Electron heating by intense short-pulse lasers propagating through near-critical plasmas

A Debayle\textsuperscript{1,2}, F Mollica\textsuperscript{3}*, B Vauzour\textsuperscript{1}, Y Wan\textsuperscript{4}, A Flacco\textsuperscript{5}, V Malka\textsuperscript{1}, X Davoine\textsuperscript{6} and L Gremillet\textsuperscript{1}

\textsuperscript{1} CEA, DAM, DIF, F-91297 Arpajon, France
\textsuperscript{2} LRC MESO, Ecole normale supérieure de Cachan—CMLA, 61 av. du Président Wilson, F-94235 Cachan, France
\textsuperscript{3} Laboratoire d’Optique Appliquée, ENSTA-CNRS-Ecole Polytechnique, UMR 7639, F-91761 Palaiseau, France

E-mail: arnaud.debayle@cea.fr and laurent.gremillet@cea.fr

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Abstract

We investigate the electron heating induced by a relativistic-intensity laser pulse propagating through a near-critical plasma. Using particle-in-cell simulations, we show that a specific interaction regime sets in when, due to the energy depletion caused by the plasma wakefield, the laser front profile has steepened to the point of having a length scale close to the laser wavelength. Wave breaking and phase mixing have then occurred, giving rise to a relativistically hot electron population following the laser pulse. This hot electron flow is dense enough to neutralize the cold bulk electrons during their backward acceleration by the wakefield. This neutralization mechanism delays, but does not prevent the breaking of the wakefield: the resulting phase mixing converts the large kinetic energy of the backward-flowing electrons into thermal energy greatly exceeding the conventional ponderomotive scaling at laser intensities \( \gtrsim 10^{21} \text{ W cm}^{-2} \) and gas densities around 10\% of the critical density. We develop a semi-numerical model, based on the Akhiezer–Polovin equations, which correctly reproduces the particle-in-cell-predicted electron thermal energies over a broad parameter range. Given this good agreement, we propose a criterion for full laser absorption that includes field-induced ionization. Finally, we show that our predictions still hold in a two-dimensional geometry using a realistic gas profile.

1. Introduction

The capability of intense lasers to create efficient accelerating plasma structures paves the way to compact, versatile platforms for research and medical applications. Such particle sources are based upon the ability of plasmas to support electric fields orders of magnitude larger than the breakdown threshold fields of conventional accelerators. Energetic electron and ion beams can be produced by focusing a relativistic-intensity \( (I_0 > 10^{18} \text{ W cm}^{-2}) \) laser pulse onto a gas [1–5] or a solid [6–14] target. Two main regimes of laser-driven particle acceleration/heating can be identified, depending on the transparency properties of the ionized medium. For electron densities much lower than the so-called critical density (i.e., \( n_e \ll n_i \equiv m_e \varepsilon_0 \omega_0^2 / e^2 \)), where \( \omega_0 \) is the laser frequency, \( e \) the electron charge, \( m_e \) the electron mass and \( \varepsilon_0 \) the permittivity of vacuum), the laser pulse can propagate large distances \( (L > \lambda_0 \equiv 2\pi / \omega_0) \) through the plasma and generate a strong wakefield [15–23]. This high-velocity \( (v \sim c) \) plasma wave can trap part of the background electrons and, for optimized laser-plasma parameters, accelerate them to ultrarelativistic energies with possibly narrow energy spectra [24–40]. The resulting electron beam, with charge in the \( \lesssim n \mu \text{C} \) range, can also act as an efficient x-ray generator [41]. In the opposite limit of a highly overcritical plasma \( (n_e \gg n_i) \), the laser is mainly reflected and/or absorbed at the plasma surface, where electrons are stochastically heated in the MeV range, with a broad energy distribution [42–49]. The high electron current \( (\sim 0.1–50 \text{ MA}) \) that ensues can then serve as a source of ultrafast (isochoric) target heating, ion acceleration at the target rear or x-ray radiation [57, 58].
Although the mean energy \( \langle e_h \rangle \) of the hot electron distribution is a complex function of the laser-plasma parameters, it is commonly assumed to obey the ponderomotive scaling, \( \langle e_h \rangle \approx m_e c^2(\sqrt{1 + a_0^2/2} - 1) \), where \( a_0 \equiv E_0/c_0 \) is the normalized laser amplitude [59, 60].

Fewer studies have addressed the case of intense short-pulse lasers interacting with near-critical \((n_e \sim n_c)\) plasmas [61–63]. This relative lack of interest is probably due to the experimental difficulty of achieving, in a controlled way, the required high gas densities \((n_e \sim 10^{20–21} \text{ cm}^{-3})\) [64] and, also, to the fact that such systems are \textit{a priori} ill-suited to efficient wakefield electron acceleration. In recent years, however, high-density gases have attracted increasing attention as promising high-energy ion sources, based on a variety of mechanisms. First, as in solid foils, the so-called target normal sheath acceleration (TNSA) [65, 66] can arise from the space-charge field set up at the plasma boundaries by the laser-driven hot electrons [2]. Second, in a dense enough plasma, the front-side ions are pushed by the 'laser piston', i.e., the space-charge field resulting from the laser ponderomotive force on the electrons. Third, under specific conditions, the laser piston or the electron pressure gradients created in an inhomogeneous plasma can launch a collisionless electrostatic shock [67–70], accelerating a fraction of the background ions to energies possibly larger than through TNSA [60, 71–74]. Further, for tightly focused lasers, ion acceleration can proceed via the electric field induced by magnetic vortices moving down density gradients [75, 76].

