PREDICTIONS FOR MICROLENSING PLANETARY EVENTS FROM CORE ACCRETION THEORY

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ABSTRACT

We conduct the first microlensing simulation in the context of a planet formation model. The planet population is taken from the Ida & Lin core accretion model for 0.3 $M_\odot$ stars. With 6690 microlensing events, we find that for a simplified Korea Microlensing Telescopes Network (KMTNet), the fraction of planetary events is 2.9%, out of which 5.5% show multiple-planet signatures. The numbers of super-Earths, super-Neptunes, and super-Jupiters detected are expected to be almost equal. Our simulation shows that high-magnification events and massive planets are favored by planet detections, which is consistent with previous expectation. However, we notice that extremely high-magnification events are less sensitive to planets, which is possibly because the 10 minute sampling of KMTNet is not intensive enough to capture the subtle anomalies that occur near the peak. This suggests that while KMTNet observations can be systematically analyzed without reference to any follow-up data, follow-up observations will be essential in extracting the full science potential of very high magnification events. The uniformly high-cadence observations expected for KMTNet also result in ~55% of all detected planets not being caustic crossing, and more low-mass planets even down to Mars mass being detected via planetary caustics. We also find that the distributions of orbital inclinations and planet mass ratios in multiple-planet events agree with the intrinsic distributions.

Key words: gravitational lensing: micro – methods: statistical – planetary systems – surveys

Online-only material: color figures

1. INTRODUCTION

Gravitational microlensing has discovered more than 50 extrasolar planets, although only about half (27) of them have appeared in the literature. Although few in number, microlensing planets occupy a unique region in the parameter space that is difficult to probe by other techniques (e.g., Gaudi 2012; Mao 2012). In fact, microlensing planets have already yielded interesting statistical results concerning the frequency of planets around M-dwarf stars (Gould et al. 2010; Cassan et al. 2012) and intriguing possibilities of free-floating planets (Sumi et al. 2011) that can only be found by microlensing.

Ever since their first discovery, extrasolar planet systems have challenged the two fashionable models of planet formation: the core accretion and gravitational instability scenarios. In particular, in the core accretion theory, the planet population synthesis models are becoming increasingly sophisticated (e.g., Ida & Lin 2010), taking into account effects such as planetesimal accretion, gas accretion, disk evolution, migration, planet-planet interactions, etc. For this reason, in this paper, we shall focus on this theory since its predictions are more quantitative and testable.

A detailed comparison between planet formation theory and microlensing observations has now become more imperative because of the emergence of next-generation microlensing experiments. In the past, the discovery of extrasolar planets often rested on a combination of work by survey teams (Optical Gravitational Lensing Experiment (OGLE; Udalski 2003) and Microlensing Observations in Astrophysics (MOA; Bond et al. 2001)) and follow-up networks with higher-cadence observations (e.g., the Microlensing Follow-up Network (Gould et al. 2006; Gaudi et al. 2008) and the Probing Lensing Anomalies Network (Albrow et al. 1998)). However, these joint operations sometimes make the selection function difficult to quantify, although for some subsamples, such as the small number of very high magnification events, the sample appears to be complete statistically (Gould et al. 2010). Such a situation is likely to change significantly with the completion of the Korea Microlensing Telescopes Network (KMTNet) by the end of 2014. KMTNet will have three telescopes sited in Chile, Australia, and South Africa (Kim et al. 2010). Each telescope will have an aperture of 1.6 m with 4 deg² of field of view, viewing about four fields with 10 minute cadence. With such high-cadence observations, KMTNet will be able to analyze the data without reference to any follow-up observations. Therefore, the selection function will be much simpler. As a result, statistical results will be easier to obtain, which will provide a more robust measurements of planet abundances and distributions, which in turn will better constrain the planet formation theories.

Shvartzvald & Maoz (2012) performed a detailed simulation for a next-generation microlensing network of four telescopes with aperture ranging from 1 to 1.8 m and cadence from 15 to 45 minutes. They use scaled solar system analogs as the lens systems and conclude that such a network can find on the order of 50 planets in 4 yr, of which one in six reveals two planets in a single lensing event. This new discovery rate is a factor of several increase over the original alert and follow-up surveys (Gaudi 2012). KMTNet, with its larger field of view and higher cadence, is therefore expected to yield more planet detections and bring down the detection limit to lower-mass planets.

