The influence of gravity load on the accuracy of mesh reflector antenna

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Abstract. Space large deployable mesh antennas are the focus of research in the aerospace field. There is a high requirement for the reflector surface accuracy of the mesh antenna in orbit. Due to the low stiffness of the truss cable net system, the deformation of the flexible mesh surface under the ground gravity load cannot be ignored. Therefore, it is the key technology to ensure the accuracy of on orbit cable network reflector that accurately predict the difference of reflector under gravity load and microgravity load. This study establishes the mechanical model of the cable-truss system. The Newton Iterative Method is used to solve the nodal nonlinear equilibrium equation in the gravity load, and the deformation of the antenna mesh surface under the gravity load is obtained.

1. Introduction
In recent years, the development of large deployable mesh reflector antennas⁴,⁵, represented by ring truss mesh antenna, has received great attention of aerospace researchers from all over the world. And it is regarded as the key technology to improve the level of communication satellites and earth observation satellites. As shown in Fig. 1, the ring truss mesh reflector is composed of cable network system (including front cable network, rear cable network and tie force cable) and truss. The metal mesh is attached to the front cable network to reflect electromagnetic signals, there is a high accuracy requirement for the cable network reflector in orbit microgravity environment⁶. The processing, assembly, adjustment and test of the antenna are all under the ground gravity environment⁴, the stiffness of the truss cable net system along the vertical direction and the radial direction of the ring truss is low, and the deformation of the whole reflector caused by the gravity of the metal net and the cable net cannot be ignored⁴,⁵. Therefore, how to eliminate the influence of gravity and ensure the on-orbit microgravity surface accuracy of the cable network has become a major problem in the process adjustment and surface accuracy measurement stage of large cable network.
This study intends to establish the mechanical model of the cable-truss system in the gravity environment. Based on the result of the geometric method of form find the cable network, the Newton Iterative Method is used to solve the balanced position of the node cable network in the gravity load. The rest of this Note is organized in the following way. Section 2 establishes the mechanical model of cable net in gravity environment by analyzing the geometric and mechanical characteristics of cable truss structure, and the Newton iteration method is proposed to solve the position of the nodes under the gravity load. Section 3 applies this method in gravity form problem problems for frontfed mesh reflectors. And Section 4 is the summary of the paper.

2. Gravity load deformation algorithm of cable network system

2.1. Compatibility equation

Suppose the cable AB topological connection numbers are \( i \) and \( j \). The original length of the cable AB is \( L_{ij}^0 \). When no external load is applied to the cable network, the node coordinates are \((x_i, y_i, z_i)\) and \((x_j, y_j, z_j)\) respectively, The pretension of cable AB is \( T_{ij}^0 \), then

\[
L_{ij} = \sqrt{x_i^2 + y_i^2 + z_i^2} = L_{ij}^0 + \frac{T_{ij}^0 L_{ij}^0}{EA}
\]

(1)

When applied gravity load, the displacements of node \( i \) and node \( j \) are \((dx_i,dy_i,dz_i)\) and \((dx_j,dy_j,dz_j)\). The length of the cable after deformation \( A'B' \) is

\[
L_{ij}^G = \sqrt{\left(x_i^2 + dx_i^2 + dy_i^2 + dz_i^2\right)^2 + \left(y_i^2 + dx_i^2 + dy_i^2 + dz_i^2\right)^2 + \left(z_i^2 + dx_i^2 + dy_i^2 + dz_i^2\right)^2}
\]

\[
= \sqrt{L_{ij}^0 \left(1 + \frac{A}{L_{ij}^2}\right)}
\]

(2)

Where, \( x_i = x_i - x_j \), \( y_i = y_i - y_j \), \( z_i = z_i - z_j \),

\[
dx_i = dx_i - dx_j, \quad dy_i = dy_i - dy_j, \quad dz_i = dz_i - dz_j
\]

\[
A = dx_i^2 + 2x_i dx_j + dy_i^2 + 2y_i dy_j + dz_i^2 + 2z_i dz_j
\]

