QCD and high energy hadronic interaction: Summary talk (theory)

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Abstract

Results presented in perturbative QCD are reviewed. The topics discussed include: structure functions, heavy flavour production, direct photon production, inclusive production at LHC/SSC, small $x$ physics, QCD jets and intermittency. For reference to the data in this talk see the summary presented by J.E. Augustin.

1. Structure functions.

New data on deep inelastic structure functions by NMC and CCFR experiments, together with a reanalysis of EMC and SLAC data have been used by the Durham and CTEQ groups [1] to construct a new 1992 set of structure function parameterization. This topic has been summarized by W.K. Tung. The importance of the new data can be appreciated by observing that in the region of $x$ between $10^{-1}$ and $10^{-2}$ the gluon density $G(x, Q)$ in the two sets are very similar. However, if one [2] takes into account also the new CDF data on $b$-production a modification of $G(x, Q)$ in the region of $x \sim 0.05$ is required. The gluon density will soon be constrained by Hera measurements down to values of $x \sim 10^{-4}$.

2. Heavy flavour productions in next-to-leading order.

These are typical short distance processes due to the large mass of the heavy quark $M_Q \gg \Lambda_{QCD}$. The canonical formula for the heavy flavour production in $pp$ hadronic collider has the form

$$ \sigma^{(pp)}(s, \cdots) \simeq \int dx_1 f_{a_1}^{(p)}(x_1, \mu) \int dx_2 f_{a_2}^{(p)}(x_2, \mu) \alpha_S^2(\mu) \sigma_{a_1a_2}(x_1x_2s, \mu, \cdots), \quad (1) $$

where $\sigma_{a_1a_2}$ is the distribution for the elementary process of heavy quark production from parton $a_1$ and $a_2$ at the factorization scale $\mu$. This distribution can be evaluated order by order in perturbative QCD

$$ \sigma_{a_1a_2} = \sigma_{a_1a_2}^{(0)} + \alpha_S(\mu) \sigma_{a_1a_2}^{(1)} + \cdots, \quad (2) $$

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where the Born $\hat{\sigma}^{(0)}$ and one loop contribution $\hat{\sigma}^{(1)}$ have been computed. The functions $f^{(p)}_j(x_i, \mu)$ are the parton $a_i$ densities with fraction $x_i$ of the incoming proton energy. By using the one loop result for the elementary distribution $\hat{\sigma}_{a_1a_2}$ and the one loop kernel for the parton density evolution equation one computes the production cross section to the so called next-to-leading order (NLO).

The physical cross section $\sigma^{(pp)}$ does not depend on the factorization scale $\mu$ (here assumed equal to the renormalization point). This means that the $\mu$ dependence in the parton densities has to be compensated by the $\mu$ dependence in the elementary distribution $\alpha_S^2(\mu)\hat{\sigma}_{a_1a_2}$. Since these functions are known only up to first loop order, the computed physical cross section has a residual $\mu$ dependence which can be used to detect the importance of higher order corrections and to estimate a “theoretical error”. Higher order corrections contain powers of $\ln \mu/Q$ with $Q$ the hard scale of the process. This implies that, in order to minimize higher order corrections one should anyway assume $\mu$ of order $Q$. Typically one explores the $\mu$ dependence taking $\mu = aQ$ with $a = 1/2 - 2$. The strong $\mu$ dependence observed when taking only the leading term (the Born contribution $\hat{\sigma}^{(0)}_{a_1a_2}$ and the first order parton density evolution) is substantially reduced in the NLO calculations. However, even in the NLO calculations, one still expects a sizable $\mu$ dependence in the region of low $p_t$ for the heavy quark jet. This is due to the fact that the one loop distribution $\hat{\sigma}^{(1)}_{a_1a_2}$ contains a contribution from a diagram with a gluon exchange which is not present in the Born approximation. This contribution grows at small $x$ and becomes more important than the Born term. Thus in the region of small $x$, which corresponds to the region of low $p_t$, there is no compensation of the residual $\mu$ dependence. Here the theoretical uncertainty it then large and estimated to be a factor two. The recent data on the low $p_t$ distribution are larger than the NLO results, but they are compatible within the experimental and theoretical uncertainties (see talk by V. Papadimitriou for possible sources of overestimation of b-production data at low $p_t$).

The status of NLO calculations of heavy flavour production have been summarized by P. Nason. The processes investigated and the associated elementary distributions are: 1) single inclusive hadro-production process, $a_1a_2 \rightarrow Q + X$ with $X$ any system of partons. [9]; 2) single inclusive photo-production process, $\gamma g \rightarrow Q + X$ and $\gamma q \rightarrow Q + X$. [10]; 3) tow-inclusive hadro-production, process $a_1a_2 \rightarrow QQ + X$ [11]; 4) tow-inclusive photo-production, $\gamma q \rightarrow QQ + X$ and $\gamma g \rightarrow QQ + X$. [12]; 5) single inclusive lepto-production, $\gamma^* g \rightarrow Q + X$ and $\gamma^* q \rightarrow Q + X$. [13]; 6) resummation of higher order contributions at small $x$. [14, 15].

