Search for Kilogram-scale Dark Matter with Precision Displacement Sensors

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Search for dark matter has been performed mainly for weakly-interacting massive particles and massive compact halo objects, and intermediate mass region has not been investigated experimentally. A method to search dark matter with precision displacement sensors is suggested for this mass range. The search is performed by detecting a characteristic motion of a test mass when it is attracted by a dark matter particle through gravity. Two different types of displacement sensors are examined: optically levitated microspheres and laser interferometers for gravitational wave detection. The state-of-the-art detectors' sensitivity is several orders of magnitude lower to put constraints on dark matter particles. Among the two types of detectors, gravitational wave detectors have higher sensitivities, and a sensitivity 10 times more than the next generation detector can potentially address the existence of dark matter particles of a few kilograms.

I. INTRODUCTION

There have been multiple independent astronomical observations that established the existence of dark matter (DM) \[^1\] \[^2\], and different candidates of DM have been searched intensively: massive halo objects (MACHOs) \[^3\], and weakly interacting massive particles (WIMPs) \[^4\] \[^5\], as well as even lighter particles such as axions and axion-like particles (ALPs) \[^6\] \[^7\]. So far, there is no convincing discovery of the constituent of the DM, and experiments and observations mainly set more and more stringent constraints on parameter spaces, as the sensitivity of detectors increases.

Search methods are different according to the mass and the characteristics of DM candidates. MACHOs, which have a mass range around solar mass (\(\sim 10^{30}\) kg), is searched by astronomical observations using gravitational lensing \[^4\]. The detection of WIMPs with the mass around 1–100 GeV/c\(^2\) (\(10^{-27}–10^{-25}\) kg) is performed by detecting recoils of nuclei by the scattering with the DM particle \[^4\] \[^8\]. Mass of the axions and ALPs is typically assumed to be a few GeV or less, and these are typically detected by conversion from photons at accelerator beam dumps for relatively massive case. Light mass ones (\(\lesssim 1\) eV) are typically searched through the conversion to photons by magnetic field. \[^10\] \[^11\]. The intermediate mass scale, which is between \(10^{-25}\) kg and \(10^{30}\) kg, has not been intensively searched. This mass range includes various interesting particles and objects, such as grand unification theory scale \((1.78 \times 10^{-11}\) kg), Planck mass \((2.2 \times 10^{-8}\) kg), and primordial black holes \((10^{13}–10^{33}\) kg) \[^12\]. In this paper, a new method to search DM particles of this intermediate range is suggested. A precision displacement sensor works as a detector to observe a motion of a test mass, which can move by an attraction by DM particles, and two different kind of displacement sensors are analyzed: optically levitated spheres, and gravitational wave detectors.

II. DISPLACEMENT OF A FREE TEST MASS BY A DARK MATTER PARTICLE

To think of how a test mass behaves when a DM particle interacts with it, we start from an analysis of a simple system consisting of a test mass and a DM particle. The DM particle, whose mass is \(m\), is assumed to be a point particle, or a particle of a size significantly smaller than its impact parameter \(b\). The DM particle moves at a velocity of \(v_0\) from infinitely distant place towards the test mass at rest, with an impact parameter \(b\). The spherical test mass has mass \(M\) and radius \(r_0\), whose displacement is measured by a displacement sensor. The test mass is trapped around the origin by a harmonic trap and the finite size of the test mass are discussed later. The DM particle and the test mass interacts only through the Newtonian gravity following \(F(t) = GMm r(t)/r^3(t)\), where \(r(t) = r_{DM}(t) - r_{det}(t)\), and \(r_{DM}(t)\) and \(r_{det}(t)\) are the position of the DM particle and the test mass at time \(t\), and \(G\) is the gravitational constant.

