Distributed Consensus in Multi-agent Nonholonomic Systems Without Position Measurement

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Abstract. A novel consensus control protocol for multi-agent nonholonomic systems has been proposed in this paper. With the aid of non-smooth analysis and algebraic graph theory, the proposed linear protocol can ensure leader-follower results and three flocking rules without position measurement. Besides, it is proved that there exist a range of control gain to preserve the connectedness of the communication network and realize consensus. Finally, some numerical simulations are given to validate the above theoretical results.

1. Introduction

Recently, much attention has been attracted to the study of multi-agent systems due to its wide applications in the real-world [1]-[4]. Flocking behaviors are characterized by distributed control, local interactions and self-organization. Considerable efforts have been made for a better understanding of how to design suitable distributed strategies to perform complex tasks without centralized control.

In 1980s, Reynolds firstly simulated behavior of birds swarm though computer and proposed three rules for flocking: (1) cohesion; (2) separation; (3) alignment [5]. From then on, Reynolds’ three rules have been used to study flocking motion and distributed control[6]. Olfati-Saber[7] suggested that an additional feedback term to track a virtual leader was necessary to avoid fragmentation for a generic set of initial states. Using fully distributed consensus protocols[8], an adaptive controller were designed for the linear multi-agent systems with directed graph. As in [9], a distributed consensus algorithm using partial velocity measurement was proposed for second-order multi-agent systems.

However, most agents such as UAV and autonomous vehicle [10]-[12] are moving in 3D space. Therefore, it is necessary to simulate the behavior of the actual agent by using 3D nonholonomic model which is more closed to real world. In addition, connectivity preservation and collision avoidance as two important specifications of flocking, are less considered in the exist literature. Thus, it is more practical to design a control protocol to guarantee the connectivity of the network between agents.

In this paper, a distributed consensus protocol without position information for multiply three dimension nonholonomic agents is designed via the proximity graph. On the one hand, the 3D nonholonomic nonlinear model is analyzed and considered, which is more closed to real world. On the other hand, the proposed distributed consensus protocol without position information can ensure the three flocking rules and the connectedness of the graph.

2. Preliminary knowledge

Consider the following multi-agent system composed of $n$ nonholonomic agents which are moving in the 3D space. Let the position and forward velocity of each agent at time $t$ be
\((x_i(t), y_i(t), z_i(t))\) and \(v_i(t)\), \((i = 1, 2, \ldots, n)\). Furthermore, as shown in Fig.1, the attitude of agent \(i\) can be defined by \(\theta_i(t)\) (pitch angle with \(-\pi/2 < \theta_i(t) < \pi/2\)) and \(\varphi_i(t)\) (yaw angle with \(-\pi < \varphi_i(t) < \pi\)). Therefore, the mathematical model of the \(i^{th}\) nonholonomic agent can be defined by [14]

\[
\dot{q}_i = \begin{bmatrix}
  \dot{x}_i \\
  \dot{y}_i \\
  \dot{z}_i
\end{bmatrix} = \begin{bmatrix}
  v_i \cos \theta_i \cos \varphi_i \\
  v_i \cos \theta_i \sin \varphi_i \\
  v_i \sin \theta_i
\end{bmatrix} = f(v_i, \varphi_i, \theta_i)
\]

where \(f(v_i, \varphi_i, \theta_i)\) is the nonlinear dynamics of the model while \(u_i = (u_{i1}, u_{i2}, u_{i3})\) denotes the input.

**Figure 1.** Geometric interpretation of the nonholonomic model

The neighbor of agent \(i\) are those which lie in a ball of radius \(r > 0\) centered at \((x_i(t), y_i(t), z_i(t))\).

Let \(q = (q_1, q_2, \ldots, q_n)\) be the position of the whole system. Besides, the sensing radius is \(r\) while the distance between agents \(i\) and \(j\) is described by \(d_{ij} = \|q_i - q_j\|\). Thus, the neighbor set of agent \(i\) is defined as follows

\[
N_i(t) = \{j \neq i | d_{ij}(t) < r\}
\]

the relationship of the nearby agents can be described by an undirected graph \(\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})\), where the vertex set \(\mathcal{V} = \{1, 2, \ldots, n\}\) is the agents, then, the edge set can be defined by

\[
\mathcal{E} = \{(i, j) \subset \mathcal{V} | d_{ij} < r, i \neq j\}
\]

The weight adjacency matrix of \(\mathcal{G}\) is defined by \(\mathcal{A} = [a_{ij}]_{n \times n} \in \mathbb{R}^{n \times n}\), where \(a_{ii} = 0\) and \(a_{ij} = a_{ji} \geq 0\) for \(i \neq j\). Moreover, \(a_{ij} = a_{ji} > 0\) if and only if there exists a link between agent \(i\) and agent \(j\). The degree matrix of \(\mathcal{G}\) is \(\mathcal{D} = \text{diag}\{d_1, d_2, \ldots, d_n\} \in \mathbb{R}^{n \times n}\), where \(d_i = \sum_{j=1}^{n} a_{ij}\) for \(i = 1, 2, \ldots, n\). Then, the Laplacian matrix of \(\mathcal{G}\) is defined as \(\mathcal{L} = \mathcal{D} - \mathcal{A}\).

