The proof of a conjecture concerning the intersection of $k$-generalized Fibonacci sequences

Diego Marques$^1$

Departamento de Matemática, Universidade de Brasília, Brasília, 70910-900, Brazil

Abstract

For $k \geq 2$, the $k$-generalized Fibonacci sequence $(F_{n}^{(k)})_n$ is defined by the initial values $0, 0, ..., 0, 1$ ($k$ terms) and such that each term afterwards is the sum of the $k$ preceding terms. In 2005, Noe and Post conjectured that the only solutions of Diophantine equation $F_{m}^{(k)} = F_{n}^{(\ell)}$, with $\ell > k > 1, n > \ell + 1, m > k + 1$ are

$$(m, n, \ell, k) = (7, 6, 3, 2) \text{ and } (12, 11, 7, 3).$$

In this paper, we confirm this conjecture.

Keywords: $k$-generalized Fibonacci numbers, linear forms in logarithms, intersection

2010 MSC: 11B39, 11J86

1. Introduction

Let $k \geq 2$ and denote $F_{n}^{(k)} := (F_{n}^{(k)})_{n \geq - (k-2)}$, the $k$-generalized Fibonacci sequence whose terms satisfy the recurrence relation

$$F_{n+k}^{(k)} = F_{n+k-1}^{(k)} + F_{n+k-2}^{(k)} + \cdots + F_{n}^{(k)},$$

(1)

with initial conditions $0, 0, ..., 0, 1$ ($k$ terms) and such that the first nonzero term is $F_{1}^{(k)} = 1$. 

$^1$Supported by DPP-UnB, FAP-DF, FEMAT and CNPq-Brazil
The above sequence is one among the several generalizations of Fibonacci numbers. Such a sequence is also called \textit{k-step Fibonacci sequence}, the \textit{Fibonacci k-sequence}, or \textit{k-bonacci sequence}. Clearly for \( k = 2 \), we obtain the well-known Fibonacci numbers \( F_n^{(2)} = F_n \), and for \( k = 3 \), the Tribonacci numbers \( F_n^{(3)} = T_n \).

Several authors have worked on problems involving \( k \)-generalized Fibonacci sequences. For instance, Togbé and the author [13] proved that only finitely many terms of a linear recurrence sequence whose characteristic polynomial has a simple positive dominant root can be \textit{repdigits} (i.e., numbers with only one distinct digit in its decimal expansion). As an application, since the characteristic polynomial of the recurrence in (1), namely \( x^k - x^{k-1} - \cdots - x - 1 \), has just one root \( \alpha \) such that \( |\alpha| > 1 \) (see for instance [24]), then there exist only finitely many terms of \( F^{(k)} \) which are repdigits, for all \( k \geq 2 \). F. Luca [12] and the author [15] proved that 55 and 44 are the largest repdigits in the sequences \( F^{(2)} \) and \( F^{(3)} \), respectively. Moreover, the author conjectured that there are no repdigits, with at least two digits, belonging to \( F^{(k)} \), for \( k > 3 \). In a recent work, Bravo and Luca [3] confirmed this conjecture.

Here, we are interested in the problem of determining the intersection of two \( k \)-generalized Fibonacci sequences. It is important to notice that Mignotte (see [17]) showed that if \((u_n)_{n \geq 0}\) and \((v_n)_{n \geq 0}\) are two linear recurrence sequences then, under some weak technical assumptions, the equation

\[ u_n = v_m \]

has only finitely many solutions in positive integers \( m, n \). Moreover, all such solutions are effectively computable (we refer the reader to [1, 20, 21, 23] for results on the intersection of two recurrence sequences). Thus, it is reasonable to think that the intersection \( F^{(k)} \cap F^{(\ell)} \) is a finite set for all \( 2 \leq k < \ell \). In 2005, Noe and Post [18] gave a heuristic argument to show that the expected cardinality of this intersection must be small. Furthermore, they raised the following conjecture

\textbf{Conjecture 1 (Noe-Post).} \textit{The Diophantine equation}

\[ F_m^{(k)} = F_n^{(\ell)}, \tag{2} \]

\textit{with} \( \ell > k \geq 2, n > \ell + 1 \) and \( m > k + 1 \), \textit{has only the solutions}:

\[ (m, n, \ell, k) = (7, 6, 3, 2) \text{ and } (12, 11, 7, 3). \tag{3} \]

