Anomalous non-Markovian effect in controllable open quantum systems

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Memory effect of non-Markovian dynamics in open quantum systems is often believed to be beneficial for quantum tasks. In this work, we employ an experimentally controllable two-photon open quantum system, with one photon experiencing a dephasing environment and the other being free from noise, to show that non-Markovian effect may have a negative impact on quantum tasks such as the remote state preparation: For certain period of controlled time interval, stronger non-Markovian effect yields lower fidelity for remote state preparation, as opposed to the common wisdom that more information leads to better performance. Criteria for non-Markovian positive/negative effect, the relevance for a real environment, and the physical interpretations are further discussed.

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Non-Markovianity is a ubiquitous and remarkable feature in open quantum systems with structured reservoirs or strong system-environment couplings [1, 2]. Due to the recent development of experimental techniques, non-Markovian effect has been observed and engineered in a variety of systems, such as ultracold neutral plasma [3], high-Q cavities [4], solid state systems [5], and quantum biology [6]. Several theoretical methods have been developed to describe non-Markovian dynamics and to construct phenomenological descriptions [7–14].

It is found that non-Markovian effect usually plays a positive role in many quantum tasks such as entanglement preservation [15], quantum key distribution [16], and quantum metrology [17]. This is because non-Markovian environment could be taken as a memory device which partially records the information of the open systems and then returns it back later. How to control the non-Markovian behavior of open systems is important in both theory and experiments, and several non-Markovian control experiments have been realized [18–20].

However, a basic problem in open systems is largely overlooked: Does non-Markovian effect always play a positive role in quantum tasks? This issue is significant not only for the understanding of non-Markovian dynamics, but also for the engineering of open systems. In this work, by considering some controllable models of open systems, we show that non-Markovian effect is quite a mixed blessing, which may have negative effect on certain fundamental quantum tasks. We illustrate this anomalous phenomenon by an instance of non-Markovian negative effect on the fidelity for remote state preparation (RSP), in terms of several non-Markovianity measures. Moreover, some conditions for determining the dual characters of non-Markovian effect are discussed, and relevance for a real environment is also analyzed in the Supplemental Material.

A controllable open system.—The controllable physical model of our open system is the polarization freedom of an entangled photon pair $AB = aa' : bb'$ created in spontaneous parametric down-conversion, with the frequency freedom $a'b'$ serving as the environment. The polarization system $a$ of photon $A$ will experience a dephasing interaction with environment $a'$, which is simulated by a quartz plate, and photon $B$ is free from noise. The dephasing of photon $A$ in a quartz plate is caused by local interaction between the polarization system $a$ (open system pertaining to $A$) and the frequency freedom $a'$ (environment pertaining to $A$) with the interaction Hamiltonian ($\hbar = 1$) [21]

$$H^{aa'} = - (n_V |V\rangle_a \langle V| + n_H |H\rangle_a \langle H|) \int \omega |\omega\rangle_{a'} \langle \omega| d\omega,$$

where $|H\rangle_a$ and $|V\rangle_a$ are the polarization, and $|\omega\rangle_{a'}$ the frequency, states of photon $A$, $n_V$ and $n_H$ are the refraction indexes without considering the dispersion effect. The interaction only occurs in the quartz plate, and the total controllable Hamiltonian is

$$H_c^{aa'bb'}(t) = \mu(t) H^{aa'} \otimes 1^{bb'},$$

where $\mu(t) = 1$ if $t \in [t_0, t_c]$ and $\mu(t) = 0$ otherwise, $t_0(t_c)$ denotes the time of photon $A$ entering (leaving) the quartz plate, and $1^{bb'} = 1^b \otimes 1^b$ is the identity operator for photon $B$ with $1^b$ and $1^b$ the identity operators for its polarization and frequency freedom, respectively. For simplicity, we take $t_0 = 0$ hereafter.

