ON ELECTROWEAK MOMENTS OF BARYONS AND SPIN–FLAVOUR STRUCTURE OF THE NUCLEON

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The phenomenological sum–rule–based approach is used to discuss the quark composition dependence of some static and quasi–static electroweak characteristics of nucleons. The role of nonvalence degrees of freedom, the nucleon sea partons and/or peripheral meson currents, is shown to be important to select and make use of the relevant symmetry parametrization of hadron observables. With our preferable universal value of the SU(3)-symmetry parameter \( \alpha_D = D/F + D = .58 \), taken for both magnetic moments and axial-vector constants entering into the semi-leptonic baryon decays, we obtain the following values for moments \( \Delta q \) of the spin-dependent structure function of the proton: \( \Delta u \simeq .84(.82), \Delta d \simeq -.42(-.44), \Delta s = -.22 \pm .05(-.10 \pm .03) \), where the values in parentheses correspond to the widely used "standard" value of \( \alpha_D^{\text{axial}} = .63 \). The estimations of the strange sea contributions to the nucleon magnetic moments and rms are also presented.

1. The magnetic moments of the lowest baryon octet, being one of the most accurately measured spin–dependent quantities in hadron physics, may serve as a useful means to verify the spin–flavour symmetry predictions and different model calculations of the nucleon structure characteristics. This report aims at discussing, on the basis of confrontation of data on the magnetic moments and axial–vector coupling of baryons, some alternative, to the usually discussed, inference for the \( \Delta q \)'s of the polarized DIS and the (hidden) strangeness–dependent characteristics of the nucleon. In (3, 4) (and the references to earlier works therein), the following parametrization was introduced for magnetic moments \( \mu(B) \) of baryons:

\[
\mu(B) = \mu(q_e)g_2 + \mu(q_o)g_1 + C(B) + \Delta, \quad (1)
\]

\[
\mu(\Lambda) = \mu(s)(\frac{2}{3}g_2 - \frac{1}{3}g_1) + (\mu(u) + \mu(d))(\frac{1}{6}g_2 + \frac{2}{3}g_1) + \Delta. \quad (2)
\]

