Identifying universality in warm inflation

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Abstract. Ideas borrowed from renormalization group are applied to warm inflation to characterize the inflationary epoch in terms of flows away from the de Sitter regime. In this framework different models of inflation fall into universality classes. Furthermore, for warm inflation this approach also helps to characterise when inflation can smoothly end into the radiation dominated regime. Warm inflation has a second functional dependence compared to cold inflation due to dissipation, yet despite this feature, it is shown that the universality classes defined for cold inflation can be consistently extended to warm inflation.

Keywords: inflation, physics of the early universe, particle physics - cosmology connection

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1 Introduction

Observational cosmology, as demonstrated by the recent Planck results, has reached an impressive level of precision that can set constrains on many cosmological models, including inflation [1–3]. However, despite the level of accuracy achieved by Planck, the degeneracy problem of inflationary model building still persists, in that many inflationary models can produce predictions, like for the tensor-to-scalar ratio and for the spectral tilt, that are very similar and compatible with the data. In ref. [4] the idea of universality classes was suggested as a means to classify a wide range of inflation models, and thus subsumes a large number of them in terms of their salient properties relevant to observation. This approach borrows ideas from the renormalization group (RG) methods of quantum field theory (QFT), such as the concept of flow away from a fixed point, here corresponding to the exact de Sitter (dS) geometry, and the use of an analog to the renormalization group equation (RGE) for the \( \beta \)-function.

The \( \beta \)-function formalism was introduced in ref. [4] and further developed and extended in refs. [5–9], to identify universality among the wide zoology of inflationary models. This formalism is based on the application of the Hamilton-Jacobi (HJ) approach to cosmology [10]. It relies on a formal analogy between the equation describing the evolution of a scalar field in an expanding background and a RGE of QFT. As will be explained below, this analogy is not coincidental but has underpinnings with holography. In this framework the near scale invariance experienced by the Universe during inflation is interpreted as a departure of the corresponding RGE from a fixed point corresponding to an exact dS spacetime. A single parametrization of the \( \beta \)-function, close to the dS fixed point, thus defines an universality class of models that can be grouped together, sharing a single asymptotic behavior. As a consequence, arbitrary potentials can be classified into a small set of classes according to the behavior of their associated \( \beta \)-function in the neighborhood of the fixed point.
This approach has some direct advantages. First of all, by grouping different potentials into a small set of classes, it significantly reduces the number of relevant cases to consider. Furthermore, as the formalism relies on intrinsic properties of inflation, it is completely general and in particular, it does not assume slow-roll, since for example it has been successfully applied to constant-roll inflation [9]. Finally, as mentioned already, this formalism has deep theoretical motivations arising from the holographic description of the Early Universe (see, for example, refs. [11–13]). Within the (A)dS-CFT correspondence of Maldacena [14], the flow away from the dS fixed point, which is realized during inflation, is dual to a deformation of the associated conformal field theory (CFT) due to relevant or marginal operators. By applying these methods to describe the early Universe, and in particular inflation, it is both possible to shed a new light on some of its problematic aspects and to provide an alternative interpretation of the observational constraints [15–22].

The aim of the present work is to formulate warm inflation in terms of the $\beta$-function formalism. Warm inflation [23, 24] differs from the usual paradigm of cold inflation in the fact that dissipative processes can lead to a sustainable radiation production throughout the inflationary expansion. Warm inflation will happen for regimes of parameters such that the inflaton interactions with other field degrees of freedom are not negligible and they generate dissipation terms, such that a small fraction of vacuum energy density can be converted to radiation. When the magnitude of these dissipation terms are strong enough to compensate the redshift of the radiation by the expansion, a steady state can be produced, with the inflationary phase happening in a thermalized radiation bath. There have been many constructions based on particle physics models demonstrating the viability of this special regime of inflation, see, for example, refs. [25–27] and for a review, see also ref. [28]. Recently a first principles warm inflation model was constructed from QFT which involves just a few fields [27], thus convincingly demonstrating that warm inflation models are on an equal footing to cold inflation as model building prospects. Moreover, the dissipative effects and the presence of a non-vanishing radiation bath are able to change both the inflationary dynamics at the background and at the fluctuation levels [29–37], such that there can be distinctive differences between the two paradigms which could be testable. As such, it is useful to understand the dynamical structure of warm inflation through different perspectives, which is a motivation of this paper.

By applying the $\beta$-function formalism to warm inflation, we show that there are two intervening characteristic functions regulating the dynamics. One of them is the function already identified in ref. [4], which was defined in the cold inflation case, and which controls the way the inflaton drives the departure from the dS fixed point. In the warm inflation context, we show that another function controlling the level of radiation production naturally emerges. By following the evolution of these two functions, we are able not only to fully characterize the dynamics, but also to determine when the end of warm inflation smoothly connect with the radiation dominated regime. Furthermore, these two functions allow us to group different forms of inflationary potentials in certain universality classes. Since this description sets direct control on the dynamics by using parameters which are different from the usual slow-roll coefficients, it offers an extremely powerful method to describe the inflationary evolution (and its end) in an independent and novel way.

In this work we make use of the generalized framework offered by the $\beta$-function formalism to obtain an analytical understanding of warm inflation. We first show that in some toy models a full analytical description of warm inflation can be derived and then we focus on more realistic scenarios. In particular, we show that it is possible to derive a relatively accu-
rate description of both the weak and strong dissipative regimes. Among the main results of the paper there is the observation that, despite a second functional dependence is introduced, a universal description of inflation, similar to the one of ref. [4], can still be consistently formulated. This allows then to study the effect of the various forms of the dissipation terms commonly considered in warm inflation on the classes of universality and on their predictions for the scalar spectral index and the tensor-to-scalar ratio. Remarkably, we show that within the $\beta$-function formalism it is easy to identify the degeneracy in the inflationary observables for some models with different dissipation coefficient forms.

This work is organized as follows. In section 2 we give the basic equations describing the dynamics of warm inflation along also the expression of the scalar curvature power spectrum for some representative examples of warm inflation dissipative forms. In section 3 we describe how the $\beta$-function formalism is applied to warm inflation. We provide details on the method and present explicit examples in section 4. The results are presented and discussed in section 5. Our concluding remarks and future perspectives are given in section 6. An appendix is included to provide further useful formulae and present some different inflation classes.

2 Warm inflation basics

Let us restrict to the case of a single inflaton field with pressure and energy density respectively given by

\[ p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi), \]
\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), \]  

(2.1)

where $V(\phi)$ is the scalar potential. In this case the dynamics of warm inflation at the background level is governed by the equation of evolution for the inflaton field

\[ \ddot{\phi} + (3H + \Upsilon) \dot{\phi} + V_{,\phi} = 0, \]

(2.2)

where $\Upsilon$ is the dissipation coefficient, which in general can be a function of both the temperature and the background inflaton field $\phi$, by the equation for the evolution for the radiation energy density $\rho_r$

\[ \dot{\rho}_r + 4H \rho_r = \Upsilon \dot{\phi}^2, \]  

(2.3)

and by the Einstein equations (in units of reduced Planck mass $M_P/\sqrt{8\pi} = m_P = \kappa^{-1} = 1$)

\[ 3H^2 = \rho_\phi + \rho_r, \]
\[ -2\dot{H} = \rho_\phi + \rho_r + p_\phi + p_r = \frac{4}{3} \rho_r + \dot{\phi}^2. \]  

(2.4)  

(2.5)

Notice that one of these equations is redundant. Moreover, it would be equivalently possible to use the equation of continuity for the inflaton energy density $\rho_\phi$

\[ \dot{\rho}_\phi + 3H \dot{\phi}^2 = -\Upsilon \dot{\phi}^2. \]  

(2.6)

In the following, we will be working mainly with the dissipation coefficient ratio $Q$, defined as $Q \equiv \Upsilon/3H$. Typically, the study of warm inflation assumes the radiation to be thermalized, i.e.

\[ \rho_r = \frac{\pi^2 g_* T^4}{30} = 3p_r, \]  

(2.7)
where \( g_* \) being the number of relativistic degrees of freedom for the radiation bath. Without knowing the details on the microphysics of the system the condition to attain this situation is normally \( Q \gtrsim 1 \). Namely, we require the radiation fields, generically associated with relativistic degrees of freedom, e.g., gauge or other fields, to have at least one interaction mediated by the inflaton in a Hubble time. This is typically the necessary condition for thermalization. However, there could be other mechanisms beyond the radiation fields/inflaton interaction, which can dramatically modify this setup. For example, if the radiation fields are coupled to the standard model (SM), the Schwinger process \([38]\) should provide an extremely efficient mechanism to reach thermalization, see for example refs. \([39–41]\). However, in this work we are not interested in building models where these mechanisms are taken into account and we assume that some process ensures that radiation thermalizes. For a detailed quantification of the thermalization process relevant for warm inflation and done in the context of the Botzmann equation, see, e.g., ref. \([42]\). Therefore, we assume eq. (2.7) to hold. Note also that the specific form for the dissipation coefficient \( \Upsilon \) in the above equations can only be determined by the details of the microphysics during inflation. Different forms of dissipation coefficients derived from QFT have been derived explicitly e.g. in refs. \([43, 44]\). It is also worth mentioning that warm inflation helps in easing the \( \eta \)-problem \([45, 46]\) since in the strong dissipative regime \( Q \gg 1 \) the inflaton mass is larger than \( H \).

