Experimental and numerical study of fatigue crack propagation in a thick-walled cylinder under cyclic hoop stress

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Abstract. In present work, after finding the anisotropy resulting in dissimilar properties in different orientations of a thick-walled cylinder, experimental and numerical study was performed to reveal the fatigue crack growth behavior of the cylinder under cyclic hoop stress. Fatigue crack growth experiments were conducted on middle tension M(T) samples prepared in an orientation to simulate the hoop stress on the cylinder. The tests were conducted under constant amplitude loading at R ratio 0.1. The fatigue crack growth data was compiled and applied to simulate and predict the crack growth process using two dimensional parametric finite element technique. The fatigue crack propagation was simulated, based on linear elastic fracture mechanics and stress intensity factor determination. Both the experimental and numerical results of crack growth data, at stress levels of 10 to 40 per cent of the yield stress of the material, were found in close agreement. The disparity observed was concluded in the range of statistical scatter in the experimental data. The crack growth rate and the fatigue life of the samples obtained from the experiments and the simulation were also in good agreement at all the stress levels analyzed.

1. Introduction
Reliability of materials and structures in the form of thick-walled cylinders (TWC) is of critical importance. Thick-walled cylinders in form of boilers, gun barrels, pipelines and high-pressure containers are essential structural members for many industries including power, chemical, armament, and food processing industries [1, 2]. In many applications the cylinders are prone to cyclic stresses during their normal operation and large internal pressures produce high tension hoop stresses along the inner surface of the cylinder. The latter may result in the nucleation of the internal surface cracks which propagate due to cyclic action of high-pressure pulses. If the primary crack growth mechanism is slow, the cracks will be detected during routine maintenance by non-destructive testing (NDT) so that corrective measures can be taken before crack growth moves into a high risk regime. After the crack reaches a critical size, the ultimate failure may be catastrophic and result at pressures which are even lower than the design capacity of the cylinder. For this reason, it is quite necessary to analyze the crack propagation behavior in TWC under cyclic hoop stress to ensure the integrity of the cylinder against the fatigue failure. Many researchers have done valuable work to analyze the TWC under fatigue loading [3-5].

The three major fatigue life methods used in design and analysis are the stress-life method, the strain-life method, and the linear-elastic fracture mechanics (LEFM) method [6]. The numerical techniques, based on linear-elastic fracture mechanics [7] are frequently used to establish fatigue failure criteria. Many
researchers used numerical approaches for the study of fatigue crack propagation [8-10] and a crack tip node-release scheme was suggested in Ref. [9] for the same. In present work, experimental and numerical study was performed to reveal the fatigue crack growth behavior of the cylinder under cyclic hoop stress. Since the full-scale fatigue crack growth test of the thick-walled cylinder subjected to internal cyclic pressure involves a significant amount of time and cost, M(T) samples taken from the cylinder in the transverse direction (CR) were alternatively used for the fatigue crack growth simulation [10].

The fatigue crack growth experiments were conducted on M(T) specimens, prepared in CR direction, at various stress levels. The experimental work was supplemented through modeling and simulation with the help of commercially available software. The data collected from the experiments was utilized to predict the fatigue life of the samples with the help of numerical technique based on LEFM. The finite element analysis (FEA) was conducted using ANSYS structural analysis software [11] with two dimensional finite element model and a code in ANSYS Parametric Design Language (APDL) was written to replicate the crack growth process. The results obtained from the two techniques are compared.

2. Experimental procedure

2.1 Fatigue crack growth test - Sample preparation

Fatigue crack growth tests were performed on middle tension M(T) specimens. Samples were cut from the cylinder in CR orientation, figure 1. The inner and outer diameters of cylinder were 100 mm and 150 mm, respectively. This orientation refers to the standard ASTM E 399 which provides the crack plane orientation code for bar and hollow cylinder. In this orientation the samples were perpendicular to the longitudinal axis of the cylinder ‘L’ while the notch direction was such that the crack propagation was along the radial direction ‘R’ as shown in figure 1. A TWC under pressure experiences three principal stresses. Among these stresses the tangential or hoop stress is the maximum and has about twice the value of the axial stress [12]. Hence the samples prepared for the experimental study were in the orientation that experiences the maximum stress present in the cylinder. Samples were machined according to the standard ASTM E 647 and the dimensions of the final M(T) sample are shown in figure 2. The machined samples were subjected to mechanical grinding and subsequently fine polishing.

