Abstract. We have measured the effect of colliding two phonon sheets together at different angles. At small angles they interact strongly and a hot line is formed along the line of intersection of the two sheets. At angles between ~13° and ~27° the interaction becomes weaker and less energy goes into the hot line. At angles greater than ~27° there is no interaction between the sheets and they pass through each other. By delaying one sheet with respect to the other, the path of the hot line can be shifted laterally. Using this behaviour, we show that the sensitive area of the bolometer is around 10^{−2} mm². We have measured the profile of the hot line and find that its width, typically 1 mm, varies as 1/sin(α/2), where α is the angle between the two sheets. We analyse the data and estimate that the typical angle between two phonons interacting by the three phonon process varies between 8.4° and 12.5°, depending on their energy. We model the formation of the hot line when the sheets are strongly interacting. From the model and the measured data, we find that the temperature of the hot line decreases, and the cone angle and energy density of the hot line increases, with both increasing energy density in the sheets and increasing angle α.
1. Introduction

Liquid $^4$He is a particularly good medium to study phonons. Phonons are well defined quasiparticles propagating in a medium without defects and essentially without impurities, so that their intrinsic behaviour can be readily observed. Furthermore their interaction with surfaces and other scatterers gives information which cannot easily be obtained in other ways. For example, evaporation at the free liquid surface of $^4$He has been shown to be a quantum process in which one quasiparticle is annihilated and a free atom created [1]; also phonon spectroscopy of magnetorotons in the fractional quantum Hall state showed that the dissociation of magnetorotons is due to phonons [2]. The quasiparticles can be very long lived, for example phonons with energy greater than 10 K in cold liquid $^4$He show no decay [3]–[5]; also other quasiparticles can spontaneously decay on time scales as short as 1 ns, for example phonons with energy around 8 K in liquid $^4$He [6]. The interactions between phonons are considered in this paper, in particular the three-phonon process and the four-phonon process. We use the collision between phonon sheets to probe these scattering processes.

A phonon sheet is a system of phonons which occupy a sheet-like volume in cold liquid helium, and a narrow cone in momentum space. The dimensions of the sheet are typically $1 \text{ mm} \times 1 \text{ mm} \times 24 \mu \text{m}$ and the phonon energy density is constant in the central area of the sheet [7]. The ambient temperature of the liquid helium is sufficiently low that there are negligible ambient phonons, so the sheet exists in a phonon vacuum. The sheet propagates in the direction of the normal to the sheet at very nearly the velocity of sound, $c = 238 \text{ m s}^{-1}$ at zero pressure. However the dispersion curve is nonlinear and there is a small upward dispersion at low wavevectors [9, 10]. This increases the velocity but is compensated by the finite angular spread of the phonon momenta in the sheet, which decreases the velocity [8].

The phonons in a sheet are predominately low energy phonons (l-phonons) with a typical energy $\epsilon \sim 1.5$ K. They are strongly interacting through the three phonon process (3pp) [11, 12], and rapidly come into equilibrium in a time which is short compared to all other time scales in the experiment. The phonons in the sheet only occupy a small solid angle in momentum space. To a first approximation this is a narrow cone of states, with the cone axis along the propagation direction, cut from an isotropic distribution with temperature $T_p$.

A phonon sheet can be created in cold liquid helium by injecting phonons with a short current pulse, $t_p \sim 1 \times 10^{-7}$ s, to a thin film heater [7]. The initial lateral dimensions of the sheet
are those of the heater, and the thickness of the sheet is $ct_p$. The helium should be at around 50 mK so that thermal phonons play no role.

When two phonon sheets collide, there can be a strong interaction that creates a hot line in the liquid helium, [13], see figure 1. The hot line creates high energy phonons, h-phonons, with energy $\epsilon/k_B \geq 10$ K as it propagates and the bolometer detects these time dispersed h-phonons. In contrast, the l-phonon signal is a measure of their energy at the instant the hot line reaches the bolometer. It has been found that if the angle, $\alpha$, between the normals to the two sheets is not too large ($\alpha \leq 13^\circ$ at $P = 0$), there is a strong interaction along the line of intersection of the two sheets, see figure 2. This prevents the two sheets passing through each other. Instead, the parts of the sheets that would have passed through the line of intersection are annihilated. Hence the sheets decrease in area and the energy associated with the loss of sheet area stays near the line of intersection and forms the hotline.

The phonons in the hot line form a new anisotropic and strongly interacting system, which propagates along the symmetry axis between the two heaters. The hot line can be detected by a bolometer and the signals from it are considerably larger than the signals from the separate sheets, see figure 2. This is due to the energy, which would otherwise miss the bolometer, being concentrated onto the bolometer in the form of the hot line.

In this paper, we present detailed measurements of the dependence of the hot line on the angle between the two sheets. Powers in the range 3.2 to 25 mW were used in heaters with area 1 mm$^2$. During this experiment we discovered that the bolometer only has a small sensitive area relative to its total area of 1 mm$^2$. This was found by slightly delaying one sheet with respect to the other, which has the effect of laterally shifting the line of travel of the hot line, see figure 3. We found that small shifts of 100 ns give a measurable change in the bolometer response. This leads us to conclude that the sensitive region has an area $90 \mu m \times 80 \mu m$.

The angular measurements show that the energy of the l-phonons in the hot line increases approximately linearly with angle between the sheets for $\alpha < 13^\circ$. For angles up to this value there is a very strong interaction between the two sheets, but for larger angles the energy in the
Figure 2. Typical signals for 12.5 mW, 100 ns heater pulses at zero pressure, after propagating 12.9 mm. Curves H6 and H8 are single pulses from heaters 6 and 8 respectively, and curve H68 is from nearly simultaneous pulses to heaters 6 and 8 (the pulse to heater 8 is delayed by 0.4 µs with respect to the pulse to heater 6 in order to maximize the signal). Note that curve H68 is much larger than the sum of H6 and H8.

hot line decreases, showing that the interaction decreases with increasing angle. From the values of the angle $\alpha$ at the peak value of the energy in the l-phonons, we can estimate the typical values of the 3pp angle, $\theta_{3pp,typ}$. This is the first time that this has been obtained from measured data.

