Dynamics of Dirichlet-Neumann Open Strings on D-branes

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Abstract
Method for computing scattering amplitudes of open strings with Dirichlet boundary on one end and Neumann boundary condition on the other is described. Vertex operator for these states are constructed using twist fields which have been studied previously in the context of Ashkin-Teller model and strings on orbifolds. Using these vertex operators, we compute the three- and four-point scattering amplitudes for (5,9) strings on 5-branes and 9-branes. In the low energy limit, these amplitudes are found to be in exact agreement with the field theory amplitudes for supersymmetric Yang-Mills coupled to hypermultiplets in 6-dimensions. We also consider the 1-brane 5-brane system and compute the amplitude for a pair of (1,5) strings to collide and to escape the brane as a closed string. (1,5) strings are found to be remarkably selective in their coupling to massless closed strings in NS-NS sector; they only couple to the dilaton.

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1 Introduction

String solitons and non-perturbative dynamics of string theory have been a subject of fascination for some time. Remarkable progress has been made recently in the understanding of this subject following the realization that certain solitons in string theory admit explicit description in terms of D-branes \[1\]. These solitons are charged with respect to the Ramond-Ramond fields \[2\] and have been identified as the source of various non-perturbative phenomena\[1\].

One of the useful properties of D-branes is the fact that they admit a perturbative description at weak coupling in terms of the open strings attached to them. These open strings can interact amongst themselves, or interact with closed string degrees of freedom outside. Their dynamics can be described using traditional string perturbation theory with slight modifications. Techniques for studying this kind of physics were developed recently in \[3, 4, 5, 6\]. These studies revealed string scale as the dynamical length scale governing the physics of D-brane fluctuations\[2\] and rich dynamics on the D-brane world volume.

In this article, we extend the results of \[3, 4, 5, 6\] to open strings which have Neumann boundary condition on one end and Dirichlet boundary condition on the other end (the ND strings). These strings can exist when several D-branes of differing dimensionality, say, a 1-brane and a 5-brane, are simultaneously present. There are reasons to be interested in the perturbative dynamics of ND strings. For instance, bound states of 1-branes and 5-branes have been well studied in the context of black hole entropy counting \[12, 13, 14\]. When both 1-branes and 5-branes are abundant, it is the (1,5) strings which dominates entropically, and their dynamics is of interest in attempts to continue weak coupling calculations to the strong coupling region \[15\].

The need for calculational techniques for ND strings becomes even more acute when one attempts to study emission and absorption cross sections, as was done in \[16, 17, 18\]. For simple D-brane configurations, one can rely on the Dirac-Born-Infeld action \[19\] to teach us how the world volume degrees of freedom on the D-brane couple to the space-time fields. For complicated D-brane bound states, such as the ones involving multiple 1-branes and 5-branes, no analogous description of the effective coupling is known. We are therefore forced to perform string theory calculations along the lines of \[1\] to deduce the effective coupling and to attempt to reconstruct the generalized Dirac-Born-Infeld action. In this article, we will develop the techniques necessary to carry out this program.

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\[1\] For recent review and references, see \[1\] and references therein.

\[2\] There is however, growing evidence for sub-string length scales arising from D-brane interactions \[6, 8, 9, 10, 11\].
The first step in the perturbative study of string amplitudes is the construction of vertex operators. When an ND vertex is inserted, boundary conditions on the world sheet change. The ND vertex operator therefore necessarily contains boundary condition changing operators. We will show that twist fields, familiar from previous studies in the Ashkin-Teller model [20], off-shell string dynamics [21], and orbifolds [22] play the necessary role. We will use these twist fields to construct a vertex operator in conformal gauge which satisfies all the requirements [23]. We will find that this vertex operator closely resembles the vertex operator for twisted states in $Z_2$ orbifolds of type II theories [22].

In the remainder of the article, we will use these vertex operators to compute tree-level scattering amplitudes. We will follow the general strategy of [6] and relate the physics of open string sector alone and that of open/closed interactions. For concreteness, we will focus on type IIB theories. The 1-brane 5-brane system will always be in the back of our mind as a concrete example. In discussing open string sector, however, we will consider the 5-brane 9-brane system instead, as on-shell polarizations for gauge particles are absent on the 1-brane world volume. They are related simply by T-duality and/or dimensional reduction.

A comment is in order regarding the choice of vacuum. We will work in a background where all the fields have vanishing vacuum expectation value. This is the simplest background from the point of view of worldsheet conformal field theory. (For more general backgrounds, we must consider non-linear sigma models.) This is a special point in the moduli-space of vacua where branes are bound at threshold. They are free to fluctuate in shape and size [24, 25].

Let us also note in passing that techniques developed in this article are applicable to a wider variety of D-brane configurations. For example, the 1-brane 5-brane configuration featured in this article is T-dual to a configuration of intersecting D-branes [26]. Intersecting branes have been found to play an important role in the D-brane construction of 4-dimensional black holes [27, 28]. We hope to say more about open string dynamics on intersecting branes in the future.