The objective of this work is to illuminate the processes of electron energization during the interaction of high-intensity, ultrashort (<100 fs) laser pulses with near-critical plasmas. Developing such an understanding is a prerequisite for modeling the above ion acceleration processes that closely depend on electron pressure gradients and the level of electron heating. Such is the case of TNSA, collisionless shock acceleration (CSA) and magnetic vortex acceleration, all necessitating high mean electron energies so as to optimize the energy of the accelerated ions. Regarding particularly CSA, a relatively large plasma density is also needed to ensure a short enough shock formation time, of the order of the inverse ion plasma frequency, \( \omega_{pi}^{-1} \) [71]. Such conditions imply strong absorption of the laser through the plasma, and therefore entail a laser depletion length shorter than caused by wakefield excitation in dilute plasmas [19, 20, 77]. For the above reasons, an in-depth insight into the electron heating mechanisms at play in near-critical plasmas is desirable. In the following, through extensive 1D particle-in-cell (PIC) simulations performed with the CALDER code [78] and a reduced model, we clarify the dependence of the final mean electron energy with the laser intensity and plasma density. Most notably, we find that very high electron energization levels (i.e. well above the ponderomotive scaling) can be achieved provided (i) the laser has propagated a distance large enough that it has developed a steep rising front; (ii) breaking of the wakefield and phase mixing between the trapped electrons and return electrons have occurred. Moreover, as this interaction regime involves efficient propagation of the laser pulse through an initially neutral gas jet, we derive a simple criterion for laser propagation that includes field ionization. Finally, we show that our 1D simulation and model results are also relevant to laser pulses focused to moderate spot sizes into ionized gases.

This paper is organized as follows. In section 2.1 is presented a tutorial 1D PIC simulation serving to illustrate the dominant interaction processes leading to efficient electron heating in near-critical plasmas. In section 2.2, using a set of reduced fluid equations and several assumptions supported by PIC simulations, we estimate the final mean electron energy as a function of the laser–plasma parameters. In section 2.3, the predictions of this model are compared with a 1D-simulation scan over a broad parameter space. In section 3, we provide a criterion for full laser absorption, taking account of field-induced ionization. In section 4, 2D PIC simulations are carried out to investigate the effects of a finite laser width. For completeness, we consider the case of a laser interacting with a realistic high-density gas jet produced at the Laboratoire d’Optique Appliquée (LOA). Finally, our main results are summarized in section 5.

2. Collisionless electron heating

While multidimensional kinetic simulations are essential to reproduce experimental conditions, a 1D description is most convenient to highlight the main physical processes and to extract some tendencies and scalings. In this section, a typical 1D PIC simulation of a laser pulse interacting with a near-critical gas is presented to reveal the basic electron heating mechanisms. Ion acceleration—via TNSA or an electrostatic shock wave—is also shown to be a promising application of such interaction regimes. We then propose a simple model that reproduces the salient features of electron energization. Finally, extensive 1D simulations are performed to support the model results and, in particular, the scaling of the electron energy with laser intensity and plasma density.

2.1. Basic mechanisms

Detailed simulations of laser-plasma interaction at near-critical densities and short pulse durations are quite scarce in the literature [4, 19, 62, 63, 79]. For this reason, and because of the nontrivial interplay of the various
underlying mechanisms, we deem it worthwhile to devote the present section to an illustrative 1D PIC simulation. The latter describes the interaction of a laser pulse, of maximum normalized amplitude $a_0 = 19.6$ (corresponding to a peak intensity $I_0 = 8.3 \times 10^{20}$ W cm$^{-2}$ at a wavelength $\lambda_0 = 0.8$ μm) and FWHM duration $\tau_0 = 70\omega_0^{-1}$ (30 fs), with a fully ionized hydrogen plasma. The laser pulse, linearly polarized along the $y$ axis, is injected through the left-hand side ($x = 0$) of the simulation domain. The initial proton density profile (blue solid line in figure 1(a)) consists of a 25πc/ω0-long (10 μm) plateau of density $n_i = n_e = 0.64n_0$ ($1 \times 10^{21}$ cm$^{-3}$), surrounded by two exponential ramps of scale lengths of 25πc/ω0 and 50πc/ω0 at the front and rear sides, respectively. The initial electron and ion temperatures are $T_i = T_e = 10$ eV. The space and time resolutions are $\omega_0\Delta t = 0.0625$ and $\omega_0\Delta x/e = 0.09375$, and each cell initially contains 100 electrons and ions. Absorbing boundary conditions are used for both fields and particles, while Coulomb collisions are neglected.

