Highly correlated two-dimensional viscous electron fluid in moderate magnetic fields

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Magnetotransport phenomena often provide critically important information about two-dimensional (2D) electron systems. For example, the independence of magneto-photo-resistance of 2D electrons in best-quality quantum wells on the polarization helicity of incident radio-frequency radiation have been treated as a puzzling effect, which is important for characterization of these systems, but had no well-established explanation up to now. Here we develop a phenomenological model of dynamics of a highly correlated 2D electron fluid in moderate magnetic fields, in which shear viscosity and the memory effects in inter-particle interaction control flow dynamics. In this system, successive collisions of electrons joined in pairs (that is, the pair correlations in time) turn out to be as important as uncorrelated collisions of statistically independent electrons. The resulting photoresistance exhibits an irregular shape of magnetoooscillations, the absence of the dependence on the helicity of the circular polarization of radiation, and a giant peak near the doubled cyclotron frequency. All these effects were observed in experiments on best-quality GaAs quantum wells in moderate magnetic fields at low temperatures. Although the most general conditions of applicability of the developed phenomenological model of the fluid dynamics is not fully clarified at now, this coincidence can point out that 2D electrons in such systems form the highly correlated viscous fluid.

1. INTRODUCTION

Microwave-induced resistance oscillations (MIRO) of 2D electrons in moderate magnetic fields were initially observed [1, 2] and then extensively studied [3] in GaAs/AlGaAs quantum wells and then in other 2D nanostructures. This is an intriguing phenomena, whose correct explanation is, apparently, crucial for understanding of physics of 2D electrons. Originally, MIRO were attributed to transitions of non-interacting 2D electrons at high Landau levels in external dc and ac electric fields [3–8]. Such “displacement mechanism” [8, 9] is based on taking into account the radiation-assisted scattering of 2D electrons in quantized states on disorder, that result in unequal probabilities of electron transitions in the opposite directions at a dc field. Theories [8, 9] yield the correct profile of the magnetoooscillations of photoresistance in not too clean samples. In Ref. [7] the “inelastic mechanism” was proposed in which the crucial role is played by radiation-induced redistribution of electrons by energy. Its contribution to photoresistance explains the temperature dependence of MIRO. In Ref. [10] it was shown that the classical memory effects in scattering of 2D electrons on disorder also can lead to MIRO. In Ref. [11] was developed a different powerful approach to theory of MIRO, which is based on a phenomenological description of memory effects in classical dynamics of non-interacting 2D electrons, scattered on disk defects in magnetic field. This approach describes the essence and properties of MIRO in a lucid transparent manner.

Theories [3–8, 11, 12] have a substantial problem which is the inconsistency between an almost full lack of dependence of MIRO on the sign of the circular polarization of radiation in some experiments [13, 14] and the need to involve special factors of experimental setup [4, 15] or very special models of electron dynamics [16, 17] to explain this lack in theory. Moreover, in best-quality GaAs quantum well and graphene samples were observed peculiar features in transport effects, in particular, a non-sinusoidal profile of MIRO and similar oscillating photo-magneto-conductivity as well as a giant peak near the doubled cyclotron frequency in photoresistance [13, 14, 15, 22]. All these effects has been remained unexplained within disorder-based models up to now.

In parallel with the studies of MIRO, the giant negative magnetoresistance was observed in the same best-mobility GaAs quantum wells (see, for example, Refs. [18, 21, 24, 28]). In Ref. [29] it was explained as the result of formation of a viscous fluid from 2D electrons in the quantum wells [31]. Later, a very similar huge negative magnetoresistance was detected in other high-quality conductors: 2D metal PdCoO$_2$ [31], the 3D Weyl semimetal WP$_2$ [32], and single-layer graphene [33, 34], for which other bright evidences of the hydrodynamic electron transport were also discovered [33, 35]. In Ref. [40] it was pointed out that the high-frequency viscosity coefficients of 2D electrons exhibit a single resonance at the twice cyclotron frequency being a characteristic sign of electron hydrodynamics. In the case of a strongly non-ideal fluid, such “viscoelastic” resonance manifests itself via the excitation of the transverse magnetosonic waves [41, 42]. Later, many other hydrodynamic effects were observed in high-quality GaAs quantum wells and graphene samples. For example, in Ref. [39] it was demonstrated that in graphene samples with constrictions THz radiation induces a strong heating of electrons leading to the hydrodynamic regime.

In view of these results, a question arises whether the giant peak and the polarization-sign-independent magnetoooscillations in high-purity 2D electron systems are explained within some model of a viscous electron fluid?

In this work we propose a phenomenological model of
a highly correlated viscous 2D electron fluid in defectless samples in moderate magnetic fields. We start from the Landau Fermi-liquid model with the strong Landau interparticle interaction. The two key points of our model are: the viscoelasticity effect leading to excitation of the shear-stress waves and the magnetic-field-induced memory effects in the inter-particle scattering. Based on the approaches and ideas of Refs. [12] [41]–[43], we formulate the viscoelastic motion equations of the fluid containing the memory terms. These equations account for multiple repeated collisions of the electrons joined in pairs, that is, of the strong pair correlations in time. The strong interparticle interaction and presence of moderate magnetic fields are the sufficient conditions of applicability of formulated model. Resulting photoconductivity exhibits magnetoooscillations of an irregular shape, a large peak at the doubled cyclotron frequency, and is independent of the helicity of polarization of radiation. All these properties qualitatively coincide with the ones observed in experiments on magnetotransport in ultra-high-quality GaAs quantum wells and graphene [47].

Apparently, the strength of interparticle interaction is not too large in the structures examined in [13] [14] [18]–[21], and the above sufficient condition of applicability of our model (the strong Landau interaction) is not fulfilled in them. However, we discuss that more general applicability conditions can depend on particular electron flow parameters and be much wider. The demonstrated good qualitative agreement of our theory and the experiments can be a promising evidence of formation of a highly correlated 2D electron fluid in such systems and of suitability of our model for their description.

2. MODEL OF ELECTRON FLUID IN MODERATE MAGNETIC FIELDS

We develop a phenomenological hydrodynamic theory of a highly-correlated viscous 2D electron fluid in classical magnetic fields starting from the Landau Fermi-liquid model. Our theory is surely applicable for 2D electrons with low densities \( n_0 \) and corresponding interparticle interaction parameter \( r_s \gg 1 \) for dc and ac flows in sufficiently wide (not ballistic) samples. Such systems were realized in inversion layers on Si crystals, ZnO-based structures, low-density GaAs quantum wells, where metal-insulator transition, renormalization of the quasiparticle effective mass, and other correlation effects were observed [48] [51].

As for the systems with moderate interaction, \( r_s \lesssim 1 \), the widest conditions of applicability of our phenomenological model are not fully clarified at now, but can include also the systems with \( r_s \lesssim 1 \) for particular flow parameters [51] [54]. Indeed, for example, electron hydrodynamics for stationary flows in moderate magnetic fields is realized in bulk samples at any inter-particle scattering rates [74] [73]. Note also that if there are no sources of small-scale disturbances in the system, then kinetic and hydrodynamic approaches often give qualitatively similar results for flow distributions with a minimum scale equal to the microscopic length [54] [74].

At high frequencies, \( \omega \gg 1/\tau_2 \), and long wavelengths, \( \Delta x \gg v_F/\omega \), a flow of a strongly non-ideal viscous 2D electron fluid in magnetic field consists of the two components [here \( \tau_2 \) is the shear stress relaxation time of electron-like quasiparticles; \( \Delta x \) is the characteristic scale of inhomogeneities, \( v_F \) is the Fermi velocity]. The first one consists of magnetoplasmon modes related to perturbations of the electron density, \( n(\mathbf{r}, t) \), and the hydrodynamic velocity \( \mathbf{V}(\mathbf{r}, t) \). The second one is formed by the transversal magnetosonic waves related to per-
turbations of the shear stress $\sigma_{ij}(r, t)$ and the velocity $V(r, t)$ \[41, 46\]. Such waves can be excited in Fermi-liquids at large dissipativeless interparticle interaction, described by the Landau interaction term \[42, 46-76\]. The inter-particle scattering leads to weak relaxation of both the components.

The Navier-Stokes-like non-stationary equations of the 2D electron Fermi-liquid at a strong interparticle interaction were derived in Refs. \[43, 45\] for zero magnetic field and in Ref. \[42\] for a nonzero classical magnetic fields. The sufficient criteria of applicability of those viscoelastic equations can be formulated as follows \[41, 42, 46\]:

\[ F_1 \gg 1, \quad F_m \ll F_1 \text{ at } m \geq 3; \quad R_e \ll \Delta x \text{ or } v_F/\omega \ll \Delta x, \]

where $F_m$ are the Landau interaction parameters, $F_2$ can be arbitrary, $R_e = v_F/\omega_c$ is the cyclotron radius of electron-like quasiparticles, and $\omega_c = eB/(mc)$ is the cyclotron frequency. These criteria do not imply, generally speaking, a thermodynamical equilibrium of quasiparticles in a frame moving with the flow velocity $V(r, t)$ (see discussion in Refs. \[41, 44, 45\]).

Memory effects in an electron fluid in magnetic field, from the classical-mechanical point of view, are induced by subsequent collisions [1] of some electrons joined in pairs due to the cyclotron rotation [see Fig. 1(a,b)]. The probability for electron to pass a cyclotron circle without collisions with other electrons is:

\[ P(B) = e^{-T/\tau_q}, \]

where $\tau_q$ is the interparticle scattering departure time and $T = 2\pi/\omega_c$ is the cyclotron period. The value $P$ increases up from zero to unity with the increase of magnetic field. Herewith we do not study the regimes of too high magnetic fields when quantization of the density of states can become substantial and high-order memory effects, related to many recollisions, $N_r \gg 1$, within an extended collision, appears. Apparently, these simplifications takes place when $\omega_c \lesssim 2\pi/\tau_q$. In Fermi systems, the time $\tau_q$ is much shorter than the shear stress relaxation time $\tau_2$ \[58, 59\]. So we consider the diapason of magnetic fields: $1/\tau_2 \ll \omega_c \lesssim 2\pi/\tau_q$, when the electron dynamics remain classical and $N_r \sim 1$.

Due to extended collisions, the dynamic of the fluid in the current moment $t$ is determined by the statistical properties of the system in the moments $t$ and

\[ t' = t - N_r T, \quad N_r = 1, 2, 3, ... \]

of previous collisions of paired particles, which are scattering in the moment $t$ (see Fig. 1(a,b) and details in \[54\]). As a result, paired electrons leads to the appearance of some retarded non-linear terms in the fluid dynamic equations of Refs. \[22, 41, 42\]. Such “memory terms” can be of the different types. The terms of one type are the retarded relaxation terms due to the inter-particle scattering, being partly similar to the retarded relaxation terms due to the extended collisions of non-interacting electrons with localized defects \[12\]. The contributions of another type are related to retarded corrections of the Landau parameters $F_m$, reflecting non-local in time perturbation of the elastic part of the interparticle interaction.

3. EQUATIONS OF FLUID DYNAMICS

We formulate the following motion equations for the particle density $n$, the hydrodynamic velocity $V$, and the in-equilibrium momentum flux $\Pi = -\delta$, accounting the described above viscoelastic and memory effects:

\[ \partial n/\partial t + n_0 \text{div} V = 0, \]

\[ \partial V_i/\partial t = eE_i/m + \omega_c \epsilon_{ikr}V_k - (1/m) \partial \Pi_{ij}/\partial x_j, \]

\[ (1 - \tilde{\delta} F_{2, ij}) \partial \Pi_{ij}/\partial t = 2 \omega_c \epsilon_{ikr} F_{kl} - \Pi_{ij}/\tau_2 - m (v_F^n)^2 V_{ij}/4 - \Gamma_{ijkl} \Pi_{kl}(r, t - T). \]

These equations are an extension of (i) the Navier-Stokes-like equations of highly viscous electron fluid \[11, 12\] and (ii) the retarded relaxation equations for scattering of independent electrons on disk defects \[12\]. In them, the electric field $E = E(r, t)$ consists of the applied field and the internal field induced by a non-equilibrium charge density $e \delta n(r, t)$: $E(r, t) = E^{ext(t)} + E^{int(r, t)}$; $e$ and $m$ are the electron charge and the quasiparticle effective mass; $\epsilon_{ikr}$ is the antisymmetric unit tensor; $V_{ij} = \partial V_i/\partial x_j + \partial V_j/\partial x_i$; $\tau_2$ is the shear stress relaxation time without the memory effects; $v_F^n$ is the parameter defining the magnitude of the viscosity $\eta$ of a highly viscous fluid at zero magnetic field \[41, 42\]; $\eta_0 = (v_F^n)^2\tau_2/4$, and the values $\tilde{\delta} F_{2, ij}$ are proportional to the retarded perturbations of the Landau parameter $F_2$. The values $m, \tau_2, v_F^n$, and $\omega_c$ (in the equation for $\partial \Pi/\partial t$) are expressed via the Landau parameters $F_0, F_1$, and $F_2$ \[42\]. The parameter $v_F^n$ in a strongly non-ideal Fermi liquid is much greater than the Fermi velocity $v_F$ \[41, 42\].

The term $-\Gamma_{ijkl}(r, t)\Pi_{kl}(r, t - T)$ in Eq. \[1\] describes the retarded relaxation of $\Pi_{ij}$ due to the extended collisions of pairs of quasiparticles with one return to the same point [see Fig. 1(b)]. Such extended collisions are sensitive to the macroscopic motion of the fluid via the forces acting on quasiparticles from the internal field $E_{int}(r, t)$ and from the elastic tension $\sigma(r, t) = -\Pi(r, t)$. The latter one is induced by perturbations of the quasiparticle energy spectrum by nonzero $V$, which is expressed via the term $-m(v_F^n)^2 V_{ij}/4$ \[42, 43\]. As a result, the tensor $\Gamma(r, t)$ depends on the fluid variables in the past, $t' < t$. 
At rare interparticle collisions, when \( \omega_c \tau_2 \gg 1 \), the tensor \( \Gamma_{ijkl}(r, t) \) depends mainly on the shift
\[
\Delta_{mn}(r, t) = \varepsilon_{mn}(r, t) - \varepsilon_{mn}(r, t - T)
\]
of the strain tensor, \( \varepsilon_{mn} = \partial u_m / \partial x_n + \partial u_n / \partial x_m \), in one cyclotron period \{ Here \( u(r, t) \) is the displacement of fluid elements; see Fig. 1(b) and qualitative justification of this property of \( \Gamma_{ijkl} \). \}
\]
Far from the resonance frequency \( \omega = 2 \omega_c \), the fluid dynamics is almost elastic (combination of magnetosonic waves) and the value \( u(r, t) \) is the proper value fully describing states of the system. For the frequencies near the resonance, the value \( u(r, t) \) characterizes the inelastic displacement of fluid elements. In both cases the mismatch of the strain tensor is expressed via the velocity gradient \( V_{mn}(r, t') \):
\[
\Delta_{mn}(r, t) = \int_{t-T}^{t} dt' V_{mn}(r, t') .
\]
For a small-amplitude flow, the retarded relaxation tensor \( \Gamma(\Delta) \) is expanded into power series by \( \Delta = \Delta(r, t) \):
\[
\Gamma_{ijkl}(u(r', t')) = \Gamma^{(0)}_{ijkl} + \alpha_{ijkl}^{mnop} \Delta_{mn} \Delta_{op} ,
\]
where the coefficients \( \Gamma^{(0)}_{ijkl} \) and \( \alpha_{ijkl}^{mnop} \) are proportional to the probability \( P \) \( \text{S2} \), like as the coefficients \( \Gamma_{ijkl} \). We note that Eq. (8) at fixes ac frequency \( \omega \) leads to the elastic terms in equations (4) playing the role of the non-local and non-linear by \( V_i \) and \( \Pi_{ij} \) corrections to the cyclotron terms \( 2 \omega_c \varepsilon_{ijk} \Pi_{kj} \).

The necessary condition of applicability of the above memory terms has the form \( \text{S4} \): \( V_{char} T R / \Delta x \ll a_p \), where \( V_{char} \) is the character flow amplitude and \( \Delta x \) is its character space scale. This is the condition on the electric field magnitudes \( E_0/1 \) determining the magnitude of \( V_{char} \) in a given flow geometry.

### 4. Fluid Flow in Long Sample

We consider a Poiseuille flow in a defectless long sample as a minimal model to study magneto-photo-transport in a highly correlated electron fluid [see Fig. 1(a)].

Note that the high-quality GaAs quantum wells often contain macroscopic defects, those can appear during the growth process \( \text{S7} \) or can be made artificially \( \text{S7} \). Their appearance leads to a complication of the flow of the 2D electron fluid and an effective reduction in the flow width \( \text{S0} \). So, when describing a real flow in a sample by our model, the value \( W \) is to be considered as a certain effective width \( W_{eff} \) of the conducting channel, which can be significantly less than the actual sample width.

The external electric field \( E^{ext}(t) = E_0 + E_1(t) \) consist of the dc field \( E_0 = E_0 e_z \) induced by an applied electric bias and the radiation field \( E_1(t) = E_1^+ e^{-i \omega t} + c.c. \) with the left or the right circular polarizations: \( E_1^\pm = E_1 (e_x \pm i e_y)/2 \). For simplicity, the sample edges are supposed to be rough, thus the diffusive boundary conditions,
\[
V_{|y=\pm W/2|} = 0 ,
\]
are applicable. The internal field \( E^{int}(y, t) \) in this geometry is directed along the \( y \) axis, being the Hall field. For simplicity, the sample is considered to be sufficiently narrow: \( W \ll l_p \), where \( l_p \) is the characteristic plasmon wavelength. In such case, the field \( E^{int}(y, t) \) screens the \( y \) component of the incident ac field \( E_1(t) \). As a result, the ac plasmonic flow component, related to \( e \delta n(y, t) \), is suppressed and the ac flow is formed mainly by the viscoelastic eigenmodes \( \text{S4} \). Herewith only the \( x \) component of the hydrodynamic velocity \( V \) and the \( xy \) component of the tensors \( V_{ij} \) and \( \varepsilon_{ij} \) present.

In order to find photoconductivity \( \sigma_\omega = j_0 [E_0, E_1(t)]/E_0 \), first, we calculate the linear responses \( V_0 \) and \( V_1 \) of the fluid on the dc \( E_0 \) and the ac
\(E_1(t)\) fields \{here \(j_0[E_0, E_1(t)]\) is the dc current density at incident radiation, averaged over the sample\}. The dc response \(V_0 = V_0e_x\) is a conventional 2D Poiseuille flow:

\[
V_0(y) = eE_0 \left[ (W/2)^2 - y^2 \right] / (2mn_{\text{dc}}) .
\]

Here \(n_{\text{dc}} = [(v_F^0)^2/2(1 + 4\omega_0^2)]\) is the dc diagonal viscosity. The velocity \(V_0\) depends on magnetic field via the dc diagonal viscosity \(\tilde{n}_{\text{dc}} \sim 1/B^2\) \cite{29}. The linear responses \(V_{1x}^\pm(t, y, t) = V_{1x}^\pm(y, t)\) are identical that reflects the screening of the component \(E_{1y}(t)\). The velocity takes the form \cite{41}:

\[
V_1(y, t) = V_1(y) e^{-i\omega t} + \text{c.c.},
\]

where \(\lambda = \sqrt{-i\omega/\eta_{\text{dc}}(\omega)}\) is the eigenvalue of the transverse magnetosonic waves.

\[
\eta_{\text{dc}}(\omega) = \frac{[(v_F^0)^2/2(1 + i\omega\tau_2)]}{1 + (4\omega_0^2 - \omega^2)^2\tau_2^2 - 2i\omega\tau_2} ,
\]

is the ac diagonal viscosity \cite{40, 41}, and the function

\[
f_\lambda(y) = \cosh(\lambda y)/\cosh(\lambda W/2)
\]

describes the flow profile. In the viscoelastic regime, \(\omega \sim \omega_e \gg 1/\tau_2\), the imaginary part dominates in \(\eta_{\text{dc}}\) far from the resonance at \(\omega = 2\omega_e\), therefore Navier-Stokes equations \cite{41} turns out into Hooke’s equations. The wavelength and the decay length of the shear waves, related to the value \(\lambda\), at \(\omega > 2\omega_e\) are estimated as \(l_x = v_F^0/\omega\) and \(l_s = v_F^0/\tau_2\) (see details in Refs. \cite{41} and \cite{54}). We consider here that \(l_x, l_s \ll l_p\). The parameter \(v_F^0\) within the model of a strongly non-ideal Fermi liquid is much greater than the Fermi velocity \(v_F\), thus a flow with characteristic spacescales \(\Delta x \gtrsim l_x = v_F^0/\omega \gg v_F/\omega\) can be described hydrodynamically \cite{41, 42}.

