RiskRank: Measuring interconnected risk

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The Financial Threats That Machines Can See

By Mark Buchanan

Humans have a terrible track record of predicting financial crises in time to fend them off. Some computer scientists think that algorithms might help.

So can computers see what humans cannot? In a new study, computer scientists Samuel Rönnqvist and Peter Sarlin explore what can be achieved with so-called Deep Learning algorithms -- similar to those that Google used to beat the world Go champion or to play 1980s Atari video games at expert human level. Such algorithms work by studying data and learning to recognize patterns. Unleashed on a database of

Sarlin and colleague Markus Holopainen, for example, have already examined how algorithms do at spotting the conditions that make crises more likely, as opposed to recognizing actual distress at banks. Using macroeconomic data for 15 European nations since the 1980s, the machine-learning methods predicted banking crises more accurately -- and, importantly, with fewer false warnings -- than did any of a wide range of more commonly used statistical methods.
Risk: nodes, links or both?
How to aggregate nodes & links?

- Interconnected risk
  - Nodes are vulnerable to shocks...
  - ...and shocks spread through links
- Node vulnerability an old task
  - LDA, Z-score, PD, EWM...
- Network connectivity more recent concern
  - Closeness, IC, PageRank, DebtRank...
- How to aggregate node vulnerability & link connectivity?
**4D data**

- **Data spaces**
  - Entities
  - Time
  - Variables
  - Links

- **Data slices**
  - A multivariate cross-section
  - A cross-sectional time series
  - A multivariate time series
  - A cross-section of interlinkages

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**Diagram**

- **Entities** (e.g. countries)
- **Time** (e.g. years)
- **Variables** (e.g. credit gap)
- **Links** (e.g. equity exposures)
RiskRank: Measuring interconnected risk

- Aggregate indicators & linkages
- We assume a hierarchical & interconnected system of nodes
- RiskRank: Node vulnerability & impact across the system

Key features

- A general-purpose measure of interconnected risk
- Allows disentangling individual, direct & indirect effects
- Allows multiple indirect effects and feedback loops
- Beyond entities, aggregates upward in the hierarchy
Case of systemic risk

- Financial crises triggered by various shocks (unpredictable)...
- ...but widespread imbalances build-up ex ante (identifiable)
- Analytics to identify systemic risk at early stages
  - Nodes (time): Build-up of macro-financial imbalances
  - Links (cross-section): Connected nature of the financial system
- How to assess vulnerability when risk is interconnected?
Related literature

Early-warning models

- **Univariate**: Kaminsky & Reinhart 1999, El-Shagi et al 2013
- **Logit**: Lo Duca & Peltonen, 2013, Betz et al 2014
- **Machine learning**: Holopainen & Sarlin 2015

Interconnectedness

- **Estimated**: Poon et al 2005, Billio et al 2011, Diebold & Yilmaz 2014
- **Real**: Upper & Worms 2004, van Lelyveld & Liedorp 2006, Poledna et al, 2015

Early-warning models & interconnectedness

- **Central**: Rose & Spiegel 2009, Minoiu et al 2013, Rancan et al 2015
- **Pass-through**: Peltonen et al 2015, Hale et al 2015
- **Integrated**: Puliga et al 2014 (≠ stress testing)
Systemic risk aggregation

- From risk indicators to probability
  - Signaling: Monitor univariate indicators
  - Non/linear approaches for combining indicators
  - Ensemble learning for model aggregation

- From interlinkages to connectivity
  - In, out & total strength/degree
  - Betweenness, closeness & eigenvector centrality

- How to combine probabilities and links?
Aggregation operators

- Conventional aggregation operators
  - Min/max: con-/disjunctive operators
  - Weighted mean: fix trade-off & compensatory
  - Quadratic/geometric/harmonic/power $\alpha$ mean
  - Ordered Weighted Average (OWA) (Yager 1988)

- Choquet (1953) integral as a general aggregation operator
  - Includes the above (and more) as special cases
  - Generalizes to non-linearity and non-additivity
  - Extends conventional operators with interactions
Choquet integral

Definition
Fuzzy measure $\mu$ on the finite set $N = \{1, 2, \ldots, n\}$ is a set function $\mu: P(N) \rightarrow [0, 1]$ (where $P(N)$ is the power set of $N$) satisfying the following two conditions:

- $\mu(\emptyset) = 0$, $\mu(N) = 1$;
- Monotonic, non-decreasing: $A \subseteq B$ implies that $\mu(A) \leq \mu(B)$.

