DOA Estimation Based on Average Processing of Redundant Virtual Array Elements for Coprime MIMO Radar

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Abstract. The redundant virtual array elements formed by the sum and difference coarray (SDCA) are always discarded in the existing DOA estimation methods, which could cause effective information loss and poor performance. In this paper, a new technology based on average processing of redundant virtual array elements is proposed for coprime MIMO radar direction of arrival (DOA) estimation. Then, Toeplitz matrix reconstruction based on MUSIC algorithm is employed to validate the effectiveness of the proposed technology for DOA estimation.

1. Introduction

Multiple Input Multiple Output (MIMO) radar [1] has great space, frequency and waveform diversity characteristics, which can form an effective large observation aperture. It is of great significance of increasing the aperture of virtual array elements [2], optimization of detection and tracking performance [3], improving direction finding resolution and precision. Scholars have successively carried out MIMO radar direction finding research from the aspects of direction of arrival (DOA) estimation [4], multi-parameter joint estimation [5-6] and array parameters estimation under non-ideal array conditions [7]. Traditional MIMO radar usually adopts uniform linear array (ULA) as the transmitting and receiving arrays for DOA estimation, and the number of virtual array elements in their echo signal model is limited by the number of physical array elements, which results in poor performance of DOA estimation.

The huge performance advantages of the sparse array have further promoted the innovation of MIMO radar. The combination of sparse array and MIMO radar to carry out target direction finding research has important theoretical significance and application prospects, which has both diversity and virtual
aperture extension characteristics. By using the concept of sum and difference coarray (SDCA) to further improve the degree of freedom [8], thereby improving the target resolution.

The minimum redundancy MIMO radar [9] usually takes the minimum redundancy array [10] as the transmitting array and the receiving array. In order to improve the utilization rate of the array elements, the optimal array spacing is designed by optimizing the number of virtual array elements. However, the location of the array elements requires complicated computational search and the closed expression of the virtual aperture is missing, which is difficult to be realized in engineering. The nested MIMO radar [11-12] generally uses nested subarrays or two-level nested arrays as transmitting arrays and receiving arrays. The positions and numbers of virtual arrays have uniform closed solutions. However, there is large mutual coupling between adjacent arrays, which will reduce the performance of DOA estimation.

The coprime MIMO radar [13-14], as a new array structure, usually adopts coprime subarrays or two-level coprime arrays as the transmitting array and receiving array. Its larger elements spacing can further reduce the mutual coupling and improve the performance of DOA estimation. Li et al. [13] used unitary ESPRIT method for DOA estimation of coprime subarrays MIMO (coprime MIMO) radar, which performance is greatly improved compared with traditional MIMO radar. Shi et al. [15] proposed a symmetric coprime MIMO radar structure and derived the closed-form solution of its degree of freedom, which targets detection number and DOA estimation performance are better than coprime MIMO radar and two-level coprime MIMO (CPA-TR) radar. In addition, on the one hand, they used the $\ell_1$-SVD algorithm [16] to perform the coherent targets under the concept of sum coarray, on the other hand, they used the CS algorithm [17] to perform the uncorrelated targets under the concept of sum and difference coarray. However, the virtual array elements formed by the vectorized covariance matrix have a lot of redundancy, so it is impossible to directly use subspace algorithms such as MUSIC [18] and ESPRIT [19]. The prior art generally removes the repeated rows of the virtual array elements and follows the virtual array elements. Reordering the positions from small to large will undoubtedly lose part of the arrival information, so as to reduce DOA estimation performance.

As a result, this paper presents a method of averaging redundant virtual arrays. Firstly, the virtual array information at the same position formed by vector-quantized covariance matrix is averaged and sorted from small to large to make full use of all virtual arrays formed by sum and difference coarray, so as to increase data utilization. Then, the Toeplitz matrix reconstruction method is used to overcome the influence of single snapshot. Finally, the proposed algorithm is combined with MUSIC algorithm for DOA estimation. The remainder is given as follows:

1) Section II outlines the basic array signal model of the symmetric coprime MIMO radar.
2) Section III analyzes the problem of redundant virtual array elements under the sum and difference coarray as well as proposes a new DOA estimation technology for the average processing of redundant virtual array elements.
3) Numerical simulations and the conclusions of this paper are presented in Section IV and Section V, respectively.

