Gravitational wave in Lorentz violating gravity

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By making use of the weak gravitational field approximation, we obtain a linearized solution of the gravitational vacuum field equation in an anisotropic spacetime. The plane-wave solution and dispersion relation of gravitational wave is presented explicitly. There is possibility that the speed of gravitational wave is larger than the speed of light and the causality still holds. We show that the energy-momentum of gravitational wave in the anisotropic spacetime is still well defined and conserved.

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I. INTRODUCTION

Lorentz Invariance is one of the foundations of the Standard model of particle physics. The constraints on possible Lorentz violating phenomenology are quite severe, see for example, the summary tables that provided by Kostelecky et al.[1]. The gravitational interaction is far more weak, compare to other fundamental interactions. This allows one to study the possible Lorentz violating effects on certain gravity theories, such as Einstein-aether theory [2] and Horava-Lifshitz theory [3]. One feature of Lorentz invariance violation is that the speed of light differ from the one in special relativity. The gravity theories with Lorentz violation could have the feature that the speed of graviton or the speed of gravitational wave differ from the one in general relativity. Studying the speed of gravitational wave in a Lorentz violating gravity theory will give different perspective on quantum gravitational phenomena.

One of the most important prediction of Einstein’s general relativity is gravitational radiation. Many pioneer works [4–6] have discussed the gravitational radiation in both theoretical properties and experimental approaches of detections. Currently, the most sensitive measurement is provided by ground-based Laser Interferometer Gravitational-Wave Observatory (LIGO) detector [7]. Another sensitive measurement, which is in progress, is the Laser Interferometer Space Antenna (LISA) that detect and accurately measure gravitational waves from astronomical sources. The primordial gravitational waves [8, 9] could be of interest to cosmologists as they provide a new and unique window on the earliest moments in the history of the universe and on possible new physics at energies many orders of magnitude beyond those accessible at particle accelerators.

In general relativity, the effects of gravitation are ascribed to spacetime curvature instead of a force. However, up to now, general relativity still faces problems.

First, the recent astronomical observations [10] found that our universe is accelerated expanding. This result can not be obtained directly from Einstein’s gravity and his cosmological principle. Since normal matters only provide attractive force. The most widely adopted way to resolve it is involving the so called dark energy which provides the repulsive force.

Second, the flat rotation curves of spiral galaxies violate the prediction of Einstein’s gravity [11]. The most widely adopted way to resolve it is involving the so called dark matter which provides enough attractive force such that the discrepancy is restored.

The above astronomical phenomena occur at very large cosmological scale. The following anomalies occur in solar system which imply the Newton’s inverse-square law of universal gravitation and general relativity need modifications.

The third one, two Pioneer spacecrafts suffer an anomalous constant sunward acceleration, \( a_p = (8.74 \pm 1.33) \times 10^{-10} \text{m/s}^2 \) [12].

The fourth one, it has been observed at various occasions that satellites after an Earth swing-by possess a significant unexplained velocity increase by a few mm/s [13].

The fifth one, from the analysis of radiometric measurements of distances between the Earth and the major planets including observations from Martian orbiters and landers from 1961 to 2003 a secular trend of the Astronomical Unit...
of $15 \pm 4$ m/cy has been reported \[14\].

The sixth one, a recent orbital analysis of Lunar Laser Ranging (LLR) \[15\] shows an anomalous secular eccentricity variation of the Moon’s orbit $(0.9 \pm 0.3) \times 10^{-11}$/yr.

All the facts imply that the Einstein’s theory should be modified. By mimicking Einstein, we have proposed that the modified gravitational theory should correspond to a new geometry which involves Riemann geometry as its special case. Finsler geometry \[16\] as a nature extension of Riemann geometry is a good candidate to solve the problems mentioned above. A new geometry (Finsler geometry) involves new spacetime symmetry. The Lorentz violation is intimately linked to Finsler geometry. Kostelecky \[17\] have studied effective field theories with explicit Lorentz violation in Finsler spacetime.

Finsler geometry really gives better description for the nature of gravity: the flat rotation curves of spiral galaxies can be deduced naturally without invoking dark matter \[18\]; a Finslerian gravity model could account for the accelerated expanding universe without invoking dark energy \[19\]; a special Finsler space-Randers space \[20\] could account for the anomalous acceleration \[12\] in solar system observed by Pioneer 10 and 11 spacecrafts; the Finsler spacetime could give a modification on the gravitational deflection of light \[21\], which may account to these observations without adding dark matter in Bullet Cluster \[22\]; the result based on the kinematics with a special Finsler spacetime is in good agreement with secular trend of the Astronomical Unit and secular eccentricity variation of the Moon’s orbit \[23\].

