PRESENT STATUS OF NEUTRINO MIXING

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Abstract

A short review of the status of neutrino mixing and neutrino oscillations is given. The basics of neutrino mixing and oscillations is discussed. The latest evidences of neutrino oscillations obtained in the Super-Kamiokande and the SNO solar neutrino experiments and in the Super-Kamiokande atmospheric neutrino experiment are considered. The results of solar and atmospheric neutrino experiments are discussed from the point of view of the three-neutrino mixing.

1 Neutrino mixing

Strong evidences in favor of neutrino masses and neutrino oscillations were obtained in atmospheric [1] and solar [2, 3, 4, 5, 6, 7, 8] neutrino experiments. We will discuss here the latest experimental data and implications for neutrino mixing that can be inferred from the existing data.

Investigation of neutrino oscillations is based on two fundamental experimental facts

1. Interaction of neutrino with matter is described by the Standard charged current (CC) and neutral current (NC) Lagrangians

\[ \mathcal{L}_I^{\text{CC}} = -\frac{g}{2\sqrt{2}} j_\alpha^{\text{CC}} W^\alpha + \text{h.c.}, \] (1)

and

\[ \mathcal{L}_I^{\text{NC}} = -\frac{g}{2\cos\theta_W} j_\alpha^{\text{NC}} Z^\alpha. \] (2)

1 A lecture at the Advanced Study Institute "Symmetries and Spin", Praha-Spin-2001, Czech Republic, July 15-28, 2001

2 Indications in favor of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations which were obtained in the accelerator LSND experiment [9], require confirmation. We will not consider LSND data here.
Here $g$ is the electroweak interaction constant, $\theta_W$ is weak (Weinberg) angle, $W^\alpha$ and $Z^\alpha$ are the fields of $W^\pm$ and $Z^0$ vector bosons and
\[ j^\text{CC}_\alpha = \sum_i \bar{\nu}_i l \gamma_\alpha l \; ; \; j^\text{NC}_\alpha = \sum_i \bar{\nu}_i l \gamma_\alpha \nu_i \]
are the leptonic charged and neutral currents.
The CC and NC interactions (1) and (2) conserve flavor lepton numbers and determine the notion of flavor neutrinos and antineutrinos. For example, neutrino that is produced together with $\mu^+$ in the decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$ is the left-handed muon neutrino $\nu_\mu$, antineutrino that is produced together with electron in the decay $n \rightarrow p + e^- + \bar{\nu}_e$ is the right-handed electron antineutrino $\bar{\nu}_e$ etc.

2. Three flavor neutrinos exist in nature. The number of the flavor neutrinos $n_{\nu_f}$ was determined from experiments on the measurement of the width of the decay $Z \rightarrow \nu_l + \bar{\nu}_l$ (SLC, LEP). In the LEP experiments it was obtained that
\[ n_{\nu_f} = 3.00 \pm 0.06. \]

According to the neutrino mixing hypothesis [10, 11] masses of neutrinos are different from zero and fields of massive neutrinos $\nu_i$ enter into the CC and the NC Lagrangians (1) and (2) in the mixed form
\[ \nu_{iL} = \sum_i U_{li} \nu_{iL}, \]
where $\nu_i$ is the field of neutrino with mass $m_i$ and $U$ is the unitary PMNS mixing matrix.

The relation (3) leads to a violation of flavor lepton numbers. The effects of the violation of flavor lepton numbers can be revealed in neutrino oscillation experiments. We will discuss neutrino oscillation experiments later. Now we will consider different general possibilities of the neutrino mixing (see, for example [12, 13])

If neutrino masses are different from zero, there is a neutrino mass term in the total Lagrangian. The structure of the mass term depends on the mechanism of neutrino mass generation. Only left-handed neutrino fields $\nu_{iL}$ enter into the Lagrangian of the weak interaction (1) and (2). In the
neutrino mass term both $\nu_L$ and singlet $\nu_R$ fields can enter. If $\nu_L$ and $\nu_R$ enter into the mass term in such a form that the total lepton number $L$ is conserved, in this case fields of massive neutrinos are four-component Dirac fields and neutrino $\nu_i$ and antineutrino $\bar{\nu}_i$ have opposite lepton numbers. The corresponding mass term is called the Dirac mass term. The number of the massive neutrinos in the case of the standard Dirac mass term is equal to the number of flavor neutrinos. The Dirac mass term can be generated by the Standard Higgs mechanism with a Higgs doublet.

If the lepton number is not conserved, only left-handed components $\nu_L$ can enter into the neutrino mass term. The corresponding mass term is called the Majorana mass term. It is a product of left-handed components $\nu_L$ and right-handed components $(\nu_L)^c$, determined by the relation

$$ (\nu_L)^c = C\bar{\nu}_L^T, $$

where $C$ is the matrix of the charge conjugation that satisfies the conditions

$$ C\gamma^a C^{-1} = -\gamma_a; \quad C^T = -C. $$

In the case of the Majorana mass term the fields $\nu_i$ in (5) are two-component Majorana fields that satisfy the condition

$$ \nu_i = \nu_i^c. $$

The condition (8) means that neutrinos and antineutrinos, quanta of the Majorana field $\nu_i$, are identical particles. The number of the massive neutrinos in the case of the Majorana mass term is equal to three. The Majorana mass term requires a beyond the Standard Model mechanism of neutrino mass generation with Higgs triplets.

