Coupling Coefficients of Different Disk Microresonators with Whispering Gallery Modes

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The coupling coefficients of the disk microresonators, consisting of different dielectrics, are presented. The analytical expressions for the coupling coefficients are obtained. The basic regularities of the coupling coefficient changing on the structure parameter variations are considered. The coupling coefficients as functions of main structure parameters are studied. The calculation results of the transmission coefficients as well as the reflection coefficients of the bandpass filters, building up on different disk microresonators with different modes are presented. The S-matrix frequency dependences of the bandpass filters on different disk microresonators in the infrared wavelength range are calculated. The dielectric Q-factor values, necessary for acceptable measure of the scattering are determined. The amplitude-frequency characteristics of the bandpass filters on the vertically coupled as well as laterally coupled disc microresonators are investigated. Most optimal configurations of coupled microresonators, allowing to achieve the best scattering characteristics are determined. It’s showed that using of microresonators with whispering gallery modes allow relatively easy obtain frequency-symmetrical characteristics of the filters.

Key words: microresonator; coupling coefficient; whispering gallery mode; S-matrix; filter

Introduction

Disk dielectric microresonators with whispering gallery (WG) modes inscribes in the planar integral circuits quite naturally. Today, ones are being actively studied for purpose of their application in the different devices of the optical, infrared and terahertz wavelength ranges [1–7]. Eigenoscillations of two disk dielectric microresonators were considered in [4–7], but its coupling coefficients were not calculated and not studied in full measure. For calculation and optimization of the device parameters, it’s convenient to carry out on basis of electrodynamic modeling with using coupling coefficients [8].

The goal of the present work is the analytical calculation of the coupling coefficients of different disk microresonators with WG modes in the open space. In this article, we also use the scattering theory for S-matrix coefficients calculation of different microresonator bandpass filters.

1 Eigenoscillation field calculation of the disk microresonator

For the coupling coefficients calculation, information about microresonator eigenoscillation fields is necessary. Most simple analytical presentation the field within dielectric cylinder can be obtained in the form of so-called one-wave approximation [9]. Toward this end, an electromagnetic field is written in the cylindrical coordinate system \((\rho, \alpha, z)\) (see fig. 1a), located in the microresonator center, approximately in the form of hybrid standing wave of the circular dielectric waveguide section:

\[
e^\rho = e_1 \frac{\beta_2}{\beta} J'_m (\beta \rho) + h_1 \frac{i \omega \mu_0}{\beta} \cdot \frac{J_m (\beta \rho)}{\beta} \cdot \begin{bmatrix}
\sin \alpha \\
\cos \alpha
\end{bmatrix} \cdot \begin{bmatrix}
\cos \beta z \\
-\sin \beta z
\end{bmatrix}
\]

\[
e^\alpha = e_1 \frac{\beta_2}{\beta} \cdot J'_m (\beta \rho) + h_1 \frac{i \omega \mu_0}{\beta} \cdot \frac{J_m (\beta \rho)}{\beta} \cdot \begin{bmatrix}
\cos \alpha \\
-\sin \alpha
\end{bmatrix} \cdot \begin{bmatrix}
\cos \beta z \\
-\sin \beta z
\end{bmatrix}
\]

\[
e^z = e_1 J_m (\beta \rho) \begin{bmatrix}
\sin \alpha \\
\cos \alpha
\end{bmatrix} \begin{bmatrix}
\sin \beta z \\
\cos \beta z
\end{bmatrix}
\]

\[
h_\rho = e_1 \frac{i \omega \varepsilon_1}{\beta} \cdot \frac{J_m (\beta \rho)}{\beta} - h_1 \frac{\beta_2}{\beta} \cdot J'_m (\beta \rho) \cdot \begin{bmatrix}
\cos \alpha \\
-\sin \alpha
\end{bmatrix} \begin{bmatrix}
\sin \beta z \\
\cos \beta z
\end{bmatrix}
\]

(1)
The $n$ defines the number of half-waves, located in the radial direction inside dielectric cylinder, the defines half-waves number, located in the direction of $z$-axis in the microresonator material, the is integer. Sign $+$ ($-$) in the cases of $HE_{nml}^{±}$ mode corresponds to an even (odd) mode distribution of $z$-component of the magnetic field; and the $(-)$ ($+$) for the $EH_{nml}^{±}$ mode corresponds to an even (odd) mode distribution of $z$-component of the electric field in the microresonator (see fig.1) relative to the plane of symmetry $z = 0$ (fig. 1a).

Equations (3) and (4) must be solved simultaneously.

