$\pi^+ + d \rightarrow p + p$ reaction between 18 and 44 MeV

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(March 30, 2022)

Abstract

A study of the reaction $\pi^+ + d \rightarrow p + p$ has been performed in the energy range of 18 – 44 MeV. Total cross sections and differential cross sections at six angles have been measured at 15 energies with an energy increment of 1 – 2 MeV. This is the most systematic data set in this energy range. No structure in the energy dependence of the cross section has been observed within the accuracy of this experiment.

PACS number(s): 25.80.Ls, 13.75.Cs 13.75.Gx, 14.20.Pt

Typeset using REVTEX
I. INTRODUCTION

Total and differential cross sections for the reaction $\pi^+d \rightarrow pp$ have been measured with high accuracy at pion energies above 100 MeV. Currently, experimental efforts in this energy range are aimed at measuring spin dependent observables. The situation at pion energies below 100 MeV is less complete, especially for energies of a few tens of MeV [1]. The disagreement between the experimental data for the total cross section in some cases exceeds 20%, much larger than the quoted uncertainties. The experimental data show a deviation from smooth behavior in the total cross section of the reaction $\pi^+d \rightarrow pp$ close to $T_\pi \sim 30$ MeV ($\sqrt{s} = 2.04 \text{ GeV/c}^2$) [2]. More recent measurements of the partial total cross section for the inverse reaction at SATURNE have verified this effect [3]. In a recent experiment of the Dubna-Gatchina group on pion absorption on carbon [4] a dip in the energy dependence of the quasi-deuteron component of absorption near $T_\pi=28$ MeV has been observed. The measurements of pion absorption on the deuteron [5] performed by the same group also indicated possible structure at a pion energy of 30 MeV. A more recent experiment at LAMPF [6] did not observe any dip on carbon, but these data suggest a change in the angular distribution of protons at pion energies near 25 MeV. Some possible alternative explanations of the structure in this energy range are given in Ref. [2,4]. One of them attributed the structure to the excitation of a diproton resonance in the $^3P_2$ $NN$ state. However, these experiments are not accurate enough to reach a conclusion regarding the structure under the discussion. A satisfactory solution fitting the available database has not been found in partial wave analysis, suggesting underlying systematic problems with the data base.

The experiment described here was intended to resolve the discrepancies in existing data and to verify the observed structure in the excitation function of pion absorption on the deuteron at low pion energies. The differential and total cross sections of the $\pi^+d \rightarrow pp$ reaction have been measured with fine steps in incident pion energy.
II. EXPERIMENT

A. Pion Beam and Target

The experiment was performed at the Low Energy Pion channel (LEP) of the Clinton P. Anderson Meson Physics Facility (LAMPF). Positive pions with energies of 21, 23, 25, 26, 27, 28, 29, 30, 31, 33, 35, 37, 39, 41 and 45 MeV were used. The pion beam had an average intensity of a few $10^4$/sec, allowing the beam particles to be counted while not overloading the BGO detectors. The pion fraction varied from 78% for 45 MeV beam to 33% for 21 MeV beam. The momentum bite of the LEP channel was set to 1% for the highest energies and increased to 4% for the lowest ones in order to increase the pion flux.

The target was composed of CD$_2$ with a cross sectional area of $1 \times 1$cm$^2$ and an areal density of 0.469 g/cm$^2$. It was attached to a thin paper pipe and placed in the center of the BGO ball. The supporting pipe was aligned along the beam axis. The diameter of the pipe was big enough to keep its walls out of the beam. A 0.25-mm-thick plastic scintillator S1 with a cross section of $6 \times 6$ mm$^2$ was located just before the target. Downstream of the target a rectangular array of nine CsI scintillators DA1–DA9 was preceded by a 10-mm-thick plastic scintillator, S2. DA1–DA9 and S2 could be used for the detection of the most forward going reaction products as well as for a determination of beam composition. In this experiment only the central detector, DA5, of the array was used. A coincidence between S1 and the central detector, DA5, of the downstream array was used as a beam monitor. Every 1000'th beam event (event triggered by a $S1 \cdot DA5$ coincidence) was read out for further analysis to determine the pion fraction which was needed for absolute normalization of the cross sections.

