Q-Balls and Baryogenesis in the MSSM

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Abstract

We show that Q-balls naturally exist in the Minimal Supersymmetric Standard Model (MSSM) with soft SUSY breaking terms of the minimal N=1 SUGRA type. These are associated with the F- and D-flat directions of the scalar potential once radiative corrections are taken into account. We consider two distinct cases, corresponding to the "$H_u L$" (slepton) direction with L-balls and the "$u^c d^c d^c$" and "$u^c u^c d^c e^c$" (squark) directions with B-balls. The L-ball always has a small charge, typically of the order of 1000, whilst the B-ball can have an arbitrarily large charge, which, when created cosmologically by the collapse of an unstable Affleck-Dine condensate, is likely to be greater than $10^{14}$. The B-balls typically decay at temperatures less than that of the electroweak phase transition, leading to a novel version of Affleck-Dine baryogenesis, in which the B asymmetry comes from Q-ball decay rather than condensate decay. This mechanism can work even in the presence of additional L violating interactions or $B-L$ conservation, which would rule out conventional Affleck-Dine baryogenesis.

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Introduction

Flat directions in the scalar potential of the Minimal Supersymmetric Standard Model (MSSM) [1] have long been understood to have potentially important consequences for cosmology [2, 3, 4]. In particular, a very natural mechanism for the generation of the observed baryon asymmetry in SUSY models is the decay of a charged scalar field condensate (the Affleck-Dine (AD) mechanism [2]). Developments in the cosmology of N=1 supergravity models, in particular the generation of order $H$ contributions to the soft SUSY breaking parameters of scalar fields in the early Universe [5], have sparked a revival of interest in flat directions and the AD mechanism [3]. This mechanism is particularly important as the other natural possibility for baryogenesis in the MSSM, namely electroweak baryogenesis, appears to succeed for only a very limited range of MSSM parameters [6].

A second idea has recently been reconsidered in the context of SUSY models with flat directions in their scalar potentials, namely Q-balls [7-11]. A Q-ball is a stable, charge $Q$ soliton in a scalar field theory with a spontaneously broken global $U(1)$ symmetry [7]. The connection between Q-balls and SUSY models lies in the natural existence of scalars carrying a global $U(1)$ charge, corresponding to $B$ or $L$ for the case of squarks and sleptons. Thus, in principle, one could have B- or L-balls in the MSSM. Whether this happens or not depends on whether the scalar potential of the squarks and sleptons can support a Q-ball solution. If it can, then the next questions are whether Q-balls can be created in the early Universe and whether the resulting Q-balls could be sufficiently long-lived to have interesting consequences for cosmology.

In reference [9] it was shown, in the context of a SUSY model with gauge mediated SUSY breaking, that the scalar potential along a flat direction of the MSSM can support a Q-ball solution. In addition, it was suggested that Q-balls with a very large charge (and so possibly stable) could be formed by the growth of perturbations in a charged condensate of the kind associated with the AD mechanism.

In this letter we will reconsider these ideas in the context of the "classical" MSSM, i.e. the MSSM with soft SUSY breaking terms due to SUSY breaking in a gravitationally coupled hidden sector [1]. We will first show that the scalar potential along the flat "$H_u L$" (slepton) and "$u^c d^c \bar{c}^c$" and "$u^c u^c d^c \bar{e}^c$" (squark) directions can indeed support a Q-ball solution once radiative corrections are taken into account. The form of the potential and the associated Q-balls will be seen to be quite different in the slepton and squark cases. We will then discuss the formation of Q-balls in the early Universe in the context of the simplest cosmological scenario, in which the AD field begins coherently oscillating during the inflaton matter dominated era following an initial period of inflation. Finally, we will discuss the lifetime of the Q-balls and their possible consequences for cosmology, in particular their role in baryogenesis.
2 Q-balls and radiatively corrected flat directions in the MSSM

2.1 Flat directions in the MSSM

For a globally $U(1)$ symmetric complex scalar field $\phi$, the condition for the scalar potential $U(\phi)$ to be able to support a Q-ball solution is that $U(\phi)/|\phi|^2$ should have a global minimum at a non-zero value of $|\phi|$. In this it is assumed that $\phi$ is defined such that $\phi = 0$ corresponds to the vacuum in which the Q-ball is formed, which in practice corresponds to our vacuum with non-zero Higgs expectation values and zero squark and slepton expectation values. We will be interested in whether Q-balls can be formed along flat directions of the MSSM. The flat directions can be defined by the scalar field operators which have non-zero expectation values along the directions in question. These have been classified generally for the case of the MSSM in reference [3]. The most interesting flat directions from the point of view of conventional AD baryogenesis are those with non-zero B-L and unbroken R-parity, which can give a baryon asymmetry once anomalous B+L violation is taken into account without introducing dangerous renormalizable B and L violating operators in the MSSM [1, 12]. These flat directions correspond to the ”$H_u L$” direction, along which the magnitude of the expectation value of $H_u^0$ and $\nu_L$ are equal, and the ”$u^c d^c d^c$” direction, along which $u_c$, $d_c$ and $d'_c$ are non-zero, where the squarks have different colour indices and where $d_c$ and $d'_c$ correspond to orthogonal combinations of down squark generations. (There are also the ”$d^c Q L$” and ”$e^c L L$” directions. However, since we expect these to be phenomenologically similar to the ”$u^c d^c d^c$” direction, we will not consider them seperately here). R-parity conservation allows the d=4 non-renormalizable superpotential term $(H_u L)^2$ and the d=6 term $(u^c d^c d^c)^2$. In addition, we will consider the d=4 B-L conserving $u^c u^c d^c e^c$ direction (there is also the phenomenologically similar $QQQL$ direction). Although this could not produce a B asymmetry via AD condensate decay once anomalous $B+L$ violation is taken into account, we will see that it can generate a baryon asymmetry via Q-ball decays occuring after the electroweak phase transition.

