Inducing non-trivial qubit coherence through a controlled dispersive environment

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We show how the dispersive regime of the Jaynes-Cummings model may serve as a valuable tool to the study of open quantum systems. We employ it in a bottom-up approach to build an environment that preserves qubit energy and induces varied coherence dynamics. We then present the derivation of a compact expression for the qubit coherence, applied here to the case of thermally populated modes. We discuss the monotonic decay of coherence for short times and its potential use to produce Markovian dephasing. Our results provide a broadly applicable platform for the investigation of energy-conserving open system dynamics which is fully within the grasp of current quantum technologies.

Studying and understanding the role of decoherence in open quantum systems has been a major topic in quantum technology. At the same time that decoherence is harmful to quantum information by washing out superposition aspects of quantum states [1], it can also be helpful in other tasks such as energy transport in quantum networks [2–4]. In other scenarios, it might be desirable to engineer it for multiple applications [5, 6]. Several experiments have also unveiled the essential aspects of decoherence in controlled quantum systems [7–11]. In a more fundamental level, decoherence is expected to be involved in the emergence of the classical world from within the set of quantum rules [12].

In the simplest case, decoherence of a two-level system (qubit) follows from its linear coupling to a thermal reservoir consisting of a collection of an infinite number of independent (non-interacting) quantum harmonic oscillators [13] or two-level systems [14]. In such descriptions, it is assumed the lacking of control and accessibility to the degrees of freedom of the environment. In this work, we propose to study qubit dephasing, which is a form of decoherence, in a fully controllable environment built from the bottom up. In other words, we blend together the advances in controlled quantum systems and open systems theory to investigate qubit decoherence in a fully controlled environment whose number of degrees of freedom can be progressively increased.

Our approach is based on the multimode version of the dispersive regime of the Jaynes-Cummings model [15], where a qubit and a single mode of the electromagnetic field are considerably out-of-resonance, preventing transitions between energy states of the free Hamiltonians. The qubit-mode coupling in this regime manifests itself through induced energy shifts in such energy levels. The dispersive limit of the Jaynes-Cummings model has been employed in a myriad of tasks. Important examples include the generation of superpositions of coherent states of opposite phases (“Schrodinger’s cat” states), non-demolition measurements in cavity QED [16], and more recently qubit readout in circuit QED [17], just to name a few. However, much less attention has been given for its use in the context of open quantum systems. This is precisely the proposal we put forward in this work: an environment consisting of N modes dispersively coupled to the qubit, as depicted in Fig. 1. Interesting enough, the extension of the dispersive condition to N modes induces a structure in the environment which now consists of coupled modes in contrast to the canonical models of decoherence mentioned before. The interplay between structure, frequencies, number of modes and temperature promotes a very rich scenario where energy-conserving non-Markovian dynamics can be studied and applied to the simulation of Markovian dephasing for short times. In particular, given the lack of energy transitions which is inherit to the model, the dispersive qubit-mode interaction might serve as a building block for a qubit dephasing model with the distinct advantage of being fully controllable in several setups as given evidence by the aforementioned applications.

Let us consider a single qubit interacting with N bosonic modes. Under the dipole and rotating-wave approximations, the total Hamiltonian of the system is thus described by the extended (multimode) Jaynes-Cummings model ($\hbar = 1$) [18]:

$$\hat{H} = \frac{\omega_0}{2} \hat{\sigma}_z + \sum_{j=1}^{N} \omega_j \hat{a}_j \hat{a}_j^\dagger + \sum_{j=1}^{N} g_j \left( \hat{\sigma}_+ \hat{a}_j + \hat{\sigma}_- \hat{a}_j^\dagger \right), \quad (1)$$

Figure 1. (Color online) Pictorical representation for the extended dispersive regime. Each mode corresponds to a distinct single-mode resonator.
where $\omega_0$ is the frequency of the qubit, $\hat{\sigma}_i$ ($i = x, y, z$) are the Pauli matrices, $\omega_j$ is the frequency of the $j$-th mode described by the annihilation operator $\hat{a}_j$, and $g_j$ is the coupling constant. The operators $\hat{\sigma}_\pm = \frac{1}{2}(\hat{\sigma}_x \pm \hat{i}\hat{\sigma}_y)$ are the ladder operators for the qubit.

