Curvature and Force Estimation for a Soft Finger using an EKF with Unknown Input Optimization

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Abstract: Sensory data such as bending curvature and contact force are essential for controlling soft robots. However, it is inconvenient to measure these variables because sensorizing soft robots is difficult due to their inherent softness. An attractive alternative is to use an observer/filter to estimate the variables that would have been measured by those sensors. Nevertheless, an observer/filter requires a model which can be analytically demanding for soft robots due to their high nonlinearity. In this paper, we propose an Unknown Input Extended Kalman Filter (UI-EKF) consisting of an EKF interconnected with a UI-optimizer to respectively estimate the state (curvature) and unknown input (contact force) for a pneumatic-based soft finger based on an identified nonlinear model. We also prove analytically that the estimation errors are bounded. Experimental results show that the UI-EKF can perform the estimation with high accuracy, even when the identified system model is not accurate and the sensor measurement is noisy. In other words, the proposed framework is able to estimate proprioceptive (internal) and exteroceptive (external) variables (curvature and contact force respectively) of the robot using a single sensor (flex), which is still an open problem in soft robotics.

Keywords: Extended Kalman filters; Unknown input estimation; Neural-network models; Stochastic systems; Lyapunov Stability; Robotics

1. INTRODUCTION

Inspired by the inherent softness and compliance found in biological organisms, soft robots are designed to have these characteristic for adaptive operation in unstructured environments (Nurzaman et al., 2014). For control purposes, soft robots should ideally have access to sensory data just like their rigid counterparts. For example, curvature angle is useful in describing the bending of soft actuators for feedback control (Webster and Jones, 2010). External force is also important because soft robots generally have to interact with objects and obstacles in real applications. However, integrating sensors in soft robots to measure these variables is not as straightforward as for their rigid counterparts (Rus and Tolley, 2015). For instance, attaching multiple load cells on a soft robot for force measurement introduces extra weight which could stiffen it, whilst cameras, which are used for measuring the curvature of a soft robot, are impractical to setup especially in unstructured environments. Due to this difficulty, many soft robots are still open-loop systems (Wang et al., 2017). To reduce the need of integrating sensors in soft robots, an alternative is to employ estimation methods.

A convenient estimation approach is using an empirical model identified by experimental data. For example, Elgeneidy et al. (2018) developed data-driven models using linear regression and neural networks to estimate the curvature angles of a pneumatic actuator. Thuruthel et al. (2019) trained a long short-term memory (LSTM) network as the kinematic and force model to estimate the tip position and contact force of their soft finger. However, these empirical methods heavily depend on sensor measurements which could be noisy, thereby reducing the estimation accuracy. Another estimation method is through optimization, for example Wang and Hirai (2017) estimated the grasping force of their soft robot by minimizing the discrepancy between the model estimates and sensor measurements. Recent work by Venkiteswaran et al. (2019) developed a pseudo-rigid body model with parameter optimization to estimate the forces applied on a magnetically-actuated manipulator. Nonetheless, the optimization accuracy is adversely affected by modelling error, which can be significant in soft robots due to their high nonlinearity. Hence, it can be seen that the estimation approaches in the aforementioned works are open-loop, and to get a good estimate would require accurate sensor measurements and a high-fidelity model, both of which are difficult or expensive to attain in soft robots. An observer/filter, that processes sensor measurements based on a model in closed-loop, could be a promising alternative to these methods.

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There are only few studies that utilize an observer/filter for state estimation in soft robots. A Luenberger observer was used to estimate the position and velocity using a reduced order steady-state model (Thieffry et al., 2019). Lunni et al. (2018) used an Adaptive Extended Kalman Filter on a simplified quasi-static model to estimate the angle of rotation at each flexural joint of a tendon-driven soft robot. Loo et al. (2019a,b) proposed an Extended Kalman Filter (EKF) to estimate states in the actuator and configuration space of both planar and non-planar pneumatic-based soft tentacles. However, none of these filter-based approaches estimated the contact force, which is an external signal (or unknown input). To the best of our knowledge, discrete-time filters by themselves cannot estimate unknown inputs. A possible solution is to integrate the filters with an additional component such as an optimizer for unknown input estimation. In the field of structural control for example, Huang et al. (2010) and Liu et al. (2016) combined Kalman Filters with least square optimization to estimate unknown input in nonlinear systems, but only for a specific class of systems in which the measurement model must be a function of the unknown input.

