The formation of supermassive black holes and the evolution of supermassive stars

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Received 5 July 2001
Published 17 September 2001
Online at stacks.iop.org/CQG/18/3965

Abstract

The existence of supermassive black holes is supported by a growing body of observations. Supermassive black holes and their formation events are likely candidates for detection by proposed long-wavelength, space-based gravitational wave interferometers like LISA. However, the nature of the progenitors of supermassive black holes is rather uncertain. Supermassive black hole formation scenarios that involve either the stellar dynamical evolution of dense clusters or the hydrodynamical evolution of supermassive stars have been proposed. Each of these formation scenarios is reviewed and the evolution of supermassive stars is then examined in some detail. Supermassive stars that rotate uniformly during their secular cooling phase will spin up to the mass-shedding limit and eventually contract to the point of relativistic collapse. Supermassive stars that rotate differentially as they cool will probably encounter the dynamical bar mode instability prior to the onset of relativistic collapse. A supermassive star that undergoes this bar distortion, prior to or during collapse, may be a strong source of quasiperiodic, long-wavelength gravitational radiation.

PACS numbers: 0440D, 0430D, 9760

1. Supermassive black holes

There is a large body of observational evidence that supermassive black holes (SMBHs) exist in the centres of many, if not most galaxies (see, e.g., the reviews of Rees \cite{Rees1} and Macchetto \cite{Macchetto}). The masses of SMBHs in the centres of more than 45 galaxies have been estimated from observations \cite{Kormendy} and there are more than 30 galaxies in which the presence of a SMBH has been confirmed \cite{Ferrarese}. The properties of a few confirmed SMBHs are given in table 1.

Several properties have been deduced from these observations \cite{Kormendy, Ferrarese, Gebhardt, Macchetto}. A correlation between the SMBH mass and both the mass and the velocity dispersion of the bulge of the host
Table 1. The host galaxies, masses and radii of selected confirmed SMBHs.

| Galaxy   | Mass ($M_\odot$) | Radius (pc) | Reference |
|----------|------------------|-------------|-----------|
| NGC 4258 | $\sim 3.6 \times 10^7$ | $<13$ | [5] |
| M87      | $\sim 3.2 \times 10^9$ | $<3.5$ | [6] |
| NGC 4151 | $\sim 10^9$ | $<60$ | [7] |
| Milky way | $2.6-3.3 \times 10^6$ | $<8$ | [8] |

galaxy is observed. These results suggest that the formation and evolution of SMBHs and the bulge component of their host galaxies may be closely related. The largest SMBHs are found in elliptical galaxies and thus may result from galaxy mergers, as ellipticals themselves are thought to form via merger.

Because of their size and mass, SMBHs are expected to be sources of long-wavelength, low-frequency gravitational radiation. Thus, phenomena involving SMBHs might be detectable with proposed space-based gravitational wave detectors, such as the laser interferometer space antenna (LISA) [13]. For example, LISA might be able to detect the collapse of a supermassive star to a SMBH, the coalescence of two SMBHs, or the coalescence of a compact star and a SMBH [14–18]. The event rates of these phenomena are uncertain due to the uncertainty in the nature of the progenitors of SMBHs. However, the rates could be appreciable because SMBHs are present in many galaxies.

1.1. Formation mechanisms

The possible formation mechanisms for SMBHs involve either the stellar dynamics of dense star clusters or the hydrodynamics of supermassive stars (SMSs).

1.1.1. Stellar dynamical routes. In general terms, dense star clusters are formed via a conglomeration of stars, produced by fragmentation of the primordial gas. In one cluster scenario for SMBH formation, massive stars form via stellar collisions and mergers in the cluster and then evolve into stellar-mass black holes. The merger of these holes, as they grow and settle to the cluster centre, leads to the build up of one or more SMBHs [16, 19, 20].

An alternative cluster scenario centres on the catastrophic collapse of the cluster core. Shapiro and Teukolsky have numerically demonstrated that cluster cores can become dynamically unstable to relativistic collapse [21]. They also showed that if the core is made up of compact stars, it will evolve to the onset of collapse on a relaxation timescale [22]. As the compact core evolves to this point of relativistic instability, the mass of its black holes will increase through collisions and gravitational radiation-driven mergers. This will accelerate the approach to relativistic instability, leading to SMBH formation [23, 24].

