Nucleon mass and pion loops: Renormalization †

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Abstract

Using Dyson–Schwinger equations, the nucleon propagator is analyzed nonperturbatively in a field–theoretical model for the pion–nucleon interaction. Infinities are circumvented by using pion–nucleon form factors which define the physical scale. It is shown that the correct, finite, on–shell nucleon renormalization is important for the value of the mass–shift and the propagator. For physically acceptable forms of the pion–nucleon form factor the rainbow approximation together with renormalization is inconsistent. Going beyond the rainbow approximation, the full pion–nucleon vertex is modelled by its bare part plus a one–loop correction including an effective ∆. It is found that a consistent value for the nucleon mass–shift can be obtained as a consequence of a subtle interplay between wave function and vertex renormalization. Furthermore, the bare and renormalized pion–nucleon coupling constant are approximately equal, consistent with results from the Cloudy Bag Model.

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1 Introduction

The interest in a consistent analysis of the effects of the pion cloud on nucleon properties is nurtured by two areas of recent research. For one, the extrapolation of nucleon lattice data, quenched or unquenched, to physical values of the quark (or pion) mass is mainly determined by pionic effects [1, 2]. Furthermore, these effects will pose constraints on the development of covariant nucleon models such as those presented in Refs. [3, 4]. The most basic observable of interest is the nucleon mass or, more precisely, the nucleon mass shift associated with the pion cloud. In a previous study [5], the connection of covariant Euclidean nucleon mass shift calculations to extant results from the Cloudy Bag Model (CBM) [6] and Chiral Perturbation Theory ($\chi$PT) [8, 9] was analyzed. A nonperturbative analysis using the Dyson–Schwinger (DS) equation in rainbow approximation (with no renormalization) suggested that almost all of the nucleon mass shift can be attributed to the covariant one–loop pion dressing. We will extend this analysis by supplementing the DS equation with correct on–shell renormalization conditions (Section 2).

To properly implement those, one has to forgo an angular approximation employed in Ref. [5]. In Section 3 we calculate a one–loop correction to the $\pi NN$ vertex which is subsequently employed in the DS equation. In the last section, we summarize and present our conclusions.

2 The model

We consider a pseudovector Lagrangian for the $\pi N$ interaction which reads in Euclidean space

\[
\mathcal{L}_{\pi NN} = \frac{g}{2M_N} \bar{\Psi} i\gamma^\mu \gamma_5 \tau \cdot (\partial_\mu \pi) .
\]  

The DS equation for the nucleon propagator $G$ is given by

\[
G^{-1}(p) = G_0^{-1}(p) + i\hat{\pi} \Sigma_V(p^2) + \Sigma_S(p^2) ,
\]

\[
i\hat{\pi} A(p^2) + B(p^2) = Z_2 (i\hat{\pi} + Z_M M) + Z_1 \int \frac{d^4k}{(2\pi)^4} \Gamma^0 G(k) \Gamma D(p - k) ,
\]

\[
\Gamma^0 = (\hat{\pi} - \hat{k})\gamma_5 \tau g .
\]  

Because of the small mass of the pion, we can approximate the full pion propagator $D(q)$ by the free scalar propagator, $D^0(q) = (q^2 + m_\pi^2)^{-1}$. $\Gamma^0$ and
\( \Gamma \) stand for the free and the full \( \pi NN \) vertex. The renormalization constants \( Z_1, Z_2 \) and \( Z_M \) refer to the \( \pi NN \) vertex, nucleon wave function and nucleon mass respectively. The relation to bare quantities is given by

\[
g = \frac{Z_2}{Z_1} \, g_{\text{bare}}, \quad M_N = \frac{M_{\text{bare}}}{Z_M}. \tag{4}
\]

To account for the compositeness of the particles, we introduce a form factor at each \( \pi NN \) vertex into this idealized field–theoretic model,

\[
\Gamma[0](p, k) \to \Gamma[0](p, k) \, u((p - k)^2). \tag{5}
\]

In the case where both nucleons are on shell \((p^2 = k^2 = -M_N^2)\), this form factor is most naturally related to the axial form factor of the nucleon. It is commonly parameterized as a dipole,

\[
u(q^2) = \left( \frac{\lambda^2 - m^2}{\lambda^2 + q^2} \right)^2, \tag{6}
\]

but we will also investigate (as in Ref. [5]) the exponential form

\[
u_{\text{exp}}(q^2) = \exp \left( -\frac{q^2 + m^2}{\Lambda^2} \right). \tag{7}
\]

From neutrino scattering experiments, the dipole width parameter (in the on–shell case) is determined as \( \lambda = 1.03 \pm 0.04 \) GeV [10]. This sets roughly the scale for our considerations.

