J/Ψ production, χ polarization and Color Fluctuations

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The hard contributions to the heavy quarkonium-nucleon cross sections are calculated based on the QCD factorization theorem and the nonrelativistic quarkonium model. We evaluate the nonperturbative part of these cross sections which dominates at $\sqrt{s_{NN}} \approx 20$ GeV at the Cern Super Proton Synchrotron (SPS) and becomes a correction at $\sqrt{s_{NN}} \approx 6$ TeV at the CERN Large Hadron Collider (LHC). J/Ψ production at the CERN SPS is well described by hard QCD, when the larger absorption cross sections of the χ states predicted by QCD are taken into account. We predict an $A$-dependent polarization of the χ states. The expansion of small wave packets is discussed.
The aim of the present paper is to investigate color coherent phenomena in the propagation of hadronic states with hidden charm in \( pA \) and \( AB \) collisions. We restrict the consideration to the central rapidity region of nuclear beams at \( \sqrt{s} \approx 20 \) AGeV (CERN SPS) where formation time effects in \( J/\Psi \)-production are corrections. Qualitatively new effects for \( \sqrt{s} \approx 200 \) AGeV (RHIC) and \( \sqrt{s} \approx 6 \) ATeV (LHC) energies will be discussed also. We assume that hadronic states with hidden flavor are produced predominantly in hard parton collisions and calculate the absorption factor \( S \) of the total yield of heavy quarkonia which is not distorted by the initial state interaction, cf. [1].

Under these assumptions \( S \) can be directly measured by the ratio of the yields of heavy quarkonium to the dimuon yield [2]. We want to emphasize that the agreement of our calculations of the \( J/\Psi \) production in \( pA \) and in \( AB \) collisions with the data strongly supports the idea that hidden charm states are perturbatively produced in \( NN \) collisions.

The QCD factorization theorem is used to evaluate the PQCD cross sections of heavy quarkonium interactions with ordinary hadrons. However, the charmonium states (here denoted \( X \)) are not sufficiently small to ignore nonperturbative QCD physics. Thus, we evaluate the nonperturbative QCD contribution to the cross sections of charmonium-nucleon interaction by using an interpolation between known cross sections. The \( J/\Psi-N \) cross section evaluated in this paper is in reasonable agreement with SLAC data [3].

PQCD predictions for the cross section of \( \Upsilon-N \) scattering should be more reliable since (differently from the \( J/\Psi \)'s) the spatial size of \( \Upsilon \)'s is significantly smaller than the typical size for soft hadron interactions.

These \( X-N \) cross sections are used to evaluate nuclear effects for the production of \( X \) states. We find that the suppression of the \( J/\Psi \) in \( pA \) and \( AB \) collisions is reasonably well described within the present approach.

We predict a new QCD effect – the polarization of produced \( \chi_c \) states: \( c\bar{c} \)-states with nonzero orbital momentum will be polarized due to the longitudinally directed nuclear (color filter) target. Their polarization is evaluated quantitatively. Random interactions with comovers will depolarize the \( \chi \) states. Thus, measuring the \( \chi \) polarization may turn out to
be an effective method to investigate their interactions with comovers.

The production of charmonium states in $AB$ collisions has been proposed (via $J/\Psi$ suppression) as signal for Debye screening in deconfined matter [4]. However, secondary collisions of $J/\Psi$'s with hadrons have been suggested early on as a competing origin of $J/\Psi$ suppression [3,4]. This topic has been considered also in [4,5], basing on QCD motivated models (but not on the QCD factorization theorem as in our paper). In [4] (but not in [5]) it is also concluded that PQCD predicts a too small $J/\Psi$-N cross section to explain the observed absorption of $J/\Psi$ at CERN energies. In contrast to the ”preresonance” hypothesis suggested in [4] and to the model in [5] we explain the $J/\Psi$ suppression observed in $AB$ collisions as a consequence of the predicted large absorption of $\chi$ states. We found that at RHIC energies perturbative and nonperturbative QCD contributions to the $J/\Psi$-N cross section become comparable in variance to [4]. At LHC energies the hard contribution will dominate (we ignore in this paper the unitarity corrections which will be important at LHC, cf. [6,7]). The $\Upsilon$-N interaction cross section is calculated for the first time in this paper.

