Mathematical model for assessing environmental risk

I V Sukhorukova and N A Chistyakova
Department of Higher Mathematics, Plekhanov Russian University of Economics, Stremyanny lane, 36, Moscow, 117997, Russia

E-mail: suhorukovaira@yandex.ru

Abstract. The article considers the urgent problem of assessing environmental risks. The problem of environmental protection is now becoming global. This is primarily due to the growth of industrial production and large-scale mining. To quantify damage, the concept of environmental insurance is introduced. This means that generally accepted insurance risks are directly applied in the field of environmental protection. In addition to the general formula for a comprehensive risk assessment to predict harmful environmental impacts, it would be natural to take into account the results of these impacts over time. In this study, the authors proposed more detailed mathematical models of risk factors. We got a calculated and analytical expression in which the probability of an insured event by risk factors varies with time. Examples of models for predicting harmful effects in the environment are given. In solving this problem, probabilistic mathematical models were used. The proposed methods and tools allow taking into account possible environmental risk factors at all stages of decision-making. This will allow in practice to avoid the occurrence of adverse economic consequences in the implementation of economic activity. This contributes to the prospects of implementing business strategies for integrating environmental requirements into their activities. Keywords - environmental insurance, environment, mathematical model, economic damage, integrated risk assessment

1. Introduction
The problem of environmental protection is currently becoming increasingly widespread and becoming global. This is primarily due to the constant influence of energy and the growth of industrial production, large-scale extraction of natural resources. The tasks of environmental protection and the problems of environmental risks over the past decade are among the priorities for the leadership of the Russian Federation. At the state level, programs are being adopted to preserve the country's environmental safety. The Federal Law of the Russian Federation “On Environmental Protection” provides for a whole range of measures for economic regulation in the field of environmental protection. Based on the accumulated knowledge and experience of both Russian scientists and foreign researchers [1-4], the use of environmental insurance and environmental audit is proposed as the basis for these measures. Their joint use is aimed at the development of economic regulation in the field of environmental protection. When deciding on environmental insurance, well-known insurance risks should be directly applied in the field of environmental protection. Environmental risks are understood as risks caused by the negative impact of a person on the environment. Of particular danger are man-
made disasters, as a result of which significant damage is caused not only to nature, but also to people, their health and property.

2. **Subject, objectives and method of research**

When deciding on environmental insurance, well-known insurance risks should be directly applied in the field of environmental protection. Environmental risks are understood as risks caused by the negative impact of a person on the environment. Of particular danger are man-made disasters, as a result of which significant damage is caused not only to nature, but also to people, their health and property. The fundamental principles of environmental management that are currently in force are based on administrative management methods, such as issuing licenses, issuing permits for emissions, discharges, waste disposal, determining the maximum amount of resource withdrawal. At the same time, they also contain economic, market mechanisms for environmental management [5-8] Payments for damage caused by pollution were calculated, payments for the development of natural resources were made, and compensation for damage was calculated. Accordingly, environmental insurance and the environmental audit procedure are additional progressive sources of economic regulation of environmental activities. Calculation of environmental insurance tariffs allows you to develop a mechanism for accounting for liability for damage caused as a result of accidental environmental pollution. Promotes the use of reserves of insurance companies to develop preventive measures for the implementation of technological and environmental measures that reduce environmental risk. This fact is beneficial to all participants in the insurance process: insurance company, company, third parties and the environment.

It should be noted that the existing system of eliminating the consequences of pollution prompts the local leadership, farmers, land users to the most intensive development of these territories. This is due to the fact that the allocated budget funds are mainly spent on the restoration of facilities located in high-risk areas, which leads to a further increase in damage [9-12]. In this situation, to reduce the amount of damage, it is necessary to use not only traditional measures. The adoption at the state level of a policy of regulating the economic use of these territories is required. Economic methods of regulation should be combined with a complex of administrative and legal restrictions. Environmental audits and environmental insurance are aimed at implementing measures stipulated by the state program of the Russian Federation “Environmental Protection” for 2012-2020. The proposed methodology will ensure the rights of the state as the owner of natural resources to maintain an acceptable quality of the environment. Promotes conservation and restoration of the natural environment. Ensure rational use and the necessary level of reproduction of natural resources. The joint mechanism of the environmental audit procedure and environmental insurance helps prevent the negative impact of economic and other activities on the environment and helps to eliminate its consequences.

