Improving the performance of structure-embedded acoustic lenses via gradient-index local inhomogeneities

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We investigate the use of graded inhomogeneities in order to enhance the focusing and collimation performance of structure-embedded acoustic metamaterial lenses. The type of inhomogeneity exploited in this study consists in axial symmetric exponential-like gradients of either material or geometric properties that create gradient-index inclusions able to bend and redirect propagating waves. In particular, we exploit the concept of gradient index inclusions to achieve focusing and collimation of ultrasonic beams created by embedded drop-channel lenses in both bulk and thin-walled structures. In the latter, the implementation is possible thanks to geometric exponential tapers known as Acoustic Black Holes (ABH). ABH tapers allow accurate control of the characteristics of the acoustic beam emanating from the lens channel which in the conventional design is severely affected by diffraction. The concept of beam control via graded inclusions is numerically illustrated and validated by using a combination of methodologies including geometric acoustics, finite difference time domain, and finite element methods.

Keywords: acoustic lens; acoustic black hole; gradient index inhomogeneity; drop-channel; acoustic metamaterials

1. Introduction

The ability to generate highly directional and focused ultrasonic excitation has become a major area of interest for the structural health monitoring (SHM) field since it enables selective damage interrogation and leads to improved accuracy and resolution. The possibility to send ultrasonic energy in a preferential direction can also help counteracting the highly directional character of certain material systems, such as orthotropic layered composites [1] where the direction of energy propagation can be largely different from the initial direction of the emitted wave. In recent years, phased array (PA) technology has emerged as one of the most promising approaches to the generation of directional and focused ultrasonic excitation. PA exploits a network of transducers that emit mostly omnidirectional waves with prescribed phase differences. Although a robust and efficient technology, PA has two major limitations that prevent its extensive use in practical applications: (1) it requires a large number of transducers, and (2) it cannot produce collimated beams. Both limitations can be traced back to the same cause that is PA operates based on principles of constructive/destructive interference of the multiple

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omnidirectional wavefronts generated by the array of transducers. An effective PA system typically requires dozens of transducers, which is a major downside from the SHM perspective because it leads to higher system complexity and to increased probability of malfunctions and false alarms.

In an effort to develop novel technologies able to address these important limitations of PA, previous studies have explored the design of embedded metamaterial lenses [3,4]. This approach was proposed as a possible alternative to achieve selective beam-forming and beam-steering via a single ultrasonic transducer. The embedded lens design exploited the host structure in order to mold the wavefronts generated by a single ultrasonic transducer into a highly directional beam; that is the structure itself was used as a mechanical filter to produce selective excitation. The lens was initially developed according to a locally resonant [4] design approach. It was shown that a proper design of the equi-frequency-contour characteristics of the lens would result in beaming, steering, focusing, and collimation properties, all achievable by simply tuning the excitation frequency of a single acoustic source. The locally resonant design, however, presented some fabrication complexities that made it not suitable for certain applications, particularly those involving the design of structural materials (that is materials exhibiting load-bearing capabilities). For these reasons, a nonresonant design [3] was proposed in an effort to retain the advantages of the embedded lens concept while reducing the fabrication complexity. The working mechanism was based on the well-known concept of drop-channel [5–7], which allows creating preferential paths for wave propagation inside a periodic material. In this approach, the lens was created by embedding periodic homogeneous inclusions in the host structure while a network of carefully engineered drop-channels was used to produce and deliver the acoustic signal in prescribed directions. The individual channels were activated by simply tuning the frequency of excitation of a single transducer thanks to a coupling mechanism based on locally resonant defects. While this design allowed achieving very effective beam-forming and beam-steering characteristics, the overall performance was more limited than in the locally resonant case. In particular, the ultrasonic beam was subjected to diffraction at the exit of the acoustic channel that ultimately degraded the performance by preventing the generation of focused or collimated beams. In fact, the diffraction process occurring at the exit of the channel resulted in large angle of apertures that produced a diffused ultrasonic beam not unlike that generated by a plane wave source through a slit.

