Cross-correlation limit of a SQUID-based noise thermometer of the pMFFT type

To cite this article: A Kirste and J Engert 2018 J. Phys.: Conf. Ser. 969 012083

View the article online for updates and enhancements.
Cross-correlation limit of a SQUID-based noise thermometer of the pMFFT type

A Kirste, J Engert
Physikalisch-Technische Bundesanstalt (PTB), Abbestraße 2-12, 10587 Berlin, Germany
E-mail: alexander.kirste@ptb.de

Abstract. The primary magnetic field fluctuation thermometer (pMFFT) is a SQUID-based noise thermometer for temperatures below 1 K, which complies with metrological requirements. It combines two signal channels in order to apply the cross-correlation technique, but it requires statistically independent noise signals for proper operation. In order to check the limit of the cross-correlation readout, we have performed zero measurements in the millikelvin range in a setup that is identical to the pMFFT, except for the removed temperature sensor. We examined the influence of different parameters such as SQUID working point or flux-lock loop parameters on the minimum cross-correlation signal down to 24 mK and below 100 kHz. Depending on the configuration, typical minimum SQUID-referred cross-power spectral densities of $1.5 \times 10^{-15}$ $\Phi_0^2$/Hz or even smaller values were observed. For the pMFFT, considering its thermal noise spectrum, these flux densities correspond to a device noise temperature of $\leq 2.5 \mu$K, thereby ensuring a negligible uncertainty contribution at the lower end of the PLTS-2000 (0.9 mK).

1. Introduction
The magnetic field fluctuation thermometer (MFFT) [1] is a noise thermometer, which derives temperature from the thermal motion of the electrons in a metallic body – the temperature sensor. For this purpose it measures the noise currents inside the temperature sensor by detecting the corresponding fluctuating magnetic field just above its surface using superconducting quantum interference devices (SQUIDs) [2]. The MFFT is based on the linear relationship between the power spectral density (PSD) $S_\Phi$ of the measured thermal magnetic flux noise $\Phi$ and thermodynamic temperature $T$, analogous to the well-known Nyquist relation for the Johnson noise in a resistor [3]. The pMFFT is an advanced version of the MFFT, which works in absolute primary mode, so that it does not require calibration at a known temperature [4].

If the thermal noise proportional to $T$ is interfered with any other added noise, this introduces an error (bias) to the measured temperature, leading to increased temperature values. This deviation gets worse for lower temperatures and finally sets a limit to the lowest measurable temperature. Often, that bias temperature is synonymous with the device noise temperature $T_N$, which is reached if the averaged PSD from the thermal noise is equal to the averaged PSD from added non-thermal noise.

The cross-correlation technique (CCT) can provide a solution to this problem. It is applied in the pMFFT and uses two separate signal channels. Provided the two signal channels are completely independent, the CCT allows the measurement of the pure thermal noise identical in both detection coils. In the pMFFT, the background noise introduced in the signal channels is typically smaller than the thermal noise to be measured, though not completely negligible. The benefit of the CCT here is to achieve a lower measurement uncertainty by reducing the bias caused by any background noise. In real measurement setups, however, the two signal channels
are not perfectly independent, and this will affect the ultimate performance (limit) of the CCT. Reasons for lifting the statistical independence of both channels are, for example, crosstalk or environmental fluctuations (due to external fields or temperature) acting simultaneously on both channels. The corresponding limit of the CCT caused by residual interactions is called hardware limit. Following first measurements of [5], we have characterized crosstalk and examined the hardware limit of the current pMFFT setup depending on different working conditions.

2. The cross-correlation technique
Here we introduce the basic concept of the cross-correlation method, the notation of quantities and selected properties used in the paper. An extensive treatment can be found in [6].

