/\Gamma_{\text{sph}} = \exp(-\Delta F/T)$, where $\Delta F$ is the free energy difference between the two vacua. Because of a finite $B + L$ density, there is a net chemical potential $\mu_{B+L}$. Therefore one obtains (Bochkarev and Shaposhnikov, 1987)

$$\frac{dn_{B+L}}{dt} = \Gamma_{\text{sph}} - \Gamma_{\text{sph}} - \sim N_g \frac{\Gamma_{\text{sph}}}{T^3} n_{B+L}. \quad (127)$$

It then follows that an exponential depletion of $n_{B+L}$ due to sphaleron transitions remains active as long as

$$\frac{\Gamma_{\text{sph}}}{T^3} \geq H \Rightarrow T \leq \alpha_2^2 \frac{M_P}{g_s^{1/2}} \sim 10^{12} \text{ GeV}. \quad (128)$$

This result implies that below $T = 10^{12}$ GeV, the sphaleron transitions can wash out any $B + L$ asymmetry being produced earlier in a time scale $\tau \sim (T^3/N_g \Gamma_{\text{sph}})$. This seems to wreck GUT baryogenesis based on $B - L$ conserving groups such as the minimal $SU(5)$.

C. Mechanisms for baryogenesis

There are several scenarios for baryogenesis, the main contenders being GUT baryogenesis, electroweak baryogenesis, leptogenesis, and baryogenesis through the decay of a field condensate, or Affleck-Dine baryogenesis. Here we give a brief description of these various alternatives.
1. GUT-baryogenesis

This model relied on out-of-equilibrium decays of heavy GUT gauge bosons $X, Y \rightarrow q_{l} \bar{q}$, and $X, Y \rightarrow \bar{q}q$, for reviews see [Dolgov, 1992; Kolb and Turner, 1988]. The decay rate of the gauge boson goes as $\Gamma_{X} \sim \alpha_{X}^{1/2} M_{X}$, where $M_{X}$ is the mass of the gauge boson and $\alpha_{X}^{1/2}$ is the GUT gauge coupling. Assuming that the universe was in thermal equilibrium at the GUT scale, the decay temperature is given by

$$T_{D} \approx g_{*}^{-1/4} \alpha_{X}^{1/2} (M_{X} M_{P})^{1/2}, \quad (129)$$

which is smaller than the gauge boson mass. Thus, at $T \approx T_{D}$, one expects $n_{X} \approx n_{X} \approx n_{\gamma}$, and hence the net baryon density is proportional to the photon number density $n_{B} = \Delta B n_{\gamma}$. However below $T_{D}$ the gauge boson abundances decrease and eventually they go out-of-equilibrium. The net entropy generated due to their decay heats up the universe with a temperature which we denote here by $T_{R}$. Let us naively assume that the energy density of the universe at $T_{D}$ is dominated by the $X$ bosons with $\rho_{X} \approx M_{X} n_{X}$, and their decay products lead to radiation with an energy density $\rho = (\pi^{2}/30) g_{*} T^{4}$, where $g_{*} \sim (100)$ for $T \geq M_{GUT}$. Equating the expressions for the two energy densities one obtains

$$n_{X} \approx \frac{\pi^{2}}{30} g_{*} T_{R}^{4} / M_{X}. \quad (130)$$

Therefore the net baryon number comes out to be

$$B \equiv \frac{n_{B}}{s} \approx \frac{\Delta B n_{X}}{g_{*} n_{\gamma}} \approx \frac{3}{4} \frac{T_{R}}{M_{X}} \Delta B. \quad (131)$$

$T_{R}$ is determined from the relation $\Gamma_{X}^{2} / (\pi^{2}/90) g_{*} T_{R}^{3} / M_{X}^{2}$. Thus,

$$B \approx \left( \frac{g_{*}^{-1/2} \alpha_{X} M_{P}}{M_{X}} \right)^{1/2} \Delta B. \quad (132)$$

Uncertainties in $C$ and $CP$ violation are now hidden in $\Delta B$, but can be tuned to yield total $B \sim 10^{-10}$ in many models.

Above we have assumed that the universe is in thermal equilibrium when $T \geq M_{X}$. This might not be true, since for $2 \leftrightarrow 2$ processes the scattering rate is given by $\Gamma \sim \alpha^{2} T$, which becomes smaller than $H$ at sufficiently high temperatures. Elastic $2 \rightarrow 2$ processes maintain thermal contact typically only up to a maximum temperature $\sim 10^{14}$ GeV, while chemical equilibrium is lost already at $T \sim 10^{12}$ GeV (Elmfors et al., 1994; Enqvist and Eskola, 1990).

2. Electroweak baryogenesis

A popular baryogenesis candidate is based on the electroweak phase transition, during which one can in principle meet all the Sakharov conditions. There is the sphaleron-induced baryon number violation above the critical temperature, various sources of $CP$ violation, and an out-of-equilibrium environment if the phase transition is of the first order. In that case bubbles of broken $SU(2) \times U(1)_{Y}$ are nucleated into a symmetric background with a Higgs field profile that changes through the bubble wall [Kuzmin et al., 1985; 1987].

There are two possible mechanisms which work in different regimes: local and non-local baryogenesis. In the local case both $CP$ violation and baryon number violation takes place near the bubble wall. This requires the velocity of the bubble wall to be greater than the speed of the sound in the plasma (Ambjorn et al., 1990; Turok and Zadrożny, 1990, 1991), and the electroweak phase transition to be strongly first order with thin bubble walls.

In the non-local case the bubble wall velocity speed is small compared to the sound speed in the plasma. In this mechanism the fermions, mainly the top quark and the tau-lepton, undergo $CP$ violating interactions with the bubble wall, which results in a difference in the reflection and the transmission probabilities for the left and right chiral fermions. The net outcome is an overall chiral flux into the unbroken phase from the broken phase. The flux is then converted into baryons via sphaleron transitions inside the unbroken phase. The interactions are taking place in a thermal equilibrium except for the sphaleron transitions, the rate of which is slower than the rate at which the bubble sweeps the space (Cohen et al., 1993; Nelson et al., 1992).

For a constant velocity profile of the bubble, $v_{w}$, the net baryon asymmetry is generated by:

$$n_{B} \simeq -\frac{F_{\text{sph}}}{\tau} \int dt \mu_{B}, \quad (133)$$

where $\mu_{B}$ is the chemical potential, which determines the tilt in the free energy of the sphaleron transitions, and numerically it is equivalent to: $\mu_{B} \equiv \rho(z - v_{w} t) / (2N + 5/3) T^{2}$. Here $\rho$ determines the profile of the bubble, and $N$ denotes the number of Higgs doublets. The net baryon asymmetry can be calculated by following Eq. (125):

$$\frac{n_{B}}{s} \simeq \frac{\kappa \alpha_{s}^{2}}{s} \left( \frac{100}{\pi^{2} g_{*}} \right) \left( \frac{F_{z}}{v_{w} T^{3}} \right) \tau T, \quad (134)$$

where $\tau$ is the transport time of the scattered fermions off the bubble wall, and $F_{z} \equiv \int_{-\infty}^{\infty} dz \rho(z)$. For the maximum wall velocity $v_{w} \sim \sqrt{3} / 3$ and a typical: $\tau T \sim 10 - 1000$ for top quarks, the maximum baryon asymmetry is given by: $n_{B} / s \sim 10^{-3} F_{z} / (v_{w} T^{3}) \sim 10^{-6}$. The details of the transport equations can be found in Refs. (Kainulainen et al., 2001; Nelson et al., 1992).

One great challenge for the electroweak baryogenesis is the smallness of $CP$ violation in the SM at finite temperatures. It has been pointed out that an additional Higgs doublet (McLerran et al., 1991; Turok and Zadrożny, 1991).
would provide an extra source for CP violation in the Higgs sector. However, the situation is much improved in the MSSM where there are two Higgs doublets $H_u$ and $H_d$, and two important sources of CP violation (Ellis et al., 1982). The Higgses couple to the chargedinos and neutralinos at one loop level leading to a CP violating contribution. There is also a new source of CP violation in the mass matrix of the top squarks which can give rise to considerable CP violation (Huet and Nelson, 1996).

Bubble nucleation depends on the thermal tunneling rate, and the expansion rate of the universe. The tunneling rate has to overcome the expansion rate in order to have a successful phase transition via bubble nucleation at a given critical temperature $T_c > T_I > T_0$. The effective potential for the Higgs at finite temperatures can be computed, which takes the form:

$$V_{cf}(\phi, T) = (-\mu^2 + \alpha T^2)\phi^2 - \gamma T\phi^3 + (\lambda/4)\phi^4$$  \hspace{1cm} (135)$$

The order parameter is given by the ratio of $(\phi(T_c)/T_c) \sim \gamma/\lambda$, which has to be larger than one for first order phase transition. For $T_c \sim 100$ GeV, one obtains the condition for the sphaleron energy (Rubakov and Shaposhnikov, 1987). Shaposhnikov, 1996; Shaposhnikov, 1987)

$$\frac{E_{sph}(T_c)}{T_c} \geq 7 \log \left[ \frac{E_{sph}(T_c)}{T_c} \right] + 9 \log(10) + \log(\kappa).$$ \hspace{1cm} (136)$$

which implies (Bochkarev et al., 1991)

$$\frac{E_{sph}(T_c)}{T_c} \geq 45 \text{ for } \kappa = 10^{-1},$$ \hspace{1cm} (137)$$

In terms of the Higgs field value at $T_c$,

$$\frac{\phi(T_c)}{T_c} = \frac{g}{4\pi B(\gamma/g2)} \frac{E_{sph}(T_c)}{T_c} \sim \frac{1}{36} \frac{E_{sph}(T_c)}{T_c},$$ \hspace{1cm} (138)$$

where $g$ is gauge coupling of $SU(2)_L$, and $B \sim 1.87$. Then the bounds in Eqs. (137) translate to

$$\frac{\phi(T_c)}{T_c} \geq 1.3,$$ \hspace{1cm} (139)$$

which implies that the phase transition should be strongly first order in order that sphalerons do not wash away all the produced baryon asymmetry. This result is the main constraint on electroweak baryogenesis.

Lattice studies suggest that in the SM the phase transition is strongly first order only below Higgs mass $m_H \sim 72$ GeV (Kajantie et al., 1996; Rummukainen et al., 1998). Above this scale the transition is just a cross-over. Such a Higgs mass is clearly excluded by the LEP measurements (Nakamura et al., 2010), thus excluding electroweak baryogenesis within the SM. However, this opens up a possibility to include new physics beyond the SM.

Electroweak baryogenesis induced by new physics:

it was pointed out that by modifying the SM Higgs self-interactions, especially the cubic term, it is possible to enhance the first order phase transition (Anderson and Hall, 1992). One such example has been considered in (Grojean et al., 2005; Mohapatra and Zhang, 1992) where non-renormalizable contribution to the Higgs potential has been considered of type:

$$V(\Phi) = \lambda \left( \Phi^\dagger \Phi - v^2 \right)^2 + \frac{1}{\Lambda^2} \left( \Phi^\dagger \Phi - v^2 \right)^3$$ \hspace{1cm} (140)$$

where $\Phi$ is the SM electroweak Higgs doublet and $\Lambda$ is the scale of new physics which induces the corrections below the energy scale of $\Lambda \sim O(1)$ TeV. At zero temperature the CP-even scalar state can be expanded in terms of its zero-temperature VEV, $(\phi) = v_0 \simeq 246$ GeV, and the physical Higgs boson $H$: $\Phi = \phi/\sqrt{2} = (H + v_0)/\sqrt{2}$.

The finite temperature effects are taken into account by adding a thermal mass to the potential:

$$V(\phi, T) = c T^2 \phi^2/2 + V(\phi, 0)$$ \hspace{1cm} (141)$$

where $c$ is given in the high-temperature expansion of the one-loop thermal potential:

$$c = \frac{1}{16} \left( 4g^2 + 3g'^2 + g'^2 + 4\frac{m_H^2}{v_0^2} - 12\frac{v_0^2}{\Lambda^2} \right),$$ \hspace{1cm} (142)$$

where $g$ and $g'$ are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, and $y_t$ is the top Yukawa coupling. Note that there is no trilinear term in the effective potential.

The critical temperature $T_c$, at which the minima $\phi \neq 0$ and $\phi = 0$ are degenerate is given by

$$T_c^2 = \frac{\Lambda^4 m_H^2 + 2\Lambda^2 m_H v_0^2 - 3v_0^4}{16c v_0^2}.$$ \hspace{1cm} (143)$$

The VEV of the Higgs field at the critical temperature in terms of $m_H$, $\Lambda$ and $v_0$ is

$$\langle \phi^2(T_c) \rangle = v_c^2 = \frac{3}{2} v_0^2 - \frac{m_H^2 \Lambda^2}{2v_0^2}.$$ \hspace{1cm} (144)$$

From Eqs. (143) and (144), one finds that for any given $m_H$ there is an upper bound on $\Lambda$ to make sure that the phase transition is always first order ($v_c^2 > 0$), and there is a lower bound on $\Lambda$ to make sure that the $T = 0$ minimum at $\phi \neq 0$ is a global minimum ($T_c^2 > 0$). These two combinations chart out a region where the phase transition is indeed first order:

$$\max \left( \frac{v_0^2}{m_H}, \frac{\sqrt{3}v_0^2}{m_H^2 + 2m_c^2} \right) < \Lambda < \frac{\sqrt{3}v_0^2}{m_H}$$ \hspace{1cm} (145)$$

where $m_c = v_0 \sqrt{(4g^2 + 3g'^2 + g'^2)/8} \approx 200$ GeV. In order to ensure that the thermal mass correction is positive: $c > 0 \rightarrow \Lambda > \sqrt{3v_0^2}/\sqrt{m_H^2 + 2m_c^2}$. For these ranges of $\Lambda$ the ratio $v_c/T_c > 1$, ensuring a successful sphaleron transition for the Higgs mass $m_H \geq 115$ GeV. One nice aspect of this model is that the non-renormalizable scale $\Lambda$ can also be constrained from the precision electroweak observable, which can be tested in near future by the LHC (Grojean et al., 2005).
3. Electroweak baryogenesis in MSSM

In the MSSM the ratio $\Phi(T_c)/T_c$ can be increased by virtue of the scalar loops which can make the cubic term in the temperature dependent Higgs potential large: $V_{eff}(\varphi, T) = (-\mu^2 + \alpha T^2)\varphi^2 - \gamma T \varphi^3 + (\lambda/4)\varphi^4$. In particular the right handed stop $t_R$ coupling to the Higgs with a large Yukawa coupling. This leads to a strong first order phase transition – as the ratio of $\Phi(T_c)/T_c \sim \gamma/\lambda \geq 1$, where $\gamma$ determines the order parameter (Carena et al., 1998; Cline et al., 1998; 2000; Laine, 1996; Laine and Rummukainen, 1998).

The finite temperature cubic term is given by: $\gamma T \varphi^3 \approx (T/4\pi)[m_{tR}^2 (\varphi, T)]^{3/2}$, where the lightest right handed stop mass

$$m_{tR}^2 \approx m_{1/2}^2 + \xi T^2 + 0.15M_Z^2 \cos(2\beta) + m_{1/2}^2 \left(1 - \frac{\tilde{A}_t^2}{m_{1/2}^2}\right),$$

(146)

where $\tilde{A}_t = A_t - \mu/\tan(\beta)$ is the stop mixing parameter, $A_t$ is the trilinear term in the MSSM superpotential, and $\mu$ is the soft-SUSY breaking mass parameter for the right-handed stop. The coefficient $\gamma$ of the cubic term $\gamma T \varphi^3$ in the effective potential reads

$$\gamma_{MSSM} \approx \gamma_{SM} + \frac{h_s^3 \sin^3(\beta)}{4\sqrt{2}\pi} \left(1 - \frac{\tilde{A}_t^2}{m_{1/2}^2}\right)^{3/2},$$

(147)

and can be at least one order of magnitude larger than $\gamma_{SM}$. The implications for the particle spectrum are:

- A light right-handed stop: 120 GeV $\leq m_{tR} \leq 170$ GeV $\leq m_{1/2}$.
- A heavy left-handed stop: $m_{Q_3} \geq 2$ TeV.
- A light SM-like Higgs: $m_H \leq 120$ GeV, for $5 < \tan(\beta) < 10$.

The present LEP constraint on the lightest CP-even Higgs mass is $m_{H} \geq 115$ GeV (Nakamura et al., 2010). Note that within MSSM, the lightest Higgs mass is bounded by: $m_{H}^2 \leq M_Z^2 \cos^2(2\beta)$. Hence, even an MSSM-based electroweak baryogenesis may be at the verge of being ruled out.

MSSM also provides new CP violating complex phases in the Higgsino sector, i.e. $arg(\mu M_{1/2}) \geq 10^{-2}$, with $\mu$, $M_{1/2} \leq 400$ GeV. The CP-violating phases are also constrained by the electric dipole moments. To match the observational limit on $|d_e| < 1.6 \times 10^{-27}$ e cm (Regan et al., 2002), one requires first and second generation sfermion masses greater than 10 TeV, while the 2-loop electron dipole moment contribution comes out to be: $|d_e| > 2 \times 10^{-28}$ e cm.

