A Decision-making Approach for Choosing a Reliable Product under the Hesitant Fuzzy Environment via a Novel Distance Measure

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In the contemporary time, the reliability of any product has become a big issue from the customer’s perspective due to exponentially mushrooming markets of electronics and digital gadgets. Since the use of digital equipment is tremendously increasing, as a consequence, the production and availability of products are also increasing rampantly. Due to the flooding of digital products, customers often end up in a dilemma regarding the abundant choice and subsequently, become very much dependent upon the reviews of experts and fellow customers as well. In many cases, unfortunately, it is encountered that the products are not reliable enough as suggested by the reviewers. Besides, it is often seen that the manufacturing companies provide almost similar types of features and facilities for products and customers usually end up in a dilemma. The confusion gets triggered when varieties of commodities are manufactured and supplied by different manufacturers bearing almost the same features nearly at the same price. In such situations, the reviews of experts and customers already using the product become essential. The reliability of a product relies upon the reviews of the previous customers of the same product. In this article, fuzzy multi-criteria decision-making methodology has been employed to find the reliability of a product considering different features of the product based on the reviews of customers and experts. This paper presents a neo distance measure on hesitant fuzzy set which is found on the notion of score function and mean deviation. Explanatory instances are provided to reveal the distinctiveness and merit of our proposed idea on distance measure over existing distance measures. After that, the proposed distance measure is applied in the decision-making approach for taking up the best electronic products. It is evidenced that the proposed distance measure is beneficial to measure distance degree between two unequal Hesitant Fuzzy Elements (HFEs) without putting extra elements in the shorter HFE. The proposed distance measures can be utilized in the decision-making field in the near future under diverse conditions to display undetermined particulars in a much-clarified manner.
In this era of technology, anyone who desires to purchase a new product, surfs the internet to get information and reviews of the product. In such situations, to handle uncertainty, the fuzzy set (Zadeh, 1965) plays a significant role. Bellman and Zadeh (1970) first initiated the concept of solving the decision-making problem using fuzzy set followed by Yagger (1978), Zimmermann (1985). Hereafter, Atanassov (1986) formulated the concept of an intuitionistic fuzzy set as an extension of the fuzzy set. To choose a product, the fuzzy set was applied in marketing strategies by Tang et al. (1999), Kaymak (2001), Sari and Kahraman (2012), Guo et al. (2012), Aghdaie et al. (2013), Lin and Yang (2015), Relich and Pawlewski (2015), Suresh (2015), Klos (2015), Bingham (2015) and Mohagheghi et al. (2016). Multi-criteria decision-making (MCDM) analysis was investigated by different researchers by using TOPSIS method (Hwang & Yoon, 2012; Yoon & Hwang, 1995), fuzzy preference information (Fan et al., 2002), ideal and ant-ideal solutions (Liang, 2006), distance measure (Garg & Kaur, 2018a; Rani & Garg, 2017), aggregation operators (Garg & Kaur, 2018b; Xia et al., 2013) and so on. It has been observed that in practical life, it is not an easy task to define one membership grade for one element. In some situations, different experts may assign different membership grades according to their level of hesitance, and the classical definition of a fuzzy set fails to describe such situations. In order to tackle such a situation, the concept of the fuzzy set was proposed by Torra and Narukawa (2009) and is known as a hesitant fuzzy set. Torra (2010) and Torra and Narukawa (2009) introduced hesitant fuzzy set as the latest expansion of fuzzy sets. With development, many research scholars have applied hesitant fuzzy set in different fields. For example, the decision-making problems under hesitant fuzzy environment has been solved by using hesitant fuzzy linguistic numbers (Wu et al., 2014), hesitant fuzzy soft sets (Wang et al., 2014), aggregation operators (Wu et al., 2014; Xia & Xu, 2011), score function (Yao & Li, 2014), correlation coefficients of hesitant fuzzy sets (Chen et al., 2013) and so on.

Verma and Sharma (2013) introduced some operations on hesitant fuzzy sets. Hesitant fuzzy information measures were applied in MCDM problems by Hu et al. (2016). Verma (2015) presented a number of properties on hesitant fuzzy sets. Hu et al. (2018) discussed similarity and entropy measure on hesitant fuzzy sets. Verma (2017) developed some operations on hesitant interval-valued fuzzy sets and studied their properties.

In the fuzzy set theory, distance measure plays an important role, and these distance measures have been applied successfully applied in different fields. Afterwards, researchers have introduced a distance measure concept in the hesitant fuzzy set theory. For example, Xu and Xia (2011) investigated distance measure and similarity measure on hesitant fuzzy sets. Recently, a new distance measure on hesitant fuzzy sets has been developed by Goala and Dutta (2018) and applied in fuzzy multi-criteria decision-making to help crime linkage analysis.

