Broken conformal invariance and spectrum of anomalous dimensions in QCD.

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Abstract

Employing the operator algebra of the conformal group and the conformal Ward identities, we derive the constraints for the anomalies of dilatation and special conformal transformations of the local twist-2 operators in Quantum Chromodynamics. We calculate these anomalies in the leading order of perturbation theory in the minimal subtraction scheme. From the conformal consistency relation we derive then the off-diagonal part of the anomalous dimension matrix of the conformally covariant operators in the two-loop approximation of the coupling constant in terms of these quantities. We deduce corresponding off-diagonal parts of the Efremov-Radyushkin-Brodsky-Lepage kernels responsible for the evolution of the exclusive distribution amplitudes and non-forward parton distributions in the next-to-leading order in the flavour singlet channel for the chiral-even parity-odd and -even sectors as well as for the chiral-odd one. We also give the analytical solution of the corresponding evolution equations exploiting the conformal partial wave expansion.

Keywords: conformal Ward identities, conformal anomalies, anomalous dimensions, evolution equations

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1 Introduction.

Quantum Chromodynamics (QCD) — a relativistic local quantum field theory based on the colour gauge group $SU_c(3)$ — is well established nowadays as a microscopic theory of strong interaction. However, due to presently unsolved bound state problem, which remains a challenge for future studies, it is not possible to predict measurable observables from first principles. The standard approach which overcomes this difficulty is based on factorization theorems [1] which allow to separate soft and hard physics in a given process. The soft part is described by expectation values of appropriate operators sandwiched between hadronic states supplied with appropriate evolution equations which govern their scale dependence. Due to lack of any reliable non-perturbative machinery, the former part can be fixed only from experimental data. At the same time the hard subprocess and the evolution of the soft function can be systematically calculated with the help of perturbation theory. The success of perturbative QCD in predicting the momentum dependence of hadronic observables serves as the main and the most important argument for the correctness of the theory. Therefore, one is interested in increasing the accuracy of the theoretical predictions for their scale violation.

In general the calculations carried out beyond leading order in the coupling constant are very difficult and require substantial computer power. There are continuous advances in higher loop calculations of inclusive processes. For instance, the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution kernels [2, 3, 4] for the twist-2 parton distribution functions (see Refs. [5, 6] for transversity) are known in next-to-leading order (NLO) [7, 8, 9] and the calculations for the next-to-next-to-leading order (NNLO) are in progress. For exclusive processes no comparable technical effort was invested so far.

For exclusive reactions at large momentum transfer the soft physics is contained in distribution amplitudes (DA) [10, 11, 12] or more general non-forward parton distributions [13, 14, 15, 16], which depend on the momentum fraction $x$ carried by the struck parton, and possibly other kinematical variables such as the skewedness parameter of the process. The scale dependence of the mesonic DA is governed by the Efremov-Radyushkin-Brodsky-Lepage (ER-BL) evolution equation [11, 17, 18]

$$\mu^2 \frac{d}{d\mu^2} \phi(x, \mu) = \int_0^1 dy V(x, y|\alpha_s(\mu)) \phi(y, \mu),$$

(1)

where the evolution kernel $V(x, y|\alpha_s) = \frac{\alpha_s}{2\pi} V^{(0)}(x, y) + \left(\frac{\alpha_s}{2\pi}\right)^2 V^{(1)}(x, y) + \ldots$ is given as a series in the coupling, and each expansion coefficient is calculable in perturbation theory. Note that due to the fact that the same light-ray operators enter the definition of DA’s and non-forward

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2Or more general correlation functions of the elementary field operators in cases when the operator product expansion does not exist.
distributions, the corresponding evolution kernels are in one-to-one correspondence [14]. In leading order (LO) the calculations are quite straightforward since there exist different reliable formalisms for the evaluation of non-forward evolution kernels in the light-cone position as well as light-cone fraction representations. On the other hand the direct calculation beyond leading order turns out to be far from being a simple task which has only been solved for the evolution kernel in the non-singlet sector [19, 20, 21] and was checked numerically in Ref. [22]. The most promising approach for two-loop calculations developed in Ref. [20] can be used for future explicit NLO computation in the singlet channel. However, the analytical solutions of the evolution equations for the DA [17, 24, 25, 26] and non-forward distributions [16, 27, 28] are known in terms of the anomalous dimensions $\gamma_{jk}$ of the, so-called, twist-2 conformal composite operators. They can be obtained from the evolution kernels by forming moments with respect to Gegenbauer polynomials

$$\int_0^1 dx \, C_j^\nu (2x - 1) V(x, y|\alpha_s) = -\frac{1}{2} \sum_{k=0}^j \gamma_{jk}(\alpha_s) C_k^\mu (2y - 1),$$

where the numerical value of the indices $\mu$ and $\nu$ depends on the channel under consideration. Unfortunately, beyond LO this calculation is very cumbersome and could not be performed analytically (only the first few entries of this matrix have been evaluated numerically [24, 23]).

Owing to the difficulties we have sketched, every field-theoretical idea is welcome which can help us to face the above mentioned problems in a more economical way. On the first glance one could think that the field-theoretical background of QCD has already been well studied in the early days. However, after more than two decades of computation experience it is desirable to have a new look on certain issues. In this paper we apply an old idea of conformal symmetry [1], which arises from the generalization of the Poincaré group up to the fifteen parameter conformal group isomorphic to $SO(4, 2)$ — the largest possible group of transformations which leave light cone invariant. Besides translations and four-dimensional rotations it contains dilatations and special conformal transformations [29, 30].

In the massless case the classical QCD action is invariant under infinitesimal conformal transformations. The restrictions coming from conformal symmetry in massless scalar and gauge field theories were studied already more than three decades ago when it was believed that this symmetry is broken only softly [29, 30, 31, 32]. To give an example for the power of conformal symmetry, we remind the reader that in a conformal invariant theory the functional form of both elementary Green functions and current correlators is severely constrained: two- and three-point functions are fixed completely up to their normalization while $n$-point ($n \geq 4$) correlators are expressed in terms of a single function of anharmonic ratios [33].

3Here we warn the reader from simple minded understanding of this terminology. From the consequent discussion it will be clear that our point of view is more sophisticated. It provides us with a powerful machinery, which enables us to predict certain quantities in a very simple manner.
Unfortunately, after quantization the theory is not conformal invariant even in the massless limit. The symmetry is broken due to anomalies related to the ultraviolet (UV) divergencies \cite{34, 35, 36, 37, 38} which arise when one goes beyond the tree level. The regularization of these divergencies introduces a dimensional parameter which breaks conformal symmetry and this breaking still remains after renormalization and removing of the regularization. It shows up in the trace anomaly of the energy-momentum tensor \cite{39, 40, 41, 42}. Getting rid of the unphysical contributions in the trace (see below Eq. (20)) such as BRST-exact operators, which originate from the gauge fixing and ghost Lagrangians, and equations of motion we end up finally with a symmetry breaking term proportional to the $\beta$-function. If one assumes the existence of a non-perturbative fixed point $g^*$ with $\beta(g^*) = 0$, the theory would be conformal invariant. Formally we speak about the hypothetical conformal limit of the theory and set $\beta$ equal to zero. Thus, the trace anomaly controls the conformal symmetry breaking in the Ward identities of the elementary fields operators \cite{35}. However, this is by no means true when one is interested in more complicated Green functions, for instance, with composite operator insertion, since this one-to-one correspondence between both anomalies is lost. This means that in the conformal Ward identities (CWIs) the breaking of the special conformal symmetry is not controlled by the dilatation anomaly alone. Instead, new anomalies enter the game.

This should sound a warning from the naive use of the conformal symmetry arguments. Indeed, in the past its application has led to a conflict between its predictions and explicit calculations beyond leading order \cite{43}. For instance, in the above mentioned example of the renormalization of the conformal Ward identities with a composite operator insertion there arises a new symmetry breaking term induced by the UV divergencies occurring in the operator product of the composite operator and the trace anomaly. Such a term is not proportional to the $\beta$-function.

To study this breaking, we will construct Ward identities for the Green functions of the elementary fields with conformal operator insertion which will serve as a main tool for the derivation of constraints for the anomalous dimensions beyond leading order. In general the renormalization matrix of the twist-2 conformal operators is triangular due to Lorentz invariance. In leading order it is diagonal so that conformally covariant operators do not mix under renormalization, i.e. $\gamma_{jk}^{(0)} = \delta_{jk}\gamma_j^{(0)}$ \cite{11, 24, 44, 45, 46}. Its eigenvalues $\gamma_j$ coincide with the Mellin moments of the DGLAP kernels. Therefore, the diagonal part\footnote{The formula given below is very schematic and valid for direct (QQ or GG) channels. For the exhaustive treatment of this problem the reader is referred to the paper \cite{47}.} of the exclusive evolution kernel can be obtained by the transformation \cite{47}

$$V^D(x, y) = \int_0^1 dz P(z) \sum_{j=0}^{\infty} \frac{w(y|\nu)}{N_j(\nu)} C_j^{(x)}(2x - 1) z^j C_j^{(y)}(2y - 1),$$  

(3)
where \( N_j(\nu) = 2^{-4\nu+1} \frac{\Gamma^2(\frac{1}{2})\Gamma(2\nu+j)}{\Gamma(\nu)\Gamma(\nu+j)j!} \) and \( w(y|\nu) = (y\bar{y})^{\nu-1/2} \) are the normalization and weight factors, respectively. Thus, conformal covariance at tree level is sufficient for the diagonality of the one-loop anomalous dimension matrix. However, beyond tree level conformal symmetry is broken even for \( \beta = 0 \). Of course, the scaling dimensions of the operators are changed by the anomalous ones but this shift does not ensure the covariance of the renormalized operators under special conformal transformation in a given renormalization scheme \[48, 49, 50, 47\]. This fact is easy to understand, while the breaking of dilatation is determined in LO only by the leading logarithmic term, the breaking of special conformal symmetry is caused by both the logarithmic term and the remaining constant one. Since the logarithmic independent term is fixed by the normalization condition, which is implicitly given in the MS scheme, we can require a normalization which ensures conformal covariance of the renormalized operators\[50, 51\]. This procedure is equivalent to a finite renormalization and provides the, so-called, conformally covariant scheme. Therefore, the renormalized conformally covariant operators at LO possess a diagonal anomalous dimension matrix in two-loop approximation, up to a term proportional to the \( \beta \)-function. Obviously, the non-diagonal anomalous dimension matrix in the MS scheme in NLO can be obtained by \( \mathcal{O}(\alpha_s) \) finite renormalization of the two-loop forward anomalous dimensions. When translated into the language of evolution kernels the whole result reads now:

\[
V(x, y|\alpha_s) = V^D(x, y|\alpha_s) + V^{\text{ND}}(x, y|\alpha_s), \quad V^{\text{ND}}(x, y|\alpha_s) = \left(\frac{\alpha_s}{2\pi}\right)^2 V^{\text{ND}(1)}(x, y) + \mathcal{O}(\alpha_s^3). \tag{4}
\]

In other words the off-diagonal part \( V^{\text{ND}(1)} \) of the kernel is a quasi-one-loop object and can be calculated in this order. This entry will be evaluated in the present paper.

While in previous studies we restricted ourselves to the Abelian case, we extend in this paper the conformal machinery to the flavour singlet sector in QCD. Consequent presentation runs as follows. In the next section we summarize some basic formulae. In section 3 we derive the renormalized CWI analogous to the ones considered in Refs. \[36, 37, 38\] for the composite operators in the dimensionally regularized theory. We do not rely there on any assumption about symmetry breaking. The commutator constraints for special conformal and anomalous dimension matrices are established in section 4. Section 5 is devoted to calculation of the special conformal anomaly kernels in both light-cone position and fraction representations. Their transformation to the language of Gegenbauer moments is given in section 6. The subsequent section contains explicit formulae for two-loop anomalous dimensions as well as ER-BL kernels. Then we give the general formalism for the solution of the evolution equation in the required approximation. In the final section 8 we give our conclusions. To clarify the presentations given in the body of the paper

\[5\] Exploiting this fact together with the conformal operator product expansion we had, recently, the opportunity to give several predictions for some non-forward processes \[50, 51, 52\].

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we add three technical appendices. In the first one we list all Feynman rules required in the calculation of the conformal anomalies. In appendix B we sketch the light-cone position formalism for the computation of the leading order evolution kernels. We mainly focus our attention on the most complicated gluonic sector. The last appendix summarizes formulae for the hypergeometric functions which appear in the evaluation of the Gegenbauer moments of the LO evolution kernels.

2 Preliminary remarks.

To start with, let us recall some basic equations of QCD. The dimensionally regularized and renormalized Lagrangian is given by

\[
\mathcal{L} = \bar{Z}_2 \bar{\psi} i \partial_\mu \psi + \bar{Z}_1 \mu^\epsilon g \bar{\psi} B^a t^a \psi - \frac{Z_3}{4} \left( F^a_{\mu\nu} \right)^2 - \frac{1}{2} \mu^\epsilon g Z_1 f^{abc} F^a_{\mu\nu} B^b_{\mu} B^c_{\nu} - \frac{Z_4}{4} \mu^2 g^2 \left( f^{abc} B^b_{\mu} B^c_{\nu} \right)^2 - \frac{1}{2 \zeta} \left( \partial_\mu B^a_\mu \right)^2 + \bar{Z}_3 \partial_\mu \omega^a \partial_\nu \omega^a + \mu^\epsilon g \bar{Z}_1 f^{abc} \partial_\mu \omega^a B^b_{\mu} \omega^c,
\]

where \( d = 4 - 2 \epsilon \) is the space-time dimension and \( \mu \) is a mass parameter with dimension one introduced in order to keep the coupling constant dimensionless. We introduce here the symbol \( F^a_{\mu\nu} \) as a shorthand notation for the Abelian part of the full QCD field strength tensor \( G^a_{\mu\nu} = F^a_{\mu\nu} + \mu^\epsilon g X f^{abc} B^b_{\mu} B^c_{\nu} \). The canonical dimension of the elementary fields are the following: \( d^\text{can}_\psi = \frac{3}{2} - \epsilon \) for fermions, \( d^\text{can}_G = 1 - \epsilon \) for gluons, \( d^\text{can}_\omega = d - 2 \) and \( d^\text{can}_\bar{\omega} = 0 \) for the anti-ghost and ghost fields, respectively. This Lagrangian is invariant under the following renormalized BRST-transformations:

\[
\delta^{\text{BRST}} \psi = -i \mu^\epsilon g \bar{Z}_1 \omega^a t^a \psi \delta \lambda, \quad \delta^{\text{BRST}} B^a_\mu = \bar{Z}_3 D_\mu \omega^a \delta \lambda,
\]

\[
\delta^{\text{BRST}} \omega^a = \frac{1}{2} \mu^\epsilon g \bar{Z}_1 f^{abc} \omega^b \omega^c \delta \lambda, \quad \delta^{\text{BRST}} \bar{\omega}^a = \frac{1}{\zeta} \partial_\mu B^a_\mu \delta \lambda,
\]

where \( \delta \lambda \) is a renormalized Grassman variable. The covariant derivative is defined as follows

\( D_\mu = \partial_\mu - i \mu^\epsilon g X T^a B^a_\mu, \)

where \( T^a \) is the generator in the fundamental \((T^a \phi_i = t^a_{ij} \phi_j)\) or adjoint \((T^b \phi_i = if^{abc} \phi_c)\) representations depending on the object it is acting on. The Ward-Takahashi identities imply the following relations between the renormalization constants \( Z_1 Z_3^{-\frac{3}{2}} = Z_1^2 Z_3^{-1} = \bar{Z}_1 \bar{Z}_3^{-1} Z_3^{-\frac{3}{2}} = \bar{Z}_1 Z_2^{-1} Z_3^{-\frac{3}{2}} = \bar{Z}_1 Z_2^{-1} Z_3^{-1} \). In the MS scheme Z-factors are defined as Laurent series in \( \epsilon \): \( Z(g, \epsilon) = 1 + Z^{[1]}(g, \epsilon)/\epsilon + Z^{[2]}(g, \epsilon)/\epsilon^2 + \ldots \). For our consequent discussion we take the renormalization constants \( Z_2, Z_3, \bar{Z}_3 \) and \( X \equiv Z_1 Z_3^{-1} = \bar{Z}_1 \bar{Z}_3^{-1} = \bar{Z}_1 Z_2^{-1} \) as independent ones.

The renormalization group coefficients and the anomalous dimensions of the fields are

\[
\beta_\epsilon(g) = \mu \frac{dg}{d\mu} = -\epsilon g + \beta(g), \quad \sigma = \mu \frac{d}{d\mu} \ln \xi = -2 \gamma_G, \quad \gamma_\phi = \frac{1}{2} \mu \frac{d}{d\mu} \ln Z_\phi,
\]

\( \xi = \frac{Z_1 Z_2}{Z_1 Z_3} \)
(φ runs over parton species entering the Lagrangian {φ = ψ, G, ω}) where Zψ = Z2, ZG = Z3, Zω = Z3. These coefficients are determined by the residues of the corresponding Z-factors \[55, 55\] and in lowest order approximation, \( \beta = \frac{4\pi}{4\pi} \beta_0 + \left( \frac{4\pi}{4\pi} \right)^2 \beta_1 + \ldots, \gamma_\phi = \frac{4\pi}{4\pi} \gamma_\phi^{(0)} + \ldots, \) they are given by

\[
\beta_0 = \frac{4}{3} T_F N_f - \frac{11}{3} C_A, \quad \gamma_\phi^{(0)}(\xi) = \frac{2}{3} T_F N_f + \frac{C_A}{4} \left( \xi - \frac{13}{3} \right), \quad \gamma_\psi^{(0)}(\xi) = \frac{\xi}{2} C_F,
\]

where the group theoretical factors are \( C_A = 3, C_F = 4/3 \), and \( T_F = 1/2 \) for \( SU_c(3) \).

