Two circumbinary planets in RR Cae eclipsing binary system

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ABSTRACT

We present the binary model and the eclipse timing variations of an eclipsing binary RR Cae, which consists of a white dwarf eclipsed by an M-type dwarf companion. The multiwavelength optical photometry from the 0.7-m Thai Robotic Telescope at Spring Brook Observatory, the 0.6-m PROMPT-8 telescope and Transiting Exoplanet Survey Satellite (TESS), combining with archive H-alpha radial velocities from the Very Large Telescope (VLT). The TESS light curves show a large spot which might be caused by the temperature variation of the M-dwarf companion. Using 430 new time of minima, the O-C curve shows two cyclic variations with amplitude of $12.1\pm 1.5$ s and $20.5\pm 5$ s, respectively. The periodic variations can be caused from the gravitational influence of the third and the fourth companions in the system. The mass of RR Cae b is revised to be $M_3 \sin i_3 = 3.0 \pm 0.3 M_{\text{Jup}}$ orbiting around the binary with a period of $15.0\pm 0.5$ yr. The fourth companion is a newly found planet orbiting a star with a period of $39 \pm 5$ yr. The second planet is a Jupiter mass planet $M_4 \sin i_4 = 2.7 \pm 0.7 M_{\text{Jup}}$.

Key words: Stars: binaries : close – Stars: binaries : eclipsing – Stars: individuals (RR Cae) – Stars: starspots – Stars: planetary system

1 INTRODUCTION

Eclipsing binaries are important for analysing physical parameters and evolution of stars. The Post Common Envelope Binary (PCEB) or pre-cataclysmic variable is a binary consisting of a white dwarf and a main-sequence star. As the main sequence component transfers mass to white dwarf companion, it is the evolution phase of binaries before they evolve to cataclysmic variables (CVs) (Warner 1995). In this evolution phase, the stellar component loses angular momentum via magnetic braking (Ribet et al. 2013).

The PCEB primary component is either a sub-dwarf O/B (sdOB) or a white dwarf (WD), while the secondary component is a main-sequence (MS). The large differences between effective temperatures and masses of hot primary and cold secondary affect the depth of primary eclipses of the system to be clearly seen. Therefore, the primary eclipse time of PCEB can be accurately measured which can be used to create the accurate O-C diagram. The O-C diagram can be used to study evolution of eclipsing binaries and search for circumbinary brown dwarfs or planets (Qian et al. 2015) (e.g. HW Vir (Beuermann et al. 2012), NY Vir (Lee et al. 2014), V2051 Oph (Qian et al. 2015), DE Cvn (Han et al. 2018)).

Nowadays, more than 4,000 exoplanets have been discovered, including over a hundred planets in binary, in around both stars (circumbinary) or orbit only one star (circumstellar) configurations 1. From few tenths of circumbinary planets, most of them are discovered via transit (e.g. Kepler-16 b (Doyle et al. 2011), Kepler-1647 b (Kostov et al. 2016)) and eclipsing binary timing techniques (e.g. HW Vir b (Beuermann et al. 2012), DP Leo b (Qian et al. 2010)). These circumbinary planet will provide a better understanding of planetary formation and evolution, especially planets around eclipsing binary where the precise mass and the radius of stellar component can be obtained from the double-line spectrum of the binary (Qian et al. 2012).

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The Extrasolar Planets Encyclopedia: http://exoplanet.eu/
Table 1. Summarized of RR Cae observation data.

| Telescope   | Observation date       | Exposure time (s) | Filters | Numbers of data | Numbers of epoch |
|-------------|------------------------|-------------------|---------|-----------------|------------------|
| TRT-SBO     | 4/10/2019 – 12/12/2020 | 30                | R       | 5,373           | 14               |
| TRT-SBO     | 1/11/2020 – 22/12/2020 | 30                | I       | 2,057           | 6                |
| PROMPT-8    | 22/9/2018 – 06/10/2018 | 120               | R       | 199             | 4                |
| TESS        | 20/9/2018 – 16/12/2020 | 120               | TESS    | 88,881          | 406              |

Figure 1. RR Cae light curves. The Upper panel: light curves in R-band from the PROMPT-8 (Dark red) and the TRT-SBO (Red). The middle panel: light curves in I-band from the TRT-SBO (Orange). The lower panel: light curves from TESS (Teal). Dashed lines show normalised baseline.

The eclipsing binary system RR Cae ($\alpha_{2000} = 04^h 21^m 5^s 56$, $\delta_{2000} = -48^\circ 39' 7''$ 06, $V=14.4$) is a detached binary which consists of a cool white dwarf and a late-type main-sequence star with a 7.289 hours orbital period (Bruch 1999) discovered by Krzeminski (1984). Bruch & Diaz (1998) provided the first photometric and spectrum report and found that the primary and secondary mass were 0.356 $M_\odot$ and 0.089 $M_\odot$, respectively. Bruch (1999) presented the double-lined spectrum of RR Cae which can be used to directly calculate the mass of components. The result shows that the primary mass was 0.467 $M_\odot$ and secondary mass was 0.095 $M_\odot$ with a secondary radius of 0.189 $R_\odot$. Bruch (1999) also showed that RR Cae is in previous common envelope phase.