The CCT is based on a two-channel measurement of the common noise signal c(t), where each channel adds its own background noise a(t) and b(t), respectively, cf. (1), (2). It is assumed that c(t), a(t) and b(t) are statistically independent, t is the time. The Fourier transform (FT) provides access to the PSD $S_x(f) \equiv S_{xx}(f)$ of a zero-mean finite-power process x(t). For the FT – inverse FT pair we use the notation $x(t) \leftrightarrow X(f)$ with an upper case for the FT and suppressing the frequency $f$ wherever possible.

\[\text{Time domain } \leftrightarrow \text{ Frequency domain}\]
\[x(t) = c(t) + a(t) \leftrightarrow X = C + A \quad (1)\]
\[y(t) = c(t) + b(t) \leftrightarrow Y = C + B \quad (2)\]

In an actual experiment, the average $\langle \cdot \rangle_m$ on a suitable number of $m$ spectrum samples replaces the expectation operator $E\{\cdot\}$. For the cross-power spectral density (CPSD) $S_{xy}(f)$ this reads
\[
\langle S_{xy} \rangle_m = \tau^{-1} \langle XY^* \rangle_m, \quad (3)
\]

where an available truncated version $x_\tau(t) \leftrightarrow X_\tau(f)$ obtained in the finite measurement time $\tau$ is used for the entire process x(t) and the superscript '*' marks the complex conjugate. $S_{xy}$ is complex and can be decomposed into real and imaginary parts: $S_{xy} = S'_{xy} + is''_{xy}$. Evaluating (3), the cross terms decrease proportional to $1/\sqrt{m}$ owing to the statistical independence:
\[
\langle S_{xy} \rangle_m = \tau^{-1} [\langle CC^* \rangle_m + \langle CB^* \rangle_m + \langle AC^* \rangle_m + \langle AB^* \rangle_m] = S_{cc} + O(\sqrt{1/m}). \quad (4)
\]

Hence, for a large number of averages $m$, $\langle S_{xy} \rangle_m$ converges to its limit $E\{S_{xy}\} = S_{cc}$, while for small $m$ the cross terms can be the dominating part in (4) and cover small $S_{cc}$. The latter case is described by the statistical limit, assuming $c = 0$, $S_{cc} = 0$:
\[
\langle S_{xy}^{\text{stat. lim.}} \rangle_m = \tau^{-1} \langle AB^* \rangle_m \approx \sqrt{\langle S_{xx} \rangle_m \langle S_{yy} \rangle_m / m}. \quad (5)
\]

In order to observe the hardware limit, the statistical limit must be made smaller by averaging over long enough times (⇒ large $m$). Assuming white Gaussian noise for c(t), a(t) and b(t) in (1), (2), we can find expressions for the expectation values and variances (operator $V\{\cdot\}$):

\[
E\{S'_{xy}/m\} = E\{S''_{xy}\} = S_{cc}, \quad V\{S'_{xy}/m\} = \frac{1}{2m} (E\{S'_{xy}\}^2 + E\{S_{xx}\}E\{S_{yy}\}), \quad (6)
\]
\[
E\{S_{xx}/m\} = S_{cc} + S_{aa}, \quad V\{S_{xx}/m\} = \frac{1}{m} E\{S_{xx}\}^2. \quad (7)
\]

A subtle difference between (6) and (7) originates from the fact that $S_{xx}$ is always real and non-negative, whereas the real part $S'_{xy} = R\{S_{xy}\}$ can be both positive or negative. We use this estimator ($R\{S_{xy}/m\}$) since it contains all the useful information but discards the imaginary
Figure 1. Setup for the zero measurements (left) in comparison to the pMFFT body (right). Except for the removed temperature sensor beneath the detection coils, both setups are identical. The chip carrying the detection coils is shown semi-transparently with interconnections drawn in different colours. The SQUID sensor chips next to it are mounted on PCBs (not shown).

part of the background noise. This improves the signal-to-noise ratio of the estimator and avoids a bias. Opposite to this, $\mathbb{E}\{\langle S_{xy} \rangle_m\}$ is a good estimator for the background. Expressions (6) and (7) given here generalize those with a particular normalization chosen in [6].

