Tests of Gravity Theories Using Supermassive Black Holes

Jeremy Sakstein¹, Bhuvnesh Jain¹, Jeremy S. Heyl², and Lam Hui³

¹ Department of Physics and Astronomy, Center for Particle Cosmology, University of Pennsylvania, 209 S. 33rd Street, Philadelphia, PA 19104, USA
² Department of Physics and Astronomy, University of British Columbia, 6224 Agricultural Road, Vancouver, BC V6T 1Z1, Canada; heyl@phas.ubc.ca
³ Department of Physics, Center for Theoretical Physics, Columbia University, New York, NY 10027, USA; lhui@astro.columbia.edu

Abstract

Scalar-tensor theories of gravity generally violate the strong equivalence principle, namely that compact objects have a suppressed coupling to the scalar force, which causes them to fall slower. A black hole is the extreme example where such a coupling vanishes, i.e., black holes have no scalar hair. We explore observational scenarios for detecting strong equivalence principle violation, focusing on galileon gravity as an example. For galaxies infalling toward galaxy clusters, the supermassive black hole can be offset from the galaxy center away from the direction of the cluster. Well-resolved images of galaxies around nearby clusters can therefore be used to identify the displaced black hole via the star cluster bound to it. We show that this signal is accessible with imaging surveys, both ongoing ones such as the Dark Energy Survey and future ground- and space-based surveys. Already, the observation of the central black hole in M 87 places new constraints on the galileon parameters, which we present here. O(1) matter couplings are disfavored for a large region of the parameter space. We also find a novel phenomenon whereby the black hole can escape the galaxy completely in less than one billion years.

Key words: black hole physics – dark energy – gravitation

1. Introduction

The study of infrared modifications of general relativity (GR) has undergone a renaissance in the last decade, driven partly by the dark energy mystery and partly by recent developments in the construction of healthy higher-derivative scalar-tensor theories (see Clifton et al. 2012; Joyce et al. 2015; Berti et al. 2015; Koyama 2016; Joyce et al. 2016; Bull et al. 2016; Burrage & Sakstein 2016 for recent reviews) that have opened up a new realm of possibilities for driving the acceleration of the cosmic expansion. On small scales, consistency with tests of GR is achieved by utilizing screening mechanisms, which use nonlinear effects to hide the modifications of gravity in the solar system. One particularly interesting and well-studied paragon for these theories is the galileon (Nicolis et al. 2009), which self-accelerates cosmologically, but screens locally using the Vainshtein mechanism (Vainshtein 1972). On large scales, the galileon can be tested by the modified rate of structure growth (Barreira et al. 2012, 2013); on small scales, the Vainshtein mechanism renders its predictions largely indistinguishable from those of GR.

One interesting exception is that of Hui & Nicolis (2012), who have pointed out that the no-hair theorem (Hui & Nicolis 2013) implies that black holes do not couple to the galileon and therefore there is a violation of the strong equivalence principle (SEP) whereby they fall at different rates to non-relativistic matter in external fields. In particular, if part of a galaxy’s motion is due to an external galileon field, then the supermassive black hole (SMBH) that lies at its center does not feel this; therefore, the SMBH lags behind as the galaxy moves. Hui & Nicolis (2012) have argued that for constant density cores, the restoring force due to the dark matter and baryons in the galaxy will eventually compensate for the lack of the galileon force, giving rise to an offset in the opposite direction to the galaxy’s acceleration. Hui & Nicolis (2012) considered galaxies moving in the cosmic field and predicted a small offset, which is difficult to look for observationally.

In this letter, we look for novel scenarios for testing galileon gravity using the predicted SEP violation that are more amenable to observational testing. We consider two new effects:

1. Galaxy clusters. Satellite galaxies infalling into galaxy clusters pass through a partially unscreened regime where the galileon field generated by the cluster pulls on the galaxies but not their resident black holes. This can result in an offset that is accessible to ongoing and planned imaging surveys. In this work, we use the analysis of M 87 presented in (Asvathaman et al. 2015) to place new constraints.