The present work is the first one that introduces the planet population synthesis model into microlensing simulations. Unlike previous works that use systems with only one planet or simplified solar system analogs as the lens system, our simulation is performed fully in the context of the Ida & Lin core accretion model. Our results will be presented in two papers.
The purpose of this paper is to provide a simple and yet somewhat realistic assessment of the fractions of extrasolar planets expected from KMTNet from the planet populations predicted by the core accretion theory; we will also explore how the KMTNet planet population differs from that of the current survey plus follow-up mode of discovery. In Paper II, we will focus on the multiple-planet events that are detected in our simulation; we will discuss the detection dependence of one planet on the other, the influence of the undetected planets on the recovery of the parameters of detected planets, and the double/triple degeneracy (Gaudi et al. 1998; Song et al. 2014).

In Section 2, we present the Ida & Lin (2010) core accretion model, the basics of microlensing, and how we simulate microlensing data. In Section 3, we describe our method of selecting events with extrasolar planets. In Section 4, we present our main statistical results on the expected detection rates of extrasolar planet populations, and finally, in Section 5, we discuss further our results and implications for future observations.

2. SIMULATION INGREDIENTS

2.1. Ida & Lin Core Accretion Model

The planetary systems placed around our lenses are drawn from the Ida & Lin (2004a, 2004b, 2005, 2008a, 2008b, 2010) core accretion planet population synthesis model. Their model generates protoplanetary disks with various surface densities and depletion timescales on the basis of observational constraints. Protoplanetary seeds are randomly selected in each disk and integrated to protoplanets by accreting planetesimals, which are assumed to be formed from dust grains in the disk. Upon reaching a threshold mass, protoplanets begin to accrete the gas around them. Type I and Type II migrations and interactions between planets are also included to determine the final positions of the planets.

To compare with microlensing observations and simplify our computations, we place each system around a 0.3 \( M_\odot \) star, which is the most likely lens mass of microlensing (Gould & Loeb 1992; Gaudi et al. 2008). To avoid excessive calculations, we include only planets more massive than 0.1\( M_\odot \). We finally extract 669 planetary systems from 1000 Ida & Lin systems. We assume that all the planets in each system are coplanar and randomly place their positions within the orbital plane according to their semimajor axes and eccentricities, and then we choose a random isotropic orientation and project the system onto the sky.

2.2. Microlensing

Microlensing is most sensitive to planets that are close to the angular Einstein ring radius of the host lens (Mao & Paczyński 1991; Gould & Loeb 1992),

\[
\theta_E = \sqrt{\kappa M_\star \mu_{\text{rel}}}, \quad \kappa = \frac{4G}{c^2 \Delta u}, \quad \frac{4G}{c^2 \Delta u} = 8.14 \text{ mas} \quad M_\odot^{-1}.
\]

Here \( M_\star \) is the mass of the host lens, \( \mu_{\text{rel}} = \Delta u (D_L^{-1} - D_S^{-1}) \) is the lens source relative parallax, and \( D_L \) and \( D_S \) are the distances to the lens and source, respectively. The typical timescale of a microlensing event, the Einstein ring crossing time, is

\[
t_E = \frac{\theta_E}{\mu_{\text{rel}}},
\]

where \( \mu_{\text{rel}} \) is the relative proper motion between the source and lens.

The morphology of microlensing light curves is strongly influenced by the caustic structures on the source plane. The caustic curve refers to the set of points on the source plane where the magnification of a point-like source is infinite. Caustics in the planetary microlensing case can be divided into three subclasses: (1) central caustic, referring to a caustic that is small and located close to the central star; (2) planetary caustic, referring to a caustic located far away from the star; and (3) resonant caustic, referring to a large but relatively weak caustic close to the central star when the planetary lens is close to the Einstein ring \( \theta_E \).

