Skyrme Branes

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Abstract

We obtain static selfgravitating solitonic 3-brane solutions in the Einstein-Skyrme model in 7D. These solitons correspond to a smooth version of the previously discussed cosmic p-brane solutions. We show how the energy momentum tensor of the Skyrme field is able to smooth out the singularities found in the thin wall approximation and falls fast enough with the distance from the core of the object so that asymptotically approaches the flat cosmic p-brane metric.

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I. INTRODUCTION

Models with extra dimensions have become intensively studied in the last few years, not only in string theory but in many other extensions of the standard model. Branes have played a major role on many of these models and have therefore been studied from several different perspectives. Some of these branes have a description in terms of solitons in the low energy theories as either domain walls, strings, monopoles, or even higher codimension objects. One of the earliest models of particle physics to incorporate solitons to its spectrum, the Skyrme model, has so far not been investigated in connection to the braneworld scenarios. The purpose of this letter is to study braneworld models based on the higher dimensional generalization of the Einstein-Skyrme model.

One of the most interesting properties of the Skyrme model that makes it different from all the other topological defects mentioned above, is the presence of higher order corrections to the kinetic term. Recently there has been several studies on topological defects with non-canonical kinetic terms. Most of those models have been focusing on the differences introduced in the soliton solutions by the new term in the lagrangian. On the other hand, our model does not have a potential term in contrast to all the other cases studied in the literature of braneworld scenarios. Furthermore, in the absence of the Skyrme term, the theory does not present any smooth stable configuration even though there is a topological charge that one can define. It is only because of this new term that one is able to have a finite size stable configuration avoiding in this way the straightforward extension of Derrick’s theorem. It is then clear that the higher order terms in our Lagrangian are crucial for the solutions presented here and are not just a small correction to the model.

On the other hand, the particular structure of the theory that we study here gives rise to solitonic solutions that are very much localized in the transverse directions. This property makes them good candidates to describe a smooth version of extended solutions previously studied only in the their thin wall limit. In fact, we will show that our thick brane solutions in the Einstein-Skyrme model asymptotically match the higher dimensional vacuum solutions of the pure Einstein’s equations previously found.
II. THE EINSTEIN-SKYRME MODEL IN 7D

The action for our model is given by,

\[ S_{ES} = \int d^7x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R + \mathcal{L}_S \right], \tag{1} \]

where \( R \) is the Ricci scalar, \( \kappa^2 = 1/M_7^2 \), with \( M_7 \) denoting the 7-dimensional Planck mass, and \( \mathcal{L}_S \) is the Skyrme Lagrangian density,

\[ \mathcal{L}_S = \frac{F_0^2}{4} Tr(L_AL^A) + \frac{1}{32e^2} Tr([L_A, L_B][L^A, L^B]), \tag{2} \]

where \( F_0 \) and \( e \) are two free parameters of the model with units of \( [M]^{5/2} \) and \( [M]^{-3/2} \) and

\[ L_A \equiv U^\dagger \partial_A U, \tag{3} \]

is the left chiral current and \( U \in SU(2) \).

We are interested in finding the smooth solution for a 3-brane that is spherically symmetric along the transverse directions in the bulk. We will also restrict ourselves to the four dimensional flat brane solutions. Taking these constraints into account we can now write the most general metric of this form in the isotropic gauge as,

\[ ds^2 = B^2(r)\eta_{\mu\nu}dy^\mu dy^\nu + C^2(r) (dr^2 + r^2 d\Omega^2), \tag{4} \]

where \( \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1) \).

We also impose the hedgehog ansatz for the chiral field, which is the natural spherically symmetric ansatz for Skyrme model; that is, we assume the following form for \( U(r) \),

\[ U(r) = \cos f(r) + i \left( \frac{r^j}{r} \right) \tau^j \sin f(r), \tag{5} \]

where \( f(r) \) is the profile function to be solved for and \( \tau^j \) with \( j = 1, 2, 3 \), are the Pauli matrices.