Even after the introduction of form factors, we assume that it is sensible to extract nonperturbative information from the DS equation (2). We note that now the loop integral in the equation is convergent and all renormalization constants will therefore be finite. A commonly used approximation to it is the rainbow approximation \( \Gamma = \Gamma^0 \). The full \( \pi NN \) vertex \( \Gamma \) fulfills its own DS equation which takes the symbolic form:

\[
\Gamma = Z_1 \, \Gamma^0 + \Gamma^0 \left( G^{[1]} G^{[2]} \right) \, K_4. \tag{8}
\]

Here, \( K_4 \) is the full, off-shell, amputated \( N - N \) scattering matrix. One sees that the rainbow approximation neglects the last term and enforces \( Z_1 = 1 \).

We employ on–shell renormalization for the unknown nucleon propagator, \( G \). This requires that \( G \) has a pole at \( p^2 = -M_N^2 \) (the physical mass),

\[
M_N(Z_2 + \Sigma_V(-M_N^2)) = \Sigma_S(-M_N^2) + Z_2 Z_M M_N, \tag{9}
\]

\[
M_{\text{bare}} - M_N = M_N(Z_M - 1) = \frac{1}{Z_2} (M_N \Sigma_V(-M_N^2) - \Sigma_S(-M_N^2)). \tag{10}
\]
The renormalization constant $Z_2$ is determined by the condition that the residue at the pole be unity,

$$\left. \frac{\partial G^{-1}}{\partial \bar{p}} \right|_{\bar{p} = -M_N} = 1 \quad \to \quad (11)$$

$$-\Sigma_V(-M_N^2) + 2M_N^2 \frac{\partial \Sigma_V}{\partial p^2} \bigg|_{p^2 = -M_N^2} - 2M_N \frac{\partial \Sigma_S}{\partial p^2} \bigg|_{p^2 = -M_N^2} = Z_2 - 1. \quad (12)$$

Mass renormalization ensures that the self–energy has cuts starting from the physical thresholds. Furthermore one can show that the spectral densities of the solution $G$ multiplied by $Z_2$ are properly normalized (i.e. $Z_2$ can be interpreted as the probability of finding a “bare” nucleon inside the pion–dressed one).

In the exploratory study [5] the DS equation, Eq. (2), was solved in rainbow approximation after putting $Z_1 = Z_2 = Z_M = 1$ and disregarding the above renormalization conditions. The mass of the dressed nucleon was then found as the solution of $-M_B^2 A(-M_B^2) + B(-M_B^2) = 0$, $M_D < M_N$. In this simplified scenario it was possible to employ a certain angular approximation to the loop integral in eq. (4) to calculate all angular integrals analytically. However, if one incorporates the renormalization conditions (10) and (12), this angular approximation fails because it underestimates the slopes of $\Sigma_V$ and $\Sigma_S$ considerably (and these enter the expression for $Z_2$). As a result, we must resort to a numerical computation of one angular integral. Since we have to evaluate the renormalization conditions on the nucleon mass shell, it is necessary to continue the DS equation to complex momenta. This intricate procedure is outlined in Appendix A.

We illustrate the results for the mass shift in Fig. 1, employing (as in Ref. [3]) as the renormalized coupling constant $g = M_N/f_\pi$, i.e. $g_A = 1$. We chose a dipole form factor with cut-offs $\lambda$ in a range compatible with the measured axial form factor. Indeed, the mass shift for the unrenormalized rainbow treatment and the one in one–loop approximation are very close. In contrast, the mass shift in the renormalized rainbow treatment is larger by a factor which rises from 1.8 ($\lambda = 0.96$ GeV) to 2.9 ($\lambda = 1.2$ GeV). The explanation for this peculiar behaviour can be found in the right panel of Fig. 1 where the ratio of bare to renormalized coupling is plotted. This ratio equals $1/Z_2$ in the rainbow treatment and is considerably larger than 1. If one divides the DS equation, Eq. (2), by $Z_2$, one sees that the loop integral is proportional