At each value of angle and heater power we have measured the energy in the l- and h-phonon signals. We use these, together with a model of the formation of the hot line, to calculate the temperature and solid angle of occupied states of the l-phonons in the hot line. We assume that a hot line is in a dynamic equilibrium, with a constant temperature and solid angle. Using the theoretically predicted h-phonon creation rates we find that the solid angle of occupied phonon states increases with heater power, and the temperature decreases with increasing heater power, perhaps a counter-intuitive result. As a function of angle between the sheets, we find that the occupied solid angle increases and the temperature decreases slightly with increasing angle, at small angles.

This paper is organized as follows. In section 2 we describe the experiments. In section 3, we present the delay data and discuss its implications for the bolometer and the width of the hot line. In section 4, we present the angular measurements and discuss them. In section 5, we describe a model of the hot line, and draw conclusions in section 6.

2. Experimental method

To measure the energy in the hot line as a function of the energy density in the individual sheets and the angle $\alpha$ between the sheets, pairs of heaters are required with well-defined angles between their normals. The energy in the phonon sheets is determined by the size of the electrical pulse which is applied simultaneously to the two heaters. We fabricated accurately positioned heaters
Figure 3. Panel (a) is a schematic of two intersecting phonon sheets showing the offset, $\Delta d$, caused by a delay between the two pulses. With no delay the propagation line is $AA'$ and with a delay is $BB'$. Panel (b) is a schematic of the zinc bolometer which is scratched to create a serpentine current path. The hot line is scanned across the bolometer as the delay between the two pulses is increased. The bolometer only has a small sensitive region somewhere on the serpentine track. Panels (c) and (d) show the measured l- and h-phonon integrals as a function of delay between the heater pulses for various values of angle $\alpha$, for 12.5 mW, 100 ns heater pulses at zero pressure.

by evaporating gold film heaters onto a cylindrical glass lens and then defining the individual heaters by lines scratched through the gold film, see figure 1(a). The lens had a radius of curvature of 12.9 mm and the line of the heaters was perpendicular to the cylinder’s axis. The heaters were $1 \times 1$ mm and the angle between the normals of adjacent heaters was 4.4°. The bolometer detector was positioned at the centre of curvature of the lens and in the plane of the arc of heaters. The bolometer was a zinc film $1 \times 1$ mm, cut into a serpentine track. The zinc film was in a constant magnetic field, parallel to its plane, and held at its superconducting transition edge by a feedback circuit [14]–[16], which maintains a constant bolometer resistance at $\sim 0.1$ of its normal value at 4.2 K. The corresponding bolometer temperature is $\sim 0.35$ K. The lens and bolometer were held in a brass support which maintained their relative positions inside a brass cell. This is the same apparatus that was used in [17]. The cell was cooled to $\sim 50$ mK with a dilution refrigerator and filled with isotopically pure $^4$He [18].
Heaters were pulsed in pairs with currents for 50 or 100 ns from a LeCroy 9210 pulse generator with two independent but synchronous outputs. Four different heater powers are used, 3.125, 6.25, 12.5 and 25 mW in heaters with area 1 mm\(^2\). The signals from the bolometer were amplified and digitally recorded with a Tektronix DSA 601A. Many signals were averaged to obtain a good signal to noise ratio. The responsivity of the detection system was \(1.3 \times 10^7\) V per watt absorbed by the bolometer. A small delay could be applied to one of the heater pulses and this was used to scan the hot line across the bolometer and so measure the spatial sensitivity. The angular data are taken with a delay that maximizes the hot-line signal. This compensates for small differences in distance from the heaters to the sensitive region of the bolometer. Such delays were \(<1\) \(\mu\)s.

Typical signals from a central pair of heaters are shown in figure 2. Each trace shows the short l-phonon signal at \(\sim 54\) \(\mu\)s after the heater pulses and the broad h-phonon signal which peaks at \(\sim 70\) \(\mu\)s. The lower two traces show the separate signals from the individual heaters. The top trace is the signal from simultaneously pulsing the heaters. The l-phonon signal is much greater than twice the sum of the l-phonon signals from individual heater pulses due to the hot line concentrating energy onto the bolometer.

3. Delay results: size of hot line and detector

When two heaters are simultaneously pulsed, two phonon sheets are created which propagate along the normals to the heaters. As the heaters are at an angle to each other, the sheets will eventually intersect, with the line of intersection perpendicular to the axis of symmetry, shown as AA’ in figure 3(a). If one heater pulse is delayed by a time \(\Delta t\) with respect to the other, the line of intersection of the two sheets is displaced laterally, which is shown as BB’ in figure 3(b). The lateral displacement \(\Delta d\) is given by

\[
\Delta d = \frac{c\Delta t}{2\sin(\alpha/2)},
\]

where \(c\) is the velocity of the sheets and \(\alpha\) is the angle between the two heaters. For example, for \(\alpha = 8.8^\circ\), a delay of 0.65 \(\mu\)s causes the centre of the hot line to move 1 mm.

The measured response to delays for several angles is shown in figure 3(c). We see that the bolometer signal varies very rapidly with delay, around the peak response. We will now argue that this shows that the width of the sensitive region of the bolometer is much smaller than 1 mm. The response curve, as a function of delay, is the convolution of the spatial profile of the energy density in the hot line and the spatial dependence of the sensitivity of the bolometer. Were the bolometer only sensitive at one infinitely small point, then the response curve would be just the energy density profile of the hot line. For a larger sensitive area on the bolometer, sharp peaks on the response curve would be broadened by roughly the width of the sensitive area.

