Pairing effects on the collectivity of quadrupole states around $^{32}\text{Mg}$

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Abstract
The first $2^+$ states in $N=20$ isotones including neutron-rich nuclei $^{32}\text{Mg}$ and $^{30}\text{Ne}$ are studied by the Hartree-Fock-Bogoliubov plus quasiparticle random phase approximation method based on the Green’s function approach. The residual interaction between the quasiparticles is consistently derived from the hamiltonian density of Skyrme interactions with explicit velocity dependence. The $B(E2,0^+_1 \rightarrow 2^+_1)$ transition probabilities and the excitation energies of the first $2^+$ states are well described within a single framework. We conclude that pairing effects account largely for the anomalously large $B(E2)$ value and the very low excitation energy in $^{32}\text{Mg}$.
I. INTRODUCTION

The pioneering observation in 1975 of the anomalous binding energy gain in very neutron rich Na isotopes revealed the breaking of the N=20 shell closure and the possibility of deformation [1]. The evidences of the breaking of N=20 shell closure in neutron-rich Mg and Ne isotopes are more clearly seen from the observations of E2 properties, the large B(E2) value in $^{32}$Mg [2] and the low excitation energies of the first 2$^+$ states in $^{32}$Mg [3] and $^{30}$Ne [4].

Several theoretical studies have been done to describe the anomalous binding energy and E2 properties in neutron-rich nuclei around N=20. Constrained Hartree-Fock (HF) calculations of Na isotopes [5] have been performed and $^{31}$Na was suggested as deformed. Early studies made by Wildenthal and Chung showed that shell model calculations within the sd shell model space cannot explain the extra binding energies in this region. Subsequent shell model calculations [6, 7, 8, 9] have demonstrated that the inclusion of the fp shell into the shell model active space is essential. The effects of the breaking of the N=20 shell closure are clearly shown in the description of the B(E2) values and the excitation energies in $^{32}$Mg and $^{30}$Ne [10, 11, 12]. The neutron 2p-2h configurations across the N=20 shell imply deformation of these nuclei. However, in the framework of the mean-field approximation with pairing correlations, like Skyrme Hartree-Fock-Bogoliubov (HFB) calculations, the calculated ground states in $^{32}$Mg and $^{30}$Ne turn out to be spherical (see, e.g., [13, 14]). One possible way to describe the (dynamical) deformation is to include correlations beyond the mean field. Generator coordinate method (GCM) [15, 16, 17] and antisymmetrized molecular dynamics calculations [18] have been done in this direction.

Nevertheless, the experimental evidence of deformation in $^{32}$Mg is not well established. The energy ratios of the first 4$^+$ and 2$^+$ states, $E(4^+)/E(2^+)$, are 3.0 in $^{24}$Mg and 3.2 in $^{34}$Mg [19], and these values are undoubtedly close to the rigid rotor limit of 3.3. On the other hand, the ratio is 2.6 in $^{32}$Mg, and this value is in between the rigid rotor limit and the harmonic vibration limit 2.0 [14, 20]. Moreover, the B(E2) value (in single-particle units) is $15.0\pm2.5$ in $^{32}$Mg. This value is larger than in the other stable N=20 isotones but smaller than in other deformed Mg isotopes ($21.0\pm5.8$ in $^{24}$Mg, and $19.2\pm3.8$ in $^{34}$Mg).

Generally speaking, the neutron 2p-2h configurations can originate not only from deformation effects but also from neutron pairing correlations. In the $^{32}$Mg nucleus these two effects may coexist and make the large B(E2) value and the low excitation energy of the 2$^+$ state. In shell model studies it is not clear which effect is more essential to describe these anomalous properties.

Recent angular-momentum projected GCM calculations with the Gogny force [16, 17] were successful to reproduce the systematic trend of the B(E2) values and the excitation energies of the first 2$^+$ and 4$^+$ states in Mg and Ne isotopes. However, the calculated excitation energies of the first 2$^+$ states are somewhat higher than the experimental data (e.g., about 1.5 MeV in $^{32}$Mg and about 2.1 MeV in $^{30}$Ne). These discrepancies may be explained by the weakness of neutron pairing correlations around the spherical ground states in $^{32}$Mg and $^{30}$Ne in the corresponding HFB mean fields.

