Self-induced neutrino flavor conversion without flavor mixing

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Abstract. Neutrino-neutrino refraction in dense media can cause self-induced flavor conversion triggered by collective run-away modes of the interacting flavor oscillators. The growth rates were usually found to be of order a typical vacuum oscillation frequency $\Delta m^2/2E$. However, even in the simple case of a $\nu_e$ beam interacting with an opposite-moving $\bar{\nu}_e$ beam, and allowing for spatial inhomogeneities, the growth rate of the fastest-growing Fourier mode is of order $\mu = \sqrt{2G_F} n_{\nu}$, a typical $\nu-\nu$ interaction energy. This growth rate is much larger than the vacuum oscillation frequency and gives rise to flavor conversion on a much shorter time scale. This phenomenon of “fast flavor conversion” occurs even for vanishing $\Delta m^2/2E$ and thus does not depend on energy, but only on the angle distributions. Moreover, it does not require neutrinos to mix or to have masses, except perhaps for providing seed disturbances. We also construct a simple homogeneous example consisting of intersecting beams and study a schematic supernova model proposed by Ray Sawyer, where $\nu_e$ and $\bar{\nu}_e$ emerge with different zenith-angle distributions, the key ingredient for fast flavor conversion. What happens in realistic astrophysical scenarios remains to be understood.

Keywords: neutrino theory, supernova neutrinos, core-collapse supernovas

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1 Introduction

Most of the energy liberated in stellar core collapse or in neutron-star mergers appears in
the form of neutrinos and antineutrinos of all flavors, but with fluxes and spectra that differ
strongly between $\nu_e$, $\bar{\nu}_e$ and the other species, collectively referred to as $\nu_x$. The subsequent
flavor evolution of these neutrinos influences energy deposition beyond the decoupling region,
neutrino-driven nucleosynthesis, and detection opportunities of the neutrino signal from the
next nearby supernova or the diffuse supernova neutrino flux from all past core-collapse
events [1–3]. However, a true understanding of flavor evolution in dense environments has
remained elusive because of many complications engendered by the nonlinear nature of collec-
tive flavor oscillations [3, 4]. We study a new item on this list which has eluded most workers
in this field with the notable exception of Ray Sawyer [5, 6], i.e., the surprising insight that
collective flavor conversion need not depend on neutrino mixing parameters.

Collective neutrino flavor oscillations manifest themselves in the form of two generic
phenomena. One is the effect of synchronisation: different modes of the neutrino mean
field oscillate in lockstep even though they have different vacuum oscillation frequencies
$\omega = \Delta m^2/2E$ [7–11]. The other is the phenomenon of self-induced flavor conversion, corre-
sponding to collective run-away modes [12–18]. Surprisingly, the growth rate in the linear
regime and the overall evolution need not depend on $\Delta m^2/2E$ and therefore the effect can
occur even for unmixed neutrinos if given an appropriate seed to grow from [5, 6].

Collective “flavor conversion” actually does not represent any change of flavor in the
overall ensemble, but a reshuffling among different modes. In the simplest case, a gas of $\nu_e$ and
$\bar{\nu}_e$ can convert to $\nu_\mu$ and $\bar{\nu}_\mu$ without change of lepton number or flavor-lepton number. Such
tax processes certainly occur in the form of non-forward scattering with a rate proportional
to $G_F^2$, but can also occur on the refractive level with a rate proportional to $G_F$. For most
cases studied in the literature, the conversion rate was actually found to be of order $\Delta m^2/2E$
instead, i.e., driven by the frequency $\omega$. Another possible driving frequency is the neutrino-neutrino interaction energy $\mu = \sqrt{2} G_F n_\nu$. The very definition of a “dense” neutrino gas is precisely that $\mu \gg \omega$. However, this dominant scale cancels when the neutrino and antineutrino angle distributions are too similar. On the other hand, with sufficiently different angle distributions the conversion rate can be driven by $\mu \gg \omega$, corresponding to much faster conversion. Moreover, these fast conversions can exist even without any vacuum frequency $\omega$ and thus in the absence of neutrino masses.

In general, therefore, self-induced flavor conversion — in the sense of flavor reshuffling among modes — can occur without flavor mixing, provided there exist fluctuations in flavor space to seed the unstable modes. One may speculate that even quantum fluctuations of the mean-field quantities could suffice as seeds. However, in practice ordinary neutrino oscillations driven by their masses and mixing parameters exist, so disturbances in flavor space to seed self-induced flavor conversion always exist even on the mean-field level.

The main purpose of our paper is to present a few simple examples which illustrate these general points and which are even more basic than those presented by Sawyer. We begin in section 2 with the simplest possible case, two colliding beams of neutrinos and antineutrinos, which shows fast flavor conversion if we allow for inhomogeneities. In section 3 we also construct a homogeneous example, consisting of four modes in the form of two beams intersecting with a nonvanishing angle. We finally turn in section 4 to the example of a spherical source which emits neutrinos and antineutrinos with different zenith-angle distributions in analogy to the schematic supernova model proposed by Sawyer [6]. We conclude with a discussion and outlook in section 5.

