Enhancement of the space of solutions of the kinetic-conformal Hořava theory

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Abstract. We show that in the Horava theory at the kinetic-conformal point there are more solutions of the field equations in the Hamiltonian formalism than in the original Lagrangian formalism. To hold this we add all the constraints, primary and secondary, to the Hamiltonian. There are certain configurations for the Lagrange multipliers associated to the secondary constraints that lead to solutions that cannot be found in the original Lagrangian formulation. We show specific examples in vacuum and with a source. The solution with the source has homogeneous and isotropic spatial hypersurfaces. Here we relax one condition on one Lagrange multiplier, showing the existence of more special solutions on the Hamiltonian side.

1. Introduction

The kinetic-conformal formulation \cite{1} of the (nonprojectable) Hořava theory \cite{2,3} is defined in 3 + 1 dimensions by setting the value $\lambda = 1/3$, where $\lambda$ is the coupling constant arising in the kinetic term of the Lagrangian. The particularity of this value is that, when it is set, two additional second-class constraints arise, implying that the extra scalar mode is eliminated. Therefore, the theory shares the same number of degrees of freedom with General Relativity, but with different dynamics and symmetries. The name kinetic-conformal is inspired by the fact that at the $\lambda = 1/3$ value the kinetic term of the Lagrangian gets an anisotropic conformal symmetry \cite{2}, although the whole theory is not conformally invariant.

Previous studies on the Hamiltonian formulation of the theory can be found in Refs. \cite{1,4}. Within the features of the Hamiltonian formulation, in this paper we point out that in the kinetic-conformal theory there are more classical solutions in the Hamiltonian formulation than in the original Lagrangian formulation. In the Hamiltonian formulation we add all the constraints, primary and secondary, to the Hamiltonian with Lagrange multipliers. An special configuration for a Lagrange multiplier allows the enhancement of the space of solutions. We give concrete examples by studying vacuum (i. e., purely gravitational) configurations and configurations with matter sources. In the first case we find examples with static solutions. In the second case we incorporate a homogeneous and isotropic perfect fluid. We present a solution that has homogeneous and isotropic spatial slices with a scale factor for the spatial metric that is time dependent. These solutions are given by fixing another Lagrange multiplier different to the one that leads to the special Hamiltonian solutions. Here we relax this, and also discuss a more general setting for the Lagrange multipliers within the special Hamiltonian solutions. In this case we show the existence of more special Hamiltonian solutions.
The enhancement of the space of solutions on the Hamiltonian side may have incidence on the problem of cosmological backgrounds in the kinetic-conformal theory. It is known that a fully homogeneous and isotropic spacetime coupled to an homogeneous and isotropic perfect fluid admits as its only solution an empty fluid, i. e., zero density and pressure (for the flat case) [3, 5, 6] (see also [7, 8] for an analogous result in the Einstein-aether theory). However, this result is based on the Lagrangian field equations. The result we present here leaves open the possibility that more homogeneous and isotropic configurations can be found in the Hamiltonian side.

2. Hamiltonian of the kinetic-conformal Hořava theory

The Hořava theory is written in terms of the ADM variables $N$, $N_1$ and $g_{ij}$. We consider the nonprojectable version where the lapse function $N$ is a function of the time and the space. We denote by $\pi^{ij}$ the canonically conjugate momentum of $g_{ij}$ and by $P_N$ the one of $N$, which is zero. We consider only the terms that are of second order in spatial derivatives (the theory is of second order in time derivatives).

The complete Hamiltonian of the kinetic-conformal theory, with all the constraints added, is

$$ H = \int d^3x \left[ \sqrt{g} N \left( \frac{1}{g} \pi^{ij} \pi_{ij} - \beta R - \alpha_0 a^i \right) + N_i \mathcal{H}^i + \sigma P_N + \mu \pi + A_1 C_1 + A_2 C_2 \right] $$

where $R$ is the spatial scalar curvature and $a_i = \partial_i \ln N$. $\beta$ and $\alpha$ are coupling constants. $\sigma$, $\mu$, $A_1$ and $A_2$ are the Lagrange multipliers of the constraints

$$
P_N = 0, \quad \pi = 0,
$$

$$
\frac{1}{\sqrt{g}} C_1 = \frac{2}{g} \pi^{ij} \pi_{ij} - \frac{\sqrt{g}}{N} \gamma_1 \frac{\nabla^2 N}{N} = 0,
$$

$$
\frac{1}{\sqrt{g}} C_2 = \gamma_2 \frac{\sqrt{g}}{N} \frac{\nabla^2 N}{N} - \alpha_0 a^i - \beta R = 0,
$$

where

$$
\gamma_1 \equiv 2\beta - \alpha, \quad \gamma_2 \equiv \beta + \frac{3\alpha}{2}.
$$

