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Reply to Comment on ‘Fully covariant radiation force on a polarizable particle’

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Abstract
We argue that the theories of Volokitin and Persson (2014 New J. Phys. 16 118001), Dedkov and Kyasov (2008 J. Phys.: Condens. Matter 20 354006), and Pieplow and Henkel (2013 New J. Phys. 15 023027) agree on the electromagnetic force on a small, polarizable particle that is moving parallel to a planar, macroscopic body, as far as the contribution of evanescent waves is concerned. The apparent differences are discussed in detail and explained by choices of units and integral transformations. We point out in particular the role of the Lorentz contraction in the procedure used by Volokitin and Persson, where a macroscopic body is ‘diluted’ to obtain the force on a small particle. Differences that appear in the contribution of propagating photons are briefly mentioned.

Keywords: applied classical electromagnetism, fluctuation phenomena, random processes, noise, Brownian motion, mechanical effects of light

1. Force per particle of Volokitin and Persson

In their Comment [1], Volokitin and Persson (VP) summarize an alternative calculation of the electromagnetic force on a neutral particle moving parallel to a planar half-space. Their approach is ‘macroscopic’ in the sense that the starting points are two half-spaces (1 and 2, say)
sliding one against the other. The focus of the present discussion is the lateral force (per unit area) given by a component of the electromagnetic stress tensor, evaluated at the surface of body 1. To arrive at the force between a single, moving particle and a surface, the moving body 2 is ‘diluted’ by taking the limit (notation of VP, cgs units):

\[ \epsilon_2(\omega) - 1 \to 4\pi n_2 \alpha(\omega), \quad |4\pi n_2 \alpha(\omega)| \ll 1 \] (1)

where \( n_2 \) is the number density of the constituent atoms (‘particles’ in the following) and \( \alpha(\omega) \) their electric polarizability. The resulting force on body 1 (in the frame where it is at rest, while body 2 moves in the \( x \)-direction with velocity \( v \)) can be written as an integral over electromagnetic waves. The focus of the discussion is the contribution of evanescent waves that takes the form (equation (27) of reference [1]):

\[
\text{VP: } f^{\text{part, ev}}_x = -\frac{8\pi\hbar}{\gamma} \int_0^\infty \frac{d\omega}{2\pi} \int_{q>\omega/c}^\infty \frac{d^2q}{(2\pi)^2} \frac{q_x}{\kappa} e^{-2\kappa z} \text{Im} \alpha(\omega') \times \left[ N_1(\omega) - N_2(\omega') \right] \sum_\mu \phi_\mu \text{Im} R_{1\mu} \] (2)

where \( \omega' = \gamma(\omega - q_v) \) is the frequency of a photon mode in the frame co-moving with the particle. We follow the notation of reference [1] except for: \( k_z \) is denoted \( \kappa = (q^2 - \omega^2/c^2)^{1/2} \), and the Bose–Einstein distribution is written \( N_i(\omega) = \frac{1}{2} \text{coth} \left( \frac{\hbar \omega}{2k_B T_i} \right) - 1 \) (\( i = 1, 2 \)), with \( T_1 \) the local temperature of the body at rest and \( T_2 \) the particle’s temperature (evaluated in its co-moving frame). The polarization-dependent weight functions and reflection amplitudes are:

\[
\phi_s = (\omega'/c)^2 + 2\gamma^2 \beta^2 q_y^2 \frac{\kappa^2}{q_x^2}, \quad R_{1s} = \frac{\kappa - k_1}{\kappa + k_1} \] (3)

\[
\phi_p = (\omega'/c)^2 + 2\gamma^2 \left( q^2 - \beta^2 q_x^2 \right) \frac{\kappa^2}{q_x^2}, \quad R_{1p} = \frac{\epsilon_1 \kappa - k_1}{\epsilon_1 \kappa + k_1} \] (4)

where \( \beta = v/c \), and the medium propagation constant \( k_1 = \sqrt{\kappa^2 - (\epsilon_1 - 1)\omega^2/c^2} \).