Figure 1(b) illustrates the electron $x - p_x$ phase space during the laser propagation through the gas at time $\omega_0 t = 1250$. The laser is then located in the density down-ramp (1000 $\lesssim \omega_0 x/e \lesssim 1100$). The ponderomotive force of the laser has caused a strongly nonlinear response of the plasma electrons behind the laser head. Two main electron structures can be distinguished. The first one corresponds to those background cold electrons suddenly pushed forward by the laser pulse (forming the pile-up structure visible at $\omega_0 x/e \approx 1100$, which extends up to $p_x/m_e c \approx 50$) and pulled back by the background ions at ultra-relativistic negative velocities ($p_x/m_e c \approx -50$ around $\omega_0 x/e \approx 1000$). This cold-fluid structure is the signature of the nonlinear electrostatic field induced by the laser pulse. The associated normalized electrostatic potential ($-e\phi_0/m_e c^2$) is overlaid as a white solid curve in figure 1(b). In the absence of wave breaking, and assuming stationary propagation in the laser’s coordinate system ($x - v_g t$, where $v_g$ is the laser group velocity), this wakefield would be described by the well-known Akhiezer–Polovin (AP) equations [80, 81], and hence would make the plasma electrons oscillate around $p_x = 0$ [16, 19, 20]. In the present case, however, wave breaking and phase mixing have occurred, giving rise to a broad distribution ($-70 \lesssim p_x/m_e c \lesssim 150$) of energetic electrons behind the laser head. This hot electron population is dense enough to neutralize the cold-fluid electron flow during its backward acceleration by the electrostatic field. This process, referred to as beam loading in wakefield accelerators [82], is well illustrated by the electrostatic potential, which, after abruptly dropping behind the laser front ($-e\phi_0/m_e c^2 \sim -150$ at $\omega_0 x/e \sim 1000$), exhibits limited variations ($e\phi_0/m_e c^2 \sim 20$) when the cold and hot electron flows meet. As a result of these variations, the cold electrons escape the laser-plasma interaction region with a very high residual longitudinal momentum ($p_x/m_e c \sim -70$).

Further behind the laser beam, the nonlinear wakefield initially sustained by the cold electron flow, finds itself in the presence of the counterstreaming cold and hot electron beams. The hot electron flow initially acts to neutralize the wakefield, hence preventing its rapid breaking (i.e., after just one oscillation). The occurrence of wavebreaking, however, is only delayed for a few plasma periods, probably as a consequence of the generic unstable behavior of relativistic plasma waves [81, 83]. The perturbations responsible for this wavebreaking result from the hot electron stream but, also, from the non-stationary character of the wave itself (which, in contrast to a pure AP solution, does not propagate at a uniform phase velocity). This phenomenon is further discussed in section 2.2 B.450. Through phase mixing, the drift energy of the cold-fluid electrons ends up being...
converting into heat, with a mean energy η(γ − 1), blue solid line in figure 1(b) in the ultra-relativistic range, η(γ − 1) ∼ 40.

As a consequence of the electron heating, the ions are set into motion through TNSA in the density down-ramp, as illustrated in figure 2 where the ion x − π phase space is plotted at successive times. At times ω0t = 1450 and ω0t = 1450, the ion phase space takes the form of a self-similar profile (i.e., roughly scaling as x/t) interrupted by a velocity spike, increasing in time as vi/c ≈ m(ec)(γ − 1)t/Lp, (Lp is the initial electron density scale length). This structure is reminiscent of the wave steepening occurring in an expanding non-uniform plasma [84]. Further away, a secondary self-similar expansion takes place due to charge separation (here the local Debye length exceeds the ion density scale length) and the rarefaction wave coming from the front of the initially truncated ion profile (at ω0X/c = 1400). At ω0t ∼ 5000, the velocity spike has evolved into a collisionless shock [68, 69]. The associated moving potential barrier reflects part of the TNSA-expanding upstream ions, thereby boosting their non-relativistic velocity by a factor of ∼2 [60]. Since the shock velocity roughly scales as the sound speed of the upstream plasma, the final ion velocity is proportional to (Z2mec(γ − 1)/m1)1/2. Note, however, that in a multidimensional geometry and for a small laser spot size, the formation of a magnetic dipole vortex in the plasma down-ramp [75, 85] may change the scaling of the accelerated ions, as observed in previous simulation studies [76, 86].

2.2. 1D model for the bulk electron dynamics

A self-consistent analytic modeling of the collisionless electron heating, which comprehends laser propagation effects, is a challenging task, well beyond the objectives of this study. However, some insight can be gained from a simple model supported by numerical simulations.

As noted above, the bulk electron heating proceeds through a sequence of wavebreaking, two-stream and phase mixing processes. These phenomena are illustrated by a 1D simulation similar to that discussed in the previous section, except that the plasma profile is now chosen to be uniform (n1 = n∞ = 0.032n1). Figure 3 presents the electron x − π phase space at different times. Early in time (ω0t = 937.5), the nonlinear wakefield induced by the laser pulse is already strong enough to have trapped a few electron bunches, and triggered phase mixing near the vacuum/plasma interface (ω0X/c ∼ 300–500). Despite these growing kinetic processes, the first nonlinear oscillations of the bulk electron flow roughly obey the cold-fluid equations of [16, 19, 20]. This remains true at ω0t = 1250, even though phase mixing is now well developed after a few wakefield oscillations.

