Microscale testing and modelling for damage tolerant composite materials and structures

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Abstract. There is a need for damage tolerant composite material for very large engineering structures such as wind turbine rotor blades. The development of improved composite materials with higher damage tolerance will be most efficiently guided by models of the relevant damage modes. Two damage modes are considered in the present paper: Fatigue damage due to in-plane tensile stresses in the fibre direction and cyclic delamination crack growth from a ply drop. For both failure modes the damage evolution is considered at macro- and microscale. Micromechanical models are used to illustrate how changes in the mechanical properties of fibre, matrix and the fibre/matrix interface can lead to an increased damage tolerance material. Also, micromechanical testing methods for characterizing the relevant micromechanical parameters are discussed.

1. Introduction
Composites based on long aligned high-stiffness fibres in a polymer matrix are widely used in large load-carrying structures where low weight and long fatigue life are of major importance. Examples of use are wind turbine rotor blades, which are to be in service for 20-25 years, while being subjected to varying cyclic loads (aerodynamic and gravitational) and varying environmental exposure [1, 2].

However, structures made of composite materials can fail by a number of complicated failure modes, since they typical possess weak interfaces at multiple length scales. Damage is a multiscale and progressive phenomenon: Damage can start as one failure mode at the microscale and then develop into other failure modes at the macro- and structural scales and prediction of progressive damage growth is an ongoing research field [3, 4]. Typically, damage prediction has been accomplished for a limited number of interacting failure modes.

Therefore, many engineering structures built using composites are designed quite conservatively against damage. Or more precisely, they are designed using criteria that does not allow damage growth. The manufacturing process then aims at making ”perfect” structures, i.e. without defects that can lead to damages [1, 2]. Structures are inspected after manufacturing and defects are identified. Some defects are deemed insignificant, while other leads to repairs. Severe manufacturing defects can lead to structures being discarded.

The traditional approach of obtaining structures with a safe life by enforcing strict quality control and discard parts with manufacturing defects is an expensive strategy for components made of very large parts since any discarded part is expensive simply due to the cost of materials used.
The present paper considers a way to go beyond the "no growth" approach for damage. Instead, the philosophy will be to assume that damage is present, but the materials and structures should be designed such that damage growth is stable (i.e., propagates slowly or even stops growing) and the damage growth is predictable.

In aerospace, damage tolerance concepts (allowing damage initiation and grow) combined with regular inspection interval are established for structures made of metals but not yet for composites [5]. The present paper considers how damage tolerant materials behaviour can be obtained by control of material properties at the microscale, i.e., by controlling the mechanical properties of the fibre, matrix and the fibre/matrix interface. Then, it is essential to have micromechanical models that can predict which microscale properties that give the optimal macroscale damage tolerance. It is likewise essential to have microscale experimental testing methods to establish what the microscale mechanical properties are. Combined, micromechanical modelling and microscale testing should be efficient tools in the development of future damage tolerant materials.

In the present paper we will discuss two important damage types for composite structures: In-plane fatigue and delamination crack growth from ply-drops. Both damage types are of great relevance for the load-carrying laminate (main spar) in wind turbine rotor blades. The focus will be on the development of composites materials with enhanced damage tolerance to fatigue, i.e., damage growth under cyclic loads.

The paper is organised as follows. In Section 2, concepts of damage tolerance will be given. Next, in Section 3, so-called "fatigue damage mechanisms maps" are introduced to visualize damage evolution. Examples of micromechanical models for the prediction of in-plane fatigue damage are presented in Section 4. Section 5 discusses delamination from ply-drops, Section 6 considers microscale experiments and Section 7 gives a brief perspective for the future.

2. Concepts of damage tolerance

2.1. Definition of damage tolerance

In the present paper, damage tolerance relates to a structures ability to undergo distributed damage and crack growth. The first mode of damage must not lead to imminent failure. Instead, damage should grow in a stable manner while being detectable and predictable, such that damage can be found well before it becomes critical. There should be plenty of time to react, e.g. stop using the component, make inspection and decide if it is necessary make repairs to the damaged structure. A high damage tolerance indicates that the damage or crack growth occurs stably over a large physical size (e.g., a long crack length) before the damage growth becomes unstable, or that the component undergoes a very high number of load cycles before unstable crack growth occurs.

Let us clarify what the statement above means. Under static loading, "stable manner" means that damage growth must arrest under fixed loading and a higher applied load is required to develop further damage. Under cyclic loading, fatigue damage should advance slowly, i.e., over many load cycles.

"Damage being detectable" means that the the presence of damage should result in changes in the materials stiffness locally (e.g. being detectable by strain measurements by strain gauges, digital image correlation and/or embedded sensors such as fibres with Fibre Bragg Grating) or other types of sensors, e.g. acoustic emission sensors. Preferably, damage should be identified by visual inspection, but this is often not possible for composite structures. We should be able to identify that damage is developing, what kind of damage it is, its physical location within a structure as well as its size and depth by the use of various non-destructive techniques (NDT).

"Predictable" means that the damage state and the load-carrying capability can be predicted for static loading and the damage growth rate and remaining number of cycles to failure can be predicted for cyclic loading. The predictions should be made using reliable models and the
relevant mechanical materials properties can be determined by independent experiments.

When the connection between microscale and macroscale has been obtained, micromechanics modelling then opens the possibility to tailor-make composites by microstructural optimization. As mentioned, we aim to make "damage controlled" materials. By this we mean to control the material properties such that the progressive damage evolution behaves in a pre-determined, damage tolerant manner.

The tailor-making of composites links micromechanics to materials processing: Modifications of the processing steps or processing conditions are needed for making more damage tolerant composite materials. The "optimized" processing condition that give the desired macroscale mechanical properties can then be obtained in iterations, where each iteration consists of (a) materials or components testing, (b) identification of damage types, and (c) micromechanical modelling to find the best microscale parameters. Then follows iterations of (d) microscale experiments to characterize the current microscale mechanical properties, and (e) change processing conditions in a direction towards the "optimal" processing conditions.

It should be noted that it is possible to design the geometry of structures such that they become more damage tolerant. Damage tolerance design is a combined material-structure property. This will be detailed in the next section.

2.2. Damage tolerance as a combined materials-structure property

As an illustration on how damage tolerance depends on both materials properties and structural properties (shape and size), we consider the growth of a crack that experiences crack bridging. Crack bridging can occur by fibres that connect the crack faces and create tractions that act against the opening of the crack. In mechanics terms, the crack bridging can be described by so-called cohesive laws, where the local bridging tractions are taken to be a function of the local crack opening displacements. For plane problems, a crack can have two tractions, a normal traction, $\sigma_n$, and a shear traction, $\tau_s$, see Fig. 1. In the general case they will both be functions of the local normal opening displacement $\delta_n$ and the local tangential opening displacement, $\delta_t$,

$$\sigma_n = \sigma_n(\delta_n, \delta_t) \quad \text{and} \quad \tau_s = \tau_s(\delta_n, \delta_t).$$

The bridging tractions contribute to the fracture resistance of the material. This can be understood by an application of the path-independent J integral [6] locally around the crack tip and bridging zone. The result becomes [7]
\[ J_{\text{loc}} = \int_0^{\delta^*_n} \sigma_n(\delta_n, \delta_t) d\delta_n + \int_0^{\delta^*_t} \tau_s(\delta_n, \delta_t) d\delta_t + J_{\text{tip}}, \] 

(2)

where \( \delta^*_n \) and \( \delta^*_t \) are the end-opening and end-sliding of the bridging zone, respectively and \( J_{\text{tip}} \) is the J integral evaluated around the crack tip. When \( J_{\text{tip}} \) equals the crack tip fracture energy (denoted \( J_0 \)), \( J_{\text{loc}} \) becomes equal to the fracture resistance, \( J_R \). The fracture resistance \( J_R \) thus increases with increasing end-opening and end-sliding. Such rising fracture resistance is called R-curve behaviour. \( J_{\text{loc}} \) can be considered as being a function of end-opening and end-sliding of the bridging zone, \( J_{\text{loc}} = J_{\text{loc}}(\delta^*_n, \delta^*_t) \).