The Ida & Lin systems are normally multiple-planet systems. Therefore, we use the multiple-lens microlensing theory in our simulation. The multiple-lens equation can be written as (Witt 1990)

\[
z_s = z - \sum_{k=1}^{N} \frac{q_k}{\bar{z} - \bar{z}_k},
\]

where \( z_s \) is the complex position of the source, \( q_k \) is the mass ratio of the \( k \)th lens relative to the mass of the massive lens, in our case, the central star \( M_\star \), \( \bar{z}_k \) is the conjugate position of the \( k \)th lens, and \( z \) and \( \bar{z} \) are the positions of the corresponding image in complex and conjugate form, respectively. Distances in Equation (1) are scaled by the Einstein radius of the host star at the lens plane,

\[
R_E = D_t \theta_E.
\]

For simplicity, we treat the lens system as static (i.e., no orbital motion) and ignore the microlens parallax. The ray-shooting method (Schneider & Weiss 1986, 1987) is used to generate theoretical light curves for multiple-lens microlensing. In reality, the timescale of microlensing events, \( \theta_E \), may vary from event to event, but to avoid a complicated ray-shooting process and also to get comprehensive statistical results in a reasonable computation time, we fix \( \theta_E \) to 15.7 days, which is for a typical bulge microlensing event with the lens system at \( D_L = 7.4 \text{kpc} \), the source star at \( D_S = 8.6 \text{kpc} \), the lens star with mass \( M_\star = 0.3 \text{M}_\odot \), and the relative proper motion between the source and the lens \( \mu_{rel} = 5 \text{mas/yr} \). These also yield \( R_E = 1.59 \text{AU} \). Additionally, we adopt a typical turnover source star with radius \( R_* = 1.6 R_\odot \), which corresponds to a scaled source size \( \rho = \theta_E / \theta_E = 0.004 \); a uniform surface brightness profile (i.e., no limb darkening effect) is also employed. The density of rays used in the ray-shooting program is determined according to Dong et al. (2006). Based on the above settings, we finally shoot \( 4.9 \times 10^9 \) rays over the area from \((-4, -4) \) to \((4, 4) \) to obtain an average accuracy of \(-0.1\% \) when the source is unmagnified. This calculation error is much smaller than the simulated photometric error (see below), so we do not account for it in \( \chi^2 \).

The impact parameter \( u_0 \) for each event is randomly chosen from \(-0.3 \) to \( 0.3 \), meaning that the maximum magnification of each event is above 3.4. We do this to ensure an interesting number of planet detections for the available computing time and a reasonable number of high-magnification events. Our simulation covers the time \(-1.5 t_0 \leq t - t_0 \leq 1.5 t_0 \), where \( t_0 \) is the time of closest approach relative to the host star in the lens system. The total baseline \( I \)-band magnitudes \( I_{bl} \) and blending fractions \( f_{bl} \) are drawn from Smith et al. (2007), where the blending effects of typical Galactic bulge fields with high stellar density are simulated for OGLE. The cumulative distributions of \( I_{bl} \) and the source baseline magnitude \( I_0 \), which is related to \( f_{bl} \) by \( I_0 = I_{bl} - 2.5 \log_{10} f_{bl} \), are displayed in Figure 1. The photometry is simulated using the same code.
as Penny et al. (2011) but employing parameters that better match the KMTNet survey (Kim et al. 2010). Particularly, we assume a larger telescope diameter of 1.6 m, a worse mean seeing of 1.4 arcsec, and a larger systematic error floor of 0.5%.

The chosen seeing is therefore worse than that used to estimate the blending statistics, so the amount of blending is slightly underestimated.

Each planetary system is used to generate 10 light curves, so we end up with 6690 microlensing events in total.

3. “OBSERVING” SIMULATED DATA

We employ the following procedures to determine how many planets are detected in each microlensing event. Simulated light curves are first fitted with a single-lens model with six parameters, \( t_0, t_E, u_0, I_{bl}, f_{bl}, \) and \( \rho \). We minimize \( \chi^2 \) using the MINUIT routine from CERNLIB (James & Roos 1975), with the first three parameters free and the last three parameters fixed to the true value to simplify the calculations. This is conservative in the sense that allowing the last three parameters to vary can only increase \( \Delta \chi^2 \) and thus improve the derived robustness of the detection. If the difference in \( \chi^2 \) between the best-fit single-lens model and the theoretical model used to simulate the light curve, \( \Delta \chi^2_{\text{single}} \), is larger than 200, this event is considered a potential planetary event. From 6690 microlensing events, we find 313 events with \( \Delta \chi^2_{\text{single}} > 200 \).

