Space–time isogeometric analysis of car and tire aerodynamics with road contact and tire deformation and rotation

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Abstract
We present a space–time (ST) isogeometric analysis framework for car and tire aerodynamics with road contact and tire deformation and rotation. The geometries of the computational models for the car body and tires are close to the actual geometries. The computational challenges include (1) the complexities of these geometries, (2) the tire rotation, (3) maintaining accurate representation of the boundary layers near the tire while being able to deal with the flow-domain topology change created by the road contact, (4) the turbulent nature of the flow, (5) the aerodynamic interaction between the car body and the tires, and (6) NURBS mesh generation for the complex geometries. The computational framework is made of the ST Variational Multiscale (ST-VMS) method, ST Slip Interface (ST-SI) and ST Topology Change (ST-TC) methods, ST Isogeometric Analysis (ST-IGA), integrated combinations of these ST methods, NURBS Surface-to-Volume Guided Mesh Generation (NSVGMG) method, and the element-based mesh relaxation (EBMR). The ST context provides higher-order accuracy in general, the VMS feature of the ST-VMS addresses the challenge created by the turbulent nature of the flow, and the moving-mesh feature of the ST context enables high-resolution flow computation near the moving fluid–solid interfaces. The ST-SI enables moving-mesh computation with the tire rotating. The mesh covering the tire rotates with it, and the SI between the rotating mesh and the rest of the mesh accurately connects the two sides of the solution. The ST-TC enables moving-mesh computation even with the TC created by the contact between the tire and the road. It deals with the contact while maintaining high-resolution flow representation near the tire. Integration of the ST-SI and ST-TC enables high-resolution representation even though parts of the SI are coinciding with the tire and road surfaces. It also enables dealing with the tire–road contact location change and contact sliding. By integrating the ST-IGA with the ST-SI and ST-TC, in addition to having a more accurate representation of the tire geometry and increased accuracy in the flow solution, the element density in the tire grooves and in the narrow spaces near the contact areas is kept at a reasonable level. The NSVGMG enables NURBS mesh generation for the complex car and tire geometries, and the EBMR improves the quality of the meshes. The car and tire aerodynamics computation we present shows the effectiveness of the analysis framework we have built.

Keywords Car and tire aerodynamics · Road contact · ST Variational Multiscale (ST-VMS) method · ST Slip Interface (ST-SI) method · ST Topology Change (ST-TC) method · ST Isogeometric Analysis (ST-IGA) · NURBS Surface-to-Volume Guided Mesh Generation (NSVGMG) method

1 Introduction
Computational challenges encountered in car and tire aerodynamics with road contact and tire deformation and rotation are rather formidable, especially when the computational-model geometries for the car body and tires are close to the actual geometries. The challenges include (1) the complexities of these geometries, (2) the tire rotation, (3) maintaining...
accurate representation of the boundary layers near the tire while being able to deal with the flow-domain topology change created by the road contact, (4) the turbulent nature of the flow, (5) the aerodynamic interaction between the car body and the tires, and (6) NURBS mesh generation for the complex geometries if we want to do the computations with isogeometric discretization. We are presenting a space–time (ST) isogeometric analysis framework that will address these challenges.

The computational framework is made of the ST Variational Multiscale (ST-VMS) method [1–3], which subsumes its precursor “ST-SUPS” (see Sect. 1.1). ST Slip Interface (ST-SI) method [4,5], ST Topology Change (ST-TC) method [6,7], ST Isogeometric Analysis (ST-IGA) [1,8,9], integrated combinations of these ST methods, such as the “ST-SI-TC-IGA” [10–12], NURBS Surface-to-Volume Guided Mesh Generation (NSVGMG) method, which we are introducing here, and the element-based mesh relaxation (EBMR) [13].

The ST context provides higher-order accuracy in general, the VMS feature of the ST-VMS addresses the challenge created by the turbulent nature of the flow, and the moving-mesh feature of the ST context enables high-resolution flow computation near the moving fluid–solid interfaces. The ST-SI enables moving-mesh computation with the tire rotating. The mesh covering the tire rotates with it, and the SI between the rotating mesh and the rest of the mesh accurately connects the two sides of the solution. The ST-TC enables moving-mesh computation even with the TC created by the contact between the tire and the road. It deals with the contact while maintaining high-resolution flow representation near the tire. Integration of the ST-SI and ST-TC enables high-resolution representation even though parts of the SI are coinciding with the tire and road surfaces. It also enables dealing with the tire–road contact location change and contact sliding. By integrating the ST-IGA with the ST-SI and ST-TC, in addition to having a more accurate representation of the tire geometry and increased accuracy in the flow solution, the element density in the tire grooves and in the narrow spaces near the contact areas is kept at a reasonable level. The NSVGMG enables NURBS mesh generation for the complex car and tire geometries, and the EBMR improves the quality of the meshes.

