A spring-supported fine particle impact damper to reduce harmonic vibration of cantilever beam

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Abstract

Spring-supported fine particle impact damper which integrates the effects of elastic deformation and the plastic deformation performs excellently on the attenuation of vibration in cantilever beam. This article studies the damping performance of spring-supported fine particle impact damper experimentally and establishes a dynamic model for understanding its mechanism. Results of the modeling are compared with conducted experiments based on the defined dimensionless structure parameters. The effects of chamber clearance ratio, stiffness ratio, and power ratio are analyzed with the model. As a result, it is shown that the spring-supported fine particle impact damper reduces 80% of the maximum amplitude of cantilever beam at the resonance point which is better compared with the 40% reduction of single impact damper; the dynamic model of the spring-supported fine particle impact damper is reliable, and there exists optimal structure parameters which are 0.15 of clearance ratio and 0.007 of stiffness ratio for achieving the best damping performance.

Keywords

Impact damper, particle, plastic deformation, support, dynamic model

Introduction

As one of the passive damping techniques, impact damper controls the response of the primary system by utilizing the impact between the free mass (the impactor) and the primary system during a vibration process. Due to its simple structure, low cost, easy implement without external power supply, adaptation to harsh environment, and good damping effects, studies on impact damper have been rapidly developed.

The early impact damper is the single-mass impact damper (SMID), which controls the vibration by the impact between the single impactor and the primary system.\(^1\) It is demonstrated that a properly designed SMID is effective in reducing the response of the primary system. However, the high level of noise and strong contact force which may cause local damage to the structures has limited the application of SMID.

Many efforts have been made to overcome these shortcomings and improve the performance of impact dampers. Popplewell and Semercigil\(^2\) found that the bean bag impact dampers were more effective to reduce the acceleration, contact force, and peak response. Masri,\(^3\) Bapat and Sankar,\(^4\) and Saeki\(^5\) proposed a concept of multunit impact damper to decrease the velocity discontinuity of the primary system in impact. Particle impact damping (PID), which substitutes single...
impactor with filled particles, is a derivative of SMID.\textsuperscript{6–9} Nonobstructive particle damping (NOPD)\textsuperscript{10} is a special form of PID which introduces damping to a structure by filling particles into cavity created within the structure. Li and Darby\textsuperscript{11} introduced a buffer region between the mass and the stops to reduce both acceleration and contact force in collisions. Du and colleagues\textsuperscript{12–14} proposed a new fine particle impact damper which introduces plastic deformation of fine particles to vibration system as perpetual energy dissipation. More and more attentions of researchers have been attracted to investigate the performance of PID.\textsuperscript{15–20}

It is widely reported that during the energy-consuming process coefficient of restitution is a key parameter, the variation of which directly influences the damping effect. However, different viewpoints on whether the relation between them is positive or negative have brought about two different directions of research: (1) to maximize the coefficient of restitution based on nearly elastic collision for achieving better damping effects via effectively reducing the acceleration during impact process; one classic example is Li and Darby’s\textsuperscript{11} buffered impact damper, and (2) to minimize the coefficient of restitution based on nearly plastic collision, such as using fine particle impact damper,\textsuperscript{13} to perform more excellent damping effects under low-frequency vibration (below 50 Hz) than SMID and PID.

This article proposes a new spring-supported fine particle impact damper (SSFPID) as shown in Figure 1. The damper employs a double damping structure: the external is a spring damper with higher coefficient of restitution, while the internal is a fine particle impact damper with a lower coefficient of restitution. The fine particle impact damper,\textsuperscript{13} consisted of a sphere impactor and a certain quantity of fine particles, is supported by springs at its two ends. The new damper is an integration of elastic deformation damper and plastic impact damper. The damping effect is maximized as the amplification effect of springs not only enables more momentum exchange but also does more strong impacts in inner fine particle impact damper. This article carries out the experimental study on the damping performance of SSFPID in a cantilever beam. A dynamic model of SSFPID is established and verified by the experiment data. In this article, the model is also used to analyze the effect of stiffness ratio, chamber clearance ratio, and power ratio for understanding the basic principles of the new damper.