1.1.2. Hydrodynamical routes. SMSs may contract directly out of the primordial gas, if radiation and/or magnetic field pressure prevent fragmentation [25–27] (see also [28–30]). Alternatively, they may build up from fragments of stellar collisions in clusters [31, 32].

The evolution of SMSs will ultimately lead to the onset of relativistic instability [33–36]. If the mass of the star exceeds $10^6 M_\odot$, the star will then collapse and possibly form a SMBH. If the star is less massive, nuclear reactions may lead to explosion instead of collapse.
2. Supermassive stars

Identifying the scenario by which SMBHs form is of fundamental importance to a number of areas of astrophysics. The remainder of this paper examines one of the possible progenitors of SMBHs discussed above, SMSs.

Supermassive stars are radiation dominated, isentropic and convective [28, 36, 37] and are thus well represented by an $n = 3$ polytrope. As mentioned previously, if the star’s mass exceeds $10^6 M_\odot$, nuclear burning and electron/positron annihilation are not important.

After formation, an SMS will evolve through a phase of quasistationary cooling and contraction. If the SMS is rotating when it forms, conservation of angular momentum requires that it spin up as it contracts. The evolutionary path taken by the SMS during this cooling phase depends on the strength of its viscosity and magnetic fields and on the nature of its angular momentum distribution.

2.1. Cooling evolution of uniformly rotating supermassive stars

There are two possible evolutionary regimes for a cooling SMS. In the first regime, viscosity or magnetic fields are strong enough to enforce uniform rotation throughout the star as it contracts.

Baumgarte and Shapiro [38] have recently studied the evolution of a uniformly rotating SMS up to the onset of relativistic instability. They demonstrated that a uniformly rotating, cooling SMS will eventually spin up to its mass shedding limit. The mass shedding limit is encountered when matter at the star’s equator rotates with the Keplerian velocity. The limit can be represented as a ratio $\beta_{\text{shed}} = (T/|W|)_{\text{shed}}$ of the star’s rotational kinetic energy $T$ to gravitational potential energy $W$. In this case, $\beta_{\text{shed}} = 9 \times 10^{-3}$. The star will then evolve along a mass shedding sequence, losing both mass and angular momentum. It will eventually contract to the point of relativistic instability.

Baumgarte and Shapiro used both a second-order, post-Newtonian approximation and a fully general relativistic numerical code to determine that the onset of the instability occurs at a ratio of $R/M \sim 450$, where $R$ is the star’s radius. Here and throughout this paper $G = c = 1$. Note that a second-order, post-Newtonian approximation was needed because rotation stabilizes the destabilizing role of nonlinear gravity at the first post-Newtonian level.

The major result of Baumgarte and Shapiro’s work is that the following universal ratios exist for the critical configuration at the onset of relativistic instability: $T/|W|$, $R/M$ and $J/M^2$. Here $J$ is the total angular momentum of the star. These ratios are completely independent of the mass of the star or its prior evolution. Because uniformly rotating SMSs will begin to collapse from a universal configuration, the subsequent collapse and the resulting gravitational waveform will be unique.

2.1.1. Collapse outcome and gravitational radiation emission. The outcome of SMS collapse can only be determined with numerical, fully relativistic three-dimensional hydrodynamics simulations. To date, such simulations have only been published for nearly spherical collapse. The numerical simulations of Shapiro and Teukolsky [39] demonstrate that this type of collapse is nearly homologous.

In this case the collapse time $\tau_{\text{coll}}$ is roughly the free-fall time at the horizon (where $R = 2M$)

$$
\tau_{\text{coll}} = \left( \frac{R^3}{4\pi M} \right)^{1/2} = 14 \text{ s} \left[ M/10^6 M_\odot \right]^{-1}.
$$
The peak gravitational wave frequency $f_{GW} = \tau_{col}^{-1}$ is then $10^{-2}$ Hz, if the mass of the star is $10^6 M_\odot$. This is in the middle of LISA’s frequency band of $10^{-4} – 1$ Hz [13, 15].

The amplitude $h$ of this burst signal can be estimated roughly in terms of the star’s quadrupole moment

$$h \leq \epsilon \frac{2 M^2}{R d} \leq 5 \times 10^{-17} \frac{[M/10^6 M_\odot]}{[d/1 \text{ Gpc}]}.$$  \hfill (2)

Here $d$ is the distance to the star and $\epsilon \sim T/|W|$ is a measure of the star’s deviation from spherical symmetry. In this case, $\epsilon$ will be much less than one even near the horizon, since the collapse is nearly spherical.