2 Calculation of coupling coefficient

The fields and frequencies of several microresonator coupled oscillations defines by values of the coupling coefficients. In the common case, the coupling coefficient can be determined as a surface integral:

$$
\kappa_{sn} = \frac{i}{2 \omega_0 w_n (1 + \delta_{sn})} \oint \left\{ \bar{e}_s \bar{H}_n + \bar{e}_n \bar{H}_s \right\} \tilde{n} ds,
$$

expressed via the eigemode field $(\bar{e}_s, \bar{H}_n)$ of one $(s$-th) microresonator on the surface of another $(n$-th) microresonator. Here $s, n = 1, 2$; and $\tilde{n}$ is the normal to the surface $s_n$ of $n$-th microresonator; $\omega_0$ is the resonance frequency; $w_n$ is the energy, stored in the dielectric of $n$-th microresonator.

As follow from [8], eigencoscillations of two identical microresonators take on form of even- and odd- modes spatial field distributions with respect to symmetry plane, located between ones (fig. 1b). In this case (5): $\kappa_{11} = \kappa_{22} = i k; \quad \kappa_{12} = \kappa_{21} = \kappa$ and $b_1^+ = +b_1; b_2^- = -b_2^-$; $\lambda_{1,2} = i k \pm \kappa$, are corresponding to the sphaxed, or even, and antiphased, or odd, field distribution of the coupling oscillations.

The real and imaginary parts of the coupling coefficient can take positive values as well as negative ones. As follow from (5), the coupling coefficients of two different microresonators are not equal to one another: $\kappa_{12} \neq \kappa_{21}$.

The integral (5) can be calculated, based on early known analytical expression for the coupling coefficients, of the Cylindrical DRs in the Rectangular metal waveguide. In this case, required analytical expressions for mutual coupling coefficients $\kappa_{12}$, can be received by transferring the waveguide walls to the infinity. Using necessary expressions, after simplifications, we obtain:

2.1 Case A.

In the case of two different microresonators, with the same parity of each field on $\alpha$ and on the indexes $m_s$.
\[(s = 1, 2), \text{relative to the plane of symmetry: } y_s - y = 0\]

where the top sign of (6) corresponds to even-mode field distribution, relative to the plane of symmetry and the bottom one corresponds to odd-mode field distribution (see (1)); \(\Delta \alpha = \Delta z/\rho z; \Delta \rho = \sqrt{\Delta y^2 + \Delta z^2}; \Delta x = x_2 - x_1; \Delta y = y_2 - y_1; \Delta z = |z_2 - z_1|;\)

\[
\begin{align*}
F_{1s}(\xi) & = \frac{\varepsilon_0 s}{\beta_{s2}} \left[ k_x^2 \sqrt{1 - \xi^2} J_m(p_s\beta_s J_{m_s}(q_s \sqrt{1 - \xi^2}) - k_0 \beta_s J_m(p_sJ_{m_s}(q_s \sqrt{1 - \xi^2}) \right] \\
& + \frac{1}{\beta_{s2}^2 - (k_0\xi)^2} \left[ \beta_{s2} \cos p_{s2} \sin q_s \xi - k_0 \xi \sin p_{s2} \cos q_s \xi \right] + \left[ \beta_{s2} \cos p_{s2} \cos q_s \xi - k_0 \xi \cos p_{s2} \sin q_s \xi \right] \\
& \cdot \left[ k_x^2 \sqrt{1 - \xi^2} J_{m_s}(q_s \sqrt{1 - \xi^2}) - k_0 \sqrt{1 - \xi^2} J_{m_s}(q_s \sqrt{1 - \xi^2}) \right] \right] - h_{0s} m_s \left[ \beta_{s2} \sin p_{s2} \cos q_s \xi - k_0 \xi \cos p_{s2} \sin q_s \xi \right] \\
& \cdot \left[ \beta_{s2} \cos p_{s2} \sin q_s \xi - k_0 \xi \sin p_{s2} \cos q_s \xi \right] \\
F_{2s}(\xi) & = (k_s^2 - k_{s2}^2) \left[ \varepsilon_0 m_s \frac{1}{p_s \sqrt{1 - \xi^2}} J_m(p_sJ_{m_s}(q_s \sqrt{1 - \xi^2}) - k_0 \beta_{s2} \sqrt{1 - \xi^2} J_{m_s}(q_s \sqrt{1 - \xi^2}) \right] \\
& - \left[ \beta_{s2} \cos p_{s2} \sin q_s \xi - k_0 \xi \cos p_{s2} \sin q_s \xi \right] - h_{0s} m_s \left[ \beta_{s2} \cos p_{s2} \sin q_s \xi - k_0 \xi \sin p_{s2} \cos q_s \xi \right] \\
\end{align*}
\]

