Large Eddy Simulation of a wind tunnel wind farm experiment with one hundred static turbine models

Juliaan Bossuyt\textsuperscript{1,2}, Charles Meneveau\textsuperscript{2} and Johan Meyers\textsuperscript{1}

\textsuperscript{1} Department of Mechanical Engineering, KU Leuven, Celestijnenlaan 300, Leuven, Belgium.
\textsuperscript{2} Department of Mechanical Engineering and Center for Environmental and applied Mechanics, Johns Hopkins University, 3400 North Charles Street, Baltimore, MD21218, USA.

Abstract. A common challenge for the validation of wind farm simulations with field data is the complex, variable and not precisely defined inflow conditions. In this study, we make use of published wind tunnel data for a scaled farm with 100 porous disk models, and for highly controlled wind conditions, to validate a large eddy simulation. We present a simple methodology to simulate the spatially developing turbulent boundary layer from the experiment. The method makes it possible to match the vertical profiles of mean velocity and turbulence intensity in front of the farm. The mean-row power from the simulations shows a good agreement with the experimental results. We find that simulating the spatially developing boundary layer in a channel with the same height as the wind tunnel, instead of the common approach of a fully developed half-channel flow, is essential to match the experimental results. It is concluded that experiments with a naturally developing boundary layer and a clean flow above, provide well-defined conditions that can be simulated accurately in large eddy simulations and are useful for the validation of wind farm models.

1. Introduction

Analytical and numerical models are essential to study the performance, design or control of wind farms. A large variety of models have been developed, with different levels of approximations, ranging from the simplest analytical wake models to computationally more expensive turbulence resolving flow simulations. Today, even the most complex methods that are still practical for large wind farms in a highly turbulent atmospheric boundary layer, use approximations and assumptions. Large Eddy Simulations (LES) in particular solve the spatially filtered Navier-Stokes equations [1], use a turbine model to represent the blade forces [2], a turbulence model to represent the subgrid-scale turbulent stresses [3] and a wall model for the wall shear stress [4]. As wind farm studies rely more and more on numerical models such as LES, it becomes increasingly important to verify the accuracy of these methods. Validation with field experiments are typically challenging due to changing wind or weather conditions, difficult flow measurements and complex inflow conditions. A comparison of mean farm power from LES and field measurements is therefore often challenging [5]. Wind tunnel experiments, on the other hand, can provide detailed and statistically converged measurements for highly controlled test cases. However, because of scaling issues, and experimental challenges, wind tunnel studies have so far mainly focused on smaller farms and only a few simplified layouts [6, 7, 8, 9]. In the first wind tunnel study of a scaled wind farm with 80 static porous disk models of wind turbines, the integral farm thrust force was measured [9]. While this study introduced the idea of very small
Figure 1. Schematic of the wind tunnel experimental setup with a spatially developing boundary layer and a wind farm with twenty spanwise rows and five streamwise column of porous disk models.

scale porous disk models, it would be especially valuable to measure also the flow field and the individual turbine outputs.

A recent study [10] has provided the first mean-row power data for a wind tunnel experiment of a scaled farm with 100 static and individually instrumented turbine models, and presents a unique opportunity for the validation of LES. For this purpose, it is important to simulate the wind tunnel conditions correctly. Consistent with actuator-disk modeled turbines in LES, the wind turbines are represented in the experiment by static porous disk models and the wind farm operates in a spatially developing boundary layer. The goal of this work is to present a methodology for the accurate simulation of such wind tunnel conditions with the SP-Wind LES code [11], and to use this experimental dataset to verify the accuracy of the simulations.

