Predictive torque control of SMPMSM drive system based on extended control set and double gradient descent method

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Abstract. Model predictive torque control (MPTC) has become an effective control method for the permanent magnet synchronous machine (PMSM) drive system in recent years. However, large torque and flux ripples due to the limited numbers of inverter voltage vector deteriorate the steady-state performance. In this paper, a novel model predictive torque control based on extended control set (ECS-MPTC) is proposed. First, an innovative extended control set is generated by the proposed partition method of quasi longitude and latitude lines, in which, many virtual voltage vectors are generated. Then, based on the characteristic of cost function without weighting factor, the optimal inverter voltage vector can be selected quickly by using the double gradient descent methods, and the invalid enumeration can be avoided. Thus, the computation burden can be kept in a low level. Finally, the simulation results of conventional MPTC and proposed ECS-MPTC validate that the proposed control can achieve excellent steady-state performance of PMSM drive system and low computation burden simultaneously.

1. Introduction

Permanent magnet synchronous motors (PMSM) have been widely used in electric vehicles and other various industrial fields due to its high efficiency and power density [1]. In recent years, the model predictive torque control becomes a promising algorithm and receives more and more attention in PMSM drives. Especially, the Finite control set model predictive torque control (FCS-MPTC) has a simple and intuitive structure, and it is easy to include multiple constraints and nonlinearity through the flexible design of cost function [2]. For the two-level voltage-sourced inverter (2L-VSI)-fed PMSM drive system, the conventional FCS-MPTC applies only one voltage vector in a control period, resulting in large motor flux linkage and torque ripple under the steady state operations [3]. In order to improve the steady-state performance of FCS-MPTC, several algorithms are proposed in recent years.

In [4] [5], the torque and flux control ripples are reduced by using duty cycle control, which applies an active and a zero voltage vector during one control period. In [6], the double vector control method is proposed, in which the second voltage vector can be either the active voltage vector or the zero voltage vector. In [7], the optimal voltage vector is selected by evaluating all the optional five groups of three vector combinations, and the coverage range of candidate voltage vectors is expanded.

The discrete space vector modulation (DSVM) is another effective way to improve the steady-state performance of PMSM drive system, in which many virtual voltage vectors can be introduced. In [8], the virtual voltage vectors are generated by using "concentric hexagon" partition, and the optimal voltage vector is selected based on the deadbeat predictive control. In [9], it subdivides the inverter voltage vector plane by angle, generates virtual voltage vector as control set, and then combines them with duty cycle control. In [10], the voltage vector plane is divided evenly into several triangle areas,
and the optimal voltage vector is selected from the three vertices of the triangle based on the deadbeat predictive control. In [11] the virtual voltage vector are divided into six sectors by the midline between active voltage vectors and in [12] the virtual voltage vectors are divided into four regions according to the voltage vector’s effect on motor torque and flux linkage. In this way, the computation burden is reduced to some extent.

Although the generation methods [8]-[12] of virtual voltage vectors are different, with the increase of the number of virtual voltage vectors, it will inevitably cause too much computational burden. In this paper, the predictive torque control based on extended control set and double gradient descent method is proposed. First, the partition method of "quasi longitude and latitude lines" is proposed to generate virtual voltage vectors as extended control set. Then, the weighting factor is eliminated by using the flux linkage reference vector [13], the weighting factor tuning process can be avoided. Moreover, based on the distribution of the rotating parabolic surface in the cost function [11], the double gradient descent method is used to enumerate and optimize the virtual voltage vectors in longitude and latitude lines direction, in which the efficient enumeration can be achieved. Finally, the comparative researches are achieved between the proposed method and the conventional MPTC method, the conclusions are shown.

2. Model of SMPMSM drive system

In a two-phase stationary coordinate system, the voltage and flux linkage equations of SMPMSM are expressed as:

\[ u_s = R_s i_s + \frac{d\psi_s}{dt} \]  
\[ \psi_s = L_s i_s + \psi_r \]  

where \( u_s, i_s \) and \( \psi_s \) are the stator voltage, current and flux vector. \( \psi_r \) is the rotor flux vector. \( R_s \) and \( L_s \) are the motor resistance and stator inductance.

The electromagnetic torque equation of the PMSM is expressed as

\[ T_s = \frac{3p}{2L_s} |\psi_r \times \psi_s| = \frac{3p}{2L_s} |\psi_r||\psi_s| \sin \theta_{rs} \]  

where \( p \) is the number of pole pairs. \( \theta_{rs} \) is the angle between \( \psi_r \) and \( \psi_s \).