\(e_0; h_0\) are the normalized amplitudes, defined from (2):

\[
w(s) = \left[ (e_0 - h_0) J_{m_s-1}(p_s) - J_{m_s}(p_s) J_{m_s-2}(p_s) \right] + \left[ e_0 + h_0 \right] \left[ J_{m_s+1}(p_s) - J_{m_s}(p_s) J_{m_s+2}(p_s) \right] + 2 \left( \frac{\beta_{s2}}{\beta_{s1} h_0} \right) \left[ J_{m_s}(p_s) - J_{m_s+1}(p_s) J_{m_s+1}(p_s) \right] \left[ 2p_{s2} + \sin 2p_{s2} \right] + 2 \left( \frac{k_s}{k_{s2}} e_0 \right) \left[ J_{m_s}(p_s) - J_{m_s-1}(p_s) J_{m_s+1}(p_s) \right] \left[ 2p_{s2} - \sin 2p_{s2} \right] + 2 \left( \frac{\beta_{s2}}{\beta_{s1} h_0} \right) \left[ J_{m_s+1}(p_s) - J_{m_s}(p_s) J_{m_s+2}(p_s) \right] + 2 \left( \frac{k_s}{k_{s2}} e_0 \right) \left[ J_{m_s}(p_s) - J_{m_s-1}(p_s) J_{m_s+1}(p_s) \right] \left[ 2p_{s2} + \sin 2p_{s2} \right]
\]
In the case of different disk microresonators with equal parity of the field distribution (see fig. 3a) relative to symmetry plane $AA'$ (fig. 1b):

in the area: $\Delta z > L_1/2 + L_2/2$ obtains

$$
\kappa_1^{1,2} = i\kappa_0^{1,2} \int_0^\infty \frac{e^{-\sqrt{1-k^2}\Delta z}}{\sqrt{1-k^2}} \left\{ \left(k_1^2 - k_0^2\right) \left(k_2^2 - k_0^2\right) \cdot
C_1(\kappa)C_2(\kappa)^* \left[ \cos(m_1 - m_2)\Delta\psi J_{m_1-m_2}(k_0\Delta \rho) \pm (-1)^{m_1+m_2} \cos(m_1 + m_2)\Delta\psi J_{m_1+m_2}(k_0\Delta \rho) \right] +
D_1(\kappa)D_2(\kappa)^* \left[ \cos(m_1 - m_2)\Delta\psi J_{m_1-m_2}(k_0\Delta \rho) \mp (-1)^{m_1+m_2} \cos(m_1 + m_2)\Delta\psi J_{m_1+m_2}(k_0\Delta \rho) \right] \right\} \kappa d\kappa
$$

where $\sin \Delta\psi = \Delta y/\Delta \rho$, $\Delta \rho = \sqrt{\Delta x^2 + \Delta y^2}$.

$$
C_s(\kappa) = \left\{ \epsilon s_0 m_s \frac{1}{p_{s\perp} k_0 \kappa} J_{m_s}(p_{s\perp}) J_{m_s}(q_{s\perp} \kappa) - \frac{1}{r_s} \frac{1}{\beta_s^2 - k_0^2 \kappa^2} \right\},
$$

$$
\cdot \left[ p_{s\perp} J_{m_s}(p_{s\perp}) J'_{m_s}(q_{s\perp} \kappa) - q_{s\perp} \kappa J_{m_s}(p_{s\perp}) J_{m_s}(q_{s\perp} \kappa) \right],
$$

$$
D_s(\kappa) = \frac{\epsilon s_0}{\beta_s q_{s\perp}} \left\{ \left[k_0^2 p_{s\perp} J_{m_s}(p_{s\perp}) J'_{m_s}(q_{s\perp} \kappa) - k_0 q_{s\perp} \kappa J_{m_s}(p_{s\perp}) J_{m_s}(q_{s\perp} \kappa) \right] \right\},
$$