It is known that in the Russian Federation as of 2017, the largest volume of generation of production and consumption waste was accounted for by the type of economic activity “mining” - 93% of the total amount of generated waste. This is due to the fact that the extraction and enrichment of minerals generates the largest amount of waste, the basis of which is overburden. Therefore, when issuing licenses for the operation of hazardous production facilities, including agricultural enterprises on contaminated lands, it is necessary to present a contract of insurance of civil liability risk for causing harm during the operation of a hazardous production facility. In turn, the contract of insurance of civil liability risk for causing harm during the operation of this facility is concluded on the basis of an environmental audit conclusion regarding identification of environmentally hazardous production facilities, as well as a conclusion on environmental risk assessment.

The object of insurance is the environmental risk of civil liability, expressed in the presentation to the policyholder of property claims by individuals and legal entities for compensation for damage from pollution of land, water or air in the territory of the specific insurance contract. Tariff rates for insurance payments may be set as a percentage of the annual turnover of the enterprise. It should be noted that the terms of environmental liability insurance provide for the establishment of limit
amounts for insurance compensation payments (liability limits) and the insurer's own participation in
the payment of losses. Insurance compensation includes compensation for damage caused by damage
or loss of property; the amount of losses associated with the deterioration of living conditions and the
environment; expenses for cleaning up the contaminated area and bringing it into a state that complies
with the standards.

An environmental audit in the environmental management system should be carried out when
developing a plan of preventive measures while reducing environmental risks and assessing damage in
the event of an insured event. Environmental auditing requires an answer to questions about assessing
the likelihood of environmental risk and negative effects, the principles for determining the size of
losses that occur in the national economy, and methods for representative ranking of enterprises
according to their environmental hazard.

3. Results and discussion
The most important issue, the solution of which is relevant to date, is the form of environmental
insurance. To effectively take into account environmental risks, a compulsory insurance procedure for
all potentially hazardous industries is required. At the level of adoption of federal laws, for the first
time, this mandatory form of environmental insurance is mentioned in Article 18 of the Federal Law
“On Environmental Protection”. In the mandatory form of environmental insurance, it is also
necessary to take into account the features of liability insurance in terms of compensation for damage
caused. Since when concluding an environmental insurance contract, it is possible that there is an
injured party in addition to two parties to the process: the insurer and the insured. The mandatory form
of the insurance contract guarantees the assistance to this third injured party in the event of an
insurance event. Due to the mandatory insurance procedure, insurance rates will be calculated to
compensate for the damage caused. When calculating compensation for damage, it is also possible to
take into account the different contributions of each individual risk of the potential hazard. When
solving the problem with the form of environmental insurance, one can rely on the experience of
foreign countries, in particular European countries. Companies carrying out activities with possible
environmental consequences receive a license for a specific type of activity related to potential health
and property risks of third parties. The way to ensure financial guarantees for licenses obtained is of
various types. Most often, in practice, insurance coverage is provided by the reserves of the insurance
company itself, concluding an insurance contract. Also, a fixed amount on the Company's deposit
account may act as a financial guarantee. In certain cases, the solvency of the Company is confirmed
by the bank by providing a targeted loan. The priority remains the mandatory conclusion of an
insurance contract. It is a mandatory form that serves as a fundamental condition for the effective
operation of the Company. The insurance contract should be concluded only with leading insurance
companies guaranteeing insurance coverage in the event of an insurance event. In the mandatory form
of concluding an insurance contract, it is necessary at the legislative level to fix the total compensation
paid to the injured party. It should also be borne in mind that damage can be caused not only to
individuals but also to legal entities. The compensation received upon the occurrence of an insurance
event takes into account the level of damage to property or its total loss. Financial coverage allows you
to compensate for damage caused to air, water, forests, soil. It is also necessary to provide financial
resources to eliminate the consequences of pollution. These are quite significant expenses for the
budget of our country. So far, there is clearly no certainty in the financial support of the environmental
insurance fund. Previously, the state insurance company Gosstrakh functioned in our country. All
mandatory types of insurance, functioning at that time, were financed from the budget of the
Gosstrakh. After the collapse of the USSR, the company Gosstrakh ceased to exist. Therefore, in
modern Russia, the formation of an environmental fund is one of the most important tasks. To assess
the value of this fund, it is necessary to take into account insurance payments from enterprises,
organizations and institutions. When calculating environmental insurance tariffs, the state will receive
certain negative socio-economic consequences. Mandatory payments of enterprises will affect the
increase in the value of their products. This will cause discontent among many people. There are
certain risks associated with filling the state environmental insurance fund. To solve the problem of assessing the necessary funds, it is necessary to calculate the optimal value of insurance coverage. Using mathematical methods allows us to assess the risk of environmental management. For this purpose, the amount of damage is first calculated, associated with adverse economic, social, environmental and economic reasons. The integrated risk [13] assessment currently used is determined by the following formula:

$$R = \sum_{i} p_i \cdot D_i,$$

(1)

where \( p_i \) - probability of occurrence of the \( i \)-th risk factor;
\( D_i \) - damage from the implementation of the \( i \)-th risk factor, rub.

