Lorentz Invariance Violation and its Role in Quantum Gravity Phenomenology

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Abstract

The notion that gravitation might lead to a breakdown of standard space-time structure at small distances, and that this might affect the propagation of ordinary particles has led to a program to search for violations of Lorentz invariance as a probe of quantum gravity. Initially it was expected that observable macroscopic effects caused by microscopic violations of Lorentz invariance would necessarily be suppressed by at least one power of the small ratio between the Planck length and macroscopic lengths. Here we discuss the implications of the fact that this expectation is in contradiction with standard properties of radiative corrections in quantum field theories. In normal field theories, radiative corrections in the presence of microscopic Lorentz violation give macroscopic Lorentz violation that is suppressed only by the size of Standard Model couplings, in clear conflict with observation. In general, this conclusion can only be avoided by extreme fine tuning of the parameters of the theory.

1.1 Introduction

Although there is enormous uncertainty about the nature of quantum gravity (QG), one thing is quite certain: The commonly used ideas of space and time should break down at or before the Planck length is reached. For example, elementary scattering processes with a Planck-sized center-of-mass energy create large enough quantum fluctuations in the gravitational field that space-time can no longer be treated as
a classical continuum. It is then natural to question the exactness of the Lorentz invariance (LI) that is pervasive in all more macroscopic theories. Exact LI requires that an object can be arbitrarily boosted. Since the corresponding Lorentz contractions involve arbitrarily small distances, there is an obvious tension with the expected breakdown of classical space-time at the Planck length. Indeed, quite general arguments are made that lead to violations of LI within the two most popular approaches towards QG: string theory (Ellis et al., 2000) and loop quantum gravity (Gambini & Pullin, 1999; Alfaro et al., 2000, 2002).

This has given added impetus to the established line of research dedicated to the investigation of ways in which fundamental symmetries, like LI or CPT, could be broken (Kostelecký et al., 1989a, 1989b, 1991, 1996). It was realized that extremely precise tests could be made with a sensitivity appropriate to certain order of magnitude estimates of violations of LI (Amelino-Camelia et al., 1998).

The sensitivity of the tests arises because there is a universal maximum speed when LI holds, and even small modifications to the standard dispersion relation relating energy and 3-momentum give highly magnified observable effects on the propagation of ultra-relativistic particles. One possible modification is

\[ E^2 = P^2 + m^2 + \frac{\xi}{M_{\text{Pl}}} E^3. \]  

(1.1)

Here \( E \) and \( p \) are a particle’s energy and momentum in some preferred frame, \( m \) is its mass, while \( \xi \) is a dimensionless parameter arising from the details of the QG effects on the particular particle type. \( \xi \) could depend on the particle species and its polarization. The dispersion relation can be written in a covariant fashion:

\[ P^\mu P_\mu = m^2 + \frac{\xi}{M_{\text{Pl}}} (P^\mu W_\mu)^3, \]  

(1.2)

where \( P^\mu \) is the particle’s four momentum, and \( W^\mu \) is the 4-velocity of the preferred frame. Amelino-Camelia et al. (1998) noted that photons \((m = 0)\) with different energies would then travel with different velocities. For a gamma ray burst originating at a distance \( D \) from us, the difference in time of arrival of different energy components would be \( \Delta t = \xi D \Delta E/M_{\text{Pl}}. \) If the parameter \( \xi \) were of order 1 and \( D \sim 100 \text{ Mpc}, \) then for \( \Delta E \sim 100 \text{ MeV}, \) we would have \( \Delta t \sim 10^{-2} \text{ s}, \) making it close to measurable in gamma ray bursts.

A second possible modification is that the parameter normally called the speed of light, \( c, \) is different for different kinds of particle. This
is implemented by a non-universal particle-dependent coefficient of $P^2$ in Eq. (1.1). The differences in the maximum speeds of propagation also gives sensitive tests: vacuum Cerenkov radiation etc (Coleman & Glashow, 1999).

There are in fact two lines of inquiry associated with modified dispersion relations. One is the initial approach, where the equivalence of all reference frames fails, essentially with the existence of a preferred frame. A second popular approach preserves the postulate of the equivalence of all frames, but tries to find modifications of the standard Lorentz or Poincaré symmetries. The most popular version, with the name of Doubly Special Relativity (DSR), replaces the standard Poincaré algebra by a non-linear structure (Amelino-Camelia, 2002; Magueijo & Smolin, 2002; Kowalski-Glikman & Nowak, 2002; Lukierski & Nowicki, 2003). Another line of argument examines a deformed algebra formed by combining the Poincaré algebra with coordinate operators one (Vilela Mendez, 1994; Chryssomalakos & Okon, 2003, 2004). Related to these are field theories on non-commutative space-time (Chaichian et al., 2004; Aschieri et al., 2005; Douglas & Nekrasov, 2001; Szabo, 2003); they give a particular kind of LIV at short distances that fits into the general field theoretic framework we will discuss.