Maxted et al. (2007) performed period change study of RR Cae and reported that its orbital period was not change by more than $5 \times 10^{-12}$ over a time-scale of a decade. In 2010, Parsons et al. found that the orbital period variation of RR Cae has a sinusoidal variation in the O-C diagram which has been confirmed by Qian et al. (2012). Qian et al. (2012) showed that the orbital period of the system increasing at rate $4.18 \pm 0.20 \times 10^{-12}$ s/s and the O-C diagram has a variation due to a third body component with a period 11.9 yr and an amplitude of 14.3 s. The third body component was suggested to be a circumbinary planet with mass $4.2 \pm 0.4 M_{\text{Jup}}$ and orbital semi-major axis $5.3 \pm 0.6$ AU. The spectroscopic study of Ribeiro et al. (2013) showed the primary star has the effective temperature of $7.260 \pm 250$ K with the metals spectrum caused by the secondary star wind.

In 2018, the NASA’s Transiting Exoplanet Survey Satellite (TESS) was launched in order to monitor nearby bright stars (Ricker et al. 2014). TESS provides a long-term photometric observation of RR Cae during 2018-2020. These data can be used to obtain an accurate sinusoidal variation model of the RR Cae’s O-C diagram, which leads to the precise mass and orbital parameters of circumbinary object. Moreover, combining with photometric data obtained from Thai Robotic Telescope Network (TRTN) and published spectroscopic data from the Ultraviolet and Visual Echelle Spectrograph (UVES) at the Very Large Telescope (VLT) (Maxted et al. 2007), the physical parameters of the binary can be revised.

In this paper, the physical parameters and orbital period variation analyses from additional observations of RR Cae are presented. The photometric data are obtained from TRTN and the published NASA’s TESS data in 2018-2020. These data are combined with the published spectrum from the European Southern Observatory.
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2 OBSERVATION AND DATA ANALYSIS

2.1 Photometric observation

In this work, the photometric data of RR Cae obtained from three telescopes: Thai Robotic Telescope at Spring Brook observatory (TRT-SBO), PROMPT-8 and NASA’s Transiting Exoplanet Survey Satellite (TESS) are presented. The observations were conducted between 2018 and 2020. A total of 430 primary eclipses were obtained from 96,510 data points. The observed photometric data are shown in Tables 1, 2 and Figure 1.

2.1.1 Thai Robotic Telescope at Spring Brook Observatory (TRT-SBO)

The Thai Robotic Telescope at Spring Brook observatory (TRT-SBO) is a 0.7-m telescope located at Spring Brook observatory, Australia. The telescope is a part of Thai Robotic Telescope Network operated by the National Astronomical Research Institute of Thailand (NARIT). RR Cae was observed with the 4096 × 4096 pixels ProLine PL16803 Monochrome CCD camera on the 0.7-m telescope in R-band between 2019 October and 2019 November, and I-band between 2020 October and 2020 November with exposure time 30 s for both filters. In total, 5,373 images covered 14 primary eclipses in R-band and 2,057 images covered 6 primary eclipses in I-band were obtained.

We used the IRAF package, Astrometry.net (Lang et al. 2010) and Sextractor (Bertin & Arnouts 1996) for reduction, astrometric calibration and photometry the images. The light curves were obtained using aperture photometry with an adaptive scaled aperture based on the seeing in an individual image.

2.1.2 PROMPT-8

We observed RR Cae with the CCD camera array size 2048 × 2048 pixel with a scale of 0.624 arcsec/pixel on the PROMPT-8 telescope, a 0.6-m robotic telescope at Cerro Tololo Inter-American Observatory (CTIO), Chile, in R-filter between 2018 September and 2018 October. The exposure time was set to be 120 s, 4 primary eclipses from 199 images were obtained. In order to obtain light curves, the same procedure as the TRT-SBO was performed.

2.1.3 Transiting Exoplanet Survey Satellite (TESS)

RR Cae has been monitored by the Transiting Exoplanet Survey Satellite (TESS) in Sector 3, 4, 5, 30 and 32 between 2018 and 2020. The calibrated data were obtained from Mikulski Archive for Space Telescopes². The exposure time of a image was 120 s in TESS filter (600-1,000 nm). The out-of-eclipse data above 3 standard deviation are cut. After the cut, the 88,881 data are used for the analyses.

Table 2. The photometric data of RR Cae.

| BJD     | Normalized flux | Error | Filters | Observatory |
|---------|----------------|-------|---------|-------------|
| 2458761.02008 | 1.006 | 0.013 | R       | TRT-SBO     |
| 2458761.02057 | 0.989 | 0.013 | R       | TRT-SBO     |
| 2458761.02621 | 1.003 | 0.014 | R       | TRT-SBO     |
| 2458761.02668 | 1.004 | 0.013 | R       | TRT-SBO     |
| ...       | ...    | ...   | ...     | ...         |
| 2458761.03362 | 0.998 | 0.006 | I       | TRT-SBO     |
| 2458761.03415 | 1.013 | 0.006 | I       | TRT-SBO     |
| 2458761.03476 | 1.005 | 0.006 | I       | TRT-SBO     |
| 2458761.03525 | 1.016 | 0.006 | I       | TRT-SBO     |
| ...       | ...    | ...   | ...     | ...         |
| 2459155.03615 | 0.982 | 0.005 | TESS    | TESS        |
| 2459155.03754 | 0.997 | 0.005 | TESS    | TESS        |
| 2459155.03893 | 0.992 | 0.005 | TESS    | TESS        |
| 2459155.04032 | 0.979 | 0.005 | TESS    | TESS        |
| ...       | ...    | ...   | ...     | ...         |
| 2458383.66410 | 0.996 | 0.003 | R       | PROMPT-8    |
| 2458383.66566 | 1.009 | 0.003 | R       | PROMPT-8    |
| 2458383.66725 | 1.011 | 0.003 | R       | PROMPT-8    |
| 2458383.66882 | 1.013 | 0.003 | R       | PROMPT-8    |
| ...       | ...    | ...   | ...     | ...         |
| 2459155.03899 | 0.992 | 0.005 | TESS    | TESS        |
| 2459155.04032 | 0.979 | 0.005 | TESS    | TESS        |

Note. The full table is available in electronic format in the full form.