B. BGO Ball Spectrometer

A large solid angle detector, the LAMPF BGO ball, was used to detect the reaction products in this study. Detailed information on the BGO ball can be found in Ref. [7,8]. The
BGO ball consists of 30 phoswich detectors. The detectors of the array were of pentagonal and hexagonal shape and tightly packed to form a truncated icosahedron of 32 sides. Two of the 32 sides are opened for the beam entry and exit. The detectors were distributed about an inner radius of 6.1 cm from the center of the array to the center of each crystal face, and were arranged in six groups centered at laboratory scattering angles of $\theta = 37^\circ, 63^\circ, 79^\circ, 102^\circ, 116^\circ$, and $142^\circ$. Each detector had a solid angle of about $\frac{1}{62} \times 4\pi$ sr and was supported in a 0.5-mm-thick electro-formed nickel can which had a 0.05-mm-thick entrance window. Each detector consisted a 3-mm-thick NE102 plastic scintillator optically coupled to the front of a 5.6-cm-thick bismuth germanate (BGO) crystal, with a 7.62-cm-diameter photomultiplier tube on the back. Since the decay constant of the BGO scintillator is much longer than that of the plastic scintillator (250 ns vs 1.5 ns), the anode signal was time sliced to provide both $\Delta E$ (fast) and $E$ (slow) signals for charged particle identification (pions, protons, deuterons, etc.), and for identification of neutrons and gamma rays. The crystals were thick enough to stop up to 185-MeV protons and 90-MeV pions. The time resolution of the detectors was about 1 ns, sufficient to eliminate events with hits from different beam bursts (the LAMPF beam has a 5-ns microstructure). The light output of BGO scintillator depends significantly on the temperature of BGO material [9]. To minimize fluctuations in temperature of the BGO, a tent-like structure was built to isolate the BGO ball from its surroundings.

The event trigger consisted of a coincidence between the target detector, S1, and at least one BGO crystal in anti-coincidence with DA5.

III. DATA ANALYSIS PROCEDURE

A. Raw Data Processing

The raw data for each event contains information about the energy deposited in the plastic and the BGO for all detectors and timing information with respect to the beam counter. In the first step of the analysis a time gate was applied to all of the signals to remove
accidentals from further analysis. The next step was the determination and application of the constants used for unmixing the $\Delta E - E$ information from the phoswich detectors. The signal from each phoswich detector was integrated in two different ADC channels with different time gates, 50 and 250 ns long. The $\Delta E$ and $E$ information was separated using:

$$\Delta E_i = (dE_i - E1_i \cdot R_{i,1} - Z_{i,1}) \cdot DEGAIN_i$$

$$E_i = (E1_i - dE_i \cdot R_{i,2} - Z_{i,2}) \cdot EGAIN_i$$

where $dE_i$ and $E1_i$ represent the raw data from the ADC’s with the short and long gates, respectively, and $DEGAIN_i$, $EGAIN_i$ are the coefficients for conversion of raw ADC data to energy in MeV. The relative fraction of a long signal in a short gate and vice versa are given by the mixing parameters, $R_{i,1}$ and $R_{i,2}$, respectively. The $R_{i,j}$ and offsets, $Z_{i,j}$, were determined using a least square fit for the events along $\Delta E$ and $E$ axes, where only one component of a signal (short or long) exists. These are produced by charge particles which stop in the plastic and leave a $\Delta E$ with no $E$ signal, or by neutral particles which interact in the BGO and leave a $E$ with no $\Delta E$ signal. An example of a $\Delta E - E$ distribution from a phoswich is shown in Fig. 1. One can clearly see regions corresponding to protons and pions. The outlined region along the $E$ axis is due to neutral particles and the outlined region along the $\Delta E$ axis is due to short range charged particles stopped in the plastic scintillator.

