1 Introduction

The cooling of neutron stars have been used by many authors [1 2 3 4 5 6 7 8 9] as a way of probing the internal composition of these objects. Such studies rely on the fact that the physical quantities relevant for the cooling (specific heat, thermal conductivity, and neutrino emissions) strongly depends on the microscopic composition, and thus different models lead to different thermal evolution. The predicted thermal evolution is compared with observed data, with the ultimate goal of constraining the microscopic model [2 3]. Recent studies along those lines [8 9] has linked the observed thermal behavior of the compact object in CasA [10] to the possible onset of hadronic superfluidity in the core of neutron stars. A possible alternative explanation to the cooling behavior of CasA has been proposed by the authors in [11], where the
observed data was explained as the late onset of the Direct Urca process, triggered by the compression that accompanies a spinning down neutron star.

In the usual approach, studies of the thermal evolution of neutron stars are performed assuming a spherically symmetric object. As pointed out in [11], however, rotation may play an important role on the thermal evolution of neutron stars. Whereas in [11] the effects of rotation in the microscopic composition (and its consequences to cooling) were considered, in the work presented here we complement that study [11], and investigate the macroscopic aspects of the cooling of rotating neutron stars.

In this work we consider the thermal evolution of rigidly rotating neutron stars. In order to perform such study we first calculate the structure of rotating objects, which is considerably more complicated than that of spherical objects. The structure of rotating neutron stars is obtained by solving Einstein’s equation for a rotationally deformed fluid distributions [12, 13]. The numerical method used is based on the the KEH method [14, 15, 16]. The equation of state used for computing the neutron star structure and composition is a simple relativistic mean field (RMF) model, with parameter set G300 [17]. With the structure of rotating neutron stars computed, we calculate the thermal evolution of these objects. In order to do so, we re-derive the thermal evolution equations to account for the metric of a rotating object. The cooling of neutron stars with different frequencies is then calculated. We show that the cooling of the star strongly depends on the frequency of the object, with higher frequencies stars showing a substantial temperature difference between the equator and poles.

2 2D thermal evolution

We review here the thermal evolution equations for a rotating neutron star, as discussed in [18]. The metric of a rotationally deformed fluid can be written as [12],

$$ds^2 = -e^{2\nu}dt^2 + e^{2\phi}(d\varphi - N^e dt)^2 + e^{2\omega}(dr^2 + r^2 d\theta^2),$$

where $e^{2\phi} \equiv e^{2(\alpha + \beta)}r^2 \sin^2 \theta$ and $e^{2\omega} \equiv e^{2(\alpha - \beta)}$. The quantities $\nu$, $\phi$ and $\omega$ denote metric functions, and $N^e$ accounts for frame dragging caused by the rotating fluid. All these functions are to be computed self-consistently from Einstein’s field equation, $G^\alpha\beta = 8\pi T^\alpha\beta$, where $T^\alpha\beta$ denotes the fluid’s energy momentum tensor.

For a uniformly rotating compact star ($\Omega = \text{const}$), the equations of energy balance and transport can be reduce to

$$\partial_t \tilde{T} = - e^{2\nu} \frac{\epsilon}{C_V} + \frac{1}{r^2 \sin \theta} \frac{e^{3\nu - \gamma - 2\xi}}{\Gamma} \frac{1}{C_V} \times$$

$$\left( \partial_r \left( r^2 \kappa \sin \theta e^\gamma \left( \partial_r \tilde{T} \right) \right) + \frac{1}{r^2} \partial_\theta \left( r^2 \kappa \sin \theta e^\gamma \left( \partial_\theta \tilde{T} \right) \right) \right),$$

