Intrasubband plasmons in a finite array of quantum wires placed into an external magnetic field

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Abstract

The paper deals with the theoretical investigation of intrasubband plasmons in an array of quantum wires, consisting of a finite number of quantum wires, arranged at an equal distance from each other and placed into an external magnetic field. Two types of quantum wire array are under consideration: an ordered array of quantum wires with equal electron densities in all quantum wires and weakly disordered array of quantum wires, which is characterized by the fact that the density of electrons of one defect quantum wire was different from that of other quantum wires. For the ordered array of quantum wires, placed into the external magnetic field, the nonmonotonical dependence of plasmon frequency upon the 1D density of electrons in quantum wires is predicted. For high magnetic field the existence of 1D electron density ranges, in which plasmon modes do not exist, is shown. For the weakly disordered array of quantum wires the existence of the local plasmon modes, which properties differ from those of usual modes, is found. At high magnetic field the disappearance of the local plasmon modes at certain ranges of 1D electron density in defect quantum wire is shown.

1 Introduction

Quasi one-dimensional electron systems (1DES) or quantum wires (QW) are artificial structures in which the motion of charge carriers is confined in two transverse directions but is essentially free (in the effective mass sense) in the longitudinal direction. Usually QW are produced by adding an additional one-dimensional confinement of a two-dimensional electron system (2DES). This additional confinement is, in general, weaker than the strong confinement of original 2DES. One of motivations to study QW is the fact that the mobility of charge carriers is higher than in 2DES on which they are built. The reason for this is that the impurity content and distribution around QW can be selectively controlled, producing enhanced mobility.

Collective charge-density excitations, or plasmons in quantum wires (QW) are of great interest to physicists. Earlier plasmons in QW were investigated both theoretically and experimentally. In those papers it was shown that plasmons in QW possess some new unusual dispersion properties. Firstly, the plasmon spectrum strongly
depends on the width of QW. Secondly, 1D plasmons are free from the Landau damping in the whole range of wavevectors.

From the point of view of practical application the so-called weakly disordered arrays of low-dimensional systems are the objects of interest. Recently the plasmons in weakly disordered superlattice, formed of a finite number of equally spaced two-dimensional electron systems, have been theoretically investigated in cases where the external magnetic field is absent or present. The weakly disordered superlattice is characterized by the fact that all of two-dimensional systems possess the equal density of electrons except one ("defect") two-dimensional system, whose density of electrons differs from that of other two-dimensional systems. It was found that the plasmon spectrum of such an array contains the local plasmon mode, whose properties differ from those of other plasmon modes. The existence of local plasmon mode is completely analogous to the existence of local phonon mode, first obtained by Lifshitz in 1947 for the problem of the phonon modes in a regular crystal containing a single isotope impurity. Notice that practically all flux of electromagnetic energy of plasmons, which correspond to the local mode, are concentrated in the vicinity of defect 2DES. At the same time the paper indicated the opportunity of using the plasmon spectrum peculiarities to determine the parameters of defects in the superlattice.

Plasmons in a finite weakly disordered array of QWs without external magnetic field have been investigated theoretically in paper. It has been supposed that the defect QW can occupy an arbitrary position in the array. It was shown in paper that the position of the defect QW in the array does not affect strongly the spectrum of the local plasmon mode but it exerts an significant influence on the spectrum of other plasmon modes. At the same time, when the defect QW is arranged inside the array, the plasmon spectrum contains modes, whose dispersion properties do not depend on the value of the electron density in the defect QW.

The external magnetic field is known to cause considerable changes in plasmon spectrum of low-dimensional structures. So, earlier plasmons in single two-dimensional electron system placed into external magnetic field directed perpendicularly to 2DES, was investigated both theoretically and experimentally. It was shown, that the dispersion relation for plasmons in 2DES placed into the external magnetic field can be expressed as

$$\omega_H^2 = \omega_c^2 + \omega^2,$$

where $\omega_H$ is the frequency of plasmon in presence of external magnetic field, $\omega_c = eB/m^*c$ is the frequency of cyclotron resonance, $\omega$ is the frequency of plasmons in the case where external magnetic field is absent.

Plasmons in single 1DES was also investigated theoretically and experimentally. As it was shown experimentally, the dispersion law for one-dimensional plasmon in the presence of magnetic field also can be described by the dependence. Nevertheless in paper another one-dimensional plasmon mode was found experimentally. The later mode possesses the negative magnetic field dispersion. At the same time paper shows theoretically, that the above-mentioned negative magnetic field dispersion in the case of one dimensionality occurs in high magnetic field only. At weak magnetic field the properties of intrasubband plasmons in single QW depend considerably upon the value of one-dimensional electrons density in QW. Thus, if density of electrons in QW exceeds a certain critical value, the intrasubband plasmon frequency increases as the magnetic field increases. In the opposite case, when density of electrons in QW is smaller then the critical value, the intrasubband plasmon frequency decreases as the magnetic field increases.
In this paper we investigate intrasubband plasmons in a finite array of QWs, placed into an external magnetic field. We consider two types of QW array: an array in which the 1D electron densities are equal in all QWs (ordered array of QWs) and an array in which the 1D electron density of one defect QW differs from that of other QWs (weakly disordered array of QWs). The paper is organized as follows. In section 2 we derive the dispersion relation for plasmons in the finite array of QWs. In section 3 we present the results of the numerical solution of the dispersion relation for intrasubband plasmons in ordered array of QWs. In section 4 we discuss the dispersion properties of intrasubband plasmons in the weakly disordered array of QWs. We conclude the paper with a brief summary of results and possible applications (section 5).

