Spin-liquid model of the sharp resistivity drop in $La_{1.85}Ba_{0.125}CuO_4$.

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We use the phenomenological model proposed in our previous paper [Phys. Rev. Lett. 98, 237001 (2007)] to analyse the magnetic field dependence of the onset temperature for two-dimensional fluctuating superconductivity $T^{**}(H)$. We demonstrate that the slope of $T^{**}(H)$ progressively goes down as $H$ increases, such that the upper critical field progressively increases as $T$ decreases. The quantitative agreement with the recent measurements of $T^{**}(H)$ in $La_{1.85}Ba_{0.125}CuO_4$ is achieved for the same parameter value as was derived in our previous publication from the analysis of the electron self energy.

Recent experiments on $La_{1-x}Ba_xCuO_4$ at $x = 1/8$ revealed a complex hierarchy of energy scales in this material. It displays a charge ordering transition at $T_{oc} = 54K$, a spin ordering transition at $T_{spin} = 42K$ with a subsequent one order of magnitude drop in the in-plane resistivity, the Berezinskii-Kosterlitz-Thouless (BKT) transition to a two-dimensional superconductivity at $T_{BKT} = 16K$, a crossover from 2D to 3D regime around 10K, and a transition to a true 3D superconductivity at 4K. This hierarchy is summarized and discussed in detail in [2].

It turns out that the temperature $T^{**}$ where the resistivity crossover occurs is sensitive to the c-axis magnetic field which separates this phenomenon separately from the spin ordering. In this paper, we address the issue of this crossover. The measurements performed in a magnetic field $H_c$ revealed that (i) $T^{**}$ marks the onset of fluctuational diamagnetism, and (ii) $T^{**}$ decreases with the field. These two effects and the fact that the resistivity sharply drops $T^{**}$ are consistent with the idea that $T^{**}$ marks the onset of a fluctuational pairing regime without (quasi-) long-range superconducting order. The details of the system behavior near $T^{**}$, however, depend on the underlying model. The authors of [2] considered a model of weakly coupled parallel superconducting stripes. Within this model, $T^{**}$ is the temperature at which the inter-stripe coupling becomes strong, and a vortex liquid is formed.

We propose another explanation, based on the model with a flat Fermi surface in the antinodal regions near $(0, \pi)$ and $(\pi, 0)$ points in the Brillouin zone [3]. Fermions in these regions form two quasi-1D spin liquids coupled by Josephson-type interaction. In this model, the pairing amplitudes in the antinodal regions are developed at $T^{**}$ due to the attractive interactions in the spin-liquid state, however, phase fluctuations at $T >> T^{**}$ are effectively one-dimensional, and are pinned by the defects. At $T^{**}$, the Josephson coupling becomes sufficiently strong to lock the relative phase of the two order parameters at $\pi$, and the system response becomes two-dimensional. This leads to depinning of the phase fluctuations resulting in the drop in the resistivity. Still, because of vortices in the 2D regime, the (quasi)-long-range superconducting order develops only at a smaller $T_c < T^{**}$.

Just like the model of parallel stripes [2], our model of “crossed stripes” near $(0, \pi)$ and $(\pi, 0)$ explains qualitatively the resistivity drop, the absence of fluctuational diamagnetism above $T^{**}$, and the sensitivity of $T^{**}$ to a magnetic field. [2]. However, the measurements of $T^{**}(H)$ put an additional constraint on the theory – not only $T^{**}$ decreases with the field, but $|dT^{**}/dH|$ also decreases as $H$ goes up, i.e., at very low $T$, the critical field below which the system response is two-dimensional, becomes very large. The data for $H < 9T$ can be well fitted by the exponential dependence (see Fig. 1):

$$T^{**}(H) = T^{**}(0) \exp\left(-H/H_0\right), \quad H_0 \approx 7.5T$$

For such $T^{**}(H)$, $|dT^{**}/dH|$ exponentially decreases as $H$ increases. If this trend continued to higher $H$, the critical field $H_{c2}(T)$ defined as $T^{**}(H_{c2}) = T$ would become infinite at $T = 0$.

