A Two stage Adaptive Knowledge Transfer Evolutionary Multi-tasking Based on Population Distribution for Multi/Many-Objective Optimization

Zhengping Liang, Weiqi Liang, Xiuju Xu, and Zexuan Zhu, Member, IEEE

Abstract—Multi-tasking optimization can usually achieve better performance than traditional single-tasking optimization through knowledge transfer between tasks. However, current multi-tasking optimization algorithms have some deficiencies. For high similarity problems, the knowledge that can accelerate the convergence rate of tasks has not been utilized fully. For low similarity problems, the probability of generating negative transfer is high, which may result in optimization performance degradation. In addition, some knowledge transfer methods proposed previously do not fully consider how to deal with the situation in which the population falls into local optimum. To solve these issues, a two stage adaptive knowledge transfer evolutionary multi-tasking optimization algorithm based on population distribution, labeled as EMT-PD, is proposed. EMT-PD can accelerate and improve the convergence performance of tasks based on the knowledge extracted from the probability model that reflects the search trend of the whole population. At the first transfer stage, an adaptive weight is used to adjust the step size of individual’s search, which can reduce the impact of negative transfer. At the second stage of knowledge transfer, the individual’s search range is further adjusted dynamically, which can increase the diversity of population and beneficial for jumping out of local optimum. Experimental results on multi-tasking multi-objective optimization test suites show that EMT-PD is superior to six state-of-the-art optimization algorithms. In order to further investigate the effectiveness of EMT-PD on many-objective optimization problems, a multi-tasking many-objective test suite is designed. The experimental results on it also demonstrate that EMT-PD has obvious competitiveness.

Index Terms—Multi-objective Optimization, Many-objective Optimization, Evolutionary multi-tasking, Population distribution, Knowledge transfer

I. INTRODUCTION

MULTI-OBJECTIVE optimization problems (MOPs) widely exist in real world [1-5]. A MOP can be described as follows:

\[
\min_x (F(x) = (f_1(x), \ldots, f_n(x)))
\]

subject to \(x \in \Omega^D\) \hspace{1cm} (1)

where \(x = (x_1, \ldots, x_D)\) represents a \(D\)-dimensional decision vector in search space \(\Omega^D\). \(F(x) = (f_1(x), \ldots, f_n(x))\) denotes the objective function vectors with \(n\) objectives. If \(n > 3\), a MOP is also called a many-objective optimization problem (MaOP) [4, 5]. Given two different solutions \(x\) and \(y\), if \(x\) is superior to \(y\) in terms of all the objective functions and \(F(x) \neq F(y)\), then \(x\) is said to dominate \(y\). A solution not dominated by any other solutions is called a Pareto optimal solution. The set of all Pareto optimal solutions is referred to as the Pareto optimal set (PS). The corresponding projection of PS in the objective space is called Pareto front (PF).

Evolutionary algorithms can solve MOPs and MaOPs effectively, which can be roughly divided into three categories, indicator-based algorithms [6-9], decomposition-based algorithms [10-13] and domination-based algorithms [14-17]. However, those algorithms only can solve one MOP at a time. Inspired by the capability of human brain to process transactions in parallel, Gupta et al. [18] proposed a new paradigm called evolutionary multi-tasking (EMT) to deal with multiple optimization tasks simultaneously. EMT can accelerate the convergence rate and improve the whole performance of optimization. The kernel module of EMT algorithm is the knowledge transfer.

In recent years, a large number of multi-objective EMT algorithms have been proposed. Gupta et al. [19] proposed a multi-objective EMT algorithm (MOMFEA), which optimized multiple multi-objective problems simultaneously through assortative mating and vertical cultural transmission. The diversity and convergence of the population can be improved effectively. Yang et al. [20] presented a two stage assortative mating method, which firstly divides decision variables into two types, namely, diversity variables and convergence variables, then both of which undergo assortative mating with different parameters to balance the diversity and convergence. Feng et al. [21] proposed an EMT algorithm with explicit genetic transfer (EMT-EGT), providing multiple search mechanisms to improve the search ability of the population. Chen et al. [22] proposed a memetic EMT framework for knowledge transfer between subpopulations. The framework can maintain the diversity of population effectively. Tuan et al. [23] proposed an EMT algorithm employing local search strategy, which can improve the convergence rate of current population by information of previous population.

EMT also has been widely used in real-world problems recently. Chandra et al. [24] presented an EMT algorithm to optimize neural networks. In the algorithm, each task optimizes different numbers of hidden neurons, which improves the stability of neural networks. Yuan et al. [25] proposed an EMT algorithm with two novel mechanisms, that is unified
representation and survivor selection procedure, which can optimize multiple real-world problems simultaneously. Sagarna et al. [26] applied EMT to solve the branch testing problem. Computational cost can be reduced by exploiting inter-branch knowledge. Li et al. [27] proposed an EMT framework to optimize multiple sparse reconstruction tasks simultaneously. Rauniyar et al. [28] proposed an EMT algorithm to solve multiple pollution-routing problems simultaneously.

Although EMT algorithms proposed previously have made great progress, there is still room for improvement. For high similarity problems, the knowledge of high quality solutions not fully used to improve the convergence rate of population. For low similarity problems, the population distributions of two tasks are different, which easily generates negative transfer with high probability between tasks [29]. Besides, some knowledge transfer methods proposed previously do not fully consider how to deal with the situation where the population falls into local optimum.

In order to improve the effectiveness of knowledge transfer, this paper proposes a two stage adaptive knowledge transfer EMT algorithm based on population distribution (EMT-PD). EMT-PD firstly builds probability models for each task, and then obtains knowledge from the product of different probability models. Those knowledge can help to accelerate the convergence rate of population. At the first stage of knowledge transfer, the step size of individual’s search is adjusted by adaptive weight, which can reduce the probability of generating negative transfer. At the second stage of knowledge transfer, the search range of individual is further adjusted dynamically, which can increase the diversity of population and beneficial for jumping across the area of local optima.

The main contributions of this paper are highlighted as follows:

1) A multi-tasking multi/many-objective evolutionary optimization algorithm with novel knowledge extract and transfer is proposed to improve the efficiency and performance of optimization.

2) A multi-tasking many-objective optimization test suite is designed based on a representative many-objectives test suite MaF [30].

3) Based on three test suites, EMT-PD is fully analyzed by comparing with state-of-the-art algorithms.

The rest of this paper is constructed as follows. Section II introduces related work and motivation. Section III describes the details of EMT-PD. Section IV discusses experimental design and analysis. Section V tests the performance of EMT-PD with different probability models. In the end, section VI concludes future works.

II. RELATED WORK AND MOTIVATION

This section introduces the proposed knowledge extract and transfer methods proposed previously and the motivation of this paper.