The definitive test of the MSSM based electroweak baryogenesis will obviously come from the Higgs and the stop searches at the LHC (Carena et al., 2003; Chung et al., 2009).

4. Electroweak baryogenesis beyond MSSM

Some of these problems of MSSM can be resolved in nMSSM (next-to minimal SUSY SM), with the help of introducing an extra singlet in the MSSM superpotential: $W = m^2 S + \lambda S H_u H_d + W_{MSSM}$. The $S$ field gets a VEV to explain the $\mu$ $\equiv \lambda(S)$-term, but it also generates a singlet tadpole – its contribution to the vacuum energy, $\delta V = t_s S \sim (1/16\pi^2)(S/M_F)^2$, can be suppressed with the help of discrete symmetries, $Z_2^F$ or $Z_2^L$, where $F_s \sim m_{soft} M_F$ (Abel et al., 1995; Panagiotakopoulos and Tamvakis, 1999). As a result the soft-SUSY breaking Higgs potential becomes:

$$V_{soft} = ts(S+h.c.) + m_s^2 |S|^2 + a_{\lambda}(S H_u H_d + h.c.) + V_{MSSM},$$

(148)

Note that the trilinear, $a_{\lambda} S H_u H_d$ now contributes to the $\gamma$-term at the tree level, indicating potentially stronger first order phase transition even without a light stop and for $m_H > 120$ GeV. The CP phases are distributed in gaugino masses as well as in the singlet, but not in the tree level of $a_{\lambda}$.

One can similarly proceed with 4 SM singlets, and the Higgs doublet as in the case of $U(1)'$ electroweak baryogenesis discussed in Ref. (Kang et al., 2005), for a review see (Kang et al., 2009), where the superpotential contains:

$$W = h S H_u H_d + \lambda S_1 S_2 S_3 + W_{MSSM}. $$

(149)

It is assumed that the $U(1)'$ is broken at higher VEVs, such as 1 – 2 TeV, and then the electroweak symmetry is broken at lower scales. The singlets $S_1$, $S_2$, $S_3$ have VEVs greater than those of $S$ and, $H_u$ and $H_d$. The mass of $Z'$ bosons are $M_{Z'} \sim O(1)$ TeV. The tree level Higgs potential can now contain CP violating contributions from the phases $\beta_1$, $\beta_2$ (Kang et al., 2009, 2005):

$$V_{soft} = V_{MSSM} + m_{s_i}^2 |S_i|^2 + \sum_{i=1}^3 m_{S_i}^2 |S_i|^2$$

$$- 2 A_h h |S||H_u^0||H_d^0| \cos \beta_3 - 2 A_1 \lambda |S_1||S_2||S_3| \cos \beta_4$$

$$- 2 m_{S_i}^2 |S_i||S_i| \cos \beta_1 - 2 m_{S_i S_j}^2 |S_i||S_j| \cos \beta_2$$

$$- 2 |m_{S_i S_j}|^2 |S_i||S_j| \cos(-\beta_1 + \beta_2 + \gamma) $$

(150)

The potential can yield strong first order phase transition without large stop masses, and the new contributions to electron dipole moments can be tamed by tuning the Yukawa sector (Kang et al., 2009, 2005).

5. Thermal Leptogenesis

At temperatures $10^{12}$ GeV $\geq T \geq 100$ GeV, the $B + L$ is completely erased by the sphaleron transitions, a net baryon asymmetry in the universe can still be generated from a non-vanishing $B - L$ (Fukugita and Yanagida, 1986; Harvey and Kolb, 1984; Luty, 1992), even if there
were no baryon number violating interactions. The lepton number violating interactions can produce baryon asymmetry, a process which is known as leptogenesis, for recent reviews, see Buchmuller et al. 2005, Davidson et al. 2008.

The lepton number violation requires physics beyond the SM. The most attractive mechanism arises in SO(10) which is left-right symmetric (for details, see Langacker, 1981), and has a natural foundation for the see-saw mechanism (Gell-Mann and Slansky, 1980; Minkowski, 1977; Mohapatra and Senjanovic, 1980; Yanagida, 1979) as it incorporates a singlet right-handed Majorana neutrino \( N_R \) with a mass \( M_R \). A lepton number violation appears when the Majorana right handed neutrino decays into the SM lepton doublet and Higgs doublet, and their CP conjugate state through

\[ N_R \rightarrow H + l, \quad N_R \rightarrow \bar{H} + \bar{l}, \]  

where \((H)\) \( l \) is the SM (Higgs) lepton. The relevant L violating interaction is then given by

\[ \mathcal{L} \supset \frac{1}{2} (M_N)_{ij} N_i N_i \gamma_5 \gamma_j \sqrt{2} H^* \gamma^5 + \text{h.c.}, \]  

where \( i, j = 1, 2, 3 \). The above interaction is also responsible for generating the observed neutrino masses via the canonical seesaw mechanism (Akhmedov et al. 2003, Buchmuller et al. 2002, Buchmuller and Plumacher, 1998, 2000), as required by the neutrino oscillation data (Gonzalez-Garcia et al. 2010). This mass turns out to be \( m_\nu \approx |y|^2 v^2/M_N \) with \( v = 174 \text{ GeV} \), what implies right-handed neutrino mass scale of \( M_N \sim O(10^{14}) \text{ GeV} \) for \( |y| \sim 1 \) and \( m_\nu \sim 0.1 \text{ eV} \).

Assuming a normal hierarchy in the heavy right handed neutrino sector, \( M_1 \ll M_2, \, M_3 \) (corresponding to \( N_1, \, N_2, \, N_3 \)). The CP asymmetry can be estimated from the \( N_1 \) decay, the asymmetry is generated through the interference between tree level and one-loop diagrams, which is given by (Covi et al. 1996, Flanz et al. 1995, Fukugita and Yanagida, 1986, Luu, 1992, Plumacher, 1998)

\[ \epsilon = \frac{1}{8\pi} \Im[yy^\dagger] \sum_{i=1,2,3} f(M_i^2/M_i^2), \]  

where \( f \) is a function which represents radiative corrections. In the case of SM, \( f(x) = \sqrt{x(x-2)/(x-1)}(x-1)\ln(1+1/x) \), and in the case of MSSM, \( f(x) = \sqrt{x^2/(x-1)} + \ln(1+1/x) \).

Let us take an example of the SM where the CP phase can be labeled by, \(|\epsilon| = 3M_1/(16\pi^2)\sqrt{\Delta m^2_{atm}} \sin\delta \), where \( \Delta m^2_{atm} \) is the atmospheric mass scale of light neutrinos (Gonzalez-Garcia et al. 2010) and \( \delta \) is the effective CP violating phase. The total lepton asymmetry is then given by

\[ \eta_L = |\epsilon| Y_{N_1} \kappa, \]  

where \( Y_{N_1} \) is the abundance of the right handed Majorana neutrino \( N_1 \) and \( \kappa \) is a thermal wash-out factor, which takes into account that the scatterings such as \( lH \leftrightarrow \ell\bar{H} \) tend to wash out any lepton asymmetry being created.

In order to process the total lepton asymmetry into baryons, we need to know the chemical potentials (Khlebnikov and Shaposhnikov, 1988)

\[ B = \sum_i (2\mu_{qi} + \mu_{d_i}), \quad L = \sum_i (2\mu_{ti} + \mu_{e_i}), \]  

where \( i \) denotes three leptonic generations. The Yukawa interactions establish an equilibrium between the different generations \((\mu_{ti} = \mu_t \text{ and } \mu_{qi} = \mu_q \text{, etc.})\), and one obtains expressions for \( B \) and \( L \) in terms of the number of colors \( N = 3 \), and the number of charged Higgs fields \( N_H \)

\[ B = -\frac{4N}{3} \mu_t, \quad L = \frac{14N^2 + 9NN_H}{6N + 3N_H} \mu_t, \]  

together with a relationship between \( B \) and \( B - L \) (Khlebnikov and Shaposhnikov, 1988)

\[ B = \left( \frac{8N + 4N_H}{22N + 13N_H} \right) (B - L), \]  

The final asymmetry is then given by \( B = (28/79)(B - L) \) in the case of SM and \( B = (8/23)(B - L) \) for the MSSM (Khlebnikov and Shaposhnikov, 1988).

The baryon asymmetry based on the decays of right handed neutrinos in a thermal bath has been computed within MSSM (Buchmuller et al. 2002, 2005a, Giudice et al. 2004), where besides the right handed neutrinos the right handed (s)neutrinos also participate in the interactions. The decay of a RH (s)neutrino with mass \( M_1 \) results in a lepton asymmetry via one-loop self-energy and vertex corrections, see Eq. (113). If the asymmetry is mainly produced from the decay of the lightest right handed states, and assuming hierarchical right handed (s)neutrinos \( M_1 \ll M_2, \, M_3 \), we will have (Davidson and Ibarra, 2002)

\[ \eta \simeq 3 \times 10^{-10} \kappa \left( \frac{m_3 - m_1}{0.05 \text{ eV}} \right) \left( \frac{M_1}{10^9 \text{ GeV}} \right), \]  

for \( O(1) \) CP-violating phases \((m_{\nu_1} < m_{\nu_2} < m_{\nu_3}) \) are the masses of light mostly light handed neutrinos. Here \( \kappa \) is the efficiency factor accounting for the decay, inverse decay and scattering processes involving the right handed states (Buchmuller et al. 2002, 2003, 2005a, Giudice et al. 2004).

A decay parameter \( K \) can be defined as

\[ K \equiv \frac{\Gamma_1}{H(T = M_1)}, \]  

where \( \Gamma_1 \) is the decay width of the lightest right handed (s)neutrino. If \( K < 1 \), the decay of right handed
states will be out of equilibrium at all times. In this case the right handed states, which are mainly produced via scatterings of the left handed (s)leptons off the top (s)quarks and electroweak gauge/gaugino fields, never reach thermal equilibrium. The cross-section for producing the right handed (s)neutrinos is $\propto T^{-2} (M_2^2)$, when $T > M_1$ ($< M_1$), and hence most of them are produced when $T \sim M_1$. The efficiency factor reaches its maximum value for $\kappa \simeq 0.1$ when $m_{\nu} = 10^{-3}$ eV. For larger values of $m_{\nu}$ it drops again, because the inverse decays become important and suppress the generated asymmetry. Producing sufficient asymmetry then sets a lower bound, $M_1 > 10^9$ GeV [Buchmuller et al. 2002, 2003, 2005a; Giudice et al. 2004]. Successful thermal leptogenesis therefore requires that $T_R \geq 10^9$ GeV.

a. Resonant leptogenesis: If the mass splitting between, say $M_1$, $M_2$ is comparable to their decay widths, the $CP$ asymmetry resonantly gets enhanced, see Eq. (155). For example, let us consider $N_1$ and $N_2$. The dominant contribution to the $CP$ asymmetry arises in the mixing of $N_1$ and $N_2$, and it is given by [Pilaftsis and Underwood, 2004, 2005; Pluhar, 1998].

$$\epsilon_1 = \frac{\text{Im}(y^l y^r) \Gamma}{8 \pi (y^l y^r)^{2/3}} (M_2^2 - M_1^2) M_1 M_2 (M_2^2 - M_1^2)^2 (M_2^2 - M_3^2)^2 (M_1^2 - M_1^2)^2 .$$

(161)

Now, assuming $M_1 \sim M_2$ and $M_3 - M_2 \sim \Gamma_1 - \Gamma_2$, there is a large $L$ asymmetry can be produced even if the initial abundance of $N_1$ and $N_2$ is small.

b. Flavored leptogenesis: So far we have assumed that all the leptonic flavors, i.e. $\tau$, $\mu$, $e$, behave alike in a thermal bath. Especially in a non-SUSY case, where we can imagine a thermal bath of SM lepton number equally between left handed and right handed particles with the net lepton number zero. A specific example will be when the decay gives rise to a negative lepton number in left-handed neutrinos, and a positive lepton number of equal magnitude in right-handed neutrinos. If the observed neutrinos are Dirac in nature with a small Yukawa couplings $h \sim 10^{-12}$, then the left and right handed neutrinos will not come to thermal equilibrium before the electroweak scale, $T \sim T^{2/3} M_1 \sim h^2 T$. Since the spherons interact with the left-handed neutrinos, violating $B + L$ and conserving $B - L$, part of the lepton number in left handed neutrinos get converted into baryon number. At lower temperatures, the universe contains a total positive baryon number, total positive lepton number, and $B - L = 0$.

$$h_{ia} l_i N_a \phi + h_{ka} l_i k N_a \phi' + \frac{1}{2} M_{ab} N_a N_b + \text{H.C.}$$

(163)

where the Yukawa interactions are given by $h_{ia}$ and $h_{ka}$. After integrating out the heavy neutrinos $N$ with a mass $M_4 = g_4 M$, where $M$ being the overall mass scale and $g_4$ are order one real constants, we obtain an effective 5 operators

$$\frac{A_{ij} l_i l_j \phi + D_{ia} l_i k N_a \phi' + A_{il} l_i \eta \phi'}{2 M} + \frac{N_a}{2 M} + \text{H.C.},$$

(164)

with coupling constant matrices of the form $A = h g^{-1} h^*$, $A' = h' g^{-1} h'^*$ and $D = h g^{-1} h^*$, and the reheating temperature is below $M$. Let us suppose that the reheate temperature in both hidden, $T_R$, and visible sector, $T_R$, are below $M$. The only way the two sectors can interact via the lepton number violating scatterings mediated by the heavy neutrinos $N$ which stay out of equilibrium, since $T_R \ll M$. The $CP$ phase can be obtained in $l \phi \leftrightarrow l' \phi'$ and $\ell \phi \leftrightarrow \ell' \phi'$. The net asymmetry is given by $\Delta \sigma = 3 J S / 32 \pi^2 M^4$, where $J = \text{Im} \text{Tr} (h l h') g^{-2} (h l h') g^{-1} (h l h') g^{-1}$ is the $CP$-violation parameter and $S$ is the c.m. of energy square. The final $B - L$ asymmetry of the universe is given by [Bento and Bereziani, 2001]

$$B - L = \frac{n_{B - L}}{s} = \left[ \frac{\Delta \sigma n_\nu^2}{4 H s} \right] R ,$$

$$\approx 10^{-8} J \left( \frac{10^{12} \text{ GeV}}{M} \right)^4 \left( \frac{T_R}{10^8 \text{ GeV}} \right)^3$$

(165)

where $s$ is the entropy density, and for Yukawa constants spreading in the range $0.1 - 1$ can achieve the right lepton asymmetry.
e. Non-thermal leptogenesis: There exist various scenarios of non-thermal leptogenesis [Allahverdi and Mazumdar, 2003; Asaka et al. 1999; Giudice et al. 1999; Lazarides and Shafi 1991; Murayama et al. 1993] which can work for $T_R \lesssim M_N$. One classic example is when the right handed sneutrino, a scalar field, with mass $M_N$, dominates the energy density of the universe and decays into the SM leptons and Higgs to reheat the universe and simultaneously creating the lepton asymmetry. The CP asymmetry can be created again from the interference between a tree level and one-loop quantum corrections, which yields the net asymmetry:

$$\frac{\eta}{s} \sim \frac{n_L}{s} \sim \frac{\rho_N}{s M_N} \sim \frac{3}{4} \frac{\epsilon T_R}{M_N}. \tag{166}$$

A similar expression can be used if any sneutrino condensate decays after inflation [Berezhiani et al. 2001; Mazumdar, 2004a,b; Mazumdar and Perez-Lorenzana, 2004a,b; Postma and Mazumdar, 2004], in which case $T_R$ is replaced by the decay temperature of the sneutrino condensate, i.e. $T_D$. The right handed neutrinos and sneutrinos could also be excited non-thermally during preheating if they couple to the inflaton, which would generate non-thermal leptonogenesis [Giudice et al. 1999a].

f. Soft leptogenesis: In a perfect SUSY preserving limit the mass and the width of the right-handed neutrino and sneutrino would be the same. Let us consider a single generation, where the mass is $M_N$, and their width is given by $\Gamma = Y^2 M_N 4\pi = m M_N^2 / 4\pi v^2$, $m \equiv Y^2 v^2 / M_N$, where $v \sim 174$ GeV is the Higgs VEV, and $Y_N$ is the Yukawa coupling. However, in a realistic scenario we would expect soft SUSY breaking terms which would be relevant for soft-leptogenesis [Allahverdi et al. 2003; D’Ambrosio et al. 2003; Grossman et al. 2003 (2005)]:

$$\mathcal{L}_{\text{soft}} = \frac{B M_N}{2} \tilde{N} \tilde{N} + A Y \tilde{L} \tilde{N} H + \text{h.c.} \tag{167}$$

This model has one physical CP violating phase given by: $\phi = \arg(AB^*)$. The soft SUSY breaking terms introduce mixing between the sneutrino $\tilde{N}$ and the anti-sneutrino $\tilde{N}$ in a similar fashion to the $B^0 - \bar{B}^0$ and $K^0 - \bar{K}^0$ systems. The mass and width difference of the two sneutrino mass eigenstates are given by

$$\Delta m = |B|, \quad \Delta \Gamma = \frac{2|A| \Gamma}{M_N}. \tag{168}$$

The CP violation in the mixing is responsible for generating the lepton-number asymmetry in the final states of the $\tilde{N}$ decay. This lepton asymmetry is converted into the baryon asymmetry through the sphaleron process. The baryon to entropy ratio is given by [D’Ambrosio et al. 2003]:

$$\frac{n_B}{s} = -10^{-3} \alpha \left[ \frac{4|A| |B|}{4|B|^2 + \Gamma^2} \right] \frac{|A|}{M_N} \sin \phi, \tag{169}$$

where the efficiency parameter $\alpha$ depends on the mechanism that produces the right-handed sneutrinos. In a thermal production, the largest conceivable value could be of order $\alpha \sim 0.1$, for the light neutrino mass $m_\nu \sim 10^{-3}$ eV [D’Ambrosio et al. 2003]. It may be slightly challenging to fix the parameters to obtain the right lepton asymmetry either making $|B|/\Gamma$ or $\Gamma/|B|$ small. The above requirement gives a non-trivial constraint on the parameters [D’Ambrosio et al. 2003; Grossman et al. 2003]:

$$A \sim 10^2 \text{ GeV}, \quad M_N \lesssim 10^8 \text{ GeV}, \quad B \lesssim 1 \text{GeV}, \quad \phi \sim 1. \tag{170}$$

Small value of $|B|$ cannot be obtained in gravity mediated SUSY breaking scenarios, but it might be possible to arrange within gauge mediated SUSY breaking [Grossman et al. 2004].