The first evaluations of $\sigma_{tot}(J/\Psi$-N) have been obtained by applying the Vector Dominance Model to $J/\Psi$ photoproduction data. This leads to $\sigma_{J/\Psi N} \sim 1$ mb for $E_{inc} \sim 20$ GeV. However, the application of the VDM leads to a paradox [4] – one obtains $\sigma_{tot}(\Psi'$-N) $\approx 0.7 \cdot \sigma_{tot}(J/\Psi$-N), although, on the other hand, $r_{\Psi'} \approx 2r_{J/\Psi}$. This clearly indicates that the charmonium states produced in photoproduction are in a smaller – than average – configuration. Therefore, the VDM significantly underestimates $\sigma_{tot}(J/\Psi$-N) [4].

Indeed, the $A$-dependence of the $J/\Psi$ production studied at SLAC at $E_{inc} \sim 20$ GeV exhibits a significant absorption effect [3] leading to $\sigma_{abs}(J/\Psi$-N) $= 3.5 \pm 0.8$ mb. It was demonstrated in [12] that, in the kinematic region at SLAC, the color coherence effects are still small on the internucleon scale for the formation of $J/\Psi$’s and lead only to a small increase of the value of $\sigma_{abs}(J/\Psi$-N). So, in contrast to the findings at higher energies, at intermediate energies this process measures the genuine $J/\Psi$-N interaction cross section at energies of $\sim 15$-20 GeV [12].

To evaluate the nonperturbative QCD contribution we use an interpolation formula for
the dependence of the cross section on the transverse size $b$ of a quark-gluon configuration. Three reference points are used to fix our parametrization of the cross sections (cf. Tab. I):

- based on the observation that the $\phi$-$N$ cross section is nearly half of the $\rho$-$N$ cross section we impose $\sigma(b_0 \cdot m_{\rho}/m_{\phi}) = 10$-$12$ mb, where $b_0$ is 0.6 fm.
- $b_0$ is approximately the transverse size of a $\pi$, so we choose $\sigma(b_0) = 25$ mb, as known from $\pi$-$N$ collisions.
- for configurations with $b \geq 1$ fm we set $\sigma(1$ fm $) = 40$ mb, the value reached when the two constituent quarks split to form two open charm mesons.

Thus, the $X$-$N$ cross sections is calculated via:

$$\sigma = \int \sigma(b) \cdot |\Psi(x, y, z)|^2 dx \, dy \, dz \ .$$

(1)

where $\Psi(x, y, z)$ is the charmonium wave function. In our calculations we use the wave functions from a non-relativistic charmonium model with a Cornell confining potential, $V(r) = -\frac{\kappa}{r} + \frac{r}{a^2}$, see [13] and refs. therein.

The calculated cross sections of the charmonium states are shown in Tab. I. For the $J/\Psi$ we found $\sigma_{J/\Psi N} = 3.6$ mb. This is in good agreement with the data [3], $\sigma = 3.5 \pm 0.8$ mb, for the $J/\Psi$-photoproduction!

Note the factor of $\approx 2$ between the $<b^2>$ values of the $\chi_{10}$ versus the $\chi_{11}$, respectively: the transverse size of a fast $\chi$ depends drastically on the polarization of the $\chi$ states. Following the above discussion one therefore can expect significant fluctuations of the strength of the interaction due to this geometrically directed filtering, even in those kinematic regions where the color coherent phenomena seem to be at first sight only a small correction. The values of the $\chi$-$N$ cross sections are given for different magnetic quantum numbers.