2.2 Spectroscopic data

Between 2005 October 15th and 2005 November 16th, RR Cae have been observed with the Ultraviolet and Visual Echelle Spectrograph (UVES) located at the Nasmyth B focus of UT2 of the Very Large Telescope (VLT) (Dekker et al. 2000; Maxted et al. 2007). The published data are obtained from the European Southern Observatory (ESO) Science Archive Facility³. The exposure time of each frame were set to be 2,720 sec. The data provide the spectrum resolution of 21,000 covered wavelength between 328.1 and 668.8 nm with signal-to-noise ratio (S/N) between 15 and 64.

From the published spectra, both emission and absorption lines of Ca II (H line, 3968Å) and Ca II (K line, 3934Å) are detected with the emission lines of Hα (6563Å), Hγ (4861Å), Hγ (4340Å), Hδ (4102Å) and absorption lines of Ca I (4227Å). The spectra are used for computing the radial velocity of each component, which can be derived from the relation,

\[ v_r = \frac{\lambda - \lambda_0}{\lambda_0} \times c, \]

where, \( \lambda \) is an observed wavelength, \( \lambda_0 \) is the reference wavelength of each line above and \( c \) is the speed of light (299,792.458 km s⁻¹).

In order to find the observed wavelength of the spectra, the emission and absorption spectra are fitted by the Gaussian function. The calculated radial velocities are performed with barycentric correction, simultaneously. In Figure 2, the phase curves of the spectra

² Mikulski Archive for Space Telescopes: https://mast.stsci.edu/
³ http://archive.eso.org/scienceportal/home

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Figure 2. The spectrum of RR Cae of (A.) $H_\alpha$, (B.) $H_\beta$, (C.) $H_\gamma$, (D.) $H_\delta$, (E.) Ca II (H line), (F.) Ca II (K line) and (G.) Ca I. The blue dashed lines show the fitting of primary component spectrum and the red dashed-dot line show the fitting of secondary component spectrum (See Section 2.2).

are shown. The phases of system were calculated from the linear ephemeris equation of Maxted et al. (2007),

$$\text{Min.I} = \text{BJD} \ 245 \ 1523.048 \ 567 + 0.303 \ 703 \ 6366 \ d \times E + 0.303 \ 703 \ 6366 \ d \times E,$$

where $E$ is an observed epoch. The radial velocities of the spectra are shown in Table 3.

In order to find the semi-amplitude, $K$, and the systemic velocity, $\gamma$, the system is assumed to be a circular orbit. The semi-amplitude and the systemic velocity are obtained from

$$v_r = \gamma + K \sin(2\pi \phi),$$

where $\phi$ is an observed orbital phase. The fitted values are shown in Table 4. The spectra show that the average semi-amplitudes of the primary component and the secondary component are $72.9 \pm 0.1$ km s$^{-1}$ and $195.5 \pm 0.2$ km s$^{-1}$, respectively. The obtained semi-amplitude of the primary component is smaller than the semi-amplitude $79.3 \pm 3.0$ km s$^{-1}$ combined from $H_\beta$, $H_\gamma$ and $H_\delta$ lines by Maxted et al. (2007). However, the obtained value is compatible with the semi-amplitude value of Ribeiro et al. (2013), $74.1 \pm 2.3$ km s$^{-1}$. For the secondary component, Maxted et al. (2007) computed the cross-correlation in the region between 8440 and 8930 Å and obtained the semi-amplitude of the secondary which is $190.2 \pm 3.5$ km s$^{-1}$. Moreover, they also obtained the semi-amplitude of the emission lines of $196.3 \pm 1.4$ km s$^{-1}$. Ribeiro et al. (2013) also reported the value of the secondary component semi-amplitude of $194.4 \pm 0.7$ km s$^{-1}$. The fitted semi-amplitudes of the emission lines of secondary component in Table 4 are agreeable with the values from the works by Maxted et al. (2007) and Ribeiro et al. (2013).

In this work, there are only three spectra: $H_\alpha$, Ca II (K line) and Ca II (H line), which can be used to calculate radial velocity solutions for both primary and secondary components. From these three spectra, the average mass ratio of the RR Cae system, $q = K_{\text{primary}}/K_{\text{secondary}}$, is $0.373 \pm 0.001$. The mass ratio is smaller than the ratio computed by Maxted et al. (2007), $0.41 \pm 0.03$ km s$^{-1}$, but the computed ratio is compatible with the ratio of Ribeiro et al. (2013), $0.376 \pm 0.005$ km s$^{-1}$.
Table 3. Radial velocities with errors of the RR Cae spectra from the VLT.