1.1 ST-VMS and ST-SUPS

This subsection, included for completeness, is mostly from [14]. The Deforming-Spatial-Domain/Stabilized ST (DSD/SST) method [15–17] was introduced for computation of flows with moving boundaries and interfaces (MBI), including fluid–structure interaction (FSI). In MBI computations the DSD/SST functions as a moving-mesh method. Moving the fluid mechanics mesh to follow an interface enables mesh-resolution control near the interface and, consequently, high-resolution boundary-layer representation near fluid–solid interfaces. Because the stabilization components of the original DSD/SST are the Streamline-Upwind/Petrov–Galerkin (SUPG) [18] and Pressure-Stabilizing/Petrov–Galerkin (PSPG) [15] stabilizations, it is now called “ST-SUPS.” The ST-VMS is the VMS version of the DSD/SST. The VMS components of the ST-VMS are from the residual-based VMS (RBVMS) method [19–22]. The ST-VMS has two more stabilization terms beyond those in the ST-SUPS, and the additional terms give the method better turbulence modeling features. The ST-SUPS and ST-VMS, because of the higher-order accuracy of the ST framework (see [1,2]), are desirable also in computations without MBI.

As a moving-mesh method, the DSD/SST is an alternative to the Arbitrary Lagrangian–Eulerian (ALE) method, which is older (see, for example, [23]) and more commonly used. The ALE-VMS method [24–27] is the VMS version of the ALE. It succeeded the ST-SUPS and ALE-SUPS [28] and preceded the ST-VMS. To increase their scope and accuracy, the ALE-VMS and RBVMS are often supplemented with special methods, such as those for weakly-enforced Dirichlet boundary conditions [29,30], “sliding interfaces” [31,32], and backflow stabilization [33]. The ALE-SUPS, RBVMS and ALE-VMS have been applied to many classes of FSI, MBI and fluid mechanics problems. The classes of problems include ram-air parachute FSI [28], wind turbine aerodynamics and FSI [34–40], more specifically, vertical-axis wind turbines (VAWTs) [37,41], floating wind turbines [42], wind turbines in atmospheric boundary layers [37,43–45], and fatigue damage in wind turbine blades [46], patient-specific cardiovascular fluid mechanics and FSI [47–49], biomedical-device FSI [50–54], ship hydrodynamics with free-surface flow and fluid–object interaction [55], hydrodynamics and FSI of a hydraulic arresting gear [56], hydrodynamics of tidal-stream turbines with free-surface flow [57], passive-morphing FSI in turbomachinery [58], bioinspired FSI for marine propulsion [59], bridge aerodynamics and fluid–object interaction [60], stratified incompressible flows [61,62], compressible flows with emphasis on gas turbine modeling [63,64] and hypersonic flows [65], cavitating flows [66,67], additive manufacturing [68], particle-laden gravity currents [69], flows with phase change [70–72], interfacial flows [73], free-surface flows in marine engineering [74], shallow-water flows [75], tidal turbines [76], immersogeometric FSI and flow analysis [77–79], and mixed ALE-VMS/Immersogeometric computations [51,52,80–86] in the framework of the Fluid–Solid Interface-Tracking/Interface-Capturing Technique [87].

The ST-SUPS and ST-VMS have also been applied to many classes of FSI, MBI and fluid mechanics problems (see [88] for a comprehensive summary of the work prior to July 2018). The classes of problems include spacecraft parachute analysis for the landing-stage parachutes
[13, 26], cover-separation parachutes [89] and the drogue parachutes [90], wind turbine aerodynamics for horizontal-axis wind turbine (HAWT) rotors [26], full HAWTs [91] and VAWTs [4, 37–39, 92], HAWT long-wake flow analysis [93–95], flapping-wing aerodynamics for an actual locust [8, 26, 96], bioinspired MAVs [97] and wing-clapping [6, 98], blood flow analysis of cerebral aneurysms [99], stent-blocked aneurysms [100, 101], aortas [53, 54, 102–105], heart valves [6, 7, 10, 11, 53, 54, 103, 106, 107] and ventricle-aorta sequences [105, 108], spacecraft aerodynamics [109], thermo-fluid analysis of ground vehicles and their tires [3, 44, 45, 106], thermo-fluid analysis of disk brakes [5], flow-driven string dynamics in turbomachinery [38, 39, 110, 111], flow analysis of turbocharger turbines [9, 112, 113], flow around tires with road contact and deformation [12, 14, 106, 114, 115], fluid films [14, 116], aerodynamic analysis of ram-air parachutes [44, 45, 117], compressible-flow spacecraft parachute aerodynamics [118, 119], U-duct turbulent flow [120], and Taylor–Couette flow [121, 122].

1.2 Mesh moving methods

This subsection, included for completeness, is mostly from [93]. In flow computations with FSI or MBI, the ST-SUPS, ALE-SUPS, ALE-VMS, and ST-VMS require mesh update as the boundaries or interfaces move. Mesh update has two components: moving the mesh for as long as it is possible, which is the core component, and full or partial remeshing when the element distortion becomes too high. The key objectives of a mesh moving method should be to maintain the element quality near solid surfaces and to minimize remeshing frequency. A number of well-performing mesh moving methods were developed in conjunction with the ST-SUPS and ST-VMS. The first one, introduced in [121, 123], was the Jacobian-based stiffening, which is now called, for reasons explained in [105], “mesh-Jacobian-based stiffening.” The most recent ones are the EBMR (see Sect. 1.8), where the mesh motion is determined by using the large-deformation mechanics equations and an element-based zero-stress state (ZSS), a mesh moving method [124] based on fiber-reinforced hyperelasticity and optimized ZSS, and a linear-elasticity-based mesh moving method with no cycle-to-cycle accumulated distortion [105, 125].

