The ILC as a natural SUSY discovery machine and precision microscope: from light higgsinos to tests of unification

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Abstract

The requirement of electroweak naturalness in simple supersymmetric models implies the existence of a cluster of four light higgsinos with mass ∼ 100 − 300 GeV, the lighter the better. While such light compressed spectra may be challenging to observe at LHC, the International Linear Collider (ILC) with √s > 2m_higgsino would serve as both a SUSY discovery machine and a precision microscope. We study higgsino pair production signatures at the ILC based on full, Geant4-based simulation of the ILD detector concept. We examine several benchmark scenarios that may be challenging for discovery at HL-LHC due to mass differences between the higgsino states between 20 and 4 GeV. Assuming √s = 500 GeV and 1000 fb$^{-1}$ of integrated luminosity, the individual higgsino masses can be measured to 1 − 2% precision in case of the larger mass differences, and at the level of 5% for the smallest mass difference case. The higgsino mass splittings are sensitive to the electroweak gaugino masses and allow extraction of gaugino masses to ∼ 3 − 20% (depending on the model). Extrapolation of gaugino masses via renormalization group running can test the hypothesis of gaugino mass unification. We also examine a case with natural generalized mirage mediation where the unification of gaugino masses at an intermediate scale apparently gives rise to a natural SUSY spectrum somewhat beyond the reach of HL-LHC.

This work was carried out in the framework of the ILD detector concept group
1 Introduction

The Standard Model (SM) of particle physics has been spectacularly confirmed across a broad array of measurements and often to very high precision at the LHC. The crowning achievement was to establish the existence of a physical scalar (Higgs) boson with mass \( m_h = 125.09 \pm 0.24 \text{ GeV} \) [1]. In spite of this impressive success, the narrative brings with it cause for concern: quantum mechanical contributions to the Higgs mass rapidly exceed \( m_h \) for energy fluctuations of order \( \Lambda \sim 1 \text{ TeV} \) [2]. These quadratic divergences necessitate ever more incredulous fine-tunings to maintain \( m_h \simeq 125 \text{ GeV} \) as the excluded energy scale of new physics increases. In addition, the SM is lacking the necessary ingredients to explain e.g. cosmic inflation, the existence of dark matter and dark energy in the universe, the origin of the matter-antimatter asymmetry and a suppression of CP-violation in the strong interactions.

A rather minimal extension of the SM – moving from the Poincaré group to the more general super-Poincaré group (supersymmetry or SUSY) of space-time symmetries – allows for solutions or improvements of all these problems. The added spacetime supersymmetry guarantees cancellation of the offending quadratic divergences to all orders in perturbation theory thus rendering the Higgs field natural. The allowance for a vast assortment of scalar fields in SUSY, as expected from string theory, allows for many possible inflaton candidate fields and for a non-zero minimum of the scalar potential, yielding a cosmological constant. The lightest SUSY particle and/or the inclusion of an axion (necessary for solving the strong CP problem) yields dark matter while scalar field flat direction (Affleck-Dine) baryogenesis and various other thermal and non-thermal leptogenesis mechanisms seem automatic in SUSY. In addition, SUSY receives indirect support from (1) the measured values of gauge couplings which unify under Minimal Supersymmetric Standard Model (MSSM) renormalization group evolution, (2) the measured value of the top mass, which is just right to produce a radiative breakdown of electroweak gauge symmetry, and (3) the measured value of \( m_t \simeq 125 \text{ GeV} \) which lies squarely within the prediction of \( m_h \lesssim 135 \text{ GeV} \) required by the MSSM.

In spite of these theoretical successes, many physicists have developed a large degree of skepticism regarding the eventual emergence of SUSY at experimental facilities. This arises due to (1) the lack of evidence for superpartners at LHC and (2) the rather large value of \( m_h \) that has been found. The first of these is exemplified by the latest gluino mass limits: that \( m_{\tilde{g}} \gtrsim 2 \text{ TeV} \) in many simplified models, which may be compared against early projections by Barbieri-Giudice (BG) [3, 4] where electroweak naturalness requires \( m_{\tilde{g}} \lesssim 400 \text{ GeV} \) for fine-tuning parameter \( \Delta_{BG} \lesssim 30 \). Secondly, a value of \( m_h \simeq 125 \text{ GeV} \) requires [6] within the MSSM the existence of highly mixed (large trilinear soft parameters \( A_t \)) TeV-scale top squarks \( \tilde{t}_1, \tilde{t}_2 \). This may be contrasted with Dimopoulos-Giudice naturalness [5] that \( m_t \lesssim 350 \text{ GeV} \) for \( \Delta_{BG} < 30 \) or that \( m_{\tilde{h}_1, \tilde{b}_1} \lesssim 500 \text{ GeV} \) from requiring \( \Delta \Sigma_{H_u}^2 \lesssim m_h^4 \) [7, 8]. Thus, in the LHC era, the question of electroweak naturalness has been elevated to one of prime importance which can serve as a guide for construction of future experimental facilities.

The most direct connection between the weak scale, as exemplified by the weak gauge and Higgs boson masses \( m_W, Z, h \) and the SUSY Lagrangian parameters, arises from the scalar potential minimization condition [19]

\[
\frac{1}{2} m_Z^2 = \frac{(m_{H_u}^2 + \Sigma_u^2) - (m_{H_u}^2 + \Sigma_u^2) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \approx -m_{H_u}^2 - \Sigma_u^2 - \mu^2 \tag{1}
\]

where the latter partial equality arises for moderate to large values of the ratio of Higgs vevs \( \tan \beta \equiv v_u/v_d \). The \( \mu \) term arises as a mass term in the MSSM superpotential; thus, it is supersymmetry conserving and feeds mass both to the SM particles \( W, Z \) and \( h \) and also the SUSY higgsinos. The weak scale soft SUSY breaking term \( m_{H_u}^2 \) feeds mass just to \( W, Z \) and \( h \) (and other SUSY Higgs via suppressed mixing). The \( \Sigma_u^2 \) are radiative corrections (for a full listing, see Ref. [10]), the largest of which typically arise from the top-squark contributions. The MSSM may be considered as natural if there are no large, unnatural cancellations (fine-tunings) on the right-hand-side of Eq. 1. A naturalness measure \( \Delta_{EW} \) has been proposed which considers the ratio of the largest element on the right-hand-side (RHS) of Eq. 1 to \( m_Z^2/2 \). Fine-tuning of \( m_Z \) sets in for values of \( \Delta_{EW} \gtrsim 20 - 30 \) and is visually displayed in Fig. 1 of Ref. [11].

The validity of the early naturalness estimates using the BG measure has been challenged in that these calculations are performed using multiple-soft-parameter effective theories instead of more fundamental theories in which the soft parameters are all related [12]. Using correlated soft parameters, the BG measure reduces to the EW

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1 In Ref. [9], it is declared that “Settling the ultimate fate of naturalness is perhaps the most profound theoretical question of our time … and will largely dictate the future of fundamental physics in this century.”
measure [18]. The validity of naturalness estimates using \(\delta m_H^2/m_H^2\) has been challenged in that, in an effort to simplify, several contributions to \(m_H^2\) and \(\delta m_H^2\) have been set to zero. By including these pieces, then one allows for radiatively-driven naturalness (RNS) [27] wherein large, seemingly unnatural high scale values of \(m_H^2\) are driven to natural values at the weak scale. The revised measure is thus brought into accord with \(\Delta_{EW}\) [12, 18].

From Eq. 1, the requirements for electroweak naturalness are then

- The superpotential \(\mu\) parameter, bounded from below by \(\mu \gtrsim 100\) GeV due to chargino searches at LEP2, is not too far from \(m_2\): \(\mu \sim 100 - 300\) GeV, the lower the better. This immediately implies the existence of several higgsino-like electroweakinos in SUSY with \(m_{\tilde{\chi}_1^\pm} \sim m_{\tilde{\chi}_1^0} \sim |\mu|^2\).
- The soft term \(m_{\tilde{H}_d}^2\), which must be driven to negative values to initiate a breakdown of electroweak symmetry, is driven to small and not large negative values.

- The radiative corrections \(\Sigma_g^\mu(\tilde{t}_1,\tilde{t}_2)\) are actually minimized for TeV-scale highly mixed top squarks. These same conditions lift the Higgs mass to \(m_h \simeq 125\) GeV. Detailed evaluations require \(m_{\tilde{t}_1} \lesssim 3\) TeV for \(\Delta_{EW} < 30\) [10].
- The gluino mass contributes at two-loop level to Eq. 1 via the \(\Sigma_g^\mu(\tilde{t}_1,\tilde{t}_2)\). Detailed evaluations require \(m_{\tilde{g}} \lesssim 6\) TeV for \(\Delta_{EW} < 30\). This may be compared to the ultimate reach of HL-LHC which extends to about \(m_{\tilde{g}} \sim 2.8\) TeV (at the 5 \(\sigma\) discovery level [29]).

Thus, HL-LHC will probe only a portion of natural SUSY parameter space via gluino and top squark pair production searches.

The naturalness-required light higgsinos may be produced at decent rates at LHC but their relatively compressed spectra imply low visible energy release from their decays. Thus, light higgsinos are very challenging to see at LHC [32, 29]. In contrast, the International Linear Collider (ILC) with \(\sqrt{s} > 2m_{(higgsino)}\) would be a higgsino factory in addition to being a Higgs factory. The reactions \(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0\) and \(\tilde{\chi}_2^0\tilde{\chi}_1^\pm\) should occur at rates comparable to muon pair production and at rates exceeding \(Zh\) production [30]. The expected mass gaps \(m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} \sim m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0} \sim 3 - 20\) GeV lead to events which are easily identified at ILC: see Fig. 1 for a simulated \(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^\pm\) with \(\tilde{\chi}_2^0 \rightarrow \mu^+\mu^-\tilde{\chi}_1^0\) event display with light higgsinos in the ILD detector.

The cleanliness of ILC higgsino pair production events along with tunable beam energy and beam polarization should allow for a rich program of higgsino measurements. While the higgsino masses should be comparable to the superpotential \(\mu\) parameter, thus allowing for a determination of \(\mu\), the higgsino mass splittings depend sensitively on the weak scale gaugino masses \(M_1\) (bino) and \(M_2\) (wino). Thus, precision measurements of \(m_{\tilde{\chi}_1^\pm}\) and \(m_{\tilde{\chi}_1^0}\) should allow for an extraction of \(M_1\) and \(M_2\) to good precision. Once the soft breaking gaugino masses are known, then the physical masses of the heavier neutralinos and charginos can also be found. The fitted values of \(M_1\) and \(M_2\) can be extrapolated to high energies to test the hypothesis of gaugino mass unification. If gluinos are discovered at LHC, then \(M_1\) (gluino) may be extracted [33] and unification of all three gaugino masses may be tested.

In this paper, we first present in Sec. 2 two natural SUSY benchmark models labeled ILC1 and ILC2 that arise from the non-universal Higgs model (NUHM2) [24]. These models allow for \(\mu\) as an input parameter so that SUSY spectra with a low value of \(\Delta_{EW}\) can easily be generated. The NUHM2 model incorporates gaugino mass unification so that under renormalization group (RG) evolution \(M_1\) and \(M_2\) should unify at the scale \(m_{GUT} \simeq 2 \times 10^{16}\) GeV. We also propose a natural generalized mirage mediation (nGMM) benchmark model which instead has gaugino mass unification at the mirage scale \(\mu_{\text{mir}} = m_{GUT}e^{-\delta \frac{\alpha}{\pi}}/\alpha \sim 5 \times 10^7\) GeV where \(\alpha = 4\) parametrizes the relative amounts of modulus-mediation versus anomaly-mediation in SUSY breaking. In this case, by determining the mirage unification scale \(\mu_{\text{mir}}\), ILC can measure the strength \(\alpha\) of moduli- vs. anomaly-mediation. If the gaugino masses are extrapolated beyond the mirage scale to the GUT scale, then ILC can also indirectly measure the underlying gravitino mass \(m_{3/2}\) [34]! Thus, in such cases ILC would allow for a window into the nature of the laws of physics at energy scales far beyond \(\sqrt{s} \sim 0.5 - 1\) TeV.

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1It is possible that non-holonomic soft terms may arise allowing for higher mass higgsinos without compromising naturalness [13]. Such “semi-soft” mass terms are expected to be of order \(m_\text{top}/m_W[14]\) but in the case where the mediation scale is arranged to be far lower than the usual Planck scale (as expected for gravity-mediation), then these terms can become much larger.

2In the case of natural anomaly-mediated SUSY breaking [28], the gluino mass bound increases to \(m_{\tilde{g}} \lesssim 9\) TeV.
2 Benchmark models

In this Section, we present three natural SUSY benchmark points which have been used for the detailed ILD studies described in Sec. 3 and Sec. 4.

2.1 ILC1 benchmark model

The ILC1 benchmark point, whose parameters are listed in Tab. 1, has been presented previously [27] and has been used for some detailed ILC studies using a toy detector simulation [30]. The ILC1 benchmark point was generated within the NUHM2 model with input parameters and output masses as listed. While the various matter scalars are essentially decoupled for LHC and ILC physics, the spectrum does contain light higgsinos with mass $m_{\tilde{\chi}_1} = 117.3$ GeV and $m_{\tilde{\chi}_2} = 102.7$ and 124 GeV, respectively, so that higgsino pair production should already turn on for ILC with $\sqrt{s} > 227$ GeV. The associated mass gaps, which play a central role in these analyses, are $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1} = 14.6$ GeV and $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0} = 21.3$ GeV. The model contains a light Higgs scalar with $m_h = 125.3$ GeV due to highly mixed TeV-scale top squarks with $m_{\tilde{t}_1} = 1893.3$ GeV (well beyond the reach of HL-LHC [22] where
2 Benchmark models

Figure 2: The $M_{1/2}$ vs. $\mu$ plane in the NUHM2 model for $\tan\beta = 15$, $M_0 = 5$ TeV, $A_0 = -8$ TeV and $M_A = 1$ TeV. We show contours of $\Delta_{EW}$ along with some limits from LHC13 and the future reach of ILC with $\sqrt{s} = 500$ and 1000 GeV and HL-LHC (via same-sign diboson production labeled SSdB (3000) and via neutralino associated production labeled $\tilde{Z}_i \tilde{Z}_j (3000)$ in the nomenclature of Ref. [26] where $\tilde{Z}_i$ denotes neutralino eigenstate $i$). Location of benchmark points is indicated in green. To aid the reader, we note that $m_{\tilde{g}} \sim 2.5 M_{1/2}$. The figure is adapted from Ref. [26].