In this article we will concentrate on the first issue, actual violations of LI. Regarding DSR and its relatives, we refer the reader to a contribution in this volume and to critiques by Schützhold & Unruh (2003), by Rembieliński & Smoliński (2003), and by Sudarsky (2005). A problem that concerns us is that the proposed symmetry algebras all contain as a subalgebra the standard Poincaré algebra, and thus they contain operators for 4-momentum that obey the standard properties. The DSR approach uses a modified 4-momentum that are non-linear functions of what we regard as the standard momentum operators. This of course raises the issue of which are the operators directly related to observations. In the discussion section we will summarize a proposal by Liberati, Sonego and Visser (2004) who propose that it is the measurement process that picks out the modified 4-momentum operators as the measurable quantities.

We will also touch on an aspect with important connections to the general field of QG: the problem of a physical regularization and construction of quantum field theories (QFT).
1.2 Phenomenological models

Methodical phenomenological explorations can best be quantified relative to a definite theoretical context. In our case, of Lorentz invariance violation (LIV) at accessible energies, the context should minimally incorporate known microscopic physics, including quantum mechanics and special relativity (in order to consider small deviations therefrom). This leads to the use of a conventional interacting quantum field theory but with the inclusion of Lorentz violating terms in the Lagrangian.

One proposal is the Standard Model Extension (SME) of Colladay & Kostelecký (1998) and Coleman & Glashow (1999). This incorporates within the Standard Model of particle physics all the possible renormalizable Lorentz violating terms, while preserving SU(3) × SU(2) × U(1) gauge symmetry and the standard field content. For example, the terms in the free part of the Lagrangian density for a free fermion field $\psi$ are:

$$
L_{\text{free}} = i\bar{\psi}(\gamma_\mu + c_{\mu\nu}\gamma_\nu + d_{\mu\nu}\gamma_5\gamma_\nu + e_\mu + if_\mu\gamma_5 + \frac{1}{2}g_{\mu\nu\rho}\sigma^{\nu\rho})\partial^\mu \psi - \bar{\psi}(m + a_\nu\gamma_\nu + b_\nu\gamma_5\gamma_\nu + \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu})\gamma_\mu \gamma_5 \partial_\nu \partial_\rho \psi.
$$

(1.3)

Here the quantities $a_\mu$, $b_\mu$, $c_{\mu\nu}$, $d_{\mu\nu}$, $e_\mu$, $f_\mu$, $g_{\mu\nu\rho}$ and $H_{\mu\nu}$ are numerical quantities covariantly characterizing LIV, and can be thought of as arising from the VEV of otherwise dynamical gravitational fields. The interacting theory is then obtained in the same way as in the usual, with SU(3) × SU(2) × U(1) gauge fields and a Higgs field. The expected renormalizability was shown by Kostelecký and Mewes (2001) and Kostelecký et al. (2002).

A second approach, as used by Myers and Pospelov (2003), is to take the LIV terms as higher dimension non-renormalizable operators. This is a natural proposal if one supposes that LIV is produced at the Planck scale with power suppressed effects at low energy; it gives modified dispersion relations at tree approximation. For example, there are dimension-5 terms with $1/M_{\text{Pl}}$ suppression in the free part of the Lagrangian, such as

$$
\frac{1}{M_{\text{Pl}}} W^\mu W^\nu W^\rho \bar{\psi}(\xi_f + \xi_f\gamma_5)\gamma_\mu \partial_\nu \partial_\rho \psi,
$$

(1.4)

where $W^\mu$ specifies a preferred frame. Similar terms can be written for scalar fields and gauge fields. Dimensionless parameters $\xi$ in these terms specify the degree of LIV in each sector.

Each of the proposed Lagrangians can be regarded as defining an effective low-energy theory. Such a theory systematically provides an approximation, valid at low energies, to a more exact microscopic theory.
In Secs. 1.4 and 1.5, we will analyze the applicability of LIV effective theories. But first, we will make some simple model calculations, to illustrate generic features of the relation between microscopic LIV and low-energy properties of a QFT.

### 1.3 Model calculation

The central issue is associated with the UV divergences of conventional QFT. Even if the actual divergences are removed because of the short-distances properties of a true microscopic theory, we know that QFT gives a good approximation to the true physics up to energies of at least a few hundred GeV. So at best the UV divergences are replaced by large finite values which still leave observable low energy physics potentially highly sensitive to short-distance phenomena.

Of course, UV divergences are normally removed by renormalization, i.e., by adjustment of the parameters of the Lagrangian. The observable effects of short-distance physics now appear indirectly, not only in the values of the renormalized parameters, but also in the presence in the Lagrangian of all terms necessary for renormalizability.

