Experimental emulation of quantum non-Markovian dynamics and coherence protection in the presence of information back-flow

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We experimentally emulate the non-Markovian dynamics of a pure dephasing spin-boson model at zero temperature. Specifically, we use a randomized set of external radio-frequency fields to engineer a desired ohmic noise power-spectrum to effectively realize a non-Markovian environment for a single NMR qubit. The information back-flow characteristic to the non-Markovianity is captured in the non-monotonicity of decoherence function and von Neumann entropy of the system. Using such emulated non-Markovian environments, we experimentally show the inefficiency of Carr-Purcell-Meiboom-Gill (CPMG) dynamical decoupling sequence to inhibit the loss of coherence. Finally, we design an optimized dynamical decoupling sequence which utilizes the information back-flow to maximize coherence protection for non-Markovian environments.

Keywords: Non-Markovian dynamics, information back-flow, reservoir engineering, dynamical decoupling, quantum control

Despite promising to outperform their classical counterparts by miles, quantum technologies are inherently plagued by the inevitable interactions with the surrounding environment leading to decoherence [1], which limits their utilization to full potential. Rescue techniques developed to counter the detrimental effect of the decoherence must be quantitatively benchmarked to ensure their robustness against various kinds of environments encountered in realistic experimental scenarios. One way to achieve this task is by engineering artificial environments since natural environments are generally beyond experimental control [2–6]. However, such techniques developed from benchmarking perspectives till now have mainly concentrated on engineering Markovian or memoryless environments [2]. It is crucial to take into account of strong environmental memory effects, characteristic of quantum non-Markovian dynamics [7–9], for robust implementations of quantum control. Understanding the impact of memory effects is of utmost importance also because they introduce information back-flow which is often considered to be a resource for quantum technologies [10–13]. In the quest to protect quantum information, the idea of combining control protocols with information back-flow has received a significant attention [14–16]. However the environmental memory effects can detrimentally impact quantum controls [17–19], and more so for coherence protection schemes such as dynamical decoupling (DD) [20–22]. DD, one of the most successful coherence protection methods, is based on controlled modulations of the quantum system to effectively isolate it from the environment. Although numerous DD sequences have been developed [23–30] and tested experimentally for Markovian open quantum systems [31–35], so far there is hardly any experimental investigation of such DD sequences for non-Markovian environments. Moreover, presently there is no universal protection scheme which can efficiently handle the information back-flow.

In this work, using $^1$H nuclear spins of water molecules in liquid-state nuclear magnetic resonance (NMR) setup as a qubit-ensemble, we experimentally mimic the non-Markovian dynamics of a pure dephasing quantum spin-boson model via injection of classically colored noise. We utilize amplitude and phase-modulated external radio-frequency (RF) fields to produce a desired noise power spectrum [2]. The signature of non-Markovianity is captured in terms of non-monotonicity of the decay of coherences as well as the behavior of von Neumann entropy of the system. Using engineered non-Markovian environments, we experimentally verify the inefficiency of Carr-Purcell-Meiboom-Gill (CPMG) DD sequence [21, 22]. Further, we design and experimentally demonstrate an optimized DD sequence that achieves a superior coherence protection by utilizing the information back-flow in the presence of a specific non-Markovian environment.

Emulation of non-Markovian dynamics.— Although various measures have been proposed to quantify the amount of non-Markovianity [13, 36–38], here we use the one based on the contractive property of trace distance $D(\rho_1, \rho_2) = ||\rho_1 - \rho_2||/2$, where $||\rho|| = Tr(\sqrt{\rho^\dagger \rho})$ [39]. Under a Markovian dynamical map $\Phi : \rho(0) \rightarrow \rho(t)$, the trace distance is always contractive, i.e., $D(\rho_1(t), \rho_2(t)) \leq D(\rho_1(0), \rho_2(0))$ for all pairs of initial states $(\rho_1(0), \rho_2(0))$. On the other hand, a dynamical map is non-Markovian if there exists a pair of initial states for which the trace distance shows a non-monotonic behavior. Such a non-monotonicity of the trace distance is associated with information back-flow from the environment to the system [7, 8, 36]. Accordingly, the BLP (Breuer-Laine-Piilo) measure [36] of non-Markovianity is defined as

$$\mathcal{N} = \max_{\rho_1(0), \rho_2(0)} \int_{\sigma > 0} \sigma(t) dt,$$

where, $\sigma(t) = D(\rho_1(t), \rho_2(t))$. 

