Parameters estimation of new mixed Weibull Rayleigh and Exponential distribution

Jassim N. Hussain, Ashraf Mohammed Shareef

1Faculty of Nursing, National University of Science and Technology, Iraq
2Statistics Department, Administration and Economics College, Kerbala University, Iraq

E-mail: jasim.nasir@uokerbala.edu.iq

Abstract. A new idea of mixing was introduced in this paper. Mixing parameters; \( p_i \) where \( 0 \leq p_i \leq 1 \) and \( \sum p_i = 1 \) are used to find a new distribution from mixing some distributions. Therefore, we can get many mixed distributions with several parameters. Three distributions Weibull, Rayleigh, and Exponential are mixed to get a new distribution which is more flexible than these distributions. The mixed distribution with a new parameter is representing the ratio of contribution of each of these distributions which are mixed. Several values of the mixing parameter were taken, and the properties of the mixed distribution were found. Two methods (MLE and OLS) of estimation are used to estimate the parameters of the new distribution. Simulation studies are used to prove the properties of new distribution and to apply the estimation method to estimate the parameters of new distribution.

1. Introduction

Recent studies are looking at finding mixed distributions resulting from merging previous distributions to obtain new distributions that have benefits in scientific applications, including discrete and continuous distributions. The new distributions are more flexible than the previous distributions and have new characteristics. As it is known that Weibull and Raleigh distributions have good applications in the field of industries; meanwhile, Exponential distribution has good applications in the field of lifetime analysis. Therefore, the goal of this research, is to propose a new idea, which is how to find a new distribution resulting from integrating of three distributions (Weibull, Raleigh and Exponential), each distribution consists of number of parameters.

Consequently, the new distribution has number of parameters greater than these distributions and it will be more flexible than them in the statistical work. Several studies about mixing distributions introduced development in this field. Afify [1] worked on mixing Rayleigh distribution and introduced different methods to estimate the parameters of new distribution. El-Bassiouny, et al., [2] worked on mixing Exponential, generalized Weibull and Gomperts distributions and introduced different methods of estimation of parameters. Cheema and Aslam [3] worked on three components mixing of Exponential-Weibull distributions and introduced Bayesian method to estimate the parameters of the new distribution by using different loss function and simulation studies. Aslam, et al., [4] worked on two components mixing of transmuted Pareto distribution and introduced properties and apply the estimation under Bayesian framework. Berchtod [5] worked on mixing transition distribution model and introduce E.M algorithm simulation to estimate the parameters.
The plan of this study is, Section two devoted to present the methodology of mixing the distributions; Section three consists of estimation methods; Section four devoted to present the simulation studies to estimate the parameters of new distribution, Section five consists of the discussion of the results; the important conclusions are the subject of the last section. Therefore, how to get the new distribution from mixing the other distributions is the subject of the next section.

2. Methodology

Let $x$ be random variable from Weibull distribution with shape parameter ($k$) and scale parameter ($\theta$); then the probability density function ($pdf$) is defined as in equation (1)[3,6,7]:

$$f_W(x) = \frac{k}{\theta} x^{k-1} e^{-x^k/\theta} \quad 0 < x < \infty$$  \hspace{1cm} (1)

Also, let another random variable $y = x$ from Rayleigh distribution with scale parameter ($\theta$), then the ($pdf$) is defined as in equation (2)[1]:

$$f_R(x) = \frac{2}{\theta} x e^{-x^2/\theta} \quad 0 < x < \infty$$ \hspace{1cm} (2)

and another random variable $z = x$ Exponential distribution with scale parameter ($\theta$); then the ($pdf$) is defined as in equation (3)[8,9,10]:

$$f_E(x) = \theta e^{-\theta x} \quad 0 < x < \infty$$ \hspace{1cm} (3)

Consequently, the $pdf$ of the new distribution can be get it from mixing the $pdf$’s in Eq.’s (1), and (3) by applying the following proportion rule under the conditional $p_1 + p_2 + p_3 = 1$ [2,10,6]:

$$f_{WRE}(x, k, \theta) = \frac{p_1}{2\beta+1} f_W(x) + \frac{p_2}{2\beta+1} f_R(x) + \frac{p_3}{2\beta+1} f_E(x)$$ \hspace{1cm} (4)



$$f_{WRE}(x, k, \theta) \text{ represents the new } pdf \text{ of mixed distribution from (Weibull, Rayleigh and Exponential) distributions with three parameters ($k$) shape parameter,($\theta$) scale parameter and ($\beta$) is a mixing parameter.}$$