In this paper, we investigate the possibility to achieve focusing and collimation of the ultrasonic beam emitted by the embedded drop-channel lens by using gradient index (GRIN) inclusions implemented via local inhomogeneity. GRIN [8,9] materials exploit a spatially variable distribution of the physical parameters to produce a position-dependent phase velocity. The existence of phase velocity gradients results in acoustic rays having curved trajectories and, ultimately, in wavefronts that are largely distorted upon propagation. In recent years, phononic crystals and/or acoustic metamaterials have been extensively used to tailor the physical parameters of the host material in order to control the propagation of acoustic and elastic waves. In the long wavelength limit approximation, effective parameters [10] can be used to characterize the behavior of these composites and to design refraction index and phase velocity profiles [11]. Note that the conventional metamaterial design of GRIN, which relies on multiple phase inclusions, cannot provide a smooth and continuous variation of the refraction index but only an approximation via a step-like distribution of the material properties. At high frequency, this characteristic yields limited performances and very restrictive requirements
on fabrication precision. An alternative approach to tailoring the phase velocity profile is based on graded elastic properties of the host material (e.g. density, modulus, etc.), the so-called functionally graded materials [12,13] (FGM). Although these materials could in principle achieve a continuous variation of the elastic parameters, such profiles are very challenging to obtain in practice. An alternative approach, conceptually analogous to FGM but limited to applications on structural waveguides, is based on tailoring the geometric properties. In structural waveguides, spatial gradients of the flexural phase velocity can be enforced by tailoring the geometric parameters of the guide such as, for example, the thickness. In previous studies, this approach has been applied to achieve extreme damping for flexural waves by exploiting the concept of Acoustic Black Hole (ABH) [14]. The fundamental physical principle exploited in the ABH tapers was first observed by Pekeris [15] for waves propagating in stratified fluids and later extended to acoustics in solids by Mironov [16]. Afterwards, Krylov [13,17] applied this concept to achieve passive vibration control of structural elements. The intrinsic wave-focusing characteristic of the ABH tapers was also exploited in the design of high-performance vibration-based energy harvesting systems [18].

The ABH consists of a circular taper with exponentially variable thickness that is embedded into the supporting structural element and able to produce smooth gradients of both the phase and group velocities. The existence of these gradients has two main effects: (1) it steers flexural waves towards the center of the ABH and (2) it gradually reduces the phase and group velocities that, in the ideal case, will gradually approach a zero value at the ABH center. A typical ABH thickness profile is described by the relation $h(x) = h_r + \varepsilon x^m$, where $h_r$ is the residual thickness, $m \geq 2$ is the taper coefficient and $\varepsilon$ is a real constant. Both $m$ and $\varepsilon$ must be chosen to satisfy the smoothness condition [15,19]. Examples of applications of geometric tailoring to thin plate structures were recently reported by Dehesa [20] and Zhu [21] who designed, respectively, refractive lenses and phononic plates based on embedded thickness profiles.

The paper is structured as follows: we first briefly review the concept of drop-channel acoustic lens, and then we illustrate via numerical computations the concept of beam focusing and collimation achievable via a graded elastic modulus in bulk structures. Successively, we extend the design to thin plate structures where the control on the beam properties is achieved via embedded ABH tapers. The performance of the graded design is explored using a combination of geometric acoustics, finite difference time domain (FDTD), and finite element methods. It will be shown that, compared with the initial design studied by Zhu [3] without tapered inclusions, the proposed approach provides a practical methodology to control the characteristics of the ultrasonic excitation especially in terms of beam collimation (i.e. angle of aperture), reduction of side-lobes, and adjustment of the beam width.