In this paper, we present a general observer/filter-based framework for state and unknown input estimation of a pneumatic soft finger. Using a single flex sensor that can be conveniently embedded in the finger, we estimate the curvature angle and contact force, which are difficult to measure. Due to the complexities involved in analytical modelling of soft robots (Rus and Tolley, 2015), we employ system identification to model the soft finger, using a NARX model with wavelet network to represent its state dynamics, and nonlinear regression model to represent its output equation. The models that we use are a class of non-recurrent methods, that are less complex compared to recurrent network structure such as that in (Thuruthel et al., 2019). In the model, the curvature angle is formulated as an internal state because it relates to proprioception (produced and perceived within the soft finger), whereas the contact force, being an exteroceptive stimulus, is modelled as an unknown input. Based on the identified models, we propose an Unknown Input Extended Kalman Filter scheme (UI-EKF) consisting of an EKF interconnected with an UI-optimizer to estimate the curvature and contact force respectively. We also prove that the estimation errors are bounded. Experimental results show that the UI-EKF is capable of estimating the curvature and contact force to a high degree of accuracy. In the model that we identified, the unknown input (contact force) does not appear in the measurement equation, and hence the methods by Huang et al. (2010) and Liu et al. (2016) are not applicable to our system. In other words, we have demonstrated the ability of the framework in estimating both proprioceptive (internal) and exteroceptive (external) variables using a single sensor, which is still a significant challenge in soft robotics.

This paper is organized as follows. Section 2 describes the soft robot fabrication method and experimental setup. Section 3 details the framework of the proposed UI-EKF. Results are presented and discussed in Section 4. Lastly, we conclude our work in Section 5.

2. MATERIALS

2.1 Fabrication of the Pneumatic Based Soft Robot

The soft robot used was a pneumatic-based soft finger (PSF). We casted silicone (EcoFlex 0050, Smooth-On Inc.) using 3D-printed molds to form the main body and a base layer. After that, we embedded a flex sensor (4.5 inches, SparkFun) in the base layer together with a piece of paper to make it inextensible. Then, the base layer and the main body were joined together with more silicone. Finally, we soldered the connections of the flex sensor and prodded the PSF with a sharp-end pneumatic pipe as air inlet. The cross-sectional view of the PSF is shown in Fig. 1.

2.2 Experimental Setup

Our pneumatic system consisted of a pneumatic supply connected in series to a pressure regulator followed by a solenoid valve (VQD1151U-5L, SMC Pneumatics), which was used to modulate the pressure by high-speed switching using PWM signals (Memarian et al., 2015). Finally, the modulated supply was connected to the PSF inlet and a pressure sensor (MPXH6400A, NXP) in parallel. The pressure inside the PSF was assumed to be the same as the measured pressure. We used a microcontroller (PSOC@5LP) to manipulate the duty cycle (control input) of the PWM signal that controls the solenoid valve. The amplitude and pulse width of the signals followed a pseudorandom sequence.

To capture the motion of the PSF, we attached it to a holder and placed reflective ball markers on its base, mid and tip position. During actuation, 3 cameras (OptiTrack Flex 13, NaturalPoint Inc.) continuously measured the coordinates of the markers to be used as ground-truth...
in the identification and estimation process. In addition, a sensing circuit was used to convert the change in resistance of the embedded flex sensor to voltage readings. For tip contact force measurements, we used a load cell (FX1901, TE Connectivity) to obtain the ground-truth by positioning it near the PSF’s tip. The experimental setup is shown in Fig. 2.

2.3 Data Collection

The sensors (for pressure, flex and force) were connected to the analog pins of an Arduino Mega 2560 board for data sampling whereas the markers’ coordinates were provided by OptiTrack’s Motive software. We collected the measurements at a rate of 11.5 Hz using MATLAB. 11 sets of 10-minute data and 4 sets of 5-minute data were collected as identification and validation data respectively. Each set corresponded to a different location of the load cell in front of the tip; this was to simulate the unstructuredness of real world environment in which the obstacles could be in any arbitrary location when in contact with the tip.

