Remarks on the Unruh effect with mixed neutrinos

M Blasone\textsuperscript{1,2}, G Lambiase\textsuperscript{1,2}, G G Luciano\textsuperscript{2} and L Petruzziello\textsuperscript{1,2}

\textsuperscript{1} Dipartimento di Fisica, Università di Salerno, Via Giovanni Paolo II, 132 I-84084 Fisciano (SA), Italy.
\textsuperscript{2} INFN Sezione di Napoli, Gruppo collegato di Salerno, Italy

E-mail: blasone@sa.infn.it, lambiase@sa.infn.it, gluciano@sa.infn.it, lpetruzziello@na.infn.it

Abstract. We discuss some recent results on the inverse $\beta$-decay of an accelerated proton in the presence of neutrino mixing. By comparing different approaches, we conclude that a consistent treatment preserving general covariance should be based on flavor neutrino states. We also comment on the issue of thermality of the Unruh effect for mixed neutrinos.

1. Introduction

The analysis of decay processes in accelerated frames [1] provides a useful framework to test the consistency of Quantum Field Theory (QFT) in non-trivial backgrounds. In particular, the inverse $\beta$-decay of accelerated protons has been addressed, showing the necessity of Unruh effect to preserve the general covariance of QFT [2, 3].

Recently, it has been argued that the introduction of neutrino mixing in the above process could lead to theoretical ambiguities [4]. In Refs. [5, 6], it has been shown that the equality of decay rates calculated in the inertial and comoving frames holds also in the presence of mixing, at least in a specific approximation. Subsequently, in the paper by Cozzella \textit{et al.} [7], it has been claimed that no problem arises at all when neutrino mixing is involved, and that the two decay rates exactly match.

We remark that the fundamental difference between the approach of Refs. [5, 6] and the one of Ref. [7] relies in the choice of asymptotic states for mixed neutrinos. Indeed, whilst in Refs. [5, 6] such a rôle is played by flavor eigenstates, in Ref. [7] asymptotic neutrinos are required to be mass eigenstates. Therefore, it appears to be an irremediable dichotomy in the above two treatments, which somehow needs to be cured.

In the present paper, we show that the statements of Ref. [7] are unjustified, being based on incorrect assumptions. Before studying this, in Sec. 2 we review some general considerations on the quantization of neutrino fields with flavor mixing, which are useful for the subsequent analysis. In Sec. 3 we discuss the results of Ref. [7] in light of our approach [5]. Sec. 4 is devoted to conclusions.

2. Neutrino mixing in QFT

Neutrino mixing [8] is embedded in the Standard Model in analogy to quark mixing, at least for the case of Dirac neutrinos. Considering for simplicity the case of two flavors, we have that the Lagrangian associated to neutrino fields with definite flavors $\nu_e$, $\nu_\mu$ exhibits off-diagonal terms.
in the mass matrix, which can be diagonalized by means of the mixing transformations:
\[
\nu_e(x) = \cos \theta \nu_1(x) + \sin \theta \nu_2(x), \\
\nu_\mu(x) = -\sin \theta \nu_1(x) + \cos \theta \nu_2(x),
\]
where \(\nu_1, \nu_2\) are fields with definite masses.

Quantization of flavor fields is achieved [9] by identifying the algebraic generator of the rotation Eq. (1)
\[
\hat{\nu}_e(x) = \hat{G}^{-1} \hat{\nu}_1(x) \hat{G}, \\
\hat{\nu}_\mu(x) = \hat{G}^{-1} \hat{\nu}_2(x) \hat{G},
\]
with
\[
\hat{G} = \exp \left\{ \theta \int d^3 x \left[ \hat{\nu}_1(x) \hat{\nu}_2(x) - \hat{\nu}_2(x) \hat{\nu}_1(x) \right] \right\},
\]
which is easy to check using the canonical anticommutation relations. The expansions for the flavor fields are obtained by inserting into Eq. (2) the corresponding expansions for the free fields. A straightforward calculation leads to
\[
\hat{\nu}_\alpha(x) = \sum_r \int d^3 k e^{i k \cdot x} \left[ \hat{\sigma}^r_{k,\alpha} u^r_{k,\alpha}(t) + \hat{\sigma}^{r\dagger}_{k,\alpha} v^r_{k,\alpha}(t) \right],
\]
where \((\alpha, \sigma) = (e, 1), (\mu, 2)\) and \(r\) is the polarization. In this way, flavor ladder operators are consistently defined [9].

