High-fidelity local addressing of trapped ions and atoms by composite sequences of laser pulses

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A vital requirement for a quantum computer is the ability to locally address, with high fidelity, any of its qubits without affecting their neighbors. We propose an addressing method using composite sequences of laser pulses, which reduces dramatically the addressing error in a lattice of closely spaced atoms or ions, and at the same time significantly enhances the robustness of qubit manipulations. To this end, we design novel high-fidelity composite pulses for the most important single-qubit operations. In principle, this method allows one to beat the diffraction limit, for only atoms situated in a small spatial region around the center of the laser beam are excited, well within the laser beam waist. © 2013 Optical Society of America

Scalable quantum computers depend critically on the ability to perform local addressing of their individual qubits \cite{1}. In a Paul ion trap, which is one of the most promising scalable platforms for the future quantum computer \cite{2}, local addressing is the ability to operate on a single ion by focused laser light while keeping the neighboring ions unaffected. When the number of ions increases, the distance between them diminishes and local addressing becomes one of the principal experimental challenges. For example, in a recent experimental demonstration of the Toffoli gate \cite{3} most of the error was attributed to addressing error, as the neighboring ions were seeing 7% of the central Rabi frequency.

In this Letter, we propose a method for high-fidelity local addressing applicable to various types of atomic qubits: trapped ions, atoms in optical lattices, quantum dots, etc. To this end, we present new narrowband (NB) and passband (PB) composite pulses\(^1\), which are specially designed for local addressing. The excitation profiles of such pulses allow one to manipulate only a single qubit, as the outer parts of the spatial laser beam profile practically do not excite its neighbors, although the latter may be subjected to significant laser intensity. Moreover, with a PB pulse one enhances the robustness of qubit manipulations, thereby eliminating errors due to imperfectly calibrated and fluctuating laser intensity and laser beam pointing instability.

The technique of composite pulses was introduced in nuclear magnetic resonance (NMR) \cite{4-6} as a powerful tool for manipulation of spins by magnetic fields. A composite pulse compensates the imperfections of a single pulse, which is the traditional tool used to drive a quantum transition, and it consists of a sequence of pulses, each with a well-defined phase. The composite phases are determined from the conditions imposed on the desired overall excitation profile. In particular, in a NB composite pulse only the qubits seeing pulse areas within a narrow range around some value \(\mathcal{A}\) are subjected to trans-

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\(^1\)We follow the usual NMR terminology, which is related to the features of the excitation profile, rather than the radiation.
We consider here an odd number of pulses, \( N = 2n + 1 \) phased resonant pulses of area \( A: A_0A_1A_2\phi_1A_3\cdots A_{n+1}A_{n+2}A_0 \). We set \( n_1 = n, n_2 = 0 \) for all NB pulses \( N_{2n+1}(A) \); \( n_1 = 2, n_2 = 1 \) for \( P_{9}(\pi) \) and \( P_{9}(A) \); \( n_1 = 6, n_2 = 3 \) for \( P_{17}(\pi) \); \( n_1 = 4, n_2 = 3 \) for \( P_{17}(\pi/2) \). For all sequences we set \( n_3 = 0 \). For \( A \neq \pi \) we also impose Eq. (4a). The pulse area is \( A = \pi \) for \( N_3(\pi) \), \( N_5(\pi) \), \( N_{13}(\pi) \), \( N_{21}(\pi) \), \( P_7(\pi) \) and \( P_{17}(\pi) \); \( A = 3\pi/7, \pi/2, 3\pi/8 \), \( 3\pi/4 \) or \( \pi/2 \), respectively, for \( N_5(\pi/2) \), \( N_7(\pi/2) \), \( N_7(\pi/\sqrt{2}) \) and \( N_7(\pi/\sqrt{2}) \); \( A = 2\pi/3 \) for \( P_{7}(\pi/2) \), \( P_9(\pi/2) \) and \( P_{17}(\pi/2) \), \( P_9(\pi/\sqrt{2}) \) and \( P_{17}(\pi/\sqrt{2}) \). With \( n_1 + n_2 + n_3 = 6 \) for a composite sequence we add the conditions

\[
\frac{\partial A_{11}^{(N)}}{\partial A_{11}^{(N)}} |_{A=\hat{A}} = \cos(\pi/2),
\]

(4a)

\[
\left[ \frac{\partial A_{11}^{(N)}}{\partial \phi_1} \right] |_{A=0} = 0 \quad (k = 2, 4, \ldots, 2n_1),
\]

(4b)

with \( n_1 \) as in (4a) and \( \partial A_{11}^{(N)} / \partial A^k \). For a \( n_2 \)-fold flat-top PB sequence we add the conditions

\[
\left[ \frac{\partial A_{11}^{(N)}}{\partial \phi_1} \right] |_{A=\hat{A}} = 0 \quad (k = 1, 2, \ldots, n_2),
\]

(5)

with \( n_1 + n_2 + n_3 = 6 \). For a sequence with a target phase \( \varphi \) we must also have

\[
\arg A_{11}^{(N)} |_{A=\hat{A}} = \varphi.
\]

(6)

This phase can be stabilized with PB pulses, for which

\[
\arg \left[ \frac{\partial A_{11}^{(N)}}{\partial \phi_1} \right] |_{A=\hat{A}} = 0 \quad (k = 1, 2, \ldots, n_3).
\]

(7)

Thus a phase-stable PB sequence consists of \( n_1 + n_2 + n_3 + 2 \) pulses. For target area \( A = \pi \), Eqs. (4a) and (5) for even \( k \) and (7) for odd \( k \) are fulfilled identically.