The motion of two bodies interacting by a central force is well analyzed \[^14\]\[^1\] \[^1\], and the analytical solution of the displacement \(r(t) = (x(t), y(t))\) at the center of mass frame is parametrized by \(\xi\) in the following form:

\[
\begin{align*}
x &= a (\epsilon - \cosh \xi) \\
y &= a \sqrt{\epsilon^2 - 1} \sinh \xi \\
t &= \sqrt{\frac{\mu \alpha^3}{\alpha}} (\epsilon \sinh \xi - \xi),
\end{align*}
\]

where \(\mu = mM/(m + M)\) is the reduced mass, \(\alpha = GMm,\ a = \alpha/2E,\ \epsilon = \sqrt{1 + 2EL^2/\mu a^2}\) is the eccentricity of the trajectory, \(E = \mu v_0^2/2\) is the total energy of the system, and \(\mu = \mu_{inv} b\) is the total angular momentum of the system. Note that the x and y axes of the coordinate system are set in the plane of motion, and the directions of two axes are defined so as for the initial relative velocity to be \(v_0 = v_0/\epsilon(1, \sqrt{\epsilon^2 - 1})\). To get the position...
of the test mass in the laboratory frame \( \mathbf{r}_M = (x_M, y_M, z_M) \),
Galilei transformation of \( \mathbf{r}_M = \mathbf{r}_{CM} - (m/(M + m)) \mathbf{r} \) is applied, where \( \mathbf{r}_{CM} \) is the position of the origin of the center of mass frame. The position of the test mass in the laboratory frame is therefore described as
\[
\begin{align*}
x_M &= \frac{m}{m + M} (e^\xi - \epsilon - \frac{\xi}{\epsilon}) , \\
y_M &= -\frac{m}{m + M} \sqrt{\epsilon^2 - 1} a \xi , \\
t &= \frac{a}{v_0} (\epsilon \sinh \xi - \xi) ,
\end{align*}
\]
This satisfies the initial velocity of DM particle as \( v_0 = v_0/\epsilon (1, \sqrt{\epsilon^2 - 1}) \). The asymptotic behavior of \( \mathbf{v}_M = \partial \mathbf{r}_M/\partial t \) at \( t \to \pm \infty \) is
\[
\begin{align*}
\mathbf{v}_-\infty &= (0, 0) , \\
\mathbf{v}_+\infty &= \left( \frac{2m}{M + m} \epsilon , 0 \right) ,
\end{align*}
\]
which implies that the total momentum transfer \( \Delta \mathbf{p} = (\Delta p_x, \Delta p_y) \) from the DM particle to the test mass at the end of the collision is only in \( x \) direction, and \( \Delta p_x = 2mMv_0/(M + m) \epsilon \). In the case of \( \epsilon \gg 1 \), this is simplified to \( \Delta p_x = 2GMm/v_0b \), which is the quantity same as the momentum transfer calculated by an assumption that \( m \gg M \) and therefore the DM particle flies straight.

This motion can be detected in two different situations: (i) observing the relaxation of the test mass displacement in the harmonic trap after it receives a momentum kick of \( \Delta p_x \) (damped oscillation measurement). (ii) performing real-time detection while the test mass is being accelerated by the attraction from the DM particle (transient measurement). Damped oscillation measurement happens when a DM particle passes by the test mass so quickly that the collision process happens within the minimum time step \( \Delta t_{aq} \) of the displacement measurement of the test mass. This condition is \( \Delta t_{aq} > b/v_0 \) within a factor of \( O(1) \). The detection is performed through observing the damped oscillation of the test mass in the harmonic trap with the initial displacement of \( y_M = 0 \) and the velocity of \( v_{Mx0} = \Delta p_x/M \). The criterion for this to be detected is the maximum amplitude of this oscillation \( A \) is larger than a minimum displacement \( A_{\text{min}} \) that can be detected on top of the noise of the detector:
\[
A = \frac{Cv_{Mx0}}{\omega_0} = \frac{2Gm}{\omega_0 b v_0} \alpha \geq A_{\text{min}} ,
\]
where \( C \) is a numerical factor that is a function of \( \omega_0 \) and \( \gamma \). This results in a condition for the collision parameter \( b \) that signal can be detected when \( b < b_{\text{max}} = 2GMC/A_{\text{min}}v_0\omega_0 \). The total volume \( V \) that is scanned over by an observation for time \( t_{ob} \) is \( V = \pi b_{\text{max}}^2 v_0 t_{ob} \). When there are \( N \) signals of DM particles during the observation period, number density of the DM particle \( n \) is
\[
n = \frac{N}{V} = \frac{N A_{\text{min}}^2 v_0 \omega_0^2}{4 \pi t_{ob} G^2 m^2 C^2} ,
\]
\( N = 3.69 \) should be chosen to address the sensitivity with 95 % confidence level when there is no signal.