2. Escaping black holes. Realistic dark matter haloes whose density falls with distance from the center have a maximum restoring force. Galileon forces larger than this drive the SMBH to escape the galaxy in an observable timescale (less than one billion years).

2. Galileon Gravity Theories

Galileons are scalar-tensor extensions of GR that are invariant under the Galilean transformation $\phi(x^i) \rightarrow \phi(x^i) + b_i x^i + c$ (Nicolis et al. 2009). This symmetry restricts the form of their action to four unique terms (in four dimensions): the so-called quadratic, cubic, quartic, and quintic galileons Nicolis et al. (2009), and ensures that the equations of motion are second-order so that the Ostrogradski ghost instability is absent. In what follows, it will be sufficient to restrict our attention to the simplest model, the cubic galileon, although we note that the results presented here apply equally to all galileon theories including their curved space generalizations (Deffayet et al. 2009, 2011; Gleyzes et al. 2015a, 2015b). In more general cases, there will be additional parameters and the tests we discuss later will constrain combinations of these. The equation of motion for the cubic
galileon field sourced by a static non-relativistic object is
\[
\nabla^2 \phi + \frac{r^2}{3} \left[ (\nabla^2 \phi)^2 - \nabla_i \nabla_j \phi \nabla^i \nabla^j \phi \right] = 8\pi \alpha G \rho,
\]
where \(i, j = 1, 2, 3\). There are two constant parameters:\footnote{4} a dimensionless coupling constant \(\alpha \sim \mathcal{O}(1)\), which parameterizes the non-minimal coupling to matter, and the crossover scale \(r_c\), which determines the size of the nonlinear galileon self-interactions. The coupling scale \(\Lambda_\phi = (6M_\phi/r_c^2)^{1/3}\) is often used instead of \(r_c\). For a point-mass \(M\), one can define the Vainshtein radius \(r_V^2 = \frac{4}{3} \alpha G M r_c^2\), which is the scale where the nonlinear galileon terms become important. The total (Newtonian plus galileon) force profile inside the Vainshtein radius is
\[
\frac{d\Phi}{dr} = \frac{GM}{r^2} \left[ 1 + 2\alpha^2 \left( \frac{r}{r_c} \right)^2 \right]
\]
so that deviations from GR are suppressed by a factor \((r/r_c)^{3/2}\).

Outside the Vainshtein radius, the galileon force becomes unsuppressed, leading to a total force profile that matches the Newtonian one except for an overall enhancement by a factor of \((1 + 2\alpha^2)^{3/2}\).

A solar mass object has \(r_c \sim \mathcal{O}(100 \text{ pc})\) (Khoury 2013), so the solarsystem lies well within the Sun’s Vainshtein radius. For extended objects, such as the dark matter haloes we discuss in this work, the transition between the screened and unscreened regimes is a rather gradual one (see Schmidt 2010 and Figure 1 below). This is one reason why the outskirts of galaxy clusters are a prime testing ground for galileon theories.

In this theory, the equation of motion for an object of mass \(m\) moving in an external Newtonian and scalar potential, \(\Phi_N^{\text{ext}}\) and \(\Phi^{\text{ext}}\), respectively, sourced by some other object is
\[
m\ddot{x} = -m\nabla \Phi_N^{\text{ext}} - \alpha Q \nabla \phi^{\text{ext}},
\]
where \(Q\) is the scalar charge that characterizes the strength of the object’s coupling to the galileon field. One can show (Hui et al. 2009) that \(Q = \int d^3x \, T_{00}^\phi\) i.e., the scalar charge is equal to the baryonic mass\(^5\) of the object. This implies that \(Q = m\) for non-relativistic objects, but that \(Q < m\) for compact objects, whose mass receives a significant contribution from the gravitational binding energy. A black hole is the extreme example where \(Q = 0\). Hui & Nicolis (2013) showed that a black hole cannot source galileon hair and therefore also does not couple to a galileon external field. Because our primary interest concerns a black hole residing in a galaxy, one might worry that the Vainshtein mechanism is enough to greatly diminish the galileon force on the stars (and dark matter) of the host galaxy, leading to essentially no difference between the falling motion of the stars and that of the black hole. A key feature of the galileon comes to the rescue: the galileon equation of motion is invariant under the galileon symmetry, so that given any solution, one can always add a component with a constant field gradient to obtain a second solution. For instance, for a galaxy located within a cluster, the galileon field sourced by the cluster behaves as a constant-gradient field, since its scale/wavelength is much longer than the size of the galaxy (Hui et al. 2009). The resident stars and dark matter of the galaxy thus respond to this cluster-sourced galileon field, while the black hole does not, setting up an astronomical version of the Eötvös experiment. Note that in this setup, it is important to account for the effect of Vainshtein mechanism on the cluster-sourced galileon field itself.