From the aforementioned analysis, it can be evidently stated that distance measure is one of the important measures for MCDM problems under a hesitant fuzzy domain. However, further investigation revealed that there are some limitations of the existing distance measures. In Xu and Xia’s (2011) approach, one needs to add extra elements in hesitant fuzzy elements (HFEs) using the method for finding distance between HFEs if they have a different number of elements, which may lead to wrong results. Although Goala and Dutta (2018) tried to overcome this limitation, in their approach, they were unable to show that the distance between full HFE and empty HFE is 1. These limitations directed us to reflect on the following main objectives:

1. To propose a novel distance between two HFEs.
2. To develop an algorithm to solve MCDM under hesitant fuzzy domain by using our proposed distance measure.
3. To present a comparative analysis with the help of examples to predict the advantage of our proposed distance measure.

In this article, an attempt has been made to devise a novel distance measure on hesitant fuzzy sets initially. The proposed distance measure has been applied in the decision-making methodology for choosing their liable product under hesitant fuzzy environment considering the customer’s and expert’s reviews well. In addition, a comparative analysis has been carried out to throw ample light on the advantages of the proposed distance measure over existing distance measure. Finally, a case study has been carried out to demonstrate the applicability of the proposed distance measure and decision-making methodology.

**PRELIMINARY**

In this section, a discussion on some preliminaries of fuzzy set, hesitant fuzzy set and distance between hesitant fuzzy sets has been presented.
**Definition 1:** Let $X$ be the universal set, then a fuzzy set $A$ defined by its membership function is as follows (Zadeh, 1965):

$$
\mu_A : X \rightarrow [0, 1].
$$

**Definition 2:** A hesitant fuzzy set $A$ on the universal set $X$ can be defined in terms of a function which gives a subset of $[0, 1]$ when applied to $X$ (Torra & Narukawa, 2009). Symbolically, hesitant fuzzy set was given as follows:

$$
A = \{(x, h_i(x)) \mid x \in X\}.
$$

Here, $h_i(x)$ is a set of values lying between 0 and 1, representing the membership grades of the element $x \in X$ to the set $A$ obtained by different experts. In general, $h_i(x)$ is called hesitant fuzzy element.

**Definition 3:** The score function $s(h)$ for the hesitant fuzzy elements $h$ is defined as follows (Xia, 2013):

$$
s_h = \frac{1}{l_h} \sum_{j=1}^{l_h} h_j.
$$

Here $l_h$ is the number of elements in the hesitant fuzzy element $h$.

Using the score function of HFES, a method for comparison proposed by Xia (2013), between two HFES is as follows:

Suppose $h_i$ and $h_2$ be two HFES, then

- $s(h_i) > s(h_2)$ implies that $h_i$ is superior to $h_2$, symbolized by $h_i > h_2$
- $s(h_i) = s(h_2)$ implies that $h_i$ is indifferent to $h_2$, symbolized by $h_i = h_2$.

**Definition 4:** The deviation degree of an HFE is defined as follows (Chen et al., 2013):

Using the deviation degree, Chen et al. (2013) defined the comparison rule between $h_i$ and $h_2$ as follows:

Let us consider two HFES $h_i$ and $h_2$, their score functions $s(h_i)$ and $s(h_2)$, and their deviation degrees, then

1. $s(h_i) > s(h_2)$ then $h_i$ is superior to $h_2$, denoted by $h_i > h_2$
2. If $s(h_i) = s(h_2)$
   a. If $\bar{h}_i = \overline{s(h_i)}$, then $h_i = h_2$
   b. If $\bar{h}_i > \overline{s(h_i)}$, then $h_i > h_2$
   c. If $\bar{h}_i < \overline{s(h_i)}$, then $h_i < h_2$.

**Definition 5:** The mean deviation among the elements of the HFES from their mean is as follows (Chen et al., 2013):

In this article, deviation function has been utilized to develop a novel distance measure on hesitant fuzzy set.

**Distance Measures**

Distance measures are an integral part of decision-making theory. Some studies have been done on decision-making theory using hesitant fuzzy distance measures.

In this section, some existing hesitant fuzzy distance measures are presented.

Some well-known hesitant fuzzy distance measures are as follows:

**Distance Measure (Xu & Xia, 2011):**

Let us consider two HFES $A = \{(x, h_i(x)) \mid x \in X\}$ and $B = \{(x, h_2(x)) \mid x \in X\}$.