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\[
\mu(\Lambda\Sigma) = \frac{1}{\sqrt{3}}(\mu(u) - \mu(d))(\frac{1}{2}g_2 - g_1) + C(\Lambda\Sigma),
\]
\[
\Delta = \sum_{q=u,d,s} \mu(q)\delta(N),
\]
where \(\mu(q)\) are the effective quark magnetic moments defined without any nonrelativistic approximations, \(g_{2(1)}\) are "reduced" dimensionless coupling constants obeying exact SU(3)–symmetry and related with the SU(3) \(F\)– and \(D\)– type constants via \(g_2 = 2F\) and \(g_1 = F - D\), \(\Delta(B)\) is a matrix element of the OZI–suppressed \(\bar{q}q\)–configuration for a given hadron: \(\Delta(B) = < B|\bar{q}q|B >\), where \(q \not\subset \{q_s^2, q_o\}\), e.g. \(\delta(N) = < N|\bar{s}s|N >\), etc. The \(C(P) = -C(N)\) and \(C(\Lambda\Sigma)\) are representing the isovector contributions of the charged–pion exchange current to \(\mu(P), \mu(N)\) and the \(\Lambda\Sigma\)–transition moment \(\mu(\Lambda\Sigma)\). In Refs. [3, 4] the use was made of two pictures of the baryon internal composition. In the first one, all baryons are considered as consisting of three massive, ”dressed” constituent quarks, locally coupled with lightest goldstonions – the pseudoscalar octet fields. In the second picture only fundamental QCD quanta, the quarks and gluons, are there, the meson component of the baryon state vectors being represented by the properly correlated ”current” quarks and gluons. The use of one picture or another will be reflected in a particular parametrization of contributions due to corresponding nonvalence degrees of freedom. Here, we concentrate on two of the earlier discussed [3, 4] sum rules (we use the particle and quark symbols for corresponding magnetic moments):
\[
\alpha = \frac{D}{F + D} = \frac{g_2 - 2g_1}{2(g_2 - g_1)} = \frac{1}{2} \left(1 - \frac{\Xi^0 - \Xi^-}{\Sigma^+ - \Sigma^- + \Xi^0 + \Xi^-}\right),
\]
\[
\frac{u}{d} = \frac{\Sigma^+(\Sigma^+ - \Sigma^-) - \Xi^0(\Xi^0 - \Xi^-)}{\Sigma^-(\Sigma^+ - \Sigma^-) - \Xi^-(\Xi^0 - \Xi^-)} = \frac{P + N + \Sigma^+ - \Xi^0 - \Xi^-}{P + N - \Sigma^+ + \Sigma^- - \Xi^0 + \Xi^-},
\]
Eqs.(6,7) were obtained provided \(\delta(N) = 0\), hence they are related to the chiral constituent quark model where a given baryon consists of three ”dressed”, massive constituent quarks. Eqs.(6,7) also show that owing to the virtual transitions \(q \leftrightarrow \pi(\eta) + q, q \leftrightarrow K + s\) the ”magnetic anomaly” is developing, i.e. \(u/d = -1.80 \pm 0.02 \neq Q_u/Q_d = -2\). Evaluation of the one–loop, quark–meson diagrams gives: \(u/d = (Q_u + \kappa_u)/(Q_d + \kappa_d) \simeq -1.85\), the \(\kappa_q\) being the quark anomalous magnetic moment in natural units, if we take the \(SU(3)\) - invariant quark-pseudoscalar- meson couplings, the physical masses for the \(\pi, \eta, K\)–mesons and the \(m_{q(s)} \simeq 300(460)\) MeV. In the \(SU(3)\)–limit, when \(m_q = m_s\) and \(m_\pi = m_q = m_K\), we return to \(u/d = -2\), the ratio pertinent to the structureless ”current” quarks.
Normalizing the "strange charge" coupling constant for the $s$–quark ($K$–meson) to $-\frac{1}{3}(+\frac{1}{3})$, i.e. the values coinciding with the electric charge of the $s$– and $\overline{s}$–quarks, we obtain, in the one–loop approximation, the contribution of strange quarks to the anomalous magnetic moments (a.m.m.) of the $u$– and $d$– constituent quarks, which also coincides approximately with the corresponding contribution to nucleon magnetic moments

$$\kappa_{u(d)}(s, \overline{s}) \simeq \mu_{P(N)}(s, \overline{s}) = 0.065 n.m. \tag{8}$$

Recalling the negative electric charge of the strange quark, we conclude that the spin of strange quark appearing inside the polarized nonstrange quark is antiparallel to that of the "parent" constituent $u$– or $d$–quark. The average polarization of the $\overline{s}$–quark, forming the $K$–meson, is zero.

With the normalization of the mentioned coupling constants to the values $+1(-1)$, we obtain the (hidden) strange quark contributions to the Dirac "strange charge" radius and the a.m.m. of nonstrange quarks the values:

$$\kappa_{u(d)}(s, \overline{s}) \simeq -0.195 n.m.$$

$$< r_{1}^{2} >_{u(d)} (s, \overline{s}) \simeq \frac{1}{3} < r_{1}^{2} >_{P(N)} (s, \overline{s}) \simeq 0.013 fm^{2} \tag{9}$$

We stress, however, that the virtual $K$–mesons were treated as point–like particles. The intrinsic strange antiquark distribution in the virtual $K$– mesons may not be negligible, e.g. for the on–shell $K$–mesons, with the adopted above–mentioned normalization, one can obtain the estimate:

$$< r_{K}^{2} > (\overline{s}) = - < r_{ch}^{2} >_{K^+(K^0)} -2 < r_{ch}^{2} >_{K}= -0.3 fm^{2}$$

where the one–loop calculation for $< r_{ch}^{2} >_{K^+(K^0)}$ were taken from [3]. Therefore even the sign of the whole value of $< r_{K}^{2} >_{P(N)} (s, \overline{s})$ may be reversed after a proper inclusion of the $< r_{K}^{2} > (\overline{s})$ as a part of a still missing two–loop calculation.

