monitoring condition of soils will speed up the process of collecting and predicting analysis of parameters of the state of soils, thereby optimizing the procedure of their certification and technical supervision over their use. The development of information database for monitoring the pollution of soils through the use of operational methods of determining their characteristics ensures an appropriate level of their control and, therefore, is absolutely relevant [4–6]. In this work, we conducted an analysis of possibilities of applying admittance method to assess one of the most important parameters of soil, namely, its acidity.

2. Literature review and problem statement

There are no generally accepted guidelines for the parameters of soils and the methods of their study for operational provision of the needs of functioning of monitoring information systems. Classic physical and chemical methods

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**Mathematical Modeling of Soil Acidity by the Admittance Parameters**

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are usually implemented in laboratories and unsuitable for field conditions [7]. The methods using bioindicators are worth noting as they rapidly develop at present [8, 9]. However, they are considered to be labour intensive and not suitable for operational control. That is why it is expedient to use fast electrical methods, one of which is a method of measuring electrical conductivity of an object as it is a source of information about the properties of soil. Most authors prefer to control the humidity of soil. For this purpose it was even established an international network of soil humidity [10], which helps its users to receive necessary information about this important indicator of soil. Some authors [11, 12] measure humidity based on the control of its electrical conductivity. Other authors [13, 14] propose to determine contaminants of soil by conductivity, which also does not fully reflect the main components on which its quality depends. There were also studies conducted, in which authors present results of measurement of humidity and temperature of soil, but using certain specialized sensors [15, 16] that belong to wireless sensors networks. However, in addition to the above described parameters of soil, very important is the degree of its acidity and salinity, which must be also operationally determined. The values of these characteristics are usually determined by the use of physical-chemical methods of conductometry [17] and potentiometry [18, 19], respectively. These studies in real life are an extremely labour-consuming task. Thus it is promising to examine a possibility of estimation of the values of acidity and soil salinity using operational admittance method by establishing a dependence of the acidity of soil and the degree of its salinity on the admittance parameters (complex conductivity). It comes down to building a mathematical model that takes into account the inherent characteristics of soil. Therefore, a study of the mathematical model will allow us to discover, analyze and substantiate essential characteristics of the examined system of primary transducer and control object (soil).

3. The aim and tasks of the study

For the study, we set the task to assess a possibility of determining the acidity of different types of soils by the admittance parameters.

To achieve this aim, the following tasks were to be solved:
- to conduct experimental research into a possibility of using admittance method for the evaluation of soil acidity;
- based on the results of experimental research, to build a mathematical model for the evaluation of chernozem soil acidity, loamy and sandy soil;
- to estimate errors of the obtained mathematical models for operational evaluation of soil acidity using the admittance method.

4. Object and methods of research

The object of research is the level of acidity of black soil, loamy and sandy soil, which is necessary to create maps of admittance. The maps of admittance of soils (taking into account frequency dependence of both its active and reactive component) will highlight contrast zones according to the type and composition of the soil. Although the absolute values of admittance will increase with increasing soil humidity, but the relative values will remain adequate over certain time periods.

Therefore, the admittance mapping of soils is a comprehensive description of change in soil conditions within certain areas, the consideration of which is important when making such decisions as:
- determining the boundaries of soil contours that are especially important for organic production;
- establishing homogeneous field areas by the acidic properties of soil in accordance with the values of admittance;
- specifying the contours of potentially acidified field areas;
- the possibility to control correctness of the application of fertilizers.

Experimental studies were conducted using an admittance measuring device with a primary capacitive transducer. The range of the examined frequencies was: 50 50 Hz–100 kHz, voltage level of the test signal: 0,01–2 V, material of the electrodes is stainless steel, the distance between electrodes was 10 mm, the length of the working part of the transducer is 60 mm.

A basic characteristic of the examined soil was the level of acidity of soil H. An application of the admittance method is to use a certain test signal of voltage U, the feedback of which was assessed at the fixed values of frequency F. Parameters of a primary converter of capacitive type can be represented by its geometrical dimensions, namely constant K, which is determined by the ratio of distance between the electrodes l to the area of these electrodes S. Thus, we will explore a possibility of evaluation the acidity of the soil by the magnitude of voltage of the test signals U, their frequency F and constant K that characterizes geometric dimensions of the electrodes.

5. The results of research into acidity of soils by the admittance parameters

Conducted experimental research demonstrated [20] that a change in response signal for the system does not significantly depend on the changing values of test voltage U. In addition, a research is primarily performed with the same primary transducer, that is, the constant K, which characterizes geometric dimensions of the electrodes, may be considered unchanged. In this work, using the method of admittance, we present the results of examining a dependence of the response signal on the acidity of soil and the frequency of the test signal.