The 95% CL exclusion reach extends to $m_{\tilde{t}_1} \sim 1400$ GeV [23]). The model is highly electroweak natural with $\Delta_{EW} = 14$ corresponding to just 7% fine-tuning. This benchmark is now likely excluded by LHC: The gluino mass $m_{\tilde{g}} = 1563.5$ GeV is excluded by LHC13 searches with $\sim 36$ fb$^{-1}$. The searches for higgsinos with small mass-splittings performed by CMS [21] and ATLAS [20] are done for a spectrum different from our benchmark - in particular the mass of $\tilde{\chi}_1^\pm$ is assumed to be exactly halfway between those of $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ - but are nevertheless likely to exclude it. However, we retain the point for comparison with previous work. If one adopts modest gaugino mass non-universality, then a small increase in $M_3(weak) > 2$ TeV would bring the point in accord with LHC searches. The relic density of thermally produced higgsino-like LSPs is a factor 13 below the measured value. Requiring also naturalness in the QCD sector, then one must bring the axion into the model, and axionic dark matter may constitute the bulk of the dark matter [25]. The location of the ILC1 benchmark point is denoted by one of the green stars in the $\mu$ vs. $M_{1/2}$ parameter space plane of the NUHM2 model shown in Fig. 2.

2.2 ILC2 benchmark

The ILC2 benchmark point is also generated within the NUHM2 SUSY model with parameter values as listed in Tab. 1. The location of ILC2 is indicated by the other green star in Fig. 2 and is found to lie just beyond the HL-LHC reach for the same-sign diboson signature arising from wino pair production [31]. The higgsino pair signature from $pp \to \tilde{\chi}_1^0 \tilde{\chi}_2^0 j$ production followed by $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \ell^+ \ell^-$ decay should be viable since the cluster of higgsinos lies in the vicinity of $\mu = 150$ GeV [32]. The value of $m_{\tilde{g}} = 2832$ GeV appears just beyond the HL-LHC reach for gluino pair production (where the $5\sigma$ reach extends to $m_{\tilde{g}} \sim 2800$ GeV[33]). The higher value of gaugino masses in ILC2 – as compared to benchmark point ILC1 – is reflected in the reduced inter-higgsino mass gaps where we find $m_{\tilde{\chi}_1} - m_{\tilde{\chi}_1^0} = 10.2$ GeV and $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0} = 9.7$ GeV. The naturalness measure $\Delta_{EW} = 28$ leads to $\sim 3\%$ electroweak fine-tuning. The thermally-produced abundance of dark matter $\Omega_{h^2} \sim 0.007$, well below the measured value of 0.12. So, as for ILC1, the axions needed to solve the strong CP problem can be expected to make up the remainder.
Table 1: Input parameters and mass spectrum and rates for benchmark points ILC1, ILC2 and nGMM1. All masses and dimensionful parameters are in GeV units. All values have been obtained with ISASUGRA.

| units   | ILC1   | ILC2   | nGMM1  |
|---------|--------|--------|--------|
| $M_0$   | 7025.0 | 5000   | –      |
| $M_{1/2}$ | 568.3  | 1200   | –      |
| $A_0$   | -10427 | -8000  | –      |
| $m_{1/2}$ | –      | –      | 75000  |
| $M_1, M_2, M_3$ | –  | – | 3382.5, 2124.4, 1225.8 |
| $\tan \beta$ | –  | 10    | 15     |
| $a_3$   | –      | –      | 3      |
| $c_m$   | –      | –      | 6.9    |
| $\alpha$ | –      | –      | 4      |
| $m_h$   | 125.3  | 125.4  | 124.9  |
| $m_A$   | 1000.0 | 1000   | 2000   |
| $m_H$   | 1006.8 | 1006.7 | 2013.3 |
| $m_{H^\pm}$ | 1003.2 | 1003.2 | 2001.6 |
| $m_{\tilde{g}}$ | 1563.5 | 2832.6 | 2856.5 |
| $m_{\tilde{Q}^i_1, \tilde{Q}^i_2}$ | 117.3, 513.0 | 158.3, 1017.5 | 158.7, 1791.6 |
| $m_{\tilde{Q}^0_1, \tilde{Q}^0_2}$ | 102.7, 124.0 | 148.1, 157.8 | 151.4, 155.8 |
| $m_{\tilde{B}_1, \tilde{B}_2}$ | 267.0, 524.2 | 538.7, 1031.1 | 1526.9, 1799.4 |
| $m_{\tilde{b}_1, \tilde{b}_2}$ | 7021, 7254 | 5440, 5566 | 5267, 5399 |
| $m_{\tilde{t}_1, \tilde{t}_2}$ | 1893, 4919 | 1774, 3878 | 1433, 3732 |
| $m_{\tilde{d}_1, \tilde{d}_2}$ | 7022, 6999 | 5441, 5384 | 5267, 5229 |
| $m_{\tilde{u}_1, \tilde{u}_2}$ | 4959, 6893 | 3903, 5204 | 3770, 5124 |
| $m_{\tilde{e}_1, \tilde{e}_2}$ | 7152, 6759 | 5149, 4817 | 5128, 4825 |
| $m_{\tilde{\tau}_1, \tilde{\tau}_2}$ | 6657, 7103 | 4652, 5072 | 4749, 5094 |
| $\Omega_T^{\tilde{h}}/m_h^2$ | 2.2 x 10^{-25} | 2.9 x 10^{-25} | 3.1 x 10^{-25} |
| $\langle \sigma v \rangle (v \rightarrow 0)$ | 10^{-25} | 10^{-25} | 10^{-25} |
| $\sigma_{SI}(\tilde{\chi}^0_1 p)$ | 6.8 | 1.5 | 0.3 |
| $\Delta_{EW}$ | 14 | 28 | 15 |

Table 2: Higgsino and gaugino fractions of the lightest neutralino $\tilde{\chi}^0_1$. The fractions are expressed so that they satisfy the relation $R_H^2 + R_W^2 + R_B^2 = 1$.

|       | ILC1   | ILC2   | nGMM1  |
|-------|--------|--------|--------|
| $R_H$ | 0.97   | 0.99   | 0.999  |
| $R_W$ | -0.14  | 0.07   | 0.04   |
| $R_B$ | 0.19   | 0.08   | 0.02   |
2.3 Natural mirage mediation (NMM) benchmark

Mirage mediated (MM) SUSY breaking models are motivated by string model compactifications with moduli fields stabilized by fluxes and where an uplifted scalar potential leads to a de Sitter vacuum (as required by cosmology) with a small breaking of supersymmetry[15, 16, 17]. In such cases, it is expected that the SUSY breaking soft terms arise with comparable moduli-mediated and anomaly-mediated contributions. In the gaugino sector (and in the scalar sector for particular modular weight choices) the GUT scale soft masses are offset from each other by contributions containing their gauge group beta functions. As a consequence, the running of the gaugino masses exactly compensates the high scale mass splitting leading to an apparent unification at the intermediate (mirage) scale $\mu_{\text{mir}} = m_{\text{GUT}} e^{-8\pi^2/\alpha}$ where the parameter $\alpha$ is introduced to parametrize the relative amounts of anomaly-versus moduli-mediation. A value $\alpha = 0$ corresponds to pure anomaly-mediation (with tachyonic sleptons) while, as $\alpha$ gets large, the soft terms become increasingly universal.

Initially, simple MM models predicted scalar masses involving discrete values of scalar field modular weights which depend on the compactification geometry and upon which branes harbored the various visible sector fields. This class of models, over a wide range of choices for modular weights, was found to be unnatural when $m_0 \simeq 125$ GeV was required[18]. However, in more general compactification and stabilization schemes, then the previously discrete parameter choices $c_m$, $c_{H_u}$, $c_{H_d}$ and $d_3$ become continuous, allowing for the construction of models with low values of $\Lambda_{\text{EW}}[34]$.

In Tab. 1, we show one such example point from natural generalized mirage mediation or nGMM in column 4. The gaugino masses unify at the mirage scale $\mu_{\text{mir}} \sim 5 \times 10^3$ GeV. The rather large value of $m_\xi = 2856.5$ GeV means the winos and bino are also rather massive so that both gluino pair production and wino pair production appear out of reach of HL-LHC. The higgsinos are clustered with masses around $\mu \sim 150$ GeV but with even smaller mass splittings than ILC2: $m_{\tilde{\chi}_1} - m_{\tilde{\chi}_0} = 7.3$ GeV and $m_{\tilde{\chi}_0} - m_{\tilde{\chi}_4} = 4.4$ GeV. Such small neutralino mass splittings may also push the soft dilepton plus jet signature from $\tilde{\chi}_4^0 \tilde{\chi}_4^0$ production out of reach of HL-LHC [29]. Nonetheless, the model is highly natural with $\Lambda_{\text{EW}} = 15$ or 6.7% fine-tuning. The LSP is more purely higgsino-like than the ILC1 or ILC2 benchmarks leading to a reduced thermally-produced dark matter relic density and reduced direct dark matter detection rates. Thus, direct detection of WIMP dark matter from the nGMM1 benchmark may require multi-ton noble liquid detectors for discovery.

3 Software Tools and Observables

In this section we describe the main features of the ILD detector as used for the simulation study and introduce the characteristics of the higgsino signal on which the strategies for event selection and reconstruction will be based.

3.1 Event generation

The physical masses for the three benchmark points have been calculated by ISASUGRA. The SUSY and SM events have been generated using WHIZARD 1.95 [38], which considers both resonant and non-resonant production, as well as their interference. WHIZARD also generates the amount and spectrum of ISR appropriate for each considered channel, and takes beam-polarisation fully into account. The dedicated setup of the generator provided by the ILC Generator Group was used, and all types SM $e^+e^-$ interactions yielding up to six fermions in the final state were considered. In addition, all $e\gamma$ interactions yielding three or five fermions and all $\gamma\gamma$ interactions yielding up to four fermions were also included. The initial photons in the latter cases might be virtual (in which case the beam remnants come in addition to the final fermions), or real from the photon component of the beams (in which case there is no beam remnants). The electron and positron beams have an initial energy-spread, which is further smeared by the effects of beamstrahlung. The resulting spectra as well as the flux and energy spectra of the beam photons are simulated according to the parameters in the ILC Technical Design Report (TDR) [37], using GuineaPig[36].

Pure left-handed or right-handed beam polarisations are used for the event generation. These samples are then weighted according to the nominal ILC beam polarisations for our simulation study. We introduce the following
notation for beam polarizations, \( \mathcal{P} \equiv (\mathcal{P}_-, \mathcal{P}_+) \), and define the pure beam polarisations as \( \mathcal{P}_{LR} \equiv (-1, +1) \) and \( \mathcal{P}_{RL} \equiv (+1, -1) \). The nominal beam polarisations for the ILC are defined as \( \mathcal{P}_{-+} \equiv (-0.8, +0.3) \) and \( \mathcal{P}_{+-} \equiv (+0.8, -0.3) \).

Table 3 shows the production cross sections for chargino and neutralino pairs for the three benchmark models introduced in Sec. 2.1-2.3 for 100% polarised beams at several center-of-mass energies. Table 4 shows the decay branching ratios in the same three benchmarks.

The results of the simulation study assume a center-of-mass energy of \( \sqrt{s} = 500 \) GeV and an integrated luminosity of \( L = 500 \) fb\(^{-1} \) for each beam polarisation; these results are then scaled according to the operation scenarios listed in Tab. 5 for the parameter fit. In case of ILC1, where all three higgsinos would already be kinematically accessible at \( \sqrt{s} = 250 \) GeV, the assumed integrated luminosities correspond to the H20 operating scenario, while for the other two benchmarks the I20 scenario was assumed, since in these cases the higgsinos are only accessible at \( \sqrt{s} = 350 \) GeV.

Table 3: Chargino and neutralino production cross sections for the three benchmark points calculated using WHIZARD at various center-of-mass energies. The ILC beam energy spectrum and ISR effects are included. Pure beam polarizations are assumed.

| \( \sqrt{s} \) | Process | \( \mathcal{P} \) | ILC1 | ILC2 | nGMM1 |
|---|---|---|---|---|---|
| 250 GeV | \( e^+ e^- \rightarrow \tilde{\chi}^+ \tilde{\chi}^- \) | \( \mathcal{P}_{LR} \) | 2618 | – | – |
| 3094 | 1044 | – | – |
| 804.8 | – | – |
| 350 GeV | \( e^+ e^- \rightarrow \tilde{\chi}^0 \tilde{\chi}^0 \) | \( \mathcal{P}_{LR} \) | 1044 | 897.0 | 538.8 |
| 1044 | 897.0 | 538.8 |
| \( \mathcal{P}_{RL} \) | 397.1 | 691.5 | 444.1 |
| 302.8 | 444.1 |
| 301.4 | 444.1 |
| 500 GeV | \( e^+ e^- \rightarrow \tilde{\chi}^+ \tilde{\chi}^- \) | \( \mathcal{P}_{LR} \) | 2618 | 1801 | 1531 |
| 1801 | 1801 | 1531 |
| \( \mathcal{P}_{RL} \) | 397.1 | 334.8 | 307.2 |
| 397.1 | 334.8 | 307.2 |
| 309.2 | 307.2 |

Table 4: Chargino and neutralino decay branching ratios for the three benchmark points calculated using ISASUGRA. For the final-state leptons, only the electrons and muons are included (\( \ell = e, \mu \)).

| \( \ell \rightarrow q\ell \bar{\nu} \) | \( \ell \rightarrow q\ell \bar{\nu} \) | \( \ell \rightarrow q\ell \bar{\nu} \) |
|---|---|---|
| ILC1 | ILC2 | nGMM1 |
| \( \tilde{\chi}_1 \rightarrow \tilde{\chi}_1 q\ell \) | 67% | 67% | 66% |
| \( \tilde{\chi}_1 \rightarrow \tilde{\chi}_1 \ell^+ \ell^- \) | 22% | 22% | 22% |
| \( \tilde{\chi}_2 \rightarrow \tilde{\chi}_2 q\ell \) | 58% | 63% | 51% |
| \( \tilde{\chi}_2 \rightarrow \tilde{\chi}_2 \ell^+ \ell^- \) | 7.4% | 8.0% | 7.5% |

### 3.2 The ILD detector model

The ILD concept is one of the two detectors being designed for the ILC. ILD employs a hybrid tracking system comprised of a time projection chamber and silicon strip sensors for tracking, and silicon pixel sensors as vertex detectors. Outside of the tracking system sits a highly granular calorimeter system optimized for particle flow reconstruction. A superconducting solenoid with a magnetic field of 3.5 T encases the calorimeters. An iron yoke outside the solenoid coil returns the magnetic flux, and is instrumented with scintillator-based muon detectors. In the low angle region, charged particles will be efficiently tracked down to 7 degrees. Dedicated calorimeters are placed in the forward region for detecting particles at even lower angles to the beam [50]. The most forward
Table 5: Total integrated luminosities for various operation scenarios for the ILC [68]. H20 is assumed for ILC1, while I20 is assumed for ILC2 and nGMM1. See Ref. [68] for the assumed timelines and machine upgrades.

| Scenario | $\sqrt{s}$ [GeV] | $\mathcal{P}$ | $\mathcal{L}$ [fb$^{-1}$] |
|----------|------------------|----------------|-----------------|
| H20      | 250              | $\mathcal{P}^+ + \mathcal{P}^+$ | 900             |
|          | 350              | $\mathcal{P}^+ + \mathcal{P}^+$ | 90              |
|          | 500              | $\mathcal{P}^+ + \mathcal{P}^+$ | 1600            |
| I20      | 250              | $\mathcal{P}^+ + \mathcal{P}^+$ | 225             |
|          | 350              | $\mathcal{P}^+ + \mathcal{P}^+$ | 765             |
|          | 500              | $\mathcal{P}^+ + \mathcal{P}^+$ | 1600            |

The simulation and reconstruction tools used in this study are part of the iLCSoft framework (v01-16-02) [35]. The beam crossing angle of 14 mrad and the response of the ILD detector in its version ILD_o1_v05 as used for the ILC TDR [39] are simulated using Mokka [42] based on GEANT4. The event reconstruction is performed using the Marlin [43] framework, including the particle flow algorithm PandoraPFA [44] for calorimeter clustering and the analysis of track and calorimeter information.