\textit{That is},

\[ u_n = v_m \]
The intersection of $k$-generalized Fibonacci sequences

$$13 = F^{(2)}_7 = F^{(3)}_6 \quad \text{and} \quad 504 = F^{(3)}_{12} = F^{(7)}_{11}$$

Since the first nonzero terms of $F^{(k)}$ are $1, 1, 2, \ldots, 2^{k-1}$, then the above conjecture can be rephrased as

**Conjecture 2.** Let $2 \leq k < \ell$ be positive integer numbers. Then

$$F^{(k)} \cap F^{(\ell)} = \begin{cases} 
\{0, 1, 2, 13\}, & \text{if } (k, \ell) = (2, 3) \\
\{0, 1, 2, 4, 504\}, & \text{if } (k, \ell) = (3, 7) \\
\{0, 1, 2, 8\}, & \text{if } k = 2 \text{ and } \ell > 3 \\
\{0, 1, 2, \ldots, 2^{k-1}\}, & \text{otherwise}
\end{cases}$$

We remark that this intersection was confirmed for $(k, \ell) = (2, 3)$, by the author [14]. Also, Noe and Post used computational methods to study this intersection (see Section 5). Let us state their result as a lemma, since we shall use it throughout our work.

**Lemma 1.** The only solutions $(m, n, \ell, k)$ in positive integers of Diophantine equation (2), with $\ell > k > 1, n > \ell + 1, m > k + 1$ and $\max\{m, n, k, \ell\} < 2^{2000}$, are listed in (3).

In this paper, we shall use transcendental tools to prove the Noe-Post conjecture. For the sake of preciseness, we stated it as a theorem.

**Theorem 1.** Conjecture 1 is true.

Let us give a brief overview of our strategy for proving Theorem 1. First, we use a Dresden formula [6, Formula (2)] to get an upper bound for a linear form in three logarithms related to equation (2). After, we use a lower bound due to Matveev to obtain an upper bound for $m$ and $n$ in terms of $\ell$. Very recently, Bravo and Luca solved the equation $F^{(k)}_n = 2^m$ and for that they used a nice argument combining some estimates together with the Mean Value Theorem (this can be seen in pages 72 and 73 of [2]). In our case, we must use two times this Bravo and Luca approach to prove our main theorem. In the final section, we present a program for checking the “small” cases. The computations in the paper were performed using Mathematica®.

We remark some differences between our work and the one by Bravo and Luca. In their paper, the equation $F^{(k)}_n = 2^m$ was studied. By applying a key method, they get directly an upper bound for $|2^m - 2^{n-2}|$. In our case, the equation $F^{(k)}_m = F^{(\ell)}_n$ needs a little more work, because it is necessary
to apply two times their method to get an upper bound for \( |2^n - 2^m| \). Moreover, they used a reduction argument due to Dujella and Pethő to solve small cases. In our work, we use a Noe and Post program to deal with these cases. Our presentation is therefore organized in a similar way that the one in the papers [2, 3], since we think that those presentations are intuitively clear.

2. Upper bounds for \( m \) and \( n \) in terms of \( \ell \)

In this section, we shall prove the following result

**Lemma 2.** If \((m,n,\ell,k)\) is a solution in positive integers of Diophantine equation (2), with \( \ell > k \geq 2 \), \( n > \ell + 1 \) and \( m > k + 1 \). Then

\[
n < m < 4.4 \cdot 10^{14} \ell^8 \log^3 \ell.
\]

Before proceeding further, we shall recall some facts and properties of these sequences which will be used after.

We know that the characteristic polynomial of \((F^{(k)}_n)\) is

\[
\psi_k(x) := x^k - x^{k-1} - \cdots - x - 1
\]

and it is irreducible over \( \mathbb{Q}[x] \) with just one zero outside the unit circle. That single zero is located between \( 2(1 - 2^{-k}) \) and \( 2 \) (as can be seen in [24]). Also, in a recent paper, G. Dresden [6, Theorem 1] gave a simplified “Binet-like” formula for \( F^{(k)}_n \):

\[
F^{(k)}_n = \sum_{i=1}^{k} \frac{\alpha_i - 1}{2 + (k+1)(\alpha_i - 2)} \alpha_i^{n-1},
\]