6. Affleck-Dine Baryogenesis

As we discussed already, within MSSM there exists cosmologically flat directions [Dine et al. 1996b; Gherghetta et al. 1996]. Field fluctuations along such flat directions are smoothed out by inflation [Enqvist and Mazumdar, 2003], which effectively stretches out any gradients, and only the zero mode of the scalar condensate remains. Baryogenesis can then be achieved by the perturbative decay of a condensate [Allahverdi and Mazumdar, 2007a, 2008] that carries baryonic charge, as was first pointed out by Affleck and Dine (AD) [Affleck and Dine, 1985]. As we will discuss, the flat direction condensate can get dynamically charged with a large $B$ and/or $L$ by virtue of CP-violating self-couplings.

In the original version [Affleck and Dine, 1985] baryons were produced by a direct decay of the condensate. It was however pointed out that in the case of gauge mediated SUSY breaking [Kusenko, 1997b; Kusenko and Shaposhnikov, 1998], and in the case of gravity mediated SUSY breaking [Enqvist et al. 2000; Enqvist and McDonald, 1998, 1999, 2000], that the AD flat direction condensate in most cases is not stable but fragments and eventually forms non-topological solitons called Q-balls [Coleman, 1985]. In gauge mediated SUSY breaking scenarios these Q-balls can be made a long lived dark matter candidate [Kusenko et al. 1998; Kusenko and Shoemaker, 2003]. For reviews see [Dine and Kusenko, 2004; Enqvist and Mazumdar, 2003].

Since, SUSY is broken by the finite energy density of the inflaton, the AD condensate receives corrections in the case of F-term inflation. Let us consider a generic superpotential for the AD field given by Eq. (29), then the effective potential for the AD field will be given by
The first and the third terms are the Hubble-induced and low-energy soft mass terms, respectively, while the second and the fourth terms are the Hubble-induced and low-energy A terms. The last term is the contribution from the non-renormalizable superpotential. The coefficients $|C_I|$, $a$, $\lambda_d \sim O(1)$, and the coupling $\lambda \approx 1/(d-1)$!. Note that low-energy $A_\phi$ term has a mass dimension. The $a$, $A$-terms in Eq. (171) violate the global $U(1)$ symmetry carried by $\phi$. If $|a|$ is $O(1)$, the phase $\theta$ of $\langle \phi \rangle$ is related to the phase of $a$ through $n\theta + \theta_a = \pi$; otherwise $\theta$ may take some random values, which will generally be of $O(1)$. This is the initial CP-violation which is required for baryogenesis/leptogenesis. The AD baryogenesis is quite robust and can occur even in presence of positively large Hubble-induced corrections [Kasuya and Kawasaki 2006].

At large VEVs the first term dictates the dynamics of the AD field if $C_I < 0$, the absolute minimum of the potential is $\phi = 0$ and during inflation the condensate will evolve to its global minimum in one Hubble time. On the other hand if $C_I > 0$, the absolute value of the AD field settles during inflation to the minimum given by $|\phi| \approx (H_t M_p^{d-3} |C_I|)^{1/2}$. After the end of inflation the minimum of the condensate evolves from its initial large VEV to its global minimum $\phi = 0$, note that the dynamics is non-trivial when the condensate starts oscillating when $H(t) \sim m_\phi \sim O(10^2)$ GeV. The dynamics of the AD condensate is non-trivial as shown in Fig. 4.

If inflation is driven by D-term, one does not get the Hubble induced mass correction to the flat direction so that $C_I$, $a = 0$. Also the Hubble induced $a$-term is absent. However the Hubble induced mass correction eventually dominates once D-term induced inflation comes to an end.

The baryon/lepton number density is related to the dynamics of the AD field by

$$n_{B,L} = \beta i (\phi^* \phi - \phi \phi^*)$$

(172)

where $\beta$ is corresponding baryon and/or lepton charge of the AD field. The equation of motion for the AD field is given by

$$\ddot{\phi} + 3H \dot{\phi} + \partial V(\phi)/\partial \phi^* = 0$$

(173)

The above two equations give rise to

$$\dot{n}_{B,L} + 3H n_{B,L} = 2\beta \Im \left[ \frac{\partial V(\phi)}{\partial \phi^*} \right]$$

$$= 2\beta \frac{m_\phi}{M_p^2} \Im (a \phi^d)$$

(174)

The net baryon and/or lepton number can be obtained by integrating the above equation

$$a^3(t) n_{B,L}(t) = 2\beta |a| \frac{m_\phi}{M_p^2} \int_0^t a^3(t') |\phi(t')|^d \sin(\theta) \, dt'$$

(175)

Note that $"a"$ introduces an extra CP phase which can be parameterized by $\sin(\delta)$. Note that the asymmetry is not governed by the Hubble induced $A$ term, the amplitude of the oscillations will be damped and so the $A$-term, which is proportional to a large power of $\phi$ will become gradually negligible. The net baryon and/or lepton asymmetry is given by [Dine et al. 1996b]

$$n_{B,L}(t_{osc}) = \frac{\beta}{3(d-2)} \frac{m_\phi}{M_p^2} \sin \theta \sin \delta$$

$$\approx \frac{\beta}{3(d-2)} \frac{m_\phi}{M_p^2} \left( m_\phi M_p^{2(d-3)} \right)^{2/(d-2)}$$

(176)

where $\sin \delta \sim \sin 2\beta \approx O(1)$. When the inflaton decay products have completely thermalized with a reheat temperature $T_R$, the baryon and/or lepton asymmetry is given by

$$n_{B,L}/s = \frac{1}{4} \frac{T_R}{H(t_{osc})^2} n_{B,L}(t_{osc})$$

$$= \frac{d-2}{6(d-3)} \beta \frac{T_R}{M_p^2 m_\phi} \left( m_\phi M_p^{d-2} \right)^{2(d-2)}$$

(177)

where we have used $H(t_{osc}) \approx m_\phi$, and $s$ is the entropy density of the universe at the time of reheating. For $d = 4$, the baryon-to-entropy ratio is

$$n_{B,L}/s \approx 1 \times 10^{-10} \times \beta \left( \frac{1 \text{ TeV}}{m_\phi} \right) \left( \frac{T_R}{10^2 \text{ GeV}} \right)$$

(178)

and for $d = 6$

$$n_{B,L}/s \approx 10^{-10} \times \beta \left( \frac{1 \text{ TeV}}{m_\phi} \right)^{1/2} \left( \frac{T_R}{100 \text{ GeV}} \right)$$

(179)
where we have taken the net $CP$ phase to be $\sim O(1)$. The asymmetry remains frozen unless there is additional entropy production afterwards.

The lepton asymmetry calculated above in Eqs. (178, 179) can be transformed into baryon number asymmetry via sphalerons $n_B/s = (8/23)n_L/s$. AD leptogenesis has important implications in neutrino physics also, because in the MSSM, the $LH_u$ direction is lifted by the $d = 4$ non-renormalizable operator which also gives rise to neutrino masses [Asaka et al. 2000a; Dine et al. 1996b]:

$$W = \frac{1}{2M_i} (L_i H_u)^2 = \frac{m_{\nu_i}}{2} \langle H_u^2 \rangle^2 (L_i H_u)^2,$$  

(180)

where we have assumed the see-saw relation $m_{\nu_i} = \langle H_u^2 \rangle^2 / M_i$ with diagonal entries for the neutrinos $\nu_i$, $i = 1, 2, 3$. The final $n_B/s$ can be related to the lightest neutrino mass since the flat direction moves furthest along the eigenvector of $L_i L_j$ which corresponds to the smallest eigenvalue of the neutrino mass matrix [Asaka et al. 2000a; Dine et al. 1996b).

$$\frac{n_B}{s} = 1 \times 10^{-10} \times 6 \left( \frac{m_{\nu_i}}{m_{\nu_i}} \right) \left( \frac{T_R}{10^8 \text{ GeV}} \right) \left( \frac{10^{-6} \text{ eV}}{m_{\nu_i}} \right),$$  

(181)

where $m_{\nu_i}$ denotes the lightest neutrino. Similarly the $d = 6$ case corresponds to the flat direction of an $L_i$.

7. Baryogenesis below the electroweak scale

At temperatures below the electroweak scale, the sphaleron transitions are rather inactive. If the universe reheats below the electroweak scale then it is a challenge to generate the required baryon asymmetry. The leptogenesis based scenarios are hard to implement below the electroweak scale. However within SM there exists a possibility of realizing cold electroweak baryogenesis as discussed in [Cornwall and Kusenko, 2001; Cornwall and Kusenko, 2000; Enqvist and McDonald, 2000; Enqvist et al., 2001; Garcia-Bellido et al., 1999; Krauss and Trodden, 1999; Tranberg et al., 2001; Tranberg and Smit, 2003; Tranberg et al., 2010]. SUSY further opens a door to realize baryon asymmetry at temperatures even close to the BBN via R-parity violating interactions [Cline and Rabl, 1994; Kitano et al., 2008; Kohri et al., 2009; Scherrer et al., 1991]. Here we will discuss both the scenarios.

a. Cold electroweak baryogenesis: There are mechanisms to obtain cold electroweak baryogenesis where it is assumed that the SM $d.o.f$ are not in thermal equilibrium. Moreover in a cold environment the CP-violation in SM is much larger than at the electroweak temperatures, and most of the baryon asymmetry is produced at the initial quench when the Higgs field is rolling down the potential. Baryon production essentially stops after the first few oscillations, after which the coherent Higgs field will start decaying, thereby reheating the universe. However, the hurdle is to obtain this fast quench without the presence of strong first order phase transition.

There are couple of possibilities of realizing cold initial condition and out of equilibrium condition. There could be a very low scale of inflation which might not be responsible for generating the seed perturbations, or the universe could be simply trapped in a vacuum where the SM $d.o.f$ are not even excited. The out-of-equilibrium condition can be obtained during the coherent oscillations of the scalar fields. In order to realize this idea, we would require a scalar field coupled to the SM Higgs, $\sigma^2 H^2$. During the coherent oscillations, it is possible to have the baryon number violating sphaleron transitions [Cornwall and Kusenko, 2000; Garcia-Bellido et al., 1999; Tranberg and Smit, 2003]. This can happen since the Higgs oscillations can excite the electroweak gauge bosons from the time dependent vacuum fluctuations with a very large occupation number, similar to case of preheating. These long wavelength fluctuations of the gauge fields are responsible for overcoming the sphaleron barriers which leads to the baryon number violation. Furthermore, there could be extra sources of CP-violations during the oscillations as pointed out in [Cornwall et al., 2001; Cornwall and Kusenko, 2000; Tranberg et al., 2010].

b. R-parity violation and baryogenesis: The current limits on some of the R-parity violating interactions are poorly understood. Let us now consider a scenario where B and L are violated within MSSM, with a superpotential:

$$W = \mu_i L_i H_u + \lambda_{ijk} L_i L_j e_k + \lambda'_{ijk} L_i Q_j d_k + \lambda''_{ijk} u_i d_j d_k,$$  

(182)

where $L_i = (\nu_i, e_i)$, $Q_i = (u_i, d_i)$, $H_u = (h^+_u, h^0_u)^T$, $H_d = (h^+_d, h^0_d)^T$, etc are $SU(2)_L$ doublets and $u_i^c, d_i^c$ are $SU(2)_L$ singlet quarks. In Eq. (182), the first three terms violate lepton number by one unit ($\Delta L = 1$), while the last term violates baryon number by one unit ($\Delta B = 1$). For the stability of proton we assume that $\lambda_{ijk} = \lambda'_{ijk} = 0$. This can be accomplished if there exists any conservation of lepton number, which then forces $\mu_i^c$ to be zero. However, the electric dipole moment of neutron gives [Barbier et al., 2005]

$$\text{Im} \left( \lambda'_{1212} \lambda''_{3322} \right) < 0.03 \left( \frac{0.01}{V_{td}} \right) \left( \frac{\tilde{M}}{\text{TeV}} \right)^2,$$  

(183)

and the non-observation of $n-\bar{n}$ oscillation gives an upper bound on $\lambda''_{111k}$ to be [Barbier et al., 2005]

$$|\lambda''_{111k}| < (10^{-6} - 10^{-5}) \left( \frac{10^8 \text{s}}{\tau_{\text{osc}} \tilde{M}} \right)^{5/2}.$$  

(184)

While $\lambda''_{3322}$ is hardly constrained and can be taken to be as large as $O(1)$. Let us consider that a scalar field $\phi$ decays to MSSM $d.o.f$ right before BBN primarily into
gauge bosons and gauginos via $R$-parity violating couplings $\lambda_{ijk}$. Let us assume that the gauginos are heavier than the quarks and squarks. As a result their decay to a pair of quark and squark through one loop quantum correction gives rise to a net CP violation. The magnitude of CP violation in the decay: $\tilde{g} \to \tilde{t}\tilde{c}$ can be estimated as \cite{Cline and Raby 1999}:

$$\epsilon = \frac{\Gamma (\tilde{g} \to \tilde{t}\tilde{c}) - \Gamma (\tilde{g} \to \tilde{t}\tilde{t})}{\Gamma^\text{tot}_{\tilde{g}}} \approx \frac{\lambda_{323}^2 \text{Im} (A_{323} m_{\tilde{g}})}{16\pi} \frac{m_{\tilde{g}}^2}{|m_{\tilde{g}}|^2}$$ \hspace{1cm} (185)

where $A_{323}$ is the trilinear SUSY breaking term and we also assume a maximal CP violation. As a result the decay of gauginos produce more squarks (antisquarks) than squarks (squarks). The baryon number violating ($\Delta B = 1$) decay, induced by $\lambda_{323}$ of squarks (antisquarks) to quarks (antisquarks) then gives rise to a net baryon asymmetry. Note that the decay of squarks (antisquarks) are much faster than any other processes that would erase the produced baryon asymmetry. Hence the $B$-asymmetry can simply be given by:

$$\eta_B \sim B_\tilde{g} \frac{n_\gamma}{s} \sim \frac{3}{4} B_\tilde{g} \frac{T_R}{m_\phi},$$ \hspace{1cm} (186)

where $B_\tilde{g} \sim 0.5$ is the branching ratio of the decay of $\phi$ to $\tilde{g}\tilde{g}$, and in the above equation $s$ is the entropy density resulted through the decay of $\phi$. For $T_R/m_\phi \sim 10^{-7}$ and $m_\phi \sim 10^6$ GeV. Therefore a reasonable CP violation of order $\epsilon \sim 0.01 - 0.001$ could accommodate the desired baryon asymmetry of $O(10^{-10})$ close to the temperature of $T \sim 10 - 1$ MeV \cite{Cline and Raby 1999, Kohri et al. 2003}.

### IV. DARK MATTER

There is a conclusive evidence that a considerable fraction of the current energy density is in the form of a non-baryonic dark matter. The dynamical motions of astronomical objects such as rotation curves for spiral galaxies \cite{Begeman et al. 1991}, velocity dispersion of individual galaxies in galaxy clusters, large x-ray temperatures of clusters \cite{Flores et al. 2007}, bulk flows and the peculiar motion of our own local group \cite{Dressler et al. 1987}, all implies the presence of a dark matter. The mass of galaxy clusters inferred by their gravitational lensing of background images is also consistent with the large dark-to-visible mass ratios \cite{Bolton et al. 2004}. Perhaps the most compelling evidence, at a statistical significance of $\sigma \geq 5$ comes from the two colliding clusters of galaxies, known as the Bullet cluster \cite{Clowe et al. 2006, Markevitch et al. 2004}. It was found that the spatial offset of the center of the total mass from the center of the baryonic mass peaks cannot be explained with an alteration of gravitational force law. Furthermore, the large scale structure formation from the initial seed perturbations from inflation requires a significant non-baryonic dark matter component \cite{Abazajian et al. 2009}. In terms of the critical density, $\rho_c = 3H_0^2 M_\odot^2/8\pi = 1.88 \times 10^{-29} g \text{ cm}^{-3}$ and with Hubble constant $H_0 \equiv 100h \text{ km sec}^{-1}\text{Mpc}^{-1}$, the dark matter density inferred from WMAP and large scale structure data is $\Omega_{DM} \equiv \rho_{DM}/\rho_c \sim 0.22$ \cite{Komatsu et al. 2011}.