The $H$ dependence of $T^{**}$ for Josephson-coupled stripes running parallel to each other in the 2D plane,
δT ≪ v

the magnetic field is weak, i.e., χ

Thus we show that the slope of dT**/dH decreases with increasing H for any value of the scaling dimension d of the superconducting order parameter. To achieve a quantitative agreement with the experimental fit [1] we have to set d ≈ 1/2. We have to remind the reader that in [3]

\[ \chi_0(k) = \frac{2}{\Delta^2} \left[ \sin \pi d \frac{\Gamma(2-d)}{\Delta} \right]^{-2+2d} \left| \frac{\Gamma(d/2+i\nu q/4\pi T)}{\Gamma(1-d/2+i\nu q/4\pi T)} \right|^2 \left[ \frac{\pi}{1-d} \right] \]  

Here Γ(...) are Γ–functions, d < 1 is the scaling dimension of the superconducting order parameter, v is the velocity of the phase mode, and Δ is the ultraviolet cut-off. The last term in χ₀ can be neglected as we will only consider T ≪ Δ, when the first term in [3] dominates. Parameters v and d are free parameters of our theory and should be extracted from the experiments in the T region where the superconducting phase fluctuations are essentially one-dimensional (that is, at T below the spin gap, but larger than T**). In [3] we found that the best agreement with the photoemission experiments is obtained when d ≈ 1/2. As we will see, this value is also favored by the observed T**(H) dependence.

Taking a Fourier transform over kₓ, but leaving kᵧ intact, we obtain from [2]:

\[ \chi_{kₓ}(x-x₁) = \chi₀(x-x₁) + J² \int dx'\chi₀(kᵧ)\chi₀(x-x')\chi_{kₓ}(x'-x₁) \]  

In a magnetic field, kₓ → kᵧ + Hx' (we set 2e/c = 1). Setting kᵧ = 0 and x₁ = 0, we obtain integral equation for \( \chi(x) = \chi_{kₓ=0}(x) \) in the form

\[ \chi(x) = \chi₀(x) + J² \int dx'\chi₀(x-x')\chi'(x')\chi₀(Hx') \]  

where \( \chi₀(Hx') \) is given by [3] for k = Hx', and \( \chi₀(x) \) is the Fourier transform of \( \chi₀(k) \). The temperature T**(H) is the one at which \( \chi(x) \) diverges.

Weak fields. Consider first the case when the magnetic field is weak, i.e., T**(H) = T**(0)/(1 − δT), and δT ≪ 1. A simple analysis shows that the parametrical condition for a weak field is \( v^2H/T << 1 \). Expanding the same value of d was postulated on the basis of analysis of the electron self energy. This gives an important check for self-consistency of the theory.

We associate T**(H) with the instability of a 2D pairing susceptibility in the random phase approximation (RPA). Fluctuations beyond RPA transform the instability into a crossover [3]. In zero field, the RPA expression for the susceptibility reads, in momentum space

\[ \chi(kₓ, kᵧ) = \chi₀(kₓ) + J²\chi(kₓ, kᵧ)\chi₀(kₓ)\chi₀(kᵧ) \]  

where \( \chi₀(k) \) is the 1D static pairing susceptibility [3]:

\[ \chi₀(Hx') \text{ in } H, \] we obtain from [4]

\[ \chi₀(Hx') = B_d \left( \frac{2\pi T}{\Delta} \right)^{2d-2} \left[ 1 - A_d \left( \frac{\nu Hx'}{\pi T} \right)^2 \right] \]  

where

\[ A_d = \frac{1}{16} \left[ (d/2) - (d-1)d/2 \right], \]

\[ B_d = \frac{2}{\Delta^2} \sin \pi d \Gamma(1-d) \frac{\Gamma^2(d/2)}{\Gamma^2(1-d/2)} \]  

and \( \psi^{(1)}(x) \) is the derivative of the diGamma function.

Substituting [4] into [5], we obtain an integral equation for \( \chi(x) \) in the form

\[ \chi(x) = \chi₀(x) + J² \int dx'\chi₀(x-x')\chi'(x')\chi₀(0) \]

\[ -J²\chi₀(0)A_d \frac{v²H²}{(\pi T)²} \int dx'\chi₀(x-x')\chi'(x')(x')² \]  

where \( \chi₀(0) = \chi₀(k = 0) \). Taking Fourier transform back to momentum space \( x → kₓ = k \), and integrating by parts, we re-write the integral equation for \( \chi \) as

\[ \chi(k) \left[ 1 - J²\chi₀(k)\chi₀(0) \right] - J²\chi₀(k)\chi₀(0) \frac{A_d v²H²}{(\pi T)²} \chi''(k) = \chi₀(k) \]  

