A NEW METHOD FOR MEASURING WEAK GRAVITATIONAL LENSING SHEAR USING HIGHER ORDER SPIN-2 HOLICs

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ABSTRACT

We investigate the possibility of using higher order moments of gravitational lensed images in the weak-lensing analysis. For this purpose, we employ spin-2 component of HOLICs (Higher Order Lensing Image’s Characteristics) developed by us. We test the weak-lensing analysis with spin-2 HOLICs using actual, ground-based Subaru observations of the massive galaxy cluster A1689 (z = 0.183). It turns out that spin-2 HOLICs of order up to 8 are sufficiently applicable for weak-lensing analysis after correcting point-spread function (PSF) anisotropy as well as isotropic PSF smearing.

Key words: cosmology: theory – dark matter – galaxies: clusters: individual (A1689) – gravitational lensing

1. INTRODUCTION

Weak gravitational lensing is now regarded as a powerful tool to reveal the distribution of dark matter of clusters of galaxies and large-scale structure. It is also useful for measuring the equation-of-state parameter of dark energy by observing cosmic shear.

In the usual treatment of the weak-lensing analysis, the quadrupole moment of background galaxy images is used to quantify the image ellipticity. In recent years, there have been theoretical efforts to include the next-order distortion effects as well as the usual quadrupole distortion effects in the weak-lensing analysis (Goldberg & Natarajan 2002; Goldberg & Bacon 2005; Bacon et al. 2006; Irwin & Shmakova 2006; Goldberg & Leonard 2007). We propose to use some combinations of octopole/higher multipole moments of background images with definite spin properties, which we call the Higher Order Lensing Image’s Characteristics (HOLICs), and show that simple relations exist between HOLICs with spin-1 (spin-3) and the third-order weak-lensing effects, or gravitational flexion with spin-1 (spin-3) (Goldberg & Bacon 2005). Thus, HOLICs serve as a direct measure for the flexion (Okura et al. 2007; hereafter OUF2007). We have also developed a realistic method for measuring flexion by the HOLICs approach by fully extending the KSB formalism to take into account higher order point-spread function (PSF) anisotropy as well as isotropic PSF smearing (Okura et al. 2008; hereafter OUF2008). The method has been applied to actual, ground-based Subaru observation of A1689 and has been able to obtain a bimodal feature in the central region of the cluster, which is not seen in the usual quadrupole weak-lensing analysis.

In this paper, we consider a possibility of applying the HOLICs method to measure shear, particularly cosmic shear. Weak-lensing shear analysis has suffered from various sort of noises. Here, we concern ourselves with the intrinsic noise, which has nothing to do with the observational condition and the method of shape measurement. The intrinsic noise can be reduced by averaging a sufficient number of background images. Since cosmic shear induces at most of the order of 1% distortion, it is absolutely necessary to reduce any source of noises. Thus, not only a large number density of background images but also a wide area of survey is necessary for the cosmic shear analysis, which is not always possible in the actual observation. Therefore, it would be very useful to have a method to reduce the intrinsic noise. We will try to do this by using information of high order, non-dimensional, and spin-2 shapes, which are called “spin-2 HOLICs.” The shear distortion has no dimension and is spin-2. Thus, the shear strongly affects not only complex ellipticity but also spin-2 HOLICs. From the differences of inner and outer regions of image, spin-2 HOLICs of different orders have different intrinsic noises, at least partially. In this way, the intrinsic noise can be reduced by averaging spin-2 HOLICs of different orders. In this paper, we define spin-2 HOLICs of various orders, and calculate the reduced shear and PSF correction. Then we evaluate the relative precision of spin-2 HOLICs of different orders by testing them using STEP 1 and find that spin-2 HOLICs up to order 8 (which is a combination of 2\textsuperscript{8} multipole moments of shape) are useful in the shear analysis. Finally, we apply the method to Abell 1689 cluster.