for \( \alpha = \alpha_1, \ldots, \alpha_k \) being the roots of \( \psi_k(x) \). There are many other ways of representing these \( k \)-generalized Fibonacci numbers, as can be seen in [7, 8, 9, 10]. Also, it was proved in [3, Lemma 1] that

\[
\alpha^{n-2} \leq F^{(k)}_n \leq \alpha^{n-1}, \text{ for all } n \geq 1,
\]

where \( \alpha \) is the dominant root of \( \psi_k(x) \). Also, the contribution of the roots inside the unit circle in formula (4) is almost trivial. More precisely, it was proved in [6] that

\[
|F^{(k)}_n - g(\alpha, k)\alpha^{n-1}| < \frac{1}{2},
\]
where we adopt throughout the notation \( g(x, y) := (x - 1)/(2 + (y + 1)(x - 2)) \).

As a last tool to prove Lemma 2, we still use a lower bound for a linear form logarithms à la Baker and such a bound was given by the following result of Matveev (see [16] or Theorem 9.4 in [4]).

**Lemma 3.** Let \( \gamma_1, \ldots, \gamma_t \) be real algebraic numbers and let \( b_1, \ldots, b_t \) be nonzero rational integer numbers. Let \( D \) be the degree of the number field \( \mathbb{Q}(\gamma_1, \ldots, \gamma_t) \) over \( \mathbb{Q} \) and let \( A_j \) be a positive real number satisfying
\[
A_j \geq \max \{ Dh(\gamma_j), |\log \gamma_j|, 0.16 \} \text{ for } j = 1, \ldots, t.
\]

Assume that
\[
B \geq \max \{|b_1|, \ldots, |b_t|\}.
\]

If \( \gamma_1^{b_1} \cdots \gamma_t^{b_t} \neq 1 \), then
\[
|\gamma_1^{b_1} \cdots \gamma_t^{b_t} - 1| \geq \exp(-1.4 \cdot 30^{t+3} \cdot t^{4.5} \cdot D^2(1 + \log D)(1 + \log B)A_1 \cdots A_t).
\]

As usual, in the above statement, the *logarithmic height* of an \( s \)-degree algebraic number \( \gamma \) is defined as
\[
h(\gamma) = \frac{1}{s}(\log |a| + \sum_{j=1}^{s} \log \max \{1, |\gamma(j)|\}),
\]
where \( a \) is the leading coefficient of the minimal polynomial of \( \gamma \) (over \( \mathbb{Z} \)) and \((\gamma(j))_{1 \leq j \leq s}\) are the conjugates of \( \gamma \) (over \( \mathbb{Q} \)).

2.1. The proof of Lemma 2

First, the inequality \( n < m \) follows from the facts that the sequences \((F_{n(\ell)}^\ell)_n\) and \((F_{n(\ell)}^k)_\ell\) are nondecreasing together with (2), \( n > \ell + 1 \) and \( m > k + 1 \). By the way, to find an upper bound for \( m \) in terms of \( n \), we combine (2) and (5) to obtain
\[
2^{n-1} \phi^{n-1} \geq F_n = F_m \geq \alpha^{m-2} > (\sqrt{2})^{m-2} \text{ and so } 2n > m,
\]
where in the last inequality we used that \( \alpha > 3/2 > \sqrt{2} \).

Now, we use (6) to get
\[
|F_m^{(k)} - g(\alpha, k)\alpha^{n-1}| < \frac{1}{2} \text{ and } |F_n^{(\ell)} - g(\phi, \ell)\phi^{n-1}| < \frac{1}{2},
\]
where $\alpha$ and $\phi$ are the dominant roots of the recurrences $(F^{(k)}_m)_m$ and $(F^{(\ell)}_n)_n$, respectively. Combining these inequalities, we obtain

$$| g(\phi, \ell) \phi^{n-1} - g(\alpha, k) \alpha^{m-1} | < 1$$

and so

$$\left| \frac{g(\phi, \ell) \phi^{n-1}}{g(\alpha, k) \alpha^{m-1}} - 1 \right| < \frac{1}{g(\alpha, k) \alpha^{m-1}} < \frac{4}{\alpha^{m-1}},$$

where we used that $g(\alpha, k) > 1/4$, since $\alpha > 3/2$ (for $k \geq 2$) and $2 + (k + 1)(\alpha - 2) < 2$. Thus (9) becomes

$$| e^{\Lambda} - 1 | < \frac{4}{\alpha^{m-1}},$$

where $\Lambda := (n-1) \log \phi + \log(g(\phi, \ell)/g(\alpha, k)) - (m-1) \log \alpha$.

