Possible hadronic molecules composed of the doubly charmed baryon and nucleon

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Abstract. We perform a systematical investigation of the possible deuteron-like bound states with configuration $\Xi_{cc}N(N)$, where $N(N)$ denotes the nucleon (anti-nucleon), in the framework of the one-boson-exchange-potential model. In the spin-triplet sector we take into account both the $^{3}S_{1}$ and $^{3}D_{1}$ channels due to non-vanishing tensor force. There exist several candidates of the loosely bound molecular states for the $\Xi_{cc}N$ and $\Xi_{cc}\bar{N}$ systems, which lie below the threshold of $\Lambda\Lambda$, $\Lambda\bar{N}$, or $\Lambda\Lambda$. We also investigate the possible loosely bound states with configurations $\Lambda N(N)$ and $\Sigma N(N)$. These molecular candidates may be searched for at Belle II and LHC in the near future.

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1 INTRODUCTION

Since the charmonium-like state $X(3872)$ was discovered by the Belle Collaboration in 2003 \cite{1}, more and more charmonium-like/bottomonium-like states have been reported by the experimental collaborations, such as $Y(4260)$ \cite{2}, $Y(10610)$, $Y(10650)$ \cite{3}, and $Z_{c}(3900)$ \cite{4,5}. Most recently, the LHCb collaboration reported two hidden-charm pentaquark states, $P_{c}^{}(4380)$ and $P_{c}^{}(4450)$, in Ref. \cite{6}. Such states are normally called “exotic” states since it is very difficult to interpret them using the conventional hadron configurations, i.e., $qqq$ for a meson or $qqqq$ for a baryon in the quark model. Thus searching for the new hadronic states beyond the quark model has become a hot topic. One can refer to Refs. \cite{7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30} for recent reviews on the experimental and theoretical progresses about the exotic states in the past decades.

Since the masses of many “exotic” states are close to the threshold of two hadrons, the hadronic molecule provides us an appealing picture to interpret them. A hadronic molecule is a loosely bound state formed by the color-singlet hadrons and the force is generated by exchanging light bosons. Since the meson-exchange force is some kind of residual force of the color force generated by exchanging gluons between quarks, the hadronic molecule may have a much larger size than the meson or baryon. For example, the loosely bound deuteron is formed by the neutron and proton and its root-mean-square radius is around 2 fm. The hadronic molecule composed of heavy mesons was first proposed by Voloshin and Okun about forty years ago. They investigated the possible molecular states formed by one charmed meson and one charmed antimeson \cite{12}. De Rujula et al tried to interpret $\psi(4040)$ as a $D^{*}D^{*}$ molecular state in Ref. \cite{13}. Tornqvist calculated the possible deuteron-like $DD^{*}$ and $D^{*}D^{*}$ states in Refs. \cite{14,15}. Recently, many other calculations were performed in the hadronic molecular picture, such as the molecular states formed by two light baryons \cite{16,17,18,19,20,21,22,23}, by two heavy baryons \cite{24,25,26,27,28,29,30}, and by two heavy mesons \cite{31,32,33,34,35,36,37,38}

In 2002, the SELEX collaboration reported a doubly charmed baryon with mass 3520 MeV, named $\Xi_{cc}^{+}$, which contains two charm quarks and one down quark \cite{39}. Later this structure was confirmed by the same collaboration \cite{40}. In the conference report \cite{41}, another doubly charmed baryon, $\Xi_{cc}^{++}$, containing two charm quarks and one up quark was reported at 3780 MeV. Most recently, the doubly charmed baryon $\Xi_{cc}^{++}$ was observed by the LHCb collaboration at 3621.40 $\pm$ 0.72(stat) $\pm$ 0.27(syst) $\pm$ 0.14($\Lambda_{cc}^{+}$) MeV/c$^{2}$ \cite{42}. Many theoretical work has been performed to calculate the mass of the doubly charmed baryons \cite{43,44,45,46,47,48,49,50,51,52}. With more and more information on $\Xi_{cc}$, it is very interesting to investigate the structures formed by $\Xi_{cc}$ and other mesons or baryons. In Refs. \cite{53}, the authors studied the hadronic molecule with configuration $\Xi_{cc}N$, and in \cite{30} we investigated the
possible deuteron-like bound states of \( \Xi}\Xi (\Xi\Xi). With the new mass reported by LHCB, \( \Xi}\Xi N(N) \) is about 13 MeV below the threshold of \( \Lambda_c\Lambda_c\). The possible \( \Xi}\Xi N(N) \) molecular states do not decay into \( \Lambda_c\Lambda_c \) due to the limit phase space. Moreover, the annihilation effect in the \( \Xi\Xi \) channel is strongly suppressed compared with the nucleon anti-nucleon case since the intermediate states are either the doubly charmed exotic teraquark states or two charmed mesons which are rather heavy. In the current work, we investigate the possible hadronic molecules with configurations \( B_c N(N) \) or \( \Xi\Xi N(N) \), where \( B_c \) denotes \( \Sigma_c \) or \( \Lambda_c \) where \( N(N) \) denotes the nucleon (anti-nucleon).

We organize the paper as follows. We give the theoretical formalism in Sec. 2, where the flavor wave functions, Lagrangians, coupling constants and interaction potentials are presented. In Sec. 3, we show the numerical results of the doubly charmed baryon and nucleon systems and give some discussions. We summarize our results in Sec. 4. Finally, we collect some useful formulae and the results of the singly charmed baryon and nucleon systems in the Appendix.

2 FORMALISM

In the present work, we concentrate on the investigation of the possible molecular states with configurations of \( \Xi\Xi N(N) \), \( \Sigma_c N(N) \) and \( \Lambda_c N(N) \), where \( \Lambda_c \) and \( \Sigma_c \) is the spin-\( \frac{1}{2} \) singly charmed baryons and \( \Xi\Xi \) is the spin-\( \frac{3}{2} \) doubly charmed baryon. Since we only consider the \( u \) and \( d \) quarks in the light quark sector, the SU(2) symmetry should be applied.

\( \Lambda_c \) is the isospin singlet and \( \Sigma_c \) is isospin triplet. To be convenient, we construct matrices as

\[
\Lambda_c = \begin{pmatrix}
\Lambda_c^+ & 0 \\
0 & \Lambda_c^-
\end{pmatrix}, \quad \Sigma_c = \begin{pmatrix}
\Sigma_c^+ + \frac{1}{\sqrt{2}} \Sigma_c^0 \\
\frac{1}{\sqrt{2}} \Sigma_c^+ & \Sigma_c^0
\end{pmatrix}.
\]

The doubly charmed baryon is simpler since only one light quark is involved. We adopt \( \Xi\Xi = (\Xi^u, \Xi^d)^T \) to denote the isospin-doublet of the doubly charmed baryons, where the superscripts, \( u \) and \( d \), denote the light quark in the baryon while \( "^T" \) means the transpose of matrix. We also perform an investigation of the possible molecular states with configuration of \( \Xi\Xi N(N) \). We list the flavor wave functions in Table 1.

