Identification of faulty sensor using relative partial decomposition via independent component analysis

Z Wang and S T Quek
Department of Civil & Environmental Engineering, National University of Singapore,
1 Engineering Drive 2, #E1A-07-03, Singapore 117576, Tel: 65 6516 6283
E-mail: a0092164@u.nus.edu; ceeqst@nus.edu.sg

Abstract. Performance of any structural health monitoring algorithm relies heavily on good measurement data. Hence, it is necessary to employ robust faulty sensor detection approaches to isolate sensors with abnormal behaviour and exclude the highly inaccurate data in the subsequent analysis. The independent component analysis (ICA) is implemented to detect the presence of sensors showing abnormal behaviour. A normalized form of the relative partial decomposition contribution (rPDC) is proposed to identify the faulty sensor. Both additive and multiplicative types of faults are addressed and the detectability illustrated using a numerical and an experimental example. An empirical method to establish control limits for detecting and identifying the type of fault is also proposed. The results show the effectiveness of the ICA and rPDC method in identifying faulty sensor assuming that baseline cases are available.

1. Introduction
Various structural health monitoring techniques have been developed during the past decades such as vibration-based global health monitoring methods focusing on finding shifts in resonant frequencies as well as changes in structural mode shapes [1]. These techniques hinge on the availability of reliable data, which remains an issue to be addressed. Methods for detecting sensors with abnormal behavior in a sensor network have been proposed, such as latent variable techniques within the multivariate statistical framework [2]. Principal component analysis (PCA) and independent component analysis (ICA) are two commonly employed latent variable techniques. ICA has proven to be more efficient than PCA especially when the measured variables exhibit non-Gaussian distributions [3]. In this study, the ICA-based fault detection method is adopted, following which the cause of the anomaly needs to be identified using an appropriate fault diagnosis method. Partial decomposition contribution (PDC) has been used to isolate the group of faulty sensors [4, 5]. However, using only the magnitudes of PDCs may lead to misleading conclusions since faulty sensors with small contributions may not be properly diagnosed [5]. The current ICA-based method is unable to detect multiplicative type of fault with scaling factor smaller than one and hard-fault-type faults (such as constant responses or complete loss of signal) [5]. In order to enhance the performance of the ICA-based method, the relative partial decomposition contributions (rPDCs) and its normalized form is proposed herein for faulty sensor detection and isolation. An empirical method to establish control limits for detecting and identifying the type of fault is also proposed. The ability of the modified ICA-based method in detecting two basic categories of faults, namely, additive-type and multiplicative-type, is illustrated through both numerical and experimental examples.
2. Independent component analysis
Under ICA, a set of measured process variables, $x$, may be decomposed as a linear combination of statistically independent non-Gaussian latent variables, $s$, called independent components, of smaller or equal dimension compared to $x$ [6]. This model can be expressed as

$$ x = As + e $$

(1)

where $A$ is a mixing matrix and $e$ is a residual matrix to account for imperfection in the model, known as reconstruction error. A demixing matrix $W$ may thus be defined as

$$ s = Wx $$

(2)

where $WA = I$, the identity matrix.

To determine $A$ and $W$, a fixed-point iteration approach called FastICA algorithm is adopted [7], where the number of independent components is chosen based on the eigenvalues of the covariance matrix $E(xx^T)$. Components corresponding to larger eigenvalues are deemed to contain more information while those with small eigenvalues are usually discarded to reduce the dimension of $s$ [7].

In equation (1), $s$ can be interpreted geometrically as points on the hyper-plane defined by $A$, and coincide with $x$ if the linear combination model is perfect, whereas $e$ represents the perpendicular distance of the actual point $x$ to the hyper-plane. $s$ can be interpreted as the systematic component of $x$ and $e$ the residual based on the linear combination model [8]. For the purpose of monitoring the out-of-control condition in the systematic component, the D-statistic, denoted as $I^2$, is adopted, defined as

$$ I^2 = s^T s = x^T W^T W x $$

(3)

Similarly, the Q-statistic, denoted as $SPE$, is defined to monitor the out-of-control condition in the residual component as

$$ SPE = e^T e = x^T (I - W^T A^T)(I - AW)x $$

(4)

To establish the confidence limits for the D- and Q-statistics, kernel density estimation is used to extract the percentile points from the underlying probability density function [3, 9]. For the data-driven process monitoring technique adopted for detection, the baselines for normal operating data are required. If bias or drift type of fault is present, at least one of the two statistics should exceed the control limits [5]. However, this may not be the case if a multiplicative type of fault is present depending on whether the scaling factor is smaller or greater than 1.