| BJD         | Phase | Hα,1,abs | Hα,2,abs | Hβ,1,abs | Hβ,2,abs | Hγ,1,abs | Hγ,2,abs | CaII(H)1,abs | CaII(H)2,abs | CaII(K)1,abs | CaII(K)2,abs | CaI1,abs |
|-------------|-------|----------|----------|----------|----------|----------|----------|--------------|--------------|--------------|--------------|----------|
| 2453658.75604 | 0.20910 | 15.7 ± 0.9 | 261.8 ± 0.2 | 293.7 ± 0.3 | 306.8 ± 0.5 | 256.3 ± 0.7 | 61.1 ± 0.3 | 310.2 ± 0.8 | 1.0 ± 0.5 | 248.1 ± 0.4 | 7.7 ± 0.4 |
| 2453658.76087 | 0.22499 | 13.9 ± 0.9 | 266.2 ± 0.2 | 298.7 ± 0.3 | 312.7 ± 0.5 | 258.9 ± 0.7 | 59.5 ± 0.3 | 314.4 ± 0.8 | -1.6 ± 0.4 | 250.8 ± 0.4 | 6.2 ± 0.5 |
| 2453658.76561 | 0.24059 | 13.4 ± 0.8 | 268.1 ± 0.1 | 299.8 ± 0.3 | 313.2 ± 0.5 | 259.2 ± 0.8 | 58.7 ± 0.2 | 314.9 ± 0.8 | -1.6 ± 0.4 | 252.3 ± 0.4 | 6.0 ± 0.5 |
| 2453658.77043 | 0.25646 | 13.9 ± 0.8 | 267.9 ± 0.1 | 298.6 ± 0.3 | 312.7 ± 0.6 | 259.8 ± 0.9 | 59.4 ± 0.3 | 315.2 ± 0.8 | -1.2 ± 0.5 | 252.1 ± 0.4 | 4.9 ± 0.4 |
| 2453658.77523 | 0.27228 | 14.4 ± 0.9 | 266.4 ± 0.2 | 297.2 ± 0.3 | 312.2 ± 0.6 | 256.8 ± 0.8 | 59.9 ± 0.4 | 314.2 ± 1.0 | -0.7 ± 0.4 | 249.5 ± 0.5 | 5.6 ± 0.4 |
| 2453658.78005 | 0.28815 | 15.8 ± 0.8 | 262.7 ± 0.2 | 293.8 ± 0.3 | 308.5 ± 0.6 | 253.0 ± 1.0 | 61.6 ± 0.3 | 310.5 ± 1.0 | 0.7 ± 0.5 | 247.5 ± 0.5 | 8.3 ± 0.4 |
| 2453658.78488 | 0.30406 | 18.8 ± 0.8 | 256.8 ± 0.2 | 287.4 ± 0.3 | 301.8 ± 0.7 | 246.1 ± 1.0 | 63.7 ± 0.3 | 301.8 ± 1.3 | 2.8 ± 0.5 | 240.6 ± 0.5 | 10.7 ± 0.4 |
| 2453658.78970 | 0.31992 | 21.3 ± 0.6 | 249.5 ± 0.2 | 279.7 ± 0.3 | 293.9 ± 0.7 | 239.0 ± 1.1 | 67.1 ± 0.4 | 298.2 ± 1.4 | 5.7 ± 0.4 | 233.7 ± 0.5 | 13.4 ± 0.4 |

Note. The (1, abs) is the primary component and absorption line. The (1, emi) is the primary component and emission line. The (2, emi) is the secondary component and emission line.
In this work, the average gravitational redshift of three aforementioned lines: Hα, Ca II (K line) and Ca II (H line), is around 16.2 ± 0.4 km s⁻¹. In order to eliminate the systemic velocity variations, only the H alpha lines spectra are used in the following work as the highest signal-to-noise spectrum lines.

The radial velocity solutions of the systemic velocities show the variations. However, there is no relation between the variations and the wavelength of the spectra, as shown in Figure 3. Maxted et al. (2007) suggested that these variations might be caused by:

- The gravitational redshift effect from the primary component (Ribeiro et al. 2013).
- The asymmetry for the higher Balmer lines caused the pressure shifts from the Stark effect (Grabowski et al. 1987).
- The tilt in the continuum will result in a systematic error in the value of primary component measured from such broad lines (Maxted et al. 2004).

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### Table 4. The absolute radial velocity solutions of features spectra of RR Cae.

| Features | Wavelength (Å) | K (km s⁻¹) | y (km s⁻¹) |
|----------|----------------|------------|------------|
| **Primary component** | | | |
| Hα | 6594.9 | 72.7 ± 0.4 | 87.0 ± 0.2 |
| Ca II (K line) | 3934.9 | 73.1 ± 0.2 | 70.4 ± 0.2 |
| Ca II (H line) | 3969.7 | 73.1 ± 0.2 | 131.7 ± 0.2 |
| Ca I (4227Å) | 4228.1 | 72.6 ± 0.2 | 77.9 ± 0.1 |
| **Secondary component** | | | |
| Hα | 6594.5 | 195.4 ± 0.3 | 70.3 ± 0.2 |
| Hδ | 4862.6 | 195.2 ± 0.4 | 100.6 ± 0.3 |
| Hγ | 4341.6 | 195.7 ± 0.5 | 112.4 ± 0.4 |
| Hδ | 4102.8 | 195.6 ± 0.4 | 60.5 ± 0.4 |
| Ca II (K line) | 3934.7 | 195.7 ± 0.5 | 54.8 ± 0.4 |
| Ca II (H line) | 3969.5 | 195.2 ± 0.7 | 115.5 ± 0.4 |

The gravitational redshift effect from the primary component might cause the variation. Intriguingly, Ribeiro et al. (2013) suggested that a polar feature in the brightness distribution of the secondary component, similar to that observed in other fast-rotating stars, is the origin of the variation. However, the asymmetric shape of the data taken in 2018 differs from the data in 2020, which can indicate the spot evolution on the dwarf. Therefore, two spot models corresponding to different years on the secondary component are modelled in this work. A spot model consists of four parameters: spot temperature in unit of stellar temperature ($\delta T_{\text{eff}}$), spot radius ($\psi$), spot co-latitude ($\beta$) and spot longitude ($\lambda$).

