Gauge Mediated Proton Decay
in a Renormalizable SUSY SO(10) with Realistic Mass Matrices

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Abstract

Proton decay via $d=5$ operators is excluded by now not only in the framework of SUSY SU(5) but also its extensions like SUSY SO(10) are on the verge of being inconsistent with $d=5$ decays. It is reasonable therefore to suppress, e.g. by a symmetry, the $d=5$ operators and to consider gauge boson induced $d=6$ decays. This is suggested in several recent papers in the framework of models with a lighter $M_X$. We discuss here explicitly the fermionic sector of such a renormalizable SUSY SO(10) with realistic mass matrices. We find that the recently “observed” large leptonic mixing leads to an enhancement of the nucleon decay channels involving $\mu$’s and in particular the $\mu^+\pi^0$, $\mu^+\pi^−$ modes.

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Nothing is known directly at present about the supersymmetric (SUSY) partners, except for the experimental lower bounds on their masses. The main indication for (low energy) SUSY is the unification of the coupling constant of the minimal SUSY Standard Model (MSSM) at a high scale \( \approx 10^{16} \) GeV. However, this unification speaks even more for SUSY-Grand Unified Theories (GUTs), which predict also the observed partial Yukawa unification (i.e. \( m_\tau = m_b \)) at those energies. Yet proton decay, the other main prediction of GUTs, was not observed till now. Is this consistent with the expectation from SUSY GUTs?

It is generally assumed that the leading proton decay modes in SUSY-GUTs are due to \( d=5 \) operators, as the \( d=4 \) contributions must be suppressed via a symmetry (like R-parity) to avoid a much too fast decay. Those \( d=5 \) decay modes are induced using the, not yet observed, SUSY particles and involve therefore many unknown parameters. As a consequence the predicted rates are highly model dependent. This large freedom does not save SUSY-SU(5) from being practically ruled out if the needed threshold corrections are used to limit the mass of the Higgsinos (\( \tilde{H} \)) which mediate proton decay. Even extensions of SUSY SU(5) like SUSY SO(10) are on the verge of being excluded by similar arguments. To save it one must push several parameters to their allowed limit and in particular \( \tilde{M}_{\tilde{H}} \) is required in some papers to be much larger than \( M_{GUT} \) and even larger than \( \tilde{M}_{Planck} = 2.4 \times 10^{18} \) GeV. This leads to very large couplings in the superpotential or the corresponding non-renormalizable contributions.

In this letter we will assume, as was already put forward by several people, to suppress not only the \( d=4 \) but also the \( d=5 \) contribution by a symmetry and consider the gauge mediated \( d=6 \) operators.

- Those involve only known coupling constants and masses and therefore their predicted rates are far more reliable than the \( d=5 \) ones.
- They are the real test of the GUTs because \( d=5 \) induced proton decay is allowed in non-GUT theories as well.
- Although \( d=6 \) contributions are suppressed by \( 1/M_X^4 \) they could yet be observable in the near future if the relevant gauge bosons mass \( M_X \) is somewhat lower than \( 10^{16} \) GeV.

This gauge-boson induced proton decay does not require non-renormalizable contributions or particles with masses above the GUT scale. We will apply therefore the “renormalized see-saw mechanism” using \( \Phi_{126} \) to give large masses to the right-handed (RH) Majorana neutrinos. This has many advantages, in particular R-parity invariance, that is needed to avoid the catastrophic \( d=4 \) contributions, is automatically obeyed in this case. This invariance leads also to a stable neutralino as a natural candidate for dark matter. On top of that, a way to lower the GUT scale in terms of a fully renormalizable gauge theory was suggested recently by Aulakh, Bajc, Melfo, Rašin and Senjanović. They introduce an intermediate Pati-Salam gauge group \( [11] \) at a scale \( M_I \). Such an intermediate scale is needed anyhow to explain the fact that the masses of the heavy Majorana neutrinos required to give the light neutrinos masses consistent with the observed oscillations.
are considerably smaller than the unification scale \( M_{\text{GUT}} \). In this SUSY-SO(10) model, due to the absence of trilinear terms in the superpotential, some particles acquire masses of \( O(M_{\text{GUT}}^2/M_{\text{GUT}}^2) \) via mixing. Those particles affect the renormalization group equations in a way that lowers the unification mass. The model can solve the doublet-triplet problem a la Dimopoulos-Wilczek [15] and can suppress naturally the d=5 operators i.e. Higgsino mediated proton decay. 