\footnote{For technical reasons we consider only $\lambda > M_N$. Otherwise poles from the form factor enter the integration domain, adding technical complications which are of no interest here.}
Figure 1: Left panel: nucleon mass shift in the renormalized treatment (defined as $\delta M = M_N - M_{\text{bare}}$) and in the unrenormalized treatment (defined as $\delta M = M_D - M_N$) of the rainbow approximation. Right panel: the ratio $g_{\text{bare}}/g = Z_1/Z_2$.

to the bare coupling and therefore drives the mass shift to larger absolute values.

A repetition of the calculations with the physical $g_A = 1.26$ only aggravates the difference between renormalized and unrenormalized results. Whereas the mass shift in the unrenormalized rainbow treatment scales with $g_A^2$ (as the one-loop result), the mass shift in the renormalized case shoots up to values between 400 MeV and 1.9 GeV (depending on $\lambda$), because of the nonlinear nature of the DS equation.

The question arises whether such a strong renormalization of the pion–nucleon coupling constant, as visible in the rainbow solutions, is physically reasonable. An analysis of pionic corrections to nucleons in the CBM to one–loop order \[3, 11\] reveals that $Z_1 \approx Z_2$ (and therefore $g_{\text{bare}} \approx g$) if one includes the $\Delta$ resonance in the analysis of the self–energies and the pion–nucleon vertex \[6\]. This is a strong indication that the rainbow approximation is unreliable for this problem: there $Z_1 = 1$ is set artificially and $Z_2 \sim 0.6 \ldots 0.7$ for physical $\pi NN$ form factors.

Therefore, we will model the full $\pi NN$ vertex $\Gamma$ by the bare one plus a one-

\[^2\text{The analysis there corresponds to a non–relativistic expansion of positive–energy graphs in time–ordered perturbation theory.}\]
loop correction that includes the $\Delta$ in an effective manner, i.e. a spin–3/2 particle described by Rarita–Schwinger spinors. Ideally, we would like to employ self–consistent propagators of $N$ and $\Delta$ in this study, but solving a coupled system of two–loop DS equations for $N$ and $\Delta$ is beyond the scope of this study. Instead, we pursue a modest but nevertheless resource–consuming modification: the one–loop vertex correction to $\Gamma$ will be calculated using free $N$ and $\Delta$ propagators and the result will be inserted back into the DS eq. (3). By that token, the problem becomes numerically tractable, and we expect the bulk of the effect on the solutions to be buried in a reasonable estimate for $Z_1$.

3 Corrections to the $\pi NN$ vertex

We calculate the covariant one–loop correction to the $\pi NN$ vertex by including the $\Delta$ as an effective degree of freedom. Pictorially the equation for the vertex is displayed in Fig. 2 and as mentioned above we take for the nucleon and $\Delta$ propagators free spin–1/2 and spin–3/2 propagators, respectively,

$$G^{0}(p^2) = \frac{-i\not{p} + M_N}{p^2 + M_N^2},$$ (13)

$$G^{\mu\nu}_\Delta(p^2) = \frac{-i\not{p} + M_\Delta}{p^2 + M_\Delta^2} \not{p}^{\mu\nu},$$ (14)

$$\not{p}^{\mu\nu} = \delta^{\mu\nu} - \frac{1}{3} \gamma^{\mu} \gamma^{\nu} + \frac{2}{3} \frac{p^{\mu} p^{\nu}}{M_\Delta^2} - \frac{i}{3} \frac{p^{\mu} \gamma^{\nu} - p^{\nu} \gamma^{\mu}}{M_\Delta},$$ (15)

with $M_N = 0.94$ GeV and $M_\Delta = 1.23$ GeV are the physical masses of both particles. For the $\pi N\Delta$ and $\pi \Delta \Delta$ interactions we employ tree level vertices derived from the simplest covariant interaction Lagrangians, i.e.,

$$\mathcal{L}_{\pi N\Delta} = \frac{g_{\pi N\Delta}}{2M_N} \bar{\Psi}^\mu T_{1/2}^{3/2} \Psi \cdot (\partial_\mu \pi) + \text{h.c.} \rightarrow \Gamma_{\pi N\Delta}^0 = \frac{g_{\pi N\Delta}}{2M_N} iq^\mu T_{1/2}^{3/2},$$ (16)