### 3.3 Signal processes and key observables

We study the pair production of the two light charginos ($\tilde{\chi}_1^\pm$, $\tilde{\chi}_1^0$) and two light neutralinos ($\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$). In our benchmark models, the higgsino component is strongly dominant for these four light states. Their masses are shown in Tab. 1. The charginos and neutralinos are both produced dominantly via the $s$-channel exchange since the sleptons are assumed to be heavy. The chargino pair production proceeds as $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$ through $\gamma/Z$ exchange, while the neutralino associated production $e^+e^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$ undergoes via the $Z$ boson exchange. While in a real analysis at ILC more decay modes of the $\tilde{\chi}_2^0$ and the $\tilde{\chi}_1^+$ can be utilized, we focus here on the semi-leptonic channel for the charginos, i.e. $\tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow q\bar{q}' \tilde{\chi}_1^0 \tilde{\chi}_1^0$, and on the leptonic channel for the neutralinos, i.e. $\tilde{\chi}_2^0 \tilde{\chi}_1^0 \rightarrow l^+l^- \tilde{\chi}_1^0$. We restrict $l = e, \mu$ in this study.

The key target observables are the three masses ($M_{\tilde{\chi}_1^0}$, $M_{\tilde{\chi}_2^0}$, and $M_{\tilde{\chi}_1^+}$) and, in this study, four polarized cross sections: chargino and neutralino production for the two opposite-sign beam polarization configurations. In a real ILC analysis, the like-sign combinations would be included as well, at least to serve as background-enriched control samples.

The three masses can be extracted from the maximum endpoints of the kinematic distributions shown at generator-level in Fig. 3(a),(b),(d) and (e). Specifically, we will rely on the maximum invariant mass and energy of the visible decay products of the $\tilde{\chi}_1^0 \rightarrow l^+l^- \tilde{\chi}_1^0$ and $\tilde{\chi}_1^+ \rightarrow q\bar{q}' \tilde{\chi}_1^0$. We find that the minimum endpoints, typically used in other SUSY studies, are too small to be useful in this study, since the resulting detector response is challenging to model in the soft spectrum and, in the case of the neutralino channel, it has large overlap with irreducible backgrounds.
The maximum energy $E_{\text{max}}$ of the di-jet (or di-lepton) system seen in the laboratory frame is given by

$$E_{\text{max}} = \frac{\gamma (1 + \beta)}{2} \left( 1 + \frac{M}{M'} \right) \Delta M,$$

(2)

where $M$ is the LSP ($\tilde{\chi}_1^0$) mass and $M'$ is the mass of $\tilde{\chi}_1^0$ ($\tilde{\chi}_2^0$) for the chargino (neutralino) channel. The mass difference is given by $\Delta M = M' - M$. As the decays studied are three body decays, it follows that the Lorentz-invariant mass of any pair of final-state particles has a maximum equal to the mass-difference between the decaying particle and the mass of the third decay product, see e.g. sect. 47.4.4.1 of [45]. In other words, the maximum of the di-jet (or di-lepton) mass is a direct measure of $\Delta M$. The boost factors $\beta$ and $\gamma$ are given according to

$$\gamma = \left( 1 - \beta^2 \right)^{-\frac{1}{2}}$$

and

$$\beta = \frac{p}{\sqrt{M^2 + p^2}},$$

where the maximum momentum $p$ in the laboratory frame is given by:

$$p = \frac{\sqrt{3}}{2} \sqrt{1 - 2 \left( \left( \frac{M}{\sqrt{3}} \right)^2 + \left( \frac{M'}{\sqrt{3}} \right)^2 \right) + \left( \left( \frac{M}{\sqrt{3}} \right)^2 - \left( \frac{M}{\sqrt{3}} \right)^2 \right)^2}.$$

(3)

For any given channel, the measurements of $\Delta M$ and $E$ yield the masses $M$ and $M'$ by numerically solving the relations above. In our study, we obtain several measurements of $\Delta M$ and $E$, specifically for the two different lepton final states ($\ell = e$ and $\mu$), and for the two different beam polarizations $\mathcal{P}_{++}$ and $\mathcal{P}_{+-}$. These measurements can be readily combined individually for the chargino and neutralino channels. Because the chargino and neutralino measurements both include the LSP mass $M_{\tilde{\chi}_1^0}$ in the observables, a final combination is performed using a fit to extract the uncertainty of the three masses.

Figure 3: Generator-level distributions, given for the beam polarization with $\mathcal{P}_{LR}$. The distributions for the other beam polarization $\mathcal{P}_{RL}$ are similar, up to the normalization due to the cross section.

Figure 3 shows the generator-level distributions of the mass and energy distributions for the di-jet (di-lepton) system for the chargino (neutralino) channel. The three benchmark points, ILC1, ILC2, and nGMM1, give progressively softer distributions due to the smaller mass gaps. The visible part of the chargino/neutralino decay will be very soft; for example, most of the jets and leptons have energies less than 20 GeV in the case of the ILC2 benchmark, and less than 10-15 GeV in the case of the nGMM1 benchmark.

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4 The value for $E_{\text{max}}$ given in formula Eq. 2 is attained when the dilepton or dijet invariant mass is zero. The complete formulae relating the maximum and minimum energy to the invariant mass may be found on p. 439 of Ref. [19]. The sensitivity of the analysis could even be improved by evaluating the maximum energy on an event-by-event basis, taking into account the dijet / dilepton invariant mass measured in each event individually.
3.4 Parameter Fitting

In the final step of the study we investigate the possibility to extract SUSY parameters from the projected measurement precisions obtained from the detector simulation. This will be addressed using a Markov chain technique as implemented in the program Fittino. Unless stated otherwise, the length of the Markov chains is $10^6$ for each fitted configuration. While the MC samples used in the full detector simulation were based on Isajet, Fittino employs SPheno as a spectrum calculator during the fit. More details about the fitting procedure can be found in [66].

In addition to the mass and cross-section projections from this study, which will be described in detail in Sec. 4, a standard set of projected Higgs precision observables from the ILC was used to constrain the fit: These projections assume the H20 running scenario for the ILC [68], and include the Higgs mass (with a precision of 15 MeV as obtained in a ILD full simulation study [69]), and a set of Higgs branching ratio precisions obtained from the model-independent coupling fit results in [67], based on the so-called $\kappa$-framework.\(^5\) The resulting precisions are shown in Fig. 4 in comparison to the expected deviations from the SM branching ratios in our three benchmarks. The Higgs mass and branching-ratio values are taken from FeynHiggs2.10.4 [53] for each of the SUSY models in question. For the NUHM2-based benchmarks, the most significant deviations would be observed in the Higgs couplings to the W and Z bosons, although they would be hardly convincing as a discovery on their own. In case of nGMM1, all Higgs precision measurements agree perfectly with the SM.

![](Images/figure4.png)

Figure 4: Deviations of the branching fractions of the SUSY light Higgs from the Standard Model expectations, as obtained with FeynHiggs2.10.4. The uncertainty bands illustrate the expected measurement precisions after the full 250 and 500 GeV ILC program, assuming the H20 scenario [67]. Note that these are somewhat more conservative than the most recent estimates from [78].

4 Full Detector Simulation Study

For each benchmark point, we select separately the chargino and neutralino channels. These are considered as background to each other, including all decay modes. A common event selection is performed and provides sufficient sensitivity in all three benchmark cases. In a real experiment, once a signal has been discovered, the selection could be optimized based on initial mass estimates, see e.g. [54] for a discussion of how to boot-strap a selection of an a priori unknown signal. In the following sections, we describe the event selection for the chargino and neutralino channels.

4.1 Chargino channel

For the chargino pair production, we study the semileptonic final state $e^-e^+ \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0qq\ell\nu$, where $\ell = e$ or $\mu$. The strategy here is to look for a single isolated lepton accompanied by two jets and large missing energy. We reconstruct the invariant mass and energy distributions of the di-jet system and extract their endpoints.

\(^5\)Note that this includes the cross section measurements which in effect dominate the coupling precisions for the weak gauge bosons. For technical reasons the coupling precisions could not be used directly in the fit.
First, an isolated lepton candidate is identified according to the following criteria. Electron identification requires that the total energy measured in the calorimetric system $E_{\text{tot}}$ is consistent with the momentum measured in the tracker $p_{\text{trk}}$, such that they satisfy $0.5 < E_{\text{tot}}/p_{\text{trk}} < 1.3$. In addition, the ECAL energy deposit $E_{\text{ECAL}}$ must be dominant over the HCAL energy deposit $E_{\text{HCAL}}$, so that we have $E_{\text{ECAL}}/(E_{\text{ECAL}} + E_{\text{HCAL}}) > 0.9$. For muon identification, we require that a charged track is associated with signals in the muon detector. In addition, lepton candidates with large impact parameter significance ($>5\sigma$) are rejected in order to suppress backgrounds due to $\tau$ or heavy quark decays. For the isolation requirement, we define an isolation cone around the lepton candidate with half-angle $\alpha$ such that $\cos \alpha = 0.95$. We require that the total energy of charged particles within the cone (not including the candidate itself) is less than 0.2 GeV. The isolated lepton candidate with the highest transverse momentum is selected as the isolated lepton in the event.

Next, we deal with high cross section $\gamma\gamma$ processes that produce soft hadrons that overlap with our signal. Jet finders are used in two steps, following the procedures in [39]. We apply the $k_t$ jet finder algorithm with the jet radius parameter $R = 1.4$, forcing all reconstructed particles of the event apart from the isolated lepton into two jets, plus two additional beam jets; particles that are clustered into the beam jets are removed in the remainder of the event reconstruction [47, 48]. The value $R = 1.4$ was chosen to yield dijet mass distributions which are optimal for the extraction of the kinematic endpoint. The remaining particles are used to reconstruct the chargino that decayed hadronically. They are forced into two jets using the Durham jet finding algorithm [49].

The event selection proceeds as follows. We select events with exactly one isolated lepton candidate, and its lepton type is identified. We reject events containing particles that are reconstructed in the BeamCal [50]. The transverse momentum of the lepton is required to be 5 GeV or greater. The number of reconstructed charged particles in each jet must be 2 or greater. It was tested whether a tighter cut on the track multiplicity would help to reject background from e.g. 3-prong $\tau$ decays, but due to low jet energies, especially in the ILC2 and nGMM1 benchmarks, c.f. Fig. 3(c), the resulting loss in signal was too severe. Both of the reconstructed jets should not be very forward, so that the polar angle of each jet $\theta_j$ is such that $|\cos \theta_j| < 0.98$. We require the coplanarity of the two jets as defined by the difference of the azimuthal angle to be $\Delta \phi = |\phi_1 - \phi_2| < 1.0$. The angle between the two jets $\theta_{jj}$ is required to satisfy $|\cos \theta_{jj}| < 0.2$. The visible energy in the event is required to be less than 80 GeV. The missing energy in the event is required to be greater than 400 GeV. The polar angle of the missing momentum $\theta_{\text{miss}}$ is required to satisfy $|\cos \theta_{\text{miss}}| < 0.99$. The expected number of signal and background events after the event selection is shown in Tab. 6. Very few background events survive after the event selection. An example of the resulting distributions is shown for the $q\bar{q}\mu\nu$ channel with beam polarization $\mathcal{P}_{\pmb{+}}$ in Fig. 5. The number of events at various steps of the event selection and the distributions for all studied channels can be found in Appendix A.

![Image](image.png)

Table 6: Expected number of events after the event selection for the chargino signal and major backgrounds, normalized to an integrated luminosity of 500 fb$^{-1}$. For each benchmark model the SUSY background is given in the column “Bkg.”.

| Process | ILC1 | ILC2 | nGMM1 | SM bkg. | $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ | $e^+e^- \to 2\gamma$ | $e^+e^- \to 4\gamma$ | $\gamma\gamma \to 2\gamma$ | $\gamma\gamma \to 3\gamma$ | $\gamma\gamma \to 4\gamma$ |
|---------|------|------|--------|---------|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| q$\bar{q}e\nu$ | $\mathcal{P}_{\pmb{-}}$ | 1463 | 85 | 392 | 23 | 283 | 15 | 5.9 | 64 | 0.0 | 22 | 2.0 |
| | $\mathcal{P}_{\pmb{+}}$ | 404 | 23 | 96 | 4.6 | 73 | 5.1 | 7.4 | 16 | 0.0 | 8.0 | 2.0 |
| q$\bar{q}\mu\nu$ | $\mathcal{P}_{\pmb{-}}$ | 1862 | 108 | 509 | 28 | 389 | 29 | 33 | 37 | 0.0 | 0.0 | 7.0 |
| | $\mathcal{P}_{\pmb{+}}$ | 524 | 34 | 127 | 8.5 | 101 | 8.2 | 8.2 | 7.2 | 0.0 | 0.0 | 7.0 |

The maximum endpoints of the dijet energy ($E_{jj}$) and mass ($M_{jj}$) distributions are extracted using a fit. Figures 7-6 show some examples of such a fit. Although the samples are almost free of backgrounds, the signal distribution has a tail which is caused by the failure to properly reconstruct the energy of soft neutral particles. We use an exponential curve to model such an effect, combined with a linear function to model the steep drop leading to the kinematic endpoint. The point where the two functions meet after the fit was used to estimate the kinematic endpoint. The statistical uncertainty of this value is estimated by performing toy Monte-Carlo experiments that repeatedly fit statistically fluctuated versions of these parent distributions. The extracted maximum endpoint of the dijet mass distributions is seen to have a systematic shift from the actual mass difference, which requires a correction at the level of 10-20% before they are used input to the final fit for the masses. It is assumed here that such a calibration procedure does not add any significant systematic uncertainties to our results, as described in [63]. The endpoints from the dijet energy distributions are used without corrections in the mass fit.
Figure 5: Example of reconstructed distributions in the chargino channel $e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 q\bar{q} \mu \bar{\nu}_\mu$ with beam polarization $P_{-+}$. The contributions from SUSY and SM backgrounds are very small. The signal histograms are stacked on top of the backgrounds.

