Neutrino mass and mass hierarchy in various dark energy

Zhenjie Liu\textsuperscript{a} Haitao Miao\textsuperscript{a,1}

\textsuperscript{a}School of Physics and Astronomy, Sun Yat-sen University, 2 Daxue Road, Tangjia, Zhuhai, People’s Republic of China

E-mail: liuzhj26@mail2.sysu.edu.cn, miaoht3@mail2.sysu.edu.cn

Abstract. Combined with various cosmology observations, one can obtain constraints on the sum of the active neutrino masses $M_\nu$. However, the bounds on the sum of neutrino masses $M_\nu$ depend on the dark energy (DE) models. We consider three dark energy models, the cosmological constant ($\Lambda$CDM) model, a phenomenological emergent dark energy (PEDE) model and a model-independent quintessential parametrization (HBK). Based on these models with cosmic microwave background (CMB) data from Planck 2018, Baryon Acoustic Oscillations (BAO) measurements and Type Ia supernovae (SNe Ia) data, we obtain the bounds on total neutrino masses with the approximation of degenerate neutrino masses. In the HBK model, we conform the conclusion from a few pioneer works that the quintessence prior of dark energy tends to tighten the cosmological constraint on $M_\nu$. On the other hand, in the PEDE model, we get a larger $M_\nu$ and a nonzero lower bound. Besides, we also explore the correlation between three different neutrino hierarchies and dark energy models.

Keywords: neutrino masses, dark energy, neutrino mass hierarchy

\textsuperscript{1}Corresponding author.
1 Introduction

The standard model of particle physics predicts that neutrinos are massless, whereas the discovery of neutrino oscillations, a phenomenon where neutrinos can switch their flavour to others, suggests that they are massive. The neutrino oscillation experiments only can accurately measure the squared mass differences between two types of individual neutrino instead of their absolute masses. From neutrino oscillation data, we know the values of mass-squared splittings: $\Delta m_{21}^2 \equiv m_2^2 - m_1^2 \approx 7.54^{+0.26}_{-0.22} \times 10^{-5}$ eV$^2$, $|\Delta m_{31}^2| \equiv |m_3^2 - m_1^2| \approx 2.46^{+0.06}_{-0.06} \times 10^{-3}$ eV$^2$. As we do not know whether $\Delta m_{31}^2$ is positive or negative, two kinds of neutrino mass ordering, normal hierarchy (NH, $m_3 \gg m_2 > m_1$) and inverted hierarchy (IH, $m_2 > m_1 \gg m_3$), are possible. Additionally, the degenerate hierarchy (DH, $m_1 = m_2 = m_3$) is also widely used in the cosmological parameter estimations while it is not physical. Hence, which ordering is more favoured by the nature has become a hot topic. Furthermore, different orderings of neutrino mass have different minimums of their total masses ($M_\nu = \sum_i m_i$): for NH scheme given by $(M_\nu)_{\text{min}} = \sqrt{\Delta m_{21}^2} + \sqrt{|\Delta m_{31}^2|} \approx 0.06$ eV, while for IH scheme we have $(M_\nu)_{\text{min}} = \sqrt{|\Delta m_{31}^2|} + \sqrt{|\Delta m_{31}^2| - \Delta m_{21}^2} \approx 0.1$ eV. So, the absolute neutrino mass scale is also unknown.

Nowadays, cosmology has been an effective tool to detect neutrinos, which can provide the most robust bounds on the neutrino masses. Massive neutrino play an important role in some cosmic phenomena, such as the formation of the large-scale structure (LSS), the big bang nucleosynthesis (BBN), the anisotropy of the cosmic microwave background (CMB) etc [1, 2]. Neutrinos have distinct effects on the evolution of the universe and leave traces on the CMB power spectrum and LSS power spectrum, so we can find their signatures in cosmological observations. Gerstein and Zeldovich were the first to derive the cosmological upper limit on the total neutrino masses [1]. Since then, further investigations have been carried out in the literature [2–10]. Up to now, current cosmological observations are primarily sensitive to the sum of neutrino masses. However, the bounds on the sum of neutrino masses depend on dark energy model [11–13]. According to the recent works [14–16], the constraints on $M_\nu$ in quintessence model are found to be tighter than those obtained in $\Lambda$CDM model. There are also studies talk about neutrino hierarchy from cosmology [3, 8, 14, 17–26]. In
most studies, though the current cosmological data is not sensitive enough to distinguish the two hierarchies, normal hierarchy and inverted hierarchy, there is a slight preference to NH [3, 8, 12, 14, 19, 20, 22, 25, 27].

