The contributions of $K^*_0(1430)$ and $K^*_0(1950)$ in the three-body decays $B \to K\pi h$

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Abstract

We study the contributions of the resonant states $K^*_0(1430)$ and $K^*_0(1950)$ in the three-body decays $B \to K\pi h$ (with $h = \pi, K$) in the perturbative QCD approach in this work. The crucial nonperturbative factor $F_{K\pi}(s)$ in the distribution amplitudes of the $S$-wave $K\pi$ system is derived from the matrix element of vacuum to $K\pi$ pair. The $CP$ averaged branching fraction of a quasi-two-body decay process $B \to K^*_0(1950)h \to K\pi h$ is about one order smaller than the corresponding decay $B \to K^*_0(1430)h \to K\pi h$. We compare our predictions with the results in literature. And the perturbative QCD predictions in this work for the relevant decays agree well with the existing experimental data.

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I. INTRODUCTION

The charmless three-body hadronic \( B \) meson decay processes provide us a field to appraise different dynamical models of strong interaction, to investigate hadronic final-state interactions and analyze hadron spectroscopy, to determine the fundamental quark mixing parameters and understand \( CP \) asymmetries. In order to extract the significative information from experimental results and present the effective and accurate predictions for the three-body \( B \) decays, some methods have been adopted in abundant works, such as the U-spin, isospin and flavor SU(3) symmetries in [1–10], the QCD factorization (QCDF) in [11–25] and the perturbative QCD (PQCD) approach in [26–28]. The three-body decays \( B \to K\pi h \), with \( h \) is the pion or kaon, have been extensively studied by Belle [29–35], BaBar [36–43] and LHCb [44–51] Collaborations in recent years. These decays especially the \( B \to K\pi\pi \) were found to be a clean source for the extraction of the Cabibbo-Kobayashi-Maskawa (CKM) [52, 53] angle \( \gamma \) [54–61]. The relevant processes also provide the new possibilities for the \( CP \) violation searches in \( B \) decays [30, 45–47].

The total decay amplitude for the \( B \) meson decays into three light mesons \( K, \pi \) and \( h \) as the final states can be described as the coherent sum of the nonresonant and resonant contributions in the isobar formalism [62–64]. The nonresonant contributions are spread all over the phase space and play an important role in the corresponding decay processes [65–67]. The resonant contributions from low energy scalar, vector and tensor resonances are known experimentally, in most cases, to be the dominated portion of the related decays and could be studied in the quasi-two-body framework [68–70] when the rescattering effects [71] and three-body effects [72, 73] are neglected. For the \( B \to K\pi h \) decays, we have the resonant contributions from the \( K\pi \), \( \pi h \) and \( Kh \) pairs which are originated from different intermediate states and as well containing the two-body final state interactions, while the \( J^P = 0^+ \) component of the \( K\pi \) spectrum, denoted as \((K\pi)^0\), are always found very important for the relevant physical observables.

The primary source of the information on \( I = 1/2 \) \( S \)-wave \( K\pi \) system comes from the LASS experiment for the reaction \( K^- p \to K^-\pi^+ n \) [74]. The \( K\pi \) \( S \)-wave amplitude has also been studied in detail in the decays \( D^+ \to K^-\pi^+\pi^+ \) by E791 [75], FOCUS [76, 77] and CLEO [78], \( \eta_c \to K\bar{K}\pi \) by BaBar [79] and \( \tau^- \to K_S\pi^-\nu \) by Belle [80] with the methods of Breit-Wigner functions [81], K-matrix formalism [82–84] or model-independent partial-wave analysis. To describe the slowly increasing phase as a function of the \( K\pi \) mass, the scalar \( K\pi \) scattering amplitude was written as the relativistic Breit-Wigner term [81] for the resonance \( K^*_0(1430) \) in the LASS parametrization together with an effective range nonresonant component in [74], and the effective range term has been applied a cutoff to the slowly varying part close to the charm hadron mass at about 1.8 GeV for the three-body \( B \) decays in the experimental studies in [38, 40, 42, 43]. At about 1.95 GeV one will find the presence of the resonance \( K^*_0(1950) \) which be assigned as a radial excitation of the \( 0^+ \) member of the \( L = 1 \) triplet in the LASS analysis [74] and also in the \( \eta_c \) decays [79, 85]. The lowest-lying broad component of the \( S \)-wave \( K\pi \) system the \( K^*_0(700) \) [86], also named as \( \kappa \) or \( K^*_0(800) \) in literature [75, 77, 78, 87–92], has commonly been placed together with the states \( \sigma, f_0(980) \) and \( a_0(980) \) into an SU(3) flavor nonet, which have been suspected to be exotics [93–99].

In this work, we will focus on the contributions of the resonant state \( K^*_0(1430) \) in the \( B \to K\pi h \) decay processes in the PQCD approach based on the \( k_T \) factorization theorem [100–103]. The contributions of the resonant state \( K^*_0(1950) \) in the three-body \( B \) decays involving
FIG. 1: Typical Feynman diagrams for the decay processes $B \to K^*_0 h \to K\pi h$, $h = (\pi, K)$. The symbol $\otimes$ is the weak vertex, $\times$ denotes possible attachments of hard gluons and the rectangle represents the vector states $K^*_0$.

$K\pi$ pair have been ignored in the relevant theoretical studies and only be noticed by LHCb Collaboration very recently in the works [104, 105]. We will systematically estimate, for the first time, the contributions from the state $K^*_0(1950)$ for the $B \to K\pi h$ decays in this work. As for the resonance $K^*_0(700)$, we shall leave to the future studies in view of its ambiguous internal structure and the accompanying complicated results for the three-body $B$ decays [106], in addition, the corresponding contributions have been covered up by the effective range part of LASS line shape for the experimental results [38–40, 42, 43].

For the quasi-two-body decays $B \to K^*_0(1430, 1950)h \to K\pi h$, the intermediate state $K^*_0$, as demonstrated in the Fig. 1, is generated in the hadronization of quark-antiquark pair including one $s$ or $\bar{s}$-quark, the subprocess $K^*_0 \to K\pi$ which can not be calculated in the PQCD approach is always absorbed in the twist-2 and twist-3 light-cone distribution amplitudes of the scalar mesons [106–108] in the studies of two-body $B$ meson decays involving the scalar mesons $K^*_0(700)$ and $K^*_0(1430)$ [109]. The quasi-two-body framework based on PQCD has been discussed in [68] and has been adopted in some studies on the quasi-two-body $B$ decay processes recently [110–119].

This work is organized as follows. In Sec. II, we give a brief introduction for the theoretical framework. In Sec. III, we show the numerical results and give some discussions. Conclusions are presented in Sec. IV. The factorization formulas and functions for the related quasi-two-body decay amplitudes are collected in the Appendix.

II. FRAMEWORK

In the rest frame of $B$ meson, we define its momentum $p_B$ and its light spectator quark momentum $k_B$ as

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_T), \quad k_B = \left( \frac{m_B}{\sqrt{2}} x_B, 0, k_{BT} \right),$$

in the light-cone coordinates, where $x_B$ is the momentum fraction and $m_B$ is the mass. For the resonant states $K^*_0$ and the $K\pi$ pair generated from it by the strong interaction as revealed in the Fig. 1, we define their momentum $p = \frac{m_B}{\sqrt{2}}(\zeta, 1, 0)$. It’s easy to validate $\zeta = s/m_B^2$, where the invariant mass square $s = p^2 = m_{K\pi}^2$ for the $K\pi$ pair. The light spectator quark comes from $B$ meson and goes into intermediate states in the hadronization of $K^*_0$ as shown in Fig. 1 (a) has the momentum $k = (0, m_B^2 z, k_T)$. For the bachelor final state $h$ and its spectator quark, their momenta $p_3$ and $k_3$ have the definitions as

$$p_3 = \frac{m_B}{\sqrt{2}}(1 - \zeta, 0, 0_T), \quad k_3 = \left( \frac{m_B}{\sqrt{2}}(1 - \zeta)x_3, 0, k_{3T} \right).$$
Where $x_s$ and $z$, which run from 0 to 1, are the corresponding momentum fractions.

The matrix element from the vacuum to the $K^+\pi^-$ final state is given by [120]

$$
\langle K^+ (p_1) \pi^- (p_2) | \bar{d} \gamma_\mu (1 - \gamma_5) s | 0 \rangle = \left[ (p_1 - p_2) \mu - \frac{\Delta K}{p^2} p_\mu \right] F^K_{K\pi} (s) + \frac{\Delta K}{p^2} p_\mu F^K_{K\pi} (s),
$$

(3)

with the $p_1(p_2)$ is the momentum for kaon(pion) in the $K\pi$ system, $\Delta K_{K\pi} = (m_{K}^2 - m_{\pi}^2)$ and $m_K(m_\pi)$ is the mass of $K(\pi)$ meson. The $F^K_{K\pi} (s)$ is the vector form factor which has been discussed in detail in the Refs. [80, 121–127]. The the scalar form factor $F^K_{0\pi} (s)$ is defined as [128–130]

$$
\langle K\pi | \bar{q} s | 0 \rangle = C_X \frac{\Delta K_{K\pi}}{m_s - m_q} F^K_{0\pi} (s) = B_0 C_X F^K_{0\pi} (s),
$$

(4)

where $q$ is the light quark $u$ or $d$, the isospin factor $C_X = 1$ for $X = \{K^+\pi^-, K^0\pi^+\}$ and $C_X = 1/\sqrt{2}$ for $X = \{K^+\pi^0, K^0\pi^0\}$. The constant $B_0$ equals to $\Delta K_{K\pi}/(m_s - m_q)$. The form factor $F^K_{0\pi} (s)$ above is suppose to be one when $s$ is zero. When the $K^+\pi^-$ pair originated from the resonant state $K^*_0(1430)^0$, we have [129]

$$
\langle K^+\pi^- | \bar{d}s | 0 \rangle \approx \langle K^+\pi^- | K^*_0 \rangle \frac{1}{\mathcal{D}_{K^*_0}} \langle K^*_0 | \bar{d}s | 0 \rangle = \Pi_{K^*_0 K\pi} \langle K^*_0 | \bar{d}s | 0 \rangle,
$$

(5)

and

$$
\Pi_{K^*_0 K\pi} = \frac{g_{K^*_0 K\pi}}{\mathcal{D}_{K^*_0}} \approx \frac{B_0}{\bar{f}_{K^*_0} m_{K^*_0}} F^K_{0\pi} (s),
$$

(6)

with $\bar{f}_{K^*_0} = \frac{m_{K^*_0}}{m_s - m_d} \cdot f_{K^*_0}$, the decay constants defined by $\langle K^*_0 | \bar{d}s | 0 \rangle = m_{K^*_0} \bar{f}_{K^*_0}$ and $\langle K^*_0 | \bar{d}\gamma_\mu s | 0 \rangle = f_{K^*_0} p_\mu$ [106], and the mass $m_{K^*_0}$ could be replaced by the invariant mass $\sqrt{s}$ for the off-shell $K^*_0$. One can find different values of $f_{K^*_0}$ for $K^*_0(1430)$ in [131], we employ $f_{K^*_0(1430)} m_{K^*_0(1430)} = 0.0842 \pm 0.0045 \text{ GeV}^3$ [132] and $f_{K^*_0(1590)} m_{K^*_0(1590)} = 0.0414 \text{ GeV}^3$ [133] in this work. The Breit-Wigner formula for the denominator $\mathcal{D}_{K^*_0} = m_{K^*_0}^2 - s - im_{K^*_0} \Gamma(s)$, with the mass-dependent decay width $\Gamma(s) = \Gamma_0 \frac{q_0 m_{K^*_0}}{\sqrt{s}}$ and $\Gamma_0$ is the full width for resonant state $K^*_0$. In the rest frame of the resonance $K^*_0$, its daughter kaon or pion has the magnitude of the momentum as

$$
q = \frac{1}{2} \sqrt{\left[ s - (m_K + m_\pi)^2 \right] \left[ s - (m_K - m_\pi)^2 \right] / s}.
$$

(7)

The $q_0$ in $\Gamma(s)$ is the value for $q$ at $s = m_{K^*_0}^2$. The coupling constant $g_{K^*_0 K\pi} = \langle K^+\pi^- | K^*_0 \rangle$, one has [18]

$$
g_{K^*_0 K\pi} = \sqrt{\frac{8 \pi m_{K^*_0} \Gamma_{K^*_0 \rightarrow K\pi}}{q_0}},
$$

(8)

where the $\Gamma_{K^*_0 \rightarrow K\pi}$ is the partial width for $K^*_0 \rightarrow K\pi$.

The $S$-wave $K\pi$ system distribution amplitudes are collected into [106, 130, 134, 135]

$$
\Phi_{K\pi}(z, s) = \frac{1}{\sqrt{2} N_c} \left[ \sqrt{\theta \phi(z, s)} + \sqrt{s} \phi^*(z, s) + \sqrt{s} (\Psi - 1) \phi^*(z, s) \right],
$$

(9)
with the \( v = (0, 1, 0_T) \) and \( n = (1, 0, 0_T) \) being the dimensionless vectors. The twist-2 light-cone distribution amplitude has the form \([106, 130, 134]\)

\[
\phi(z, s) = \frac{F_{K \pi}(s)}{2\sqrt{2N_c}} \left\{ 6z(1-z) \left[ a_0(\mu) + \sum_{m=1}^{\infty} a_m(\mu)C_m^{3/2}(2z-1) \right] \right\},
\]

with \( C_m^{3/2} \) the Gegenbauer polynomials, \( a_0 = (m_s(\mu) - m_q(\mu))/\sqrt{s} \) for \( (K_0^-, \bar{K}_0^0) \) and \( a_0 = (m_q(\mu) - m_s(\mu))/\sqrt{s} \) for \( (K_0^+, \bar{K}_0^*) \) according to Ref. \([134]\). The \( a_m \) are scale-dependent Gegenbauer moments, with \( a_1 = -0.57 \pm 0.13 \) and \( a_3 = -0.42 \pm 0.22 \) at the scale \( \mu = 1 \) GeV for the resonance \( K_0^*(1430) \), and the contributions from the even terms could be neglected \([106]\). There is no available Gegenbauer moments for the state \( \bar{K}_0^* \) and \( K_0^* \). In Appendix A, we obtain the masses, decay constants, full widths of \( K^0 \) and Table

\[
\begin{align*}
TABLE I: & \text{ Masses, decay constants, full widths of } K_0^*(1430) \text{ and } K_0^*(1950) \text{ (in units of GeV) and Wolfenstein parameters} \ [86]. \\
& m_{B^0} = 5.280 \quad m_{B^+} = 5.279 \quad m_{B^0} = 5.367 \quad m_{\pi^0} = 0.140 \quad m_{\pi^0} = 0.135 \\
& m_{K^+} = 0.494 \quad m_{K^0} = 0.498 \quad f_K = 0.156 \quad f_{\pi} = 0.130 \\
& m_{K_0^*}(1430) = 1.425 \pm 0.050 \quad \Gamma_{K_0^*}(1430) = 0.270 \pm 0.080 \\
& m_{K_0^*}(1530) = 1.945 \pm 0.010 \pm 0.020 \quad \Gamma_{K_0^*}(1530) = 0.201 \pm 0.034 \pm 0.079 \\
& \lambda = 0.22453 \pm 0.00044 \quad A = 0.836 \pm 0.015 \quad \bar{\rho} = 0.122^{+0.018}_{-0.017} \quad \bar{\eta} = 0.355^{+0.012}_{-0.011}.
\end{align*}
\]

The factor \( F_{K \pi}(s) \) is related to scalar form factor \( F_{0}^{K \pi}(s) \) by \( F_{K \pi}(s) = \frac{B_0}{m_{K_0^*}} F_{0}^{K \pi}(s) \).

