A Model for Luminous and Long Duration Cosmic Gamma Ray Bursts

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We present here a simple and generic model for the luminous \( Q > 10^{52} \) erg and long duration \( (t \sim 10^s) \) Gamma Ray Bursts (GRBs) based on the fundamental fact that the General Theory of Relativity (GTR) suggests the existence of Ultra Compact Objects (UCOs) having surface gravitational red-shift \( z_s \lesssim 0.615 \) even when most stringent constraint is imposed on the equation of state. This simple model may explain the genesis of an electromagnetic fireball (FB) of energy as high as \( Q_{FB} \sim 5 \times 10^{54} \) erg and an initial bulk Lorentz Factor as high as \( \eta \sim 10^3 \).

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It is now clear that a large number of Gamma Ray Bursts (GRB) involve emission of \( \gamma \)-rays as large as \( Q_\gamma \sim 10^{52} - 10^{53} \) erg under condition of isotropy. Afterglow observations of GRB970228, 970508 and 980703 show that they indeed have quasi spherical morphology. In fact, if GRB9901023 were also isotropic, one would infer a value of \( Q_\gamma \approx 3.4 \times 10^{54} \) erg \( \text{[1]} \). However, in this paper we shall not consider the unique case of GRB9901023, which might be anisotropic \( \text{[2]} \), and focus attention on the (other) most luminous events recorded so far. We explain them below as events related to the formation of UCOs whose existence is suggested by GTR irrespective of the details of the EOS of the collapsing matter. Since this is a spherical model, unlike the irregular non-spherical models, the liberated energy will be in the form of photons and neutrinos alone, and, not in gravitational radiation.

As was first shown by Schwarzschild in 1916 \( \text{[3]} \), GTR yields an absolute upper limit on the value of the surface gravitational redshift of a static relativistic spherical star irrespective of the details of the Equation of State (EOS):

\[
z_s = \left(1 - \frac{2GM(R)}{Rc^2}\right)^{-1/2} - 1 \leq z_s = 2 \quad (1)
\]

Here the subscript “s” refers to the respective “surface” values, \( R \) is the invariant circumference radius, \( c \) is the speed of light, and \( M \) is the gravitational mass inclosed within \( R = R \)

\[
M(R) = \int_0^R \rho dV = \int_0^R dM \quad (2)
\]

where \( \rho \) is the total mass-energy density, \( dV = 4\pi R^2 dR \) is coordinate volume element, and the symbol \( dM \) is self-explanatory. Schwarzschild obtained this limit for homogeneous stars by demanding that the central pressure of

the star does not blow up. This result is actually valid even for non-homogeneous stars (see pp.333 of Weinberg \( \text{[4]} \)) and is obtained when the EOS is allowed to have a causality violating sound speed \( c_s = (dp/d\rho)^{1/2} > c \). When the EOS is constrained to obey causality, it follows from Eq.(9.5.19) of Shapiro & Teukolsky (pp.261) \( \text{[6]} \), that one would have a tighter limit on \( z_s = 1.22 \). If one constrains the EOS further so as to have \( c_s \leq c/\sqrt{3} \), it follows \( \text{[5]} \) that one has an even tighter bound on \( z_s = 0.615 \).

To present a realistic model, in the following we shall work with the tightest GTR and EOS bound on \( z_s = z_s = 0.615 \). It may be also noted that this limit on \( z_s \) is independent on the precise value of \( M \) itself. Thus, this limit may be applicable to both stellar mass compact objects like Neutron Stars (NSs) or even supermassive stars, and, hence, it was debated after the discovery of quasars, whether their redshifts were of gravitational origin \( \text{[7]} \).