The eigenvalues of Laplacian matrix \(\mathcal{L}\) can be defined by

\[
0 = \lambda_1(\mathcal{L}) \leq \lambda_2(\mathcal{L}) \leq \ldots \leq \lambda_n(\mathcal{L})
\]

\(\lambda_2(\mathcal{L})\) is usually called algebraic connectivity[13] and the graph \(\mathcal{G}\) is said to be connected if and only if \(\lambda_2(\mathcal{L})\) is positive.

**Assumption 1:** We assume that the virtual leader’s states, which are called the desired states are bounded and available to all of agents. Let \(v_r\) be the velocity of the virtual leader while \(\theta_r\) and \(\varphi_r\) are orientations respectively.

Under the assumption and above description, the consensus control of multi-agent nonholonomic systems considered in this paper is defined as follows:
Definition 1: Under the initial connected graph $\mathcal{G}(0)$, the consensus control problem for the above 3D nonholonomic system (1) is to design controller $u_i$ with local information such that
\[
\lim_{t \to \infty} (v_i(t) - v_j) = 0, \lim_{t \to \infty} (\gamma_i(t) - \gamma_j) = 0, \lim_{t \to \infty} (\theta_i(t) - \theta_j) = 0
\]
(5)
Besides, if no agents collide at the initial time, i.e., $d_{ij}(0) > r_0$ for $\forall i \neq j$, then $d_{ij}(t) > r_0$ for all $t \geq 0$, where $r_0$ is the minimum threshold between agents.

Remark 1: In this paper, we assume that the communication network described by the graph $\mathcal{G}$ can only transmit the neighbors’ velocity, pitch and yaw angles. Thus, agent $i$ can only use above information and the states of itself instead of positions and relative distances. The problem of connectivity preservation and collision avoidance will be discussed in the next section.

3. Main Results
In this section, motivated by the results of [15]-[17], we discuss the consensus problem of multi-agent systems (1). Some typical conditions will be given to ensure that the velocities and orientations of all agents exponentially converge to the desired states, respectively, while ensuring there are no collision between agents.

Before beginning the main results of this paper, some lemmas are introduced as follows.
Lemma 1: [14] For any $\varphi_i \in [-\pi/2, \pi/2], \varphi_j \in [-\pi, \pi], i \in V$ in the systems (1)
\[
\begin{bmatrix}
\cos \theta_i \cos \varphi_i - \cos \theta_j \cos \varphi_j \\
\cos \theta_i \sin \varphi_i - \cos \theta_j \sin \varphi_j \\
\sin \theta_i - \sin \theta_j
\end{bmatrix}
\leq \frac{\sqrt{2} + 2}{2} \left(|\theta_i - \theta_j| + |\varphi_i - \varphi_j|\right)
\]
(6)
where $\|\|$ denotes Euclidean norm of a vector.

Lemma 2: If $\mathcal{G}$ is a connected graph with order $n$, for any $\xi, \eta \in \mathbb{R}^n$ satisfying $1_n^T \xi = 0$,
\[
\lambda_2(\mathcal{L}) \xi^T \xi \leq \xi^T \mathcal{L} \xi \leq \lambda_n(\mathcal{L}) \xi^T \xi
\]
(7)

Lemma 3: For graph $\mathcal{G}_i$ with $n$ order, $i = 1, 2, \ldots, n$ while $\mathcal{L}_i$ is their Laplacian matrices, respectively. If $\mathcal{G}_i \subset \mathcal{G}_j$, then $\lambda_k(\mathcal{G}_i) \leq \lambda_k(\mathcal{G}_j)$ for $k = 1, 2, \ldots, n$.