The orientation of the hot line is perpendicular to the long zinc tracks of the bolometer as shown in figure 3(b). If the bolometer detected uniformly over the whole of the zinc track, then the response curves would have a flat top with a minimum width of 0.65 \(\mu\)s, which corresponds to the 1 mm nominal width of the bolometer. As this flat top is not seen, it means that the bolometer has one or more small sensitive regions along the zinc track. However, it can be seen from the response curves in figure 3(c) that there is only one peak as a function of delay which implies
that, either there is only one sensitive spot or, if there are more than one, they fall on a straight line parallel to the hot line. As this latter possibility is unlikely, we shall assume that there is only one small sensitive spot.

The track width of the zinc in the bolometer is \( \sim 0.09 \) mm wide; (there are 10 scratched lines per mm to form the serpentine track). We envisage that there is some imperfection in the track that causes a slightly higher current density at this point, and this determines the position of the normal-superconducting boundary, i.e. the zinc track is normal up to some line across its width, and superconducting beyond it. This boundary is the sensitive detecting region; in the absence of electronic feedback, heat from an incident phonon flux is absorbed by the zinc, and the normal region increases its length along the track. This changes the position of the boundary and the resistance of the track. In our system, the feedback changes the voltage across the bolometer, and hence the Joule heating, to keep the resistance constant. The decrease in joule heat exactly compensates for the absorbed heat from the incident phonons. As the resistance is constant, the position of the normal-superconducting boundary is constant. The change in feedback power is proportional to the change in voltage, for small changes relative to the standing voltage across the bolometer.

To estimate the length of the sensitive detecting region we must estimate the width of the top of the response curve. From the response curves in figure 3(c), we see that the response at the peak has a noticeable change for a change in delay of 100 ns. Using equation (1) and the response curve for \( \alpha = 17^\circ \), which gives the minimum value of the length and therefore represents an upper bound, the length is \( 238 \text{ms}^{-1} \times 10^{-7} \text{s}/2 \sin(17/2^\circ) = 81 \mu\text{m} \). From this value and the width of the zinc track, we estimate the sensitive area of the bolometer is \( 80 \mu\text{m} \times 90 \mu\text{m} \).

The response curves in figure 3(c) therefore represent the profile of the hot line, broadened by convolution with a top-hat function \( 0.08 \) mm wide. We see that the time delay profiles of the hot lines, at different angles, are very similar; the full widths at half height, FWHH, all lie in the range \( 0.55 \pm 0.05 \mu\text{s} \). From equation (1), we see that this means that the FWHH of the hot line in space, varies as \( 1/\sin(\alpha/2) \). For \( \alpha = 13.2^\circ \), this corresponds to a FWHH of \( 238 \times 0.55 \times 10^{-6} \times 10^3/2 \sin(6.6^\circ) = 0.57 \mu\text{m} \), for \( \alpha = 8.8^\circ \) the FWHH is \( 0.85 \mu\text{m} \) and for \( \alpha = 4.4^\circ \), the FWHH is \( 1.7 \mu\text{m} \). This dependence of the width is the same as that of the length of the diamond formed by the overlap of two sheets. However this is a coincidence as we find much the same hot line widths for pulse lengths twice as long, i.e. 100 ns, whereas the length of the overlap diamond scales as the pulse length. Although the width of the hot line is independent of pulse length, the thickness of the hot line, in the direction of propagation, is proportional to the pulse length. As the time constant of the detector is long compared with these pulse lengths, the l-phonon signal heights are proportional the pulse length.

We see in figure 3(c) that the peaks occur at larger delays for larger values of \( \alpha \). This means that the offset of the sensitive spot on the bolometer is offset from the centre of curvature of the lens. If the offset of the sensitive spot is \( \delta x_{pk} \) then, from equation (1) we find the delay corresponding to the peak, \( \delta t_{pk} \), is given by

\[
\delta t_{pk} = \frac{2 \delta x_{pk} \sin(\alpha/2)}{c}.
\]

A plot of \( \delta t_{pk} \) versus \( \sin(\alpha/2) \), for the data in figure 3(c) gives a straight line. The gradient gives \( \delta x_{pk} = 0.56 \) mm. This is \( \approx 0.26 \) mm from the centre of the bolometer as the centre of the bolometer was inadvertently offset by 0.3 mm from the centre of curvature of the lens.
In figure 3(d), we show the result of delay on the $h$ integral. The behaviour is similar to the $l$ integral showing that there are $h$-phonons created by the hot line.

The main conclusion of this section is that the bolometer has a small detecting area so that when one phonon sheet is delayed with respect to the other, the hot line is swept across the sensitive spot. This means that the response curve is the transverse energy profile of the hot line, broadened by $\sim0.1$ mm due to the finite size of the bolometer’s sensitive area. The width of the hot line is typically 0.6 mm wide at $\alpha = 13.2^\circ$, and it varies with angle as $1/\sin(\alpha/2)$.

4. The hot line as a function of angle and power

We first review the behaviour of single phonon sheets so that we can discuss the collision between two sheets. At zero pressure, the phonons in a sheet strongly interact with each other through 3pp scattering, in which two phonons scatter to form one phonon or one phonon scatters into two phonons. The scattering only occurs with small angles between the two incoming or two outgoing phonons, due to the need to conserve energy and momentum. The combination of strong interactions and high anisotropy in phonon sheets gives them unique and interesting properties. One of the most surprising is that a sheet of phonons with an initial temperature $T_p \sim 0.9$ K, converts some of its energy to phonons with energy $\epsilon/k_B \geq 10$ K, [19, 20]. This is due to phonons in the sheet also scattering by the much weaker four-phonon process, 4pp, in which phonon number is conserved [21, 22].