Moreover, the TESS data in 2018 and 2020 show different primary eclipse depths which might be caused by the large TESS pixel size. As the large TESS pixel size, the flux background source might be blended in the RR Cae’s photometric pixels. In order to

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### 3 RR CAE PHYSICAL PARAMETERS

In order to obtain physical parameters of RR Cae, the PHOEBE (PHysics Of Eclipsing BinariEs) version 2.3 (Prša et al. 2016; Conroy et al. 2020) was used to analyse the light curves. The PHOEBE code is based on the Wilson-Devinney code (Wilson & Devinney 1971). The encee (Foreman-Mackey et al. 2013) python module that implements a Markov Chain Monte Carlo (MCMC) sampling algorithm was used with the PHOEBE in order to estimate the best-fit values of the parameters. The log-likelihood function is given by

\[
\ln L(D|M) = -\frac{1}{2} \sum \ln(2\pi \sigma^2) + \sum \ln(2\pi \sigma^2) + \chi^2.
\]

The $\chi^2$ function is calculated by

\[
\chi^2 = \left( \sum \left( D_{\text{obs},LC}^n - M_n^{\text{model},LC}(\theta)^2 \right) \frac{1}{\sigma^2} \right) + \left( \sum \left( D_{\text{obs},RV}^n - M_n^{\text{model},RV}(\theta)^2 \right) \frac{1}{\sigma^2} \right),
\]

where $D_{\text{obs}}$ is the observed flux density, $M_{\text{model}}$ is the modeled flux density and $\sigma^2$ is the variance of flux measurement.

Due to the quality of data from the PROMPT-8 and TRT-SBO, only the TESS light curves are selected to perform the fitting along the radial velocity data from the VLT. In order to reduce the computation time, the light curves are phased-binned, with a sampling orbital period in the MCMC, over 0.001 phase unit. The fitting parameters and their priors are shown in Table 5. The limb-darkening coefficients table of the TESS filters for the primary and the secondary are obtained from Claret et al. (2020a) and Claret (2018), respectively. The effective temperature ($T_{\text{eff}}$) and the surface gravity ($\log g$) of each component are used to interpolate the limb-darkening coefficients.

The TESS phased-binned light curve shows asymmetric maxima (Figure 5). The out-of-eclipse variations in the light curve has been discussed in Bruch & Diaz (1998) and Bruch (1999). The variations might be caused by:

- Tidal variations of a component that is not small compared with its Roche lobe.
- The reflection of light of one component of the other component.

Maxted et al. (2007) suggested that the spot on the primary component might cause the variation. Intriguingly, Ribeiro et al. (2013) suggested that a polar feature in the brightness distribution of the secondary component, similar to that observed in other fast-rotating stars, is the origin of the variation. However, the asymmetric shape of the data taken in 2018 differs from the data in 2020, which can indicate the spot evolution on the dwarf. Therefore, two spot models corresponding to different years on the secondary component are modelled in this work. A spot model consists of four parameters: spot temperature in unit of stellar temperature ($\delta T_{\text{eff}}$), spot radius ($\psi$), spot co-latitude ($\beta$) and spot longitude ($\lambda$).

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Figure 4. RR Cae phase-folded light curves and Hα radial velocity curves with the best fit models from the PHOEBE code. The light curves are obtained in R-band from PROMPT-8 (Dark red), R-band from TRT-SBO (Red), I-band from TRT-SBO (Orange), TESS in 2018 (Green) and 2020 (Purple). The Hα radial velocity curves of primary and secondary components are obtained from the VLT data of Maxted et al. (2007). The solid lines represent the fitted model from the PHOEBE code. The dashed lines show normalised baseline.

reduce the blending effect, the model with a blending background flux is adopted:

\[ F_{\text{normalized}} = F_{\text{binary}} + F_{\text{background}} \]

where \( F_{\text{normalized}} \) is a normalized observed flux, \( F_{\text{binary}} \) is a flux from binary system modelled by PHOEBE code and \( F_{\text{background}} \) is a blending factor in the unit of the normalized observed flux.

As the system show a large eclipse timing variation (Qian et al. 2012), the time of minima in TESS data that observed in 2018 and 2020 differ from time of minima in 2005, when the spectra are obtained from the VLT. The reference time of minimum, \( T_0 \), is obtained from the fitting of only TESS light curves. The radial velocities data from the VLT use another reference time, \( T_{0,rv} \).

The burn-in and production chains are set to be 2,000 and 3,000 steps, respectively. The primary component temperature is fixed as the obtained temperature from Ribeiro et al. (2013). The physical parameters results for the MCMC analyses are shown in Figures 6.
Table 5. The physical parameters of RR Cae.

| Parameters          | Prior range          | Fitted value          | Literature value                          |
|---------------------|----------------------|-----------------------|------------------------------------------|
|                     |                      |                       | Ribeiro et al. (2013)                    |
|                     |                      |                       | Maxted et al. (2007)                     |
|                     |                      |                       | Bruch (1999)                             |
|                     |                      |                       | Bruch & Diaz (1998)                      |
| **System parameters** |                     |                       |                                        |
| $T_0$ (BJD-2450000) | 1523.04 – 1523.04    | 1523.0493$^{+0.0003}_{-0.0004}$ | 1523.048567 ± 0.000019                   |
| $P_0$ (d)           | 0.303700 – 0.303708  | 0.30370361$^{+1.6E-8}_{-1.8E-8}$ | 0.303703686 ± 0.0000000047                |
| $a$ (R$_\odot$)     | 1.5 – 1.7            | 1.62$^{+0.003}_{-0.003}$ | 1.370 ± 0.007                           |
| $T_2$ (K)           | Fixed                | 7.260$^{+0.000}_-$    | 3.500$^\dagger$                         |
| $q$                 | 0.30 – 0.45          | 0.371$^{+0.005}_{-0.005}$ | 0.43 ± 0.02                             |
| $M_1$ (M$_\odot$)   | –                    | 0.45$^{+0.002}_{-0.002}$ | 0.440 ± 0.023                           |
| $M_2$ (M$_\odot$)   | –                    | 0.166$^{+0.000}_{-0.000}$ | 0.15                                    |
| $R_1$ (R$_\odot$)   | 0.0001 – 0.1         | 0.020$^{+0.000}_{-0.000}$ | (0.015 – 0.016) ±0.0004                  |
| $R_2$ (R$_\odot$)   | 0.15 – 0.40          | 0.24$^{+0.006}_{-0.006}$ | 0.21                                    |