Now, we shall apply Lemma 3. To this end, take $t := 3$,

$$\gamma_1 := \phi, \quad \gamma_2 := \frac{g(\phi, \ell)}{g(\alpha, k)}, \quad \gamma_3 := \alpha$$

and

$$b_1 := n - 1, \quad b_2 := 1, \quad b_3 := m - 1.$$

For this choice, we have $D = [\mathbb{Q}(\alpha, \phi) : \mathbb{Q}] \leq k\ell < \ell^2$. Also $h(\gamma_1) = (\log \phi)/\ell < (\log 2)/\ell < 0.7/\ell$ and similarly $h(\gamma_3) < 0.7/k$. In [2, p. 73], an estimate for $h(g(\alpha, k))$ was given. More precisely, it was proved that

$$h(g(\alpha, k)) < \log(k + 1) + \log 4.$$

Analogously,

$$h(g(\phi, \ell)) < \log(\ell + 1) + \log 4.$$

Thus

$$h(\gamma_2) \leq h(g(\phi, \ell)) + h(g(\alpha, k)) \leq \log(\ell + 1) + \log(k + 1) + 2 \log 4,$$

where we used the well-known facts that $h(xy) \leq h(x) + h(y)$ and $h(x) = h(x^{-1})$. Also, in [2] was proved that $|g(\alpha_i, k)| < 2$, for all $i = 1, \ldots, k$.

Since $\ell > k$ and $m > n$, we can take $A_1 = A_3 := 0.7\ell$, $A_2 := 2\ell^2 \log(4\ell+4)$ and $B := m - 1$.

Before applying Lemma 3, it remains us to prove that $e^\Lambda \neq 1$. Suppose, towards a contradiction, the contrary, i.e., $g(\alpha, k) \alpha^{m-1} = g(\phi, \ell) \phi^{n-1} \in \mathbb{Q}(\phi)$. So, we can conjugate this relation in $\mathbb{Q}(\phi)$ to get
The intersection of $k$-generalized Fibonacci sequences

$g(\alpha_{s_i}, k)\alpha_{s_i}^{m-1} = g(\phi_i, \ell)\phi_i^{n-1}$, for $i = 1, ..., \ell$,

where $\alpha_{s_i}$ are the $\ell$ conjugates of $\alpha$ over $\mathbb{Q}(\phi)$. Since $g(\alpha, k)\alpha^{m-1}$ has at most $k$ conjugates (over $\mathbb{Q}$), then each number in the list $\{g(\alpha_{s_i}, k)\alpha_{s_i}^{m-1} : 1 \leq i \leq \ell\}$ is repeated at least $\ell/k > 1$ times. In particular, there exists $t \in \{2, ..., \ell\}$, such that $g(\alpha_{s_1}, k)\alpha_{s_1}^{m-1} = g(\alpha_{s_t}, k)\alpha_{s_t}^{m-1}$. Thus, $g(\phi, k)\phi^{n-1} = g(\phi_t, \ell)\phi_t^{n-1}$ and then

$$\left(\frac{7}{4}\right)^{n-1} < \phi^{n-1} = \left|\frac{g(\phi_t, \ell)}{g(\phi, \ell)}\right| \phi_t^{n-1} < 8,$$

where we used that $\phi > 2(1 - 2^{-\ell}) \geq 7/4$, $|g(\phi_t, \ell)| < 2 < 8|g(\phi, \ell)|$ and $|\phi_t| < 1$ for $t > 1$. However, the inequality $(7/4)^{n-1} < 8$ holds only for $n = 1, 2, 3, 4$, but this gives an absurdity, since $n > \ell + 1 \geq 3 + 1 = 4$. Therefore $e^A \neq 1$.