With the neglect of the nonvalence contribution, i.e. with $C(B's) = \Delta(B's) = 0, u/d = -2$ we obtain for magnetic moments of baryons the results coinciding almost identically with the results of the $SU(6)$–based nonrelativistic quark model, taking account of the $SU(3)$– breaking due to the quark–mass differences [4]. We stress, however, that no NR assumption or explicit $SU(6)$-wave function are used this time.

The ratio $\alpha_D$ equals 0.61 in this case (cf. Eq.(5), giving the value 0.57 ), thus demonstrating a substantial influence of the nonvalence (i.e. the meson) degrees of freedom on this important parameter.
We turn now to a complementary view of the nucleon structure, absorbing $C(N's)$ into products of the corresponding $\mu(q)$ and $g(N's)$, keeping the constraint $u/d = -2$, and $\Delta(B's)$ non-zero. We shall refer to this approach [3, 4] as a correlated current quark picture of nucleons. In this case we have

$$\Delta(N) = \sum_{q=u,d,s} \mu(q)\delta(N) = \frac{1}{6}(3(P + N) - \Sigma^+ + \Sigma^- - \Xi^0 + \Xi^-) = -0.062 \pm 0.01 \text{n.m.},$$

(10)

$$\mu_N(\bar{s}s) = \mu(s)\langle N|\bar{s}s|N \rangle = (1 - \frac{d}{s})^{-1}\Delta = 0.11 \text{n.m.},$$

(11)

$$(\alpha_D)_N = \frac{3(N - \Delta(N))}{2(N - P)} = 0.59,$$

(12)

where independence of the sum $P + N$ of the $C(N's)$ and the ratio $d/s = 1.55$ [3, 4] have been used.

By definition, $\mu_N(\bar{s}, s)$ represents the contribution of strange (“current”) quarks to nucleon magnetic moments. Numerically, Eq.(11) agrees fairly well with other more specific models (see, e.g. [8]) but exceeds the value given by Eq.(8) by factor of about 2, indicating on other possible mechanisms of the transition $q \leftrightarrow q + s + \bar{s}$. The different values $(\alpha_D)_N = 0.59$ and $(\alpha_D)_Y = 0.57$, Eq.(5), give the hint of the difference of the nucleon and hyperon wave functions that would lead to their mismatch and corresponding influence on the $Y \rightarrow Nl\nu$-transitions. Even bigger difference of $\alpha_D^{mag} = 0.59$ or 0.57 and the average value $\alpha_D^{axial} = 0.635$ also lead to difficulty, when interpreted [1] via the admixture of the higher representation of the $SU(6) \otimes O(3)$-basis functions to the ground state. In that case we would have an unacceptably large admixture, with the probability of about 0.2, of the D-wave in the nucleon wave function (see the next section).

In this respect we wish here to recall a less popular inference from the semi-leptonic hyperon decays and its impact on a description of the polarized DIS.

As is known [10], to obtain the contributions of the $u-, d-$ and $s$-flavoured quarks to the proton spin, denoted by $\Delta u(p), \Delta d(p)$ and $\Delta s(p)$, the use is usually made of baryon semileptonic weak decays treated with the help of the exact $SU(3)$-symmetry. It has been shown earlier [11, 12] that when both the strangeness-changing ($\Delta S = 1$) and strangeness-conserving ($\Delta S = 0$) transitions are taken for the analysis, then $(D/F + D)_{ax}^{\Delta S=0.1} = 0.635 \pm 0.005$ while $(D/F + D)_{ax}^{\Delta S=0} = 0.584 \pm 0.035$, which is close to the mean value $(D/D + F)_{mag} \simeq 0.58$, according to Eqs.(5,11). We list below two sets of the $\Delta q$-values, we have obtained from the data with inclusion of the QCD radiative corrections (e.g. [10] and references therein) : $\Delta u(p) \simeq 0.82(0.84)$, $\Delta d(p) \simeq -0.44(-0.42)$, $\Delta s = -0.10 \pm 0.03(-0.22 \pm 0.05)$,
where the values in the parentheses correspond to $\alpha_D = (D/D + F) = .58$. At the same time, the problem of difference of the following two expressions

