SCATTERING POLARIZATION IN THE Ca\textsc{ii} INFRARED TRIPOLET WITH VELOCITY GRADIENTS

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ABSTRACT

Magnetic field topology, thermal structure, and plasma motions are the three main factors affecting the polarization signals used to understand our star. In this theoretical investigation, we focus on the effect that gradients in the macroscopic vertical velocity field have on the non-magnetic scattering polarization signals, establishing the basis for general cases. We demonstrate that the solar plasma velocity gradients may have a significant effect on the linear polarization produced by scattering in chromospheric spectral lines. In particular, we show the impact of velocity gradients on the anisotropy of the radiation field and on the ensuing fractional alignment of the Ca\textsc{ii} levels, and how they can lead to an enhancement of the zero-field linear polarization signals. This investigation remarks on the importance of knowing the dynamical state of the solar atmosphere in order to correctly interpret spectropolarimetric measurements, which is important, among other things, for establishing a suitable zero-field reference case to infer magnetic fields via the Hanle effect.

Key words: polarization – radiative transfer – scattering – Sun: chromosphere

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1. INTRODUCTION

Over the last few years it has become increasingly clear that the determination of the magnetic field in the “quiet” solar chromosphere requires measuring and interpreting the linear polarization profiles produced by scattering in strong spectral lines, such as H\textalpha and the 8542 Å line of the infrared (IR) triplet of Ca\textsc{ii} (e.g., see reviews by Trujillo Bueno 2010 and Uitenbroek 2011). In these chromospheric lines, the maximum fractional linear polarization signal occurs at the center of the spectral line under consideration, where the Hanle effect (i.e., the magnetic-field-induced modification of the scattering line polarization) operates (Stenflo 1998). Since the opacity at the center of such chromospheric lines is very significant, it is natural to find that the response function of the emergent scattering polarization to magnetic field perturbations peaks in the upper chromosphere (Štěpán & Trujillo Bueno 2010). This contrasts with the circular polarization signal caused by the Zeeman effect whose response function peaks at significantly lower atmospheric heights (Socas-Navarro & Uitenbroek 2004). Of particular importance for developing the Hanle effect as a diagnostic tool of chromospheric magnetism is to understand and calculate reliably the linear polarization profiles corresponding to the zero-field reference case.

The physical origin of the scattering line polarization is atomic level polarization (that is, population imbalances and/or coherence between the magnetic sublevels of a degenerate level with total angular momentum $J$). Atomic polarization, in turn, is induced by anisotropic radiation pumping, which can be particularly efficient in the low-density regions of stellar atmospheres where the depolarizing role of elastic collisions tends to be negligible. The larger the anisotropy of the incident radiation field, the larger the induced atomic level polarization, and the larger the amplitude of the emergent linear polarization.

The degree of anisotropy of the spectral line radiation within the solar atmosphere depends on the center-to-limb variation (CLV) of the incident intensity. In a static model atmosphere, the CLV of the incident intensity is established by the gradient of the source function of the spectral line under consideration (Trujillo Bueno 2001; Landi Degl’Innocenti & Landolfi 2004). However, stellar chromospheres are highly dynamic systems with shocks and wave motions (e.g., Carlsson & Stein 1997). The ensuing macroscopic velocity gradients and Doppler shifts might have a significant impact on the radiation field anisotropy and, consequently, on the emergent polarization profiles. Therefore, it is important to investigate the extent to which macroscopic velocity gradients may modify the anisotropy of the spectral line radiation and, therefore, the emergent scattering line polarization. We aim also at evaluating, with the help of ad hoc velocity fields introduced in a semi-empirical solar model atmosphere, their possible impact on the scattering polarization of the IR triplet of Ca\textsc{ii}. A recent investigation by Manso Sainz & Trujillo Bueno (2010), based on radiative transfer calculations in static model atmospheres, shows why the differential Hanle effect in these lines is of great potential interest for the exploration of chromospheric magnetism.

2. FORMULATION OF THE PROBLEM AND RELEVANT EQUATIONS

2.1. The Atomic Model and the Statistical Equilibrium Equations (SEE)

We assume an atomic model consisting of the five lowest energetic, fine structure levels of Ca\textsc{ii} (see Figure 1). The excitation state of the atomic system is given by the populations of its 18 magnetic sublevels and the coherences among them. We neglect coherences between different energy levels of the same term (multilevel approximation). Moreover, since the problem we consider here (plane-parallel, non-magnetic atmosphere with vertical velocity fields) is axially symmetric around the vertical
direction, no coherences between different magnetic sublevels exist when the symmetry axis is taken for quantizing the angular momentum. We use the multipolar components of each \( J \)-level,

\[
\rho^K_J = \sum_{M=-J}^{+J} (-1)^{J-M} \sqrt{2K+1} \begin{pmatrix} J & M & K \\ J & -M & 0 \end{pmatrix} N_M,
\]

where \( K = 0, \ldots, 2J \), \( N_M \) is the population of the sublevel with magnetic quantum number \( M \), and the symbol between brackets is the Wigner \( 3j \)-symbol (e.g., Brink & Satchler 1968). Due to the symmetry of the scattering process (no magnetic field, no polarized incident radiation in the atmosphere’s boundaries) in a given level \( N_{+M} = N_{-M} \), and the excitation state of the system is described by just nine independent sublevel populations. Consequently, odd-\( K \) elements (orientation components) in Equation (1) vanish for all levels, and the only independent variables of the problem in the spherical components formalism are the total populations of the five levels (\( \sqrt{2J+1} \rho^K_J \)); the alignment components (\( \rho^0_J \)) of levels 2, 3, and 5; and \( \rho^0_J \) of level 3, whose role is negligible for our problem.

The statistical equilibrium equations (SEEs) accounting for the radiative and collisional excitations and deexcitations in the five-level system of Figure 1 are given explicitly in Manso Sainz & Trujillo Bueno (2010; hereafter MSTB2010). We have particularized them to the no-coherence case (only \( \rho^K_J \) elements) in Appendix B. The SEEs for the \( \rho^K_J \) components contain terms that are equal to those appearing in the SEEs for the populations in a standard (no magnetization) NLTE problem (e.g., Mihalas 1978), plus higher order terms \( \sim J^2 \rho^K_J \) (see Equations (B1)–(B5)). The SEEs for the alignment (\( \rho^0_J \) components) are formally similar to the ones for the populations with additional terms \( \sim J^2 \rho^0_J \) accounting for the generation of alignment from the anisotropy of the radiation field, and (negligible) higher order terms \( \sim J^2 \rho^0_J \) and \( J^2 \rho^0_J \) (see Equations (B6)–(B8)). These equations are expressed in the atom reference frame (comoving system).