The risk of environmental management from the perspective of insurance can be considered in several sections:
- the risk of the occurrence of a specific phenomenon that is spontaneous in nature, such as floods and floods, etc.;
- the risk in the operation of a particular environmental management facility when unforeseen events are correlated with a specific environmental management facility;
- the risk associated with the likelihood of damage or death of the natural resource object (for example, the risk of the death of valuable ecological systems, loss of soil fertility, etc.).

In addition to the general formula for integrated risk assessment for predicting harmful environmental impacts, it would be natural to take such impacts into account in dynamics. To do this, we can propose more detailed mathematical models of risk factors and use formula (1), in which the probability of occurrence of an insured event by risk factors varies over time. For different risk factors, different probabilistic models of exposure may be natural. Before constructing specific models, we consider [14] the random variable \( T \) - the time before the occurrence of the insurance event by the risk factor and the concept of the survival function that is natural for such a probability model

$$S(t) = P(T > t), \quad t > 0$$

In addition, since the insured event is associated with a point in time, which is characteristic of the fact that before that everything worked in a normal mode, and at that moment an emergency situation arose, it is natural to introduce a function, which we will call a risk intensity function, which depends on the time \( x \),

$$r_x = \lim_{\varepsilon \to 0^+} \frac{1}{\varepsilon} P(T < x + \varepsilon | T > x), \quad x > 0$$

It is easy to obtain a connection between the introduced probabilistic characteristics. On the one hand, we express a risk intensity function across survival function:

$$r_x = \lim_{\varepsilon \to 0^+} \frac{1}{\varepsilon} P(T < x + \varepsilon | T > x) = \lim_{\varepsilon \to 0^+} \frac{1}{\varepsilon} \frac{P(x < T < x + \varepsilon)}{P(T > x)} =$$

$$= \lim_{\varepsilon \to 0^+} \frac{1}{\varepsilon} \frac{S(x) - S(x + \varepsilon)}{S(x)} = -S'(x), \quad x > 0$$

(2)

On the other hand, solving a differential equation (2) for survival function and given that at zero it is equal to unity, we obtain the expression survival function across risk intensity function

$$d \ln S(x) = -r_x \quad \Rightarrow \quad \ln S(x) \bigg|_{0}^{x} = -\int_{0}^{x} r_x \, dx \quad \Rightarrow$$

(3)
\[ \Rightarrow \ln \frac{S(t)}{S(0)} = - \int_0^t r_x dx \quad \Rightarrow S(t) = e^{-\int_0^t r_x dx}, \quad t > 0 \]

The upper bounds for \( x \) and \( t \) in equalities (2) and (3) are not indicated here, since for specific statements of the problems they can be either finite or conditionally infinite. Here are some examples of models for predicting harmful effects in the environment.

The simplest example is when the risk factor acts with constant intensity \( r \). Then from (3) we obtain the forecast for the survival function

\[ S(t) = e^{-\int_0^t r_x dx} = e^{-rt}, \quad t > 0 \]

Another example is when the intensity of risk increases over the interval of influence of the factor \((0, \Delta)\). If it grows by law \( r_x = \frac{1}{\Delta-x}, \quad 0 < x < \Delta \), then the survival function is equal to

\[ S(t) = e^{-\int_0^t r_x dx} = \frac{\Delta-t}{\Delta} = 1 - \frac{t}{\Delta}, \quad 0 < t < \Delta \]

A rather natural model can be an example of risk intensity when equipment wear leads to an exponential increase in risk intensity. In this case

\[ r_x = ce^{ax}, \quad x > 0, \quad a > 0, \quad c > 0, \]

\[ S(t) = e^{-\int_0^t r_x dx} = e^{-\int_0^t ce^{ax} dx} = e^{-\frac{c}{a}(e^{ax} - 1)}, \quad t > 0 \]

Constants are selected based on statistical observations when monitoring the influence of a harmful factor.

Having obtained a survival function for a specific environmental risk model, it is possible to predict the probability of the occurrence of an insured event at any time interval.

4. **Conclusions**

The proposed mathematical model for assessing environmental risk and calculating the amount of damage caused to the environment due to pollution as a result of anthropogenic activities determines the procedure and the procedure for applying environmental audit to assess the amount of harm. The authors of this scientific study have proposed a formula for integrated risk assessment for predicting the dynamics of harmful effects on the environment. More detailed mathematical models of risk factors were developed and the probability of an insured event on risk factors changing over time was calculated. The calculated economic damage in accordance with the calculations can be used to determine the insured amount for civil liability insurance of enterprises and organizations - sources of an increased environmental hazard.

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