In the transient measurement, the motion of the test mass under an influence of a DM particle is monitored continuously. In the general case, it is difficult to predict analytically the motion of a test mass in a harmonic trap attracted by a DM particle. Here, for simplicity, we assume the time scale of the oscillation and the damping is significantly longer than that of the force on the test mass by a DM particle, which is justified when we consider gravitational wave detectors. This simplifies the situation to the observation of Eq. 2 As \( x_M \) has larger displacement than \( y_M, x_M \) is used for this analysis. The sensitivity of the detector is typically described by a power spectral density as a function of frequency of noises, and if the signal is larger than the noise at some frequency range, the signal can be detected. To estimate the sensitivity, the position of the test mass is Fourier transformed. This gives \( 1/f \) behavior, where \( f \) is frequency of the signal, and the largest \( b \) that makes the signal of the test mass displacement cross the noise spectral density of the detector gives \( b_{\text{max}} \). With the same argument as the damped oscillation measurement, the sensitivity is described as
\[
n = \frac{N}{\pi v_0 t_{ob} b_{\text{max}}^2} ,
\]
\( b_{\text{max}} \) is used for this analysis.

III. APPLICATION TO ACTUAL SYSTEMS

The calculation so far ignored certain aspects of actual detectors, such as finite size of the test mass and the detector. To consider them, application to two kinds of detectors are discussed: optically levitated microspheres \([15, 20]\) and interferometers for gravitational wave detection \([27, 29]\). The calculation in the idealized system is modified according to the properties of the detectors, and the sensitivity to the DM particle density is numerically derived.

For numerical calculations, the velocity of the DM particle is assumed to be \( v_0 = 2.2 \times 10^5 \) m/s \([30]\), and the density of DM \( \rho_{DM} = 0.39 \) GeV c\(^{-2}\) cm\(^{-3}\) \([31, 32]\). The number density \( n \) of DM particles of mass \( m \) is therefore
\[
n = \frac{\rho_{DM}}{m} = \frac{0.695 \times 10^{-21}}{m} \text{[kg]} \text{[m}^{-3} \text{]},
\]
with an assumption that all DM is made of the particle of mass \( m \).

A. Optically levitated microspheres

Optically levitated microspheres are used for force sensors, and their method to measure the force is to convert the displacement of a sphere into the force by using its mass and resonant frequency of a harmonic trap.
Thus, this system works as a precision displacement sensor of microspheres. The resonant frequency of the sphere ranges from a few hundred hertz \(16\) to a few kilohertz \(23\), and to trap the sphere stably, displacement feedback with a bandwidth of an order of magnitude larger than the resonant frequency is applied. Here, for simplicity, \(C = 1\) is assumed, which holds as far as quality factor \(Q\) of the resonance is much larger than 1. \(C\) decreases as \(Q\) decreases, but even when \(Q = 1\), \(C \approx 0.4\), which means even highly damped situation the difference in \(C\) is at most a factor of a couple. In fact, the system in \(16\) has \(Q \sim 1\) when the feedback cooling to reduce the noise is implemented. Also, it is possible to make a sequence such that for a certain amount of time the feedback cooling is turned on and off to alternate the high that for a certain amount of time the feedback cooling is implemented. Here, for simplicity, \(23\), and to trap the sphere stably, displacement feedback ranges from a few hundred hertz \(16\) to a few kilohertz \(s\) of microspheres. The resonant frequency of the sphere is assumed, as the size of vacuum chamber, inside of which only the last aspheric lens to tightly focus the trapping laser beam is located in an experiment in Ref. \(16\), is on the order of 0.1 m.