For sufficiently large $|\phi|$ along the D-flat direction the scalar potential is of the form

$$U(\phi) = m_S^2 |\phi|^2 + \frac{\lambda^2 |\phi|^{2(d-1)}}{M_p^{2(d-3)}} + \left(\frac{A_\lambda \phi^d}{M_p^{d-3}} + \text{h.c.}\right),$$

(1)

where $m_S^2$ is the soft SUSY breaking mass squared term and $A_\lambda$ the A-term, and we take the non-renormalizable terms to be suppressed by the Planck scale $M_p$. Thus, if $m_S$ has no $|\phi|$ dependence, then $U(\phi)/|\phi|^2$ will not have a global minimum at non-zero $|\phi|$ and so no stable Q-ball will exist. In order to have such a $|\phi|$ dependence, Kusenko and Shaposhnikov [3] considered the possibility that $m_S^2$ is generated by gauge mediated...
SUSY breaking. In this case, for $|\phi|$ larger than the mass of the messenger quarks, $m_S$ becomes proportional to $|\phi|^{-2}$ and the potential becomes essentially completely flat up to where the non-renormalizable terms become important. However, gauge mediated SUSY breaking is not the most commonly considered form of SUSY breaking. The most common SUSY breaking mechanism is that where soft SUSY breaking terms are generated by SUSY breaking in a hidden sector, not coupled directly to the observable MSSM fields in the Kähler potential of N=1 supergravity [1]. This results in soft SUSY breaking terms which are constant at tree level. However, once radiative corrections are taken into account, they become scale dependent. Indeed, it is widely believed that radiative corrections to the soft SUSY breaking mass squared term of the Higgs scalar giving masses to the up-type quarks, $H_u$, when summed from the GUT or Planck scale using the renormalization group (RG), are responsible for driving its mass squared negative at renormalization scales of the order of the weak scale, so breaking the electroweak symmetry [1, 13]. It is well-known that the effect of radiative corrections on the effective potential as a function of $|\phi|$ can be summed by replacing the tree-level masses and couplings by their value run by the RG equations to a renormalization scale $\mu \approx |\phi|$ [14]. Thus, for sufficiently small values of $|\phi|$, the scalar potential along the D-flat direction will have the form $U(\phi) \approx m_S^2(|\phi|)|\phi|^2$. A necessary condition for this potential to be able to support a Q-ball solution is that $m_S^2(|\phi|)$ decreases over at least some range of $|\phi|$ as $|\phi|$ increases from zero.

### 2.2 $H_uL$ direction

In general we will be considering models with several scalar fields. To correctly describe the Q-balls, which correspond to L-balls in this case, we would have to solve the coupled equations of motion of the scalars to find a Q-ball solution. However, in practice, we will be able to describe the nature of the Q-balls reasonably accurately by considering a single scalar degree of freedom. This can be understood as follows. Along the D-flat direction, for large values of the scalar field compared with the mass scale of the soft SUSY breaking terms (which are typically of the order of 100 GeV), the degree of freedom in the direction orthogonal to the D-flat direction will be much more massive than 100 GeV and so will play no role in the dynamics of the Q-ball. Thus we need only consider the scalar field with non-zero value along the D-flat direction. However, at sufficiently small values of the scalar field, the orthogonal scalar mass becomes of the same order as the soft SUSY breaking masses and so all the scalar fields can play a role in the dynamics of the Q-ball. However, in this limit, all the fields will typically have similar masses and couplings and so we can simply consider one of the scalars to be decoupled from all the others and calculate the Q-ball dynamics for this scalar. This will give the correct order of magnitude for the properties of the Q-ball in this regime.
Let us first consider the potential at large enough values of $|\phi|$ that the $H_uL$ D-flat direction describes the minimum of the potential. In this case the scalar field along the minimum of the potential corresponds to a linear combination of the $H_u$ and $\nu_L$ fields, $\phi = \frac{1}{\sqrt{2}}(H_u^0 + \nu_L)$, with mass $m_\phi^2 = \frac{1}{2}(\mu_H^2 + m_{H_u}^2 + m_{L}^2)$, where $\mu_H$ corresponds to the SUSY $\mu$-term in the superpotential, $\mu_H H_u H_d$. In general, the renormalization group equations for the soft SUSY breaking mass terms will have the form

$$\mu \frac{\partial m_i}{\partial \mu} = \alpha_i m_i^2 - \beta_\alpha M_\alpha^2,$$  \hspace{1cm} (2)

where the $m_i^2$ represent the soft SUSY breaking scalar mass terms as well as the $A$-terms, and $M_\alpha$ are the gaugino masses. The full RG equations are given in reference [1]. The only important $\alpha_i$ terms are those associated with the top quark Yukawa coupling. These appear in the RG equation for $m_{H_u}$ but not for $m_L$. In the absence of a large $\alpha_i$, the masses increase monotonically with decreasing $\mu$. However, with a large $\alpha_i$, for sufficiently small $\mu$, the masses can start to decrease with decreasing $\mu$.

The form of the solutions depends on the initial values of the parameters of the SUSY breaking terms at the initial large scale, which we will take in the following to be the GUT scale $M_X$. However, two features are important for the existence and cosmology of L-balls. First, in general, the value of $m_\phi^2$ becomes negative for scales typically smaller than $10^8$ GeV or so. Second, depending on the parameters, there can be a "hill" in the plot of $m_\phi^2$ versus $|\phi|$, such that $m_\phi^2$ starts decreasing with increasing $|\phi|$ for sufficiently large values of $|\phi|$.

The running of the mass in the $H_uL$-direction as obtained from solving the coupled one-loop RG equations is illustrated in Fig. 1a for different initial values of the SUSY parameters, chosen so that they give radiative breaking of electroweak symmetry at the scale $\mu \simeq M_W$ (we took $\alpha_s(M_W) = 0.11$ and the top Yukawa $h_t(M_W) = 1.05$; no threshold corrections were augmented). We have not made a systematic exploration of the parameter space, but it appears that typically the "hill" in the plot of $m_\phi^2$ versus $|\phi|$ arises for $m_0$ smaller than the gaugino masses or for $A(M_X) < 0$. Examples of both can be seen in Fig. 1a.