The generalized hesitant normalized distance is defined as follows:

$$
d(A, B) = \sum_{i=1}^{n} \left[ \frac{1}{l_i} \sum_{j=1}^{l_i} \left( \bar{h}_i^m(x_i) - \bar{h}_2^m(x_i) \right)^{\frac{1}{\lambda}} \right], \lambda > 0.
$$

For $\lambda = 1$, generalized hesitant normalized distance becomes the hesitant weighted hamming distance and is given by

$$
d(A, B) = \sum_{i=1}^{n} \left[ \frac{1}{l_i} \sum_{j=1}^{l_i} \left( \bar{h}_i^m(x_i) - \bar{h}_2^m(x_i) \right) \right].
$$

For $\lambda = 2$, the generalized hesitant normalized distance becomes hesitant weighted Euclidean distance measure and is given by

$$
d(A, B) = \sum_{i=1}^{n} \left[ \frac{1}{l_i} \sum_{j=1}^{l_i} \left( \bar{h}_i^m(x_i) - \bar{h}_2^m(x_i) \right)^{\frac{1}{2}} \right].
$$

Here $l_i$ is the number of elements in the $i$th HFES, $\bar{h}_i^m(x_i)$ is the $j$th largest value in HFE in $A$ and $\bar{h}_2^m(x_i)$ is the $j$th largest value in HFE in $B$. To find the $j$th largest values, HFES are ordered in decreasing order.

Now, the generalized weighted Hausdorff distance measure is defined as follows:

$$
d(A, B) = \sum_{i=1}^{n} \left( \max \left| \bar{h}_i^m(x_i) - \bar{h}_2^m(x_i) \right| \right)^{\frac{1}{\lambda}}, \lambda > 0.
$$
For \( \lambda = 1 \), the generalized weighted Hausdorff distance measure becomes the weighted Hausdorff hamming distance and is given by

\[
d(A, B) = \sum_{i=1}^{n} \frac{1}{2} \left[ \sum_{j=1}^{l} \left( \left\| h_{x_i}^{\omega_1}(x) - h_{y_j}^{\omega_2}(x) \right\| + \max \left\{ h_{x_i}^{\omega_1}(x) - h_{y_j}^{\omega_2}(x) \right\} \right)^2 \right]^{1/2}.
\]

For \( \lambda = 2 \), the generalized weighted Hausdorff distance measure becomes the weighted Hausdorff Euclidean distance and is given by

\[
d(A, B) = \sum_{i=1}^{n} \left[ \sum_{j=1}^{l} \left( \left\| h_{x_i}^{\omega_1}(x) - h_{y_j}^{\omega_2}(x) \right\| + \max \left\{ h_{x_i}^{\omega_1}(x) - h_{y_j}^{\omega_2}(x) \right\} \right)^2 \right]^{1/2}.
\]

Afterwards, Xu and Xia (2011) put forward a generalized hybrid hesitant weighted distance measure which is as follows:

\[
d(A, B) = \sum_{i=1}^{n} \frac{1}{2} \left[ \sum_{j=1}^{l} \left( \left\| h_{x_i}^{\omega_1}(x) - h_{y_j}^{\omega_2}(x) \right\| + \max \left\{ h_{x_i}^{\omega_1}(x) - h_{y_j}^{\omega_2}(x) \right\} \right)^2 \right]^{1/2}.
\]

Similarly, for \( \lambda = 1 \), the generalized hesitant weighted Hausdorff distance is known as the hybrid weighted hesitant hamming distance and is given by

\[
d(A, B) = \sum_{i=1}^{n} \frac{1}{2} \left[ \sum_{j=1}^{l} \left( \left\| h_{x_i}^{\omega_1}(x) - h_{y_j}^{\omega_2}(x) \right\| + \max \left\{ h_{x_i}^{\omega_1}(x) - h_{y_j}^{\omega_2}(x) \right\} \right)^2 \right]^{1/2}.
\]

For \( \lambda = 2 \), the generalized hesitant weighted Hausdorff distance is known as the hybrid weighted hesitant Euclidean distance and is given by

\[
d(A, B) = \sum_{i=1}^{n} \left[ \sum_{j=1}^{l} \left( \left\| h_{x_i}^{\omega_1}(x) - h_{y_j}^{\omega_2}(x) \right\| + \max \left\{ h_{x_i}^{\omega_1}(x) - h_{y_j}^{\omega_2}(x) \right\} \right)^2 \right]^{1/2}.
\]

Distance Measure (Goala & Dutta, 2018)

Let us consider two HFEs \( A = \{ (x_i, h_{x_i}(x)) \} \subseteq X \) and \( B = \{ (x_i, h_{x_i}(x)) \} \subseteq X \).

The distance measure between the HFEs \( A \) and \( B \) is defined as follows:

\[
d(A, B) = \frac{1}{L} \sum_{i=1}^{L} \left[ \left\| h_{x_i}(x) - h_{y_j}(x) \right\| + \left\| \sigma(h_{x_i}(x)) - \sigma(h_{y_j}(x)) \right\| \right]
\]

where \( L \) is the number of elements in the hesitant fuzzy sets, \( h_{x_i}(x) \) and \( h_{y_j}(x) \) are the HFEs or membership grades of the element \( x \), \( s(h_{x_i}(x)) \) and \( s(h_{y_j}(x)) \) are the score functions, \( \sigma(h_{x_i}(x)) \) and \( \sigma(h_{y_j}(x)) \) are the variation functions of the HFEs \( h_{x_i}(x) \) and \( h_{y_j}(x) \) respectively.