4pp scattering is important when it creates phonons that cannot be created by faster scattering processes, this occurs for phonons with energy $\epsilon/k_B \geq 10$ K at zero pressure. The 4pp scattering can create a pair of phonons, one with low energy and the other with high energy, $\epsilon/k_B \geq 10$ K, a so called ‘$h$-phonon’. This $h$-phonon is extremely stable because it cannot spontaneously decay, unlike a phonon with $\epsilon/k_B \leq 10$ K which decays rapidly into two or more phonons [23]. As the group velocity of $h$-phonons is $\leq 189$ ms$^{-1}$, which is less than $c$, the $h$-phonons are lost from a thin sheet, by being left behind soon after they are created. The $l$- and $h$-phonons separate from each other and produce two distinct signals in the detector, for sufficiently long path lengths, typically $>10$ mm, [19]. The theory of $h$-phonon creation has been presented in a number of papers [24]–[27].

As the sheet propagates and loses $h$-phonons, the solid angle of occupied states in momentum space, $\Omega_p$, ($\Omega_p = 2\pi\xi_p = 2\pi(1 - \cos\theta_p)$ where $\theta_p$ is the cone angle of the occupied states) increases because there is overall conservation of momentum in the phonon system [24]. At the same time the temperature, $T_p$, of the sheet decreases as it propagates, due to energy lost from the sheet in the form of $h$-phonons. It can also decrease if the sheet expands laterally by 3pp interactions, even when there is no $h$-phonon creation [28]. Hence the energy of the phonons in the sheet depends on the distance from the heater, as well as on the heater power.

The final energy density in a sheet is thought to arise from the strong temperature dependence of the $h$-phonon creation rate. This causes the sheets to lose energy until each part of the sheet falls to a temperature where the $h$-phonon creation essentially stops [26]. For $\theta_p = 11.4^\circ$ ($\Omega_p = 0.124$ sr) this temperature is $\sim0.7$ K. In this way, the sheets develop an area of uniform energy density with dimensions larger than those of the heater [7]. This area increases with heater power.

As described in the previous section, two sheets can interact and form a hot line and a delay between the two sheets causes a lateral displacement of the hot line. The data in this section
were taken with a delay between the heater pulses that maximizes the l-phonon signal from the hotline. We plot the integrals of the signals, which give values of energy which are independent of the time constant, typically 1 $\mu$s, of the detecting system. The l-phonon signals are integrated over 3.7 $\mu$s from the start of the signal at 58.3 $\mu$s, and will be called the l integral. The h-phonons are integrated from 62.0 to 98.3 $\mu$s; 98.3 $\mu$s is chosen to exclude the tail of the signal which is due to the heater substrate being glass and emitting phonons stored from the heater pulse.

In figure 4(a), we show the l integral from pairs of heaters as a function of the angle $\alpha$ between the two heaters. Both heater pulses are 50 ns. The results for four powers are shown. This l-phonon energy comes from the hot line as the sensitive spot on the bolometer is much narrower than the hot line so that none of the remaining parts of the sheets, outside the hot line, are detected. We see that the l integral rises rapidly with angle, reaches a peak between 8° and 12°, depending on the heater power and then falls rapidly between 14° and 25°, to a small value, depending on heater power. The initial rise of the l integral appears to extrapolate through the origin for all powers, although $\alpha < 4.4°$ is not possible with 1 mm wide heaters and a radius of curvature of 12.9 mm.

In figure 4(b), we show the h integral as a function of angle between each pair of heaters. The peak energy is similar to that of the l integral. However there are significant differences in the behaviour. The detected peak energy occurs at $\sim$9° for all heater powers, so at higher heater powers the peak of the l integral occurs at a larger angle than the peak of the h integral. At large angles, the h integral does not fall as much as the l integral, but the angles where the fall ends, are similar to those for the l integral, at the same power. The initial rise of the curves does not appear to extrapolate back through the origin.

In figures 5(a) and (b), we show similar measurements for 100 ns pulse lengths for $\alpha \leq 30°$. Generally these curves show twice the energy of the 50 ns data, but otherwise their behaviour is similar but with small quantitative differences in the angular behaviour.
Figure 5. Panel (a) shows the integral of the l-phonon bolometer signal from the hot line as a function of angle between the heater normals, for four heater powers and 100 ns heater pulses. Panel (b) shows the integral of the h-phonon bolometer signal from simultaneous pulses as a function of angle between the heater normals, for four heater powers and 100 ns heater pulses.

We interpret the shape of the curves as follows. We recall that the signal from the l-phonons measures their energy density when they reach the bolometer. The initial rise with angle is primarily due to the increase in the rate of energy being fed into the hot line from the two sheets, as the angle increases. The rate of energy going into the unit length of the hot line from the loss of area of the two interacting sheets, $\frac{dE_{hl}}{dt}$, as a function of angle $\alpha$ is given by

$$\frac{dE_{hl}}{dt} = 2E_s c \tan(\alpha/2)$$

where $E_s$ is the energy per unit area of one sheet. Equation (3) only applies while there is a strong interaction between sheets. We see from equation 3 that $dE_{hl}/dt$ is proportional to the angle $\alpha$ for small $\alpha$. If the rate of loss of energy by the hot line is proportional to $E_{hl}$ and if the hot line is in dynamic equilibrium, then $E_{hl} \propto \alpha$, a result which is found experimentally, see figures 4(a) and 5(a).

When the interaction is strong, each sheet interacts with the other. We expect that there will be maximum interaction when the axes of the cones in the sheets make an angle, $\theta_{3pp,typ}$, that allows the maximum number of phonons in the two systems to interact by 3pp. So in the first approximation, the peak values of the 1 integrals occur at angles, $\alpha_{pk}$, given by

$$\alpha_{pk} \sim \theta_{3pp,typ}.$$  (4)

From equation (4), we find the typical angle for 3pp, $\theta_{3pp,typ}$. These values are tabulated in table 1 for 50 ns and 100 ns pulse lengths. These values are near to those calculated for $T_p \leq 0.8$ K and $\zeta_p = 0.011$, corresponding to $\theta_p = 8.5^\circ$, [29] (note in [29] and here, the 3pp angle is defined as twice that used in [27] figure 2).

