Effective Chiral Theory for Pseudoscalar and Vector Mesons

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We consider the vector meson mixing scheme and mass splitting within the framework of an extended $U(3)_L \otimes U(3)_R$ chiral effective field theory based on the hidden local symmetry approach where, the pseudoscalar and vector meson nonets play the role of dynamic variables. Unlike other variants of this model, we show that the diagonalization of the vector meson mass matrix and the assumption that its eigenvalues are identical with the physical meson masses, determines the mixing scheme as well as the model free parameters. We show that VMD can be derived from our lagrangian and that for electromagnetic processes at low momenta the VMD model is a good first approximation. The model reproduces nicely the radii of the charged pions and kaons.

I. INTRODUCTION

In a recent contribution [1] it was indicated that, for any effective field theory (EFT) of colorless meson fields, the mixing schemes of particle states and decay constants are determined in a unique way by the kinetic and mass lagrangian densities. In a general case, these densities are bilinear in terms of the intrinsic fields, involving nondiagonal kinetic and mass matrices. However, they can be reduced into a standard quadratic form by transforming the intrinsic fields into the physical ones in three consecutive steps. These steps include: (i) the diagonalization of the kinetic matrix, (ii) rescaling of intrinsic fields to restore the standard normalization of the kinetic term, and (iii) the diagonalization of the resulting mass matrix. In such case where the dimensions of the nondiagonal kinetic and mass submatrices are respectively, $k \times k$ and $n \times n$, this procedure leads to schemes which involve $[k(k−1)/2] + [n(n−1)/2]$ mixing angles and $k$ field rescaling parameters. The commonly used mixing schemes correspond to specific choice of the kinetic and mass matrices. In particular, $\eta - \eta'$ mixing requires one mixing angle scheme, if and only if, the kinetic term for the intrinsic fields has a quadratic form. Such are the traditional scheme [2] and the one proposed by Feldmann et al. [3], the so called quark flavor basis (QFB) scheme. For bilinear kinetic and mass lagrangian densities (i.e. with nondiagonal kinetic and mass matrices) mixing schemes involve two mixing angles similar to the ones proposed by Escribano and Frère [4].

There have been several attempts to incorporate within a single effective lagrangian a wide range of different electroweak and strong processes [5–10]. Particularly interesting are the Bando, Kugo and Yamawaki (BKY) model [5], based on the hidden local symmetry (HLS) approach, and variants of this model [6–9], where the vector mesons ($\rho, \omega, K^*, K^+, \phi$) are identified with the dynamical gauge bosons of the HLS nonlinear chiral lagrangian. It was indicated by Benayoun et al. [6], that the original BKY model neither reproduces the observed vector mass splitting nor preserves current conservation for the pseudoscalar octet-singlet sector. A variant of this model proposed by Bramon et al. [7] also does not predict the masses correctly, although it guarantees current conservation, explicitly. Yet another variant of this model [6] includes the $\eta'$ meson also, and to some extent cures these deficiencies. However, once the HLS lagrangian is extended to include the $\eta'$ meson, the trace of the pseudoscalar (and vector) nonet field matrix no longer vanishes, and therefore, a general expression for the lagrangian must include additional terms which are quadratic in the traces of the nonet field matrices. More recently, a most general lagrangian based on local $U(3)_L \otimes U(3)_R$ symmetry was constructed [11,12], by combining the HLS approach with a general procedure of including the $\eta'$ meson into a chiral theory [13,14,15,16]. Both the symmetric parts of the lagrangian, as well as the symmetry breaking companions, include terms quadratic in the traces of the field matrices. Likewise, the lagrangian includes an explicit $U(3)_L \otimes U(3)_R$ symmetry violating mass term for the pseudoscalar mesons [14,15]. These additional terms influence the meson mass spectra and more importantly, by means of the diagonalization procedure of Ref. [1], not only the mixing scheme is defined uniquely but also, taking the eigenvalues of the resulting mass matrix to be equal to the physical masses, it was possible to fix a number of the model free parameters. All observables are calculated from the lagrangian straightforwardly without additional assumptions. Albeit, the lagrangian constructed reflects the
fundamental symmetries of the QCD lagrangian and provides a coherent theoretical framework where the pseudoscalar and vector meson nonets are treated on equal footing.

Our main interest in the present work is to further develop and explore the $U(3)_L \otimes U(3)_R$ chiral theory of Ref. [12] by considering the vector meson sector. It is demonstrated that in this case the relevant model parameters can all be fixed through the diagonalization procedure of Ref. [1]. It has been illustrated already [7] that the vector meson dominance (VMD) model emerges as a natural consequence of the basic assumption of the HLS approach where vector mesons play the role of gauge bosons. Particularly, the VMD model becomes a dynamical consequence for a specific choice of the HLS model parameter $a = 2$. Detailed analyses of data argue for departures from this value [9,17], and that in addition to the vector meson poles in the charged meson form factors, a direct photon- pseudoscalar meson coupling may also contribute significantly. More recently, Harada and Yamawaki [18] have shown that in an effective field theory based on HLS, VMD may be badly violated. We demonstrate in this work that VMD derived from the lagrangian constructed in Ref. [12] is a good first approximation for electromagnetic processes at low momenta. Our paper is organized as follows. In Section II we present briefly the lagrangian. In section III we discuss vector mixing and in the more evolved case of isospin symmetry breaking. In Section IV we consider the photon interaction with hadronic matter and discuss the relation between the lagrangian constructed and the traditional VMD model. We summarize and conclude in Section V.

II. THE EFFECTIVE LAGRANGIAN

As in Ref. [12], the lagrangian is written in the form,

$$L = L_A + L_{\bar{A}} + L_m + a(L_V + \bar{L}_V) - \frac{1}{4} Tr(V_{\mu \nu}V^{\mu \nu}) + L_{WZW} + \ldots,$$

(1)

where $L_A(L_{\bar{A}})$ and $L_V(\bar{L}_V)$ are the symmetric parts (symmetry breaking companions) for the pseudoscalar and vector mesons, $L_m$ is a most general $U(3)_L \otimes U(3)_R$ symmetry violating mass term [13,14,16], $L_{WZW}$ stands for the well known Wess-Zumino-Witten term [19,20], and the ellipsis “...” represents terms accounting for the regularization of one loop contributions [22,23,13]. All terms in the expression above are constructed from boson gauge fields $V_\mu$, and vector and axial-vector covariants defined as $\Gamma_\mu$ [21,22,23],

$$\Gamma_\mu = \frac{i}{2} \left[ \xi^\dagger, \partial_\mu \xi \right] = \frac{1}{2} \left( \xi^\dagger r_\mu \xi + \xi l_\mu \xi^\dagger \right),$$

(2)

$$\Delta_\mu = \frac{i}{2} \left[ \xi^\dagger, \partial_\mu \xi \right] = \frac{1}{2} \left( \xi^\dagger l_\mu \xi - \xi r_\mu \xi^\dagger \right).$$

