Magnetic Field Induced Phase Transitions in YBa$_2$Cu$_4$O$_8$

T. Schneider and J. M. Singer

Physikinstitut, Universität Zürich, Winterthurerstr. 190, CH-8057 Zürich, Switzerland

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Abstract. The c-axis resistivity measurements in YBa$_2$Cu$_4$O$_8$ from Hussey et al. for magnetic field orientations along the c-axis as well as within the ab-plane are analyzed and interpreted using the scaling theory for static and dynamic classical critical phenomena. We identify a superconductor to normal conductor transition for both field orientations as well as a normal conductor to insulator transition at a critical field $H_c$ with a dynamical critical exponent $z = 1$, leading to a multicritical point where superconducting, normal conducting and insulating phases coexist.

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Recently it has been demonstrated, that the doping tuned superconductor to insulator (SI) transition in cuprates can be understood in terms of quantum critical phenomena in two dimensions. Zero temperature magnetic field driven SI transitions have also been observed in ultrathin Bi films, and successfully interpreted in terms of the scaling theory of quantum critical phenomena. Nevertheless, three important questions concerning the physics of insulating and superconducting cuprates remain open. One is the nature and dimensionality of the normal state revealed when superconductivity is suppressed by a magnetic field and the second is the role of disorder. The third issue concerns the dynamical universality classes of SI and superconductor to normal conductor (SN) transitions at finite temperatures.

We address these three issues through an analysis and interpretation of recent out-of-plane resistivity measurements $\rho_c$ of YBa$_2$Cu$_4$O$_8$ in magnetic fields by Hussey et al. and, using the scaling theory of static and dynamic classical critical phenomena. Since this material is stoichiometric and, therefore, can be synthesized with negligible disorder, we consider the pure case. As shown below, the experimental data for $\rho_c(T, H \parallel c)$ are consistent with a magnetic field and $\rho_c(T, H \parallel a)$ and $\rho_c(T, H \parallel b)$ and the second is the role of disorder. The third issue concerns the dynamical universality classes of SI and superconductor to normal conductor (SN) transitions at finite temperatures.

The basic scaling argument, which amounts to a dimensional analysis, states that the two terms on the right hand side must have the same scaling dimension, (Length)$^{-1} \equiv L^{-1}$. The dimensionality of the magnetic and electric field are then expressed as

$$\mathbf{H} = \nabla \times \mathbf{A} \propto L^{-2}, \quad \mathbf{E} = \frac{\partial \mathbf{A}}{\partial t} \propto (Lt)^{-1}. \quad (2)$$

In a superconductor the order parameter $\Psi$ is a complex scalar,

$$\Psi = \text{Re}(\Psi) + i\text{Im}(\Psi),$$

corresponding to a vector with two components. Consequently, the dimensionality of the order parameter is $n = 2$. Based upon the dimensional statement

$$\xi = \xi_0 |\epsilon|^\nu \propto L, \quad \epsilon = \frac{T - T_c}{T_c}, \quad (3)$$

where $\pm = \text{sign}(\epsilon)$, we obtain in $D$ dimensions for the free energy density the scaling form

$$f = \frac{F}{(V k_B T)} \propto L^{-D} \propto (\xi^{\pm})^{-D}, \quad (4)$$
and for \( H \neq 0 \)

\[
f = (\xi^\pm)^{-D} G(Z), \quad Z = \frac{H(\xi^\pm)^2}{\Phi_0},
\]

(5)
due to Eq. (2). \( G \) is an universal scaling function of its argument \( Z \). An extension to 3D anisotropic materials, such as cuprates, is straightforward:

\[
f = (\xi^\pm c^\pm)^{-1} G(Z),
\]

(6)
where the indices \( x, y, z \) denote the corresponding crystallographic \( b, a, c \)-axes of the cuprates, and

\[
\begin{align*}
\mathbf{H} &= H(0, \sin \delta, \cos \delta), \\
Z &= \left( \frac{\xi_x^2}{\Phi_0} \right)^2 H_x^2 + \left( \frac{\xi_y^2}{\Phi_0} \right)^2 H_y^2 + \left( \frac{\xi_z^2}{\Phi_0} \right)^2 H_z^2,
\end{align*}
\]

(7)

Using this scaling form of the free energy density magnetization \([10,11]\) and magnetic torque data \([9,12]\) have been successfully analyzed.