A useful quantity describing the normalized measure of association is the coherency $C_{xy}(f)$:

$$C_{xy} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}.$$  \hspace{1cm} (8)

Its magnitude is referred to as coherence, and looking at its real part, $C'_{xy} = \Re\{C_{xy}\}$, it holds $-1 \leq C'_{xy} \leq +1$. Transforming (8) into $S_{xy} = C_{xy}\sqrt{S_{xx}S_{yy}}$ makes clear that for absolutely low values of $S_{xy}$ both a low residual correlation (coherence) and low background noise in both signal channels are required.

3. Experiment

3.1. Experimental setup

The new zero measurements in the millikelvin range were performed in a dry dilution refrigerator, using the same setup and environment that is otherwise used for regular temperature measurements with the pMFFT. The only difference is a modified copper body of the pMFFT, in which the temperature sensor located directly beneath the detection coils is removed. Figure 1 depicts the setup for the zero measurement in comparison to that of the pMFFT.

Each signal channel consists of a detection coil, a SQUID sensor, wiring to the room-temperature SQUID electronics and an analogue-to-digital converter (ADC). The gradiometric detection coils have an estimated total inductance of $\sim 400$ nH. Details are described in [4]. SQUID current sensors with an input inductance of 400 nH (single-stage, double-transformer SQUIDs [7] of size "L") are used for the readout. Each input circuit is equipped with an additional RC damping ($R = 3.9 \Omega$, $C = 47 \text{nF}$). The flux-lock loop (FLL) based readout scheme is depicted in figure 2. In FLL mode, $S \equiv S_{ph}$ is measured.

A 3 m long, shielded cable (type Magnicon Cryocable CC-1/3 [8]) connects the SQUID sensors with the room temperature SQUID electronics (type Magnicon XXF-1/3 [8]). This cable contains three channels, each consisting of two twisted pairs and one twisted triple. All strands are combined in a common stainless steel braid. For the data acquisition we use a two-channel dynamic signal analyzer card (type National Instruments PXI-4461, 24 Bit, 204.8 kSa/s [8]). This enables us to measure spectra up to $\sim 100$ kHz, exceeding by far the typical $\sim 5$ kHz evaluated in the pMFFT for temperature determination.

3.2. Results and Discussion

The spectra $S_{1} \equiv \langle S_{11} \rangle_m$, $S_{2} \equiv \langle S_{22} \rangle_m$ and $S'_{12} \equiv \langle S'_{12} \rangle_m$ were measured for various numbers of averages $m$ using the Welch periodogram [9]. If not otherwise stated, $T = 24 \text{mK}$. In view of the scaling of its variance, it is useful to compare $S'_{12}$ with a reference spectrum that is normalized
with respect to \( m \). We choose \( S_{\text{ref}} = \sqrt{(S_{11})_m(S_{22})_m/(2m)} \), which is equal to \( \text{dev}\{\langle S_{12}'\rangle_m\} \) in case of vanishing coherence (cf. (6) with \( E\{S_{xy}'\} = 0 \) and \( \text{dev}\{\cdot\} = (V\{(\cdot\})^{1/2} \).

### 3.2.1. Zero measurements
In order to distinguish between interferences from the pMFFT’s SQUID readout and those from other origins (wiring to room temperature, SQUID electronics, ADCs), measurements of \( S_{12}' \) were performed without SQUID operation. The SQUIDs were made insensitive to flux by a large bias current or by using an insensitive working point (\( dV_{\text{SQUID}}/d\Phi = 0 \)), and the SQUID electronics acted simply as a preamplifier (AMP mode, gain 2000). Results are identical, no matter which method is used. Both channels have a background white noise of \( \approx 0.4 \text{nVHz}^{-1/2} \) with pronounced interference peaks at the mains frequency and odd harmonics up to 1050 Hz. For \( S_{12}' \), it turns out that there are only few interference peaks above 1 kHz, apart from a pronounced peak centered at 59 kHz. Below 1 kHz, there is pronounced power only at the mains frequency and odd harmonics. The important outcome is that there is no obvious sign of a bias to \( S_{12}' \) apart from the few discrete peaks.