2. Experimental setup
For the purpose of this study we make use of the experimental data published by Bossuyt et al. [10]. Figure 1 visualizes the setup of the experiment. The experiments were performed with a clean inflow from the inlet contraction of the wind tunnel, without the use of any turbulence grids. The boundary layer develops naturally over the wind tunnel floor, after being tripped at the entrance by three chains. Once the boundary layer reaches the wind farm, the boundary layer height is $\delta_9 = 0.16 \text{m}$, the roughness length $z_0 = 0.9 \times 10^{-2} \text{mm}$, and the friction velocity $u_\tau = 0.6 \text{m/s}$. The wind farm consists of 100 static porous disk models. The diameter of the models is 0.03 m and the hub height is 0.023 m. The porous disks are spaced with a distance of seven rotor diameters in the streamwise direction ($s_x = 0.21 \text{m}$) and five rotor diameters in the spanwise direction ($s_y = 0.15 \text{m}$). Each model is instrumented with strain gages, and the individual time-dependent force is measured for the sixty models in the central three columns. The porous disk models have a realistic thrust coefficient of $C_T = 0.75 \pm 0.04$ and produce wakes that are representative for a wind turbine in the far-wake region. The measurement time was 5 minutes or more, to ensure excellent statistical convergence. For more details about the experimental setup, the calibration, measurement uncertainty (as estimated from an error propagation analysis of the individual strain measurements) and a description of the signal reconstruction methodology, we refer to Ref. [10].

3. Numerical setup
LES studies of wind farms often simulate a pressure driven, fully developed half-channel flow, by making use of the horizontal periodicity of pseudo-spectral codes and a symmetry boundary condition at the top of the domain, which is assumed to be somewhere at or slightly above

\[ \delta_9 = 0.16 \text{m}, \quad z_0 = 0.9 \times 10^{-2} \text{mm}, \quad u_\tau = 0.6 \text{m/s}. \]
Figure 2. The numerical setup of the simulations with a spatially developing boundary layer.

the boundary layer [5, 11]. However, such a setup does not correctly represent the wind tunnel conditions in which a turbulent boundary layer develops over the bottom wall and interacts with a clean freestream above. A spatially developing boundary layer can be simulated by making use of a rescaling method [12, 13]. In the current paper we present a different approach, following the simplified method by [14], and which can be implemented more easily in existing LES codes that already employ a concurrent precursor simulation technique [15] to condition the inflow for the main domain. The proposed approach adds a vertical damping layer above a boundary layer in the precursor simulation, so that inlet conditions with a desired boundary-layer height are generated. The damping layer forces all velocity fluctuations to zero and the mean velocity towards a symmetry condition at the interface. In this regard, the damping layer is implemented similarly to the fringe region technique [15], which enforces the inflow conditions in a select part of the domain by adding a body force term to the momentum equation. Specifically, in the fringe region, the body force $F_i$ is added for each velocity component $i$, according to $F_i(x, y, z) = -\lambda(x)(u_i(x, y, z) - u_{p,i}(x, y, z))$ [16], with $\lambda(x)$ a location dependent and smoothly varying weighting function which reduces to zero outside the fringe region, $u_i$ the velocity in the simulation domain, and $u_{p,i}$ the new velocity which is enforced by the fringe region, and for instance sampled in a concurrent precursor simulation. In the main domain, no damping layer is simulated such that the boundary layer is able to develop freely. The simulation setup is illustrated in figure 2, and the simulated wind farm layouts are shown in figure 3.

In this study, we make use of the LES code SP-Wind, developed at KU Leuven [17, 18, 16]. The pseudo spectral code is periodic in horizontal directions, and uses a standard Smagorinsky model [19] with a Smagorinsky coefficient $C_s = 0.14$, which is damped with a wall damping function [16]. The flow is forced through a constant pressure gradient in the precursor simulation. The simulations are normalized by a reference length scale $L$, a velocity scale $u_*$ as defined by
the imposed pressure gradient in the precursor domain $-u_*^2/L$, and the corresponding time scale $L/u_*$. Because the simulations are run specifically to compare with an experimental setup, length scales of the simulation domain will be provided in physical units throughout this paper. The wind turbines are represented by actuator disk models, for which the thrust coefficient is set to the same value as in the experiment, $C_T = 0.75$, by setting $C'_T = 4/3$ [18].