Since the laser pulse is longer than the local plasma wavelength (τp = 70c/ω0 > 30c/ω0), the wakefield is generated in the laser beam head [19]. The sustained energy transfer from the laser to the wakefield depletes the laser beam head, so that the laser front profile becomes increasingly steep during its propagation [19, 23, 87]. This self-steepening effect is clearly seen in the insets of figures 3(a)−(c), where the normalized transverse electric field (aγ = eEγ/mecω0) is plotted at the same times as the electron phase spaces. The initially Gaussian laser beam progressively develops a short gradient at its front, down to roughly one laser wavelength (inset of figure 3(c)). This ‘optical shock’ increases the longitudinal ponderomotive force ∝∂x√1 + (π/mec)2 + aγ2,
and so strengthens the forward push experienced by the background electrons, which reach \( n_x \approx \frac{40}{40} \) at \( \omega b/c \approx 1800 \) (see figure 3(c)). These electrons are subsequently drawn back by the ions with a negative drift momentum of similar amplitude \( (- \frac{40}{40}) \). Due to the space charge associated with the injected hot electrons, these cold-fluid electrons are left with a highly negative drift momentum, as illustrated at times \( \omega b t = 1875 \) (figure 3(c)) and \( \omega b t = 3125 \) (figure 3(d)). Both electron flows then support the longitudinal wake field. When the counterstreaming cold and hot flows are fully developed, the former escape the interaction region with a very high momentum, \( -\frac{100}{100} \) at \( \omega b t = 3125 \) (figure 3(d)). In this regime, the energy loss rate of the laser pulse largely exceeds the depletion rate derived in the ‘classical’ wake field regime [19, 20, 77].

Both the mean drift momentum/energy of the cold electron flow and the mean thermal energy of the hot electrons are correlated with the electrostatic potential gap set up behind the laser beam head. The latter can be estimated by a simple extension of the well-known quasi-stationary wake field model [19]. In our model, we assume that (i) the laser pulse propagates at the speed of light; (ii) the laser front has already been depleted, so that its sharp intensity gradient extends over \( 1 \lambda_0 \); (iii) The cold-fluid electrons are acted upon by both the laser ponderomotive force and the hot electron flow resulting from trapping/wavebreaking and moving at the velocity of light behind the laser front. From now on, to simplify notations, space and time are normalized by \( c/\omega_0 \) and \( 1/\omega_0 \), electric fields by \( m_e c \omega_0 / e \), magnetic fields by \( m_e \omega_0 / \omega e \), electrostatic potentials by \( m_e c^2 / \omega e \), velocities by \( c \), momenta by \( m_e c \), energies by \( m_e^2 c^2 \) and densities by \( n_e \). Introducing \( \xi = x - t \) and \( \eta_0 = \pi (\tau_\xi + 1) \), the laser field profile is taken to be of the following form.

![Figure 3](https://example.com/fig3.png)

Figure 3. Electron \( x - p_x \) phase space (in log scale) at times \( t = 937.5 \) (a) \( \omega b t = 1250 \) (b) \( \omega b t = 1875 \) (c) and \( \omega b t = 3125 \) (d). The white solid line plots the normalized electrostatic potential, \( -e\varphi / m_e c^2 \). In each figure, the inset plots the corresponding normalized transverse electric field, \( \varphi = eE_y / m_e c \). The plasma occupies the \( x > 300 \) region and has a uniform density profile at \( n_e = n_i = 0.03 \). A video of the \( x - p_x \) phase space in the supplementary material is available online at stacks.iop.org/NJP/19/123013/mmedia.
\[ a_y = \begin{cases} 
\frac{a_0}{\gamma} \sin \left( \frac{\xi}{2} \right)^2 \sin (\xi) & \text{if } -\pi < \xi < 0, \\
\frac{a_0}{\gamma} \sin \left( \frac{\xi + \eta_0}{2\gamma} \right)^2 \sin (\xi) & \text{if } -\eta_0 < \xi < -\pi, \\
0 & \text{else.}
\end{cases} \] (1)

The dynamics of the cold-fluid electrons is governed by the equations of mass and momentum conservation, complemented by Poisson’s equation:

\[ \frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x} \left( \frac{n_e p_x}{\gamma} \right) = 0, \] (2)

\[ \frac{\partial p_x}{\partial t} + \frac{p_x}{\gamma} \frac{\partial p_x}{\partial x} = \frac{\partial \phi}{\partial x} - \frac{a_y}{\gamma} \frac{\partial n_e}{\partial x}, \] (3)

\[ \frac{\partial^2 \phi}{\partial x^2} = n_e + \frac{n_0}{2} H(-x + t + \eta) - n_0. \] (4)

Here, the Lorentz factor writes \( \gamma = \sqrt{1 + \frac{p_x^2}{\gamma^2} + a_y^2} \) due to conservation of the transverse canonical momentum (valid in a 1D geometry). The Heaviside term \( \times H(-x + t + \eta) \) in the right-hand side of equation (4) represents the hot electron flow. The latter is supposed to be a rigid beam sited behind the laser front, at the position, \( \eta \), where the electric field is neutralized: \( \partial \phi(t, x - t = \eta) = 0 \). The (uniform) hot electron beam density is chosen to be equal to half the initial plasma density \( (n_0/2) \) owing to conservation of the total electron density, as justified below. Looking for quasi-stationary solutions depending on \( \xi = x - t \) only, equations (2)–(4) can be recast as

\[ \frac{\partial}{\partial \xi} [n_e(v_e - 1)] = 0, \] (5)

\[ \left( \frac{p_x}{\gamma} - 1 \right) \frac{\partial p_x}{\partial \xi} = \frac{\partial \phi}{\partial \xi} - \frac{a_y}{\gamma} \frac{\partial n_e}{\partial \xi}, \] (6)

\[ \frac{\partial^2 \phi}{\partial \xi^2} = n_e + \frac{n_0}{2} H(\eta - \xi) - n_0. \] (7)