### 2.1 The scalar spectrum of perturbations in warm inflation

Given the complexity of the warm inflation dynamics, which involves a system of coupled fluids associated with the inflaton and radiation, alongside perturbations, which in general are also coupled \([47, 48]\), an analytical treatment for the spectrum of perturbations is in general difficult. In what follows, we briefly present this analysis. For a full discussion we send the reader to the refs. \([29, 50]\). The dimensionless scalar power spectrum \( \Delta_s^2(k, \tau) \) at horizon crossing, meaning \( k\tau = 1 \) where \( \tau \) is used to denote the conformal time, is a sum of thermal and vacuum contributions

\[
\Delta_s^2(k, \tau) = \Delta_s^2,\text{th}(k, \tau) + \Delta_s^2,\text{vac}(k, \tau).
\]

In the simplest realization of inflation \( \Delta_s^2,\text{vac}(k, \tau) \) at horizon crossing can be simply expressed as

\[
\Delta_s^2,\text{vac}(k, \tau) \bigg|_{\tau=k^{-1}} = \frac{1}{4\pi^2} \frac{H^4}{\dot{\phi}^2}.
\]

On the other hand, the thermal contribution in general is something that depends on the microphysics of the model. Nevertheless, it is known that semi-analytical expressions for the full spectrum of scalar perturbations can be derived. In particular the spectrum at horizon crossing can be expressed as \([23, 45, 51]\)

\[
\Delta_s^2(k, \tau) \bigg|_{\tau=k^{-1}} = \left( \frac{H^2}{2\pi \dot{\phi}} \right)^2 \left[ 1 + \frac{\sqrt{12\pi Q} T_{\delta \phi}}{\sqrt{3} + 4\pi Q \frac{T}{H}} G(Q) \right] G(Q) \bigg|_{\tau=k^{-1}},
\]

where \( n_{\text{BE}} = [\exp(H/T_{\delta \phi}) - 1]^{-1} \) is the Bose-Einstein distribution and \( G(Q) \) is a function of \( Q \) that accounts for the fact that the radiation fluctuations are in general coupled to the inflaton which is thus leading to a growing mode in the inflaton fluctuations \([47–49]\). Moreover, the temperature \( T_{\delta \phi} \) inside \( n_{\text{BE}} \) corresponds to the temperature of the inflaton fluctuations and is not necessarily the same as \( T \), corresponding to the temperature of the
thermal bath. For a recent discussion based on solutions of the Boltzmann equation relevant during the warm inflation dynamics, see, e.g., ref. [52]. In the following we assume thermal equilibrium with $T_{\delta \phi} = T$. Typically, $G(Q)$ reduces to 1 for $Q = 0$ and in most of the known models it is well approximated by a fraction of polynomials in $Q$ with numerically fitted coefficients [47, 48]. Notice that for $Q = 0$, which also implies $T = T_{\delta \phi} = 0$, we recover the usual cold inflation spectrum given in eq. (2.9) as expected for consistency.

As a matter of fact the presence of radiation is thus inducing a series of modifications in the typical CMB observables, namely $n_s$ and $r$. In particular, we expect two competing effects:

1. The decay of the inflaton into radiation is effectively playing the role of an additional friction term for the inflaton beyond the usual Hubble friction. As a consequence we expect, similarly to [53], a shift in the point of the potential probed by CMB observations. In particular this effect is expected to produce a decrease of $n_s$ and an increase in $r$.

2. The radiation will play the role of a source term for scalar field fluctuations which induces an amplification in the scalar power spectrum. Indeed, this can be noticed by eq. (2.10). However, in general we do not expect a similar coupling between radiation and tensor fluctuations. As a consequence this effect induces an increase in $n_s$ and a decrease in $r$.

If thermal effects are already important (or at least not completely negligible) at CMB scales, the first of these two effects happens to be subdominant with respect to the second one, meaning that typically $n_s$ is increasing and $r$ is decreasing with respect to the cold case.

For completeness, we should mention that an analysis of non-Gaussianities has been performed for warm inflation for the weak and strong dissipation regimes, see, e.g., refs. [32, 33]. In both cases the predictions are generally in good agreement with the Planck constraints in ref. [2].

3 \textbf{\textbeta-function formalism and warm inflation}

Let us start this section with a brief review of the main ideas behind the formulation of the \textbeta-function formalism for inflation introduced in ref. [4]. In the simplest realization of inflation, i.e. a single field, with a standard kinetic term and no interactions with other particles, the evolution of the Universe during inflation is completely specified by the system of equations

\begin{align}
3H^2 &= \rho, \\
-2 \dot{H} &= p + \rho, \\
\ddot{\phi} + 3H \dot{\phi} + V_{,\phi} &= 0,
\end{align}

i.e., by the Einstein equations and the equation of motion for the scalar inflaton $\phi$. In the spirit of the HJ formalism, the solution of this system is assumed to exist and, in particular, the time evolution of the inflaton field $\phi$ is assumed to be piecewise monotonic. Under this assumption it is possible to invert $\phi(t)$ to get $t(\phi)$ and the field can be directly used as the clock describing the evolution of the system.
By introducing the so-called superpotential\(^1\) \(W(\phi) \equiv -2H(\phi)\) and using Raychauduri’s equation (3.2) we easily obtain that
\[
\dot{\phi} = W_{,\phi}.
\] (3.4)

This equation clearly shows that by exploiting the HJ formalism it is possible to express \(\dot{\phi}\) as a function of \(\phi\) only. The \(\beta\)-function is then defined as
\[
\beta(\phi) \equiv \frac{d\phi}{d\ln a} = \frac{\dot{\phi}}{H} = -2\frac{W_{,\phi}}{W}.
\] (3.5)

Notice that, in analogy with the RGE, the scalar field is playing the role of the coupling constant and the scale factor is playing the role of the renormalization scale. Using the definition of eq. (3.5), the equation of state becomes
\[
\frac{p + \rho}{\rho} = 4\frac{W_{,\phi}^2}{3\ W^2} = \frac{\beta^2(\phi)}{3}.
\] (3.6)

This equation implies that an exact dS geometry corresponds to a fixed point with \(\beta(\phi) = 0\) and thus a phase of nearly exponential expansion of the Universe is realized by departing from a region where \(\beta \ll 1\). As a consequence, in the framework of the \(\beta\)-function formalism, a class of models of inflation is defined by specifying the asymptotic parameterization of \(\beta(\phi)\) in the neighborhood of a point where \(\beta(\phi) \ll 1\). Moreover while the leading contribution around the fixed point sets the main properties of the universality class, higher order contributions to \(\beta(\phi)\), which are negligible in the neighborhood of fixed point, break the universality at different scales.

For warm inflation a model is not only specified by the inflationary potential, but also by the dissipation coefficient ratio \(Q\), which in general is a function of both \(\phi\) and \(T\). Once these two functions are specified, the evolution is completely determined by the set of equations (2.2)–(2.5). By solving these equations, we can express all the relevant quantities, i.e., \(H(t)\), \(\phi(t)\), \(Q(t)\), and \(T(t)\), as functions of time. Once again the problem can be studied in the framework of the HJ formalism and, assuming the evolution of \(\phi(t)\) to be piecewise monotonic, it is possible to compute, at least locally, \(t(\phi)\) and express all the relevant quantities as functions of the field only.

In analogy with the treatment carried out in the cold case, we introduce a superpotential
\[
W(\phi) \equiv -2H(\phi).
\]
Assuming the radiation energy density to be quasi-stable,\(^2\) meaning 
\(\dot{\rho}_r \ll 4H\rho_r\), and using the Raychauduri’s equation (2.5) we thus obtain
\[
\dot{\phi} = \frac{W_{,\phi}}{1 + Q}.
\] (3.7)

By using this equation and eq. (2.5) we get from eq. (2.7)
\[
T^{-4} = \frac{45}{2\pi^2 g_*} \frac{Q}{(1 + Q)^2} W_{,\phi}^2.
\] (3.8)

\(^1\)This choice for the name is justified by the formal analogy with the parameterization of the scalar potential in SUSY (for a review, see for example, ref. [54]).

\(^2\)More on this approximation is said below eq. (3.13) where we re-express this condition in terms of the typical quantities of the formalism.
To find the temperature as a function of \( \phi \) only, eq. (3.8) needs to be solved for \( T \). Note that since in general \( Q \) depends both on \( T \) and \( \phi \), the solution of this equation might exist only numerically. Then, once \( T(\phi) \) is known, the dissipation coefficient ratio is expressed as a function of \( \phi \) only as \( Q(T(\phi), \phi) \equiv Q(\phi) \). Note that a different notation, \( Q \), is used here to stress the difference in the functional dependence on \( \phi \).

We proceed our discussion by introducing the cosmological \( \beta \)-function as defined in eq. (3.5), \( \beta(\phi) \equiv d\phi/d\ln a = \dot{\phi}/H \). Note that the analogy with a RGE still holds. The equation of state reads
\[
-\frac{2\dot{H}}{3H^2} = \frac{(1 + Q)\dot{\phi}^2}{3H^2} = \frac{(1 + Q)\beta^2(\phi)}{3}.
\] (3.9)
Interestingly, eq. (3.9) shows that an exact dS geometry is again realized in correspondence to the zeros of \( \beta(\phi) \) and the phase of accelerated expansion of the Universe stops when \((1 + Q)\beta^2(\phi) \) is of order one. This is a crucial difference with respect to the cold case in eq. (3.6). As \( Q \) is always positive, the fixed point is only attained by a vanishing \( \beta \)-function, but in general, unless we have \( \beta \) exactly equal to zero, \( \beta^2(\phi) \ll 1 \) is not sufficient to ensure that the Universe is inflating. In particular, the Universe may stop to inflate because \((1 + Q) \gg \beta^{-2}(\phi) \), while \( \beta \ll 1 \). Another original and strictly warm realization of inflation is the case in which, departing from the dS fixed point, \( \beta(\phi) \) reaches a constant value smaller than one. In such a scenario, the last part of the inflationary phase is thus driven and, in particular, is concluded by the evolution of \( Q \). As inflation can only be realized for \( \beta(\phi) \ll 1 \), its parametrization can still be used to fix the flow in the neighborhood of the fixed point. Once again it is thus possible to use \( \beta(\phi) \) to define a set of universality classes as in the cold inflation case.