$\mathcal{H}^i$ is the momentum constraint,

$$
\mathcal{H}^i \equiv -2\nabla_i \pi^{ij} + P_N \partial_j N = 0.
$$

By applying Dirac’s procedure for the time preservation of the constraints, we obtain that the preservation of the four second-class constraints (2 - 4) leads to the following four equations for the Lagrange multipliers,

$$
0 = \nabla^2 B + 2\alpha a^k \nabla_k \left( \frac{A_2}{N} \right) + \frac{\gamma_1}{2} \nabla^2 N \left( 2A_1 - A_2 \right),
$$

$$
0 = \frac{4\beta}{N} \nabla^2 A_2 - a_k \nabla^2 a^k - \frac{\nabla^2 N}{N^2} \left( 3\gamma_1 A_1 + \gamma_2 A_2 \right),
$$

$$
0 = \frac{\gamma_1}{\sqrt{g}} \left( \nabla^2 \sigma - \frac{\nabla^2 N}{N} \sigma \right) + \left( \frac{3}{\sqrt{g}} \pi^{ij} \pi_{ij} + \frac{\gamma_1}{2} \sqrt{g} a_k a^k \right) \mu
$$

$$
- \left\{ C_1, \left( N + 2A_1 \right) \frac{\pi^{ij} \pi_{ij}}{\sqrt{g}} - \sqrt{g} \left( N + A_2 \right) \left( \beta R + \alpha_0 a^k \right) + \sqrt{g} B \nabla^2 N \right\},
$$

$$
0 = \frac{\gamma_2}{\sqrt{g}} \left( \nabla^2 \sigma - \frac{\nabla^2 N}{N} \sigma \right) + 2\alpha \nabla_k a^k \left( \partial_k \sigma + a_k \sigma \right) + 2\beta \sqrt{g} \nabla^2 \mu
$$

$$
- \frac{\sqrt{g}}{2} \left( \beta R + (\alpha - \gamma_2) a_k a^k \right) \mu + \left\{ C_2, \left( N + 2A_1 \right) \frac{\pi^{ij} \pi_{ij}}{\sqrt{g}} \right\}. 
$$

2
where

$$B \equiv -\frac{1}{N}(\gamma_1A_1 - \gamma_2A_2) .$$

We now present the evolution equations. Taking variations with respect to $N$ reproduces exactly the Eq. (7). Variations with respect to $\pi^{ij}$, $g_{ij}$, and $P_N$ yield, respectively, the equations

$$\dot{g}_{ij} = \frac{2}{\sqrt{g}}(N + 2A_1)\pi_{ij} + \mu g_{ij} ,$$

$$\dot{\pi}^{ij} = -\frac{1}{g}(N + 2A_1)(2\pi^{ik}\pi^{kj} - \frac{1}{2}g^{ij}\pi^k\pi_k) - \frac{\pi^{ij}}{\sqrt{g}} - \nabla^{(i}N\nabla^{j)}B + \frac{1}{2}g^{ij}\nabla_kN\nabla^kB + \left[\beta(\nabla^{ij} - g^{ij}\nabla^2) - \beta(R^{ij} - \frac{1}{2}g^{ij}R) - \alpha(a^ia^j - g^{ij}a_ka^k)\right](N + A_2) ,$$

$$\dot{N} = \sigma .$$

In these equations we have fixed the gauge condition $N_i = 0$.

