Three family $Z_3$ orbifold trinification, MSSM and doublet-triplet splitting problem

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Abstract

A $Z_3$ orbifold compactification of $E_8 \times E_8'$ heterotic string is considered toward a trinification $SU(3)^3$ with three light families. The GUT scale VEV’s of the $SU(2)_W \times U(1)_Y \times SU(3)_c$ singlet chiral fields in two sets of the trinification spectrum allow an acceptable symmetry breaking pattern toward MSSM. We show that a doublet-triplet splitting is related to the absence of a $\Delta B$ nonzero operator.

[Key words: $Z_3$ orbifold, superstring, trinification, three families]

12.10.-g, 11.25.Mj, 12.60.-i
I. INTRODUCTION

It seems that the family structure of the standard model (SM) is completed with three light ones. This observation stems from the recent experiments toward understanding neutrino oscillation, Big Bang nucleosynthesis, and experiments saturating the unitarity triangle. For a long time, the question, “Why are there three light families?”, has been the heart of the family problem. In 4 dimensional (4D) field theories, the grand unification idea with a big gauge group was suggested toward this family structure, which is called the grand unification of families (GUF) [1]. For the GUF idea to work from a bottom-up approach, the three different gauge coupling constants observed at the electroweak scale should meet at a grand unification (GUT) scale $M_{GUT}$. With the three light families and one Higgs doublet scalar fields, they do not meet. But one can make them meet by introducing a number of particles beyond the three family structure of the SM. One interesting possibility is the particle spectrum of the minimal supersymmetric SM (MSSM) [2].

With the advent of superstring models, the GUF idea seems to be automatically implemented. In particular, the 10 dimensional (10D) heterotic string models need big gauge groups, $E_8 \times E_8'$ or $SO(32)$ [3]. Among these, the $E_8 \times E_8'$ has attracted a particular attention. However, the big gauge group is given in 10D, and one has to hide six internal spaces to contact with our 4D world. This process of hiding six internal spaces is known as “compactification”, accompanies the breaking of the big 10D gauge group, and also generates multi-families in 4D [4,5]. The most serious objective in this compactification has been to obtain the MSSM in 4D. For an $N = 1$ supersymmetry, the internal space with an $SU(3)$ holonomy has been suggested first [4]. But a more interesting and easily soluble case is the orbifold compactification [5]. In particular, the $Z_3$ orbifold models with two Wilson lines attracted a great deal of attention because of the multiplicity 3 in the spectrum [6]. Along this line, the standard-like models, which allow three families and $SU(3)_c \times SU(2) \times U(1)^n$ groups, have been extensively studied [6,7].

The standard-like models, however, suffered from the following two problems:
(i) the $\sin^2 \theta_W$ problem, and

(ii) the problem of too many Higgs doublets.

With the MSSM spectrum, it is necessary to assume that the unification value of $\sin^2 \theta_W$ is $\frac{3}{8}$ to reconcile with the low energy data on $\alpha_{QCD}$, $\alpha_{em}$ and $\sin^2 \theta_W$. The $\sin^2 \theta_W$ problem (i) is that it is generally difficult to obtain $\frac{3}{8}$ for the unification value of $\sin^2 \theta_W$. The problem of too many Higgs doublets is that the standard-like models have many pairs of Higgs doublets while the MSSM needs just one pair. To solve the above problems, recently it was suggested to unify the standard model in a semi-simple gauge group at the compactification scale so that the electroweak hypercharge is not leaked to $U(1)^n$ factors [8]. In [8], the motivation has been to embed the electroweak hypercharge in semi-simple groups with no need for the adjoint representation (HESSNA). In the HESSNA, the QCD gauge group must be already factored out so that an adjoint representation is not needed. The simplest HESSNA is the $SU(3)^3$ gauge group with the so-called trinification [9] spectrum for one family,

$$(\bar{3}, 3, 1) + (1, \bar{3}, 3) + (3, 1, \bar{3}).$$

This leads us to search for simple $SU(3)^3$ models for HESSNA. In this paper, we present a $Z_3$ orbifold model which leads to a model close to the MSSM below a GUT scale. We also show a correlation between the doublet-triplet splitting and the $\Delta B$ nonzero operator $u^cd^e d'^c$.