(2)
with the definitions \( r \sin \theta e^{-\nu + \gamma} = e^{\phi} \) and \( e^{-\nu + \xi} = e^{\alpha - \beta} \). In the above equation \( T \) is the temperature, \( \tilde{T} \equiv e^{\nu T}/\Gamma \), \( \kappa \) is the thermal conductivity, \( C_V \) is the specific heat, \( \epsilon \) is the neutrino emissivity, and the Lorentz factor \( \Gamma \equiv (1 - U^2)^{-1/2} \) where \( U \) is the four velocity. The standard cooling equations of spherically symmetric, non-rotating neutron stars are obtained from Eq. (3) for \( \Omega = 0 \) and \( \partial_\theta \tilde{T} = 0 \) \cite{12}. In this work we solve Eq. (3) for the temperature distribution \( T(r, \theta; t) \) of non-spherical rotating neutron stars. The boundary condition are given by defining the heat flux at \( r = 0, R \), and at \( \theta = 0, \pi/2 \), with \( R \) denoting the stellar radius. The star’s initial temperature, \( T(r, \theta; t = 0) \), is chosen as \( \tilde{T} \equiv 10^{11} \) K. Eq. (3) is the equation that needs to be solved numerically, where the macroscopic input (metric functions and radius) is provided by solution of the structure of the rotating neutron stars, and the microscopic input (specific heat, thermal conductivity and neutrino emissivity) is derived from the equation of state.

In this work we consider the following neutrino processes for the stellar core: the direct Urca, modified Urca and bremsstrahlung processes. A detailed review of the emissivities of such processes can be found in reference \cite{19}. In addition to the core we also consider the standard processes that take place at crust of a neutron star \cite{19}.

### 3 Surface Temperature Evolution

We now present the results for the thermal evolution of rotating neutron stars. In order to evaluate the impact of the 2D structure, we studied the thermal evolution of three different stars, with the same central density, and different frequencies. The properties of these stars are listed in table 1, where \( M \) denotes the gravitational mass, \( R_e \) the equatorial radius, \( R_p \) the polar radius, \( r = R_p/R_e \) is the ratio between polar and equatorial radii, and \( \Omega \) the rotation frequency.

Equation 3 was solved numerically for each star of table 1, and the results are shown in figs. 1, 2. In these figures we plot the redshifted temperature of the poles and of the equator of the star.

| \( \frac{M}{M_{\odot}} \) | \( R_e \) (km) | \( R_p \) (km) | \( r \) | \( \Omega \) (hz) |
|----------------|-------------|-------------|------|------|
| 1.28           | 13.25       | 13.11       | 0.99 | 148  |
| 1.34           | 13.85       | 12.49       | 0.90 | 488  |
| 1.48           | 15.21       | 11.50       | 0.75 | 755  |

Table 1: Properties of the different stars whose thermal evolution was investigated. All stars have a central density of 350 MeV/fm\(^3\), and were computed using a Relativistic Mean Field model with parameter set G300 \cite{17}.  

3
The cooling curves shown in Fig. 1 indicate that there is no difference between the polar and equatorial temperatures. As indicated in Table 1 for the frequency of this star (148 hz) the ratio between polar and equatorial radii $\sim 0.99$. The matching of the polar and equatorial temperatures indicates that the cooling of this object is the same as that of a spherical object. As the frequency of the star increases (or equivalently as $r$ decreases) one can see more significant differences between the polar and equatorial temperatures, with the pole being slightly warmer than the equator. Another notable difference can be seen at the sharp temperature drop at $\sim 100$ years. Such temperature drop is associated with the thermal coupling between the core and the crust of the star, and its magnitude depends on whether or not fast neutrino emission processes are taking place. The microscopic model used in this work allows the Direct Urca process for stars with masses above $1.0 \, M_\odot$, which explains the sharp temperature drop observed. Since this "knee" is associated with the thermal coupling between core and crust [20], it is only natural that it happens at different times for the polar and equatorial direction. This behavior is due to the deformation of the star, which causes it to be flattened at the poles (leading to a thinner crust in the polar direction), and elongated at the equator (leading to a thicker crust). The

Table 2: Same as Fig. 1 but for the 488 hz (right) and 755 hz (left) star.
thinner crust, combined with the low fluid velocity in the polar region, allows the cold front (that originates in the core) to reach this region before the equator, as observed in Figs. 2. For lower frequencies (Fig. 1) the structure approaches that of a spherically symmetric stars and thus this effect vanishes.