The purpose of this paper is to emphasize how neutron pairing correlations play an essential role in the description of E2 properties in $^{32}$Mg and $^{30}$Ne. The existence of neutron pairing correlations means the breaking of the N=20 shell closure. As we will see, the appearance of neutron pairing correlations is related to a special mechanism in loosely bound systems. We study the first 2$^+$ states in N=20 isotones in the framework of self-consistent
quasiparticle random phase approximation (QRPA) with Skyrme interactions \[21\]. The QRPA equations are solved in coordinate space by using the Green’s function method \[22\]. Spherical symmetry is assumed for simplicity. The residual interaction between the quasiparticles is self-consistently derived from the Hamiltonian density of Skyrme interaction that has an explicit velocity dependence. We will show that the B(E2) values and the excitation energies of the first $2^+$ states in N=20 isotones, from the stable nucleus $^{38}\text{Ar}$ to the neutron-rich nuclei $^{32}\text{Mg}$ and $^{30}\text{Ne}$ are well described within a single framework and a fixed parameter set. The paper is organized as follows. In Sect. II we briefly describe the HFB plus QRPA calculations that we have done. In Sect. III we present the general results for the ground states of the N=20 isotones studied here. In Sect. IV we discuss the calculated and experimental $E2$ properties of these nuclei. Conclusions are drawn in Sect. V.

II. HFB-QRPA CALCULATIONS

A. Formulation

We use the approach of self-consistent HFB-QRPA calculations with Skyrme interactions \[21, 22\]. By self-consistent we mean that the HFB mean fields are determined self-consistently from an effective force and the residual interaction of the QRPA problem is derived from the same force. The QRPA problem is solved by the response function method in coordinate space. A detailed account of the method can be found in Ref.\[22\]. Here, we just recall the main steps of the calculation. The QRPA Green’s function $G$ is solution of a Bethe-Salpeter equation,

$$G = G_0 + G_0 V G . \tag{1}$$

The knowledge of $G$ allows one to construct the response function of the system to a general external field, and the strength distribution of the transition operator corresponding to the chosen field is just proportional to the imaginary part of the response function.

In Eq.(1) the unperturbed Green’s function $G_0$ is defined as

$$G_0^{\alpha\beta}(r\sigma, r'\sigma'; \omega) = \sum_{i,j} \frac{W_{ij}^{\alpha\beta}(r\sigma)[W_{ij}^{\beta\alpha}(r'\sigma')]_-}{\hbar \omega - (E_i + E_j) + i\eta} - \frac{W_{ij}^{\alpha\beta}(r\sigma)[W_{ij}^{\beta\alpha}(r'\sigma')]_-}{\hbar \omega + (E_i + E_j) + i\eta} , \tag{2}$$

where the functions $W(r\sigma)$ are introduced as

$$W_{ij}(r\sigma) = \begin{pmatrix} U_i(r\sigma)V_j(r\sigma) & V_i(r,\sigma)U_j(r\sigma) \\ U_i(r\sigma)\bar{U}_j(r\sigma) & V_i(r\sigma)V_j(r\sigma) \\ -V_i(r\sigma)\bar{V}_j(r\sigma) & -U_i(r\sigma)\bar{U}_j(r\sigma) \end{pmatrix} . \tag{3}$$

Here, the $U(r), V(r)$ are quasiparticle wave functions, the index $\alpha$ ($\alpha=1,2,3$) stands for particle-hole (ph), particle-particle (pp) and hole-hole (hh) channels. The notation $f(r\sigma) \equiv -2\sigma f(r - \sigma)$ indicates time-reversal and $[W_{ij}]_- = W_{ij} - W_{ji}$.

The residual interaction $V$ between quasiparticles is derived from the Hamiltonian density $\langle H \rangle$ of Skyrme interaction by the so-called Landau procedure,

$$V_{\alpha\beta}(r\sigma, r'\sigma') = \frac{\partial^2 \langle H \rangle}{\partial \rho_{\alpha}(r\sigma) \partial \rho_{\beta}(r'\sigma')} . \tag{4}$$
The notation $\bar{\alpha}$ means that whenever $\alpha$ is pp (hh) then $\bar{\alpha}$ is hh (pp). The normal and abnormal densities are defined as

\[
\begin{pmatrix}
\rho_{ph}(r\sigma) \\
\rho_{hp}(r\sigma) \\
\rho_{hh}(r\sigma)
\end{pmatrix} = \begin{pmatrix}
\rho(r\sigma) \\
\kappa(r\sigma) \\
\bar{\kappa}(r\sigma)
\end{pmatrix} = \begin{pmatrix}
<0|\psi_{\sigma}^\dagger(r\sigma)\psi(r\sigma)|0> \\
<0|\psi_{\bar{\sigma}}(r\bar{\sigma})\psi(r\sigma)|0> \\
<0|\psi_{\sigma}^\dagger(r\sigma)\psi_{\bar{\sigma}}(r\bar{\sigma})|0>
\end{pmatrix} .
\tag{5}
\]