In the following we are interested in the renormalization of the conformal operators in the flavour singlet sector, which we define as two-dimensional vector:

\[
\mathcal{O}_{jl} = \begin{pmatrix} Q^{jl} \\ \phi^{jl} \end{pmatrix}.
\]

The group-theoretical construction of these local operators was given sometimes ago in Refs. \[55, 55\] (see also Ref. \[55\] for a non-local version of conformal operators)

\[
\begin{pmatrix} Q^{O^V} \\ Q^{O^A} \end{pmatrix}_{jl} = \bar{\psi}(i\partial_+) \begin{pmatrix} \gamma_+ \\ \gamma_+ \gamma_5 \end{pmatrix} C_\frac{3}{2} \begin{pmatrix} \frac{D_+}{\partial_+} \\ \frac{D_+}{\partial_+} \end{pmatrix} \psi, \quad \begin{pmatrix} Q^{O^V} \\ Q^{O^A} \end{pmatrix}_{jl} = G_{+\mu}(i\partial_+)^{\mu-1} \begin{pmatrix} g_{\mu\nu} \\ i\epsilon_{\mu\nu-} \end{pmatrix} C_\frac{3}{2} \begin{pmatrix} \frac{D_+}{\partial_+} \\ \frac{D_+}{\partial_+} \end{pmatrix} \mathcal{G}_{\nu+},
\]

where \( \partial = \nabla + \nabla \) and \( \nabla \nabla = \nabla - \nabla \). The + and − components are obtained by contraction with the two light-like vectors \( n \) and \( n^* \), such that \( n^2 = n^*2 = 0 \) and \( nn^* = 1 \). The indices of the Gegenbauer polynomials are determined by the canonical dimensions \( d_{\phi}^{\text{can}} \) and the spins \( s_{\phi} \) of the fields: \( \nu_{\phi} = d_{\phi}^{\text{can}} + s_{\phi} - 1/2 \).

The composite operators \[10\] form an infinite representation of the collinear conformal algebra at the tree level. This subalgebra \( SU(1, 1) \) of the whole conformal algebra contains the generators

\[
\mathcal{P}_+ = n_\mu \mathcal{P}_\mu, \quad \mathcal{M}_{-+} = n_\mu^* \mathcal{M}_{\mu\nu} n_\nu, \quad \mathcal{D}, \quad \mathcal{K}_- = n_\mu^* \mathcal{K}_\mu.
\]

The total momentum \( \mathcal{P}_+ \) is a raising, the special conformal generator \( \mathcal{K}_- \) is a lowering and the dilatation \( \mathcal{D} \) and the Lorentz \( \mathcal{M}_{-+} \) generators are diagonal operators:

\[
\delta^\mathcal{P}_+ \mathcal{O}_{jl} = i[\mathcal{O}_{jl}, \mathcal{P}_+]_-= i\mathcal{O}_{jl+1}, \quad \delta^\mathcal{M}_+ \mathcal{O}_{jl} = i[\mathcal{O}_{jl}, \mathcal{M}_{-+}]_- = -(l+1)\mathcal{O}_{jl},
\]

\[
\delta^\mathcal{S} \mathcal{O}_{jl} = i[\mathcal{O}_{jl}, \mathcal{D}]_- = -(l+3)\mathcal{O}_{jl}, \quad \delta^\mathcal{C} \mathcal{O}_{jl} = i[\mathcal{O}_{jl}, \mathcal{K}_-]_- = i a(j, l)\mathcal{O}_{jl-1},
\]

with \( a_{jk}(l) = \delta_{jk} \cdot a(j, l) \) and \( a(j, l) = 2(j-l)(j+l+3) \). There is one Casimir operator of the collinear conformal algebra, which is given by \( J^2 = \frac{1}{2} \mathcal{P}_+ \mathcal{K}_- - \frac{1}{4} (\mathcal{D} + \mathcal{M}_{-+})^2 - \frac{1}{2} (\mathcal{D} + \mathcal{M}_{-+}) \). The operators introduced above possess conformal spin \( j + 1 \): \( J^2 \mathcal{O}_{jl} = (j+1)(j+2)\mathcal{O}_{jl} \), spin \( l + 1 \) and scale dimension \( l + 3 \).
To study the conformal anomalies we consider the CWI for the connected Green functions defined by the formula
\[ \langle [O_{jl}]X \rangle = \frac{\int D\phi [O_{jl}] \phi(x_1) \ldots \phi(x_k) e^{iS}}{\int D\phi e^{iS}}, \] (14)

\( X \) is a product of elementary fields \( X = \prod_{i=1}^{N} \phi(x_i) \) at different space-time points. The integration is weighted with the factor \( e^{iS} \), where \( S \) is the QCD action determined in terms of the Lagrangian
\[ S = \int d^d x L, \]
and
\[ [O_{jl}] = \sum_{k=0}^{j} Z_{jk}(\epsilon, g) O_{kl} \] (15)
are the renormalized composite operators. Here we neglect possible gauge-variant counterterms, since they give no contribution to the physical sector of the theory. The four entries of the matrix \( ABZ_{jk}(\epsilon, g) \) with \( A, B = \{Q, G\} \) are triangular and independent of \( l \) due to Poincaré invariance. In the MS scheme the expansion \( ABZ_{jk}(\epsilon, g) = \delta^{AB}\delta_{jk} + ABZ_{jk}[1](g)/\epsilon + \cdots \) is valid \[53, 54\]. Since the regularization is manifest in the action, the true conformal Ward identities, i.e. those which are valid in the interacting theory, can be obtained by a formal conformal variation of Eq. (14):
\[ \langle [O_{jl}]\delta X \rangle = -\langle \delta [O_{jl}]X \rangle - \langle i [O_{jl}]\delta S X \rangle. \] (16)

Here the LHS contains a differential operator acting on a renormalized Green function and, therefore, it is finite by itself. Thus, the same property should be satisfied by the RHS. Because of Poincaré invariance, the variations \( \delta^P S \) and \( \delta^M S \) vanish identically and thus, with Eq. (12) the corresponding Ward identities read:
\[ \langle [O_{jl}]\delta^P X \rangle = -i \langle [O_{jl+1}]X \rangle \quad \langle [O_{jl}]\delta^M X \rangle = (l + 1) \langle [O_{jl}]X \rangle. \] (17)

In the case of conformal transformations, i.e. \( \delta = \{\delta^S, \delta^C\} \), the variations of the action do not vanish anymore. The operator product \([O_{jl}]\delta S\) appearing on the RHS of Eq. (17) is infinite. The removal of the UV divergencies induces the conformal anomalies, i.e. anomalous dimension and special conformal anomaly matrices.

Before we start our study of the conformal variation of the action, we write down the variation of the composite operators when the interaction is switched on. Even in the interacting theory it is convenient to deal with scale dimensions which are equal to the canonical ones in 4-dimensional space-time, i.e. \( d_G = 1, d_\psi = \frac{3}{2} \). Therefore, the infinitesimal conformal transformations of the renormalized operators (13) are simply given by
\[ \delta^S [O_{jl}] = -(l + 3)[O_{jl}], \quad \delta^C [O_{jl}] = i \sum_{k=0}^{j} \left\{ \hat{Z} \hat{a}(l) \hat{Z}^{-1} \right\}_{jk} [O_{kl-1}]. \] (18)
The choice $d_G = 1$, $d_\psi = \frac{3}{2}$, $d_\omega = d - 2$ and $d_{\bar{\omega}} = 0$ ensures that the conformal variation of the action can be expressed by well classified operators: gauge invariant operators, BRST exact operators, and equation of motion (EOM) operators. A further advantage of this set of scale dimensions is that it allows to use Jackiw’s conformal covariant transformation law for the gauge fields [56]. After some algebra, the final result can be written as

$$\delta S = \int d^4x \Delta(x), \quad \delta C = \int d^4x 2x_\nu \Delta(x),$$

where the integrand $\Delta(x)$ can be expressed in terms of the regularized but unrenormalized trace anomaly of the energy-momentum tensor $\Theta_{\mu\nu}$:

$$\Delta(x) = -\Theta_{\mu\nu}(x) - d_G \Omega_G(x) - d_\psi \Omega_{\bar{\psi}}(x) - d_{\bar{\omega}} \Omega_{\bar{\omega}}(x)$$

$$= \epsilon \left\{ O_A(x) + O_B(x) + \Omega_\omega(x) - \Omega_{\bar{\psi}}(x) \right\} + (d - 2) \partial_\mu O_{B\mu}(x).$$

Here we introduce the following set of operators

$$O_A(x) = \frac{Z_3}{2} (G_{\mu\nu})^2, \quad O_B(x) = \frac{\delta BRST}{\delta \lambda} \bar{\omega}^a \partial_\mu B^a_\mu, \quad O_{B\mu}(x) = \frac{\delta BRST}{\delta \lambda} \bar{\omega}^a B^a_\mu,$$

as well as EOM operators

$$\Omega_G(x) = B^a_\mu \frac{\delta S}{\delta B^a_\mu}, \quad \Omega_{\bar{\psi}}(x) = \frac{\delta S}{\delta \bar{\psi}} \psi + \bar{\psi} \frac{\delta S}{\delta \bar{\psi}}, \quad \Omega_\omega(x) = \bar{\omega}^a \frac{\delta S}{\delta \bar{\omega}^a}.$$

It is desirable to express these variations of the action in terms of renormalized operators as this allows to neglect then terms proportional to $\epsilon$. For this reason the renormalization problem of the above mentioned operators have to be solved [57]. The only gauge invariant operator $O_A$ (class A) is of twist-4 and needs counterterms of class B operators, given as BRST-exact ones, and equation of motion operators (class C). The twist-4 class B operators are $O_B$ and the total derivative of the twist two operator $\partial_\mu O_{B\mu}$. Since there is no other class B operator which they can mix with, only class C operators are needed as counterterms for the class B operators. This set of twist-4 and twist-2 operators is closed under renormalization. First, we discuss the renormalization of the twist-2 operator $O_{B\mu}$. Since there is no further class B or class C operator with dimension three and the same quantum numbers we may conclude that this operator is renormalized $O_{B\mu}(x) = [O_{B\mu}(x)]$. It has vanishing physical matrix elements at non-zero momentum transfer [58] while at zero momentum transfer it vanishes identically as it contains no poles. The renormalization problem of the twist-4 operators at zero momentum can be solved by a direct
calculation of the following differential vertex operator insertions:

\[
[\Delta^g] \equiv \xi \frac{\partial}{\partial \xi} S = \frac{1}{2} \left\{ [O_B] + \Omega_\omega \right\}, \quad [\Delta^\epsilon] \equiv g \frac{\partial}{\partial g} S = [O_A] + [O_B] + \Omega_G + \Omega_\omega. \tag{23}
\]

We tentatively expressed in these equations the derivatives of the renormalization constants in terms of the derivatives of independent set. From the finiteness of the differential vertex operator insertions the counterterms of \([O_A(x)]\) and \([O_B(x)]\) have been fixed up to total derivatives (here \(Z_g = X Z_3^{-\frac{1}{2}}\)):

\[
[O_A(x)] = \left( 1 + g \frac{\partial \ln Z_g}{\partial g} \right) O_A(x) + \frac{1}{2} \left( g \frac{\partial \ln X}{\partial g} - 2 \xi \frac{\partial \ln X}{\partial \xi} \right) \{O_B(x) + \Omega_\omega(x) + \Omega_G(x)\} + \left( \xi \frac{\partial \ln \tilde{Z}_2}{\partial \xi} \right) \Omega_\psi(x) + \partial_\mu \mathcal{R}_A^\mu(x),
\]

\[
[O_B(x)] = \left( 1 + 2 \xi \frac{\partial \ln X}{\partial \xi} \right) O_B(x) + 2 \xi \frac{\partial \ln X}{\partial \xi} \{\Omega_G(x) + \Omega_\omega(x)\} + \xi \frac{\partial \ln \tilde{Z}_2}{\partial \xi} \Omega_\psi(x) + 2 \xi \frac{\partial \ln \tilde{Z}_2}{\partial \xi} \Omega_\omega(x) \tag{25}
\]

+ \partial_\mu \mathcal{R}_B^\mu(x).

Making use of the representation

\[
\epsilon = -\frac{\xi_1(g)}{g} \left( 1 + g \frac{\partial \ln Z_g}{\partial g} \right) \tag{26}
\]

we can express \(\epsilon \{O_A(x) + O_B(x)\}\) in terms of the renormalized operators only

\[
\epsilon\{O_A(x) + O_B(x)\} = -\frac{\beta_\epsilon}{g} [O_A(x)] - \left( \frac{\beta_\epsilon}{g} - \gamma_G \right) [O_B(x)] + \left( \gamma_G - \frac{\beta}{g} \right) \{\Omega_G(x) + \Omega_\omega(x)\} + \gamma_\psi \Omega_\psi(x) + 2 \gamma_\omega \Omega_\omega(x). \tag{27}
\]

Thus, inserting our findings into Eq. (20) we obtain the renormalized anomaly:

\[
\Delta(x) = -\frac{\beta_\epsilon}{g} [O_A(x)] - \left( \frac{\beta_\epsilon}{g} - \gamma_G \right) \{[O_B(x)] + \Omega_\omega(x)\} + \left( \gamma_G - \frac{\beta}{g} \right) \Omega_G(x) + \gamma_\psi \Omega_\psi(x) + 2 \gamma_\omega \Omega_\omega(x) + (d - 2) \partial_\mu [O_{B\mu}(x)]. \tag{28}
\]

### 3 Renormalization of the Ward identities.

Combining the results we have obtained so far in Eqs. (16), (18), (19) and (28), we will come to the desired conformal Ward identities. However, since the products of composite operators possess

\footnote{Throughout the paper, to simplify notations, we use the following conventions: for unintegrated operator insertions and EOM operators we keep the dependence on the space-time point explicit \(\mathcal{O}(x)\); for the integrated quantities with weight function 1 we just omit this dependence \(\mathcal{O} \equiv \int d^d x \mathcal{O}(x)\), while for weight \(2 x\) we use \(\mathcal{O}^- \equiv \int d^d x 2 x_\omega \mathcal{O}(x)\).}
UV divergencies, we have to take care of their subtractive renormalization. The main problem here is that no theorems for the renormalization of (gauge invariant) operator products exist [53].

This section is devoted to the solution of this problem for the operator products appearing in the CWI [16]. It is worth to mention that in our previous work on Abelian gauge theory [47] we discriminate a rigorous treatment in favour of a symmetrical handling of the quark and gluon sectors. Due to the non-Abelian character of the present consideration it is helpful to deal with the well classified operators $O_i$. Of course, both approaches — the one which we have used there and the other we describe below — lead to the same results for a theory with $U(1)$ symmetry group.

Fortunately, due to the fact that the products involved are composed of gauge invariant conformal operators and differential vertex operator insertions the form of the gauge-variant part can be fixed from the study of the Green function (14) with the help of the action principle. So that finally we have for the unintegrated operators:

\[
i[O_A(x)][O_{jl}] = i[O_A(x)O_{jl}] - \delta^{(d)}(x) \sum_{k=0}^{j} \{ \hat{Z}_A \}_{jk} [O_{kl}] - \frac{i}{2} \partial_+ \delta^{(d)}(x) \sum_{k=0}^{j} \{ \hat{Z}_A \}_{jk} [O_{kl-1}] - \ldots
\]

\[
- \left( g \frac{\partial \ln X}{\partial g} - 2\xi \frac{\partial \ln X}{\partial \xi} \right) B^a_{\mu}(x) \frac{\delta}{\delta B^a_{\mu}(x)} [O_{jl}], \tag{29}
\]

\[
i[O_{B\ast}(x)][O_{jl}] = i[O_{B\ast}(x)O_{jl}] - \delta^{(d)}(x) \sum_{k=0}^{j} \{ \hat{Z}_B \}_{jk} [O_{kl}] - \frac{i}{2} \partial_+ \delta^{(d)}(x) \sum_{k=0}^{j} \{ \hat{Z}_B \}_{jk} [O_{kl-1}] - \ldots
\]

\[
- \left( 2\xi \frac{\partial}{\partial \xi} \ln X \right) B^a_{\mu}(x) \frac{\delta}{\delta B^a_{\mu}(x)} [O_{jl}], \tag{30}
\]

The ellipses stand for the higher derivatives counterterms which are not of relevance for our present study. For convenience we introduced here the operator combination $[O_{B\ast}(x)] = [O_B(x)] + \Omega_\omega(x)$, which on tree level is the gauge fixing term and thus require only counterterms proportional to $\xi$. In the Landau gauge the operator product $[O_{B\ast}(x)][O_{jl}]$ is finite, since all counterterms will vanish. Obviously, the term containing the variation of the composite operators with respect to the gauge field is gauge-variant (it cannot be decomposed in terms of A, B or EOM operators). In the following subsection we show that the appearance of this term ensures the correct renormalization group equation for the composite operators. We also will fix the $Z$-matrices $\hat{Z}_A$ and $\hat{Z}_B$ in terms of known renormalization group coefficients.

