High-fidelity quantum gate operations are essential for achieving scalable quantum circuits. In spin qubit quantum computing systems, metallic gates and antennas which are necessary for qubit operation, initialization, and readout, also cause detriments by enhancing fluctuations of electromagnetic fields. Therefore, evanescent wave Johnson noise (EWJN) caused by thermal and vacuum fluctuations becomes an important unmitigated noise, which induces the decay of spin qubits and limits the quantum gate operation fidelity. Here, we first develop a macroscopic quantum electrodynamics theory of EWJN to account for the dynamics of two spin qubits interacting with metallic circuitry. Then we propose a numerical technique based on volume integral equations to quantify EWJN strength in the vicinity of nanofabricated metallic gates with arbitrary geometry. We study the limits to two spin-qubit gate fidelity from EWJN-induced relaxation processes in two experimentally relevant quantum computing platforms: (a) silicon quantum dot system and (b) NV centers in diamond. Finally, we introduce a Lindbladian engineering method to optimize the control pulse sequence design and show its enhanced performance over Hamiltonian engineering in mitigating the influence of thermal and vacuum fluctuations. Our work leverages advances in computational electromagnetics, fluctuational electrodynamics, and open quantum systems to suppress the effects of thermal and vacuum fluctuations and reach the limits of two-spin-qubit gate fidelity.

I. INTRODUCTION

Quantum gate fidelity measures closeness between physically implemented quantum gate operations and theoretically ideal counterparts. Realizing high-fidelity quantum gate operations is necessary for any practical application of quantum computing. Recent demonstrations of quantum gate operations have observed increasing fidelity by mitigating various noise sources and shown spin qubits in multiple solid-state systems as promising candidates for future quantum computers [1–8]. The current record for two-spin-qubit gate fidelity has exceeded 99.5% [1–3, 5, 8].

Despite these significant achievements, pushing the current state-of-the-art quantum gate fidelity to higher records is still central for quantum computing research. The complexity of scalable quantum circuits is sensitive to the fidelity of underlying two-qubit quantum gate operations. This arises since lower gate fidelity causes the need for a larger number of qubits in the error correction process. Error correction is critical for eliminating uncertainty in the final results of quantum circuits. For the case of one popular error correction method - surface code [9–11], increasing gate fidelity from 99.9% to 99.999% could decrease the number of qubits required for building the same quantum circuits by a factor of 10 [10].

Environmental noise from various origins is the roadblock to achieving high gate fidelity. Figure 1 shows the schematic of a spin qubit quantum computing device with different types of noise. Among all the noise sources, thermal and vacuum fluctuations are important but are relatively less explored. Previous works have quantified the influence of various other noise sources and demonstrated corresponding optimization methods (table I).

For instance, nuclear spin noise originating from non-zero nuclear spins of silicon isotopes $^{29}$Si can decohere spin qubits [12, 13]. However, with recent improvements in materials and fabrication technology, nuclear spin noise is suppressed in isotopically enriched $^{28}$Si samples [14].

Another important fluctuation is induced by instability in microwave pulses used for spin qubits control. One potential solution is redesigning the amplitudes and lengths of microwave pulses [8]. Fluctuating charges in semiconductor quantum dots can lead to charge noise that induces spin qubits’ decoherence [15, 16]. Biasing spin qubits symmetrically reduces their sensitivity to charge noise, thus decreasing the impacts of charge noise in quantum gate operations [17–19]. In contrast, the influence of thermal and vacuum fluctuations of electromagnetic (EM) fields on quantum gate fidelity is not well evaluated or mitigated.

In the quantum computing system based on spin qubits, metallic lines, gates, and antennas are necessary for initialization, control, and readout [2–4, 6]. Error correction protocols rely on metallic lines for large-scale
FIG. 1. Schematic of a spin qubit quantum computing system with different noise sources. Four kinds of noise are shown: nuclear spin bath noise, noise in control pulses, charge noise, and evanescent wave Johnson noise (EWJN). Red arrows represent interactions between spin qubits and enhanced vacuum and thermal fluctuations of electromagnetic fields.

qubit manipulations as well [20, 21]. Evanescent surface and bulk waves in these metals inside the quantum computing devices intensify the fluctuations of EM fields [22]. Therefore, evanescent wave Johnson noise (EWJN) originating from thermal and vacuum fluctuations in the vicinity of metals is inevitable in quantum computing devices.

Multiple experimental and theoretical results show EWJN as an important and prevalent type of noise that induces spin qubit relaxation processes in various devices. In the semiconductor quantum dot system, theoretical studies have shown signatures of EWJN effects on the relaxation time (T1) [23–25]. Experiments in silicon quantum dot qubits display a deviation of the measured spin relaxation rate from the expected phonon relaxation rate at low magnetic fields [26], which can be explained by Johnson noise [27]. A recent study on donor qubits in metal oxide semiconductor (MOS) devices also shows the measured spin relaxation rate deviates from the typical B^2 dependency, caused by spin-orbit mediated phonon relaxation, and follows a linear B dependency at low magnetic fields (B < 3 T) [28]. Meanwhile, EWJN has been proposed to give this linear B dependency [28]. In the diamond NV center system, the measured relaxation time of NV electron spins in the vicinity of metals matches well with the Ford-Weber EWJN theory [22, 29]. The Ford-Weber theory of understanding EWJN and the non-local dielectric response of metals can also be used to derive bounds for the qubit relaxation rate near thin films. However, current studies are generally limited to a single qubit and fail to provide accurate EWJN-induced decay rates in the vicinity of metallic gates with realistic geometry. Furthermore, quantum dynamics of the qubit system including EWJN effects cannot be captured by conventional Ford-Weber theory. Thus moving beyond Ford-Weber EWJN theory is necessary for understanding two-qubit quantum dynamics and optimizing realistic gate geometries.

Thermal and vacuum fluctuations require a careful treatment distinct from aspects like nuclear spin noise to mitigate their influence on quantum gate fidelity. Metallic systems that enhance thermal and vacuum fluctuations are essential and nearly irreplaceable for quantum gate operations. Lowering operation temperature reduces the influence of thermal fluctuations; however, vacuum fluctuations persist even at T = 0 K. A feasible solution is optimizing microwave pulse sequences used for qubit control. Hamiltonian engineering and dynamical decoupling are popular techniques to seek optimal control of quantum systems [30–36]. In Hamiltonian engineering, unwanted interactions represented by Hermitian Hamiltonian operators are effectively suppressed in rotating frames corresponding to microwave pulses. However, thermal and vacuum fluctuations and other Markovian noise are usually captured in a non-Hermitian superoperator [37, 38]. Apart from that, implementations of Hamiltonian engineering and dynamical decoupling commonly focus on reducing noise from the spin bath instead of general Markovian noise including EWJN.

In this paper, we first provide a macroscopic quantum electrodynamics (QED) theory of EWJN. Based on macroscopic QED [39–41], we obtain the Lindblad master equation, which describes the non-unitary dynamics of the two-spin-qubit system coupled to vacuum and thermal fluctuations. We consider contributions from all spontaneous, stimulated, and cooperative decay processes induced by EWJN. The cooperative decay appears naturally in the multi-qubit system. In the quantum computing system based on spin qubits, cooperative decay could also happen at a comparable rate as stimulated and spontaneous decay, indicating it as an important part of EWJN that has not been well studied in previous works. Similar methods have been used to study quantum cooperative effects near metamaterials and single photon pulse induced transient entanglement near graphene [40, 41].

Previous studies have considered the thin film geometry of metal gates and its effect on spin qubits. Here, we overcome this stumbling block and consider realis-

| noise source                  | optimization method                        |
|-------------------------------|--------------------------------------------|
| nuclear spin bath             | isotopic enrichment [14]                   |
| imperfect MW pulses           | Hamiltonian engineering [8]                |
| fluctuating charges           | symmetric operation [17–19]                |
| thermal and vacuum fluctuations| Lindbladian engineering (this work)        |

* Lindbladian engineering is also applicable to general Markovian noise
tic gate geometry by combining computational electromagnetics approaches with the above-mentioned quantum theory. We introduce the volume integral equations (VIEs) method as an efficient and accurate numerical technique to calculate spontaneous and cooperative decay rates of spin qubits in the vicinity of metallic gates with arbitrary geometry. The VIE-based method has great flexibility in modeling complicated geometry in open-region settings. Nonlocal effects also have important contributions to decay rates when the operating temperature is low, and the distance between qubits and metallic gates is small [23, 29]. As a result, we take nonlocal effects into calculations when applicable.