2 Colliding beams

The current-current structure of the low-energy neutrino-neutrino interaction implies that we need at least two different propagation directions to obtain any effects at all. Therefore, the simplest possible example is an initially homogeneous gas of neutrinos and antineutrinos, allowing only for two opposing directions of motion, i.e., a system that is one-dimensional in momentum space and that we can view as two colliding beams (figure 1). This type of simple model was recently used by several groups to study the impact of spontaneously breaking various symmetries [19–25].

2.1 Linearized equations of motion

On the refractive level, the interacting neutrino system is best represented in terms of the mean-field $\varrho_i$ for every momentum mode $i$. The diagonal components of this matrix in flavor space are phase-space densities of the different flavor states, whereas the off-diagonal elements represent flavor correlations. Antineutrinos are represented by negative energies and we use the “flavor isospin convention,” where the $\varrho$ matrices for antineutrinos correspond to negative phase-space densities. The advantage is that we do not need to distinguish explicitly between neutrino and antineutrino modes. The $\varrho$ matrices of $N$ modes evolve according to [26]

$$i (\partial_t + \mathbf{v}_i \cdot \nabla) \varrho_i = [H_i, \varrho_i] , \quad \text{where} \quad H_i = \frac{M^2}{2 E_i} + \mu \sum_{j=1}^N (1 - \mathbf{v}_i \cdot \mathbf{v}_j) \varrho_j ,$$  \hspace{1cm} (2.1)

where $M^2$ is the matrix of neutrino mass-squares. We assume neutrinos to be ultra-relativistic so that the velocities $\mathbf{v}_i$ are unit vectors giving the directions of the individual modes. The
We are looking for exponentially growing solutions, i.e., eigenvalues \( \Omega \) with a nonvanishing imaginary part.

In figure 1. Initially homogeneous ensemble of four neutrino modes (“colliding beams” of neutrinos and antineutrinos). The system is taken to be infinite in all directions. The normalized \( \nu \) flux is \( 1 + a \), the \( \bar{\nu} \) flux \( 1 - a \) with the asymmetry parameter \( a \) in the range \( -1 \leq a \leq +1 \). The left-right asymmetry is parametrized by \( b \) such that the upper beam in this figure has normalized strength \( 1 + b \), the lower beam \( 1 - b \) with \( -1 \leq b \leq +1 \). The relation of parameters \( r, \bar{r}, l \) and \( \bar{l} \) to parameters \( a \) and \( b \) can be found in equation (2.11).

The neutrino-neutrino interaction energy is \( \mu = \sqrt{2} G_F n_\nu \) with the effective neutrino density \( n_\nu = \frac{1}{2} (n_\nu + n_{\bar{\nu}} - n_{\bar{\nu}_e} - n_{\bar{\nu}_x}) \). We always consider two-flavor oscillations between \( \nu_e \) and another flavor \( \nu_x \) which is a suitable combination of \( \nu_\mu \) and \( \nu_\tau \). These conventions follow reference [27] and are chosen such that a fixed \( \mu \) corresponds to a fixed density of \( \nu_e \) plus \( \bar{\nu}_e \), even when we modify, for example, their relative abundance. In much of the previous literature, instead either the number of \( \nu_e \) or of \( \bar{\nu}_e \) was held fixed, but we here prefer a more symmetric definition.

In order to identify unstable modes, we consider a linearized version of equation (2.1). We first note that \( \text{Tr} \rho_i \) is conserved by flavor conversion and, if the system was initially homogeneous, it is not modified by the transport term in equation (2.1). It is convenient to define traceless normalized \( g \) matrices in the form

\[
\rho_i - \frac{1}{2} \text{Tr} \rho_i = \frac{g_i}{2} \left( s_i S_i - s_i^* S_i^* \right),
\]

where \( s_i \) is a real and \( S_i \) a complex number with \( s_i^2 + |S_i|^2 = 1 \). Moreover, if neutrinos are initially prepared in \( \nu_e \) or \( \bar{\nu}_e \) eigenstates (our usual example), then initially \( s_i = +1 \). The “spectrum” \( g_i \) gives the actual density of neutrinos in mode \( i \) and is positive for an initial \( \nu_e \) and negative for an initial \( \bar{\nu}_e \), corresponding to our flavor-isospin convention. Our definition of the effective neutrino density \( n_\nu \) corresponds to the normalization \( \sum_{i=1}^N |g_i|^2 = 2 \). To linear order, \( s_i = 1 \) remains constant, whereas the off-diagonal elements evolve according to

\[
i (\partial_t + \mathbf{v}_i \cdot \nabla) S_i = \left[ \omega_i + \mu \sum_{j=1}^N (1 - \mathbf{v}_i \cdot \mathbf{v}_j) g_i \right] S_i - \mu \sum_{j=1}^N (1 - \mathbf{v}_i \cdot \mathbf{v}_j) g_j S_j.
\]

We have assumed a very small vacuum mixing angle and use \( \omega_i = \Delta m^2 / 2 E_i \) with \( \Delta m^2 \) positive and the convention that \( \omega_i \) is positive for neutrinos and negative for antineutrinos.