As the angle between the sheets increases beyond $\alpha_{pk}$, the interaction between the two sheets weakens and the energy in the hot line decreases. According to [30] there is no interaction when...
the crossing time of the sheets is much less than the inverse of the scattering rate. This 3pp rate depends on the temperature $T_p$ of the sheet and the angle between the sheets. Our measurements show that the hot line rapidly falls to zero with angle and there is a well defined angle for no interactions between the sheets. We call this angle $\alpha_{\text{max}}$ and the values at the four powers are tabulated in Table 1.

The data for 50 and 100 ns pulses are very similar, and both $\alpha_{pk}$ and $\alpha_{\text{max}}$ increase with heater power. This probably indicates that the temperature of a sheet increases a little with heater power, which increases the energy of a typical phonon. For low energy phonons, $\epsilon < 5$ K, the 3pp angle for creating two equal phonons, increases with phonon energy (see figure 2 in [27]), so $\theta_{3\text{pp,typ}}$ increases with temperature. The energy of a sheet increases with heater power mainly through the increase in the occupied solid angle in momentum space, $\Omega$.

The data for 50 and 100 ns pulses give slightly different values of $\alpha_{pk}$; the angles are larger for the longer pulse length. In our current picture of the hot line, its characteristics are independent of pulse length, except for its dimension in the direction of propagation. So the slight dependence on pulse length suggests that a sheet requires time, of around 10 ns, to form due to the rise time of the current pulse, 5 ns, and the thermal time constant of the heater. For 50 ns this is a larger fraction of the pulse than for 100 ns.

The two sheets do not interact if the time that the two parts of the sheets overlap is less than the time for scattering between phonons in different sheets. This time of overlap is called the crossing time, $t_{\text{cross}}$, and is given in [30]

$$t_{\text{cross}} = \frac{t_p}{2 \sin^2(\alpha/2)}$$

so for $\alpha = 27^\circ$, $t_{\text{cross}} = 0.92 \mu s$. This is much more than the inverse of the 3pp scattering rate previously assumed; the scattering rate between two cones with cone angle of 12.3° and $T_p = 0.7$ K is $\nu_{3\text{pp}}^{-1} = 25$ ns [30]. However there is no direct evidence for the assumed value of the cone angle. The scattering rate reduces rapidly with cone angle and to make the crossing time equal to the scattering time, the cone angle must be reduced to $\sim 7^\circ$ at $T_p = 0.7$ K, a value found by interpolating between the graphs 1 and 2, which are for $\zeta_p = 0.011$ ($\theta_p = 8.5^\circ$) and 0.0046 ($\theta_p = 5.5^\circ$) respectively, in figure 1(b), and using the temperature data in figure 1(a), both in [30]. This suggests that the cone angle is about half that we have assumed in previous papers. Furthermore the scattering rate drops very rapidly with $\alpha$ around $\alpha = 27^\circ$ and $T_p = 1.0$ K [30], so when the crossing time is halved by halving the pulse length from 100 to 50 ns, the value of $\alpha_{\text{max}}$ is only reduced by $\sim 2^\circ$, which is what we find experimentally, see Table 1. However, for a lower sheet temperature, the scattering rate drops more slowly and the angle $\alpha$ is reduced more when the pulse length is halved. More work is needed to resolve these differences.

| $W_H$ (mW) | $t_p$ (μs) | $\alpha_{pk}^\circ$ | $\alpha_{\text{max}}^\circ$ |
|------------|------------|----------------------|-----------------------------|
| 3.125      | 50         | 8.4 ± 1.5            | 14 ± 1                      |
| 6.25       | 50         | 9.9 ± 1.3            | 18 ± 1                      |
| 12.5       | 50         | 11.2 ± 1             | 23 ± 1                      |
| 25.0       | 50         | 11.9 ± 1             | 25 ± 1                      |
Figure 6. Panel (a) shows the integral of the l-phonon bolometer signals as a function of angle $\alpha$, at large angles, between heater normals. The solid line joins points which are from the sum of bolometer signals from separate pulses from two heaters. The outline points are the bolometer signals from nearly simultaneous pulses (maximized with a small delay) to the same two heaters. Results for four powers are shown. All pulse lengths are 50 ns. Note there is little difference between the sum of the separate pulses and simultaneous pulses at these high angles. Panel (b) shows the hotline contribution to the h-phonon integral as a function of angle $\alpha$, for pulse lengths of 100 ns.

For angles $\leq \alpha_{pk}$, the sheets strongly interact but when $\alpha > \alpha_{pk}$, an increasing fraction of the sheets pass through each other, and when $\alpha \geq \alpha_{\text{max}}$ the sheets do not interact at all. In this last case the l integral for simultaneous pulses to a pair of heaters, should be exactly equal to the sum of the integrals from the two heaters measured separately. This is verified in figure 6(a) where we show the sum of the two separate contributions, measured at the same angles, as filled symbols and joined by a line, and the l integral from simultaneous pulses, as outline symbols. We see that within the measurement error, the two data sets agree. We therefore conclude that there is no interaction between the sheets at values of $\alpha$ where the l integral has stopped rapidly decreasing in figures 4(a) and 5(a). At large angles, when $\alpha \geq \alpha_{\text{max}}$, the signal slowly decreases, this is due to the bolometer response decreasing at large angles of incidence [31].

We now consider the h integrals. In contrast to the l-phonon signal which measures the instantaneous l-phonon energy at the bolometer, the h-phonon signal is due to stable high-energy phonons continuously created in the helium. The creation of h-phonons is at the expense of energy from the l-phonons.