2 Dispersion relation

We consider the array of QWs consisting of a finite number $M$ of QWs, arranged at planes $z = ld$ ($l = 0, \ldots, M - 1$ is the number of QW, $d$ is the distance between adjacent QWs). At the same time we suppose that the 1D density of electrons in $l$-th QW is equal to $N_{l \text{QW}}$. QWs are considered to be placed into the uniform dielectric medium with the dielectric constant $\varepsilon$. We use such a simple model (in which the dielectric constants of the media inside and outside the array are equal) to avoid the appearance of a surface plasmon mode. We consider the movement of electrons to be free in the $x$-direction and is considerably confined in the directions $y$ and $z$. We assume that the array of QWs is built on an ideal 2DES by application an additional confining potential along $y$-direction, which are parabolic: $U_{\text{conf}}(x,y) = \frac{1}{2} m^* \omega_0^2 y^2$. Here $m^*$ is the effective mass of electron, $\omega_0$ is the classical oscillation frequency of electron, placed in potential $U_{\text{conf}}$. At the same time we suppose that the width of all QWs is equal to zero in $z$-direction. The external constant magnetic field is considered to be directed perpendicularly to plane $xy$ along axis $z$.

To obtain the single-particle wave-function of the electron in QW we write down the expression for vector potential $A$ in Landau gauge: $A = (-By, 0, 0)$. So, in that case the single-particle Hamiltonian of the electrons can be represented as follows

$$\hat{H} = \frac{1}{2m^*} \left( \hat{p} + \frac{e}{c} A \right)^2 + U_{\text{conf}}(x, y),$$

where $\hat{p} = -i\hbar \nabla$ is the operator of the momentum of electron. In expression (2) we neglect the spin splitting in the magnetic field.

We seek an explicit form for the electron wave-function: $\psi(x, y) = \exp(ikx)\phi(y)$. In this case the Schrödinger equation $\hat{H}\psi(x, y) = E\psi(x, y)$ after some algebra can be written as

$$-\frac{\hbar^2}{2m^*} \frac{\partial^2 \phi}{\partial y^2} + \frac{1}{2} m^* \Omega^2 [y - \alpha k]^2 \phi(y) = \left\{ E - \frac{\hbar^2 k^2}{2m^*} \frac{\omega_0^2}{\Omega^2} \right\} \phi(y),$$

where $\alpha = \hbar \omega_c / m^* \Omega^2$, $\Omega^2 = \omega_c^2 + \omega_0^2$. The solution of equation (3) is a shifted harmonic oscillator wave function. So, the expression for energy subbands and a single-particle wave function for the electron in $l$-th QW can be written in the form (4):

$$E_m(k) = E_m + (1/2m^*) (\omega_0 / \Omega)^2 \hbar^2 k^2,$$

$$\psi_{l,m,k}(r) = (1/2\pi)^{1/2} e^{ikx} \phi_m(y - \alpha k) [\delta(z - ld)]^{1/2}.$$
Here
\[ E_m = \hbar \Omega (m + 1/2), \quad \phi_m(y) = (2^m m! \pi^{1/2} l_\Omega)^{-1/2} \exp \left( -\frac{y^2}{2l_\Omega^2} \right) H_m(y/l_\Omega), \] (6)
m is the number of energy subband, \( H_m(y) \) is an Hermite polynomial, \( l_\Omega = (\hbar/m*\Omega)^{1/2} \) is a typical width of wave function (which is simply a magnetic length, if \( \omega_0 = 0 \)).

As it can be seen from the expression (4), in the presence of confining potential in \( y \)-direction the degeneracy of Landau-levels is broken and each Landau-level forms subband. At the same time the wave function (5) in \( y \)-direction depends on the wavevector \( k \) in \( x \)-direction.

So, in the presence of confining potential and external magnetic fields the directions \( x \) and \( y \) are coupled.