For the case of an environment being composed of a continuum of electromagnetic modes, Hamiltonian (1) has been exhaustively used to model dissipative qubit dynamics or spontaneous emission [18, 19]. This problem has been tackled with perturbative [18, 20, 21] and Markov approximation [18] and been slightly modified to accommodate structured environments [22, 23]. These approaches have in common the fact that the environment formed by the modes has a frequency distribution which is essentially centered around the qubit frequency. In this way, whenever the resonant or quasi-resonant Hamiltonian (1) is employed to build a reservoir model, the result is dissipative dynamics. In the present work, however, we take a different route which aims at producing a non-dissipative open system dynamics, i.e., pure dephasing. In order to do it, we will consider that the modes are far from resonance with the qubit and then take the dispersive limit of Hamiltonian (1). Not only that, we take full advantage of the current high level of experimental control over dispersive interactions to propose a bottom up approach. We provide analytical results for the coherence dynamics in our model.

In the interaction picture with respect to the free part of Hamiltonian (1), the dynamics follows from

$$\dot{\hat{H}}^I(t) = \sum_{j=1}^N g_j \left( \hat{\sigma}_+ \hat{a}_j e^{i\Delta_j t} + \hat{\sigma}_- \hat{a}_j^\dagger e^{-i\Delta_j t} \right),$$

with $\Delta_j = \omega_0 - \omega_j$ the detuning between the qubit and mode $j$. The requirement

$$\left| \frac{g_j}{\Delta_k} \right| \ll 1 \quad (j, k = 1, 2, ..., N),$$

allows one to perform a Magnus expansion [24] on the time-evolution operator $\hat{U}^I(t)$ associated to $\dot{\hat{H}}^I(t)$ which up to second order produces

$$\hat{U}^I(t) \approx e^{-i\dot{\hat{H}}^I_{\text{eff}}(t)}, \quad \dot{\hat{H}}^I_{\text{eff}}(t) = \Lambda_N t\hat{\sigma}_+ \hat{\sigma}_- + \frac{\hat{\sigma}_z}{2} \hat{M}(t),$$

where $\Lambda_N = \sum_{j=1}^N \frac{g_j^2}{\Delta_j}$ is the resulting energy shift on the qubit, and the bosonic part of Eq. (4) is given by

$$\hat{M}(t) = \sum_{j,k=1}^N m_{jk}(t)\hat{a}_j^\dagger \hat{a}_k,$$

with

$$m_{jk}(t) = i \frac{g_j g_k}{\Delta_j \Delta_k} (\Delta_j + \Delta_k) \left( \frac{1 - e^{i(\omega_j - \omega_k)t}}{\omega_j - \omega_k} \right).$$

The complete derivation of Eq. (4) is shown in the supplemental material. It follows from Eq. (5) that, apart from energy shifts, the dispersive condition stated in Eq. (3) also promotes interaction among the modes, see Fig. 1. Quite importantly, this interaction is dependent on the state of the qubit through $\hat{\sigma}_z$ in Hamiltonian (4). Also, given that $\langle \omega \hat{\sigma}_z, \dot{\hat{H}}^I_{\text{eff}}(t) \rangle = 0$, there are no population changes in the eigenstates of $\hat{\sigma}_z$ so that the resulting dynamics will be energy-conserving and only changes in the qubit coherence will be observed.

