\section*{$B^+$ and $B^0$ Direct CP Asymmetries difference in a sequential Fourth Generation scenario}

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\textbf{Abstract.} Direct CP violation in $B^0 \to K^+\pi^−$ decay has emerged at $−10\%$ level, but the asymmetry in $B^+ \to K^+\pi^0$ mode is consistent with zero. This difference points towards possible New Physics in the electroweak penguin operator. We point out that a sequential fourth generation, with sizable $V_{ts}'V_{tb}$ and near maximal phase, could be a natural cause. We compare $t'$ effects on direct CP violation in $B^+ \to K^+\pi^0$ with $b \to s\ell^+\ell^−$ and $B_s$ mixing. Such large effects in $b \to s$ transitions would affect $s \to d$ transitions, as kaon constraints would demand $V_{td} \neq 0$. Using $\Gamma(Z \to bb)$ to bound $|V_{ts}|$, we infer sizable $|V_{ts'}| \lesssim |V_{ts}| \lesssim |V_{us}|$. Imposing $\varepsilon_K$, $K^+ \to \pi^+\nu\bar{\nu}$ and $\varepsilon'/\varepsilon$ constraints, we find $V_{ts}'V_{ts} \sim \text{few } \times 10^{-4}$ with large phase, enhancing $K_L \to \pi^0\nu\bar{\nu}$ to $5 \times 10^{-10}$ or even higher. Interestingly, $\Delta m_{B_s}$ and $\sin 2\Phi_{B_s}$ are not much affected, as $|V_{td}V_{ts}| \ll |V_{td}V_{tb}| \sim 0.01$.

Direct CP violation (DCPV) in $B^0 \to K^+\pi^−$ decay has recently been observed \cite{1, 2} at the B factories. The combined asymmetry is $A_{K\pi} = -0.114 \pm 0.020$. However, the asymmetry in $B^+ \to K^+\pi^0$ decay is found to be \cite{2, 3} $A_{K\pi^0} = +0.049 \pm 0.040$, which differs from $A_{K\pi}$ by

$$A_{K\pi^0} - A_{K\pi} = +0.163 \pm 0.045,$$

with 3.6$\sigma$ significance. All existing models have predicted $A_{K\pi^0} \sim A_{K\pi}$, as this basically follows from isospin symmetry. The large difference of Eq. (1), if it persists, could indicate isospin breaking New Physics (NP), likely \cite{4, 5, 6, 7, 8, 9} through the electroweak penguin (EWP) operator or a large tree level color suppressed amplitude $C$ \cite{10}.

Following Ref. \cite{11} and \cite{12} we review how the existence of a 4th generation can be a natural source for EWP effects. The $t'$ quark can modify the EWP coefficients, but leave the strong and electromagnetic penguin coefficients largely intact. Eq. (1) can be accounted for, provided that $m_{t'} \sim 300$ GeV, and the quark mixing elements $V_{ts}'V_{tb}$ is not much smaller than $V_{ts}$ and has near maximal CP phase. Independently, $b \to s\ell^+\ell^−$ and $B_s$ mixing constraints can allow large $t'$ effects only if \cite{13} the associated CP phase is near maximal.

Such large effects in $b \to s$ transitions would affect $s \to d$ transitions. In view of the large $r_{sb}$ and $\phi_{sb}$ values, $V_{td} \neq 0$ is required, and one must explore $s \to d$ and $b \to d$ implications. The reasoning is as follows. Since a rather large impact on $V_{td}'V_{tb}$ is implied by a value of $V_{td}'V_{tb} \sim V_{cb}$ with a near maximal CP phase, if one sets $V_{td}' = 0$, then $V_{td}'V_{ts}$ would still be rather different from SM3 case. With our current knowledge of $m_{t}$, the $\varepsilon_K$ parameter would deviate from the well measured experimental value. Thus, a finite $V_{td}'$ is needed to tune for $\varepsilon_K$.

We find that the kaon constraints that are sensitive to $t'$ (i.e. $P_{EW}$-like, viz. $K^+ \to \pi^+\nu\bar{\nu}$, $\varepsilon_K$, and $\varepsilon'/\varepsilon$) can all be satisfied. Interestingly, once kaon constraints are satisfied, we find little impact is implied for $b \to d$ transitions, such as $\Delta m_{B_d}$ and $\sin 2\Phi_{B_d}$. That is, $V_{td}' \to 0$ works
approximately for $b \to d$ transitions, for current level of experimental sensitivity. The main outcome for $s \to d$ and $b \to d$ transitions is the enhancement of $K_L \to \pi^0\nu\bar{\nu}$ mode by an order of magnitude or more, to beyond $5 \times 10^{-10}$.