A. Extract and Transfer Knowledge from Single Individual

In this paper, knowledge transfer methods based on single individual (KTS) refer extracting knowledge from single individual of one task, and transferring knowledge to other tasks. EMT algorithms with KTS include MFEA [18], M-BLEA [31], LDA-MFEA [32], S&M-MFEA [33], MO-MFEA [19], GMFEA [34], TMO-MFEA [20], MTO-DRA [35], MFEA-II [36] and MFEA-GHS [37]. All above algorithms transfer knowledge through assortative mating and vertical cultural transmission [38]. In assortative mating, two individuals are selected from population randomly, and then generate offspring by simulated binary crossover (SBX) and polynomial mutation. In vertical cultural transmission, each offspring is randomly assigned to a task. In KTS, each individual provides different knowledge for tasks, the diversity of population is maintained effectively. However, EMT algorithms with KTS cannot fully utilize the knowledge of high quality solutions to accelerate the convergence rate of population due to the randomness in knowledge transfer.

B. Extract and Transfer Knowledge from Multiple Individuals

Knowledge transfer methods based on multiple individuals (KTM) refer to extracting knowledge from multiple individuals of one task, and transferring knowledge to other tasks. EMT algorithms with KTM can be divided into three categories: 1) EMT algorithms with particle swarm optimization (PSO). Feng et al. [39] proposed multi-tasking PSO algorithm. The convergence of population is accelerated by optimal solutions of multiple tasks. Tang et al. [40] proposed an adaptive multi-tasking PSO algorithm. It designs a self-adaptation strategy to adjust the inter-task knowledge transfer probability, which reduces the probability of negative transfer effectively. Song et al. [41] proposed a multi-tasking multi-swarm optimization algorithm. The quality of solutions are improved by crossover between optimal individuals of all tasks. 2) EMT algorithms with differential evolution (DE). Liu et al. [42] proposed SaM-MA employing three different mechanisms, i.e., DE algorithm, predicting optimal solution via surrogate model, and local searching strategy. The mixed use of three mechanisms can balance diversity and convergence of population. Zhou et al. [43] proposed a new mutation strategy called DE/best/1+ρ, gradually increasing the weight of knowledge transfer in the process of evolution, which improves the diversity of population. 3) EMT algorithms with explicit knowledge transfer. Feng et al. [21] first proposed an EMT algorithm EMT-EGT with explicit knowledge transfer, which allows the incorporation of multiple search mechanisms with different biases. It can improved the search ability of population. Shang et al. [44] proposed a credit assignment approach, which selects proper individuals for explicit knowledge transfer. The efficiency of knowledge transfer of it can be improved. In conclusion, KTM is beneficial to improve the convergence of population, but it also has the probability of extracting knowledge from inferior individuals. Besides, the search of entire population at one generation is guided by the same solutions in some KTM, which increase the probability of population falling into local optimum.

C. Motivation

For high similarity problems, knowledge extracted from the high quality solutions of one task can accelerate the
Fig. 1: The illustration of knowledge transfer and search direction, where x1 and x2 are the two dimensions of decision variable. Triangles and squares represent high quality solutions and inferior solutions, respectively. Hollow represents solutions of task1, and solid represents solutions of task2. The hollow circle represents an solution selected from task1. (a) and (b) are KTS and KTM in high similarity scenario respectively. (c) and (d) are KTS and KTM in low similarity scenario respectively.

convergence of another task effectively [44]. However, no matter KTS or KTM, knowledge may be extracted from inferior individuals. Fig. (a) shows an example of KTS in high similarity scenario. In the search space, high quality solutions of task1 and task2 converge in the same area. p1 is a solution of task1. p2 is a high quality solution of task2. p3 is an inferior solution of task2. If knowledge is transferred from p2 to p1, then p1 will be guided to search the area of high quality solutions aggregation. However, KTS may transfer knowledge from inferior solution p3 to p1, which would slow down the convergence of task1. Fig. (b) shows an example of KTM in high similarity scenario. p1 is an solution of task1. p2 and p3 are two high quality solutions of task2, respectively. p4 is a inferior solution of task2. If knowledge is transferred from p2 and p3 to p1, p1 will search the area of high quality solutions aggregation. However, KTS may also transfer the knowledge of inferior solution p4 to p1, which would causes p1 to deviate from the ideal search direction.

For low similarity problems, the population distribution of task1 and task2 is very different [45]. Whether in KTS or KTM, population of different tasks are difficult to guide each other to search with ideal search direction. There would be a high probability of generating negative transfer between those tasks. Fig. (c) shows an example of KTS in low similarity scenario, high quality solutions of different tasks are distributed in different areas. p1 represents an individual of task1. p2 is a high quality solution of task2. When knowledge is transferred from p2 to p1, p1 will deviate severely from the convergence area of high quality solutions of task1. Fig. (d) shows an example of KTM in low similarity scenario, which is similar to the situation of KTS.

Furthermore, in KTMs with PSO or with DE, the search of entire population at one generation is only guided by the best solutions of two tasks, which easily leads the population falling into local optimum.

To address the above issues, this paper presents a novel EMT algorithm with two stage adaptive knowledge transfer based on population distribution. Specifically, the probability models are firstly built for the population of each task, and the knowledge used for transfer is extracted from the maximum point of the probability models’ product. Then, at the first stage of knowledge transfer, the step size of individual’s search is adaptively adjusted to reduce the impact of negative transfer. At the second stage of knowledge transfer, the search range of individual is adjusted again dynamically, which can increase the diversity of population and reduces its probability of falling into local optimum.

III. PROPOSED ALGORITHM

In this section, we first introduce the main framework of EMT-PD. Then the process of building probability model and extracting knowledge is proposed. After that, the details of two stage adaptive knowledge transfer are explained. Finally, computational complexity analysis of EMT-PD is discussed.

A. The Main Framework of EMT-PD

The main framework of EMT-PD is summarized in Algorithm 1. Firstly, the population P is initialized, and divided into two sub-populations pop1 and pop2 for different tasks. At each evolutionary generation, the probability model is built for each task by Algorithm 2 in line 4. In line 5, the maximum point mp of the product of probability models is obtained by Formula 5 which is used as the common knowledge to guide the following knowledge transfer. The two stage adaptive knowledge transfer and offspring generation carried out by Algorithm 3 in line 6. It is worth noting that EMT-PD is a general algorithm, which supports to adopt various types of probability model.