If a light curve is assessed as possibly containing planetary perturbations, we employ the following method to identify which planets are responsible for these perturbations. We first rank the planets in the system in decreasing order by the width of their planetary caustics (Han 2006)

\[
\omega_k = \begin{cases} 
q_k^{1/2} |z_k|^3 & \text{if } |z_k| < 1 \\
q_k^{1/2} |z_k|^{-2} & \text{if } |z_k| \geq 1
\end{cases}
\]  

(2)

Theoretical double-lens (i.e., the host star plus a single planet) light curves are then generated for the six most highly ranked planets individually, using the same \( u_0, I_{bl}, f_{bl} \) and under the same simulated photometric conditions. Such double-lens light curves are also fitted with a single-lens model.

We examined by eye each of the 313 candidates of planetary event generated by the automatic selection criterion. We rejected 21 of these as not real detections. Of these, 19 were events with very bright sources and hence extremely small error bars, for which the numerical precision of our light curve modeling (designed for more typical events) was not adequate. Hence, \( \Delta \chi^2 \) was simply due to numerical noise. For the other two, \( \Delta \chi^2 \) was contributed by more than one planet, but a single-planet model yielded \( \Delta \chi^2 < 200 \), and the signal-to-noise ratio was not adequate to claim more than one planet. Such a “planet detection” would be rejected in practice, so these two events were excluded. Finally, we confirm 292 planetary events, of which 23 have two detectable planets and none has more than two. In Figure 2, we show an example single-planet event that contains an Earth-mass planet.

That two planets can be detected individually does not guarantee that they can be detected in the same event because of the degeneracy existing between light curves arising from multiple- and single-planet events (e.g., Gaudi et al. 1998; Song et al. 2014). Thus, a double-lens model is used to fit the 23 multiple-planet candidates. We first fit each light curve
from 74,560 planets in 6690 lens systems, each with at least one planet. The baseline magnitudes of these planetary events are shown in cumulative function form in Figure 1, together with that of all our simulated microlensing events for comparison.

Masses and separations of all detected planets are shown in the bottom panel of Figure 4, with the histogram of the separations shown on the top. Here we use the true masses and separations, not the mass ratios and separations obtained by fitting the light curves. Although the latter is what we should use in order to compare with real observations, we notice that 85% of all planetary events can be very well reproduced by the single-planet light curve derived from the true mass and separation. Colors and symbols in Figure 4 encode the caustic that was encountered and the number of detected planets, respectively. As is expected, planets are mostly detected near $R_E$, although high-mass planets ($M_p > M_{Neptune}$) can be detected in a broader range than the low-mass planets ($M_p < M_{Neptune}$). We also find that low-mass planets are more often detected via their planetary caustics, while high-mass planets are more often detected via central caustics. The planets detected via resonant caustics are located within a more narrow region around $R_E$, compared to the overall distribution. We notice that all the sub-earths ($M_p < M_\oplus$) are detected via planetary caustics, which is reasonable since for fixed $s$ the diameter of the planetary caustic scales with $\sqrt{q}$, while the diameter of the central caustic scales with $q$. In particular, we notice that even Mars-mass planets can be detected in this KMT-like simulation. Figure 4 also shows that planets in double-planet events are mostly high mass and detected via central caustics. We will give our explanation of this in Section 5.

We show the distribution of masses and semimajor axes of Ida & Lin planets in Figure 5, with colors in the bottom panel encoding the detection frequency within 10 simulated events based on the same planetary system. As has been seen in Figure 4, the high-mass planets are more often detected than those of low mass. As expected, the distribution of semimajor axes $a$ is shifted upward relative to the Einstein radius by $\Delta \log a \sim 0.5 \log 1.5 = 0.09$, while the distribution of $\log a_\perp$ ($a_\perp$ is the projected separation $s$ in Figure 4) peaks right at the Einstein ring. What is more surprising is that both distributions are approximately symmetric even though the underlying distribution of Ida & Lin planets rises strongly toward closer separations, which means that wide planets have larger detection efficiencies than close ones. Such a detection bias needs further study in order to recover the underlying distribution of planets on the basis of real microlensing observations.