2. Comparison to Pieplow and Henkel

Using the fact that \( \kappa^2 = q^2 - (\omega/c)^2 \), the weight functions \( \phi_\mu \) become identical to ours (equations (65) and (66) of reference [2]). In the sector of evanescent waves, \( \kappa \) is real and positive so that:

\[
\text{Im} \frac{e^{-2\kappa z}}{\kappa} = \frac{e^{-2\kappa z}}{\kappa} \text{Im} R_{1\mu}. \] (5)

The evanescent contribution to the friction force by Pieplow and Henkel (PH, equations (67) and (69) of reference [2]) can therefore be written in the form:

\[
\text{PH: } f^{\text{part, ev}}_x = \frac{\hbar}{\gamma} \int_{-\infty}^\infty \frac{d\omega}{2\pi} \int_{q>\omega|c} \frac{d^2q}{(2\pi)^2} \frac{q_x}{\kappa} e^{-2\kappa z} \text{Im} \alpha(\omega') \times \left[ N_1(\omega) - N_2(\omega') \right] \sum_\mu \phi_\mu \text{Im} R_{1\mu}. \] (6)

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The frequency integral can be reduced to the range \( \omega \geq 0 \), using the fact that the integrand is even under the transformation \((\omega, q_x) \mapsto (-\omega, -q_x)\): the expressions \( q_x, N_1(\omega) - N_2(\omega') \), \( \text{Im} \alpha(\omega') \), and \( \text{Im} R_{1\mu} \) are all odd under this transformation. The resulting factor 2 in front of \( \int_{0}^{\infty} d\omega \) brings equation (6) into the form derived by VP, except that equation (2) contains an additional prefactor \(-4\pi\). We now suggest an explanation for this factor.

The minus sign is due to the fact that VP calculate the force on body 1 (it is dragged along by the moving particle), while PH consider the force on the moving particle (a friction force). Provided the latter is evaluated in the rest frame of body 1 (as done by PH), the two forces are opposite by Newton’s *actio = reactio*.

The factor \( 4\pi \) is due to the choice of units: in the cgs units used by VP, the displacement field in the dilute limit of body 2 is given by (see equation (1)):

\[
\text{VP: } D = (1 + 4\pi n_2 \alpha) E
\]

while the same quantity is, in the units used by PH (vacuum permittivity \( \varepsilon_0 = 1 \)),

\[
\text{PH: } D = (1 + n_2 \alpha) E.
\]

The factor \( 4\pi \) can therefore be attributed to the different unit for the polarizability.

The factor \( 1/\gamma \) in front of equation (2) is, of course, impossible to check by taking the non-relativistic limit. It arises from the following relativistic argument in the ‘dilute medium’ procedure used by VP. The starting point is the lateral stress \( \sigma_{xz} \) on body 1 at rest, a force per unit area. One takes a slice of thickness \( dz \) of medium 2 that is centered at a distance \( z \) from body 1. This slice increases the force on that body by an amount:

\[
dF_x = A \, dz \, \frac{d\sigma_{xz}}{dz}
\]

where \( A \) is the area of the body. In the dilute limit, forces are additive so that we convert this into a force per particle (in medium 2) by dividing by the number of particles in that slice:

\[
f_{x}^{\text{part}} = \frac{dF_x}{dn_2} = \frac{A \, dz \, d\sigma_{xz}/dz}{A \, n \, dz}.
\]

This is the first formula in equation (27) of reference [1].

