Lepton Electric Charge Swap at the 10 TeV Energy Scale

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Abstract We investigate the nature of the dark matter by proposing a mechanism for the breaking of local rotational symmetry between ordinary third family leptons and proposed non-regular leptons at energy scales below 10 TeV. This symmetry breaking mechanism involves electric charge swap between ordinary families of leptons and produces highly massive non-regular leptons of order O (1 TeV) mass unobservable at energy scales below 10 TeV (the scale of LEP I, II and neutrino oscillation experiments). Electric charge swap between ordinary families of leptons produces heavy neutral non-regular leptons with order O (1 TeV) masses, which may form cold dark matter. The existence of these proposed leptons can be tested once the Large Hadron Collider (LHC) becomes operative at 10 TeV energy-scales. This proposition may have far reaching applications in astrophysics and cosmology.

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1 Introduction

The nature of dark matter, proposed in 1933 to explain why galaxies in some clusters move faster than their predicted speed if they contained only baryonic matter,[1] is one of the intriguing questions of modern physics. Several candidate dark matter particles have been suggested, including Light Supersymmetric Particles,[2–7] heavy fourth generation neutrinos,[8–9] Q-Balls,[10–11] mirror particles,[12–16] and axion particles, the latter introduced in an attempt to solve the Charge–Parity (CP) violation problem in particle physics.[17–18]

Recently, the Brane world idea has been used to furnish new solutions to old problems in particle physics and cosmology.[19–33] Scenarios in which all fields are allowed to propagate in the bulk are called Universal Extra Dimensions (UED) models.[34–35] UED models provide a viable dark matter candidate, namely the Lightest Kaluza Klein particle (LKP).[36–37] Gauge–Higgs unification models, based on grand unified gauge theories defined on six-dimensional space-time, have interesting properties. In these models, the extra-dimensional space has the topological structure of a two-sphere orbifold $S^2/Z_2$.[38–40]

Furthermore, (thin) braneworlds with conical singularities in six-dimensional Einstein-Gauss-Bonnet gravity with a bulk cosmological constant have been investigated.[41] For axially symmetric bulk, however, this model does not provide isotropic braneworld cosmological solutions.[41]

Other stable or quasi-stable particles could emerge in the string theory spectrum and have been suggested in this context: modulinos,[42] exotic gauge-charged matter,[5] hidden sector matter composites,[2] hidden sector gauge composites[43] and wrapped D-branes.[44] It could be that one or more of these states (or others not yet imagined) contribute to the dark matter of the cosmos.

The original formulation of the superstring theory was in 10 space-time dimensions.[45–47] Indeed, most superstring theories have an underlying interpretation involving more than four space-time dimensions. It was only realized quite recently that strings necessarily contain D-branes (Polchinski 1995, 1996[48–49]), structures which exist in $(D + 1)$-space-time dimensions, less than or equal to the number of dimensions of the full string theory. Open string states may be confined to a lower-dimensional space because their end points are constrained to lie on D-branes through Dirichlet boundary conditions (hence the term D-brane). Thus, if the states of the Standard Model (SM)[50–53] are described by open string states, they may live in a lower dimension than the closed string states, which, having no end points, are not confined to D-branes. The graviton is necessarily a closed string state, and so string theories naturally lead to the possibility that gravity may live in a larger number of dimensions than the states of the SM. Of course, having more space-time dimensions than four is in conflict with observation unless there is some form of compactification which leads to a world that is effectively four-dimensional at large distance scales.

In this paper we investigate the nature of dark matter by beginning with the simplest set-up where only the third family of leptons exists in the four-dimensional part of six-dimensional space-time. This proposition provides a global rotational symmetry between ordinary third family of leptons and proposed non-regular leptons. The latter
is tested in various numbers of lepton families and space-time dimensions. Furthermore, some properties of lepton families may be explained from the provided symmetry, within the framework of superstring theories.\textsuperscript{[45–49]} This local symmetry breaks at energy scale below 10 TeV. We propose a symmetry breaking mechanism that can make the unobserved non-regular leptons highly massive, “explaining” thereby their inobservability by Large Electron Positron (LEP) I, II and neutrino oscillations experiments at energy scales below 10 TeV.\textsuperscript{[54–60]} Electric charge swap between ordinary families of leptons produces heavy neutral non-regular leptons of masses of order O (1 TeV). These particles may form dark matter. The existence of these proposed leptons can be tested once the Large Hadron Collider (LHC) becomes operative at 10 TeV energy-scales.\textsuperscript{[61–63]} This proposition may have far reaching applications in astrophysics and cosmology.

2 Solution of Six-Dimensional Einstein Equations

We begin with the simplest set-up, where only the third family of leptons exists in the four-dimensional part of six-dimensional space-time.

Following, Gogberashvili \textit{et al.},\textsuperscript{[64]} we consider a six-dimensional spacetime with signature $(+,−,−,−,−,−)$. Einstein’s equations in this spacetime have the form

$$R_{AB} − \frac{1}{2} g_{AB} R = \frac{M}{4} (g_{AB} \Lambda + T_{AB}), \quad (1)$$

where $M$ is the six-dimensional fundamental scale, $\Lambda$ is the cosmological constant and $A, B$ are capital indices equal 0, 1, 2, 3, 4, 5.

To split the six-dimensional space-time into four-dimensional and two-dimensional parts, we use the metric ansatz:\textsuperscript{[64]}

$$ds^2 = \phi^2(\theta) g_{\mu\nu}(x^a) dx^\mu dx^\nu - \varepsilon^2 (d\theta^2 + b^2 \sin^2 \theta d\varphi^2), \quad (2)$$

where $\varepsilon$ and $b$ are constants and $\phi(\theta)$ is the warp factor. This warp factor equals one at brane location ($\theta = 0$) and decreases to zero in the asymptotic region ($\theta = \pi$), at the south pole of the extra two-dimensional sphere. Here the metric of the ordinary four-dimensional $g_{\mu\nu}(x^a)$ has signature $(+,−,−,−)$, with $\alpha, \mu, \nu = 0, 1, 2, 3$. The extra compact 2-manifold is parameterized by the spherical angles $\theta, \phi$ ($0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi$). This 2-surface is attached to the brane at point $\theta = 0$. When $\theta$ changes from 0 to $\pi$, therefore, the geodesic distance into the extra dimensions shifts from the north to the south pole of the 2-spheroid. For $b = 1$ in Eq. (2), the extra 2-surface is exactly a 2-sphere with radius $\varepsilon$ (0.1 TeV$^{-1}$).