3. Sensor fault diagnosis

3.1. Partial decomposition contributions
Once a faulty sensor is detected in the system, a diagnosis method should be used to identify the location and nature of the fault. One basis is to look at the contribution of individual variables to the D- or Q-statistics known as partial decomposition contribution (PDC) [4]. The PDCs are defined as

$$ PDC^2_j = x^T M^2_j \zeta_j^T x $$

$$ PDC^{SPE}_j = x^T M^{SPE}_j \zeta_j^T x $$

(5)

(6)

where $PDC^2_j$ and $PDC^{SPE}_j$ are the contributions of $x_j$ to $I^2$ and $SPE$ respectively, $M^2 = W^T W$, $M^{SPE} = (I - W^T A^T)(I - AW)$ and $\zeta_j$ is the $j$th column of the identity matrix.

The basic assumption is that the variable with the largest component is the primary contributor of the fault [4]. Some researchers evaluated the PDCs at one point in time [5] whereas others summed up each PDC over a certain time period [8] to determine the out-of-control situation. However, this approach is not suitable for multiplicative type of fault with scaling factor smaller than one since such fault results in a decrease in PDC. Moreover, when an additive type of fault (or a multiplicative type of fault with scaling factor larger than one) is mainly attributed to a variable with relatively small absolute contributions, as the faulty sensor may not exhibit a much larger contribution than others.

3.2. Relative partial decomposition contribution
As an improvement, the relative partial decomposition contribution (rPDC) is proposed, given by
where the superscript \( \text{index} \) refers to either to \( I \) and \( \text{SPE} \), and \( E\{ \} \) is the expectation over the same realization. The assumption is that the contribution of each variable should be statistically unchanged throughout the realization if there is no change in the condition of the sensors or structure [4]. However, a fault-free variable may exhibit a relative large change in contribution at different point in time due to the input excitation. Hence, a baseline set of representative realizations is used for the normalization, giving:

\[
\text{rPDC}_{i}^{\text{Index}} = \frac{\text{PDC}_{i}^{\text{Index}}}{E[\text{PDC}_{i}^{\text{Index}}]} \tag{8}
\]

where \( \text{PDC}_{i}^{\text{Index}} \) are the PDCs of the variables in the baseline and \( \text{PDC}_{i}^{\text{Index}} \) are the PDCs of the variables in a particular set of the measurement. Instead of using the rPDCs at individual point in time, the expectation over the measured realization is proposed, given by

\[
E[\text{rPDC}_{i}^{\text{Index}}] = E\left[ \frac{\text{PDC}_{i}^{\text{Index}}}{E[\text{PDC}_{i}^{\text{Index}}]} \right] = \frac{E[\text{PDC}_{i}^{\text{Index}}]}{E[\text{PDC}_{i}^{\text{Index}}]} \tag{9}
\]

When no fault is present, \( E[\text{rPDC}_{i}^{\text{Index}}] \) for all variables should be close to 1. If a fault is mainly caused by one variable, the contributions of that variable is expected to experience the largest change compared to other variables, its \( E[\text{rPDC}_{i}^{\text{Index}}] \) will change according to the type of fault. It will increase if an additive type or multiplicative type of fault with scaling factor larger than one is present.

4. Application to case studies
To illustrate the performance of the ICA-based detection and diagnosis method using the proposed rPDC technique, a numerically simulated two-dimensional truss and an actual steel cantilever three-dimensional frame were used.