1.3 ST-SI

This subsection, included for completeness, is mostly from [14]. The ST-SI was introduced in [4], in the context of incompressible-flow equations, to retain the desirable moving-mesh features of the ST-VMS and ST-SUPS in computations involving spinning solid surfaces, such as a turbine rotor. The mesh covering the spinning surface spins with it, retaining the high-resolution representation of the boundary layers, while the mesh on the other side of the SI remains unaffected. This is accomplished by adding to the ST-VMS formulation interface terms similar to those in the version of the ALE-VMS for computations with sliding interfaces [31, 32]. The interface terms account for the compatibility conditions for the velocity and stress at the SI, accurately connecting the two sides of the solution. An ST-SI version where the SI is between fluid and solid domains was also presented in [4]. The SI in that case is a “fluid–solid SI” rather than a standard “fluid–fluid SI” and enables weak enforcement of the Dirichlet boundary conditions for the fluid. The ST-SI introduced in [5] for the coupled incompressible-flow and thermal-transport equations retains the high-resolution representation of the thermo-fluid boundary layers near spinning solid surfaces. These ST-SI methods have been applied to aerodynamic analysis of ram-air parachutes with fabric and geometric porosities. That enabled weak enforcement of the Dirichlet conditions for the fluid on its two sides. Compressible-flow porosity models were also introduced in [118], including the version where the SI is between a thin porous structure and the fluid on its two sides. Compressible-flow porosity models were also introduced in [118]. These, together with the compressible-flow ST SUPG method [131], extended the ST computational analysis range to compressible-flow aerodynamics of parachutes with fabric and geometric porosities. That enabled ST computational flow analysis of the Orion spacecraft drogue parachute in the compressible-flow regime [118, 119].

1.4 ST-TC

This subsection, included for completeness, is mostly from [14]. The ST-TC [6, 7] was introduced for moving-mesh computation of flow problems with TC, such as contact between solid surfaces. Even before the ST-TC, the ST-SUPS and ST-VMS, when used with robust mesh update methods, have proven effective in flow computations where the solid surfaces are in near contact or create other near TC, if the nearness is sufficiently near for the purpose of solving the
problem. Many classes of problems can be solved that way with sufficient accuracy. For examples of such computations, see the references mentioned in [6]. The ST-TC made moving-mesh computations possible even when there is an actual contact between solid surfaces or other TC. By collapsing elements as needed, without changing the connectivity of the “parent” mesh, the ST-TC can handle an actual TC while maintaining high-resolution boundary layer representation near solid surfaces. This enabled successful moving-mesh computation of heart valve flows [6,7,10,11,53,54,103,106,107,132], ventricle-valve-aorta flows [105], wing clapping [6,98], flow around a rotating tire with road contact and prescribed deformation [12,14,106,114,130], and fluid films [14,116].

1.5 ST-SI-TC

This subsection, included for completeness, is mostly from [14]. The ST-SI-TC is the integration of the ST-SI and ST-TC. A fluid–fluid SI requires elements on both sides of the SI. When part of an SI needs to coincide with a solid surface, which happens for example when the solid surfaces on two sides of an SI come into contact or when an SI reaches a solid surface, the elements between the coinciding SI part and the solid surface need to collapse with the ST-TC mechanism. The collapse switches the SI from fluid–fluid SI to fluid–solid SI. With that, an SI can be a mixture of fluid–fluid and fluid–solid SIs. With the ST-SI-TC, the elements collapse and are reborn independent of the nodes representing a solid surface. The ST-SI-TC enables high-resolution flow representation even when parts of the SI are coinciding with a solid surface. It also enables dealing with contact location change and contact sliding. This was applied to heart valve flow analysis [10,11,53,54,103,107], ventricle-valve-aorta flows [105], tire aerodynamics with road contact and deformation [12,14,106,114,130], and fluid films [14,116].

1.6 ST-IGA

This subsection, included for completeness, is mostly from [14]. The success with IGA basis functions in space [24,31,47,133] motivated the integration of the ST methods with isogeometric discretization, which is broadly called “ST-IGA.” The ST-IGA was introduced in [1]. Computations with the ST-VMS and ST-IGA were first reported in [1] in a 2D context, with IGA basis functions in space for flow past an airfoil, and in both space and time for the advection equation. Using higher-order basis functions in time enables deriving full benefit from using higher-order basis functions in space. This was demonstrated with the stability and accuracy analysis given in [1] for the advection equation.