3. METHODS

3.1 Planar Bending Modelling

The motivation behind choosing curvature angle to characterize the PSF bending is the fact that the angle subtended by the bending arc of a single segment, \( \phi_i \), is the total change in tangential direction along that arc as illustrated in (Loo et al., 2019a). Here we use a piece-wise constant curvature (PCC) analysis to calculate the angle \( \phi_i \) between any two consecutive segments using method in (Wang and Hirai, 2017), given the nodal point coordinates as the measured markers’ coordinates, as shown in Fig. 3. We also considered the case when the 2\(^{nd}\) and 3\(^{rd}\) markers are to the left of the base marker by placing the origin at the most leftward marker and calculate \( \phi_i \) as before so that the bending arc always extends out from the origin marker, parallel to Y axis. With that, we approximate the segmental curvature angle using the relationship \( \phi_i = 2 \phi_i \) (Webster and Jones, 2010). Finally, we quantify the curvature of the whole PSF as sum of the discrete segmental curvature angles, \( \theta = \sum_{i=1}^{3} \phi_i \).

3.2 System Identification

The dynamics of the curvature angle that characterizes the bending of the soft finger can be represented using a general nonlinear state model and a nonlinear measurement model as follows

\[
\dot{x} = \Phi(x, u_1, u_2) \quad , \quad y = \Gamma(x) \tag{1}
\]

where \( x \) is the system state (curvature angle), \( u_1 \) is the known system input (PSF’s internal pressure), \( u_2 \) is the unknown system input (tip contact force), \( y \) is the measurement output (flex sensor voltage).

Using the Forward-Euler scheme, we discretize the continuous models (1) by approximating the curvature angular velocity, \( \dot{x}_k \) at each time step as

\[
\dot{x}_k \approx \frac{x_{k+1} - x_k}{t_s} \tag{2}
\]

where \( t_s \) is the sampling time step, to get the following discrete-time models:

\[
x_{k+1} = x_k + \Phi(x_k, u_{1k}, u_{2k}) t_s = f(x_k, u_{1k}, u_{2k}) \tag{3}
\]

\[
y_k = h(x_k) \tag{4}
\]

where \( f(\cdot) \) is the system difference equation function and \( h(\cdot) \) is the discretized version of \( \Gamma(\cdot) \). Fig. 4 illustrates this configuration.

Since the soft robot is slowly-actuated, we can disregard its dynamics, and assume \( \dot{x}_k \approx 0 \), i.e.

\[
\Phi(x_k, u_{1k}, u_{2k}) \approx 0 \tag{5}
\]

from which we can get the following quasi-static model:

\[
x_k = g_2(u_{1k}, u_{2k}) \tag{6}
\]

Following our previous approach (Loo et al., 2019a), the state model is identified as a NARX model using a wavelet network with the modification of \( \mathbf{x}_r = [x_k u_{ps} u_{fs}] \) as the regressor input. Similarly, the measurement model is identified as a fourth order polynomial function using polynomial regression.

3.3 Unknown Input Extended Kalman Filter

Consider a nonlinear discrete system with additive noises, represented by state and measurement models as follows

\[
x_{k+1} = f(x_k, u_{1k}, u_{2k}) + G_k w_k \tag{7}
\]

\[
y_k = h(x_k) + D_k v_k \tag{8}
\]

Here, \( w_k \) and \( v_k \) are modelling error and measurement noise of (3),(4), both assumed to have zero mean and unit covariance. In this paper, we introduce a novel observation-based filter structure termed as UI-EKF (Fig. 4), in which the conventional EKF is interconnected with a least square optimization scheme to simultaneously estimate the state (curvature) and unknown input (contact force) of

Fig. 3. Piece-wise constant curvature model of the PSF.

Fig. 4. Block diagram of pneumatic soft finger system and UI-EKF.

