Two-component quark-gluon plasma in stringy models

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Abstract. The two-component liquid model reproduces the basic properties of the quark-gluon plasma as observed in heavy-ion collisions. The key dynamic element of the model is the existence of a light scalar. We argue that existence of such a scalar is a generic feature of stringy models of quantum chromodynamics. The lattice data provide evidence for a condensed, three-dimensional scalar field as well. We outline a possible crucial check of the model on the lattice.

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Observations on the quark-gluon plasma at RHIC have led to the discovery of a quantum relativistic liquid at temperatures higher than the deconfinement phase transition, \( T > T_c \). A crucial observation is the importance of quantum effects, as follows from the low value of the ratio of the viscosity to the entropy density,

\[
\frac{\eta}{s} \approx \frac{1}{4\pi},
\]

for an analysis of the data see [3]. There exist not many models of quantum liquids, and superfluidity is a natural first candidate. And, indeed, the two-component model, with one component being superfluid, explains ‘naturally’ the basic observations on the plasma [4].

The phenomenology of a two-component liquid has been thoroughly discussed in the literature, see in particular [5, 6]. In the hydrodynamic approximation and neglecting dissipation effects one has the following basic expressions for the current and energy-momentum tensor:

\[
j^\mu = nu^\mu + f^2 \partial^\mu \phi \quad (2)
\]

\[
T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P\eta^{\mu\nu} + f^2 \partial^\mu \phi \partial^\nu \phi
\]

\[
u^\mu \partial_\mu \phi = \mu
\]

where we used the notations of Ref [6]. Here, \( u^\mu \) is the 4 velocity of an element of the liquid, representing the normal component, \( \phi \) is a scalar field, representing the superfluid component, and \( \mu \) is the chemical potential.

It is worth emphasizing, however, that the two-component picture has never been applied, to our knowledge, directly to the actual data on the quark-gluon plasma. The resulting low value of the viscosity, see (1), is then, in a way, in contradiction with the classical (not quantum) nature of the hydrodynamic model used. It would be very interesting to check, whether inclusion of terms containing derivatives from the scalar field into \( j^\mu \) and \( T^{\mu\nu} \) changes significantly fits to the data.

In this paper we consider the possibility that a variation of the famous two-component model of superfluidity applies directly to the quark-gluon plasma. The crucial issue is whether the QCD dynamics might produce a light scalar field entering (2). The answer we get is rather in the affirmative. The stringy models of QCD tend to predict light scalars. A crucial phenomenological test of the model is mentioned.

SCALAR CONDENSATE

General constraints

Dynamically, the validity of the superfluidity scenario depends strongly on the existence of an (effective) scalar field \( \phi \). The only known way to keep a (real) scalar field massless is to assume condensation of a complex field \( \phi \), condensed into the thermal vacuum

\[
\langle \phi \rangle_{\text{ground state}} \neq 0.
\]

The phase \( \phi \) of the field \( \phi \) corresponds then to a new light degree of freedom.

The condition (3) looks very restrictive and, in more detail, assumes a number of constraints:

a) The field \( \phi \) is a complex field:

\[
\phi^* \neq \phi.
\]

b) There should be then a charge which distinguishes the field \( \phi \). The thermal-vacuum expectation value \( \langle \phi \rangle \)
breaks spontaneously the corresponding symmetry. The problem is that in QCD we seemingly do not have any symmetry of this type.

c) In the case of superfluidity, one thinks rather in terms of a three-dimensional field \( \phi(\mathbf{r}) \) while the time derivative of the corresponding phase is determined by the chemical potential \( \mu : \partial_t \phi = \mu \). A relativistic generalization of this condition is included into \( \mathbf{2} \).

**Thermal scalar**

At first sight the conditions a)-c) above amount to a kind of no-go theorem for superfluidity. However, it is striking that a 3d field with similar properties arises naturally \( \mathbf{3} \) within a string model and is commonly called the thermal scalar. For a concise review and further insights see \( \mathbf{8} \).

One considers temperatures \( T \) below and close to the temperature of the (Hagedorn) phase transition \( T_H \). In the string picture \( \beta_H \equiv 1/T_H = 4\pi \alpha'^{1/2} \) (where \( 1/(2\pi \alpha') \sim 1/l_s^2 \) is the string tension). At \( T = T_H \) the statistical sum over the states diverges. The main observation is that at small \( |T - T_H| \) the sum is dominated by the contribution of a single degree of freedom, that is a complex scalar meson with the mass

\[
m^2 = \frac{\beta_H (\beta_H - \beta)}{2\pi^2 (\alpha')^2} ,
\]

In other words, at \( T = T_H \) the mass would become tachyonic. There exist various, dual interpretations of the thermal scalar. One way to visualize it is that \( \mathbf{4} \) refers to the mass of the mode once wrapped around the compact, Euclidean time direction.

To consider the plasma we should address temperatures above the phase transition, \( T > T_H \) where \( \mathbf{3} \) does not apply. Imagine, however, that the thermal scalar becomes tachyonic and condenses at \( T > T_H \). Then, remarkably enough, the conditions we formulated above are satisfied. Indeed:

a) The thermal scalar is a complex field. This is because the string can be wrapped around the Euclidean time coordinate in both directions, a typical \( U(1) \) situation.

b) Thus, the thermal scalar is associated with the topological quantum number which is the wrapping number around the compactified time direction. This is a quantum number specific for strings. And this is a remarkable resolution of the puzzle that we do need a symmetry to be spontaneously broken, on one hand, and we cannot identify such a symmetry in the field-theoretic language on the other hand. The dual-model language does allow for such an identification!

c) The thermal scalar is a 3d scalar field in the sense that the time derivative of its phase is fixed:

\[
|\partial_t \phi_{\text{thermal scalar}}| = 2\pi T ,
\]

Note an important change compared to the third equation in \( \mathbf{2} \).