Within this ansatz, the Lagrangian density then becomes,

\[ \mathcal{L}_S = \frac{-F_0^2}{2} \left[ \frac{1}{C^2(r)} \left( \frac{df}{dr} \right)^2 \left( 1 + \frac{2\sin^2 f}{e^2F_0^2C^2(r)r^2} \right) + \frac{\sin^2 f}{C^2(r)r^2} \left( 2 + \frac{\sin^2 f}{e^2F_0^2C^2(r)r^2} \right) \right]. \]

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1 We use the following notation. The upper case latin indices \( A, B \) run over \( 0, ..., 6 \) and the greek indices \( \mu, \nu = 0, ..., 3 \) denote the four dimensional spacetime coordinates. We use the the mostly positive signature and the Riemann tensor conventions of the form, \( R_{BCD}^A = \partial_C \Gamma_{BD}^A - \partial_D \Gamma_{BC}^A + ... \).
It is convenient at this point to rescale the radial coordinate $e F_0 r \to x$ and define,

\[ u \equiv \frac{1}{C^2(x)} \left( 1 + \frac{2 \sin^2 f}{C^2(x) x^2} \right), \]

\[ v \equiv \frac{\sin^2 f}{C^2(x) x^2} \left( 2 + \frac{\sin^2 f}{C^2(x) x^2} \right), \]

to obtain

\[ \mathcal{L}_S = \frac{-e^2 F_0^4}{2} (u f'^2 + v). \]  \hspace{1cm} (6)

Having simplified the Lagrangian, the action for the Skyrme field becomes,

\[ S_S = \int d^7 X \sqrt{-g} \mathcal{L}_S = \frac{-2\pi F_0}{e} \int (u f'^2 + v) B^4(x) C^3(x) x^2 dxdy. \]  \hspace{1cm} (7)

It is now straightforward to obtain from this action the equation of motion for the field $f(x)$ in the static case which is given by,

\[ f''(x) = \frac{1}{2u} \left( u f'^2(x) + v_f \right) - \left[ \frac{4 B'(x)}{B(x)} + \frac{3 C'(x)}{C(x)} + \frac{u'}{u} + \frac{2}{x} \right] f'(x), \]  \hspace{1cm} (8)

where

\[ u_f \equiv \frac{\delta u(x)}{\delta f(x)}, \]  \hspace{1cm} (9)

\[ v_f \equiv \frac{\delta v(x)}{\delta f(x)}, \]  \hspace{1cm} (10)

and the primes denote the derivatives with respect to $x$. On the other hand, varying the Skyrme action with respect to the metric tensor $g_{AB}$ yields

\[ T_{AB} = -\frac{2}{\sqrt{-g}} \frac{\delta S_S}{\delta g^{AB}}, \]  \hspace{1cm} (11)

where $T_{AB}$ is the energy-momentum tensor for the scalar field given by

\[ T_{AB} = g_{AB} \mathcal{L}_S - \frac{F_0^2}{2} Tr(L_A L_B) - \frac{1}{8e^2} g^{MN} Tr([L_A, L_M][L_B, L_N]). \]

In our ansatz it becomes,
In the following, we will consider the single charged soliton solution (G equations of the form, using this energy-momentum tensor and our ansatz for the metric, (4), we obtain Einstein’s that can be written in terms of an integral over the extra-dimensional space as,

\[
T^\mu_\nu = -\frac{e^2 F_0^4}{2} (uf^2 + v) \delta^\mu_\nu \\
T^x_x = \frac{e^2 F_0^4}{2} (uf^2 - v) \\
T^\theta_\theta = \frac{e^2 F_0^4}{2C^2(x)} \left(-f'^2 + \frac{\sin^4 f}{C^2(x)x^4}\right) \\
T^{\phi}_\phi = T^\phi_\phi.
\]