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a nonlinear stochastic system. The formulations are as follows:

**EKF Predict phase:**

\[
\hat{x}_{k+1} = f(\hat{x}^+_k, u_{1k}, \hat{u}_{2k}) \\
P_{k+1}^- = F_k P_k^+ P_k^+ + Q_k
\]

where

\[
P_k = A_k + \eta B_k
\]

**EKF Correct phase:**

\[
\begin{align*}
K_{k+1} &= P_{k+1}^- C_k^T \left[ C_{k+1} P_{k+1}^- C_k^T + R_{k+1} \right]^{-1} \\
\hat{x}^+_{k+1} &= \hat{x}^-_{k+1} + K_{k+1} \left( y_{k+1} - h(\hat{x}^-_{k+1}) \right) \\
P^+_{k+1} &= \left[ I - K_{k+1} C_{k+1} \right] P^-_{k+1}
\end{align*}
\]

where \( A_k, B_k \), and \( C_k \) are the state, input and measurement Jacobians respectively, obtained as

\[
A_k = \frac{\partial f}{\partial x} \bigg|_{\hat{x}^+_k, u_{1k}, \hat{u}_{2k}} \quad B_k = \frac{\partial f}{\partial u_2} \bigg|_{\hat{x}^+_k, u_{1k}, \hat{u}_{2k}} \quad C_{k+1} = \frac{\partial h}{\partial x} \bigg|_{\hat{x}^+_{k+1}}
\]

**Unknown Input Optimization:**

\[
\begin{align*}
\hat{u}_{2k} &= \arg\min_{\hat{u}_{2k}} \left\| \hat{x}^-_{k+1} - g(u_{1k}, \hat{u}_{2k}) \right\|^2 \\
&=: \psi(\hat{x}^-_k, u_{1k}, \hat{u}_{2k})
\end{align*}
\]

Define the estimation errors to be

\[
e_k = x_k - \hat{x}_k \quad z_k = x_k - \hat{x}_k \quad \xi_k = u_{2k} - \hat{u}_{2k}
\]

Then

\[
\xi_k = \psi(x_k, u_{1k}, \epsilon_k) - \psi(\hat{x}_k, u_{1k}, \hat{u}_k) := \psi(x_k, \hat{x}_k
\]

The Taylor’s expansion of \( f(\cdot) \) at \( \hat{x}_k^+ \) and \( h \circ f(\cdot) \) at \( \hat{x}_k^- \) are

\[
\begin{align*}
f(x_k, u_{1k}, \hat{u}_{2k}) - f(\hat{x}_k^+, u_{1k}, \hat{u}_{2k}) &= J_k \xi_k + \varphi(x_k, \hat{x}_k^+, \hat{u}_{2k}) \\
h\left( f(x_k, u_{1k}, \hat{u}_{2k}) + G_k w_k \right) - h\left( f(\hat{x}_k^+, u_{1k}, \hat{u}_{2k}) \right) &= C_{k+1} J_k \xi_k + \varphi(x_k, \hat{x}_k^+, \hat{u}_{2k}) + G_k w_k \\
&+ \chi(x_k, \hat{x}_k^+, \hat{u}_{2k})
\end{align*}
\]

where we have used chain rule for \( h \circ f(\cdot) \) and

\[
J_k = [A_k, B_k] \quad \xi_k = \begin{bmatrix} e_k \\ \xi_k \end{bmatrix}
\]

Introduce the a posteriori estimation error as

\[
\begin{align*}
z_{k+1} &= x_{k+1} - \hat{x}^+_{k+1} \\
&= f(x_k, u_{1k}, \hat{u}_{2k}) - f(\hat{x}^+_k, u_{1k}, \hat{u}_{2k}) + G_k w_k \\
&= J_k \xi_k + \varphi(\cdot) + G_k w_k
\end{align*}
\]