Since the radiation field is axially symmetric, just two radiation field tensor elements (\( J^0_0 \) and \( J^0_2 \)) are necessary to describe the symmetry properties of the spectral line radiation. Let \( I(v, \mu) \) and \( Q(v, \mu) \) be the Stokes parameters expressed in the observer’s frame at a given height \( z \), where \( v \) is the frequency, \( \mu = \cos \theta \) and \( \theta \) is the angle that the ray forms with the vertical direction. Then, the corresponding values seen by a comoving frame with vertical velocity \( v_z \) with respect to the observer’s frame are \( I'(v', \mu) = I(v, \mu) \) and \( Q'(v', \mu) = Q(v, \mu) \), where \( v' = v(1-v_z/\mu/c) \) and \( v' = v'(1+v_z/\mu/c) \) (to first order in \( v_z/\mu/c \)). Therefore, the mean intensity at the considered height can be expressed from one or another reference frame as

\[
\bar{J}^0_0 = \frac{1}{2} \int dv \int_{-1}^{1} d\mu \phi^0_{Is}(v, \mu) I(v, \mu) = \frac{1}{2} \int dv' \int_{-1}^{1} d\mu \phi^0_{Is}(v', \mu) I(v'(1+v_z/\mu/c), \mu),
\]

where \( \phi^0_{Is}(\nu) \) is the absorption profile (e.g., for a Gaussian profile, we would have \( \phi^0_{Is}(\nu) = \pi^{-1/2} \Delta v_{D}^{-1} \exp(-\nu v_0^2/\Delta v_D^2) \), with \( v_0 \) the central line frequency and \( \Delta v_D \) the Doppler width) and \( \phi^0_{Is}'(v, \mu) = \phi^0_{Is}(v(1-v_z/\mu/c)), \) with \( v_z > 0 \) for upflowing material. Analogously, the anisotropy in the observer’s frame is

\[
\bar{J}^2_0 = \frac{1}{4\sqrt{2}} \int dv \int_{-1}^{1} d\mu \phi^2_{Is}(\mu, v)
\times [(3\mu^2 - 1)I(v, \mu) + 3(1 - \mu^2)Q(v, \mu)],
\]

The important quantity that controls the ability of an anisotropic radiation field to generate atomic polarization is the line anisotropy factor for each transition, which can be calculated as

\[
w_{\text{line}} = \sqrt{2\bar{J}^2_0}/\bar{J}^0_0.
\]

Its range goes from \( w_{\text{line}} = -0.5 \) (for a radiation field coming entirely from the horizontal plane) to \( w_{\text{line}} = 1 \) (for a collimated vertical beam).

2.2. The Radiative Transfer Equations (RTEs)

Due to symmetry, in a non-magnetized plane-parallel medium with a vertical velocity field, light can only be linearly polarized parallel or perpendicularly to the stellar limb. Therefore, choosing the reference direction for positive \( Q \) parallel to the limb, the only non-vanishing Stokes parameters are \( I \) and \( Q \), and they satisfy the following radiative transfer equations (RTEs):

\[
\frac{d}{ds} I = \epsilon_I - \eta_I I - \eta_Q Q,
\]

\[
\frac{d}{ds} Q = \epsilon_Q - \eta_Q I - \eta_I Q,
\]

where \( s \) is the distance along the ray. The absorption and emission coefficients are (MSTB2010)

\[
\epsilon_I = \epsilon^\text{cont}_I + \epsilon^\text{line}_I = \eta^\text{cont}_I B_v + \epsilon_0 \times \left[ \rho^0_J(u) + w^{(2)}_{J\mu}(1 - 3\mu^2 - (1 - \rho^0_J(u))^2) \right],
\]

\[
\eta_I = \eta^\text{cont}_I + \eta^\text{line}_I = \eta^\text{cont}_I + \eta_0 \times \left[ \rho^0_J(\ell) + w^{(2)}_{J\mu}(1 - 3\mu^2 - (1 - \rho^0_J(\ell))^2) \right].
\]
where $\eta^\text{cont}$ and $\epsilon^\text{cont}$ are the continuum absorption and emission coefficients for intensity, respectively. Likewise, $\eta^\text{line}$ and $\epsilon^\text{line}$ are the line absorption coefficients for Stokes $I$ and $Q$, respectively, while $\epsilon^\text{line}$ and $\epsilon^\text{line}$ are the line emission coefficients for Stokes $I$ and $Q$, respectively. The coefficients $w_{J_0 J_0}^{(2)}$ and $w_{J_0 J_0}^{(2)}$ depend only on the transition and are detailed in Table (1). The subscripts $u$ and $\ell$ refer to the upper and lower level of the transition considered, respectively, and $B$ is the Planck function at the central frequency $\nu_0$ of the transition. Note also that

$$
\epsilon_Q = \epsilon_Q^{\text{line}} = \epsilon_0 w_{J_0 J_0}^{(2)} \frac{3}{2\sqrt{2}} (1 - \mu^2) \rho_0^2(\ell),
$$

$$
\eta_Q = \eta_Q^{\text{line}} = \eta_0 w_{J_0 J_0}^{(2)} \frac{3}{2\sqrt{2}} (1 - \mu^2) \rho_0^2(\ell),
$$

(7a)

(7b)

where $\eta^\text{cont}$ and $\epsilon^\text{cont}$ are the continuum absorption and emission coefficients for intensity, respectively. Likewise, $\eta^\text{line}$ and $\epsilon^\text{line}$ are the line absorption coefficients for Stokes $I$ and $Q$, respectively, while $\epsilon^\text{line}$ and $\epsilon^\text{line}$ are the line emission coefficients for Stokes $I$ and $Q$, respectively. The coefficients $w_{J_0 J_0}^{(2)}$ and $w_{J_0 J_0}^{(2)}$ depend only on the transition and are detailed in Table (1). The subscripts $u$ and $\ell$ refer to the upper and lower level of the transition considered, respectively, and $B$ is the Planck function at the central frequency $\nu_0$ of the transition. Note also that

$$
\epsilon_Q = \frac{h \nu}{4\pi} A_{\nu u} \phi'_{\nu u}(\mu, \nu) N \sqrt{2 J_u + 1},
$$

(8a)

$$
\eta_Q = \frac{h \nu}{4\pi} B_{\nu u} \phi'_{\nu u}(\mu, \nu) N \sqrt{2 J_\ell + 1},
$$

(8b)

where $N$ is the total number of atoms per unit volume.

With the total absorption coefficient for the intensity, the line of sight (los) optical depth for each frequency is calculated by the following integral along the ray:

$$
\tau^\text{los} = - \int \mu \eta(\mu, \nu) \, \frac{dz}{\mu^\text{los}}.
$$

(9)

2.3. Numerical Method

The solution to the non-LTE problem of the second kind considered here (the self-consistent solution of the SEE's for the density matrix elements together with the RTE for the Stokes parameters) is carried out by generalizing the computer program developed by Manso Sainz & Trujillo Bueno (2003a), to allow for radial macroscopic velocity fields. For integrating the RTE, a parabolic short-characteristics scheme (Kunasz & Auer 1988) is used. At each iterative step, the RTE is solved, and $J_0^0$ and $J_0^\ell$ are computed and used to solve the SEE following the accelerated Lambda iteration method outlined in the Appendix of MSTB2010. Once the solution for the multipolar components of the density matrix is consistently reached, the emergent Stokes parameters are calculated for the desired los, which in all the figures of this paper is $\mu = 0.1$. This final step is done increasing the frequency grid resolution to a large value in order to correctly sample the small features and peaks of the emergent profiles.