$$\mathcal{L}_{\pi \Delta \Delta} = \frac{g_{\pi \Delta \Delta}}{2M_N} \bar{\Psi}^\rho i\gamma^\mu \gamma_5 T_{3/2}^{3/2} \Psi^\rho \cdot (\partial_\mu \pi) \rightarrow \Gamma_{\pi \Delta \Delta}^0 = \frac{g_{\pi \Delta \Delta}}{2M_N} \not{q} \gamma_5 T_{3/2}^{3/2}. $$ (17)
Here \( q \) is the momentum of the incoming pion. For the isospin \( 1/2 - 3/2 \) transition matrix we adopt the convention

\[
T^{3/2}_{1/2} = C^{3/2}_M^{1/2} m, \quad (18)
\]

As indices in the Clebsch–Gordan coefficient, \( M, m, k \) stand for the isospin–z components of \( \Delta, N \) and \( \pi \). Furthermore, the isospin matrices \( T^{3/2} \) are just the ones for the 4–dimensional \( SU(2) \) representation.

For the numerical values of the coupling constants \( g_{\pi N\Delta} \) and \( g_{\pi\Delta\Delta} \) we relate them to \( g \) via \( SU(6) \) quark model expressions. This of course leaves room to fine–tuning which is not the main interest here. To apply the quark model, we note the non–relativistic limit of both vertices,

\[
\Gamma^0_{\pi N\Delta} \to i\frac{g_{\pi N\Delta}}{2M_N} T^{3/2}_{1/2} (S^{3/2}_{1/2} \cdot q), \quad (19)
\]

\[
\Gamma^0_{\pi\Delta\Delta} \to -\frac{2i}{3} \frac{g_{\pi\Delta\Delta}}{2M_N} T^{3/2}_{3/2} (S^{3/2}_{3/2} \cdot q). \quad (20)
\]

The transition matrix \( S^{3/2}_{1/2} \) is identical to \( T^{3/2}_{1/2} \); it just refers to the spin degree of freedom. Comparing to the expressions in Ref. [6], we readily find

\[
g_{\pi N\Delta} = \sqrt{\frac{72}{25}} g, \quad (21)
\]

\[
g_{\pi\Delta\Delta} = \frac{6}{5} g. \quad (22)
\]

Here, we employ for the renormalized coupling constant the physical value \( g = g_A M_N / f_\pi \) with \( g_A = 1.26 \). Having fixed the strength of our interactions, we proceed now to the calculation of the renormalization constant \( Z_1 \). We choose to fix it at the virtual point where both nucleons and the pion are on shell, i.e.

\[
\bar{u}(k) \Gamma u(p) = \frac{g}{2M_N} \bar{u}(k) (\not{k} - \not{p}) \gamma_5 u(p)
\]

\[
[p^2 = k^2 = -M_N^2, (p - k)^2 = -m_\pi^2] \quad (23)
\]

This has the advantage that the \( \pi NN \) form factor should reduce to 1 at this point.

The resulting \( Z_1 \) as a function of the dipole cut–off is plotted in Fig. 3. From the right panel one can see that the loop–nucleons contribute less than 0.05 to
1 − Z_1. If this were the whole story, the rainbow treatment of the DS equation would seem to be justified, yielding mass-shifts > 500 MeV as demonstrated before! The bulk of the difference 1 − Z_1 comes from the two graphs with one intermediate Δ. The resulting bare coupling is somewhat lower than the renormalized one. If the Δ were also included in the nucleon self–energy, we would expect a somewhat larger bare coupling since its contribution lowers Z_2 additionally. Keeping that in mind, we expect the present results for the mass shift in the improved DS equation to be a lower bound.