Next, we evaluate limits to controlled-NOT (CNOT) gate fidelity due to vacuum and thermal fluctuations induced spin qubit relaxation processes in two popular quantum computing systems: NV center in diamond and quantum dot in silicon. CNOT gate is a representative two-qubit quantum gate involving interaction between two qubits. Its fidelity is commonly used to assess the performance of quantum computing devices [4, 42]. We calculate the average CNOT gate fidelity through system dynamics simulations based on the Lindblad master equation.

Finally, we propose a Lindbladian engineering method as an effective approach to reduce the influence of general Markovian noise, including EWJN. Our Lindbladian engineering approach searches for optimal control pulse sequences via dynamics determined by the Lindblad master equation. We provide optimized control protocols robust against Markovian relaxation and dephasing processes through Lindbladian engineering targeting two-spin-qubit systems.

The paper’s outline is as follows. In section II, we introduce the macroscopic QED theory of EWJN and important numerical techniques. Then, in section III and section IV, we evaluate EWJN’s influence on CNOT gate fidelity in the silicon quantum dot and NV center system. Next, in section V, we present the application of Lindbladian engineering in both systems with Markovian relaxation and dephasing processes of spin qubits. Finally, in section VI, we indicate future prospects for further research.

II. MACROSCOPIC QUANTUM ELECTRODYNAMICS THEORY OF THERMAL AND VACUUM FLUCTUATIONS AND LINDBLADIAN ENGINEERING

In this section, we first discuss the dynamics of the two-spin-qubit system and the Lindblad master equation. We adopt macroscopic quantum electrodynamics (QED) approaches [39, 43] to study EWJN effects in the vicinity of metal gates. We present the analytical and numerical methods to obtain spontaneous and cooperative decay rates of spin qubits induced by thermal and vacuum fluctuations. Finally, we demonstrate the Lindbladian engineering for control pulse optimization to mitigate the influence of EWJN.

A. Macroscopic QED theory of EWJN

In our model, the system consists of two spin qubits coupled to each other via dipolar or exchange interaction. Two-qubit quantum gate operations are realized by driving the qubits using resonant microwave pulses. Vacuum and thermal fluctuations of EM fields induce spontaneous, stimulated, and cooperative decay processes through the interaction between the system and the Markovian photon bath environment. Macroscopic QED theory valid in arbitrary dissipative media [39–41] provides a more accurate description of the system-bath interaction and system dynamics compared to semiclassical methods such as Fermi’s golden rule and Ford-Weber theory, which fail to adequately describe cooperative decay processes [44, 45]. In the Schrodinger picture, we can obtain the following Lindblad master equation governing the time evolution of the two-spin-qubit density matrix $\rho_q(t)$ (see Appendix. A for derivations):

$$
\frac{d\rho_q(t)}{dt} = \frac{1}{i\hbar}[\hat{H}(t), \rho_q(t)] + \hat{L}_r \rho_q(t) = \frac{1}{i\hbar}[\hat{H}(t), \rho_q(t)] + \sum_{i,j} \gamma_{ij}[\hat{\sigma}^\pm_i \rho_q(t)\hat{\sigma}^\mp_j - \frac{1}{2}\rho_q(t)\hat{\sigma}^\pm_i \hat{\sigma}^\mp_j - \frac{1}{2}\hat{\sigma}^\pm_i \hat{\sigma}^\mp_j \rho_q(t)]
$$

$$
+ \sum_{i,j} \mathcal{N}_{ij} \gamma_{ij}[\hat{\sigma}^\mp_i \rho_q(t)\hat{\sigma}^\pm_j - \frac{1}{2}\rho_q(t)\hat{\sigma}^\mp_i \hat{\sigma}^\pm_j - \frac{1}{2}\hat{\sigma}^\mp_i \hat{\sigma}^\pm_j \rho_q(t)],
$$

(1)

where $i,j \in \{1, 2\}$. $\hat{H}(t)$ is the Hamiltonian governing the unitary evolution of $\rho_q(t)$. It consists of Hamiltonians corresponding to control pulses $H_{mw}$ and the coupling between spin qubits. Coupling mechanisms of spin qubits can be dipole-dipole interaction $H_{dd}$ dominant in the NV center system, or exchange coupling Hamiltonian $H_{xx}$ dominant in the silicon quantum dot system. $\hat{L}_r$ is the Lindblad super-operator describing the nonunitary relaxation processes induced by thermal and vacuum fluctuations. $\hat{\sigma}^\pm_i$ is the raising (lowering) operator for the $i$th spin qubit. $\gamma_{ij}$ represents the spontaneous ($i = j$) and cooperative ($i \neq j$) decay rates, which will be discussed in detail in the next subsection. $\mathcal{N}_{ij}$ is the mean photon number at thermal bath temperature $T$ and average spin qubit resonance frequency $\omega_{\pm} = \frac{\omega_{+} + \omega_{-}}{2}$.

$$
\mathcal{N}(\omega_{\pm}, T) = \frac{1}{e^{\frac{\hbar \omega_{\pm}}{k_B T}} - 1}.
$$

(2)

Here, we briefly discuss Eq. (1) and different terms in $\hat{L}_r$. Eq. (1) is valid for two qubits with resonance fre-
quencies $|\omega_i - \omega_j| \ll \omega_i + \omega_j$. We have taken a Born-Markovian approximation to obtain Eq. (1), where we assume that the two-spin-qubit system is weakly coupled to the thermal photon bath (Born approximation) and the bath correlation time $\tau_c$ is much smaller than the relaxation times of the system $\tau_i \ll \gamma_i^{-1}$ (Markovian approximation) [37] (see Appendix. A for more details). The second term on the right-hand side of Eq. (1) describes the spontaneous ($i = j$) and cooperative decay ($i \neq j$) processes due to vacuum fluctuations. The third and fourth terms on the right-hand side of Eq. (1) correspond to thermally stimulated activated emission and absorption induced by thermal fluctuations. It also shows that when cooperative decay rates $\gamma_{ij}$ ($i \neq j$) are comparable to spontaneous decay rates $\gamma_{ii}$, cooperative decay processes should not be neglected in EWJN effects. Including both spontaneous and cooperative decay rates in EWJN is one of the main contributions of this work.

B. Spontaneous and cooperative decay rates: computational electromagnetics simulations

From macroscopic QED theory of EWJN, the spontaneous ($i = j$) and cooperative ($i \neq j$) decay rates $\gamma_{ij}$ are:

$$\gamma_{ij} = \frac{2\mu_0}{\hbar} m_i \cdot \left[ \nabla r_i \times \text{Im} \begin{array}{c} G_m \end{array} \left( r_i, r_j, \omega_+ \right) \times \nabla r_j \right] \cdot m_j^\dagger, \quad \gamma_{ij} = \frac{2\mu_0}{\hbar} (\omega_+)^2 m_i \cdot \text{Im} \begin{array}{c} G_m \end{array} \left( r_i, r_j, \omega_+ \right) \cdot m_j^\dagger,$$

where $\mu_0$ is the vacuum permeability, $\omega$, $r_i$, $\gamma_i$ and $m_i = [\hbar \gamma_i/2, -i\hbar \gamma_i/2, 0]$ are the resonance frequency, position, gyromagnetic ratio and spin magnetic moment of the $i$th qubit separately. $\begin{array}{c} G \end{array}$ and $\begin{array}{c} G_m \end{array}$ are the electric and magnetic dyadic Green’s functions.

To solve the system dynamics governed by Eq. (1), we need to calculate the EWJN induced spontaneous and cooperative decay rates $\gamma_{ij}$ (Eq. (3)). Dissipative metallic materials greatly enhance fluctuations of EM fields, which are captured by $\begin{array}{c} G_m \end{array}$ [22]. Many factors including positions of the spin qubits $r_i$ and geometries and material properties of metallic contacts, determine $\text{Im} \begin{array}{c} G_m \end{array}$. In the following, we discuss $\text{Im} \begin{array}{c} G_m \end{array}$ calculations in two different scenarios: (a) simplified gate geometries in some proposed quantum processor architectures, where $\text{Im} \begin{array}{c} G_m \end{array}$ can be calculated analytically; (b) realistic metal gate geometries in experimentally relevant quantum computing devices, where one needs to employ advanced computational electromagnetics methods based on VIEs.