3. Existence of special Hamiltonian solutions

The key for the special solutions is the vanishing of the coefficient of $\pi^{ij}$ in the evolution Eq. (12). This leads to the lacking of the invertibility of the Legendre transformation. Therefore, these special configurations are characterized by the relation $A_1 = -\frac{1}{2}N$. Let us analyze the existence of them. We first require that also $A_2$ is proportional to $N$, $A_2 = k_2N$ (further details on this assumption can be found in [9]). With these settings for $A_{1,2}$ the variable $B$ becomes constant, and we obtain that the two Eqs. (7) and (8) are completely solved if $k_2 = -1$, i.e., $A_{1,2}$ are given in terms of $N$ by

$$N + 2A_1 = N + A_2 = 0 .$$

The Eqs. (9 - 10) result

$$0 = \frac{\gamma_1}{N}\left(\nabla^2 - \frac{\nabla^2N}{N}\right)\sigma + \left(\frac{3}{g}\pi^{ij}\pi_{ij} + \frac{\gamma_1}{2}a_ka^k\right)\mu ,$$

$$0 = \frac{\gamma_2}{N}\left(\nabla^2 - \frac{\nabla^2N}{N}\right)\sigma + \frac{2\alpha}{N}a^k(\partial_k + a_k)\sigma + 2\beta\nabla^2\mu - \frac{1}{2}\left(\beta R + (\alpha - \gamma_2)a_ka^k\right)\mu .$$

These two equations form a system of homogeneous partial differential equations for $\sigma$ and $\mu$. A particular solution, without requiring any further condition, is $\sigma = \mu = 0$. We shall use this solution on the next section. On more general grounds, the system (16 - 17) becomes elliptic if the matrix of coefficients of the terms of second order in derivatives is positive definite. This sets two conditions on the space of coupling constants, namely $\beta > 0$, $\gamma_1 = 2\beta - \alpha > 0$. Therefore, if we assume these condition holds, there exist solutions of the Eqs. (16 - 17), each one corresponding to a given boundary condition. The equations for the evolution of the canonical variables evaluated on these configurations take the simple form

$$\dot{g}_{ij} = \mu g_{ij} , \quad \dot{\pi}^{ij} = -\mu \pi^{ij} , \quad \dot{N} = \sigma .$$

Therefore, we have that a class of interesting special solutions exists. They represent a flow of the three-dimensional metric on a given topology. These solutions are characterized by $2A_1 = A_2 = -N$. The remaining Lagrange multipliers $\sigma$ and $\mu$ are determined by the equations (16 - 17), and the initial data on the canonical variables must solve the constraints (2 - 4) and (6). The evolution equations are (18). Notice that the evolution flows of $g_{ij}$ and $\pi^{ij}$ are coupled.
each other due to the factor $\mu$. This class of solutions cannot be obtained from the original Lagrangian field equations since there is no direct relation between $\dot{g}_{ij}$ and $\pi^{ij}$.

We may study the existence of more general solutions on the Hamiltonian side by relaxing the condition on the Lagrange multiplier $A_2$. In this case we focus on the constraints and the equations that determine the stationary points of the canonical action; these are the Eqs. (2 - 4), (6), (7) and (12 - 14) [Equations (8 - 10) are needed only for the time preservation of the second-class constraints]. By substituting the condition $A_1 = -\frac{1}{2}N$ that leads to the noninvertibility of the Legendre transformation, equation (7) takes the form

$$\gamma_2 \nabla^2 \left( \frac{A_2}{N} \right) + 2\alpha \frac{\nabla^k N \nabla_k \left( \frac{A_2}{N} \right)}{N} - \frac{\gamma_1}{2} \frac{\nabla^2 N}{N} \left( \frac{A_2}{N} \right) = \frac{\gamma_1}{2} \frac{\nabla^2 N}{N}$$ \hspace{1cm} (19)