To make the generalization from cold inflation more evident, let us define
\[
\beta_{CI}(\phi) \equiv -2\frac{W_{\phi}}{W} = (1 + Q)\beta(\phi),
\] (3.10)
which has the exact same dependence on \( W \) as the beta function of the cold inflation, eq. (3.5). Note that with this definition, the superpotential \( W \) can be readily expressed as
\[
W(\phi) = W_f \exp \left[ -\frac{1}{2} \int_{\phi_f}^{\phi} d\phi' \beta_{CI}(\phi') \right],
\] (3.11)
where the subscript \( f \) is used to denote quantities evaluated at the end of inflation. Moreover, using the definition given in eq. (3.10), it is easy to prove that the equation of state can be expressed as
\[
-\frac{2\dot{H}}{3H^2} = \frac{\beta_{CI}^2(\phi)}{3(1 + Q)}. \tag{3.12}
\]
Again, the fixed point is reached when \( \beta_{CI} \) goes to zero and we see that inflation ends when \( \beta_{CI}^2 \sim 1 + Q \). Notice that according to eq. (3.11) \( \beta_{CI} \) is directly associated with the superpotential and thus with the inflationary potential. This equation makes clear that for \( Q \) sufficiently large, the Universe is inflating for \( \beta_{CI} \gg 1 \). In this sense, the dissipation

\footnote{In principle, it could also be possible to start by directly fixing a parameterization for \( Q(\phi) \). More on this will be commented in section 6.}
coefficient can be interpreted as a friction term that slows down the evolution of the inflaton field and this is potentially allowing for inflation in regions of the potentials that are steeper than the ones usually considered in the cold case. As already mentioned in the previous section, this could provide a mechanism to ease the $\eta$-problem. In order to generalize the universality classes defined for cold inflation in ref. [4], in this work we will simply use the $\beta$-functions associated with these classes as choices for $\beta_{CI}$. We then observe how the different dissipation coefficient ratios will affect the predictions of any classes, this analysis is carried out in section 4 and 5.

At this point we can translate the quasi-stable assumption of the radiation energy density in the language of the $\beta$-function formalism

$$\left| \frac{\beta}{4} \frac{d \ln \rho_r}{d \phi} \right| = \left| \frac{\beta_{CI}}{4(1 + Q)} \left[ \frac{Q}{Q} \left( \frac{1 - Q}{1 + Q} \right) + 2 \frac{\beta_{CI,\phi}}{\beta_{CI}} - \beta_{CI} \right] \right| \ll 1.$$  (3.13)

The validity of this condition has to be checked for each choice of $\beta_{CI}$ and $Q$. However it is possible to show that for all the cases discussed in this paper, this assumption is satisfied.

The expression of the number of e-foldings $N$ in this formalism reads

$$N(\phi) \equiv - \int_{a_f}^{a} d \ln a = - \int_{\phi_f}^{\phi} d \phi' = - \int_{\phi_f}^{\phi} \frac{1 + Q(\phi')}{\beta_{CI}(\phi')} d \phi',$$  (3.14)

where $\phi_f$ is the field value at the end of inflation, fixed by $\beta_{CI}(\phi_f) = 1 + Q(\phi_f)$, and the expression of the inflationary potential which is derived by using eq. (2.4),

$$V(\phi) = \frac{3}{4} W^2(\phi) \left[ 1 - \frac{1}{6} \frac{(1 + 3Q/2)}{(Q + 1)^2} \beta_{CI}^2(\phi) \right].$$  (3.15)

As for the physically relevant cases, we expect both $\beta_{CI}$ and $Q$ to be negligible while the Universe is deep into the inflationary phase, i.e., for large values of $N$, the parameterization of the inflationary potential is typically mainly determined by the superpotential $W(\phi)$. It is worth mentioning that the formalism is not only valid at the background level, but rather it can also be used to describe cosmological perturbations.

To grasp a better understanding of the competing influences of $\beta_{CI}$ and $Q$ during the phase of inflation, it is worth defining the complementary function $\beta_T$ as

$$\beta_T(\phi) \equiv \frac{T}{H} = - \frac{2}{W}.$$  (3.16)

Using eq. (2.7) we express

$$\frac{\rho_r}{H^2} = \frac{\pi^2 g_\ast T^4}{30} = \frac{\pi^2 g_\ast}{30} |T\beta_T(\phi)|^2,$$  (3.17)

which makes manifest the interpretation of $\beta_T$. This function captures the amount of radiation produced during warm inflation. In particular, by considering the full equation of state,

$$- \frac{2H}{3H^2} = \frac{\dot{\phi}^2 + \frac{4}{3} \rho_r}{3H^2} = \frac{\beta^2(\phi)}{3} \frac{2\pi^2 g_\ast}{45} \frac{|T\beta_T(\phi)|^2}{3}$$

$$= \frac{1}{3} \left[ \frac{\beta_{CI}(\phi)}{Q + 1} \right]^2 + \frac{2\pi^2 g_\ast}{45} \frac{|T\beta_T(\phi)|^2}{3},$$  (3.18)
it is clear that (as $T \neq 0$) $\beta_T$ parameterizes the flow from the dS fixed point induced by radiation. Interestingly, using the definitions of $\beta_{CI}$ and $\beta_T$ we can represent the phase of inflation in a two-dimensional plot depicting the departure from the usual cold inflation case. In figure 1 the phase of inflation is represented as a trajectory starting from, or close to, the dS fixed point at the origin and reaching the circle of unitary radius, where $(1 + Q_f)\beta_f^2 = 1$, which corresponds to the end of inflation. From the equation of state (3.18) we note that the axes in figure 1 are proportional the square roots of the fractional kinetic and thermal energy densities. The flow along the different inflationary trajectories can be directly parametrized by the value of the inflaton field $\phi$, or equivalently by the number of e-foldings $N$ defined in eq. (3.14). A motion in the horizontal direction is due to $\beta_{CI}$, whereas a vertical motion is an effect of production of radiation. Since we have that

$$T\beta_T(\phi) = \sqrt{\frac{45}{2\pi^2 g_*}} \frac{Q}{(Q + 1)^2} |\beta_{CI}(\phi)|,$$

(3.19)

we observe that the shape of the trajectory is mostly defined by the dissipation coefficient ratio $Q$. Note that in these kind of plot, any model of cold inflation is represented as an horizontal line with $T\beta_T = 0$. Conversely, warm inflation models are expected to be represented as curves departing from this line. As all the inflationary trajectories are expected to end on the solid black line, large values of $\beta_f$, i.e., for values closer to $T_f\beta_f = 0$, imply a small radiation contribution to the equation of state at the end of inflation. Conversely, small values for $\beta(\phi_f)$, imply a non-negligible radiation contribution to the equation of state at the end of inflation.

Apart from the de Sitter fixed point at the origin, there are two other special points on figure 1. The first is $(1,0)$, where cold inflation usually ends. When a trajectory crosses this point, the Universe stops inflating and it must then enter into the (p)reheating phase. The second point, which is not appearing on our plots, should be $(0,2)$. This corresponds to the Universe being in the radiation-dominated era, i.e. $\rho_r/(3H^2) \simeq 1$. Notice that all trajectories describing viable cosmological models, which consistently include the evolution of the Universe after inflation, must cross this point. However, since in $(0,2)$ we have $\dot{\rho}_r = 4H\rho_r$, for sure the assumption of quasi-stable radiation must be violated, implying that our treatment cannot be extended all the way up to this point.

Figure 1. Some possible inflationary trajectories corresponding to the flow from the dS fixed point to the solid black line and showing the departure from the usual cold inflation case (red dashed line). The curves shown in this plot are illustrative examples which are not corresponding to any concrete model.
A model which touches the solid black line for large (order 1) values of $\sqrt{2\pi^2 g_*/45 T}\beta_T$, for example the vertical dotted curve in figure 1, implies that the RG flow in the last part of inflation is mainly induced by the radiation. This does not imply that the Universe is dominated by the radiation, but rather that it is rapidly approaching the moment where the transition from inflation to a radiation dominated Universe takes place. Since in these models the radiation energy density at the end of inflation is already sizable and the inflaton kinetic energy is small, an explosive (p)reheating may not be required and the transition from inflation to radiation may be smooth. This has to be checked model by model. Since in warm inflation it is possible to unify the treatment of inflation and (p)reheating, a self-consistent computation of $N_{\text{CMB}}$, the value of $N$ at which CMB observables leave the horizon, could in principle be carried out. However, in order to perform this analysis, we need to study the trajectory until the point (0,2) is reached, which requires to violate the assumption of quasi-stable radiation. As this goes beyond the scope of this work, where required to adopt $N_{\text{CMB}} = 60$ as a representative value.

For completeness the remaining relevant quantities in the description of inflation expressed in terms of the $\beta$-function formalism can be found appendix A, in particular, the scalar-spectral index and the tensor-to-scalar ratio.