As a next step, we transform this linear equation of the space-time coordinates \( (t, \mathbf{r}) \) into Fourier space \( (\Omega, \mathbf{k}) \) and we write \( S_i(t, \mathbf{r}) = Q_i(\Omega, \mathbf{k}) e^{-i(\bar{\Omega}t - \mathbf{k} \cdot \mathbf{r})} \). The linearized equations of motion in Fourier space are

\[
\Omega Q_i = \left[ \omega_i + \mathbf{v}_i \cdot \mathbf{k} + \mu \sum_{j=1}^N (1 - \mathbf{v}_i \cdot \mathbf{v}_j) g_j \right] Q_i - \mu \sum_{j=1}^N (1 - \mathbf{v}_i \cdot \mathbf{v}_j) g_j Q_j.
\]

We are looking for exponentially growing solutions, i.e., eigenvalues \( \Omega \) with a nonvanishing imaginary part.
We finally turn to a system which is one-dimensional in momentum space (a beam) so that \( \mathbf{v}_i \to v_i \) and \( \mathbf{k} \to k \). The current-current factors \((1 - \mathbf{v}_i \cdot \mathbf{v}_j)\) are 0 for parallel-moving modes or 2 for opposite moving ones. We consider four modes as in figure 1 and use the vacuum oscillation frequency \(+\omega\) for neutrinos and \(-\omega\) for antineutrinos. We denote the amplitudes \(Q_i\) for the different modes with \(R\) for right-moving neutrinos, \(\bar{R}\) for right-moving antineutrinos, and \(L\) and \(\bar{L}\) for the left movers. Likewise, we denote the mode occupations \(g_i\) with \(r, \bar{r}, l\) and \(\bar{l}\), respectively. Equation (2.4) becomes

\[
\Omega \begin{pmatrix} R \\ L \\ \bar{L} \\ \bar{R} \end{pmatrix} = \begin{pmatrix} \omega + k & 0 & 0 & 0 \\ 0 & -\omega - k & 0 & 0 \\ 0 & 0 & \omega - k & 0 \\ 0 & 0 & 0 & -\omega + k \end{pmatrix} + 2\mu \begin{pmatrix} l + \bar{l} & -\bar{r} & -\bar{l} & 0 \\ -r & 0 & r + \bar{r} & -\bar{r} \\ 0 & -\bar{r} & 0 & l + \bar{l} \end{pmatrix} \begin{pmatrix} R \\ L \\ \bar{L} \\ \bar{R} \end{pmatrix}. \tag{2.5} \]

This is the most general one-dimensional four-mode case and the starting point for the discussion in the rest of this section.

### 2.2 Two modes

The possible existence of unstable modes for \(\omega = 0\) is most easily understood in a yet simpler case consisting only of right-moving neutrinos and left-moving antineutrinos, i.e., \(r = 1 + a\), \(\bar{l} = -(1 - a)\), and \(\bar{r} = l = 0\). The parameter \(-1 \leq a \leq 1\) codifies the neutrino-antineutrino asymmetry of our system. Equation (2.5), reduced to the two occupied modes, becomes

\[
\Omega \begin{pmatrix} R \\ \bar{L} \end{pmatrix} = \begin{pmatrix} \omega + k & 0 \\ 0 & -\omega - k \end{pmatrix} + 2\mu \begin{pmatrix} -1 + a, 1 - a \\ -1 - a, 1 + a \end{pmatrix} \begin{pmatrix} R \\ \bar{L} \end{pmatrix}. \tag{2.6} \]

Without further calculation we can see that the role of the vacuum oscillation frequency \(\omega\) is here played by \(\omega + k\). If we consider vanishing neutrino masses (\(\omega = \Delta m^2 / 2E = 0\)) and a spatial Fourier mode \(k > 0\), the role usually played by \(\omega\) will be taken over by \(k\). The reason for this behavior is that we have constructed our system such that neutrinos (vacuum frequency \(+\omega\)) move right so that the spatial Fourier term \(\mathbf{v} \cdot \mathbf{k}\) enters as \(+k\), and the other way round for the beam of left-moving antineutrinos.

For completeness we provide the explicit eigenvalues for this two-mode case. Using the notation \(\tilde{\omega} = \omega + k\) we find

\[
\Omega = 2a\mu \pm \sqrt{(2a\mu)^2 + \tilde{\omega}(\tilde{\omega} - 4\mu)}. \tag{2.7} \]

For \(\tilde{\omega} = 0\) the eigenvalues are purely real. The eigenfrequencies have an imaginary part for

\[
1 - \sqrt{1 - a^2} < \frac{\tilde{\omega}}{2\mu} < 1 + \sqrt{1 - a^2}. \tag{2.8} \]

Notice that in this system, the solutions for positive or negative \(k\) are different. Because \(\mu\) is defined to be positive, we have unstable solutions only for \(\tilde{\omega} > 0\), i.e., for \(k > -\omega\). The imaginary part has its maximum for \(\mu = \tilde{\omega} / (2a^2)\) and is

\[
\text{Im } \Omega|_{\text{max}} = \tilde{\omega} \sqrt{\frac{1}{a^2} - 1}. \tag{2.9} \]