3. Galaxy Clusters

A cluster carries sufficient mass that, despite some Vainshtein suppression, the galileon force sourced by it is large enough to give interesting observational effects at distances \(D \gtrsim 0.1 R_{200}\) (Schmidt 2010). As an example, consider a model of the Virgo cluster with mass \(M_{200} = 10^{15} M_\odot\). We model the mass distribution of the cluster with a Navarro-Frenk-White (NFW) profile (Navarro et al. 1996) (with concentration \(c = 5\)) inside \(R_{200}\); outside it we model the “2-halo” mass distribution using the fits to N-body simulations of Diemer & Kravtsov (2014). We define \(R_{200}\) using the critical density, which corresponds to \(R_{200} \approx 3\, H_0^{-1}\) in Diemer & Kravtsov (2014); we take \(H_0 = 72\) km s\(^{-1}\) Mpc\(^{-1}\).

Note that we consider haloes at \(z = 0\) since the tests described presently are intended to be applied to low-redshift \((z < 0.05)\) clusters. Any potential time evolution of \(\alpha\) over this range will be negligible, although we note that more detailed modeling may be required if a sample of useful clusters at redshifts \(z \gg 0.1\) were to be found. Previous studies have consistently found that the properties of halos are largely unmodified in galileon cosmologies (Barreira et al. 2014b, 2015), especially inside the Vainshtein radius. Outside, at distances larger than \(\sim 2\) Mpc, more detailed modeling may be required, although M 87, which we consider below, lies well within the Vainshtein radius of the Virgo cluster so this is not a caveat to the results we obtain here.

The Newtonian, galileon, and cosmological root mean square (rms) galileon forces (the scenario considered by Hui & Nicolis (2013)) are plotted in Figure 1 for models with

\footnote{5} We use the term “baryons” as a proxy for anything that contributes to the matter energy-momentum tensor (as opposed to the pseudo-energy-momentum tensor, which includes the contribution from gravitational binding energy). This could include actual baryons as well as dark matter.

Figure 1. Newtonian, galileon, and cosmological forces for a cluster with mass \(M_{200} = 10^{15} M_\odot\). The galileon force falls below the Newtonian force inside the Vainshtein radius as expected, but it remains flat even well inside the cluster. For black hole offsets, the true amplitude of the galileon force matters (unlike other tests that rely on its amplitude relative to the Newtonian force).
The galileon force was calculated by solving the field equations for the cubic galileon exactly using spherical symmetry. One can see that there is a large galileon force for $r \gtrsim 0.1 R_{200} \sim 0.3$ Mpc. Satellite galaxies are typically infalling toward the cluster within $< R_{200} \sim 13$ Mpc (Diemer & Kravtsov 2014). These galaxies would feel a large galileon force that their central SMBHs would not, thus allowing the effects of the SEP violation to manifest. Note that the galileon force is suppressed relative to the Newtonian force when $r \lesssim R_{200}$, but it is the amplitude of the galileon force that is important for SEP violations (not this ratio) and therefore large effects are still expected at distances above 0.1$R_{200}$. The cluster forces are at least an order of magnitude larger than rms cosmological force. One therefore expects offsets of $\mathcal{O}(\text{kpc})$, which are accessible to imaging surveys of nearby galaxy clusters.