The ansatz for the energy-momentum tensor of the bulk matter fields is:

$$T_{\mu\nu} = -g_{\mu\nu} E(\theta), \quad T_{ij} = -g_{ij} P(\theta), \quad T_{\mu} = 0. \quad (3)$$

Small Latin indices in Eq. (3) correspond to the two extra coordinates. The source functions $E$ and $P$ depend only on the extra coordinate $\theta$. For these ansätze, Einstein’s equations (1) take the following form:

$$3 \frac{\partial^n}{\partial \phi} + 3 \frac{\partial^{n-2}}{\partial \phi} \frac{\phi}{\phi^2} \cot \theta - 1 = \frac{\varepsilon^2}{M^4} [E(\theta) - \Lambda],$$

$$6 \frac{\partial^2}{\partial \phi} - 4 \frac{\phi}{\phi^2} \cot \theta = \frac{\varepsilon^2}{M^4} [P(\theta) - \Lambda],$$

$$4 \frac{\partial^3}{\partial \phi} + 6 \frac{\partial^2}{\partial \phi} \frac{\phi}{\phi^2} = \frac{\varepsilon^2}{M^4} [P(\theta) - \Lambda], \quad (4)$$

where the prime denotes differentiation $d/d\theta$.

For the four-dimensional space-time, we have assumed zero cosmological constant. Einstein’s equations take the form:

$$R^{(4)}_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R^{(4)} = 0, \quad (5)$$

where $R^{(4)}_{\alpha\beta}$ and $R^{(4)}$ are four-dimensional Ricci tensor and scalar curvature, respectively. In Ref. [65] Gogberashvili and Singleton found a non-singular solution of Eq. (4) for boundary conditions $\phi(0) = 1, \phi'(0) = 0$. This solution was given by:

$$\phi(\theta) = 1 + (a - 1) \sin^2 (\theta/2), \quad (6)$$

where $a$ is the integration constant. The source terms for this solution were given by:

$$E(\theta) = \Lambda \left[ \frac{3(a + 1)}{5\phi(\theta)} - \frac{3a}{5\phi^2(\theta)} \right],$$

$$P(\theta) = \Lambda \left[ \frac{4(a + 1)}{5\phi(\theta)} - \frac{3a}{5\phi^2(\theta)} \right], \quad (7)$$

with the radius of the extra 2-spheroid given by $\varepsilon^2 = 10 M^4 / \Lambda$.

For simplicity, in this paper we take $a = 0$ so that the warp factor takes the form:

$$\phi(\theta) = 1 - \sin^2 (\theta/2) = \cos^2 (\theta/2). \quad (8)$$

This warp factor equals one at the brane location ($\theta = 0$) and decreases to zero in the asymptotic region $\theta = \pi$, i.e., at the south pole of the extra two-dimensional spheroid. The expression for the determinant of our ansatz (2) used in this paper is given by:

$$\sqrt{-g} = \sqrt{-g^{(4)}} \varepsilon^2 \phi^4(\theta) \sin \theta, \quad (9)$$

where $\sqrt{-g^{(4)}}$ is determinant of four-dimensional space-time.

3 Non-Regular Leptons in Six Dimensions

Here we assume that the zero mode corresponds to the non-regular leptons, which are copies of the third family of leptons. Although uncertain, this assumption is not physically implausible: it is reasonable to expect that, when entering the six-dimensional bulk, third family leptons change their properties profoundly and lose, so to say, their individuality (e.g. their observable masses) to their bare masses, spin and magnetic moment.

Let us now consider spinors in the six-dimensional space-time (2), where the warp factor $\phi(\theta)$ has the form
Of course, there are also very massive Kaluza Klein (KK) tons. Higher mass and are distinct from the third family of leptons. These massive KK modes, therefore, have a much higher mass and are distinct from the third family of leptons.

The six-dimensional spinor is given by:
\[ \Psi(x^A) = \left( \begin{array}{c} \psi \\ \xi \end{array} \right). \] (12)

This six-dimensional spinor has eight components and is equivalent to a pair of four-dimensional Dirac spinors \( \psi, \xi \). The representation of the flat (8 x 8) gamma-matrices is given by Ref. [64] as:
\[ \Gamma_\nu = \left( \begin{array}{cc} \gamma_\nu & 0 \\ 0 & -\gamma_\nu \end{array} \right), \quad \Gamma_\theta = \left( \begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right), \]

where 1 denotes the four-dimensional unit matrix and \( \gamma_\nu \) are ordinary (4 x 4) gamma-matrices. The representation (13) gives the correct space-time signature (+,−,−,−,−). The generalization of \( \gamma_5 \) matrix is:
\[ \Gamma_7 = \left( \begin{array}{cc} \gamma_5 & 0 \\ 0 & \gamma_5 \end{array} \right). \] (14)

The variation of action (10) yields the following six-dimensional massless Dirac equation
\[ (h^\mu_A \Gamma^\mu B_\mu + h^\phi_A \Gamma^\phi \partial_\phi + h^\theta_A \Gamma^\theta \partial_\theta) \Psi(x^A) = 0, \] (15)
with the sechbein for our background metric (2) is given by
\[ h^\mu_A = \left( \frac{1}{\phi} \frac{\delta}{\partial \phi}, \frac{1}{\epsilon} \frac{\delta}{\partial \theta}, \frac{1}{\epsilon \sin \theta} \frac{\delta}{\partial \varphi} \right). \] (16)

