Lower limit on the ultra-high-energy proton-to-helium ratio from the measurements of the tail of $X_{\text{max}}$ distribution

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Abstract. There are multiple techniques to determine the chemical composition of the ultra-high-energy cosmic rays. While most of the methods are primarily sensitive to the average atomic mass, it is challenging to discriminate between the two lightest elements: proton and helium. In this paper, the proton-to-helium ratio in the energy range from $10^{18.3}$ eV to $10^{19.3}$ eV is estimated using the tail of the distribution of the depth of the shower maximum $X_{\text{max}}$. Using the exponential decay scale $\Lambda$ measured by the Telescope Array experiment we derive the constraint on the proton-to-helium ratio $p/He > 0.43$ (68% CL). The result is conservative with respect to the admixture of heavier elements. We evaluate the impact of the hadronic interaction model uncertainty. The implications for the astrophysical models of the origin of cosmic rays and the safety of the future colliders are discussed.

1. Introduction
The mass composition of the ultra-high-energy cosmic rays lies among the key tasks of major present-day and upcoming experiments. The precise knowledge of the composition is important for understanding the cosmic-ray production mechanism in the sources and its population [1]. Moreover, composition at the highest energies is the decisive factor for the observable flux of cosmogenic photons [2, 3] and neutrinos [4, 5], see [6] for a review. The photons and neutrinos are most efficiently produced by the primary protons due to the highest energy per nucleon compared to heavier elements. The diversity of the models may be illustrated with the two antipodal examples namely the dip model [7, 8, 9] and the disappointing model [10]. The dip model has purely proton composition and as a consequence predicts observable fluxes of the cosmogenic photons and neutrino. The model is named after the dip spectral feature which is naturally explained with the electron-positron pair production by protons. The disappointing model includes both protons and nuclei in the source and assumes that the acceleration of primary nuclei in the sources is rigidity-dependent with relatively low maximum energy of acceleration. It was named “disappointing”, because in this case there are no pion photo-production on CMB photons in extragalactic space and consequently no high-energy cosmogenic neutrino fluxes. Disappointing model predicts no GZK-cutoff [11, 12] in the spectrum and shows no correlation with nearby sources due to nuclei deflection in the galactic magnetic fields up to highest energies.

Another implication of the mass composition at ultra-high energies is the investigation of safety of the future colliders. In certain theoretical models characterized by additional spatial...
dimensions, the production of non-evaporating microscopic black holes becomes possible. This phenomenon was taken into consideration in the framework of the Large Hadron Collider (LHC) safety analysis [13, 14]. The proof of the LHC safety is based on the constraints on the black hole production derived from the stability of dense astrophysical objects, such as white dwarfs and neutron stars. The latter interact with the ultra-high-energy cosmic rays with the center of mass energies larger than ones achieved at LHC. One may ascertain the safety of the future 100 TeV colliders by studying the interaction of the cosmic rays of the highest energies. The primary protons again play an important role as the production of the black holes is determined by the energy per nucleon. It was shown that the charged stable microscopic black hole production may be excluded already, while the exclusion of the neutral black holes would require a precise knowledge of the proton fraction at the ultra-high energy [15].

One of the most common approaches is the measurement of the longitudinal shape of the extensive air showers (EAS). The depth of a shower maximum, or $X_{\text{max}}$, is used as a composition-sensitive variable [16]. The measurements of the mean $X_{\text{max}}$ gives the estimate of the average atomic mass, while the study of the $X_{\text{max}}$ distribution and its moments may resolve the multicomponent composition, see [17, 18, 19] for review.

The tail of the $X_{\text{max}}$ distribution may be studied independently on the main part of the distribution. It may be fit with an exponential function $\exp(-X_{\text{max}}/\Lambda)$, where $\Lambda$ is called the attenuation length. The attenuation length is found to be sensitive to the proton-air interaction cross-section. The first results by this method were obtained by the Fly’s Eye Collaboration [20, 21] followed by the results of the Pierre Auger and Telescope Array Collaborations [23, 22, 24]. It was shown in [25] that the attenuation length may be used to estimate the proton-to-helium ratio $p/He$. The latter estimate has only minor dependence on the hadronic interaction models and $X_{\text{max}}$ experimental systematic uncertainties. The proton-to-helium ratio is directly measured below the knee and it allows to constrain different astrophysical models of the origin of cosmic rays [26, 27]. The measurements of the proton-to-helium ratio at the ultra-high energies may be used similarly to discriminate between different source models. As a recent example, a modified dip model [28] agrees with the measured spectrum of the ultra-high-energy cosmic rays for the value of proton-to-helium ratio $p/He = 5$. Furthermore, the value of $p/He$ used jointly with the other composition studies will allow to pinpoint the flux of the primary protons. The latter is an important quantity as discussed above.