**Three dimensional scalar at \( T > T_c \)**

Nowadays, it is common to consider dual models of Yang-Mills theories in terms of strings living in extra dimensions with non-trivial geometry. The thermal scalar at temperatures below and close to \( T_c \) is generic to such models as well, see \( \mathbf{3} \) and references therein. However, the phase transition is treated now as a change of geometry in the extra dimensions and the information on the scalar at \( T < T_c \) does not directly help to approach physics at \( T > T_c \). This is ‘bad news’. ‘Good news’ is that the 3d scalars are resurrected at \( T > T_c \) in another guise.

Very briefly, for details and classical references see, e.g., \( \mathbf{9} \), there are two compact directions, the Euclidean time and an extra one associated with the \( \theta \) dependence, or topological charge. As always, there is also the scale-related \( z \) direction, with a horizon, \( 0 < z < z_H \). At low temperatures the radius of the time direction is independent of \( z \) while the radius of the \( \theta \)-coordinate tends to zero at the horizon:

\[
R_\tau(z) = \text{const} , \quad R_\theta(z_H) = 0 ; T < T_c .
\]

At temperatures above the phase transition the geometry of the two compact coordinates is interchanged so that:

\[
R_\tau(z_H) = 0 , \quad R_\theta(z) = \text{const} ; T > T_c .
\]

As a result, one predicts that the defects become time-oriented at \( T > T_c \). In particular, the magnetic strings, well studied on the lattice at \( T < T_c \), \( \mathbf{10} \), are becoming time-oriented. The phenomenon can be readily understood from the example of the thermal scalar. Indeed, the corresponding stringy mode corresponds to wrapping around the time direction which fixes the time dependence and, as a result, the thermal scalar is a 3d field \( \mathbf{3} \).

The intersection of time-oriented strings with the spatial 3d volume is a set of trajectories which are predicted to percolate in 3d. In field theoretic language, the 3d trajectories correspond to a 3d scalar field, let us call it magnetic field \( \phi_{\text{magn}} \). The percolation, in turn, corresponds to

\[2\] The temperatures of the Hagedorn and deconfinement phase transitions coincide only for critical dimensions, \( d = 26 \) for the bosonic strings, see \( \mathbf{3} \). For us, however, only generic features of the thermal scalar are important. We do not rely literally on existence of the thermal scalar, see below.
a non-vanishing vacuum expectation value:
\[ \langle \phi_{\text{magn}} \rangle^2 \sim \Lambda_{\text{QCD}} , \] (8)
which is a prerequisite for superfluidity of the gluon plasma (see above). The independent lattice results do support the validity of the prediction [9], for detailed analysis see [11].

POSSIBLE CRUCIAL TEST OF THE MODEL

So far, we listed arguments in favor of the two-component model of the plasma. However, one cannot claim, of course, that the data validate the model. A crucial test of the model could be performed through lattice measurements of a correlator of components of the energy-momentum tensor \( T^{ij} \), \( i=1,2,3 \). In more detail, consider the retarded Green’s function defined as:
\[ G_R^{ij}(k) = i \int d^4xe^{-ikx} \langle \langle T^{ij}(x)T^{ij}(0) \rangle \rangle . \] (9)
Moreover, concentrate on the case of vanishing frequency, \( k_0 = 0 \). There are two independent form factors, corresponding to transverse and longitudinal waves.
\[ G_R^{ij}(0,k) = \frac{k^i k^j}{k^2} G_T^L(k) + \left( \delta^{ij} - \frac{k^i k^j}{k^2} \right) G_T^T(k) \] (10)
In the absence of the superfluidity \( G_T^L = G_T^R \) which means that there is no non-analyticity at \( k \to 0 \).

Contribution of the superfluid component to the \( G_T^L \) has been discussed in many papers and textbooks, see, in particular, Ref. [6], which includes also relativistic corrections. The result is
\[ \left( \lim_{k \to 0} [G_T^R(k) - G_T^L(k)] \right)_{\text{superfluidity}} = \rho_s \mu , \] (11)
where \( \rho_s \) is the density of the superfluid component, \( \mu \) is the chemical potential.

In case of the gluonic plasma which are considering a similar result is expected to hold. An educated guess is:
\[ \left( \lim_{k \to 0} [G_T^R(k) - G_T^L(k)] \right)_{\text{plasma}} \sim T(2\pi T)^2 \langle \phi_{\text{magn}} \rangle^2 , \] (12)
where \( T \) is temperature and \( \langle \phi_{\text{magn}} \rangle^2 \) is the vacuum expectation value of the magnetic scalar field discussed above. Again, a non-analytical term at \( k \to 0 \) is predicted.

Note that the proposed crucial test of the model [12] refers to static quantities. Since there is no time dependence, the continuation from the Euclidean to Minkowski space is straightforward and the prediction of the model, [12], can be tested on the lattice.

CONCLUSIONS

In more general terms, the stringy approach reveals mechanisms of generating dynamical (i.e. not seen in the QCD Lagrangian) \( U(1) \) symmetries. They are directly related to the topology of extra dimensions and specific for strings. Another lesson concerns applications of the holographic models. According to conventional wisdom, the probability to find a defect is exponentially suppressed in the limit of large \( N_c \):
\[ W_{\text{defect}} \sim \exp(-S_{\text{defect}}) , \quad S_{\text{defect}} \sim N_c . \] (13)
However, in the confining models it is generic that some radii of extra dimensions are vanishing, see [6], [7]. Then, in the classical approximation there are defects whose action is vanishing:
\[ S_{\text{defect}} \sim N_c \cdot 0 , \]
for examples see [9]. The effect of such defects should be added ‘by hand’ to the standard formalism.

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