\( M_X \) and/or the unification scale have smaller values also in other recent models [9] and in particular in those using large extra dimensions. As a specially interesting example let us mention here the paper of Hall and Nomura [16] based on the model of Kawamura [17]. The idea is to use a five dimensional SU(5) GUT compactified to four dimensions on the orbifold \( S^1/(Z_2 \times Z_2') \). This yields the MSSM with doublet-triplet splitting and a vanishing proton decay from d=5 and d=4 operators by a \( U(1)_R \) symmetry. The model gives a compactification scale \( (M_c = M_X) \) somewhat lower than the four dimensional unification scale. This model is not a 4d GUT, but it involves all the properties we use for the fermionic mass matrices and proton decay.

When d=6 contributions are discussed in the literature one refers always to the proton decay into \( e^+\pi^0 \), that is the dominant decay mode only when the mixing is neglected. This is in contrast with recent papers about d=5 proton decay [8] [4], where the effects of realistic mass matrices are explicitly considered.

We present here a model for the explicit realization of the fermionic sector of a SUSY-SO(10) GUT with realistic fermionic mass matrices. We will in particular show that the observed large leptonic mixing leads to the enhancement of the branching ratios of the nucleon decay into muons with respect to those with \( e^+ \) in the final state. The observed large leptonic mixing will be coupled in our model with large quark mixing. Note, that the CKM matrix gives the difference between the mixing angles of the up and down quarks. Therefore, only the difference between the lefthanded (LH) mixing angles of the quarks must be small while the righthanded (RH) rotations are unobservable in the framework of the SM or the MSSM and can be large. The nucleon decay in GUTs is one of the few observables in which all mixing angles are involved.

The aim of our effective model is not to get as many as possible “predictions” for the known observables of the SM. It is to calculate all the “non-observable” mixing angles in terms of the observables of the SM in order to predict the proton decay.

The model uses a scheme developed in a series of papers [19] [21]. All mass matrices have a non-symmetric Fritzsch texture

\[
M = \begin{pmatrix}
0 & A & 0 \\
B & 0 & C \\
0 & D & E
\end{pmatrix}.
\]  

(1)

This texture and the contributions of the Higgs representations are fixed by a global \( U_F(1) \) (or \( Z_n \)). We use only renormalizable contribution à la Harvey, Ramond and Reiss [24].

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2This scale is needed also for the invisible-axion [13] and the baryon asymmetry induced via the leptogenesis [13].

3Partial information on the “non-observable” angles can be obtained by looking for RH currents, scalar leptoquark interactions, baryon asymmetry due to leptogenesis e.t.c.
This has some interesting advantages in our opinion, as was mentioned before, compared to the method of Froggatt and Nielsen [23] that uses broken $U_F(1)$’s with non-renormalizable contributions [4].

The SO(10) symmetry as well as $U_F(1)$ give relations between the different entries of the mass matrices. We need one large VEV in $\Phi_{126}$ to give the RH Majorana neutrinos masses the corresponding mass matrix must have the texture [19]

\[ M_{\nu_R} = \begin{pmatrix} 0 & a & 0 \\ a & 0 & 0 \\ 0 & 0 & b \end{pmatrix}. \tag{2} \]

Because our main interest lies in the nucleon decay, CP violation is neglected in this letter and the parameters are taken to be real, for simplicity [5].

We use, as is discussed in [21], one heavy $\Phi_{126}$ to give the RH neutrinos a mass. To generate the light mass matrices one $\Phi_{126}$ and pairs of $\Phi_{10}$ and $\Phi_{120}$ are needed [6]. Those Higgs representations involve several $SU_L(2)$ doublets. Note, however that as is usually done in SUSY GUT models [18] [19] broken into the Minimal SUSY Standard Model(MSSM) at the TeV scale, only two combinations of those doublets remain “light”. All other combinations acquire heavy masses and therefore do not affect the gauge unification [7]. The explicit combinations are not important because the effective VEVs are anyhow free parameters in our model. For the ratio of the light VEVs we use in this letter the commonly used value $\tan \beta = 5$ [8]. The gauge coupling becomes non-perturbative soon above the unification point but we do not use physics above the GUT-scale.