### 3.2.2. Crosstalk, mutual inductances
Crosstalk between both channels can, in principle, take place via different mechanisms and elements. If the cold-to-warm wiring of both channels is involved (each one consisting of a twisted triple and a twisted pair), several dual combinations exist. We confine our analysis to parasitic mutual induction between the feedback line of channel \( i \) (current \( I_i \)) and a flux sensitive circuit of the other channel \( j \) (causing the flux \( \Phi_j \) in SQUID \( j \)), described by effective values \( M_{12} \) and \( M_{21} \), cf. figure 2. The feedback is described by \( M_{i1} \) and \( M_{i2} \).

\[
\Phi = M I_i , \quad M = \begin{pmatrix} M_{i1} & M_{i2} \\ M_{21} & M_{22} \end{pmatrix} = M_0 \begin{pmatrix} M_{i1}/M_0 & M_{i2}/M_0 \\ M_{21}/M_0 & M_{22}/M_0 \end{pmatrix} \quad (9)
\]

The restriction to mutual inductances is consistent with the finding that the coupling is virtually independent of frequency when comparing results for 59 Hz, 1070 Hz and 4130 Hz. We can express our results relative to the scaling value \( M_0^{-1} = 63.10 \mu \text{A}/\Phi_0 \) (\( \Phi_0 = h/2e \) is the flux quantum). For the parasitic coupling we obtained \( M_{12}/M_0 = 7.4 \times 10^{-6} \) and \( M_{21}/M_0 = 2.2 \times 10^{-5} \), which is almost 5 orders of magnitude smaller than the feedback, \( M_{i1}/M_0 = 0.9986, M_{i2}/M_0 = 1.0016 \).

### 3.2.3. Influence of the SQUID working point
Although the SQUID working point does not affect the flux-to-voltage transfer coefficient of the FLL, it has an influence on noise properties and dynamic properties (bandwidth) of the FLL by means of the derivative of the SQUID characteristics (\( V_\Phi \equiv dV_{\text{SQUID}}/d\Phi \)). Hence, it can be expected that the working point affects
the limit of $S'_{12}$ and possibly also the coherence $|C'_{12}|$. We have checked this at the four working points (WP-x) given in table 1. The corresponding unity-gain frequencies $f_1$ of the FLL based on the given gain-bandwidth products (GBP) and on a feedback resistance of $R_f = 30 \, \text{k}\Omega$ vary between 620 kHz and 1.9 MHz.

For further evaluation we define a frequency band $B$ in order to determine the statistical quantities of (6), (7) and (8). An averaging $\langle \cdot \rangle_f$ is done over all $n_{\text{bin}}$ bins within this band. By choosing $B = (1 \ldots 12) \, \text{kHz}$, we obtain the results summarized in table 2 with $n_{\text{bin}} = 451$ and $m$(WP-B, WP-D) = 8788 or $m$(WP-A, WP-C) = 24413. If the mean value $E\{\langle S'_{12}\rangle_f\}$ is larger than the corresponding statistical limit ($= \sqrt{2} \langle S_{\ref{f}}\rangle_f$), it reflects the current hardware limit. This is clearly the case for WP-A, WP-C and WP-C/3. For the other working points (WP-B, WP-C/2, WP-D), longer measurement times (larger $m$) are necessary to reveal the hardware limit, and smaller absolute values ($\langle S'_{12}\rangle$) can be expected. In comparison, WP-A, is constricted by the relatively large background noise, caused by the low $V_\Phi$. Although this results in a substantially increased $S_{12}$ for equal $m$, the corresponding $C'_{12}$ as a normalized quantity is only moderately increased. With two exceptions (WP-C, WP-C/2), the coherence ($C'_{12}$) varies between $\approx 0.009$ and $\approx 0.003$, and a correlation with increasing $V_\Phi$ is apparent. The same correlation is apparent for $S'_{12}$. Consequently, working points with large $V_\Phi$ are favorable.