2. Drop-channel lens design and numerical model

In the following, we briefly review the main design and modeling approach for the drop-channel-based acoustic lenses [3]. The lens design relies on the use of periodic distributions of inclusions embedded in the surrounding structure. These inclusions effectively create a metamaterial structure that is designed to provide full bandgaps [22,23] in the frequency range of interest for the operation of the lens. In this frequency range, the metamaterial acts as a stopband mechanical filter therefore preventing energy from propagating through. Figure 1(a) shows the conceptual
schematic and the geometry of such a lens. The lens is mostly divided in three sections: (I) an inner section without inc-
clusions, (II) an intermediate section with embedded point defects, and (III) an outer section containing a set of radial wave-
guides at prescribed azimuthal locations. The inner section is intended to host the source of the ultrasonic excitation. Wave propagation at frequencies inside the band-
gap can be achieved by creating a network of line defects (i.e. waveguides) associated with either localized or spatially confined modes \[24,25\]. The waveguides are dyna-
mically coupled to the inner section I by a network of point defects that are engi-
neered to exhibit resonant modes at desired coupling frequencies. The dynamic response of the lens embedded in a bulk material is described by the Navier’s equations. The process is illustrated on a semicircular lens made of cylindrical nickel inclusions distributed in an epoxy background [3]. The Navier’s equations for the bulk material are:

\[
\rho(x,y,z)\ddot{u}_3(x,y,z,t) = \partial_j \left[ C_{ijkl}(x,y,z) \frac{\partial u_l}{\partial x} \right]
\]

where \(\rho(x,y,z)\) and \(C_{ijkl}(x,y,z)\) are the space-dependent density and elastic tensor, respectively. For the sake of simplicity, in the numerical simulations, we consider only the shear vertical (SV) mode (i.e., particle displacement parallel to the axis of the inclusions) that is uncoupled from other bulk modes. From Equation (1), the corresponding governing equation for the SV mode becomes:

\[
\rho(x,y) \frac{\partial^2 u_3}{\partial t^2} = \frac{\partial \tau_{13}}{\partial x} + \frac{\partial \tau_{23}}{\partial y}
\]

\[
\tau_{13} = C_{44}(x,y) \frac{\partial u_3}{\partial x}
\]

\[
\tau_{23} = C_{44}(x,y) \frac{\partial u_3}{\partial y}
\]

Figure 1. (a) Conceptual schematic of the drop-channel based lens with four embedded channels. The lens is divided in three main sections: (I) an inner section without cylindrical inclusions, (II) an intermediate section with embedded point defects, and (III) an outer section containing a set of radial wave-
guides at prescribed azimuthal locations. (b) Shows the magnitude of the nondimensional displacement field generated when the lens is excited by a single-tone harmonic input at \(f_z = 24.95\) kHz. \(U_0\) indicates the maximum displacement at the excitation point.
Equations (2)–(4) can be solved by using the FDTD \[26\] approach that yields the following set of algebraic equations:

\[
{u_3(\mathit{i}, \mathit{j}, \mathit{t} + 1) = 2u_3(\mathit{i}, \mathit{j}, \mathit{t}) - u_3(\mathit{i}, \mathit{j}, \mathit{t} - 1) + \frac{\Delta t^2}{\rho(i,j)\Delta x} \left[ \tau_{13}\left(i + \frac{1}{2}, \mathit{j}, \mathit{t}\right) - \tau_{13}\left(i - \frac{1}{2}, \mathit{j}, \mathit{t}\right) + \tau_{23}\left(\mathit{i}, \mathit{j} + \frac{1}{2}, \mathit{t}\right) - \tau_{23}\left(\mathit{i}, \mathit{j} - \frac{1}{2}, \mathit{t}\right) \right]}
\]

\[
\tau_{13}(\mathit{i} + \frac{1}{2}, \mathit{j}, \mathit{t}) = \frac{C_{44}(\mathit{i} + \frac{1}{2}, \mathit{j}, \mathit{t})}{\Delta x} [u_3(\mathit{i} + 1, \mathit{j}, \mathit{t}) - u_3(\mathit{i}, \mathit{j}, \mathit{t})]
\]

\[
\tau_{13}(\mathit{i}, \mathit{j} + 1/2, \mathit{t}) = \frac{C_{44}(\mathit{i}, \mathit{j} + 1/2, \mathit{t})}{\Delta x} [u_3(\mathit{i}, \mathit{j} + 1, \mathit{t}) - u_3(\mathit{i}, \mathit{j}, \mathit{t})]
\]

where \(u_3\), \(\tau_{13}\), \(\tau_{23}\) are the out-of-plane displacement and two components of the shear stress, while \(i\), \(j\), and \(t\) are the space and time step indices, respectively.