Our one–loop improved model for the vertex can now be inserted into the DS eq. (2). Technically, we proceed by projecting the Dirac structure of the vertex onto basic covariant matrices,

\[ \Gamma_{\text{Dirac}}(p, k, l) = (i\gamma_5) V_1(p^2, k^2, p \cdot k) + (\not{p}\gamma_5) V_2(p^2, k^2, p \cdot k) + (\not{k}_T\gamma_5) V_3(p^2, k^2, p \cdot k) + (i\not{k}_T\not{p}\gamma_5) V_4(p^2, k^2, p \cdot k) , \]

\[ k_T = k - p \frac{\not{k} \cdot \not{p}}{p^2} , \]

from which the scalar functions V_{1...4} can be readily traced out, using the program FORM [12]. The three remaining scalar integrals are evaluated numerically. We calculate the functions V_i on a three–dimensional grid in the variables p^2, k^2 and \( \not{p} \cdot \not{k} \), needed for the solution of the DS equation.
The results for mass-shift and the wave–function renormalization constant $Z_2$ are depicted in Fig. 4. In the physically interesting region $\lambda \in [0.95, 1.05]$ GeV the mass shift stays rather flat, at a value around 200 MeV. For a harder form factor it actually drops, which can again be explained by looking at the ratio $g_{\text{bare}}/g$: beyond $\lambda \sim 1.1$ GeV $g_{\text{bare}}/g$ becomes less than 1/2. As explained before, this ratio enters the pion loop integral in the DS equation through $Z_1$. Since $Z_1$ itself is very low there, the one–loop treatment of the $\pi NN$ vertex can be questioned.

The feature of a plateau in the mass shift for a certain range of cut–off parameters (which happens to coincide with the experimental results for the axial form factor) is actually a desired one in effective models with cut–off functions, since it indicates the relative independence of the results on the specific choice of cut–off. Whether that still holds after self–consistent inclusion of the $\Delta$ resonance is an open question. Certainly, by the arguments given above, its proper inclusion should lead to $g_{\text{bare}}/g > 1$ and a larger mass shift. Keeping this in mind, a value $-\delta M \approx -\delta M_{1\text{-loop}} \approx 200$ MeV constitutes a lower bound.

In Fig. 5 we show the results for an exponential $\pi NN$ form factor. Since the exponential is an entire function, we can investigate also somewhat softer form factors without encountering analytical problems. One–loop mass shifts are the same for dipole and exponential, if $\lambda \approx 1.2\Lambda$. The result for the self–
consistent mass–shift looks overall very similar to the previous case. Its curve flattens around $\Lambda = 0.8$ GeV at a value of approximately 190 MeV and then drops because of the rapidly decreasing bare coupling constant.

### 4 Summary and Conclusions

We have investigated the covariant Dyson–Schwinger equation for the nucleon in a field–theoretic model for the pion–nucleon interaction with a pseudovector Lagrangian. Its treatment in the commonly used rainbow approximation, including proper renormalization, leads to a strong renormalization of the pion–nucleon coupling constant, $g_{\text{bare}}/g > 1$, in contradiction to perturbative one–loop calculations. We were therefore led to calculate the one–loop perturbative correction to the vertex, including the $\Delta$ resonance, and to resolve the DS equation. For physically reasonable values for the cut–off in the dipole pion–nucleon form factor, $\lambda \approx 1$ GeV, we find $g_{\text{bare}}/g \lesssim 1$ and a rather stable value for the nucleon mass–shift of around $-200$ MeV, consistent with one–loop results in both covariant treatments and semi–relativistic approaches such as the cloudy bag model. The fully self–consistent inclusion of the $\Delta$ resonance is expected to raise the absolute value of the mass shift even further.
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A Solving the Dyson–Schwinger equation for complex momenta

Here we present a more detailed account of the procedure to solve the DS eq. (2),

\[ i\dot{\phi} A(p^2) + B(p^2) = Z_2 (i\dot{\phi} + Z_M M) + i\dot{\phi} \Sigma_V(p^2) + \Sigma_S(p^2) , \quad (27) \]

\[ i\dot{\phi} \Sigma_V(p^2) + \Sigma_S(p^2) = Z_1 \int \frac{d^4k}{(2\pi)^4} \Gamma^0(p - k) \times \]

\[ \frac{1}{ik A(k^2) + B(k^2)} \Gamma(p, k) D^0(p - k) . \quad (28) \]

In order to evaluate the renormalization conditions (10,12), we will need (as shown below) the functions \( A(p^2) \) and \( B(p^2) \) for \( p^2 \in [-M_N - m_\pi]^2, \infty \) – i.e. if \( p = (0, p_4) \), \( p_4 \) may assume imaginary values.