It is interest to investigate the gravitational wave in Finsler spacetime. It is well known that the gravitational wave propagates with the speed of light in general relativity. This is due to the fact that the spacetime metric is close to the Minkowski metric in the weak gravitational field approximation, and the causal speed of Minkowski spacetime is just the speed of light. However, in Finsler spacetime the causal speed is generally different with the speed of light \[24\].

In this paper, we will present the solution of linearized gravitational vacuum field equation in Finsler spacetime. It is shown that there is possibility that the causal speed of it is larger than the speed of light.

II. VACUUM FIELD EQUATION IN FINSLER SPACETIME

Instead of defining an inner product structure over the tangent bundle in Riemann geometry, Finsler geometry is based on the so called Finsler structure $F$ with the property $F(x, \lambda y) = \lambda F(x, y)$ for all $\lambda > 0$, where $x$ represents position and $y \equiv \frac{dx}{d\tau}$ represents velocity. The Finsler metric is given as \[16\]

$$g_{\mu\nu} \equiv \frac{\partial}{\partial y^\mu} \frac{\partial}{\partial y^\nu} \left( \frac{1}{2} F^2 \right).$$

(1)

Finsler geometry has its genesis in integrals of the form

$$\int_s^r F(x^1, \cdots, x^n; \frac{dx^1}{d\tau}, \cdots, \frac{dx^n}{d\tau}) d\tau.$$  

(2)

The Finsler structure represents the length element of Finsler space.

The parallel transport has been studied in the framework of Cartan connection \[25\]. The notation of parallel transport in Finsler manifold means that the length $F \left( \frac{dx}{d\tau} \right)$ is constant. The geodesic equation for Finsler manifold is given as \[16\]

$$\frac{d^2 x^\mu}{d\tau^2} + 2G^\mu = 0,$$

(3)

where

$$G^\mu = \frac{1}{4} g^{\mu\nu} \left( \frac{\partial^2 F^2}{\partial x^\lambda \partial y^\nu y^\lambda} - \frac{\partial F^2}{\partial x^\nu} \right).$$

(4)

is called geodesic spray coefficient. Obviously, if $F$ is Riemannian metric, then

$$G^\mu = \frac{1}{2} \hat{\gamma}_{\nu\lambda} y^\nu y^\lambda,$$

(5)

where $\hat{\gamma}_{\nu\lambda}$ is the Riemannian Christoffel symbol. Since the geodesic equation \[43\] is directly derived from the integral length

$$L = \int F \left( \frac{dx}{d\tau} \right) d\tau,$$

(6)
the inner product \( \left( \sqrt{g_{\mu \nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}} \right) \) of two parallel transported vectors is preserved.

In Finsler manifold, there exists a linear connection - the Chern connection [23]. It is torsion free ness and almost metric-compatibility,

\[
\Gamma^\alpha_{\mu \nu} = \gamma^\alpha_{\mu \nu} - g^{\alpha \lambda} \left( A_{\lambda \mu \beta} \frac{N^\beta}{F} - A_{\mu \lambda \beta} \frac{N^\beta}{F} + A_{\nu \lambda \beta} \frac{N^\beta}{F} \right),
\]

(7)

where \( \gamma^\alpha_{\mu \nu} \) is the formal Christoffel symbols of the second kind with the same form of Riemannian connection, \( N^\mu \) is defined as \( N^\mu \equiv \gamma^\mu_{\nu \alpha} y^\alpha - A^\mu_{\nu \lambda} \gamma^\alpha_{\mu \lambda} y^\beta y^\beta \) and \( A_{\lambda \mu \beta} \equiv \frac{F}{\sqrt{F}} \frac{\partial^2 F}{\partial y^\lambda \partial y^\mu \partial y^\beta} + \frac{F}{\sqrt{F}} \frac{\partial^2 F}{\partial y^\mu \partial y^\beta \partial y^\alpha} \) is the Cartan tensor (regarded as a measurement of deviation from the Riemannian Manifold). In terms of Chern connection, the curvature of Finsler space is given as

\[
R^\lambda_{\kappa \mu \nu} = \frac{\delta \Gamma^\lambda_{\kappa \nu}}{\delta x^\mu} - \frac{\delta \Gamma^\lambda_{\kappa \mu}}{\delta x^\nu} + \Gamma^\lambda_{\alpha \mu} \Gamma^\alpha_{\kappa \nu} - \Gamma^\lambda_{\alpha \nu} \Gamma^\alpha_{\kappa \mu},
\]

(8)

where \( \frac{\delta}{\delta y^\mu} = \frac{\partial}{\partial y^\mu} - N^\mu_{\nu \lambda} \frac{\partial}{\partial y^\lambda} \).