The organization of this article is as follows. In section 2, we study world sheets with mixed Neumann and Dirichlet boundary conditions, and construct the vertex operator for ND strings. In section 3, we compute the three and four-point amplitude for these ND strings and compare with low energy effective theory. In section 4, we compute the Hawking emission amplitude for colliding (1,5) and (5,1) strings. We conclude in section 5.
2 World-sheets with Dirichlet and Neumann boundaries

The goal of this section is to construct the vertex operator for the ND strings. Because these states are open string states, the vertex operators are inserted along the boundary of the world sheet. The fact that these states are ND strings means that the boundary condition of the world sheet changes from Dirichlet to Neumann and vice versa at the insertion point of its vertex operator (see figure 1). The ND open string vertex operator must necessarily contain a boundary condition changing operator. To learn how one constructs such an operator, it is useful to study the scalar Green’s function on a disk with mixed Dirichlet-Neumann boundary conditions. By bringing the charges near the point where the boundary condition changes, we can infer the singularity structure of the operator product expansion of the twist operator and the scalar field.

Calculation of scalar Green’s function is a simple exercise in electrostatics. It is simplest to map the disk onto a strip parameterized by complex variable \( \{ \zeta : -\infty < \text{Re} \zeta < +\infty, 0 < \text{Im} \zeta < \pi \} \). The strip can be mapped onto half-plane by \( z = e^\zeta \), and the half plane can be mapped on to the disk as usual. We impose Neumann boundary condition at \( \text{Im} \zeta = 0 \) and Dirichlet boundary condition at \( \text{Im} \zeta = \pi \). On the half-plane, this corresponds to Dirichlet boundary condition on the negative real-axis and Neumann boundary condition on the positive real axis. The change of boundary condition occurs at \( z = 0 \) and \( z = \pm \infty \).

We can determine the potential at \( \zeta \) on a strip due to a charge at \( \omega \) by placing image charges of charge \( (-1)^n \) at \( (\omega + 2\pi n i) \) and \( (\bar{\omega} + 2\pi n i) \). These image charges ensure that
appropriate boundary conditions are satisfied. The potential at \( \zeta \) is therefore given by

\[
G(\zeta, \bar{\zeta}; \omega, \bar{\omega}) = \left( \sum_n (-1)^n \left\{ \ln [\zeta - (\omega + 2\pi n i)] + \ln [\zeta - (\bar{\omega} + 2\pi n i)] \right\} \right) + (\zeta \leftrightarrow \bar{\zeta})
\]

\[
= \left\{ \ln \left[ \frac{\prod_{n \text{ even}} (\zeta - \omega + 2\pi n i)}{\prod_{n \text{ odd}} (\zeta - \omega + 2\pi n i)} \right] + (\omega \leftrightarrow \bar{\omega}) \right\} + (\zeta \leftrightarrow \bar{\zeta}). \tag{1}
\]

The infinite product in the above expression can be evaluated explicitly in terms of matching expression with appropriate residues and poles. Mapping to half plane using \( z = e^\zeta, \ w = e^\omega \), we find

\[
G(z, w) = \prod_{n \text{ even}} \left[ \frac{1 - e^{\frac{\zeta - \omega}{2}}}{1 + e^{\frac{\zeta - \omega}{2}}} \right] = \ln \left[ \frac{1 - \sqrt{z} \sqrt{w}}{1 + \sqrt{z} \sqrt{w}} \right].
\]

This expression corresponds to scalar potential on a half plane where boundary condition changing operator is inserted at \( z = 0 \) and \( z = \pm \infty \). We might as well conformally deform the boundary condition changing points to arbitrary points along the boundary of the half plane. This is straightforward to do, and we find

\[
G(z_3, z_4) = \ln \left[ \frac{1 - \sqrt{z_3 z_4}}{1 + \sqrt{z_3 z_4}} \right],
\]

where the boundary condition changing points are \( z_1 \) and \( z_2 \), while \( z_3 \) and \( z_4 \) are the arguments of the Green’s functions. The electrostatic Green’s function is essentially the two-point correlation function of a scalar boson. However, in practice, it is the electric field, not the electrostatic potential, which makes a good conformal field. We are therefore interested in the correlation function of the primary field \( \partial X \).

\[
\frac{\langle \sigma(z_1)\sigma(z_2)\partial X(z_3)\partial X(z_4) \rangle}{\langle \sigma(z_1)\sigma(z_2) \rangle} = \partial_{z_3} \partial_{z_4} G(z_3, z_4) \tag{2}
\]

Here we have introduced a yet to be identified boundary condition changing operator \( \sigma(z) \) inserted at \( z_1 \) and \( z_2 \). Now let us probe the singularity structure of operator product expansion of \( \partial X \) and \( \sigma \) by studying the \( z_3 \rightarrow z_1 \) limit. By explicit calculation, we find that

\[
\partial_{z_3} \partial_{z_4} G(z_3, z_4) = \frac{1}{z_3^{1/2}} F(z_1, z_2, z_4) + \ldots
\]

where \( F \) is some function. What this suggests is that we should look for an operator algebra which includes \( \partial X \) and \( \sigma \) such that their leading operator product expansion is of the form

\[
\partial X(z)\sigma(0) = \frac{1}{z^{1/2}} \tau + \ldots
\]
where $\tau$ is another operator with dimension $1/2$ greater than the dimension of $\sigma$. It just so happens that a closed algebra consisting of $\partial X$, $\sigma$, and $\tau$ alone exists as a conformal limit of Ashkin-Teller model $[20, 21]$. These fields have dimensions

$$[\sigma] = \frac{1}{16}; \quad [\tau] = \frac{9}{16}$$

and their operator algebra is given by

$$\partial X^\alpha(z)\sigma^\beta(w) = \delta^{\alpha\beta}(z - w)^{-1/2}\tau^\beta(w) + \text{Reg}$$

$$\partial X^\alpha(z)\tau^\beta(w) = \frac{1}{2}\delta^{\alpha\beta}(z - w)^{-3/2}\sigma^\beta(w) + 2\delta^{\alpha\beta}(z - w)^{-1/2}\partial w\sigma^\beta(w) + \text{Reg}.$$ 

