The study of the implementation of research-based learning model to improve the students’ proving skills in dealing with the resolving efficient dominating set problem

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Abstract. Research-based learning (RBL) is one of a student-centered learning (SCL) method that involves research in the learning process. One of skills that can be developed through RBL model is proving skill. The student’s mathematics proving skill needs to be trained and developed because mathematics is a science that includes definitions and statements that need to be proved. This research was conducted to determine the effect of RBL implementation to improve the student’s proving skill in dealing with resolving efficient dominating set problems. The research was used mixed methods by combining qualitative and quantitative methods. The subjects of the study were divided into two classes, namely 23 students in the control class and 20 students in the experimental class. The homogeneity test of the pretest results for two classes showed \( p - value \) of 0.4309, so that the variants of the classes was homogeneous. The independent t-test on the posttest results of two classes showes \( p - value \) of 0.008615 \((p \leq 0.05)\). It can be concluded that there is an effect of RBL implementation to improve the students' proving skills in dealing with resolving efficient dominating set problems.

1. Introduction
One of the efforts made to reform education in Indonesia is related to the quality of learning. Currently, the learning approach is being transformed into Student Centered Learning (SCL), as students are no longer learning objects, but as learning subjects. The learning carried out must allow students to develop creativity, capabilities, needs and independence in the advancement of knowledge in order to obtain the required skills. This can be supported by motivating students to learn actively, so that they can think more, learn new ideas, solve problems, and apply what they have learned [1]. Therefore, we need an effective learning that can actively involve students in the development of their own knowledge.

Implementation of learning can also be supported by mathematics learning instrument. Mathematics learning instrument that have been developed are student worksheets. The student worksheet is an assignment sheet given to students, containing instructions and completion steps. The existence of a student worksheet would encourage students to take an active role in learning and help students develop their skills so that problems can be solved [2].
Research-Based Learning (RBL) is a learning model that provides students with opportunities to learn and develop knowledge based on research steps such as seeking information, formulating hypotheses, collecting data, analyzing, making conclusions and compiling reports [3]. The learning that used the RBL model is carried out under constructivism which includes four aspects: learning to create understanding of students, learning through the development of prior knowledge, learning through social experiences, and substantive learning through direct experience [4].

One of the skills that can be developed through the RBL model is students’ proving skill. The implementation of RBL can provide students’ independence in constructing mathematical proof so that students will better understand the concepts being studied. It is important to train and improve the mathematics proving skills of the students because mathematics is a science that contains definitions and statements that need to be proved [5]. There are several indicators of students proving skills that have been developed, namely 1) identifying problems, 2) proposing conjectures, 3) calculating arguments, 4) compiling proof, and 5) checking the validity of arguments [6].

The study about RBL implementation have been carried out by several researchers. One of them was stated that the implementation of RBL had a significant impact in improving students' combinatorial thinking skills in solving local irregularity vertex r-dynamic coloring problems [7]. Apart from the study about the RBL implementation, research on students’ proving skills have also been carried out. One of the results states that the development of mathematics learning instrument based on inquiry-based learning affects students skills to prove the dominating metric dimension numbers problems [8]. Based on previous research, researcher interested to determine the effect of RBL model to improve students’ proving skills in dealing with the resolving efficient dominating set problems.

There are two concepts that are combined from the problems in this study, namely the concept of efficient dominating set and the concept of resolving set. The efficient dominating set was first describe in Deng, et al [9]. For a graph \( G = (V, E) \), the vertex set \( D \subseteq V(G) \) is called an efficient set of graph \( G \) so that \( D \) is an independent set and all vertex of \( v \in V(G) - D \) is adjacent to exactly one vertex in \( D \) [10][11]. Meanwhile, the resolving set has been explain by Slater in different terminology, namely locating number [12]. The vertex set \( W = \{w_1, w_2, ..., w_k\} \subseteq V(G) \) is called resolving set of graph \( G \) if \( r(v|W) \neq r(u|W) \), which \( r(v|W) \) is a vertex representation of vertex \( v \) in respect of \( W \)[13].

2. Research Methods

In this study, a mixed method has been used. The mixed method is research method design that involves the combination of qualitative and quantitative methods to collect and analyze research data. In this study, a sequential exploratory design was used. In the sequential exploratory design, the initial stage of research uses qualitative methods and quantitative methods is used at the second stage[14].