From the definition of spin connection:
\[ \omega^{\bar{A}}_A = \frac{1}{2} h^{\bar{N}A} (\partial_M h^{N\bar{A}} - \partial_N h^{M\bar{A}} - \frac{1}{2} h^{QS} (\partial_M h_{Q\bar{A}} - \partial_Q h_{M\bar{A}}) h^{\bar{N}M} - \frac{1}{2} h^{\bar{A}Q} \delta h_{Q\bar{A}}) h^{\bar{N}M}. \] (17)

The non-vanishing components of the spin connection are:
\[ \omega^{\bar{A}}_\varphi = -\sin \varphi, \quad \omega^{\bar{A}}_\theta = -\omega^{\bar{A}}_\varphi = -\frac{\omega^{\bar{A}}_\varphi}{\epsilon \sin \varphi} = -\frac{\sin \theta}{2 \epsilon}. \] (18)

The covariant derivatives of the spinor field take the form:
\[ D_\mu \Psi(x^A) = \left( \partial_\mu + \sin \frac{\theta}{4 \epsilon} \Gamma_G \partial_\nu \right) \Psi(x^A), \]
\[ D_\theta \Psi(x^A) = \partial_\theta \Psi(x^A), \]
\[ D_\varphi \Psi(x^A) = \left( \partial_\varphi - \frac{\cos \theta}{2} \Gamma_G \partial_\varphi \right) \Psi(x^A). \] (19)

The Dirac equation takes the form:[66–67]
\[ \left[ \Gamma^\theta \left( \frac{\partial}{\partial \theta} + \cot \theta \right) + \frac{\cot \theta}{2 \epsilon} \Gamma^\varphi \frac{\partial}{\partial \varphi} \right] \Psi(x^A). \] (20)

This system of first-order partial differential equations has the following solutions:
\[ \Psi(x^A) = \frac{1}{\sqrt{2 \pi \phi^2(\theta)}} \left( \frac{a_0(\theta) \psi_0(x^\nu)}{\beta_0(\theta) \xi_0(x^\nu)} \right). \] (21)

\( \psi_0(x^\nu), \xi_0(x^\nu) \) are the four-dimensional Dirac spinors.

Here we note that since the dimension of \( \Psi(x^A) \) in six dimensions is \( m^5/2 \), the dimensions of \( a_0(\theta), \beta_0(\theta) \) and \( \psi_0(x^\nu), \xi_0(x^\nu) \) should be \( m \) and \( m^{5/2} \), respectively.

We are looking for four-dimensional leptonic zero modes. To this end, we consider the conditions under which Eq. (21) obeys the four-dimensional, massless Dirac equations
\[ \gamma^\mu \partial_\mu \psi_0(x^\nu) = \gamma^\mu \partial_\mu \xi_0(x^\nu) = 0. \] (22)

Of course, there are also very massive Kaluza Klein (KK) modes of masses \( u/\epsilon \). However, we assume that \( 1/\epsilon \approx 10 \) TeV. These massive KK modes, therefore, have a much higher mass and are distinct from the third family of leptons.

For the massless case, the 4 spinors \( \psi_0(x^\nu), \xi_0(x^\nu) \) are indistinguishable from the four-dimensional point of view, and we can write \( \psi(x^\nu) = \xi_0(x^\nu) \). Inserting Eqs. (21) and (22) into Eq. (20) converts the bulk Dirac equation into:
\[ \left[ \Gamma^\theta \left( \frac{\partial}{\partial \theta} + \cot \theta \right) + \frac{\cot \theta}{2 \epsilon} \Gamma^\varphi \frac{\partial}{\partial \varphi} \right] \left( \frac{a_0(\theta)}{\beta_0(\theta)} \right) = 0. \] (23)

Using the representation for \( \Gamma^\theta, \Gamma^\varphi \) gives the following system of equations for \( a_0(\theta) \) and \( \beta_0(\theta) \)
\[ \left( \frac{\partial}{\partial \theta} + \frac{\cot \theta}{-2} \right) a_0(\theta) = 0, \quad \left( \frac{\partial}{\partial \theta} + \frac{\cot \theta}{\epsilon} \right) \beta_0(\theta) = 0. \] (24)

The solutions of these equations are:
\[ a_0(\theta) = \frac{A_0}{\sqrt{\sin \theta}}, \quad \beta_0(\theta) = \frac{B_0}{\sqrt{\sin \theta}}, \] (25)
where \( A_0 \) and \( B_0 \) are integration constants with the dimension of mass. The normalizable modes are those for which:
\[ \int \sqrt{-g} \text{d}^6 x \bar{\psi} \Psi = \int \sqrt{g^{(4)}} \text{d}^4 x (\bar{\psi}_0 \psi_0 + \bar{\xi}_0 \xi_0). \] (26)

In other words, we want the integral over the extra coordinates, \( \varphi \) and \( \theta \) are equal to 1. Inserting Eqs. (21), (25) and
the determinant (9) into Eq. (26), the latter requirement gives:
\[ \pi \varepsilon^2 (A_0^2 A_0 + B_0^2 B_0) = 1. \]  
(27)
Explicitly, the expressions for the three normalizable 8-spinors (21) that solve the six-dimensional Dirac equations (20) are:
\[ \psi_0(x^4) = \frac{1}{\sqrt{2\pi \sin \theta \varepsilon^2}} \left( \begin{array}{c} A_0 \\ B_0 \end{array} \right) \psi_0(x^\nu), \]  
(28)
where constants $A_0$ and $B_0$ obey the relations (27).

4 Non-Regular Leptons Coupling with Higgs Field

Brane solutions with different gauge fields and fermion localization mechanisms have been investigated in the literature.\[64,68–71]\] The mass of the zero mode is given via the Higgs mechanism.\[64,68–76]\]

Following Neronov, Aguilar, and Singleton,\[77]\] we address both outstanding issues by introducing a coupling between non-regular leptons and the bulk scalar field $\Phi_0(x^4)$ (with dimensions (mass)$^2$). We do this by adding to the action an interaction term of the form:
\[ S_{int} = \frac{1}{F} \int d^4x d\varphi d\theta \sqrt{-g} \Phi_0 \bar{\psi}_0 \psi_0. \]  
(29)
$F$ is the coupling constant between the scalar and spinor fields and has the dimensions of mass.

