Jet Quenching with $T$-Dependent Running Coupling

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We perform an analysis of jet quenching in heavy ion collisions at RHIC and LHC energies with the temperature dependent running QCD coupling. Our results show that the $T$-dependent QCD coupling largely eliminates the difference between the optimal values of $\alpha_s$ for the RHIC and LHC energies. It may be viewed as direct evidence of the increase in the thermal suppression of $\alpha_s$ with rising temperature.

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1. INTRODUCTION

It is accepted that the strong suppression of the high-$p_T$ particle spectra in $AA$ collisions (usually called the jet quenching) observed at RHIC and LHC, is due to parton energy loss (radiative and collisional) in the quark–gluon plasma (QGP). The jet quenching is one of the major signals of the QGP formation in relativistic $AA$ collisions. The main contribution to the parton energy loss comes from the radiative mechanism due to induced gluon emission [1–5]. The effect of the collisional energy loss turns out to be relatively weak [6, 7].

The available pQCD approaches to the radiative energy loss [1–5] are limited to the one gluon emission. The effect of multiple gluon radiation is usually taken into account in the approximation of independent gluon emission [8]. Altogether, the pQCD calculations within this approximation give a rather good agreement with the jet quenching data from RHIC and LHC (see, e.g., [9] and references therein). However, it was found that, in the formulation with a unique temperature independent QCD coupling, the simultaneous description of the RHIC and LHC data requires to use somewhat smaller $\alpha_s$ at the LHC energies [9–12] (in [13, 14] a similar difference between jet quenching at RHIC and LHC energies has been found in terms of the transport coefficient $\hat{q}$). In [9–12] this fact has been demonstrated within the light-cone path integral (LCPI) approach to induced gluon emission [2], using the method developed in [15, 16], for running $\alpha_s$ which is frozen at low momenta at some value $\alpha_s^{fr}$. There it was found that the RHIC data support a significantly larger value of $\alpha_s^{fr}$ than the LHC data. One of the reasons for this difference may be somewhat stronger thermal suppression of the effective QCD coupling in a hotter QGP at the LHC energies. To draw a firm conclusion on this possibility it is highly desirable to perform calculations with a temperature dependent $\alpha_s$. Obviously, it is clear that, even without respect to the problem with a joint description of the RHIC and LHC jet quenching data, an observation of the temperature dependence of $\alpha_s$ from the jet quenching data would be of great importance on its own. The case of the $T$-dependent coupling has not been discussed so far in the literature on jet quenching. The purpose of this work is to perform such an analysis. We adapt the LCPI formalism to the case of the $T$-dependent running $\alpha_s$, and perform a joint analysis of the jet quenching data from RHIC on 0.2 TeV $Au + Au$ collisions and from the LHC on 5.02 TeV $Pb + Pb$ collisions.

2. THEORETICAL FRAMEWORK

We will consider the central rapidity region around $y = 0$. Our method for calculating the nuclear modification factor $R_{AA}$ is similar to the one used in our previous jet quenching analyses [9, 12, 16]. Therefore, we only outline its main points. We write the nuclear modification factor $R_{AA}$ for given impact parameter $b$ of $AA$ collision, the hadron transverse momentum $p_T$ and rapidity $y$ as

$$R_{AA}(b, p_T, y) = \frac{dN(A + A \rightarrow h + X)/dp_T dy}{T_{AA}(b)d\sigma(N + N \rightarrow h + X)/dp_T dy},$$

where $T_{AA}(b) = \int d\mathbf{p}_A (\sqrt{\mathbf{p}_T^2 + z^2})$ is the nuclear thickness function.
(with $\rho_A$ the nuclear density). The nominator of (1) is the differential yield of the process $A + A \rightarrow h + X$ (for clarity we omit the arguments $b$ and $p_T$). It can be written in terms of the medium-modified hard cross section $d\sigma_m/dp_Tdy$ as

$$\frac{dN(A + A \rightarrow h + X)}{dp_Tdy} = \int dp_Td(p + b/2)T_A(p - b/2) \times \frac{d\sigma_m(N + N \rightarrow i + X)}{dp_Tdy}.$$  \hspace{1cm} (2)