Figure 6: Example of the endpoint extraction of the di-jet invariant mass system in the chargino channel $e^- e^+ \rightarrow \tilde{\chi}_1^- \tilde{\chi}_1^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 q\bar{q} \mu \bar{\nu}_\mu$ with beam polarization $P_{-+}$ for the three benchmarks.

Figure 7: Example of the endpoint extraction of the di-jet energy system in the chargino channel $e^- e^+ \rightarrow \tilde{\chi}_1^- \tilde{\chi}_1^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 q\bar{q} \mu \bar{\nu}_\mu$ with beam polarization $P_{-+}$ for the three benchmarks.
4.2 Neutralino channel

For the neutralino mixed production, we choose the clean leptonic decay of $\tilde{\chi}_1^0$ as the final state: $e^-e^+ \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0 \rightarrow e^- \ell^+ \ell^- \rightarrow e^- e^+ \rightarrow \ell^+ \ell^-$. The strategy is to look for a pair of isolated leptons with large missing energy. The invariant mass and the energy of the di-lepton system provides information about the neutralino masses. The isolated leptons are selected in the same way as in the chargino channel. This time, we require two oppositely-charged leptons of the same flavor, instead of one.

The expected number of signal and major background events are summarized in Tab. 7; the full tables of event selection can be found in Appendix A. At the preselection stage, we require that two oppositely charged leptons are found, each having a transverse momentum of at least 2 GeV. Then, the lepton flavor is required to be either an electron or a muon pair, and the total number of reconstructed charged particles (including the leptons) in the event is required to be exactly two. We reject events containing particles that are reconstructed in the BeamCal. The requirement on the transverse momentum of both leptons are further tightened to 2.3 GeV or greater. The requirement on the transverse momentum of each lepton’s momentum $p_T$ is required to satisfy $|\cos \theta| < 0.95$. The coplanarity of the two leptons $\Delta \phi = |\phi_1 - \phi_2|$ is required to satisfy $\Delta \phi < 0.8$. The visible energy of the event is required to be less than 25 GeV. The missing energy in the event must be greater than 300 GeV. The polar angle of the missing momentum angle $\theta_{\text{miss}}$ must satisfy $|\cos \theta_{\text{miss}}| < 0.98$.

An example of the $M_\ell$ and $E_\ell$ distributions after this selection is shown in Fig. 8; the full distributions can be found in Appendix A. In contrast to the chargino channel, the neutralino channel has sizable SM backgrounds after the event selection, since due to the much smaller number of signal events the cuts cannot be as tight as in the chargino channel. The dominant backgrounds are the $e^+e^- \rightarrow \ell^+ \ell^- \nu\nu \nu\nu$ processes, where $\ell$ is the same lepton flavor as the final state leptons of the signal. The SUSY backgrounds remain negligible.

The maximum endpoints of the energy ($E_\ell$) and mass ($M_\ell$) distributions of the di-lepton system are extracted using a fit. We show some examples of the fit in Figs. 9-10. We use an exponential curve to model the background near the endpoint. A linear function is used to model the signal part. The intersection of the two functions is used to extract the kinematic endpoint. Again, the uncertainty of this value is estimated using toy Monte-Carlo experiments. Fitting the invariant mass distribution in the nGMM1 benchmark point requires special care due to the $J/\psi$ resonance from the neutralino decay, which sits on the falling end of the distribution. The fit is done in two steps. First, a Gaussian distribution with a narrow width is used to fit the narrow peak in the small window of the $J/\psi$ resonance. The fitted yield and width of the resonance are fixed in the second, overall fit, which extracts the maximum endpoint. As was the case for the chargino channel, the extracted maximum endpoint of the dilepton mass distributions requires a correction at the level of 10-20% before they are used input to the final fit for the masses. The end points of the dilepton energy distributions are used without corrections.

Table 7: Expected number of events after the event selection for the neutralino signal and major backgrounds, normalized to an integrated luminosity of 500 fb$^{-1}$. For each benchmark model the SUSY background is given in the column “Bkg.”.

| $e^+e^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$ Process | ILC1 | ILC2 | nGMM1 | SM bkg. |
|----------------------------------|------|------|-------|---------|
|                                  | Sig. | Bkg. | Sig. | Bkg. | Sig. | Bkg. | Sig. | Bkg. | $e^+e^- \rightarrow 2f$ | $e^+e^- \rightarrow 4f$ | $\gamma\gamma \rightarrow 2f$ | $\gamma\gamma \rightarrow 3f$ | $\gamma\gamma \rightarrow 4f$ |
| $ee$                            | 1621 | 185  | 1250 | 226  | 490  | 207  | 14   | 3875 | 14   | 371  | 19   |
| $e^+e^-$                        |   1284 | 69   | 1017 | 111  | 409  | 119  | 13   | 508  | 14   | 83   | 19   |
| $\mu\mu$                       | 1939 | 176  | 1496 | 197  | 640  | 91   | 0.0  | 5506 | 77   | 100  | 9.6  |
| $\mu^+\mu^-$                    | 1521 | 49   | 1222 | 67   | 516  | 40   | 0.0  | 672  | 77   | 100  | 9.6  |

4.3 Results from the full detector simulation study

We present the combined result of the mass measurements in Tab. 8. Assuming an integrated luminosity of 500 fb$^{-1}$ at each of the two beam polarizations $P(e^+, e^-) = (\pm 30\% , \pm 80\%)$, it is shown that the chargino and neutralino masses can be measured to about 0.5-0.7% for benchmarks with mass gaps of 10 GeV or larger, and better than 1% for benchmarks with mass gaps of a few GeV.

In the last column, the relative precisions on the masses have been scaled to the full luminosity foreseen to be collected at $\sqrt{s} = 500$ GeV according to the H20 and I20 running scenarios of the ILC [68]. These values are
Figure 8: Example of reconstructed distributions in the neutralino channel $e^- e^+ \rightarrow \tilde{\chi}_1^1 \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \chi_1^0 e^+ e^-$ with beam polarization $P_{+-}$. The signal histograms are stacked on top of the backgrounds. The remaining background is fully dominated by the SM contribution.

Figure 9: Example of the endpoint extraction of the di-lepton invariant mass system in the neutralino channel $e^- e^+ \rightarrow \tilde{\chi}_1^1 \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \chi_1^0 e^+ e^-$ with beam polarization $P_{+-}$ for the three benchmarks. For the nGMM1 benchmark, the $J/\psi$ peak is included in the fit.

Figure 10: Example of the endpoint extraction of the di-lepton energy system in the neutralino channel $e^- e^+ \rightarrow \tilde{\chi}_1^1 \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \chi_1^0 e^+ e^-$ with beam polarization $P_{+-}$ for the three benchmarks.
The fits include the following inputs based on ILC simulation studies: SPheno
Sec. 3.4, is zero, apart from effects of finite numerical precision and the finite length of the Markov chains. As discussed in χ randomly around their true values. Thus, in all cases where the correct underlying model is fitted, the expected along with their projected uncertainties. It should be noted that the asummed measurements have not been varied sections for the P to lower center-of-mass energies based on cross section and luminosity scaling. In most cases, the cross sections 15, and 5 GeV are selected for the ILC1, ILC2, and nGMM1 benchmarks, respectively. The acepted region of the 5 Fitting fundamental parameters

| √s = 500 GeV only | ILC1 | ILC2 | nGMM1 |
|-------------------|------|------|-------|
| Model mass [GeV]  | m_{\tilde{\chi}^\pm_1} | m_{\tilde{\chi}^\pm_2} | m_{\tilde{\chi}^\pm_3} |
|                   | 102.7 | 148.1 | 151.4 |
|                   | 124.0 | 157.8 | 155.8 |
|                   | 117.3 | 158.3 | 158.7 |
| Precision         | \delta m_{\tilde{\chi}^\pm_1}/m_{\tilde{\chi}^\pm_1} | 0.5% | 0.7% | 1.0% |
|                   | \delta m_{\tilde{\chi}^\pm_2}/m_{\tilde{\chi}^\pm_2} | 0.5% | 0.7% | 1.0% |
|                   | \delta m_{\tilde{\chi}^\pm_3}/m_{\tilde{\chi}^\pm_3} | 0.5% | 0.7% | 1.0% |
| Scaled precision  | \delta m_{\tilde{\chi}^\pm_1}/m_{\tilde{\chi}^\pm_1} | 0.3% | 0.4% | 0.5% |
|                   | \delta m_{\tilde{\chi}^\pm_2}/m_{\tilde{\chi}^\pm_2} | 0.3% | 0.4% | 0.5% |
|                   | \delta m_{\tilde{\chi}^\pm_3}/m_{\tilde{\chi}^\pm_3} | 0.3% | 0.4% | 0.5% |

considered to be conservative as they neglect further improvements from data sets at lower center-of-mass energies or from dedicated threshold scans. The relative precisions on the masses range from 0.3% in case of ILC1 with the largest mass differences to about 0.6% for nGMM1 as the case with the smallest mass differences.

The precision expected for the cross section times branching ratio measurements at √s = 500 GeV is estimated from the statistical significance computed from the number of signal and background events using the di-lepton and di-jet energy distributions. In the case of the neutralino channel, an additional cut on the di-lepton mass distribution is applied in order to remove the SM backgrounds in the high mass region; namely, events with M_{M_0} less than 25, 15, and 5 GeV are selected for the ILC1, ILC2, and nGMM1 benchmarks, respectively. The accepted region of the energy distribution is optimized to yield the best statistical significance for each channel. These were extrapolated to lower center-of-mass energies based on cross section and luminosity scaling. In most cases, the cross sections can be measured with a precision of a few %, as summarized in Tab. 9. Notorical exception are the chargino cross sections for the P_{+-} case, which are typically a factor two worse than the other precisions.

5 Fitting fundamental parameters

In this section we will pursue the question of whether the projected precisions on the physics observables will be sufficient to discriminate between different SUSY models, and to determine the parameters of the correct model. To this purpose, assumed measurements of Higgsino masses and polarized cross sections are presented to Fittino along with their projected uncertainties. It should be noted that the asummed measurements have not been varied randomly around their true values. Thus, in all cases where the correct underlying model is fitted, the expected \chi^2 is zero, apart from effects of finite numerical precision and the finite length of the Markov chains. As discussed in Sec. 3.4, SPheno had to be used instead of Isajet as a spectrum calculator in the fitting step.\footnote{Here we switch to the spectrum generator SPheno since there exists a direct interface between SPheno and Fittino while no such interface exists for Fittino and Isajet. The mass spectra generated from Isajet and SPheno are slightly different due to different algorithms used by the code authors.}

The fits include the following inputs based on ILC simulation studies:

- The higgsino masses obtained with SPheno 3.3.9beta as listed in Tab. 10, together with their estimated precisions based on a preliminary version of the full simulation study at √s = 500 GeV, listed in Tab. 7.11 of Ref. [66]. Note that these are between 30% and 100% more conservative than the results given in Tab. 8 of this paper. We will study and discuss the relevance of these differences in Sec. 6.4.
- The polarised total cross sections for chargino and neutralino production at all relevant center-of-mass energies with precisions as given in Tab. 7.12-7.14 of Ref. [66]. As for the masses, these are between 20% and
Table 9: Estimated experimental precisions for the three benchmark points, for the four different final states and the two beam polarizations. The full simulation results, performed for $\sqrt{s} = 500$ GeV, are given for $\mathcal{L} = 500$ fb$^{-1}$. The scaled precisions for the various center-of-mass energies are shown assuming the H20 scenario for ILC1 and the I20 scenario for ILC2 and nGMM1.

| $\Delta(\sigma \times BR)$ [%] | $\sqrt{s} = 500$ GeV | $\sqrt{s} = 250$ GeV | $\sqrt{s} = 350$ GeV |
|---------------------------------|----------------------|----------------------|----------------------|
|                                 | $\mathcal{P}_{++}$ | $\mathcal{P}_{+-}$ | $\mathcal{P}_{++}$ | $\mathcal{P}_{+-}$ | $\mathcal{P}_{++}$ | $\mathcal{P}_{+-}$ |
| ILC1                            |                     |                      |                      |                     |                      |
| $\tilde{\chi}^0_1\tilde{\chi}^0_1 \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 ee$ | 3.98                | 3.13                | 2.22                | 1.75                | 2.04                | 1.60                | 6.94                | 5.47                |
| $\tilde{\chi}^0_1\tilde{\chi}^0_1 \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 \mu\mu$ | 3.81                | 2.97                | 2.13                | 1.66                | 1.95                | 1.52                | 6.66                | 5.18                |
| $\tilde{\chi}^0_1\tilde{\chi}^0_1 \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 q\bar{q}e_e$ | 2.59                | 4.94                | 1.45                | 2.76                | 1.61                | 3.22                | 4.66                | 9.04                |
| $\tilde{\chi}^0_1\tilde{\chi}^0_1 \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 q\bar{q}\mu\nu_\mu$ | 2.27                | 4.30                | 1.27                | 2.40                | 1.41                | 2.80                | 4.09                | 7.87                |
| ILC2                            |                     |                      |                      |                     |                      |
| $\tilde{\chi}^0_1\tilde{\chi}^0_1 \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 ee$ | 3.92                | 3.50                | 2.19                | 1.96                | -                   | -                   | 2.82                | 2.52                |
| $\tilde{\chi}^0_1\tilde{\chi}^0_1 \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 \mu\mu$ | 3.90                | 3.33                | 2.18                | 1.86                | -                   | -                   | 2.81                | 2.40                |
| $\tilde{\chi}^0_1\tilde{\chi}^0_1 \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 q\bar{q}e_e$ | 5.17                | 10.30               | 2.89                | 5.76                | -                   | -                   | 4.09                | 8.28                |
| $\tilde{\chi}^0_1\tilde{\chi}^0_1 \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 q\bar{q}\mu\nu_\mu$ | 4.39                | 8.84                | 2.45                | 4.94                | -                   | -                   | 3.47                | 7.10                |
| nGMM1                           |                     |                      |                      |                     |                      |
| $\tilde{\chi}^0_1\tilde{\chi}^0_1 \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 ee$ | 5.30                | 4.98                | 2.96                | 2.78                | -                   | -                   | 3.84                | 3.61                |
| $\tilde{\chi}^0_1\tilde{\chi}^0_1 \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 \mu\mu$ | 5.05                | 4.64                | 2.82                | 2.59                | -                   | -                   | 3.66                | 3.36                |
| $\tilde{\chi}^0_1\tilde{\chi}^0_1 \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 q\bar{q}e_e$ | 6.20                | 11.73               | 3.47                | 6.56                | -                   | -                   | 4.94                | 9.48                |
| $\tilde{\chi}^0_1\tilde{\chi}^0_1 \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 q\bar{q}\mu\nu_\mu$ | 4.99                | 9.90                | 2.79                | 5.53                | -                   | -                   | 3.98                | 8.00                |

100% more conservative than the full simulation results listed in Tab. 9, and we will study and discuss the relevance of these differences in Sec. 6.4. Note that in ILC1, higgsino production is kinematically accessible at center-of-mass energies as low as 250 GeV, while for ILC2 and nGMM1 $\sqrt{s} = 350$ GeV is the lowest ILC energy stage which allows higgsino production. Therefore, we consider the alternative running scenario I20 [68] for these benchmarks.

