Gossamer superconductivity and the mean field approximation of a new effective Hubbard model

Yue Yu
Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100080, China
(November 21, 2018)

We construct a new effective two-dimensional Hubbard model by taking the different electron occupancy on site into account. The mean field state of the new Hamiltonian gives rise to the gossamer superconducting state proposed by Laughlin recently [1].

PACS numbers:74.20.-z, 74.20.Mn, 72.-h, 71.10.Fd

Since the high temperature superconductor of Cu-O cuprate was discovered, the mechanism of the superconductivity has been attracting much research interesting due to various unusual properties of the high $T_c$ superconductor. A well-known model trying to describe the normal state properties is the 2-dimensional Hubbard model or t-J model [2,3]. Many analytical and numerical investigations on these two models have been carried out [4]. Anderson proposed a resonant valence bond (RVB) state as a mean field state of the models to explain the undoped cuprate as a Mott insulator while the superconductor is thought as a doped Mott insulator [5]. Although many progresses have been made along the clues of the Hubbard model and t-J model, there is no persuasive evidence that the theory of the high $T_c$ superconductivity of the cuprates can firmly based on these two models and various extended models of them. Especially, the quantum antiferromagnetic state may always be favorable in these two-dimensional models for a small doping.

In a recent work, Laughlin proposed a new scenario for the high $T_c$ superconducting cuprate, called the gossamer superconductivity [1]. The basic notion is that, instead of the full projection in the RVB state, which forbids the double-occupancy of electrons on a site, a partial projection acting on a BCS-type wave function is introduced. If the partial projection is not far from the full projection, one has a very tiny superfluid density. This is called the gossamer superconductivity. Laughlin’s explicit microscopic wave function for the gossamer superconductor reads

$$
|\Psi_G\rangle = \prod_i (1 - \alpha n_{i\uparrow} n_{i\downarrow}) |\Psi_{BCS}\rangle,
$$

$$
|\Psi_{BCS}\rangle = \prod_k (u_k + \xi_k c_k\uparrow c_k\downarrow |0\rangle),
$$

where $c_{k\sigma}$ is the Fourier component of the electron creation operator $c_{i\sigma}^\dagger$ on a two-dimensional lattice; $n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$ is the electron number operator at site $i$. The projection operator $\Pi_\alpha = \prod_i (1 - \alpha n_{i\uparrow} n_{i\downarrow})$ for $0 \leq \alpha \leq 1$ is the partial Gutzwiller projection which has an inverse $\Pi_{\alpha}^{-1} = \prod_i (1 + \beta n_{i\uparrow} n_{i\downarrow})$ if $\alpha < 1$ and $\beta = \alpha/(1-\alpha)$. The BCS superconducting state $|\Psi_{BCS}\rangle$ is defined as usual by

$$
u_k^2 = \frac{1}{2} (1 - \frac{\xi_k}{E_k}),
$$

$$u_k v_k = \frac{\Delta_k}{2E_k},
$$

where $\xi_k = c_k - \mu$ and $E_k = \sqrt{\xi_k^2 + \Delta_k^2}$ for the electron dispersion $c_k$, chemical potential $\mu$ and the superconducting gap $\Delta_k$. It has been pointed out that the state $|\Psi_G\rangle$ may be superconducting even at half filling. Furthermore, Laughlin has found that the state $|\Psi_G\rangle$ is the exact ground state of the model Hamiltonian

$$H_G = \sum_k E_k \tilde{b}_k \tilde{b}_k^\dagger,
$$

where $\tilde{b}_k = \Pi_\alpha b_k \Pi_\alpha^{-1}$ for $b_k\uparrow = u_k c_k\uparrow + v_k c_{-k}\downarrow$ and $b_k\downarrow = u_k c_k\downarrow - v_k c_{-k}\uparrow$ annihilate the BCS state. Explicitly, $\tilde{b}_k$ reads

$$
\tilde{b}_k\uparrow = \frac{1}{\sqrt{N}} \sum_j e^{ikr} [u_k (1 + \beta n_{j\downarrow}) c_j\uparrow + v_k (1 - \alpha n_{j\uparrow}) c_j\downarrow],
$$

$$
\tilde{b}_k\downarrow = \frac{1}{\sqrt{N}} \sum_j e^{ikr} [u_k (1 + \beta n_{j\uparrow}) c_j\downarrow - v_k (1 - \alpha n_{j\downarrow}) c_j\uparrow].
$$