1. $B \to K\pi$ with 4th generation
Adding a fourth generation modifies short distance coefficients. Defining $\lambda_q = V_{qs}^* V_{qb}$, the effective Hamiltonian relevant for $B \to K\pi$ can be written as

$$H_{\text{eff}} \propto \lambda_u (C_{1} O_{1} + C_{2} O_{2}) + \sum_{i=3}^{10} (\lambda_i C_i^t - \lambda_i^* \Delta C_i) O_i,$$

where $O_{1,2}$ are the tree operators, $\lambda_i C_i^t$ are the usual SM penguin terms, and $-\lambda_i^* \Delta C_i$ with $\Delta C_i \equiv C_i^t - C_i^l$ is the 4th generation effect. We have used $\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0$, simplified by ignoring $|\lambda_u| \lesssim 10^{-3}$, such that $\lambda_i \approx -\lambda_c - \lambda_{t'}$ [14]. The penguin coefficients $\lambda_i C_i^t + \lambda_{t'} C_i^t$ at scale $\mu$ are then put in the form of Eq. (2), which respect the SM limit for $\lambda_{t'} \to 0$ or $m_{t'} \to m_t$. Explicit forms for $C_i$ and $O_i$ can be found, for example, in Ref. [15].

The $K\pi$ amplitudes are dominated by $C_{4,6}^t$. To illustrate $t'$ sensitivity, in Fig. 1 we plot $-\Delta C_i / |C_i^t|$ at $m_b$ scale vs $m_{t'}$. The effect is clearly most prominent for the EWP $C_9$ coefficient, with linear $x_{t'} = m_{t'}^2 / M_W^2$ dependence arising from $Z$ and box diagrams [14]. $\Delta C_7$ has similar dependence but has weaker strength. For the strong penguin $\Delta C_{4,6}$, the $t'$ effect in the QCD penguin loop is weaker than logarithmic [16] and is very mild. As we shall see, the $B^0 \to K^+\pi^-$ amplitude does not involve the EWP. In contrast, the $B^+ \to K^+\pi^0$ amplitude is sensitive to the EWP via $\Delta C_9 - \Delta C_7$ (virtual $Z$ materializing as $\pi^0$).

We see that it is natural for the 4th generation to show itself through the EWP. The effect depends also on the quark mixing matrix parameterized as [13]

$$\lambda_{t'} = V_{t's}^* V_{t'b} = r_{sb} e^{i\phi_{sb}}.$$

(3)

The phase $\phi_{sb}$ is needed to affect the CPV observables, Eq. (1).

Let us first see how $A_{K\pi} < 0$ can be generated. In the usual QCD factorization (QCDF) approach [17], strong phases are power suppressed, while strong penguin $C_4$ and $C_6$ coefficients

![Figure 1. The $t'$ correction $-\Delta C_i$ normalized to strength of strong penguin coefficient $|C_i^t|$ (both at $m_b$ scale) vs. $m_{t'}$.](image-url)
pick up perturbative absorptive parts. Thus, the predicted $A_{K\pi}$ is small, and turns out to be positive. For the perturbative QCD factorization (PQCD) [18] approach, one has an additional absorptive part coming from the annihilation diagram, which arises from a cut on the two quark lines in $B \to \bar{s}q \to K\pi$ decay. In this way, the PQCD approach predicted [18] the sign and order of magnitude of $A_{K\pi}$. By incorporating annihilation contributions as in PQCD, however, QCDF can also [19] give negative $A_{K\pi}$.

We adopt PQCD as a definite calculational framework. The $B^0 \to K^-\pi^+$ amplitude for the 3 generation SM is roughly given by

$$M_{K^-\pi^0}^{SM} \propto \lambda_u f_K F_e + \lambda_c (f_K F_e^P + f_B F_a^P),$$

where $F_e^{(P)}$ is the color-allowed tree (strong penguin) contribution and is real, and $F_a^{(P)}$ is the strong penguin annihilation term that has a large imaginary part. We have dropped subdominant non-factorizable effects. Factorizable and non-factorizable contributions can be computed by following Ref. [18]. We give the SM numbers for $F_e$, $F_e^P$ and $F_a^P$ in Table I, which leads to $A_{K\pi} = -0.16$ for $\phi_3 \equiv \arg \lambda_u = 60^\circ$, compared to the experimental value of $-0.114 \pm 0.020$.

For $B^- \to K^0\pi^0$, the difference with $K^-\pi^+$ is

$$\sqrt{2} M_{K^-\pi^0}^{SM} - M_{K^-\pi^+}^{SM} \propto \lambda_u f_K F_e + \lambda_c f_K F_e^P,$$

where $F_{ek}$ is the color suppressed tree term, while $F^P_{ek}$ is the color allowed EWP, and both are real. A negligible tree annihilation term $\lambda_u f_B F_e$ has been dropped. Since both the $F_e$ and $F_{ek}$ terms are subdominant compared to $F^P_{e}$ in the 3 generation SM, $A_{K^0\pi^0}$ and $A_{K\pi}$ cannot be far apart. From the values of $F_{ek}$ and $F_{ek}^P$ given in Table I, we get $A_{K^0\pi^0} = -0.10$, which is less negative than $A_{K\pi}$, but at some variance with Eq. (1).