We write $d\sigma_m/dp_Tdy$ in the form

$$\frac{d\sigma_m(N + N \rightarrow i + X)}{dp_Tdy} = \sum_k \int \frac{dz D_{k,i}^m(z, Q)}{p_T'dy} \frac{d\sigma(N + N \rightarrow i + X)}{dp_T'dy},$$  \hspace{1cm} (3)

where $p_T' = p_T/z$ is the transverse momentum of the initial hard parton, $d\sigma(N + N \rightarrow i + X)/dp_T'dy$ is the ordinary hard cross section, and $D_{k,i}^m(z, Q)$ is the medium-modified fragmentation function for transition of a parton $i$ with the virtuality $Q$ to the final particle $h$. The fragmentation functions $D_{k,i}^m(z, Q)$ accumulate the medium effects. They depend crucially on the QGP fireball density profile along the hard parton trajectory. We use somewhat improved method of [16] for evaluation of $D_{k,i}^m(z, Q)$ via the one gluon induced spectrum in the approximation of independent gluon emission [8]. We refer the interested reader to [9] for details. In addition, there, a detailed description of the technical aspects of the implementation of formulas (1)–(3) can be found.

We turn now to the method for incorporation of the temperature dependent coupling in calculating the induced gluon spectrum. Let us consider the case of $q \rightarrow gg$ process. In the LCPI formalism [2] the gluon spectrum in $x = E_g/E_q$ for $q \rightarrow gg$ process can be written as [15]

$$\frac{dP}{dx} = \int dz n(z) \frac{d\sigma_{BH}^m(x, z)}{dx},$$  \hspace{1cm} (4)

where $n(z)$ is the medium number density, $d\sigma_{BH}^m/dx$ is an effective Bethe–Heitler cross section including both the Landau–Pomeranchuk–Migdal and the finite-size effects. Note that for the midrapidity region $y = 0$, the longitudinal coordinate $z$ in (4) coincides with the proper time $\tau$ for evolution of the QGP fireball. At fixed coupling $d\sigma_{BH}^m/dx$ can be written as [15]

$$\frac{d\sigma_{BH}^m(x, z)}{dx} = -\frac{P_q^s(x)}{\pi M} \text{Im} \left[ \frac{\xi}{p} \frac{d\xi}{dp} \frac{F(\xi, p)}{\sqrt{p}} \right]_{p=0}.$$  \hspace{1cm} (5)

Here, $P_q^s(x) = C_F[1 + (1 - x)^2]/x$ is the usual splitting function for $q \rightarrow gg$ process, $M = E_x(1 - x)$ is the reduced “Schrödinger mass,” $F$ is the solution to the radial Schrödinger equation for the azimuthal quantum number $m = 1$

$$\frac{i}{\xi} \frac{dF(\xi, p)}{d\xi} = \left[ -\frac{1}{2M} \left( \frac{\partial}{\partial p} \right)^2 + \frac{1}{8M^3} \right] F(\xi, p),$$  \hspace{1cm} (6)

with the boundary condition $F(\xi = 0, p) = \sqrt{\rho} \sigma_3(\rho, x) e^{K_f(\epsilon p)}$ ($K_1$ is the Bessel function), $L_f = 2M/e^2$ with $e^2 = m_q^2 + m_g^2(1 - x)^2$, $\sigma_3(\rho, x)$ is the cross section of interaction of the $q\bar{q}g$ system with a medium constituent located at $z$. The potential $\nu$ in (6) reads

$$\nu(\rho, x, z) = -i \frac{n(z)\sigma_3(\rho, x, z)}{2}. $$  \hspace{1cm} (7)

The $\sigma_3$ is given by [17]

$$\sigma_3(\rho, x, z) = 9 \left[ \sigma_{qg}(\rho, z) + \sigma_{qg}(1 - x, \rho, z) - \frac{1}{8} \sigma_{qg}(x\rho, z) \right],$$  \hspace{1cm} (8)

where

$$\sigma_{qg}(\rho, z) = C_F^2 \int dqq_3^2 \frac{1 - \exp(q_3^2)}{[q_3^2 + \mu_D^2(z)]^2},$$  \hspace{1cm} (9)

is the local dipole cross section for the color singlet $q\bar{q}$ pair, $C_{F,T}$ are the color Casimir for the quark and thermal parton (quark or gluon), $\mu_D$ is the local Debye mass.