In the more general case both $\nu_L$ and $\nu_R$ fields enter into the mass term and there are no conserved lepton numbers (the Dirac and Majorana mass term). Because the lepton numbers are not conserved there is no possibility to distinguish neutrino and antineutrino and fields of neutrinos with definite masses $\nu_i$ in the case of the Dirac and Majorana mass term are two-component Majorana fields. If three left-handed fields $\nu_L$ and three right-handed fields $\nu_R$ enter into the mass term, the number of massive Majorana neutrinos is equal to 6. For the mixing we have

$$ \nu_{iL} = \sum_{i=1}^{6} U_{li} \nu_{iL} \quad (l = e, \mu, \tau) $$

(9)
\( (\nu_{iR})^c = \sum_{i=1}^{6} U_{iL} \nu_{iL}, \)  

(10)

where \( U \) is the \( 6 \times 6 \) unitary matrix and the fields \( \nu_i \) satisfy the condition \( (8) \).

In the framework of the Dirac and Majorana mass term there exist a plausible mechanism of neutrino mass generation, which is called the see-saw mechanism [14]. This mechanism is based on the assumption that lepton number is violated by the right-handed Majorana mass term at the scale \( M \), which is much larger than the electroweak scale \( \simeq 300 \text{ GeV} \). In the see-saw case in the spectrum of masses of Majorana particles there are three light masses \( m_k \) (masses of neutrinos) and three very heavy masses \( M_k \) (\( k=1,2,3 \)). Masses of neutrinos are connected with the masses of the heavy Majorana particles by the see-saw relation

\[
m_k \simeq \frac{(m^l_k)^2}{M_k} \ll m^l_k.
\]

(11)

where \( m^l_k \) is the mass of lepton or quark in \( k \)-family. The see-saw mechanism connects the smallness of neutrino masses with respect to the masses of all other fundamental fermions with a new physics at a large scale.

The fields \( \nu_{iR} \) do not enter into the Lagrangian of the standard electroweak interaction and are called sterile. The nature and the number of sterile fields depend on model. They can be not only singlet right-handed neutrino fields but also SUSY fields and so on. Thus, in the most general case for the mixing we have

\[
\nu_{iL} = \sum_{i=1}^{3+n_s} U_{iL} \nu_{iL},
\]

(12)

and

\[
\nu_{sL} = \sum_{i=1}^{3+n_s} U_{siL} \nu_{iL},
\]

(13)

where \( n_s \) is the number of sterile fields and \( U \) is a \((3+n_s) \times (3+n_s)\)

unitary matrix.
Neutrino oscillations

Neutrino oscillations is the most important consequence of neutrino mixing. Neutrino oscillations were first considered by B. Pontecorvo in 1957-58 [10]. It was very courageous conjecture, made at the time when two-component theory for massless neutrino was proposed [15] and it was established in the Goldhaber et al experiment [16] that neutrino is left-handed particle. Only electron neutrino was known at that time. B. Pontecorvo considered transition of $\nu_e$ into sterile left-handed state $\bar{\nu}_{eL}$.

If there is neutrino mixing and neutrino mass- squared differences are much smaller than square of neutrino energy, the normalized state of flavor (sterile) neutrino with momentum $\vec{p}$ is given by

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle.$$  \hspace{1cm} (14)

Here $|\nu_i\rangle$ is the vector of the state of neutrino with mass $m_i$, momentum $\vec{p}$, energy

$$E_i = \sqrt{\vec{p}^2 + m_i^2} \simeq \vec{p} + \frac{m_i^2}{2\vec{p}}$$ \hspace{1cm} (15)

and negative helicity (up to the terms $\frac{m_i^2}{\vec{p}^2}$)

Thus, if there is neutrino mixing, the state of flavor (sterile) neutrino is a superposition of states of neutrinos with definite masses. The phenomenon of neutrino oscillations is based on the relation (14). This relation is similar to the well known relations that connect the states of $K^0$ and $\bar{K}^0$ mesons with the states of $K_S$ and $K_L$ mesons, particles with definite masses and times of life.

Let us consider now the evolution in vacuum of a mixed state given by Eq. (14). If at the initial time $t=0$ the state of neutrino is $|\nu_\alpha\rangle$, at the time $t$ for the neutrino state we have

$$|\nu_\alpha\rangle_t = \sum U_{\alpha i}^* e^{-iE_i t} |\nu_i\rangle = \sum_{\alpha'} A_{\nu_\alpha \rightarrow \nu_{\alpha'}}(t) |\nu_{\alpha'}\rangle,$$  \hspace{1cm} (16)

where

$$A_{\nu_\alpha \rightarrow \nu_{\alpha'}}(t) = \sum_i U_{\alpha i} U_{\alpha' i}^* e^{-iE_i t}$$ \hspace{1cm} (17)

is the amplitude of the transition $\nu_\alpha \rightarrow \nu_{\alpha'}$ during the time $t$. 

5
The expression (17) for the transition amplitude has a simple meaning. The term \( U^*_\alpha i \) is the amplitude of the transition from the state \( |\alpha\rangle \) to the state \( |i\rangle \); the term \( e^{-iE_it} \) describes evolution in the state with energy \( E_i \); the term \( U_{\alpha' i} \) is the amplitude of the transition from the state \( |i\rangle \) to the state \( |\alpha'\rangle \).

It follows from (17) that transition between different neutrinos is an effect of neutrino masses differences and neutrino mixing. In fact, if all neutrino masses in (17) are the same and/or \( U = 1 \) in this case
\[
A_{\nu_\alpha \rightarrow \nu_{\alpha'}}(t) = e^{-iEt} \delta_{\alpha' \alpha}
\]
and there are no transitions between different types of neutrinos in a neutrino beam.