Fortunately, by employing BRST invariance and naive power counting we are able to show in general that the operator product $[O_{jl}][O_{B\mu}(x)]$ is renormalized:

\[
[O_{jl}][O_{B\mu}(x)] = [O_{jl}O_{B\mu}(x)]. \tag{31}
\]

Finally, we have also to deal with the product of composite operators and EOM operators. For these products we do not use the MS prescription, rather we perform a mere integration in
the path integral which results in equation:

\[ i[O_{jl}]\Omega_\phi(x) = i[O_{jl}O_\phi(x)] - \phi(x)\frac{\delta}{\delta \phi(x)}[O_{jl}], \tag{32} \]

where it is obvious that

\[ \langle [O_{jl}O_\phi(x)]X \rangle = i\langle [O_{jl}]\phi(x)\frac{\delta}{\delta \phi(x)}X \rangle. \tag{33} \]

For instance, for the counterterms of the products containing the EOM operator of the quark fields we have

\[ \int d^4x \left( \bar{\psi} \frac{\delta}{\delta \bar{\psi}} + \frac{\delta}{\delta \psi} \psi \right) [O_{jl}] = 2 \sum_{k=0}^j \{ \hat{Z} \hat{\mathcal{P}} \hat{Z}^{-1} \} \eta_{jk} [O_{kl}], \tag{34} \]

\[ \int d^4x 2x_+ \left( \bar{\psi} \frac{\delta}{\delta \bar{\psi}} + \frac{\delta}{\delta \psi} \psi \right) [O_{jl}] = 2i \sum_{k=0}^j \{ \hat{Z} \hat{\mathcal{P}} \hat{Z}^{-1} \} \eta_{jk} [O_{kl-1}]. \tag{35} \]

We have used above the projectors on the quark and gluon sectors

\[ \hat{\mathcal{P}}_Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \hat{\mathcal{P}}_G = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{36} \]

and the \( \hat{b} \)-matrix with elements given by

\[ b_{jk}(l, \nu) = 2 \int_0^1 dx \frac{w(x|\nu)}{N_k(\nu)} C^\nu_k(2x-1) \left[ l - \frac{1}{2}(2x-1) \frac{\partial}{\partial x} \right] C^\nu_k(2x-1) \]

\[ = 2 \theta_{jk} \left\{ (l+k+2\nu)\delta_{jk} - \left[ 1 + (-1)^{j-k} \right](k+\nu) \right\}. \tag{37} \]

Due to the definition of the conformal operators \( (\Pi) \), the same matrix \( b_{jk}(l) \equiv b_{jk}(l,3/2) = b_{j-1k-1}(l-1,5/2) \) appears in the quark as well as in the gluon channels.

### 3.1 Scale Ward identity.

The renormalization of the dilatation Ward identity is quite straightforward in terms of the differential vertex operator insertions \( (23) \). Nevertheless, we present here an alternative way, which, however, can also be used for the renormalization of the special conformal Ward identities. Afterwards we show that the appearing dilatation anomaly is just the anomalous dimension matrix of the composite operators.

Taking into account the subtractive renormalization procedures in Eqs. \( (29), (30) \) and \( (32) \), we find with Eqs. \( (21) \) and \( (28) \) the following equation for the operator product appearing in the Ward identity \( (16) \) (with \( \delta \equiv \delta^S \)) expressed via renormalized insertions:

\[ i[O_{jl}]\delta^S S = \sum_{k=0}^j \left\{ \frac{\beta_\epsilon}{g} \hat{Z}_A + \left( \frac{\beta_\epsilon}{g} - \gamma_G \right) \hat{Z}_B - 2(\gamma_\psi - \epsilon)\hat{Z}_Q \hat{Z}^{-1} \right\} \eta_{jk} [O_{kl}] \]

\[ - \frac{\beta_\epsilon}{g} i[O_{jl}O_A] - \left( \frac{\beta_\epsilon}{g} - \gamma_G \right) i[O_{jl}O_{B^*}] + \left( \gamma_G - \frac{\beta_\epsilon}{g} \right) i[O_{jl}O_G] \]

\[ + (\gamma_\psi - \epsilon) i[O_{jl}O_{\hat{\psi}\psi}] + 2\gamma_\omega i[O_{jl}O_{\bar{\omega}}]. \tag{38} \]
Inserting this result in the Ward identity (16) we get with Eq. (13) in the limit $\epsilon \to 0$:

$$
\langle [O_{ji}] \delta^S \mathcal{X} \rangle = \sum_{k=0}^{j} \left\{ (l+3) \hat{1} + \hat{\gamma} \right\}_{jk} \langle [O_{kl}] \mathcal{X} \rangle + \frac{\beta}{g} \langle i[O_{ji}O_{A}] \mathcal{X} \rangle + \left( \frac{\beta}{g} - \gamma_G \right) \langle i[O_{ji}O_{B^*}] \mathcal{X} \rangle
$$

$$
- \gamma_\psi \langle i[O_{ji}O_{\psi\psi}] \mathcal{X} \rangle - \left( \gamma_G - \frac{\beta}{g} \right) \langle i[O_{ji}O_{G}] \mathcal{X} \rangle - 2 \gamma_\omega \langle i[O_{ji}O_{\omega}] \mathcal{X} \rangle,
$$

where the finiteness of the LHS of the Ward identity ensures the existence of the matrix

$$
\hat{\gamma} = \lim_{\epsilon \to 0} \left\{ -\hat{\beta}_g \hat{Z}_A - \left( \frac{\beta}{g} - \gamma_G \right) \hat{Z}_B + 2(\gamma_\psi - \epsilon) \hat{P}_Q \hat{Z}^{-1} \right\}. 
$$

It remains to show that the dilatation Ward identity (39) is nothing else than the familiar Callan-Symanzik equation [60, 61] and that $\hat{\gamma}$ is the anomalous dimension matrix defined in the MS scheme. By differentiation of the renormalized Green function with respect to the parameters $g$ and $\xi$ we define the differential vertex operator insertions $[O_{ji}\Delta^i]$, so that

$$
i[O_{ji}][\Delta^g] = i[O_{ji} \Delta^g] - \sum_{k=0}^{j} \left\{ g \frac{\partial \hat{Z}}{\partial g} \hat{Z}^{-1} + \left( 1 + g \frac{\partial \ln X}{\partial g} \right) \hat{Z} \left( \hat{1} \int d^d x B_\mu^a(x) \frac{\delta}{\delta B_\mu^a(x)} - 2 \hat{P}_G \right) \hat{Z}^{-1} \right\}_{jk} [O_{kl}],
$$

$$
i[O_{ji}][\Delta^\xi] = i[O_{ji} \Delta^\xi] - \sum_{k=0}^{j} \left\{ \xi \frac{\partial \hat{Z}}{\partial \xi} \hat{Z}^{-1} + \xi \frac{\partial \ln X}{\partial \xi} \hat{Z} \left( \hat{1} \int d^d x B_\mu^a(x) \frac{\delta}{\delta B_\mu^a(x)} - 2 \hat{P}_G \right) \hat{Z}^{-1} \right\}_{jk} [O_{kl}].
$$

We learn from Eqs. (23, 29, 30, 41, 42) that the renormalization constants $\hat{Z}_i$ can be expressed in terms of the renormalization matrix of conformal operators and read

$$
\hat{Z}_A = \left( \frac{\partial \hat{Z}}{\partial g} - 2 \xi \frac{\partial \hat{Z}}{\partial \xi} \right) \hat{Z}^{-1} - 2 \hat{Z} \hat{P}_G \hat{Z}^{-1} - 2 \left( g \frac{\partial \ln X}{\partial g} - 2 \xi \frac{\partial \ln X}{\partial \xi} \right) \hat{P}_G \hat{Z}^{-1}, 
$$

$$
\hat{Z}_B = 2 \xi \frac{\partial \hat{Z}}{\partial \xi} \hat{Z}^{-1} - 4 \left( \xi \frac{\partial \ln X}{\partial \xi} \right) \hat{P}_G \hat{Z}^{-1}.
$$

Inserting these findings into the renormalized dilatation Ward identity (39) provides us the well-known form of this Ward identity:

$$
\langle [O_{ji}] \delta^S \mathcal{X} \rangle = \sum_{k=0}^{j} \left\{ (l+3) \hat{1} + \hat{\gamma} \right\}_{jk} \langle [O_{kl}] \mathcal{X} \rangle + \frac{\beta}{g} \langle i[O_{ji}O_{A}] \mathcal{X} \rangle + \sigma \langle i[O_{ji}O_{g}] \mathcal{X} \rangle
$$

$$
- \gamma_\psi \langle i[O_{ji}O_{\psi\psi}] \mathcal{X} \rangle - \gamma_G \langle i[O_{ji}O_{G}] \mathcal{X} \rangle - 2 \gamma_\omega \langle i[O_{ji}O_{\omega}] \mathcal{X} \rangle,
$$

and from Eq. (40) follows the familiar definition for the anomalous dimension matrix:

$$
\hat{\gamma} \equiv \hat{\gamma}_Z + 2 \gamma_G \hat{P}_G + 2 \gamma_\psi \hat{P}_Q, \quad \text{with } \hat{\gamma}_Z = -\mu \frac{d}{d\mu} \hat{Z} \hat{Z}^{-1} = g \frac{\partial}{\partial g} \hat{Z}^{[1]}.
$$
3.2 Special conformal Ward identity.

The renormalization of the special conformal Ward identity can now be performed along the same line as in the previous subsection for the dilatation. For the renormalization of the product $[O_{jl}]\delta^C\mathcal{S}$ we obtain from Eqs. (19), (28) together with (29), (30), (31) and (32):

$$i[O_{jl}]\delta^C\mathcal{S} = -i\sum_{k=0}^j \left\{ \frac{\beta_k}{g} \hat{Z}_A - \left( \frac{\beta_k}{g} - \gamma_G \right) \hat{Z}_B + 2(\gamma_\psi - \epsilon) \hat{b} \hat{P}_Q \hat{Z}^{-1} \right\}_k [O_{kl}]$$

$$- \frac{\beta_e}{g} i[O_{jl}O_A] - \left( \frac{\beta_e}{g} - \gamma_G \right) i[O_{jl}O_{B^*}] + \left( \gamma_G - \frac{\beta}{g} \right) i[O_{jl} \Omega^-_G]$$

$$+ (\gamma_\psi - \epsilon) i[O_{jl} \Omega^-_\psi] + 2\gamma_\omega i[O_{jl} \Omega^-_\omega] + (d - 2) i[O_{jl} \Delta^-_{ext}],$$

where $[\Delta^-_{ext}] = \int d^d x \, 2x_\mu \partial_\mu [O_{B\mu}(x)]$. Inserting this equation together with the special conformal variation of the composite operator, given in Eq. (18), into the Ward identity (16) (with $\delta \equiv \delta^C$) we obtain the desired CWI

$$\langle [O_{jl}]\delta^C\mathcal{X} \rangle = -i \sum_{k=0}^j \{ \hat{a}(l) + \hat{\gamma}^c(l) \}_k \langle [O_{kl}]\mathcal{X} \rangle + \frac{\beta}{g} \langle i[O_{jl}O_A^*] \mathcal{X} \rangle + \left( \frac{\beta}{g} - \gamma_G \right) \langle i[O_{jl}O_{B^*}] \mathcal{X} \rangle$$

$$- 2\langle i[O_{jl} \Delta^-_{ext}] \mathcal{X} \rangle - \gamma_\psi \langle i[O_{jl} \Omega^-_\psi] \mathcal{X} \rangle - \left( \gamma_G - \frac{\beta}{g} \right) \langle i[O_{jl} \Omega^-_G] \mathcal{X} \rangle - 2\gamma_\omega \langle i[O_{jl} \Omega^-_\omega] \mathcal{X} \rangle. \tag{48}$$

The anomaly appearing here is called special conformal anomaly matrix and is defined by

$$\hat{\gamma}^c(l) = \lim_{\epsilon \to 0} \left\{ \hat{Z} \left[ \hat{a}(l) - 2(\gamma_\psi - \epsilon) \hat{P}_Q \hat{b} \right] \hat{Z}^{-1} - \frac{\beta_e}{g} \hat{Z}_A - \left( \frac{\beta_e}{g} - \gamma_G \right) \hat{Z}_B - \hat{a}(l) \right\}$$

$$= -2\gamma_\psi P_Q \hat{b} + 2 \left[ \hat{Z}^{[1]}_A, \hat{P}_Q \hat{b} \right] + \hat{Z}_A^{[1]} - \hat{Z}_B^{[1]}, \tag{49}$$

where the second equality holds due to the finiteness of $\hat{\gamma}^c$. Note that this definition looks similar to the anomalous dimension matrix defined in Eq. (40).

4 Commutator constraints.

A priori there is no way to relate the new renormalization matrices $\hat{Z}_A$ and $\hat{Z}_B$ to the renormalization group constants. Obviously, the special conformal anomaly matrix $\hat{\gamma}^c(l)$ contains therefore new information. However, from the conformal algebra it is possible to derive the following constraints for the conformal anomaly matrices:

$$[\mathcal{K}_{-}, \mathcal{P}_{+}] = -2i(\mathcal{D} + \mathcal{M}_{++}) \quad \Rightarrow \quad \hat{\gamma}^c(l + 1) - \hat{\gamma}^c(l) = -2 \left( \hat{\gamma} - 2\frac{\beta}{g} \hat{P}_G \right), \tag{50}$$

$$[\mathcal{D}, \mathcal{K}_{-}] = i\mathcal{K}_{-} \quad \Rightarrow \quad \left[ \hat{\gamma}, \hat{a}(l) + \hat{\gamma}^c(l) + 2\frac{\beta}{g} \hat{b}(l) \hat{P}_Q \right]_{-} = 0. \tag{51}$$
The constraint \([\mathcal{P}_-, \mathcal{P}_+] = -2(i\mathcal{D} + \mathcal{M}_{++})\) tells us that the breaking of translation invariance by the infinitesimal special conformal transformation is controlled by the commutator relation between \(\mathcal{K}_-\) and \(\mathcal{P}_+\) providing the spin dependence of the special conformal anomaly matrix. The constraint \([\mathcal{I}, \mathcal{O}_{\gamma\beta}]\) relates the off-diagonal matrix elements of \(\hat{\gamma}\) to the matrix \(\hat{\gamma}(l) + 2\frac{g}{2}\hat{b}(l)\hat{P}_Q\). Since \(\hat{a}(l)\) is a diagonal matrix, which is independent of the coupling \(\alpha_s\), the \(n\)-loop approximation of \(\hat{\gamma}_{\text{ND}}\) is determined by the \((n-1)\)-loop approximation of \(\gamma(l)\) and \(\beta\)-function.

### 4.1 Constraint for the special conformal anomaly matrix.

From the definitions \([12,13]\) it follows that the commutator \([\mathcal{K}_-, \mathcal{P}_+] = -2i(\mathcal{D} + \mathcal{M}_{++})\) corresponds to the following relation for the variations \([\delta^P_\gamma, \delta^C_\gamma] = 2(\delta^S + \delta^M_+).\) We will employ this identity in order to prove the constraint \([\mathcal{I}],\) which comes from the conformal algebra and the Ward identities \([17,39]:\)

\[
\langle [\mathcal{O}_{jl}] [\delta^P_\gamma, \delta^C_\gamma] \mathcal{X} \rangle = \langle [\mathcal{O}_{jl}] 2(\delta^S + \delta^M_+) \mathcal{X} \rangle
\]

\[
= \sum_{k=0}^j \{4(l+2)\hat{1} + 2\hat{\gamma}\} \langle [\mathcal{O}_{kl}] \mathcal{X} \rangle + 2\frac{\beta}{g} \langle [i[\mathcal{O}_{jl}\mathcal{A}]\mathcal{X}] \rangle + 2\left(\frac{\beta}{g} - \gamma_G\right) \langle [i[\mathcal{O}_{jl}\mathcal{B}^+]\mathcal{X}] \rangle
\]

\[
-2\gamma_\psi \langle [i[\mathcal{O}_{jl}\Omega]\mathcal{X}] \rangle - 2\left(\gamma_G - \frac{\beta}{g}\right) \langle [i[\mathcal{O}_{jl}\mathcal{G}]\mathcal{X}] \rangle - 2\gamma_\omega \langle [i[\mathcal{O}_{jl}\Omega]\mathcal{X}] \rangle.
\]

With the help of the Ward identities \([17,38]\) we find immediately for the first term in the commutator on the LHS of Eq. \([52]:\)

\[
\langle [\mathcal{O}_{jl}] \delta^P_\gamma \delta^C_\gamma \mathcal{X} \rangle = -\sum_{k=0}^j \{\hat{a}(l+1) + \hat{\gamma}(l+1)\} \langle [\mathcal{O}_{kl}] \mathcal{X} \rangle + \frac{\beta}{g} \langle [\mathcal{O}_{jl+1}\Omega^-] \mathcal{X} \rangle
\]

\[
+ \left(\frac{\beta}{g} - \gamma_G\right) \langle [\mathcal{O}_{jl+1}\mathcal{B}^+] \mathcal{X} \rangle - 2\langle [\mathcal{O}_{jl}\mathcal{D}_{\psi\psi}^-] \mathcal{X} \rangle - \gamma_\psi \langle [\mathcal{O}_{jl+1}\Omega^-]\mathcal{X} \rangle
\]

\[
- \left(\gamma_G - \frac{\beta}{g}\right) \langle [\mathcal{O}_{jl+1}\mathcal{G}^-] \mathcal{X} \rangle - 2\gamma_\omega \langle [\mathcal{O}_{jl+1}\Omega^-]\mathcal{X} \rangle.
\]

While for the second term we get

\[
\langle [\mathcal{O}_{jl}] \delta^C_\gamma \delta^P_\gamma \mathcal{X} \rangle = -\sum_{k=0}^j \{\hat{a}(l) + \hat{\gamma}(l)\} \langle [\mathcal{O}_{kl+1}] \mathcal{X} \rangle + \frac{\beta}{g} \langle [i[\mathcal{O}_{jl}\mathcal{A}]\delta^P_\gamma \mathcal{X}] \rangle
\]

\[
+ \left(\frac{\beta}{g} - \gamma_G\right) \langle [i[\mathcal{O}_{jl}\Omega]\delta^P_\gamma \mathcal{X}] \rangle - 2\langle [i[\mathcal{O}_{jl}\mathcal{D}_{\psi\psi}^-]\delta^P_\gamma \mathcal{X}] \rangle - \gamma_\psi \langle [i[\mathcal{O}_{jl}\Omega^-]\delta^P_\gamma \mathcal{X}] \rangle
\]

\[
- \left(\gamma_G - \frac{\beta}{g}\right) \langle [i[\mathcal{O}_{jl}\mathcal{G}^-]\delta^P_\gamma \mathcal{X}] \rangle - 2\gamma_\omega \langle [i[\mathcal{O}_{jl}\Omega^-]\delta^P_\gamma \mathcal{X}] \rangle.
\]

To proceed further we have to get rid of the variation sign on the field monomial, i.e. \(\delta^P_\gamma \mathcal{X}\).
Using the definition of the subtractive renormalization (29), (30) for the product of two composite operators we have

\[ \langle i[O_{jl}O_i^{-}]\delta_+^{P}\mathcal{X} \rangle = -\langle i\delta_+^{P}[O_{jl}O_i^{-}]\mathcal{X} \rangle \text{ for } i = \{A, B^*\} = \langle [O_{jl+1}O_i^{-}]\mathcal{X} \rangle - 2\langle i[O_{jl}O_i^{-}]\mathcal{X} \rangle - 4\delta_iA\hat{P}_G\langle [O_{jl}]\mathcal{X} \rangle. \] (55)

Note that the last term arises due to the non-minimal nature of the subtraction: the renormalization matrix \( \hat{Z}_A \) consists of a finite term apart from divergent contributions \( \propto \frac{1}{\epsilon} \).