Introduce the a posteriori estimation error as

\[
\begin{align*}
e_{k+1} &= x_{k+1} - \hat{x}^-_{k+1} \\
&= x_{k+1} - \left( \hat{x}^-_{k+1} + K_{k+1} \left( y_{k+1} - h(\hat{x}^-_{k+1}) \right) \right) \\
&= f(x_k, u_{1k}, u_{2k}) - f(\hat{x}^-_k, u_{1k}, \hat{u}_{2k}) - K_{k+1} \left( \left( f(x_k, u_{1k}, u_{2k}) + G_k w_k \right) + D_{k+1} v_{k+1} \right) \\
&- h(\hat{x}^-_k, \hat{u}_{2k})) \\
&= J_k \xi_k - K_{k+1} C_{k+1} J_k \xi_k + r_k + s_k \\
&= [I - K_{k+1} C_{k+1}] J_k \xi_k + r_k + s_k
\end{align*}
\]

where

\[
r_k = [I - K_{k+1} C_{k+1}] \varphi(\cdot) - K_{k+1} \chi(\cdot) \\
s_k = [I - K_{k+1} C_{k+1}] G_k w_k - K_{k+1} D_{k+1} v_{k+1}
\]

Here, we make the following global boundedness assumptions

\[
\begin{align*}
\|A_k\| \leq \pi , & \quad \|B_k\| \leq \bar{b} , \quad \|C_k\| \leq \tau \\
\|\varphi\| \leq Q_k \leq \pi I , & \quad \|\chi\| \leq R_k \leq \pi I \\
\|\xi_k\| \leq P_k \leq \pi I
\end{align*}
\]

and the following local Lipschitz assumptions:

\[
\begin{align*}
\|\xi_k\| \leq \|\Psi(x_k, \hat{x}_k)\| \leq \|\eph\| \|e_k\| \\
\|\varphi(x_k, \hat{x}_k, \hat{u}_{2k})\| \leq \bar{\kappa}_\varphi \|\xi_k\|^2 \\
\|\chi(x_k, \hat{x}_k)\| \leq \bar{\kappa}_\chi \|z_k\| \|\xi_k\| \\
\|J_k\| \leq \|P_k\| \leq \pi I
\end{align*}
\]

**Proof.** See Appendix A.1.

**Lemma 2.** Under the Lipschitz assumptions (29)-(31) and let \( r_k \) be given by (24), there exist positive real numbers \( \kappa_{nonl_1}, \kappa_{nonl_2} \) and \( \kappa_{nonl_w} \) such that \( \Pi_k = P_k^{+1} \) satisfies

\[
E \left[ P_k^{+1} \left( 2[I - K_{k+1} C_{k+1}] J_k \xi_k + r_k \right) \right] \\
\leq \kappa_{nonl_1} \|e_k\| + \kappa_{nonl_2} \|e_k\| \|\delta_w\| + \kappa_{nonl_w} \|\delta_v\|$
\]

**Proof.** See Appendix A.2.

**Lemma 3.** Let \( s_k \) be given by (25), there exist positive real numbers \( \kappa_{noise_w} \) and \( \kappa_{noise_v} \) such that \( \Pi_k = P_k^{+1} \) satisfies

\[
E \left[ s_k^{+1} \Pi_k s_k \right] \leq \kappa_{noise_w} \|\delta_w\| + \kappa_{noise_v} \|\delta_v\|$
\]

**Proof.** See Appendix A.3.
bounded in mean square with probability one, provided that the initial state error is small i.e.
\[ \|e_0\| \leq \epsilon \]
and the model error and sensor noise are bounded i.e.
\[ G_k G_k^\top \leq \delta_v I, \quad D_k D_k^\top \leq \delta_w I \]

**Proof.** See Appendix A.4.

**Remark:** Assuming that our identified system (3),(4) is locally observable (i.e. two distinct states, \( x^1, x^2 \) are U-distinguishable for every admissible system input, \( u \), implying that the nonlinear observability rank condition is satisfied) (Hermann and Krener, 1977) and their nonlinear functions, \( f(\cdot), h(\cdot) \) are twice differentiable, then from Lemma 4.1 and Lemma 4.2 in (Reif et al., 1999), \( A_k, C_k \) in (15) satisfy the uniform observability condition, thus condition (28) is valid if the noise covariances are bounded i.e. (27) holds. Lipchitz assumptions (29) generally holds if the objective function in (16) is convex and the model error, \( \epsilon \) of (6) is small. According to (Reif et al., 1999, Proof of Theorem 4.1), \( \kappa_n, \kappa_s \) in (30), (31) are given by the spectral norm of the respective Hessian matrices of \( f(\cdot), h(\cdot) \).