**A NOVEL HESITANT FUZZY DISTANCE MEASURE**

In this section, an effort has been made to define a novel distance measure for HFEs and investigate its properties. Let us consider two HFSs \( A = \{ (x_i, h_{x_i}(x)) \} \subseteq X \) and \( B = \{ (x_i, h_{x_i}(x)) \} \subseteq X \), then the new distance measure between \( A \) and \( B \) is defined as a function:

\[
d(A, B) = \frac{1}{n} \sum_{i=1}^{n} \left[ \left\| s(h_{x_i}(x)) - s(h_{y_j}(x)) \right\| + \left\| \sigma(h_{x_i}(x)) - \sigma(h_{y_j}(x)) \right\| \right]^{1/2}
\]

\[
d(A, B) : A \times B \to [0, 1] \text{ such that }
\]

Here \( n \) is the number of elements in the hesitant fuzzy sets, \( h_{x_i}(x) \) and \( h_{y_j}(x) \) are the HFEs of the element \( x \), \( s(h_{x_i}(x)) \) and \( s(h_{y_j}(x)) \) are the score functions, \( \sigma(h_{x_i}(x)) \) and \( \sigma(h_{y_j}(x)) \) are the mean deviation of the HFEs.

\( \lambda > 0 \).

**Theorem:** The proposed distance measure is a valid distance measure.

**Proof:** To prove the theorem, we need to verify the following three conditions for HFEs \( A \) and \( B \):

\( 0 \leq d(A, B) \leq 1 \)

\( d(A, B) = d(B, A) \)

\( d(A, B) = 0 \iff A = B \)

Now, we have

\( 0 \leq d(A, B) \leq 1 \)

**Proof:**

Let \( x_i \in X \) then \( s(h_{x_i}(x)) = \frac{1}{l_{h_{x_i}(x)}} \sum_{x \in l_{h_{x_i}(x)}} \gamma \) and \( s(h_{y_j}(x)) = \frac{1}{l_{h_{y_j}(x)}} \sum_{x \in l_{h_{y_j}(x)}} \gamma \).

Also \( m(h_{x_i}(x)) = \frac{1}{l_{h_{x_i}(x)}} \sum_{x \in l_{h_{x_i}(x)}} \gamma - s(h_{x_i}(x)) \) and \( m(h_{y_j}(x)) = \frac{1}{l_{h_{y_j}(x)}} \sum_{x \in l_{h_{y_j}(x)}} \gamma - s(h_{y_j}(x)) \).

\( \therefore \ 0 \leq s(h_{x_i}(x)) \leq 1 \) and \( 0 \leq s(h_{y_j}(x)) \leq 1 \).

Similarly, \( 0 \leq m(h_{x_i}(x)) \leq 1, \ 0 \leq m(h_{y_j}(x)) \leq 1 \).
\[ 0 \leq \|h'_s(x) - s(h_s(x))\| \leq 1 \text{ and } \]
\[ 0 \leq \|m(h'_s(x)) - \bar{m}(h_s(x))\| \leq 1 \]
\[ \Rightarrow 0 \leq \|h'_s(x) - s(h_s(x))\|^2 \leq 1 \text{ and } \]
\[ 0 \leq \|m(h'_s(x)) - \bar{m}(h_s(x))\|^2 \leq 1 \text{ for any } \lambda > 0 \]
\[ \therefore 0 \leq \|h'_s(x) - s(h_s(x))\|^2 \leq 1 \]
\[ s(h'_s(x)) \|m(h'_s(x)) - \bar{m}(h_s(x))\|^2 \leq 1 \]
\[ \Rightarrow 0 \leq \frac{1}{n} \sum_{i=1}^{n} \|h'_s(x) - s(h_s(x))\|^2 \leq 1 \]
\[ \Rightarrow \sum_{i=1}^{n} \|h'_s(x) - s(h_s(x))\|^2 \leq 1 \]
\[ s(h'_s(x)) \|m(h'_s(x)) - \bar{m}(h_s(x))\|^2 \leq 1 \]
\[ \Rightarrow 0 \leq d(A, B) \leq 1 \]
\[ d(A, B) = d(B, A) \]

**Proof:**
\[ d(A, B) = \sum_{i=1}^{n} \|h'_s(x) - s(h_s(x))\|^2 \]
\[ s(h'_s(x)) \|m(h'_s(x)) - \bar{m}(h_s(x))\|^2 \]
\[ = d(B, A) \]
\[ d(A, B) = 0 \iff A = B \]

Advantages of our proposed distance over the distance measure presented by Xu and Xia (2011):

In this section, we cite some numerical examples for displaying the advantages of our proposed distance measure over the existing distance measures. Major drawbacks are encountered in the existing approaches presented by Xu and Xia (2011) and Goala and Dutta (2018). In Xu and Xia (2011) approach, extra elements are added in HFEs with a view to balance the order of HFEs. Due to this reason, decision-makers are unable to give accurate results. Besides, in Goala and Dutta (2018) process, the distance between the whole HFE and empty HFE is 0.5, which is not reasonable. Such type of result affects the ranking in the decision-making problems.