Since the operator \( [\Delta^{-}_{\text{ext}}] = -2n^*_\mu \int d^d x \ [O_{B\mu}] \) is translation invariant we can write

\[ \langle i[O_{jl}\Delta^{-}_{\text{ext}}]\delta_+^{P}\mathcal{X} \rangle = \langle [O_{jl+1}\Delta^{-}_{\text{ext}}]\mathcal{X} \rangle. \] (56)

For the terms containing the EOM we find immediately

\[ \langle i[O_{jl}\Omega^{-}\phi]\delta_+^{P}\mathcal{X} \rangle = 2\langle [O_{jl}]\int dx\phi(x)\frac{\delta}{\delta\phi(x)}\mathcal{X} \rangle - \langle i[O_{jl}]\delta_+^{P}\int dx2x\phi(x)\frac{\delta}{\delta\phi(x)}\mathcal{X} \rangle - 2\langle i[O_{jl}\Omega\phi]\mathcal{X} \rangle. \] (57)

Inserting our findings (53-56) in equation (52) and using the definition of the elements of the \( \hat{a} \)-matrix yields

\[ \sum_{j,k=0}^j \left\{ \hat{\gamma}^c(l+1) - \hat{\gamma}^c(l) + 2\left( \hat{\gamma} - 2\hat{P}_G \right) \right\} \langle [O_{kl+1}]\mathcal{X} \rangle = 0. \] (58)

Owing to linear independence of the Gegenbauer polynomials we can safely omit the Green function and get finally the matrix equation (57).

**4.2 Constraint for the off-diagonal anomalous dimension matrix.**

Now we prove the commutator constraint (51). Since \( \delta^S = -\mu \frac{\partial}{\partial\mu} \), instead of dealing with the corresponding commutator we apply the renormalization group operator

\[ D = \mu \frac{d}{d\mu} \hat{1} + \hat{\gamma} \]

on both sides of the special CWI (48) without taking into account the differentiation of the field monomial \( \mathcal{X} \) with respect to \( \mu \) since these terms vanish by means of the special conformal Ward identity. The LHS vanishes identically since \( D[O_{jk}] = 0 \). To calculate the RHS we employ formally the renormalization group equations of the appearing operator insertions. For this purpose we need the following commutators

\[ \left[ \mu \frac{d}{d\mu} , g \frac{\partial}{\partial g} \right] = -\left( g \frac{\partial}{\partial g} \beta_c \right) g \frac{\partial}{\partial g} + 2\left( g \frac{\partial}{\partial g} \gamma_c \right) \xi \frac{\partial}{\partial \xi} , \quad \left[ \mu \frac{d}{d\mu} , \xi \frac{\partial}{\partial \xi} \right] = 2\left( \xi \frac{\partial}{\partial \xi} \gamma_c \right) \xi \frac{\partial}{\partial \xi}. \] (59)
Due to Eq. (33) the following relation holds

\[ \mu \frac{d}{d\mu} [O_A] = - \left( g \frac{\partial}{\partial g} \frac{\beta}{g} \right) [O_A] - \left( g \frac{\partial}{\partial g} - 2\xi \frac{\partial}{\partial \xi} \right) \left( \frac{\beta}{g} - \gamma_G \right) [O_{B^*}] \]

\[ + \sum_{\phi} \left( g \frac{\partial}{\partial g} - 2\xi \frac{\partial}{\partial \xi} \right) F_\phi(g) \Omega^- \phi, \]

\[ \mu \frac{d}{d\mu} [O_{B^*}] = \left( 2\xi \frac{\partial}{\partial \xi} \gamma_G \right) [O_{B^*}] + \sum_{\phi} \left( 2\xi \frac{\partial}{\partial \xi} F_\phi(g) \right) \Omega^- \phi, \]

where

\[ F_G(g) = \gamma_G - \frac{\beta}{g}, \quad F_\psi(g) = \gamma_\psi, \quad F_\omega(g) = 2\gamma_\omega. \]

Now we list the action of the operator \( D \) on every term appearing on the RHS of the special conformal Ward identity. The first equation is obvious:

\[ D \sum_{k=0}^{j} \{ \hat{a}(l) + \hat{c}(l) \} \langle [O_{kl^{-1}}] \chi \rangle = \sum_{k=0}^{j} \left\{ \left[ \hat{\gamma}, \hat{a} + \hat{c} + 2\beta \hat{P}_\psi \hat{b} \right] \right\} \]

\[ + \beta \frac{\partial}{\partial g} \left[ \hat{Z}_A^{-1} + \hat{Z}_B^{-1} - 2\gamma_\psi \hat{P}_\psi \hat{b} \right] \langle [O_{kl^{-1}}] \chi \rangle. \]

Here we have taken into account that the special conformal anomaly matrix is a gauge independent quantity. Next we calculate the action of \( D \) on the renormalized operator products by using the subtractive renormalization prescriptions (29) and (30) as well as the renormalization group equations derived above:

\[ D \frac{\beta}{g} \langle [O_{j} O_{\bar{a}}] \chi \rangle \]

\[ = \beta \left( g \frac{\partial}{\partial g} - 2\xi \frac{\partial}{\partial \xi} \right) \left( \gamma_G - \frac{\beta}{g} \right) \langle [O_{j} O_{\bar{a}}] \chi \rangle + \beta \sum_{\phi} \left( g \frac{\partial}{\partial g} - 2\xi \frac{\partial}{\partial \xi} \right) F_\phi(g) \langle [O_{j} \Omega^- \phi] \chi \rangle \]

\[ + \beta \sum_{k=0}^{j} \left\{ \left( g \frac{\partial}{\partial g} \hat{Z}_A^{-1} \right) - 2 \left( g \frac{\partial}{\partial g} - 2\xi \frac{\partial}{\partial \xi} \right) \gamma_\psi \hat{P}_\psi \hat{b} \right\} \langle [O_{kl^{-1}}] \chi \rangle, \]

\[ D \left( \frac{\beta}{g} - \gamma_G \right) \langle [O_{j} O_{B^*}] \chi \rangle \]

\[ = -\beta \left( g \frac{\partial}{\partial g} - 2\xi \frac{\partial}{\partial \xi} \right) \left( \gamma_G - \frac{\beta}{g} \right) \langle [O_{j} O_{B^*}] \chi \rangle + \beta \left( \frac{\beta}{g} - \gamma_G \right) \sum_{\phi} \left( 2\xi \frac{\partial}{\partial \xi} F_\phi(g) \right) \langle [O_{j} \Omega^- \phi] \chi \rangle \]

\[ + \left( \frac{\beta}{g} - \gamma_G \right) \sum_{k=0}^{j} \left\{ \left( g \frac{\partial}{\partial g} \hat{Z}_B^{-1} \right) - 4 \left( \xi \frac{\partial}{\partial \xi} \gamma_\psi \right) \hat{P}_\psi \hat{b} \right\} \langle [O_{kl^{-1}}] \chi \rangle. \]

Due to Eq. (33) the following relation holds

\[ D F_\phi(g) \langle [O_{j} \Omega^- \phi] \chi \rangle = \left( \mu \frac{d}{d\mu} F_\phi(g) \right) \langle [O_{j} \Omega^- \phi] \chi \rangle. \]
In the above equations we have omitted the divergent contributions on the RHS (which cancel each other) since the LHS is finite. All other terms in the special conformal Ward identity vanish when the $D$-operator acts on them.

Collecting all terms listed above, we find the desired constraint

$$\sum_{k=0}^{j} \left\{ \hat{\gamma}, \hat{a}(l) + \hat{\gamma}^c(l) + 2 \frac{\partial}{\partial l} \hat{b}(l) \hat{P}_Q \right\}_{jk} [\mathcal{O}_{kl-1}]\mathcal{X} = 0, \quad (66)$$

which implies finally the matrix equation (51).

We have also found the following expression for the gauge-dependent renormalization constant $\hat{Z}_B$:

$$g \frac{\partial}{\partial g} \hat{Z}_B^[1]^- = 4 \left( \xi \frac{\partial}{\partial \xi} \gamma_\psi \right) \hat{P}_Q \hat{b}. \quad (67)$$

5 Leading order conformal anomalies.

In this section we compute the conformal anomalies to LO in a general covariant gauge. Since we are interested in the present paper in the NLO approximation for the anomalous dimension matrix of conformal operators, we restrict ourselves to the LO approximation of the special conformal anomaly which reads

$$\hat{\gamma}^c = \begin{pmatrix} 2 \left[ QQ\hat{Z}_B^[1]^- \right. + QQ\hat{Z}_A^[1]^- \cdot \frac{-2 \hat{b} QQ\hat{Z}_A^[1]^-}{QQ\hat{Z}_A^[1]^-} & \left. \cdot \frac{-2 \hat{b} QQ\hat{Z}_A^[1]^-}{QQ\hat{Z}_A^[1]^-} \end{pmatrix} \end{pmatrix} \cdot \frac{-2 \hat{b} QQ\hat{Z}_A^[1]^-}{QQ\hat{Z}_A^[1]^-}. \quad (68)$$

Here we have taken into account that in one-loop approximation the following equalities hold

$$QQ\hat{Z}_B^[1]^- = 2 \gamma_\psi \hat{b} \quad \text{and} \quad QQ\hat{Z}_A^[1]^- = 0. \quad (69)$$

For technical reason we perform the calculation in the light-cone position formalism. Then it is quite straightforward to transform these results to any other representation. We define the relevant light-ray operators as follows:

$$\mathcal{O}(\kappa_1, \kappa_2) = \begin{pmatrix} QQ(\kappa_1, \kappa_2) \\ G\mathcal{O}(\kappa_1, \kappa_2) \end{pmatrix}, \quad (70)$$

where

$$\begin{pmatrix} QQ(\kappa_1, \kappa_2) \\ G\mathcal{O}(\kappa_1, \kappa_2) \end{pmatrix} = \begin{pmatrix} \bar{\psi}(\kappa_2 n) \left\{ \begin{array}{c} \gamma_+ \\ \gamma_+ \gamma_5 \end{array} \right\} \Phi[\kappa_2 n, \kappa_1 n] \psi(\kappa_1 n) \mp \bar{\psi}(\kappa_1 n) \left\{ \begin{array}{c} \gamma_+ \\ \gamma_+ \gamma_5 \end{array} \right\} \Phi[\kappa_1 n, \kappa_2 n] \psi(\kappa_2 n), \quad (71) \\
G\mathcal{O}(\kappa_1, \kappa_2) = G_+(\kappa_2 n) \begin{pmatrix} g_{\mu\nu} \\ \epsilon_{\mu\nu-} \end{pmatrix} \Phi[\kappa_2 n, \kappa_1 n] G_+(\kappa_1 n). \quad (72) \end{pmatrix}$$
Figure 1: One-loop graphs contributing to the renormalization constant \( \hat{Z}_A \).

The path-ordered link factor \( \Phi[x_2, x_1] \) ensures gauge invariance and reads

\[
\Phi[x_2, x_1] = P \exp \left( i \mu \left( g X(x_2 - x_1) \mu \int_0^1 d\tau T^a B^a_\mu(x(\tau)) \right) \right),
\]

where the path is parametrized as \( x(\tau) = \tau x_2 + \bar{\tau} x_1 \). The local conformal composite operators (10) are obtained by differentiation with respect to \( \kappa_1 \) and \( \kappa_2 \), namely:

\[
\mathcal{O}_{jl} = \left. \left( \frac{1}{2} (i \partial_+)^l C^{\frac{3}{2}}_j \left( \frac{\partial_+}{\partial_\nu} \right)^C \mathcal{O}(\kappa_1, \kappa_2) \right) \right|_{\kappa_1 = \kappa_2 = 0},
\]

where \( \partial_+ = \partial_{\kappa_1} + \partial_{\kappa_2} \) and \( \dot{\partial}_+ = \partial_{\kappa_1} - \partial_{\kappa_2} \).

In the next subsection we derive the special conformal anomaly matrix for the quark sector including chiral-odd (transversity) twist-2 operators. This matrix turns out to be universal in a sense that it is the same for all Dirac structures. Then we comment shortly our results for the mixed \( QG \) and \( GQ \) channels, which were derived previously in Abelian theory. In the last subsection we evaluate the special conformal anomaly matrix for the \( GG \) channel for parity even and odd operators. Again in all channels we observe the universality of these kernels, which implies that the additional special conformal symmetry breaking is the same in vector and axial-vector channels: the difference arises from the different scale anomalies, i.e. the one-loop anomalous dimensions of composite operators.

### 5.1 Once more about quarks: chiral-even and -odd sectors.

Although the quark sector was considered by us in detail in previous studies, nevertheless, it is worthwhile to address this issue from our present point of view. Moreover, we complete the list of two-loop anomalous dimensions by adding those of the chiral-odd (transversity) sector. Thus we should evaluate the renormalization constant \( \hat{Z}_A \) from diagrams in Fig. 1. This calculation is
where we have introduced the plus-prescription for singularities. This kernel is the same for all quark operators irrespective of the Dirac structure: quark anomalous dimension vanishes, \( \gamma_w \).

When Eq. (77) is transformed into the light-cone fraction representation (see Ref. [62]) we reproduce the known expression for the \( w(x, y) \)-function [19]:

\[
QQ_w(x, y) = -2C_F \left[ \frac{x \theta(y-x)}{y(x-y)^2} + \left( \frac{x}{y} \rightarrow \tilde{x}, \frac{y}{x} \rightarrow \tilde{y} \right) \right].
\]

This kernel is the same for all quark operators irrespective of their Dirac structure: \( \Gamma = \gamma_+, \gamma_+ \gamma_5 \) or \( \sigma_+ \gamma_5 \). Therefore, the special conformal anomaly matrix reads \( QQ_\gamma^{(0)}(0) = -\hat{b} QQ_\gamma^{(0)} + QQ_\hat{w} \).

### 5.2 Mixed channels.

Note, that in the present approach there is no need to perform any calculation for the mixing \( QG \) channel. The \( w \)-kernel for the \( QG \)-channel was calculated by us in Ref. [47] with the same result for the parity even and odd channels

\[
QQ^{w}(y, z) = CF[\delta(z) - \delta(y)].
\]

The momentum space kernel reads

\[
QQ^{w}(x, y) = 2C_F \left\{ \frac{1}{y} \theta(y-x) - \left( \frac{x}{y} \rightarrow \tilde{x}, \frac{y}{x} \rightarrow \tilde{y} \right) \right\}.\]

Note the extra factor of 2 in front of the light-cone fraction kernel which comes as a result of the transformation from the coordinate to the momentum space according to Eqs. (A.1,A.2).

\[\text{The last argument of the kernel stands for the value of the gauge fixing parameter.}\]
Figure 2: One-loop diagrams which give rise to divergencies in the product of the renormalized operator insertions $i[\mathcal{O}_A][^G\mathcal{O}_{jl}]$. The symmetry factors $s_\sigma$ for the diagrams are the following: $s_\sigma = 1$ for $\sigma = a, b, c, f, g, h, i$ and $s_\sigma = \frac{1}{2}$ for $\sigma = d, e, j, k, l$. The diagram (m) is identically zero in dimensional regularization.

