Building a “holographic dual” to QCD in the AdS$_5$: instantons and confinement

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Abstract

Recent attempts to find a “holographic dual” to QCD-like theories included a suggestion by Karsh et al (below referred to as KKSS) to incorporate confinement via a potential quadratically increasing into the 5-th direction of the AdS$_5$ space. We show that the same conclusion follows from completely different line of arguments. If instantons are promoted into the 5d space by identifying the instanton size $\rho$ at the 5-th coordinate, the background geometry necessarily should be the AdS$_5$. As I argued already in 1999, confinement described via “dual superconductivity” leads to a factor $e^{\exp(-2\pi\sigma\rho^2)}$, where $\sigma$ is a string tension, which is nearly exactly identical to that suggested by KKSS. This expression is also well supported by available lattice data. At the end of the paper we propose a IR potential generalized to the nonzero temperatures.

1. AdS/CFT correspondence is a conjectured duality between the conformal $\mathcal{N}$=4 supersymmetric Yang-Mills theory in d=4 dimensions (to be called CFT for short below) and the string theory in AdS$_5 \times O_5$, which passed an impressive number of particular tests (see review [1]). It is clearly the most interesting development at the interface of string/field theory. It is
especially fruitful in the large $N$ (number of colors), in which quantum CFT in a strongly coupled regime is dual to classical and weakly coupled bulk supergravity.

The AdS/CFT correspondence made it clear that (contrary to beliefs of many people) a strong coupling regime does not necessarily imply confinement. Indeed, its most direct applications to QCD are related to deconfined high-$T$ phase of QCD, known as quark-gluon plasma. It has been recently argued that just above deconfinement transition it seems to be in a strongly coupled regime, because it is a near-perfect liquid as seen from analysis of RHIC data (see e.g. [2]). In order to understand confinement one has to move from conformal $N=4$ theory to more QCD-like ones, which still have a gravity dual. Such approaches, known as a “top-down” approach is very interesting, but it is not discussed in this letter.

Another direction of current active research, the “down-up” approach sometimes called AdS/QCD, is a search for a “holographically dual” description of QCD. It is done by incorporating various phenomenological aspects of QCD, promoted into a 5-dimensional settings. Such models for QCD spectroscopy [3] have started with simple infrared cut-off of the $AdS_5$ space. Recently Karch et al [4] have proposed a more refined effective model, based on the conjecture that confinement can be implemented by a potential, quadratically growing with the 5-th coordinate into the IR. KKSS argued that such behavior reproduce correctly the behavior of Regge trajectories at high excitations in QCD.

In this letter we point out that the KKSS suggestion is in fact (nearly exactly) the same as a conjecture suggested in my paper [5]. The correspondence between these two works is is especially interesting since they originate in quite different parts of the QCD dynamics: in [5] the IR probe is done by instantons rather than via hadronic spectroscopy.

In order to elucidate this correspondence we will first show how the instanton dynamics can be promoted into the 5-dimensional space-time, with the 5-th coordinate naturally identified with the instanton size $\rho$. We will then argue that the metric of it must indeed necessarily be that of the $AdS_5$. After that is done, we will see that a scalar potential is indeed needed to append purely semiclassical calculation, and will see that its behavior in infrared is nearly exactly what KKSS have proposed. Not only our line of arguments is completely different from KKSS one, we will also speculate at the end that this potential can arise from “dual superconductor” picture of confinement by ’t Hooft and Mandelstam and is thus proportional to the
condensate of some magnetically charged objects.

2. **Instantons** play a very important role in many aspects of QCD, including meson and baryon spectroscopy, for a general review see [6]. Instantons of course are also known to play important role in other gauge theories: see e.g. a review by Dorey et al [12] about $\mathcal{N}=4$ theory and AdS/CFT and recent instanton-based derivation [7] of the celebrated Seiberg-Witten solution of the $\mathcal{N}=2$ SUSY YM theory.

In QCD interaction between instantons and anti-instantons cannot be treated analytically\(^1\). Therefore the so called “instanton liquid” model (ILM) which addresses many-body aspects of instanton ensemble is usually studied numerically. For details and references to papers which has lead to its development the reader should consult reviews such as [6]. It turned out to be very successful phenomenologically, predicting large set of correlation functions in quantitative agreement with lattice measurements. Furthermore, predicted dominance of instanton-zero-mode states for lowest eigenstates Dirac eigen-values was directly confirmed by lattice studies: those quark states do dominate quark propagators at distances relevant for the lowest hadronic states. In this letter we will not discuss any of these applications: we will only touch some building blocks of the ILM to the extent to show that they can be promoted to AdS space.

The partition function of ILM can be schematically written as

$$Z = \frac{1}{N_+ N_-} \prod_{i} \int d\Omega_i d^4 x_i d\rho_i \exp\left(-S_{\text{glue}} + N_f \log det(\bar{\rho}D) - \Phi\right)$$

where we have identified integration over collective coordinates, the instanton 4d position $x_i$, size $\rho$ and orientation in color space $\Omega$. We will discuss different effects related to the exponent subsequently.