In front of the laser field \( (\xi > 0) \) the initial conditions are \( p_x(0) = 0, n_e(0) = n_0 \) and \( \partial \phi(0) = 0 \). The equation of mass conservation can be readily integrated as \( n_e = n_0/(1 - v_e) \), so that \( n_e \to n_0/2 \) when \( v_e \to -1 \), as fulfilled in the ultra-relativistic scenario under consideration. Within the assumption of full charge neutralization between the two electron flows (at \( \xi = \eta \)), this justifies the choice of a hot electron density equal to \( n_0/2 \). The well-known longitudinal momentum conservation equation can be deduced from equation (6):
Hence, in the limit $\phi^2 \ll a_0^2$, the longitudinal momentum oscillates at $2\omega_0$ as is visible in figures 3(a), (b). In the limit $\phi^2 \gg a_0^2$, corresponding to the ‘optical shock’ regime, the longitudinal electron momentum simplifies to $p_x \sim -\frac{\phi}{2}$, as observed in figures 3(c), (d).

Figure 5 presents two sets of numerical solutions for the system (5)–(7), for $(a_0, n_0) = (38.4, 1.28)$ (figure 5(a)) and $(a_0, n_0) = (4.8, 0.1600)$ (figure 5(b)). The longitudinal cold-fluid electron momentum, $p_{x0}$, and the electrostatic potential, $-\phi$, are plotted as dashed white and red lines, respectively. Because of the strong longitudinal ponderomotive force ($-a_0/2\pi$) exerted by the steep laser front, the bulk electrons pile up until the electrostatic field overcomes the ponderomotive force and pulls them back at a longitudinal velocity $v_f \sim -1$, so that their density is approximately halved. At the neutralization point ($\xi = \eta$) between the cold and hot electron flows, the cold-fluid electron momentum scales with the local electrostatic potential as $p_x \sim -\Delta \phi/2$, where $\Delta \phi = \phi (\xi = \eta)$ is obtained by numerically solving equations (5)–(7).

In figures 5(a), (b), our model predictions, $p_x$ (red dashed line) and $-\phi$ (white dashed line), are overlaid on the electron $x - p_x$ phase spaces extracted from the corresponding 1D PIC simulations. The simulated electrostatic potential is also plotted as a white solid line. Qualitative agreement is found between the early shapes of the theoretical and PIC curves, i.e., prior to the occurrence of phase mixing effects. The differences between the model predictions and the PIC curves can be ascribed to the non-stationary character of the cold electron response.

The oscillations in the cold electron flow, discernible in the phase spaces of figures 3(c), (d) and figures 5(a), (b), can be attributed to the non-stationarity of the laser field in the coordinate system $\xi = x - t$. More precisely, owing to the strong discontinuity of the electron bulk density—increasing from $n_0$ to $n_0/(1 - \psi_0)$ over a laser wavelength—at the laser front, the latter propagates at a velocity $v_f$ significantly different from the laser phase velocity $v_f$. This difference induces oscillations in the laser ponderomotive force, which causes the bulk electrons to leave the interaction region with an initially oscillating negative momentum, of period $\sim 2\pi/(v_f - v_\phi)$. The wakefield can thus be viewed as an AP wave perturbed by large oscillations in velocity and density, further enhanced by the collective interaction with the counterstreaming hot electrons. Such a modulated AP wave is known to be prone to fast wave breaking, at amplitudes well below the limiting amplitude of a pure AP wave [81, 83]. Under the present conditions, we find that phase mixing occurs after several tens of relativistic plasma periods $\sqrt{m_0/\gamma_f}$. We point out that the sole two-stream instability between the cold and hot electron flows (both supposed of uniform density and velocity profiles) cannot account for the fast phase mixing observed in the simulations. The maximum two-stream growth rate ($\Gamma_{\text{max}}$) can be estimated assuming a simple waterbag distribution function, $f_{s}(p_x) = \frac{2H}{\sqrt{2\pi}} (p_x - \gamma_f p_0)$, for the hot electrons, and a monoenergetic distribution, $f_{c}(p_x) = \frac{n_0}{\sqrt{2\pi}} \delta(p_x + \gamma_f p_0)$, for the cold electrons. In the limit $p_x \sim \gamma_f \gg 1$ and $\Delta p \ll p_f$, the electrostatic dispersion relation of the system can be approximately solved [88, 89]. One finds $\Gamma_{\text{max}} \approx 2^{-1/4} \sqrt{3m_0 \gamma_f^2} \gamma_h^{-1/2}$, where $\gamma_h^2 = (2/M_0) \int_{-\infty}^{\infty} dp_x f_x / \gamma - 1 \approx p_f^{-2} (1 - \Delta p^2/p_f^2)^{-1/2}$. For the simulation parameters corresponding to figure 3, we obtain $\gamma_{\text{max}} \sim 10^3$, a value well above the observed phase-mixing time ($\sim 10^3$). A tentative description of the observed phase mixing in terms of the two-stream instability should therefore include the nonlinear wakefield that modulates both electron flows. A similar calculation, yet
restricted to the nonrelativistic limit, has demonstrated that $\Gamma_{\text{max}}$ strongly increases with the amplitude of the preexisting electrostatic wave \[90, 91\]. The extension of this problem to the relativistic regime is left for a future work.