Fields $\sigma$ and $\tau$ are referred to as “twist fields” and “excited twist fields,” respectively. These fields have also been used in the vertex operators for type II twisted states on a $Z_2$ orbifold $[22]$. In the context of orbifolds, the square root branch cut in the operator product expansion implied that as $\partial X$ circumscribes $\sigma$, it picks up a minus sign, implying that $X$ and $-X$ are identified (i.e. orbifolded). In open strings, there is no such thing as a twisted state. Because the branch point is inserted at the boundary of the world sheet, we can maintain single valued operator algebra on the world sheet without any space-time identifications by simply pushing the branch cut to the outside the world sheet. It is therefore not necessary to orbifold the spacetime. Instead, the square-root branch cut exchanges the boundary condition between Dirichlet and Neumann. This can also be seen from the mode expansion of twisted states

$$X(z) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} a_r z^r.$$ 

which is identical to the mode expansion of ND strings.

Correlation functions involving these twist fields have been studied in $[20, 21, 22]$ using conformal field theory techniques. They are in agreement with equation (2) computed using the electrostatic method. By now, it is clear that these twist fields implement the change in boundary conditions.

Having found the operator responsible for changing the boundary condition, let us proceed to construct the vertex operators for ND string states. To be concrete, we will focus on Neveu-Schwarz states on 5-branes and 9-branes, oriented as follows

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
5\text{-brane:} & N & N & N & N & N & D & D & D & D \\
9\text{-brane:} & N & N & N & N & N & N & N & N & N \\
\end{array}
\]  

which breaks $SO(1,9)$ down to $SO(1,5) \times SO(4)$. At times, we will refer to ND strings as (5,9) strings to indicate the brane on which the strings end. In order to implement the
appropriate boundary condition on the world sheet, we need to insert a boundary condition changing operator for directions \{6789\}. Supersymmetry requires that we also change the boundary condition for the world sheet fermions. In the \{6789\} directions, Neveu-Schwarz states have Ramond boundary conditions. Therefore, the vertex operator for the ND state must also contain spin fields for \{6789\} directions. This is consistent with the oscillator analysis \[1\] where the spectrum was shown to contain a massless state which is singlet under \(SO(1, 5)\) and a chiral spinor of definite chirality under \(SO(4)\). The simplest natural candidate for the ND vertex operator then is

\[
V(z) = \lambda_{iJ} u_a e^{-\phi} S^\alpha \Delta e^{i2pX} (z). \tag{4}
\]

The factor of \(\lambda_{iJ}\) describes the Chan-Paton degrees of freedom which label the brane on which the strings end. The indices \(i\) and \(J\) transform under fundamental and anti-fundamental representations of \(U(Q_5)\) and \(U(Q_9)\) respectively. The \(SO(4)\) polarization of this state is given by \(u_a\). The spin field \(S(z)\) is given by the usual bosonization rule, except it only involves the \{6789\} directions

\[
S(z) = e^{\pm i \phi/2} \left( e^{\pm i \phi/2} \right)
\]

and has dimension 1/4. \(\Delta\) is the boundary condition changing operators for the directions \{6789\}:

\[
\Delta(z) = \sigma^6 \sigma^7 \sigma^8 \sigma^9 (z)
\]

and has dimensions 1/16 \(\times 4 = 1/4\).

The charge conjugate of (5,9) string is a (9,5) string. Charge conjugation flips the index structure of the Chan-Paton factor and changes the chirality of the spinor. The vertex operator then has the structure

\[
V(z) = \lambda_{iJ} u_\beta e^{-\phi} S_\beta \Delta e^{i2pX} (z). \tag{5}
\]

The superghost contribution to the conformal weight was fixed by the requirement that the total vertex operator be of conformal weight one. Not surprisingly, an identical expression was derived in the context of orbifolds using superfield techniques and picture changing \[22\]. Furthermore, it is amusing to note that we can reproduce the formula for the zero-point energy \[1\]

\[
-\frac{1}{2} + \frac{1}{8\nu}
\]

by simply requiring the vertex operator to have conformal weight one. Here, \(\nu\) is the number of ND dimensions. The \(-1/2\) comes from the dimension of ghosts, and \(1/8\) comes from
conformal weight per embedding dimension of 1/16 for the twist field and 1/16 for the spin field.

We have found an explicit expression (4) for the vertex operator of ND states. This is the main result of this article. In principle, one can compute any amplitudes involving ND states along this line. In the following sections, we will work out a few examples.