Experimental studies were carried out for various combinations of values of the specified parameters, namely: acidity was changed from 4.8 to 8.5 pH, frequency – from 0.05 kHz to 100 kHz. An analysis of dependence of the response signal was conducted at a fixed temperature value. For convenience of processing the results of the experiment, the value of frequency of the test signal was expressed through logarithm – $\log F$.

Fig. 1 displays dependence of the active component of admittance G and the reactive component of admittance B, respectively, on the frequency of the test signal F for different fixed values of acidity H for black soil.

Fig. 2, 3 demonstrate dependences of the active and reactive components of admittance, respectively, on the frequency of the test signal F for different fixed values of acidity H for sandy and loamy soils.
It follows from the frequency dependences of the components of admittance $G$ and $B$ for the fixed acidity values $H$ for different types of soil, represented in Fig. 1–3, that with the increasing frequency of signals, the response signals for different values of acidity, starting from the frequency of 5 kHz, are almost the same. That is why, for the assessment of soil acidity using the admittance method, we will refer to the value of the active component of admittance $G$.

According to the results of experimental data from Fig. 1, a–3, a, we obtained a dependence of the active component of admittance $G$ on the acidity $H$ and the frequency of the test signal $F$

$$G = f(H, F)$$

for different types of soil.

As a result of the performed research, to describe dependences (1) by the method of uniform approximation of functions of two variables [21], we obtained, with satisfactory accuracy, models for black soil, loamy and sandy soil. The use of the uniform approximation method is predetermined by the fact that this method provides for the possibility of receiving a model, which, at the lowest possible error, reproduces the required dependence on the given set of points. An error of the model, obtained by the method of uniform approximation, is less than the error of the same model determined by the method of least squares on the given set of points [22].

To describe dependence of the active component of admittance $G$ for black soil, we obtained a model by the method of uniform approximation

$$G = a_1 + a_2 \log(F) + a_3 H + a_4 H \log(F) + a_5 H^2,$$

where

$$a_1 = -0.0002360279.$$
\[ a_2 = 0.000733464, \]
\[ a_3 = -0.0000197652, \]
\[ a_4 = -0.000025562, \]
\[ a_5 = -0.0000693467. \]

The accuracy of description by this model of dependence of the active component of admittance \( G \) is 7.3%. The surface of distribution of relative error of model (2) depending on the frequency of the signal and acidity, respectively, is depicted in Fig. 4.

In this Fig. 4, \( \Delta \) is the relative error, which equals \( \Delta = \delta / 100 \), where \( \delta \) is the relative error as a percentage.

It follows from Fig. 4 that the error of model (2) is evenly distributed among all points of observation. In this case, the largest by module positive and negative deviations of the obtained approximation from the values of the active component of admittance \( G \) almost match. This peculiarity is characteristic for uniform approximation. For comparison, let us note that the error of model of the form (2), defined by the method of least squares, amounted to 10.3%.

To describe dependence of the active component of admittance \( G \) for loamy soil, we received model by the uniform approximation method

\[ G = a_1 + a_2 \lg(F) + a_3 H + a_4 H \lg(F) + a_5 H^2, \]  

where

\[ a_1 = 0.0017015822, \]
\[ a_2 = 0.0001979279, \]
\[ a_3 = -0.0015917013, \]
\[ a_4 = -0.00002563384, \]
\[ a_5 = 0.00009102326. \]

The accuracy of description by this model of dependence of the active component of admittance \( G \) is 8%. An error of model of the form (3), determined by the method of least squares, was 10.1%.

To describe dependence of the active component of admittance \( G \) for sandy soil, we received model by the uniform approximation method

\[ G = a_1 + a_2 \lg(F) + a_3 H + a_4 H \lg(F) + a_5 H^2, \]  

\[ H = f_5(G, F) \]  

for different types of soil.

A graph of dependence of the soil acidity \( H \) on the value of the active component of admittance \( G \) and the signal frequency \( F \) for black soil is depicted in Fig. 5. It follows from this figure that black soil acidity is a strictly monotonously increasing function from the active component of admittance and the signal frequency. Similar character of acidity is also inherent for loamy and sandy soils.

As a result of the conducted research, for the evaluation of value of soil acidity, by the method of uniform approximation of functions of two variables [22], we obtained, with satisfactory accuracy, models for six to five parameters for black soil, loamy and sandy soils. To evaluate the value of acidity of black soil by the active component of admittance \( G \) and the signal frequency \( F \), by the uniform approximation method, we obtained model

\[ H = b_1 + b_2 \lg(F) + b_3 G + b_4 \lg(F)G + b_5 \lg(F)^2 + b_6 G^2, \]  

where

\[ b_1 = 0.0001220975, \]
\[ b_2 = 0.0105903364, \]
\[ b_3 = -0.00016815525, \]
\[ b_4 = -0.00014772322, \]
\[ b_5 = 0.0000106396. \]

The obtained models (2)–(4) confirm the dependence of the active component of admittance \( G \) on the soil acidity and the signal frequency. Therefore, the value of the active component of admittance \( G \) may be applied to estimate the level of acidity of the soil. For this purpose, it is necessary to get dependence of the soil acidity \( H \) on the value of the active component of admittance \( G \) and the signal frequency \( F \) for different types of soil.
where

\[ b_1 = 3.165639347, \]
\[ b_2 = 6.15834065, \]
\[ b_3 = -15008.46390, \]
\[ b_4 = -979.0720272, \]
\[ b_5 = -0.6483059431, \]
\[ b_6 = 6512820.841. \]

The accuracy of description by this particular model of the estimation of acidity of black soil depending on the active component of admittance \( G \) and the signal frequency is 10.2 %.