- The mass of the lightest CP-even Higgs boson, with the ILC precision according to [69], as discussed in Sec. 3.4.
- The branching ratios $BR(h \rightarrow b\bar{b})$, $BR(h \rightarrow c\bar{c})$, $BR(h \rightarrow \tau^+\tau^-)$, $BR(h \rightarrow gg)$, $BR(h \rightarrow W^+W^-)$, $BR(h \rightarrow ZZ)$ and $BR(h \rightarrow \gamma\gamma)$, with the ILC precisions according to [68], as discussed in Sec. 3.4.

In total, these amount to 25 observables. In the following, we will start by discussing fits of different GUT-scale models in Sec. 5.1, and before proceeding to the determination of weak-scale parameters in Sec. 5.2. Finally, we will address the predictions of the LSP’s relic density in Sec. 5.3.

Table 10: ILC1, ILC2 and nGMM1 MSSM model masses from SPheho3.3.9beta together with the input precisions assumed in the fit. The assumed input precisions are given for $\sqrt{s} = 500$ GeV and $\mathcal{L} = 1600$ fb$^{-1}$. Data sets with other center-of-mass energies in the H20 and I20 operating scenarios are neglected.

| $\sqrt{s} = 500$ GeV only | ILC1 | ILC2 | nGMM1 |
|---------------------------|------|------|-------|
| Model mass [GeV]          |      |      |       |
| $m_{\tilde{\chi}^0_1}$   | 104.8| 151.3| 154.9 |
| $m_{\tilde{\chi}^0_1}$   | 127.5| 162.4| 160.2 |
| $m_{\tilde{\chi}^0_1}$   | 116.0| 157.0| 157.4 |
| Assumed precision $\delta m_{\tilde{\chi}^0_1}/m_{\tilde{\chi}^0_1}$ [%] | 0.5% | 0.7% | 1.0% |
| $\mathcal{P}_{++}$, $\mathcal{L} = 1600$ fb$^{-1}$ | $\delta m_{\tilde{\chi}^0_1}/m_{\tilde{\chi}^0_1}$ [%] | 0.4% | 0.7% | 1.0% |
| $\mathcal{P}_{++}$, $\mathcal{L} = 1600$ fb$^{-1}$ | $\delta m_{\tilde{\chi}^0_1}/m_{\tilde{\chi}^0_1}$ [%] | 0.5% | 0.7% | 1.0% |
5 Fitting fundamental parameters

5.1 Fitting GUT-scale parameters

The fitting of GUT-scale parameters requires strong assumptions on the underlying SUSY breaking scheme. Since our benchmarks cover two very different approaches to unification, it is interesting to study whether these can be distinguished by directly fitting different GUT-scale models.

5.1.1 Results of fitting NUHM2

In case of NUHM2, we fit the parameters $M_1/2$, $\mu$, $\tan\beta$, $M_0$, $A_0$ and $m_A$ to the observables described in Sec. 4.3. Table 11 shows the best fit point and its 1 and 2\(\sigma\) confidence intervals obtained in the case of the ILC1 benchmark, in comparison to the input model parameters. Tables 12 and 13 give the analogous information for the ILC2 and nGMM1 benchmarks, respectively. In case of the ILC1 and ILC2 benchmarks, where NUHM2 is the correct underlying model, the $\chi^2$ of the best fit point is very small, and all fitted parameters agree well with their true input values. The 1\(\sigma\) uncertainties for $M_1/2$, $\mu$, $\tan\beta$ are typically 10\% or better, while $M_0$, $A_0$ and $m_A$, which enter only at loop-level into the considered observables, are still determined within about 20\%.

| parameter | ILC1 NUHM2 true | best fit point | 1\(\sigma\) CL | 2\(\sigma\) CL |
|-----------|-----------------|----------------|-----------------|----------------|
| $M_{1/2}$ | 568.3           | 556.7          | +24.3           | +37.7          |
| $\mu$     | 115.0           | 105.3          | +12.8           | +14.0          |
| $\tan\beta$ | 10.0             | 11.4          | +5.6            | +11.4          |
| $m_A$     | 1000            | 968            | +167            | +288           |
| $M_0$     | 7025            | 7685           | +1243           | +2311          |
| $A_0$     | -10427          | -11064         | +2695           | +2927          |
| $\chi^2$  | 0.0013          | 0.0011         |

Table 11: Fitted parameters in the fit of NUHM2 parameters to ILC1 observables in the H20 scenario. All values in GeV apart from $\tan\beta$. Note that the $\chi^2$ value of the model point is increased from 0 by the rounding errors of the observables in the inputs.

| parameter | ILC2 NUHM2 true | best fit point | 1\(\sigma\) CL | 2\(\sigma\) CL |
|-----------|-----------------|----------------|-----------------|----------------|
| $M_{1/2}$ | 1200            | 1194           | +107            | +164           |
| $\mu$     | 150.0           | 150.7          | +4.3            | +7.2           |
| $\tan\beta$ | 15.0             | 16.0          | +26.2           | +28.8          |
| $m_A$     | 1000            | 1008           | +141            | +256           |
| $M_0$     | 5000            | 4788           | +2546           | +3566          |
| $A_0$     | -8000           | -7663          | +3817           | +3817          |
| $\chi^2$  | 0.0007          | 0.02848        |

Table 12: Best fit point and confidence intervals of the NUHM2 parameters fitted to ILC2 SUSY and Higgs measurements in the I20 operating scenario. Note that the $\chi^2$ value of the model point is increased from 0 by the rounding errors of the observables in the inputs.

In case of the nGMM1 benchmark, the $\chi^2$ of the best fit point is somewhat larger than for the other two benchmarks, but still so small that an NUHM2 interpretation of this benchmark cannot be rejected. This is not surprising as it has been constructed to have the physical observables very similar to ILC2. However, the best fit point is found for $M_{1/2}$ about a factor two bigger, and $M_A$ about 60\% larger, than in ILC2. This implies that a direct observation of the heavy Higgs bosons and the electroweakinos could distinguish the two models. However this also raises the question of whether the weak-scale fits based on input from the higgsino properties alone will be able to identify
the nGMM1 benchmark unambiguously as a non-NUHM2 model, with a completely different underlying SUSY breaking mechanism. We will investigate this in the next section.

| parameter | best fit point | 1σ CL | 2σ CL |
|-----------|----------------|-------|-------|
| $M_{1/2}$ | 2407           | +150  | +356  |
|           |                | −135  | −215  |
| $\mu$    | 155.6          | +1.5  | +4.0  |
|           |                | −1.9  | −2.9  |
| $\tan\beta$ | 10.0    | +2.1  | +2.4  |
|           |                | −0.5  | −0.7  |
| $m_A$    | 1603           | +528  | +1026 |
|           |                | −279  | −469  |
| $M_0$    | 3422           | +3309 | +4435 |
|           |                | −820  | −1196 |
| $A_0$    | −7409          | +666  | +887  |
|           |                | −3756 | −5304 |

| $\chi^2$ | 0.233 |

Table 13: Best fit point and confidence intervals of the NUHM2 parameters fitted to nGMM1 SUSY and Higgs measurements in the I20 operating scenario.

![Figure 11](image.png)

Figure 11: Predicted mass ranges for all the unobserved sparticles from the NUHM2 fit the observables of the three benchmark models. The green star indicates the true model mass, while the black dot shows the best fit point.

Based on the fitted NUHM2 parameters and their uncertainties, the mass spectrum of the unobserved sparticles can be predicted for all three benchmark cases. This is illustrated in Fig. 11. In the cases of ILC1 and ILC2, clear predictions for the masses of the electroweakinos and the heavy Higgs bosons are obtained in excellent agreement with the true model masses, providing motivation and an energy scale for further upgrades of the ILC. Due to the modelling of all gauginos by a single $M_{1/2}$ parameter, the gluino mass can also be firmly predicted, which can give important inputs to LHC analyses. If the predicted gluino were not observed, or observed at a very different mass, this would give strong support to the idea that nature does not describe all gauginos by one mass parameter at the GUT scale. The other sfermions are less well constrained than the Higgs bosons and gauginos, but upper limits on their masses can still be obtained. Such information would provide important motivation — and a target energy scale — for a future hadron collider.

Even in case of the nGMM1 benchmark, the NUHM2 fit predicts the general pattern of the mass spectrum correctly, albeit with less precision and significantly worse agreement between true model masses and the best fit point. The worse agreement is not surprising as we’re fitting a wrong model hypothesis in this case. Nonetheless, upper limits on all sparticle masses are obtained, which shows that even in the case that the wrong model is assumed such important information for the planning of future colliders or upgrades can be obtained.
5.1.2 Results of fitting NUHM1 and CMSSM

Before turning to the weak-scale fits, we investigate whether the three benchmarks could also be described by other widely used constrained models, in particular NUHM1 [65] and the CMSSM, which have one or even two fewer parameters to model the Higgs and higgsino sectors: in NUHM1, instead of $M_A$ and $\mu$ (or $M_{1/2}$ and $M_{3/2}$) only one parameter, $M_{1/2}$, is used to describe the Higgs and higgsino sectors, while in the CMSSM this reduces further to only the sign of $\mu$ being a free choice, while its absolute value is derived from the other model parameters. Table 14 gives the best fit point obtained when fitting NUHM1 and CMSSM to the ILC1 and ILC2 benchmarks, which give very large values of $\chi^2$/dof. These interpretations could be ruled out at the 95% C.L. already with about 0.1% the total integrated luminosity.

| best fit points | ILC1 CMSSM | ILC2 CMSSM | ILC1 NUHM1 | ILC2 NUHM1 |
|-----------------|------------|------------|------------|------------|
| $M_{1/2}$ [GeV] | 128.8      | 155.9      | 136.9      | 159.9      |
| $M_0$ [GeV]     | 3585       | 5631       | 1796       | 4264       |
| $A_0$ [GeV]     | -6873      | -10873     | -4396      | -10086     |
| $\tan \beta$   | 13.7       | 16.2       | 16.2       | 14.8       |
| $M_{3/2}$ [GeV] | -          | -          | 47659      | 56553      |
| $\chi^2_{\min}$| 238046     | 93155      | 235014     | 85174      |
| $\chi^2$/dof   | 11336      | 4436       | 11751      | 4259       |

Table 14: Best fit points in CMSSM and NUHM1 fits of ILC1 and ILC2 observables, including SUSY and Higgs measurements in the H20 operation for ILC1 and I20 operation for ILC2.

5.2 Weak scale fit results

In the following, the results of various weak scale fits to the ILC1, ILC2 and nGMM1 observables are discussed. The most general model considered is pMSSM-10, the MSSM with 10 weak scale input parameters: $M_1$, $M_2$, $M_3$, $\mu$, $\tan \beta$, $m_A$, $A_t = A_b = A_f$ and $M_{H_u}$, $M_{H_d}$, $M_L = M_{E_{1,2,3}} = M_{U_{1,2,3}} = M_{D_{1,2,3}} = M_{U_{1,2,3}} = M_{D_{1,2}}$. We use this model to test whether it is possible to constrain a comprehensive set of parameters from the observables of the higgsino sector alone, and to study the influence of the parameters in which the higgsino sector enters only at loop level. If the pMSSM-10 fit is successful and reproduces the input parameters at a satisfactory level, we proceed to investigate the precision achievable when fitting only tree-level higgsino parameters. For this we use the phenomenological MSSM with a reduced number of 4 free parameters, the pMSSM-4, which fixes the squark, slepton, heavy Higgs boson and gluino parameters to their true values and so only depends on the four weak scale parameters $M_{1,2}$, $\mu$ and $\tan \beta$. In a real analysis, the “true” values are of course unknown, but instead the best fit point of pMSSM-10 fit could be used, which agrees with the model point to usually much better than 1σ. The possible bias from fixing the 6 non-higgsino parameters was tested explicitly in case of ILC1 by fixing them to some point in the 2σ region. The best fit gaugino masses are nearly the same in the two pMSSM-4 fits irrespective of the fixed parameters. Their difference is only 2.5 GeV for $\tilde{\chi}^0_2$ and 0.8 GeV for $\tilde{\chi}^0_1$ and $\tilde{\chi}^0_2$. The two sets of best fit masses agree within the 1σ uncertainties [66]. In the following, the results for the individual benchmarks will be presented.

5.2.1 ILC1 Benchmark

Figure 12 shows the minimum $\chi^2$ as a function of $M_1$, $M_2$ and $\mu$ and $\tan \beta$ in the pMSSM-10 and pMSSM-4 fits. Due to the much smaller parameter space to be sampled in case of the 4-dimensional fit, the resulting curve is much smoother than in the 10-dimensional case. The precision on $M_1$ is nearly identical in both cases. $M_2$ is somewhat better constrained in the pMSSM-4 fit, while the determination of $\mu$ and $\tan \beta$ improves drastically.

The resulting best fit values for the pMSSM parameters and their 1 and 2σ intervals are compared to the input values in Tab. 15, quantifying the effect which could already be seen qualitatively in Fig. 12. In the case of the pMSSM-10 fit, it should be noted that also for the parameters of the coloured sector some constraints, and especially upper bounds, can be obtained. This even applies for the squark mass parameters, which might seem
surprising at the first glance, but is due to the 2-loop RGEs included in SPheno. If a hypothetical gluino mass measurement with 11% uncertainty from the LHC\cite{33} is included in the fit, the constraint on $M_3$ improves accordingly to about 10%. All other parameters, including the squark mass parameters, show only minor improvements.

Figures 13(a) and 13(b) illustrate the precisions obtained on the pMSSM-10 parameters, without and with assuming a gluino mass measurement from the LHC, respectively. Thereby, $\tan \beta$ is displayed as if it were in GeV. It can clearly be seen that the precision on $M_3$ is improved considerably by the gluino mass measurement, while the precision on all the other parameters don’t change significantly.