Algorithm 1 Main Framework of EMT-PD

Input: R, type of probability model.
Output: a series of non-dominated solutions.
1: Initialize population P;
2: Split P into two subpopulations pop1 and pop2;
3: while stopping conditions are not satisfied do
4: Build probability model for pop1 and pop2 by Algorithm 2
5: Calculate the maximum points of product of two probability models mp according to Formula 5
6: Conduct two stage adaptive knowledge transfer and generating offspring C by Algorithm 3
7: Evaluate offspring;
8: Environmental selection;
9: end while
Algorithm 2: Build Probability Model

**Input:** pop, the subpopulation of task; \( R \), the type of probability model.

**Output:** \( M \), the probabilistic model; \( m \), maximum point of \( M \).

1: for each dimension of decision variable do
2: Generate the log-likelihood \( LL(\theta_j) \) by Formula (2).
3: Build probability model \( M_j \) for \( pop \) by Formula (3).
4: Calculate the maximum point \( m_j \) of \( M_j \) by Formula (4).
5: end for

### B. Build Probability Model

In this paper, the Maximum Likelihood Estimation (MLE) is used to estimate the parameters of probability model according to population. And the maximum point of probability model is obtained.

Suppose \( M \) is the probability models of population, \( \theta_j \) is the parameter of \( M \) in \( j \)-th dimension of decision variable. The MLE is used to find the parameter value \( \hat{\theta}_j \), which can maximize the log-likelihood \( LL(\theta_j) \). Firstly, \( LL(\theta_j) \) is calculated as follows:

\[
LL(\theta_j) = \sum_{i=1}^{N} \ln f(p_{i,j}|\theta_j)
\]

(2)

where \( p_{i,j} \) represents the \( j \)-th variance of \( i \)-th individual, \( i = 1, 2, ..., N \). \( N \) denotes the population size. The type of probability model \( f(p_{i,j}|\theta_j) \) is \( R \). \( \ln \) denotes the natural logarithm.

Then, \( \hat{\theta}_j \) is calculated as follows:

\[
\hat{\theta}_j = \arg\max_{\theta_j} LL(\theta_j)
\]

(3)

Based on above calculations, the probability model of \( j \)-th decision variable \( M_j \) can obtained, that is \( f(p_{i,j}|\theta_j) \).

As the probability model learned from population reflects the search trend of current population, the maximum point \( m \) of \( M \) is the most representative point which comprehensively reflects the search trend of entire population. The formula of calculating \( m_j \) is as follows:

\[
m_j = \arg\max_{x} M_j
\]

(4)

Algorithm 2 shows the process of building probability models by MLE. For each dimension of decision variable, the log-likelihood \( LL(\theta_j) \) is calculated by the Formula (2) in line 2. Then the parameter of probability model \( M_j \) are calculated by Formula (3) in line 3. After that, the maximum point \( m_j \) of \( M_j \) is obtained according to Formula (4) in line 4.

Finally, the probability model \( M \) and the maximum point \( m \) of \( M \) is obtained, where \( M = (M_1, M_2, ..., M_j, ..., M_D) \), \( m = (m_1, m_2, ..., m_j, ..., m_D) \). \( D \) is the dimension of decision variable.

### C. Extract Knowledge from Population Distribution

The product of two populations’ probability models reflects the common search trend of related populations. In our algorithm, the maximum point of the product is used as the common knowledge to guide the search of each task. The maximum point of the product comprehensively reflects the largest search trend of those two population. It is helpful to accelerate convergence rate and decrease the probability of falling into local optimum.

According to Algorithm 2, the two probability models \( M_1 \) and \( M_2 \) can be learned from population of task1 and task2, respectively, where \( M_1 = (M_{1,1}, M_{1,2}, ..., M_{1,j}, ..., M_{1,D}) \) and \( M_2 = (M_{2,1}, M_{2,2}, ..., M_{2,j}, ..., M_{2,D}) \). The maximum point \( mp \) of the product of \( M_1 \) and \( M_2 \) can be calculated as follows:

\[
mp_j = \arg\max_{x} (M_{1,j} \times M_{2,j})
\]

(5)

As shown in Fig. 2, the green line is the population distribution of task1. The blue line is the population distribution of task2. \( m_1 \) and \( m_2 \) are the maximum point of task1 and task2 respectively. The red line represents the product of population distributions of task1 and task2, which reflects the common search trend of task1 and task2. The maximum point \( mp \) of red line reflects the max common search trend of task1 and task2, which is used as the common knowledge to guide the search of each population.
Algorithm 3 Two stage Adaptive Knowledge Transfer and Offspring Generation

Input: pop, the subpopulation of task; mp, the maximum point of product of two probability models; m, the maximum point of the probability model of pop.

Output: C, offspring.

1: Calculating the Euclidean distance $d_1$ of m and mp according to Formula [1] for each individual $p ∈ pop$ do
2: Calculating the Euclidean distance $d_2$ of m and $p$ according to Formula [7]
3: Generating an intermediate individual $p'$ according to Formula [8]
4: Generating an offspring $c$ according to $p'$ by Formula [10]
5: $c' = $ Polynomial mutation($c$);
6: $C = C ∪ c'$
7: end for

$D. Two stage Knowledge Transfer and Offspring Generation$

For high similarity problems, populations of task1 and task2 will converge to the approximate area, the convergence rate of task1 and task2 can be accelerated by mp directly. But for low similarity problems, populations of task1 and task2 will converge to different area. If mp is taken directly as the guide to the search of population, there may be a lot of negative transfer. In addition, it is necessary to balance the convergence and the diversity of populations no matter for high or low similarity problems. To solve those issues, we propose a two stage adaptive knowledge transfer based on population distribution. At the first stage, the step size of individual’s search is adjusted adaptively to reduce the impact of negative transfer. At the second stage, the search range of individual is further adjusted dynamically, which increases the diversity of population and helps to jumping out of local optimum.

Fig. 3 displays the process of two stage knowledge transfer, where $p$ is a individual selected from population of certain task. mp is the maximum point of product of probability models. At the first stage of knowledge transfer, the knowledge is transferred from mp to p. The search of p is guided by mp, and an intermediate individual $p'$ is generated by adaptive weight. At the second stage, the search range of the intermediate individual $p'$ is adjusted again dynamically to generate offspring $c$.

