Weak convergence of the intersection point process of Poisson hyperplanes.

**Summary:** This paper deals with the intersection point process of a stationary and isotropic Poisson hyperplane process in $\mathbb{R}^d$ of intensity $t > 0$, where only hyperplanes that intersect a centred ball of radius $R > 0$ are considered. Taking $R = t^{-\frac{d}{d+1}}$ it is shown that this point process converges in distribution, as $t \to \infty$, to a Poisson point process on $\mathbb{R}^d \setminus \{0\}$ whose intensity measure has power-law density proportional to $\|x\|^{-(d+1)}$ with respect to the Lebesgue measure. A bound on the speed of convergence in terms of the Kantorovich-Rubinstein distance is provided as well. In the background is a general functional Poisson approximation theorem on abstract Poisson spaces. Implications on the weak convergence of the convex hull of the intersection point process and the convergence of its $f$-vector are also discussed, disproving and correcting thereby a conjecture of L. Devroye and G. Toussaint [J. Algorithms 14, No. 3, 381–394 (1993; Zbl 0778.68088)] in computational geometry.

**MSC:**
52A22 Random convex sets and integral geometry (aspects of convex geometry)
53C65 Integral geometry
60D05 Geometric probability and stochastic geometry
60F05 Central limit and other weak theorems
60G55 Point processes (e.g., Poisson, Cox, Hawkes processes)
68U05 Computer graphics; computational geometry (digital and algorithmic aspects)
68Q25 Analysis of algorithms and problem complexity

**Keywords:**
convex hull; integral geometry; intersection point process; Poisson hyperplane process; Poisson point process approximation; rate of convergence; weak convergence

**Full Text:** DOI arXiv

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