The learning instrument was used in the form of student worksheets. The learning instrument is developed based on the 4-D development model by Thiagarajan. This research used two class, namely the experimental class and the control class with non equivalent control group design. The RBL model has been applied to both classes, but the learning instrument that has been developed is only being used by the experimental class. The learning model used in both classes is same so the effect of the RBL implementation with an instruments that have been developed can be seen. The pretest already done to students of both classes to find out the students initial skills at the beginning of learning. At the end of the lesson, a post test was given to both classes to determine the effect of the treatment given. The research design using a non equivalent control group can be found out in Table 1.

| Experimental class | \( O_1 \) | \( X \) | \( O_2 \) |
|--------------------|---------|-------|-------|
| Control class      | \( O_3 \) | \( - \) | \( O_4 \) |
Information:

\( O_1, O_3 \) : Pre-test
\( O_2, O_4 \) : Post-test

\( X \) : The treatment is given by using the mathematics instrument based on RBL model

2.1 Population
This study was conducted in Mathematics Education at the University of Jember in the odd semester of the 2020/2021 academic year. The sampling technique used by randomly selecting two classes, and its classified as experimental class which consisted of 20 students and control class which consisted of 23 students.

2.2 Instrument
The instruments used in this study were tests, interviews, and observations. The test was given to determine the students’ proving skill. Observations are made to observe the implementation of RBL model and observe the students activities. Meanwhile, the interview is used to find out the phase portrait of the student in solving the problems. The research method is illustrated in Figure 1.

![Research Method Diagram]

**Figure 1.** The flow chart of the mixed method.
2.3 Data Collection and Data Analysis
The data consisted of qualitative data and quantitative data. Qualitative data was obtained based on observations and interviews. Quantitative data was obtained from the results of pre-test and post-test which provided to students. The pre-test is given at the beginning of the lesson, and the post-test is given at the end of the lesson. The homogeneity test and the normality test were done on the pre-test and post-test results of both classes. If the data was normally distributed and homogeneous, the independent t-test was carried out using R-shiny program. The independent t-test was conducted to determine the effect of the implementation of RBL model with the learning instrument to improve the students’ proving skills based on post-test results of the control class and experimental class. There are two hypotheses in this research, namely the null hypothesis ($H_0$) and the alternative hypothesis ($H_a$), formulated as follows.

$H_0$ : There is no effect of the learning instrument based on RBL model to the students' proving skills

$H_a$ : There is an effect of the learning instrument based on RBL model to the students' proving skills

If the $p-value > 0.05$, then $H_0$ was accepted, while if the $p-value < 0.05$, then $H_0$ was rejected.

2.4 Task
The task given are pre-test, post-test, and student worksheets. The students are asked to find the resolving efficient domination number and its function. The task given are as follows:
1. Determine the graph that will be searched for the resolving efficient domination number.
2. Label the vertices and the edges of the graph.
3. Write the cardinality of the graph.
4. Determine the efficient dominating set
5. Determine the resolving efficient dominating set and resolving efficient domination number.
6. Prove the resolving efficient domination number.
7. Make a conclusion

The following is an example in solving the resolving efficient dominating set problem on the $K_n \triangleright P_3$ graph.

![Diagram](image.png)

**Figure 2.** The illustration in solving the resolving efficient dominating set problem.

After finding the efficient dominator on the graph, the next step is determine the vertices representation on the $K_n \triangleright P_3$ graf against the efficient dominator to determine the resolving set as the following.

- **Vertex and Edges Function**
  
  \[
  V(G) = \{x_i ; \quad 1 \leq i \leq 4 \} \\
  \{x_{ij} ; \quad 1 \leq i \leq 4, j = 1,2 \}
  \]
  
  \[
  |V(G)| = 12
  \]

- **Efficient Dominating Set**
  
  \[
  E(G) = \{x_i x_j ; \quad 1 \leq i \leq 3, (i + 1) \leq j \leq 4 \} \\
  \{x_i x_{i+1} ; \quad 1 \leq i \leq 4, j = 1 \}
  \]
  
  \[
  |E(G)| = 14
  \]

- **Efficient Dominating Set**
  
  \[
  D(G) = \{x_{11}, x_{21}, x_{31}, x_{41} \}
  \]
  
  \[
  \gamma_e(G) = 4
  \]