4 Applying the formalism to explicit examples

In this section we provide a general procedure for computing the predictions in the $\beta$-function formulation of warm inflation. In particular, we start by explaining our numerical methods for examining models, and then focus on some special cases that admit an analytical treatment. As already explained in section 3, the model is completely specified by fixing a $\beta$-function, either $\beta(\phi)$ or $\beta_{\text{CI}}(\phi)$, and by a dissipation coefficient ratio $Q(T, \phi)$. In order to generalize the classes of universality for cold inflation [4], we choose to start by fixing a parameterization for $\beta_{\text{CI}}(\phi)$. The dissipation coefficient $\Upsilon(T, \phi)$ is derived explicitly by QFT methods, see, e.g., refs. [43, 44]. In this work, we focus on a rather general parameterization for the dissipation ratio $Q = T/(3H)$ that is motivated by the previous warm inflation models developed in the literature [25, 27, 30, 43, 44, 55],

$$Q = \frac{C T^n}{H \phi^n} = -\frac{2 C T^n}{W \phi^n},$$

where $C$ is a constant. This example will also facilitate the illustration of the methodology. When a complete specification of the model is required, i.e. an explicit choice for $\beta_{\text{CI}}(\phi)$, for simplicity we will restrict our analysis to the chaotic class

$$\beta_{\text{CI}}(\phi) = -\frac{\alpha}{\phi},$$

where $\alpha$ is a positive constant. The generalisation to other classes of models can be carried out analogously.

In general it is unlikely to have a complete analytical description of the model and therefore numerical methods are required. The procedure we have used to derive numerical solutions is the following: having $\beta_{\text{CI}}(\phi)$, $Q(T, \phi)$ and an initial guess value for the constant $W_f$, which fixes the normalization of the inflationary potential, as inputs, the value of the scalar field at the end of inflation $\phi_f$ and the corresponding temperature $T_f$ are computed.
using

\[ \beta_{CI,f}^2 = 1 + Q_f, \]

\[ T_f^4 = \frac{45}{8\pi^2 g_*} \frac{Q_f}{1 + Q_f} W_f^2. \]

(4.3)

where we recognize eq. (3.8) in second of these equations. They can be recasted as

\[ T_f = \left( \frac{45 C - W_f}{4\pi^2 g_* \beta_{CI,f}^2 \phi_f^n} \right)^{\frac{1}{4-m}}, \]

\[ \beta_{CI,f}^2 = 1 + \left( \frac{45}{8\pi^2 g_* \beta_{CI,f}^2 \phi_f^n} \right)^{\frac{4}{4-m}} \left( \frac{2 C}{\phi_f^n} \right)^{\frac{4}{4-m}} (W_f)^{\frac{2m-4}{4-m}}. \]

(4.4)

The solution of the above system of equations is obtained by first solving for \( \phi_f \) and then computing \( T_f \). The inflaton field then serves as a clock for the evolution of the system. We evolve the field from \( \phi_f \) to \( \phi_f \pm \Delta \phi \) with \( \Delta \phi \ll \phi_f \) being an infinitesimal step. The sign of the increment is fixed by the position of the fixed point, i.e., whether the value of the field increases or decreases during inflation. At this point the relevant quantities \( T, Q \) and \( N \) are evaluated at \( \phi_f \pm \Delta \phi \) using eq. (3.8), the definition of \( Q \) and eq. (3.14), respectively. The procedure is then repeated until the value \( \phi_{\text{CMB}} \) is reached. The latter is defined as the value of \( \phi \) which gives \( N(\phi_{\text{CMB}}) = N_{\text{CMB}} \) where in this work we assume \( N_{\text{CMB}} = 60 \) as the value of \( N \) at which CMB observables leave the horizon. As a consequence, the evolution is solved for all the scales between the end of inflation and CMB scales. Finally, by comparing the amplitude of the scalar power spectrum with the COBE normalization [1, 3], it is possible to adjust the constant \( W_f \) in order to satisfy this constrain. The predictions for the scalar spectral index and tensor-to-scalar ratio are then computed from eqs. (A.8) and (A.9) for the values of \( \phi \) corresponding to \( N_{\text{CMB}} \). These quantities can finally be compared with the observational constraints [1, 3].

### 4.1 Analytical methods

In this subsection we focus on some cases where a complete (or partial) analytical treatment can be performed. In order to carry out this treatment we have to

1. Compute \( \phi_f \) and \( T_f \), the values of the inflaton field and of the temperature at the end of inflation using eqs. (4.3);
2. Derive the superpotential and its derivative using \( \beta_{CI} \) and eq. (3.11);
3. Compute \( T(\phi) \) by solving eq. (3.8) with the dissipation coefficient ratio \( Q(T, \phi) \) written explicitly in terms of \( T \) and \( \phi \). Having \( T \) as a function of the field, we can also write \( Q(T(\phi), \phi) \) as a function of \( \phi \) only;
4. Finally, we express \( \beta_T \) as a function of \( \phi \) using eq. (3.19).

Note that in general, the third step cannot be carried out analytically for non-trivial forms of the dissipation coefficient. Typically, it is also useful to derive all the relevant quantities as functions of the number of e-folds. For this purpose, we thus compute the number of e-folding \( N(\phi) \) from eq. (3.14) and invert it to find \( \phi(N) \). Note also that once again we fix the constant \( W_f \) in order to be consistent with the COBE normalization \( \Delta_s^2(N_{\text{CMB}} = 60) = 2.2 \times 10^{-9} \). In particular this is done by solving eq. (A.6) for \( W(N) \) at \( N = N_{\text{CMB}} \). Let us now illustrate the method with some examples where a partial (or complete) analytical treatment exists.
4.1.1 Constant Q — full analytical treatment

Let us restrict to the simplest case possible, that of a constant dissipation coefficient ratio $Q(T, \phi) = Q$. We first consider a generic $\beta_{CI}$ and then restrict to the specific example of the chaotic class specified by eq. (4.2). For a constant $Q$, eq. (3.8) admits the solution

$$T(\phi) = \left[ \frac{45}{8\pi^2 g_s} \frac{Q}{(1+Q)^2} W^2(\phi) \beta_{CI}^2(\phi) \right]^{1/4},$$

(4.6)

where $W(\phi)$ is directly set by eq. (3.11). To check the consistency of the model, we can compute $\rho_r$, by substituting eq. (4.6) into eq. (3.17),

$$\rho_r = \frac{3}{16} \frac{Q}{(1+Q)} W^2(\phi) \beta_{CI}^2(\phi).$$

(4.7)

Interestingly, this can be compared with the result

$$\rho_\phi = \frac{3}{4} W^2 - \rho_r = \frac{3}{4} W^2 \left[ 1 - \frac{Q}{(1+Q) \frac{\beta_{CI}^2(\phi)}{4}} \right],$$

(4.8)

to conclude that, independently on the value of $Q$, when we approach the dS fixed point, $\beta_{CI}(\phi) \ll 1$, we always consistently get $\rho_r \ll \rho_\phi$.

To further proceed we need to precise a parameterization for $\beta_{CI}$ and therefore we restrict ourselves to the case of the chaotic class of eq. (4.2). In this case, the superpotential and the temperature, respectively, read

$$W = W_f \left( \frac{\phi}{\phi_f} \right)^{\frac{\alpha}{2}},$$

(4.9)

$$T(\phi) = \left[ \frac{45}{8\pi^2 g_s} \frac{Q}{(1+Q)^2} \frac{\alpha^2 W^2_f}{\phi_f^2} \phi^{\alpha-2} \right]^{1/4}.$$

(4.10)

For completeness, we also derive, using eq. (3.15), the potential

$$V = \frac{3}{4} W_f^2 \left( \frac{\phi}{\phi_f} \right)^{\alpha} \left[ 1 - \frac{\alpha^2}{12\phi^2} \frac{2 + 3Q}{(1+Q)^2} \right] \approx \frac{3}{4} W_f^2 \left( \frac{\phi}{\phi_f} \right)^{\alpha},$$

(4.11)

where the approximation in the last step relies on $\phi \gg \alpha$, which is valid deep in the inflationary phase. The value of the field at the end of inflation is

$$\phi_f = \sqrt{\frac{\alpha^2}{1+Q}},$$

(4.12)

and the number of e-foldings $N$ as a function of $\phi$ reads

$$N = \frac{1 + Q}{2\alpha} \left( \phi^2 - \frac{\alpha^2}{1+Q} \right),$$

(4.13)

which implies

$$\phi = \sqrt{\frac{2\alpha N + \alpha^2}{1+Q}}.$$

(4.14)
At this point we can also compute $\beta_{C1}(N)$, $T(N)$ and $\beta_T(N)$, whose expressions are given, respectively, by

$$
\beta_{C1}(N) = -\sqrt{\frac{(1 + Q)\alpha}{2N + \alpha}},
$$

(4.15)

$$
T(N) = \frac{45}{8\pi^2 g^*} \frac{Q}{1 + Q} \alpha^{2-\alpha} W_f^2 \left(2\alpha N + \alpha^2\right)^{\alpha/2-1},
$$

(4.16)

$$
\beta_T(N) = \frac{90}{\pi^2 g^*} \frac{Q}{1 + Q} \alpha^{2+\alpha} W_f^2 \left(2\alpha N + \alpha^2\right)^{-\alpha/2-1}.
$$

(4.17)

Notice that for $\alpha = 2$ the temperature is constant during inflation. Finally, we fix $W_f$ using the COBE normalization and we compute the spectral tilt $n_s$ and the tensor-to-scalar ratio $r$ using eqs. (A.8) and (A.9). As $Q$ is positive, we expect a slightly increased value of $n_s$ and a slightly reduced value of $r$ with respect to the cold inflation case.

### 4.1.2 Weak and strong dissipative limits

For the general choice of $Q(T, \phi)$, eq. (4.1), an analytical description does not exist in all regimes. However, similarly to the treatment of ref. [56], an analytical description of these models can be achieved both in the strong $Q \gg 1$ and in the weak $Q \ll 1$ dissipative limits.

In particular, it is possible to derive analytical expressions for $Q$ and $T$ as function of $\phi$ only, which we do next.