If the condition $\gamma_2 > 0$ is satisfied, this is an elliptic equation for the variable $A_2/N$. The numerical bounds on the coupling constants can be extracted from the discussion in Ref. [10]. Observational constraints relative to the solar-system tests demands that $\alpha$ is strongly close to the expression $\alpha = 2(\beta - 1)$. Hence we expect that the value of $\gamma_2$ (5) is close to $\gamma_2 = 4\beta - 3$. Cherenkov radiation imposes the bound $\beta \geq 1$. With this information we can take $\gamma_2$ to be positive safely on physical grounds. Therefore, Eq. (19) is an inhomogeneous elliptic partial differential equation for $A_2$. It can be solved by giving boundary conditions. As can be easily seen from (19), one of these solutions is the case $A_2 = -N$ we discussed above, but there are more solutions, and each of them leads to a special Hamiltonian configuration since they cannot be found in the Lagrangian formalism. These configurations must satisfy the constraints (2 - 4) and (6), as well as the evolution equations (12 - 14), which take the form

$$\dot{g}_{ij} = \mu g_{ij},$$ \hspace{1cm} (20)

$$\frac{\pi^{ij}}{\sqrt{g}} = -\mu \frac{\pi^{ij}}{\sqrt{g}} - \gamma_2 \nabla^{(i}N \nabla^{j)} \left( \frac{A_2}{N} \right) + \frac{\gamma_1}{2} \frac{\nabla^k N \nabla^k \left( \frac{A_2}{N} \right)}{N} + \left[ \beta(\nabla^{ij} - g^{ij}\nabla^2) - \beta(R^{ij} - \frac{1}{2}g^{ij}R) - \alpha(a^j a^i - g^{ij} a_k a^k) \right](N + A_2),$$ \hspace{1cm} (21)

$$\dot{N} = \sigma,$$ \hspace{1cm} (22)

where $B = \gamma_2 \frac{A_2}{N} + \frac{\gamma_1}{2}$. The conditions on the couplings constants $\alpha$ and $\beta$ that are necessary to have elliptic equations for $\mu$ and $\sigma$, which we discussed above, are also satisfied once the observational bounds are imposed.

4. Example: static solutions

We start with the equations for the Lagrange multipliers $\sigma$ and $\mu$, Eqs. (16 - 17). Since these equations are homogeneous, an obvious solution is $\sigma = \mu = 0$. With this the right-hand sides of Eqs. (18) vanish, which implies that the configurations have static canonical fields, $\dot{g}_{ij} = \pi^{ij} = \dot{N} = 0$.

It is convenient to use the constraint $C_2$ to solve $N$. To this end the following change of variables is useful,

$$N = W^\gamma, \quad \gamma \equiv \frac{2\gamma_2}{2\beta + \alpha}.$$ \hspace{1cm} (23)

With this change the constraint $C_2$ given in (4) becomes

$$\nabla^2 W = \chi_1 R W, \quad \chi_1 \equiv \frac{\beta(2\beta + \alpha)}{2\gamma_2^2},$$ \hspace{1cm} (24)
which we regard as an equation for \( W \). We now use (24) in the constraint \( C_1 \) given in (3), obtaining
\[
\frac{1}{g} \pi^{ij} \pi_{ij} = 2 \chi_2 \frac{\partial_i W \partial^i W}{W^2} + 2 \chi_3 R ,
\]
where
\[
\chi_2 \equiv \frac{\alpha \gamma_1 \gamma_2}{(2 \beta + \alpha)^2} , \quad \chi_3 \equiv \frac{\beta (2 \beta + \alpha)}{4 \gamma_2} .
\]

To further advance, we assume that we are in a region on which the static spatial metric \( g_{ij} \) is a flat metric. Then the Eq. (24) takes the form
\[
\nabla^2 W = 0 ,
\]
where now \( \nabla^2 \) is the flat Laplacian. Therefore, any harmonic function \( W \) on the given region satisfying some boundary condition solves the constraint (27). There remain the constraints on \( \pi^{ij} \), which we summarize here,
\[
\begin{align*}
\pi &= 0 , \\
\nabla_i \pi^{ij} &= 0 , \\
\frac{1}{g} \pi^{ij} \pi_{ij} &= 2 \chi_2 \frac{\partial_i W \partial^i W}{W^2} .
\end{align*}
\]

We may give a class of static solutions of the system (27 - 30) with the flat metric in the following way. We implement Cartesian coordinates on the spatial slices. The solutions start with the assumption that one is given with a harmonic function \( W \) that can be expressed in separate variables,
\[
W = X^1(x^1)X^2(x^2)X^3(x^3) .
\]