Diagrammatically, the effective Bethe–Heitler cross section (5), for any partonic process $a \rightarrow bc$, is given by the graph shown in Fig. 1 where the left and right parts correspond to the dressed and bare Green’s functions describing $z$-evolution of the $abc$ three-body system (i.e., $\bar{a}gg$ for $q \rightarrow gg$ process). The central black blob in Fig. 1 describes interaction of the three-body state with a medium constituent. For RHIC and LHC conditions, the dominating contribution to the effective Bethe–Heitler cross section comes from $N = 1$ scattering. This means that the dressed Green’s function in Fig. 1 is close to the bare one. Therefore, in this regime the average $z - z_i$ is close to $z_2 - z$. For generalization of the above formulas to the case of running $T$-dependent coupling one
should modify $\alpha_s$ that appears on the right hand side of (5), which comes from product of the QCD couplings in the decay vertices at $z_1$ and $z_2$ in Fig. 1, and $\alpha_s^2$ in formula for the dipole cross section (9). In the latter case, it is natural simply to replace the fixed $\alpha_s$ by the local running coupling $\alpha_s(q, T(z))$. However, the situation is more complicated for $\alpha_s$ on the right hand side of (5). In terms of the variable $\xi$ in (5) $z_1 = z - \xi$. As we said above, for the dominating $N = 1$ scattering term on the average $z - z_1 \sim z_2 - z$. We will use this approximation for the whole effective Bethe–Heitler cross section. Then, for the temperatures at the decay vertices one can take $T(z \pm \xi)$. This approximation should be reasonable due to a smooth dependence of $T$ on the proper time ($T \propto \tau^{-\frac{1}{3}}$) and a smooth (logarithmic) dependence of $\alpha_s$ on the QGP temperature. Since we work in the coordinate space the virtualities at the decay vertices do not appear in our formulas. Qualitatively, from the uncertainty relation, one can obtain that $Q^2 \sim \rho^2$, where $\rho$ is the transverse size of the three-body state at $z$. Similarly to our previous analyses of jet quenching with a unique $T$-independent running $\alpha_s$, we determine the virtuality for these vertices as $Q^2(\xi) = a^2/\xi^2$ with $a = 1.85$ [7]. This parametrization takes into account the Schrödinger diffusion relation, that gives $\rho^2 \sim \xi/M$, and the value of the parameter $a$ has been adjusted to reproduce the $N = 1$ scattering contribution evaluated in the ordinary Feynman diagrammatic approach [18]. Thus, for calculations with the $T$-dependent running coupling we replace the fixed $\alpha_s$ on the right-hand side of (5) by $\sqrt{\alpha_s(Q_0^2, T(z - \xi))\alpha_s(Q_0^2, T(z + \xi))}$. We checked that the version with $T(z \pm \xi)$ replaced by $T(z)$ gives practically the same results, i.e., the effect of the finite separation between the decay vertices is small. This occurs because the dominating contribution to the radiative energy loss comes from gluons with the formation length considerably smaller than the QGP size.

First principle calculations of the $\alpha_s(Q, T)$ in the QGP are not yet available. In the lattice analysis [19], via calculation of the free energy of a static heavy quark–antiquark pair, there have been obtained an effective in-medium coupling $\alpha_s(r, T)$ in the coordinate representation. The results of [19] show that $\alpha_s(r, T)$ at $r \ll 1/T$ becomes close to the ordinary vacuum QCD coupling $\alpha_s(Q)$ with $Q \sim 1/r$. In the infrared region $\alpha_s(r, T)$ reaches maximum at $r \sim 1/\kappa T$ with $\kappa \sim 4$ and then with increasing $r$ it falls to zero. With identification $r \sim 1/Q$, this pattern is qualitatively similar to that obtained for $\alpha_s(Q, T)$ in the momentum representation within the functional renormalization group calculations [20]. Motivated by the results of [19, 20], we use parametrization of $\alpha_s(Q, T)$ in the form

$$
\alpha_s(Q, T) = \begin{cases} 
\frac{4\pi}{9 \log (Q^2/\Lambda_{QCD}^2)} & \text{if } Q > Q_p(T), \\
\alpha_s^f(T) & \text{if } Q < Q_p(T),
\end{cases}
$$

