Jet multiplicities as the QGP thermometer

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Abstract

It is proposed to use the energy behavior of mean multiplicities of jets propagating in a nuclear medium as the thermometer of this medium during the collision phases. The qualitative effects are demonstrated in the framework of the fixed coupling QCD with account of jet quenching.

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1 Introduction

The properties of quark and gluon jets are firmly established and carefully studied in $e^+e^-$, $ep$ and hadron collision experiments. They are well described by quantum chromodynamics (QCD) equations (see, e.g., reviews [1, 2, 3]). The jets in $e^+e^-$ annihilation evolve in the QCD vacuum as a cascade process by stretching and breaking strings between color charges. The high energy behavior of mean jet multiplicities is uniquely determined by the QCD coupling strength $\alpha_S$. In the leading perturbative approximation it looks like

$$\langle n \rangle \propto \exp\left(\int_{0.5\ln s}^{\infty} \sqrt{2N_c\alpha_S(y')/\pi} dy'\right). \quad (1)$$

In case of high energy nucleus-nucleus collisions the similar hard jets propagate in a medium with properties different from those of vacuum. Between confined initial and final states, the system can pass the deconfined state of color charges (QGP) characterized by various temperature regimes. During its motion, the jet initiated by a high energy parton meets different (local) temperature conditions from cold to hot phases. Thus the energy (distance) scale behavior of the effective coupling strength differs from the traditional $1/\ln s$-law. In particular, it does not increase at large distances but flattens

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off and even decreases there at high temperatures. This has been shown by lattice calculations for heavy quarks in \([4, 5, 6]\) (see also \([7]\) and Fig. 3 below). Therefore, with the temperature-dependent coupling, mean multiplicities of jets can be used for measuring the QGP temperature as follows from (1).

For qualitative estimates, we will use two simplifying proposals. According to the lattice results, the energy (distance) scale dependence of the coupling strength becomes quite mild. It depends however on the temperatures. Averaging over different temperature regimes would flatten it even stronger. Therefore, in first approximation, it can be effectively replaced by a temperature-dependent constant at the distances most important for jet evolution. It simplifies the treatment of QCD integrodifferential equations for the generating functions of jet multiplicities because in this case they possess the scaling property and can be reduced to the system of algebraic equations which are exactly solved \([8]\).

Another well known property of ”in-medium” jets which should be taken into account is jet quenching. Several different theoretical approaches to the non-Abelian radiative energy loss of partons in dense QCD matter have been proposed \([9, 10, 11, 12]\). For our qualitative estimates we use the QCD interpretation of the medium-induced modification of single inclusive hadron spectra for such jets which was recently considered in \([13]\). The kernels of the QCD equations were distorted so that the role of soft emissions was enhanced by introducing some new parameter in the infrared terms. This can be interpreted as the phenomenological description of the experimental fact of softening of jets spectra in nucleus-nucleus collisions. It implies that only soft partons with long wavelengths feel the neighboring deconfined partons of the medium surrounding the jets and multiply by rescattering. The leading perturbative terms of the QCD kernels play now even more important role. From the theoretical point of view it could be considered as a result of some yet unknown effective lagrangian which would be responsible for processes in the complicated nuclear medium and, probably, unite the above approaches. It accounts for the fact that jets lose their energy spending a part of it in the strong field of the surrounding quark-gluon matter.

Thus it is assumed that the hadronic medium changes both the coupling strength (the vertices) and the functional behavior of the kernels of QCD equations. We show how these two modifications of QCD equations influence mean multiplicities of hard jets piercing through the nucleus. This can be used for measuring the medium temperature.
2 QCD equations and their solution

Any moment of the parton multiplicity distribution $P_n$ of gluon and quark jets in QCD can be obtained from the equations for the generating functions

\[
G'_G = \int_0^1 dx K_G^G(x) \gamma_0^2 \left[ G_G(y + \ln x) G_G(y + \ln(1 - x)) - G_G(y) \right] \\
+ n_f \int_0^1 dx K_G^F(x) \gamma_0^2 \left[ G_F(y + \ln x) G_F(y + \ln(1 - x)) - G_F(y) \right],
\]