3.2.4. Influence of FLL parameters ($R_f$, GBP) Feedback resistance $R_f$ and GBP have a direct influence on the FLL bandwidth ($f_1 \propto \text{GBP} / R_f$) provided the cable delay is still negligible. It is known that the background noise can increase due to down mixing if the FLL bandwidth increases. A similar influence can be supposed to happen on $S'_{12}$. If down mixing is involved, the curvature of the SQUID characteristics ($d^2V_{\text{SQUID}}/d\Phi^2$) at the working point plays a role. For WP-C we tested two values for $R_f$: 30 kΩ (WP-C) and 100 kΩ (WP-C/2, $m = 48827$), keeping the GBP constant at 0.30 GHz. Results are given in table 2. For larger $R_f$, significantly reduced values for both $S'_{12}$ and $C'_{12}$ are found that have an even reversed sign. Although this finding fits into the simple picture outlined above, another, apparently contradicting measurement exists. Testing WP-C with an elevated GBP of 2.8 GHz (and thus increased FLL bandwidth) yields data very close to those of WP-C. For a comprehensive description of the influence of the FLL parameters on $S'_{12}$ and $C'_{12}$, further investigations are necessary.

| Table 1. Characteristic properties of working points (WP-x) and FLL parameters. |
| Parameter | WP-A | WP-B | WP-C | WP-D | GBP (Ch. 1 & Ch. 2) |
|---|---|---|---|---|---|
| $dV_{\text{SQUID}}/d\Phi$ (Ch. 1 | Ch. 2) | 220 | 190 | 840 | 690 | 4300 | 4200 | 12000 | 7400 | µV/Φ₀ |
| GBP | 6.2 | 2.8 | 0.30 | 0.30 | GHz |

| Table 2. Cross-correlation results at different working points (WP-x) and other conditions evaluated for $(1 \ldots 12)$ kHz. If no other values are given, $R_f = 30 \, \text{k}\Omega$ and $T = 24 \, \text{mK}$ apply. |
| WP/Condition | WP-A | WP-B | WP-C | WP-C/2 | WP-C/3 | WP-D | Unit |
|---|---|---|---|---|---|---|
| $R_f=100 \, \text{k}\Omega$ | $R_f=100 \, \text{k}\Omega$ | $T=100 \, \text{mK}$ | |
| $E\{\langle S'_{12}\rangle_f\}$ | 320 | 23 | 15 | -0.10 | 5.5 | 2.2 | $10^{-16} \Phi_0^2$/Hz |
| $\langle S_{\ref{f}}\rangle_f$ | 160 | 26 | 3.8 | 2.7 | 3.5 | 5.4 | $10^{-16} \Phi_0^2$/Hz |
| $\text{dev}\{\langle S'_{12}\rangle_f\}$ | 350 | 41 | 12 | 7.1 | 8.1 | 10 | $10^{-16} \Phi_0^2$/Hz |
| $E\{\langle C'_{12}\rangle_f\}$ | 9.1 | 6.6 | 18 | -0.14 | 5.2 | 3.1 | $10^{-3}$ |
| $\text{dev}\{\langle C'_{12}\rangle_f\}$ | 9.0 | 11 | 11 | 7.1 | 6.5 | 12 | $10^{-3}$ |
3.2.5. Influence of temperature In order to check if there is any influence of temperature, we performed two measurements under otherwise identical conditions. The two results for 24 mK (WP-C/2) and 100 mK (WP-C/3) are given in table 2. It is found that \( S'_{12} \) and consequently \( C'_{12} \) are increased at 100 mK in comparison to 24 mK. Between these two temperatures we can certainly assume that the SQUID working point does not change (because \( T \) is much smaller than the critical temperature of the SQUID) and that the intrinsic SQUID noise does not change (because the actual SQUID temperature decouples already below \( \sim 300 \) mK). Since no direct influence of temperature on \( C'_{12} \) is known, an obvious reason for this result is thermal noise sensed from the remaining copper body of the pMFFT. In this case we can assume a linear relationship for the background thermal noise: 

\[
E\{\langle S'_{12,T}\rangle_f\} = S'_{12,0} + \alpha_{12}T.
\]