**a) Weak dissipative regime.** Let us consider the parameterization of $Q(T, \phi)$ given in eq. (4.1). In the limit $Q \ll 1$ we can immediately use eq. (3.8) to compute the temperature to obtain

$$
T(\phi) = \left[\frac{45C}{4\pi^2 g^*} \frac{\beta_{C1}^2(-W)}{\phi^n}\right]^{1/m},
$$

(4.18)

and then, by substituting this expression into eq. (4.1), we find

$$
Q(\phi) = 2C \left(\frac{45C}{4\pi^2 g^*}\right)^{m/m} (-W)^{2(m-2)/m} \phi^{4n/m} \beta_{C1}^{2m/m}.
$$

(4.19)

To completely specify the model, we need to substitute an explicit parameterization for $\beta_{C1}$.

For the example of chaotic class eq. (4.2), we find that

$$
T(\phi) \approx \left[\frac{45C\alpha^2(-W\phi_j^{-2})}{4\pi^2 g^*}\right]^{1/m} \phi^{\alpha-2n/(4-4m)},
$$

(4.20)

$$
Q(\phi) \approx 2C \left(\frac{45C\alpha^2}{4\pi^2 g^*}\right)^{m/m} \left(W_j^2 \phi_j^{-\alpha}\right)^{(m-2)/(4-4m)} \phi^{n(m-2)-2m-4n/(4-4m)}.
$$

(4.21)

It is worth to note that this regime can only be attained dynamically for a certain set of values for $\alpha, C, n$ and $m$. Recall that $W_f$ is fixed by the COBE normalization and, thus, it should not be considered a free parameter. In particular, as the chaotic class describes large field models, meaning that inflation takes place for large values of $\phi$, this can only be attained if $\alpha(m-2) - 2m - 4n < 0$. It is interesting that since in the chaotic class both the superpotential and the $\beta$-function have the form of a power law, the temperature and $Q(\phi)$ must have a
**Dissipation Coefficient Ratio** | $Q(\phi) \sim \phi^\beta$ | $T(\phi) \sim \phi^\alpha$
---|---|---
Cubic ($m = 3$, $n = 2$) | $\alpha - 14$ | $(\alpha - 8)/2$
Linear ($m = 1$, $n = 0$) | $-(\alpha + 2)/3$ | $(\alpha - 4)/6$
Inverse ($m = -1$, $n = 0$) | $(-3\alpha + 2)/5$ | $(\alpha - 4)/10$

*Table 1.* Power-law behaviors of $Q(\phi)$ and $T(\phi)$ for the chaotic class in the weak dissipative limit.

power law dependence as well. This behavior can actually change for different classes.\(^4\) The dependence of $T$ and $Q$ on $\phi$ for some particular choices of $m$ and $n$ are written in Table 1.

A graphic representation of these behaviors for particular sets of $m$ and $n$ can be seen in Figure 3 shown in section 5. Assuming that the model stays in the weak dissipative regime (this assumption has to be checked model by model) for the whole period of inflation, we can proceed further with the computation of the number of $e$-foldings,

$$N(\phi) = \frac{1}{2\alpha} (\phi^2 - \alpha^2). \quad (4.22)$$

Using eq. (4.3), it is now possible to compute the value of the inflaton field at the end of inflation as given by $\phi_f = \alpha$. At this point, in order to check the consistency of the approximation, we should verify that $Q(\phi_f) \ll 1$

$$2C \left( \frac{45C}{4\pi^2 g_*} \right)^{m} \left( -W_f \right)^{2(m-2)} \frac{(4-4n)^{-4n}}{4m} \ll 1. \quad (4.23)$$

Finally, by inverting eq. (4.22), we obtain

$$\phi(N) = \sqrt{2\alpha N + \alpha^2}. \quad (4.24)$$

Having derived $Q(\phi)$, $T(\phi)$ and $\phi(N)$, we can immediately compute the predictions for $n_s$ and $r$ using eq. (A.8) and eq. (A.9).

(b) **Strong dissipative limit.** Let us follow a procedure for the strong dissipative regime analogous to the one carried out above for the weak dissipative limit. As a first step, we compute the temperature and the dissipation coefficient as functions of $\phi$ only, such that we have

$$T(\phi) = \left[ \frac{45}{16\pi^2 g_* C} \phi^2 \right]^{\frac{1}{4+m}} \beta_{CI}^{2m+1} \left( -W \right)^{2(4m-2)} \phi^{-4n}. \quad (4.25)$$

$$Q(\phi) = 2C \left( \frac{45}{16\pi^2 g_* C} \right)^{\frac{m}{4+m}} \beta_{CI}^{2m} \left( W \right)^{2(4m-2)} \phi^{-4n}. \quad (4.26)$$

Once again, restricting to the chaotic class gives

$$T(\phi) = \left[ \frac{45\alpha^2}{16\pi^2 g_* C} \left( \phi_{f}^{-3\alpha/2} \right) \right]^{\frac{1}{4+m}} \phi^3 \left( -W_{f} \right)^{\frac{3\alpha + 2n - 4}{2(4+m)}} \left( \phi_{f}^{-\alpha/2} \right)^{\frac{2m - 2 + 2n - 4}{4 + m}}. \quad (4.27)$$

$$Q(\phi) = 2C \left( \frac{45\alpha^2}{16\pi^2 g_* C} \right)^{\frac{m}{4+m}} \left( -W_{f} \phi_{f}^{-\alpha/2} \right)^{\frac{2m - 2 + 2n - 4}{4 + m}} \phi^{\frac{\alpha(m - 2) - 2 - 4n}{(4 + m)}}. \quad (4.28)$$

\(^4\)For both the monomial and inverse classes (see eq. (A.10) and eq. (A.12)) $\beta_{CI}(\phi)$ is still a power law, but $W(\phi)$ are respectively given by eq. (A.11) and eq. (A.13). As a consequence an approximate power law behavior can only be attained in regions where $W$ is nearly constant, i.e., where $\phi$ is very close to the fixed point (meaning deep in the inflationary phase). Conversely, for the exponential class (see eq. (A.14)) the $\beta$-function is not a power law and, thus, the power law behavior is never approached.
Dissipation Coefficient Ratio & $Q(\phi) \sim \phi^#_T(\phi) \sim \phi^#$

| Cubic ($m = 3, n = 2$) | $\alpha/7 - 2$ | $3\alpha/14$
| Linear ($m = 1, n = 0$) | $-(\alpha + 2)/5$ | $(3\alpha - 4)/10$
| Inverse ($m = -1, n = 0$) | $(-3\alpha + 2)/3$ | $(3\alpha - 4)/6$

Table 2. Power-law behaviors of $Q(\phi)$ and $T(\phi)$ for the chaotic class in the strong dissipative limit.

Recall that this regime is only attained for a particular set of values for $\alpha$, $C$, $n$ and $m$ and therefore the consistency of the condition $Q \gg 1$ has to be checked explicitly model by model. The dependence of $T$ and $Q$ on $\phi$ for different choices of $m$ and $n$ are presented in table 2.

Once again, a graphic representation of these behaviors can be seen in figure 3 shown in section 5. Similar to the weak dissipation limit case (in particular see footnote 4), different scalings can be obtained by considering different classes of models, in particular choosing a different parameterization of $\beta_{CI}$, which implies different expressions for $W(\phi)$. In principle, by assuming that the strong dissipative regime holds during the last 60 e-foldings it could be possible to derive equations similar to eq. (4.22), however, in most of the cases this would not be physically relevant since we typically want $Q \ll 1$ at CMB scales.

5 Discussion of the results

In this section we present and discuss the results of the numerical analysis carried out by following the procedure outlined in the previous section. While all the results shown are obtained by considering $\beta_{CI}$ of the chaotic class, eq. (4.2), a similar analysis can be performed for any other class, such as the monomial ones, eq. (A.10), the inverse type of potential, eq. (A.12), or the exponential forms, eq. (A.14). Although we restrict to a single choice for $\beta_{CI}$, we consider the four different cases introduced in section 4, namely, the constant $Q = C$, cubic $Q = CT^3/(H\phi^2)$, linear $Q = CT/H$ and inverse $Q = C/(HT)$ forms of the dissipation coefficient ratio $Q$.

Let us start by discussing the evolution in the plane ($\beta_{CI}, T\beta_T$), shown in figure 2. The motivation for this kind of plots was explained in section 3. As expected, different parameterizations of the dissipation coefficient ratio lead to different inflationary trajectories. Consistently with our expectations, all the curves start from the neighborhood of the dS fixed point ($\beta_{CI}, T\beta_T) = (0, 0)$ and end onto the solid black curve, which represents the points in the plane ($\beta_{CI}, T\beta_T$) where inflation ends. One notes that the straight trajectories of the constant case, among which we have the standard cold case with $T\beta_T = 0$, are perfectly consistent with the theoretical expectations; indeed from eq. (3.19) we see that $(T\beta_T)^2 \propto Q\beta_{CI}^2/(1+Q)^2$. Interestingly, in many of these models inflation ends with $\sqrt{2\pi^2 g_*/45} T\beta_T = 4\rho_r/(9H^2) \approx 1$. This implies that in these scenarios the amount of radiation present in the Universe at the end of inflation is already sufficiently large to quickly take over the inflaton energy density. As a consequence, already mentioned in section 3, these models are not expected to require an explosive (p)reheating to trigger the transition from inflation to the radiation dominated phase.

\footnote{It is fair to stress that, according to the discussion of section 4.1.2, for different classes we expect qualitatively different results for the results shown in figure 3.}
Figure 2. 2D plots to show the evolution of $\beta_{C1}$ and $T\beta_{T}$ for different dissipation coefficients. In particular, we show the evolution of models with (from top left to bottom right) $Q$ constant, cubic, linear and inversely proportional to $T$.