(10)

where $Q_p = \Lambda_{QCD} \exp \left[ \pi/9 \alpha_s^f \right]$ (in the present analysis we take $\Lambda_{QCD} = 200$ MeV), and $c < 1$. The parameter $c$ defines the width of the plateau where $\alpha_s$ equals its maximum value $\alpha_s^f$. For our basic version we take $c = 0.8$. We have also performed calculations for $c = 0$. The case $c = 0$ is similar to the model with a frozen QCD coupling in the infrared region at $T = 0$ [21, 22]. In [22] it was called the $F$-model. For $c \sim 1$ the parametrization (10) is qualitatively similar to the $G$-model of the vacuum $\alpha_s$ of [22]. We take $Q_p = \kappa T$, and perform fit of the free parameter $\kappa$ using the data on the nuclear modification factor $R_{AA}$. From the lattice results of [19] one can expect that $\kappa \sim 4$. However, since the relation $r \sim 1/Q$ is of qualitative nature, our parameter $\kappa$ may differ from that in the lattice calculations in the coordinate space. In addition, one should bear in mind that in the infrared region the effective coupling becomes process dependent [19]. We also present the results for a unique $\alpha_s$ in the whole QGP with the $T$-independent free parameter $\alpha_s^f$.

3. NUMERICAL RESULTS

We perform calculations for the QGP fireball with purely longitudinal Bjorken’s $1 + 1$ expansion [23], which gives proper time dependence of the entropy density $s(\tau)/s(\tau_0) = \tau_0/\tau$, where $\tau_0$ is the QGP thermalization time. We take $\tau_0 = 0.5$ fm. As in [9] we take a linear $\tau$-dependence $s(\tau) = s(\tau_0)\tau/\tau_0$ for $\tau < \tau_0$. We
Table 1. Optimal values of $\kappa$ and $\alpha_s^{fr}$ (for $c = 0.8$ in Eq. (10)) and corresponding $\chi^2/d.p.$ for different data sets. For LHC the results are presented for fits for the data points with $9 \text{ GeV} < p_T < 120 \text{ GeV}$ and $9 \text{ GeV} < p_T < 22 \text{ GeV}$ (the numbers in brackets). For RHIC the fits are performed for the data points with $p_T > 9 \text{ GeV}$. The numbers in brackets for $\chi^2/d.p.$ for RHIC give $\chi^2/d.p.$ obtained with the LHC optimal parameters $\kappa/\alpha_s^{fr}$ obtained for the LHC fits for $9 \text{ GeV} < p_T < 22 \text{ GeV}$ and $9 \text{ GeV} < p_T < 120 \text{ GeV}$

| $\alpha_s(Q,T)$ | $\alpha_s^{fr}$ |
|-----------------|-----------------|
| $\kappa$  | $\chi^2/d.p.$ | $\chi^2/d.p.$ |
| PHENIX Au + Au 0.2 TeV | 2.65 | 0.167 (0.71, 0.81) | 0.698 | 0.157 (4.4, 4.75) |
| ALICE Pb + Pb 5.02 TeV | 3.19 | 0.46 (0.68) | 0.464 (0.464) | 0.56 (0.88) |
| ATLAS Pb + Pb 5.02 TeV | 3.48 (3.46) | 0.37 (0.22) | 0.439 (0.439) | 0.33 (0.2) |
| CMS Pb + Pb 5.02 TeV | 3.99 (3.81) | 0.58 (0.25) | 0.403 (0.412) | 0.46 (0.21) |
| All LHC Pb + Pb 5.02 TeV | 3.33 (3.28) | 1.04 (0.96) | 0.451 (0.455) | 0.93 (0.96) |

