Abstract. Knowledge of the parameters of dynamic force transducers is required to determine the response of the transducers under dynamic conditions. We describe a model representation of dynamic force transducers with different transfer functions that cover all measurement probabilities, and we introduce three identification tools depending on least-square fit with the modeling of noise. In applying these methods to different measurements, it is demonstrated that stable results can be obtained for the parameter of dynamic force transducers.

1. Introduction
Dynamic calibration of force transducers is carried out by a couple of national metrology institutes [1–4]. Parametric identification of dynamic force transducers is a part of the calibration process. The stability of the dynamic parameters, especially the dimpling coefficient, of force transducers face challenges with the application of different load masses [5, 6], in addition to torque transducers [7]. In contrast to the other publications, the focus of this work is to gain a more reliable estimation of the damping coefficient of force transducers. In addition, the analysis shown in this paper is based on time-domain data, not the usual frequency-domain data [8].

2. Measurement setup
A detailed schematic diagram of the experimental setup is shown in figure 1. The setup described here is the same system that presented earlier in [9]. A periodic chirp excitation is provided by an electrodynamic shaker. The force transducer is arranged on top of the shaker armature by means of mechanical adaptation. The adaptation is designed to be stiff and provides a point to mount an accelerometer on top of the shaker armature on the longitudinal axis of the force transducer. Different load masses are attached on the top of the force transducer using a thread joint without intermediate couplings. The surface of the load masses is illuminated by a Laser from a scanning vibrometer. The measurement concept of the scanning vibrometer enables the serial measurement of acceleration on a finite number of scan points on top of the load mass. The software records three signals for each scan point; force signal in mV/V, accelerometer signal in m/s^2, and the local acceleration on the point using the laser scanning vibrometer in m/s^2. The three measurement signals are collected in a junction box which is connected to a computer through a 12-bit, 5 MHz ADC DAQ card.
3. Identification of force transducer

The dynamic response of a force transducer can be described within linear differential equations that describe the equation of motion of a one degree-of-freedom mas-spring-damper system[2]. In this case as illustrated in figure 2, the force transducer itself can be modelled as a head mass which is connected to its base mass by a spring, with a spring constant, \( k \), and a damper with a damping coefficient, \( b \). The mass of the force transducer is mounted to a load mass. The combination of the two masses; load and head masses; is represented as mass, \( m \). Particularly, the electrical output signal of the force transducer is relative to the reference \( z = y - u \) by a factor \( \rho \), where \( y \) represents the displacement of the mass, \( m \), while \( u \) is the displacement of the base mass of the force transducer which is rigidly connected on top of the shaker armature.

3.1. Mathematical model

The equation of motion of the system shown in figure 2 and figure 1 can be represented by the following differential equation:

\[
 m\ddot{y} = -k_f(y - u) - b_f(\dot{y} - \dot{u}).
\]  

(1)
Assuming that all acquired signals are harmonic, and we can measure three signals; the force transducer output, the acceleration on the top of the shaker armature and the acceleration on top of the load mass: we can apply three transfer functions from the above equation.

\[
G_1(s) = \frac{y(s)}{u(s)} = \frac{b_f}{m} s + \frac{k_f}{m} s^2 + \frac{b_f}{m} s + \frac{k_f}{m} . \tag{2}
\]

The parameters in the transfer function \(G_1\) can be identified by the measurement of the two accelerations; on top of the load mass and on top of the shaker armature. The Laplace variable \(s = \sigma + j \cdot \omega\) represents the transformation of continuous time domain to frequency domain equations. The second transfer function is:

\[
G_2(s) = \frac{F(s)}{s^2 D(s)} = \frac{\rho}{s^2} s + \frac{b_f}{m} s + \frac{k_f}{m} . \tag{3}
\]

The transfer function \(G_2\) represents the relationship between the output of the force transducer and the measured acceleration on top of the shaker armature, while the third transfer function, \(G_3\), represents the relation between the force transducer output and the measured acceleration on top of the load mass.

\[
G_3(s) = \frac{F(s)}{s^2 Y(s)} = \frac{\rho}{s^2} s + \frac{k_f}{m} . \tag{4}
\]