The interesting and generic consequences in the presence of Lorentz violation we now illustrate in a simple Yukawa theory of a scalar field and a Dirac field. Before UV regularization the theory is defined by

\[
\mathcal{L} = \frac{1}{2} (\partial \phi)^2 - \frac{m_0^2}{2} \phi^2 + \bar{\psi} (i \gamma^\mu \partial_\mu - M_0) \psi + g_0 \phi \bar{\psi} \psi.
\]

We make the theory finite by introducing a cut-off on spatial momenta (in a preferred frame defined by a 4-velocity \(W^\mu\)). We use a conventional real-time formalism, so that the cutoff theory is within the framework of regular quantum theory in 3 space dimensions. The cutoff is implemented as a modification of the free propagators:

\[
\frac{i}{\gamma^\mu p_\mu - m_0 + i\epsilon} \rightarrow \frac{i f(|p|/\Lambda)}{\gamma^\mu p_\mu - m_0 + \Delta(|p|/\Lambda) + i\epsilon},
\]

\[
\frac{i}{p^2 - M_0^2 + i\epsilon} \rightarrow \frac{i \tilde{f}(|p|/\Lambda)}{p^2 - M_0^2 + \tilde{\Delta}(|p|/\Lambda) + i\epsilon}.
\]

Here, the functions \(f(|p|/\Lambda)\) and \(\tilde{f}(|p|/\Lambda)\) go to 1 as \(|p|/\Lambda \rightarrow 0\), to reproduce normal low energy behavior, and they go to zero as \(|p|/\Lambda \rightarrow \infty\), to provide UV finiteness. The functions \(\Delta\) and \(\tilde{\Delta}\) are inspired by concrete proposals for modified dispersion relations, and they should go...
to zero when $|p|/\Lambda \to 0$. But in our calculations we will set $\Delta$ and $\tilde{\Delta}$ to exactly zero. We will assume $\Lambda$ to be of order the Planck scale.

Corrections to the propagation of the scalar field are governed by its self-energy $\Pi(p)$, which we evaluate to one-loop order. We investigate the value when $p^\mu$ and the physical mass $m$ are much less than the cutoff $\Lambda$. Without the cutoff, the graph is quadratically divergent, so that differentiating three times with respect to $p$ gives a convergent integral (i.e., one for which the limit $\Lambda \to \infty$ exists). Therefore we write

$$\Pi(p) = A + p^2 B + p^\mu p^\nu W_{\mu\nu} \tilde{\xi} + \Pi^{(LI)}(p^2) + \mathcal{O}(p^4/\Lambda^2), \quad (1.8)$$

in a covariant formalism with $p^2 = p^\mu p^\nu \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the space-time metric. The would-be divergences at $\Lambda = \infty$ are contained in the first three terms, quadratic in $p$, so that we can take the limit $\Lambda \to \infty$ in the fourth term $\Pi^{(LI)}(p^2)$, which is therefore Lorentz invariant. The fifth term is Lorentz violating but power-suppressed. The coefficients $A$ and $B$ correspond to the usual Lorentz-invariant mass and wave function renormalization, and the only unsuppressed Lorentz violation is in the third term. Its coefficient $\tilde{\xi}$ is finite and independent of $\Lambda$, and explicit calculation (Collins et al., 2004) gives:

$$\tilde{\xi} = \frac{g^2}{6\pi^2} \left[ 1 + 2 \int_0^\infty dx x f'(x)^2 \right]. \quad (1.9)$$

Although the exact value depends on the details of the function $f$, it is bounded below by $g^2/6\pi^2$. Lorentz violation is therefore of the order of the square of the coupling, rather than power-suppressed. The LIV term in (1.8) behaves like a renormalization of the metric tensor and hence of the particle’s limiting velocity. The renormalization depends on the field and the size of the coupling, so that we expect different fields in the Standard Model to have limiting velocities differing by $\sim 10^{-2}$. The rough expected size depends only on UV power counting and Standard-Model couplings.

The expected size is in extreme contrast to the measured limits. To avoid this, either Lorentz-violation parameters in the microscopic theory are extremely fine-tuned, or there is a mechanism that automatically removes low-energy LIV even though it is present microscopically. More exact calculations would use renormalization group methods. But we know from the running of Standard-Model couplings, that this can produce changes of one order of magnitude, not twenty.

† In perturbation theory, the sum over one-particle-irreducible two-point graphs.
We could also perform the same calculation in conventional renormalization theory. We would use a Lorentz-invariant UV regulator followed by renormalization and removal of the regulator. The results would be of the same form, except that that coefficients $A$ and $B$ would change in value and $\xi$ would be zero. If we regard our theory with the spatial-momentum cutoff as an analog of a true Lorentz-violating microscopic theory, we deduce that it agrees with conventional Yukawa theory with suitable values of its parameters provided only that an explicitly Lorentz violating term proportional to $(W \cdot \partial \phi)^2$ is added to its Lagrangian.

1.4 Effective long-distance theories

Normally, the details of physical phenomena on very small distance scales do not directly manifest themselves in physics on much larger scales. For example, a meteorologist treats the atmosphere as a continuous fluid on scales of meters to many kilometers, without needing to know that the atmosphere is not a continuum but is made up of molecules.