The dark matter is assumed to be a weakly interacting massive particle (WIMP), yet undiscovered. There are many well motivated particle physics candidates, e.g. \cite{Bertone et al. 2005, Jungman et al. 1995, Kusenko 2009, Taoso et al. 2008}, all of which arise from beyond the SM physics. The dark matter is assumed to be stable on the scale of cosmological structure formation. By virtue of new symmetries, for example R-parity conservation in SUSY allows the lightest SUSY particle (LSP) to be absolutely stable \cite{Ellis et al. 1984a, Goldberg 1983}, or in the case of extra dimensions, the Kaluza-Klein (KK) parity leaves the lightest KK particle (LKP) stable \cite{Servant and Tait 2003}.

In many cases some of these symmetries which protect the dark matter particle from decaying are broken by sufficiently suppressed higher-dimensional operators, such that the dark matter might as well have a finite time comparable to the age of the universe. In the context of SUSY grand unification, operators with mass dimension 6 are expected to make SUSY dark matter unstable, with a time-scale

$$\tau \sim 8\pi \left(\frac{M_{\text{GUT}}}{m_X}\right)^5 \sim 10^{27}\text{ sec} \left(\frac{\text{TeV}}{m_X}\right)^5 \left(\frac{M_{\text{GUT}}}{2 \times 10^{16} \text{ GeV}}\right)^4$$ \hspace{1cm} (187)

where $M_{\text{GUT}} \sim 10^{16}$ GeV, and $m_X$ is the dark matter particle. The lower dimensional operators would yield much shorter time scale, as it would lead to dark matter decay long before the structure formation. Within SUSY one compelling candidate could be the gravitino with R-parity weakly broken in the hadronic sector, yielding the required baryon asymmetry also in the process \cite{Kohri et al. 2009}.

The widely accepted lore is that after radiation-matter equality, when the universe becomes matter dominated, the density perturbations in the dark matter begin to grow, and drive the oscillations of the photon-baryonic fluid around the dark matter gravitational potential wells. Immediately after the epoch of recombination the baryons kinematically decouple from photons, which then free-stream through the universe; the baryons on the other hand slowly fall into the potential wells created by the dark matter particles, eventually becoming light emitting galaxy, see for more details \cite{Dodelson 2003, Kolb and Turner 1988, Peebles 1994}. There are three broad categories of dark matter which have been central to our discussion.
A. Types of dark matter

1. Hot Dark Matter

If the dark matter particle is collisionless, then they can damp the fluctuations from higher to lower density regions above the free-streaming scale. This hot dark matter consists of particles which are relativistic at the time of structure formation and therefore lead to large damping scales \cite{BondsandSzalay1983}.

The SM neutrinos are the simplest examples of hot dark matter. In the early universe they can be decoupled from a relativistic bath at \( T \sim 1 \text{ MeV} \), leading to a relic abundance today that depends on the sum of the flavor masses:

\[
\Omega_{\nu} h^2 = \sum \frac{m_{\nu_i}}{90 \text{ eV}} \tag{188}
\]

Various observational constraints combining Ly-\( \alpha \) forest, CMB, SuperNovae and Galaxy Clusters data leads to \cite{Fogli2008, Seljaketal2006}: \( \sum m_{\nu} < 0.17 \text{ eV (95\% CL)} \). Similar limits can be applied to any generic hot dark matter candidate, such as axions \cite{Hannestadetal2010} or to hot sterile neutrinos \cite{Dodelsonetal2006, Kusencko2009}. The free-streaming length for neutrinos is \cite{KolbandTurner1988}:

\[
\lambda_{FS} \sim 20 \left( \frac{30 \text{ eV}}{m_{\nu}} \right) \text{ Mpc}. \tag{189}
\]

For instance, the universe dominated by the eV neutrinos would lead to suppressed structures at 600 Mpc scale, roughly the size of supercluster. Furthermore, hot dark matter would predict a top-down hierarchy in the formation of structures, with small structures forming by fragmentation of larger ones, while observations show that larger galaxies have formed from the mergers of the initially small galaxies.

2. Cold Dark Matter

The standard theory of structure formation requires cold dark matter (CDM), whose free-streaming length is such that only fluctuations roughly below the Earth mass scale are suppressed \cite{Bertschinger2003, Greenetal2001, 2003, Hofmannetal2001, LoebZaldarriaga2003}. The CDM candidates are heavy and non-relativistic at the time of their freeze-out from thermal plasma. The current paradigm of LCDM is falsifiable whose predictive power can be used to probe the structures at various cosmological scales, such as the abundance of clusters at \( z \leq 1 \) and the galaxy-galaxy correlation functions have proven it a successful and widely accepted cosmological model of large scale structure formation.

The N-body simulations based on \( \Lambda \)CDM provide a strong hint of a universal dark matter profile, with the same shape for all masses, and initial power spectrum. The halo density can be parametrized by:

\[
\rho(r) = \frac{\rho_0}{(r/R_s)^\gamma [1 + (r/R_s)^\alpha]^{(\beta-\gamma)/\alpha}}, \tag{190}
\]

where \( \rho_0 \) and the radius \( R_s \) vary from halo to halo, the parameters \( \alpha, \beta, \gamma \) vary slightly from one profile to other. The four most popular ones are:

- **Navarro, Frenk and White** (NFW) profile \cite{NavarroFrenkWhite1997}, where \( \alpha = 1, \beta = 3, \gamma = 1, \text{ and } R_s = 20 \text{ Kpc} \).
- **Moore profile** \cite{Mooreetal1999}, where \( \alpha = 1.5, \beta = 3, \gamma = 1.5, \text{ and } R_s = 28 \text{ Kpc} \).
- **Kra profile** \cite{Kravtsovetal1998}, where \( \alpha = 2, \beta = 3, \gamma = 0.4, \text{ and } R_s = 10 \text{ Kpc} \).
- **Modified Isothermal profile** \cite{Bergstrometal1998}, where \( \alpha = 2, \beta = 3, \gamma = 0, \text{ and } R_s = 3.5 \text{ Kpc} \).

Amongst all the four profiles, the scales where deviations are most pronounced (the inner few kiloparsecs) are also the most compromised by numerical uncertainties. The power-law index value, \( \gamma \), in the inner most regions is part of the numerical uncertainties and still under debate, as all four simulations provide different numbers. The simulations hint towards a cuspy profile, as the density in the inner regions becomes large, while from the rotation curves of low surface brightness (LSB) galaxies point towards uniform dark matter density profile with constant density cores \cite{Gentiletal2004}. In our own galaxy the situation is even more murky, as the observations of the velocity dispersion of stars near the core suggests a supermassive black hole at the center of our Galaxy, with a mass \( M_{\text{SMBH}} \approx 2.6 \times 10^6 M_{\odot} \) \cite{Ghezetal1998}. Many galaxies have been found to host supermassive blackholes of \( 10^6 - 10^8 M_{\odot} \). It has been argued that if supermassive blackhole exists at the galactic center, the accretion of dark matter by the blackhole would enhance the dark matter density \cite{Peebles1994}. To alleviate some of these problems, dark matter with a strong elastic scattering cross section \cite{Daveetal2001, SpergelandSteinhardt2000}, or large annihilation cross sections \cite{Kaplinghatetal2000} have been proposed.

There are further discrepancies between observations and numerical simulations. The number of satellite halos as predicted by simulations exceeds the number of observed Dwarf galaxies in a typical galaxy like Milky-Way \cite{Klypineletal1999, Mooreetal1999}. However recent hydrodynamical simulations with \( \Lambda \)CDM, including the supernovae induced outflows suggest a fall in the dark-matter density to less than half of what it would otherwise be within the central Kpc.
3. Warm Dark Matter

Besides hot and cold dark matter, the early universe can also provide warm dark matter (WDM) candidates whose velocity dispersion lies between that of hot and CDM. The presence of WDM reduces the power at small scales due to larger free-streaming length compared to that of a CDM (Bode et al. 2001; Sommer-Larsen and Dolgov 2001).

The origin of WDM can be found within sterile states. For instance, the see-saw mechanism for the active neutrino mass from the SM singlet states (Gell-Mann and Slansky 1980; Minkowski 1977; Mohapatra and Senjanovic 1981; Yanagida 1979) would naturally generate masses to the active $m(\nu_{i,2,3}) \sim y^2(H)^2/M_N$, and sterile neutrinos $m(\nu_s) \sim M_N (a > 3)$ in Eq. (152), if we take $i, j = 1, \ldots n + 3$. The typical mixing angles in this case are: $\theta_{ai} \sim y^2 a(H)^2/M_N^2$. In order to explain the neutrino masses from atmospheric and solar neutrino data, $n = 2$ is sufficient, however for pulsar kicks (Kusenko 2006; Kusenko and Segre 2001; Asaka et al. 2005; Dodelson and Widrow 1994), supernovae explosion (Fryer and Kusenko 2006; Hidaka and Fuller 2006, 2007), as well as sterile neutrino as a dark matter candidate (Abazajian et al. 2001; Asaka et al. 2005; Dodelson and Widrow 1994; Dolgov and Hansen 2002; Petraki and Kusenko 2008; Shi and Fuller 1999), we require at least $n = 3$, so in total 6 sterile Majorana states, for a review on all these effects, see (Kusenko 2009). The presence of such extra sterile neutrinos is also supported by $\nu_{\mu} \rightarrow \nu_{\tau}$ oscillations observed at LSND (Aguilar et al. 2001), and the recent results by MiniBoone (Aguilar-Arevalo et al. 2010).

A sterile neutrino with a KeV mass can be an ideal WDM candidate which can be produced in the early universe by oscillation/conversion of thermal active neutrinos, with a momentum distribution significantly suppressed from a thermal spectrum (Abazajian et al. 2001; Dodelson and Widrow 1994). A typical free-streaming scale is given by, see (Abazajian and Koussiapnas 2006)

$$\lambda_{FS} \approx 840 \text{ Kpc h}^{-1} \left(\frac{1 \text{ KeV}}{m_s}\right) \left(\frac{<p/T>}{3.15}\right), \quad (191)$$

where $m_s$ is the mass of the sterile flavor eigenstate, $0.9 \geq <p/T>/3.15 \geq 1$ is the mean momentum over temperature of the neutrino distribution and ranges from 1 (for a thermal) to $\sim 0.9$ (for a non-thermal) distribution. Very stringent bounds on the mass of WDM particles have been obtained by different groups. Typically, the bounds range from $m_s \geq 10 - 20$ KeV (95 % CL) ($m_{WDM} \geq 2 - 4$ KeV), see (Kusenko 2009). It is quite plausible to imagine a mixed dark matter scenario, where more than one species contributed to the total dark matter abundance. If there is a fraction of sterile neutrinos or WDM, then the above bounds can even be relaxed.

B. WIMP production

1. Thermal relics

At early times it is assumed that the dark matter particle, denoted by $X$ is in chemical and kinetic equilibrium, i.e. in local thermodynamic equilibrium. The dark matter will be in equilibrium as long as reactions can keep $X$ in chemical equilibrium and the reaction rate can proceed rapidly enough as compared to the expansion rate of the universe, $H(t)$. When the reaction rate becomes smaller than the expansion rate, then the particle $X$ can no longer be in its equilibrium, and thereafter its abundance with respect to the entropy density becomes constant. When this occurs the dark matter particle is said to be “frozen out.”

The equilibrium abundance of $X$ relative to the entropy density depends upon the ratio of the mass of the particle to the temperature. Let us define the variable $Y \equiv n_X/s$, where $n_X$ is the number density of $X$ with mass $m_X$, and $s = 2n_e^2 g_s T^3/45$ is the entropy density, where $g_s$ counts the number of relativistic d.o.f. The equilibrium value of $Y$, $Y_{EQ} \propto \exp(-x)$ for $x = m_X/T > 1$, while $Y_{EQ} \propto$ constant for $x \ll 1$.

The precise value of $Y_{EQ}$ can be computed exactly by solving the Boltzmann equation (Kolb and Turner 1988):

$$\dot{n}_X + 3Hn_X = -\langle \sigma v \rangle (n_X^2 - (n_{eq} X)^2), \quad (192)$$

where dot denotes time derivative, $\sigma$ is the total annihilation cross section, $v$ is the velocity, bracket denotes thermally averaged quantities, and $n_{eq} X$ is the number density of $X$ in thermal equilibrium:

$$n_{eq} X = \frac{g}{3} \frac{m_X}{T^2} e^{-m_X/T}, \quad (193)$$

where $T$ is the temperature. In terms of $Y = n_X/s$ and $x = m_X/T$, and using the conservation of entropy per comoving volume ($s a^3$ = constant), we rewrite Eq. 192 as:

$$\frac{dY}{dx} = \frac{-\langle \sigma v \rangle s}{H X} \left( Y^2 - (Y_{eq})^2 \right). \quad (194)$$

In the case of heavy $X$, the cross section can be expanded with respect to the velocity in powers of $v^2$, $\langle \sigma v \rangle = a + b(v^2) + O(v^4) + \cdots \approx a + b/vx$, where $x = m_X/T$ and $a, b$ are expressed in GeV$^{-2}$. Typically $a \neq 0$ for s-wave annihilation, and $a = 0$ for p-wave annihilation. We can rewrite Eq. 194 in terms of a new variable: $\Delta = Y - Y_{eq}$

$$\Delta' = -\frac{\langle \sigma v \rangle}{45} m_X M_P(a + 6b/vx)x^{-2}, \quad (195)$$

where prime denotes $d/dx$, and

$$f(x) = \frac{\pi g_s}{45} m_X M_P(a + 6b/vx)x^{-2}. \quad (196)$$
One can find a simple analytic solution for Eq. (195) for two extreme regimes

\[ \Delta = -\frac{Y^{eq}}{f(x)(2Y^{eq} + \Delta)}, \quad x \ll \frac{m_X}{T_f}, \quad \Delta' \ll Y^{eq} \]  

\[ \Delta^{-2}\Delta' = -f(x), \quad x \gg \frac{m_X}{T_f}, \quad \Delta' \gg Y^{eq}. \]  

Integrating the last equation for \((x_f, \infty)\), and using \(\Delta(x_f) \gg \Delta_{\infty}\), we find

\[ \Delta_{\infty}^{-1} \approx Y_{\infty}^{-1} = \sqrt{\frac{\pi d_{\ast}}{45}} M_p \left( \frac{m_X}{x_f} \right) \left( a + \frac{3b}{x_f} \right). \]  

In terms of the present energy density, \(\rho_X = m_X n_X = m_X s_0 Y_{\infty}\), where \(s_0 = 2889.2 \text{ cm}^{-3}\) is the present entropy density, the relic abundance of dark matter particle in terms of the critical energy density is given by:

\[ \Omega_X h^2 \approx \frac{1.07 \times 10^9}{M_p} \frac{x_f}{\sqrt{\left( g_{\ast}(a + 3b/x_f) \right)}} \text{ GeV}^{-1}, \]  

where the freeze-out temperature is defined by solving this equation \(\Delta(x_f) = c Y^{eq}(x_f)\) iteratively for early and late time solutions, for \(c \sim O(1)\)

\[ x_f = \ln \left[ c(c + 2)\sqrt{\frac{d_{\ast}}{2\pi^2}} \frac{m_X M_p(a + 6b/x_f)}{g_{\ast}^{1/2}} \sqrt{x_f} \right]. \]  

An approximate order of magnitude estimation of the abundance can be written as:

\[ \Omega_X h^2 \approx \frac{3 \times 10^{-27} \text{ cm}^3 \text{s}^{-1}}{\langle \sigma v \rangle} \sim 0.1 \text{ pbarn} / \langle \sigma v \rangle. \]  

For a WIMP interacting with a heavy gauge boson, would naturally yield an upper bound on \(m_X\). From the above Eq. (202), on dimensional grounds, \(\Omega \sim 1/(\sigma v) \sim 1/m_X^2\) for \(\langle \sigma v \rangle \sim a^2(m_X/M^2)^2\), where \(M\) is the mass of the new gauge boson. For \(m_X \sim M \sim 1 \text{ TeV}\), abundance of the dark matter becomes of order of unity, \(\Omega_X h^2 \sim O(1)\), for \(a \sim 0.1\). Actually, the unitarity bound limits the dark matter mass to be below \(m_X \leq 300 \text{ TeV} \) \cite{Griest1990}. For a realistic scenario \(a \sim 0.01\), the unitarity bound would yield \(m_X \lesssim 3 \text{ TeV}\).

Note that the above non-relativistic expansion of \(\langle \sigma v \rangle \sim a + 6b/x\) may not hold universally. When a mass of second particle becomes nearly degenerate with the dark matter particle \(X\) as in the case of coannihilation \cite{Binetruy1984, Griest1991}, or the cross section is strongly varying function of the center of mass energy as in the case of a resonant annihilation \cite{Griest1991}. In the latter case, \(\sigma\) gets a boost by resonant annihilation when \(m_X \approx m_A/2\), where \(X\) annihilates with an exchange of particle, \(A\), with a mass \(m_A\).