2. Array Signal Model
A monostatic symmetric coprime MIMO radar system is shown in Figure 1. Two ULAs are arranged symmetrically with the origin as the center. The transmit array is composed of $M_t = 4M - 1$ dense ULAs with adjacent interval being $Nd_1$, and the receive array is composed of $N_r = 2N - 1$ sparse ULAs with adjacent interval being $Md_2$. $d_1$ is the unit array element spacing, which is generally equal to $\frac{\lambda}{2}$, and $\lambda$ is the signal wavelength. $M$ and $N$ are two coprime integer factors.
Assume that there are \( K \) far-field uncorrelated sources from angels \( \theta = \{ \theta_k | k = 1, 2, \ldots, K \} \), and the reflection coefficient of the \( k \)-th target is \( \beta_k \). Then, the echo signal model of the monostatic symmetric coprime MIMO radar can be expressed as

\[
x(t) = \sum_{k=1}^{K} \beta_k \langle \alpha_k(\theta_k) \rangle \alpha_k^*(\theta_k) b(t) + w(t)
\]

where \( b(t) = [b_{(2M-1)}(t), \ldots, b_{(N-1)}(t), b_{(N)}(t), b_{(N+1)}(t), \ldots, b_{(2M-1)}(t)]^T \) is the transmitted signal; \( w(t) \) is an additive white Gaussian noise (AWGN) at the receiving elements; \( \alpha_k(\theta_k) \) and \( \alpha_k^*(\theta_k) \) are the transmitting steering vector and receiving steering vector of the \( k \)-th target, respectively, which can be expressed as

\[
\alpha_k(\theta_k) = [e^{j2\pi M (2M-1) \sin \theta_k / \lambda}, \ldots, e^{j2\pi M (2M-1) \sin \theta_k / \lambda}]^T
\]

\[
\alpha_k^*(\theta_k) = [e^{j2\pi N (N-1) \sin \theta_k / \lambda}, \ldots, e^{j2\pi (2M-1) \sin \theta_k / \lambda}]^T
\]

Utilizing the characteristics of MIMO radar transmitting waveforms as mutually orthogonal pulse signals, that is \( R_c = E[b(t)b(t)^H] = I_{M \times M} \), the echo signal is subjected to generalized matched filtering to obtain as

\[
x(t) = \sum_{k=1}^{K} \beta_k \langle \alpha_k(\theta_k) \otimes \alpha_k^*(\theta_k) \rangle s(t) + m(t)
\]

\[
= [\alpha_k(\theta_k) \otimes \alpha_k(\theta_k), \ldots, \alpha_k(\theta_k) \otimes \alpha_k(\theta_k)] s(t) + m(t)
\]

\[
= (A \otimes A) s(t) + m(t)
\]

where \( A = [\alpha_1(\theta_1), \ldots, \alpha_K(\theta_K)] \); \( A = [\alpha_1(\theta_1), \ldots, \alpha_K(\theta_K); \alpha_1(\theta_1), \ldots, \alpha_K(\theta_K)] \); \( s(t) = [\beta_1, \beta_2, \ldots, \beta_K]^T \); \( m(t) \) is an AWGN vector; \( \otimes \) and \( \otimes \) denote Kronecker product and Khatri-Rao product, respectively.

According to formula (3), the covariance matrix of the echo signal can be obtained as

\[
R = E[x(t)x(t)^H] = (A \otimes A)(A \otimes A)^H + \sigma_n^2 I_{M,N}
\]

\[
= A R_c A^H + \sigma_n^2 I_{M,N}
\]

where \( R_c = E[s(t)s(t)^H] = \text{diag}[\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2] \) is the targets covariance matrix; \( \sigma_i^2 \) is the signal energy of the \( i \)-th target; \( A = A \otimes A \); \( \sigma_n^2 \) is the noise variance.