In a classical field theory or the tree approximation of a QFT, the transition from a discrete approximation to a continuum is a simple matter of replacing discrete derivatives by true derivatives, without change of parameters. But in QFT, the situation is much less trivial, and is formalized in the concept of a “long-distance effective theory”. This provides an approximation to a more exact microscopic theory, and the errors are a power of $l/D$, where $l$ is the intrinsic distance scale associated with the microscopic theory, while $D$ is the much larger distance scale of the macroscopic phenomena under consideration.

The effective field theory approach has become particularly important because of the repeated discovery of particles corresponding to fields with ever higher mass. To the extent that gravity is ignored so that we can stay within the framework of QFT, the relation between effective theories appropriate for different scales has become extremely well understood (e.g., Rothstein, 2003). The basic theorems build from the decoupling theorem of Appelquist and Carazzone (1975). (See also Weinberg (1996).)

Both the ideas of an effective field theory and the complications when the microscopic theory is Lorentz violating were illustrated by our calculation in the previous section. For phenomena at low energies relative to some large intrinsic scale $\Lambda$ of a complete theory, we have agreement, up to power-suppressed corrections, of:
(i) Calculations in the exact microscopic theory. This theory, as concerns quantum gravity, is not yet known.

(ii) Calculations in a renormalized low-energy continuum field theory whose Lagrangian contains only renormalizable terms, i.e., of dimension four or less, possibly supplemented by power-suppressed higher-dimension non-renormalizable terms.

A basic intuition is obtained by the use of Wilsonian methods, where the most microscopic degrees of freedom are integrated out. At the one-loop level, these give unsuppressed contributions to low-energy phenomena of a form equivalent to vertices in a renormalizable Lagrangian, as with the first three terms in Eq. (1.8). This and its generalizations to all orders of QFT show that a renormalized effective QFT gives the dominant low energy effects of the microscopic theory. A renormalizable low-energy effective theory is self-contained and self-consistent: it contains no direct hints that it is an approximation to a better theory. In constructing candidate approximate theories of physics, we now treat renormalizability not as an independent postulate but as a theorem.

In our model calculations, the theory with a cutoff stands in for the true microscopic theory. Our calculations and their generalizations show that the low energy effective theory is an ordinary renormalizable QFT but with a LIV Lagrangian, just like the Standard Model Extension.

Higher power corrections, in \( p/\Lambda \) can be allowed for by including higher-dimension non-renormalizable terms in the Lagrangian of the effective theory, as in Eq. (1.4). Loop corrections derived from the non-renormalizable terms involve a series of counterterm operators in the Lagrangian with ever higher dimension. But these also correspond to a suppression by more inverse powers of \( \Lambda \), so it is consistent to truncate the series. The natural sizes of the coefficients in the Lagrangian are set in the Wilsonian fashion by integrals in the effective theory with cutoffs of order the intrinsic scale \( \Lambda \) of the full theory.

However, the phenomenological use of non-renormalizable terms does imply a definite upper limit on the energies where it is appropriate to use them. A classic case is the four-fermion form of weak interactions, where the limit is a few hundred GeV. The form of the interaction gave enough hints to enable construction of the full Standard Model. The four-fermion interaction (with some additions) now arises as the low-energy limit of processes with exchange of \( W \) and \( Z \) bosons.

An issue very important to the treatment of LIV and quantum gravity is that, normally, the terms in the Lagrangian a low-energy effective
theory must be all those consistent with the unbroken symmetries of the microscopic theory. If some of the terms are observed to be absent, that gives strong implications about the microscopic theory. A good example is given by QCD. At short distances, weak interactions lead to violations of electromagnetic strength of symmetries such as parity. But at energies of a few GeV, it is measured that these symmetries are much more exact; that is why the weak interactions are called weak. As Weinberg (1973) showed, a generic unified theory would not give this weak parity violation. He then observed that if the strong-interaction group commutes with the weak-interaction group, then the unobserved symmetry violation can be removed by a redefinition of the fields. This leads essentially uniquely to QCD as the strong-interaction part of the Standard Model.

In one respect, the situation with gravity is different from the usual kinds of effective field theory. Low energy gravitational physics is described by a non-renormalizable Lagrangian but is not power suppressed. The reasons are that the graviton has zero mass and that macroscopic classical gravitational fields occur, with coherent addition of the sources. The standard power-law suppression of gravitation occurs for quantum interactions of small numbers of elementary particles. Unsuppressed gravitational phenomena involve macroscopic classical fields, which need not be treated by quantum theoretic methods.

Modulo this qualification, we get the standard result that the total (leading-power) effect of the microscopic (Plank-scale) physics on GeV-scale physics is in determining the values of the renormalized parameters of the theory, and in changing them from the values obtained from the naive classically motivated considerations. This accounts for the folklore that macroscopic manifestations of Planck-scale physics are to be found only in power-suppressed phenomena.