Experiment of damping performance

Experimental device

The purpose of the experiment is to investigate the damping effects of the SSFPID proposed by this article. Figure 2 illustrates the schematic diagram of experimental device, and the experimental device is shown in Figure 3. An electromagnetic vibration exciter is used to excite the root of a cantilever beam with sinusoidal signal. The damper is placed at the end of the cantilever beam, and an accelerometer is equipped to measure the amplitude of the end of cantilever beam. The damper is consisted of inner cylindrical cavity and outer spring. An impactor and a certain amount of fine particles are placed in the cavity. Up to eight groups of spring are connected to both ends of the cavity, as shown in Figure 3. The parameters of the primary system and the damper are shown in Table 1.

Experimental scheme

Experimental studies are performed on the cantilever beams with different types of impact dampers. Based on different components in damper cavities (shown Figure 1) in Table 2, the experiment is grouped into four cases to test the vibration attenuation effects on cantilever beam. In these four different cases, the vibration damping effects of dampers are investigated via the vibration status of cantilever beam within the scope of the first flexural mode.
Experimental results and discussion

Figure 4 exhibits the experimental results of all four cases with dampers in Table 2 under harmonic excitation. Due to the effect of momentum exchange, the maximum amplitude of the single impact damper at the free end of cantilever beam decreases to 60% of the maximum amplitude with no damper applied; the maximum amplitude of the impact damper with fine particles is reduced by nearly 60% compared with no damper because of the plastic deformation of fine particles caused by the collision between impactor and damper cavity; the maximum amplitude of the SSFPID is reduced by 80% accordingly attributed to the double damping effect of the elastic deformation of outer spring and the plastic deformation of fine particles in inner fine particle impact damper. It can be concluded that the SSFPID brings a highest amplitude attenuation ratio and shows the most flat curve in Figure 4, which indicates the stability and adaptability of maximum amplitude to different frequencies.

Figure 5 shows the damping ratio of the damped cantilever beam under free decay vibration. In the experiment of free decay vibration, the free end of cantilever beam is given an initial displacement of 20 mm and then the displacement time history is measured under the condition of free vibration. The damping ratio of the cantilever beam with different dampers in Table 2 is calculated by the testing result of free decay vibration. As shown in Figure 5, the SSFPID has the

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**Table 1.** Parameters of primary system and damper.

| Component              | Parameter       | Value                                      |
|------------------------|-----------------|--------------------------------------------|
| Cantilever beam        | Size            | Length 315 mm, width 45 mm, and height 3 mm |
|                        | Material        | A3 steel                                   |
| Impactor               | Size            | Steel ball with diameter 15 mm             |
|                        | Material        | GCr15 bearing steel                        |
| Cavity of inner damper | Size            | Cylinder with internal diameter 18 mm and height 80 mm |
|                        | Material        | #45 Steel                                  |
| Outer spring           | Specification   | Single-spring stiffness 0.0324 N/mm and 0.34 N/mm |
|                        | Material        | Carbon spring steel                        |

**Table 2.** Components in damper cavities under different cases.

| Damper type                           | Impactor           | Zinc powder | Spring                               |
|---------------------------------------|--------------------|-------------|--------------------------------------|
| No damper                             | No                 | No          | No                                   |
| Single impact damper                  | A Φ15-mm steel ball| No          | No                                   |
| Fine particle impact damper           | A Φ15-mm steel ball| 90 g        | No                                   |
| Spring-supported fine particle impact damper | A Φ15-mm steel ball| 90 g        | Stiffness 0.0648 N/mm                |

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**Figure 3.** Picture of experimental device.