Let us enumerate neutrino masses in such a way that \( m_1 < m_2 < \ldots \). From (17) for the probability of the transition \( \nu_\alpha \rightarrow \nu_{\alpha'} \) in vacuum we have the following expression
\[
P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \left| \sum_i U_{\alpha' i} U^*_{\alpha i} e^{-i \Delta m^2_{i1} \frac{L}{2E}} \right|^2, \quad (18)
\]
where \( \Delta m^2_{i1} = m_i^2 - m_1^2 \) and \( L \simeq t \) is the distance between a neutrino source and a neutrino detector and \( E \) is neutrino energy.

The simplest case of neutrino oscillations is the oscillations between two types of neutrinos. In this case the index \( i \) in (18) takes values 1 and 2. Taking into account the unitarity relation
\[
U_{\alpha' 1} U^*_{\alpha 1} = \delta_{\alpha' \alpha} - U_{\alpha' 2} U^*_{\alpha 2} \quad (19)
\]
for the transition probability we have
\[
P(\nu_\alpha \rightarrow \nu_{\alpha'}) = |\delta_{\alpha' \alpha} + U_{\alpha' 2} U^*_{\alpha 2} (e^{-i\Delta m^2_{21} \frac{L}{2E}} - 1)|^2, \quad (20)
\]
where \( \Delta m^2_{21} \equiv \Delta m^2 \). The neutrino mixing matrix for the \( 2 \times 2 \) case has the following general form
\[
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix},
\quad (21)
\]
where \( \theta \) is the mixing angle.

For \( \alpha' \neq \alpha \) from (21) we have
\[
P(\nu_\alpha \rightarrow \nu'_{\alpha}) = \frac{1}{2} A_{\alpha' : \alpha} \left( 1 - \cos \Delta m^2 \frac{L}{2E} \right). \quad (22)
\]
Here

\[ A_{\alpha',\alpha} = 4|U_{\alpha'2}|^2|U_{\alpha2}|^2 = \sin^2 2\theta . \]  

(23)

For the survival probability from Eq. (20) we have

\[ P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \frac{1}{2} B_{\alpha;\alpha} \left( 1 - \cos \frac{\Delta m^2 L}{2E} \right) , \]  

(24)

where

\[ B_{\alpha;\alpha} = 4|U_{\alpha2}|^2 \left( 1 - |U_{\alpha2}|^2 \right) = \sin^2 2\theta \]  

(25)

Thus, in the case of transitions between two types of neutrinos all transition probabilities are characterized by the two oscillation parameters \( \sin^2 2\theta \) and \( \Delta m^2 \). Notice that due to the unitarity of the mixing matrix in the two-neutrino case \( A_{\alpha',\alpha} = B_{\alpha;\alpha} \). The formulas (22) -(25) are the standard ones. They are usually used for an analysis of experimental data.

From (22) and (24) it is obvious that neutrino oscillations can be observed if the condition

\[ \frac{\Delta m^2 L}{2E} \gtrsim 1 \]  

(26)

is satisfied. We can rewrite this condition in the form

\[ \frac{\Delta m^2 (eV^2) L(m)}{E(MeV)} \gtrsim 1 \]  

(27)

The values of the parameter \( \frac{L}{E} \) depend on conditions of an experiment. The larger \( \frac{L}{E} \) the more sensitive a neutrino oscillation experiment to the presumably small values of \( \Delta m^2 \). The average values of the parameter \( \frac{L}{E} \) for experiments with accelerator neutrinos, reactor antineutrinos, atmospheric neutrinos and solar neutrinos are, correspondingly, in the ranges \( 10^{-1} - 10^3 \) , \( 10^2 - 10^3 \), \( 10 - 10^4 \) and \( 10^{10} - 10^{11} \). Thus, neutrino oscillation experiments are sensitive to the parameter \( \Delta m^2 \) in a wide range from \( \Delta m^2 \approx 10^{-11} eV^2 \) to \( \Delta m^2 \approx 10^{-11} eV^2 \).

3 Neutrino oscillation experiments

There exist at present compelling evidence in favor of neutrino oscillations that were obtained in all solar and atmospheric neutrino experiments. We will discuss here only the latest results of the Super-Kamiokande [7] and
SNO [8] solar neutrino experiments and the result of the Super-Kamiokande atmospheric neutrino experiment [1].

3.1 Solar neutrinos

The energy of the sun is generated in the reactions of the thermonuclear pp and CNO cycles. From thermodynamical point of view the energy of the sun is produced in the transition

\[ 4p + 2e^- \rightarrow 4He + 2\nu_e, \]  

in which the energy

\[ Q = 4m_p + 2m_e - m_{4He} \simeq 26.7\,\text{MeV} \]  

is released. From (28) we can easily obtain a model independent relation

\[ \int \frac{1}{2}(Q - 2E) \Phi_{\nu_e}^{tot}(E) dE = \frac{L_\odot}{4\pi R^2}, \]  

which connect the luminosity of the sun \( L_\odot \) with the initial total flux of the solar electron neutrinos

\[ \Phi_{\nu_e}^{tot}(E) = \sum_i \Phi_{\nu_e}^i(E), \]  

where \( \Phi_{\nu_e}^i \) is the flux of \( \nu_e \) from the source \( i \). In Eq. (30) \( R \) is the distance between the sun and the earth.

The main source of solar neutrinos is pp reaction

\[ p + p \rightarrow d + e^- + \nu_e. \]  

Low energy neutrinos with the maximum neutrino energy 0.42 MeV is produced in this reaction. The total flux of the pp neutrinos, predicted by the Standard Solar Model BP2000(SSM) [17] and determined mainly by the luminosity relation (30), is equal to \( \Phi_{\nu_e}^{pp} = 5.94 \cdot 10^{10}\,\text{cm}^{-2}\,\text{s}^{-1} \).