However, for our purposes, the folklore is wrong because it ignores the price of the low-energy effective theory: that its Lagrangian must contain all renormalizable terms consistent with the symmetries of the microscopic theory. If Lorentz symmetry is violated by Planck-scale physics, then we are inexorably led not the Lorentz-invariant Standard Model, but to its Lorentz-violating extension. Observe that because logarithmic divergences are momentum-independent they are not associated with Lorentz violation. It is the self-energy (and related graphs) with higher divergences that are associated with Lorentz violation. Note that the true microscopic theory might well be UV finite. The UV divergences concern the ordinary continuum limit for the low-energy effective
1.5 Difficulties with the phenomenological models

The expected sizes of the Lorentz-violating parameters in the models summarized in Sec. 1.2 raise some serious difficulties, which we now discuss. We assume that on appropriate distance scales, presumably comparable to the Planck length, there is considerable Lorentz violation. This is the kind associated with space-time granularity, and leads in classical theory or tree approximation to modified dispersion relations like \( (1.1) \).

In the case of the SME, which contains only renormalizable terms, the natural size of the LIV parameters is then that of a one-loop Standard-Model correction. Although this appears to have been recognized by Kostelecký and Potting (1995), the point is quite obscured in that paper. The conflict with data means either that there is also very small Lorentz violation at the Planck scale or that quantum gravity contains a mechanism for automatically restoring macroscopic Lorentz invariance. In either case, it is unjustified to adhere to the naive expectation that Lorentz violation is expected to be suppressed by a power of energy divided by \( M_{\text{Pl}} \), as in \( (1.1) \).

The scheme of Myers and Pospelov (2003) at first appears more natural. The renormalizable part of their effective low-energy Lagrangian is the usual Lorentz-invariant one, to which is added a 5-dimensional operator suppressed by \( 1/M_{\text{Pl}} \) coefficient.

But as noted by these authors, consistent use of the effective theory requires that radiative corrections are needed; insertion of a dimension-5 operator in a self energy generically leads to large Lorentz-violation from the same power counting as in our model calculation. In general it even gives dimension-3 operators enhanced with a factor of \( M_{\text{Pl}} \). They found that they could avoid these problems by postulating a certain antisymmetry structure for the tensor coefficient in the dimension-5 operator.

This is still not sufficient. Consistent use of the theory also requires iteration of the physical effects that give the dimension-5 operators, and hence, within the effective theory, multiple insertions of these operators. As shown by Perez and Sudarsky (2003), this leads back to the LIV dimension-4 operators that one was trying to avoid.

The overall result is simply a set of particular cases of the general rule that the terms in the renormalizable part of the Lagrangian are all...
those not prohibited by symmetries of the microscopic theory. Lorentz
symmetry is, by the initial hypothesis of all this work, not among the
symmetries. Starting with Lorentz-violating modifications of disper-
sion relations that by themselves are only large at Planck-scale energies,
bringing in virtual loop corrections in QFT generates integrals over all
momenta up to the Planck scale, complete with the hypothesized Lorentz
violation. This is a direct consequence of known properties of relativistic
QFT, of which the Standard Model is only one example, and must be
obeyed by any theory of quantum gravity that reproduces known Stan-
dard Model physics in Standard Model’s domain of validity. Extreme
fine tuning of the parameters of the microscopic theory could be used to
evade the conclusion, but this is generally considered highly inapprop-
riate for a fundamental microscopic theory of physics.

Thus a very important requirement of a theory of QG is that it should
ensure the absence of the macroscopic manifestation of effects of any
presumed Lorentz-violating microscopic structure of space-time. This
feature should be robust, without requiring any fine tuning. Note that
such overriding general considerations have played a critical role in the
discovery of key physical theories in the twentieth century, from rela-
tivity to QCD. As to experimental data, it can be seen in retrospect
that only a relatively very small set of experimental data was essential
in determining the course of these developments.

1.6 Direct Searches

We now give a short account of some of the methods that have yielded
the most important bounds on Lorentz violation. These experimental
results are important independently of our critiques of their theoretical
motivations. For a very complete summary of the situation we refer the
reader to the recent review by Mattingly (2005).

In the introduction, we have already mentioned the idea of Amelino-
Camelia et al. (1998) to search for energy-dependent differences in the
times of arrival of gamma rays from gamma bursts.

Another interesting source of information relies on the expected parity-
violating nature of some of the natural proposals for LIV effects in the
propagation of photons (Gambini & Pullin, 1999, Myers & Pospelov,
2003). This would lead to differences in the propagation velocity for
photons with different helicities. It was observed that the effects would
lead to a depolarization of linearly polarized radiation as it propagates
towards the Earth. Therefore the observation of linearly polarized ra-
diation from distant sources could be used to set important bounds on such effects. For instance, Gleiser and Kozameh (2001) found a bound of the order $10^{-4}$ for the parameter $\xi$ for the photon.