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In this work, we shall consider the Hamiltonian
\[ H = \omega_0 \sigma_z/2 + \sum_k \omega_k a_k^\dagger a_k + \sum_k \sigma_z (g_k a_k + g_k^* a_k^\dagger), \]
consisting of precession of a single-qubit with a frequency \( \omega_0 \) and Pauli z-operator \( \sigma_z \) (1st term), a bosonic environment with creation (annihilation) operator \( a_k^\dagger (a_k) \) (2nd term) and the mutual interaction with coupling constant \( g_k \) (3rd term). This Hamiltonian model is exactly solvable and leads to the decay of coherences without affecting the populations. The decoherence function is of the form \( \Gamma_0(t) = e^{-\chi_0(t)} \),
\[ \chi_0(t) = 2 \int_0^\infty d\omega J(\omega) \coth(\omega/2k_B T) \frac{\sin^2(\omega t/2)}{\omega^2}, \quad (1) \]
where \( k_B T \) and \( J(\omega) \) describe the thermal energy and the spectral density of the environment respectively. In this case non-Markovianity measure \( N \) is maximized for any pair of antipodal initial states on Bloch sphere [40] and \( \sigma(t) = \Gamma_0(t) \). Accordingly, non-Markovianity measure takes a simple form, \( N = \sum_k [\Gamma_0(t_k^l) - \Gamma_0(t_k^r)] \), considering all the intervals \([t_k^l, t_k^r] \) wherein \( \Gamma_0 > 0 \). The functional forms of \( N \) for the widely considered case of Ohmic spectral density
\[ J(\omega) = \lambda \exp(-\omega/\omega_c) \omega^s/\omega_c^{s-1}, \quad (2) \]
versus the dimensionless coupling constant \( \lambda \), the ohmicity parameter \( s \), and the cut-off frequency \( \omega_c \), have been investigated in [41, 42]. The semi-classical limit of interaction Hamiltonian is \( \xi(t)\sigma_z/2 \), where \( \xi(t) \) is a stationary Gaussian stochastic process with zero mean and with a correlation function \( \langle \xi(t_1)\xi(t_2) \rangle = g(t_1 - t_2) \). The Fourier transform \( S(\omega) \) of time averaged \( g(t) \) is called noise power spectrum which replaces \( J(\omega) \coth(\omega/2k_B T) \) in Eq. 1, so that the decoherence function in this limit reduces to
\[ \chi_0^c(t) = \frac{2}{\pi} \int_0^\infty d\omega S(\omega)|F_0(\omega, t)|^2, \quad (3) \]
where the free-evolution filter-function \( |F_0(\omega, t)|^2 = \sin^2(\omega t/2)/\omega^2 \).

It suggests that we can mimic the non-Markovian dynamics of a single qubit coupled to a bosonic environment with a synthetic noise power spectrum. We engineer such a power spectrum via a temporal average over a set of stochastic fields of the form,
\[ \xi(t) = \gamma \sum_{k=1}^M a(k) \cos(k\omega_b t + \phi), \quad (4) \]
where \( \gamma \) is strength of noise, \( a(k) \) is the amplitude of the \( k \)th Fourier component, \( \omega_b \) is the base frequency, and \( \phi \in [-\pi, \pi] \) is a random number with an uniform distribution.