\[
n = \frac{N}{\pi v_0 t_{\text{ob}} r_D^2}
\]  

for \(b_{\text{max}} > r_D\). For the numerical calculation, \(r_D = 0.1\) m is assumed, as the size of vacuum chamber, inside of which only the last aspheric lens to tightly focus the trapping laser beam is located in an experiment in Ref. \(16\), is on the order of 0.1 m.

### B. Laser interferometers for gravitational wave detection

Laser interferometers for gravitational wave detection have had significant improvement in past decades. They have two arms each of which has an optical cavity to enhance the effective path length. The mirrors for the cavities serve as test masses. If a DM particle interacts with only a single mirror or have larger effect on one mirror than the other, the displacement of the mirror is recorded as a signal. Advanced LIGO \(27\), and Advanced VIRGO \(28\) are currently in operation, and KAGRA \(29\) is under construction. These detectors have similar sensitivities, and in this analysis, Advanced LIGO is used as a representative. The arm is 4 km long, and the most sensitive frequency range is 10-1000 Hz, which means both the transient measurement and the damped oscillation measurement is possible.

For the damped oscillation measurement, the highest resonant frequency is at 9 Hz \(23\), and the noise level at this resonant frequency sets \(A_{\text{min}}\) as \(5 \times 10^{-17}\) m/\(\sqrt{\text{Hz}}\) \(27\). The fact that this resonance is one of the two undamped resonant mode in the mirror suspension system makes it suitable for the damped oscillation measurement. It should be noted that specific data processing or analysis method might need to be developed for this DM search, and there might be decent amount of backgrounds that can be difficult distinguish signal by DM particles from, as this frequency is low end of the frequency that is paid attention to by the gravitational wave observations. Based on their mirror size, \(r_D \approx 0.1\) m. The detector size \(r_D\) is set as \(r_D = 2\) km, which is a half of the length of an interferometer arm. This is because if the impact parameter \(b_{\text{max}}\) is larger than this, the two circle of radius \(b_{\text{max}}\) centered on two cavity mirrors start to overlap, which results in the volume covered by two mirrors smaller than \(V = 2\pi b_{\text{max}}^2 v_0 t\). Another justification is that when the DM particle passes a few kilometers away outside of the cavity, the force on the two mirrors becomes close, and amount of signal is reduced. Additional factor to be considered in the case of gravitational wave detector is that it is primarily for the detector for one dimensional displacement, and therefore the sensitivity of the detector to the DM particle oscillates on daily basis according to the relative angle \(\theta\) between \(v_0\) and the sensitive direction.
of the detector. The reduction in the amount of motion of the test mass is a factor of $\cos \theta$, and time average of this is $\frac{1}{2\pi} \int_0^{2\pi} |\cos \theta| d\theta = 2/\pi$. Also, the fact that there are totally four mirrors in one interferometer needs to be taken into account. Each mirror can be regarded as a free test mass, and naively the enhancement by the four mirrors would be four. However, two of the four mirrors are reasonably close to the input optics, compared to the mirrors at the other end of 4 km long arm. Therefore, this factor should be three, because two mirrors on the input side are so close to each other that the circles of radius $b_{\text{max}}$ centered at the mirrors overlap with each other at large $b_{\text{max}}$. The argument that each mirror is viewed as a test mass implies that the sensitivity becomes higher proportionally to the total number of interferometers, assuming that each interferometer are farther apart than $b_{\text{max}}$.

The performance of the transient measurement is estimated by comparing the $f^{-1}$ curve and the sensitivity curve. The minimum signal curve that is tangent to the sensitivity curve is $3 \times 10^{-18}/f$, and therefore $b_{\text{max}}$ is given by $b$ that induces this amount of signal. The transient measurement also has the limitation due to the detector size $r_D$ and the test mass size $r_0$.