(3)

Here $r_\mu$ and $l_\mu$ represent the standard model external gauge fields; $r_\mu = v_\mu + a_\mu$ and $l_\mu = v_\mu - a_\mu$, with $v_\mu$ and $a_\mu$ being the vector and axial vector external electroweak fields, respectively. The non-linear representation of the pseudoscalar nonet fields is taken as [13,14,14],

$$U(P, \eta_0 + F_0 \vartheta) \equiv \xi^2(P, \eta_0 + F_0 \vartheta) \equiv \exp \left\{ \frac{\sqrt{2}}{F_8} P + i \sqrt{\frac{2}{3}} \frac{F_0}{F_8} \eta_0 \frac{1}{2} \right\},$$

(4)

with $\eta_0(x)$ being the pseudoscalar singlet and $P(x)$ the pseudoscalar Goldstone octet matrix,

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & \pi^+ \\ -\frac{1}{\sqrt{2}} \pi^- + \frac{1}{\sqrt{6}} \eta_8 & K^0 \\ K^- & -\frac{2}{\sqrt{6}} \eta_8 \end{pmatrix},$$

(5)

with obvious notations. The vacuum angle $\vartheta(x)$ is an auxiliary field which renders the variable $X(x) \equiv \sqrt{6} \eta_0(x)/F_0 + \vartheta(x)$ to be invariant under $U(3)_L \otimes U(3)_R$ transformations [21]. In this representation, the unimodular part of $U$ contains the octet degrees of freedom while the phase $det U = \exp(i\sqrt{6}\eta_0/F_0)$ involves the singlet only. To lowest order (i.e., smallest number of derivatives) the lagrangian is built from traces of the covariants $\Delta_\mu$, $\Gamma_\mu - gV_\mu$, $\Delta_\mu \Delta^\mu$, $(\Gamma_\mu - gV_\mu)(\Gamma^\mu - gV^\mu)$, $D^\mu \vartheta$, and arbitrary functions of the variable $X(x)$, all being invariant under $U(3)_L \otimes U(3)_R$ transformations. Explicit expressions of various lagrangian parts are listed for convenience in the Appendix. Their derivation can be found in Ref. [12]. Although the lagrangian, Eqn.1 appears similar to that of Bando et al. [7] and
other variants of this model, the terms \( L_i \) and their symmetry breaking companions \( \tilde{L}_i (i = A, V) \) are different, including additional terms proportional to \( Tr(\Delta \mu)Tr(\Delta \nu) \), \( Tr(\Delta \mu)D^{\mu} \partial \) and \( Tr(\Gamma^\mu - gV^\mu)Tr(\Gamma_\mu - gV_\mu) \). Clearly, such terms must be included in any \( U(3)_L \otimes U(3)_R \) symmetry based HLS theory, where the traces of \( \Delta \mu \) and \( \Gamma_\mu - gV_\mu \) do not vanish. It is demonstrated below that these terms are needed in order to reproduce the meson masses from the lagrangian without imposing additional constraints. We add also that the matrix used to generate the symmetry breaking companions is a universal Hermitian matrix defined as,

\[
B = \xi b \xi^\dagger ; \quad b = \text{diag}(b_u, b_d, b_s) ,
\]

where \( b_i = m_i / (m_u + m_d + m_s) \). This form guarantees that the ratio of \( SU(3)_F \) symmetry breaking scales to isospin symmetry breaking scales be identical to the once of the underlying QCD. Note that, the lagrangian involves coefficient functions \( W_i(X) \) which are absent in the \( SU(3) \) symmetry limit. Again such functions must be included in the process of extending the \( SU(3)_L \otimes SU(3)_R \) based model into a general \( U(3)_L \otimes U(3)_R \) theory. As a final remark we note that the symmetry violating mass term in Eqn. (3) has the exact form derived by Herera-Siklodi et al. [16]. As in Refs. [8,9], the terms \( a(L + \bar{L}) \) contains, amongst other contributions, a vector meson mass term \( \sim V_\mu V^\mu \), a vector-photon conversion factor \( \sim V_\mu A^\mu \), and the coupling of pseudoscalar pairs to both vectors and photons. The latter coupling can be eliminated by choosing \( a = 2 \) allowing to incorporate the strict VMD lagrangian [20]. We shall attempt to determine the value of this parameter from data in order to assess the validity of the traditional VMD model. Altogether, the lagrangian Eqn. (3) involves symmetry breaking scales \( c_i, d_i (i = A, V) \), a general coupling constant \( g \) and a free parameter \( a \). The symmetry breaking scales \( c_A, d_A \) and \( r \) are determined from global fit analysis of radiative decay widths of pseudoscalar and vector mesons [12]. The other symmetry breaking scales \( c_V \), \( d_V \) and \( ag^2 \) are determined through the diagonalization of the vector mass matrix.

**III. VECTOR MESON MASSES AND MIXING**

We now turn to consider mixing and masses of vector mesons. The vector part \( a(L + \bar{L}) \) of the lagrangian involvement a vector meson mass term \( \sim V_\mu V^\mu \) which determines as already indicated the vector mixing scheme, uniquely. Generally speaking, the vector nonet matrix \( V \) is the sum of an octet matrix \( V_8 \), involving the \( \rho^0, \rho^0, K^{*0}, K^{*0}, K^{*\pm} \) and \( \omega_8 \) meson fields, and an \( SU(3)_F \) singlet \( \omega_0 \) matrix, e.g.,

\[
V = V_8 + r_V \frac{1}{\sqrt{3}} \omega_0 \times 1 .
\]

Here a departure of the parameter \( r_V \) from one accounts for possible nonet symmetry breaking. However, unlike the case of pseudoscalar mesons, the light vector meson spectrum shows no clear evidence for such a departure. In the discussion to follow we take \( r_V = 1 \), assuming exact nonet symmetry. Then, the vector meson nonet matrix has the form (for brevity the Lorentz indices of the vector fields are omitted),

\[
V = \begin{pmatrix}
\rho^0 + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_0}{\sqrt{3}} & \rho^+ & K^{*+} \\
\rho^- & -\rho^0 + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_0}{\sqrt{3}} & K^{*0} \\
K^{*-} & \frac{K^{*-}}{\sqrt{6}} + \frac{\omega_0}{\sqrt{3}} & -2\bar{\omega}_0 + \frac{\omega_0}{\sqrt{3}}
\end{pmatrix} .
\]

As was already indicated in the introduction, the kinetic and mass terms of the lagrangian can be reduced to a standard quadratic form by first diagonalizing the kinetic matrix via a unitary transformation \( Y \), rescale the fields through a transformation \( R \) to restore the standard normalization of the kinetic term, and finally diagonalizing the resulting mass matrix through another unitary transformation \( \Omega \). These three steps are sufficient to define the state mixing scheme [6].

