Floquet topological magnons

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Abstract
We introduce the concept of Floquet topological magnons—a mechanism by which a synthetic tunable Dzyaloshinskii–Moriya interaction (DMI) can be generated in quantum magnets using circularly polarized electric (laser) field. The resulting effect is that Dirac magnons and nodal-line magnons in two-dimensional and three-dimensional quantum magnets can be tuned to magnon Chern insulators and Weyl magnons respectively under circularly polarized laser field. The Floquet formalism also yields a tunable intrinsic DMI in insulating quantum magnets without an inversion center. We demonstrate that the Floquet topological magnons possess a finite thermal Hall conductivity tunable by the laser field.

Quantum magnets without an inversion center allow an intrinsic Dzyaloshinskii–Moriya interaction (DMI) [1, 2], which is a consequence of spin–orbit coupling and it is usually fixed in different magnets. The associated magnon bands in the magnetically ordered systems have a nontrivial topology with Chern number protected chiral magnon edge modes in two-dimensional (2D) systems and magnon surface states in three-dimensional (3D) systems. They are dubbed topological magnon Chern insulators [3–9] and Weyl magnons [10–13] respectively. They are the analogs of electronic topological (Chern) insulators [14–16] and Weyl semimetals [17–19]. However, due to the charge-neutral property of magnons, topological magnonic materials are believed to be potential candidates for low-dissipation magnon transports in insulating quantum magnets and they are applicable to magnon spintronics and magnetic data storage [20]. Topological magnonic materials (i.e. magnon Chern insulators and Weyl magnons) also possess a thermal Hall effect as predicted theoretically [21–25] and observed experimentally [26–28]. To date, topological magnon bands have been realized only in a quasi-2D kagomé ferromagnet [29].

Generally, every quantum ferromagnetic material does not have a strong intrinsic DMI necessary for topological magnons to exist. For instance, the single crystals of the ferromagnetic honeycomb compounds CrX₃ (X = Br, Cl, and I) show no evidence of DMI [30–35], and kagomé haydeeite also does not have a finite (topological) energy gap in the observed spin-wave spectra [36], suggesting that the DMI does not play a significant role in haydeeite. These 2D ferromagnetic materials with negligible DMI are candidates for Dirac magnons [37], and the 3D ferromagnetic counterparts are candidates for nodal-line magnons [13, 38]. By applying a circularly polarized laser field in these ‘topologically trivial’ systems, one can generate topological magnons (i.e., magnon Chern insulators and Weyl magnons) via a tunable synthetic laser-induced DMI. This is particularly important as it offers a way to tune the DMI in magnetic materials.

The Floquet theory [39] of laser-driven systems provides a theoretical as well as experimental method to engineer such topologically nontrivial systems. This formalism is mostly dominated by electronic systems [40–59] and also optical bosonic systems [60–65]. Moreover, an applied laser field provides a means for coherent control of the magnetization in 1D quantum magnets [66–70].

In this Letter, we introduce the Floquet formalism to 2D and 3D ferromagnetic quantum magnets in the presence of a circularly polarized electric (laser) field. We show that the underlying charge-neutral magnons acquire a time-dependent Aharonov–Casher phase [71], which generates a tunable synthetic DMI in magnetic
systems with negligible intrinsic DMI. This leads to emergent nontrivial magnon Chern insulators and Weyl magnons with finite thermal Hall conductivity, which can be manipulated by the laser field. Our results provide a novel platform to engineer topological magnons in topologically trivial insulating quantum magnets. We hope that these results will extend the experimental search for topological magnons to a broader class of insulating quantum magnetic materials even without an intrinsic DMI.

Let us consider 2D and 3D ferromagnetic spin systems described by the pristine Hamiltonian

$$\mathcal{H}_0 = -J \sum_{(\alpha, \beta)} S_\alpha \cdot S_\beta,$$

where $\langle \alpha, \beta \rangle$ denotes summation over nearest-neighbor (NN) sites and $S_\alpha$ are the magnetic spin vectors at the lattice sites $\alpha$ located at $r_\alpha$, and $J$ is the ferromagnetic interaction. For now we take the intrinsic DMI to be negligible, which is the case in most ferromagnetic compounds. Therefore, the goal is to generate a synthetic DMI by periodic modulation of the lattice using a time-dependent electric field of a laser light.