The determined parameters can be used to predict the masses of the yet unobserved sparticles, as shown for the pMSSM-10 fit in Tab. 16 and Fig. 14, again without and with assuming a gluino mass measurement from the LHC. As expected from Fig. 13, the effect of the gluino measurement on the other predicted masses is small.

Figure 15 illustrates the result of the corresponding pMSSM-4 fit with $M_1$, $M_2$, $\mu$ and $\tan \beta$ only. All four parameters can be determined accurately as also shown in Tab. 15. This results in predictions for the masses of the heavier electroweakinos with precisions between 1.6 and 3\%.

Figure 12: ILC1: Minimum $\chi^2$ as a function of $M_1$, $M_2$ and $\mu$ and $\tan \beta$ in pMSSM-4 fit (dashed black line) and pMSSM-10 fit (solid grey line). For each bin, the minimum $\chi^2$ of all Markov chain points which have the x axis quantity in that bin is plotted.
In the case of the ILC2 benchmark, the overall situation is similar to the case of ILC1. The minimum $\chi^2$ as a function of $M_1$, $M_2$ and $\mu$ and tan$\beta$ is displayed in Fig. 16 for the pMSSM-10 and pMSSM-4. However, this time the I20 running scenario was assumed, c.f. Sec. 4.3. Also here, the resulting curve for the 4-parameter fit is much smoother than for the 10-parameter version due to the much smaller parameter space to be sampled. Like for ILC1, the precision on $M_2$, $\mu$, and tan$\beta$ are significantly better constrained.

The resulting best fit values for the pMSSM parameters and their 1 and 2$\sigma$ intervals are compared to the input values in Tab. 17, quantifying the effect which could already be seen qualitatively in Fig. 16. Again constraints on the sfermion sector can be derived due to their loop contributions. In contrast to the perfect agreement of the best fit point with the input parameter values in ILC1, the best fit point for ILC2 visibly overestimates the sfermion mass parameters. However the true values still remain within the 1$\sigma$ interval. Figure 17(b) displays the precisions obtained on the pMSSM-10 parameters. In case of ILC2, the gluino is most likely outside the reach of LHC, therefore $M_3$ is only constrained via its loop effects on the higgsino sector.

| parameter | ILC1 pMSSM true | pMSSM-4 best fit point | 1$\sigma$ CL | 2$\sigma$ CL | pMSSM-10 best fit point | 1$\sigma$ CL | 2$\sigma$ CL |
|-----------|----------------|-----------------------|-------------|-------------|-----------------------|-------------|-------------|
| $M_1$     | 250            | 250.2                 | +8.2        | +17.1       | 251.3                 | +8.6        | +17.2       |
| $M_2$     | 463            | 463.3                 | +8.0        | +16.2       | 465.8                 | +24.2       | +31.4       |
| $\mu$     | 115.0          | 115.0                 | -8.1        | -14.9       | 115.7                 | -4.7        | -6.2        |
| tan$\beta$| 10.0           | 10.0                  | +0.1        | +0.2        | 9.7                   | +8.8        | +45.3       |
| $m_A$     | 1000           | 1000                  |            |             | 1050                  | +310        | +607        |
| $M_3$     | 1270           | 1270                  |            |             | 1412                  | +1791       | +1411       |
| $M_L$     | 7150           | 7150                  |            |             | 7603                  | -1004       | -2843       |
| $M_{U(3)}$| 1670           | 1751                  |            |             | 1751                  | +2411       | +4998       |
| $M_{Q(3)}$| 4820           | 4951                  |            |             | 4951                  | +2324       | +3858       |
| $A_{t=\mu=\tau}$| -4400 | -4591 | +1371 | +1647 |
| $\chi^2$ |               | 0.0011                |             |             | 0.1360                |             |             |

Table 15: Fitted parameters in ILC1 pMSSM-4 and pMSSM-10. All units in GeV except for tan$\beta$ and $\chi^2$.

Table 16: True and fitted masses as well as their uncertainties from a pMSSM-10 fit to ILC1 observables. All values in GeV.

### 5.2.2 ILC2 Benchmark

In the case of the ILC2 benchmark, the overall situation is similar to the case of ILC1. The minimum $\chi^2$ as a function of $M_1$, $M_2$ and $\mu$ and tan$\beta$ is displayed in Fig. 16 for the pMSSM-10 and pMSSM-4. However, this time the I20 running scenario was assumed, c.f. Sec. 4.3. Also here, the resulting curve for the 4-parameter fit is much smoother than for the 10-parameter version due to the much smaller parameter space to be sampled. Like for ILC1, the precision on $M_2$ improves somewhat in the pMSSM-4 fit, while $\mu$ and tan$\beta$ are significantly better constrained.

The resulting best fit values for the pMSSM parameters and their 1 and 2$\sigma$ intervals are compared to the input values in Tab. 17, quantifying the effect which could already be seen qualitatively in Fig. 16. Again constraints on the sfermion sector can be derived due to their loop contributions. In contrast to the perfect agreement of the best fit point with the input parameter values in ILC1, the best fit point for ILC2 visibly overestimates the sfermion mass parameters. However the true values still remain within the 1$\sigma$ interval. Figure 17(b) displays the precisions obtained on the pMSSM-10 parameters. In case of ILC2, the gluino is most likely outside the reach of LHC, therefore $M_3$ is only constrained via its loop effects on the higgsino sector.
Figure 13: Predicted SUSY parameter ranges from the pMSSM-10 fit to ILC1. The magenta star indicates the true model values, while the black dot shows the best fit point.

Figure 14: Predicted mass ranges from the pMSSM-10 fit to ILC1. The green star indicates the true model values, while the black dot shows the best fit point.

As in the ILC1 case, the determined parameters can be used to predict the masses of the as-yet unobserved particles, as shown for the pMSSM-10 fit in Tab. 18 and Fig. 17(a). Finally, Fig. 18(a) shows the result of the 4 parameter fit of $M_1, M_2, \mu, \tan \beta$. Again, the remaining parameters fixed to model values, based on the assumption that the best fit point of the 10-parameter fit is sufficiently close to the true point that the effect of fixing to the true values is negligible.
5 Fitting fundamental parameters

Figure 15: Predicted mass and SUSY parameter ranges from the pMSSM-4 fit to ILC1. The green/magenta star indicates the true model values, while the black dot shows the best fit point.

| parameter | ILC2 pMSSM true | pMSSM-4 best fit point | 1σ CL | 2σ CL | pMSSM-10 best fit point | 1σ CL | 2σ CL |
|-----------|-----------------|------------------------|-------|-------|------------------------|-------|-------|
| $M_1$     | 520.3           | 520.7                  | +38.6 | +79.1 | 502.1                  | +91.3 | +130.1|
| $M_2$     | 957.2           | 959.42                 | +55.4 | +124.1| 941.0                  | +145.4| +229.2|
| $\mu$     | 150.0           | 150.0                  | +0.4  | +0.7  | 154.4                  | +24.7 | +36.6 |
| $\tan\beta$ | 15.0           | 15.0                   | +0.7  | +1.7  | 14.8                   | +38.4 | +48.2 |
| $m_A$     | 1000            |                        |       |       | 1043                   | +135  | +240 |
| $M_3$     | 2607            |                        |       |       | 2684                   | +9900 | +5670 |
| $M_L$     | 5146            |                        |       |       | 5797                   | +2402 | +3511 |
| $M_{U(3)}$ | 1395            |                        |       |       | 2073                   | +3518 | +4716 |
| $M_{Q(3)}$ | 3757            |                        |       |       | 4871                   | +3680 | +5030 |
| $A_t$     | -4714           |                        |       |       | -5948                  | -3933 | -4608 |
| $\chi^2$ |                | 0.0026                 |       |       | 0.1627                 |       |       |

Table 17: Fitted parameters in ILC2 pMSSM-4 and pMSSM-10. All units in GeV except for $\tan\beta$ and $\chi^2$.

5.2.3 nGMM1 Benchmark

Finally, Fig. 19 shows the minimum $\chi^2$ as a function of $M_1$, $M_2$ and $\mu$ and $\tan\beta$ in the pMSSM-10 and pMSSM-4 fits to the nGMM1 observables. Also here, the much smaller parameter space to be sampled in case of the 4-dimensional fit leads to much smoother curves than in the 10-dimensional case. Again, the determinations of $\mu$ and $\tan\beta$ improve significantly. However $M_1$ and $M_2$ exchange their roles compared to the other benchmarks, so that now $M_1$ is somewhat better constrained in the pMSSM-4 fit, while the precision on $M_2$ is nearly identical in the two fits. However, it should be noted that $M_1$ and $M_2$ are less well constrained than in the cases of the ILC1 and ILC2 benchmarks. This results from a combination of the worse experimental resolutions and the larger absolute values of $M_1$ and $M_2$ in case of nGMM1. In this most challenging case, the mass splitting between $\tilde{\chi}_1^0$ and $\tilde{\chi}_1^\pm$ is only 2.5 GeV, which corresponds to less than 2σ of the experimental resolution. As discussed in Sec. 3.3, the mass differences $m_{\tilde{t}^0} - m_{\tilde{t}^0}$ and $m_{\tilde{t}^\pm} - m_{\tilde{t}^0}$ are directly accessible experimentally as the endpoint of the di-lepton or di-jet invariant mass spectrum. Therefore, we consider in this case as alternative input these mass differences...
in addition to the $\tilde{\chi}_1^0$ mass, which also presents a set of observables with minimal correlations. The corresponding precisions are summarized in Tab. 19.

As can be seen in Fig. 20, the determination of $M_2$ in the 10-parameter fit improves significantly when instead of the absolute masses the mass differences are used as fit input, especially the upper bound. There is no significant effect on $M_1$, $\mu$ or $\tan\beta$.

The resulting best fit values for the pMSSM parameters and their 1 and 2\(\sigma\) intervals are compared to the input values in Tab. 20, quantifying the effect which could already been seen qualitatively in Fig's. 19 and 20. As before, constraints on the sfermion sector can be derived due to their loop contributions. In contrast to the perfect agreement of the best fit point with the input parameter values in ILC1, the best fit point for nGMM1 visibly overestimates the sfermion mass parameters. However the true values still remain within the 1\(\sigma\) interval. Using the mass differences as input instead of the absolute higgsino masses notably improves the precision on $M_2$, as expected from the $\chi^2$ distribution, but it also significantly improves the agreement of the best fit point with the true model parameters. These improvements can also be seen in Fig. 21(b) and 21(d), in particular the better agreement in $m_A$ and $A_t$.

Again, the determined parameters can be used to predict the masses of the yet unobserved sparticles, as shown for the pMSSM-10 fit in Tab. 21 and in Fig. 21(a) and 21(c) with standard input and when using the mass differences
Table 18: True and fitted masses as well as their uncertainties from a pMSSM-10 fit to ILC2 observables. All values in GeV.

| observable | pMSSM-10 fit to ILC2 | nGMM1 model value [GeV] | precision I20 precision |
|------------|----------------------|--------------------------|-------------------------|
| $m_{\tilde{\chi}_1^0}$ | 534.6 | 154.9 | 1.7% | 1.0% |
| $m_{\tilde{\chi}_2^0}$ | 1026 | 5.3 | 2.1% | 1.4% |
| $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ | 1000 | 2.4 | 2.5% | 1.2% |

Table 19: Experimental precision on the higgsino mass differences in nGMM1 combined from 500 GeV 500 fb for both $\mathcal{P}(\pm0.8, \pm0.3)$, as well as scaled to 1600 fb$^{-1}$ for both polarisations at $\sqrt{s} = 500$ GeV, ignoring the data sets with other centre-of-mass energies in the I20 operating scenario. Again, it is assumed that the precisions obtained from the simulations based on the Isajet spectrum can be transferred to the SPheno3.3.9beta spectrum.
Figure 18: Predicted mass and SUSY parameter ranges from the pMSSM-4 fit to ILC2. The green/magenta star indicates the true model values, while the black dot shows the best fit point.

Table 20: Fitted parameters in nGMM1: pMSSM-4, pMSSM-10 and pMSSM-10 with mass differences as input. All units in GeV except for $\tan \beta$ and $\chi^2$.

Instead, as expected, the improved precision on $M_2$ when using the mass differences as input leads to improved predictions of the $\tilde{\chi}^0_3$ and $\tilde{\chi}^0_2$ masses. In addition, the agreement between the best fit predictions for the heavy Higgs boson masses as well as for all the sfermion masses with their true value improves significantly due to the better agreement in $m_A$ and $\Lambda_t$.

Finally, Fig. 22(a) shows the result of the 4 parameter fit of $M_1$, $M_2$, $\mu$, $\tan \beta$. Again, the remaining parameters are fixed to their model values, based on the assumption that the best fit point of the 10-parameter fit is sufficiently close to the true point that the effect of fixing to the true values is negligible. The masses of the heavier electroweakinos are predicted within an 1 $\sigma$ uncertainty of about 150 GeV. This fit has only been run with the standard input, further improvements could be expected when using the mass differences as input also in this fit.

5.3 Dark Matter in Higgsino Fits

An additional benefit from our fits to MSSM parameters is that it is possible to extract various WIMP dark matter related observables\[^{[70]}\]. These include 1. the thermally-produced WIMP relic density $\Omega_{\chi}^{TP} h^2$, 2. the spin-
5 Fitting fundamental parameters

![Diagram](image)

Figure 19: nGMM1: Minimum $\chi^2$ as a function of $M_1$, $M_2$ and $\mu$ and $\tan\beta$ in pMSSM-4 fit (dashed black line) and pMSSM-10 fit (solid grey line). For each bin, the minimum $\chi^2$ of all Markov chain points which have the x axis quantity in that bin is plotted.