### 4.2. Dependence on Impact Parameter or Maximum Magnification

We correct the impact parameter to the center of magnification ($u_0^*$), rather than that to the host star ($u_0$), which was randomly chosen in the simulation,

$$u_0^* = u_0 - \sum_k \frac{q_k \sin \alpha_k}{|z_k| + |z_k^*|^{-1}},$$  \hspace{1cm} (3)

where $\alpha_k$ is the angle between the source trajectory and the $k$th planet (Chung et al. 2005). Figure 6 shows the cumulative distribution function of this impact parameter $u_0^*$ for different groups of events in our simulation. Compared to the uniformly distributed input impact parameters, relatively small impact parameters, which correspond to relatively high magnification events ($A_{\text{max}} \approx 1/u_0^*$), are favored in planet detections,
especially multiple-planet detections, which is consistent with previous theoretical expectations (Griest & Safizadeh 1998).

However, we notice that events with extremely small impact parameters ($u_0^* \lesssim 0.005$) are less sensitive to planets than moderately small $u_0^*$. To clarify this, we list the number of events within each $A_{\text{max}}$ range in Table 1. Events with $A_{\text{max}} > 200$, which can be regarded as extremely high magnification events in our simulation due to the relatively large source size we use, are significantly less sensitive to planets than the lower-magnification ones. This seems to conflict with previous studies based on ongoing observations (Gould et al. 2010). The reason might come from the different observing strategies used in current observations and in our simulation. We will give a detailed discussion of this in Section 5.

4.3. Dependence on Inclination and Planet Mass Ratios

For microlensing planets, the orbital inclination is important in converting the projected separation to semimajor axis of the
planet and therefore understanding the physical properties of planetary systems. Thus, the distribution of orbital inclinations of planetary events in realistic simulations is worthy of investigation.

Our simulation shows that the distribution of orbital inclinations of detected planetary events is statistically consistent with the input distribution of inclinations, as Figure 7 shows, although the distribution for double-planet events deviates slightly from the input distribution. This implies that the planet detection is not biased on any typical orbital inclinations, which is reasonable since the projected position is determined not only by the inclination but also by the orbital phase of the planet.

Figure 8 shows the comparison between the mass ratios of planets detected in double-planet events and those of the two most massive planets in all planetary events. A two-sample KS test gives a confidence level $\alpha = 34\%$, meaning the two distributions are consistent with being drawn from the same distribution. This indicates that multiple-planet systems detected via microlensing are representative of all multiple-planet systems.

4.4. Detection Efficiency

We estimate the detection efficiency of microlensing for planets located between $0.5R_E$ and $2R_E$. The numbers of planets with different masses and the fraction of detected planets are listed in Table 2 and displayed in Figure 9 in more bins with statistical error bars. In particular, we notice that the detection efficiency for super-Jupiters is 20 times higher than that for Earth-mass planets. However, the low detection efficiency of low-mass planets is compensated by the larger number of such planets, which yields almost equal numbers of super-Earths, super-Neptunes, and super-Jupiters detected in our simulation. This is consistent with the result of C. B. Henderson et al. (2014, in preparation), where a more realistic simulation for KMTNet is performed and the distribution of detected planets is estimated on the basis of the planetary mass function given by Cassan et al. (2012).

As a result of this difference in detection efficiency, the fraction of massive planets detected in our simulation exceeds the fraction of such planets given by the planet formation model. Within 292 planetary events, we find 114 systems holding a super-Jupiter planet. This fraction, 39%, is $\sim 8$ times higher than the prediction of the Ida & Lin model, which only contains 5.3% of such systems.
5. DISCUSSION

We conducted a simple and yet realistic microlensing simulation for a KMTNet-like microlensing survey. The planet population is taken from the Ida & Lin core accretion model for $0.3 M_\odot$ lenses. Our simulation results in 292 planetary events, including 16 double-planet events, from 6690 microlensing events for which the lens system has at least one planet more massive than $0.1 M_\oplus$. With the frequency of such planetary systems considered, we find the fraction of planetary events is 2.9%, out of which 5.5% show multiple-planet detections.