The initial state of the global system is assumed to be in the form $\hat{\rho}(0) = \hat{\rho}_q \otimes \hat{\rho}_E$ and we focus on the reduced dynamics of the qubit described by $\hat{\rho}_q(t) = \text{Tr}_E \hat{\rho}(t)$. The time evolution of the coherence in the original Schrodinger picture is

$$\rho_{01}(t) := \langle 0|\hat{\rho}_S(t)|1 \rangle = \rho_{01}(0) e^{-i\Lambda_N^t} r_N(t),$$

with

$$r_N(t) = \text{Tr}_E \left[ \hat{\rho}_E e^{-i \hat{M}(t)} \right].$$

This quantity completely characterizes the open dynamics of the qubit in our model, regardless the number of modes. Furthermore, $|r_N(t)|$ is proportional to the well-known $I_1$-norm measure of coherence [25]. If the environment is initially at the zero temperature vacuum state $\hat{\rho}_E = |01, \cdots, 0_N\rangle \langle 01, \cdots, 0_N|$, Eq. (8) reveals that the dispersive condition inhibits correlations between the qubit and the modes, so that $r_N(t) = 1$. This is valid regardless the frequency distribution of the modes as long as the dispersive condition Eq. (3) is observed.

Now we consider a thermal environment where $\hat{\rho}_E$ is the product of $N$ Gibbs states, one for each mode, and equal temperature $T$ for all modes. In this case, the thermal occupation of mode $k$ is $\bar{n}_k = (e^{\omega_k/T} - 1)^{-1}$, where we considered the Boltzmann constant $k_B = 1$. We start the analysis of our model by first considering the degenerate case, where all modes have the same angular frequency $\omega_j = \omega$ ($j = 1, 2, ..., N$) and therefore the same thermal occupation $\bar{n}_j = \bar{n} = (e^{\omega/T} - 1)^{-1}$ ($j = 1, 2, ..., N$). In this case, the coherence will depend only on the total shift $\Lambda_N$ and the thermal occupation number $\bar{n}$ through (see supplemental material)

$$r_N(t) = \frac{e^{i\Lambda_N t}}{\cos (\Lambda_N t) + i \left(2\bar{n} + 1\right) \sin (\Lambda_N t)}.$$
The behavior of the coherence is enriched when non-degenerate modes are used. We now detail the derivation of analytical results for the coherence which include this case. Since $\hat{M}(t)$ defined in Eq. (5) is hermitian, there might be a time-dependent unitary operator $\hat{V}(t)$ which diagonalizes it. Given that Hamiltonian (5) does not promote squeezing, i.e., there are no terms in the form $\hat{a}_j \hat{a}_k$ or $\hat{a}_j^\dagger \hat{a}_k^\dagger$, we can write

$$\hat{M}_d(t) = \hat{V}(t) \hat{M}(t) \hat{V}^\dagger(t) = \sum_{j=1}^N \epsilon_j(t) \hat{a}_j^\dagger \hat{a}_j. \quad (10)$$

In what follows, $\hat{M}(t)$ is the $N \times N$ hermitian matrix whose entries are $m_{jk}(t)$ given by Eq. (6). Also, $\hat{V}(t)$ is an unitary matrix such that $\hat{V}(t) \hat{M}(t) \hat{V}(t) = \text{Diag} [\epsilon_1(t), ..., \epsilon_N(t)]$. Then,

$$\hat{V}(t) \hat{a}_j \hat{V}^\dagger(t) = \sum_{k=1}^N \hat{V}_{jk}(t) \hat{a}_k \quad (j = 1, ..., N). \quad (11)$$

The evaluation of Eq. (8) is facilitated by evaluating the trace in the coherent states basis and by expressing $\hat{\rho}_k$ in the $P$-representation [26, 27], i.e., $\hat{\rho}_k = \int d^2N \alpha P(\alpha) |\alpha\rangle \langle \alpha|$, where $|\alpha\rangle = |\alpha_1, ..., \alpha_N\rangle$ is a $N$-mode coherent state. One finds

$$r_N(t) = \int d^2N \alpha P(\alpha) |\alpha\rangle \langle \alpha| \hat{V}(t) e^{-i\hat{M}(t) \hat{V}(t)} |\alpha\rangle, \quad (12)$$

and further analytical progress depends now on how we deal with the action of $\hat{V}(t)$ on $|\alpha\rangle$. Notice that Eq. (11) implies that $\hat{V}(t)|\alpha\rangle$ is an eigenvector of $\hat{a}_j$ with eigenvalue $\alpha'_j(t)$ given by