**Figure 4.** Maximum amplitude of free end of cantilever beam in resonance point region.
highest damping ratio, which further indicates the good performance of the damper.

The SSFPID performs two damping mechanisms: the external spring support has a higher coefficient of restitution, which reduces the vibration by the elastic deformation of spring; the internal fine particle impact damper has a lower coefficient of restitution, which can completely consume the kinetic energy by the plastic deformation of fine particles. The double structure makes full use of the characteristics of both damping mechanisms, maximizing the damping performance to a level that could be hardly achieved by traditional impact damper.

A dynamic model of SSFPID

Establishment of systematic differential equation

A 2-degree-of-freedom model is developed to represent the vibration model of a cantilever beam damped by the SSFPID in Figure 6. This system consists of three mass bodies, \( m_1, m_2, \) and \( m_3 \); two stiffness, \( k_1 \) and \( k_2 \); and two damping, \( c_1 \) and \( c_2 \). The principal mass \( m_1 \) vibrates under the external harmonic exciting force \( F(t) \). \( k_1 \) and \( c_1 \) are the stiffness and damping of principal system. \( m_2 \) represents the mass of the damper, and \( m_3 \) is the mass of the impactor. The impact from \( m_3 \) to \( m_2 \) consumes a given value of energy each period; for better analysis, energy approach is used to simplify it into the equivalent damping \( c_2 \), which acts on the cavity \( m_2 \). \( k_2 \) is the stiffness of the outer spring.

For the convenience of calculation, assumptions are as follows:

1. The friction force among \( m_1, m_2, \) and \( m_3 \) is negligible;
2. This impact is inelastic, and the relation of the before and after impact is simulated by coefficient of restitution \( e \);
3. Only vibration in the horizontal direction is considered.

The motion differential equation is

\[
\begin{bmatrix}
  m_1 & 0 \\
  0 & m_2
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
  c_1 + c_2 & -c_2 \\
  -c_2 & c_2
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
  k_1 + k_2 & -k_2 \\
  -k_2 & k_2
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  F \sin \omega t
\end{bmatrix}
\]  

(1)

Provided that

\[
M = \begin{bmatrix}
  m_1 & 0 \\
  0 & m_2
\end{bmatrix}
\]

is the mass matrix

\[
C = \begin{bmatrix}
  c_1 + c_2 & -c_2 \\
  -c_2 & c_2
\end{bmatrix}
\]

is the damping matrix

\[
K = \begin{bmatrix}
  k_1 + k_2 & -k_2 \\
  -k_2 & k_2
\end{bmatrix}
\]

is the stiffness matrix

\[
x = \begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix}
\]

is the displacement vector

\[
\dot{x} = \begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2
\end{bmatrix}
\]

is the velocity vector

\[
\ddot{x} = \begin{bmatrix}
  \ddot{x}_1 \\
  \ddot{x}_2
\end{bmatrix}
\]

is the acceleration vector and
The main stiffness in the nate vibrating followed the inherent frequency of the calculated. The square roots of both eigenvalues are the vector of exciting force. 

where $M_{p}$ is the main mass in the $i$th order and $K_{pi}$ is the main stiffness in the $i$th order.

So, the motion differential equation of the system can be represented as

$$M \ddot{x} + C \dot{x} + Kx = F \sin \omega t$$  

(2)

The system kinetic energy is

$$T = \frac{1}{2} (m_{1} \dot{x}_{1}^{2} + m_{2} \dot{x}_{2}^{2})$$  

(3)

The system potential energy is

$$U = \frac{1}{2} (k_{1}x_{1}^{2} + k_{2}x_{2}^{2})$$  

(4)

The system energy consumption function is

$$E = \frac{1}{2} (c_{1}\dot{x}_{1}^{2} + c_{2}\dot{x}_{2}^{2})$$  

(5)

### Modal analysis

Suppose the main vibration is

$$x = \phi (\sin \omega t + \varphi)$$  

(6)

where $\phi = [\phi_1, \phi_2]$ is the eigenvector and $\phi_i$ is the main vibration mode in the $i$th order.

