Error Identification in Problem Solving on Multivariable Calculus

Reni Untarti, Anggun Badu Kusuma
Universitas Muhammadiyah Purwokerto, Jl. Raya Dukuhwaluh POX 202 Purwokerto, Indonesia
E-mail: reniuntarti@gmail.com

Abstract. Knowing the students errors in solving the problem became one of the important role of the lecturer. By knowing the problem, the lecturer would be able to anticipate it, so that the learning outcomes could be reached. The research aimed at identifying the errors done by the students in solving the problem. The study aimed at identifying the errors that were made by the students in solving the problem in multivariable calculus class. The study belongs to quantitative and qualitative descriptive research. The subject of the research was the mathematics education study program students who joined the multivariable calculus subject consisting of 41 students. The instrument was the mathematical problem solving test. The conclusion was the sequence of the error type that have been done mostly by the students, such as the first is the concept error, the second one is the procedural error, and the last is the factual error.

1. Introduction
The problem solving ability takes the very important role in mathematics learning process [1-5]. Solving the problem means the problem solving process by finding the solution from the obstacle or the difficulty in order to obtain the objective that can’t be achieved directly [1, 6-7]. By solving the problem, the students learn how to apply the mathematics ability in a new situation and to develop the understanding of the mathematical idea [1, 8]. In mathematics, the problem is considered as the situation or the question that requires the thinking ability to solve it [9] and it is non-routine [8, 10]. A problem that can be solved easily using the common way is a daily problem [11]. A non-routine task belongs to relativity for the students, in which the task can be the problem recently, but it will be not in the other time [12]. In the fact, the student seems have the error in solving the problem frequently. The students’ error can be interpreted manifestation of the difficulties in the mathematics learning process that can be the minimum performance of the mathematics, the low skill of mathematical concept application, the mathematical skill definition, and the problem solving students’ skill which is inefficient [5,13]. This error made the students could not solve the problem perfectly. It is the important thing that the teacher or the lecturer knows the errors that have been done by the students in the problem solving process [14-15]. By realising the errors, the lecturer is able to: 1) measure the skill of the students, so that the lecturer knows the learning process difficulties and the causative factors [14]. The result can be used as the evaluation tool in the learning process, so that the lecturer can take the decision to achieve the goal of the learning process optimally, 2) identify the steps of the students that they can show their ability appropriately, 3) determine the type of the students’ error, 4) determine the most happened errors, 5) select the effective learning strategy that will be no more error happen [14, 16-17].
The error types in solving the problems are: the factual error, the procedure error, and the conceptual error [17]. The factual error is the error caused by the students’ inability to use the factual information, for example: the inability in vocabulary, the inability in identifying decimal value [17]. The procedural error is the error caused by the students’ inability to show their ability in the mathematical problem solving process, for instance the counting error [14, 17]. The conceptual error is the error caused by the students who do not understand the important principal and the idea related to the given mathematical problem, for example: the students do not understand the geometry characteristics.

The previous research showed that the most of the error happened in mathematics were factual error [18-19], the procedure error [14, 20-22], and the concept error [18,23]. Despite the similar research has been done before, this research is still significant. This research is significant because the most of previous researches focus on the high school mathematics, not the college mathematics. This research material focuses on the multivariable calculus. The result of this research will be very useful to help in deciding the appropriate learning process in the next time.

2. Method
This research belongs to quantitative and qualitative descriptive researches. Those types were used because the researcher wanted to get the information about the most happened type of errors that were done by the students in solving the multivariable calculus problem, and to describe the students’ work and the interview data as well. The result of this research was not going to be generalised, but it was going to be used as the input for the lecturer of the better learning process of multivariable calculus in the next year. The research subjects were 41 students of mathematics education study program who joint the multivariable calculus subject in the academic year 2017/2018.