In the first case, when the distance between spin qubits and metallic control systems is much smaller than the characteristic size of metallic control systems, as shown in some quantum network and processor architecture designs [20, 21, 46, 47], one can simplify the actual metal geometry to a metal thin film. Here, at extremely low temperatures and when qubits are very close to metal gates, non-local effects of metals become important since electron-electron scatterings are dominant and amplify the effects of non-locality [29]. Due to the translational symmetry of the metal thin film geometry, analytical expressions of $\text{Im} \begin{array}{c} G_m \end{array}$ exist and we consider Lindhard theory of non-local dielectric function $\varepsilon(q, \omega)$ in the calculation (see Appendix. B for equations and details) [22, 48, 49].

In the second case, we consider a realistic quantum computing device reported in a recent reference [6]. Here, we model the actual gate geometry in numerical simulations based on the volume integral equations (VIEs) method [50, 51]. The VIE method formulates a VIE in inhomogeneous materials that are different from the background material. With a tetrahedron-element-based discretization, it offers great flexibility in modeling arbitrarily shaped conductors and dielectrics. With vector basis functions, it is capable of capturing both conduction and displacement currents flowing along an arbitrary direction inside the conductor and dielectric materials. Fast solvers [52, 53] have also been developed to accelerate VIE computation. The relative error of our VIE simulations is controllable and estimated to be less than 5%. (see Appendix. C for more details).

C. Gate infidelity $\Delta F$ induced by EWJN

With the calculated decay rates $\gamma_{ij}$, quantum dynamics is simulated in the Liouville space (see Appendix. D). To this end, we define the average CNOT gate fidelity as closeness between actual final density matrices $\rho$ of CNOT gate operations and theoretically ideal counterparts $\rho_0^i$:

$$F = \sum_{i,j=0,1} \text{Tr}[\rho_{ij}^i \rho_{ij}^0]/4,$$

where $\rho_{ij}^i$ represents one of the four ideal final density matrices (corresponding to $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$ two-qubit states), $\rho_{ij}^0$ represents actual final density matrices of the two qubits after physical CNOT gate operations realized by MW control pulses. We calculate $\rho_{ij}^i$ via Eq. (1) with experimental parameters of MW control pulses. By comparing the associated average CNOT gate fidelity $F$ with the ideal counterpart $F_0$ where EWJN is ignored, we evaluate quantum gate infidelity $\Delta F = F_0 - F$ induced by EWJN.

D. Lindbladian engineering for pulse optimization

Finally, we present a Lindbladian engineering method that suppresses the influence of Markovian noise on two-spin-qubit quantum gate operations. In this part, we extend the noise source to include general Markovian noise where the Born-Markovian approximation is still justified. We also consider both the relaxation and dephasing
In this section, we study CNOT gate infidelity $\Delta F$ induced by thermal and vacuum fluctuations in the silicon double quantum dot (DQD) system. We employ experimental parameters of the two-spin-qubit systems [6] in these simulations.

The silicon DQD system consists of two electron spins trapped by hyperbolic potentials of quantum dots. The two spin qubits interact through exchange coupling, which dominates their unitary evolution. Both spontaneous and cooperative decay processes have important contributions to the non-unitary relaxation processes.

The detailed representation of $\hat{H}_{ex}$, $\hat{H}_{mw}$, and $\hat{L}_r$ in a rotating frame can be found in Appendix A.

Figure 2(a) displays a schematic of the silicon DQD system. Here, considering distance between spin qubits and metals is much smaller than the characteristic size of metals in some quantum processor architectures [20, 46], we simplify the metal lines and gates to a metal film with 100 nm thickness on top of the silicon substrate. CNOT gate operation is realized by MW pulse generated from the metal electron spin resonance (ESR) stripline in the two-qubit processor. In the silicon DQD system, the operation temperature is around 30 mK [6], and electron spins trapped in gate-defined quantum dots can be in close proximity to the metal gates. As a result, EWJN is important for spin qubit relaxation processes and the nonlocal dielectric response of metals is significant at such low temperatures and small distances. Decay rates $\gamma_{ij}$ are calculated based on the dyadic Green’s function calculations presented in Appendix B.

Aluminum is commonly used for building metal gates and antennas in quantum computing devices based on silicon DQD. Aluminum plasma frequency $\omega_p = 1.75 \times 10^{16}$ Hz and Fermi velocity $v_f = 2.02 \times 10^6$ m/s are constants in the range of temperature and material properties in our study [55]. In the following, we first study the influence of EWJN induced by vacuum fluctuations on CNOT gate fidelity, then add thermal fluctuations and study the temperature depen-
FIG. 3. Limits to CNOT gate fidelity from vacuum and thermal fluctuations in a realistic quantum computing device based on silicon double quantum dot (DQD). (a) Schematic of a quantum computing device using silicon DQD electron spin qubits. Red double-arrow lines represent all the spontaneous, stimulated, and cooperative decay processes of the two-qubit system induced by thermal and vacuum fluctuations. Aluminum electron spin resonance (ESR) antennas and gates (gray) are on top of the silicon quantum dot. (b) Metal gate geometry with a tetrahedral mesh for computational electromagnetics simulations. Detailed dimensions of this gate geometry used in VIE simulations are depicted in Fig. 8 in Appendix C. (c) CNOT gate infidelity $\Delta F$ induced by vacuum fluctuations as a function of different metal dielectric properties $\text{Im} \varepsilon$ and distance $d$ between metal gates and qubits. (d) Dependence of $\Delta F$ on temperature $T$ and distance $d$ at fixed $\text{Im} \varepsilon$. Red lines in (c) and (d) represent constant contour lines of $\Delta F$.

dence. We consider $\omega_i = \omega_j = 39.33$ GHz [6], electron gyromagnetic ratio $\gamma_e = -1.76 \times 10^{11}$ rad/(s · T) in our calculations.

The electron collision frequency $\nu$ and spin qubits’ distance $d$ from the metal film are crucial in determining vacuum fluctuations in the vicinity of aluminum. $\nu$ depends on material quality and fabrication process and is related to nonlocal dielectric response of aluminum. In Fig. 2(b), we plot quantum gate infidelity $\Delta F$ induced by vacuum fluctuations for a range of $\{\nu, d\}$ values. Decreasing $d$ and $\nu$ will lead to an increasingly significant infidelity $\Delta F$ at $T = 0$ K. At small $d$ and low electron collision frequency $\nu$, maximum $\Delta F$ can exceed 0.005%. Considering the size of a logic qubit is sensitive to the fidelity of physical CNOT gates, this limits the minimum size and complexity of a practical silicon quantum processor with high-fidelity quantum logic operations.

In Fig. 2(c), we demonstrate the dependence of $\Delta F$ caused by EWJN on distance $d$ and environment temperature $T$. Both thermal and vacuum fluctuations contribute to EWJN in this case. Here, we keep electron collision frequency $\nu = 3.65 \times 10^{13}$ Hz as a constant. $\Delta F$ increases with temperature because thermal fluctuations gradually dominate the decay processes of the two spin qubits. At $T = 10$ K, for quantum gate operations based on shallow quantum dots close to the aluminum film, gate infidelity $\Delta F$ induced by thermal and vacuum fluctuations can exceed 0.03%. This analysis shows that to build a processor based on silicon DQD with high-fidelity quantum logic operations and minimum size, it is necessary to mitigate the influence of EWJN to achieve higher physical gate fidelity.

Computational electromagnetics: realistic device geometries for silicon quantum dots

In this subsection, we demonstrate the influence of EWJN on CNOT gate fidelity in a realistic silicon DQD device.

Figure 3(a) shows a schematic of the silicon DQD device considered in our model, which is obtained from the device reported in reference [6]. Here, we consider the thickness of the ESR antenna and metal gates to be 100 nm and obtain accurate $\text{Im} \varepsilon$ values using the VIE method (Appendix C). The range of distance $d$ between electron spin qubits and metal gates in the simulations is from 15 nm to 100 nm. In this distance range, we neglect nonlocal effects for the simplicity of VIE simulations.