2.1 The Lagrangian

The Lagrangians for the nucleon interacting with the light mesons are,

\[
\mathcal{L}_{\pi NN} = \sqrt{2} g_{\pi NN} N \bar{g}_\gamma \gamma_5 M_\pi N, \tag{2}
\]

\[
\mathcal{L}_{\rho NN} = \sqrt{2} g_{\rho NN} N \bar{g}_\gamma \gamma_5 V_\rho N + \frac{f_{\rho NN}}{\sqrt{2} m_N} N \bar{g}_\sigma \partial \nu V_\rho N, \tag{3}
\]

\[
\mathcal{L}_{\omega NN} = \sqrt{2} g_{\omega NN} N \bar{g}_\gamma \gamma_5 V_\omega N + \frac{f_{\omega NN}}{\sqrt{2} m_N} N \bar{g}_\sigma \partial \nu V_\omega N, \tag{4}
\]

\[
\mathcal{L}_{\sigma NN} = g_{\sigma NN} N \bar{g}_\sigma N, \tag{5}
\]

where \( N = (p, n)^T \) is the nucleon doublet. \( m_N \) is the nucleon mass and \( g_{\pi NN}, g_{\rho NN}, f_{\rho NN}, \rho, \omega, \sigma \) are the coupling constants, the values of which are given in Tables 2 and 3 respectively. The light mesons are introduced with the following notations,

\[
\mathcal{M}_\pi = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} & \pi^+ \\
\pi^- & -\frac{\pi^0}{\sqrt{2}}
\end{pmatrix}, \tag{6}
\]

\[
\mathcal{V}_\rho = \begin{pmatrix}
\frac{\rho^0}{\sqrt{2}} & \rho^+ \\
\rho^- & -\frac{\rho^0}{\sqrt{2}}
\end{pmatrix}, \quad \mathcal{V}_\omega \mathcal{V}_\rho = \begin{pmatrix}
\frac{\omega}{\sqrt{2}} & 0 \\
0 & \frac{\omega}{\sqrt{2}}
\end{pmatrix}, \tag{7}
\]

where \( \mathcal{M}_\pi, \mathcal{V}_\rho \) and \( \mathcal{V}_\omega \) are the \( \pi, \rho \) and \( \omega \) fields, respectively. Following the Ref. [51], the interactions between heavy baryons and vector mesons can be introduced. Here we do not show the coupling between the nucleon and \( \eta \) because it is rather small. Similarly, the Lagrangians for \( \Xi\Xi \) interacting with the light mesons read,

\[
\mathcal{L}_{\pi \Xi\Xi} = \sqrt{2} g_{\pi \Xi\Xi} \Xi\Xi \bar{g}_\gamma \gamma_5 M_\pi \Xi\Xi, \tag{8}
\]

\[
\mathcal{L}_{\rho \Xi\Xi} = \sqrt{2} g_{\rho \Xi\Xi} \Xi\Xi \bar{g}_\gamma \gamma_5 V_\rho \Xi\Xi + \frac{f_{\rho \Xi\Xi}}{\sqrt{2} m_{\Xi\Xi}} \Xi\Xi \bar{g}_\sigma \partial \nu V_\rho \Xi\Xi, \tag{9}
\]

\[
\mathcal{L}_{\omega \Xi\Xi} = \sqrt{2} g_{\omega \Xi\Xi} \Xi\Xi \bar{g}_\gamma \gamma_5 V_\omega \Xi\Xi + \frac{f_{\omega \Xi\Xi}}{\sqrt{2} m_{\Xi\Xi}} \Xi\Xi \bar{g}_\sigma \partial \nu V_\omega \Xi\Xi, \tag{10}
\]

\[
\mathcal{L}_{\sigma \Xi\Xi} = g_{\sigma \Xi\Xi} \Xi\Xi \bar{g}_\sigma \Xi\Xi, \tag{11}
\]

The masses of the doubly charmed baryons, \( m_{\Xi\Xi} \), and the values of the coupling constants, \( g_{\pi \Xi\Xi}, g_{\rho \Xi\Xi}, \) etc., are given in Tables 2 and 3 respectively.

For the interactions between \( \Sigma_c \) (\( \Lambda_c \)), the corresponding Lagrangians read,

\[
\mathcal{L}_{\pi \Sigma_c} = g_{\pi \Sigma_c} \Sigma_c \bar{g}_\gamma \gamma_5 M_\pi \Sigma_c, \tag{12}
\]

\[
\mathcal{L}_{\rho \Sigma_c} = g_{\rho \Sigma_c} \Sigma_c \bar{g}_\gamma \gamma_5 V_\rho \Sigma_c + \frac{f_{\rho \Sigma_c}}{2 m_{\Sigma_c}} \Sigma_c \bar{g}_\sigma \partial \nu V_\rho \Sigma_c, \tag{13}
\]

\[
\mathcal{L}_{\omega \Sigma_c} = g_{\omega \Sigma_c} \Sigma_c \bar{g}_\gamma \gamma_5 V_\omega \Sigma_c + \frac{f_{\omega \Sigma_c}}{2 m_{\Sigma_c}} \Sigma_c \bar{g}_\sigma \partial \nu V_\omega \Sigma_c, \tag{14}
\]

\[
\mathcal{L}_{\sigma \Sigma_c} = g_{\sigma \Sigma_c} \Sigma_c \bar{g}_\sigma \Sigma_c. \tag{15}
\]

For the isospin-singlet baryon \( \Lambda_c \), it only interacts with the light isospin-singlet mesons. Namely, there are only two interaction vertices, \( \Lambda_c \omega, \Lambda_c \rho \) and \( \Lambda_c \sigma \Lambda_c \). The corresponding Lagrangians are similar to Eqs. (14) and (15).

2.2 Coupling Constants

The coupling constants for the nucleons interacting with the light mesons are well-known. They are either extracted
Table 1. The flavor wave functions for the systems with configurations of $\Sigma_cN$, $\Lambda_cN$ and $\Xi_{cc}N$. $I$ denotes the isospin.

| Systems | Flavor wave functions |
|---------|----------------------|
| $[\Sigma_cN]^I=3/2$ | $\Sigma_c^0+p$ |
| $[\Xi_{cc}N]^I=1$ | $\Xi_{cc}^0$ |
| $[\Lambda_cN]^I=1/2$ | $\Lambda_c$ |

Table 2. The masses of the one-charm baryon and light mesons are taken from [33] while the mass for the doubly charmed baryon is taken from [42].

| Baryons | Mass (MeV) | Mesons | Mass (MeV) |
|---------|------------|--------|------------|
| Nucleon | 939        | $\pi$  | 137.27     |
| $\Sigma_c$ | 2454       | $\rho$ | 775.49     |
| $\Lambda_c$ | 2287       | $\omega$ | 782.65   |
| $\Xi_{cc}$ | 3621       | $\sigma$ | 600       |

from the experimental data or calculated by the theoretical models. We take the values from Refs. [50,57,58]. The other coupling constants used in the current calculation can be derived from those of the nucleons interacting with the light mesons through the quark model. The same method as in Ref. [30] is adopted. For the doubly charmed baryon case, we make use of the following relations,

$$\langle p \uparrow | L_{mNN}| p \uparrow \rangle = \langle p \uparrow | L_{mqq}| p \uparrow \rangle, \quad (16)$$

$$\langle \Xi_{cc}^u \uparrow | L_{mbb}| \Xi_{cc}^u \uparrow \rangle = \langle \Xi_{cc}^u \uparrow | L_{mqq}| \Xi_{cc}^u \uparrow \rangle, \quad (17)$$

where $\uparrow$ means the third component of the spin is $+\frac{1}{2}$. The matrix elements are calculated at both hadronic and quark level. At the quark level, we adopt the following Lagrangians [54],

$$\mathcal{L}_q = g_{qqq} \bar{q} \gamma_5 M_q q + g_{qqq} \bar{q} \gamma_\mu V_\mu q + g_{qqq} \bar{q} \gamma_q q$$

(18)

where $q = (u, d)^T$ is the light quark doublet. Here we adopt the non-relativistic chiral quark model proposed by Manohar and Georgi [59], in which the pseudoscalar goldstone octet as well as the vector and scalar mesons are introduced as fundamental fields. Finally, we can obtain the coupling constants used in the current calculation in terms of those of the nucleon interacting with the light meson. The specific expressions read,

