Abstract: In this paper we calculate the one loop contributions to the CP violating three gauge boson couplings in two-Higgs doublet and Left–Right symmetric models. In the two-Higgs doublet model only a P conserving and CP violating coupling is generated, and it can be large as $10^{-3}$. In the Left–Right symmetric model both P conserving and violating couplings are generated. Due to constraints on the $W_L$–$W_R$ mixing, these couplings are small.
At $e^+e^-$ colliders with enough energy $W$ pairs can be produced. Such facilities can be used to detect the three gauge boson couplings and provide tests for the Standard Model (SM). An important feature of $e^+e^-$ colliders is that they give very nice opportunities to look for CP violation because the initial state is CP eigenstate. There have been many studies on the anomalous CP conserving couplings of three gauge bosons[1,2]. In this paper we will concentrate on possible CP violating $VWW$ (here $V$ stands for $Z$ or $\gamma$) three gauge boson couplings in some models. Under $U(1)_{em}$-invariance the CP violating $VW^+W^-$ vertex for $VWW$ can be written with form factors (we use the same convention as in [1]):

$$\Gamma_{\mu_1\mu_2\nu} = if_4^V (q_{\mu_2} g_{\mu_1\nu} + q_{\mu_1} g_{\mu_2\nu}) + f_6^V \varepsilon_{\mu_1\mu_2\nu\rho} q^\rho + \frac{1}{m_W^2} f_7^V \varepsilon_{\mu_1\mu_2\sigma\rho} (k_1 - k_2)_\nu (k_1 - k_2)^\sigma q^\rho$$  \hspace{1cm} (1)

where $q$, $k_1$ and $k_2$ are the momenta of $V$, $W^+$ and $W^-$ bosons respectively. The $W$ bosons are on shell. The $f_4^V$ are the form factors depending on $q^2$ in general. Of these, $f_4^V$ is P conserving and CP violating whereas $f_6^V$ and $f_7^V$ are P and CP violating.

The form factors $f_6^\gamma$ and $f_7^\gamma$ can be constrained by using low energy data[3,4,5]. The best constraints on these form factors at $q^2 = 0$ are obtained by relating them to electric dipole moments (EDM) of electron and neutron. From the experimental upper bound on the electron EDM, one obtains [4]: $f_6^\gamma \leq 4.1 \times 10^{-2}$, $f_7^\gamma \leq 1.1 \times 10^{-2}$. The constraints from the experimental bound on neutron EDM are less certain, the bound varies from $10^{-4}$ to $O(1)$ depending on the method used. Constraints on $f_6^Z$ have also been obtained[4]. So far, there is no constraint on $f_7^Z$ from CP odd effects in experiments, CP even effect one gets $f_4^Z \leq 0.24$[5]. Needless to say much effort should be spent to measure these important form factors to find CP violation somewhere other than $K_L$ meson system.

In the minimal SM, CP violating three gauge boson couplings are zero up to the two loop level at least and therefore are too small to be seen. If CP violation were to be detected experimentally in the $VWW$ vertex, this would imply new CP violating interactions beyond the minimal SM. We will study the CP violating three gauge boson couplings in two extensions of the SM, the two-Higgs doublet and the Left-Right symmetric models. The form factors $f_6^\gamma$ at $q^2 = 0$, which are related to the electric dipole moment of $W$ boson, have been considered in multi-Higgs doublet models and Left-Right symmetric models[7,8,9]. We will calculate all the form factors in eq. (1) with arbitrary $q^2$.