Second, to calculate the photoconductivity in the second by \(E_1\) order, \(\sim E_1^2\), we account the following contributions in the hydrodynamic velocity: \(V = V_0 + V_1 + V_c\), where \(V_0 = V_0(y)\) and \(V_1 = V_1(y, t)\) are the linear responses presented above and \(V_c = V_c(y)\) is the nonlinear dc component being proportional to \(E_0E_1^2\). The latter one is calculated in Supplemental material \cite{54} on the base of Eqs. \cite{1} by the perturbation theory by the nonlinear memory inelastic, \(\sim \Gamma_{ijkl}\), and elastic, \(\delta F_{2,ij}\), terms. In this calculation we account that, in the lowest approximation, in the nonlinear memory terms one should account the flow characteristics \(V_{xy}, V_{xx}, X_{xx}, V_{xy}, \Delta_{xx}, \text{and } \Pi_{xy} \) corresponding to the linear responses \(V_0\) and \(V_1\).

Based on the nonlinear dc velocity \(V_c(y)\), we find the mean sample photoconductivity \(\Delta\sigma_{\omega} = \sigma_{\omega} - \sigma_0\) (here \(\sigma_0\) is the dc mean conductivity that is \(\sigma_{\omega}\) at \(E_1 = 0\)):

\[
\Delta\sigma_{\omega} = e\eta_0 \int_{-W/2}^{W/2} V_c(y) dy / (WE_0) .
\]

The non-linear values \(V_{\omega}\) and \(\Delta\sigma_{\omega}\) within our model consists of the inelastic (relaxational) and the elastic contributions: \(V_{\omega} = V_{\omega}^{\text{rel}} + V_{\omega}^{\text{el}}, \Delta\sigma_{\omega} = \Delta\sigma_{\omega}^{\text{rel}} + \Delta\sigma_{\omega}^{\text{el}}\). We will see that the relaxation and the elastic contribution have different magnetic field dependences.

The result for the relaxational contribution to the nonlinear dc velocity can be presented in the form:

\[
V_{\omega}^{\text{rel}}(y) = e^3E_0E_1^2\alpha u + 2Re \frac{v}{\omega} J(y) ,
\]

where \(\alpha\) is a linear combination of the coefficients \(\Gamma_{ijkl}\) corresponding to the Poiseuille flow geometry, \(J(y)\) is the dimensionless factor determining the profile of the nonlinear flow component:

\[
J(y) = \frac{|\lambda|^2}{\left| \cosh(\lambda W/2)^2 \right|} \int_{|y|}^{W/2} d\tilde{y} \tilde{y} \left| \sinh(\lambda \tilde{y}) \right|^2 ,
\]

and the dimensionless values \(u\) and \(v\) contain the dependence on the frequencies \(\omega\) and \(\omega_e\):

\[
u = 2\pi/\omega_e (1 + 4\omega_e^2/\omega^2) ,
\]

\[
u = (2\pi/\omega_e) \left[ \sin(2\pi/\omega_e) - 2i \sin^2(\pi/\omega_e) \right] \times
\]

\[
\left( -1 + i\omega_\tau_2 + 4\omega_e^2/\omega^2 \right) (1 + 4\omega_e^2/\omega^2) ,
\]

\[
1 + 4\omega_e^2/\omega^2 - 2i\omega_\tau_2 .
\]
For the elastic contribution to the nonlinear dc velocity it was obtained \[54\]:

\[
V^{\text{el}}_{\omega}(y) = \frac{e^3 E_0 E^2_{\omega}}{m_0^2 (v_F^0)^2} \frac{|\eta_{xx}(\omega)|^2}{\omega \eta_{xx}} \Phi J(y),
\]

where the factor \( J(y) \) is the same as for the inelastic contribution [Eq. \(570]\) and the factor \( \Phi = \Phi(\omega, \omega_c) \) has the form:

\[
\Phi = 4 \left\{ -a \omega_c \tau_2 + 2d (\omega_c/\omega)^2 \sin(2\pi \omega/\omega_c) + 
+ [-2b \omega_c \tau_2 + c_2] (\omega_c/\omega) \cos(2\pi \omega/\omega_c) \right\}.
\]

(19)

Here \( a, b, c, d \) are combinations of the coefficient \( \beta_{ij}^{\text{kips}} \) from Eq. \(3\) for the case of the Poiseuille flow geometry. We remind that the coefficients \( a \) in Eq. \(570\) as well as \( a, b, c, d \) in Eq. \(5108\) are proportional to the probability \( P = e^{-T_f/\tau_2} \) for two quasiparticles in a pair to make a complete cyclotron rotation without a collision with a third quasiparticle.

The profiles of the linear and nonlinear parts of the dc responses, \( V_1(y) \) and \( V_\omega(y) \), for considered relatively narrow samples are plotted in Fig. 1(c,d). The ac-field-induced correction to the dc velocity, \( V_\omega(y) \), inherits both the oscillations of \( V_\omega(y) \) by \( y \), related to standing shear waves [see Eq. \(13\) and Fig. 1(c)], as well as the parabolic profile of the dc Poiseuille flow \( V_0(y) \).

In Fig. 2 we plot the relaxation contribution to photoconductivity, \( \Delta \sigma_{\omega}^{\text{rel}} \sim V_\omega^{\text{rel}} \sim \Gamma_{ijkl} \), for the cases of wide, medium, and narrow samples. It is seen that the shape of magnetoooscillations of \( \Delta \sigma_{\omega}^{\text{rel}}(B) \) is regular for the wide and the narrow samples, while it is irregular for medium-size samples with the widths \( W \sim L_\omega \), \( W > l_s \). This property of \( \Delta \sigma_{\omega}^{\text{rel}}(B) \) reflects the formation of smeared standing waves inside the sample with relatively big amplitudes, which takes place when:

\[
\lambda(B) W \approx i \pi (1 + 2N_w), \quad N_w = 0, \pm 1, \pm 2, \ldots
\]

(20)

At such conditions the amplitude of the ac response \( V_\omega(y) \) exhibits the resonance [see Eq. \(19\)], and consequently \( V_\omega(y) \) and \( \Delta \sigma_{\omega} \) also exhibits positive or negative peaks [see Fig. 2(b)]. Note that the value \( \Delta \sigma_{\omega}^{\text{rel}} \) is not proportional just to the absorbed power \( W(B) \) neither for wide, nor for narrow samples (see the plot of \( V_\omega(B) \) in Fig. S3 of Supplemental material \[54\]), but is related with both the real and imaginary parts of \( V_1(y) \).

The magnitude of the calculated response strongly depends on magnetic field, frequency, and sample width. For example, let us make the estimate for the photoconductivity magnitude at the magnetic field and frequency corresponding to the regime: \( \omega_c \sim \omega \gg 1/T_2 \), \( |\text{Im}(\lambda)| \gg |\text{Re}(\lambda)| \), and the sample width in the intervals, \( |\text{Im}(\lambda)|^{-1} \ll W \ll |\text{Re}(\lambda)|^{-1} \). In this case, the standing shear waves are well formed. For the relaxational contribution in the limiting case \( F_1 \sim 1 \) \( (v_F \sim v_F^0) \) and far from the resonances appearing at condition \(511\), the formulas in Supplemental material \[54\] yield:

\[
\frac{\Delta \sigma_{\omega}^{\text{rel}}}{\sigma_0} \sim P(\omega_c) \left( \frac{eE_1}{m} \right)^2 \frac{\tau_2}{a_B^2 \omega^2} \omega^2.
\]

(21)

Here we imply that the magnitude \( E_1 \) satisfies the linear by \( E_1^2 \) regime, when \( |\Delta \sigma_{\omega}^{\text{rel}}| \ll \sigma_0 \). This result is based on consideration in \[54\] of the process of an extended collision in detail, providing the estimate for the parameters \( \alpha_{ijkl} \) in Eqs. \(7\) via the screening radius \( \sim a_B \). Note that there is no a direct dependence on the sample width in Eq. \(21\). The estimate of the relative photoresistance of the Ohmic samples, being analogous to the value \( \Delta \sigma_{\omega}^{\text{rel}}/\sigma_0 \) in a Poiseuille flow, within the model \[12\] of magnetotransport due to the memory effects at scattering of non-interacting electrons in smooth defects yields:

\[
\frac{\Delta \varrho_{xx,0}}{\varrho_{xx,0}} \sim P_{\text{imp}}(\omega_c) \left( \frac{eE_1}{m} \right)^2 \frac{1}{r_0^2 \omega^2}.
\]

(22)

Here \( \varrho_{xx,0} \) is the Drude dc conductivity, \( r_0 \) is the defect radius, \( P_{\text{imp}} \) is the probability for electron make a rotation without scattering on defects, being similar to \( P(B) \) \[52\], and \( r_{\text{imp}} \) is the impurity scattering departure time. We see from Eqs. \(21\) and \(22\) that for reasonable size of the defect radius, \( a_B \sim r_0 \), and at considerable scattering times, \( r_{\text{imp}} \sim r_0 \), in the hydrodynamic regime a much larger magnitude of the relative photoconductivity, \( \Delta \sigma_{\omega}^{\text{rel}}/\sigma_0 \), is attained than the similar value, \( \Delta \sigma_{\omega}/\sigma_D \), within the theory \[12\] for Ohmic flows. Finally, we note that in the hydrodynamic samples with some very low defect density at sufficiently large magnetic fields, when \( \omega_c \tau_2 \) is larger than some threshold value depending on sample width and the density of residual weak defects, the scattering of quasiparticles on these defects becomes important, and the flow acquires a mixed hydrodynamic-Ohmic character.

The resulting mean sample photoconductivity \( \Delta \sigma_{\omega} \) does not depend on the sign \( \pm \) of the circular polarization of the ac field \( E_1(t) \). Indeed, as we mentioned above, in narrow samples, \( W \ll l_p \), the ac linear response \( V_1(y,t) \), which is mainly formed by the viscoelastic contribution, is independent of the sign \( \pm \) due to the screening of the \( y \) component of the incident field \( E_1(t) \) by the internal field \( E^{\text{int}} \sim (d\delta n/dy) e_y \). Thus the nonlinear components of the velocity, \( V_{\omega}^{\text{rel}} \) and \( V_{\omega}^{\text{rel}} \), stemming from the retarded relaxation term, \( \sim \Gamma_{ijkl} \), and from the retarded elastic terms, \( \sim \delta F_{2,ij} \), are also independent of the polarization sign \( \pm \). So the independence of \( \Delta \sigma_{\omega} \) of the sign \( \pm \) is a necessary (but not sufficient) evidence of the formation of a high-frequency hydrodynamic flow.

In the regime linear by \( E_1^2 \) and \( E_0 \), the photoresistance \( \Delta \varrho_{\omega} = \varrho_{\omega} - \varrho_0 \) is proportional to \( -\Delta \sigma_{\omega} \) (here \( \varrho_{\omega} = 1/\sigma_{\omega} \) and \( \varrho_0 = 1/\sigma_0 \) are the sample mean resistivities at nonzero and at zero \( E_1 \)). Therefore in Fig. 2 and Fig. 4 below we have presented the value \( -\Delta \sigma_{\omega} \).
In Fig. 3(A) we plot the relaxational contribution to photoconductivity $\Delta \sigma_{\omega}^{rd}$ for smaller values of the parameters $\omega \tau_2$ and $W/L_s$ than in Fig. 2 (that is, for a “smoother” viscoelastic system; the used value of $\tau_v$ lies at the edge of the model applicability condition $1/\tau_2 \ll \omega \leq 2\pi/\tau_v$). It is seen that the magnetooscillations of $\Delta \sigma_{\omega}^{rd}$ are regular (sinusoidal with damping) in relatively wide samples: $W \gg L_s$, and become irregular (some oscillations have peculiar values of amplitudes) in the narrower samples, whose widths $W$ are comparable with the transversal sound decay length $L_s$. As like for the curves $\Delta \sigma_{\omega}^{rd}(B)$ in Fig. 2, these irregularities are manifestations of the “geometric” resonances related to coincidence of $W$ with a half-integer numbers of the magnetosonic wavelengths [see Eq. (S11)]. For the samples shown in Fig. 3(A) these resonances are much more smeared as compared with the ones shown in Fig. 2 because of not too large value of $\omega \tau_2$.

In Fig. 4(A) we plot the elastic contribution $\Delta \sigma_{\omega}^{el} \sim V_{\omega}^{el} \sim \delta F_{2,ij}^{(1)}$ to the photoconductivity $\Delta \sigma_{\omega}$ from the $F_2$-parameter memory term, $\sim \delta F_{2,ij}^{(1)}[V(t,t')]$, in equations (4). It is seen that this contribution, along with magneto-oscillations, exhibits a strong resonance at the doubled cyclotron frequency $\omega = 2\omega_c$. This is the viscoelastic resonance stemming from the resonance in $\eta_{xx}(\omega)$ and reflected in the dispersion law of magnetosonic waves [11]. In the calculated photoconductivity $\Delta \sigma_{\omega}^{el}$ it originates from the resonance at $\omega = 2\omega_c$ in the linear response $V(y,t)$ the interaction-induced term $-\delta F_{2,ij}^{(1)}[V(y,t-t')]\partial \Pi_{ij}/\partial t$.

## 5. DISCUSSION

In above consideration, we have implied the Landau Fermi-liquid model with the large parameter $F_1 \gg 1$ [see Eq. (1)] as the simplest model of a highly correlated electron liquid where the formulated equations (4) are surely applicable. In realistic GaAs quantum wells samples, which we will discuss in next section, such strong interaction is not apparently realized, therefore the Fermi-liquid starting model of Ref. [2] not directly applicable to them. However, the above phenomenological consideration of flow dynamics, based on solution of Eqs. (4), leads to reasonable results even in the limit $F_1 \ll 1$ (corresponding to $r_s \lesssim 1$ or $r_s \ll 1$), when the fluid parameters tends to their Fermi-gas values, in particular, $v^e_F \approx v_F$, $I_s \sim R_c \sim v_F/\omega$, and $L_s \sim v_F \tau \gg R_c$. Although shear magnetosonic waves are no longer formed, Eqs. (11)-(13) and (S75)-(S108) in this limit yield finite reasonable flow characteristics in the bulk region, $W/2 - |y| \geq R_c$ (provided $W \gg R_c$). Indeed, provided that there is no source of small-scale disturbances of sizes $\Delta x \ll R_c$ in a sample, the kinetic and hydrodynamic approaches can yield similar results [54, 74]. This situation corresponds to the edge of applicability of any hydrodynamic-like equations for the flow with the minimal scale: $\Delta x \sim R_c$.

A generalized model, being valid for any $F_m$, should be based on the kinetic equation for time-dependent distributions of quasiparticles, in which the hydrodynamic contribution as well as the ballistic contribution, containing the angular harmonics by the quasiparticle velocity of the third and higher orders, are comparable. Herewith non-linear memory terms in the collision integral and in the Landau interaction with any unperturbed parameters $F_m$ should be accounted. Instead of the standing shear waves, some quasi-periodical by 2$R_c$ distribution of the flow characteristics can be formed due to the ballistic effects of commensurability of $W$ and 2$R_c$ (see, for example, consideration of Ref. [73] for stationary flows).

As it was mentioned above [51], a quantum reconstruction of the ground and excited Fermi-liquid states of 2D electrons in magnetic field also can be very important for the form and the conditions of applicability of the macroscopic equations of sort of Eqs. (41) even at $r_s \lesssim 1$. Beside this, for particular flow parameters, the widest
conditions of literal applicability of our phenomenological model, based on solution of the balance equations \(4\), can include also the systems with \(r_s \lesssim 1\). For example, it was demonstrated in Refs. \cite{22, 23} that the electron hydrodynamics for stationary flows in pure samples at any inter-particle scattering rates, \(1/\tau_2\), is eventually forming with the increase of magnetic field, when \(W > 2R_c\).

From this discussion, the following conclusion can be made. In ultra-pure samples, even at small interaction parameter, \(F_1 \lesssim 1\), the 2\(\omega_c\)-resonance in viscosity coefficient \(12\) and the memory effects in Eq. \(4\) due to formation of quasiparticle pairs do retain. In bulk samples with the widths \(W \gg R_e, \nu_F/\omega\) or/and under some other conditions on the parameters \(R_e, \omega, \tau_q\), a kinetic equation model accounting the above two effects as well as ballistic effects, apparently, can lead to the electron fluid flows and photoresponses, being similar to the ones derived here on base of Eqs. \(4\).

6. COMPARISON WITH EXPERIMENT

In Fig. 3 we compare the calculated inelastic contribution in photoconductivity \(\Delta\sigma_{el}^{\exp}\) with experiments \cite{13} \cite{14} on measurements of the MIRO effect in high-quality GaAs quantum wells.

First, it is seen from panels \(B\) and \(C\) of Fig. 3 that for the structure with the lower “experimental averaged sample mobility” (sample \#A in panel \(C\)) there is some substantial dependence of the MIRO-like photoconductivity on the polarization sign of radiation, whereas this dependence is almost absent for MIRO (panel \(B\) and for the MIRO-like photoconductivity (sample \#B in panel \(C\)) in the ultra-high and the moderately high mobility samples. Second, it is also seen that for the lower-mobility sample the shape of oscillation is regular (sinusoidal with a slow damping with the increase of \(1/B\)), whereas for the two higher-mobility samples the shape of oscillations is irregular: the amplitudes of some oscillations are too large or too small, but the other oscillations are sinusoidal with a slow damping with \(1/B\) (see the bottom subpanels of the both panels \(B\) and \(C\)). The irregular shape of magnetooscillations, as it was discussed above, is explained within our theory by the formation of smeared standing magnetosonic waves inside the sample.

The independence of \(\Delta\sigma_{\omega}\) of the polarization sign \(\pm\) and the irregularities of oscillations were unexplained within theories \(4\) \cite{8} \cite{11} \cite{12} for independent electrons in bulk disordered samples.

In the samples studied in Ref. \cite{13} and Ref. \cite{14} the 2D electron densities were \(2.3 \cdot 10^{11}\) cm\(^{-2}\) and \(12.0, 9.3 \cdot 10^{11}\) cm\(^{-2}\) (for samples \#A, \#B). Such densities lead to \(r_s = 1.1, 0.50, 0.56\) and, apparently, to not large values of the Landau parameters \(F_1\). So the sufficient conditions \(4\) of the big magnitude of the Landau interparticle interaction, which surely leads to the developed viscoelastic model, seem to be not fulfilled. However, provided some most general conditions of applicability of Eqs. \(4\), discussed in Section 5, can be met for the flow regimes realized in experiments \cite{13} \cite{14}, the explanation of the photoresponses observed in them within our theory can evidence about formation of flows of a viscous electron fluid with the strong memory effects.

In Fig. 4 we compare the calculated elastic contribution to photoconductivity, \(\Delta\sigma_{el}^{\exp}(B)\), with the magneto-photoresistance measured in the GaAs quantum wells of record quality \cite{18} \cite{19} and exhibiting a strong peak at \(\omega = 2\omega_c\). Rather similar results on observation of magneto-photoresistance with the strong \(\omega = 2\omega_c\)-peak were obtained in Ref. \cite{22} \cite{23} for high-quality graphene samples.

In experimental data shown in panels \(B\) and \(C\) of Fig. 4 a large narrow peak is observed near the doubled cyclotron frequency, \(\omega = 2\omega_c\), and much smaller peculiarities (or oscillations) at higher cyclotron harmonics.

