Optimization of costs of residual deficit regulation in production system

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Abstract. The article considers the production subsystem as an element of the technological chain of a production system of any structure. Improvement of business efficiency involves optimizing key integration processes and coordinating the interaction of subsystems and links in technological chains. Each subsystem participates in the production of final products and is both a consumer and a supplier of products. The influence of various factors on the production process, including random ones, is the cause of violations of the time standards for product movement, which is expressed in deviations of the actual timing of the launch and release of planned accounting units from the planned standards. The concepts are introduced: late start-up in the subsystem-input deficit, delay transfer to the time of planned start-up in a neighboring subsystem along the technological chain-residual deficit. The solution of the subsystem is analyzed taking into account the amount of deficit transmitted along the technological chain. The function of costs for regulating external instability is defined, which includes two parameters: the value of the input deficit and the value of the residual deficit. If there is a residual deficit, compensation for the cost of reserving a neighboring subsystem along the chain is carried out in accordance with the linear function of the penalty and should be sufficient to neutralize it in this subsystem. A solution to the problem was obtained and a policy for regulating the remaining deficit was formulated.

1. Introduction
The studied production systems have a complex multi-level structure with a huge number of economic, organizational and communication links [1], so the stability of their functioning depends on the activity of their constituent elements, each of which in turn is subject to various disturbances that are random in relation to a separate division. The connection of elements of a production system can be characterized by a different structure: it can be a linear straight-through technological chain, with elements of a fan structure both at the input of a separate link in the system and at the output. The main structural element of the production system is always the production link (subject-closed or technological section, group or separate work centre, group of jobs). In the article, the object of consideration is a link (subsystem) of the production system.

In accordance with the structure of production links, each subsystem, having completed the finished part of the production process, transfers products to the next "supplier – consumer" subsystem in the specified time frame. Under the conditions of random disturbances (both external and internal) in discrete production, which are random in relation to a separate subsystem, there is a mismatch between the actual delivery dates of products and the planned ones. Because in the actual production
to coordinate the supply of products is possible only up to a certain time interval, then the supply of "just in time" is understood as adopted in the current system of operational production planning time period - the time interval coordination of product delivery just in time. This standard shows the accuracy (just in time) with which production units set delivery times for products to adjacent production subsystems in the process chain. Thus, if production units ensure the receipt of products in the supplier-consumer chain within the approval interval, then we can assume that the system meets the specified criteria for efficiency and reliability [2, 3, 4, 5, 6].

2. Materials and methods
The main regulatory document for drawing up calendar schedules for the movement of planned accounting units (PUS) for process operations is the launch-release schedule. In figure 1 a fragment of the launch-release schedule of two adjacent batches for a specific name is given. The schedule shown in the figure for the launch and release of batches characterizes the operation of production system (a subject-closed section, work centers, or a group of jobs) with a different or uniform rhythm for all parts batch names - \( R_0 \). The planned and actual dates of launch and release of parts \( D_{pl}^p, D_{al}^p, D_{pr}^p, D_{ar}^p \) of the j-th name are indicated by the index \( p \) – the serial number of the batch; the difference between the dates \( D_{jd}^{pl} - D_{jd}^{al} \) is equal to the planned duration of the production cycle of manufacturing parts in the production system.

![Figure 1. Planned and actual schedule start of production of batches of parts](image)

Due to random factors, both internal and external, as well as objective reasons due to production conditions, the actual execution of the launch-release schedule for this fragment is determined by the actual launch and release dates of parts batches. The shaded fragments represent, respectively, the delay in delivery of the planned accounting unit from the production subsystem adjacent to the process chain, which is equal to the difference between the actual launch date of the PUS and the planned date \( D_{pl}^p - D_{al}^p \ ) and which is the input deficit; the difference between the actual release date of the PUS and the planned date \( D_{ar}^p - D_{pr}^p \ ) is the residual deficit.