According to Taylor series expansion, we can get
Therefore, the change of the rope length is,

\[
\begin{align*}
L_{ij}^G & \approx L_{ij} \left[ 1 + \frac{1}{2} \left( \frac{A}{L_{ij}^2} \right)^2 + \ldots \right] \\
& = L_{ij} + \frac{dx_{ij}^2}{2L_{ij}} + \frac{dy_{ij}^2}{2L_{ij}} + \frac{dz_{ij}^2}{2L_{ij}} + \frac{x_y dx_{ij}}{L_{ij}} + \frac{y_y dy_{ij}}{L_{ij}} + \frac{z_y dz_{ij}}{L_{ij}} - \frac{x_y^2 dx_{ij}^2}{2L_{ij}^3} \\
& \quad - \frac{y_y^2 dy_{ij}^2}{2L_{ij}^3} - \frac{z_y^2 dz_{ij}^2}{2L_{ij}^3} - \frac{x_y y_y dx_{ij} dy_{ij}}{L_{ij}^3} - \frac{x_y z_y dx_{ij} dz_{ij}}{L_{ij}^3} - \frac{y_y z_y dy_{ij} dz_{ij}}{L_{ij}^3} \\
\end{align*}
\]

(3)

Therefore, the change of the rope length is,

\[
\begin{align*}
e &= \frac{dx_{ij}^2}{2L_{ij}} + \frac{dy_{ij}^2}{2L_{ij}} + \frac{dz_{ij}^2}{2L_{ij}} + \frac{x_y dx_{ij}}{L_{ij}} + \frac{y_y dy_{ij}}{L_{ij}} + \frac{z_y dz_{ij}}{L_{ij}} - \frac{x_y^2 dx_{ij}^2}{2L_{ij}^3} \\
& \quad - \frac{y_y^2 dy_{ij}^2}{2L_{ij}^3} - \frac{z_y^2 dz_{ij}^2}{2L_{ij}^3} - \frac{x_y y_y dx_{ij} dy_{ij}}{L_{ij}^3} - \frac{x_y z_y dx_{ij} dz_{ij}}{L_{ij}^3} - \frac{y_y z_y dy_{ij} dz_{ij}}{L_{ij}^3} \\
\end{align*}
\]

(4)

2.2. Equilibrium equations
The expressions of cosine $\cos \theta_x$, $\cos \theta_y$ and $\cos \theta_z$ of the angles between the cable and the axes of the design coordinate system are:

\[
\begin{align*}
\cos \theta_x &= \frac{x_y + dx_{ij}}{L_{ij}^G} = \frac{x_y + dx_{ij}}{L_{ij} + \frac{x_y dx_{ij} + y_y dy_{ij} + z_y dz_{ij}}{L_{ij}^2}} = \frac{x_y + dx_{ij}}{\frac{x_y dx_{ij} + y_y dy_{ij} + z_y dz_{ij}}{L_{ij}^2}} + \ldots \\
& \approx \left( \frac{x_y + dx_{ij}}{L_{ij}^G} \right) \left( 1 - \frac{x_y dx_{ij} + y_y dy_{ij} + z_y dz_{ij}}{L_{ij}^2} \right) + \ldots \\
& \approx \frac{x_y + dx_{ij}}{L_{ij}^G} - \frac{x_y^2 dx_{ij} + x_y y_y dy_{ij} + x_y z_y dz_{ij}}{L_{ij}^3} \\
& \approx \frac{x_y + dx_{ij}}{L_{ij}} - \frac{x_y^2 dx_{ij} + x_y y_y dy_{ij} + x_y z_y dz_{ij}}{L_{ij}^3} \\
\end{align*}
\]

