Isospin dependence of liquid-gas phase transition in hot asymmetric nuclear matter

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Abstract

By using the Furnstahl, Serot and Tang’s model, the effect of density dependence of the effective nucleon-nucleon-\(\rho\)-meson (NN\(\rho\)) coupling on the liquid-gas phase transition in hot asymmetric nuclear matter is investigated. A limit pressure \(p_{\text{lim}}\) has been found. We found that the liquid-gas phase transition cannot take place if \(p > p_{\text{lim}}\). The binodal surface for density dependent NN\(\rho\) coupling situation is addressed.

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It is generally recognized that the liquid-gas (LG) phase transition of one component system is of first order. The chemical potential continues at the phase transition point but its first order derivatives, namely, entropy and volume, are discontinuous. But for a multi-components or multi-conserved charges system, as was pointed out by Müller and Serot [1], because of the greater dimensionality of the binodal surface, the LG phase transition can be of second order, i.e., the entropy continues but the second order derivatives of chemical potential (for example, capacity) are discontinuous. An asymmetric nuclear matter has two components of proton and neutron, and two conserved charges of baryon number and the third component of isospin, will undergo a continuous second order phase transition.

Obviously, because of charge independence, the basic difference between proton and neutron be isospin. The isospin dependent interactions of nucleon-nucleon-isovector mesons play the key role to address the LG phase transition. As was pointed out by our previous papers [2–4], if one employed the isospin independent model, for example, Welacka model [5] or Zimanyi-Moszkowski model [6], to investigate asymmetric nuclear matter, a lot of difficulties, e.g. Coulomb instability and negative asymmetric parameter in the vapor phase, will emerge. To overcome these difficulties, the isospin vector \( \rho \)-meson must be introduced. It can be shown that the chemical potentials of proton and neutron may depend on the third component of isospin when NN\( \rho \) interaction exists. A model without isospin vector \( \rho \)-meson, or even if it has \( \rho \)-meson, but the chemical potentials of the proton or neutron are still independent of the NN\( \rho \) interaction because the third component \( I_3 \) of isospin be zero such as in a symmetric nuclear matter, the LG phase transition is still of first order.

In fact, the chemical potentials of proton and neutron not only depend on \( I_3 \) but also on the effective NN\( \rho \) coupling \( g_\rho \). Then the effective coupling \( g_\rho \) is also essential for studying the LG phase transition because the chemical potentials determine the binodal surface directly. In our previous papers [7–9], we have shown that the effective couplings depend on the density and temperature in a hot and dense nuclear matter. By using the Thermo Field Dynamics [5] to calculate the three-lines vertices Feynman diagrams of NN\( \pi \), NN\( \sigma \), NN\( \omega \) and NN\( \rho \) interactions, we have found that the effective couplings of \( g_\pi \), \( g_s \), \( g_v \) and \( g_\rho \) all decrease as the nucleon density increases. In an asymmetric nuclear matter, one can easily prove that the chemical potentials of nucleons have a term which is proportional to \( g_\rho^2 \) and \( I_3 \). This term has opposite signs for proton and neutron due to their different third component of isospin. Obviously, if \( g_\rho \) depends on density, this term will change and then the chemical potentials of proton and neutron, as well as the binodal surface of LG phase transition will also be changed. The objective of the present paper is to investigate the effect of the density dependence of \( g_\rho \) on LG phase transition. We will prove that if \( g_\rho \) is a decreasing function of density, a limit pressure \( p_{\text{lim}} \) will occur, when \( p > p_{\text{lim}} \), the coexistented equations have no solution and the LG phase transition cannot be existed. The chemical potential of neutron will become a monotoneous function of asymmetry \( \alpha \) in this case.