The research procedure consisted of: the first was giving the test for all the research subjects. The test consisted of 5 items of multivariable calculus problem solving. The test validation was based on the logical validation that was matching among the number item test, the indicator of the problem solving ability, and the learning outcomes. The second was analysing the students’ answer and identifying the errors that had been done by the students. The third was classifying the errors into 3 groups; they were factual, procedure, and concept error. The factual error was the error that happened when the students were not able to use the factual information, the procedure error was the error in the process of problem solving, and the concept error happened when the students had misunderstanding about the important principle or the idea related to the mathematical matter which was given. The fourth was choosing 3 students to be interviewed by using the purposive sampling method. The consideration in choosing the students was based on the errors which were done, the hand writing neatness, and the students communication skill orally. This process was done in order to know the deeper information about the students. The questions which were used in the interview were the open ended question based on the type of the error of the students. The fifth was using the descriptive statistics method to analyse the quantitative test of the problem solving ability of the students, meanwhile using the qualitative descriptive method to describe the students’ error type and the interview result.

3. Result and Discussion
Table 1 showed the result after the students’ work had been analysed and grouped based on the type of the error.
Table 1. The Type of the Students’ Error in Mathematics Problem Solving

| No. | Type of Error                                      | Amount of Error | Error Percentage |
|-----|---------------------------------------------------|----------------|-----------------|
| 1.  | Factual Error                                     |                |                 |
|     | a. symbol writing error                           | 1              | 14.85%          |
|     | b. error of the discriminant meaning unknown      | 1              |                 |
|     | c. writing error                                  | 2              |                 |
|     | d. the error of Linier of System Equation in two  | 8              |                 |
|     | Variables solution identification                  |                |                 |
|     | e. error of information using                      | 5              |                 |
|     | **Total**                                         | **17**         |                 |
| 2.  | Procedure Error                                   |                |                 |
|     | a. The error in doing the algebra procedure (such as: | 17             | 17.82%          |
|     | adding, subtraction, multiplying, dividing, etc.)  |                |                 |
|     | b. The error of differentiate process             | 1              |                 |
|     | **Total**                                         | **18**         |                 |
| 3.  | Concept Error                                     |                |                 |
|     | a. the error of limitation of integrated area      | 13             | 67.33%          |
|     | b. the error of the integration area drawing concept | 8              |                 |
|     | c. the error of multiple integral application      | 3              |                 |
|     | d. the error of multivariable derivative application concept | 6              |                 |
|     | e. the error of partial derivative concept         | 22             |                 |
|     | f. the error of partial derivative concept in implicit function | 14             |                 |
|     | g. the error of integration concept in multivariable function | 2              |                 |
|     | **Total**                                         | **68**         |                 |
|     | **Error amount**                                  | **103**        |                 |

Based on Table 1 above, it can be concluded that the errors which were done by the student mostly were the concept error. It generally happened to the students who learnt mathematics [24-26]. The concept is the basic idea of the mathematics, and the by using the idea we can classify the mathematics objects based on the basic idea and explain the concept as well [27-28]. In order to solve the problem, the students first must understand the concept of the given problem. The students will not be able to compile the strategy in solving the problem when they do not understand the concept then finally they can solve it appropriately. The other errors which were done by the students were the factual and procedure error. This error happened when they felt very anxious, so they became careless and inattentive. This conclusion was based on the interview to the students. The factual and the procedure errors typically happened when the students were careless or they forgot some of the information, instead of misunderstanding [16]. Although it was considered as the slighter error than the concept error, the students’ disability in the concept and the procedure caused the misleading to the right solution of the given problem. Geary [13] state that the concept understanding ability and the procedure knowledge are the essential ability in solving the problem.

There were some the errors which were done by the students in solving the multivariable calculus problem.
3.1 The Factual error

3.1.1 The symbol writing error

In Figure 1, the students wrote the symbol “$F(u) = x \ln(x + 2y)$”, so that they wrote a formula $\frac{du}{dt} = \frac{\partial F}{\partial x} \frac{dx}{dt} + \frac{\partial F}{\partial y} \frac{dy}{dt}$. The symbol writing “$F(u) = x \ln(x + 2y)$” was not correct, because it was known only “$u = x \ln(x + 2y)$” in the question, so the appropriate formula was $\frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt}$.