From Eqs. (8) and (2) the vector meson kinetic and mass terms amounts to,

\[
L_{km} = -\frac{1}{4}(\partial_\mu V_\nu - \partial_\nu V_\mu)(\partial^\mu V^\nu - \partial^\nu V^\mu) + \frac{1}{2}(V_\mu M_\nu^2 V^\mu) ,
\]

with,

\[
\frac{1}{2}V_\mu M_\nu^2 V^\mu = a F_0^2 g^2 [Tr(V_\mu V^\mu) + \bar{w}_4 Tr(V_\mu) Tr(V^\mu) + c_V (Tr(BV_\mu V^\mu) + \bar{w}_4 Tr(BV_\mu) Tr(V^\mu)) + d_V (Tr(BV_\mu BV^\mu) + \bar{w}_4 Tr(BV_\mu Tr(BV^\mu))] .
\]

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Thus the kinetic term has the standard normalized quadratic form, and following [1] the mixing scheme is determined through the diagonalization of the mass matrix only and involves one mixing angle. The nonvanishing elements of the vector meson mass matrix read,

$$
\mu^2(\rho^0\rho^0) = \mu^2_V \left( 1 + \frac{c_V}{2} (b_u + b_d) + \frac{d_V}{2} [b_u^2 + b_d^2 + \bar{\omega}_4 (b_u - b_d)^2] \right),
$$

$$
\mu^2(\rho^+\rho^+) = \mu^2_V \left( 1 + \frac{c_V}{2} (b_u + b_d) + \frac{d_V}{2} [b_u^2 + b_d^2 + \bar{\omega}_4 (b_u - b_d)^2] \right),
$$

$$
\mu^2(\omega_8\omega_8) = \mu^2_V \left( 1 + \frac{c_V}{6} (b_u + b_d + 4b_s) + \frac{d_V}{6} [(b_u^2 + b_d^2 + 4b_s^2) + \bar{\omega}_4 (b_u + b_d - 2b_s)^2] \right),
$$

$$
\mu^2(\omega_0\omega_0) = \mu^2_V \left( 1 + 3\bar{\omega}_4 + \frac{c_V}{3} (b_u + b_d + b_s)(1 + 3\bar{\omega}_4) + \frac{d_V}{3} [b_u^2 + b_d^2 + b_s^2 + \bar{\omega}_4 (b_u + b_d + b_s)^2] \right),
$$

$$
\mu^2(\rho^0\omega_0) = \mu^2_V \left( \frac{1}{2\sqrt{3}} (b_u - b_d) (c_V + d_V [b_u + b_d + \bar{\omega}_4 (b_u + b_d - 2b_s)]) \right),
$$

$$
\mu^2(\rho^0\omega_0) = \mu^2_V \left( \frac{1}{2\sqrt{3}} \sqrt{\frac{1}{6}} (b_u - b_d) (c_V + d_V [b_u + b_d + \bar{\omega}_4 (b_u + b_d + b_s)]) \right),
$$

$$
\mu^2(\omega_8\omega_0) = \mu^2_V \left( \frac{1}{3\sqrt{2}} \left( c_V (b_u + b_d - 2b_s)(1 + \frac{3}{2} \bar{\omega}_4) + d_V [b_u^2 + b_d^2 - 2b_s^2 + \bar{\omega}_4 (b_u + b_d - 2b_s)(b_u + b_d + b_s)] \right) \right),
$$

$$
\mu^2(K^+K^-) = \mu^2_V \left( 1 + \frac{c_V}{2} (b_u + b_s) + \frac{d_V}{2} b_s b_s \right),
$$

$$
\mu^2(K^0\bar{K}^0) = \mu^2_V \left( 1 + \frac{c_V}{2} (b_d + b_s) + \frac{d_V}{2} b_d b_s \right),
$$

with,

$$
\mu^2_V = 2\alpha g^2 F_8^2,
$$

We stress here that even in the limits of exact chiral symmetry and nonet symmetry, i.e. $c_V = d_V = 0$, $r_V = 1$, the singlet vector meson mass, $m_{\omega^0} = \mu V \sqrt{1 + 3 \bar{\omega}_4}$, differs from that of the octet $m_{\omega^0} = \mu V$. This distinct property occurs due to the term $\bar{W}_4(X)\text{Tr}(\Gamma_{\mu} - g\nu_{\mu})\text{Tr}(\Gamma_{\mu} - g\nu_{\mu})$ in the lagrangian Eqn. [1]. Only for $\bar{\omega}_4 = 0$ (corresponding to lagrangians previously used) all nonet meson masses are equal. Clearly, the mass matrix is nondiagonal due to the $\omega_8 - \omega_0$, $\rho^0 - \omega_8$ and $\rho^0 - \omega_0$ mixing. Notice though that the $\rho^0 - \omega_8$ and $\rho^0 - \omega_0$ admixtures are extremely small and vanish in the limit of exact isospin symmetry ($b_u = b_d$).

First we diagonalize the mass matrix in the limit of exact isospin symmetry. In this limit, taking $b_s = 1$ and neglecting terms of the order of $c_V b_u, d_V b_u (i = u, d)$, the matrix mass assumes a particularly simple form with elements,

$$
\mu^2(\mu\mu) = \mu^2_V ,
$$

$$
\mu^2(\omega_8\omega_8) = \mu^2_V \left( 1 + \frac{2c_V}{3} + \frac{2d_V}{3} (1 + \bar{\omega}_4) \right),
$$

$$
\mu^2(\omega_0\omega_0) = \mu^2_V \left( (1 + \frac{c_V}{3})(1 + 3\bar{\omega}_4) + \frac{d_V}{3} (1 + \bar{\omega}_4) \right),
$$

$$
\mu^2(\omega_8\omega_0) = -\mu^2_V \left( \frac{\sqrt{2}}{3} (c_V + \frac{3}{2} \bar{\omega}_4) + \frac{d_V}{3} (1 + \bar{\omega}_4) \right),
$$

$$
\mu^2(K^*) = \mu^2_V \left( 1 + \frac{c_V}{2} \right),
$$

which we diagonalize using the unitary transformation,

$$
\begin{pmatrix}
\omega_8 \\
\omega_0
\end{pmatrix} = \begin{pmatrix}
\cos \theta_V & \sin \theta_V \\
-\sin \theta_V & \cos \theta_V
\end{pmatrix} \begin{pmatrix}
\omega \\
\phi
\end{pmatrix} .
$$