To illustrate our result, we employ a model suggested by Furnstahl, Serot and Tang (FST) [11–14] recently. This model is an extension of quantum hadrodynamics and has been proven to be successful to explain many experimental properties of both nuclear matter and the finite nuclei in mean field approximation. The Lagrangian density of FST model under mean field approximation is

\[
L_{MFT} = \bar{\Psi} \left[ i \gamma^\mu \partial_\mu - (M - g_s \phi_0) - g_v \gamma^0 V_0 - \frac{1}{2} g_\rho \tau_3 \gamma^0 b_0 \right] \Psi
\]  

(1)
\[+\frac{1}{2}m_v^2 v_0^2 \left(1 + \frac{\phi_0}{S_0}\right) + \frac{1}{4!} \zeta (g_v V_0)^4 + \frac{1}{2} m_\rho^2 b_0^2
\]

\[-H_q \left(1 - \frac{\phi_0}{S_0}\right) \frac{4}{d} \left[\frac{1}{d} \ln \left(1 - \frac{\phi_0}{S_0} - \frac{1}{4}\right)\right]\]

where \(g_s, g_v, g_\rho\) are, respectively, the couplings of light scalar meson \(\sigma\), vector meson \(\omega\) and isovector meson \(\rho\) fields to the nucleon, \(\phi_0, V_0, b_0\) are the expectation values \(\phi_0 \equiv <\phi>, <V_\mu \equiv \delta_{\mu0} V_0, <b_\mu \equiv \delta_{\mu3} b_0\). The scalar fluctuation field \(\phi\) is related to \(S\) by \(S(x) = S_0 - \phi(x)\) and \(H_q\) is given by \(m_\sigma^2 = 4H_q/(d^2 S_0^2)\), \(d\) the scalar dimension. By using the standard technique of statistical mechanics, we get the thermodynamic potential \(\Omega\) as

\[\Omega = V \left\{H_q \left[\left(1 - \frac{\phi_0}{S_0}\right) \frac{4}{d} \left[\frac{1}{d} \ln \left(1 - \frac{\phi_0}{S_0} - \frac{1}{4}\right)\right]\right] - \frac{1}{2} m_\rho^2 b_0^2 - \frac{1}{2} \left(1 + \eta \frac{\phi_0}{S_0}\right) m_v^2 V_0^2 - \frac{1}{4!} \zeta (g_v V_0)^4\right\}

\[\quad - 2k_B T \left[\sum_{k,\tau} \ln \left(1 + e^{-\beta (E^*(k) - \nu_\tau)}\right) + \sum_{k,\tau} \ln \left(1 + e^{-\beta (E^*(k) + \nu_\tau)}\right)\right]\]

where \(\beta = 1/k_B T\) and the quantity \(\nu_i (i = n, p)\) is related to the usual chemical potential \(\mu_i\) by the equations

\[\nu_n = \mu_n - g_v V_0 + \frac{g_\rho^2 \rho_3}{4m_v^2}\]

\[\nu_p = \mu_p - g_v V_0 - \frac{g_\rho^2 \rho_3}{4m_v^2}\]

where \(\rho_3 = \rho_p - \rho_n\) and the third component of isospin \(I_3 = (N_p - N_n)/2 = V \rho_3/2\). The third term of the right hand side of Eq(4) and Eq(4) depends on \(\rho_3\) and \(g_\rho^2\). They have opposite signs and play the essential role to determine the LG phase transition.

Having obtained the thermodynamic potential, all other thermodynamic quantities, for example, pressure \(p = -\Omega/V\), can be calculated. The two-phase coexistence equations are

\[\mu_i^L (T, \rho_i^L) = \mu_i^V (T, \rho_i^V)\]

\[p^L (T, \rho_i^L) = p^V (T, \rho_i^V)\]

where subscripts of one phase L and V stand for liquid and vapor, respectively. The stability conditions are given by

\[\rho \frac{\partial p}{\partial \rho} \left|_{T,\alpha} \right. = \rho^2 \left(\frac{\partial^2 F}{\partial \rho^2}\right)_{T,\alpha} > 0\]


\[ \left( \frac{\partial \mu_p}{\partial \alpha} \right)_{T,p} < 0 \text{ or } \left( \frac{\partial \mu_n}{\partial \alpha} \right)_{T,p} > 0 \]  

(8)

where \( \mathcal{F} \) is the density of free energy, \( \alpha = \frac{(\rho_n - \rho_p)}{\rho} \) the asymmetric parameter, and \( \rho = \rho_n + \rho_p \).