However, we have found that the LHC fits for $p_{T,\text{max}} = 120$ and 22 GeV give very similar results. We calculate $\chi^2$ as

$$\chi^2 = \sum_i \left( \frac{f_{\text{exp}}^i - f_{\text{th}}^i}{\sigma_i^2} \right)^2,$$

where $N$ is the number of the data points, the squared errors include the systematic and statistic errors $\sigma_i^2 = \sigma_{i,\text{stat}}^2 + \sigma_{i,\text{sys}}^2$. In calculating $\chi^2$ as functions of free parameters $\kappa$ and $\alpha_s^{fr}$ we have used the theoretical $R_{AA}$ obtained with the help of a cubic spline interpolation from the grids calculated with steps (in $\kappa$ and $\alpha_s^{fr}$)

$\Delta\kappa/\kappa$, $\Delta\alpha_s^{fr}/\alpha_s^{fr} \sim 0.05$. The optimal values of $\kappa$ and $\alpha_s^{fr}$ with corresponding values of $\chi^2/d.p.$ ($\chi^2$ per data point) are summarized in Table 1. In Table 1, we show the results for RHIC and LHC separately. We also performed fitting for the combined RHIC plus LHC data set (not shown). In this case, the results are very close to that for the LHC data set alone. This occurs because the number of the data points for the LHC data set is much bigger than for the RHIC data set. From Table 1, one can see that the LHC data give somewhat bigger value of the optimal parameter $\kappa$. However, the difference is not very big. To illustrate better the difference between RHIC and LHC, in Table 1 for the PHENIX data set besides $\chi^2/d.p.$ for $\kappa$ and $\alpha_s^{fr}$ fitted to the PHENIX data we also give $\chi^2/d.p.$ for the optimal $\kappa$ and $\alpha_s^{fr}$ obtained for the LHC data set. One case see that for the $T$-dependent $\alpha_s$ the LHC value of $\kappa$ gives rather good fit quality $\chi^2/d.p. \approx 0.7–0.8$, while for the $T$-independent $\alpha_s$ for the LHC optimal parameter $\alpha_s^{fr}$ we have $\chi^2/d.p. \approx 4.4–4.8$, that says about a rather strong disagreement with the PHENIX data.
In Figs. 2, 3 we compare our results for $R_{AA}$ of $\pi^0$ for 0.2 TeV Au + Au collisions for different centrality bins from our calculations for $T$-dependent $\alpha_s$ (solid and dashed) and $T$-independent $\alpha_s$ (dotted and dash-dotted) compared to data from PHENIX [30]. The solid and dotted curves are for $\kappa = 2.65$ and $\alpha_s^{fr} = 0.698$ obtained by fitting the PHENIX $R_{AA}$ data set for $p_T > 9$ GeV. The dashed and dash-dotted lines are for $\kappa = 3.28$ and $\alpha_s^{fr} = 0.455$ obtained by fitting the LHC $R_{AA}$ data sets for 9 GeV $< p_T < 22$ GeV.

Fig. 2. $R_{AA}$ of $\pi^0$ for 0.2 TeV Au + Au collisions for different centrality bins from our calculations for $T$-dependent $\alpha_s$ (solid and dashed) and $T$-independent $\alpha_s$ (dotted and dash-dotted) compared to data from PHENIX [30]. The solid and dotted curves are for $\kappa = 2.65$ and $\alpha_s^{fr} = 0.698$ obtained by fitting the PHENIX $R_{AA}$ data set for $p_T > 9$ GeV. The dashed and dash-dotted lines are for $\kappa = 3.28$ and $\alpha_s^{fr} = 0.455$ obtained by fitting the LHC $R_{AA}$ data sets for 9 GeV $< p_T < 22$ GeV.

In Figs. 2, 3 we compare our results for $R_{AA}$ with the RHIC data from PHENIX for $\pi^0$-meson in 0.2 TeV Au + Au collisions [30] and the LHC data [31–33] for $h^2$ in 5.02 TeV Pb + Pb collisions. One can see that for the optimal parameters (separately for RHIC and LHC) agreement with the data is quite good for both the versions. However, the situation with a joint description of the RHIC and LHC data is very different for the $T$-dependent and $T$-independent couplings. To visualize better this difference in Fig. 2, in addition to predictions for $R_{AA}$ in Au + Au collisions obtained with the optimal parameters fitted to the PHENIX data, we also plot the results for the optimal parameters fitted to the LHC data. As one can see, for the version with $T$-dependent coupling the LHC value of $\kappa$ leads to not bad agreement with the PHENIX data. While for the version with $T$-independent coupling the curves for the LHC value of $\alpha_s^{fr}$ overshoot the data considerably at $p_T \leq 15$ GeV. For the $T$-dependent version some overshoot at $p_T \leq 15$ GeV also exists, but it is rather small. We conclude from comparison with experimental data shown in Figs. 2, 3 and results of our fits given in Table 1, that the $T$-dependence of the QCD coupling may strongly reduce the difference between the optimal $\alpha_s$ for RHIC and LHC.\footnote{Note that for the LHC data on 2.76 TeV Pb + Pb collisions [36–38] the situation is the same. Our calculations show that the optimal parameters (and quality of the fits) in this case are very close to that for 5.02 TeV Pb + Pb collisions.}