The three fermionic families in $16_i$ i=1,2,3 and the Higgs representations $\Phi_k$ transform under the global $U(1)_F$ as follows:

\[ 16_j \rightarrow \exp(i\alpha_j \theta) 16_j \]
\[ \Phi_k \rightarrow \exp(i\beta_k \theta) \Phi_k. \tag{3} \]

Invariance of the Yukawa coupling terms $16_i \Phi_k 16_j$ under $U(1)_F$ requires $\alpha_i + \alpha_j = \beta_k$. Hence, the most general structure of the Yukawa matrices is:

\[ Y^{(1)}_{10} = \begin{pmatrix} 0 & x_1 & 0 \\ x_1 & 0 & 0 \\ 0 & 0 & \bar{x}_1 \end{pmatrix}; \quad Y^{(h)}_{126} = \begin{pmatrix} 0 & y_1 & 0 \\ y_1 & 0 & 0 \\ 0 & 0 & \bar{y}_1 \end{pmatrix}; \quad Y^{(1)}_{120} = \begin{pmatrix} 0 & z_1 & 0 \\ -z_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \]
\[ Y^{(2)}_{10} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x_2 \\ 0 & x_2 & 0 \end{pmatrix}; \quad Y^{(2)}_{126} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_2 \\ 0 & y_2 & 0 \end{pmatrix}; \quad Y^{(2)}_{120} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & z_2 \\ 0 & -z_2 & 0 \end{pmatrix}. \tag{5} \]

These Yukawa matrices give explicit expressions for the $M_d$, $M_u$, $M_e$ and $M^{(Dir)}_\nu$ mass matrices, in terms of a set of 14 parameters (combinations of the Yukawa couplings and

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4The method of Froggatt and Nielsen is useful to explain the hierarchy in the quark mass matrices but not for the neutrinos. This is because the matrix elements are fixed in this method only up to unknown $O(1)$ factors and the see-saw matrix is a product of three matrices. The neutrino matrix elements are given in this case only up to corrections of $[O(1)]^3$ which can be large.

5Complex parameters will be used in a forthcoming detailed paper [21].

6The model involves pairs like $\mathbf{126} + 126$ e.t.c. to avoid breaking of the SUSY at high energies, but only part of those are relevant for the fermionic mass matrices.

7Some light VEVs are then induced ones.

8In Ref. [21] we shall discuss a set of different values of $\tan \beta$. 


the corresponding VEVs). These matrices are diagonalized and fitted to the the known masses and mixing of the quarks and leptons as will be described in the following.

We start by taking diagonal mass matrices for the charged fermions at $M_Z$. Than the full two-loop renormalization group equations (RGEs) of the Minimal Supersymmetric Standard Model (MSSM), with $\tan\beta = 5$, are used to get the following masses at $M_{GUT} = 2 \times 10^{16}$:

|                | $m_u(M_{GUT})$ | $m_d(M_{GUT})$ | $m_s(M_{GUT})$ |
|----------------|----------------|----------------|----------------|
| $1.04 \text{ MeV}$ | $1.33 \text{ MeV}$ | $26.5 \text{ MeV}$ | - |
| $302 \text{ MeV}$ | $1 \text{ GeV}$ | $129 \text{ GeV}$ | - |
| $0.32502032 \text{ MeV}$ | $68.59813 \text{ MeV}$ | $1171.4 \text{ MeV}$ | - |

In our model only 12 independent parameters are needed to specify charged fermion matrices at the GUT scale, exactly the number of the underlying masses and CKM mixing angles (the CKM matrix changes only slightly from $M_Z$ to $M_{GUT}$). By fitting these parameters, all the LH and RH mixing angles of the quarks and leptons are fixed. This procedure involves a set of non-linear equations:

$$
U_L^t M_u U_R = M_u^{(D)}, \quad D_L^t M_d D_R = M_d^{(D)},
$$
$$
E_L^t M_e E_R = M_e^{(D)}, \quad U_L^t D_L = V_{CKM}. \quad (6)
$$