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1 In case of broken nonet symmetry it is still possible to maintain $m_{\omega^0} \neq \mu V$ but with the nonet symmetry breaking parameter fixed arbitrary to be $r^2_V = 1/(1 + 3\bar{\omega}_4)$
It is straightforward to show that the resulting physical meson masses and mixing angle are,

\[
m_\omega^2 = \frac{1}{2} \left( \mu^2 (\omega_8 \omega_8) + \mu^2 (\omega_7 \omega_7) - \sqrt{(\mu^2 (\omega_7 \omega_7) - \mu^2 (\omega_8 \omega_8))^2 + 4(\mu^2 (\omega_7 \omega_7))^2} \right),
\]

\[
m_\phi^2 = \frac{1}{2} \left( \mu^2 (\omega_8 \omega_8) + \mu^2 (\omega_7 \omega_7) + \sqrt{(\mu^2 (\omega_7 \omega_7) - \mu^2 (\omega_8 \omega_8))^2 + 4(\mu^2 (\omega_7 \omega_7))^2} \right),
\]

\[
\tan 2\theta_V = -\frac{2\mu^2 (\omega_7 \omega_7)}{(\mu^2 (\omega_7 \omega_7) - \mu^2 (\omega_8 \omega_8))} = -\frac{(m_\phi^2 - m_\omega^2)^2}{(m_\rho^2 + m_\phi^2 - 2\mu^2 (\omega_8 \omega_8))^2} - 1.
\]

From Eqns. 13 with \( \tilde{w}_4 = 0 \) the mixing angle reduces to the ideal mixing angle value \( \theta_{V\text{ideal}} = -54.7^\circ \), and more importantly does not depend on the symmetry breaking scales \( c_V \) and \( d_V \). Correspondingly, the \( \omega \), \( K \) and \( \phi \) meson masses become,

\[
m_\omega^2 = m_\rho^2, \quad m_K^2 = m_\rho^2 (1 + \frac{1}{2} c_V), \quad m_\phi^2 = m_\rho^2 (1 + c_V + d_V).
\]

Clearly, assuming now \( d_V = c_V^2 / 4 \) leads to the BKY mass relations,

\[
\frac{m_K^2}{m_\rho^2} = \frac{m_\phi^2}{m_\rho^2} = 1 + \frac{1}{2} c_V.
\]

Otherwise, taking \( d_V = 0 \) leads to the mass relations of Bramon et al. 8

\[
\frac{m_K^2}{m_\rho^2} = 1 + \frac{1}{2} c_V, \quad \frac{m_\phi^2}{m_\rho^2} = 1 + c_V.
\]

It should be stress though that in all of these private cases, the \( \rho \) and \( \omega \) masses are equal and \( \theta_V = \theta_{V\text{ideal}} \). To describe the actual vector meson masses we should keep \( d_V \) as a free parameter and \( \tilde{w}_4 \neq 0 \). Based on the proximity of the \( \rho \) and \( \omega \) masses, the parameter \( \tilde{w}_4 \) should be very small, and to first order the \( \omega \) and \( \phi \) meson masses are,

\[
m_\omega^2 = m_\rho^2 (1 + 2\tilde{w}_4), \quad m_\phi^2 = m_\rho^2 (1 + c_V + d_V)(1 + \tilde{w}_4).
\]

Thus, the parameter \( \tilde{w}_4 \) determines the \( \rho - \omega \) mass splitting while \( (c_V + d_V) = 2(m_\rho^2 - m_\phi^2)/(m_\rho^2 + m_\phi^2) \). At this order, using the expressions above and Eq.13 for \( m_K^2 \), along with the experimental meson mass values 2 one obtains, \( c_V = 0.69 \pm 0.01 \), \( d_V \approx 0.03 \) and \( \tilde{w}_4 \approx 0.01 \). Indeed, \( \tilde{w}_4 \) is very small but it plays an important role in reproducing the \( \rho - \omega \) mass splitting. Note also that the actual value of the parameter \( d_V \) is far smaller than \( c_V^2 / 4 \approx 0.12 \) favoring the Bramon et al. 8 relations Eq.13. More accurate values of these parameters can be deduced from the exact expressions for the vector meson masses. These are, \( \tilde{w}_4 = 0.009 \pm 0.0001 \) and \( d_V = 0.025 \pm 0.01 \). Correspondingly, the mixing angle for the vector meson states is \( \theta_V = -(52 \pm 1)^\circ \), rather close to ideal mixing. In turn, this value constrains the field admixtures due to nonideal mixing to be very small. With nonideal mixing taken into account, the matrix of the physical vector fields reads,

\[
V = \begin{pmatrix}
\frac{\rho^0 + \omega^+ + \phi}{\sqrt{2}} & \rho^+ & K^{*+} \\
\rho^- & \frac{\rho^0 + \omega^+ + \phi}{\sqrt{2}} & K^{*0} \\
K^{*-} & K^{*0} & \phi - \epsilon \omega
\end{pmatrix},
\]

where \( \epsilon = 0.047 \pm 0.001 \), being a measure of a non-strange (strange) admixture in the physical \( \phi \) and \( \omega \) fields. We stress again that this parameter like the symmetry breaking scales \( c_V \), \( d_V \) and \( \tilde{w}_4 \) are all determined through the diagonalization of the mass matrix and the requirement that the eigenvalues of this matrix should reproduce the physical masses.

We now consider the effects of isospin symmetry breaking on the mass splitting and mixing angle. The nondiagonal part of the mass matrix now reads

\[
\mathcal{M}^2(\rho^0, \omega_8, \omega_0) = \begin{pmatrix}
\mu^2(\rho^0 \rho^0) & \mu^2(\rho^0 \omega_8) & \mu^2(\rho^0 \omega_0) \\
\mu^2(\rho^0 \omega_8) & \mu^2(\omega_8 \omega_8) & \mu^2(\omega_8 \omega_0) \\
\mu^2(\rho^0 \omega_0) & \mu^2(\omega_8 \omega_0) & \mu^2(\omega_0 \omega_0)
\end{pmatrix}
\]

It is convenient to write the transformation \( \Omega \) which diagonalize this submatrix in the form
\[ \Omega = \Omega(\omega_8, \omega_0) \Omega^0(\rho^0, \omega, \phi) \]  

where the transformation \( \Omega(\omega_8, \omega_0) \) diagonalize the \( \omega_8, \omega_0 \) submatrix

\[ \Omega(\omega_8, \omega_0) = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta_V & \sin \theta_V \\
0 & -\sin \theta_V & \cos \theta_V 
\end{pmatrix} \]  

and the transformation \( \Omega(\rho^0, \omega, \phi) \) diagonalize the reduced mass matrix

\[ \Omega^{-1}(\omega_8, \omega_0) M^2(\rho^0, \omega_8, \omega_0) \Omega(\omega_8, \omega_0) = \begin{pmatrix}
m^2_{\rho} & m^2_{\rho\omega} & m^2_{\rho\phi} \\
m^2_{\rho\omega} & m^2_{\omega} & 0 \\
m^2_{\rho\phi} & 0 & m^2_{\phi}
\end{pmatrix}, \]  

where,

\[ m^2_{\rho\omega} = \frac{1}{2} m^2_{\rho} (b_u - b_d) c_V, \quad m^2_{\rho\phi} = \frac{1}{2} \sqrt{2} m^2_{\rho} (b_u - b_d) c_V \tilde{w}_4. \]  