Of particular interest in the present context is the conductivity \( \sigma \). From the dimension of the current,

\[
\mathbf{J} = \nabla \chi \times L^{-D+1},
\]

(8)
and the electric field (see Eq. (4)), we obtain for the electric conductivity

\[
\sigma = \frac{J}{E} \propto t L^{2-D} \propto \xi^{2-D+z}, \quad t \propto \xi^z,
\]

(9)
since the scaling dimension of time is fixed by

\[
t \propto L^2 \propto \xi^z.
\]

The relaxation time \( \tau \) describes the rate at which the system relaxes to equilibrium. \( \tau \) diverges at the transition and the dynamic critical exponent \( z \) is defined as

\[
\tau \propto \xi^z \propto |\epsilon|^{-2\nu}.
\]

For \( H \neq 0 \) the scaling expression for the conductivity reads as

\[
\sigma(T, H) = \xi^{2-D+z} G(Z), \quad Z = \frac{H(\xi^\pm)^2}{\Phi_0}.
\]

(10)
Supposing then that there is a critical point at \( Z = Z_c \) with

\[
2 - D + z = 0,
\]

(11)
the curves \( \sigma \) versus \( H \) recorded at different temperatures \( T \) will cross at \( H = H_c \), where \( Z = Z_c \). The extension to anisotropic systems reads as

\[
\sigma_{ii} \propto \frac{\xi_i \xi_j}{\xi_j \xi_k} G(Z),
\]

(12)
with \((i, j, k) \equiv (x, y, z)\) and \( Z \) given by Eq. (10).

We are now prepared to analyze the resistivity data of Hussey et al. \([7]\). At \( H = 0 \), the material is supposed to undergo a continuous SN transition belonging to the 3D-XY universality class, and accordingly

\[
\xi_{x,y} \equiv |\xi_{0,x,y}| |\epsilon|^{-\nu}, \quad \xi_z \propto |\epsilon|^{-\nu}, \quad \nu \approx 2/3.
\]

(13)
If the transition occurs in finite fields, then \( Z = Z_c \), and close to \( T_c = T_c(H = 0) \) the phase transition line is given by

\[
H_{c,i} = \frac{Z_c \Phi_0}{\xi_{0,j} \xi_{0,k}} \left( \frac{T - T_c}{T_c} \right)^{2\nu}, \quad (i, j, k) \equiv (x, y, z).
\]

(14)
From Fig. 3 it is seen that the resulting behavior for \( H_{c,z} \) (i.e. \( \mathbf{H} \parallel c \)), namely

\[
T_c(H) = T_c(H = 0) \left( 1 - 0.078 H^{3/4} \right),
\]

(15)
agrees remarkably well with the experimental data. Similarly, for \( H_{c,y} \) (\( \mathbf{H} \parallel a \)) we obtain the estimate (using the
two points \(T_c(H = 0) = 80K, T_c(H = 35T) = 65K\) measured by \([7]\)

\[
T_c(H) \approx T_c(H = 0) \left(1 - 0.013H^{3/4}\right), \quad (16)
\]

which has been included in the phase diagram shown Fig. 4. Combining Eqs. (14), (15) and (16), we obtain for the

of the \(c\)-axis resistivity for a field oriented along the \(a\)-axis

differs drastically from the \((H \parallel c)\) data. Indeed, it is seen that in the normal state \(T_c < T < 200K\) for \((H \parallel a)\) a NI transition occurs. In particular, all \(\rho_c(H \parallel a)\) curves go through a single crossing point (Fig. 3) at

\[
H_c \approx 19T, \quad \rho_{c,c} \approx 8m\Omega cm,
\]

which is the value where \(\rho_c(T)\) becomes essentially temperature independent (Fig. 3). Nevertheless, there is still a transition into a superconducting regime beyond \(H_c \approx 19T\) for \(H \parallel a\) (e.g. \(T_c(35T) = 65K\)). One clearly observes in Fig. 2 that at low magnetic fields \((H < H_c \approx 19T)\), as the temperature is reduced, \(\rho_c(T)\) shows a drop from its normal state value \(\rho_c \approx 8m\Omega cm\). For \(H > H_c \approx 19T\) and \(T > T_c(H)\), \(\rho_c\) rises with decreasing temperature, signalling the onset of insulating behavior. The resulting phase diagram, showing SN, SI and NI transitions as well as a multicritical point, where the superconducting, normal conducting and insulating phases can coexist, is drawn in Fig. 4. On physical grounds one expects that the NI transition line will have a critical endpoint in the normal state, too.