Therefore, in the homogeneous case (\(k = 0\)), the growth rate is indeed proportional to the vacuum oscillation frequency times a numerical factor which depends on the neutrino-antineutrino asymmetry. Conversely, for vanishing \(\omega\), the maximum growth rate is proportional to the spatial Fourier wave number \(k\).
We can turn this discussion around and ask which Fourier modes $k$ are unstable for a fixed $\mu$ value. The maximum growth rate occurs for $\tilde{\omega} = 2\mu$. The fastest-growing $k$ mode grows with the rate

$$\text{Im} \Omega_{\text{max}} = 2\mu \sqrt{1 - a^2}.$$  
(2.10)

This rate is indeed “fast” in the sense that it is proportional to $\sqrt{2G_Fn_\nu}$. Of course, we assume that $-1 < a < 1$ is not fine-tuned to be very close to $\pm 1$ which would correspond to having only neutrinos or only antineutrinos in the system.

### 2.3 Four modes

The results of the previous section came about because the system was constructed with maximal left-right asymmetry: neutrinos were only right-moving, antineutrinos left-moving. On the other hand, previous one-beam examples [19–25] had been constructed to be left-right symmetric, although spontaneous symmetry breaking of the solution was allowed. In all previous cases, the colliding beams were stable against self-induced flavor conversion in the $\omega = 0$ limit even for nonvanishing $k$. Therefore, the system must be prepared with some amount of left-right asymmetry to be unstable for vanishing $\omega$.

To study this condition, we now turn to a four-mode system with left- and right-moving neutrinos and antineutrinos. As before, we use the parameter $a$ to denote the neutrino-antineutrino asymmetry, and in addition the parameter $-1 \leq b \leq 1$ to denote the left-right asymmetry. Specifically, we use the beam occupations

$$r = \frac{1}{2}(1 + a)(1 + b),$$
(2.11a)

$$\bar{l} = -\frac{1}{2}(1 - a)(1 + b),$$
(2.11b)

$$l = \frac{1}{2}(1 + a)(1 - b),$$
(2.11c)

$$\bar{r} = -\frac{1}{2}(1 - a)(1 - b).$$
(2.11d)

The two-mode example of the previous section corresponds to $b = 1$. The neutrino-neutrino interaction matrix in equation (2.5) becomes

$$
\mu = \begin{pmatrix}
2(a-b) & (1-a)(1+b) & - (1+a)(1-b) & 0 \\
-(1+a)(1+b) & 2(a+b) & 0 & (1-a)(1-b) \\
-(1+a)(1+b) & 0 & 2(a+b) & (1-a)(1-b) \\
0 & (1-a)(1+b) & - (1+a)(1-b) & 2(a-b)
\end{pmatrix}.
$$

(2.12)

The eigenvalue equation is now quartic and the explicit solutions provide little direct insight. However, in several special cases there are simple solutions. In the previous literature, one always used a system which was set up in a left-right symmetric configuration, meaning $b = 0$. Considering the homogeneous case ($k = 0$), one finds the eigenvalues

$$\Omega = a\mu \pm \sqrt{(a\mu)^2 + \omega(\omega - 2\mu)} \quad \text{and} \quad \Omega = 3a\mu \pm \sqrt{(a\mu)^2 + \omega(\omega + 2\mu)}.$$  
(2.13)

The first solution corresponds to the usual “flavor pendulum” for inverted neutrino mass ordering, the second to the left-right symmetry breaking solution for normal ordering as discussed previously [19]. Notice that changing the mass ordering corresponds to $B \rightarrow -B$ in equation (2.1) and thus to $\omega \rightarrow -\omega$ in these expressions.
The corresponding inhomogeneous case ($|k| > 0$) was studied in references [21, 22]. The system is stable for $\omega = 0$ and the growth rate is proportional to $\omega$. Explicit results can be derived in the $k \to \infty$ limit [21].

Turning now to the left-right asymmetric case, one can actually find explicit solutions for $\omega = 0$,

$$\Omega = 2a\mu \pm (k - 2b\mu) \quad \text{and} \quad \Omega = 2a\mu \pm \sqrt{(2a\mu)^2 + k(k - 4b\mu)}.$$  \hfill (2.14)

All solutions are real in the homogeneous case ($k = 0$) and for any $k$ in the left-right symmetric system ($b = 0$). In the general case, the second eigenvalue has an imaginary part if the expression under the square root is negative, which occurs for

$$b - \sqrt{b^2 - a^2} < \frac{k}{2\mu} < b + \sqrt{b^2 - a^2}.$$  \hfill (2.15)

An instability exists only for $a^2 < b^2$, i.e., the left-right asymmetry must exceed the neutrino-antineutrino asymmetry. For fixed $\mu$, the Fourier mode with the largest growth rate is $k = 2b\mu$, growing with a rate

$$\text{Im} \Omega_{\text{max}} = 2\mu \sqrt{b^2 - a^2}.$$  \hfill (2.16)

Again, this growth is of order a typical neutrino-neutrino interaction energy and thus “fast”.