$-\pi$-exchange,

$$g_{\pi \Sigma_c \Sigma_c} = \frac{4\sqrt{2}}{5} \frac{m_{\Sigma_c}}{m_N} g_{\pi NN}, \quad g_{\pi \Lambda_c \Lambda_c} = 0, \quad (19)$$

$g_{\pi \Xi_{cc} \Xi_{cc}} = \frac{1}{5} \frac{m_{\Xi_{cc}}}{m_N} g_{\pi NN}, \quad (20)$

$-\rho$-exchange,

$$g_{\rho \Sigma_c \Sigma_c} = 2\sqrt{2} g_{\rho NN}, \quad g_{\rho \Xi_{cc} \Xi_{cc}} = g_{\rho NN}, \quad (21)$$

$$g_{\rho \Sigma_c \Xi_{cc}} + f_{\rho \Sigma_c \Xi_{cc}} = \frac{4\sqrt{2}}{5} \frac{m_{\Sigma_c}}{m_N} (g_{\rho NN} + f_{\rho NN}), \quad (22)$$

$$g_{\rho \Xi_{cc} \Xi_{cc}} + f_{\rho \Xi_{cc} \Xi_{cc}} = -\frac{1}{5} (g_{\rho NN} + f_{\rho NN}) \frac{m_{\Xi_{cc}}}{m_N}, \quad (23)$$

$-\omega$-exchange,

$$g_{\omega \Sigma_c \Sigma_c} = \sqrt{\frac{2}{3}} g_{\omega NN}, \quad g_{\omega \Lambda_c \Lambda_c} = \sqrt{\frac{2}{3}} g_{\omega NN}, \quad (24)$$

$$g_{\omega \Xi_{cc} \Xi_{cc}} = \frac{1}{3} g_{\omega NN}, \quad g_{\omega \Lambda_c \Lambda_c} + f_{\omega \Lambda_c \Lambda_c} = 0, \quad (25)$$

$$g_{\omega \Sigma_c \Xi_{cc}} + f_{\omega \Sigma_c \Xi_{cc}} = \frac{4\sqrt{2}}{3} \frac{m_{\Sigma_c}}{m_N} (g_{\omega NN} + f_{\omega NN}), \quad (26)$$

$$g_{\omega \Xi_{cc} \Xi_{cc}} + f_{\omega \Xi_{cc} \Xi_{cc}} = -\frac{1}{3} (g_{\omega NN} + f_{\omega NN}) \frac{m_{\Xi_{cc}}}{m_N}, \quad (27)$$

$-\sigma$-exchange,

$$g_{\sigma \Sigma_c \Sigma_c} = \frac{2}{3} g_{\sigma NN}, \quad g_{\sigma \Lambda_c \Lambda_c} = \frac{1}{3} g_{\sigma NN}, \quad (28)$$

$$g_{\sigma \Xi_{cc} \Xi_{cc}} = \frac{1}{3} g_{\sigma NN}, \quad (29)$$

the numerical values of which are given in Table 3

2.3 The Interaction Potentials

With the Lagrangians given in Sec. [24], we calculate the scattering amplitude of the process $B + N \rightarrow B + N$, where $B$ and $N$ represent the heavy baryon and nucleon respectively. The potentials have the following forms,

$$V(Q) = V_{cen}(Q) + V_{SS}(Q)O_{SS} + V_{LS}(Q)O_{LS} + V_T(Q)O_T$$

(30)

where $O_{SS} = \sigma_1 \cdot \sigma_2$, $O_{LS} = \frac{\sigma_1 + \sigma_2}{2} \cdot (Q \times k)$, and $O_T = \sigma_1 \cdot \Omega_{\sigma_2} \cdot Q - 3\sigma_1 \cdot \sigma_2$ are the spin-spin, spin-orbit, and tensor operators, respectively. $Q = p' - p$ is the momenta transfer while $k = \frac{p' + p}{2}$ is the averaged momenta between the incoming and outgoing nucleons. By
performing the Fourier transformation, $k$ is replaced by $-i\mathbf{\nabla}$ which provides the only nonlocal force in the current calculation. Other nonlocal interactions, such as the recoil effect, are very small and neglected. In the non-relativistic limit, we expand $V(Q)$ as series of $\frac{Q}{m_\pi}$ and truncate at order $O(Q^2/m_\pi^2)$, which coincides with the Bonn model [58].

At each vertex, we introduce a monopole form factor

$$F(Q) = \frac{A^2 - m_\pi^2}{A^2 - Q^2} = \frac{A^2 - m_\pi^2}{\lambda^2 + Q^2}$$  \hspace{1cm} (31)

to suppress the high-momentum contribution. $A$ is the cutoff parameter used to adjust the high-momentum contribution. $m_\pi$ is the exchanged meson mass and $\lambda^2 = A^2 - Q_0^2$. Additionally, the form factor also accounts for the physics that the constituent hadrons should be viewed as point-like particles in a molecular state since they are well separated. Through the Fourier transformation,

$$V(r) = \frac{1}{(2\pi)^3} \int dQe^{iQ\cdot r}V(Q)F^2(Q),$$  \hspace{1cm} (32)

we can obtain the interaction potentials in the coordinate space. The specific expressions will be given later.

If the form factor were not introduced, there would be terms with a delta function, $\delta^{(3)}(r)$, in the potentials after the Fourier transformation. Such terms account for the very short interaction and we call them contact interaction or delta interaction. More details can be found in Appendix A. In order to investigate the role of the delta interaction in the formation of the loosely bound states, we make calculations both with and without the delta interaction. Our potentials with the delta interaction included in the coordinate space are,

- Pseudoscalar exchange:

$$V^p_{SS}(r; \sigma) = C_0^p g_{1s}g_{2s} \frac{m_\pi^3}{4\pi} \frac{m_\pi^2}{M_1 M_2} H_{12} \sigma_1 \cdot \sigma_2, \hspace{1cm} (33)$$

- Scalar exchange:

$$V^p_{LS}(r; \sigma) = -C_0^p g_{1s}g_{2s} \frac{m_\pi^2}{4\pi} \left[ H_{00} - \frac{m_\pi^2}{16M_1^2} H_{11} - \frac{m_\pi^2}{16M_2^2} H_{11} \right], \hspace{1cm} (34)$$

$$V^v_{LS}(r; \sigma) = -C_0^v g_{1s}g_{2s} \frac{m_\pi}{4\pi} \left[ M_1 M_2 \left( \frac{m_\pi^2}{M_1^2} + \frac{m_\pi^2}{M_2^2} \right) H_3 \mathbf{L} \cdot \mathbf{S} \right]. \hspace{1cm} (35)$$

In the above expressions, the superscripts $p$, $s$ and $v$ denote the pseudoscalar, scalar and vector mesons, respectively. $\alpha = \pi$ while $\beta = \omega$ and $\rho$. $H_{00}$, $H_{11}$, $H_{12}$ and $H_3$ are some functions with arguments, $r$, $A$ and $m_\pi$, refer to Appendix A for their special definitions. One can obtain the potentials without the short-range delta interaction straightforwardly by making the simple replacement, $H_1 \rightarrow H_{00}$, $C_0^p$, $C_0^v$ and $C_0^s$ are the isospin factors, the values of which are given in Table 4. The isospin-isospin factor is already included in $C_0^p$, etc. That is why the operators $\tau_1 - \tau_2$ does not appear in Eq. (30).

The spin-isospin factor is obtained in Appendix B. In Appendix B, $L S (L S_1, L S_2)$ is the spin-orbit operator which provides the nonlocal force while $S_{12}(r) = \sigma_1 \cdot \sigma_2 \cdot \hat{r} - 3\sigma_1 \cdot \sigma_2$ is the tensor force which has a non-vanishing matrix element between the $S$ and D waves. Thus, we also take into account the $S - D$ mixing effect which plays a critical role in the formation of the loosely bound deuteron. The matrix elements of the operators are given in Appendix B.