- **Cardinality of Graph** ($G = K_n \triangleright P_3$)
  
  \[
  r(x_1|W) = (1, 2, 2, 2) \\
  r(x_2|W) = (2, 1, 2, 2) \\
  r(x_3|W) = (2, 2, 1, 2) \\
  r(x_4|W) = (2, 2, 2, 1)
  \]

  \[
  r(x_{11}|W) = (0, 3, 3, 3) \\
  r(x_{12}|W) = (1, 4, 4, 4) \\
  r(x_{21}|W) = (3, 0, 3, 3) \\
  r(x_{22}|W) = (4, 1, 4, 4) \\
  r(x_{31}|W) = (3, 3, 0, 3) \\
  r(x_{32}|W) = (4, 4, 1, 4) \\
  r(x_{41}|W) = (3, 3, 3, 0) \\
  r(x_{42}|W) = (4, 4, 4, 1)
  \]
3. Research Finding

3.1 Instrument Validity

In this study, the validity and reliability tests were conducted to determine the accuracy level of the tests being developed. The validity and reliability tests are carried out on the test that will be given to students. The results of the validity and reliability tests that have been carried out are as follows.

The validity result of the test can be found out in Table 2. The $r_{count}$ value of question number 1 was 0.325, the $r_{count}$ value of question number 2 was 0.598, the $r_{count}$ value of question number 3 was 0.784, the $r_{count}$ value of question number 4 was 0.755, and the $r_{count}$ value of question number 5 was 0.714. Meanwhile, the $r_{table}$ value for $N = 43$ for question number 1 was 0.301 and the $r_{table}$ value for $N = 43$ for question number 2 until 4 was 0.389. It can be concluded that all question were valid because $r_{count} > r_{table}$ for question number 1 until 5.

| Correlations | Q1 | Q2 | Q3 | Q4 | Q5 | SCORE |
|--------------|----|----|----|----|----|-------|
| Pearson      | .329* | .027 | .053 | .115 | .105 | .325* |
| Sig. (2-tailed) |    | .728 | .454 | .491 | .030 |       |
| N            | 45  | 45  | 45  | 45  | 45  |       |
| Pearson      |    | .387** | .288 | .387** | .598** |       |
| Sig. (2-tailed) |    |     | .009 | .009 | .000 |       |
| N            | 45  | 45  | 45  | 45  | 45  |       |
| Pearson      |    |    |     | .656** | .571** | .784** |
| Sig. (2-tailed) |    |     |     |     |     |       |
| N            | 45  | 45  | 45  | 45  | 45  |       |
| Pearson      |    |    |     | .538** | .755** |       |
| Sig. (2-tailed) |    |     |     |     |     |       |
| N            | 45  | 45  | 45  | 45  | 45  |       |
| Pearson      |    |    |     | 1   | .714** |       |
| Sig. (2-tailed) |    |     |     |     |     |       |
| N            | 45  | 45  | 45  | 45  | 45  |       |
| Pearson      |    |    |     |    | .714** |       |
| Sig. (2-tailed) |    |     |     |     |     |       |
| N            | 45  | 45  | 45  | 45  | 45  |       |
| Pearson      |    |    |     |    | 1   |       |
| Sig. (2-tailed) |    |     |     |     |     |       |
| N            | 45  | 45  | 45  | 45  | 45  |       |

*Correlation is significant at the 0.05 level (2-tailed).
**Correlation is significant at the 0.01 level (2-tailed).

The reliability result of the test can be seen in Table 3. Based on Table 4, the reliability value was 0.740 and the $r_{table}$ value was 0.304 for $dk = N - 1 = 42$. The value of $r_{count} > r_{table}$, so it can be concluded that the test is reliable.
Table 3. The reliability result of the test.

| Cronbach’s Alpha | N of Items |
|------------------|------------|
| 0.740            | 5          |

3.2 Results

The research was conducted by giving pre-test and post-test to the students in the control and experimental class. Besides that, observations were made during the learning process to determine student activity. The interview was conducted at the end of the lesson to find out the phase portrait of the students.

3.2.1 Pre-test Results. The pre-test was given to the control class and the experimental class at the beginning of the lesson. The normality test is carried out on the pre-test results of both classes in order to see if the data is normally distributed or not. Based on the Table 4, the p-value was 0.19 ≥ 0.05, it can be concluded that the pretest data of both classes were in normal distribution.