The residual interaction $\mathbf{V}$ has an explicit momentum dependence,

\[
\mathbf{V}(r, r') = \mathbf{F}[\bar{\Delta}_U + \bar{\Delta}_V, \bar{\Delta}_U + \bar{\Delta}_V, \bar{\nabla}_U \pm \bar{\nabla}_V, \bar{\nabla}_U \pm \bar{\nabla}_V]|\mathbf{r} - \mathbf{r}'| .
\tag{6}
\]

The explicit form of the form factor $\mathbf{F}$ is shown in Ref. [21]. The operators with $\leftarrow (\rightarrow)$ act on the coordinate $\mathbf{r}$ ($\mathbf{r}'$), and the operators with the index $U$ ($V$) operate on the quasiparticle wave functions $U(\mathbf{r})$ ($V(\mathbf{r})$) only. These momentum dependence are explicitly treated in our calculation. Because we calculate only natural parity (non-spin-flip) excitations, we drop the spin-spin part of the residual interaction. The Coulomb and spin-orbit residual interactions are also dropped.

We have studied the influence of these momentum dependent terms and we have found that they can be important [21]. In a fully consistent calculation the spurious center-of-mass state should come out at zero energy. In practice, the full velocity dependence of the residual interaction [10] is replaced by a Landau-Migdal form to reduce the computational efforts [22]. Then, the self-consistency between the mean-field and the residual interaction is broken. To recover the self-consistency approximately, the residual interaction has to be renormalized by a factor adjusted so as to have the spurious state at zero energy. For example, the renormalization in Ref. [22] is about 20%. If the residual interaction in Eq. [10] is used, the renormalization factor is very close to 1 and the self-consistency is well recovered. For comparison, if one drops all momentum-dependent terms in Eq. [10] the renormalization factor would be about 0.6.

Another important aspect is related to the description of the low-lying states. The B(Eλ) transition probability is very sensitive to the treatment of the residual interaction. For example, the $B(E2, 0^-_1 \rightarrow 2^+_1)$ in $^{20}$O calculated with SkM* parameter with (without) these momentum dependent terms is 34.1 (20.9) $e^2 fm^4$. The B(E2) value increases by 64% [21]. The experimental data is $28 \pm 2 e^2 fm^4$ [23] which is close to the value calculated with the momentum dependent terms. Thus, the full residual interaction (6) is important for describing quantitatively the low-lying states and for comparison with experimental data.

### B. Inputs

We apply the above formalism to study the first $2^+$ states in N=20 isotones, $^{30}$Ne, $^{32}$Mg, $^{34}$Si, $^{36}$S and $^{38}$Ar. The ground states are given by Skyrme-HFB calculations. The HFB equation is diagonalized on a Skyrme-HF basis calculated in coordinate space with a box boundary condition [24, 25, 26]. Spherical symmetry is imposed on quasiparticle wave functions. The quasiparticle cut-off energy is taken to be $E_{cut} = 50$ MeV, and the angular momentum cut-off is $l_{max} = 7\hbar$ in our HFB and QRPA calculations.

The Skyrme parameter sets SkM*[27] and SkP [28] are used for the HF mean-field, and the density-dependent, zero-range pairing interaction

\[
V_{pair}(\mathbf{r}, \mathbf{r}') = V_{pair} \left[1 - \left(\frac{\rho(\mathbf{r})}{\rho_c}\right)^\alpha\right] \delta(\mathbf{r} - \mathbf{r}') ,
\tag{7}
\]
is adopted for the pairing field. The parameters $\alpha$ and $\rho_c$ are fixed as $\alpha = 1$ and $\rho_c = 0.16$ fm$^{-3}$. The strength $V_{pair}$ is determined so as to reproduce the experimental neutron pairing gap in $^{30}$Ne, $\Delta_{n,exp}^{(30)}Ne = 1.26$ MeV. $^{30}$Ne is the lightest mass even-even N=20 nucleus. The experimental pairing gaps are extracted by using the 3-point mass difference formula\[20\], $\Delta_n(N) = \Delta_n^{(3)}(N-1) = \frac{(-1)^N}{2} [E(N-2) + E(N) - 2E(N-1)]$. On the other hand, the average pairing gap in HFB calculations is defined as the integral of the pairing field, $\Delta_n = \int d\vec{r} \rho_n(\vec{r}) \Delta_n(\vec{r}) / \int d\vec{r} \rho_n(\vec{r})$\[30\]. The pairing strength adopted for SkM* is $V_{pair} = -418$ MeV fm$^{-3}$, and for SkP, $V_{pair} = -400$ MeV fm$^{-3}$. Fig\[1\] shows the experimental and the calculated pairing gaps in $^{26,28,30}$Ne. With these Skyrme parameters and pairing strengths, we get finite pairing gap in $^{30}$Ne (vanishing of N=20 shell gap) and zero pairing gap in $^{26}$Ne (appearance of N=16 shell gap) at the same time.