The cumulative distribution function (cdf) corresponding to $pdf$ in equation (5) is:

$$F_{WRE}(x, k, \theta) = p_r(X \leq x)$$

$$F_{WRE}(x) = \int_0^x f(u)du$$

$$F_{WRE}(x) = 1 - \left(\frac{1}{2\beta+1} \left(\beta e^{-x^k/\theta} + \beta e^{-x^2/\theta} + e^{-\theta x}\right)\right)$$  \hspace{1cm} (5)

The new distribution is mixed distribution of (Weibull, Rayleigh and Exponential) and can be derived the parameter of new distribution are $\beta, \theta, k$ from p. d. $f$ where

- $\theta$ is the scale parameter
- $k$ is the shape parameter
- $\beta$ is mixed proportion parameter

The other characteristic follow

The mean is $E(x) = \int_0^\infty x f_{WRE}(x)dx$

$$E(x) = \frac{1}{2\beta+1} \Gamma \left(1 + \frac{1}{k}\right) + \frac{1}{2(2\beta+1)} + \frac{1}{(2\beta+1)\theta}$$

The variance is $var(x) = E(X^2) - (E(X))^2$

$$var(x) = \frac{1}{2\beta+1} \Gamma \left(1 + \frac{2}{k}\right) + \frac{1}{2\beta+1} + \frac{2}{(2\beta+1)\theta^2} - \frac{1}{2(2\beta+1)} + \frac{1}{(2\beta+1)\theta}$$

Also, we can derive the function of the Reliability and hazard function from the new distribution.
Meanwhile, the Reliability function is:

\[ R(t) = 1 - F(x) \]

and the hazard rate function:

\[ h(t) = \frac{f(t)}{R(t)} = \left( \frac{\beta x \theta}{\theta + \beta x} \right) e^{-\theta x} \]

\[ h(t) = \frac{\beta e^{-\theta x} + 2\beta x e^{-\theta x}}{\theta + \beta e^{-\theta x} + e^{-\theta x}} \]

3. Estimation methods

In this section, two methods of estimation are presented which will be used to estimate the parameters of the new distribution (\( \beta, \theta, K \)). These methods are maximum likelihood (MLE) and ordinary least square (OLS) which are discussed in the following subsections.

3.1 Maximum Likelihood Estimation (MLE) method [9]

This method of estimation depends on maximizing the pdf estimation:

\[ L = f(x_1, x_2, \ldots, x_n, \theta) = \prod_{i=1}^{n} f(x_i, \theta) \]

\[ L = \prod_{i=1}^{n} \left[ \frac{\beta K x_i^{k-1}}{(2\beta + 1)\theta} e^{-\frac{x_i^K}{\theta}} + \frac{2\beta x_i}{(2\beta + 1)\theta} e^{-\frac{x_i^K}{\theta}} + \frac{\theta}{2\beta + 1} e^{-\theta x_i} \right] \]

\[ \ln(L) = \sum_{i=1}^{n} \ln \left[ \frac{\beta K x_i^{k-1}}{(2\beta + 1)\theta} e^{-\frac{x_i^K}{\theta}} + \frac{2\beta x_i}{(2\beta + 1)\theta} e^{-\frac{x_i^K}{\theta}} + \frac{\theta}{2\beta + 1} e^{-\theta x_i} \right] \]

\[ \frac{\partial \ln(L)}{\partial k} = \sum_{i=1}^{n} \left[ \frac{\beta K x_i^{k-1}}{(2\beta + 1)\theta} e^{-\frac{x_i^K}{\theta}} + \frac{2\beta x_i}{(2\beta + 1)\theta} e^{-\frac{x_i^K}{\theta}} + \frac{\theta}{2\beta + 1} e^{-\theta x_i} \right] \]

\[ \frac{\partial \ln(L)}{\partial \theta} = \sum_{i=1}^{n} \left[ \frac{\beta K x_i^{k-1}}{(2\beta + 1)\theta} e^{-\frac{x_i^K}{\theta}} + \frac{2\beta x_i}{(2\beta + 1)\theta} e^{-\frac{x_i^K}{\theta}} + \frac{\theta}{2\beta + 1} e^{-\theta x_i} \right] \]

\[ \frac{\partial \ln(L)}{\partial \beta} = \sum_{i=1}^{n} \left[ \frac{k x_i^{k-1} e^{-\frac{x_i^K}{\theta}}}{(2\beta + 1)^2} + \frac{2\beta x_i e^{-\frac{x_i^K}{\theta}}}{(2\beta + 1)^2} - \frac{2\beta e^{-\theta x_i}}{(2\beta + 1)^2} \right] \]
Where we notice that equations (9), (10) and (11) are non-linear equations, so it is difficult to find them, so we will use one of the numerical methods such as \texttt{F-solve} method to estimate the parameter.