Figure 3 shows the evolution of $T(\phi)$ and $Q(\phi) \equiv Q(\phi, T(\phi))$ for two illustrative cases, given by the values $\alpha = 2$ and $C = 10^5$, with a dissipation coefficient ratio cubic in $T$ and $\alpha = 4$, $C = 2 \times 10^{-3}$ with a dissipation coefficient ratio linear in $T$. During inflation the field monotonically evolves from large to small values and conversely the dissipation coefficient ratio (top panels of figure 3) monotonically evolves from small to large values. As a consequence, we expect the models to switch from the weak dissipative regime $Q \ll 1$ to the strong dissipative limit $Q \gg 1$ discussed in section 4.1.2. We expect the dissipation coefficient $Q(\phi)$ to be monotonically growing with $\phi$ during the phase of inflation. On the contrary, the radiation temperature $T(\phi)$ tends to approach the temperature of the “thermal bath” of the inflaton energy density. As the latter is expected to slightly decrease during inflation, the expected behavior of radiation temperature $T(\phi)$ is to be decreasing towards the end of inflation after a possible initial phase of growth. The top and bottom panels of figure 3 clearly reproduce these behaviors for $Q(\phi)$ and $T(\phi)$.

One notes that in both the plots of $Q$ and $T$, the curves are asymptotically approaching (both for $Q \ll 1$ and for $Q \gg 1$) the power law behaviors predicted in section 4.1.2. The transition from the weak to the strong dissipative limit appears to be sharper in the Cubic case. This is a direct consequence of the different dependences of $Q$ on $\phi$ in the asymptotic behaviors. Hence, the good agreement between theory and numerical simulations confirms the robustness of the numerical methods.
The sole exception to this behavior is the cubic case with $\alpha = 4$ excluded by the Planck constraints in the cold case can be recovered in the warm scenario. For each model considered, a unique prediction for the scalar-spectral index and the tensor-to-scalar ratio is obtained. Figure 4 shows the evolution of the predictions for $n_s$ and $r$ for the chaotic class with different types of dissipation coefficient ratio. As expected, for very small values of $Q$ at CMB scales, the CMB observables are, as expected, matching the predictions of the usual cold inflation case, which in the plots shown in figure 4, are represented by a red star. For larger values of $Q$ the predictions are modified as typically happens in warm inflation. It is worth pointing out that the modification of the predictions, see in particular the linear and cubic cases with $\alpha = 4$, are qualitatively in agreement with the results of ref. [51]. The small difference, at around the 1% level, in the predicted values of $n_s$ is mainly due to slightly different values of $T$ in the numerical evolution and the chosen value of $N_{\text{CMB}}$ used in the present work. As expected, the value of $n_s$ increases and the value of $r$ decreases with $Q$ and $T$ and, thus, models which are in tension with (or even excluded by) the Planck constraints in the cold case can be recovered in the warm scenario. The sole exception to this behavior is the cubic case with $\alpha = 2$ of figure 4. In this case the values of $Q$ and $T$ are small at CMB scales implying that the spectrum is not modified by thermal/dissipative effects. However, as at smaller scales the production of radiation induces a friction that slows down the evolution of the inflaton field, we see, similarly to ref. [53], a decrease of $n_s$ and an increase in $r$ due to shifting of the point of the potential probed by

**Figure 3.** Loglog plots of the evolution of $Q(\phi) \equiv Q(\phi, T(\phi))$ (top plots) and $T(\phi)$ (bottom plots) during the last 60 e-folds of inflation for $\alpha = 4$, $C = 10^6$ (left plots) with a dissipation coefficient ratio cubic in $T$ and $\alpha = 4$ and $C = 2 \times 10^{-3}$ (right plots) with a dissipation coefficient ratio linear in $T$ compared with the analytical predictions of section 4.1.2. In these models $\phi$ decreases during inflation.

For each model considered, a unique prediction for the scalar-spectral index and the tensor-to-scalar ratio is obtained. Figure 4 shows the evolution of the predictions for $n_s$ and $r$ for the chaotic class with different types of dissipation coefficient ratio. As expected, for very small values of $Q$ at CMB scales, the CMB observables are, as expected, matching the predictions of the usual cold inflation case, which in the plots shown in figure 4, are represented by a red star. For larger values of $Q$ the predictions are modified as typically happens in warm inflation. It is worth pointing out that the modification of the predictions, see in particular the linear and cubic cases with $\alpha = 4$, are qualitatively in agreement with the results of ref. [51]. The small difference, at around the 1% level, in the predicted values of $n_s$ is mainly due to slightly different values of $T$ in the numerical evolution and the chosen value of $N_{\text{CMB}}$ used in the present work. As expected, the value of $n_s$ increases and the value of $r$ decreases with $Q$ and $T$ and, thus, models which are in tension with (or even excluded by) the Planck constraints in the cold case can be recovered in the warm scenario. The sole exception to this behavior is the cubic case with $\alpha = 2$ of figure 4. In this case the values of $Q$ and $T$ are small at CMB scales implying that the spectrum is not modified by thermal/dissipative effects. However, as at smaller scales the production of radiation induces a friction that slows down the evolution of the inflaton field, we see, similarly to ref. [53], a decrease of $n_s$ and an increase in $r$ due to shifting of the point of the potential probed by
in this work and some power potentials of the form

\[ V(\phi) = V_0 \phi^\alpha. \]  

Note that the amplitudes are always fixed in order to respect the COBE normalization. For \( \alpha = 2 \) the \( \phi \) dependence is the same as in the well known case of chaotic inflation [57]. As expected, the two sets of curves are perfectly matching for large values of \( \phi \), meaning deep in the inflationary phase, where \( \beta_{CI} \) is much smaller than one and the potentials predicted by eq. (3.15) are well approximated by power laws. Conversely, for small values of \( \phi \), higher order corrections induce a deviation in \( V(\phi) \) from the power law behavior observed at large scales. This type of analysis might be of particular use in the problem of reconstructing potentials in warm inflation [58].
Scalar field $\phi$

Potential $V(\phi)$

Figure 5. Comparison between the inflationary potentials set by eq. (3.15) (for $\alpha = 4$) and power laws potentials $V(\phi) = V_0\phi^4$.

6 Conclusions and future perspectives

In this work we have discussed the application of the $\beta$-function formalism for inflation to the case of warm inflation. We have shown in section 3 that a consistent treatment of warm inflation can be carried out in the language of this $\beta$-function formalism. Interestingly, we have found that despite the presence of an additional functional freedom with respect to the cold case, a universal description still exists. For example, we have demonstrated that models with different functional forms for the dissipation coefficient ratios can give rise to very similar cosmological observables. Moreover, we have shown that this formalism naturally offers an interesting graphical representation of the inflationary phase in terms of bidimensional plots in a plane of the variables $(\beta_{CI}, T\beta_T)$, depicting the departure from the usual cold inflation case. A peculiar property of these results is that they provide a clear insight on the Universe energy budget in the last part of inflation, which in turns allows us to infer some of the necessary properties of (p)reheating.

We have also discussed in section 4 the definition of both numerical and analytical techniques used to perform a systematic study of warm inflation within this framework. The results of the numerical analysis were then presented and discussed in section 5. All the plots show an extremely good agreement between numerical results and theoretical predictions. In particular, we stress the accuracy of the predictions for the power law behaviors of the dissipation ratio $Q$ and temperature $T$ in both the small and large $Q$ limits. These analytical approximations could provide an extremely useful tool for further studies on the topic. For example, by studying the consistency of the conditions $Q \ll 1$ and $Q \gg 1$ with the analytical expressions, it is possible to understand at a fully analytical level whether a given model could or could not access the cold or warm regime respectively.

While in this paper our interest was mainly focused on the chaotic class of potential, the generalization of the analysis to different classes would be an interesting subject for future works on this topic and should follow similarly the steps put forward in this work. In particular, as already explained in section 4, different scaling solutions (for small and large $Q$) are expected to be obtained for different classes. These analyses would be extremely useful in expanding and strengthening our understanding of warm inflation. Moreover, the deepening of our comprehension on the effects of interactions between the inflaton and radiation could
result in a definite step towards the formulation of a theory of inflation which is somehow connected with the rest of the fundamental interactions.

Finally, it is worth mentioning that in order to keep a direct connection with previous works on this topic (and also with theory), we always proceeded by first specifying $\beta_{CI}$ and $Q(\phi, T)$ and then computing $T(\phi)$ (and thus $Q(\phi) \equiv Q(\phi, T(\phi))$) by numerically solving eq. (3.8). However, it could also be equivalently possible to start by fixing $Q(\phi)$ and then identifying the parameterizations of $Q(\phi, T)$ which correspond to this choice. While formally these two possibilities are exactly equivalent, the latter presents some computational advantages and has theoretical interest, namely

- By starting with a fixed parameterization for $Q(\phi)$ it could be possible to solve eq. (3.8) analytically. This implies that it could be possible to provide a full analytical treatment of some models of warm inflation;

- As a single parameterization of $Q(\phi)$ corresponds to several parameterizations of $Q(\phi, T)$, by specifying $Q(\phi)$ we are not restricting our analysis to a single model but rather to a class of models sharing the same properties. In this sense such an analysis would be more general than the one obtained by specifying $Q(\phi, T)$. Interestingly, the universality which is manifest at the background level is not expected to be broken by quantum perturbations. In particular this can be directly seen from eq. (A.6)–(A.7), where it is manifest that all the quantities appearing in the expressions of the spectra can be directly computed once $\beta_{CI}$ and $Q$ are specified.

Such an analysis would be an extremely interesting topic for future studies on warm inflation. In particular, it would be interesting to understand how, given a parameterization of $\beta_{CI}$, it could be possible to reproduce the usual parameterizations of $Q$ given, e.g., by eq. (4.1), using $Q(\phi)$.