Some technical considerations have to be kept in mind for the treatment of velocity fields. Due to the presence of Doppler shifts, the frequency axis used to compute $J_0^0$ and $J_0^\ell$ must include the required extension and resolution, because the spectral line radiation may be now shifted and asymmetric. In our strategy for the frequency grid, the resolution is larger in the core than in the wings, keeping the same frequency grid for all heights.

The cutoff frequency for the core (where resolution is appreciably higher) is dictated by the maximum expected Doppler shift. Thus, the core bandwidth is estimated allowing for a range of $2 V_{\text{max}}$ around the zero-velocity central frequency of the spectral lines, with $V_{\text{max}}$, the maximum velocity found in the atmosphere (in Doppler units). Apart from that, a minimum typical resolution for the core is set to two points per Doppler width ($\Delta V_D$). Then, the height with the smallest Doppler width determines the core resolution, and the height with the maximum macro-velocity states its bandwidth.

Furthermore, as frequencies and angles are inextricably entangled (through terms $\nu - v, \mu v/c$ appearing in the absorption/emission profiles due to the Doppler effect, like in Equation (2)), the maximum angular increment ($\Delta \ell_{\text{max}}$) is restricted by the maximum frequency increment ($\Delta \nu_{\text{max}} \approx 1/2$, in Doppler units). Thus, it must occur that $\Delta \mu_{\text{max}} \cdot V_{\text{max}} < 1/2$. In the worst case, the maximum allowed angular increment will be smaller (more angular resolution needed) when the maximum vertical velocity increases. Besides this consideration, the maximum angular increment could be even more demanding because of the high sensitivity of the polarization profiles to the angular discretization.

Finally, the depth grid must be fine enough, in such a way that the maximum difference in velocity between consecutive points is not too large, the typical difference being equal to half the Doppler width ($|V(z_i) - V(z_{i-1})| \leq 1/2$). If the difference is larger, the absorption/emission profiles would change abruptly with height, producing imprecisions in the optical depth increments (Mihalas 1978).

3. EFFECT OF A VELOCITY GRADIENT ON THE RADIATION FIELD ANISOTROPY AND THE MEAN INTENSITY

As we shall see, the presence of a vertical velocity gradient in an atmosphere enhances the anisotropy of the radiation field and, hence, of the scattering line polarization patterns. The fundamental process underlying this mechanism can be simply understood with the following basic examples.

3.1. Anisotropy Seen by a Moving Scatterer

Consider an absorption spectral line with a Gaussian profile emerging from a static atmosphere with a linear limb darkening law,

$$
I(v, \mu) = I^{(0)}(1 - u + u \mu) \left[ 1 - a \exp \left( - \left( \frac{v - v_0}{w} \right)^2 \right) \right],
$$

(10)

where $I^{(0)}$ is the continuum intensity at disk center, $u$ is the limb darkening coefficient, $a < 1$ measures the intensity depression of the line, and $w$ its width. In this approximation, we assume that all the parameters are constant. Now imagine that, at the top
of the atmosphere, there is a thin cloud scattering the incident light given by Equation (10) and moving radially at velocity $v_z$, with respect to the bottom layers of the atmosphere, supposed static. We will assume that the absorption profile is Gaussian (dominated by Doppler broadening), with width $\Delta v_D$. When $\Delta v_D \ll w$, the incident spectral line radiation is much broader than the absorption profile (in fact, for $\alpha = \Delta v_D / w = 0$ the absorption profile is formally a Dirac-$\delta$ function). Then, from Equations (2) and (3), we can derive explicit expressions for the mean intensity and anisotropy of the radiation field as a function of the scatterer velocity (see Appendix A): $\bar{J}_0 = I_0(0; \xi)/2$, $\bar{J}_0^2 = I_0(0; \xi)/4\sqrt{2}$, where the $I_0$ functions are defined by Equations (A4) and (A5). The behavior of $\bar{J}_0^0$ and $\bar{J}_0^2$ with the adimensional velocity $\xi = v_z v_0/(cw)$ (Figure 2) is most clearly illustrated in their asymptotic limits at low velocities:

$$\bar{J}_0^0 = \frac{1}{4} \left(1 - \frac{a}{1 + a^2}\right)(2 - u) + \frac{a(4 - u)}{12(1 + a^2)^{3/2}} \xi^2 + O(\xi^3),$$

(11)

$$\sqrt{2} \bar{J}_0^2 = \frac{u}{4(2 - u)} + \frac{a(64 - 56u + 7u^2)}{120(2 - u)^2(1 + a^2)(\sqrt{1 + a^2} - a)} \xi^2 + O(\xi^3).$$

(12)

Equation (11) shows that, for an absorption line ($a > 0$), $\bar{J}_0^0$ is always increasing with the velocity since the coefficient of $\xi^2$ is positive, and this, regardless of the sign of $v_z$ (i.e., regardless of whether the scatterers move upward or downward); if the line is in emission ($a < 0$), $\bar{J}_0^0$ monotonically decreases. These are the Doppler brightening and Doppler dimming effects (e.g., Landi Degl’Innocenti & Landolfi 2004). An analogous analysis applies to $\bar{J}_0^2$ (Equation (12)). Note that, in the absence of limb darkening ($u = 0$), the anisotropy vanishes in a static atmosphere, while the mere presence of a relative velocity between the scatterers and the underlying static atmosphere induces anisotropy in the radiation field—hence, a polarization signal. A real atmosphere could then be understood as a superposition of scatterers that modify the anisotropy depending on the local velocity gradient and the illumination received from lower shells. An interesting discussion on the effect of velocities with directions other than radial can be found in Landi Degl’Innocenti & Landolfi (2004, Section 12.4).

Clearly, all the above discussion depends on the Doppler shift induced by the velocity $v_z$, normalized to the width of the spectral line, i.e., on $\xi$. A large velocity gradient on a broad line has the same effect as that of a smaller velocity gradient on a narrow line. This is important to keep in mind since the response of different spectral lines to the same velocity gradient will be different, which can help us decipher the velocity stratification. Even different spectral lines belonging to the same atomic species may have very different widths, as for example, the CaII IR triplet and the UV doublet studied in the next section.