The spin of the systems composed of two spin-$\frac{1}{2}$ baryons are 0 (spin singlet) or 1 (spin triplet). For the spin-singlet case we focus on the $^1S_0$ channel while for the spin-triplet case we take into account both the $^3S_1$ and $^3D_1$ channels due to the existence of the tensor force. The wave function of the $^1S_0$ channel reads,

$$\Psi(r, \theta, \phi)\chi_{ss} = y_s(r)^1S_0,$$  \hspace{1cm} (36)
Table 4. Values of the isospin factors for $B_c N$ and $\Xi_{cc} N$. The isospin factors of the $B_c N$ and $\Xi_{cc} N$ systems can be obtained easily by multiplying the G-parity of the exchanged mesons. The G-parity of the exchange mesons are given in the square brackets.

| Systems                  | $C_p^0 [-1]$ | $C_p^1 [-1]$ | $C_p^0 [1]$ | $C_p^1 [1]$ |
|--------------------------|--------------|--------------|-------------|-------------|
| $[\Sigma_{cc}N]^I=3/2$   | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | 1           |
| $[\Sigma_{cc}N]^I=1/2$   | $-\sqrt{2}$  | $-\sqrt{2}$  | $\frac{1}{\sqrt{2}}$ | 1           |
| $[\Lambda_{cc}N]^I=1/2$  | 0            | 0            | $\sqrt{2}$  | 2           |
| $[\Xi_{cc}N]^I=3$        | 1            | 1            | 1           | 1           |
| $[\Xi_{cc}N]^I=0$        | -3           | -3           | 1           | 1           |

while that of the spin-triplet channel is

$$\Psi(r, \theta, \phi)^T \chi_s^T \chi_s^\ast = \begin{pmatrix} T_S(r) & 0 \\ 0 & T_D(r) \end{pmatrix} |^3S_1\rangle + \begin{pmatrix} 0 \\ T_D(r) \end{pmatrix} |^3D_1\rangle,$$

(37)

where $y_S(r)$ is the radial wave function for the $^1S_0$ channel while $T_S^2(r)$ and $T_D^2$ are the radial wave functions for the $^3S_1$ and $^3D_1$ channels respectively.

3 Numerical results

We solve the Schrödinger equation with the potentials derived in the previous section and obtain the binding energy (B.E.) and the radial wave function, with which we further calculate the root-mean-square radius $r_{rms}$. The binding energy and the root-mean-square radius provide us the information to judge whether a bound state exists. For the coupled channels, we additionally calculate the individual probability for each channel.

A reasonable range of the cutoff in the study of the deuteron with the OBEP model is $0.80 - 1.50$ GeV. Since the charmed baryon is much heavier than the nucleon, we take a wider range, $0.80 - 2.50$ GeV, for the cutoff parameter. Notice that in Ref. 61 an even wider range, around $0.80 - 3.60$ GeV, was used for the cutoff parameter and different values were used for different exchanged mesons.

The binding energy (B.E.) and root-mean-square radius ($r_{rms}$) are physical observables, from which we can judge if there are loosely bound states. The large B.E. and small $r_{rms}$ are not consistent with the molecular scheme. Recall the B.E. and $r_{rms}$ of deuteron are $2.22$ MeV and $1.97$ fm [58], respectively, we choose $0.5 < \text{B.E.} < 15$ MeV and $0.8 < r_{rms} < 5$ fm as criteria for good candidates of molecular states. In the numerical analysis, we list some results with B.E. and $r_{rms}$ beyond these ranges in order to see the cutoff dependence of our results.

We show the numerical results for the spin-singlet $\Xi_{cc} N$ system in Table 5 and those for the spin-triplet $\Xi_{cc} N(N)$ system in Table 6.

3.1 Spin-singlet ($S = 0$)

For the state $[\Xi_{cc}N]^{[0,0]}$, with the full potential we obtain binding solutions which depend sensitively on the cutoff parameter. After removing the delta interaction, the bound state disappears. Thus, this state is not supported to be a candidate of the hadronic molecule. For the state $[\Xi_{cc}N]^{[0,0]}$, we obtain a loosely bound state with binding energy $0.49 < \text{B.E.} < 10.39$ MeV for the cutoff to be $2.20 < \Lambda < 2.80$ GeV. Without the delta interaction, the binding energy becomes $4.94 < \text{B.E.} < 61.79$ MeV for the cutoff to be $1.00 < \Lambda < 1.20$ GeV. The root-mean-square radius is a few fm. $[\Xi_{cc}N]^{[0,0]}$ can form a hadronic molecule with the meson-exchange potential.

The $\Xi_{cc} N$ system is particularly interesting. We obtain loosely bound states for both states with $(S, I) = (0, 0)$ and $(0, 1)$. We show the interaction potentials in Fig. 1. From the plots, one can see clearly that both the $\sigma$- and $\rho$-exchanges generate the attractive force for the state $(S, I) = (0, 1)$ while both the $\sigma$- and $\pi$-exchanges provide the attractive force for the states $(S, I) = (0, 0)$. In addition, the $\omega$-exchange supplies the attractive force at long range for both of the two states. For the state $[\Xi_{cc}N]^{[0,0]}$, the binding energy is $4.98 < \text{B.E.} < 34.86$ MeV for the cutoff parameter to be $0.85 < \Lambda < 1.00$ GeV. The corresponding root-mean-square radius is $0.97 < r_{rms} < 1.90$ fm. Thus, the system $[\Xi_{cc}N]^{[0,0]}$ is favored to be a candidate of the hadronic molecule. More interestingly, we obtain a loosely bound state for $[\Xi_{cc}N]^{[0,1]}$ both with and without the delta interaction. The binding energy is $1.98 < \text{B.E.} < 44.15$ MeV for the cutoff to be $1.05 < \Lambda < 1.20$ GeV. With the same cutoff, the binding energy becomes $3.73 < \text{B.E.} < 35.18$ MeV after neglecting the delta interaction. In both cases, the root-mean-square radius is around $0.90 < r_{rms} < 2.5$ fm. The $[\Xi_{cc}N]^{[0,1]}$ should be viewed as an ideal candidate of the hadronic molecule.

3.2 Spin-triplet ($S = 1$)

For the state $[\Xi_{cc}N]^{[1,1]}$, we obtain no binding solutions. Although we obtain bound state for $[\Xi_{cc}N]^{[1,0]}$, the results depend strongly on the cutoff parameter. Neither of the two states is favored to be candidates of the loosely bound states.