![Image](image-url)
It is noteworthy that the stronger is the negative magnetoresistance, the sharper and bigger is the peak in magnetophoto-resistance (see discussions in [41, 54]). It is seen from experiments as well as from the theoretical model that, depending on the specific parameters of the sample and the electron fluid, the resonance changes its shape. In other ultra-high-quality graphene and GaAs quantum well samples the $2\omega_c$-resonance can disappear, depending on the sample width [21, 23]. In lower magnetic fields, $\omega_c < \omega/2$, magnetoooscillations appear on the theoretical curves, qualitatively corresponding to peculiarities on the experimental curves (see Fig. 4). Note that there are no resonance at $\omega = 2\omega_c$ in the relaxation contribution $\Delta\sigma_{rel}^{el}$ [see Figs. 2 and 3(A)].

The magnitudes of the cited in Fig. 4 experimental data on the resistance change due to radiation, $\rho_{\omega} - \rho_0$, are relatively large, comparable with $\rho_0$ at $B = 0$. As we have described in Section 4, we perform the calculations of $\rho_{\omega} - \rho_0$ in the lowest, $E_{rel}^2$, order by the ac field amplitude. So it is important for comparison of theory and experiments that in Ref. 19 an eventual appearance of the $2\omega_c$-resonance in photoresistance with the increase of radiation power was observed and the measurements of the dependance of its amplitude on $E_{rel}^2$ were performed. The resonance appears already at relatively small $E_1$, when $|\rho_{\omega} - \rho_0| \ll \rho_0$, and the value $(\rho_{\omega} - \rho_0)$ turns out linear by $E_{rel}^2$ (see Fig. 3 in Ref. 19).

In recent experiment [21] on ultra-pure GaAs quantum well samples of various widths the giant negative magnetoresistance and the peak in photoresistance near $\omega = 2\omega_c$, similar to the ones presented at Fig. 4, were observed. For medium sample widths and at low radiation powers, the irregular shape of MIRO and the giant $2\omega_c$-peak at were seen very well {see Fig. 4(b) in Ref. 21].

In this way, with the same precaution about the strength of interparticle interaction in the discussed GaAs and graphene samples, the appearance of a large distinct peak at $\omega = 2\omega_c$ in magnetoo-photo-resistance of GaAs and graphene samples can be related to the viscoelastic resonance in the values $V_1$ and $\Delta\sigma_{rel}^{el}$.

7. CONCLUSION

We have proposed and developed a simple phenomenological theory of non-linear magnetotransport in the highly correlated 2D electron fluid. The $2\omega_c$-resonance in the viscosity coefficient and the ac memory effects in the inter-particle interaction are crucial in the theory. The sufficient conditions of the applicability of the developed theory is the large magnitudes of Landau interaction parameters for the considered 2D electron systems. The calculated photoconductivity exhibits an independence on the helicity of the polarization of incident radiation, irregular magnetoooscillations, and a large peak at the doubled cyclotron frequency. Although the most general conditions of applicability of the developed theory are not fully understood now, observation of all these effects on best-quality GaAs quantum wells can be the evidence of formation of a highly correlated 2D viscous electron fluid in such structures and of suitability of our model (or it close analog) for their description.

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Supplemental material to the manuscript “Highly correlated two-dimensional viscous electron fluid in moderate magnetic fields”

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Here we present the details of our theoretical model of the highly correlated electron fluid, of the solution of its equations, and of derivation of the properties of the resulting photoconductivity. We discuss possible extensions of our theoretical model to more general hydrodynamic-like systems in disordered samples. We also compare our results with preceding theoretical works and the related experiments of photo-magneto-transport in the best-quality GaAs quantum wells and graphene.

1. Memory effects in scattering of 2D non-interacting electrons on localized defects

Memory effects in classical ac magnetotransport of non-interacting 2D electrons in disordered samples were considered, for example, in Refs. [11, 12, 55–57, 60].

For the scattering of 2D electrons on defects with a localized potential (“impurities”), the memory effects in a perpendicular magnetic field $\mathbf{B}$ are due to the appearance of (i) the electrons not scattering on defects (their trajectories are located between the impurities) and of (ii) the so-called “extended collisions”. The last events consist of several returns of an electron to the same impurity after a first scattering on it because of cyclotron rotation (see Fig. 2 in Ref. [12]). Both these effects (i) and (ii) become substantial in the classically strong magnetic fields, when the electron mean free path relative to the scattering on defects becomes comparable with or longer than the length of the cyclotron circle [55, 56].

Within the approach of Ref. [12], such events are accounted in a phenomenologically transparent way by the retardation term $-\hat{\Gamma}(t)(\mathbf{V}(t-T))$ in the Drude-like equation for the mean velocity $\mathbf{V}(t)$:

$$\frac{\partial \mathbf{V}}{\partial t} = \frac{e}{m} \mathbf{E}(t) + \left[ \mathbf{V} \times \omega_c \right] - (1 - P) \left[ \frac{\mathbf{V}}{\tau_{tr}} + \hat{\Gamma}(t) \mathbf{V}(t-T) \right],$$  

(S1)

where $e$ and $m$ are the electron charge and the electron mass, the electric field $\mathbf{E}(t)$ can contain dc and ac components; $\omega_c = \omega_0 e_z$ is the vector along the magnetic field, $\omega_0 = eB/(mc)$ is the electron cyclotron frequency, $\tau_{tr}$ is the momentum relaxation time due to the scattering on disorder in the absence of the memory effect,

$$P = e^{-T/\tau_{tr}}$$  

(S2)

is the probability for an electron to make a full cyclotron rotation without collisions with defects, $\tau_{tr}^{def}$ is the departure scattering time due to collisions with defects, $T = 2\pi/\omega_c$ is the cyclotron period, and $\hat{\Gamma}(t)$ is the retarded relaxation tensor due to double extended collisions.

The tensor $\hat{\Gamma}(t)$ depends on the dynamics of individual electrons in the past, $t' < t$. At sufficiently weak flows, such dependence is accounted by a nonlinear contribution in $\hat{\Gamma}$ [being additional to the unperturbed value $\hat{\Gamma}^{(0)}$ corresponding to the limit $E, V \to 0$]. This contribution is proportional to the vector $\Delta$ characterizing the deviation of the electron trajectories $\mathbf{r}_0(t)$ from the exact cyclotron circles:

$$\Gamma_{ij}(\Delta) = \Gamma^{(0)}_{ij} + \alpha \Delta^2 \delta_{ij} + \beta \Delta_i \Delta_j .$$  

(S3)

Here the coefficients $\hat{\Gamma}^{(0)}$, $\alpha$, and $\beta$ are the microscopic characteristics of the electron gas and the defects, being proportional to the probability $P$ [S2]. The vector $\Delta$ for an Ohmic flow is the mismatch $\Delta(t) = \mathbf{q}(t) - \mathbf{q}(t-T)$ of the impact scattering parameters $\mathbf{q}(t) = \mathbf{r}_0(t) - \mathbf{R}_0$ and $\mathbf{q}(t-T) = \mathbf{r}_0(t-T) - \mathbf{R}_0$ of an electron in two successive collisions with the same defect [here $\mathbf{R}_0$ is the position of the defect center; it follows from the above formulas that $\Delta(t) = \mathbf{r}_0(t) - \mathbf{r}_0(t-T)$]. Such form of relation [S3] of the part of the tensor $\hat{\Gamma}$ related to the perturbation of the trajectories between collisions is valid when the size of defects is small: $a \ll R_c$. Provided this inequality, the mismatch $\Delta(t)$ is directly related with the forces from the dc and the ac electric fields $\mathbf{E}_0$ and $\mathbf{E}_1(t)$ acting on an electron during a cyclotron period $t - T < t' < t$. Herein the approximation linear by $\mathbf{E}_0$ and $\mathbf{E}_1(t)$, the corresponding value of the mismatch:

$$\Delta(t) = \Delta_0 + \Delta_1(t) ,$$  

(S4)

does not depend on particular parameters of a trajectory $\mathbf{r}_0(t)$.

Such model allowed to analytically calculate the photoconductivity of non-interacting 2D electrons in bulk samples [12]. The obtained magnetooscillations of the photoconductivity are induced by the described extended collisions and are similar in many properties to the ones of MIRO observed in experiments.
2. Memory effects in relaxation due to interparticle scattering in highly viscous 2D electron fluid

Below we construct phenomenological dynamic equations for 2D interacting electrons forming a viscous fluid in samples with no defects (or, more generally, in the sample regions where defects are not substantial) in classical magnetic fields. On the one hand, these equations are based on the Landau Fermi-liquid model which is a simplest model accounting strong many-particle correlation effects in electron systems. On the other hand, the formulated below equations contains similar memory terms to the ones presented in previous Section 1 for non-interacting electrons in sample with localized defects.

For a viscous electron fluid, the motion equations describes the mean hydrodynamic velocity $\mathbf{V}(r, t)$, the perturbed electron density $n(r, t)$ and the stress tensor $\sigma(r, t)$. The hydrodynamic Navier-Stokes-like equations of evolution of these values for a viscous 2D electron fluid without accounting for the memory effects were formulated and derived in Refs. [29, 40–45, 58, 59]. In a strongly non-ideal electron liquid, the viscoelastic effects (first of all, propagation of the shear-stress waves) become possible within those hydrodynamic equations, and such fluid can be referred as highly viscous electron fluid [77]. For a high-frequency flow in such system, the Navier-Stokes-like equations are transformed into Hooke’s equations of an almost elastic dynamics of an electron media with a weak damping due to the interparticle scattering [42, 43].

In Hooke’s equations, the only physical value that characterizes the state of the system is the displacement vector $\mathbf{u}(r, t)$, related to the strain tensor $\varepsilon_{ij}(r, t)$:

$$
\varepsilon_{ij} = \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i}.
$$

(S5)

The connection between Hooke’s and the Navier-Stokes equations is based on the following relation for the hydrodynamic velocity in an almost fully elastic flow:

$$
\mathbf{V}(r, t) = \frac{\partial \mathbf{u}(r, t)}{\partial t}.
$$

(S6)

This formula reflects the absence of slipping between neighbor layers of the flow.

Generally, the dynamics of a viscous fluid with taking into account the interparticle scattering, resulting in slipping between fluid layers, is described by all the three independent variables: $n$, $\mathbf{V}$, and $\sigma_{ij} = -\Pi_{ij}$. Equation (S5) in this case is a formula for calculation, in the linear by $\mathbf{V}$ approximation, of the displacement $\mathbf{u}(r, t)$ of fluid elements. Note that the value $\mathbf{u}$ for a flow with the slipping of neighbour layers is not the proper variable sufficient for the description of the fluid dynamics.

Analogously to the dynamics of independent electrons in a sample with small-size defects, the memory effects for the electron fluid in a defectless sample in a magnetic field are due to the “extended collisions” between electron-like Fermi-liquid quasiparticles [see Fig. S1(a) and Fig. 1 in the main text]. Indeed, if the mean free path of quasiparticles relative to their collisions is of the order of the length of the cyclotron circle $2\pi R_c$, two quasiparticles can suffer several successive collisions with slow subsequent changes of their relative impact scattering parameter [see Fig. S1(a)]: $q(t') = r_{0,1}(t') - r_{0,2}(t')$, $t' \approx t - N_r T$. Here $N_r = 1, 2, \ldots$ is a number of successive rotations in an extended collision.

Such events lead to the dependence of the relaxation rate of the momentum flux, $\partial \Pi/\partial t$, in the moment $t$ on the characteristic of the fluid in the moments $t' = t - N_r T$ of previous scattering events. The physical reason for the dependence of the fluid relaxation rate at the present moment $t$ on the fluid characteristics at previous moments, $t' < t$, consists in a rather long collisionless motion of quasiparticles before moment $t$. Indeed, when describing the motion of a fluid by a one-particle distribution function and the corresponding quantities $\mathbf{V}(r, t)$ and $\Pi(r, t)$, their average values, used in the theory, are actually formed after last collisions of quasiparticles, that is, at times $t' < t$, significantly preceding moment $t$. Accordingly, in the interval $t - (N_r + 1)T < t' < t - N_r T$ the evolution of the fluid is quasi-deterministic. Such motion type is described by the retarded terms resulting from averaging of many determinististic electron trajectories.

The corresponding phenomenological dynamic equations of the fluid, extending the viscoelastic equations from Refs. [42, 43] in order to account only double extended collisions ($N_r = 1$), can be formulated as follows:

$$
\frac{\partial n}{\partial t} + n_0 \text{div} \mathbf{V} = 0
$$

$$
\frac{\partial V_i}{\partial t} = \frac{e}{m} E_i(r, t) + \omega_c \varepsilon_{ikz} V_k - \frac{1}{m} \frac{\partial \Pi_{ij}}{\partial x_j} \frac{\partial \Pi_{ij}}{\partial t} + \frac{\delta F_{2,ij}(r,t)/(1 + F_2)}{1 + \delta F_{2,ij}(r,t)/(1 + F_2)} = 2\omega_c \varepsilon_{ikz} \Pi_{kj} - \frac{\Pi_{ij}}{\tau_2} - \frac{m(v_0^2)}{4} V_{ij} - \Gamma_{ijkl}(r, t) \Pi_{kl}(r, t - \tau_2).
$$

(S7)

Here $n_0$ is the quasiparticle characteristic being analogous to the equilibrium density $n_0 = n - \delta n$ in the case of a Fermi gas; $e$ is the electron charge; $m$ is the electron renormalized mass; the electric field $\mathbf{E}(r, t)$ contains the components from dc and ac external fields induced by the electric bias and the microwave radiation as well as from the internal dc and ac field related to the non-equilibrium charge density $e \delta n(r, t)$: $\mathbf{E}(r, t) = \mathbf{E}_0(r, t) + \mathbf{E}_1(r, t)$,

$$
\mathbf{E}_0(r, t) = E_0^{\text{ext}}(r) + E_0^{\text{int}}(r),
$$

$$
\mathbf{E}_1(r, t) = E_1^{\text{ext}}(t) + E_1^{\text{int}}(r, t);
$$

(S8)
FIG. S1. (a) Sketch of the microscopic mechanism of the memory effect in collisions of electron-like quasiparticles in a 2D electron Fermi-liquid in a classical magnetic field \( \mathbf{B} = B \mathbf{e}_z \). Motion of the fluid, averaged by dynamics of quasiparticles, is described by the hydrodynamic velocity \( \mathbf{V}(r, t) \). It contains the components which are linear by the applied electric fields, \( \mathbf{V}_0(r) \sim E_0 \) and \( \mathbf{V}_1(r, t) \approx \partial \mathbf{u}(r, t) / \partial t \sim \mathbf{E}_1(t) \), as well as the nonlinear by \( \mathbf{E}_0 \) and \( \mathbf{E}_1(t) \) components. Red solid lines are the quasiparticle trajectories at \( \mathbf{E}_0 \), \( \mathbf{E}_1 \), \( \mathbf{V} \equiv 0 \), which are exact cyclotron circles. Blue dashed lines are the trajectories accounting the elastic force, microscopically related to space-dependent perturbations of the quasiparticle energy spectrum and leading to the term \( \sim V_i(r, t) \) in Eq. (S7), as well as to the forces from the full dc and the ac fields \( \mathbf{E}_0(r, t) \) and \( \mathbf{E}_1(t) \) [S5]. The profiles of \( \mathbf{V}(r, t) \) and \( \mathbf{u}(r, t) \) are changing weakly by \( r \) on the scale of \( R \). The value of the relative impact scattering parameter, \( g(t') = r_{0,1}(t') - r_{0,2}(t') \), at the moment \( t' = t - T \) differ from its value at \( t' = t \) on the difference \( \delta(t) = \delta_1(t) - \delta_2(t) \) of the trajectory mismatches by a period: \( \delta_1(t) = r_{0,1}(t) - r_{0,1}(t - T) \), \( p = 1, 2 \). The values \( \delta_1(t) \) are estimated as the mean shifts of the whole fluid in the centers \( r_p \) by a period: \( \delta_p(t) \sim [\mathbf{u}(r_p, t) - \mathbf{u}(r_p, t - T)] \).

(b) The screened Coulomb energy \( U_C(g(t)) \) with the characteristic screening radius \( a_B \) two probe interacting quasiparticles in the time moments \( t < 0 \) preceding the current collision at \( t = 0 \) at some nonzero ac field \( \mathbf{E}_1(t) \) with \( \omega = 2.4 \omega_c \) (schematic). Green and red curves correspond to a larger and a smaller distances \( g(t) \sim a_B \) between the two electrons [the value \( g(t) = r_{0,1}(t) - r_{0,2}(t) \) is the two-particle scattering parameter, \( a_B \) is the character radius of Debye screening in 2D case being the Bohr radius]. It is seen that the interaction energy for a given pair of quasiparticles has maximums at the moments \( t_0 = -NT \), where \( N = 0, 1, 2 \), and so on. The local maximums \( U_C(g(t)) \) are nonmonotonic with the increase of \( N \) due to the effect from the ac field \( \mathbf{E}_1(t) \) in the electron trajectories \( r_{0,1/2}(t) \) [see panel (a)].

\( \epsilon_{ikm} \) is the antisymmetric unit tensor; \( V_{ij} = V_{ij}(r, t) \) is the tensor of the gradients of the velocity \( \mathbf{V} \):

\[
V_{ij} = \frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} ;
\]

\( \tau_2 \) is the shear stress relaxation time without the memory effects; the parameter \( \nu^2_F \gg \nu_F \) defines the viscosity \( \eta \) of a highly viscous fluid at zero magnetic field [41]:

\[
\eta_0 = n_B = (\nu_F^2) \tau_2 / 4 ;
\]

\( \Gamma_{ijkl}(r, t) \) is the tensor describing the retarded relaxation of \( \Pi_{ij} \) due to the extended collisions [see Fig. S1(a)]; \( T = 2 \pi / \omega_c \) is the cyclotron period; and \( \delta F_{2,ij}(r, t) \) are the perturbations of the coefficient \( F_2 \) of the Landau function. The values \( \delta F_{2,ij} \) describe the effect of the viscous motion of the fluid on the quasiparticle energy spectrum in a non-linear by \( V_i \) and \( F_{pk} \) approximation.

The Navier-Stokes-like non-stationary equations of the 2D electron liquid at a strong interparticle interaction without the memory effects were derived in Refs. [43–45] for zero magnetic field and in Ref. [42] for a nonzero classical magnetic fields. Those equation are Eqs. (S7) with omitted the \( F_{2,ij} \)– and \( \Gamma_{ijkl} \)-terms. Their criteria of applicability can be formulated as follows [41, 42]:

\[
F_1 \geq 1 , \quad \text{and} \quad F_m \ll F_1 \ \text{at} \ \ m \geq 3 ;
\]

\( R_c \ll \Delta x \) or \( \nu_F / \omega \ll \Delta x \),

where \( F_m \) are the angular harmonics of the Landau interaction function \( F_{p,p'} \); \( \Delta x \) is the characteristic space scale of flow inhomogeneities; \( R_c = \nu_F / \omega_c \) is the cyclotron radius of electron-like quasiparticles; and \( \nu_F \) is the Fermi velocity. Provided conditions (S11) are fulfilled, the parameter \( F_2 \) can be moderate, small, or large.

In view of a phenomenological qualitative type of our theory, we do not presented in Eq. (S7) the exact Fermi-liquid renormalizations of all the parameters \( \nu_F, \ m , \ \omega_c , \ \tau_2 \). They were studied and presented in Ref. [42] at the absence of memory effects. In this connection, we make some related simplifications: do not distinguish between the two cyclotron frequencies in equations for \( \partial \mathbf{V} / \partial t \) and \( \partial \mathbf{P} / \partial t \); make the change:

\[
\delta F_{2,ij} / (1 + F_2) \rightarrow \delta F_{2,ij} ;
\]

and keep only the renormalization of the viscosity amplitude via \( \nu^2_F \gg \nu_F \) [as it is substantial for applicability of Eqs. (S7) for description of the shear stress waves].