To obtain the collective excitations spectrum we start with a standard linear-response theory in an random phase approximation. Let us consider \( \delta n(\mathbf{r}) \) which is the deviation of the electron density from its equilibrium value. After using the standard linear-response theory and the random phase approximation, the matrix element of the electron density deviation from its equilibrium value \( \delta n_{\alpha,\alpha'} = \langle \alpha | \delta n | \alpha' \rangle = \int d\mathbf{r} \psi^*_\alpha(\mathbf{r}) \psi_{\alpha'}(\mathbf{r}) \delta n(\mathbf{r}) \) can be related to the perturbation as
\[ \delta n_{\alpha,\alpha'} = \frac{f_{\alpha'} - f_\alpha}{E_{\alpha'} - E_\alpha + \hbar \omega} V_{\alpha\alpha'}. \] (7)

Here \( \alpha = (l, m, k) \) is a composite index, \( f_\alpha \) is the Fermi distribution function, \( V_{\alpha,\alpha'} = \langle \alpha | V | \alpha' \rangle \) are the matrix elements of the perturbing potential \( V = V^{ex} + V^H \), \( V^{ex} \) and \( V^H \) are the external and Hartree potentials, respectively.

Note, that the matrix elements of Hartree potential can be expressed through the perturbation (6) as
\[ V_{\alpha\alpha'}^{H} = \frac{\epsilon^2}{\varepsilon} \int d\mathbf{r} \psi^*_\alpha(\mathbf{r}) \psi_{\alpha'}(\mathbf{r}) \int \frac{d\mathbf{r}_1}{|\mathbf{r} - \mathbf{r}_1|} \delta n(\mathbf{r}_1). \] (8)

Taking into account, that
\[ \delta n(\mathbf{r}_1) = \sum_{\beta,\beta'} \delta n_{\beta,\beta'} \psi^*_\beta(\mathbf{r}_1) \psi_\beta(\mathbf{r}_1), \quad \beta = (n, s, k_1) \]
we obtain
\[ V_{\alpha\alpha'}^{H} = \sum_{\beta,\beta'} W_{\alpha\beta\alpha',\beta'} \delta n_{\beta,\beta'}. \] (9)

Here
\[ W_{\alpha\beta\alpha',\beta'} = \frac{\epsilon^2}{\varepsilon} \int d\mathbf{r} \psi^*_\alpha(\mathbf{r}) \psi_{\alpha'}(\mathbf{r}) \int \frac{d\mathbf{r}_1}{|\mathbf{r} - \mathbf{r}_1|} \psi^*_\beta(\mathbf{r}_1) \psi_\beta(\mathbf{r}_1) = \frac{\delta(q - q_1) 2e^2}{2\pi} W_{i,n|m,m',s,s'}(k_{1}', k; k_{1}', k_1) \delta_{n,n'} k_{1}', \] (10)

where
\[ W_{i,n|m,m',s,s'}(k_{1}', k; k_{1}', k_1) = \int \phi_m(y - \alpha k) \phi_{m'}(y - \alpha k') \phi_{s'}(y_1 - \alpha k_{1}') \phi_{s}(y_1 - \alpha k_1) \times \times K_0 \left( q((y - y_1)^2 + (l - n)^2 d^2)^{1/2} \right) dy dy_1, \] (11)
\( q = k' - k \), \( q_1 = k'_1 - k_1 \), \( K_0(x) \) is the zeroth-order modified Bessel function of the second kind. From the equations (7), (9), (11), after some algebra we get

\[
\delta n_{lmk,lm'k'+q} = \frac{f_{lm'k'+q} - f_{lmk}}{E_{lm'k'} - E_{lmk} + \hbar \omega} \left( V_{lmk,lm'k'}^{ex} + \frac{2e^2}{\varepsilon} \sum_{n,s,s'} \int dk_1 W_{l,n|m,m',s,s'} (k + q, k, k_1 + q, k_1) \delta n_{nsk_1,ns'k_1+q} \right). \tag{12}
\]

The factor of 2 before summation comes from the spin degeneracy.

Collective excitations of QW array exist when equation (12) has a nonzero solution \( \delta n \) in the case where the external perturbation \( V^{ex} = 0 \). Since the parameter \( \alpha k \) is the small value \cite{22}, we can expand the wave function in terms of \( \alpha \) as \( \phi_m(y - \alpha k) = \phi_m(y) - \alpha k \phi_m''(y) + \frac{1}{2} \alpha^2 k^2 \phi_m''(y) \). Besides that, taking into account, that when \( q \to 0 \), we can admit \( \alpha(k + q) \approx \alpha k \), \( \alpha(k_1 + q) \approx \alpha k_1 \). Under this assumption we can represent expression (11) in the form

\[
W_{l,n|m,m',s,s'} (k + q, k, k_1 + q, k_1) \approx C_{l,n|m,m',s,s'}^{(0)}(q) + \alpha k C_{l,n|m,m',s,s'}^{(1)}(q) + \alpha^2 k^2 C_{l,n|m,m',s,s'}^{(2)}(q) + k_1 \left\{ \alpha B_{l,n|m,m',s,s'}^{(1)}(q) + \alpha^2 k B_{l,n|m,m',s,s'}^{(2)}(q) \right\} + \alpha^2 k^2 A_{l,n|m,m',s,s'}(q), \tag{13}
\]

where

\[
C_{l,n|m,m',s,s'}^{(0)}(q) = \int \phi_m(y) \phi_{m'}(y) \phi_s(y_1) \phi_{s'}(y_1) K_0 \left( q((y - y_1)^2 + (l - n)^2 d^2) \right) dy dy_1,
\]