Algorithm 3 is the pseudo code of the two stage adaptive knowledge transfer and offspring generation for one task. Firstly, the Euclidean distance $d_1$ between mp and the maximum point of the task’s probability model is calculated in line 1. Then for each individual $p$, the Euclidean distance $d_2$ between p and m is calculated in line 3. $d_1$ and $d_2$ are calculated as follows:

$$d_1 = \sqrt{\text{tr}((m - mp)^T(m - mp))}$$ \hspace{1cm} (6)

where $\text{tr}(\cdot)$ denotes the trace operation of a matrix.

$$d_2 = \sqrt{\text{tr}((m - p)^T(m - p))}$$ \hspace{1cm} (7)

In line 4, the intermediate individual $p'$ is generated via the first stage of knowledge transfer. $p'$ is calculated is as follows:

$$p' = p + w(mp - p)$$ \hspace{1cm} (8)

where $w$ is the adaptive weight of knowledge transfer. $w$ is defined as follows:

$$w = d_2/(d_1 + d_2)$$ \hspace{1cm} (9)

As m reflects the search trend of population which $p$ belongs to, if mp is close to m, it means that the population distribution of the two tasks are similar, and the knowledge from mp can effectively guide the search of p. Therefore, it is suitable to increase the step size from p to $p'$ when mp is close to m. In Formula [9] $d_1$ is the distance of mp and m, the smaller of $d_1$ is, the larger of $w$ is. On the contrary, if m is far away from p, there are significant discrepancy between the population distributions of two tasks, and the knowledge from mp may leads negative transfer between tasks. It should decrease the step size from p to $p'$ in the situation. In Formula [9] the larger of $d_1$ is, the smaller of $w$ is. In addition, $d_2$ is used to measure the distance between current individual $p$ and the aggregation region of high quality solutions. If $d_2$ is small, it means that the step size from p to $p'$ also only needs small adjustment. In Formula [9] the smaller of $d_2$ is, the smaller of $w$ is. If $d_2$ is large, that is to say, there are large distance from p to the convergence area of high quality solutions, the step size from p to $p'$ should be increased by a large margin. In Formula [9] the larger of $d_2$ is, the larger of $w$ is.

In line 5, the offspring $c$ is generated via the second stage of knowledge transfer as follows:

$$c = p' + v$$ \hspace{1cm} (10)

where $v$ is a search vector defined as follows:

$$v = \frac{1}{D} * F * Q * (d_1 + d_2)$$ \hspace{1cm} (11)

where $D$ is the dimension of decision variable, $Q$ is the $D$-dimensional Gaussian White Noise. $F$ is the scale factor.

The reason to introduce the second stage knowledge transfer is to better balance the convergence and the diversity of populations due to all individuals are searched in the same direction guided by mp at the first stage. When $d_1$ is small, the knowledge transfer at the first stage may effectively due to the high similarity between two tasks. It is not necessary to substantially adjust the range of search at the second stage in Formula [11]. On the contrary, if $d_1$ is large, it means the low similarity between two tasks. The adjustment of search range at the second stage should be reasonably increased. As for $d_2$, if it is small, the individual p is close to the convergence area. It is also not necessary to substantially adjust again at the second stage. If $d_2$ is large, the adjustment of search range at the second stage also should be reasonably increase. Polynomial mutation is performed in line 6. Offspring $c'$ is combined into population C in line 7.
E. Computational Complexity Analysis

In this section, the computational complexity of EMT-PD within a generation is discussed. EMT-PD mainly consists of three parts: 1) building probability model; 2) two stage knowledge transfer and offspring generation; and 3) environmental selection. The maximum likelihood estimation is used to build probability model, which requires computational complexity of $O(DN)$. In two stage knowledge transfer and offspring generation, a computational complexity of $O(DN)$ is needed. In environmental selection, computational complexity of non-dominated sorting and crowding distance are $O(nN^{2})$ and $O(nN\log(N))$, respectively. To sum up, the computational complexity of EMT-PD is $O(nN^{2})$, where $D$ and $N$ represent the dimension of decision variable and population size respectively. $n$ is the number of objectives.

IV. EXPERIMENT AND ANALYSIS

The performance of EMT-PD is firstly evaluated and compared to several state-of-the-art optimization algorithms on multi-tasking multi-objective test suites. Then a novel multi-tasking many-objective test suite is proposed, and the performance of EMT-PD on it is also evaluated and compared to state-of-the-art optimization algorithms.

A. Experiment and Analysis on Multi-objective Problems

1) Test Problems and Compared Algorithms: To assess the performance of EMT-PD, two multi-objective test suites are applied.

The classical multi-tasking multi-objective test suite M-TMOPs [47], which can be split into three groups according to the degree of intersection, i.e., complete intersection (CI), partial intersection (PI), and no intersection (NI). Moreover, these groups can be further partitioned into high similarity (HS), medium similarity (MS), and low similarity (LS). Therefore, there are nine problems, namely, CIHS, CIMS, CILS, PIHS, PIILS, PIIMS, PLS, NIHS, NIMS, and NILS.

The complex multi-tasking multi-objective optimization test suite CEC2019-CMO was proposed in IEEE CEC2019 competition on evolutionary multi-tasking optimization. CEC2019-CMO contains ten multi-tasking multi-objective problems. The details of CEC2019-CMO are summarized in [48].

Six state-of-the-art algorithms are used in comparison with EMT-PD, including three multi-objective EMT algorithms, i.e., MO-MFEA [19], TMO-MFEA [20], EMT-EGT [21], and three representative multi-objective evolutionary algorithms, i.e., NSGA-II [14], AR-MOEA [6] and CMOPSO [49]. NSGA-II is a basic multi-objective evolutionary algorithms serving as the baseline here. AR-MOEA is an indicator-based multi-objective optimization algorithms with an enhanced inverted generational distance indicator. CMOPSO is a multi-objective particle swarm optimization algorithm with competitive mechanism.

2) Performance Metric: The Inverted Generational Distance (IGD) [50] is widely used to evaluate the performance of multi-objective algorithms. IGD is calculated as follows:

$$IGD(P^*, A) = \frac{1}{|P^*|} \sum_{z \in P^*} \min d(z, A)$$  \hspace{2cm} (12)

where $d(z, A)$ refers to the Euclidean distance between a reference point $z$ and a solution $A$ in the objective space, and $P^*$ represents a predefined set of reference points on the PF. The smaller IGD is, the better the convergence and diversity of population is.

3) Parameter Settings: In EMT-PD, the type of probability model $R$ is set to Gaussian probability model, and the scale factor $F$ is set to 0.01. The parameters of TMO-MFEA are set according to [20], i.e., $rmp$ is set to 1 for the diversity variable and 0.3 for the convergence variable. According to the description of EMT-EGT in [21], SPEA2 is used for one task and NSGA-II is used for the other task. The interval of explicit transmission is set to 10. The size of the elite population $\lambda$ of CMOPSO is set to 10 according to [49]. Common parameter settings of algorithms are summarized in Table I.

4) Results and Analysis on MTMOPs: Table II shows the experimental results on MTMOPs. The best result is highlighted. The Wilcoxon rank sum test is performed at a significance level of 5%. Symbols ‘+’, ‘−’, ‘≈’ denote that the result is significantly better, significantly worse, or comparable with that of EMT-PD, respectively. Next, the experimental results are analyzed from the degree of similarity.