3 Open string dynamics of ND strings

In this section, we will study the tree-level dynamics of ND open strings. All external states in the amplitudes we consider in this section will be open strings. The low energy dynamics of these open strings are given by $N = 1$ super Yang-Mills with gauge fields $U(Q_5)$ and $U(Q_9)$ dimensionally reduced to six dimensions, and coupled to a hypermultiplet in fundamental and anti-fundamental representations of $U(Q_5)$ and $U(Q_9)$, respectively. Their interactions are mostly determined by supersymmetry and gauge invariance. The bosonic part of the action is given by [29, 30]

$$S = \int d^6x \left( \frac{1}{4} \text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{1}{2} \text{Tr}[(\partial_\mu A_M + g[A_\mu, A_M])^2] 
\quad + \frac{1}{4} \text{Tr}(F'_{\mu\nu}F'^{\mu\nu}) + \frac{1}{2} \text{Tr}[(\partial_\mu A'_M + g'[A'_\mu, A'_M])^2] 
\quad + \left| \left( \partial_\mu + g A_\mu T^a + g' A'_\mu T^a \right) \chi \right|^2 + \frac{1}{4} g^2 \sum_{aMN} D_{MN}^a + \frac{1}{4} g'^2 \sum_{aMN} D_{MN}^{a'} \right).$$

(6)

Gauge fields $A$ and $A'$ transform under $U(Q_5)$ and $U(Q_9)$ respectively. Indices $\mu$ and $\nu$ run from 0 to 5. Indices $M$ and $N$ runs from 6 to 9. Fields $A_M$ arise from dimensional reduction of gauge fields and are scalars. The hypermultiplet is described by $\chi$. In addition to gauge coupling, hypermultiplets couple to the gauge field by $D$-terms in the action. They are given by

$$D_{MN}^a = \left( f_{bc} A_M^b A_N^c + \chi^{\dagger} T^a \Gamma_{MN} \chi \right).$$

In the remainder of this section we will compute some tree level amplitudes involving the ND strings and verify that they are consistent with (6) in the low energy limit.

3.1 Three point functions

One simplest kind of amplitudes one can compute in string theory are the three point tree amplitudes. Let us first focus on these. A moment’s thought should convince the reader that
an even number of boundary condition changing operators is necessary along the boundary. Indeed, terms cubic in $\chi$'s are absent in the action (6). On the other hand, there is nothing inconsistent with a coupling of $A_\mu$ or $A_M$ with two $\chi$'s from the world sheet point of view. Let us consider each case separately.

3.1.1 $\chi\chi A_\mu$ coupling

The relevant vertex operators are

\[ V_1 = \lambda_1^I u_{1\alpha} e^{-\phi} \Delta S^\alpha e^{i2k_1X}(z_1) \]
\[ V_2 = \lambda_2^J u_{2\beta} e^{-\phi} \Delta S^\beta e^{i2k_2X}(z_1) \]
\[ V_3 = \Lambda_3^k \xi_{3\mu} (\partial X^\mu + (i2k_3\cdot \psi)\psi^\mu) e^{i2k_3X}(z_3). \] (7)

The total superghost charge is $-2$. The amplitude is given by

\[ A = \int \frac{dz_1 dz_2 dz_3}{V_{CKG}} (V_1(z_1)V_2(z_2)V_3(z_3)) \]
\[ = \text{Tr}(\lambda_1 \lambda_2 A_3)(z_{12}z_{13}z_{23}) \cdot \langle e^{-\phi}(z_1)e^{-\phi}(z_2) \rangle \cdot \xi_{3\mu} \langle e^{i2k_1X}(z_1)e^{i2k_2X}(z_2)\partial X^\mu e^{i2k_3X}(z_3) \rangle \]
\[ \cdot u_{1\alpha} u_{2\beta} \langle S^\alpha(z_1)S^\beta(z_2) \rangle \cdot \langle \Delta(z_1)\Delta(z_2) \rangle. \] (8)

All of the correlation functions are standard except possibly for

\[ \langle \Delta(z_1)\Delta(z_2) \rangle = \frac{1}{z_{12}^{1/2}} \]

which follows from the fact that $\Delta$ has conformal weight $1/4$. The amplitude evaluates to

\[ A = \text{Tr}(\lambda_1 \lambda_2 A_3) u_{1\alpha} u_{2\alpha} \xi_{3\mu} (k_1^\mu - k_2^\mu) \]

which is in agreement with the coupling due to the gauge interaction term

\[ S = |(\partial_\mu + gA_\mu)\chi|^2 \]

in the action (3).

3.1.2 $\chi\chi A_M$ coupling

The vertex operators are the same as in the $\xi\xi A_\mu$ case, but the polarization vector $\xi_M$ will now point in the $\{6789\}$ direction. Just as in the previous case, the amplitude is given by

\[ A = \int \frac{dz_1 dz_2 dz_3}{V_{CKG}} (V_1(z_1)V_2(z_2)V_3(z_3)) \]
\[ = \text{Tr}(\lambda_1 \lambda_2 A_3)(z_{12}z_{13}z_{23}) \cdot \langle e^{-\phi}(z_1)e^{-\phi}(z_2) \rangle \cdot \langle e^{i2k_1X}(z_1)e^{i2k_2X}(z_2)e^{i2k_3X}(z_3) \rangle \]
\[ \cdot u_{1\alpha} u_{2\beta} \langle S^\alpha(z_1)S^\beta(z_2) \rangle \cdot \xi_{3M} \langle \Delta(z_1)\Delta(z_2)\partial X^M(z_3) \rangle. \] (9)
Here, we encounter a new correlation function

$$\langle \Delta(z_1)\Delta(z_2)\partial X^M(z_3) \rangle.$$ 

In the electrostatic language, however, this amounts to measuring the electric field in the presence of boundary condition changing operators. Since no sources of electric fields are present, this correlation function must vanish. This implies that $\chi\chi A_M$ three point function also vanishes. This is in agreement with the absence of $\chi\chi A_M$ coupling term in the low-energy effective action (7).