P. Nason presented the calculation of tow-inclusive photo-production. In this process there are two contributions: the ”photon point-like” and ”photon resolved” term. In the first one the photon is directly involved in the elementary process ($\gamma g \rightarrow QQ + X$ and $\gamma q \rightarrow QQ + X$), while in the second term the parton distribution inside the photon is involved. This process, presently studied at Hera, gives informations both on the gluon density at $x$ as small as $10^{-4}$ and on the parton densities into the photon.

NLO calculations of inclusive lepto-production has been discussed by E. Leanen. In this case the “photon resolved” contribution is negligible for large off-shell photon mass $Q^2 > 4 \text{ GeV}^2$. Therefore this process is suitable for obtaining constraints on the gluon distribution at small $x$. The calculation can be done to NLO by using the one loop elementary distribution and the NLO parton densities. For instance the $F_2$ or $F_L$ structure functions for
heavy flavour leptoproduction can be cast in the form

\[ F(x, Q^2, M_{Q\bar{Q}}) = \int dx f_a(z, \mu) C_a \left( \frac{x}{z}, M_{Q\bar{Q}}, Q^2, \mu \right) \]  

(3)

where \( C_a \) are the Wilson coefficient function computed from the elementary distribution of the processes \( \gamma^* g \to Q + X \) and \( \gamma^* q \to Q + X \).

The structure function \( F \) should be independent of the factorization/renormalization scale \( \mu \). The importance of higher order contributions can be studied by analyzing the \( \mu \) dependence of the NLO results. For values of \( x \) away from \( x = 0 \) and \( x = 1 \) the result is quite independent of \( \mu \) for \( \mu \) near the hard scale \( \mu = a \sqrt{4M_Q^2 + Q^2} \) with \( a = 1/2 - 2 \).

For large \( x \), there are large higher order corrections involving powers of \( \ln(1-x) \) which are due to soft gluon emission. The leading contributions \( \alpha_s^n \ln^n(1-x) \) (and some next to leading corrections) can be easily resummed by taking into account the coherence of the soft gluon radiation \([11]\).

For small \( x \) there are higher order corrections involving powers of \( \ln x \). As discussed in Ref. \([9]\), the leading contributions \( \alpha_s^n \ln^n x \) can be resummed by using the “Lipatov” anomalous dimension (see later) and the generalized \( k_t \)-factorization theorem which is based on the fact that at small \( x \) one should take into account the gluon and the photon off-shell mass in the elementary process \( \gamma^* g \to Q + X \). Therefore the heavy flavour structure function for small \( x \) is given by an expression of the type

\[ F(x, Q^2, M_{Q\bar{Q}}) \simeq \int dx dk_t^2 G(z, k_t^2) \hat{\sigma} \left( \frac{x}{z}, M_{Q\bar{Q}}, Q^2, k_t^2 \right) \]  

(4)

where \( \hat{\sigma} \) is the off-shell Born distribution with \(-Q^2\) and \(-k_t^2\) the photon and gluon virtual mass squared (only physical polarizations contribute at small \( x \) thus this quantity is gauge invariant). In this formula the function \( G(x, k_t^2) \) is the generalized gluon density giving the probability (per unit of \( \ln x \)) of finding a gluon at longitudinal momentum fraction \( x \) and transverse momentum \( k_t \). Integrating this distribution over \( k_t < \mu \) one obtains the gluon density \( G(x, \mu^2) \)

\[ G(x, \mu) = \int_{\mu^2}^{\mu^2} dk_t^2 G(x, k_t^2) \]  

(5)

By studying \( \hat{\sigma}(z, M_{Q\bar{Q}}, k_t^2) \) at Hera energy one has \([10]\) that the hard scale is typically \( \mu^2 = 4M_Q^2 + Q^2 \). However at small \( Q^2 \) or at c.m. energy \( W \sim M_Q \), the dynamical suppression in \( k_t \) is at a significantly smaller scale.

In the conventional calculation, the heavy flavour cross section is obtained by convoluting the on-shell elementary cross section and the gluon density \( G(x, \mu^2) \). This procedure has two main effects: (i) for \( k_t^2 < \mu^2 \) the elementary cross section is overestimated by its on-shell value; (ii) the ‘tail’ of the cross section at \( k_t^2 > \mu^2 \) is ignored. Asymptotically, the second effect dominates and the cross section is expected \([5]\) to be larger than the conventional on-shell Born approximation. At subasymptotic energies the first effect is important and one overestimates the cross section. Monte Carlo simulation based on the coherent branching algorithm \([10]\) predicts that the b-quark leptoproduction cross section at Hera energy is lower than the one obtained by a conventional one-loop calculation \([12]\).
3. Direct photon production.