The dimensions of the main domain are chosen to represent the wind tunnel test section, $L_x = 6.9$ m, $L_y = 1.2$ m and $L_z = 0.8$ m. A fringe region is used to smoothly enforce the inflow conditions from a concurrent precursor simulation into the main domain, and is located at the final 10% of the main domain. The simulation time-step was on the order of $3 \times 10^{-4}L/u_*$ and the maximum damping coefficient for the fringe region was set to $\lambda = 1500u_*/L$, while a much lower value of $\lambda = 5u_*/L$ was found to give good results for the damping layer. The boundary layer can develop over a distance of 1.2 m in the main domain (i.e. approximately six times the boundary layer height), before it reaches the first row of the wind farm. The simulations are run until a statistically stationary state is reached, and then averaged over a time that equals approximately 4.4 seconds in physical time. To prevent that very long streamwise elongated structures would lock in place in the precursor simulation, we employ shifted periodic boundary conditions as developed and implemented in SP-Wind by Munters et al. [16]. The velocity for the fringe regions is sampled at a location of 60% of the precursor domain length, and shifted in the spanwise direction over a length equal to half the boundary layer height, for a minimal spatial correlation. It is important to note that the shifted boundary conditions are not essential for this study. However, this technique greatly improves the spanwise homogeneity of the flow.

We performed a simulation of the aligned and staggered layout with the spatially developing

Figure 3. Top view of the aligned and staggered wind farm layouts, with $S_x/D = 7$ and $S_y/D = 5$. The porous disk models for which the thrust forces are measured in the experiment, and similarly acquired in the LES, to calculate the mean row power, are indicated in blue.
Figure 4. A snapshot of the instantaneous (a) and time averaged (b) streamwise velocity in the main domain of a simulation with a spatially developing boundary layer. The fringe region of the main domain is not included.

boundary layer setup, as indicated in figure 2. For comparison, we also simulate the aligned layout in a fully developed half-channel flow. For this simulation the domain height was 0.2 m, which was chosen slightly larger than the incoming boundary layer height. For each simulation, the streamwise and spanwise grid spacing is: $\Delta x = 0.0066$ m and $\Delta y = 0.006$ m. For the fully developed case the uniform vertical grid spacing is $\Delta z = 0.003$ m. In the simulations with a spatially developing boundary layer, we use the same grid spacing of $\Delta z = 0.003$ m for the bottom 0.3 m of the domain. Above this height, we smoothly increase the vertical grid spacing to $\Delta z = 0.032$ m. The resulting resolution of the simulations are $N_x \times N_y \times N_z = 1000 \times 200 \times 67$ for the fully developed and $N_x \times N_y \times N_z = 1000 \times 200 \times 124$ for the spatially developing case. The concurrent precursor simulation domain extended over the same respective $y$ and $z$ resolutions, and $N_x = 672$ in the streamwise direction. The total domain length of the precursor is thus approximately 4.4 m, and the effective length, considering the shifted periodic boundary conditions, approximately 3.3 m. The roughness length is set to the value measured in the experiment $z_0 = 0.9 \times 10^{-5}$ m.

Because of the large simulation domain covering twenty rows of turbines and a concurrent precursor simulation, the resolution was limited by the computational cost, and corresponds to five grid cells per wind turbine diameter in the spanwise direction. Similar grid sizes are commonly used for LES studies of large farms with the actuator disk implementation, and have been found to provide acceptable relative row-power estimates [20, 21, 22, 23]. As indicated by Ref. [20, 24], using a higher resolution is expected to improve the overall accuracy of the simulations, as smaller turbulent scales are resolved by the grid and actuator disk forces are less smeared out spatially by the Gaussian filter kernel. Ref. [20] found that specifically the relative power of the second row, up to the seventh row of an aligned wind farm, improves by using a finer resolution. We therefore expect that a finer resolution may further improve the agreement between the simulations and the experiments, which is discussed in the next section.

