Field Equation of Correlation Function of Mass Density Fluctuation for Self-Gravitating Systems

Yang Zhang*, Qing Chen †
Department of Astronomy, CAS Key Laboratory for Researches in Galaxies and Cosmology, University of Science and Technology of China, Hefei, Anhui, 230026, China

Abstract

We study the mass density distribution of Newtonian self-gravitating systems. Modeling the system as a fluid in hydrostatical equilibrium, we obtain from first principle the field equation and its solution of correlation function $\xi(r)$ of the mass density fluctuation itself. We apply this to studies of the large-scale structure of the Universe within a small redshift range.

The equation tells that $\xi(r)$ depends on the point mass $m$ and the Jeans wavelength scale $\lambda_0$, which are different for galaxies and clusters. It explains several longstanding, prominent features of the observed clustering: that the profile of $\xi_{cc}(r)$ of clusters is similar to $\xi_{gg}(r)$ of galaxies but with a higher amplitude and a longer correlation length, and that the correlation length increases with the mean separation between clusters as a universal scaling $r_0 \simeq 0.4d$. Our solution $\xi(r)$ also yields the observed power-law correlation function of galaxies $\xi_{gg}(r) \simeq (r_0/r)^{1.7}$ valid only in a range $1 < r < 10h^{-1}\text{Mpc}$. At larger scales the solution $\xi(r)$ breaks below the power law and goes to zero around $\sim 50h^{-1}\text{Mpc}$, just as the observational data have demonstrated.

With a set of fixed model parameters, the solutions $\xi_{gg}(r)$ for galaxies, the corresponding power spectrum, and $\xi_{cc}(r)$ for clusters, simultaneously, agree with the observational data from the major surveys of galaxies, and of clusters.

Key words.
galaxies:clusters:general, large-scale structure of Universe, gravitation, cosmology:theory

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1 Introduction

It is one of the major goals of modern cosmology to understand the matter distribution in the Universe on large scales. The large-scale structure is determined by self gravity of galaxies and clusters. Since the number of galaxies is enormous, one needs statistics to study their distribution. In this regard, the 2-point correlation functions \( \xi_{gg}(r) \) of galaxies and \( \xi_{cc}(r) \) of clusters serve as a powerful statistical tool \([10, 32, 40]\). It not only provides statistical information, but also contains underlying dynamics mainly due to gravitational force. Therefore, we would like to investigate the correlation functions of self gravitating systems in approximation of hydrostatical equilibrium.

Over the years, various observational surveys have been carried out for galaxies and for clusters, such as the Automatic Plate Measuring (APM) galaxy survey \([27]\), the Two-degree-Field Galaxy Redshift Survey (2dFGRS) \([30]\), Sloan Digital Sky Survey (SDSS) \([1]\), etc. All these surveys suggest that the correlation of galaxies has a power-law form \( \xi_{gg}(r) \propto (r_0/r)^\gamma \) with \( r_0 \sim 5.4h^{-1}\text{Mpc} \) and \( \gamma \sim 1.7 \) in a range \((0.1 \sim 10)h^{-1}\text{Mpc}\) \([21, 32, 37, 40]\). The correlation of clusters is found to be of a similar form: \( \xi_{cc}(r) \sim 20\xi_{gg}(r) \) in a range \((5 \sim 60)h^{-1}\text{Mpc}\), with an amplified magnitude \([4, 26]\). For quasars, the correlation is \( \xi_{qq}(r) \sim 5\xi_{gg}(r) \) \([36]\). Numerical computations have been extensively employed to study clustering of galaxies and of clusters, and significant progresses have been made. To understand physical mechanisms behind the clustering, analytical studies are important. In particular, references \([33, 34]\) used thermodynamics whereby the power-law form of \( \xi_{gg}(r) \) was introduced as modifications to the energy and pressure. Similarly, references \([15, 16]\) used the grand partition function of a self-gravitating gas to study a possible fractal structure of the distribution of galaxies. However, the field equation of \( \xi \) was not given in these studies. In this paper we use units that the speed of light is \( c = 1 \) and the Boltzmann constant is \( k_B = 1 \).