In case the bridging tractions are derived from a potential function \( \Phi(\delta_n, \delta_t) \)

\[ \sigma_n(\delta_n, \delta_t) = \frac{\partial \Phi(\delta_n, \delta_t)}{\partial \delta_n}, \quad \text{and} \quad \tau_s(\delta_n, \delta_t) = \frac{\partial \Phi(\delta_n, \delta_t)}{\partial \delta_t}, \]

(3)

and with \( J_{\text{tip}} = J_0 \), eq. (2) becomes

\[ J_R(\delta^*_n, \delta^*_t) = \Phi(\delta^*_n, \delta^*_t) + J_0. \] 

(4)

Now we are ready to consider the stability of growth of cracks with bridging. For materials with R-curve behaviour, a criterion for stable crack growth is [8]

\[ J_{\text{ext}} = J_R \quad \text{and} \quad \frac{dJ_{\text{ext}}}{d\ell} \leq \frac{dJ_R}{d\ell}, \]

(5)

where \( J_{\text{ext}} \) is the J integral evaluated around the external boundaries (sometimes called the "applied energy release rate" in linear elastic fracture mechanics) of the specimen, and \( \ell \) is the crack length. Eq. (5) expresses that stable crack growth happens if the fracture resistance, \( J_{\text{loc}} \) (the right hand side), increases more rapid than the applied energy release rate (the left hand side). By eq. (4)

\[ \frac{dJ_{\text{ext}}}{d\ell} \leq \frac{d\Phi(\delta^*_n, \delta^*_t)}{d\ell}. \]

(6)

From the chain rule we have

\[ \frac{d\Phi(\delta^*_n, \delta^*_t)}{d\ell} = \frac{\partial \Phi(\delta^*_n, \delta^*_t)}{\partial \delta^*_n} \frac{\partial \delta^*_n}{d\ell} + \frac{\partial \Phi(\delta^*_n, \delta^*_t)}{\partial \delta^*_t} \frac{\partial \delta^*_t}{d\ell}. \]

(7)

or by eq. (3)

\[ \frac{d\Phi(\delta^*_n, \delta^*_t)}{d\ell} = \sigma_n^* \frac{\partial \delta^*_n}{d\ell} + \tau_s^* \frac{\partial \delta^*_t}{d\ell}, \]

(8)

where \( \sigma_n^* \) and \( \tau_s^* \) are the normal traction and shear traction at the end of the bridging zone, \( \sigma_n^* = \sigma_n(\delta^*_n, \delta^*_t) \) and \( \tau_s^* = \tau_s(\delta^*_n, \delta^*_t) \). Inserting eq. (8) into eq. (6) leads to the following criterion for stable crack growth:

\[ \frac{dJ_{\text{ext}}}{d\ell} \leq \sigma_n^* \frac{\partial \delta^*_n}{d\ell} + \tau_s^* \frac{\partial \delta^*_t}{d\ell}. \]

(9)

Equation (9) expresses that crack growth will be stable if the right hand side of the equation exceeds the left hand side.

In eq. (9), the terms \( \sigma_n^* \) and \( \tau_s^* \) obviously depend on the cohesive law and are thus material properties of the fracture process zone. The term \( \frac{\partial \delta^*_n}{d\ell} \) expresses how the normal end-opening increases as the crack length increases. Likewise, \( \frac{\partial \delta^*_t}{d\ell} \) expresses how the end-sliding (tangential opening displacement) increases as the crack length increases. Both these
two terms depends on the specimen geometry and stiffness and are thus structural properties. In conclusion, the criterion for stable crack growth, eq. (9), depends on both material and structural parameters.

3. Fatigue damage mechanisms maps

Since damages often evolve sequentially in composite structures, it is possible to understand each damage mechanism within progressive damage evolution one by one. If each damage mechanism is understood (and modelled), damage controlled materials may be realized by considering how the evolution of each damage mechanism can be slowed down or completely stopped.

In the following we will introduce a graphical way of representing the interaction of various fatigue damage mechanisms with the purpose of identifying which individual fatigue damage models are required and how they should interact. We will call this representation "fatigue damage mechanisms maps" and will illustrate this concept by two examples: in-plane fatigue in a unidirectional laminate and delamination fatigue crack growth from a ply-drop.

We contemplate a situation where a composite specimen is subjected to cyclic tension-tension loading between a fixed maximum strain value, $\bar{\varepsilon}_{\text{max}}$, and a minimum strain value, $\bar{\varepsilon}_{\text{min}}$. A fatigue damage mechanisms map is a representation with the number of load cycles, $N$, on the horizontal axis and the applied maximum cyclic strain level, $\bar{\varepsilon}_{\text{max}}$, on the vertical axis, similar to S-N graphs. However, while S-N graphs only contain the relationship between the maximum applied strain (or maximum applied stress) and the associated number of cycles to failure, $N_f$, a fatigue damage mechanisms map shows the most important fatigue damage mechanisms from virgin specimen to complete specimen failure. The central idea is that a fatigue damage mechanisms map should be constructed from experimental observation of microscale damage state. In order to make consistent model predictions, a model should be formulated for each damage mechanism and these models can then be linked together. This illustration is to some extend inspired by the concepts of "Fatigue Life Diagram" introduced by Talreja [9] and the damage scenario illustration of Carraro and Quaresimin [10].

3.1. A fatigue damage mechanisms map for in-plane fatigue

Fig. 2 shows a fatigue damage mechanisms map for in-plane tension-tension fatigue of a uniaxial fibre composite. There are no scales on the axis to illustrate that the concepts are generic. Never the less, Fig. 2 is constructed from literature observation of fatigue damage in unidirectional fibre composites with polymer matrix [11, 12, 13, 14]. For low values of $\bar{\varepsilon}_{\text{max}}$, the first mode of damage is isolated fibre failures accompanied by limited fibre/matrix debonding. The location of the fibre failure is "random" (unpredictable) in the sense that it occurs at the position along the fibre that possesses the worst defect. With increasing number of load cycles, a second damage mechanism evolves: an increase in the debond length with increasing number of load cycles. Microscale observations indicates that this situation can be stable (i.e., constant debond length, no further fibre failures [11, 12, 15]), indicating that a fatigue limit might exist [11]. For higher values of $\bar{\varepsilon}_{\text{max}}$, a second damage mechanism is activated: Breakage of neighbour fibres as a result of the stress field from the debond crack tip stress field of broken and debonded fibres [11, 12, 13, 14]. This establishes a progressive failure mode with a "damage front" growth rate, $da/dN$, where $a$ is the size of the damage zone consisting of broken and debonded fibres [16]. When a damage zone reaches a certain size, a third damage mechanism can be activated: Splitting cracks (cracks extend from the edge of the damage zone parallel to the fibre direction) [16, 17] propagate along the fibre direction to other fibre damage zones, resulting in fatigue failure of the specimen. For in-plane fatigue, the main composite properties to predict are: (i) the maximum applied cyclic strain that does not lead to fatigue failure (the fatigue limit), $\bar{\varepsilon}_{\text{fl}}$, and when $\bar{\varepsilon}_{\text{max}} > \bar{\varepsilon}_{\text{fl}}$, (ii) the fatigue damage growth rate $da/dN$ and the (iii) the associated fatigue life $N_f$. Both $da/dN$ and $N_f$ are expected to depend on $\bar{\varepsilon}_{\text{max}}$. 
Figure 2. Fatigue damage mechanisms map for in-plane fatigue of unidirectional fibre composites.