(5)

\[
\begin{align*}
\cos \theta_y &= \frac{y_y + dy_{ij}}{L_{ij}^G} = \frac{y_y + dy_{ij}}{L_{ij} + \frac{x_y dx_{ij} + y_y dy_{ij} + z_y dz_{ij}}{L_{ij}^2}} = \frac{y_y + dy_{ij}}{\frac{x_y dx_{ij} + y_y dy_{ij} + z_y dz_{ij}}{L_{ij}^2}} + \ldots \\
& \approx \left( \frac{y_y + dy_{ij}}{L_{ij}^G} \right) \left( 1 - \frac{x_y dx_{ij} + y_y dy_{ij} + z_y dz_{ij}}{L_{ij}^2} \right) + \ldots \\
& \approx \frac{y_y + dy_{ij}}{L_{ij}^G} - \frac{y_y^2 dy_{ij} + y_y y_y dy_{ij} + y_y z_y dz_{ij}}{L_{ij}^3} \\
& \approx \frac{y_y + dy_{ij}}{L_{ij}} - \frac{y_y^2 dy_{ij} + y_y y_y dy_{ij} + y_y z_y dz_{ij}}{L_{ij}^3} \\
\end{align*}
\]

(6)

\[
\begin{align*}
\cos \theta_z &= \frac{z_y + dz_{ij}}{L_{ij}^G} = \frac{z_y + dz_{ij}}{L_{ij} + \frac{x_y dx_{ij} + y_y dy_{ij} + z_y dz_{ij}}{L_{ij}^2}} = \frac{z_y + dz_{ij}}{\frac{x_y dx_{ij} + y_y dy_{ij} + z_y dz_{ij}}{L_{ij}^2}} + \ldots \\
& \approx \left( \frac{z_y + dz_{ij}}{L_{ij}^G} \right) \left( 1 - \frac{x_y dx_{ij} + y_y dy_{ij} + z_y dz_{ij}}{L_{ij}^2} \right) + \ldots \\
& \approx \frac{z_y + dz_{ij}}{L_{ij}^G} - \frac{x_y z_y dx_{ij} + y_y z_y dy_{ij} + z_y^2 dz_{ij}}{L_{ij}^3} \\
& \approx \frac{z_y + dz_{ij}}{L_{ij}} - \frac{x_y z_y dx_{ij} + y_y z_y dy_{ij} + z_y^2 dz_{ij}}{L_{ij}^3} \\
\end{align*}
\]

(7)

Since the resultant force of the three directions of the nodes is 0. The force balance equation of the structure is
2.3. constitutive relationship
The relationship between cable elongation \( e \) (caused by gravity load) and rope tension change \( \Delta T \) is,

\[
\Delta T = \frac{EA}{L_0} e
\]

(9)

In Eqn. 9, \( \Delta T \) is a nonlinear function of nodes displacement, so the equation is a typical nonlinear equation, and the matrix form can be expressed as,

\[
\psi^e(X^e) = D^e(dX^e) - P^e = 0
\]

(10)

Where the element of matrix \( D^e(X^e) \) is the nonlinear function of the displacement of the cable nodes, \( dX^e = [dx_i, dy_i, dz_i, dx_j, dy_j, dz_j]^T \) is the cable nodes displacement vector, and \( P^e \) is the external load vector of the node. Eqn. 10 can be solved by the Newton Iteration Method, and its Jacobian matrix is

\[
J(dX^e) = \frac{\partial \psi^e}{\partial dX^e} = \begin{bmatrix}
\frac{\partial \psi^e_1}{dx_i} & \frac{\partial \psi^e_1}{dy_i} & \cdots & \frac{\partial \psi^e_1}{dz_j} \\
\frac{\partial \psi^e_2}{dx_i} & \frac{\partial \psi^e_2}{dy_i} & \cdots & \frac{\partial \psi^e_2}{dz_j} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial \psi^e_{ij}}{dx_i} & \frac{\partial \psi^e_{ij}}{dy_i} & \cdots & \frac{\partial \psi^e_{ij}}{dz_j}
\end{bmatrix}
\]

(11)

Displacement of the nodes of the initial iterative is

\[
dX^e = [dx, dy, dz, dx, dy, dz]^T = [0, 0, 0, 0, 0, 0]^T
\]

(12)

Then the Jacobian matrix of the initial iteration is,

\[
J_0(dX^e) = EA \cdot L_0 \cdot L^2
\]

(13)

The nonlinear equation of the overall cable network structure is

\[
\psi(dX) = D(dX) - P = 0
\]

