Dilaton Supersymmetry Breaking

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ABSTRACT

We argue that dilaton supersymmetry breaking in string derived supergravity requires an effective superpotential which is not separable as a function of the dilaton times a function of the moduli. We show that in a simple model with hidden sector matter condensation and a dilaton independent term one can easily obtain $|F_S| \neq 0$. For a wide range of realistic model parameters $|F_S| >> |F_T|$ and supersymmetry is mainly broken in the dilaton direction.

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1. Introduction

It is well-known that supersymmetry (SUSY) breaking in the dilaton direction (i.e. due to a nonzero dilaton F–term) has a number of attractive features. In this scenario, due to the universal dilaton coupling to all matter, the soft SUSY breaking scalar and gaugino masses as well as the A–terms are universal[1]. In addition, these only depend on one parameter, the gravitino mass, \( m_{3/2} \), which parametrizes the nonperturbative SUSY breaking effects. The results for the soft SUSY breaking parameters are the universal boundary conditions often used in supergravity (SUGRA) models. In this case, the parameter space is reduced by a large amount which makes the low–energy calculations much more predictive. Moreover, the universal soft scalar masses obtained in this scenario eliminate the danger of flavor changing neutral currents at the weak scale even after running effects are taken into account [1]. Thus, it is highly desirable to obtain dilaton SUSY breaking in string derived SUGRA models.

In conventional SUSY breaking mechanisms obtained by hidden sector gaugino condensation[2] in string derived SUGRA models there are two problems related to the dilaton. First, the resulting nonperturbative effective superpotentials give effective scalar potentials which are not stable in the dilaton direction; i.e. the dilaton runs to infinity and there is no stable vacuum. (A possible solution to this problem is given by racetrack models[3,4] which have more than one condensing hidden gauge group but these require fine tuning of the coefficients.) In this letter, we do not try to solve this problem. Instead, we add a dilaton independent term to the nonperturbative superpotential which stabilizes the dilaton potential[5]. The origin of such a term may be the condensation of the antisymmetric field tensor present in all string models[6]. In any case, this term parametrizes the effects of the unknown nonperturbative mechanism which stabilizes the dilaton potential.

Second, even though nonzero VEVs for moduli F–terms \( (F_T) \) are very easy to obtain in this scenario, the dilaton F–term \( F_S \) always vanishes. This has been shown for hidden sectors with a pure gauge group with or without a dilaton in-
dependent term[5]. The reason for the vanishing $F_S$ in the models considered so far can be traced back to the separability of the nonperturbative superpotential. That is $W_{np} = g(S)h(T)$ for the cases considered so far; the superpotential is a product of a function of the dilaton $S$ and a function of the overall modulus $T$. (We consider only the overall modulus case but the generalization to the multimodulus case is trivial.) In this case, it can be shown that the minimum of the effective potential in the dilaton direction occurs where $F_S = 0$, i.e. there is no dilaton SUSY breaking[4,5]. Therefore, in order to have dilaton breaking one should look at effective superpotentials which are not separable.

In this letter, we investigate the case with a nonseparable superpotential. This is obtained by considering a model with hidden matter in the vector representation of the hidden gauge group and a dilaton independent term in the effective superpotential which stabilizes the dilaton. We assume that target space duality[7] is unbroken by nonperturbative effects such as condensation in the hidden sector. Invariance under target space duality fixes the moduli dependence of different terms in the superpotential. We show that generically $F_S \neq 0$ in addition to $F_T \neq 0$. The ratio $|F_S|/|F_T|$ depends on the parameters of the model such as the hidden gauge group, the hidden matter content, their masses and the coefficient of the dilaton independent term. We find that for a large range of realistic values of parameters, $S \sim O(1)$ and $|F_S| >> |F_T|$ which gives dominantly dilaton SUSY breaking. In addition by properly choosing the magnitude of the constant term we obtain $|F_S| \sim 10^{-15}M_P^2$ which gives a $TeV$ scale soft scalar masses. The cosmological constant is $O(m_{3/2}^2M_P^2)$ and negative as usual. We make no attempt to solve this problem but note that it can be made to vanish by fine tuning the constant term which stabilizes the dilaton.