3.2. Fatigue damage mechanisms map for delamination from ply-drops

A fatigue damage mechanisms map for delamination cracks originating from two ply-drops is shown in Fig. 3. The first mode of damage is the formation of a so-called tunneling crack at the end of the ply drop, i.e., a crack that is limited in growth in one direction (the thickness direction) but can increase in width direction. The initiation is "random" in the sense that it depends on the worst defect across the width of the specimen. For tunneling cracks it is well-known [18, 19] that once the length of a tunneling crack exceeds a few times the ply thickness, the energy release rate at the crack front attains a steady-state value that is independent of the crack length. For low values of $\bar{\varepsilon}_{\text{max}}$, the steady-state energy release rate can be so low (below a threshold) that no cyclic crack propagation can occur. For a higher value of $\bar{\varepsilon}_{\text{max}}$, the tunneling crack can propagate at a constant rate, in accordance with a Paris-Erdogan law [20]. For even higher value of $\bar{\varepsilon}_{\text{max}}$, the tunneling crack can occur as static crack propagation in the very first load cycle.

The second damage mechanism that can evolve after the formation of a tunneling crack is the initiation and growth of a delamination crack at the tips of the tunneling crack. For low values of $\bar{\varepsilon}_{\text{max}}$, the energy release rate of the delamination crack can propagate at a constant rate, in accordance with a Paris-Erdogan law [20]. We now consider the situation of a laminate made of layers with identical thicknesses and several ply-drops. There is experimental evidence that indicates that tunneling cracks and delamination cracks develop at all ply-drops but do not form simultaneously and that they propagate with different growth rates [21].

Thus, models are needed to be able to predict the delamination growth rate $d\ell/dN$ as a function of applied strain levels and the ratio between the ply thickness, $h_1$, and the thickness $h_2$ of the underlying layers.

3.3. Microscale parameters

Although these damage modes appear as macroscale damages and cracks, we will analyse them from the perspective of both macroscale and micromechanics in the next section. The basic microscale parameters for fibre, matrix and interface are listed in Table 1 and additional composite parameters are listed in Table 2.

Some comments to the choice of parameters. They represent a balance between accuracy and simplicity, sufficient to describe the most important phenomena while still allow micromechanical
Figure 3. Fatigue damage mechanisms map delamination cracks growing from multiple ply-drops in a unidirectional fibre composites.

Table 1. Microscale parameters for fibres, matrix and interface

| Parameter | Description |
|-----------|-------------|
| $E_f$     | Young’s modulus of the fibre |
| $\alpha_f$ | Thermal expansion coefficient of the fibres |
| $r$       | Fibre radius |
| $\sigma_0(L_0)$ | Characteristic strength (Weibull model) |
| $m$       | Weibull modulus (Weibull model) |
| $L_0$     | Reference length (Weibull model) |
| $E_m$     | Young’s modulus of the matrix |
| $\alpha_m$ | Thermal expansion coefficient of the matrix |
| $G_{IC}^m$ | Mode I fracture energy of matrix |
| $G_{IIc}^m$ | Mode II fracture energy of matrix |
| $\phi_i^c$ | Interface fracture energy (debond energy) |
| $\tau_s$ | Interfacial frictional sliding shear stress |

Table 2. Composites and processing parameters

| Parameter | Description |
|-----------|-------------|
| $V_f$     | Fibre volume fraction |
| $T_0$     | Stress-free temperature |
| $T$       | Ambient temperature |
| $\Delta \varepsilon^T$ | Misfit strain associated with processing |
models to be formulated as relative simple closed-form analytical solutions. For instance we disregard Poisson’s effects of fibres and matrix. We take the fibre distribution to be uniform. Assuming the fibres to be distributed in a hexagonal array, the inter-fibre spacing $d^∗$ can be calculated as \[d^* = \frac{2\pi}{\sqrt{3}V_f} - 2,\]

The fibre/matrix interface is particular challenging to model in terms of mechanics. In the present paper we characterise the mechanics of the fibre/matrix interface by two parameters: The interfacial fracture energy, $G^{i}$, represents the breakage of chemical and physical bonds (covalent bonds, ionic bonds, hydrogen bonds) while a frictional sliding shear stress $\tau_s$ represents the stress-transfer between debonded fibres and the surrounding matrix as they slip relative to each other. This discrimination between debonding and friction sliding are considered important in the present paper.

Debonding, being cracking along a bimaterial interface involves a more complicated crack tip stress field than for cracks in homogeneous materials [23]. There is empirical evidence that shows that the interfacial fracture energy depends on the mode mixity $\psi$ [18]. Usually the interfacial fracture energy is lowest for normal crack opening dominated fracture and highest for tangential crack opening displacements. Obviously, a more rigorous description should take $G^{i}$ as being a function of the mode mixity, $G^{i} = G^{i}(\psi)$. However, with the present state of relative simple measurement methods for fibre/matrix interfaces, this is left for the future.

The present formulation of $\tau_s$ can be taken to be a combination of friction due to fibre roughness asperities and Coulomb friction [24]

$$\tau_s = \tau_s^0 - \mu \sigma_{rr} \text{ for } \sigma_{rr} < 0,$$

where $\tau_s^0$ is a friction stress, $\mu$ is a friction coefficient and $\sigma_{rr}$ is the normal stress (taken positive in tension and negative in compression) acting normal to the fibre/matrix interface. In eq. (11) the first term represents friction due to roughness asperities of the fibre surface and the second term in a Coulomb-like friction. Note from eq. (11) that there will be a frictional shear stress due to roughness of the fibre/matrix interfaces even when there is zero normal pressure across the fibre/matrix interface ($\sigma_{rr} = 0$).

The misfit strain $\Delta \varepsilon^T$ is the difference in the in-elastic strains of the fibre and matrix occurring during processing of the composite. The major contribution to $\Delta \varepsilon^T$ is likely to be the differences in thermal expansion coefficients,

$$\Delta \varepsilon^T = (\alpha_f - \alpha_m)(T - T_0),$$

where $\alpha_f$ and $\alpha_m$ are the linear thermal expansion coefficients of fibre and matrix, respectively, $T$ the present temperature and $T_0$ the stress-free temperature.

4. Models of in-plane fatigue

In this section we review recent models for the prediction of fatigue damage growth and fatigue failure [25]. As indicated in the fatigue damage mechanisms map for in-plane fatigue (Fig. 2), there are three mechanisms that should be modelled independently: First fibre failures (damage initiation), progressive fibre/matrix debonding and growth of clusters of broken fibres, and splitting along fibres (longitudinal splitting). We will ignore the very first mechanisms (isolated fibre failures) and consider only the two latter progressive mechanisms: Growth of the damage front by progressive fibre failure and longitudinal splitting. In the present work, we utilize a recent model by Sørensen et al. [25]. In this model, the fibre-to-fibre breakage is attributed to
the stress field of the debond crack tip - it induces an extra stress to un-broken neighbour fibres. The progressive increase in the fibre/matrix debond length during cyclic loading is controlled by a decrease in the interfacial friction stress, $\tau_s$, originating from abrasion of interfacial asperties due to repeated forward and reverse slip. The decrease in $\tau_s$ from its initial (first forward slip) value $\tau_s^0$ to a final value $\tau_s^c$ takes place over $N_c$ load cycles (counting from the number of cycles the fibre broke and experienced it first slip) and is controlled by a shape parameter denoted $n$.