(14)
According to the principle of superposition of finite element method, the Jacobian matrix of cable network structure is

\[ J(dX) = \frac{\partial \psi}{\partial dX} = \sum_{i,j \in C} \frac{\partial \psi^e}{\partial dX^e} \]

Where \( C \) is the set of connection topologies of the cable-truss system. Since the Newton method recalculates the Jacobian matrix accordingly after each iteration, the calculation cost of this process is relatively large for cable-truss antennas with a large number of cables and trusses. In order to improve the calculation efficiency, you can use the initial instead of the Jacobian matrix of the \( n \)-th iteration, this is the simplified Newton method. The initial Jacobian matrix can be used to replace the Jacobian matrix of the \( n \)-th iteration, which is the simplified Newton method.

3. Gravity load deformation analysis example of Mesh Reflectors

The case in this article takes the result of geometric method form finding\(^6\) as input to calculate the gravitational load deformation. The frontfed mesh reflectors has a circular aperture of \( D = 12 \) m and a focal length of \( f = 7.5 \) m. The reflection surface is meshed with 30 border nodes and 121 inner nodes on the paraboloid. In engineerin, in order to reduce the height of the cable net antenna, the arch height of the rear cable network is usually reduced, as shown in Fig.2, the front and rear cable network are not completely symmetrical. The commonly method to reduce the arch height of the rear cable network is to extend the focal length of the paraboloid of the rear cable network. In this example, the arch-to-height ratio of the front and rear cable network is 4, that is, the rear cable network is a standard parabola with a focal length of 30m. The same geometric method is used to calculate the balance pretension of rear cable network, the front and rear network balance pretension is shown in the Fig.3.

![Fig.2 the front and rear cable network](image)

![Fig.3 the front and rear network balance pretension](image)

Usually, different types of cables are used in the front and rear cable networks, so that the stiffness of the two nets along the Z direction is basically the same and the nodes displacement of front and rear cable networks are approximately the same under the gravity load. Assume that the cable net in this article uses carbon fiber cable with a modulus of elasticity of 20Gpa. However, the diameter of the front network cables is 1mm and the rear network cable is 2mm. Both ring truss and vertical truss are made
of carbon fiber tube with diameter of 30mm and wall thickness of 1mm, and the modulus of elasticity is 130Gpa.

The cable network form-finding design is the tension in the orbit without gravity, and the original length of the rope is calculated by substituting the above material parameters. The original length of the cables is calculated by substituting the above material parameters. The simplified Newton Iterative Method is used to solve the cable net deformation under gravity load, and the cable tension of form finding design is used as the initial iterative tension. The relationship between the number of iterations and the maximum resultant force of cable net nodes is shown in Fig.4. The iteration termination condition is that the maximum resultant force of nodes is $10^{-6}$N. Solve the displacements of the nodes when the reflector port of the front network is placed upward under the gravity load. Take the ring truss plane of the front network as the $xy$ plane, the ring center as the origin, the line connecting the ring center and the 122 nodes as the positive x direction, and the direction of the aperture surface perpendicular to the focal length is the z axis to establish a Cartesian coordinates. The displacement of the front cable network nodes under gravity load is shown in Fig. 5. The displacement in the $xy$ direction of the node is small, less than ±1mm, and the displacement in the $z$ direction of the node is larger, the average displacement of the node the $z$ direction is 4mm, and the maximum displacement reaches 5.35mm.

![Fig.4 convergence of Simplified Newton Iteration method](image1)

- (a) x-direction displacement
- (b) y-direction displacement
4. Conclusions
In this paper, by analyzing the geometric nonlinear geometrical nonlinear characteristics of the space cable truss system, the mechanical nonlinear model of deployable cable truss antenna under gravity load is established. The simplified Newton iterative algorithm is used to solve the displacement of the node. For the frontfed cable network form-finding design case, the displacement of the nodes is solved when the front cable network is placed upward under the gravity load. After calculation, under gravity load mainly along the z direction, and the displacement in the xy direction of the node is small. This article provides a method for solving the deformation of the cable-truss system subjected to external force loads.

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