We are interested in the underlying magnon excitations of the quantum spin Hamiltonian (1) as described by the Holstein–Primakoff transformation: $S^z_{\alpha} = S - a^+_{\alpha} a_{\alpha}, S^+_{\alpha} = \sqrt{2S} a_{\alpha} = (S^z_{\alpha})^*, \left( S^+_{\alpha} = S^z_{\alpha} \pm i S^y_{\alpha} \right)$ where $a_{\alpha}^+ (a_{\alpha})$ are the bosonic creation (annihilation) operators, and $S^z_{\alpha} = S^z_{\alpha} \pm i S^y_{\alpha}$ denote the spin creation and annihilation operators which correspond to the hopping terms. However, we will retain the spin operators in order to show the explicit form of the laser-induced DMI. As magnons are charge-neutral bosonic quasi-particles they do not interact with electromagnetic field except through their magnetic dipole moment, which we assume to be in the $z$-direction $\mu = g \mu_B \hat{z}$, where $\mu_B$ is the Bohr magneton and $g$ the Landé $g$-factor. Now suppose that a circularly polarized laser (electromagnetic) field with dominant electric field components $E(t) = E_0 (\tau \cos \omega t, \sin \omega t, 0)$ is irradiated perpendicular to the magnetic material lying on the $x$–$y$ plane, where $\tau = \pm$ for right and left circularly polarizations respectively. The magnetic dipole moment of magnon quasi-particles hopping in such an electric field background will acquire a phase factor given by

$$A_{\alpha, \beta}(t) = \frac{g \mu_B}{\hbar c^2} \int_{r_\alpha}^{r_\beta} A(t) \cdot d\ell,$$

where $A(t) = A_0 (\tau \sin \omega t, -\cos \omega t, 0)$ is the vector potential with amplitude $A_0 = \lambda = E_0 / \omega$ due to the electric field of the laser beam $E(t) = -\partial A(t) / \partial t$. The phase factor (2) can be regarded as the time-dependent Aharonov–Casher phase [71] acquired by the hopping of charge-neutral magnon quasi-particles in the presence of a time-varying electric field. The corresponding time-dependent Hamiltonian is given by

$$\mathcal{H}(t) = -\sum_{\langle \alpha, \beta \rangle} \left[ \frac{J}{2} (S^+_{\alpha} S^z_{\beta} S^z_{\alpha} S^+_{\beta}) e^{i A_{\alpha, \beta}(t)} + \text{h.c.} \right] + J S^z_{\alpha} S^z_{\beta},$$

Noticing that the direction of the vector pointing from $\alpha$ to $\beta$ defines a relative angle $\phi_{\alpha, \beta}$, we get $A_{\alpha, \beta}(t) = \lambda (\sin \omega t - \phi_{\alpha, \beta})$ and $g \mu_B / \hbar c^2$ is absorbed in $\lambda$.

The Floquet theory is a standard mechanism to study driven quantum systems in the likes of equation (3) [40–43]. It enables one to transform a time-dependent model into a static effective model governed by what is called the Floquet Hamiltonian. Now, we proceed with this formalism. The static time-independent effective Hamiltonian can be expanded in the power series of $\omega^{-1}$ and it is given by $\mathcal{H}_{\text{eff}} = \sum_{n=0}^\infty \omega^{-n} \mathcal{H}^{(n)}$. We calculate the series expansion of the effective Hamiltonian using the discrete Fourier component of the time-dependent Hamiltonian $\mathcal{H}^{(n)} = \frac{1}{T} \int_0^T dt e^{-i n \omega t} \mathcal{H}(t)$ with period $T = 2\pi / \omega$, where

$$\mathcal{H}^{(n)} = -\sum_{\langle \alpha, \beta \rangle} \left[ \frac{J}{2} (S^+_{\alpha} S^z_{\beta} S^z_{\alpha} S^+_{\beta}) e^{-i \omega n t} + \text{h.c.} \right] + J \mathfrak{b}_{n, \alpha} S^z_{\alpha},$$

with $\mathfrak{b}_{n, \alpha} = J f_\ell (\lambda)$, and $f_\ell (\lambda)$ is the Bessel function of order $n \in \mathbb{Z}$, $f_\ell (\lambda) = 1$ for $n = \ell$ and zero otherwise. We have used the standard relation $e^{i \omega \sin \omega t} = \frac{1}{2} \sum_{m=-\infty}^{\infty} f_m (\lambda) e^{i m \omega t}$. In the high frequency limit $\omega \gg J$ the leading order contribution is the zeroth order effective Hamiltonian given by $\mathcal{H}_{\text{eff}}^{(0)} = \mathcal{H}^{(0)}$, where