Advantages of our proposed distance over the distance measure presented by Xu and Xia (2011):

To display the distinctiveness, originality and advantage of our proposed distance measure, the results are computed by adding different elements to the shorter HFEs and are summarized in Table 1. Here, there are two HFEs of different lengths. For computing the distance degree between them, we need to add extra elements to balance the length of HFEs, which are shown clearly in Table 1.
Table 1: Different Distance Measures after Adding Different Elements in Deficit HFEs

| h₁ = {1, 0.8, 0.8} | Ĥ₁ = {1, 0.8, 0.8, 0.6, 0.8} | Hamming Distance Xu and Xia (2011), λ = 1 | Hamming Distance Xu and Xia (2011), λ = 2 | Hausdorff Distance Xu and Xia (2011), λ = 1 | Hausdorff Distance Xu and Xia (2011), λ = 2 | Hybrid Distance Xu and Xia (2011), λ = 1 | Hybrid Distance Xu and Xia (2011), λ = 2 | Proposed Distance for Taking (No Need of Adding Extra Elements), λ = 1 |
|-------------------|-------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 1 Add 0.87 and 0.87 in h₁ = {1, 0.8, 0.8} | 0.01424 | 0.0998 | 0.04 | 0.04 | 0.02712 | 0.0699 |
| 2 Add 1 and 0.8 in h₁ = {1, 0.8, 0.8} | 0.04 | 0.1265 | 0.04 | 0.04 | 0.028 | 0.08325 |
| 3 Add 1 and 1 in h₁ = {1, 0.8, 0.8} | 0.024 | 0.1549 | 0.04 | 0.04 | 0.032 | 0.0975 |
| 4 Add 0.8 and 0.8 in h₁ = {1, 0.8, 0.8} | 0.008 | 0.0894 | 0.04 | 0.04 | 0.024 | 0.0647 | 0.074963 |
| 5 Add 0.8 and 0.7 in h₁ = {1, 0.8, 0.8} | 0.04 | 0.002 | 0.02 | 0.01 | 0.012 | 0.006 |
| 6 Add 0.7 and 0.7 in h₁ = {1, 0.8, 0.8} | 0.008 | 0.0632 | 0.04 | 0.01 | 0.024 | 0.0365 |
| 7 Add 1 and 0.7 in h₁ = {1, 0.8, 0.8} | 0.012 | 0.1 | 0.04 | 0.04 | 0.026 | 0.07 |

From the aforementioned table, in case of an unbalanced length of HFEs it is seen that the proposed distance measure has the capability to measure distance degree between two HFEs without adding extra elements in the shorter HFE. As a result, it can decrease the chance of intentional or unintentional biases of the decision-makers during decision-making problems.

Furthermore, let us consider the following three hesitant fuzzy sets: h₁ = {0.1, 0.3}, h₂ = {0.1, 0.1, 0.3}, h₃ = {0.1, 0.3, 0.3}

Now, the distance measure (Hamming/Hausdorff/Hybrid) between h₁ and h₂ defined by Xu and Xia (2011) via pessimistic approach is (i.e., adding the least element in the shorter HFE as h₁ and h₂ are of different lengths)

\[ d(h₁, h₂) = 0. \] But it is obvious that
\[ h₁ = \{0.1, 0.3\} \neq h₂ = \{0.1, 0.1, 0.3\} \]

Thus, the distance measure presented by Xu and Xia (2011) violates the elementary norms of a distance measure in some cases. But \( d(h₁, h₂) \neq 0 \) and \( d(h₁, h₂) \neq 0 \) by our proposed distance measure.

Advantages of our proposed distance over the distance measure presented by Goala and Dutta (2018):

Let us consider the HFEs \( h₁ = \{1, 1\} \) (Full HFE)and \( h₂ = \{0, 0\} \) (Empty HFE). Then the distance degree between full HFE and empty HFE is \( d(h₁, h₂) = 0.5 \) by the distance measure presented by Goala & Dutta (2018). But obviously it is a well-known fact that distance between the full HFE and empty HFE should be 1 and the same fact is corroborated by the proposed distance, that is, \( d(h₁, h₂) = 1 \), which is more logical.
From the above deliberations, we have exhibited the uniqueness, novelty and advantage of our proposed distance measure. Thus, we can say that the proposed distance measure is a better indicator.

DECISION-MAKING METHODOLOGY

Let us consider a set of features of a specific kind of product in the market be $F = \{F_1, F_2, \ldots, F_m\}$ and $P = \{P_1, P_2, \ldots, P_n\}$ be the set of available products of that specific kind in that market. The importance or weight of the features be $w = \{w_1, w_2, \ldots, w_m\}^T$.