2. BASIS OF WEAK LENSING

The gravitational deflection of light rays can be described by the lens equation

$$\beta = \theta - \nabla \psi(\theta),$$

where $\theta$ and $\beta$ are the angular positions of the image and source, respectively, and $\psi(\theta)$ is the effective lensing potential, which is defined by the two-dimensional Poisson equation as $\nabla^2 \psi(\theta) = 2\kappa(\theta)$, with the lensing convergence. Here, the convergence $\kappa = \Sigma_{m} \Sigma_{\text{crit}}^{-1}$ is the dimensionless surface mass density projected on the sky, normalized with respect to the critical surface mass density of gravitational lensing $\Sigma_{\text{crit}} = (c^2 D_s)/(4\pi G D_d D_{ds})$, where $D_d$, $D_s$, and $D_{ds}$ are the angular diameter distances from the observer to the deflector, from the observer to the source, and from the deflector to the source, respectively. By introducing the complex gradient operator, $\partial = \partial_1 + i \partial_2$, that transforms as a vector, $\partial' = \partial e^{i\phi}$, with $\phi$ being the angle of rotation, the lens equation can be expressed as

$$\beta = \theta - \partial' \psi,$$

and the lensing convergence $\kappa$ is expressed as

$$\kappa = \frac{1}{2} \partial \partial^* \psi,$$

where * denotes the complex conjugate. Similarly, the complex gravitational shear of spin-2 is defined as

$$\gamma \equiv \gamma_1 + i\gamma_2 = \frac{1}{2} \partial \partial^* \psi.$$
Note that a quantity is said to have spin-$s$ if it has the same value after rotation by $2\pi/s$.

3. HOLICs

HOLICs are introduced by us (OUF2007) as particular combinations with a definite spin of multipole moments of images. Here, we briefly review the HOLICs formalism. It is useful to use complex moment instead of usual moment of images to define HOLICs (OUF2008).

First, we define complex displacement as

$$X_1^N = d\theta_1 + i d\theta_2,$$

$$X_M^N = (d\theta_1 + i d\theta_2)^{(N+M)/2} (d\theta_1 - i d\theta_2)^{(N-M)/2},$$

where $d\theta_i$ is displacement vector from the image center and $N$ and $M$ are order of $d\theta$ and the spin number, respectively. Similarly, we define $Y_M^{(s)}$ as a complex displacement of the source image defined by the displacement vector $(d\beta_i)$ from the source center.

Complex moments of images having brightness distribution $I(\theta)$ are defined as follows:

$$Z_M^N = \int d^2\theta X_M^N q[I(\theta)].$$

where $q[I(\theta)]$ is an appropriate weight function and is taken, for simplicity, as the brightness distribution $I(\theta)$. In this notation, the spin-2 HOLICs of order $N$ is defined as $Z_2^N/Z_0^N$ ($N$ is an even number). For example, $N = 2$ is complex ellipticity $\chi$, and $N = 4$ is $\eta$, and $N = 6$ is $\upsilon_{II}$ according to the notation in OUF2008. Similarly, we define $Z_M^{N(s)}$ as complex moments of the source image using $d\beta_i$ and $Y_M^{(s)}$.

In the shear dominant field, the local expanded lens equation

$$d\beta = d\theta - \frac{1}{2} (d\theta^* \delta + d\theta \delta^*) \partial \psi$$

is expressed using the complex moments as

$$Y_1^N = (1 - \kappa) (X_1^N - g X_1^{N*}),$$

where $g$ is reduced shear $\gamma/(1 - \kappa)$. Using the above form of the lens equation we have the following simple relation between
the intrinsic and the observed spin-2 HOLICs:

$$\frac{Z_{2}^{\text{IN}}}{Z_{0}^{\text{IN}}} \approx \frac{Z_{2}^{N}}{Z_{0}^{N}} - \frac{N + 2}{2} g,$$

(10)

where we have used the approximation

$$Y_{2}^{N} = Y_{2}^{2} Y_{0}^{N-2} \approx (1 - \kappa)^{N} \left( X_{2}^{N} - \frac{N + 2}{2} g X_{0}^{N} \right).$$

(11)

If we assume the average of intrinsic spin-2 HOLICs to vanish, the reduced shear is obtained directly by observing spin-2 HOLICs as

$$g \approx \left( \frac{2}{N + 2} \frac{Z_{2}^{N}}{Z_{0}^{N}} \right).$$

(12)

Since the maximum value of absolute spin-2 HOLICs is 1, the maximum value of $|g|$ is $2/(2 + N)$ in the this approximation.