Another type of bound can be obtained by noting that is quite unlikely that the Earth would be at rest in the preferred rest frame associated with the sought-for LIV. Thus in an Earth-bound laboratory Lorentz-violation could appear as violation of the isotropy of the laws of physics. Using the prescription for the expected effects on fermions which arise in the loop quantum gravity scenarios (Alfaro et al., 2000, 2002), one arrives at an effective SME description. Measurements rely on the extreme sensitivity of the Hughes driver type of test of the isotropy of physics using nuclear magnetic clocks (Chupp et al., 1989; Bear et al., 2000). The bounds obtained this way are of the order $10^{-5}$ and $10^{-9}$ on parameters that were originally expected to be of order unity. Then one obtains very stringent bounds on the parameters characterizing the state of the quantum geometry (Sudarsky et al., 2002). Similar constraints can be placed on the effects that arise in the string theory scenarios (Sudarsky et al., 2003).

A further source of severe constraints uses the possibility that different particle species have different values of their limiting velocity, as in the SME. Tests are made by examining the resulting changes in thresholds and decay properties of common particles. Coleman and Glashow (1999) obtained a dimensionless bound of $10^{-23}$ on this kind of Lorentz violation. Other related arguments connected to the existence of a bound to the propagation velocity of particles for modified dispersion relations have been used by Jacobson et al. (2002, 2003). These authors noted that the 100 MeV synchrotron radiation from the Crab nebula requires extremely high energy electrons. They combined the upper bound on the frequency of synchrotron radiation for electrons with a given velocity in a given magnetic field with the fact that there would be an upper bound for any electron’s velocity if $\xi$ for the electron had a particular sign. In fact the analysis, carried out within the Myers and Pospelov framework, indicates that at least for one of the electron’s helicities a corresponding $\xi$ parameter, if it had a particular sign, could not have a magnitude larger than about $10^{-7}$.

Finally there is the reported detection of cosmic rays with energies beyond the GZK cutoff. We recall that these ultrahigh energy cosmic rays are thought to be protons whose interaction with the photons of the cosmic microwave background would prevent them from traveling more than about 50 Mpc, while the likely sources are located much fur-
ther away. This anomaly is often presented as candidate observational evidence for LIV (Bird et al., 1995; Elbert and Sommers, 1995; Takeda et al., 1998; Abu-Zayyad et al., 2002; Bergman, 2003; Bahcall & Waxman, 2003). Our own feeling is that the list of unexplored alternative explanations of this anomaly, even if one needs to go beyond established physics, is much too broad at this time, and thus its interpretation as a signature of a LIV — given the difficulties we discussed here — is at best premature. Fortunately the Auger Experiment will become fully operational soon and its results should help clarify the situation.

1.7 Evading the naturalness argument within QFT

Several proposals have been made to evade the naturalness problem for Lorentz violation.

One argument relies essentially on the possibility that a fiducial symmetry would protect Lorentz symmetry. Jain and Ralston (2005) and Nibbelink and Pospelov (2005) argue that supersymmetry could be such symmetry. At the one-loop level this indeed works: contributions to self-energy graphs with particles and their superpartners have the same couplings but opposite signs. This cancellation is very reminiscent of the one for the cosmological constant in the same theories. However the authors note that, as the Lorentz algebra is a subalgebra of the supersymmetry algebra, invoking the latter to protect the former is not entirely consistent. They then observe that they would actually need only the translation subalgebra of the Poincaré algebra to be unbroken. However, it is hard to envision a situation in which a granular space-time would have the full translation group as a full continuous symmetry. Moreover as it is well known, even if it is there at some level, supersymmetry must be broken at low energies. Then it is difficult to understand how could it protect the low energy phenomena from the LIV we have been discussing, while allowing at the same time for violations to be observable at higher energy scales that are closer to that energy regime where super-symmetry is presumably unbroken.

Liberati et al. (2005) treat a condensed matter model of two component Bose-Einstein condensate as a model system. LI is associated with monometricity in the propagation of the two types of quasi-particles. The authors show that LI can, under certain conditions, be violated at high energies while being preserved at low energies. This is achieved by fine tuning a certain parameter in the model (the interaction with an
(external laser source). The fine tuning is in agreement with our general results.

The authors also conclude that their results are a hint that effective theories in emergent spacetimes could be unreliable beyond the tree approximation. Addressed to effective field theories themselves we do not think that this is correct, since it contradicts the meaning of an effective field theory. The Standard Model is an effective field theory relative to some more complete microscopic theory, and it most definitely must be used beyond tree approximation with non-trivial UV renormalization to get its phenomenological successes. The real issue, as is apparent from their next sentence, concerns the issue of the relation between the EFT and the microscopic theory beyond tree approximation, in the approach of interacting out short distance degrees of freedom. However, if the microscopic theory does actually violate LIV in an essential way at the Planck scale, then a EFT of the conventional kind derived from Planck-scale consideration will have LIV operators obeying the usual power counting rules. The applicability of EFT is then at all smaller momentum scales, and low energy phenomena have an expected size of normal one-loop corrections. Moreover the microscopic BEC model is a conventional quantum theory.