Very interestingly, we obtain loosely bound states for $[\Xi_{cc}N]^{[1,0/1]}$, no matter the delta interaction is included or not. We show the interaction potentials in Fig. 2. From the plots, one can see clearly that for the state $(S, I) = (1, 0)$, the attractive force is generated by the $\rho$-, $\omega$- and $\sigma$-exchanges while for the state $(S, I) = (1, 1)$, the attractive force is provided by the $\pi$-, $\omega$- and $\sigma$-exchanges. With the full potential, the binding energy of the state $[\Xi_{cc}N]^{[1,0]}$ is $0.94 < \text{B.E.} < 64.58$ MeV for the cutoff to be $0.95 < \Lambda < 1.10$ GeV. With the same cutoff, the binding energy becomes $4.81 < \text{B.E.} < 77.01$ MeV after one removes the delta interaction. For this state, the contribution of the $D$ wave is around $2\% - 5\%$, which is similar to the deuteron case, about $4\%$. Based on our results, $[\Xi_{cc}N]^{[1,0]}$ can be viewed a candidate of the hadronic molecule. The state
Table 5. The binding solutions for the spin-singlet $\Xi_{cc}N(\bar{N})$ systems. “A” is the cutoff parameter. “B.E.” means the binding energy while $r_{rms}$ is the root-mean-square radius.

|                  | With contact interaction | Without contact interaction |
|------------------|--------------------------|-----------------------------|
|                  | A (GeV)                  | B.E. (MeV)                  | $r_{rms}$ (fm) | A (GeV) | B.E. (MeV) | $r_{rms}$ (fm) |
| $[\Xi_{cc}N]_{0}^{(0,1)}$ | 1.28                     | 8.51                        | 1.17          | ×       | ×          | ×              |
|                  | 1.30                     | 21.08                       | 0.98          | ×       | ×          | ×              |
|                  | 1.36                     | 88.30                       | 0.43          | ×       | ×          | ×              |
| $[\Xi_{cc}N]_{0}^{(0,0)}$ | 2.20                     | 0.49                        | 5.07          | 1.00    | 4.94       | 2.06           |
|                  | 2.40                     | 2.36                        | 2.95          | 1.10    | 25.75      | 1.07           |
|                  | 2.50                     | 5.67                        | 2.09          | 1.20    | 61.79      | 0.78           |
| $[\Xi_{cc}N]_{0}^{(0,1)}$ | 1.05                     | 1.98                        | 2.97          | 1.05    | 3.73       | 2.26           |
|                  | 1.10                     | 9.78                        | 1.52          | 1.10    | 10.80      | 1.46           |
|                  | 1.20                     | 44.15                       | 0.85          | 1.20    | 35.18      | 0.93           |
| $[\Xi_{cc}N]_{0}^{(0,0)}$ | 0.85                     | 4.98                        | 1.90          | 0.90    | 0.47       | 4.73           |
|                  | 0.90                     | 13.35                       | 1.31          | 0.95    | 24.73      | 1.06           |
|                  | 1.00                     | 34.86                       | 0.97          | 1.00    | 80.35      | 0.71           |

![Fig. 1](image1.png)

**Fig. 1.** The interaction potentials for the spin-singlet ($S=0$) $\Xi_{cc}\bar{N}$. “w/o delta” means the delta interaction is removed from the total potential.

$[\Xi_{cc}\bar{N}]_{1}^{(1,1)}$ can form an even more loosely bound state. The binding energy is $0.31 < \text{B.E.} < 38.99$ MeV for the cutoff to be $0.95 < A < 1.10$ GeV. If one neglects the delta interaction, the binding energy changes by a few MeV. The root-mean-square radius of this state is about $0.90 - 4.5$ fm and the contribution of the $D$ wave is tiny, around $0.2\% - 0.5\%$. $[\Xi_{cc}\bar{N}]_{1}^{(1,1)}$ should also be taken as an ideal candidate of the hadronic molecule.

Finally, we show the radial wave functions for the state $[\Xi_{cc}\bar{N}]_{1}^{(1/1)}$ in Fig. 3.

### 4 DISCUSSIONS AND CONCLUSIONS

We have performed a systematic investigation of the possible deuteron-like bound states with configurations $B_{c}N(\bar{N})$ and $\Xi_{cc}N(\bar{N})$, where $B_{c}$ means $\Sigma_{c}$ (or $A_{c}$) while $N(\bar{N})$ denotes the nucleon (anti-nucleon). In our calculation, the one-boson-exchange potential model is applied. Since in the hadronic molecule picture the constituent hadrons should be taken as point-like particles, we apply one monopole form factor for each vertex. By tuning the cutoff in the form factor one can control the high-momentum contribution. Additionally, to investigate the the effect of the very short-range interaction we give the numerical results both with and without the delta interaction.
Table 6. The binding solutions for the spin-triplet \((S = 1)\) \(\Xi_{cc}N(\bar{N})\). “\(A\)” is the cutoff parameter. “B.E.” means the binding energy while \(r_{rms}\) is the root-mean-square radius. \(P_s\) is the probability (%) of the S wave.

| \(\Xi_{cc}N^{(1,1)}\) | With contact interaction | Without contact interaction |
|-------------------------|---------------------------|----------------------------|
| \((\bar{N})\)          | \((N)\)                   | \((\bar{N})\)              |
| \(A\) (GeV)            | B.E (MeV)                 | \(r_{rms}\) (fm)          | \(P_s\) (%)          | \(A\) (GeV)            | B.E (MeV)                 | \(r_{rms}\) (fm)          | \(P_s\) (%)          |
| 1.97                   | 13.50                     | 0.64                       | 27.51               | 2.00                   | 7.13                     | 0.78                       | 25.77               |
| 1.98                   | 33.12                     | 0.50                       | 26.01               | 2.01                   | 26.98                     | 0.53                       | 17.77               |
| 1.99                   | 54.83                     | 0.48                       | 18.14               | 2.02                   | 49.04                     | 0.48                       | 15.20               |
| 1.00                   | 6.02                      | 1.82                       | 99.66               | 1.10                   | 15.81                     | 1.26                       | 99.54               |
| 1.10                   | 38.99                     | 0.89                       | 99.59               | 1.20                   | 48.30                     | 0.83                       | 99.47               |
| 0.95                   | 0.31                      | 5.31                       | 98.48               | 0.95                   | 4.81                      | 2.09                       | 97.67               |
| 1.00                   | 10.17                     | 1.57                       | 96.77               | 1.00                   | 18.59                     | 1.24                       | 96.66               |
| 1.10                   | 64.58                     | 0.80                       | 94.88               | 1.10                   | 77.01                     | 0.75                       | 95.03               |

Fig. 2. The interaction potentials for the spin-triplet \((S = 1)\) \(\Xi_{cc}\bar{N}\). Since the S wave plays the dominant role, we only plot the potential for the \(^3S_1\) channel. “w/o delta” means the delta interaction is removed from the total potential.

For the spin-singlet \((S = 0)\) case, we focus on the \(^1S_0\) channel while in the spin-triplet \((S = 1)\) sector we take into account both the \(^3S_1\) and \(^3D_1\) channels because of the tensor force. Thus for the spin-triplet channels, a two-dimensional coupled-channel Shrödinger Equation is solved and the probability of the S wave is also given accordingly.

In the one-charm sector, without the coupled channel effects in the flavor space we obtain bound states for neither spin-singlet or spin-triplet \(A_cN\). One can refer to Ref. [62] for a coupled-channel analysis of this state. Our results indicate that \([\Sigma_cN]^{(0,\frac{1}{2})}\) and \([\Sigma_cN]^{(1,\frac{1}{2})}\) can be taken as the candidates of the hadronic molecule. The \(A_c\bar{N}\) system is very interesting. Because of the vanishing tensor force for the spin-triplet case, the spin-triplet states has the same numerical results as those of the spin-singlet state. Both states can be viewed as the candidates of the deuteron-like bound states. The \([\Sigma_c\bar{N}]\) system is also very interesting and all the spin and isospin multiplets can form the loosely bound states by the meson-exchange potentials. We also find that the delta interaction has big influence on the \(\Sigma_c\bar{N}\) system but it plays a small role in the formation of the loosely bound \(A_c\bar{N}\) state.