Vectorizing \( R \) yields

\[
r = \text{vec}(R)
\]

\[
= (A \otimes A) s(t) + \sigma_n^2 \text{vec}(I_{M,N})
\]

\[
= (A \otimes A) s(t) + \sigma_n^2 \text{vec}(I_{M,N})
\]

\[
= Bp + \sigma_n^2 \text{vec}(I_{M,N})
\]
where $\text{vec}()$ ($\cdot$) represents vectorized operation; $B = A^* \odot A = [a_1^* (\theta) \odot a_1 (\theta) \odot a_n (\theta), \cdots, a_1^* (\theta_k) \odot a_n^* (\theta_k) \odot a_n (\theta_k)]$ is the virtual array direction matrix of the single snapshot vector $r$; $p = [\sigma^2_1, \sigma^2_2, \cdots, \sigma^2_K]^T$; $^*$ represents the complex conjugation of the matrix.

It can be known from formula (5) that the virtual element positions of $A^* \odot A$ is composed of sum and difference coarray of actual element positions. And the consecutive virtual array elements [15] in the range $[-(5M - M - N - 1), 5M - M - N - 1]$.

### 3. Proposed Technology

#### 3.1. Review Traditional Technology Based on SDCA

Remove the duplicate rows of the consecutive virtual array elements in $B$ and reorder them according to the position of the virtual array elements from small to large, and a $(10MN - 2M - 2N - 1) \times K$ dimensional array flow matrix $B_0$ can be constructed. According to formula (5), remove the corresponding rows in vector $r$ and sort them to construct a new echo signal vector as

$$r_0 = B_0 p + \sigma^2_e e$$

where $e = [0, \cdots, 0, 1, 0, \cdots, 0]^T$, that is the vector whose middle position is 1 and the rest positions are 0. $B_0$ can be expressed as

$$B_0 = \begin{bmatrix}
    e^{-jP \sin \theta_1} & e^{-jP \sin \theta_2} & \cdots & e^{-jP \sin \theta_K} \\
    \vdots & \vdots & \ddots & \vdots \\
    e^{-jP \sin \theta_1} & e^{-jP \sin \theta_2} & \cdots & e^{-jP \sin \theta_K} \\
    1 & 1 & \cdots & 1 \\
    \vdots & \vdots & \ddots & \vdots \\
    e^{-jP \sin \theta_1} & e^{-jP \sin \theta_2} & \cdots & e^{-jP \sin \theta_K}
\end{bmatrix}$$

where $P = 5M - M - N - 1$ denotes the maximum position of the extended continuous virtual aperture elements.

#### 3.2. Proposed Technology by Average Processing of Redundant Virtual Array Elements of SDCA

In order to improve the utilization of data, we can take the average value of the virtual elements in the repeated positions as the new virtual elements. Then, we can obtain a new covariance matrix as

$$r = \bar{B} p + \sigma^2_e e$$

In order to understand the technology in more detail, we assume that $M = 2$, $N = 3$. The SDCA location can be showed in Figure 2.

![Figure 2: The SDCA location](image-url)
As shown in Figure 2, the positions of the consecutive virtual array elements are distributed between -24 and 24. Traditional technology usually uses one virtual element in the same position, and then deletes other elements in the same position, which obviously lose part of the covariance matrix data. However, the proposed technique performs a weighted average of all virtual array elements information at each position, making full use of the covariance matrix data.

Next, we can use the Toeplitz matrix reconstruction method to overcome the impact of single snapshot. According to formula (8), assuming that \( \tilde{\mathbf{f}}_i (\tilde{M} \leq i \leq \tilde{M}) \) represents the \( i + \tilde{M} \)-th row of vector \( \mathbf{f} \), then \( \mathbf{R}_i = \mathbf{R}_i^* \) can be obtained. Consequently, the Toeplitz matrix is established as follows

\[
\mathbf{R} = \begin{bmatrix}
\mathbf{f}_0 & \mathbf{f}_1 & \cdots & \mathbf{f}_{\tilde{M}} \\
\mathbf{f}_1 & \mathbf{f}_0 & \cdots & \mathbf{f}_{\tilde{M} - 1} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{f}_{\tilde{M}} & \mathbf{f}_{\tilde{M} - 1} & \cdots & \mathbf{f}_0
\end{bmatrix}
\]

(9)

Perform eigenvalue decomposition on the new covariance matrix \( \mathbf{R} \) obtained in equation (9), and perform DOA estimation with MUSIC algorithm.