Apparently, the consideration of Refs. [41, 42] and extending them equations (S7) are qualitatively valid also for a moderately non-ideal electron liquid, in which the Landau parameters \( F_m \) are greater than the critical values \( F_{m,c} \sim 1 \), when the shear-stress mode becomes possible in zero magnetic field [75, 76].
In magnetic field, the strong quantum many-particle correlation effects, namely a reconstruction of the Fermi-liquid ground state and quasiparticles of 2D interacting electrons, should appear. The electron systems becomes substantially different from the conventional Fermi-liquid. This may be a quantum form of description of interparticle pair correlations related to the extended collisions with classical mechanics picture [see Fig. S1]. For example, the effects of reconstruction of the ground and excited states for 2D electrons in quantizing magnetic fields were studied in Refs. [52, 53]. Similar reconstruction in moderate magnetic fields can change the structure of the macroscopic Navier-Stokes-like equations of the type of [S7] and substantially change the applicability condition $F_1 \gg 1$ [S11].

Finally, we note that the resulting formulas for the flow characteristics, which we will obtain below from Eq. (S7), correspond to the Fermi gas, and their solutions vary on scales $\Delta x$, which is the edge of applicability of any hydrodynamic-like equations:

$$\Delta x \sim R_c \sim v_F/\omega.$$  

(S13)

This may point out, that, even for weakly non-ideal Fermi gas, the consideration based on the distribution function of quasiparticles and solution of the kinetic-equation, at least at some flow parameters, leads to the results being similar to ones following from Eqs. (S7). Indeed, for example, for stationary flows in magnetic fields, electron hydrodynamics is realized at any inter-particle scattering rates and values of $F_1$ in the bulk regions of sufficiently wide samples, $W \gg 2R_c$ [72, 73]. Moreover, if a system does not contain objects with sizes smaller than the minimum microscopic length, say, $R_c$ or $l_2 = v_F\tau_2$, then the kinetic and hydrodynamic descriptions often give qualitatively similar flow distributions with minimal scales $R_c$ or $l_2$ (see discussion in Ref. [74] and references therein).

Below in Section 2 we construct the relaxational contribution to the memory effects, described by the term $\tilde{\Gamma}_{ijkl}(r, t)\Pi_{kl}(r, t - T)$. The elastic contribution to the memory effects, described by the term $\sim \delta F_{2,ij}(r, t)$, will be studied in next Section 3.

Analogously to the coefficients $\alpha$ and $\beta$ in Eq. (S3), the memory coefficients $\delta F_{2,ij}$ and $\Gamma_{ijkl}$ in Eq. (S7) should be proportional to the probability to make a cyclotron rotation of a quasiparticle in magnetic field without collisions with other quasiparticles:

$$P = e^{-T/\tau_0},$$  

(S14)

where the probability $P$ is not too close to unity, $(1 - P) \sim 1$. For such $\omega_c$, in rough approximation, we can replace the factor $(1 - P)$ by unity [it could appear in the memory terms in Eqs. (S7) and describes the probability of any scattering for a particle during one cyclotron rotation; compare Eqs. (S1) and (S7)]. Apparently, in diapason (S15) the quantization of the density of states and appearance of many returns, $N_r \gg 1$, in extended collisions (and/or more complex not-pair correlation effects) are not substantial. Therefore, we will consider it as the range of applicability by magnetic field of the developed theory of the fluid dynamics.

The characteristic radius $a_2$ of the interparticle interaction potential is much smaller than the cyclotron radius. In this case, each extended collision consists of two (or several) successive collisions of two quasiparticles in a small region [a blue star in Fig. S1(a)] and of almost collisionless motions of these two quasiparticles far from the region of scattering. This motion is mainly determined by the magnetic field, but is also substantially affected by the full electric field $E(r, t)$ (S8) as well as by the elastic force, related to the space-dependent change of the quasiparticle energy spectrum (42, 43) and leading to the term

$$- \left[ m(v_F^2)^2 / 4 \right] V_j(r, t)$$  

(S16)

in Eq. (S7). The elastic force and the components $E_{0i}^{\text{int}}$ and $E_i^{\text{int}}$ of field (S8) are inhomogeneous by $r$ and are determined by the formation of the flow in the whole sample. These inhomogeneities lead to the difference of the full forces acting on two quasiparticles “1” and “2” [see Fig. S1(a)]. This difference defines the dependence of the probability of the extend collisions on the flow magnitude and shape in the current moment and in the past. For example, the probability to return to two quasiparticles is much greater for a stationary homogeneous flow than for a fast ac flow, in which the memory about positions of two quasiparticles can be lost with a large probability in one cyclotron rotation [see Fig. S1(a)].

To describe such memory effects quantitatively, one should use the Boltzmann-like kinetic equation with a generalized collision operator of the inter-quasiparticle scattering, being nonlocal by time. Such collision operator, leading to the motion equation of the type of Eqs. (S7), could be derived within some sophisticated procedure from the classical or the quantum Liouville equations for the interacting Fermi-liquid electron-like quasiparticles. Herewith some other retarded terms in the resulting effective transport equations, except the term $-\tilde{\Gamma}_{ijkl}(r, t)\Pi_{kl}(r, t - T)$ accounted in Eq. (S7), may be possible, for example: $A_{ij}E_i(r, t - T)$, $B_{ijkl}E_j(r, t - T)\Pi_{kl}(r, t - T)$, and so on [61]. However we hope that the term $-\tilde{\Gamma}_{ijkl}(r, t)\Pi_{kl}(r, t - T)$ is sufficient to account the main part of the memory effects in interparticle collisions and below describe its origin and estimate its magnitude.
Quantitatively, the extended inter-particle collisions are characterized by the following values. Two successive collisions are controlled by the impact relative scattering parameter of particles “1” and “2” [see Fig. S1(a)]:
\[ \mathbf{g}(t') = \mathbf{r}_{0,1}(t') - \mathbf{r}_{0,2}(t'), \ t' = t, t - T. \] (S17)
During one cyclotron period \( t - T < t' < t \), two scattered quasiparticles are moving along the trajectories \( \mathbf{r}_{0,1}(t) \) and \( \mathbf{r}_{0,2}(t) \) with the centers near the points \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \). Their impact scattering parameter \( \mathbf{g}(t') \) (S17) differs in the moments \( t' = t \) and \( t' = t - T \) on the value \( \delta(t) = \delta_1(t) - \delta_2(t) \), where \( \delta_1(t) \) and \( \delta_2(t) \) are the mismatches of the trajectories \( \mathbf{r}_{0,1}(t) \) and \( \mathbf{r}_{0,2}(t) \) by a cyclotron period:
\[ \delta(t) = \mathbf{g}(t) - \mathbf{g}(t - T) = \delta_1(t) - \delta_2(t), \] (S18)
\[ \delta_p(t) = \mathbf{r}_{0,p}(t) - \mathbf{r}_{0,p}(t - T), \ p = 1, 2. \]

Nonzero values of the mismatches \( \delta_p(t) \) are induced by the action of the elastic force \( V \) and the electric fields \( \tilde{F} \), as it was discussed above [see also Fig. S1(a)].

In accordance with the essence of the extended collisions, the tensor \( \Gamma_{ijkl}(r, t) \) in the retarded relaxation term in Eq. (S7) is determined by the mismatches \( \delta(t) \) (S18) of the scattering parameters \( \mathbf{g}(t') \) (S17), averaged by all the quasiparticle pairs near the given point \( r \).

The averaged mismatch \( \langle \delta(t) \rangle \) is apparently expressed via the macroscopic variables of the fluid in the past, generally speaking, via all variables: \( n, V, \tilde{F} \). Thus the tensor \( \Gamma_{ijkl}(r, t) \) depends on these variables as an operator:
\[ \Gamma_{ijkl}(r, t) = \hat{\Gamma}_{ijkl}[n(r', t'), V(r', t'), \tilde{F}(r', t')], \] (S19)
where \( |r - r'| \lesssim R_c \) and \( t - T \lesssim t' < t \).

We consider high-frequency regime when that inter-particle are rare as compared with cyclotron period. \( \omega, \omega_c \gg 1/\tau_2 \). In this case, a collision of one of two particles during an extended collision with a third particle is a relatively rare event, thus \( P \sim 1 \). Thus the only macroscopic variable determining the mismatches \( \delta_1(t) \) and \( \delta_2(t) \) is the displacement vector \( \mathbf{u}(r', t') \) in the centers of the particle trajectories “1” and “2” (see Fig. S1):
\[ \delta_p(t) \sim \mathbf{u}(r_p, t) - \mathbf{u}(r_p, t - T), \ p = 1, 2, \] (S20)
where \( r_p \) are the centers of the two neighbor unperturbed trajectories. For the resulting dependence of \( \Gamma \) on the shape and the magnitude of the flow we have:
\[ \hat{\Gamma}_{ijkl}[\mathbf{u}(r', t')] = \Gamma_{ijkl} \left( \langle \delta \mathbf{u}(r_1, r_2, t) \rangle_{r_1, r_2} \right), \] (S21)
where the brackets \( \langle . \rangle_{r_1, r_2} \) denote the averaging by \( |r - r_p| \lesssim R_c, p = 1, 2, \) and the vector \( \delta \mathbf{u} \) is:
\[ \delta \mathbf{u}(r_1, r_2, t) = [\mathbf{u}(r_1, t) - \mathbf{u}(r_1, t - T)] - [\mathbf{u}(r_2, t) - \mathbf{u}(r_2, t - T)]. \] (S22)
Here the displacement \( \mathbf{u}(r, t) \) can consist of the ac near-elastic as well as the dc dissipative parts, both related to the ac and dc components of \( V(r, t) \) by Eq. (S6).

In the hydrodynamic regime, the characteristic space scale of the inhomogeneity of the flow, \( W \), is much larger than \( R_c \), therefore we have:
\[ u_n(r_1, t') - u_n(r_2, t') \approx (x_{1n} - x_{2n}) \frac{\partial u_m(r_1, t')}{\partial x_n}, \] (S23)
where the summation by \( n \) is supposed. Thus the expression \( \delta \mathbf{u}(r_1, r_2, t) \) (S22), averaged by \( r_1 \) and \( r_2 \), is expressed via the shift (the “mismatch”) \( \Delta_{mn}(r, t) \) of the strain tensor \( \varepsilon_{mn}(r, t) \) in a cyclotron period. For the resulting \( \Gamma \) we obtain:
\[ \hat{\Gamma}_{ijkl}[\mathbf{u}(r', t')][r, t] = \Gamma_{ijkl} \left( \Delta_{mn}(r, t) \right), \] (S24)
\[ \Delta_{mn}(r, t) = \varepsilon_{mn}(r, t) - \varepsilon_{mn}(r, t - T). \]

Expansion of \( \Gamma_{ijkl} \) by \( \Delta_{mn} \) up to the quadratic order is:
\[ \Gamma_{ijkl}(\Delta_{mn}) = \Gamma_{ijkl}^{(0)} + a_{ijkl}^{mnop} \Delta_{mn} \Delta_{op}, \] (S25)
where the tensors \( \Gamma_{ijkl}^{(0)} \) and \( a_{ijkl}^{mnop} \) are determined by the parameters of the fluid and depend on temperature and magnetic field. The linear term \( a_{ijkl}^{mnop} \Delta_{mn} \) is absent in Eq. (S25) due to the symmetry of the relaxation processes relative to the inversion of the direction of flows.

From Eq. (S6) we obtain the expression for the mismatch of the strain tensor via the velocity gradient \( V_{mn}(r, t') \):
\[ \Delta_{mn}(r, t) \approx \int_{t - T}^{t} dt' \ V_{mn}(r, t'). \] (S26)

Here, similarly to Eqs. (S5), the tensor \( V_{mn} \) can contain both the ac component corresponding to an almost elastic dynamics and the dc component corresponding to a dissipative viscous motion.

One should expect that the tensors \( \Gamma_{ijkl}^{(0)} \) and \( a_{ijkl}^{mnop} \) are proportional to the probability \( P \) (S14) [or, possibly, \( P^2 \)] to make a full rotation for a quasiparticle in a pair without collisions with other quasiparticles and to some interparticle scattering rate, apparently, the rate of the relaxation of the shear stress \( 1/\tau_2 \).

According to Fig. S1(a), the relative value of the correction to the probability of an extended collision from the deviation of the trajectories \( r_{0,p}(t) \) from exact cyclotron circles is proportional to the squared ratio \( |\delta(t)|/\Delta B \ll 1 \). Here the value \( \delta(t) = \delta_1(t) - \delta_2(t) \),
\[ |\delta(t)| \sim ||\Delta \mathbf{r}(r, t)|| R_c \] (S27)
is the characteristic difference of the mismatches of the trajectories of two quasiparticles near the point \( r \) at the moments \( t \) and \( t - T \), and \( \Delta B \) is the Bohr radius, being.
the estimate for the size of the interparticle interaction potential.

In this way, from inequality $|\delta(t)|/a_B \ll 1$ we obtain of the condition of applicability of the memory terms:

$$V_{\text{char}} TR_c/\Delta x \ll a_B,$$

where $V_{\text{char}}$ is the character flow amplitude and $\Delta x$ is its character scalescale. Equation (S28) is the condition on the the electric fields $E_{0,1}$ determining the magnitude of $V_{\text{char}}$. According to the definition of the tensors $\Gamma^{(0)}_{ijkl}$ and $\alpha_{ijkl}^{mnop}$ in Eq. (S25), we obtain from Eq. (S27):

$$\Gamma^{(0)}_{ijkl} = A_{ijkl} \frac{e^{-T/\tau_q}}{\tau_2},$$
$$\alpha_{ijkl}^{mnop} = B_{ijkl}^{mnop} \frac{R_c}{a_B^2} \frac{e^{-T/\tau_q}}{\tau_2},$$

(S29)

where $A_{ijkl}$ and $B_{ijkl}^{mnop}$ are the numeric constants independent on the fluid parameters. We remind that for the case of the electron scattering on soft impurities, analogous formulas for the memory constants $\alpha$ and $\beta$ in the tensor (S29) were derived in Ref. [12] and lead to the estimations of $\alpha$ and $\beta$ analogous to Eq. (S29).

Formulas (S29) lead to a definite dependence of the retarded relaxation terms on the magnetic field (via $T$ and $R_c$) and on the temperature $T_e$ of the electron fluid via the times $\tau_2$ and $\tau_q$. As we mentioned above, the departure time $\tau_q$ is usually much smaller than the stress relaxation time $\tau_2$, $\tau_q \ll \tau_2$, but they has similar temperature dependencies for 2D degenerate electrons at any strength of interparticle interaction [58, 59]. Up to the logarithmic factor $\ln(\varepsilon_F/T_e)$, being weakly dependent on the temperature $T_e$, both the rates $1/\tau_q$ and $1/\tau_2$ are the quadratic functions of $T_e$ [58, 59]:

$$\frac{\hbar}{\tau_{q,2}} = C_{q,2}(T_e) \frac{T_e^2}{\varepsilon_F}. \quad (S30)$$

Here $\varepsilon_F$ is the Fermi energy of the electron fluid and $C_{q,2}(T_e)$ are the dimensionless coefficients, which are determined by the interparticle interaction parameter $r_s$, related to the electron density $n_0$, and can weakly depend on temperature via the logarithm $\ln(\varepsilon_F/T_e)$ (see discussion in Ref. [58]).

3. Memory effects in elastic part of interparticle interaction in highly viscous 2D electron fluid

The memory effect due to the formation of pairs of quasiparticles can also lead to non-dissipative “elastic” retarded terms in Eqs. (S27). Microscopically, elastic retarded terms are related to a collisionless motion of quasiparticles in pairs during several cyclotron periods, a formation of a statistical distribution of quasiparticles before the current moment (after collisions preceding the collisionless motion of pairs), and the dependence of the energy of pairs, containing the long-range term $U_C(|r|_{0,1}(t) - r_{0,2}(t))$ as well as the short-range terms with the Landau function $F_{p_0,1}(t), F_{p_0,2}(t)$, on the electric and stress fields during the almost free rotation between collisions [see Fig. S1(a,b)].

In this section we develop a phenomenological description of this effect within the Navier-Stokes-like hydrodynamic equations (S7). Namely, we construct the qualitative form of the corrections:

$$\delta F_{2,ij}(r, t) = \delta F_{2,ij} \{ V(r', t'), \hat{\Pi}(r', t') \}, \quad (S31)$$

to the Landau parameter $F_2$, appearing in Eqs. (S7),

The energy of the interparticle interaction in the fluid and the resulting elastic forces depend on correlations in the positions $r_0,p(t)$ and $v_0,p(t)$ the velocities of all quasiparticles with numbers $p$ in time and space. Evolution of correlations between the positions and velocities of particles in pairs are controlled by the cyclotron rotation, leading to the approach and the removal of quasiparticles one from another, accompanied by the increase and the decrease of the magnitudes of of the electrostatic long-range Coulomb interaction and the Fermi-liquid short-range many-particle effects. For a known evolution of the distributions and correlations of quasiparticles, the energy of the interparticle interaction, containing, in particular, the long-range term $\sum_{p_1,p_2} U_C(|r_{p_1} - r_{p_2}|)$, is determined at each time moment $t$ primarily by the pairs of quasiparticles located at minimal distances one from each other:

$$\min_{\text{all pairs } (p_1,p_2) ; t’ < t} \{ |r_{0,p_1}(t’) - r_{0,p_2}(t’)| \} \sim a_B. \quad (S32)$$

At the moments of time $t’ = t - N_c T$ these quasiparticles approach each other at distances of the order of the radius of their interaction, $a_B$. In Fig. 1(b) we illustrate the energy of interaction of two quasiparticles in a pair in a retarded time moments.

If during one period many pairs of quasiparticles did not destroyed due to collisions with “third” quasiparticles, their configurations in the moments of time before one or several cyclotron revolutions ago, $t’ = t - N_c T$, are similar to the configuration at the current moment of time $t$ [see Figs. S1(a,b)]. So the interaction energy of the quasiparticles, the correlation between which is impor-
tant at $t$, depends mainly on the distributions of quasiparticles at the moments $t$ and $t' = t - N_k T \ (N_k \geq 1)$, when also had especially closely approached one to another, on the distances $\sim a_B$.

Within the Fermi-liquid theory for 2D electrons, the effect of the distribution of quasiparticles on their energy spectrum is described by the wavevector-dependent effective Landau function, containing both the short-range as well as the long-range electrostatic contributions. We account the perturbation of the elastic part of the interparticle interaction via a nonlocal (in time) perturbation of the second harmonic of the Landau function $F_2$, as it is related with the non-equilibrium shear stress related characteristics of the fluid. Indeed, the parameters $F_0$ and $F_1$ determine compressibility and the quasiparticle mass in the Fermi liquid [67], while the parameter $F_2$ determine the viscoelastic cyclotron frequency and the magnitude of the viscosity coefficients [12].

For simplicity, we consider only the corrections to $F_2$ for the flow in a long [see Fig. 1(a) in the main text] and account only the current-time and the $T$-retarded contributions:

$$\delta F_{2,xx/xy} = \delta F_{2,xx/xy}^{(0)} + \delta F_{2,xx/xy}^{(1)}, \quad (S33)$$

where the values $\delta F_{2,xx/xy}^{(0)}(y, t)$ are expressed via the values $V(y, t)$ and $\Pi_{xx/xy}(y, t)$, while the values $\delta F_{2,xx/xy}^{(1)}(y, t)$ refer to this values in the retarded moment, $t' = t - T$: $V(y, t - T)$ and $\Pi_{xx/xy}(y, t - T)$. Here we imply that such perturbation of the parameter $F_2$ are relatively small:

$$\left| \delta F_{2,xx/xy}^{(1), (0)}(y, t) \right| \ll 1. \quad (S34)$$

The difference between the $xx$- and $xy$-components of the corrections in $F_2$ is due to a nonzero shear stress leading to a breaking of symmetry of the Fermi surface and the Landau function $F(\varphi)$ with respect to the quasiparticle velocity angle $\varphi$. For simplicity, we omit the current time contribution in Eq. (S33) and account only the most nontrivial retarded term, $\delta F_{2,xx/xy}^{(1)}(y, t)$.