2 Field equation of the 2-pt correlation function of density fluctuations

Galaxies, or clusters, distributed in the Universe can be described as fluids at rest in gravitational fields. This modeling is an approximation since the cosmic expansion is not considered. We apply hydrostatics to systems of galaxies within a small redshift range. For these self-gravitating systems, the field equation of mass density is \([45, 46]\)

\[
\nabla^2\psi - \frac{1}{\psi}(\nabla\psi)^2 + k_J^2\psi^2 + J\psi^2 = 0, \tag{1}
\]

where \( \psi(r) \equiv \rho(r)/\rho_0 \) with \( \rho_0 = mn_0 \) being the mean mass density of the system, \( k_J \equiv \sqrt{4\pi G\rho_0/c_s} \) is the Jeans wavenumber, and \( J \) is a Schwinger type of external source introduced for taking functional derivative \([35]\). The effective Hamiltonian density is

\[
\mathcal{H}(\psi, J) = \frac{1}{2}\left(\frac{\nabla\psi}{\psi}\right)^2 - k_J^2\psi - J\psi. \tag{2}
\]

The generating functional for the correlation functions of \( \psi \) is

\[
Z[J] = \int D\psi e^{-\alpha \int d^3r\mathcal{H}(\psi, J)}, \tag{3}
\]
where $\alpha \equiv c_s^2/4\pi Gm$, $c_s$ is the sound speed and $m$ is the mass of a single particle.

Since the distribution of galaxies, or clusters, can be viewed as fluctuations of the mass density in the Universe, we consider the fluctuation field $\delta \psi(r) \equiv \psi(r) - \langle \psi(r) \rangle$, where the statistical ensemble average is defined as $\langle \psi(r) \rangle = \frac{1}{a b J(r)} \log Z[J] \mid_{J=0}$, and, in our case, $\langle \psi(r) \rangle = \psi_0$ is a constant. The connected $n$-point correlation function of $\delta \psi$ is defined as

$$G^{(n)}(r_1, \ldots, r_n) = \langle \delta \psi(r_1) \ldots \delta \psi(r_n) \rangle = \alpha^{-n-1} \frac{\delta^{n-1}(\psi(r_1))}{\delta J(r_1) \ldots \delta J(r_{n-1})} \mid_{J=0}$$

for $n \geq 2$. One can take $G^{(2)}(r_1, r_2) = G^{(2)}(r_{12})$ for the homogeneous and isotropic Universe. To derive the field equation of $G^{(2)}(r)$ [19], one takes functional derivative of the ensemble average of Eq. 4 with respect to $J(r_1)$. In doing this, $G^{(3)}$ occurs in the equation of $G^{(2)}(r)$ hierarchically. We adopt the Kirkwood-Groth-Peebles ansatz [21, 25] $G^{(3)}(r_1, r_2, r_3) = Q[G^{(2)}(r_{12})G^{(2)}(r_{23}) + G^{(2)}(r_{23})G^{(2)}(r_{31}) + G^{(2)}(r_{31})G^{(2)}(r_{12})]$, where $Q$ is a dimensionless parameter. Then, after a necessary renormalization, we obtain the field equation of the 2-point correlation function

$$(1 - b\xi)\xi'' + ((1 - b\xi)\frac{2}{x} + a)\xi' + \xi - b\xi'^2 - c\xi^2 = -\frac{1}{\alpha} \frac{\delta(x)k_0}{x^2},$$

where $\xi = \xi(r) \equiv G^{(2)}(r)$, $\xi' \equiv \frac{d}{dr}\xi$, $x \equiv k_0r$, $k_0 \equiv \sqrt{2J}$, and $a$, $b$, and $c$ are three independent parameters. Eq. 5 extends that in the earlier work [40]. The special case of $a = b = c = 0$ is the Gaussian approximation. The terms containing $a$, $b$, and $c$ represent the nonlinear contributions beyond the Gaussian approximation. The nonlinear terms with $b$ and $c$ in Eq. 5 can enhance the amplitude of $\xi$ at small scales and increase the correlation length. The term containing $a$ plays a role of effective viscosity. The value of $a$ should be large enough to ensure $1 + \xi(r) \geq 0$ for the whole range $0 < r < \infty$.