## 3 Intersecting beams

The crucial ingredient to obtain fast flavor conversion appears to be a sufficient difference between the direction distribution of neutrinos and antineutrinos. However, if the momentum distribution is one dimensional we need spatial inhomogeneities. As a next step we construct the simplest homogeneous system ($k = 0$) that shows fast flavor conversion. We consider four modes with directions which intersect at a relative angle $\theta$ as shown in figure 2. We continue to denote the modes as “left- and right-moving” in an obvious sense. The mode occupations are taken to be symmetric between left and right, but we include a neutrino-antineutrino asymmetry $a$, i.e., the mode occupations are taken to be

$$r = l = \frac{1}{2}(1 + a),$$

$$\bar{r} = \bar{l} = -\frac{1}{2}(1 - a).$$  \hfill (3.1a)

As before, we use the normalization $|r| + |l| + |\bar{r}| + |\bar{l}| = 2$. The current-current factors $(1 - \mathbf{v}_i \cdot \mathbf{v}_j)$ are equal to 2 for opposite moving modes, and $1 \pm \cos \theta$ for the other pairs in obvious ways.

The symmetries of this setup suggest to combine the neutrino and antineutrino amplitudes in a symmetric and antisymmetric form, $A_\pm = \frac{1}{2}(L \pm R)$ and $\bar{A}_\pm = \frac{1}{2}(\bar{L} \pm \bar{R})$. Indeed, the linearized equations of motion decouple and we find with $c = \cos \theta$

$$\Omega \begin{pmatrix} A_+ \\ A_+ \end{pmatrix} = \begin{pmatrix} \omega & 0 \\ 0 & -\omega \end{pmatrix} + \frac{\mu(3-c)}{2} \begin{pmatrix} -1 + a & 1 - a \\ -1 - a & 1 + a \end{pmatrix} \begin{pmatrix} A_+ \\ A_+ \end{pmatrix},$$

$$\Omega \begin{pmatrix} A_- \\ A_- \end{pmatrix} = \begin{pmatrix} \omega & 0 \\ 0 & -\omega \end{pmatrix} + \frac{\mu(5+c)}{2} + \frac{\mu}{2} \begin{pmatrix} -(1 - 3c) & -(1 + c)(1 - a) \\ -(1 + c)(1 + a) & 1 - 3c \end{pmatrix} \begin{pmatrix} A_- \\ A_- \end{pmatrix}.$$  \hfill (3.2)
Figure 2. Homogeneous ensemble of four neutrino modes (two beams with relative angle $\theta$.) The system is taken to be infinite in all directions.

Figure 3. Growth rate in units of $\mu$ for neutrino-antineutrino beams intersecting at an angle $\theta$ and a neutrino-antineutrino asymmetry $-1 < a < 1$. The mode occupations are taken to be symmetric between left and right. The analytic expression is given in equation (3.4). A fast growth rate occurs for $\cos \theta > 0$ and it is symmetric between positive and negative $a$.

The first equation again corresponds to the usual flavor pendulum. Indeed, for $c = -1$ we return to the situation of a completely left-right asymmetric system of all neutrinos moving right and all antineutrinos moving left and we reproduce equation (2.6). As observed earlier, we then need a nontrivial spatial variation to obtain fast flavor conversion.

The second case with left-right symmetry breaking, on the other hand, provides nontrivial eigenvalues even for $\omega = 0$ which are found to be

$$\Omega = \frac{a\mu(5 + c)}{2} \pm \frac{\mu}{2} \sqrt{(1 + c)^2 a^2 - 8c(1 - c)}. \quad (3.4)$$

In figure 3 we show the imaginary part as a contour plot in the $\cos \theta$-$a$ plane. A fast growth rate occurs only for $\cos \theta > 0$ and it is symmetric between positive and negative $a$. The absolute maximum obtains for $a = 0$ and $\cos \theta = \frac{1}{2}$ and is found to be $\text{Im}\Omega_{\text{max}} = \mu/\sqrt{2}$.

We have performed a numerical solution of the full nonlinear equations for typical parameters $\cos \theta$ and $a$ in the unstable regime. We find the usual behavior of an inverted oscillator. Given a small perturbation, there is a long phase of exponential growth of the
Figure 4. Zenith-angle distribution for neutrinos (blue) and antineutrinos (orange) implied by the two-bulb supernova emission model.

transverse component, followed by a deep dip of the flavor content of type $\nu_e \bar{\nu}_e \rightarrow \nu_x \bar{\nu}_x$ and back to $\nu_e \bar{\nu}_e$ and so on, similar to the usual flavor pendulum.

4 Two-bulb supernova model

4.1 Setting up the model

As a final example we consider the model proposed by Sawyer [6] which is motivated by typical supernova emission characteristics. Neutrinos are taken to emerge from a spherical surface, the “neutrino bulb,” with a blackbody-like angular characteristic, i.e., isotropically into the outer half space [37]. In this case, a distant observer measures a zenith-angle distribution which is uniform in the variable $\sin^2 \theta$ up to a maximum which is determined by the angular size of the neutrino bulb at the observation point. The species $\nu_e$ and $\bar{\nu}_e$ decouple in different regions. Therefore, as a simple approximation one can assume that they are emitted from neutrino surfaces of different radii, which we call a two-bulb emission model. This setup leads to $\nu_e$ and $\bar{\nu}_e$ zenith-angle distributions of the type illustrated in figure 4. In a supernova, one usually expects the $\nu_e$ flux to exceed that of $\bar{\nu}_e$. However, the recently discovered LESA phenomenon (lepton-emission self-sustained asymmetry) implies that the relative fluxes show a strong hemispheric asymmetry [38]. Moreover, in neutron-star mergers, very different distributions may occur which also depend strongly on direction [39, 40].

