Invisible Z decay width bounds on active-sterile neutrino mixing

in the $$(3+1)$$ and $$(3+2)$$ models

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Abstract

In this work we consider the standard model extended with singlet sterile neutrinos with mass in the eV range and mixed with the active neutrinos. The active-sterile neutrino mixing renders new contributions to the invisible Z decay width which, in the case of light sterile neutrinos, depends on the active-sterile mixing matrix elements only. We then use the current experimental value of the invisible Z decay width to obtain bounds on these mixing matrix elements for both $$(3+1)$$ and $$(3+2)$$ models.

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I. INTRODUCTION

Right-handed neutrinos can be introduced in the standard model in the singlet form only. Consequently, they do not interact with the standard gauge bosons. Thus people usually refers to these neutrinos as sterile neutrinos. In this context the left-handed neutrinos, those that compose the leptonic standard doublets, are called the active ones.

Sterile neutrinos with mass in the eV range became popular since that LSND experiment released a report claiming the detection of electronic anti-neutrinos in a beam of muonic anti-neutrinos. When the LSND signal is explained in terms of neutrino oscillations, then at least one sterile neutrino is required with mass around eV and mixed with the active neutrinos with a mixing angle about $\sin^2(2\theta) \approx 10^{-3}$. It happens that such range of values for mass and mixing angle are strongly disfavored by cosmological and astrophysical data as well as by other short-baseline (SBL) data.

In order to solve this dilemma, an experiment at Fermilab, called MiniBooNE (MB), was projected exclusively to confirm or refute LSND signal. Recently MB collaboration released a report which refutes the LSND signal with 98% CL. However, according to the first global analysis after MB report, what is, in fact, being refuted by MB data is the possibility of explaining the LSND signal through neutrino oscillation in the context of one sterile neutrino. Moreover they showed that if CP phase is included in scenarios with two or three sterile neutrinos then MB results can be conciliated with LSND appearance.

On the other hand, we know that the mixing of active neutrinos with the sterile neutrinos gives rise to interactions of these neutrinos with the standard neutral gauge bosons Z. Consequently, they will contribute to the invisible Z decay ($\Gamma_{\text{inv}}$). In the case of light sterile neutrinos, the $\Gamma_{\text{inv}}$ will present dependence on the active-sterile neutrino mixing only. The proposal of this work is to get bounds on the active-sterile neutrino mixing for both $(3+1)$ and $(3+2)$ models by using the current experimental value of the invisible Z decay width.

This work is organized as follows. In Sec. (II) we settle the framework of our approach in the case of three singlet sterile neutrinos. Next, in Sec. (III) we obtain the bounds for the case of one sterile neutrino and in Sec. (IV) we do the same for the case of two sterile neutrinos. In Sec. (V) we summarize our results.
II. THE (3 + 3) MODEL

We develop the framework for the calculation of the $\Gamma_{\text{inv}}$ for the case of three singlet sterile neutrinos added to the standard model[11]. We refer to these neutrinos as $\nu_{s1R}$, $\nu_{s2R}$ and $\nu_{s3R}$. The case of interest arises when we allow these singlet neutrinos to develop mixing with the active neutrinos. Such mixing is generated by mass terms[1]. The neutrino mixing matrix $U$ in this case is of dimension $6 \times 6$ and relates the flavor eigenstates, which we consider in the base $(\nu _{L}, \nu _{C_{sL}}) = (\nu _{1L}, \nu _{2L}, \nu _{3L}, \nu _{C_{s1L}}, \nu _{C_{s2L}}, \nu _{C_{s3L}})$, with the mass eigenstates, which we consider in the base $(\nu _{L}, \nu _{sL}) = (\nu _{1L}, \nu _{2L}, \nu _{3L}, \nu _{4L}, \nu _{5L}, \nu _{6L})$. The relation among these bases is given by

$$
\begin{pmatrix}
\nu _{L} \\
\nu _{C_{sL}}
\end{pmatrix} = U_{6 \times 6}
\begin{pmatrix}
\nu _{L} \\
\nu _{sL}
\end{pmatrix},
$$

(1)

The neutrinos $\nu_1$, $\nu_2$ and $\nu_3$ are the physical active neutrinos while $\nu_4$, $\nu_5$ and $\nu_6$ are the physical sterile neutrinos. In this work we neglect CP phases, which means $U_{6 \times 6}$ is a real mixing matrix.

The mixing in Eq. (1) automatically generates interactions involving the standard gauge bosons and sterile neutrinos. Here we are interested only in interactions of these light sterile neutrinos with the neutral gauge boson $Z$. Below we present the Lagrangian that describes such interactions for the case of three sterile neutrinos

$$
\mathcal{L}^\nu _Z = \frac{g}{2 c_W} (\bar{\nu} _{jL} \gamma ^\mu \nu _{jL} + \bar{\nu} _{jL} \gamma ^\mu U _{ji} U _{ia} \nu _{aL} + \bar{\nu} _{aL} \gamma ^\mu U _{ai} U _{i\alpha} \nu _{\alpha L}) Z _\mu ,
$$

(2)

where $i, j = 1, 2, 3$, $\alpha = 1, 2, 3, 4, 5, 6$ and $a = 4, 5, 6$. The Lagrangian for the case of one or two sterile neutrinos is obtained from Eq. (2) by taking the corresponding values of $\alpha$ and $a$.