![Diagram](image)

Figure 20: nGMM1 $\chi^2$ parabola for $M_2$ in the 10-parameter fit with mass differences (black, dashed) and the same fit with masses as observables (grey, solid).
Table 21: nGMM1: True and fitted masses and their uncertainties from pMSSM-10 fits with the standard set of observables as well as with the higgsino mass differences replacing the $\tilde{\chi}^0_2$ and $\tilde{\chi}^0_1$ masses as observables. All values in GeV.

| $m_{\chi^0_2}$ | $m_{\chi^0_1}$ | $m_{\chi^+_{10}}$ | $m_{\chi^0_{10}}$ | $m_{\chi^0_{12}}$ | $m_{\chi^0_{13}}$ | $m_{\chi^0_{14}}$ | $m_{\chi^0_{15}}$ | $m_{\chi^0_{16}}$ | $m_{\chi^0_{17}}$ | $m_{\chi^0_{18}}$ | $m_{\chi^0_{19}}$ | $m_{\chi^0_{20}}$ | $m_{\chi^0_{21}}$ | $m_{\chi^0_{22}}$ | $m_{\chi^0_{23}}$ | $m_{\chi^0_{24}}$ | $m_{\chi^0_{25}}$ | $m_{\chi^0_{26}}$ | $m_{\chi^0_{27}}$ | $m_{\chi^0_{28}}$ | $m_{\chi^0_{29}}$ | $m_{\chi^0_{30}}$ |
|-------------|-------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1522        | 1412        | 1603            | 1602            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            | 1801            |
| 2652        | 2649        | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            | 2649            |
| 2655        | 2655        | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            | 2655            |
| 2663        | 2663        | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            | 2663            |
| 2675        | 2675        | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            | 2675            |
| 2681        | 2681        | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            | 2681            |
| 3029        | 3029        | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            | 3029            |
| 3120        | 3120        | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            | 3120            |
| 3168        | 3168        | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            | 3168            |
| 3264        | 3264        | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            | 3264            |
| 3361        | 3361        | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            | 3361            |

5 Fitting fundamental parameters

The fitted and scaled relic density is plotted and the 2σ confidence interval has been extracted. The centre of the 2σ confidence level is calculated and used as the mean. The width of the 2σ range is divided by two to obtain the 1σ width assuming the $\Delta \chi^2$ distribution is parabolic. The distributions are more flat than parabolic so this procedure gives a conservative estimate of the 1σ width. The relic density distribution from each fit is plotted, assuming a gaussian distribution, in Fig. 23. In case of the pMSSM-10 fit without any further inputs, the relic density is not sufficiently constrained. However this has been traced to be due to fit solutions with extremely low gluino masses of less than 200 GeV. Excluding these points, the blue dashed curves are obtained, which show a very good determination of the relic density agreeing quite well with the theoretical value. The precision improves even further when the pMSSM4 fit is run after the pMSSM10 fit. Such a measurement of the relic density would clearly confirm a possible underabundance of higgsino-like WIMPs.

We also fit the expected values of $\sigma^{SI}(\chi p)$ and $\langle \sigma v \rangle$ which are listed in Tab. 22 (these theory values are somewhat higher than the results). The $\sigma^{SI}$ values can be fit to an accuracy typically better than 1% with the $\langle \sigma v \rangle$ values typically fit to $\approx 10\%$ or worse. By comparing the direct detection rates from WIMP detection experiments to the ILC fitted values for a measured higgsino mass $m_{\chi_1^0}$, a direct measurement of WIMP relic density can be made since the WIMP direct detection rates are actually sensitive to $\xi \sigma^{SI}$ where $\xi$ is the ratio of actual WIMP abundance divided by the total measured abundance $\Omega_{\chi^0_1}h^2$. Such interplay between

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Figure 21: Predicted mass and SUSY parameter ranges from the pMSSM-10 fit to nGMM1 observables - including either $\tilde{\chi}_0^2$ and $\tilde{\chi}_1^+1$ masses or their mass differences with the LSP. The green/magenta star indicates the true model values, while the black dot shows the best fit point.

ILC results and direct detection results offer direct confirmation that WIMPs would comprise only a portion of dark matter. In addition, indirect WIMP detection rates are proportional to $\xi^2$ since they search for WIMP-WIMP annihilation. The interplay of ILC results with indirect WIMP detection rates could offer further confirmation for multicomponent dark matter.

6 Testing gaugino mass unification

The pMSSM parameters which were extracted in Sec. 5.2 were fitted at the energy scale $Q = 1$ TeV. The scale dependence of the parameters is governed by their renormalization group equations or RGEs. Using the MSSM RGEs, the fitted parameters can be evolved to higher energy scales in order to check hypotheses regarding uni-
Testing gaugino mass unification

Specifically, we will test unification of the various gaugino masses which are assumed to unify at $Q = m_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV (the scale at which gauge couplings unify) in models like NUHM2 and NUHM3 but which would unify at a lower scale in models such as nGMM1. Since in this work we do not subscribe to any particular GUT or string theory, GUT scale threshold corrections to gauge and Yukawa couplings and soft SUSY breaking terms are not imposed.

This section continues the program initiated by Blair et al. of extracting tests of high scale unification from weak scale measurements of SUSY particle properties at ILC\cite{76, 77}. Since the estimates of the achievable precision for the experimental observables used in Sec. 5 are somewhat more pessimistic than the results obtained in Sec. 4, we also discuss the expected impact of the experimental improvements taking the nGMM1 benchmark as an example.

Figure 22: Predicted mass and SUSY parameter ranges from the pMSSM-4 fit to nGMM1. The green/magenta star indicates the true model values, while the black dot shows the best fit point.

Figure 23: Fitted relic densities in ILC1, ILC2 and nGMM1 fits.
Testing gaugino mass unification

| observable | ILC1 | ILC2 | nGMM1 |
|------------|------|------|-------|
| $\sigma^{SI}$ model [$10^{-9}$ pb] | 259.3 | 316.9 | 328.5 |
| $\sigma^{SI}$ best fit [$10^{-9}$ pb] | $260.7^{+4.1}_{-6.9}$ | $317.0^{+2.1}_{-2.1}$ | $328.5^{+1.5}_{-0.9}$ |
| $\langle \sigma v \rangle$ model [$10^{-27}$ cm$^3$s$^{-1}$] | 15.36 | 3.439 | 0.597 |
| $\langle \sigma v \rangle$ best fit [$10^{-27}$ cm$^3$s$^{-1}$] | $15.01^{+1.53}_{-0.88}$ | $3.501^{+5.74}_{-0.523}$ | $0.621^{+0.994}_{-0.165}$ |

Table 22: Relic density from MicrOMEGAs and AstroFit, and direct and indirect detection cross sections from AstroFit in the pMSSM-10 fits to ILC1, ILC2 and nGMM1 observables (without the gluino mass measurement). $\Omega_{Planck}$ is taken to be 0.1199 [75].

6.1 Method

The running pMSSM-10 weak scale parameters and error bars are extracted using Fittino and SPheno3.3.9beta at $Q = 1$ TeV. Then a random scan of $10^4$ samples of the 10 parameters is performed, approximating the parameter PDFs as either Gaussian or flat within $\pm 1\sigma$, depending on the shape of their $\chi^2$ distribution, see e.g. Fig. 19. For each of the sampled points, SPheno was used to calculate the running parameters at each of 21 energy scales between 91 and $10^{19}$ GeV. The mean and standard distribution of these parameters’ distributions at each energy scale were used to define confidence bands, as shown in e.g. Fig. 22(b). The unification scale $Q_{unif}$ is determined by fitting linear functions the running parameters in a range close to the visible intersection and extracting the intersection point. With each value for $Q_{unif}$, a corresponding estimate of $M_{1/2}$ is determined. Gaussian functions can be fitted to the distributions of the resulting values for $Q_{unif}$ and $M_{1/2}$ in order to obtain central values and uncertainties.

For the gluino mass, several scenarios are considered: the determination from loop contributions to the higgsino observables only, a direct observation at the LHC resulting in a precision of 10% on the physical gluino mass, or simply by assuming gaugino mass unification. In the latter case, the extracted mean $M_{1/2}$ and $Q_{unif}$ values can be used to determine the value of $M_3(Q = 1$ TeV) and consequently the physical gluino mass. In this case, predictions for the expected value of $m_{\tilde{g}}$ may be made which can serve as a target for future hadron collider searches or compared to the mass of an already-discovered gluino.

6.2 Running gaugino masses for ILC1

The weak scale ILC1 parameters are sampled according to Gaussian distributions for $M_1$ and $M_2$ and uniformly within the $1\sigma$ range for $M_3$, motivated by the shape of the $\chi^2$ distributions obtained in the pMSSM-10 fits discussed in Sec. 5.2.1. The resulting running of the gaugino mass parameters in the ILC1 pMSSM-10 fit is plotted in Fig. 24(a). From the plot, it can be seen that $M_1$ and $M_2$ cross near $10^{16}$ GeV which would verify the prediction of a SUSY GUT model. The uncertainty band for $M_3$ is quite wide but is consistent with the hypothesis of unification of all three gaugino masses at the same energy scale. The extracted unification scale $Q_{unif}$ for $M_1$ and $M_2$ is plotted in Fig. 25(a) from which it can be seen that the distribution follows a Gaussian. The gaugino mass unification scale is found to be $Q_{unif} = 3.8 \times 10^{16}$ GeV with a 68% confidence range of $[3.0 \times 10^{15}, 4.9 \times 10^{17}]$ GeV. From Fig. 25(b), the unified gaugino mass parameter is found to be $M_{1/2} = 583 \pm 40$ GeV in agreement with the GUT scale model model fit.

If it is then assumed that the unification is due to an NUHM2 model, and true model parameter values are assumed for parameters other than $M_{1/2}$, then instead $M_3$ can be extracted by running down in energy to find the running value of $M_3(Q = 1$ TeV). From Fig. 24(b) we obtain $M_3(Q = 1$ TeV) = $1216 \pm 76$ GeV (which agrees with the the weak scale fitted value). Consequently, a prediction for the physical gluino mass can be obtained: $m_{\tilde{g}} = 1467 \pm 80$ GeV which could then be checked against results from hadron collider searches.
6 Testing gaugino mass unification

Figure 24: The running gaugino masses $M_i$ based on the pMSSM-10 fit to ILC1 observables. The bands correspond to one standard deviation. (a) Using $M_3$ at the weak scale as constrained from ILC measurements (b) $M_3$ is assumed to unify with $M_1$ and $M_2$ and then run to the weak scale to gain a prediction for $M_3(Q = 1$ TeV).

Figure 25: Distribution of the unification scale $Q_{\text{unif}}$ and unified gaugino mass $M_{1/2}$ obtained from the running parameters $M_1$ and $M_2$ and their uncertainties from the pMSSM-10 fit to ILC1 observables.

6.3 Running gaugino masses for ILC2

The uncertainties of the weak scale gaugino mass fit parameters are larger in the case of ILC2 as compared to ILC1. Still, the weak scale ILC2 parameters are sampled according to Gaussian distributions for $M_1$ and $M_2$ and uniformly within the 1σ range for $M_3$, motivated by the shape of the $\chi^2$ distributions obtained in the pMSSM-10 fits discussed in Sec. 5.2.2. The larger uncertainties are reflected in the running gaugino mass plots in Fig. 26(a) and 26(b). Nevertheless, it is still possible to verify that $M_1$ and $M_2$ unify near the GUT scale. For ILC2, the fitted weak scale error band for $M_3$ is so wide that it is consistent with unification with $M_1$ or $M_2$ at almost any
scale.

![Graph](image1.png)

**Figure 26:** The running gaugino masses \( M_i \) based on the pMSSM-10 fit to ILC2 observables. The bands correspond to one standard deviation. (a) Using \( M_3 \) at the weak scale as constrained from ILC measurements (b) \( M_3 \) is assumed to unify with \( M_1 \) and \( M_2 \) and then run to the weak scale to gain a prediction for \( M_3(Q = 1 \text{ TeV}) \).

![Graph](image2.png)

**Figure 27:** Distribution of the unification scale \( Q_{unif} \) and unified gaugino mass \( M_{1/2} \) obtained from the running parameters \( M_1 \) and \( M_2 \) and their uncertainties from the pMSSM-10 fit to ILC2 observables.

Using the same methodology as for ILC1, the unification scale for ILC2 where \( M_1 = M_2 \) is found to be Gaussian with a mean of \( Q_{unif} = 1.5 \times 10^{16} \text{ GeV} \) with a 68% confidence interval of \([5.4 \times 10^{13}, 4.4 \times 10^{18}] \text{ GeV}\) as shown in Fig. 27(a). The unified value of \( M_{1/2} \) is found in Fig. 27(b) to be Gaussian with \( M_{1/2} = 1222 \pm 173 \text{ GeV} \) which corresponds to the GUT scale fit model value. If \( M_3 \) is instead assumed to unify with \( M_1 \) and \( M_2 \) at \( Q_{unif} \) and the NUHM2 model is adopted, then the extrapolated value of \( M_3 \) at 1 TeV is found to be \( M_3(Q = 1 \text{ TeV}) = 2616 \pm 582 \text{ GeV} \) while the physical gluino mass is found to be \( m_{\tilde{g}} = 2872 \pm 605 \text{ GeV} \). Such a large value may serve as a target for gluino pair searches at upgraded hadron colliders.
6.4 Running gaugino masses for nGMM1

The running of the gaugino mass parameters in the nGMM1 benchmark model differs from the running in the ILC1 and ILC2 models. There are two reasons: 1. the underlying model is now a mirage unification model where the gaugino mass parameters unify at an intermediate energy scale and 2. the determination of $M_1$ and $M_2$ from the weak scale fits is much less accurate in nGMM1 as compared to the ILC1 and ILC2 benchmark models.

Figure 28(a) shows the running gaugino masses resultant from the pMSSM-10 fit with absolute masses as input as described in Sec. 5.2.3. Even in this most conservative case, the plot is certainly inconsistent with any sort of GUT scale unification of gaugino masses. From a closer look we notice that the hierarchy between $M_1$ and $M_2$ at $Q = 1$ TeV is not well defined, and that actually the $M_1$ band seems to start above the lower rim of the $M_2$ band. This effect occurs since, motivated by the shape of the $\chi^2$ landscape of the pMSSM-10 fit (c.f. Fig. 19), $M_1$ and $M_3$ are sampled from a uniform distribution and only $M_2$ is treated with a Gaussian. In addition the 1σ interval for $M_1$ is very asymmetric around the best fit point (c.f. Tab. 20). In combination with the flat sampling, the 1σ band for $M_1$ seems to start much higher than the best fit value for $M_1$ would indicate.

A substantial improvement of the precision can be seen in Fig. 28(b), which shows the analogous result obtained when using the improved experimental precisions presented in Sec. 4.3 plus a 10% measurement of the gluino mass from HL-LHC (or other future hadron collider). In addition to the improved inputs, the parameter extraction has also been refined: the estimates of $M_1$, $M_2$ and $M_3$ at the weak scale are obtained from a fit of only the pMSSM-4 parameters and $M_3$ as a fit parameter, is run, as discussed at the beginning of Sec. 5.2. The Markov chain had a length of $10^5$ points.