Note that, the presumed canonical NS has a value of \( M \sim 1M_\odot \) and \( R \sim 10^6 \) km with \( 2GM/Rc^2 \sim 0.26 \) and \( z_s \sim 0.16 \). However, actually, many existing EOSs easily allow a value of \( M = (2 - 3)M_\odot \) and \( R \approx 7Km \) (for degenerate matter, there is inverse relationship between \( M \) and \( R \) \( \text{[8]} \)). This may result in a value of \( 2GM/Rc^2 \approx 0.63 \) or \( z_s \approx 1.0 \). Thus, we can very well have an UCO, in lieu of a so-called NS. And our UCO is nothing but a NS having a compactness, though, higher than the canonical (i.e., assumed) value, very much allowed not only GTR but also by all existing EOSs. So, all that GTR tells here is that for a non-trivial \( (p \neq 0) \) EOS, gravitational collapse process may end with static objects \( \text{[9]} \) having \( z \leq z_s \). Beyond, \( z > z_s \), there would be no stable static configuration. However, light can escape from the collapsing body and the collapse is, in principle, reversible, until one crosses a deadline of \( 2GM/Rc^2 = 1 \) or \( z = \infty \), when the collapse becomes irreversible. However, because of inaccuracies, the present day numerical computations have a blurred vision about \( z_s \), and further, erroneously, they conclude that the collapse becomes irreversible immediately after \( z = z_c \approx 0.16 \).

Recall that the 10 GTR collapse equations form a set of highly complicated coupled non-linear partial differential equations. Even, numerically, they can be solved to obtain a unique result only for the homogeneous dust \( (p = 0) \). Other cases might also be solved up to a certain extent but there could be hundreds of solutions depending on a number of explicit or implicit assumptions one makes, like self-similarity, adiabaticity, polytropic EOS, the variation of the polytropic index, nature of opacities,
radiation transport properties etc, etc. Equally important is the question of the initial conditions one explicitly or implicitly assumes. And then depending on the expectations, one might get the desired result. The genesis of a high \( z_s \) object would be marked by emission of energy flux \( Q \approx M c^2 \) and then it becomes practically impossible to handle the most complex coupled energy transport problem in a precise manner. By definition, in such cases, one requires to work in the strong gravity limit where most of the inherent assumptions break down. For stellar mass objects, although, the high density cold EOS is known with relatively more certainty, our knowledge about finite temperature EOS of nuclear matter at arbitrary high \( T \) is, at present, at is infancy. Also note that for numerical computations, for a total accumulated uncertainty of few percentage, arising from either present theoretical inputs and intrinsic simplifications, a potential result like \( 2GM/Rc^2 \approx 8/9 \) (corresponding to \( z_s \approx 2 \), with finite gravitational acceleration) may precipitate to a “\( 2GM/Rc^2 \approx 1 \)” \( (z \approx \infty \) with infinite gravitational acceleration) signalling the apparent formation of an early “event horizon” or a “trapped surface”. Even if we consider the infinitely simpler problem of collapse of an inhomogeneous dust, there could be varied numerical results, and, in particular, there is a raging debate whether such collapse gives rise to a Black Hole or a “naked singularity”. Such gross uncertainties may, at present, obfuscate the signals of formation of more compact NSs. In any case, as discussed before, both the nuclear EOSs and GTR actually suggest the existence of more compact NSs.

The self-gravitational energy of a static relativistic star is given by

\[
E_g = \int \rho c^2 dV \left( 1 - \left[ 1 - \frac{2GM(R)}{Rc^2} \right]^{-1/2} \right)
\]

Then recalling the definition of \( z \) from Eq.(1), we may write

\[
E_g = - \int z(R)c^2 dM \approx -\alpha z_s M c^2 \sim -z_s Mc^2
\]

where \( \alpha \approx 1 \) is a model dependent parameter. The binding energy, i.e., the energy liberated in the formation of the eventually cold stellar mass compact object, is given by virial theorem to be \( E_B \approx (1/2) | E_g | \). Most of this binding energy is expected to be radiated in the form of \( \nu - \bar{\nu} \) during the final stages of formation of the UCO:

\[
Q_\nu \approx E_B \approx -\frac{z_s Mc^2}{2}
\]

So, given the most restricted limit \( z_s = 0.615 \) the maximum value of \( Q_\nu \approx 0.6 M_c c^2 M_2 \approx 1.2 \times 10^{32} M_2 \text{ erg} \) where \( M = M_2 M_9 \). This is in agreement with our similar previous crude estimate. The value of \( Q_\nu \) measured near the compact object will be higher by a factor \((1 + z_s)\): \( Q_\nu' = z_s(1 + z_s)M/2 \) (now we set \( c = 1 \)).

For the NS-formation case, the neutrinos diffuse out of the hot core in a time \( t_\nu < 10s \) and we may expect a somewhat longer time scale for the diffusion of neutrinos from the nascent hot UCO. However, here note that, the rather long value of \( t_\nu \approx 10 s \) occurs because of coherent scattering of neutrinos by the heavy (Fe) nuclei if the Fe-nucleons are already partially dissociated by an immediately preceding heating, the rise in the value of \( t_\nu \) for the UCO formation need not be much larger.