The gravity in Finsler spacetime has been investigated for a long time [29–32]. In this paper, we introduce vacuum field equation by the way discussed first by Pirani [33, 34]. In Newton’s theory of gravity, the equation of motion of a test particle is given as

\[
\frac{d^2 x^i}{dt^2} = -\eta^{ij} \frac{\partial \phi}{\partial x^j},
\]

(9)

where \( \phi = \phi(x) \) is the gravitational potential and \( \eta^{ij} \) is Euclidean metric. For an infinitesimal transformation \( x^i \rightarrow x^i + \epsilon \xi^i(\epsilon \ll 1) \), the equation (9) becomes, up to first order in \( \epsilon \),

\[
\frac{d^2 x^i}{dt^2} + \epsilon \frac{d^2 \xi^i}{dt^2} = -\eta^{ij} \frac{\partial \phi}{\partial x^j} + \epsilon \eta^{ij} \xi^k \frac{\partial^2 \phi}{\partial x^j \partial x^k}.
\]

(10)

Combining the above equations (9) and (10), we obtain

\[
\frac{d^2 \xi^i}{dt^2} = \eta^{ij} \xi^k \frac{\partial^2 \phi}{\partial x^j \partial x^k} \equiv \xi^k H_k^i.
\]

(11)

In Newton’s theory of gravity, the vacuum field equation is given as \( H_i^j = \nabla^2 \phi = 0 \). It means that the tensor \( H_k^i \) is traceless in Newton’s vacuum.

In general relativity, the geodesic deviation gives similar equation

\[
\frac{D^2 \xi^\mu}{D\tau^2} = \xi^\nu \tilde{R}_{\mu \nu}^\nu,
\]

(12)

where \( \tilde{R}_{\mu \nu}^\nu = \tilde{R}_{\mu \nu \rho} \frac{dx^\rho}{d\tau} \frac{dx^\nu}{d\tau} \). Here, \( \tilde{R}_{\mu \nu \rho} \) is Riemannian curvature tensor, \( D \) denotes the covariant derivative along the curve \( x^\mu(\tau) \). The vacuum field equation in general relativity gives \( \tilde{R}_{\mu \lambda \nu \sigma} = 0 \). It implies that the tensor \( \tilde{R}^\nu_{\mu} \) is also traceless, \( \tilde{R} = \tilde{R}^\mu_{\mu} = 0 \).

In Finsler spacetime, the geodesic deviation gives [16]

\[
\frac{D^2 \xi^\mu}{D\tau^2} = \xi^\nu R^\nu_{\mu},
\]

(13)

where \( R^\nu_{\mu} = R^\nu_{\lambda \mu \rho} \frac{dx^\lambda}{d\tau} \frac{dx^\mu}{d\tau} \frac{dx^\rho}{d\tau} \). Here, \( R^\nu_{\lambda \mu \rho} \) is Finsler curvature tensor defined in [8], \( D \) denotes covariant derivative \( \frac{Dx^\nu}{D\tau} = \frac{dx^\nu}{d\tau} + \xi^\nu \frac{\partial \Gamma^\nu_{\lambda \rho}}{\partial y^\lambda} \). Since the vacuum field equations of Newton’s gravity and general relativity have similar form, we may assume that vacuum field equation in Finsler spacetime hold similar requirement as the case of Newton’s gravity and general relativity. It implies that the tensor \( R^\nu_{\mu} \) in Finsler geodesic deviation equation should be traceless, \( R = R^\mu_{\mu} = 0 \). We have proved that the analogy from the geodesic deviation equation is valid at least in Finsler spacetime of Berwald type [18]. For this reason, we may suppose that this analogy is valid in general Finsler spacetime.