The general consensus control for multi-agent nonholonomic systems is to design a distributed consensus protocol which use the full state information or the potential function. However, considering that some state information such as position information may not accurate or available, we design a novel consensus control protocol to solve the consensus problem of systems (1) as follows:
\[
\begin{aligned}
u_{ii} &= -k \sum_{j \in N_i} a_{ij} (v_i(t) - v_j(t)) - l(v_i(t) - v_j) \\
u_{ij} &= -k \sum_{j \in N_i} a_{ij} (\theta_i(t) - \theta_j(t)) - l(\theta_i(t) - \theta_j) \\
u_{ji} &= -k \sum_{j \in N_i} a_{ij} (\varphi_i(t) - \varphi_j(t)) - l(\varphi_i(t) - \varphi_j)
\end{aligned}
\]
(8)
where $k, l > 0$. Thus, the closed-loop system with control protocol (8) is a nonlinear multi-agent nonholonomic system described by
\[
\begin{align*}
\dot{x}_i &= \left[ \begin{array}{c} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \\
\end{array} \right] = \left[ \begin{array}{c} v_i \cos \theta_i \cos \phi_i \\ v_i \cos \theta_i \sin \phi_i \\ v_i \sin \theta_i \\
\end{array} \right] \triangleq f(v_i, \theta_i, \phi_i) \\
\dot{v}_i &= -k \sum_{j \in N_i} a_{ij} \left( v_j(t) - v_i(t) \right) - l \left( v_i(t) - v_r \right) \\
\dot{\theta}_i &= -k \sum_{j \in N_i} a_{ij} \left( \theta_j(t) - \theta_i(t) \right) - l \left( \theta_i(t) - \theta_r \right) \\
\dot{\phi}_i &= -k \sum_{j \in N_i} a_{ij} \left( \phi_j(t) - \phi_i(t) \right) - l \left( \phi_i(t) - \phi_r \right)
\end{align*}
\]

(9)

Let \( v = (v_1, v_2, \ldots, v_n) \), \( \theta = (\theta_1, \theta_2, \ldots, \theta_n) \) and \( \phi = (\phi_1, \phi_2, \ldots, \phi_n) \), then, denote

\[
\delta_v = v - 1_n \otimes v, \delta_\theta = \theta - 1_n \otimes \theta, \delta_\phi = \phi - 1_n \otimes \phi
\]

(10)

where \( \otimes \) denotes the Kronecker product.

**Theorem 1:** For multi-agent nonholonomic system (9) with distributed consensus protocol, it is supposed that the initial communication network \( \mathcal{G}(0) \) is connected and there are no collision between agents, namely, the algebraic connectivity \( \mu = \lambda_2(\mathcal{G}(0)) > 0 \) and \( d_j(0) > r_0 \) for all \( i \neq j \).

If

\[
(k \mu + l) > \frac{1}{d} \left( 1 + \sqrt{2} \right) \left( \| \delta_v (0) \| + v \left( \| \delta_\theta (0) \| + \| \delta_\phi (0) \| \right) + \sqrt{2} \| \delta s (0) \| \right)
\]

(11)

where \( d = \min \{ \sigma_1, \sigma_2 \} \), \( \sigma_1 = r - \max_{i,j \in \mathcal{E}_0} d_{ij}(0) \), \( \sigma_2 = \min_{i,j \in \mathcal{E}_0} d_{ij}(0) - r_0 \). Then the consensus problem of multi-agent non-holonomic systems (1) can be solved.

Before the beginning of the proof of above Theorem, some lemma are necessary to be introduced and analyzed.

**Lemma 4:** For closed-loop system (9), if the initial communication network \( \mathcal{G}(0) \) is connected, then there is a constant \( 0 < h \leq \infty \) such that

\[
\mathcal{G}(t) = \mathcal{G}(0), \| \delta_v (t) \| \leq e^{-(k \mu + l) t} \| \delta_v (0) \|, s = v, \theta, \phi
\]

(12)

for all \( t \in [0, h) \), \( \mu = \lambda_2(\mathcal{G}(0)) \).

**Proof:** Because of the system (9) is a smooth continuous system, then, there must exist a continuous solution for any giving initial. Thus, there must be a constant \( 0 < h \leq \infty \) such that \( \mathcal{G}(t) = \mathcal{G}(0) \) for \( t \in [0, h] \). Furthermore, it follows from (9) that

\[
\dot{\delta}_v = -k L(t) \delta_v (t) - l \delta_v (t), s = v, \theta, \phi
\]

(13)

Consider the following Lyapunov function candidates:

\[
V(\delta_v) = \frac{1}{2} \delta_v^T \delta_v, s = v, \theta, \phi
\]

(14)