The distribution amplitudes for \( B \) meson and the bachelor final state \( h \) in this work are the same as those widely employed in the studies of the hadronic \( B \) meson decays in the PQCD approach, one can find their expressions and parameters in the Appendix.

### III. RESULTS AND DISCUSSIONS

In the numerical calculation, we adopt the decay constants \( f_B = 0.189 \), \( f_{B_s} = 0.231 \) GeV \([136]\), the mean lifetimes \( \tau_{B^0} = (1.520 \pm 0.004) \times 10^{-12} \) s, \( \tau_{B^+} = (1.638 \pm 0.004) \times 10^{-12} \) s and \( \tau_{B^0_s} = (1.509 \pm 0.004) \times 10^{-12} \) s \([86]\) for the \( B^0, B^+ \) and \( B^0_s \) mesons, respectively. The masses and the decay constants for the relevant particles in the numerical calculation in this work, the full widths for \( K_0^*(1430) \) and \( K_0^*(1950) \), and the Wolfenstein parameters of the CKM matrix are presented in Table I.

Utilizing the differential branching fraction Eq. (A8) and the decay amplitudes collected in Appendix A, we obtain the \( CP \) averaged branching fractions \( (B) \) and the direct \( CP \) asymmetries \( (A_{CP}) \) in Table II and Table III for the concerned quasi-two-body decay processes including the resonances \( K_0^*(1430) \) and \( K_0^*(1950) \), respectively, as the intermediate
The results for those quasi-two-body decays with one daughter of the $K_0^*$ is the neutral pion are omitted. One will get a half value of the $B$ and the same value of the $A_{CP}$ of the corresponding result in Tables II, III for a decay with the subprocesses $K_0^* \rightarrow K\pi^0$ considering the isospin relation. For example, we have

$$B(B^+ \rightarrow K_0^*(1430)^+\pi^0 \rightarrow K^+\pi^0\pi^0) = \frac{1}{2}B(B^+ \rightarrow K_0^*(1430)^+\pi^0 \rightarrow K^0\pi^+\pi^0), \quad (12)$$

while these two decay processes have the same direct $CP$ asymmetry.

For the PQCD predictions in Tables II, III, the shape parameters $\omega_B = 0.40 \pm 0.04$ or $\omega_{B_{\pi}} = 0.50 \pm 0.05$ in Eq. (A3) for the $B^{+}\pi^0$ or $B^{0}_{\pi}$ contribute the first error. The second error for each PQCD result comes from the Gegenbauer moments $a_1$ and $a_2$ in the Eq. (10). The third one is induced by the chiral masses $m_h^b$ and the Gegenbauer moment $a_2^b = 0.25 \pm 0.15$ of the bachelor final state pion or kaon. The large uncertainties of the decay widths for $K_0^*(1430)$ and $K_0^*(1950)$ in Table I result in quite small errors for these quasi-two-body predictions because the most variation effect of decay width in the denominator $D_{K_0^*}$ of Eq. (6) is offset by the $\Gamma$ in its numerator $gK^*_0K\pi$. For instance, the corresponding errors for the decay process $B^+ \rightarrow K_0^*(1430)^0\pi^+ \rightarrow K^+\pi^-\pi^+$ are $0.04 \times 10^{-5}$ and 0.2% for its branching fraction and direct $CP$ asymmetry, respectively, while for $B^+ \rightarrow K_0^*(1500)^0\pi^+ \rightarrow K^+\pi^-\pi^+$, the two errors are $0.01 \times 10^{-6}$ and 0.1%. There are other errors, which come from the uncertainties of the Wolfenstein parameters of the CKM matrix, the parameters in the distribution amplitudes for bachelor pion or kaon, the masses and the decay constants of the initial and final states, etc. are small and have been neglected. One can find that for those decay modes with the main contributions come from the annihilation diagrams of Fig. 1, their branching fraction errors generated from the variations of the $a_1$ and $a_3$ could be larger than the corresponding errors from $\omega_B$ or $\omega_{B_{\pi}}$, because there is no shape parameter for $B$ meson in the factorizable annihilation diagrams.

![FIG. 2: Differential branching fractions from threshold of $K\pi$ pair to 3 GeV for the $B^+ \rightarrow K_0^*(1430)^0\pi^+ \rightarrow K^+\pi^-\pi^+$ and $B^+ \rightarrow K_0^*(1500)^0\pi^+ \rightarrow K^+\pi^-\pi^+$ decays.](image)

For a quasi-two-body decay process with the resonance $K_0^*(1500)$ involved, its branching fraction is predicted to be roughly one order smaller than the corresponding decay mode including the resonant state $K_0^*(1430)$. Or rather, the decays in Table III with the factorizable emission diagrams of Fig. 1 (c), for their $CP$ averaged branching ratios, will be about 12%-15% ($R_1$) of the corresponding values in Table II, and the others will be about 6%-9% ($R_2$) for the corresponding branching fractions in Table II. The difference between $R_1$ and
TABLE II: PQCD predictions of the $CP$ averaged branching ratios and the direct $CP$ asymmetries for the quasi-two-body $B \to K_0^*(1430)h \to K\pi h$ decays.

| Decay modes | Quasi-two-body results |
|-------------|------------------------|
| $B^+ \to K_0^*(1430)^0\pi^+ \to K^+\pi^-\pi^+$ | $\mathcal{B}(10^{-5})$ | $2.27 \pm 0.59(\omega_B) \pm 0.17(a_{3+1}) \pm 0.34(m_0^a + a_5^s)$ |
| $A_{CP}(\%)$ | $-1.3 \pm 0.2(\omega_B) \pm 0.4(a_{3+1}) \pm 0.2(m_0^a + a_5^s)$ |
| $B^+ \to K_0^*(1430)^+\pi^0 \to K^0\pi^+\pi^0$ | $\mathcal{B}(10^{-6})$ | $7.86 \pm 2.16(\omega_B) \pm 0.55(a_{3+1}) \pm 1.36(m_0^a + a_5^s)$ |
| $A_{CP}(\%)$ | $1.5 \pm 0.4(\omega_B) \pm 0.8(a_{3+1}) \pm 0.4(m_0^a + a_5^s)$ |
| $B^+ \to K_0^*(1430)^+\bar{K}^0 \to K^0\pi^+\bar{K}^0$ | $\mathcal{B}(10^{-7})$ | $2.33 \pm 0.04(\omega_B) \pm 1.29(a_{3+1}) \pm 0.34(m_0^K + a_5^K)$ |
| $A_{CP}(\%)$ | $-18.4 \pm 5.8(\omega_B) \pm 2.7(a_{3+1}) \pm 5.4(m_0^K + a_5^K)$ |
| $B^+ \to \bar{K}_0^*(1430)^0K^+ \to K^-\pi^+K^+$ | $\mathcal{B}(10^{-6})$ | $2.86 \pm 0.54(\omega_B) \pm 0.51(a_{3+1}) \pm 0.42(m_0^K + a_5^K)$ |
| $A_{CP}(\%)$ | $17.9 \pm 0.4(\omega_B) \pm 8.0(a_{3+1}) \pm 0.9(m_0^K + a_5^K)$ |
| $B^0 \to K_0^*(1430)^+\pi^- \to K^0\pi^+\pi^-$ | $\mathcal{B}(10^{-6})$ | $2.07 \pm 0.54(\omega_B) \pm 0.14(a_{3+1}) \pm 0.30(m_0^a + a_5^s)$ |
| $A_{CP}(\%)$ | $0.3 \pm 0.5(\omega_B) \pm 0.8(a_{3+1}) \pm 0.1(m_0^a + a_5^s)$ |
| $B^0 \to K_0^*(1430)^0\pi^0 \to K^0\pi^+\pi^0$ | $\mathcal{B}(10^{-5})$ | $1.39 \pm 0.35(\omega_B) \pm 0.11(a_{3+1}) \pm 0.18(m_0^a + a_5^s)$ |
| $A_{CP}(\%)$ | $-1.8 \pm 0.4(\omega_B) \pm 0.2(a_{3+1}) \pm 0.1(m_0^a + a_5^s)$ |
| $B^0 \to K_0^*(1430)^+K^- \to K^0\pi^+K^-$ | $\mathcal{B}(10^{-8})$ | $5.77 \pm 2.38(\omega_B) \pm 2.92(a_{3+1}) \pm 0.62(m_0^K + a_5^K)$ |
| $A_{CP}(\%)$ | $4.9 \pm 6.4(\omega_B) \pm 3.7(a_{3+1}) \pm 3.6(m_0^K + a_5^K)$ |
| $B^0 \to K_0^*(1430)^-K^+ \to \bar{K}_0\pi^-K^0$ | $\mathcal{B}(10^{-7})$ | $3.84 \pm 1.48(\omega_B) \pm 1.95(a_{3+1}) \pm 0.09(m_0^K + a_5^K)$ |
| $A_{CP}(\%)$ | $-5.0 \pm 2.6(\omega_B) \pm 6.7(a_{3+1}) \pm 3.0(m_0^K + a_5^K)$ |
| $B^0 \to K_0^*(1430)^0K^0 \to K^+\pi^-\bar{K}^0$ | $\mathcal{B}(10^{-7})$ | $3.04 \pm 0.15(\omega_B) \pm 2.04(a_{3+1}) \pm 0.36(m_0^K + a_5^K)$ |
| $A_{CP}(\%)$ | - |
| $B^0 \to \bar{K}_0^*(1430)^0K^0 \to K^-\pi^+K^0$ | $\mathcal{B}(10^{-6})$ | $2.89 \pm 0.53(\omega_B) \pm 0.65(a_{3+1}) \pm 0.41(m_0^K + a_5^K)$ |
| $A_{CP}(\%)$ | - |

$R_2$ mainly originated from the $(S - P)(S + P)$ current amplitude the Eq. (A43), which possess the intermediate state invariant mass dependent factor $m_B\sqrt{s} (\equiv \sqrt{s})$. Take the decays $B^+ \to K_0^*(1430)^+\pi^- \to K^+\pi^-\pi^+$ as an example, if we neglect the factorizable contributions from Fig. 1 (c), the ratio between two branching fractions of $B^+ \to K_0^*(1950)^0\pi^+ \to K^+\pi^-\pi^+$ and $B^+ \to K_0^*(1430)^0\pi^+ \to K^+\pi^-\pi^+$ will be 0.08, which is in the range of $R_2$. The
diagram of the differential branching fractions for $B^+ \to K_0^*(1950)^0\pi^+ \to K^+\pi^0\pi^+$ and $B^+ \to K_0^*(1430)^0\pi^+ \to K^+\pi^-\pi^+$ is shown in the Fig. 2. From which one can find that the main portion of the branching fractions lies in the region around the corresponding pole mass of the intermediate states. This feature makes that the proportion of the contribution of the branching ratio from Eq. (A43) will be larger for a decay mode including the $K_0^*(1950)$ than the corresponding decay process including a $K_0^*(1430)$ as the intermediate state.

TABLE III: PQCD predictions of the CP averaged branching ratios and the direct CP asymmetries for the quasi-two-body $B \to K_0^*(1950)h \to K\pi h$ decays.