EMT algorithms can work well on high similarity problems CI+HS, PI+HS and NI+HS. The overall performance of EMT algorithms are better than single-tasking algorithms. These results confirm that knowledge transfer across tasks in EMT is capable of accelerating convergence and finding better solutions.

For medium similarity problems CI+MS, PI+MS and NI+MS, EMT algorithms except for EMT-PD are not very competitive. CI+MS, PI+MS and NI+MS are comprised of unimodal and multimodal functions. The unimodal function only has a single global optimal solution. With the development of evolution, the population of unimodal function will gradually converge to a small area of search space. On the contrary, multimodal function has multiple global optimal solutions or local optimal solutions, the population of it will gradually converge to multiple different areas of search space. Due to populations’ convergence areas of unimodal and multimodal functions are quite different, the probability of negative transfers is high. MO-MFEA, TMO-MFEA and EMT-EGT cannot effectively handle those kinds of negative transfer, which makes the performance of them degradation. However, EMT-PD can reduce the probability of generating negative transfer and increase the diversity of population through the two stages adaptive knowledge transfer, which makes a good

| Parameter                      | Value |
|--------------------------------|-------|
| Size of population ($N$)       | 200   |
| Maximal iteration ($G$)        | 1000  |
| Maximal function evaluations ($EF's$) | 200000 |
| Crossover probability ($p_c$)  | 0.3   |
| Mutation probability ($p_m$)   | $1/N$ |
| Distribution index for crossover ($\eta_c$) | 20 |
| Distribution index for mutation ($\eta_m$) | 20 |

TABLE I: COMMON PARAMETER SETTINGS OF ALGORITHMS
EMT algorithms still show obvious competitiveness on NI+LS owing to the step size of search can be adjusted by adaptive weight at the first stage of knowledge transfer.

5) Results and Analysis on CEC2019-CMO: Table III shows the experimental results on CEC2019-CMO. The overall performance of EMT-PD is better than other algorithms obviously. CMO2, CMO4 and CMO10 are composed of functions with the same PS. Therefore, the diversity and convergence of population can be maintained well via knowledge transfer between tasks. Comparing with single-task algorithms,
the performance of EMT algorithms is excellent. For CMO1, CMO3 and CMO6, objective functions of two tasks are very similar, which means the two tasks have similar properties. EMT algorithms can effectively optimize this kind of problems. For CMO7, CMO8 and CMO9 with different PS and objective functions, the performances of EMT algorithms proposed previously are not obvious. However, base on adaptive step size of search at the first stage of knowledge transfer, EMT-PD can effectively reduce the probability of negative transfer. The PFs of two tasks in CMO5 are very complex. To achieve better performance, the diversity of population needs to be carefully maintained in the optimization process. EMT-EGT provides multiple search mechanisms for the population, which is conducive to maintain the diversity of population to obtain best performance on CMO5.

Fig. 4 shows the average performance score of all algorithms on MTMOPs and CEC2019-CMO. The calculation method of average performance score can refer to [51]. The lower average score is, the better performance of IGD the algorithm gets. EMT-PD has achieved the best results on MTMOPs and CEC2019-CMO, which demonstrates the proposed knowledge extract and transfer method can effectively improve the performance of algorithm.

B. Experiment and Analysis on Many-objective Problems

1) Test Problems and Compared Algorithms: In order to further validate the performance of EMT-PD on MaOPs, we designs a novel multi-tasking many-objective test suite based on MaF test suite [30], labelled as MTMaOPs. The proposed MTMaOPs includes six problems, as shown in Table IV. MaF-HS1 is composed of MaF3 and MaF4. MaF-HS2 is composed of MaF4 and MaF6. MaF-MS1 is composed of MaF1 and MaF5*, where MaF5* is a shifted MaF5. The shifted individual is \( z = (p - r) \), where \( r = (0.05)^D \) and the original individual \( p \in (0, 1)^D \). MaF-MS2 is composed of MaF5 and MaF6*, where MaF6* uses the same shift operation as MaF5*. MaF-LS1 is composed of MaF4 and MaF5. MaF-LS2 is composed of MaF3 and MaF6. The value of Spearman rank correlation coefficient (\( Rs \)) for each problem of MTMaOPs is calculated using 1,000,000 randomly generated points in the unified search space as [47]. The \( Rs \) lying in \((1/3, 2/3, 1), (2/3, 1)\) is regarded as low, medium, and high similarity, respectively. According to the degree of similarity, MTMaOPs is divided into high similarity problems MaF-HS1 and MaF-HS2, medium similarity problems MaF-MS1 and MaF-MS2, low similarity problems MaF-LS1 and MaF-LS2. In addition, each problem of MTMaOPs is set to contain three instances of different objective dimensions, that is, \( n = 10, n = 20, \) and \( n = 30 \).

Six state-of-the-art algorithms are used in comparison with EMT-PD on MTMaOPs, including three EMT algorithms, i.e., MO-MFEA [19], TMO-MFEA [20], EMT-EGT [21], and three many-objective optimization algorithms, i.e., NSGA-III [4], VaEA [52] and DDEANS [53]. NSGA-III is a basic many-objective evolutionary algorithms serving as the baseline here. VaEA is characterized by novel selection strategies. DDEANS is characterized by novel dynamical decomposition strategy.

2) Performance Metric: The Modified Inverted Generational Distance (IGD+) [54] is adopted in this paper as the performance evaluation measure on MTMaOPs. IGD+ considers the Pareto dominance relation between reference vector and solution. It can accurately evaluate the performance of many-objective optimization algorithms. IGD+ is calculated as follows:

\[
IGD_+(P^*, A) = \frac{1}{|P^*|} \sum_{z \in P^*} \min \left[ \sum_{k=1}^{n} (\max(\{z_k - A_k\}, 0))^2 \right] 
\]

where \( z = (z_1, z_2, ..., z_n) \) represents a reference vector. \( A = (A_1, A_2, ..., A_n) \) represents a solution set. \( n \) is the dimension of objectives. \( P^* \) represents a predefined set of reference points on the PF. In this experiment, the number of reference points for calculating IGD_+ is set to 100,000. The smaller value of IGD_+ is, the better convergence and diversity of population gets.