**B. Calibration and Stabilization of the BGO Ball**

An initial energy calibration of the BGO ball ($DEGAIN_i$ and $EGAIN_i$) was made by using two proton coincidences from the $\pi^+d \rightarrow pp$ reaction. An alternate method used elastically scattered pions from $^{12}C$. Pions from this reaction give a strong peak (Fig. 1) with well defined energy for each ring of detectors in the BGO ball. This eliminated the strong kinematic energy dependence in the former procedure. Use of the CD$_2$ as a target and a single hit trigger made this possible. The $\Delta E$ and $E$ gains were fitted continuously
during data taking and off-line data analysis. This continuous stabilization was important because of instability in the BGO ball detectors. The parameters $R_{i,j}$ and $Z_{i,j}$ also require continuous stabilization because both the gain and the decay constant of the BGO scintillator is temperature dependent. We used the following stabilization algorithm. For sequential subsets of raw events the complete set of parameters was fitted. The weighted average of old and new values of the parameters was used for the next subset of events. This procedure was completely automated. Fig. 2 (described below) shows the energy resolution and accuracy of this calibration procedure. The width of the two proton total energy peak from the $d(\pi^+, p)p$ reaction, summed over entire ball is $\sigma = 3 - 4\%$ MeV. After the system has been calibrated one can easily determine the event multiplicity, identify detected particles and measure their energies.

IV. RESULTS

A. Event Selection Criteria

We selected events using following criteria:

(1) the multiplicity must be equal 2;

(2) both particles must be protons; and

(3) the opening angle between the two protons and their total energy must satisfy the
kinematics of the reaction $\pi^+d \rightarrow pp$.

Fig. 2 is the energy spectrum for two proton events. The events are summed over all
kinematically allowed combinations of BGO ball detectors. The narrow peak at the higher
energy corresponds to the reaction $\pi^+d \rightarrow pp$. The lower energy broad peak with a long tail
results from pion absorption on carbon. One observes that there is practically no background
under the $\pi^+d \rightarrow pp$ reaction peak. Measurements with no target but with the supporting
pipe in its normal position and with a pure carbon target show that the background under the peak does not exceed 0.1% so no background subtraction was necessary.

**B. Pion Fraction Determination**

Because of the low beam flux in this experiment it was possible to directly count the beam particles. Pulse height and timing information from S1, S2 and DA5 detectors was used to determine the pion fraction. First, a cut was placed on the time between S1 and S2 to eliminate accidental coincidences between particles from different beam bursts. Another cut was applied to a two dimensional $\Delta E - E$ distribution (pulse height from S2 and DA5) to eliminate positrons. The resulting pulse height spectrum from S2 is shown in Fig. 3. The left peak corresponds to muons, the right one to pions. Two modified Moyal functions: 

$$F(E) = P_1 e^{-P_3(E-P_2)} - e^{-P_4(E-P_2)},$$

(3)

one for pions and one for muons, were fitted to the energy loss distribution. Here $E$ is the energy loss in S2 and $P_1 - P_4$ are adjusted parameters. The pion distribution was integrated to obtain the pion fraction. At the two lowest energies (21 MeV and 23 MeV) some pions lose all their energy in S2 and do not hit DA5 and the corresponding pion peak has a different, and more complicated shape. In this case only the muon peak was fitted and the pion fraction was determined by subtracting the muon fraction from the integral number of beam particles. A Monte Carlo simulation of beam particles passing through the setup based on the GEANT code [12] has been done. The corrections for energy losses, straggling, multiple scattering and decay were calculated from this simulation. The resulting correction to the total number of beam pions is 7% for the highest beam energy and 44% for the lowest one. For beam energies of 21 and 23 MeV this correction accounts for pions, which passed through the target but did not hit DA5 and were lost from the beam trigger. This simulation also was used for calculation of the pion energy in the center of a target. The uncertainty in the number of beam pions, mainly due to fitting errors, is 3 – 5%, with the contribution from the statistical error less than 1%.
C. Efficiency Calculation

