Dependence of $\tan^2 \theta_{12}$ on Dirac CP phase $\delta$ in tri-bimaximal neutrino mixing under charged lepton correction

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Abstract

We consider charged lepton correction to Tri-bimaximal(TBM) neutrino mixing, defined by the relation $U_{PMNS} = U_l^\dagger U_{TB}$ and find possible form of $U_l$ which can impart non-zero value of $\sin \theta_{13}$ as well as $\tan^2 \theta_{23} < 1$, consistent with latest global analysis data. We adopt a new parametrization, other than the standard PDG parametrization, to introduce Dirac CP violating phase $\delta$ in the PMNS matrix which is discussed by Fritzsch. Under such charged lepton correction pattern we note that $\tan^2 \theta_{12}$ becomes dependent on the CP phase $\delta$ from where constraints on $\delta$ phase can be obtained after employing experimental range of mixing angles. To compute the values of mixing angles we assume the charged lepton correction to be of Cabibbo-Kobayashi-Maskawa(CKM) like. Since all the mixing matrices, involved in the calculation, are derived from three dimensional rotation matrices they satisfy unitarity condition.

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1 Introduction

Recent precision measurements[1-4] and latest global 3ν oscillation analysis of nuetron mixing parameters by two individual groups[5,6], have confirmed the non-vanishing value of $\theta_{13}$, and also predict a bestfit value of $\sin \theta_{13}$ which lies near $\frac{\lambda}{\sqrt{2}}$, $\lambda$ being the Wolfenstein parameter. Further, both groups of the global analysis provide an indication for $\theta_{23}$ to lie in the first octant($\theta_{23} < \frac{\pi}{4}$) for normal hierarchy (NH) upto 1σ range of data. One of the important aspects of neutrino physics is to understand such mixing patterns[7]. Tri-bimaximal(TBM)[8] neutrino mixing is the most popular mixing pattern of neutrinos among several special mixings obeying $\mu - \tau$ symmetry, whose predictions are attractively close to global data. It is therefore likely to believe that recent global data could have been accomodated within TBM mixing under certain perturbation to itself. Charged lepton corrections[9,10,11] in this context, is an attractive tool which can generate desired results.

The lepton mixing matrix known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix[12], is usually expressed as

$$U_{PMNS} = U_l' U_\nu,$$

where $U_l$ and $U_\nu$ are the diagonalizing matrices for charged lepton and left-handed Majorana neutrino mass matrices respectively. They are defined through the relations: $m_l = U_{lL} m^\text{diag}_{lL} V^\dagger_{lR}$ and $m_\nu = U_{\nu l} m^\text{diag}_{\nu l} U_l^\dagger$, where $m^\text{diag}_{lL} = \text{Diag}(m_e, m_\mu, m_\tau)$ and $m^\text{diag}_{\nu l} = \text{Diag}(m_1, m_2, m_3)$. In the basis where charged lepton mass matrix $m_l$ is diagonal, $U_{PMNS} = U_\nu$, $U_l$ being identity matrix, and the left-handed Majorana mass matrix is then expressible as[15], $m_\nu' = U_{lL}^\dagger m_\nu U_{lL}$. The PMNS matrix is also analogous to the CKM matrix, $V_{CKM} = U_{uL} U_{dL}$ for quark sector[13,14], where $U_{uL}$ and $U_{dL}$ are the diagonalizing matrices for up-type and down-type quark mass matrices.

In the standard Particle Data Group (PDG) parametrization[14], PMNS matrix can be parametrized as

$$U_{PMNS} = R_{23}, U_{13}, R_{12}, P,$$

where,

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix},$$

$$U_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} \text{e}^{-i \delta} \\ 0 & 1 & 0 \\ -s_{13} \text{e}^{i \delta} & 0 & c_{13} \end{pmatrix},$$

and $P = \text{diag}(1, \text{e}^{i \alpha}, \text{e}^{i \beta})$. Here $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ with $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$ being the solar angle, atmospheric angle and the reactor angle respectively. $\delta$ is the Dirac CP violating phase while $\alpha$ and $\beta$ are the two Majorana CP violating phases. Then eq.(2) yields the following standard form of the PMNS matrix:

$$U_{PMNS} = \begin{pmatrix} c_{12} c_{13} & s_{12} s_{23} s_{13} \text{e}^{i \delta} & s_{12} c_{23} - s_{12} s_{23} s_{13} \text{e}^{i \delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} \text{e}^{i \delta} & c_{12} s_{23} - s_{12} s_{23} s_{13} \text{e}^{i \delta} & s_{12} c_{23} - s_{12} s_{23} s_{13} \text{e}^{i \delta} \\ s_{13} e^{i \delta} & s_{23} c_{13} & c_{23} c_{13} \end{pmatrix}.P.$$