Then, as the debond crack tip moves along the broken fibre, it exposes a larger length of neighbour fibres to the debond crack tip stress field (the so-called K-field [26]), which then can induce failure in a neighbour fibre. A just-broken fibre will also undergo progressive debonding during the subsequent load cycles (due to the gradual decrease in $\tau_s$). The stresses of the debond crack tip (the K-field) can induce failure in one of its neighbours as it gradually moves up along the fibre. In this manner, microscale damage in the form of progressive fibre failures at a damage front can propagate across the cross section of a specimen (see [25] for details). When the damage zone is sufficiently large, another failure mechanism becomes favourable: Longitudinal splitting. Longitudinal splitting is the growth of mode II like cracks [17]. Longitudinal splitting will be possible when the energy release rate at the tip of the splitting crack is equal to (or larger than) the Mode II fracture energy of the composite. Longitudinal cracks can connect damage zones of clusters of broken fibres, leading to specimen separation.

4.1. Prediction of the damage growth rate, $da/dN$

The model by Sørensen et al. [25] predicts the damage growth rate of a straight damage front. Denote $a$ as the damage size (length), the fatigue damage growth rate is found as

$$\frac{da}{dN} = \sqrt{\frac{3\pi}{2V_f}} N^*,$$

(13)

where $N^*$, the number of cycles it takes (on average) to cause breakage of the next row of fibres, is found as:

$$N^* = N_c \left\{ \frac{\tau_s^0}{\tau_s^0 - \tau_s^c} \left[ 1 - (1 - R) \left\{ \frac{2}{\varepsilon_{max}} \left\{ \frac{2}{E_f} \frac{\ell^*_d}{r} + \frac{\sigma_0}{E_c} \right\} - (1 + R) \right\} \right] \right\}^{1/n}.$$  

(14)

In eq. (14), the parameter $R$ is the so-called R-ratio ($R = \bar{\varepsilon}_{min}/\bar{\varepsilon}_{max}$), $E_f$ the Young’s modulus of the fibres, $r$ is the fibre radius, $\ell^*_d$ is the debond length after $N^*$ cycles, given by

$$\frac{\ell^*_d}{r} = \frac{3ln2 L_0}{2} \left[ \left( \frac{\sigma^f + \sigma^K}{\sigma_0} \right)^m + 2 \left( \frac{\sigma^f}{\sigma_0} \right)^m \right]^{-1},$$

(15)

where $\sigma^f$ is the stress in the fibre far ahead of the debond crack tip at an applied strain level corresponding to the maximum applied strain:

$$\frac{\sigma^f(\varepsilon_{max})}{E_f} = -\Delta\varepsilon^T (1 - V_f) \frac{E_m}{E_c} + \varepsilon_{max},$$

(16)

with $\Delta\varepsilon^T$ being the misfit strain.

Also, in eq. (15) $\sigma_0$ and $m$ are the characteristic strength and Weibull modulus associated with the gauge length $L_0$ (parameters of the Weibull model [27]), $\sigma^K$ is the the stress induced by the K-field of the debond crack tip to the neighbour fibres, given by [22]

$$\sigma^K = 0.435 \frac{E_f}{E_m} \sqrt{\frac{d^* E^*}{d^*}},$$

(17)
with $d^*$, the fibre spacing, given by (10) and $E_m$ is the Young’s modulus of the matrix and $E^*$ is (plane stress)

$$\frac{1}{E^*} = \frac{1}{2} \left( \frac{1}{E_f} + \frac{1}{E_m} \right).$$  

Furthermore, in (14), the so-called macroscopic debond initiation stress, $\sigma_i$, is given by [22]

$$\sigma_i \frac{E_m}{E_c} = (1 - V_f) E_m \Delta \varepsilon^T + 2 \sqrt{\frac{(1 - V_f) E_m}{E_c}} \left( \frac{G_{ic}}{E_f} \right)^{\frac{1}{2}},$$  

and the parameter $E_c$, used in (16) and (19), is the Young’s modulus of the composite which is (assuming no porosity) is given by the rule of mixtures

$$E_c = V_f E_f + (1 - V_f) E_m.$$  

4.2. Number of cycles to fatigue failure, $N_f$

We now model the final fatigue damage mechanism, the growth of longitudinal splitting cracks. As a first approximation, we assume that the splitting occurs as the growth of a sharp crack tip, so that linear-elastic fracture mechanics is applicable. It is thus implicitly assumed that the crack faces are traction-free (no friction and no fibre bridging). When the length of the splitting cracks is a few times the damage zone size, splitting becomes a steady-state problem where the energy release rate takes a constant value. Then, splitting can occur as static crack growth at the maximum applied strain, $\varepsilon^{\max}$. This can happen when the steady-state energy release rate equals the mode II fracture energy of the composite, $G_{IIc}$. The critical size for growth of splitting cracks around a hexagonally shaped column is [17]

$$g_s = \frac{4 G_{IIc}}{E_c \varepsilon_{\max}^2},$$  

where $g_s$ is the half of the width of a hexagonal-shaped column forming during splitting (see [17] for more details).

Using fracture area arguments we can express $G_{IIc}$ in terms of microscale parameters as [17],

$$G_{IIc} = \left( 1 - \sqrt{\frac{2\sqrt{3} V_f}{\pi}} \right) \frac{G_m}{\pi} \frac{2 \sqrt{\sqrt{2} \sqrt{3} V_f}}{\pi} G_c,$$  

where $G_m$ is the Mode II fracture energy of the matrix.

In case the fatigue damage growth rate, $da/dN$, is determined, the number of cycles to failure can be estimated as

$$N_f = \frac{g_s}{da/dN}.$$  

This reasoning is based on the assumption that the damage zone extends normal to its damage front at a constant rate, $da/dN$, from failure of the first fibres to the critical cluster size. Inserting $g_s$ from (21) and $da/dN$ from (13) into (23) gives

$$N_f = \frac{4 G_{IIc}}{E_c \varepsilon_{\max}^2} \sqrt{\frac{2 V_f}{\sqrt{3} \pi} N^*},$$  

where $N^*$ is given by (14).
As an example, we show the effect of varying the variation in the fibre strength, by varying the Weibull modulus, $m$. Fig. 4 shows the damage growth rate $da/dN$, calculated from eqs. (13) and (14) as a function of the maximum applied strain, $\bar{\varepsilon}_{\text{max}}$. The curves have similar form: They all start at a low strain value giving the same minimum damage growth rate, increasing with increasing $\bar{\varepsilon}_{\text{max}}$, reaching a maximum value of $\bar{\varepsilon}_{\text{max}}$ where the damage growth rate increases asymptotically (corresponding to infinite growth rate, resulting in failure in the very first load cycle). Increasing $m$ (corresponding to a narrower fibre strength variation) increases the damage growth quite significantly or equivalently decreases the strain range at which fatigue occurs (between the two strain values, $\bar{\varepsilon}_{\text{max}}$ and $\bar{\varepsilon}_{\text{min}}$) and narrows the range between the strain values of slowest and first cycle failure.