This can be re-expressed as

\[ \left( \epsilon + c₁k² - c₂\frac{∂²}{∂k²} \right) \chi(k) = \chi₀(k) \]  

where \( \epsilon = 1 - (T**(0)/T)^{4-4d} \), \( c₁ = A_d v²/(\pi T)² \), \( c₂ = A_d v²H²/(\pi T)² \), and we defined \( T**(0) = \)
\( (\Delta/2\pi) (B_d J)^{1/(2-2d)} \). This agrees with the zero-field transition temperature in \([3]\). Expanding now in the eigenvalues of the differential equation as

\[
\chi(k) = \sum_n a_n \chi_n(k), \quad \chi_0(k) = \sum_n a_n^{(0)} \chi_n(k) \tag{11}
\]

where \( \chi_n(k) \) are the solutions of

\[
\left( c_1 k^2 - c_2 \frac{\partial^2}{\partial k^2} \right) \chi_n(k) = \epsilon_n \chi_n(k) \tag{12}
\]

we obtain

\[
a_n = \frac{a_n^{(0)}}{\epsilon + \epsilon_n} \tag{13}
\]

The eigenvalues of Eq. (12) can be easily obtained as \[12\] can be re-expressed as a harmonic oscillator

\[
-\frac{1}{2M} \frac{\partial^2 \chi_n(k)}{\partial k^2} + \frac{M \omega^2 k^2}{2} \chi_n(k) = \epsilon_n \chi_n(k) \tag{14}
\]

where \( \omega^2 = 4c_1 c_2 \) and \( M^{-1} = 2 A_d (v/\pi T)^2 \). The eigenfunctions of \([13]\) are \( \epsilon_n = \omega (n + 1/2) \), the lowest one is \( \epsilon_0 = \omega/2 = A v T (\pi T)^2 \). From \([13]\), the instability in the field occurs when \( \epsilon + \epsilon_0 = 0 \), i.e., when \( T = T^{*}(H) = T^{*}(0)(1- \delta T) \), where

\[
\delta T \approx \frac{1}{4(1-d)} \frac{A_d v^2 H}{(\pi T^{*}(0))^2} \tag{15}
\]

We see that at small fields, \( T^{*}(H) \) decreases linearly with \( H \). The linear dependence at small fields is also present in the model of parallel stripes \([3]\). If we formally extrapolate the small-field result to \( T = 0 \), we obtain the upper critical field

\[
H^{ext}_{c2}(T = 0) = \left( \frac{\Delta}{v} \right)^2 (J B_d)^{1/(1-d)} \frac{1-d}{A_d} \tag{16}
\]

The actual \( H_{c2}(T = 0) \) is somewhat smaller in the model of parallel stripes \([3]\), but, as we will see, is much larger than \([10]\) in our model of crossed stripes.

**Strong fields.** Consider now the opposite limit of vanishing \( T \), when \( v^2 H/T >> 1 \), i.e., the expansion in the field is no longer possible. In this limit, we have from \([3]\)

\[
\chi_0(Hx') = \frac{B_d}{|Hx'|^{2-2d}} \tag{17}
\]

where

\[
B_d = (8/\Delta)^2 \sin(\pi d) \Gamma^2 (1-d) (v^2/4\Delta^2)^d \tag{18}
\]

\[
= B_d (2\Delta/v)^{2-2d} \left( T^2 (1-d/2)/T^2 (d/2) \right). \tag{19}
\]

Instead of Eq. \([9]\), we now have

\[
\chi(k) = \chi_0(k) \left[ 1 + J^2 \frac{B_d (2\Delta/v)^{2-2d} \Gamma^2 (1-d/2)}{H^2 - 2d^2} \int dq \chi(q) \right] \tag{20}
\]

Using

\[
\int dx' e^{i(k-q)x'} |x'|^{2-2d} = \frac{\Gamma(2d-1) \sin \pi d}{|k-q|^{2d-1}} \tag{21}
\]

and introducing

\[
\hat{\chi}(k) = \frac{B_d}{|k|^{2-2d}} \tilde{\chi}(k) \tag{22}
\]