The power of thermal energy dissipated at a given point $y$ of the viscous fluid flow in the moment $t'$ is:

$$\mathcal{W}(y, t') = - (V_{ik} \Pi_{ik})_{y, t'} / 2, \quad (S35)$$

where summation over same indices is supposed. The Landau functional of the total fluid energy and, thus, the Landau parameters $F_n$ are expressed via the integral of powers $\mathcal{W}^k$. Similarly, in a viscous inhomogeneous flow the relationship between the total fluid energy and the inhomogeneous part of the distribution function of quasiparticles, $\delta f_p$, contains not only the linear and quadratic Fermi-liquid terms, but also high-order by $\delta f_p$ terms. Correspondingly, the perturbation of the quasiparticle spectrum as well as of the Landau parameters acquire correction proportional to powers of $\delta f_p$, therefore to the powers of $V_{ik}$ and $\Pi_{ik}$.

By analogy with Eq. (S35), for the perturbation of the second harmonic of the Landau function $F_2$ in a Poiseuille flow, in which only the derivative $\partial V_{xx}/\partial y$ is nonzero, we write:

$$\delta F_{2,xx}^{(1)}(y, t) = \left[ (a \Pi_{xx} + b \Pi_{xy}) \frac{\partial V}{\partial y} \right]_{y, t'} \, (y, t'), \quad (S36)$$

$$\delta F_{2,xy}^{(1)}(y, t) = \left[ (c \Pi_{xx} + d \Pi_{xy}) \frac{\partial V}{\partial y} \right]_{y, t'} \, (y, t').$$

In this work we do not construct the full form of the non-linear perturbations of the parameter $F_2$, but propose and use only the simplest relation (S36) for a Poiseuille flow the presence of magnetic field. Within the proposed above mechanism of perturbations of the elastic part of the interparticle interaction energy due to formation of pairs, the coefficients $a, b, c, d$ are proportional to the probability $P(B)$ [Eq. (S23)] for a quasiparticle to make a complete cyclotron circle without collisions with a third quasiparticle:

$$a, b, c, d \sim e^{-T / \tau_e}. \quad (S37)$$

It is noteworthy that appearance of the coefficients $(1 - \delta F_{2,ij}) - \partial F_{2,ij}/\partial t$ in Eq. (S7) for a given flow frequency $\omega$ is effectively equivalent to the renormalization of the coefficients $2\omega_e$ in the right-hand side of the equation for $\partial F_{2,ij}/\partial t$. That is, the memory effects in the interparticle interaction energy for an electron fluid is mainly a non-linear elastic (non-dissipative) effect, being the flow-dependent shift of the frequency $2\omega_e$ of the own dynamic of the shear stress of the fluid:

$$2\omega_e \to 2\omega_e + \Delta \omega_{e, t}, \quad \Delta \omega_{e, t} = \sum_{l} M_{lij} \delta F_{2,ij} \quad (S38)$$

where the tensor $M_{lij}$ stems from the form of Eqs. (S7) for a given flow.

For the just described elastic memory effects, the inequality $|\delta(t)|/a_B \ll 1$, which guarantee sufficiently small amplitudes of the change of the scattering parameter after one return in the extended collisions due to the external and internal field [see Fig. S1(b)], also should be fulfilled, thus the condition of applicability of the memory terms (S28) also takes place.

We emphasize that we have developed a “minimal” phenomenological model of a non-linear dynamic of an electron fluid, in which some proper phenomenologically justified terms are accounted to describe photosresponse of the fluid. Others more complex (not pair) types of correlations and corresponding memory terms can be possible in macroscopic dynamic equations of the type of Eqs. (S7).
4. Linear responses of highly viscous electron fluid on dc and ac electric fields

In order to study the photoresistance effect within the proposed model, first of all, we need to find the linear responses of the fluid on a dc and an ac electric fields \( E_0 \) and \( E_1(t) \).

Both these fields can be written as \( E(t) = E(\omega) e^{-i\omega t} + \text{c.c.} \) with \( \omega = 0 \) and \( \omega \neq 0 \). The linear responses should be calculated by linearized equations (S7) applied to the \( \omega \) harmonics of the hydrodynamic velocity \( V(r, t) = V(r, \omega)e^{-i\omega t} + \text{c.c.} \), the density perturbation \( \delta n(r, t) = \delta n(r, \omega)e^{-i\omega t} + \text{c.c.} \), and the shear stress tensor \( \Pi(r, t) = \Pi(r, \omega)e^{-i\omega t} + \text{c.c.} \), corresponding to the harmonics of \( E(t) \).

First, we formulate the resulting equations for the amplitudes \( V(r, \omega) \sim \Pi(r, \omega) \sim E(\omega) \) at a general geometry of a flow.

A solution of the last of equations (S7) without the memory effect term \(-\Gamma_{\text{slm}}(r, t)\Pi_{\text{lm}}(t, -T)\) leads to the following linear relation between the time harmonics of the momentum flux \( \Pi_{ij} = \Pi_{ij}(r, \omega) \) and of the gradients of the harmonics of the velocity \( V_{ij}(r, \omega) = \partial V_i/\partial x_j + \partial V_j/\partial x_i \): \( \Pi_{ij} = m [ \eta_{xx}(\omega) V_{ij} + \epsilon_{ijk} \eta_{xy}(\omega) V_{kj} ] \), where the ac viscosity coefficients \( \eta_{xx} = \eta_{xx}(\omega) \) and \( \eta_{xy} = \eta_{xy}(\omega) \) have the form \( \text{[40]} \):

\[
\eta_{xx} = \frac{(v_p^0)^2 \tau_2/4}{1 + (4v_p^0 - \omega^2)^2 - 2i\omega^2/2}\quad \text{and} \quad \eta_{xy} = \frac{\omega \tau_2}{2}\text{.} \tag{S40}
\]

The second of equations (S7) with \( \Pi(r, \omega) \) from Eq. (S39) is transformed into the Naiver-Stokes equation for the harmonic of the velocity \( V = V(r, \omega) \):

\[
-i\omega V = \frac{e}{m} E(\omega) + [V \times \omega_e] + \eta_{xx}(\omega) \Delta L V + \eta_{xy}(\omega) \Delta L V \times e_z , \tag{S41}
\]

where \( \Delta L = \partial^2/\partial x^2 + \partial^2/\partial y^2 \) is the Laplace operator.

The role of the retarded relaxation term in the last of equations (S7) for the linear response is as follows. If we choose the main part \( \Gamma^{(0)} \) of the relaxation tensor \( \Gamma(r, t) \) in the simplest form:

\[
\Gamma^{(0)}_{ijkl} = \delta_{ik} \delta_{jl} \frac{1}{\tau_2} , \tag{S42}
\]

we arrive to a redefinition of the relaxation rate \( 1/\tau_2 \) which enters Eq. (S40) as compared with its value in the absence of the memory effects:

\[
\frac{1}{\tau_2} \to \frac{1}{\tau_2} + \frac{e^{2\pi i\omega/\omega_c}}{\tau_2} . \tag{S43}
\]

For the time \( \tau_2^* \) here we should use estimate \( [29] \): \( 1/\tau_2^* \sim e^{-T/\tau_c}/\tau_2 \).

Further in this work we do not take into account redefinition (S43) of \( \tau_2 \) by the two reasons. First, in view of estimate (S29), the second term in this formula is much smaller than the first one on the factor \( e^{-T/\tau_c} \), which is substantially smaller than unity at \( \omega_c \sim \omega \gg 1/\tau_2, 1/\tau_0 \). Second, our analysis shows that redefinition (S43) leads only to some subtle distortion of the sinusoidal dependence of a factor in the resulting photocconductivity on the ratio \( \omega/\omega_c \). For the current work, being a first proposal of the hydrodynamic mechanism of MIRO, such detail is not substantial.

We remind that the parameter \( v_p^0 \) in Eq. (S40) for a strongly non-ideal electron Fermi liquid with large Landau parameters, \( F_m \gg 1 \), is much greater than the actual Fermi velocity \( v_F \) \( \text{[41]} \). \( \text{[42]} \).

Second, we use equations (S40) and (S41) to calculate the linear response of the electron fluid on the applied dc and the ac fields, \( E_0 \) and \( E_1(t) \), in a Poiseuille-flow like geometry: a long straight sample with rough edges [see Fig. 1(a) in the main text].

We consider that the sample width \( W \) is much smaller than the characteristic plasmon wavelength \( l_p \):

\[
W \ll l_p \quad \text{and} \quad l_p = s/\omega . \tag{S44}
\]

Here we use the estimate of \( l_p \) for a quantum well with a metallic gate. The value \( s \) is the plasmon velocity in such structure which is usually much larger than \( v_F \) and \( v_p^0 \). In defectless samples of such widths the dc and ac linear responses are formed mainly by a dissipative viscous flow and by standing magnetosonic waves, respectively, whereas the plasmonic contribution to the ac flow component is suppressed to the extent of the small parameter \( v_p^0/s \ll 1 \) \( \text{[41]} \).

In such flow geometry, only the \( x \) component of the velocity \( V \) and the \( xy \) component of the velocity gradient tensor \( V_{ij} \) are substantial in both the ac and dc components \( \text{[41]} \). For the ac flow component, the density perturbation \( \delta n(y, t) \) and the \( y \) component of the velocity \( V_{y,1}(y, t) \) are relatively small quantities being proportional to \( (v_p^0/s)^2 \ll 1 \) \( \text{[41]} \). However, such \( \delta n \) determines the not small ac component \( E_{y,1}^{\text{int}}(y, t) \) of the internal Hall field, which screens the \( y \) component of the radiation fields \( E_{y,1}(t) \) [see Eqs. (S3)]. For the dc flow component, \( V_{y,0} \equiv 0 \) and the density perturbation \( \delta n(y) \) determines the \( x \) part of the Hall field \( E_{y,0}^{\text{int}}(y) \).

Thus, in order to find the velocity \( V = V_0 + V_1 \) we need only the \( x \)-component of equation (S11). For each component \( V_0 \) and \( V_1 \) it takes the form:

\[
-i\omega V = \frac{e}{m} E_x(\omega) + \eta_{xx}(\omega) \frac{d^2V}{dy^2} . \tag{S45}
\]

The both fields \( E_{y,0}^{\text{int}}(y) \) and \( E_{y,1}^{\text{int}}(y, t) \) are calculated from the \( y \)-components of equation (S11) \( \text{[29]} \).

The resulting response of the fluid in such sample on a dc electric field \( E_0^{dc} = E_0 e_x \) is the dc Poiseuille flow with
The reciprocal imaginary and real parts of this eigenvalue, $1/\text{Im} \lambda$ and $1/\text{Re} \lambda$, provide the wavelength and the decay length of the magnetosonic waves, respectively. Their dependencies on $\omega_c$ at a fixed frequency $\omega \gg 1/\tau_2$ are drawn in Fig. S2.

Far above from the viscoelastic resonance ($\omega - 2\omega_c \gg 1/\tau_2$) we obtain from Ref. [S52] the following estimates for the length of decay and the wavelength of magnetosonic waves:

$$\frac{1}{\text{Re} \lambda} \sim L_s = v_F^p \tau_2, \quad \frac{1}{\text{Im} \lambda} \sim l_s = \frac{W}{\omega}$$

which lead to the relation $1/\text{Re} \lambda \ll 1/\text{Im} \lambda$. In this case the velocity profile $V_1(y,t) \sim e^{-i\omega t + \lambda y}$ is formed by exponentially decaying non-oscillating eigenmodes, and the imaginary part of $\lambda$ is not substantial.

It follows from Eqs. (S51)-(S53) that above the resonance, $\omega > 2\omega_c$, the standing magnetosonic waves $V_1(y,t) \sim e^{-i\omega t + \lambda y}$ are formed. They are localized in the whole sample at the intermediate sample widths:

$$l_s \ll W \ll L_s, \quad \frac{1}{\text{Re} \lambda} \sim l_s, \quad \frac{1}{\text{Im} \lambda} \sim L_s,$$

and in the near-edge regions, $W/2 - |y| \lesssim l_s$, in the samples with the large widths (provided $L_s \ll l_p$):

$$L_s \ll W \ll l_p.$$  

In the last case, the flow in the central part of the sample, $W/2 - |y| \gg L_s$, is a trivial response, $V_1 \sim i e^{E_1^+/(2\omega)}$, of a dissipativeless homogeneous electron media on $E_1^+(t)$ [see Eq. (S51)].

In the samples with the widths $W \gg l_s$ and at the frequencies below the resonance, $\omega < 2\omega_c$, the ac response $V_1(y,t)$ is located in the narrower near-edge regions, $W/2 - |y| \lesssim l_s$ and have non-oscillation exponential profile [see Eq. (S54)].

In the narrowest samples:

$$W \ll l_s,$$

as it follows from Eqs. (S51)-(S54), the velocity profile $V_1(y)$ is parabolic in both the cases $\omega > 2\omega_c$ and $\omega < 2\omega_c$, and its amplitude $V_1(0)$ decreases with the decrease of $W$ as $\sim |\lambda|^2 W^2$. We note that last equation (S57) is consistent with the condition $W = \Delta x \gg v_F/\omega$ of the macroscopic hydrodynamic description of the flow as for a highly correlated fluid one should imply that $v_F^p \gg v_F$ (see Section 2 and Refs. [42][41]).
In Fig. S3 we plot the energy absorbed by the Poiseuille flow \( W_{\text{tot}} = \int_{-W/2}^{W/2} \int_{-W/2}^{W/2} dt dy \mathcal{W}(y, t) \), where \( \mathcal{W}(y, t) \) is given by Eq. (S33). The evolution of the flow described above with a change in the relations between the frequencies \( \omega \) and \( \omega_c \), on the one hand, and the sample width \( W \) and the eigenvalue \( \lambda \), on the other hand, is reflected in the change in the dependence \( W_{\text{tot}}(\omega_c) \). In particular, oscillations in \( W_{\text{tot}}(\omega_c) \) at \( \omega_c < \omega/2 \) for not very wide samples are associated with the appearance of standing magnetosonic waves.

Now we can calculate the shift (mismatch) of the \( xy \) component of strain tensor, \( \Delta_{xy}(r, t) \) in a cyclotron period, that enters the retardation relaxation term \(-\Gamma_{ijkl}(\Delta(r, t))\Pi_{kl}(r, t - T)\) in the motion equation (S7) for the Poiseuille flow. In the presence of both the dc and ac fields \( E_0, E_1(t) \), as it takes place in the experiments of photoresistance, the shift \( \Delta_{xy}(r, t) \equiv \Delta(y, t) \) in the linear approximation by \( E_0 \) and \( E_1 \) contains the two contributions:

\[
\Delta(y, t) = \Delta_0(y) + \Delta_1(y, t), \tag{S58}
\]

from the dc dissipative and the ac almost elastic components of the linear response: \( V_{\text{lin}}(y, t) = V_0(y) + V_1(y, t) \). Equations (S20), (S47), and (S51) yield:

\[
\Delta_0(y) = \frac{2\pi}{\omega_c} \frac{dV_0}{dy} \tag{S59}
\]

and \( \Delta_1(y, t) = \Delta_1(y)e^{-i\omega t} + c.c. \), where

\[
\Delta_1(y) = \frac{i}{\omega} \left[ 1 - e^{2\pi i \omega/\omega_c} \right] \frac{dV_1}{dy}. \tag{S60}
\]

It is noteworthy that this formula contain the factor \( e^{2\pi i \omega/\omega_c} \) being periodic by the reciprocal magnetic field.

5. Non-linear response of fluid on dc and ac electric fields: memory effects in interparticle scattering

In this section we calculate the nonlinear correction to the dc flow component \( V_0(y) \) (S47), induced by the ac component \( V_1(y, t) \) (S51) and the memory relaxation term, \(-\Gamma_{ijkl}\Pi_{kl}\) in Eqs. (S7), and the resulting mean photoconductivity \( \Delta \partial \delta_{\omega} \) of the sample.

We continue to consider the flow in a long sample with the width \( W \) which is much smaller than the characteristic plasmon wavelength \( l_p \) [see Eq. (S14)]. Similarly as for the linear component, only the \( x \) component of the velocity \( \mathbf{V} \) and the \( xy \) components of the tensors \( V_{ij} \) and \( \varepsilon_{ij} \) present in the non-linear component [see Fig. 1(a) in the main text]. In such geometry, equations (S7) take the simplified form:

\[
\begin{align*}
\frac{\partial V}{\partial t} &= \frac{e}{m} E^{\text{ext}}(t) - \frac{1}{m} \frac{\partial \Pi_{xx}}{\partial y} \\
\frac{\partial \Pi_{xx}}{\partial t} &= 2\omega_c \Pi_{xy} - \frac{\Pi_{xy}}{\tau_2} \\
&\quad - \Gamma(\Delta) \Pi_{xx}(y, t - T) \tag{S61} \\
\frac{\partial \Pi_{xy}}{\partial t} &= -2\omega_c \Pi_{xx} - \frac{\Pi_{xy}}{\tau_2} \\
&\quad - \frac{m(v_y^p)^2}{4} \frac{\partial V}{\partial y} \tag{S62} - \Gamma(\Delta) \Pi_{xy}(y, t - T)
\end{align*}
\]

where \( E^{\text{ext}}(t) = E_0 + E_{\text{ac,1}}(t) \) and we used the simplest form of the retarded relaxation tensor \( \Gamma_{ijkl} = \Gamma_{\delta i} \delta_{jl} \), analogous to Eq. (S42). For the dependence of \( \Gamma \) on \( \Delta \), according to Eq. (S25), we write:

\[
\Gamma(\Delta) = 1/\tau_2^\alpha - \alpha \Delta^2, \quad \alpha > 0 . \tag{S62}
\]

The positiveness of \( \alpha \) corresponds to the decrease of the probability for a particle “1” to scatter again on a particle “2” with the increase of the amplitudes of the velocity \( V(y, t) \), of the strain \( \varepsilon_{xy}(y, t) \), and, thus, of the trajectories mismatches \( \delta_{1,2}(t) \) during a cyclotron period (see Fig. S1). Note that the same sign of the analogous parameters \( \alpha \) and \( \beta \) in the tensor (S3) of the memory effect in the scattering of non-interacting electrons on defects was obtained in Ref. [12] for weak localized scatters with a smooth potential.

The mismatch \( \Delta = \Delta(y, t) \) in Eqs. (S61) and (S62) is expressed via the integral of \( \partial V(y, t)/\partial y \) by Eq. (S25).

The photoconductivity and the photoresistance at small ac powers are given by the correction \( I_0 \) to the dc current \( I_0 \) (S49), linear by the ac power \( W \sim E_0^2 \). Thus, together with the linear responses \( L_0 \sim V_0 \sim E_0 \) and \( V_1 \sim E_1 \), one needs to calculate the time-independent nonlinear component \( V_\omega(y) \) of the dc flow of the order of \( \sim E_0^2/\tau_2^2 \):

\[
V(y, t) = V_0(y) + V_1(y, t) + V_\omega(y), \tag{S63}
\]
and the corresponding contribution $\hat{\Pi}_\omega \sim E_0 E_1^2$ to the momentum flux tensor:

$$\hat{\Pi}(y, t) = \hat{\Pi}_0(y) + \hat{\Pi}_1(y, t) + \hat{\Pi}_\omega(y).$$  \hfill (S64)

Here $\hat{\Pi}_0$ and $\hat{\Pi}_1$ are the linear contributions in $\hat{\Pi}$, related to $V_0$ and $V_1$ by Eq. (S39).

The equations for $\Pi_\omega$ and $V_\omega$ are derived by the substitution of the dc and the ac linear components, $\Pi_0$, $\Delta_0$ and $\hat{\Pi}_1$, $\Delta_1$, into the nonlinear parts of the retarded relaxation terms in Eqs. (S61) with $\Gamma_2(\omega)$ (S62) and by the integration of the resulting equations for $\partial V_\omega/\partial t$ and $\partial \hat{\Pi}/\partial t$ by one cyclotron period. After such procedure, we obtain:

$$\begin{aligned}
\frac{d\Pi_{\omega,xy}}{dy} &= 0 \\
\frac{\Pi_{\omega,xx}}{\tau_2} - 2\omega_c\Pi_{\omega,xy} &= \\
&= \alpha \langle \Delta^2(y, t) \Pi_{xx}(y, t - T) \rangle_\omega.
\end{aligned}$$  \hfill (S65)

Here the angular brackets with the subscript “$\omega$” denote the operation of taking the contribution of the order of $\sim E_0 E_1^2$, averaged by the last period of the ac field:

$$\langle f(t) \rangle_\omega = \frac{1}{T} \int_{t - T_\omega}^t dt' f_\omega(t'),$$  \hfill (S66)

where $T_\omega = 2\pi/\omega$ is the period of the ac field and $f_\omega(t) = f_{12}(t) E_0 E_1^2$ denotes the term “$12$” in the decomposition of the function $f(t)$ in power series by the amplitudes of the dc and the ac fields: $f(t) = \sum_{m,n=1}^{\infty} f_{mn}(t) E_0^m E_1^n$.