The bosonic action $S_{\text{glue}}$ should in principle include the determinant of the moduli space metric (the overlaps of bosonic modes), but in practice more important are instanton-antiinstanton interactions. Those were determined from the so called “streamline” method: the specific function would not be important, all we want to point out here is that for an instanton-antiinstanton

\(^{1}\)A simple analytical limits are possible, e.g. at high density, when instanton ensemble is exponentially dilute and is a weakly coupled Coulomb plasma [8], with light $\eta'$ as its “photon”. In the large $N$ limit and zero density the $\eta'$ is still light, but the vacuum is still a strongly interacting liquid [9]. In the real $N = 3, N_f = 2 - 3$ QCD the instanton ensemble is very strongly correlated, as seen e.g. from a very large value of the $\eta'$ mass.
\((IA)\) pair their \(4N + 4N\) collective variables appear in form of only two combinations

\[
z = (1/N) Tr[Ω_IΩ_A^†(\hat{R}_μτ_μ)], \quad d_{IA}^2 = \frac{(x_I - x_A)^2}{ρ_Iρ_A}
\]

(2)

The former is called relative orientation factor, the orientation matrices Ω live in SU(\(N\)) while the \(τ_μ\) is the usual 4-vector constructed from Pauli matrices: it is nonzero only in the 2*2 corner of the \(N * N\) color space. The \(\hat{R}\) is the unit vector in the direction of 4d inter-particle distance \(x_I - x_A\). The second combination of position and sizes appears due to conformal invariance of the classical YM theory.

The most complicated part of the effective action is the term which takes into account fermionic exchange described by the determinant of the Dirac operator in the basis spanned by the zero modes

\[
\begin{pmatrix}
0 & T_{IA} \\
T_{AI} & 0
\end{pmatrix}
\]

(3)

The overlap matrix elements \(T_{IA}\) are defined by

\[
T_{IA} = \int d^4x \psi_0^\dagger(x - z_I)i\slashed{D}\psi_0(x - z_A),
\]

(4)

where \(ψ_0\) is the fermionic zero mode. Note that each matrix element has the meaning of an amplitude of a “jump” of a quark from one pseudoparticle to another. Furthermore, the determinant of the matrix (3) is also equal to a sum of all closed loop diagrams, and thus ILM sums all orders in ’t Hooft effective \(2N_f\)-fermion effective interaction.

The matrix element can be written as \((\bar{ρ} = \sqrt{ρ_Iρ_A})\) a function of the same two variables (2) as the gauge action

\[
\bar{ρ}T_{IA} = z_{IA}f(d_{IA}) \quad f \approx \frac{4d_{IA}}{(2 + d_{IA}^2)^2}.
\]

(5)

where the expression is an approximate simple parameterization of a more complicated exact result\(^2\). The reason for that is again a conformal symmetry of zero modes with massless quarks.

3.Promotion of instantons into AdS space is natural for \(\mathcal{N}=4\) theory, see [12]. The instantons are identified as some point-like objects positions at

\(^2\)We have shown it to remind that at large distances it decays as the distance cube, corresponding to a fermion exchange in 4d, but not in AdS\(_5\).
distance $\rho$ from the $D_3$ brane. Its image at the brane, readily calculated from the dilaton bulk-to-brane propagator, gives precisely correct $G^{2}_{\mu\nu}(x)$. Furthermore, remaining 8 supersymmetries near the instanton solution nicely relate the fermionic zero modes to bosonic ones. In fact a demonstration by Dorey et al. that in this theory instantons fit so comfortably into AdS/CFT correspondence was one of its early spectacular conformations.

For QCD-like theories, with fundamental (rather than adjoint) fermions and no scalars or supersymmetries, the situation is quite different. We will however still argue that one can actually lift an instanton ensemble from the brane to bulk in this case as well, which we will do in two steps.

Step one, same as in $\mathcal{N}=4$, is identifying the factor in the instanton measure as that in the $AdS_5$ with the metric

$$ds^2 = \frac{(d\rho^2 + dx^2)}{\rho^2}$$

$$d^5x\sqrt{g} = \frac{d^4x d\rho}{\rho^5}$$

These standard coordinates naturally relate the $\rho \to 0$ limit with the ultraviolet (UV) and $\rho \to \infty$ with the infrared (IR) directions.

Step two requires a good look at the variable $d$ in (2) for bosonic and zero-mode interactions between instantons and antiinstantons: in fact it is nothing else as the invariant distance between 2 points in $AdS_5$.

3. **Scale-depending potential** $\Phi$ in the partition function (1) contains all effects that break conformal symmetry of classical YM. In the UV it is defined by the asymptotic freedom of QCD$^3$:

$$exp(-\Phi)|_{\rho^{-}0} = (\rho * \Lambda_{QCD})^{(11/3)N -(2/3)N_f}$$

Its IR behavior is of course non-perturbative and is the main issue discussed in this work. It was the main subject my paper [5], which argued that in the dual superconductor model, in which there is a nonzero Higgs VEV of magnetically charged objects, one can rely on selfduality of instantons and thus go into dual “magnetic” description. If so, the steps dual to ’t Hooft derivation of instanton measure in Higgs models (such as the standard model of electroweak interactions) lead to a result that the cutoff is clearly done by a quadratic potential, $\Phi(\rho) \sim \rho^2$.