With the interactions given in Eq. (2), we obtain the following expression for $\Gamma_{\text{inv}}$ for the case of three active neutrinos and three sterile neutrinos,

$$
\Gamma_{\text{inv}} = \frac{G_F m_e^2}{4 \sqrt{2} \pi} \left\{ \frac{1}{3} + \frac{1}{3} \sum _{a=4} ^6 \left[ \frac{6}{3} \sum _{j=1} ^3 (U _{ji} U _{ia})^2 + \frac{6}{3} (U _{ai} U _{ia})^2 \right] \right\}.
$$

(3)

Note that the invisible Z decay can constrain exclusively the active-sterile neutrino mixing. In the next sections we use this expression in both $(3 + 1)$ and $(3 + 2)$ models to extract bounds on such mixing matrix elements.
III. THE \((3 + 1)\) MODEL

The \((3 + 1)\) model is the simplest sterile neutrino model. In it the mixing matrix \(U\) is of dimension \(4 \times 4\) which, in the case of CP invariance, can be parameterized by six independent free parameters. Our considerations on these free parameters are the followings. Only two of them are really known, which are the angles, \(\theta_{23}\) and \(\theta_{12}\), involved in the atmospheric and solar neutrino oscillation, respectively. The current best fit values for these angles are \(\theta_{23} = 45^o\) and \(\theta_{12} = 34^o\) [12]. The third parameter we consider is the angle \(\theta_{13}\). Direct searches at reactor experiments give the upper bounds \(\theta_{13} \leq 12^o\), but its best fit value is \(\theta_{13} = 0\) [12]. The other three free parameters are responsible for the mixing among active and sterile neutrinos. It is expected that these free parameters be small such that their effects on \(\theta_{12}, \theta_{23}\) and \(\theta_{13}\) could be neglected. Moreover we will make use of the fact that, in the SBL experiments, the relevant mixing matrix elements are \(U_{14}\) and \(U_{24}\). For example, the effective angle probed by MB and LSND experiments is \(\sin^2(2\theta) = 4U_{14}^2U_{24}^2\) [5, 6]. Thus we will use a \(U_{4 \times 4}\) Pontecorvo-Maki-Nakagawa-Sakata mixing matrix whose parameterization focus exclusively on these mixing matrix elements. One parameterization of interest for us is given by [13]

\[
U_{4 \times 4} \approx \begin{pmatrix}
  c & s & 0 & \delta \\
  \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \kappa \\
  -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \kappa \\
  -\delta c + \sqrt{2}\kappa s & -\delta s - \sqrt{2}\kappa c & 0 & 1
\end{pmatrix}, \tag{4}
\]

with \(c = \cos \theta_{12}\) and \(s = \sin \theta_{12}\). For \(\theta_{12} = 34^o\) we have \(c = 0.83\) and \(s = 0.56\). Throughout this paper we use these values for \(c\) and \(s\). This parameterization is interesting for our proposal because the bounds on \(\delta\) and \(\kappa\) fall directly on \(U_{14}\) and \(U_{24}\).

After these considerations we are ready to extract the bounds that \(\Gamma_{\text{inv}}\) put on \(\delta\) and \(\kappa\). For this, we substitute the elements of \(U_{4 \times 4}\) given above in the expression for \(\Gamma_{\text{inv}}\) given in Eq. (3) for the particular case of one sterile neutrino. The current experimental value for the invisible \(Z\) decay width is \(\Gamma_{\text{inv}}^{\exp} = 499 \pm 1.5\) MeV [14]. In this work we use \(m_Z = 91.1875\) GeV and \(G_F = 1.16637 \times 10^{-5}\) GeV\(^{-2}\).

The bounds on \(\delta\) and \(\kappa\) with 95% CL are showed in FIG. 1. As we can see in that picture, the maximum value \(\delta\) and \(\kappa\) can develop is 0.116 and 0.08, respectively. This is
a very restrictive bound, which get clear when we translate it to the effective angle that arises in appearance experiments, \( \sin^2(2\theta) = 4\delta^2\kappa^2 \). According to FIG. 1, the upper bound required by the invisible Z decay width on this effective angle is of order of \( \sin^2(2\theta) \leq 10^{-5} \).

Let us confront this bound with the LSND data. Remember that LSND signal requires \( \sin^2(2\theta) \approx 10^{-3} \). Such value is two order of magnitude above the upper bound on this effective angle coming from the invisible Z decay width. Thus the invisible Z decay width bound on the effective angle \( \theta \) enter in conflict with the possibility that LSND signal be explained by neutrino oscillation in the context of one sterile neutrino.