The numerical calculations have been done by adopting the parameters set T1 of FST model [11–13]. The parameters of set T1 are

\[
\begin{align*}
g_s^2 &= 99.3, & g_v^2 &= 154.5, & g_\rho^2 &= 70.2 \\
m_s &= 509\text{MeV}, & S_0 &= 90.6\text{MeV} \\
\zeta &= 0.0402, & \eta &= -0.496, & d &= 2.70
\end{align*}
\]  

(9)

Our results for \( g_\rho = (70.2)^{1/2} \) = constant are shown in Fig.1 and Fig.2 by solid curves. The Gibbs conditions (5) and (6) for phase equilibrium demand equal pressures and chemical potentials for two phase with different concentrations. The collection of all such pairs \( \alpha_1(T, p) \) and \( \alpha_2(T, p) \) form the binodal surface. In Fig.1, the chemical isobar vs. \( \alpha \) curves at fixed temperature \( T=10\text{MeV} \) and \( p=0.100\text{MeV}(fm)^{-3} \) are labeled by A and A’ for neutron and proton respectively. The two desired solutions form the edges of a rectangle and can be found by means of the geometrical construction shown in Fig.1 [1]. The critical curves with \( T=10\text{MeV} \) and \( p_{\text{crit}} = 0.165\text{MeV}(fm)^{-3} \) are shown in Fig.1 by B and B’ where the chemical potential curve arrive at a inflection point and the rectangle is degenerate to a line vertical to the \( \alpha \) axis. The behaviour of the nuclear matter under isothemal compression, or in other words, the section of binodal surface at finite temperature \( T=10\text{MeV} \) are shown in Fig.2. The physical behaviour of this processes has been discussed by ref. [1]. Assume that the system is initially prepared with \( \alpha = 0.6 \) (gas), during the compression, the two-phase region is encountered at point A, and the liquid phase emerges at point B. The gas phase evolves from A to D, while the liquid phase evolves from B to C. The system leaves the region of instability at point C, while the original gas phase is about to disappear. The critical point (CP), the point of equal concentration (EC) and the maximal asymmetry (MA) are indicated in Fig.2.

Now we are in a position to extend our discussion to the case of \( g_\rho \) density dependence. In fact, the effective masses of nucleons, effective masses or screening masses of mesons, and the effective couplings of NN-mesons are all dependent on density and temperature. We can sum the tadpole diagrams and the exchange diagrams for nucleon, the vacuum polarization diagrams for mesons and the three-lines vertex diagrams for effective couplings to get their density and temperature dependence [4,7–9,15–17]. But in order to illustrate our result more transparently, instead of the exact calculation of three-lines vertex, we introduce an ansatz

\[ g_\rho' = g_\rho \left[ 1 - A\rho + B\rho^2 \right] \]  

(10)

where \( A, B \) are two adjust parameters. The reason for our choice are: at first, the three lines vertex calculations are model dependent, but we hope that our investigation can be more general; secondly adjust the values of \( A, B \) can make \( g_\rho' \) be decreased or increased with density, then we can study the LG phase transition for two different cases. We can imagine that the Eq(10) is an density expansion of effective coupling \( g_\rho' \) at low density regions.