A subtle point in the above quantization procedure arises when considering the action of the mixing generator \(\hat{G}\) on the free field vacuum \(|0\rangle_m\) (mass vacuum). This indeed generates a non-trivial state \(|0\rangle_f\) (flavor vacuum), defined as:
\[
|0\rangle_f \equiv \hat{G}^{-1} |0\rangle_m.
\]

It is possible to show that the Fock spaces for neutrinos with definite mass and definite flavor (\(\mathcal{H}_m\) and \(\mathcal{H}_f\), respectively) are unitarily inequivalent to each other in the infinite volume limit [9]. Indeed, one has
\[
\lim_{V \to \infty} m \langle 0 | 0 \rangle_f = 0.
\]

The flavor vacuum has the following properties:

- it is an eigenstate of the flavor charge operators \(\hat{Q}_\alpha\), obtained by means of Noether’s theorem [10];
- it is a generalized Gilmore-Perelomov coherent state [11];
- it is a condensate of neutrino-antineutrino pairs both with same and different masses [9].

Flavor neutrino states are then consistently defined as eigenstates of flavor charges
\[
\hat{Q}_\alpha |\nu_\alpha\rangle = |\nu_\alpha\rangle, \quad \alpha = e, \mu.
\]
In the relativistic limit, these states reduce to the usual Pontecorvo flavor states [8],
\[
|\nu_\alpha\rangle_P = \sum_i U_{\alpha,i} |\nu_i\rangle,
\]
where \(U_{\alpha,i}\) is the generic element of the mixing matrix
\[
U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}.
\]
3. Remarks on the Unruh effect with mixed neutrinos

In what follows, we discuss the results of Ref. [7] in view of the framework developed in Sec. 2.

In the Introduction of Ref. [7], the authors consider the recent investigation of Ref. [4] about the inequality of the inertial and the comoving decay rates for the process $p \rightarrow n \ e^+ \ \nu_e$ with mixed neutrinos, asserting that in principle no discrepancy should exist between the calculations done in the two reference frames. On the basis of this, they then affirm that “either the Unruh effect is wrong (contradicting several previous results [7], including what we consider to be a virtual observation of it [8]) or some mistake was made in the previously mentioned analysis”.

We basically agree with the authors of Ref. [7] on the fact that general covariance should be taken as a guiding principle in the analyzed problem and thus the decay rates in the two frames should coincide also when mixed neutrinos are involved. On the other hand, we do not agree that a mismatch of the above decay rates in the presence of neutrino mixing undermines the validity of Unruh effect as proved in Refs. [2], for the simple reason that in these works mixing is not taken into account at all.

Rather, we think that it is possible that corrections to the Unruh effect (in the form of non-thermal contributions) may arise in connection with neutrino mixing. This has indeed been shown explicitly in Refs. [12, 13], where the quantization of both bosonic and fermionic mixed fields was performed in accelerated frame.

In this connection, we stress that our analysis of the proton decay rates with mixed neutrinos carried out in Ref. [5], demonstrates that general covariance holds in the approximation in which Pontecorvo states may be used to represent flavor neutrinos. Beyond such an approximation, general covariance must hold, but Unruh effect may be somehow modified.

* * *

We now turn the attention to another important debated point. Although in the Appendix B of Ref. [7] the authors show that Pontecorvo states are well-defined in the limit of small mass difference with respect to the intrinsic uncertainties in the neutrino momenta, they never use such states in their treatment, even in the aforementioned approximation. This allows us to conclude that their result should be equal to our outcome in the regime in which Pontecorvo states are correct, since we only deal with such states\footnote{Note that these states can be efficiently generalized also to the case of curved backgrounds, and in particular of accelerated frames [14, 15].}. However, this is not the case: indeed, the use of Pontecorvo states produces “off-diagonal” terms (namely, contributions proportional to $\cos^2 \theta \sin^2 \theta$) in the expressions of the inertial ($\Gamma_{\text{in}}^{p \rightarrow n}$) and comoving ($\Gamma_{\text{acc}}^{p \rightarrow n}$) decay rates [5, 6],