Three-dimensional, general relativistic simulations of non-spherical, rotating collapse are underway [40]. The results are not available as yet. However, there are two possible outcomes of this type of collapse that can be discussed.

The first outcome is direct collapse to a SMBH, from the onset of instability. In this case $\epsilon$ will be on the order of one near the horizon. Thus the peak amplitude (see equation (2)) of the burst signal will be $h \sim 5 \times 10^{-17} \frac{[M/10^6 M_\odot]}{[d/1 \text{ Gpc}]}$.

Alternatively, the star may encounter the dynamical bar mode instability. The bar mode is the strongest of a set of global non-axisymmetric instabilities that may be encountered by a rapidly rotating object. This instability will deform the star into a bar-shaped configuration, making it a strong source of long-wavelength, quasiperiodic gravitational radiation. During the development of the instability, angular momentum and mass will be transported outwards [41–43]. This could hasten the star’s eventual collapse. Previous linear and nonlinear analyses indicate that $\beta_{\text{bar}} \sim 0.27$ is likely to be an upper limit for the onset of this instability, for a wide range of polytropic equations of state and rotation laws (see, e.g., [44, 45]).

Baumgarte and Shapiro [38] have estimated that a uniformly rotating SMS will reach $\beta \sim 0.27$ when $R/M = 15$. This estimate awaits confirmation from three-dimensional, general relativistic hydrodynamical simulations.

The frequency of the quasiperiodic gravitational radiation emitted by the bar can be estimated in terms of its rotation frequency to be

$$f_{GW} = 2 f_{\text{bar}} \sim 2 \left( \frac{GM}{R^3} \right)^{1/2} \sim 2 \times 10^{-3} \text{ Hz} \frac{[M/10^6 M_\odot]}{[d/1 \text{ Gpc}]}^{-1}.$$ \hfill (3)

when $R/M = 15$. The corresponding amplitude of the radiation, again estimated in terms of the star’s quadrupole moment, is

$$h \sim \frac{2 M^2}{R d} \sim 6 \times 10^{-18} \frac{[M/10^6 M_\odot]}{[d/1 \text{ Gpc}]}.$$ \hfill (4)

The emission of gravitational radiation will continue as long as the collapsing SMS’s bar shape persists.

2.2. Cooling evolution of differentially rotating supermassive stars

In the opposite evolutionary regime, neither viscosity nor magnetic fields are strong enough to enforce uniform rotation throughout the cooling SMS as it contracts. In this case it has been shown that the angular momentum distribution is conserved on cylinders during
The formation of supermassive black holes and the evolution of supermassive stars 3969

contraction [46]. Because viscosity and magnetic fields are weak, there is no means of redistributing angular momentum in the star. So even if the star starts out rotating uniformly, it cannot remain so.

The star will then rotate differentially as it cools and contracts. In this case, the subsequent evolution depends on the star’s initial angular momentum distribution, which is largely unknown.

One possible outcome is that the star will spin up to mass shedding (at a different value of $\beta_{\text{shed}}$ than a uniformly rotating star) and then follow an evolutionary path that may be similar to that described by Baumgarte and Shapiro [38].

The alternative outcome is that the star will encounter the dynamical bar instability prior to reaching the mass shedding limit. As discussed above, the bar shape induced by this instability will make the star a source of quasiperiodic, long-wavelength gravitational radiation. The outward transport of angular momentum that occurs during the instability could hasten the eventual collapse of the star.

New and Shapiro [47] have investigated the evolution of differentially rotating SMSs. Because the angular momentum distribution of these stars is unknown, they examined SMS models with several different initial angular momentum distributions. The goal of their work was to determine whether the bar instability or mass shedding limits are reached by these SMS models.

New and Shapiro’s strategy was to examine equilibrium sequences of SMS models, each of which was constructed with a different rotation law. The individual models on each sequence were constrained to have the same $M$ and $J$, since these quantities are conserved during the cooling evolution of an SMS. However, the models along a sequence have decreasing entropy and thus a decreasing axis ratio $R_p/R_e$, where $R_p$ is the polar radius and $R_e$ is the equatorial radius. A sequence is thus representative of the quasistatic, cooling/contracting evolution of a single SMS. This quasistatic approximation is appropriate because the cooling timescale is much longer than the hydrodynamic timescale for $M \lesssim 10^{13} M_\odot$ [38]. New and Shapiro examined each of the sequences to determine whether the limits $\beta_{\text{bar}}$ or $\beta_{\text{shed}}$ are reached for an SMS model with the given rotation law.