4. WEIGHTED HOLICs AND PSF CORRECTION FOR REAL OBSERVATION

In the actual observation, the data have random noise and images are smeared by isotropic PSF and distorted by anisotropic PSF. In order to correct these effects, we introduce the weighted HOLICs and PSF correction. The general treatment of the PSF correction may be found, for example, in Kaiser et al. (1995).
and Bartelmann & Schneider (2001). Technical details of the PSF corrections in measuring HOLICs are found in OUF2008.

We redefine the weighted HOLICs by using an extra weight function $W(X_0^2/\sigma^2)$ with a characteristic scale $\sigma$ as

$$Z_M^N \equiv \int d^2 \theta I(\theta) X_M^N W \left( \frac{X_0^2}{\sigma^2} \right).$$

(13)

The weighted HOLICs of source $Z_M^{(s)N}$ are defined similarly in the source plane using $d\beta_i$ and $Y_M^N$. Using the lens equation as well as the expansion of the weight function

$$W \left( \frac{Y_0^2}{\sigma^2} \right) = W \left( \frac{X_0^2 - Re[\delta^*X_2]}{\sigma^2} \right) \approx W \left( \frac{X_0^2}{\sigma^2} \right) - \frac{Re[\delta^*X_2]}{\sigma^2} W' \left( \frac{X_0^2}{\sigma^2} \right),$$

(14)

the reduced shear effect for spin-2 weighted HOLICs becomes

$$\frac{Z_2^{(s)N}}{Z_0^{(s)N}} \approx \frac{Z_2^N}{Z_0^N} \left( \frac{N + 2}{2} + \frac{Z_0^{N+2}}{\sigma^2 Z_0^N} \right) g - \left( \frac{N - 2}{2} \frac{Z_2^N}{Z_0^N} + \frac{Z_4^{N+2}}{\sigma^2 Z_0^N} \right) g^* \times \frac{Z_2^N}{Z_0^N} - C_0^N g - C_4^N g^*, \quad \text{(15)}$$
where HOLICs with $'$ are measured with $W'(x) = \partial W(x)/\partial x$ instead of $W(x)$.

Next, the anisotropic PSF is corrected as

$$Z_0^{\text{Viso}} \approx Z_0^{\text{Nobs}} - P_0^N \chi_q - P_4^N \chi_q^*,$$  \hspace{1cm} (16)

where

$$P_0^N \equiv \frac{1}{Z_0^N} \left(\frac{N(N + 2)}{8} Z_0^{N-2} + \frac{(N + 2)}{2\sigma^2} Z_0^{N'} + \frac{1}{2\sigma^4} Z_0^{N+2} \right),$$  \hspace{1cm} (17)

$$P_4^N \equiv \frac{1}{Z_0^N} \left(\frac{(N - 2)(N - 4)}{8} Z_0^{N-2} + \frac{(N - 2)}{2\sigma^2} Z_4^{N'} + \frac{1}{2\sigma^4} Z_4^{N+2} \right),$$  \hspace{1cm} (18)

where the quantities with superscripts “obs” and “iso” mean the quantities before and after making the anisotropic PSF correction, respectively, and HOLICs with $''$ are measured with $W''(x) = \partial^2 W(x)/\partial^2 x$ instead of $W(x)$. In the actual observation, higher order corrections using higher order spin quantities such as $P_6^N$ (spin-6) might be necessary for accurate shape measurement. However, one can see in Section 5 that the constructed mass distribution is accurate enough even in the approximation using up to $P_4^N$.