The importance of this process is based on the fact that “photons do not hadronize”. Thus hadronization, a corrections beyond perturbative QCD, does not affect the calculation. In principle this is then a nice way to measure, for instance, the gluon density. However there are various points which should be understood before a complete reliable calculation could be used. J. Qiu presented the status of the calculation to NLO. This includes NLO hard elementary distribution, NLO parton densities and NLO photon fragmentation function. This last quantity is the first source of uncertainty. In purely inclusive data the photon fragmentation function gives the most important contribution. From perturbative QCD we can compute the dependence of the photon fragmentation function on the factorization scale $\mu$, which has to be taken of the order of hard jet transverse energy. This is given by the Altarelli-Parisi evolution equation and the kernel is known to NLO. However, as in the case of parton densities, to compute the photon fragmentation function we need to implement the initial condition, i.e. a distribution at a fixed scale $\mu_0$ which should be obtained from the data (see Ref. for a discussion at LEP).

For experimental reasons the measure is done by selecting “isolated photons”, i.e. by requiring that within a given angle $\delta$ the photon is accompanied by an energy fraction less than $\epsilon$. The isolation has the effect of reducing by a large fraction the size of the contribution from the photon fragmentation function, thus reducing the size of the uncertainty. However, we know that in perturbative QCD when we select events as with these isolation requirements, the distribution contains double logarithmic contributions $\alpha_S \ln \delta \ln \epsilon$ giving rise to Sudakov form factors. These quantities are well understood in perturbative QCD. Since these Sudakov corrections are large, one needs to define an algorithm to isolate the photon which must be the same in experiments and in the NLO perturbative QCD resummation. At present the CDF data agree with the QCD calculation within the errors and the presence of the residual theoretical uncertainties.

4. Inclusive production at LHC/SSC

With the advent of very energetic hadron colliders a precise estimation of inclusive production of neutral particles is important in order to pin down signals due to Higgs particles or new physics. For large transverse momentum $p_t$ the distribution can be computed in perturbative QCD. A consistent NLO calculation of inclusive $\pi^0$ production has been presented by P. Chiappetta. The distribution can be written as a convolution

$$
\sigma^{(pp)}(x, \ldots) = \int \frac{dz}{z} \hat{\sigma}_{pp \to a} \left( \frac{x}{z}, \alpha_S(\mu), \mu, \ldots \right) D_\pi^{a^0}(z, \mu)
$$

where $\hat{\sigma}_{pp \to a}$ is the inclusive distribution for emitting a parton $a$ from the two incoming protons. The function $D_\pi^{a^0}(z, \mu)$ is the $\pi^0$ fragmentation function. The determination of $\hat{\sigma}_{pp \to a}$ can be done to NLO accuracy by using the one loop elementary distributions and the NLO parton densities. As described in the previous case of prompt photon production, at present, the NLO evolution kernel for the fragmentation function is known. However we need to implement $D_\pi^{a^0}(z, \mu)$ at some scale $\mu_0$. In the calculation presented the distribution has been evaluated by using the Monte Carlo simulation HERWIG at $\mu_0 = 30\text{GeV}$ and compared with Cello data. The distribution is then evolved with NLO
corrections at ISR and UA2 energies and checked. Finally a prediction is given at LHC. The theoretical uncertainty can be analyzed by changing the factorization scale $\mu$ of the order of the hard scale given by the large $p_t$ of the emitted $\pi^0$. The theoretical uncertainty is estimated to be within a factor two.

Nice data for total photoproduction cross section at Hera are expected. This is a typical “soft physics” problem since no hard scale is involved. First attempts of computing the part of the cross section due to hard collisions with emission of high $p_t$ jets have been presented by I. Sarcevic and G. Schuler. The uncertainty here is mostly on the extrapolation of perturbative QCD results to low $p_t$ scale.

5. Small $x$ physics

One of the most arduous problems in perturbative QDC is the the analysis of processes involving incoming hadrons in the region $\Lambda \ll Q \ll \sqrt{s}$ where $Q$ is the hard scale and $\sqrt{s}$ the c.m. energy of the process. In this region the Bjorken variable $x \simeq Q^2/s$ is small. An example of these processes is heavy flavour leptoproduction at Hera, where the heavy quark mass $M$ sets the scale of the hard process. This region, which is now available at Hera for the first time, has been the subject of intensive studies in perturbative QCD. Recent new results allow one to formulate the problem in a complete form at least to leading order. Namely one is now able to attempt the analysis not only of the behaviour of the cross sections, of its size, but also of the structure of the radiation associated to these processes. The main results have been reported by L. Lipatov, M. Ryskin.