Using this energy-momentum tensor and our ansatz for the metric, [4], we obtain Einstein’s equations of the form,

\[
G^\mu_\nu = e^2 F_0^2 \delta^\mu_\nu \left[ \frac{3B''(x)}{B(x)C^2(x)} + \frac{2C''(x)}{B^2(x)C^2(x)} + \frac{3B^2(x)}{C^2(x)} - \frac{C'^2(x)}{C^4(x)} - \frac{6B'(x)}{B(x)C^2(x)x} + \frac{4C'(x)}{C^3(x)x} + \frac{3B'(x)C'(x)}{B(x)C^3(x)} \right] \\
= -\frac{k^2 e^2 F_0^4}{2} (uf'^2 + v) \delta^\mu_\nu \\
C^x_x = e^2 F_0^2 \left[ \frac{8B'(x)}{B(x)C^2(x)x} + \frac{6B^2(x)}{B^2(x)C^2(x)} + \frac{2C'(x)}{xC^3(x)} + \frac{8B'(x)C'(x)}{B(x)C^3(x)} + \frac{C'^2(x)}{C^4(x)} \right] \\
= \frac{k^2 e^2 F_0^4}{2} (uf'^2 - v) \\
C^\theta_\theta = e^2 F_0^2 \left[ \frac{4B''(x)}{B(x)C^2(x)} + \frac{C''(x)}{C^3(x)} + \frac{6B^2(x)}{B^2(x)C^2(x)} - \frac{C'^2(x)}{C^4(x)} + \frac{4B'(x)}{B(x)C^2(x)x} + \frac{C'(x)}{C^3(x)x} \right] \\
= \frac{k^2 e^2 F_0^4}{2C^2(x)} \left(-f'^2 + \frac{\sin^4 f}{C^2(x)x^4}\right)
\]

Eqs. (13) and (5) constitute the equations of motion for Einstein-Skyrme model consistent with the restrictions imposed by our ansatz.

III. NUMERICAL RESULTS

We want to find solutions for solitonic objects characterized by a topological charge Q that can be written in terms of an integral over the extra-dimensional space as,

\[
Q = \frac{\epsilon^{ijk}}{24\pi^2} \int \text{tr} \left( L_i L_j L_k \right) C^5(r) r^2 dr d\Omega_2 = -\frac{2}{\pi} \int \sin^2 f df = -\frac{2}{\pi} \left[ \frac{f}{2} - \frac{\sin 2f}{4} \right]_{f(0)}^{f(\infty)}.
\]

One can see that fixing the charge specifies the boundary conditions for our function f(r). In the following, we will consider the single charged soliton solution (Q = 1), which means
that we will take the following boundary conditions for the scalar field function \( f(r) \),

\[
f(0) = \pi \quad f(\infty) = 0. \tag{15}
\]

We want to integrate our equations of motion starting from the core of the defect, so we still need to specify the conditions for the metric functions at \( x = 0 \). We demand that our initial data at the origin does not have any singularity which in turn means that the most general expansion for the metric functions at \( x = 0 \) should be of the form,

\[
B(x) = B_0 + B_2 x^2 + O(x^4)
\]

\[
C(x) = C_0 + C_2 x^2 + O(x^4)
\]

\[
f(x) = \pi + f_1 x + f_3 x^3 + O(x^4). \tag{16}
\]

Using this expansion in Einstein's equations, \([13]\), one can see that the higher order coefficients can be obtained in terms of the parameters \( f_1, B_0, C_0, \hat{\kappa}^2 \equiv \kappa^2 F_0^2 \) and are given at the lowest order by,

\[
B_2 = \frac{B_0 f_1^4 \hat{\kappa}^2}{10 C_0^2}
\]

\[
C_2 = -\frac{f_1^2(5C_0^2 + 11 f_1^2)\hat{\kappa}^2}{40 C_0}
\]

\[
f_3 = -\frac{f_1^3(162 f_1^4 \hat{\kappa}^2 - 5 C_0^4(-16 + 3 \hat{\kappa}^2) + 5 C_0^2 f_1^2(8 + 9 \hat{\kappa}^2))}{600(C_0^4 + 2 C_0^2 f_1^2)}
\]

We numerically solve the system of equations \([13]\) and \([8]\) using the shooting method, i.e. adjusting \( f_1, B_0 \) and \( C_0 \) such that the asymptotical solutions satisfy \( f(\infty) = 0 \) and \( B(\infty) = C(\infty) = 1 \). We show in Figs. 1-3, a sample of the numerical solutions found by this procedure.\(^2\)

We found that similarly to what happens to the Einstein-Skyrme model in 4D \([10, 11]\) there is a critical value of \( \hat{\kappa} \), \( \hat{\kappa}_c \), beyond which no more regular solutions exist. Our numerical investigation reveals that this critical value is around the order of \( \hat{\kappa}_c^2 \sim \frac{1}{20} \).