The ST-IGA with IGA basis functions in time enables a more accurate representation of the motion of the solid surfaces and a mesh motion consistent with that. This was pointed out in [1,2] and demonstrated in [8,96]. It also enables more efficient temporal representation of the motion and deformation of the volume meshes, and more efficient remeshing. These motivated the development of the ST/NURBS Mesh Update Method (STNMUM) [8,96], with the name coined in [91]. The STNMUM has a wide scope that includes spinning solid surfaces. With the spinning motion represented by quadratic NURBS in time, and with sufficient number of temporal patches for a full rotation, the circular paths are represented exactly. A “secondary mapping” [1,2,8,26] enables also specifying a constant angular velocity for invariant speeds along the circular paths. The ST framework and NURBS in time also enable, with the “ST-C” method [134], extracting a continuous representation from the computed data and, in large-scale computations, efficient data compression [3,5,106,110,111,134]. The STNMUM and the ST-IGA with IGA basis functions in time have been used in many 3D computations. The classes of problems solved are flapping-wing aerodynamics for an actual locust [8,26,96], bioinspired MAVs [97] and wing-clapping [6,98], separation aerodynamics of spacecraft [89], aerodynamics of horizontal-axis [26,91] and vertical-axis [4,37–39,92] wind turbines, thermo-fluid analysis of ground vehicles and their tires [3,44,45,106], thermo-fluid analysis of disk brakes [5], flow-driven string dynamics in turbomachinery [38,39,110,111], and flow analysis of turbocharger turbines [9,112,113].

The ST-IGA with IGA basis functions in space enables more accurate representation of the geometry and increased accuracy in the flow solution. It accomplishes that with fewer control points, and consequently with larger effective element sizes. That in turn enables using larger time-step sizes while keeping the Courant number at a desirable level for good accuracy. It has been used in ST computational flow analysis of turbocharger turbines [9,112,113], flow-driven string dynamics in turbomachinery [38,39,110,111], ram-air parachutes [44,45,117], spacecraft parachutes [119], aortas [53,54,103,104], heart valves [10,11,53,54,103,107], ventricle-valve-aorta sequences [105,108], tires with road contact and deformation [12,14,114,115], fluid films [14,116], VAWTs [4,37–39,92], HAWT long-wake flow analysis [93–95], U-duct turbulent flow [120], and Taylor–Couette flow [122]. The image-based arterial geometries used in patient-specific arterial FSI computations do not come from the ZSS of the artery. Using IGA basis functions in space is now a key part of some of the newest ZSS estimation methods [53,135–137] and related shell analysis [138]. The IGA has also been successfully applied to structural analysis and design [139–148], including the structural analysis of wind turbine blades and heart valves.
1.7 ST-SI-IGA and ST-SI-TC-IGA

This subsection, included for completeness, is mostly from [14]. The ST-SI-IGA is the integration of the ST-SI and ST-IGA, and the ST-SI-TC-IGA is the integration of the ST-SI, ST-TC, and ST-IGA. The turbocharger turbine flow [9,112,113,128,129] and flow-driven string dynamics in turbomachinery [110,111] were computed with the ST-SI-IGA. The IGA basis functions were used in the spatial discretization of the fluid mechanics equations and also in the temporal representation of the rotor and spinning-mesh motion. That enabled accurate representation of the turbine geometry and rotor motion and increased accuracy in the flow solution. The IGA basis functions were used also in the spatial discretization of the string structural dynamics equations. That enabled increased accuracy in the structural dynamics solution, as well as smoothness in the string shape and fluid dynamics forces computed on the string.

The ram-air parachute analysis [117] and spacecraft parachute compressible-flow analysis [119] were conducted with the ST-SI-IGA, based on the ST-SI version that weakly enforces the Dirichlet conditions and the ST-SI version that accounts for the porosity of a thin structure. The ST-IGA with IGA basis functions in space enabled, with relatively few number of unknowns, accurate representation of the parafoil and parachute geometries and increased accuracy in the flow solution. The volume mesh needed to be generated both inside and outside the parafoil. Mesh generation inside was challenging near the trailing edge because of the narrowing space. The spacecraft parachute has a very complex geometry, including gores and gaps. Using IGA basis functions addressed those challenges and still kept the element density near the trailing edge of the parafoil and around the spacecraft parachute at a reasonable level.

The heart valve flow analysis [10,11,53,103,107] was conducted with the ST-SI-TC-IGA. The method, beyond enabling a more accurate representation of the geometry and increased accuracy in the flow solution, kept the element density in the narrow spaces near the contact areas at a reasonable level. When solid surfaces come into contact, the elements between the surface and the SI collapse. Before the elements collapse, the boundaries could be curved and rather complex, and the narrow spaces might have high-aspect-ratio elements. With NURBS elements, it was possible to deal with such adverse conditions rather effectively.

In computational analysis of flow around tires with road contact and deformation [12,14,114], the ST-SI-TC-IGA enabled a more accurate representation of the geometry and motion of the tire surfaces, a mesh motion consistent with that, and increased accuracy in the flow solution. It also kept the element density in the tire grooves and in the narrow spaces near the contact areas at a reasonable level. In addition, we benefit from the mesh generation flexibility provided by using SIs.

In computational analysis of fluid films [14,116], the ST-SI-TC-IGA enabled solution with a computational cost comparable to that of the Reynolds-equation model for the comparable solution quality [116]. With that, narrow gaps associated with the road roughness [14] can be accounted for in the flow analysis around tires.

An SI provides mesh generation flexibility in a general context by accurately connecting the two sides of the solution computed over nonmatching meshes. This type of mesh generation flexibility is especially valuable in complex-geometry flow computations with isogeometric discretization, removing the matching requirement between the NURBS patches without loss of accuracy. This feature was used in the flow analysis of heart valves [10,11,53,103,107], turbocharger turbines [9,112,113,128,129], and spacecraft parachute compressible-flow analysis [119].