2. Coannihilating WIMPs

If there are \(N\) particles, \(X_i (i = 1, \ldots, N)\) with the lightest one, \(X_1\), which have nearly degenerated masses \(m_i\), such that \(m_1 \leq m_2 \leq \cdots \leq m_{N-1} \leq m_N\), and internal \(d.o.f\) (statistical weights) \(g_i\). The next to lightest dark matter particle will be \(N_2\). In this case the above calculation of relic density, Eq. (192), gets modified \cite{Binetruy1984, Griest1991, Servant2003}.

\[ \frac{d \rho}{dt} = -3Hn - \sum_{i,j=1}^N \langle \sigma_{ij} v_{ij} \rangle (n_i n_j - n_i n_j^eq), \]  

where \(n = \sum_{i=1}^N n_i\) is the number density of the relic particle, since all other particles decay much before the long-lived \(X_1\). The total annihilation rate for \(X_i - X_j\) into a SM particle is given by:

\[ \langle \sigma_{ij} v_{ij} \rangle = \sum_x \sigma(X_i X_j \rightarrow X SM), \quad \text{and} \quad v_{ij} = \sqrt{(p_i \cdot p_j)^2 - m_i^2 m_j^2/E_i E_j}, \]

is the relative particle velocity, with \(p_i\) and \(E_i\) are the four-momentum and energy of particle \(i\). One requires to define a thermal averaged \(\langle \sigma_{ij} v_{ij} \rangle\), which is defined by:

\[ \langle \sigma_{ij} v_{ij} \rangle = \int d^3 p_i d^3 p_j f_i f_j \sigma_{ij} v_{ij} \]  

where \(f_i\) are distribution functions in the Maxwell-Boltzmann approximation.

Typically, when the scattering rate of particles off SM particles in a thermal background is much faster than their annihilation rate, then in the above Eq. (192), \(\langle \sigma v \rangle\) is replaced by:

\[ \langle \sigma_{eff} v \rangle = \sum_{ij} \frac{g_i g_j}{g_{eff}} (1 + \Delta_i)^3/2 (1 + \Delta_j)^3/2 e^{-x(\Delta_i + \Delta_j)} \]  

where \(\Delta_i = (m_i - m_1)/m_1\), and \(g_{eff} = \sum_i g_i (1 + \Delta_i)^3/2 \exp(-x \Delta_i)\). In the case of co-annihilation, the freeze-out temperature is determined by

\[ x_f = \ln \left[ c(c + 2)\sqrt{\frac{g_i g_j}{2\pi^2}} \frac{m_X M_p(a_{eff} + 6b_{eff}/x_f)}{g_{ast}^{1/2}} \right]. \]  

where \(a_{eff}\) and \(b_{eff}\) are the coefficients of the Taylor expansion of \(\sigma_{eff}\). The relic abundance for \(N_1\) is now given by

\[ \Omega_{N_1} h^2 \approx \frac{3 \times 10^{-27} \text{ cm}^3 \text{s}^{-1}}{g_{ast}^{1/2} x_f^4 (I_a + 3I_b/x_f)} \]  

where \(\Omega_{N_1} = x_f \int_{x_f}^\infty a_{eff} x^{-2} dx, \quad I_a = 2\pi^2 \int_{x_f}^\infty b_{eff} x^{-3} dx, \)
In any realistic framework there are many particles which interact with the dark matter candidate $X$. They all eventually decay into $X$, and at the time of freeze-out the density of all heavy particles is exponentially suppressed except when there is mass degeneracy occurs between heavy particles and the $X$. The details of coannihilation has been studied extensively within SUSY (Edsjo and Gondolo, 1997), and publicly available numerical codes include coannihilations with all SUSY particles (Gondolo et al., 2004).

3. Non-thermal relics

The dark matter particle, $X$, can also be created in an out of equilibrium condition, i.e. $X$ must not have been equilibrium when it froze out. A sufficient condition for non-equilibrium is that the annihilation rate (per particle) must be smaller than the expansion rate of the universe: $n_X(\eta_v) < H$.

Let us assume that $X$ were non-relativistic at the time of production and they were never in local thermodynamical equilibrium. The largest dark matter density would thus be determined by the largest freeze out temperature, which can be attainable in the universe. Assuming this to be the reheat temperature, $T_R$ and the universe follows a radiation domination, then the ratios of energy densities will be given by (Kolb et al., 1998):

$$\frac{\rho_X(t_R)}{\rho_r(t_R)} \approx \frac{\rho_X(t_0)}{\rho_r(t_0)} \left( \frac{T_R}{T_0} \right)^{3 - 2},$$

(210)

where $T_0$ is the present temperature and $t_0$ corresponds to the present time, $\rho_r$ is the energy density in radiation, and $\rho_X = m_X n_X$ denotes the energy density in the dark matter with the number density $n_X$. If we further assume that $X$ particles were created at time $t = t_* < t_0$, sometime during the coherent oscillations of the inflaton and before the completion of reheating, then both the $X$ particle energy density and the inflaton energy density would redshift approximately at the same rate until reheating is completed. Therefore,

$$\frac{\rho_X(t_R)}{\rho_r(t_R)} \approx \frac{\rho_X(t_0)}{\rho_r(t_0)} \left( \frac{T_R}{T_0} \right)^{3 - 2},$$

(211)

assuming that the inflaton energy density dominated the universe. Since, $\Omega_X = \rho_X(t_0)/\rho_r(t_0)$, where $\rho_r(t_0) = 3H_0^2M_p^2$ and $H_0 = 100$ km sec$^{-1}$ Mpc$^{-1}$, then using Eq. (210), one obtains (Kolb et al., 1998):

$$\Omega_X h^2 \approx \Omega_r h^2 \left( \frac{T_R}{T_0} \right)^{3 - 2} \left( \frac{M_X}{M_p} \right) \frac{n_X(t_0)}{3 \rho_r(t_0)} H^2 R(t_0),$$

(212)

$$\approx 10^{17} \left( \frac{T_R}{10^9 \text{GeV}} \right) \frac{\rho_X(t_0)}{\rho_n f(t_0)},$$

(213)

where $\Omega_r h^2 \approx 4.31 \times 10^{-5}$ is the fraction of critical energy density in radiation today, and $T_0 \sim 2.3 \times 10^{-13}$ GeV. The above expression tells us that a non-thermal creation would require a very small fraction of the inflaton energy density to be transferred to the dark matter particle $X$, otherwise the universe would be dominated by the dark matter particles.

For a singlet hidden sector dark matter, it is really a challenge not to overproduce them directly from the decay of the inflaton. If the inflaton sector belongs to the hidden sector, then it is natural to have inflaton couplings to such hidden sector dark matter field. There are three possible ways to obtain a small fraction of $\rho_X(t_0)/\rho_n f(t_0)$ in order to match the current observations.

(a) Gravitational production

The dark matter can be created from the transition of the equation of state of the universe from inflation to matter domination or radiation domination, due to non-adiabatic evolution of the vacuum (Chung et al., 1999b, 2000; Kolb et al., 1998). The underlying mechanism is similar to the metric fluctuations which seed the structure formation, except now the excitations can create massive particles. The gravitational production of dark matter is universal, and it can occur even if the dark matter coupling to the inflaton is vanishingly small.

Let us consider a simple action for $X$ field with a metric $ds^2 = dt^2 - a^2(t) d\mathbf{x}^2 = a^2(\eta) \left[ d\eta^2 - d\mathbf{x}^2 \right]$, where $\eta$ is a conformal time.

$$S = \int \frac{dt}{2} \int d^3 x a^3 \left( \ddot{X} - \frac{\nabla X^2}{a^2} - m_X^2 X^2 - \xi R X^2 \right),$$

(214)

where $R$ is the Ricci scalar. Let us expand the $X$ field in terms of creation and annihilation operators which obey: $[a_k, a_k^\dagger] = \delta^{(3)}(k_1 - k_2)$, and

$$X = \int \frac{d^3 k}{(2\pi)^{3/2} a(\eta)} \left[ a_k u_k(\eta) e^{ik \cdot \mathbf{x}} + a_k^\dagger u_k^*(\eta) e^{-ik \cdot \mathbf{x}} \right],$$

(215)

where the mode functions obey the identity $u_k u_k^* - u_k^* u_k = i$, and prime denotes derivative w.r.t. $\eta$. The mode equation is given by:

$$u_k''(\eta) + \left[ k^2 + m_X^2 a^2 + (6\xi - 1)a''/a \right] u_k(\eta) = 0,$$

(216)

The parameter $\xi = 1/6$, for conformal and $\xi = 0$ for minimal coupling. Here we will consider the conformal coupling for simplicity. The number density of $X$ particles can be estimated by a Bogoliubov transformation:

$$u_k^\dagger(\eta) u_k(\eta) = \alpha_k u_k^{\dagger 0}(\eta) + \beta_k u_k^{\dagger 0}(\eta),$$

(217)

where $\eta_0 = -\infty$, and $\eta_1 = +\infty$. The energy density of produced particles is given by (Chung et al., 1999a):

$$\rho_X(\eta_1) = m_X H_{m,f}^2 (\tilde{a}(\eta_1))^{-3} \int_0^{\infty} \frac{dk}{2\pi} k^2 |\beta_k|^2,$$

(218)

where the number operator is defined at $\eta_1$. Assuming that the transition from inflation-radiation or matter
domination is smooth, the largest energy density can be obtained if $m_X/H_{inj} \sim 1$. If $0.04 \leq m_X/H_{inj} \leq 2$. If $H_{inj} \sim m_\phi \sim 10^{13}$GeV and $m_\phi$ is the mass of the inflaton, then $X$ particles produced gravitationally can match the density today of the order of the critical density provided they are long lived. Such super heavy massive dark matter particle $X$ is known as Wimpzillas!

(b) Direct decay of the inflaton

The dark matter can also be created from direct inflaton decay if $m_X < m_\phi/2$, with a rate $\Gamma_X \sim h_X^2 m_\phi/8\pi$, where $h_X$ is the interaction strength. The total inflaton decay rate is given by $\Gamma_d \sim \sqrt{1/3} T_R^2 M_p$, while the inflaton number density at the time of decay is given by $n_\phi \sim T_R^4/m_\phi$. This constrains the overall coupling to

$$h_X^2 \leq 32\pi \sqrt{\frac{1}{3} \frac{T_R}{M_p} \frac{10^{-9}}{m_X}},$$

where $m_X$ is in units of GeV. This is required due to the fact that the produced $X$ must not overclose the universe which, for $\Omega_X \leq 0.22$ and $H_0 = 70$ km sec$^{-1}$ Mpc$^{-1}$, reads

$$n_X/n_\gamma \leq 4 \times 10^{-9} m_X^{-1},$$

when $m_X$ is expressed in units of GeV (Allahverdi et al. 2002). It is evident from the overclosure bound that $h_X$ needs to be very small.

(c) Creation during reheating

If the process of reheating is slow and not instantaneous, then it is possible to create WIMP from the ambient plasma which is in the process of acquiring thermalization via scatterings. The Boltzmann equations for inflaton energy density, $\rho_\phi$, radiation energy density, $\rho_R$, and dark matter energy density $\rho_X$ are given by (Chung et al. 1999a; Kolb and Turner 1988):

$$\dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi \rho_\phi = 0$$

$$\dot{\rho}_R + 4H\rho_R - \Gamma_\phi \rho_\phi - \frac{\langle |\sigma|v| \rangle}{m_X} (\rho_X^e - \rho_X^q) = 0$$

$$\dot{\rho}_X + 3H\rho_X + \frac{\langle |\sigma|v| \rangle}{m_X} (\rho_X^q - \rho_X^e) = 0,$$

where dot denotes time derivative, and thermal averaged cross section is given by: $\langle |\sigma|v| \rangle$. The equilibrium energy density for the $X$ particles, $\rho_X^e$, is determined by the radiation temperature, $T = (30PR/\pi^2 g_*)^{1/4}$. Following (Chung et al. 1999a; Kolb and Turner 1988), it is useful to introduce two dimensionless constants, $\alpha_\phi$ and $\alpha_X$, defined in terms of $\Gamma_\phi = \alpha_\phi m_\phi$, $\langle |\sigma|v| \rangle = \alpha_X m_X^2$, and $\Phi \equiv \rho_\phi m_\phi^{-1} a^3$; $R \equiv \rho_\phi a^4$; $X \equiv \rho_X m_X^{-1} a^3$. With these parameters the Boltzmann equations are:

$$\Phi' = -c_1 \frac{x}{\sqrt{\Phi x + R}} \Phi$$

$$R' = c_1 \frac{x^2}{\sqrt{\Phi x + R}} \Phi + c_2 \frac{x^{-1}}{\sqrt{\Phi x + R}} (X^2 - X_{eq}^2)$$

$$X' = -c_3 \frac{x^{-2}}{\sqrt{\Phi x + R}} (X^2 - X_{eq}^2).$$

where $x = am_\phi$, prime denotes $d/dx$, and the constants $c_1$, $c_2$, and $c_3$ are given by

$$c_1 = \sqrt{\frac{3}{8\pi m_\phi}} \alpha_\phi, \quad c_2 = c_1 m_\phi \alpha_\phi/m_X, \quad c_3 = c_2 m_\phi/m_X.$$

The equilibrium value of $X$ is given in terms of the temperature $T$ and Eq. (193): $X_{eq} = x R x_{eq} = (m_X/m_\phi)(1/2\pi)^{3/2} x^3 (T/M_X)^{3/2} \exp(-M_X/T)$.

It is straightforward to solve the system of equations in Eq. (222) with initial conditions at $x = x_1$, $R(x_1) = X(x_1) = 0$, and $\Phi(x_1) = \Phi_1 = (3/8\pi)(M_\phi/m_X^2)(H_0^2/m_X^2)$. At early time solution for $R$ can be easily obtained:

$$R \sim 0.4c_1 \left( x_1^{5/2} - x_I^{5/2} \right) \Phi_1^{1/2}$$

By maximizing the above equation, which is obtained at $x/x_I = (8/3)^{2/5} = 1.48$, the largest temperature of the ambient plasma can be even larger than the reheat temperature (Kolb and Turner 1988)

$$T_{MAX}/T_R = 0.77 \left( 9/5 \pi^3 g_* \right)^{1/8} (H_{inj} M_p/T_R^2)^{1/4}.$$

From Eq. (224) when $x/x_I > 1$, $T$ scales as $a^{-3/8}$, which implies that entropy is created in the early-time regime. For the choices of $m_\phi$, $g_*$, $x_I$, and $\alpha_X$, and $\Omega_X h^2 = 0.22$ (Chung et al. 1999a):

$$\Omega_X h^2 = \alpha_X^2 \left( \frac{200}{g_*} \right)^{3/2} \left( \frac{2000 T_R}{m_X} \right)^7.$$

(d) Non-perturbative creation of dark matter

The dark matter can be created non-perturbatively during the coherent oscillations of the inflaton.

a. Superheavy dark matter during preheating: If the dark matter couples to the inflaton directly, then it is more efficient to excite them from the coherent oscillations of the inflaton during preheating. One of the most interesting applications of preheating is the copious production of particles which have a mass greater than the inflaton mass $m_\phi$. Such processes are impossible in perturbation theory and in the theory of narrow parametric resonance.

Following Eq. (97), let us suppose that the dark matter $X$ is coupled to the inflaton with an interaction term:
(1/2)g^2X^2\phi^2$. During the broad resonance regime, as we have discussed in section [1], superheavy X-particles with mass $m_X \gg m_\phi$ can be produced. The momentum dependent frequency $\omega_k(t)$ violates the adiabatic condition of time dependent vacuum, see Eq. (103), when

$$k^2 + m_X^2 \lesssim (g^2 m_\phi \hat{\phi})^{2/3} - g^2 \hat{\phi}^2,$$

(226)

where $\hat{\phi}$ is the amplitude of the inflaton oscillations. The maximal range of momenta for which particle production occurs corresponds to $\phi(t) = \phi_*$, where $\phi_* \approx 1/2 \sqrt{(m_\phi \hat{\phi})/g}$. The maximal value of momentum for particles produced at that epoch can be estimated by $k_{\text{max}}^2 + m_X^2 = (g m_\phi \hat{\phi})/2$. The resonance becomes efficient for $g m_\phi \hat{\phi} \gtrsim 4m_X^2$. Thus, the inflaton oscillations may lead to a copious production of superheavy particles with $m_X \gg m_\phi$ if the amplitude of the field $\phi$ is large enough, $g \phi \gtrsim 4m_X^2/m_\phi$ (Kofman et al. 1997).