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1 Note that the deviation of $\sigma(b)$ from the estimate of PQCD occurs for $b \geq 0.3$ fm which is the distance close to the Chiral Phase Transition scale. We thank J. Bjorken for emphasizing this point.
We calculate $\sigma(X-N)$ using the assumption that PQCD is applicable in this case. Here we will ignore the differences between bare quarks of the QCD Lagrangian and the constituent quarks. Then the expression for $\sigma(X-N)$ follows unambiguously from the QCD factorization theorem [10]. The numerical calculation shown in Tab. I yields at CERN energies $\sigma(X-N)$ values which are significantly smaller than in [3]. The $\sigma_{\text{hard}}$ values in Tab. I are also calculated with eq. [1] but here we integrate only over the region $0.1 \text{ fm} < b < 0.2 \text{ fm}$, because $\alpha_s$ increases with $b$. Here $b$ is the transverse distance between the heavy quarks transverse to the momentum of the heavy quarkonium. Thus, the region $b > 0.2 \text{ fm}$ cannot be calculated within PQCD. We choose the lower limit for the integral, $b = 0.1 \text{ fm}$, because the quarkonium states will have a finite size already at the production point. For the calculation of the hard cross section the $\sigma(b)$ in eq. [1] is given by the function [10]

$$\sigma(b) = \frac{\pi^2}{3} b^2 \alpha_s(Q^2) \cdot xG(x, Q^2) .$$

The Bjorken $x$ needed for the calculation of $\sigma_{\text{hard}}(X - N)$ is calculated by $x = \frac{Q^2}{2m_N \nu}$, where $Q^2 = \frac{9}{4} b^2$, $m_N$ is the nucleon mass and $\nu$ is the energy of the heavy Quarkonium state $X$ in the rest frame of nucleon. The calculation in Tab. I was done for a state $X$ produced in midrapidity at SPS energy and in the target fragmentation region at RHIC and LHC. One can see that the hard contribution to the cross section is just a correction at SPS energies, but at RHIC energies both contributions become comparable and at LHC it dominates (we neglect here that the DGLAP equation (Dokshitser-Gribov-Lipatov-Altarelli-Parisi) should be probably violated [11]).

We follow the analysis of [7] (and refs. therein) to evaluate the fraction of $J/\Psi$’s (in $pp$ collisions) that come from the decays of the $\chi$ and $\Psi'$. No experimental information is available on the contribution from the $\chi(1^{+-})$, nor are any contributions of the higher

\[2\] The calculated $\sigma_{\text{hard}}$ reflects the effective cross section for the interaction of a $Q\bar{Q}$ pair with nucleons of the residual nucleus, which will transform into the corresponding $Q\bar{Q}$ state only after it has traversed the nucleus.
$c\bar{c}$ states (D-wave, radial excitations) known. Thus, the fraction of the directly produced $J/\Psi$'s used in eq. 3 is an upper limit for the actual fraction of directly produced $J/\Psi$'s. Furthermore, the ratio of directly produced $J/\Psi$'s to the yield of $J/\Psi$'s due to $\Psi'$ decays is $J/\Psi : \Psi' = 0.92 : 0.08$ in $pp$ collisions. So, the suppression factor $S$ of $J/\Psi$'s produced in the nuclear medium is calculated as:

$$S = 0.6 \cdot (0.92 \cdot S_{J/\Psi} + 0.08 \cdot S_{\Psi'}) + 0.4 \cdot S^X.$$  

Here $S^X$ are the respective suppression factors of the different pure charmonium states $X$ in nuclear matter. The $S^X$ are for minimum bias $pA$ collisions within the semiclassical approximation (cf. [15]):

$$S_A^X = \frac{\sigma(pA \rightarrow X)}{A \cdot \sigma(pN \rightarrow X)} = \frac{1}{A} \int d^2B \, dz \, \rho(B, z) \cdot \exp \left( -\int_z^{\infty} \sigma_X \rho(B, z')dz' \right).$$  

Here $\rho(B, z)$ is the local nuclear groundstate density (we used the standard parametrization from [14]).