5.3 Gluonic sector.

Using the representation of the conformal operators in terms of the derivatives of non-local string operators we can write the divergent part of the operator product $i[\mathcal{O}_A][^G\mathcal{O}(\kappa_1, \kappa_2)]$ via the following relation in leading order of the coupling constant:

$$
i[\mathcal{O}_A][^G\mathcal{O}(\kappa_1, \kappa_2)] = \frac{\alpha_s}{2\pi} \epsilon \int_0^1 dz \int_0^z dy \left\{ ^G\mathcal{K}_A(y, z)[^G\mathcal{O}(\bar{y}\kappa_1 + y\kappa_2, z\kappa_1 + \bar{z}\kappa_2)] + ^G\tilde{\mathcal{K}}_A(y, z) \int d^d x 2x - B^b_\mu(x) \frac{\delta}{\delta B^b_\mu(x)}[^G\mathcal{O}(\bar{y}\kappa_1 + y\kappa_2, z\kappa_1 + \bar{z}\kappa_2)] \right\}. \quad (82)$$

Here the kernels $\mathcal{K}(y, z)$ are the subjects of the calculations. The diagrammatic representation of the corresponding graphs is given in Fig. 2 and the corresponding Feynman rules can be found in Appendix A. The details of the calculations together with the general approach to the evolution equations for non-local string operators are given in Appendix B.
Note that the factor in front of the operator $B\frac{\delta}{\delta B}[O]$ is gauge independent in leading order. It can be conveniently represented as
\[
\left( g \frac{\partial \ln X}{\partial g} - 2\xi \frac{\partial \ln X}{\partial \xi} \right) = \frac{1}{\epsilon} \frac{\alpha_s}{2\pi} \left( \frac{\beta_0}{2} - \gamma_G(0)(\xi = 0) \right).
\] (83)

The result of the calculations given in detail in Appendix B can be summarized as follows
\[
GGK_A^{-}(y, z) = -i2(\kappa_1 + \kappa_2)GGK(y, z) - i(\kappa_1 + \kappa_2)\beta_0\delta(y)\delta(z) + GGK^w(y, z),
\] (84)
\[
GGK_A^{+}(y, z) = i\left( \frac{\beta_0}{2} - \gamma_G(0)(\xi = 0) \right) \delta(y)\delta(z),
\] (85)
where the parity-even and -odd gluonic kernels read
\[
GGK^V(y, z) = C_A\left\{ 4(1 - y - z + 3yz) + \left\{ y - 2 + \left[ \frac{1}{y} \right]_+ \right\} \delta(z)
\right.
\]
\[
\left. + \left\{ z - 2 + \left[ \frac{1}{z} \right]_+ \right\} \delta(y) \right\} - \frac{\beta_0}{2} \delta(y)\delta(z),
\] (86)
\[
GGK^A(y, z) = GGK^V(y, z) - 12C_Ayz,
\] (87)
and
\[
GGK^w(y, z) = \frac{C_A}{k_2+} \left\{ 4 \left[ \frac{1}{z} \right]_+ - 2 \left[ \frac{1}{z^2} \right]_+ \right\} \delta(y) + \frac{C_A}{k_1+} \left\{ 4 \left[ \frac{1}{y} \right]_+ - 2 \left[ \frac{1}{y^2} \right]_+ \right\} \delta(z),
\] (88)
which is the same in both channels. The light-cone fraction function reads from this
\[
GGw(x, y) = -2C_A \left\{ \frac{x^2 \theta(y - x)}{y^2 (x - y)^2} \right\} + \frac{1}{y^2} \delta(y - x) + \frac{1}{y} \delta(y - x) + \left( x \rightarrow \bar{x}, y \rightarrow \bar{y} \right)
\] (89)

Using the identity
\[
(i\partial_+)^lC_j^\nu\left( \frac{\partial}{\partial_+} / \partial_+ \right) \{ (\kappa_1 + \kappa_2)O(\kappa_1, \kappa_2) \}\bigg|_{\kappa_i = 0} = i \sum_{k=0}^{j} b_{jk}(l, \nu)O_{jl-1},
\] (90)
with the $\hat{b}$-matrix being given by Eq. (37), we can represent the above result (84) in the usual matrix form
\[
GGz_c^{(0)} = -\hat{b} GGz^{(0)} + \beta_0 \hat{b} + GG\hat{w},
\] (91)
where the $\hat{w}$-matrix possesses elements defined as Gegenbauer moments of the kernel (89). This quantity will be evaluated in the next section. Note also that Eq. (91) as well as other conformal anomalies fulfill the constraint equality (50).

6 Gegenbauer moments of the evolution kernels.

This section is mainly technical so that rather experienced reader can skip this presentation. Here we intend to give the method for evaluation of the moments of the evolution kernels we have
derived in the preceding section in the basis of Gegenbauer polynomials. We deal here only with gluonic kernels.

Let us demonstrate the main technical steps for the integration of the functions with prescription, e.g. the first term in Eq. (39). Namely, \( G^G_{\omega_1} (x, y) = [x^2 / y^2 \theta(y-x)/(x-y)^2]_+ + (x \to \bar{x}, y \to \bar{y}) \). Integrating by parts over \( x \) we decrease the power of the singularity and end up with equation (according to Eq. (C.3) we have replaced the Gegenbauer polynomial by \( \frac{\Gamma(j+4)\Gamma(k+4)}{\Gamma^2(5)\Gamma(j)\Gamma(k)} \) where we have used analytical regularization of the singular distribution at the end we get the result in terms of hypergeometric functions and their derivatives with respect to the lower index, the simplicity of reduction to elementary functions depends on the handling of the integrals \( J \). Therefore, at this step it is instructive to expand the integrand of \( J \) with respect to a complete set of Gegenbauer polynomials. Namely,

\[
J_1(y) = \int_0^1 dx \left[ \frac{x^2}{1-x} \right] + \frac{d}{dy} 2F_1 \left( \frac{-j+1, j+4}{3} \right) \ ,
\]

\[
J_2(y) = \int_0^1 dx \left[ \frac{x}{1-x} \right] \frac{1}{xy} \left[ 2F_1 \left( \frac{-j+1, j+4}{3} \right) - 2F_1 \left( \frac{-j+1, j+4}{3} \right) \right] = \sum_{l=1}^{\infty} f_{jl} 2F_1 \left( \frac{-l+1, l+4}{3} \right) .
\]

The integrals (33) can be easily evaluated with the result

\[
J_1 = \sum_{l=1}^{\infty} e_{jl} \frac{\partial}{\partial \epsilon} \left. 2F_1 \left( \frac{-l+1, l+4}{3} \right) \right|_{\epsilon=0} \ ,
\]

\[
J_2 = \sum_{l=1}^{\infty} f_{jl} \left. \left( \frac{\partial}{\partial \epsilon} - \frac{1}{2} \right) \right|_{\epsilon=0} 2F_1 \left( \frac{-l+1, l+4}{3} \right) .
\]

where we have used analytical regularization of the singular distribution

\[
\int_0^1 \frac{dx}{[1-x]_+} \, 2F_1 \left( \frac{-l+1, l+4}{3 + \epsilon} \right) = \int_0^1 \frac{dx}{(1-x)^{1+\epsilon}} \{ F(x) - F(1) \} .
\]
Now these matrices can be easily computed. The main steps are reduced to the use of the representation \((C.4)\) and integration by parts until all derivatives will act on the right hypergeometric function. Then with the help of Eqs. \((C.5)\) and \((C.2)\), we obtain the result in terms of hypergeometric functions of argument \(1\). The reduction to elementary functions is accomplished according to the formulae \((C.9)-(C.10)\). In this way we get

\[
e_{jl} = -[1 - (-1)^{j-l}]\theta_{j-1,l}(3 + 2l)\frac{\Gamma(j)\Gamma(l + 4)}{\Gamma(j + 4)\Gamma(l)},
\]

\[
f_{jl} = -\frac{1}{2}[1 - (-1)^{j-l}]\theta_{j-1,l}(3 + 2l)\frac{\Gamma(j)\Gamma(l + 4)}{\Gamma(j + 4)\Gamma(l)} \left[\frac{\Gamma(j + 4)\Gamma(l)}{\Gamma(j)\Gamma(l + 4)} - 1\right],
\]

\[
h_{lk}(j) = \delta_{jk} \frac{\Gamma(j + 4)\Gamma(l)}{\Gamma(j)\Gamma(l + 4)},
\]

\[
g_{lk}(j) = \theta_{l,k} \frac{\Gamma(j + 4)\Gamma(l)}{\Gamma(j)\Gamma(l + 4)} \left\{ \psi(3) - \psi(k + 2) \right\} \delta_{jk} + [1 - (-1)^{l-k}] \frac{(3 + 2k)}{(l - k)(l + k + 3)} \right\}.
\]

Combining these results together we have

\[
GG_{w_{jk}}^{(1)} = [1 + (-1)^{j-k}] \theta_{j-2,k}(3 + 2k) \left\{ \psi(3) - \psi(k + 2) \right\} \left[ \frac{\Gamma(j + 4)\Gamma(k)}{\Gamma(j)\Gamma(k + 4)} - 1\right] + \sum_{l=k+1}^{j-1} [1 - (-1)^{l-k}] \frac{(3 + 2l)}{(l - k)(l + k + 3)} \left[ \frac{\Gamma(j + 4)\Gamma(k)}{\Gamma(j)\Gamma(k + 4)} - 1\right].
\]

The final summation can be easily performed with help of the formulae

\[
\sum_{l=k+1}^{j-1} [1 - (-1)^{l-k}] \frac{(3 + 2l)}{(l - k)(l + k + 3)} \frac{\Gamma(l)}{\Gamma(l + 4)} = A_{jk} + \psi(j + 2) - \psi(k + 2),
\]

\[
\sum_{l=k+1}^{j-1} [1 - (-1)^{l-k}] \frac{(3 + 2l)}{(l - k)(l + k + 3)} \frac{\Gamma(l)}{\Gamma(l + 4)} = \frac{\Gamma(k)}{\Gamma(k + 4)} \left\{ A_{jk} - \psi(j + 2) + \psi(k + 2) - \frac{(j-k)(j+k+3)}{(j+1)(j+2)} \right\}.
\]

Adding the contributions from the last two terms in Eq. \((89)\)

\[
GG_{w_{jk}}^{(2)} = [1 + (-1)^{j-k}]\theta_{j-2,k}(3 + 2k) \frac{\Gamma(k)}{\Gamma(k + 4)} (j-k)(j+k+3) [(j-k)(j+k+3) + 2],
\]

\[
GG_{w_{jk}}^{(3)} = -[1 + (-1)^{j-k}]\theta_{j-2,k}(3 + 2k) \frac{\Gamma(k)}{\Gamma(k + 4)} \left[\frac{\Gamma(j + 4)\Gamma(k)}{\Gamma(j)\Gamma(k + 4)} - 1\right],
\]

we get the quantity in question

\[
GG_{w_{jk}} = -2C_{A} \sum_{i=1}^{3} GG_{w_{jk}}^{(i)} = -2C_{A} [1 + (-1)^{j-k}]\theta_{j-2,k}(3 + 2k) \frac{\Gamma(k)}{\Gamma(k + 4)} \left\{ 2A_{jk} + (A_{jk} - \psi(j + 2) + \psi(1)) \left[\frac{\Gamma(j + 4)\Gamma(k)}{\Gamma(j)\Gamma(k + 4)} - 1\right] + 2(j-k)(j+k+3) \frac{\Gamma(k)}{\Gamma(k + 4)} \right\},
\]

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where we have introduced the matrix $\hat{A}$ the elements of which are defined by

$$A_{jk} = \psi \left( \frac{j+k+4}{2} \right) - \psi \left( \frac{j-k}{2} \right) + 2\psi(j-k) - \psi(j+2) - \psi(1). \quad (110)$$

The matrices for the $QQ$ and $GQ$ cases can be found analogously and were derived in our previous studies [49, 47]:

$$QQ_{\gamma_{jk}} = -2C_F\left[1 - (-1)^{-j+k}\right] \theta_{j-2,k}(3 + 2k) \times \left\{ 2A_{jk} + (A_{jk} - \psi(j + 2) + \psi(1)) \frac{(j - k)(j + k + 3)}{(k + 1)(k + 2)} \right\}, \quad (111)$$

$$GQ_{\gamma_{jk}} = -2C_F\left[1 - (-1)^{-j+k}\right] \theta_{j-2,k}(3 + 2k) \frac{1}{6} \frac{(j - k)(j + k + 3)}{(k + 1)(k + 2)}. \quad (112)$$

In addition $GQ_{\psi} = 0$.

### 7 Two-loop anomalous dimensions and evolution kernels.

From the consistency relation (50) we can now derive the equation which defines the off-diagonal elements of the two-loop anomalous dimension matrix via the one-loop quantities evaluated in the preceding sections. Namely, we get ($j > k$)

$$a(j,k)\hat{\gamma}^{ND(1)}_{\gamma_{jk}} = \hat{\gamma}^{(0)}_{\gamma_{jk}} - \gamma^{(0)}_{\gamma_{jk}} + \gamma^{(0)}_{\gamma_{jk}} + \beta_0 \left( \hat{\gamma}^{(0)}_{\gamma_{jk}} + \hat{\gamma}^{(0)}_{\gamma_{jk}} \right) b_{jk}. \quad (113)$$

The complete entry which governs the evolution in NLO is defined by the sum $\hat{\gamma}^{(1)}_{\gamma_{jk}} = \hat{\gamma}^{(1)}_{\gamma_{jk}} + \gamma^{(1)}_{\gamma_{jk}}$, with $\gamma^{(1)}_{\gamma_{jk}}$ being the two-loop forward anomalous dimensions [7, 8, 9]. Defining new matrices $d_{jk} = b_{jk}/a(j,k)$ and $g_{jk} = w_{jk}/a(j,k)$, we can rewrite the above equality in components

$$QQ^{\gamma_{\gamma_{jk}}}^{ND(1)} = \left( QQ^{\gamma_{\gamma_{jk}}}^{(0)} - QQ^{\gamma_{\gamma_{jk}}}^{(0)} \right) \left\{ d_{jk} \left( \beta_0 - QQ^{\gamma_{\gamma_{jk}}}^{(0)} \right) + QQ^{\gamma_{\gamma_{jk}}} \right\} \quad (114)$$

$$GQ^{\gamma_{\gamma_{jk}}}^{ND(1)} = \left( GQ^{\gamma_{\gamma_{jk}}}^{(0)} - GQ^{\gamma_{\gamma_{jk}}}^{(0)} \right) \left\{ d_{jk} \left( \beta_0 - GQ^{\gamma_{\gamma_{jk}}}^{(0)} \right) + GQ^{\gamma_{\gamma_{jk}}} \right\} \quad (115)$$

$$GQ^{\gamma_{\gamma_{jk}}}^{ND(1)} = \left( GQ^{\gamma_{\gamma_{jk}}}^{(0)} - GQ^{\gamma_{\gamma_{jk}}}^{(0)} \right) \left\{ d_{jk} \left( \beta_0 - GQ^{\gamma_{\gamma_{jk}}}^{(0)} \right) + GQ^{\gamma_{\gamma_{jk}}} \right\} \quad (116)$$

$$GQ^{\gamma_{\gamma_{jk}}}^{ND(1)} = \left( GQ^{\gamma_{\gamma_{jk}}}^{(0)} - GQ^{\gamma_{\gamma_{jk}}}^{(0)} \right) \left\{ d_{jk} \left( \beta_0 - GQ^{\gamma_{\gamma_{jk}}}^{(0)} \right) + GQ^{\gamma_{\gamma_{jk}}} \right\} \quad (117)$$

The matrices for the two-loop anomalous dimensions and evolution kernels.
Here the leading order anomalous dimensions of conformal operators read [65, 66, 67, 47]

\[
QQ_{\gamma_j}^{(0)} = -C_F \left( \frac{2}{(j+1)(j+2)} - 4\psi(j+2) + 4\psi(1) \right) \tag{118}
\]

\[
QG_{\gamma_j}^{(0)} = -\frac{24N_f T_F}{j(j+1)(j+2)(j+3)} \times \begin{cases} 
  j^2 + 3j + 4, & \text{for even parity} \\
  j(j+3), & \text{for odd parity}
\end{cases} \tag{119}
\]

\[
GQ_{\gamma_j}^{(0)} = \frac{-C_F}{3(j+1)(j+2)} \times \begin{cases} 
  j^2 + 3j + 4, & \text{for even parity} \\
  j(j+3), & \text{for odd parity}
\end{cases} \tag{120}
\]

\[
GG_{\gamma_j}^{(0)} = -C_A \left( 4\psi(j+2) + 4\psi(1) - \frac{\beta_0}{C_A} \right) \\
\quad - \frac{8C_A}{j(j+1)(j+2)(j+3)} \times \begin{cases} 
  j^2 + 3j + 3, & \text{for even parity} \\
  j(j+3), & \text{for odd parity}
\end{cases} \tag{121}
\]

and for the transversity sector [51, 68]

\[
QQ_{\gamma_j}^{(0)} = -C_F \left( 3 - 4\psi(j+2) + 4\psi(1) \right). \tag{122}
\]

Note that Eqs. (118-121) fulfill the Dokshitzer SUSY relation [4, 5] (here \( C_A = C_F = 2T_F \))

\[
QQ_{\gamma_j}^{(0)} + \frac{6}{j} QG_{\gamma_j}^{(0)} = \frac{j}{6} QQ_{\gamma_j}^{(0)} + QG_{\gamma_j}^{(0)}. \tag{123}
\]

The prefactors \( j/6 \ (6/j) \) result from the conventional normalization of the Gegenbauer polynomials.