The efficiency was calculated using a Monte Carlo code which incorporates the actual geometry of the experimental setup. The efficiency accounts for the effective solid angle (geometrical solid angle reduced by the requirement of both protons detection), reaction losses, and dead time. The effective solid angles for all kinematically allowed pairs of the detectors have been calculated. The missing solid angle due to the nickel cans surrounding the crystals is about 6%. Reaction loss correction due to the interaction of protons with the BGO were obtained using data from Ref. [10]. The typical value of this correction is 10%. Finally, the data were corrected for dead time. A typical value of the dead time in this experiment was 5%. We estimate the total uncertainty in the efficiency of the setup is about 5%.

D. Cross Sections

The differential cross section has been calculated for the selected events at each specific scattering angle. The data for the backward angles were translated to the forward hemisphere since the angular distribution in the center of mass frame is symmetrical around 90° due to indistinguishability of the two protons. The resulting differential cross sections are presented in Table I. To obtain the total error, we added the statistical and normalization errors in quadrature. An overall systematic uncertainty of about 5% in the efficiency calculation is not included in the total error. This affects the global normalization of all 90 differential cross section points and does not change their relative value.

We fitted the cross sections using a simple parameterization given by the Legendre polynomial series:

\[
\frac{d\sigma}{d\Omega_{c.m.}} = a_0 + a_2 P_2(\cos \theta_{c.m.}).
\]  

At each energy we used only the statistical errors of the cross sections but not the normalization error because it does not change the relative shape of the angular distribution.
The total cross sections were determined by:

$$\sigma_{tot} = \int_{2\pi} d\sigma d\Omega_{c.m.}$$

using the parameterization of Eq. (4). The integration over $2\pi$ instead of $4\pi$ is necessary because there are two indistinguishable protons in the final state. The fitted coefficients, total cross sections, and $\chi^2$ are presented in Table II. Errors for $a_0$ and $a_2$ are obtained from the fit. For the total cross section errors we added the normalization errors in quadrature. Overall systematic uncertainty of 5% is not included for the same reason as for differential cross section.

V. DISCUSSION

The differential cross sections measured in this experiment are plotted in Fig. 4. Solid lines show the best fit results with Eq. (4). The results listed in Table II show that only S- and P- pionic waves are important in this energy region. The absence of a need for higher partial waves agrees with previous measurements [13,14]. Fixed angle excitation functions are presented in Fig. 5. Fig. 6 shows the behavior of the angular distribution as a function of energy by plotting the ratio of the coefficients $a_2$ to $a_0$. The shape of the angular distribution changes smoothly with pion energy. No structure, within the accuracy of the experiment, is observed in the differential cross section as a function of energy. The total cross section is presented in Fig. 7 along with the rest of the world’s data. The current data follows the general trends. The biggest disagreement in the total cross section data, which existed around $T_\pi = 30$ MeV is resolved by the new data.

We compared our cross sections with the predictions of the recent partial wave analysis of Ref. [15]. The new VPI solution, SM95, that includes the current data is compared with the previous SP94 results which do not include these data.

Table III and Figs. 5,7 show the level of agreement between our data and the phase shift fits. The normalization factor averaged over all 15 energies are 1.04 and 1.01 for SP94 and
SM95 solutions respectively. The values of the normalization factors for both solutions show that our estimate of systematic uncertainties is reasonable. Table IV presents a comparison of our data with other partial wave analysis. The recent VPI results, which include our data set, give better agreement than the previous results by the Queen Mary College group (9 – 256 MeV) [10] and by the Hiroshima University group (6 – 256 MeV) [11]. The good agreement between the predictions of SP94 and recent SM95 VPI solutions with the experimental data support the absence of any anomalous behavior of the cross sections in this energy region within the accuracy of the present experimental results.