1. Lipatov equation.

In perturbative QCD the field of small $x$ physics started with the fundamental works by Fadin, Kuraev and Lipatov (FKL) in the framework of soft type of physics, namely, without any hard scale. Originally they considered a theory with a massive gauge field and they evaluated the total cross section as function of $s$ by resumming terms of type $\alpha^n S \ln^n s/m_g^2$ with $m_g$ the gluon mass. One finds that the various terms can be arranged within a ladder type of kinematical topology of the multi-gluon emission. The contribution of the exchange of a gluon with a squared sub-energy $\hat{s}$ grows proportionally to $\hat{s}$. It was found that resumming the mentioned contributions one should ”reggeize” the gluon by substituting

$$\frac{\hat{s}}{m_g^2} \equiv \frac{1}{z} \rightarrow \frac{1}{z} \Delta_{FKL}(z, k_t, m_g),$$

$$\Delta_{FKL}(z, k_t, m_g) = \exp\left\{-\hat{\alpha}_S \int_z^1 \frac{dz'}{z'} \int_{m_g^2}^{k_t^2} \frac{dq_t^2}{q_t^2}\right\} = \left(\frac{\hat{s}}{m_g^2}\right)^{-\hat{\alpha}_S \ln k_t^2/m_g^2}$$

where $\hat{\alpha}_S = C_A \alpha_S / \pi$ and $k_t$ is the transverse momentum of the exchanged gluon. The total cross section is obtained by integrating over $k_t$ a distribution $\mathcal{G}(x = m_g^2/s, k_t)$ which satisfies the Lipatov equation. The presence of the ”reggeized” gluon turned out to be crucial in order to satisfy a fundamental property of cancellation of singularities for $k_t \rightarrow 0$, so that one can avoid the cutoff given by the gluon mass $m_g$.

2. All loops anomalous dimension
A further fundamental contribution to this field is due to L.V. Gribov, Levin and Ryskin in Ref. [17]. Here the problem was formulated in the framework of hard processes and the Lipatov equation was rederived and discussed. The distribution \( G(x, k_t) \), corresponds to a generalized gluon density giving the probability (per unit of \( \ln x \)) of finding a gluon at longitudinal momentum fraction \( x \) and transverse momentum \( k_t \). Integrating this distribution over \( k_t < \mu \) one finds the gluon density at scale \( \mu \) as in (5) (see also Ref. [11]). The gluon density is given in term of the space-like anomalous dimension \( \gamma_{S,N}(\alpha_S) \) (the limit \( x \to 0 \) corresponds to \( N \to 1 \), where \( N \) is the energy moment index). The leading contributions in \( \gamma_{S,N}(\alpha_S) \) are given by an expansion in powers of \( \alpha_S/(N-1) \). The first terms of the “Lipatov” anomalous dimension are

\[
\gamma_{S,N}(\alpha_S) = \frac{\alpha_S}{N-1} + 2\zeta_3 \left( \frac{\alpha_S}{N-1} \right)^4 + 2\zeta_5 \left( \frac{\alpha_S}{N-1} \right)^6 + 12\zeta_3^2 \left( \frac{\alpha_S}{N-1} \right)^7 + \cdots
\]

(8)

where \( \alpha_S = C_A \alpha_S/\pi \) and \( \zeta_i \) is the Riemann zeta function. There are no leading terms of order \( \alpha_S^2 \), \( \alpha_S^3 \), and \( \alpha_S^5 \). Although each term is singular only at \( N = 1 \), this expansion develops a square root singularity at \( N = 1 + (4 \ln 2)\bar{\alpha}_S \). The presence of this singularity at \( N > 1 \) implies that the behaviour of the gluon density for \( x \to 0 \) is more singular than that given by any finite number of loops. For fixed \( \alpha_S \) and small \( x \) the behaviour of the one loop gluon density is

\[
G^{(1)}(x, Q) \sim \exp \sqrt{a \ln(1/x)}, \quad a = 4\bar{\alpha}_S \ln(Q^2/Q_s^2).
\]

(9)

By summing the all loop result in Eq. (8) one finds instead the following behaviour

\[
G^{(all)}(x, Q) \sim x^{-p}, \quad p = (4 \ln 2)\bar{\alpha}_S \simeq 0.5,
\]

(10)

which is much more singular for \( x \to 0 \).