\[
C_{l,n|m,m',s,s'}^{(1)}(q) = - \int \{ \phi_m'(y) \phi_{m'}(y) + \phi_{m'}'(y) \phi_m(y) \} \times
\times \phi_s(y_1) \phi_{s'}(y_1) K_0 \left( q((y - y_1)^2 + (l - n)^2 d^2) \right) dy dy_1,
\]

\[
C_{l,n|m,m',s,s'}^{(2)}(q) = \frac{1}{2} \int \{ \phi_m''(y) \phi_{m'}(y) + \phi_{m'}''(y) \phi_m(y) + 2 \phi_m'(y) \phi_{m'}'(y) \} \times
\times \phi_s(y_1) \phi_{s'}(y_1) K_0 \left( q((y - y_1)^2 + (l - n)^2 d^2) \right) dy dy_1,
\]

\[
B_{l,n|m,m',s,s'}^{(1)}(q) = - \int \{ \phi_m'(y) \phi_{m'}(y_1) + \phi_{m'}'(y) \phi_s(y_1) \} \times
\times \phi_m(y) \phi_{m'}(y) K_0 \left( q((y - y_1)^2 + (l - n)^2 d^2) \right) dy dy_1,
\]

\[
B_{l,n|m,m',s,s'}^{(2)}(q) = \int \{ \phi_m'(y) \phi_{m'}(y) + \phi_{m'}'(y) \phi_m(y) \} \times
\times \{ \phi_s(y_1) \phi_{s'}(y_1) + \phi_{s'}(y_1) \phi_s(y_1) \} K_0 \left( q((y - y_1)^2 + (l - n)^2 d^2) \right) dy dy_1,
\]

\[
A_{l,n|m,m',s,s'}(q) = \frac{1}{2} \int \{ \phi_m''(y) \phi_{m'}(y_1) + \phi_{m'}''(y) \phi_s(y_1) + 2 \phi_m'(y) \phi_{m'}'(y_1) \} \times
\times \phi_m(y) \phi_{m'}'(y) K_0 \left( q((y - y_1)^2 + (l - n)^2 d^2) \right) dy dy_1.
\]

Substituting the expression (13) into (12), we obtain:

\[
\delta n_{lmk,lm'k+q} = \frac{2e^2}{\varepsilon} \frac{f_{lm'k+q} - f_{lmk}}{E_{lm'k+q} - E_{lmk} + \hbar \omega} \times
\times \sum_{n,s,s'} \int dk_1 \delta n_{nsk_1,ns'k_1+q} \left[ C_{l,n|m,m',s,s'}^{(0)}(q) + \alpha k C_{l,n|m,m',s,s'}^{(1)}(q) + \alpha^2 k^2 C_{l,n|m,m',s,s'}^{(2)}(q) + k_1 \left\{ \alpha B_{l,n|m,m',s,s'}^{(1)}(q) + \alpha^2 k B_{l,n|m,m',s,s'}^{(2)}(q) \right\} + \alpha^2 k^2 A_{l,n|m,m',s,s'}(q) \right]. \tag{14}
\]
After multiplication both left and right hand side of equation (14) by \(2k^i\) \((i = 0, 1, 2)\) and integration, we get:

\[
\chi^{(0)}_{l|m',m'} = \frac{2e^2}{\varepsilon} \sum_{n,s,s'} \left[ C^{(0)}_{l,n|m',s,s'}(q)\Pi^{(0)}_{l|m',m'} + \alpha C^{(1)}_{l,n|m',s,s'}(q)\Pi^{(1)}_{l|m',m'} + \right.
\]
\[
+ \alpha^2 C^{(2)}_{l,n|m',s,s'}(q)\Pi^{(2)}_{l|m',m'} \right] \chi^{(0)}_{n|s,s'} +
\]
\[
+ \left\{ \alpha B^{(1)}_{l,n|m',s,s'}(q)\Pi^{(1)}_{l|m',m'} + \alpha^2 B^{(2)}_{l,n|m',s,s'}(q)\Pi^{(2)}_{l|m',m'} \right\} \chi^{(1)}_{n|s,s'} +
\]
\[
+ \alpha^2 A_{l,n|m',s,s'}(q)\Pi^{(0)}_{l|m',m'} \chi^{(2)}_{n|s,s'} \right]
\]

\[
\chi^{(1)}_{l|m',m'} = \frac{2e^2}{\varepsilon} \sum_{n,s,s'} \left[ C^{(0)}_{l,n|m',s,s'}(q)\Pi^{(1)}_{l|m',m'} + \alpha C^{(1)}_{l,n|m',s,s'}(q)\Pi^{(2)}_{l|m',m'} + \right.
\]
\[
+ \alpha^2 C^{(2)}_{l,n|m',s,s'}(q)\Pi^{(3)}_{l|m',m'} \right] \chi^{(0)}_{n|s,s'} +
\]
\[
+ \left\{ \alpha B^{(1)}_{l,n|m',s,s'}(q)\Pi^{(1)}_{l|m',m'} + \alpha^2 B^{(2)}_{l,n|m',s,s'}(q)\Pi^{(2)}_{l|m',m'} \right\} \chi^{(1)}_{n|s,s'} +
\]
\[
+ \alpha^2 A_{l,n|m',s,s'}(q)\Pi^{(0)}_{l|m',m'} \chi^{(2)}_{n|s,s'} \right]
\]