The present work is dedicated to the determination of proton-to-helium ratio of ultra-high-energy cosmic rays in the energy range from $10^{18.3}$ eV to $10^{19.3}$ eV based on the Telescope Array measurements of the attenuation length [22]. The data are compared to the Monte-Carlo simulations using the CORSIKA package [29] along with the QGSJET II-04 [30, 31] and EPOS-LHC [32] hadronic interaction models.

2. Method and Monte-Carlo simulations

The method generally follows the work of Yushkov et. al [25] to derive the proton-to-helium ratio using the measurements of the attenuation length by the Telescope Array collaboration [22].

At first, the simulated sets of extensive air showers initiated by primary protons, helium and carbon are produced with the use of the CORSIKA package [29]. Simulations are performed separately with QGSJET II-04 [31] and EPOS-LHC [32] hadronic interaction models. 17 354 events are simulated for each species in the energy range from $10^{18.3}$ eV to $10^{19.3}$ eV with the spectrum obtained by the Telescope Array collaboration defined by the spectral index $-3.226$ for $E < 10^{18.72}$ eV and $-2.66$ for $E > 10^{18.72}$ eV [33].

At the second step, the simulated sets are “mixed” in different proportions from $p/He = 0.01$ to $p/He = 100.0$. For each mixture the $X_{\text{max}}$ distribution slope is fitted exponentially to derive the attenuation length for a mixed composition model.

An important constituent of this method is the choice of the starting point of the fit: it can
Figure 1. $X_{\text{max}}$ distributions for $10^{18.3}$ eV < $E$ < $10^{19.3}$ eV for the Telescope Array for proton (magenta), helium (orange) and carbon (blue) Monte-Carlo distributions simulated with QGSJET II-04. $X_{\text{max}}$ distribution’s tail exponential fit exp($-X_{\text{max}} / \Lambda$) is shown for each Monte-Carlo with a line of the corresponding color.

be defined in many different ways. In the initial papers [20, 21] the lower range of $X_{\text{max}}$ fit was fixed at the constant values $X_{\text{max}} = 760$ g/cm$^2$ and $X_{\text{max}} = 830$ g/cm$^2$, respectively. Yushkov et. al [25] have proposed another determination of lower fit range, which involves carbon $X_{\text{max}}$ distribution: the lower limit is defined as a value at which only $\approx 0.5\%$ of the carbon-initiated showers get into the fitting range. In the present work we follow the method implemented by Abbasi et al. [22], where the lower limit is defined as the $X_i = \langle X_{\text{max}} \rangle + 40$ g/cm$^2$, where $\langle X_{\text{max}} \rangle$ is the average value of a given distribution.

In the present analysis, the implemented method leads to the following starting points for protons, helium and carbon nuclei: $X_{\text{max,start}}^p = 804$ g/cm$^2$, $X_{\text{max,start}}^{He} = 772$ g/cm$^2$ and $X_{\text{max,start}}^C = 745$ g/cm$^2$. For each MC-set, ending point for $X_{\text{max}}$ fit is 1000 g/cm$^2$.

Finally, after performing the fit of each mixture’s $X_{\text{max}}$ distribution, $\Lambda$ for the Telescope Array values are obtained as a function of $p$/He ratio. The constraints on the proton-to-helium ration are then obtained by comparing these values with the experimental $\Lambda$ values [22]. Lower limit on the proton to helium ratio at 68% CL corresponds to the lower limit of the experimentally measured $\Lambda$ value derived with the use of measured experimental uncertainties. Expansion of the experimental $\Lambda$ range to $2\sigma$ than leads to the derivation of proton-to-helium ratio with the 95% CL.
Figure 2. $X_{\text{max}}$ distributions for $10^{18.3}$ eV $< E <$ $10^{19.3}$ eV for the Telescope Array for mixtures $p : C = 3 : 1$ (magenta) and $p : C = 1 : 1$ (orange). $X_{\text{max}}$ distribution’s tail exponential fit $\exp(-X_{\text{max}}/\Lambda)$ is shown for each mixture with a line of the corresponding color.