While $t'$ quark, one finds $M_{K^-\pi^0} \approx M_{K^-\pi^+}^{SM}$. The difference is proportional to $\lambda_u (f_K \Delta F_e^P + f_B \Delta F_a^P)$, which is small unless $\lambda_u$ is very large. This is because $F_{e,a}^P$ are strong penguins, hence $\Delta F_{e,a}^P$ depends very weakly on $m_{t'}$, as can be seen from Table I (for $m_{t'} = 300$ GeV) and Fig. 1. Thus, $A_{K^0\pi^0}$ is insensitive to the 4th generation. For $K^-\pi^0$, one finds

$$\sqrt{2} M_{K^-\pi^0} - \sqrt{2} M_{K^-\pi^+} \propto -\lambda_u f_K \Delta F_{ek}^P,$$

where again $\Delta F_{e,a}^P$ terms have been dropped, and $\Delta F_{ek}^P$ is the $t'$ correction to the EWP, which is generated by $\Delta C_9 - \Delta C_7$ at short distance.

Performing a detailed calculation following Ref. [18], we plot $A_{K\pi}$ and $A_{K^0\pi^0}$ in Fig. 2(a) for $m_{t'} = 300, 350$ GeV and $r_{ab} = 0.01$ and 0.03. We see that, indeed, $A_{K\pi}$ is almost independent of $t'$, while it is clear that the largest impact on $A_{K^0\pi^0}$ is for $\phi_{ab} \sim \pm \pi/2$ and large $m_{t'}$ and $r_{ab}$. To maximize $A_{K^0\pi^0} - A_{K\pi} > 0$, $\phi_{ab} \sim +\pi/2$ is selected, and Eq. (1) can in principle be accounted for.

To entertain a large EWP effect in CPV in $b \to s\ell^+\ell^-$ and $B_s$ mixing constraints, as well as the usually stringent $b \to s\gamma$ constraint.

### Table 1. Factorizable contributions for $B^0[+ \to K^+\pi^-[0]$ in Standard Model, and for $m_{t'} = 300$ GeV. The difference between the $t'$ and $t$ penguin contributions gives $\Delta F_{t}^{P}_{e}$. “N.A.” stands for “not applicable”.

|       | tree          | $t$ penguin | $t'$ penguin |
|-------|---------------|-------------|--------------|
| $F_e^{(P)}$ | 0.841 [0.843] | -0.074 [-0.075] | -0.076 [-0.078] |
| $F_a^{(P)}$ | N.A. [0.001 + 0.002 i] | 0.003 + 0.026 i [0.003 + 0.026 i] | 0.003 + 0.026 i [0.003 + 0.026 i] |
| $F_{ek}^{(P)}$ | N.A. [-0.105] | N.A. [-0.014] | N.A. [-0.029] |
We have checked that the $b \to s\gamma$ rate constraint is well satisfied for the range of parameters under discussion. This is because on-shell photon radiation is generated by the $b \to s$ transition operator $O_{7\gamma}$, and the associated coefficient $\Delta C_{7\gamma}$ has weaker $m_t'$ dependence than $\Delta C_7$ shown in Fig. 1. However, $b \to s\ell^+\ell^-$ is generated by EWP [14] operators very similar to $O_{7-10}$ in Eq. (2) for $b \to s\bar{q}q$. The difference is basically just in the $Z$ charge of $q$ vs. $\ell$, hence with same $m_t'$ dependence. The box diagram for $B_s$ mixing also has similar $m_t'$ dependence. Taking the formulas from Ref. [13], we plot $b \to s\ell^+\ell^-$ rate ($m_{\ell\ell} > 0.2$ GeV) and $\Delta m_{B_s}$ vs. $\phi_{sb}$ in Figs. 2(c) and (d), for $m_{\ell'} = 300, 350$ GeV and $r_{sb} = 0.01$ and 0.03.

We can understand the finding of Ref. [13] that $\phi_{sb} \sim 90^\circ$ is best tolerated by the $b \to s\ell^+\ell^-$ and $\Delta m_{B_s}$ constraints. For $\cos \phi_{sb} < 0$, the $b \to s\ell^+\ell^-$ rate gets greatly enhanced [14], and would run against recent measurements. One is therefore forced to the $\cos \phi_{sb} > 0$ region, where $t'$ effect is destructive against SM $t$ effect. For $\Delta m_{B_s}$, the effect gets destructive for $\cos \phi_{sb} > 0$ when $r_{sb}$ is sizable. Since one just has a lower bound [20] of $14.4 \, \text{ps}^{-1}$ [21], $\Delta m_{B_s}$ tends to push one away from the $\cos \phi_{sb} > 0$ region. The combined effect is to settle around $\phi_{sb} \sim \pm \pi/2$, i.e. imaginary [13]. This result is independent of the discrepancy of Eq. (1).

We see that for a range of parameter space roughly around $m_{\ell'} \sim 300$ GeV and $0.01 < r_{sb} \lesssim 0.03$, solutions to Eq. (1) can be found that do not upset $b \to s\ell\ell$ and $\Delta m_{B_s}$. Both large $t'$ mass

![Figure 2](image-url)

**Figure 2.** (a) $A_{K\pi}$ and $A_{K^{*}\pi^0}$, (b) $2\Phi_{B_s}$, (c) $B(b \to s\ell^+\ell^-)$ and (d) $\Delta m_{B_s}$ vs. $\phi_{sb} = \text{arg} V_{tb}^* V_{ts}$. The solid and dashed curves are for $m_{\ell'} = 300$ and 350 GeV, respectively, and for $r_{sb} = |V_{tb}^* V_{ts}| = 0.01$ and 0.03. Horizontal solid band in (c) corresponds to 1$\sigma$ experimental range, and solid line in (d) is the lower limit, both from Ref. [20]. The experimental range for (c) is outside the plot.
and sizable $V_{ts}$ mixing are needed.