The dynamics of this situation are easily exemplified by considering a galaxy where the central density $\rho_0$ is approximately constant. In this case, the black hole is subject to the galileon acceleration $a_{\text{BH}}$ away from the center and a linear restoring force given by $\frac{4\pi G r \rho_0}{3}$, in which case the equation of motion for the black hole (in the rest frame of the galaxy) is

$$\ddot{r} = -\frac{4\pi}{3} G \rho_0 r + a_{\text{BH}}$$  \hspace{1cm} (3)

so that the black hole undergoes oscillations about the equilibrium point $\ddot{r} = 3 a_{\text{BH}} / 4\pi G \rho_0$ with a time-period $T_0 \sim (G \rho_0)^{-1/2}$. In practice, the galileon force turns on slowly over a time $\sim a_{\text{BH}} / v \sim v / \mathcal{O}(10^{10} \text{ years})$ (where $v \sim 100 \text{ km s}^{-1}$ is the typical infall velocity and $d$ is the distance from the cluster center), whereas the oscillation period is $\mathcal{O}(10^7 \text{ years})$ so that the amplitude of these oscillations is small and the black hole tracks the equilibrium position adiabatically. One then expects the black hole to be offset from the center by a distance

$$\bar{r} = 1 \text{ kpc} \left( \frac{a_{\text{BH}}}{2000 \text{ (km/s)}^2 / \text{kpc}} \right) \left( \frac{\rho_0}{0.1 \text{ M}_\odot \text{ pc}^{-3}} \right)^{-1}.$$  \hspace{1cm} (4)

Clearly, the size of the offset depends on the central density $\rho_0$ and the external force; we used a fiducial external force that is typical for the outskirts of a galaxy cluster similar to the one modeled in Figure 1. As the galaxy infalls, the galileon force increases and so does the offset of the black hole (up to around $R_{200}$); the direction of the offset would be in the opposite direction to the galaxy’s direction of acceleration i.e., away from the center of the cluster.

An example of this (taking $\alpha = 1$) is shown in Figure 2; one can see that observable offsets of $\mathcal{O}$(kpc) can develop at a variety of distances from the cluster’s center. They are at least an order of magnitude larger than one would expect for

$${}^6$$ The latter corresponds to self-accelerating cosmological galileons, while the former is often used as a paradigm for Vainshtein screening (Schmidt 2010). These models are not ruled out by local observational constraints (Dvali et al. 2003; Khoury 2013). The constraints of Schmidt (2009) and Barreira et al. (2014a) coming from cosmological probes do not directly apply to our model as they are derived for a sub-class of models where the constant contribution to $\alpha$ is absent. Future efforts to constrain more general galileon models, in particular those with a linear coupling to matter, will be able to use the results we obtain below as a consistency check.

$${}^7$$ Masses in the range $M_{200} = 1 \times 10^{13} M_\odot$ have been reported (Fouque et al. 2001; Peirani & de Freitas Pacheco 2006), which is less uncertain than effects of modeling the complex 3D structure (Mei et al. 2007) with an NFW profile so we adopt the conservative value of $10^{13} M_\odot$. The Astrophysical Journal Letters, 844:L14 (5pp), 2017 July 20

Figure 2. Offset of the SMBH for a cored galaxy infalling toward a cluster. The central densities of the infalling satellites and the cluster mass ($M_{200}$) are indicated in the figure. In all cases we set $c = 5$, $r_c = 500$ Mpc, and $\alpha = 1$; we used the same modeling techniques as in Figure 1. The curves show that the outskirts of clusters are promising for this test, as the offsets are large, and most of the infalling galaxies have not had interactions with other galaxies that may perturb the SMBH.

Galaxies moving in the cosmological field, so this scenario is well suited to testing galileon gravity. As is evident from the figure, as the galaxy infalls, the offset becomes larger due to an increased galileon acceleration. This reaches a maximum and begins to fall off as the galaxy approaches the cluster’s center, where the galileon modifications are more screened.