$$\Gamma_{\text{in}}^{p \rightarrow n} = \cos^4 \theta \Gamma_1^{p \rightarrow n} + \sin^4 \theta \Gamma_2^{p \rightarrow n} + \cos^2 \theta \sin^2 \theta \Gamma_{12}^{p \rightarrow n}, \quad (10)$$

$$\Gamma_{\text{acc}}^{p \rightarrow n} = \cos^4 \theta \tilde{\Gamma}_1^{p \rightarrow n} + \sin^4 \theta \tilde{\Gamma}_2^{p \rightarrow n} + \cos^2 \theta \sin^2 \theta \tilde{\Gamma}_{12}^{p \rightarrow n}. \quad (11)$$

Notice that these off-diagonal contributions are completely absent in Eqs. (49) and (53) of Ref. [7], thus showing the internal inconsistency of that formulation.

The origin of this discrepancy can be readily understood: in Ref. [7], the authors consider the decay process

$$p \rightarrow n \ \bar{l}_\alpha \ \nu_i, \quad (12)$$

with $l_\alpha = \{ e, \mu, \tau \}$ and $\nu_i = \{ \nu_1, \nu_2, \nu_3 \}$. It is clear that, with such a choice, the aforementioned off-diagonal terms can never appear in any calculation of the decay rates. Thus, the outcome of Ref. [7] - which is claimed to be exact - cannot reproduce the one of Refs. [5, 6] within the relativistic approximation, where Pontecorvo states are universally recognized to be well-defined.
We further point out that in Ref. [7] mixing is correctly taken into account at the level of fields in the weak interaction action

$$\hat{S}_I = \int d^4x \sqrt{-g} \left( \sum_\alpha \bar{\psi}_\alpha \gamma^\mu \hat{P}_\mu \phi_\alpha + h. c. \right),$$

but not at the level of the states\(^2\), not even in the particular situations in which Pontecorvo states are known to be valid.

Furthermore, it must be emphasized that the combined use in Ref. [7] of the action Eq. (13) with the neutrino states for the process Eq. (12) does not satisfy normalization conditions. Fields and states adopted in the evaluation of transition amplitudes should indeed be “compatible” in the sense of normalization. Namely, if the action is written in terms of a generic field \(\phi\) as\(^3\)

$$\hat{S} = \int d^4x \mathcal{L} \left( \phi, \partial \phi \right),$$

then one expects to work with states \(|\phi\rangle\) that satisfy the relation \([16, 17]\)

$$\langle 0 | \hat{\phi}(0) | \phi(p) \rangle = 1.$$

Expanding the field \(\hat{\phi}\) as

$$\hat{\phi}(x) = \int \frac{d^3k}{(2\pi)^3 2 \omega_k} \left[ a_k e^{-ik \cdot x} + \tilde{a}_k e^{ik \cdot x} \right],$$

then the commutation relations for the ladder operators read \([16, 18]\)

$$[a_k, \tilde{a}_p] = [\tilde{b}_k, b_p] = (2\pi)^3 2 \omega_k \delta(k-p),$$

which validate the use of Eq. (15).

If now we consider the case involving mixing, we have

$$|\phi_A\rangle = \cos \theta |\phi_1\rangle + \sin \theta |\phi_2\rangle,$$
$$|\phi_B\rangle = -\sin \theta |\phi_1\rangle + \cos \theta |\phi_2\rangle,$$

with a similar relation holding for fields, in analogy to Eq. (1). Then, in the expansion Eq. (16), for each field \(\hat{\phi}_i, i = \{1, 2\}\), we have \(\omega_k \rightarrow \omega_{k,i}\), \(\hat{a}_k \rightarrow \hat{a}_{k,i}\) and \(\tilde{b}_k \rightarrow \tilde{b}_{k,i}\).

Assuming for the mixed fields \(\phi_a, a = \{A, B\}\), an action analogous to that in Eq. (13) and considering a decay process involving \(|\phi_i\rangle\) similar to the one in Eq. (12), we get

$$\sum_a \langle 0 | \hat{\phi}_a | \phi_i \rangle = \sum_{a,i} U_{a,i} \neq 1.$$

thus showing that the states \(|\phi_i\rangle\) are at odds with the fields \(\hat{\phi}_a\) in the sense of the normalization Eq. (15).