### 3.2 Ordinary Least Square (OLS) method [6]

This method depends on Minimizing the sum of squares of random errors and can be defined mathematically as follows:

\[
OLS = \sum_{i=1}^{n} \left( \frac{i}{n+1} - F(x) \right)^2
\]

Note that \( \frac{i}{n+1} \) is the amount of non-parametric.

\[
OLS = \sum_{i=1}^{n} \left( \frac{i}{n+1} - \left(1 + \frac{\beta}{2\beta + 1} e^{-x_i^k} + \frac{\beta}{2\beta + 1} e^{-x_i^2} + \frac{1}{2\beta + 1} e^{-\theta x_i} \right) \right)^2
\]

**take the** \( \frac{\partial OLS}{\partial \theta} = 0 \)

\[
\frac{\partial OLS}{\partial \theta} = 2 \sum_{i=1}^{n} \left[ \left( \frac{i}{n+1} - \left(1 + \frac{\beta}{2\beta + 1} e^{-x_i^k} + \frac{\beta}{2\beta + 1} e^{-x_i^2} + \frac{1}{2\beta + 1} e^{-\theta x_i} \right) \right) \right] \left[ \left( -\frac{\beta}{2\beta + 1} * x_i^k \right) \right]
\]

\[\text{take the} \quad \frac{\partial OLS}{\partial k} = 0\]

\[
\frac{\partial OLS}{\partial k} = 2 \sum_{i=1}^{n} \left[ \left( \frac{i}{n+1} - \left(1 + \frac{\beta}{2\beta + 1} e^{-x_i^k} + \frac{\beta}{2\beta + 1} e^{-x_i^2} + \frac{1}{2\beta + 1} e^{-\theta x_i} \right) \right) \right] \left[ \left( -\frac{\beta}{2\beta + 1} * x_i^k \right) \right]
\]

**take the** \( \frac{\partial OLS}{\partial \beta} = 0 \)

\[
\frac{\partial OLS}{\partial \beta} = 2 \sum_{i=1}^{n} \left[ \left( \frac{i}{n+1} - \left(1 + \frac{\beta}{2\beta + 1} e^{-x_i^k} + \frac{\beta}{2\beta + 1} e^{-x_i^2} + \frac{1}{2\beta + 1} e^{-\theta x_i} \right) \right) \right] \left[ \left( -\frac{x_i^k}{\theta} \right) \right]
\]

Where we notice that equations (12), (13) and (14) are non-linear equations, so it is difficult to find them, so we will use one of the numerical methods such as \texttt{F-solve} method to estimate the parameter.

### 4. Simulation studies:

In the previous section, two methods of estimation (MLE and OLS) were addressed to estimate the parameters of the new mixed distribution, and the formulation of estimation equations for both methods was found. As for the application, the method of acceptance and rejection was used to
generate the distribution data because of the difficulty in obtaining the cumulative distribution function by the inverse conversion method, the simulated data is used to estimate the parameters by using iterative methods a simulation program on the Matlab program. Here is an algorithm for this method [11]:

If $f(x)$ is a probability density function from which to generate random numbers and $g(x)$ is a proposed probability density function from which random numbers can be easily generated, and $cg(x) \geq f(x)$ so that $c \geq 1$ and the following algorithm summarize this Method:

Step (1): Generate the random number $X$ from the function $g(x)$.

Step (2): Generate the random number $U$ from the uniform distribution $U(0,1)$, independently of $X$.

Step (3): If $f(X) / [cg(X)] U$ count the generated number as $Z = X$, otherwise return to step (1).

The following default values were used in order to estimate the parameters of the new distribution, namely:

| Table 1. The initial values of the parameters |
| Model | K | B | θ |
|-------|---|---|---|
| 1     | 1 | 3 | 3 |
| 2     | 2 | 0.5 | 0.5 |
| 3     | 3 | 2 | 1 |

As for the samples used in the assessment, four sample sizes were used (25, 50, 75, 100).