3 General predictions of field equation

We inspect Eq. 6 to see its predictions on general properties of the correlation function $\xi(r)$.

1. The equation contains a point mass $m$ and a characteristic wavenumber $k_0$. It applies to the system of galaxies, as well as to the system of clusters, but with different $m$ and $k_0$ in each respective case. Thus, as solutions of Eq. 6, $\xi_{gg}$ for galaxies, but will differ in amplitude and in scale determined by different $m$ and $k_0$. Indeed, the observations show that both $\xi_{gg}$ and $\xi_{cc}$ have a power-law form: $c r^{-1.8}$ in their respective, finite range, but $\xi_{cc}$ has a higher amplitude [41, 26].

2. The $\delta^3(r)$ source in Eq. 6 has a coefficient $1/\alpha = 4\pi Gm/c_s^2$, which determines the overall amplitude of a solution $\xi$. The mass $m$ of a cluster can be $10 \sim 10^3$ times that of a galaxy [7], while $c_s$ regarded as the the peculiar velocity is around several hundreds km/s for galaxies and clusters. Therefore, $1/\alpha \propto m$, and a greater $m$ will yield a higher amplitude of $\xi$. This general prediction naturally explains a whole chain of prominent facts of observations, that luminous galaxies are more massive and have a higher correlation amplitude than ordinary galaxies [41], that clusters are much more massive and have a much higher correlation than galaxies, and that rich clusters have a higher correlation than poor clusters since richness is proportional to mass [41][8][17][18]. These phenomena
The observed perspectives. The property 3 also agrees with the observed fact from a variety of surveys. Therefore, the properties 2 and 3 reflect the same physical law of clustering from different perspectives. The property 3 also agrees with the observed fact from a variety of surveys. The observed \( P(k) \) of clusters is much higher than that of galaxies, and the observed \( P(k) \) of rich clusters is higher than poor clusters, etc. This is because that \( n_0 \) of clusters is much lower than that of galaxies, and \( n_0 \) of rich clusters is lower than that of poor clusters [24].

4. The characteristic length \( \lambda_0 = 2\pi/k_0 \propto \sqrt{n/m} \) appears in Eq. 5 as the only scale that underlies the scale-related features of clustering. Cluster surveys extend over larger spatial volumes, including those very dilute regions. The mass density \( \rho_{0c} \) of the region covered by cluster surveys is lower than \( \rho_{0g} \) for galaxy surveys, as implied in the references [5, 7]. Thus, \( \lambda_0 \) for cluster surveys will be longer than that for galaxy surveys. As will be seen in Sects. 4 and 5, in using one solution \( \xi(r) \) to match the data of both galaxies and clusters, one needs to take \( k_0 \) smaller for clusters than for galaxies.

4 Confronting observational data of galaxy surveys

Now we give the solution \( \xi_{gg}(r) \) for a fixed set of parameters \((a, b, c)\), and confront the observed correlation from major galaxy surveys.

1. The correlation function \( \xi_{gg}(r) \)

Fig. 1 shows the solution \( \xi_{gg}(r) \) and the observational data by the galaxy surveys of APM [29], SDSS [44], and 2dFGRS [23]. The theoretical \( \xi_{gg}(r) \) matches the data in the range of \( r = (2 \sim 50) \) h\(^{-1}\)Mpc. The usual power-law \( \xi_{gg} \propto r^{-1.7} \) is valid only in an interval \((0.1 \sim 10) \) h\(^{-1}\)Mpc. On large scales, the solution \( \xi_{gg}(r) \) deviates from the power law, decreases rapidly to zero, and becomes negative around 50 h\(^{-1}\)Mpc. However, on small scales \( r \leq 1 \) h\(^{-1}\)Mpc, the solution \( \xi_{gg}(r) \) is lower than the data, even though it has already improved the Gaussian approximation [45]. This insufficiency at \( r \leq 1 \) h\(^{-1}\)Mpc should be due to neglect of high order nonlinear terms such as \((\delta \psi)^3\) in our perturbation. We remark that the equation of \( \xi_{gg}(r) \) has been derived assuming \( \delta \psi < 1 \). Thus, it is only an approximation to extrapolate our calculated \( \xi_{gg}(r) \) down to smaller scales \( r \leq 5h^{-1}\)Mpc.