When translated into the language of the evolution kernels their off-diagonal parts in NLO read:

\[
QQ V_{\text{ND}(1)} = -(\mathcal{I} - \mathcal{D}) \left\{ QQ \hat{V} \otimes \left( QQ V^{(0)} + \frac{\beta_0}{2} I \right) \right\} \tag{124}
\]

\[
+ \left\{ QQ g \otimes QQ V^{(0)} - QQ V^{(0)} \otimes QQ g + QQ \hat{V} \otimes QQ V^{(0)} - QQ V^{(0)} \otimes QQ g \right\},
\]

\[
QG V_{\text{ND}(1)} = -(\mathcal{I} - \mathcal{D}) \left\{ QG \hat{V} \otimes \left( QG V^{(0)} + \frac{\beta_0}{2} I \right) \right\} \tag{125}
\]

\[
+ \left\{ QG \hat{V} \otimes QG V^{(0)} + QG g \otimes QG V^{(0)} - QG V^{(0)} \otimes QG g \right\},
\]

\[
GQ V_{\text{ND}(1)} = -(\mathcal{I} - \mathcal{D}) \left\{ GQ \hat{V} \otimes \left( GQ V^{(0)} + \frac{\beta_0}{2} I \right) \right\} \tag{126}
\]

\[
+ \left\{ GQ \hat{V} \otimes GQ V^{(0)} - GQ V^{(0)} \otimes GQ g + GQ g \otimes GQ V^{(0)} - GQ V^{(0)} \otimes GQ g \right\},
\]

\[
GG V_{\text{ND}(1)} = -(\mathcal{I} - \mathcal{D}) \left\{ GG \hat{V} \otimes \left( GG V^{(0)} + \frac{\beta_0}{2} I \right) \right\} \tag{127}
\]

\[
+ \left\{ GG \hat{V} \otimes GG V^{(0)} - GG V^{(0)} \otimes GG g + GG \hat{V} \otimes GG V^{(0)} + GG g \otimes GG V^{(0)} \right\}.
\]
Here we use a shorthand $\otimes = \int_0^1 dz$ so that for any two test functions $\tau_1 \otimes \tau_2 = \int_0^1 dz \tau_1(x,z) \tau_2(z,y)$.

As usual, the $(I - D)$-projector extracts the off-diagonal matrix elements of any function in the basis of the Gegenbauer polynomials [17].

Let us mention that the $\gamma_5$-problem which might arise in dimensional regularization does not show up in the off-diagonal elements in NLO. This follows from the fact that the finite renormalization constant which restores the chiral invariance broken in the t' Hooft-Veltman-Breitenlohner-Maison scheme is diagonal in the basis of the Gegenbauer polynomials and thus cannot affect the results given above [19].

The normalization of the Gegenbauer moments is defined as follows

$$\int_0^1 dx C_j^{\nu(A)} (2x - 1)^{AB} K(x, y) = \sum_{k=0}^j AB K_{jk} C_k^{\nu(B)} (2y - 1),$$

so that we have the correspondence $K = V$ and $K_{jk} = -\frac{1}{2} \gamma_{jk}$, $K = (I - D)g$ and $K_{jk} = \theta_{j-2,k} g_{jk}$, $K = (I - D)V$ and $K_{jk} = \theta_{j-2,k}(AB^{\gamma(0)}_j - AB^{\gamma(0)}_k) d_{jk}$. The leading order ER-BL kernels have the following form

$$AB V^{(0)}(x, y) = \theta(y - x)^{AB} F(x, y) \pm \left( x \rightarrow \bar{x}, \quad y \rightarrow \bar{y} \right),$$

with the following kernels for the parity-even [10, 11, 65, 16, 15, 64, 47] and -odd [67, 70, 16, 15, 64, 47] channels

$$QQ F(x, y) = C_F \frac{x}{y} \left[ 1 + \frac{1}{(y - x)_+} + \frac{3}{2} \delta(x - y) \right],$$

$$QG F(x, y) = 2N_f T_F \frac{x}{y^2 y} \left\{ 2x - y - 1, \quad \text{for even parity} \right\} \left\{ -\bar{y}, \quad \text{for odd parity} \right\},$$

$$GQ F(x, y) = C_F \frac{x}{y} \left\{ 2y - x, \quad \text{for even parity} \right\} x, \quad \text{for odd parity},$$

$$GG F(x, y) = C_A \frac{x^2}{y^2} \left[ \frac{1}{(y - x)_+} - \frac{1}{2} C_A \delta(x - y) \right] + 2C_A \frac{x^2}{y^2} \left\{ \bar{x} + y(1 + 2\bar{x}), \quad \text{for even parity} \right\} \left\{ 1, \quad \text{for odd parity} \right\}.$$  

While for the chiral odd sector we have [41] (see also [18])

$$QQ F(x, y) = C_F \frac{x}{y} \left[ \frac{1}{(y - x)_+} - \frac{1}{2} \delta(x - y) \right].$$

For the dotted kernels we get [17]

$$AB \dot{V}(x, y) = \theta(y - x) \left[ AB \dot{F}(x, y) \ln \frac{x}{y} + \Delta AB \dot{F}(x, y) \right] \pm \left( x \rightarrow \bar{x}, \quad y \rightarrow \bar{y} \right),$$

for $\{ A = B, \quad A \neq B \}$. (135)

Note that the terms with $\delta$-function are understood in the following way $\delta(x-y) \left[ \theta(y - x) + \theta(x - y) \right] = \delta(x-y).$
Figure 3: This diagram generates the $N_f$-dependent part of the term $\beta_0^{\text{GG}}(x,y)$ in the two-loop ER-BL kernel.

with

$$
\Delta^{GQ}\hat{F}^V = 2C_F [x \ln y - \bar{x} \ln \bar{x}], \quad \Delta^{QG}\hat{F}^V = 4T_F N_f \frac{1}{yy} [x \ln y - \bar{x} \ln \bar{x}],
$$

(136)

$$
\Delta^{GG}\hat{F}^V = 2C_A \frac{x^2}{y^2} (y-x).
$$

(137)

In all other cases $\Delta^{AB}\hat{F}(x,y) = 0$.

A consistency check for the $\beta_0$-dependent part of the mixed $GQ$ and $QG$ channels was given in Ref. [47] by an explicit calculation of the simplest two-loop diagrams with fermion bubble insertion. Here we mention that a similar analysis can be easily done for the pure gluonic sector. First we should mention that there is no net $N_f$-dependence of the off-diagonal kernel $GGV^{ND(1)}(x,y)$. However, one can argue that among other graphs the diagram in Fig. 3 produces the $N_f$-part of the complete QCD $\beta$-function in Eq. (127). Since we know that the dotted kernel for the parity odd sector, which arises from the logarithmic modification of the leading order kernel, is correct, we can just consider the difference between the vector and axial vector channels. Then, neglecting in the expression for diagram in Fig. 3 the piece which comes, after subtraction of subdivergencies from the $O(\epsilon)$-effects, (the latter generate only diagonal contributions to the kernel), we get the following result for the light-cone position dotted kernel:

$$
GG\hat{k}^{V-A}(y, z) = 12C_A y z \ln(1 - y - z).
$$

(138)

This result is quite obvious, since the only effect of the fermion bubble insertion into the gluon line is the shift of the power of the denominator by the amount $\epsilon$. Due to the fact that the weight factor associated with this line is $(1 - y - z)^{-\epsilon}$, the first non-trivial term in the Taylor expansion gives us the result (138), where the factor $12C_A y z$ is just the difference between the leading order gluonic kernels (86,87). By Fourier transformation to the momentum space we easily obtain

$$
GG\hat{V}^{V-A}(x, y) = \theta(y-x) \left[ \Delta^{GQ}\hat{F}^V(x,y) + (GGF^V(x,y) - GGF^A(x,y)) \left( \ln \frac{x}{y} \frac{x - \frac{11}{6}}{y - \frac{11}{6}} \right) \right] + \left( x \rightarrow \bar{x} \right).
$$

(139)
Getting rid of the remaining diagonal piece, we obtain the result displayed in Eqs. (135,136). It is a trivial task to check that the moments of this kernel is equal to \((\GG_{\gamma}^{(0)})_j - \GG_{\gamma}^{(0)}) d_{jk}.

The last but not the least ingredients for the off-diagonal part of the two-loop evolution kernels are the \(g\)-functions, which are given by the formulae

\[
\begin{align*}
Q^Q g(x, y) &= -C_F \left[ \theta(y - x) \ln \left( \frac{1 - \frac{x}{y}}{y - x} \right) + \left( x \rightarrow \bar{x}, y \rightarrow \bar{y} \right) \right], \\
Q^G g(x, y) &= 0, \\
G^Q g(x, y) &= -C_F \left[ \theta(y - x) \ln \left( 1 - \frac{x}{y} \right) - \left( x \rightarrow \bar{x}, y \rightarrow \bar{y} \right) \right], \\
G^G g(x, y) &= C_A C_F Q^Q g(x, y).
\end{align*}
\]

These results were obtained by solving the second order differential equations for the \(QQ\) and \(GG\) channels and by explicit summation of the infinite series of the Gegenbauer polynomials for the \(GQ\) one.

8 Solution of the two-loop evolution equations.

So far we have determined the matrix elements of the anomalous dimension matrix of the conformal operators in NLO. In order to solve the renormalization group equation one should determine its eigenvectors (the eigenvalues are given by the diagonal elements and coincide with known forward anomalous dimensions). This section is devoted to the solution of this problem. It will be shown here that they are determined again by the special conformal anomaly matrix introduced previously.

Beyond leading order one has to perform an additional shuffling of operators in order to get the eigenstates of the renormalization group equation. Therefore, let us consider the set of operators \(\tilde{O}_{jl}\)

\[
[\mathcal{O}_{jl}] = \sum_{k=0}^{j} \hat{B}_{jk}(g)[\tilde{\mathcal{O}}_{kl}],
\]

where the \(\hat{B}\)-matrix defines the rotation to the diagonal basis. This new set of operators satisfies the renormalization group equation

\[
\mu \frac{d}{d\mu} [\tilde{O}_{jl}] = -\tilde{\gamma}_j^{D} [\tilde{O}_{jl}], \quad \text{and} \quad \tilde{\gamma}_j^{D} = \begin{pmatrix} QQ_{\gamma}^{D} & GQ_{\gamma}^{D} \\ GQ_{\gamma}^{D} & GG_{\gamma}^{D} \end{pmatrix},
\]

with the following solution

\[
[\tilde{O}_{jl}(\mu^2)] = \mathcal{T} \exp \left\{ -\frac{1}{2} \int_{\mu_0^2}^{\mu^2} \frac{d\tau}{\tau} \tilde{\gamma}_j^{D}(\alpha_s(\tau)) \right\} [\tilde{O}_{jl}(\mu_0^2)].
\]

Since the matrices \(\tilde{\gamma}_j^{D}\) do not commute with each other, we have introduced the \(\mathcal{T}\)-ordered exponential.
Substituting Eq. (141) into the conformal spin expansion

$$\phi(x) = \sum_{j=0}^{\infty} \phi_j(x) \langle h'|[\mathcal{O}_{jj}]|h \rangle,$$  \hspace{1cm} (144)

for the wave function \(\phi(x)\) which is defined as a two-dimensional vector

$$\phi(x) = \begin{pmatrix} Q \phi(x) \\ G \phi(x) \end{pmatrix},$$  \hspace{1cm} (145)

and with the conformal waves being Gegenbauer polynomials

$$\phi_j(x) = \begin{pmatrix} w(x|\frac{3}{2})/N_j(\frac{3}{2})C_{\frac{3}{2}}^j(2x-1) \\ w(x|\frac{5}{2})/N_{j-1}(\frac{5}{2})C_{\frac{5}{2}}^{j-1}(2x-1) \end{pmatrix},$$  \hspace{1cm} (146)

we obtain the expansion in terms of the eigenfunctions \(\phi_j(x|\alpha_s)\) of the ER-BL evolution equation beyond leading order

$$\phi(x) = \sum_{j=0}^{\infty} \phi_j(x|\alpha_s) \langle h'|[\tilde{\mathcal{O}}_{jj}]|h \rangle, \quad \text{with} \quad \phi_j(x|\alpha_s) = \sum_{k=j}^{\infty} \phi_k(x) \hat{B}_{kj}(g).$$  \hspace{1cm} (147)

As it is seen the eigenfunctions are generalized to non-polynomial functions and corrections to the leading order partial waves (146) are determined entirely by the \(\hat{B}\)-matrix which satisfies the following differential equation:

$$\beta(g) \frac{\partial}{\partial g} \hat{B}(g) + [\hat{\gamma}^D, \hat{B}(g)]_- + \hat{\gamma}^{ND} \hat{B}(g) = 0.$$  \hspace{1cm} (148)

As will be shown in the following two sections, the formalism we have described allows to find the corrections to the eigenfunctions analytically, since the \(\hat{B}\)-matrix is defined in terms of the conformal anomaly \(\hat{\gamma}^c\) and the \(\beta\)-function. Below we treat two different cases separately: the hypothetical conformal limit and the case with running coupling constant.

\subsection*{8.1 \(\beta(g) = 0\).}

As we have observed the problem of finding the corrections to the eigenfunctions is reduced to the evaluation of the \(\hat{B}\)-matrix which satisfies the first order differential equation (148). In the conformal limit of the theory it simplifies to

$$[\hat{\gamma}^D, \hat{B}(g)]_- + \hat{\gamma}^{ND} \hat{B}(g) = 0.$$  \hspace{1cm} (149)

We can construct a recursive perturbative solution of this equation, \(\hat{B} = \sum_{\ell=0} \hat{B}^{(\ell)}\). Since in leading order the operators with different conformal spin \(j\) do not mix with each other, it provides
us with condition \(\hat{B}^{(0)} = 1\). Combining these results give us an inhomogeneous equation for \(\hat{B}^{(\ell)}\) with the source \(\hat{\gamma}^{\text{ND}} \hat{B}^{(\ell-1)}\) treated as perturbation

\[
[\hat{\gamma}^D, \hat{B}^{(\ell)}(g)]_-(g) = -\hat{\gamma}^{\text{ND}} \hat{B}^{(\ell-1)}(g), \quad (\ell \geq 1).
\]

(150)

For the first non-trivial term \(\hat{B}^{(1)}\), which is the only one we need to the accuracy we are limited for, we have the following solution

\[
\hat{B}^{(1)}_{jk} = -\frac{1}{a(j, k)} \hat{\gamma}^c_{jk},
\]

(151)

where on the RHS of Eq. (150) we insert the expression for \(\hat{\gamma}^{\text{ND}}\) in terms of the special conformal anomaly matrix. Thus, the new two-loop eigenfunctions read

\[
\phi_j(x|\alpha_s) = \sum_{k=j}^{\infty} \phi_k(x) \left\{ 1_{jk} - \frac{1}{a(k, j)} \hat{\gamma}^c_{kj} \right\}.
\]

(152)

This result can be conveniently written in the form of a convolution, so that we have

\[
\left( \begin{array}{c} Q_{\phi_j} \\ G_{\phi_j} \end{array} \right)(x|\alpha_s) = \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \delta(x - y) + \frac{\alpha_s}{2\pi} \left( \begin{array}{cc} QQ_{\Phi} & QG_{\Phi} \\ GG_{\Phi} & GG_{\Phi} \end{array} \right)(x, y) \otimes \left( \begin{array}{c} Q_{\phi_j} \\ G_{\phi_j} \end{array} \right)(y).
\]

(153)

Here the matrix-valued function \(\Phi(x, y)\) is expressed in terms of kernels derived in the preceding sections

\[
\Phi(x, y) = -(I - D) \left\{ S(x, z) \otimes V^{(0)}(z, y) + g(x, y) \right\},
\]

(154)

which thus define the complete set of corrections to the eigenfunctions of NLO evolution equation. Here the \(S\)-operator generates the shift in the index of the Gegenbauer polynomials:

\[
S(x, y) \otimes [yy]^{\nu-\frac{1}{2}} C_j^\nu(2y - 1) = \frac{d}{d\rho}\bigg|_{\rho=0} [x\bar{x}]^{\nu-\frac{1}{2}+\rho} C_j^{\nu+\rho}(2x - 1).
\]

(155)

We can also recover the dependence on \(\beta_0\) which is related to the shift of the index of Gegenbauer polynomials and not to the effects of a running \(\alpha_s\). From the definition of the conformal anomaly matrix it follows that this reduces to a mere substitution

\[
V^{(0)}(x, y) \Rightarrow V^{(0)}(x, y) + \frac{\beta_0}{2} \delta(x - y).
\]

(156)

Note that the results for the \(QQ\) and \(QG\) sectors were used by us previously for the evaluation of NLO corrections to the amplitudes of the non-forward processes making use of the conformal covariant operator product expansion [51, 52].

Let us now address the case with \(\beta(g) \neq 0\).
8.2 $\beta(g) \neq 0$.

In the case of a running coupling constant the solution of the corresponding differential equation (148) is more complicated. First, we should specify the boundary condition for $\hat{B}(g)$. Note that in the previous case of a fixed coupling we are naturally led to a scheme with constant $\hat{B}$. The minimization of the radiative corrections corresponds to the choice $\hat{B}(g_0) = 1$ that means the absence of the former at the reference point $\mu_0$ ($g_0 = g(\mu_0^2)$). The most important advantage of this requirement is that contrary to our previous discussion the initial condition for the solution of the RG equation (143) can be easily determined by composing ordinary Gegenbauer moments of the initial wave function since

$$ [\mathcal{O}_{jl}(\mu_0)] = [\bar{\mathcal{O}}_{jl}(\mu_0)]. $$

To the required two-loop accuracy we can write the equation for the recursive functions $\hat{B}^{(1)}_{jk}(g) = \hat{1}_{jk} + \hat{B}^{(1)}_{jk}(g)$

$$ \beta(g) \frac{\partial}{\partial g} \hat{B}^{(1)}_{jk}(g) + [\gamma^D, \hat{B}^{(1)}_{jk}(g)] + \gamma^{ND} = 0. $$

The solution can be written in the form

$$ \hat{B}^{(1)}_{jk}(g) = -G \int_{g_0}^{g} \frac{dg'}{\beta(g')} \exp \left\{ - \int_{g_0}^{g} \frac{dg''}{\beta(g'')} \gamma^D_{j}(g'') \right\} \gamma^{ND}_{jk}(g') \exp \left\{ - \int_{g}^{g'} \frac{dg''}{\beta(g'')} \gamma^D_{k}(g'') \right\}, $$

where we have introduced $G$-ordering along the integration path.

Recapitulating the results derived here we have found the analytical solution of the two-loop ER-BL evolution equation and thus can study explicitly the effects of NLO corrections to the wave functions as well as to the non-forward parton distribution functions.