As prediction, we find $\sin 2\Phi_{B_d} < 0$ for CPV in $B_d$ mixing, which is plotted vs $\phi_{ab}$ in Fig. 2(b). We find $\sin 2\Phi_{B_s}$ in the range of $-0.2$ to $-0.7$ and correlating with $\mathcal{A}_{K^0\pi^0} - \mathcal{A}_{K^0\pi^-}$. Three generation SM predicts zero.

It is of interest to predict the asymmetries for the other two $B \to K\pi$ modes. $K^0\pi^-$ is analogous to $\mathcal{M}_{K^-\pi^+}$ except tree contribution is absent. We find $\sin 2\Phi_{B_d} \approx 300$ GeV, $V_{ct} V_{tb}$ is of order suggested by Eq. (1).

We adopt the parametrization in Ref. [22] where the third column and fourth row is kept as real and of order 1, one immediately finds the strength and complexity of $V_{tb}$ as given in Eq. (7), we depict Eq. (8) in Fig. 3(a).

Using SM3 values for $V_{us}^*V_{ub}$, $V_{cs}^*V_{cb}$ (validated later by our $b \to d$ study), since they are probed in multiple ways already, and taking $V_{ts}^*V_{tb}$ as given in Eq. (7), we depict Eq. (8) in Fig. 3(a).

The solid, rather squashed triangle is the usual $V_{us}V_{ub} + V_{cs}V_{cb} + V_{ts}V_{tb} = 0$ in SM3. Given the size and phase of $V_{ts}V_{tb}$, one sees that the invariant phase represented by the area of the quadrangle is rather large, and $V_{ts}V_{tb}$ picks up a large imaginary part, which is very different from SM3 case. Such large effect in $b \to s$ would likely spill over into $s \to d$ transitions, since taking $V_{tb}$ as real and of order 1, one immediately finds the strength and complexity of $V_{td}^*V_{ts}$ would be rather different from SM3, and one would need $V_{td}^*V_{ts} \neq 0$ to compensate for the well measured value for $\varepsilon_K$.

We adopt the parametrization in Ref. [22] where the third column and fourth row is kept simple. This is suitable for $B$ physics, as well as for loop effects in kaon sector. With $V_{cb}, V_{tb}$ and $V_{ts}$ defined as real, one keeps the SM3 phase convention for $V_{ub}$, now defined as $\arg V_{ub}^* = \phi_{ub}$, which is usually called $\phi_3$ or $\gamma$ in SM3. We take $\phi_{ub} = 60^\circ$ as our nominal value [23]. The two additional phases are associated with $V_{ts}$ and $V_{td}$, and for the rotation angles we follow the

![Figure 3](image-url)

**Figure 3.** Unitarity quadrangles of (a) Eq. (8), with $|V_{us}^*V_{ub}|$ exaggerated; (b) Eq. (23), where actual scale is $\sim 1/4$ of (a). Adding $V_{ts}^*V_{tb}$ (dashed) according to Eq. (7) drastically changes the invariant phase and $V_{ts}^*V_{tb}$ from the SM3 triangle (solid), but from Eq. (22), the dashed lines for $V_{td}^*V_{tb}$ and $V_{td}^*V_{tb}$ can hardly be distinguished from SM3 case.
PDG notation [20]. To wit, we have

$$V_{td} = -c_{24}c_{34}s_{14}e^{-i\phi_{db}},$$

$$V_{ts} = -c_{34}s_{24}e^{-i\phi_{ub}},$$

$$V_{tb} = -s_{34},$$

while $V_{t'\beta} = c_{14}c_{24}c_{34}$, $V_{tb} = c_{13}c_{23}c_{34}$, $V_{ob} = c_{13}c_{34}s_{23}$ are all real. With this convention for rotation angles, from Eq. (9) we have $V_{ub} = c_{34}s_{13}e^{-i\phi_{ub}}$.

Analogous to Eq. (7), we also define

$$V_{t'd}V_{t'b} \equiv r_{db}e^{i\phi_{db}}, \; V_{t'd}V_{t's} \equiv r_{ds}e^{i\phi_{ds}},$$

as these combinations enter $b \rightarrow d$ and $s \rightarrow d$ transitions. Inspection of Eqs. (7), (10–12) gives the relations

$$r_{db}r_{sb} = r_{ds}s_{34}^2, \; \phi_{ds} = \phi_{db} - \phi_{ab}. \quad (14)$$

As we shall see, $s \rightarrow d$ transitions are much more stringent than $b \rightarrow d$ transitions, hence we shall turn to constraining $r_{ds}$ and $\phi_{ds}$.

Before turning to the kaon sector, we need to infer what value to use for $s_{34} = |V_{t'b}|$, as this can still affect the relevant physics through unitarity. Fortunately, we have some constraint on $s_{34}$ from $Z \rightarrow bb$ width, which receives special $t$ (and hence $t'$) contribution compared to other $Z \rightarrow q\bar{q}$, and is now suitably well measured.