**IV. THE \((3 + 2)\) MODEL**

In considering CP invariance the \( U_{5 \times 5} \) mixing matrix can be parameterized by ten free parameters. Here also we consider that three of them are the angles \( \theta_{12}, \theta_{23} \) and \( \theta_{13} \) whose current best fit values are shown in the previous section. The other seven free parameters are responsible by the active-sterile neutrino mixing. As in the previous case, we consider that these free parameters are small such that their effects on the angles \( \theta_{12}, \theta_{23} \) and \( \theta_{13} \) are neglected. Moreover we consider the fact that the expression for the relevant appearance probability in SBL experiments involve the elements \( U_{14}, U_{15}, U_{24} \) and \( U_{25} \) only \[8, 9, 15\]. Thus, following the arguments of the previous section, the simplest parameterization for the \( U_{5 \times 5} \) Pontecorvo-Maki-Nakagawa-Sakata mixing matrix of interest for us here is given by \[13\]

\[
U_{5 \times 5} \approx \begin{bmatrix}
\begin{array}{cccc}
c & s & 0 & \delta & \epsilon \\
-\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \kappa & \xi \\
-\frac{s}{\sqrt{2}} & \frac{-c}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \kappa & \xi \\
-\frac{\delta c + \sqrt{2}\kappa s}{\kappa} & -\frac{\delta s - \sqrt{2}\kappa c}{\kappa} & 0 & 1 & 0 \\
-\frac{\epsilon c + \sqrt{2}\xi s}{\xi} & -\frac{\epsilon s - \sqrt{2}\xi c}{\xi} & 0 & 0 & 1
\end{array}
\end{bmatrix}
\]

With this mixing matrix in hand, what we have to do now is to substitute its elements in the expression for \( \Gamma_{\text{inv}} \) given in Eq. (3) for the case of two sterile neutrinos. We proceed by attributing some specific values for \( \xi \) and then varying \( \delta, \kappa \) and \( \epsilon \). We would like to stress that \( \xi = 0.08 \) is the maximal value that \( \Gamma_{\text{inv}}^{\text{exp}} \) allows this parameter develop with 95 % CL. The graphics in FIG. 2 show the allowed values \( \delta, \kappa \) and \( \epsilon \) can develop for four different
values of the parameter $\xi$. As we can see in these figures, the maximum value $\kappa$ and $\epsilon$ can develop is about $10^{-2}$ while $\delta$ can attain $10^{-1}$ as maximum value.

Just for effect of comparison, let us confront our bounds with the best fit points for these mixing matrix elements presented in the global analysis of the Refs. [8, 9, 15]. The global analysis in Refs. [8, 15] present best fit points for the elements $U_{14}$, $U_{24}$, $U_{15}$ and $U_{25}$ while Ref. [9] presents best fit points for the combinations $|U_{14}U_{24}|$ and $|U_{15}U_{25}|$. As we can see in such references, the order of magnitudes of such best fit points lie in the range $10^{-1}$ to $10^{-2}$. Looking at the graphics displayed in FIG. 2, we see that the values $\delta$, $\kappa$, $\xi$ and $\epsilon$ can develop is in complete disagreement with such best fit points. This indicate that, even with two sterile neutrinos, the invisible Z decay width enters in conflict with the possibility of explaining LSND signal through neutrino oscillation.

V. SUMMARY

To summarize, we have considered scenarios with one and two light sterile neutrinos and obtained bounds on the active-sterile neutrino mixing elements from the invisible Z decay width. For the case of one sterile neutrino, the bound is such that the maximum values $\delta$ and $\kappa$ can develop are 0.116 and 0.08, respectively. This translates in the following bound on the effective mixing angle that arises in appearance experiments, $\sin^2(2\theta) = 4\delta^2\kappa^2 \leq 10^{-5}$. For the case of two sterile neutrinos, the bounds on the mixing matrix elements $\delta$, $\kappa$, $\epsilon$ for four different values of $\xi$ are displayed in FIG. 2. The bounds are such that the maximum values $\kappa$, $\epsilon$ and $\xi$ can develop are about $10^{-2}$ while $\delta$ can attain $10^{-1}$. To finalize, in both $(3+1)$ and $(3+2)$ models, the invisible Z decay width bounds on the active-sterile mixing enter in conflict with the possibility of explaining LSND signal through neutrino oscillation.

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FIG. 1: Possible values that $\delta$ and $\kappa$ can develop allowed by the invisible $Z$ decay width with 95% CL in the $(3 + 1)$ model.
FIG. 2: Possible values that $\delta$, $\kappa$ and $\epsilon$ can develop allowed by the invisible Z decay width with 95% CL in the (3 + 2) model for the following values of $\xi$: $\xi = 0.08$(upper left), $\xi = 0.06$(upper right), $\xi = 0.04$(lower left), $\xi = 0.02$(lower right).