On the redefinition of the kelvin

A Peruzzi
VSL, Postbus 654, 2600 AR, Delft, NL
E-mail: aperuzzi@vsl.nl

Abstract. The current and the forthcoming definition of the base unit kelvin are critically analysed. Using basic concepts of statistical thermodynamics, an attempt is made to elucidate the physical meaning of the revised definition of the kelvin. The consistency between the revised and the current thermodynamic scales is evaluated and the impact of the redefinition on the measurement infrastructures is speculated on.

1. Introduction
In May 2019 a major and unprecedented revision of the International System of Units (SI) will take place: four of the seven base units, the kelvin, the kilogram, the ampere and the mole, will be redefined. The new definitions will be based on abstract prescriptions that relate the units to fundamental constants of nature. The rationale behind such radical change in the SI is to disconnect the definitions of the SI base units from material artefacts, which may vary uncontrollably with time and location, and to link them to fundamental constants of nature, which are by definition space- and time-invariant.

Whenever an SI unit is revised, extreme attention must be devoted to the consistency of the new definition to the previous one, because the measurements performed in the past, when the previous definition was in force, must remain valid and the new definition should not cause any disruption on the everyday measurement practice. More specifically, in the case of the kelvin redefinition, the transition is further complicated by the existence of two temperature scales: the thermodynamic temperature scale, of which the revised kelvin is the unit, and the International Temperature of 1990 (ITS-90), which is the scale used in all practical measurements of temperature.

In this paper we recall the current definition of the kelvin (Section 2), we introduce the new definition (Section 3), that will come into force in May 2019, and we clarify its physical meaning. We then analyse the consistency of the new thermodynamic temperature scale, generated by the kelvin redefinition, with the present thermodynamic temperature scale (Section 4) and finally we discuss the potential impact of the kelvin redefinition on the user community (Section 5).

2. The current definition of the kelvin
Since 1954, the unit of thermodynamic temperature, the kelvin (symbol K) is defined as “the fraction 1/273.16 of the thermodynamic temperature of the triple point of water” [1]. In other words, the kelvin is defined by assigning a numerical value of 273.16 kelvins to the thermodynamic temperature of the triple point of water (TPW): $T_{TPW} = 273.16$ K.

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1 To whom any correspondence should be addressed.
The TPW is the unique physical state of water in which all three phases (solid, liquid and vapour) coexist at thermodynamic equilibrium. It is realized in practice in TPW cells [2], which are sealed fused-silica envelopes containing approximately half litre of high-purity water having approximately the isotopic composition of Vienna Standard Mean Ocean Water (VSMOW). When a fraction of the liquid is frozen by cooling the cell and the cell is accommodated in a thermostat bath controlled at a temperature close to 273.16 K, the equilibrium between the three phases is automatically established within the cell and can be maintained for many months.

The definition of the kelvin is thus currently linked to the practical realization of an artefact, the TPW cell. However, differently from the present kilogram definition, the kelvin definition is not linked to a specific artefact, in the sense that there is no international prototype of the kelvin: any laboratory can realize the kelvin by manufacturing its own TPW cell.

Apart from the uncertainty associated with the practical realization of the TPW (for example, the water in the TPW cell cannot have exactly the isotopic composition of VSMOW water and cannot be completely free from chemical impurities), which is estimated to be, at best, approximately 10 µK, no thermodynamic uncertainty is attributed to its numerical value.

It is worth mentioning that, because based on the principles of thermodynamics, the thermodynamic temperature scale, besides the TPW has in fact another fundamental fixed point: the thermodynamic temperature of the physical state “absolute zero” is given a numerical value of 0 K.

In the present SI (before the redefinition that will take place in 2019), the Boltzmann constant $k$ is a measured quantity that has been measured very accurately in recent years with fundamentally different methods: Acoustic Gas Thermometry (AGT), Dielectric Constant Gas Thermometry (DCGT), Johnson Noise Thermometry (JNT) [3]. According to the CODATA 2017 Special Adjustment of fundamental constants [4], $k = 1.38064903(51) \times 10^{-23}$ JK$^{-1}$, with a relative standard uncertainty of $3.7 \times 10^{-7}$.

