Long-range percolation
on the hierarchical lattice

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Abstract

We study long-range percolation on the hierarchical lattice of order $N$, where any edge of length $k$ is present with probability $p_k = 1 - \exp(-\beta^{-k} \alpha)$, independently of all other edges. For fixed $\beta$, we show that the critical value $\alpha_c(\beta)$ is non-trivial if and only if $N < \beta < N^2$. Furthermore, we show uniqueness of the infinite component and continuity of the percolation probability and of $\alpha_c(\beta)$ as a function of $\beta$. This means that the phase diagram of this model is well understood.