Generation of arbitrary Fock states via resonant interactions in cavity QED

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We propose a scheme to generate arbitrary Fock states $|N\rangle$ in a cavity QED using $N$ resonant Rydberg atoms. The atom-field interaction times are controlled via Stark-shifts adjusted in a way that each atom transfers a photon to the cavity, turning atomic detections useless. Fluctuations affecting the control of the atom-field interactions are also considered.

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Fock states have various potential applications, as in secure quantum communication [1, 2, 3, 4], quantum cryptography [5], optimal capacity coding in quantum channels [6], high-precision quantum interferometry [7], etc. However, their generation in laboratories is not a trivial task, mainly concerning with highly excited fields. Recent experimental results for one-photon [8] and two-photon [9] Fock states have been obtained in a cavity QED taking advantage of the high level control of the matter-field interaction [10]. Proposals for the generation of highly excited Fock states using a large number of atoms have been presented [11, 12]. Ref. [11] employs a generation of highly excited Fock states using a large number of atoms to generate Fock states; they also need highly efficient atomic detectors.

Pursuing the same goal, there are also proposals that employ a single atom escaping the detection efficiency problem at price of complications in atomic level schemes [22, 23] or in successive atom-field operations [24]. In Ref. [22] a three-level atom driven by three classical fields via a two-channel Raman interaction transfers photons to a cavity mode to prepare it in Fock states; in Ref. [24] a simplified scheme prepares Fock states using a single two-level atom which undergoes a controlled succession of interactions with two modes of a cavity and transfer photons from one of them to the other. The procedure in [24] has a good accuracy and could achieve Fock states $|N\rangle$ with $N \approx 5$.

In the present report, inspired by the work of Krause et al. [11], we present a scheme for generation of the arbitrary Fock states in a cavity QED using resonant atom-field interactions. The underlying idea is to send a set of Rydberg atoms with the same atomic velocity and interaction times adjusted via Stark effect [25] in such a way that each atom transfers a photon to a cavity mode. In accord to the sudden approximation [26], we will neglected the system evolution between the active and frozen atom-field interactions. From the experimental QED-cavity point of view this approximation is supported by the $1 \mu$s time switch spent by the atom between the electric fields $0.26 \text{ V/cm}$ and $1.1 \text{ V/cm}$ available in laboratories [27]. Hence, our procedure differs from Ref. [11] which employs Rydberg ions whose interaction times with the cavity field are determined by the control of ionic velocities via an accelerating electric field. So, although following the same fundamental idea, the present procedure differs from Ref. [11] in the control of the atom-field interaction times.

The simplicity of our scheme makes it attractive experimentally, being feasible with the present status of QED-cavity technology [23]. The proposal requires the experimental setup shown in Fig. 1: the source $S$ ejects rubidium atoms, which are velocity selected and prepared in circular Rydberg state, one at a time, by appropriated laser beams. The relevant atomic levels $|g\rangle$ and $|e\rangle$, with the principal quantum numbers 50 and 51, have the atomic transition of 51.1 GHz. After $S$ one obtains a known atomic position $r(t)$ at any time during the experiment. The high-Q superconducting cavity $C$ is a Fabry-Perot resonator made of two spherical niobium mirrors with a Gaussian geometry (waist $w = 6 \text{ mm}$) and photon damping times of $130 \text{ ms}$ [28]. The cavity is prepared at a low temperature ($T \approx 0.6 \text{ K}$) to reduce the average number of thermal photons; before the beginning of the experiment the thermal field is erased; $D_e$ ($D_g$) represents the atomic ionization detector for the state $|e\rangle$ ($|g\rangle$).

To describe atom-field interaction in the cavity we employ the Jaynes-Cummings model [29], also including variation of the strength of the local atom-field coupling. In the rotating-wave-approximation this model is represented by the Hamiltonian:

$$\hat{H} = \hbar \omega_0 \hat{\sigma}_z + \hbar \omega \hat{a}^\dagger \hat{a} + \hbar \Omega(t)(\hat{a}^\dagger \hat{\sigma}_- + \hat{a} \hat{\sigma}_+), \quad (1)$$

where $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\hat{\sigma}_+ = |e\rangle\langle g|$, and $\hat{\sigma}_- = |g\rangle\langle e|$ are the atomic operators of the two-level atom with the atomic transition frequency $\omega_0$; $\hat{a}$ ($\hat{a}^\dagger$) is the annihilation (creation) operator of the single-mode field of frequency $\omega$; $\Omega(t)$ is the atom-cavity interaction strength with a Gaussian mode profile [10]

$$\Omega(t) = \Omega_0 \exp \left[ -\frac{r^2(t)}{w^2} \right], \quad (2)$$

where $\Omega_0$ stands for the vacuum Rabi oscillation at the center of the cavity and the atomic position is described classically,$r(t) = r_0 + vt$, since the kinetic energy of an atom is much larger than the height and the depth of the optical potential...
The effective resonant Hamiltonian for the atom-field system in the interaction picture is

$$\hat{V}_{JC}(t) = \hbar \Omega(t) (\hat{a}^\dagger \sigma_- + \hat{a} \sigma_+).$$  \hspace{1cm} (3)$$