Table 4. The results of normality test on pre-test by using R shiny.

| statistic | p.value | method               |
|-----------|---------|----------------------|
| 0.96      | 0.19    | Shapiro-Wilk normality test |
| 0.96      | 0.19    | Shapiro-Wilk normality test |

The homogeneity test was also carried out to determine the variances of the pre-test data were homogeneous or not. Based on the Figure 3, the p-value was 0.4309 (p ≥ 0.05), it can be concluded that the pretest data variances of both classes is homogeneous.

Figure 3. The results of homogeneity test on pre-test by using R-shiny.

The post-test data were normally distributed and homogeneous, so an independent t-test was performed to determine the mean score of both class. Based on the independent t-test in Figure 4, we obtained the p – value of 0.792. The p – value was higher than 0.05, so that $H_0$ was accepted. It can be concluded that there was no differences in the mean score of pre-test result in both classes.

Figure 4. The results of independent t-test on pre-test by using R-shiny.
The students were grouped based on their pre-test results, which consisted of three levels, namely the high level of proving skills, the middle level of proving skills, and the low level of proving skills. Based on the pretest results on 23 students in the control class, it was found that 23% of students were at high level, 36% of students were at middle level, and 41% of students were at low level. The distribution of students’ proving skills based on pre-test result in the control class can be seen in Figure 5.

![Figure 5](image)

**Figure 5.** The graphic of students’ proving skill based on pre test result in the control class.

The pretest results on 20 students in the experimental class showed that 24% of students were at high level, 37% of students were at middle level, and 39% of students were at low level. The distribution of students’ proving skills based on pre test result in the experimental class can be seen in Figure 6.

![Figure 6](image)

**Figure 6.** The graphic of students’ proving skill based on pre test result in the experimental class.

### 3.2.2 Post-test Results

The post-test was given to the control class and the experimental class at the end of the lesson. The normality test is carried out on the post-test results of both classes in order to see if the data is normally distributed or not. Based on the Table 5, the p-value was $0.22 \geq 0.05$, it can be concluded that the post-test data of both classes were in normal distribution.

| statistic | p.value | method                        |
|-----------|---------|-------------------------------|
| 0.97      | 0.22    | Shapiro-Wilk normality test   |
| 0.97      | 0.22    | Shapiro-Wilk normality test   |

The homogeneity test was also carried out to determine the variances of the post-test data were homogeneous or not. Based on the Figure 7, the p-value was $0.7488 (p \geq 0.05)$, it can be concluded that the post-test data variances of both classes is homogeneous.
Figure 7. The results of homogeneity test on post-test by using R-shiny.

The post-test data were normally distributed and homogeneous, so an independent t-test was performed to determine the impact of the learning instrument created. Based on the independent t-test in Figure 8, we obtained the $p-value$ of 0.008615. The $p-value$ was lower than 0.05, so that $H_0$ was rejected and $H_a$ was accepted. It can be concluded that there is an effect of the implementation of the learning instrument based on RBL to improve the students’ proving skills. The mean score of the control class was 18.04348 and the mean score of the experimental class was 20.05.

Figure 8. The results of independent t-test on post-test by using R-shiny.

The students were grouped based on their post-test results, which consisted of three levels, namely the high level of proving skills, the middle level of proving skills, and the low level of proving skills. Based on the posttest results on 23 students in the control class, it was found that 36% of students were at high level, 36% of students were at middle level, and 28% of students were at low level. The distribution of students’ proving skills based on post-test result in the control class can be seen in Figure 9.

Figure 9. The graphic of students’ proving skill based on post test result in the control class.
While the posttest results on 20 students in the experimental class showed that 41% of students were at a high level, 37% of students were at middle level, and 22% of students were at low level. The distribution of students’ proving skills based on post-test result in the experimental class can be found in Figure 10.

![Distribution of students' proving skills](image)

**Figure 10.** The graphic of students’ proving skill based on post-test result in the experimental class.

Observations of student activities are also carried out during the learning process in the experimental class. Based on the observation results in Figure 11, it is shown that 28% of students are in the very active category, 30% of students are in the active category, 18% of students are in the hesitate category, 14% of students are in the less active category, and 10% of students are in the inactive category. It can be concluded that student activities show the positive results.