Fig. 5 shows predicted S-N-curves for various values of $m$, calculated using eq. (24): The maximum applied strain is shown as a function of the number of cycles to failure, $N_f$. The curves have a horizontal part that asymptotically (for $N_f \to 1$) approaches the predicted static strength. For lower values of $\bar{\varepsilon}_{\text{max}}$, the predicted S-N-curves turn downwards in a near-linear form. An increase in $m$ moves the curves down (lower strain value for the same fatigue life, $N_f$) and correspondingly lower fatigue limit.

5. Delamination from ply-drops

5.1. Delamination growth under static, monotonic loading for brittle interface

A simple model of delamination of a symmetric tri-layer (a laminate consisting of three layers) is developed in Appendix A. The two surface layers have a Young’s modulus, $E_1$ and a thickness, $h_1$, and the middle layer as a Young’s modulus, $E_2$ and thickness $h_2$. An interface crack exist along both interfaces.

We consider first the case without bridging tractions ($\tau_s = 0$). Only the first term in eq. (45) remains. Then, debonding will happen at a constant applied stress value, given by
Figure 5. Predicted S-N-curves for in-plane fatigue of unidirectional fibre composites - effect of Weibull modulus, $m$.

\[
\bar{\sigma} = \sqrt{\frac{2(1 + 2\Sigma\eta)}{\Sigma\eta} \frac{G_{Ic}}{E_2}} = \sqrt{\frac{2(E_2h_2 + 2E_1h_1)}{E_1h_1} \frac{G_{Ic}}{E_2h_2}},
\]

irrespective of the debond length $\ell_d$. Likewise, the J integral result, eq. (44), becomes

\[
J_{tip} = \frac{\Sigma\eta}{2(1 + 2\Sigma\eta)} \frac{\sigma^2h_2}{E_2} = \frac{E_1h_1}{2(E_2h_2 + 2E_1h_1)} \frac{\sigma^2h_2}{E_2}.
\]

Fig. 6 shows the normalized J integral result (energy release rate) as a function of the thickness ratio and stiffness ratio. The normalisation is done such that the properties (stiffness and thickness) of the middle layer is held constant and properties of the surface layers are varied. It is seen in Fig. 6 that the energy release rate increases with increasing thickness and stiffness of the surface layers. For softer surface layers ($E_1 < E_2$), the energy release rate increases almost linearly with the thickness of the surface layer - a linear relation applies for thickness ratio up to about 0.3. When the surface and middle layers have identical stiffness, the relationship is more non-linear; the linear relation only applies for small thickness ratios (say, less than 0.1). For stiffer surface layers, the relationship is non-linear.

As another example, consider a laminate with two ply-drops as shown in Fig. 7a). All plies have the same thickness, $t$. There are pre-existing delamination at both ply-drops. We wish to use the model to predict which of the two delaminations, at the thin end (position A) or the thick end (position B) will propagate first under monotonically increasing load. We use (26) for the two cases. For simplicity, we take $E_1 = E_2$ so that $\Sigma = 1$. For Case A (Fig. 7b)), $h_1 = t$ and $h_2 = 2t$ so that $\eta = 1/2$. The stress in the thin end of the delaminated laminate, $\bar{\sigma}_A$ is equal to $\bar{\sigma}_0$. Then (26) gives $J_{tip} = \bar{\sigma}_0^2t/4E_2$. For Case B (Fig. 7c)), $h_1 = t$ and $h_2 = 4t$ so that $\eta = 1/4$. Moreover, the stress in the middle layer, $\bar{\sigma}_B = \bar{\sigma}_0/2$. Then (26) gives $J_{tip} = \bar{\sigma}_0^2t/12E_2$. It is clear that $J_{tip}$ is lowest for Case B. Conversely, it can be expected that under a monotonically increasing load, the delamination at position A will start propagating at a lower value of $\bar{\sigma}_0$. 

![Figure 5](image_url)
than required for growth of the delamination at position B. The $J_{\text{tip}}$ value is independent of crack length, so the crack will propagate quasi-stable at a constant value of $\bar{\sigma}_0$. As the crack tip of delamination A propagates past the tip of delamination B, then (as shown in Fig. 7d) effectively $h_1 = 2t$ and $h_2 = 2t$, so that $\eta = 1$. With $\bar{\sigma}_A = \bar{\sigma}_0$ we then get $J_{\text{tip}} = \bar{\sigma}_0^2 t / 3E_2$. This value of $J_{\text{tip}}$ is higher than $J_{\text{tip}}$ was when the delamination was shorter (Case A). Conversely, as the delamination crack tip A reaches the crack tip of delamination B at under quasi-static growth, delamination crack A will then propagate unstably along the remainder of the specimen. In contrast, delamination B will not be able to propagate under static loading (if the fracture energy of the two interfaces are the same) since its value of $J_{\text{tip}}$ is lower than that of delamination A.

5.2. Prediction of crack growth rate, $d\ell/dN$ under cyclic loading for brittle interface

Now we consider cyclic loading applied to the ply-drop specimen shown in Fig. 7. The applied stress is assumed to alternate between a fixed maximum applied tensile stress, $\bar{\sigma}_{\text{max}}$, and a fixed minimum tensile stress, $\bar{\sigma}_{\text{min}}$. Also, we assume that the cyclic crack advancement of the delamination crack tip follows a Paris-Erdogan law [20, 26]:

$$\frac{d\ell}{dN} = C (\Delta K - (\Delta K)_{th})^{\tilde{m}},$$  \hspace{1cm} (27)

where $\Delta K$ is the stress intensity range, $(\Delta K)_{th}$ is a threshold value of the stress intensity factor range and $C$ and $\tilde{m}$ are constants.

At the microscale, the crack advancement of crack tip can be considered as consisting of a debond crack growth along the fibre/matrix interface and cyclic crack growth in the matrix. The local crack front of the debond crack and the matrix crack front will adjust themselves (and their local stress intensity ranges) such that they both advance in the same rate, equal to the macroscale delamination crack growth rate, $d\ell/dN$. 

**Figure 6.** Normalized J integral for the traction-free case shown as a function of ply thickness ratio for several stiffness mismatch.
Figure 7. Example of symmetric laminate with two ply-drops with delaminations. The overall problem (a) can be analysed as three "elementary" cases, Case A (b), Case B (c) and Case C (d).

Returning to the macroscale, let us consider the specimen with two ply-drops, Fig. 7. For simplicity, assume that \((\Delta K)_{th} = 0\) and take \(\bar{\sigma}_{min} = 0\) (so that the minimum J integral value, \(J_{min}\), is zero) and (again) \(E_1 = E_2\). Then, with the Irwin relationship \([8, 26]\) we have

\[
\Delta K = \sqrt{J_{max}E_2},
\]

so that we find from eq. (27) that the ratio between the crack growth rate of delamination crack tip B \((d\ell/dN)\)\(_B\) to the crack growth rate of delamination crack tip A \((d\ell/dN)\)\(_A\) becomes: \((d\ell/dN)\)\(_B\)/(\(d\ell/dN)\)\(_A\) = \(1/(3)^{\tilde{m}}/2\). As an illustrative example, with \(\tilde{m} = 2.5\), we get \((d\ell/dN)\)\(_B\)/(\(d\ell/dN)\)\(_A\) \(\approx 0.25\). Thus, during cyclic loading, both delamination crack tips A and B propagate during cyclic loading, but delamination B propagates significantly slower than delamination A. Then the crack tip of delamination A can catch up and propagate past the crack tip of delamination B. Then the situation becomes Case C. We find \((d\ell/dN)\)\(_C\)/(\(d\ell/dN)\)\(_A\) = \((2/\sqrt{3})^\tilde{m}\). Again, as an example, with \(\tilde{m} = 2.5\), we find \((d\ell/dN)\)\(_C\)/(\(d\ell/dN)\)\(_A\) \(\approx 1.4\). Thus, once the delamination crack tip A catches delamination crack tip B, its growth rate increases. From the perspectives of avoiding a faster delamination crack growth, a criterion could be to ensure that crack tip A does not extend to crack tip B. We progress to set up such a criterion.