The charmonium states are produced as small ($r_{\text{init}} \approx 0.2$ fm) configurations predominantly in gluon-gluon-fusion, then they evolve to their full size. Therefore, if the formation length of the charmonium states, $l_f$, becomes larger than the average internucleon distance ($l_f > r_{NN} \approx 1.8$ fm), one has to take into account the evolution of the cross sections with the distance from the production point [12].

The formation length of the $J/\Psi$ is given by the energy denominator $l_f \approx \frac{2p}{m_{J/\Psi}^2 - m_{J/\Psi}^2}$, where $p$ is the momentum of the $J/\Psi$ in the rest frame of the target. With $p = 30$ GeV, the momentum of a $J/\Psi$ produced at midrapidity at SPS energies ($E_{\text{lab}} = 200$ AGeV), this yields $l_f \approx 3$ fm. Due to the lack of better knowledge, we use the same $l_f \approx 3$ fm for the $\chi$. For the $\Psi'$ we use $l_f \approx 6$ fm, i.e. we introduced an additional factor of 2, because it is not a small object, but has the size of a normal hadron, i.e. the pion. For $E_{\text{lab}} = 800$ AGeV we get another factor of two for the formation lengths due to the larger Lorentz factor.

However, this has a large impact on the $\Psi'$ to $J/\Psi$-ratio depicted in Fig. 4 which shows the ratio $0.019 \cdot S_{\Psi'}/S_{J/\Psi}$ calculated with (squares (200 GeV) and triangles (800 GeV)) and
without (crosses) expansion. The factor 0.019 is the measured value in \( pp \) collisions, because the experiments do not measure the calculated value \( S_{\Psi'}/S_{J/\Psi} \) but \( \frac{B_{\mu\mu}\sigma(\Psi')}{B_{\mu\mu}\sigma(J/\Psi)} \). \( B_{\mu\mu} \) are the branching ratios for \( J/\Psi, \Psi' \rightarrow \mu\mu \). The calculations which take into account the expansion of small wave packages show better agreement with the data (circles) (taken from [2]) than the calculation without expansion time, i.e. with immediate \( J/\Psi \) formation, \( l_f = 0 \). We calculated this effect both at \( E_{lab} = 200 \text{ AGeV} \) and \( 800 \text{ AGeV} \). The data have been measured at different energies \( (E_{lab} = 200, 300, 400, 450, 800 \text{ GeV} \) and \( \sqrt{s} = 63 \text{ GeV} \)).

The charmonium survival probability \( S_{AB}^X \) in minimum bias \( AB \) collisions can be calculated by [15] \[ S_{AB}^X = S_A^X \cdot S_B^X \]. \( S_A^X \) and \( S_B^X \) are defined by eq. [4]. The neglect of the stopping of nucleons due to the energy loss seems safe: Drell-Yan processes do not suffer from this energy loss (see [16,17] and refs. therein).

The strong spin dependence of the \( \chi-N \) cross section is due to the angular dependence of the wave functions, which leads to different transverse quark separation for different states. The S-states (\( J/\Psi \) and \( \Psi' \)) in contrast, do not exhibit any dependence on the angles.

However, the P-states yield two vastly different cross sections (see Tab. [I]) for \( \chi_{10} \) and \( \chi_{11} \), respectively. This leads to a higher absorption rate of the \( \chi_{11} \) as compared to the \( \chi_{10} \). This new form of color filtering is predicted also for the corresponding states of other hadrons; e.g. for the bottomium states which are proposed as contrast signals to the \( J/\Psi \)'s at RHIC and LHC! (For a detailed review of color coherence effects see [18].) Fig. 2a depicts the \( A \) dependence of the suppression of the \( \chi_{11} \) and \( \chi_{10} \) in \( pA \) and in minimum bias \( AB \) collisions.

