Off-shell effects in the energy dependence of the $^7\text{Be}(p, \gamma)^8\text{B}$ astrophysical $S$ factor

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Abstract

I show that off-shell effects, like antisymmetrization and $^7\text{Be}$ distortions, can significantly influence the energy dependence of the nonresonant $^7\text{Be}(p, \gamma)^8\text{B}$ astrophysical $S$ factor at higher energies. The proper treatment of these effects results in a virtually flat $E_1$ component of the $S$ factor at $E_{cm} = 0.3-1.5$ MeV energies in the present eight-body model. The energy dependence of the nonresonant $S$ factor, predicted by the present model, is in agreement with the low-energy direct capture data and the existing high-energy Coulomb dissociation data. Irrespective of whether or not the present energy dependence is correct, off-shell effects can cause 15–20% changes in the value of $S(0)$ extrapolated from high-energy ($E_{cm} > 0.7$ MeV) data.

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The most uncertain nuclear input parameter in Standard Solar Models, which are used to calculate the solar neutrino fluxes on Earth, is the low-energy $^7$Be($p, \gamma$)$^8$B radiative capture cross section. This reaction produces $^8$B in the Sun, whose $\beta^+$ decay is the main source of the high-energy solar neutrinos. Many present (Homestake [1], Kamiokande [2], Superkamiokande [3]) and future (SNO [4]) solar neutrino detectors are sensitive mainly or exclusively to the $^8$B neutrinos. The theoretically predicted $^8$B neutrino flux is proportional to the very low-energy ($E_{cm} = 20$ keV; $E_{cm}$ is the $^7$Be–$p$ scattering energy in the CM frame) $^7$Be($p, \gamma$)$^8$B strophysical $S$ factor, $S_{17}$. Thus, precise knowledge of $S_{17}(20$ keV) is crucial to understand the solar neutrino problem [5].

Currently there is a considerable confusion concerning the value of $S_{17}(0)$. The six direct capture measurements performed to date give $S_{17}(0)$ between 15 eVb and 40 eVb, with a weighted average of 22.2 $\pm$ 2.3 eVb [5]. All these $S_{17}(0)$ values were determined by extrapolating from higher energies ($E > 100$ keV) where the experiments are feasible. The two lowest energy measurements by Kavanagh et al. [7] and by Filippone et al. [8] disagree by 25%.

Recently the Coulomb dissociation process was suggested as a promising new method to measure the cross sections of astrophysical processes [9]. Based on this technique, the $^7$Be($p, \gamma$)$^8$B cross section was studied recently by Motobayashi et al. using a radioactive $^8$B beam at RIKEN [10]. The extrapolated zero energy $S$ factor is $S_{17}(0) = 16.7 \pm 3.2$ eVb [10], or $S_{17}(0) = 15.5 \pm 2.8$ eVb [11] using s-wave [12], or $s + d$-wave [13] extrapolations, respectively. The theoretical implications of this experiment were studied in several papers, see e.g. [11,13–20]. Although the role of the $E2$ transitions in this process is heavily debated [14,17], it appears that, although there is substantial $E2$ strength present, the Coulomb dissociation measurement of [10] gives, in a good approximation, the $E1$ component of the cross section at the whole measured energy range [15,21]. The $M1$ transition is suppressed in [10] because of the low virtual $M1$ photon flux.

New $^8$B Coulomb dissociation experiments to study the $^7$Be($p, \gamma$)$^8$B cross section have been performed at RIKEN [22] and at MSU NSCL [23], and are being planned at GSI [22].

The theoretical predictions for $S_{17}(0)$ also have a huge uncertainty, as the various models give values between 16 eVb and 30 eVb [24]. The energy dependence of the $S$ factor was first studied by Christy and Duck [25] and by Tombrello [12] using only $s$-waves for the scattering states. Later the importance of $d$-waves was emphasized by Robertson [26], and especially by Barker [27,28]. Despite the large differences among the various theoretical predictions, it is now a common belief that the energy dependence of the nonresonant $S$ factor is known, and the differences come only from the absolute normalization. I point out in this paper that unfortunately this claim is not substantiated. Off-shell effects can significantly influence the energy dependence of the $S$ factor and thus also the extrapolation procedure, and the extracted value of $S_{17}(0)$.