$$F - D = (g_a/g_v)^{\exp}(\Sigma^- \to N) = -.34 \pm .02, \quad (13)$$

$$F - D = (g_a/g_v)^{\exp}(N \to P) - \sqrt{6}g_a^{\exp}(\Sigma \to \Lambda) = -.19 \pm .04, \quad (14)$$
of which we prefer the second one when we postulate $\alpha_{ax}^D = \alpha_{mag}^D$, remains open. The intriguing possibility can, however, be mentioned that the numerical value of the $(g_a/g_v)^{\exp}(\Sigma^- \to N)$, coinciding with Eq.(14), was in fact found in [13], if the ”weak–electricity” form factor, $g_{w.el} \neq 0$, referred as one of the second class current effects, is included in the joint analysis of all experimental data.

2. In this section, we consider the possible difference between $\alpha_m$, Eq. (5), and the $\alpha_{ax}'s$, given in [11, 12], as originating from the higher $SU(6) \times O(3)$ three–quark configurations and/or the exotic $(3q + g)$–admixture in the ground state wave function:

$$\Psi = A_0\Psi_0(\{56\}_S, L_q = 0, S_q = 1/2) + A_1\Psi_1(\{70\}_M, L_q = 0, S_q = 1/2)$$

$$+ A_2\Psi_2(\{70\}_M, L_q = 2, S_q = 3/2) + A_3\Psi_3(\{20\}_A, L_q = 1, S_q = 1/2)$$

$$+ A_g\Psi_g(\{3q\}_8c + g_{8c}). \quad (15)$$

The coefficients $A_i$ and $A_g$ satisfy the normalization condition

$$\sum_{i=0}^{3} A_i^2 + A_g^2 = 1. \quad (16)$$

In Eq. (13), $L_q(S_q)$ is the quark orbital (spin) moment, and the index ”8c” stands for the color–octet states. To specify different cases, we keep for the gluon angular momentum two simplest possibilities $J^P_g = 1^{\pm}$ which are the $M1$– or $E1$– gluon modes. Different components of the total wave function are built of the antisymmetrized products of the flavor ($\Phi$), spin ($\chi$) color ($\omega$) and orbital/radial ($\Psi$) wave functions:

$$\Psi = \Phi \times \chi \times \omega \times \Psi(\vec{\rho}, \vec{\lambda}), \quad (17)$$

$\vec{\rho}$ and $\vec{\lambda}$ being the Jacobi coordinates of the 3–quark system. Of many considered possibilities for $\Psi_g$ we present two examples, one for the $M1$– and one for the $E1$– gluon mode (the $M1$-case has been considered also in [13], but with all higher orbital configurations neglected, $A_i = 0, i = 1, 2, 3$)

$$\Psi_g^{M1} = \frac{1}{2}[(\Phi^\rho \omega^\rho - \Phi^\lambda \omega^\lambda) \chi^\lambda + (\Phi^\rho \omega^\rho + \Phi^\lambda \omega^\lambda) \chi^\rho] \Psi_{sym}, \quad (18)$$

$$\Psi_g^{E1} = \frac{1}{2}[(\Phi^\rho \chi^\rho + \Phi^\lambda \chi^\lambda) (\omega^\rho \Psi^\lambda - \omega^\lambda \Psi^\rho)], \quad (19)$$
where $\Psi_{\text{sym}}(L_\rho = L_\lambda = 0) = \Psi_0(\rho^2 + \lambda^2), \Psi^{(\lambda)}(L_\rho(\lambda) = 1, L_\lambda(\rho) = 0) = \vec{\rho}(\vec{\lambda}) \cdot \Psi_1(\rho^2 + \lambda^2), \Psi_{0,1}(\rho^2 + \lambda^2)$ are unspecified radial wave functions, $\Phi^{(\lambda)}, \omega^{(\lambda)}$, etc are familiar, octet–type wave functions (see, e.g., Ref. [14] and earlier citations therein). Then we find expectation values of the magnetic moment ($\hat{\mu}$) and axial charge ($\hat{A}$) operators