3. Unified branching

Recently [18] the technique of soft gluon factorization theorems [11] has been extended to the field of small \( x \) physics. It was found that the resummation of the leading \( \alpha_S^2 \ln^p x \) terms gives rise to multi-gluon emission with a ladder type of kinematical diagrams. One finds coherence phenomena arising from interference among emitted gluons. The result is that the emission can be described by a branching process in which gluons are successively emitted in a phase space ordered in angles. This is the same phase space valid also in the complementary region in which \( x \to 1 \). Therefore one can formulate a branching process which is valid in all region of \( x \) even for \( x \to 0 \) or 1. One finds then a unified picture for parton emission which for finite \( x \) gives rise to the usual light-cone evolution equation while for small \( x \) one finds the Lipatov equation. This unified branching process can be used to construct a Monte Carlo simulation program [10] in which many features can be studied quantitatively.

In spite of the quite different behaviours of the gluon densities (9) and (10), it turns out that the all-loop and the conventional one-loop formulations give similar results. This is partially due to the fact that the first correction to the one-loop expression of the anomalous dimension is to order \( \alpha_S^3 \). Thus the steeper behaviour of the gluon density function is seen only for very low \( x \). Moreover, as pointed out in the talk of M. Ryskin, the steeper behaviour for \( \alpha_S \) running is even more asymptotic than for fixed \( \alpha_S \). This is due to the presence of
the cutoff in the exchanged transverse momenta \( k_t > Q_0 \). Although this condition is asymptotically negligible, it has some effect in reducing the evolution of the branching in the first steps. As a result the distribution at small \( x \) is somewhat reduced.

In the study of heavy flavour leptoproduction \([10]\), the most important differences between the improved all-loop branching and the conventional one-loop branching are seen in the final state gluon distributions. These differences arise from the additional phase space available for primary gluon emission in the all-loop evolution. The number of emitted gluons is enhanced, especially at small \( x \) and large angles, i.e. in the low-rapidity region. At present, these differences are small compared with uncertainties due to our lack of knowledge of the input gluon distribution. This underlines the importance of determining the gluon density experimentally down to the lowest possible values of \( x \).

It may be surprising that one can extend the soft gluon theorems from the region \( x \to 1 \) to the region \( x \to 0 \). In the first case one has that all emitted gluons are soft, while in the second case one has that the exchanged gluons are soft instead. Actually for small \( x \) one finds that \( x \) plays the role of an infrared cutoff. Due to the LNK theorem all gluons emitted with energy fractions smaller than \( x \) do not give singular contributions and therefore can be neglected to leading order. Thus the limit \( x \to 0 \) is again a soft physics problem. The most important new effect in the region of small \( x \) is the presence of new virtual singularities negligible for finite \( x \). They can be exponentiated and give rise to a new form factor called the "non-Sudakov" form factor. It is given by

\[
\Delta_{ns}(z, q_t, k_t) = \exp \left\{ -\alpha_S \int_{z'}^1 \frac{dz'}{z'} \int \frac{k_t^2}{(z'q_t)^2} \frac{d\vec{q}_t^2}{\vec{q}_t^2} \right\}
\]

where \( z, k_t \) are the energy fraction and transverse momentum of the exchanged gluon, while \( q_t \) is the transverse momentum of the emitted gluon. Comparing with (7) we see that this form factor corresponds to the "reggeization" of the gluon in the Lipatov calculation.

The calculation done so far are for the leading terms \( \alpha^n_S \ln^n x \) or in terms of moments \( \alpha^n_S/(N-1)^n \). It has been pointed out in this meeting that the next to leading calculations of terms \( \alpha^n_S/(N-1)^{n-1} \) are of crucial importance. First of all they are required in order to control the argument of the running coupling. Observe that changing the scale one finds

\[
\frac{\alpha_S(\rho Q)}{N-1} = \frac{\alpha_S(Q)}{N-1} - \beta_0 \ln \frac{\alpha_S^2(Q)}{N-1} + \cdots
\]

where \( \beta_0 \) is the first coefficient of the beta function. The present level of accuracy does not allow one to control these corrections. Even most importantly one finds that in the limit \( x \to 0 \) the generalized gluon density \( G(x, k_t) \) depends only on \( k_t \) while the hard scale itself is lost. This is just the Regge type of factorization. In this way however one does not have the property of renormalization group, a property typical of hard scattering physics. This can be obtained only by an accurate control of the hard scale \( Q \). The scale enters in the calculation for \( x \to 0 \) only to next to leading order. It was then pointed out in this meeting the urgency of these NLO calculations.