3.2. Calculations in a Milne–Eddington Model

The discussion above explains the basic mechanism by means of which a velocity gradient enhances the anisotropy of the radiation field. Now we can get further insight into the structure of the radiation field within an atmosphere with velocity gradients from just the formal solution of the RT equation for the intensity (e.g., Mihalas 1978). As before, we neglect effects due to polarization and $\bar{J}_0^0$ and $\bar{J}_0^2$ are calculated from Stokes $I$ alone. We consider a semi-infinite, plane-parallel atmosphere with a source function $S = S_0(1 + \beta \tau_0)$, where $\tau_0$ is the integrated line optical depth in the static limit (hence, the element of optical depth $d\tau_0 = r + \beta \phi\{v(1 - v_z(\tau_0)\mu/c)/c\}d\tau$, where $r = \kappa_c/\kappa_l$ is the ratio of continuum to line opacity). We begin by considering a vertical velocity field $v_z(\tau_0) = v_0/[1 + (\tau_0/\tau_0)]$ (positive outward the star), shown in Figure 3. Equivalently, we may express the velocity in adimensional terms by using $\xi = v_0/c v_z/\Delta v_D$ (the width $\Delta v_D$ of the Gaussian absorption profile is assumed to be constant with depth). The parameter $\tau_0$ fixes the position of the maximum velocity gradient. Note that the frequency dependence of the Doppler effect $\Delta v = v_0 v_z/c$ is canceled in the adimensional problem, where velocities are measured in Doppler units. It is easy to calculate $I_1(\nu, \mu)$ numerically at every point in the atmosphere and thus, the mean intensity and anisotropy of the radiation field (Figure 4).

The rise in $\bar{J}_0^0$ in higher layers with respect to the static case corresponds to the Doppler brightening discussed above. Thanks
to the Doppler shifts, the atoms see more and more of the brighter continuum below, which enhances $J_0^0$. When the maximum velocity gradient takes place at optically thick enough layers ($\tau_0 \gtrsim 0$), $J_0^0$ is also larger than for the static case, but it decreases monotonically with height in the atmosphere ($\tau_0 = 10^2$, dotted lines in upper panel of Figure 4). Note that the important quantity that modulates the increase in $J_0^0$ is not the maximum velocity but the velocity gradient (difference in velocity between optically thick and optically thin parts of the atmosphere). The larger the gradient, the more pronounced the radiative decoupling is between different heights. An extreme example of such radiative decoupling could be found in supernovae explosions, where the vertical velocity gradients are huge.

In our case (vertical motions), the Doppler brightening implies an enhancement of the contribution of vertical radiation to Equation (3) with respect to the horizontal radiation, with the latter remaining almost equal to the static case (no horizontal motions, no horizontal Doppler brightening). This velocity-induced limb darkening is the origin of the anisotropy enhancement.

However, note that the maximum anisotropy does not rise indefinitely when increasing the maximum velocity. If the velocity gradient in units of the Doppler width is larger than $\xi_{\text{max}}$, the anisotropy at the surface saturates and decreases (even below the curves corresponding to shorter velocity gradients). It is accompanied by a bump around $\tau_1 = 1$ when the maximum velocity gradient is taking place at low-density layers ($\tau_0 \lesssim 1$). This behavior can be understood using Equation (3). When $\xi_{\text{max}} \lesssim 3$, an increment in $\xi_{\text{max}}$ entails a rise in $J_0^0$, $J_2^0$, and $J_0^2/J_0^0$ ($w_{\text{line}}$) in the upper atmosphere, which means that the velocity gradients enhance the imbalance between vertical and horizontal radiation. However, if $\xi_{\text{max}}$ is above that threshold, $J_0^0$ and $J_2^0$ rise, but the ratio $J_0^2/J_0^0$ saturates and diminishes. The reason is that a large velocity gradient makes the absorption profiles associated to almost horizontal outgoing rays ($0 < \mu < 1/\sqrt{3}$) shifted so much that they also capture the background continuum radiation. Their contributions are negative to the angular integral of $J_0^2$ but positive for $J_0^0$.

Separating the contributions of rays with angles in the range $0 < |\mu| < 1$ (which we refer to with the label $+$) and angles in the range $0 < |\mu| < 1/\sqrt{3}$ (which we refer to with the label $-$), the line anisotropy can be written as $w_{\text{line}} = w_{\text{line}}^+ + w_{\text{line}}^- = J_0^2/J_0^0 - |J_2^0|^2/|J_0^0|^2$. Here, $J_0^0$ and $J_2^0$ always grow with $\xi_{\text{max}}$, but $|J_2^0|^2/|J_0^0|^2$ only grows appreciably when $\xi_{\text{max}} \gtrsim 3$. Therefore, although $w_{\text{line}}$ increases for all velocity gradients, its enhancement is smaller for large velocity gradients than for smaller ones. This effect occurs as well when motions take place deeper ($\tau_0 \gtrsim 1$) but it is less important and the anisotropy bump and saturation are reduced.

For the considered velocity fields (with a negligible gradient in the upper atmosphere), $J_0^0$ and $J_2^0$ reach an asymptotic value in optically thin regions. We have verified that this effect does not occur if the velocity gradient is not zero in those layers. In any case, the presence of a large anisotropy in optically thin heights barely affects the emergent linear polarization profiles.

3.3. Two-level Model Atom in Dynamic Atmospheres

Before considering a more realistic case, a final illustrative example is considered. In this case, we assume the same parameterization of the velocities than in the previous example,
but now we solve the complete iterative RT problem with a two-level atom model and a specific temperature stratification. Consequently, the source function and the anisotropy are consistently obtained in a moving atmosphere. The intensity source function is

\[ S_I = r_{vis} S^{\text{loc}} + (1 - r_{vis}) B \]

(e.g., Rybicki & Hummer 1992), with \( r_{vis} = \phi_{\text{ta}}(\nu, \mu)/(\nu_z + \phi_{\text{ta}}(\nu, \mu)) \) and the expression for the line source function remains formally equal to that of the static case, being

\[ S_{\text{line}}^\tau = (1 - \epsilon) J_{\text{B}}^0 + \epsilon B, \]

where \( B \) is the imposed Planck function, \( \epsilon \) is the inelastic collisional parameter, and \( J_{\text{B}}^0 \) is calculated with Equation (2).

The qualitative behavior explained in the previous subsection is maintained in these two-level atom calculations. For small \( \epsilon \) values (large NLTE effects), the source function \( S_I \approx J_{\text{B}}^0 \) shows Doppler brightening effects and its surface value depends on the maximum velocity gradient and on the maximum background continuum set by the photospheric conditions (upper panel in Figure 5). The anisotropy rises proportionally to the velocity gradient until a saturation occurs (lower panel in Figure 5). A similar behavior is found when the maximum velocity gradient occurs higher in the atmosphere (see Figure 10).

In a static atmosphere, the radiation field anisotropy is dominated by the presence of gradients in the intensity source function (Trujillo Bueno 2001; Landi Degl’Innocenti & Landolfi 2004), which can be modified via the Planck function (equivalently, the temperature). In the dynamical case that we are dealing with, the slope of the source function is also modified due to the existence of velocity gradients thanks to the frequency decoding caused by relative motions between absorption profiles (Doppler brightening). In general, both mechanisms act together (velocity-induced and temperature-induced modification of the source function gradient) and the ensuing anisotropy and the emergent linear polarization profiles are modified accordingly.