For the momentum field we assume that all its diagonal components vanish, \( \pi^{11} = \pi^{22} = \pi^{33} = 0 \), such that the constraint (28) is automatically solved. The momentum constraint (29) is completely solved if the off-diagonal components have the following dependence on the coordinates:
\[
\pi^{12}(x^3) , \quad \pi^{13}(x^2) , \quad \pi^{23}(x^1) .
\]

Finally, the constraint (30) is solved if these components of the momentum are given in terms of the components of the harmonic function in the form
\[
\begin{align*}
\pi^{12} &= \frac{\sqrt{\chi_2}}{X^3} \frac{dX^3}{dx^3} , \\
\pi^{13} &= \frac{\sqrt{\chi_2}}{X^2} \frac{dX^2}{dx^2} , \\
\pi^{23} &= \frac{\sqrt{\chi_2}}{X^1} \frac{dX^1}{dx^1} .
\end{align*}
\]

5. Coupling to sources: homogeneity and isotropy
Parallizing the standard approach of GR, we consider a perfect fluid that has a relativistic energy-momentum tensor,
\[
T_{\mu \nu} = \rho u_\mu u_\nu + P (g_{\mu \nu} + u_\mu u_\nu) .
\]

We also consider that the perfect fluid is at rest in the chosen reference frame, \( u^\mu = (N^{-1}, 0, 0, 0) \), such that the constraint \( u_\mu u^\mu = -1 \) is satisfied.

We summarize the resulting system of field equations,
\[
\begin{align*}
\pi &= 0 , \\
\nabla_i \pi^{ij} &= 0 , \\
\frac{1}{\sqrt{g}} C_1 &= \frac{2}{g} \pi^{ij} \pi_{ij} - \gamma_1 \frac{\nabla^2 N}{N} + \rho + 3P = 0 ,
\end{align*}
\]
\[
\frac{1}{\sqrt{g}} C_2 \equiv \gamma_2 \frac{\nabla^2 N}{N} - \alpha a^i a^i - \beta R + \frac{3}{2} (\rho - P) = 0 ,
\]
(38)

\[
\dot{g}_{ij} = \frac{2}{\sqrt{g}} (N + 2A_1) \pi_{ij} + \mu g_{ij} ,
\]
(39)

\[
\frac{\dot{\pi}^{ij}}{\sqrt{g}} = -\frac{1}{g} (N + 2A_1) (2\pi^{ik} \pi_{kj} - \frac{1}{2} g^{ij} \pi_{kl} \pi_{kl}) - \mu \frac{\pi^{ij}}{\sqrt{g}} - \nabla (N \nabla^i \pi^j) B + \frac{1}{2} \pi^{ij} \nabla_k N \nabla_k B + \beta \left( \nabla_j (g^{ij} \nabla^2) - \beta (R^{ij} - \frac{1}{2} g^{ij} R) - \alpha (a^i a^j - g^{ij} a^k a^k) \right) (N + A_2)
\]

\[
- \frac{1}{2} g^{ij} \left[ (A_1 + 3) A_2 \rho - (2N - 3A_1 + \frac{3}{2} A_2) P \right] ,
\]
(40)

\[
0 = \nabla^2 B + 2\alpha a^k \nabla_k \left( \frac{A_2}{N} \right) + \frac{\gamma_1}{2} \frac{\nabla^2 N}{N^2} (2A_1 - A_2) .
\]
(41)

We fix again the Lagrange multipliers \( A_1 \) and \( A_2 \) according to \( 2A_1 = A_2 = -N \), such that Eq. (41) is completely solved. We consider that the source variables \( \rho \) and \( P \) depend only on the time coordinate. We introduce the ansatz on which the spatial metric is homogeneous and isotropic. For the sake of simplicity we only consider the flat case. In Cartesian coordinates, this ansatz is impleted by the spatial metric

\[
d s^2_{(3)} = a^2(t) (dx^1)^2 + (dx^2)^2 + (dx^3)^2 .
\]
(42)

Notice that we restrict the conditions of homogeneity and isotropy to the spatial metric, but not to the lapse function. We can do so because in this theory the diffeomorphisms that preserve the foliation do not mix the lapse function with the spatial metric, and observationally these conditions are restricted to the spatial metric. We also consider that the momentum \( \pi^{ij} \) depends only on the time coordinate. With these settings the momentum constraint (36) is automatically solved.