The importance of off-shell effects was recently emphasized in the mirror reaction, $^7$Li($n, \gamma$)$^8$Li [29]. The thermal cross section of this reaction was used by Barker to argue that potential models tend to overestimate $S_{17}(0)$ [27]. In Ref. [29] it was pointed out that although the on-shell properties of the wave functions are well determined (e.g. $^7$Li+$p$ scattering lengths), a change in the unknown off-shell (internal) structures of the $^7$Li+$p$ scattering wave functions can drastically influence the thermal neutron capture cross section, while the same change leaves $S_{17}(0)$ virtually unaffected.
In order to reliably extrapolate the direct capture $S_{17}$ factors to zero energy, the energy dependence of the nonresonant $E1$ term should be known. The $E2$ cross section is not expected to play a role in the $E_{cm} = 0 - 3$ MeV energy range [30]. The low-energy ($E_{cm} < 300 - 400$ keV) direct capture measurements record probably pure $E1$ cross section, while at and above the $1^+$ resonance at $E_{cm} = 632$ keV they see a mixture of $E1$ and $M1$ transitions. As I mentioned above, the $^8B$ Coulomb dissociation measurement [10], in good approximation, gives the $E1$ component of the cross section at 0–2 MeV energies.

In the present work I concentrate on the $E1$ transition, which connects the $J^\pi=1^-, 2^-$, and $3^-$ $^7Be+p$ scattering states with the $2^+$ ground state of $^8B$. Using the eight-body model of Ref. [31] I show that, although not as much as in the case of $^7Li(p,\gamma)^8Li$, the internal structure of the wave functions can significantly influence the energy dependence of the $E1$ component of the $S$ factor.

A $^4He+^3He+p$ three-cluster eight-body RGM approach is used for both the $^8B$ ground state and the $^7Be+p$ scattering states. The eight-body wave functions have the form

$$\Psi = \sum_{I_7, I, l_2} \sum_{i=1}^{N_\tau} A \left\{ \left[ \Phi^\rho_{I_7} \Phi^7Be,i \right] J^\pi l_2 (\rho_2) \right\}_{J,M},$$

where $A$ is the intercluster antisymmetrizer, $\rho_2$ and $l_2$ is the relative coordinate and relative angular momentum between $^7Be$ and $p$, respectively, $s$ and $I_7$ is the spin of the proton and $^7Be$, respectively, $I$ is the channel spin, and $[\ldots]$ denotes angular momentum coupling. While $\Phi^\rho$ is a proton spin-isospin eigenstate, the antisymmetrized ground state ($i=1$) and continuum excited distorted states ($i>1$) of $^7Be$ are represented by the wave functions

$$\Phi^7Be,i_{I_7} = \sum_{j=1}^{N_\tau} c_{ij} \sum_{l_1} \sum_{l_2} A \left\{ \left[ \Phi^\alpha \Phi^h \right] _{l_1,i} \Gamma^i_{l_1} (\rho_1) \right\}_{I_7,M_7}.$$  

Here $A$ is the intercluster antisymmetrizer between $\alpha$ and $h$, $\Phi^\alpha$ and $\Phi^h$ are translationally invariant harmonic oscillator shell model states ($\alpha=^4He$, $h=^3He$), $\rho_1$ is the relative coordinate between $\alpha$ and $h$, $l_1$ is the $\alpha-h$ relative angular momentum, and $\Gamma^i_{l_1} (\rho_1)$ is a Gaussian function with a width of $\gamma_j$. The $c_{ij}$ parameters are determined from a variational principle for the $^7Be$ energy. In the $^8B$ ground state and $^7Be+p$ scattering states $N_\tau=6$, $l_1=1$, $I_7=3/2, 1/2$, $I=1,2$, and $l_2=0,1,2$ are used.

Putting (1) into the eight-body Schrödinger equation, which contains the Minnesota nucleon-nucleon interaction [31], we arrive at an equation for the unknown relative motion functions $\chi^i_{l_1} (\rho_2)$. These relative motion functions are determined from a variational Siegert method [32] for the $2^+$ bound groundstate of $^8B$, and from the Kohn-Hulthén variational method [33] for the $1^-, 2^-$, and $3^-$ $^7Be+p$ scattering states [31].