\[ \alpha_s(Q,T) = \alpha_s(Q) + (1 - \Lambda_{QCD}^2) \exp(-\kappa T) \]
LHC data one can significantly improve agreement with the RHIC data in the low $p_T$ region. Such an increase in $\alpha_s$ is not unrealistic; e.g., it may mimic an enhancement of the induced gluon emission at $T \sim T_c$ [39, 40] in the presence color-magnetic monopoles [41].

We have also calculated the azimuthal asymmetry $v_2$. Although we have fitted the optimal parameters to the data on $R_{AA}$, for both the $T$-dependent and $T$-independent versions we have obtained a quite reasonable agreement with the $v_2$ data as well. For the $T$-dependent version, we obtained a bit bigger $v_2$. This occurs due to some enhancement of the contribution to the energy loss from the later, low temperature, stage of the QGP evolution, which has a bigger initial fireball azimuthal asymmetry.

The above results have been obtained for parametrization (10) with $c = 0.8$. The results from calculation for $c = 0$ in (10), i.e., for flat $\alpha_s$ at $Q < Q_p$, turn out to be very similar to that for $c = 0.8$. We obtained rather good agreement with the RHIC and LHC data for the versions with $T$-dependent and $T$-independent coupling. However, similarly to the case $c = 0.8$, the latter version leads to a considerable disagreement between free parameters for RHIC and LHC. While the former version largely eliminates this disagreement.

4. SUMMARY

We have studied the influence of the temperature dependence of running coupling on the variation of jet quenching from the RHIC to LHC energies within the LCPI [2] approach to the induced gluon emission. The calculations are performed using the method suggested in [15, 16]. For our basic version, we use parametrization of running coupling $\alpha_s(Q,T)$, which has a short plateau around $Q \sim Q_p$, turn out to be very similar to that for $c = 0.8$. We obtained rather good agreement with the RHIC and LHC data for the versions with $T$-dependent and $T$-independent coupling. However, similarly to the case $c = 0.8$, the latter version leads to a considerable disagreement between free parameters for RHIC and LHC. While the former version largely eliminates this disagreement.

\begin{equation}
\chi^2/d.p. \approx 0.7-0.8.
\end{equation}

This differs drastically from the results for the $T$-independent $\alpha_{T}^{\rho}$, which leads to rather strong disagreement with the RHIC data ($\chi^2/d.p. \approx 4.4-4.8$) for the optimal value $\alpha_{T}^{\rho}$ fitted to the LHC data. Thus, our anal-

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**Fig. 3.** $R_{AA}$ of charged hadrons for 5.02 TeV Pb + Pb collisions for different centrality bins. Solid: calculations for $T$-dependent $\alpha_s$ with $\kappa = 3.33$. Dotted: calculations for $T$-independent $\alpha_s$ with $\alpha_s^{\rho} = 0.451$. $\kappa$ and $\alpha_s^{\rho}$ are obtained by fitting $R_{AA}$ in the range $9 \text{ GeV} < p_T < 120 \text{ GeV}$. Data points are from ALICE [31], ATLAS [32], and CMS [33].
ysis shows that the $T$-dependent $\alpha_s$ may largely eliminate the problem of different optimal QCD coupling for the RHIC and LHC energies. For parametrization with flat $\alpha_s$ at $Q < Q_F$ with $Q_F = \kappa T$, we obtained very similar results. Our results may be viewed as the first direct evidence of the increase in the thermal suppression of $\alpha_s$ with rising QGP temperature.

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