To see whether future experiments have any benefits for the DM search, Einstein Telescope (ET) is also analyzed, though it is only for the transient measurement. As far as analyzing the effect on the single mirror, important parameters are the same as LIGO case. The minimum $f^{-1}$ curve tangent to the sensitivity curve is assumed to be $1 \times 10^{-19}/f$, based on the sensitivity curve and the arm length of 10 km. Note that the sensitivity curve is different between ET-B and ET-C, and ET-C has a better sensitivity in low frequency region, which can lower the minimum $f^{-1}$ curve by a factor of $\sim 2$. It is assumed that $r_D$ equals 5 km, because of 10 km long arm, and $r_0 = 0.1$ m is assumed, as the size or the mirror is in the order of 10 cm.

C. Current and future sensitivities

The sensitivity to the number density of DM particles with the 95% confidence level is summarized in Fig. 1. Shaded region (UN Reno) is the currently available constraint based on Ref. 23. The fact that their force measurement graph averages down over $10^5$ s proportionally to the inverse of the square root of the measurement time means no extra feature in addition to the noise was observed, leading to the conclusion that they did not observe any DM particle.

Stanford curves are expected performance based on Ref. 18 with the measurement time of $10^7$ s. The solid line is sensitivity with the current performance of the detector, and the dotted line assumes an improved position sensitivity limited by the shot noise. When the measurement time is the same as UN Reno curve, the Stanford setup have four orders of magnitude higher sensitivity than UN Reno setup as long as the sensitivity is not limited by the detector size. This is simply because the Stanford setup has better position sensitivity. This trend is the same when microsphere setups and gravitational wave detectors are compared. Advanced LIGO can put nine orders of magnitude better constraint than shot noise limited performance of the Stanford system, mainly because it has a smaller amount of noise at the mechanical resonance compared to the microsphere setups. Still, the sensitivity by the damped oscillation detection is five orders of magnitude above the number density of DM particles.

As for the transient measurement, the sensitivity for a particle of the same mass is an order of magnitude higher than the resonant measurement, because the noise level relevant to the transient measurement is much better than that at the mechanical resonance at 9 Hz. Note that transient measurement has to detect smaller motion than overall amplitude at the resonant measurement, which prevents the sensitivity from being improved by the same amount of the noise ratio between two measurements. The sensitivity itself is still five orders of magnitude lower than the number density of the DM particles at $m = 5$ kg. This is consistent with the analysis in Ref. 37. In their analysis, 10 kg and 1000 kg DM particles have a cumulative rate of $\sim 10^{-5}$ yr$^{-1}$ and $\sim 4 \times 10^{-4}$ yr$^{-1}$, respectively, at a signal-to-noise ratio (SNR) of 1. Fig. 1 suggests that 3.69 hits are expected at a SNR of 1 over an observation of a third of a year, if the DM particle density were $10^{10}$ m$^{-3}$, which is four orders of magnitude larger than the actual DM particle density. With a similar calculation to the 10 kg case, this is equivalent to a cumulative rate of $8.5 \times 10^{-4}$ yr$^{-1}$. Thus, the analysis shown here matches that in Ref. 37 within a factor of $O(1)$.

ET is analyzed only for the transient measurement, because of an unavailability of detailed information on the mechanical resonance. Thanks to the lower noise and longer arm, both sensitivity at the same mass and the detector size limited sensitivity is improved compared to the LIGO case. However, it is still three orders of magnitude away from the DM particle density even at the closest point of 5 kg. When both the noise level is reduced by a factor of 10 and the detector size is increased by a factor of 10, the sensitivity improves by a factor of 100. With all three mirror sites taken into account and an observation is performed for three years, the sensitivity can reach the DM particle density at $m = 5$ kg.
The DM particles in the mass range of few kilograms has never been searched before. If these particles are point particle, or particles that is smaller than its Schwarzschild radius $r_s = 2GM/c^2$, these are black holes. Such small mass black holes are usually discussed in the context of the primordial black hole [13], but primordial black holes of gram scale mass evaporates quicker than the lifetime of universe, and therefore theoretically it has already been excluded. In case such small mass black holes can be generated at some time other than the birth of the universe, this will be the first search for such light mass black holes. If the size of the DM particles are larger than $r_s$, the DM can be an unknown particles that interact with other matters only through gravity.