The effect of the negative value of $m_\phi^2$ at small enough $|\phi|$ is to generate a minimum of $U(|\phi|)/|\phi|^2$, typically at $|\phi_o| \approx 1$ TeV. To see this, we first note that the definition of the L-ball field $\phi$ will not correspond to the D-flat direction once $H_u^0 \lesssim m_{L}/g$. The $\nu_L$ expectation value vanishes at the minimum of the potential as a function of $H_u^0$ in this case. In this regime the L-ball field may be taken to be $\nu_L$. This will have a positive mass squared, corresponding to our L-conserving vacuum. Thus we see that the effective L-ball field will be characterized by a positive mass squared for $|\phi| \lesssim 1$ TeV and a negative mass squared for $|\phi| \gtrsim 1$ TeV. The rate by which the positive mass switches on is given by the rate by which the Higgs VEV develops. This mass turns positive over a very narrow range in $\phi$. This is illustrated in Fig. 1b. We need not
Figure 1: a) The running of the mass in the $H_uL$ direction (lower set of curves) for the susy parameters (a) $m_0 = 0$, $A(M_X) = 0$, $\mu_H = 2.55$, (b) $m_0 = 0$, $A(M_X) = -3$, $\mu_H = 2.4$ and (c) $m_0 = 1$, $A(M_X) = -1$, $\mu_H = 2.5$, all in units of the common gaugino mass; for comparison, the running of the physical Higgs mass is also shown (upper set of curves). b) The Q-ball potential for the case (a).

be concerned with the details of the potential; the L-ball properties will be essentially determined by this rapid change of the sign of the effective mass squared of the L-ball field, which is responsible for $U(|\phi|)/|\phi|^2$ having a minimum at $|\phi_o| \approx 1$ TeV.

The potential importance of the ”hill” feature in Fig. 1b is that if a condensate in the early Universe starts to oscillate coherently with its initial amplitude in the range of $|\phi|$ where $m_S^2$ is decreasing with increasing $|\phi|$, then, as we will show later, there could, in principle, be a negative pressure associated with the condensate, which would lead to the exponential growth of perturbations in the condensate field and ultimately to the formation of L-balls.

2.3 $u^c d^c d^c$ direction

In this case the D-flat direction essentially describes the B-ball field $\phi$ for all values of $|\phi|$. The corresponding mass term will have the form $m_S^2 \approx \alpha_i m_{u_i}^2 + \beta_i m_{d_i}^2 + \gamma_i m_{d_i'}^2$, where the sum is over different generations $i$ and where $d$ and $d'$ correspond to different colours and orthogonal combinations of down squark generations, in order to have a non-zero value for the operator $\epsilon^{ijk} u^c_i d^c_j d^c_{k'}$, where i, j and k are colour indices. The values of $\alpha_i$, $\beta_i$ and $\gamma_i$ will be determined by the initial conditions following inflation, which will determine the direction in colour and flavour space of the flat direction. On average, in the sense of allowing equal contributions from all squark species, we expect that the effective mass will be of the form $m_S^2 \approx \frac{1}{9}(m_{t^c}^2 + 2m_{u^c}^2 + 6m_{d^c}^2)$ (where we assume
a common mass for first and second generation up squarks and for all down squarks), although it is quite possible to have, for example, a much smaller contribution from $m^2_c$ (which can decrease with decreasing $|\phi|$ for small enough $|\phi|$) for particular initial conditions. We generally expect $m^2_S$ to decrease with increasing $|\phi|$ without any hill feature or change of sign, in complete contrast with the case of the $H_u L$ direction. This leads to a correction to the $\phi$ mass which we can roughly model as a logarithmic correction

$$m^2_S \approx m^2_o \left( 1 + K \log \left( \frac{|\phi|^2}{M_X^2} \right) \right),$$

where $K < 0$ and $M_X$ is a large mass scale at which $m^2_S$ is defined to have the value $m^2_o$. Perturbatively, the value of $K$ due to SUSY breaking gaugino masses will be of the form

$$K \approx - \sum_{\alpha, \text{gauginos}} \frac{\alpha_{\alpha} M^2_{\alpha}}{8\pi m^2_S}$$

which, taking $\alpha_{\alpha} \approx 0.1$ for $SU(3)_c$ and summing over the gauginos gaining a $\phi$ dependent mass from mixing with the $u^c$ and $d^c$ squarks, will typically give $|K|$ in the range 0.01 to 0.1. The minimum of the potential will then be determined by the non-renormalizable terms in the potential. The form of the non-renormalizable terms will depend on the flat direction in question. Since we are considering R-parity to be unbroken, the lowest-order non-renormalizable superpotential term lifting the scalar potential along the $u^c d^c d^c e^c$ direction will be of the form $\frac{\lambda}{M_p} (u^c d^c d^c)$. Thus the full scalar potential in this direction will have the form

$$U(\phi) \approx m^2_o \left( 1 + K \log \left( \frac{|\phi|^2}{M_X^2} \right) \right) |\phi|^2 + \frac{\lambda^2}{M_p^6} |\phi|^6.$$  (5)

In what follows we will always take $\lambda \approx 1$. Thus the value of $|\phi|$ at the minimum of $U(|\phi|)/|\phi|^2$ is given by $|\phi|_o \approx (|K|m^2_S M_{Pl}^6)^{1/8} \approx 5 \times 10^{14} |K|^{1/8} \text{ GeV for } m_S \approx 100 \text{ GeV}.$

### 2.4 $u^c u^c d^c d^c e^c$ direction

This will be similar to the $u^c d^c d^c e^c$ direction, except that the non-renormalizable terms will correspond to the d=4 superpotential term $\frac{\lambda}{M_p} u^c u^c d^c e^c$. Thus the scalar potential in this case will have the form

$$U(\phi) \approx m^2_o \left( 1 + K \log \left( \frac{|\phi|^2}{M_X^2} \right) \right) |\phi|^2 + \frac{\lambda^2}{M_p^2} |\phi|^6.$$  (6)

The value of $|\phi|$ at the minimum of $U(|\phi|)/|\phi|^2$ is given by $|\phi|_o \approx (|K|m^2_S M_{Pl}^2)^{1/4} \approx 3 \times 10^{10} |K|^{1/4} \text{ GeV for } m_S \approx 100 \text{ GeV}.$
3 Q-ball solutions

3.1 Q-ball equation of motion

From the point of view of cosmology and phenomenology, the important quantities are
the energy and radius of the Q-ball as a function of its charge $Q$. The Q-ball solution
is of the form

$$\phi = \frac{\phi(r)}{\sqrt{2}} e^{i\omega t}.$$  (7)