3) Parameter Settings: Because decomposition-based algorithms NSGA-III and VaEA are limited by reference vector, the population size of NSGA-III and VaEA can not be set optionally. For fair comparison, the same setting of population size are employed for all algorithms. The setting of population size can refer to [55]. The details of parameters

| Problem  | Task | Function  | \( Rs \) with \( m = 10 \) | \( Rs \) with \( m = 20 \) | \( Rs \) with \( m = 30 \) |
|----------|------|-----------|-------------------|-------------------|-------------------|
| MaF-HS1  | T1   | MaF3      | 1                 | 1                 | 1                 |
|          | T2   | MaF4      |                   |                   |                   |
| MaF-HS2  | T1   | MaF4      |                   |                   |                   |
|          | T2   | MaF6      |                   |                   |                   |
| MaF-MS1  | T1   | MaF1      | 0.3703            | 0.3756            | 0.4236            |
|          | T2   | MaF5*     |                   |                   |                   |
| MaF-MS2  | T1   | MaF5      | 0.3865            | 0.3866            | 0.4396            |
|          | T2   | MaF6*     |                   |                   |                   |
| MaF-LS1  | T1   | MaF4      | 0.0038            | 0.0051            | 0.0044            |
|          | T2   | MaF5      |                   |                   |                   |
| MaF-LS2  | T1   | MaF3      | 0.0038            | 0.0052            | 0.0051            |
|          | T2   | MaF6      |                   |                   |                   |
TABLE V: PARAMETERS SETTING FOR DIFFERENT OBJECTIVE DIMENSION ON MTaMOPs

| $n$ | $H$     | $N$ | $G$   | FEs  |
|-----|---------|-----|-------|------|
| 10  | (3,1)   | 230 | 300   | 69000|
| 20  | (2,2)   | 420 | 300   | 126000|
| 30  | (2,1)   | 465 | 300   | 139500|

TABLE VI: PARAMETER SETTINGS FOR CROSSOVER AND MUTATION ON MTaMOPs

| Parameter                          | Value          |
|------------------------------------|----------------|
| Crossover probability ($p_c$)      | 0.3            |
| Mutation probability ($p_m$)       | $1/N$          |
| Distribution index for crossover ($\eta_c$) | 20            |
| Distribution index for mutation ($\eta_m$) | 20            |

for different objective dimension are summarized in Table V where $n$ indicates the number of objectives, $H$ is the number of divisions considered along each objective coordinate, $N$ represent the size of population, $G$ is the maximal iteration, FEs is maximal function evaluations. The common parameters of crossover and mutation are summarized in Table VI.

4) Results and Analysis on MTMaOPs: Table VII lists the statistical results on MTMaOPs. Except for MaF-HS2 and MaF-MS1, EMT-PD performs better than other algorithms. This result shows that the knowledge extract and transfer of EMT-PD is efficient on many-objective optimization problems. As following, the results of MaF-HS2 and MaF-MS1 are detailed analyzed.

MaF-HS2 is a high similarity problem and composed of MaF4 and MaF6. The property of high similarity can promote the cooperation between two tasks, and improve the diversity and convergence of population for EMT algorithms. It makes all EMT algorithms have obvious competitiveness on MaF-HS2 comparing with single-tasking many-objective algorithms. MaF4 is a badly scaled many-objective function. The traditional non-dominated sorting employed by EMT-PD cannot normalize the value of objective function, which leads the population of MaF4 is easy to converge to one side of PF, and cannot guide the search of population toward to convergence area effectively. It makes the poor performance of EMT-PD compared with EMT-EGT and TMO-MFEA.

MaF-MS1 is a medium similarity problem and composed of MaF1 and MaF5*. MaF1 is a linear problem. Its Pareto optimal solutions mainly concentrate on a very small area of search space. In the early stage of evolution, only few individuals can reflect the true search trend of population for MaF1. VaEA adopts maximum-vector-angle-first principle in environmental selection to ensure the diversity of population. DDEANS can balance dynamically the diversity and convergence of population according to the Euclidean distance between reference vector and population. The diversity mechanism of TMO-MFEA and EMT-EGT are also superior to EMT-PD, which brings better performance of TMO-MFEA and EMT-EGT than EMT-PD on MaF-MS1.

Fig. 5 shows the average performance score of all algorithms on MTMaOPs. The smaller the score is, the better IGD$^+$ the algorithm gets. It can be seen that single-tasking many-objective optimization algorithms achieves better performance than EMT algorithms on most of the test problems, which illustrates that EMT algorithm proposed previously cannot effectively deal with MaOPs. EMT-PD is still competitive on MaOPs, because the search direction of each variable is guided by population distribution, which makes the knowledge transfer is more accurate.

Fig. 6 shows the average performance score of all algorithms over 10, 20 and 30 objective dimensions for MTMaOPs in term of IGD$^+$. It can be seen that with the increase of objective dimension, the average performance score of EMT-PD becomes lower, which indicates that EMT-PD has more advantages in handling optimization problems with larger objective dimensions.

V. THE PERFORMANCE OF EMT-PD WITH DIFFERENT PROBABILITY MODELS

EMT-PD supports different probability models to fit population distribution. This section discusses the performance of EMT-PD with different probability models.
A. The Error of Fitting Population Distribution with Different Probability Models

Fig. 7 shows the average error of fitting population distribution with different probability models on CI+MS, PI+MS, CEC2019-CMO9, MAF-HS2 and MAF-LS2. The average error $e_{avg}$ is calculated as follows:

$$ e_{avg} = \log\left(\frac{1}{G} \sum_{g=1}^{G} e_g\right) $$

where $e_g$ represents the error at the $g$-th generation, $g = 1, 2, ..., G$, and $G$ represents the maximum number of iterations. $e_g$ is calculated as follows:

$$ e_g = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{D} \left| 1 - M_g(x_{i,j}) \right| $$

where $x_{i,j}$ represents the $j$-th dimension of $i$-th individuals, $M_g(x)$ represents the probability model calculated by Algorithm 2 at the $g$-th generation. $N$ is the population size, and $D$ is the dimension of decision variable.

It can be seen from Fig. 7 that the average error of EMT-PD with Gaussian probability model is very small, especially for CI+MS, PI+MS, CEC2019-CMO9, MAF-HS2 and MAF-LS2. For CEC2019-CMO5, the average error of Exponential probability model, Gamma probability model and Beta probability model are also small. In summary, Gaussian probability model is the most versatile model for fitting all kinds of test suites, and other probability models may also suit for specific problems.
B. IGD and IGD\+

of EMT-PD with Different Probability Models

Fig. 7 (a)-(h) shows the average IGD of EMT-PD with different probability models on 30 independent running on PI+MS, NI+LS, 2019CEC-CMO5 and 2019CEC-CMO8. Fig. 8 (i)-(l) shows the average IGD of EMT-PD with different probability models on 30 independent running on MAF-HS2 and MAF-MS1. It can be seen that EMT-PD with Gaussian probability model has better performance than EMT-PD with other probability models. EMT-PD with exponential model and Beta model shows poor performance on NI+LS, MAF-HS2 and MAF-MS1, which indicates that the universality of exponential model and Beta model are weak. It is important to note that the convergence speed of EMT-PD with Gamma model is faster than with Gaussian model on NI+LS in Fig. 8(d) which means that a strategy with adaptive selecting probability model is more suitable for the improvement of performance.