All of the sequences New and Shapiro examined were constructed with Hachisu’s self-consistent field (HSCF) technique [48]. The HSCF method builds individual models in hydrostatic equilibrium, such that their pressure, gravitational and centrifugal forces are in balance.

The HSCF method requires the choice of a barotropic equation of state $P = P(\rho)$. All of the SMS models examined by New and Shapiro were constructed with a polytropic equation of state for which

$$P = K \rho^{\frac{1}{n+1}}. \tag{5}$$

As mentioned above, the structure of an SMS is well represented by an $n = 3$ polytrope. In this equation of state, the polytropic constant $K$ is a measure of the specific entropy of the model.

The selection of appropriate rotation laws was constrained by the fact that each rotation law chosen had to enforce conservation of the contracting star’s specific angular momentum profile [46].

One rotation law that satisfies this constraint is the so-called $n' = 3$ law [46]. Its specific angular momentum profile $j(m)$ is

$$j(m) = a_1 + a_2 \left(1 - \frac{m(\sigma)}{M}\right)^{a_2} + a_3 \left(1 - \frac{m(\sigma)}{M}\right)^{a_3}, \tag{6}$$
where $M$ is the total mass of the system, $m$ is the mass interior to cylindrical radius $\sigma$ and the numerically determined constants are $a_1 = 13.27$, $a_2 = 163.3$, $a_3 = -176.5$, $a_4 = 0.2353$ and $a_3 = 0.2222$. This angular momentum profile is identical to that of a uniformly rotating, spherical $n = 3$ polytrope. Thus an equilibrium sequence constructed with this rotation law is representative of the evolution of an SMS that rotates uniformly prior to its cooling phase. New and Shapiro [47] have constructed an equilibrium sequence with this rotation law.

Because the initial rotation profiles of SMSs are unknown, New and Shapiro also examined sequences constructed by Hachisu, Tohline and Eriguchi (see [49]; hereafter, HTE). These sequences were built with the following parametrized angular momentum profile:

$$ j(m) = (1 + q)(J/M)\left[1 - (1 - m(\sigma)/M)^{1/q}\right]. $$

(7)

Here, the index $q$ specifies the rotation law. Note that the limiting case of $q = 0$ corresponds to the $j$-constant rotation law.

Nearly spherical models built with these ‘$q$-laws’ are differentially rotating. Thus, HTE’s $n = 3$, $q$-law sequences are representative of the evolution of SMSs, with a wide range of initial differential rotation profiles.

2.2.1. Evolutionary scenarios. A detailed discussion of the properties of the $n' = 3$ sequence, constructed by New and Shapiro and the $q$-law sequences of HTE can be found in [47]. In what follows we summarize these properties as they relate to the evolutionary scenarios of differentially rotating SMSs.

Density contour plots of selected models from the $n' = 3$ sequence are shown in figure 1. The $n' = 3$ sequence terminates due to mass shedding at $\beta_{\text{shed}} \gtrsim 0.30$. $\beta_{\text{shed}}$ exceeds the likely upper limit for the bar instability $\beta_{\text{bar}} \lesssim 0.27$. Thus we expect that an SMS with this rotation law should never reach the mass shedding limit, but will instead encounter the dynamical bar instability near $R_p/R_e \sim 0.004$. This is the axis ratio of the model with $\beta = 0.27$ (see figure 1(d)).

No mass shedding limits exist on HTE’s $q$-law sequences. Each of these sequences makes a continuous transition from spheroidal to toroidal configurations at values of $\beta_{\text{trans}} > 0.33 > 0.27 \gtrsim \beta_{\text{bar}}$. Thus, $n = 3$ models with these $q$-indexed laws would probably encounter the bar mode as spheroids.

We note that the hydrodynamical study of [44] indicates that $\beta_{\text{bar}}$ may be less than 0.27 for rotation laws that place a significant amount of angular momentum in equatorial mass elements. Their results predict that the $m = 2$ stability limit may be less than $\sim 0.20$ for models with the $n' = 3$ rotation law. Their simulations, of $n = 1.5$ polytropes, also suggest that a one-armed spiral, $m = 1$ mode may become increasingly dominant over the $m = 2$ mode as the equatoral concentration of angular momentum increases. Note that the grid resolutions used in [44] were probably not sufficient to accurately model the development of instabilities in models with these extreme differential rotation laws [45]. However, the results of [44] and the linear and nonlinear stability analyses of [45] confirm that 0.27 is likely to be an approximate upper limit to $\beta_{\text{bar}}$, for a variety of polytropic indices and rotation laws. The analysis presented in [47] assumes that the $m = 2$ bar mode is the dominant mode and that $\beta_{\text{bar}} \lesssim 0.27$.