From the Eq.20 it is clear that effects of isospin symmetry breaking on the \( \rho \) and \( \omega \) masses are negligibly small \( \approx 0.1\% \). Taking \( \tilde{w}_4 = 0.009 \) as above, it is rather easy to show that the matrix element \( m^2_{\rho\phi} \) is negligibly small and consequently the \( \rho^0 - \phi \) mixing angle is practically zero. Therefore we may write the \( \Omega(\rho^0, \omega, \phi) \) transformation in the simple form

\[ \Omega(\rho^0, \omega, \phi) = \begin{pmatrix}
\cos \theta_{\rho\omega} & \sin \theta_{\rho\omega} & 0 \\
-\sin \theta_{\rho\omega} & \cos \theta_{\rho\omega} & 0 \\
0 & 0 & 1
\end{pmatrix}, \]  

where the \( \rho - \omega \) mixing angle is given by the expression,

\[ \tan 2\theta_{\rho\omega} = -2 \frac{m^2_{\rho\omega}}{m^2_{\rho} - m^2_{\omega}}. \]  

With this in mind, we may now write the vector meson matrix in the form,

\[ V = \begin{pmatrix}
X_1 \rho^0 + X_2 \omega + \epsilon \phi & \rho^+ & K^{*+} \\
\rho^- & -X_1 \rho^0 + X_2 \omega + \epsilon \phi & K^{*0} \\
K^{*-} & K^{*0} & \phi + \epsilon (\rho^0 \sin \theta_{\rho\omega} - \omega \cos \theta_{\rho\omega})
\end{pmatrix}, \]  

with,

\[ X_1 = \cos \theta_{\rho\omega} - \sin \theta_{\rho\omega}, \quad X_2 = \cos \theta_{\rho\omega} + \sin \theta_{\rho\omega}. \]  

To evaluate the \( \rho^0 - \omega \) mixing angle, we need an estimate of the quark masses ratios \( b_i, (i = u, d, s) \). We recall that,

\[ m^2_{K^+} = \frac{1}{2(b_u + b_d)} m^2_{\pi^0} (b_u + b_s)(1 + \frac{1}{2} c_A) \],

\[ m^2_{K^0} = \frac{1}{2(b_u + b_d)} m^2_{\pi^0} (b_d + b_s)(1 + \frac{1}{2} c_A) \].

Then, we may estimate \( b_u - b_d \) from the kaon masses \( K \) according to,

\[ \frac{b_u - b_d}{1 - (b_u + b_d)/2} = 2 \frac{m^2_{K^+} - m^2_{K^0}}{m^2_{K^+} + m^2_{K^0}}, \]  

which yields,

\[ b_u - b_d = -0.0160 \pm 0.0005. \]
With this value of $b_u - b_d$ the calculated $\rho^0 - \omega$ mixing angle reads,

$$\theta_{\rho\omega} = (-8.9 \pm 0.3)\ ^\circ.$$  \hfill (37)

For the parameter $b_v$ we have \cite{12},

$$\frac{m_s}{m} = \frac{2m_s}{m_u + m_d} = \frac{2b_v}{1 - b_v} = 2 \left(1 + \frac{1}{2}c_A\right) \left(\frac{m_K}{m_{\pi}}\right)^2 - 1 ,$$ \hfill (38)

which yields for $c_A = 0.64 \pm 0.06$ (Alternative I of Ref. \cite{2})

$$b_v = 0.940 \pm 0.01 ,$$ \hfill (39)

and consequently $b_u = 0.022 \pm 0.005$, $b_d = 0.038 \pm 0.005$. For the Alternative II ($c_A = 0.2 \pm 0.05$) an QFB mixing scheme ($c_A = 1.4 \pm 0.1$) we obtain $b_u = 0.928$, $b_d = 0.208$, $b_d = 0.044$ and $b_s = 0.953$, $b_u = 0.0155$, $b_d = 0.0315$ respectively. This complete defining the mixing scheme in the case of isospin symmetry breaking. Here as in the analysis of the pseudoscalar meson sector \cite{12}, the exact form of the matrix $B$ used to generate flavor symmetry breaking (see Appendix), guarantees that the ratios of $SU(3)$ symmetry breaking scales to isospin symmetry breaking scales are identical to the ones of the underlying QCD.

IV. VECTOR MESON DOMINANCE MODEL AND PSEUDOSCALAR MESON FORM FACTORS

A particularly interesting aspect of the theory presented above is the interaction between the photon and hadronic matter. It was already illustrated by Bando et al. \cite{3} and O’Connell et al. \cite{26} that the HLS lagrangian is closely related to the well known VMD type 2 lagrangian and that by suitable transformation it can be reduced into the VMD type 1 lagrangian. (See also Ref. \cite{9} on this matter). The lagrangian, Eqn. 4. contains terms proportional to $\sim V_\mu V^\mu$ and $A_\mu A^\mu$ and $\sim V_\mu A^\mu$, corresponding respectively to a vector meson and photon mass terms and photon-vector meson interactions. likewise, it contains the coupling of both vector mesons and photons to pairs of pseudoscalar mesons. By following arguments similar to those of Ref. \cite{26} we derive in this section explicit expressions for the photon, vector, and pseudoscalar parts of the lagrangian, Eqn. 4. We recall first that the covariants $\Gamma_\mu$ and $\Delta_\mu$, Eqns. 40 can be expanded as,

$$\Gamma_\mu = -eA_\mu Q + \frac{i}{4F_0^2}[P, \partial_\mu P] - \frac{ie}{2F_0^2} A_\mu \{\{P^2, Q\} - PP\} + \ldots ,$$ \hfill (40)

$$\Delta_\mu = -\frac{1}{F_0 \sqrt{2}} \partial_\mu P + i \frac{e}{F_0 \sqrt{2}} A_\mu (Q, P) + \ldots ,$$ \hfill (41)

where $A_\mu$ stands for the photon field, $Q = \text{diag}(2/3, -1/3, -1/3)$ the quark charge operator, and $P$ is the pseudoscalar nonet matrix expressed in terms of the physical fields (see Ref. \cite{12}). The ellipsis “...” denotes terms involving higher powers of the fields. The covariant $\Gamma_\mu - gV_\mu$ contains then terms proportional to $(eA_\mu Q - gV_\mu)$. By substituting the expression, Eqn. 40 into the lagrangian part $L_V + L_V$ we obtain the following photon and vector meson terms:

(i) a direct photon-vector meson coupling,

$$aL(V, \gamma) = aF_0^2 \bar{e} A^\mu \text{Tr} \{\{V_\mu, Q\}(1 + c_V B)\} + c_V \text{Tr}(V_\mu) \text{Tr}(BQ) + d_V \text{Tr}(\{BV_\mu, BQ\}) + 2d_V \text{Tr}(BQ) \text{Tr}(bV_\mu) ,$$ \hfill (42)