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A Compendium of useful formulae

In this appendix we give some of the formula necessary to connect the analyses carried out in the framework of the $\beta$-function formalism for inflation with the rest of the literature on (warm) inflation, which is typically expressed in terms of usual techniques relying on the specification of the inflationary potential. As already discussed in section 3, any model of warm inflation can be completely specified by a choice of $\beta_{CI}(\phi)$ and $Q(T, \phi)$. The parameterizations of $\beta_{CI}$ are constrained by the fact that inflation is realized by departing from a
region with $\beta_{\text{CI}} \ll 1$ and the parameterizations of $Q(T, \phi)$ are typically fixed by theory. Once these two quantities are specified, all the observables can be consistently expressed in term of $\beta_{\text{CI}}(\phi)$ and $Q(T, \phi)$. It is important to stress that the formalism not only applies to the background but also to perturbations.

The Hubble slow-roll parameters\(^6\) $\epsilon_1 \equiv -\dot{H}/H^2$, $\epsilon_{i+1} \equiv \dot{\epsilon}_i/(H\epsilon_i)$ in terms of $\beta_{\text{CI}}$ and $Q$, read

\[
\epsilon_1 = \frac{1}{2} \frac{\beta_{\text{CI}}^2}{1 + Q} = \frac{1}{2} (1 + Q) \beta^2(\phi),
\]

\[
\epsilon_2 = \frac{2\beta_{\text{CI},\phi}}{1 + Q} - \frac{\beta_{\text{CI},\phi}}{1 + Q},
\]

\[
\epsilon_3 = \frac{\beta_{\text{CI}}}{1 + Q} \frac{2\beta_{\text{CI},\phi}}{1 + Q} - \frac{3\beta_{\text{CI},\phi}Q}{(1+Q)^2} - \frac{\beta_{\text{CI},\phi}Q}{(1+Q)^2} + 2\beta_{\text{CI},Q}^2,
\]

\[
\epsilon_{i+1} = \frac{\beta_{\text{CI}}}{1 + Q} \frac{d\ln \epsilon_i}{d\phi}.
\]

In order to have a better connection with the literature on warm inflation, it is also useful to define

\[
\epsilon \equiv \frac{1}{2} \frac{\beta_{\text{CI}}^2}{1 + Q}, \quad \eta \equiv \frac{2\beta_{\text{CI},\phi}}{1 + Q}, \quad \sigma \equiv -\frac{\beta_{\text{CI},Q}}{1 + Q},
\]

such that $\epsilon_1 = \epsilon$ and $\epsilon_2 = \eta + \sigma$.

Regarding cosmological perturbations, using the definitions given in section 3 and in this appendix we translate the expressions of section 2.1 in terms of the typical quantities of the $\beta$-function formalism. We start by expressing the scalar spectrum in terms of $\beta_{\text{CI}}$, $\beta_T$ and $Q$,

\[
\Delta_s^2(k, \tau)_{\tau = k^{-1} = -\frac{2}{3\pi\epsilon}} = \left( \frac{(1 + Q)W}{4\pi\beta_{\text{CI}}} \right)^2 \left( 1 + \frac{2}{\eta} \frac{1}{\eta} - 1 + \frac{\sqrt{12\pi Q}}{3 + 4\pi Q} \beta_T \right) G(Q).
\]

This expression for the scalar spectrum is used to fix $W_T$, the value of the superpotential at the end of inflation, in order for the model to agree with the COBE normalization [1, 3]. In particular, we first derive $W(\phi)$ and then we impose $W_T = W(\phi(N_{\text{CMB}}))$ with $N_{\text{CMB}} = 60$.

For completeness, let us proceed by expressing the tensor power spectrum as

\[
\Delta_t^2(k)_{\tau = k^{-1} = -\frac{2}{3\pi\epsilon}} = \frac{W^2}{2\pi^2},
\]

which has exactly the same expression as in the cold case.

Finally, we provide the predictions for $n_s$ and $r$, with expressions given by

\[
n_s - 1 = \frac{\beta_{\text{CI}}}{1 + Q - \frac{1}{2} \beta_{\text{CI}}} \left[ \frac{2Q}{Q + 1} - \beta_{\text{CI}} - \frac{2\beta_{\text{CI},\phi}}{\beta_{\text{CI}}} + \frac{G_{\phi}}{G} \right.
\]

\[
\left. + \frac{2n^2 \epsilon_2}{\beta_T} \frac{\beta_{T,\phi}}{\beta_T} + \frac{\sqrt{12\pi Q}}{\sqrt{3 + 4\pi Q}} \beta_T - \frac{\sqrt{3\pi Q}}{(3 + 4\pi Q)^{3/2}} (4\pi Q_{\phi} \beta_T) + \frac{\sqrt{12\pi Q}}{\sqrt{3 + 4\pi Q}} \beta_{T,\phi} \right], \quad (A.8)
\]

\[
r = \frac{8\beta_{\text{CI}}^2(\phi)}{(1 + Q)^2} \left( 1 + 2n + \frac{2\sqrt{3\pi Q}}{\sqrt{3 + 4\pi Q}} \beta_T \right) G(Q).
\]

\(^6\) Also known as Hubble flow functions (HFF) [1].
It has been a basic feature of the fluctuation-dissipation dynamics, intrinsic to warm inflation, that the tensor-to-scalar ratio in general is lower as compared to cold inflation. For the $\phi^4$ model, it was predicted from warm inflation in [45, 49], well before the CMB data, that this ratio would be lower. Here we present a compact expression for the tensor-to-scalar ratio simply written as the expression that appears for cold inflation $r_{CI} = 8\beta_{CI}^2(\phi)$ multiplied by a correction factor, the denominator of eq. (A.9). In agreement with the literature [45, 49] and as discussed in section 2.1 this correction term lowers the prediction for $r$ with respect to cold inflation when the dissipation coefficient ratio is of order of unity or when $\beta_T$ is larger than one.

### A.1 Some additional universality classes

Let us conclude this appendix by briefly presenting some of the classes introduced in ref. [4], starting with the so-called monomial class, where

$$\beta_{CI} = \alpha \phi^q,$$

(A.10)

with $\alpha$ and $q$ being positive constants. This class describes small field models, i.e., inflation take place for $\phi \ll 1$, with

$$W(\phi) = W_f \exp \left[ -\frac{\alpha}{2(q+1)} \left( \phi^{q+1} - \phi_f^{q+1} \right) \right],$$

(A.11)

implying that at the lowest order models of this class feature a hilltop potential.

We can also consider the so-called inverse class, where

$$\beta_{CI} = -\frac{\alpha}{\phi^q},$$

(A.12)

with $\alpha$ and $q$ being positive constants. This class describes large field models, i.e., inflation take place for $\phi \gg 1$, with

$$W(\phi) = W_f \exp \left[ \frac{\alpha}{2(q-1)} \left( \frac{1}{\phi^{q-1}} - \frac{1}{\phi_f^{q-1}} \right) \right],$$

(A.13)

implying that at the lowest order models of this class feature an algebraically flat plateau potential.

Finally, is the so-called exponential class, where

$$\beta_{CI} = -\alpha \exp(-\gamma \phi),$$

(A.14)

with $\alpha$ and $\gamma$ being positive constants. This class describes large field models, with

$$W(\phi) = W_f \exp \left\{ -\frac{\alpha}{2\gamma} \left[ \exp(-\gamma \phi) - \exp(-\gamma \phi_f) \right] \right\},$$

(A.15)

implying that at the lowest order models of this class feature an exponentially flat plateau potential.
References

[1] PLANCK collaboration, P.A.R. Ade et al., Planck 2015 results. XX. Constraints on inflation, *Astron. Astrophys.* 594 (2016) A20 [arXiv:1502.02114] [SPIRE].

[2] PLANCK collaboration, P.A.R. Ade et al., Planck 2015 results. XVII. Constraints on primordial non-Gaussianity, *Astron. Astrophys.* 594 (2016) A17 [arXiv:1502.01592] [SPIRE].

[3] PLANCK collaboration, P.A.R. Ade et al., Planck 2015 results. XIII. Cosmological parameters, *Astron. Astrophys.* 594 (2016) A13 [arXiv:1502.01589] [SPIRE].

[4] P. Binétruy, E. Kiritsis, J. Mabillard, M. Pieroni and C. Rosset, Universality classes for models of inflation, *JCAP* 04 (2015) 033 [arXiv:1407.0820] [SPIRE].

[5] M. Pieroni, $\beta$-function formalism for inflationary models with a non minimal coupling with gravity, *JCAP* 02 (2016) 012 [arXiv:1510.03691] [SPIRE].

[6] M. Pieroni, Classification of inflationary models and constraints on fundamental physics, Ph.D. Thesis, APC, Paris (2016) [arXiv:1611.03732] [SPIRE].

[7] P. Binétruy, J. Mabillard and M. Pieroni, Universality in generalized models of inflation, *JCAP* 03 (2016) 060 [arXiv:1611.07019] [SPIRE].

[8] F. Cicciarella and M. Pieroni, Universality for quintessence, *JCAP* 08 (2017) 010 [arXiv:1611.10074] [SPIRE].

[9] M. Pieroni, New perspectives on constant-roll inflation, *JCAP* 01 (2018) 024 [arXiv:1709.03527] [SPIRE].

[10] D.S. Salopek and J.R. Bond, Nonlinear evolution of long wavelength metric fluctuations in inflationary models, *Phys. Rev.* D 42 (1990) 3936 [SPIRE].

[11] K. Skenderis and P.K. Townsend, Hidden supersymmetry of domain walls and cosmologies, *Phys. Rev. Lett.* 96 (2006) 191301 [hep-th/0602260] [SPIRE].