Definition
Discrete Choquet integral with respect to a monotone measure $\mu$ is

$$C_\mu(x_1, \ldots, x_n) = \sum_{i=1}^{n} \left( \mu \left( C(i) \right) - \mu \left( C(i+1) \right) \right) x(i)$$

where $x(i)$ denotes a permutation of the $x_i$ values such that $x(1) \leq x(2) \leq \ldots \leq x(n)$ and $C(i) = \{c(i), c(i+1), \ldots, c(n)\}$. 
Additive Choquet integral

The additive Choquet integral is the weighted sum

\[ C_\mu(x_1, \ldots, x_n) = \sum_{i=1}^{n} \mu(c(i)) x(i) \]

2-additive case covers pairwise interactions and individual effects

\[ C_\mu(x_1, \ldots, x_n) = \sum_{i=1}^{n} (v(c_i) - \frac{1}{2} \sum_{j \neq i} l(c_i, c_j)) x_i + \sum_{l(c_i, c_j) > 0} l(c_i, c_j) \min(x_i, x_j) + \sum_{l(c_i, c_j) < 0} |l(c_i, c_j)| \max(x_i, x_j) \]

where \( v(c_i) \) stands for the Shapley-index (average contribution of fixed element \( x_i \) in any subset) and \( l(c_i, c_j) \in [-1, 1] \) for the interaction. This relies on the Möbius transformation of \( \mu \) and that it equals 0 on any subset with cardinality above 2 (Grabisch, 1997)
Case of utility theory

\[ C_\mu(x_1, \ldots, x_n) = \sum_{i=1}^{n} (v(c_i) - \frac{1}{2} \sum_{j \neq i} l(c_i, c_j)x_i + \sum_{l(c_i, c_j) > 0} l(c_i, c_j) \min(x_i, x_j) + \sum_{l(c_i, c_j) < 0} |l(c_i, c_j)| \max(x_i, x_j) \]

![Diagram with criteria and interaction types](attachment:diagram.png)

- **Positive interaction**
  - Complements
  - \( l(c_i, c_j) > 0 \)
  - \( \min \) operator

- **Negative interaction**
  - Substitutes
  - \( l(c_i, c_j) < 0 \)
  - \( \max \) operator

- **No interaction**
  - Independent
  - \( l(c_i, c_j) = 0 \)
  - no operator
From Choquet to RiskRank

For risk levels $x_i$ and links $l(c_i, c_j)$, 2-additive RiskRank is

$$RR_c = w(c)x_c + \frac{1}{2} \sum_{j \neq i} l(c_i, c_j)x_i + \sum_{i=1}^{n} (v(c_i) - \frac{1}{2} \sum_{j \neq i} l(c_i, c_j))x_i +$$

- Individual effect of entity $c$
- Direct effects of entity $i$ on $c$
- Indirect effects of $j$ via $i$ on $c$

which

- separates effects
- allows multiple indirect effects via $k$-additivity
- allows simulated feedback through dynamic iteration
Measuring systemic risk

- EU early-warning model & BIS exposures [App]
  - Sample: 15 EU countries, 1980Q1-2015Q1
  - Domestic risk: EWM, 14 macro-financial indicators
  - Linkages: BIS foreign claims, immediate borrower
  - Output: Country & EU-level measures of interconnected risk
Measuring systemic risk

- **Usefulness** $U_a$: \(\text{[loss of disregarding the model]} - \text{[model loss]}\)
  - $U_r(\mu = 0.8)$: Relative usefulness with imbalanced preferences
  - AUC: Model usefulness for all $\mu$

| $\mu$ | $U_r(\mu)$ | AUC  | $U_r(\mu)$ | AUC  |
|-------|------------|------|------------|------|
| 0.0   | 0 %        | 0.915| 0 %        | 0.934|
| 0.1   | -6 %       | 0.915| 1 %        | 0.934|
| 0.2   | -3 %       | 0.915| 3 %        | 0.934|
| 0.3   | 6 %        | 0.915| 14 %       | 0.934|
| 0.4   | 12 %       | 0.915| 28 %       | 0.934|
| 0.5   | 15 %       | 0.915| 37 %       | 0.934|
| 0.6   | 25 %       | 0.915| 47 %       | 0.934|
| 0.7   | 44 %       | 0.915| 59 %       | 0.934|
| 0.8   | 60 %       | 0.915| 69 %       | 0.934|
| 0.9   | 73 %       | 0.915| 78 %       | 0.934|
| 1.0   | 0 %        | 0.915| 0 %        | 0.934|
Measuring systemic risk

Germany

Europe
Measuring systemic risk
Comparing networks: Direct vs common exposures

Giudici et al. ('16)
News co-occurrences & sentiment

Forss & Sarlin (prel., ’16)
Conclusion

- RiskRank as a measure of interconnected risk
  - Nodes are vulnerable to shocks...
  - ...and shocks spread through links
  - RiskRank aggregates node vulnerability across links

- General properties of RiskRank
  - Allows disentangling individual, direct & indirect effects
  - Allows multiple indirect effects and feedback loops
  - Beyond entities, aggregates upward in the hierarchy
  - Could be used for measuring any interconnected risk
Thanks for your attention!