Equation (39) reduces to

\[
2a \dot{a} \delta_{ij} = \mu a^2 \delta_{ij} ,
\]
(43)

and its solution is

\[
\mu = \frac{2a}{a} .
\]
(44)

Equation (40) becomes

\[
\frac{\dot{\pi}^{ij}}{a} + \frac{2 \dot{a}}{a} \pi^{ij} = \delta^{ij} a N (\rho + P) .
\]
(45)

Since constraint (35) demands that \( \pi^{ij} \) is traceless, from (45) we extract the equation of state \( P = -\rho \). In turn, this implies that the right hand side of Eq. (45) is equal to zero. This equation can be directly integrated, its solution is

\[
\pi^{ij} = \frac{m^{ij}}{a^2} ,
\]
(46)

where \( m^{ij} \) is a symmetric traceless constant matrix, such that the constraint (35) is automatically solved.

There remain the constraints \( C_1 \) and \( C_2 \), given in (37) and (38), to be solved. Under the conditions we are considering, these constraints take, respectively, the form

\[
\frac{2m^2}{a^4} - (2\beta - \alpha) \frac{\partial^2 N}{N} = 2a^2 \rho ,
\]
(47)

\[
(\beta + \frac{3\alpha}{2}) \frac{\partial^2 N}{N} - \alpha \frac{\partial_i N \partial_j N}{N^2} = -3a^2 \rho ,
\]
(48)
where \( m^2 \equiv m^{ij}m_{ij} \) and \( \partial^2 \) is the flat Laplacian in Cartesian coordinates, \( \partial^2 = \partial_i \partial_i \). We introduce the following ansatz for \( N \),

\[
N = \exp \left( f_0(t) + f_1(t)x^1 + f_2(t)x^2 + f_3(t)x^3 \right),
\]

(49)

where \( f_{0,1,2,3}(t) \) are arbitrary functions of time. Then the Eq. (48) reduces to

\[
f_i f_i = -\frac{6a^2 \rho}{2\beta + \alpha},
\]

(50)

where \( f_if_i = f_1^2 + f_2^2 + f_3^2 \). Consistency of this equation requires that the coupling constants satisfy the bound \( 2\beta + \alpha < 0 \) (assuming \( \rho > 0 \)). By using Eq. (50) in Eq. (47), we obtain that (47) becomes an algebraic equation for \( a \) and \( \rho \), whose solution is

\[
\rho = ka^{-6}, \quad k \equiv \frac{|2\beta + \alpha|m^2}{4(\beta - \alpha)}.
\]

(51)

(If we assume \( \beta > 0 \) and \( 2\beta + \alpha < 0 \), then \( \beta - \alpha > 0 \)).

Conclusions
We have shown how in the kinetic-conformal formulation of the nonprojectable Hořava theory the space of solutions of the Hamiltonian formulation is bigger than the corresponding space in the original Lagrangian formulation. A key role is played by one of the Lagrange multipliers associated to the secondary constraints, namely \( A_1 \), since there is a configuration of this multiplier for which the Legendre transformation cannot be inverted.

We have presented a large class of evolving solutions governed by the flow of a three-dimensional metric, new static vacuum solutions and a new solution with a source. The solution with the source, which specifically is a homogeneous and isotropic perfect fluid with a relativistic energy-momentum tensor, has the interesting feature of possessing a homogeneous and isotropic spatial metric with nontrivial (time dependent) scale factor. This solution signals how more homogeneous and isotropic configurations arise when, first, the solutions are studied in the Hamiltonian formalism and, second, the condition of homogeneity and isotropy is restricted to the spatial metric, whereas the lapse function is allowed to have a more general dependence on the space. All these examples are characterized by a specific setting of one of the Lagrange multipliers, namely \( A_2 \). This is a condition additional to the setting that leads to the noninvertibility of the Legendre transformation, which is a condition on \( A_1 \). We have extended the discussion by relaxing the restriction on \( A_2 \), and we have shown that more special Hamiltonian solutions exists in this more general case.

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