| Decay modes | Quasi-two-body results |
|-------------|------------------------|
| $B^+ \to K_0^*(1950)^0\pi^+ \to K^+\pi^-\pi^+$ | $\mathcal{B}(10^{-6})$; $3.36 \pm 0.86(\omega_B) \pm 0.24(a_{3+1}) \pm 0.51(m_0^+ + a_2^+)$; $\mathcal{A}_{CP}(\%)$; $1.5 \pm 0.3(\omega_B) \pm 0.2(a_{3+1}) \pm 0.3(m_0^+ + a_2^+)$ |
| $B^+ \to K_0^*(1950)^+\pi^0 \to K^0\pi^+\pi^0$ | $\mathcal{B}(10^{-6})$; $1.19 \pm 0.32(\omega_B) \pm 0.08(a_{3+1}) \pm 0.21(m_0^+ + a_2^+)$; $\mathcal{A}_{CP}(\%)$; $3.5 \pm 0.1(\omega_B) \pm 0.4(a_{3+1}) \pm 0.2(m_0^+ + a_2^+)$ |
| $B^+ \to K_0^*(1950)^+\bar{K}^0 \to K^0\pi^+\bar{K}^0$ | $\mathcal{B}(10^{-8})$; $1.86 \pm 0.04(\omega_B) \pm 0.60(a_{3+1}) \pm 0.38(m_0^+ + a_2^+)$; $\mathcal{A}_{CP}(\%)$; $-9.2 \pm 5.3(\omega_B) \pm 4.0(a_{3+1}) \pm 2.8(m_0^K + a_2^K)$ |
| $B^+ \to \bar{K}_0^*(1950)^0K^+ \to K^-\pi^+K^+$ | $\mathcal{B}(10^{-7})$; $3.59 \pm 0.66(\omega_B) \pm 0.54(a_{3+1}) \pm 0.54(m_0^K + a_2^K)$; $\mathcal{A}_{CP}(\%)$; $19.2 \pm 0.1(\omega_B) \pm 7.4(a_{3+1}) \pm 1.4(m_0^K + a_2^K)$ |
| $B^0 \to K_0^*(1950)^+\pi^- \to K^0\pi^+\pi^-$ | $\mathcal{B}(10^{-6})$; $2.99 \pm 0.77(\omega_B) \pm 0.20(a_{3+1}) \pm 0.45(m_0^+ + a_2^+)$; $\mathcal{A}_{CP}(\%)$; $1.9 \pm 0.5(\omega_B) \pm 0.5(a_{3+1}) \pm 0.1(m_0^+ + a_2^+)$ |
| $B^0 \to K_0^*(1950)^0\pi^- \to K^0\pi^-\pi^0$ | $\mathcal{B}(10^{-6})$; $2.01 \pm 0.50(\omega_B) \pm 0.15(a_{3+1}) \pm 0.26(m_0^+ + a_2^+)$; $\mathcal{A}_{CP}(\%)$; $0.4 \pm 0.6(\omega_B) \pm 0.3(a_{3+1}) \pm 0.3(m_0^+ + a_2^+)$ |
| $B^0 \to K_0^*(1950)^+\pi^- \to K^0\pi^-\pi^-$ | $\mathcal{B}(10^{-9})$; $5.14 \pm 1.90(\omega_B) \pm 1.66(a_{3+1}) \pm 0.29(m_0^K + a_2^K)$; $\mathcal{A}_{CP}(\%)$; $-2.8 \pm 10(\omega_B) \pm 10.6(a_{3+1}) \pm 3.3(m_0^K + a_2^K)$ |
| $B^0 \to \bar{K}_0^*(1950)^0K^- \to K^-\pi^-\bar{K}^0$ | $\mathcal{B}(10^{-8})$; $2.36 \pm 0.95(\omega_B) \pm 1.10(a_{3+1}) \pm 0.06(m_0^K + a_2^K)$; $\mathcal{A}_{CP}(\%)$; $-1.0 \pm 2.4(\omega_B) \pm 8.5(a_{3+1}) \pm 1.3(m_0^K + a_2^K)$ |
| $B^0 \to \bar{K}_0^*(1950)^0K^- \to K^-\pi^-\bar{K}^0$ | $\mathcal{B}(10^{-8})$; $2.22 \pm 0.08(\omega_B) \pm 1.05(a_{3+1}) \pm 0.35(m_0^K + a_2^K)$; $\mathcal{A}_{CP}(\%)$; $-48(\omega_B) \pm 0.58(a_{3+1}) \pm 0.48(m_0^K + a_2^K)$ |
| $B^0 \to K_0^*(1950)^0\pi^- \to K^-\pi^-\pi^+$ | $\mathcal{B}(10^{-6})$; $3.36 \pm 0.59(\omega_B) \pm 0.37(a_{3+1}) \pm 0.01(m_0^+ + a_2^+)$; $\mathcal{A}_{CP}(\%)$; $12.9 \pm 7.0(\omega_B) \pm 3.1(a_{3+1}) \pm 0.8(m_0^+ + a_2^+)$ |
| $B^0 \to \bar{K}_0^*(1950)^0\pi^- \to K^-\pi^-\pi^0$ | $\mathcal{B}(10^{-8})$; $3.74 \pm 0.35(\omega_B) \pm 1.01(a_{3+1}) \pm 0.48(m_0^+ + a_2^+)$; $\mathcal{A}_{CP}(\%)$; $57.1 \pm 4.0(\omega_B) \pm 8.1(a_{3+1}) \pm 5.5(m_0^+ + a_2^+)$ |
| $B^0 \to K_0^*(1950)^+\pi^- \to K^0\pi^-\pi^-$ | $\mathcal{B}(10^{-6})$; $2.03 \pm 0.31(\omega_B) \pm 0.19(a_{3+1}) \pm 0.32(m_0^K + a_2^K)$; $\mathcal{A}_{CP}(\%)$; $0.6 \pm 0.2(\omega_B) \pm 1.0(a_{3+1}) \pm 0.9(m_0^K + a_2^K)$ |
| $B^0 \to K_0^*(1950)^0K^- \to K^-\pi^-\bar{K}^+$ | $\mathcal{B}(10^{-6})$; $1.26 \pm 0.15(\omega_B) \pm 0.54(a_{3+1}) \pm 0.20(m_0^K + a_2^K)$; $\mathcal{A}_{CP}(\%)$; $-45.1 \pm 1.3(\omega_B) \pm 4.6(a_{3+1}) \pm 5.7(m_0^K + a_2^K)$ |
| $B^0 \to \bar{K}_0^*(1950)^0K^- \to K^-\pi^-\bar{K}^0$ | $\mathcal{B}(10^{-6})$; $2.13 \pm 0.33(\omega_B) \pm 0.19(a_{3+1}) \pm 0.33(m_0^K + a_2^K)$; $\mathcal{A}_{CP}(\%)$; $-54(\omega_B) \pm 0.54(a_{3+1}) \pm 0.48(m_0^K + a_2^K)$ |
| $B^0 \to \bar{K}_0^*(1950)^0K^- \to K^-\pi^-\bar{K}^0$ | $\mathcal{B}(10^{-7})$; $7.65 \pm 0.54(\omega_B) \pm 4.56(a_{3+1}) \pm 1.48(m_0^K + a_2^K)$; $\mathcal{A}_{CP}(\%)$; $-76(\omega_B) \pm 0.54(a_{3+1}) \pm 0.48(m_0^K + a_2^K)$ |
We must stress that the ratios $R_1$, $R_2$, and also the branching fractions in Table III for the quasi-two-body decays involving $K_0^* (1950)$ are strongly dependent on the relation $f_{K_0^* (1950)}m_{K_0^* (1950)}^2 = 0.0414 \text{ GeV}^3$ [133]. If the value 0.0414 becomes two times larger, the $R_1$, $R_2$ and the branching fractions in Table III will become four times larger than their current values. In Ref. [105], there are two branching fractions measured by LHCb to be

$$B(B^0 \to \eta_c K_0^*(1950)^0 \to \eta_c K^+ \pi^-) = (2.18 \pm 1.04 \pm 0.04 \pm 0.80_{-1.43}^{+0.80} \pm 0.25) \times 10^{-5}, \quad (13)$$

$$B(B^0 \to \eta_c K_0^*(1430)^0 \to \eta_c K^+ \pi^-) = (14.50 \pm 2.10 \pm 0.28 \pm 2.01 \pm 1.67) \times 10^{-5}. \quad (14)$$

The two central values above give us the ratio $R = 0.15$ which is in the range of $R_1$ for these two branching fractions, but there is no diagrams like Fig. 1 (c) for $B^0 \to \eta_c K_0^0$ decays. Because of the large errors for $B^0 \to \eta_c K_0^*(1950)^0$, we can not extract the decay constant $f_{K_0^*(1950)}$ from this measurement. While from the data of the fit fractions for $\eta_c \to K_0^* K^\pm \pi^\mp$ in [79] and $\eta_c \to K^+ K^- \pi^0$ in [85] both from BaBar, one can expect a larger value than 0.0414 GeV$^3$ for the $f_{K_0^*(1950)}m_{K_0^*(1950)}^2$.

**TABLE IV:** Comparison of the extracted predictions with the experimental measurements for the relevant two-body branching fractions (in units of $10^{-6}$). The first error for the theoretical results is added in quadrature from the errors in Table II, the second error comes from the uncertainty of $B(K_0^*(1430) \to K \pi) = 0.93 \pm 0.10$ [86].

| Two-body decays | This work       | Data  | Ref.          |
|-----------------|----------------|-------|---------------|
| $B^+ \to K_0^*(1430)^0 \pi^+$ | 36.6 $\pm$ 11.3 $\pm$ 3.9 | 34.6 $\pm$ 3.3 $\pm$ 4.2 $^{+1.9}_{-1.5}$ | BaBar [43] |
|                 |                | 32.0 $\pm$ 1.2 $\pm$ 2.7 $^{+0.9}_{-1.4}$ | BaBar [38] |
|                 |                | 51.6 $\pm$ 1.7 $\pm$ 6.8 $^{+1.8}_{-3.1}$ | Belle [30]  |
| $B^+ \to K_0^*(1430)^+ \pi^0$ | 12.7 $\pm$ 4.2 $\pm$ 1.4 | 11.9 $\pm$ 1.7 $\pm$ 1.0 $^{+0.0}_{-1.3}$ | BaBar [43] |
| $B^0 \to K_0^*(1430)^+ \pi^-$ | 33.4 $\pm$ 10.2 $\pm$ 3.6 | 29.9 $^{+2.3}_{-1.7}$ $\pm$ 1.6 $\pm$ 0.6 $\pm$ 3.2 | BaBar [40] |
|                 |                | 49.7 $\pm$ 3.8 $\pm$ 6.7 $^{+1.2}_{-1.8}$ | Belle [31]  |

The two-body results for the branching fractions of $B \to K_0^* h$ can be extracted from the quasi-two-body predictions in this work with the relation

$$\Gamma(B \to K_0^* h \to K \pi h) = \Gamma(B \to K_0^* h) \times B(K_0^* \to K \pi). \quad (15)$$

In Ref. [137], a parameter $\eta$ was defined to measure the violation of the factorization relation in the $D$ meson decays. For the $B \to K_0^*(1430) h$ and $B \to K_0^*(1430) h \to K \pi h$ decays, we have

$$\eta = \frac{\Gamma(B \to K_0^*(1430) h \to K \pi h)}{\Gamma(B \to K_0^*(1430) h) \times B(K_0^* \to K \pi)}$$

$$\approx \frac{m_{K_0^*(1430)}^2}{4\pi m_B} \frac{\Gamma_{K_0^*(1430)}}{\hat{q}_h q_0} \int^{(m_B - m_h)^2}_{(m_K + m_h)^2} \frac{ds}{s} \frac{\lambda_{1/2}(m_B^2, s, m_h^2)\lambda_{1/2}(s, m_{K_0^*}^2, m_h^2)}{(s - m_{K_0^*(1430)}^2 + (m_{K_0^*(1430)}^2 + (m_{K_0^*(1430)}^2 + (m_{K_0^*(1430)}^2 + \hat{q}_h^2)^2)^2,} \quad (16)$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, the $\hat{q}_h$ is the expression of Eq. (A9) in the rest frame of $B$ meson and fixed at $s = m_{K_0^*(1430)}^2$. With Eq. (16), we have $\eta = 0.90$.
for the decays $B^+ \to K^*_0(1430)^0\pi^+$, which means the violation of the factorization relation is not large when neglecting the effect of the invariant mass $s$ in the decay amplitudes of the quasi-two-body decays. In order to check this conclusion, we calculate the decay $B^+ \to K^*_0(1430)^0\pi^+$ in the two-body framework of the PQCD approach, and we have $\mathcal{B}(B^+ \to K^*_0(1430)^0\pi^+) = 35.2 \times 10^{-6}$, which is about 96.2% of the result in Table IV extracted with Eq (15), and $A_{CP}(B^+ \to K^*_0(1430)^0\pi^+) = -1.0\%$ is consistent with the $-1.3\%$ in Table II.

The comparison of the PQCD branching fractions with the experimental measurements for the two-body decays $B^+ \to K^*_0(1430)^0\pi^+$, $B^+ \to K^*_0(1430)^+\pi^0$ and $B^0 \to K^*_0(1430)^+\pi^-$ are shown in the Table IV, with the first error is added in quadrature from the errors in Table II and the second error comes from the uncertainty of $\mathcal{B}(K^*_0(1430) \to K\pi) = 0.93\pm0.10$ [86] for these theoretical results. The branching fraction and direct $CP$ asymmetry for $B^+ \to K^*_0(1430)^0\pi^+$ in Review of Particle Physics [86] averaged from the results in [30, 38, 43] are $39^{+6}_{-5} \times 10^{-6}$ and $0.061 \pm 0.032$, respectively, which are consistent with the predictions $(36.6 \pm 11.3 \pm 3.9) \times 10^{-6}$ in Table IV and $(-1.3 \pm 0.5)\%$ in Table II. Because of the large uncertainty of the $A_{CP} = 0.26^{+0.18}_{-0.14}$ for $B^+ \to K^*_0(1430)^+\pi^0$ in [86], we can not evaluate the significance of the prediction $(1.5 \pm 1.0)\%$, but our branching fraction agree very well with BaBar’s result in [43] for this decay mode. For the decay $B^0 \to K^*_0(1430)^+\pi^-$, one has two results as listed in Table IV from BaBar and Belle Collaborations, its average $\mathcal{B}$ is presented to be $(33 \pm 7) \times 10^{-6}$ in Review of Particle Physics [86], this value agree well with the PQCD prediction $(33.4 \pm 10.2 \pm 3.6) \times 10^{-6}$. There is an upper limit of $2.2 \times 10^{-6}$ for the decay $B^+ \to K^*_0(1430)^0K^+$, which is below the expectation. Our predictions will be tested by future experiments. In the very recent work, LHCb Collaboration presented the branching fractions for the combined decays $B^0_s \to \overline{K}^0\pi^+K^+$ as [51]

\[
\mathcal{B}(B^0_s \to K^*_0(1430)^0\pi^+K^+) = (19.4 \pm 1.4 \pm 0.4 \pm 15.6 \pm 2.0 \pm 0.3) \times 10^{-6},
\]

\[
\mathcal{B}(B^0_s \to \overline{K}^0(1430)^0\overline{K}^0 \to K^+\pi^0\overline{K}^0) = (20.5 \pm 1.6 \pm 0.6 \pm 5.7 \pm 2.2 \pm 0.3) \times 10^{-6},
\]

which are in agreement with the PQCD predictions in Table V.

| Decay modes | Quasi-two-body results |
|-------------|------------------------|
| $B^0_s \to K^*_0(1430)^0\pi^+K^+$ | $\mathcal{B}(B^0_s \to K^*_0(1430)^0\pi^+K^+) = 1.97 \pm 0.45(\omega_B) \pm 0.10(a_{3+1}) \pm 0.43(m_0^K + a_2^K)$ |
| $B^0_s \to K^*_0(1430)^0\pi^+K^+$ | $A_{CP}(%) = -7.7 \pm 1.5(\omega_B) \pm 1.3(a_{3+1}) \pm 4.3(m_0^K + a_2^K)$ |
| $B^0_s \to K^*_0(1430)^0\pi^+K^+$ | $B(10^{-6}) = 1.50 \pm 0.36(\omega_B) \pm 0.09(a_{3+1}) \pm 0.40(m_0^K + a_2^K)$ |
| $B^0_s \to K^*_0(1430)^0\pi^+K^+$ | $A_{CP}(%) = -$ |

TABLE V: PQCD predictions of the $CP$ averaged branching ratios and the direct $CP$ asymmetries for the $B^0_s \to \overline{K}^0\pi^+K^+$ decays, with $K^*_0(1430)$ and $K^*_0(1950)$ as the intermediate states.