Consider an initial two-photon state $\rho^{ab} \otimes \rho^{a'b'}$, where $\rho^{ab}$ is the polarization state representing the open system, and $\rho^{a'b'} = \int d\omega d\omega' F(\omega) F^*(\omega') |\omega\rangle_{a'} \langle \omega'| \otimes \rho^{b'}$ is the environmental state with $f(\omega) = |F(\omega)|^2$ the probability density of finding photon $A$ with frequency $\omega$, and $\rho^{b'}$ any state of the frequency freedom of photon $B$. The frequency probability density $f(\omega)$, as well as the time $t_c$, are under our control. The two-photon polarization state at time $t$ can be expressed as

$$\rho^{ab}(t) = \Lambda_t \rho^{ab} := \text{tr}_{a'b'} \left\{ U(t)(\rho^{ab} \otimes \rho^{a'b'}) U(t)^\dagger \right\} \quad (1)$$

where $\Lambda_t$ is a controllable unitary operator.
The transition point from non-Markovian to Markovian regime.

\[ \tau_c = (n_V - n_H)t_c \in [\pi/\Delta \omega, 2\pi/\Delta \omega] \] for the control time parameter \( \xi \in [0, \pi/2] \) of a two-photon system with photon 1 suffering from dephasing noise (simulated by a quartz plate with \( \Delta \omega = 10 \) and \( \sigma = 1 \)) and photon 2 being free from noise. The red curve sliced by the light red plane \( \tau_c = 3\pi/(2\Delta \omega) \) depicts the non-Markovianity for this specific control time with \( \xi_1(\tau_c) \) the sudden transition point from Markovian non-Markovian regime, and \( \xi_2(\tau_c) \) the sudden transition point from non-Markovian to Markovian regime.

with \( \Lambda = \{ \Lambda_t \} \) our quantum dynamics, \( U(t) = e^{-i \int_0^t H_{\text{env}}(t') dt'} \). For \( t \in [0, t_c] \), we have \( T = \int_0^t \mu(t') dt' \) and

\[ \rho^{ab}(t) = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix}, \tag{2} \]

where \( \rho^{ab} = (\rho_{ij}) \) is the initial open system state, and \( \kappa(t) = \int f(\omega) e^{i(\omega - n_H)H} d\omega \). We consider a controllable two-peaked Gaussian frequency distribution \( f(\omega) = \frac{\cos^2 \xi}{\sqrt{2\pi} \sigma} e^{-\frac{(\omega - \omega_1)^2}{2\sigma^2}} + \frac{\sin^2 \xi}{\sqrt{2\pi} \sigma} e^{-\frac{(\omega - \omega_2)^2}{2\sigma^2}} \)

for the environment \( \omega' \), with \( \xi \in [0, \pi/2] \) controlling the relative weight of the two peaks, which are centered at \( \omega_1 \) and \( \omega_2 \), respectively, with the same width \( \sigma \). Then the dephasing rate takes the form

\[ |\kappa(t)| = e^{-\frac{\Delta \omega t^2}{2}} \sqrt{1 - \sin^2 2\xi \cdot \sin^2 \frac{\Delta \omega \tau}{2}} \tag{3} \]

with \( \tau = (n_V - n_H)t \) and \( \Delta \omega = \omega_2 - \omega_1 \).

Non-Markovianity of the controllable dynamics. To characterize the non-Markovian effect of the above model, we first employ the information-flow method [12], in which the non-Markovianity of \( \Lambda = \{ \Lambda_t \} \in [0, t_c] \) is defined as the total amount of information flowing back from the environment. The information is quantified by the trace distance \( D(\Lambda_1 \rho_1^{ab}, \Lambda_1 \rho_2^{ab}) = (1/2) \text{tr} |\Lambda_1 \rho_1^{ab} - \Lambda_1 \rho_2^{ab}| | \) of a pair of evolved quantum states, which describes the distinguishability between them [23]. The direction of information flow depends on the gradient \( g(t) = \partial_t D(\Lambda_1 \rho_1^{ab}, \Lambda_1 \rho_2^{ab}) \), with positive gradient indicating information flowing back to the system. The non-Markovianity of \( \Lambda = \{ \Lambda_t \} \in [0, t_c] \) is quantified as [12]

\[ N(\Lambda) := \max \int g(t) dt. \]