In this approach, products are considered as the alternatives, and the features of the products are considered as the attributes of the fuzzy decision situation. Therefore, a fuzzy decision situation can be expressed by the following matrices (Klir & Yuan, 1995):

\[
F = \begin{bmatrix}
F_1 & F_2 & \cdots & F_m \\
F_1 & F_2 & \cdots & F_m \\
\vdots & \vdots & \ddots & \vdots \\
F_1 & F_2 & \cdots & F_m \\
\end{bmatrix}
\]

Here $h^C_i$ and $h^E_i$ are HFEs that represent the quality of the feature $F_i$ contained by the product $P_j$ from the customer's or user's point of view and the professional reviewer, respectively. Thus, the products can be represented as hesitant fuzzy sets in two ways, as follows:

\[
P_j^C = \{(F_i, h^C_i(F_i)) | F_i \in F\} \text{ where } h^C_i \to [0,1]; \quad i = 1, 2, \ldots, m
\]

and

\[
P_j^E = \{(F_i, h^E_i(F_i)) | F_i \in F\} \text{ where } h^E_i \to [0,1]; \quad i = 1, 2, \ldots, m
\]

Obviously, the minimum value of distance measures, that is, $\min\{d(P_j^C, P_j^E)\}$ will correspond to the most reliable product among the given available products.

The algorithm of the decision-making model can be described as follows:

**Step 1.** Choose features of the product $F = \{F_1, F_2, \ldots, F_m\}$

**Step 2.** Consider the products $P = \{P_1, P_2, \ldots, P_n\}$ among which the reliable product has to be selected

**Step 3.** Select the importance or weight of the features be $w = \{w_1, w_2, \ldots, w_m\}^T$

**Step 4.** Represent the products as HFSs $P_j^C = \{(F_i, h^C_i(F_i)) | F_i \in F\}$

$h^C_i$ is HFEs representing the quality of the feature $F_i$ contained by the product $P_j$ from the customer's or user's point of view

**Step 5.** Represent the products as HFSs $P_j^E = \{(F_i, h^E_i(F_i)) | F_i \in F\}$

$h^E_i$ is HFEs representing the quality of the feature $F_i$ contained by the product $P_j$ from the expert's point of view

**Step 6.** Find $\min\{d(P_j^C, P_j^E)\}$ and corresponding product will be chosen as reliable.

In this section, a case study will be carried out to exhibit the consistency, rationality and usability of our proposed distance measure. Our methodology will be applied to choose the most reliable product considering different features of the product based on the reviews of customers and experts.

Let us consider a set of four smartphones having almost the same type of features. Feedback is then collected from technical and experienced experts. After that, the review of the smartphone is collected from some real users or customers based on the features of the smartphone. In this case, we have taken into consideration the following features:

1. Speed and performance of the smartphone: The higher speed quad-core will operate apps quicker and is usually less time consuming for everyday tasks, but octa-core is better for weighty tasks, such as gaming and video editing.

2. Display quality: The display quality depends upon the following parameters:
   a. Pixel density: Pixel density is the most deciding parameter for quality of the screen and the real resolution of the display. The higher the number, the better the quality of the display.
   b. Brightness: Brightness is another component or parameter that affects the degree of excellence of
a screen. Better brightness capabilities mean our smartphone is more operational in bright places.

c. Display protection: One of the most vital features that confers safeguards to the smartphone is the Gorilla glass protection, used extensively in order to guarantee screen dependability.

d. Glare resistance: Many phones offer glare resistance technology so that we do not have to escape our own shadow when using our phone in a bright room. Not all phones offer this, but it is something to look for.

e. Size: Another factor that can come down to personal preference is the size of a screen. If we are a techno freak or we really prefer digitalized mode for reading, we always opt for a phone with a greater screen.

3. Video and photo quality: The following are the parameters affecting the video and photo quality:

a. Sensor type: The most significant element of a camera is its sensor as the sensor dictates the image size, rotation, focal range, lenses compatible and overall size of the body.

b. Sensor size: Larger sensor magnitudes assistances in producing high-quality images.

c. Pixel size: When we have a small pixel in our sensor, it does not accept light precisely and tends to yield an unclear photo. So, pixel size is also a significant factor for getting good quality photo.

d. Image stabilization: Image stabilization also plays a vital role in producing sharp images.

e. Post-processing techniques: Lastly, there are handsets that have their own post-processing techniques as a final touch to their images. So, to produce highly saturated photos we need post-processing techniques.

In the above case, the specifications of the product are explained in terms of their parameters so that decision-makers are able to judge how the change in parameters can affect the specifications of the product. Furthermore, the specifications are selected for the case study due to the fact that most of the online shopping websites such as Amazon and Flipkart display comparisons among phones depending mostly upon specifications and people tend to buy products by looking the review of customers and experts depending upon these three specifications.