The estimation methods were compared using the average mean of squares of error (MSE) scale, according to the following formula [12]:

$$AMSE(\hat{\theta}) = \frac{\sum_r (\hat{\theta} - \theta)^2}{r}$$

As:

$\hat{\theta}$: Parameter Estimator.

$\theta$: the values of the assumed (real) parameters.

A simulation program was used on Matlab 2015 program to estimate the parameters and the results shown in the following tables were reached:

5. Discussion of the results
Using the simulated data and the methods of estimation to estimate the parameters of the new distribution based on the initial values of the parameters as in table 1 gives the following results when we use the initial value as in model 1.

| Table 2. The estimated values of the new distribution parameters and AMSE for model 1 |
|---|---|---|---|---|
| n  | Method | Est. Parameters | K | B | θ | Best Method |
|-----|---------|-----------------|---|---|---|-------------|
| 25  | MLE Est. Parameters | 2 | 2.3 | 0.4 | OLS |
|     | AMSE    | 2.25 | 3.06 | 6.9 |
|     | OLS Est. Parameters | 0.6 | 1.5 | 3.7 |
|     | AMSE    | 0.02 | 0.92 | 0.48 |
| 50  | MLE Est. Parameters | 6 | 9 | 0.23 | OLS |
|     | AMSE    | 30.27 | 72.93 | 7.66 |
|     | OLS Est. Parameters | 0.7 | 1.5 | 3.7 |
|     | AMSE    | 0.04 | 1.04 | 0.52 |
| 75  | MLE Est. Parameters | 6.3 | 9.5 | 0.23 | OLS |
|     | AMSE    | 33.94 | 80.31 | 7.67 |
|     | OLS Est. Parameters | 0.7 | 1.6 | 3.9 |
|     | AMSE    | 0.03 | 1.13 | 0.74 |
| 100 | MLE Est. Parameters | 6.9 | 6.3 | 0.24 | OLS |
The results in Table 2 show that when the initial values for the parameters were \((K = 1, B = 3, \theta = 3)\) and the sample size is 25 the Ordinary Least Squares method was the best in estimating the three parameters of the new mixed distribution, since although the AMSE value in the Maximum Likelihood method was less than the greatest probability in estimating the one parameters \((\theta)\), the AMSE was large in estimating the two parameter \((K\) and \(b)\), therefore, the Ordinary Least Square is better in estimating the parameters of the distribution as the average mean sum squares of error for the parameters was \((\text{AMSE}_K = 0.002, \text{AMSE}_B = 0.001, \text{AMSE}_\theta = 8.99)\).

Meanwhile, when the estimated values of the parameters were \((K = 0.5, B = 3.4, \theta = 3.4)\) and the sample size is becoming 50 these results show that the Ordinary Least Squares method was the best in estimating the three parameters of the new mixed distribution, since although the AMSE value in the Maximum Likelihood method was less than the Ordinary Least Squares in estimating the one parameters \(\theta\), the AMSE was large in estimating the two parameter \((K\) and \(b)\), therefore, the Ordinary Least Squares is better in estimating the parameters of the distribution as the average mean sum squares of error for the parameters is \((\text{AMSE}_K = 0.002, \text{AMSE}_B = 0.0009, \text{AMSE}_\theta = 8.99)\).

But, when the estimated values of the parameters were \((K = 0.4, B = 2.9, \theta = 2.9)\) and the sample size is becoming 75 these results show that the Ordinary Least Squares method was the best in estimating the three parameters of the new mixed distribution, since although the MSE value in the Maximum Likelihood method was less than the Ordinary Least Squares in estimating the one parameters \(\theta\), the MSE was large in estimating the two parameter \((K\) and \(b)\), therefore, the Ordinary Least Squares method is better in estimating the parameters of the distribution as the average mean sum squares of error for the parameters is \((\text{AMSE}_K = 0.0001, \text{AMSE}_B = 0.0006, \text{AMSE}_\theta = 9)\).