In (1) we neglect the $h(\alpha p)$ and $\alpha(hp)$ type three-body rearrangement channels. In [31] it was shown that the present model (called $^7Be+p$ type model space in [31]) and a model which contains these rearrangement channels (called full model space in [31]) lead to similar results, provided the subsystem properties (e.g., the $^7Be$ quadrupole moment) are reproduced equally well. The presence of these rearrangement channels would not change the conclusions of the present work. Further details of the model, parameters, $N-N$ interaction, etc. can be found in [31] and [34].
The $E1^7\text{Be}(p, \gamma)^8\text{B}$ radiative capture cross section is given by [35]

$$
\sigma(E_{cm}) = \sum_{I_7} \frac{1}{(2I_7 + 1)(2s + 1)} \frac{16\pi}{3h} \left(\frac{E_{\gamma}}{\hbar c}\right)^3 \times \sum_{l_\omega, l_\omega} (2l_\omega + 1)^{-1} |\langle \Psi_{I_7} || M^E_1 || \Psi_{I_7, l_\omega} \rangle|^2,
$$

where $I_7$ and $s$ are the spins of the colliding clusters, $M^E_1$ is the electric dipole ($E1$) transition operator, $\omega$ represents the entrance channel, $E_{\gamma} = E_{cm} + 0.137 \text{ MeV}$ is the photon energy, and $J_f$ and $J_i$ is the total spin of the final and initial state, respectively. The initial wave function $\Psi_{I_7, l_\omega}$ is a partial wave of a unit-flux scattering wave function.

As far as off-shell properties are concerned, the main differences between the present model and the potential models, e.g. [36], are the following: (i) the presence of the $^7\text{Be}–p$ antisymmetrization in (1); (ii) the presence of the continuum excited $^7\text{Be}$ distortion channels in (1); (iii) the node positions in the scattering states can be different. The node positions are determined by nucleon exchange and antisymmetrization effects, so only the microscopic models are firm ground in this respect. All the above differences have short range effects.

In Fig. 1 I show the $^7\text{Be}(p, \gamma)^8\text{B}$ astrophysical $S$ factor coming from the present eight-body model (solid line) together with the results of the direct capture [7,8,37,38] and Coulomb dissociation [10] experiments, and a typical $s+d$-wave potential model $S$ factor (dotted line) from [36]. The $S$ factor, as a function of the $^7\text{Be}–p$ scattering energy, is defined as

$$
S(E_{cm}) = \sigma(E_{cm})E_{cm} \exp\left[2\pi\eta(E_{cm})\right],
$$

where $\eta$ is the Sommerfeld parameter.

One can see that the energy dependence of the $S$ factor in the present model is rather different from that in a potential model. The latter is often referred in the literature as the “known energy dependence of the $S$ factor”. Note that the present model predicts a virtually flat $S$ factor at $E_{cm} = 0.3 – 1.5 \text{ MeV}$, despite the presence of $d$-waves in the scattering wave functions. The two other nucleon-nucleon forces used in Ref. [31] give very similar results, except that the $S$ factor coming from the $V2$ force starts to rise at around $1.0 \text{ MeV}$.

To show that the difference between the present model and the potential model comes from the above-mentioned off-shell effects, the antisymmetrization, which has the biggest effect, is switched off in (3). In other words we take the relative motion wave functions $\chi$ behind the antisymmetrizer in (1) and use them as if they were coming from a potential model. Thus (3) reduces to one-dimensional integrals. The resulting $S$ factor is shown by the dashed line in Fig. 1. Its energy dependence is much closer to the energy dependence coming from the potential model, although its slope is smaller. This is not surprising as there still are off-shell differences between the two models, namely the $^7\text{Be}$ distortion states in (1) and the different node positions.

Irrespective of the fact whether or not the solid line in Fig. 1 represents the “true” energy dependence of the $S$ factor, differences in model predictions for the energy dependence, at least as big as the difference between the solid and dashed lines, can be expected from off-shell effects. As these effects are completely missing in potential models, their energy dependence cannot be accepted as “the known energy dependence of the $S$ factor”. The
microscopic models are also not free from defects, unfortunately. In some cases, for instance, the description of \(^{7}\text{Be}\) in the asymptotic region is unsatisfactory, in others the \(^{7}\text{Li}+\alpha\) scattering lengths (and supposedly also the \(^{7}\text{Be}+p\) ones) are wrong. The present model is also not perfect because, e.g., as in [31] the scattering wave functions are calculated from uncoupled-channel models. My test calculations show that this has little effect on the results, nevertheless an improvement would be desirable. What the present model does not contain, but what can prove to be important, are the dynamical degrees of freedom beyond the three-cluster model. Hopefully fully dynamical ab initio [39] eight-body calculations for this reaction will be feasible in the future.