For more on the ST-SI-TC-IGA, see [11,12]. In the computations presented here, the ST-SI-TC-IGA is used for the reasons given and as described in the earlier paragraphs of this section.

1.8 EBMR, ZSS, and fiber-reinforced hyperelasticity

This subsection, included for completeness, is mostly from [124] and provides a short description of the EBMR, ZSS, and fiber-reinforced hyperelasticity concepts mentioned in Sect. 1.2.

1.8.1 EBMR

The EBMR was introduced in 2013 [13]. It restores, without resorting to remeshing, the mesh integrity lost during the mesh motion. The loss of mesh integrity, though not frequent because of the advanced mesh moving methods used with the ST-SUPS and ST-VMS, can happen in FSI computations with a high level of complexity. A recent example of such complexity is FSI computation of clusters of spacecraft parachutes with modified geometric porosity [13,149–153]. When faced with a loss of mesh integrity, it was proposed in [13] to use the EBMR, where the mesh is “relaxed” without altering the mesh at the fluid–structure interface and thus the mesh integrity is somewhat restored. As commented in [13], this is of course a less costly and less disruptive alternative to remeshing.

In the EBMR, the new mesh has the same number of nodes and elements as before, but some of the nodes are moved slightly to improve the quality of some of the elements. The motion is determined by using the large-deformation mechanics equations and an element-based ZSS (EBZSS). This is essentially a shape generated for each element. By design, the undeformed shape, made of “target elements,” is
the shape we want to obtain from solving the solid mechanics equations. There are several options for constructing the target element shapes, and those options can be found in [13]. The EBMR was successfully used in FSI computation of clusters of spacecraft parachutes with modified geometric porosity (see [13]).

1.8.2 Locally-defined ZSS

Locally-defined ZSS originated as arterial ZSS estimation (see [53,135–137,154–156]). It was formulated first as the EBZSS in the finite element discretization context [154,155], then as the EBZSS in the isogeometric discretization context [135,136], and then as the integration-point-based ZSS (IPBZSS) in the isogeometric discretization context [137,156].

In the EBZSS, the ZSS is defined with a set of positions for each element. Positions of nodes (control points) from different elements mapping to the same node in the mesh do not have to be the same. In the reference configuration, all elements are connected by nodes, and the displacement is measured from that connected configuration. This way of formulating the structural mechanics problem was named “element-based total Lagrangian” (EBTL) method in [154]. The EBTL was used in the EBMR [13].

In the IPBZSS, the way the EBZSS is defined is extended to integration-point counterpart of that. The ZSS is represented in terms of the metric tensor. Converting the EBZSS representation to IPBZSS representation is straightforward and the conversion will be exact. Converting the IPBZSS representation to EBZSS representation will, in general, not be exact because the IPBZSS has more parameters than the EBZSS. This way of formulating the structural mechanics problem was named “integration-point-based total Lagrangian” (IPBTL) method in [124].

1.8.3 Mesh relaxation and mesh moving based on fiber-reinforced hyperelasticity and optimized ZSS

Mesh relaxation and mesh moving methods based on fiber-reinforced hyperelasticity and optimized ZSS [124] were introduced targeting isogeometric discretization but are also applicable to finite element discretization. With the fibers placed in multiple directions, the element is stiffened in those directions for the purpose of reducing the distortion during the mesh deformation. The ZSS is optimized by seeking orthogonality of the parametric directions, by mesh relaxation, and by making the ZSS time-dependent as needed. The objective of the mesh relaxation is to improve the quality of the mesh after its initial creation and to have an equilibrium state with the optimized ZSS, boundary conditions and the constitutive law. The NURBS mesh used in the computational flow analysis of the tsunami-shelter VAWT [92] was created with the mesh relaxation method.

1.9 Stabilization parameters and element lengths

This subsection, included for completeness, is mostly from [93]. In all the semi-discrete and ST stabilized and VMS methods discussed in Sect. 1.1, an embedded stabilization parameter, known as “τ,” plays a significant role (see [26]). This parameter involves a measure of the local length scale (also known as “element length”) and other parameters such as the element Reynolds and Courant numbers. The interface terms in the ST-SI also involve element length, in the direction normal to the interface. Various element lengths and τs were proposed, starting with those in [18,157] and [158–160], followed by the ones introduced in [161,162]. In many cases, the element length was seen as an advection length scale, in the flow-velocity direction. The τ definition introduced in [162], which is for the advective limit and is now called “τSUGN1” (and the corresponding element length is now called “hSUGN”), automatically yields lower values for higher-order finite element basis functions.

Calculating the τs based on the element-level matrices and vectors was introduced in [163] in the context of the advection–diffusion equation and the Navier–Stokes equations of incompressible flows. These definitions are expressed in terms of the ratios of the norms of the matrices or vectors. They automatically take into account the local length scales, advection field and the element Reynolds number. The definitions based on the element-level vectors were shown [163] to address the difficulties reported at small time-step sizes. A second element length scale, in the solution-gradient direction and called “hRGN,” was introduced in [16,164]. Recognizing this as a diffusion length scale, a new stabilization parameter for the diffusive limit, “τSUGN3,” was introduced in [16,165], to be used together with “τSUGN1” and “τSUGN2,” the parameters for the advective and transient limits. For the stabilized ST methods, “τSUGN12,” representing both the advective and transient limits, was also introduced in [16,164].