We would now like to drop the Majorana phase matrix $P$ in our discussion. Then from the PMNS matrix in eq.(5) we obtain the following useful expressions for mixing angles:

$$\frac{\sin^2 \theta_{13}}{U_{e3}} = |U_{e1}|^2,$$

$$\tan^2 \theta_{12} = \frac{|U_{e2}|^2}{|U_{e1}|^2},$$

$$\tan^2 \theta_{23} = \frac{|U_{e3}|^2}{|U_{\tau3}|^2}.$$
which are free from the Dirac CP violating phase $\delta$.

The TBM mixing matrix is now followed from eq.(2) with $s_{12} = \frac{1}{\sqrt{3}}$, $s_{23} = \frac{1}{\sqrt{2}}$ and $s_{13} = 0$ and is given by

$$U_{TB} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (9)$$

The CP phase $\delta$ disappears along with $s_{13}$ in eq.(9). However it can be restored by adopting another parametrization\[16\] where $U_{13}$ in eq.(4) is replaced by

$$U'_{13} = \begin{pmatrix} c_{13}e^{i\delta} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13}e^{-i\delta} \end{pmatrix}. \quad (10)$$

Under this parametrization, the new TBM matrix becomes

$$U'^{T}_{TB} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}}e^{i\delta} & \frac{1}{\sqrt{3}}e^{i\delta} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}e^{-i\delta} \end{pmatrix}, \quad (11)$$

and the general PMNS matrix looks like

$$U'_{PMNS} = \begin{pmatrix} c_{12}c_{13}e^{i\delta} & s_{12}c_{13}e^{i\delta} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}s_{23} - s_{12}c_{23}s_{13} & s_{23}c_{13}e^{-i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13}e^{-i\delta} \end{pmatrix}. \quad (12)$$

The phase $\delta$ in eqs. (11) and (12) has no physical significance. We mention them only in the context of the parametrization (10) as we would like to adopt this parametrization in our future calculations.

The paper is organized as follows: Section 2 is divided into three subsections. In subsection 2.1 we begin the discussion of charged lepton correction to TBM mixing in the absence of CP violation. In subsection 2.2 we introduce the Dirac CP violating phase into the discussion and present the central issue of the present paper. Finally section 3 is devoted to summary and discussion.

## 2 Charged lepton correction to TBM mixing

### 2.1 Without Dirac CP phase

We begin with eq.(1) where $U_\nu$ is to be given by $U_{TB}$ in eq.(9) for our case. We then consider the following form of the charged lepton mixing matrix:

$$U_l = \tilde{R}_{23}.\tilde{R}_{12}, \quad (13)$$

with

$$\tilde{R}_{12} = \begin{pmatrix} \tilde{c}_{12} & \tilde{s}_{12} & 0 \\ -\tilde{s}_{12} & \tilde{c}_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \tilde{R}_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \tilde{c}_{23} & \tilde{s}_{23} \\ 0 & -\tilde{s}_{23} & \tilde{c}_{23} \end{pmatrix}. \quad (14)$$

The structure of $U_l$ defined by eq. (13) is analogous to the that of $U_{TB}$ in eq.(9) in the sense that $U_{TB}$ is also given by $U_{TB} = R_{23}.R_{12}$ with $s_{12} = \frac{1}{\sqrt{3}}$ and $s_{23} = \frac{1}{\sqrt{2}}$. Then eqs. (13) and (14) yield