It should be noticed that \( H \) is called the Ricci scaler, which is a geometrical invariant. For a tangent plane \( T_x M \) and a non-zero vector \( y \in T_x M \), the flag curvature is defined as

\[
K(\Pi, y) = \frac{g_{\lambda \mu} R^\mu_{\rho \nu} u^\rho u^\lambda}{F^2 g_{\rho \theta} u^\rho u^\theta - (g_{\rho \theta} y^\rho y^\theta)^2},
\]

(14)
where \( u \in \Pi \). The flag curvature is a geometrical invariant that generalizes the sectional curvature of Riemannian geometry. It is clear that the Ricci scaler \( R \) is the trace of \( R_{\mu}^{\lambda} \), which is the predecessor of flag curvature. Therefore, the value of Ricci scaler \( R \) is invariant under the ordinate transformation. Furthermore, the predecessor of flag curvature could be written in terms of the geodesic spray coefficient

\[
R_{\mu}^{\nu} = 2 \frac{\partial G_{\mu}^{\lambda}}{\partial x^{\nu}} - y^{\lambda} \frac{\partial^{2} G_{\mu}^{\lambda}}{\partial x^{\nu} \partial y^{\rho}} + 2 G_{\lambda}^{\rho} \frac{\partial^{2} G_{\mu}^{\lambda}}{\partial y^{\rho} \partial y^{\nu}} - \frac{\partial G_{\nu}^{\lambda}}{\partial y^{\rho}} \frac{\partial G_{\mu}^{\lambda}}{\partial y^{\rho}}.
\]

(15)

Thus, the Ricci scaler \( R \) is insensitive to connection that one is using, it only depends on the length element \( F \). The gravitational vacuum field equation \( R = 0 \) is universal in any types of theories of Finsler gravity. Pfeifer et al. have constructed gravitational dynamics for Finsler spacetimes in terms of an action integral on the unit tangent bundle. Their researches also show that the gravitational vacuum field equation in Finsler spacetime is \( R = 0 \).

III. GRAVITATIONAL WAVE IN FINSLERIAN VACUUM

It is hard to find a non trivial solution of the gravitational vacuum field equation \( (R = 0) \) in Finsler spacetime. Here, we study the weak field radiative solution of the Finslerian vacuum field equation \( R = 0 \). It is well known that the Minkowski spacetime is a trivial solution of Einstein’s vacuum field equation. In the Finsler spacetime, the trivial solution of Finslerian vacuum field equation is called locally Minkowski spacetime. A Finsler spacetime is called a locally Minkowski spacetime if there is a local coordinate system \( (x^{\mu}) \), with induced tangent space coordinates \( y^{\mu} \), such that \( F \) depends only on \( y \) and not on \( x \). Using the formula (18), one knows obvious that locally Minkowski spacetime is a solution of Finslerian vacuum field equation.

We suppose that the metric is close to the locally Minkowski metric \( \eta_{\mu\nu}(y) \),

\[
g_{\mu\nu} = \eta_{\mu\nu}(y) + h_{\mu\nu}(x, y),
\]

(16)

where \( |h_{\mu\nu}| \ll 1 \). In the rest of this section, the lowering and raising of indices are carried out by \( \eta_{\mu\nu} \) and its matrix inverse \( \eta^{\mu\nu} \). To first order in \( h \), the geodesic spray coefficient is

\[
G_{\mu}^{\lambda} = \frac{1}{4} \eta_{\rho\sigma} \left( 2 \frac{\partial h_{\sigma\nu}}{\partial x^{\lambda}} y^{\rho} y^{\lambda} - \frac{\partial h_{\rho\nu}}{\partial y^{\sigma}} y^{\rho} y^{\lambda} \right).
\]

(17)

We have already used the Euler’s theorem for homogeneous functions to obtain the above equation. And the Ricci scaler is

\[
R = R_{\mu}^{\mu} = 2 \frac{\partial G_{\mu}^{\lambda}}{\partial x^{\mu}} - y^{\rho} \frac{\partial^{2} G_{\mu}^{\lambda}}{\partial x^{\rho} \partial y^{\mu}}.
\]

(18)

where

\[
2 \frac{\partial G_{\mu}^{\lambda}}{\partial x^{\mu}} = \frac{1}{2} \eta_{\rho\sigma} \left( 2 \frac{\partial^{2} h_{\sigma\nu}}{\partial x^{\lambda} \partial x^{\rho}} y^{\rho} y^{\lambda} - \frac{\partial^{2} h_{\rho\nu}}{\partial x^{\lambda} \partial y^{\rho}} y^{\rho} y^{\lambda} \right)
\]