$$
\cdot \left[ \frac{1}{\beta_s^2 - k_0^2 (1 - \kappa^2)} \left[ k_0 \sqrt{1 - \kappa^2} \sin q_{s\perp} \sqrt{1 - \kappa^2} - \beta_{s\perp} \cos q_{s\perp} \sqrt{1 - \kappa^2} \cos p_{s\perp} \sin q_{s\perp} \sqrt{1 - \kappa^2} \right] -
\frac{1}{\beta_s^2 - k_0^2 (1 - \kappa^2)} \left[ \beta_{s\perp} \cos q_{s\perp} \sqrt{1 - \kappa^2} \sin p_{s\perp} \cos q_{s\perp} \sqrt{1 - \kappa^2} \cos q_{s\perp} \sin q_{s\perp} \sqrt{1 - \kappa^2} \right] \right\}.
$$
The real part of the coupling coefficients has shown imaginary part of the coupling coefficients. It's known, possessed a highest possible quality. The greatest amount of coupling between microresonators appears on its coaxial arrangement (fig. 3b, c). The sign of the coupling coefficients extreme values is determined both by the azimuth numbers and mutual microresonators position (fig. 2c, e).

Imaginary parts of the coupling coefficients are more smooth functions on coordinates (fig. 2b, d; fig. 3b; fig. 4a, c). For selected dielectric permittivity, its values are approximately one tenth degrees to the real parts.

The Fig. 2b is showing difference between two resonator coupling coefficients.

3 Coupling coefficient analysis

Discovered relationships valid for any oscillations of the disk microresonators in the open space, but a greatest interest presents the WG modes, as it’s well known, possessed a highest possible quality.

Fig. 2, 3, 4 are showing coupling coefficient dependences on Disk microresonator centers, composed of the dielectric with the relative permittivity \( \varepsilon_{1r} = 16; \varepsilon_{2r} = 9.6 \) and the comparative dimensions \( \Delta_s = L_s/2r_s = 0.2 \). Right ordinate axes on fig. 2e - e; fig. 3, 4 shows imaginary part of the coupling coefficients. It’s clear, that the real part of the coupling coefficients has sufficiently visible values only in a region, where the resonator surfaces are close to each other (fig. 2b - e; fig. 3b - c; fig. 4). Increase of the distance between resonator centers is accompanied by rapid coupling coefficient decrease. According to that, relative motion in the tangent direction leads to complex interference (fig. 2b, d; fig. 3b; fig. 4a, c) of their mutual influence, determining by significant eigenmode field variation nearby their surface.

The integral theorem can be obtained as:

\[
\kappa_s = \frac{32i}{\varepsilon_{sr}w(s)} \frac{p_{sz}}{q_{sz}} \int_0^1 \left\{ (k_s^2 - k_0^2)^2 |C_s(\sqrt{1 - \kappa^2})|^2 + |D_s(\sqrt{1 - \kappa^2})|^2 \right\} dk. \tag{11}
\]

4 Filter parameters calculation

Obtained results allow us to create electromagnetic models of various filters in the millimeter, terahertz or infrared wavelength ranges. As can be seen the coupling between not adjacent microresonators in the filters will...
be very small in comparison with coupling between adjacent ones, that allows simply to build filters with symmetrically parameters of the scattering. Different microresonator using in this case will allow to obtain much clean stop-bands.

The transmission T and the reflection coefficient of different DR system for the bandpass filter configuration in the transmission line can be obtained by using perturbation theory:

\[ T = T_0 - \frac{1}{B(\omega)} \sum_{s=1}^{N} B_s^+(\omega); R = R_0 - \frac{1}{B(\omega)} \sum_{s=1}^{N} B_s^-(\omega) \]

(12)

where \( T_0, R_0 \) are the transmission and reflection coefficients of the transmission line without DRs;

\[ B_s^\pm(\omega) = \det \begin{bmatrix} b_1^sQ_{11}(\omega) & ... & Q_{1N}^D & \tilde{k}_{11}^s & ... & b_N^sQ_{N1}(\omega) \\ b_2^sQ_{12}(\omega) & ... & 0 & ... & b_N^sQ_{N2}(\omega) \\ . & ... & ... & ... & . \\ b_N^sQ_{1N}(\omega) & ... & 0 & ... & b_N^sQ_{NN}(\omega) \end{bmatrix} \]

(13)

\[ B(\omega) = \det \begin{bmatrix} b_1^1Q_{11}(\omega) & b_1^1Q_{21}(\omega) & ... & b_N^1Q_{N1}(\omega) \\ b_2^1Q_{12}(\omega) & b_2^2Q_{22}(\omega) & ... & b_N^2Q_{N2}(\omega) \\ . & ... & ... & ... & . \\ b_N^1Q_{1N}(\omega) & b_N^2Q_{2N}(\omega) & ... & b_N^NQ_{NN}(\omega) \end{bmatrix} \]

For simplicity we proposed that \( \tilde{k}_{11}^+=\tilde{k}_{11}^-=k_L \); where \( k_L \) is the coupling coefficient of the 1-th and the \( N \)-th microresonator with the transmission input (output) line of the filter.