[12] P. McFadden and K. Skenderis, Holography for Cosmology, *Phys. Rev.* D 81 (2010) 021301 [arXiv:0907.5542] [SPIRE].

[13] P. McFadden and K. Skenderis, The Holographic Universe, *J. Phys. Conf. Ser.* 222 (2010) 012007 [arXiv:1001.2007] [SPIRE].

[14] J.M. Maldacena, The large $N$ limit of superconformal field theories and supergravity, *Int. J. Theor. Phys.* 38 (1999) 1113 [hep-th/9711200] [SPIRE].

[15] P. McFadden and K. Skenderis, Holographic Non-Gaussianity, *JCAP* 05 (2011) 013 [arXiv:1011.0452] [SPIRE].

[16] A. Bzowski, P. McFadden and K. Skenderis, Holography for inflation using conformal perturbation theory, *JHEP* 04 (2013) 047 [arXiv:1211.4550] [SPIRE].

[17] J. Garriga and Y. Urakawa, Holographic inflation and the conservation of $\zeta$, *JHEP* 06 (2014) 086 [arXiv:1403.5497] [SPIRE].

[18] J. Garriga, K. Skenderis and Y. Urakawa, Multi-field inflation from holography, *JCAP* 01 (2015) 028 [arXiv:1410.3290] [SPIRE].

[19] N. Afshordi, C. Corianò, L. Delle Rose, E. Gould and K. Skenderis, From Planck data to Planck era: Observational tests of Holographic Cosmology, *Phys. Rev. Lett.* 118 (2017) 041301 [arXiv:1607.04878] [SPIRE].

[20] N. Afshordi, E. Gould and K. Skenderis, Constraining holographic cosmology using Planck data, *Phys. Rev. D* 95 (2017) 123505 [arXiv:1703.05385] [SPIRE].

[21] S.W. Hawking and T. Hertog, A Smooth Exit from Eternal Inflation?, *JHEP* 04 (2018) 147 [arXiv:1707.07702] [SPIRE].
[22] G. Conti, T. Hertog and Y. Vreys, *Squashed Holography with Scalar Condensates*, arXiv:1707.09663 [INSPIRE].

[23] A. Berera and L.-Z. Fang, *Thermally induced density perturbations in the inflation era*, Phys. Rev. Lett. 74 (1995) 1912 [astro-ph/9501024] [INSPIRE].

[24] A. Berera, *Warm inflation*, Phys. Rev. Lett. 75 (1995) 3218 [astro-ph/9509049] [INSPIRE].

[25] A. Berera, M. Gleiser and R.O. Ramos, *A first principles warm inflation model that solves the cosmological horizon/flatness problems*, Phys. Rev. Lett. 83 (1999) 264 [hep-ph/9809583] [INSPIRE].

[26] A. Berera and R.O. Ramos, *Construction of a robust warm inflation mechanism*, Phys. Lett. B 567 (2003) 294 [hep-ph/0210301] [INSPIRE].

[27] M. Bastero-Gil, A. Berera, R.O. Ramos and J.G. Rosa, *Warm Little Inflaton*, Phys. Rev. Lett. 117 (2016) 026401 [arXiv:1604.08838] [INSPIRE].

[28] A. Berera, I.G. Moss and R.O. Ramos, *Warm Inflation and its Microphysical Basis*, Rept. Prog. Phys. 72 (2009) 026901 [arXiv:0808.1855] [INSPIRE].

[29] R.O. Ramos and L.A. da Silva, *Power spectrum for inflation models with quantum and thermal noises*, JCAP 03 (2013) 032 [arXiv:1302.1554] [INSPIRE].

[30] S. Bartrum, M. Bastero-Gil, A. Berera, R. Cerezo, R.O. Ramos and J.G. Rosa, *The importance of being warm (during inflation)*, Phys. Lett. B 732 (2014) 116 [arXiv:1307.5868] [INSPIRE].

[31] M. Bastero-Gil, A. Berera, I.G. Moss and R.O. Ramos, *Cosmological fluctuations of a random field and radiation fluid*, JCAP 05 (2014) 004 [arXiv:1401.1149] [INSPIRE].

[32] M. Bastero-Gil, A. Berera, I.G. Moss and R.O. Ramos, *Theory of non-Gaussianity in warm inflation*, JCAP 12 (2014) 008 [arXiv:1408.4391] [INSPIRE].

[33] I.G. Moss and C. Xiong, *Non-Gaussianity in fluctuations from warm inflation*, JCAP 04 (2007) 007 [astro-ph/0703307] [INSPIRE].

[34] G.S. Vicente, L.A. da Silva and R.O. Ramos, *Eternal inflation in a dissipative and radiation environment: Heated demise of eternity*, Phys. Rev. D 93 (2016) 063509 [arXiv:1509.08983] [INSPIRE].

[35] R. Arya, A. Dasgupta, G. Goswami, J. Prasad and R. Rangarajan, *Revisiting CMB constraints on warm inflation*, JCAP 02 (2018) 043 [arXiv:1710.11109] [INSPIRE].

[36] M. Bastero-Gil, S. Bhattacharya, K. Dutta and M.R. Gangopadhyay, *Constraining Warm Inflation with CMB data*, JCAP 02 (2018) 054 [arXiv:1710.10008] [INSPIRE].

[37] R. Rangarajan, *Current Status of Warm Inflation*, in 18th Lomonosov Conference on Elementary Particle Physics, Moscow, Russia, August 24–30, 2017 (2018) [arXiv:1801.02648] [INSPIRE].

[38] J.S. Schwinger, *On gauge invariance and vacuum polarization*, Phys. Rev. 82 (1951) 664 [INSPIRE].

[39] T. Hayashinaka, T. Fujita and J. Yokoyama, *Fermionic Schwinger effect and induced current in de Sitter space*, JCAP 07 (2016) 010 [arXiv:1603.04165] [INSPIRE].

[40] W. Tangarife, K. Tobioka, L. Ubaldi and T. Volansky, *Dynamics of Relaxed Inflation*, JHEP 02 (2018) 084 [arXiv:1706.03072] [INSPIRE].

[41] R.Z. Ferreira and A. Notari, *Thermalized Axion Inflation*, JCAP 09 (2017) 007 [arXiv:1706.00373] [INSPIRE].

[42] I.G. Moss and C.M. Graham, *Particle production and reheating in the inflationary universe*, Phys. Rev. D 78 (2008) 123526 [arXiv:0810.2039] [INSPIRE].
M. Bastero-Gil, A. Berera and R.O. Ramos, *Dissipation coefficients from scalar and fermion quantum field interactions*, *JCAP* **09** (2011) 033 [arXiv:1008.1929] [insPIRE].

M. Bastero-Gil, A. Berera, R.O. Ramos and J.G. Rosa, *General dissipation coefficient in low-temperature warm inflation*, *JCAP* **01** (2013) 016 [arXiv:1207.0445] [insPIRE].

A. Berera, *Warm inflation at arbitrary adiabaticity: A Model, an existence proof for inflationary dynamics in quantum field theory*, *Nucl. Phys. B* **585** (2000) 666 [hep-ph/9904409] [insPIRE].

M. Bastero-Gil, A. Berera, R.O. Ramos and J.G. Rosa, *General dissipation coefficient in low-temperature warm inflation*, *JCAP* **01** (2013) 016 [arXiv:1207.0445] [insPIRE].

A. Berera, *Warm inflation solution to the eta problem*, *PoS*(AHEP2003)069 [hep-ph/0401139] [insPIRE].

C. Graham and I.G. Moss, *Density fluctuations from warm inflation*, *JCAP* **07** (2009) 013 [arXiv:0905.3500] [insPIRE].

M. Bastero-Gil, A. Berera and R.O. Ramos, *Shear viscous effects on the primordial power spectrum from warm inflation*, *JCAP* **07** (2011) 030 [arXiv:1106.0701] [insPIRE].

M. Bastero-Gil and A. Berera, *Warm inflation model building*, *Int. J. Mod. Phys. A* **24** (2009) 2207 [arXiv:0902.0521] [insPIRE].

A.N. Taylor and A. Berera, *Perturbation spectra in the warm inflationary scenario*, *Phys. Rev. D* **62** (2000) 083517 [astro-ph/0006077] [insPIRE].

M. Benetti and R.O. Ramos, *Warm inflation dissipative effects: predictions and constraints from the Planck data*, *Phys. Rev. D* **95** (2017) 023517 [arXiv:1610.08758] [insPIRE].

M. Bastero-Gil, A. Berera, R.O. Ramos and J.G. Rosa, *Adiabatic out-of-equilibrium solutions to the Boltzmann equation in warm inflation*, *JHEP* **02** (2018) 063 [arXiv:1711.09023] [insPIRE].

V. Domcke, M. Pieroni and P. Binétruy, *Primordial gravitational waves for universality classes of pseudoscalar inflation*, *JCAP* **06** (2016) 031 [arXiv:1603.01287] [insPIRE].

P. Binétruy, *Supersymmetry: Theory, experiment and cosmology*, Oxford University Press, Oxford, U.K. (2006) [insPIRE].

Y. Zhang, *Warm Inflation With A General Form Of The Dissipative Coefficient*, *JCAP* **03** (2009) 023 [arXiv:0903.0685] [insPIRE].

K. Sayar, A. Mohammadi, L. Akhtari and K. Saaidi, *Hamilton-Jacobi formalism to warm inflationary scenario*, *Phys. Rev. D* **95** (2017) 023501 [arXiv:1708.01714] [insPIRE].

A.D. Linde, *Chaotic Inflation*, *Phys. Lett. B* **129** (1983) 177 [insPIRE].

R. Herrera, *Reconstructing warm inflation*, *Eur. Phys. J. C* **78** (2018) 245 [arXiv:1801.05138] [insPIRE].