(2)

\[
G'_F = \int_0^1 dx K_F^G(x) \gamma_0^2 \left[ G_G(y + \ln x) G_F(y + \ln(1 - x)) - G_G(y) \right],
\]

(3)

where $G_i$ are the generating functions of the multiplicity distributions $P_n^{(i)}$ of gluon ($i = G$) and quark ($i = F$) jets, defined by

\[
G_i(y, z) = \sum_{n=0}^{\infty} (z + 1)^n P_n^{(i)} = \sum_{q=0}^{\infty} \frac{z^q}{q!} \langle n_i \rangle_q F_q^{(i)}.
\]

(4)

In these expressions and below, $\langle n_i \rangle = \sum_{n=0}^{\infty} n P_n^{(i)}$ is the average multiplicity, $z$ is an auxiliary variable, $y = \ln(p \Theta / Q_0)$ is the evolution variable, $p, \Theta$ are the momentum and the opening angle of a jet, $Q_0 = \text{const.}$, $G'(y) = dG/dy$, $n_f$ is the number of active flavors, $N_c$ is the number of colors, and $C_F = (N_c^2 - 1) / 2N_c = 4 / 3$ in QCD.

According to the above discussion, the coupling strength

\[
\alpha_s = \frac{\gamma_0^2 \pi}{2N_c}
\]

(5)

is set to be energy independent. Its temperature dependence will be however crucial for further consideration.

The modified kernels of the equations have been chosen similar to those used in [13]. They differ from the common QCD kernels [3] by the nuclear QCD factor $N_s$ which replaces 1 in soft infrared parts of the kernels. It changes the relative role of soft and hard splittings of partons.

\[
K_G^G(x) = \frac{N_s}{x} - (1 - x) [2 - x (1 - x)],
\]

(6)

\[
K_F^G(x) = \frac{1}{4N_c} [x^2 + (1 - x)^2],
\]

(7)
For $N_s = 1$, they reduce to QCD kernels used for jets in $e^+e^-$ annihilation. For $N_s > 1$, the role of soft gluon emissions is enhanced (therefore, the label $s$) that accounts for their rescattering and multiplication in the nuclear medium of deconfined partons. We do not show explicitly the $N_s$-dependence in the kernels’ arguments.

The normalized factorial moment of any rank $q$ can be obtained by differentiation

$$F^{(i)}_q = \frac{1}{\langle n_i \rangle^q} \frac{d^q G_i}{dz^q} \bigg|_{z=0},$$

or, equivalently, by using the series (4) and collecting the terms with equal powers of $z$ on both sides of the equations (2), (3). In particular, the equations for mean multiplicities look like

$$\langle n_G(y) \rangle' = \int dx \gamma^2 \left[ K^G_G(x)(\langle n_G(y+\ln x) \rangle + \langle n_G(y+\ln(1-x)) \rangle - \langle n_G(y) \rangle) + n_f K^F_G(x)(\langle n_F(y+\ln x) \rangle + \langle n_F(y+\ln(1-x)) \rangle - \langle n_F(y) \rangle) \right],$$

$$\langle n_F(y) \rangle' = \int dx \gamma^2 \left[ K^G_F(x)(\langle n_G(y+\ln x) \rangle + \langle n_G(y+\ln(1-x)) \rangle - \langle n_G(y) \rangle) + n_f K^F_F(x)(\langle n_F(y+\ln x) \rangle + \langle n_F(y+\ln(1-x)) \rangle - \langle n_F(y) \rangle) \right].$$

Their solutions can be looked for [8] as

$$\langle n_G \rangle \propto \exp(\gamma y), \quad \langle n_F \rangle \propto r^{-1} \exp(\gamma y),$$

i.e. the increase in multiplicities with energy follows a power law $\langle n \rangle \propto s^{\gamma/2}$. Both $\gamma$ and $r$ are energy independent. They depend however on the temperature because $\alpha_s$ (5) depends on it and on $N_s$-factor in the kernels.