Neglecting the ion plasma expansion, the conversion from electron kinetic to thermal energy can be assumed to be isochoric. Therefore, from energy conservation, the final bulk electron energy should approximately verify $g_\epsilon \approx n_0 (\langle \gamma \rangle_{\text{h}} / 2 - \delta\phi)$, where $\langle \gamma \rangle_{\text{h}}$ is the mean energy of the hot electrons (we assume $\langle \gamma \rangle_{\text{h}} | p_0 | \gg 1$). As the electron energies are significantly higher than the potential variations $\delta\phi$ in the phase-mixing region, and the bulk electron energy is expected to converge to the hot electron energy well behind the laser pulse ($\langle \gamma \rangle \approx \langle \gamma \rangle_{\text{h}}$), one obtains for the final bulk electron energy:

$$\langle \gamma \rangle \approx \frac{\Delta\phi}{2}.$$  

The mean electron energy predicted by equation (9) is plotted in figure 6(a) over the parameter space $5 \leq a_0 \leq 100$ and $0.1 \leq n_0 \leq 2$. Some numerical studies \[60, 92\] suggest that the mean electron energy follows a ponderomotive-like scaling (i.e., $\langle \gamma - 1 \rangle \propto a_0$) in the case of a laser pulse interacting with a marginally overcritical plasma. To compare our results with the ponderomotive scaling, we display in figure 6(b) the ratio $\Delta\phi / 2 (\sqrt{1 + a_0^2} / 2 - 1)$. According to our calculations, the mean bulk electron energy could reach the 100 MeV range (i.e., $\langle \gamma \rangle > 200$, several times in excess of the ponderomotive energy) at near-critical densities $n_0 > 0.2$ and laser amplitudes $a_0 > 50$.

### 2.3. 1D PIC simulations: electron energy scaling

To test the accuracy of the above scaling for the electron heating, we performed an extensive PIC simulation scan over the parameter space of figures 6(a), (b). In all (3600) 1D simulations, the ions were kept fixed to avoid electron cooling from plasma expansion. The laser temporal profile was taken to be a 10$\lambda_0$, long plateau surrounded by two 1$\lambda_0$, long ramps so as to mimic optical shock formation. The plasma length was adjusted as a function of the (uniform) plasma density in order to allow the laser pulse to propagate through the whole gas length. Each cell, of fixed size $\Delta x = 0.075$, was filled by 100 electrons and 100 protons in the plasma region. Absorbing boundary conditions were applied to both particles and fields.

**Figure 6.** Electron mean energy, $\langle \gamma - 1 \rangle$, as a function of the normalized laser amplitude, $a_0$, and electron density, $n_0$: (a) model predictions based on equations (5)–(9), (c) 1D PIC simulation results. In panels (b) and (d), the electron mean energy is normalized to the ponderomotive potential $\sqrt{1 + a_0^2} / 2 - 1$. 

(8)
To reduce the statistical fluctuation level, the mean electron energy, $\langle \gamma - 1 \rangle$, was computed over the whole simulated gas length, at the time when the laser pulse reached the end of the plasma. Figures 6(c), (d) display the resulting values of $\langle \gamma - 1 \rangle$ and $\langle \gamma - 1 \rangle / (\sqrt{1 + n_0^2}/2 - 1)$ in the same parameter space ($a_0$, $n_0$) as in figures 6(a), (b). The PIC results are observed to corroborate the model predictions. The quantitative agreement between figures 6(a)–(d) as well as the scalings that these maps evidence constitute the main results of this paper.

The electron heating process under consideration relies on enhanced phase mixing in the plasma wakefield as a result of optical shock formation and beam loading. The latter phenomena should then occur fast enough that the electron heating takes place over most of the gas length and the laser pulse is efficiently absorbed. The minimum gas length (or areal density) for complete laser absorption can be determined from the above scalings as done in the next section.

### 3. A criterion for complete laser absorption

Given the final electron energy behind the laser beam, one can infer the minimum plasma areal density, $\sigma_{\text{abs}} = Z^* n_a L_{\text{abs}}$ (where $n_a$ is the atomic density, $Z^*$ is the final ionization degree and $L_{\text{abs}}$ is the absorption gas length), that yields complete laser absorption. In principle, the ionization dynamics of the gas should be included in the wakefield model of section 2. Since, for simplicity, we have assumed a plasma of fixed ionization degree, a knowledge of the dependence of $Z^*$ with the laser-plasma parameters is required. Here we propose a simple model that predicts, for a few-cycle laser pulse, the final ionization state as a function of the laser intensity $I_z$, the atomic number $Z$ and the atomic density $n_a$.