From the last two equations in system (S65), we obtain the expression for the nonlinear part $\Pi_\omega$ of the momentum flux tensor. In particular, for its $xy$-component we have:

$$\Pi_{\omega,xy}(y) = -m \eta_{xy} \frac{dV_\omega}{dy} +$$

$$+ \frac{\alpha \pi^2}{1 + 4\omega_c^2 \tau_2} \left[ \langle \Delta^2(y, t) \Pi_{xx}(y, t - T) \rangle_\omega \right].$$  \hfill (S67)

Here each nonlinear term in the brackets $\langle .. \rangle_\omega$ contain two parts with different combinations of the dc, $V_0 \sim \Pi_{0,ij} \sim \Delta_0$, and ac, $V_1 \sim \Pi_{1,ij} \sim \Delta_1$, linear responses:

$$\begin{aligned}
\langle \Delta^2(y, t) \Pi_{ij}(y, t - T) \rangle_\omega &= \\
&= \langle \Delta^2_1(y, t) \rangle \Pi_{0,ij}(y) + \\
&+ \langle \Delta_1(y, t) \Pi_{1,ij}(y, t - T) \rangle \Delta_0(y).
\end{aligned}$$  \hfill (S68)

Here $ij = xx, xy$ and the angular brackets without a subscript denotes the operation of averaging over the last period of the ac field:

$$\langle g(t) \rangle = \frac{1}{T} \int_{t - T_\omega}^t dt' g(t').$$  \hfill (S69)

The linear components of the momentum flux tensor in Eq. (S68) have the form [see Eq. (S39)]:

$$\Pi_{0,(xx/xy)}(y) = -m \eta_{xx/xy} \frac{dV_0}{dy}$$

and

$$\Pi_{1,(xx/xy)}(y) = -m \eta_{xx/xy} \frac{dV_1}{dy} e^{-i\omega t} + c.c.$$  \hfill (S70)

In view of Eq. (S60), the squared shift of the deformation $\Delta_1^2$ in Eq. (S68), averaged by $t - T_\omega < t' < t$, is:

$$\langle \Delta_1^2(y, t) \rangle = \frac{4}{\omega^2} \sin^2 \left( \frac{\pi \omega}{\omega_c} \right) \left| \frac{dV_1}{dy} \right|^2.$$  \hfill (S71)

Due to Eqs. (S60) and (S71), the expression under the angular brackets in the last term in Eq. (S68), averaged by $t - T_\omega < t' < t$, takes the form:

$$\langle \Delta_1(y, t) \Pi_{1,(xx/xy)}(y, t - T) \rangle =$$

$$= \frac{2}{\omega} \mathrm{Re} \left[ i \eta_{xx/xy}(\omega) (e^{2\pi i\omega/\omega_c} - 1) \right] \left| \frac{dV_1}{dy} \right|^2.$$  \hfill (S72)

Substitution of formulas (S67), (S73) for $\Pi_{\omega,xy}$ into the first of equations (S65) leads to the final equation for the nonlinear part of the flow profile $V_\omega = V_\omega(y)$:

$$\frac{d^2V_\omega}{dy^2} = G(y),$$  \hfill (S74)

where $G(y)$ is given by formulas (S69), (S60), and (S67)-(S73). Equation (S74) should be solved with the diffusive boundary conditions on the component $V_\omega(y)$: $V_\omega|_{y = \pm W/2} = 0$. The result takes the form:

$$V_\omega(y) = \frac{e^3 E_0 E_1^2 \alpha}{mv^2} \left[ u + 2 \mathrm{Re} \nu \right] J(y),$$  \hfill (S75)

where $J(y)$ is the dimensionless factor determining the profile of the nonlinear flow component:

$$J(y) = \frac{|\lambda|^2}{\cosh(\lambda W/2)^2} \int_{|y|}^{W/2} d\tilde{y} \tilde{y} |\sinh(\lambda \tilde{y})|^2$$  \hfill (S76)

and the dimensionless values $u$ and $v$ contain the dependence on the frequencies $\omega$ and $\omega_c$:

$$u = 4 \sin^2 \left( \frac{\pi \omega}{\omega_c} \right) (-1 + 4\omega_c^2 \tau_2^2),$$  \hfill (S77)
\[ v = \frac{2\pi \omega}{\omega_c} \left[ \sin \left( \frac{2\pi \omega}{\omega_c} \right) - 2i \sin^2 \left( \frac{\pi \omega}{\omega_c} \right) \right] \times \]
\[ \times \left( -1 + i\omega \tau_2 + 4\omega_c^2 \tau_2^2 \right) \left( 1 + 4\omega_c^2 \tau_2^2 \right) \left( 1 + 4\omega_c^2 - \omega^2 \right) \tau_2^2 - 2i\omega \tau_2 \].

(S78)

A direct calculation of the factor \( J(y) \) yields:

\[ J(y) = \frac{\nu^2 + \mu^2}{8 \nu^2 \mu^2 \cos[(\mu - i\nu)W/2]^2} \times \]
\[ \times \left\{ \mu^2 \left[ -\cos(\nu W) + \cos(2\nu y) - \nu W \sin(\nu W) + 2\nu y \sin(2\nu y) \right] + \right\} \]
\[ + \mu^2 \left[ -\cosh(\mu W) + \cosh(2\mu y) + \nu W \sinh(\mu W) - 2\nu y \sinh(2\mu y) \right] \} . \]

(S79)

where \( \mu = \text{Re} \lambda \) and \( \nu = -\text{Im} \lambda \). It is seen from Fig. S2 that both the values \( \mu \) and \( \nu \) are positive at any \( \omega \) and \( \omega_c \).

In the limiting cases of the narrow and the wide samples, \( W \ll \min(1/\mu, 1/\nu) \) and \( W \gg \max(1/\mu, 1/\nu) \), we obtain from Eq. (S79):

\[ J(y) = \frac{(\nu^2 + \mu^2)^2}{4} \left[ \left( \frac{W}{2} \right)^4 - y^4 \right] \] (S80)

and

\[ J(y) = \frac{W (\nu^2 + \mu^2)}{4\mu} \left[ 1 - e^{-2\nu (W/2 - |y|)} \right] . \] (S81)

In this way, in the very narrow samples the profile of the flow perturbation \( V_\omega(y) \) is a fourth-order parabola, while in the very wide samples it is almost flat in the central region, \( W/2 - |y| \gg 1/\mu \), and exponentially goes to zero in the near-edge regions, \( W/2 - |y| \lesssim 1/\mu \). For the samples with the intermediate widths, \( \min(1/\mu, 1/\nu) < W < \max(1/\mu, 1/\nu) \), the component \( V_\omega(y) \) contains comparable decaying and oscillating parts [see Eq. (S79)].

The profiles \( V_\omega(y) \) and \( V_1(y) \) for the medium and the narrow samples are drawn in Fig. S4.

From the resulting expressions (S75), (S79) and estimate (S28) for the coefficient in the memory term, \( \alpha = B(Re/aB) e^{-T/\tau_0} / \tau_2 \), one can find the magnetooscillations of the velocity \( V_\omega(y) \) at any \( y \). Note that \( R_e = v_F/\omega_c \), contains the actual Fermi velocity, unlike the viscosity coefficients containing the parameter \( v_h^2 \), \( v_h^2 \gg v_F \). At the strong magnetic fields and the high ac frequencies, \( \omega \sim \omega \gg 1/\tau_2 \), and far from the resonance, \( |\omega - 2\omega_c| \gg 1/\tau_2 \), for the point \( y = 0 \), corresponding to the typical values of \( V_\omega(y) \) (see Figs. S4), we have:

\[ V_\omega(0) = \sin^2 \left( \frac{\pi \omega}{\omega_c} \right) V_{\omega \text{max},1} + \sin \left( \frac{2\pi \omega}{\omega_c} \right) V_{\omega \text{max},2} \] (S82)

\[ V_{\omega \text{max},1} = 16 \frac{e^3 E_0 E_1^2 v_F^2 r_2}{m^3 (v_F^2)^2 a_B^2} \omega^2 e^{-T/\tau_0} \right|_{\gamma} \] (S83)

\[ V_{\omega \text{max},2} = -12 \frac{4\pi \omega \omega_c}{\omega^2 - 4\omega_c^2} V_{\omega \text{max},1} \] (S84)

and the dimensionless factor \( J_e \equiv J(0) \) is:

\[ J_e = \frac{|\lambda|^2}{\cosh^2(\lambda W/2)} \int_0^{W/2} d\gamma \sinh(\lambda \gamma) \gamma^2 . \] (S85)

From Eq. (S79) we obtain that above the viscoelastic resonance, \( \omega > 2\omega_c \), when \( \mu \ll \nu \), the factor \( J \) for
different sample width is estimated as:

\[ J_c \propto \frac{\nu^2 W^2}{W^\mu}, \quad W \gg 1/\mu \]
\[ \nu^2 W^2, \quad W \ll 1/\nu \]  \hspace{1cm} (S86)

This formula for the intermediate sample widths, \( 1/\nu \ll W \ll 1/\mu \), is valid for the sample width far from the following condition: the coincidence of the sample width \( W \) with a half-integer number of the wavelengths of transversal magnetosound \( [41] \):

\[ \nu W = \pi + 2pl, \]  \hspace{1cm} (S87)

where \( l = 0, \pm 1, \pm 2, \ldots \). At this condition acoustic-like resonances related to standing magnetosonic modes inside the sample occur in the flow. For the sample widths, given by Eq. (S87), the denominator of \( J_c \) in Eq. (S86) becomes zero. Therefore in the vicinities of such \( \omega \) and \( \omega_c \) one should use the exact expression for the denominator:

\[ \nu W = \nu W/2 - (\pi/2 + pl), \quad |\Delta w| \ll 1, \]

and provided \( \mu W \ll 1 \) we have:

\[ J_c \approx \frac{\nu^2 W^2}{\Delta w^2 + (\mu W/2)^2}. \]  \hspace{1cm} (S88)

It follows from this formula that the resonances at the peculiar values of \( \nu W \) (S87) can be more or less sharp depending on parameters \( \omega_2 \) and \( W/L_s \).

Below the viscoelastic resonance, \( \omega < 2\omega_c \), when \( \nu \gg \nu \) (see Fig. S2), one obtains from Eq. (S79):

\[ J_c \approx \mu W \begin{cases} 1, & W \gg 1/\mu \\ \mu^2 W^3, & W \ll 1/\mu \end{cases}. \]  \hspace{1cm} (S89)

From Eqs. (S53), (S54), (S55), and (S89) we estimated the amplitudes \( V_{\omega \max,1/2}^{\text{visc}} \) (S83), (S84) in an exact form for different regimes: above the viscoelastic resonance \( \omega > 2\omega_c \) for the wide \( [W \gg 1/\mu(\omega_c)] \), the medium \( [1/\nu(\omega_c) \ll W \ll 1/\mu(\omega_c)] \), and the narrow \( [W \ll 1/\nu(\omega_c)] \) samples as well as below the viscoelastic resonance \( \omega < 2\omega_c \) for the wide \( [W \gg 1/\mu(\omega_c)] \) and the narrow \( [W \ll 1/\mu(\omega_c)] \) samples.

We do not present all the resulting formulas for \( V_{\omega \max,1/2}^{\text{visc}} \) for these cases because of their cumbersome nature. Let us discuss in detail only the result for the most interesting regime: above the viscoelastic resonance, \( \omega > 2\omega_c \gg 1/\tau_2 \), for the medium sample width, \( 1/\nu \ll W \ll 1/\mu \), when the magnetooscillations acquire irregular shape [see Fig. S5]. In this case and far from the magnetosonic resonances (S87), the velocity amplitudes (S83) and (S84) in this regimes take the form:

\[ V_{\omega \max,1} = (\omega^2 - 4\omega_c^2) V_\omega^0, \]  \hspace{1cm} (S90)

\[ V_{\omega \max,2} = -4\pi \omega \omega_c V_\omega^0, \]  \hspace{1cm} (S91)

where

\[ V_\omega^0 = \frac{e^3 E_0 E_1^2 v_F^2}{m^3} S(\nu) \frac{W^2 \frac{\tau_2}{\omega^4} e^{-\tau/\tau_2}}{\omega^4}, \]  \hspace{1cm} (S92)

and \( S(\nu) \sim 1/\cos^2(\nu W/2) \), when the value \( \nu W \) is not too close to \( (1 + 2N_r) \pi \) with integer \( N_r \).

Now we can find the dependencies of the radiation-induced correction \( \Delta \sigma_\omega \) to the dc conductivity, \( \sigma = \sigma_0 + \Delta \sigma_\omega \), on temperature, ac frequency, and magnetic field far from the magnetosonic resonances (S87). The value \( \Delta \sigma_\omega \) is defined as:

\[ \Delta \sigma_\omega = \frac{I_\omega}{E_0 W} \]  \hspace{1cm} (S93)

\[ I_\omega = e n_0 \int_0^{W/2} dy V_\omega(y). \]

Equations (S85)-(S82) and (S90)-(S92) yield for the photoconductivity \( \Delta \sigma_\omega \) in the considered case, \( 1/\nu \ll W \ll 1/\mu \), when the standing magnetosonic waves are well-formed:

\[ \Delta \sigma_\omega = (U + V) \Delta \sigma_\omega^{\max}, \]  \hspace{1cm} (S94)

where the factors \( U = U(\omega/\omega_c) \) and \( V = V(\omega/\omega_c) \) are:

\[ U = \left[ 1 - \left( \frac{2\omega_c}{\omega} \right)^2 \right] \sin^2 \left( \frac{\pi \omega}{\omega_c} \right), \]  \hspace{1cm} (S95)

\[ V = \frac{4\pi \omega_c}{\omega} \sin \left( \frac{2\pi \omega}{\omega_c} \right), \]  \hspace{1cm} (S96)
and the amplitude \( \Delta \sigma_{\omega}^{max} = \Delta \sigma_{\omega}^{max}(\omega, \omega_c \tau_q) \) is:

\[
\Delta \sigma_{\omega}^{max} \sim \frac{e^4 E_0^2 n_0 v_F^2}{m^4 (v_F^2)^4 a_B^2} \frac{W^2 \tau_2}{\omega^2} e^{-2\pi/(\omega_c \tau_q)} .
\]

(S96)

Here the relaxation times \( \tau_q = \tau_q(T_c) \) and \( \tau_2 = \tau_2(T_c) \) are given by Eqs. (S20). It follows from this result that the oscillation of photoconductivity are suppressed with the increase of the temperature \( T_c \) and the ac frequency \( \omega \).

One can see from Eq. (S88) that in the vicinities of the magnetosonic resonances in the ac linear response \( \nu = \omega_c \tau_q \) the factor \( J_c \) increases as compared with Ref. (S96) in the large factor \( \nu^2/\mu^2 \gg 1 \).

In Fig. S5 we present the averaged photoconductivity \( \Delta \sigma_{\omega} \) multiplied on \(-1\), as this value is presented as photoresistance in linear by the ac power \( W \) approximation:

\[
\Delta \varphi_{\omega} = - \Delta \sigma_{\omega}/\sigma_0^2 .
\]

(S97)

The curves are plotted at a fixed ac frequency \( \omega \) and the sample widths \( W \) smaller, comparable, and larger than the characteristic decay length \( L_s \). The dependence \(- \Delta \sigma_{\omega}(\omega_c) \) exhibits oscillations by the inverse magnetic field. The shape of the obtained oscillations is sinusoidal in the limiting cases \( W \gg L_s \) and \( W \ll L_s \), while for intermediate sample widths, \( W \sim L_s \), it is irregular. The last property is related to the appearance of the magnetosonic resonances in the ac linear response \( V_1(y, t) \) at the sample width satisfying conditions (S87). These resonances manifest themselves most clearly when \( W \sim L_s \) and, thus a not too many wavelengths fit in the sample, but the damping of waves is relatively weak.

6. Non-linear response of fluid on dc and ac electric fields: memory effects in elastic part of interparticle interaction

In this section, we calculate the “elastic” contribution in the nonlinear correction \( V_\omega(y) \) to the dc velocity \( V_0(y) \), which originates from the nonlinear by the perturbations \( \delta F_{2,ij} \) and \( \delta F_{2,kl} \) of the Landau parameter \( F_2 \). This contribution is additive with the relaxational contribution in \( V_\omega \) from retarded relaxation due to the memory effect in extended collisions, calculated in previous Section 5. The resulting contribution in photoconductivity \( \Delta \sigma_{\omega} \) has the properties being partially different from the ones of the relaxational ones.

The equations for \( \Pi_{\omega} \) and \( \delta F_{\omega} \) are derived by the substitution of the dc and the ac linear components, \( \Pi_0 \), \( \Delta_0 \) and \( \Pi_1 \), \( \Delta_1 \), into the nonlinear parts of the retarded elastic terms in Eqs. (S7) with \( \delta F_{2,ij} \) and \( \delta F_{2,kl} \). As we mentioned in Section 2, for brevity we omit the amplitude of the unperturbed Landau parameter writing just \( F_2 = 0 \) in the left part of Eq. (S7).

For the geometry of Poiseuille flow, the resulting equations for \( \partial V/\partial t \) and \( \partial \Pi/\partial t \) are similar to Eqs. (S61) accounting the retarded relaxation via the terms \( \sim T \). In order to calculate the dc non-linear contributions to the velocity and the stress tensor, one should again integrate the motion equations by one cyclotron period, as it was done in Section 5 for the inelastic contribution due to the term \( \Gamma_{ijkl} \Pi_{kl} \). We obtain:

\[
\begin{align*}
\frac{1}{m} d\Pi_{\omega,xy}/dy &= 0 \\
\Pi_{\omega,xx}/\tau_2 - 2\omega_c \Pi_{\omega,xy} &= \left< F_{2,xx}(y, t) \frac{\partial \Pi_{xx}(y, t)}{\partial t} \right>_{\omega}, \\
2\omega_c \Pi_{\omega,xx}/\tau_2 + \Pi_{\omega,xy}/\tau_2 &= - \frac{m_0 (v_F^2)^2}{4} \frac{dV_\omega}{dy} + \\
&+ \left< \delta F_{2,xy}(y, t) \frac{\partial \Pi_{xy}(y, t)}{\partial t} \right>_{\omega}. 
\end{align*}
\]

(S98)

Here all the notations correspond to the ones in Section 5.