For simplicity, we take the massless, real scalar field to be of the form
\[ \Phi_0(x^4) = k_0 \Phi_0(\theta), \]  
(30)
i.e. we take the scalar field as only depending on the bulk coordinates $\theta$, $\varphi$ and not on the brane coordinates $x^\mu$.

The equation of motion of a massless real scalar field in six dimensions has the form:
\[ \frac{1}{\sqrt{-g}} D_A \left[ \sqrt{-g} g^{AB} D_B \Phi_0(x^4) \right] = 0. \]  
(31)
Using the form of Laplace operator on our 2-sphere:
\[ \Delta_2 = \frac{1}{\varepsilon^2} \left( \frac{\partial^2}{\partial \theta^2} + \cot \theta \frac{\partial}{\partial \theta} + \frac{4\Phi'}{\Phi} \frac{\partial}{\partial \theta} + \frac{1}{\sin \theta \partial \theta^2} \right), \]  
(32)
with $\Phi$ derived from Eq. (8), Eq. (31) can be written as:
\[ \Phi'' + \left( \cot \theta - \frac{4 \sin \theta}{1 + \cos \theta} \right) \Phi' = 0. \]  
(33)
To make meaningful estimates of the masses of non-regular leptons we use approximate solutions. Close to the origin ($\theta \to 0$), when $\sin \theta \to 0$ and $\varphi \to 1$, Eq. (33) can be approximated as:
\[ \Phi'' + \cot \theta \Phi' = 0, \]  
(34)
with the following solution
\[ \Phi_0(\theta) = D_0 \left[ 1 + \ln[\tan(\theta/2)] \right]. \]  
(35)
We determine the constants $D_0$ by requiring that the scalar field is normalized over the extra coordinates; i.e. by using Eq. (9), we require:
\[ 2\pi^2 \int_0^\pi d\theta \sin \theta \phi^4(\theta) \Phi_0^2(\theta) = 1. \]  
(36)
From Eq. (36) we derive $D_0 = 10$ TeV.

Substituting Eqs. (21) and (30) into Eq. (29), we find:
\[ S_{int} = U_{0,0}^2 \int d^4x \sqrt{-g} g^{AB} D_B \Phi_0(x^4), \]  
and
\[ U_{0,0}^2 = \frac{\varepsilon^2 k_0}{2\pi F} \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta \Phi_0(\theta) \times [A_0^2 A_0(\theta) \alpha_0(\theta) + B_0^2 B_0(\theta) \beta_0(\theta)]. \]  
(37)
Below, we use the new definition $f_0 = k_0/F$ for the ratios of the four-dimensional constant values of Higgs field derived from Eq. (30). The mass term is, then, as follows:
\[ U_{0,0}^2 f_0 D_0 = 0, 1 \text{ eV}, \]  
(38)
where $f_0 = 10^{-14}$ and $D_0 = 10$ TeV.

Neutral non-regular leptons remain massless, since they are not mixed to the states localized on the brane. Violation of the lepton number $L_s$, can be achieved by introducing operators of six dimension in the number of fields, $(\tilde{l}_0 \Phi)^2/M_{L_s}$, where $\tilde{l}_0$ is the non-regular lepton doublet, $\lambda_{ij}$ are the dimensionless couplings and $M_{L_s}$ is the violation energy scale. Such operators may be generated through gravity effects.\[78]\] Violation of the lepton number $L_s$ generates neutral non-regular lepton masses:
\[ M_{ij} = \lambda_{ij} (\tilde{l}_0 \Phi)^2/M_{L_s}. \]  
(39)
For $\lambda_{ij} \approx 1$ and $M_{L_s} = M_c = 10$ TeV, we find $m_{ij} \approx 1784$ MeV.

5 The Set-Up of the Electric Charge Swap (ECS) Symmetry in Six-Dimensional Space-Time

In the four-dimensional part of six-dimensional space time non-regular leptons have the same mass as ordinary third family leptons. Hypothetical non-regular leptons are, (i) A zero-charged version of the tau, $\tilde{\nu}_\tau^0$ (1784 MeV) and, (ii) A positive charged version of the tau neutrino, $\tilde{\nu}_\tau^+$ (0, 1 eV). non-regular leptons may, therefore, be obtained by the swap of electric charges between tau and tau neutrino particles in the six-dimensional part. We call these proposed non-regular leptons, electric charge swap (ECS) leptons.

Although ECS leptons have the same mass as ordinary third family leptons, they are distinguished from the latter by their different lepton numbers ($L_s = 1$ for ordinary leptons and $L_s = -1$ for ordinary antileptons, respectively) and by their electric charges (positive or neutral for ordinary leptons; negative or neutral for ordinary antileptons, respectively). We hypothesize that ECS leptons are produced by third family leptons when these enter the six-dimensional bulk: in these conditions, the properties of third family leptons change profoundly as these leptons lose, so to say, their individuality and swap their electric charge.
To formulate the swap of electric charge between ordinary leptons, we have to look for symmetry that characterizes swap process in the framework of 2-extra dimensions with compactification scale 10 TeV.

We consider the 2-sphere $S^2$ as a quotient space $S^2 \equiv SU(2)_L / U(1)_Y$ and express the latter in terms of the new symmetry between the original lepton and the new, ECS lepton doublets.

We do this through the following steps:

First, we observe that both the ordinary lepton doublet $l_0(x^\nu) = (\tau^-_L, \nu_e)$ and the ECS lepton doublet $\tilde{l}_0(x^\nu) = (\tilde{\tau}^0_L, \tilde{\nu}^+_e)$ can form the fundamental representation of $SU(2)_L$.[79] This fundamental representation is given by:

$$[I_j, I_k] = i \varepsilon_{jkl} I_l.$$  \hfill (40)

The generators are denoted as:

$$I_i = \frac{1}{2} \tau_i,$$  \hfill (41)

where

$$\tau_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \hfill (42)$$

are the isospin versions of Pauli matrices.