Moreover, a specific Abelian model for dual superconductor, identifying QCD strings with Abrikosov vortices, allows to express the coefficient in

$^3$By the way, it is surprising that this aspect was not included in spectroscopic applications [3, 4].
Figure 1: (a) The instanton density $dn/d\rho d^4x$, $[\text{fm}^{-5}]$ versus its size $\rho$ [fm].
(b) The combination $\rho^{-6}dn/d\rho d^4z$, in which the main one-loop UV behavior drops out for $N = 3$, $N_f = 0$, thus presumably giving only the IR part of the potential we discuss. The points are from the lattice study [11], for pure gauge theory, with $\beta = 5.85$ (diamonds), 6.0 (squares) and 6.1 (circles). (Their comparison should demonstrate that results are rather lattice-independent.) The line corresponds to the expression $\sim \exp(-2\pi\sigma\rho^2)$, see text.

Terms of the string tension, namely

$$\exp(-\Phi)|_{IR} = \exp(-2\pi\sigma\rho^2)$$

(8)

It is the same as proposed by KKSS, except for numerical factor (which is $\pi/2$ in their case, as far as I can tell). It was shown in [5] (see Fig.1 borrowed from it) that this expression very nicely explains the instanton size distribution measured on the lattice. (We mention this fact to stress that we dont see any possibility that the factors of two in the derivation got confused.)

Different coefficients of the quadratic term with otherwise identical parametric dependence is puzzling. One question one might ask is whether the potential $\Phi$ in the instanton size distribution is renormalized by mutual effective repulsion of instantons and antiinstantons\(^4\). However it is easy to

\(^4\)In fact in the mean field approach, convergence of the rho integration was first attributed to it entirely [10].
see that this cannot happen in a context just described. Indeed, if all the
interactions in an instanton liquid are conformal, instantons would be just
floating in the $AdS_5$ and their mutual repulsion would obviously be related to
their density gradient, leading to homogeneous population of all $AdS_5$ space,
as it happens in conformal theories such as $\mathcal{N}=4$. Thus $\Phi$ potential should
be external to a conformal version of ILM outline above, including quantum
effects like asymptotic freedom or confinement.

4. **IR cutoff at finite temperatures** is the last issue we address. If the
“soft cutoff” is provided by a quadratic potential proportional to the string
tension, as argued above, this cutoff should vanish at the deconfinement as
$\sigma(T) \to 0$ as $T \to T_c$ and remains zero above it.

In the opposite limit of very high $T$ the instanton suppression should be
given by (a perturbatively calculated) Pisarski-Yaffe [13] factor, its main part
is

$$
\exp(\Phi)_{|large\ T} \sim \exp \left[ -\frac{1}{3}(2N + N_f)(\pi \rho T)^2 \right]
$$

Note that a combination of color and flavors are the same as the perturbative
Debye screening mass. It is not a coincidence: as shown in [14], one can
derived it using the universal gluon/quark forward scattering amplitudes on
a (small size) instanton and the perturbative thermal factors. This leads to
natural generalization of this factor

$$
(N/3 + N_f/6)T^2 \to M_E^2(T)/g^2
$$

where the electric screening mass in the r.h.s. in not necessarily the lowest
order one. It may e.g. include in the thermal weight the nonzero effective
quasiparticle masses and can be independently determined from lattice
simulations. Unlike its perturbative counterpart, the screening mass should
vanish at and below $T_c$, where there is no quark-gluon plasma of free charges.
Thus, we have two IR factors so far, one acting below and one above $T_c$; while
both have to vanish at the transition point.

We would now argue that the there must be the third term in the $O(\rho^2)$
potential, which would be nonzero (and in fact peaked) at $T_\text{c}$: the **magnetic**
screening term. Indeed, apart of Bose condensed monopoles there should be
also thermally excited magnetically charged quasiparticles. Their presence
can be monitored via a magnetic screening mass $M_M$, which is absent in the
perturbation theory but is measured the lattice (see e.g. [15]) and seem to
have a maximum at $T_c$. Because our probe - instantons - are selfdual, let me
suggest that the IR cutoff should also respect an electric-magnetic duality. We thus suggest the IR suppression factor in the form

$$e^{\Phi(\rho, T)}|_{IR} = e^{[ -2\pi \rho^2 (\sigma(T) + \frac{\pi M^2_E(T)}{g^2} + \frac{\pi M^2_M(T)}{g^2_M}) ]} \quad (11)$$

where $g_M$ is the magnetic charge. Three terms in the exponent can all be independently measured on the lattice at any $T$: we remind that they originate from condensed monopoles, thermal (non-condensed) electric and magnetic quasiparticles, respectively.

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