Recently, a simple phenomenological emergent dark energy (PEDE) model has been proposed to resolve the Hubble Tension [28, 29]. This model assumes that dark energy does not exit effectively in the past but emerges at later time. The equation of state (EOS) \(w_{de}\) of dark energy goes from \(-\frac{2}{3}\ln 10\) to \(-1\) in the past and to \(-1\) in the future. Additionally, an analytic approximation of EOS has been derived based on a minimally coupled and slowly or moderately rolling quintessence field \(\phi\) with a smooth potential \(V(\phi)\) (HBK model) [30]. Based on the PEDE model, the HBK model and the \(\Lambda\)CDM model with CMB data from Planck 2018, BAO measurements and SNe Ia, we investigate the bounds on the sum of neutrino masses \(M_\nu\) with the approximation of degenerate neutrino masses. We also explore the correlation between three different neutrino hierarchies and dark energy models.

This article is organized as follows. Sec. 2 describes the methodology and the observational data used in our analysis. We discuss our results in Sec. 3. In Sec. 4, conclusion and discussion are given.

## 2 Methodology and Datasets

In our analysis, we apply three dark energy models with different types of dark energy evolution. To perform bayesian analysis of the cosmological dataset, we modify the publicly available markov chain monte carlo (MCMC) package CosmoMC [31]. The priors on the main cosmological parameters used for all models are listed in Tab. 1. We also study the impacts of neutrinos on the CMB temperature spectrum and the matter power spectrum, using the Boltzmann solver CAMB [32]. Here is an introduction to the dark energy models and datasets.

### 2.1 Dark Energy Models

The \(\Lambda\)CDM + \(M_\nu\) model. The parameter space of the \(\Lambda\)CDM model is

\[
\mathcal{P} \equiv \{\Omega_b h^2, \Omega_c h^2, 100\Theta_{MC}, \tau, n_s, \ln(10^{10} A_s), M_\nu\},
\]

### Table 1. Priors on the main cosmological parameters included in this paper.

| Parameters | Prior |
|------------|-------|
| \(\Omega_b h^2\) | [0.005, 0.1] |
| \(\Omega_c h^2\) | [0.01, 0.99] |
| \(10^3 \Theta_{MC}\) | [0.5, 10] |
| \(\tau\) | [0.1, 0.8] |
| \(n_s\) | [0.8, 1.2] |
| \(\ln(10^{10} A_s)\) | [1.6, 3.9] |
| \(M_\nu\) (eV) | [0, 3] |
| \(\epsilon_s\) | [0, 0.5] |
| \(\epsilon_{\phi, \infty}\) | [0, 1] |
| \(\zeta_s\) | [-1, 1] |
| \(H_0\) (km/s/Mpc) | [40, 100] |
which has the minimum number of parameters compared to other models. $\Omega_b h^2$ and $\Omega_c h^2$ describe the baryon and cold dark matter densities today. $\Theta_{MC}$ is an approximation to the angular size of sound horizon at the time of decoupling. $\tau$ is the optical depth due to reionization. $n_s$ and $A_s$ refer to the spectral index and the amplitude of initial power spectrum related to early universe cosmology. The EOS for $\Lambda$CDM model remains a constant, i.e. $w_{de} = -1$.

**The PEDE model.** It is a phenomenological model of emergent dark energy, which did not exist effectively in the past until later time [28, 29]. It has the same parameter space $\mathcal{P}$ as the minimal $\Lambda$CDM cosmology, namely it does not use any additional degrees of freedom. The EOS are given by

$$w_{de} = -1 - \frac{1}{3 \ln 10} \times [1 - \text{tanh}(\log_{10} a)],$$

(2.2)

The dark energy evolves as

$$\rho_{de} = \rho_{de,0} \times [1 + \text{tanh}(\log_{10} a)],$$

(2.3)

where $a$ is a scale factor normalized to unity today. From the Eq.2.2, we can derive that the dark energy state equation goes from $-\frac{2}{3 \ln 10} - 1$ in the past to $-1$ in the future.