Following Ref. [24] and using $m_t = 300$ GeV, we find

$$|V_{tb}|^2 + 3.4|V_{t'b}|^2 < 1.14. \quad (15)$$

Since all $c_{ij}s$ except perhaps $c_{34}$ would still likely be close to 1, we infer that $s_{34} \lessapprox 0.25$. We take the liberty to nearly saturate this bound by imposing

$$s_{34} \simeq 0.22, \quad (16)$$

to be close to the Cabibbo angle, $\lambda \equiv |V_{ub}| \simeq 0.22$. Combining it with Eq. (7), one gets $|V_{ts}| \sim 0.11 \sim \lambda/2$. Its strength would grow if a lower value of $s_{34} \lesssim \lambda$ is chosen, which would make even greater impact on $s \rightarrow d$ transitions.

Using current values [20] of $V_{cb}$ and $V_{ub}$ as input and respecting full unitarity, we now turn to the kaon constraints of $K^+ \rightarrow \pi^+\nu\bar{\nu}$, $\varepsilon\K$, and $\varepsilon'/\varepsilon$. The first two are short-distance (SD) dominated, while the last two suffer from long-distance (LD) effects.

Let us start with $K^+ \rightarrow \pi^+\nu\bar{\nu}$. The first observed event [25] by E787 suggested a sizable rate hence hinted at NP. The fourth generation would be a good candidate, since the process is dominated by the Z penguin. Continued running, including E949 data has yielded overall 3 events, and the rate is now $\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) = (1.47^{+1.30}_{-0.89}) \times 10^{-10}$ [26]. This is still somewhat higher than the SM3 expectation of order $0.8 \times 10^{-10}$.

Defining $\lambda_{qs}^d \equiv V_{qs}V_{qs}^*$ and using the formula [27]

$$\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) = \kappa_+ \left| \frac{\lambda_{cs}^d}{|V_{us}|} P_{\ell c} + \frac{\lambda_{qs}^d}{|V_{us}|^5} q_\ell X_0(x_\ell) \right|^2 + \frac{4}{|V_{us}|} q_\ell X_0(x_\ell)^2, \quad (17)$$

we plot in Fig. 4 the allowed range (valley shaped shaded region) of $r_{ds} - \phi_{ds}$ for the 90% confidence level (C.L.) bound of $\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) < 3.6 \times 10^{-10}$. We have used [27] $\kappa_+
Figure 4. Allowed region from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ (valley shaped shaded region), $\varepsilon_K$ (simulated dots) and $\varepsilon'/\varepsilon$ (elliptic rings) in $r_{ds}$ and $\phi_{ds}$ plane, as described in text, where $V_{td}^* V_{ts} \equiv r_{ds} e^{i\phi_{ds}}$. For $\varepsilon'/\varepsilon$, the rings on upper right correspond to $R_6 = 2.2$, and $R_8 = 0.8, 1.1$ (bottom to top), and on upper left, $R_6 = 1.0, 1.2$ (bottom to top), $R_8 = 1.2$.

$(4.84 \pm 0.06) \times 10^{-11} \times (0.224/|V_{us}|)^4$ and $P_c = (0.39 \pm 0.07) \times (0.224/|V_{us}|)^4$. We take the QCD correction factors $\eta_{t'c} \sim 1$, and $X_0(x_{t'})$ evaluated for $m_t = 166$ GeV and $m_{t'} = 300$ GeV. We see that $r_{ds}$ up to $7 \times 10^{-4}$ is possible, which is not smaller than the SM3 value of $4 \times 10^{-4}$ for $|V_{td}^* V_{ts}|$

The rather precisely measured CPV parameter $\varepsilon_K = (2.284 \pm 0.014) \times 10^{-3}$ [20] is predominantly SD. It maps out rather thin slices of allowed regions on the $r_{ds}$-$\phi_{ds}$ plane, as illustrated by dots in Fig. 4, where we use the formula of Ref. [24] and follow the treatment. Note that $r_{ds}$ up to $7 \times 10^{-4}$ is still possible, for several range of values for $\phi_{ds}$. This is the aforementioned effect that extra CPV effects due to large $\phi_{sb}$ and $r_{sb}$ now have to be tuned by $t'$ effect to reach the correct $\varepsilon_K$ value. We have checked that $\Delta m_K$ makes no additional new constraint.

The DCPV parameter, Re $(\varepsilon'/\varepsilon)$, was first measured in 1999 [28], with current value at $(1.67 \pm 0.26) \times 10^{-5}$ [20]. It depends on a myriad of hadronic parameters, such as $m_s$, $\Omega B$ (isospin breaking), and especially the non-perturbative parameters $R_6$ and $R_8$, which are related to the hadronic matrix elements of the dominant strong and electroweak penguin operators. With associated large uncertainties, we expect $\varepsilon'/\varepsilon$ to be rather accommodating, but for specific values of $R_6$ and $R_8$, some range for $r_{ds}$ and $\phi_{ds}$ is determined.