\(^2\) Note that we integrate the solutions to a much longer range in \( x \) where we clearly see the convergence of the different functions to their asymptotic values. We only plot a limited range in order to show the smooth structure of the soliton in the core region.
Another interesting result is that for each value of $\hat{\kappa} < \hat{\kappa}_c$ we have two branches of solutions. At $\hat{\kappa} = \hat{\kappa}_c$, the two branches merge (See Fig. 4) such that beyond this point we always find a singularity at some value of $x$. This is another feature that is shared by the $4D$ system [11].

The numerical solutions we found asymptotically approach flat space, and it is therefore possible to identify the form of the energy momentum tensor that sources these metrics, in a similar way to what one does in $4D$ spacetime [12]. Following [13, 14] we obtain,
FIG. 3: Typical form of the $f(x)$ function for the smooth 3-brane solutions.

FIG. 4: Two fundamental branches of soliton solutions. The shooting parameter $f_1$ is plotted as a function of $\hat{\kappa}^2$. We see how the two solutions merge at $\hat{\kappa} = \hat{\kappa}_c$.

\[ T_{\mu\nu}^{ADM} = \lim_{r \to \infty} \frac{1}{2\kappa^2} \int \hat{r}^i [\eta_{\mu\nu} (\partial_i h_{\sigma}^\sigma + \partial_i h_j^j - \partial_j h_i^i) - \partial_i h_{\mu\nu}] r^2 d\Omega_2 \tag{17} \]

where $h_{AB} = g_{AB} - \eta_{AB}$, denotes the deviation of our metric from flat space, $\hat{r}^i$ is the radial unit vector in the transverse 3-dimensional space and $\mu, \nu, \sigma = 0, ..., 3$ and $i, j = 4, 5, 6$. Using the expression of our ansatz we obtain in our case,
\[
T^{ADM}_{\mu\nu} = \frac{8\pi}{\kappa^2} \eta_{\mu\nu} \lim_{r \to \infty} \left[ r^2 (3B(r)B'(r) + 2C(r)C'(r)) \right] = -T_{ADM} \eta_{\mu\nu}
\]

It is then clear from this calculation that we can read off the value of the tension of the 3-brane source from the asymptotic behaviour of the metric. We show in Table I, our numerical results for the tension \(T_{ADM}\) computed from the asymptotic form of the numerical functions \(B(x)\) and \(C(x)\), together with the values of the shooting parameters \(B_0, f_1, C_0\), for a range of \(\kappa\) values. The subscript \(u\) denotes the upper branch in Fig. 4.

The occurrence of the two branches in 4D has been claimed to be linked to the existence of similar solutions in the Einstein-Yang-Mills system in 4D [15]. It is likely that a similar situation may arise in our higher dimensional case.

### IV. COSMIC 3-BRANES IN THE ISOTROPIC GAUGE

As we discussed in the introduction, our brane solutions have the same symmetry as the cosmic 3-brane gravity solutions discussed by Gregory [9]. Furthermore our branes are not charged with respect to any long range field, and therefore their energy momentum tensor is very well localized. It is then tempting to identify our solutions with the smooth
out version of the cosmic 3-brane examples. This possibility was in fact already suggested by Gregory in [9]. In the following we will prove that this is indeed the case, by showing that the metric solutions found numerically in the previous section match asymptotically the thin wall vacuum solutions found by Gregory. Let us briefly review the cosmic 3-brane geometries. The solutions of vacuum Einstein equations found in [9] relevant for us are of the form,

\[ ds^2 = F(\hat{r})^2 \eta_{\mu\nu}dy^\mu dy^\nu + F(\hat{r})^2 d\hat{r}^2 + \hat{r}^2 F(\hat{r})^2 d\Omega^2 \tag{19} \]

where

\[ F(\hat{r}) = 1 - \frac{\hat{r}_0}{\hat{r}} \tag{20} \]

and \( \alpha = \frac{1}{2\sqrt{10}}, \beta = -\frac{2}{\sqrt{10}} \) and \( \gamma = \frac{1+2\beta}{2} \).