**The HBK model.** We use an analytic approximation of $w_{de}$ in quintessence models proposed by Huang et al. [30]. It fits well the ensemble of trajectories for a wide class of potentials $V(\phi)$ with three additional parameters compared with the $\Lambda$CDM model. The EOS takes the following form

$$w_{de} = -1 + \frac{2}{3} \left\{ \sqrt{\epsilon_{\phi,\infty}} + \left( \sqrt{\epsilon_s} - \sqrt{\frac{2 \epsilon_{\phi,\infty}}{1 - \Omega_k}} \right) \times \left[ F\left(\Omega_k a_{eq}, a_{eq}, a_{eq}\right) + \zeta_s F_2\left(\frac{a}{a_{eq}}\right)\right] \right\}^2,$$

(2.4)

where the parameter $\epsilon_s$ describes the slope of the potential $V(\phi)$ at $a = a_{eq}$, i.e. the dark energy density is equal to matter density. The tracking parameter $\epsilon_{\phi,\infty}$ characterizes the curvature of the scalar-field logarithm potential at the pivot, and the running parameter $\zeta_s$ is the initial velocity of the scalar field. The functions $F$ and $F_2$ are given by

$$F(\lambda, x) \equiv \frac{3}{x^3} \int_0^x \sqrt{\frac{t^7}{1 + \lambda t + t^3}} dt,$$

(2.5)

$$F_2(x) \equiv \sqrt{2}\left[1 - \ln(1 + x^3)\right] - \frac{\sqrt{1 + x^4}}{x^{3/2}} + \frac{\ln[x^{3/2} + \sqrt{1 + x^4}]}{x^3},$$

(2.6)

respectively. The detailed derivation can be seen in Refs. [30, 33].

### 2.2 Dataset

We process the most recent datasets from Planck 2018 [34] in combination with other low-redshift observations. We use the Planck 2018 CMB low-l ($2 \leq l \leq 29$) and high-l ($30 \leq l \leq 2508$) TT likelihood, high-l E mode polarization and temperature-polarisation cross correlation likelihood, and low-l E mode polarization likehood. We also include the CMB lensing data. The low-redshift observations contain the Baryon acoustic oscillations (BAO) measurements and Type Ia supernovae (SNe Ia) data. BAO measurements cover 6dFGS [35], SDSS-MGS [36] and BOSS DR12 [37] surveys. SNe Ia data are taken from the latest Pantheon Sample [38], including the information of 1048 type Ia supernovae in the range of redshift $(0.01 < z < 2.3)$. 

-- 3 --
3 results

In this section, we present the bounds on neutrino masses for three models and we analyse the correlation between three different neutrino hierarchies and dark energy models. We use three data combinations mentioned above, CMB + BAO + SNe Ia, in all the models.
### 3.1 Bounds on neutrino masses

Here, we talk about the sum of neutrino masses within the assumption of DH. In Tab. 2, we list the bounds on the sum of the neutrino masses for three dark energy models. When using Planck 2018 data for $\Lambda$CDM model, we find that the upper bound on $M_\nu$ is $M_\nu < 0.114$ eV at 95% confidence level (CL), which is more stringent than the result from Vagnozzi et al. [15] with Planck 2015 data. Moreover, we find a tighter upper bound on $M_\nu < 0.087$ eV for HBK model, which conforms the conclusion that the constraints on the sum of neutrino masses in quintessence models are tighter than the results in $\Lambda$CDM model. In particular, for PEDE model, we find the constraints on the sum of neutrino masses $M_\nu$ is $0.2123^{+0.1293}_{-0.1367}$ at 2$\sigma$. We note that PEDE model tends to have a larger value of the sum of neutrino masses and it even gives a nonzero lower bound. As is studied in Ref.[39], there is an anti-correlation between $M_\nu$ and $w_{de}$, a smaller $w_{de}$ tends to a larger $M_\nu$.

In Fig. 3, we plot the theoretical prediction on the CMB temperature spectrum $C_l^{TT}$ and the matter power spectrum $P(k)$ to explore the impacts of $w_{de}$. For HBK model, we take $\epsilon_s = 0, 0.5, 1$, with $\epsilon_{\phi\infty} = 0$, $\zeta_s = 0$ and other parameters kept fixed, as examples. Notice that, when $\epsilon_s = 0$, $\epsilon_{\phi\infty} = 0$ and $\zeta_s = 0$, the HBK model corresponds to $\Lambda$CDM model. We also display the case of PEDE model for comparison. According to Eq. 2.4, $w_{de}$ would increase as the increase of $\epsilon_s$. Therefore, we can see that the significant variations for low-l tail ($2 < l < 50$) of CMB temperature spectrum from left panel and $P(k)$ is gradually suppressed from the right panel. We can also know from the Refs. [4, 16, 40] that, a larger $M_\nu$ can also cause the suppression on the CMB temperature spectrum at the low multipoles (late Integrated Sachs-Wolfe effect) and the matter power spectrum. Therefore, it implies that, as $w_{de}$ gets smaller, $M_\nu$ would become larger to compensate the variation caused by $w_{de}$ on power spectrum.