$$\mathcal{H}_{\text{eff}}^{(0)} = -\sum_{\langle \alpha, \beta \rangle} \left[ \mathfrak{b}_{0, \alpha} S^z_{\alpha} S^z_{\beta} + S^z_{\alpha} S^z_{\beta} \right] + J S^z_{\alpha},$$

and $\mathfrak{b}_{0, \alpha} = J f_0 (\lambda)$. Thus, the zeroth order term yields an XXZ ferromagnetic Hamiltonian. For quantum spin-$1/2$ ferromagnetic systems, equation (5) is equivalent to the Bose–Einstein condensation of hardcore bosons studied by Matsubara and Matsuda [72], but in this case $J \gg |\mathfrak{b}_{0, \alpha}|$. By lowering the frequency the first order contribution to the effective Hamiltonian is non-negligible. It is given by

$$\mathcal{H}_{\text{eff}}^{(1)} = \sum_{n=1}^\infty \frac{1}{n} [\mathcal{H}^{(n)} - \mathcal{H}^{(n-1)}].$$

The commutator of the spin operators results in a product of three spins reminiscent of the scalar spin chirality. In order to see this we note that the $z$-component of the spins vanishes for $n \geq 1$, therefore the commutation
relation in the first order term involves only \( [S_x^+ S_y^-, S_y^+ S_x^-] = 2(\delta_{xx'} S_x^+ S_y^+ \delta_{yy'} - \delta_{xx'} S_x^- S_y^- \delta_{yy'}) \). We use this relation together with the identity \( \mathcal{J}_\alpha(z) = (-)^z \mathcal{J}_\alpha(z) \) and obtain

\[
\mathcal{H}_{\text{eff}} = \sum_{\Delta / \nabla} J_{\alpha \beta} S_x \cdot (S_x \times S_y),
\]

where

\[
J_{\alpha \beta} = -\sum_{n=1}^{\infty} 2(-)^n J_{m \downarrow} \sin(n\Phi_{\alpha \beta}),
\]

with \( \Phi_{\alpha \beta} = \phi_{\alpha \gamma} - \phi_{\beta \gamma}, S_\gamma = S_2^x \) and \( \gamma \) is the intermediate lattice site between \( \alpha \) and \( \beta \) and the sum is over the triangular plaquettes of the chosen lattice geometry.

The main result of this Letter is the induced synthetic DMI by circularly polarized laser field as given in equation (7). The DMI points along the \( z \)-direction perpendicular to the \( x-y \) plane. On the kagomé and pyrochlore lattices the synthetic DMI lies within the bonds of the NN sites, whereas on the honeycomb lattice it lies within the bonds of the next-nearest neighbor (NNN) sites. The DMI is the primary source of magnon Chern insulators and Weyl magnons in insulating quantum ferromagnets. Consequently, Dirac magnons in 2D ferromagnetic systems and nodal-line magnons in 3D ferromagnetic systems with negligible intrinsic DMI will be driven to magnon Chern insulators and Weyl magnons respectively by the circularly polarized laser field.

Now, we exemplify this method by utilizing the 2D honeycomb ferromagnets for simplicity (see figure 1). In this case the relative angle is given by \( \phi_{\alpha \gamma} - \phi_{\beta \gamma} = 2\pi \nu_{\alpha \beta} [57, 58] \), where \( \nu_{\alpha \beta} = +/ - \) for hopping within \( \Delta / \nabla \) respectively. Hence \( J_{m \downarrow} \approx \sqrt{3} J_{0 \beta} \). The total effective Floquet Hamiltonian up to first order is given by

\[
\mathcal{H}_{\text{eff}} = -\sum_{\langle \alpha \beta \rangle} \left[ J_{0 \beta} (S_x^\alpha S_y^\beta + S_x^\beta S_y^\alpha) + JS_x^\alpha S_z^\alpha \right]
+ D_F \sum_{\Delta / \nabla} \nu_{\alpha \beta} S_\gamma \cdot (S_\alpha \times S_\beta),
\]

where \( D_F = \sqrt{3} J_{0 \beta} / \omega \). We note that \( |J_{0 \beta}| \leq J_1 \) so that the spins would prefer to order magnetically along the \( z \)-direction. The synthetic DMI does not affect the ferromagnetic ordering of the spins. We study the underlying magnon excitations of equation (9) by implementing the linear order Holstein–Primakoff transformation mentioned above on the two sublattices \( A \) and \( B \) of the honeycomb lattice. In the bosonic representation the synthetic DMI is imaginary. It generates a synthetic magnetic gauge flux within the NNN sites. The momentum space Hamiltonian is given by

\[
\mathcal{H}_{\text{eff}}(k) = h_0 I_{2 \times 2} + h(k) \cdot \sigma,
\]

where \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \) are Pauli matrices and \( I_{2 \times 2} \) is an identity \( 2 \times 2 \) matrix. \( h_0 = 3JS_1 \) and \( h(k) = (h_1, h_2, h_3) \) are the Hamiltonian parameters, where

\[
h_1(k) = -J_{0 \beta} \sum_j \cos k \cdot \delta_j, h_2(k) = -J_{0 \beta} \sum_j \sin k \cdot \delta_j, h_3(k) = 2D_F S_z^\alpha \sin k \cdot b_j.
\]