The integral is over \( [0, t_c] \) \{t : g(t) > 0\}. Employing the orthogonal condition in Ref. [24], after some calculations, we find that the optimal state pairs \( \rho_{1,2}^{ab} \) for the above maximization problem in our one-side local noise model turn out to be the following orthonormal state pairs \( \rho_{1,2}^{ab} = |\zeta^\pm \rangle \langle \zeta^\pm | \) or \( |\eta^\pm \rangle \langle \eta^\pm | \) with

\[ |\zeta^\pm \rangle = \sqrt{\alpha} |\phi^\pm \rangle + \sqrt{1 - \alpha} |\varphi^\pm \rangle, \]

\[ |\eta^\pm \rangle = \sqrt{\alpha} |\phi^\pm \rangle + \sqrt{1 - \alpha} |\varphi^\pm \rangle, \]

where \( \alpha \in [0, 1] \), \( |\phi^\pm \rangle = (|HH\rangle_{ab} \pm e^{i\theta} |VV\rangle_{ab})/\sqrt{2} \), \( |\varphi^\pm \rangle = (|HV\rangle_{ab} \pm e^{i\theta} |VH\rangle_{ab})/\sqrt{2} \) with \( \theta \in [0, 2\pi] \) [25]. All the above state pairs yield the same trace distance

\[ D(t) = |\kappa(t)| \]

with \( D(t) \) denoting the optimal trace distance of non-Markovianity.

Obviously, the non-Markovianity \( N(\Lambda) \) depends on the control time \( t_c \) which characterizes the dephasing duration. For \( \tau_c = (n_V - n_H)t_c \in [\pi/\Delta \omega, 2\pi/\Delta \omega] \), we have

\[ N(\Lambda) = |\kappa(\tau_c)| - \delta \quad \text{with} \quad \delta = |\cos 2\xi| e^{-\frac{\Delta \omega \tau_c^2}{2}}. \]

The non-Markovianity is depicted in Fig. 1. Clearly, for a fixed control time \( \tau_c \), the non-Markovianity can be adjusted by the parameter \( \xi \), the relative weight of the two frequency peaks. There exist two critical points of sudden transition between Markovian and non-Markovian regime for the open system, which take the form

\[ \xi_1(\tau_c) = \arctan \sqrt{p - q}, \quad \xi_2(\tau_c) = \arctan \sqrt{p + q}, \]

where

\[ p = \frac{u + v \cos (\Delta \omega \tau_c)}{u - v}, \quad u := e^{\pi \sigma / \Delta \omega} \]

\[ q = \frac{\sqrt{2uv(1 + \cos (\Delta \omega \tau_c))} - v^2 \sin^2 (\Delta \omega \tau_c)}{u - v}. \]

This sudden transition is similar to the one photon case observed in laboratory [18].

Anomalous non-Markovian effect. The above dephasing model reveals an interesting phenomenon: From Fig. 1, for \( \tau_c \in [\pi/\Delta \omega, 2\pi/\Delta \omega] \), we see that when \( \xi \) varies from \( \xi_1(\tau_c) \) to \( \pi/4 \), the non-Markovianity increases, but the trace distance \( D(t) = |\kappa(t)| \) decreases in view of
Eq. (3) for any fixed $t$, i.e., the stronger the non-Markovian effect, the smaller the trace distance for a pair of evolved quantum states. In other words, here the non-Markovianity renders two quantum states less distinguishable, which turns out to be the very reverse of the folklore intuition of non-Markovian effect making quantum states more distinguishable due to the back flow of information. Consequently, the illustrated non-Markovianity becomes a negative effect in trace distance related quantum tasks. However, we note that for $\tau_c = 2\pi/\Delta\omega$, the trace distance $D(t) = |\kappa(t_c)| = e^{-2(\pm\xi)^2}$ is independent of the parameter $\xi$, and thus trace distance related quantum tasks will be immune from the non-Markovian effect in this instance.

We illustrate the implication of the above new phenomenon on the RSP fidelity [26]. RSP is a variation of quantum teleportation requiring only one cbit (classical bit) and a single qubit measurement. In RSP, Alice and Bob initially share a maximally entangled state. A qubit $|\psi\rangle$ to be sent from Alice to Bob is initially known to Alice but unknown to Bob. Alice performs a measurement on her party of the shared entangled state along the basis $\{|\psi\rangle, |\psi_\perp\rangle\}$. The outcome of Alice’s measurement is sent to Bob with one cbit communication through a classical channel, Bob then performs a unitary operation on his own party of the shared state according to the cbit received. The state $|\psi\rangle$ can be reconstructed with 100% by Bob. However, if the shared state is not maximally entangled, the success probability will decrease.