Some new options for the stabilization parameters used with the SUPS and VMS were proposed in [3,8,91,166]. These include a fourth τ component, “τSUGN4” [3], which was introduced for the VMS, considering one of the two extra stabilization terms the VMS has compared to the SUPS. They also include stabilization parameters [3] for the thermal-transport part of the VMS for the coupled incompressible-flow and thermal-transport equations.

Some of the stabilization parameters described in this subsection were also used in computations with other SUPG-like methods, such as the computations reported in [167,168].

The stabilization parameters and element lengths discussed in this subsection so far were all originally intended
for finite element discretization but quite often used also for isogeometric discretization. The element lengths and stabilization parameters introduced in [169] target isogeometric discretization but are also applicable to finite element discretization. They were introduced in the context of the advection–diffusion equation and the Navier–Stokes equations of incompressible flows. The direction-dependent element length expression was outcome of a conceptually simple derivation. The key components of the derivation are mapping the direction vector from the physical ST element to the parent ST element, accounting for the discretization spacing along each of the parametric coordinates, and mapping what has been obtained in the parent element back to the physical element. The test computations presented in [169] for pure-advection cases, including those with discontinuous solution, showed that the element lengths and stabilization parameters proposed result in good solution profiles. The test computations also showed that the “UGN” parameters give reasonably good solutions even with NURBS basis functions. The stabilization parameters given in [12], which were mostly from [169], were the latest ones designed in conjunction with the ST-VMS.

In general, we decide what parametric space to use based on reasons like numerical integration efficiency or implementation convenience. Obviously, choices based on such reasons should not influence the method in substance. We require the element lengths, including the direction-dependent element lengths, to have node-numbering invariance for all element types, including simplex elements. The direction-dependent element length expression introduced in [170] meets that requirement. This is accomplished by using in the element length calculations for simplex elements a preferred parametric space instead of the standard integration parametric space. The element length expressions based on the two parametric spaces were evaluated in [170] in the context of simplex elements. It was shown that when the element length expression is based on the integration parametric space, the variation with the node numbering could be by a factor as high as 1.9 for 3D elements and 2.2 for ST elements. It was also shown that the element length expression based on the integration parametric space could overestimate the element length by a factor as high as 2.8 for 3D elements and 3.2 for ST elements.

Targeting B-spline meshes for complex geometries, new direction-dependent element length expressions were introduced in [171]. These latest element length expressions are outcome of a clear and convincing derivation and more suitable for element-level evaluation. The new expressions are based on a preferred parametric space, instead of the standard integration parametric space, and a transformation tensor that represents the relationship between the integration and preferred parametric spaces. We do not want the element splitting to influence the actual discretization, which is represented by the control or nodal points. Therefore, the local length scale should be invariant with respect to element splitting. That invariance is a crucial requirement in element definition, because unlike the element definition choices based on implementation convenience or computational efficiency, it influences the solution. It was proven in [172] that the local-length-scale expressions introduced in [171] meet that requirement.

The direction-dependent local-length-scale expressions introduced in [169,171] have been used in computational flow analysis of turbocharger turbines [113], compressible-flow spacecraft parachutes [119], tires with road contact and deformation [12,14,115], fluid films [14,116], heart valves [107], ventricle-valve-aorta sequences [105,108], tsunami-shelter VAWTs [92], HAWT long-wake flow analysis [93–95], U-duct turbulent flow [120], and Taylor–Couette flow [122]. They have also been used in [64], in the context of gas turbine computational flow analysis with isogeometric discretization, in calculating the Courant number based on the NURBS mesh local length scale in the flow direction.

1.10 Prior ST computations of vehicle and tire aerodynamics

Vehicle and tire aerodynamics computations with the ST-VMS started with multidomain, multiscale thermo-fluid analysis of a freight truck and its tires [3,44,45,106], with a higher-resolution, local-domain thermo-fluid analysis of the rear set of tires. The computations were based on finite element discretization with tetrahedral elements, layered near the tire surface. The tread patterns were not taken into account, and therefore the mesh around the tire did not rotate with it.

The ST-SI-TC was used in [106], with the ST-SUPS, in a 2D computation where the model was a section of a tire and included grooves. The mesh around the tire was made of manually-generated quadrilateral elements and rotated with the tire. The rotating mesh was connected, with the ST-SI, to the rest of the mesh, which was made of one more layer of manually-generated quadrilateral elements and automatically-generated triangular elements beyond that. A detailed report on that 2D computation was presented in [130], together with a 3D computation where the tire model included transverse grooves. The 3D computation was also based on the ST-SUPS and ST-SI-TC. The mesh around the tire was made of manually-generated hexahedral elements and rotated with the tire. The rest of the mesh was made of one more layer of manually-generated hexahedral elements, a layer of manually-generated pyramid elements after that, and automatically-generated tetrahedral elements beyond that.