The coupled dynamics of the variously charged ion species is ruled by the following set of continuity equations:

$$\frac{\partial n_j}{\partial \xi} = \nu_j n_j^{j-1} - \nu_{j+1} n_j^j \quad 0 \leq j \leq Z,$$

where $n_j^j$ is the density of the $j$th charged ions and $\nu_j$ is the ionization rate by an oscillating laser field [93–95], given by (in atomic units):

$$\nu_j(E) = \frac{2^{18/5}(j\pi)^{2/5}}{n_j^j (2n_j^j)^{1/5}} \frac{1}{E^{2n_j^j - 1}} e^{-22j/5},$$

where $n_j^j = j/\sqrt{2U_j^j}$ is the effective principal quantum number, $l_j$ the orbital quantum number, $E$ the electric field and $U_j^j$ the ionization energy of the $j$th electron [96]. By convention, we define $\nu_0 = \nu_{Z+1} = 0$. The initial conditions of equation (10) are $n_j^j (\xi = 0) = n_a$ and $n_j^{j=0} (\xi = 0) = 0$.

Following [97], the final ionization state can be deduced as follows. Starting from a weak laser intensity, the first atomic shell can be considered to be fully ionized over one laser cycle if the ionization rate is comparable with the laser frequency, i.e., $\nu_1 \approx 1$. Hence, solving $\nu_1(E) = 1$ yields the threshold laser amplitude $E_t$ required to ionize the first atomic shell. Similarly, the solution of $\nu_1(E) = 1$ gives the minimum laser amplitude $E_f$ leading to the ionization state $Z^*(E_f) = j$. One can define the corresponding threshold laser intensity $I_t = \varepsilon_0 c |E_t|^2 / 2$ (in physical units), which is plotted in figure 7 as a function of $Z^*$ for various gases. Note that the ionization state reached after $N$ laser cycles can also be estimated by solving $\nu_j(E) = 1/N$. Yet, the threshold intensity, varying as $\log(N)$, is weakly sensitive to this correction.

From the knowledge of the electron density and mean energy following the laser interaction and phase mixing in the plasma wakefield, we can infer the minimum electron areal density, $\sigma_{\text{abs}}$, that yields full absorption of the laser pulse. The ion kinetic and electrostatic field energies being negligible in this regime, the laser energy is essentially converted into electron internal energy. The minimum areal density required for complete laser absorption can therefore be approximated as

$$\sigma_{\text{abs}} \approx \frac{a_0^2 \tau_l}{2 \langle \gamma \rangle},$$

where $\tau_l$ is the laser pulse duration. Substituting $\langle \gamma \rangle$ from equation (9), one can calculate $\sigma_{\text{abs}}$ in terms of $a_0$ and $n_0 \equiv Z^* n_a$, assuming a constant laser amplitude during 10 laser cycles (i.e., $\tau_l = 20\pi$ in normalized units). Figure 6(a) displays the variations of $\sigma_{\text{abs}}$ in the parameter range of figures 6(a)–(d). Owing to the strong electron heating occurring at low plasma densities and high laser intensities, $\sigma_{\text{abs}}$ decreases with decreasing plasma density. This somewhat counterintuitive result holds insofar as the lengths required for optical shock formation and phase mixing are much smaller than the gas length. Obviously, the minimum gas length for full laser absorption is always a decreasing function of the electron density, as illustrated in figure 8(b).
4. 2D simulations with realistic gas profiles

Multidimensional processes stemming from the finite size of the laser focal spot are likely to affect the electron energization mechanisms previously discussed. In the range of laser intensities under consideration, and for a focal spot larger than the relativistic plasma skin depth, the pulse may break up into small-scale filaments [98, 99]. In the small focal spot limit, the plasma electrons can be transversely expelled by the laser ponderomotive force, leading to ion-filled cavities in the wake of the beam [21, 23].

To explore these effects, we performed 2D Cartesian PIC simulations using a typical gas profile obtained at LOA and presented in figure 9(a). In figure 9(b), the electron areal densities for fully ionized Helium ($Z^* = 2$, blue solid line) and partially ionized Argon ($Z^* = 16$, black solid line) gases are plotted against the distance ($y$) from the nozzle. Note that a fully ionized Ar gas would require laser intensities much higher than those considered in this study.

In our 2D simulations, the laser pulse has a Gaussian temporal and spatial profile, with a normalized field strength $a_0 = 19.6$ and a pulse duration $\tau_p = 20\pi$ (33 fs). Under such conditions, the expected electron areal densities crossed by the laser are plotted in the same panel, assuming a constant atomic density $n_0 = 0.05 \pm 0.03$. According to our 1D scaling, the laser should cross the He gas but should be stopped in the Ar gas.

Two 2D simulations were first performed for an initially neutral He gas irradiated by a laser pulse with $\lambda_0 = 1 \mu$m and two different focal spot values: $w_L = 48\pi (24 \lambda_0)$ and $w_L = 12\pi (6 \lambda_0)$. Field and collisional ionization were both included [100, 101]. For the sake of simplicity, the gas profile was assumed uniform in the transverse direction, with a longitudinal Gaussian profile represented by the red dashed line in the inset of figure 9(a).