The key point is here: \( n \) is the number density of body 2 as observed in the rest frame of body 1. This is the only way that an observer fixed to body 1 can define a force per particle. The density \( n \) differs from the number density in the co-moving frame due to the Lorentz–Fitzgerald contraction. Hence, we have:

\[
n = \gamma n_2, \quad f_{x}^{\text{part}} = \frac{d\sigma_{xz}}{\gamma n_2 \, dz}
\]

where the number density in the co-moving frame is precisely the density \( n_2 \) that appears in equation (1) above. Indeed, the dielectric response \( \varepsilon_2(\omega) \) is the one in the rest frame of body 2, as required by the way VP and PH formulate the relativistic description: the field incident on body 2 is transformed into its local rest-frame, where \( \varepsilon_2(\omega) \) can be applied. The Lorentz

1 To see this for \( N_1(\omega) - N_2(\omega') \), write it as a difference of coth functions. For \( \alpha(\omega) \) and \( R_{1\mu}(\omega) \), this is a property of Fourier transforms of real-valued response functions. Specifically in \( R_{1\mu} \), we use that \( \kappa \) is real and positive for all \( \omega, q_x \), in the evanescent sector. The medium propagation constant is extended according to \( \kappa_1(-\omega) = \kappa_1^{*}(\omega) \) (real \( \omega \)), ensuring a retarded solution to the reflection and transmission problem for waves of negative frequencies.
contraction of the particle density is thus the explanation why equations (2) and (6) contain the factor $1/\gamma$.

### 3. Comparison to Dedkov and Kyasov

In equation (13) of reference [3], Dedkov and Kyasov (DK) give the following expression for the evanescent contribution to the friction force on a moving particle:

$$f_{\text{part, ev}}^\text{DK} = \frac{16\pi \hbar}{\gamma} \int_0^\infty \frac{d\omega}{2\pi} \int_{q_x, q_y > 0} \frac{d^2q}{(2\pi)^2} q_x \ e^{-2\kappa} \omega \left[ \phi(q_x, \omega) \right] \Im \left\{ \alpha(q_x, \omega) \right\} R_{\mu}$$

where $\omega = \gamma(\omega + q, \nu)$. We have used the translation table 1 for the transcription into the notation of VP (except for $\kappa$ and $N(\omega)$, as mentioned after equation (2)). Note that for a fair comparison, we have neglected the contribution from the magnetic polarizability $\alpha_m$ and written $\alpha = \alpha_e$. equation (12) uses an integration range over only one quadrant in the $q$-plane.

Since the integrand is even in $q_y$, a prefactor 2 can be removed and the integral extended over the entire $q_y$-axis (restricted to evanescent waves, of course). The two lines in equation (12) involving $\omega^- = \omega'$ and $\omega^+$ only differ by the sign of $q_x$ and can therefore be combined into one integral over the $q_x$-axis (in the evanescent sector). After these manipulations, we arrive at equation (2), except for a factor $-1$. This sign is explained as above. If one includes the Lorentz-contracted density in the procedure for taking the dilute limit, as outlined above, the formulas by VP and by DK are thus in full agreement.

### 4. Propagating sector

VP do not discuss in their Comment the contribution from propagating photons. A quick glance at their equation (22), first term, suggests that the ‘dilution procedure’ gives a result that is qualitatively different. The rules spelled out after equation (26) give to the leading order a contribution to the stress (force per area) on body 1 given by:

$$\sigma_{xz}^{pr} = -\hbar \int_0^\infty \frac{d\omega}{2\pi} \int_{q \leq \omega/c} \frac{d^2q}{(2\pi)^2} q_x \left(2 - \left| R_{1p}\right|^2 - \left| R_{1s}\right|^2\right) \left( N_1(\omega) - N_2(\omega') \right).$$

Note that does not allow for a dilute limit, because it is not proportional to the density $n_2$. (It only depends on the temperature $T_2$ of the diluted body 2.) A detailed comparison to the result given by our approach would go beyond the purpose of this Reply, as there are also physical reasons to expect a difference.