The existence of the crossing point (critical field) in the bulk material \((D = 3)\) (Fig. 3) implies according to Eq. (11) that the dynamical critical exponent of the NI transition is \(z = 1\). As a consequence, the scaling form of the conductivity (Eq. (12)) reduces for \(H \parallel a, b\) to

\[
\sigma_{zz} \propto G(Z_c) \quad \text{for } z = 1, \quad D = 3, \quad (18)
\]

where

\[
Z_c = \frac{\xi_y \xi_z H_c}{\Phi_0} \sqrt{\cos^2(\phi) + \left(\frac{\xi_y}{\xi_x}\right)^2 \sin^2(\phi)}. \quad (19)
\]

Thus, curves \(\rho_c\) vs. \(H\) \((H \parallel a)\), recorded at different temperatures \(T\), exhibit a crossing point at \(H = H_c\), where \(Z = Z_c\), in agreement with the experiment (Fig. 3). Moreover, if the data for \(H \parallel a\), as shown in Fig. 3.

According to Figs. 2 and 3 showing \(\rho_c\) vs. temperature for various fields \((H \parallel a)\) as well as \(\rho_c\) vs. \(H\) \((H \parallel a)\) for various temperatures between 80K and 200K, the behavior

Fig. 3. \(\rho_c\) versus \(H, H \parallel a\) for various temperatures, taken from \([7]\).

yz-anisotropy of the correlation lengths the estimate

\[
\frac{\xi_y}{\xi_z} \approx \left(\frac{0.078}{0.013}\right)^{4/3} \approx 11, \quad (17)
\]

which is close to the value obtained from magnetic torque measurements \([3]\).
are replotted according to

$$\sigma_{zz} = \mathcal{F}\left(\frac{h}{t^{2\nu}}\right),$$  \hspace{1cm} (20)

with $h = |H - H_c|/H_c$ and $t = |T - T_c(H_c)|/T_c(H_c)$, they collapse, as shown in Fig. 5, onto two branches (using $T_c(H_c) \approx 70.5K$, $H_c \approx 19T$ and $\nu \approx 2/3$).

Finally, close to the NI transition ($Z = Z_c$) we obtain

$$\sigma_{zz} \approx \sigma_{zz}(Z_c) + \frac{\partial \sigma_{zz}}{\partial Z} \bigg|_{Z=Z_c} (Z - Z_c),$$  \hspace{1cm} (21)

where

$$Z - Z_c = (H - H_c)\frac{\xi_y}{\xi_y} \cos^2 \phi + \frac{\xi_x^2}{\xi_y^2} \sin^2 \phi.$$  \hspace{1cm} (22)

Thus, provided that $\xi_y \neq \xi_x$, both, out-of-plane conductivity and resistivity will depend on the angle $\phi$, so that close to the multicritical point

$$\Delta \rho_{zz} = \frac{1}{\sigma_{zz} - \sigma_{zz}(Z_c)} = \left(\frac{\partial \sigma_{zz}}{\partial Z} \bigg|_{Z=Z_c} (H - H_c)\right)^{-1} \frac{\xi_x \xi_y}{\phi_0} \cos^2 \phi + \frac{\xi_x^2}{\xi_y^2} \sin^2 \phi.$$  \hspace{1cm} (23)

Fig. 6 shows a fit of the Y-124 data to Eq. (23), this expression describes the experimental data very well, yielding $\xi_z/\xi_y \approx 1.27$. Given the estimate $\xi_y/\xi_z \approx 11$ (Eq. (17)) and noting that at the crossing point the magnetic field has to scale (Eq. (24)) as

$$H_c,z/H_c,y = \xi_y/\xi_z,$$  \hspace{1cm} (24)

it is readily seen that with $H_c,y \approx 19T$ and $H || (z = c)$ a magnetic field driven NI transition might occur for rather high fields only. Indeed, according to the experimental phase diagram shown in Fig. 6 there is no such NI transition up to $H = 35T$, where the superconducting phase dissappears even at $T = 0$ and a magnetic field driven SN quantum phase transition occurs.

To summarize, we have shown that the magnetic field and temperature dependence of the $c$-axis resistivity in YBa$_2$Cu$_4$O$_8$, recorded for various magnetic field orientations, can be understood in terms of the scaling theory of static and dynamic classical critical phenomena. For $H \parallel c$ we identified a SN transition. For $H \parallel a$ we provide considerable evidence for the occurrence of a finite temperature NI transition in $D = 3$ with the dynamical critical exponent $z = 1$, as well as for a multicritical point, where superconducting, normal conducting and insulating phases can coexist. The occurrence of this multicritical point must be attributed to the comparatively small anisotropy, characterized by the correlation length ratio $\xi_y/\xi_z (\xi_y/\xi_z \approx 11)$, rendering the critical field to accessible values. For this reason, the multicritical point and the associated NI transition are expected to be generic for sufficiently clean, homogeneous and almost optimally doped cuprates with moderate anisotropy $\xi_y/\xi_z$ and $\xi_z/\xi_y$.

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