In the condition of structural and technological connectivity of subsystems by the sequence of production processes performed, the delivery of products in the "just in time" mode is regulated by direct methods of production process control. However, in order to be more effective, direct methods of production management are always supplemented by methods of economic regulation, the focus and content of which is ultimately reduced to taking into account the economic interests of subsystems. The organizational form of such accounting is the system of material remuneration and compensation for damages [2, 7]. This system provides a function of stabilizing planned standards for the implementation of the production process by embedding the economic interests of subsystems in direct management methods. With an effective system of economic regulation, the effect of self-regulation is observed, i.e. ensuring a stable flow of the production process by neutralizing deviations from the supply of products between subsystems. This management principle is important in terms of pulling production and ensuring its competitiveness through rapid response [8].
Based on the statistical model of the movement of labour items over time along the "supplier – consumer" technological chain, the instability of the subsystem can be described using average delays in the delivery of products to related units [2, 9]. It is the size of these delays that characterizes the level of redundancy, as well as the size of the losses that occur in the division that consumes products. Losses from instability determine the amount of additional costs in other divisions throughout the supplier-consumer chain. These costs perform a compensatory function for their intended purpose and are intended to completely or partially neutralize the consequences of instability, which is transmitted through technological links to production system units. The amount of costs for the full or partial elimination of the consequences of instability is determined by the level of reserves created in each production unit, as well as between them. The most used methods of reservation in practice are volume and temporary reservation [10, 11, 12].

Further consideration of the behavior of the production link of the system requires determining the values of input and residual deficits. Deviations of the actual moments of launching batches into production from the planned ones are random values. Let us denote the average duration of the delay (input deficit) in the production unit for the set of planned accounting units through $\Theta_0$.

Note that there may be several approaches to determining the average value of the input deficit. Let all the set of names of planned accounting units be processed in the production link in the considered period be equal to m. The actual start date of production of a planned accounting unit in the production system can be deferred from the planned date for a time multiple of the planned control period $- \theta_{oi}$. 

We introduce a random variable $\Theta^0 = \max(\theta_0^1, \theta_0^2, ..., \theta_0^m)$, where $\theta_0^i$ is a discrete random variable that takes the values $\theta_0^j = j$, $j = 1(1)N$ with a step equal to the value of $1kpp$ and with statistical probabilities $p_{ij}$, $i = 0, ..., m$.

The length of the time interval for approving the delivery of a batch on time may vary depending on the production conditions and the operational scheduling system. For example, $1kpp = 0.25$ the duration of the work shift or $1kpp= 0.5$ the duration of the work shift, etc. We calculate the probability values and get the distribution series of a discrete random variable $\Theta^0$ [13]:

$$P(\theta_0^i = \Theta_0^j) = \sum_{i=1}^{m-1} P(i, \Theta_0^j) + P(\theta_0^i = \Theta_0^j, i = 1, ..., m), j = 1, ..., N,$$

where

$$P(i, \Theta_0^j) = \sum_{v_1=v_{i+1}}^{m-i+1} \sum_{v_2=v_{i+1}}^{m-i+2} \sum_{v_3=v_{i+1}}^{m} \sum_{v_{i+1}}^{m} P(\theta_{0v_1} = \Theta_0^j, \theta_{0v_2} = \Theta_0^j, ..., \theta_{0v_{i+1}} = \Theta_0^j, \mu < \Theta_0^j, \mu \in [1, ..., m] \setminus \{v_1, ..., v_i\}).$$

The description of random variables as discrete is explained by the greater adequacy of the process under consideration. Using continuous random variables in calculations is more convenient, but it requires justifying the accepted types of distributions and obtaining their convolutions. We also believe that the assumption of independence of random variables is justified by the required accuracy of calculations for practical purposes.

The average delay duration (input deficit) in the production unit for the set of planned accounting units is defined as the mathematical expectation of a random variable $\Theta^0$: 

$$\mathbb{E}(\Theta^0) = \sum_{j=1}^{N} \Theta_0^j P(\theta_0^i = \Theta_0^j).$$
\[ \theta_0 = \sum_{j=1}^{N} P(\theta^0 = \theta^0_j) \cdot \theta^0_j. \]

Similarly, the average value of the residual (output) deficit is determined. The availability of statistical data is ensured by the simulation of calendar schedules, taking into account the provision of additional time to the production link (temporary redundancy) to reduce its own instability and compress the input deficit.