For example, an infinitely thick half-space does not show any transmission for radiation emitted by body 1, while a single particle does. The expression $1 - |R_{1s}|^2$ gives the absorption of a half-space and appears in the analog of equation (19), first line, to calculate the emission from body 2. If body 2 were a thin layer, however, its transmission also would appear here and even become significant in the dilute limit.
Table 1. Translated notations from Dedkov and Kyasov [3] to Volokitin and Persson [1].

| Temperatures | Photon modes | Occupation | Polarization weights |
|--------------|--------------|------------|---------------------|
| DK [3]       | $T_1$, $T_2$ | $k$, $q_0$ | $\tilde{q}_0$, $\gamma \omega^\pm$ | $W(\omega/T_2, \omega^\mp/T_1)$, $\gamma^2 \chi^\pm (\omega, k)$ | $\Delta_e$, $\Delta_m$ |
| VP [1]       | $T_2$, $T_1$ | $q$, $k_z$, $\kappa$ | $q_z$, $\omega^\prime_\pm$ | $2[N_1(\omega) - N_2(\omega^\prime_\pm)]$, $\phi_p(\omega^\prime_\pm)$ | $R_{1p}$, $R_{1s}$ |
Let us compare in the following the results of DK and PH in the propagating sector. Equation (13) of [3] by DK provides an integral representation whose first line actually corresponds to a free-space (fs) contribution (taking only the electric polarizability):

\[ f_x^{\text{part, pr}} \Big|_{fs} = -\frac{4\hbar \gamma_c^4}{c^4} \int_0^\infty \frac{d\omega}{2\pi} \omega^4 \int_{-1}^1 dx \frac{1 + \beta x}{c^2} \times \bar{\omega}(\omega) \left[ N_1(\omega) - N_2(\omega) \right] \]

(14)

where \(\omega_1 = \gamma \omega (1 + \beta x)\). The force in free space, filled with blackbody radiation at temperature \(T_1\), is apparent from equation (56) in PH's reference [2]. Equations (52, 54) in that paper translate into the present notation as follows:

\[ f_x^{\text{part, pr}} \Big|_{fs} = \frac{2\hbar \gamma}{\pi c^3} \int_0^\infty \frac{d\omega}{2\pi} \int d\omega \left( \omega - \beta q_x \right)^2 \times \bar{\omega}(\omega') \left[ N_1(\omega) - N_2(\omega') \right] \]

(15)

where the symmetry manipulations mentioned after equation (6) have been used for the \(\omega\)-integral. We integrate over the directions of photon wave vectors (solid angle \(d\Omega\)), their length being fixed to \(\omega_c\). By rotational symmetry around the x-axis, this integral can be reduced to (substitution \(q_x = (\omega/c)x\)):

\[ f_x^{\text{part, pr}} \Big|_{fs} = \frac{\hbar \gamma}{\pi c^4} \int_0^\infty \frac{d\omega}{2\pi} \omega^4 \int_{-1}^1 dx \frac{1 - \beta x}{c^2} \times \bar{\omega}(\omega') \left[ N_1(\omega) - N_2(\omega') \right] \]

(16)

where now \(\omega' = \gamma \omega (1 - \beta x)\). Flipping the sign of \(x\), we arrive at equation (14), up to a factor 4\(\pi\) that arises again from the choice of units for the polarizability (see above).

The surface-dependent part of DK, equation (13), involves the reflection coefficients and reads:

\[ f_x^{\text{part, pr}} \Big|_{surf} = \frac{16\pi h}{\gamma} \int_0^\infty \frac{d\omega}{2\pi} \int_{q \in q_{\text{m}}, q_{\text{c}}} \frac{d^2q}{(2\pi)^2} q_x \left( -\sin 2q_z z \right) \times \left\{ \text{Im} \left( \omega' \right) \left[ N_1(\omega) - N_2(\omega') \right] \sum_{\mu} \phi_\mu \text{Im} R_{1\mu}(\omega') \right. \]

\[ + \frac{16\pi h}{\gamma} \int_0^\infty \frac{d\omega}{2\pi} \int_{q \in q_{\text{m}}, q_{\text{c}}} \frac{d^2q}{(2\pi)^2} q_x \cos 2q_z z \times \left\{ \text{Im} \left( \omega' \right) \left[ N_1(\omega) - N_2(\omega') \right] \sum_{\mu} \phi_\mu \text{Re} R_{1\mu}(\omega') \right. \]

\[ \left. - (\omega' \rightarrow \omega'_+ \left\} \right. \right. \]