4.1. Numerical simulation using a two-dimensional truss
A seven degrees-of-freedom (d.o.f.) two-dimensional truss, with seven accelerometers monitoring each d.o.f. as shown in figure 1, was analysed. All members of the structure are 2 m long hot-finished circular hollow sections with outer diameter of 88.9 mm and thickness of 5 mm. Damping ratio of 5% was assumed for all modes. The natural frequencies of the structure ranges from 4.56 to 19.19 Hz.

A band-limited (4 to 20 Hz) white-noise excitation was applied at d.o.f. 5. The responses were sampled at a frequency of 1000 Hz and 2 sets of data both consisting of 20001 points (duration of 20 s) were simulated for each type of fault. One data set under normal condition was used as the baseline to build the latent-variable model. Different types of faults were introduced to sensor 2 from time of 5 to 20 s (i.e. points 5001 to 20001), four examples are shown in figure 2.

![Figure 1. Two-dimensional simulated truss.](image)

4.1.1. Additive-type fault
After the ICA model is built, the 99% confidence control limits for D- and Q-statistics were obtained. For the different amount of bias introduced to sensor 2 data, the detection rates (defined as percentage of samples outside the control limits) were computed. Figure 3 shows the rates for both statistics increased with amount of shift in the mean. The mean of rPDCs for sensor 2 changed significantly with bias compared with other sensors, which had values close to 1 as shown in figure 4, indicating that sensor 2 was faulty. The negative values of rPDCs for SPE was because initially the mean of PDC for sensor 2 was negative and it increased to a positive value after an additive-type fault was introduced.
**Figure 2.** Simulated fault: (a) bias, (b) drift, (c) multiplicative (SF >1) and (d) multiplicative (SF < 1)

**Figure 3.** Detection rates for $I^2$ and $SPE$ for different amount of bias.

**Figure 4.** Variation in mean of $rPDC$ for $I^2$ and $SPE$ with bias for each sensor. Dashed lines correspond to faulty sensor.
To illustrate the advantages of using rPDC instead of PDC values at a point in time, the PDC values for \( I^2 \) at 13 s (data point 13001) are shown in figure 5 when a relative small amount of bias (0.5 m²/s) was introduced. Although sensor 2 was the only faulty sensor, it did not have a much larger PDC. Instead, sensor 6 had the largest PDC. This might lead to wrong identification of the faulty sensor. In contrast, rPDC values, based on relative changes, correctly identified the faculty sensor.

![Figure 5. PDCs for \( I^2 \) at 13 s and means of rPDC for \( I^2 \).](image)

Similar results were observed when a drift type of fault was introduced. The increased in both D- and Q-statistics were exhibited and by examining the rPDCs, sensor 2 was identified (not presented).

4.1.2. Multiplicative-type fault

Two cases of multiplicative-type fault were analysed. For scaling factor (SF) > 1, the rPDC results are shown in figure 6. Sensor 2, with the largest mean of rPDC, was diagnosed as faulty. The detection results were similar to the bias case and not presented here.

![Figure 6. Mean of rPDC for \( I^2 \) and \( SPE \) for different scaling factors (> 1). Dashed lines correspond to faulty sensor.](image)

For SF < 1, the distance of each point along the model to the origin decreased. Hence, the number of points crossing the upper control limit for D-statistic will decrease, leading to decrease in detection rate. As the Q-statistic measures the distance perpendicular to the ICA model, it is hard to predict whether the detection rate will increase or decreases as SF decreases. An empirical method to help distinguish this type of fault from other types will be suggested in the experimental example.
For multiplicative-type fault with SF < 1, the rPDC for D-statistic is more useful. Due to the decrease in $I^2$ values, the faulty sensor should experience the largest reduction in contribution, resulting in a smaller mean of rPDC compared to other sensors. Alternatively, the reciprocal of the mean of rPDCs can be used to identify the faulty sensor. Figure 7 shows that the faulty sensor had the smallest mean of rPDC (and the largest reciprocal of mean).

![Figure 7](image)

**Figure 7.** Mean of rPDC for $I^2$ and its reciprocal for different SF (< 1). Dashed lines correspond to faulty sensor.