Based on the interview data, the reason of students wrote the symbol “$F(u)$” was that they were influenced by the example given by the lecturer in learning process. The students felt difficulty to understand that the symbol which was used changed, so they wrote “$F(u)$”. The students had the difficulty to express the algebra symbol [18]. Although the answer was correct, but the symbol writing was not correct.

3.1.2 The error of discriminant meaning unknowing

Figure 2 and the interview data indicated that the students did not understand yet the discriminant function when they defined the extrem value in the 2 variable functions. The students argued that the discriminant value in a certain point was the extrem value in that point. This arguments was absolutely not correct because the discriminant value would only show that that point was the extrem point, it was not the extrem value. It was not enough because beside they had to observe the discriminant value, they had to observe the value of the second derivation of $x(f_{xx})$. It means that the students didn’t understand the keyword of the gien problem [19].
3.1.3 The writing error

Figure 3. The writing error

Figure 3 showed that the students did the error in writing the solution of the given questions. Although the answer was correct, but from the figure 3, it would give the misunderstanding for the readers to understand the idea of the student. The writing above was not communicative. The students had the difficulty to write the expression and the mathematics equation [18]. The student should write as follows:

\[
\begin{align*}
\text{known: } xy^2 + yz^2 + zx^2 - 3 &= 0, \\
\text{then } \frac{\partial F}{\partial x} &= y^2 + 2xz \quad \text{and } \frac{\partial F}{\partial z} = 2yz + x^2, \quad \text{so } \frac{\partial z}{\partial x} = -\frac{y^2 + 2xz}{2yz + x^2}.
\end{align*}
\]

3.1.4 The error in Linier of System Equation in two Variables solution identification

Figure 4. The error in Linier of System Equation in two Variables solution identification

In figure 4, it showed that when the students were asked to define the critical point, then they got 2 condition, they were “\( y = 0 \) or \( x = 3 \)”. The students should define the point when “\( y = 0 \)” and “\( x = 0 \)”, in which those points were not (3,0).

3.1.5 The error of Information using

Figure 5. The error of Information using
Figure 5 indicated that the students were not careful in using the information which was given by the lecturer (Lai, 2012) [16]. The students wrote that implicit function which was known was \( xy^2 + yz^2 + zx^3 - 3 = 0 \), in which the appropriate one was \( xy^2 + yz^2 + zx^3 - 3 = 0 \). Although the error was not so many but it influenced on the incorrectness of the students in solving the problem.

### 3.2 The Procedure error

![Figure 6. Error of using algebra procedure](image)

![Figure 7. Error of differentiate process](image)

Figure 6 showed that the student was not correct in defining the value “\( z \)”. When the question were \( \frac{x^2}{4} + \frac{y^2}{9} + z = 1 \) so \( z = 1 - \frac{x^2}{4} - \frac{y^2}{9} \), not \( z = \frac{x^2}{4} + \frac{y^2}{9} - 1 \) (it was like the students’ work). Based on the interview data, it could be known that the students were not careful in defining “\( z \)”. From the figure 7, we knew the students were not careful in defining the derivation of the function \( f(x, y) = xy^2 - 6x^2 - 3y^2 \) toward \( x \). The student wrote \( f_x(x, y) = y^2 - 2x \) that it should be \( f_x(x, y) = y^2 - 12x \). The procedure error happened when the students had the wrong steps of the problem solving [14, 21-22].