Laughlin has shown that for any magnitude of $\alpha$, the quasiparticle energies remain at $E_k$, which indicates the pseudogap phenomenon. Right following up Laughlin’s work, Zhang has checked the gossamer superconductivity in a more realistic effective Hubbard model [4]. It was found that the gossamer superconducting state is similar to the RVB superconducting state, except that the chemical potential is approximately pinned at the mid of the two Hubbard bands away from the half filling.

Ref. [1] indicated that the gossamer superconducting state may possibly be a good variational state of the effective Hubbard model. Nevertheless, it is still in question to make an explicit relation between the Hamiltonian [1] and that of the effective Hubbard model used in ref. [4]. In this paper, we try to provide a new effective Hubbard model which is the extension of the Hubbard model and t-J model. Our extending method is other than all the previous extended models of those two models. We consider the case that the lattice sites are allowed to be
where we have included the chemical potential term and Hamiltonian of the effective Hubbard model there is a gossamer superconducting phase in the system is that the Laughlin gossamer superconducting state may be new by employing such a new effective Hubbard model the hopping may be dependent on the occupancy of the sites (Fig. 2) while cause of the on-site Coulomb repulsion between electrons, double-occupied by electrons with opposite spins. Be-

To extend the Hubbard model, we first write down the Hamiltonian of the effective Hubbard model

\[ H_{ch} = - \sum_{\sigma \neq j, \sigma} (t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i, \sigma} n_{i\sigma} + \sum_{\sigma \neq j} J_{ij} (S_i \cdot S_j - \frac{1}{4} n_i n_j), \] 

where we have included the chemical potential term and \( S = \frac{1}{2} c^\dagger \tau_{\sigma\sigma'} c_{\sigma'} \) is the local spin operator. One can rewrite this Hamiltonian as

\[ H_{ch} = - \sum_{\sigma \neq j, \sigma} (t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + h.c.) - \frac{1}{2} \sum_{i \neq j} \hat{D}_{ij} J_{ij} \hat{D}_{ij} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i, \sigma} n_{i\sigma}, \]

where \( \hat{D}_{ij} = c_{i\uparrow} c_{j\downarrow} - c_{i\downarrow} c_{j\uparrow} \) is the pairing operator. Now, we extend this model according to our consideration mentioned above. Due to the different occupant situations as shown in Fig. 1, one can extend \( t_{ij} \) to be an operator \( \hat{t}_{ij, \sigma} \):

\[ \hat{t}_{ij, \sigma} = t_{ij} + \frac{(1)}{2} n_{i\sigma} + \frac{(2)}{2} n_{i\sigma} n_{j\sigma}, \]

where the first term is the common site-dependent hopping probability; the second term is corresponding to the correction to the hopping process for the occupancy like Fig. 1(b) and the third term to that like Fig. 1(c). This kind of ‘hopping’ terms has been met when we deal with the magnetic impurity problem. The exact version of \( t_{ij}^{(a)} \) via a microscopic calculation is not obtained in the present work. However, we can estimate them through the following physical consideration. It is known that \( t_{ij} = -\frac{1}{2N} \sum_k \epsilon_k e^{i \epsilon_k \kappa (r_i - r_j)}. \) Notice that the chemical potential and the gap-dependence, we can estimate \( \delta^{(a)} t_{ij} \) by