We address the limitations of our simulation here before discussing the implications of our results. (1) We admit that our simulation is not fully realistic in the sense that the Einstein timescale $t_E$, which involves distances $D_S$ and $D_L$; lens mass $M_L$; relative proper motion $\mu_{\text{rel}}$; and the source size $\rho$ are fixed to some typical values. This does prevent us from making precise predictions for KMTNet, but predicting the yields of KMTNet or any other specific microlensing experiment is not the main purpose of our work. Our simulation aims to address more general questions, which are not easily clarified if too many observational factors are considered. Moreover, a microlensing simulation with 12 lenses in each system on average will become extremely complicated if all the parameters that we have held fixed (i.e., $D_L$, $D_S$, $M_L$, and $\mu_{\text{rel}}$) are set free. (2) The planet population given by Ida & Lin’s model is produced for stars in the Galactic disk, but in our simulation the lens system is placed in the bulge. Planets forming around bulge stars may well have very different distributions from those forming around disk stars, not only because of the different metallicity but also because of the very dense environment (Thompson 2013). However,
there is as yet no model available to quantitatively predict the planet population in the Galactic bulge. Therefore, using planet population predictions for disk stars is our only choice. One positive outcome of this approach is that comparing the results of our simulation with real observations may tell us how different the planet populations are in the bulge and in the disk, which is a question that can only be answered by microlensing. (3) When randomly placing the planets on their orbit, we do not take mean motion resonances into account. This may not be correct in the case of resonant systems. However, we notice that Ida & Lin’s model does not show strong resonance signatures, as is shown in Figure 10. Therefore, the orbit of each planet is mostly unaffected by others in the same system, so randomly placing them on their orbit is acceptable.

Our simulation yields more multiple-planet events than our naive expectation. Given that the total number of planets in 6690 systems is 74,560, the probability for one planet to be detected is therefore $p = 0.0041$ if we naively assume such detection does not depend on the characteristics of the microlensing event or properties of the planet. Then the number of single-planet events we would expect to detect in our simulation is

$$N_1 = \sum_j n_j p,$$

and the number of double-planet events is

$$N_2 = \sum_j n_j (n_j - 1) \frac{1}{2} p^2,$$

where $n_j$ is the number of planets in the $j$th system. Given $N_1 = 276$ in our simulation, we would expect $N_2$ to be 7 if our assumption holds, which is significantly lower than what we do detect, $N_2 = 16$. This is reasonable since the detectability of planets depends not only on the physical properties of the planet but also on the impact parameter of that microlensing event. High-magnification events and massive planets are more favored in multiple-planet microlensing, as is shown in Figures 4 and 6. Our simulation therefore predicts that multiple-planet events will be detected more often than our naive expectation, but they are strongly biased toward massive planets and high-magnification events.

In Section 4.2, we have shown that extremely high-magnification events are less sensitive to planet detections than the moderately high-magnification ones in such a KMTNet-like survey program. This apparently conflicts with previous theoretical predictions (Griest & Safizadeh 1998) as well as ongoing observations (Gould et al. 2010). The reason might come from the different observing strategies used in our simulation and in current observations. The survey plus follow-up mode used in current microlensing observations can achieve very high cadence (e.g., more than one observation per minute after accounting for multiple observatories) during the peak of high-magnification events, although the survey teams typically obtain only a few observations per night. These intensive observations during the peak make high-magnification events extremely sensitive to planet perturbations. Our simulation is conducted using a strategy similar to that expected for KMTNet, which uses a constant cadence (10 minutes) of observations everywhere. Therefore, for extremely high-magnification events, the planet perturbation is so weak that it may be missed by such a 10 minute cadence observing strategy. This argues that even in the era of next-generation surveys, there is still a need for follow-up of high-magnification events, which will require the next-generation surveys to process their data in real time and produce high-magnification alerts, as is done for current surveys. With $\sim 20\%$ of high-magnification planet detections yielding multiple planets, such follow-ups are important for measuring the number of multiple-planet systems (Gaudi et al. 1998).