It is important to note that the adimensional velocity \( \xi \) depends both on the velocity and also on the line Doppler width because \( \delta \nu = \delta \nu_{\text{D}} = v/\sqrt{2k_B T/m} \), with \( k_B \) the Boltzmann’s constant, \( T \) the temperature, and \( m \) the mass of the atom. In the photosphere, where velocities are much lower than in the chromosphere, \( \xi \) is expected to be negligible. In the chromosphere, plasma motions are important and the temperature is still comparable to that of the photosphere, inducing \( \xi \) to be controlled by the velocity field. However, for layers in the transition region and above, the high temperatures reduce the value of \( \xi \). In any case, at these heights, the density is so low that, although \( \xi \) (and consequently the anisotropy) could have a highly variable behavior, the emergent polarization profiles of chromospheric lines will not be sensitive to them.

4. RESULTS FOR THE Ca\textsc{ii} IR TRIPLET

Now, we study the effect of the velocity field on a multilevel atomic system in a realistic atmospheric model, within the more general framework described in Section 2. We consider the formation of the scattering polarization pattern of the Ca\textsc{ii} infrared triplet in a semi-empirical model atmosphere (FAL-C model of Fontenla et al. 1993) in the presence of vertical velocity fields \( \mathbf{v} = v_z(\mathbf{z}) \mathbf{k} \), with \( \mathbf{k} \) being the unit vector along the vertical pointing upwards). We will assume a constant microturbulent velocity field of 3.5 km s\(^{-1}\), a representative value for the region of formation which gives a realistic broadening of the triplet profiles.

4.1. Behavior of the Anisotropy in the Ca\textsc{ii} IR Triplet

For simplicity, we set linear velocity fields (constant velocity gradient along \( z \)) between \( z = -100 \) and 2150 km (see Figure 6). Consequently, the adimensional velocity field \( \xi_z \) has a non-monotonic behavior due to its dependence on the temperature (upper right panel in Figure 6). In the chromosphere, where the Ca\textsc{ii} triplet lines form, \( \xi_z \) is dominated by the macroscopic motions. Here, the velocity gradients produce variations in the anisotropy of the triplet lines that agree with the behavior outlined in the previous sections. Namely, an amplification and a subsequent saturation of the anisotropy factor due to the significant velocity gradient at those heights (see the lower panels and middle right panel in Figure 6). Above the chromosphere, on the contrary, the temperature dominates (\( \xi_z \) stabilizes and diminishes) and the anisotropy slightly decreases with height.

If the (adimensional) velocity gradient is negligible where the line forms (around \( v_{\text{th}}^\text{lim} \sim 1 \)), the anisotropy remains unaffected. Otherwise, if a spectral line forms at very hot layers, where...
Figure 6. Amplification of the line anisotropy ($w_{\text{line}} = \sqrt{\bar{J}_0^2 / \bar{J}_0^2}$) due to vertical velocity gradients. Upper left panel: linear velocity fields vs. height, with velocity gradients going from 0 (darker lines) to 20 m s$^{-1}$ km$^{-1}$ (light blue lines) in steps of 2 m s$^{-1}$ km$^{-1}$. Upper right panel: corresponding adimensional velocity fields ($\xi_z$) for an FALC temperature stratification and a constant microturbulent velocity of 3.5 km s$^{-1}$. The horizontal axis is in units of the K-line optical depth along the line of sight (los). The vertical lines mark the position of $\tau_{\text{los}}^{\nu_0} = 1$ for the transitions 8498 Å (blue), 8542 Å (green), 8662 Å (red), and the K line (black). Remaining panels: corresponding line anisotropy factors plotted against $\tau_{\text{los}}^{\nu_0}$ for each line.

(A color version of this figure is available in the online journal.)

the absorption profiles are wider and their sensitivity to the velocity gradients is lower, the Doppler brightening will not be so efficient amplifying the anisotropy. This is the case of the anisotropy of the Ca II K line (middle left panel in Figure 6). Compare how the slope of $\xi_z$ is smaller where the Ca II K line forms (black line on Figure 6) than where the triplet lines do. Consequently, the enhancement of the line anisotropy through the presence of velocity gradients in this line is reduced.

All our calculations demonstrate that the anisotropy in the Ca II IR triplet can be amplified through chromospheric vertical velocity gradients. This results suggest that the same occurs with the ensuing linear polarization profiles.

4.2. The Impact on the Polarization of the Emergent Radiation

To investigate the effect of vertical velocity fields on the emergent fractional polarization profiles we perform the following numerical experiments. First, we impose velocity gradients with the same absolute value but opposite signs (top left panel of Figure 7). The resulting emergent $Q/I$ profiles (remaining left
Figure 7. Left panels: calculation at $\mu = 0.1$ of the emergent $Q/I$ polarization signals of the Ca $\text{ii}$ IR triplet when four different choices for the vertical velocity gradients are imposed. Positive velocities imply upflowing plasma. Gradient “a”/“b” simulates an atmosphere where the plasma is entirely moving toward the observer increasing/decreasing linearly the velocity along the outgoing $z$-axis. Gradients “c” and “d” are the same for plasma moving away from the observer. The black dotted line is the solution for the static reference case. Each curve is computed on the converged solution of the multilevel NLTE problem described in Section 2. Right panels: same calculations than in the left-hand panels, but with different velocity gradient values varying from 0 to 16.3 m $\text{s}^{-1}$ km $\text{s}^{-1}$ in steps of 2.3 m $\text{s}^{-1}$ km $\text{s}^{-1}$ (see top right panel). These results show that the polarization signals are increased and shifted with respect to the static case depending only on the absolute value of the vertical velocity field gradient and independently of the sign of the velocity field.

(A color version of this figure is available in the online journal.)
panels of Figure 7) are magnified by a significant factor (>2 for all the transitions) with respect to the static case (black dotted line). The linear polarization profiles have the same amplitude, independently of the sign of the velocity gradient. Another remarkable feature is the asymmetry of the profiles, having a higher blue wing in those cases in which the velocity gradient is positive and a higher red wing when the velocity gradient is negative, independently of the sign of the velocity gradient. Note also that the $Q/I$ profile is shifted in frequency due to the relative velocity between the plasma and the observer.

As a second experiment, we consider different velocity fields with increasing gradients (right upper panel in Figure 7). In the ensuing $Q/I$ profiles we see that the larger the velocity gradient, the larger the frequency shift of the emergent profiles and the larger the amplitude. In all transitions, one of the lateral lobes of the signal remains almost constant. Thus, what really changes is the central part of the profiles, being a “valley” in the $\lambda$8498 line and a “peak” in the other two transitions. To quantify these variations, we define $(Q/I)_{\text{pp}}$ (peak-to-peak amplitude of $Q/I$) as the difference between the lowest and the highest value of the emergent $Q/I$ signal, which is also a measure of its contrast. Note that, as expected from the first experiment, $(Q/I)_{\text{pp}}$ depends only on the absolute value of the gradient. Figure 8 summarizes these results.