We found five solutions to those equations all of which have several large mixing angles [20]. These fix the neutrino mass matrices $M_{\nu}^{\text{Dir}}$ and $M_{\nu}^{\text{R}}$ also up to two parameters (and the overall scale $M_R$). Hence, the see-saw matrix

$$
M_{\nu}^{\text{light}} \simeq - M_{\nu}^{\text{Dir}} (M_{\nu}^{\text{Maj}})^{-1} (M_{\nu}^{\text{Dir}})^T \quad (7)
$$

as well as the leptonic LH mixing $U_{MNS} = E_L^t N_\nu$ are also known. We varied than the two free parameters and looked for solutions that reproduced the neutrino data with LMA-MSW or SMA-MSW for the solar neutrinos [20]. The details will be given in a forthcoming paper [20]. The best fit is obtained for the a LMA-MSW solution with the following properties:

1. The Quark LH and RH mixing angles at the GUT scale:

$$
\theta_{L12}^u = -0.077, \quad \theta_{L23}^u = -1.48, \quad \theta_{L13}^u = -4 \times 10^{-8}.
$$
$$
\theta_{R12}^u = -0.045, \quad \theta_{R23}^u = -2.2 \times 10^{-4}, \quad \theta_{R13}^u = -1.1 \times 10^{-3}.
$$
$$
\theta_{L12}^d = 0.15, \quad \theta_{L23}^d = -1.44, \quad \theta_{L13}^d = 1 \times 10^{-5}.
$$
$$
\theta_{R12}^d = -0.33, \quad \theta_{R23}^d = -3 \times 10^{-3}, \quad \theta_{R13}^d = 6 \times 10^{-2}.
$$

2. The mixing angles of the Charged Leptons:

$$
\theta_{L12}^\ell = -1.17, \quad \theta_{L23}^\ell = 1.44, \quad \theta_{L13}^\ell = 0.0002.
$$
$$
\theta_{R12}^\ell = 0.002, \quad \theta_{R23}^\ell = -0.003, \quad \theta_{R13}^\ell = 0.002.
$$

\footnote{If only one parameter is taken to be complex, one can use its phase to account for the observed CP violation and again all mixing angles will be fixed [20].}
3. The Neutrino masses:

\[ M_R = 5.2 \times 10^{13} \text{ GeV} \]

\[ m_{\nu_e} = 1.88 \times 10^{-3} \text{ eV}, \quad m_{\nu_\mu} = 5.89 \times 10^{-3} \text{ eV}, \quad m_{\nu_\tau} = 5.85 \times 10^{-2} \text{ eV}. \]

4. The LH Leptonic (Neutrino) mixing angles:

\[ \theta_{12}^\nu = 0.55, \quad \theta_{23}^\nu = 0.74, \quad \theta_{31}^\nu = -0.0053. \]

Using these results one can calculate the proton and neutron decay branching ratios. We use the method of Gavela et al. \cite{27} as it was extended in a series of papers \cite{28, 29} to models with large fermionic mixing (especially RH ones). In this work it is once more generalized into a SUSY GUT.