2. Separable and nonseparable superpotentials

In all gaugino condensation scenarios considered so far the effective nonpertur-
bative superpotential is a function of the dilaton times a function of the moduli,

\[ W_{np} = g(S)h(T) \]  \hspace{1cm} (1)

For example for a hidden pure $SU(N)$ gauge group $W_{np}$ is given by\[8,9\]

\[ g(S) = -N \exp(-32\pi^2S/N) \]  \hspace{1cm} (2)

and

\[ h(T) = (32\pi^2e)^{-1}\eta(T)^{-6} \]  \hspace{1cm} (3)

Here $\eta(T)$ is the Dedekind eta function whose power is determined by target space duality invariance. In this case, one can add a dilaton independent term $c\eta(T)^{-6}$ to $W_{np}$ (where $c$ is a constant) in order to stabilize the dilaton potential[5]. $W_{np}$ is still separable because both $g(S)$ and $c$ have the same (i.e. vanishing) modular weights. The dilaton and moduli F–terms are obtained from ($i = S, T$)

\[ F_i = e^{K/2}(W_i + K_iW) \]  \hspace{1cm} (4)

and are given by

\[ F_S = e^{K/2}(g_S h(T) + K_S g(S)h(T)) \]  \hspace{1cm} (5)

and

\[ F_T = e^{K/2}(g(S)h_T + K_T g(S)h(T)) \]  \hspace{1cm} (6)

with the Kahler potential

\[ K(S, \bar{S}, T, \bar{T}) = -\log(S + \bar{S}) - 3\log(T + \bar{T}) \]  \hspace{1cm} (7)

The effective nonperturbative scalar potential is given by

\[ V_{eff} = |F_i|^2 G_{ii}^{-1} - 3e^K |W|^2 \]  \hspace{1cm} (8)

where $G = K + log|W|^2$. Minimizing the effective scalar potential obtained by using Eqs. (1-8) one finds that there is a minimum at $S$ which satisfies $g(S) -$
From Eq. (5) we see that this is exactly the condition for a vanishing dilaton F-term, \( F_S = 0 \). Thus we find generically that if the superpotential is separable \( \partial V/\partial S \propto F_S \). (There is another condition for an extremum of \( V_{eff} \) but this corresponds to a maximum[4].)

One possible way to obtain a nonseparable superpotential is to include effects of hidden matter condensation in \( W_{np} \). Hidden sectors of string models generically contain matter in vector–like representations of the hidden gauge group. When the hidden gauge group condenses at a scale \( \Lambda_H \sim M_P exp(8\pi^2/bg^2) \) hidden matter condensates (\( \Pi \)) form in addition to gaugino condensates (\( Y^3 \)). Here \( b \) is the coefficient of the \( \beta \)–function and \( g \) is the coupling constant at \( M_P \) which is about 0.7. Integrating out \( \Pi \) and \( Y^3 \) one obtains \( W_{np} \) as a function of the dilaton and moduli and \( detA \) where \( A \) is the hidden matter mass matrix which has to be nonsingular for a stable vacuum[10]. In order to stabilize the dilaton we can add a dilaton independent term \( c\eta(T)^{-6} \) to \( W_{np} \) as mentioned above. One can choose \( c \) so that the dilaton VEV is \( O(1) \) in order to get \( g^2 \sim 1/2 \) for successfull gauge coupling unification[11]. This term can arise from the condensation of the antisymmetric tensor which is present in all string models[6]. In any case, we can view it as a parametrization of the unknown nonperturbative physics which stabilizes the dilaton. As we will see, this new term renders \( W_{np} \) nonseparable if it appears in a model with hidden matter condensation.

The effective nonperturbative superpotential for an \( SU(N) \) gauge group with \( M \) matter multiplets in the vector representations \( N + \bar{N} \) is given by[9]

\[
W_{np} = g(S)h(T)[detA]^{1/N} + ck(T) 
\] (9)

where

\[
g(S) = -N exp(-32\pi^2 S/N) \] (10)

and

\[
h(T) = (32\pi^2 e)^{M/N-1}\eta(T)^{2M/N-6} \] (11)
with the determinant of the hidden mass matrix \( \text{det} A = k \phi^s \eta(T)^t \) [13]. \( k \) is a constant of \( O(1) \), \( \phi \) denote generic gauge singlet fields with modular weights \(-1\) [16] and whose VEVs give hidden matter mass [14]. The powers \( s \) and \( t \) are models dependent but positive and of \( O(5-10) \). The powers of \( \eta(T) \) in \( h(T), k(T) \) and \( \text{det} A \) are determined from target space duality invariance. \( k(T) = \eta(T)^{-6} \) since \( c \) has vanishing modular weight. The power of \( \eta(T) \) in \( h(T) \) arises from the \( M \) multiplets of hidden matter with modular weights \(-1\). \( \text{det}A \) has exactly the modular weight to compensate this factor of \( 2M/N \). The superpotential in Eq. (9) is not separable due to the different powers of \( \eta(T) \) in the two terms. This is a result of the modular weights of hidden matter and that of \( \text{det}A \) which arises from the modular weights of \( \phi \).