\(^2\) This is the reason why the total decay rate Eq. (48) in Ref. [7] contains second powers of the mixing matrix elements, while in Eq. (38) of Ref. [5] they appear with the fourth power.

\(^3\) To make the analysis as transparent as possible, in what follows we refer to scalar fields. Analogous considerations hold for fermionic fields.
A proper choice for the process Eq. (12) should instead be
\[ p \rightarrow n \bar{l}_\alpha \nu_\alpha, \]  
with \[ \langle 0 \mid \hat{\phi}_\alpha \mid \phi_\alpha \rangle = \cos^2 \theta + \sin^2 \theta = 1, \] which is the result one would expect in analogy with Eq. (15).

We stress that, with this normalization, the transition probabilities between the states \( \mid \phi_A \rangle \) and \( \mid \phi_B \rangle \) are slightly different from the usual ones. In fact, it is possible to derive that
\[ |\langle \phi_A(t) \mid \phi_A \rangle|^2 = 4(2\pi)^6 \omega_A^2 \left[ 1 - \frac{\omega_1 \omega_2}{\omega_A^2} \sin^2 2\theta \sin^2 \left( \frac{\Delta \omega t}{2} \right) \right], \] with \( \omega_A = \omega_1 \cos^2 \theta + \omega_2 \sin^2 \theta \) and \( \Delta \omega = \omega_2 - \omega_1 \). By defining
\[ \mid \tilde{\phi}(p) \rangle = \frac{\mid \phi(p) \rangle}{(2\pi)^3 \omega_p}, \] the expression Eq. (22) becomes
\[ \left| \langle \tilde{\phi}_A(t) \mid \tilde{\phi}_A \rangle \right|^2 = 1 - \frac{\omega_1 \omega_2}{\omega_A^2} \sin^2 2\theta \sin^2 \left( \frac{\Delta \omega t}{2} \right), \] which, in the relativistic limit, reduces to the usual Pontecorvo oscillation formula.

\[ * \ * \ * \]

Finally, we comment on the effort of the authors of Ref. [7] to prove that mixed neutrinos do not spoil the thermality of the Unruh effect. Similarly to what done in Ref. [4], they assume a thermal bath of neutrinos with definite masses. The probability of detecting a neutrino with flavor \( \alpha \) per unit proper time is then given by Eq. (34) of Ref. [7]:
\[ \frac{dP_\alpha}{d\tau} \mid \Delta m^2_{ij} \sim 0 \approx \sum_i |U_{\alpha,i}|^2 \frac{dP_{\text{exc},i}}{d\tau} \mid \Delta m^2_{ij} \sim 0 \approx \frac{dP_{\text{exc},i}}{d\tau} \mid m_i \approx \text{const}, \] with \( \Delta m^2_{ij} = |m_i^2 - m_j^2| \).

The last step of Eq. (25) practically erases mixing, which is not equivalent to consider it in the relativistic regime. Hence, their claim that the thermality of the Unruh effect is preserved also for flavor neutrino is unfounded. A rigorous analysis of field mixing in accelerated frame has been given in Refs. [12, 13], where nonthermal contributions have been found, which correctly vanish when the relativistic limit is taken into account. Of course, such contributions also vanish in the trivial equal-masses limit considered by Cozzella et al.

We want to remark once again that the assumption of Ref. [7] about thermal states for mixed neutrinos is the same adopted in Ref. [4] (where it is motivated by the requirement of KMS condition). Such an observation has been shown to violate general covariance in Refs. [5, 6].

4. Conclusions
In this paper we have discussed some claims of Ref. [7], where it is stated that the standard Unruh effect is perfectly valid for mixed neutrinos and that the decay rates in the inertial and comoving frames do indeed match also in the presence of mixing.