3.2 Four point functions

The next simplest thing to the three-point functions are the four-point functions. These amplitudes are often written in terms of relativistic invariants

$$s = 4k_1 \cdot k_2 = 4k_3 \cdot k_4, \quad t = 4k_1 \cdot k_4 = 4k_2 \cdot k_3, \quad u = 4k_1 \cdot k_3 = 4k_2 \cdot k_4.$$ 

Generically, these four point amplitudes exhibit Regge-pole behavior, indicating exchange of a tower of massive intermediate string states. The low energy limit is gotten by expanding the amplitude to leading order in derivatives. In this approximation, the amplitude gets contributions only from the leading $s$ and $t$-channel pole, as well as contact terms in the low energy effective theory.

As was noted in the previous section, consistency requires that there be even number of external $\chi$’s in a given process. Amplitudes containing exactly two $\chi$’s are particularly simple. There are three such amplitudes: $\chi\chi A_\mu A_\nu$, $\chi\chi A_M A_N$, and $\chi\chi A_\mu A_M$. Let us consider each case separately.

3.2.1 $\chi\chi A_\mu A_\nu$ coupling

The relevant vertex operators are

$$V_1 = \lambda_1^{IJ} u_{1\alpha} e^{-\phi} \Delta S^\alpha e^{i2k_1 X}(z_1)$$

$$V_2 = \lambda_2^{JK} u_{2\beta} e^{-\phi} \Delta S^\beta e^{i2k_2 X}(z_1)$$

$$V_3 = \Lambda^{k\mu}_3 \xi_{3\mu}(\partial X^\mu + (i2k_3 \cdot \psi)\psi^{(\mu)}) e^{i2k_3 X}(z_3)$$

$$V_4 = \Lambda^{k\nu}_4 \xi_{4\nu}(\partial X^\nu + (i2k_4 \cdot \psi)\psi^{(\nu)}) e^{i2k_4 X}(z_4).$$ (10)

The total superghost charge is $-2$. The amplitude is given by

$$A = \int \frac{dz_1 dz_2 dz_3 dz_4}{V_{CKG}} \langle V_1(z_1)V_2(z_2)V_3(z_3)V_4(z_4) \rangle.$$
Although generally speaking one must compute these correlation functions in a usual manner, in this particular case there is a short-cut. The correlation function factorizes into parts involving world sheet fields with space-time index $\{012345\}$ and parts involving space-time index $\{6789\}$. Similar factorization takes place in a computation of $A_M A_N A_\mu A_\nu$. We can therefore cut the $\{6789\}$ part of the $\chi A_\mu A_\nu$ correlation function and paste it into the more familiar $A_M A_N A_\mu A_\nu$ amplitude which takes the form

$$A = \frac{\Gamma(s)\Gamma(t)}{\Gamma(1+s+t)} K(\xi_1, k_1; \xi_2, k_2; \xi_3, k_3; \xi_4, k_4).$$

The kinematic factor $K$ is the familiar open string kinematic factor [31] except for the fact that in our dimensionally reduced system, $\xi_1$ and $\xi_2$ are orthogonal to $\xi_3$, $\xi_4$, and all the $k_i$'s

$$K(\xi_1, k_1; \xi_2, k_2; \xi_3, k_3; \xi_4, k_4) = \left(\frac{tu}{4} \xi_3 \cdot \xi_4 + u k_1 \cdot \xi_4 k_2 \cdot \xi_3 + t k_1 \cdot \xi_3 k_2 \cdot \xi_4\right) \xi_1 \cdot \xi_2.$$

Upon comparison of $\{6789\}$ part of the correlation function, we note that the only difference between $\chi A_\mu A_\nu$ and $A_M A_N A_\mu A_\nu$ is that $\xi_1 \cdot \xi_2$ is replaced by $u_1 u_2$. Therefore, the amplitude of interest is given by

$$A = \sum_{3 \leftrightarrow 4} \frac{\Gamma(s)\Gamma(t)}{\Gamma(1+s+t)} \left(\frac{tu}{4} \xi_3 \cdot \xi_4 + u k_1 \cdot \xi_4 k_2 \cdot \xi_3 + t k_1 \cdot \xi_3 k_2 \cdot \xi_4\right) u_1 u_2 \text{Tr}(\lambda_1 \lambda_2 [A_3, A_4]).$$