\[ t_{ij}^{(1)} = \frac{\alpha}{N} \sum_k \left[ a_1 \epsilon_k + b_1 (E_k - \xi_k) \right] e^{i \epsilon_k \kappa (r_i - r_j)}, \] 

\[ t_{ij}^{(2)} = \frac{\alpha^2}{N} \sum_k \left[ a_2 \epsilon_k + b_2 (E_k - \xi_k) \right] e^{i \epsilon_k \kappa (r_i - r_j)}, \]

where the coefficients \( a_1 \) and \( b_1 \), in principle, depend on \( U \) and \( J_{ij} \) but we are unable to calculate them at present. We leave them to be determined later. The \( \alpha \)-dependence in eq. (8) is because in the limit \( U \to 0, \alpha \to 0 \) while the hopping probability becomes independent of the site-occupancy.

The coupling between the pairing operators \( \hat{D}_{ij} \) and \( \hat{D}_{ij}^\dagger \) may also depend on the occupancy of the sites \( i \) and \( j \) (See Fig. 2):

\[ \hat{J}_{ij} = J_{ij}^{(1)} (n_i + n_j) + J_{ij}^{(2)} n_i n_j, \]

where

\[ J_{ij}^{(1)} = A_1 \alpha J_{ij}, \quad J_{ij}^{(2)} = A_2 \alpha^2 J_{ij}, \]

with the undetermined coefficients \( A_i \). The extended effective Hubbard model we are considering is given by

\[ H_{eff} = \sum_{\sigma \neq j, \sigma} (\hat{t}_{ij, \sigma} c^\dagger_{i\sigma} c_{j\sigma} + h.c.) - \frac{1}{2} \sum_{\sigma \neq j} \hat{D}_{ij} \hat{J}_{ij} \hat{D}_{ij} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i, \sigma} n_{i\sigma}. \]

To see the gossamer superconducting state, we consider the mean field state of the Hamiltonian (11). Using the gap order parameter \( D_{ij} = \langle \hat{D}_{ij} \rangle \) to replace the pairing operator \( \hat{D}_{ij} \) in the Hamiltonian (11), one has, up to a chemical potential re-definition, the mean field Hamiltonian is given by

\[ H_{MF} = \sum_{\sigma \neq j, \sigma} (\hat{t}_{ij, \sigma} c^\dagger_{i\sigma} c_{j\sigma} + h.c.) - \frac{1}{2} \sum_{\sigma \neq j} \hat{D}_{ij} \hat{\Delta}_{ij} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu_R \sum_{i, \sigma} n_{i\sigma}, \]

where \( \hat{\Delta}_{ij} = \hat{D}_{ij}^\dagger \hat{J}_{ij} \). Now we make a special choice of the parameters such that

\[ a_1 = 2 \beta / \alpha, \quad b_1 = (\alpha + \beta) / \alpha, \]

\[ a_2 = \frac{\alpha^2 - \beta^2}{2 \alpha^2}, \quad b_2 = - \frac{\alpha^2 + \beta^2}{2 \alpha^2}, \]

\[ J_{ij}^{(1)} = (\alpha + \beta) J_{ij}, \quad J_{ij}^{(2)} = \alpha \beta J_{ij}, \]

and \( U = U_G \) and \( \mu_R = \mu + \mu_G \) with

\[ U_G = \frac{1}{2N} \sum_k [(\beta - \alpha) E_k + (\beta + \alpha) \xi_k], \]

\[ \mu_G = \frac{1}{N} \sum_k [2(\alpha - 1) \xi_k + 2 \alpha E_k]. \]