**II. A $Z_3$ ORBIFOLD MODEL FROM $E_8 \times E_8'$**

The heterotic string theory has $N = 4$ supersymmetry from the 4D viewpoint. To obtain chiral fermions in 4D, we have to reduce $N = 4$ supersymmetry down to $N = 1$. The $Z_3$ orbifold reduces $N = 4$ down to $N = 1$ when we compactify the six internal spaces [5]. The six internal spaces are split into a direct product of three two-dimensional tori $(y_1 - y_2; y_3 - y_4; y_5 - y_6)$. A $Z_3$ orbifolding of two dimensional torus gives three fixed points; thus three $Z_3$ orbifolded tori have 27 fixed points. The 27 fixed points are not
distinguishable unless one introduces Wilson lines. The shift vector $V$ and the six Wilson lines $a_i (i = 1, \cdots, 6)$ are embedded in the gauge group $E_8 \times E_8'$. The $a_1$ is transformed to $a_2$ by a $Z_3$ transformation, and we consider only three independent Wilson lines: $a_1 = a_2, a_3 = a_4, a_5 = a_6$ [10].

The model we study here is

\[
V = (0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{2}{3})(0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{2}{3})
\]

\[
a_1 = (\frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ 0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ 0)(\frac{1}{3} \ \frac{1}{3} \ 0 \ 0 \ 0 \ \frac{1}{3} \ \frac{1}{3})
\]

\[
a_3 = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{2}{3} \ 0 \ 0 \ 0)
\]

with $a_5 = (0 \ \cdots)(0 \ \cdots)$. Eq. (2) is allowed in superstring orbifolds. For the conditions to be satisfied, see Ref. [10]. The unbroken gauge group is $[SU(3)^3 \times U(1)^2] \times [SU(3)^2 \times U(1)^4]'$. Here, however, we assume that six $U(1)$’s are broken by VEV’s of $SU(3)^5$ singlet fields at the string scale. Below the string scale, the effective gauge group is $SU(3)^3 \times [SU(3)^2]'$, and hence the invariance under the nonabelian gauge group is our main concern in this paper. In HESSNA, one does not have to know the extra $U(1)$ quantum numbers to pinpoint the electroweak hypercharge.

Thus in the observable sector, this compactification leads at low energy to an $N = 1$ effective field theory $SU(3)^3$ with three copies of trinification spectrum (1). The massless chiral fields are presented in Table I with the well-known method [10,6,8]. Because there are nine twisted sectors, the multiplicity in one twisted sector is 3. Because of $Z_3$, the chiral fields of the untwisted sector also have the multiplicity 3. These are the bases for three chiral families. Note that the fields in the nine twisted sectors of Table I form vectorlike representations which can be removed at a GUT scale. Therefore, we will be interested in the 3 copies of the trinification spectrum appearing in the untwisted sector.

In many aspects for low energy physics, it is similar to an $E_6$ model with three families of $27$. In the present model, however, the electroweak gauge group and $SU(3)_c$ are already

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1A precursor of the present model with $V$ and $a_1$ was already given before [8,11].
split and we do not need an adjoint representation for the symmetry breaking [8].

When one blows up the fixed points and obtain a smooth Calabi-Yau manifold with an $SU(3)$ holonomy, one $SU(3)$ factor from the orbifold is identified with the $SU(3)$ holonomy and is removed from the low energy gauge group [5]. We can identify one of $SU(3)$'s in the hidden sector for this purpose if we wish.