Now suppose that for RSP, the shared entangled state between Alice and Bob is the decohered state $\rho^{ab}_t = \Lambda_t \rho^{ab}$ of the Bell-diagonal state

$$\rho^{ab} = \frac{1}{4} \left( 1^a \otimes 1^b + \sum_{j=1}^{2} c_j \sigma_j^a \otimes \sigma_j^b \right),$$

under the quantum dynamics $\Lambda = \{\Lambda_t\}$ described by Eq. (1). Here $\sigma_j$ are the Pauli matrices and $c_j$ are real constants [27]. Then the decohered state takes the form

$$\rho^{ab}(t) = \frac{1}{4} \left( 1^a \otimes 1^b + \sum_{j,j'}^{3} c_{jj'}(t) \sigma_j^a \otimes \sigma_j^b \right),$$

with the correlation matrix

$$C = (c_{jj'}) = \begin{pmatrix} c_1 \text{Re} \kappa(t) & c_2 \text{Im} \kappa(t) & 0 \\ -c_1 \text{Im} \kappa(t) & c_2 \text{Re} \kappa(t) & 0 \\ 0 & 0 & c_3 \end{pmatrix}.$$  

The RSP fidelity can be calculated as $F = F(C^T C)$ [28], which is equal to the mean value of the two lowermost eigenvalues of the matrix $C^T C$ (the superscript $T$ represents transposition). The three eigenvalues of $C^T C$ are $c_1^2 |\kappa(t)|^2, c_2^2 |\kappa(t)|^2$ and $c_3^2$. Since non-Markovianity of $\Lambda$ is $N(\Lambda) = |\kappa(t)| - \delta$, the fidelity reads

$$F \propto (N(\Lambda) + \delta)^2.$$  

When $c_3 \geq c_1, c_2$, we have $F = c_1^2 + c_2^2 (N(\Lambda) + \delta)^2$. At first glance, it seems that the RSP fidelity is proportional to non-Markovianity, however, exactly the opposite is true in some cases since the RSP fidelity is also related to $\delta$. This can be seen from Fig. 2(a), in which we plot the dependence of the RSP fidelity on non-Markovianity with a specific control time $\tau_c = 3\pi/(2\Delta\omega)$ for two initially entangled states $\rho^{ab}$ with parameters $(c_1, c_2, c_3) = (1, -1, 1), (-0.5, 0.4, 0.8)$, respectively. A counter-intuitive and interesting phenomenon can be seen in Fig. 2(a): In the non-Markovian region, i.e., $\xi \in [\xi_1(\tau_c), \xi_2(\tau_c)]$, stronger non-Markovian effect will induce lower RSP fidelity, which implies that the information flowed back from the environment is not always beneficial. We also note that for some specific control time, e.g., $\tau_c = 2\pi/\Delta\omega$, the RSP fidelity is given by $F = c_1^2 + c_2^2 e^{-2(\pm\xi)^2}$, which is totally uncorrelated with the parameter $\xi$, as shown in Fig. 2(b). The reason is that, in this situation, no matter how strong non-Markovianity is, the trace distance evolves to the same value, and the RSP fidelity therefore keeps unchanged.

Other measures for non-Markovianity.—Apart from the information-flow based non-Markovianity measure, there are several other measures quantifying non-Markovianity [13, 14], and they do not coincide in general [29]. The question arises: Does the anomalous non-Markovian effect arise solely due to information-flow based non-Markovianity measure, or is it a rather general feature which also survives for other non-Markovianity measures? To answer this question, we further consider three other popular measures for non-Markovianity and their implications for the anomalous non-Markovian effect in our model [25].