We consider the two Higgs doublet model first. In this model the gauge group is the same as the minimal SM, i.e., $SU(3)_C \times SU(2)_L \times U(1)_Y$ and has two Higgs representations transforming under $SU(2)_L$ as doublet. It is possible to have CP violation in the Higgs sector in this model. If such CP violation is allowed, in general there will be flavour
changing neutral current (FCNC) at the tree level. There are very strong experimental constraints on FCNCs. These FCNCs can be avoided by imposing certain symmetries such that only one Higgs doublet generates masses for one class of given charge fermions. However, if these symmetries are exact for the complete theory, CP violation in the Higgs sector will also be eliminated. We will take the approach of allowing soft symmetry breaking terms in the Higgs potential[10]. Thus we have a theory consistent with experiment. Let \( \phi_i^T = (\phi_i^+, \phi_i^0 + v_i e^{i\varphi_i}) \) with \( i = 1, 2 \) be the two Higgs doublets. Here \( v_i e^{i\varphi_i} \) are their vacuum expectation values. The CP violating phases can be chosen such that \( \varphi_1 = 0 \). In this model, there are three neutral and one charged physical Higgs particles. It is convenient to work in a basis where the would-be Goldstone bosons are decoupled from physical ones. To this end, we first rotate the Higgs field in a new basis, 

\[
U = \begin{pmatrix}
\cos \beta & \sin \beta e^{-i\varphi_2} \\
-\sin \beta & \cos \beta
\end{pmatrix}
\]

where \( \cos \beta = \frac{v_1}{\sqrt{v_1^2 + v_2^2}} \) and \( \sin \beta = \frac{v_2}{\sqrt{v_1^2 + v_2^2}} \). \( H^+ \) is the mass eigenstate of the physical charged Higgs particle. In general \( H_2^0 \) and \( I_2^0 \) are not mass eigenstates. There are mixings, and if CP is violated there are mixings between \( I_2^0 \) and \( H_1^0 \). We will parametrize the mass eigenstates \( \varphi_{mi} \) in terms of \( G_2^0 \) and \( H_1^0 \) by a orthogonal matrix \( d_{ij} \), we have

\[
\begin{pmatrix}
H_1^0 \\
H_2^0 \\
I_2^0
\end{pmatrix} =
\begin{pmatrix}
\cos \beta & \sin \beta e^{-i\varphi_2} \\
-\sin \beta & \cos \beta
\end{pmatrix}
\begin{pmatrix}
d_{11} & d_{12} & d_{13} \\
d_{21} & d_{22} & d_{23} \\
d_{31} & d_{32} & d_{33}
\end{pmatrix}
\begin{pmatrix}
\phi_{m1} \\
\phi_{m2} \\
\phi_{m3}
\end{pmatrix}
\]

At the one loop level, we find that only P conserving and CP violating \( ZW+W^- \) coupling will be generated. There are two diagrams contribute which are shown in Fig. 1. Evaluating these diagrams we obtain

\[
f_4^2(q^2) = \frac{e^2}{64\pi^2 \sin^2 \theta_W \cos^2 \theta_W} \epsilon_{ijk} d_{1i} d_{1j} d_{1k} \int [d\alpha] (\alpha_1 - \alpha_2) \ln \left| \frac{-\alpha_1 \alpha_2 q^2 - \alpha_3 (1 - \alpha_3) m_W^2 + \alpha_1 m_j^2 + \alpha_2 m_i^2 + \alpha_3 m_C^2}{-\alpha_1 \alpha_2 q^2 - \alpha_3 (1 - \alpha_3) m_W^2 + \alpha_1 m_j^2 + \alpha_2 m_i^2 + \alpha_3 m_C^2} \right|
\]

where \( q \) is the momentum carried by \( Z \), \( m_W \), \( m_i \) and \( m_C \) are the \( W \), neutral Higgs and charged Higgs masses respectively, and \( \int [d\alpha] = \int_0^\infty d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3) \).
When $q^2$ is large enough an absorptive part of the form factor will be generated, which is given by

$$\text{Im} f_4^Z = \frac{e^2}{2 \sin^2 \theta_W \cos^2 \theta_W} \epsilon_{ijk} d_{1j} d_{1k}$$

$$\times \theta(q^2 - (m_i + m_j)^2) \left[ f(m_i^2, m_j^2, m_C^2) - f(m_i^2, m_j^2, m_W^2) \right]$$