### III. THE MINIMAL SUPERSYMMETRIC STANDARD MODEL

To obtain the low energy effective theory MSSM, we must break the $SU(3)^3$ gauge symmetry down to the MSSM group $SU(2)_W \times U(1)_Y \times SU(3)_C$ at a GUT scale $M_{GUT}$. Let us represent the trinification fields of (1) as

\[
\begin{align*}
(\bar{3}, 3, 1) & \rightarrow \Psi_{l=(\bar{M}, I, 0)} = \Psi_{(\bar{1}, i, 0)}(H_d)_{-\frac{1}{2}} + \Psi_{(\bar{2}, i, 0)}(H_u)_{+\frac{1}{2}} + \Psi_{(\bar{3}, i, 0)}(l)_{-\frac{1}{2}} \\
& \quad + \Psi_{(1,3,0)}(N_5)_{0} + \Psi_{(2,3,0)}(e^+)_{+1} + \Psi_{(3,3,0)}(N_{10})_{0} \quad (3) \\
(1, \bar{3}, 3) & \rightarrow \Psi_{q=(0, i, \alpha)} = \Psi_{(0, \bar{i}, \alpha)}(q)_{+\frac{1}{2}} + \Psi_{(0, 3, \alpha)}(D)_{-\frac{1}{2}} \quad (4) \\
(3, 1, \bar{3}) & \rightarrow \Psi_{a=(M, 0, \alpha)} = \Psi_{(1, 0, \alpha)}(d^c)_{+\frac{1}{3}} + \Psi_{(2, 0, \alpha)}(u^c)_{-\frac{2}{3}} + \Psi_{(3, 0, \alpha)}(D)_{+\frac{1}{3}} \quad (5)
\end{align*}
\]

where $M, I, \alpha$ are the $SU(3)_1, SU(3)_2 \equiv SU(3)_W$, and $SU(3)_3 \equiv SU(3)_C$ indices. Under the SM gauge group, $I = i = \{1, 2\}$ and $\alpha = \{red, green, blue\}$ represent $SU(2)_W$ and $SU(3)_C$ indices, and we appropriately represented the well-known notations for the SM fields in the parenthesis. The $U(1)_Y$ charges are shown with subscripts. Let us call the three representations given in (3), (4) and (5), as three different *humors* and name them as *lepton-* , *quark-* , and *antiquark-humors* because leptons, doublet quarks, and $u^c, d^c$ quarks appear there. In (3) there are two fields which are neutral under the SM gauge group: $N_5$ and $N_{10}$. Therefore, GUT scale vacuum expectation values of these fields break down the $SU(3)^3$ gauge group down to the SM gauge group,

\[
SU(3)^3 \xrightarrow{(N_{10})} SU(2)_1 \times SU(2)_W \times U(1)_a \times SU(3)_C \xrightarrow{(N'_5)} SU(2)_W \times U(1)_Y \times SU(3)_C \quad (6)
\]
The symmetry breaking is achieved by giving VEV’s to the scalar partners of the three family trinification fields. In the first step of symmetry breaking (6), 9 Goldstone bosons are absorbed through the Higgs mechanism to the gauge bosons. These are contained in $H_d, H_u, l, N_5, e^+$, and $N_{10}$. In the second step of (6), 3 further Goldstone bosons are absorbed to gauge bosons through the Higgs mechanism. The resulting gauge group is the SM gauge group and must be anomaly free. The study of this symmetry breaking pattern is not trivial and one must consider two steps of (6) together. With only one $(\bar{3}, 3, 1)$ representation, we cannot break $SU(3)_1 \times SU(3)_2$ down to $SU(2)_W \times U(1)_Y$. We need at least two $(\bar{3}, 3, 1)$ representations which are supposed to be scalar partners of two out of three copies of (3). After the Higgs mechanism, the remaining SM fields are linear combinations of the fields arising in (3). Then we can redefine the fields so that SM fields are renamed. The remaining fields from two sets of (3) must include two sets of $\{l_{-\frac{1}{2}}, e^+\}$. If $H$ fields are removed, they must be vectorlike representations. Otherwise, there appear anomalies. Note that Eqs. (4) and (5) lead to three quark families, and hence the anomaly free condition dictates to have three lepton families. Thus, after the Higgs mechanism there appear two sets of $\{l_{-\frac{1}{2}}, e^+\}$ from two sets of $(\bar{3}, 3, 1)$ for the spontaneous symmetry breaking. These $l_{-\frac{1}{2}}$’s are the renamed fields from the linear combinations of the original fields $H_d^{(1)}, H_d^{(2)}, l_{-\frac{1}{2}}^{(1)}$, and $l_{-\frac{1}{2}}^{(2)}$.