$$\alpha'_j(t) = \sum_{k=1}^N \hat{V}_{kj}(t) \alpha_k. \quad (13)$$

This means that $\hat{V}(t)|\alpha\rangle = |\alpha'(t)\rangle = |\alpha'_1(t), ..., \alpha'_N(t)\rangle$ is itself a factorized multimode coherent state. Using now $e^{-i\epsilon_j(t) \hat{a}_j^\dagger \hat{a}_j} |\alpha'_j(t)\rangle = |\alpha'_j(t)\rangle e^{-i\epsilon_j(t)}$, and the thermal distribution

$$P(\alpha) = \frac{e^{-\sum_{j=1}^N |\alpha_j|^2 / \bar{n}_j}}{\pi^N \prod_{j=1}^N \bar{n}_j}, \quad (14)$$

we end up with

$$r_N(t) = \frac{1}{\bar{n}} \frac{1}{\sqrt{\det \hat{A}(t)}} \quad (15)$$

where $\hat{A}(t)$ is the $2N \times 2N$ complex matrix

$$\hat{A}(t) = \begin{pmatrix} y_1(t)I & W_{12}(t) & \cdots & W_{1N}(t) \\ W_{12}^\dagger(t) & y_2(t)I & \cdots & W_{2N}(t) \\ \vdots & \vdots & \ddots & \vdots \\ W_{1N}^\dagger(t) & W_{2N}^\dagger(t) & \cdots & y_N(t)I \end{pmatrix}. \quad (16)$$

The compact and analytical form of the function $r_N(t)$ in Eq. (15) results from the evaluation of a $2N$-dimensional Gaussian integral. Still, in Eq. (16), $I$ denotes the $2 \times 2$ identity matrix and

$$W_{jk}(t) = \begin{pmatrix} -u_{jk}(t) & -v_{jk}(t) \\ v_{jk}(t) & -u_{jk}(t) \end{pmatrix}, \quad (17)$$

with

$$y_j(t) = \bar{n}_j^{-1} (\bar{n}_j + 1) - \sum_{l=1}^N e^{-i\epsilon_l(t)} |V_{jl}(t)|^2, \quad (18)$$

$$u_{jk}(t) = \sum_{l=1}^N e^{-i\epsilon_l(t)} \text{Re} [V_{jl}(t)V_{kl}(t)], \quad (19)$$

$$v_{jk}(t) = \sum_{l=1}^N e^{-i\epsilon_l(t)} \text{Im} [V_{jl}(t)V_{kl}(t)].$$

Note that we are not bound to the case of identical thermal occupations for each mode since $\bar{n}_j$ is a free parameter in (14).

We start our analysis with the first non-trivial case which is $N = 2$. For this choice, Eqs. (15)-(18) allow one to obtain

$$r_2(t) = e^{i\epsilon_+(t)} \left[ \bar{n}_1 \bar{n}_2 e^{-i\epsilon_+(t)} + (\bar{n}_1 + 1) (\bar{n}_2 + 1) e^{i\epsilon_+(t)} + \bar{n}_1 \bar{n}_2 |V_{11}(t)|^2 + \bar{n}_2 |V_{12}(t)|^2 e^{-i\epsilon_-(t)} + \bar{n}_1 \bar{n}_2 |V_{12}(t)|^2 + \bar{n}_2 |V_{11}(t)|^2 e^{-i\epsilon_-(t)} \right]^{-1},$$