From the last two equations in system (S98) we obtain the expression for the nonlinear part \( \Pi_{\omega} \) of the momentum flux tensor. For the \( xy \)-component we have:

\[
\Pi_{\omega,xy}(y) = -m_0 \eta_{xx} \frac{dV_\omega(y)}{dy} + \\
\frac{\tau_2}{1 + 4\omega_c^2 \tau_2^2} \left[ \left< \delta F_{2,xy}(y, t) \frac{\partial \Pi_{xy}(y, t)}{\partial t} \right>_{\omega} - 2\omega_c \tau_2 \right].
\]

(S99)
Here the nonlinear terms in the brackets \( \langle \ldots \rangle_\omega \) again contain the two contributions with the different combinations of the dc and ac linear responses:

\[
\begin{align*}
\left\langle \delta F_{2,xx}(y, t) \frac{\partial \Pi_{xx}(y, t)}{\partial t} \right\rangle_\omega &= 0 \\
&= \left[ a \left\langle \Pi_{1,xx}(y, t - T) \frac{\partial \Pi_{1,xx}(y, t)}{\partial t} \right\rangle \right. + \left. b \left\langle \Pi_{1,xy}(y, t - T) \frac{\partial \Pi_{1,xx}(y, t)}{\partial t} \right\rangle \right] dV_0(y) \frac{dy}{dy} \\
&+ c \left\langle \Pi_{1,xx}(y, t - T) \frac{\partial \Pi_{1,xy}(y, t)}{\partial t} \right\rangle + d \left\langle \Pi_{1,xy}(y, t - T) \frac{\partial \Pi_{1,xy}(y, t)}{\partial t} \right\rangle dV_0(y) \frac{dy}{dy}.
\end{align*}
\]

At the considered regime of high frequencies and magnetic fields, \( \omega > \omega_c \gg 1/\tau_2 \), the viscosity coefficients are related as:

\[
-i \omega \eta_{xy}(\omega) = 2 \omega_2 \eta_{xx}(\omega).
\]  

(S102)

After substitution of the expression for \( \Pi_1 \) via \( V_1 \) and averaging over the interval \( t - T < t' < t \), the values in Eqs. \( \text{(S100)} \) and \( \text{(S101)} \) takes the form:

\[
\left\langle \Pi_{1,ij}(y, t - T) \frac{\partial \Pi_{1,kl}(y, t)}{\partial t} \right\rangle = Q_{ij,kl} R(y)
\]  

(S103)

where

\[
\begin{align*}
Q_{xx,xx} &= \frac{2}{\omega} \sin\left(\frac{2\pi \omega}{\omega_c}\right) \left\{ \frac{4 \omega_c^2}{\omega^2} \right. \\
Q_{xy,xy} &= \frac{4 \omega_c \cos\left(\frac{2\pi \omega}{\omega_c}\right)}{\omega}
\end{align*}
\]

(S104)

\[
Q_{xx,xy} = Q_{xy,xx} = 4 \omega_c \cos\left(\frac{2\pi \omega}{\omega_c}\right),
\]

(S105)

and

\[
R(y) = \left| \eta_{xx}(\omega) \right|^2 \left\{ \frac{dV_1(y)}{dy} \right\}^2
\]

(S106)

Substitution of formulas \( \text{(S99)-(S106)} \) for \( \Pi_{\omega,xy} \) into the first of equations \( \text{(S98)} \) leads to the final equation for the nonlinear component \( V_\omega = V_\omega(y) \) of the dc velocity:

\[
\frac{d^2 V_\omega}{dy^2} = H(y),
\]

(S107)

where \( H(y) \) is given by Eqs. \( \text{(S99)-(S105)} \). Equation \( \text{(S107)} \) should be solved with the diffusive boundary conditions on the component \( V_\omega(y) \): \( V_\omega|_{y=\pm W/2} = 0 \). The result takes the form:

\[
V_\omega(y) = \frac{e^3 E_0 E_f^2}{m_0 (v_F^2) \omega} \eta_{xx}(\omega) \Phi(\omega, \omega_c) J(y),
\]

(S108)

where the factor \( J(y) = J(y, \lambda) \), determining the shape of \( V_\omega(y) \), is the same [Eq. \( \text{(S70)} \)], as for the inelastic contribution and the factor \( \Phi(\omega, \omega_c) \) has the form:

\[
\Phi = 4 \left\{ \left[ -a \omega_c \tau_2 + 2d \left( \frac{\omega_c}{\omega} \right)^2 \right] \sin\left(\frac{2\pi \omega}{\omega_c}\right) + \\
+ \left[ -2b \omega_c \tau_2 + c \right] \frac{\omega_c}{\omega} \cos\left(\frac{2\pi \omega}{\omega_c}\right) \right\}.
\]

(S109)

We remind that the coefficients \( a, b, c, d \) are proportional to the probability \( P \) \( \text{(S14)} \) for two quasiparticles in a pair to make a complete cyclotron rotation without a collision with a third quasiparticle.

The resulting dependence of the elastic contribution to the sample photoconductivity on magnetic field at fixed \( \omega \) exhibits MIRO-like oscillations at \( \omega_c < \omega / 2 \) and a huge peak near \( \omega_c = \omega / 2 \) [see Fig. 4(A) in the main text]. By the last feature, it differs from the inelastic contribution, which has no peak at \( \omega_c = \omega / 2 \) [see Fig. 3(A) in the main text].
7. Discussion of results and model

7.1. Independence of photoresistance on the sign of circular polarization of radiation

In experiments [13, 14] it was demonstrated that the dependence of photoresponses $\Delta \sigma_{\omega}$ and $\Delta \varphi_{\omega}$ on the polarization helicity "±" is absent in ultra-high quality GaAs quantum well samples with low defect densities and very large experimental values of mobilities (see discussion in the main text). In previous Sections 5 and 6, we have shown that for purely hydrodynamic flows in narrow samples, $W < l_{p}$ [S1], the viscoelastic contribution dominates in the ac flow, the component $E^{ext}_{1,y}$ of the external electric field is screened inside the sample, and the velocity $V_{1}||e_{x}$ becomes independent of $E^{ext}_{1,y}$ and the helicity of polarization. Thus within our theory photoresponses $\Delta \sigma_{\omega}$ and $\Delta \varphi_{\omega}$ also are independent on the polarization helicity "±".

We remind that for Ohmic flows in wide bulk samples the situation is totally different. In theory [12] and other theories for disordered samples it is implied that the ac electric field acting on independent electrons in Ohmic samples is just the external radiation field $E_{1}(t)$. Unlike the current hydrodynamic model, the memory term due to extended collisions of electrons with localized defects in the equation for $\partial V/\partial t$ contains not the mismatch of the deformation, $\Delta(r,t)$, but the mismatch of the electron position, $r_{0}(t)$, after one cyclotron rotation, $\Delta(t) = r_{0}(t) - r_{0}(t - T)$. Such mismatch $\Delta \neq 0$ arises due to the force $eE_{0} + eE_{1}(t)$ from external electric fields, directly acting on electrons between their collisions with defects. Obviously, $\Delta(t)$ and the resulting photoconductivity strongly depend on the helicity ± of the polarization of $E_{1}(t)$ [4, 12].

Thus, the independence of the photoresistance of the sign ± of circular polarization is a necessary (but not sufficient) evidence of the formation of a high-frequency hydrodynamic flow of a viscous electron fluid. The dependence on ± can appear in sufficiently wide samples, $W \gtrsim l_{p}$, owing to the formation of a substantial plasmonic component in $V_{1}$, or at a sufficiently large strength of disorder when the main contribution to photoconductivity becomes related to the scattering of single quasi-particles on defects or sample edges.

In Ref. [4] the following idea about the most typical origin of the absence of the dependence of MIRO on the helicity of the circular polarization of an incident radiation field $E^{ext}_{1}(t)$ was formulated. The difference of the polarization of the actual ac field $E_{1}(r,t) = E^{ext}_{1}(t) + E^{inst}_{1}(r,t)$, which is "felt" by 2D electrons, and the polarization of the incident field $E^{ext}_{1}(t)$ can be induced by some elements of an experimental setup. This can result in an independence (more exactly, a weak dependence) of MIRO on the sign of the polarization of $E^{ext}_{1}(t)$. However, it was not indicated in Ref. [4] that this modification of the polarization can be a consequence of the screening of one of the components of the incident field $E^{ext}_{1}(t)$ by the internal ac field $E^{inst}_{1}(r,t)$ originating from the redistribution of the very 2D electron fluid density $\delta n(r,t)$ and, thus, can be inextricably related with the mechanism of MIRO. It is noteworthy that in Ref. [13] it was experimentally demonstrated that particular setups of experiments (shapes of samples, designs of ac radiation sources) actually can sufficiently change the sensitivity of the MIRO effect relative to the polarization of the external radiation $E^{ext}_{1}(t)$. However, in experiment [13] an almost complete independence of photoresistance on the helicity of the circularly polarized radiation, which was observed, for example, in Ref. [13], was not achieved [66]. In this way, an experimental setup and other external factors, apparently, can strongly weaken the dependence of photoresistance on the sign ±, but not almost absolutely eliminate it, as it was observed in Ref. [13] and other works.

In Refs. [16, 17] theories of another type for 2D electron magnetotransport were developed, in which the photoresistance is associated with the ac-field-induced redistribution of the electron density near sample contacts [16] or around localized defects inside a sample [17]. Although these theories lead to magnetooscillations of photoresistance with a weak dependence on the sign of the circular polarization of the incident radiation, there exist substantial problems in the comparison of other predictions of those theories with experiments on MIRO. For example, there is a discrepancy between the shapes of the predicted oscillations and of the oscillations observed in experiments. The latter ones are often sinusoidal, whereas in Refs. [16, 17] the sinusoidal shape of magnetooscillations was not obtained.

7.2. Mixed hydrodynamic-Ohmic flows

Below, in this and next Subsections 7.2 and 7.3, we discuss how the particular violation of the "maximal" conditions of applicability of the developed purely hydrodynamic model (the absence of bulk disorder and a very large strength of the interparticle interaction) may be relaxed so that the obtained above key results on photoresistance of a highly viscous fluid remain qualitative valid.

First, in this subsection, we discuss the case of a disordered sample in which the hydrodynamic contributions to the ac linear in $E_{1}$ and the dc nonlinear in $E_{1}$ components of the flow components still persist, despite of the dominance of the Ohmic contribution in the main dc flow component, linear in $E_{0}$. We will estimate the parameters corresponding to such "mixed" regime.

At high magnetic fields, $\omega_{c}T_{2} \gg 1$, the magnitude of a purely hydrodynamic dc flow unlimitedly increases, being proportional to $\omega_{c}^{2}$ [Eqs. (S17) and (S18)]. Thus, the purely hydrodynamic dc averaged resistance, $\varrho_{0} = E_{0}W/I_{0}$, strongly decreases with the increase of $\omega_{c}$.
and $W$:

$$g_0 \propto 1/(W\omega_c)^2. \quad (S110)$$

Such divergence is limited by the formation of an Ohmic flow in central region of the sample where the scattering of quasiparticles on disorder (and/or phonons) becomes to dominate and to determine the magnitude of the current and the resistance $g_0$ (see Refs. [29, 59] and Fig. S5(a)).

The Ohmic contribution in the dc current $I_0$ dominates if the sample width is much larger than the Gurzhi length $l_G = l_G(\omega_c)$:

$$W \gg l_G, \quad l_G = \sqrt{\eta_{xx}(\omega_c)/\omega}, \quad (S111)$$

where $\tau_{tr}$ is the total momentum relaxation time due to the scattering of quasiparticles on bulk disorder and acoustic phonons. In this case, the Ohmic part of the dc flow is located in the central sample region:

$$-W/2 + l_G \leq y \leq W/2 - l_G, \quad (S112)$$

or, more precisely, $W/2 - |y| \gg l_G$. The resulting hydrodynamic-Ohmic magnetoresistance has the form (see Fig. S5(a) and discussion in Ref. [29]):

$$g_0(B) \sim \eta_{xx}(B) W^2 + \frac{1}{\tau_{tr}}, \quad (S113)$$

One can show that at the considered high-frequency regime, $\omega_c \sim \omega \gg 1/\tau_{tr}$, the inequality (S111) is consistent with inequality (S44), which ensures the dominance of the viscoelastic part in the ac component, provided the following condition is fulfilled:

$$s \gg v_F^0 \sqrt{\frac{\tau_{tr}}{2\tau}}, \quad (S114)$$

Here $s \sim \sqrt{n_0}$ is the velocity of plasmons for the structures with 2D electrons near a metallic gated. Although for literal applicability of the presented above hydrodynamic description of the viscoelastic component one needs $v_F^0 \gg v_F$, the last two parameters, apparently, do not differ too much at realistic electron densities $n_0 \sim 10^{11} - 10^{12}$ cm$^{-2}$ in GaAs quantum wells [21, 59] (see discussion in the main text). Herewith the plasmon velocity $s \sim 1/\tau_{tr}$ is usually far greater than $v_F$ [12]. Thus condition (S114) can be typically fulfilled in such GaAs structures. It follows from the above conditions that for the frequencies $\omega \sim \omega_c$ in the interval:

$$v_F^0 \sqrt{\frac{\tau_{tr}}{W}} \ll \omega \ll \frac{s}{W}, \quad (S115)$$

the dc current component $I_0$ is predominantly due to the Ohmic flow $V_0(y) \approx \text{const}$ in the central (bulk) region (S112), whereas the ac flow component $V(y, t)$ is still formed mainly by the magnetosonic waves in the whole sample.

For such flows, only in the near-edge regions:

$$W/2 - |y| \lesssim l_G, \quad (S116)$$

both the dc and ac flow components $V_0(y)$ and $V_1(y, t)$ are controlled by viscosity and viscoelasticity. In the bulk region (S112) the dc component of the flow is homogeneous, $V_0(y) = \text{const}$, thus values $\sigma_0$, (S100) and $\sigma_{\text{rel}}$, determining the relaxational and the elastic parts of $V_\omega(y)$, are zero, and the bulk region provides no hydrodynamic contributions to photoresistance.

Now we are able to estimate the magnitude of the hydrodynamic contribution to photoconductivity $\Delta\sigma_\omega$ from the near-edge regions (S110), originating only from the memory effect in the interparticle scattering (see Section 2).

For a rough estimate, we can use the derived above formulas for $\Delta\sigma_\omega$ of a Poiseuille flow, changing in them the sample width $W$ on the characteristic width of the near-edge regions, $l_G$ (S111). Hereafter we neglect the contribution to photoconductivity from the bulk region (S112), which is controlled by disorder. Our estimations, based on Eqs. (S90) for $\Delta\sigma_\omega|_{W=\ell_G}$ and on the formulas from Refs. [4] and [12] for photoconductivity due to the memory effects in the scattering on disorder, show that such assumption can be realistic.

In this way, when the Ohmic contribution in the dc conductivity $\sigma_0$ and the hydrodynamic contribution $\Delta\sigma_\omega$ of the near-edge layers (S110) in the nonlinear photoconductivity dominate, the averaged photoresistivity in the linear by ac power regime, $\Delta\sigma_\omega \sim E_1^2$, takes the form:

$$\Delta\sigma_\omega = -\Delta\sigma_\omega/\sigma_0^2, \quad (S117)$$

where $\Delta\sigma_\omega$ is calculated by Eqs. (S94)–(S96), (S108), applied to the near-edge layers ($W = \ell_G$), while the averaged dc conductivity $\sigma_0$ of a sample is close to the Ohmic conductivity of the bulk region (S112):

$$\sigma_0 = e^2 n_0 \tau_{imp,tr}/m. \quad (S118)$$

Note that the last value has no dependence on magnetic field, the electron temperature, and the sample width.

Equation (S117) leads to an estimate for the dependence of the photoresistivity on magnetic field $B \propto \omega_c$, ac frequency $\omega$, and the electron temperature $T_e$.

For example, for the inelastic relaxation contribution $\Delta\sigma_{\omega}^{\text{rel}}$ due to the extended collisions of quasiparticle pairs in the interval of the magnetic fields corresponding to the inequality $1/\nu \ll l_G(\omega_c) \ll 1/\mu$ [and, thus, to the applicability of Eq. (S90), equations (S94)–(S96), (S111), (S117), and (S118) yield:

$$\Delta\sigma_{\omega}^{\text{rel}} = \sin^2 \left(\frac{2\pi\omega}{\omega_c}\right) \Delta\sigma_0^{(1)} + \sin \left(\frac{2\pi\omega}{\omega_c}\right) \Delta\sigma_0^{(2)}, \quad (S119)$$
where

$$\Delta q^{(1)}_\omega = -\frac{A_O}{\omega^2 \omega_c^2} \left( 1 - \frac{\omega_c^2}{\omega^2} \right) e^{-A_k T_e^2/\omega_c} \quad (S120)$$

$$\Delta q^{(2)}_\omega = \frac{4\pi A_O}{\omega^3 \omega_c} e^{-A_k T_e^2/\omega_c}. \quad (S121)$$

Here the coefficients $A_O$ and $A_k$ are independent of $\omega$, $\omega_c$, $T_e$ and are easily calculated from Eqs. (S10), (S11), and (S18). It is seen from Eqs. (S120) and (S121) that at $1 < \omega/\omega_c < 4\pi$ the first amplitude, $\Delta q^{(1)}_\omega$, is numerically smaller than the second one, $\Delta q^{(2)}_\omega$.

We note that there may be many sources of other temperature dependencies of photoresistance as compared with the last result (S119)-(S121). One of the them, proposed in Ref. [7], is related with heating of the 2D electron system by microwave radiation. Others can be related to, for example, with the thermoelectric effects (without or with the heating of the electron system).

The last effects for the case of hydrodynamic viscoelastic flows can be studied on base of the theories of thermoelectric magnetotransport and heat release without memory effects in space-inhomogeneous electron flows (see, for example, Refs. [62, 63]) as well as of the electron energy relaxation in quantum wells due to the interaction with acoustic phonons (see, for example, Refs. [65, 75]).

Beside this, the ballistic contributions to the observed photoresponses, induced by the scattering of quasiparticles on the edges and/or macroscopic defects can reduce the temperature dependencies of all types (for the consideration of the ballistic contribution in dc hydrodynamic magnetotransport see, for example, Refs. [72, 74]).

### 7.3. Case of weak interparticle interaction

In this subsection, we discuss the possibility of weakening of the condition (S11) of a very large strength of the interparticle interaction, which allows using simple viscoelastic equations [77] to describe high-frequency dynamics of the fluid.

In Refs. [41, 42] transverse shear-stress waves in the presence of magnetic field were studied within the hydrodynamic Navier-Stokes-like equations (See Sections 2 and 4) for a strongly non-ideal electron Fermi liquid, in which $F_1 \gg 1$ and $v_F \ll v_F^T$. (S11). We remind that the hydrodynamic description of flows by the variables $n$, $V$, and $\Pi$ implies the predominance of the zero, the first, and the second harmonics by the velocity variable $v$ in the quasiparticle distribution function $f(r, v, t)$. Possibly, the results of Refs. [41, 42] remains qualitatively valid also for the electron fluid with moderate interparticle interaction, when $F_m \sim 1$.

Indeed, it is known that shear stress transversal sound can be excited in an electron fluid in zero magnetic field not only at the very low densities $n_0$ corresponding to $r_s \gg 1$ and to large Landau parameters: $F_m \gg 1$, but also at moderate densities, when $F_m \sim 1$ [67]. Here $r_s$ is the dimensionless parameter characterizing the relative magnitude of the interparticle Coulomb interaction energy.

Namely, the transverse waves in 3D and 2D Fermi cases in zero magnetic field can be excited if the Landau parameters $F_m$ are greater than some critical values $F_{m,c}^{3D}/F_{m,c}^{2D} \approx 1$, herewith $F_{m,c}^{2D} > F_{m,c}^{3D} \approx 10^4$ [67]. In the latter case, an ac flow is described not by the hydrodynamic variables $n$, $V$, and $\Pi$, but by the whole distribution function $f(r, v, t)$ of the Fermi-liquid quasiparticles. A decomposition of such $f(r, v, t)$ by the angle $\varphi$ of the quasiparticle velocity $v$ contains many comparable angular harmonics $e^{im\varphi}$.

However, we note that the interaction parameter $r_s$ is near unity for many of GaAs quantum wells (for example, for the structures studied in experiments [13, 19, 21]), thus the Landau parameter $F_1$ turn out to be not large, possibly, much smaller than unity. Therefore a question of the microscopic structure of the highly correlated fluid and the maximally general condition of applicability of the Navier-Stokes-like equations of the form of equations (S7) remains open.

As it is discussed in the main text, one solution of the problem of finding the applicability conditions of the developed theory to these structures with $r_s \sim 1$ may be as follows.

It is possible that for some ranges of sample widths $W$, magnetic fields $B$, and frequencies $\omega$, equations (S7) remain qualitatively applicable even at $r_s \approx 1$, that is, for the Fermi gas systems. For arbitrary relationships between $R_e$, $\omega$, and at $W \gg R_e$, $v_F/\omega$ the flows of weakly interacting electrons calculated within the kinetic equation can turn out to be qualitatively similar to the predictions of the hydrodynamic theory, provided there is no source of small-scale disturbances with sizes $\Delta x \ll R_e$.