**Light curve parameters**

| $F_{\text{background,TESS2018}}$ | 0.0 – 0.5 | 0.0$^{+0.04}_{-0.02}$ | –                                        |
| $F_{\text{background,TESS2020}}$ | 0.0 – 0.5 | 0.09$^{+0.04}_{-0.02}$ | –                                        |
| $\sigma_{\text{TESS}}$        | 0.0 – 0.2 | 1 × 10$^{-7}$          | –                                        |

**Radial velocity parameters**

| $T_0$ (BJD-2450000) | 1523.04 – 1523.06 | 1523.0486$^{+0.0001}_{-0.0001}$ | –                                        |
| $\gamma_{\text{primary}}$ (km s$^{-1}$) | 85 – 95 | 87.0$^{+0.3}_{-0.2}$ | 95.1 ± 0.3                               |
| $\gamma_{\text{secondary}}$ (km s$^{-1}$) | 65 – 75 | 70.1$^{+0.2}_{-0.2}$ | 77.9 ± 0.4                               |
| $\sigma_{\text{primary}}$ (km s$^{-1}$) | 0 – 10 | 4.3$^{+0.3}_{-0.3}$ | 4.3 ± 0.3                                |
| $\sigma_{\text{secondary}}$ (km s$^{-1}$) | 0 – 10 | 3.8$^{+0.6}_{-0.4}$ | 3.8 ± 0.6                                |

Remarks: $^\dagger$: The primary component temperature is obtained from Ribeiro et al. (2013). $^*$: Fixed value.
The dwarf over a time span of two years. However, the decrease in the effective temperature of the whole dwarf ($T_2$) can be explained such the evolution. The change in the effective temperature can be also explained the difference in eclipse depths of the data in the same filter in two difference years: TESS data in 2018 and 2020, and R-band data in 2018 (PROMPT-8) and 2019 (TRT-SBO).

4 RR CAE TIME OF MINIMA AND ITS CIRCUMBINARY PLANETS

In Section 2, 20, 4, and 406 epoch light curves from the TRT-SBO, PROMPT-8 and TESS are provided. In order to obtain the minimum time of each eclipse, the synthetic light curves with 0.1 s time resolution are simulated in three filters: R-, I- and TESS bands, with PHOEBE using the parameters in Table 5. The light curves in R- and I-bands are simulated with the interpolated limb-darkening coefficients from Claret et al. (2020b) and Claret (1998) for primary and secondary stars, respectively. The R- and I-bands light curves are not added a spot in the secondary, as the spot in varying with the time. The R-band data from PROMPT-8 and TRT-SBO show different eclipse depths, therefore difference $F_1$ is adopted. The simulated light curve are interpolated with the first order spline interpolation and performed MCMC fitting to find the minimum time of each eclipse. Due to the poor quality of out-of-eclipse in R- and I- bands data, only the data with phases between -0.1 and 0.1 are used. The fitted times of minima are shown in Table 7.

The times of minima are combined with primary eclipse times from Parsons et al. (2010) and Qian et al. (2012). The times of minima are fitted with the binary period change model combining with light travel time effect from a circumbinary object as follow,

$$t = t_0 + P \cdot E + \frac{1}{2} \frac{dP}{dE} E^2 + \sum_{n=1}^{N} \tau_n,$$

where $t$ is the time of minimum, $t_0$ is the reference time of minimum, $P$ is the orbital period of the binary system, $E$ is the observed epoch and $\tau$ is the sinusoidal term for light travel time (LTT) effect by the third component (Irwin 1952).

$$\tau_n = K_n \sin\left(\frac{2\pi}{P_n} E + \phi_n\right) = \frac{a_n \sin i_n}{c} \left[\frac{1 - e_n^2}{1 + e_n \cos f_n \sin(f_n + \omega_n)}\right],$$

where $K_n$ is the amplitude of sinusoidal variation, $P_n$ is the orbital period $\phi_n$ is the orbital phase, $a_n \sin i_n$ is the projected semi-major axis, $e_n$ is the eccentricity, $f_n$ is the true anomaly of the binary orbit around the center of mass of the system and $\omega_n$ is the longitude of the periastron of the $n^{th}$ body. $c$ is the speed of light.

In this work, two cases: (i) one circumbinary object case and (ii) two circumbinary objects case, are focused. In both cases, the circumbinary objects are assumed to be circular orbits ($e = 0$). The emcee module is used to perform the fitting with the MCMC sampling (Foreman-Mackey et al. 2013). The fitting results are shown in Table 8. The O-C diagram of RR Caes with the modelled time of minima from both models are presented in Figure 8.

From the parameters in Table 8, the system shows the orbital decay with a rate of $-1.2^{+0.1}_{-0.3} \times 10^{-12}$ d/cycle for one circumbinary object model and $0.5^{+0.8}_{-0.7} \times 10^{-12}$ d/cycle for two circumbinary objects.
objects model. The cause of the orbital period decreasing in the one circumbinary object model may be due the angular momentum loss gravitational radiation or/and magnetic braking. However, it might be only a part of a long-period cyclic variation, revealing the presence of the second planet in the planetary system (Qian et al. 2011).