And the locally measured duration of the burst would be \( t'_\nu = (1 + z_s)^{-1} t_\nu \). Therefore, the mean (local) \( \nu - \bar{\nu} \) luminosity will be

\[
L'_\nu = \frac{Q'_\nu}{t'_\nu} = \frac{z_s(1 + z_s)^2 M}{2t_\nu}
\]

\[\approx 2 \times 10^{53} z_s(1 + z_s)^2 M_2 t^{-1}_{10} \text{ erg/s} \]

where \( t_\nu = t_{10}10s \). It may be noted that this value of \( L'_\nu \) is well below the corresponding \( \nu \)-Eddington luminosity. The luminosity in each flavour will be \( L'_i = (1/3)L'_\nu \). By assuming the radius of the neutrinosphere to be \( R_i \approx R \), the value of effective local neutrino temperature \( T' \) (assumed to be same for all the flavors), is obtained from the condition

\[
L'_\nu = \frac{21}{8} 4\pi R^2 \sigma T'^4
\]

where \( \sigma \) is the Stephan-Boltman constant. Therefore, we have,

\[
T' = \left( \frac{2z_s(1 + z_s)^2 M c^2}{21\pi\sigma R^2 t_\nu} \right)^{1/4}
\]

\[\approx 13.3 \text{MeV} \] \( z_s 0.25 \) \( (1 + z_s)^{0.5} \) \( M_2^{0.25} R_6^{-0.5} t_{10}^{-0.25} \]

where \( R = R_6 10^6 \). For a Fermi-Dirac distribution, the mean (local) energy of the neutrinos is \( E_\nu \approx 3.15T' \approx 48 \text{ MeV} \) (for \( z_s = 0.6 \)). The various neutrinos will collide with their respective antiparticles to produce electromagnetic pairs by the \( \nu + \bar{\nu} \rightarrow e^+ + e^- \) process. The rate of energy generation by pair production per unit volume per unit time, at a distance \( r \) from the center of the star, is given by

\[
\dot{q}_\pm(r) = \sum_i K_{\nu i} G^2 \xi L_i^2(r) \frac{1}{12 \pi^2 c^2 R_5^3} \varphi(r)
\]

where, \( L'_i(r) \approx r^{-2} \) is the \( \nu \)-luminosity of a given flavour above the \( \nu \)-sphere, \( G^2 = 5.29 \times 10^{-44} \text{ cm}^2 \text{ MeV}^{-2} \) is the universal Fermi weak coupling constant squared, \( K_{\nu i} = 2.34 \) for electron neutrinos and has a value of 0.503 for muon and tau neutrinos. Here the geometrical factor \( \varphi(r) \) is

\[
\varphi(r) = (1 - x)^4 (x^2 + 4x + 5); \quad x = [1 - (R_\nu/r)^2]^{1/2}
\]
Now, considering all the 3 flavours, a simple numerical integration yields the local value of pair luminosity produced above the neutrinosphere:

\[ L'_\pm = \int_R^\infty \frac{q_\pm 4\pi r^2 dr}{2\sigma T_r r} \approx \sum_i K_{\nu_i} G F_{\nu_i} E_{\nu_i} L_{\nu_i} \approx 7 \times 10^{51} \text{ erg/s} \]

This estimate is obtained by assuming rectilinear propagation of neutrinos near the UCO. Actually, in the strong gravitational field near the UCO surface the neutrino orbits will be curved with significant higher effective interaction cross-section. Since, most of the interactions take place near the \( r \)-sphere, for a modest range of \( z_s \), we may tentatively try to incorporate this nonlinear effect by inserting a \( (1 + z_s)^2 \) factor in the above expression. On the other hand, the value of this electromagnetic luminosity is found to be much larger.

\[ q_\nu = 0.15 \times 10^{52} \text{ erg/s} \]

Thus, the efficiency for conversion of \( Q_{\nu} \) into \( Q_{FB} \) is

\[ \epsilon_\pm = \frac{Q_{FB}}{Q_{\nu}} \approx 3.3\% \]