On the experimental side, the LASS parametrization [36, 74]

\[
R(s) = \frac{\sqrt{s}}{q\cot\delta_B - i q} + e^{2i\delta_B} \frac{m_0\Gamma_0 m_q}{m_0^2 - s - im_0\Gamma_0} \frac{m_q}{m_q},
\]

(19)
are employed in most cases to describe the $S$-wave $K\pi$ system, where $m_0$ and $\Gamma_0$ are now the pole mass and full width for $K^*_0(1430)$, and $\cot\delta_B = \frac{a}{aq} + \frac{1}{2}rq$ with the parameters $a = 2.07 \pm 0.10$ GeV$^{-1}$ and $r = 3.32 \pm 0.34$ GeV$^{-1}$ [36]. The relativistic Breit-Wigner term of Eq. (19) is different from the Eq. (6). Before the $F_{K_\pi}(s)$ in Eqs. (10-11) be replaced by the LASS expression, a coefficient is needed for $R(s)$. We have the replacement

$$F_{K_\pi}(s) \rightarrow \hat{R}(s) = \frac{q_0}{m_0^2}g_{K^*_0(1430)K\pi}\hat{f}_{K^*_0(1430)}R(s)$$ \quad (20)$$
onumber

on the theoretical side. With $\hat{R}(s)$ in the concerned quasi-two-body decay amplitudes, one will have the results for the decays $B \rightarrow (K\pi)_0^*h$, including the contributions from the nonresonant effective range term, and the contributions from the resonance $K^*_0(1430)$ which are the same as in the Table II. As the examples, we listed the results for $B^+ \rightarrow (K\pi)_0^0\pi^+ \rightarrow K^+\pi^-\pi^+$, $B^+ \rightarrow (K\pi)_0^{0*}\pi^0 \rightarrow K^0\pi^+\pi^0$ and $B^0 \rightarrow (K\pi)_0^{0*}\pi^- \rightarrow K^0\pi^+\pi^-$ in the Table VI with the columns NERT for the nonresonant effective range term, BW for the Breit-Wigner term and Total for the whole LASS formula. The errors have the same sources as the results in Table II and are added in quadrature. The percentages of the branching ratios in the column NERT are about 49% of the total results with the cutoff at 1.8 GeV, which are close to the percentages for the nonresonant effective range term in Refs. [38, 40, 43], while the total branching fractions from the LASS formula in Table VI are smaller than those values in [38, 40, 43]. We argue that, it’s not really good for the effective range part of Eq. (19) to be studied in the quasi-two-body framework with the same expressions of the decay amplitudes in Appendix A, because the nonresonant term of a three-body decay amplitude should not be included in Eq. (9) and the effective range term hides the possible contributions from the exotic $K^*_0(700)$.

| Decay modes | NERT \(\mathcal{B}(10^{-5})\) | BW \(\mathcal{B}(10^{-5})\) | Total \(\mathcal{B}(10^{-5})\) |
|-------------|-----------------|-----------------|-----------------|
| $B^+ \rightarrow (K\pi)_0^0\pi^+ \rightarrow K^+\pi^-\pi^+$ | $0.98 \pm 0.31$ | $2.27 \pm 0.70$ | $2.04 \pm 0.64$ |
| $\mathcal{A}_{CP}(\%)$ | $-2.1 \pm 1.0$ | $-1.8 \pm 1.0$ | $-1.6 \pm 0.76$ |
| $B^+ \rightarrow (K\pi)_0^{0*}\pi^0 \rightarrow K^0\pi^+\pi^0$ | $3.51 \pm 1.14$ | $7.86 \pm 2.61$ | $7.21 \pm 2.42$ |
| $\mathcal{A}_{CP}(\%)$ | $0.8 \pm 2.4$ | $2.5 \pm 1.6$ | $3.0 \pm 1.9$ |
| $B^0 \rightarrow (K\pi)_0^{0*}\pi^- \rightarrow K^0\pi^+\pi^-$ | $0.93 \pm 0.27$ | $2.07 \pm 0.63$ | $1.89 \pm 0.56$ |
| $\mathcal{A}_{CP}(\%)$ | $3.6 \pm 1.6$ | $3.0 \pm 1.1$ | $3.2 \pm 1.2$ |

The two-body decays $B^+ \rightarrow K^*_0(1430)^0\pi^+$, $B^+ \rightarrow K^*_0(1430)^+\pi^+$, $B^0 \rightarrow K^*_0(1430)^+\pi^-$ and $B^0 \rightarrow K^*_0(1430)^0\pi^0$ have been studied in Ref. [106] and updated in [134] in the QCDF with $K^*_0(1430)$ being the first excited states of $K^*_0(700)$ (scenario 1) or the lowest lying scalar state (scenario 2), and in the scenario 2 $K^*_0(700)$ is treated as a four-quark state. In view of the discussions for $K^*_0(700)$ in [93–99], we will consider only the results for the $K^*_0(1430)$ in the scenario 2 in this work. The results of the branching fractions in [106, 134] for the four decays involving the $K^*_0(1430)$ are all smaller when comparing with the measurements and our results but with quite large errors as shown in Table VII. In the PQCD approach, the two-body decays $B \rightarrow K^*_0(1430)\pi$ were studied in [138], with the branching fractions are
all larger than the corresponding results of this work except the decay $B^0 \rightarrow K^*_0(1430)\pi^0$ which is $18.4^{+4.4+1.5-4.0}_{-3.9-1.4-2.9} \times 10^{-6}$ in [138] as listed in Table VII. The result $28.8^{+6.8+1.9+3.2}_{-8.7-4.8-3.94} \times 10^{-6}$ in [138] is about two times of our prediction and BaBar’s measurement [43] for the decay $B^+ \rightarrow K^*_0(1430)^+\pi^0$. The decays $B \rightarrow K^*_0(1430)\bar{K}$ have been studied in the QCDF in [139]. One can find the comparison of relevant branching fractions in Table VIII. The $A_{CP} = -22.51^{+9.0+5.63+19.61}_{-7.57-9.36-22.86} \%$ for the decay $B^+ \rightarrow K^*_0(1430)^+\bar{K}$ in [139] is consistent with the result $(-18.4 \pm 5.8 \pm 2.7 \pm 5.4) \%$ in Table II, while the $A_{CP} = -2.60^{+1.61+0.59+3.52}_{-1.76-0.59-5.47} \%$ for $B^+ \rightarrow K^*_0(1430)^0\bar{K}^+$ in [139] is smaller than the PQCD prediction $(17.9 \pm 0.4 \pm 8.0 \pm 0.9) \%$ in this work and with an opposite sign.

TABLE VII: Comparison of the extracted predictions with the results in literature for the relevant two-body branching fractions (in units of $10^{-6}$). The sources of the errors of our results are the same as in Table IV.

| Two-body decays | This work | Theory | Ref. |
|-----------------|-----------|--------|------|
| $B^+ \rightarrow K^*_0(1430)^0\pi^+$ | $36.6 \pm 11.3 \pm 3.9$ | $11.0^{+10.3+7.5+49.9}_{-6.0-3.5-10.1}$ | [106] |
| $B^+ \rightarrow K^*_0(1430)^+\pi^0$ | $12.7 \pm 4.2 \pm 1.4$ | $5.3^{+4.7+1.6+22.3}_{-2.8-1.7-4.7}$ | [106] |
| $B^0 \rightarrow K^*_0(1430)^+\pi^-$ | $33.4 \pm 10.2 \pm 3.6$ | $11.3^{+9.4+3.7+45.8}_{-5.8-3.7-9.9}$ | [106] |
| $B^0 \rightarrow K^*_0(1430)^0\pi^0$ | $22.4 \pm 6.6 \pm 2.4$ | $6.4^{+5.4+2.2+26.1}_{-3.3-2.1-5.7}$ | [106] |

TABLE VIII: Comparison of the extracted predictions with the QCDF results in [139] for the relevant two-body branching fractions (in units of $10^{-7}$). The sources of the errors of our results are the same as in Table IV.

| Two-body decays | This work | QCDF [139] |
|-----------------|-----------|------------|
| $B^+ \rightarrow K^*_0(1430)^0\bar{K}$ | $3.76 \pm 2.16 \pm 0.40$ | $1.14^{+0.84+1.40+1.17}_{-0.38-0.56-0.92}$ |
| $B^+ \rightarrow K^*_0(1430)^0\bar{K}$ | $39.9 \pm 13.8 \pm 4.3$ | $33.70^{+10.3+5.5+3.37}_{-8.47-4.82-3.94}$ |
| $B^0 \rightarrow K^*_0(1430)^+\bar{K}^-$ | $0.93 \pm 0.61 \pm 0.10$ | $1.07^{+0.72+0.03+2.27}_{-0.47-0.04-0.97}$ |
| $B^0 \rightarrow K^*_0(1430)^-\bar{K}^+ | $6.19 \pm 3.95 \pm 0.67$ | $0.58^{+0.45+0.02+0.14}_{-0.29-0.03-0.05}$ |
| $B^0 \rightarrow K^*_0(1430)^0\bar{K}$ | $4.90 \pm 3.34 \pm 0.53$ | $2.39^{+1.20+1.95+2.67}_{-0.85-0.90-2.00}$ |
| $B^0 \rightarrow \bar{K}^*_0(1430)^0\bar{K}$ | $46.1 \pm 15.0 \pm 5.0$ | $40.47^{+13.36+6.09+6.06}_{-10.77-5.38-6.16}$ |

With $m_{K\pi}$ in the region $(0.64 \sim 1.76)$ GeV, the decay processes $B^- \rightarrow [\bar{K}^*_0(1430)^0 \rightarrow K^-\pi^+]\pi^-$ and $\bar{B}^0 \rightarrow [K^*_0(1430)^- \rightarrow K^0\pi^-]\pi^+$ were calculated in Ref. [13] with the predictions $(11.6 \pm 0.6) \times 10^{-6}$ and $(11.1 \pm 0.5) \times 10^{-6}$, respectively, for the branching ratios.
in QCDF. These two decay processes have also been calculated with $m_{K\pi}$ in the region $(1.0 \sim 1.76)$ GeV in Ref. [14] with the results $(12.11 \pm 0.32) \times 10^{-6}$ and $(11.05 \pm 0.25) \times 10^{-6}$, respectively. In the PQCD approach we have $(16.6 \pm 5.3) \times 10^{-6}$ and $(15.2 \pm 4.7) \times 10^{-6}$ in the region $m_{K\pi} \in (0.64 \sim 1.76)$ GeV, $(16.4 \pm 5.1) \times 10^{-6}$ and $(15.0 \pm 4.6) \times 10^{-6}$ in the region $m_{K\pi} \in (1.0 \sim 1.76)$ GeV for the decays $B^+ \rightarrow K^*_0(1430)^0\pi^+ \rightarrow K^+\pi^-\pi^+$ and $B^0 \rightarrow K^*_0(1430)^+\pi^- \rightarrow K^0\pi^+\pi^-$, respectively, which are consistent with the results in Refs. [13, 14] within errors. The three-body decays $B \rightarrow K\pi h$ have been discussed in detail in Refs. [18, 21] in QCDF. The comparison of PQCD predictions in this work with the related results in [18, 21] are listed in Table IX. From Table IV and Table IX, one can find that the PQCD predictions are totally larger than the QCDF results [18, 21] but closer to the available data.

TABLE IX: Comparison of the PQCD predictions with the theoretical results for the relevant quasi-two-body branching fractions (in units of $10^{-6}$). The errors of this work have been added in quadrature.

| Two-body decays | This work | Theory | Ref. |
|-----------------|-----------|--------|------|
| $B^+ \rightarrow K^*_0(1430)^0\pi^+ \rightarrow K^+\pi^-\pi^+$ | 22.7 ± 7.0 | $11.3^{+0.0+3.3+0.1}_{-0.0-2.8-0.1}$ | [18] |
| $B^+ \rightarrow K^*_0(1430)^+\pi^0 \rightarrow K^0\pi^+\pi^0$ | 7.86 ± 2.61 | $5.4^{+0.0+1.6+0.1}_{-0.0-1.4-0.1}$ | [18] |
| $B^+ \rightarrow K^*_0(1430)^0K^+ \rightarrow K^-\pi^+K^+$ | 2.86 ± 0.85 | $1.0^{+0.0+0.2+0.0}_{-0.0-0.2-0.0}$ | [18] |
| $B^0 \rightarrow K^*_0(1430)^+\pi^- \rightarrow K^0\pi^+\pi^-$ | 20.7 ± 6.3 | $10.3^{+0.0+2.9+0.0}_{-0.0-2.5-0.0}$ | [18] |
| $B^0 \rightarrow K^*_0(1430)^0\pi^0 \rightarrow K^+\pi^-\pi^0$ | 13.9 ± 4.1 | $4.1^{+0.0+1.4+0.0}_{-0.0-1.2-0.0}$ | [18] |

FIG. 3: Differential direct $CP$ asymmetry for the decay $B^0 \rightarrow K^*_0(1430)^-K^+ \rightarrow \bar{K}^0\pi^-K^+$.

There is no direct $CP$ asymmetries for the decays $B^0 \rightarrow K^*_0\bar{K}^0$ and $B^0 \rightarrow \bar{K}^0K^0$ in Tables II, III, because these processes have only contributions from the penguin operators for their decay amplitudes. For the processes $B^0 \rightarrow K^*_0(1430)^+\pi^- \rightarrow K^0\pi^+\pi^-$ and $B^0 \rightarrow K^*_0(1430)^0\pi^0 \rightarrow K^+\pi^-\pi^0$ via the $b \rightarrow sq\bar{q}$ transition at quark level, the very
small fraction of the total branching ratio for the contributions from the current-current operators led to the small direct CP asymmetries for these two decays as shown in Table II. The same pattern will appear for the decays \( B^+ \to K_0^*(1430)^0\pi^+ \to K^+\pi^-\pi^+ \) and \( B^+ \to K_0^*(1430)^+\pi^0 \to K^0\pi^+\pi^0 \), and also for the relevant decays with the \( K_0^*(1950) \) replace \( K_0^*(1430) \) as the intermediate, but not for the decays \( B_s^0 \to K_0^*(1430)^-\pi^+ \to K^0\pi^-\pi^+ \) and \( B_s^0 \to K_0^*(1430)^0\pi^0 \to K^-\pi^+\pi^0 \) via the \( b \to d\bar{q}q \) transition. The interference between weak and strong phase of the decay amplitudes from current-current and penguin operators results in the large direct CP asymmetries for the \( B_s^0 \to K_0^*(1430)^+\pi^- \to K^0\pi^-\pi^+ \) and \( B_s^0 \to K_0^*(1430)^0\pi^0 \to K^-\pi^+\pi^0 \) decays. The differential distribution curve of the \( A_{CP} \) in \( m_{K\pi} \) for the decay process \( B_s^0 \to K_0^*(1430)^-K^+ \to K^0\pi^-K^+ \) is displayed in Fig. 3.