The main point of this supernova-motivated setup is the neutrino velocity distribution in the transverse direction. One can formulate this problem in terms of velocities in the transverse plane [21] and it is then very similar to the colliding-beam examples of the previous sections, with different velocity distributions for $\nu_e$ and $\bar{\nu}_e$. Therefore, this case is conceptually quite similar to our previous ones.

We consider a stationary two-flavor neutrino flux and assume stationarity of the solution, i.e., we study the flavor evolution as a function of radius. We ignore small-scale effects in the transverse direction, i.e., the solution is constrained to depend on radius alone. The neutrino radiation field at some observation point beyond the emitting surface is described by the azimuth angle $\varphi$ and the zenith-angle variable $u \propto \sin^2 \theta$. The range of occupied zenith angles is normalized to some chosen reference radius, so the $u$-range does not depend
on the test radius where we perform the stability analysis. The emission spectrum $g(\omega, u)$ has the same meaning as $g_i$ in our earlier sections, except that we use the continuous labels $\omega$ and $u$. We assume axial symmetry of emission so that $g(\omega, u)$ does not depend on $\varphi$.

### 4.2 Eigenvalue equation

We use the eigenvalue equation in the form developed in reference [28] for the case of axially symmetric neutrino emission. As input for the eigenvalue equation we need the integrals

$$I_n = \mu \int d\omega \, du \, u^n g(\omega, u) \left( \omega u \bar{\lambda} - \Omega \right)$$

for $n = 0, 1$ and 2. We have dropped a possible $\varphi$ dependence because we assume axially symmetric emission. The parameter $\bar{\lambda} = \lambda + \epsilon \mu$ describes the effective multi-angle matter effect where $\lambda = \sqrt{2} G_F n_e$ and

$$\epsilon = \int d\omega \, du \, g(\omega, u).$$

In contrast to reference [28] we here normalize the spectrum in the same way as in the previous sections, i.e., $\int du \, d\omega \, \text{sign}(\omega) \, g(\omega, u) = 2$ which also influences the meaning of $\mu$. The only physically relevant quantity is the product $\mu g(u, \omega)$ and it is somewhat arbitrary how to normalize the two quantities separately. The main point is to define a quantity $\mu$ which has the meaning of a typical neutrino-neutrino interaction energy. The eigenvalues $\Omega$ are found as solutions of one of the equations

$$(I_1 - 1)^2 - I_0 I_2 = 0 \quad \text{and} \quad I_1 + 1 = 0.$$  (4.3)

The first equation corresponds to those solutions which respect axial symmetry, whereas the second corresponds to spontaneous axial symmetry breaking.

We look for instabilities in the limit $\omega = 0$. In this case the contributions to $g(\omega, u)$ from emitted $\nu_x$ and $\bar{\nu}_x$ drop out exactly if their emission characteristics are the same. Notice, however, that the $\nu_x$ distribution enters indirectly through the definition of the effective neutrino density $n_\nu$ and the definition of $\mu$ and $\epsilon$. However, for simplicity we assume that no $\nu_x$ or $\bar{\nu}_x$ are emitted. We denote the $\omega$-integrated zenith-angle distributions as $h_{\nu_\ell}(u) = \int_0^\infty d\omega \, g(\omega, u)$ for neutrinos (positive $\omega$) and $h_{\bar{\nu}_\ell}(u) = -\int_0^\infty d\omega \, g(\omega, u)$ for antineutrinos (negative $\omega$). In this notation we have

$$\int du \left[ h_{\nu_\ell}(u) + h_{\bar{\nu}_\ell}(u) \right] = 2 \quad \text{and} \quad \int du \left[ h_{\nu_\ell}(u) - h_{\bar{\nu}_\ell}(u) \right] = \epsilon.$$  (4.4)

After performing the trivial $\omega$ integration, the above integrals are

$$I_n = \int du \, \frac{u^n}{u(\epsilon + m) - w} \left[ h_{\nu_\ell}(u) - h_{\bar{\nu}_\ell}(u) \right],$$

where $w = \Omega/\mu$ is the normalized eigenvalue and $m = \lambda/\mu$ represents the matter effect.

The two-bulb model of neutrino emission implies the top-hat $u$ distributions shown in figure 4. We express the occupied $u$-ranges in terms of a width parameter $-1 < b < +1$ in the form $u_{\nu_\ell} = 1 + b$ and $u_{\bar{\nu}_\ell} = 1 - b$. In the supernova context, the $\nu_\ell$ interact more strongly, thus decouple at a larger distance, and hence correspond to $b > 0$. Moreover, we describe the normalized neutrino densities as $n_{\nu_\ell} = 1 + a$ and $n_{\bar{\nu}_\ell} = 1 - a$ in terms of an “asymmetry parameter” $-1 < a < +1$. In the supernova context, deleptonization implies

\[ \text{...} \]
Figure 5. Growth rate in units of $\mu$ for the axial-symmetry breaking solution of the two-bulb supernova model without matter. The normalized $\nu_e$ density is $1 + a$, for $\bar{\nu}_e$ it is $1 - a$. The $\nu_e$ zenith-angle distribution is nonzero on the range $0 < u < 1 + b$ and on $0 < u < 1 - b$ for $\bar{\nu}_e$. The results show no instability in the SN motivated parameters ($a > 0$ and $b > 0$), i.e., the first quadrant.