![Distribution of students' activities](image)

**Figure 11.** The graphic of the students activities during the implementation of research based learning instrument.

### 3.3 Phase Portrait

The phase portrait is a description of the process of student thinking in solving a problem. In this study, the student phase portrait is based on the flow of students’ proving skill in solving the resolving efficient dominating set problem. The phase portrait was obtained by conducting interviews with 3 subjects who were selected from each level of proving skills. The results obtained are as follows.

#### 3.3.1 Students 1 at the high level of proving skills.

Student 1 solve the resolving efficient dominating set problems with determining the graph to be used, labeling the graph, determining the efficient dominator on the graph, determining the resolving efficient dominator on the graph, determining the vertex and edges functions of the graph, determining the efficient dominator function and resolving efficient domination number on a graph, proving the resolving efficient domination number function and making a conclusion. The results of student 1 can be seen in Figure 12.
The Cardinality of Graph ($G = P_n \Rightarrow \rho_3$)

Vertices and Edges Function

\[ V(G) = \begin{cases} 
 x_i ; & 1 \leq i \leq n \\
 y_i ; & 1 \leq i \leq n \\
 z_i ; & 1 \leq i \leq n \\
 x_i y_{i+1} ; & 1 \leq i \leq n - 1 \\
 y_i z_i ; & 1 \leq i \leq n 
\end{cases} \]

\[ |V(G)| = 3n \]

Efficient Dominating Set

\[ D(G) = \{ y_i ; 1 \leq i \leq n \} \]

Resolving Efficient Dominating Set

\[ D(G) = \{ y_i ; 1 \leq i \leq n \} \]

Resolving Efficient Dominating Set Function

\[ \gamma_{re}(G) = n \]

The Vertices Representation in Respect of D

\[ r(x_i|x) \begin{cases} 
 1, 4, \ldots, (n - i), (n - i + 1) & n \geq 3 \\
 i, (i - 1), \ldots, 2, 1, 2, \ldots, (n - i), (n - i + 1) & i \geq 2, n \geq 3 \\
 n(n - 1), \ldots, 2, 1 & n \geq 3 \\
 0, 3, \ldots, (n - i + 1), (n - i + 2) & i \geq 2, n \geq 3 \\
 (i + 1), i, \ldots, 2, 1, 2, \ldots, (n - i + 1), (n - i + 2) & n \geq 3 \\
 (n + 1), n, \ldots, 3, 0 & n \geq 3 \\
 1, 4, \ldots, (n - i + 3), (n - i + 4) & i \geq 2, n \geq 3 \\
 (i + 2), (i + 1), \ldots, 4, 1, 4, \ldots, (n - i + 3), (n - i + 4) & n \geq 3 \\
 (n + 2), n, \ldots, 4, 1 & n \geq 3 
\end{cases} \]

Conclusion:

Because the set $D$ is an efficient dominating set and resolving set, so that $D$ is resolving efficient dominating set with $\gamma_{re}(G) = n$.

Figure 12. The results of students 1

Interviews were conducted on student 1 to find out the portrait phase of student 1 in solving the problems given. Figure 13 shows that student 1 solved the resolving efficient dominating set problem by determining the graph to be used (1a), followed by finding the efficient dominator (2a) and labelling the graph (1b), then continue with determining the resolving efficient dominator on graph (2b), determining the cardinality of the graph which includes point and edge functions (3a), determining the efficient dominating set function (3b) and resolving efficient dominating set function on the graph (3c). Then, student 1 proved the resolving efficient domination number function found (4a) and made a conclusion (4b). The interview results is in the following.

Researcher : How did you thought after reading the problems?
Student 1 : At first I felt confused in completing it, but after finishing worksheet, I began to understand the concept of resolving efficient dominating set and how to solve it.
Researcher : How do you solve this problem?
Student 1 : First, I determined the graph I would use, then I looked for the efficient dominator of graph, after that I labeled the graph and determined the resolving
efficient dominator of graph. Then, I determined the cardinality of graph, the function of efficient dominating set, and the function of resolving efficient dominating set. After that, I proved the resolving efficient domination number and make a conclusion.

Researcher: Did you have trouble resolving the problem?

Researcher: Yes Miss, I had trouble finding the graph I would use at first, even in deciding the representation of the point up to the order $n$.