An error of model of the form (6), defined by the method of least squares, was 15.8 %.

To assess the value of acidity of loamy soil by the active component of admittance \( G \) and the signal frequency, by the method of uniform approximation, we obtained model

\[ H = b_1 + b_2 \log(F) + b_3G + b_4\log(F)G + b_5G^2, \] (7)

where

\[ b_1 = 9.016382468, \]
\[ b_2 = 0.1493079751, \]
\[ b_3 = -10762.61881, \]
\[ b_4 = -150.7089874, \]
\[ b_5 = 8910312.049. \]

The accuracy of description by this model of the evaluation of acidity of loamy soil depending on the active component of admittance \( G \) and the signal frequency is 1.7 %.

An error of model of the form (7), defined by the method of least squares, was 2.2 %.

To assess the value of sandy soil acidity by the active component of admittance \( G \) and the signal frequency, by the uniform approximation method, we obtained model

\[ H = b_1 + b_2 \log(F) + b_3G + b_4\log(F)G + b_5G^2, \] (8)

where

\[ b_1 = 8.778029461, \]
\[ b_2 = 0.02221437932, \]
\[ b_3 = -6650.751506, \]
\[ b_4 = 103.9815459, \]
\[ b_5 = 4806411.277. \]

The accuracy of description by this model of the evaluation of sandy soil acidity depending on the active component of admittance \( G \) and the signal frequency is 1.5 %. An error of model of the form (8), defined by the method of least squares, amounted to 1.7 %.

6. Discussion of results of research into mathematical models of dependence of the acidity of soils on the admittance parameters

Based on the processing of results of experimental research into parameters of admittance, it was established that the active component \( G \) is the informative parameter in a wide frequency range on the level of acidity of soil. In accordance with this, we obtained mathematical models (2)–(4), which describe with sufficient accuracy dependence of the active component of admittance \( G \) on the signal frequency and acidity of different types of soils.

Namely, mathematical models (2)–(4) describe the dependences of the active component of admittance on the signal frequency and acidity of black soil, sandy and loamy soils. Based on this, we solved a direct problem, namely, we obtained models (6)–(8) to assess the acidity of soil depending on the value of the active component of admittance \( G \) and the frequency of test signal \( F \). Model (6) provides for receiving the evaluation of black soil acidity with accuracy of 10.2 %, model (7) provides for obtaining the evaluation of loamy soil acidity with accuracy of 1.7 %, and model (8) provides for getting the evaluation of sandy soil acidity with accuracy of 1.5 %.

The use of models (6)–(8) makes it possible to establish effective operational control over the acidity of soils through the use of admittance mapping. The admittance maps provide for the possibility of operational monitoring of soils under field conditions while the classic physical and chemical methods of determining the level of acidity of soil are usually applied in laboratories and are not suitable to support information systems of monitoring in the field.

In addition to acidity, a not less important indicator of condition of the soil is the degree of its salinity. That is why the consequent research will address the dependence of parameters of admittance on the salinity of various types of soils.

7. Conclusions

1. An analysis of experimental results obtained using the admittance method confirmed the possibility of its application for the evaluation of acidity of soil; it is demonstrated that the frequency dependences of parameters of admittance are the important informative parameter to control the acidity of soil.

2. Using the results of experimental research, we constructed mathematical models which with sufficient accuracy describe dependence of the active component of admittance \( G \) on the frequency of the test signal and the acidity of black soil, sandy and loamy soils. We solved a practical problem, in particular, mathematical models were received for the evaluation of acidity of soils depending on the value of the active component of admittance \( G \) and the frequency of the test signal. These analytical dependences take the form of polynomials of the corresponding order.

3. In the paper, we performed an assessment of errors of the derived mathematical models for the operational assessment of soil acidity using the admittance method. When
constructing the models, we used the method of uniform approximation since it provides for obtaining models with lower errors than while using the method of least squares. Namely, the accuracy of model for obtaining the evaluation of the acidity of black soil is 10.2%, the accuracy of model for receiving the assessment of loamy soil acidity is 1.7%, and the accuracy of model for getting the evaluation of sandy soil acidity is 1.5%. Low values of the errors of these models confirm the possibility to apply the admittance method to control the acidity of soils.

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