Now, the conditions to apply Lemma 3 are fulfilled and hence

$$|e^A - 1| > \exp(-1.5 \cdot 10^{11} \ell^8 (1 + 2 \log \ell) \log(4\ell + 4)(1 + \log(m - 1)))$$

Since, $1 + 2 \log \ell \leq 3 \log \ell$ and $4\ell + 4 < \ell^{2.6}$ (for $\ell \geq 3$), we have that

$$|e^A - 1| > \exp(-2.4 \cdot 10^{12} \ell^8 \log^2 \ell \log(m - 1)) \tag{11}$$

By combining (10) and (11), we get

$$\frac{m - 1}{\log(m - 1)} < 6.1 \cdot 10^{12} \ell^8 \log^2 \ell,$$

where we used that $\log \alpha > 0.4$. Since the function $x/\log x$ is increasing for $x > e$, it is a simple matter to prove that

$$\frac{x}{\log x} < A \text{ implies that } x < 2A \log A. \tag{12}$$

A proof for that can be found in [2, p. 74].

Thus, by using (12) for $x := m - 1$ and $A := 6.1 \cdot 10^{12} \ell^8 \log^2 \ell$, we have that

$$m - 1 < 2(6.1 \cdot 10^{12} \ell^8 \log^2 \ell) \log(6.1 \cdot 10^{12} \ell^8 \log^2 \ell).$$

Now, the inequality $30 + 2 \log \log \ell < 28 \log \ell$ for $\ell \geq 3$, yields

$$\log(6.1 \cdot 10^{12} \ell^8 \log^2 \ell) < 30 + 8 \log \ell + 2 \log \log \ell < 36 \log \ell.$$  

Therefore

$$m < 4.4 \cdot 10^{14} \ell^8 \log^3 \ell \tag{13}$$

The proof is then complete. \qed
3. Upper bound for $\ell$ in terms of $k$

**Lemma 4.** If $(m, n, \ell, k)$ is a solution in positive integers of equation (2), with $\ell > k > 1, n > \ell + 1$ and $m > k + 1$, then

$$\ell < 1.8 \cdot 10^{16} k^3 \log^3 k. \quad (14)$$

**Proof.** If $\ell \leq 239$, then the inequalities (13) yields $m < 8 \cdot 10^{35}$. In particular, $\max\{m, n, \ell, k\} < 10^{36} < 2^{2000}$. So, by Lemma 1, the only solutions of equation (2) with the conditions in the statement of Theorem 1 are $(m, n, \ell, k) = (7, 6, 3, 2)$ and $(12, 11, 7, 3)$.

Thus, we may assume that $\ell > 239$. Therefore

$$n < 4.4 \cdot 10^{14} \ell^8 \log^3 \ell < 2^{\ell/2} \quad (15)$$

where we used (13) and the fact that $n < m$. By using a key argument due to Bravo and Luca [2, p. 72-73], we get

$$|2^{n-2} - g(\alpha, k)\alpha^{m-1}| < \frac{5 \cdot 2^{n-2}}{2^{\ell/2}} \quad (16)$$

or equivalently,

$$|1 - g(\alpha, k)\alpha^{m-1}2^{-(n-2)}| < \frac{5}{2^{\ell/2}}. \quad (17)$$

For applying Lemma 3, it remains us to prove that the left-hand side of (17) is nonzero, or equivalently, $2^{n-2} \neq g(\alpha, k)\alpha^{m-1}$. To obtain a contradiction, we suppose the contrary, i.e., $2^{n-2} = g(\alpha, k)\alpha^{m-1}$. By conjugating the previous relation in the splitting field of $\psi_k(x)$, we obtain $2^{n-2} = g(\alpha_i, k)\alpha^{m-1}_i$, for $i = 1, \ldots, k$. However, when $i > 1$, $|\alpha_i| < 1$ and $|g(\alpha_i, k)| < 2$. But this leads to the following absurdity

$$2^{n-2} = |g(\alpha_i, k)||\alpha_i|^{m-1} < 2,$$

since $n > 4$. Therefore $g(\alpha, k)\alpha^{m-1}2^{-(n-2)} \neq 1$ and then we are in position to apply Lemma 3. For that, take $t := 3$,

$$\gamma_1 := g(\alpha, k), \quad \gamma_2 := \alpha, \quad \gamma_3 := 2$$

and

$$b_1 := 1, \quad b_2 := m - 1, \quad b_3 := -(n - 2).$$
By some calculations made in Section 2, we see that $A_1 := k \log(4k + 4)$, $A_2 = A_3 := 0.7$ are suitable choices. Moreover $D = k$ and $B = m - 1$. Thus

$$|1 - g(\alpha, k)\alpha^{m-1}2^{-(n-2)}| > \exp(-C_1k^3(1 + \log k)(1 + \log(m - 1)) \log(4k + 4)),$$

where we can take $C_1 = 0.75 \cdot 10^{11}$. Combining (17) and (18) together with a straightforward calculation, we get

$$\ell < 4.7 \cdot 10^{12} k^3 \log^2 k \log m$$

(19)