Since the time dependence of this Hamiltonian comes from a parameter, $\Omega(t)$, then $[\hat{V}_{JC}(t), \hat{V}_{JC}(t')] = 0$ and the time evolution operator has the form $U_{JC}(t) = \exp\left(-i/\hbar \int_{t'}^{t} \hat{V}_{JC}(t')dt'\right)$. Thus, considering the cavity mode in the Fock state $|n-1\rangle$ and the atom in the excited state $|e\rangle$, this evolution operator $U_{JC}$ produces

$$\hat{U}_{JC}(\tau_n)|n-1\rangle|e\rangle = \cos\left(\sqrt{n}\theta(\tau_n)\right)|n-1\rangle|e\rangle - i \sin\left(\sqrt{n}\theta(\tau_n)\right)|n\rangle|g\rangle,$$  \hspace{1cm} (4)

with

$$\theta(\tau_n) = \int_{0}^{\tau_n} \Omega(t)dt,$$  \hspace{1cm} (5)

where $\tau_n$ stands for the atom-field interaction time concerning with the $n$-th atom.

Next, to describe our procedure to generate arbitrary Fock states consider a first atom prepared in the excited state $|e\rangle_1$, which enters the cavity and interacts resonantly with a field mode in a vacuum state $|0\rangle$. According to the Eq. (4), the initial state $|0\rangle|e\rangle_1$ evolves to the state $|1\rangle|g\rangle_1$ after an interaction time $\tau_1$ obtained from the Eq. (5) plus the condition $\theta(\tau_1) = \pi/2$.

Next, we send a second atom prepared in state $|e\rangle_2$ which interacts with the field in the state $|1\rangle$ obtained in the previous step. During the interaction time $\tau_2$ obtained from the condition $\theta(\tau_2) = \pi/(2\sqrt{2})$ the atom-field system evolves to the state $|2\rangle|g\rangle_2$. Proceeding further in this way, after the passage of the $N$-th atom one obtains the desired Fock state inside the cavity

$$|\psi(\tau_N)\rangle_{AF} = |N\rangle|g\rangle_N,$$  \hspace{1cm} (6)

where $\tau_N$ stands for the interaction time in last step.

To calculate the interaction time for the creation of the Fock state $|N\rangle$ we take typical values [10, 27] for the coupling constant $\Omega_0 \approx 2\pi \times 47$ kHz and velocity $\nu = 500$ m/s for all atoms. Using the Stark effect one can adjust all interaction times in such a way that each atom has 100% probability for emitting a photon inside the cavity. For example, when the first atom enters the cavity the atom-field interaction is frozen by a 1.1 V/cm field. When the atom is 1.4 mm before the cavity axis, it is rapidly tuned in resonance with cavity mode by a $0.26$ V/cm field. During the next $5.4 \mu$s ($2.8$ mm path) the atom emits a photon to the cavity. After that the atom-field interaction is canceled out again. The same procedure is repeated for all atoms. The instant of an atom entering the cavity coincides with that of the previous atom exiting the cavity, so the total interaction time is $N\tau = N(l/\nu)$, $l$ being the length of the cavity. For example, assuming $\tau = 10$ $\mu$s the creation of the number state $|6\rangle$ requires the total interaction time 60 $\mu$s, much lesser than the decoherence time $t_d = t_{cav}/N \approx 12.3$ ms, with damping time $t_{cav} \approx 123$ ms [28]. So the scheme is experimentally feasible within the realm of microwave.