### III. GROUND STATE PROPERTIES

Fig\[2\] shows the neutron single-particle levels in N=20 isotones calculated in HF with the SkM* force. Results with the SkP force are qualitatively the same. The N=20 shell gaps change from 4.2 MeV in $^{40}$Ca to 3.4 MeV in $^{30}$Ne. The N=16 shell gaps change from 2.4 MeV in $^{40}$Ca to 4.0 MeV in $^{30}$Ne. Within HF we can describe the vanishing of N=20 magicity and the appearance of N=16 magic number at the same time. The change of N=20 shell gaps looks moderate in comparison with the effective single-particle energies in shell model calculations\[22\]. However, the definitions are different. In shell model calculations the single-particle energies are inputs of calculations and they are determined so as to reproduce the neutron separation energies and the one-particle spectra of $^{17}$O and $^{41}$Ca, and the change of the effective single-particle energies according to proton number are due to the change of the many-body correlations. On the other hand, the change of the single-particle energies in mean-field calculations reflects the self-consistent change of the mean-field potential.

An important feature in Fig\[2\] is the behavior of low-$l$ orbits, $2p_{3/2}$ and $2p_{1/2}$ in the $fp$ shell. As the proton number decreases the single-particle energies of the high-$l$ orbit $1f_{7/2}$ change almost linearly while the changes of $2p_{3/2}$ and $2p_{1/2}$ energies become very slow around zero energy. Moreover, the spin-orbit splitting of $2p_{3/2}$ and $2p_{1/2}$ states becomes smaller. As pointed out by Hamamoto et al.\[31\], these effects can be understood by different $l$-dependences of the kinetic energy and the spin-orbit form factor as the single-particle energy comes close to zero. Because of these different $l$-dependences of the single-particle energies, the level density in the $fp$ shell becomes higher with decreasing proton number, and the three orbits $1f_{7/2}$, $2p_{3/2}$ and $2p_{1/2}$ become almost degenerate in $^{30}$Ne. We can describe this behavior naturally by solving the HF and HFB equations in coordinate space but it is difficult to get this property by the methods based on the harmonic oscillator basis.

Fig\[3\] shows the HFB neutron and proton pairing gaps in N=20 isotones calculated with SkM* and SkP. The pairing strengths are adjusted so as to reproduce the experimental neutron pairing gap in $^{30}$Ne. As the proton number increases, the neutron pairing gaps decrease monotonically and eventually, the neutron pairing gap becomes zero (for both SkM* and SkP) in $^{38}$Ar as expected in stable N=20 nuclei. The interesting point is that the N=20 shell gap itself changes very moderately but the calculated neutron pairing gap changes considerably from 1.26 MeV in $^{30}$Ne to zero in $^{38}$Ar. The mechanism can be understood by the increase of the level density in the $fp$ shell when the proton number decreases, as noted above. Since the neutron pairing gap is adjusted in $^{30}$Ne it remains close in $^{32}$Mg for both interactions, but large differences are seen in $^{34}$Si and $^{36}$S. On the other hand, the calculated
proton pairing gaps do not depend on the Skyrme force.

Fig. 4 shows the average number of neutrons $N_{fp}$ in the $fp$ shell in N=20 isotones, calculated in HFB with SkM* and SkP. According to the change of the neutron pairing gaps, $N_{fp}$ decreases monotonically from $\simeq 0.8$ in $^{30}$Ne to $\simeq 0.5$ in $^{32}$Mg. These values are very different from the prediction of the "island of inversion", $N_{fp} = 2$ [9] and Monte Carlo shell model, $N_{fp} \geq 2$ in $^{30}$Ne and $^{32}$Mg [12].