2.2 Testing procedure

Fatigue crack growth experiments were performed on a servo-hydraulic testing machine in accordance with ASTM standard E 647. Tests were conducted in the tension-tension mode under constant amplitude loading with $R$ ratio 0.1. A sinusoidal waveform was applied at a loading frequency of 10 Hz. Tests were conducted at stress levels of 10 to 40 per cent of the yield strength of the material. A total of 12 samples were tested and fatigue crack growth data and SN curves were achieved. Crack length was measured with the help of a traveling microscope, at a magnification of 100x. Tests were conducted in air at a temperature 20°C and approximately 50 per cent relative humidity.

3. Finite element modeling

Fatigue crack growth analysis was performed using ANSYS structural software by repeatedly loading the geometry, recording stress intensity factor $K_I$ at crack tip, advancing the crack and then unloading. Two dimensional finite element analysis of M(T) sample geometry was conducted using four-noded quadrilateral elements under plane-strain conditions. The symmetry in loading and geometry of the M(T) sample was taken advantage of and a solid model for a quarter section of the sample was created in the ANSYS pre-processor. Figure 3 shows the geometry of the quarter model. The quarter model has an initial crack length ‘a’ of 3 mm and $a/W = 0.4$, where W is the sample half width.
Figure 1. Orientation of the CR sample representing tangential (T) and radial (R) stress under internal pressure (Pi)

Figure 2. Dimensions (in mm) of the M(T) sample

3.1 Element selection and meshing
The M(T) sample was modeled using two dimensional four-noded, PLANE42 solid elements. The element possesses two degrees of freedom at each node, translation in the nodal x and y directions and does not have rotational degrees of freedom. It also has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities [13]. Other researchers have also conducted fatigue crack growth
analysis by two-dimensional finite element analyses of CT and M(T) geometries. They used the four-noded quadrilateral elements and three-noded triangular elements for the simulation [14-17]. In order to predict the crack growth from the crack tip, the crack advancing region was mapped meshed [17]. The spacing between the consecutive nodes allowed the crack to advance in steps of equal size by node-release scheme [18]. The mesh was optimized by correlation with selected experiments prior to detailed analysis. For the model used in this study the total number of elements and nodes were 12000 and 12261, respectively.

![Figure 3. (a) Quarter model of the M(T) sample – element plot with the applied boundary conditions and (b) enlarged crack tip region showing mapped meshing](image)

### 3.2 Boundary conditions and solution

The boundary conditions applied on the quarter model are shown in the figure 3. The model was constrained applying symmetry boundary conditions along the left and the bottom edges. A 3 mm long crack was modeled by removing the symmetry constraints from 0 to 3 mm along the x direction at the bottom edge, thus providing the crack tip node at 3 mm. The model was loaded by applying tractions at the upper edge in the y direction, simulating mode I loading. After applying the boundary conditions and getting the solution, the value of $K_{I_{max}}$ at the crack tip was obtained, thereby $\Delta K$ calculated. This value of $\Delta K$ was used along with the experimental data to obtain the crack growth rate, using the Paris equation. The crack size was increased by releasing the crack tip node, which was equal to the distance between the two consecutive nodes along the line of crack advancement. The number of cycles to move to the next node (one step) was calculated using crack growth rate, and the process was repeated. During crack propagation, $\Delta K$ value was monitored and the process stopped when the parameter attained the fracture toughness of the material. In order to validate the FEA results, analysis was conducted by simulating loads similar to those applied during the experimental tests.
4. Results and discussion