VI. CONCLUSION

This paper proposes a two stage adaptive knowledge transfer EMT algorithm based on population distribution. The knowledge extracted from the maximum point of probability model of population distribution can effectively guide the search of population for convergence. The first stage of knowledge transfer is characterized by a novel adaptive weight, which can effectively reduce the probability of generating negative transfer. At the second stage of knowledge transfer, the search range of individuals is adjusted dynamically again to balance the diversity and the convergence of population and to help jumping across local optimum. At the same time, in order to further study the performance of EMT-PD on MaOPs, a novel test suite MTMaOPs based on MaF test suite is proposed. EMT-PD is compared with state-of-the-art algorithms on MT-MOPs, CEC2019-CMO and MTMaOPs. The experimental results show that EMT-PD is competitive.

Although EMT-PD has shown superior performance on various test suites, there are still some further works worth doing. For example, all kinds of real-world problems are very different and complex. Gaussian probability model may not be the best model to fit specific real-world problem. It is valuable to propose an adaptive selection strategy of probability model according to the characteristics of problem. In addition, many-tasking optimization is also an interesting field worth investigation. We will try to expand EMT-PD to many-tasking optimization in the future.

REFERENCES

[1] J. H. Wang, Y. Zhou, Y. Wang, J. Zhang, C. L. P. Chen, and Z. B. Zheng, “Multiobjective vehicle routing problems with simultaneous delivery and pickup and time windows: Formulation, instances, and algorithms,” IEEE Transactions on Cybernetics, vol. 46, no. 3, pp. 582–594, March 2016.
[2] F. Sarro, F. Ferrucci, M. Harman, A. Manna, and J. Ren, “Adaptive multi-objective evolutionary algorithms for overtime planning in software projects,” IEEE Transactions on Software Engineering, vol. 43, no. 10, pp. 898–917, Oct 2017.
[3] L. Z. Cui, C. Xu, S. Yang, J. Z. Huang, J. Q. Li, X. Z. Wang, Z. Ming, and N. Lu, “Joint optimization of energy consumption and latency in mobile edge computing for internet of things,” IEEE Internet of Things Journal, vol. 6, no. 3, pp. 4791–4803, June 2019.
[4] K. Deb and H. Jain, “An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part i: Solving problems with box constraints,” IEEE Transactions on Evolutionary Computation, vol. 18, no. 4, pp. 577–601, Aug 2014.
[5] M. Li, S. Yang, and X. Liu, “Diversity comparison of pareto front approximations in many-objective optimization,” IEEE Transactions on Cybernetics, vol. 44, no. 12, pp. 2568–2584, Dec 2014.
[6] Y. Tian, R. Cheng, X. Y. Zhang, F. Cheng, and Y. C. Jin, “An indicator-based multiobjective evolutionary algorithm with reference point adaptation for better versatility,” IEEE Transactions on Evolutionary Computation, vol. 22, no. 4, pp. 609–622, Aug 2018.
[7] W. J. Hong, K. Tang, A. M. Zhou, H. Ishibuchi, and X. Yao, “A scalable indicator-based evolutionary algorithm for large-scale multiobjective optimization,” IEEE Transactions on Evolutionary Computation, vol. 23, no. 3, pp. 525–537, June 2019.
[8] L. G. de la Fraga and E. Tlelo-Cuautle, “Optimizing an amplifier by a many-objective algorithm based on r2 indicator,” in 2015 IEEE International Symposium on Circuits and Systems (ISCAS), May 2015, pp. 265–268.
[9] Y. N. Sun, G. G. Yen, and Z. Yi, “IGD indicator-based evolutionary algorithm for many-objective optimization problems,” IEEE Transactions on Evolutionary Computation, vol. 23, no. 2, pp. 173–187, April 2019.
[10] Q. F. Zhang and H. Li, “MOEA/D: A multiobjective evolutionary algorithm based on decomposition,” IEEE Transactions on Evolutionary Computation, vol. 11, no. 6, pp. 712–731, Dec 2007.
M. Y. Wu, K. Li, S. Kwong, Y. Zhou, and Q. F. Zhang, “Matching-based selection with incomplete lists for decomposition multiobjective optimization,” IEEE Transactions on Evolutionary Computation, vol. 21, no. 4, pp. 554–568, Aug 2017.

R. Cheng, Y. C. Jin, M. Olhofer, and B. Sendhoff, “A reference vector guided evolutionary algorithm for many-objective optimization,” IEEE Transactions on Evolutionary Computation, vol. 20, no. 5, pp. 773–791, Oct 2016.

X. Y. He, Y. R. Zhou, Z. F. Chen, and Q. F. Zhang, “Evolutionary many-objective optimization based on dynamical decomposition,” IEEE Transactions on Evolutionary Computation, vol. 23, no. 3, pp. 361–375, June 2019.

K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, “A fast and elitist multiobjective genetic algorithm: NSGA-II,” IEEE Transactions on Evolutionary Computation, vol. 6, no. 2, pp. 182–197, Apr 2002.

X. F. Zou, Y. Chen, M. Z. Liu, and L. S. Kang, “A new evolutionary algorithm for solving many-objective optimization problems,” IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), vol. 38, no. 5, pp. 1402–1412, Oct 2008.

Y. Yuan, H. Xu, B. Wang, and X. Yao, “A new dominance relation-based evolutionary algorithm for many-objective optimization,” IEEE Transactions on Evolutionary Computation, vol. 20, no. 1, pp. 16–37, Feb 2016.

V. Palakonda, S. Ghorbanpour, and R. Mallipeddi, “Pareto dominance-based moea with multiple ranking methods for many-objective optimization,” in 2018 IEEE Symposium Series on Computational Intelligence (SSCI), July 2018, pp. 958–964.

A. Gupta, Y. S. Ong, and L. Feng, “Multifactorial evolution: Toward evolutionary multitasking,” IEEE Transactions on Evolutionary Computation, vol. 20, no. 3, pp. 343–357, June 2016.

A. Gupta, Y. S. Ong, L. Feng, and K. C. Tan, “Multiobjective multifactorial optimization in evolutionary multitasking,” IEEE Transactions on Cybernetics, vol. 47, no. 7, pp. 1652–1665, July 2017.

C. E. Yang, J. L. Ding, K. C. Tan, and Y. C. Jin, “Two-stage assortative mating for multi-objective multifactorial evolutionary optimization,” in 2017 IEEE 56th Annual Conference on Decision and Control (CDC), Dec 2017, pp. 76–81.