In the double-charm sector, none of the states \([\Xi_{cc}N]^{(0,1)}\), \([\Xi_{cc}N]^{(1,0)}\), \([\Xi_{cc}N]^{(1,1)}\) are supported to be the candidates of the hadronic molecular states. However, the state \([\Xi_{cc}N]^{(0,0)}\) can form a loosely bound state with the meson-exchange potentials. The \(\Xi_{cc}\bar{N}\) is particularly interesting. We obtain loosely bound states for all the four states: \((S,I) = (0,0), (0,1), (1,0)\) and \((1,1)\), with the meson-exchange potentials. For the \([\Xi_{cc}N]^{(0,0/1)}\) systems, the delta interaction changes the binding energy by a few
to tens of MeV. The root-mean-square radius is around $0.8 - 2.5$ fm, which is comparable to that of the deuteron, about 2 fm. For the $[\Xi_{cc}N]^{(1,0/1)}$ system, the delta interaction has small effects, changing the binding energy by a few MeV. The contribution of the $D$ wave to the state $[\Xi_{cc}N]^{(1,1)}$ is tiny, around $0.2\% - 0.5\%$ while for the $[\Xi_{cc}N]^{(1,0)}$ system the probability of the $D$ wave is about $2\% - 5\%$. All of the four states are supported to be the candidates of the hadronic molecule by the one-boson-exchange potential model.

For the molecules composed of hadron and anti-hadron, there exit strong decay modes. The possible loosely bound states with configuration of $[B_cN]$, can decay into one $D/D^\ast$ together with some photons or light mesons. The $[\Xi_{cc}N]$-type hadronic molecules may decay into two $D/D^\ast$ plus some photons or light mesons.

For the molecules composed of a heavy baryon and nucleon, they can decay via the decay of their daughter particles. For example, $[\Sigma_cN] \to \Lambda_cN\pi$, $\Lambda_c \to pK^-\pi^+$. Thus, the deuteron-like bound states of $[\Sigma_cN]$ can be searched by searching for its invariant mass in the final states of $npK^-\pi^+\pi$. The strong decay of $[\Lambda_cN]$ is forbidden. Because the weak decay model $\Lambda_c \to pK^-\pi^+$ takes the largest fraction ratio, the loosely bound state of $[\Lambda_cN]$ can also be searched by searching for its invariant mass in the final state of $npK^-\pi^+$. The threshold of $[\Xi_{cc}N]$ is below that of $\Lambda_c\Lambda_c$, the strong decay of $[\Xi_{cc}N]$ is forbidden either. The favorable decay modes is $[\Xi_{cc}N]^{(0,0)} \to \Lambda_cK^-\pi^+\pi^+, \ [25, 33]$. Some of these possible deuteron-like bound states may be searched for at BelleII and LHCb in the near future.

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A Definitions of some functions and Fourier transform formulæ

The definitions of the functions $H_i$ are [25, 33],

$$
H_0(A, m, r) = Y(ur) - \frac{\lambda^2}{2u} Y(\lambda r) - \frac{r^2\beta^2}{2u} Y(\lambda r),
$$

$$
H_1(A, m, r) = Y(ur) - \frac{\lambda^2}{2u} Y(\lambda r) - \frac{r^2\lambda^2\beta^2}{2u^3} Y(\lambda r),
$$

$$
H_2(A, m, r) = Z_1(ur) - \frac{\lambda^3}{u^3} Z_1(\lambda r) - \frac{\lambda^2\beta^2}{2u} Z_2(\lambda r),
$$

$$
H_3(A, m, r) = Z(ur) - \frac{\lambda^3}{u^3} Z(\lambda r) - \frac{\lambda^2\beta^2}{2u} Z_2(\lambda r),
$$

(38)
where, 
\[ \beta^2 = \Lambda^2 - m^2, \quad u^2 = m^2 - Q_0^2, \quad \lambda^2 = \Lambda^2 - Q_0^2, \]
and
\[ Y(x) = \frac{e^{-x}}{x}, \quad Z(x) = \left( 1 + \frac{3}{x} + \frac{3}{x^2} \right) Y(x), \]
\[ Z_1(x) = \left( \frac{1}{x} + \frac{1}{x^2} \right) Y(x), \quad Z_2(x) = (1 + x)Y(x). \]

In our case all heavy hadrons have the same masses, we have
\[ Q_0^2 = \left( \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + p_2^2} \right)^2 \approx \frac{(p_1 + p_2)^2 Q^2}{4m_{cc}^2}. \] (39)

Thus \( Q_0^2 \) is a high-order term and can be directly dropped out.

Without the form factor, one makes Fourier transformation and obtains
\[ \frac{1}{u^2 + Q^2} \rightarrow e^{-ur} \frac{u}{4\pi} Y(ur), \] (40)
\[ \frac{Q}{u^2 + Q^2} \rightarrow i\nabla \left( \frac{u}{4\pi} Y(ur) \right) = i \frac{u^3}{4\pi} Z_1(ur)r, \] (41)
\[ \frac{Q^2}{u^2 + Q^2} \rightarrow - \frac{u^3}{4\pi} Y(ur) + \delta^{(3)}(r), \] (42)
\[ \frac{Q_i Q_j}{u^2 + Q^2} \rightarrow - \frac{u^3}{12\pi} [Z(ur)k_{ij} + Y(ur)\delta_{ij}] + \frac{\delta_{ij}}{3} \delta^{(3)}(r), \] (43)
where \( k_{ij} = 3r_i r_j - \delta_{ij}. \) Clearly, there exist terms with a delta function \( \delta^{(3)}(r) \) in Eqs. (42)-(43). In the current work, we call these terms the contact interaction or delta interaction.

After introducing the form factor, the Fourier transformation formulae read
\[ \frac{1}{u^2 + Q^2} F^2(Q) \rightarrow \frac{u^3}{4\pi} H_0(1, A, m, r), \]
\[ \frac{Q}{u^2 + Q^2} F^2(Q) \rightarrow - \frac{u^3}{4\pi} H_1(1, A, m, r), \]
\[ \frac{Q^2}{u^2 + Q^2} F^2(Q) \rightarrow i \frac{u^3}{4\pi} r H_2(1, A, m, r), \]
\[ \frac{Q_i Q_j}{u^2 + Q^2} F^2(Q) \rightarrow - \frac{u^3}{12\pi} [H_3(1, A, m, r)k_{ij} + \delta_{ij} \delta^{(3)}(r)] + H_1(1, A, m, r) \delta_{ij}. \] (44)

One can also get the results without the contact interaction term by a simple replacement in the above equations,
\[ H_1(1, A, m, r) \rightarrow H_0(1, A, m, r). \] (45)

### B Matrix elements of the operators

In the present work, we encounter the following operators,

- Spin-spin operator:
  \[ \sigma_1 \cdot \sigma_2, \] (46)

- Spin-orbit operator:
  \[ L \cdot S, \quad L \cdot S_1, \quad L \cdot S_2, \] (47)

- Tensor operator:
  \[ S_{12}(\hat{r}) = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2. \] (48)

The derivation of matrix elements of these operators are given in the Ref. [30]. We give the results as follows,

- Spin-singlet \((S = 0)\):
  \[ \sigma_1 \cdot \sigma_2 = -3, \quad S_{12}(\hat{r}) = 0, \quad L \cdot S = 0, \quad L \cdot S_1 = 0, \quad L \cdot S_2 = 0, \] (49)

- Spin-triplet \((S = 1)\):
  \[ \sigma_1 \cdot \sigma_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad S_{12}(\hat{r}) = \begin{pmatrix} 0 & \sqrt{3} \\ \sqrt{3} & -2 \end{pmatrix}, \]
  \[ L \cdot S = 2L \cdot S_1 = 2L \cdot S_2 = \begin{pmatrix} 0 & 0 \\ 0 & -3 \end{pmatrix}, \] (50)

### C The \( B_cN \) and \( B_c \bar{N} \) systems

Throughout the work, we use \( B_c \) to denote the one-charm baryon, \( \Sigma_c \) or \( \Lambda_c \). For the \( \Sigma_cN(\bar{N}) \) systems, the \( \pi-, \rho-, \omega- \) and \( \sigma \)-exchanges are permitted whereas only the \( \omega \)- and \( \sigma \)-exchanges are allowed for the \( \Lambda_cN(\bar{N}) \) systems. We present the binding solutions for the spin-singlet \( B_cN(\bar{N}) \) systems in Table 7 and those for the spin-triplet \( B_cN(\bar{N}) \) systems in Table 8.