\[
\chi^{(2)}_{l|m',m'} = \frac{2e^2}{\varepsilon} \sum_{n,s,s'} \left[ C^{(0)}_{l,n|m',s,s'}(q)\Pi^{(2)}_{l|m',m'} + \alpha C^{(1)}_{l,n|m',s,s'}(q)\Pi^{(3)}_{l|m',m'} + \right.
\]
\[
+ \alpha^2 C^{(2)}_{l,n|m',s,s'}(q)\Pi^{(4)}_{l|m',m'} \right] \chi^{(0)}_{n|s,s'} +
\]
\[
+ \left\{ \alpha B^{(1)}_{l,n|m',s,s'}(q)\Pi^{(1)}_{l|m',m'} + \alpha^2 B^{(2)}_{l,n|m',s,s'}(q)\Pi^{(2)}_{l|m',m'} \right\} \chi^{(1)}_{n|s,s'} +
\]
\[
+ \alpha^2 A_{l,n|m',s,s'}(q)\Pi^{(0)}_{l|m',m'} \chi^{(2)}_{n|s,s'} \right]
\]

where

\[
\chi^{(i)}_{n|s,s'} = 2 \int dk_1 k^i \delta n_{sk_1,ns'k_1+q} \quad \chi^{(i)}_{l|m,m'} = 2 \int dk k^i \delta n_{lmk,l'm'k+q}.
\]

\[
\Pi^{(i)}_{l|m,m'} = \frac{1}{\pi} \int dk \frac{f_{lm'k+q} - f_{lmk}}{E_{lm'k+q} - E_{lmk} + \hbar \omega}.
\]

We restrict our consideration to the case of intrasubband plasmons. So, we consider that the intersubband transistions of charge carriers are absent. In this case \(\Pi^{(i)}_{l|m,m'} = 0\), if \(m \neq m'\) and the system of equations (15)–(17) can be rewritten as follows

\[
\chi^{(p)}_{l|m,m} = \frac{2e^2}{\varepsilon} \sum_{n,s,p,t} U_{l,n,m,s,p,t} \chi^{(t)}_{n|s,s},
\]

where

\[
U_{l,n,m,s,p,1} = C^{(0)}_{l,n|m,m,s,s}(q)\Pi^{(p)}_{l|m,m} + \alpha C^{(1)}_{l,n|m,m,s,s}(q)\Pi^{(p+1)}_{l|m,m} + \alpha^2 C^{(2)}_{l,n|m,m,s,s}(q)\Pi^{(p+2)}_{l|m,m}.
\]

\[
U_{l,n,m,s,p,2} = \alpha B^{(1)}_{l,n|m,m,s,s}(q)\Pi^{(p)}_{l|m,m} + \alpha^2 B^{(2)}_{l,n|m,m,s,s}(q)\Pi^{(p+1)}_{l|m,m}.
\]

\[
U_{l,n,m,s,p,3} = \alpha^2 A_{l,n|m,m,s,s}(q)\Pi^{(p)}_{l|m,m}, \quad p = 0, \ldots, 2.
\]

Equation (18) is a set of linear equation and it has nonearo solution in the case where its determinant is equal to zero. So, the plasmon dispersion relation can be written in the form:

\[
\det \left| \delta_{l,s} \delta_{m,s} \delta_{p,t} - \frac{2e^2}{\varepsilon} U_{l,n,m,s,p,t} \right| = 0.
\]
Note, that as $M = 1$, the dispersion relation (19) coincides with the dispersion relation for plasmons in single QW in the presence of external magnetic field obtained in [22].

At a zero temperature and in the long-wavelength limit (where $q \to 0$) function $\Pi_{l|m,m}^{(i)}$ can be written as

\[
\Pi_{l|m,m}^{(0)} = \frac{2g_m^l q^2}{\pi m_r \omega^2}, \quad \Pi_{l|m,m}^{(1)} = -\frac{a}{b} \Pi_{l|m,m}^{(0)},
\]

\[
\Pi_{l|m,m}^{(2)} = \left(\frac{a}{b}\right)^2 \Pi_{l|m,m}^{(0)} - \frac{q}{b} \frac{2g_m^l}{\pi}, \quad \Pi_{l|m,m}^{(3)} = -\left(\frac{a}{b}\right)^3 \Pi_{l|m,m}^{(0)} + \frac{q}{b} \frac{2g_m^l}{\pi} \left(q + \frac{a}{b}\right),
\]

\[
\Pi_{l|m,m}^{(4)} = \left(\frac{a}{b}\right)^4 \Pi_{l|m,m}^{(0)} - \frac{q}{b} \frac{2g_m^l}{\pi} \left[q^2 + \left(\frac{2g_m^l}{\pi}\right)^2 + \frac{a}{b}q + \left(\frac{a}{b}\right)^2\right],
\]

where

\[
a = \frac{\hbar^2 q^2}{2m_r} + \hbar \omega, \quad b = \frac{\hbar^2 q}{m_r}, \quad m_r = m^*(\Omega/\omega_0)^2, \quad g_m^l = \frac{1}{\hbar} \sqrt{2m_r (E_F^l - E_m)},
\]