We introduce the two component vectors \( \Sigma(p^2) = (\Sigma_S(p^2), \Sigma_V(p^2)) \) and \( S(p^2) = (A(p^2), B(p^2)) \). After applying suitable traces and doing the two trivial angular integrals we arrive at an equation of the form

\[ \Sigma_i(p^2) = \frac{3}{(2\pi)^3} \int_0^\infty \frac{k^2 dk^2}{k^2 A^2(k^2) + B^2(k^2)} \times \int_{-1}^1 \frac{1}{\sqrt{1 - z^2}} \frac{dz}{p^2 + k^2 + m_\pi^2 - 2p_4|k|z} K_{ij}(p^2, k^2, z) S_j(k^2) , \quad (29) \]

where \( z = \cos \psi \) refers to the angle between the Euclidean vectors \( p \) and \( k \). If we want to evaluate the loop integral for \( p^2 < 0 \) (\( p_4 \) imaginary), we have to note that for \( p^2 + m_\pi^2 < 0 \) poles from the pion propagator cross the real \( k_4 \) axis. These have to be avoided by a suitable deformation of the integration contour. This situation is depicted in Fig. 4. Alternatively, the original real path may be retained provided that a loop contour around the poles is added.
This leads to

$$\Sigma_i(p^2) = \frac{3}{(2\pi)^3} \int_0^\infty \frac{k^2 \, dk^2}{k^2 A^2(k^2) + B^2(k^2)} \times \int_{-1}^1 \sqrt{1 - z^2} \, dz \frac{k^2}{p^2 + k^2 + m^2_\pi - 2p_k k_4 z} K_{ij}(p^2, k^2, z) S_j(k^2) + \frac{3}{(2\pi)^3} \theta(-p^2 - m^2_\pi) \int_{\text{poles}} \frac{d^3k}{2\sqrt{k^2 + m^2_\pi}} \times \frac{K_{ij}(p^2, k^2, z)}{k^2 A^2(k^2) + B^2(k^2)} S_j(k^2) \bigg|_{(p-k)^2 = -m^2_\pi}. \quad (30)$$

The loop contour in $k_4$ picks up residues $(-2\pi i)/(-2i\sqrt{k^2 + m^2_\pi})$ from the pion propagator, and the integrand for the remaining integral in 3–space has to be evaluated at the pion poles (thus the value of $z$ to be used in $K_{ij}$ is determined). We convert the integral over $k$ into an integral over the squared four–momentum $k^2$ using

$$d k^2 \frac{|k|}{2\sqrt{k^2 + m^2_\pi}} = -d k^2 \, k^2 \, F(k^2, p^2), \quad (31)$$

$$F(k^2, p^2) = \frac{\sqrt{(k^2 + p^2 + m^2_\pi)^2 - 4k^2 p^2}}{4k^2 p^2}. \quad (32)$$

Hence, we arrive at the final expression for the self–energies, valid also for
timelike momenta up to \(-p^2 = M_N^2\): 

\[
\Sigma_i(p^2) = \frac{3}{(2\pi)^3} \int_0^\infty \frac{k^2 dk^2}{k^2 A^2(k^2) + B^2(k^2)} \times \\
\int_{-1}^1 \frac{\sqrt{1 - z^2}}{p^2 + k^2 + m^2 - 2pzikz} K_{ij}(p^2, k^2, z) S_j(k^2) - \\
\frac{3}{(2\pi)^2} \theta(p^2 - m^2) \int_{-(\sqrt{-p^2 - m^2})}^{-(\sqrt{-p^2 - m^2})} k^2 F(k^2, p^2) dk^2 \times \\
\frac{K_{ij}(p^2, k^2, z)}{k^2 A^2(k^2) + B^2(k^2)} S_j(k^2) \bigg|_{(p-k)^2 = m^2} .
\]

(33)

Now one sees clearly that in order to evaluate the renormalization conditions at \(p^2 = -M_N^2\), one needs to know the self–consistent solution to \(A(p^2)\) and \(B(p^2)\) for the interval \(p^2 \in [-M_N^2 - m^2, \infty)\). The DS equation can now be solved iteratively, performing both \(z\) and \(k^2\) integrations numerically. By virtue of the pion–nucleon form factors, the numerical treatment is not hampered by ultraviolet divergences and stable results are achieved by using a minimum of 100 mesh points for each integral.

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