Asvatham et al. (2015) considered the effect of the sub-clump of the Virgo cluster centered on M 84 and M 86 on the position of the black hole within M 87. Using the same assumptions as Asvatham et al. (2015), in particular the dynamical model of Walsh et al. (2013) and the photometry of Ferrarese et al. (2006), we find that the strength of the galileon field in the plane of the sky is less than about 700 (km/s)^2/kpc (or 1000 (km/s)^2/kpc in three dimensions). We find the SMBH to be located at the center of the light to within 0.03 arcseconds (0.03 is the upper limit at 1σ). We are only sensitive to the motion of the SMBH across the sky because we are measuring its position on the sky relative to the center of the galaxy; consequently, only the component of the galileon field in the plane of the sky is measurable. The distribution of light and mass-to-light ratio in the context of a Hernquist model for the central region of the galaxy was used to obtain an estimate of the restoring force.

Combining this with a model for the Virgo cluster as depicted in Figure 1, i.e., NFW + 2-halo modeling with $c = 5$ and $M_{200} = 10^{13} M_\odot$, we obtain constraints on the values of $\alpha$ and $r_c$ as depicted in Figure 3. These were found by scanning the parameter space to find the region where the galileon force at $R_{200}$ is smaller than 1000 (km/s)^2/kpc. The constraints therefore assume that M 87 is located within $R_{200}$ of the Virgo cluster, but that it is otherwise insensitive to its precise location because the galileon force is fairly constant for $r < R_{200}$. We checked that varying the concentration over the range $c = 5$–10 produces a minimal change in the constraints. Finer constraints could be obtained by measuring the displacements of many SMBHs relative to the centers of their galaxies and the
driven out of the galaxy by the galileon force. Such a situation might arise as a result of a merger event or by recoil from gravitational wave “kicks.”

In practice, if the infalling galaxy is initially at a position where the galileon force is smaller than the restoring force then the black hole will be offset, but as the galaxy moves closer to the cluster, the galileon force will increase and may exceed the restoring force so that there is nothing halting the SMBH’s motion. One can estimate the timescale for the black hole to escape the galaxy entirely by neglecting the restoring force. Galaxies falling into clusters experience an increasing galileon force while the restoring force remains the same and so this quickly becomes a good approximation. Treating the galileon acceleration as constant, the timescale for the black hole to move a distance \( R \) is

\[
T = 10^7 \text{ yr} \left( \frac{R}{\text{kpc}} \right)^\frac{1}{2} \left( \frac{a_{\text{BH}}}{2000 (\text{km/s})^2/\text{kpc}} \right)^\frac{1}{2}.
\]

The black hole can then escape the galaxy in less than a billion years. Note that the typical velocity of the black hole is \( \mathcal{O}(\text{km s}^{-1}) \), far less than the typical velocity dispersion of the galaxy so that dynamical friction can safely be ignored (Binney & Tremaine 2008). Precisely which galaxies would allow escape depends on the environment and the galaxy central density. We leave a systematic investigation for the future.

4. Observational Tests

The novel features identified above present new avenues for testing galileon gravity and constraining the parameters \( \alpha \) and \( r_c \) (recall that \( \alpha \) parameterizes the strength of the galileon coupling to matter and \( r_c \) parameterizes the galileon’s self-interactions).

Displaced SMBHs in nearby galaxies. The gravitational acceleration is several times larger at the outskirts of galaxy clusters than in the field and is directed toward the cluster center. The predicted offset is detectable via the star cluster bound to the SMBH, which would be offset from the centroid of the stellar light at the center of the galaxy. The correlation with the expected displacement direction from modified gravity, i.e., the opposite direction to the galaxy cluster, is critical in such an exercise as it narrows the search zone and can distinguish it from other sources of fluctuations.