The system eigenquation is

$$|K - \lambda^2 M| = 0$$  

(7)

By this equation, two eigenvalues $\lambda_1$ and $\lambda_2$ are calculated. The square roots of both eigenvalues are the inherent frequency of the $i$th order, and each coordinate vibrating followed the inherent frequency of the $i$th order is called the main vibration of the $i$th order. The superposition of two main vibrations becomes the inherent vibration of the system.

**Modal matrix of this 2-degree-of-freedom system.** $(K - \lambda^2 M)$ is the system eigenmatrix, marked as $B$, and eigenvalue could be substituted into $B$'s adjoint matrix $adj(B)$, to obtain the main vibration mode $\phi_i$. The main vibration mode has orthogonality. When the inherent frequency of the main vibration mode differs, both the mass matrix and the stiffness matrix are orthogonal to it.

Among the main vibration modes with the same inherent frequency, there is

$$\phi_i^T M \phi_i = M_{pi}$$  

(8)

$$\phi_i^T K \phi_i = K_{pi}$$  

(9)

Calculate the system dynamical matrix

$$H = M^{-1} K$$  

(10)

Substitute formula (10) into the following one

$$|\lambda I - H| = 0$$  

(11)

Obtain eigenvalue $\lambda$ and then make $|\lambda I - H| = [f(\lambda)]$ to obtain adjoint matrix $[F(\lambda)]$ of $[f(\lambda)]$. Substitute $\lambda$ into the adjoint matrix $[F(\lambda)]$ and reconstruct $[F(\lambda)]$. For each three eigenvalues, select a nonzero array of $[F(\lambda)]$ to constitute a new matrix. This is the required modal matrix.

**Coordinate exchange.** The vibration mode matrix or the modal matrix could be illustrated as

$$\Phi = [\Phi_1, \Phi_2]$$  

(12)

To change the system coordinates as follows

$$x = \Phi \eta$$  

(13)

Thus, the original vibration equation becomes

$$\Phi^T M \Phi \ddot{\eta} + \Phi^T C \Phi \dot{\eta} + \Phi^T K \Phi \eta = \Phi^T F(t)$$  

(14)

That is

$$M_p \ddot{\eta} + C_p \dot{\eta} + K_p \eta = Q(t)$$  

(15)

where $Q(t)$ is the exiting force in the principal coordinate, which is indicated as

$$Q(t) = \begin{bmatrix} Q_{1}(t) \\ Q_{2}(t) \end{bmatrix} = \Phi^T F(t) = \begin{bmatrix} \Phi^T & 0 \\ 0 & F(t) \end{bmatrix}$$  

(16)

Also

$$\Phi^T M \Phi = M_p$$  

(17)

$$\Phi^T K \Phi = K_p$$  

(18)

$$\Phi^T C \Phi = C_p$$  

(19)

So, the matrix after coordinate exchange is expressed as follows

$$M_p = \begin{bmatrix} M_{p1} & 0 \\ 0 & M_{p2} \end{bmatrix}$$  

is the main mass matrix

$$K_p = \begin{bmatrix} K_{p1} & 0 \\ 0 & K_{p2} \end{bmatrix}$$  

is the main stiffness matrix and
The damping matrix.