Note that both the contribution of hidden matter condensates and the dilaton independent term are essential to get a nonseparable superpotential. If there is no hidden matter, target space duality requires that \( h(T) = k(T) \) and \( W_{np} \) is separable. In this case the dilaton potential is stable but \( F_S = 0 \) at the minimum. On the other hand, if \( c = 0 \) then the second term in Eq. (9) vanishes and \( W_{np} \) is again separable. Only when both are present, there are two terms with different moduli dependence as dictated by target space duality and therefore the superpotential is not separable.

### 3. The effective scalar potential

One can absorb the moduli dependence of \( \text{det}A \) into \( h(T) \) and the constant parts of \( \text{det}A \) and \( h(T) \) into \( g(S) \) so that now

\[
g(S) = -(Nk\phi^s)(32\pi^2e)^{M/N-1}\exp(-32\pi^2S/N) \tag{12}
\]

and

\[
h(T) = \eta(T)^{2M/N-6-t/N} \tag{13}
\]

The superpotential in Eq. (9) is now given by

\[
W_{np} = g(S) h(T) + c k(T) \tag{14}
\]
where \( g(S) \) and \( h(T) \) are given by Eqs. (12) and (13). From the above formulas for the nonperturbative superpotential, the dilaton F–term \( F_S \) is

\[
F_S = \frac{-1}{8S_R^{3/2}T_R^{3/2}} \eta(T)^{d'} [(g(S)(1 + 2bS_R) + c\eta(T)^{d-d'}]
\]

(15)

where \( b = -32\pi^2/N \), \( d = -6 \) and \( d' = -6 + 2M/N + t/N \). The moduli F–term \( F_T \) is

\[
F_T = \frac{1}{4S_R^{1/2}T_R^{3/2}} \eta(T)^{d'} \left[ \frac{G_2(T)}{4\pi} (g(S)d' + c\eta(T)^{d-d'}) - \frac{3}{2T_R} (g(S) + c\eta(T)^{d-d'}) \right]
\]

(16)

where \( G_2(T) \) is the second Eisenstein function and arises due to \( \partial \eta(T)/\partial T = -\eta(T)G_2(T)/4\pi \). The effective scalar potential for \( S \) and \( T \) becomes

\[
V_{eff} = \frac{1}{16S_R T_R^3} |\eta(T)^{d'}|^2 \left[ |g(S)(1 + 2bS_R) + c\eta(T)^{d-d'}|^2 \right.
\]

\[
+ \left. \frac{G_2(T)}{4\pi} |g(S)d' + c\eta(T)^{d-d'}| - \frac{3}{2T_R} |g(S) + c\eta(T)^{d-d'}| \right]^2
\]

\[- 3|g(S) + c\eta(T)^{d-d'}|^2 \]

(17)

Due to the nonseparability of \( W_{np} \) \( V_{eff} \) is much more complicated than in the separable superpotential case. In particular, the condition for minimum in the dilaton direction \( \partial V_{eff}/\partial S = 0 \) is \( S \) and \( T \) dependent. This is in contrast to the case with a separable superpotential in which the same condition depends only on \( S \) and not on \( T \). This makes the analysis more difficult, e.g. one needs to minimize \( V_{eff} \) numerically as a function of four real variables \( S_R, S_I, T_R, T_I \) simultaneously.

For \( W_{np} \) given by Eq. (14), \( \partial V_{eff}/\partial S \) is not proportional to \( F_S \) due to the nonseparable superpotential. Therefore, generically one obtains \( F_S \neq 0 \) in addition to \( F_T \neq 0 \) at the minimum of \( V_{eff} \). This has been confirmed by numerical analysis of \( V_{eff} \). Thus, generically there is some amount of SUSY breaking in the dilaton direction. However, this is not important if in these cases \( |F_T| >> |F_S| \) so that moduli breaking is the dominant effect. The question we are interested in is whether
there are points in the parameter space of the model for which $|F_S| >> |F_T|$. In that case supersymmetry is mainly broken in the dilaton direction in contrast to moduli SUSY breaking one obtains from nonseparable superpotentials.