On the other hand, $m < 4.4 \cdot 10^{14} \ell^8 \log^3 \ell$ (by (13)) and so

$$\log m < \log(4.4 \cdot 10^{14} \ell^8 \log^3 \ell) < 45 \log \ell.$$  

(20)

Turning back to inequality (19), we obtain

$$\frac{\ell}{\log \ell} < 2.2 \cdot 10^{14} k^3 \log^2 k$$

which implies (by (12)) that

$$\ell < 2(2.2 \cdot 10^{14} k^3 \log^2 k) \log(2.2 \cdot 10^{14} k^3 \log^2 k).$$

Since $\log(2.2 \cdot 10^{14} k^3 \log^2 k) < 39 \log k$, we finally get the desired inequality

$$\ell < 1.8 \cdot 10^{16} k^3 \log^3 k.$$  

\[\Box\]

4. The proof of Theorem 1

If $k \leq 1655$, then $\ell < 4 \cdot 10^{28}$ (by (14)). Thus, by (13), one has that $n < m < 2 \cdot 10^{248}$. In particular, $\max\{m, n, \ell, k\} < 2 \cdot 10^{248} < 2^{2000}$. So, Lemma 1 gives the known solutions.

Therefore, we may suppose that $k > 1655$. The inequality $\ell < 1.8 \cdot 10^{16} k^3 \log^3 k$ together with (13) yield

$$m < 4.4 \cdot 10^{14}(1.8 \cdot 10^{16} k^3 \log^3 k)^8 \log^3(1.8 \cdot 10^{16} k^3 \log^3 k)$$

$$< 3 \cdot 10^{148} k^{24} \log^{27} k < 2^{k/2},$$
The intersection of $k$-generalized Fibonacci sequences

where the last inequality holds only because $k > 1655$. Now, we use again the key argument of Bravo and Luca to conclude that

$$|2^{m-2} - g(\phi, \ell)\phi^{n-1}| < \frac{5 \cdot 2^{m-2}}{2^{k/2}}.$$  \hfill (21)

Combining (16), (21) and (8), we get

$$|2^{n-2} - 2^{m-2}| \leq |2^{n-2} - g(\alpha, k)\alpha^{n-1}| + |g(\alpha, k)\alpha^{n-1} - g(\phi, \ell)\phi^{n-1}| + |2^{m-2} - g(\phi, \ell)\phi^{n-1}| < \frac{5 \cdot 2^{n-2}}{2^{k/2}} + 1 + \frac{5 \cdot 2^{m-2}}{2^{k/2}} < \frac{11 \cdot 2^{m-2}}{2^{k/2}},$$

since $n < m$, $k < \ell$ and $m > k + 1$. Therefore

$$|2^{n-m} - 1| < \frac{11}{2^{k/2}}.$$ \hfill (22)

Since $n \leq m - 1$, then

$$\frac{1}{2} \leq 1 - 2^{n-m} = |2^{n-m} - 1| < \frac{11}{2^{k/2}}.$$  \hfill (22)

Thus $2^{k/2} < 22$ leading to an absurdity, since $k > 1655$.

In conclusion, the only solutions of equation (2) with $\ell > k > 1, n > \ell + 1$ and $m > k + 1$ are those listed in (3). Thus, the proof of Theorem 1 is complete. \hfill \Box

5. The program

In this section, for the sake of completeness, we present the Mathematica program (which was kindly sent to us by Noe [19]) used to confirm Lemma 1:

\begin{verbatim}
nn = 2000;
f = 2^Range[nn] - 1;
f[[1]] = Infinity;
cnt = 0;
seq = Table[Join[2^Range[i - 1], {2^i - 1}], {i, nn}];
done = False;
While[! done, fMin = Min[f];
\end{verbatim}
The intersection of $k$-generalized Fibonacci sequences

pMin = Flatten[Position[f, fMin]]; If[Length[pMin] > 1, Print[{fMin, pMin}]]; Do[k = pMin[[i]]; s = Plus @@ seq[[k]]; seq[[k]] = RotateLeft[seq[[k]]]; seq[[k, k]] = s; f[[k]] = s, {i, Length[pMin]}]; cnt++; done = (fMin > 2^nn); cnt

The calculations took roughly 54 hours on 1.80 GHz AMD Triple-Core PC.