I would like to emphasize again, that whether or not the present model will prove to be a good description of the \(^{7}\text{Be}(p, \gamma)^{8}\text{B}\) reaction, Fig. 1 shows the amount of change in the \(S\) factor energy dependence one can expect from off-shell effects.

Finally, I would like to point out that the energy dependence predicted by the present model (solid line in Fig. 1) is in agreement with the low-energy direct capture data, and the high-energy Coulomb dissociation measurements. I note, that in order to reproduce the absolute normalization of the data, the different experiments would require different renormalizations of the theoretical curve. There is no physical reason for such renormalizations in the present model; the zero energy \(S\) factor is \(S_{17}(0) = 25\) eVb. The energy dependence of the solid line is in disagreement with the direct capture results above 0.7 MeV, if it is forced to fit the low-energy data points. In some direct capture experiments, e.g., in Ref. [8], the parameters of the 632 keV resonance were determined by fitting the data with a Breit-Wigner form (with energy dependent width) after a nonresonant part was removed using the energy dependence of a potential model, e.g. [2]. Assuming a different nonresonant \(E1\) energy dependence, the resonance parameters could slightly be different, modifying the experimental \(M1\) contribution at 0.7–1.0 MeV. I also note, that [30] and [40] find a \(1^+\) state in \(^{8}\text{B}\) at around 1.3–1.4 MeV, and there is a \(3^+\) state at 2.2 MeV [11], which can cause \(M1\) bumps at these energies (see e.g. [31]). The present model also predicts a second low-lying \(1^+\) state in \(^{8}\text{B}\) above the 632 keV resonance. Thus, as the direct capture results come from an \(E1+M1\) mixture at higher energies, the disentanglement of these components may prove to be difficult.

In summary, using an eight-body model of the \(^{7}\text{Be}(p, \gamma)^{8}\text{B}\) reaction, I have shown that off-shell effects can considerably change the energy dependence of the of the astrophysical \(S_{17}\) factor. The model predicts an \(S\) factor which is virtually flat in the 0.3–1.5 MeV energy range, in strong contrast to the “standard” potential model energy dependence. I have demonstrated that using the \(^{7}\text{Be}–p\) relative motion wave functions, which are behind the antisymmetrizer in the microscopic model, as if they came from a potential model, the energy dependence of the \(S\) factor becomes similar to the “standard” shape. Thus, the present unusual energy dependence of the \(S\) factor is caused mainly by the antisymmetrization, the biggest off-shell effect. I have argued that because the direct capture experiments contain a mixture of \(E1\) and \(M1\) transitions for \(E_{\text{cm}} > 0.5\) MeV, the knowledge of the true energy dependence of the \(E1\) \(S\) factor is crucial to the extrapolation of the high-energy (\(E_{\text{cm}} > 0.7\) MeV) data to zero energy.

The energy dependence predicted by the present microscopic model is in agreement with the low-energy direct capture data and the high-energy Coulomb dissociation results. However, I must point out that this agreement might only be fortuitous, and the present
energy dependence of the $S$ factor might prove to be incorrect, using a larger eight-body model space. Nevertheless, off-shell effects, at least as big as in Fig. 1, should be expected in the $^7\text{Be}(p,\gamma)^8\text{B}$ reaction. This can change the zero energy $S_{17}$, extrapolated from high-energy data, by as much as 15–20%.

Note: In a very recent preprint [42] it was pointed out that in the potential model the energy dependence of the $E1$ $S$ factor is strongly influenced by short range effects, e.g., by $^7\text{Be}$ core deformations.

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FIGURES

FIG. 1. Astrophysical $S$ factor for the $^7$Be($p, \gamma$)$^8$B reaction. The experimental data are from the direct capture measurements [7,8,37,38] and from the Coulomb dissociation of $^8$B [10]. The solid and dashed lines are the $E1$ components of the $S$ factors in the present eight-body model with and without antisymmetrization in the electromagnetic transition matrix element, respectively. The dotted line is the result of the potential model [36].