For the decays \( B^+ \to K_0^*(1430)^0\pi^+ \) and \( B^0 \to K_0^*(1430)^+\pi^- \), they both receive the contributions from the Fig. 1 (c, d). With the isospin limit we have \( [138] \)

\[
R = \frac{\tau_{B^0} B(B^+ \to K_0^*(1430)^0\pi^+)}{\tau_{B^+} B(B^0 \to K_0^*(1430)^+\pi^-)} \approx 1. \tag{21}
\]

With the predictions in Table VII, one has \( R = 1.017 \pm 0.003 \) from PQCD. The small error of \( R \) is because of the cancellation between the errors of two branching ratios, which means the increase or decrease of the parameters that caused the errors will result in nearly identical change of the weight for the numerator and denominator of \( R \). For the decays \( B^+ \to K_0^*(1430)^+\pi^0 \) and \( B^0 \to K_0^*(1430)^0\pi^0 \), the diagrams of Fig. 1 (a, c, d) will contribute to the branching fractions, the decay amplitudes from Fig. 1 (a) are same for both \( B^+ \to K_0^*(1430)^+\pi^0 \) and \( B^0 \to K_0^*(1430)^0\pi^0 \), but the decay amplitudes from Fig. 1 (c, d) have the opposite sign considering the difference for \( \bar{u}u \) and \( \bar{d}d \) to form a neutral pion. It is not strange for the ratio between branching fractions of \( B^+ \to K_0^*(1430)^+\pi^0 \) and \( B^0 \to K_0^*(1430)^0\pi^0 \) away from unity. Because of the amplitude pollution from Fig. 1 (a) with the tree operators, the ratio for the branching fractions between \( B^+ \to K_0^*(1430)^+\pi^0 \) and \( B^+ \to K_0^*(1430)^0\pi^+ \), and the ratio for \( B^0 \to K_0^*(1430)^0\pi^0 \) and \( B^0 \to K_0^*(1430)^+\pi^- \) could deviate from the isospin limit. The relation for direct CP asymmetries of the two-body decays \( B^+ \to K^+\pi^0 \), \( B^+ \to K^0\pi^+ \) and \( B^0 \to K^+\pi^- \) and \( B^0 \to K^0\pi^0 \) was suggested in Ref. [140] as

\[
A_{CP}(B^+ \to K^+\pi^0) \frac{2B(B^+ \to K^+\pi^0)}{B(B^0 \to K^+\pi^-)} \frac{\tau_{B^0}}{\tau_{B^+}} + A_{CP}(B^0 \to K^0\pi^0) \frac{2B(B^0 \to K^0\pi^0)}{B(B^0 \to K^+\pi^-)} \frac{\tau_{B^0}}{\tau_{B^+}} = A_{CP}(B^0 \to K^+\pi^-) + A_{CP}(B^+ \to K^0\pi^+) \frac{B(B^+ \to K^0\pi^+)}{B(B^0 \to K^+\pi^-)} \frac{\tau_{B^0}}{\tau_{B^+}}. \tag{22}
\]

Considering the same transitions at quark level, one could extend Eq. (22) to the \( B \to K_0^*(1430)\pi \) decays with the replacement \( K \to K_0^*(1430) \). This relation is satisfied within errors with the \( A_{CP}(B^0 \to K_0^*(1430)^+\pi^-) = (0.3 \pm 0.9)\% \), \( A_{CP}(B^+ \to K_0^*(1430)^0\pi^+) = (-1.3 \pm 0.5)\% \), \( A_{CP}(B^+ \to K_0^*(1430)^0\pi^+) = (1.5 \pm 1.0)\% \) and \( A_{CP}(B^0 \to K_0^*(1430)^0\pi^0) = (-1.8 \pm 0.5)\% \), and relevant branching fractions in Table II. One can find that the relation Eq. (22) will also hold for \( B \to K_0^*(1950)\pi \) decays with the values in Table III.

IV. CONCLUSION

In this work, we studied the contributions of the resonant state \( K_0^*(1430) \) and, for the first time, the resonance \( K_0^*(1950) \) in the three-body decays \( B \to K\pi h \) in the PQCD approach.
The crucial nonperturbative factor $F_{K\pi}(s)$ in the distribution amplitudes of the $S$-wave $K\pi$ system was derived from the matrix element of the vacuum to $K\pi$ final state and was related to the time-like scalar form factor $F_{K\pi}^{0}(s)$ by the relation $F_{K\pi}(s) = B_0/m_{K_0} F_{K\pi}^{0}(s)$. This relation also means that the LASS parametrization for the $(K\pi)^*_0$ system which frequently appeared in the experimental works cannot be adopted directly for the $K\pi$ system distribution amplitudes in the PQCD approach.

With $f_{K^*_0(1430)}m_{K^*_0(1430)}^2 = 0.0842 \pm 0.0045$ GeV$^3$ and $f_{K^*_0(1950)}m_{K^*_0(1950)}^2 = 0.0414$ GeV$^3$ we calculated the branching fractions and the direct $CP$ asymmetries for the concerned quasi-two-body decays $B \to K^*_0(h) \to K\pi h$. We found that the $CP$ averaged branching fraction of a quasi-two-body process with $K^*_0(1950)$ as the intermediate state is about one order smaller than the corresponding decay mode involving the resonance $K^*_0(1430)$. We compared our predictions with the related results in literature. And the predictions in this work for the relevant decays agree well with the existing experimental results from BaBar, Belle and LHCb Collaborations.

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Appendix A: DECAY AMPLITUDES

The Lorentz invariant decay amplitude $A$ for the quasi-two-body decay processes $B \to K^*_0 h \to K\pi h$ in the PQCD approach, according to Fig. 1, is given by [26, 68]

$$A = \Phi_B \otimes H \otimes \Phi_h \otimes \Phi_{K\pi}. \quad (A1)$$

The symbol $\otimes$ here means convolutions in parton momenta, the hard kernel $H$ contains one hard gluon exchange at the leading order in strong coupling $\alpha_s$ as in the two-body formalism. The distribution amplitudes $\Phi_B, \Phi_h$ and $\Phi_{K\pi}$ absorb the nonperturbative dynamics in the relevant decay processes.

The $B$ meson light-cone matrix element can be decomposed as [141–143]

$$\Phi_B = \frac{i}{\sqrt{2N_c}}(\not{p}_B + m_B)\gamma_5\phi_B(k_B), \quad (A2)$$

where the distribution amplitude $\phi_B$ is of the form

$$\phi_B(x_B, b_B) = N_B x_B^2(1 - x_B)^2\exp\left[-\frac{(x_Bm_B)^2}{2\omega_B^2} - \frac{1}{2}(\omega_Bb_B)^2\right], \quad (A3)$$

with $N_B$ the normalization factor. The shape parameters $\omega_B = 0.40 \pm 0.04$ GeV for $B^0$ and $B^\pm$, $\omega_{B_s} = 0.50 \pm 0.05$ for $B^0_s$, respectively.

The light-cone wave functions for pion and kaon are written as [144–147]

$$\Phi_h = \frac{i}{\sqrt{2N_c}}\gamma_5 \left[\not{p}_3^h \phi^A(x_3) + m_0^h \phi^P(x_3) + m_0^h (\not{p}_3 - 1)\phi^T(x_3)\right]. \quad (A4)$$
The distribution amplitudes of $\phi^A(x_3)$, $\phi^P(x_3)$ and $\phi^T(x_3)$ are

$$\phi^A(x_3) = \frac{f_h}{2\sqrt{2N_c}} 6x_3(1-x_3) \left[ 1 + a_1 h C_1^{3/2}(t) + a_2 h C_2^{3/2}(t) + a_4 h C_4^{3/2}(t) \right],$$  \hspace{1cm} (A5)

$$\phi^P(x_3) = \frac{f_h}{2\sqrt{2N_c}} \left[ 1 + (30\eta_3 - 5\rho_0^2) C_2^{1/2}(t) - 3[\eta_3 \omega_3 + \frac{9}{20} \rho_0^2 (1 + 6a_2 h)] C_4^{1/2}(t) \right],$$  \hspace{1cm} (A6)

$$\phi^T(x_3) = \frac{f_h}{2\sqrt{2N_c}} (-t) \left[ 1 + 6 \left( 5\eta_3 - \frac{1}{2}\rho_0^2 \omega_3 - \frac{7}{20} \rho_0^2 \right) (1 - 10x_3 + 10x_3^2) \right],$$  \hspace{1cm} (A7)

with $t = 2x_3 - 1$, $C_{1,2,4}^{1/2}(t)$ and $C_{3,4}^{3/2}(t)$ are Gegenbauer polynomials. The chiral masses $m_0^h$ for pion and kaon are $m_0^\pi = (1.4 \pm 0.1)$ GeV and $m_0^K = (1.6 \pm 0.1)$ GeV as they in Ref. [148]. The Gegenbauer moments $a_1^\pi = 0, a_1^K = 0.06, a_2^\pi = 0.25, a_4^\pi = -0.015$ and the parameters $\rho_0 = m_h/m_0^h, \eta_3 = 0.015, \omega_3 = -3$ are adopted in the numerical calculation.

For the the differential branching fraction, we have [86]

$$\frac{d\mathcal{B}}{d\zeta} = \tau_B \frac{q_h q}{64\pi^3 m_B} |A|^2,$$  \hspace{1cm} (A8)

The magnitude momentum for the bachelor $h$ is

$$q_h = \frac{1}{2} \sqrt{[(m_B^2 - m_h^2)^2 - 2(m_B^2 + m_h^2) s + s^2] / s},$$  \hspace{1cm} (A9)

in the center-of-mass frame of the $K_0^*$, where $m_h$ is the mass of the bachelor state. The direct $CP$ asymmetry $\mathcal{A}_{CP}$ is defined as

$$\mathcal{A}_{CP} = \frac{\mathcal{B}(\bar{B} \to \bar{f}) - \mathcal{B}(B \to f)}{\mathcal{B}(B \to f) + \mathcal{B}(B \to f)}.$$  \hspace{1cm} (A10)

The errors induced by the parameter $\mathcal{P} \pm \Delta \mathcal{P}$ for the $\mathcal{B}$ and $\mathcal{A}_{CP}$ in this work, we employ the formulas

$$\Delta \mathcal{B} = \left| \frac{\partial \mathcal{B}}{\partial \mathcal{P}} \right| \Delta \mathcal{P}, \quad \Delta \mathcal{A}_{CP} = \frac{2(\mathcal{B} \Delta \mathcal{B} - \bar{\mathcal{B}} \Delta \mathcal{B})}{(\mathcal{B} + \bar{\mathcal{B}})^2},$$  \hspace{1cm} (A11)

With the subprocesses $K_0^{*+} \to \{K^0\pi^+, \sqrt{2}K^+\pi^0\}$, $K_0^{*0} \to \{K^+\pi^-, \sqrt{2}K^0\pi^0\}$, $K_0^- \to \{K^0\pi^-, \sqrt{2}K^-\pi^0\}$ and $\bar{K}_0^* \to \{K^-\pi^+, \sqrt{2}K^0\pi^0\}$, and the $K_0^*$ is $K_0^*(1430)$ or $K_0^*(1950)$, the concerned quasi-two-body decay amplitudes are given as follows:

$$\mathcal{A} \left( B^+ \to K_0^{*0}\pi^+ \right) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us} [a_1 F_{Ah}^{LL} + C_1 M_{Ah}^{LL}] - V_{tb} V_{ts} [(a_4 - \frac{a_{10}}{2}) F_{Th}^{LL} + (a_6 - \frac{a_8}{2}) F_{Th}^{SP} + (C_3 - \frac{C_0}{2}) M_{Th}^{LL} + (C_5 - \frac{C_7}{2}) M_{Th}^{LR}] + (a_4 + a_{10}) F_{Ah}^{LL} + (a_6 + a_8) F_{Ah}^{SP} + (C_3 + C_9) M_{Ah}^{LL} + (C_5 + C_7) M_{Ah}^{LR} \right\},$$  \hspace{1cm} (A12)

$$\mathcal{A} \left( B^+ \to K_0^{*+}\pi^0 \right) = \frac{G_F}{2} \left\{ V_{ub} V_{us} [a_2 F_{TFK_0^{*+}}^{LL} + C_2 M_{TFK_0^{*}}^{LL} + a_1 (F_{Th}^{LL} + F_{Ah}^{LL}) + C_1 (M_{Th}^{LL} + M_{Ah}^{LL})] - V_{tb} V_{ts} \left[ \frac{3}{2} (a_9 - a_7) F_{TFK_0^{*}}^{LL} + \frac{3 C_{10}}{2} M_{TFK_0^{*}}^{LL} + \frac{3 C_8}{2} M_{TFK_0^{*}}^{SP} + (a_4 + a_{10}) (F_{Th}^{LL} + F_{Ah}^{LL}) + (a_6 + a_8) (F_{Th}^{SP} + F_{Ah}^{SP}) + (C_3 + C_9) (M_{Th}^{LL} + M_{Ah}^{LL}) + (C_5 + C_7) (M_{Th}^{LR} + M_{Ah}^{LR}) \right] \right\}.$$  \hspace{1cm} (A13)
\[ A(B^+ \to K^{*+} K^0) = \frac{G_F}{\sqrt{2}} \left\{ V^*_{ub} V_{td} [a_1 F_{LL}^{0K_0} + C_1 M_{LL}^{M_{LK_0}}] - V^*_{tb} V_{ts} [(a_4 - \frac{a_{10}}{2}) F_{LL}^{T_K^0}] + (a_6 - \frac{a_8}{2}) F_{T_K^0}^{SP} + (C_3 - \frac{C_9}{2}) M_{LL}^{T_K^0} + (C_5 - \frac{C_7}{2}) M_{LR}^{T_K^0} \right\}, \]  
(A14)

\[ A(B^+ \to \bar{K}^{*0} K^+) = \frac{G_F}{\sqrt{2}} \left\{ V^*_{ub} V_{td} [a_1 F_{LL}^{0K_0} + C_1 M_{LL}^{M_{LK_0}}] - V^*_{tb} V_{ts} [(a_4 - \frac{a_{10}}{2}) F_{LL}^{T_K^0}] + (a_6 - \frac{a_8}{2}) F_{T_K^0}^{SP} + (C_3 - \frac{C_9}{2}) M_{LL}^{T_K^0} + (C_5 - \frac{C_7}{2}) M_{LR}^{T_K^0} \right\}, \]  
(A15)

\[ A(B^0 \to K^{*+} \pi^-) = \frac{G_F}{\sqrt{2}} \left\{ V^*_{ub} V_{us} [a_2 F_{LL}^{0K_0} + C_2 M_{LL}^{M_{LK_0}}] - V^*_{tb} V_{ts} [(a_4 + a_{10}) F_{LL}^{T_K^0}] + (a_6 + a_8) F_{T_K^0}^{SP} + (C_3 + C_9) M_{LL}^{T_K^0} + (C_5 + C_7) M_{LR}^{T_K^0} \right\}, \]  
(A16)