We first study the influence of EWJN induced by vacuum fluctuations. In the local dielectric response regime, vacuum fluctuations of EM fields in the vicinity of alu-
Close diamond quantum system Hamiltonian \( \hat{H} \) consists of two spin qubits’ Hamiltonian \( \hat{H}_s \) and microwave Hamiltonian \( \hat{H}_{MW} \). Here \( \hat{H}_s \) is dominated by dipole-dipole interaction Hamiltonian \( \hat{H}_{dd} \). Since resonance frequency \( \omega_c \) and gyromagnetic ratio \( \gamma_c \) of the electron spin qubit are much larger than those of the \(^{13}\text{C} \) nuclear spin qubit \([47, 56]\), spontaneous decay rate \( \gamma_{ee} \) is much larger than other \( \gamma_{ij} \). As a result, the spontaneous and stimulated decay of the electron spin qubit will be dominant in the relaxation processes. We thus only consider terms related to the spontaneous and stimulated decay of the electron qubit in the Lindblad super-operator \( \hat{L}_r \). Detailed representations of \( \hat{H} \) and \( \hat{L}_r \) in the truncated Hilbert space spanned by NV electron spin states \(|0\rangle, |−1\rangle\) in the rotating frame are derived in Appendix \(A\).

For the electron spin qubit with a distance \( d \) greater than 100 nm away from top metallic contacts, and gate operations around room temperature, we can ignore the nonlocal dielectric response of metallic contacts. We consider silver as the material for metal gates. In the following, we first present CNOT gate infidelity \( \Delta F \) due to EWJN induced by vacuum fluctuations. Next, we examine \( \Delta F \) when thermal fluctuations are the dominant sources of EWJN. We consider \( \omega_c = 2.458 \text{ GHz} \), \( \gamma_c = −1.76 \times 10^{11} \text{ rad/(s} \cdot \text{T}) \) in our calculations. Related pulse and system parameters are provided in Appendix \( A\).

EWJN in the vicinity of silver is associated with the imaginary part of silver permittivity \( \text{Im} \varepsilon \). In Fig. 4(b), we present the dependence of CNOT gate infidelity \( \Delta F \) on distance \( d \) and silver permittivity \( \varepsilon \) when vacuum fluctuations are the only sources of EWJN. The permittivity of single-crystal silver and poly-crystalline silver \([29]\) are marked in this plot. CNOT gate operations suffer from high \( \Delta F \) when the electron spin is close to metallic contacts and metals have large \( \text{Im} \varepsilon \).

In Fig. 4(c), we show spatial and temperature dependence of \( \Delta F \). Silver permittivity \( \text{Im} \varepsilon = 5.8 \times 10^8 \) is taken to be a constant. The range of environment temperature

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**FIG. 4.** Limits to CNOT gate fidelity from vacuum and thermal fluctuations in the NV center system in diamond. (a) Schematic of an NV center system where electron spin and \(^{13}\text{C} \) nuclear spin are used as two qubits. Top metal film (gray) represents the plane geometry of the metallic control system considered. (b) CNOT gate infidelity \( \Delta F \) induced by vacuum fluctuations as a function of metal permittivity \( \text{Im} \varepsilon \) and distance \( d \) between metal gates and qubits. (c) Dependence of \( \Delta F \) on temperature of environment \( T \) and distance \( d \) at fixed \( \text{Im} \varepsilon \). Red lines in (b) and (c) represent constant contour lines of \( \Delta F \).
FIG. 5. Comparison of different MW pulse sequences and corresponding CNOT gate infidelity $\Delta F$ caused by Markovian noise in the diamond NV center system. (a) MW pulse sequence presented in reference [4]. (b) MW pulse sequence optimized via Hamiltonian engineering. (c) MW pulse sequence optimized via Lindbladian engineering. Top bar plots show different pulse parameters $X$ in different cases. Contour lines corresponding to gate fidelity $F = 85\%$ and $F = 90\%$ are marked in red. MW pulse sequence optimized through Lindbladian engineering results in the highest achievable CNOT gate fidelity with every $\{\gamma_r, \gamma_p\}$.

considered is from 200 K to 300 K, where thermal fluctuations are the major sources of EWJN. In this case, the maximum $\Delta F$ from EWJN exceeds 0.1% even when the electron spin qubit is over a distance of 400 nm away from metallic contacts. This poses a limit to the minimum size and complexity of a practical diamond quantum processor with high-fidelity quantum logic operations.

V. LINDBLADIAN ENGINEERING TO MINIMIZE INFLUENCE OF MARKOVIAN NOISE

In this section, we perform Lindbladian engineering in the two experimentally relevant systems to maximize quantum gate fidelity. We compare CNOT gate fidelity realized by MW pulses obtained through Lindbladian engineering and Hamiltonian engineering. Furthermore, we demonstrate that Lindbladian engineering can provide qubit driving protocols more robust against Markovian noise, including thermal and vacuum fluctuations.

A. Lindbladian engineering in two-qubit diamond quantum processor

Here, we consider Markovian relaxation and decoherence processes of the electron spin qubit in Lindbladian engineering. We examine the same qubit system shown in Sec. IV and consider a MW control pulse sequence consisting of three pulses within four delays. Pulse parameters include lengths of four delays $t_1, t_2, t_3, t_4$ and phases and lengths of three pulses $\phi_1, \phi_2, \phi_3$. Pulse parameter vector $X$ is: $X = \{t_1, t_2, t_3, t_4, \phi_1, \phi_2, \phi_3\}$.

In the following, the relaxation and decoherence rates of the electron spin qubit are denoted as $\{\gamma_r, \gamma_p\}$, and we assume Markovian approximation is appropriate for describing spin decay and dephasing processes. CNOT gate is realized at room temperature $T = 300$ K. We investigate three different pulse sequences, and their corresponding CNOT gate infidelity $\Delta F$ due to Markovian noise induced electron spin decay and dephasing, as is shown in Fig. 5. The three different pulse sequences are: original pulse sequence from [4] in Fig. 5(a), pulse sequence optimized via Hamiltonian engineering in Fig. 5(b), and pulse sequence optimized via Lindbladian engineering in Fig. 5(c). For $\{1/\gamma_r, 1/\gamma_p\} = \{1$ s, 120 $\mu$s\}, ten parameters of the optimized control pulse sequence in the three different cases are shown in the bar plots of Fig. 5. In the three colormaps of $\Delta F$, we find that the high fidelity region (low $\Delta F$ region) is the largest in Fig. 5(c) corresponding to Lindbladian engineering. This shows that it is possible to realize high fidelity quantum gate operations in the range of large $\{\gamma_r, \gamma_p\}$. This is because control pulses obtained via Lindbladian engineering suppress the influence of Markovian noise on quantum gate fidelity. These results demonstrate that Lindbladian engineering provides an optimal control protocol for this quantum computing system affected by Markovian noise.

B. Lindbladian Engineering in two-qubit silicon quantum processor

In silicon DQD system, we consider Markovian relaxation and decoherence processes of both electron spin qubits in Lindbladian engineering. We examine the same system presented in Sec. III. Since only one microwave pulse is implemented, pulse parameters $X$ consist of pulse length $t$ and $\Omega$ which is related to pulse strength. We use the interior-point method [54] to find optimal
FIG. 6. Comparison of different MW pulse sequences and corresponding CNOT gate infidelity $\Delta F$ caused by Markovian noise in the silicon quantum dot system. (a) MW pulse optimized via Hamiltonian engineering. (b) MW pulse optimized via Lindbladian engineering. Top bar plots show different pulse parameters $\chi$ in different cases. Contour lines corresponding to gate fidelity $F = 99\%$ and $F = 99.5\%$ gate fidelity are marked in red. The high fidelity region of CNOT gate operations expands in the colormap corresponding to Lindbladian engineering.

$\chi$ through Hamiltonian engineering and Lindbladian engineering for given $\{\gamma_r, \gamma_p\}$, which represent the spontaneous decay and dephasing rates of both qubits determined by Markovian noise. Figure 6 shows the optimized $\chi$ and CNOT gate infidelity $\Delta F$ incurred by Markovian noise for the two cases. We observe the expansion of the high fidelity (low $\Delta F$) region in the colormap associated with Lindbladian engineering. Limited by the small optimization space in this control protocol, Lindbladian engineering can only show that reducing pulse length can decrease $\Delta F$.