IV. B(E2) VALUES AND EXCITATION ENERGIES

We have calculated the first $2^+$ states in N=20 isotones in HFB-QRPA with Skyrme interactions, assuming spherical symmetry. At the mean-field level the ground states in N=20 isotones including $^{32}$Mg and $^{30}$Ne have been found to be spherical (see, e.g.,, [13, 14]). Our aim is to investigate whether these $2^+$ states can be described as vibrational states built on the spherical ground states.

In Fig. 5 our results of QRPA calculations with SkM* are compared with the predictions of the Monte Carlo shell model (MCSM) [12], and the available experimental data [2, 3, 4, 32, 33, 34]. Our QRPA results are in good agreement with the experimental data and they are consistent with the MCSM prediction of the B(E2) value in $^{30}$Ne which is not yet measured experimentally. The QRPA calculations have been done with the SkM* parameter set and the fixed pairing strength $V_0 = -418$ MeV fm$^{-3}$ the choice of which is explained in subsect. 2.2. The general properties of the first $2^+$ states in N=20 isotones, namely very large B(E2) values and very low excitation energies in $^{32}$Mg and $^{30}$Ne are well reproduced within a single framework.

To check the interaction dependence we have carried out QRPA calculations with the SkP interaction. Fig. 6 shows the B(E2) values and excitation energies of the first $2^+$ states with SkM* and SkP. Concerning the B(E2) values we get similar results, especially very large B(E2) values in $^{32}$Mg and $^{30}$Ne. On the other hand, large differences are seen in the excitation energies in $^{34}$Si and $^{36}$S. This can be understood by the difference in the neutron pairing correlations shown in Fig. 3. In $^{30}$Ne, $^{32}$Mg and $^{38}$Ar the neutron pairing gaps calculated with SkM* and SkP are almost the same while they are somewhat different in $^{34}$Si and $^{36}$S. Because the neutron pairing gaps are larger in SkP than in SkM*, the excitation energies become lower with SkP than with SkM*.

We now explain how the neutron pairing correlations are important to make the B(E2) values larger and the excitation energies lower. To see which two-quasiparticle configurations contribute to make the low-lying $2^+$ states, we show the unperturbed isoscalar quadrupole strength functions in N=20 isotones calculated with SkM* in Fig. 7. The peaks indicated by straight (dotted) arrows correspond to proton (neutron) two-quasiparticle configurations. All these neutron two-quasiparticle configurations appear because of the neutron pairing correlations. Many peaks of the neutron configurations are seen in $^{30}$Ne, $^{32}$Mg. On the other hand, the neutron configurations are negligible in $^{34}$Si and they completely disappear in $^{36}$S. The B(E2) values are primarily made of the proton configurations in the $sd$-shell but the neutron configurations assist to make the B(E2) values larger by coherence between protons and neutrons. Actually, if the neutron pairing correlations are neglected the B(E2) values become very small and the excitation energies are sizably higher in $^{32}$Mg and $^{30}$Ne, as shown in Fig. 8. Under these considerations, we can conclude that the very large B(E2) values and the very low excitation energies in $^{32}$Mg and $^{30}$Ne appear thanks to the presence of the neutron pairing correlations. We have seen in sect. III that around the drip line the
origin of neutron pairing correlations lies in the different behavior of the single-particle levels with different orbital angular momentum $l$ as the levels approach the separation threshold.

Generally speaking, neutron 2p-2h configurations across $N=20$ can originate from deformation effects or pairing effects. Both effects can a priori contribute in the nucleus $^{32}$Mg. We have shown that a spherical QRPA description, i.e., putting emphasis on the pairing aspects and neglecting the possible deformation effects, can give very satisfactory results. In the previous studies based on shell model calculations\[10, 11, 12\] the importance of neutron 2p-2h configurations for describing the B(E2) values in $^{32}$Mg, $^{30}$Ne was shown, but the respective roles of pairing and deformation were not clear. The angular momentum projected GCM calculations\[16, 17\] are successful in reproducing the general trend of the B(E2) values and the excitation energies in Mg and Ne isotopes. However, their predicted excitation energies are somewhat higher than experiment. This may be due to the weakness of neutron pairing correlations in these calculations.