We show in figure 4(b) the magnitudes of the h integral. These signals are due to the hot line and the h-phonons created by the sheets. To separate these two contributions we recall that sheets create h-phonons near the heater which is where their temperature is highest. So the sheets create most of their h-phonons before they collide with the other sheet, if the angle between them is not too small. So we adopt the following procedure; for $\alpha > 13.2^\circ$ we subtract from the double pulse signals, the individual-sheet h-phonon signals, and for $\alpha \leq 13.2^\circ$ we subtract the h-phonon signal in the wings of the delay scans. This leaves the h-phonon contribution from the hotline.
At low angles, the h integral for the hot line is about the same as the l integral. As the bolometer is roughly twice as sensitive to h-phonons as it is to l-phonons [7], this indicates that about a third of the energy density, propagating along the symmetry axis, which was in the l-phonon system of the hot line, has been converted to h-phonons. The creation rate of h-phonons is a very strong function of the temperature of the l-phonons, it varies as $\exp(-c_3/T)$ (the factor $c_3 = 13.18$ K is the sum of two contributions: 3.18 K from the typical energy phonon in the 4pp scattering [32], and 10 K from the integral over p which depends on the cut off momentum $p_c = 10$ K see equation (6) and (9) with (5), (8) and (10), of [31]). So the fact that the h integral peaks at a smaller angle than the l integral, indicates that the maximum temperature of the hot line occurs at a smaller angle $\alpha$ than the angle for the maximum l-phonon energy density in the hot line. This can happen if the occupied solid angle of the hot line increases with angle $\alpha$, but the temperature of the hot line decreases with $\alpha$; the increase in $\Omega$ increases the l-phonon energy and the decrease in temperature decreases the h-phonon creation rate. However the l-phonon energy also depends on temperature as $T^4$ and the h-phonon creation also depends on $\Omega$. But $\exp(-c_3/T)$ is a much stronger function of $T$ than $T^4$, so a higher $T$ increases the h integral faster than the l integral. This interpretation is supported by the detailed analysis given in section 5.

The h integral has a nearly constant value for $\alpha \geq \alpha_{\text{max}}$. The values of $\alpha_{\text{max}}$ have very similar values to those for the l integrals, for the same heater power, see figures 4(b) and 5(b). At $\alpha \geq \alpha_{\text{max}}$, the phonon sheets do not interact and hence the sum of the h integrals for the simultaneous pulses should be equal to the sum of the h integrals from separate pulses. Figure 6(b) shows this is the case.

Although the results qualitatively agree with the theory in [30], the measured values of $\alpha_{\text{max}}$ are considerably lower than predicted. This could be due to one of a number of reasons: the theory only considered the expansion of thick pulses where variations in the propagation direction are ignored. Also the theory neglected the creation of h-phonons and so underestimated the cooling rate of the phonon sheets; if the sheets cool more rapidly then the 3pp rate is reduced and interactions between the sheets will stop at smaller values of angle $\alpha$. Indeed we saw above, that if the sheets have cooled to 0.7 K by the time they interact, and the cone angle is around 7°, then the crossing time is similar to the calculated scattering time.

5. Analysis of the hot line

We now calculate the temperature $T_{hl}$ and the solid angle $\Omega_{hl}$ of the hot line from the measured l and h integrals (figures 5(a) and 6(b) respectively) and the calculated h-phonon creation rate.

The hot line has energy fed into it from the two sheets. As the sheets do not pass through each other when the interactions are strong, the rate of energy fed into the hot line, is equal to the rate of energy loss due to the decrease in area of the sheets, see equation (3). We assume that the hot line is in dynamic equilibrium, so that $\Omega_{hl} = T_{hl}$ are constant as it propagates. We also assume that the energy densities of the sheets is independent of the distance they have propagated after the start of the interaction with the other sheet. This is an approximation as a sheet loses energy by the creation of h-phonons, and so the energy given to the hot line decreases with propagation time. However, the sheets cool most rapidly near the heater and hence before they interact, so it is a reasonable approximation.

As the loss of energy by the hot line due to h-phonon creation is a function of $\Omega_{hl}$ and $T_{hl}$, we can find an equation that relates the h integrals to $\Omega_{hl}$ and $T_{hl}$. Also the l integral is another
function of $\Omega_{hl}$ and $T_{hl}$. Using the measured values of $l$ and $h$ integrals, these two equations can be solved for the two unknowns $\Omega_{hl}$ and $T_{hl}$ at different values of $\alpha \leq \alpha_{\text{max}}$ and heater power.

The time-integrated energy fluxes per unit area $\Phi_l$ and $\Phi_h$ from the hot line (the normal to this area is the propagation direction) can be written in terms of the time-integrated bolometer signals as

$$\Phi_l = \frac{I_l}{\alpha_l A_h R},$$

(6)

$$\Phi_h = \frac{I_h}{\alpha_h A_h R},$$

(7)

where $I_l$ and $I_h$ are the time integrals (volts seconds) of the $l$- and $h$-phonons from the hotline respectively. $R$ is the responsivity of the detecting system (volts per watt absorbed by the bolometer), and $\alpha_l$ and $\alpha_h$ are the fractions of the incident energy that is transmitted into the bolometer and $A_h$ is the area of the sensitive part of the bolometer.