VI. CONCLUSIONS

We have presented new data on the total and differential cross section for the reaction \( \pi^+d \rightarrow pp \) for pion energies from 18 MeV to 44 MeV. In this experiment, 90 experimental data points for the differential cross section and 15 points for the total cross section have been measured with the same systematic uncertainty. The number of the experimental data points in this energy range is almost doubled by these results. The data generally follow the trends of previous results and are in a good agreement with the recent SM95 solution of the VPI partial wave analysis. The previous disagreement in total cross section data is resolved by this experiment. We do not observe any structure either in total cross section or in differential cross section.

ACKNOWLEDGMENTS

The authors wish to acknowledge valuable discussions with R. A. Arndt, R. A. Giannelli, R. D. Ransome, and B. G. Ritchie. This work was supported in part by Russian Fundamental Research Foundation Grant 93-02-3995 and by the U.S. Department of Energy.
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communication (1993).
FIGURES

FIG. 1. Contour plot of the two dimensional distribution of $\Delta E$ (in plastic) vs. $E$ (in BGO) obtained from BGO ball phoswich detectors. Regions corresponding to different particle types are labeled on the plot.

FIG. 2. Observed total energy spectrum for two protons produced in pion absorption on a CD$_2$ target at $T_\pi = 43.5$ MeV.

FIG. 3. Pulse height spectrum from the S2 detector for a 30 MeV pion beam. The histogram is a measured spectrum, dashed and dotted lines represent the fit results with Eq. (3) for muons and pions respectively. Solid line is a sum of these two functions. Positrons have been rejected.

FIG. 4. Differential cross sections determined in this experiment. Solid lines are the best fits with Eq. (4).

FIG. 5. Energy dependence of the differential cross section at fixed scattering angles. Solid lines are the predictions from the recent VPI solution SM95 [15].

FIG. 6. Ratio of the coefficients of a Legendre polynomial expansion of the differential cross section with Eq. (4).

FIG. 7. Total cross section. Black circles are the data from this experiment. Open circles represent previous experimental data set for this energy range (taken from SAID [15] data base. Solid line is the VPI SM95 global fit to data in the range 0 – 550 MeV [15].
TABLE I. Differential cross sections. Overall systematic uncertainty of 5% is not included in the total error.