(19)

and

\[
- y^{\rho} \frac{\partial^{2} G_{\mu}^{\lambda}}{\partial x^{\rho} \partial y^{\mu}} = - \frac{y^{\rho}}{4} \eta_{\rho\sigma} \frac{\partial}{\partial x^{\sigma}} \left( 2 \frac{\partial h_{\mu\nu}}{\partial x^{\lambda}} y^{\lambda} + 2 \frac{\partial h_{\rho\nu}}{\partial x^{\lambda}} y^{\rho} - 2 \frac{\partial h_{\rho\mu}}{\partial x^{\nu}} y^{\rho} \right) - \frac{y^{\rho} \eta_{\rho\sigma}}{4} \frac{\partial}{\partial y^{\sigma}} \left( \frac{2 \partial h_{\mu\nu}}{\partial x^{\lambda}} y^{\lambda} - \frac{\partial h_{\rho\nu}}{\partial x^{\lambda}} y^{\rho} \right).
\]

(20)

Since the value of Ricci scaler \( R \) is invariant under the coordinate transformation, we must fix the gauge symmetry to yield unique solution. Under a coordinate transformation

\[
\bar{x}^{\mu} = x^{\mu} + \epsilon^{\mu}(x),
\]

(21)

the metric \( h_{\mu\nu} \) transforms as

\[
\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{\partial \epsilon_{\mu}}{\partial x^{\lambda}} \eta_{\mu}^{\lambda} - \frac{\partial \epsilon_{\nu}}{\partial x^{\lambda}} \eta_{\nu}^{\lambda}.
\]

(22)

By performing the coordinate transformation with

\[
\eta^{\mu\lambda} \frac{\partial^{2} \epsilon_{\nu}}{\partial x^{\mu} \partial x^{\lambda}} = \frac{\partial h_{\mu}^{\nu}}{\partial x^{\mu}} - \frac{1}{2} \frac{\partial h_{\mu}^{\nu}}{\partial x^{\nu}},
\]

(23)
we find that $\tilde{h}_{\mu\nu}$ satisfies
\[
\frac{\partial \tilde{h}_{\mu\nu}}{\partial x^\nu} = \frac{1}{2} \frac{\partial \tilde{h}_{\mu\nu}}{\partial x^\nu}.
\] (24)
This choice of gauge (24) has the same form with the Lorentz gauge in general relativity, due to the fact that the locally Minkowski metric $\eta_{\mu\nu}$ does not depend on $x$.

By making use of the Finslerian gauge (24), and noticing that $\eta_{\mu\nu}$ does not depend on $x$, we rewrite the Ricci scaler as
\[
R = -\frac{\eta^{\mu\nu}}{2} \frac{\partial^2 h_{\alpha\beta}}{\partial x^\mu \partial x^\nu} y^\alpha y^\beta + \frac{1}{4} \frac{\partial \eta^{\mu\nu}}{\partial y^\nu} \frac{\partial^2 h_{\alpha\beta}}{\partial x^\mu \partial x^\nu} y^\lambda y^\beta.
\] (25)
We find from (25) that the solution of $R = 0$ has following properties
\[
h_{\mu\nu}(x, y) = \epsilon_{\mu\nu} \exp(ik\lambda x^\lambda) + h.c.
\] , (26)
where
\[
k_\mu k_\nu \eta^{\mu\nu} - \frac{1}{2} \frac{\partial y^{\mu\nu}}{\partial y^\nu} k_\nu k_\mu y^\lambda = 0,
\] (27)
$k = k(y)$ is function of $y$ and $\epsilon_{\mu\nu}$ is the polarization tensor. The term $\frac{\partial y^{\mu\nu}}{\partial y^\nu}$ could be written as
\[
\frac{\partial y^{\mu\nu}}{\partial y^\nu} = -2A_\mu^{\nu}/\tilde{F} = -\eta^{\nu\lambda} \frac{\partial \ln |\det(\eta)|}{\partial y^\lambda},
\] (28)
where $\tilde{F}^2 = \eta_{\mu\nu} y^\mu y^\nu$. Substituting the equation (28) into (27), we obtain
\[
k_\mu k_\nu \eta^{\mu\nu} = -\eta^{\nu\lambda} \frac{\partial \ln |\det(\eta)|}{\partial y^\lambda} k_\nu k_\mu y^\mu.
\] (29)
It is obvious that $k_\mu k_\nu \eta^{\mu\nu} \neq 0$ while the Finsler spacetime $\eta_{\mu\nu}$ is not Minkowskian. It implies that the wave vectors $k_\mu$ of gravitational waves is not null in Finsler spacetime $\eta_{\mu\nu}$.