4.1 Fatigue crack propagation

After finding the anisotropy in the material, experiments were conducted to reveal the fatigue crack growth behavior of the cylinder in CR direction which shows inferior mechanical properties. In addition, the numerical simulation of fatigue crack growth was performed using Paris law and linear elastic fracture mechanics. The element size in the crack growth region was fixed to 0.05 mm [19, 20]. The detailed FE crack growth analysis was performed at a constant crack growth rate of 0.05 mm/cycle. Figure 4 shows the increasing von-Mises stress at the crack tip node as the crack grows from 3 mm to 4.5 mm under a stress of 40 per cent of the material’s yield strength. Von-Mises stress value is normally used in both fatigue and static load design of such cylinders. The stress intensity factor was determined at the crack tip node by defining the path and using KCALC command. Figure 5 shows the plots for the crack length versus the number of cycles and compares the data obtained from the two techniques at different stress levels. The plots provide the number of cycles in the crack growth region and do not include the cycles to initiation the crack. The data covers the entire range from start of the crack at the notch up to the specimen failure.

It was concluded from the results that crack grows faster at higher stress level and vice versa, this observation was validated by literature [6]. Comparison of experimental and FE results shows close agreement at all stress levels analyzed. The small deviation of the FE results from the experimental is within the statistical scatter that was observed in the experimental findings.

![Figure 4](image-url)

**Figure 4.** Increasing von-Mises stress (MPa) as the crack grows from a) 3 mm to b) 4.5 mm under a stress of 40 per cent of the material’s yield strength. Inset shows enlarged crack tip region.
Figure 5. Experimental and FEA results of the crack length versus the number of cycles in the crack growth region at stress levels in per cent of the yield strength: a) 40 b) 35 c) 30 d) 25 e) 20 f) 15 and g) 10. Legend shows the experimental data (EXP-1 and EXP-2) and the FEA results.
4.2 Fatigue crack growth rate – Experimental vs. FEA

Figure 6 shows the fatigue crack growth rate versus $\Delta K$ plot. The experimental and FEA data is plotted to compare the results obtained from the two methods. The experimental data on log-log scale shows almost linear relation. The experimental results show that the crack growth rate increases almost linearly with $\Delta K$. The experimental values of the Paris constants, $m$ and $C$, were determined by curve fitting of the data through Excel. The values of $m$ obtained from the experimental data and that used for the numerical analysis are 3.40 and 3.35, respectively. A minor adjustment in the constant $m$ was carried out to optimize the modeling results. The value of $C$ was 4E-11 in both the cases. A smooth crack growth rate was achieved in FEA using Paris equation. The fatigue crack growth rate determined by the FEA lied within the upper and lower bounds of the crack growth rate achieved from the experimental data. It can be seen from the plot that the crack growth rates obtained from the two techniques are in close proximity and the predicted results are within the bounds of the experimental data. This concluded that at all the stress levels analyzed the crack growth rates obtained from the two techniques are in close proximity and the predicted results are within the bounds of the experimental data. This concluded that at all the stress levels analyzed the crack growth rate follows the Paris equation and remains in the region II of the crack growth curve.

4.3 Fatigue life analysis

The experimental and the FE results of $\Delta S$ versus $N_f$ are shown in the plots in figure 7. $\Delta S$ vs. $N_f$ relations for the two data sets was obtained through curve fit and are also given in the figure. The number of cycles to failure includes the number of cycles to initiate the crack and its growth up to the specimen failure. The number of cycles to crack initiation was incorporated in the FE results, from the experimental data. The fatigue life analysis results obtained from both the techniques shows that the fatigue lifetime increases as the stress range decreases. It can be seen that the results obtained from the two techniques are in good agreement at all the stress levels analyzed. Thus the simulation model and the crack growth technique used in the present study provide excellent results and can be utilized satisfactorily in the prescribed stress range.