Further details on the lens configuration can be found in Ref. \[3\]. The different waveguides can be activated by changing the excitation frequency, therefore achieving directional ultrasonic excitation via a single transducer. Figure 1(b) shows an example of the displacement field produced by the lens using a single tone frequency excitation at \(f_z = 24.95\) kHz, which corresponds to the activation frequency of the channel oriented at 120°. Perfectly matched layers \[27\] are also used all around the computational domain so to avoid backscattering from the boundaries.

The performance of the different lens designs will be evaluated based on two main metrics: (1) the angle of aperture of the main beam, and (2) the side lobe amplitude ratio \(U_{si}\). The angle of aperture is defined as the central angle \(\angle AOB\), including the portion of the beam where the amplitude drops less than 90% with respect to the peak value in the propagation direction (Figure 1(b) red arrow). The side lobes amplitude ratio \(U_{si}\) is defined as \(U_{si} = U_{pm}/U_{psl}\), where \(U_{pm}\) and \(U_{psl}\) represent the peak value of the displacement field for the main and the side lobes, respectively. A zero value of this ratio indicates that no side lobes exist.

Results from the above simulations clearly indicate that as the ultrasonic beam exits the channel, it undergoes a diffraction process that ultimately produces a large beam aperture (\(\theta \approx 101°\) in the present example). Under these conditions, the waveguide behaves not unlike a point source through a slit, therefore drastically reducing the ability of the lens in generating directional excitation. This characteristic was found to be one of the main limitations of this lens design. New or alternative design approaches are needed to limit this diffraction effect and to improve the overall performance of the lens.

### 3. Design based on GRIN inclusions: geometric acoustic analysis

In order to alleviate the effects of diffraction, we explore possible modifications to the design of the lens that are based on the use of GRIN inclusions. The ability of GRIN materials (or, equivalently, of ABH tapers) to bend incoming waves is determined by the existence of a phase velocity gradient produced by the local inhomogeneity. In the case of a geometric ABH taper, the characteristics of the profile are identified by two main parameters: the residual thickness \(h_r\) and the coefficient \(m\) of the exponential term. The former determines the lower bound of the local phase velocity while the latter determines
how rapidly, in space, the phase velocity changes (i.e., it determines the spatial gradient of
the phase velocity). In order to study the wave propagation in inhomogeneous media, it is
convenient to use a geometric acoustic approach which provides a clear assessment of the
wave trajectory when affected by the inhomogeneity. The trajectory of the individual ray
for the uncoupled elastic wave is given by:

\begin{align}
\frac{dx}{dt} &= c^2 s \\
\frac{ds}{dt} &= -\frac{1}{c} \nabla s
\end{align}

where \( x \) is the position vector along the ray trajectory, \( s \) is the slowness vector that can be
expressed as \( s = \frac{n}{c} \), where \( c \) is the local phase velocity, and \( n \) is the unit vector normal to
the wavefront. Once the phase velocity distribution and the initial conditions (in terms of
the source location and type) are specified, Equations (6)–(7) can be numerically inte-
grated to obtain the ray trajectories across the inhomogeneous medium. In the case under
study, the phase velocity \( c_0 \) of the background area is constant while the phase velocity \( c \)
of the tapered area satisfies the relation \( \frac{c}{c_0} = \sqrt{\frac{h(r)}{h_0}} = \sqrt{\frac{h+\varepsilon m}{h_0}} \).

We observe that similar considerations are applicable if we assume the GRIN inclusion
made of an exponentially variable elastic modulus as \( \frac{c}{c_0} = \sqrt{\frac{G(r)}{G_0}} = \sqrt{\frac{G_0+\varepsilon m}{G_0}} \). This
latter approach produces in bulk materials phase velocity profiles that are equivalent, in
principle, to those produced by an exponential-like taper in thin-walled structures.