#### C.1 Spin-singlet \((S = 0)\)

For the spin-singlet \((S = 0)\) systems, we focus on the ground state, namely the \( ^3S_0 \) channel. The isospin of the \( \Lambda_cN(\bar{N}) \) system is \( \frac{3}{2} \), while that of the \( \Sigma_cN(\bar{N}) \) system is either \( \frac{1}{2} \) or \( \frac{3}{2} \). We denote the \( \Lambda_cN \) system by \([\Lambda_cN]^{(0,1;\frac{1}{2})}\), where \( S \) and \( I \) denote the spin and isospin respectively. The other systems are denoted in a similar way. For the \([\Lambda_cN]^{(0,1;\frac{3}{2})}\) system, a systematic coupled-channel analysis has already been given in Ref. [32]. We present them again in the current work to be self-consistent. Without the coupled-channel effects to the \( \Sigma_cN(\Sigma_cN^*) \) channels, one could not obtain binding solutions. For the \([\Sigma_cN]^{(0,1;\frac{1}{2})}\) system, the possible binding solutions depend strongly on the cutoff and the delta interaction so we omit the results. For the \([\Sigma_cN]^{(0,1;\frac{3}{2})}\), we obtain no binding solutions and a loosely bound state is produced if one removes the delta interaction. This loosely bound state has binding...
energy $1.08 < \text{B.E.} < 34.82$ MeV for the cutoff to be $0.90 < \Lambda < 1.10$ GeV. The root-mean-square radius is $1.01 < r_{rms} < 4.05$ fm, which indicates that it might be a candidate of the molecular state.

For the $[\Lambda_cN]^{(0,\frac{1}{2})}$ system, only the $\omega$- and $\sigma$-exchanges are allowed by the symmetry. The corresponding interaction potentials are shown in Fig. 4 from which one can see clearly that the $\omega$- and $\sigma$-exchanges offer attractive force. Very interestingly, we obtain a loosely bound state of $[\Lambda_cN]^{(0,\frac{1}{2})}$, no matter the delta interaction is included or not. We also find that the delta interaction plays a minor role in the formation of the loosely bound $[\Lambda_cN]^{(0,\frac{1}{2})}$, i.e., only changing the binding energy by a few MeV. For example, the binding energy is $3.09 < \text{B.E.} < 56.51$ MeV for the cutoff to be $0.90 < \Lambda < 1.00$ GeV with the total potential. With the same cutoff, the binding energy is $2.51 < \text{B.E.} < 54.12$ MeV if one neglects the delta interaction. The $[\Lambda_cN]^{(0,\frac{1}{2})}$ should be taken as a promising candidate of the hadronic molecule.

For $[\Sigma_cN]^{(0,\frac{1}{2})}$, the possible bound state is also very sensitive to the cutoff. After removing the delta interaction, no binding solutions are obtained. Thus $[\Sigma_cN]^{(0,\frac{1}{2})}$ is not a good candidate of the molecular state.

The $[\Sigma_cN]^{(0,\frac{1}{2})}$ system is very interesting. We obtain a loosely bound state both with and without the delta interaction. We show the interaction potentials in Fig. 4 from which one can see clearly that the $\tau$-, $\omega$- and $\sigma$-exchanges generate the attractive force. With the full interaction, the binding energy is $10.78 < \text{B.E.} < 70.82$ MeV for the cutoff to be $0.80 < \Lambda < 0.90$ GeV. If one neglects the delta interaction, the binding energy is $0.56 < \text{B.E.} < 52.17$ MeV for the cutoff to be $0.90 < \Lambda < 1.00$ GeV. The corresponding root-mean-square radius is $0.73 < r_{rms} < 1.44$ fm and $0.85 < r_{rms} < 4.63$ fm respectively, which is comparable to that of the loosely bound deuteron, around 2 fm. The $[\Sigma_cN]^{(0,\frac{1}{2})}$ system should be taken as a candidate of the loosely bound state.

### C.2 Spin-triplet ($S = 1$)

In the spin-triplet sector, both the $\frac{3}{2}S_1$ and $\frac{3}{2}D_1$ channels are considered because of the tensor force. Different from the spin-singlet case, we additionally calculate the probability of the $S$ wave. We present the binding solutions in Table 5. Similar to the spin-singlet case, one can refer to Ref. 62 for a systematicatical coupled-channel analysis of $[\Lambda_cN]^{(1,\frac{1}{2})}$. Without the coupled-channel effects in the flavor space, one could not obtain binding solutions. For the $[\Sigma_cN]^{(1,\frac{1}{2})}$ system, we could not find binding solutions with the full interaction and obtain a loosely bound state after removing the delta interaction from the total potential. The binding energy is $1.57 < \text{B.E.} < 29.86$ MeV with the cutoff parameter $0.80 < \Lambda < 1.20$ GeV. The root-mean-square radius is $1.15 < r_{rms} < 3.62$ fm, which indicates that $[\Sigma_cN]^{(1,\frac{1}{2})}$ can form a hadronic molecules with the meson-exchange potential. For this state, the probability of the $D$ wave is round 5%. For the system $[\Sigma_cN]^{(1,\frac{3}{2})}$, with the full potential the binding solutions depend strongly on the cutoff whereas no binding solutions are obtained without the delta interaction. Thus $[\Sigma_cN]^{(1,\frac{3}{2})}$ is not supported to be a candidate of the hadronic molecule.

The results of $\Lambda_cN$ do not depend on the spin because the tensor force is vanishing for the spin-triplet case. The results of $[\Lambda_cN]^{(1,\frac{1}{2})}$ are exactly the same as those of $[\Lambda_cN]^{(0,\frac{1}{2})}$. The $[\Sigma_cN]$ system is very interesting. For both states ($S,I = (1,\frac{1}{2})$ and $(1,\frac{3}{2})$), we obtain loosely bound states, no matter the delta interaction is considered or not. We show the interaction potentials in Fig. 5. From the plots, one can see clearly that the $\pi$- and $\sigma$-exchanges generate the attractive force for $[\Sigma_cN]^{(1,\frac{1}{2})}$ while the $\rho$- and $\sigma$-exchange provide the attractive force for $[\Sigma_cN]^{(1,\frac{3}{2})}$. The $\omega$-exchange provides the repulsive force in the short range and attractive force in the long range for both of the two states. The contribution of the $D$ is around $10\% - 15\%$ for $[\Sigma_cN]^{(1,\frac{1}{2})}$ and $2\% - 3\%$ for $[\Sigma_cN]^{(1,\frac{3}{2})}$. Both of these two systems can form hadronic molecules with the meson-exchange potential.