As Liberati, Sonego and Visser (2004) discuss in another paper, which we will summarize in the discussion section 1.9, it is possible that more fundamental issues come into play, perhaps concerned with measurement in a theory with a dynamical space-time. These issues would of course make even the principles of the derivation of an EFT quite different than in normal QFTs. But they would also remove the rationale for simple estimates for the sizes of higher dimension Lorentz-violating operators in an EFT.

Another proposal was made by Alfaro (2005) for a way to generate naturally small Lorentz violations. His general idea is to generate LIV in the integration measure for Feynman graphs. The proposal involves two concrete schemes. One uses a Lorentz violating cutoff that contains a parameter which when set to zero recovers a Lorentz invariant situation; the scheme thus has a parametrizably small LIV. The second scheme involves a Lorentz violating dimensional regularization scheme, where the standard Minkowski metric \( \eta_{\mu\nu} \) is replaced by \( g_{\mu\nu} = \eta_{\mu\nu} + \alpha \epsilon W_\mu W_\nu \) where \( \epsilon = n - 4 \) is the small parameter in the dimensional regularization scheme.

In the first scheme the regularization of a one-loop integral is to modify
it by multiplying the integrand by

\[ R(k) = \frac{-\Lambda^2}{k^2 - \Lambda^2 + ak_0^2 + i\epsilon}. \] (1.10)

where \( a = 0 \) is the Lorentz-invariant case. This suffers from a routing dependence and is therefore not well-defined, certainly not as a complete theory. Furthermore, in the Lorentz-invariant case \( a = 0 \), the regulator factor has a pole at \( k^2 = \Lambda^2 \). This is very similar to Pauli-Villars regularization, which gives negative metric states and therefore the regulated theory cannot be considered a normal quantum theory. This scheme therefore does not address the actual situation we are concerned with in quantum gravity.

The second scheme uses dimensional regularization and modifies the metric in a way that depends on the \( \epsilon = 0 \) pole in the integral being calculated. This graph-dependent modification of the metric does not correspond to any normal definition of a QFT, and no rationale is given.

1.8 Cutoffs in QFT and the physical regularization problem

Our results also have important implications for the definition of QFT. Given the well-known complications of renormalization, it is sensible to try defining a QFT as the limit of an ordinary quantum mechanical theory defined on a lattice of points in real space. One could also make time a discrete variable, but this is unnecessary. Continuum field theory is defined by taking the limit of zero lattice spacing, with appropriate renormalization of the bare parameters of the theory. However, if the cutoff theory is defined on an ordinary spatial lattice, boost invariance is completely broken by the rest frame of the lattice. Therefore all the issues discussed in this paper apply to the construction of the renormalized continuum limit, and fine-tuning is needed to get Lorentz invariance. This is acceptable for a mathematical definition of a QFT, but not in a theory that has a claim on being a fundamental theory.

Normal methods of calculation avoid the problem, but in none of them is the regulated theory a normal quantum mechanical model. For instance, the functional integral, as used in lattice gauge theories, is defined in Euclidean space-time. The regulated theory on a lattice is a purely Euclidean construct. Discrete symmetries under exchange of coordinate axes are enough to restrict counterterms to those that give SO(4) invariance in the continuum limit. Continuum QFT in Minkowski space is obtained by analytic continuation of the time variable, and the
compact SO(4) symmetry group of the Euclidean functional integral corresponds to the non-compact Lorentz group in real space-time.

On the other hand, a Pauli-Villars regulator can preserve LI in the regulated theory, but only at the expense of negative metric particles. That is, the regulated theory is not a normal quantum mechanical model.

Finally, dimensional regularization does preserve LI and many other symmetries. In this method, space is treated as having a non-integer dimension. Technically, space is made infinite dimensional, and this allows nonstandard definitions to be made of the integrals used in Feynman graphs so that they behave as if space has an arbitrary complex dimension (Collins, 1984). However, it is not even known how to formulate quantum field theories non-perturbatively within this framework.

Therefore we pose the problem of whether there exists a physical regularization of QFT in which LI is preserved naturally. A physical regularization means that the regulated theory is a normal quantum theory whose existence can be taken as assured.

One proposal of this kind was made by Evens et al. (1991), and it uses a nonlocal regularization. However Jain and Joglekar (2004) argue that the scheme violates causality and thus is physically unacceptable.

So one is left with a spatial lattice, or some variant, as the only obvious physical regulator of a QFT.