$$U_l = \begin{pmatrix} \tilde{c}_{12} & \tilde{s}_{12} & 0 \\ -\tilde{s}_{12}\tilde{c}_{23} & \tilde{c}_{12}\tilde{c}_{23} & \tilde{s}_{23} \\ \tilde{s}_{12}\tilde{s}_{23} & -\tilde{c}_{12}\tilde{s}_{23} & \tilde{c}_{23} \end{pmatrix}. \quad (15)$$
Table 1: Best fit, 1σ and 3σ ranges of parameters for NH obtained from global analysis by Fogli et al.[5] and Forero et al.[6]

| parameter | best fit Ref[5] | Ref[6] | 1σ range Ref[5] | Ref[6] | 3σ range Ref[5] | Ref[6] |
|-----------|-----------------|--------|-----------------|--------|-----------------|--------|
| \(\tan^2 \theta_{12}\) | 0.443 | 0.470 | 0.410-0.481 | 0.435-0.506 | 0.350-0.560 | 0.370-0.587 |
| \(\tan^2 \theta_{23}\) | 0.629 | 0.745 | 0.575-0.695 | 0.667-0.855 | 0.495-1.755 | 0.563-2.125 |
| \(\sin^2 \theta_{13}\) | 0.0241 | 0.0246 | 0.0216-0.0266 | 0.0218-0.0275 | 0.0169-0.0313 | 0.017-0.033 |

Figure 1: Variation of \(\tan^2 \theta_{23}\) with \(U^2_{e3}\) for TBM mixing under charged lepton correction. Dotted and dashed lines represents 1σ and 3σ bounds respectively, obtained from the global analysis[6]

With this \(U_l\) we get the PMNS matrix, from the relation \(U_{PMNS} = U^\dagger_l U_T\), as

\[
U_{PMNS} = \begin{pmatrix}
\sqrt{\frac{1}{3}}[\hat{c}_{12} + \frac{1}{2}\hat{s}_{12}(\hat{c}_{23} + \hat{s}_{23})] & \frac{1}{\sqrt{3}}[\hat{c}_{12} - \hat{s}_{12}(\hat{c}_{23} + \hat{s}_{23})] & -\frac{1}{\sqrt{2}}\hat{s}_{12}(\hat{c}_{23} - \hat{s}_{23}) \\
-\frac{1}{\sqrt{6}}[\hat{c}_{12}(\hat{c}_{23} + \hat{s}_{23}) - 2\hat{s}_{12}] & \frac{1}{\sqrt{3}}[\hat{s}_{12} + \hat{c}_{12}(\hat{c}_{23} + \hat{s}_{23})] & \frac{1}{\sqrt{6}}[\hat{s}_{12}(\hat{c}_{23} + \hat{s}_{23})] \\
\frac{1}{\sqrt{6}}(\hat{c}_{23} - \hat{s}_{23}) & -\frac{1}{\sqrt{2}}(\hat{c}_{23} - \hat{s}_{23}) & \frac{1}{\sqrt{2}}(\hat{c}_{23} + \hat{s}_{23})
\end{pmatrix}.
\] (16)

This PMNS matrix predicts

\[
\sin^2 \theta_{13} = \frac{\hat{s}_{12}^2(\hat{c}_{23} - \hat{s}_{23})^2}{2},
\] (17)

\[
\tan^2 \theta_{12} = \frac{1}{2} \left[ \frac{\hat{c}_{12} - \hat{s}_{12}(\hat{c}_{23} + \hat{s}_{23})}{\hat{c}_{12} + \frac{1}{2}\hat{s}_{12}(\hat{c}_{23} + \hat{s}_{23})} \right]^2,
\] (18)

\[
\tan^2 \theta_{23} = \frac{\hat{c}_{12}^2(\hat{c}_{23} - \hat{s}_{23})^2}{(\hat{c}_{23} + \hat{s}_{23})^2}.
\] (19)

To compute the numerical predictions let us now assume that the charged lepton corrections are Cabibbo-Kobayashi-Maskawa(CKM) like[14], which allows us to take

\[
\hat{s}_{12} = \lambda \text{ and } \hat{s}_{23} = A\lambda^2,
\] (20)

where \(\lambda\) is the Wolfestein parameter and is related to the Cabibbo angle \((\theta_C)\) by \(\lambda = \sin \theta_C\). \(A\) is a constant. Taking \(\hat{s}_{23} \approx 0.041\) with \(\lambda = 0.232\) and \(A = 0.759\) we get \(\sin^2 \theta_{13} = 0.0247\), \(\tan^2 \theta_{12} = 0.224\) and \(\tan^2 \theta_{23} = 0.80\). The prediction on solar angle \(\tan^2 \theta_{12}\) is significantly smaller than the global best fit value(Table-1). This is the problem with TBM mixing under the charged lepton correction pattern considered.
2.2 Dirac CP violation