Let us check that the approximation of hydrostatical equilibrium can be applied in the quasi-linear regime in the expanding Universe. The density fluctuation behaves as \( \delta \psi \propto a(t)^{0.3} \) approximately, where \( a(t) \) is the scale factor in the present stage of accelerating expansion. So the time-evolving correlation function \( \xi_{gg}(r, t) = \langle \delta \psi \delta \psi \rangle \propto a^{0.6}(t) = 1/(1 + z)^{0.6} \). For the sample of \( \sim 200,000 \) galaxies of SDSS [44], the redshift range is \( z = (0.02 \sim 0.167) \). Taking its maximum \( z = 0.167 \) into the ratio gives \( \xi_{gg}(r)/\xi_{gg}(r, t) \approx (1 + 0.167)^{0.6} \sim 1.097 \), and the error is \( 0.6\% \approx 0.1 \). The conclusion of this analysis has also been supported by studies of numerical simulations [22, 39, 42].

2. The power spectrum \( P(k) \)

The power spectrum \( P(k) \) is the Fourier transform

\[
P(k) = 4\pi \int_0^\infty \xi(r) \frac{\sin(kr)}{kr} r^2 dr \tag{6}
\]
Figure 1: The solution $\xi_{gg}(r)$ confronts the data of galaxies by APM [29], 2dFGRS [23], and SDSS [44]. Here $k_0 = 0.055 \, \text{hMpc}^{-1}$ is taken in calculation.
of the correlation function $\xi(r)$. It measures the mass density fluctuation in $k$-space. In principle, $P(k)$ and $\xi(r)$ contain the same information if both are complete on their respective space, $k = (0, \infty)$, and $r = (0, \infty)$. Actually, the observed $\xi_{gg}(r)$ is not complete, limited to a finite range, say $r \leq 50$ Mpc. If the observed power-law $\xi_{gg}(r) = (r_0/r)^{1.8}$ were plugged in Eq. 6, one would have $P(k) \propto k^{-1.2}$, which does not comply with the observed $P(k) \propto k^{-1.6}$ [31]. Our $P(k)$ is obtained from the solution $\xi_{gg}(r)$ given on the whole range $r = (0, \infty)$. Fig. 2 shows the theoretical $P(k)$ converted by Eq. 6 from the solution $\xi_{gg}(r)$ with the same set $(a, b, c)$ and $k_0$ as those in Fig. 1. Also shown in Fig. 2 are the observational data of $P(k)$ from APM [29], 2dFGRS [12], and SDSS [11]. It is seen that the theoretical $P(k)$ agrees well with the data $P(k) \propto k^{-1.6}$ in the range of $k = (0.04 \sim 0.7)$ hMpc$^{-1}$. However, at large $k$, the theoretical $P(k)$ is lower than the data. This insufficiency of $P(k)$ corresponds to that of $\xi_{gg}(r)$ at small scales $r \leq 1h^{-1}$Mpc shown in Fig. 1.