VI. CONCLUSION

In conclusion, we combined macroscopic quantum electrodynamics theory, computational electromagnetics, and fluctuational electrodynamics to study the effects of thermal and vacuum fluctuations in a two-spin-qubit quantum computing system. We examine limits to quantum gate fidelity from thermal and vacuum fluctuations in two experimentally relevant systems: diamond NV center and silicon quantum dot systems. We provided detailed calculations of CNOT gate infidelity due to EWJN as a function of distance, temperature, and dielectric properties of metallic contacts necessary for quantum gate operations. Even with the rapid progress of current technology, this influence is relatively less explored and can limit the minimum size and complexity of the spin-qubit based quantum processor with high-fidelity quantum logic operations. Further, we propose Lindbladian engineering to mitigate the influence of EWJN on quantum gate fidelity, which can also suppress other Markovian noise impacts. We compare Hamiltonian engineering and Lindbladian engineering and demonstrate that control pulses optimized by Lindbladian engineering can realize higher CNOT gate fidelity by overcoming effects of Markovian noise. Our findings help to reach the limits of two-spin-qubit quantum gate fidelity and accelerate the practical application of quantum computing.

VII. ACKNOWLEDGEMENTS

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Appendix A: Macroscopic Quantum Electrodynamics Theory of EWJN

In this appendix, we use a macroscopic quantum electrodynamics (QED) method to study the evolution of two spin qubits in the presence of EWJN. Following the quantization framework in macroscopic QED [39–41, 43], the total Hamiltonian can be written as:

$$\hat{H} = \hat{H}_q + \hat{H}_f + \hat{H}_{\text{int}} + \hat{H}_{\text{ex}}, \quad (A1)$$

$$\hat{H}_q = \sum_i \hbar \omega_i \hat{\sigma}_i^+ \hat{\sigma}_i^-, \quad (A2)$$

$$\hat{H}_f = \int d^3 r \int_0^\infty \hbar \omega \hat{f}(r, \omega) \hat{f}(r, \omega), \quad (A3)$$

$$\hat{H}_{\text{int}} = -\sum_i (m_{i, eg} \hat{\sigma}_i^+ + m_{i, ge} \hat{\sigma}_i^-) \cdot \hat{B}(r_i). \quad (A4)$$
Here, $\hat{H}_q$ represents the Hamiltonian of the two spin qubits, $\hat{H}_f$ represents the Hamiltonian of the electromagnetic bath in a non-magnetic media, $\hat{H}_{ex}$ represents the Hamiltonian of the exchange interaction, and $\hat{H}_{int}$ represents the interaction between spin qubits and the electromagnetic bath. $\omega_i$ ($i = 1, 2$) represents the resonance frequency of the $i$th spin qubit. $\hat{\sigma}^{+(-)} = \ket{1}\bra{0}\ket{0}\bra{1}$ is the qubit raising (lowering) operator. $\hat{f}^\dagger$ and $\hat{f}$ are photon/polariton creation and annihilation operators satisfying the following relation:

$$[\hat{f}^\dagger(r, \omega), \hat{f}_\beta(r', \omega')] = \delta_{\alpha\beta}\delta(r - r')\delta(\omega - \omega'),$$

(A5)

where $\alpha, \beta = x, y, z$. Magnetic field operator $\hat{B}(r)$ can be expressed in terms of $\hat{f}^\dagger$, $\hat{f}$ and dyadic Green’s function $\hat{G}(r,r',\omega)$:

$$\hat{B}(r) = \int_0^{\infty} d\omega [\hat{B}(r, \omega) + \hat{B}^\dagger(r, \omega)],$$

(A6)

$$\hat{B}(r, \omega) = (i\omega)^{-1} \int d^3r' \nabla \times \hat{G}(r,r',\omega) \cdot \hat{f}(r', \omega).$$

(A7)

Exchange interaction Hamiltonian $\hat{H}_{ex}$ can be approximated as [6, 57]:

$$\hat{H}_{ex} = -J (\ket{10}\bra{10} + \ket{01}\bra{01} - \ket{10}\bra{01} - \ket{01}\bra{10}),$$

(A8)

where $J$ describes the strength of exchange interaction. In silicon quantum dot system, two spin qubits are coupled dominantly through $\hat{H}_{ex}$. In NV center system, two spin qubits are coupled through dipole-dipole interaction and we can take $J = 0$.

In the interaction picture with respect to $\hat{H}_0 = \hat{H}_q + \hat{H}_f$, we have the Liouville–von Neumann equation (superscript $I$ represents the interaction picture):

$$\frac{d\rho^{\dagger}_{\text{tot}}(t)}{dt} = \frac{1}{i\hbar} [\hat{H}^{\dagger}_{\text{int}}(t) + \hat{H}_{ex}(t), \rho^{\dagger}_{\text{tot}}(t)].$$

(A9)

Integral form of Eq. (A9) is:

$$\rho^{\dagger}_{\text{tot}}(t) = \rho^{\dagger}_{\text{tot}}(0) + \frac{1}{i\hbar} \int_0^t d\tau [\hat{H}^{\dagger}_{\text{int}}(\tau) + \hat{H}_{ex}(\tau), \rho^{\dagger}_{\text{tot}}(\tau)].$$

(A10)

Substitute Eq. (A10) back into Eq. (A9), we can obtain:

$$\frac{d\rho^{\dagger}_{\text{tot}}(t)}{dt} = \frac{1}{i\hbar} [\hat{H}^{\dagger}_{\text{int}}(t) + \hat{H}_{ex}(t), \rho^{\dagger}_{\text{tot}}(0)]$$

$$- \frac{1}{\hbar^2} \int_0^t d\tau [\hat{H}^{\dagger}_{\text{int}}(\tau) + \hat{H}_{ex}(\tau), [\hat{H}^{\dagger}_{\text{int}}(\tau) + \hat{H}_{ex}(\tau), \rho^{\dagger}_{\text{tot}}(\tau)]]].$$

(A11)

Total density matrix $\rho^{\dagger}_{\text{tot}}(t) = \rho^\dagger_q(t) \otimes \rho^\dagger_f(t)$. Two-spin-qubit density matrix $\rho^\dagger_q(t)$ can be obtained by tracing off the field part:

$$\frac{d\rho^\dagger_q(t)}{dt} = \frac{1}{i\hbar} [\hat{H}^{\dagger}_{\text{ex}}(t), \rho^\dagger_q(0)] - \frac{1}{\hbar^2} \int_0^t d\tau \text{Tr}_f [\hat{H}^{\dagger}_{\text{int}}(t), [\hat{H}^\dagger_{\text{int}}(\tau), \rho^\dagger_{\text{tot}}(\tau)]] - \frac{1}{\hbar^2} \int_0^t d\tau [\hat{H}^\dagger_{\text{ex}}(t), [\hat{H}^\dagger_{\text{ex}}(\tau), \rho^\dagger_q(\tau)]]$$

$$- \frac{1}{\hbar^2} \int_0^t d\tau \text{Tr}_f [\hat{H}^{\dagger}_{\text{int}}(t)\hat{H}^\dagger_{\text{int}}(\tau)\rho^\dagger_q(\tau) - \hat{H}^\dagger_{\text{int}}(t)\rho^\dagger_q(\tau)\hat{H}^\dagger_{\text{int}}(\tau) - \hat{H}^\dagger_{\text{int}}(\tau)\rho^\dagger_q(\tau)\hat{H}^\dagger_{\text{int}}(t)]]$$

(A12)

where the dipolar interaction Hamiltonian in the interaction picture $\hat{H}^\dagger_{\text{int}}(t)$ is:

$$\hat{H}^\dagger_{\text{int}}(t) = - \sum_{j=1,2} \int_0^\infty d\omega (i\omega)^{-1} \int d^3r' \left( m_{ie,ge}\hat{\sigma}^+_i e^{i\omega t} + m_{ge,ie}\hat{\sigma}^-_i e^{-i\omega t} \right) \cdot \nabla \times \frac{\hat{G}}{\hbar}(r_i, r', \omega) \cdot \hat{f}(r', \omega) e^{-i\omega t} + h.c.]$$

(A13)
In the following, we consider a Markovian environment and take the Born-Markovian approximation. We assume that the influence of the two-spin-qubit system on the electromagnetic bath is small (Born approximation) [37]. As a result, we have \( \rho_f(t) = \rho_J(t) \otimes \rho_J(0) \) because \( \rho_J(t) \) is only negligibly affected by the two-spin-qubit system. We also assume that the bath correlation time \( \tau_c \) is much smaller than the relaxation times of the system (Markovian approximation) [37]. For two spin qubits with resonance frequencies \( \omega_i, \omega_j \) satisfying \( |\omega_i - \omega_j| \ll \omega_i + \omega_j \), under the Markovian approximation, we have:

\[
\int_0^t ds \ e^{-i\omega(t-s)} e^{\mp i\omega s} = e^{\pm 2i\omega t} \int_0^\infty d\tau \ e^{-i(\omega \mp \omega_s)\tau} = e^{\pm 2i\omega t} \left[ \pi \delta(\omega \mp \omega_s) - i \mathcal{P} \left( \frac{1}{\omega \mp \omega_s} \right) \right],
\]

\[
\int_0^t ds \ e^{-i\omega(t-s)} e^{\mp i\omega s} = e^{\pm 2i\omega t} \int_0^\infty d\tau \ e^{-i(\omega \mp \omega_s)\tau} = e^{\pm 2i\omega t} \left[ \pi \delta(\omega \pm \omega_s) + i \mathcal{P} \left( \frac{1}{\omega \pm \omega_s} \right) \right],
\]

where \( \tau = t - s \), \( \omega_- = (\omega_i - \omega_j)/2 \), \( \omega_+ = (\omega_i + \omega_j)/2 \).