\[ A(B^0 \to K^{*0} \pi^0) = \frac{G_F}{2} \left\{ V^*_{ub} V_{us} [a_2 F_{LL}^{0K_0} + C_2 M_{LL}^{M_{LK_0}}] - V^*_{tb} V_{ts} [(\frac{3}{2} (a_9 - a_7) F_{LL}^{T_K^0} + 3C_{10} M_{LL}^{T_K^0} + 3C_8 M_{LL}^{T_K^0} - (4 - \frac{a_{10}}{2}) (F_{T_K^0}^{LL} + F_{T_K^0}^{SP}) - (a_6 - \frac{a_8}{2}) (F_{T_K^0}^{SP} + F_{T_K^0}^{LL}) - (C_3 - \frac{C_9}{2}) (M_{LL}^{T_K^0} + M_{LL}^{SP}) \right\}, \]  
(A17)

\[ A(B^0 \to K^{*+} K^-) = \frac{G_F}{\sqrt{2}} \left\{ V^*_{ub} V_{ud} [a_2 F_{LL}^{0K_0} + C_2 M_{LL}^{M_{LK_0}}] - V^*_{tb} V_{ts} [(a_3 + a_9 - a_5 - a_7) F_{LL}^{T_K^0}] + (C_4 + C_{10}) M_{LL}^{M_{LK_0}} + (C_6 + C_8) M_{LL}^{M_{LK_0}} + (a_3 - \frac{a_9}{2} - a_5 + \frac{a_7}{2}) F_{LL}^{T_K^0} \right\}, \]  
(A18)

\[ A(B^0 \to K^{*0} K^+) = \frac{G_F}{\sqrt{2}} \left\{ V^*_{ub} V_{ud} [a_2 F_{LL}^{0K_0} + C_2 M_{LL}^{M_{LK_0}}] - V^*_{tb} V_{ts} [(a_3 + a_9 - a_5 - a_7) F_{LL}^{T_K^0}] + (C_4 + C_{10}) M_{LL}^{M_{LK_0}} + (C_6 + C_8) M_{LL}^{M_{LK_0}} + (a_3 - \frac{a_9}{2} - a_5 + \frac{a_7}{2}) F_{LL}^{T_K^0} \right\}, \]  
(A19)
\[ A(B^0 \to K_s^0 \bar{K}^0) = -\frac{G_F}{\sqrt{2}} \left( V_{tb}^* V_{td} \left[(a_4 - \frac{a_{10}}{2}) F_{T K_0^0}^{LL} + (a_6 - \frac{a_8}{2})(F_{T K_0^0}^{SP} + F_{A K_0^0}^{SP}) \right. \right. \]
\[ \left. \left. + \left( C_3 - \frac{C_9}{2} \right) M_{T K_0^0}^{LL} + \left( C_5 - \frac{C_7}{2} \right) (M_{T K_0^0}^{LR} + M_{A K_0^0}^{LR}) + \frac{4}{3} (C_3 + C_4) \right. \right. \]
\[ \left. \left. - \frac{C_9 + C_{10}}{2} - a_5 + \frac{a_7}{2} F_{A K_0^0}^{LL} + (C_3 + C_4 - \frac{C_9 + C_{10}}{2}) M_{A K_0^0}^{LL} \right) \right. \]
\[ \left. + \left( C_6 - \frac{C_8}{2} \right)(M_{A K_0^0}^{SP} + M_{A h}^{SP}) + (a_3 - \frac{a_9}{2} - a_5 + \frac{a_7}{2}) F_{A h}^{LL} \right) \]
\[ \left. \left. + \left( C_4 - \frac{C_{10}}{2} \right) M_{A h}^{LL} \right] \right) \right), \tag{A20} \]

\[ \begin{align*}
A(B^0 \to \bar{K}_0^0 K^0) & = -\frac{G_F}{\sqrt{2}} \left( V_{tb}^* V_{td} \left[(a_4 - \frac{a_{10}}{2}) F_{T h}^{LL} + (a_6 - \frac{a_8}{2})(F_{T h}^{SP} + F_{A h}^{SP}) \right. \right. \\
& \left. \left. + \left( C_3 - \frac{C_9}{2} \right) M_{T h}^{LL} + \left( C_5 - \frac{C_7}{2} \right) (M_{T h}^{LR} + M_{A h}^{LR}) + \frac{4}{3} (C_3 + C_4) \right. \right. \\
& \left. \left. - \frac{C_9 + C_{10}}{2} - a_5 + \frac{a_7}{2} F_{A h}^{LL} + (C_3 + C_4 - \frac{C_9 + C_{10}}{2}) M_{A h}^{LL} \right) \right. \]
\[ \left. \left. + \left( C_6 - \frac{C_8}{2} \right)(M_{A h}^{SP} + M_{A K_0^0}^{SP}) + (a_3 - \frac{a_9}{2} - a_5 + \frac{a_7}{2}) F_{A K_0^0}^{LL} \right) \right) \]
\[ \left. \left. + \left( C_4 - \frac{C_{10}}{2} \right) M_{A K_0^0}^{LL} \right] \right), \tag{A21} \]

\[ A(B_s^0 \to K_s^0 \pi^+ \pi^-) = \frac{G_F}{\sqrt{2}} \left( V_{ub}^* V_{ud} \left[a_1 F_{T K_0^0}^{LL} + C_1 M_{T K_0^0}^{LL} \right] - V_{tb}^* V_{td} \left[(a_4 + a_{10}) F_{T K_0^0}^{LL} \right. \right. \]
\[ \left. \left. + (a_6 + a_8) F_{T K_0^0}^{SP} + (C_3 + C_9) M_{T K_0^0}^{LL} + (C_5 + C_7) M_{T K_0^0}^{LR} \right) \right. \]
\[ \left. \left. + (a_4 - \frac{a_{10}}{2}) F_{A K_0^0}^{LL} + (a_6 - \frac{a_8}{2}) F_{A K_0^0}^{SP} + (C_3 - \frac{C_9}{2}) M_{A K_0^0}^{LL} \right) \right. \]
\[ \left. \left. + (C_5 - \frac{C_7}{2}) M_{A K_0^0}^{LR} \right] \right) \right), \tag{A22} \]

\[ A(B_s^0 \to \bar{K}_0^0 \pi^0 \pi^0) = \frac{G_F}{\sqrt{2}} \left( V_{ub}^* V_{ud} \left[a_2 F_{T K_0^0}^{LL} + C_2 M_{T K_0^0}^{LL} \right] - V_{tb}^* V_{td} \left[(-a_4 - \frac{3a_7}{2}) \right. \right. \]
\[ \left. \left. + \frac{5C_9}{3} + C_{10} \right) F_{T K_0^0}^{LL} - (a_6 - \frac{a_8}{2}) F_{T K_0^0}^{SP} + (a_4 - \frac{a_{10}}{2}) M_{T K_0^0}^{LL} \right. \]
\[ \left. \left. + 3C_8 \right) F_{T K_0^0}^{SP} + (C_5 - \frac{C_7}{2}) M_{A K_0^0}^{LL} + (C_5 - \frac{C_7}{2}) M_{A K_0^0}^{LR} \right] \right), \tag{A23} \]
\[ \mathcal{A}(B_s^0 \to K_0^{*+} K^-) = \frac{G_F}{\sqrt{2}} \left[ V_{ub}^* V_{us} \left[ a_1 F_{T}^{LL} + C_1 M_{TK_0}^{LL} + a_2 F_{AK_0}^{LL} + C_2 M_{AK_0}^{LL} \right] \right. \\
- V_{ub}^* V_{ts} \left[ (a_4 + a_{10}) F_{T}^{LL} + (a_6 + a_8) F_{T}^{SP} + (C_3 + C_9) M_{TK_0}^{LL} \right. \\
+ (C_5 + C_7) M_{TK_0}^{LR} + \left( \frac{4}{3}(C_3 + C_4 - \frac{C_9 + C_{10}}{2}) \right) - a_5 + \frac{a_7}{2} \right) F_{A}^{LL} \\
+ \left( a_6 - \frac{a_8}{2} \right) F_{A}^{SP} + (C_3 + C_4 - \frac{C_9 + C_{10}}{2}) M_{A}^{LL} + (C_5 - \frac{C_7}{2}) M_{A}^{LR} \\
\left. \left. + \left( a_6 - \frac{a_8}{2} \right) F_{T}^{SP} + (C_3 + C_4 - \frac{C_9 + C_{10}}{2}) M_{A}^{LL} + (C_5 - \frac{C_7}{2}) M_{A}^{LR} \right) \right] \\
+ (C_6 + C_8) M_{A}^{LL} \}, \quad (A24) \]

\[ \mathcal{A}(B_s^0 \to K_0^{*-} K^+) = \frac{G_F}{\sqrt{2}} \left[ V_{ub}^* V_{us} \left[ a_1 F_{T}^{LL} + C_1 M_{TK_0}^{LL} + a_2 F_{A}^{LL} + C_2 M_{A}^{LL} \right] \right. \\
- V_{ub}^* V_{ts} \left[ (a_4 + a_{10}) F_{T}^{LL} + (a_6 + a_8) F_{T}^{SP} + (C_3 + C_9) M_{TK_0}^{LL} \right. \\
+ (C_5 + C_7) M_{TK_0}^{LR} + \left( \frac{4}{3}(C_3 + C_4 - \frac{C_9 + C_{10}}{2}) \right) - a_5 + \frac{a_7}{2} \right) F_{A}^{LL} \\
+ \left( a_6 - \frac{a_8}{2} \right) F_{A}^{SP} + (C_3 + C_4 - \frac{C_9 + C_{10}}{2}) M_{A}^{LL} + (C_5 - \frac{C_7}{2}) M_{A}^{LR} \\
\left. \left. + \left( a_6 - \frac{a_8}{2} \right) F_{T}^{SP} + (C_3 + C_4 - \frac{C_9 + C_{10}}{2}) M_{A}^{LL} + (C_5 - \frac{C_7}{2}) M_{A}^{LR} \right) \right] \\
+ (C_6 + C_8) M_{A}^{SP} \}, \quad (A25) \]

\[ \mathcal{A}(B_s^0 \to K_0^{*-} K^+) = -\frac{G_F}{\sqrt{2}} \left[ V_{ub}^* V_{ts} \left[ (a_4 - \frac{a_{10}}{2}) F_{T}^{LL} + (a_6 - \frac{a_8}{2}) F_{T}^{SP} \right. \right. \\
+ (C_3 - \frac{C_9}{2}) M_{TK_0}^{LL} + (C_5 - \frac{C_7}{2}) (M_{TK_0}^{LR} + M_{A}^{LR}) + \left( \frac{4}{3}(C_3 + C_4 \right. \right. \\
- \frac{C_9 + C_{10}}{2}) - a_5 + \frac{a_7}{2} \right) F_{A}^{LL} + (C_3 + C_4 - \frac{C_9 + C_{10}}{2}) M_{A}^{LL} \\
\left. \left. + (C_6 - \frac{C_8}{2}) (M_{A}^{SP} + M_{A}^{SP}) + (a_3 - \frac{a_9}{2} - a_5 + \frac{a_7}{2} F_{A}^{LL} \right) \right] \\
+ (C_4 - \frac{C_{10}}{2}) M_{A}^{LL} \}, \quad (A26) \]

\[ \mathcal{A}(B_s^0 \to K_0^{*-} K^+) = -\frac{G_F}{\sqrt{2}} \left[ V_{ub}^* V_{ts} \left[ (a_4 - \frac{a_{10}}{2}) F_{T}^{LL} + (a_6 - \frac{a_8}{2}) F_{T}^{SP} + F_{A}^{SP} \right. \right. \\
+ (C_3 - \frac{C_9}{2}) M_{TK_0}^{LL} + (C_5 - \frac{C_7}{2}) (M_{TK_0}^{LR} + M_{A}^{LR}) + \left( \frac{4}{3}(C_3 + C_4 \right. \right. \\
- \frac{C_9 + C_{10}}{2}) - a_5 + \frac{a_7}{2} \right) F_{A}^{LL} + (C_3 + C_4 - \frac{C_9 + C_{10}}{2}) M_{A}^{LL} \\
\left. \left. + (C_6 - \frac{C_8}{2}) (M_{A}^{SP} + M_{A}^{SP}) + (a_3 - \frac{a_9}{2} - a_5 + \frac{a_7}{2} F_{A}^{LL} \right) \right] \\
+ (C_4 - \frac{C_{10}}{2}) M_{A}^{LL} \}, \quad (A27) \]
in which $G_F$ is the Fermi coupling constant, $V$’s are the CKM matrix elements. The combinations $a_i$ of Wilson coefficients are defined as

$$a_1 = C_2 + \frac{C_6}{3}, \quad a_2 = C_1 + \frac{C_8}{3}, \quad a_3 = C_3 + \frac{C_6}{3}, \quad a_4 = C_4 + \frac{C_8}{3}, \quad a_5 = C_5 + \frac{C_8}{3}, \quad (A28)$$

$$a_6 = C_6 + \frac{C_8}{3}, \quad a_7 = C_7 + \frac{C_6}{3}, \quad a_8 = C_8 + \frac{C_8}{3}, \quad a_9 = C_9 + \frac{C_8}{3}, \quad a_{10} = C_{10} + \frac{C_8}{3}. \quad (A29)$$

It should be understood that the Wilson coefficients $C$ and the amplitudes $F$ and $M$ for the factorizable and nonfactorizable contributions, respectively, appear in convolutions in momentum fractions and impact parameters $b$.