As an example, let us consider one of the methods mentioned above, that were used to accurately determine the value of the Boltzmann constant: AGT. AGT exploits the relationship between the zero-pressure limit of the speed of sound $u_0$ in a gas and the thermodynamic temperature $T$: 

$$kT = \frac{m u_0^2}{\gamma_0}$$

where $m$ is the mass of one molecule of the gas and $\gamma_0$ is the zero-pressure limit of the ratio of the constant-pressure specific heat capacity to the constant-volume specific heat capacity. In the current SI, an AGT measurement performed at the $T_{TPW}$, being $T_{TPW}$ exactly known (apart from the uncertainty in its practical realization), provides a measurement of $k$.

3. The future definition of the kelvin

From May 2019 on, the kelvin will be defined “by taking the fixed numerical value of the Boltzmann constant $k$ to be $1.380649 \times 10^{-23}$ when expressed in the units JK$^{-1}$, which is equal to kg m$^2$ s$^{-1}$ K$^{-1}$, where the kilogram, metre and second are defined in terms of $h$, $c$ and $\Delta v_C$.” [5]. In other words, the kelvin is chosen in such a way that makes the Boltzmann constant exactly equal to $1.380649 \times 10^{-23}$ JK$^{-1}$.

For a clear understanding of the physical meaning of this new definition of the kelvin, we need to resort to statistical thermodynamics, which provides the connection between the macroscopic properties of a thermodynamic system and the microscopic behaviour of the particles that constitutes the system. A standard proof of statistical thermodynamics is that, for an ideal gas of monoatomic non-interacting molecules at thermodynamic equilibrium, the mean kinetic energy $\langle E_k \rangle$ of each molecule is given by:

$$\langle E_k \rangle = \frac{3}{2} kT$$

which can then be generalized into the equipartition theorem: at thermodynamic equilibrium any degree of freedom contributes $\frac{1}{2} kT$ to the mean energy per molecule.
A direct consequence of this result is that the Boltzmann constant $k$ is essentially the conversion factor between the mean energy of the molecules and the thermodynamic temperature of the system (and the link between the concepts of statistical mechanics and thermodynamics).

Enlightened by the light of statistical thermodynamics, the new definition of the kelvin gets easier to grasp: the kelvin is the change of thermodynamic temperature that results in a change of thermal energy $kT$ by $1.380649 \times 10^{-23}$ J.

The advantage of the new definition is that thermodynamic temperature can be realized directly at any point in the scale without referring to the singular temperature of the TPW.

Let us consider again the AGT experiment that we introduced at the end of Section 2. With the new definition, the Boltzmann constant $k$ is fixed and has no uncertainty, so the same AGT experiment performed at the TPW is in fact a new determination of the thermodynamic temperature of the TPW. The new numerical value obtained for the TPW can be compared to the value at the time of the kelvin redefinition: $273.16 \pm 101 \mu$K. Such experiment will provide a new numerical value for the TPW, probably different from exactly 273.16, but hopefully compatible with $273.16 \pm 101 \mu$K (confidence level $k = 1$) [7]. In the future, the uncertainty of the AGT in determining $T_{TPW}$ will improve to values lower than 101 µK, but will always include a component from the realization of the TPW (currently 10 µK at best).

4. Consistency of the revised thermodynamic scale with the current thermodynamic scale

As clarified in the previous section, from 2019 on, the Boltzmann constant will be an exact quantity with no uncertainty. The relative uncertainty of the Boltzmann constant in the current definition ($3.7 \times 10^{-7}$) will be, after the redefinition, transferred to the numerical value of the TPW: a relative uncertainty of $3.7 \times 10^{-7}$ in $T_{TPW} = 273.16$ K, corresponding to a standard uncertainty of 101 µK.