The next important source is the reaction

\[ e^- + ^7Be \rightarrow ^7\text{Li} + \nu_e. \]  

In this reaction monochromatic neutrinos with energy 0.86 MeV are produced. The flux of \( ^7\text{Be} \) neutrinos, predicted by the SSM, is given by \( \Phi_{\nu_e}^{^7Be} = 4.8 \cdot 10^9\,\text{cm}^{-2}\,\text{s}^{-1} \).
In the Super-Kamiokande (S-K) and the SNO experiments because of high energy thresholds mainly high energy neutrinos from the decay

$$^{8}B \rightarrow ^{8}Be^{*} + e^{+} + \nu_{e}.$$  \hspace{1cm} (34)

are detected. The maximum energy of the $^{8}B$ neutrinos is equal to 15 MeV and the flux, predicted by the SSM, is given by $\Phi_{\nu_{e}}^{^{8}B} = 5.1 \cdot 10^{6} cm^{-2} s^{-1}$.

In the S-K experiment large water Cherenkov detector is used (50 ktons of H$_{2}$O). The solar neutrinos are detected by the observation of the elastic (ES) neutrino-electron scattering

$$\nu_{x} + e \rightarrow \nu_{x} + e.$$  \hspace{1cm} (35)

All flavor neutrinos $\nu_{e}$, $\nu_{\mu}$ and $\nu_{\tau}$ are detected in the S-K experiment. However, the sensitivity to $\nu_{\mu}$ and $\nu_{\tau}$ is much lower than sensitivity to $\nu_{e}$: the cross section of NC $\nu_{\mu}$ ($\nu_{\tau}$) - $e$ scattering is about six times smaller than the cross section of CC+NC $\nu_{e} - e$ scattering.

During 1258 days of running, in the S-K experiment $18464 \pm 677 - 590$ events with energy of the recoil electrons larger than 5 MeV was observed.

The total ES event rate is given by

$$R^{ES} = <\sigma_{\nu_{e}e}> \Phi_{\nu_{e}}^{ES},$$  \hspace{1cm} (36)

where

$$\Phi_{\nu_{e}}^{ES} = \Phi_{\nu_{e}}^{ES} + <\sigma_{\nu_{\mu}e}> <\sigma_{\nu_{e}e}> \Phi_{\nu_{\mu},\tau}^{ES}.$$  \hspace{1cm} (37)

Here $\Phi_{\nu_{e}}^{ES}$ ($\Phi_{\nu_{\mu},\tau}^{ES}$) is the flux of solar $\nu_{e}$ ($\nu_{\mu,\tau}$) on the earth and $<\sigma_{\nu_{e}e}>$ and $<\sigma_{\nu_{\mu}e}>$ are the cross sections of the processes $\nu_{e}e \rightarrow \nu_{e}e$ and $\nu_{\mu}e \rightarrow \nu_{\mu}e$, averaged over initial spectrum of $^{8}B$ neutrinos. Notice that the spectrum of neutrinos from the decay \cite{4} is determined by the weak interaction and is known. We have

$$<\sigma_{\nu_{\mu}e}> \approx 0.154.$$  \hspace{1cm} (38)

From the data of the S-K experiment it was obtained

$$(\Phi_{\nu_{e}}^{ES})_{SK} = (2.32 \pm 0.03 \pm 0.08) \cdot 10^{6} cm^{-2} s^{-1},$$  \hspace{1cm} (39)

The S-K data alone do not allow to obtain an information separately on the fluxes of $\nu_{e}$ and $\nu_{\mu,\tau}$ on the earth. It became possible only after the data of the SNO experiment \cite{8} appeared.
In the SNO experiment heavy water Cherenkov detector is used (1 kton of D$_2$O). Solar neutrinos were detected in the experiment via the observation of CC reaction
\[ \nu_e + d \to e^- + p + p \] (40)
and elastic scattering (ES) reaction
\[ \nu_x + e \to \nu_x + e. \] (41)

The electron kinetic energy threshold in the experiment was 6.75 MeV. From November 1999 till January 2001 it was observed 975.4±39.7 CC events and 106.1±15.2 ES events. For the effective flux of ES events it was obtained the value
\[ (\Phi^{ES}_{\nu_x})_{SNO} = (2.39 \pm 0.34 \pm 0.16) \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1}, \] (42)
which is in agreement with the S-K value (39).

The CC events rate is given by
\[ R^{CC} = \langle \sigma_{\nu_e d} \rangle \Phi^{CC}_{\nu_e}, \] (43)
where \( \langle \sigma_{\nu_e d} \rangle \) is the cross section of the process (40) averaged over initial spectrum of the $^8$B neutrinos and \( \Phi^{CC}_{\nu_e} \) is the flux of $\nu_e$ on the earth. In the SNO experiment it was found
\[ \Phi^{CC}_{\nu_e} = (1.75 \pm 0.07 \pm 0.12 \pm 0.05(\text{theor})) \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1}. \] (44)

Let us compare now (39) and (44). For the fluxes of $\nu_e$ on the earth in Eq. (37) and Eq.(43) we have, respectively
\[ \Phi^{CC}_{\nu_e} = \langle P(\nu_e \to \nu_e) \rangle_{CC} \Phi^0_{\nu_e}; \quad \Phi^{ES}_{\nu_e} = \langle P(\nu_e \to \nu_e) \rangle_{ES} \Phi^0_{\nu_e}, \] (45)
where \( \Phi^0_{\nu_e} \) is the total initial flux of the $^8$B neutrinos.