The work reported in [114] included computations with the ST-SI-TC-IGA and two models of flow around a rotating tire with road contact and prescribed deformation. One was a simple 2D model for verification purposes and had
no grooves, and one was a 3D model with an actual tire geometry, with longitudinal and transverse grooves, and a deformation pattern provided by the tire company. The meshes were made of manually-generated quadratic NURBS elements, and the mesh around the tire rotated with it. Meshes with two different resolutions were used in both the 2D and 3D computations: preliminary mesh and refined mesh. The 3D computation with the preliminary mesh was conducted with the ST-SUPS, and the other three computations with the ST-VMS. In [12], which is essentially the journal version of [114], all four computations were with the ST-VMS.

It was mentioned in Sect. 1.7 that in computational analysis of fluid films, the ST-SI-TC-IGA, as deployed in [116], enables solution with a computational cost comparable to that of the Reynolds-equation model for the comparable solution quality. This was accomplished with the computational flexibility to go beyond the limitations of the Reynolds-equation model. With the ST-IGA, even with just one quadratic NURBS element across the gap of the fluid film, we reach a solution quality comparable to that of the Reynolds-equation model. The work reported in [116] included detailed 2D test computations. The computations showed how the ST computational method performs compared to the Reynolds-equation model, and also compared to finite element discretization. The test cases included different circumferential and normal mesh refinement levels. Also included were test cases with an SI in the mesh and cases where the no-slip boundary conditions are enforced weakly.

With the computational analysis of fluid films enabled in that fashion, the computational challenges associated with the road roughness and the fluid film between the tire and the road were addressed in [14]. To accomplish that, some new methods were added in [14] to the ST computational methods described in this section so far. The added methods included a remedy for the trapped fluid and a method for reducing the number of control points as a space occupied by the fluid shrinks down to a narrow gap. They also included a method for representing the road roughness. The work reported in [14] included computations with the ST-SI-TC-IGA and two models of flow around a rotating tire with road contact and prescribed deformation. One was a 2D model with grooves, and one was a 3D model with an actual tire geometry, with longitudinal and transverse grooves, and a deformation pattern provided by the tire company. The meshes were made of manually-generated quadratic NURBS elements, and the mesh around the tire rotated with it. Both the 2D and 3D computations were conducted with the ST-SUPS. The 2D computations were detailed studies on different ways of modeling the fluid trapped in the grooves. The 3D computation was a demonstration of the effectiveness of the ST methods in addressing the computational challenges associated with road roughness and fluid films, beyond the challenges associated with the longitudinal and transverse grooves, road contact, and tire deformation.

1.11 Car and tire models

In this article, we go beyond the computational challenges summarized in Sect. 1.10. We address the challenges associated with the aerodynamic interaction between the car body and the tires and the task of generating good-quality NURBS meshes for model geometries (see Fig. 1) close to the actual geometries.

1.12 Outline of the remaining sections

The NURBS mesh generation methods, including the NSVGMG, are described in Sect. 2. The computation settings are described in Sect. 3, the results are presented in Sect. 4, and the concluding remark are given in Sect. 5.

2 Mesh generation

Here we explain some of the mesh generation techniques used in the computations. We combine the techniques we explain with using SIs, so that the two adjacent subdomains can have different knot vectors. In this way, for example, changing the tire geometry would not affect the other parts of the mesh.

2.1 Mesh refinement

One of the most important parts of multi-patch NURBS mesh generation is how to partition the domain in the most effective way. For that reason, we typically first generate a coarse mesh, which makes partitioning easier, and then refine the mesh by knot insertion. This strategy was explained also in [128].

Fig. 1 Car body and tire models

Car and tire models

Outline of the remaining sections

Mesh generation

Mesh refinement
2.2 NURBS Surface-to-Volume Guided Mesh Generation (NSVGMG) method

A NURBS patch can be generated from six surfaces, where each pair of surfaces facing each other have the same knot vectors and consequently the same number of control points. We use a commercial software, Rhinoceros, to generate the surfaces, which are based on NURBS discretization. Following that, we determine the internal control points. We determine one control point at a time. To determine an internal point \( \mathbf{x}_c \), we use a total of \( 3^{n_{pd}} - 1 \) control points, where \( n_{pd} \) is the number of parametric dimensions (see a 2D example in Fig. 2). In each parametric direction, we use \( 3^{n_{pd}} - 1 \) side “guides” with three control points each, and one inner guide with only two control points. The side guides are defined by points \( (B^\alpha_2)_1 \), \( (B^\alpha_2)_2 \), and \( (B^\alpha_2)_3 \), where \( \alpha \) and \( k \) denote the parametric direction and the guide index (see Fig. 3). The center guide is defined by the points \( (C^{\alpha})_1 \) and \( (C^{\alpha})_3 \) (see also Fig. 3). All those points come from the surface (edge in 2D) meshes. With them, we define

\[
\begin{align*}
\mathbf{b}^\alpha_k &= \frac{(B^\alpha_2)_3 - (B^\alpha_2)_1}{\left\| (B^\alpha_2)_3 - (B^\alpha_2)_1 \right\|}, \\
\mathbf{s}^\alpha_k &= \frac{\mathbf{b}^\alpha_k \cdot ((B^\alpha_2)_2 - (B^\alpha_2)_1)}{\left\| (B^\alpha_2)_3 - (B^\alpha_2)_1 \right\|}.
\end{align*}
\]