A comparison has been made to choose the reliability of products on the basis of reviews of technical and experienced experts with reviews of real users or customers of that product. Since our objective is to choose the best product from the available products, therefore we will consider or take only positive reviews of the products from the experts. But there will be no such restriction about consideration for the reviews of the customers or real users of the products in this case study. Here, we are taking the review of the smartphone from three technical and experienced experts and five customers, and consider that the weight of each feature is equal, that is, \( w_1 = w_2 = w_3 \). The steps of the proposed methods are executed here to select the best alternative:

Step 1. The rating information of smartphones under the different criteria is accessed by the technical experts and expressed as HFEs.

Step 2. The rating information of smartphones under the different criteria is accessed by the customers and expressed as HFEs.

Step 3. The decision matrix for technical expert’s review is constructed with HFEs.

Step 4. The decision matrix for the customer’s review is constructed with HFEs.

Step 5. The distance between the technical expert’s review and customer’s review of the smartphones expressed as HFEs are computed.

Step 6. The reliable smartphone is selected with the help of the ranking of the alternatives for distance measures.

The overall scenario is explained by the following Table 2 and Table 3:

In the following tables, we consider the following:

- \( F_1 \): Speed and performance of the smartphone
- \( F_2 \): Display quality
- \( F_3 \): Video and photo quality
- \( P_1 \): Product 1
- \( P_2 \): Product 2
- \( P_3 \): Product 3
- \( P_4 \): Product 4
### Table 2: Expert’s Review of Smartphones Against the Features

| Product | Experts | $F_1$   | $F_2$   | $F_3$   |
|---------|---------|---------|---------|---------|
| $P_1$   | $e_1$   | Average | Good    | Better  |
|         | $e_2$   | Good    | Better  | Better  |
|         | $e_3$   | Average | Better  | Good    |
| $P_2$   | $e_1$   | Good    | Better  | Better  |
|         | $e_2$   | Better  | Average | Good    |
|         | $e_3$   | Good    | Good    | Good    |
| $P_3$   | $e_1$   | Average | Better  | Best    |
|         | $e_2$   | Good    | Best    | Better  |
|         | $e_3$   | Average | Best    | Better  |
| $P_4$   | $e_1$   | Average | Average | Good    |
|         | $e_2$   | Good    | Average | Good    |
|         | $e_3$   | Average | Average | Average |

### Table 3: Customer’s Review of Smartphones against the Features

| Product | Customers | $F_1$   | $F_2$   | $F_3$   |
|---------|-----------|---------|---------|---------|
| $P_1$   | $c_1$     | Average | Bad     | Average |
|         | $c_2$     | Good    | Average | Good    |
|         | $c_3$     | Average | Bad     | Average |
|         | $c_4$     | Good    | Average | Average |
|         | $c_5$     | Better  | Average | Better  |
| $P_2$   | $c_1$     | Bad     | Worst   | Better  |
|         | $c_2$     | Average | Bad     | Good    |
|         | $c_3$     | Average | Bad     | Good    |
|         | $c_4$     | Average | Worst   | Average |
|         | $c_5$     | Average | Bad     | Average |

*Table 3 continued*
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(Table 3 continued)

| Product | Customers | \( F_1 \) | \( F_2 \) | \( F_3 \) |
|---------|-----------|------------|------------|------------|
| \( c_1 \) | Average   | Better     | Best       |
| \( c_2 \) | Good      | Best       | Better     |
| \( c_3 \) | Average   | Best       | Better     |
| \( c_4 \) | Average   | Better     | Good       |
| \( c_5 \) | Good      | Better     | Better     |

For the case study, the following membership grades (Table 4) are considered for linguistic variables.

| Linguistic Variable | Membership Grades |
|---------------------|--------------------|
| Worst               | 0                  |
| Bad                 | 0.2                |
| Average             | 0.4                |
| Good                | 0.6                |
| Better              | 0.8                |
| Best                | 1                  |

From the above information, the following decision matrix for expert’s review can be constructed with HFEs:

\[
P^E = \begin{bmatrix}
F_1 & F_2 & F_3 & F_4 \\
0.4,0.6,0.4 & 0.6,0.8,0.6 & 0.4,0.6,0.4 & 0.4,0.6,0.4 \\
0.6,0.8,0.8 & 0.8,0.4,0.6 & 0.8,1.1 & 0.4,0.4,0.4 \\
0.8,0.8,0.6 & 0.8,0.6,0.6 & 1.0,8,0.8 & 0.6,0.6,0.4 \\
\end{bmatrix}
\]