The two key observational parameters are the size of the offset relative to the point-spread function (PSF), which determines whether it is resolved, and the change in flux due to the star cluster bound to the SMBH. At \( z \simeq 0.05 \), an offset of 1 kpc corresponds to 1 arcsecond on the sky which is larger than the (full width half maximum) PSF of the best ground based telescopes. Out to this distance, hundreds of clusters can be found for which the offsets in infalling galaxies are resolved. A large fraction of these are already imaged by the SDSS and DES optical surveys and by X-Ray surveys. By identifying suitable galaxies at the outskirts of each cluster, a large sample of galaxies can be assembled. These galaxies could be followed up with high-resolution imaging, ideally from the Hubble Space Telescope, and multi-wavelength observations. The sample size may be essential in handling the second observational challenge: detecting the star cluster given the typical flux variations across a galaxy, and other sources of error. For the central parts of elliptical galaxies, these are at the percent level (e.g., Bernardi et al. 2017), which in many cases
is smaller than the displaced SMBH’s star cluster (e.g., Asvathamam et al. 2015’s study of M 87). Another challenge discussed by Asvathamam et al. (2015) is centroiding the galaxy light using the outer isophotes. Finally, we note that the predicted offset is sensitive to the central density profile of the host galaxy, which is challenging to determine. A range of offset values must therefore be considered, and for lower values only the most nearby clusters may be suitable for our test.

**Perturbations to the galaxy light profile.** While we have not investigated morphological features in this study, a SMBH that is displaced or drifting through the stellar disk will produce characteristic distortions that can be measured by analyzing the images of a large sample of galaxies. Even for galaxies above $z \sim 0.1$, where the offset is no larger than the PSF, the model-fitting approach described above can be attempted. Weak lensing studies that measure the low-order moments of the surface brightness of galaxies may also be well suited to extracting the skewness or “flexion” in the light distribution, and correlating it with the direction of the external force vector on the galaxy.

**Missing SMBHs.** For larger galion force or if initially displaced, the SMBH would leave the visible galaxy in less than a billion years, the timescale over which galaxies move with coherent velocities. Hence, a fraction of galaxies would not have central SMBHs in such a scenario. During galaxy mergers, SMBHs are displaced from their center and then occupy steeper parts of the density profile until they lose energy to dynamical friction and sink to the center. The timescales may be sufficient for the modified gravity effect to act on the black hole. The observational finding that the overwhelming majority of galaxies above a certain mass have central SMBHs may provide new limits on $\alpha$ and $r_\alpha$ for typical values of the density profile.

Note that for many of these observable tests, we can estimate the direction of the effect from observations of the galaxies surrounding the cluster of interest by estimating the direction of the local gravity acceleration vector. This direction vector is more reliable for galaxies infalling into galaxy clusters. This is crucial for distinguishing modified gravity effects from other astrophysical processes such as acceleration due to asymmetric jets, recoil from gravitational wave emission, Brownian motion, gravitational slingshot due to mergers, and perturbations due to massive objects such as globular clusters. All of these can displace the central black hole for varying periods of time, but in a direction uncorrelated with the galaxy’s acceleration. These processes are important to study even in the absence of modified gravity and may shed light on the dynamics of galaxy mergers (Merritt & Milosavljevic 2005).

**5. Conclusions.**

To summarize, we identified novel methods for testing gravity using the equivalence principle violations for black holes first noted by Hui & Nicolis (2012). Galaxies infalling into massive clusters may host SMBHs that are offset from the center by $\mathcal{O}(kpc)$ and may even be absent altogether. The offset is at least an order of magnitude larger than for galaxies moving in the cosmological field and is in the opposite direction to the galaxy’s acceleration, which can help distinguish this effect from other astrophysical displacement mechanisms. We discussed how a sample of nearby galaxy clusters can be used to obtain bounds on the model parameters $\alpha$ and $r_\alpha$ that would have strong implications for models of modified gravity that try to explain cosmic acceleration. Indeed, using observations of the central black hole in M 87 we have been able to place new constraints on the parameter space that push into the interesting region for self-accelerating models.

We are grateful to Eric Baxter, Mariangela Bernardi, Gary Bernstein, Chihway Chang, Neal Dalal, Marla Geha, Jenny Green, Mike Jarvis, Gordon Richards, Ed Moran, Nadia Zakamska, Justin Alising and colleagues at the Center for Computational Astrophysics. J.S. is supported by grants provided to the Center for Particle Cosmology by the University of Pennsylvania. B.J. is supported in part by the US Department of Energy grant DE-SC0007901. J.H. is supported by a Discovery Grant from the Natural Sciences and Engineering Research Council of Canada. L.H. is supported in part by the NASA grant NNX16AB27G and the DOE grant DE-SC0011941.