If the $\nu_e$ survival probability depends on energy, the average probabilities \( \langle P(\nu_e \to \nu_e) \rangle_{CC} \) and \( \langle P(\nu_e \to \nu_e) \rangle_{ES} \) are in principle different. However, no indications of significant energy dependence of the $\nu_e$ survival probability in the S-K and SNO energy ranges were obtained. In fact, in both

\[ \text{It was shown in [13] that it is possible to choose the S-K and the SNO thresholds in such a way that these quantities will be practically equal at any } P(\nu_e \to \nu_e). \]
experiments spectra of electrons were measured. If survival probability is a constant the shapes of the spectra can be predicted in a model independent way. No sizable deviations from the predicted spectra were found in both experiments. Thus, we have

$$\Phi^{CC}_{\nu_e} \approx \Phi^{ES}_{\nu_e}.$$  \hspace{1cm} (46)

Taking into account this relation from (37), (39) and (44) for the flux of $\nu_\mu$ and $\nu_\tau$ on the earth we obtain

$$\Phi^{ES}_{\nu_\mu,\tau} = (3.69 \pm 1.13) \cdot 10^6 \text{cm}^{-2} \text{s}^{-1}$$  \hspace{1cm} (47)

Thus, the results of the SNO and the S-K experiments give us the first model independent evidence (at $\simeq 3\sigma$ level) of the presence of $\nu_\mu$ and $\nu_\tau$ in the flux of solar neutrinos on the earth. The flux of $\nu_\mu$ and $\nu_\tau$ is approximately two times larger than the flux $\nu_e$.

From (44) and (17) for the total flux of all flavor neutrinos on the earth we have

$$\Phi^{\nu_e,\mu,\tau}_{\nu_e} = (5.44 \pm 0.99) \cdot 10^6 \text{cm}^{-2} \text{s}^{-1}$$  \hspace{1cm} (48)

This value is in agreement with the total flux of $^8B$ neutrinos

$$\Phi^{SSM}_{\nu_e} = 5.05 \cdot 10^6 \text{cm}^{-2} \text{s}^{-1},$$  \hspace{1cm} (49)

predicted by the SSM BP 2000 [17].

The data of all solar neutrino experiments can be described if we assume that two-neutrino oscillations, which are characterized by the two parameters $\Delta m^2_{\text{sol}}$ and $\tan^2 \theta_{\text{sol}}$, take place. From the analysis of all existing data, made under the assumption that initial fluxes are given by the SSM, the several allowed regions (solutions) in the plane of these parameters were obtained. After the new S-K and SNO data were obtained the large mixing angle MSW allowed regions (LMA, LOW) became the preferable ones (see [18, 20]) For the best-fit values of the oscillation parameters in the LMA region in ref. [19] it was found

$$\Delta m^2_{\text{sol}} = 4.5 \cdot 10^{-5} \text{eV}^2; \quad \tan^2 \theta_{\text{sol}} = 4.1 \cdot 10^{-1}.$$  \hspace{1cm} (50)
3.2 Atmospheric neutrinos

The decays of charged pions

$$\pi^+ \to \mu^+ + \nu_\mu, \quad \pi^- \to \mu^- + \bar{\nu}_\mu,$$

produced in interaction of cosmic rays with the atmosphere, and subsequent decays of muons

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu, \quad \mu^- \to e^- + \bar{\nu}_e + \nu_\mu,$$

are the main source of the atmospheric neutrinos.

In the Super-Kamiokande experiment muons and electrons, produced in interaction of the atmospheric $\nu_\mu$ and $\nu_e$ with nuclei, are detected in the the large water Cherenkov detector. The first compelling evidence in favor of neutrino oscillations was obtained by the S-K collaboration in 1998.

For the high-energy neutrinos the distance $L$ between the region where neutrinos are produced and the detector is determined by the zenith angle $\theta_z$.

Down-going neutrinos ($\cos \theta_z = 1$) pass the distance about 20 km. Up-going neutrinos ($\cos \theta_z = -1$) pass the distance about 13000 km. If there is no neutrino oscillations for the number of muon (electron) events we have

$$N_l(\cos \theta_z) = N_l(-\cos \theta_z) \quad (l = e, \mu).$$

The S-K collaboration measured $\cos \theta_z$ dependence of the number of electron and muon events. For the electron events no $\cos \theta_z$ asymmetry was observed. The data are in a good agreement with the Monte Carlo prediction, obtained under the assumption of no oscillations. For the muon neutrinos in the Multi-GeV region (neutrinos with energies larger than 1.3 GeV) strong $\cos \theta_z$ asymmetry was observed. For the ratio of the total numbers of up-going and down-going high-energy muons it was found

$$\left( \frac{U}{D} \right)_\mu = 0.54 \pm 0.04 \pm 0.01$$

The data of the S-K atmospheric neutrino experiment can be described if we assume that $\nu_\mu \to \nu_\tau$ oscillations take place. From the data of the S-K experiment the following best-fit values of the oscillation parameters were found

$$\Delta m^2_{atm} \simeq 2.5 \cdot 10^{-3} \text{eV}^2; \quad \sin^2 2 \theta_{atm} \simeq 1$$

$$21$$

$12$
4 Oscillations of solar and atmospheric neutrinos from the point of view of three-neutrino mixing

The data of all solar and atmospheric neutrino oscillation experiments are described by the two-neutrino oscillations. We will discuss here the origin of such a picture in the framework of the minimal scheme of the mixing of three massive neutrinos (see, for example [13]).

The probability of the transition $\nu_\alpha \rightarrow \nu_{\alpha'}$ in vacuum is given by the general expression (18). Taking into account (50) and (55), we will assume that the following hierarchy relation holds

$$\Delta m_{21}^2 \ll \Delta m_{31}^2. \quad (56)$$

Let us consider first neutrino oscillations in atmospheric and long baseline (LBL) reactor and accelerator experiments. In these experiments

$$\Delta m_{21}^2 \frac{L}{2E} \ll 1 \quad (57)$$

and we can neglect the contribution of $\Delta m_{21}^2$ to the transition probability (18).