Similarly, the following decision matrix for customer’s review can be constructed with HFEs:

\[
P^C = \begin{bmatrix}
F_1 & F_2 & F_3 & F_4 \\
0.4,0.6,0.4,0.6,0.8 & 0.2,0.4,0.4,0.4,0.4 & 0.8,1.0,8,0.8,0.8 & 1.0,8,0.8,0.8,0.8 \\
0.8,0.6,0.4,0.4,0.4,0.4,0.4 & 0.2,0.4,0.4,0.4,0.4 & 0.8,1.0,8,0.8,0.8 & 1.0,8,0.8,0.8,0.8 \\
0.4,0.6,0.4,0.6,0.8 & 0.4,0.6,0.4,0.6,0.8 & 0.6,0.6,0.4 & 0.6,0.6,0.4 \\
0.4,0.8,0.4,0.6,0.8 & 0.4,0.8,0.4,0.6,0.8 & 0.6,0.6,0.4 & 0.6,0.6,0.4 \\
\end{bmatrix}
\]

Similarly, the products can be represented as hesitant fuzzy sets from information of expert’s review and can be expressed as follows:

\[
P^E = \{F_1[0.4,0.6,0.4], F_2[0.6,0.8,0.8], F_3[0.8,0.8,0.6]\}
\]

\[
P^E = \{F_1[0.6,0.8,0.6], F_2[0.8,0.4,0.6], F_3[0.8,0.6,0.6]\}
\]

\[
P^E = \{F_1[0.4,0.6,0.4], F_2[0.8,1.1], F_3[1.0,8,0.8]\}
\]

\[
P^E = \{F_1[0.4,0.6,0.4], F_2[0.4,0.4,0.4], F_3[0.6,0.6,0.4]\}
\]

Similarly, the products can be represented as hesitant fuzzy sets from information of customer’s review and can be expressed as follows:

\[
P^C = \{F_1[0.4,0.6,0.4,0.6,0.8], F_2[0.2,0.4,0.4,0.4,0.4], F_3[0.8,0.6,0.6,0.4,0.4]\}
\]

\[
P^C = \{F_1[0.2,0.4,0.4,0.4,0.4], F_2[0.2,0.2,0.2,0.2]\}
\]

\[
P^C = \{F_1[0.8,0.6,0.6,0.4,0.4]\}
\]

\[
P^C = \{F_1[0.4,0.6,0.4,0.4,0.6], F_2[0.8,1.1,0.8,0.8], F_3[1.0,8,0.8,0.8]\}
\]

\[
P^C = \{F_1[0.8,0.8,0.6,0.8]\}
\]

\[
P^C = \{F_1[0.4,0.6,0.4,0.6,0.8], F_2[0.4,0.6,0.4,0.6,0.8]\}
\]

\[
P^C = \{F_1[0.6,0.6,0.4,0.6,0.6]\}
\]
Table 5: Ranking of Alternatives for Distance Measures for Different Parameter $\lambda$

| $\lambda$ | $d(P_C, P_E)$ | $d(P_C, P_S)$ | $d(P_C, P_I)$ | $d(P_C, P_A)$ | $d(P_C, P_D)$ |
|-----------|---------------|---------------|---------------|---------------|---------------|
| 1         | 0.267662222   | 0.321647407   | 0.051792593   | 0.171124938   | 0.157092593   |
| 2         | 0.159696      | 0.19407       | 0.029148      | 0.085401      | 0.085401      |
| 3         | 0.14444       | 0.173399      | 0.025578      | 0.069202      | 0.069202      |
| 4         | 0.140257      | 0.166374      | 0.024223      | 0.062878      | 0.062878      |
| 5         | 0.13879       | 0.163286      | 0.02352       | 0.05963       | 0.05963       |

Now, the distance between expert’s review and customer’s review of the products expressed in hesitant fuzzy sets are obtained for different parameters $\lambda$:

(Here $\lambda$ is taken as $\lambda = 1, 2, 3, 4, 5$)

From Table 5, it is observed that $d(P_C, P_E)$ is $0.051792593, 0.029148, 0.025578, 0.024223$ and $0.02352$ for $\lambda = 1, 2, 3, 4, 5$ respectively, that is, the distance degree $d(P_C, P_E)$ between the review of the technical expert and review of customers for the third smartphone is minimum. So, it can be decided that the most reliable smartphone will be the third one, that is, $P_3$ depending upon the selected specifications, that is, speed and performance of the smartphone ($F_1$), display quality ($F_2$), and video and photo quality ($F_3$).

**CONCLUSION**

In this research work, a novel distance measure on hesitant fuzzy set has been developed based on the concept of score function and mean deviation. Illustrative examples are taken to exhibit the uniqueness and advantage of our proposed distance measure over existing distance measures. The proposed distance measure has an advantage over existing distance measure because the latter requires the addition of extra elements in HFE in case of the deficient number of membership grades in HFEs. This may lead to biased results intentionally or unintentionally. We have applied the proposed distance measure in the decision-making approach for choosing the best electronic products. In the forthcoming days, this distance measure can be extended to the decision-making field under diverse environment to represent uncertain information in a much-refined manner.

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