Comparing the obtained LTT effect results with the RR Cae Ab model of Qian et al. (2012), the model of a circumbinary planet with the planet mass of $4.2 \, M_{\text{Jup}}$ and orbital period of 11.9 yr provides the reduced chi-squared ($\chi^2$) of $1.42 \times 10^{23}$. While, the $\chi^2$ of our one circumbinary object model and two circumbinary objects model are 8.55 and 4.82, respectively. Therefore, the variation of minimum times indicate that there are two circumbinary objects in the system. The model shows that the $a_n \sin i_n$ values of the third component is $0.024 \pm 0.002 \, \text{AU}$ and the fourth component is $0.040 \pm 0.009 \, \text{AU}$, which can infer to the orbital periods of $15.0 \pm 0.6 \, \text{yr}$ and $39 \pm 5 \, \text{yr}$, respectively. The mass function can be written as

$$f'(m, n) = \frac{(M_n \sin i_n)^3}{(M_1 + M_2 + \sum_{n=1}^{N} M_n)^2} = \frac{4\pi^2}{GP_n} \times (a_n \sin i_n)^3$$

where $G$ is gravitational constant. By considering the $M_1 = 0.453 \pm 0.002 \, M_{\odot}$ and $M_2 = 0.168 \pm 0.001 \, M_{\odot}$ from Section 3, the third and the fourth bodies have the mass function $f(m, 3) = 6 \times 10^{-8} \, M_{\odot}$ and $f(m, 4) = 4 \times 10^{-8} \, M_{\odot}$. The calculated mass of the third and the fourth bodies are $M_3 \sin i_3 = 3.0 \pm 0.3 \, M_{\text{Jup}}$ and $M_4 \sin i_4 = 2.7 \pm 0.7 \, M_{\text{Jup}}$. From the orbital period $39 \pm 5 \, \text{yr}$
of the second planet, it is the known circumbinary planet with the longest orbital period discovered by the eclipse timing technique.

5 CONCLUSIONS

In this work, we observed and studied a detached binary system, RR Cae. A 4.2 Jupiter mass planet with orbital period 11.9 yr was discovered by Qian et al. (2012) using the light travel time effect from the circumbinary planet. Optical multimeter observations of the binary were obtained with the 0.7-m Thai Robotic Telescope at Spring Brook Observatory in R-band I-bands, the 0.6-m PROMPT-8 telescope in R-band and TESS in 2018-2020. 430 primary eclipses were obtained.

In order to revise the RR Cae parameters, we use the photometric data combining with the spectra from the VLT (Maxted et al. 2007). There are only three spectra: H α, Ca II (K line) and Ca II (H line), which can be used to calculate radial velocity solutions for both primary and secondary components, with the average gravitational redshift around 16.2 ± 0.4 km s\(^{-1}\). Only the H alpha lines spectra are used in this work as a highest signal-to-noise spectrum lines. The data are analysed with the PHOEBE code (Prša et al. 2016) and performed the fitting with the MCMC method. The derived data from the analysis provides an orbital period \( P = 0.30370361^{+0.0000005}_{-0.0000005} \) d, orbital inclination \( i = 82.9^{+0.9}_{-0.8} \) deg and mass ratio \( q = 0.371^{+0.002}_{-0.003} \), which is corresponding to the primary component mass, \( M_1 = 0.453^{+0.002}_{-0.002} \) M\(_\odot\) and the secondary component mass, \( M_2 = 0.168^{+0.001}_{-0.001} \) M\(_\odot\). The TESS data show the evolution of a spot on the secondary component. The TESS data in 2018 shows a hot spot near the stellar pole. However, a cold spot at the moderate latitude is preferred from the data in 2020. The model can be explained by the variation of the secondary effective temperature.

The eclipse timing variation of RR Cae shows that there are cyclic variations on the observed timing. The times of minima are fitted with binary period change model combining with light travel time effect from two circumbinary objects. The O-C diagram of RR Cae shows the periodic variations that have the amplitudes of 12±1 s.
### Table 7. The lists of primary eclipse time of minima of RR Cae.

| Observatory | Filter | $t_0$ (+240000 BJD) | Error (d) | Ref |
|-------------|--------|---------------------|-----------|-----|
| LCO         | B      | 45927.916650        | 0.000116  | (1) |
| SAAO        | W      | 49721.47852         | 0.000003  | (2) |
| SAAO        | W      | 49722.389675        | 0.000003  | (2) |
| SAAO        | W      | 49726.337828        | 0.000004  | (2) |
| LNA         | R      | 50681.789580        | 0.000116  | (3) |
| LNA         | R      | 50684.827030        | 0.000116  | (3) |
| LNA         | R      | 50687.863610        | 0.000116  | (3) |
| LNA         | R      | 50688.774700        | 0.000116  | (3) |
| LNA         | R      | 50688.774770        | 0.000116  | (3) |
| SAAO        | W      | 50700.619201        | 0.000004  | (2) |
| SAAO        | W      | 50750.426547        | 0.000002  | (2) |
| SAAO        | W      | 50753.463587        | 0.000002  | (2) |
| SAAO        | W      | 50756.500622        | 0.000002  | (2) |
| SAAO        | W      | 51045.626463        | 0.000002  | (2) |
| SAAO        | I      | 51523.35226         | 0.00003   | (2) |
| SAAO        | I      | 51524.56706         | 0.00003   | (2) |
| CASLEO      | N      | 55891.82585         | 0.00002   | (5) |
| CASLEO      | V      | 55891.82585         | 0.00002   | (5) |
| CASLEO      | V      | 55892.73695         | 0.00002   | (5) |
| PROMPT-8    | R      | 58383.71392         | 0.00001   | (6) |
| PROMPT-8    | R      | 58390.69004         | 0.00001   | (6) |
| PROMPT-8    | R      | 58393.73611         | 0.00001   | (6) |
| PROMPT-8    | R      | 58397.68424         | 0.00001   | (6) |
| TRT-SBO     | R      | 58833.19527         | 0.00002   | (6) |
| TRT-SBO     | R      | 58762.12863         | 0.00003   | (6) |
| TRT-SBO     | R      | 58765.16564         | 0.00002   | (6) |
| TRT-SBO     | R      | 59206.14337         | 0.00005   | (6) |
| SAAO        | I      | 59155.12114         | 0.00003   | (6) |
| SAAO        | I      | 59168.18041         | 0.00005   | (6) |
| TESS        | TESS   | 58382.19539         | 0.00003   | (6) |
| TESS        | TESS   | 58382.49913         | 0.00002   | (6) |
| TESS        | TESS   | 58383.71393         | 0.00003   | (6) |