### 3.3 The concept error

#### 3.3.1 The error of defining the limitation of integrated area

![Figure 8. The error of defining the limitation of integrated area](image)

Based on the figure 8, it showed that the students did the error when they defined the limitation of “\( y \)”. The students wrote the limitation of “\( 4 \leq y \leq 4x \)”, whereas it should be \( 4x \leq y \leq 4 \). The students wrote the integrated limitation toward “\( y \)” inversely. Based on the interview data, the student was still confused to define the limitation of integrated area, especially when the limitation was a function.
3.3.2 The error of Integrated area drawing

Figure 9. The error of integrated area drawing

Figure 9 reported that students’ error happened when they were asked to draw the integrated area. When the students drew the limitation area incorrectly, it would cause the incorrectness as well to define the limitation; finally they would do the error to give the final answer. From the question, it could be seen that the integrated area was the triangle with the point angles were (0,0); (0,4); and (1,4). The students incorrectly wrote the point (0, 4) with the point (4, 0). Based on the interview data, the error happened because the students were still confused to define the ordinate and axis [18, 23].

3.3.3 The error of partial derivation

Figure 10. The error of the partial derivation

Figure 12 reported that the students did not understand how to define the partial derivation from \( u = x \ln(x + 2y) \), toward \( x \) and also \( y \). When the students defined the partial derivation toward \( x \), they did not realize that function \( u \) was the multiple of two function that consisted of variable \( x \). The students only observe the variable \( x \) in \( \ln(x + 2y) \). So, it should be \( \frac{\partial u}{\partial x} = \ln(x + 2y) + \frac{x}{x + 2y} \). Based on the interview, it could be concluded that the students often were confuse when they faced question related to the partial derivation from the multiple of two functions.
3.3.4 The error the partial derivation concept toward the implicit function

![Figure 11](image)

In order to finish the question in figure 13, the students used the derivation toward the implicit function without using the formula. From the figure, it could be seen that the students did an error; the students did not understand that variable \( z \) contained variable \( x \) implicitly. Besides, the students did not realize that in the third term, there was the multiple of two functions that consisted of variable \( x \), they were \( 2x^2 \) and \( z \). So, the derivation toward \( x \) from the function above should be \( y^2 + 2yz \frac{\partial z}{\partial x} + 2xz + x^2 \frac{\partial z}{\partial x} = 0 \).

3.3.5 The error of the multivariable function integration

![Figure 12](image)

Figure 14 showed that the students did the error in the process of integration. The integration of \( \frac{11}{12} \) toward \( y \) should be \( \frac{11}{12} y \). Based on the interview, it could be concluded that the students still did not understand completely about the concepts of integral. They thought that the term that should have the process of integral was only the term that had the variable \( x \) or \( y \).

4. Conclusion

Based on the research result, it can be acknowledge the sequence of the error type that have been done mostly by the students, such as the first is the conceptual error. The conceptual error consists of: the conceptual error in defining the limitation of integrated area; the conceptual error in drawing the integrated area; the conceptual error in multiple integrated application; conceptual error in multivariable derivative application; the conceptual error partial derivation; the conceptual error in partial derivation concept toward the implicit function; and the conceptual error in multivariable function integration. The
second is the procedural error. The procedural error consists of: errors in doing the algebra procedure, such as: adding, subtraction, multiplying, dividing, etc. and errors in differentiate process. The third is the factual error. The factual error consists of: errors in symbol writing; errors because of the unknowing the discriminative meaning; errors in writing; errors in identifying the Linier of System Equation in Two Variables solution; and errors in using the information.

5. Acknowledgments
The authors thank to LPPM Universitas Muhammadiyah Purwokerto for giving us the research grant so that this research can be success.