An excess of $\nu_e$ over $\bar{\nu}_e$ so that $a > 0$. In other words, the traditional supernova-motivated situation corresponds to the first quadrant $a, b > 0$ in the parameter space of our model. In terms of these parameters, the zenith-angle distributions are

$$h(u) = \frac{1 \pm a}{1 \pm b} \times \begin{cases} 1 & \text{for } 0 \leq u \leq 1 \pm b, \\ 0 & \text{otherwise,} \end{cases}$$

(4.6)

where the upper sign refers to $\nu_e$, the lower sign to $\bar{\nu}_e$. So finally the integrals are

$$I_n = \frac{1 + a}{1 + b} \int_0^{1+b} du \frac{u^n}{u(2a + m) - w} - \frac{1 - a}{1 - b} \int_0^{1-b} du \frac{u^n}{u(2a + m) - w};$$

(4.7)

where we have used $\epsilon = 2a$. These integrals can be performed analytically without problem, but the eigenvalues can be found only numerically.

One special case is $b = 0$ where the zenith-angle distributions for $\nu_e$ and $\bar{\nu}_e$ are the same, yet their number density is different ($a \neq 0$). In this case the integrals are

$$I_n = 2a \int_0^1 du \frac{u^n}{u(2a + m) - w}. $$

(4.8)

Numerically it appears that in this case the eigenvalues are always real, i.e., fast flavor conversion indeed requires the top-hat distributions to have different widths. However, we have not tried to prove this point mathematically.

### 4.3 Solution without matter effect

As a first nontrivial case we ignore matter ($m = 0$) and find that the first case in equation (4.3), the axially symmetric solution, does not show any instabilities in a numerical
scan over the space $-1 < a < 1$ and $-1 < b < 1$. The second equation (axial symmetry breaking) provides solutions and hence allows fast flavor conversion. We show the imaginary part of $w$, i.e., the growth rate in units of $\mu$ as a contour plot in figure 5. The first and third quadrants are stable, i.e., when $a$ and $b$ have the same sign. These results suggest that fast flavor conversion requires that the species $\nu_e$ or $\bar{\nu}_e$ with the broader zenith-angle distribution must have a smaller flux. This particular conclusion appears to be opposite of what Sawyer has found in his recent study [6].

Of course, in accretion disks arising from neutron-star mergers or black hole-neutron star mergers, the flux is dominated by $\bar{\nu}_e$, not $\nu_e$, so that $a < 0$. Also LESA can be another interesting scenario spanning parameters other than the traditional supernova-motivated case. Therefore, the main point is the possible existence of fast flavor conversion if nontrivial zenith-angle distributions are used.

4.4 Including matter

These above results change drastically in the presence of matter. A substantial matter effect is expected when $\lambda$ is at least of order $\mu$, so as a specific example we use $m = \lambda/\mu = 1$ and show the growth rates in figure 6. We find fast growth rates for both the axially symmetric and the axial-symmetry breaking cases. While the latter (bottom panel) is simply a modified version of the matter-free case, we now find run-away solutions even in the axially symmetric case (upper panel). In particular, there are unstable solutions for supernova-motivated parameters, where the $\nu_e$ distribution is the broader one ($b > 0$) and there are more $\nu_e$ than $\bar{\nu}_e$ ($a > 0$), i.e., the first quadrant of our parameter space.

If we had used instead a background of antimatter ($m < 0$), the unstable range would lie in the other half where $b < 0$. For $b = 0$, when the two zenith-angle distributions are the same, no fast instability seems to occur as remarked earlier.

If the matter effect is very large ($\lambda \gg \mu$, corresponding to $m \gg 1$), the axially symmetric solution disappears, so it exists only for some range of matter density. For example, during the supernova accretion phase, this instability would be suppressed in analogy to the “slow” instabilities [41–45]. Of course, we have here only considered the $k = 0$ case as well as stationarity of the solution. Therefore, what all of this means in practice remains to be understood.

4.5 Previous studies

Flavor-dependent angle distributions were previously investigated by Mirizzi and Serpico [29, 30]. These authors have not reported fast flavor conversion in any of their cases. They have used forward-peaked distributions of the form $(1 - u)^{\beta/2}$ where $\beta = 0$ provides a top-hat distribution on the interval $0 < u < 1$ and for $\beta > 0$ a distribution which is more concentrated for smaller $u$-values. However, these authors assumed equal $\beta$ for both $\nu_e$ and $\bar{\nu}_e$ and a different one for $\nu_x$ which was the same for $\bar{\nu}_x$, i.e., they only studied angle differences between the $x$-flavor and the $e$-species. As we remarked earlier, if $\nu_x$ and $\bar{\nu}_x$ have the same distribution, they drop out of the equation in our limit of $\omega = 0$. In other words, the distributions used in references [29, 30] indeed do not spawn fast flavor conversion.