**References**

Asvathamam, A., Heyl, J. S., & Hui, L. 2015, MNRAS, in press (arXiv:1506.07607)
Barreira, A., Li, B., Baugh, C., & Pascoli, S. 2013, JCAP, 11, 056
Barreira, A., Li, B., Baugh, C., & Pascoli, S. 2014a, JCAP, 1408, 059
Barreira, A., Li, B., Baugh, C. M., & Pascoli, S. 2012, PhRvD, 86, 124016
Barreira, A., Li, B., Hellwing, W. A., et al. 2014b, JCAP, 1404, 029
Barreira, A., Li, B., Jennings, E., et al. 2015, MNRAS, 454, 4085
Bernardi, M., Fischer, J. L., Sheth, R. K., et al. 2017, MNRAS, in press (arXiv:1702.08527)
Berti, E., Barausse, E., Cardoso, V., et al. 2015, CQGra, 32, 243001
Binney, J., & Tremaine, S. 2008, Galactic Dynamics (2nd ed.; Princeton, NJ: Princeton Univ. Press)
Bull, P., Akrami, Y., Adamek, J., et al. 2016, PDU, 12, 56
Burrage, C., & Sakstein, J. 2016, JCAP, 1611, 045
Clifton, T., Ferreira, P. G., Padilla, A., & Skordis, C. 2012, PhR, 513, 1
Deffayet, C., Espinosa-Farèse, G., & Vikman, A. 2009, PhRvD, 79, 084003
Deffayet, C., Gao, X., Steer, D. A., & Zahariade, G. 2011, PhRvD, 84, 064039
Diemer, B., &Kravtsov, A. V. 2014, ApJ, 789, 1
Dvali, G., Gruzinov, A., & Zaldarriaga, M. 2003, PhRvD, 68, 024012
Ferrarese, L., Côté, P., Jordán, A., et al. 2006, ApJS, 164, 334
Fouque, P., Solanes, J. M., Sanchis, T., & Balkowski, C. 2001, A&A, 375, 770
Gleyzes, J., Langlois, D., Piazza, F., & Vernizzi, F. 2015a, PhRvL, 114, 211101
Gleyzes, J., Langlois, D., Piazza, F., & Vernizzi, F. 2015b, JCAP, 02, 018
Hui, L., & Nicolis, A. 2012, PhRvL, 109, 051104
Hui, L., & Nicolis, A. 2013, PhRvL, 110, 241104
Hui, L., Nicolis, A., & Stubbs, C. 2009, PhRvD, 80, 104002
Joyce, A., Jain, B., Khoury, J., & Trodden, M. 2015, PhR, 568, 1
Joyce, A., Lombriser, L., & Schmidt, F. 2016, ARNPS, 66, 95
Khoury, J. 2013, arXiv:1312.2006
Koyama, K. 2016, RPPh, 79, 046902
Mei, S., Blakeslee, J., Cote, P., et al. 2007, ApJ, 655, 144
Merritt, D., & Milosavljevic, M. 2005, LRR, 8, 8
Navarro, J. F., Frenk, C. S., & White, S. D. 1996, ApJ, 462, 563
Nicolis, A., Rattazzi, R., & Trinchieri, E. 2009, PhRvD, 79, 064036
Peirani, S., & de Freitas Pacheco, J. A. 2006, NewA, 11, 325
Peirani, S., & de Freitas Pacheco, J. A. 2006, PhRvD, 79, 084003
Peirani, S., & de Freitas Pacheco, J. A. 2006, NewA, 11, 325
Schmidt, F. 2009, PhRvD, 80, 123003
Schmidt, F. 2010, PhRvD, 81, 103002
Trujillo, I., Erwin, P., Asensio Ramos, A., & Graham, A. W. 2004, AJ, 127, 1917
Vainshtein, A. 1972, PhLB, 39, 393
Walsh, J. L., Barth, A. J., Ho, L. C., & Sarzi, M. 2013, ApJ, 770, 86

Sakstein et al.