The integrodifferential equations (2), (3) become the algebraic equations

$$\gamma = \gamma_0^2 \left[ M_1^G + n_f (r^{-1} M_1^F - M_0^F) \right],$$

$$\gamma = \gamma_0^2 (L_2 - L_0 + r L_1).$$

Here

$$M_1^G = \int_0^1 dx K^G_G[x^\gamma + (1-x)^\gamma - 1],$$

$$M_1^F = \int_0^1 dx K^F_G[x^\gamma + (1-x)^\gamma].$$
\[
M^F_0 = \int_0^1 dx K^F_G, \tag{17}
\]
\[
L_1 = \int_0^1 dx K^G_F x^\gamma, \tag{18}
\]
\[
L_2 = \int_0^1 dx K^G_F (1-x)^\gamma, \tag{19}
\]
\[
L_0 = \int_0^1 dx K^G_F. \tag{20}
\]

Finally, these equations can be represented as the equation for \( \gamma \) and the relation of \( r \) to \( \gamma \):

\[
\left( \frac{\gamma}{\gamma_0} - a(\gamma) \right) \left( \frac{\gamma}{\gamma_0} - d(\gamma) \right) = b(\gamma)c(\gamma), \tag{21}
\]

\[
r(\gamma) = b(\gamma) \left( \frac{\gamma}{\gamma_0} - a(\gamma) \right)^{-1}, \tag{22}
\]

where

\[
a = N_s(\psi(1) - \psi(\gamma+1) + B(\gamma, 1)) - 2B(\gamma+1, 2) - 2B(\gamma+2, 1) + 0.5B(\gamma+2, 2) + \frac{11}{12} \frac{n_f}{6N_c} \tag{23}
\]
\[
b = \frac{n_f}{2N_c} [B(\gamma+3, 1) + B(\gamma+1, 3)], \tag{24}
\]
\[
c = \frac{C_F}{N_c} [N_s B(\gamma, 1) - B(\gamma+1, 1) + 0.5B(\gamma+2, 1)], \tag{25}
\]
\[
d = \frac{C_F}{N_c} [N_s(\psi(1) - \psi(\gamma+1)) - B(\gamma+1, 1) + 0.5B(\gamma+1, 2) + 0.75]. \tag{26}
\]

Here psi-functions and Euler beta-functions are used.

3 Results and their discussion

Eqs (21), (22) determine the experimentally measurable power in energy increase of jet multiplicities \( \gamma \) and their ratio \( r \) in gluon to quark jets as functions of the coupling strength \( \alpha_S \) and the nuclear QCD factor \( N_s \).

Let us fix \( \gamma \). Then both \( \alpha_S \) and \( r \) can be found from (21), (22) as functions of a single variable \( N_s \). These dependences are shown in Figs 1 and 2 for
\( \gamma = 0.4, 0.5, 0.6 \). The nearby values of \( \gamma \) are chosen to show the sensitivity to \( \alpha_S \) and \( N_s \). They are grouped near \( \gamma = 0.5 \) simply because it is the only theoretically known power regime (obtained in relativistic hydrodynamics). Other powers are admissible in theory but, finally, the value of \( \gamma \) should be chosen according to experimental data. Our aim here is to demonstrate within this simplified model that, when measured, the definite values of \( \gamma \) will reveal the temperature regimes of the matter during nuclear collisions, and, therefore, serve as a thermometer for the states of this matter during the collision. This conclusion is however more general as follows from (1).

The effective range of \( N_s \) was estimated in [13] as changing from 1.6 to 1.8 for fits of the rapidity distributions. According to Fig. 1, it implies that the effective values of the coupling strength \( \alpha_S \) are between 0.14 and 0.122 for \( \gamma = 0.5 \). Then the average temperature of the regions inside nuclei which determine the jet evolution can be estimated by comparison of these values with the lattice values of the coupling strength \( \alpha_S \) calculated at different temperatures in [4, 5, 6] and presented also in [7]. To demonstrate this, the band \( 0.122 < \alpha_S < 0.14 \) is imposed in Fig. 3 on curves for distance dependence of \( \alpha_S \) at temperatures \( 3T_c \) and \( 6T_c \) (see Fig. 1 in [7], and also Fig. 5 in [4], Fig. 2 in [5]). Herefrom, we conclude that the effective temperature is surely above \( 3T_c \) and close to \( 6T_c \). This would be the argument in favor of the statement that the jets penetrate through the deconfined state. The average values of \( \alpha_S \) become smaller at higher temperatures because the strings are broken and the coupling strength decreases at large distances.