$E_F^l$ is the Fermi level in $l$-th QW.

### 3 Intrasubband plasmons in the ordered QW array

Fig.1 presents the dispersion curves for intrasubband plasmons in finite ordered array of QWs (in which 1D electron densities are equal in all QWs), placed into the external magnetic field. The $y$-axis gives the dimensionless frequency $\omega/\omega_0$, and the $x$-axis gives the dimensionless wavevector $ql_0$. As the model of QW we use heterostructure GaAs with the effective mass of electrons $m^* = 0.067m_0$ ($m_0$ is the mass of free electron) and the dielectric constant $\varepsilon = 12$. For comparison the dispersion curve for the plasmons in a single QW with the same parameters is depicted in Fig.1 by dashed curve 1. As seen from Fig.1, the intrasubband plasmon spectrum in the finite ordered array of QWs contains $M$ modes. Thus, the number of modes in the spectrum is equal to the number of QWs in the array [14] (it should be mentioned that in the case under consideration the value of plasma frequency of the electrons in QWs is chosen so, that in each QW only the lowest energy subband is occupied by electrons). Notice that with an increase of wavenumber $q$ the plasmon frequency $\omega$ increases monotonically likewise. It should be emphasized that in the limit $qd \to \infty$, when the Coulomb interaction between electrons in adjacent QWs is negligible, the dispersion curves for plasmon modes are gradually drawn together and are close to the dispersion curve for the plasmon in the single QW with the same density of electrons (dashed curve 1).

Now we consider the influence of the electron density value on the properties of intrasubband plasmons in ordered array of QWs. Fig.2 presents the dependence of plasmon frequency upon the plasma frequency of electrons in QWs in the case of the fixed value of wavevector $q$ and for different values of cyclotron frequency of electrons. The $y$-axis gives the dimensionless frequency $\omega/\omega_0$, and the $x$-axis gives the dimensionless plasma frequency of electrons in QWs $\omega_p/\omega_0$. We consider first the case when $\omega_c = 0$ (Fig.2a), i.e. when the external magnetic field is absent. As Fig.2a shows, in this case the frequency of intrasubband plasmons increases when the value of plasma frequency of electrons in QWs $\omega_p$ is increased. At the same time at small values of $\omega_p$, when the Fermi energy is below the bottom of first subband ($E_F^l < E_1$) and only the lowest (zero) subband in each QW is occupied by electrons, the intrasubband plasmon spectrum contains $M$ modes (curves 1). Nevertheless
when the value of plasma frequency of electrons in QWs exceeds the value of \( \omega_p \approx 2.68\omega_0 \), the intrasubband plasmon spectrum contains \( 2M \) modes (curves 1 and 2). In this case the Fermi energy is above the bottom of first subband but below the bottom of second subband and consequently there are already two subbands (zero and first) in each QW, which are occupied by electrons. With further increasing of \( \omega_p \), new subbands become occupied by electrons, and each occupied subband in each QW supports its own intrasubband plasmon. Hence the general number of intrasubband plasmon modes in finite ordered array of QWs without an external magnetic field is equal to \( nM \) (\( n \) is the quantity of subbands in each QW, occupied by electrons).

The properties of intrasubband plasmons somewhat change, when the ordered array of QWs is placed into the external magnetic field. So, at weak magnetic field (Fig.2b), the frequency of intrasubband plasmons, supported by the lowest subband (curves 1), is increased monotonically with the increasing of \( \omega_p \). At the same time when the Fermi energy exceeds the bottom of first subband (and it becomes populated by the electrons), the intrasubband plasmons, supported by first subbands in each QW, arises in the spectrum (curves 2). The frequency of these plasmons (as distinct from the case of zero magnetic field, see Fig.2a) increases nonmonotonically when the value of \( \omega_p \) is increased. So, starting with some value of \( \omega_p \) (in our case starting with \( \omega_p \approx 3.55\omega_0 \)) the frequency of intrasubband plasmons, supported by first subbands, is decreased with the increasing of \( \omega_p \) and when \( \omega_p \approx 4.0\omega_0 \) these plasmons disappear. Notice that intrasubband plasmons, supported by second subbands (curves 3), possess the same properties.