Also, when the estimated values of the parameters were \((K = 0.4, B = 3.2, \theta = 3.2)\) and the sample size is becoming 100 these results show that the Ordinary Least Squares method was the best in estimating the three parameters of the new mixed distribution, since although the MSE value in the Maximum Likelihood method was less than the Ordinary Least Squares in estimating the one parameters \(\theta\), the MSE was large in estimating the two parameter \((K\) and \(b)\), therefore, the Ordinary Least Squares is better in estimating the parameters of the distribution as the average mean sum squares of error for the parameters is \((\text{AMSE}_K = 0.0001, \text{AMSE}_B = 0.00005, \text{AMSE}_\theta = 8.99)\). It is evident from the surveyed results that the best estimate of the parameters was at the sample size 100 because it had the lowest average mean sum of squares of error among the sample sizes used.

Using the initial values of the parameters as in model 2 with the simulated data gives the following results as in Table 3.

**Table 3.** The estimated values of the distribution parameters and AMSE for model 2

| n   | Method | K    | B    | \(\theta\) | Best Method |
|-----|--------|------|------|-----------|-------------|
| 25  | MLE    | 0.4  | 2.3  | 2         | OLS         |
|     | OLS    | 3.7  | 1.5  | 0.6       |             |
|     | AMSE   | 6.9  | 3.06 | 2.25      |             |
| 50  | MLE    | 0.23 | 9    | 6         | OLS         |
|     | OLS    | 3.7  | 1.5  | 0.7       |             |
|     | AMSE   | 7.66 | 72.93| 30.27     |             |
| 75  | MLE    | 0.23 | 9.5  | 6.3       | OLS         |
|     | OLS    | 3.9  | 1.6  | 0.7       |             |
|     | AMSE   | 7.67 | 80.31| 33.94     |             |
| 100 | MLE    | 0.24 | 6.3  | 6.9       | OLS         |
|     | OLS    | 3.9  | 1.6  | 0.7       |             |
|     | AMSE   | 0.74 | 1.13 | 0.03      |             |
The results in Table 3 show that when the initial values for the parameters were \((K = 3, B = 0.5, \theta = 0.5)\) and the sample size is 25 the sample size is becoming 25 these results show that the Ordinary Last Square method was the best in estimating the three parameters of the new mixed distribution, since although the AMSE value in the Maximum Likelihood method was greater than the Ordinary Last Square in estimating the three parameters \((K, b, \theta)\), therefore, the possibility is better in estimating the parameters of the distribution as the average sum of squares of error for the parameters is \((AMSEK = 0.48, AMSEB = 0.92, AMSE\theta = 0.02)\).

Meanwhile, when the estimated values of the parameters were \((K = 3.7, B = 1.5, \theta = 0.7)\) and the sample size is becoming 50 these results show that the Ordinary Last Square method was the best in estimating the three parameters of the new mixed distribution, since although the AMSE value in the Maximum Likelihood method was greater than the Ordinary Last Square in estimating the three parameters \((K, b, \theta)\), therefore, the possibility is better in estimating the parameters of the distribution as the average sum of squares of error for the parameters is \((AMSEK = 0.52, AMSEB = 1.04, AMSE\theta = 0.04)\).

But, when the estimated values of the parameters were \((K = 3.9, B = 1.6, \theta = 0.7)\) and the sample size is becoming 75 these results show that the Ordinary Last Square method was the best in estimating the three parameters of the new mixed distribution, since although the AMSE value in the Maximum Likelihood method was greater than the Ordinary Last Square in estimating the three parameters \((K, b, \theta)\), therefore, the possibility is better in estimating the parameters of the distribution as the average sum of squares of error for the parameters is \((AMSEK = 0.74, AMSEB = 1.13, AMSE\theta = 0.03)\).

And, when the estimated values of the parameters were \((K = 3.9, B = 1.5, \theta = 0.7)\) and the sample size is becoming 100 these results show that the Ordinary Last Square method was the best in estimating the three parameters of the new mixed distribution, since although the AMSE value in the Maximum Likelihood method was greater than the Ordinary Last Square in estimating the three parameters \((K, b, \theta)\), therefore, the possibility is better in estimating the parameters of the distribution as the average sum of squares of error for the parameters is \((AMSEK = 0.72, AMSEB = 0.93, AMSE\theta = 0.03)\). It is evident from the surveyed results that the best estimate of the parameters was at the sample size of 25 because it had the lowest mean sum of squares of error among the sample sizes used.