From now on $\phi$ will refer to $\phi(r)$, which is real and positive. The energy and charge
of the Q-ball are then given by

$$E = \int d^3x \left[ \frac{1}{2} \left( \frac{\partial \phi}{\partial r} \right)^2 + U(\phi) \right] + \frac{1}{2} \omega Q$$  (8)

and

$$Q = \int d^3x \omega \phi^2.$$  (9)

The equation of motion for a Q-ball of a fixed value of $\omega$ is given by

$$\phi'' + \frac{2}{r} \phi' = \frac{\partial U(\phi)}{\partial \phi} - \omega^2 \phi$$  (10)

where $\phi' = d\phi/dr$. We require a solution such that $\phi(0) = \phi'(0) = 0$ and $\phi \to 0$ as
$r \to \infty$. This corresponds to a tunnelling solution for the potential $-U(\phi)$, where

$$U(\phi) = U(\phi) - \frac{\omega^2}{2} \phi^2.$$  (11)

In practice we vary $\phi(0)$ with these boundary conditions until the correct form of
solution is obtained for a given $\omega$. The energy and charge of the solution are then
calculated using the above expressions.

3.2 $H_uL$ direction

The scalar potential which describes the L-ball solution for the $H_uL$ direction will be
that due to the rapid change in the sign of the mass squared term of the effective
L-ball field at $|\phi| \lesssim 1$ TeV. We will model this by

$$U(\phi) \approx \frac{m^2}{2} (2e^{-s\phi} - 1)\phi^2,$$  (12)

where $s \approx 1 TeV^{-1}$. It is straightforward to show (and confirmed by numerical solution
of the Q-ball equation, Eq. (7)) that the L-balls in this case will correspond to thick-
walled L-balls, with radius $r_o \approx m^{-1}$ and charge $L \lesssim (sm)^{-2}$, which, for typical value
of $s$ and $m$ will not be much larger that $10^3$. This can be understood by noting that the effective tunnelling potential in this case is given by

$$-\mathcal{U}(\phi) = \left(\frac{\omega^2}{2} + \frac{m^2}{2} - m^2 \exp(-s\phi)\right)\phi^2.$$  \hfill (13)

Thus as $\phi \to 0$, this tends to $\frac{1}{2}(\omega^2 - m^2)\phi^2$. A tunnelling solution can only exist if $w^2 \leq m^2$. In addition, the largest value of $\phi$ cannot be much larger than $s^{-1}$. From this we can see that the largest charge of a L-ball of volume $V$ has an upper bound $L \lesssim ms^{-2}V$. From the equation of motion it is easy to see that any solution of the L-ball equation will tend to zero once $r \gtrsim m^{-1}$. Thus $L \lesssim (sm)^{-2}$ in general. Therefore the L-balls have a maximum charge and an essentially fixed radius in this case, with the scale set by the mass scale of the soft SUSY breaking terms. This is in contrast with the case of Q-balls which can have a thin-walled solution, for which the Q-ball charge is proportional to its volume for sufficiently large charge \[.\]

### 3.3 $u^c d^c d^c$ and $u^c u^c d^c e^c$ directions

In these cases we consider the solution of the Q-ball equation for the potentials of Eq. (3) and Eq. (6). We find that the B-balls can have a thin-wall solution for sufficiently large charge, with a wall thickness of the order of $\omega_o^{-1}$. This can be understood as follows. For the case of the $u^c d^c d^c$ potential, the equation for the B-ball is given by

$$\phi'' + \frac{2}{r} \phi' = -\omega_o^2 \phi + m_o^2 \phi K \log \left(\frac{\phi^2}{\phi_c^2}\right) + \left(\frac{10\lambda^2}{32}\right) \frac{\phi^9}{M_p^6},$$  \hfill (14)

where $\omega_o$ is defined by

$$\omega_o^2 = \omega^2 - m_o^2 \left[1 + K \left(1 + \log \left(\frac{\phi_c^2}{M_X^2}\right)\right)\right]$$  \hfill (15)

and $\phi_c$ is the value of $\phi$ for which the right-hand side of Eq. (14) vanishes. If the initial value of $\phi$ is equal to $\phi_c$, then $\phi$ will remain constant for all $r$. However, if $\phi$ is slightly smaller than $\phi_c$, then $\phi$ will decrease slowly with increasing $r$ until the change $\delta \phi$ is of the order of $\phi_c$, after which $\phi$ will quickly decrease to zero as $r$ changes by $\delta r \approx \omega_o^{-1}$ (this gives the thickness of the wall of the B-ball). The value of $r$ at which $\phi$ begins to rapidly decrease, $r_c$, may be estimated by perturbing the B-ball equation around $\phi_c$. We find that

$$r_c \approx \frac{1}{2} \omega_o \log \left(\frac{\phi_c}{\delta \phi(0)}\right).$$  \hfill (16)

Thus, for sufficiently small $\delta \phi(0)$, the B-ball can be made as large as we wish. This corresponds to the thin-wall Q-ball property that the volume of the Q-ball is proportional to its charge. The value of $\omega_o$ is determined by the solution of the B-ball equation for
a given B. From solving numerically for the B-ball we find that \( \omega_o \) is approximately \( 4|K|^{1/2}m_o \) for \( |K| \lesssim 0.1 \). Therefore, so long as the charge of the B-ball is sufficiently large that its radius in the thin-wall limit, Eq. (16), is large compared with \( \omega_o^{-1} \), we can use the thin-wall expressions. These are given by [7]

\[
\frac{E}{Q} = \left( \frac{2U(\phi_o)}{\phi_o^2} \right)^{1/2} \\
V = \frac{Q}{(2\phi_o^2 U(\phi_o))^{1/2}},
\]

and

where \( \phi_o \) is the value of \( \phi \) at the minimum of \( U(\phi)/\phi^2 \). If the charge of the B-ball is such that the radius from equation Eq. (16) is not larger than \( \omega_o^{-1} \), then the B-ball will be thick-walled. In this case, from numerical solutions of the B-ball equation, we find that as B decreases its radius stays more or less constant at roughly \( 4/\omega_o \), but that the value of \( \phi \) inside the B-ball decreases. This is as we would expect, since the width of a bubble wall is purely determined by the mass of the associated scalar and we are effectively solving for a bubble here. The energy per unit charge in the thick-walled case is found numerically to be larger than in the thin-walled case. (In both cases it is not very much smaller than the free \( \phi \) mass, in contrast with the case of gauge-mediated SUSY breaking where it is proportional to \( Q^{-1/4} \) [9]). We may estimate the charge \( Q_c \) at which a Q-ball becomes thin-walled by using the thin-wall expression for the Q-ball volume as a function of its charge, equation Eq. (18), with the radius equal to \( (|K|^{1/2}m_o)^{-1} \). For the d=6 u\( \phi \) potential Eq. (3), taking \( m_o \approx 100 \text{ GeV} \), we find that \( Q_c \approx 10^{26}|K|^{-5/4} \), whilst for the d=4 u\( \phi \) potential, Eq. (3), we find that \( Q_c \approx 10^{17}|K|^{-1} \).