In the case of higher magnetic field (Fig.2c) the dependence of intrasubband plasmon frequency upon the value of plasma frequency of electrons in QWs possesses the following properties. So, in this case the frequency of intrasubband plasmons, supported by zero (curves 1), first (curves 2) and second (curves 3) subbands depends nonmonotonically upon the value of \( \omega_p \). At the same time there are a certain intervals of values of \( \omega_p \) (in this case \( 2.7\omega_0 < \omega_p < 3.2\omega_0 \), \( 4.45\omega_0 < \omega_p < 4.9\omega_0 \)), in which the intrasubband plasmons don’t exist.

4 Intrasubband plasmons in a weakly disordered array of quantum wires

We consider now the spectrum of intrasubband plasmons in weakly disordered array of QWs, in which all QWs possess the equal 1D density of electrons \( N \) except one defect QW whose density of electrons is equal to \( N_d \). So, the density of electrons in \( l \)-th QW can be expressed as \( N_l = (N_d - N)\delta_{pl} + N \). Here \( p \) is the number of defect QW arranged at the plane \( z = pd \), \( \delta_{pl} \) is the Cronecker delta.

Fig.3 presents the spectrum of intrasubband plasmons (solid curves) in weakly disordered array of QWs for zero external magnetic field. For comparison the dispersion curves for the intrasubband plasmons in a single QW with the electron density \( N \) and \( N_d \) are depicted by dashed curves 1 and 2, correspondingly. As seen from Fig.3, the propagation of intrasubband plasmons in the weakly disordered array of QWs is characterized by the presence of the local plasmon mode (LPM). At zero external magnetic field when the density of electrons in defect QW is less than the density of electrons in other QW (\( N_d < N \)), the LPM lies in the lower-frequency region in comparison with the usual plasmon modes (Fig.3a). Accordingly, if \( N_d > N \), the LPM lies in the higher-frequency region in comparison with the usual ones (Fig.3b) [11]. It should be emphasized that in the limit \( qd \rightarrow \infty \), when the Coulomb interaction between electrons in adjacent QWs is negligible, the LPM dispersion curve is close
to the dispersion curve for the plasmons in single QW with the density of electrons $N_d$ (curve 2). Meanwhile, the dispersion curves for usual plasmon modes in the limit $qd \to \infty$ are gradually drawn together and are close to the dispersion curve for the plasmon in the single QW with the density of electrons $N$ (curve 1).

Now we consider the dependence of intrasubband plasmon spectrum upon the value of 1D electron density in the defect QW. Fig.4 depicts the dependence of intrasubband plasmon frequency upon the ratio $N_d/N$ for the fixed value of wavevector $q$ and for different positions of the defect QW in the array. As seen from Fig.4, at zero external magnetic field the number of LPM (depicted by bold solid curves) in the intrasubband plasmon spectrum is equal to the number of subbands in defect QW, occupied by the electrons. As can be seen from the comparison of Fig.4a,b,c, the LPM spectrum is weakly dependent upon the position of defect QW in the weakly disordered array of QWs. That phenomenon can be explained by the fact, that practically the whole flux of the LPM electromagnetic energy is localized in the vicinity of the defect QW [19]. However, the spectrum of usual plasmon modes is more sensitive to the position of defect QW in the array. Note that the frequency of LPM increases when the value of ratio $N_d/N$ is increased. At the same time the usual plasmon modes spectrum is characterized by these features. As $p = 0$ (Fig.4a) when the value of ratio $N_d/N$ is increased, the frequency of all usual plasmon modes increases as well. It should be noted that the frequencies of intrasubband plasmons supported by first subbands of QWs (curves $1'–4'$) are less sensitive to the value of ratio $N_d/N$ in comparison with the frequencies of intrasubband plasmons supported by zero subbands of QWs (curves $1–4$). However, when $p = 1$ (Fig.4b) the frequencies of two of the usual plasmon modes (curves 2 and $2'$) does not practically depend upon the value of ratio $N_d/N$. In the case where $p = 2$ (Fig.4c) there are already four intrasubband plasmon modes (curves $1,1',3,3'$) which possess such a distinctive feature. The spatial distribution of the Hartree potential for those modes has a feature, that the absolute value of the Hartree potential in the vicinity of the defect QW is negligible. Therefore, the defect QW does not exert a significant influence on the dispersion properties of plasmon modes [19].

The properties of intrasubband plasmons change, if weakly disordered array of QWs is placed into the external magnetic field. Fig.5 presents the dependence of plasmon frequency upon the value of ratio $N_d/N$ for fixed value of wavenumber $q$ and for different positions of the defect QW in the array. As seen from Fig.5, at external magnetic field the dependence of LPM frequency upon ratio $N_d/N$ is nonmonotonic. So, the frequency of LPM, supported by defect QW zero subband (curve LMP1), increases as the value of ratio $N_d/N$ is increased in the range $0 < N_d/N < 0.39$. At the same time as the value of $N_d/N$ increases in the range $0.39 < N_d/N < 0.51$, the frequency of LMP, supported by defect QW zero subband, is decreased. Meanwhile, the frequencies of LPM, supported by defect QW first and second subbands (curves LPM2 and LPM3, correspondingly) also depend nonmonotonically upon the value of $N_d/N$. Notice that when weakly disordered array of QWs is placed into an external magnetic field, there are certain ranges of 1D electron density in defect QW (in Fig.5, e.g. $0.51 < N_d/N < 0.82$ and $1.46 < N_d/N < 1.98$), in which the LPMs don’t exist. As seen from Fig.5 at external magnetic field (as in the case of zero external magnetic field) when $p = 1$ and when $p = 2$, the spectrum of usual plasmon modes contains intrasubband modes (curve 2 in Fig.5b, curves 1 and 3 in Fig.5c), which frequencies don’t practically depend upon ratio $N_d/N$. 