![Figure 13. The phase potrait of student 1 at the high level.](image)

**3.3.2 Students 2 at the middle level of proving skills.** Student 2 solve the problems with determining the graph to be used, labeling the graph, determining the efficient dominator on the graph, determining the vertex and edges functions of the graph, determining the function of efficient dominating set, proving the resolving efficient domination number of the graph and making a conclusion. The students 2 can’t make a conclusion correctly. The results of student 2 can be seen in Figure 14.

**The Cardinality of Graph ($G = P_n \Rightarrow S_3$)**

- Vertices and Edges Function

  \[
  \begin{align*}
  V(G) &= \{ x_i; 1 \leq i \leq n \} \\
  |V(G)| &= 4n \\
  E(G) &= \{ x_ix_{i+1}; 1 \leq i \leq n-1 \} \\
  |E(G)| &= 4n-1 \\
  \end{align*}
  \]

- Efficient Dominating Set

  \[
  D(G) = \{ y_i; 1 \leq i \leq n \} \\
  \gamma_{er}(G) = n
  \]

**The Vertices Representation in Respect of D**

\[
\begin{align*}
  r(x_1|D) &= \left( \frac{1,2,3,...}{n-1} \right) \\
  r(x_2|D) &= \left( \frac{i,(i-1),...,1,2,...}{i-1} \right) \\
  r(x_n|D) &= \left( \frac{1,2,...}{n-2} \right) \\
  r(y_1|D) &= \left( \frac{1,3,4,5,...}{n-1} \right) \\
  r(y_i|D) &= \left( \frac{...4,3,0,3,4,...}{i-1} \right) \\
  r(y_n|D) &= \left( \frac{0,3,4,...}{n-1} \right) \\
  \end{align*}
\]

**Conclusion:**

The set $D$ is resolving efficient dominating set. Thus $\gamma_{er}(G) = n$.

![Figure 14. The results of students 2](image)
Interviews were conducted on student 2 to find out the portrait phase of student 2 in solving the problems given. Figure 15 shows that student 2 solved the resolving efficient dominating set problem by determining the graph to be used (1a), labelling the graph (1b), finding the efficient dominator (2a) and finding the resolving efficient dominator (2b). Then, student 2 solve the problem by determining the cardinality of the graph which includes point and edge functions (3a). Then, student 2 continue with determining the function of efficient dominating set of graph (3b), proving the resolving efficient domination number function (4a) and it ends with creating a conclusion (4b). The interview results is in the following.

Researcher : How did you thought after reading the problems?
Student 2 : At first I felt confused, but after being given an explanation, I began to understand a little about the meaning of the problem ma'am.

Researcher : How do you solve this problem?
Student 2 : I look for a graph first, then I label it, then I look for the efficient dominator of graph, and the resolving efficient dominator of the graph. After that I determine the cardinality of the graph, and the function of efficient dominating set of graph. Then, I prove the resolving efficient domination number function and make a conclusion.

Researcher : Did you have trouble resolving the problem?
Student 2 : Yes Miss, I had difficulty to solve these problems.

Figure 15. The phase portrait of student 2 at the middle level.

3.3.3 Students 3 at the low level of proving skills. Student 3 solve the resolving efficient dominating set problems with determining the graph to be used, labeling the graph, and write the vertex and edge function of the graph. The students 3 can’t determine the efficient dominator on the graph correctly, students 3 also can’t determine the resolving efficient domination number of the graph and prove the function. The results of student 3 can be seen in Figure 16.

Interviews were conducted on student 3 to find out the portrait phase of student 3 in solving the problems given. Figure 17 shows that student 3 solved the resolving efficient dominating set problem by determining the graph to be used (1a), labelling the graph (1b), and determining the cardinality of the graph which includes point and edge functions (3a). Then, student 2 solve the problem by determining finding the efficient dominator (2a) and the resolving efficient dominator of graph (2b) and. Then, student 2 continue with determining the efficient dominating set functions (3b) and it ends with proving the resolving efficient domination number of graph (4b). The interview results is in the following.

Researcher : How did you thought after reading the problems?
Student 3 : I feel confused to solve it, Miss.
Researcher : How do you solve this problem?
Student 3 : I looked for the graph first, then I labeled the graph and wrote the cardinality of the graph. After that, I determined the efficient dominator and resolving efficient dominator of the graph. Then, I determined the function of efficient dominating
set and proved the resolving efficient domination number of graph.

Researcher: Did you have trouble resolving the problem?
Student 3: Yes Miss, I'm having trouble finding a graph.