In particular, for \( z_s = 0.615 \), \( M_2 = 1 \), \( R_6 = 1 \) and \( t_{10} = 1 \), we obtain a large \( \epsilon_\pm \approx 15.5\% \), and it may be reminded here that the value of \( \epsilon_\pm \) should saturate to a limiting value of \( \sim 40\% \), corresponding to a local statistical equilibrium between the 3 flavours of \( \nu, \bar{\nu} \) and \( e^+, e^- \). This highest value of efficiency may be attained, for instance, for \( R = 7 \text{km} \) and \( M = 2.5 M_\odot \). Correspondingly, we obtain a best estimate of \( Q_{FB} \approx 4.8 \times 10^{53} \text{ erg} \) in this model. And thus we may explain the energy budget of GRB971214, \( Q_e \approx 3 \times 10^{53} \text{ erg} \) without overstretched any theory or making any unusual assumption or invoking any unconfirmed exotic physics (like “strange stars”).

Now, we shall address the question of baryonic pollution: \( \eta = Q_{FB}/\Delta M > 10^2 \). In general all models involving collision and full/partial disruption of compact object(s) will spew out thick and massive debris (few \( M_\odot > M_* > 0.1 M_\odot \)). Part of this debris is likely to settle into a torus and an uncertain small fraction (\( \Delta M \)) may hang around the system and get accreted on a long time scale or may even be unbound. It is practically, impossible to simulate the latter fraction dynamically even in a Newtonian theory. On the other hand, spherical implosion models are completely free from the presence of such unaccountable and intractable thick collisional debris. However, in a normal SN event (assumed to be basically spherical implosion), the ejection of baryonic mass \( \sim 0.1 M_\odot \) occurs probably because of shock mediated hydrodynamic process. Since, by definition, the system is gravitationally bound, any normal hydrodynamic attempt of mass ejection cannot be much successful in a spherical model. But the shock generates additional entropy and heat in its vicinity and might be able to effect the mass ejection. Yet, the shock is constantly depleted of energy and gets stalled because of \( \nu \)-losses, and disintegration of heavy nuclei [8]. Probably, the shock might be rejuvenated by the “shock reheating mechanism” [8]. The energy transfer between neutrinos and matter behind the shock is mediated primarily by the charged current reactions \( \nu_e + n \rightarrow p + e^- \) and \( \bar{\nu}_e + p \rightarrow n + e^+ \). When these reactions proceed to the right, the matter heats up, and conversely, the matter cools. To have a successful and sufficient net heating is a critical phenomenon, and present day (realistic) SN codes are unable to find the shock mediated mass-ejection (explosion) even in a relatively weak nascent-NS gravitational field [10]. It is not surprising then that the same numerical calculations, at present, do not find existence of UCO, whose study involves strong gravity, finite temperature EOS and complex physics. Probably, only for a narrow range of initial conditions and modestly deep gravitational potential well this mechanism of shock ejection is successful. Thus the real issue is how to explain the non-ejection of mass by direct hydrodynamic processes by defying the extremely deep relativistic potential well. On the contrary, the meaningful question is how, for a weak Newtonian potential well, for certain range of initial conditions, there could be successful hydrodynamic mass ejection. Note that an UCO with a modest value of \( z_s \sim 0.5 \) has a potential well which is \( \sim 300\% \) deeper than the one associated with a canonical NS, \( z_s \sim 0.16 \). Again the basic reason that a critical phenomenon like shock heated mass ejection might be successful for the SN case is that as one moves from a relativistic potential well (high \( z_s \)) to a Newtonian well \( (z_s \leq 0.2) \) is that while the local temperature due to \( \nu \)-heating may decrease slowly \( T^\prime \sim z_s^{-2.25} \) and the \( \nu \)-matter interaction cross-section \( \sigma_{\nu,m} \sim T^2 \sim z_s^{-0.5} \), the depth of the potential well drops rapidly \( \sim z_s \). Even then, it is far from clear how the hydrodynamic mass ejection can really occur. In fact there are ideas that departure from spherical symmetry induced by rotation and magnetic field might be important in effecting the SN mass-ejection.