The Floquet magnon bands are given by

**Figure 1.** The schematic of the honeycomb lattice with laser-field-induced DMI. The blue dotted and crossed circles denote the direction of the synthetic DM vectors pointing in and out of the lattice plane on the NNN bonds. Here, \( b_i \) are the vectors connecting the NN sites and \( b_i \) connect the NNN sites. The different colors of the solid circles red(green) are used to denote the two sublattices \( A \) (\( B \) ).

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It is easily shown that the Floquet Chern numbers are proportional to the synthetic DMI:

\[ C_\eta^F = \frac{1}{2\pi} \int_{BZ} d^2k \, \Omega_{\eta k}^F, \]  

(11)

where \( \eta = \pm \) labels the top and the bottom bands respectively. Henceforth we set \( S = 1/2 \). The Floquet magnon bands are depicted in figure 2 for zeroth order (i) and first order (ii) contributions to the effective Hamiltonian. In the former (i), the system exhibits Dirac magnon points, but they are different from the Dirac magnon points in the undriven systems\,[37] because the Floquet Dirac magnons can be manipulated by anisotropic laser field amplitude, i.e., \( A_0 = (A_x, A_y, 0) \). In this case the Dirac magnon points can move away from the high symmetry points of the Brillouin zone at \( \pm K = (\pm 4\pi/3\sqrt{3}, 0) \) and can also be gapped out by fine-tuning the anisotropic amplitudes. However, the system will still remain topologically trivial at the zeroth order as time-reversal symmetry is preserved.

A nontrivial band topology arises by lowering the frequency and the first order correction becomes important, which breaks time-reversal symmetry. This leads to a synthetic DMI which induces a gap \((\Delta_{\text{gap}} \approx 2|D_2|)\) at the Dirac points as shown in figure 2(ii). The Floquet magnon bands now acquire a nonzero Berry curvature which has the form \( \Omega_{\eta k}^F = (\nabla \times \mathbf{A}_{\eta k})_2 \) where \( \mathbf{A}_{\eta k}^F = i\langle \psi_{\eta k}^F | \nabla | \psi_{\eta k}^F \rangle \) is the Berry connection and \( \psi_{\eta k}^F \) are the Floquet eigenvectors of \( \mathcal{H}_{\text{eff}}(k) \). The Floquet Chern numbers are given by the integration of the Berry curvatures over the Brillouin zone,

\[ C_\eta^F = \frac{1}{2\pi} \int_{BZ} d^2k \, \Omega_{\eta k}^F. \]  

(12)

It is easily shown that the Floquet Chern numbers are proportional to the synthetic DMI: \( C_\eta^F \approx \eta \, \text{sgn}[D_2] \), which can be controlled by the laser field.

The nontrivial topology of the Floquet magnon bands has an important transport consequences. It can lead to Floquet thermal magnon Hall effect. This refers to the generation of a transverse heat current upon the application of a longitudinal temperature gradient. In this case, the synthetic laser-field-induced Berry curvature due to the DMI appears in the equations of motion of a magnon wave packet in the same mathematical structure as a magnetic field in the Lorentz force. In other words, the Berry curvature due to the DMI acts like an effective magnetic field in momentum space on the magnons. The resulting effect is the production of a transverse thermal Hall conductivity, which can derived from linear response theory\,[22]. The Floquet thermal Hall conductivity is given by

\[ \kappa_{\eta}^F(\lambda, T) = -\frac{k_B T}{2\pi} \sum_{\eta = \pm} \int_{BZ} d^2k \, c_2(n_{\eta}) \Omega_{\eta k}^F, \]  

(13)

where \( n_{\eta} \equiv n_{b}(\epsilon_{\eta k}) = (e^{\epsilon_{\eta k}/k_B T} - 1)^{-1} \) is the Bose function close to thermal equilibrium, \( c_2(x) = (1 + x)(\ln 1 + x)^2 - (\ln x)^2 - 2Li_2(-x) \), and \( Li_2(x) \) is a polylogarithm. In figure 3 we have shown the plots of \( \kappa_{\eta}^F(\lambda, T) \) as functions of the parameters in units of \( k_B / T \). For the two-band honeycomb ferromagnets the Floquet thermal Hall conductivity is negative and it is tunable by the amplitude (frequency) of the laser field. In particular, \( \kappa_{\eta}^F(\lambda, T) \) can be tuned off at the zeros of \( J_{\text{eff}}(\lambda) \). This is usually not possible in materials with a strong intrinsic DMI in the absence of a laser field.