(ii) a photon mass term,

$$aL_{m\gamma} = a \frac{F_0^2 e^2}{3} A^2_\mu \left(2 + \frac{1}{3}(c_V + 2d_V)\right) ,$$ \hfill (43)

(iii) a photon-vector meson-pseudoscalar meson meson interactions,

\footnote{Strictly speaking, $\Gamma_\mu - gV_\mu$ involves the terms $-e(A_\mu - \tan \theta_W Z_\mu^0)Q - \frac{e}{2\sin \theta_W \cos \theta_W} T_Z Z_\mu^0 - \frac{e}{2\sin \theta_W \cos \theta_W} W_\mu - gV_\mu$ which give rise to direct $V - A$, $V - Z^0$ and $V - W$ couplings.}

\footnote{Strictly speaking, $\Gamma_\mu - gV_\mu$ involves the terms $-e(A_\mu - \tan \theta_W Z_\mu^0)Q - \frac{e}{2\sin \theta_W \cos \theta_W} T_Z Z_\mu^0 - \frac{e}{2\sin \theta_W \cos \theta_W} W_\mu - gV_\mu$ which give rise to direct $V - A$, $V - Z^0$ and $V - W$ couplings.}
\[ aL(V, \gamma, P) = -\frac{a e}{2} \rho_{\pi}^2 + A_\mu \{ \text{Tr}(\{Q, [P, \partial^\mu P]\}) + c_\gamma \text{Tr}(B\{Q, [P, \partial^\mu P]\}) \} + d_\gamma \{ \text{Tr}(\{BQ, B[P, \partial^\mu P]\}) + 2\text{Tr}(BQ)\text{Tr}(B[P, \partial^\mu P]) \} + \ldots , \]  
\[ aL(V, P) = -\frac{a g_{\pi}^2}{2} \rho_{\pi}^2 \{ \text{Tr}(\{V_\mu, [P, \partial^\mu P]\}) + c_\gamma \text{Tr}(B\{Q, [P, \partial^\mu P]\}) \} + \text{Tr}(V_\mu)\text{Tr}(B[P, \partial^\mu P]) \] 
\[ d_\gamma \{ \text{Tr}(\{BQ, B[P, \partial^\mu P]\}) + 2\text{Tr}(BQ)\text{Tr}(B[P, \partial^\mu P]) \} + \ldots . \]  
(44)

and (iv) a vector-pseudoscalar meson interaction term,
\[ aL(V, P) = -\frac{a g_{\pi}^2}{2} \rho_{\pi}^2 \{ \text{Tr}(\{V_\mu, [P, \partial^\mu P]\}) + c_\gamma \text{Tr}(B\{Q, [P, \partial^\mu P]\}) \} + \text{Tr}(V_\mu)\text{Tr}(B[P, \partial^\mu P]) \] 
\[ d_\gamma \{ \text{Tr}(\{BQ, B[P, \partial^\mu P]\}) + 2\text{Tr}(BQ)\text{Tr}(B[P, \partial^\mu P]) \} + \ldots . \]  
(45)

Similarly, substituting the expansion of \( \Delta_\mu \), Eqn.11 into the pseudoscalar lagrangian part, \( L_A + \bar{L}_A \), leads to the photon-pseudoscalar meson interactions,
\[ L(A, \gamma, P) = -\frac{i e}{2} \rho_{\pi}^2 A_\mu \{ \text{Tr}(\{Q, [P, \partial^\mu P]\}) + c_A \text{Tr}(B\{Q, [P, \partial^\mu P]\}) \} + \ldots . \]  
(46)

Here the ellipsis \( \ldots \) denote terms involving \( \partial PP^n \), \( n \geq 2 \). To summarize the photon-meson part corresponding to the lagrangian, Eqn.1 is given by,
\[ L_{VMDII} = -\frac{1}{4} \text{Tr}(V_{\mu\nu}V^{\mu\nu}) + aL(V, \gamma) + aL_{m\gamma} + aL(V, \gamma, P) + aL(V, P) + L(A, \gamma, P) . \]  
(47)

This last expression, with its photon mass \( L_{m\gamma} \) term is rather similar to the popular VMDII model \cite{26}. By suitable transformation of the fields this term can be removed but prior to doing that some comments are in order. First, in complete analogy with the discussion of Ref. 26, we state here that the dressed photon propagator has the proper behavior of \(-i/q^2\) at small photon momenta but significantly modified away from \(q^2 = 0\). This certainly may be important in describing processes at sufficiently high momenta. Secondly, it is clear that the neutral \( \rho, \omega \) and \( \phi \) meson mix with the photon, spontaneously breaking symmetry down to the \( U(1)_{em} \). We may exploit the diagonalization procedure described in section III to introduce physical photon and mesons. In terms of these physical fields, the lagrangian has no explicit coupling between the photon and neutral mesons, whilst there is a direct coupling between the photon to hadronic currents in the limit of exact symmetry. Thirdly, so far the value of the parameter \( a \) is left free and in order to assign its value one needs considering the dynamics underlying QCD. Taking \( a = 2 \) at the instance of exact symmetry, \( i.e. c_A = d_A = c_\gamma = d_\gamma = 0 \), leads successfully to the phenomenological KSFR relations \cite{27,28} \( m_{\rho}^2 = 4F_\rho^2 g_\rho^2 \), \( g_\rho = 2g_{\rho\pi\pi}F_\rho^2 \) with \( g_\rho \) and \( g_{\rho\pi\pi} \) are the \( \rho \)\gamma and \( \rho \pi \pi \) effective coupling constants, respectively. Furthermore, with \( a = 2 \) the symmetric lagrangian part \( L(A, \gamma, P) + aL(V, \gamma, P) = 0 \) so that the effective \( \gamma \pi \pi \) coupling constant vanishes. Thus in the limit of exact symmetry the pion form factor is saturated by the \( \rho \) meson contribution. All departures from the KSFR relations, nonvanishing effective \( \gamma \pi \pi \) coupling \( g_{\gamma \pi \pi} \neq 0 \), and the \( \rho \)-meson dominance for the pion form factor are due to contributions from the symmetry breaking companions. The value \( a = 2.13 \pm 0.06 \) deduced from the \( \rho \) meson mass and its decay width into two pions, indicates that VMD provides, a good first approximation for the electromagnetic part of our lagrangian.\(^3\) With this value of \( a \) our lagrangian includes in addition to the usual VMD corrections due to direct photon-pseudoscalar meson coupling.