where $\epsilon_{\pm}(t) = [\epsilon_1(t) \pm \epsilon_2(t)] / 2$, and $\epsilon_{1,2}(t)$ are the eigenvalues of $\hat{M}(t)$. Moreover, following the described protocol, the entries of $\hat{V}(t)$ satisfy (omitting the time-dependence) $|V_{11}|^2 = |V_{22}|^2 = |m_{12}(t)|^2 \left[ (\epsilon_1 - m_{11}(t))^2 + |m_{12}(t)|^2 \right]^{-1}$ and $|V_{21}|^2 = |V_{21}|^2 = |(\epsilon_1 - m_{11}(t))^2 + |m_{12}(t)|^2 \right]^{-1}$. One can easily check that Eq. (19) reduces to Eq. (9) when $\bar{n}_1 = \bar{n}_2$ and $\omega_1 = \omega_2 = \omega$. For non-degenerate modes, one finds $\epsilon_{\pm}(t) = \Lambda_2 t$ and $\epsilon_{\mp}(t) = \frac{g_{12}^2}{\Delta_1 \Delta_2} t \left[ (\frac{\Delta_2}{\Delta_1}) - \frac{\Delta_2}{\Delta_1} \right]^2 + (\Delta_1 + \Delta_2)^2 f^2(t) \right]^{1/2}$ with $f(t) = \sin \left[ (\omega_1 - \omega_2) t / 2 \right] / (\omega_1 - \omega_2) t / 2$. Therefore, the time-dependence on the induced mode-mode coupling tends to disturb the periodicity of $r_2(t)$ since $\epsilon_{\pm}(t)/t$ are in general dynamically incommensurate. This confers on the coherence of $\hat{\rho}_S(t)$ an interesting and non-trivial dynamics which can be engineered through the choices of the system parameters.

Fig. 2 shows $|r_2(t)|$ as function of the dimensionless time $\omega_0 t$ for different choices of frequencies $\omega_j$, coupling constants $g_j$, and temperature $T$. As indicated in the left panels, keeping the modes slightly detuned from each other is sufficient to make $|r_2(t)|$ exhibit irregular oscillations due to $\epsilon_+(t) \neq \epsilon_-(t)$ and $\bar{n}_1 \neq \bar{n}_2$ in Eq. (19).
In the center panels, for fixed coupling constants $g_1$ and $g_2$, and also for fixed $\omega_1$, we emphasize the sensibility of $|r_2(t)|$ with the variation of $\omega_2$. For engineering of qubit dephasing, the non-degenerate case [see Figs. 2(c) and 2(e)] allows us to delay recurrence times in the coherence and to change its amplitudes of oscillation. In all plots, the parameters $g_j/\Delta_j$ essentially dictate how fast the oscillations are, whereas the thermal occupations $\bar{n}_j$ determine their amplitudes.

Besides its suitability to the study of decoherence, the function $|r_2(t)|$ also reveals non-Markovianity in the dynamics through its attempts to recur. For a pure dephasing model, such as the one considered here, there is a one-to-one correspondence between $|r_2(t)|$ and the distinguishability of the pair of initial states consisting of eigenstates of $\hat{\sigma}_z$. Any dynamical increment of distinguishability between these two initial states indicates non-Markovianity as it is a consequence of information backflow from the environment to the system [28–30].

All plots in the left and center panels of Fig. 2 signalizes non-Markovianity what is a direct consequence of the finiteness of the environment [1]. The short-time regime is particularly interesting as it can be used to emulate a Markovian environment. Even for very small environments ($N = 2$), one can see that proper adjustments of $g_j/\Delta_j$, and $T$ may lead to an approximate monotonic decay of coherences, typical of Markovian dynamics [Fig. 2(f)]. In particular, the temperature $T$ radically affects the quality of the simulation. It is important to remark that the dynamical map as a whole is non-Markovian and there will be long-time attempts of recurrence. The point here, however, is the simulation of Markovian monotonic decay for short-time dynamics. This might be useful in tests of principle where Markovian dephasing is needed.

Our simulation of Markovian maps acquires extra power when larger non-degenerate environments are considered. More modes result in additional oscillating terms in $r_2(t)$ [see Eqs. (16)-(18)] with amplitudes consisting of products of $\bar{n}_j$. This favours a more steep depletion of $|r_2(t)|$ which resembles an exponential decay. This is presented in Fig. 3, which also shows that the presence of more (distinguished) frequencies in the environment attenuates the revivals of $|r_2(t)|$. Consequently, it takes longer times to recur what indicates that a Markovian behavior is expected in the limit of large $N$, in constrast to what is observed in the complete degenerate case [see Eq. (9) and Fig. 2(d)].