$$f(x, y, z) = \frac{y - x}{256 \pi q^6 \beta^3_W} \left\{ \left[ \beta_W^4 q^4 + \beta_W^2 (8zq^2 - 2q^4 + 4q^2(x + y) - 4(x - y)^2) \right] \right.$$

$$+ 16z^2 + 8zq^2 - 16z(x + y) + q^4 - 4q^2(x + y) + 4(x + y)^2 \ln \frac{a + b}{a - b}$$

$$+ 8b(\beta_W^2 q^2 + 4z + q^2 - 2(x + y)) \right\}$$

where

$$\beta_W = \sqrt{1 - \frac{4m_W^2}{q^2}}$$

$$a = m_W^2 + \frac{1}{2}(x + y) - z - \frac{1}{2}q^2$$

$$b = \frac{1}{2} \beta_W \sqrt{(q^2 - x + y)^2 - 4yq^2}$$

It is easy to see from eq.(5) that the CP violating form factor is proportional to $d_{11}d_{12}d_{13}$ as indicated in [11]. Further, if the charged Higgs has the same mass as $W$, $f_4^Z$ is zero. For our numerical results for the form factor we assume that the Higgs particle $\phi_{m3}$ is too heavy to give sizeable contribution. The form factor is then a function of $m_1, m_2, q^2$ and $m_C$. In fig.2, we show some typical values for $f_4^Z$ for different Higgs masses, where $m_1 = 60$GeV. The magnitude also depends on the CP violating parameter $d_{11}d_{12}d_{13}$. The maximum possible value for this quantity is $1/3\sqrt{3}$. So far the best constrain on these CP violating parameters in two Higgs doublet model are from fermion dipole moment constraint. One of the biggest contributions to the neutron EDM is from the quark colour dipole moment. The contribution from up and down quark color dipole moments is given in the $SU(6)$ approximation[12] by:

$$D_n \approx 2.5 \times 10^{-26} \text{ecm} \left\{ (d_{1j} - \tan \beta d_{2j}) \tan \beta d_{3j} \left[ f\left(\frac{m_i^2}{m_j^2}\right) + g\left(\frac{m_i^2}{m_j^2}\right) \right] \right.$$

$$+ \frac{m_d}{2m_u} ((d_{1j} - \tan \beta d_{2j}) \cot \beta d_{3j} f\left(\frac{m_i^2}{m_j^2}\right) + (d_{1j} + \cot \beta d_{2j}) \tan \beta d_{3j} g\left(\frac{m_i^2}{m_j^2}\right)) \right\}$$

which has to be less than the experimental upper bound of $10^{-25}$ecm.[13]. The functions $f(z)$ and $g(z)$ are defined in [12]. The strange quark color dipole moment gives a similar constraint[14]. The maximum value for $d_{11}d_{12}d_{13}$ is not ruled out. The P conserving and CP violating form factor in the two Higgs model can be as large as $10^{-3}$. 

3
Let us now study the CP violating three gauge boson form factors in Left-Right symmetric models. The gauge group of the Left-Right symmetric models is $SU(3)_C \times U(2)_L \times SU(2)_R \times U(1)_{B-L}$. In these models the gauge bosons from $SU(2)_L$ and $SU(2)_R$ will in general mix. Due to this mixing at the one loop level CP violating three gauge boson form factors will be generated. The diagrams which contribute are shown in Fig. 3. For the lighter W and Z gauge bosons and the photon couplings to fermions can be parametrized as

$$L = -e\bar{f}_i Q_i \gamma_{\mu} f_i A^\mu - \frac{e}{2 \sin \theta_W \cos \theta_W} \bar{f}_i (g^i_V - g^i_A \gamma_5) \gamma_{\mu} f_i Z^\mu$$

$$+ \bar{u}_i (f^{ij}_V - f^{ij}_A \gamma_5) \gamma_{\mu} d_j W^{+\mu} + H.C. \tag{9}$$