**Note:**
Observation: LCO: Las Campanas Observatory, Cerro Las Campanas, Chile. SAAO: South African Astronomical Observatory, Sutherland, South Africa. LNA: Laboratorio Nacional de Astrofísica, Pico dos Dias, Brazil. VLT: Very Large Telescope. CASLEO: Jorge Sahade telescope at Complejo Astronomico El Leoncito, San Juan, Argentina.

**References:**

(1) Krzeminski (1984), (2) Maxted et al. (2007), (3) Bruch & Diaz (1998), (4) Parsons et al. (2010), (5) Qian et al. (2012) and (6) this study

and 20 ± 5 s with period 15.0 ± 0.5 yr and 39 ± 5 yr, respectively. The variation can be caused by two circumbinary planets with the masses $M_3 \sin i_3 = 3.0 \pm 0.3 M_{\text{Jup}}$ and $M_4 \sin i_4 = 2.7 \pm 0.2 M_{\text{Jup}}$. In order to confirm the spot evolution and the presence of second planet in the system, long-term continuously monitoring of the RR Cae is still needed in the future.

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**DATA AVAILABILITY**

The TRT-SBO, PROMPT-8 and TESS photometric data, the VLT radial velocities data and the primary eclipse time of minima of RR Cae are available in the article and in the online supplementary material.

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Table 8. The parameters of the circumbinary planets in the RR Cae system.

| Parameters | Qian et al. (2012) | One object model | Two objects model |
|------------|--------------------|------------------|-------------------|
| $t_0$ (+2451523.0485 d) | $-0.4(\pm 0.5) 	imes 10^{-5}$ | $8.6^{+0.8}_{-0.6} 	imes 10^{-5}$ | $17^{+2}_{-1} 	imes 10^{-5}$ |
| $P$ (+0.3037036 d) | $5.79(\pm 0.08) 	imes 10^{-8}$ | $6.5^{+0.3}_{-0.2} 	imes 10^{-8}$ | $3.0^{+0.5}_{-0.4} 	imes 10^{-8}$ |
| $dP/dE$ (d/cycle) | $1.27(\pm 0.06) 	imes 10^{-12}$ | $-1.2^{+0.1}_{-0.2} 	imes 10^{-12}$ | $0.0^{+0.5}_{-0.3} 	imes 10^{-12}$ |
| $\omega_n$ (rad/epoch) | $-3.78 \pm 0.13$ | $-3.78^{+0.02}_{-0.02}$ | $-3.86^{+0.04}_{-0.04}$ |
| $\nu_n$ (rad) | $4.25(\pm 0.02) 	imes 10^{-4}$ | $3.16^{+0.04}_{-0.04} 	imes 10^{-4}$ | $3.5^{+0.1}_{-0.1} 	imes 10^{-4}$ |
| $P_n$ (yr) | $2.95 \pm 0.01$ | $-3.06^{+0.08}_{-0.08}$ | $2.9^{+0.1}_{-0.1}$ |
| $a_n$ ($i_n = 90^\circ$, AU) | $5.3 \pm 0.6$ | $5.55 \pm 0.05$ | $5.2 \pm 0.1$ |
| $M_n \sin i_n$ (M_Jup) | $4.2 \pm 0.4$ | $3.4 \pm 0.2$ | $3.0 \pm 0.3$ |

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**APPENDIX A: POSTERIOR PROBABILITY DISTRIBUTIONS OF MCMC FITTING PARAMETERS.**
Figure A1. Posterior probability distributions of the radial velocity parameters of RR Cae using the PHOEBE code with the MCMC fitting. The dashed-lines mark the 16th, 50th, and 84th percentiles.

Figure A2. Posterior probability distributions of the spot parameters on the RR Cae TESS light curves in 2018 using the PHOEBE code with the MCMC fitting. The dashed-lines mark the 16th, 50th, and 84th percentiles.
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Figure A3. Posterior probability distributions of the spot parameters on the RR Cae TESS light curves in 2020 using the PHOEBE code with the MCMC fitting. The dashed-lines mark the 16th, 50th, and 84th percentiles.
Figure A4. Posterior probability distributions of the O-C MCMC fitting parameters of the one circumbinary object model. The dashed-lines mark the 16th, 50th, and 84th percentiles.
Figure A5. Posterior probability distributions of the O-C MCMC fitting parameters of the two circumbinary objects model. The dashed-lines mark the 16th, 50th, and 84th percentiles.