Consider the situation that tunneling cracks have formed across the full specimen width at both ply-drop A and ply-drop B in the very first load cycle. Assume further that both delamination cracks A and B propagate under the subsequent load cycles with crack growth rates \((d\ell/dN)\)\(_A\) and \((d\ell/dN)\)\(_B\), respectively. With the distance between the ply-drops given by \(L\), the critical number of load cycles for crack tip A to catch crack tip B can be estimated as:
Figure 8. Crack length of two delamination cracks starting from different ply-drops (separated a distance $L$) in a symmetric laminate as a function of number of load cycles.

\[ N_c = \frac{L}{(\frac{d\ell}{dN})|_A - (\frac{d\ell}{dN})|_B}. \]  

Fig. 8 shows predicted crack growth using (26), (27) and (28). Crack A starts from a ply-drop at the thinnest part of the laminate and crack B starts from the next ply-drop, a distance $L$ away from ply-drop A. Both cracks grow with constant growth rates (crack A grows faster than crack B, $(\frac{d\ell}{dN})|_A > (\frac{d\ell}{dN})|_B$) until crack A has caught up crack B after $N_c$ cycles. Thereafter, crack A continues to grow with a faster crack growth rate $(\frac{d\ell}{dN})|_C$ while crack B stops growing, since it is now unloaded. These predictions are qualitatively in agreement with experimental findings [21].

It is clear from these considerations that delaminations from ply-drops do not possess a damage tolerant behaviour in case the interface between plies are brittle and crack growth is controlled by a Paris-Erdogan law.

Now we turn our attention to bridging fibres which can bring in damage tolerant behaviour.

5.3. Increasing damage tolerance by crack bridging - microscale aspects

In some cases, unidirectional fibre composites can experience crack bridging by single fibres or ligaments of multiple fibres that connect crack faces and create tractions at the fracture surfaces. As the crack openings these tractions perform work that contributes to the fracture resistance, as described in Section 2.2. This creates damage tolerance, as will be shown in the following.

For a delamination crack that initiate from a ply-drop, the crack openings are dominated by tangential crack opening displacements. Rectangular ligaments (width $b$ and height $h$) being peeled-off under pure tangential crack opening displacement creates a shear traction [28]

\[ \tau_s = \left[ \frac{2G_{IIc}E_c}{b^2} \right]^{1/2} \eta bh E_c, \]
where $G_{IIc}$ is the Mode II fracture energy of the composite, and $\bar{\eta}$ is the number of bridging ligaments per unit cracked area. For bridging by single fibres a similar relation is found:

$$\tau_s = \left[ \frac{2G_{IIc}}{E_f r} \right]^{1/2} \bar{\eta} \pi r^2 E_f,$$

where $G_{IIc}$ is the fracture energy of the fibre/matrix interface. Note that for both cases, the shear traction is independent of the tangential opening displacement, but increases proportionally with the square root of the fracture energy. Thus, for a crack undergoing pure tangential opening displacements, bridging ligaments or bridging fibres can create a constant shear traction along the bridging zone (failure of ligaments and fibres is disregarded for simplicity). This relationship is shown schematically in Fig. 9a). Under cyclic opening/closure the ligaments are expected to unload linear-elastically, with a linear relationship between opening and traction as indicated in Fig. 9b).

5.4. Delamination growth under static, monotonic loading under large-scale bridging

We take up the model of delamination of a symmetric tri-layer presented in Appendix A. In this model, the bridging shear tractions are represented in terms of a constant shear stress $\tau_s$. Equation (45) gives the relationship between debond length $\ell_d$ and the applied stress, $\bar{\sigma}$. The applied stress required to start the debond crack is identical to that of a non-bridged crack, eq. (25) and then increases linearly with debond length. This implies that under monotonically increasing loading, the debond crack growth is stable: Although debond cracking has started, a higher value of the applied stress is needed to grow the crack further; the delamination crack growth is stable. This is a damage tolerant behaviour.

5.5. Prediction of crack growth rate, $d\ell/dN$ under cyclic loading under large-scale bridging

We now consider the ply-drop specimen subjected to cyclic stress between a maximum tensile stress, $\bar{\sigma}_{\text{max}}$ and minimum applied tensile stress, $\bar{\sigma}_{\text{min}}$. In the following, we take for simplicity $\bar{\sigma}_{\text{min}} = 0$ (so that the minimum J integral value, $J_{\text{min}}$ at $\bar{\sigma}_{\text{min}}$, is zero) and $(\Delta K)_{\text{th}} = 0$.

Replacing the value of the applied stress, $\bar{\sigma}$, with the maximum value of the applied stress, $\bar{\sigma}_{\text{max}}$, during cyclic loading, eq. (45) can also be useful in connection with cyclic loading:

$$\frac{\bar{\sigma}_{\text{max}}}{E_2} = \left( 1 + 2\Sigma \eta \right) \frac{\tau_s \ell_d}{E_2 h_2} + \sqrt{\frac{2(1 + 2\Sigma \eta) J_{\text{max}}}{\Sigma \eta E_2 h_2}},$$

where we have used $J_{\text{max}}$ (the value of $J_{\text{tip}}$ at the maximum applied stress) instead of $G_{IIc}$. Now (32) can be understood as the relationship between $J_{\text{max}}$ and $\ell_d$ at $\bar{\sigma}_{\text{max}}$. With fixed, $\bar{\sigma}_{\text{max}}$, an increase in $\ell_d$ during cyclic loading leads to a decrease in $J_{\text{max}}$. Thus, during a cyclic experiment, $J_{\text{max}}$ decreases with increasing $\ell_d$. The decrease in $J_{\text{max}}$ decreases $\Delta K$ (according
to eq. (28)) and thus a decreasing crack growth rate from the Paris-Erdogan law, eq. (27). Thus, with sufficiently long debond crack length, \( J_{\text{max}} = 0 \) so that \( \Delta K = 0 \) and thus the cyclic crack growth will stop (crack arrest).

Setting \( J_{\text{max}} = 0 \) in (32) we can find the associated debond length as

\[
\ell_a^d = \frac{\Sigma \eta}{\bar{\sigma}_{\text{max}} h_2 (1 + 2 \Sigma \eta) \tau_s}.
\]

(33)

where superscript "a" has been used to indicate crack arrest.

A criterion for ensuring crack arrest under cyclic loading will then be:

\[
\ell_a^d < L,
\]

(34)

where \( L \) is the distance between the ply-drops. Combined, eqs. (34) and (33) can be used to design the maximum applied stress \( \bar{\sigma}_{\text{max}} \) and the spacing between ply-drops \( L \) and laminate dimensions such that delamination cracks originating from ply-drops will arrest, in the case that the delamination occurs with fibre bridging. From eq. (33) it is apparent that a critical parameter in such a design approach is the maximum cyclic shear traction \( \tau_s \) acting along the delaminated interface.