Now, some comments on the feasibility of observation of our results in a real setup. In circuit QED, tempera-
tures about a few dozens or hundreds of mK are easily achieved using dilution refrigerators [31]. Such temperatures are consistent with our choice $T \approx \omega_0$, i.e., thermal mode energy comparable to the typical qubit energies $\omega_0 \approx 10$ GHz [32]. This is the temperature regime adopted in Figs. 2 and 3. Also, in these architectures, for the qubit coherence valid for its simulation for short times. A closed-form expression is fully within the grasp of current quantum technologies. We discussed issues such as Markovianity and its simulation for short times. A closed-form expression for the qubit coherence valid for $N$ environmental modes was obtained. These modes are assumed to be single-mode resonators. Our investigation is fully within the grasp of current quantum technologies. We discussed issues such as Markovianity and its simulation for short times. A closed-form expression for the qubit coherence valid for $N$ environmental modes was obtained. These modes are assumed to be single-mode resonators and we are only considering finite $N$. There is an ongoing discussion about potential divergencies in a multimode resonator as $N \to \infty$ [34, 35]. Finally, a possible extension of our work may focus on the ultrastrong coupling regime where $g_j/\omega_j > 1$ [36]. In this case, a generalized dispersive regime beyond the rotating-wave approximation will have to be considered.

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SUPPLEMENTAL MATERIAL

A. Derivation of the effective dynamics

Here we detail the derivation of Eq. (4) from the main text, which describes the effective evolution of a qubit interacting dispersively with $N$ electromagnetic modes.

The time-evolution operator for the interaction-picture Hamiltonian (2) is expressed as a Magnus series [24, 37] in terms of the anti-hermitian operator $\hat{\Omega}(t)$, i.e.,

$$\hat{U}^I(t) = \exp \left[ \hat{\Omega}(t) \right], \quad \hat{\Omega}(t) = \sum_{n=1}^{\infty} \hat{\Omega}_n(t). \quad (S1)$$

Conveniently, the Magnus expansion perturbatively produces a unitary operator for any desired order, which is not true for the usual Dyson series [37]. Following [24, 37] and using the Hamiltonian (2), the first two terms of this expansion are

$$\hat{\Omega}_1(t) = -i \int_0^t dt_1 \hat{H}^I(t_1)$$

$$= \sum_{j=1}^{N} \frac{g_j}{\Delta_j} \left[ (1 - e^{i\Delta_j t}) \hat{\sigma}_+ \hat{a}_j - H.C. \right], \quad (S2)$$

$$\hat{\Omega}_2(t) = -\frac{1}{2} \int_0^t dt_1 \int_0^{t_1} dt_2 \left[ \hat{H}^I(t_1), \hat{H}^I(t_2) \right]$$

$$= -i \Lambda_{\sigma} t \hat{\sigma}_+ \hat{\sigma}_- - i \frac{\hat{\sigma}_z}{2} \hat{M}(t) + \mathcal{O}^2 \left( \frac{g_j}{\Delta_j} \right), \quad (S3)$$

where $\Lambda_{\sigma}$ and $\hat{M}(t)$ are both defined in the main text, respectively below Eq. (4) and in Eq. (5). The generalized dispersive condition $|g_j/\Delta_k| \ll 1$ in (3) allows one to neglect the higher order terms in (S3). The same applies to the terms $\hat{\Omega}_n(t)$ with $n > 2$ in (S1), since they involve higher order commutators [24, 37] which give rise to products of $g_j/\Delta_k$. Therefore, up to second order, the time-evolution operator in (S1) for the interaction picture Hamiltonian (2) is well approximated by $\hat{U}^I(t) \approx e^{\hat{\Omega}_1(t)+\hat{\Omega}_2(t)}$. Now, another important assumption is to neglect the influence of $\hat{\Omega}_1(t)$ on the dynamics, and this can be understood as follows. Considering an initial state in the interaction-picture $|\psi\rangle$, the suppression of $\hat{\Omega}_1(t)$ on the dynamics would demand that $\langle \psi | e^{\hat{\Omega}_1(t)+\hat{\Omega}_2(t)} | \psi \rangle \approx \langle \psi | e^{\hat{\Omega}_2(t)} | \psi \rangle$. Roughly speaking, this is obtained if $|\langle \psi | \hat{\Omega}_1(t) | \psi \rangle|$ is sufficiently small, which is assured in our case, since $\hat{\Omega}_1(t)$ is an oscillatory function of the time with amplitudes $|g_j/\Delta_j| \ll 1$, see Eq. (S2). To some extent, dropping out such term is what witnesses the dispersive feature of the model: it inhibits energy exchange between the qubit and the modes due to the elimination of terms with $\hat{\sigma}_+ \hat{a}_j$ and $\hat{\sigma}_- \hat{a}_j$ in (S2). The same is found using other perturbative approaches for the study of the dispersive regime, e.g., [38].