To discuss the light spectrum more concisely, let us utilize the $N = 1$ supersymmetry explicitly. Possible cubic terms among the untwisted sector fields are [13],

$$-\mathcal{L}_Y = \frac{1}{3!} f_{abc} \Psi^a \Psi^b \Psi^c$$  \hspace{1cm} (7)

where $a, b, c$ are the family indices. Note that we consider only the $SU(3)^3$ symmetry. [At the fundamental level, $f_{abc}$ are the coupling constant times ratios of singlet VEV’s to the string scale.] Note that $f_{abc}$ is completely symmetric. To distinguish the third family from the first two families participating in the GUT symmetry breaking, we postulate that $f_{ab3} = 0$ if $a$ or $b$ is in $\{1, 2\}$. Therefore, let us study the GUT symmetry breaking sector with $a, b, c \subset \{1, 2\}$ first. Assigning VEV’s as $\langle \Psi^{(1)}_{(3,3,0)} \rangle = \tilde{V}_1$, $\langle \Psi^{(2)}_{(1,3,0)} \rangle = \tilde{V}_2$, we note that one $H_a$ and one combination $H_d'$ (composed of $H_d$’s and $l_{-\frac{1}{2}}$’s) form Dirac particles at a GUT scale.
Therefore, out of 18 chiral fields we can figure out ten fields first: four from massive $H_u$ and $H_d'$ and six from two sets of $\{l_{-\frac{1}{2}}, e^+\}$. Thus, we can identify 12 Goldstone bosons among the remaining 8 complex (or 16 real) scalar fields. After the Higgs mechanism (removing 12 real fields), the remaining fields are two complex fields: $N_5$ and $N_{10}$. If we consider $SU(3)^3$ singlets $S$’s with GUT scale VEV’s, these singlet neutrinos can obtain large masses. In this case, we obtain only two sets of $\{l_{-\frac{1}{2}}, e^+\}$ from two sets of the trinification spectrum. The third set of the trinification spectrum contains one pair of $H_u$ and $H_d$ which is the needed light Higgs doublet pair in the MSSM.

Out of the three sets of the trinification spectrum (1), thus we obtain three fermion families, and their superpartners. For the number of Higgs doublets, see below.

IV. DOUBLET-TRIPLET SPLITTING

For the MSSM, we need a pair of Higgs doublets. But if the coupling (7) is completely general, we cannot achieve this objective since $H_u$ and $H_d$ in the third family, not participating in the GUT group breaking, will be heavy. We need a fine-tuning to keep them light. But this fine-tuning is correlated with a $\Delta B \neq 0$ operator.

Before showing the doublet-triplet splitting explicitly, we point out that the resolution of this doublet-triplet splitting problem in the flipped $SU(5)$ model [12] heavily assumes the absence of $H_d H_u$ coupling. It is the familiar $\mu$ problem, and can be solved by introducing a Peccei-Quinn symmetry [14]. But in string theory, we can see that the $H_d H_u$ term cannot arise at the tree level. Since both $H_d$ and $H_u$ belong to (3) in our compactification, a guessed term for $H_d H_u$, i.e. the term among the light fields $(\bar{3}, 3, 1) \cdot (\bar{3}, 3, 1)$ is forbidden from the gauge symmetry. In addition, however, the coupling $(\bar{3}, 3, 1) \cdot (\bar{3}, 3, 1) \cdot (\bar{3}, 3, 1)$ among the light fields, must be forbidden to remove the $H_d H_u$ coupling at a GUT scale because $H_d H_u$ can arise after giving a VEV to $N_5$ or $N_{10}$. Below we show that this can be realized by a fine-tuning but this fine-tuning must be dictated from a $\Delta B$ nonzero operator.