However, applying circularly polarized laser field in magnetic materials with a strong intrinsic DMI can induce a tunable DMI. Using the simple honeycomb ferromagnetic model as an example, the suitable DMI due to inversion symmetry breaking of the lattice is of the form\,[7] \( \mathcal{H}_{\text{DMI}} = D_0 \sum_{(\alpha, \beta)} \nu_{\alpha\beta} \hat{z} \cdot \mathbf{S}_\alpha \times \mathbf{S}_\beta \) where the summation is taken over the triangular plaquettes of the NNN sites. In the presence of a circularly polarized laser field with sufficiently high frequency, the zeroth order correction to the effective Hamiltonian gives

\[ \mathcal{H}_{\text{eff}} = D_0 J_{\text{eff}}(\lambda) \sum_{(\alpha, \beta)} \nu_{\alpha\beta} \hat{z} \cdot \mathbf{S}_\alpha \times \mathbf{S}_\beta. \]  

Now the value of the intrinsic DMI can be manipulated by varying \( \lambda \). For \( \lambda < \lambda_c \approx 2.4048 \) a gap exists at the Dirac points and it closes near the first zero of the Bessel function \( \lambda \approx \lambda_c \) and reopens for \( \lambda > \lambda_c \). Consequently, the sign of the Chern numbers (Berry curvatures) of the two bands changes as shown in figure 4. Hence, a sign change emerges in \( \kappa_{\eta}^F(\lambda, T) \) (not shown). We note that a sign
change is not possible in the undriven system on the honeycomb lattice \([8, 9]\). The complete topological phase transition including a trivial insulating regime with zero Chern number can be found by including NNN Heisenberg interaction as we have shown in the supplemental material\(^1\).

It is important to note that the circularly polarized laser field is necessary to induce a tunable synthetic DMI in quantum ferromagnets with negligible intrinsic DMI. However, in frustrated magnets with a coplanar magnetic order, a static magnetic field applied perpendicular to the plane of the magnets can induce a tunable synthetic scalar spin chirality due to non-coplanar spin configurations. In this case tunable topological magnons can be produced even in the absence of an intrinsic DMI \([73–75]\).

In conclusion, we have shown that the time-dependent Aharonov–Casher phase acquired by charge-neutral magnons in a time varying circularly polarized electric (laser) field induces a synthetic DMI, leading to nontrivial topological magnons in the absence of an intrinsic DMI. Therefore, Dirac magnons and nodal-line magnons become magnon Chern insulators and Weyl magnons respectively under radiation. Our results also showed that the intrinsic DMI in insulating quantum magnets without an inversion symmetry can be manipulated with circularly polarized laser field. Hence, the emergent thermal Hall effect can be turned. It is experimentally feasible to investigate the proposed phenomena in magnetic insulators using ultrafast terahertz spectroscopy \([76]\). This formalism may find application in the switching of magnetization and magnon spin current by a laser

\(^1\) See the supplemental material is available online at stacks.iop.org/JPCO/1/021002/mmedia.

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**Figure 3.** Floquet tunable thermal Hall conductivity. (i) \(\kappa_{xy}^F\) versus \(\lambda\) for different temperatures. (ii) \(\kappa_{xy}^F\) versus \(T\) for different amplitudes. Here, \(J = 1\) meV and \(\hbar \omega = 2.5 J\).

**Figure 4.** Evolution of sign change in the Floquet Chern numbers of honeycomb ferromagnets with tunable intrinsic DMI for different values of \(\lambda\) and \(D_{xy}/J = 0.15\). The black lines are the magnon bulk bands and blue lines denote the chiral magnon edge modes gapless at \(\varepsilon(k_y = \pi/\sqrt{3}) = \epsilon_0\) as indicated by the dashed line.
field as an important step towards magnon spintronics and magnetic data storage [77, 78]. We believe that these results will extend the study of topological magnons to different magnetic materials without any restrictions. In an inhomogeneous static electric field a time-independent Aharonov–Casher phase can be acquired by magnons and leads to magnonic Landau levels in insulating magnets [79].

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