In practice, we use \( P_{k+1}^* = A_k P_k^* A_k^\top + Q_k \) instead of (10) for estimate covariance propagation in EKF while setting a higher \( Q \) to compensate for the term \( \eta^2 B_k P_k^* B_k^\top \) at each time step, so that we circumvent the difficulty of finding the global Lipchitz constant, \( \eta \) in (29).

### 4. RESULTS & DISCUSSIONS

#### 4.1 System Identification

With the identification data, the state model \( f(\cdot) \) is identified as a wavelet-based NARX model using \( x_k \) (curvature), \( u_{k-1} \) (pressure) and \( u_{2k} \) (force) as inputs and \( x_{k+1} \) as output, whereas the measurement model \( h(\cdot) \) uses a 4\(^{\text{th}}\) order polynomial to fit \( x_k \) and \( y_k \) (flex voltage) as input and output respectively. The model is then tested against the validation data, and its normalized root-mean-squared error (NRMSE) is used to indicate the quality of fit. Fig. 5 shows the validation results of the state model (top) and measurement model (bottom), with a NRMSE of 7.47% and 2.72% respectively. The validation results also show that both the open-loop models (state and measurement) are able to capture the soft robot’s dynamics well and also estimate the curvature angles, but only to a certain accuracy with the respective margin of errors. In addition, the identified model requires the knowledge of contact force for estimation, and it cannot be used for real-time testing as contact force is an unknown quantity.

#### 4.2 State and Unknown Input Estimation

Using the identified models, the UI-EKF closed-loop framework is employed to estimate both the curvature and contact force. Fig. 6 (top) shows the curvature estimation result with a much-improved NRMSE of 2.94% compared to the open-loop estimation (Fig. 5-top). Even without the knowledge of contact force, the EKF is able to estimate the curvature with a high accuracy. In addition, the EKF, being a filter, reduces the effect of noise from the measurement output onto the estimated state.

Fig. 6 (bottom) shows the result for contact force estimation, with a NRMSE of 4.05%. This indicates that our proposed UI-EKF framework is successful in estimating not only the state, but also the unknown input. Furthermore, the force estimate is much less affected by measurement noise because it is obtained by the UI optimizer using the filtered curvature angle.

### 5. CONCLUSION

In this work, we present a novel Unknown Input Extended Kalman Filter (UI-EKF) to estimate the state (curvature) and unknown input (contact force) for a pneumatic soft finger based on an identified nonlinear model, and measurements of pressure and flex voltage (which are more convenient to obtain in our setting). We also prove that the estimation errors are bounded. The results show that
the estimates from the UI-EKF are very accurate. Finally, the paper has also shown that the proposed framework can estimate both proprioceptive (internal) and exteroceptive (external) variables using only a single sensor, resolving a significant challenge in soft robotics. Future work includes estimation in more complex soft robotic systems.

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Appendix A. STABILITY PROOFS

The proofs follow closely the results of (Reif et al., 1999) who considered a priori estimation of a one-step EKF: in this paper, we also include unknown input estimation by considering the conventional two-step EKF in cascade with an unknown input optimization and prove for bounded error in both a posteriori state and unknown input estimates. Hence, Lemma 1, Lemma 2, Lemma 3 and Proposition 1 follow from Lemma 3.1, Lemma 3.2, Lemma 3.3 and Theorem 3.1 in (Reif et al., 1999), but with the necessary modifications.

A.1 Proof of Lemma 1. The posteriori estimate covariance propagation equation in (14) can be reformulated into

\[
P_{k+1}^+ = \left[ I - K_{k+1}C_{k+1} \right] P_{k+1}^- \left[ I - K_{k+1}C_{k+1} \right]^T + K_{k+1}R_{k+1} K_{k+1}^T
\]