4.2. **Experiment using a steel cantilever frame**

A steel cantilever frame shown in figure 8 was used to illustrate the performance of the faulty sensor identification method. Its first 15 natural frequencies ranged from 6 to 40 Hz. A shaker attached at the free end imposed a vertical white-noise excitation with a bandwidth of 0 to 50 Hz. Three wireless accelerometers, each measuring both horizontal and vertical accelerations, were placed as indicated.

![Figure 8](image)

**Figure 8.** Location of accelerometers on steel cantilever frame.

An empirical method is proposed to distinguish different types of fault. Twelve sets of responses with duration of 20s operating under normal conditions were sampled. One set of response (set 1) was used to build the ICA model while another set of response (set 2) was used to generate various abnormal conditions. The remaining sets (3 to 12) were used to generate the statistical bounds. Specifically, detection rates for D-statistics and Q-statistics tested against set 1 were computed. These detection rates will not be identical due to variation in ambient temperature and environmental noise, amongst other factors. The mean ($\mu$) and standard deviation ($\sigma$) of these rates were used to establish
upper and lower bounds to aid monitoring, typically set at $\mu + 3\sigma$ and $\mu - 3\sigma$, respectively, as illustrated in Table 1. If the detection rates for both statistics exceed $\mu + 3\sigma$, an additive type of fault or a multiplicative type of fault with $SF > 1$ may be present in the system. If the detection rate is below $\mu - 3\sigma$ for $I^2$, a multiplicative type of fault with $SF < 1$ may probably be present in the system.

| Baseline set no. | 3 | 4 | 5 | … | Mean (µ) | Std deviation (σ) | $\mu+3\sigma$ | $\mu-3\sigma$ |
|------------------|---|---|---|----|---------|------------------|--------------|--------------|
| D-statistic detection rate (%) | 1.42 | 1.20 | 1.25 | … | 1.30 | 0.13 | 1.70 | 0.90 |
| Q-statistic detection rate (%) | 0.97 | 0.95 | 1.10 | … | 1.00 | 0.09 | 1.26 | 0.74 |

Different types of fault were introduced to sensor 2 by modifying the data in set 2 from 5 s to 20 s. The results for different magnitude of additive bias are shown in figures 9 and 10. The bias fault can be effectively detected by both D- and Q-statistics, but the latter is less sensitive due to the much smaller contributions of sensor 2 to $SPE$. However, the faulty sensor can still be identified from the rPDC plots of both statistics. The results for drift type of fault were similar and presented here.

Figure 9. Detection rates for $I^2$ and $SPE$ for different amount of bias. Dashed lines are empirical upper and lower bounds.

Figure 10. Mean of rPDC for $I^2$ and $SPE$ for different amount of bias.

The detection rates and rPDC for multiplicative type of fault with $SF > 1$ were investigated and the performance was similar to the additive type of fault. Although the detection rates of $SPE$ were relatively small, the rapid change in contribution can be seen in both rPDC plots (not presented). The
detection rates for multiplicative type of fault with SF < 1 are shown in figure 17. The Q-statistic was unable to detect the presence of fault while the abnormal reduction in detection rate of D-statistic can be observed for SF < 0.9 using the empirical lower bound. By examining the rPDC (or its inverse) plot for $I^2$, the faulty sensor can be effectively identified.

**Figure 17.** Detection rates for $I^2$ and SPE for SF (<1). Dashed lines are empirical bounds.

5. Conclusions
A normalized relative partial decomposition contribution (rPDC) method in conjunction is proposed for sensor fault detection and identification. Both numerical simulation and experimental examples suggested that for additive type of fault and multiplicative type of fault with scaling factor (SF) larger than one, the rPDC provides a more definite and accurate identification compared to using the magnitude of PDC. For a multiplicative type of fault with SF < 1, the fault causes a decrease in the detection rate of $I^2$ and the sensor is identified through the smallest mean of rPDC for $I^2$. It is proposed that upper and lower bounds for detection rates be established empirically to better account for the likely operating environmental conditions.

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