Substituting Eq. (A14, A15) into Eq. (A12), we can obtain the open quantum system dynamics of the two-spin-qubit system. The Lindblad super-operator \( \hat{L}_r \) captures the dissipation of the two-spin-qubit system induced by thermal and vacuum fluctuations of electromagnetic fields. In the interaction picture, \( \hat{L}_r \) describing the effects of vacuum fluctuations on the two-spin-qubit is (consider \( \rho_J(t) = |0\rangle \langle 0| \)):

\[
\hat{L}_r \rho_J(t) = \sum_{i,j} \gamma_{ij} [\hat{\sigma}_i^+ \rho_J(t) \hat{\sigma}_j^- e^{-i(\omega_i - \omega_j)t} - \frac{1}{2} \hat{\sigma}_i^+ \hat{\sigma}_j^- \rho_J(t) e^{i(\omega_i - \omega_j)t}],
\]

(A16)

where the spontaneous and cooperative decay rates \( \gamma_{ij} \) are:

\[
\gamma_{ij} = \frac{2\mu_0}{\hbar} \mathbf{m}_{i,eq} \cdot \left[ \nabla_{r_i} \times \text{Im} \mathcal{G}(\mathbf{r}_j, \mathbf{r}_j, \mathbf{r}_+ \times \nabla_{r_j}) \right] \cdot \mathbf{m}_{i,eq}.
\]

Transforming back to the Schrodinger picture, we finally obtain the following Lindblad master equation:

\[
\frac{d\rho_J(t)}{dt} = \frac{1}{\hbar} [\hat{H}_J(t), \rho_J(t)] + \sum_{i,j} \gamma_{ij} [\hat{\sigma}_i^- \rho_J(t) \hat{\sigma}_j^+] - \frac{1}{2} \rho_J(t) \hat{\sigma}_i^+ \hat{\sigma}_j^- - \frac{1}{2} \hat{\sigma}_i^+ \hat{\sigma}_j^- \rho_J(t)].
\]

(A18)

The first term on the RHS of Eq. (A18) describes the Hermitian evolution of the two-spin-qubit system. In the NV center system, \( \hat{H}_J \) is dominated by \( \hat{H}_d \) which represents the dipole-dipole coupling between the NV electron spin and \( ^{13}\text{C} \) nuclear spin. In the silicon dot system, \( \hat{H}_J \) is dominated by \( \hat{H}_e \) which represents the exchange coupling between the two electron spins in silicon DQD. The second term on the RHS of Eq. (A18) describes the system relaxation process due to the coupling between two-spin-qubit system and photon bath. Taking the influence of thermal fluctuations into consideration, at finite temperature, we have:

\[
\frac{d\rho_J(t)}{dt} = \frac{1}{\hbar} [\hat{H}_J(t), \rho_J(t)] + (\mathcal{N} + 1) \sum_{i,j} \gamma_{ij} [\hat{\sigma}_i^- \rho_J(t) \hat{\sigma}_j^+] - \frac{1}{2} \rho_J(t) \hat{\sigma}_i^+ \hat{\sigma}_j^- - \frac{1}{2} \hat{\sigma}_i^+ \hat{\sigma}_j^- \rho_J(t)] + \mathcal{N} \sum_{i,j} \gamma_{ij} [\hat{\sigma}_i^+ \rho_J(t) \hat{\sigma}_j^- - \frac{1}{2} \rho_J(t) \hat{\sigma}_i^- \hat{\sigma}_j^+ - \frac{1}{2} \hat{\sigma}_i^- \hat{\sigma}_j^+ \rho_J(t)],
\]

(A19)

where \( \mathcal{N} \) is the mean photon number given by Eq. (2).

In a rotating frame with respect to the unitary operator \( \hat{U}(t) \), Eq. (A19) can be transformed into the rotating frame by replacing \( \hat{H}(t) \) with \( \hat{H}'(t) = i\hbar \frac{d}{dt} \hat{U}(t) + \hat{U}(t) \hat{H}(t) \hat{U}(t)^\dagger \) and \( \sigma_i^\pm(t) = \hat{U}(t) \sigma_i^\pm \hat{U}(t)^\dagger \), and \( \rho'(t) = \hat{U}(t) \rho(t) \hat{U}(t)^\dagger \).

1. Rotating Frame for the Silicon Quantum Dot System

For the silicon quantum dot system, our simulations are performed in a rotating frame defined by the following two unitary transformation sequentially [6]:

\[
\hat{U}^{(1)} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(\theta/2) & \sin(\theta/2) & 0 \\
0 & -\sin(\theta/2) & \cos(\theta/2) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
\]

\[
\hat{U}^{(2)} = \begin{bmatrix}
e^{i\hat{E}_z t} & 0 & 0 & 0 \\
0 & e^{(\delta \hat{E}_z - J/2)t/2} & 0 & 0 \\
0 & 0 & e^{-i(\delta \hat{E}_z + J)/2t} & 0 \\
0 & 0 & 0 & e^{-i\hat{E}_z t}
\end{bmatrix},
\]

where we take the experiment parameters \( \theta = 0.097, J = 1.59 \text{ MHz}, \hat{E}_z = 39.33 \text{ GHz}, \delta \hat{E}_z = 13.35 \text{ MHz} \) [6].

In the following, we use superscript \( \text{R} \) to denote the operators in the rotating frame. The unitary evolution
part in Eq. (A19) is governed by $\hat{H}_R(t)$ from reference [6],:

$$\hat{H}_R = \frac{\hbar}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega \\ 0 & 0 & 0 & 0 \\ 0 & \Omega^* & 0 & 0 \end{pmatrix} + \frac{\hbar}{2} \frac{1-\tan(\frac{\theta}{2})}{1+\tan(\frac{\theta}{2})} \begin{pmatrix} 0 & 0 & \Omega e^{-iJt} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \Omega e^{-iJt} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$  \hspace{1cm} (A20)

where $\Omega = 0.41 \text{MHz}$. The control pulse sequence for CNOT gate operations considered in section III is presented in the bar plot of Fig. 6(a). The non-unitary evolution part of Eq. (A19) is captured by $\hat{L}_R(t)$, which can be obtained by substituting $\sigma_i^{R,\pm}(t)$:

$$\sigma_1^{R,-} = e^{-iE_z t} \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^\dagger, \hspace{1cm} \sigma_2^{R,-} = e^{-iE_z t} \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^\dagger \hspace{1cm} (A21)$$

2. Rotating Frame for the NV Center System

For the NV center system, we consider a rotating frame defined by the following operator:

$$\hat{U} = e^{i\omega t \hat{z}_{x,e}}. \hspace{1cm} (A23)$$

Here, $\omega$ is the frequency of the microwave pulse, $\hat{z}_{x,e}$ is the $z$ component of Pauli matrix in the truncated Hilbert space corresponding to electron spin qubit.