The sensitivity of the linear polarization to the velocity gradient can be measured approximately as commented in Section 3.1, using a parameter $\alpha = \Delta v_{\ell}/u$ that accounts for the difference in width of the absorption profile with respect to the emergent intensity profile. If $\alpha \sim 1$, small adimensional velocities will produce large changes in shape; if $\alpha \ll 1$, much larger $\xi$ values are needed for the same effect. In the case of the IR triplet lines, $\alpha \sim 0.355$ in the formation region of $\lambda$8498 and around 0.29 and 0.285 in the formation region of $\lambda$8542 and $\lambda$8662 (having wider profiles), respectively. Then, the former is more sensitive to velocity variations in its formation region (Figure 8). Finally, the K line has $\alpha (\tau_{\text{los}} = 1) \sim 0.015$, a low value due to its wider spectral wings.

The enhancement of the polarization signals is a consequence of the increase in the anisotropy. Therefore, since this increase is produced by the presence of velocity fields, the polarization signals of the Ca II IR triplet are sensitive also to the dynamic state of the chromosphere.

4.3. Variations on the Atomic Alignment Due to Velocity Gradients

In order to get physical insight on the formation of the emergent polarization profiles, we use an analytical approximation. Following Trujillo Bueno (2003), the emergent linear polarization for a strong line at the central wavelength can be approximated with

$$\frac{Q}{I} \approx \frac{3}{2\sqrt{2}} (1 - \mu^2) \left[ w_{J\ell,J_0}^{(2)} \sigma_0^2(J_\ell) - w_{J\ell,J_0}^{(2)} \sigma_0^2(J_0) \right]. \quad (13)$$

The symbols $w_{J\ell,J_0}^{(2)}$ are numerical coefficients already introduced in Section 2. The quantities $\sigma_0^2(J_\ell)$ and $\sigma_0^2(J_0)$ are the fractional alignment coefficients ($\sigma_0^2 = \tau_0^2 / \tau_0^2$) evaluated at $\tau_{\text{los}} = 1$ for the upper and lower level of the transition, respectively. This is the generalization of the Eddington–Barbier approximation to the scattering polarization and establishes that changes in linear polarization (for a static case) are induced by changes in the atomic alignment of the energy levels.

Our calculations show that vertical velocity fields with moderate gradients ($\lesssim 10 \text{ m s}^{-1} \text{ km}^{-1}$) in a linear velocity field, as the ones shown in the figures) do indeed produce variations in the fractional alignment, which are small for $|\sigma_0^2(J_\ell)|$ and significant for $|\sigma_0^2(J_0)|$ (see Figure 9). The lower level alignment is the main driver of the changes produced in the $Q/I$ signals. This is strictly true for the $\lambda$8662 line, whose upper level with $J = 1/2$ cannot be aligned (zero-field dichroism polarization; Manso Sainz & Trujillo Bueno 2003b). In the other transitions, a certain influence of the upper level alignment becomes important only for large gradients. The reason is that the K transition is so strong in comparison with the IR triplet that it is dictating the common upper level 5 alignment (Figure 1). In fact, $\sigma_0^2(J_5)$ is driven by the K-line anisotropy which, at chromospheric heights, is almost unaffected for the considered velocity gradients, as we discussed in Section 4.1 (Figure 6). Thus, the strong H and K lines feed population to the upper levels and the K line controls the alignment of the $^2P_{3/2}$ level (see Figure 1), while the polarization signals of the IR triplet change with velocity fields affecting $\sigma_0^2(J_5)$ (through the anisotropy enhancement).

To illustrate the well-known link between the alignment and the anisotropy we can follow the next reasoning. For the Ca II model atom that we deal with in this work, it is possible to derive a simple analytic expression that relates the anisotropy and the alignments for the $\lambda$8542 transition. Making use of Equations (B3) and (B7) and neglecting second-order terms and collisions, we find that

$$2\sigma_0^2(J_5) - \sqrt{7} \sigma_0^2(J_3) \simeq w_{\text{line}}(3 \to 5). \quad (14)$$

As before, we can roughly assume that $\sigma_0^2(J_5) \sim$ constant at the chromosphere because it is controlled by the K line. Then, Equation (14) suggests that, if the radiation anisotropy increases at that heights, an amplification of $|\sigma_0^2(J_3)|$ occurs (note that $\sigma_0^2(J_5)$ is negative for these lines). A more aligned atomic population produces a more intense scattering polarization signal.
5. CONCLUSIONS

When vertical velocity gradients exist, the polarization profiles are always shifted in wavelength, asymmetrized and enhanced in amplitude with respect to the constant velocity case. The reason is that increments in the absolute value of the velocity gradient increase the source function (Doppler brightening) and enhance the anisotropy of the radiation field (Sections 3 and 4), that in turn modify the fractional alignment (Section 4.3) and amplify the scattering polarization profiles (Section 4.2).

For this very reason, all calculations assuming static models in the formation region might underestimate the scattering polarization amplitudes and not capture the right shape of the profiles. In particular, it must be taken into account that the Ca II IR triplet lines form under non-LTE conditions in chromospheric regions, where velocity gradients may be significantly large due to the upward propagation of waves in a vertically stratified atmosphere (e.g., Carlsson & Stein 1997). Probably, in photospheric and transition region lines the effect of velocities on polarization can be safely neglected (they will be predominantly amplified by temperature gradients as discussed in Section 3.3), but not necessarily in the chromosphere. In our study we see that the λ8498 line is more sensitive to macroscopic motions in the low chromosphere, while the polarization amplitudes of the λ8542 and λ8662 lines may be significantly amplified when strong velocity gradients are found at heights around 1.5 Mm and higher in our model.

At the light of these results, it is obvious that the effect of the velocity might be of relevance for measuring chromospheric magnetic fields. In particular, the described mechanism might turn out to be important for the correct interpretation of polarization signals in the Sun with the Hanle effect. Given that weak chromospheric magnetic fields are inferred with the Hanle effect using the difference between the observed linear polarization signal and the signal that would be produced in the absence of a magnetic field, it is crucial to correctly compute the reference no-magnetic signal. In the Ca II IR triplet, it could be possible to break the degeneracy of the combined effects by taking into account the different sensitivities of the three lines of the triplet to the magnetic field and the thermodynamics. As stated by MSTB2010, the λ8498 line is very sensitive to the thermal structure of the atmosphere. Likewise, the response of this line to the velocity gradient is also higher than in the other two. Then, a first step could be to characterize the response of all the lines to combined variations of the temperature, velocity, and magnetic field and find observables (i.e., amplitude ratios) that are as insensitive to the temperature and velocity as possible and as sensitive to the magnetic field as possible. We are currently carrying out this study on realistic velocity fields, including shocks and temporal variations.