In order to evaluate the consistency of the new definition with the old definition, we can reason as follows. The exactly known defining constant in the current SI is the temperature of the TPW: $T_{TPW} = 273.16 \text{ K}_{cur}$, where the subscript to the kelvin symbol emphasizes that we mean the unit kelvin in the current SI. In the revised SI, the numerical value X of the temperature will be inexacty known: $T_{TPW} = X \text{ K}_{rev}$, where $K_{rev}$ is the unit kelvin in the revised SI. Of course, the temperature of the TPW does not depend on the SI unit adopted so: $273.16 \text{ K}_{cur} = X \text{ K}_{rev}$. The consistency factor $f$ is defined as the ratio of the numerical value of $T_{TPW}$ in the revised SI to the numerical value of $T_{TPW}$ in the current SI:

$$f = \frac{X}{273.16} = \frac{T_{TPW}/K_{rev}}{273.16} = \frac{k_{cur}}{k_{rev}} = \frac{1.38064901 \times 10^{-23}}{1.380649 \times 10^{-23}} = 1.000000007$$

Where $k_{cur}$ and $k_{rev}$ are the numerical values of the Bolzmann constant in the current and in the revised SI, respectively. Such consistency, when expressed in temperature, corresponds to approximately 2 µK at the TPW.

Clearly the change from the current thermodynamic scale to the revised thermodynamic scale will not be perceptible: 2 µK difference at 273.16 K (TPW) and 9 µK difference at 1234.93 K (Ag fixed point).

5. Future implications of the redefinition of the kelvin

We have shown that the thermodynamic temperature scale arising from the redefinition of the kelvin will be, for any practical purpose, indistinguishable from the current thermodynamic temperature scale. Although the thermodynamic scale is the fundamental scale of temperature to which all temperature measurements should be related, the measurement of thermodynamic temperature is extremely complex, time consuming, expensive, highly specialized personnel-intensive and poorly reproducible. Consequently, until the present days, the measurement of thermodynamic temperature has always been confined to few National Metrology Institutes (NMIs) in the world.

To meet the needs of worldwide uniformity of temperature measurements, practical International Temperature Scales have been defined (ITS-27, IPTS-48, IPTS-68 and ITS-90, named after the year of
promulgation), which are essentially recipes for the realizations of highly reproducible and precise temperature standards, that agree with the thermodynamic scale as closely as current knowledge permits. As consequence, the quantity determined in all present-day temperature measurements is not the thermodynamic temperature $T$ but the international temperature $T_{90}$, as defined by the ITS-90 [6].

In the ITS-90, the TPW has a fundamental role, because any ITS-90 temperature $T_{90}$ must be referenced (in the standard platinum resistance thermometers range, from 13.8033 K to 1234.93 K) to the $T_{TPW}$, which is assigned a value of 273.16 K by definition, with no thermodynamic uncertainty.

In the present thermodynamic scale, the TPW has in a way the same fundamental role, because the TPW is its fundamental fixed point and is assigned the same numerical value of 273.16 K (in fact, it is the assignment of this numerical value that defines the unit kelvin of the thermodynamic scale) with no thermodynamic uncertainty.

After the redefinition of the kelvin in 2019, the TPW will lose its fundamental role in the thermodynamic scale, but will preserve its fundamental role in the ITS-90 scale and, as we have seen in Section 3, it may be possible that in the future, the best estimate for the TPW (within the thermodynamic temperature scale) will be different from 273.16.

After nearly 30 years of worldwide use, the ITS-90 is now deeply implemented in the hardware and in the software of the user community (not only NMIs and calibration laboratories, but also industrial laboratories) and it is expected that it will continue to be used, at least in the next future, for nearly all practical measurements.

At the same time, the redefinition of the kelvin is expected to promote the increased use of primary thermometers, that can realize and disseminate thermodynamic temperatures directly through the redefined kelvin [8]. In the long term, primary thermometers will become competitive with the practical thermometers traceable to the ITS-90, because the future progress in the techniques will improve their reproducibility, uncertainty and ease of use.

Clearly, for many years to come, two temperature scales will continue to coexist, the thermodynamic temperature scale and the ITS-90. Although this coexistence is not a news and does not generate ambiguities among the NMI thermometrists, it could be of some concern that, in the coming years, for the first time both scales will start to coexist in a much wider environment than just a few NMIs. The continuous update of the estimated differences between $T$ and $T_{90}$ ($T - T_{90}$) will alleviate this potential ambiguity.

6. References

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