In both simulations, the optical shock forms and causes strong backward acceleration of the bulk electrons, as illustrated by the $x = P_x$ phase spaces of figures 10(a), (c). For $w_L = 24\lambda_0$, the bulk electrons are accelerated to $P_x \approx -70$ (figure 10(a)), consistent with the model prediction $P_x \approx 60$ for a constant electron density.
0.160 (corresponding to $n_e / n_0$). For $w_L = 6 \lambda_0$, the negative electron drift momentum is reduced to $p_x \sim -40$. This decrease probably originates from the transverse breakup of the laser into three sub-beams. The ponderomotive expulsion of the plasma electrons that results from the filamentary laser field is clearly seen in figure 10(d). Interestingly, such a laser breakup is not observed in the weaker focusing conditions of figures 10(a), (b).

Figures 10(b), (d) show the development of electron cavitation, which tends to modify the scalings (of the electrostatic potential and of the electron energy gain) derived above in a 1D geometry. By contrast, in a 2D/3D geometry and in case of laser filamentation or cavitation, the electrons are transversely ejected while being pushed forward by the ponderomotive force and subsequently drawn back by the ions [23]. As a result, they

$n_0 \sim 0.1 \sim 0.16$ (corresponding to $n_e / n_0 \sim 0.05 \sim 0.08$). For $w_L = 6 \lambda_0$, the negative electron drift momentum is reduced to $p_x \sim -40$. This decrease probably originates from the transverse breakup of the laser into three sub-beams. The ponderomotive expulsion of the plasma electrons that results from the filamentary laser field is clearly seen in figure 10(d). Interestingly, such a laser breakup is not observed in the weaker focusing conditions of figures 10(a), (b).

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reach weaker longitudinal momenta than predicted in the 1D framework of section 2. Note that their backward motion may be affected by the magnetic field induced by the forward-moving hot electrons, which is predicted to scale as $B \propto n_0^{-1} L_0^{-2}$ [76]. While this field may help confine the hot electrons in the laser wake, we do not expect that the final mean electron energy will be significantly modified compared to the 1D picture. Indeed, our simulations indicate that the scalings of $\gamma$ displayed in figures 6(a)–(d) remain qualitatively valid for laser spot sizes in the range $6 \leq w_L \leq 24 \lambda_0$. This is illustrated in figures 11(a), (b) which display the mean electron energy maps at time $t = 3750$ (after the laser has escaped the gas) for (a) $w_L = 24 \lambda_0$ and (b) $w_L = 6 \lambda_0$. In both cases, the mean electron energy in the channel produced by the laser field reaches values consistent with the theoretical prediction of $\gamma \sim 60$. For this set of parameters, $(n_0 \sim 0.1, a_0 = 19.6)$, the mean electron energy is 4 times higher than the ponderomotive scaling.

One should note that the relativistic nonlinearities responsible for laser focusing and filamentation effects in our 2D simulations are probably underestimated in our 2D simulations compared to a realistic 3D configuration [102, 103]. The 3D radial focusing or filamentation, depending on the initial beam waist, can substantially increase the laser intensity, and hence modify the nonlinear plasma wakefield. A more refined model for the electron energization should also take account of the 3D laser beam deformation and the quasistatic fields induced by partial electron depletion in the laser channel. Working out such a sophisticated model is well beyond the scope of the present paper.

Finally, to assess the robustness of our criterion for full laser absorption, we also conducted a simulation in an Ar gas with a small laser spot size, $w_L = 12 \pi (6 \lambda_0)$. The other parameters are identical to those described above. Figure 12(a) shows that the laser-driven electron density perturbations extend only over the first half of the plasma length. This means that the laser has been fully absorbed, as predicted from figure 9(b). By contrast, and, as expected, figure 12(b), which is recorded at time $t = 6750$ (i.e., much later than figure 10(d)), evidences partial laser transmission through the He gas. Note that the thin electron density shell that extends up to $x \approx 3000$ is the signature of an electrostatic shock accompanied by a magnetic dipole vortex [76].
The above condition for efficient laser absorption may be used in the future to determine the occurrence and/or efficiency of the ion acceleration mechanisms (TNSA or CSA) resulting from the electron energization.

5. Conclusions and perspectives

We have revealed an efficient, as yet overlooked, electron heating mechanism induced during the propagation of an ultra-intense laser pulse through a near-critical plasma. This mechanism arises once the laser pulse has developed a steep (\(~\lambda_0\) front profile, and results from the interaction between the bulk electrons and a population of hot electrons originating from the breaking of the laser-driven wakefield. The former experience a strong backward acceleration before being thermalized. Mean thermal energies several times higher than the ponderomotive scaling can be reached—of the order of \(~100\ MeV\) and beyond—at laser field strengths \(a_0 \sim 50–100\) and plasma densities \(~0.1–0.2n_e\). We have worked out a simple model, based on modified AP equations, which predicts the final mean electron energy in the \((a_0, n_e/n_i)\) parameter space, in good agreement with extensive PIC simulations. From these results, we have derived a practical criterion for efficient laser absorption through an initially neutral gas jet, taking account of field-induced ionization. Our study is of particular interest for applications necessitating strong electron energization. Such is the case of laser-induced absorption through an initially neutral gas jet, taking account of electrostatic shocks, which are believed to form when driving strong electron pressures within inhomogeneous plasma profiles.

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ORCID iDs

F Mollica @ https://orcid.org/0000-0003-2953-237X
A Flacco @ https://orcid.org/0000-0003-0067-4155

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