The fact that the B-balls in these cases can be thin-walled is important, as it implies that there is no upper bound on the B-ball charge. This allows for the possibility of large charge and so relatively stable B-balls, which may have significant consequences for cosmology.

4 Q-Ball Cosmology

4.1 AD baryogenesis and Q-ball formation

In general, we expect that the soft SUSY breaking mass squared term will receive an order \( H^2 \) correction, coming from non-minimal kinetic terms in the supergravity Kähler potential [3]. Such terms would be expected in realistic unified theories such as string-type theories. The resulting potential for the complex AD field is then of the
form
\[ U(\phi) \approx (m_S^2 - c_S H^2)|\phi|^2 + \frac{\lambda^2|\phi|^{2(d-1)}}{M_p^{2(d-3)}} + \left(\frac{A_\lambda \phi^d}{M_p^{d-3}} + h.c.\right), \quad (19) \]

where \( A_\lambda = A_{\lambda o} + a_\lambda H \) is the corrected A-term and \( c_S \) and \( a_\lambda \) are typically of the order of 1. In this we are choosing the \( H^2 \) correction to have a negative sign in order to have condensate formation and AD baryogenesis.

The minimal cosmological scenario for AD baryogenesis begins with a period of inflation, during which the Hubble parameter \( H \) has a value \( H_I \approx 10^{14} \) GeV, in accordance with COBE perturbations [14]. The inflaton subsequently oscillates coherently about the minimum of its potential until it completely decays and reheats the Universe to a temperature \( T_R \), which should be less than around \( 10^9 \) GeV in order not to thermally regenerate too many gravitinos [17]. The mass squared term of the AD scalar becomes positive at \( H \approx m_S \), at which time it begins to oscillate and the B or L asymmetry is formed by the influence of the phase dependent A-terms at this time. Throughout this period the Universe is matter-dominated by the energy of the coherently oscillating inflaton. For completeness, let us note that during inflaton oscillation domination there will be a thermal background of particles, with temperature \( T \approx 0.4(M_p H T_R^2)^{1/4} \), coming from inflaton decays [3].

The basic picture for the formation of Q-balls within this scenario is as follows. As first observed by Kusenko and Shaposhnikov [9], the AD condensate can be unstable with respect to growth of space-dependent perturbations of the condensate field. This is due to the fact that when the potential can lead to the formation of Q-balls, it must be increasing less rapidly than \( \phi^2 \) for some range of \( \phi \). If the condensate starts oscillating with an initial amplitude lying within this range of \( \phi \), then, on averaging over the coherent oscillations, it will behave as matter with a negative pressure. This negative pressure will cause any small perturbations to grow exponentially and go non-linear. It is assumed that the further collapse of the condensate in the non-linear regime will lead to the formation of Q-balls, once the non-zero charge (baryon or lepton asymmetry) of the condensate comes to stabilize the collapsing condensate field [4]. This leaves the question of the origin of the initial "seed" perturbations. Since the AD field should be out of thermal equilibrium in order to avoid thermalizing the condensate before the baryon or lepton asymmetry can be created, the only source of initial perturbations in this scenario are quantum fluctuations of the AD field created during inflation.

Thus, in order to discuss the length scale and charge of the regions of condensate which first go non-linear and collapse to produce Q-balls, we must first find the spectrum of perturbations of the condensate field at \( H \approx m_S \) due to quantum fluctuations and then consider the growth of perturbations of the condensate once \( \phi \) begins to oscillate coherently at \( H \approx m_S \).
4.2 Seed perturbations from quantum fluctuations during inflation

Let $\lambda$ be the length scale of a perturbation at $H \approx m_S$, when the AD condensate begins to oscillate coherently. Let $\lambda_e$ be the corresponding length scale at the end of inflation. Since the Universe is matter dominated after inflation, we have

$$\frac{\lambda}{\lambda_e} = \left(\frac{H_I}{m_S}\right)^{2/3}. \tag{20}$$

The corresponding length scale at $\Delta N_e$ e-folds before the end of inflation will be

$$\lambda_{\Delta N_e} = \lambda_e e^{-\Delta N_e}. \tag{21}$$

This length scale will leave the horizon during inflation when $\lambda_{\Delta N_e} \approx H_I^{-1}$. Quantum perturbations of the A-D field become classical on leaving the horizon, with the magnitude of the perturbation, treating $\phi$ as a massless scalar, being given by

$$\delta \phi \approx \frac{H_I}{2\pi}. \tag{22}$$

Here $\delta \phi$ refers to a perturbation about the minimum of the $\phi$ potential with $H > m_S$, when $\phi$ has a negative mass squared term of the order of $H^2$. Thus the physical $\phi$ scalar will have a mass of the order of $H$. Although this is not really an effectively massless scalar, which would require a mass small compared with $H$, it is not very massive relative to $H$ and so we expect the massless result to be roughly correct.