The $\chi_q$ is the spin-2 anisotropic component of PSF (OUF2008), and is obtained from HOLICs measured from the star images as follows:

$$\chi_q \approx \frac{1}{P_0^N} \frac{Z_0^{\text{Nobs}}}{Z_0^{\text{Nstar}}},$$  \hspace{1cm} (19)

Finally, we obtain the relation between spin-2 HOLICs and the reduced shear from each order of HOLICs after isotropic PSF correction as follows:

$$\frac{Z_0^{\text{Viso}}}{Z_0^{\text{Viso}}} \approx \left(\frac{C_0^N - \left(\frac{C_{\text{Olfar}}}{P_0^N}\right) P_0^N}{Z_0^{\text{Nobs}}} \right) \times \left(\frac{C_4^N - \left(\frac{C_{\text{Olfar}}}{P_4^N}\right) P_4^N}{Z_0^{\text{Nobs}}} \right) \times \left(\frac{\sigma}{\sigma^*}\right)^2.$$  \hspace{1cm} (20)

We have tested our shear estimation by HOLICs using the ready-made simulation from STEP, and have obtained the input value of the shear when we use higher order PSF corrections ($C_0^N$ and $C_4^N$) in all orders of HOLICs measurement and do not use faint images in the higher order HOLICs shear measurement. This can be seen in Figure 1 where the results of the analysis of STEP1 simulations are shown in each order of HOLICs shear measurement. In this simulation, we used about 10$^5$ brighter images with 20.3 < MAG < 23.3 and 8.6 arcmin$^{-2}$. In order to use fainter images for higher order HOLICs shear measurement, we need higher order polarization matrices to correct PSF appropriate for higher order HOLICs shear measurement, which will be presented in the forthcoming paper.

5. A1689 ANALYSIS

In this section, we apply the weak-lensing analysis using spin-2 HOLICs with order $N = 2, 4, 6, 8$ to Subaru imaging observations of the massive galaxy cluster Abell 1689 at $z = 0.183$. Abell 1689 is one of the best-studied clusters (e.g., Tyson & Fisher 1995; King et al. 2002; Bardeau et al. 2005; Broadhurst et al. 2005a, 2005b; Halkola et al. 2006; Leonard et al. 2007; Limousin et al. 2007; Umetsu et al. 2007; Umetsu & Broadhurst 2008), and therefore is an ideal target for testing the new method.

We use Subaru Suprime-Cam $i'$-band data, which have 30'×25' field and 0'0202 pixel$^{-1}$. We analyzed 3000×3000 pixel (or about 10'×10'), which is the center of the field, and the seeing is 0'088 by FWHM.

We used IMCAT (Kaiser et al. 1995) and some scripts (K. Umetsu 2006, private communication) for image detection and measure position, Gaussian radius "r$_g$", half-light radius "r$_h$", and magnitude of detected objects in some stage of the analysis.

We used 81 star images, which have 2.2 < $r_h$ < 2.5 and 21 < MAG < 22.5 for PSF correction, and 3366 galaxies (about 33 arcmin$^{-2}$) with 2.5 < $r_h$ < 10 and 22 < MAG < 25.5. In Figure 2, we plot the average of spin-2 HOLICs before and after anisotropic PSF correction.

Figure 3 shows the reconstructed mass distributions measured with HOLICs of order $N = 2, 4, 6, 8$. The smoothing is 0'15 by Gaussian radius and the interval of successive contours is $\Delta \kappa = 0.2$ and the lowest contour is $\kappa = 0.2$. Reconstructed $B$-modes of each order have dispersion $\sigma_B \approx 0.065, 0.060, 0.062, 0.073$, respectively. Figure 4 presents the average of these four reconstructed mass distributions, and the interval of successive contours is $\Delta \kappa = 0.2$ and the lowest contour is $\kappa = 0.2$. Figure 5 presents the $B$-mode of Abell 1689 measured with HOLICs of order $N = 2, 4, 6, 8$; the mapping status is the same as in Figure 3. The contours separate $\kappa_B = 0.2$ and purple is the contour with $\kappa_B = 0$. These figures show that the absolute values of the $B$-mode are less than 0.2 almost everywhere in the fields. Figure 6 shows the average of four $B$-modes, the contours separate $\kappa_B = 0.2$, and purple is $\kappa_B = 0$. Averaged $B$-modes have dispersion $\sigma_B \approx 0.059$.