The need to treat gravity quantum mechanically provides the known limits to the physical applicability of the concepts and methods of QFT. Therefore the observed Lorentz invariance of real phenomena indicates that a proper theory of quantum gravity will provide a naturally Lorentz invariant physical regulator of QFT. So perhaps a discovery of a better method of defining a QFT in Minkowski space might lead to important clues for a theory of QG.

1.9 Discussion

It is well-known that nontrivial space-time structure is expected at the Planck scale, and this could easily lead to Lorentz-violating phenomena. The simplest considerations suggest that the observable Lorentz violation is suppressed by at least one power of particle energy divided by the Planck energy; this small expectation has led to an ingenious set of sensitive measurements, with so far null results.

However, an examination of field theoretic loop corrections shows that the expectation is incorrect, in general. Standard theorems in quantum field theory show that the low-energy effects of Planck-scale phenomena
can be summarized in an effective low-energy QFT whose Lagrangian contains all renormalizable terms compatible with the symmetries of the microscopic theory and the appropriate low-energy field content; this is the Standard Model Extension. If there is Lorentz violation in the fundamental theory, then in the effective theory, then the Lorentz violating parameters are, as we have shown, of the size of normal one-loop corrections in the Standard Model, in violent contradiction with data. Without some special mechanism, extreme fine tuning is needed.

It is already known (Susskind, 1979; Weinberg, 1989) that there are fine-tuning problems with the Standard Model, involving at least the cosmological constant, mass hierarchies and the Higgs mass term. These, of course, suggest to many physicists that the Standard Model is not the ultimate microscopic theory, but is a low-energy approximation to some more exact theory where fine-tuning is not needed. Our results show that Lorentz invariance should be added to the list of fine tuning problems that should be solved by a good theory that includes quantum gravity, or alternatively by a new theory that supersedes currently known ideas. We thus suggest that a search for a physically meaningful, Minkowskian space-time, Poincaré and gauge invariant regulator for the Standard Model could be intimately connected with the search for a theory of QG and with its possible phenomenological manifestations. The lack of a physical regularization for QFT besides the lattice makes the non-naturalness of Lorentz invariance a particularly important problem even when gravity is left out of the discussion.

We conclude by mentioning some intriguing ideas.

Some ideas regarding how a discrete nature of space-time can be made consistent with Lorentz invariance are explored by Rovelli & Speziale (2003) and by Dowker, Henson, and Sorkin (2004). In particular Dowker et al. show that by using a random lattice or causal set methods one can evade the problem that regular spatial lattices prevent a physical realization of Lorentz contraction.

There are also considerations of other possible types of manifestations of QG. For instance there are proposals regarding nonstandard couplings to the Weyl tensor (Corichi & Sudarsky, 2005), fundamental quantum decoherence (Gambini et al., 2004), and QG induced collapse of the wave function (Penrose, 1989; Perez et al., 2005).

Finally, there are proposals invoking fundamental modifications of the Lorentz or Poincaré structures. This is the subject of doubly special relativity (DSR) which we discussed briefly in our introduction, Sec. together with some critiques of the physical significance of DSR.
An interesting idea, with more general applicability, is the proposal by Liberati, Sonego and Visser (2004) for resolving the problem in DSR that the measurable momentum operators differ from the operators, also present in DSR, that obey the standard commutation relations with the Lorentz generators. They suggest that the modifications of the momentum operators are a non-trivial effect of quantum mechanical measurement when quantum gravity effects are important. To our mind, this impinges on an important foundational problem in QFT and QG as compared with elementary quantum mechanics, including the issue of the relation between an effective field theory and an underlying theory in which space-time is genuinely dynamical.

In simple quantum mechanical theories of systems like the Schrödinger equation for a single atom, measurement involves an external apparatus. But with an interacting QFT, the theory is sufficiently broad in scope that it describes both the system being measured and the experimental apparatus measuring it. If the Standard Model is valid, it accurately governs all strong, electromagnetic and weak interactions, and therefore it includes particle detectors as well as particle collisions. An interacting QFT has a claim on being a theory of everything (in a certain universe-wide domain) in a way that a few-body Schrödinger equation does not. Measurement theory surely has a different status in QFT. This point is exemplified by the analysis by Sorkin (1993). This should apply even more so when quantum gravity is included. A localized measurement of a sufficiently elementary particle of sufficiently super-Planck energy could have a substantial effect on the local space-time metric and thus on the meaning of the energy being measured.

The emergence of the field known as QG phenomenology is certainly a welcome development for a discipline long considered as essentially removed from the empirical realm. However one should avail oneself of all the other established knowledge in physics, in particular, the extensive development both at the theoretical and experimental level of QFT. Ignoring the lessons it provides, and the range of its successful phenomenology is not a legitimate option, unless one has a good substitute for it. The unity of physics demands that we work to advance in our knowledge by seeking to expand the range covered by our theories, therefore we should view with strong skepticism, and even with alarm any attempt to extrapolate in one direction — based essentially on speculation — at the price of having to cede established ground in any other.
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