| $T_{\pi}^a$ | $\theta_{\text{c.m.}}$ | $\frac{d\sigma}{d\Omega_{\text{c.m.}}}$ | statistical error | normalization $^b$ | total $^c$ |
|------------|----------------|-----------------|------------------|----------------|----------|
| (MeV)      | (deg)          | (mb/sr)         |                  |                 |          |
| 18.8       | $36^{+7}_{-9}$ | 0.891           | 0.045            | 2.9            | 0.052    |
|            | $41^{+9}_{-12}$ | 0.793           | 0.035            |                | 0.042    |
|            | $58^{+9}_{-9}$ | 0.669           | 0.038            |                | 0.042    |
|            | $66^{+10}_{-10}$ | 0.600           | 0.033            |                | 0.038    |
|            | $73^{+10}_{-10}$ | 0.460           | 0.030            |                | 0.033    |
|            | $83^{+11}_{-11}$ | 0.430           | 0.028            |                | 0.031    |
| 20.9       | $36^{+7}_{-9}$ | 0.860           | 0.043            | 4.2            | 0.056    |
|            | $41^{+9}_{-12}$ | 0.797           | 0.034            |                | 0.048    |
|            | $58^{+9}_{-9}$ | 0.638           | 0.036            |                | 0.045    |
|            | $66^{+10}_{-10}$ | 0.533           | 0.030            |                | 0.038    |
|            | $73^{+10}_{-10}$ | 0.437           | 0.028            |                | 0.034    |
|            | $83^{+11}_{-11}$ | 0.365           | 0.025            |                | 0.029    |
| 22.9       | $36^{+7}_{-9}$ | 0.795           | 0.040            | 4.5            | 0.054    |
|            | $41^{+9}_{-12}$ | 0.710           | 0.031            |                | 0.045    |
|            | $57^{+9}_{-9}$ | 0.592           | 0.033            |                | 0.043    |
|            | $66^{+10}_{-10}$ | 0.490           | 0.028            |                | 0.036    |
|            | $73^{+10}_{-10}$ | 0.456           | 0.028            |                | 0.035    |
|            | $84^{+11}_{-11}$ | 0.422           | 0.026            |                | 0.032    |
| 24.0       | $36^{+7}_{-9}$ | 0.906           | 0.049            | 4.3            | 0.062    |
|            | $41^{+9}_{-12}$ | 0.800           | 0.038            |                | 0.051    |
|            | $57^{+9}_{-9}$ | 0.676           | 0.040            |                | 0.050    |
|            | $67^{+10}_{-10}$ | 0.621           | 0.036            |                | 0.044    |
| 73^{10}_{-10} | 0.458 | 0.032 | 0.037 |
| 84^{11}_{-11} | 0.418 | 0.029 | 0.034 |
| 25.0 | 36^{7}_{-9} | 0.874 | 0.045 | 4.1 | 0.057 |
| 41^{9}_{-12} | 0.808 | 0.035 | 0.048 |
| 58^{9}_{-9} | 0.674 | 0.037 | 0.047 |
| 68^{10}_{-10} | 0.557 | 0.031 | 0.039 |