The polarization of the \( \chi \) shown in Fig. 2a may be difficult to observe. However, it is manifested in the production of all P-states (except in the \( J^{PC} = 0^{++} \) state, which in any case has a very small \( \chi \rightarrow J/\Psi + \gamma \) branching ratio). Significant effects are expected for both \( 1^{++} \) and \( 2^{++} \) states, which give the major contribution to the \( J/\Psi \) production, and also for the \( S=0 \) state \( 1^{+-} \). This state was observed to date only in \( p\bar{p} \) collisions [19]. The predicted effect is present for the \( 1^{++} \) state, since the probability of the m=0 state is 1/2 for helicity +1 and is 0 for helicity 0.

There are other promising \( \chi \) states with the total angular momentum \( J=2 \). Fig. 2b shows
the different suppressions for these \( \chi \)-states, i.e. with \( J=2 \) and \( J_z = 0, 1(-1) \) and \( 2(-2) \).

These states are mixtures of \( \chi_{10}, \chi_{11} \) and \( \chi_{1-1} \):

\[
\chi(J, J_z) = a_1 \cdot \chi_{10} + a_2 \cdot \chi_{11} + a_3 \cdot \chi_{1-1}.
\]

The \( a \)'s are given by the Clebsch-Gordan coefficients.

The measured \( J/\Psi \)-suppression up to sulphur-uranium collisions seems to be described reasonably well with the present simple model, in view of the neglect of comoving hadrons \([5,6]\). Note that the physically most reasonable scenario, namely the one including the expansion actually underestimates the suppression for \( SU \) by about 10 \%. And also the apparently strong \( J/\Psi \) suppression found at CERN in going from \( SU \) to \( PbPb \) collisions \([20]\) can not be explained in our calculations. This leaves space for more physics. Density fluctuations and the energy loss due to gluon radiation have also been neglected. Higher statistics for the \( pA \)-data are needed to clarify the importance of the expansion effects. For example the ratio of \( \Psi' \)'s to \( J/\Psi \)'s should be measured at one energy (e.g. \( \sqrt{s} = 20 \) GeV, because here the Lorentz factor is not too large) and for different systems. This can prove that this ratio is not constant. One should also measure the rapidity distribution of this ratio to see the dependence on the Lorentz factor.

We recommend to measure the \( A \)-dependent polarization of the \( \chi \) also in \( pA \) reactions at \( \sqrt{s} = 20 \) GeV and in the midrapidity region, because the above described expansion effects and the interaction with secondaries will decrease this polarization. The polarization, once found, can be used to measure the interaction of these charmonium states with secondaries in \( AB \) collisions. The quantitative evaluation of the interaction with comovers, nucleon correlations and density fluctuations is postponed to the future.

The other new effect predicted by the QCD factorization theorem is the fast increase of the cross section of spatially small configurations with energy, see Tab. I. Thus, QCD predicts a large nuclear absorption of charmonium in \( AB \) collisions at the RHIC and, in particular, at LHC energies, although the effective size of \( c\bar{c} \) pairs will be \( \sim \frac{1}{m_c} \sim 0.1 \) fm due to the strong increase of the parton distribution functions at small \( x \). The absorption depends also on the rapidity of the produced \( c\bar{c} \) pair.
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[1] G.T. Bodwin, S.J. Brodsky, G.P. Lepage: Phys. Rev. Lett. 47, 1799 (1981)

[2] C. Lourenco: Nucl. Phys. A610, 553c (1996)

[3] R. L. Anderson et al.: Phys. Rev. Lett. 38, 263 (1977)

[4] T. Matsui and H. Satz: Phys. Lett. B 178, 416 (1986)

[5] D. Neubauer, K. Sailer, B. Müller, H. Stöcker, W. Greiner: Mod. Phys. Lett. A 4, 1627 (1989)

[6] S. Gavin, R. Vogt: Nucl. Phys. B345, 104 (1990)

[7] D. Kharzeev, C. Lourenco, M. Nardi, H. Satz: Z. Phys. C 74, 307 (1997)

[8] B. Kopeliovich: procs. of Hirschegg 1997, QCD phase transitions 281-292, hep-ph/9702365

[9] L. Frankfurt and M. Strikman: Nucl. Phys. B250, 147 (1985)