The action of the latter on the new leptons states is represented by:

$$\tilde{\tau}^0_L = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \tilde{\nu}^+_e = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \hfill (43)$$

To link the two distinct sectors, ordinary and ECS leptons, we assume that neither ordinary $L$ nor ECS $L_\text{ECS}$ lepton numbers are conserved, while the overall lepton number is conserved oblivatorily.

$$L_{\text{overall}} = L_s + L = 0,$$  \hfill (44)

$$L_s = \tilde{L}, \quad L_s(\tilde{\nu}^+_e) = \tilde{L}(\tau^+) = -1,$$  \hfill (45)

$$\tilde{L}_s = L, \quad \tilde{L}_s(\tilde{\tau}^0_L) = L(\nu_e) = 1. \hfill (46)$$

The quantum numbers of the new ECS leptons of mass 1784 MeV and 0, 1 eV respectively, are given in Table 1.

**Table 1**  

| New lepton | $M$ | $I$ | $I - z$ | $Q$ | $Y_{S_3}$ | $L$ |
|------------|-----|-----|---------|-----|-----------|-----|
| $\tilde{\nu}^+_e$ | 0, 1 eV | 1/2 | 1/2 | 1 | 1 | -1 |
| $\tau^-_L$ | 1784 MeV | 1/2 | -1/2 | 0 | 1 | -1 |

The next step is to define the group transformation that can account for the swap of electric charges between the tau and tau neutrino particles. The ECS transformation must be derived from a transformation from

(i) $SU(2)_L / U(1)_Y$, in which the fundamental representation of $SU(2)_L$ is $l_0(x^\nu) = (\tau^-_L, \nu_e)$ and $U(1)_Y$ is the symmetric group generated by hypercharge $Y = -1$.

(ii) $SU(2)_L / U(1)_{Y_5}$, in which the fundamental representation of $SU(2)_L$ is $\tilde{l}_0(x^\nu) = (\tilde{\tau}^0_L, \tilde{\nu}^+_e) \quad \text{and} \quad U(1)_{Y_5}$ is the symmetric group generated by swap hypercharge $Y_5 = 1$.

The quotient space $SU(2)_L / U(1)$ is diffeomorphic to the unit 2-sphere $S^2$. Consequently, the swap of electric charges between the tau and neutrino of tau particles must be an automorphism of the 2-sphere to itself.

Here, since the two extra dimensions are endowed with the Fubini–Study metric,[80–81] not all Möbius transformations (e.g. dilations and translations) are isometries. Therefore, the automorphism from the $S^2 \equiv SU(2)_L / U(1)$ to itself, which brings the electric charge swap between the tau and neutrino of tau particles, is given by the isometries that form a proper subgroup of the group of projective linear transformations $PGL_2(\mathbb{C})_{(\text{Charge})}$, namely $PSU_{2\text{(Charge)}}$. Subgroup $PSU_{2\text{(Charge)}}$ is isomorphic to the rotation group $SO(3)_{(\text{ECS})}$,[80–81] which is the isometric group of the unit sphere in three-dimensional real space $R^3$. The automorphism of the Riemann sphere $\hat{C}$ is given by:

$$\text{Rot}_{(\text{ECS})}(\hat{C}) = PSU_{2\text{(Charge)}} = SO(3)_{(\text{ECS})},$$

$$\hat{C} = \mathbb{C} \cup \infty = S^2,$$  \hfill (47)

where $\hat{C}$ is the extended complex plane, $PSU_{2\text{(Charge)}}$ is the proper subgroup of the projective linear transformations, and swap symmetry, $SO(3)_{(\text{ECS})}$ is the group of rotations in three-dimensional vector space $R^3$.

The universal cover of $SO(3)_{(\text{ECS})}$ is the special unitary group $SU(2)_{(\text{ECS})}$. This group is also differomorphic to the unit 3-sphere $S^3$.

We regard ordinary and ECS leptons as different electric charge states of the same particle analogous, that is, to the proton-neutron isotopic pair.

Finally, in terms of rotational symmetry between the original lepton and the proposed ECS leptons, the ECS two-extra dimensional sphere $S^2_{\text{ECS}}$ is given by:

$$S^2_{\text{ECS}} \equiv SU(2)_{(\text{ECS})} / U(1)_{Y_5(\nu_e)}.$$

where $SU(2)_{(\text{ECS})}$ is the special unitary group, and $U(1)_{Y_5(\nu_e)}$ is the symmetric group generated by hypercharge $Y_5(\nu_e)$.

**6 The ECS Symmetry in Various Numbers of Lepton Families and Space Time Dimensions**

ECS leptons of $F$ number of families of leptons ($F$-families) can be introduced analogously to ECS leptons of

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1 The round metric of the 2-extra dimensional sphere can be expressed in stereographic coordinates as $g = (dy_1^2 + dy_2^2)/(1 + \varepsilon^2)^2$, where $\varepsilon = \sqrt{y_1^2 + y_2^2}$. The metric $g$ is Fubini–Study metric of the 2-sphere.[80–81]
third family given by Eq. (48), at the cost of two extra dimensions for each family of leptons:

\[ S_{1-ECS}^2 \times S_{2-ECS}^2 \times S_{3-ECS}^2 \times \cdots \times S_{n-ECS}^2. \]  

(49)

Each of these ECS 2-spheres corresponds to one lepton family and its ECS copy.