The polarization amplification mechanism that we have discussed in this paper is not limited to plane-parallel atmospheres, although its effect is surely more important in plane-parallel atmospheres than in three-dimensional ones. The reason is that (1) vertical velocity gradients in a three-dimensional atmosphere are weaker given the increased degrees of freedom and (2) non-resolved motions or large variations in velocity direction along the medium mix the contribution of different layers and broaden the profiles, diminishing them in amplitude. In any case, strong three-dimensional velocity gradients might create preferred directions along which the plasma becomes more optically thin through the radiative uncoupling mechanism discussed in this paper.

The amplification of the radiation field anisotropy through vertical velocity gradients is a general and interesting phenomenon that improves our understanding of the stellar atmospheres. With the present investigation, we have obtained some feeling about the formation of the Ca II IR triplet in dynamic situations.

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APPENDIX A

SPECIAL FUNCTIONS IN SECTION 3

Introducing Equation (10) into Equation (2) gives

\[ \bar{J}_0^0 = \frac{1}{2} \int_0^\infty dv' \int_0^1 dt \exp \left\{ - \frac{1}{\Delta v_D} \int_0^1 \left[ \left( \frac{v' - v_0}{\Delta v_D} \right)^2 - a \exp \left\{ - \frac{\left( v' (1 + v_c \mu / c) - v_0 / w \right)^2}{w^2} \right\} \right] \right\}. \] (A1)

Introducing the variables \( x = (v' - v_0)/w, \alpha = \Delta v_D/w, \) and \( \xi = v_c v_0/(cw), \) then the mean intensity in the comoving frame is

\[ \bar{J}_0^0 = \int_0^\infty \frac{dx}{\sqrt{\pi \alpha}} e^{-x/\alpha^2} \int_0^1 d\mu \int_0^1 d\mu (1 - u + u \mu) (1 - ae^{-(x + \xi \mu)^2}). \] (A2)

In passing from Equation (A1) to Equation (A2), we have extended the integration limit on \( x \) to \( \infty. \) Analogously for the anisotropy in the comoving frame

\[ \bar{J}_0^2 = \int_0^\infty \frac{dx}{\sqrt{\pi \alpha}} e^{-x/\alpha^2} \int_0^1 d\mu (3 \mu^2 - 1) (1 - u + u \mu) (1 - ae^{-(x + \xi \mu)^2}). \] (A3)

The following integrals are easily evaluated (see Spiegel & Liu 1998):

\[ I_0(\alpha; \xi) = \int_0^\infty \frac{dx}{\sqrt{\pi \alpha}} e^{-x/\alpha^2} \int_0^1 (1 - u + u \mu) \left[ 1 - ae^{-(x + \xi \mu)^2} \right] d\mu \]
\[ = \frac{1}{2} \left[ 2 - u + a(u - 1) \sqrt{\pi} \frac{1}{\xi} \text{Erf} \left( \frac{\xi}{\sqrt{1 + \alpha^2}} \right) + au \sqrt{1 + \alpha^2} \frac{1}{\xi^2} \left( \exp \left\{ - \frac{\xi^2}{1 + \alpha^2} \right\} - 1 \right) \right], \] (A4)

\[ I_2(\alpha; \xi) = \int_0^\infty \frac{dx}{\sqrt{\pi \alpha}} e^{-x/\alpha^2} \int_0^1 (3 \mu^2 - 1) (1 - u + u \mu) \left[ 1 - ae^{-(x + \xi \mu)^2} \right] d\mu \]
\[ = \frac{1}{4} \left[ 2 - 2a(u - 1) \sqrt{\pi} \frac{1}{\xi} \text{Erf} \left( \frac{\xi}{\sqrt{1 + \alpha^2}} \right) + 2u \sqrt{1 + \alpha^2} \frac{1}{\xi^2} \left[ u + (3 - u) \exp \left\{ - \frac{\xi^2}{1 + \alpha^2} \right\} \right] \right] \]
\[ + 3a(u - 1)(1 + \alpha^2) \sqrt{\pi} \frac{1}{\xi^3} \text{Erf} \left( \frac{\xi}{\sqrt{1 + \alpha^2}} \right) + 6a(1 + \alpha^2)^{1/2} \frac{1}{\xi^4} \left( \exp \left\{ - \frac{\xi^2}{1 + \alpha^2} \right\} - 1 \right) \], (A5)

where we have made use of

\[ \int_0^\infty \frac{dx}{\sqrt{\pi \alpha}} e^{-x/\alpha^2} \left[ 1 - ae^{-(x + \xi \mu)^2} \right] = 1 - \frac{a \mu e^{\xi^2}}{\sqrt{1 + \alpha^2}} e^{\xi^2/(1 + \alpha^2)} \] (A6)

From them, the values for \( \bar{J}_0^0 \) and the anisotropy \( \sqrt{\bar{J}_0^2} / \bar{J}_0^0 \) are trivially derived.

In the high velocity limit (\( \xi \to \infty \)), \( I_0(\alpha; \xi) = (2 - u)/2, \) and \( I_2(\alpha; \xi) = u/4 \) (regardless of \( \alpha \)). In the low velocity limit,

\[ I_0(\alpha; \xi) = \frac{1}{2} \left[ (2 - u) + \frac{a(4 - u)}{12(1 + \alpha^2)^{3/2}} \xi^2 + O(\xi^3) \right], \] (A7)

\[ I_2(\alpha; \xi) = \frac{u}{4} \left[ (2 - u) + \frac{a(16 - u)}{60(1 + \alpha^2)^{3/2}} \xi^2 + O(\xi^3) \right]. \] (A8)

APPENDIX B

STATISTICAL EQUILIBRIUM EQUATIONS

The rate equations for the considered problem are as follows. They have been obtained by particularizing to the no-coherence case (only \( \rho^0_{\alpha} \) elements) the equations given by MSTB2010 for the model atom of Figure 1:

\[ \frac{d}{dt} \rho^0_{\alpha}(1) = - \sum_{u=1}^{5} B_{1u} \rho^0_{\alpha}(1 \to u) + \sum_{i \neq 1} C_{1i} \rho^0_{\alpha}(1) + A_{41} \rho^0_{\alpha}(4) + \sqrt{\bar{2}A_{51}} \rho^0_{\alpha}(5) + \sum_{i \neq 1} C_{1i} \sqrt{\frac{2J_i + 1}{2}} \rho^0_{\alpha}(i), \] (B1)
\[
\frac{d}{dt}\rho_0^0(2) = -\left[ \sum_{i=4}^5 B_{2i} \bar{J}_0^0(2 \rightarrow u) + \sum_{i \neq 2} C_{2i} \right] \rho_0^0(2) - \left( \frac{1}{\sqrt{2}} B_{24} \bar{J}_0^0(2 \rightarrow 4) - \frac{2\sqrt{3}}{5} B_{25} \bar{J}_0^0(2 \rightarrow 5) \right) \rho_0^0(2) \\
+ \frac{1}{\sqrt{2}} A_{42} \rho_0^0(4) + A_{52} \rho_0^0(5) + \sum_{i \neq 2} C_{2i} \frac{2J_i + 1}{2} \rho_0^0(i), 
\] (B2)