The resulting noise power spectrum is of the form [2],
\[ S(\omega) = \frac{\pi\gamma^2}{2} \sum_{k=1}^M a^2(k)|\delta(\omega - k\omega_b) + \delta(\omega + k\omega_b)|. \quad (5) \]
Comparing Eqs. 1, 2 and 5, we find that for ohmic spectral density
\[ a^2(k) = \frac{(k\omega_b)^s}{\omega_c^{s-1}} e^{-k\omega_b/\omega_c} \coth(\frac{k\omega_b}{2k_B T}), \quad \gamma^2 = 2\lambda/\pi, \quad (6) \]
which implies we can emulate pure dephasing dynamics with a non-Markovian behavior of a bosonic reservoir by properly tuning \( s, \gamma, T \), and \( \omega_c \).

The NMR sample consists of 20% H$_2$O in 80% D$_2$O, with a trace of CuSO$_4$ that shortens ¹H longitudinal and transverse relaxation times to \( T_1 \approx 200\) ms and \( T_2 \approx 180\) ms respectively. The experiments are carried out in a Bruker 500 MHz NMR spectrometer at an ambient temperature of 300 K. Here the two Zeeman levels of the spin-1/2 ¹H nucleus forms the qubit and stochastic controls required to engineer a desired \( S(\omega) \) are realized by transverse radio-frequency (RF) fields whose amplitude and phase are modulated according to Eq. 4. The corresponding NMR pulse-sequence requires an initial \( \pi/2 \) pulse to prepare coherence followed by a stochastic longitudinal control field \( U_z(t) = e^{-i\xi(t)\sigma_z/2} \). However, as illustrated in Fig. 1, \( U_z(t) \) is implemented by a transverse stochastic unitary \( U_z(t) \) sandwiched between \( (\pi/2)_y \) and \( (\pi/2)_y \) pulses, wherein the \( (\pi/2)_y \) pulse is nullified with the initial \( (\pi/2)_y \) pulse. Finally, an effective dephasing dynamics is achieved by temporally averaging NMR signals over \( N = 1000 \) independent realizations of the stochastic process \( \xi(t) \) (Eq. 4). We tuned the strength of injected noise \( \lambda \in [10\pi, 100\pi] \), cut-off frequency \( \omega_c = 2\pi \times 320 \) rad/s, and the ohmicity parameter \( s \in [1, 6] \) so that the signal decays out in 2.5 ms. Fig. 2(a) contrasts the Markovian temporal aver-
The experimentally obtained non-Markovianity measure $\mathcal{N}$ versus ohmicity parameter $s$ for $\lambda = 10\pi$ and (c) coupling constant $\lambda$ for $s = 4$. The shaded regions for simulations and error bars in experiments. Non-Markovianity measure $\mathcal{N}$ for Markovian ($s = 1$) and non-Markovian ($s = 4$) emulated environments. Non-Markovianity measure $\mathcal{N}$ versus ohmicity parameter $s$ for $\lambda = 10\pi$ and (c) coupling constant $\lambda$ for $s = 4$. The shaded regions for simulations and error bars in experiments correspond to standard deviations over 10 distinct bins each of 900 realizations, and they capture the finite-ensemble effects. (d) Variation of von-Neumann entropy of the system $S(t)$ which provides a thermodynamic perspective on the transition from Markovian to non-Markovian dynamics. For a single qubit with an initial state $\rho = 1/2 + c\sigma_x/2$, 

$$S(t) = -\text{Tr}[\rho(t) \log_2 \rho(t)] \approx 1 - e^2 \Gamma_0^2(t)/2,$$  

(7)

(blue-line) along with corresponding simulated (red-line) and experimental (symbols) entropies are also shown in Fig. 2 (d). While the monotonic growth of entropy for $s = 1$ indicates Markovian behavior, the slight drop of entropy from $t \approx 0.5$ ms for $s = 4$ is a signature of non-Markovianity.