Even if the actual value of $\beta_{\text{bar}}$ is less than 0.27, the qualitative nature of the results of New and Shapiro would not change. That is, the sequences they examined would still have models that are unstable to the bar mode. In addition, their quantitative estimates of the characteristics of the gravitational radiation emission presented below would only change by a numerical factor of order 1–10, even if $\beta_{\text{bar}}$ were as low as 0.22.
The formation of supermassive black holes and the evolution of supermassive stars

Figure 1. Snapshots of a cooling SMS. Density contours of selected models on the $n' = 3$ equilibrium sequence are shown in the ($x > 0, z > 0$) plane. The maximum density is normalized to unity. The highest-density contour level is 0.9; subsequent contour levels range from $10^{-4}$ to $10^{-10}$ and are separated by a decade. The axis ratios $R_p/R_e$ of the models displayed are: (a) 1.00, (b) 0.700, (c) 0.300, (d) 0.004, (e) 0.002. The model with $R_p/R_e = 0.004$ shown in (d) has $\beta = \beta_{\text{bar}} \sim 0.27$.

2.2.2. Gravitational radiation emission. The results of New and Shapiro [47] indicate that a bar mode phase is likely to be encountered by differentially rotating SMSs with a wide range of initial angular momentum distributions. Hydrodynamical simulations are needed to follow the evolution of the star as the instability develops, to compute the gravitational radiation waveforms emitted and to determine the fate of an SMS that undergoes the bar instability.

Previous hydrodynamical simulations of the bar instability in models with other rotation laws and a different polytropic equation of state indicate that the outcome of this instability is a persistent bar-like structure that emits quasiperiodic gravitational radiation over many cycles [40, 42, 43, 50]. If a similar outcome results from the bar instability in an SMS, the quasiperiodic, long-wavelength gravitational radiation emitted could be detected by LISA.
The frequency of these quasiperiodic gravitational waves can be estimated from the expected bar rotation rate $\Omega_{\text{bar}}$. The model on New and Shapiro’s $n' = 3$ sequence with $\beta \sim \beta_{\text{bar}} = 0.27$ has a central rotation rate $\Omega_{c} = 1.02 \times 10^{-1} \, \text{Hz} [M_6\beta_{-5}^{-3}R_{17}^{-1}]^{1/2}$. Here $M_6 \equiv M/10^6 \, M_{\odot}$, $R_{17} \equiv (R_e)/10^{17}$ cm and $\beta_{-5} \equiv \beta_0/10^{-5}$. The subscript 0 denotes the value for the (nearly) spherical progenitor star, prior to the start of its cooling phase. In previous hydrodynamics simulations of the bar mode instability [42], $\Omega_{\text{bar}}$ was $\sim 0.4/\Omega_{c}$. With this relation between $\Omega_{\text{bar}}$ and $\Omega_{c}$, the gravitational wave frequency $f_{GW}$ can be estimated to be

$$f_{GW} = 2f_{\text{bar}} = 2\frac{\Omega_{\text{bar}}}{2\pi} \sim \frac{0.4}{\pi} \Omega_{c}$$

$$\sim 1 \times 10^{-2} \, \text{Hz} [M_6\beta_{-5}^{-3}R_{17}^{-1}]^{1/2}. \quad (8)$$

For an SMS of $10^6 \, M_{\odot}$, which begins as a slowly rotating star of radius $10^{17}$ cm with $\beta_0 = 10^{-5}$, this yields a frequency of $1 \times 10^{-2} \, \text{Hz}$. This frequency is in the range in which LISA is expected to be most sensitive, $10^{-4}$–1 Hz. The choice $\beta_0 = 2 \times 10^{-4}/R_{17}$ is the maximum value of $\beta_0$ for which $f_{GW} (= 10^{-4} \, \text{Hz})$ is still in LISA’s range of sensitivity.