Steady-state response

In this way, there is a decoupling of formula (15), for the th equation is

\[ M_p \ddot{\eta}_i + C_p \dot{\eta}_i + K_p \eta_i = Q_i(t) \]  (20)

or

\[ \ddot{\eta}_i + 2\xi_i \omega_i \dot{\eta}_i + \omega_i^2 \eta_i = \frac{Q_{0i}(t)}{M_p} \sin \omega t \]  (21)

where \( \xi_i \) is the damping ratio of the vibration mode in the th order. It is easy to determine the steady-state response of this main vibration mode as

\[ \eta_i(t) = \frac{Q_{0i}}{K_p} \beta_i \sin(\omega t - \Phi_i) \]  (22)

For \( \Phi_i \) is the phase difference in the th order, and \( \beta_i \) is the amplification factor. The definition is

\[ \beta_i = \frac{1}{\sqrt{(1 - \lambda_i^2)^2 + (2\xi_i \lambda_i)^2}} \]  (23)

\[ \Phi_i = \tan^{-1} \frac{2\xi_i \lambda_i}{1 - \lambda_i^2} \]  (24)

where \( \lambda_i \), the ratio of the excitation frequency and the inherent frequency of the th order, is called the frequency ratio of the th order and is shown as

\[ \lambda_i = \frac{\omega}{\omega_i} \]  (25)

The system equation is

\[ M \ddot{x} + C \dot{x} + K x = F(t) \]  (26)

By Laplace transformation, the above equation is considered with a starting condition as 0

\[ (s^2 M + sC + K)x(s) = F(s) \]  (27)

where \( x(s) \) and \( F(s) \) are the corresponding Laplace transformations and \( s \) is the complex variable.

Given that

\[ G(s) = (s^2 M + sC + K)^{-1} \]  (28)

Say it as the transfer function matrix and then

\[ x(s) = G(s)F(s) \]  (29)

To transform \( G(s) \) could lead to its modal expansion formula

\[ G(s) = \Phi \Phi^\dagger \left( \begin{array}{c} s^2 M + sC + K \end{array} \right)^{-1} \]  (30)

\[ = \sum_{i=1}^{n} \frac{\phi_i}{K_p\left(1 - \lambda_i^2 + i\omega \xi_i \lambda_i\right)} \]  (31)

where \( \omega \) and \( F(s) \) are the Fourier transformations of steady-state response and the exciting force vector, respectively, and thus, the modal expansion formula of \( H(\omega) \) is

\[ H(\omega) = \sum_{i=1}^{n} \frac{\beta_i e^{-i\Phi_i}}{K_p} \phi_i \phi_i^\dagger \]  (32)

Substituting formulae (23) and (24), the result is

\[ H(\omega) = \sum_{i=1}^{n} \frac{\beta_i e^{-i\Phi_i}}{K_p} \phi_i \phi_i^\dagger \]  (33)

To express the steady-state response of this vibration system as the following complex number form

\[ x = \bar{B}e^{i\omega t} \]  (34)

To change the \( F \sin \omega t \) in formula (2) into a form of complex number as \( F e^{i\omega t} \) and then substitute formula (34) into it, the result is

\[ B = K - \omega^2 M + i\omega C^{-1} F = H(\omega)F \]  (35)

Based on the above formula and formula (33), the result is as follows

\[ x = \sum_{i=1}^{n} \beta_i e^{-i\Phi_i} \phi_i \phi_i^\dagger F e^{i\omega t} \]  (36)

This system is of 2 degrees of freedom, and therefore, \( n = 2 \), and the steady-state response of the system is illustrated as

\[ x = \sum_{i=1}^{2} \beta_i \phi_i \phi_i^\dagger F \sin(\omega t - \Phi_i) \]  (37)

Parameter calculation

The mass of the cantilever beam system, \( m_1 \), is 1.531 kg; the mass of the damper, \( m_2 \), is 0.359 kg; and the mass of the impactor steel ball, \( m_3 \), is 0.024 kg. The sectional inertia moment of the cantilever beam is
\[ I = \frac{bh^3}{12} = 153.6 \text{ mm}^4 \]. The stiffness of the cantilever beam is \( k_1 = \frac{3EI}{l^3} = 7.8 \text{ N/mm} \). The damping coefficient of the cantilever beam is calculated by vibration attenuation test. The damping ratio of the cantilever beam is \( \xi = 0.00577 \), and the damping coefficient is \( c_1 = 2\xi\sqrt{k_1m_1} = 0.397 \).