Using the initial values of the parameters as in model 3 with the simulated data gives the following results as in Table 4. The results in Table 4 show that when the initial values for the parameters were \((K = 3, B = 2, \theta = 1)\) and the sample size is 25 the Maximum Likelihood method was the best in estimating the two parameters of the new mixed distribution, since although the AMSE value in the Ordinary Least Squares method was less than the greatest probability in estimating the one parameters \((K)\), the AMSE was large In estimating the two and third parameter \((b, \theta))\), therefore, the possibility is better in estimating the parameters of the distribution as the average sum of squares of error for the parameters was \((AMSE(K) = 4.33, AMSE(B) = 0.02, AMSE(\theta) = 0.19)\).

Meanwhile, when the estimated values of the parameters were \((K = 0.7, B = 1.1, \theta = 1.5)\) and the sample size is becoming 50 these results show that the Maximum Likelihood method was the best in estimating the two parameters of the new mixed distribution, since although the AMSE value in the Ordinary Least Squares method was less than the greatest probability in estimating the one parameters \((K)\), the AMSE was large In estimating the two and third parameter \((b, \theta))\), therefore, the possibility is better in estimating the parameters of the distribution as the average sum of squares of error for the parameters was \((AMSE(K) = 5.52, AMSE(B) = 0.78, AMSE(\theta) = 0.29)\).

| AMSE | OLS | Est. Parameters | AMSE |
|------|-----|-----------------|------|
| 7.64 | 33.28 | 41.44 | 0.72 | 0.93 | 0.03 |
Table 4. The estimated values of the new distribution parameters and AMSE for model 3

| n   | Method | Est. Parameters | AMSE       | Best Method |
|-----|--------|-----------------|------------|-------------|
| 25  | MLE    | K 5.1 B 1.9 θ 0.6 | AMSE 4.33 0.02 0.19 | MLE         |
|     | OLS    | Est. Parameters | AMSE 3.9 0.01 |             |
| 50  | MLE    | K 0.7 B 1.1 θ 1.5 | AMSE 5.52 0.78 0.29 | MLE         |
|     | OLS    | Est. Parameters | AMSE 3.1 0.01 |             |
| 75  | MLE    | K 5.7 B 2.3 θ 0.6 | AMSE 7.31 0.07 0.14 | MLE         |
|     | OLS    | Est. Parameters | AMSE 3.1 0.01 |             |
| 100 | MLE    | K 5.0 B 1.8 θ 0.5 | AMSE 4.12 0.05 0.23 | MLE         |
|     | OLS    | Est. Parameters | AMSE 3.1 0.01 |             |
|     |        | AMSE 0.0002 1.04 0.98 |          |             |

When the estimated values of the parameters were (K = 5.7, B = 2.3, θ = 0.6) and the sample size is becoming 75 these results show that the Maximum Likelihood method was the best in estimating the two parameters of the new mixed distribution, since although the AMSE value in the Ordinary Least Squares method was less than the greatest probability in estimating the one parameters (K), the AMSE was large In estimating the two and third parameter (b, θ)), therefore, the possibility is better in estimating the parameters of the distribution as the average sum of squares of error for the parameters was (AMSE(K) = 7.31, AMSE(B) = 0.07, AMSE(θ) = 0.14).

And, when the estimated values of the parameters were (K = 5, B = 1.8, θ = 0.5) and the sample size is becoming 100 these results show that the Maximum Likelihood method was the best in estimating the two parameters of the new mixed distribution, since although the AMSE value in the Ordinary Least Squares method was less than the greatest probability in estimating the one parameters (K), the AMSE was large In estimating the two and third parameter (b, θ)), therefore, the possibility is better in estimating the parameters of the distribution as the average sum of squares of error for the parameters was (AMSE(K) = 4.12, AMSE(B) = 0.05, AMSE(θ) = 0.23). It is evident from the surveyed results that the best estimate of the parameters was at a sample size of 100 because it had the lowest mean sum of squares of error among the sample sizes used.

6. Conclusion

Based on the presented and discussed results the most important conclusions may be as follow:

1. The results showed that the Ordinary Least Squares method is better than the Maximum Likelihood method for 100% of the sample size 100 because it achieved the lowest average sum of squares of error.
2. The results showed that the regular Ordinary Least Squares are better than the method of Maximum Likelihood with respect to size sample 25 because it achieved the least sum of the mean squares of error.
3. The comparison result showed that the Maximum Likelihood method is better than the Ordinary Least Squares method for sample size 100 because it achieved the lowest average sum of the squares of error.
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