It is convenient to re-express this equation in the operator form, as \( \hat{L} \chi(k) = 1 \), and expand in the eigenfunctions of the operator \( \hat{L} \), which we label as \( \tilde{\chi}_m(k) \). We get

\[
\tilde{\chi}_m(k) = \sum_m a_m \tilde{\chi}_m(k) \tag{23}
\]

where \( a_m \) are constants. The eigenvalues \( \lambda_m \) are the solutions of

\[
\hat{L} \tilde{\chi}_m(k) = (1-\lambda_m) \tilde{\chi}_m(k) \tag{24}
\]

where

\[
\hat{L} \tilde{\chi}_m(k) = \tilde{\chi}_m(k) - \frac{J^2 B_d^2 \cos \pi \epsilon/2 \Gamma(\epsilon)}{H^{1-\epsilon}} \int dq \frac{\tilde{\chi}_m(q)}{|q|^{1-\epsilon} |k-q|^{\epsilon}} \tag{25}
\]

Eq. \([26]\) was studied in the context of non-BCS superconductivity (with frequency instead of momentum) \([7]\). A similar equation has been studied in the content of superconductivity in graphene \([8]\). For \( \epsilon > 0 \), the normalized solution of \([26]\) with the largest eigenvalue is

\[
\tilde{\chi}_m(k) = \frac{1}{|k|^\epsilon} \tag{26}
\]

and the eigenvalue is

\[
\lambda_0 = \frac{J^2 B_d^2}{H^{1-\epsilon}} \Psi_\epsilon, \quad \Psi_\epsilon = \frac{\pi^2}{2} \frac{1}{\Gamma^2 (1-\epsilon/2)(\sin \pi \epsilon/4)^2} \tag{27}
\]

The critical field \( H_{c2}(T = 0) \) is determined from \( \lambda_0 = 1 \) and is given by

\[
H_{c2}(T = 0) = [J^2 B_d^2 \Psi_\epsilon]^{1/(1-\epsilon)} \tag{28}
\]

In explicit form, we have
\[ H_{c2}(T = 0) = (J\bar{B}_d)^{1/(1-d)} \left( \frac{2\Delta}{v} \right)^2 \left( \frac{8}{(2d-1)^2} \right)^{1/(1-d)} \left[ \frac{\Gamma(1-d/2)}{\Gamma(d/2)} \right]^{2/(1-d)} \]

\[ = H_{c2}^{extr}(T = 0) \left[ \left( \frac{4A_d}{1-d} \right) \left( \frac{8}{(2d-1)^2} \right)^{1/(1-d)} \left[ \frac{\Gamma(1-d/2)}{\Gamma(d/2)} \right]^{2/(1-d)} \right] \] (29)

Substituting into (31), we re-write it as a differential equation

\[ \partial^2 \chi + \frac{4(J\bar{B}_{c=0})^2}{H} \chi = -2\partial^2 \log(T|e^\epsilon - 1|) \] (33)

where \( \epsilon = \log|x| \). The analysis of this equation shows that the susceptibility diverges at \( H = H_{c2}(T) \propto |\log T| \). This is equivalent to \( T^{**}(H) \propto exp\frac{H}{H_0} \), in agreement with Eq. (1). We see therefore that the high field dependence is well captured by our model with \( d \approx 1/2 \) – the same as we used in the previous work to fit the normal state self-energy.

To summarize, we analyzed the behavior of \( T^{**}(H) \) (or, equivalently \( H_{c2}(T) \)) in the model of two one-dimensional spin liquids near \((0, \pi)\) and \((\pi, 0)\) coupled by Josephson-type interaction. For weak fields we found that \( T^{**} \) decreases linearly with \( H \). Extrapolating this dependence down to zero temperature yields the extrapolated field \( H_{c2}^{extr}(T = 0) \). Considering the strong fields we found that the actual \( H_{c2}(T = 0) \) is always larger than the extrapolated value. The ratio \( H_{c2}(T = 0)/H_{c2}^{extr}(T = 0) \), characterizing the convexity of the \( H_{c2}(T) \)-curve, increases when \( d \) decreases and becomes infinite at \( d \leq 1/2 \). This convex behavior is consistent with the data, and has to be contrasted with the concave behavior for the model of parallel stripes. As a further evidence in support of our model, we found that the experimental \( H_{c2}(T) \) are well described by the theoretical formula with the scaling dimension of the 1D superconducting order parameter \( d \approx 1/2 \). The same \( d \) provides the best fit to the photoemission data, as we argued earlier.

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