Taking into account the unitarity relation

$$\sum_{i=1,2} U_{\alpha'i} U_{\alpha'i}^* = \delta_{\alpha\alpha'} - U_{\alpha'3} U_{\alpha3}^* \quad (58)$$

for the transition probability in the leading approximation we obtain the following relation

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) \simeq \left| \delta_{\alpha\alpha'} + U_{\alpha'3} U_{\alpha3}^* \left( e^{-i\Delta m_{31}^2 \frac{L}{2E}} - 1 \right) \right|^2. \quad (59)$$

Thus, if inequality (57) is satisfied, the probabilities of transition $\nu_\alpha \rightarrow \nu_{\alpha'}$ in the atmospheric (LBL) experiments are determined by $\Delta m_{31}^2 \equiv \Delta m^2_{atm}$ and the elements of the mixing matrix $U_{\alpha3}$, which connect flavor neutrinos with the heaviest neutrino $\nu_3$.

For $\alpha' \neq \alpha$ from (59) we have

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \frac{1}{2} A_{\alpha'\alpha} \left( 1 - \cos \Delta m_{31}^2 \frac{L}{2E} \right), \quad (60)$$
where
\[ A_{\alpha'\alpha} = 4|U_{\alpha'3}|^2|U_{\alpha3}|^2. \] (61)

For the survival probability from Eq. (59) we obtain
\[ P(\nu_\alpha \rightarrow \nu_\alpha) \simeq 1 - \frac{1}{2}B_{\alpha;\alpha} \left( 1 - \cos \frac{\Delta m^2_{\odot}}{2E} \right), \] (62)

where
\[ B_{\alpha;\alpha} = 4 \left| U_{\alpha3} \right|^2 \left( 1 - \left| U_{\alpha3} \right|^2 \right). \] (63)

Thus, due to the hierarchy (56) oscillations of atmospheric (LBL) neutrinos are described by the two-neutrino type formulas with the same \( \Delta m^2_{\odot} \) for all channels. The quantities \( A_{\alpha'\alpha} \) and \( B_{\alpha;\alpha} \) are oscillation amplitudes. From the unitarity of the mixing matrix it follows that they are connected by the relation
\[ \sum_{\alpha \neq \alpha'} A_{\alpha'\alpha} = B_{\alpha;\alpha} \] (64)

and satisfy the inequalities
\[ 0 \leq B_{\alpha;\alpha} \leq 1; \quad 0 \leq A_{\alpha'\alpha} \leq 1 \] (65)

The oscillation amplitudes depend on the two parameters \( |U_{\mu3}|^2 \) and \( |U_{\tau3}|^2 \) (due to the unitarity of the mixing matrix \( |U_{e3}|^2 = 1 - |U_{\mu3}|^2 - |U_{\tau3}|^2 \)).

It is important to stress that the phase of the matrix elements \( U_{\alpha3} \) does not enter into expression (60) for the transition probability. Thus, if there is hierarchy (56), the relation
\[ P(\nu_\alpha \rightarrow \nu_{\alpha'}) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}) \] (66)
is satisfied automatically and (in the leading approximation) CP violation in the lepton sector can not be revealed by the investigation of neutrino oscillations in LBL (atmospheric) neutrino experiments.

The hierarchy of neutrino mass squared differences (56) is the reason why in the leading approximation the results of the atmospheric and the LBL neutrino oscillation experiments can be described by the standard two-neutrino formulas.
Let us consider now solar neutrinos. The probability of solar $\nu_e$ to survive in vacuum is given by the expression

$$ P_{\text{sol}}(\nu_e \rightarrow \nu_e) = \left| \sum_{i=1,2} |U_{ei}|^2 e^{-i \Delta m_{i1}^2 \frac{L}{E}} + |U_{e3}|^2 e^{-i \Delta m_{31}^2 \frac{L}{E}} \right|^2. $$

(67)

We are interested in the survival probability averaged over the region, where neutrinos are produced, over neutrino energies etc. Because of the hierarchy \[\text{(56)}\] the interference between the first and the second term in (67) disappears due to averaging and for the averaged survival probability we have

$$ P_{\text{sol}}(\nu_e \rightarrow \nu_e) = |U_{e3}|^4 + (1 - |U_{e3}|^2)^2 P^{1,2}(\nu_e \rightarrow \nu_e), $$

(68)

where $P^{1,2}(\nu_e \rightarrow \nu_e)$ is two-neutrino survival probability which depend on $\Delta m_{21}^2$ and the angle $\theta_{12}$ that is determined by the relations

$$ \cos^2 \theta_{21} = \frac{|U_{e1}|^2}{\sum_{i=1,2} |U_{ei}|^2}, \quad \sin^2 \theta_{21} = \frac{|U_{e2}|^2}{\sum_{i=1,2} |U_{ei}|^2}. $$

(69)

It was shown \[\text{(56)}\] that the relation (68) is valid also in the case of matter. In this case the electron density $\rho_e$ in the effective matter potential must be replaced by $(1 - |U_{e3}|^2)\rho_e$.

From data of the long baseline reactor experiments CHOOZ \[\text{(23)}\] and Palo Verde \[\text{(24)}\] and from the data of the S-K atmospheric neutrino experiment \[\text{(1)}\] it follows that the element $|U_{e3}|^2$ is small. The best limit on $|U_{e3}|^2$ can be obtained from the results of the CHOOZ experiment. In this experiment $\bar{\nu}_e$'s from two reactors at the distance of about 1 km from the detector were detected. No indications in favor neutrino oscillations were found.