4. Unitarity corrections

The growth of the gluon density at small \( x \) in \([9]\) and \([10]\) are violating unitarity. There are then two important questions: (i) how unitarity is restored; (ii) at what value of small
should one find experimental indications of this violation. To obtain such indications one needs a model of unitarity restoration. The most practical model has been suggested by Gribov Levin and Ryskin in their important Ref. [17]. They proposed the GLR equation for the gluon density at small $x$

$$\frac{\partial^2 G(x, Q)}{\partial \ln x \partial \ln Q^2} = \frac{\alpha_S C_A}{\pi} G(x, Q) - \frac{81 (\alpha_S G(x, Q))^2}{16 R^2 Q^2},$$

(13)

where unitarity is implemented by the second term in the r.h.s. The equation with only the first term is the Altarelli Parisi equation for small $x$ (the anomalous dimension is given by the first term in (8)). The second term, coming from higher twist Feynman diagram contributions, slows the growth of the parton distribution and leads to the saturation of the parton density. The observability of the saturation has been discussed by Ryskin and depends on the value for the parameter $R$. A uniform distribution of partons corresponds to $R \sim 1$fm. If the partons are clustered in hot spots a value of $R \sim 0.2$fm would be more appropriate. An unambiguous observation of unitarity saturation seems to be very difficult. As discussed before, even at the very small values for $x$ which will be explored at Hera it will be difficult to detect the difference between the one and all loops behaviour in (4) and (10).

As discussed by Ryskin, recently [19] it has been found that there are Feynman diagrams contributions at the required level of accuracy which are not included in the GLR equation. The modifications required are under study. These contributions seems to be small since they are non planar thus suppressed by $1/C^2_A$.

A more systematic way to implement unitarity has been discussed by L. Lipatov. Taking into account that in the small $x$ region the multi-gluon emission takes place as a two dimensional random walk in the transverse momentum, Kirschner, Lipatov and Szymanowski propose to describe the multi-gluon emission by an effective field theory in two dimension. This theory should resum all relevant higher twist diagrams, includes absorption and should satisfy unitarity. It is interesting that one can study the solution of this effective lagrangian with the modern techniques of conformal field theory.

One of the necessary consequences of this study is the presence of the odderon. This has been discussed by B. Nicolescu which reported a recent estimation of the odderon singularity. Recalling that the anomalous dimension in (8) is at $N = N_F = 1 + (4 \ln 2)\alpha_S C_A/\pi \simeq 1.5$, one finds that the odderon singularity is at $N \gtrsim 1 + 0.13 (4 \ln 2)\alpha_S C_A/\pi \gtrsim 1.07$. This would predict that the difference $\sigma_{pp} - \sigma_{p\bar{p}}$ grows with energy.

C.I. Tan reported a nice attempt [20] to merge the hard perturbative QCD results at small $x$ and some important phenomenological properties of soft physics. The model is based on the following two features: (i) in hard processes the multi-gluon emission at small $x$ takes place as a diffusion process in $t = \ln(k_t/\Lambda)$; (ii) in soft physics the emission takes place as a diffusion process in impact parameter $b$. In the model then one assumes (i) diffusion in $t$ for any $b$; (ii) diffusion in $b$ only in the soft physics region $t < t_0$ with $t_0$ a cutoff corresponding to a transverse momentum of the order of a GeV.
6. QCD jets.

Hard processes are characterized by the presence of jets. They are not elementary degree in the lagrangian, so they need an operatorial definition, a "jet-finding" algorithm. The main ingredients for a "jet-finding" algorithm are the rule for clustering particles and the jet resolution. Within perturbative QCD one should make on the algorithm the following requirements: (i) it should involve the proper QCD hard scale which is dictated by coherence. In $e^+e^-$ the scale is the total c.m. energy $Q$, the photon virtuality $Q$ in deep inelastic scattering, $E_t$ in hadron-hadron processes with large transverse energy processes; (ii) it should be easy to use theoretically (and experimentally of course). In particular it should allow higher order calculations and resummation when required such as in the case of small jet resolutions; (iii) it should be not very sensitive to hadronization corrections. Recall that the fact that colour is neutralized locally [21] in the transverse momentum of partons is consistent with perturbative QCD. Phenomenologically the size of the hadronization could be estimated by Monte Carlo simulations involving colour coherence and local colour neutralization.

The "Snowmass–accord" algorithm, introduced few years ago for defining jets in hadron colliders, has been summarized by S. Ellis. This has been extensively used for instance by the CDF collaboration. For each particle or calorimeter cell with rapidity, azimuthal angle and transverse energy $\eta_i, \phi_i$ and $E_{ti}$, one defines a radius

$$r_i = \sqrt{(\eta_i - \eta_J)^2 + (\phi_i - \phi_J)^2}$$

where $\eta_J, \phi_J$ and $E_{tJ}$ are the corresponding variables for a jet. The particle or calorimeter cell belong to the jet if $r_i$ is smaller than a given resolution $R$ which is typically of order one.