In Ref. [13], a divisibility based non-Markovianity measure is defined as

$$N_D(\Lambda) := \int_0^\infty \hat{h}(t)dt$$

![FIG. 2: (Color online) Anomalous non-Markovian effect: a two-photon system with photon A suffering from dephasing noise (simulated by quartz plate with $\Delta\omega = 10$ and $\tau = 1$) and photon B free from noise. The dotted curve depicts the non-Markovianity with control time $\tau_c = 3\pi/(2\Delta\omega)$ in (a) and $\tau_c = 2\pi/\Delta\omega$ in (b) versus the parameter $\xi \in [\xi_1(\tau_c), \xi_2(\tau_c)]$ (non-Markovian regime). The purple (dark) and orange (light) curves represent the RSP fidelity $F_1$ and $F_2$ with initial Bell state $(c_1, c_2, c_3) = (1, -1, 1)$ and a mixed entangled state $(c_1, c_2, c_3) = (-0.5, 0.4, 0.8)$, respectively.](image)
with \( h(t) = \lim_{\epsilon \to 0} \frac{\text{tr}((\Lambda_{t+\epsilon,t} \otimes \mathds{1})(\rho^{ss'} \cdot \rho^{ss'} \cdot 1))}{\epsilon} \). Here \( \Lambda_{t+\epsilon,t} \) is defined via \( \Lambda_{t+\epsilon,t} = \Lambda_{t+\epsilon,t} \Lambda_t \), \( \epsilon \geq 0 \). \( \rho^{ss'} = \ketbra{\Psi}{\Psi} \) with \( \ket{\Psi} = \frac{1}{\sqrt{d}} \sum_{j=1}^{d} \ket{j}_s \ket{j}'_s \) is a maximally correlated state of the \( d \)-dimensional open system \( s \) (= \( ab \) in this Letter) and an ancillary system \( s' \). According to Eq. (2), we have \( h(t) = \partial_t \ln |\kappa(t)| = \partial_t \ln \ln |\kappa(t)| \) for \( \partial_t \ln |\kappa(t)| > 0 \), otherwise \( h(t) = 0 \). Since the function \( \ln \ln |\kappa(t)| \) has the same monotonicity as \( |\kappa(t)| \), this divisibility based non-Markovianity measure will yield the same anomalous non-Markovian effect as that for the information-flow based measure.

There is also an entanglement based non-Markovianity measure defined as [13]

\[
\mathcal{N}_E(\Lambda) := \int_{\partial t > 0} \partial_t E(\rho^{ss'}(t)) dt
\]

with \( \rho^{ss'}(t) = (\Lambda_t \otimes \mathds{1}) \rho^{ss'} \) and \( E(\cdot) \) a measure of entanglement. If we take \( E(\cdot) \) as the negativity [30], then in our model, \( E(\rho^{ss'}(t)) = |\kappa(t)| + 1/2 \), and \( \partial_t E(\rho^{ss'}(t)) = \partial_t |\kappa(t)| \), which will also yield the same anomalous non-Markovian effect as that for the information-flow based measure.

We further consider the correlations based non-Markovianity measure defined [14]

\[
\mathcal{N}_I(\Lambda) := \int_{\partial t > 0} \partial_t I(\rho^{ss'}(t)) dt,
\]

where \( \rho^{ss'}(t) = (\Lambda_t \otimes \mathds{1}) \rho^{ss'} \), \( I(\rho^{ss'}(t)) = S(\rho(t)) + S(\rho'(t)) - S(\rho^{ss'}(t)) \) is the quantum mutual information, \( \rho(t) = tr_s' \rho^{ss'}(t), \rho'(t) = tr_s \rho^{ss'}(t) \), and \( S(\rho^{ss'}(t)) := -tr \rho^{ss'}(t) \log_2 \rho^{ss'}(t) \) is the von Neumann entropy. After simple calculations, we have \( I(\rho^{ss'}(t)) = 4 - H\left(\frac{1 - |\kappa(t)|}{2}\right) \), where \( H(x) := -x \log_2 x - (1 - x) \log_2(1 - x) \). Since \( 0 \leq 1 - \frac{|\kappa(t)|}{2} \leq \frac{1}{2} \), the monotonicity of \( I(\rho^{ss'}(t)) \) is the same as \( |\kappa(t)| \). In addition, \( \partial_t I(\rho^{ss'}(t)) = \frac{\partial_t |\kappa(t)|}{2} \log_2 \frac{1 + |\kappa(t)|}{1 - |\kappa(t)|} \), which implies the same anomalous non-Markovian effect as that for the information-flow based measure.

Obviously, \( h(t) \), \( E(\rho^{ss'}(t)) \) and \( I(\rho^{ss'}(t)) \) have the same monotonicity as \( |\kappa(t)| \), and the above three different non-Markovianity measures are all equivalent to the information-flow measure for identifying non-Markovianity in our model. This fact implies that the illustrated anomalous non-Markovian effect is a rather generic feature.