The VEV’s of $N_5$ and $N_{10}$ allow the following two types of nonvanishing mass terms.
The first possibility is coming from $SU(3)^3$ singlets by taking three different *humors*, and the second possibility is coming from $SU(3)^3$ singlets by picking up the same *humor* from $\Psi^a$, $\Psi^b$, and $\Psi^c$. In general, these two possibilities are present. In the discussion on the GUT symmetry breaking, we allowed both of these couplings. Below, we mainly focus on the couplings of the third family.

The first possibility gives masses to $D$ and $\bar{D}$. For example, for $\langle N_{10}(3rd \ family) \rangle = \tilde{V}_1$,\(^2\) we obtain $DM_D\bar{D}$ where

$$M_D = \tilde{V}_1 \begin{pmatrix} f_{113} & f_{123} & 0 \\ f_{213} & f_{223} & 0 \\ 0 & 0 & f_{333} \end{pmatrix}.$$  

Note that $\det M_D$ is nonzero, and three pairs of $D$ and $\bar{D}$ are removed at a GUT scale. Let us focus on the $f_{333}$ coupling below.

The second possibility allows a $u^cd^cd^c$ coupling, considering the *antiquark humor*. It violates the $R$-parity, and is dangerous for proton stability. Therefore, we choose a fine-tuning such that the second possibility from $f_{333}$ is excluded.

Let us try to implement a permutation symmetry $S_3$ in the $SU(3)^3$ model for a simpler discussion of the couplings. The three humor sets (3), (4), and (5), i.e. *lepton–*, *quark–*, and *antiquark–humors*, $\Psi_l, \Psi_q$, and $\Psi_a$ are represented as a singlet and a doublet of the permutation of $\{l, q, a\}$ [15],

$$\Psi_0 = \frac{1}{\sqrt{3}}(\Psi_l + \Psi_q + \Psi_a)$$

$$\Psi_+ = \frac{1}{\sqrt{3}}(\Psi_l + \omega \Psi_q + \omega^2 \Psi_a)$$

$$\Psi_- = \frac{1}{\sqrt{3}}(\Psi_l + \bar{\omega} \Psi_q + \bar{\omega}^2 \Psi_a)$$

\(^2\)Before, we assigned VEV’s only to the first two families. Since we have figured out the light spectrum before with two sets of (1), now we can also assign a VEV to the third family member. The composition of the new light fields will be more complicated, but the number of light degrees will be intact.
where $\omega$ and $\bar{\omega}$ are the cube roots of unity $\omega = e^{2\pi i/3}$, $\bar{\omega} = e^{4\pi i/3}$. Note that $\Psi_0$ is a singlet under the permutation of $l, q, a$. On the other hand $\Psi_\pm$ goes into a multiple of $\Psi_\mp$. Thus, $\Psi_+$ and $\Psi_-$ form a doublet under permutation, which we can represent as $\Psi_{\text{doublet}} \equiv (\Psi_+, \Psi_-)^T$.

The $S_3$ invariant cubic couplings are $\Psi_3^0$ and $\Psi_0 \Psi^+ \Psi^-$. In terms of humors, these are

\[
\Psi_0^3 = \frac{1}{3\sqrt{3}} (\Psi_l^3 + \Psi_q^3 + \Psi_a^3 + 3\Psi_l^2 \Psi_q + 3\Psi_l^2 \Psi_a + 3\Psi_q^2 \Psi_l + 3\Psi_a^2 \Psi_l + 3\Psi_a^2 \Psi_q + 6\Psi_l \Psi_q \Psi_a)
\]

\[
\Psi_0 \Psi_+ \Psi_- = \frac{1}{3\sqrt{3}} (\Psi_l^3 + \Psi_q^3 + \Psi_a^3 - 3\Psi_l \Psi_q \Psi_a)
\]