The importance of this algorithm is that it allowed for the first time the possibility of a comparison of data with NLO calculations of $p_t$ distributions [3, 15]. In the presentation made by S. Ellis some difficulties were pointed out: a separation cut $E_0$ is needed and this makes difficult to study the problem of merging two jets and changing the resolution; when the resolution $R$ becomes small the distributions develop $\ln R$ powers which become large and they need to be resummed; the algorithms is not suited for higher order resummations; underlying radiation, which can not be studied by perturbative QCD, is not easy to isolate by this algorithm.

S. Catani and M. Seymour discussed recent theoretical developments of a "jet-finding" algorithms applied to Lep, Hera, and hadronic colliders hard processes. This is similar to the "Jade" algorithm which was originally proposed for $e^+e^-$ annihilation. Schematically it can be defined as follows: For each pair of particle momenta $p_i$ and $p_j$ one constructs a "resolution" variable $y_{ij}$ (to be defined later). Consider the two momenta with minimum value of $y_{ij}$. If $y_{ij} < y_c$ the two particle momenta are combined to give a single momentum. The parameter $y_c$ is the jet resolution parameter. The clustering is stopped when non pair of momenta satisfies the bound. The resulting number of jet of a given event is then a function of $y_c$. The key quantity in this algorithm is the proper definition of the "resolution" variable $y_{ij}$. One needs a definition which would allow simple higher order calculations and $\alpha_s^n \ln^n y_c$ resummations when the resolution becomes small. To this end one should takes into account that perturbative QCD is characterized by the presence of
collinear singularities for small angle $\theta_{ij}$ between two partons and infrared singularities when a parton become soft. This observation is the basis of the recently proposed the $k_t$/Durham algorithm and resolution variable $y^\alpha_{ij}$. To see the main difference with the old "Jade" resolution variable consider the case of two partons with ordered energy $E_i \ll E_j$ and small relative angles. In this limit, the Jade and $k_t$ resolution variables tends to

$$y_{ij}^{\text{Jade}} = \frac{2E_iE_j(1 - \cos \theta_{ij})}{Q^2} \Rightarrow \frac{E_iE_j}{Q^2} \theta^2_{ij}$$

$$y_{ij}^{k_t} = \frac{2\min(E_i^2, E_j^2)(1 - \cos \theta_{ij})}{Q^2} \Rightarrow \left(\frac{E_i\theta_{ij}}{Q}\right)^2 \approx \left(\frac{k^{(j)}_{t\alpha}}{Q}\right)^2$$

(15)

where $k^{(j)}_{t\alpha}$ is the transverse momentum of the soft parton $i$ with respect to parton $j$. We see then that in the limit of soft and collinear partons the $k_t$ algorithm exposes the proper kinematical variable angle time soft energy. For small resolutions, this fact allows one to exponentiate and resum the leading and next to leading powers of $\ln y_c$. Moreover the $k_t$ algorithm, as expected, is less sensitive to hadronization corrections, as can be phenomenologically estimated by Monte Carlo simulations. At small resolution parameter the exponentiation is obtained from soft gluon factorization theorems which are at the basis of the coherent branching process.

M. Seymour discussed the extension of the $k_t$-jet algorithm to hard processes with incoming hadrons, i.e. at Hera or hadron colliders. Also in these processes a prominent feature is the presence of hard QCD jets. Together with these jets one has also radiation coming from the remnants of the incoming hadron. This radiation is not described by perturbative QCD since no hard scale is here involved. In order to measure and compute distributions of QCD jets involving the hard scale one has to find an operative definition of the jets which does not involve soft physics radiation. To this end one observes that the characteristic phenomenological feature of soft physics is that the radiation is bounded to be within a fixed values of transverse energy independent of the hard scale. Consider for instance the jets emitted in deep inelastic process at Hera with $Q$ the photon virtuality. The $k_t$ algorithm in this process is preceeded by an algorithm which is intended to isolate the soft physics radiation. This isolation algorithm is defined at a scale $E_c$ with $\Lambda \ll E_c \ll Q$. From the emitted particles or calorimeter cells with energies $E_i$ with angles $\theta_{ip}$ with respect to the incoming proton and with relative angle $\theta_{ij}$ between two particles one computes the resolution parameters

$$Y_{ip} = 2E^2_p \frac{E^2_i}{E^2_c}(1 - \cos \theta_{ip})$$

$$Y_{ij} = 2\min(E^2_i, E^2_j)(1 - \cos \theta_{ij})$$

(16)

We have one of the following situations: (i) the minimum resolution parameter is $Y_{ip} < 1$. In this case particle $i$ is associated to the remanent; (ii) the minimum resolution parameter is $Y_{ij} < 1$. In this case the two particles are combined into a single "particle"; (iii) no resolution parameter is smaller than one. In this case the combination ends. After this algorithm is applied one has two sets of particles, for a given cutoff $E_c$: Particles which belong to the proton remanent and hard jet particles we can analyse by the same $k_t$-algorithm introduced in $e^+e^-$ annihilation. The algorithm here depends on two resolution parameters $E_c$ and $y_c$. The same algorithm can be generalized to the case of two incoming hadrons.
On the theoretical point of view, the important feature of this algorithm is that no subtraction of underlying event is necessary thus avoiding one of the most important difficulties of the Snowmass–accord. The resummation at small resolution of the large $\ln y_c$ powers can be done as in $e^+e^-$ to NLO accuracy. Moreover one can show that collinear singularities are factorized. This is important in order to take into account the NLO corrections in the elementary distributions and parton densities. In the analysis of jets great importance plays the scale of the hard process, as emphasized by S. Catani. In general one finds that the proper scale for jet emission is given by coherence.