The energy density, $E_{l,hl}$, of the $l$-phonons in the hot line is given by

$$E_{l,hl} = \frac{\Omega_{hl} \pi k^4 B T_{hl}^4}{120 \hbar^3 c^3}.$$  

(8)

To find the time-integrated energy flux of the $l$-phonons at the bolometer, we need the thickness $b$ of the hotline in the direction of propagation. It is the width of the crossing area of the two sheets, and depends on the pulse length and the angle $\alpha$: it is given by

$$b = \frac{ct_p}{\cos(\alpha/2)}.$$  

(9)

Then

$$\Phi_l = b E_{l,hl}$$

(10)

and combining equations (6), (8)–(10) we find

$$\frac{I_l}{\alpha_l A_h R} = \frac{\Omega_{hl} \pi k^4 B T_{hl}^4}{120 \hbar^3 c^3} \frac{ct_p}{\cos(\alpha/2)}.$$  

(11)

The $h$-phonons are created at a rate $dE_{h,hl}/dt$ in unit volume of the hot line, and they are created for the lifetime of the hot line, which is the time between the instants when the sheets start intersecting and when they reach the bolometer. For a sheet with side $l$ at an angle $\alpha/2$ to the symmetry axis, see figure 1, this time is

$$t_{hl} = \frac{l}{2c \tan(\alpha/2)}.$$  

(12)

Hence from equations (9) and (12) we have

$$\Phi_h = \frac{dE_{h,hl}}{dt} b t_{hl} = \frac{dE_{h,hl}}{dt} \frac{l}{2c \tan(\alpha/2) \cos(\alpha/2)} \frac{ct_p}{c}.$$  

(13)
The thickness of the hot line increases a little with time because the apex of the crossed area travels faster than \( c \), while the phonons in the hotline travel at \( c \). However, the crossing thickness \( b \) of the hotline is the relevant thickness for the creation of h-phonons because this instantaneous thickness has a higher temperature than parts of the hot line created earlier which have already cooled by creating h-phonons.

The rate of increase of the energy density increment \( \delta E_h \), for the momentum increment \( \delta p \), in a system at temperature \( T \), is given by [27],

\[
\frac{d(\delta E_h)}{dt} = \frac{\Omega_{hl}\epsilon p^2 n_h \delta p}{(2\pi\hbar)^3} v_{b1}(p, T)
\]  

(14)

where \( \epsilon \) and \( p \) are the energy and momentum of the h-phonons and \( v_{b1} \) is the creation rate of h-phonons in the quantum state with momentum \( p \) from l-phonons at temperature \( T \). The occupation number, \( n_h \), for h-phonons is

\[
n_h = \frac{1}{\exp(\epsilon/k_B T) - 1}
\]

(15)

and \( v_{b1} \) is given by [32]

\[
v_{b1} = c_1 \left( \frac{\Omega_{hl}}{\Omega_p} \right)^a \exp \left( -\frac{c_2}{T} \right) \exp \left( -\frac{(p - p_c) c}{k_B} \right)
\]

(16)

where \( c_1 = 1.22 \times 10^9 \text{ s}^{-1}, a \approx 1.5 \) is a parameter obtained from the numerical evaluation of the theory in [32], \( \Omega_p = 0.124 \text{ sr}, c_2 = 3.18 \text{ K} \), and \( p_c \) is the cut off energy for any spontaneous decay (i.e. the 4pp is dominant for \( p > p_c \)) and \( c_p k_B = 10 \text{ K} \) at zero pressure.

Substituting equations (15) and (16) into (14) and integrating over \( p \), we find

\[
\frac{dE_h}{dt} = \frac{c_4 k_B^4 \Omega_{hl}^{(1+a)} l_{p}^2}{(2\pi\hbar)^3 c^3 \Omega_p^a} \exp \left( -\frac{c_2 k_B + c p_c}{k_B T_{hl}} \right)
\]

(17)

where \( c_4 = 7.39 \times 10^{11} \text{ K}^4 \text{ m}^{-1} \text{ s}^{-2} \). Hence from equations (7), (13) and (17) for the hot line, we obtain

\[
\frac{I_{hl}}{\alpha_h A_b R} = \frac{c_4 k_B^4 \Omega_{hl}^{(1+a)} l_{p}^2}{2(2\pi\hbar)^3 c^3 \Omega_p^a \sin(\alpha/2)} \exp \left( -\frac{c_3}{T_{hl}} \right)
\]

(18)

where \( c_3 = 13.18 \text{ K} \). This equation assumes that the area of the hot line is larger than the sensitive area of the bolometer, which applies in our experiment.

Equations (11) and (18) are two simultaneous equations with two unknowns; \( \Omega_{hl} \) and \( T_{hl} \), which can be evaluated from the measured data. We use the following constants; \( A_b = 7.2 \times 10^{-9} \text{ m}^2 \), \( l = 1 \times 10^{-3} \text{ m} \), \( R = 1.3 \times 10^7 \text{ V W}^{-1} \), \( \alpha_l = 3 \times 10^{-3} \), \( \alpha_h = 6 \times 10^{-3} \). The value of \( \alpha_h \) is estimated by noting that the transmission probabilities for h-phonons by the background and peak channels are \( 5 \times 10^{-3} \) and \( 8 \times 10^{-3} \) respectively [33]. For a rough bolometer surface, the background channel will predominate, so we take the weighted average value \( 6 \times 10^{-3} \). For l-phonons the background channel probability is \( 2.3 \times 10^{-3} \) for phonons with temperature \( T = 0.8 \text{ K} \), and the peak channel probability is \( 8 \times 10^{-3} \). Again we weight towards the background channel and take the rounded value of \( 3 \times 10^{-3} \).
Figure 7. Panel (a) shows the calculated cone angle, $\theta_p$, for the l-phonons in a hot line as a function of heater power. Panel (b) shows the calculated temperature, $T_p$, of the l-phonons in a hot line as a function of heater power. Panel (c) shows the calculated value of $T^4 \Omega$ (K$^4$sr), which is proportional to the energy density in a hot line, as a function of heater power. For all plots, diamonds, triangles and squares are for $\alpha = 4.4^\circ$, $8.8^\circ$ and $13.2^\circ$ respectively; the same heater power is applied to both heaters, but the graphs are plotted against the power to one heater.