**b. Dark matter from the fragmentation of a scalar condensate:** Let us assume that coherent oscillations of a scalar condensate in a potential $U(\phi)$ has a frequency which is large compared to the expansion rate of the universe. The equation of state is obtained by averaging, $p/\rho = (|\phi|^2/\rho) - 1$, over one oscillation cycle $T$. The result is: $p = (\gamma - 1)\rho$, where $\gamma = (2/T)\int_0^T (1 - U(\phi)/\rho) \, dt$ (Turner 1983). For the case $U \sim m^2_\phi \phi^2$, one finds $\gamma = 1$, so that one effectively obtains the usual case of pressureless, non-relativistic cold matter. When the motion of the condensate is not simply oscillatory, such as in the case of a rotating trajectory with a phase, one can generalize the above calculation by integrating over the orbit of the condensate. Let us consider the potential

$$U(\phi) = \frac{1}{2} m_\phi^2 |\phi|^2 (\phi^2/\mu^2)^{x},$$

(227)

one finds that $\gamma = (1 + x)/(1 + x/2)$, $p = x/(2 + x)$. There arises a negative pressure whenever $x < 0$. This is a sign of an instability of the scalar field under arbitrarily small perturbations. The quantum fluctuations in the condensate grow according to when effective mass of the scalar field is much larger than the expansion

$$\delta_k = -Kk^2 \delta_k.$$  

(228)

If $K = 2x < 0$, quantum fluctuations of the condensate at the scale, $\lambda = 2\pi/|k|$, will grow exponentially in time:

$$\delta \phi_k(t) = \delta \phi(0) \exp(-Kk^2t).$$

(229)

In reality the onset of non-linearity sets the scale at which the spatial coherence of the field can no longer be maintained and the condensate fragments. The initial fluctuations in the condensate owes to the inflationary perturbations. If the condensate carries a global charge, due to charge conservation the energy-to-charge ratio changes as the condensate fragments. This is what happens in the case of MSSM.

The AD condensate which was responsible for generating the baryon asymmetry at the first instance could fragment (Kasuya and Kawasaki 2000a,b; Kusenko et al. 2009). The ground state of these fragmented lumps is a non-topological soliton with a fixed charge, called the Q-ball (Coleman 1985; Kusenko 1997b; Lee and Pang 1992). In gauge mediated SUSY breaking scenarios the Q-balls can absolutely stable and can be a candidate for CDM (Kusenko et al. 1998; Kusenko and Shaposhnikov 1998; Kusenko and Shoenmacker 2009). The Q-balls can also be formed from the fragmentation of the inflaton (Enqvist et al. 2002a,b). The slow surface evaporation of a Q-ball will also create SUSY LSP, which we will discuss below.

**C. Candidates**

In this subsection we will discuss some of the dark matter candidates which are well motivated, and the challenges they face.

1. Primordial blackholes

The primordial blackholes (PBH) can be created in the early universe (Carr and Hawking 1974; Hawking 1971), and they can survive the age of the universe with a typical lifetime of an evaporating blackhole which is given by:

$$\tau \approx \left(\frac{M}{10^{15} \, \text{grams}}\right)^3,$$

(230)

If the initial mass $M \approx 10^{15}$ g, the blackhole will be evaporating now, for heavier blackholes the Hawking evaporation is negligible, and they can be a CDM candidate. When $M \approx 10^9$ g, the blackholes would decay at the time of BBN. The PBHs are formed from the collapse of order one perturbations, $\delta \equiv \delta \rho/\rho \sim O(1)$, inside the Hubble patch. The detailed numerical simulation suggest $\delta_0 \sim 0.7$ (Jedamzik and Niemeyer 1998, 1999; Niemeyer and Jedamzik 1998, 1999). In a radiation dominated epoch the blackhole mass is bounded by the Hubble mass

$$M_H \approx 10^{18} \, g \left(\frac{10^7 \, \text{GeV}}{T}\right)^2,$$

(231)

where $T$ is temperature of the thermal bath during radiation. In spite of novelty in this idea, the detailed calculations suggest that it is hard to form primordial blackholes just from the collapse of sub-Hubble over denser regions - one requires more power on small scales $n < 1.25 - 1.30$ in $P_s(k) = Ak^{n-1}$ (Carr and Lidsey 1993; Drees and Erfani 2011; Green and Liddle 1997), while the CMB data points towards $n \sim 0.96$. It was shown in a hydrodynamical simulation (Jedamzik and Niemeyer 1999) that primordial blackholes can also be produced
in a first order phase transition, and during preheating (Green and Malik 2001).

The abundance of PBH contains many uncertainties, as the details of the initial gravitational collapse and the initial number density of \( n_{PBH} \) depends on many physical circumstances. These uncertainties can however be encoded in terms of the ratio determined by the initial time, \( t_i: \rho_{PBH}(t_i)/\rho(t_i) = M_{PBH}(t_i)/\rho(t_i) = 4M_{PBH}/3T_1s(T_1) \), by assuming \( \rho = 3sT^4/4 \),

\[
\beta'(M) \approx 7.98 \times 10^{-29} \left( \frac{M}{M_{\odot}} \right)^{3/2} \left( \frac{n_{PBH}(t_0)}{1 \text{ Gpc}^{-3}} \right). 
\] (232)

where \( t_0 \) corresponds to present time. In terms of this fraction \( \beta' \), the PBH abundance is given by (Carr 1975, Green and Liddle 1999)

\[
\Omega_{PBH}h^2 \approx 0.5 \left( \frac{\beta'(M)}{1.15 \times 10^{-8}} \right) \left( \frac{M}{M_{\odot}} \right)^{-1/2}. 
\] (233)

The value of \( \beta'(M) \) can be constrained from \( \Omega_{PBH} \leq \Omega_{CDM} \), which yields \( \beta'(M) < 2.04 \times 10^{-18}(\Omega_{CDM}/0.25)(M/10^{15} \text{ g})^{1/2} \), for mass \( M \geq 10^{15} \text{ g} \). A tighter constrain on \( \beta' \) arises from a range of astrophysical observations, such as BBN, CMB anisotropy, and \( \gamma \)-ray backgrounds for \( M \leq 10^{15} \text{ g} \), the bound weakens for larger mass blackholes (Carr et al. 2010).

2. Axions

The axions were introduced to solve the strong CP problem (Peccei and Quinn 1977ab) which requires a new global chiral symmetry \( U(1)_{PQ} \) that is broken spontaneously at the Peccei-Quinn (PQ) scale \( f_a \) (for reviews, see (Kim 1987, Raffelt 1990, Sikivie 2008, Turner 1990)). The corresponding pseudo-Nambu-Goldstone boson is the axion \( a \) (Weinberg 1978, Wilczek 1978), which couples to the gluons

\[
\mathcal{L} = \frac{a}{f_a/N} \frac{g^2}{32\pi^2} G^a G^a, 
\] (234)

where \( N \) is the color anomaly of the PQ symmetry depends on the interactions. This interaction term compensates the vacuum contribution in the QCD Lagrangian \( \mathcal{L}_0 = \Theta(g^2/32\pi^2)G^a G^a \), in a way that \( \Theta \rightarrow \Theta + \text{Arg} \det(M) < 10^{-9} \) (Nakamura et al. 2010), in order to match the electric dipole moment of the neutron. The dynamical solution yields \( \langle a \rangle = -\Theta f_a/N \), at which the effective potential for the axion has its minimum.

The axion can interact via heavy quark while all other SM fields do not carry any PQ charge, in which case \( N = 1 \) (Kim 1979, Shifman et al. 1980). The axion can directly couple to the SM, and at the lowest order it will induce non-renormalizable coupling with the gluons, where \( N = 6 \) (Dine et al. 1981, Zhitnitsky 1980).

The axion searches, various astrophysical and cosmological observations suggest that the PQ scale (Nakamura et al. 2010, Raffelt 2008, Sikivie 2000) must be large,

\[
f_a/N \gtrsim 6 \times 10^6 \text{ GeV}, \tag{235}
\]

and the axion mass must be very small, \( m_a \lesssim 0.01 \text{ eV} \). The cosmological constraints on \( f_a > 2 \times 10^7 \text{ GeV} \) \( (m_a \lesssim 0.3 \text{ eV}) \) arises from BBN, accounting for the current bound on the relativistic species, \( N_e^\text{eff} = 3.1_{-1.4}^{+1.4} \) (Iocco et al. 2009). Another interesting bound arises from isocurvature perturbations from CMB. At best one can allow less than 10% of the total perturbations to arise from sources other than the inflaton fluctuations the axions being massless during inflation can account for such fluctuations, which limits \( f_a \gtrsim 10^{12} - 10^{13} \text{ GeV} \), however, it depends on the scale of inflation (Beltran et al. 2007, Steffen 2009).

The axion life time depends on the axion-photon interaction, which gives a long life time compared to the age of the universe, i.e., \( \tau_a = \Gamma_a^{-1} \gamma \gamma = \frac{64\pi}{g^2_a m_a^3} \sim \frac{10^{10}}{m_a N - 1} \frac{10^{10}}{10^{10} \text{ GeV}} \), for \( m_a/m_a \sim 0.56 \) (Kob and Turner 1988).

Axion is massless for \( T \gtrsim 1 \text{ GeV} \) \( \lesssim \Lambda_{QCD} \) and it acquires mass only through instanton effects for \( T \lesssim \Lambda_{QCD} \). For \( f_a/N \lesssim 3 \times 10^7 \text{ GeV} \) (corresponding to \( m_a \gtrsim 0.2 \text{ eV} \)), the axion is a thermal relic that decouples after the quark–hadron transition, \( T_f \lesssim 150 \text{ MeV} \). The axion is kept in thermal equilibrium with \( \pi \pi \leftrightarrow \pi a \). The relic thermal abundance is given by \( \Omega_a h^2 = 0.077 \left( 10/g_a(T_f) \right) (m_a/10 \text{ eV}) \), where \( g_a \) denotes the number of effectively massless degrees of freedom.

In an opposite limit, when \( f_a/N \) is very large, the axions are never in thermal equilibrium, and in particular when \( T_R < f_a \) the PQ symmetry is never restored. The main production mechanism is due to the coherent oscillations of the axion due to the initial misalignment angle \( \Theta_i \) of the axion. At \( T \sim \Lambda_{QCD} \), the axion obtains a temperature dependent effective mass and oscillate coherently around its minimum when \( m_a(T_{osc}) \sim H(T) \). These oscillations of the axion condensate behaves as cold dark matter (Abbott and Sikivie 1983, Dine and Fischler 1983, Preskill et al. 1983) with a relic density that is governed by the initial misalignment angle \( -\pi < \Theta_i \leq \pi \) (Beltran et al. 2007, Sikivie 2008):

\[
\Omega_a h^2 \approx 0.15 \xi f(\Theta_i^2) \Theta_i^2 \left( \frac{f_a/N}{10^{10} \text{ GeV}} \right)^{7/6} 
\] (236)

with \( \xi = O(1) \) parameterizing theoretical uncertainties related to details of the quark–hadron transition and \( f(\Theta_i^2) \) accounting for anharmonicity in \( \Theta_i - f(\Theta_i^2) \rightarrow 1 \) for \( \Theta_i^2 \rightarrow 0 \). For \( 10^{10} \text{ GeV} \lesssim f_a/N \lesssim 10^{13} \text{ GeV} \), this “misalignment mechanism” can provide the correct dark matter abundance.

D. SUSY WIMP

The most general gauge invariant and renormalizable superpotential would also include baryon number \( B \) or...
lepton number $L$ violating terms, with each violating by one unit: $W_{\Delta L=1} = \frac{1}{2} \lambda^{ijk} L_i L_j e_k + \lambda'^{ijk} L_i Q_j d_k + \mu'' L_i H_u$ and $W_{\Delta B=1} = \frac{1}{2} \lambda^{ijk} u_i d_j d_k$, where $i = 1, 2, 3$ represents the family indices. The chiral supermultiplets carry baryon number assignments $B = +1/3$ for $Q_i$, $B = -1/3$ for $u_i, d_i$, and $B = 0$ for all others. The total lepton number assignments are $L = +1$ for $L_i$, $L = -1$ for $e_i$, and $L = 0$ for all the others. Unless $\lambda'$ and $\lambda''$ terms are very much suppressed, one would obtain rapid proton decay which violates both $B$ and $L$ by one unit.

There exists a discrete $Z_2$ symmetry, which can forbid baryon and lepton number violating terms, known as $R$-parity (Fayet, 1979). For each particle:

$$P_R = (-1)^{(B-L)+2s}$$

with $P_R = +1$ for the SM particles and the Higgs bosons, while $P_R = -1$ for all the sleptons, squarks, gauginos, and Higgsinos. Here $s$ is spin of the particle. Besides forbidding $B$ and $L$ violation from the renormalizable interactions, $R$-parity has interesting phenomenological and cosmological consequences. The lightest sparticle with $P_R = -1$, the LSP, must be absolutely stable. If electrically neutral, the LSP is a natural candidate for dark matter (Dimopoulos and Hall, 1988; Ellis et al., 1984a). The advantage here is that their cross sections are governed by the SM gauge group – and therefore the dark matter paradigm is embedded within a visible sector. However, there are some exceptions which we will discuss first.

1. Gravitino

The gravitino is a spin-3/2 supersymmetric partner of the graviton, which is coupled to all the sectors universally, e.g. visible and hidden sectors, with a Planck suppressed interaction. In this respect gravitino is truly a singlet dark matter candidate, if it happens to be the LSP. The gravitino mass can vary $(m_{3/2} \sim \mathcal{O}(100) \text{ GeV} - \mathcal{O}(1) \text{ KeV})$ depending on the details of the SUSY breaking schemes, for instance, in gauge and gravity-mediated SUSY breaking scenarios (Dine et al., 1996a; 1995a; Giudice and Rattazzi, 1999). Indeed, without considering the SUSY breaking mechanisms and the SUSY breaking scale, we can treat the gravitino mass as a free parameter.

**Production:** Gravitinos with both the helicities can be produced from a thermal bath, they are never in thermal equilibrium. They are produced mainly through the scatterings–within MSSM there are many scattering channels which include fermion, sfermion, gauge and gaugino quanta all of which have a cross-section $\propto 1/M_{\tilde{g}}^2$.

The thermal abundance is given by (up to a logarithmic correction) for $g_*=228.75$ as in the case of MSSM:

$$Y_{\pm 3/2} \simeq (T_R/10^{10} \text{ GeV}) \times 10^{-12} \quad (Ellis et al., 1984b),$$

and

$$Y_{\pm 1/2} \simeq [1 + M_{\tilde{g}}^2(T_R)/12m_{3/2}^2](T_R/10^{10} \text{ GeV}) \times 10^{-12} \quad (Ellis et al., 1984b).$$

Thermally produced gravitinos have a negligible free-streaming velocity today. However, the gravitinos created from decays can be warm or hot dark matter.

Besides thermal production, gravitino can be produced non-thermally from the decay of the NLSP (next-to-lightest SUSY particle). Obviously different NLSP’s give slightly different abundances. For sneutrino, see Refs. (Arina and Fornengo, 2007; Ellis et al., 2008; Feng et al., 2004), for stop NLSP, see (Berger et al., 2008; Diaz-Cruz et al., 2007; Kang et al., 2008). A simple approximation yields (Pospelov et al., 2008; Steffen, 2006)

$$\Omega_{3/2}h^2 \simeq 0.32 \left( \frac{10 \text{ GeV}}{m_{3/2}} \right) \left( \frac{m_{1/2}}{1 \text{ TeV}} \right)^2 \left( \frac{T_R}{10^{6} \text{ GeV}} \right)$$

![Graph](image_url)

**FIG. 5:** The bound on reheat temperature $T_R$ with respect to an unstable gravitino mass $m_{3/2}$, where neutralino is the LSP with a mass 117 GeV (indicated by the shaded light-orange region in which $m_{3/2} \leq m_{\tilde{\chi}^0}$ for $(m_{1/2} , m_0) = (300, 141) \text{ GeV}$, $A_0 = 0$, $\tan \beta = 30$). The thermal relic density is given by: $\Omega_{\chi^0}h^2 = 0.111$. Above the dotted line labeled as $\Omega_{3/2}$, the $\tilde{\chi}^0_1$ density from decays of thermally produced gravitinos exceeds $\Omega_{\chi^0}h^2 = 0.118$, see (Kawasaki et al., 2008).

Note that for $M_{\tilde{g}} \leq m_{3/2}$ both the helicity states have essentially the same abundance, while for $M_{\tilde{g}} \gg m_{3/2}$ production of helicity $\pm 1/2$ states is enhanced due to their Goldstino nature. The net abundance $\Omega_{3/2}h^2$ can be approximated by the convenient expression for the universal gaugino masses $M_{1,2,3} = m_{1/2}$ at $M_{\text{GUT}}$ and $m_{3/2} \ll M_{1,2,3}$ (Pradler and Steffen, 2008)

$$\Omega_{3/2}h^2 \simeq 0.32 \left( \frac{10 \text{ GeV}}{m_{3/2}} \right) \left( \frac{m_{1/2}}{1 \text{ TeV}} \right)^2 \left( \frac{T_R}{10^{6} \text{ GeV}} \right)$$

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which holds not only for a charged slepton NLSP but also for the sneutrino NLSP. Similar expressions for the lifetimes for the neutralino NLSP (Feng et al. 2004) and the stop NLSP can be found in Diaz-Cruz et al. (2007). If the NLSP decays into the gravitino LSP after BBN, the SM particles emitted in addition to the gravitino can affect the abundances of the primordial light elements. Also the presence of a long-lived negatively charged particle, champ, i.e. $\tilde{l}^-$, can lead to bound states that catalyze BBN reactions (De Rujula et al., 1990; Dimopoulos et al., 1990). The new acceptable limits on champs: $10^3(\eta_{X}/e)^2 \leq m_X \leq 10^6(\eta_{X}/e)^2$ TeV, virtually ruled out any low scale SUSY champs (Chuzhoy and Kolb, 2009). It was suggested in Ref. (Pospelov, 2007) that bound-state formation of champ with $He^4$ can lead to a substantial production of primordial $Li^6$, which puts the constraint $\tau_{\tilde{e}} \lesssim 5 \times 10^3$ sec. (Bird et al., 2008; Hamaguchi et al., 2007; Pospelov, 2007; Pospelov et al., 2008; Pradler and Steffen, 2008; Takayama, 2008).