On the other hand, there is a genuine possibility, that all models of cosmological GRBs, irrespective of whether they explicitly invoke the \( \nu + \bar{\nu} \rightarrow e^+ + e^- \) process or not, should involve strong direct electromagnetic or \( \nu \)-heated mass loss. Even if an unusual pulsar is assumed to emit \( \sim 10^{52} \text{ erg/s} \) rather than \( 10^{38-40} \text{ erg/s} \), the superstrong return current impinging back on the pulsar may drive a catastrophic wind, a possibility overlooked so far. On the other hand, for the thin outermost layers of the object
(UCO or an hot accretion torus) emitting the neutrinos, well above the \( \nu \)-sphere, the \( \nu \)- flux \( S_\nu \) may induce a super-Eddington photon flux \( S_{\text{ph}} \) [1].

Also, though, for a torus with uncertain dynamically changing geometry, it is difficult to make any semi-analytical or numerical estimate of such a process, in general this effect is expected to be much more pronounced because its gravitational self-binding (\( z_s \)) is much weaker than that for a spherical UCO surface. And even if a steady state model calculation yields a high value of \( \eta \), the eventual value of \( \eta \) might be very low if the jet is intercepted by this debris. In fact, (by ignoring such unmanageable real life uncertainties and difficulties), detailed Newtonian and crude post Newtonian calculations for the NS-NS collision case have been presented by several authors [12]; and the conclusion is that, it is difficult to understand a value of \( \eta \) higher than few.

For non-spherical configurations, it is difficult to ensure that a small fraction of the accreted matter itself is not contaminating the FB. And the estimate of \( \Delta M \) may be made with much larger confidence only for a spherical model, where by definition, the entire matter, in general, is moving inwardly. Here, a certain fraction of the matter lying above the neutrinosphere may be ejected out by the \( \nu \)-heating and it may be possible to crudely estimate the baryonic mass lying above the \( \nu \)-sphere independent of the details of the problem. The mean cross section for \( \nu_- \)-matter interaction is approximately given as [3]

\[
\sigma_{\nu,m} \approx 9 \times 10^{-44} \left( E_\nu / \text{1 MeV} \right)^2 \text{cm}^2
\]

so that, given our range of \( E_\nu \approx 40 - 50 \text{ MeV} \), the value of \( \sigma_{\nu,m} < 10^{-40} \text{ cm}^2 \). Since the \( \nu \)-optical thickness of the layer above the \( \nu \)-sphere is \( \sim 2/3 \), the surface density of this layer \( \delta \sim 2m_p/(3\sigma_{\nu,m}) \sim 10^{16} \text{ g/cm}^2 \), where \( m_p \) is the proton rest mass. Therefore, the mass of the matter above the \( \nu \)-sphere is \( \Delta M \sim \pi R^2 \delta \sim 10^{59} p \sim 10^{-4} M_\odot R^2 \). Probably, the most detailed work on this problem of \( \nu_- \)-driven mass ejection from a hot nascent NS is due to Duncan, Shapiro \& Wasserman [1]; and the Table 5 of it shows that for \( R \approx 10^6 \text{ cm} \), \( M = 2 M_\odot \), we have \( \Delta M \approx 10^{-4} M_\odot \), if \( T' = 20 \text{ MeV} \). On the other hand, for \( T' = 30 \text{ MeV} \), one has, \( \Delta M \approx 7 \times 10^{-4} M_\odot \). These estimates were made in the framework of Newtonian gravity, and a GTR calculation, if possible, would certainly yield, lower values of \( \Delta M \). Even considering these Newtonian values of \( \Delta M \), we find that the value of \( \eta \) could be easily lie between \( 10^3 > \eta > 10^2 \). Now the occurrence of luminous and long GRBs can be understood by using the existing ideas [3].

Previously, accretion induced collapse of a White Dwarf to a NS was suggested as a model for GRBs [1]. The difficulty of this model was that (i) for a canonical NS with \( M = 1 M_\odot \) and \( z_s \approx 0.16 \), Eq.(12) yields a very low value of \( Q_{\text{FB}} \approx 4 \times 10^{50} \text{ erg} \), (ii) further, the value of \( \eta \) is seen to be too low, and (iii) in the correspond-

ing \textit{weak} gravitational well, it can not be ensured that supernova shock is not launched.

And it is probable that less luminous and short GRBs may occur by various other scenarios too including the so-called “collapsar” or “hypernova” type models [4], which, may explain the origin of a value of \( Q_\gamma \sim 10^{39} - 10^{51} \text{ erg} \) for a duration of \( \tau_s < 1 \text{ s} \) (though in extreme cases they go beyond this range).

\[\text{References}\]

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