Crack bridging thus provides the possibilities to have delaminations from ply-drops propagate in a stable manner under increasing applied load and propagate slowly and potentially arrest. Thus, crack bridging is a promising mechanism to incorporate in materials to create damage tolerance behaviour.

6. Microscale experiments

The spirit of the present paper is to consider composites from the microscale, acknowledging that the macroscale behavior of composites is controlled by the microscale parameters. An advantage of such multiscale approach is that the effects of actual physical processing parameters can be understood by micromechanical modelling. Linking to the microscale should thus be an advantage primarily from the perspective of the development of new composite materials with improved properties, such as enhanced damage tolerance.

However, this pushes the mechanical characterisation down to the microscale. In order to be useful in materials development, mechanical characterisation of microscale parameters should become a key activity of future materials development.

The models used in the present paper are relatively simple and have analytical solutions. Obviously, they have less accuracy than more advanced computational models that require numerical solutions (e.g. by the finite element method). However, analytical models provide clear insight into the effects of model parameters, something that can be more difficult or time consuming to obtain from numerical models. Also, the simple models balance well with the present state of relative simple experimental methods used for microscale characterisations.

Within the last few years, X-ray tomography has emerged as a very useful tool to study fatigue damage in composites at the microscale. This provides a lot of input and clarification to microscale damage evolution [16]. More such studies are expected in the coming years.

6.1. Characterisation of fibre properties

The major microscale parameters for fibres are stiffness and strength properties, see Table 1. These are determined from single-fibre tensile testing experiments. This used to be tedious and difficult work. But within recent years, automatized fibre testing equipment, which enables fast and reproducible tensile tests of fibres has been commercialized. This enables the tensile testing of larger batches of fibres increasing the reliability of the parameters for fibre strength variation, the characteristic strength \( \sigma_0(L_0) \) and the Weibull modulus, \( m \).
6.2. Characterisation of matrix properties

Stiffness and Mode I fracture energy of matrix materials should be relatively easy to obtain from standard materials testing (tensile testing and fracture mechanics testing). The Mode II fracture energy of matrix, $G_{IIc}$, can be more challenging but can be measured using sandwich specimens consisting of a thin resin layer between stiff beams [29].

6.3. Characterisation of fibre/matrix interface properties

Experimentally, the most challenging area is the characterisation of the mechanics of the fibre/matrix interface. Microscale experiments are required for this purpose. As mentioned, debonding is essentially cracking along a bimaterials interface. For some problems like in-plane fatigue, the fibre/matrix debonding takes place due to a tensile load in the fibre direction and fibre/matrix debonding is predominantly "Mode II" (debonding is truly a mixed mode cracking problem [23]) and is influenced by frictional sliding along the interface.

There is a relative large number of experimental techniques for the characterisation of the fibre/matrix interface under "Mode II" debonding. Examples are fibre pull-out, single fibre fragmentation test, microbond testing and fibre push-out [30, 31]. The analysis of the experiments are sometimes rather simple, so that only one parameter (Mode II debond energy or friction sliding shear stress) but not both are determined. In other cases, complicated numerical models are used to determine debond energy $G_i$ and a friction shear stress $\tau_s$ (or in connection with assumed Coulomb friction, a friction coefficient, $\mu$). However, the parameter determination is far from simple, and there are examples of different "advanced" models that - applied to the same experimental data - have given debond fracture energy values that differ by a factor of ten [32, 33, 34].

There are relatively few experimental studies of the cyclic behaviour of fibre/matrix interfaces, experiments where the debond crack length has been recorded as a function of the number of cycles of forward and reverse slip [12, 15]. Such studies, which can be conducted under light microscopy, are essential for the formulation of the evolution laws for $\tau_s$ as function of number of slip cycles etc. This deserves much more research in the future.

In contrast to in-plane fatigue, fibre/matrix debonding in connection with crack bridging of cracks parallel to the fibre direction (delamination and splitting) usually involves fibre debonding without friction. Also, such "peel"-like debonding occurs under mixed mode (involving both normal and shear stresses at the debond crack tip) [35]. Other types of single fibre peel-off experiments could be useful, both under static and cyclic loading.

7. Perspectives

7.1. Measurements of mechanical material properties

Currently, the macroscale is the length scale where the majority of materials testing takes place. There are a large number of test specimens (so-called "coupons") established to characterize quite a number of engineering strength and failure parameters that are considered necessary to describe the fairly large number of complicated failure modes that exist for composite materials and structures. For some of these, there are several test specimen geometries and test standards. A full materials characterisation at the macroscale can thus be quite costly and involving. Still, all these macroscale strength properties must be controlled by the microscale properties of the fibres, the matrix and the fibre/matrix interface. It is thus anticipated that in a long term perspective, the mechanical behaviour at the macroscale can be predicted from micromechanical models with microscale properties of fibre, matrix and the fibre/matrix interface as the "basic" material properties; materials characterisation then moves to the microscale.
7.2. NDT detection of fatigue damage mechanisms
NDT methods can play an important role in the detection of fatigue damage at various length scales. The use of fatigue damage mechanism maps could be useful in linking mechanisms and sensor signals. For instance, sensor signals e.g. from acoustic emission sensors, can be recorded and characterised for individual damage mechanism at the microscale. These signals can then help identify the damage evolution in coupon specimens at the macroscale and elements at the substructural scale. This could help uncovering invisible or barely visible damages and thus help the establishment of generic description of damage evolution in terms on fatigue damage mechanism maps at a coarser length scale.

7.3. Multiscale modelling and multiscale testing
The multiscale nature of the damage evolution in composite materials and composite structures has lead to a multiscale approach to modelling of damage evolution. Clearly, some sort of averaging and loss of details are needed when transferring material laws from a small length scale to a coarser length scale. It remains a non-trivial challenge to establish rigorous correct connections between models at various length scales.

In parallel, as new materials testing methods will be established at the microscale, the testing at the macroscale by coupons and elements could shift from the measurement of "strength" properties and load-carrying capacity to check if the actual damage evolution is similar to the predicted damage evolution. This may require other, new generic test specimens designed for the study of damage evolution.

8. Conclusions
The development of very large composites structures such as wind turbine blades requires materials with high damage tolerance, such that manufacturing defects are of less importance. The use of models of fatigue damage evolution holds the potential to lead the development of future composite materials with enhanced damage tolerance. It is however still a challenge to measure the relevant mechanical properties at the microscale.

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Appendix A - J integral analysis of delamination from ply-drop of a tri-layer

8.1. Problem description

The problem to be analysed is shown in Fig. 10. It is a symmetric, three-layer specimen consisting of two surface layers enclosing a central layer, all with the width, $B$. All layers are taken to be made of linear-elastic materials. The surface layers are made of material #1 that has a Young’s modulus $E_1$ and a thickness $h_1$, and the Young’s modulus of the central layer (material #2) is $E_2$ and its thickness is $h_2$. Pre-existing debond cracks of identical lengths, $\ell_d$, exist from the left hand side. The Mode II interfacial fracture energy of the debond crack tip is denoted $\Gamma_{IIc}$. It is here assumed that the second Dundurs’ parameter is zero, so that the complex stress intensity factor decouples into traditional Mode I and Mode II stress intensity factors, so the pure Mode I and Mode II readily exists for the interface crack [18]. A shear stress, $\tau_x$, is present along the debonded interface; it can represent friction or crack bridging by fibres. A Cartesian coordinate system, $x_i$, is placed such that its origin is located at the tip of the upper delamination crack (see Fig. 10). A tensile stress, $\sigma$, is applied to the left hand end of the central layer. The problem will be analysed under plane stress condition (but can easily be extended to a plane strain condition). We aim to establish a relationship between $\sigma$ and $\ell_d$ during delamination crack growth by the use of the J integral [6].