(17)

where the prescription \(\left( R_+^\pm, R_-^\pm \rightarrow \tilde{R}_+^\pm, \tilde{R}_-^\pm \right)\) has been applied as explained after equation (25) of reference [3]. We have used the notation \(q_z = (\omega/c)(1 - (cq/\omega)^2)^{1/2}\), which is real.
We extend the $q$ integral from one quadrant to the entire circle $q \leq \omega/c$, using the manipulations described after equation (12), and obtain:

\[
\int \int \sum_{\pi} \omega \pi \alpha \omega \omega \omega \phi \omega \omega = \hbar' - \frac{1}{2} \times\frac{\omega}{\mu} \mu \mu \infty \sum_{\mu} \frac{\phi_{\mu} \left( \text{Re} R_{1\mu} (\omega') \cos 2q_{z} - \text{Im} R_{1\mu} (\omega') \sin 2q_{z} \right)}{q_{z}}. \tag{18}
\]

The result of PH can be found from equations (67) and (69) in reference [2] and is an integral identical to equation (6), with the $q$-range restricted to $q \leq \omega/c$ (propagating waves), and the replacement:

\[
\frac{e^{-2\pi \varepsilon}}{\kappa} \text{Im} R_{1\mu} \mapsto \frac{\text{Re} \left( R_{1\mu} e^{2q_{z}} \right)}{q_{z}}. \tag{19}
\]

recalling that $q_{z}$ is real. The manipulations mentioned after equation (6) bring this expression to a positive-frequency integral of the form:

\[
\int \int \sum_{\pi} \omega \pi \alpha \omega \omega \omega \phi \omega \omega = \hbar' - \frac{1}{2} \times\frac{\omega}{\mu} \mu \mu \infty \sum_{\mu} \frac{\phi_{\mu} \left( \text{Re} R_{1\mu} \cos 2q_{z} - \text{Im} R_{1\mu} \sin 2q_{z} \right)}{q_{z}}. \tag{20}
\]

Up to the familiar $4\pi$, this is identical to equation (18) because $\omega' = \omega$.

5. Discussion

The word ‘covariant’ in the title of our paper [2] may have led to the impression that this is the only way to formulate a fully relativistic theory. This is, of course, wrong: it is just a convenient formulation, and other approaches that do not work with four vectors and metric tensors, etc., give equally valid results, even for relativistic velocities. The calculations of VP and DK are examples of these. The advantage of the ‘manifestly covariant’ formulation is that transformation properties are relatively easy to identify. For example, the transformation properties of the electromagnetic field and the polarization field both arise from tensor fields, namely $F_{\mu\nu}$ and $M^{\mu\nu}$.

We have provided some technical details to show that DK and PH get the same electromagnetic force for the particle + surface scenario, as mentioned in [2]. The agreement holds for both propagating and evanescent waves and for arbitrary temperatures. The approach of VP apparently differs by a factor $-4\pi$ for evanescent waves. We have argued that this factor disappears when the same units are used and have observed that the procedure of diluting the moving body takes into account the relativistic contraction of densities.

For propagating photons, a disagreement between VP and PH arises. We have argued that it is not obvious how to combine thermal equilibrium in a medium with the dilution procedure: indeed, as long as body 2 is infinitely thick, there can be no contribution ‘from its back side’ to the electromagnetic stress between bodies 1 and 2. It is quite possible that a calculation where body 2 is a slab of finite thickness, which is then diluted, will retrieve the particle + surface case.
in full, provided the photons incident on the ‘back side’ of the slab are in equilibrium in the same frame and temperature as body 1. Otherwise, a drag stress must be expected on body 1, similar to the force on a particle that moves relative to the frame where a thermal radiation field is in equilibrium [4].

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