With the ratio $r_0 = m_0^H/m_B$, the amplitudes from Fig. 1 (a) are written as

$$F_{TK_0}^{LL} = 8\pi C_F m_B^4 f_{K(\pi)}(\zeta - 1) \int dx_B dz \int b_B db_B bdb \phi_B(x_B, b_B)$$

$$\times \left\{ \left[ \sqrt{2}(z - 1)(\phi^* + \phi^T) - (z + 1)\phi \right] E_{a12}(t_{a1}) h_{a1}(x_B, z, b_B, b) + \left[ \phi(2\zeta - x_B) - 2\sqrt{2}\phi^s(\zeta - x_B + 1) \right] E_{a12}(t_{a2}) h_{a2}(x_B, z, b_B, b) \right\}, \quad (A30)$$

$$F_{TK_0}^{LR} = -F_{TK_0}^{LL}, \quad (A31)$$

$$F_{TK_0}^{SP} = 16\pi C_F m_B^4 r_0 f_{K(\pi)} \int dx_B dz \int b_B db_B bdb \phi_B(x_B, b_B)$$

$$\times \left\{ \left[ \phi(2\zeta - x_B) - 2\sqrt{2}\phi^s(\zeta - x_B + 1) \right] E_{a12}(t_{a2}) h_{a2}(x_B, z, b_B, b) \right\}, \quad (A32)$$

$$M_{TK_0}^{LL} = 32\pi C_F m_B^4 \sqrt{2N_c}(\zeta - 1) \int dx_B dz dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B) \phi^A$$

$$\times \left\{ \left[ \phi(1 - x_3 - z) + x_B + x_3 - 1] \phi + \sqrt{2}\zeta^s(\phi^s - \phi^t) \right] E_{a34}(t_{a3}) h_{a3}(x_B, z, x_3, b_B, b_3) + \left[ \phi(2\zeta - x_B) - 2\sqrt{2}\phi^s(\zeta - x_B + 1) \right] E_{a12}(t_{a2}) h_{a2}(x_B, z, b_B, b) \right\}, \quad (A33)$$

$$M_{TK_0}^{LR} = 32\pi C_F m_B^4 r_0 \sqrt{2N_c} \int dx_B dz dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B)$$

$$\times \left\{ \left[ \phi(1 - x_3 - z) + x_B + x_3 - 1] \phi + \sqrt{2}\zeta^s(\phi^s - \phi^t) \right] E_{a34}(t_{a3}) h_{a3}(x_B, z, x_3, b_B, b_3) + \left[ \phi(2\zeta - x_B) - 2\sqrt{2}\phi^s(\zeta - x_B + 1) \right] E_{a12}(t_{a2}) h_{a2}(x_B, z, b_B, b) \right\}, \quad (A34)$$

$$M_{TK_0}^{SP} = 32\pi C_F m_B^4 \sqrt{2N_c}(\zeta - 1) \int dx_B dz dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B) \phi^A$$

$$\times \left\{ \left[ \phi(1 - x_3 - z) + x_B + x_3 - 1] \phi + \sqrt{2}\zeta^s(\phi^s - \phi^t) \right] E_{a34}(t_{a3}) h_{a3}(x_B, z, x_3, b_B, b_3) + \left[ \phi(2\zeta - x_B) - 2\sqrt{2}\phi^s(\zeta - x_B + 1) \right] E_{a12}(t_{a2}) h_{a2}(x_B, z, b_B, b) \right\}, \quad (A35)$$
with the color factor $C_F = 4/3$. The amplitudes from Fig. 1 (b) are written as

$$F_{AK_0}^{LL} = 8\pi C_F m_B^4 f_B \int dz dx_3 \int b db_b db_b$$

$$\times \left\{ [(1 - \zeta)(z - 1)\phi^A + 2\sqrt{\zeta}r_0[(2 - z)\phi^* + z\phi^T]E_{bL}(t_{bL})h_{bL}(z, x_3, b, b_3) + \left[(1 - \zeta)[x_3(1 - \zeta) + \zeta]\phi^A + 2\sqrt{\zeta}r_0\phi^*[(\zeta(x_3 - 1) - x_3)(\phi^P + \phi^T) - (\phi^P - \phi^T)]\right] \times E_{bL}(t_{bL})h_{bL}(z, x_3, b, b_3) \right\},$$

(E36)

$$F_{AK_0}^{LR} = -F_{AK_0}^{LL},$$

(E37)

$$F_{AK_0}^{SP} = 16\pi C_F m_B^4 f_B \int dz dx_3 \int b db_b db_b$$

$$\times \left\{ [(\zeta - 1)\sqrt{\zeta}(z - 1)(\phi^A + \phi^T)]E_{bL}(t_{bL})h_{bL}(z, x_3, b, b_3) + \right\},$$

(E38)

$$M_{AK_0}^{LL} = 32\pi C_F m_B^4 / \sqrt{2N_c} \int dx_B dz dx_3 \int b db_b db_b\phi_B(x_B, b_B)$$

$$\times \left\{ [(\zeta^2(1 - z - x_3) + \zeta(x_B + 2x_3 + z - 1) - (x_B + x_3))\phi^A + \right\},$$

(E39)

$$M_{AK_0}^{LR} = 32\pi C_F m_B^4 / \sqrt{2N_c} \int dx_B dz dx_3 \int b db_b db_b\phi_B(x_B, b_B)$$

$$\times \left\{ [(\zeta - 1)(z + 1)\sqrt{\zeta}(\phi^A + \phi^T)]E_{bL}(t_{bL})h_{bL}(x_B, z, x_3, b, b) + \right\},$$

(E40)

$$M_{AK_0}^{SP} = 32\pi C_F m_B^4 / \sqrt{2N_c} \int dx_B dz dx_3 \int b db_b db_b\phi_B(x_B, b_B)$$

$$\times \left\{ [(z\zeta + z - 1)(\zeta - 1)\phi^A + \right\},$$

(E41)
The amplitudes from Fig. 1 (c) are

\[
F_{T_h}^{LL} = 8 \pi C_F m_B^4 F_{K^\pi}(s)/\mu_s \int d x_B d x_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B) \\
\times \left\{ [(1 - \zeta)((\zeta - 1)x_3 - 1)\phi^A - r_0[(2x_3(\zeta - 1) + \zeta + 1)\phi^P + (\zeta - 1)(2x_3 - 1)\phi^T]] \\
+ E_{c12}(t_{c1}) h_{c1}(x_B, x_3, b_B, b_3) + [\zeta(\zeta - 1)x_B \phi^A + 2r_0(\zeta x_B + \zeta - 1)\phi^P] E_{c12}(t_{c2}) \\
+ h_{c2}(x_B, x_3, b_B, b_3) \right\}, \quad (A42)
\]

\[
F_{T_h}^{SP} = 16 \pi C_F m_B^4 \sqrt{s} F_{K^\pi}(s) \int d x_B d x_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B) \\
\times \left\{ [(\zeta - 1)\phi^A + r_0[x_3(\zeta - 1)(\phi^P - \phi^T) - 2\phi^P]] E_{c12}(t_{c1}) h_{c1}(x_B, x_3, b_B, b_3) \\
+ [(\zeta - 1)x_B \phi^A + 2r_0(\zeta + x_B - 1)\phi^P] E_{c12}(t_{c2}) h_{c2}(x_B, x_3, b_B, b_3) \right\}, \quad (A43)
\]

\[
M_{T_h}^{LL} = 32 \pi C_F m_B^4 \sqrt{2N_c} \int d x_B d z d x_3 \int b_B db_B b 3 db_3 \phi_B(x_B, b_B) \\
\times \left\{ [(\zeta^2 - 1)(x_B + z - 1)\phi^A + r_0[\zeta(x_B + z)(\phi^P + \phi^T) + x_3(\zeta - 1)(\phi^P - \phi^T)] \\
- 2\zeta\phi^P] E_{c34}(t_{c3}) h_{c4}(x_B, z, x_3, b_B, b) + [(1 - \zeta)x_3[(\zeta - 1)\phi^A + r_0(\phi^P + \phi^T)] \\
- (x_B - z)[(\zeta - 1)\phi^A + \zeta r_0(\phi^P - \phi^T))] E_{c34}(t_{c4}) h_{c4}(x_B, z, x_3, b_B, b) \right\}, \quad (A44)
\]

\[
M_{T_h}^{LR} = 32 \pi C_F m_B^4 \sqrt{s}/\sqrt{2N_c} \int d x_B d z d x_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B) \\
\times \left\{ [(\zeta - 1)(x_B + z - 1)(\phi^s + \phi^o)\phi^A + r_0[(\zeta(1 - x_3) + x_3)(\phi^s - \phi^t)(\phi^P + \phi^T)] \\
+ (x_B + z - 1)(\phi^s + \phi^t)(\phi^T - \phi^P)] E_{c34}(t_{c3}) h_{c4}(x_B, z, x_3, b_B, b) \\
+ [(z - x_B)(\phi^s - \phi^t)[(\zeta - 1)\phi^A + r_0(\phi^T - \phi^P)] + (\zeta - 1)r_0 x_3(\phi^s + \phi^t) \\
\times (\phi^P + \phi^T)] E_{c34}(t_{c4}) h_{c4}(x_B, z, x_3, b_B, b) \right\}, \quad (A45)
\]

The amplitudes from Fig. 1 (d) are

\[
F_{Ah}^{LL} = 8 \pi C_F m_B^4 f_B \int d z d x_3 \int b db_b b_3 db_3 \\
\times \left\{ [[(\zeta - 1) [(\zeta - 1)x_3 + 1] \phi \phi^A - 2 \sqrt{s} r_0 \phi^s [(\zeta - 1)x_3 (\phi^P - \phi^T) + 2 \phi^P]] E_{d12}(t_{d1}) \\
+ h_{d1}(z, x_3, b, b_3) + [z^2 2 \sqrt{s} r_0 (\phi^s + \phi^t) \phi^P + (1 - \zeta)(\phi^o \phi^A) - 2(\zeta - 1) \sqrt{s} r_0 (\phi^s - \phi^t) \\
\times \phi^P] E_{d12}(t_{d2}) h_{d2}(z, x_3, b, b_3) \right\}, \quad (A46)
\]
\[ F_{Ah}^{LR} = -F_{Ah}^{LL}, \quad (A47) \]

\[ F_{Ah}^{SP} = 16\pi C_F m_B^4 f_B \int dz dx_3 \int b db dB \phi_B(x_B, b_B) \]

\[ \times \left\{ [2(1 - \zeta)\sqrt{\zeta} \phi^s \phi^A + r_0 \phi[\zeta((z - 1) x_3 + 1)](\phi^P + \phi^T) + \zeta(\phi^P - \phi^T)] E_{d12}(t_{d1}) \right. \]

\[ \times h_{d1}(z, x_3, b, b_3) + \left. \left[ (1 - \zeta)\sqrt{\zeta} \phi^s \phi^A + 2r_0 \zeta(\zeta - 1) + 1\phi^P \right] E_{d12}(t_{d2}) \right. \]

\[ \times h_{d2}(z, x_3, b, b_3) \right\}, \quad (A48) \]

\[ M_{Ah}^{LL} = 32\pi C_F m_B^4 / \sqrt{2N_c} \int dx_B dz dx_3 \int b db dB \phi_B(x_B, b_B) \]

\[ \times \left\{ [(x_B + z - 1)^2 + (x_B + z)\phi^A + \sqrt{\zeta} r_0[(x_3 - \zeta(\zeta - 1))\phi^s - \phi^t](\phi^P + \phi^T) \right. \]

\[ + (\phi^T) + (x_B + z - 1)(\phi^s + \phi^t)(\phi^P - \phi^T) - 4\phi^t \phi^P] E_{d34}(t_{d3}) h_{d3}(x_B, z, x_3, b_B, b) \]

\[ + [(1 - \zeta)\sqrt{\zeta} (x_3 - x_B + z - 1) - x_3 + 1\phi^A - \sqrt{\zeta} r_0[(x_B - z)(\phi^s - \phi^t)(\phi^P + \phi^T) \right. \]

\[ + (1 - x_3)(\zeta - 1)(\phi^s + \phi^t)(\phi^P - \phi^T)] E_{d34}(t_{d4}) h_{d4}(x_B, z, x_3, b_B, b) \right\}, \quad (A49) \]

\[ M_{Ah}^{LR} = 32\pi C_F m_B^4 / \sqrt{2N_c} \int dx_B dz dx_3 \int b db dB \phi_B(x_B, b_B) \]

\[ \times \left\{ [(\zeta - 1)\sqrt{\zeta} (x_B + z - 2)(\phi^s + \phi^t)\phi^A + r_0 \phi[\zeta(x_B + z - 1)](\phi^P + \phi^T) \right. \]

\[ + (x_3 - x_B - 1)(\phi^P - \phi^T) - 2\zeta \phi^P] E_{d34}(t_{d3}) h_{d3}(x_B, z, x_3, b_B, b) \]

\[ + [(\sqrt{\zeta} - 1)(x_B - z)(\phi^s + \phi^t)\phi^A + r_0 \phi[\zeta(x_B - z)(\phi^P + \phi^T) \right. \]

\[ + (1 - x_3)(\zeta - 1)(\phi^P - \phi^T)] E_{d34}(t_{d4}) h_{d4}(x_B, z, x_3, b_B, b) \right\}, \quad (A50) \]

\[ M_{Ah}^{SP} = 32\pi C_F m_B^4 / \sqrt{2N_c} \int dx_B dz dx_3 \int b db dB \phi_B(x_B, b_B) \]

\[ \times \left\{ [(1 - \zeta)[\zeta(x_B + z + x_3 - 2) - x_3 + 1]\phi^A + \sqrt{\zeta} r_0[(x_B + z - 1)(\phi^s - \phi^t)(\phi^P \right. \]

\[ + (x_B + z - 1)(\phi^P - \phi^T) + (x_3 - x_B - 1)(\phi^s + \phi^t)(\phi^P - \phi^T) + 4\phi^s \phi^P] E_{d34}(t_{d3}) h_{d3}(x_B, z, x_3, b_B, b) \]

\[ + [(1 - \zeta)\sqrt{\zeta} (1 - x_3)(\zeta - 1)(\phi^s - \phi^t)(\phi^P + \phi^T) \right. \]

\[ + (x_B - z)(\phi^s + \phi^t)(\phi^P - \phi^T)] E_{d34}(t_{d4}) h_{d4}(x_B, z, x_3, b_B, b) \right\}, \quad (A51) \]

**Appendix B: PQCD Functions**

In this section, we group the functions which appear in the factorization formulas of this work.
With \( \zeta = (1 - \zeta) \), \( \bar{x}_3 = (1 - x_3) \) and \( z = (1 - z) \), the involved hard scales are chosen as

\[
\begin{align*}
&t_{a1} = \max \left\{ m_B \sqrt{z}, \frac{1}{b_B}, \frac{1}{b} \right\}, \quad t_{a2} = \max \left\{ m_B \sqrt{x_B - \zeta}, \frac{1}{b_B}, \frac{1}{b} \right\}, \quad (B1) \\
&t_{a3} = \max \left\{ m_B \sqrt{x_B z}, m_B \sqrt{z(\zeta \bar{x}_3 - x_B)}, \frac{1}{b_B}, \frac{1}{b_3} \right\}, \quad (B2) \\
&t_{a4} = \max \left\{ m_B \sqrt{x_B z}, m_B \sqrt{z(x_B - \zeta \bar{x}_3)}, \frac{1}{b_B}, \frac{1}{b_3} \right\}, \quad (B3) \\
&t_{b1} = \max \left\{ m_B \sqrt{1 - z}, \frac{1}{b}, \frac{1}{b_3} \right\}, \quad t_{b2} = \max \left\{ m_B \sqrt{\zeta + x_3 \zeta}, \frac{1}{b}, \frac{1}{b_3} \right\}, \quad (B4) \\
&t_{b3} = \max \left\{ m_B \sqrt{z(\zeta + x_3 \zeta)}, m_B \sqrt{1 - z(\bar{x}_3 \zeta - x_B)}, \frac{1}{b_B}, \frac{1}{b} \right\}, \quad (B5) \\
&t_{b4} = \max \left\{ m_B \sqrt{z(\zeta + x_3 \zeta)}, m_B \sqrt{z(x_B - \zeta - x_3 \zeta)}, \frac{1}{b_B}, \frac{1}{b} \right\}, \quad (B6) \\
&t_{c1} = \max \left\{ m_B \sqrt{x_3 \zeta}, \frac{1}{b_B}, \frac{1}{b_3} \right\}, \quad t_{c2} = \max \left\{ m_B \sqrt{x_B \zeta}, \frac{1}{b_B}, \frac{1}{b_3} \right\}, \quad (B7) \\
&t_{c3} = \max \left\{ m_B \sqrt{x_B x_3 \zeta}, m_B \sqrt{1 - x_B - z}|x_3 \zeta + \zeta|, \frac{1}{b_B}, \frac{1}{b} \right\}, \quad (B8) \\
&t_{c4} = \max \left\{ m_B \sqrt{x_B x_3 \zeta}, m_B \sqrt{|x_B - z|x_3 \zeta}, \frac{1}{b_B}, \frac{1}{b} \right\}, \quad (B9) \\
&t_{d1} = \max \left\{ m_B \sqrt{1 - x_3 \zeta}, \frac{1}{b}, \frac{1}{b_3} \right\}, \quad t_{d2} = \max \left\{ m_B \sqrt{z \zeta}, \frac{1}{b}, \frac{1}{b_3} \right\}, \quad (B10) \\
&t_{d3} = \max \left\{ m_B \sqrt{\bar{x}_3 \bar{z} \zeta}, m_B \sqrt{1 - (x_3 \zeta + \zeta)(1 - x_B - z)}, \frac{1}{b_B}, \frac{1}{b} \right\}, \quad (B11) \\
&t_{d4} = \max \left\{ m_B \sqrt{\bar{x}_3 \bar{z} \zeta}, m_B \sqrt{|x_B - z|\bar{x}_3 \zeta}, \frac{1}{b_B}, \frac{1}{b} \right\}. \quad (B12)
\end{align*}
\]