5 Conclusion

In conclusion, we calculated the intrasubband plasmon spectrum of the finite array of QWs placed into an external magnetic field. Two types of QW arrays were under consideration: an ordered array of QWs (in which all the QWs possess equal 1D density of electrons) and weakly disordered array of QWs (in which the 1D densities of electrons are equal in all QWs except one defect QW). It is found that in the ordered array of QWs at zero magnetic field in each QW every subbands, filled by electrons, support their own intrasubband plasmons. Hence the total quantity of intrasubband plasmon modes in ordered QW array is equal to the number of QWs in the array multiplied by the number of filled subbands in QW. Nevertheless, at nonzero external magnetic fields the quantity of intrasubband plasmon modes depends upon the value of magnetic field and the 1D electron density of QWs. In particular, at high enough external magnetic field there are certain ranges of 1D electron densities, in which none of intrasubband plasmon modes exists in the spectrum.

In the case of weakly disordered array of QWs the LPMs whose properties differ from those of other modes exist in the plasmon spectrum. We point out that as distinct from the case of zero magnetic field, at high enough magnetic field the dependence of LPM frequency upon defect QW 1D density of electrons is of nonmonotonical character. Moreover, at high magnetic field there are certain ranges of 1D electron density in defect QW, in which LMPs don’t exist. At the same time it is found that the intrasubband plasmon modes, whose spectrum does not depend upon the density of electrons of the defect QW [19], exist also in the case of external magnetic field.

To conclude, it should be emphasized that the above-mentioned features of plasmon spectra can be used for the diagnostics of defects in QW structures. Hence, the LPM properties can be used for the determination of the electron density in the defect QW. At the same time the properties of usual plasmon modes can be used to define the position of the defect QW in the array.
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Figures

Figure 1. Dispersion curves of intrasubband plasmons in an ordered array of QWs with parameters: $M = 5$, $d = 15.0l_0$, $N_l = N = \text{const}$ ($l = 0, \ldots, M - 1$), $\omega_p = \left(\frac{2e^2N/\varepsilon m^*l_0^2}{1}\right)^{1/2} = 1.5\omega_0$, $\omega_c = 0.75\omega_0$.

Figure 2. The dependence of intrasubband plasmon frequency upon the plasma frequency of electrons in QW in the case where $M = 5$, $q_l = 0.04$, $d = 15.0l_0$, $N_l = \text{const}$ ($l = 0, \ldots, M - 1$) and for three values of cyclotron frequency of electrons in QWs: $\omega_c = 0$ (a), $\omega_c = 0.25\omega_0$ (b), $\omega_c = 0.75\omega_0$ (c).

Figure 3. Dispersion curves of intrasubband plasmons in weakly disordered array of QWs for parameters $M = 5$, $\omega_p = 1.5\omega_0$, $\omega_c = 0$, $d = 15.0l_0$, $p = 0$ and for two values of 1D density of electrons in defect QW: $N_d/N = 0.5$ (a), $N_d/N = 1.5$ (b). The values of parameters are chosen in a manner that in all QWs only one (zeroth) subband is occupied by electrons.

Figure 4. The dependence of intrasubband plasmon frequency upon the ratio $N_d/N$ in the case where $M = 5$, $q_l = 0.04$, $d = 15.0l_0$, $\omega_p = 3.0\omega_0$, $\omega_c = 0$ and for three different positions of defect QW in the array: $p = 0$ (a), $p = 1$ (b) and $p = 2$ (c). At these values of parameters there are two subbands in all QWs (except the defect QW), filled by electrons. Meanwhile, in defect QW the number of filled subbands is determined by the value of $N_d$.

Figure 5. The dependence of intrasubband plasmon frequency upon the ratio $N_d/N$ in the case where $M = 5$, $q_l = 0.04$, $d = 15.0l_0$, $\omega_p = 3.2\omega_0$, $\omega_c = 0.75$ and for different positions of defect QW in the array: $p = 0$ (a), $p = 1$ (b) and $p = 2$ (c). These values of parameters correspond to the fact that in all QWs (except the defect one) two subbands (zeroth and first) are occupied by electrons. The number of filled subbands in defect QW is determined by the value of $N_d$. 
Figure 1.
Figure 2.
Figure 3.
Figure 4.
Figure 5.