By letting \( \Pi_k = P_{k+1}^- \),

\[
\text{Taking the inverse on both sides of (A.1),}
\]

\[
\begin{align*}
P_{k+1}^+ & \leq \left[ I - K_{k+1}C_{k+1} \right] P_{k+1}^- F_{k+1}^T \times \\
& \times P_{k+1}^+ \left( 1 + \frac{1}{\rho(a + \eta b)^2} \frac{p^2 c^2}{r(1 + p^2 c^2)\frac{1}{2}} \right) \times F_{k+1} \left[ I - K_{k+1}C_{k+1} \right]^T \\
& \leq P_{k+1}^+ \left( 1 + \frac{1}{\rho(a + \eta b)^2} \frac{p^2 c^2}{r(1 + p^2 c^2)\frac{1}{2}} \right)^{-1}
\end{align*}
\]

By letting \( \Pi_k = P_{k+1}^- \),
\[ F_k^T \left[ I - K_{k+1}C_{k+1} \right]^T \Pi_{k+1} \left[ I - K_{k+1}C_{k+1} \right] F_k \]
\[ \leq \Pi_k + \frac{1}{p} \left[ \frac{\eta^2}{\tau^2 + \eta^2 \tau^2} - \frac{1}{p^2} \right] = \Pi_k (1 - \beta) \]

**A.2 Proof of Lemma 2.** From (24), (30), (31), (22) we have

\[ \|r_k\| \leq \left\| \left[ I - K_{k+1}C_{k+1} \right] \nu(\cdot) + \left[ K_{k+1} + 1 \right] \nu(\cdot) + G_k w_k \right\| \]
\[ \leq (1 + \frac{p^2}{2}) \kappa_\nu \|q_k\| + \frac{p^2}{2} \kappa_\nu \|J_k z_k + \varphi(\cdot) + G_k w_k\| \]
\[ \leq \left( 1 + \frac{p^2}{2} \kappa_\nu + \frac{p^2}{2} \kappa_\chi (p_\nu + \eta) \right) \|q_k\| \]
\[ + \frac{p^2}{2} \kappa_\chi \|q_k\|^2 + \frac{p^2}{2} \kappa_\nu \|q_k\|^2 \]
\[ \leq \kappa_\nu (1 + \eta^2) \|q_k\|^2 + \kappa_\nu (1 + \eta^2) \|q_k\|^2 + \kappa_\chi \|G_k w_k\|^2 \]
\[ = \kappa_\nu \|q_k\|^2 + \kappa_\nu \|q_k\|^2 + \kappa_\chi \|G_k w_k\|^2 \]

where we have used the facts that from (21) and (29), \[ \|J_k z_k\| \leq \left\| \left[ \|A_k\| + \eta B_k \right] \right\| \|q_k\|. \]
Given (33) holds and the fact that \( w_k \) is white, we have

\[ E_q \left[ \left( 2 \left[ I - K_{k+1}C_{k+1} \right] J_k z_k + r_k \right) \right] \]
\[ \leq E_q \left[ \kappa_\nu \|q_k\|^2 + \kappa_\nu \|q_k\|^2 + \kappa_\chi \|G_k w_k\|^2 \right] \]
\[ \leq \kappa_\nu \|q_k\|^2 + \kappa_\nu \|q_k\|^2 + \kappa_\chi \|G_k w_k\|^2 \]

where \( \kappa_\nu \) and \( \kappa_\chi \) are polynomial functions of \( \|q_k\| \).

We also used the matrix identity for trace operation \( tr(AB) = tr(BA) \), and the fact that

\[ tr(G_k G_k^T) \leq g \delta_w \]  
\[ tr(D_k D_k^T) \leq d \delta_v \]  

where \( g \) and \( d \) are the number of rows of \( G_k \) and \( D_k \). Note that taking the trace of a scalar does not change its value.

**A.3 Proof of Lemma 3.** Taking the expectation of \( s_k^T \Pi_{k+1} s_k \), we get

\[ E_q \left[ s_k^T \Pi_{k+1} s_k \right] \]
\[ = E_q \left[ w_k^T G_k^T \left[ I - K_{k+1}C_{k+1} \right] \right] \]
\[ = E_q \left[ w_k^T G_k^T \left[ I - K_{k+1}C_{k+1} \right] \right] \]
\[ + v_k^T D_k^T (k+1) K_k+1 k+1 (I+1) D_k+1 \]