Since the present scheme involves no atomic detection, in the ideal case the desired Fock state is obtained with 100% success rate and fidelity. However, to be more realistic we have taken into account variations of the atomic velocity coming from the size of the excitation laser beams and the residual velocity dispersion [10]. Nowadays, the best accuracy in the variance of the velocity is $\Delta v = \pm 2 m/s$ which yields atomic position with $\pm 1$ mm accuracy. Also, there is no fundamental problem to get a more accurate velocity and sufficiently well known atomic position, since even improving the accuracy in the velocity for $\Delta v = \pm 2 \times 10^{-3}$ m/s the Heisenberg uncertainty would furnish $\Delta x \approx \hbar/m\Delta v \approx 0.35 \mu m$, $m$ standing for rubidium mass. Accordingly, the uncertainty in the atomic velocity leads to the impossibility of sharply fixing the atom-field interaction times. Following the Ref. [31] we introduce the probability density $f_i(t_i, \bar{t}_i)$ where $t_i$ is the true atom-field interaction time. For a Gaussian distribution centered around the average interaction time $\bar{t}_i$, the probability density reads

$$f_i(t_i, \bar{t}_i) = \frac{1}{\Delta_i \sqrt{2\pi}} \exp\left\{-\frac{(t_i - \bar{t}_i)^2}{2\Delta_i^2}\right\},$$  \hspace{1cm} (7)

where $\Delta_i = \gamma \bar{t}_i$. The spread parameter $\gamma$ characterizes the control of the atom-field interaction time and it usually fluctuates from 0 to 0.1 [31]. When $\Delta_i \rightarrow 0$ the function $f_i(t_i, \bar{t}_i)$ becomes a Dirac distribution $\delta(t_i - \bar{t}_i)$ corresponding to the ideal control of the atom-field interaction time. The effective density operator $\rho_{AF}^N$, which describes the whole atom-field state during the generation of the Fock state $|N\rangle$, including unavoidable influences of fluctuations upon the interaction time, may be represented as

$$\rho_{AF}^N = \prod_{i=1}^{N} \int_{-\infty}^{\infty} dt_i f_i(t_i, \bar{t}_i) \rho_{AF}(t_1, \ldots, t_N),$$  \hspace{1cm} (8)

where $\bar{t}_i = \pi/(2\sqrt{\lambda})$, $i = 1, 2, \ldots, N$ and

$$\rho_{AF}(t_1, \ldots, t_N) = \hat{U}_{JC}(t_N) \hat{U}_{JC}(t_{N-1}) \ldots \times \hat{U}_{JC}(t_1) \rho_{AF}(0) \hat{U}_{JC}(t_1) \ldots \times \hat{U}_{JC}(t_{N-1}) \hat{U}_{JC}(t_N).$$  \hspace{1cm} (9)
with

\[
\hat{\rho}_{AF}(0) = |e\rangle_1|e\rangle_2 \cdots |e\rangle_N|0\rangle_F F F(0)|N\rangle|e\rangle\cdots|e\rangle_1|e\rangle. \tag{10}
\]

Now, to obtain the success rate \( P \) and the fidelity \( F \), considering fluctuations affecting the atom-field interaction time, we use the definitions

\[
P = \left\{ \prod_{i=1}^{N} \langle i | \langle g | e \rangle \langle N | \right\} \text{Tr}_F[\hat{\rho}_{AF}^N] \left\{ \prod_{j=1}^{N} \langle j | \langle g | e \rangle \langle N | \right\},
\]

and

\[
F = \langle N | \text{Tr}_A[\hat{\rho}_{AF}^N] | N \rangle, \tag{11}
\]

where \( \text{Tr}_F \) and \( \text{Tr}_A \) stand for the trace on the field and atomic states, respectively. Here the success rate and fidelity coincide and they are given by

\[
F(\gamma, N) = \left( 1 + e^{-\frac{2 \gamma^2}{N}} \right)^2 / 2^N.
\]

Note that, as expected, for \( \gamma = 0 \) corresponding to the ideal case, the fidelity is 100\%. Fig. 2 shows the fidelity of the Fock state \( |N\rangle \) obtained in the present scheme, for the interval \( N \in [1, 10] \). Accordingly the minimum value of fidelity is 79\%, for \( \gamma = 0.1, N = 10 \).

In summary, we have employed a set of \( N \) Rydberg atoms to create an arbitrary Fock state \( |N\rangle \) inside a microwave cavity via resonant atom-field interactions. The variation of the Rabi frequency due to the atomic motion across the Gaussian cavity mode was taken into account to calculate the generation time; for example, it takes 0.2 ms to create the number state \( |6\rangle \) with the success probability of 100\% in the ideal case. We have also investigated the loss of fidelity (cf. Fig. 2) due to the variation of the atomic velocity caused by the size of the excitation laser beams and the residual velocity dispersion [10]. Finally, a recent result by the Haroche’s group [28] allows us to neglect the decoherence of the state during the generation process.

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