In our calculation, we adopt the CD-Bonn potential model. Specifically, the $\pi$ exchange provides the long-range force, the $\sigma$ exchange supplies the medium-range potential and the heavier vector meson ($\rho$, $\omega$ and $\phi$) exchanges account for the short-range forces. In addition to the CD-Bonn one-boson-exchange potential (OBEP) model, there are also other OBEP approaches applied to the studies of the charmed-baryon-nucleon systems. For example, the Nijmegen group proposed an OBEP model with phenomenological hard-core potentials at short distance and applied it to the nucleon-nucleon and hyperon-nucleon systems 61–65. Following the Nijmegen OBEP approach, Dove and Kahana studied the possibility of the charmed baryons, $B_c(A_c, \Sigma_c, \Xi_c, \Xi'_c)$, bind to a nucleon in 66. Bhammathit analyzed the three-body system $B_cNN$ as well as $B_cN$ in 67, and Bando and Nagata investigated bindings of the $B_c - \alpha$ systems 68. In these three calculations, the SU(3) symmetry used in the Nijmegen potential was extended to SU(4) in order to include the charmed baryons and mesons, although the charmed $D$ and $D^*$ exchanges are not important because of their heavy masses. Recently, Maeda et al. constructed a potential model (called “CTNN”) in which the long-range force arises from the $\tau$ and $\sigma$ exchanges while the short-range repulsion is evaluated by a quark cluster potential. Additionally, a monopole type form factor is introduced to the long-range potential to reflect the extended structure of hadrons 69. We take the coupling constants $C_\sigma$ for $\sigma$ exchange and parameter $b$ for Gaussian potential in the quark cluster model as two running parameters. In our calculations, we take four sets of parameters,

- Set a: $C_\sigma = -67.58$, $b = 0.6$fm;
- Set b: $C_\sigma = -77.50$, $b = 0.6$fm;
- Set c: $C_\sigma = -60.76$, $b = 0.5$fm;
- Set d: $C_\sigma = -70.68$, $b = 0.5$fm.
Table 7. The binding solutions for the spin-singlet $B_cN$ and $B_c\bar{N}$ systems. “A” is the cutoff parameter. “B.E.” means the binding energy while $r_{rms}$ is the root-mean-square radius.

| Systems                  | With contact interaction | Without contact interaction |
|--------------------------|--------------------------|-----------------------------|
|                          | A (GeV)                  | B.E. (MeV)                  | $r_{rms}$ (fm) | A (GeV)                  | B.E. (MeV)                  | $r_{rms}$ (fm) |
| $[\Sigma_cN]^{(0,\frac{3}{2})}$ | ×                        | ×                           |                      | 0.90                     | 1.08                         | 4.05             |
|                          |                          | 1.00                        |                      | 1.00                     | 12.06                       | 1.52             |
| $[\Sigma_c\bar{N}]^{(0,\frac{3}{2})}$ | 0.80                     | 10.78                       | 1.44                 | 0.90                     | 0.56                         | 4.63             |
|                          |                          | 0.90                        | 70.82                | 0.73                     | 1.00                         | 52.17            | 0.85             |
| $[\Sigma_c\bar{N}]^{(0,\frac{1}{2})}$ | 1.00                     | 1.85                        | 3.38                 | ×                        | ×                            | ×                |
|                          |                          | 1.05                        | 58.06                | 0.78                     |                               |                  |
| $[\Lambda_c\bar{N}]^{(0,\frac{1}{2})}$ | 0.90                     | 3.09                        | 2.58                 | 0.90                     | 2.51                         | 2.81             |
|                          | 1.00                     | 56.51                       | 0.85                 | 1.00                     | 54.12                        | 0.86             |

Fig. 4. The interaction potentials for the spin-singlet ($S = 0$) $B_cN$ and $B_c\bar{N}$ systems. “w/o” means the delta interaction is removed from the total potential.

Fig. 5. The interaction potentials for $[\Sigma_c\bar{N}]^{(1,\frac{3}{2})}$ and $[\Sigma_cN]^{(1,\frac{3}{2})}$. Since the $S$ wave plays the dominant role, we only plot the interaction for the $^3S_1$ channel. “w/o” means the delta interaction is removed from the total potential.
Table 8. The binding solutions for the spin-triplet $B_cN$ and $B_c\bar{N}$ systems. “A” is the cutoff parameter. “B.E.” means the binding energy while $r_{rms}$ is the root-mean-square radius. $P_S$ is the probability (%) of the S wave.

| Systems       | $A$ (GeV) | B.E (MeV) | $r_{rms}$ (fm) | $P_S$ (%) | $A$ (GeV) | B.E (MeV) | $r_{rms}$ (fm) | $P_S$ (%) |
|---------------|-----------|-----------|----------------|-----------|-----------|-----------|----------------|-----------|
| $[\Sigma_cN]^{(1,\frac{3}{2})}$ | 1.12      | 3.70      | 1.97           | 99.47     | ×         | ×         | ×              | ×         |
| $[\Sigma_cN]^{(1,\frac{3}{2})}$ | 1.15      | 17.51     | 0.92           | 99.81     | 1.00      | 17.85     | 1.40           | 93.94     |
|                | 1.18      | 45.87     | 0.60           | 99.92     | 1.20      | 29.86     | 1.15           | 94.30     |
|                | ×         | ×         | ×              | ×         | 0.80      | 1.57      | 3.62           | 96.32     |
| $[\Sigma_cN]^{(1,\frac{3}{2})}$ | 1.00      | 17.85     | 1.40           | 93.94     | 1.00      | 17.85     | 1.40           | 93.94     |
| $[\Sigma_c\bar{N}]^{(1,\frac{3}{2})}$ | 0.90      | 1.17      | 3.91           | 98.51     | 0.90      | 6.24      | 1.98           | 97.85     |
|                | 1.00      | 36.70     | 1.03           | 96.91     | 1.00      | 54.76     | 0.88           | 97.13     |
| $[\Sigma_c\bar{N}]^{(1,\frac{3}{2})}$ | 0.80      | 17.31     | 1.39           | 90.54     | 0.80      | 1.66      | 3.43           | 92.27     |
|                | 0.90      | 88.98     | 0.83           | 85.11     | 0.90      | 81.77     | 0.86           | 84.63     |
| $[\Lambda_c\bar{N}]^{(1,\frac{3}{2})}$ | 0.90      | 3.09      | 2.58           | 100       | 0.90      | 2.51      | 2.81           | 100       |
|                | 1.00      | 56.51     | 0.85           | 85.11     | 1.00      | 54.12     | 0.86           | 84.63     |

A coupled-channel effect, $\Lambda_cN \leftrightarrow \Sigma_cN \leftrightarrow \Sigma_c^*N$, is also included in this calculation. We make a comparison of the present results with those of the previous studies, see Table 9. From the comparison, one can see clearly that the results of $\Lambda_cN$ are model-dependent in the “CTNN” approach even if the coupled-channel effect, $\Lambda_cN - \Sigma_cN - \Sigma_c^*N$, as well as the coulomb potential are included whereas the CD-Bonn OBEP model does not support $\Lambda_cN$ to form a bound state without coupling to $\Sigma_cN$ and $\Sigma_c^*N$. From our results, the system $[\Sigma_cN]^{(0, \frac{3}{2})}$ can be viewed as a candidate of the hadronic molecule, although the results depend slightly on how the delta potential is dealt with. This is consistent with the results from [66].

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Binding energies, in units of MeV, of the charmed-baryon-nucleon systems with different OBEP approaches. “—” denotes no relevant results while “×” means no bound states or the binding solutions is not stable at all.

| Channels  | CTNN   | this work |
|-----------|--------|-----------|
|           | a      | b | c     | d     | with δ | without δ |
| \([Σ_cN]^{(0,\frac{1}{2})}\) | – | × | × | 1.72 × 10^{-3} | 1.37 | × | × |
| \([Σ_cN]^{(1,\frac{1}{2})}\) | – | × | 2.62 × 10^{-4} | 1.97 × 10^{-2} | 1.57 | × | × |
| \([Σ_cN]^{(0,\frac{1}{2})}\) | – | – | – | – | – | × | × |
| \([Σ_cN]^{(1,\frac{1}{2})}\) | – | – | – | – | – | 1.08 – 12.06 |
| \([Σ_cN]^{(1,\frac{1}{2})}\) | – | – | – | – | – | 1.57 – 29.86 |

3.70 – 45.87 ×

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