4 Conclusions

It was the objective of this work to investigate in details the thermal evolution of rotating neutron stars. Such investigation is extremely important if one wants to understand the thermal evolution of systems in which the spherical symmetry is broken like in highly magnetized neutron stars, in spinning-down neutron stars, and/or of accreting objects. In order to achieve our objective we studied the relatively simple case of a rotating neutron stars with rigid rotation, and constant frequency. The metric of rotating neutron stars is significantly different than that of spherically symmetric objects, hence the general relativistic equations that govern the thermal evolution were re-derived to account for the new metric.

We have studied the thermal evolution of neutron stars with a purely hadronic composition (the whole baryon octet) and that allows for the presence of the direct Urca process. The microscopic model used leads to the so-called ”enhanced” cooling. This choice of equation of state was intentional, since it allow us to study the energy transport inside the rotating neutron star more clearly. After obtaining a clear understanding of the thermal evolution of rotating compact stars, our study can be applied to system with more sophisticated features like hadronic superfluidity, and quark matter for instance.

Our study shows that the 2D thermal evolution of neutron stars can in fact be significantly different than that of spherically symmetric objects. We have found that the deformation in a neutron star (in the case studied here due to rotation) leads to non-uniform surface gravity, which in turns leads to non-uniform surface temperatures. We have also found that the deformation of the crust plays a major role for the thermal evolution of these objects. Now that we have a better understanding of the heat transport inside a deformed neutron star, we intend to extend this research to account for the effects described in [1], where spin-down compression, and its consequences to the cooling of a neutron stars were discussed. We also plan to include magnetic field effects, since those also contribute to braking the spherical symmetry of the object. It is also our intention to explore more sophisticated microscopic models, that account for hadronic superfluidity, and possibly quark matter for instance.
References

[1] C. Schaab, F. Weber, M. K. Weigel, and N. K. Glendenning, Nuclear Phys A, 605, 531, (1996).

[2] D. Page, J. Lattimer, M. Prakash, and A. W. Steiner, The Astrophysical Journal Supplement Series, 155, 623 (2004).

[3] D. Page, U. Geppert, and F. Weber, Nuclear Physics A, 777, 497 (2006).

[4] D. Page, J. Lattimer, M. Prakash, and A. W. Steiner, The Astrophysical Journal, 707, 1131 (2009).

[5] D. Blaschke, T. Klahn, and D. N. Voskresensky, The Astrophysical Journal, 533, 406 (2000).

[6] H. Grigorian, D. Blaschke, and D. Voskresensky, Physical Review C, 71, 045801 (2005).

[7] D. Blaschke, D. Voskresensky, and H. Grigorian, Nuclear Physics A, 774, 815 (2006).

[8] D. Page, M. Prakash, J. Lattimer, and A. W. Steiner, Physical Review Letters, 106, 081101 (2011a).

[9] D. G. Yakovlev, W. C. G. Ho, P. S. Shternin, C. O. Heinke, and A. Y. Potekhin, Monthly Notices of the Royal Astronomical Society, 411, 1977 (2011).

[10] C. O. Heinke and W. C. G. Ho, The Astrophysical Journal, 719, L167 (2010).

[11] R. Negreiros, S. Schramm, and F. Weber, Phys. Lett. B, 718, 1176 (2013).

[12] F. Weber, Pulsars as astrophysical laboratories for nuclear and particle physics (Institute of Physics, Bristol, 1999), 1st ed.

[13] N. K. Glendenning, Compact stars: nuclear physics, particle physics, and general relativity (Springer, 2000), 1st ed.

[14] H. Komatsu, Y. Eriguchi, and I. Hachisu, Royal Astronomical Society, Monthly Notices, 237, 355 (1989).

[15] G. B. Cook, S. L. Shapiro, and S. A. Teukolsky, The Astrophysical Journal, 398, 203 (1992).

[16] N. Stergioulas and J. L. Friedman, The Astrophysical Journal, 444, 306 (1995).
[17] N. K. Glendenning, Nuclear Physics A, 493, 521 (1989).

[18] R. Negreiros, S. Schramm, and F. Weber, Physical Review D, 85, 104019 (2012)

[19] D. G. Yakovlev, A. D. Kaminker, O. Y. Gnedin, and P. Haensel, Physics Reports, 354, 1 (2001).

[20] O. Y. Gnedin, D. G. Yakovlev, and A. Y. Potekhin, Monthly Notices of the Royal Astronomical Society, 324, 725 (2001).