Geometric acoustics was used to guide the design of the GRIN inhomogeneity and
perform a preliminary assessment of the performance versus some of the design para-
ters. We observe that the use of a large taper coefficient \( m \) would result in a very steep
variation of the phase velocity therefore increasing the reflection coefficient and prevent-
ing a progressive bending of the wavefronts. In fact, a high taper coefficient produces an
inclusion mostly divided into two main regions: an outer region with low impedance
mismatch with respect to the background, and an inner region with large impedance
mismatch. For this reason, we fixed the taper coefficient to \( m = 2.2 \), which is a typical
value for ABH \([13–15]\) satisfying the smoothness criterion \([15,18]\) and producing a
smooth variation of the phase velocity, and explored the effect of a variable residual
thickness \( h_r \).

The exponential-like taper was first simulated in bulk materials by tailoring the
elastic properties, in particular the local shear modulus. Figure 2(b)–(e) shows the
displacement fields, obtained by FDTD simulations, where a single circular taper
with fixed radius \( r = 4.32 \text{ cm} \) and taper exponent \( m = 2.2 \) is placed at the exit of the
waveguide. We explored the performance of tapers having different values of the
lower bound of the shear modulus \( G_r \). In Figure 2(b)–(e), the shear modulus \( G_r \)
increases from left to right (i.e. from (b) to (e)) according to the following pre-
defined values \( G_r = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2} G_0 \). Consequently, also the phase velocity gradient
increases (from right to left). Figure 2(a) provides the reference case where no
taper is placed at the exit of the channel. The insets in the upper part of each figure
show the corresponding ray acoustic solutions that are in good agreement with the
full wave solutions. As expected from Equations (5)–(6), larger phase velocity
gradients provide more bending ability showing how an omnidirectional source
can be progressively scattered (b), focused (c), or collimated (d) by different taper designs.

4. Functionally graded arrays of GRIN inclusions

The results shown in the previous section illustrated the main operating principle of the graded inclusions. In fact, a GRIN inclusion operates much like an optical lens which can scatter, focus, or collimate waves depending on its geometric design. Following a similar analogy, we can expect that an array of GRIN inclusions can be used, much like optical multi-lens devices, to produce a progressive alignment of the ultrasonic beam further improving the level of control on the excitation.

We consider a distribution of three ABH tapers forming a sequence of functionally graded inclusions $r_i = (0.2880i - 0.1440)a$ for $i = 1, 2, 3$ (Figure 3(a)). The geometric parameters are: $r_1 = 0.432a$, $r_2 = 0.72a$, $r_3 = 1.008a$, $\Delta R_1 = 1.44a$, and $\Delta R_2 = 2.016a$, where $a = 0.03$ cm is the lattice constant which defines the unit cell of the lens geometry [3]. The residual thickness ratios for all the holes are set to $\frac{h}{h_0} = \frac{5}{9}$, the exponential taper is $m = 2.2$ and therefore the coefficient can be determined as $\epsilon = \frac{4h_0}{9a\pi^2}$. Equations (5)–(6) were numerically integrated to identify the trajectories of the rays emanated by a point source placed in front of the first ABH. From the ray-tracing results (Figure 3(b)), it can be seen that the graded ABH sequence provides larger flexibility in creating focused or collimated beams from initially omnidirectional sources. Note that the functionally graded array also provides control on the beam width allowing either expansion or compression (depending on the gradient of the radius profile $r_i$) of the incident beam. The beam width at the exit of the graded array can be, at most, the size of the ABH diameter. Figure 3(b) shows that, in the selected design, the beam width increases from $w_1 = 2.72$ cm to $w_2 = 3.96$ cm.