L. Feng, L. Zhou, J. H. Zhong, A. Gupta, Y. S. Ong, K. Tan, and A. K. Qin, “Evolutionary multitasking via explicit autoencoding,” IEEE Transactions on Cybernetics, vol. 49, no. 9, pp. 3457–3470, Sep. 2019.

Y. L. Chen, J. H. Zhong, and M. K. Tan, “A fast memetic multiobjective differential evolution for multi-tasking optimization,” in 2018 IEEE Congress on Evolutionary Computation (CEC), July 2018, pp. 1–8.

N. Q. Tuan, T. D. Hoang, and H. T. Thanh Binh, “A guided differential evolutionary multi-tasking with powell search method for solving multi-objective continuous optimization,” in 2018 IEEE Congress on Evolutionary Computation (CEC), July 2018, pp. 1–8.

R. Chandra, A. Gupta, Y. S. Ong, and C. K. Goh, “Evolutionary multi-task learning for modular knowledge representation in neural networks,” Neural Processing Letters, vol. 47, no. 3, pp. 993–1009, June 2018.

Y. Yuan, Y. S. Ong, A. Gupta, P. S. Tan, and H. Xu, “Evolutionary multitasking in permutation-based combinatorial optimization problems: Realization with TSP, QAP, LOP, and JSP,” in 2016 IEEE Region 10 Annual International Conference (TENCON), Nov 2016, pp. 3157–3164.

R. Sagarna and Y. S. Ong, “Concurrently searching branches in software tests generation through multitask evolution,” in 2016 IEEE Symposium Series on Computational Intelligence (SSCI), Dec 2016, pp. 1–8.

H. Li, Y. S. Ong, M. G. Gong, and Z. K. Wang, “Evolutionary...
multitasking sparse reconstruction: Framework and case study,” in *IEEE Transactions on Evolutionary Computation*, vol. 23, no. 5, pp. 733–747, Oct 2019.

[28] A. Rauniyar, R. Nath, and P. K. Muhuri, “Multi-factorial evolutionary algorithm based novel solution approach for multi-objective pollution-routing problem,” *Computers and Industrial Engineering*, vol. 130, no. 5, pp. 757–771, Apr 2019.

[29] Y. S. Ong and A. Gupta, “Evolutionary multitasking: A computer science view of cognitive multitasking,” *Cognitive Computation*, vol. 8, no. 2, pp. 125–142, Apr 2016.

[30] M. Q. Li, Y. Tian, X. Y. Zhang, S. X. Yang, Y. C. Jin, and X. Yao, “A benchmark test suite for evolutionary many-objective optimization,” *IEEE Computational Intelligence Magazine*, vol. 3, no. 1, pp. 67–81, Mar 2017.

[31] A. Gupta, J. Mańdziuk, and Y. S. Ong, “Evolutionary multitasking in bi-level optimization,” *Complex & Intelligent Systems*, vol. 1, no. 1, pp. 83–105, Dec 2015.

[32] K. K. Bali, A. Gupta, L. Feng, Y. S. Ong, and P. S. Tan, “Linearized domain adaptation in evolutionary multitasking,” in *2017 IEEE Congress on Evolutionary Computation (CEC)*, June 2017, pp. 1295–1302.

[33] B. S. Da, A. Gupta, Y. S. Ong, and L. Feng, “Evolutionary multitasking across single and multi-objective formulations for improved problem solving,” in *2016 IEEE Congress on Evolutionary Computation (CEC)*, Jul 2016, pp. 1695–1701.

[34] J. L. Ding, C. Yang, Y. C. Jin, and T. Y. Chai, “Generalized multitasking for evolutionary optimization of expensive problems,” *IEEE Transactions on Evolutionary Computation*, vol. 23, no. 1, pp. 44–58, Feb 2017.

[35] M. G. Gong, Z. D. Tang, H. Li, and J. Zhang, “Evolutionary multitasking with dynamic resource allocating strategy,” *IEEE Transactions on Evolutionary Computation*, vol. 23, no. 5, pp. 858–869, Oct 2019.

[36] K. K. Bali, Y. S. Ong, A. Gupta, and P. S. Tan, “Multifactorial evolutionary algorithm with online transfer parameter estimation: MFEEA-II,” IEEE Transactions on Evolutionary Computation, pp. 1–1, Mar 2019.

[37] Z. P. Liang, J. Zhang, L. Feng, and Z. X. Zhu, “A hybrid of genetic transform and hyper-rectangle search strategies for evolutionary multi-tasking,” *Expert Systems with Applications*, vol. 138, no. 30, Dec 2019.

[38] D. N. Liu, A. Gupta, Y. S. Ong, and L. Feng, “The boon of gene-culture interaction for effective evolutionary multitasking,” *Lecture Notes in Computer Science*, vol. 9592, pp. 54–65, Feb 2016.

[39] L. Feng, W. Zhou, L. Zhou, S. W. Jiang, J. H. Zhong, B. S. Da, Z. X. Zhu, and Y. Wang, “An empirical study of multifactorial PSO and multifactorial DE,” in *2017 IEEE Congress on Evolutionary Computation (CEC)*, June 2017, pp. 921–928.

[40] Z. D. Tang and M. G. Gong, “Adaptive multifactorial particle swarm optimization,” *CAAI Transactions on Intelligence Technology*, vol. 4, no. 1, pp. 37–46, Mar 2019.

[41] H. Song, A. K. Qin, P. Tsai, and J. J. Liang, “Multitasking multi-swarm optimization,” in *2019 IEEE Congress on Evolutionary Computation (CEC)*, June 2019, pp. 1937–1944.

[42] D. N. Liu, S. J. Huang, and J. H. Zhong, “Surrogate-assisted multi-tasking memetic algorithm,” in *2018 IEEE Congress on Evolutionary Computation (CEC)*, Jul 2018, pp. 1–8.

[43] L. Zhou, L. Feng, K. Liu, C. Chen, S. J. Deng, T. Xiang, and S. W. Jiang, “Towards effective mutation for knowledge transfer in multifactorial differential evolution,” in *2019 IEEE Congress on Evolutionary Computation (CEC)*, June 2019, pp. 1541–1547.

[44] Q. Shang, L. Zhang, L. Feng, Y. Hou, J. Zhong, A. Gupta, K. C. Tan, and H. L. Liu, “A preliminary study of adaptive task selection in explicit evolutionary many-tasking,” in *2019 IEEE Congress on Evolutionary Computation (CEC)*, June 2019, pp. 2153–2159.

[45] A. Gupta, Y. S. Ong, B. Da, L. Feng, and S. D. Handoko, “Landscape synergy in evolutionary multitasking,” in *2016 IEEE Congress on Evolutionary Computation (CEC)*, July 2016, pp. 3076–3083.