The stiffness of the cantilever beam is
\[ k_1 = \frac{3EI}{l^3} = 7.8 \text{ N/mm} \]

The damping specific volume created by the damper is
\[ \psi = \frac{\Delta T}{T} \]  
where \( \Delta T \) represents how much kinetic energy transforms into thermal energy in a circulation, and \( T \) is the maximum kinetic energy during the circulation
\[ \Delta T = \frac{1}{2} \left( 1 - e^2 \right) \frac{m_1 m_2}{m_3 + m_2} \left( v_i^2 - v_i^- \right)^2 \]

Since the mass of \( m_1 \) is relatively large, the maximum kinetic energy of this system could be approximately known as the maximum kinetic energy of \( m_1 \) and, therefore, could be considered as follows
\[ T = \frac{1}{2} m_1 v_1^2 \]

The impact acting on \( m_2 \) could also be approximately seen as the damping
\[ c_2 = \frac{\psi}{2\pi} \sqrt{k_2 m_2} \]

**Calculation procedures**

This article uses MATLAB to program the mentioned calculation procedures. The flowchart of MATLAB simulation procedure is shown in Figure 7.

**Comparison between simulation results and experimental data**

Several dimensionless parameters are defined below for better analyses:

1. Amplitude ratio: the ratio of system stable amplitude to amplitude without damper;
2. Frequency ratio: the ratio of excitation frequency to natural frequency without damper;
3. Clearance ratio: the ratio of clearance to amplitude without damper;
4. Stiffness ratio: the ratio of stiffness of additional springs to stiffness of the main system;
5. Power ratio: the ratio of input power to system rated power;
6. Amplitude decay rate
\[ \sigma = \left( \frac{A_R - X_R}{A_R} \right) \times 100\% \]

In this formula, \( A_R \) represents the amplitude without damper and \( X_R \) is the amplitude with damper.

Experiments take the clearance 36 mm; the copper powder as filled particles; the filling rate 20%; the mass of the impactor steel ball 0.024 kg; and the spring stiffness 0.34, 0.68, 1.02, and 1.36 N/mm, respectively. The comparison between modeling results and experimental results is shown in Figure 8. Figure 8 shows a favorable consistency between MATLAB simulation results and experiment results, which verifies the validity of the established dynamic model. The discrepancy between simulation and experiment results might be caused by the simplification of the model in which the plastic deformation of fine particles is simplified into the equivalent damping \( c_2 \) by energy approach.

**Model calculation results and analyses**

On the basis of established dynamic model, a numerical simulation is employed on the new damper to explore the basic principle and effect of structure parameters and find out the optimal parameters for achieving best damping performance.

**Effect of clearance ratio**

In order to determine the optimal chamber clearance value, the influence of clearance ratio on amplitude ratio is measured under the conditions of different stiffness values. It can be observed from Figure 9 that there is an optimal clearance ratio of 1.5. As the clearance ratio increases, the amplitude ratio first decreases and then increases.
Effect of stiffness ratio

The correlation between amplitude ratio and stiffness ratio is shown in Figure 10. There is an optimal stiffness ratio of 0.007 where the minimum of amplitude ratio is obtained. When the stiffness ratio is below 0.044, the amplitude ratio initially drops and then

Figure 8. Comparison between simulation results and experiment results under spring stiffness: (a) 0.34 N/mm, (b) 0.68 N/mm, (c) 1.02 N/mm, and (d) 1.36 N/mm.

Figure 9. Influence of clearance ratio on amplitude ratio.

Effect of stiffness ratio

The correlation between amplitude ratio and stiffness ratio is shown in Figure 10. There is an optimal stiffness ratio of 0.007 where the minimum of amplitude ratio is obtained. When the stiffness ratio is below 0.044, the amplitude ratio initially drops and then
slowly increases. As the stiffness ratio is above 0.044, the amplitude ratio rapidly increases. The damper shows favorable performance below stiffness ratio of 0.044.