We now turn to derive a more elegant form of the lagrangian Eqn.1 with manifestly a massless photon. We do that by transforming the vector meson fields into \( \tilde{V} \),
\[ V_\mu = V_\mu - \frac{e}{g} A_\mu Q , \]  
(48)

and define \( \Gamma_\mu - gV_\mu = \hat{\Gamma}_\mu - gV_\mu \) where \( \hat{\Gamma}_\mu \) does not involve the pure photon field contributions. In terms of these new fields, the lagrangian part \( L_V + \bar{L}_V \) neither contains a photon mass term nor an explicit coupling of the photon to pseudoscalar currents, \( j_\mu \sim [\partial_\mu P, P] \). Thus the terms \( L_{m\gamma} \) and \( aL(V, \gamma, P) \) are removed from \( L_V + \bar{L}_V \). An explicit photon coupling to pseudoscalar currents, is now confined to the \( L(A, \gamma, P) \) term resulting from the pseudoscalar \( L_A + \bar{L}_A \) part which is left unaffected by the transformation Eqn.48. Clearly, since the \( \Gamma_\mu \) as before involves terms like \(-ieA_\mu(PQP - \{P^2, Q\})/2F_\rho^2 \) etc. the lagrangian \( L_V + \bar{L}_V \) still contains the photon – vector meson – pseudoscalar meson couplings. Correspondingly, since

\(^3\)Note that the same way the momentum independent direct \( V_\mu - Z_\mu^0 \) and \( V_\mu - W_\mu \) couplings may be transformed into momentum dependent couplings
\[ V_{\mu\nu} = V_{\nu\mu} - \frac{e}{g} A_{\mu\nu} Q \, , \] (49)

the vector meson kinetic term transforms into,
\[ -\frac{1}{4} Tr(V_{\mu\nu} V^{\mu\nu}) = -\frac{1}{4} \left( Tr(V_{\mu\nu} V^{\mu\nu}) - 2 \frac{e}{g} A_{\mu\nu} Tr(Q V^{\mu\nu}) + \frac{2 e^2}{3 g^2} A_{\mu\nu} A^{\mu\nu} \right) \, , \] (50)

where as usual \( A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). The first term on rhs of Eqn. 50 is the common kinetic term of the vector fields \( V \) while the second term describes the direct photon-vector meson coupling. The last term is associated with the kinetic term of the photon \(-A_{\mu\nu} A^{\mu\nu}/4\) and can be removed by rescaling the charge \( e \) and the photon field according to \( [26] \),
\[ A_{\mu\nu}' = \sqrt{1 + 2 e^2/3 g^2} A_{\mu\nu}; \quad e' = e/\sqrt{1 + 2 e^2/3 g^2} \, , \] (51)

Inserting all of these modifications into the lagrangian Eqn. [15] leads to,
\[ L_{VMDI} = -\frac{1}{4} \left( Tr(V_{\mu\nu} V^{\mu\nu}) - 2 \frac{e}{g} A_{\mu\nu} Tr(Q V^{\mu\nu}) \right) + \\
\quad a F^2 S [Tr(\nabla V) + \tilde{a}_4 Tr(V)Tr(V) + c V (Tr(BVV) + \tilde{a}_4 Tr(BV)Tr(BV)) + d V (Tr(BBV) + \tilde{a}_4 Tr(BV)Tr(BV)) + \\
\quad \frac{a g z_2^2}{2} \{ Tr([V_{\mu\nu}, [P, \partial^\mu P]]) + c V [Tr(B\{Q, [P, \partial^\mu P]\}) + Tr(V_\mu)Tr(B[P, \partial^\mu P])] + \\
\quad d V [Tr([B_{\mu\nu}, B[P, \partial^\mu P]]) + 2 Tr(B_{\nu\mu})Tr(B[P, \partial^\mu P])] + \\
\quad \frac{ie}{2} g^2 z_{\pi} A_{\mu} \{ Tr([Q, P]) + c A Tr(B\{Q, [P, \partial^\mu P]\}) \} \right], \] (52)

where for convenience we have omitted the ’ from \( A_{\mu\nu}' \) and \( e' \). The last expression is closely related to the VMDI model with the characteristics of (i) a massless photon, (ii) a momentum dependent direct photon-vector meson coupling \( V_{\mu\nu} A^{\mu\nu} \), and (iii) a direct photon-pseudoscalar meson coupling as well as the coupling through direct photon-vector meson transitions.

To conclude this section we now use the lagrangian \( L_{VMDI} \), Eq. [52] to calculate the pion and kaon form factors. In the three diagram approximation (see Fig. 1) one obtains,
\[ F_\pi(q^2) = 1 + \frac{q^2 a}{2 m^2 - q^2} \, , \] (53a)
\[ F_{K+}(q^2) = 1 + \frac{3 a}{2} z_K^2 \left[ \frac{1}{m^2 - q^2} + \frac{1 - c_v}{3} - \frac{1}{m^2 - q^2} + \frac{2(1 + c_v + d_v)}{3} \right] \, , \] (53b)
\[ F_{K^0}(q^2) = q^2 \frac{3 a}{2} z_K^2 \left[ \frac{1}{m^2 - q^2} - \frac{1}{3} \frac{1 - c_v}{m^2 - q^2} - \frac{2(1 + c_v + d_v)}{3} \right] \, , \] (53c)

where \( z_K = 1/\sqrt{1 + c_A/2} \) is a rescaling parameter defined from applying the diagonalization procedure to the pseudoscalar meson sector. It must be stressed that the three diagrams involving \( V \rightarrow A \) transition vertex contribute to those terms proportional to \( Q^2 \), only. The meson charge radii are completely determined by diagrams involving the conversion of \( \rho \), \( \omega \), \( \phi \) mesons into a photon (diagram 1b). We have
\[ < r^2 >_\pi = \frac{3 a}{2} \frac{1}{m^2} \, , \] (54a)
\[ < r^2 >_{K^\pm} = 3 a \frac{1}{2} z_K \left[ \frac{1}{m^2} + \frac{1 - c_v}{3} \frac{1}{m^2} + \frac{2(1 + c_v + d_v)}{3} \right] \, , \] (54b)
\[ < r^2 >_{K^0} = 3 a \frac{1}{2} z_K \left[ -\frac{1}{m^2} + \frac{1 - c_v}{3} \frac{1}{m^2} + \frac{2(1 + c_v + d_v)}{3} \right] \, . \] (54c)

The calculated radii of the pions and kaons along with data are listed in Table [1]. In the first three columns we list the results from our model for the Alternative I \( (c_A = 0.64 \pm 0.06) \), Alternative II \( (c_A = 0.20 \pm 0.05) \) and QFB scheme \( (c_A = 1.4 \pm 0.1) \). The results from the Alternative I and Alternative II explain equally well the pion and kaon radii although the \( \chi^2 \) criteria is slightly better for the former. The results from the QFB scheme are by far inferior. For comparison we show the radii calculated with \( a = 2.4 \) as determined by Benayoun and O’Connell \[14\]. Clearly, this value can not explain the charged pion radius.
V. SUMMARY AND CONCLUSIONS