Using the method described above, we have constructed new NB and PB composite pulses; examples are presented in Table 1 for the most often used areas. Figure 1 illustrates the excitation profiles for different composite pulses of target area \( \pi \), and \( \pi/2 \) (right frames). At the center of the laser spot (origin) the

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Table 1. Phases \( \phi_k \) (in units \( \pi \)) for some NB (\( N \)) and PB (\( P \)) sequences of \( N = 2n + 1 \) phased resonant pulses of area \( A: A_0A_1A_2\phi_1A_3\cdots A_{n+1}A_{n+2}A_0 \). We set \( n_1 = n, n_2 = 0 \) for all NB pulses \( N_{2n+1}(A) \); \( n_1 = 2, n_2 = 1 \) for \( P_9(\pi) \) and \( P_9(A) \); \( n_1 = 6, n_2 = 3 \) for \( P_{17}(\pi) \); \( n_1 = 4, n_2 = 3 \) for \( P_{17}(\pi/2) \). For all sequences we set \( n_3 = 0 \). For \( A \neq \pi \) we also impose Eq. (4a). The pulse area is \( A = \pi \) for \( N_3(\pi) \), \( N_5(\pi) \), \( N_{13}(\pi) \), \( N_{21}(\pi) \), \( P_7(\pi) \) and \( P_{17}(\pi) \); \( A = 3\pi/7, \pi/2, 3\pi/8 \), \( 3\pi/4 \) or \( \pi/2 \), respectively, for \( N_5(\pi/2) \), \( N_7(\pi/2) \), \( N_7(\pi/\sqrt{2}) \) and \( N_7(\pi/\sqrt{2}) \); \( A = 2\pi/3 \) for \( P_7(\pi/2) \), \( P_9(\pi/2) \) and \( P_{17}(\pi/2) \), \( P_9(\pi/\sqrt{2}) \) and \( P_{17}(\pi/\sqrt{2}) \). With \( n_1 + n_2 + n_3 = 6 \) for a composite sequence we add the conditions

\[
\left[ \frac{\partial A_{11}^{(N)}}{\partial A_{11}^{(N)}} \right] |_{A=\hat{A}} = \cos(\pi/2),
\]

(4a)

\[
\left[ \frac{\partial A_{11}^{(N)}}{\partial \phi_1} \right] |_{A=0} = 0 \quad (k = 2, 4, \ldots, 2n_1),
\]

(4b)

with \( n_1 \) as in (4a) and \( \partial A_{11}^{(N)} / \partial A^k \). For a \( n_2 \)-fold flat-top PB sequence we add the conditions

\[
\left[ \frac{\partial A_{11}^{(N)}}{\partial \phi_1} \right] |_{A=\hat{A}} = 0 \quad (k = 1, 2, \ldots, n_2),
\]

(5)

with \( n_1 + n_2 + n_3 = 6 \). For a sequence with a target phase \( \varphi \) we must also have

\[
\arg A_{11}^{(N)} |_{A=\hat{A}} = \varphi.
\]

(6)

This phase can be stabilized with PB pulses, for which

\[
\arg \left[ \frac{\partial A_{11}^{(N)}}{\partial \phi_1} \right] |_{A=\hat{A}} = 0 \quad (k = 1, 2, \ldots, n_3).
\]

(7)
Table 2. Phases $\phi_k$ (in units $\pi$) for some NB ($N_N(A, \varphi)$) and PB ($P_N(A, \varphi)$) sequences of $N = 2n + 1$ phased resonant pulses of area $A$: $A_0 A_2 A_3 \cdots A_{n+1} \cdots A_3 A_2 A_0$, which produce phased rotations of angle $\varphi$ at area $A$. We set $n_2 = n_3 = 0$ for all NB pulses $N_{2n+1}(A, \varphi)$. We have $n_2 = 1, n_3 = 4$ for $P_{11}(A, \varphi)$; $n_2 = n_3 = 1$ for $P_{13}(A, \varphi)$. For all sequences we set $n_1 = 2$ and we impose Eq. (6). For $A \neq \pi$ we also impose Eq. (4a). The pulse area is $A = \pi$ for $N_7(\pi, 3\pi/2)$ and $P_{11}(\pi, 3\pi/2)$; $A = 3\pi/4$ for $N_9(\pi/2, \pi/2)$ and $P_{13}(\pi/2, \pi/2)$; $A = 3\pi/5$ for $N_9(\pi/\sqrt{2}, 3\pi/2)$ and $P_{13}(\pi/\sqrt{2}, 3\pi/2)$.