As a cross check we have also considered angle distributions of this form, but taking different $\beta$ for $\nu_e$ and $\bar{\nu}_e$ as well as different abundances. We find fast instabilities which qualitatively agree with our earlier cases of top-hat distributions. Therefore, fast flavor conversion is not an artifact of the top-hat distribution.
Figure 6. Growth rate in units of $\mu$ in analogy to figure 5, but now with matter $m = \lambda/\mu = 1$. Top: axially symmetric solution. Bottom: axial symmetry spontaneously broken.
A stability analysis was also performed by Saviano et al. [31], using realistic energy and zenith-angle distributions taken from a few specific numerical supernova simulations. The growth rates reported in their figure 2 are always of order the vacuum oscillation frequency and thus not fast. The used numerical angle distributions are shown in the lower panels of their figure 1. The $\nu_e$ and $\bar{\nu}_e$ (dotted and solid curves) look visually very similar except for the overall normalization which represent the different fluxes. The matter effect was taken into account, but not the possibility of axial symmetry breaking. However, the fast conversion will remain absent when the ordinary matter effect is significantly larger than the effect of background neutrinos. Indeed, in the examples of reference [31] the matter effect $\lambda$ is almost an order of magnitude larger than $\mu$. Also, probably in these specific models, the angle distributions were not different enough to spawn fast flavor conversion. In a later study [32], these authors analyzed the same models for the axial symmetry breaking case. However, again the matter effects were large and the angle distributions were similar. The lepton asymmetry was supernova inspired, i.e., the first quadrant in figure 5 and bottom panel of figure 6. Unsurprisingly, fast flavor conversion did not show up in this case either.

In summary, while a number of previous studies have considered nontrivial zenith-angle distributions, the chosen examples could not have found fast flavor conversion.

5 Conclusions

We have studied a few simple examples of interacting neutrino systems which show the phenomenon of “fast flavor conversion,” i.e., they have unstable modes in flavor space which grow with rates of order the neutrino-neutrino interaction energy $\mu = \sqrt{2} G_F n_\nu$ instead of the much smaller vacuum oscillation frequency $\omega = \Delta^2 m^2 / 2E$. In these cases, self-induced flavor conversion in the sense of flavor shuffling among modes does not depend on $\Delta m^2$ or the vacuum mixing angle except for providing disturbances as seeds for the run-away modes. In other words, the main conceptual point is that self-induced flavor conversion does not depend on flavor mixing. In the supernova context, neutrino flavor evolution on the refractive level would have had to be considered even if flavor mixing among neutrinos did not exist.

Notice that to lowest order, neutrino-neutrino interactions are of neutral-current type and thus flavor blind. We ignore radiative corrections which introduce a flavor dependence in neutrino-neutrino refraction [33]. In this approximation, the overall flavor content of the ensemble remains conserved by the action of neutrino-neutrino refraction, i.e., self-induced flavor conversion corresponds to flavor reshuffling among modes which however can lead to flavor decoherence if neighboring modes become effectively uncorrelated.

The principle of fast flavor conversion was discovered ten years ago by Ray Sawyer [5] in a three-flavor setup of a small number of modes. He speculated that supernova neutrinos might flavor-equilibrate over very short distances, meters or even centimeters, in their decoupling region. With hindsight it is difficult to understand why the conceptual and practical points raised in this paper were completely lost on the community.

Fast flavor conversion by definition does not depend on the vacuum oscillation frequencies and thus not on neutrino energy. The energy spectrum plays no role, fast flavor conversion is driven by nontrivial angle distributions. In several of our examples, the spontaneous breaking of initial symmetries was also important. However, the crucial condition is that the initial angle distribution must not be too symmetric or too simple, although we cannot provide a general mathematical condition.
In the context of astrophysical applications in supernovae or neutron-star mergers, the main question is if neutrinos emerging from the decoupling region maintain spectral fluxes which strongly depend on species or if self-induced flavor conversion and its interplay with matter effects and vacuum oscillations leads to quick flavor decoherence. The effects of spatial [22–24, 34] and temporal [4, 35, 36] symmetry breaking as well as the possibility of fast flavor conversion [5, 6] have been taken as evidence for quick decoherence. Still, the breaking of spatial homogeneity may be suppressed by the multi-angle matter effect [21], and the breaking of stationarity depends on a narrow resonance condition.

Actually, our stability studies as well as numerical solutions of the full equations in the free-streaming limit may not be appropriate to capture the realistic evolution at or near the neutrino decoupling region of a compact object. In this region, the description of the neutrino mean field in terms of a freely outward streaming neutrino flux is not appropriate: neutrinos flow in all directions, but with different intensity. Even at larger distances, the re-scattered neutrino flux plays an important role [46, 47].

Therefore, the toy examples studied here and in the recent literature leave crucial questions open and do not yet provide clear-cut conclusions concerning realistic flavor evolution in core-collapse or neutron-star merger events. Neutrino flavor evolution in dense media remains a challenging subject where different pieces of the jigsaw puzzle keep showing up, but do not yet form a complete picture.

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