![Figure 1](image1)

**Figure 16.** The results of students 3 at the low level of proving skills.

**The Cardinality of Graph (G = Hₙ)**

- Vertices and Edges Function
  
  \[ V(G) = \begin{cases} 
  x & ; 1 \leq i \leq n \\
  y_{i1} & ; 1 \leq i \leq n 
  \end{cases} \]

  \[ |V(G)| = 2n + 1 \]

  \[ E(G) = \begin{cases} 
  xy_i & ; 1 \leq i \leq n \\
  y_{i1}y_{i2} & ; 1 \leq i \leq n \\
  y_{ni}y_{n1} & ; 1 \leq i \leq n 
  \end{cases} \]

  \[ |E(G)| = 3n - 1 \]

- Efficient Dominating Set
  
  \[ D(G) = \{x,y_{11},y_{21},y_{31},y_{41},y_{(n-1)1},y_{n1}\} \]

  \[ = n + 1 \]

![Figure 17](image2)

**Figure 17.** The phase portrait of student 3 at the low level.

\[ r(x_1|D) = \begin{pmatrix} 0, 2, 2, \ldots, \frac{n-1}{n-1} \end{pmatrix} \quad r(y_{11}|D) = \begin{pmatrix} 2, 0, 3, 4, \ldots, 4, 3 \end{pmatrix} \]

\[ r(y_{11}|D) = \begin{pmatrix} 2, 3, 0, 3, 4, \ldots, 4 \end{pmatrix} \]

\[ r(y_{21}|D) = \begin{pmatrix} 2, 4, 3, 0, 3, 4, \ldots, 4 \end{pmatrix} \]

\[ r(y_{31}|D) = \begin{pmatrix} 2, 4, \ldots, 4, 3, 0, 3, 4, \ldots, 4 \end{pmatrix} \]

\[ r(y_{11}|D) = \begin{pmatrix} 2, 4, \ldots, 4, 3, 0, 3, 4, \ldots, 4 \end{pmatrix} \]

\[ r(y_{(n-1)1}|D) = \begin{pmatrix} 2, 4, \ldots, 4, 3, 0, 3 \end{pmatrix} \]

\[ r(y_{n1}|D) = \begin{pmatrix} 2, 3, 4, \ldots, 4, 3 \end{pmatrix} \]

\[ r(y_{n1}|D) = \begin{pmatrix} 2, 3, 4, \ldots, 4, 3 \end{pmatrix} \]

\[ r(y_{n1}|D) = \begin{pmatrix} 2, 3, 4, \ldots, 4, 3 \end{pmatrix} \]
4. Discussion
This study was shown that the implementation of RBL model can improve students’ proving skills in dealing with resolving the efficient dominating set problems. The independent t-test of the pre-test showed that there is no differences at the mean score of both classes. Meanwhile, the independent t-test of the post-test obtained the \( p - \text{value} \) of 0.008615 (\( p < 0.05 \)), so that there is a significant effect of the implementation of the learning instrument based on RBL model on the students’ proving skills. The mean score of the control class was 18.04348 and the mean score of the experimental class was 20.05. It revealed that the mean score of students’ proving skills in the experimental was higher than the control class, so it can be concluded that research-based learning implementation with the learning instrument has affected students’ proving skills significantly. This is in line with the research conducted by Suntusia, et al [15]. The results of the research show that the implementation of RBL model is more effective to improve students’ achievement in solving two dimensional arithmetic sequence problems.

In addition, the implementation of RBL model also affects the students activities throughout the learning process. Based on the observation of student activities, it is shown that the implementation of RBL model give the positive impact to increase the students activities during the learning process. This is in line with research conducted by Maylisa [7]. The results show that RBL give a positive learning process to the students on solving the problem and it can improve the students’ combinatorial thinking.

5. Conclusion
Based on the results of the study, it is found that the implementation of RBL has a significant effect to improve the students' proving skills. The results showed that the mean score of post-test in the experimental class was higher than the control class. Thus, the implementation of RBL model with learning instrument has improved students' proving skills significantly. The implementation of RBL model can also affects student activity during the learning process. The student phase portrait shows the flow of student thinking to solve resolving efficient dominating set problems. Based on the results of the interview, it is understood that there is a different flow of thought for each students. However, we need to develop this research, so we offer the open problem to determine the implementation of RBL model in improving another students’ thinking skills.

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