![Figure 2. Magnitude of the displacement field produced by FDTD simulations showing the response of an acoustic waveguide filtered through a graded index inclusion (red circle). (a) typical diffraction pattern occurring at the exit of a waveguide without graded inclusion. (b)–(e) Magnitude of the displacement field obtained by placing a single GRIN taper at the exit of the waveguide. The tapers have all the same radius and taper coefficient while the residual shear modulus $G_0$ at the center increases from (b) to (e). The inset provides the corresponding geometric acoustic solution which helps understanding the effect of the graded index inclusion on the ray trajectory.](image)
Note that the above results for either single or multiple tapers were not obtained following an optimization process, and therefore these results are not indicative of the maximum performance of the GRIN inclusions.

5. Application to the acoustic lens

As initially mentioned, the main objective of this study is to integrate the concept of graded ABH tapers into the design of the acoustic lens in order to improve the focusing and collimation performance. Here below, this concept is first investigated for bulk materials and then extended to thin-walled structures.

5.1. Functionally graded tapers in bulk materials

We consider the semicircular lens as in Section 2 and place a graded array of three circular tapers (see Figure 4(a), gray circles) at the exit of the 120° waveguide. The array has the same parameters as that discussed in the previous section. The first hole of the array was centered on the outer perimeter of the lens. The dynamic response of the ABH-enhanced lens was evaluated by using a single-tone harmonic input at frequency $f_z = 24.95$ kHz (i.e., the activation frequency of the channel). Results are shown in Figure 4(b) and (c). A direct comparison of the dynamic response of the enhanced lens (Figure 4(b)) versus the conventional design is presented in Figure 1(b).
Results show large improvement in terms of beam formation and an evident reduction in the beam aperture, which is now $\theta \approx 36.5^\circ$ (about 65% improvement with respect to the original drop-channel design). The performance of the two designs is also compared in Figure 4(c) in terms of directivity plots. The plots are extracted from the displacement field produced at a distance $L = 34.5$ cm (see dashed white line) from the center of the lens which coincides with the location of the acoustic source. The plot shows the magnitude of the displacement field normalized by the amplitude at the source location $U_0$. The comparison of the directivity functions clearly indicates the improvement in terms of beam formation as well as a drastic reduction of the side lobes. The improved control on the beam aperture results in a consistent increase (about 30%) of the vibrational energy delivered in the selected direction of excitation. The reduction of the side lobes can be quantified using the ratio $U_{sl} = U_{pm}/U_{psl}$, where $U_{pm}$ and $U_{psl}$ represent the peak value of the displacement field for the main and the side lobes, respectively. For the bulk material, the proposed ABH-enhanced design delivers a ratio $U_{sl} = 0.47$ that represents a reduction of about 40% with respect to the case without lens (Figure 1(b)) that achieved $U_{sl} = 0.78$.

It is expected that by a proper optimization of the design parameters, further reduction of the side lobes as well as a fully collimated beam could be achieved.

5.2. Geometrically graded tapers in thin-walled structures

It was previously pointed out that fabricating structural materials with smoothly varying elastic properties (such as density and elastic moduli) is a challenging task. In addition, due to integrity and durability issues it is often preferred to avoid the use of multiple material interfaces. Thin-walled structures offer an interesting alternative approach for the integration of graded inclusions. In these structural components, GRIN inhomogeneities can be obtained exploiting the concept of ABH geometric taper. According to the ABH design, the inhomogeneous phase velocity distribution can be achieved by tailoring the local thickness according to an exponential-like profile, as discussed in Section 3.

The concept is illustrated using a rectangular thin silicon plate where a single drop-channel is realized following the same design procedure previously used for the bulk
acoustic lens. In this case, the inclusions of the lens were obtained via through-holes (following the approach in Ref. [3]) arranged in a square lattice geometry.

In particular, the plate was built out of a square array of $11 \times 11$ circular through-holes having radius ratio $r/a = 0.46$, thickness $h_0/a = 0.4$, and lattice constant $a = 10$ cm. The corresponding infinite and perfectly periodic plate based on this configuration yields a narrow full bandgap [28] between $\Delta f = 28–31$ kHz, as shown in Figure 6(a). A drop-channel was created removing an entire array of inclusions in the center so as to create defect modes inside the bandgap that could be exploited to support the directional excitation. The channel was coupled to the area hosting the acoustic source by using one point cavity defect implemented via a missing hole. The defect exhibited a fundamental resonance frequency at $f = 29$ kHz.