5 Confronting the observational data of clusters

Clusters are believed to trace the cosmic mass distribution on even larger scales, and the observational data cover spatial scales farther than that of galaxies. Now we shall apply the solution with the same two sets of $(a, b, c)$ as in Sect. 4 to the system of clusters. A
cluster has a mass $m$ greater than that of a galaxy. This leads to a higher overall amplitude of $\xi_{cc}(r)$. Besides, to match the observational data of clusters, a small value $k_0 = 0.03$ Mpc$^{-1}$ is required, smaller than that for galaxies. In Fig. 3 for each set of values of $(a, b, c)$, two solutions $\xi_{cc}(r)$ with different amplitudes are given, and are compared with two sets of data of richness $N > 10$ and $N > 20$ from the SDSS [8]. Interpreted by Eq. 5, the $N > 20$ clusters have a greater $m$ than the $N > 10$ clusters. The solutions match the data on the whole range $r = (4 \sim 100)h^{-1}$Mpc.

It has long been known that there is a scaling behavior of cluster correlation. The correlation scale increases with the mean spatial separation between clusters [3,7,13,20,38]. If a power-law $\xi_{cc} = (r_0/r)^{1.8}$ was used to fit the data, the “correlation length” would be of a form $r_0 \simeq 0.4d_i$, where $d_i = n_i^{-1/3}$ and $n_i$ is the mean number density of clusters of type $i$. Simulations have also produced this $r_0 - d_i$ dependence [2]. For SDSS, the scaling can also be fitted by $r_0 \simeq 2.6d_i^{1/2}$ [8], and for the 2df galaxy groups by $r_0 \simeq 4.7d_i^{0.32}$ [43]. This kind of universal scaling of $r_0 - d_i$ has been a theoretical challenge [6], and was thought to be either caused by a fractal distribution of galaxies and clusters [38], or by the statistics of rare peak events [24]. The difference in the scaling slope was attributed to different richness of clusters [6]. In our theory the scaling is fully embodied in the solution $\xi_{cc}(k_0r)$ with the characteristic wavenumber $k_0 = (8\pi Gmn/c_s^2)^{1/2} \sim d^{-3/2}$. To comply with the
empirical power-law, we take the theoretical “correlation length” as \( r_0(d) \propto \xi_{cc}^{1/1.8} \), where \( \xi_{cc} \) is the theoretical solution and depends on \( d \). Fig. 4 shows that the solution \( \xi_{cc} \) with \( k_0 = 0.03 \text{ hMpc}^{-1} \) gives the universal scaling \( r_0(d) \simeq 0.4d \), agreeing well with the observation [7]. If a greater \( k_0 = 0.055 \text{ hMpc}^{-1} \) is taken, the solution \( \xi_{cc} \) would yield a flatter scaling \( r_0(d) \simeq 0.3d \), which fits the data of APM clusters better [8]. Our analysis based on the solution \( \xi_{cc} \) reveals that a higher background density \( \rho_0 \) predicts a flatter slope of the scaling \( r_0(d) \). Conclusively, the universal \( r_0 - d \) scaling is naturally explained by the solution \( \xi_{cc}(r) \).

6 Conclusions and discussions

We have presented a field theory of density fluctuations of Newtonian gravitating systems, and applied it to the study of correlation functions of galaxies and of clusters. Starting from the field equation Eq. 1 of mass density, under the condition of hydrostatic equilibrium, and keeping to the nonlinear order of \((\delta \psi)^2\) in perturbation, by taking functional derivative, we have obtained the field equation Eq. 5 of the 2-point correlation function as a main result. In deriving Eq. 5, we have adopted the Kirkwood-Groth-Peebles ansatz necessary to cut off the hierarchy in n-point correlation functions, and have done renormalization to absorb divergences into the parameters. This analytic approach from first principle is different from those using gravitational potential. Eq. 5 explains the observational data of clustering of galaxies, and of clusters well on large scales. To extend the analytic study further, higher-order nonlinear terms should be included to describe small-scale clusterings better, and the cosmic evolution should be taken into consideration as a more realistic model.

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Figure 4: The solution $\xi_{cc}(r)$ with $k_0 = 0.03$ hMpc$^{-1}$ gives the universal scaling $r_0 \approx 0.4d$. If a greater $k_0 = 0.055$ hMpc$^{-1}$ is taken, $\xi_{cc}$ would give a flatter scaling $r_0 \approx 0.3d$, which fits the data of APM clusters better.
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