| 72^{10}_{-10} | 0.443 | 0.029 | 0.034 |
| 84^{11}_{-11} | 0.374 | 0.026 | 0.030 |
| 26.1 | 36^{7}_{-9} | 0.984 | 0.048 | 3.6 | 0.060 |
| 41^{9}_{-12} | 0.897 | 0.037 | 0.049 |
| 57^{9}_{-9} | 0.728 | 0.039 | 0.047 |
| 67^{10}_{-10} | 0.615 | 0.033 | 0.040 |
| 72^{10}_{-10} | 0.483 | 0.031 | 0.035 |
| 84^{11}_{-11} | 0.414 | 0.027 | 0.031 |
| 27.1 | 36^{7}_{-9} | 0.940 | 0.046 | 3.7 | 0.058 |
| 41^{9}_{-12} | 0.901 | 0.036 | 0.050 |
| 57^{9}_{-9} | 0.757 | 0.039 | 0.048 |
| 67^{10}_{-10} | 0.607 | 0.032 | 0.039 |
| 72^{10}_{-10} | 0.530 | 0.031 | 0.037 |
| 84^{11}_{-11} | 0.476 | 0.029 | 0.034 |
| 28.1 | 36^{7}_{-9} | 0.948 | 0.049 | 3.7 | 0.060 |
| 41^{9}_{-12} | 0.931 | 0.039 | 0.052 |
| 57^{9}_{-9} | 0.767 | 0.042 | 0.051 |
| 67^{10}_{-10} | 0.632 | 0.035 | 0.042 |
| 72^{10}_{-10} | 0.504 | 0.032 | 0.037 |
| 84^{11}_{-11} | 0.421 | 0.029 | 0.033 |
| 29.2 | 36^{7}_{-9} | 0.972 | 0.052 | 3.6 | 0.062 |
|      |   |   |   |   |
|------|---|---|---|---|
| 41 $^+_{-12}$ | 0.965 | 0.041 | 0.054 |
| 57 $^{+9}_{-9}$ | 0.755 | 0.043 | 0.051 |
| 67 $^{+10}_{-10}$ | 0.557 | 0.034 | 0.039 |
| 72 $^{+10}_{-10}$ | 0.508 | 0.034 | 0.038 |
| 84 $^{+11}_{-11}$ | 0.512 | 0.033 | 0.038 |
| 31.2 | 36 $^{+7}_{-9}$ | 1.032 | 0.053 | 3.5 | 0.064 |
| 42 $^{+9}_{-12}$ | 0.997 | 0.042 | 0.054 |
| 57 $^{+9}_{-9}$ | 0.815 | 0.045 | 0.053 |
| 67 $^{+10}_{-10}$ | 0.608 | 0.035 | 0.041 |
| 72 $^{+10}_{-10}$ | 0.490 | 0.033 | 0.037 |
| 85 $^{+11}_{-11}$ | 0.415 | 0.029 | 0.033 |
| 33.3 | 36 $^{+7}_{-9}$ | 1.043 | 0.043 | 4.2 | 0.053 |
| 42 $^{+9}_{-12}$ | 0.921 | 0.032 | 0.042 |
| 56 $^{+9}_{-9}$ | 0.703 | 0.033 | 0.039 |
| 67 $^{+10}_{-10}$ | 0.539 | 0.027 | 0.031 |
| 72 $^{+10}_{-10}$ | 0.455 | 0.025 | 0.029 |
| 85 $^{+11}_{-11}$ | 0.409 | 0.023 | 0.026 |
| 35.4 | 36 $^{+7}_{-9}$ | 1.032 | 0.043 | 4.0 | 0.060 |
| 42 $^{+9}_{-12}$ | 0.929 | 0.032 | 0.049 |
| 56 $^{+9}_{-9}$ | 0.768 | 0.035 | 0.047 |
| 68 $^{+10}_{-10}$ | 0.611 | 0.028 | 0.037 |
| 71 $^{+10}_{-10}$ | 0.470 | 0.026 | 0.032 |
| 85 $^{+11}_{-11}$ | 0.435 | 0.024 | 0.030 |
| 37.4 | 36 $^{+7}_{-9}$ | 1.101 | 0.068 | 4.9 | 0.087 |
| 42 $^{+9}_{-12}$ | 0.930 | 0.048 | 0.067 |
| 56 $^{+9}_{-9}$ | 0.711 | 0.051 | 0.062 |
| 68 $^{+10}_{-10}$ | 0.534 | 0.040 | 0.048 |
| Energy (MeV) | a | b | c | d |
|-----------|---|---|---|---|
| 39.5      |    |    |    |    |
| 42 ± 9    | 1.205 | 0.055 | 3.4 | 0.068 |
| 56 ± 9    | 0.961 | 0.045 | 0.056 |
| 68 ± 10   | 0.691 | 0.035 | 0.042 |
| 71 ± 10   | 0.567 | 0.033 | 0.038 |
| 86 ± 11   | 0.456 | 0.028 | 0.032 |
| 43.5      | 1.353 | 0.048 | 3.8 | 0.070 |
| 42 ± 9    | 1.201 | 0.035 | 0.057 |
| 56 ± 9    | 0.946 | 0.037 | 0.052 |
| 68 ± 10   | 0.732 | 0.029 | 0.040 |
| 71 ± 10   | 0.606 | 0.028 | 0.036 |
| 86 ± 11   | 0.515 | 0.025 | 0.032 |