In the low energy limit, this reduces to

$$A = \frac{1}{8} \xi_3 \cdot \xi_4 \text{Tr}(\lambda_1 \lambda_2 \{A_3, A_4\})$$

$$+ \frac{1}{s} \left[\frac{1}{8} (u-t) \xi_3 \cdot \xi_4 - \xi_3 \cdot k_2 \xi_4 \cdot k_1 + \xi_3 \cdot k_1 \xi_4 \cdot k_2\right] \text{Tr}(\lambda_1 \lambda_2 [A_3, A_4])$$

$$- \frac{1}{t} \xi_3 \cdot k_2 \xi_4 \cdot k_1 \text{Tr}(\lambda_1 \lambda_2 [A_3 A_4]) - \frac{1}{u} \xi_3 \cdot k_1 \xi_4 \cdot k_2 \text{Tr}(\lambda_1 \lambda_2 A_4 A_3).$$

(11)

All terms in the above expression also arise from the low-energy effective action. For example,

$$\frac{1}{8} \xi_3 \cdot \xi_4 \text{Tr}(\lambda_1 \lambda_2 \{A_3, A_4\})$$

is due to the contact term in the gauge coupling

$$\left| (\partial_\mu + gA_\mu^a T_a) \chi \right|^2,$$

and the term proportional to $1/s$ is due to exchange of massless gluons between $\chi A$ and $A^3$ three-point vertices.
3.2.2 $\chi\chi A_M A_N$ coupling

The vertex operator is the same as in the $\chi\chi A_\mu A_\nu$ case except for the fact that polarization vectors $\xi_{3M}$ and $\xi_{4N}$ point in the {6789} direction. The correlation function of interest takes the form

$$
\langle V_1(z_1)V_2(z_2)V_3(z_3)V_4(z_4)\rangle = \{e^{-\phi(z_1)}e^{-\phi(z_2)}\} \cdot \{e^{i2k_1X(z_1)}e^{i2k_3X(z_2)}e^{i2k_3X(z_3)}e^{i2k_4X(z_4)}\}
$$

$$
\left\{4k_3^{\mu\nu}\langle\psi_\mu(z_3)\psi_\nu(z_4)\rangle \cdot \langle S_\alpha(z_1)S_\beta(z_2)\psi^M(z_3)\psi^N(z_4)\rangle \cdot \langle \Delta(z_3)\Delta(z_4)\rangle + \langle S_\alpha(z_1)S_\beta(z_2)\rangle \cdot \langle \Delta(z_1)\Delta(z_2)\partial X(z_3)\partial X(z_4)\rangle \right\}. \tag{12}
$$

Twist field correlation function $\langle \Delta(z_1)\Delta(z_2)\partial X(z_3)\partial X(z_4)\rangle$ can be deduced from (1). The spin field correlation function

$$
\langle S_\alpha(z_1)S_\beta(z_2)\psi^M(z_3)\psi^N(z_4)\rangle = (z_{14}z_{24}z_{13}z_{23})^{-1/2}(z_{12})^{-1/2}(z_{34})^{-1}
$$

$$
\times \frac{1}{2} \left\{ \delta^{MN}\delta^\alpha_\beta(z_{14}z_{23} + z_{13}z_{24}) - [\Gamma^{MN}]^\alpha_\beta z_{12}z_{34} \right\}
$$

was worked out in [32]. Fixing conformal Killing volume by fixing the location of vertex operators at $\{-z, z, 1, -1\}$, the amplitude reduces to a simple integral expression

$$
A = \int dz \left[ \frac{4z}{(1 + z)^2} \right]^s \left[ \frac{(1 - z)^2}{(1 + z)^2} \right]^t \left[ (s - 1) \frac{1 + z^2}{z^2} \delta^{MN} - \frac{2s}{z} [\Gamma^{MN}]^\alpha_\beta \right] \text{Tr}(\lambda_1\lambda_2A_3A_4). \tag{13}
$$

Performing the integral, one obtains

$$
A = \sum_{3+4} \left( (s + 2t) \xi_3 \cdot \xi_4 u_1 u_2 - s (u_1\Gamma^{MN}u_2) \xi_{3M}\xi_{4N} \right) \frac{\Gamma(s)\Gamma(\frac{1}{2} + t)}{\Gamma(\frac{1}{2} + s + t)} \text{Tr}(\lambda_1\lambda_2A_3A_4).
$$

The half integer poles in the $t$-channel may seem unusual. These states arise from modes excited in the direction for which the world-sheet boundary condition is ND, as the oscillator number of these excitations is half integral. In the low energy limit, this amplitude reduces to

$$
A = \frac{1}{s} \xi_3 \cdot \xi_4 u_1 u_2 \text{Tr}(\lambda_1\lambda_2[A_3, A_4]) + (u_1\Gamma^{MN}u_2)\xi_{3M}\xi_{4N} \text{Tr}(\lambda_1\lambda_2[A_3, A_4]).
$$

From the low energy effective field theory point of view, the first term corresponds to exchange of gluon between $\chi\chi A$ and $A^3$ three-point vertices, and the second term corresponds to contact term arising from the $D$-term. The low energy limit of $\chi\chi A_M A_N$ amplitude is found to be consistent with the low-energy effective field theory.