The advantage of conducting uniformly high cadence observations everywhere in the light curve like KMTNet does, in addition to obtaining a well-controlled planet sample for statistical studies, is the ability to detect more low-amplitude planetary perturbations and perturbations due to planetary caustics. Low-amplitude perturbations are usually produced by source-star trajectories that do not cross any caustics. If we define caustic crossings as occurring if the closest distance between the source trajectory and caustics is less than two source radii, we find that 55% of all detected planets in our simulation are not due to this caustic crossing, as is listed in Table 1. In contrast, we searched all published microlensing planets and characterized them according to this definition of caustic crossing. We find only three real microlensing planets are due to such non-caustic-crossing events within 26 published microlensing planets with very good data coverage under the current observation strategy, which are listed in Table 3. This implies that in future microlensing programs like KMTNet, the Wide-Field Infrared Survey Telescope (WFIRST; Spergel et al. 2013), and possibly Euclid (Penny et al. 2013), at least half of the microlensing planets will not be detected by crossing caustics. The non-caustic-crossing character of the event makes it more difficult to determine the physical properties of the lens system since the unknown but important quantity $\theta_E$ cannot be determined from the angular size of the source star that is derived from the source color and brightness (Yoo et al. 2004). However, in the case of WFIRST, it may be possible to measure $\theta_E$ by astrometric microlensing (Gould & Yee 2014) or (in the case that the lens is luminous) by taking high-resolution images several years before or after the event.

The number of planets detected via planetary, central, and resonant caustics are 107, 128, and 78 respectively. The fraction

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5 Planets detected via both planetary and central caustics are counted twice.
Table 3

List of All Published Microlensing Planets and How We Classify Them

| Name                  | $A_{\text{max}}$ | $q(10^{-4})$ | Caustic Type | Caustic Crossing? | References                                      | Comment |
|-----------------------|-------------------|---------------|--------------|-------------------|------------------------------------------------|---------|
| OGLE-2009-BLG-151b/   | 5                 | 4190          | R            | Yes               | Choi et al. (2013)                               | A brown dwarf, but listed as planet at http://exoplanet.eu |
| MOA-2009-232Lb       |                   |               |              |                   |                                                 |         |
| OGLE-2011-BLG-0420b   | 40                | 3770          | C            | Yes               | Choi et al. (2013)                               | A brown dwarf, but listed as planet at http://exoplanet.eu |
| OGLE-2012-BLG-358Lb   | 10                | 800           | P            | Yes               | Han et al. (2013b)                               | The host star has mass 0.02 $M_{\odot}$ |
| MOA-2011-BLG-322Lb    | 21                | 280           | C            | No                | Shvartzvald et al. (2014)                        |         |
| MOA-2009-BLG-387Lb    | 11                | 132           | R            | Yes               | Batista et al. (2011)                            |         |
| OGLE-2005-BLG-071Lb   | 42                | 71            | C            | No                | Udalski et al. (2005)                            |         |
| MOA-2008-BLG-379Lb    | 167               | 68.5          | R            | Yes               | Suzuki et al. (2014)                             |         |
| OGLE-2012-BLG-406Lb   | 2                 | 62.6          | P            | Yes               | Poleski et al. (2014)                            |         |
| MOA-2011-BLG-293Lb    | 286               | 53            | C            | Yes               | Yee et al. (2012)                                |         |
| MOA-bin-1b            | 1.1               | 49            | P            | Yes               | Bennett et al. (2012)                            | The planet has a large separation from the star |
| OGLE-2003-BLG-235Lb   | 8                 | 39            | R            | Yes               | Bond et al. (2004)                               |         |
| MOA-2007-BLG-400Lb    | 628               | 25            | C            | Yes               | Dong et al. (2009b)                              | Same for close/wide solutions |
| MOA-2010-BLG-477Lb    | 294               | 21.81         | R            | Yes               | Bachelet et al. (2012)                           |         |
| OGLE-2011-BLG-251Lb   | 18                | 19.2          | C            | No                | Kains et al. (2013)                              | Four solutions, D is favored |
| OGLE-2006-BLG-109Lb   | 289               | 13.5          | R            | Yes               | Han et al. (2013a)                               | Four solutions, D is favored |
| OGLE-2012-BLG-0026Lc  | 109               | 7.84          | R            | Yes               | Han et al. (2013a)                               | An alternate model leads to a host mass of $\sim 4 M_J$ |
| OGLE-2005-BLG-169Lc   | 289               | 4.86          | C            | Yes               | Gaudi et al. (2008)                              |         |
| MOA-2011-BLG-262Lb    | 80                | 4.7           | C            | Yes               | Bennett et al. (2014)                            |         |
| MOA-2009-BLG-319Lb    | 167               | 3.95          | R            | Yes               | Miyake et al. (2011)                             |         |
| MOA-2008-BLG-310Lb    | 400               | 3.3           | C            | Yes               | Janczak et al. (2010)                            |         |
| MOA-2010-BLG-328Lb    | 14                | 2.6           | P            | Yes               | Furusawa et al. (2013)                           |         |
| OGLE-2012-BLG-0026Lb  | 109               | 1.30          | C            | Yes               | Han et al. (2013a)                               | Four solutions, D is favored |
| OGLE-2007-BLG-368Lb   | 13                | 0.95          | P            | Yes               | Sumi et al. (2010)                               |         |
| OGLE-2005-BLG-169Lb   | 800               | 0.9           | R            | Yes               | Gould et al. (2006)                              |         |
| OGLE-2005-BLG-390Lb   | 3                 | 0.76          | P            | Yes               | Beaulieu et al. (2006)                           |         |
| MOA-2009-BLG-266Lb    | 8                 | 0.563         | P            | Yes               | Muraki et al. (2011)                             |         |
| MOA-2007-BLG-192Lb    | $\sim 270$        | ...           | ...          | ...               | Bennet et al. (2008)                             | Too few data points to constrain the planet |