4. Simulation Results

In this section, we have performed several simulations with three coprime MIMO radars to illustrate the advantages of the proposed technology for DOA estimation. We assume that the total number of arrays is equal to 12, that is \( M = 2 \), \( N = 3 \). The coprime MIMO radar consists of 10 transmit arrays and 2 receive arrays as well as the CPA-TR consists of 6 transmit arrays and 6 receive arrays.

4.1. Spatial Spectrum

In this part, we show the spatial spectrum of the three coprime MIMO radars by using traditional technology and proposed technology in Figure 3, where we suppose the \( SNR = 0 dB \), the number of snapshots \( L = 500 \), and the angle of the targets are \( [-60^\circ : 10^\circ : 60^\circ] \). The searching range is \( [-90^\circ : 0.01^\circ : 90^\circ] \).

As can be seen in Figure 3, among the three kinds of coprime MIMO radars, the spatial spectrum of the improved technology is better than that of the traditional technology, and the symmetric coprime MIMO radar has sharper spectral peaks due to its higher degree of freedom. Therefore, performing redundant averaging processing on the virtual array elements at the same position can significantly improve the angle estimation performance.
4.2. Root Mean Square Error

In this part, we compare the performance of the three coprime MIMO radars by using traditional technology and proposed technology in Figure 4 via Monte-Carlo experiments, where we suppose the $SNR = \left[ -10 : 2 : 10 \right] dB$, the number of snapshots $L = 500$ and the $SNR = 0 dB$, the number of snapshots $L = \left[ 50 : 50 : 500 \right]$, respectively. And the angle of the targets are range in $[20 ^\circ : 10 ^\circ : 20 ^\circ]$. The root mean square error (RMSE) of DOA estimation can be calculated as

$$RMSE = \sqrt{\frac{1}{TK} \sum_{j=1}^{T} \sum_{k=1}^{K} (\hat{\theta}_j - \theta_j)^2}$$

where $T$ represents the number of total Monte-Carlo simulations, $T = 200$, $\theta_j$ denote the true DOAs and $\hat{\theta}_j$ denote the estimated DOAs of the $i$-th trials.

![Figure 4](image)

(a) RMSE versus the SNR. (b) RMSE versus the number of snapshots

As described in Figure 4, the RMSE in all cases decreases as the increase of the SNR and the number of snapshots. For the same coprime MIMO radar, the proposed technology makes full use of more virtual array elements information so that its RMSE is lower than the traditional technology, even under low SNR and low snapshots. Therefore, the proposed technology combined with the coprime MIMO radars can obtain better DOA estimation performance.

4.3. Probability of Detection

In this part, we compare the probability of detection (PD) of the three coprime MIMO radars by using traditional technology and proposed technology in Figure 5 via 200 Monte-Carlo experiments, where we suppose the angle of the targets are $[20 ^\circ : 10 ^\circ : 20 ^\circ]$. Figure 5(a) depicts the PD of different cases in terms of $SNR = \left[ -10 : 2 : 10 \right] dB$, where the number of snapshots $L = 200$, Figure 5(b) depicts the PD of
different cases in terms of the number of snapshots $L=[50:50:500]$, where the $SNR=0dB$. Here, PD is defined as the ratio of the number of trials with the DOA estimation error within $\pm0.01^\circ$ to the total experiments.

![Graph](image1)

![Graph](image2)

Figure 5 (a) PD versus the SNR. (b) PD versus the number of snapshots

It can be clearly seen from Figure 5 that PD of all these cases improves with the increase of the SNR and the number of snapshots. For the same coprime MIMO radar, PD of the proposed technology is higher than that of the traditional technology, because the proposed technology makes full use of more virtual array elements information. Therefore, the proposed technology combined with symmetric coprime MIMO radar can achieve the best DOA estimation performance.

5. Conclusion

In this paper, we have proposed coprime MIMO radar DOA estimation based on the average processing of redundant virtual array elements information with high performance. It has been verified that making full use of the redundant virtual array elements formed by the sum and difference coarray can obtain superior DOA estimation performance. Simulation results verify the effectiveness of the proposed technology in terms of DOA estimation accuracy.

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