Solving for \( \alpha_{12} \) yields \( \alpha_{12} = 7.1 \times 10^{-15} \Phi_0^2/(\text{Hz K}) \). The corresponding values of both signal channels (evaluated at 4.0 kHz only) are considerably larger: \( \alpha_1 = 2.7 \times 10^{-13} \Phi_0^2/(\text{Hz K}) \) and \( \alpha_2 = 3.4 \times 10^{-13} \Phi_0^2/(\text{Hz K}) \). These values must be compared to the sensitivity of the pMFFT, \( \alpha_{\text{pMFFT}} \). At 5 kHz, \( \alpha_{\text{pMFFT}} = 2.1 \times 10^{-10} \Phi_0^2/(\text{Hz K}) \). Thus, for pMFFT operation, the temperature effect is of the order \( 10^{-5} \).

3.2.6. Device noise temperature predicted for the pMFFT Now, in knowledge of the various effects on \( S'_{12} \), we are able to estimate the device noise temperature \( T_N \) for the pMFFT. We can neglect the temperature effect (subsection 3.2.5), but have to take into account influence from the working point and from FLL parameters (subsections 3.2.3, 3.2.4). Based on the mean value of WP-C, which can be considered as routine performance of the CCT, and on the spectral sensitivity of the pMFFT, we obtain \( T_N = 2.5 \mu \text{K} \).

4. Conclusions In conclusion, we have found that our cross-correlation setup, which is characterized by a rather small crosstalk between both signal channels, exhibits a typical coherence ranging between 0.3% and 0.9%. Although the absolute performance limit of the CCT depends on concrete working conditions, 40 n\( \Phi_0/\text{Hz}^{1/2} \) can be considered as a typical average value. The best performance of the CCT is achieved for SQUID working points with large \( V_\Phi \). When operating the pMFFT, the CCT performance can easily be optimized by monitoring the deviation of the imaginary parts \( \{\langle S_{12}'\rangle_m\} \) or \( \{\langle C_{12}'\rangle_m\} \). The device noise temperature was estimated as \( T_N = 2.5 \mu \text{K} \).

Acknowledgments The authors thank D. Heyer for technical assistance as well as PTB’s Berlin Institute workshop for manufacturing the high-precision copper bodies. The research reported here has received funding from the EMPIR programme as an integrated part of Horizon 2020, the European Union’s Framework Programme for Research and Innovation (grant number 15SIB02InK2).

References
[1] Beyer J, Drung D, Kirste A, Engert J, Netsch A, Fleischmann A and Enss C 2007 IEEE Trans. Appl. Supercond. 17, 760-763
[2] Netsch A, Hassinger E, Enss C and Fleischmann A 2006 AIP Conf. Proc. 850 1593-1594
[3] Nyquist H 1928 Phys. Rev. 32 110-113
[4] Kirste A and Engert J 2016 Phil. Trans. R. Soc. A 374 20150050
[5] Kirste A, Regin M, Engert J, Drung D and Schurig Th 2014 J. Phys.: Conf. Ser. 568 032012
[6] Rubiola E and Vernotte F 2010 http://arxiv.org/pdf/1003.0113v1.pdf
[7] Drung D, Aßmann C, Beyer J, Kirste A, Peters M, Ruede F, Schurig T. 2007 IEEE Trans. Appl. Supercond. 17 699-704
[8] Identification of commercial equipment in this paper does not imply any recommendation or endorsement by the PTB.
[9] Welch PD 1967 IEEE Transactions on Audio and Electroacoustics AU-15 70-76