Acknowledgements

We would like to express our deepest gratitude to the anonymous referee for carefully examining this paper and providing it a number of important comments. In particular, he/she pointed out the useful reference [2]. This research was partly supported by FAP-DF, FEMAT and CNPq.

References

[1] M. A. Alekseyev, On the intersections of Fibonacci, Pell, and Lucas numbers, Integers 11A (2011).

[2] J. Bravo, F. Luca, Powers of two in generalized Fibonacci sequences, Rev. Colombiana Mat. 46 (2012), 67–79.

[3] J. Bravo, F. Luca, $k$-generalized Fibonacci numbers with only one distinct digit, Preprint.

[4] Y. Bugeaud, M. Mignotte, S. Siksek, Classical and modular approaches to exponential Diophantine equations. I. Fibonacci and Lucas perfect powers, Ann. of Math. 163 (2006), no. 3, 969–1018.

[5] A. Dujella and A. Pethö, A generalization of a theorem of Baker and Davenport, Quart. J. Math. Oxford Ser. (2) 49 (1998), 291–306.
The intersection of $k$-generalized Fibonacci sequences

[6] G. P. Dresden, *A simplified Binet formula for $k$-generalized Fibonacci numbers*, Preprint, arXiv:0905.0304v2 (2011). Accessed 31 December 2011.

[7] D. E. Ferguson, *An expression for generalized Fibonacci numbers*, Fibonacci Quart. 4 (1966), 270273.

[8] I. Flores, *Direct calculation of $k$-generalized Fibonacci numbers*, Fibonacci Quart. 5 (1967), 259–266.

[9] H. Gabai, *Generalized Fibonacci $k$-sequences*, Fibonacci Quart. 8 (1970), 3138.

[10] D. Kalman, *Generalized Fibonacci numbers by matrix methods*, Fibonacci Quart. 20 (1982), 7376.

[11] P. Y. Lin, *De Moivre-type identities for the Tribonacci numbers*, Fibonacci Quart. 26 (1988), 131-134.

[12] F. Luca, *Fibonacci and Lucas numbers with only one distinct digit*, Port. Math. 57 (2) (2000), 243–254.

[13] D. Marques and A. Togbé, *On terms of a linear recurrence sequence with only one distinct block of digits*, Colloq. Math. 124, p. 145-155, 2011.

[14] D. Marques, *On the intersection of two distinct $k$-generalized Fibonacci sequences*, To appear in Math. Bohem.

[15] D. Marques, *On $k$-generalized Fibonacci numbers with only one distinct digit*, To appear in Util. Math.

[16] E. M. Matveev, *An explicit lower bound for a homogeneous rational linear form in the logarithms of algebraic numbers*, Izv. Math. 64 (2000), no. 6, 1217–1269.
The intersection of $k$-generalized Fibonacci sequences

[17] M. Mignotte, *Intersection des images de certaines suites récurrentes linéaires*. Theor. Comput. Sci., 7 (1), (1978), 117-121.

[18] T. D. Noe and J. V. Post, *Primes in Fibonacci $n$-step and Lucas $n$-step sequences*, J. Integer Seq., 8 (2005), Article 05.4.4.

[19] T. D. Noe, personal communication, 27 January 2012.

[20] H. P. Schlickewei, W. M. Schmidt, *Linear equations in members of recurrence sequences*. Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) 20 (1993) 2, 219-246.

[21] H. P. Schlickewei, W. M. Schmidt, *The intersection of recurrence sequences*. Acta Arith. 72 (1995) 1-44.

[22] W. R. Spickerman, *Binet’s formula for the Tribonacci sequence*, Fibonacci Quart. 20 (1982), 118–120.

[23] S. K. Stein, *The intersection of Fibonacci sequences*. Michigan Math. J. 9 (1962) 399-402.

[24] A. Wolfram, *Solving generalized Fibonacci recurrences*, Fibonacci Quart. 36 (1998), 129–145.