Once the perturbation is larger than the horizon it will appear to be a homogeneous perturbation on sub-horizon scales and so will coherently oscillate about the minimum of its potential and red-shift like matter, $\delta \phi \propto a^{-3/2}$. This assumes that the mass is large compared with $H$, so that we can ignore the $3H\dot{\phi}$ damping term in the $\phi$ equation of motion. Again this is not strongly satisfied, but we expect that it will roughly give the correct behaviour. Thus at the end of inflation the spectrum of perturbations will be given by $\delta \phi_e \approx e^{-3\Delta N_e(\lambda)/2} H_I/(2\pi)$, where $\Delta N_e(\lambda)$ refers to the number of e-folds before the end of inflation at which the perturbation of length scale $\lambda$ first leaves the horizon. Suppose the perturbation re-enters the horizon at $a\lambda$. Once back inside the horizon, the perturbation will be space-dependent. Its subsequent evolution will depend on whether its energy density is dominated by the gradient or potential energy. The $\delta \phi$ equation of motion is

$$\delta \ddot{\phi} + 3H\dot{\phi} - \nabla^2 \delta \phi = -V'(\delta \phi), \tag{23}$$

where $V(\delta \phi) \approx H^2\delta \phi^2$. The gradient energy will dominate if $k^2 \gtrsim H^2$, where $\delta \phi \propto e^{i k \cdot x}$. This will obviously be satisfied once the perturbation enters the horizon. Since,
as the Universe expands, \( k^2 \) is proportional to \( a^{-2} \) while \( H^2 \) is proportional to \( a^{-3} \), in general the perturbations will be gradient energy dominated once they re-enter the horizon. Therefore \( \delta \phi \propto a^{-1} \). Putting all this together, we find that the spectrum of the sub-horizon sized perturbations at \( H \approx m_S \) due to quantum fluctuations during inflation is given by

\[
\delta \phi(\lambda) \approx \frac{1}{2\pi m_S H_1^{1/2} \lambda^{5/2}}. \tag{24}
\]

4.3 Negative pressure and Q-Ball formation

The growth of these perturbations once the AD field begins to oscillate coherently can be understood by considering the case of B-balls and the squark potential for values of \( \phi \) somewhat smaller than the value of \( \phi \) at the minimum of the potential (this describes the potential of the oscillating field once \( H \) is less than \( m_S \)),

\[
U(\phi) \approx \frac{m_o^2}{2} \left( 1 + K \log \left( \frac{\phi^2}{M_X^2} \right) \right) \phi^2, \tag{25}
\]

where we typically expect \( |K| \approx 0.01 - 0.1 \). For \( |K| \) small compared with 1, this has the form \( U(\phi) \propto \phi^{2+2K} \). As discussed in reference [13], the equation of state of matter corresponding to a coherently oscillating scalar in a potential of the form \( \phi^n \) is given by

\[
p = (\gamma - 1) \rho ; \quad \gamma = \frac{2n}{n + 2}. \tag{26}
\]

Thus for a scalar oscillating in the potential given by Eq. (25) the equation of state is given by [19]

\[
p = \frac{K}{2} \rho, \tag{27}
\]

which gives a negative pressure if \( K < 0 \). Perturbations in the field will grow according to

\[
\ddot{\delta}_k = -\frac{K k^2}{2} \delta_k, \tag{28}
\]

where the perturbation of the density of the AD field on the length scale \( \lambda = 2\pi/|k| \) is proportional to \( \delta_k = \delta \rho_k / \rho \). With \( \delta \rho \propto 2\phi \delta \phi \), this implies that

\[
\delta \ddot{\phi}_k = -\frac{K k^2}{2} \delta \phi. \tag{29}
\]

Thus the quantum fluctuations of the \( \phi \) field on the scale \( |k| = 2\pi / \lambda \) will grow exponentially with time according to

\[
\delta \phi_k = \delta \phi_{i_k} \exp \left( \left( \frac{|K| k^2}{2} \right)^{1/2} t \right), \tag{30}
\]
where \( t = 0 \) corresponds to the beginning of the coherent oscillations and \( \phi_i \) and \( \delta \phi_i \) are the values of the field and its perturbation respectively at \( H \approx m_S \). In this we have neglected the expansion of the Universe for simplicity. This is justified as we need only establish the value of \( H \) at which the perturbations go non-linear in an expansion time \( H^{-1} \). Since \( \delta \phi_i \) and \( \phi_i \) scale the same way due the expansion of the Universe, we can calculate their ratio at \( H \approx m_S \).

How can B-balls form as a result of this negative pressure? The naive picture is that a perturbation on some scale \( \lambda \) will go non-linear once \( t \gtrsim m_S^{-1} \) (which is the smallest time scale on which B-balls can form and on which we can average over coherent oscillations to obtain a negative pressure), causing the AD condensate to collapse into fragments of size \( \lambda \), trapping some charge \( B \), which will then form into B-balls with a charge of the order of \( B \) (depending on the efficiency of B-ball formation in the non-linearly collapsing fragments). However, by considering only the effect of negative pressure we are neglecting the effect of the charge \( B \). The charge of the B-ball serves to prevent the soliton from collapsing to a radius smaller than the B-ball radius. Therefore we do not expect perturbations to be able to grow on length scales smaller than the B-ball radius once charge is taken into account. We can use the negative pressure argument once the length scale going non-linear at a time \( t \) is larger than the final B-ball radius. We would then expect the B-balls to form quite efficiently when these B-ball size perturbations go non-linear. The charge of the B-ball will be given by the charge contained in a volume of radius of the order of the B-ball radius at this time.

The time at which a perturbation of scale \( \lambda \) goes non-linear is given by

\[
t \approx \frac{\alpha_k}{2\pi} \left( \frac{2}{|K|} \right)^{1/2} \lambda ,
\]

(31)

where

\[
\alpha_k = \log \left( \frac{\phi_i}{\delta \phi_{i,k}} \right).
\]

(32)

We find that \( \alpha_k \approx 34 (44) \) for the \( d=4 \) (\( d=6 \)) directions. In practice, the B-balls will typically turn out to be thick-walled, with radius \( r_o \approx (|K|^{1/2} m_S)^{-1} \). Perturbations on this scale, which have the largest possible growth in a time \( H^{-1} \), will go non-linear at

\[
t \approx \frac{10}{|K| m_S} ,
\]

(33)

corresponding to \( H \approx 0.1 |K| m_S \). In order to find the charge of the resulting B-balls we must find the charge density of the Universe at this time. We will assume that the charge asymmetry corresponds to the presently observed baryon asymmetry, \( \eta_B \approx 10^{-10} \). Then the charge asymmetry at a given value of \( H \) during the inflaton...
oscillation dominated era prior to reheating is given by

\[ n_B \approx \left( \frac{n_B}{2\pi} \right) \frac{H^2 M_p^2}{T_R}. \]  

(34)

Thus we find that the charge of the region with radius \( r_o \approx (|K|^{1/2} m_S)^{-1} \) is given by

\[ B \approx 10^{15} |K|^{1/2} \left( \frac{\eta_B}{10^{-10}} \right) \left( \frac{10^9 \text{ GeV}}{T_R} \right) \left( \frac{100 \text{ GeV}}{m_S} \right). \]  

(35)

Thus, with \( |K| \gtrsim 0.01 \), we see that B-balls of charge larger than \( 10^{14} \) are likely to be formed, depending on the reheating temperature and the efficiency with which this charge is trapped within the B-balls.