This is achieved in the following steps:

First, by following Liu, et al. [82–83] the number of zero-modes of the Dirac operator is decided by the index of it. The index of the Dirac operator on manifold \( K \) is defined as the difference \( n^+ - n^- \) between the number \( n^+ \) of right-handed four-dimensional ECS leptons obtained by dimensional reduction and the number \( n^- \) of left-handed four-dimensional ECS leptons. This number is a topological quantity of the manifold upon compactification and the gauge bundles the Dirac operator might be coupled to. Indeed, this index can be computed in terms of characteristic classes of the tangent and gauge bundles. The Atiyah–Singer index theorem in two dimensions gives the difference. [84–86] If we take \( K = S_{2-ECS}^2 \) with a U(1) magnetic monopole field of charge \( n \) on it, the number of chiral families of ECS leptons will then be equal to \( F_{ch}^{(ECS)} \):

\[ F_{ch}^{(ECS)} = n_s^+ - n_s^- = \frac{e^2}{4\pi} \int_M d^2q \varepsilon^{\mu\nu} F_{\mu\nu}, \]  

(50)

where \( \varepsilon^{\mu\nu} (e^{12} = 1) \) is the contravariant Levi–Civita tensor density in two dimensions, and \( F_{\mu\nu} \) is the field strength of \( A_\mu \):

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ie[A_\mu, A_\nu]. \]  

(51)

Here, we can consider zero modes of the Dirac operator in the background of Abelian gauge potentials representing Dirac strings and center vortices on the ECS-torus \( T_{2-ECS}^2 \). The result is for a two-vortex gauge potential (smeared out vortices) there is one normalizable zero mode, which has exactly one zero on the ECS-torus. [89] The probability density of the spinor field is peaked at the positions of the vortices.

Next, we relate the number of chiral families of leptons \( F_{ch} \) and the number of chiral families of ECS leptons as follows:

\[ F_{ch} = \exp \left[ \frac{e^2}{4\pi} \int_M d^2q \varepsilon^{\mu\nu} F_{\mu\nu} \right]. \]  

(52)

Finally, by assuming that the integral in Eq. (52) is vanished (\( \int_M d^2q \varepsilon^{\mu\nu} F_{\mu\nu} = 0 \)) in the \( S_{2-ECS}^2 \), we obtain one chiral family of leptons and no chiral ECS copies.

Of course, having more than four space-time dimensions is inconsistent with observation, unless there is some form of compactification which leads, at large distance scales, to a world that is effectively four-dimensional.

At large enough distances, or low energies, such extra \( n \) dimensions are hidden and the theory effectively looks four-dimensional. [46] Seen from a distance perspective, rolling up three-dimensional object appear to be one-dimensional. Assuming that space-time has the form of an \( n \)-dimensional manifold of very small size, the latter can appear to be zero-dimensional, if seen from a distance.

### 6.1 ECS Symmetry in the Superstrings Framework

The original formulation of superstring theory was in ten space-time dimensions. [45–47] Indeed, most superstring theories are underpinned by an underlying assumption of more than four space-time dimensions. Furthermore, superstring theory can only be consistent in a higher-dimensional framework. [45]

Therefore superstring theory is intrinsically a higher-dimensional theory.

Here, we study ECS symmetry in the higher-dimensional framework of superstring theory.

We assume a \( D \)-dimensional space-time of the form:

\[ M_D = M_4 \times K^n, \]  

(53)

where \( K^n \) is an \( n \)-dimensional manifold of very small size and \( M_4 \) is the four-dimensional space-time.

To ensure supersymmetry in total \( D \) dimensions space-time, extra \( n \) dimensions \( K^n \) must be an \( n \)-torus \( T^n \), or a Calabi–Yau (CY)-manifold with holonomy, \( H \subset SU(5 - D/2) \). [47]

In the presence of the ECS symmetry, compactification of extra \( n \)-dimensions is achieved by the following steps:

First, let the compact \( n \)-dimensional manifold in Eq. (53) to be an \( n \)-torus \( K^n = T^n \). The \( T^n \) in the presence of ECS symmetry for F-families of leptons is factorized as follows:

\[ T^n = T_{1-ECS}^2 \times T_{2-ECS}^2 \times T_{3-ECS}^2 \times \cdots \times T_{F-ECS}^2, \]  

(54)

where \( T_{i-ECS}^2 = SO(2)Y_1 \times SO(2)Y_1 \). [50]

The ECS 2-torus corresponds to one lepton family and its ECS copy. The ECS 2-torus can be identified with the group of \( 2 \times 2 \) complex diagonal matrices of the form

\[
\begin{pmatrix}
    e^{iY_1} & 0 \\
    0 & e^{iY_1}
\end{pmatrix},
\]  

(56)

where \( Y_1, Y_1 \in \mathbb{R} \) the ordinary hypercharges and swap hypercharges, respectively.

Next, we quotient the torus \( T^n \) (54) by constructing an \( n \)-orbifold on which the string is compactified. Using the complex coordinates (for details see Ref. [46]), we define a \( \mathbb{Z}_4 \) lattice in each of the three complex planes via generators

\[ t_i : F^i \to F^i + R_i, \]  

(57)

\[ u_i : F^i \to F^i + e^{2\pi i/3}R_i, \]  

(58)

where \( i \) runs over \( 1, \ldots, 4 \) covering the transverse modes of the string, \( F^i \) is a complex boson which mediates the ECS changing interaction between the families of leptons. Here, the \( F^1 \) boson arises from the local ECS symmetry.

Dividing by this lattice gives the \( n \)-torus

\[ T^n = \left( \frac{C_{1-ECS}}{\mathbb{Z}_4^2} \right)^F. \]  

(59)
Component $T^2_{1\text{-ECS}}$ is given by
\[ T^2_{1\text{-ECS}} = \frac{C_{1\text{-ECS}}}{Z^2}. \] (60)
We then take the orbifold point group to be the $Z_3$ group generated by
\[ a : F^i \rightarrow e^{2\pi i \phi^i} F^i, \] (61)
where
\[ \phi^2 = \phi_2 = \frac{1}{3}, \quad \phi_2 = -\frac{2}{3}. \] (62)
It is not difficult to verify that rotation (61) is symmetry of the torus defined in Eq. (62). This is most easily seen by tiling $C_{1\text{-ECS}}$ with equilateral triangles of side length $R_i$, or by observing that
\[ e^{\pi i/3} = e^{2\pi i/3} + 1. \] (63)
For a given $i$ there are three fixed points of the action:
\[ F^i = \frac{n_i}{\sqrt{3}} e^{\pi i/3} R_i, \quad n_i = 0, 1, 2. \] (64)
By sewing together the edges of the fundamental region of $T^2_{1\text{-ECS}}/Z_3$ one sees that this orbifold is topologically an ECS 2-sphere, given by Eq. (48). The orbifold fails to be a manifold at the three conical singularities around each of which the holonomy group is $Z_3$.