Note. Planets are sorted by their mass ratio; the lower two single solid lines show what mass ratio a Jupiter and a Neptune would have if they are around a 0.3 $M_{\odot}$ star.

of that detected by planetary caustics, 35%, is slightly higher than but consistent with the 27% ($= 7/26$) based on real microlensing planets. More planets being detected via planetary caustics and the high-cadence observations around the planetary anomaly lead to the detection of very low-mass planets even down to Mars mass, as the planetary caustic shrinks slower ($\sim \sqrt{q}$) than the central caustic does ($\sim q$) as the planetary mass ratio $q$ decreases.

In Figure 11, we compare the cumulative distribution of mass ratios of planets detected in our simulation with that of real microlensing planets. We notice that the two curves coincide with each other surprisingly well for $q > 10^{-3}$ but that the curve from our simulation has a long tail toward very small mass ratio, which means that future microlensing surveys will be able to explore more very low mass planets than current observations. This tendency is not changed even when we choose a larger $\Delta \chi^2$ cutoff value. To understand which events contribute to this change, we divide all events into two groups: high-magnification events ($A_{\text{max}} > 100$) and low-magnification events ($A_{\text{max}} < 100$). The two panels in Figure 12 tell us that most of these low-mass planets are detected in low-magnification events, which is understandable since they are more often detected via planetary caustics (Figure 4).

Within all 26 well-understood microlensing planets, 3 are claimed to be super-Jupiters around M-dwarf hosts (Dong et al. 2009a; Batista et al. 2011; Poleski et al. 2014). The ratio, 3 out of 26, is much higher than the estimation from either core accretion theory (e.g., Kennedy & Kenyon 2008) or other exoplanet detection techniques (e.g., Cumming et al. 2008). In our simulation, we find that the small fraction of super-Jupiter systems given by the Ida & Lin core accretion model is magnified by a factor of $\sim 8$ if observed via microlensing. Therefore, this observational bias should be taken into account when comparing the frequency of massive planets around...
M dwarfs from microlensing observations with that from planet formation theory.

Our simulation also shows that the inclination of the lens system of multiple-planet events obeys the intrinsic distribution of orbital inclinations and that the mass ratio between the two detected planets also agrees with the intrinsic mass ratio distribution of the planetary system.

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