where the cross terms of \( w_k \) and \( v_k \) vanished since they are uncorrelated. With the fact that both \( w_k \) and \( v_k \) are white, we have

\[ E_q \left[ s_k^T \Pi_{k+1} s_k \right] \leq \frac{1}{p} \left( 1 + \frac{p^2}{2} \right) \left( r^T + \frac{p^2}{2} \delta_v \right) \]
\[ + \frac{p^2}{2} \delta_v \left( r^T + \frac{p^2}{2} \delta_v \right) \]
\[ \leq \frac{q}{p} \left( 1 + \frac{p^2}{2} \right) \delta_w + \frac{p^2}{2^2} \delta_v \]
\[ \leq \frac{q}{p} \left( 1 + \frac{p^2}{2} \right) \delta_w + \frac{p^2}{2^2} \delta_v \]
\[ \leq \frac{q}{p} \left( 1 + \frac{p^2}{2} \right) \delta_w + \frac{p^2}{2^2} \delta_v \]

Then, from (29), we have

\[ V_k(e_k) = e_k^T \Pi_k e_k \]

where \( \Pi_k = P_k^{1-1} \). From (28) we have

\[ \frac{1}{p} \|e_k\|^2 \leq V_k(e_k) \leq \frac{1}{p} \|e_k\|^2 \]

Then, from (23), we have

\[ V_{k+1}(e_{k+1}) = e_{k+1}^T \Pi_{k+1} e_{k+1} \]

where we have used the facts that from (21) and (29), \[ \|J_k z_k\| \leq \left\| \left[ \|A_k\| + \eta B_k \right] \right\| \|q_k\|. \]

Given (33) holds and the fact that \( w_k \) is white, we have

\[ E_q \left[ s_k^T \Pi_{k+1} s_k \right] \leq \left( 1 + \frac{p^2}{2} \right) \left( r^T + \frac{p^2}{2} \delta_v \right) \]
\[ + \frac{p^2}{2} \delta_v \left( r^T + \frac{p^2}{2} \delta_v \right) \]
\[ \leq \frac{q}{p} \left( 1 + \frac{p^2}{2} \right) \delta_w + \frac{p^2}{2^2} \delta_v \]
\[ \leq \frac{q}{p} \left( 1 + \frac{p^2}{2} \right) \delta_w + \frac{p^2}{2^2} \delta_v \]

Taking the conditional expectation, \[ E_q[V_{k+1}(e_{k+1}) | e_k] \]

\[ = E_q \left[ 2s_k^T \Pi_{k+1} \left( I - K_{k+1}C_{k+1} \right) J_k z_k + r_k \right] \]

\[ + \frac{p^2}{2} \delta_v \left( r^T + \frac{p^2}{2} \delta_v \right) \]

vanishes due to the fact that \( \Pi_{k+1} \left( I - K_{k+1}C_{k+1} \right) \) is independent of the noise term \( s_k \) and \( z_k \). Applying Lemma 1, 2, 3 on (A.5), we get

\[ E_q \left[ V_{k+1}(e_{k+1}) \right] \]
\[ \leq - \beta V_k(e_k) + \kappa_{\nu_1}(\|e_k\|) + \kappa_{\nu_2}(\|e_k\|) \delta_w + \kappa_{\nu_3}(\|e_k\|) + \kappa_{\nu_4}(\|e_k\|) \delta_v \]

where the second inequality follows from (A.4).

Then (A.6) becomes

\[ E_q[V_{k+1}(e_{k+1}) | e_k] = V_k(e_k) \leq - \mu \alpha V_k(e_k) \]

with \( \mu = \kappa_{\nu_1}(\|e_k\|) + \kappa_{\nu_2}(\|e_k\|) \delta_w + \kappa_{\nu_3}(\|e_k\|) + \kappa_{\nu_4}(\|e_k\|) \delta_v \)

Finally, if there exist some positive constants \( \epsilon, \delta_w \) and \( \delta_v \) such that (32) and (33) holds, together with (A.4), the conditions in (Reif et al., 1999, Lemma 2.1) are fulfilled. Hence, we proved that \( e_k \) is exponentially bounded in mean square and bounded with probability one. Then from (29), the same boundedness applies to \( \xi_k \).