The hard functions are written as

\[
\begin{align*}
h_{a1}(x_B, z, b_B, b) &= K_0(m_B \sqrt{x_B z} b_B) \left[ \theta(b_B - b) K_0(m_B \sqrt{z} b_B) I_0(m_B \sqrt{z} b) \right. \\
&\left. + (b \leftrightarrow b_B) \right] S_t(z), \\
h_{a2}(x_B, z, b_B, b) &= K_0(m_B \sqrt{x_B z} b_B) S_t(x_B) \\
&\times \begin{cases} \\
\frac{1}{\pi^2} \left[ \theta(b_B - b) H_0^{(1)}(m_B \sqrt{\zeta - x_B} b_B) I_0(m_B \sqrt{\zeta - x_B} b_B) \right. \\
+ (b \leftrightarrow b_B), & \zeta < x_B \\
\left. \left[ \theta(b_B - b) K_0(m_B \sqrt{x_B - \zeta} b_B) I_0(m_B \sqrt{x_B - \zeta} b_B) \right. \\
+ (b \leftrightarrow b_B), & \zeta \geq x_B. \end{cases} \quad (B13)
\end{align*}
\]
\begin{align}
h_{a3}(x_B, z, x_3, b_B, b_3) & = \left[ \theta(b_B - b_3) K_0(m_B \sqrt{x_B z b_B}) I_0(m_B \sqrt{x_B z b_3}) + (b_B \leftrightarrow b_3) \right] \\
& \times \begin{cases} 
\frac{i \pi}{2} H_0^{(1)}(m_B \sqrt{z [\zeta x_3 - x_B]} b_3), & \zeta x_3 > x_B, \\
K_0(m_B \sqrt{z [x_B - \zeta x_3]} b_3), & \zeta x_3 \leq x_B.
\end{cases} \\
\text{(B15)}
\end{align}

\begin{align}
h_{a4}(x_B, z, x_3, b_B, b_3) & = \left[ \theta(b_B - b_3) K_0(m_B \sqrt{x_B z b_B}) I_0(m_B \sqrt{x_B z b_3}) + (b_B \leftrightarrow b_3) \right] \\
& \times \begin{cases} 
\frac{i \pi}{2} H_0^{(1)}(m_B \sqrt{z [x_3 \zeta - x_B]} b_3), & x_3 \zeta > x_B, \\
K_0(m_B \sqrt{z [x_B - x_3 \zeta]} b_3), & x_3 \zeta \leq x_B.
\end{cases} \\
\text{(B16)}
\end{align}

\begin{align}
h_{b1}(z, x_3, b, b_3) & = \left( \frac{i \pi}{2} \right)^2 H_0^{(1)}(m_B \sqrt{\bar{z} (\zeta + x_3 \bar{\zeta}) b_3}) S_i(z) \\
& \times \left[ \theta(b - b_3) H_0^{(1)}(m_B \sqrt{1 - z b}) J_0(m_B \sqrt{1 - z b_3}) + (b \leftrightarrow b_3) \right], \\
\text{(B17)}
\end{align}

\begin{align}
h_{b2}(z, x_3, b, b_3) & = \left( \frac{i \pi}{2} \right)^2 H_0^{(1)}(m_B \sqrt{\bar{z} (\zeta + x_3 \bar{\zeta}) b}) S_i(x_3) \left[ \theta(b - b_3) \\
& \times H_0^{(1)}(m_B \sqrt{\bar{z} (\zeta + x_3 \bar{\zeta}) b_3}) J_0(m_B \sqrt{\bar{z} (\zeta + x_3 \bar{\zeta}) b_3}) + (b \leftrightarrow b_3) \right], \\
\text{(B18)}
\end{align}

\begin{align}
h_{b3}(x_B, z, x_3, b_B, b) & = \frac{i \pi}{2} K_0(m_B \sqrt{1 - z (\bar{\zeta} + x_3 \bar{\zeta} - x_B) b_B}) \left[ \theta(b_B - b) \\
& \times H_0^{(1)}(m_B \sqrt{\bar{z} (\zeta + x_3 \bar{\zeta} - x_B) b_B}) J_0(m_B \sqrt{\bar{z} (\zeta + x_3 \bar{\zeta} - x_B) b_B}) + (b_B \leftrightarrow b) \right], \\
\text{(B19)}
\end{align}

\begin{align}
h_{b4}(x_B, z, x_3, b_B, b) & = \frac{i \pi}{2} \left[ \theta(b_B - b) H_0^{(1)}(m_B \sqrt{\bar{z} (\zeta + x_3 \bar{\zeta}) b}) \\
& \times J_0(m_B \sqrt{\bar{z} (\zeta + x_3 \bar{\zeta}) b}) + (b_B \leftrightarrow b) \right] \\
& \times \begin{cases} 
\frac{i \pi}{2} H_0^{(1)}(m_B \sqrt{\bar{z} (\zeta + x_3 \bar{\zeta} - x_B) b_B}), & x_B < \zeta + x_3 \bar{\zeta}, \\
K_0(m_B \sqrt{\bar{z} (x_B - \zeta - x_3 \bar{\zeta}) b_B}), & x_B \geq \zeta + x_3 \bar{\zeta},
\end{cases} \\
\text{(B20)}
\end{align}

\begin{align}
h_{c1}(x_B, x_3, b_B, b_3) & = K_0(m_B \sqrt{x_B x_3 \bar{\zeta} b_B}) \left[ \theta(b_B - b_3) K_0(m_B \sqrt{x_3 \bar{\zeta} b_B}) \\
& \times I_0(m_B \sqrt{x_3 \bar{\zeta} b_3}) + (b_3 \leftrightarrow b_B) \right] S_i(x_3), \\
\text{(B21)}
\end{align}

\begin{align}
h_{c2}(x_B, x_3, b_B, b_3) & = h_{c1}(x_3, x_B, b_B, b_3), \\
\text{(B22)}
\end{align}

\begin{align}
h_{c3}(x_B, z, x_3, b_B, b) & = \left[ \theta(b_B - b) K_0(m_B \sqrt{x_B x_3 \bar{\zeta} b_B}) I_0(m_B \sqrt{x_B x_3 \bar{\zeta} b}) + (b_B \leftrightarrow b) \right] \\
& \times \begin{cases} 
\frac{i \pi}{2} H_0^{(1)}(m_B \sqrt{1 - x_B - z} [x_3 \zeta + \bar{\zeta} b]), & x_B + z < 1, \\
K_0(m_B \sqrt{(x_B + z - 1) [x_3 \zeta + \bar{\zeta} b]}), & x_B + z \geq 1,
\end{cases} \\
\text{(B23)}
\end{align}

\begin{align}
h_{c4}(x_B, z, x_3, b_B, b) & = \left[ \theta(b_B - b) K_0(m_B \sqrt{x_B x_3 \bar{\zeta} b_B}) I_0(m_B \sqrt{x_B x_3 \bar{\zeta} b}) + (b_B \leftrightarrow b) \right] \\
& \times \begin{cases} 
\frac{i \pi}{2} H_0^{(1)}(m_B \sqrt{x_3 (z - x_B) \bar{\zeta} b}), & x_B < z, \\
K_0(m_B \sqrt{x_3 (x_B - z) \bar{\zeta} b}), & x_B \geq z,
\end{cases} \\
\text{(B24)}
\end{align}
\[ h_{d1}(z, x_3, b, b_3) = \left( \frac{i \pi}{2} \right)^2 H_0^{(1)}(m_B \sqrt{x_3 z} b) S_t(x_3) [\theta(b - b_3) \times H_0^{(1)}(m_B \sqrt{1 - x_3 \zeta^2} b) J_0(m_B \sqrt{1 - x_3 \zeta b_3}) + (b \leftrightarrow b_3)], \]  
\[ h_{d2}(z, x_3, b, b_3) = \left( \frac{i \pi}{2} \right)^2 H_0^{(1)}(m_B \sqrt{x_3 z} \zeta b_3) S_t(z) \times [\theta(b - b_3) H_0^{(1)}(m_B \sqrt{z} \zeta b) J_0(m_B \sqrt{z} \zeta b_3) + (b \leftrightarrow b_3)], \]  
\[ h_{d3}(x_B, z, x_3, b, b_3) = \frac{i \pi}{2} K_0(m_B \sqrt{1 - x_3 (1 - x_B - z) \zeta + (x_B + z - 1) \zeta b_B}) \times [\theta(b - b) H_0^{(1)}(m_B \sqrt{x_3 z} \zeta b) J_0(m_B \sqrt{x_3 z} \zeta b_3) + (b \leftrightarrow b)], \]  
\[ h_{d4}(x_B, z, x_3, b, b_3) = \frac{i \pi}{2} [\theta(b - b) H_0^{(1)}(m_B \sqrt{x_3 z} \zeta b) J_0(m_B \sqrt{x_3 z} \zeta b_3) + (b \leftrightarrow b)] \times \begin{cases} \frac{i \pi}{2} H_0^{(1)}(m_B \sqrt{x_3 (z - x_B) \zeta b}), & x_B < z, \\ K_0(m_B \sqrt{x_3 (x_B - z) \zeta b_3}), & x_B \geq z, \end{cases} \]  

where \( H_0^{(1)}(\chi) = J_0(\chi) + i Y_0(\chi) \). The factor \( S_t(\chi) \) with the expression [149]  
\[ S_t(\chi) = \frac{2^{1+2c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [\chi(1 - \chi)]^c, \]  
resums the threshold logarithms \( \ln^2 \chi \) appearing in the hard kernels to all orders, and the parameter \( c \) has its expression as \( c = 0.04 Q^2 - 0.51 Q + 1.87 \) with \( Q^2 \) the invariant mass square of the final state \( f \) in the \( B \to f \) transition \([135, 150]\).

The evolution factors in the factorization expressions are given by  
\[ E_{a12}(t) = \alpha_s(t) \exp[\frac{1}{2} S_B(t)], \]  
\[ E_{a34}(t) = \alpha_s(t) \exp[\frac{1}{2} S_{K_0}(t) - S_B(t)], \]  
\[ E_{b12}(t) = \alpha_s(t) \exp[\frac{1}{2} S_{K_0}(t) - S_{h}(t)], \]  
\[ E_{b34}(t) = \alpha_s(t) \exp[\frac{1}{2} S_{K_0}(t) - S_{K_3}(t)], \]  
\[ E_{c12}(t) = \alpha_s(t) \exp[\frac{1}{2} S_{B}(t) - S_{K_3}(t)], \]  
\[ E_{c34}(t) = \alpha_s(t) \exp[\frac{1}{2} S_{B}(t) - S_{K_0}(t)], \]  
\[ E_{d12}(t) = E_{b12}(t), \]  
\[ E_{d34}(t) = E_{b34}(t), \]  
in which the Sudakov exponents are defined as  
\[ S_B = s \left( x_B \frac{m_B}{\sqrt{2}}, b_B \right) + \frac{5}{3} \int_{1/b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \]  
\[ S_{K_0} = s \left( z \frac{m_B}{\sqrt{2}}, b \right) + s \left( 1 - z \frac{m_B}{\sqrt{2}}, b \right) + 2 \int_{b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \]  
\[ S_{h} = s \left( x_3 \frac{m_B}{\sqrt{2}}, b_3 \right) + s \left( 1 - x_3 \frac{m_B}{\sqrt{2}}, b_3 \right) + 2 \int_{1/b_3}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \]
with the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$. The explicit form for the function $s(Q, b)$ is [143]

$$s(Q, b) = \frac{A^{(1)}}{2\beta_1} \hat{q} \ln\left(\frac{\hat{q}}{\hat{b}}\right) - \frac{A^{(1)}}{2\beta_1} \left(\hat{q} - \hat{b}\right) + \frac{A^{(2)}}{4\beta_1^2} \left(\frac{\hat{q}}{\hat{b}} - 1\right) - \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1} \ln\left(\frac{e^{2\gamma_E - 1}}{2}\right)\right]$$

$$\times \ln\left(\frac{\hat{q}}{\hat{b}}\right) + \frac{A^{(1)}\beta_2}{4\beta_1^3} \hat{q} \left[\ln(2\hat{q}) + 1 - \ln(2\hat{b}) + 1\right] + \frac{A^{(1)}\beta_2}{8\beta_1^3} \left[\ln^2(2\hat{q}) - \ln^2(2\hat{b})\right],$$

(B41)

with the variables are

$$\hat{q} \equiv \ln[Q/(\sqrt{2}\Lambda)], \quad \hat{b} \equiv \ln[1/(b\Lambda)],$$

(B42)

and the coefficients $A^{(i)}$ and $\beta_i$ are

$$\beta_1 = \frac{33 - 2n_f}{12}, \quad \beta_2 = \frac{153 - 19n_f}{24}, \quad A^{(1)} = \frac{4}{3},$$

$$A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1\ln(\frac{1}{2}e^{\gamma_E}),$$

(B43)

where $n_f$ is the number of the quark flavors and $\gamma_E$ is the Euler constant.

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