### 3. Results

We present the $X_{\text{max}}$ distributions and corresponding fits of exponential tails for proton, helium and carbon Monte-Carlo simulated sets in Figure 1. $X_{\text{max}}$ distributions and corresponding fits of exponential tails for mixtures of proton and carbon with proportions $p : C = 3 : 1$ and $p : C = 1 : 1$ are shown in Figure 2.

$\Lambda$ as a function of proton-to-helium ratio in QGSJET II-04 and EPOS-LHC models is shown in Figure 3 with a black line. The plot includes the proton-to-helium ratio range from $p/He = 0.01$ to $p/He = 100$ with a step $\Delta = 10^{0.2}$. Comparing the Monte-Carlo function with the experimental value $\Lambda = 50.47 \pm 6.26$ g/cm$^2$ obtained by the Telescope Array collaboration [22] we arrive at the following lower limits on the proton-to-helium ratio for the $<E> =$ $3.6 \times 10^{18}$ eV:

\begin{align*}
    p/He & > 0.43 \ (68\% \ CL) \quad \text{QGSJET II-04}, \\
    p/He & > 0.63 \ (68\% \ CL) \quad \text{EPOS-LHC}.
\end{align*}

We note that the pure proton composition is well compatible with the measured attenuation length.

The stability of the method in respect to the admixture of the heavier elements is studied. For this reason, the analysis is repeated for three-component mixtures containing 25%, 50% and 75% of carbon and corresponding $\Lambda$ is shown in Figure 3 by red triangles and green squares respectively. One may see that the constraints 1 and 2 are conservative to addition of the heavier elements as expected in [25].
Figure 3. $\Lambda$ parameter as a function of proton-to-helium ratio for two-component mixture (p and He, black line) and three component mixtures (p, He and 25 % C – blue line; p, He and 50 % C – red line; p, He and 75 % C – orange line) of Monte-Carlo events simulated with QGSJET II-04 (left) and EPOS-LHC (right). Black solid and dashed lines correspond to the experimental value $\Lambda = 50.47 \pm 6.26$ g/cm$^2$ obtained by the Telescope Array collaboration [22].

It is also important to mention that the detector effects, such as acceptance of different types of primary particles and detector resolution may affect the measured $\Lambda$ value. It was shown in [22], that the detector effects are negligible for the $\Lambda$ value obtained by the Telescope Array.

One may further study the three-component mixture of protons, helium and carbon. By calculating $\Lambda$ for all possible combinations we arrive to the following lower limits on the fraction of protons in the three-component mixture for $\langle E \rangle = 3.6 \times 10^{18}$ eV:

$$\frac{p}{(p + He + C)} > 0.20 \ (68\% \ CL) \ \text{QGSJET II-04},$$

$$\frac{p}{(p + He + C)} > 0.23 \ (68\% \ CL) \ \text{EPOS-LHC}.$$  \hspace{1cm} (3)

One may note that the possible systematic error of the method is mainly caused by the uncertainties in the hadronic models used for the Monte-Carlo simulations, all of which are known to be inconsistent with the muonic component of the EAS [34, 35].

4. Conclusion

Let us finally discuss the possible applications of the obtained lower limit on the proton-to-helium ratio at the energies $10^{18.3}$eV < $E$ < $10^{19.3}$eV. First of all, we consider the properties of the sources of the ultra-high-energy cosmic rays in the view of the constraints 1 and 2. The present limits constrain the models with helium domination in the energy range under study, e.g. the helium version of the disappointing model [10]. These models generally include the preferential acceleration of helium or an excessive helium abundance at the acceleration region. At the same time, the result is fully compatible with the original pure proton dip model [7, 8, 9] and with the modification of the dip model with $p/\text{He} = 5$ [28] as long as with the standard disappointing model [10] with $p/\text{He} \sim 1$.

Secondly, let us discuss the safety of the future colliders. The proof of the safety relies largely on the existence of non-zero flux of the ultra-high-energy protons [15]. One may see that this work confirmed the existence of the primary protons at the $2\sigma$ level. Indeed, the following 95% CL limits may be obtained repeating the analysis done for Equations 3 and 4 for $\langle E \rangle = 3.6 \times 10^{18}$ eV:

$$\frac{p}{(p + \text{He} + C)} > 0.09 \ (95\% \ CL) \ \text{QGSJET II-04},$$

$$\frac{p}{(p + \text{He} + C)} > 0.11 \ (95\% \ CL) \ \text{EPOS-LHC}.$$  \hspace{1cm} (5)
The results are in favor of the safe operation of the future colliders. Still the importance of issue demand higher confidence which may be achieved with the future precision measurements of the attenuation length.

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