For the case of L-balls, we have seen that L-balls have a maximum charge of the order of \( 10^3 \) and field strength of the order of 1 TeV, which is much smaller than the initial amplitude of the d=4 AD field at \( H \approx m_S \). This suggests that L-balls cannot be formed by the collapse of an unstable condensate at \( H \approx m_S \) and that AD baryogenesis along the \( H_uL \) direction will be essentially unaltered from the conventional scenario.

### 4.4 Cosmological consequences of Q-balls in the MSSM

Given that B-balls with a large charge will naturally form in the AD scenario along the squark directions, what might be their consequences for cosmology? This is crucially dependent upon the temperature at which the B-balls decay. If they decay after the electroweak phase transition has occurred at \( T_{ew} \), then we will have the possibility of a new version of AD baryogenesis, in which the baryon asymmetry originates from the decay of the B-balls rather than the decay of the condensate. For example, if we were to consider L violating interactions which are in thermal equilibrium together with anomalous B+L violation at \( T \gtrsim T_{ew} \) (as may be expected in extensions of the MSSM with light Majorana neutrinos via the see-saw mechanism), then any B asymmetry coming from thermalization or decay of the AD condensate would be subsequently washed out [20]. However, the B contained in the B balls will not be erased, at least for some range of reheating temperatures, due to the fact that the large field inside the B-ball prevents thermal particles from penetrating beyond the surface of the B-ball, thus suppressing the thermalization rate [4]. To see this, we will give a conservative upper bound on the reheating temperature from requiring that the B-balls are not thermalized. Thermal particles will penetrate into the B-ball down to a stopping radius \( r_{st} \), corresponding to \( g\phi_{st} \approx T \), where \( \phi_{st} = \phi(r_{st}) \) and where the thermal particles gain a mass \( g\phi \) from interacting with the B-ball. Let us also assume that the thermal particles reflect from the ”hard” region of the B-ball corresponding to \( r < r_{st} \). Then there will be a flux of thermal particles flowing in and out of the ”soft” part of the B-ball at \( r > r_{st} \). The most effective way possible for this flux to
remove charge from the B-ball would be via B absorbing inverse decay processes with a maximum possible rate \( \Gamma_{inv.d} = k_d T \), where \( k_d \approx 10^{-2} \) for strong interactions [21]. The true inverse decay rate can be significantly smaller, depending on the details of the model. For large enough \( r \), the thick-walled B-ball will be roughly described by a Gaussian profile, \( \phi(r) \approx \phi_0 e^{-r^2/r_0^2} \). Therefore the stopping radius will be given by \( r_{st} \approx r_o \log^{1/2}(\phi_o/\phi_{st}) \). Thus, since the charge density at radius \( r \) is \( \omega \phi^2(r) \), which will be approximately constant over a change in radius \( \delta r \approx r_0^2/(2r_{st}) \), we find that the maximum rate of loss of charge by the B-ball via inverse decays is approximately given by

\[
\frac{1}{B} \frac{dB}{dt} \approx -\frac{3}{2} \frac{\phi_{st}^2}{\phi_0^2} \frac{r_{st}^2}{r_0} k_d T ,
\]

where \( r_o \approx (|K|^{1/2}m_S)^{-1} \) is the radius within which most of the Q-ball charge is concentrated. The condition for the Q-ball to survive thermalization is then that \( \dot{B}/B < H / \dot{\phi} \). This gives an upper bound on the reheating temperature

\[
T_R \lesssim \frac{2g^2 k_T \phi_o^2}{3k_d M_p \log^{1/2} \left( \frac{g_{eff}}{T} \right)} .
\]

For thick-walled B-balls, the value of the scalar field at the centre of the B-ball, \( \phi_o \), is given by \( \phi_o \approx 0.5 |K|^{3/4} m B^{1/2} \), where \( B \) is given by Eq. (33). Thus, with \( g \approx 1 \) and \( k_d \approx 10^{-2} \) for strong interactions and with \( m \approx 100 \text{GeV} \), we obtain an upper bound \( T_R \lesssim 5 \times 10^5 |K| \text{GeV} \). Thus, although a non-trivial upper bound may exist, even with relatively conservative assumptions there is still a range of reheating temperatures for which the B-balls can survive thermalization. Clearly a much more detailed analysis of thermalization is required to give the true upper bound on \( T_R \), which could easily be weaker than our conservative estimate. (We have checked that destruction of the B-ball by direct collisions of thermal particles with the hard region is less important than thermalization of the soft region [23]).

From this we may conclude that, at least for some range of \( T_R \), a B asymmetry can be preserved in the presence of rapid L violating interactions so long as the B-balls decay at \( T_d < T_{ew} \). Another interesting possibility is connected with the d=4 B-L conserving directions \( u^c \overline{u} d^c \overline{e} \) and \( QQQL \). These cannot produce a B asymmetry in conventional AD baryogenesis, since the condensate will be thermalized at temperatures large compared with \( T_{ew} \) and so anomalous B+L violation will wash out the B asymmetry. However, if the Q-balls, which in this case are charged under \( U(1)_{B+L} \), decay after the electroweak phase transition, then a non-zero B asymmetry will result, together with an equal L asymmetry. In addition, even in those cases where conventional AD baryogenesis can, in principle, account for the observed baryon asymmetry, the B-ball decay mechanism might replace the conventional mechanism if, after the collapse of the AD condensate, most of the asymmetry were trapped in the B-balls.
The Q-ball decay rate to light fermions is proportional to the area of the Q-ball. It has been estimated to have an upper bound, which is likely to be saturated for Q-balls with \( \phi_o \) much larger than \( m_o \), given by [8]

\[
\frac{dQ}{dt} = -\frac{\omega^3 A}{192\pi^2},
\]