3. ECS-MPTC based on double gradient descent method

3.1. Cost function design

In a two-phase stationary coordinate system, the voltage and flux linkage equations of SMPMSM are expressed as:

\[ g_i = |\psi_s^{ref} - \psi_s(k+1)| \]  

The reference flux vector amplitude \( \psi_s^{ref} \) takes a given value; its angle \( \angle \psi_s^{ref} \) can be expressed as:

\[ \angle \psi_s^{ref} = \theta_r + \theta_{rs} = \theta_r + \arcsin \left( \frac{T_s^{ref} L_s}{1.5p |\psi_r||\psi_s^{ref}|} \right) \]  

where \( \theta_r \) is rotor position angle, \( \theta_{rs} \) is the angle between \( \psi_r \) and \( \psi_s \).

According to equation (1), the prediction value of stator flux vector at \( k+1 \) instant is given as:

\[ \psi_s(k+1) = \psi_s(k) + T_s i_r(k) - T_s R_s i_s(k) \]  

where \( \psi_s(k) \) is the estimated stator flux linkage at \( k \) instant based on equation (2). \( u_r \) is the voltage vector of control set.
3.2. Virtual voltage vectors generation by "Quasi-Longitude and Latitude lines"

Based on the six active voltage vectors of the inverter and the midline of the adjacent active voltage vectors, the voltage hexagonal space is divided into 12 sectors, as shown in Figure 1. The paper proposes a division method of "quasi-longitude and latitude lines" (QLL), that is, in each sector, the active voltage vector is divided into N segments along the active voltage vector direction (longitude line direction), and then the active voltage vector direction is formed as a series of vertical line through the equal division point, defined as the latitude line, the latitude line will intersect with the adjacent partition at the midline of the active voltage vector. Through these intersections and make more longitude line parallel to the active vector longitude line. The longitude and latitude lines will produce many intersection points in the voltage vector space, and these intersection points are the generated virtual voltage vectors.

Figure 1 shows that when N=7, a total of 229 virtual voltage vectors are generated through the proposed "quasi-longitude and latitude line" division, and the voltage and duty cycle of each virtual voltage vector can be easily determined according to the geometric relationship.

![Figure 1. The virtual voltage vectors generation based on the proposed QLL division method](image)

3.3. Fast optimization of inverter voltage vector based on character of cost function

By means of symmetry of the cost function without weighting factor, the voltage vector at the minimum of the cost function value along the longitude line is obtained by using the gradient descent method to match the proposed division method of quasi-longitude and latitude lines, through which the minimum value of the cost function value along the latitude lines is obtained by using the second gradient descent method to obtain the optimal inverter voltage vector. Figure 2 is the structure diagram of QLL-ECS and double gradient descent method.

Taking sector I in Figure 3 as an example, the main steps of the proposed control method are described as follows:

**Step 1: select the optimal sector with cost function values of active vector**

Substitute active vectors $u'_1, u'_2, u'_3$ into the cost function, calculate $g_1, g_5, g_6$ and mark the voltage vector of $\text{min}(g_1, g_5, g_6)$ as $u_\text{min}$. Its cost function value is denoted as $g_\text{min}$. The active voltage vector opposite to $u_\text{min}$ is denoted as $u_\text{max,rev}$, and its cost function value $g_{\text{max,rev}}$. Compare $g_\text{min}$ with $g_{\text{max,rev}}$ determine the optimal sector, and record the smaller of them as $g_{\text{opt}}$.

**Step 2: determine the search direction**

After determining the optimal sector, assuming that there is a voltage vector $V_{\text{opt}}$ at the circle of the optimal sector so that the cost function is zero. It is necessary to determine the search direction to quickly locate the virtual voltage vector closest to it. Compare the zero-voltage vector cost function $g_0$ with $g_{\text{opt}}$ to determine the search direction, as shown in Figure 3, $g_{\text{opt}} = g_1$ in this sector. If
When $g_0 < g_{opt1}$, the search is performed from the of the $v_0$ arrow direction, otherwise, the search is performed from the direction of the $v_1$ arrow. Meanwhile mark the smaller of them as $g_{opt2}$.