**Uncertainties:** The main uncertainties on gravitino abundance arise from the hidden sectors. If the inflaton sector is embedded within a hidden sector then there are many more sources of gravitino production—the inflaton could decay directly into gravitino during reheating or preheating (Frey et al., 2006; Gaudence et al., 1999; Kallosh et al., 2000a,b; Maroto and Zumalacarregui, 2001b; Nilles et al., 2001a, 2001b; Nilles and Pospelov, 2001), the inflaton couplings to other hidden sectors can similarly excite gravitinos giving rise to large uncertainties in their total abundance.

$$\Omega_{3/2} h^2 = \Omega_{3/2}^{MSSM} h^2 + \Omega_{3/2}^{Inflaton} h^2 + \Omega_{3/2}^{Hidden} h^2,$$

(241)

All these contributions can easily overproduce gravitinos, i.e. $\Omega_{3/2} h^2 \sim 1$, especially the last two sectors are largely unconstrained by particle physics. These uncertainties can be minimized if the last phase of inflation occurs within MSSM, as discussed in Sect. 77, then the only predominant source of gravitino production arises from the decay of the MSSM inflaton and from the MSSM thermal bath.

Another solution has been put forward for high scale and hidden sector models of inflation – since the flat directions of MSSM can be displaced from their minimum during inflation, the flat direction VEV at early times would generate time dependent masses to the MSSM fields which are coupled to the flat direction. As a result, the inflaton might not even decay into all the MSSM d.o.f due to kinematical blocking (Allahverdi and Mazumdar, 2003, 2006b, 2007b). Furthermore, the flat direction VEV also generates masses to gauge bosons and gauginos which participate in scatterings, therefore delaying the actual thermalization process and lowering the reheating temperature, i.e. $T_r$. Both these effects address the thermal gravitino overproduction problem without any need of extra assumptions (Allahverdi and Mazumdar, 2003).

**Unstable gravitino:** An unstable gravitino decays to particle-particle pairs, and its decay rate is given by $\Gamma_{3/2} \approx m_{3/2}^3/4M_p^2$. If $m_{3/2} < 50$ TeV, the gravitino decays during or after BBN, which can ruin its successful predictions for the primordial abundance of light elements (Cyburt et al., 2003; Kawasaki et al., 2005). If the gravitinos decay radiatively, the most stringent bound, $(n_{3/2}/s) \lesssim 10^{-14} - 10^{-12}$, arises for $m_{3/2} \approx 100$ GeV to 1 TeV (Cyburt et al., 2003). On the other hand, much stronger bounds are derived if the gravitinos mainly decay through the hadronic modes. In particular, for a hadronic branching ratio $\approx 1$, and in the same mass range, $(n_{3/2}/s) \lesssim 10^{-16} - 10^{-15}$ will be required (Kawasaki et al., 2003; 2008). This puts constraint on reheat temperature of the universe, i.e. $T_R$, at which these unstable gravitinos are produced, see Fig. 5. An intriguing possibility arises if R-parity is broken. The gravitino LSP with $m_{3/2} \sim 1$ GeV can still be a long-lived cold dark matter, while evading the bounds on $T_R$ from Fig. 5. The gravitino in this case cannot decay into hadrons which is kinematically suppressed, and the three-body decay life time is typically larger than the age of the universe (Kohri et al., 2009). For a GeV mass gravitino, the present day free-streaming velocity is $\lesssim 10^{-9}$ km/s, which corresponds to that of a cold dark matter.

**Detection:** The direct detection of gravitino will be impossible at the LHC, their production will be extremely suppressed. If the NLSP is long lived (quasi-stable) as in the case of stau then they would penetrate the collider detector in a way similar to muons (Drees and Tata, 1990; Feng and Moroi, 1998; Nisati et al., 1997). If the produced staus are slow, then from the associated highly ionizing tracks and time of flight measurements one can determine their mass (Ellis et al., 2006). This might give some indirect handle on gravitinos.

2. Axino

The axino, $\tilde{a}$, is a superpartner of the axion, is another example of a gauge singlet dark matter candidate (Kim, 1984; Kim and Nilles, 1984; Nilles and Raby, 1982). It interacts extremely weakly since its couplings are suppressed by the PQ scale $f_a \gtrsim 10^8$ GeV (Raffelt, 2007; 2008; Sikivie, 2008), and its mass can range from eV to GeV. In the hadronic axion model (Kim, 1979; Shifman et al., 1980) in a SUSY setting, the axino couples to MSSM field indirectly via loops of heavy (s)quarks. Typically $\tilde{a}$ decouples early at a temperature $T_f \gtrsim 10^9$ GeV, below this temperature, they are mainly created from scatterings of MSSM fields in an kinetic equilibrium. The thermal abundance is given by (Brandenburg and Steffen, 2004; Choi et al., 2008).
Covi et al. [2001]

\[ \Omega_\tilde{a}h^2 \approx 5.5g_s^6(T_R) \ln \left( \frac{1.108}{g_s(T_R)} \frac{10^{11} \text{GeV}}{f_a/N} \right)^2 \]
\[ \times \left( \frac{m_{\tilde{a}}}{0.1 \text{ GeV}} \right) \left( \frac{T_R}{10^{12} \text{ GeV}} \right) \]

with the axion-model-dependent color anomaly \( N \) of the PQ symmetry breaking scale \( f_a \). Thermally produced axinos can be hot, warm, and cold dark matter [Covi et al. 2001] as indicated in the plot. The plot is taken from [Brandenburg and Steffen, 2004].

Non-thermal production of \( \tilde{a} \) has many uncertainties. The \( \tilde{a} \) can be created from the decay of the NLSP, direct decay from the inflaton or moduli, or any other hidden sector. The expression for the final abundance is similar to that of Eq. (241), where \( 3/2 \rightarrow \tilde{a} \). In this regard \( \tilde{a} \) faces similar challenges as that of a gravitino dark matter.

The \( \tilde{a} \) LSP is inaccessible in direct/indirect dark matter searches if R-parity is conserved. Their direct production at collider is strongly suppressed. Nevertheless, quasi-stable \( \tilde{\tau} \)’s could appear in collider detectors (and neutrino telescopes [Ahlers et al. 2006; Albuquerque et al. 2007]) as a possible signature of the \( \tilde{a} \) LSP.

3. Neutralino

In the MSSM the binos \( \tilde{B} \) (superpartner of B), winos \( \tilde{W} \) (superpartner of W) and Higgsinos \( \tilde{H}^0 \) and \( \tilde{H}^\pm \) mix into 4 Majorana fermion eigenstates, called neutralinos with 4 mass eigenstates: \( \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0 \), ordered with increasing mass. The LSP is thus denoted by \( \tilde{\chi}_1^0 = N_{11}\tilde{B} + N_{12}\tilde{W} + N_{13}\tilde{H}_u^0 + N_{14}\tilde{H}_d^0 \). The gaugino fraction, \( f_G = N_{11}/N_{12} \), and Higgsino fraction, \( f_H = N_{13}^2 + N_{14}^2 \), are determined by the mixing matrix, \( N \), which diagonalizes the neutralino mass matrix [Jungman et al. 1996, Kane et al. 1994].

The \( \tilde{\chi}^0 \)'s were in thermal equilibrium for primordial temperatures of \( T > T_f \sim m_{\tilde{\chi}^0}/20 \). At \( T_f \), the annihilation rate of the (by then) non-relativistic \( \tilde{\chi}^0 \)'s becomes smaller than the Hubble rate so that they decouple from the thermal plasma, see Sect. [IV.B.1].

It is easy to work with a limited set of parameters, the mSUGRA model is a simple model which contains only five parameters:

\[ m_0, \ m_{1/2}, \ A_0, \ \tan \beta \text{ and } \text{sign}(\mu). \]

\( m_0 \) is the universal scalar soft breaking parameter, \( m_{1/2} \) is the universal gaugino mass, \( A_0 \) is the universal cubic soft breaking mass, measures at \( M_{GUT} \), and \( \tan \beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle \) at the electroweak scale.

The model parameters are already constrained by different experimental results. (i) the light Higgs mass bound of \( M_{h^0} > 114 \text{ GeV} \) from LEP [Barate et al. 2003], (ii) the \( b \to s\gamma \) branching ratio bound of \( 1.8 \times 10^{-4} < B(B \to X_s^0\gamma) < 4.5 \times 10^{-4} \) (we assume here a relatively broad range, since there are theoretical errors in extracting the branching ratio from the data) [Alam et al. 1995], (iii) the 2\( \sigma \) bound on the dark matter relic density: \( 0.095 < \Omega_{CDM} h^2 < 0.129 \) [Komatsu et al. 2009], (iv) the bound on the lightest chargino mass of \( m_{\tilde{\chi}_2^\pm} > 104 \text{ GeV} \) from LEP [Nakamura et al. 2010], and (v) the muon magnetic moment anomaly \( \alpha_{\mu} \), where one gets a 3.3\( \sigma \) deviation from the SM from the experimental result [Bennett et al. 2004].

The allowed mSUGRA parameter space, at present, has mostly three distinct regions: (i) the stau-neutralino \( (\tilde{\tau}_1 - \tilde{\chi}^0) \) coannihilation region where \( \tilde{\chi}_1^0 \) is the lightest.
SUSY particle (LSP), (ii) the $\chi_1^0$ having a dominant Higgsino component (focus point) and (iii) the scalar Higgs ($A^0, H^0$) annihilation funnel ($2M_{\tilde{\chi}_1^0} \approx M_{A^0, H^0}$). These three regions have been selected out by the CDM constraint. There still exists a bulk region where none of these above properties is observed, but this region is now very small due to the existence of other experimental bounds. The allowed parameter space for the neutralino dark matter (blue region) for $\tan(\beta) = 40$ is shown in the left panel of Fig. 7.

Detection: In general the observable signals for SUSY at LHC are: $n$ leptons + $m$ jets + $E_T$ (missing transverse energy), where either $n$ or $m$ could be 0. The existence of missing energy in the signal will tell us the possibility of dark matter candidate. There are SM backgrounds, e.g. $W$ and $Z$ bosons decaying to neutrinos providing $E_T$. The clean signal for SUSY would be jets + $E_T$, without isolated leptons. One of the key analysis for mSUGRA is to measure the $M_{eff}$ which is the sum of the transverse momenta of the four leading jets and the missing transverse energy: $M_{eff} = p_{T,1} + p_{T,2} + p_{T,3} + p_{T,4} + E_T$. One has to further measure the masses (squarks, sleptons), $A_0$ and $\tan \beta$, and the mixing matrices which lead to the calculation of the relic density. One particularly favored parameter space is the coannihilation region where the stau and the neutralino masses are close for smaller values of $m_{\tilde{e}_L}$. The mass difference, $\Delta m$, governs the relic abundance due to the Boltzmann suppression factor $e^{-\Delta M/T}$, see section III.B.2. Therefore measuring $\Delta M$ directly gives handle on measuring the relic abundance at the LHC, see for a detailed discussion in (Nath et al. 2010).

MSSM inflation and dark matter: After considering all these bounds it was found that there exists an interesting overlap between the constraints from MSSM inflation and the $\chi_1^0$ abundance, see the right hand panel of Fig. 7 for $\tan(\beta) = 40$. The constraints on the parameter space arising from the inflation appearing to be consistent with the constraints arising from the dark matter content of the universe and other experimental results.

It is also interesting to note that the allowed region for $u_d d_d$ as an MSSM inflaton with a mass $m_{\phi}$, required by the CMB observations for $\lambda = 1$, see Fig. 8 in Sect. III.E.2 lies in the stau-neutralino coannihilation region which requires smaller values of the SUSY particle masses (Allahverdi et al. 2007a). Similar analysis were performed in (Balazs et al. 2003, 2004), where the authors studied the overlap between MSSM parameters for the electroweak baryogenesis and the $\chi_1^0$ dark matter abundance.

4. Sneutrino

The lightest right handed (RH) sneutrino $\tilde{N}$ can be a good dark matter candidate when the SM gauge group is augmented to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, with a superpotential (Allahverdi et al. 2007b, 2010b)

$$W = W_{MSSM} + W_{B-L} + yNH_u L.$$  (244)

The model introduces new gauge boson $Z'$, two Higgs fields $H'_1, H'_2$, and their superpartners, the Yukawa coupling is denoted by $y$. The spontaneous breaking of $U(1)_{B-L}$ will generate Majorana neutrino masses, or if $y \approx 10^{-12}$, then Dirac neutrino masses, or a mixture of both Dirac and Majorana (Allahverdi et al. 2011a). If the right handed sneutrino is the LSP then it provides another compelling candidate which is well motivated from particle theory and can be embedded with least unknown uncertainties (Allahverdi et al. 2007b, Arina and Fornengo 2007, Lee et al. 2007).

Scatterings via the new $U(1)$ gauge interactions bring the RH sneutrino into thermal equilibrium. In order to calculate the relic abundance of the RH sneutrino, we need to know the masses of the additional gauge boson $Z'$ and its SUSY partner $Z''$, the new Higgsinos masses, Higgs VEVs which break the new $U(1)_{B-L}$ gauge symmetry, the RH sneutrino mass, the new gauge coupling, and the charge assignments for the additional $U(1)$. The primary diagrams responsible to provide the right amount of relic density are mediated by $Z'$ in the $t$-channel.

By assuming that the new gauge symmetry is broken around TeV in Fig. 8 we show the relic density values for smaller masses of sneutrino where the lighter stop mass is $\leq 1$ TeV. The smaller stop mass will be easily accessible at the LHC. By varying new gaugino and Higgsino masses and the ratio of the VEVs of new Higgs fields, the WMAP (Komatsu et al. 2009) allowed values of the relic density, i.e., 0.094 – 0.129 is satisfied for many points. In the case of a larger sneutrino mass in this model, the correct dark matter abundance can be obtained by annihilation via $Z'$ pole (Allahverdi et al. 2007b, Lee et al. 2007).

Detection: Since the dark matter candidate, the RH sneutrino, interacts with quarks via the $Z'$ boson, it is possible to see it via the direct detection experiments. The detection cross sections are not small as the interaction diagram involves $Z'$ in the $t$-channel. The typical
cross section is about $2 \times 10^{-8}$ pb for a $Z'$ mass around 2 TeV. It is possible to probe this model in the upcoming dark matter detection experiments. The signal for this scenario at the LHC will contain standard jets plus missing energy and jets plus leptons plus missing energy. The jets and the leptons will be produced from the cascade decays of squarks and gluinos into the final state containing the sneutrino.

5. Stable and evaporating Q-ball, LSP dark matter

The AD condensate fragments to form Q-balls, for reviews see [Dine and Kusenko 2004; Enqvist and Mazumdar 2003], and a finite size Q-ball has a minimum of energy and it is stable with respect to decay into free quanta if $U(\phi)/\phi^2 = \text{min}$, for $\phi_0 > 0$.

For a sufficiently large $Q$, the energy of a soliton is then given by [Coleman 1985; Kusenko 1997b; Lee and Pang 1998; Kusenko and Shaposhnikov 1998], where

$$E \approx \frac{4\sqrt{3}}{3} \pi m_\phi Q^{3/4}, R \approx \frac{Q^{1/4}}{\sqrt{2m_\phi}}, \phi_0 \approx \frac{m_\phi}{\sqrt{2\pi}} Q^{1/4}$$

This allows for the existence of some entirely stable Q-balls with a large baryon number $Q \sim B$ (B-balls). Indeed, if the mass of a B-ball is $M_B \sim (1 \text{ TeV}) \times B^{3/4}$, then the energy per baryon number $(M_B/B) \sim (1 \text{ TeV}) \times B^{-1/4}$ is less than 1 GeV for $B > 10^{12}$. Such large B-balls cannot dissociate into protons and neutrons and are entirely stable thanks to the conservation of energy and the baryon number. If they were produced in the early universe, they would exist at present as a form of dark matter [Kusenko and Shaposhnikov 1998]. There are astrophysical and terrestrial limits [Kusenko et al. 1998; Kusenko and Shoemake, 2009], and direct searches for the Q-balls, which places a lower limit on $Q > 10^{22}$ [Arafune et al. 2000].

In the gravity mediated case the B-balls are not stable, but they evaporate via surface evaporation. In this process AD condensate can generate the required baryon asymmetry and also create dark matter. The appropriate candidate will be the $udd$ flat direction, lifted by $n = 6$ operator, which carries the baryon number and the right dark matter abundance, see Eq. (179).