Due to \( \mathcal{G}(t) = \mathcal{G}(0) \) for \( t \in [0, h) \), it follows from lemma 2 that

\[
\dot{V}(\delta_v) = -k \delta_v^T L(t) \delta_v - \delta_v^T l \delta_v \leq -k \delta_v^T \lambda_2(\mathcal{G}(0)) \delta_v - \delta_v^T l \delta_v = -2(k \mu + l) V(\delta_v)
\]

(15)

Thus, we can get

\[
V(\delta_v) \leq e^{-2(k \mu + l) t} V(\delta_v (0))\]

(16)

which implies inequality (12) with (14). This completes the proof.
Lemma 5: For nonlinear function \( f(v_i, \theta_i, \varphi_i) \), it follows that
\[
\| f(v_i, \theta_i, \varphi_i) - f(v_j, \theta_j, \varphi_j) \| \leq |v_i - v_j| + \frac{\sqrt{2} + 2}{2} \left[ (|\theta_i - \theta_j| + |\varphi_i - \varphi_j|) (\|\delta_i(0)\| + v_r) \right] \tag{17}
\]

Proof: It follows from (12) that \( \|\delta_i(t)\| \leq e^{-(k\mu t)} \|\delta_i(0)\| \) for \( t \geq 0 \). Thus, we can get
\[
|v_i - v_j| \leq |v_i - v_r| + |v_j - v_r| \leq \sqrt{2} (|v_i - v_r|^2 + (v_j - v_r)^2) \leq \sqrt{2} \|\delta_i(t)\| \leq \sqrt{2} e^{-(k\mu t)} \|\delta_i(0)\| \tag{18}
\]
Based on lemma 1, we can obtain
\[
\| f(v_i, \theta_i, \varphi_i) - f(v_j, \theta_j, \varphi_j) \| \leq |v_i - v_j| + \frac{\sqrt{2} + 2}{2} v_r (|\theta_i - \theta_j| + |\varphi_i - \varphi_j|) \tag{19}
\]
where \( s. \) represents \( \sin(\cdot) \) while \( c. \) represents \( \cos(\cdot) \). Furthermore, combined with (18), we can get
\[
v_i (|\theta_i - \theta_j| + |\varphi_i - \varphi_j|) = (v_i - v_r) (|\theta_i - \theta_j| + |\varphi_i - \varphi_j|) + v_i (|\theta_i - \theta_j| + |\varphi_i - \varphi_j|) \leq \|\delta_i(t)\| (|\theta_i - \theta_j| + |\varphi_i - \varphi_j|) + v_r (|\theta_i - \theta_j| + |\varphi_i - \varphi_j|) \tag{20}
\]
therefore, inequality (17) is established. This completes the proof.

Proof of Theorem 1: First, based on Lemma 5, if \( h = \infty \), the graph \( \mathcal{G}(t) = \mathcal{G}(0) \) for all \( t \geq 0 \).

Otherwise, there must exist a finite time \( t_1 \geq h \) such that
\[
\mathcal{G}(t) = \mathcal{G}(0), t \in [0, t_1), \mathcal{G}(t_1) \neq \mathcal{G}(0) \tag{21}
\]
Besides, we suppose that there exists two agents \( (i_1, j_1) \in \mathcal{E}(0), (i_2, j_2) \in \mathcal{E}(t_1) \), which implies \( r_0 < d_{ij}(0) < r, d_{ij}(t_1) \geq r \). For close-loop system (9), consider \( d_{ij}^2 = q_{ij}^2, q_{ij} = q_i - q_j \), based on Lemma 5, we can get
\[
\dot{d}_{ij} = \left[ x_i - x_j, y_i - y_j, z_i - z_j \right] \left[ v_i c.\theta_i c.\varphi_i - v_j c.\theta_j c.\varphi_j \right. \left. v_i c.\theta_i s.\varphi_i - v_j c.\theta_j s.\varphi_j \right. \left. v_i s.\theta_i - v_j s.\theta_j \right] \nonumber \leq \| f(v_i, \theta_i, \varphi_i) - f(v_j, \theta_j, \varphi_j) \| \tag{22}
\]
\[
\leq |v_i - v_j| + \frac{\sqrt{2} + 2}{2} v_r (e^{-(k\mu t_1)} \|\delta_i(0)\| + v_r) (|\theta_i - \theta_j| + |\varphi_i - \varphi_j|) \leq \sqrt{2} \|\delta_i(t)\| + (1 + \sqrt{2}) (\|\delta_{ij}(0)\| + v_r) (\|\delta_{ij}(t)\| + \|\delta_{ij}(0)\|)
\]
Let $\gamma(t) = \sqrt{2} \| \dot{x}_i(t) \| + \left( 1 + \sqrt{2} \right) \left( \| \dot{x}_i(0) \| + v_r \right) \left( \| \dot{\theta}_i(t) \| + \| \dot{\varphi}_i(t) \| \right)$, based on Lemma 4 and (22), due to $G(t) = G(0), t \in [0, t_1)$, we have
\[
d_{i,h}(t) = d_{i,h}(0) + \int_0^t \dot{d}_y(s) \, ds \\
\leq d_{i,h}(0) + \int_0^t \gamma(s) \, ds \\
\leq d_{i,h}(0) + \int_0^t \left[ \gamma(0) \right] e^{-(k\mu+\ell)} \, ds \\
\leq d_{i,h}(0) + \frac{1}{k\mu+\ell} \left[ \gamma(0) \right]
\] (23)
which is contradictory with (11). Thus, there is no link missing at time $t_1$, and $G(t_1) \supseteq G(0)$, since the node number of whole graph is finite, so is edge number. Thus, there much exist a finite integer $m$ such that
\[
G(t) = G(0), t \in [0, t_1), \\
G(t) = G(t_j) \supseteq G(t_{j-1}), t \in [t_j, t_{j-1}), j = 1, \ldots, m-1 \\
G(t) = G(t_m) \supseteq G(t_{m-1}), t \in [t_m, \infty)
\] (24)
which implied that the connectivity of the communication network is maintained for all time.