The above couplings include the so-called $R$-parity violating couplings of the MSSM. In particular, the $\Delta B \neq 0$ operator $u^c d^e d^c$ (the so-called $\lambda''$ coupling) is dangerous. It is contained in $\Psi_a^3$. To remove this $\Delta B \neq 0$ coupling $\Psi_a^3$, we fine-tune the $\Psi_0^3$ and $\Psi_0 \Psi_+ \Psi_-$ couplings such that they have the same magnitude but the opposite signs. Then, the $S_3$ invariant coupling is

\[
\frac{1}{\sqrt{3}} (\Psi_l^2 \Psi_q + \Psi_l^2 \Psi_a + \Psi_q^2 \Psi_l + \Psi_q^2 \Psi_a + \Psi_l^2 \Psi_l + \Psi_l^2 \Psi_q + 3\Psi_l \Psi_q \Psi_a)
\]

\[
\rightarrow \sqrt{3} \Psi_l \Psi_q \Psi_a
\]  

(8)

where in the second line we excluded the terms not allowed by the gauge invariance. Thus, the phenomenological requirement for proton stability excludes the $H_u H_u$ allowing term $\Psi_l^3$ (the second possibility), and hence $H_d$ and $H_u$ are left as light particles. Furthermore, the coupling allows the first possibility, i.e. the coupling chooses different humors in the cubic terms, and hence removes the color triplets $D$ and $\bar{D}$, realizing the doublet-triplet splitting.

If this argument is applied to the first two families, we will end up with two pairs of Higgs doublets, one pair too much. We must remove one more pair, but then we must allow a $\lambda''$ coupling. A sizable $\lambda''$ for the $t$ quark family is not forbidden very strongly phenomenologically (For proton decay, a product $\lambda'\lambda''$ is constrained.). To obtain a phenomenologically acceptable MSSM, we may require this kind of fine-tuning, forbidding the same humor cou-
pling, among the two lighter families\(^3\); but allow an O(1) same humor coupling for the \(t\) family.

V. CONCLUSION

In conclusion, we constructed a \(Z_3\) orbifold trinification model with three light families, and showed that the symmetry breaking leads to a spectrum close to the MSSM. The discussion on keeping one pair of \(H_u\) and \(H_d\) light needed a fine-tuning in this paper, but this fine tuning has been shown to be correlated with the absence of \(\Delta B\) nonzero operator \(u^c d^c d'^c\). It will be very interesting if this fine-tuning is naturally obtained.

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\(^3\)The previous discussion on the GUT symmetry breaking assumed the same humor coupling, mainly to find out the heavy fields, i.e. non-Goldstone fields.
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Every representation has multiplicity 3 because of $Z_3$ for the case of $U$ and two Wilson lines for the cases of nine $T$'s.

| sector       | fields                                                   |
|--------------|----------------------------------------------------------|
| $U$          | $(3, 3, 1)(1, 1) + (3, 1, 3)(1, 1) + (1, 3, 3)(1, 1)$     |
|              | $+ 3(1, 1, 1)(1, 3)$                                    |
| $T_0 (V)$    | nine singlets                                           |
| $T_1 (V + a_1)$ | $(1, 3, 1)(1, 1) + (3, 1, 1)(1, 1) + (1, 1, 3)(1, 1)$   |
| $T_2 (V - a_1)$ | $(1, 3, 1)(1, 1) + (3, 1, 1)(1, 1) + (1, 1, 3)(1, 1)$   |
| $T_3 (V + a_3)$ | nine singlets                                           |
| $T_4 (V - a_3)$ | $3(1, 1, 1)(1, 3)$                                     |
| $T_5 (V + a_1 + a_3)$ | $(1, 1, 3)(1, 1) + (3, 1, 1)(1, 1) + (1, 3, 1)(1, 1)$   |
| $T_6 (V + a_1 - a_3)$ | $(1, 1, 3)(1, 1) + (3, 1, 1)(1, 1) + (1, 3, 1)(1, 1)$   |
| $T_7 (V - a_1 + a_3)$ | $(1, 1, 3)(1, 1) + (3, 1, 1)(1, 1) + (1, 3, 1)(1, 1)$   |
| $T_8 (V - a_1 - a_3)$ | $(1, 1, 3)(1, 1) + (3, 1, 1)(1, 1) + (1, 3, 1)(1, 1)$   |