When a $B$-ball decays, for each unit of $B$ produced, corresponding to the decay of 3 squarks to squarks, there will be at least three units of $R$-parity produced, corresponding to at least 3 LSPs (depending on the nature of the cascade produced by the squark decay and the LSP mass, more LSP pairs could be produced). Let $N_{LSP} \geq 3$ be the number of LSPs produced per baryon number and $f_B$ be the fraction of the total $B$ asymmetry contained in $B$-balls. Then the baryon to dark matter ratio, $r_B = \rho_B/\rho_{DM}$, and the dark matter abundance are given by [Enqvist and McDonald 1998, 1999],

$$r_B = \frac{m_n}{N_{LSP} f_B m_{LSP}}, \quad \Omega_{LSP} \approx 3 f_B \left( \frac{m_{LSP}}{m_n} \right)$$

where $m_n$ is the nucleon mass and $m_{LSP}$ is the LSP mass. It is rather natural to have $r_B < 1$.

The LSPs produced in $B$-ball decays will collide with themselves and with other weakly interacting particles in the background and settle locally into a kinetic equilibrium. Thermal contact can be maintained until $T_f \sim m_{LSP}/20$, and a rough freeze-out condition for LSPs (if they were initially in thermal equilibrium) will be given by: $n_{LSP}(\sigma_{eff} T_f) \approx H f_{\chi_1^0}/T_f$, where $\sigma_{eff}$ is the LSP annihilation cross-section and the subscript $f$ refers to the freeze-out values. The thermally averaged cross section can be written as $\langle \sigma_{eff} T \rangle = a + b T/m_\chi_1^0$, where $a$ and $b$ depend on the couplings and the masses of the light fermions [Jungman et al. 1990].

Assuming an efficient LSP production, so that $f_B = 1$, one finds for the LSP density for $b \approx H m_\chi_1^0 T_f^{-2} n_{LSP}$, where $n_f \approx (m_\chi_1^0 T_f)^{3/2} \exp[-m_\chi_1^0/T_f]$. The LSPs produced in $B$-ball decays will spread out by a random walk with a rate $\nu$ determined by the collision frequency divided by a thermal velocity $\nu_s \approx (T/m_\chi_1^0)^{1/2}$. Since the decay is spherically symmetric, it is very likely that the LSPs have a Gaussian distribution. In terms of the density parameter $\Omega_{\chi_1^0}$, the neutralino abundance can be written as [Fujii and Hanaguchi 2002]

$$\Omega_{\chi_1^0} \approx 0.5 \left( \frac{m_\chi_1^0}{100 \text{ GeV}} \right) \cdot \frac{10^{-7} \text{GeV}^2}{(\sigma v)} \cdot \frac{100 \text{MeV}}{T_d} \left( \frac{10}{g_s(T_d)} \right)^{1/2}$$

where the decay temperature of the B-ball is given by

$$T_d \ll 21 \left( \frac{m_\chi_1^0}{100 \text{ GeV}} \right)^{3/16} \left( \frac{10^{20}}{N_{LSP}} \right)^{1/8} \left( \frac{100}{g(T)} \right)^{3/16} \text{GeV}$$

Below this temperature the annihilations of $\chi_1^0$ are negligible. Similar analysis can be performed for other LSP candidates. For example, if the gravitino is an LSP, the
gravitational abundance from the Q-balls decay will be given by (Shoemaker and Kusenko, 2009):

\[ \Omega_{\lambda/2}h^2 \approx 0.11 \left( \frac{m_{\lambda}/2}{1 \text{ GeV}} \right) \left( \frac{N_\lambda f_b}{3} \right) \left( \frac{\Omega_h h^2}{0.02} \right) , \]  

(249)

where \( N_\lambda = 3 \) and \( f_b \sim 1 \). The above expression is valid for temperatures below \( 10^7 \text{ GeV} \). Similar expression can be derived for the Q-balls decaying into axino dark matter (Roszkowski and Seto, 2007).

E. Detection of WIMP

The direct detection of WIMP is the cleanest way to seek the identity of the dark matter, their detection is possible through elastic collision with the nuclei at terrestrial targets (Goodman and Witten, 1983). This method is especially promising for detecting SUSY WIMP candidates, such as neutralino or sneutrino. There are also ways of inferring WIMP in the sky by using the galaxy itself as a detector in the indirect dark matter searches via studying the gamma rays, high energy neutrinos, charged leptons, proton, anti-proton background from the decay or annihilation of the dark matter particles.

1. Direct detection of WIMPs

Important quantity is the recoil energy deposited by the WIMP interaction with the nucleus of mass \( m_X \) in an elastic collision, \( E_r = m_X^2 v^2 (1 - \cos \theta) / m_X, \) where \( m_r \) is the WIMP nucleus reduced mass, \( \theta \) is the scattering angle in the dark matter-nucleus center-of-mass frame, and \( v \) is the velocity relative to the detector, and it is of the order of the galactic rotation velocity \( \sim 200 \text{ km/s}. \) Typical recoil energies are \( E_r \sim \mathcal{O}(1-100) \text{ keV}. \)

The differential rate for WIMP elastic scattering off nuclei is given by (Lewin and Smith, 1996)

\[ \frac{dR}{dE_R} = N_T \frac{\rho_0}{m_W} \int_{v_{\min}}^{v_{\max}} d\bar{v} f(\bar{v}) \sqrt{2} |q| F(q) \frac{d\sigma}{dE_R} , \]  

(250)

where \( N_T \) represents the number of the target nuclei, \( m_X \) is the dark matter mass and \( \rho_0 \sim 0.3 \text{ GeV/cm}^3 \) is the local WIMP density in the galactic halo, \( \bar{v} \) and \( f(\bar{v}) \) are the WIMP velocity and velocity distribution function in the Earth frame, which we take it to be Maxwellian, and \( d\sigma/dE_R \) is the WIMP-nucleus differential cross section. The velocity \( v_{\min} = \sqrt{(m_X E_r/2 m_r^2)} \) and \( v_{\max} \) is the escape velocity of the WIMP in the Earth frame, \( v_{\text{esc}} = 544^{+94}_{-46} \text{ km/s}. \)

In fact, the Earth velocity with respect to the dark matter halo must be written as \( v_e = v_0 (1.05 + 0.07 \cos \omega) \) where \( 1.05 v_0 = \text{ galactic velocity of the Sun and } \omega = 2\pi/1 \text{ year}. \) The 7\% modulation is due to the rotation of the Earth around the Sun (Drukier et al., 1986; Freese et al., 1988). In the above expression, \( f(\bar{v}) \) must be replaced by \( f(|\bar{v} - v_e|) \). There also exists a forward-backward asymmetry in a directional signal as first pointed out in (Copi et al., 1999; Spergel, 1988).

For a given momentum transfer \( q \), the differential cross section depends on the nuclear form factor

\[ \frac{d\sigma}{dq^2} = \frac{\sigma_0}{4 m_r^2 v^2} F^2(q) , \]  

(251)

where \( F(q) \) is a dimensionless form factor such that \( F(0) = 1 \), in which case \( \sigma_0 \) corresponds to the total cross-section. It is possible to estimate the parameters \( \sigma_0 \) and \( F(q) \), for example in the case of neutralino WIMP, which is a Majorana fermion therefore it only has axial and scalar couplings (Jungman et al., 1996).

a. Spin-dependent cross-section: The axial part of the neutralino-quark interaction is mediated via Z boson and squark exchange \( \mathcal{L}_{\chi q} \sim (\bar{X} \gamma^\mu \gamma_5 X) (q^\mu \gamma_5 q) \). At the level of neutralino-nucleon interaction by considering the nucleon matrix element \( \langle n | \bar{q} \gamma_\mu \gamma_5 \gamma_\lambda f | n \rangle \), the effective Lagrangian is given by:

\[ \mathcal{L}^{\chi}_{\text{eff}} = 2 \sqrt{2} G_F a_n (\bar{X} \gamma^\mu \gamma_5 X) (\bar{n} \gamma_\lambda n) , \]  

where \( G_F \) is the Fermi constant and \( a_n \) is a dimensionless parameter. For a nucleus of spin \( J \), with \( \langle S_p \rangle \) and \( \langle S_n \rangle \) being the average spins “carried” by protons and neutrons respectively, the cross-section at zero momentum transfer is given by (Jungman et al., 1996)

\[ \frac{d\sigma}{dq^2}(q = 0) = \frac{8}{\pi v^2} G_F^2 \Lambda^2 J(J + 1) , \]  

(252)

where \( \Lambda = (a_p \langle S_p \rangle + a_n \langle S_n \rangle) / J \). Additional corrections to the form factor is required to take into account of the non-zero momentum transfer. There are many experiments which are sensitive to spin-dependent cross section with pure proton couplings. DAMA (Bernabei et al., 2004), PICASSO (Archambault et al., 2009), KIMS (Lee et al., 2007). Recently COUPP (Belokne et al., 2011) has set the best constraint on spin-dependent cross section down to \( 7 \times 10^{-38} \text{ cm}^2 \) for a WIMP mass \( \sim 30 \text{ GeV}. \)

b. Spin-independent cross-section: The scalar part of the neutralino-quark interaction is mediated via Higgses and squark exchanges: \( \mathcal{L}_{\chi q} = f_q (\bar{q} q)(XX) \). To express the neutralino-nucleon coupling one needs the nucleon matrix element \( \langle n | \bar{q} q | n \rangle = m_n \), the effective interaction has the form:

\[ \mathcal{L}^{\chi}_{\text{eff}} = f_n (XX)(nn) , \]  

where \( f_n \) contains the information about hadron physics, and typically it is the same for proton or neutron, i.e., \( f_p \sim f_n \). In the case of right handed sneutrino also there exists no spin-dependent part as it has no axial-vector interactions, and the cross-section is dominated by the spin-independent part. One can define a single spin-independent WIMP-nucleon cross section \( \sigma_0 \equiv \sigma_{SI} \), independent of the spin of the nucleon (Jungman et al., 1996).

\[ \frac{d\sigma}{dq^2} = |Z f_p + (A - Z) f_n|^2 F^2(q) / \pi v^2 , \]  

(253)
In the spin-indipendent case, for low nuclear recoil energies \( F(q) \sim 1 \), there is a coherence effect which boosts the WIMP-nucleus cross section by a factor \( A^2 m_r^2 \). As a result, this technique is better suited to detect the WIMP candidate with heavy nucleus targets. Recently the most stringent limits on the elastic spin-independent WIMP-nucleon cross-section has been given by number of experiments, such as CDMS-II (Ahmed et al. 2010), EDELWEISS-II (Armengaud et al. 2011) and XENON100 (Aprile et al. 2011). These limits are shown in Fig. 9. The shaded gray area also shows the expected region of CMSSM for the WIMP mass and the cross section are indicated at 68% and 95% CL (Buchmuller et al. 2011). The current results also covers the 90% CL areas favored by CoGeNT (green) (Aalseth et al. 2011) and DAMA (light red) (Savage et al. 2009).

It should be noted that CoGeNT (Aalseth et al. 2011), and DAMA/NaI (Bernabei et al. 2004) and DAMA/LIBRA (Bernabei et al. 2008) collaborations have observed an annual modulation signal. The combined results of the latter group stands at greater than 8\( \sigma \) statistical significance. The modulation signal phase matches well with the expected annual signal of WIMPs, and subsequent data (Bernabei et al. 2010) has increased the statistical significance of the modulation signal. However the annual modulation claim has not been verified by any other experiments, especially the null results from CDMS, XENON10, and XENON100 data. The CDMS data (Ahmed et al. 2010) shows 2 signal events with 0.6±0.1 events expected as background, but due to low statistics this result has not provided sufficient evidence for the dark matter.

2. Indirect detection

The indirect dark matter detection depends on the nature of the WIMP. If the WIMP belongs to the visible sector, or if it has some SM interactions, then their annihilation or decay would yield to the known particle physics spectrum, for a recent review on indirect detection, see (Porter et al. 2011). However the spectrum would depend on where they are produced, their energy deposition, and what are their final states, e.g. \( \gamma, e^\pm, \cdots \). The signal could be a hard spectrum with a monochromatic line if WIMPS annihilate directly into photons (Bergstrom et al. 1998), or a continuum spectrum if they annihilate into a pair of intermediate particles \((q = u, d, c, s, t, b), \bar{q}, Z, g, W^\pm, t^\pm\) . The former process is generically suppressed compared to the latter. Most of these latter particles, i.e. \( W, Z, g \) decay into \( p, \bar{p}, \pi^0 \), and a tiny fraction of deuterium or antideuterium \( D/D \). The \( \pi^0 \)'s decay to gamma rays, while the \( \pi^\pm \)'s produce \( e^\pm \). If the final states of decay or annihilations are \( e^\pm \) or \( \mu^\pm \), they dominantly produce a hard \( e^\pm \) spectrum, with the \( \mu^\pm \) decays into \( \nu_\mu \) and \( \bar{\nu}_\mu \). If the final states have \( \tau^\pm \), they produce a softer \( e^\pm \) spectrum and a strong neutrino signal. The \( \tau^\pm \) can also decay hadronically to pions and thus can also produce a strong \( \gamma \)-ray signal.

The source spectrum is generically given by

\[
\Phi_s(E) = \frac{1}{4\pi} \frac{\langle \sigma v \rangle}{2m_X^2} \sum_f \frac{dN_f}{dE} B_{f,s}, \tag{254}
\]

where \( f \) denotes the annihilating final states, each with branching fraction \( B_{f,s} \) with \( E \) being the energy of secondary particles. The production rate per annihilation of species \( f \) is given by \( dN_f/dE \). For a decaying dark matter \( \langle \sigma v \rangle/2m_X^2 \) can be replaced by \( \Gamma/M_X \), where \( \Gamma \) is the decay rate.

The flux of such final states would depend on the annihilation or decay rate, which in turn would depend on the dark matter density \( \varphi \). Therefore, the natural sources to look at in the sky are the nearby galactic centers – where there are large astrophysical uncertainties, dwarf galaxies – which have small astrophysical background, and galactic centers – where the dark matter density is very large but distant sources would yield a local tiny flux.

Gamma rays and neutrinos are perhaps the cleanest signals if they are produced as a result of WIMP annihilations or decays as they are undeflected by magnetic fields and effectively indicating the direction to their source. The flux is given by the integral of the WIMP density-squared along the line of sight from the observer to the source, multiplied by the production spectrum \( \phi_X(E, \psi) = J(\psi) \times \Phi_s(E) \)

\[
\Phi_s(E, \psi) = \frac{dN_f}{dE} \frac{\langle \sigma v \rangle}{4\pi m_X^2} B_{f,s} \int_{s_{1,0}} d\Omega^2(r(s, \psi)), \tag{255}
\]

where \( "f" \) denotes the final states and the coordinate \( s \) runs along the line of sight, \( E \) is the gamma-ray energy, and \( \psi \) is the elongation angle with respect to the center of the source. The astrophysics related term is hidden.
in \cite{Bergstrom2008}

\[ J(\psi) = \frac{1}{8.5 \text{ Kpc}} \left( \frac{1}{0.3 \text{ GeV/cm}^2} \right)^2 \int_{\text{l.o.s.}} \rho^2(\ell) \, d\ell \]  

(256)

where the integration is in the direction \(\psi\) along the line \(\ell\). The above expression is also valid for neutrinos if the source is not far away from us.

On the other hand the charged particles do not have the directional sensitivity, they lose it in their course of path in a random motion due to the interstellar magnetic field. The motion of \(e^\pm\) are deflected by the interstellar radiation field by synchrotron radiation in presence of magnetic field. If produced at energies \(\geq 100\) GeV and if their sources are within few kiloparsecs, then they can reach the solar system. Furthermore, the cosmic rays from the primary and secondary products can also generate \(\gamma\) rays during their course through the ISM, all these effects can be captured numerically in the publicly available code GALPROP \cite{Strong1998, Strong2000, Strong2004}, for a review see \cite{Strong2007}.

The main challenge is to disentangle the WIMP signals from astrophysical backgrounds, the powerful discriminator is the spectral tilt in the power spectrum. It is quite possible to have a significant fraction of WIMP annihilation or decay into monoenergetic photons, giving rise a distinctive line in the gamma-ray spectrum \cite{Bergstrom1998}, but the likely signal would be to have a relatively hard continuum spectrum with a bump or edge near the WIMP mass that is above the astrophysical background.

In recent years there have been many new experiments which have propelled the indirect search for dark matter research vigorously. For instance, the ATIC data – which shows a significant bump in the electron flux around 300 – 800 GeV \cite{Chang2008}, where conventional astrophysical sources would have predicted a decaying power law spectrum, and the PAMELA – which shows a positron fraction which has a rising slope above 10 GeV (Adriani shows a positron fraction which has a rising slope above 10 GeV (Adriani et al. 2009) and the PAMELA – which conventional astrophysical sources would have predicted a decaying power law spectrum, and the Fermi-LAT collaboration has released a diffused galactic and extra galactic \(\gamma\)-ray background (Abdo et al. 2009). It is also possible to conceive a scenario of non-thermal dark matter production in order to evade the strict bound on cross-section, or decaying dark matter scenarios, i.e. \cite{Arvanitaki2009, Fairbairn2009}.

Finally, one would also expect a gamma ray signal from the galactic center, the typical signature for a WIMP annihilation will be a line at the WIMP mass, due to the 2\(\gamma\), or \(\gamma Z\) production channels. The Fermi-LAT collaboration has released a diffused galactic and extra galactic \(\gamma\)-ray background \cite{Abdo2010a}. The systematic uncertainty associated with the absolute energy scale is shown by the non-vertical arrow. The dashed line shows the background from secondary \(e^\pm\) in cosmic rays from GALPROP.
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