Based on lemma 4 and the connectivity preservation of the graph, we can obtain (12) which implies that the velocities and angles of each agents exponentially converge to the states of desired states.

Finally, we prove that there are no collision between agents via contradiction. Assume that there exist a collision between agents $i_0$ and $j_0$ at the time $t_0 > 0$, namely, $d_{i_0,j_0}(t_0) = r_0$. Hence, it is easy to obtain that
\[
d_{i_0,j_0}(t_0) = d_{i_0,j_0}(0) + \int_0^{t_0} \dot{d}_{i_0,j_0}(s) \, ds
\] (25)
It follows from (12), (22), and (25) that
\[
\min_{i,j} d_{i,j}(0) - r_0 \leq \int_0^{t_0} \dot{d}_{i_0,j_0}(s) \, ds \leq \int_0^{t_0} \gamma(s) \, ds \leq \int_0^{t_0} \left[ \gamma(0) \right] e^{-(k\mu+\ell)} \, ds \leq \frac{1}{k\mu+\ell} \left[ \gamma(0) \right]
\] (26)
which clearly contradics (11). Therefore, Theorem 1 is proved.

4. Numerical simulation

We consider a 6 agents system with one virtual leader in the 3D space. We set the communication topology radius of agents $r = 40$ and the threshold for collision avoidance $r_0 = 5$. We also assume that the desired reference state is $(v_r, \theta_r, \varphi_r) = (11.3, 0, 0)$. The initial states information of 5 agents are as follows:
\[
v(0) = (15.8, 10.15, 12.16), \quad \theta(0) = (0.3, 0.9, -1.1, 0.4, -0.7, -0.8), \quad q(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -30 & -15 & 15 & 15 & -15 \\ 0 & 25.5 & 25.5 & -25.5 & 25.5 \end{bmatrix}
\]
\[
\varphi(0) = (0.8, -1.1, 0.2, 0.3, -0.7, -0.9), \quad q(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -30 & -15 & 15 & 15 & -15 \\ 0 & 25.5 & 25.5 & -25.5 & 25.5 \end{bmatrix}
\]

Firstly, the initial position of 6 agents are depicted in Figure 2(a), which implies that the initial communication network $G(0)$ is connected with $\mu = 1$. According to (11) in Theorem 1, we get
Thus, we should make \( k = 1, l = 2 \) to address the consensus control problem for 6 nonholonomic agents in 3D space.

As shown in Figure 2 (a) and Figure 2 (b), the states of 6 agents (blue circle) converge quickly to that of virtual leader (red triangle) and the connectivity of communication network is always maintained. Fig. 2 (c) shows the minimum distance between agents, which implies that collision avoidance between agents can be always guaranteed. Fig. 2 (d)-(f) show that the velocity, pitch and yaw angle of each agent achieve consensus to the virtual leader as desired, respectively, which verifies the effectiveness of the theoretical results.

5. **Conclusion**

This paper has investigated the consensus problem for multi-agent nonholonomic systems. Though the theoretical analysis, it is proved that there are always distributed consensus protocols with only local velocities, pitch, and yaw angles information to realize consensus when the initial communication network is connected and no collision occur in the beginning. Finally, several numerical examples have been given to verify the obtained theoretical results. In the future work, time delay in the network of nonholonomic systems will be considered.

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