7. Intermittency

A nice summary of intermittency has been presented by A. Bialas. The most characteristic feature of intermittency is that the multiplicity moments have fractal dimensions. Consider two cones of angular apertures $\delta$ and $\Delta > \delta$. The moments of particles emitted within a cone have fractal dimension if

$$F_2(\delta) \equiv \frac{\langle n(n-1) \rangle > \delta}{\langle n > \delta^2} = \left( \frac{\Delta}{\delta} \right)^{f_2} F_2(\Delta).$$  \hspace{1cm} (17)

This behaviour can be generalized to all multiplicity moment ratios $F_s(\delta)$. The importance of this quantity in describing dynamical properties is clear if we observe that for $\delta \to 0$ Eq. (17) implies the singular behaviour $F_2(\delta) \sim \delta^{-f_2}$. Similarly also the two particle correlation is singular when the particles become parallel. We have then that the fractal dimension $f_2$ is a fundamental parameter for describing in a unified way various dynamical features. This become even more interesting in the hypothesis proposed by Fialkowski that $f_2$ is universal, independent of particles and reactions.

The prediction of fractal dimension within perturbative QCD has been described in the contributions of W. Ochs, J. Meunier and I. Dremin. One finds in general

$$F_s(\delta)/F_s(\Delta) \sim \left( \frac{\langle n > \delta}{\langle n > \Delta} \right)^{\nu(s)} = \left( \frac{\delta}{\Delta} \right)^{\omega(s)}$$ \hspace{1cm} (18)

where $\nu(s)$ and $\omega(s)$ are anomalous dimensions related to the time–like gluon anomalous dimension for $N \to 1$ which to leading order is

$$\gamma_T N^u(\alpha_s) = \sqrt{\bar{\alpha}_s + \left( \frac{N-1}{4} \right)^2} - \frac{N-1}{4}$$ \hspace{1cm} (19)

where $\alpha_s = C_A\alpha_s/2\pi$. For running $\alpha_s$ one finds

$$\nu(s) \simeq \frac{s - \omega(s)}{1 - \sqrt{\tau}}, \quad \omega(s) \simeq s\sqrt{\tau}(1 - \frac{\ln{\tau}}{2s^2}), \quad \tau = \frac{\ln(E\delta/\Lambda)}{\ln(E\Delta/\Lambda)}$$ \hspace{1cm} (20)

The coherent branching algorithm allows one to compute all these quantities to leading order. We must however be aware of the following limitations on the accuracy of the calculation:

(i) in perturbative calculation we require a large scale which in this case give a limitation on the smallest angle to consider $E\delta \gg \Lambda$, with $E$ the jet energies. Small angle could then be reached at large jet energy;
(ii) all multiplicity moments obtained by the coherent branching algorithm are to leading and next to leading order \([24]\). Since soft gluon emission is very singular, it turns out that multiplicity moments have the expansion parameter \(\sqrt{\alpha_S}\) rather than \(\alpha_S\). Usually only the first corrections in \(\sqrt{\alpha_S}\) are known. There is one case in which we know higher order terms in the \(\sqrt{\alpha_S}\) expansion. This is the KNO scaling function

\[
\begin{align*}
\langle n(n-1) \cdots (n-s+1) \rangle &= g_s(\sqrt{\alpha_S}) < n >^s, \\
g_s(\sqrt{\alpha_S}) &= g_s(0)(1-s\sqrt{\alpha_S} + \cdots c_m(s\sqrt{\alpha_S})^m \cdots),
\end{align*}
\]

(21)

where all coefficients of \(s\sqrt{\alpha_S}\) powers have been evaluated \([24]\) and one finds large modification to leading order KNO formula;

(iii) hadronization corrections should be estimated. They should factorize in the ratios \(F_s(\delta)\) and should be small for \(E\delta\) much larger than some hadronization scale of the order of a \(\text{GeV}\). It is interesting to note that, due to the universality of the fractal dimension one could phenomenologically study the behaviour of \(F_s(\delta)\) even at smaller \(E\delta\). This could be used in order to learn some constraints on the hadronization process.

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