From only CMB data, there is a strong anti-correlation between Hubble constant $H_0$ and $M_\nu$, however, the degeneracy would be broken more effectively when combined with BAO and SNe data [12]. In Fig. 1, we show the marginalized 68.3% CL and 95.4% CL constraints on $H_0$ and $M_\nu$. We can note that releasing $M_\nu$ does not help to relieve the $H_0$ tension between low-redshift measurement and high-redshift measurement. Particularly, when considering massive neutrinos for PEDE model, the Hubble tension can still be alleviated.

### 3.2 Neutrino mass hierarchy

In Tabs. 3, 4 and 5, we present the constraints of some selected parameters and the values of $\chi^2_{min}$ from MCMC analysis for $\Lambda$CDM, HBK and PEDE models, respectively. In Fig. 2, we show comparison of 1-D marginalized posterior distribution on $M_\nu$ for three dark energy models with different hierarchies, which have been normalized. We can note that, though considering different hierarchy, the results that the quintessence prior of dark energy tends to tighten the cosmological constraint on $M_\nu$ still holds for each neutrino ordering. In PEDE model, the constraints on the total neutrino masses with different neutrino hierarchies are almost uniform.
Inverted Hierarchy
0.224\pm 0.0001
0.0224\pm 0.0001
0.0225\pm 0.0001

\Theta_s
1.0140\pm 0.003
1.0410\pm 0.003
1.0410\pm 0.003

\nu
0.9671\pm 0.0038
0.9673\pm 0.0037
0.9676\pm 0.0037

\ln(10^{10}A_s)
3.0449\pm 0.0143
3.0490\pm 0.0150
3.0527\pm 0.0142

\nu\ (eV)
< 0.114
< 0.150
< 0.170

\sigma_8
0.8145\pm 0.0097
0.8044\pm 0.0085
0.7982\pm 0.0080

H_0
67.8936\pm 0.4860
67.5201\pm 0.4590
67.3298\pm 0.4334

\Omega_m0
0.3085\pm 0.0061
0.3127\pm 0.0060
0.3189\pm 0.0063

\chi^2_{min}
1910.795
1911.990
1913.325

| Table 3. | Marginalized constraints on cosmological parameters of the \( \Lambda \)CDM model for different neutrino hierarchies, which are given at 1\( \sigma \) errors except the upper bounds on \( M_\nu \) are given at 2\( \sigma \) errors. |

| Table 4. | Marginalized constraints on cosmological parameters of the HBK model for different neutrino hierarchies, which are given at 1\( \sigma \) errors except the upper bounds on \( M_\nu \) and the dark energy parameters are given at 2\( \sigma \) errors. |

Figs. 4, 5 and 6 depict the 1D marginalized posterior distributions and 2D joint contours at 68\% and 95\% CL for some selected cosmological parameters of the \( \Lambda \)CDM, HBK and PEDE models. In the \( \Lambda \)CDM and HBK models, different hierarchies lead to some slight effects on other cosmological parameters because of the degeneracy, as shown in Fig. 4 and Fig. 5. For instance, as the value of \( M_\nu \) increases from the degenerate approximation to normal hierarchy to inverted case, the value of \( H_0 \) and \( \sigma_8 \) decrease and \( \Omega_m0 \) increase. Moreover, as neutrinos masses get larger due to the different hierarchy, the dark energy parameters \( \epsilon_s \) and \( \epsilon_{\phi\infty} \) get...
| Parameter          | Degenerate Hierarchy | Normal Hierarchy | Inverted Hierarchy |
|--------------------|----------------------|-----------------|-------------------|
| $\Omega_bh^2$      | 0.0223 $^{+0.0001}_{-0.0001}$ | 0.0223 $^{+0.0001}_{-0.0001}$ | 0.0223 $^{+0.0001}_{-0.0001}$ |
| $\Omega_ch^2$      | 0.1212 $^{+0.0008}_{-0.0008}$ | 0.1212 $^{+0.0008}_{-0.0008}$ | 0.1212 $^{+0.0008}_{-0.0008}$ |
| $\Theta_s$         | 1.0407 $^{+0.0003}_{-0.0003}$ | 1.0407 $^{+0.0003}_{-0.0003}$ | 1.0407 $^{+0.0003}_{-0.0003}$ |
| $\tau$             | 0.0530 $^{+0.0075}_{-0.0075}$ | 0.0535 $^{+0.0068}_{-0.0075}$ | 0.0541 $^{+0.0074}_{-0.0074}$ |
| $n_s$              | 0.9619 $^{+0.0036}_{-0.0036}$ | 0.9622 $^{+0.0036}_{-0.0036}$ | 0.9619 $^{+0.0037}_{-0.0036}$ |
| $\ln(10^{10}A_s)$ | 3.0444 $^{+0.0148}_{-0.0148}$ | 3.0453 $^{+0.0134}_{-0.0147}$ | 3.0464 $^{+0.0151}_{-0.0151}$ |
| $M_{\nu}$ (eV)     | 0.2123 $^{+0.0666+0.1293}_{-0.0642-0.1367}$ | 0.2188 $^{+0.0647+0.0123}_{-0.0648-0.1357}$ | 0.8257 $^{+0.0774+0.1094}_{-0.0711-0.1249}$ |
| $\sigma_8$         | 0.8239 $^{+0.0164}_{-0.0164}$ | 0.8227 $^{+0.0164}_{-0.0162}$ | 0.8215 $^{+0.0174}_{-0.0145}$ |
| $H_0$              | 70.0558 $^{+0.7255}_{-0.7279}$ | 69.9905 $^{+0.7851}_{-0.7908}$ | 69.9210 $^{+0.7932}_{-0.7230}$ |
| $\Omega_{m0}$      | 0.2971 $^{+0.0081}_{-0.0081}$ | 0.2977 $^{+0.0087}_{-0.0087}$ | 0.2985 $^{+0.0077}_{-0.0076}$ |
| $\chi^2_{\text{min}}$ | 1918.044 | 1918.425 | 1917.067 |