On the other hand, the main contribution to the effective description comes from the energy exchange among the modes, these are represented by the crossing terms $\hat{a}_j^{\dagger} \hat{a}_k$ in $\hat{\Omega}_2(t)$ throughout the operator $\hat{M}(t)$ defined in (5). Such terms become essentially linear on time, provided that the mode detunings $|\omega_j - \omega_k|$ are not large, see Eq. (6). This last condition is also indirectly required for both the dispersive limit above explained and the rotating-wave approximation producing (1) to hold.

When these conditions are fulfilled, it is possible to write an effective evolution for the dispersive limit as in (4). To certify the validity of the effective description for the parameters adopted in the main text, we now provide a numerical evaluation with the exact Hamiltonian and contrast it with the results using the effective dispersive one.
B. Numerical checking of the effective dynamics

In order to check the validity of the approximations in writing the effective evolution for the qubit in the dispersive limit, one can numerically determine the evolution governed by the Hamiltonian (1). The numerical procedure consists in writing the matrix elements of that Hamiltonian with respect to the Fock basis of the modes electromagnetic modes governed by the Hamiltonian (1). The numerical procedure consists in writing the matrix elements of the Hamiltonian with respect to the Fock basis of the modes electromagnetic modes. Specifically, the case of \( N = 2 \) modes, we perform a truncation on the space dimension conveniently choosing the number of Fock states. Specifically, for the case of \( N = 2 \) modes, we calculate \( |r_2(t)| \) from Eq. (8) and the mean value of \( \langle \hat{\sigma}_z \rangle \), which are respectively related to the \( l_1 \)-norm coherence measure and population inversion for the qubit in the \( \hat{\sigma}_z \)–eigenbasis. The results are shown in Fig. S1. For the chosen parameters, same as in the main text, the function \( |r_2(t)| \) from the Magnus expansion is in close agreement with the numerical results; small deviations appear only for times \( \omega_0 t \sim 4000 \) or longer, even for the highest values of \( |g_j/\Delta_k| \) considered in this work, as in Figs. S1(a) and S1(c). Furthermore, the numerical calculations show that values of \( \langle \hat{\sigma}_z \rangle \) are practically kept constant, indicating that qubit populations are preserved. Indeed, such behavior is predicted by the dispersive model, as explained in the main text. As expected, the validity of the effective model is progressively degraded as \( |g_j/\Delta_k| \) becomes larger, and this is why the tiny oscillations in the numerical curves are more pronounced in Figs. S1(a), (d) and in Figs. S1(c), (f).

C. Calculation of \( r_n(t) \) for degenerate modes

In this section we detail the derivation of Eq. (9) presented in the main text. This equation exhibits the function \( r_n(t) \) in (8) for a set of \( N \) electromagnetic modes with the same frequency and in thermal equilibrium. The procedure is based on the Weyl-Wigner formalism and related to the tools developed in Refs. [39, 40] for the obtainment of the total phase acquired by a Gaussian state.