8.2. Upstream and downstream stresses

Stresses far ahead of the crack tip ($x_1/(h_1+h_2) >> 1$) are called ”upstream stresses” and stresses in the debonded layers left of the crack tip ($-\ell_d < x_1 < 0$) are called ”downstream stresses”. The upstream stress (in the $x_1$-direction, i.e., stress component, $\sigma_{11}$) far ahead of the debond crack tip in the surface layer, is denoted $\sigma_{11}^+$ (subscript 1 indicates layer number, superscript + indicates upstream), and the upstream stress in the centre layer, $\sigma_{22}$, can be found by assuming
Figure 10. The analysed problem: A tri-layer specimen undergoing delamination with a shear stress \( \tau_s \) in the delaminated zone.

Figure 11. The integration paths for the J integral around the external boundaries of the upper half of the problem (a) and locally around the fracture process zone (b).

the same axial strain in all the layers and by elementary force balance

\[
\sigma_1^+ = \bar{\sigma} + \frac{\Sigma}{1 + 2\Sigma \eta} \quad \text{and} \quad \sigma_2^+ = \bar{\sigma} - \frac{1}{1 + 2\Sigma \eta}, \quad (35)
\]

where the non-dimensional parameters \( \Sigma \) and \( \eta \) are given by

\[
\Sigma = \frac{E_1}{E_2} \quad \text{and} \quad \eta = \frac{h_1}{h_2}. \quad (36)
\]

Downstream stresses at the debonded zone (\(-\ell_d < x_1 < 0\)) are modelled by a shear-lag approximation as

\[
\sigma_1^- = \tau_s \frac{x_1 + \ell_d}{h_1} \quad \text{and} \quad \sigma_2^- = \bar{\sigma} - 2\tau_s \frac{x_1 + \ell_d}{h_2} \quad \text{for} \quad -\ell_d \leq x_1 \leq 0. \quad (37)
\]

8.3. J integral analysis along external boundaries, \( \Gamma_{\text{ext}} \)

Due to symmetry, we only need to analyse the upper half of the specimen (\(x_2 > 0\)). The problem to be analysed is sketched in Fig. 11. First, we calculate the J integral evaluation along the external boundaries. The integration path is divided into 6 paths as indicated in Fig. 11.

For \( \Gamma_1 \), \( \sigma_{11} = \bar{\sigma} \), the rest of the stress components \( \sigma_{ij} = 0 \), the normal vector, \( n_j = (-1; 0; 0) \), the integration path increment \( dS = -dx_2 \). Furthermore, \( u_{11} = \varepsilon_{11} \) (here, \( u_1 \) and \( \varepsilon_{11} \) are the displacement and the normal strain in the \( x_1 \)-direction respectively, and comma denotes partial derivation). Using Hooke’s law for material \#2 (\( \varepsilon_{11} = \sigma_{11}/E_2 \)) we obtain

\[
J_1 = \int_0^{-h_2/2} \left( -\frac{\sigma^2}{2E_2} \right) dx_2 = \frac{\sigma^2 h_2}{4E_2}. \quad (38)
\]
For $\Gamma_2$, $dx_2 = 0$, and due to symmetry $\sigma_{12} = 0$ and $u_{2,1} = 0$ (where $u_2$ is the displacement in the $x_2$-direction) and $n_j = (0; -1; 0)$. As a result, $J_2 = 0$.

For path $\Gamma_3$ and $\Gamma_4$ we find

$$J_3 = -\frac{\bar{\sigma}^2 h_2}{4E_2 (1 + 2\Sigma\eta)^2} \quad \text{and} \quad J_4 = -\frac{\bar{\sigma}^2 h_2}{4E_2 (1 + 2\Sigma\eta)^2}.$$

(39)

For $\Gamma_5$ and $\Gamma_6$ we find $J_5 = J_6 = 0$. Collecting all $J$ integral terms ($J_1$ to $J_6$) gives

$$J_{\text{ext}} = \frac{\bar{\sigma}^2 h_2}{2E_2 (1 + 2\Sigma\eta)}.$$

(40)

### 8.4. $J$ integral analysis locally around delamination, $\Gamma_{\text{loc}}$

An application of the $J$ integral locally around the slip zone and the Mode II crack tip gives [7]

$$J_{\text{loc}} = \int_0^{\delta^*_t} \tau_s d\delta_t + J_{\text{tip}} = \tau_s \delta^*_t + J_{\text{tip}},$$

(41)

where $\delta^*_t$ is the end-sliding (displacement difference between upper and lower crack faces at $x_1 = -\ell_d$) and $J_{\text{tip}}$ is the $J$ integral evaluated around the crack tip.

Displacements can be obtained by integration of the strains $\epsilon_{11}$ (from $x_1 = 0$ to $x_1 = -\ell_d$). The downstream strains in material #1 and #2 can be obtained from the stresses $\sigma_{1}^-$ and $\sigma_{2}^-$ (eq. (37)) using Hooke’s law. Defining integrations constants such that there is zero displacement difference at the crack tip ($x_1 = 0$) leads to

$$\delta^*_t = \frac{\bar{\sigma}}{E_2} \ell_d - \frac{\tau_s \ell_d^2}{2E_1 h_1} (1 + 2\Sigma\eta).$$

(42)

Next, inserting $\delta^*_t$ from eq. (42) into eq. (41) and using eq. (36) leads to:

$$J_{\text{loc}} = \frac{\bar{\sigma} \tau_s \ell_d}{E_2} - \frac{\tau_s \ell_d^2}{2E_2 h_2} \frac{(1 + 2\Sigma\eta)}{\Sigma\eta} + J_{\text{tip}}.$$

(43)

### 8.5. Path-independence of the $J$ integral

Due to path-independence of the $J$ integral [6], $J_{\text{ext}} = J_{\text{loc}}$. Setting eq. (40) equal to eq. (43) gives

$$J_{\text{tip}} = \frac{\Sigma\eta}{2(1 + 2\Sigma\eta)} \frac{\bar{\sigma}^2 h_2}{E_2} - \frac{\bar{\sigma} \tau_s \ell_d}{E_2} + \frac{(1 + 2\Sigma\eta)}{2\Sigma\eta} \frac{\tau_s \ell_d^2}{E_2 h_2}.$$

(44)

During cracking, $J_{\text{tip}}$ must be equal to the Mode II interfacial fracture energy, $G_{\text{f}_{\text{IC}}}$. Setting $J_{\text{tip}} = G_{\text{f}_{\text{IC}}}$ in eq. (44) leads to the desired relationship between the applied stress and debond length during delamination cracking:

$$\bar{\sigma} = \frac{(1 + 2\Sigma\eta)}{\Sigma\eta} \frac{\tau_s \ell_d}{E_2 h_2} + \sqrt{\frac{2(1 + 2\Sigma\eta)}{\Sigma\eta} \frac{G_{\text{f}_{\text{IC}}}}{E_2 h_2}}.$$

(45)

For $\ell_d = 0$ (or $\tau_s = 0$), eq. (44) reduces to eq. (26) and eq. (45) reduces to eq. (25).

It was checked that the same results can also be obtained using the potential energy loss approach of Budiansky et al. [36] for large-scale frictional sliding problems (using that approach, $\tau_s$ was taken to be a frictional sliding shear stress).