(38)

where \( A \) is the area of the Q-ball and where \( \omega \approx m_o \) for \( |K| \) small compared with 1. The decay rate will depend on whether the Q-ball is thin or thick-walled. For thin-walled Q-balls the area of the Q-ball is related to the charge by

\[
A = \left(\frac{36\pi}{k^{2/3}}\right)^{1/3}Q^{2/3},
\]

(39)

where \( k = (2\phi_o^2U(\phi_o))^{1/2} \). Thus the lifetime of the Q-ball in this case is given by

\[
\tau = 144\pi \left(\frac{4\pi}{3}\right)^{2/3} k^{2/3} Q^{1/3} \omega^{-3},
\]

(40)

For the thick-walled case, the area of the Q-ball is independent of its charge, being fixed by its radius \( r_o \approx (|K|^{1/2}m_o)^{-1} \). The Q-ball lifetime in this case is then given by

\[
\tau = \frac{48\pi Q}{r_o^2 \omega^3},
\]

(41)

For charges less than of the order of \( 10^{26}|K|^{-5/4} \) for the \( u^c d^c d^c \) direction and \( 10^{17}|K|^{-1} \) for the \( u^c u^c d^c e^c \) direction, the Q-balls will be thick-walled. The temperature at which the Q-balls decay is given by, assuming radiation domination,

\[
T_d = \left(\frac{1}{k_T}\right)^{1/2} \left(\frac{M_p}{2\tau}\right)^{1/2},
\]

(42)

where \( k_T = (\frac{4\pi^3 g(T)}{45})^{1/2} \). Thus, assuming \( \omega \approx m_o \approx 100 \text{ GeV} \), thick-walled Q-balls will decay at

\[
T_d \approx 15 \left(\frac{\omega}{100 \text{ GeV}}\right)^{1/2} \left(\frac{10^{15}}{Q}\right)^{1/2} \text{ GeV},
\]

(43)

using \( k_T \approx 17 \). Thus we see that the Q-balls will decay at a temperature less than 100 GeV if \( Q \gtrsim 2 \times 10^{13}|K|^{-1} \). For thin-walled Q-balls, the right-hand side of equation Eq. (42) has an additional factor \( (Q/Q_c)^{1/3} \), where \( Q_c \) is the value of \( Q \) at which the thin-wall limit becomes valid. Q-ball decay is summarized in Fig. 2. Since we have shown that it is likely that the B-balls will have a charge greater than \( 10^{14} \) in the MSSM, we see that it is quite natural for MSSM B-balls to decay after the electroweak phase transition has occurred. Thus it is possible to generate the observed B asymmetry via the decays of B-balls occurring after the electroweak phase transition. This is true
Figure 2: Q-ball decay temperature $T$ vs. the charge $Q$. The regions where L-balls and B-balls exist are also indicated.

even in those cases with rapid L violating interactions or B-L conservation, where the conventional AD mechanism fails.

The decay temperature depends on $B$, which in turn depends on the reheating temperature $T_R$. Nucleosynthesis requires that $T_R \gtrsim 1$ MeV, which imposes an upper limit on the possible charge of the B balls, $B \lesssim 10^{26}$. This in turn imposes a lower bound on $T_d$. For the $d=4$ case we find that $T_d \gtrsim 50$ MeV, whilst for the $d=6$ case we find $T_d \gtrsim 50$ keV (see Fig. 2). In principle, B-balls could decay at temperatures as low as that of the quark-hadron phase transition or less. This could have interesting consequences for the spatial distribution of baryons and inhomogeneous nucleosynthesis.

L-balls, which have $Q \approx 1000$, would decay at $T_d \approx 10^7$ GeV even if they could be formed primordially. It is therefore unlikely that such L-balls could have any cosmological consequences. However, it is possible that L-balls, which have a field strength of the order of 1 TeV or less, could play a role in the physics of the electroweak phase transition.

5 Conclusions

We have shown that it is quite natural for Q-balls to exist in the MSSM without requiring any special choice of SUSY breaking mechanism. For the case of B-balls associated with D-flat directions involving squark fields, these are usually sufficiently stable to decay after the electroweak phase transition has occurred. As a result, the Affleck-Dine
mechanism along these direction will have to be revised, as the baryon asymmetry will come from condensate collapse and B-ball decay rather than simply from the decay of a coherently oscillating scalar condensate. As many extensions of the MSSM introduce additional L violating interactions, which, if they are sufficiently rapid to be in thermal equilibrium when anomalous B+L violation is in thermal equilibrium, rule out conventional AD baryogenesis but not B-ball decay baryogenesis, the potential importance of B-ball decay baryogenesis in the MSSM and its extensions is clear. We have also pointed out the existence of L-balls of small charge, characterized by a field strength not much larger than a TeV at most. Although such L-balls will be very short-lived, decaying at a temperature of around $10^7$ GeV even if produced primordially, and so are unlikely to significantly alter the conventional Affleck-Dine mechanism, they might nevertheless be produced during the electroweak phase transition.

There are several points that this discussion of Q-balls in the MSSM has not addressed. The details of the formation of B-balls from the non-linear collapse of the condensate and the efficiency which which charge is trapped in B-balls needs to be clarified, in order to obtain an accurate estimate of the B-ball charge, which determines the temperature at which the B-balls decay. The physics of the thermalization of B-balls should also should be analysed in detail, in order to obtain constraints on the parameters of the models, in particular the reheating temperature. Given that the B-balls are decaying after the electroweak phase transition has occured, we should also consider what are the possible consequences of B-ball decay for other issues in cosmology. If the B-ball has a sufficiently large charge, corresponding to a low enough reheating temperature after inflation, then it might decay at a temperature close to that of the quark-hadron phase transition or nucleosynthesis. This could have consequences for inhomogeneous nucleosynthesis models. B-ball decay might also have consequences for dark matter in SUSY models, given that the B-balls are made of squarks which will produce neutralinos when they decay. For the case of L-balls, it would be interesting to consider the possible consequences of L-ball production during the electroweak phase transition. These issues will be the subject of future work.

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