Figure 4 (a) shows an instantaneous velocity contour plot of the main domain for the spatially developing simulation and an aligned layout. The growth of boundary layer when it interacts with the farm is clearly observed. Figure 4 (b) shows the time averaged streamwise velocity, with above the farm a small acceleration due to wind tunnel blockage by the increasing boundary layer height and wind farm induced wake region.
Figure 5. Profiles of mean velocity (a) and local turbulence intensity (b). Dashed lines indicate the bottom, hub, and top height of the porous disk or actuator disk models, and the boundary layer height in the experiment respectively.

4. Results
In this section we compare the simulation results with the experimental data. First we compare the simulated conditions with the inflow conditions in the experiment. Subsequently, the mean row power from the actuator disk models in the LES are compared with the experimental results.

4.1. Inflow conditions
Figure 5 (a) shows a good agreement between the LES and experimental profiles of mean velocity, at a location 0.21 m, or seven disk diameters, upstream from the wind farm. For the fully developed simulation, the log-region extends over the entire height of the simulation domain. For the experiment, the profile indicates the presence of both a log-region and a wake-region. The simulation with the spatially developing boundary layer indicates that a similar wake-region is developed. However, because of the limited distance between the main domain inlet and the wind farm (approximately six times the boundary layer height), the wake region is not yet as pronounced as in the experiment. We conclude that both simulations match the experimental profile of mean velocity relatively good, the simulation with a spatially developing boundary layer provides the best agreement, but with a more advanced rescaling method, one may be able to further improve the agreement in the wake region.

At the height of the wind turbines, the profiles of local turbulence intensity, see figure 5 (b), show an excellent agreement. Above the wind farm, the spatially developing LES shows the best agreement with the experimental data. Figures 5 (a) and (b) indicate that the incoming boundary layer height in the spatially developing simulations is slightly larger than in the experiment, e.g. approximately 0.2 m in comparison to 0.16 m for the experiment. For a future simulation, the agreement can be further improved by simply adjusting the height of the damping layer in the precursor domain correspondingly.

4.2. Wind farm performance
In figure 5 the time and row averaged power output is plotted, as normalized by the power of the first row. Some qualitative differences can be observed for the second and third rows in which the LES predict an undershoot while the experiments approach the asymptotic behavior.
Figure 6. Comparison of the mean row power from the LES and experiments for an aligned layout. Vertical bars indicate the experimental uncertainty as estimated by Ref. [10].

In more monotonic fashion. The cause for these differences are still being explored. We remark also that the error bars on the experimental results are similar to the observed differences. For the fully developed LES, it can be seen that after approximately five rows, the mean power does not change from one row to the next. On the other hand, the spatially developing LES results in lower power values, which reduce slowly as a function of row number, all the way to the end of the farm. In this regard, the developing LES matches much better the experimental results, with mean power values that are mostly within the experimental uncertainty. This comparison indicates how the conditions at the top of the boundary layer can influence the energy exchange with the wind farm and result in higher power values. Such effects have for example also been observed in LES studies that consider the presence of a capping inversion layer in the atmospheric boundary layer [25]. While it is possible that adjusting the height of the domain of a fully developed case could result in the same asymptotic power extraction, it is unclear how such a height could be chosen a-priori, thus making the simulation of developing conditions preferable in general.

In figure 6, the time and row averaged power for a spatially developing LES simulation with an aligned and staggered layout are compared with the experimental data. It can be seen that also for a staggered layout, the LES with a spatially developing boundary layer, is able to match the experimental results closely. The simulated mean power values are mostly within the uncertainty bounds of the experiment, and the agreement for the second and third row is also significantly better for the staggered layout. It is concluded that the use of the proposed simple method to simulate a spatially developing boundary layer, results in a good agreement with the experimental results for the mean power output. The agreement is especially good if we consider that the LES is run for a very high Reynolds number (by setting viscosity to zero, and only using the subgrid-scale model to represent stresses), while the experiments are performed at a relatively low Reynolds number (e.g. \( Re_D \approx 2.5 \times 10^4 \) based on the mean hub velocity and the disk diameter). This observation supports the argument that drag based porous disk models can be used at such low Reynolds numbers [26], as they prove to have a significantly less Reynolds number dependent behavior than scaled rotor models for which the wake characteristics and performance depend on the Reynolds number dependent lift and drag coefficients over the blades.
5. Conclusion
We performed simulations for the aligned and staggered layout of the wind tunnel experiments by Bossuyt et al. [10], by making use of a simple methodology to simulate a spatially developing boundary layer. With this technique we are able to reproduce the wind tunnel inflow conditions accurately, and find a good agreement between the average row power in the simulations and experiments. By comparing the results, with an LES of a fully developed boundary layer, we show the importance of simulating the spatial development of the boundary layer as it interacts with the farm. The good agreement between the simulations and experiments serve as valuable code validation of the LES tool.