We introduce into consideration the cost function for the regulation of external instability. The division, being an intermediate link in the technological sequence of manufacturing the final product, is a source of delays in deliveries to the planned date, that is, it operates with an average output deficit. The process of restoring consistency, which consists in partially or completely neutralizing the input deficit, requires additional labour and material costs in the division. By its content, the costs incurred are the costs of regulating external instability - \( Z \). The amount of these costs is a loss to the consumer, and therefore must be reimbursed in the form of penalties imposed on it, either by the unit that is the source of the deficit, or by the system. The amount of the penalty is the amount of expenses that the consumer subsystem needs for its own reservation in order to neutralize the residual deficit. We assume that for the purposes of the study, it is sufficient to determine the costs by a function that depends on two parameters – the value of the input and output deficit. For each subsystem, the cost function for regulating external instability has the form \( Z(\theta_0, \theta) \), where the residual deficit of the previous subsystem is an input deficit to the next subsystem[14].

The residual deficit is the sum of two components, the first of which is the intrinsic instability of the subsystem, and the second is the input deficit allowed by the preceding subsystems. The simulation results show that the residual deficit is a positive value. With a relative balance of equipment group throughput and low load factors, the minimum residual deficit is small and may not be taken into account when reconciling the supplier-consumer production relationship over time. However, as the load factors increase and the input deficit remains at the same level, the minimum possible amount of residual deficit increases because it is determined by the smaller amount of reserves remaining at the disposal of the division.

3. Results and Discussion

The cost function for regulating external instability is a function of two variables \( Z(\theta_0, \theta) \). The function reflects the variety of applications and the extent to which internal regulators are used to compensate for instability in the division. The general nature of the dependence of costs on variables shows that the cost function for regulating external instability has a clearly expressed nonlinear form.

When analyzing a function in order to further approximate it with simpler functions, it is most important to establish a relationship between the growths of the function of regulation costs when the residual deficit decreases. We will approximate the function of regulation costs assuming that the relative increase in costs per unit of output deficit reduction is constant

\[ \left[ \frac{dZ(\theta_0, \theta)}{Z(\theta_0, \theta)} \right] / \left( \frac{d\theta}{\theta} \right) = -\alpha. \]  

(1)

Solving equation (1), using natural boundary conditions and taking into account that the minimum cost of completely neutralizing the consequences of the deficit itself depends on the value of the input deficit, allows obtaining a function of the control costs depending on the value of the input and residual deficit in the form of an expression

\[ Z(\theta_0, \theta) = \theta_0^{1+\alpha} (c + \theta)^{-\alpha}, \quad \alpha > 0. \]  

(2)

It is obvious that the amount of expenses allocated in the division to regulate the residual deficit, which is determined by the formula (2), takes into account the most significant requirements. If the values of the residual deficit at the output of the production system are equal, then the value of the
costs of regulation will be greater if the input deficit is greater. Meaning, if the residual deficit is equal to zero, it means that the input deficit is fully compensated and the control cost function must take the maximum value

\[ Z(0_0, 0) = \max Z(0_0, \theta). \]

An exogenously set parameter \( \Theta \), equal to the cost of fully compensating the input deficit unit, a constant value of \( c \), and an exponent \( \alpha \) allow flexible change the shape of the lines of the cost function and thus ensure that they correspond to the actual values. Parameter \( c \) corresponds to the amount of the costs of regulation that is sufficient to completely neutralize the input deficit in size \( \theta_0 \).

The linear penalty model is used in the subsystem interaction model in order to establish relationships between the sizes of mutual liability claims. For a linear function of the residual deficit penalty

\[ W(\theta_0, \theta) = \beta \cdot \theta \]

the required minimum of the amount of the costs of regulation and penalties is determined by the expression:

\[ W(\theta_0, \theta) = \min \{Z(\theta_0, \theta) + \beta \cdot \theta \} \quad \text{if} \quad \theta_0 \cdot \theta \geq 0. \quad (3) \]

Finding the minimum of the function gives the following result [15]. When searching for valid values, the equation is

\[ \frac{dW(\theta_0, \theta)}{d\theta} = 0, \quad \text{if} \quad \theta = 0. \]

So

\[ -\alpha \theta_0^{1+\alpha} (c + \theta)^{-\alpha - 1} + \beta = 0, \quad \theta = 0, \]

wherefrom

\[ c + \theta = \left( \frac{\alpha \theta}{\beta} \right)^{1+\alpha} \theta_0 = k \theta_0 \quad (4) \]

and with a sufficient deficit equal to zero, we get the value of the input deficit \( \tilde{\theta}_0 \) such that for all the value \( \theta_0 \leq \tilde{\theta}_0 \) of the function (3) is the minimum for

\[ \tilde{\theta}_0 = c / k. \quad (5) \]