The data of the CHOOZ experiment were analyzed in ref. \[\text{(23)}\] under the assumption of two-neutrino oscillations and the exclusion plot in the plane of the parameters $\sin^2 2\theta \equiv B_{e,e}$ and $\Delta m^2 \equiv \Delta m_{31}^2$ was obtained. From this plot for a fixed value of $\Delta m_{31}^2$ we have

$$ B_{e,e} \leq B_{e,e}^0(\Delta m_{31}^2), $$

(70)

From (71) and (70) it follows that

$$ |U_{e3}|^2 \leq \frac{1}{2} \left( 1 - \sqrt{1 - B_{e,e}^0} \right). $$

(71)
or

$$|U_{e3}|^2 \geq \frac{1}{2} \left( 1 + \sqrt{1 - B_{e,e}^0} \right). \quad (72)$$

From the CHOOZ exclusion plot we can conclude that in the region

$$\Delta m_{31}^2 \geq 2 \cdot 10^{-3} \text{eV}^2, \; B_{e,e}^0 \leq 2 \cdot 10^{-2}.$$  If the value of $\Delta m_{31}^2$ lies in this region from (71) and (72) it follows that the element $|U_{e3}|^2$ can be small (inequality (71)) or large (inequality (72)).

This last possibility is excluded by the solar neutrino data. In fact, if $|U_{e3}|^2$ is close to one, than from Eq. (68) it is obvious that the suppression of the flux of solar $\nu_e$, observed in all solar neutrino experiments, cannot be explained by neutrino oscillations. Thus, from the results of the CHOOZ and solar neutrino experiments it follows that the upper bound of $|U_{e3}|^2$ is given by inequality (71).

At $\Delta m_{31}^2 = 2.5 \cdot 10^{-3} \text{eV}^2$ (the S-K best-fit value) we have

$$|U_{e3}|^2 \lesssim 4 \cdot 10^{-2}. \quad (73)$$

The smallness of $|U_{e3}|^2$ is the reason why in the leading approximation oscillations of solar neutrinos are described by the standard two-neutrino formula.

In the limiting case $|U_{e3}|^2 = 0$ oscillations of solar and atmospheric (LBL) neutrinos are decoupled [25]. In this approximation solar neutrino experiments allow to obtain information on the values of the parameters $\Delta m_{21}^2$ and $\theta_{12}$, that characterize oscillations $\nu_e \to \nu_{\mu,\tau}$ and the atmospheric (LBL) experiments allow to obtain information on the values of the parameters $\Delta m_{31}^2$ and $\theta_{23}$, which characterize oscillations $\nu_{\mu} \to \nu_{\tau}$.

There is, however, no general theoretical reasons for $|U_{e3}|^2$ to be equal to zero. The exact value of the parameter $|U_{e3}|^2$ is of a great interest for further investigation of neutrino mixing. If $|U_{e3}|^2$ has a nonzero value and the parameter $\Delta m_{21}^2$ is not very small there is a possibility to investigate effects of three-neutrino mixing and, in particular, fundamental effects of CP-violation in the lepton sector in the future LBL experiments with neutrinos from the Neutrino factories and the Superbeam facilities (see [26, 27] and references therein).

## 5 Conclusions

About 40 years passed from the first idea of neutrino oscillations, put forward by B. Pontecorvo in 1957-58, to the evidences for neutrino oscillations,
obtained in the atmospheric and solar experiments. The first idea of neutrino masses and mixing was based on an analogy with $K^0 - \bar{K}^0$ mixing and on the fact that there is no general principle (like gauge invariance in the case of photon) that oblige neutrino to be massless particle. In seventies neutrino mixing was considered as a natural consequence of the analogy between quarks and leptons. After the appearance of GUT and other models beyond the Standard Model and after the see-saw mechanism of neutrino mass generation was proposed neutrino masses and mixing are considered as a signature of a new physics at a scale much larger than the electroweak scale.

Today we have not only evidence in favor of neutrino oscillations but also an information about the values of parameters, which characterize neutrino oscillations. With many new ongoing and future experiments ((K2K) [28], KamLAND [29], BOREXINO [30] MINOS [31], CNGS [32] and other) evidence in favor of neutrino oscillations most probably will be more vigorous and neutrino oscillations parameters will be determined with much better accuracy than today.

There exist, however, several unsolved basic problems of neutrino masses and mixing. From our point of view they are

1. How many massive light neutrinos exist in nature?
2. Are massive neutrinos Dirac or Majorana particles?
3. What is the value of the minimal neutrino mass $m_1$?

An answer to the first question probably will be obtained in the MiniBooNE experiment [33], which will check the LSND claim.

An answer to the second question can be obtained from future experiments on the search for neutrinoless double-$\beta$ decay (see [35]).

Finally, we can hope to get some answer on the third question from the future experiment on the investigation of the high-energy part of the $\beta$ spectrum of $^3H$ [34].

Existing solar and atmospheric neutrino data are well described by practically decoupled $\nu_e \rightarrow \nu_{\mu,\tau}$ and $\nu_\mu \rightarrow \nu_\tau$ oscillations. Detailed investigation of effects of three (or may be more?) neutrino masses and mixing and in particular effects of CP-violation in the lepton sector will require such high-intensity neutrino facilities as the Superbeam facilities and the Neutrino factories (see [23, 27]).

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References

[1] Super-Kamiokande Collaboration, S. Fukuda et al., Phys. Rev. Lett. 81, 1562 (1998); S. Fukuda et al., Phys. Rev. Lett. 82, 2644 (1999); S. Fukuda et al., Phys. Rev. Lett. 85, 3999-4003 (2000).