We use the 100 ns data for $\alpha < \alpha_{\text{max}}$, where there is a strong interaction. The equations (11) and (18) were solved numerically. The values of $\Omega_{hl}$ and $T_{hl}$ as a function of angle $\alpha$ and power are shown in figures 7(a) and (b). We see that $\Omega_{hl}$ increases with heater power as it does for a single sheet and also with angle $\alpha$. This is not unexpected, as a larger angle between the sheets means a smaller momentum component along the symmetry axis. This leads to a smaller forward momentum for the hot line and gives a larger cone angle for the hot line, as observed, see figure 7(a). The temperature of the hot line decreases with heater power and with angle $\alpha$, see figure 7(a). Both energy and momentum have to be conserved overall in the interaction between the sheets to create the hot line [30]. As the phonon dispersion is nearly linear, there is an approximate rule for strongly interacting phonon sheets: first $\Omega_{hl}$ is mostly set to conserve momentum and then $T_{hl}$ is mostly set to conserve energy.

The energy density of the hot line increases with heater power as shown in figure 7(c), and the energy density of the hotline is greater than the energy density of the sheets because $\Omega$ is larger in the hotline. However, the temperature of the hot line is not necessarily higher than the temperature of the sheets; this was also found theoretically, even in the absence of h-phonon creation [30]. Of course the hot line is always hotter than the liquid helium in which it is propagating.

It is a success for the theory that the predicted h-phonon creation rate can be used quantitatively to find solutions to equations without any adjustable parameters. The dependence of temperature and occupied solid angle on heater power, which are derived from the measured data, seem reasonable, however while the qualitative behaviour of $T_{hl}$ and $\Omega_{hl}$ with $\alpha$ and $W$ are not sensitive to the values of $a$, $\alpha_1$, $\alpha_0$, $l$, $A_b$ and $R$, the quantitative values are. Furthermore the theory of the creation rate does not apply at large angles. So the results shown in figure 7 should only be considered as showing the qualitative functional behaviour.
6. Conclusion

We have measured the energy density of l- and h-phonons due to a hot line formed from the interaction between two phonon sheets as a function of angle between the sheets, for a range of heater powers. During these experiments, the effect of delaying one sheet with respect to the other was investigated. Introducing a delay laterally offsets the path of propagation of the hot line, so varying the delay sweeps the hot line across the bolometer. We found a very rapid response to small changes in delay, which shows that the bolometer has only a small sensitive region.

The delay response curves are then the energy profile of the hot line with only a relatively small broadening due to the finite size of the bolometer’s sensitive spot. The response curves, as a function of delay, had a Lorentzian-like shape which was nearly independent the angle between the two phonon sheets. The half-width of the hot line in space varied as $1/\sin(\alpha/2)$, i.e. the width decreased as the angle $\alpha$ increased. The calculation of the profile of the hot line remains a task for the future.

The variation of the energy of the l-phonons in the hot line as a function of angle shows three distinct regions. At low angles, $\alpha < \alpha_{pk}$, $\alpha_{pk} = 13^\circ$ for 25 mW mm$^{-2}$ pulses, the phonon sheets strongly interact. For $\alpha_{pk} < \alpha < \alpha_{max}$, $13^\circ < \alpha < 25^\circ$ for 25 mW mm$^{-2}$ pulses, the interaction decreases with angle and for $\alpha > \alpha_{max}$, $\alpha_{max} = 25^\circ$ for 25 mW mm$^{-2}$, 50 ns pulses, there is no interaction between the sheets. At lower powers, the three regions shift to lower angles. The l-phonon integral rises approximately linearly with angle $\alpha$ in the first region. It then decreases rapidly in the second region and, in the third region, the signal is the same as the sum of the signals from separate sheets. These measurements are in qualitative agreement with theoretical predictions [30], but the values of $\alpha_{max}$ are considerably smaller than those predicted. We suggest this is due to sheets cooling more rapidly due to the creation of h-phonons; if the temperature $T_p \sim 0.7$ K and the cone angle in the sheets is around 7$^\circ$, which is about half that previously assumed, then the crossing time is the same as the inverse of the scattering rate at $\alpha \sim 25^\circ$.

From the values of the angles where the l-phonon signal peaks, we estimate the predominant angle for 3pp scattering is around 10$^\circ$ but dependent on heater power. These angles are consistent with those calculated [29].

The h-phonons similarly have three regions, although in the first region, the h-phonon energy varies much more slowly with angle $\alpha$ than the l-phonon energy. The l and h integrated energies for 100 ns pulses are double that for 50 ns. However, we do not expect this trend to continue for much longer pulses as we would then move out of short pulse behaviour, where h-phonons are lost from the sheet, or hot line, without scattering.

We have suggested that the hot line is in dynamic equilibrium with the energy transferred to the hot line, from the decreasing area of the sheets being equal to the energy lost from the hot line due to the creation of h-phonons. With this one assumption, and the theoretically derived creation rate for h-phonons, we have developed a pair of simultaneous equations in terms of $\Omega_{hl}$ and $T_{hl}$. Substituting the measured phonon energies into these equations and using other measured quantities, we have solved these equations for $\Omega_{hl}$ and $T_{hl}$. The dependencies of $\Omega_{hl}$ and $T_{hl}$ are reasonable and are obtained without any adjustable parameters. However, these results should only be considered as qualitative because of the uncertainty in the input parameters. We find that $\Omega_{hl}$ increases with both angle $\alpha$ and heater power. However remarkably, $T_{hl}$ decreases...
with both angle and heater power. The energy density of the hotline is proportional to $\Omega_{hl} T_{hl}^4$, and increases with both angle $\alpha$ and heater power.

Studying the collisions between two phonon sheets has enabled us to investigate 3pp and 4pp in more detail than has previously been possible. It has given values of the typical angles for 3pp scattering for different energy phonons, and values of the 4pp h-phonon creation rate as a function of $T$ and $\Omega$ of the l-phonons.

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