\[
\frac{d}{dt}\rho_0^0(3) = -\left[ B_{35} \bar{J}_0^0(3 \rightarrow 5) + \sum_{i \neq 3} C_{3i} \right] \rho_0^0(3) - \frac{\sqrt{7}}{5} B_{35} \bar{J}_0^0(3 \rightarrow 5) \rho_0^0(3) + \frac{\sqrt{7}}{3} A_{53} \rho_0^0(5) + \sum_{i \neq 3} C_{3i} \sqrt{\frac{2J_i + 1}{6}} \rho_0^0(i), 
\] (B3)

\[
\frac{d}{dt}\rho_0^0(4) = -\left[ \sum_{i=1}^2 A_{4i} + \sum_{i \neq 4} C_{4i} \right] \rho_0^0(4) + \sum_{i=1}^2 B_{14} \bar{J}_0^0(l \rightarrow 4) \sqrt{\frac{2J_i + 1}{2}} \rho_0^0(l) + B_{24} \bar{J}_0^0(l \rightarrow 4) \rho_0^0(2) + \sum_{i \neq 4} C_{4i} \sqrt{\frac{2J_i + 1}{2}} \rho_0^0(i), 
\] (B4)

\[
\frac{d}{dt}\rho_0^0(5) = -\left[ \sum_{i=1}^3 A_{5i} + \sum_{i \neq 5} C_{5i} \right] \rho_0^0(5) + \sum_{i=1}^3 B_{15} \bar{J}_0^0(l \rightarrow 5) \sqrt{\frac{2J_i + 1}{2}} \rho_0^0(l) \\
- \frac{2\sqrt{2}}{5} B_{25} \bar{J}_0^2(2 \rightarrow 5) \rho_0^2(2) + \frac{\sqrt{7}}{10} B_{35} \bar{J}_0^0(3 \rightarrow 5) \rho_0^0(3) + \sum_{i \neq 5} C_{5i} \sqrt{\frac{2J_i + 1}{2}} \rho_0^0(i). 
\] (B5)

\[
\frac{d}{dt}\rho_0^2(2) = -\left[ \sum_{i=4}^5 B_{2i} \bar{J}_0^0(2 \rightarrow u) + \sum_{i \neq 2} C_{2i} + D_{2}^{(2)} \right] \rho_0^2(2) - \left( \frac{1}{\sqrt{2}} B_{24} \bar{J}_0^0(2 \rightarrow 4) - \frac{2\sqrt{3}}{5} B_{25} \bar{J}_0^0(2 \rightarrow 5) \right) \rho_0^0(2) \\
+ \frac{1}{2} A_{52} \rho_0^0(5) + \sum_{i=3,5} C_{2i} \frac{2J_i + 1}{2} \rho_0^0(i), 
\] (B6)

\[
\frac{d}{dt}\rho_0^3(3) = -\left[ B_{35} \bar{J}_0^0(3 \rightarrow 5) + \sum_{i \neq 3} C_{3i} + D_{3}^{(2)} \right] \rho_0^3(3) - B_{35} \bar{J}_0^0(3 \rightarrow 5) \sqrt{\frac{7}{5}} \rho_0^0(3) \\
+ B_{35} \bar{J}_0^2(3 \rightarrow 5) \left[ \sqrt{\frac{5}{7}} \rho_0^2(3) - \frac{9}{2} \sqrt{\frac{3}{35}} \rho_0^0(3) \right] + \frac{\sqrt{7}}{5} \sum_{i=2,5} C_{2i} \rho_0^2(5) + \frac{\sqrt{7}}{5} \rho_0^0(5) + \sum_{i \neq 5} C_{5i} \frac{2J_i + 1}{2} \rho_0^0(i). 
\] (B7)

\[
\frac{d}{dt}\rho_0^3(5) = -\left[ \sum_{i=1}^3 A_{5i} + \sum_{i \neq 5} C_{5i} + D_{3}^{(2)} \right] \rho_0^3(5) + \frac{1}{5} B_{25} \bar{J}_0^0(2 \rightarrow 5) \rho_0^0(2) + \frac{\sqrt{7}}{5} B_{35} \bar{J}_0^0(3 \rightarrow 5) \rho_0^0(3) + \frac{1}{2} B_{15} \bar{J}_0^0(1 \rightarrow 5) \rho_0^0(1) \\
- \frac{2\sqrt{7}}{5} B_{25} \bar{J}_0^0(2 \rightarrow 5) \rho_0^0(2) + \frac{\sqrt{7}}{10} B_{35} \bar{J}_0^0(3 \rightarrow 5) \rho_0^0(3) + 2\sqrt{\frac{7}{5}} B_{35} \bar{J}_0^2(2 \rightarrow 5) \rho_0^2(2) - \frac{\sqrt{7}}{5} B_{35} \bar{J}_0^2(3 \rightarrow 5) \rho_0^2(3) \\
+ \frac{9}{\sqrt{5}} B_{35} \bar{J}_0^0(3 \rightarrow 5) \rho_0^0(3) + \sum_{i=2,3} C_{2i} \frac{2J_i + 1}{2} \rho_0^0(i), 
\] (B8)

\[
\frac{d}{dt}\rho_0^3(5) = -\left[ B_{35} \bar{J}_0^0(3 \rightarrow 5) + \sum_{i \neq 3} C_{3i} + D_{3}^{(4)} \right] \rho_0^3(3) - B_{35} \bar{J}_0^2(3 \rightarrow 5) \left[ \frac{9}{2} \sqrt{\frac{3}{35}} \rho_0^0(3) + 3 \sqrt{\frac{11}{70}} \rho_0^3(3) \right], 
\] (B9)

where \( A_{ul} \) and \( B_{tu} \) are the Einstein emission and absorption coefficients; \( C_{lu} \) and \( C_{u} \) are the excitation and deexcitation inelastic collisional rates, respectively; \( C_{2i}^{(2)} \) and \( C_{2i}^{(4)} \) are the collisional transfer rates for alignment between polarizable levels (with \( J > 1/2 \)); and \( D_{i}^{(k)} \) is the depolarization rate of the \( k \)th multipole of level \( i \) due to elastic collisions with neutral hydrogen atoms. The \( \rho_k^0 \) elements are referred to a coordinate system with the quantization axis along the solar local vertical direction.
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Figure 10. Calculations in isothermal two-level atom moving atmospheres with $\xi_{\text{max}} = 5$. We assume a very strong line ($r_c = 0$) and $\epsilon = 10^{-4}$. The highest velocity gradient occurs at $\tau_l = \tau_0 = 100, 10, 1, 0.1, 0.01$ for a, b, c, d, and e, respectively. Case f corresponds to the solution in a static atmosphere. The vertical dotted line marks the position of $\tau_l = 1$.

(A color version of this figure is available in the online journal.)

APPENDIX C

TWO-LEVEL ATOM CALCULATION IN A MOVING ATMOSPHERE

Figure 10 is similar to Figure 5, but for velocity fields with $\xi = 5$ and maximum gradients occurring at different positions along the atmosphere.

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