We share with the authors of Ref. [7] the belief that general covariance should act as a guiding principle in the comparison of the two decay rates. However, we have shown that the analysis
performed by Cozzella et al. fails to pinpoint the peculiar aspects of mixing involved in the problem under consideration. In particular, this last statement is supported by the fact that calculations of Ref. [7] do not recover the outcome of Ref. [5] (where Pontecorvo states are employed to investigate the inverse $\beta$-decay) in the limit there considered. Indeed, there is no approximation that can provide the decay rates of Ref. [7] with the off-diagonal contributions arising from the study conducted in Ref. [5].

As a final remark, we would like to stress that the problem considered in Refs. [5, 7] has to do with the internal consistency of quantum field theory, and thus it is necessary to proceed as far as possible with an exact approach without assumptions based on phenomenological considerations. In our analysis [5, 6], by resorting to the concept of flavor neutrino states defined as exact eigenstates of flavor charges obtained via Noether’s theorem, we consider an approximation in which usual Pontecorvo states for flavor neutrinos can be safely used. In Ref. [5], we have shown that, in such an approximation, the decay rates in the two frames do indeed agree. It remains an open question if corrections to the Unruh effect are necessary in the more general case of exact flavor states. In any case, corrections will be small and this is to be regarded as a virtue of the formalism which allows to highlight novel, unpredicted, effects (as, for example, in the context of the Casimir effect [19], where the non-trivial nature of flavor vacuum is responsible for an unusual behavior of the attractive strength between the binding plates [20]). Further investigation is required along this line [21].

Appendix: Meaning of the approximation used in Ref. [5]

In Ref. [5], we have adopted an approximation in the evaluation of the decay rates, in particular the interference terms $\Gamma_{12}$ and $\tilde{\Gamma}_{12}$ in Eqs. (10) and (11), by taking small $\epsilon \equiv \frac{\delta m}{m_{\nu_1}}$ and then letting $m_{\nu_1} \to 0$. Thus, one may think that it is necessary to carry out a “controlled expansion” of such terms, considering $m_{\nu_1}$ and $\epsilon$ on the same footing.

In this regard, we would like here to clarify that, although the expansions in Eqs. (41) and (56) of Ref. [5] are formally performed around $\epsilon = 0$, the contributions we actually compare in the two frames are the first-order terms of the expansions in $\delta m$ (see Eqs. (42) and (57) of Ref. [5]), where $\Gamma^{(1)}$ and $\tilde{\Gamma}^{(1)}$ are divided by $m_{\nu_1}$, as it can be seen by simple calculations. These terms are clearly well-behaved for $m_{\nu_1} = 0$.

Similar considerations hold if one considers an expansion in terms of a dimensionless parameter such as $\frac{\delta m}{\omega}$ rather than $\delta m$.

We also emphasize that the particular approximation considered in Ref. [5], namely the expansion in $\delta m$ and the subsequent requirement of vanishing $m_{\nu_1}$, is adopted not only for bypassing technicalities, but also for physical reasons. Indeed, as shown in Ref. [6], in such a regime Pontecorvo states (which we use in Ref. [5]) can be identified with the exact QFT flavor states [9, 22], up to corrections of the order $\mathcal{O}(\delta m^2)$. This can be seen by observing that, in the QFT treatment of mixing, flavor neutrino states (coherent states) have schematically the form [9, 22]

$$|\nu_e\rangle = \left[ \cos \theta a_1^\dagger + \sin \theta |U| a_2^\dagger + \sin \theta |V| a_1^\dagger a_2^\dagger b_1^\dagger \right] |0\rangle_{1,2},$$

(26)

where $|0\rangle_{1,2}$ is the vacuum for free fields, $a_i^\dagger (b_i^\dagger)$, $i = 1, 2$, is the creation operator for a neutrino (antineutrino) with mass $m_i$, and $U$, $V$ are the Bogoliubov coefficients arising from the non-orthogonality of Dirac spinors with different masses, i.e.

$$|U| = \left( \frac{\omega_{\nu_1} + m_1}{2\omega_{\nu_1}} \right)^{\frac{1}{2}} \left( \frac{\omega_{\nu_2} + m_2}{2\omega_{\nu_2}} \right)^{\frac{1}{2}} \left( 1 + \frac{k^2}{(\omega_{\nu_1} + m_1)(\omega_{\nu_2} + m_2)} \right) ,$$

(27)

$$|U|^2 + |V|^2 = 1 .$$

(28)
Expanding in $\delta m$ and then setting $m_{\nu_1} = 0$ yields

$$|U| \approx 1 - \frac{\delta m^2}{8k^2} + O(\delta m^4), \quad (29)$$

From Eq. (26), it then follows that the exact QFT flavor eigenstates are equal to the Pontecorvo states up to the leading order, thus justifying the employment of the latter in Ref. [5].