We now introduce the Dirac type CP violating phase $\delta$ in the PMNS matrix (16) by adopting the parametrization described in eq.(10). Then the new PMNS matrix with Dirac CP phase is given by

$$U'_{P\text{MNS}} = U_{l}^\dagger (R_{23}U_{13}'R_{12}) = U_{l}^\dagger U'_{T\text{B}},$$

(21)

where $U_{l}$ and $U_{13}'$ are respectively given by eqs. (15) and (10) and we set $s_{12} = \frac{1}{\sqrt{3}}$, $s_{23} = \frac{1}{\sqrt{2}}$, and $s_{13} = 0$. Thus we obtain

$$U'_{P\text{MNS}} = \begin{pmatrix}
\frac{\sqrt{2}}{\sqrt{3}}&\frac{1}{\sqrt{6}}&\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\frac{1}{\sqrt{6}}&\frac{\sqrt{2}}{\sqrt{6}}&\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}
\frac{1}{\sqrt{6}}&-\frac{1}{\sqrt{6}}&\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\frac{1}{\sqrt{6}}&\frac{1}{\sqrt{6}}&\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}
\end{pmatrix}.
$$

(22)

This PMNS matrix predicts

$$\tan^2 \theta_{12} = \frac{\tilde{c}_{12}^2 + \tilde{s}_{12}^2 (\tilde{c}_{23} + \tilde{s}_{23})^2 - 2\tilde{c}_{12}\tilde{s}_{12}(\tilde{c}_{23} + \tilde{s}_{23}) \cos \delta}{2[\tilde{c}_{12}^2 + \frac{1}{2}\tilde{s}_{12}^2 (\tilde{c}_{23} + \tilde{s}_{23})^2 + \tilde{c}_{12}\tilde{s}_{12}(\tilde{c}_{23} + \tilde{s}_{23}) \cos \delta]}.$$

(23)
The predictions on $\sin^2 \theta_{13}$ and $\tan^2 \theta_{24}$ remain unaffected by the phase $\delta$ and are given by eqs. (17) and (19) respectively. For $\delta = 0$, eq.(23) necessarily yields the same analytic expression given by eq.(18). As discussed in subsection 2.1, the numerical value predicted by eq.(18) is much smaller than the global bestfit value but now the dependency of $\tan^2 \theta_{12}$ on $\cos \delta$ can lift the prediction on $\tan^2 \theta_{12}$ upto desired experimental prediction as shown in Fig.2. Fig.2 also shows that prediction on $\tan^2 \theta_{12}$ can accomodate $1\sigma$ and $3\sigma$ ranges of global data for non-zero value of $\cos \delta$. For the best fit value $\tan^2 \theta_{12} = 0.47$ we calculate $\cos \delta \approx 0.147$ from eq.(23).

The rephasing invariant quantity defined as $J_{CP} = \text{Im}\{U_{e2}U_{\mu3}U_{e2}^*U_{\mu3}^*\}$, is obtained from the new PMNS matrix $U'_{PMNS}$ in eq.(22) as

$$|J_{CP}| = \frac{1}{6} \hat{c}_{12} \hat{s}_{12} (\hat{c}_{23} + \hat{s}_{23}) (\hat{c}_{23} - \hat{s}_{23})^2 \sin \delta. \tag{24}$$

For maximal CP violation ($\delta = \frac{\pi}{2}$) and for numerical values of $\hat{s}_{12}$ and $\hat{s}_{23}$ considered in subsection 2.1, eq.(24) predicts $|J_{CP}|_{\text{max}} \approx 0.0359$ while for $\cos \delta \approx 0.147$ we get $|J_{CP}| \approx 0.0355$.