The results for density dependent \( g_\rho' \) ansatz Eq(10) are shown in Fig.3, Fig.4 and Fig.5. We see from Fig.3 and Fig.4 that the chemical potential of neutron \( \mu_n \) increases rapidly
with density. It passes through an inflection point and becomes monotoneous when pressure increases. But the shape of $\mu_p$ vs. $\alpha$ curves change slowly. Then we will get a limit pressure $p_{\text{lim}}$, when $p > p_{\text{lim}}$, the rectangle cannot be found and the coexisted equations have no solution. The last rectangle in the chemical isobar vs. $\alpha$ curves for $A=1$, $B=0$, $T=10\text{MeV}$, and $p_{\text{lim}}=0.130\text{MeV}(fm)^{-3}$ is shown in Fig.3 by dashed lines, where $\alpha_1=0.62$ and $\alpha_3=0.75$ correspond to the maximum and the minimum of $\mu_n$ respectively. The pair $\alpha_1=0.62$ and $\alpha_2=0.84$ form the end of the binodal surface, as shown in Fig.5. The curve for $A=1$, $B=0$, $T=10\text{MeV}$, but $p=0.145\text{MeV}(fm)^{-3}$ ($p > p_{\text{lim}}$) is shown in Fig.4. We see that $\mu_n$ becomes monotoneous at this pressure, and no rectangle can be found. The relation between limit pressure and parameters $A$ and $B$ for a fixed temperature $T=10\text{MeV}$ is shown in Table 1. We find from Table 1 when $A$ and $B$ increase, the effective coupling $g'_{\rho}$ decreases and the limit pressure decreases. If $A$ changes its sign to become negative, $g'_{\rho}$ will increase with density, and in this case, instead of $\mu_n$, $\mu_p$ becomes monotoneous. The limit pressure is still existed but decrease when $A$ and $B$ decrease.

The section of binodal surface for $A=1$, $B=0$, $T=10\text{MeV}$ is shown in Fig.5. We see from Fig.5 that the curve will cut off at limit temperature $p_{\text{lim}}$ clearly. The total asymmetric parameter $\alpha$ is divided into four regions, namely, $[0, \alpha_1]$, $[\alpha_1, \alpha_3]$, $[\alpha_3, \alpha_2]$ and $[\alpha_2, \alpha_{\text{max}}]$. The physical behaviour of isothermal compression in different regions are different. We find:

1) If the system is initially prepared with $0 < \alpha < \alpha_1$, the precess of isothermal compression is similar to that of the case with constant $g_{\rho}$. It begin at gas phase, suffers a second order LG phase transition and ends at liquid phase.

2) If the initial $\alpha$ is located at the region $\alpha_2 < \alpha < \alpha_{\text{max}}$, the system enters and leaves the two-phase region on the same branch, so the system remain in the same gas phase. As was pointed out by Müller and Serot [1], this retrograde condensation is unique to the binary system and dose not occur in one-component systems.

3) Suppose that the initial $\alpha$ is located at the region $\alpha_1 < \alpha < \alpha_3$. Since $\alpha_1$ and $\alpha_3$ correspond to the maximum and minimum of $\mu_n$, we find $(\partial \mu_n/\partial \alpha)_{T,p} < 0$ in this region and the stability condition Eq.(8) will be destroyed. The system begins at gas phase, enters a two-phase region and becomes instable at the limit pressure.

4) If the initial $\alpha$ is located at the region $\alpha_3 < \alpha < \alpha_2$, the behaviour of the system is similar to that of the case (3), except it will be ended to a stable phase at the limit pressure because the stability condition $(\partial \mu_n/\partial \alpha)_{T,p} > 0$ is satisfied.

In summary, we have shown that the density dependence of effective NN$\rho$ coupling is important to the LG phase transition. A limit pressure $p_{\text{lim}}$ has been found for a fixed temperature and the LG phase transition cannot take place in asymmetric nuclear matter provided $p > p_{\text{lim}}$. Of course, for a fixed pressure, we can also get a limit temperature. This conclusion is similar to that of the Coulomb instability [2,3] of nuclei. The basic difference is that instead of finite nuclei, our conclusion has been found to be suitable for asymmetric nuclear matter. Finally, we would like to emphasize that the isospin is very important for the LG phase transition of asymmetric nuclear matter.