### 3.2. Estimation of model parameters

The transfer function in frequency-domain \(G(s)\) can be transformed to \(z\)-domain transfer function \(G(z^{-1})\) which represents discrete-time linear system with a sampling time \(\delta T\) as a function of \(z = e^{\delta T}\) is given as \[10\]

\[
G(z^{-1}) = \frac{y(z^{-1})}{u(z^{-1})} = \frac{b_0 + b_1 z^{-1} + \ldots + b_m z^{-m}}{1 + a_1 z^{-1} + \ldots + a_m z^{-m}} = \frac{B(z^{-1})}{A(z^{-1})} . \tag{5}
\]

The output error signal \(v(z)\) vanishes for the adapted model only if an additional model for the noise signal is used, such that the complete model structure of figure 3 is obtained \[11\]. The different model structures used today for parameter estimation are based on this method. Here we represent two of these estimators; the first one is the extended auto regressive, moving average model (ARMAX-model) with least square (LS) estimator. The ARMAX model is represented as:

\[
A(z^{-1}) y(z) - B(z^{-1}) z^{-d} u(z) = D(z^{-1}) v(z) . \tag{6}
\]

Where \(d\) is the delay in the input \(u(z)\). The other one uses Box-Jenkin (BJ) model for the noise filter with the following model equation:

\[
A(z^{-1}) y(z) - B(z^{-1}) z^{-d} u(z) = D(z^{-1}) v(z) / C(z^{-1}) . \tag{7}
\]

Another parameter estimation method is a grey-box estimation of the transfer function using the method of least-squares rational function estimation (Vector Fit) that has been thoroughly described in \[12\].

Figure 3. Schematic illustration of the parameter estimation model with noise. The polynomials \(D(z^{-1}), C(z^{-1})\) represent the effect of the noise and disturbance while the polynomials \(A(z^{-1}), B(z^{-1})\) represent the model of main system (e.g. spring-mass-damper model).
4. Results
The proposed identification techniques have been applied to a strain gauge shear force transducer with a 27.5 kN force capacity. Three different load masses were attached to the force transducer. Figure 4 and figure 5 show a comparison between the model and measurement data of the output acceleration for two load masses; 2 and 4 kg respectively. Table 1 introduces the estimated dynamic parameters of the force transducer with three load masses.

![Figure 4. Experimental data and model fits of the output acceleration $\ddot{y}(t)$ with a 2 kg load mass.](image1)

![Figure 5. Experimental data and model fits of the output acceleration $\ddot{y}(t)$ with a 4 kg load mass.](image2)

Table 1. Summary of identified parameters of each model for the transfer function $G_1(s)$.

| Method     | Fundamental frequency, f(Hz) | Stiffness, k(N m$^{-1}$) | Damping coefficient, b(N s m$^{-1}$) | Normalised RMSE (%) |
|------------|-----------------------------|--------------------------|--------------------------------------|---------------------|
| 2 kg Load mass |                             |                          |                                      |                     |
| ARMAX      | 1813                        | $2.956 \times 10^8$      | 556.51                               | 66.1                |
| BJ         | 1811                        | $2.951 \times 10^8$      | 773.69                               | 70.21               |
| Vector Fit | 1813                        | $2.956 \times 10^8$      | 752.716                              | --                  |
| 3 kg Load mass |                             |                          |                                      |                     |
| ARMAX      | 1490                        | $2.852 \times 10^8$      | 611.225                              | 86.44               |
| BJ         | 1487                        | $2.840 \times 10^8$      | 1087.38                              | 78.37               |
| Vector Fit | 1558                        | $3.117 \times 10^8$      | 643.418                              | --                  |
| 4 kg Load mass |                             |                          |                                      |                     |
| ARMAX      | 1283                        | $2.782 \times 10^8$      | 899.193                              | 89.60               |
| BJ         | 1282                        | $2.778 \times 10^8$      | 1019.88                              | 89.95               |
| Vector Fit | 1391                        | $3.268 \times 10^8$      | 882.577                              | --                  |

* Normalised root mean square error (100% is the best case).

5. Conclusions
The dynamic behaviour of dynamic force transducers is described by three transfer functions depending on the measurement quantities during the calibration procedure. Three system identification techniques based on linear least-squares with the modelling of noise have been introduced. The application to different measurement leads to stable estimation of the dynamic parameters of force transducers.

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