In our context, the frequency degeneracy causes \( \hat{M}(t) \) in (5) to be linear on time, since Eq. (6) becomes \( m_{jk}(t) = 2g_jg_kt/(\omega_0 - \omega) \), and \( r_n(t) \) is the average value of the metaplectic operator

\[
\hat{R}(t) = e^{i\mathbf{N}_t} e^{-\frac{i}{2} \hat{x}^\dagger \mathbf{H} \hat{x}},
\]

due to the quadratic Hamiltonian \( \frac{1}{2} \hat{x}^\dagger \mathbf{H} \hat{x} \). For convenience we have defined the \( 2N \)-dimensional column vector \( \hat{x} = (\hat{q}_1, \ldots, \hat{q}_N, \hat{p}_1, \ldots, \hat{p}_N)^\dagger \), which is composed by quadrature operators given by \( \hat{q}_j = (\hat{a}_j^\dagger + \hat{a}_j)/\sqrt{2} \) and \( \hat{p}_j = i(\hat{a}_j^\dagger - \hat{a}_j)/\sqrt{2} \). Also, \( \mathbf{H} \) is the \( 2N \times 2N \) symmetric real matrix given by

\[
\mathbf{H} = \mathbf{G} \mathbf{G}^\dagger, \quad G_{ij} = 2(\omega_0 - \omega)^{-1} g_i g_j.
\]

The matrix \( \mathbf{G} \) is a dyadic and thus has a unique non-null eigenvalue which is given by 2\( \Delta_n \). Considering the orthogonal matrix \( \mathbf{O} \) that diagonalizes \( \mathbf{G} \), i.e., \( \mathbf{G} = \mathbf{OGO}^\dagger = \text{Diag}[2\Delta_n, 0, \ldots, 0] \), then one is able to write \( \mathbf{H} = \mathbf{G} \mathbf{O} \mathbf{G}^\dagger \mathbf{O}^\dagger \) as the matrix with the eigenvalues of \( \mathbf{H} \).

The metaplectic operator \( \hat{R}(t) \) is associated to the symplectic matrix \( \mathbf{S} = e^{i\mathbf{H}t} \) with

\[
J = \begin{pmatrix}
0_N & I_N \\
-I_N & 0_N
\end{pmatrix},
\]

where \( 0_N \) denotes the \( N \times N \) null matrix and \( I_N \) the \( N \times N \) identity. Under the orthogonal transformation
\( O \oplus O \), the matrix \( S_O = O \oplus OS(O \oplus O)^\top \) acquires the form
\[
S_O = \begin{pmatrix}
\cos (G_O t) & \sin (G_O t) \\
-\sin (G_O t) & \cos (G_O t)
\end{pmatrix}.
\] (S7)

In addition, the Cayley parametrization of \( S \) defined by
\[
C = J(I_{2N} - S)(I_{2N} + S)^{-1},
\] under the same transformation reads
\[
C_O = J(I_{2N} - S_O)(I_{2N} + S_O)^{-1}.
\] (S8)

Using the above definitions, considering \( \hat{\rho}_E \) a Gaussian state with covariance matrix \( V \) and null mean values, we resort to the Wigner representation to express an average value as \([39–41]\) \( \text{Tr} [\hat{A}] = \int dx W(x) A(x) \), with \( W(x) \) being the Wigner function of \( \hat{\rho}_E \) and \( A(x) \) the center symbol of \( A \). Then, one finds
\[
r_n(t) = \text{Tr} \left[ \hat{\rho}_E \hat{R}(t) \right] = \frac{e^{i\Lambda_{x}t}}{\sqrt{\det (S + I_{2N}) \det \left( \frac{1}{2}I_{2N} + iVC \right)}}
\] (S10)

If the environment state \( \hat{\rho}_E \) is a \( N \)-mode thermal state, then its covariance matrix becomes \( V = (\bar{n} + \frac{1}{2})I_{2N} \). Inserting this into (S10), using the fact that \( \det (O \oplus O) = 1 \), since \( O \) is orthogonal, Eq. (S10) becomes
\[
r_n(t) = \frac{e^{i\Lambda_{n}t}}{\sqrt{\det (S_O + I_{2N}) \det \left( \frac{1}{2}I_{2N} + i(\bar{n} + \frac{1}{2})C_O \right)}}.
\]
which leads directly to Eq. (9).

As a final comment, we have implicitly assumed that \( \det (S + I_{2N}) \neq 0 \) in (S10). However, if this happens to not be the case at some instant of time, then such a choice does not invalidate Eq. (9), but its derivation follows a complementary procedure based on symplectic Fourier transformations. This is discussed in depth in Refs. [39, 40].

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