We use the formula

$$\text{Re} \frac{\varepsilon'}{\varepsilon} = \text{Im} (\lambda_{et}^{ds}) P_0 + \text{Im} (\lambda_{et'}^{ds}) F(x_t) + \text{Im} (\lambda_{et'}^{ds}) F(x_{t'}),$$  \hspace{1cm} (18)$$

where $F(x)$ is given by

$$F(x) = P_X X_0(x) + P_Y Y_0(x) + P_Z Z_0(x) + P_E E_0(x).$$ \hspace{1cm} (19)$$

The SD functions $X_0$, $Y_0$, $Z_0$ and $E_0$ can be found, for example, in Ref. [29], and the coefficients
$P_i$ are given in terms of $R_6$ and $R_8$ as

$$P_i = r_i^{(0)} + r_i^{(6)} R_6 + r_i^{(8)} R_8,$$  \hspace{1cm} (20)

which depends on LD physics. We differ from Ref. [29] by placing $P_0$, multiplied by Im ($\Lambda_c^{d_s}$), explicitly in Eq. (18). In SM4, one no longer has the relation Im $\Lambda_c^{d_s} = -\text{Im} \lambda_c^{d_s}$ that makes Re ($\varepsilon'/\varepsilon$) proportional to Im ($\lambda_c^{d_s}$). We take the $r_i^{(j)}$ values from Ref. [29] for $\Lambda_c^{d_s} = 310$ MeV, but reverse the sign of $r_i^{(j)}$ for above mentioned reason. Note that Re ($\varepsilon'/\varepsilon$) depends linearly on $R_6$ and $R_8$. For fixed SD parameters $m_{\nu}$ and $\lambda_d^{ds} = V_{ud}V_{us}^*$, one may adjust for solutions to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $\varepsilon_K$.

For the “standard” [29] parameter range of $R_6 = 1.23 \pm 0.16$ and $R_8 = 1.0 \pm 0.2$, we find $R_8 \sim 1.2$ and $R_6 \sim 1.0 - 1.2$ allows for solutions at $r_{ds} \sim (5-6) \times 10^{-4}$ with $\phi_{ds} \sim (+35^\circ - 50^\circ)$, as illustrated by the elliptic rings on upper left part of Fig. 4. For $R_6 = 2.2 \pm 0.4$ found [30] in $1/N_C$ expansion at next-to-leading order (and chiral perturbation theory at leading order), within SM3 one has trouble giving the correct Re ($\varepsilon'/\varepsilon$) value. However, for SM4, solutions exist for $R_6 \sim 2.2$ and $R_8 = 0.8 - 1.1$, for $r_{ds} \sim (3.5 - 5) \times 10^{-4}$ and $\phi_{ds} \sim -(45^\circ - 60^\circ)$, as illustrated by the elliptic rings on upper right part of Fig. 4. We will take

$$r_{ds} \sim 5 \times 10^{-4}, \quad \phi_{ds} \sim -60^\circ \text{ or } +35^\circ, \hspace{1cm} (21)$$

as our two nominal cases that satisfy all kaon constraints.

To illustrate in a different way, we plot $\varepsilon_K$, $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and Re ($\varepsilon'/\varepsilon$) vs $\phi_{ds}$ in Figs. 5(a), (b) and (c), respectively, for $r_{ds} = 4$ and $6 \times 10^{-4}$. The current $1\sigma$ experimental range is also illustrated. In Fig. 5(c), we have illustrated with $R_6 = 1.1$, $R_8 = 1.2$ [29] and $R_6 = 2.2$, $R_8 = 1.1$ [30]. For the former (latter) case, the variation is enhanced as $R_6$ ($R_8$) drops.

It is interesting to see what are the implications for the CPV decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$. The formula for $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is analogous to Eq. (17), except [29] the change of $\kappa_+ \rightarrow \kappa_L = (2.12 \pm 0.03) \times 10^{-10} \times (|V_{us}|/0.224)^8$, and taking only the imaginary part for the various CKM products. Since $\phi_{ds} \sim -60^\circ$ or $+35^\circ$ have large imaginary part, while $r_{ds} = |V_{td}V_{ts}^*| \sim 5 \times 10^{-4}$ is stronger than the SM3 expectation of Im $V_{td}V_{ts}^* \sim 10^{-4}$, we expect the CPV decay rate of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ to be much enhanced.

We plot $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ vs $\phi_{ds}$ in Fig. 5(d), for $r_{ds} = 4$ and $6 \times 10^{-4}$. Reading off from the figure, we see that the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ rate can reach above $10^{-9}$, almost two orders of magnitude above SM3 expectation of $0.3 \times 10^{-10}$. It is likely above $5 \times 10^{-10}$, and in general larger than $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. Specifically, for our nominal value of $r_{ds} \sim 5 \times 10^{-4}$ and $\phi_{ds} \sim +35^\circ$, $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ are 6.5 and $2 \times 10^{-10}$, respectively, while for the $\phi_{ds} \sim -60^\circ$ case, they are 12 and $3 \times 10^{-10}$, respectively. The latter case is closer to the Grossman-Nir bound [31], i.e. $B(K_L \rightarrow \pi^0 \nu \bar{\nu})/B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \sim \tau_{K_L}/\tau_{K^+} \sim 4.2$, because $V_{td}V_{ts}^*$ is more imaginary. Thus, both $\pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ should be very interesting at the next round of experiments. However, for $r_{ds} \sim 3.5 \times 10^{-4}$ and $\phi_{ds} \sim -45^\circ$, which is still a solution for $R_6 \sim 2.2$, one has $B(K_L \rightarrow \pi^0 \nu \bar{\nu}) \sim 4 \times 10^{-10}$ with $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ at lower end of current range.