**DD for non-Markovian environment.**—Consider a DD-protected qubit undergoing sequential phase-flips represented by the rectangular-wave modulation function $f(t) \in \{-1, 1\}$. In this case, the effective decoherence function $\Gamma(t) = e^{-\chi(t)}$, where $\chi(t)$ has a similar form as in Eq. 3, except that the filter-function is replaced with the Fourier transform,

$$F(\omega, t) = \frac{1}{2\pi} \int_0^t f(t') e^{-i\omega t'} dt'.$$  

(8)

Construction of a DD sequence is based on engineering a filter-function $F(\omega, t)$ which minimizes its overlap with a given noise power spectrum $S(\omega)$, and thereby minimizes $\Gamma(t)$. Therefore, the performance of a DD sequence depends crucially on the timescale associated with the environmental correlation function. Since DD sequences are generally designed with Markovian approximation with memory-less environments, they under perform in the presence of environments with finite correlation times. In fact, it has been theoretically proven that the PDD sequence [20] is inefficient for non-Markovian environments [18]. We also found, using filter-function analysis, that even CPMG [21, 22] and UDD [25] sequences under perform in the presence of information back-flow. Here we show that, for a known non-Markovian environment we can engineer a DD-sequence (NDD) that maximizes coherence protection, $\mathcal{P} = \int_0^t dt' \Gamma(t')/t$ [18], by numerically optimizing the time-instants of spin-flips keeping the total number of flips to be a constant. Fig. 3 (a) shows an ohmic non-Markovian spectral density curve.
Coherence protection and filter-functions

FIG. 3. Ohmic non-Markovian spectral density (dashed line) and filter-functions $|P(\omega, t)|^2$ at $t = 2.5$ ms for various DD sequences as well as for free-evolution (a) and corresponding decoherence functions (b). (c) Theoretical (smooth lines) and experimental (symbols) decoherence functions with free-evolution, CPMG (d), or NDD (e) sequences. In (d) and (e) one particular noise realization is interleaved with $\pi$ pulses. Coherence protection $P$ for various DD-sequences and for free-evolution versus the ohmicity parameter $s$ (f) and versus coupling constant $\lambda$ (g).

(dashed-line) corresponding to the ohmicity parameter $s = 4$ and the coupling constant $\lambda = 10\pi$. It also shows filter-functions for various DD sequences including free-evolution. Note that NDD filter-function has the minimal overlap with $S(\omega)$ indicating a better coherence protection as evident from the corresponding decoherence functions plotted in Fig. 3 (b). The inefficiency of other sequences can be attributed to the localization of the noise strength at the intermediate frequencies unlike the usual scenario with low-frequency Markovian noises. Fig. 3 (c) compares the theoretical (smooth lines) experimental (symbols) performances of CPMG sequence (Fig. 3 (d)) with NDD (Fig. 3 (e)) and free-evolution. Each of the 1000 realizations for emulated environmental $x$-modulations (see Fig. 1) was interleaved with a total of ten composite $\pi_z$ pulses of width 50 $\mu$s. It is interesting to note that for durations less than 0.5 ms, CPMG shows faint improvement over free-evolution. However, once the information back-flow sets in, CPMG not only fails to protect the coherence, but also has a detrimental impact on it. In contrast, the NDD sequence should have a much better coherence protection as indicated by the theoretical decoherence function. Experimentally, there is a significant protection for up to 1 ms, and then the performance drops below free-evolution presumably due to finite pulse-widths, calibration errors, and other pulse-imperfections. Generation of numerous NDD sequences starting from random guesses revealed a general pattern involving bunching of $\pi$ pulses at the beginning and at the end of sequences [43]. This feature explains the exclusion of filter-function at the intermediate frequencies as observed in Fig. 3(a) and consequently the minimization of overlap with the spectral density. This pattern may help the NDD sequence to take advantage of the inherent information back-flow by avoiding $\pi$ pulses in the intermediate time durations. Fig. 3(f) compares the robustness of various sequences versus the ohmicity parameter $s$ at a fixed coupling constant $\lambda = 10\pi$. At low ohmicity, the environment is essentially Markovian and accordingly the standard sequences namely CPMG, PDD, and UDD perform well. Whereas, for $s > 2$, non-Markovianity sets in, and therefore NDD outperforms the standard sequences. Fig. 3 (g) compares the robustness versus the coupling constant $\lambda$ for a fixed ohmicity parameter $s = 4$. Although the NDD sequence was optimized for $\lambda = 10\pi$, it seems to perform better than all the other sequences for the entire range of the coupling constant.