**Table 5.** Marginalized constraints on cosmological parameters of the PEDE model for different neutrino hierarchies, which are given at 1σ errors and also given at 2σ errors on $M_{\nu}$.

**Figure 4.** 68% and 95% CL contour plot in the ΛCDM model.
Figure 5. 68% and 95% CL contour plot in the HBK model.

smaller since the anti-correlation between $M_\nu$ and $w_{de}$. However, these variations are not significant enough and the impact of neutrino hierarchy on dark energy parameters is only at a few percent level. In PEDE model, because the constraints on $M_\nu$ are similar in three hierarchies, all the cosmological parameters almost coincide. Therefore, with the improvement of observation accuracy, we may not be able to solve the neutrino mass hierarchy problem for PEDE model in the future. Besides, we present the value of $\chi^2_{min}$ calculated at best-fit points for each case in tables. Current cosmological observations can not provide a rigorous statistical treatment of the preference for hierarchy. The differences between the values of $\chi^2_{min}$ for normal hierarchy and for inverted hierarchy are not significant. However, there is a kind of interesting point that the PEDE model slightly prefers IH, which is different from the $\Lambda$CDM and HBK models which slightly prefer NH.

We also show the theoretical predictions on the CMB temperature spectrum and matter power spectrum for the three models with three hierarchies in Fig. 7. We assumed the best-fit values generated by MCMC analysis above. When the neutrino hierarchies are considered,
one can not find significant variations on the power spectrum. Therefore, neutrino hierarchy has very slight impacts on other cosmological parameters whichever type of dark energy we apply.

4 Conclusion

Cosmology can be used to address the problems about the total neutrino masses and the mass hierarchy. Current observations can provide constraints on the sum of the neutrino masses \( M_\nu \). However, the bounds on \( M_\nu \) from cosmology are model-dependent. In this work, we investigated cosmological constraints on \( M_\nu \) within the \( \Lambda \)CDM model, the HBK model and PEDE model. These models are well constrained by the latest observational data, CMB + BAO + SNe Ia. For ACMD model, when we use the latest Planck 2018 CMB data, we obtain tighter upper bounds on total neutrino masses. For HBK model, we find the quintessence prior of dark energy tends to tighten the cosmological constraints on \( M_\nu \), as previously stated. On the other hand, the phantom prior in PEDE model tends to make the constraints on \( M_\nu \) looser and its value larger and we can even obtain the lower bounds (95%CL) on \( M_\nu \).

In addition, we also consider different neutrino hierarchies for the three models. It leads to some impacts on cosmological parameters due to the variations of \( M_\nu \). However, those
Figure 7. The impacts of neutrino hierarchies on the CMB temperature power spectrum $C_l^{TT}$ and on the matter power spectrum for PEDE, Λ CDM, and HBK models.

variations are not significant enough. Especially, in the PEDE model, the change of neutrino hierarchy nearly has no impacts on other cosmological parameters.

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