$$\hat{\mu} = \sum_q [g_\sigma(q)\sigma_3(q) + g_l(q)\hat{l}_3(q)], \quad (20)$$

$$\hat{A} = \sum_q g_{ax}(q)\sigma_3(q)\hat{\tau}_3(q), \quad (21)$$

and define $g_i^m$ in Eq. (11) and the analogous $g_i^{ax}$ as function of $A_0, ..., A_g, g_\sigma, g_l$ and $g_{ax}$. Then we take the ratios $\alpha_m$ and $\alpha_{ax}$; from the latter the unknown (due to various renormalization effects) $g_{ax}$ will be cancelled out. First, we indicate what physics’ factors will cause deviation of $F/D_{ax}$-ratio from $2/3$ (the $SU(6)$-value) in either of two options

$$\frac{F}{D}_{ax} = \begin{cases} 
0.58 & \text{if } \Delta S = 0, 1; \quad g_{w, el} = 0, \\
0.72 & \text{if } \Delta S = 0; \quad g_{w, el} \neq 0,
\end{cases} \quad (22)$$

when in addition to $A_0$ only one of $A_i, i = 1, 2$ or $A_g(M1orE1)$ is taken into account (we put also $A_3 = 0$ in the following). Solving a system of two equations, the first one being the definition of $F/D_{ax}$ in terms of $A_0$ and $A_i$ or, alternatively, in terms of $A_0$ and $A_g$, while the second one is the normalization condition, we obtain the values collected in Table 1.

| $\frac{F}{D}$ | $A_0^2$ | $A_1^2$ | $A_2^2$ | $A_g^2(M1)$ | $A_g^2(E1)$ |
|----------------|--------|--------|--------|-------------|-------------|
| 0.58           | 0.81   | –      | 0.19   | –           | –           |
| 0.58           | 0.35   | –      | –      | 0.65        | –           |
| 0.72           | 0.86   | 0.14   | –      | –           | –           |
| 0.72           | 0.51   | –      | –      | 0.49        | –           |
| $0.72|_{mag}$    | 0.72   | 0.114  | 0.005  | –           | 0.16        |
| 0.687|_{QM}     | 0.938  | 0.059  | 0.002   | –           | –           |

Too large values of either $A_i$ or $A_g$ on the first four lines look rather difficult to accept. The entries on the 5th line correspond to solution of the enlarged system of equations[8] including $F/D_{mag}$, which is expressed in terms of $A_i, i = 0, 1, 2$ and $A_g$. Although appearing to be more attractive, they deviate from typical values of nonrelativistic quark
model (QM) represented on the 6th line. The QM-results are obtained via diagonalization of the hamiltonian containing spin-dependent potentials induced by the one-gluon exchange \[16\]. Thus, the increase of the spin-tensor (spin-spin) potential is responsible for the larger value of \(A_2\) (\(A_1\)) and, respectively, for the decrease (or increase) of \(F/D\)-ratio compared to \(2/3\). Concerning the magnitude of \(A_2^g\), we note that \(A_0^2|_{QM}\) is the sum of two terms

\[
0.938 = A_0^2(56_S) + A_0^2(56_{S'}) \simeq 0.85 + 0.09, \quad (23)
\]

Therefore, if the Roper resonance \(N^*(1440)\) and other members of lowest \(J^P = \frac{1}{2}^+\) - multiplet, traditionally considered to be the radially excited \(56_{S'}\) - multiplet, would largely to be hybrid states, we should exclude their contributions to \(A_2^0\) in favour of either of \(A_2^g\). In that case the mentioned discrepancy is diminished.

3. We conclude with the following remarks:

1) The deviation of the ratio \(F/D = 0.72\), Eq.(12), from the \(SU(6)\) –value 2/3 shows, that despite the validity of the celebrated \(SU(6)\) –ratio \(\mu(P)/\mu(N) = -3/2\), the \(SU(6)\) –symmetry is strongly broken. The importance of taking into account the nonvalence degrees of freedom in relevant parametrization of the observables within the (broken) internal symmetries is demonstrated.

2) The new more accurate angular correlation measurements in different \(Y \rightarrow Nl\nu\) –decays could give, as exemplified in \[13\], very important information on second–class currents and new values of the \(g_A\).

3) The current and forthcoming measurements of the flavour-separated \(\Delta q\)'s are of indispensable value to discriminate between different approaches to the spin phenomena at high energies.

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