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REFERENCES

[1] H. Müllerr and B. D. Serot, Phys. Rev. C52 2072 (1995)
[2] H. Q. Song, Z. X. Qian and R. K. Su, Phys. Rev. C47 2001 (1993)
[3] H. Q. Song, Z. X. Qian and R. K. Su, Phys. Rev. C49 2924 (1994)
[4] P. Wang, Z. W. Chong, R. K. Su and P. K. N. Yu, Phys. Rev. C59 928 (1999)
[5] B. D. Serot and J. D. Walecka, Advances in Nuclear Physics, edited by J. W. Negels and E. Vogt Plenum, New York, 1986), Vol. 16. P. 1
[6] J. Zimanyi and J. A. Moszkowski, Phys. Rev. C42 1416 (1990)
[7] S. Gao, Y. J. Zhang and R. K. Su, Nucl. Phys. A593 362 (1995)
[8] Z. X. Qian, C. G. Su and R. K. Su, Phys. Rev. C47 877 (1993)
[9] R. K. Su, G. T. Zheng and G. G. Siu, J. Phys. G19 79 (1993)
[10] H. Umezawa, H. Matsumoto and M. Tachiki, Thermo Field Dynamics and Condensed States (North-Holland, Amsterdam, 1982)
[11] R. J. Furnstahl, H. B. Tang and B. D. Serot, Phys. Rev. C52 1368 (1995)
[12] R. J. Furnstahl, B. D. Serot and H. B. Tang, Nucl. Phys. A598 539 (1996)
[13] R. J. Furnstahl, B. D. Serot and H. B. Tang, Nucl. Phys. A615 441 (1997)
[14] B. D. Serot and J. D. Walecka, Int. Jour. of Mod. Phys. E6 515 (1997)
[15] L. L. Zhang, H. Q. Song, P. Wang and R. K. Su, Phys. Rev. C59 3292 (1999)
[16] Y. J. Zhang, S. Gao and R. K. Su, Phys. Rev. C56 3336 (1997)
[17] S. Gao, Y. J. Zhang and R. K. Su, Phys. Rev. C53 1098 (1996)
|    | 1    | 2    | 3    | 5    | -2   | -2   | -5   |
|----|------|------|------|------|------|------|------|
| A  |      |      |      |      |      |      |      |
| B  |      |      |      |      |      |      |      |
| $p_{\text{lim}}$ (MeV fm$^{-3}$) | 0.13 | 0.115 | 0.105 | 0.90 | 0.125 | 0.125 | 0.115 |

TABLE I. relation between the limit pressure and adjust parameter A, B
FIGURES

FIG. 1. The chemical isobar as a function of \( \alpha \) at fixed temperature \( T=10\text{MeV} \) and the geometrical construction used to obtain the asymmetry parameters in the two coexisting phases. B, B’ correspond to the critical curves.

FIG. 2. The section of binodal surface at \( T=10\text{MeV} \), where CP, EC, and MA stand for critical point, equal concentration and maximal asymmetry respectively.

FIG. 3. The chemical isobar as a function of \( \alpha \) for \( A=1, B=0, T=10 \text{ MeV} \) and \( p=0.145 \text{ MeV(fm)}^{-3} \).

FIG. 4. The chemical isobar as a function of \( \alpha \) for \( A=1, B=0, T=10 \text{ MeV} \) and \( p=0.13 \text{ MeV(fm)}^{-3} \).

FIG. 5. The section of binodal surface for \( A=1, B=0, T=10\text{MeV} \). \( \alpha_1, \alpha_2 \) correspond to the maximum and minimum of \( \mu_n \) respectively.
Fig1
Fig 2
$$\alpha_1 = 0.62$$
$$\alpha_2 = 0.75$$
$$\alpha_3 = 0.84$$
Fig 4
Fig5