Substituting eqs. (13) and (14) into \( H_{MF} \), one has

\[ H_{MF}^G = \sum_{\sigma \neq j, \sigma} (\hat{t}_{ij, \sigma} c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + U_G \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_{\sigma \neq j, \sigma} (-1)^{\sigma} \hat{\Delta}_{ij, \sigma} c^\dagger_{i\sigma} c_{j\sigma} + h.c.) - \mu_R \sum_{i, \sigma} n_{i\sigma}, \]
where

\[ i_{g j, \sigma}^G = \sum_k \frac{E_k}{N} \left( \frac{1}{2} (u_k^2 - u_k^2) - (\alpha v_k^2 + \beta u_k^2) n_{i\bar{\sigma}} \right) \]
\[ + \frac{1}{2} (\alpha^2 v_k^2 - \beta^2 u_k^2) n_{i\bar{\sigma}} n_{j\bar{\sigma}} e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)}, \tag{16} \]
\[ \Delta_{ij, \sigma} = \sum_k \frac{E_k}{N} u_k v_k e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \]
\[ \times (1 + (\alpha + \beta)n_{i\bar{\sigma}} + \alpha \beta n_{i\bar{\sigma}} n_{j\sigma}). \]

Substituting the definition (4) of \( \tilde{b}_{k\sigma} \) into the gossamer superconducting Hamiltonian (3), one can directly check that the gossamer superconducting Hamiltonian (3) is exactly the same as the mean field Hamiltonian (15), i.e.,

\[ H_{MF}^G = H_G. \tag{17} \]

Thus, we find that the gossamer superconducting state is indeed a fixed point of the system described by the effective Hubbard model (11). If the parameters of the hopping, exchange, chemical potential and interaction are not far from their fixed point values, the system may exhibit a gossamer superconductivity.

In conclusions, we have constructed a new effective Hubbard model in which the different hopping probabilities and the pairing couplings are introduced due to the different site-occupancy at sites. We showed that there is a superconducting phase in such a system since the mean field Hamiltonian of this system in a proper choice of the parameters is just the gossamer superconducting Hamiltonian. Beside the superconducting phase, this new effective Hubbard model is anticipated to have a fruitful phase structure. In ref. [3], it was shown that there is a critical interaction strong \( U_c \) that separates the gossamer superconducting phase from the Mott insulator phase. In this new effective Hubbard model, we see that even the superconducting gap vanishes or the superfluid density is completely suppressed (\( \alpha = 1 \)), the dispersion of electron as well as the interaction between electrons for the latter are dependent on the occupancy of the site that electrons lying on. We expect these unusual behaviors of the electrons may cause unusual normal state properties and relate to the anomalous features in the cuprates. It is possible that the phase diagram of this model may have a better overlap to the cuprate superconductors. The further works are in progress.

The author is grateful to useful discussions with Shaojin Qin, Tao Xiang and Lu Yu. He would like to thank Shuping Feng for him to draw the author’s attention to gossamer superconductivity. This work was partially supported by the NSF of China.

[1] R. B. Laughlin, E-print: cond-mat/0209263.

[2] P. W. Anderson, Science 235, 1196 (1987); Z. Zou and P. W. Anderson, Solid State Commun. 63, 973 (1987).
[3] F. C. Zhang and T. M. Rice, Phys. Rev. B 37, 3759 (1988).
[4] For a reference list, see, e.g., Ref. [1].
[5] P. W. Anderson, The Theory of Superconductivity in the High-\( T_c \) Cuprates, Princeton University Press, Princeton (1997); P. A. Lee, Physica C 317, 194 (1999); X. G. Wen and P. A. Lee, Phys. Rev. Lett. 76, 503 (1996).
[6] F. C. Zhang, E-print: cond-mat/0209272.
[7] Yue Yu and Wan-Peng Tan, Phys. Rev. B 57, 5879 (1998).

Fig.1 Three possible hopping processes. (a) An electron at a single-occupant site hops to an empty site; (b) An electron at a double-occupant site hops to an empty site; (c) An electron with spin \( \sigma \) at a double-occupant site hops to a site which is occupied by a spin \( \bar{\sigma} \).

Fig.2 Three possible electron pairs depending on the site occupancy. (a) the pairs with both sites single-occupied; (b) the pairs with one site single-occupied and another double-occupied; (c) the pairs with both sites double-occupied.