The Randers spacetime [37] is a special kind of Finsler geometry with Finsler structure
\[
\tilde{F}(x, y) \equiv \alpha + \beta,
\] (30)
where
\[
\alpha \equiv \sqrt{\tilde{a}_{\mu\nu} y^\mu y^\nu},
\] (31)
\[
\beta \equiv \tilde{b}_\mu y^\mu,
\] (32)
and $\tilde{a}_{\mu\nu}$ is Riemannian metric. The indices on certain objects that decorated with a bar are lowered and raised by $\tilde{a}_{\mu\nu}$ and its matrix inverse $\tilde{a}^{\mu\nu}$. Substituting the Randers-Finsler structure $\tilde{F}$ into the dispersion relation of gravitational wave (29) and supposing the Randers spacetime is very close to Minkowski spacetime $\tilde{a}_{\mu\nu}$, to first order in $\tilde{b}$, we obtain
\[
k_\mu k_\nu \eta^{\mu\nu} = -\frac{5(\tilde{k} \cdot \tilde{b})}{2} \left( \frac{\beta}{\alpha} (k \cdot \tilde{b}) - \frac{\alpha}{\alpha} (k \cdot \tilde{l}) \right),
\] (33)
\[
k \cdot k = -\frac{3(\tilde{k} \cdot \tilde{b})}{2} \left( \frac{\beta}{\alpha} (k \cdot \tilde{b}) - \frac{3\beta}{\alpha} (k \cdot \tilde{l}) \right),
\] (34)
where `$\cdot$' denotes the inner product on Minkowski spacetime $\tilde{a}_{\mu\nu}$ and $\tilde{b}^\mu \equiv y^\mu/\alpha$. The causality should holds in Finsler spacetime $\eta_{\mu\nu}$, thus $k_\mu k_\nu \eta^{\mu\nu} > 0$ while the signature of Minkowski metric $\tilde{a}_{\mu\nu}$ is of the form $(+\ -\ -\ -)$. If $k \cdot k < 0$, it means that the speed of gravitational wave is larger than speed of light. It implies that the speed of gravitational wave could larger than speed of light and causality still holds. The inequalities $k_\mu k_\nu \eta^{\mu\nu} > 0$ and $k \cdot k < 0$ satisfy if
\[
\frac{3\beta}{\alpha} < \frac{k \cdot \tilde{b}}{k \cdot \tilde{l}} < \frac{\beta}{\alpha} < 0,
\] (35)
so that the speed of gravitational wave in the anisotropic spacetime is larger than the speed of light and the causality still holds.

The sketch figure of the causal structure of Finsler spacetime ($\eta_{\mu\nu}$) is shown in Fig.1. It is clear from Fig.1 that the null vectors on Finsler spacetime ($\eta_{\mu\nu}$) are spacelike vectors on Minkowski spacetime. The causal speed of Finsler spacetime could be larger than the one of Minkowski spacetime.
IV. CONCLUSIONS

In this paper, we used the weak gravitational field approximation to get a linearized solution of the gravitational vacuum field equation in Finsler spacetime. The plane-waves solution (26) of gravitational wave in an anisotropic spacetime was presented. It is shown that the gravitational wave is propagating in locally Minkowski spacetime ($\eta_{\mu\nu}$). The Killing vectors of locally Minkowski spacetime ($\eta_{\mu\nu}$) are investigated in Ref.[38]. It was shown that Finsler spacetime admits less symmetry than Minkowski spacetime, and the translation symmetry is preserved in locally Minkowski spacetime ($\eta_{\mu\nu}$). Based on the Noether theorem, the spacetime translational invariance implies that the energy-momentum is well defined and conserved in locally Minkowski spacetime ($\eta_{\mu\nu}$). The dispersion relation of gravitational wave in Finsler spacetime (29) was presented. The speed of gravitational wave could be larger than the speed of light in Randers spacetime and the causality of gravitational wave still holds, if the condition (35) is satisfied. Since the wave vector $k_{\mu}$ of gravitational wave is timelike in locally Minkowski spacetime ($\eta_{\mu\nu}$), it would not lose energy via the gravitational Cherenkov radiation.

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