### 6.2 The Number of Families of Leptons in the Presence of ECS Symmetry

All in all 12 fundamental fermions (6 leptons and 6 quarks, including the top quark) are known today. For over ten years now it has been accepted that these particles can be organized into three “families”, each containing 2 quarks and 2 leptons. These particle families (or generations) behave identically under electroweak and strong interactions and do not differ in anything but their masses. The number of particle generations will probably continue to be restricted to three, because it has been shown experimentally that at most 3 species of light neutrinos exist. A fourth family, if it existed at all, would necessarily contain a heavy neutrino and would, therefore, differ in nature from the known families.

Here, seeking a relation between the number of space-time dimensions and the number of chiral lepton families, we express the $n$-torus (59) as follows:
\[ T^n = \left( \frac{C_{1\text{-ECS}}}{Z^2} \right)^{F_{ch}} = \left( \frac{\mathbb{R}^2_{1\text{-ECS}}}{Z} \right)^{2F_{ch}} \]
\[ = S^{2F_{ch}}_{1\text{-ECS}} \cong SO(2)^{2F_{ch}}. \] (65)
where
\[ C_{1\text{-ECS}} = \mathbb{R}^2_{1\text{-ECS}}, \]
\[ S^1_{1\text{-ECS}} = \frac{\mathbb{R}^2_{1\text{-ECS}}}{Z} \cong O(2)/Y(Y). \] (67)
The ECS-orbifold $S^1_{1\text{-ECS}}$ is a smooth manifold. This is just the toroidal compactification of the ECS-real line $\mathbb{R}^1_{1\text{-ECS}}$, isomorphism to the $SO(2)/Y(Y)$ rotational group generated by hypercharge $Y(Y)$.

By substituting Eq. (65) to Eq. (53), the $D$-dimensional space-time becomes
\[ M_D = M_4 \times T^n = M_4 \times S^{2F_{ch}}_{1\text{-ECS}}, \] (68)
where $S^1_{1\text{-ECS}}$ is given by Eq. (67).

For Eq. (68), we find the following relation between space-time dimensions and the families of leptons:
\[ D(F_{ch}) = 4 + 2F_{ch}, \] (69)
$D$ is the superstring critical dimension, that is the number of space-time dimensions, and $F_{ch}$ is the number of families of leptons. Here, $D$ and $F_{ch}$ are both integers and $4 \leq D \leq 11$ by superstring theory. Values of the superstring critical dimension $D$ and chiral families of leptons $F_{ch}$ are given in Table 2.

In superstring theory, the superstring critical dimension $D$ is not a free parameter, but is fixed by the requirement that the conformal anomaly in the quantum theory is cancelled. This cancellation is obtained by demanding equivalence of the quantization in both the light-con and conformal gauges. The superstring critical dimension $D$ now equals 10. The predicted families of the leptons are, therefore, derived from Eq. (69), as has been shown experimentally.

| $D$ dimensions | $F_{ch}$ Families |
|---------------|-------------------|
| 4             | 0                 |
| 5             | -                 |
| 6             | 1                 |
| 7             | -                 |
| 8             | 2                 |
| 9             | -                 |
| 10            | 3                 |
| 11            | -                 |

### 7 Breaking of the ECS Local Symmetry at Energy Scales below 10 TeV

ECS symmetry implies that ECS leptons and ordinary leptons have the same mass. Since no mass degeneracy is observed in nature, ECS local symmetry breaking must occur below the threshold energy scale of 10 TeV. Here, we propose an ECS local symmetry breaking mechanism that can make the unobserved ECS leptons highly massive, explaining thereby their inobservability.

In this proposition, ECS is not an exact symmetry: ECS leptons and ordinary leptons have the same masses above 10 TeV but different masses below 10 TeV and SM fermions and gauge $W$, $Z$-bosons normally get their masses through the coupling with the SM Higgs particle of mass 114–200 GeV, while ECS lepton mass is generated at energy scales below 10 TeV, from the finite one-loop ECS lepton self-energy graph.

This mechanism of derivation of ECS lepton mass is more economical in assumptions, as the latter...
masses are obtained from the original massless electroweak Lagrangian by calculating ECS lepton self-energy graphs.\textsuperscript{[91–92]} An ECS lepton obeys the equation: \( p - m_{0L} + \Sigma(p) = 0, \quad p - m_L = 0. \) (70)

Here, \( m_{0L} (\tilde{L} = \tilde{e}, \tilde{\mu}, \tilde{\tau}, \tilde{\nu}_e^+, \tilde{\nu}_\mu^+, \tilde{\nu}_\tau^+) \) is the bare ECS lepton mass, \( m_L \) is the observed ECS lepton mass at energy scales below 10 TeV and \( \Sigma(p) \) is the finite proper self-energy part. We have \( m_L - m_{0L} = \Sigma(p, m_L, g, \Lambda_L) p - m_{0L} = 0. \) (71)

\[
    m_L = \frac{g^2}{4\pi^2} \exp \left( -\frac{m^2_L}{\Lambda^2_L} \right) m_L \left[ E_1 \left( \frac{2m^2_L}{\Lambda^2_L} \right) - \frac{m^2_L}{\Lambda^2_L} \right] \int_2^\infty \exp \left( \frac{\tau m^2_L}{\Lambda^2_L} \right) E_1 \left( \frac{\tau m^2_L}{\Lambda^2_L} \right) ; \tag{72}
\]

In addition to admitting a trivial solution at \( m_L = 0 \), this equation also has non-trivial solutions that can be computed numerically. We work with a single massless vector boson. A solution is obtained when \( m_L = \frac{g^2}{4\pi^2} m_L E_1 \left( \frac{2m^2_L}{\Lambda^2_L} \right) \),

\[
    m_L = \sqrt{\frac{1}{2} E_1^{-1} \left( \frac{4\pi^2}{g^2} \right) } ; \tag{73}
\]

For ECS leptons with degeneracy masses of order \( O(1 \text{ TeV}) \), the corresponding coupling constant and energy scale are \( g \sim 0.649 \) and \( \Lambda_L \approx 2 \text{ TeV} \), respectively.