With $\phi_{db} \sim 70^\circ$ and $\phi_{db} \sim -60^\circ$ (and $+35^\circ$) both sizable while the associated CKM product is larger than the corresponding SM3 top contribution, there is large impact on $b \rightarrow s$ and $s \rightarrow d$ transitions from $Z$ penguin and box diagrams. It is therefore imperative to check that one does not run into difficulty with $b \rightarrow d$ transitions. Remarkably, we find that the impact on $b \rightarrow d$ is mild. From Eqs. (7), (14), (16) and (21), we infer

$$r_{db} \sim 1 \times 10^{-3}, \quad \phi_{db} \sim 10^\circ (105^\circ). \hspace{1cm} (22)$$

Since $r_{db}$ is much smaller than $|V_{td}V_{tb}| \sim \lambda^3 \sim 0.01$ in SM3, the impact on $b \rightarrow d$ is expected to be milder, i.e. we are not far from the $V_{td} \rightarrow 0$ limit. We stress that this is nontrivial since
there is a large effect in $b \to s$; it is a consequence of imposing $s \to d$ and $Z \to b\bar{b}$ constraints. We illustrate in Fig. 3(b) the unitarity quadrangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* + V_{t'd'}V_{tb}^* = 0.$$  

(23)

In contrast to Fig. 3(a), $(V_{td}V_{tb}^* + V_{t'd'}V_{tb}^*)_{SM4}$ and $(V_{td}V_{tb}^*)_{SM3}$ can hardly be distinguished.

The $B_d^0 - \bar{B}_d^0$ mass difference and CP violation phase in mixing are respectively given by

$$\Delta m_{B_d} \equiv 2|\lambda_{t'b}^d|^2 \eta_t S(x_t) + (\lambda_{t'b}^d)^2 \eta_t' S(x_{t'})$$

$$+ 2\lambda_{t'b}^d \lambda_{t'b}^{d'} \eta_{tt'} S(x_t, x_{t'})$$

(24)

with $\kappa_{B_d} = \frac{G_F^2 m_W^2 m_{B_d} f_{B_d}^2}{12\pi^3}$. The functions $S(x)$ and $S(x,y)$ can be found in [32]. We take $\eta_t = 0.55$, $\eta_{t'} = 0.58$ and $\eta_{tt'} = 0.50$, and plot in Fig. 6(a) $\Delta m_{B_d}$ vs $\phi_{dB}$, for $r_{ds} = 8$ and $12 \times 10^{-4}$ (corresponding to $r_{ds} = 4$ and $6 \times 10^{-4}$). We have taken the experimental value of $\Delta m_{B_d} = (0.505 \pm 0.005)$ ps$^{-1}$ from PDG 2005 [20], and illustrated with the lower range of $f_{B_d} \sqrt{B_{B_d}} = (246 \pm 38)$ MeV [33]. We have scaled up the error for the latter by 1.4, since it comes from the new result on $f_{B_d}$ with unquenched lattice QCD [34], but $B_{B_d}$ is not yet

![Figure 5](https://example.com/figure5.png)

**Figure 5.** (a) $\varepsilon_K$, (b) $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$, (c) Re$(\varepsilon'/\varepsilon)$ and (d) $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})$ vs $\phi_{dB}$, for $r_{ds} = 4$ and $6 \times 10^{-4}$ and $m_{\nu} = 300$ GeV. Larger $r_{ds}$ gives stronger variation, and horizontal bands are current (1$\sigma$) experimental range [20] (the bound for (d) is outside the plot). For (c), solid (dashed) lines are for $R_6 = 2.2$, $R_8 = 1.1$ ($R_6 = 1.1$, $R_8 = 1.2$).
updated. We see from Fig. 6(a) that $\Delta m_{B_d}$ does not rule out the parameter space around Eq. (22) (equivalent to Eq. (21)). The overall dependence on $r_{db}$ and $\phi_{db}$ is mild, and error on $f_{B_d} \sqrt{B_{B_d}}$ dominates. Seemingly, a lower value of $f_{B_d} \sqrt{B_{B_d}} \sim 215$ MeV is preferred. SM3 would give $\Delta m_{B_d} = 0.44 - 0.62$ ps$^{-1}$ for $f_{B_d} \sqrt{B_{B_d}} = 208$ MeV - $246$ MeV, so the problem is not with SM4.