We would also like to analyze the structure of the PMNS matrix under the criterion when a Dirac type CP phase $\phi$ is introduced from the charged lepton sector. In this case the PMNS matrix can be parametrized as

$$U'_{PMNS} = (\hat{R}_{23}, \text{Diag}(e^{i\phi}, 1, e^{-i\phi}), \hat{R}_{12})^T U_{TB}, \tag{25}$$

where $\hat{R}_{12}$ and $\hat{R}_{23}$ are given by eq.(14). Eq.(25) then gives

$$U'_{PMNS} = \begin{pmatrix}
\sqrt{\frac{1}{6}} [c_{12} e^{-i\phi} + \frac{1}{2} \hat{s}_{12} (\hat{c}_{23} + \hat{s}_{23})] & \sqrt{\frac{1}{3}} [\hat{c}_{12} e^{-i\phi} - \hat{s}_{12} (\hat{c}_{23} + \hat{s}_{23})] & -\sqrt{\frac{1}{2}} \hat{s}_{12} (\hat{c}_{23} - \hat{s}_{23}) \\
-\frac{1}{\sqrt{6}} \hat{c}_{12} (\hat{c}_{23} + \hat{s}_{23}) - 2 \hat{s}_{12} e^{-i\phi}] & \sqrt{\frac{1}{3}} [\hat{c}_{12} e^{-i\phi} + \hat{c}_{12} (\hat{c}_{23} + \hat{s}_{23})] & \frac{1}{\sqrt{2}} \hat{s}_{12} (\hat{c}_{23} - \hat{s}_{23}) e^{i\phi} \\
\frac{1}{\sqrt{6}} (\hat{c}_{23} - \hat{s}_{23}) e^{i\phi} & -\frac{1}{\sqrt{3}} (\hat{c}_{23} - \hat{s}_{23}) e^{i\phi} & \frac{1}{\sqrt{2}} (\hat{c}_{23} + \hat{s}_{23}) e^{i\phi}
\end{pmatrix}. \tag{26}$$

This PMNS matrix leads to the following expressions for $\tan^2 \theta_{12}$ and $J_{CP}$:

$$\tan^2 \theta_{12} = \frac{c_{12}^2 + \hat{s}_{12}^2 (\hat{c}_{23} + \hat{s}_{23})^2 - 2 \hat{c}_{12} \hat{s}_{12} (\hat{c}_{23} + \hat{s}_{23}) \cos \phi}{2 [c_{12}^2 + c_{12}^2 (\hat{c}_{23} + \hat{s}_{23})^2 + \hat{c}_{12} \hat{s}_{12} (\hat{c}_{23} + \hat{s}_{23}) \cos \phi]}, \tag{27}$$

$$J_{CP} = \frac{1}{6} \hat{c}_{12} \hat{s}_{12} (\hat{c}_{23} + \hat{s}_{23}) (\hat{c}_{23} - \hat{s}_{23})^2 \sin \phi, \tag{28}$$

which are similar in structure to those given in eqs. (23) and (24). Further, from eqs. (17) and (20), eq.(24) can be approximated as

$$|J_{CP}| \approx \frac{1}{3 \sqrt{2}} \sin \theta_{13} \sin \delta \tag{29}$$
which is consistent with the result of [9].

3 Summary and Discussion

We have discussed charged lepton correction to TBM mixing with a possible form of $U_l$ which can generate $\sin \theta_{13}$ of the order of $\lambda \sqrt{2}$ and $\tan^2 \theta_{23} < 1$ under the consideration that the charged lepton correction is CKM like. The charged lepton mixing matrix $U_l$ is derived from three dimensional rotation matrices in the same manner as the TBM neutrino mixing matrix. We found that in the absence of CP violation numerical predictions on $\sin \theta_{13}$ and $\tan^2 \theta_{23}$ are consistent with latest global data but that on $\tan^2 \theta_{12}$ is significantly smaller than the global best fit value. However, when we introduce the Dirac CP violating phase $\delta$, the expression for $\tan^2 \theta_{12}$ shows that it becomes dependent on $\cos \delta$. This dependency can be employed to lift up the value of $\tan^2 \theta_{12}$ to desired experimental prediction. For the best fit value $\tan^2 \theta_{12} = 0.47$ we find $\cos \delta \approx 0.147$. Further we get expression for the rephasing invariant quantity in case of TBM mixing which is consistent with the result of [9].

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