**Figure 2.** Structure diagram of proposed model predictive torque controlled PMSM drive system

**Table 1.** Optimal sector judgment.

| min | max | max_re | gmin | gmax_re | sector |
|-----|-----|--------|------|---------|--------|
| 3   | 6   | 6      | Y    | XI      |
| 5   | 2   | N      | Y    | I       |
| 1   | 4   | N      | Y    | V       |
| 5   | 2   | N      | Y    | IV      |
| 1   | 4   | N      | Y    | VIII    |
| 3   | 6   | N      | Y    | IX      |

**Figure 3.** The search diagram based on double gradient descent method

**Step 3:** determine the optimal voltage vector of ECS based on the double gradient descent method

Taking sector I as an example, search along the longitude line $v_1$ arrow direction. Calculate A point cost function $g_A$. If $g_A < g_{opt2}$, continue searching, otherwise back to $v_1$. In Figure 3, the circle size represents the relative size of the cost function of the point, and the search order of the first gradient descent method is $g_{opt2} > g_A > g_B > g_C < g_D$. Therefore, point C is the optimal voltage vector on the longitude line. In the same way, the second gradient descent method is used to search the optimal
point on the latitude line with point C. \( g_C > g_E > g_F < g_G \), In other words, F point is the global optimal voltage vector.

Figure 3 clearly shows that circular points are enumerated points and a large number of square points are not included in the enumeration. The proposed control significantly reduces the invalid enumeration and improves the efficiency of the algorithm. In this paper, taking \( N \) to 7 as an example, the proposed method needs to enumerate the optimal voltage vectors as fast as 6 times in a control period and 13 times in the worst case. Single vectors FCS method need to be enumerated 7 times in a control period. DSVM method with \( N=3 \) in reference [11] needs to be enumerated 13 times.

4. System simulation researches

Based on MATLAB/Simulink software, the proposed SMPMSM drive system simulation model was established, and the system simulation research was carried out. The FCS-MPTC, reference [11] DSVM-MPTC and proposed ECS-MPTC are compared for same SMPMSM drive system, and SMPMSM parameters are shown as Table 2.

| Parameters                  | Value         | Parameters             | Value         |
|-----------------------------|---------------|------------------------|---------------|
| Rated torque                | 13N\(\cdot\)m | Stator resistance      | 1mH           |
| Rated speed                 | 500 rpm       | Stator inductance      | 0.0957\(\Omega\) |
| Rated current (rms)         | 19A           | Magnet flux            | 0.027Wb       |
| Number of pole-pairs        | 12            | Moment of inertia      | 0.01015kg\(\cdot\)m² |

Figure 4-6 shows the torques, flux linkage and three-phase current waveforms of SMPMSM drive system at 100rpm 5N.m, 300rpm 8N.m and 500rpm 11N.m, respectively.

(a) (b) (c)

Figure 4. Steady-state performance comparison at 100rpm 5N.m (a) Traditional FCS-MPTC (b)DSVM MPTC in reference[11] (c) The proposed ECS-MPTC.
Figure 5. Steady-state performance comparison at 300rpm 8N.m (a) Traditional FCS-MPTC (b) DSVM MPTC in reference [11] (c) The proposed ECS-MPTC.

Figure 6. Steady-state performance comparison at 500rpm 11N.m (a) Traditional FCS-MPTC (b) DSVM MPTC in reference [11] (c) The proposed ECS-MPTC.

It is clear that the traditional FCS-MPTC, DSVM-MPTC in reference [11] and proposed ECS-MPTC can achieve accurate steady-state flux and torque tracking, but the proposed ECS-MPTC scheme enjoys the lowest torque and flux ripple. Compared with the FCS-MPTC, the DSVM-MPTC, the proposed scheme reduces the torque ripple by 3.2 times, 0.5 times, and the flux ripple by 2.7 times and 0.3 times, respectively.
Table 3 shows current THD and the calculation times of cost function under different operating conditions. The proposed scheme achieves better steady-state control performance while guaranteeing lower calculation load for the controller. Therefore, system simulation studies indicate that the proposed ECS-MPTC can effectively introduce more extended voltage vectors without increasing the computing load, therefore significantly reduce the torque and flux ripple.

5. Conclusion
In this paper, the virtual voltage vector generated by the division of quasi-longitude and latitude lines is proposed as an extended control set. The cost function without weighting factor is designed. Resorting to the symmetry of the cost function without weighting factor, the optimal sector is determined quickly with the cost function value of active vector, and then the optimal virtual voltage vector of ECS was determined quickly by using the double gradient descent method in the optimal sector. By comparing the control performance of SMPMSM drive system with that of FCS-MPTC and DSVM-MPTC, it is verified that the proposed predictive torque controlled SMPMSM drive system based on ECS and double gradient descent method can effectively reduce the computational burden, improve the steady-state performance of SMPMSM drive system, and significantly reduce the torque and the flux ripple.

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