The unitary evolution part is governed by $\hat{H}_{ud}(t)$ and $\hat{H}_{mw}(t)$ from reference [4]:

$$\hat{H}_{ud}(t) = 2\pi h|0\rangle\langle 0| \otimes [-\nu_e \hat{I}_z] + 2\pi |1\rangle\langle -1| \otimes [-(\nu_e + A_{zz}) \hat{I}_z - A_{zz} \hat{I}_x], \hspace{1cm} (A24)$$

$$\hat{H}_{mw}(t) = 2\pi h\omega [\cos(\phi_i)\hat{s}_x + \sin(\phi_i)\hat{s}_y], \hspace{1cm} (A25)$$

where $\nu_e = 0.158 \text{MHz}$, $A_{zz} = -0.152 \text{MHz}$, $A_{xx} = 0.110 \text{MHz}$, $\omega = 0.5 \text{ MHz} [4]$. $\phi_i$ represents the phase of $i$th pulse. The control pulse sequence for CNOT gate operations considered in section IV is presented in the bar plot of Fig. 5(a).

Here, since $\sigma_i^{\dagger}(t) = \hat{U}(t) \sigma_i^{\dagger}(t) \hat{U}^\dagger(t) = \sigma_i^{\dagger}(t)$, Lindblad super-operator $\hat{L}_R(t)$ in the rotating frame will have the same form as $\hat{L}_r(t)$ in the lab frame.

Appendix B: Magnetic Dyadic Green’s Function

In this appendix, we present the analytical expressions of magnetic dyadic Green’s function $\vec{G}_m$ in the vicinity of metal gates with plane geometry. Without loss of generality, we assume the metal gates are perpendicular to the $\hat{z}$ direction.

Electric dyadic Green’s function $\vec{G}$ is defined by the following equation:

$$\nabla \times \nabla \times \vec{G}(x_1, x_2, \omega) = k_0^2 \delta \vec{G}(x_1, x_2, \omega) = \hat{T} \delta(x_1 - x_2), \hspace{1cm} (B1)$$

where $k_0 = \frac{\omega}{c}$ is the free space wave vector and $\hat{T}$ is the $3 \times 3$ identity matrix.

The solution to Eq. (B1) can be expressed in terms of incident and reflected fields as $\vec{G}_\text{tot} = \vec{G}_0 + \vec{G}_r$. The incident part $\vec{G}_0$ has a negligible contribution to the decay of spin qubits in the vicinity of metals. The reflected part $\vec{G}_r$ is [58]:

$$\vec{G}_r(x_1, x_2, \omega) = \frac{i}{8\pi k_0^2} \int_0^\infty \frac{qdq}{k_z} e^{2ik_zd}$$

\[\left[(k_0^2 r_p - k_z^2 r_s)(\hat{x} \otimes \hat{x} + \hat{y} \otimes \hat{y}) + 2q^2 r_s \hat{z} \otimes \hat{z}\right], \hspace{1cm} (B2)\]

where $q = |\mathbf{q}|$ is the component of the wavevector parallel to the metal plane, $d = |\mathbf{x}_1 - \mathbf{x}_2|$, $k_z = \sqrt{k_0^2 - q^2}$ is the $z$
component of the wavevector perpendicular to the metal plane, \( r_s \) and \( r_p \) are the reflection coefficients for \( s \) and \( p \) polarized light.

\[
\widehat{G}_m(x_1, x_1, \omega) = \frac{1}{k_0^2} \nabla_i \times \widehat{G}(x_1, x_1, \omega) \times \nabla_i = \frac{i}{8\pi k_0^2} \int_0^\infty \frac{qdq}{k_0^2} e^{2ikz_\parallel [(k_0^2r_\parallel^2 - k_z^2)r_s](\hat{x} \otimes \hat{x} + \hat{y} \otimes \hat{y}) + 2q^2r_z \hat{z} \otimes \hat{z}], \tag{B3}
\]

\[
\widehat{G}_m(x_i, x_j, \omega) = \frac{i}{8\pi^2} \int \frac{dq}{k_z} e^{iq(r-r')} e^{ik_z(z+z')} \begin{bmatrix} q_y^2 & -q_xq_y & 0 \\ -q_xq_y & q_x^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{r_p}{k_0^2q^2} \begin{bmatrix} -q_xk_z^2 & -q_xq_yk_z & -q_yk_z^2 \\ -q_xq_yk_z & -q_x^2k_z & -q_yk_z^2 \\ q_x^2q_yk_z & q_xq_y^2k_z & q^4 \end{bmatrix}, \tag{B4}
\]

where \( q_x = q \hat{x}, q_y = q \hat{y} \). Here, \( \widehat{G}_m(x_i, x_1, \omega) \) is related to the spontaneous and stimulated decay rates of a spin qubit, while \( \widehat{G}_m(x_i, x_j, \omega) \) is related to the cooperative decay rates of the two-spin-qubit system.

When the non-local dielectric response is neglected and metal gate geometries are simplified to the plane geometry with thickness much larger than the skin depth, \( r_s \) and \( r_p \) in Eq. (B3, B4) are given by the Fresnel reflection coefficients:

\[
r_s(q) = \frac{k_z - \sqrt{\frac{\omega^2}{c^2} - q^2}}{k_z + \sqrt{\frac{\omega^2}{c^2} - q^2}}, \tag{B5}
\]

\[
r_p(q) = \frac{\varepsilon k_z - \sqrt{\frac{\omega^2}{c^2} - q^2}}{\varepsilon k_z + \sqrt{\frac{\omega^2}{c^2} - q^2}}, \tag{B6}
\]

where \( \varepsilon = \varepsilon(\omega) \) is the permittivity of metal contacts.

Non-local dielectric response of metals can be captured by the Lindhard model [22, 59]. Reflection coefficients \( r_s \) and \( r_p \) for a metal thin film with thickness \( t \) and Lindhard non-local permittivity \( \varepsilon(\omega, k) \) are [48, 49]:

\[
r_s(q) = \frac{1}{2} \sum_{t=\epsilon,o} \frac{2\kappa_{n,t}^2}{k_{n,t}^2} \sum_{n=-\infty}^\infty \frac{1}{\varepsilon_t(\omega, k_n, \epsilon) - \frac{k_{n,t}^2}{k_0^2}} - 1, \tag{B7}
\]

\[
r_p(q) = \frac{1}{2} \sum_{t=\epsilon,o} \frac{1 - 2q^2}{1 + \frac{2q^2}{k_{n,t}^2} \sum_{n=-\infty}^\infty \frac{1}{\varepsilon_t(\omega, k_n, \epsilon) - \frac{k_{n,t}^2}{k_0^2}}}, \tag{B8}
\]

where \( k_{n,t}^2 = q^2 + P_{c,n}^2, \quad P_{c,n} = \frac{2n\pi}{t}, \quad P_{o,n} = \frac{(2n+1)\pi}{t} \).

Longitudinal and transverse Lindhard non-local permittivity \( \varepsilon(\omega, k) \) are [22, 59]:

\[
\varepsilon_t(\omega, k) = 1 + \frac{3k_0^2}{\omega} \left( \frac{u^2}{\omega + iv} \right) F_1(u), \tag{B9}
\]

Magnetic dyadic Green’s functions \( \widehat{G}_m(x_i, x_j, \omega) \) are:

\[
\varepsilon_t(\omega, k) = 1 - \frac{\omega_p^2}{\omega(\omega + iv)} F_1(u), \tag{B10}
\]

\[
u = \frac{\omega + iv}{kv_F}, \tag{B11}
\]

\[
F_1(u) = 1 - \frac{1}{2} \ln \frac{u + 1}{u - 1}, \tag{B12}
\]

\[
F_1(u) = \frac{3}{2} u^2 - \frac{3}{4} u(u^2 - 1) \ln \frac{u + 1}{u - 1}, \tag{B13}
\]

where \( \omega_p \) is the plasma frequency, \( \nu \) is the electron collision frequency, \( v_F \) is the Fermi velocity.

Appendix C: Computational Electromagnetics Simulations of Magnetic Dyadic Green’s Function

In this appendix, we present the computational electromagnetic simulations of magnetic dyadic Green’s function \( \widehat{G}_m \) in the vicinity of metal gates with arbitrary geometry. We discuss how to obtain \( \widehat{G}_m \) close to metal gates in a quantum computing device via the volume integral equations (VIEs) method. Here, due to lack of translational symmetry of metal gates, \( r_s \) and \( r_p \) become ill-defined and expressions in Appendix B is no longer applicable.