Effect of power ratio

The influence of power ratio on amplitude decay rate is illustrated in Figure 11. From Figure 11, the system amplitude decay rate is approximately proportional to the power ratio. In other words, the higher the input energy, the better the damper’s damping performance turns out to be.

Conclusion

This article carries out the experimental study and theoretical modeling for a new SSFPID. The integration of the elastic deformation of outer spring and the plastic deformation of inner particles makes the SSFPID exhibiting better damping performance than traditional impact dampers. The established dynamic model of the SSFPID is verified reliable to simulate the damping performance by the experiment. As known from the dynamic model analysis, to achieve the best damping performance, there is an optimal parameter combination for the new damper, which is clearance ratio of 0.15 and stiffness ratio of 0.007.

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References

1. Lieber P and Jensen DP. An acceleration damper: development, design and some applications. *Trans ASME* 1945; 67: 523–530.
2. Popplewell N and Semercigil SE. Performance of the bean bag impact damper for a sinusoidal external force. *J Sound Vib* 1989; 133: 193–223.
3. Masri SF. Analytical and experimental studies of multiple-unit impact dampers. *J Acoust Soc Am* 1969; 45: 1111–1117.
4. Bapat CN and Sankar S. Multounit impact damper re-examined. *J Sound Vib* 1985; 103: 457–469.
5. Saeki M. Analytical study of multi-particle damping. *J Sound Vib* 2005; 281: 1133–1144.
6. Papalou A and Masri SF. Performance of particle dampers under random excitation. *J Vib Acoust* 1996; 118: 614–621.
7. Papalou A and Masri SF. An experimental investigation of particle dampers under harmonic excitation. *J Vib Control* 1998; 4: 361–379.
8. Olson SE. An analytical particle damping model. *J Sound Vib* 2003; 264: 1155–1166.
9. Wong CX, Daniel MC and Rongong JA. Energy dissipation prediction of particle dampers. *J Sound Vib* 2009; 319: 91–118.
10. Panosimian HV. Structural damping enhancement via non-obstructive particle damping technique. *J Vib Acoust* 1992; 114: 101–105.
11. Li K and Darby AP. An experimental investigation into the use of a buffered impact damper. *J Sound Vib* 2006; 291: 844–860.
12. Du YC, Wang SL and Zhang JL. Energy dissipation in collision of two balls covered by fine particles. *Int J Impact Eng* 2010; 37: 309–316.
13. Du YC and Wang SL. Modeling the fine particle impact damper. *Int J Mech Sci* 2010; 52: 1015–1022.
14. Li HC, Wang SL, Du YC, et al. Durability of particle impact dampers. *J Vib Shock* 2013; 32: 156–160.

15. Lu Z, Lu XL, Lu WS, et al. Shaking table test of the effects of multi-unit particle dampers attached to an MDOF system under earthquake excitation. *Earthquake Eng Struct* 2012; 41: 987–1000.

16. Lu Z, Lv XL and Yan WM. A survey of particle damping technology. *J Vib Shock* 2013; 32: 1–7.

17. Ma CW and Mu QS. Passive damping effect of particle damper on free vibration of a cantilever beam. *J Lanzhou Univ Techno* 2014; 40: 165–168.

18. Trigui M, Foltete E and Bouhaddi N. Prediction of the dynamic response of a plate treated by particle impact damper. *Mech Eng Sci* 2014; 228: 799–814.

19. Hu L, Tang Z, Xu X, et al. Study on external features of particle damping loss factor. *China Mech Eng* 2015; 26: 2005–2009.

20. Gharib M and Karkoub M. Shock-based experimental investigation of the linear particle chain impact damper. *J Vib Acoust* 2015; 137: 061012-1–061011-10.