Here $Q_i$ is the electric charge of fermions in units of $e > 0$, $g^i_V$ and $g^i_A$ are the coupling constants of $Z$ to neutral and axial neutral currents, and

$$f^{ij}_V = \frac{e}{2\sqrt{2}} \left( \frac{V_{Rij} \sin \zeta}{\sin \theta \cos \theta_W} - \frac{V_{Lij} \cos \zeta}{\sin \theta_W} \right)$$

$$f^{ij}_A = \frac{e}{2\sqrt{2}} \left( -\frac{V_{Rij} \sin \zeta}{\sin \theta \cos \theta_W} - \frac{V_{Lij} \cos \zeta}{\sin \theta_W} \right) \tag{10}$$

Here $V_{R,L}$ are the right-handed and left-handed KM matrices, $\zeta$ is the $W_L - W_R$ mixing angle, and $\sin \theta = g_{B-L}/\sqrt{g^2_R + g^2_{B-L}}$ with $g_i$ being the gauge couplings. In the limit $g_R = g_L$, $\sin \theta \cos \theta_W = \sin \theta_W$. At the one loop level non vanishing $f^Z_4$ and $f^V_6$ will be generated while $f^V_7$ is still zero. Evaluating the diagrams in Fig. 3, we obtain

$$f^Z_4 = -\frac{N_C}{16\pi^2 \cos^2 \theta_W} 4m_u m_d \text{Im}(f^{ij}_V f^{ij*}_V)(g^d_A I_{1d} - g^u_A I_{1u})$$

$$f^V_6 = \frac{N_C}{16\pi^2 \cos^2 \theta_W} 4m_u m_d \text{Im}(f^{ij}_V f^{ij*}_A)(g^u_V I_{0u} + g^d_V I_{0d}) \tag{11}$$

where $N_C$ is the number of color and,

$$I_{1d} = \int d[\alpha] \frac{1 - 2\alpha_1}{-\alpha_1 \alpha_2 q^2 - (1 - \alpha_3) \alpha_3 m^2_W + (\alpha_1 + \alpha_2) m^2_d + \alpha_3 m^2_u}$$

$$I_{0d} = \int d[\alpha] \frac{1}{-\alpha_1 \alpha_2 q^2 - \alpha_3 (1 - \alpha_3) m^2_W + (\alpha_1 + \alpha_2) m^2_d + \alpha_3 m^2_u} \tag{12}$$

Exchange $m_d$ and $m_u$ in (12) to obtain $I_{1u}$ and $I_{0u}$. From the above expressions we clearly see that the heaviest internal fermions dominate the contribution. If there are only three generations of quarks and leptons, the dominant contribution is from the third generation.
Again if \( q^2 \) is big enough an absorptive part of the amplitude will be generated, we have

\[
\text{Im} f_4^Z = -\frac{N_C}{4\pi \cos^2 \theta_W q^2} m_t m_b \text{Im}(f_V^{ij} f_A^{ij*}) \]

\[
\times \left( \beta_b g_A^d \left[ -2 + \frac{A_b}{B_b} \ln \frac{A_b + B_b}{A_b - B_b}\right] \theta(q^2 - 4m_b^2) - \beta_t g_A^u \left[-2 + \frac{A_t}{B_t} \ln \frac{A_t + B_t}{A_t - B_t} \right] \theta(q_2 - 4m_t^2) \right)
\]

\[
\text{Im} f_6^V = -N_C \frac{m_t m_b \text{Im}(f_V^{ij} f_A^{ij*})}{4\pi \cos^2 \theta_W q^2 \beta_W}
\]

\[
\times \left( g_V^d \ln \frac{A_b + B_b}{A_b - B_b} \theta(q^2 - 4m_b^2) + g_V^u \ln \frac{A_t + B_t}{A_t - B_t} \theta(q^2 - 4m_t^2) \right)
\]

(13)

where \( A_b = m_b^2 + m_W^2 - m_t^2 - \frac{1}{2}q^2 \), \( B_b = \frac{1}{2}q^2 \beta_W \beta_b \) with \( \beta_i = \sqrt{1 - \frac{4m_i^2}{q^2}} \). \( A_t \) and \( B_t \) are obtained by exchanging \( m_b \) and \( m_t \). Changing \( g_V^i \) to \( 2\cos^2 \theta_W Q^i \), one obtains the expression for \( f_6^\gamma \). Setting \( q^2 = 0 \) we find that our expression agree with that obtained in [7,9].