5. Conclusions

Following conclusions can be drawn on the basis of the present study:

- The finite element technique combined with LEFM and Paris law could predict the fatigue crack growth life of the cylinder.
- Both the experimental and numerical results of crack growth data at the tested stress levels were found in close agreement. However, any difference observed was concluded in the range of statistical scatter in the experimental data.
- SN curves showed that the fatigue life of the samples obtained from the experiments and that predicted from the simulation were in good agreement at all the stress levels analyzed.
Figure 6. Comparison of the predicted crack growth rate with the experimental observation. Legend shows the total experimental data (EXP) and the FEA results at 10 per cent of the yield strength.

Figure 7. SN curves; Experimental (EXP) versus predicted (FEA) fatigue life.

6. References
[1] Alegre J M, Bravo P and Preciado M 2007 Fatigue behavior of an autofrettaged high-pressure vessel for the food industry Eng. Fail. Anal. 14 396–407
[2] Shlyannikov V N 2000 Fatigue shape analysis for internal surface flaw in a pressurized hollow cylinder Int. J. Pressure Vessels Piping 77 227–34
[3] Rees D W A 1989 Fatigue crack growth in thick-walled cylinders under pulsating internal pressure *Eng. Fract. Mech.* 33(6) 927–40

[4] Tomkins B 1973 Fatigue failure criteria for thick-walled cylindrical pressure vessels *Int. J. Pressure Vessels Piping* 1(1) 37–59

[5] Su B and Bhuyan G S 1998 Elasto-plastic fracture properties of an all-aluminium gas cylinder with different cracks *Int. J. Pressure Vessels Piping* 75(12) 879–86

[6] Shigley J E, Mischke C R and Budynas R G 2004 Mechanical Engineering Design. 7th ed. McGraw-Hill Companies Inc. New York

[7] Newman Jr J C and Ruschau J J 2007 The stress-level effect on fatigue-crack growth under constant-amplitude loading *Int. J. Fatigue* 29 1608–15

[8] Newman Jr J C and Armen Jr H 1975 Elastic–plastic analysis of a propagating crack under cyclic loading *J. AIAA* 13 1017–2023

[9] Newman Jr J C 1977 Finite-element analysis of crack growth under monotonic and cyclic loading, ASTM STP 637 Philadelphia PA 56–80

[10] Koh S K and Na E G 1999 Fatigue crack growth life of thick-walled cylinders with an external radial crack *Int. J. Fatigue* 21 135–46

[11] ANSYS 8.1 user’s Manual, Structural Analysis Guide 2004, Houston, Swanson Analysis system Inc.

[12] Staat M and Vu D K 2006 Limit loads of circumferentially flawed pipes and cylindrical vessels under internal pressure *Int. J. Pressure Vessels Piping* 83 188–96

[13] ANSYS Inc. ANSYS elements manual 2004, 7th edition, Houston, Swanson Analysis system Inc.

[14] Solanki K N, Daniewicz S R and Newman Jr J C 2003 Finite element modeling of plasticity-induced crack closure with emphasis on geometry and mesh refinement effects *Eng. Fract. Mech.* 70 1475–89

[15] Skinner J D 2001 Finite Element Predictions of Plasticity-Induced Fatigue Crack Closure In Three-Dimensional Cracked Geometries *MS thesis* Mississippi State University USA

[16] Solanki K N 2002 Two and three-dimensional finite element analysis of plasticity-induced fatigue crack closure - A comprehensive parametric study *MS Thesis* Mississippi State University USA

[17] Lei Y 2008 Finite element crack closure analysis of a compact tension specimen *Int. J. Fatigue* 30 21–31

[18] Stoychev S and Kujawski D 2008 Crack-tip stresses and their effect on stress intensity factor for crack propagation *Eng. Fract. Mech.* 75(8) 2469–79

[19] Jiang Y and Feng M 2004 In *Fracture Methodologies and Manufacturing Process*, July 25–29, San Diego, California, ASME PVP-Vol. 474, PVP2004-2297 23–31

[20] Ding F, Feng M and Jiang Y 2007 Modeling of fatigue crack growth from a notch *Int. J. Plasticity* 23 1167–88