3.2.3 $\chi\chi A_\mu A_M$ coupling

The vertex operators are the same as in the previous two subsections, except for the orientation of the polarization vectors. Upon inspection, one finds that the correlation function
takes the form

\[
\langle V_1(z_1)V_2(z_2)V_3(z_3)V_4(z_4) \rangle = \\
\langle \Delta(z_1)\Delta(z_2)\partial X^M(z_4) \rangle \cdot \langle S^\alpha(z_1)S_\beta(z_2) \rangle \times \{\text{ghosts and 012345-field correlator}\} \\
+ \langle \Delta(z_1)\Delta(z_2) \rangle \cdot \langle S^\alpha(z_1)S_\beta(z_2)\psi^M(z_4) \rangle \times \{\text{ghosts and 012345-field correlator}\}.
\]

(14)

Since \(\langle \Delta(z_1)\Delta(z_2)\partial X^M(z_4) \rangle\) and \(\langle S^\alpha(z_1)S_\beta(z_2)\psi^M(z_4) \rangle\) both vanish, it follows that \(\chi\chi A_\mu A_M\) four-point amplitude also vanishes. This too is consistent with the low-energy effective theory, as neither \(\chi\chi A_M\) and \(A_\mu A_\nu A_M\) three point coupling nor \(\chi\chi A_\mu A_N\) contact term is present in the theory.

4 Hawking radiation from decay of ND strings

So far we have focused on open string sector and have found agreement between the dynamics of ND states in string theory and the low energy effective field theory. These ND strings can also interact with the closed strings propagating outside the brane through the usual mechanism of open-closed string interaction. An example of such an interaction is a process where a pair of open string collides and escapes the brane by becoming a closed string. This kind of process mimics Hawking radiation if these D-branes are truly black holes [12] and has recently been studied more carefully in [16, 17, 18].

The near extremal black hole studied in [12, 13, 14] is a system of 1-branes and 5-branes. This is related to the system of 5-branes and 9-branes considered in the previous section by T-dualizing along \{2345\} directions. For reference, we list the orientation of these branes below:

| Direction | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------|---|---|---|---|---|---|---|---|---|---|
| 1-brane   | N | N | D | D | D | D | D | D | D | D |
| 5-brane   | N | N | D | D | D | D | N | N | N | N |

Directions \{6789\} are compactified, and the 5-brane wraps around the internal space and becomes a string parallel to the D1-brane. The compactification breaks the Lorentz group from \(SO(1,9)\) to \(SO(1,5) \times SO(4)_I\). The string further breaks \(SO(1,5) \times SO(4)_I\) down to \(SO(1,1) \times SO(4)_E \times SO(4)_I\).

We further compactify the \(\hat{1}\) direction. By taking this compactification radius larger than the compactification radius in \{6789\} directions, the 1-brane 5-brane system describes a macroscopic black string wrapped around the long direction which was discussed in [13]. On the other hand, if the \(\hat{1}\) radius is small, this system describes a black hole in 5-dimensions.
When the numbers of 1-branes and 5-branes are both large, strings stretching between 1-branes and 5-branes dominate entropically. These are precisely the ND strings we have been studying. It would be quite interesting to compute the Hawking radiation rate for the decay of (1,5) strings.

Fortunately, it was shown recently in \[6\] how amplitudes of this type can be computed quite effortlessly in terms of open-string four-point functions. These are precisely the four-point amplitudes we have been considering in the previous section. In the remainder of this section, we will apply the technique of \[6\] to compute the decay amplitude of (1,5) strings.

We will only compute the decay into massless closed string states in the Neveu-Schwarz sector. These states are gravitons, antisymmetric tensors, and the dilaton. In the context of black hole (string) entropy counting \[12, 13, 14\], these fields are dimensionally reduced to 5 + 1 dimension by compactifying along the \(\{6789\}\) directions. It is therefore convenient to consider polarizations which transform differently under \(SO(1,5) \times SO(4)\) separately. The polarization breaks down under \(SO(1,5) \times SO(4)\) into the following groups: \(\varepsilon_{\mu\nu}\), \(\varepsilon_{\mu N}\), \(\varepsilon_{MN}\).

Polarization tensor \(\varepsilon_{\mu\nu}\) describes the graviton, the antisymmetric tensor, and the dilaton in 5 + 1 dimensions. States with polarization \(\varepsilon_{\mu N}\) describes states which transform as a vector under \(SO(5,1)\) Lorentz group. Finally, \(\varepsilon_{MN}\) is a scalar under \(SO(1,5)\). Note that this grouping is precisely analogous to cases \(\chi\chi A_\mu A_\nu\), \(\chi\chi A_\mu A_N\), and \(\chi\chi A_M A_N\) which we considered separately in the previous section.

What makes these amplitudes so easy to compute is that under suitable prescription of fixing the conformal killing group, the Hawking amplitude and the open string 4-point amplitude take on an identical form. The only thing which distinguishes between these amplitudes is the contour of integration. We refer readers to \[6\] for the details concerning this trick.

We denote the momenta of incoming (1,5) and (5,1) strings by \(p_1\) and \(p_2\). Because these strings are attached to the 1-brane, momentum vectors are constrained to lie in \(\{01\}\) plane. We denote the momentum of the closed string by \(q^\mu\). There is one kinematic invariant for this process given by

\[
t = 2p_1 \cdot q = 2p_2 \cdot q = -2p_1 \cdot p_2.
\]

It will be convenient to restrict \(q^\mu\) to lie in \(\{012345\}\) space, as the open string correlation functions in section 3 were computed for this case. Since \(\{6789\}\) directions are compactified, this corresponds to considering only states neutral with respect to the Kaluza-Klein gauge fields in the context of Hawking radiation calculation.