1. Magnetic dyadic Green’s function

The magnetic field generated by current density \( j(r', \omega) \) is:

\[
H(r, \omega) = \int_V \left[ \nabla \times \widehat{G}_m(r, r', \omega) \right] j(r', \omega) dV'. \tag{C1}
\]
For a point magnetic dipole with dipole moment \( \mathbf{m} \) and oscillating frequency \( \omega \), it is equivalent to a closed electric current loop with current density:

\[
\mathbf{j}(\mathbf{r}', \omega) = -\mathbf{m}(\mathbf{r}_0, \omega) \times \nabla' \delta(\mathbf{r}' - \mathbf{r}_0). \tag{C2}
\]

Substitute Eq. (C2) into Eq. (C1):

\[
\mathbf{H}(\mathbf{r}, \omega) = \int_V [\nabla \times \hat{\mathbf{G}}(\mathbf{r}, \mathbf{r}', \omega)] : [-\mathbf{m} \times \nabla' \delta(\mathbf{r}' - \mathbf{r}_0)] dV' \tag{C3}
\]

\[
= \int_V [\nabla \times \hat{\mathbf{G}}(\mathbf{r}, \mathbf{r}', \omega) \times \nabla'] \cdot \mathbf{m} \delta(\mathbf{r}' - \mathbf{r}_0) dV' = k_0^2 \hat{\mathbf{G}}_m(\mathbf{r}, \mathbf{r}_0, \omega) \cdot \mathbf{m}(\mathbf{r}_0, \omega),
\]

where we have used the following relation to simplify Eq. (C3):

\[
\frac{d}{dx} \delta(x) f(x) = -\frac{d}{dx} f(x) \delta(x). \tag{C4}
\]

From Eq. (C3), to obtain \( \hat{\mathbf{G}}_m(\mathbf{r}, \mathbf{r}_0, \omega) \), we can place a test point magnetic dipole at \( \mathbf{r}_0 \) near the metallic gate structure (the scatterer), and use volume integral equations (VIE) to solve the the scattered magnetic field at \( \mathbf{r} \). By repeating this procedure three times with magnetic dipoles oriented along X, Y, Z axes separately, we can obtain all the 9 components of the magnetic dyadic Green’s function \( \hat{\mathbf{G}}_m(\mathbf{r}, \mathbf{r}_0, \omega) \).

2. VIE Formulation

Since the size of the scatterer (metal gates) is comparable to or smaller than the skin depth of the material and the test dipole is placed very close, we need to solve the field inside the metal gates in order to get the scattered field. Here we use the VIE method, which is robust against the low-frequency breakdown.

In the simulation, we treat the scatterer (metal gates) as a dissipative medium with permittivity \( \varepsilon(\mathbf{r}) = \varepsilon_0 (\varepsilon_r - \sigma / \omega \varepsilon_0 i) \), occupying the volume \( V \) in the space. Consider the electric field at \( \mathbf{r} \in V \), \( E(\mathbf{r}) = \frac{D(\mathbf{r})}{\varepsilon(\mathbf{r})} = E^i + E^{sc} \), where \( E^i \) and \( E^{sc} \) denote the incident and scattered field respectively. Therefore we can formulate an integral equation with an unknown D-field as [51]:

\[
\begin{align*}
E^i(\mathbf{r}) &= E(\mathbf{r}) - E^{sc}(\mathbf{r}) = \frac{D(\mathbf{r})}{\varepsilon(\mathbf{r})} - \int_V \left\{ \mu_0 \omega^2 \kappa(\mathbf{r}') \right\} D(\mathbf{r}') + \frac{1}{\varepsilon_0} \nabla' \cdot (\kappa(\mathbf{r}') D(\mathbf{r}') \nabla' \partial_g(\mathbf{r}, \mathbf{r}') dV', \tag{C5}
\end{align*}
\]

where \( g(\mathbf{r}, \mathbf{r}') = \frac{\varepsilon_{\mathbf{r}'\mathbf{r}} - \varepsilon_{\mathbf{r}\mathbf{r}}}{k_0^2 |\mathbf{r}' - \mathbf{r}|} \) is the scalar Green’s function, \( \kappa(\mathbf{r}') = \frac{\varepsilon(\mathbf{r}') - \varepsilon_0}{\varepsilon(\mathbf{r})} \) is the contrast ratio, \( \omega \) is the angular frequency, and \( k_0 \) denotes the free-space wave number.

To solve Eq. (C5), we discretize the volume \( V \) (metal gates) with a tetrahedral mesh, expand \( D(\mathbf{r}) \) using the Schaubert-Wilton-Glisson (SWG) basis, and do a standard Galerkin testing [50, 60]. If the number of unknowns \( N \) is small, we can solve it using a full matrix. Otherwise, we could employ a fast solver to compress the dense matrix and solve it in \( O(N \log N + N N_{iter}) \) with an iterative solver or \( O(N \log N) \) time with a direct inverse [53, 61].

3. Accuracy of VIE simulations

We examine the accuracy of magnetic dyadic Green’s function obtained via VIE simulations. We compare the analytical [62] and VIE simulated Green’s function \( \hat{\mathbf{G}}_m(\mathbf{r}, \mathbf{r}_i, \omega) \) in the vicinity of a silver sphere (Fig. 7(a)). Relative error \( \epsilon \) of VIE simulated Green’s function can be reduced to less than 1% with an increasing number of meshes (Fig. 7(b)). For VIE simulations of magnetic dyadic Green’s function in the vicinity of metal gates in a quantum computing device (Fig. 8), we use a...
refined mesh, and the relative error is estimated to be less than 5%.

Appendix D: Numerical Simulations of System Dynamics in the Liouville Space

In this section, we discuss simulation methods for studying the dynamics of the two-spin-qubit system in the Liouville space [63, 64]. The Liouville representation is effective for solving the Lindblad master equation. In the Liouville space, a density matrix of size \( N \times N \) is represented by a column vector of size \( N^2 \times 1 \). As a result, equations of motion for density matrix in the Liouville space can be solved by similar techniques that have been developed to solve equations of motion for state vectors in the Hilbert space. In the following, we present the density matrices and operators in the Liouville space used in our simulations. \( \otimes \) represents the Kronecker product.

In the Liouville space, the two-spin-qubit density matrix \( \rho_q(4 \times 4) \) is:

\[
|\rho_q\rangle = |\rho_{11}, \cdots, \rho_{41}, \cdots, \rho_{14}, \cdots, \rho_{44}\rangle^T,
\]

where \( \rho_{ij} \) is the element of the two-spin-qubit density matrix \( \rho_q \).

The commutator between Hamiltonian \( \hat{H} \) and the two-spin-qubit density matrix \( \rho_q \) becomes:

\[
- i[\hat{H}, \rho_q] = - i (\hat{H} \otimes \hat{I} - \hat{I} \otimes H^T) |\rho_q\rangle,
\]

where \( \hat{I} \) is the identity matrix of the same size as Hamiltonian \( \hat{H} \).

Lindblad super-operator in the Liouville space can be obtained based on the following transformations [63]:

\[
\hat{\sigma}_i \rho(t) \hat{\sigma}_j^\dagger = \hat{\sigma}_i \otimes (\hat{\sigma}_j^\dagger)^T |\rho\rangle,
\]

\[
\rho(t) \hat{\sigma}_i \hat{\sigma}_j^\dagger = \hat{I} \otimes (\hat{\sigma}_i^\dagger \hat{\sigma}_j)^T |\rho\rangle,
\]

\[
(\hat{\sigma}_i^\dagger \hat{\sigma}_j) \rho(t) = (\hat{\sigma}_i^\dagger \hat{\sigma}_j) \otimes \hat{I} |\rho\rangle.
\]

As a result, the Lindblad master equation with respect to \( |\rho_q\rangle \) is:

\[
\frac{d}{dt} |\rho_q(t)\rangle = \hat{L}(t) |\rho_q(t)\rangle,
\]

where \( \hat{L}(t) \) contains both the Hermitian evolution component (Eq. (D2)) and the decay component \( \hat{L}_r, \hat{L}_p \).

The dynamics of \( |\rho_q(t)\rangle \) can be calculated by splitting \([0, t]\) into \( N \) intervals \( \Delta t_1, \cdots, \Delta t_N \):

\[
|\rho_q(t)\rangle = e^{\hat{L}(t_N) \Delta t_N} \cdots e^{\hat{L}(t_1) \Delta t_1} |\rho_q(0)\rangle.
\]
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