Reference
[1] NCTM 2000 Principles and standards for school mathematics (Reston VA: The National Council of Teachers of Mathematics, Inc)
[2] Reiss K. and Törner G 2007 Problem solving in the mathematics classroom: the German perspective. ZDM 39 431
[3] Hino K 2007 Toward the problem-centered classroom: trends in mathematical problem solving in Japan ZDM 39 503
[4] Charles R I 2009 The role of problem solving in high school mathematics Research into Practice Mathematics
[5] Rickard A 2005 Evolution of a teacher’s problem solving instruction: A case study of aligning teaching practice with reform in middle school mathematics RMLE Online 29 1
[6] Polya G 2004 How to solve it: A new aspect of mathematical method (Princeton: Princeton university press)
[7] Aydoğdu M and Ayaz M F 2008 The Importance of Problem Solving In Mathematics Curriculum Physical Sciences 3 538
[8] Badger M, Sangwin C J, Hawkes T O, Burn R P, Mason J and Pope S 2012 Teaching problem-solving in undergraduate mathematics (West Midlands: Coventry University)
[9] Lesh R and Zawojewski J 2007 Problem-solving and modeling Second handbook of research on mathematics teaching and learning ed In F. Lester (Reston, VA: NCTM) pp. 763-804
[10] Posamentier A S and Krulik S 2009 Problem solving in mathematics, grades 3-6: powerful strategies to deepen understanding (Thousand Oaks: Corwin Press)
[11] Voskoglou M G 2008 Problem solving in mathematics education: Recent trends and development Quaderni di Ricerca in Didattica (ScienzeMatematiche) 18 22
[12] Doorman M, Drijvers P, Dekker T, van den Heuvel-Panhuizen M, de Lange J and Wijers M 2007 Problem solving as a challenge for mathematics education in The Netherlands ZDM 39 405
[13] Tambychik T and Meerah T S M 2010 Students’ difficulties in mathematics problem-solving: What do they say? Procedia-Social and Behavioral Sciences 8 142
[14] Ketterlin-Geller L R and Yovanoff P 2009 Diagnostic assessments in mathematics to support instructional decision making Practical Assessment, Research & Evaluation 14 1
[15] Herholdt R and Sapire I 2014 An error analysis in the early grades mathematics-A learning opportunity? South African Journal of Childhood Education 4 43
[16] Irvin P S, Alonzo J, Lai C F, Park B J and Tindal G 2012 Analyzing the reliability of the easy cbm reading comprehension measures: grade 7 (Eugene: University of Oregon)
[17] Brown J, Skow K, and the IRIS Center 2016 Mathematics: Identifying and addressing student errors Retrieved from http://iris.peabody.vanderbilt.edu/case_studies/scs_mathearr.pdf
[18] Rushton N 2014 Common errors in mathematics http://www.cambridgeassessment.org.uk /Images/466316-common-errors-in-mathematics.pdf
[19] Wijaya A., van den Heuvel-Panhuizen M, Doorman M and Robitzsch A 2014 Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors The Mathematics Enthusiast 11 555
[20] Mok I, Cai, J and Fung A T F 2008 Missing learning opportunities in classroom instruction:
Evidence from an analysis of a well-structured lesson on comparing fractions The Mathematics Educator 11 111

[21] Isgiyanto A 2011 Diagnosis kesalahan siswa berbasis penskoran politomus model partial credit pada matematika Jurnal Penelitian dan Evaluasi Pendidikan 15 308

[22] Zulfah Z 2017 Analisis Kesalahan Peserta Didik Pada Materi Persamaan Linear Dua Variabel Di Kelas VIII Mts Negeri Sungai Tonang Jurnal Cendekia: Jurnal Pendidikan Matematika 1 12

[23] Luneta K 2015 Understanding students' misconceptions: an analysis of final Grade 12 examination questions in geometry Pythagoras 36 1

[24] Ay Y 2017 A review of research on the misconceptions in mathematics education Education Research Highlights in Mathematics Science and Technology 2017 ed Shelley M and Pehlivan M (Ames: ISRES Publishing) pp. 21-31

[25] Ojose B 2015 Students' misconceptions in mathematics: Analysis of remedies and what research says Ohio Journal of School Mathematics 72 30

[26] Veloo A, Krishnasamy H N and Abdullah W S W 2015 Types of student errors in mathematical symbols, graphs and problem-solving Asian Social Science 11 324

[27] Arend R I 2007 Learning to teach (New York: McGraw Hill Companies)

[28] Bell F H 1978 Teaching mathematics and learning mathematics (in secondary schools) (Iowa: Brown Company Publishers)