Let us start with the \(\varepsilon_{\mu\nu}\) case. In the corresponding \(\chi\chi A_\mu A_\nu\) calculation, the correlation function was found to be identical in form to that of \(A^4\) amplitude. Without doing any work,
it follows that the $\varepsilon_{\mu\nu}$ amplitude must be of the form identical to what was found in [6], namely

$$A = \frac{\Gamma(-2t)}{\Gamma(1-t)^2} K(1, 2, 3)$$

where $K(1, 2, 3)$ is obtained from the kinematic factor we found in section 3.2.1 by imposing the kinematic constraint $-2s = t = u$ and making the substitution $\varepsilon \cdot D \rightarrow \xi_3 \otimes \xi_4$. In the low-energy limit, this amplitude reduces to

$$A = u_1 u_2 \left( p_2 \cdot \varepsilon \cdot p_1 + p_1 \cdot \varepsilon \cdot p_2 + \frac{t}{4} \text{Tr} (D \cdot \varepsilon) \right)$$

where the trace runs only over $\{012345\}$ directions. To enumerate the physical closed string states involved in this process, it is convenient to express the amplitude explicitly in terms allowed momentum and polarization vectors. Without loss of generality, we can set

$$q^\mu = \{q_0, q_1, \vec{q}\}$$

where $\vec{q}$ points in some $\{2345\}$ direction. In six dimensions, there are four physical polarizations transverse to $q^\mu$: 

$$\begin{align*}
\varepsilon_\perp &= \{0, \frac{\vec{q}}{q_0}, -\frac{q_0}{q_0} \hat{q}\} \\
\varepsilon_A &= \{0, 0, \varepsilon_A\} \\
\varepsilon_B &= \{0, 0, \varepsilon_B\} \\
\varepsilon_C &= \{0, 0, \varepsilon_C\}
\end{align*}$$

where $\{\hat{q}, \varepsilon_A, \varepsilon_B, \varepsilon_C\}$ are mutually orthogonal set of vectors in the $\{2345\}$ plane. The most general polarization for which the amplitude (16) is non-vanishing is of the form

$$\varepsilon^{\mu\nu} = C_\perp \varepsilon_\perp^{\mu} \varepsilon_\perp^{\nu} + C_A \varepsilon_A^{\mu} \varepsilon_A^{\nu} + C_B \varepsilon_B^{\mu} \varepsilon_B^{\nu} + C_C \varepsilon_C^{\mu} \varepsilon_C^{\nu}$$

(17)

where $C_\perp, C_A, C_B$, and $C_C$ are numerical coefficients. Substituting (17) back into (16), we arrive at

$$A = -\frac{t}{4} (C_\perp + C_A + C_B + C_C).$$

The only state for which this amplitude is non-vanishing is the dilaton, suggesting a low-energy effective coupling of the form

$$S = \phi \partial^\alpha \chi^\alpha \partial_\mu \chi_\mu.$$ 

(18)

Here, indices $\mu$ and $\nu$ run over the dimensions of the 1-brane world volume $\{01\}$.

For the case of $\varepsilon_{\mu N}$, we found that the correlation function vanished in the corresponding open string 4-point amplitude. This means that a pair of (1,5) strings can not decay into an $\varepsilon_{\mu N}$ state.
Finally, let us consider the $\varepsilon_{MN}$ case. The integral expression for the corresponding open-string amplitude was written down in (13). The prescription of [6] is to make a change of variables $z = ix$ and integrate over $-\infty < x < \infty$. When this is done (13) becomes

$$A = \int_{-\infty}^{\infty} dx \left[ \frac{(1 + x^2)^2}{16x^2} \right]^t \left( (2t + 1) \frac{1 - x^2}{x^2} \delta^\alpha_\beta g^{MN} - \frac{2s}{ix} \Gamma^{MN} \right).$$

This integral vanishes as first term is odd under $x \to 1/x$ and the second term is odd under $x \to -x$. (1,5) strings can not decay into $\varepsilon_{MN}$ state.

To summarize, the only massless NS-NS state to which a pair of (1,5) string can collide and decay into is the dilaton. No other fields seem to couple to the (1,5) strings.

## 5 Conclusions

In this article we studied the perturbative dynamics of open strings in the ND sector. By studying the singularity in electrostatic potential near the boundary condition changing point on a half-plane, we were led to twist fields as a necessary ingredient. Using these twist fields we constructed the vertex operators satisfying the usual requirements and carrying appropriate spacetime quantum numbers. The resulting vertex operator turned out to closely resemble the vertex operator for type II twisted states on $Z_2$ orbifolds. In rough terms, ND states are the open string analogue of twisted states [3].

Using these vertex operators, we computed several string amplitudes. The effective low-energy dynamics on the world volume was shown to be in perfect agreement with the string theory calculation.

We then followed [6] and computed the leading amplitude for a pair of (1,5) strings to collide and escape the brane. These amplitudes exhibit the same Regge-pole structure previously found for the DD strings [6]. Somewhat surprisingly, (1,5) strings are found to be quite selective in their choice of decay-particles. They appear to couple only to the dilaton.

The tools developed in this article provides us with control over perturbative dynamics of ND strings. It would be interesting to explore the consequence of string theory dynamics to black hole physics.

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3Twisted sector of open strings was considered recently in [6] from a different point of view.
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