This result seems quite reasonable for the chosen interval of \( N_s \). The slower increase of multiplicities with energy would require (Fig. 1) too weak coupling strength \( 0.07 < \alpha_S < 0.08 \) for \( \gamma = 0.4 \) and very high effective temperatures. The stronger increase can be hardly supported by experiment, and it would ask for stronger coupling \( 0.22 < \alpha_S < 0.25 \) for \( \gamma = 0.6 \) and temperatures closer to \( T_c \). Thus the energy dependence of mean multiplicities can be used for measuring the QGP temperature.

The values of the ratio of multiplicities in gluon to quark jets \( r \) increase at larger \( N_s \) as expected and seen from Fig. 2. They are larger at smaller \( \gamma \). For \( N_s \to \infty \) they tend to their asymptotical value 2.25 but are still quite far from it. In the region \( 1.6 < N_s < 1.8 \) the ratio \( r \) stays practically constant at fixed \( \gamma \) and equal approximately 1.68, 1.82, 1.95 at \( \gamma = 0.6, 0.5, 0.4 \), correspondingly. We conclude that gluon jets are still more active in producing secondary partons in the nuclear medium.
We have checked that we reproduce the limiting values of $\alpha_S$ and $r$ both for the ”vacuum” QCD $N_s = 1$ obtained in [8] and for $N_s \to \infty$ which corresponds to the leading double-logarithmic approximation.

4 Conclusions

The nuclear medium impact on jet properties has been considered in QCD with fixed (but temperature-dependent) coupling. The enhancement of soft gluon emission (jet quenching) has been presented by the factor $N_s$ in the kernels of equations.

The general qualitative tendencies invoked by this factor $N_s$ for a constant power growth of the average multiplicities with energy increase can be summarized in the following way:

1. The effective coupling strength $\alpha_S$ decreases with increase of $N_s$.

2. It is larger for larger power of multiplicity increase $\gamma$.

3. The ratio of gluon to quark jet multiplicities $r$ increases with increase of $N_s$.

4. It is larger for smaller power of multiplicity increase $\gamma$.

It follows from the first two statements that the energy increase of average multiplicities at definite values of $N_s$ can serve as the thermometer of the nuclear medium. The values of $N_s$ found in [13] are larger than 1. It implies that $\alpha_S$ is lower than its zero-temperature value, i.e., in accordance with lattice results, the nuclear medium is characterized by the non-zero temperature.

The gluon jets are still more active than quark jets in producing secondary particles but this characteristics is not very sensitive to the parameters of the medium.

The assumption of the fixed coupling can be relaxed inserting directly the lattice values for $\alpha_S$ at different temperatures in QCD equations. This complicates their computer solution. The work is in progress.

The behavior of the higher rank moments of multiplicity distributions and nuclear modification of their oscillations (compared to jets in $e^+e^-$ annihilation studied in [8, 14]) can provide further insight in the problem and will be described elsewhere.

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Figure captions.

Fig. 1. The dependence of the coupling strength $\alpha_S$ on the nuclear QCD factor $N_s$ for $\gamma=0.4$ (bottom), 0.5, 0.6 (top).

Fig. 2. The dependence of the ratio of mean multiplicities in gluon and quark jets $r$ on $N_s$ for $\gamma=0.4$ (top), 0.5, 0.6 (bottom).

Fig. 3. The band of values $0.122 < \alpha_S < 0.14$ imposed on curves for distance dependence of $\alpha_S$ at temperatures $3T_c$ and $6T_c$ (the lattice results are taken from [4, 5, 6, 7]). The values for $T=0$ are shown by the curved line.

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