References
[1] Muller R 1997 Phys. Rev. D 56 953
[2] Matsas G E A and Vanzella D A T 1999 Phys. Rev. D 59 094004; Matsas G E A and Vanzella D A T 2000 Phys. Rev. D 63 014010; Matsas G E A and Vanzella D A T 2001 Phys. Rev. Lett. 87 151301
[3] Suzuki H and Yamada K 2003 Phys. Rev. D 67 065002
[4] Ahluwalia D V, Labun L and Torrieri G 2016 Eur. Phys. J. A 52 189
[5] Blasone M, Lambiase G, Luciano G G and Petruzziello L 2018 Phys. Rev. D 97 105008
[6] Blasone M, Lambiase G, Luciano G G and Petruzziello L 2018 PoS CORFU 2017 198
[7] Cozzella G, Fulling S A, Landulfo A G S, Matsas G E A and Vanzella D A T 2018 Phys. Rev. D 97 105022
[8] Bilenyk S M and Pontecorvo B 1978 Phys. Rept. 41 225
[9] Blasone M and Vitiello G 1995 Annals Phys. 244 283
[10] Blasone M, Jizba P and Vitiello G 2001 Phys. Lett. B 517 471
[11] Gilmore R 1972 Ann. Phys. NY 74 391; Perelomov A M 1972 Commun. Math. Phys. 26 222
[12] Blasone M, Lambiase G and Luciano G G 2017 Phys. Rev. D 96 025023
[13] Blasone M, Lambiase G and Luciano G G 2018 J. Phys. Conf. Ser. 956 012021
[14] Cardall C Y and Fuller G M 1997 Phys. Rev. D 55 7900
[15] Blasone M, Lambiase G, Luciano G G and Petruzziello L 2018 EPL 124 51001
[16] Schwartz M D 2014 Quantum Field Theory and the Standard Model Cambridge UK: Cambridge University Press
[17] Cheng T P and Li L F 1984 Gauge Theory Of Elementary Particle Physics Oxford UK: Clarendon
[18] Greiner W and Reinhardt J 1996 Field quantization Berlin Germany: Springer
[19] Casimir H B G 1948 Proc. K. Ned. Akad. Wet. 51 793; Casimir H B G and Polder D 1948 Phys. Rev. 73 360; Milton K A 2001 The Casimir effect: physical manifestations of zero-point energy River Edge: World Scientific; Nesterenko V V, Lambiase G and Scarpetta G 2001 Phys. Rev. D 64 025013 (2001); Nesterenko V V, Lambiase G and Scarpetta G 2002 Ann. Phys. 298 403; Blasone M, Lambiase G, Petruzziello L and Stabile A 2018 Eur. Phys. J. C 78 976 (2018); Buoninfante L, Lambiase G, Petruzziello L and Stabile A 2019 Eur. Phys. J. C 79 41.
[20] Blasone M, Luciano G G L, Petruzziello L and Smaldone L 2018 Phys. Lett. B 786 278
[21] Blasone M, Luciano G G L, Petruzziello L and Smaldone L In preparation
[22] Blasone M, Capolupo A, Remei O and Vitiello G 2001 Phys. Rev. D 63 125015; Blasone M and Palmer J 2004 Phys. Rev. D 69 057301; Blasone M, Capolupo A, Terranova F and Vitiello G 2005 Phys. Rev. D 72 013003; Blasone M, Gargiulo M V and Vitiello G 2016 Phys. Lett. B 761 104; Blasone M, Henning P A and Vitiello G 1999 Phys. Lett. B 451 140; Blasone M, Capolupo A and Vitiello G 2002 Phys. Rev. D 66 025033; Blasone M, Pires Pacheco P and Wan Chan Tseung H 2003 Phys. Rev. D 67 073011; Blasone M, Jizba P and Luciano G G 2018 Annals Phys. 397 213