This is not even the most optimistic case, since further improvements can be expected from using the higgsino mass differences instead as input (c.f. Fig. 21(d)) and from more precise $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^0$ masses extracted from scanning the thresholds of $\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp$, respectively. In addition, the consideration of further constraints from
improved EWPOs, flavour physics, direct search limits etc. is expected to further improve the weak scale parameter determination.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure29.png}
\caption{Distribution of the unification scale $Q_{\text{unif}}$ and unified gaugino mass $M_{1/2}$ obtained from the running parameters $M_1$ and $M_2$ and their uncertainties from the pMSSM-10 fit to nGMM1 observables.}
\end{figure}

For the conservative version of the running masses in Fig. 28(a), we quantify the constraints on the unification scale in Fig. 29a. While the distribution of obtained $Q_{\text{unif}}$ values has no clear peak, it increases towards lower unification scales, away from the GUT scale. A unification at $\sim 10^{16}$ GeV is excluded with 99.9% probability. The most probable unified value of $M_{1/2}$ was found to be $1600 \pm 450$ GeV in Fig. 29b. The drastic improvement in Fig. 28(b) compared to Fig. 28, illustrates the substantial impact which can be expected from further refinements of the underlying analysis.

Due to the extracted gaugino mass unification scale not matching with the GUT scale, there would be important implications for SUSY model building. It is noteworthy that the pMSSM fit and the fit parameter evolution indicate that the underlying model does not have gaugino mass unification, even though the fit of NUHM2 parameters to the nGMM1 observables presented in Sec. 5.1 does not entirely rule NUHM2 out as a possible model.

## 7 Summary and conclusions

Supersymmetry with radiatively-driven naturalness is especially compelling in that it reconciles electroweak naturalness with (multi-TeV) LHC sparticle mass limits and Higgs boson mass measurements. The most fundamental consequence of radiatively-driven natural SUSY is the prediction of four light higgsinos $\chi_{1}^{\pm}, \chi_{0}^{0}, \chi_{2}^{0,2}$ with mass $\sim 100 - 300$ GeV (the lower the better). Such light higgsinos are difficult (but perhaps not impossible) to see at LHC, but would be easily visible at ILC operating with $\sqrt{s} > 2m(\text{higgsino})$. In this case, the ILC, initially constructed as a Higgs factory, would turn out to be a higgsino factory! Thus, for this highly motivated scenario, ILC could serve as both a SUSY discovery (or confirmation) machine, and a precision microscope.

In this paper we have examined the capability of experiments at the ILC to both discover (or confirm) supersymmetry and to make precision measurements of superparticle properties that would probe the superpotential higgsino mass parameter $\mu$ via direct sparticle mass measurements and in addition provide a measurement of SUSY-breaking gaugino mass parameters via the higgsino mass splittings.

When these measurements are combined with precision Higgs boson measurements, precision fits to both weak scale SUSY and high scale SUSY model parameters can be made. We have investigated the capability of ILC to discover light higgsinos in three natural SUSY benchmark models: two with unified gaugino masses and one with
mire unification of gaugino masses at an intermediate mass scale between $m_{\text{GUT}}$ and $m_{\text{weak}}$. Our calculations implement a detailed ILD detector simulation along with event generation from Whizard.

By measuring $e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow (\ell \nu, \tilde{\chi}_0^0) + (q \bar{q}, \tilde{\chi}_0^0)$ we are able to extract $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_0^0}$ via the $m(jj)$ and $E(jj)$ distributions, typically to percent level accuracy. By measuring the dilepton mass and energy distributions from $e^+ e^- \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_1^- \rightarrow \ell^- \tilde{\chi}_1^0$, we are able to measure $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0}$ to typically percent level accuracy. We combine the higgsino mass measurements with precision higgsino pair production cross section measurements using different beam polarizations.

When these precision higgsino measurements are combined with precision Higgs boson measurements, precision fits to both weak scale SUSY and high scale SUSY model parameters can be made. In particular, an indirect measurement of wino and bino SUSY breaking masses can be extracted from the higgsino mass splittings. When extrapolated to high energies, the hypothesis of gaugino mass unification can be tested. If combined with LHC gluino mass measurements, the unification of all three gaugino masses may be explored. Such measurements will shed light on different possibilities for SUSY breaking as may be expected in SUSY GUT models or in models with mixed moduli- and anomaly- (mirage) mediation. In addition, fits of SUSY dark matter observables may shed light on the nature of dark matter, such as confirming or ruling out multi-component dark matter as expected from natural SUSY where both higgsino-like WIMPs and axions are expected to be produced in the early universe.

Thus, in assessing the ILC capabilities in this compelling SUSY extension of the SM, we conclude that ILC can indeed serve as a SUSY discovery machine and precision microscope, offering a window into the intricacies of SUSY breaking and fundamental particle physics and providing insights into the nature of dark matter and cosmology.

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A Additional Figures and Tables

Table 23: Expected number of events for chargino signal and major backgrounds for the electron final state and beam polarization $P_{\mu \gamma}$. The integrated luminosity is assumed to be 500 fb$^{-1}$. For each benchmark model, the background refers to the other SUSY backgrounds.
Table 24: Expected number of events for chargino signal and major backgrounds for the muon final state and beam polarization $\mathcal{P}_{-+}$. The integrated luminosity is assumed to be 500 fb$^{-1}$. For each benchmark model, the background refers to the other SUSY backgrounds.

| $\chi^+ \rightarrow \chi^0 q f \bar{p} v_\nu$ | ILC1 | ILC2 | nGMM1 | SM bkg. |
|------------------------------------------|------|------|--------|---------|
| Preselection | 13276 | 16354 | 10382 | 97279 |
| Lepton selection | 1251 | 5394 | 660 | 2546 |
| BeamCal veto | 1238 | 5320 | 653 | 2518 |
| $p_T > 5$ GeV | 834 | 787 | 298 | 310 |
| $n_{\text{had}} > 2$ | 615 | 116 | 161 | 30 |
| $|\cos \theta| < 1.0$ | 400 | 113 | 157 | 29 |
| $E_{\text{vis}} < 500$ | 423 | 47 | 100 | 69 |
| $|\cos \theta| < 0.2$ | 421 | 25 | 99 | 51 |
| $E_{\text{miss}} > 400$ | 409 | 24 | 98 | 51 |
| $|\cos \theta_{\text{miss}}| < 0.99$ | 404 | 23 | 96 | 46 |

Table 25: Expected number of events for chargino signal and major backgrounds for the electron final state and beam polarization $\mathcal{P}_{-+}$. The integrated luminosity is assumed to be 500 fb$^{-1}$. For each benchmark model, the background refers to the other SUSY backgrounds.

| $\chi^+ \rightarrow \chi^0 q f e \nu_e$ | ILC1 | ILC2 | nGMM1 | SM bkg. |
|------------------------------------------|------|------|--------|---------|
| Preselection | 13222 | 16354 | 10352 | 97279 |
| Lepton selection | 1502 | 16551 | 869 | 7667 |
| BeamCal veto | 1487 | 16318 | 860 | 7574 |
| $p_T > 5$ GeV | 993 | 1108 | 388 | 521 |
| $n_{\text{had}} > 2$ | 733 | 276 | 199 | 69 |
| $|\cos \theta| < 0.98$ | 712 | 270 | 194 | 67 |
| $E_{\text{vis}} < 500$ | 555 | 134 | 134 | 20 |
| $|\cos \theta| < 1.0$ | 550 | 37 | 132 | 91 |
| $E_{\text{miss}} > 400$ | 536 | 34 | 131 | 87 |
| $|\cos \theta_{\text{miss}}| < 0.99$ | 524 | 34 | 127 | 85 |

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Table 26: Expected number of events for chargino signal and major backgrounds for the muon final state and beam polarization $\mathcal{P}_{-+}$. The integrated luminosity is assumed to be 500 fb$^{-1}$. For each benchmark model, the background refers to the other SUSY backgrounds.

| $\chi^+ \rightarrow \chi^0 q f \mu \bar{p} \nu_\mu$ | ILC1 | ILC2 | nGMM1 | SM bkg. |
|------------------------------------------|------|------|--------|---------|
| Preselection | 13222 | 16354 | 10352 | 97279 |
| Lepton selection | 1502 | 16551 | 869 | 7667 |
| BeamCal veto | 1487 | 16318 | 860 | 7574 |
| $p_T > 5$ GeV | 993 | 1108 | 388 | 521 |
| $n_{\text{had}} > 2$ | 733 | 276 | 199 | 69 |
| $|\cos \theta| < 0.98$ | 712 | 270 | 194 | 67 |
| $E_{\text{vis}} < 500$ | 555 | 134 | 134 | 20 |
| $|\cos \theta| < 1.0$ | 550 | 37 | 132 | 91 |
| $E_{\text{miss}} > 400$ | 536 | 34 | 131 | 87 |
| $|\cos \theta_{\text{miss}}| < 0.99$ | 524 | 34 | 127 | 85 |
Figure 30: Reconstructed di-jet mass in the chargino channel $e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 q\bar{q}^* \nu\bar{\nu}$ for 500 fb$^{-1}$ at $\sqrt{s} = 500$ GeV. In all cases, the background contributions are very small. The signal histograms are stacked on top of the backgrounds.

Figure 31: Reconstructed di-jet energy in the chargino channel $e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 q\bar{q}^* \nu\bar{\nu}$ for 500 fb$^{-1}$ at $\sqrt{s} = 500$ GeV. In all cases, the background contributions are very small. The signal histograms are stacked on top of the backgrounds.

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Table 27: Expected number of events for neutralino signal and major backgrounds for the electron final state and beam polarizations $P(e^-, e^+) = (-0.8, +0.3)$. The integrated luminosity is assumed to be 500 fb$^{-1}$. For each benchmark model, the benchmark refers to the other SUSY backgrounds.

| $P(e^-, e^+)$ | ILC1 | ILC2 | nGMM1 | SM bkg |
|---------------|------|------|-------|--------|
|                | sig. | bkg. | sig.  | bkg.   |
| Preselection   | 0.3  |      | 0.8   |        |
| Lepton selection, $N_{\ell\ell} = 2$ | 4028 | 4039 | 2866  | 3576   |
| BeamCal veto   | 3965 | 3977 | 2831  | 3531   |
| $p_T > 2.3$ GeV | 3822 | 3638 | 2504  | 3106   |
| $|\cos \theta| < 0.95$ | 3601 | 3443 | 2455  | 2997   |
| $|\Delta \theta| < 0.8$ | 2384 | 327  | 1696  | 371    |
| $E_{\text{vis}} < 25$ GeV | 1621 | 189 | 1253  | 235    |
| $E_{\text{miss}} > 300$ GeV | 1621 | 189 | 1253  | 235    |
| $|\cos \theta_{\ell\ell}| < 0.98$ | 1621 | 189 | 1253  | 235    |
| $M_{\ell\ell}$ selection | 1621 | 185 | 1250  | 226    |

Table 28: Expected number of events for neutralino signal and major backgrounds for the muon final state and beam polarizations $P(e^-, e^+) = (-0.8, +0.3)$. The integrated luminosity is assumed to be 500 fb$^{-1}$. For each benchmark model, the benchmark refers to the other SUSY backgrounds.

| $P(e^-, e^+)$ | ILC1 | ILC2 | nGMM1 | SM bkg |
|---------------|------|------|-------|--------|
|                | sig. | bkg. | sig.  | bkg.   |
| Preselection   | 0.3  |      | 0.8   |        |
| Lepton selection, $N_{\ell\ell} = 2$ | 4028 | 4039 | 2866  | 3576   |
| BeamCal veto   | 3965 | 3977 | 2831  | 3531   |
| $p_T > 2.3$ GeV | 3822 | 3638 | 2504  | 3106   |
| $|\cos \theta| < 0.95$ | 3601 | 3443 | 2455  | 2997   |
| $|\Delta \theta| < 0.8$ | 2384 | 327  | 1696  | 371    |
| $E_{\text{vis}} < 25$ GeV | 1621 | 189 | 1253  | 235    |
| $E_{\text{miss}} > 300$ GeV | 1621 | 189 | 1253  | 235    |
| $|\cos \theta_{\ell\ell}| < 0.98$ | 1621 | 189 | 1253  | 235    |
| $M_{\ell\ell}$ selection | 1621 | 185 | 1250  | 226    |

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Table 29: Expected number of events for neutralino signal and major backgrounds for the electron final state and beam polarizations $P(e^-, e^+) = (0.8, -0.3)$. The integrated luminosity is assumed to be 500 fb$^{-1}$. For each benchmark model, the background refers to the other SUSY backgrounds.

| $X_1^0 X_2^0 (e^-, e^+)$ | ILC1 | ILC2 | nGMM1 | SM bkg. |
|-------------------------|------|------|-------|--------|
| $P(e^-, e^+) = (0.8, -0.3)$ | | | | |
| Preselection | 3466 | 6769 | 2495 | 5364 |
| Lepton selection, $N_{\text{trk}} = 2$ | 3214 | 1042 | 2308 | 1024 |
| BeamCal veto | 3160 | 1024 | 2275 | 1014 |
| $p_T > 2.3$ GeV | 3047 | 938 | 2021 | 889 |
| $|\cos \theta_T| < 0.95$ | 2872 | 882 | 1985 | 852 |
| $\Delta \phi < 0.8$ | 1904 | 121 | 1382 | 169 |
| $E_{\text{vis}} < 25$ GeV | 1284 | 69 | 1020 | 113 |
| $E_{\text{miss}} > 300$ GeV | 1284 | 69 | 1020 | 113 |
| $|\cos \theta_{\text{miss}}| < 0.98$ | 1284 | 69 | 1020 | 113 |
| $M_{\ell_2}$ selection | 1284 | 69 | 1020 | 113 |

Table 30: Expected number of events for neutralino signal and major backgrounds for the muon final state and beam polarizations $P(e^-, e^+) = (0.8, -0.3)$. The integrated luminosity is assumed to be 500 fb$^{-1}$. For each benchmark model, the background refers to the other SUSY backgrounds.

| $X_1^0 X_2^0 (\mu^-, \mu^+)$ | ILC1 | ILC2 | nGMM1 | SM bkg. |
|-------------------------|------|------|-------|--------|
| $P(e^-, e^+) = (0.8, -0.3)$ | | | | |
| Preselection | 3586 | 6399 | 2972 | 4887 |
| Lepton selection, $N_{\text{trk}} = 2$ | 3573 | 950 | 2762 | 973 |
| BeamCal veto | 3513 | 932 | 2727 | 960 |
| $p_T > 2.3$ GeV | 3421 | 869 | 2451 | 862 |
| $|\cos \theta_T| < 0.95$ | 3209 | 824 | 2402 | 836 |
| $\Delta \phi < 0.8$ | 2099 | 73 | 1633 | 93 |
| $E_{\text{vis}} < 25$ GeV | 1522 | 50 | 1223 | 70 |
| $E_{\text{miss}} > 300$ GeV | 1522 | 50 | 1223 | 70 |
| $|\cos \theta_{\text{miss}}| < 0.98$ | 1522 | 50 | 1223 | 70 |
| $M_{\ell_1} > 0$ selection | 1521 | 49 | 1222 | 67 |

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