In these calculations, \( \Lambda_L \) plays a role similar to that of the diagonalized fermion mass matrix in the standard model.\textsuperscript{[50–53]} The number of undetermined parameters, therefore, is the same as in the standard model: for each fermion there is a corresponding \( \Lambda_L \) determining its mass. Our model permits massive ECS-neutrinos. However, as \( \Lambda_L \) corresponds to the diagonal components of a fermion mass matrix, off-diagonal terms are absent and no flavor mixing takes place. Therefore, self energy calculations alone are not sufficient to account for observed ECS-neutrino oscillations experiments.\textsuperscript{[58–60]}

8 Discussion

Experiments at LEP.II have placed very stringent bounds on charged particles lighter than about 100 GeV.\textsuperscript{[54–57]} In e + e-colliders, cross sections for the direct pair production of charged particles are quite large, allowing for limits to be placed at, or slightly below, half of the center-of-mass energy of collision. For LEP.II, which reached a center-of-mass energy of 209 GeV, limits of 87–103 GeV have been placed for such particles. The pretined charges of ECS leptons of mass (1 TeV), in particular, are not subjected to LEP.II bounds.\textsuperscript{[54–57]} Limits for charged ECS leptons can be used to indirectly limit the possible masses of neutral ECS leptons beyond the invisible Z width constraints.\textsuperscript{[54]}

The predicted neutral ECS lepton (1 TeV) is heavier than \( M_Z/2 \) and does not contribute to the invisible Z width measured at LEP.II collision energy scale. This is required to be \( (\bar{\ell}^0) \) a dark matter particle.\textsuperscript{[54]}

Our search aims at unstable ECS charged heavy lepton \( (\tilde{\nu}_L^+ ) \), decaying into heavy stable neutral ECS lepton \( (\ell^0) \):

\[
    \tilde{\nu}_L^+ \rightarrow \ell^0 W^+ W^{*-} \rightarrow \ell^0 \bar{\nu}_L q\bar{q}^* . \tag{74}
\]

Since the mass of the associated heavy neutral ECS lepton \( \ell^0 \) is about 1 TeV, the results are a large amount of missing energy and a large transverse momentum imbalance. In the limit of a vanishing mass difference between charged lepton and associated neutral lepton \((\Delta m = m_{\nu_L} - m_{\ell^0})\), the signal efficiency is limited by the trigger efficiency and the two-photon background in the L2 detector of LEP II experiment.\textsuperscript{[93]} This missing energy corresponds to the decay of the proposed ECS charged heavy lepton \((\tilde{\nu}_L^+ ) \) of around 1 TeV. Such a large amount of missing energy, however, has never been observed at the L2 detector, although such observation could be made in future LHC studies.

The relic density of this heavy stable neutral ECS lepton \((\ell^0) \) should be

\[
    \Omega_{\nu_L h^2} = \frac{0.1 \text{ pb}}{\langle \sigma v \rangle} = 0.11 , \tag{75}
\]

where \( \langle \sigma v \rangle = 0.81 \text{ pb} \).

Analysis of the three-year Wilkinson Microwave Anisotropy Probe (WMAP) data tells us that the density of dark matter is \( \Omega_{\text{dm}} h^2 = 0.102 \pm 0.009 \), where \( \Omega_{\text{dm}} \) is \( \rho_{\text{dm}} / \rho_{\text{crit}}, \rho_{\text{crit}} \) is the density corresponding to a flat universe\textsuperscript{[94]} and \( h \) is the Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\).\textsuperscript{[95]}

A cold dark matter candidate produced at the LHC should, therefore, have this annihilation cross section. This quantity leads us to the second method of measuring the coupling of dark matter from Standard Model particles: through the search for the annihilation or decay
products of dark matter coming from high-density regions of the Universe, such as the centre of galaxies.\cite{20} Since WMAP results provide good information about $\langle u_{\alpha\beta}\rangle$, the uncertainties in this approach stem from our sketchy knowledge of the exact density of dark matter in the centre of galaxies and in the difficulty of separating the signal from dark-matter annihilation from possible background signals.

The mechanism of electric charge swap proposed here predicts the occurrence of a lepton ($\ell$) with mass of 1 TeV. Lepton ($\ell$) is stable particle. Since it also interacts weakly with baryonic matter it can be a good Weakly Interacting Massive Particles (WIMP) candidate. The 1 TeV mass predicted by this model also provides the correct abundance of dark matter in the universe. This encouraging theoretical suggestion is testable through LHC studies.

9 Conclusions

We propose that, for various numbers of lepton families and space-time dimensions, superstring theory predicts that there is a global rotational symmetry between ordinary families of leptons and non-regular leptons. This local symmetry breaks at energy scales below 10 TeV. The proposed symmetry breaking mechanism, electric charge swap between ordinary families of leptons, produces heavy neutral non-regular leptons of order $O$ (1 TeV) masses. Their large mass renders these non-regular leptons the unobservable in LEP I, II and neutrino oscillation experiments at energy scales below 10 TeV. These highly massive non-regular leptons may form cold dark matter. The existence of these proposed leptons can be tested once the LHC becomes operative at the 10 TeV energy-scales. This proposition may have far reaching applications in astrophysics and cosmology.

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