9 Conclusion.

To summarize, in this paper we have generalized the formalism, developed previously for the evaluation of the two-loop corrections to the non-diagonal part of the singlet exclusive evolution kernels in an Abelian theory, to the QCD case. We stress once more that the diagonal entries are determined entirely by the known DGLAP splitting functions. An enormous simplification of the calculations we have presently performed is due to the fact that the $\mathcal{O}(\alpha_s^2)$-corrections to the evolution kernels are obtained from the evaluation of the one-loop renormalization matrices of the conformal operators. To obtain these we have used specific Feynman rules which result from the study of the special conformal Ward identities. The latter control the way how the conformal symmetry is broken in gauge theories. We would like to point out that our results can be used as a guide for a simple convolution-type representation [25] of the full evolution kernels calculated.
in the direct way. Moreover, numerical simulations previously done in the $QQ$-channel [22] can serve as a consistency check.

As a by-product of our analysis we gained some insight into the structure of the counterterms for the product of two renormalized composite operators, a problem which has not been solved so far. Evidently, the complete solution of this task deserves further study in order to put it on the same grounds as Joglekar-Lee renormalization theory of composite operators [57].

We have observed universality of the LO special conformal anomalies in the sense that they do not depend on parity and chirality but only on the particles involved. The difference in the off-diagonal elements of the NLO anomalous dimension matrices arises as a result of different dilatation anomalies, i.e. LO anomalous dimensions of the conformal operators.

Therefore, at present stage we have all the necessary perturbative input for explicit studies of evolution effects for the wave function beyond leading order in the flavour singlet channel. However, one should mention that convergence of the partial wave expansion in particular regions of the phase space is numerically difficult to achieve. A study of the magnitude of the effects due to two-loop evolution for the non-singlet non-forward distribution functions will be addressed by us elsewhere [71].

Moreover, now it is possible to perform a complete NLO analysis for deeply virtual Compton scattering in the leading twist-2 approximation, since the one-loop coefficient functions were determined earlier [72, 51, 73, 52, 74]. One can also include the effects of non-leading twist power corrections which were already estimated by us using renormalon based techniques [52].

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A Feynman rules for the operator insertions.

In this appendix we list the Feynman rules for the operator insertions which enter into the special conformal Ward identities as well as the rules for the light-cone string operators. The latter when sandwiched between appropriate hadronic states define the non-forward distributions or wave functions. Namely, these relations read ($\zeta \equiv (p - p')_+$)

$$\langle h'|Q^{F}(\kappa_1, \kappa_2)|h\rangle = \int dx \left[ e^{-i\kappa_1 x - i\kappa_2 (\zeta - x)} \mp e^{-i\kappa_2 x - i\kappa_1 (\zeta - x)} \right] Q^{F}(x, \zeta), \quad (A.1)$$
Figure 4: Feynman rules for the quark non-local string operator.

Figure 5: Feynman rules for the gluon operator insertions. The grey blob stands either for the non-local string operator (⊗), or the $i[O_A]$-vertex (◦), or the usual vertices from the QCD Lagrangian (●).

\begin{equation}
\langle h' | G O^\Gamma(k_1, k_2) | h \rangle = \frac{1}{2} \int dx \left[ e^{-i\kappa_1 x - i\kappa_2 (\zeta - x)} \pm e^{-i\kappa_2 x - i\kappa_1 (\zeta - x)} \right] G O^\Gamma(x, \zeta), \tag{A.2}
\end{equation}

where the non-local operators are defined in Eqs. (71,72). When $\zeta = 1$ and one of the hadronic states is replaced by the vacuum the $O$’s coincide with usual singlet wave functions $Q^O(x, \zeta = 1) = Q^\phi(x)$ and $G^O(x, \zeta = 1) = G^\phi(x)$, which obey the evolution equation (7).

For every gluon and quark line we associate the propagators

\begin{equation}
(-i)D^{ab}_{\mu\nu}(k) = -i\delta^{ab} \frac{k_{\mu}k_{\nu}}{k^2 + i0}, \quad \text{and} \quad iS(k) = \frac{i}{k^2 + i0}, \tag{A.3}
\end{equation}

respectively. The Feynman rules for the vertices are given in the next subsections. The resulting expression, composed out of propagators and vertices should be integrated with respect to the momentum of every internal line, i.e. multiplied by the factor

\begin{equation}
\int \prod_{\ell} \frac{d^d k_\ell}{(2\pi)^d}. \tag{A.4}
\end{equation}
A.1 Feynman rules for $Q\mathcal{O}(\kappa_1, \kappa_2)$.

For the quark string operator we have the following rules

$$\mathcal{O} = \Gamma \left[ e^{-i\kappa_1 k_1^+ - i\kappa_2 k_2^+} \mp e^{-i\kappa_1 k_2^+ - i\kappa_2 k_1^+} \right],$$  \hspace{1cm} (A.5)

$$g\mathcal{O}_\mu^a = -g t^a n_\mu \Gamma \left[ e^{-i\kappa_1 k_1^+ - i\kappa_2 k_2^+} \pm e^{-i\kappa_1 k_2^+ - i\kappa_2 k_1^+} \right] \frac{e^{-i\kappa_2 k_3^+} - e^{-i\kappa_1 k_3^+}}{k_3^+}, \hspace{1cm} (A.6)$$

which are given diagrammatically in Fig. 4. Here the upper sign in the square brackets stands for $\Gamma = \gamma_+ + \sigma_+ \gamma_5$ and the lower sign for $\gamma_+ \gamma_5$.

A.2 Feynman rules for $G\mathcal{O}(\kappa_1, \kappa_2)$.

The gluon non-local operator can be decomposed as $G\mathcal{O}(\kappa_1, \kappa_2) = G\mathcal{O}_A(\kappa_1, \kappa_2) + G\mathcal{O}_{NA}(\kappa_1, \kappa_2) + G\mathcal{O}_\Phi(\kappa_1, \kappa_2)$, where the subscripts designate the origin of the corresponding contributions, namely, $(N)A$: (non-)Abelian part of the field strength tensor; $\Phi$: path ordered exponential. In one-loop approximation the only Feynman rules we need are the following (crossed circles instead of grey blobs in the diagrams of Fig. 4)

$$A\mathcal{O}^{ab}_{\mu\nu} = \delta^{ab} \left\{ \begin{array}{c} \frac{g_{\alpha\beta}}{i\epsilon_{\alpha\beta-+}} \\ f_{+\beta, \mu}(k_1) f_{+\alpha, \nu}(k_2) \left[ e^{-i\kappa_1 k_1^+ - i\kappa_2 k_2^+} \pm e^{-i\kappa_1 k_2^+ - i\kappa_2 k_1^+} \right] \end{array} \right\}, \hspace{1cm} (A.7)$$

$$NA\mathcal{O}^{abc}_{\mu\rho} = ig f^{abc} \left\{ \begin{array}{c} \frac{g_{\alpha\beta}}{i\epsilon_{\alpha\beta-+}} \\ 1 + n_\mu \left[ g_{\beta\nu} f_{+\alpha, \rho}(k_3) e^{-i\kappa_1 k_1^+} - g_{\alpha\nu} f_{+\beta, \rho}(k_2) e^{-i\kappa_2 k_1^+} \right] e^{-i\kappa_1 k_2^+ - i\kappa_2 k_3^+} \\ + n_\nu \left[ g_{\beta\rho} f_{+\alpha, \nu}(k_1) e^{-i\kappa_1 k_2^+} - g_{\alpha\rho} f_{+\beta, \nu}(k_3) e^{-i\kappa_2 k_2^+} \right] e^{-i\kappa_1 k_3^+ - i\kappa_2 k_1^+} \\ + n_\rho \left[ g_{\beta\nu} f_{+\alpha, \nu}(k_2) e^{-i\kappa_1 k_3^+} - g_{\alpha\nu} f_{+\beta, \nu}(k_1) e^{-i\kappa_2 k_3^+} \right] e^{-i\kappa_1 k_1^+ - i\kappa_2 k_2^+} \mp (\kappa_1 \leftrightarrow \kappa_2) \end{array} \right\}, \hspace{1cm} (A.8)$$

$$g\mathcal{O}^{abc}_{\mu\nu\rho} = ig f^{abc} \left\{ \begin{array}{c} \frac{g_{\alpha\beta}}{i\epsilon_{\alpha\beta-+}} \\ 1 + n_\mu \left[ f_{+\alpha, \nu}(k_2) f_{+\beta, \rho}(k_3) \left[ e^{-i\kappa_1 k_1^+ - i\kappa_2 k_2^+} \mp e^{-i\kappa_1 k_2^+ - i\kappa_2 k_3^+} \right] \frac{e^{-i\kappa_2 k_1^+} - e^{-i\kappa_1 k_1^+}}{k_1^+} \right] \\ + n_\nu \left[ f_{+\alpha, \mu}(k_3) f_{+\beta, \mu}(k_1) \left[ e^{-i\kappa_1 k_1^+ - i\kappa_2 k_3^+} \mp e^{-i\kappa_1 k_3^+ - i\kappa_2 k_1^+} \right] \frac{e^{-i\kappa_2 k_1^+} - e^{-i\kappa_1 k_2^+}}{k_2^+} \right] \\ + n_\rho \left[ f_{+\alpha, \mu}(k_1) f_{+\beta, \nu}(k_2) \left[ e^{-i\kappa_1 k_2^+ - i\kappa_2 k_1^+} \mp e^{-i\kappa_1 k_1^+ - i\kappa_2 k_2^+} \right] \frac{e^{-i\kappa_2 k_3^+} - e^{-i\kappa_1 k_1^+}}{k_3^+} \right] \end{array} \right\}. \hspace{1cm} (A.9)$$

Here and below $f_{\alpha\beta, \mu}(k) \varepsilon_\mu = k_\alpha \varepsilon_\beta - k_\beta \varepsilon_\alpha$ is the Abelian part of the gluon field strength tensor.
A.3  Feynman rules for $i[\mathcal{O}_A^-]$.

For the operator insertion $i[\mathcal{O}_A^-]$ which appears in the special conformal Ward identity we have (empty blobs in the Feynman rules)

$$2\tilde{\mathcal{V}}_{\mu\nu}^{ab} = -i\delta^{ab} f_{\alpha\beta;\mu}(k_1) f_{\alpha\beta;\nu}(k_2) (2\pi)^4 2 i \partial_- \delta^{(4)} (k_1 + k_2),$$  \hspace{1cm} (A.10)

$$3\tilde{\mathcal{V}}_{\mu\nu\rho}^{abc} = -2g f^{abc} \{(k_1 - k_2)_{\rho} g_{\mu\nu} + (k_2 - k_3)_{\mu} g_{\nu\rho} + (k_3 - k_1)_{\nu} g_{\mu\rho}\} (2\pi)^4 2 i \partial_- \delta^{(4)} (k_1 + k_2 + k_3).$$  \hspace{1cm} (A.11)

$$4\tilde{\mathcal{V}}_{\mu\nu\rho\sigma}^{abcd} = 2i g^2 \{ f^{ab} f^{cde} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) + f^{ace} f^{bde} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) + f^{ade} f^{bce} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) \} (2\pi)^4 2 i \partial_- \delta^{(4)} (k_1 + k_2 + k_3 + k_4).$$  \hspace{1cm} (A.12)

It is instructive to compare them with the usual Feynman rules for the triple and quartic interaction vertices (full points in the graphs):

$$3\mathcal{V}_{\mu\nu\rho}^{abc} = g f^{abc} \{(k_1 - k_2)_{\rho} g_{\mu\nu} + (k_2 - k_3)_{\mu} g_{\nu\rho} + (k_3 - k_1)_{\nu} g_{\mu\rho}\} (2\pi)^4 \delta^{(4)} (k_1 + k_2 + k_3),$$  \hspace{1cm} (A.13)

$$4\mathcal{V}_{\mu\nu\rho\sigma}^{abcd} = -ig^2 \{ f^{ab} f^{cde} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) + f^{ace} f^{bde} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) + f^{ade} f^{bce} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) \} (2\pi)^4 \delta^{(4)} (k_1 + k_2 + k_3 + k_4).$$  \hspace{1cm} (A.14)

A.4  Feynman rules for $\int d^d x \ x^{-B^a_{\mu}(x)} \frac{\delta}{\delta B^a_{\mu}(x)} [G\mathcal{O}]$.

Finally, in the considered approximation we need only the part of $\int d^d x \ x^{-B^a_{\mu}(x)} \frac{\delta}{\delta B^a_{\mu}(x)} [G\mathcal{O}]$ which survives for $g = 0$

$$\int d^d x \ x^{-B^a_{\mu}(x)} \frac{\delta}{\delta B^a_{\mu}(x)} [G\mathcal{O}(\kappa_1, \kappa_2)] \big|_{g=0} = (\kappa_1 + \kappa_2) F^{a}_{\mu}(\kappa_2 n) \left\{ \frac{g_{\mu\nu}}{i\epsilon_{\mu\nu-+}} \right\} F^{a}_{\nu+}(\kappa_1 n)$$  \hspace{1cm} (A.15)

$$+ (B^a_{\mu}(\kappa_2 n) - n^* B^a_{\mu}(\kappa_2 n)) \left\{ \frac{g_{\mu\nu}}{i\epsilon_{\mu\nu-+}} \right\} F^{a}_{\nu+}(\kappa_1 n) - F^{a}_{\mu}(\kappa_2 n) \left\{ \frac{g_{\mu\nu}}{i\epsilon_{\mu\nu-+}} \right\} (B^a_{\nu}(\kappa_1 n) - n^* B^a_{\nu}(\kappa_1 n)).$$

Thus the Feynman rules for this operator are given by $(\kappa_1 + \kappa_2) A^{ab}_{\mu\nu}(k_1, k_2)$ and the following vertex for the second and third term

$$i\delta^{ab} \left\{ \frac{g_{\alpha\beta}}{i\epsilon_{\alpha\beta-+}} \right\} \left\{ (g_{\beta\mu} - n^* n_{\mu}) f_{+\alpha;\nu}(k_2) + (g_{\alpha\nu} - n^* n_{\nu}) f_{+\beta;\mu}(k_1) \right\} e^{-i\kappa_1 k_1 - i\kappa_2 k_2} \pm e^{-i\kappa_1 k_2 - i\kappa_2 k_1}.$$  \hspace{1cm} (A.16)
B Renormalization of string operators.

In this appendix we intend to review the approach for construction of evolution equations for non-local light-cone operators developed in Ref. [63]. Although a lot of results have been obtained so far using the latter, we did not find in the literature any comprehensive review which allows a pedestrian to make these calculations, especially, in the case of non-forward kinematics. To our opinion this machinery is more general than the background field formalism [55], since the former is not tied to any particular gauge and thus can be used for studying the form of gauge-variant counterterms. Next we present the details of the calculations which were cited in the main text.

The momentum integrals which have to be evaluated are of the following form after joining the propagators via Feynman parametrization

$$\int \frac{d^d q}{(2\pi)^d} e^{-iq_+(\kappa_2-\kappa_1)} \frac{P(k_i, x_i|q)}{q^2 - L} n,$$

where $P(k_i, x_i|q)$ is a polynomial function in $q$, $k_i$ and the Feynman parameters $x_i$. The divergencies can be evaluated by expanding the exponential factor in the integrand: $e^{-iq_+(\kappa_2-\kappa_1)} = 1 - iq_+(\kappa_2 - \kappa_1) + \ldots$. Since the denominator depends on $q^2$ we can average with respect to possible rotations of $q$. Due to the light-like character of the vector $n$ only the first few terms survive after integration (we maximally need to expand up to $q^3$). To reduce the result to the conventional form, we have to remove the terms proportional to $(\kappa_2 - \kappa_1)^m$ integrating by parts with respect to the Feynman parameters which play now the rôle of fractions in the light-cone position formalism. Typical integration for a test function $\tau(y, z)$ looks like

$$i(\kappa_2 - \kappa_1)k_1^+ J * \tau(y, z) = J * \left\{ \delta(y)\tau(0, z) + \frac{\partial \tau(y, z)}{\partial y} - \delta(1 - y - z)\tau(y, \bar{y}) \right\},$$

and similarly for terms with $k_2^+$. In the above equation we have introduced the following shorthand notation for the integral

$$J = \int_0^1 dz \int_0^z dy e^{-ik_1^+(\bar{y}\kappa_1 + y\kappa_2) - ik_2^+(\bar{z}\kappa_2 + z\kappa_1)}.$$ 

When triple and double integration by parts is required, it turns out that after the first integration all dangerous terms proportional to $\delta(1 - y - z)$ that might cause a problem vanish identically in all cases.

For the diagrams which originate from the expansion of the phase factor there appear in the integrand terms of the type

$$E(q, k_1, y) = \frac{1}{(q + yk_1^+)} \left[ 1 - e^{-i(\kappa_2 - \kappa_1)(yk_1^+ + q_+)} \right],$$

In fact, we have used two different formalisms for our calculations: the one mentioned above and another technique developed in Ref. [76]. Computer realizations were also implemented with two different languages: FORM and MATHEMATICA, respectively.
Figure 6: One-loop renormalization of the twist-2 gluon operator.

with \( q \) being the momentum of integration and \( y \) one of the Feynman parameters. The best way to treat them is to factorize the \( q \)-dependence using the following trivial identity

\[
E(q, k_1, y) = \frac{1}{y} e^{\frac{q \cdot k_1}{y}} - \frac{1 - e^{-i(\kappa_2 - \kappa_1)y k_{1+}}}{k_{1+}}. \tag{B.4}
\]

Expanding as previously the exponential in power series of \( q^n \), we can easily perform the final momentum integration since only a limited number of lowest order terms contribute due to the light-like character of the vector \( n \).