Conclusions.— We described experimentally emulating the non-Markovian dynamics of a pure dephasing spin boson model at zero temperature by engineering noise power spectrum with the help of a temporal averaged set of randomized external fields. We characterized the emulated non-Markovianity using BLP measure [36] and von Neumann entropy of the system. Emulating quantum non-Markovian dynamics is important not only from the fundamental point of view to understand dynamics of information back-flow [7–9, 36, 37, 42, 44] and thermodynamic properties such as flow of heat, entropy production [45–49], but also from a practical perspective of developing coherence protection protocols in presence of environmental memory effects [14, 16–19]. Using emulated non-Markovian dynamics, we experimentally verify the inefficiency of CPMG DD sequence. Moreover, using the filter function formalism [30, 31, 50–53] we designed an NDD sequence that optimizes the position of spin-flips to maximize coherence protection for a specific non-Markovian environment. The goal of achieving universal DD sequences and dynamically protected quantum gates for Markovian or any non-Markovian environments is challenging and needs further investigation.

This work was supported by DST/SJF/PSA-03/2012-
Supplemental Material

ESTIMATION OF NON-MARKOVIANITY MEASURE $N$

For emulating non-Markovian dynamics by noise power spectrum engineering using randomized control fields, a finite number of realization are possible due to experimental constraints of time required (five times longitudinal relaxation time $5 \times T_1 = 2$ s) for reinitialization after every realization of $\xi(t)$. It produces oscillation artifact on top of characteristic non monotonicity of decoherence function due to non-Markovianity Fig S1(a). Smoothing can not be used on time domain data directly because it can not differentiate between spurious oscillations and concerned non monotonicity. However, in Fourier domain these two can be separated since the artifact appears as noise on top of peak due to non-Markovianity in Fourier transform of $\Gamma_0(t)$ (Fig. S1(b)). We smoothen out these oscillation using standard data processing techniques (Fig S1(c)) keeping maximum of the peak intact and then we inverse Fourier transform to get smoothened decoherence function (Fig S1(d)).

![Figure S1. Smoothing process to estimate non-Markovianity measure $N$.](image)

OPTIMAL DD SEQUENCES

In search of a general pattern in DD sequence for non-Markovian environments, we design NDD sequence for various ohmicity parameters $s \in [1, 5]$ for fixed coupling constant $\lambda = 10\pi$ (Fig. S2(a)) by optimizing the delay between $\pi$ pulse which maximizes the parameter $P$. We observe bunching of pulse at the beginning and end of the sequence for the non-Markovian environments $s = 3, 4, 5$ in contrast to Markovian cases $s = 1, 2$. As described in the main text, this is because localization of noise power spectrum in the intermediate frequency range for $s = 3, 4, 5$. In Fig. S2 (b) and (c) we plot non-Markovianity parameter $N$ for for CPMG, PDD, UDD, and NDD sequence against various ohmicity parameter and coupling constant respectively. Note that $N$ is has significant variation due to additional information back-flow introduced due to DD sequences. Therefore, these plots indicate that one can engineer non-Markovianity by just carefully optimizing delay between spin flips.
FIG. S2. (a) Optimal DD sequences for various ohmicity parameter $s \in [0, 5]$. Corresponding values of non-Markovianity measure are also indicated. Non-Markovianity measure for CPMG, PDD, UDD, and NDD sequence for (b) various values of ohmicity parameter $s$ for fixed value of coupling constant $\lambda = 10\pi$ and (c) various values of coupling constant $\lambda$ for fixed value ohmicity parameter $s = 4$. 