An interesting mass range relatively close to the sensitivity plot is the Planck mass $2.18 \times 10^{-8}$ kg. At this mass, sensitivity is ten orders of magnitude above the number density of DM particles. Comparing "ET, $10^7$ s" line and "ET L, $S \times 10$, $10^7$ s" line, reducing the noise level by a factor of 10 moves sensitivity curve towards an order of magnitude smaller mass range. This implies that five orders of magnitude reduction of noise compared to "ET $L, S \times 10$, $10^8$ s" line is necessary to reach the Planck mass, which is unrealistic with the currently available technology.

The discussion so far put emphasis on setting constraint onto the existence of DM particles when there is no signal including background, and did not mention how the discovery of such a particle can happen. One method to narrow down the mass range is to detect the same DM particle by two test masses. In the general case, impact parameter $b$ for the two test masses are different, and this gives the information on two unknown parameters $m$ and $b$. Careful analysis of gravitational wave detectors would easily give this information, as different cavity mirrors surce as different test masses, and having two or more optically trapped microspheres nearby would give the similar information. What is more difficult to put strong constraint is to distinguish the signal and the background. When a coincidence of two nearby test mass is taken, most of the backgrounds induced by objects on the Earth should be rejected, thanks to the much faster motion of the DM particles against the Earth than objects on the earth. No typical objects on the Earth move as fast as $v_0 = 2.2 \times 10^5$ m/s, and typical sound velocity of solid, liquid, and gas, at which vibrations propagates, ranges from $10^2$ to $10^4$ m/s, which is orders of magnitude smaller than $v_0$. However, it is impossible to theoretically reject the possibility of any independent backgrounds have a timing coincidence to mimic $v_0$, or a part of a gigantic background resembles signal. Electrical noises, which propagate faster than $v_0$, could be another background difficult to remove. In such a case, taking a coincidence of more than two test masses might help. In this sense, optically levitated microspheres have an advantage that it is easier to build multiple detectors aligned in a desired way.