We use \( m_b = 4.5\text{GeV} \) and \( m_t = 130\text{GeV} \) to obtain numerical results for the form factors in (11) and (13).

For \( q^2 = (200\text{GeV})^2 \):

\[
\begin{align*}
\text{Im} f_4^Z &= N_C \text{Im}(f_V^{th} f_A^{tb*}) \cdot ( -4.5 - i2.4 ) \times 10^{-5} \\
\text{Im} f_6^Z &= N_C \text{Im}(f_V^{th} f_A^{tb*}) \cdot ( 2.0 - i7.3 ) \times 10^{-5} \\
\text{Im} f_6^\gamma &= N_C \text{Im}(f_V^{tb} f_A^{tb*}) \cdot ( 1.04 - i1.07 ) \times 10^{-3}
\end{align*}
\]

(14)

For \( q^2 = (500\text{GeV})^2 \):

\[
\begin{align*}
\text{Im} f_4^Z &= N_C \text{Im}(f_V^{th} f_A^{tb*}) \cdot ( -4.3 - i4.7 ) \times 10^{-5} \\
\text{Im} f_6^Z &= N_C \text{Im}(f_V^{th} f_A^{tb*}) \cdot ( 2.4 + i5.2 ) \times 10^{-5} \\
\text{Im} f_6^\gamma &= N_C \text{Im}(f_V^{tb} f_A^{tb*}) \cdot ( 2.5 + i11.9 ) \times 10^{-3}
\end{align*}
\]

(15)

\( \zeta \) is constrained to be less than \( 10^{-2} \)[15]. We will then have small values for \( f_4^V \) (less than \( 10^{-5} \)), and it will be hard to see these form factors in experiment. However if a heavy fourth generation exists, these form factors can be enhanced by a factor of \( m_{4u} m_{4d} / m_t m_b \). Here \( m_{4i} \) are the fourth generation fermion masses.

The form factors discussed here can be measured at \( e^+ e^- \) colliders with \( \sqrt{s} \geq 2m_W \) or at \( p\bar{p} \) colliders. Such measurements are CP tests. CP tests in the process \( e^+ e^- \rightarrow W^+ W^- \) have been discussed in [1,16,17,18]. At LEP the sensitivity for the dispersive parts of the form factors can reach to 0.1, and at the proposed NLC 10^{-2}. However, a detailed analysis
with the absorptive parts is still lacking, and there may be some observables, which can be optimized to get higher sensitivity as is done for the process $e^+e^- \rightarrow t\bar{t}$[19]. We hope to return to this point in the near future.

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Figure Caption:
Fig. 1. The Feynmann diagrams which contribute to $f_4^Z$ in the two-doublet Higgs model.
Fig. 2a. The real part of the form factor $f_4^Z$ divided by $d_{11}d_{12}d_{13}$ as function of $\Delta m = m_2 - m_1$ in GeV with $m_C = 60\text{GeV}$. The solide line is for $q^2 = (200\text{GeV})^2$, the dashed line is for $q^2 = (500\text{GeV})^2$.
Fig. 2b. The same as in Fig. 2a, but with $m_C = 1\text{TeV}$.
Fig. 2c. The imaginary part of the form factor $f_4^Z$ divided by $d_{11}d_{12}d_{13}$ as function of $\Delta m = m_2 - m_1$ in GeV with $m_C = 60\text{GeV}$. The solide line is for $q^2 = (200\text{GeV})^2$, the dashed line is for $q^2 = (500\text{GeV})^2$.
Fig. 2d. The same as in Fig. 2c, but with $m_C = 1\text{TeV}$.
Fig. 3. The Feynmann diagrams which contribute to $f_4^Z$ and $f_6^V$ in the Left-Right symmetric model.