*aEnergy in the center of target

*bSame value for all six angles at given energy

*cTotal error is the statistical and normalization error summed in quadrature.
TABLE II. The Legendre polynomial expansion coefficients of the differential cross section as fit with Eq. (4) and the associated total cross sections. Errors for $a_0$ and $a_2$ are obtained from the fit. For the total cross section error the normalization errors are added in quadrature. The overall systematic uncertainty of 5% is not included.

| $T_\pi$ (MeV) | $a_0$ (mb/sr) | $a_2$ (mb/sr) | $\chi^2$/data | $\sigma_{tot}$ (mb) |
|---------------|---------------|---------------|----------------|---------------------|
| 18.8          | 0.663±0.015   | 0.458±0.041   | 4.7/6          | 4.167±0.153         |
| 20.9          | 0.632±0.014   | 0.510±0.038   | 2.4/6          | 3.974±0.190         |
| 22.9          | 0.597±0.013   | 0.372±0.037   | 0.7/6          | 3.749±0.188         |
| 24.0          | 0.669±0.016   | 0.478±0.043   | 5.5/6          | 4.205±0.206         |
| 25.0          | 0.649±0.015   | 0.517±0.039   | 4.1/6          | 4.076±0.191         |
| 26.1          | 0.717±0.016   | 0.583±0.042   | 3.8/6          | 4.508±0.190         |
| 27.1          | 0.730±0.015   | 0.503±0.042   | 2.0/6          | 4.585±0.197         |
| 28.1          | 0.732±0.016   | 0.576±0.044   | 5.7/6          | 4.598±0.199         |
| 29.2          | 0.743±0.017   | 0.557±0.047   | 4.1/6          | 4.669±0.198         |
| 31.2          | 0.763±0.017   | 0.683±0.047   | 4.9/6          | 4.796±0.200         |
| 33.3          | 0.714±0.014   | 0.652±0.037   | 1.8/6          | 4.486±0.155         |
| 35.4          | 0.740±0.014   | 0.624±0.037   | 5.9/6          | 4.648±0.204         |
| 37.4          | 0.724±0.021   | 0.690±0.056   | 1.5/6          | 4.549±0.259         |
| 39.5          | 0.888±0.017   | 0.834±0.046   | 7.3/6          | 5.580±0.217         |
| 43.5          | 0.939±0.015   | 0.857±0.040   | 3.2/6          | 5.900±0.240         |
TABLE III. The $\chi^2/data$ for the present differential cross sections vs pion kinetic energy for recent VPI SP94 and SM95 solutions [15]. SP94 does not include the present data while SM95 includes them. Norm is a common normalization factor determined from the SP94 and SM95 solutions.

| $T_\pi$ (MeV) | Norm | $\chi^2/data$ | Norm | $\chi^2/data$ |
|---------------|------|---------------|------|---------------|
| 18.8          | 1.04 | 8/6           | 1.03 | 7/6           |
| 20.9          | 0.98 | 3/6           | 0.98 | 3/6           |
| 22.9          | 0.93 | 14/6          | 0.93 | 14/6          |
| 24.0          | 1.01 | 11/6          | 1.01 | 11/6          |
| 25.0          | 0.98 | 8/6           | 0.97 | 8/6           |
| 26.1          | 1.04 | 7/6           | 1.04 | 7/6           |
| 27.1          | 1.05 | 17/6          | 1.05 | 16/6          |
| 28.1          | 1.04 | 12/6          | 1.03 | 11/6          |
| 29.2          | 1.04 | 14/6          | 1.04 | 13/6          |
| 31.2          | 1.04 | 7/6           | 1.04 | 7/6           |
| 33.3          | 0.96 | 4/6           | 0.96 | 3/6           |
| 35.4          | 0.98 | 13/6          | 0.98 | 12/6          |
| 37.4          | 0.94 | 4/6           | 0.94 | 4/6           |
| 39.5          | 1.08 | 13/6          | 1.07 | 12/6          |
| 43.5          | 1.09 | 13/6          | 1.09 | 13/6          |

All energies 147/90 142/90
TABLE IV. Comparison of $\chi^2$/data for the present and World total and differential cross sections for the recent VPI SP94 and SM95 solutions [15] and previous solutions BU93 [16] and HI84 [17]. World data is selected for the energy range 18 – 44 MeV covered by the present experiment.

| Solution | $d\sigma/d\Omega_{c.m.}$ | $\sigma_{tot}$ |
|----------|------------------------|---------------|
|          | Present                | World        | Present | World |
| SM95     | 142/90                 | 297/98       | 12/15   | 38/21  |
| SP94     | 147/90                 | 292/98       | 12/15   | 40/21  |
| BU93     | 176/90                 | 319/98       | 52/15   | 45/21  |
| HI84     | 332/90                 | 364/98       | 93/15   | 73/21  |