The strength of the gravitational wave signal can be estimated roughly to be

$$h \sim \frac{G \dot{Q}}{c^2 d} \sim \frac{GM^2 R_{\text{bar}}^2 f_{\text{bar}}^2}{c^4 d}$$

$$\sim 4 \times 10^{-15} \left( \frac{d}{1 \, \text{Gpc}} \right)^{-1} M_6^2 \beta_{-5}^{-1} R_{17}^{-1}, \quad (9)$$

where $\dot{Q}$ is the second time derivative of the star’s quadrupole moment and $d$ is the distance, which we scale to 1 Gpc (the Hubble distance is $\sim 3$ Gpc). Here we have used $R_{\text{bar}} \sim R_e = 4.14 \times 10^{13} \beta_{-5} R_{17}$ cm for the $n' = 3$ model of [47] that reaches the point $\beta \sim \beta_{\text{bar}}$.

The bar will decay on a secular timescale due to dissipative effects. For differentially rotating SMSs, the largest source of dissipation will be gravitational radiation. The gravitational radiation damping timescale $\tau_{GW}$ is approximately

$$\tau_{GW} \sim \frac{T}{(dE/dt)_{GW}}, \quad (10)$$

where $T$ is the rotational kinetic energy and $(dE/dt)_{GW}$ is the rate at which gravitational radiation carries energy away from the system. Recall that $T = \beta |W|$. The gravitational potential energy $|W| \sim GM^2/R_{\text{bar}}$. Thus,

$$T = \frac{GM^2}{R}. \quad (11)$$

The radiation rate can be estimated as [36]

$$\left( \frac{dE}{dt} \right)_{GW} \sim \frac{G \, M}{c^3 R^2} v^6. \quad (12)$$

In this case the characteristic velocity of the system is $v = (\beta GM/R)^{1/2}$. Substitution of equations (11) and (12) into equation (10) yields

$$\tau_{GW} \sim \frac{c^5 R_{\text{bar}}^4}{G^3 \beta^2 M^8}$$

$$\sim 1 \times 10^4 \, \text{yr} \ M_6^{-3} [\beta_{-5} R_{17}]^4. \quad (13)$$
The number of cycles $N$ for which the signal will persist is

$$N \sim \tau_{GW} f_{GW} \sim 4 \times 10^9 [M_6 \beta^{-5} R_{17}]^{-5/2}. \quad (14)$$

The quasiperiodicity of such a signal will assist in its detection [18].

The fraction of the mass $(dM/M)_{GW}$ radiated via gravitational radiation over the interval $\tau_{GW}$ can be estimated as

$$\left( \frac{dM}{M} \right)_{GW} \sim \tau_{GW} \left( \frac{dE}{dt} \right)_{GW} \frac{1}{M c^2} \sim \frac{G \beta M}{c^2 R} \sim 1 \times 10^{-3} M_6 [\beta^{-5} R_{17}]^{-1}. \quad (15)$$

3. Conclusions and future work

There is strong evidence that SMBHs exist. However, the nature of their progenitors is uncertain. Proposed formation mechanisms involve the evolution of stellar clusters or SMSs.

Recent studies indicate that differentially rotating SMSs and possibly uniformly rotating SMSs as well, are likely to encounter the dynamical bar instability and thus emit quasiperiodic, long-wavelength gravitational radiation. SMSs will also emit burst gravitational wave signals as they undergo relativistic collapse.

Linear and nonlinear stability analyses are needed to determine precisely the onset of the $m = 2$ dynamical bar instability for the SMS models considered here. Such analyses are also needed to determine the relative importance of various unstable non-axisymmetric modes in SMS models (as a dominant $m = 1$ mode would change the characteristics of the gravitational radiation emission).

In addition, three-dimensional hydrodynamical simulations are necessary to study the evolution of SMSs as they undergo the bar instability and/or relativistic collapse, to compute the gravitational waveforms emitted and to determine the final fate of the star.

Future investigations involving hydrodynamical simulations of SMS models would benefit from improved knowledge of initial conditions, such as an appropriate value for $\beta_0$. These appropriate initial conditions could be determined from studies of large-scale structure and cosmology.

Most importantly, studies are required to assess the roles of viscosity and magnetic fields in rotating SMSs to judge whether they are sufficient to drive these configurations to uniform rotation prior to bar instability.

Acknowledgments

This work has been supported in part by NSF grants AST 96-18524 and PHY 99-02833 and NASA grants NAG5-7152 and NAG5-8418 to the University of Illinois at Urbana-Champaign. A portion of this work performed under auspices of the US Department of Energy by Los Alamos National Laboratory under contract W-7405-ENG-36.

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The formation of supermassive black holes and the evolution of supermassive stars

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