We plot $\sin 2\Phi_{B_d}$ vs $\phi_{db}$ in Fig. 6(b), for $r_{db} = 8$ and $12 \times 10^{-4}$. One can see that $\sin 2\Phi_{B_d}$, which is not sensitive to hadronic parameters such as $f_{B_d} \sqrt{B_{B_d}}$, is well within experimental range of $\sin 2\phi_1 = 0.73 \pm 0.04$ from PDG 2005 [20] for the $\phi_{db} \sim 10^5$ case. However, for $\phi_{db} \sim 10^5$ case, which is much more imaginary, $\sin 2\Phi_{B_d}$ is on the high side [35], and it seems that CPV in $B$ physics prefers $R_0 \sim 2.2$ over $R_0 \sim 1$. As another check, we find the semileptonic asymmetry $A_{SL} = -0.7 \times 10^{-3}$ ($-0.2 \times 10^{-3}$) for $\phi_{db} \sim 10^5$ ($10^5$), which is also well within range of $A_{SL}^3 = (-1.1 \pm 7.9 \pm 7.0) \times 10^{-3}$ [36].

With Eqs. (7), (16) and (22), together with standard (SM3) values for $V_{cb}$ and $V_{ub}$, we can get a glimpse of the typical $4 \times 4$ CKM matrix, which appears like

$$
\begin{pmatrix}
0.9745 & 0.2225 & 0.0038 \ e^{-i \ 60^\circ} & 0.0281 \ e^{i \ 61^\circ} \\
-0.2241 & 0.9667 & 0.0415 & 0.1164 \ e^{i \ 66^\circ} \\
0.0073 \ e^{-i \ 25^\circ} & -0.0555 \ e^{-i \ 125^\circ} & 0.9746 & 0.2168 \ e^{-i \ 11^\circ} \\
-0.0044 \ e^{-i \ 10^\circ} & -0.1136 \ e^{-i \ 170^\circ} & -0.2200 & 0.9688
\end{pmatrix}
$$

(25)

for $\phi_{db} \sim 10^5$ case ($V_{cd}$ and $V_{cs}$ pick up tiny imaginary parts, which are too small to show in angles). For the $\phi_{db} \sim 10^5$ case, the appearance is almost the same, except $V_{td} \simeq 0.0082 \ e^{-i \ 17^\circ}$ and $V_{td}' \simeq 0.029 \ e^{i \ 74^\circ}$. Note the “double Cabibbo” nature, i.e. the 12 and 34 diagonal $2 \times 2$ submatrices appear almost the same. This is a consequence of our choice of Eq. (16). To keep Eq. (7) intact, however, weakening $s_{34}$ would result in even large $V_{ts}$, but it would still be close to imaginary. However, note that $V_{td}^* V_{ts}$ is almost real, and CPV in $s \rightarrow d$ comes mostly from $t'$.
The entries for $V_{ub}$, $i = u, c, t$ are all sizable. $|V_{ub}| \sim 0.03$ satisfies the unitarity constraint $|V_{ub}| \sim 0.08$ from the first row, but it is almost as large as $V_{cb}$.

The element $V_{ub} \simeq V_{ts}$ is even larger than $V_{cb}$ and close to imaginary. Together with finite $V_{ub}$, $V_{ub}V_{cb}^{*} \simeq 0.0033 e^{-i\phi_{cb}} (0.0034 e^{i\phi_{cb}})$ is not negligible, and one may worry about $D^{0}\bar{D}^{0}$ mixing. Fortunately the $D$ decay rate is fully Cabibbo allowed. Using $f_{D}\sqrt{B_{D}} = 200$ MeV, we find $\Delta m_{D^{0}} \lesssim 0.05$ ps$^{-1}$ for $m_{\nu} \lesssim 280$ GeV, for both nominal cases of Eq. (22). Thus, the current bound of $\Delta m_{D^{0}} < 0.07$ ps$^{-1}$ is satisfied, and the search for $D^{0}$ mixing is of great interest.

3. summary

In summary, the deviation of direct CPV measurements between neutral and charged $B$ decays, $A_{K^{+}\pi^{0}} - A_{K^{+}\pi^{-}} \simeq 0.16$ while $A_{K^{+}\pi^{-}} \simeq -0.12$, is a puzzle that could be hinting at New Physics. A plausible solution is the existence of a 4th generation with $m_{\nu} \sim 300$ GeV and $V_{ts}^{2}V_{td}^{*} \sim 0.025 e^{i\theta_{t}}$. If so, we find special solution space is carved out by stringent kaon constraints, and the $4 \times 4$ CKM matrix is almost fully determined. $K^{+} \rightarrow \pi^{+}\nu\bar{\nu}$ may well be of order $(1 - 2) \times 10^{-10}$, while $K_{L} \rightarrow \pi^{0}\nu\bar{\nu} \sim (4 - 12) \times 10^{-10}$ is greatly enhanced by the large phase in $V_{td}^{*}V_{ts}$. With kaon constraints satisfied, $B_{d}$ mixing and sin $2\Phi_{B_{d}}$ are consistent with experiment. Our results are generic. If the effect weakens in $b \rightarrow s$ transitions, the effect on $K \rightarrow \pi^{0}\nu\bar{\nu}$ would also weaken. But a large CPV effect in electroweak $b \rightarrow s$ penguins would translate into an enhanced $K_{L} \rightarrow \pi^{0}\nu\bar{\nu}$ and sin $2\Phi_{B_{d}} < 0$.

Acknowledgement. This work is supported by HPRN-CT-2002-00292 of Israel. AS would like to thank Prof. Zoupanos and all the organizing stuff for the great hospitality during the Corfu Summer Institute on EPP.

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