[2] B. T. Cleveland et al., Astrophys. J. 496, 505 (1998).

[3] Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 77, 1683 (1996).

[4] GALLEX Collaboration, W. Hampel et al., Phys. Lett. B 447, 127 (1999).

[5] GNO Collaboration, M. Altmann et al., Phys. Lett. B 490, 16 (2000).

[6] SAGE Collaboration, J. N. Abdurashitov et al., Phys. Rev. C 60, 055801 (1999).

[7] Super-Kamiokande Collaboration, S. Fukuda et al., Phys. Rev. Lett. 86, 5651 (2001).

[8] SNO collaboration Q.R. Ahmad et al., Phys. Rev. Lett. 87, 071301 (2001).

[9] LSND Collaboration, G. Mills, Proceedings of the 19th International Conference on Neutrino Physics and Astrophysics, Neutrino 2000 (Sudbury, Canada, June 16-21, 2000).

[10] B. Pontecorvo, J. Exptl. Theoret. Phys. 34, 247 (1958) [Sov. Phys. JETP 7, 172 (1958)]; B. Pontecorvo, Zh. Eksp. Teor. Fiz. 53, 1717 (1967) [Sov. Phys. JETP 26, 984 (1968)].

[11] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. 28, 870 (1962).

[12] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. 59, 671 (1987).

[13] S.M. Bilenky, C. Giunti and W. Grimus. Prog. Part. Nucl. Phys. 43, 1 (1999); hep-ph/9812360.
[14] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, p. 315, edited by F. van Nieuwenhuizen and D. Freedman, North Holland, Amsterdam, 1979; T. Yanagida, Proc. of the *Workshop on Unified Theory and the Baryon Number of the Universe*, KEK, Japan, 1979; R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[15] L. Landau, Nucl. Phys. **3**, 127 (1957); T.D. Lee and C.N. Yang, Phys. Rev. **105**, 1671 (1957); A. Salam, Il Nuovo Cim. **5**, 299 (1957).

[16] M. Goldhaber, L. Grodzins and A.W. Sunyar, Phys. Rev. **109**, 1015 (1958).

[17] J. N. Bahcall, M. H. Pinsonneault, and S. Basu, Astrophys. J. **555**, 990 (2001).

[18] F. L. Villante, G. Fiorentini, and E. Lisi, Phys. Rev. D **59**, 013006 (1999).

[19] J. N. Bahcall, M. C. Gonzalez-Carcia, and C.Pena-Garay, [hep-ph/0106258](http://arxiv.org/abs/hep-ph/0106258).

[20] G. L. Fogli, E. Lisi, D. Montanino, and A. Palazzo, [hep-ph/0106247](http://arxiv.org/abs/hep-ph/0106247).

[21] Super-Kamiokande Collaboration, C. McGrew, Proceeding of the IXth International Workshop on ”Neutrino Telescopes” (Venice, Italy, March 6-9, 2001). Phys. Lett. B **283**, 305 (1992).

[22] S.T. Petcov, Phys. Lett. B**214** (1988) 259; X. Shi and D.N. Schramm, Phys. Lett. B **283**, 305 (1992).

[23] CHOOZ Collaboration, M. Apollonio *et al.*, Phys. Lett. B**338**, 383 (1998); M. Apollonio *et al.*, Phys. Lett. B**466** (1999) 415.

[24] F. Boehm, J. *et al.*, Phys. Rev. Lett. **84**, 3764 (2000); Phys. Rev. D**62** (2000) 072002.

[25] S.M. Bilenky and C. Giunti, Phys. Lett. B**444** (1998) 379.

[26] C. Albright *et al.* ”Physics at a Neutrino Factory” [hep-ex/0008064](http://arxiv.org/abs/hep-ex/0008064); A. Blondel *et al.* ”The Neutrino factory: beam and experiments”,

19
[27] V. Barger et al., hep-ph/0103052; J. Gomes Cadenas, Proceedings of
the 9th International Workshop "Neutrino Telescopes" (Venice, March
6-9, 2001).

[28] K2K Collaboration, K. Nakamura, Proceedings of the 19th International
Conference on Neutrino Physics and Astrophysics, Neutrino 2000
(Sudbury, Canada, June 16-21, 2000).

[29] KamLAND Collaboration, A. Piepke, Proceedings of the 19th Interna-
tional Conference on Neutrino Physics and Astrophysics, Neutrino 2000
(Sudbury, Canada, June 16-21, 2000).

[30] Borexino Collaboration, G. Ranucci, Proceedings of the 19th Interna-
tional Conference on Neutrino Physics and Astrophysics, Neutrino 2000
(Sudbury, Canada, June 16-21, 2000).

[31] MINOS Collaboration, S. Wojcicki, Proceedings of the 19th Interna-
tional Conference on Neutrino Physics and Astrophysics, Neutrino 2000
(Sudbury, Canada, June 16-21, 2000).

[32] A. Ereditato, Proceedings of the 19th International Conference on Neut-
trino Physics and Astrophysics, Neutrino 2000 (Sudbury, Canada, June
16-21, 2000).

[33] MiniBooNE Collaboration, report by A. Bazarko, Proceedings of the 19th
International Conference on Neutrino Physics and Astrophysics, Neutrino 2000
(Sudbury, Canada, June 16-21, 2000).

[34] KATRIN Collaboration, hep-ex/0109033.

[35] H.V. Klapdor-Kleingrothaus, Cambridge Monogr. Part. Phys. Nucl.
Phys. Cosmol. 14,113 (2000).