Furthermore, we conclude that when wind tunnel studies are performed of wind farms, creating realistic boundary conditions is important and challenging. For the purpose of using the experimental results to validate numerical codes, it is especially important that the inflow conditions can be easily reproduced numerically. To this end, we conclude that a naturally developing turbulent boundary layer, results in well defined inflow condition which can be accurately reproduced in numerical simulations such as LES.

Acknowledgements
The authors would like to thank W. Munters, D. Allaerts, A. Vitsas and T. Haas for their contributions to the SP-wind code and insightful discussions. Work is supported by ERC (grant no. 306471, the ActiveWindFarms project) and by NSF (grant OISE-1243482, the WINDINSPIRE project).

References
[1] Sagaut P 2006 Large eddy simulation for incompressible flows: an introduction (Springer Science & Business Media)
[2] Martínez-Tossas L, Churchfield M and Leonardi S 2015 Wind Energy
[3] Meneveau C and Katz J 2000 Annual Review of Fluid Mechanics 32 1–32
[4] Piomelli U and Balaras E 2002 Annual Review of Fluid mechanics 34 349–374
[5] Stevens R J A M and Meneveau C 2017 Annual Review of Fluid Mechanics 49 311–339
[6] Corten G P, Schaa k P and Hegberg T 2004 ECN report ECN-C-04-048
[7] Chamorro I P and Porté-Agel F 2011 Energies 4 1916 ISSN 1996-1073
[8] Cal R B, Lebrón J, Castillo L, Kang H S and Meneveau C 2010 *Journal of Renewable and Sustainable Energy* 2 013106
[9] Theunissen R, Housley P, Allen C B and Carey C 2015 *Wind Energy* 18
[10] Bossuyt J, Howland M F, Meneveau C and Meyers J 2017 *Experiments in Fluids* 58 1
[11] Goit J P and Meyers J 2015 *Journal of Fluid Mechanics* 768 5–50 ISSN 1469-7645
[12] Spalart P R and Leonard A 1987 *Turbulent Shear Flows 5* (Springer) pp 234–252
[13] Lund T S, Wu X and Squires K D 1998 *Journal of Computational Physics* 140 233–258
[14] Lund T S 1993 *Annual Research Briefs-1993* 91
[15] Stevens R J A M, Graham J and Meneveau C 2014 *Renewable Energy* 68 46–50
[16] Munters W, Meneveau C and Meyers J 2016 *Physics of Fluids* 28 025112
[17] Meyers J and Sagaut P 2007 *Physics of Fluids* 19 095105
[18] Calaf M, Meneveau C and Meyers J 2010 *Physics of Fluids* 22 015110
[19] Smagorinsky J 1963 *Monthly weather review* 91 99–164
[20] Stevens R J A M, Gayme D and Meneveau C 2014 *J. Renewable and Sustainable Energy* 6 023105
[21] Wu Y and Porté-Agel F 2015 *Renewable Energy* 75 945–955
[22] Goit J P, Munters W and Meyers J 2016 *Energies* 9 29
[23] Wu K L and Porté-Agel F 2017 *Energies* 10 2164
[24] Munters W and Meyers J 2017 *Phil. Trans. R. Soc. A* 375 20160100
[25] Allaerts D and Meyers J 2017 *Journal of Fluid Mechanics* 814 95–130
[26] Lim H C, Castro I P and Hoxey R P 2007 *Journal of Fluid Mechanics* 571 97–118