The effective baryon number violating Lagrangian of SO(10) is

\[
\mathcal{L}_{\text{eff}} = A_1 \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{e}_L^{\gamma} \gamma_\mu d_L^\mu \right) + A_2 \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{e}_L^{\gamma} \gamma_\mu d_R^\mu \right) + A_3 \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\mu}_L^{\gamma} \gamma_\mu d_L^\mu \right) + A_4 \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\mu}_L^{\gamma} \gamma_\mu d_R^\mu \right) + A_5 \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\mu}_L^{\gamma} \gamma_\mu s_L \right) + A_6 \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\mu}_L^{\gamma} \gamma_\mu s_R \right) + A_7 \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\mu}_L^{\gamma} \gamma_\mu s_L \right) + A_8 \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\mu}_L^{\gamma} \gamma_\mu s_R \right) + A_9 \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\nu}_{\epsilon R} \gamma_\mu d_L^\mu \right) + A_{10} \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\nu}_{\epsilon R} \gamma_\mu d_R^\mu \right) + A_{11} \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\nu}_{\epsilon R} \gamma_\mu s_L \right) + A_{12} \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\nu}_{\epsilon R} \gamma_\mu s_R \right) + A_{13} \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\nu}_{\epsilon R} \gamma_\mu s_L \right) + A_{14} \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\nu}_{\epsilon R} \gamma_\mu s_R \right) + A_{15} \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\nu}_{\epsilon R} \gamma_\mu s_L \right) + A_{16} \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\nu}_{\epsilon R} \gamma_\mu s_R \right) + A_{17} \left( \varepsilon_{\alpha\beta\gamma} \bar{u}_L^{\gamma\mu} u_L^\mu \right) \left( \bar{\nu}_{\epsilon R} \gamma_\mu s_L \right) + ( \text{ terms with two } s \text{ quarks } ) + ( \text{ terms with } c, b, t \text{ quarks } ) + ( \text{ terms with } \bar{\tau}_{L,R} \text{ and } \bar{\nu}_{\epsilon,\mu,\tau L} \text{ ) } + \text{ h.c. } \]

where \( A_i \) are functions of the mixing angles given in Ref. \cite{21}.

The partial decay rate for a given process nucleon \( \rightarrow \) meson + antilepton is expressed as follows \cite{27}:

\[
\Gamma_j = \frac{1}{16\pi} m_{\text{nucl}}^2 \rho_j |S|^2 |A|^2 \left( |A_L|^2 \sum_l |A_l M_l|^2 + |A_R|^2 \sum_r |A_r M_r|^2 \right), \]

where \( M_l \) and \( M_r \) are the hadronic transition matrix elements for the relevant decay process. \( l \) and \( r \) denote the chirality of the corresponding antilepton \cite{28}. \( A_l \) and \( A_r \) are the relevant coefficients of the effective Lagrangian \cite{8}. \( A, A_L \) and \( A_R \) are factors which result from the renormalization of the four fermion operators, see Ref. \cite{20}.

Using all this we obtain the nucleon decay branching ratios for our best solution. These are presented in the following table compared with the \( d=6 \) nucleon decays without mixing.
One sees clearly that the nucleon decay rates into muons are strongly enhanced. Also decays into $e^+K^0$ and $\nu K^0$ are not negligible.

The other solutions in the LMA-MSW case give similar results. Since we use the most general Yukawa matrices (in terms of the asymmetric Fritzsch texture), we expect our results to be generic in the large mixing scenario.

The absolute decay rates are much less reliable than the branching ratios given above. They depend not only on the “unknown” value of $M_X$, but also on the uncertain hadronic matrix elements [29], the short distance enhancement factors e.t.c. Using estimates for the absolute decay rates [9, 7], in terms of the recent Lattice calculations of the JLQCD collaboration [30], we obtain for our branching ratios the following result

$$\frac{1}{\Gamma(p \to \mu^+\pi^0)} \approx \frac{1}{\Gamma(p \to e^+\pi^0)} = 16 \times 10^{34} \times \left(\frac{M_X}{10^{16}\text{GeV}}\right)^4 \left(\frac{0.015\text{GeV}^3}{\alpha_H}\right)^2 \text{ yrs.}$$

One can apply now the Super-Kamiokande bound [2]

$$\frac{1}{\Gamma(p \to e^+\pi^0)} > 4.4 \times 10^{33} \text{ yrs (90% CL)}$$

to get a lower bound on effective $M_X$’s in the theory

$$M_X > 0.7 \times 10^{16} \text{ GeV}.$$
should not be neglected.

The enhancement of the muon branching ratios is a unique feature of our model because the decay mode \( p \to e^+\pi^0 \) is not negligible also in the \( d = 5 \) induced decays. In view of the fact that this enhancement is the effect of the large observed leptonic mixing on the \( d = 6 \) nucleon decay, we suggest that the observation of a considerable rate for the decay \( p \to \mu^+\pi^0 \) will be a clear indication for a gauge mediated proton decay.

One can say in general, that the branching ratios of the nucleon decay can teach us about the “fundamental” mass matrices as they depend on all mixing angles. The present huge freedom in the mass matrices would then be strongly restricted and one could better understand the origin of the fermionic masses.

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