Before closing this section we would like to make a brief comment on the stability of QRPA solutions. As it is well known, when a QRPA eigenvalue is approaching zero the solution suffers instability and the transition probability diverges. Since our QRPA solutions for $^{32}$Mg and $^{30}$Ne have very low energies, we have to check whether the calculated B(E2) values are really meaningful or just spurious results. Fig.9 shows the pairing strength $V_{\text{pair}}$ dependence of the excitation energy and the B(E2) value in $^{32}$Mg. If $|V_{\text{pair}}|$ increases, the excitation energy decreases and the B(E2) value increases. This behavior is the result of competition between two effects. First, the pairing gap and also the quasiparticle energies increase with increasing $|V_{\text{pair}}|$. Therefore, the two-quasiparticle energies and the QRPA excitation energies should increase and the B(E2) values should decrease. Second, if $|V_{\text{pair}}|$ increases, many two-quasiparticle configurations can contribute to make the $2^+$ state and the collectivity increases. In this case the QRPA excitation energies would decrease and the B(E2) values would increase. In the $^{32}$Mg case the second effect wins (cf. Fig.9). As $|V_{\text{pair}}|$ is increasing, the excitation energy becomes lower and the B(E2) value increases linearly up to $|V_{\text{pair}}| \approx 422$ MeV fm$^{-3}$. Above $|V_{\text{pair}}| \approx 426$ MeV fm$^{-3}$ the B(E2) value starts to diverge. Because our adopted pairing strength is $V_{\text{pair}} = -418$ MeV fm$^{-3}$, we confirm that our QRPA solution is meaningful.

V. CONCLUSION

We have studied the first 2$^+$ states in N=20 isotones by the HFB-QRPA model with Skyrme interactions. The residual interaction is consistently derived from the Skyrme Hamiltonian, keeping all its explicit momentum dependence.

Because of the different behaviors of the neutron 1$f_{7/2}$, 2$p_{3/2}$ and 2$p_{1/2}$ levels when the single-particle energies are approaching zero, the neutron pairing gaps have finite values. This mechanism breaks the N=20 magicity in $^{32}$Mg and $^{30}$Ne.

Within our consistent QRPA calculation with spherical symmetry the $B(E2, 0^+_1 \rightarrow 2^+_1)$ values and the excitation energies of the first 2$^+$ states in N=20 isotones including $^{32}$Mg and $^{30}$Ne are well described. The existing experimental data are reproduced quantitatively. The B(E2) value in $^{30}$Ne has not been measured yet but the QRPA value is consistent with the prediction of the MCSM. The important role of the neutron pairing correlation is emphasized. If the neutron pairing is dropped, we cannot get the correct B(E2) value and excitation energy in $^{32}$Mg and $^{30}$Ne. In the real $^{32}$Mg nucleus, both neutron pairing and deformation effects may coexist and help to make the large B(E2) value, but our calculation...
shows that neutron pairing correlations are essential.

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FIG. 1: HFB neutron pairing gaps in $^{26,28,30}$Ne calculated with SkM* and SkP. The pairing strengths $V_{\text{pair}}$ are fixed so as to reproduce the experimental neutron gap in $^{30}$Ne. The experimental pairing gaps are extracted by using the 3-point mass difference formula [29].

FIG. 2: HF neutron single-particle levels in $N=20$ isotones calculated with SkM*. Solid lines correspond to bound and resonance-like states, dashed lines to positive energy discretized states.
FIG. 3: The neutron and proton pairing gaps in $N=20$ isotones calculated in HFB with SkM* and SkP.

FIG. 4: Average number of neutrons in the $fp$ shell in $N=20$ isotones calculated in HFB with SkM* and SkP.
FIG. 5: The $B(E2, 0^+ \rightarrow 2^+)$ transition probabilities and excitation energies of the first $2^+$ states in N=20 isotones calculated in QRPA with SkM*. For comparison the predictions of MCSM[12] and the available experimental data [2, 3, 4, 32, 33, 34] are shown.

FIG. 6: The $B(E2, 0^+_1 \rightarrow 2^+_1)$ transition probabilities and excitation energies of the first $2^+$ states calculated in QRPA with SkM* and SkP interactions.
FIG. 7: The unperturbed isoscalar quadrupole strength functions in N=20 isotones calculated with SkM*. The peaks indicated by straight (dotted) arrows correspond to proton (neutron) two-quasiparticle configurations.
FIG. 8: The $B(E2, 0^+_1 \rightarrow 2^+_1)$ values and the excitation energies of the first $2^+$ states in N=20 isotones calculated with/without neutron pairing correlations. Proton pairing is included in both cases.

FIG. 9: The $B(E2)$ value and excitation energy of the first $2^+$ state in $^{32}$Mg calculated in QRPA with SkM*, as a function of the pairing strength $V_{pair}$. 