However, this is not the end of the story since some contributions from the graphs possess apart form desired structure \( k_{1+}k_{2+} \) also \( k_{i+}^2 \). Therefore, it is necessary to get rid of them by reduction \( k_{i+}^2 \rightarrow k_{1+}k_{2+} \). The general structure of these terms in the sum of all diagrams is of the form (e.g. the coefficient in front of \( k_{1+}^2 \))

\[
\tau_1(y) + \tau_2(y) \delta(z) + \tau_3(y) \delta(1 - y - z). \tag{B.5}
\]

Thus, integration by parts reduces this expression to the desired form up to terms proportional to \( \delta(1 - x - y) \). We get

\[
k_{1+}^2 \mathcal{J} \ast [\tau_1(y)] = k_{1+}^2 \mathcal{J} \ast \{[\tau_1(y)z + \tau_2(y)] \delta(1 - y - z) - \tau_2(y) \delta(z) \}
+k_{1+}k_{2+} \mathcal{J} \ast \{[\tau_1(y)z + \tau_2(y)] \delta(1 - y - z)
- [\tau_1(0)z + \tau_2(0)] \delta(y) - [\tau_1'(y)z + \tau_2'(y)]\}. \tag{B.6}
\]

Note also that the resulting function

\[
k_{1+}^2 [\tau_1(y)z + \tau_2(y) + \tau_3(y)] \delta(1 - y - z) \tag{B.7}
\]

can be discard safely because of the Bose symmetry properties of the string operators it is convoluted with. Similar equations hold for \( k_{2+}^2 \).
Since our result for the $^2GGK_{-1}$-kernel is expressed in terms of the leading order gluonic evolution kernel we find it useful to give diagram-by-diagram contributions. Namely, straightforward calculations of the graphs displayed\footnote{Of course, there exists a diagram analogous to $(d)$ with composite operator instead of the four-gluon vertex, but it is identically zero in dimensional regularization since there is no mass parameter entering the loop integral.} in Fig. 3 give for parity-odd operators

\[
^2GGK_{(a)}(y, z) = 1 - y - z + 4yz + \frac{1}{2}(1 - y)\delta(z) + \frac{1}{2}(1 - z)\delta(y) - (1 - \xi)\frac{1}{2}\delta(y)\delta(z)
\]
\[
+ \frac{k_{1+}}{k_{2+}}[2y(1 - y) - 1] + \frac{k_{2+}}{k_{1+}}[2z(1 - z) - 1],
\]
\[
(B.8)
\]

\[
^2GGK_{(b+c|N.A)}(y, z) = -(5 + \xi)\frac{1}{4}\delta(y)\delta(z)
\]
\[
+ \frac{1}{2k_{2+}}(1 - y)(2 - y)\delta(z) + \frac{1}{2k_{1+}}(1 - z)(2 - z)\delta(y),
\]
\[
(B.9)
\]

\[
^2GGK_{(b+c|F)}(y, z) = \frac{5}{2}\delta(y)\delta(z) + \frac{1}{2}\left\{y - 3 + 2\left[\frac{1}{y}\right]_{+}\right\}\delta(z) + \frac{1}{2}\left\{z - 3 + 2\left[\frac{1}{z}\right]_{+}\right\}\delta(y),
\]
\[
(B.10)
\]

\[
^2GGK_{(d)}(y, z) = -yz\delta(1 - y - z)
\]
\[
- \frac{1}{2k_{1+}}yz\delta(1 - y - z) - \frac{1}{2k_{2+}}yz\delta(1 - y - z),
\]
\[
(B.11)
\]

where the symmetry factors are included into the kernels. Making the reduction with Eq. (B.6), where \(\tau_1(y) = 2y(1 - y) - 1\) and \(\tau_2(y) = \frac{1}{2}(1 - y)(2 - y)\) (and similarly for the \(y \to z\) contribution), and summing the resulting contributions with renormalization constant of the gluonic fields

\[
^2GGK_R(y, z) = -\gamma_G^{(0)}(\xi)\delta(y)\delta(z),
\]
\[
(B.12)
\]

we come to the well known kernel \((86)\).

The contributions to the parity even kernel $^2GGK_{A}$ from diagrams in Fig. 3 read as follows

\[
^2GGK_{(a+b)}(y, z) = -4i(\kappa_1 + \kappa_2)^2GGK_{(a)}(y, z|\xi) + \mathcal{W}_{(a+b)}(y, z),
\]
\[
(B.13)
\]

\[
^2GGK_{(c)}(y, z) = -2i(\kappa_1 + \kappa_2)^2GGK_{(c)}(y, z|\xi = 0) + \mathcal{W}_{(c)}(y, z),
\]
\[
(B.14)
\]

\[
^2GGK_{(d+c)}(y, z) = 2i(\kappa_1 + \kappa_2)^2GGK_{(d+c)}(y, z|\xi) + \mathcal{W}_{(d+c)}(y, z),
\]
\[
(B.15)
\]

\[
^2GGK_{(j+g)}(y, z) = -2i(\kappa_1 + \kappa_2)\left\{^2GGK_{(b+c)}(y, z|\xi) + ^2GGK_{(b+c)}(y, z|\xi = 0)\right\} + \mathcal{W}_{(j+g)}(y, z),
\]
\[
(B.16)
\]

\[
^2GGK_{(h+i)}(y, z) = 2i(\kappa_1 + \kappa_2)^2GGK_{(a)}(y, z|\xi) + \mathcal{W}_{(h+i)}(y, z),
\]
\[
(B.17)
\]

\[
^2GGK_{(j+k)}(y, z) = -4i(\kappa_1 + \kappa_2)^2GGK_{(d)}(y, z) + \mathcal{W}_{(j+l)}(y, z),
\]
\[
(B.18)
\]

\[
^2GGK_{(l)}(y, z) = 2i(\kappa_1 + \kappa_2)^2GGK_{(d)}(y, z) + \mathcal{W}_{(j)}(y, z).
\]
\[
(B.19)
\]

From these formulae it is clearly seen how the cancellation of gauge dependence in the kernels in front of the factor \((\kappa_1 + \kappa_2)\) occurs. To be sure in the correctness of the calculation of the remaining part \(\mathcal{W}(y, z)\), we have kept gauge fixing parameter to be arbitrary. We have observed that the
cancelation of the gauge dependence occurs again between diagrams with the same structure, similar to the situation for the kernels which accompany the \((\kappa_1 + \kappa_2)\)-factor. Namely, we have

\[
W_{(a+b+c+h+i)}(y, z) = \frac{1}{k_{2+}} \left\{ 2\delta(y) - (1 - y)\delta(z) - 2yz\delta(1 - y - z) \right\} \\
+ \frac{1}{k_{1+}} \left\{ 2\delta(z) - (1 - z)\delta(y) - 2yz\delta(1 - y - z) \right\},
\]

(B.20)

\[
W_{(d+e|N.A.)}(y, z) = \frac{1}{k_{2+}} \left\{ 1 - 4y + 2y^2 - \frac{1}{2}(1 + \xi)\delta(y) \right\} \delta(z) \\
+ \frac{1}{k_{1+}} \left\{ 1 - 4z + 2z^2 - \frac{1}{2}(1 + \xi)\delta(z) \right\} \delta(y),
\]

(B.21)

\[
W_{(d+e|\Phi)}(y, z) = \frac{1}{k_{2+}} \left\{ 2z - 3 - 2 \left[ \frac{1}{z} \right]_+ + 2 \left[ \frac{1}{z^2} \right]_+ + 2\delta(z) \right\} \delta(y) \\
+ \frac{1}{k_{1+}} \left\{ 2y - 3 - 2 \left[ \frac{1}{y} \right]_+ + 2 \left[ \frac{1}{y^2} \right]_+ + 2\delta(y) \right\} \delta(z),
\]

(B.22)

\[
W_{(f+g|N.A.)}(y, z) = \frac{1}{k_{2+}} \left\{ 3y - 2y^2 + \frac{\xi}{2}\delta(y) \right\} \delta(z) \\
+ \frac{1}{k_{1+}} \left\{ 3z - 2z^2 + \frac{\xi}{2}\delta(z) \right\} \delta(y),
\]

(B.23)

\[
W_{(f+g|\Phi)}(y, z) = \frac{1}{k_{2+}} \left\{ 1 - 2z + 6 \left[ \frac{1}{z} \right]_+ - 4 \left[ \frac{1}{z^2} \right]_+ \right\} \delta(y) \\
+ \frac{1}{k_{1+}} \left\{ 1 - 2y + 6 \left[ \frac{1}{y} \right]_+ - 4 \left[ \frac{1}{y^2} \right]_+ \right\} \delta(z),
\]

(B.24)

\[
W_{(j+k)}(y, z) = \frac{1}{k_{2+}} \left\{ 4yz - 1 \right\} \delta(1 - y - z) \\
+ \frac{1}{k_{1+}} \left\{ 4yz - 1 \right\} \delta(1 - y - z),
\]

(B.25)

\[
W_{(l)}(y, z) = \frac{1}{k_{2+}} \left\{ 1 - 2yz \right\} \delta(1 - y - z) \\
+ \frac{1}{k_{1+}} \left\{ 1 - 2yz \right\} \delta(1 - y - z).
\]

(B.26)

To simplify the result given by \(W_{(a+b+c+h+i)}(y, z)\), we have used Eq. (B.10) and the following identity

\[
\mathcal{J} \ast (k_{1+} + k_{2+})\delta(1 - y - z) = \mathcal{J} \ast \left\{ k_{1+}\delta(z) + k_{2+}\delta(y) \right\}.
\]

(B.27)

Finally, the kernel which we are interested in is given by

\[
GG\mathcal{K}^w(y, z) = W(y, z) + 2 \left( \frac{1}{k_{1+}} + \frac{1}{k_{2+}} \right) \left( \frac{\beta_0}{2} - \gamma_C^{(0)}(\xi = 0) \right) \delta(y)\delta(z) \\
= \frac{C_A}{k_{2+}} \left\{ 4 \left[ \frac{1}{z} \right]_+ - 2 \left[ \frac{1}{z^2} \right]_+ \right\} \delta(y) + \frac{C_A}{k_{1+}} \left\{ 4 \left[ \frac{1}{y} \right]_+ - 2 \left[ \frac{1}{y^2} \right]_+ \right\} \delta(z),
\]

(B.28)

where we restored the colour factors omitted previously. This kernel arises as a factor convoluted with \(\mathcal{J}\). Of course, it is easy to integrate by parts to get rid of momentum factors \(k_{i+}\) and acquire instead \(i(\kappa_2 - \kappa_1)\). Adding then the contribution with \(\kappa_2 \leftrightarrow \kappa_1\) gives the resulting kernel in front of the Bose symmetrical gluon factor \((\text{A.7})\). However, we will skip this last trivial step since we can obtain the desired momentum fraction \(w\)-function from equations similar to \((\text{B.28})\) with the
substitution $k_{1+} = y$, $k_{2+} = \bar{y}$ and Fourier transformation.

Finally, let us mention that the results of the calculations for the parity odd gluonic sector are the same as given above in Eq. (B.28). The differences in the intermediate steps arise from the fact that the contributions of the type $yz\delta(1 - y - z)$ are absent by simple reason of Bose symmetry of the light-ray operators which are convoluted with this kernel.

\section*{C Properties of hypergeometric functions.}

In this appendix we list all formulae which are necessary for the evaluation of the Gegenbauer moments of the evolution kernels.

The generalized hypergeometric function is defined as infinite series

$$mF_n \left( \begin{array}{c} \alpha_1, \ldots, \alpha_m \\ \beta_1, \ldots, \beta_n \end{array} \mid x \right) = \sum_{\ell=0}^{\infty} \frac{(\alpha_1)_\ell \cdots (\alpha_m)_\ell}{(\beta_1)_\ell \cdots (\beta_n)_\ell} x^\ell, \quad (C.1)$$

where we have introduced the Pochhammer symbol $(\alpha)_\ell = \Gamma(\alpha + \ell)/\Gamma(\alpha)$. The integral representation of the hypergeometric function is (where $1F_0(\alpha|xy) = (1 - xy)^{-\alpha}$)

$$m+1F_{n+1} \left( \begin{array}{c} \alpha_1, \ldots, \alpha_{m+1} \\ \beta_1, \ldots, \beta_{n+1} \end{array} \mid x \right) = \frac{\Gamma(\beta_{n+1})}{\Gamma(\alpha_{m+1})} \oint_0^1 dy y^{\alpha_{m+1}-1} (1 - x y)^{-\beta_{n+1}-1} mF_n \left( \begin{array}{c} \alpha_1, \ldots, \alpha_m \\ \beta_1, \ldots, \beta_n \end{array} \mid xy \right). \quad (C.2)$$

When one of the upper indices $\alpha_i = -j$, where $j \in \mathbb{N}$, the infinite series (C.1) degenerates into a finite polynomial of order $j$. Therefore, the relation between the Gegenbauer polynomials and the hypergeometric function reads

$$C_j^\nu(2x - 1) = (-1)^j \frac{\Gamma(2\nu + j)}{\Gamma(\nu)\Gamma(j + 1)} 2F_1 \left( \begin{array}{c} -j, j + 2\nu \\ \nu + \frac{1}{2} \end{array} \mid x \right). \quad (C.3)$$

In the calculation of the integrals we use the following representation of the Gegenbauer polynomials ($w(x|\nu) = (x \bar{x})^{\nu-\frac{1}{2}}$)

$$\frac{w(x|\nu)}{N_j(\nu)} C_j^\nu(2x - 1) = 2^{2\nu} \frac{\Gamma(\nu)(\nu + j)}{\Gamma(\nu + j + \frac{1}{2})} (-1)^j \frac{d^j}{dx^j} (x \bar{x})^{j+\nu-\frac{1}{2}}. \quad (C.4)$$

Integrating $j$-times by parts we end up with $j$ derivatives acting on the hypergeometric function. This differentiation can be done with the help of the formula

$$\frac{d^j}{dx^j} x^k mF_n \left( \begin{array}{c} \alpha_1, \ldots, \alpha_m \\ \beta_1, \ldots, \beta_n \end{array} \mid x \right) = \frac{(\alpha_1)_j \cdots (\alpha_m)_j! (\beta_1)_j \cdots (\beta_n)_j!}{(\alpha_1-j)_k \cdots (\alpha_m-j)_k! (\beta_1-j)_k \cdots (\beta_n-j)_k!} m+1F_{n+1} \left( \begin{array}{c} \alpha_1 + j - k, \ldots, \alpha_m + j - k, j + 1 \\ \beta_1 + j - k, \ldots, \beta_n + j - k, j - k + 1 \end{array} \mid x \right), \quad \text{for } j - k \geq 0. \quad (C.5)$$

\textsuperscript{12}Do not mix this $y$ variable (which is the momentum fraction in the ER-BL kernel) with $y$ in the light cone position formalism which is integrated out with Fourier transformation.
To simplify the result of integration it is instructive to use a relation between the functions of \((m, n)\) and \((m - 1, n - 1)\) orders

\[
mF_n \left( \begin{array}{c} \alpha_1, \ldots, \alpha_m \\ \beta_1, \ldots, \beta_{n-1}, \alpha_m + 1 \end{array} \middle| x \right) = \frac{\alpha_m}{1 + \alpha_m - \beta_{n-1}} mF_{n-1} \left( \begin{array}{c} \alpha_1, \ldots, \alpha_{m-1} \\ \beta_1, \ldots, \beta_{n-1} \end{array} \middle| x \right) - \frac{\beta_{n-1} - 1}{1 + \alpha_m - \beta_{n-1}} mF_n \left( \begin{array}{c} \alpha_1, \ldots, \alpha_m \\ \beta_1, \ldots, \beta_{n-1} - 1, \alpha_m + 1 \end{array} \middle| x \right). \tag{C.6}
\]

The number of consequent iterations is governed by the condition of fulfilling of the relation

\[1 + \sum \alpha_i = \sum \beta_j.\]

Once being satisfied it allows to apply Saalschütz’s theorem [75]

\[
\begin{align*}
\binom{-j, j + \alpha, \beta}{\gamma, 1 + \alpha + \beta - \gamma} & = \frac{(\gamma - \beta)_j (1 + \alpha - \gamma)_j}{(\gamma)_j (1 + \alpha + \beta - \gamma)_j}.
\end{align*} \tag{C.7}
\]

From this relation follows immediately

\[
\binom{-j, j + \alpha, \beta}{\alpha - 1, 1 + \beta} = \frac{\beta}{\alpha - 1 (\alpha)_j} \left[ (-1)^j + \frac{\alpha - \beta - 1 (\alpha - \beta)_j}{\beta (1 + \beta)_j} \right]. \tag{C.8}
\]

For \(2F_1\) we have

\[
\begin{align*}
\binom{-j, j + \alpha}{\beta} & = (-1)^j \frac{(1 + \alpha - \beta)_j}{(\beta)_j}, \\
\binom{-j + k, j + k + 3}{2k + 4} & = \delta_{jk}. \tag{C.9}
\end{align*}
\]

Since for the regularized functions we get the derivatives of the hypergeometric function \(3F_2\) with respect to an index, we need the following expansion

\[
\binom{-j, j + \alpha, \beta}{1 + \alpha, \beta + \epsilon} = \frac{\Gamma(1 + \alpha)}{\Gamma(1 + \alpha + j)} \left\{ \delta_{j0} + \epsilon \Gamma(j) \left[ \frac{(1 + \alpha - \beta)_j}{(\beta)_j} - (-1)^j \right] \right\} + O(\epsilon^2), \tag{C.10}
\]

which can be derived making use of a fundamental identity for \(3F_2\)-functions [75] and the definition (C.1).

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