These analytic solutions are written in a different gauge from the numerical ones found in the previous section, but it is always possible to transform them into the isotropic gauge of the form,

\[ ds^2 = B(r)^2 \eta_{\mu\nu}dy^\mu dy^\nu + C(r)^2 (dr^2 + r^2 d\Omega^2) \tag{21} \]

In this gauge, the 3-brane vacuum solutions become,

\[ ds^2 = \left(\frac{4r - r_0}{4r + r_0}\right)^2 \eta_{\mu\nu}dy^\mu dy^\nu + \left(\frac{4r - r_0}{4r}\right)^4 \left(\frac{4r + r_0}{4r - r_0}\right)^{2+\frac{8}{\sqrt{10}}} (dr^2 + r^2 d\Omega^2) \tag{22} \]

Using the asymptotic form of this metric and the expression for the ADM energy-momentum tensor found in [13] we arrive at,

\[ T^{ADM}_{\mu\nu} = -\frac{\sqrt{10}\pi}{\kappa^2} r_0 \eta_{\mu\nu} \tag{23} \]

which depends on the single parameter \( r_0 \) that completely characterizes the 3-brane metric solution. We give in Table 1 the results obtained for this parameter in our numerical examples. Note that our numerical solutions are smooth everywhere contrary to the analytic expressions in Eq. (22) that have naked singularities located at \( r = r_0 \) and at \( r = 0 \). On the other hand, we will show that the analytic solutions are in fact a good approximation to our
numerical results in the asymptotic region where $r >> r_0$ but start deviating from them as $r \sim r_0$. In order to see this, we proceed in the following way.

We first identify, using Eq. (18) and Eq. (23), the value of the parameter $r_0$ by looking at the asymptotic form of the ADM stress tensor of our numerical solution. Once this parameter is fixed, it singles out a particular member within the family of solutions given by Eq.(22). Using this parameter we now plot in Figs. 5 and 6 the function $B(x)$ and $C(x)$ and compare them with the same functions in the numerical calculations. As we see the asymptotic form of the two functions, the numerical ones and the analytic vacuum solutions, are in perfect
agreement so we conclude that our numerical solutions do, in fact, match their thin wall counterparts.

V. CONCLUSIONS

We have constructed, using numerical techniques, static 3-brane solitonic solutions in the 7D Skyrme model coupled to gravity. We have shown that their asymptotic form can be well approximated by the analytic vacuum solutions of pure Einstein theory obtained in [9]. The presence of the energy momentum tensor that makes up the core of the brane is able to smooth out the singularities that show up in the analytic case, rendering these solutions completely smooth.

The Skyrme branes obtained in this paper are asymptotically flat and therefore represent a good candidate for regular braneworlds in the DGP model in 7D [16, 17]. It is also clear that one could generalize our Skyrme model to higher dimensions in order to accommodate the DGP braneworld models of higher codimension within a similar framework to the one studied here. On the other hand, the properties of the gravitational sector of the braneworld may be affected by the details of the brane core [14] so it would be interesting to test these ideas within our model.

Furthermore, we have found that there is a maximum value of the coupling constant beyond which smooth solutions are not possible anymore. It is not clear what type of objects one should obtain for larger values of the coupling constant. One possibility is to relax the staticity of the metric. This seems to suggest that the branes would start to inflate in a similar way to the solutions found in [18, 19, 20]. This is certainly a possibility although we note that the situation is slightly different from the usual defect solutions since we do not have any potential energy in our model, so the possibility of having topological inflation [21] at the core does not seem very likely.

We have also found that there are two branches of solutions. We expect, as it happens in the 4D case, that the lower (upper) branch corresponds to a stable (unstable) configuration, although the stability calculation has to be performed taking into account the new possible channels opened due to the extradimensional nature of the solutions we have.

We hope to come back to these and other interesting issues for Skyrme Branes in a future publication.
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