[10] L. Frankfurt, A. Radyushkin, M. Strikman: Phys. Rev. D 55, 98 (1997)

B. Blättel et al.: Phys. Rev. Lett. 71, 896 (1993)

L. Frankfurt, G. Miller, M. Strikman: Phys. Lett. B 304, 1 (1993)

[11] FELIX Collaboration (E. Lippmaa et al.): CERN-LHCC-97-45, Aug 1997, 197pp

[12] G. Farrar, L. Frankfurt, M. Strikman, H. Li: Phys. Rev. Lett. 64, 2996 (1990)
[13] L. Frankfurt, W. Koepf, M. Strikman: *Phys. Rev.* D **54**, 3194 (1996)

[14] C.W. deJager, H. deVries, and C. deVries: *Atomic Data and Nuclear Data Tables* **14**, 485 (1974)

[15] C. Gerschel, J. H{"u}fner: *Phys. Lett.* B **207**, 253 (1988)

[16] D. Kharzeev: nucl-th/9802037

[17] R. Baier, Yu.L. Dokshitser, A.H. Mueller, S. Peigne, D. Schiff: *Nucl. Phys.* B**483**, 291 (1997)

[18] L. Frankfurt, G. Miller, M. Strikman: *Annual Rev. Nucl. Part. Sci.* **45**, 501 (1994)

[19] T.A. Armstrong et al.: *Phys. Rev. Lett.* **69**, 2337 (1992)

[20] M. Gonin: *Nucl. Phys.* A**610**, 442c (1996)
| $c\tau/b\bar{b}$-state | $J/\Psi$ | $\Psi'$ | $\chi_{c10}$ | $\chi_{c11}$ | $\Upsilon$ | $\Upsilon'$ | $\chi_{b10}$ | $\chi_{b11}$ |
|------------------------|---------|---------|--------------|--------------|----------|----------|------------|------------|
| $<b^2>$ (fm$^2$)       | 0.094   | 0.385   | 0.147       | 0.293       | 0.027   | 0.15     | 0.059      | 0.12       |
| $\sigma_{\text{nonperturbative}}$ (mb) | 3.62 | 20.0 | 6.82 | 15.9 | 0.31 | 7.35 | 1.48 | 4.48 |
| $\sigma_{\text{hard}}$ (mb) (SPS) | 0.024 | 0.012 | 0.021 | 0.006 | 0.101 | 0.030 | 0.103 | 0.064 |
| $\sigma_{\text{hard}}$ (mb) (RHIC) | 1.73 | 0.68 | 1.23 | 0.30 | 2.30 | 0.64 | 2.17 | 1.27 |
| $\sigma_{\text{hard}}$ (mb) (LHC) | 20.8 | 8.2 | 14.7 | 3.5 | 28.4 | 7.8 | 26.3 | 15.0 |

**TABLE I.** The average square of the transverse distances of the charmonium states and the total quarkonium-nucleon cross sections $\sigma$. For the $\chi$ two values arise, due to the spin dependent wave functions ($lm = 10, 11$). $\sigma_{\text{hard}}$ are the perturbative QCD contribution at different energies (see text).
FIG. 1. The ratio $0.019 \cdot S_{\Psi'}/S_{J/\Psi}$ is shown in $pA$ with the calculated cross sections for the $J/\Psi$, $\Psi'$ and $\chi$ (crosses) in comparison to the data (circles). The squares and the triangles shows the ratio calculated with the expansion of small wave packages (see text).
FIG. 2. The survival probabilities $S$ for the different $\chi$-states are shown for $pA$ and minimum bias $AB$ collisions. a) gives the $\chi_{10}$ and the $\chi_{11(-1)}$, which differ by 0.2-0.3. The fraction of the $\chi_{10}$’s of all $\chi$’s increases to more than 50 % for $PbPb$-collisions. b) the $\chi$-states with $J=2$ and $J_z = 0, 1(-1)$ and $2(-2)$ are shown.