Here for different microresonators, the functions \( Q_{st}(\omega) \) are dependent on the partial DR and the coupled oscillation numbers:

\[ Q_{st}(\omega) = 2\iota\frac{\omega - \tilde{\omega}^s}{\omega_0}Q^D_t + \frac{\omega}{\omega_0}; \]

(14)

\[ Q^D_t = \omega_0w_t/P_t^D; P_t^D = \omega_0\frac{\epsilon''}{2V_i}\int |\epsilon_t|^2dv \] is the loss power in the dielectric of \( t \)-th DR; \( b_t^s \) is the complex amplitude of \( t \)-th resonator of the \( s \)-th mode \( \tilde{\omega}^s \) frequency of the filter [8].

The fig. 5 and 6 show results of the calculation of bandpass filter S-parameters matrix, that is built up on different disk microresonators with \( HE_{21,m,1} \) mode. It’s proposed, that coupling coefficients of the terminal microresonators \( k_L \) with transmission lines are known. Mutual coupling coefficients where obtained from (6-11).

It’s seen, that in consequence of rapidly coupling coefficients decrease, all S-parameters are symmetrical functions on the frequency. As we used a large number of resonators, the \( S_{21} \) squareness was obtained well.
It's stated, that the coupling between not adjacent microresonators in the filters is small in comparison with coupling between adjacent ones that allows simply building bandpass filters with symmetrically scattering parameters.

The filters on WG modes microresonators has acceptable frequency responses and after optimization maybe recommended for utilization on multiplexing of various communication systems.

Conclusions

Analytical relationships for the coupling coefficients of the different disk microresonators in the Open space have been obtained and investigated.

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Коефіцієнти зв'язку різних дискових мікрорезонаторів з коливаннями шепочучої галереї

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Приведено результати розрахунків коефіцієнтів взаємного зв'язку різноманітних дискових мікрорезонаторів з коливаннями шепочучої галереї, виконаних із різних матеріалів. Отримані аналітичні вирази для коефіцієнтів зв'язку. Розглянути основні закономірності зміни зв'язку при зміні параметрів структури. Приведені результати розрахунків коефіцієнтів передачі та відбиття смугових фільтрів, побудованих на різних мікрорезонаторах з різними видами коливань. Розраховані частотні залежності матриці розсіювання смугових фільтрів на різних дискових мікрорезонаторах інфрачервоного діапазону довжин хвиль. Визначено значення добротності діелектрику, необхідне для отримання прійнятних характеристик розсіювання. Досліджено амплітудно-частотні характеристики смугових фільтрів на вертикально зв'язаних, а також зв'язаних по боків стінці дисковых мікрорезонаторах. Встановлено найбільш оптимальні конфігурації зв'язаних мікрорезонаторів, які дозволяють досягати найкращих характеристик розсіювання. Показано, що застосування мікрорезонаторів з коливаннями шепочучої галереї дозволяє відносно просто отримувати частотно-симетричні характеристики розсіювання фільтрів.

Ключові слова: мікрорезонатор; коефіцієнт зв'язку; режим шепочучої галереї; S-матриця; фільтр

Коэффициенты связи различных дисковых мікрорезонаторов с колебаниями шепчущей галереи

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Приведены результаты расчета коэффициентов взаимной связи различных дисковых мікрорезонаторов с колебаниями шепчущей галереи, выполненных из разных материалов. Получены аналитические выражения для коэффициентов связи. Рассмотрены основные закономерности изменения связи при вариации параметров структуры. Приведены результаты расчета коэффициентов передачи и отражения полосовых фільтров, построенных на различных мікрорезонаторах с различными видами колебаний. Рассчитаны частотные зависимости матрицы рассеяния полосовых фільтров на различных дисковых мікрорезонаторах инфракрасного діапазона на длину волны. Определены значения добротности диэлектрика, необходимые для получения приемлемых характеристик рассеяния. Исследованы амплитудно-частотные характеристики полосовых фільтров на вертикально связанных, а также связанных по боковой стенке дисковых мікрорезонаторах. Установлены наиболее оптимальные конфигурации связанных мікрорезонаторов, позволяющие достичь наилучших характеристик рассеяния. Показано, что применение мікрорезонаторов с колебаниями шепчущей галереи позволяет относительно просто получать частотно-симметричные характеристики рассеяния фільтров.

Ключевые слова: мікрорезонатор ; коэффициент связи ; режим шепчущей галереи ; S-матрица ; фільтр