| Sequences | $A$ | $\varphi$ | $p_A$ |
|-----------|-----|-----------|-------|
| $N_7(\pi, \frac{3\pi}{2})$ | $\pi$ | (1.256; 0.792; 0.072) |
| $P_{11}(\pi, \frac{3\pi}{2})$ | $\pi$ | (0.221; 1.109; 0.753; 1.304; 1.878) |
| $N_9\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ | $\frac{3\pi}{4}$ | (1.074; 0.935; 0.173; 1.562) |
| $P_{13}\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ | $\frac{3\pi}{4}$ | (0.959; 1.048; 0.367; 1.967; 1.511; 0.860) |

The probability of excitation is $p_0 = \sin^2(A/2)$, whereas at the wings it naturally decreases. The logarithmic scale allows us to examine the fidelity of the profile against the $10^{-4}$ benchmark [1]. We assume that the laser beam is a spot with a full-width-at-half-maximum (FWHM) of intensity $\xi/\sqrt{2}$, this implies FWHM of Rabi frequency $\xi$ for single-photon transitions. Remarkably, suppression of unwanted neighbor excitation below the $10^{-4}$ benchmark is achieved at up to 15% of the peak Rabi frequency (corresponding to distance $0.89\xi$ from the center of the spot) with the $N_5$ pulse, 27% (distance 0.70$\xi$) with $P_{17}$, and 48% (distance 0.51$\xi$) with $N_{21}$. In this manner we can beat the diffraction limit as the excitation is localized in a spatial range, which, with sufficiently many ingredient pulses, can be made smaller than the beam waist. The physical reason for this suppression is the destructive interference of the ingredient pulses in the composite sequence.

The bottom frames of Fig. 1 show the infidelity of the target qubit itself and reveal that PB composite pulses can greatly enhance the robustness of manipulation of the target qubit without losing selectivity. For target area $A = \pi$ (left frame) an infidelity of $10^{-4}$ is encountered at offset 0.05$\xi$ for a single pulse, while the admissible offset reaches 0.18$\xi$ for $P_2(\pi)$ and 0.21$\xi$ for $P_1(\pi)$.

Our scheme can be further exploited, in conjunction with Eqs. (6) and (7), to generate various NB and PB sequences of specified target phases $\varphi$ at the expense of additional pulses. Table 2 lists a set of composite sequences, which produce experimentally relevant phased rotations. Larger sequences achieve stabilization of the phase $\varphi$. Figure 2 illustrates the robustness of the composite phase $\varphi = 3\pi/2$ [3] produced by two of our phase-stabilized sequences from Table 2. Remarkably, one can perform high-fidelity local addressing even with pulse area deviation of 20%. In addition, we have found that pulse area noise with relative amplitude 5% introduces an absolute error in the target phase of $2.5 \times 10^{-3} \pi$.

To conclude, the new composite sequences designed for high-fidelity local addressing in a lattice of closely spaced qubits, are of potential application to Paul ion traps [7,8] and ultracold atoms in optical lattices [9]. They allow one to stabilize both the rotation angle and the phase of the desired qubit rotation. This technique can be adapted to addressing on the vibrational sidebands, which should allow to construct high-fidelity two-qubit operations.

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References

1. M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, UK, 2000).
2. http://qist.lanl.gov/qcomp_nap.shtml
3. T. Monz, K. Kim, W. Hüsler, M. Riebe, A. S. Villar, P. Schindler, M. Chwalla, M. Hennrich, and R. Blatt, Phys. Rev. Lett. 102, 040501 (2009).
4. E. L. Hahn, Phys. Rev. 80, 580 (1950).
5. M.H. Levitt and R. Freeman, J. Magn. Reson. 33, 473 (1979); R. Freeman, S. P. Kempsell, and M. H. Levitt, J. Magn. Reson. 38, 453 (1980); H. M. Cho, R. Tycko, A. Pines, and J. Guckenheimer, Phys. Rev. Lett. 56, 1905 (1986); M.H. Levitt, Prog. NMR Spectrosc. 18, 61 (1986); R. Freeman, Spin Choreography (Spectruman, Oxford, 1997).
6. S. Wimperis, J. Magn. Reson. 109, 221 (1994).
7. H. Häffner, C.F. Roos, R. Blatt, Phys. Rep. 469, 155 (2008).
8. S. Gulde, M. Riebe, G. P. T. Lancaster, C. Becher, J. Eschner, H. Häffner, F. Schmidt-Kaler, I. L. Chuang and R. Blatt, Nature 421, 48 (2003); F. Schmidt-Kaler, H. Häffner, M. Riebe, S. Gulde, G. P. T. Lancaster, T.
Deuschle, C. Becher, C. F. Roos, J. Eschner, and R. Blatt, Nature 422, 408 (2003); N. Timoney, V. Elman, S. Glaser, C. Weiss, M. Johannng, W. Neuhauser, and C. Wunderlich, Phys. Rev. A 77, 052334 (2008).

9. I. Bloch, T. W. Hänsch, and T. Esslinger, Nature 403, 166 (2000); I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. 80, 885 (2008); C. Weitenberg, M. Endres, J. F. Sherson, M. Cheneau, P. Schauß, T. Fukuhara, I. Bloch, and S. Kuhr, arXiv:1101.2076.