**Identifying the anomalous region.**– The anomalous non-Markovian effect illustrated above shows that we should be more careful about engineering environment for quantum information processing. Thus, it is necessary to determine when the effect of non-Markovian environment is positive or negative in specific contexts. Here, we further discuss this issue.

To understand the above anomalous non-Markovian effect, it should be clarified that the current popular measures for non-Markovianity actually aim to capture the global effect of a quantum dynamics, i.e., \( \mathcal{N}(\Lambda) \) is time-integrated over the past quantified by backflow of information or indivisibility of quantum maps, while the trace-distance related quantum tasks are determined by \( D(t_e) \) which is local-in-time. If \( D(t_e) \) is in inverse proportion to \( \mathcal{N}(\Lambda) \), non-Markovian effect will be negative (and vice versa), just as it did in the case of our model.

As an example of non-Markovian positive effect, consider a two-qubit system suffering from the quantum dynamics \( \Lambda \) described by Eq. (1) with \( f(\omega) \) a resonant Lorentzian reservoir of frequency spectral width \( \Gamma \) and correlation time \( \gamma_0^{-1} \) [1]. The optimal initial state pairs for non-Markovianity are still \( \rho^{\text{in}}_{1,2} = |\zeta^1\rangle \langle \zeta^1| \) or \( |\eta^2\rangle \langle \eta^2| \) [25], and we have

\[
D(t) = |\chi(t)| \quad \text{with} \quad \chi(t) = e^{-\frac{\pi}{2} \left( \cos \frac{\epsilon t}{2} + \frac{\Gamma}{\varepsilon} \sin \frac{\epsilon t}{2} \right)},
\]

where \( \varepsilon = \sqrt{\Gamma^2 - 2\gamma_0 \Gamma} \). For simplicity, we take \( t_e = 2\pi/\varepsilon \), then \( \mathcal{N}(\Lambda) = |\chi(t_e)| = e^{-\pi\Gamma/\varepsilon} \). We still consider the RSP fidelity. If the initial state is the Bell-diagonal state described by Eq. (4), then we have \( F = F(C^3 \mathcal{C}) \) with the three eigenvalues \( c_1^2 \mathcal{N}^2(\Lambda), c_2^2 \mathcal{N}^2(\Lambda) \) and \( c_3^2 \mathcal{N}(\Lambda) \) of \( C^3 \mathcal{C} \). In Fig. 3, the fidelity with initial states \((c_1, c_2, c_3) = (1, -1, 1) \) and \((-0.5, 0.4, 0.8) \) is depicted in comparison with the non-Markovianity \( \mathcal{N}(\Lambda) \) in the non-Markovian regime \( \Gamma/\gamma_0 \in [0, 2] \) (smaller value corresponds to stronger non-Markovian effect) [1]. We see that non-Markovianity in this example is positive: stronger non-Markovianity will induce higher RSP fidelity.

**Discussion and Conclusion.**– The anomalous effect, more non-Markovianity with lower performance of quantum tasks, exposed here indicates that the currently used
measures of non-Markovianity may be flawed to account fully the subtle and complex nature of quantum non-Markovianity. In particular, the popular yet vaguely defined concept of a "flow of information from and back to the system" and related approaches need more careful investigations. Indeed, this concept has come under challenge recently [31], where it is emphasized that non-Markovian behavior may arise from systems coupled to a classical reservoir, so that the idea of information flowing from the reservoir back to the system has no dynamical basis. The physical nature of non-Markovianity and its quantification, and the role of "flow of information" and system-environment correlations in building non-Markovian dynamics, are still not fully understood and required further exploration.

In summary, we have revealed that non-Markovian effect in an experimentally controllable open system can have quite different virtues, stronger non-Markovianity may cause lower performance of quantum tasks such as the fidelity for remote state preparation. Although the noise model we consider is specific, it already indicates an anomalous phenomenon concerning non-Markovianity, which also manifests in a real environment with a macroscopic number degrees of freedom [25]. Rigorously identifying the border between the positive and negative effects of non-Markovian behavior is still an open question, which is of great importance in open systems engineering. (January 9, 2013)

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