For example, this statement was demonstrated in Ref. [74] for the near-edge regions of a Poiseuille flow of a 2D electron fluid at $\omega \tau_2 \ll 1$. This similarity is due to the fact that the distribution function harmonics $f_m$ can decrease moderately quickly with $m$, therefore the contributions of its hydrodynamic part ($m \leq 2$) and the ballistic part ($m > 2$) can give comparable contributions to the flow characteristics. As other example, we also cite Refs. [72, 73], where it was demonstrated that in the bulk region of pure sample ($W \ll l_2$), the hydrodynamic regime of two-dimensional electron transport is formed starting from magnetic fields, when the cyclotron diameter $2R_e$ becomes smaller than $W$.

There may be also another reasons of applicability of the hydrodynamic viscoelastic equations (S7).

We hypothesize that it is possible that, even at moderate and small parameter $r_s$ and corresponding small $F_m$, the emergence of pair correlations (described in the classical mechanics approach as extended collisions, see Fig. S1), may be responsible for the formation of a
viscoelastic system of 2D electron, being similar by its macroscopic properties to the Fermi liquid with $F_m$.

The reasons why the pair correlations lead to viscoelasticity may be described within two approaches. First, a rigorous microscopic description of such paired electrons (see Figs. S1) cannot be reduced to the dynamics of the quasiparticle distribution function according to the Boltzmann kinetic equation for the one-particle distribution function of quasiparticles. A possible way of description of highly correlated systems of quasiparticles is based on the dynamic equations for the two-particle correlation function. For example, such equations were derived in Ref. [21] for non-degenerate Boltzmann gas. Generally speaking, similar equations for time evolution of the two-particle correlation function of quasiparticles can be derived for an electron Fermi liquid at $r_s \sim 1$ in a non-zero magnetic field. Possibly, they may lead macroscopic hydrodynamic equations of the type of Eq. (S7) with large parameter $v_F^s$, $v_F^p \gg v_F$.

Second, formation in magnetic field of paired electrons (see Figs. S1) may dramatically enhance the degree of quantum coherence in the electron fluid. As a result, a reconstruction of the Fermi-liquid ground state and quasiparticles of 2D interacting electrons may appear. These effects for quantizing magnetic fields were studied in Refs. [52, 53]. Such reconstruction can substantially change the parameters and criteria of applicability of the Navier-Stokes-like macroscopic equations (S7). In this connection, even weakly interacting 2D electron-like quasiparticles in classically-strong magnetic fields may be described within some effective Fermi-liquid-like model with large effective Landau parameters: $\tilde{F}_m$, being not equal to the result of calculation of $F_m$ within, for example, the random phase approximation method (which underestimated the role of correlation effects).

8. Additional comparison with experiments

8.1. Simultaneous observation of peak at doubled cyclotron frequency and giant negative magnetoresistance

First of all, we note that a very pronounced giant negative magnetoresistance and the irregular shape of MIRO, including the large peak near $\omega = 2 \omega_c$, are usually observed in the same highest mobility samples up to the lowest temperatures (see experimental works [13, 14, 18, 19, 21] and Figs. 3, 4 in the main text).

This fact, apparently, point out on the realization of the hydrodynamic transport regime in both the dc and ac components of a flow in 2D electron systems in the same samples. In particular, the giant negative magnetoresistance, which was initially predicted for the flows of an ordinary viscous electron fluid [29–30], is also a characteristic property of a highly correlated viscous fluid. In this connection, we cite the experimental data on magnetoresistance and magneto-photo-resistance and present the results of our calculations for these two physical values on the neighbour panels of Figures 2-4.

8.2. Dependencies of oscillating part of photoconductivity and photoresistance on electron temperature

The temperature and magnetic field dependencies of the amplitude of MIRO observed in Ref. [68] for a high-mobility GaAs quantum very well correspond to Eq. (S121), including the absence of a temperature dependence in the pre-exponent factor $A_0$. The formula used in Ref. [68] for fitting of the MIRO amplitude was just the same as Eq. (S121).

Herewith we note that, a substantial temperature dependence of the pre-exponent amplitude of photoresistance oscillations was reported in experimental works [69, 70]. This result may not contradict to the correspondence of theoretical result (S119)-(S121) to experiment [68] by the following reasons. First, experiment [69] was performed for the GaAs structures with double quantum wells, where all the discussed effects may be quite different. Second, in Ref. [70] rather different formulas for fitting the MIRO curves were used, as compared with Eq. (S119) and the formulas used in Ref. [68], so the results of Ref. [70] cannot be directly compared with the results of Ref. [68] and the predictions of our theory.

In this way, the results on the temperature dependence of MIRO experimentally obtained in Ref. [68] can be explained by the model of a mixed Ohmic-hydrodynamic flow, developed in Section 7.2 and leading to photoresistance (S119)-(S121).

8.3. Behavior of MIRO with varying of ac frequency

With the increase of the frequency $\omega$, the shape of the oscillations of $\Delta \rho_\omega$ as the function of the ratio $\omega/\omega_c$ is retained in our theory [see Eqs. (S119)-(S121)]. The pre-
exponent factor in the larger contribution $\Delta q_0(2)$ at a fixed ratio $\omega/\omega_c$ is rapidly suppressed with $\omega$ as $\sim 1/\omega^4$:

$$
\frac{\Delta q_0(2)}{(\omega/\omega_c)} e^{-\pi A_0/\omega^2} = \frac{4\pi A_0}{\omega^2}.
$$

(S122)

Beside this, some destructive “geometric” effects from inhomogeneities of the sample edges and from possible macroscopic defects inside the sample on the formation of the standing waves in the ac flow component become more and more substantial with the increase of $\omega$. Indeed, the characteristic wavelength, $l_s = v_p/\omega$, which should be greater than the sample inhomogeneities in order to form the standing magnetosonic waves, decreases with $\omega$. So a suppression and a smearing of the ac flow component and of its contribution to magnetooscillations of $\Delta q_0$ are expected with the increase of $\omega$.

This prediction about the frequency dependence of magneto-photo-resistance of the radio frequency is in a good agreement with the measurements of MIRO in a high-mobility GaAs quantum well at different frequencies $\omega$ in Ref. [21].

8.4. Comparison of departure time $\tau_q$ with the shear stress relaxation time $\tau_2$

The departure times $\tau_q$, which appear in Eqs. (S2) and (S14), controls the amplitude of MIRO via the probability $P = e^{-T/\tau_q}$. It is of interest to compare the experimental value of $\tau_q$ with the experimental value of the shear stress relaxation time $\tau_2$, which determine the magnitude and the magnetic field dependence of the dc viscosity coefficients $\eta_{xx,xy}|_{\omega=0}$.

In Fig. S6 we present experimental data for the GaAs quantum wells in the two different samples with the close electron densities $n_0$ corresponding to the intermediate interaction parameter $r_s \approx 1.05$. In panel (a) we cite the data on the departure scattering time $\tau_q(T_e)$ obtained in Ref. [68] from the measurements of the temperature dependence of the MIRO amplitude $\Delta q_0$. This value was supposed to be proportional to $P = e^{-T/\tau_q}$ with the departure rate $1/\tau_q \propto T$. This contribution due to scattering on disorder, $1/\tau_q|_{T \to 0} = 1/\tau_{dis,q}$, and the interparticle contribution being quadratic by temperature: $1/\tau_{ee,q}(T_e) \sim T_e^2$.

In panel (b) we plot experimental data on the shear stress relaxation time $\tau_2(T_e)$ extracted from us by the giant negative magnetoresistance of 2D electron fluid measured in Ref. [27] in a GaAs quantum well. To obtain the experimental values of $\tau_q$, we used the fitting of the negative magnetoresistance by the procedure proposed in Refs. [29, 59]. Both the rates $1/\tau_q(T_e)$ and $1/\tau_2(T_e)$ are indeed approximately quadratic by the electron temperature $T_e$ and have some residual values in the limit $T_e \to 0$. It is seen that the temperature-dependent part of $1/\tau_q$ is in several times greater than the one of $1/\tau_2$.

This is consistent with the calculations of these rates, $1/\tau_{ee,q}(T_e)$ and $1/\tau_{ee,2}(T_e)$, within the kinetic equation exactly accounting the interparticle scattering [58, 59].

Comparable experimental values of $\tau_q$ and $\tau_2$ and their similar temperature dependencies as well as the inequality $\tau_q < \tau_2$ qualitatively evidence in favor of the hydrodynamic nature of the magnetoresistance and magnetooscillations of the photoresistance in the samples studied in Refs. [27, 68]. However, as it was mentioned in Section 7.3, for a quantitative description of the experimental data for samples with the interaction parameter $r_s = 1$, a rigorous accounting of the correlations due to formation of the quasiparticle pairs is needed.

We note that the values $2\pi/\omega \tau_q$ are of the order of unity [see Fig. 6(a)]. Herewith after interpolation of the dependence $2\pi/\omega \tau_q(T_e)$ in Figs. 6(a) to higher $T_e$, this value becomes greater than unity. From comparison of the plot of $2\pi/\omega \tau_2(T_e)$ in Figs. 6(b) and the interpolated dependence in Figs. 6(a) we see that the inequality $\tau_q < \tau_2$ is fulfilled at the temperatures $T_e > 8$ K, in accordance with the condition (S15) of applicability of our model of the memory effects in the interparticle scattering.

8.5. Role of disorder in narrow samples

Let us discuss the role of “residual disorder” in the relatively narrow samples, $W \ll l_p$, in the limit of lowest temperatures $T_e$, when any interparticle scattering is switched off.

At $T_e \lesssim 1$ K, the giant negative magnetoresistance as well as photoresistance of GaAs quantum wells re-
tain their characteristic properties, while the scattering times $\tau_1$ and $\tau_2$ obtained from fitting of experimental data attain some nonzero residual values (see Fig. S6 and Refs. [21, 28, 29, 59, 68]). According to the results of Refs. [28, 29, 59], it is believed that at $T_e \to 0$ the electron transport is still hydrodynamic-like, described by equations (57). In particular, in sufficiently narrow samples, $W \lesssim l_G, l_p$ [here $l_G = \sqrt{(\eta_{xx} \tau_T)}_{T_e=0}$], the viscoelastic part of the flow is to dominate. Herewith the relaxation of the momentum flux density $\Pi$, including the retarded relaxation, in this regime is determined by the scattering of electron-like quasiparticles on bulk disorder.

For applicability of hydrodynamics, one needs that the shear stress relaxation time on disorder, $\tau_{imp,2}$, is much shorter than the momentum relaxation time on the same disorder, $\tau_{imp,tr}$:

$$\tau_{imp,2} \ll \tau_{imp,tr}. \tag{S123}$$

This inequality follows, for example, from the condition $l_G|B=0, \omega=0| \ll \nu_T \tau_{imp,tr}$, which guarantees that the near-edge hydrodynamic hydrodynamic layers can be formed in wide samples, $W \gg l_G$. From the analysis of experimental data on the giant negative magnetoresistance it was concluded that this inequality, actually, is often realized in high-mobility GaAs quantum wells with a good margin. (see, for example, Refs. [28, 29, 59]).

According to the consideration of Refs. [29, 40–42], in this hydrodynamic-like regime, the same dependence of the ac viscosity coefficients $\eta_{xx/xy}(\omega)$ on the frequencies $\omega$ and $\omega_c$ as well as the possibility of the excitation of shear stress transverse waves is to retain. However the memory effects in the scattering of quasiparticles on disorder, being similar in some aspects to the ones studied in Refs. [11, 12], will affect the retarded relaxation of $\Pi$ and the nonlinear components of the flow (instead of the memory effects in the interparticle scattering studied in this work).

In this way, a microscopic justification of the possibility of such hydrodynamic-like regime, including Eq. (S123) (based on classical or quantum accounting of correlations due to the paired electrons, see Section 7.3) is needed for more deep understanding of magneto-transport in 2D electron fluids in realistic high-mobility samples as well as for a justification of the hydrodynamic explanation of the discussed above experiments in the limit $T_e \to 0$.

### 8.6. Role of disorder in wide samples

As we discussed in Introduction of the main text, in sufficiently wide samples, $W \gg l_p, l_G$, photoresistance and, in particular, MIRO, apparently, has a bulk nature and are usually controlled by the purely disorder mechanisms, related to the quantum or the classical dynamics of independent 2D electrons. These mechanisms were extensively studied to date [3, 8, 11, 12, 15]. As it was mentioned in Introduction, they explain many properties of MIRO in many experimentally studied samples. For example, the dependence of MIRO of the sign of circular polarization of radiation, corresponding to the magneto-transport theories in disordered samples, was clearly observed. In particular, in recent work [15] the dependence of photoresistance on the polarization of radiation and on the geometry of the setup, which determines the degree of polarization of radiation actually entering the sample, was studied in detail in different experiment setup.

Note that in the GaAs high-mobility quantum wells the macroscopic oval defects are often present [? ? ?]. The can appear in the growth process [?] or made artificially [? ? ?]. Apparently, they do not destroy the hydrodynamic regime in some high-quality and sufficiently wide samples (with a weak “residual” disorder between the oval defects), but strongly modify the shape of the ac and the dc viscous flows of the electron fluid.

The oval defects lead to the decrease of the actual width of dc flows of the 2D electron fluid [22]. The width becomes equal not to the sample width $W$, but is to be estimated as the mean distance between the oval defects, $W_{eff} \sim n_\phi^{-1/2}$, $W_{eff} < W$ (here $n_\phi$ is the 2D density of defects) [23]. It is natural to think that the oval defects can also significantly affect the high-frequency and the dc nonlinear components of the electron fluid flow in a similar way, namely: reducing the effective width of the conductive channel.

Possibly, such effects of shortening of the width of an electron fluid flow due to the macroscopic localized defects were also observed in recent experiment [21] for wide sections of examined samples in dc and ac components of the flow.

### 8.7. Non-hydrodynamic versus hydrodynamic mechanisms of magnetooscillations and peaks in photoresistance of 2D electrons

Let us discuss recent work [22], in which photoresistance and photovoltage of pure graphene samples in magnetic field were studied, together with recent work [21], in which magneto- and photoresistance of 2D electrons in ultra-high quality GaAs quantum wells samples of different width were examined.

In experiment [22] a larger peak near the doubled cyclotron frequency $\omega = 2\omega_c$ and smaller peculiarities near higher harmonics, $\omega = n\omega_c$, $n \geq 3$, were observed in photoresistance and photovoltage of high-quality graphene samples. These peculiarities were explained as the manifestation of the conventional Bernstein modes (“charge cyclotron Bernstein modes”) which are excited in a 2D electron gas due to nonlocality of the internal ac electric fields, related to a redistribution of the electron density. The last one was induced in experiment [22] by the sharp contacts of the samples. A quantitative theory of the Bernstein waves in a 2D electron Fermi gas in magnetic field in the samples with inhomogeneous ac electric
fields was developed in Ref. [22] in order to explain the observed photoresponses.

In experiment [21], the giant negative magnetoresistance and the peak in photovoltage near \( \omega = 2\omega_c \), similar to the ones presented at Fig. 4 in the main text, were observed in high-quality GaAs quantum wells in samples with regions having different widths. For medium sample widths and at low radiation powers, the irregular shape of MIRO and the giant \( 2\omega_c \)-peak at were seen very well (see Fig. 4(b) in Ref. [21]).

We have the following comments on this experiments [22] and [21] and their explanation.

From the one hand, although the excitation of the Bernstein modes in samples with sharp contacts leads to strong peaks and peculiarities at \( \omega = n\omega_c \) in photoresponse and photovoltage [22], the calculated relative magnitudes of the peaks at \( n \geq 3 \) (as compared with the \( n = 2 \)-peak magnitude) are sufficiently larger than it was observed in that work for graphene samples (compare Fig. 3(f) and 3(g) in Ref. [22] and see also Fig. S3 in its Supplemental material).

From the other hand, in Ref. [21] it was experimentally shown that, in high-quality GaAs quantum wells samples of medium widths, the ratios of magnitudes of the \( 2\omega_c \)-peak and the higher harmonic peaks, \( n \geq 3 \), apparently being analogous to the one observed in Ref. [22], are also very large, similar to those ratios for graphene samples studied in Ref. [22] (see Fig. 4(b) in Ref. [21]).

Section 5 of Supplemental material of Ref. [22] presents experimental results on the polarization dependence of the photovoltage \( \Delta V_{ph} \) of a graphene sample. The measured dependence of \( \Delta V_{ph} \) on magnetic field in the region of the main harmonic of the cyclotron resonance, \( \omega = \omega_c \), strongly depends on the sign \( \pm \) of the circular polarization of radiation, while in the region near the doubled harmonic, \( \omega = 2\omega_c \), it is almost independent of the sign \( \pm \). Herewith there are no visible peculiarities on the photovoltage curves at \( \omega = 3\omega_c \) and higher harmonics in Figure S5 of this section. Such behaviour of the experimental data agrees very well with the predictions of the hydrodynamic theory developed in this work.

In this connection, we also note that the sharp contacts of the samples studied in Ref. [22] can lead not only to the formation of an inhomogeneous ac electric field, leading to the excitation of the Bernstein modes, but also to the formation of the viscoelastic contribution in spatially inhomogeneous flows of the electron fluid, exhibiting the \( 2\omega_c \)-resonance in the viscosity coefficient \( \eta_{xx}(\omega) \).

Let us discuss other properties of the results of Refs. [21] [22].

When the radiation power becomes very high, the magnitudes of all the peaks \( \omega = n\omega_c \) observed in Ref. [21] turn out comparable (see Fig. 4(c,d) in Ref. [21]). Apparently, this is due to the destruction of the collective motion of the viscous electron fluid by the strong external electric forces, that allows only the magnetoplasmon and Bernstein waves to be excited in the system of almost independent electrons (as these waves are related to perturbations of charge density and the corresponding large Coulomb forces). As it is seen from Fig. 4(c,d) in Ref. [21], such regime is strongly non-linear by the radiation field.

Note also that only the \( \omega = 2\omega_c \)-peak is observed without any peculiarities at \( \omega = 3\omega_c \) and \( \omega = 4\omega_c \) in experiment [21] for the narrowest 25 \( \mu \)m sample section at low ac field power, when the viscoelastic contribution apparently dominates. This fact is well seen after vertical scaling of Fig. 4(b) in Ref. [21]. Herewith for the wider 50 \( \mu \)m sample section, in which the magnetoplasmon component in the ac flow is to be larger, both the \( \omega = 2\omega_c \)-peak and a peculiarity at \( \omega = 3\omega_c \) are seen after vertical scaling.

Apparently, for the proper explanation of the magneto-photo-conductivity of the discussed samples it is especially important to verify the truth of the following statement (or disprove it). The shapes of the magnetic field dependencies of photoresistance \( \varrho_{ph}(B) \) and photovoltage \( \Delta V_{ph}(B) \) can differ significantly from the ones of the absorption power of radiation \( W(B) \). Therefore, in order to explain the experimental dependencies of photoresistance and photovoltage, it is necessary to calculate them within some nonlinear model of magnetotransport, while calculation of only the single absorbed power \( W(B) \) can be not enough to describe the dependencies of \( \varrho_{ph}(B) \) and \( \Delta V_{ph}(B) \). Namely, the last dependencies can differ very much from the shape of \( \varrho(B) \). In our opinion, this conclusion is to be made from a comparison of these dependencies obtained in experiments [21] [22], in the hydrodynamic theory of Ref. [22], as well as in the hydrodynamic theory of Ref. [21] and of the present work (see Figs. 3, 4 in the main text and Fig. S3, S5 here). Indeed the shape of the peak can differ from sample to sample, but, as it was discussed in Section 8.1, the appearance of a “giant” peak in the magneto-photo-resistance of the sample is accompanied by the appearance of a giant negative magnetoresistance.

From all above facts, apparently, one should make the following conclusions. On the one hand, for the samples with relatively strong disorder density and/or in the regime of large electron temperatures (or strong ac power heating), excitation of the conventional Bernstein modes leads to the appearance of the peaks and peculiarities in the measured photoresistance at \( \omega = n\omega_c \) with the amplitudes, those decreases not too fast with the increase of \( n \). On the other hand, observations in some high-quality samples of the giant peak in photoresistance near \( \omega = 2\omega_c \), being especially large as compared with the peculiarities at higher cyclotron harmonics, \( n \geq 3 \), together with simultaneous observations of the irregular shape of MIRO and the giant negative temperature-dependent magnetoresistance, evidence in favor of hydrodynamic regime of electron flows and formation of ac flows of a highly correlated 2D electron fluid.