For the numerical analysis we need to choose some reasonable range for the parameters of the model. For example, the constant $c$ which is crucial for dilaton supersymmetry breaking also fixes the overall magnitudes of $F_S$ and $F_T$. Requiring $TeV$ scale soft scalar masses or $m_{3/2}$ means that $c \sim 10^{-15} M_P^3$. We took $10^{-16} M_P^3 < c < 8 \times 10^{-15} M_P^3$ in our numerical analysis. $N$ which gives the hidden gauge group $SU(N)$ lies in the range, $2 < N < 8$, and the number of hidden matter multiplets $M$ must satisfy $M < N$ for gaugino condensation to occur. $t$ which gives the power of $\eta(T)$ in the determinant of the hidden mass matrix is generically $O(10)$. Hidden matter masses generically arise from nonrenormalizable higher order terms in the superpotential and so $s \sim O(5)[14]$. The parameter $t$ only appears in $d'$ and therefore one can take $d'$ to be the parameter instead. The scalar VEVs which give masses to hidden matter are fixed by the coefficient of the anomalous D–term[12] which is generic to string models to be $O(M_P/10)[15]$.

Even though our numerical search is not exhaustive, we find a large number of points in the parameter space which gives a minimum of $V_{eff}$ with $|F_S| >> |F_T|$. There are enough points to be convinced that this happens for a large and realistic range of model parameters. For example, for $N = 3$, $d' = -2$ and $c = 5 \times 10^{-15} M_P^3$ the minimum of $V_{eff}$ is at $S = 0.29 + i0.22$, $T = 0.99 + i1.00$ which gives the ratio $R = |F_S|/|F_T| = 53$. For $N = 4$, $d' = -5$ and $c = 6 \times 10^{-15} M_P^3$ the minimum is at $S = 0.30 + i0.22$, $T = 0.99 + i1.00$ which gives $R = 713$ which is an order of magnitude larger than the previous case. There are a large number of such points in the parameter space and these two are given simply to demonstrate the case. Of course, there is also a large part of the parameter space for which $|F_S| << |F_T|$ and moduli SUSY breaking is dominant.

The location of the minimum is quite robust at least in the range of parameters we examined. For example, at the minimum of $V_{eff}$, $S_R \sim 0.3$ which is somewhat
smaller than the value required by coupling constant unification. In addition, $T_R$ and $T_I$ at the minimum turn out to be very close to 1 in all cases with large $R = \frac{|F_S|}{|F_T|}$. The dependence of the ratio $R$ on the different parameters of the model such as $N, M, \phi, c, t$ etc. is very complicated. In general it can be said that $|F_S|$ is inversely proportional to $N$ whereas it is directly proportional to $c$. (We remind that for vanishing $c$ the superpotential is separable and $F_S = 0$.) Note that $N \geq 2$ and $c$ cannot be much larger than $10^{-15} M_P^3$ for phenomenological reasons. Finally, the cosmological constant is always of $O(\frac{m_3^2}{2M_P^2})$ and negative in the cases we examined for both dilaton and moduli SUSY breaking. One can make it vanish by simply fine tuning the constant $c$ which does not affect our results.

4. Conclusions and discussion

In this letter, we argued that one needs a nonseparable effective superpotential in order to get a nonzero dilaton F–term. If the superpotential is separable, then the minimum condition for the dilaton automatically insures that $F_S = 0$. We considered a simple but realistic example of a nonseparable superpotential which was obtained by including hidden matter condensation effects and a dilaton independent term in the usual hidden gaugino condensation scenario. The superpotential is nonseparable because it contains two terms with different modulus dependence, one from condensation effects and the other the dilaton independent term. This is a result of the nonzero modular weights of hidden matter fields and their masses. Due to the nonseparability of the superpotential, $F_S \neq 0$ in vacuum for generic values of the parameters of the model. The ratio $|F_S|/|F_T|$ depends on the specific choice of the model parameters such as the hidden gauge group, the hidden matter content, the hidden matter masses and the coefficient of the dilaton independent term.

We gave two examples of such points in our results even though there are many more of them. One can easily obtain $|F_S|$ which are one or two orders of magnitude larger than $|F_T|$ and thus resulting in dominantly dilaton SUSY breaking. Even though our search of the parameter space was not exhaustive, it clearly shows that
once the effective superpotential is not separable it is fairly easy to break SUSY in the dilaton direction.

The superpotential was rendered nonseparable by including hidden matter condensation effects and a dilaton independent term. Whereas the origin of the former is evident the same is not true for the latter. It may arise from the condensation of the antisymmetric rank two tensor present in all string models or it may have some other origin. The crucial point for our purposes is the different moduli dependence of the two terms in $W_{np}$ which renders it nonseparable. As long as this is the case our general conclusions remain valid.

The same ideas can also be explored in racetrack models for which there is no need for a dilaton independent term since the dilaton is stabilized by the interplay between the two gaugino condensation effects present. If the two hidden gauge groups have different matter content with different masses as expected in generic cases, the resulting effective superpotential will not be separable. In this case, our results indicate that there will be cases with dilaton SUSY breaking for some range of the parameters of the model.
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