V. SUMMARY

A method of using precision displacement sensor of a test mass for a DM particle search is discussed. Although present and future technology by optically levitated microspheres can put marginal constraint to the number density of DM particles of ton scale, future gravitational wave detectors that have 10 times lower noise level and 10 times more detector size than ET can potentially have high enough sensitivity to detect DM particles of ~ 5 kg. This would be a first experimental search for primordial black holes, and even further improvement in the noise level by several orders of magnitude enables a search for DM particles of Planck mass.
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[1] V. C. Rubin, W. K. Ford, and N. Thonnard, Astrophys. J. 238, 471 (1980)
[2] Y. Akrami et al. (Planck Collaboration), arXiv:1807.06205 (2018)
[3] D. Clowe, M. Brada, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones, and D. Zaritsky, Astrophys. J. Lett. 648, L109 (2006)
[4] C. Alcock et al., Astrophys. J. Lett. 499, L9 (1998)
[5] E. Aprile et al. (XENON Collaboration), Phys. Rev. Lett. 121, 111302 (2018)
[6] D. S. Akerib et al. (LUX Collaboration), Phys. Rev. Lett. 118, 021303 (2017)
[7] X. Cui et al. (PandaX-II Collaboration), Phys. Rev. Lett. 119, 181302 (2017)
[8] R. Agnese et al. (SuperCDMS Collaboration), Phys. Rev. D 97, 022002 (2018)
[9] G. Angloher et al., The Eur. Phys. J. C 76, 25 (2016)
[10] S. J. Asztalos, G. Carosi, C. Hagmann, D. Kinion, K. van Bibber, M. Hotz, L. J. Rosenberg, G. Rybka, J. Hoskins, J. Hwang, et al., Phys. Rev. Lett. 104, 041301 (2010)
[11] M. Arik et al. (CAST Collaboration), Phys. Rev. Lett. 112, 091302 (2014)
[12] P. W. Graham, I. G. Irastorza, S. K. Lamoreaux, A. Lindner, and K. A. van Bibber, Annu. Rev. Nucl. Part. Sci. 65, 485 (2015)
[13] B. Carr, F. Kühnel, and M. Sandstad, Phys. Rev. D 94, 083504 (2016)
[14] L. D. Landau and E. M. Lifshitz, Mechanics Third Edition (Elsevier, 1976).
[15] T. M. Hoang, Y. Ma, J. Ahn, J. Bang, F. Robicheaux, Z.-Q. Yin, and T. Li, Phys. Rev. Lett. 117, 123604 (2016)
[16] A. D. Rider, C. P. Blakemore, G. Gratta, and D. C. Moore, Phys. Rev. A 97, 013842 (2018)
[17] P. Montiero, S. Glish, A. G. Fine, and D. C. Moore, Phys. Rev. A 96, 063841 (2017)
[18] T. Li, S. Kheifets, and M. G. Raizen, Nat. Phys. 7, 527 (2011)
[19] A. Ashkin and J. M. Dziedzic, Appl. Phys. Lett. 19, 283 (1971)
[20] J. Gieseler, B. Deutsch, R. Quidan, and L. Novotny, Phys. Rev. Lett. 109, 103603 (2012)
[21] P. Asenbaum, S. Kuhn, S. Nimmrichter, U. Sezer, and M. Arndt, Nat. Commun. 4, 2743 (2013)
[22] M. Mazilu, Y. Arita, T. Vettenburg, J. M. Auñón, E. M. Wright, and K. Dholakia, Phys. Rev. A 94, 053821 (2016)
[23] G. Ranjit, M. Cunningham, K. Casey, and A. A. Geraci, Phys. Rev. A 93, 053801 (2016)
[24] P. Z. G. Fonseca, E. B. Aranas, J. Millen, T. S. Monteiro, and P. F. Barker, Phys. Rev. Lett. 117, 173602 (2016)
[25] J. Vovrosh, M. Rashid, D. Hempston, J. Bateeman, M. Paternostro, and H. Ulbricht, J. Opt. Soc. Am. B 34, 1421 (2017)
[26] N. Kiesel, F. Blaser, U. Delić, D. Grass, R. Kaltenbaek, and M. Aspelmeyer, Proc. Natl. Acad. Sci. 110, 14180 (2013)
[27] D. V. Martynov et al., Phys. Rev. D 93, 112004 (2016)
[28] F. Acernese et al., Class. Quantum Grav. 32, 024001 (2015)
[29] K. Sumiya, Class. Quantum Grav. 29, 124007 (2012)
[30] J. Herzog-Arbeitman, M. Lisanti, P. Madan, and L. Necib, Phys. Rev. Lett. 120, 041102 (2018)
[31] R. Catena and P. Ullio, J. Cosmol. Astropart. Phys. 2010, 004 (2010)
[32] M. Tanabashi et al., Phys. Rev. D 98, 030001 (2018)
[33] S. M. Aston et al., Class. Quantum Grav. 29, 235004 (2012)
[34] M. Punturo et al., Class. Quantum Grav. 27, 194002 (2010)
[35] S. Hild, S. Chelkowski, and A. Freise, (2008), arXiv:0810.0604
[36] S. Hild, S. Chelkowski, A. Freise, J. Franc, N. Morgado, R. Flaminio, and R. DeSalvo, Classical and Quantum Gravity 27, 015003 (2010)
[37] E. D. Hall, R. X. Adhikari, V. V. Frolov, H. Müller, and M. Pospelov, Phys. Rev. D 98, 083019 (2018)