These results clearly show that we can reconstruct A1689 mass distribution using the higher order spin-2 HOLICs as well as complex ellipticity $\chi$. The dispersion of the $B$-mode is reduced by combining $B$-mode of each order. Compared with
Figure 7. Differences between spin-2 HOLICs of order \( N = 4 \) (left), 8 (right), and complex ellipticity \( \chi \). The horizontal axis shows the differences between the absolute value of spin-2 HOLICs of order \( N = 4(8) \) and the absolute value of complex ellipticity. The vertical axis shows the angles determined by spin-2 HOLICs of order \( N = 4(8) \) relative to the direction determined by complex ellipticity.

The case of using only complex ellipticity \( \chi \), the dispersion is reduced by about 10%. We can even detect a subclump (\( \kappa > 0.4 \)) with a larger value of \( \kappa \) by using higher order spin-2 HOLICs than by using only complex ellipticity. We can see the suppression of the noise in the averaged mass distribution.

A detailed analysis of Abell 1689 (e.g. radial profiles and so on) will be presented in the forthcoming paper.

6. CONCLUSION

We examined the possibility of using higher order multipole moments to measure the weak shear. For this purpose we used spin-2 HOLICs of order \( N \) larger than 2. In order to apply the actual observational data, we introduced the weighted HOLICs and investigated PSF correction. Then we apply the method to the massive galaxy cluster A1689 (\( z = 0.183 \)), and found, surprisingly, that spin-2 HOLICs of order \( N \) up to 8 may be able to measure the mass distribution with sufficient accuracy. The reconstructed mass distributions from higher order spin-2 HOLICs show the second peak in the central region, and the dispersions of \( B \)-mode measured by these HOLICs are of the order of that measured by the usual complex ellipticity. The combined \( B \)-mode is reduced by about 10% and thus, the accuracy of the combined mass distribution is improved by about 10% over that measured only by the complex ellipticity.

The improvement is obtained by the partial independence of the shear measured by spin-2 HOLICs of different orders. The higher the order, the further out the region of the image the shear is measured. Thus, spin-2 HOLICs of different orders measure the shear in the different region of the image. In general, the direction determined by the measured shear near the central region of the image is not necessarily the same with the direction determined by the measured shear in the outer part. This is the basic reason why we can improve the accuracy of the measurement of mass distribution. Of course, the weight function of the different orders of multipoles overlaps each other, and thus the shear measured by them is only partly independent. We show in Figure 7 the difference between the directions of shear determined by spin-2 HOLICs of different orders as a function of the difference between the magnitudes of the shear determined by spin-2 HOLICs of different orders. Since the difference is important to reduce intrinsic noise, we will address this point in more detail in the forthcoming paper. This figure is made by analyzing the data obtained in the Subaru imaging observation of the blank region ELAIS N1. We use 35521 galaxies in 0.28 deg\(^2 \approx 2 \times (12.6)^2 \pi \) arcmin\(^2 \), corresponding to a mean surface number density of 35.6 arcmin\(^{-2} \). This clearly suggests that the shear measured by spin-2 HOLICs of larger order is more independent of that measured by the usual complex ellipticity.

In this paper, we have shown the usefulness of the higher order spin-2 HOLICs in the shear measurement. This will open a new possibility of improving the accuracy of the weak-lensing analysis without doing further observations. Although we have tested our method to weak-lensing analysis for clusters of galaxies, we can also apply the method to the cosmic shear. It is very interesting to see how we can improve the measurement of the cosmological parameters by the cosmic shear using higher order spin-2 HOLICs. For this purpose, we need a more detailed analysis of this method, for example the test of other PSF data of STEP I and STEP II. These will be dealt with in the forthcoming paper.

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