For input deficit values \( \theta_0 > \tilde{\theta}_0 \), the optimal value of the deficit \( \hat{\theta} \) passed to the consumer unit is found from the solution of the equation

\[ \frac{dW(\theta_0, \theta)}{d\theta} = 0, \quad \text{if} \quad \theta \geq 0, \theta_0 \geq \tilde{\theta}_0 \]

where the output deficit value is calculated as

\[ \hat{\theta} = k \theta_0 - c = k (\theta_0 - \tilde{\theta}_0) \]

By combining the results we have obtained

\[ W(\theta_0, \theta) = \begin{cases} (\beta / \alpha) \theta_0^{1+\alpha} \tilde{\theta}_0^{-\alpha} k = W(\theta_0, 0), & \theta_0 \leq \tilde{\theta}_0 \\ (\beta / \alpha) \theta_0 k + \beta \cdot k (\theta_0 - \tilde{\theta}_0) = W(\theta_0, \hat{\theta}), & \theta_0 \geq \tilde{\theta}_0 \end{cases} \quad (6) \]

The general view of the function, which is monotonically increasing on each of the segments, is illustrated in Figure 2.
The optimal regulatory policy of the value of the remaining deficit is formulated on the basis of the result (Eq. 6): if the value of the input deficit in the subsystem is within the range of $0 < \theta_0 < \bar{\theta}_0$, so it is economically advantageous for the subsystem to eliminate the deficit completely using its own resources and reserves. With an input deficit $\theta_0 > \bar{\theta}_0$, the optimal policy is to partially compensate for the deficit.

\[ W(\theta_0, \bar{\theta}_0) \]

\[ W(\tilde{\theta}_0, 0) \]

\[ \tilde{\theta}_0 \]

\[ \bar{\theta}_0 \]

\[ \theta_0 \]

Figure 2. Graph of the cost function for regulating the external instability of the production system

The compression ratio of the deficit is a characteristic of the level of redundancy of the production system:

\[ \text{CRD} = \begin{cases} 1, & \theta_0 = \bar{\theta}_0, \\ 1 - k_1 \left( 1 - \frac{\tilde{\theta}_0}{\theta_0} \right) = 1 - k_1 \left( 1 - \frac{c}{\theta_0} \right), & \theta_0 > \bar{\theta}_0 \end{cases} \]  

(7)

Depending on the ratio of constants that determine the coefficient $k$, the residual deficit may increase at a higher or lower rate relative to the increasing deficit at the input. For known values $k$ and $c$ for the value of the input deficit from expression (7), the deficit compression coefficient corresponding to the minimum amount of regulation costs and penalties is easily determined.

4. Conclusion

The results obtained can be used when considering a sequential scheme of production relations "supplier-consumer" or sequential fragments in General schemes. In the case of a fan structure of links subsystems, the average input or residual deficit values are determined using a series of distributions across a set of supplies from previous subsystems or consumer subsystems. If the components of the cost management function are known, they are distributed among the subsystems in accordance with the accepted rules. The costs of controlling the residual deficit in an explicit form (6) allow them being used in tasks of optimizing the total costs of reserving production systems, as well as in managing supply chains [16].

Note an important circumstance. The concept of "reliability of the production system" and the related concepts of "redundancy", "stability", and "sustainability" make sense for stationary, relatively stable operating conditions of the production system [1, 2, 12, 17]. If parametric product changes are significant enough that the system does not detect stationary conditions, then special adaptation reserves are needed for stable operation. The results of simulation experiments show that the residual deficit when the average values of the load factors in the system exceed 0.85 in many cases exceeds the value of the input deficit due to the inherent instability of the subsystem. The most effective tools for reducing the residual deficit in practice are: earlier product launches times compared to the planned dates in the production system (advance) and increasing the working time utilization rate. A
significant reduction in the residual deficit and the cost of managing external instability is achieved by reducing the size of batches.

To determine the statistical characteristics of input and residual deficits in high-tech production, it is advisable to use the applied results of the theory of Queuing systems. The use of Queuing systems with management allows establishing an analytical relationship between the amount of residual deficits and the cost of reserving the subsystem [18].

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