We have considered the vector meson sector within the framework of an effective field theory based on local $U(3)_L \otimes U(3)_R$ symmetry, where the vector mesons play the role of the dynamical gauge bosons of the HLS nonlinear chiral lagrangian. This supplement a previous study of the pseudoscalar sector [12] within this same framework. The lagrangian is written in the most general way, including additional terms quadratic in the traces of the field matrices, which no longer vanish once singlet contributions are included. With these terms included, particularly the contribution from the small $\tilde{W}_4(X)Tr(\Gamma_\mu - g\Gamma_\nu)Tr(\Gamma_\mu - g\Gamma_\nu)$ term and its symmetry breaking companion, it is possible to reproduce the $\omega - \rho$ mass splitting. The explicit form of the lagrangian, through its kinetic and mass terms corresponding to vector mesons, determine the vector meson mixing scheme unambiguously. This observation which has been overlooked in previous studies allows us to calculate the model symmetry breaking scales and the value of $ag^2$ through the diagonalization of the mass matrix. Once the value of $g$ and $a$ are disentangled, say from using another data point like the decay width of $\rho \to \pi\pi$, the lagrangian is completely defined, and can be used to calculate all observables without any additional assumption. As an example we have calculated the form factors and radii of the pions and kaons. Our calculations reproduce straightforwardly the proper normalization of the form factors and explain nicely the charged radii. We finally indicate that VMD model emerges naturally as a first approximation of our lagrangian. This stands in marked difference with Ref. [9,18] where VMD seems to be badly violated.

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VI. APPENDIX

For convenience we quote here the axial and vector parts of the lagrangian [1] explicitly. First the symmetric parts are,

$$L_A = W_1(X)Tr(\Delta_\mu \Delta^\mu) + W_4(X)Tr(\Delta_\mu)Tr(\Delta^\mu) +$$

$$W_5(X)Tr(\Delta_\mu)D^\mu \vartheta + W_6(X)D_\mu \vartheta D^\mu \vartheta ,$$

$$L_V = \tilde{W}_1(X)Tr[\Gamma_\mu - g\Gamma_\nu][\Gamma_\mu - g\Gamma_\nu] +$$

$$\tilde{W}_4(X)Tr(\Gamma_\mu - g\Gamma_\nu)Tr(\Gamma_\mu - g\Gamma_\nu) ,$$

where

$$D_\mu \vartheta = \partial_\mu \vartheta + Tr(r_\mu - l_\mu) ,$$

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu] .$$

The symmetry breaking companions $\bar{L}_A$ are constructed in two alternative ways. The first (referred to as Alternative I) breaks $SU(3)_F$ symmetry, and corresponds to a quadratic form of the Goldstone meson kinetic energy term,

$$\bar{L}_A = W_1(X) \left( c_A Tr(B\Delta_\mu \Delta^\mu) + d_A Tr(B\Delta_\mu B\Delta^\mu) \right) +$$

$$W_4(X)d_A Tr(B\Delta_\mu)Tr(B\Delta^\mu) + W_5(X)c_A Tr(B\Delta_\mu)D^\mu \vartheta .$$

where the axial vector covariant,

$$\bar{\Delta}_\mu = \frac{i}{2} \left\{ \xi_8^\dagger \partial_\mu \xi_8 + \frac{1}{2} \left( \xi_8^\dagger \gamma_\mu \xi_8 - \xi_8 \gamma_\mu \xi_8^\dagger \right) \right\} ,$$

involves the pseudoscalar octet field matrix only. The second alternative (Alternative II) breaks $U(3)_F$ symmetry and corresponds to bilinear kinetic energy term. Namely,

$$\bar{L}_A = W_1(X) \left( c_A Tr(B\Delta_\mu \Delta^\mu) + d_A Tr(B\Delta_\mu B\Delta^\mu) \right)$$

$$+ W_4(X)(c_A Tr(B\Delta_\mu)Tr(\Delta^\mu) + d_A Tr(B\Delta_\mu)Tr(B\Delta^\mu)) +$$

$$W_5(X)c_A Tr(B\Delta_\mu)D^\mu \vartheta ,$$

with $\Delta_\mu$ involves now the nonet pseudoscalar field matrix. Similarly the asymmetric companion of $L_V$ is,
\[ \bar{\mathcal{L}}_V = \bar{W}_1(X)(c_V T r(B[\Gamma_\mu - gV_\mu][\Gamma^\mu - gV^\mu]) + d_V T r(B[\Gamma_\mu - gV_\mu]B[\Gamma^\mu - gV^\mu])) + \bar{W}_4(X)(c_V T r(\Gamma_\mu - gV_\mu)T r(B[\Gamma^\mu - gV^\mu]) + d_V T r(B[\Gamma_\mu - gV_\mu]T r(B[\Gamma^\mu - gV^\mu])) \right) . \] (62)

In the expressions above, \( c_A, d_A, c_V \) and \( d_V \) are the symmetry breaking parameters determined from data. In Ref. [12] \( c_A, d_A \) and \( r = \frac{F_8}{F_0} \) were deduced from global fit of calculated radiative decay widths. Their values as determined for the Alternative I, Alternative II and QFB mixing schemes are listed in Table II.

### TABLE I. Pseudoscalar meson squared charge radii in \( fm^2 \)

|        | Alternative I | Alternative II | QFB | \( g = 2.4 \) | Data |
|--------|---------------|----------------|-----|--------------|------|
| \( < r_{\pi^\pm}^2 > \) | 0.43 ± 0.01 | 0.43 ± 0.01 | 0.43 ± 0.01 | 0.49 ± 0.01 | 0.439 ± 0.03 |
| \( < r_{K^\pm}^2 > \) | 0.29 ± 0.02 | 0.35 ± 0.02 | 0.23 ± 0.01 | 0.33 ± 0.02 | 0.31 ± 0.05 |
| \( < r_{K^0}^2 > \) | -0.040 ± 0.003 | -0.048 ± 0.003 | -0.031 ± 0.003 | -(0.045 ± 0.003) | -0.054 ± 0.026 |
| \( \chi^2 \) | 0.5 | 0.7 | 3.3 | 2.1 |

### TABLE II. Symmetry breaking scales and \( \chi^2/dof \) from global fit to data. Values marked with an asterisk were kept fixed.

|        | \( c_W \)  | \( c_A \)  | \( d_A \)  | \( r \)  |
|--------|------------|------------|------------|----------|
| Alternative I | -(0.20 ± 0.05) | (0.64 ± 0.06) | -0.25 ± 0.04 | 0.91 ± 0.04 |
| Alternative II | -(0.27 ± 0.05) | 0.2 ± 0.05 | 0.1 ± 0.02 | 0.94 ± 0.05 |
| QFB | -(0.19 ± 0.05) | (1.4 ± 0.1) | -1.1 ± 0.1 | *1 |

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FIG. 1. Diagram contributing into pseudoscalar meson form factors. Diagram a corresponds to direct photon-pseudoscalar meson coupling, diagram b involves the photon-vector meson conversion.