A graded ABH taper was then located in front of the channel to achieve beam control. The ABH taper had radius $r = 25$ cm, residual thickness $h_r = 1.77$ cm and a taper coefficient $m = 2.2$. The schematics of the two thin plates with and without the taper are shown in Figure 5(a) and (b).

In order to test the ABH taper design, we solved the plate model by finite elements using the commercial software Comsol Multiphysics (Comsol 4.3, COMSOL Inc., Burlington, MA, USA). Both plates were excited by a harmonic uniformly distributed boundary load in the out-of-plane direction tuned to the cavity mode frequency and applied at the right boundary of the plate. Perfectly matched layers were used all around the boundary of the plate to reduce backscattering and to improve the beam visualization.

Figure 5 shows the performance of the ABH-graded design by direct comparison with the conventional design. The response of the two lenses is shown in terms of normalized out-of-plane displacement fields. Similarly to what observed for the bulk material design, the ABH-graded taper is extremely effective in controlling the

![Figure 5](image_url)

Figure 5. Numerical investigation on the performance of the (a) conventional and (b) ABH-enhanced lens. The top figures show the geometric design of the host structure with the embedded lens and the ABH taper. Results are shown in terms of out-of-plane displacement amplitude maps when the lens is excited with a harmonic force load at the frequency of the cavity mode $f = 29$ kHz. The direct comparison shows the focusing ability of the ABH taper that drastically reduces the angle of aperture and increases the energy transferred in the channel direction.
formation of the beam and reducing the diffraction at the exit of the channel. The aperture of the beam is reduced from $\theta \approx 97.8^\circ$ to $\theta \approx 38.6^\circ$ (about 60%). The directivity plots (Figure 6(b)), extracted from the response at distance $L/a = 7$ from the exit section of the channel, indicate a drastic reduction of the side lobes. The ratio $U_{sl}$ drops from 0.54 for the case without lens to approximately 0.12 (side lobes have almost completely disappeared) in the ABH-enhanced case. We also observe an outstanding increase in the amount of energy transferred in the channel direction, as shown by the nondimensional field $U/U_0$ that increases from 0.43 to 0.86 (about 100%).

6. Conclusions
We presented an approach to improve the performance of structure-embedded acoustic lenses by exploiting a network of engineered local inhomogeneities. The conventional drop-channel lens design was able to achieve selective beam-forming and beam-steering via a single ultrasonic transducer but suffered from diffraction effects that severely reduced its performance in terms of directivity and energy intensity. GRIN inclusions implemented either via material or geometric inhomogeneity were proven to provide a viable solution and to drastically improve the performance of the lens. We numerically demonstrated that either single or multiple inclusions allow an effective control of the beam aperture, drastically reducing the side lobes and consistently improving the intensity of the beam in the channel direction.

For the bulk material design where the lens is implemented via material inhomogeneity, the beam aperture was reduced of about 65% while the side lobes ratio $U_{sl}$ dropped of about 40% from 0.78 (without lens) to 0.47 (with lens).

In the case of the thin-walled structure, the beam control was achieved via ABH tapers. This design provided a reduction of the beam aperture of about 60% and of the side lobes ratio from 0.54 to about 0.12, that is, an almost complete elimination of the side lobes.
The proposed design is of particular interest for applications on thin-walled structures where the graded inhomogeneity can be achieved via geometric ABH tapers. These tapers allow a high degree of control on the beam properties while involving minimal fabrication complexity. The ABH-enhanced drop-channel lens can have critical implications in ultrasonic-based SHM systems. In particular, the outstanding reduction in the number of transducers (only one is needed versus the several dozen required by the PA technology) would reduce considerably the system complexity and the probability of malfunctions and false alarms.

The largely increased directionality of the lens will drastically improve the ability to selectively scan a structure for damage delivering more effectively ultrasonic energy in a predefined direction and reducing backscattering from unwanted sources.

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