(1)

Figure 3 also shows, as an example, how \( s^1_1 \) is determined. We average the \( s^\alpha_k \).

\[
\bar{s}^\alpha = \frac{1}{3^{n_{pd}} - 1 - 1} \sum_{k=1}^{3^{n_{pd}} - 1} s^\alpha_k.
\]

(3)

With \( \bar{s}^\alpha \), we define

\[
\begin{align*}
t^\alpha &= \frac{(C^{\alpha})_3 - (C^{\alpha})_1}{\left\| (C^{\alpha})_3 - (C^{\alpha})_1 \right\|}, \\
c^\alpha &= (1 - \bar{s}^\alpha) (C^{\alpha})_1 + \bar{s}^\alpha (C^{\alpha})_3.
\end{align*}
\]

(4)

Using the geometric relationships in all parametric directions, we form the equation system

\[
\sum_{\alpha=1}^{n_{pd}} (\mathbf{x}_c - (C^{\alpha})_1) \cdot t^{\alpha} c^{\alpha} = 0.
\]

(6)

From that, we obtain the form

\[
\sum_{\alpha=1}^{n_{pd}} t^{\alpha} c^{\alpha} \cdot \mathbf{x}_c = \sum_{\alpha=1}^{n_{pd}} t^{\alpha} t^{\alpha} \cdot c^{\alpha}.
\]

(7)

We solve this as a matrix system. We note that if the number of spatial dimensions, \( n_{sd} \), is not equal to \( n_{pd} \), we need \( n_{sd} - n_{pd} \) constraints added. Figure 4 illustrates how \( \mathbf{x}_c \) is determined, and Fig. 5 shows the resulting mesh based on the example in Fig. 2.

Remark 1 In NURBS discretization, we quite often have patches with bending deformation. We use the scalar values, as given by Eq. (2), because we want the averaging to be applicable to such patches.
Fig. 5 Resulting NURBS mesh with the elements and control points. The blue points are those determined with the NSVGMG. (Color figure online)

Remark 2 We can use weights in Eq. (3), but for simplicity we do not do that here.

Remark 3 The process can be simplified by using only two surfaces (edges in 2D) facing each other. Along the lines connecting the corresponding points of the two surfaces, the process starts with having one linear Bézier element, followed by order elevation, followed by knot insertion to generate the mesh. This simplification would be reasonable in computations where the surfaces used are those most relevant to the problem geometry.

Remark 4 We want to be able to generate wedge-shaped meshes. The NSVGMG needs to have a provision for that. When the control points of a surface-mesh edge coalesce and the edge collapses, the expressions given by Eqs. (1) and (2) become unusable for that edge. Then we just remove that edge from the averaging given by Eq. (3).

2.3 Mesh relaxation

With the NSVGMG, we can generate a volume mesh with accurate representation of the problem geometry and with a good multi-patch arrangement. Here we further improve the distribution of the interior control points. For that purpose, the mesh relaxation method described in Sect. 1.8.3 is used. We explain here how the parameters required in the method can be designed to obtain a high-quality mesh.

The method improves the mesh-line orthogonality and element size and shape. The size and shape can be controlled by the ZSS. We design the ZSS based on a number of considerations. Near the solid surfaces, we set the normal-direction thickness of the first layer of elements to be constant, which results in a high aspect ratio. Then, with a given number of layers and an expansion factor from one layer to the next, we define the ZSS for those layers of elements. Other than that, we try to have comparable sizes and aspect ratios close to 1. To achieve that, we calculate a target element size by a simple division of the estimated length in a direction with the number of elements along that direction. The target element size does not need to be very accurate but will play a role in the final mesh quality.

Remark 5 After generating a mesh with the NSVGMG, we would have the options of doing a mesh relaxation, mesh refinement by knot insertion, and a second mesh relaxation.

2.4 Volume mesh around a tire

We first generate the mesh around the tire manually, without considering the contact and deformation, as done also in [12,14]. The boundary of this part of the mesh, other than the tire surface, will be the SI that connects it to the rest of the mesh. We begin with generating a surface mesh over the tire surface, considering the tread patterns, but without actually representing the depth of the grooves. Then, we extrude the mesh outward, and also inward into the grooves. In this way, we have full control of the mesh, and we can easily prepare the information required for the ST-SI-TC. We note that we use NURBS weights to represent the circular arc geometry exactly. After that, for a given deformation of the tire in contact with the road, we update the mesh. The mesh update is based on a combination of mesh moving with a special-purpose method and element collapse on the SI plane with the ST-SI-TC.

3 Computation settings

3.1 Car and tire models

The car and tire models are shown in Figs. 6 and 7. We note that while the tire model is provided by YOKOHAMA RUBBER CO., LTD., the car body, wheel, and the disk rotor are not from the actual CAD data. The car is 4.65 m long, 1.81 m wide, and 1.16 m high. The tire, wheel, and disk rotor dimensions are given in Table 1. Figure 8 shows the zoomed view of the tire. There are no transverse grooves in this model. The groove widths are different between the two center and two side grooves. We carry out a steady-state structural mechanics computation to obtain the tire deformation, resulting in the shape shown in Fig. 7. All four tires have the same deformation.

3.2 Problem setup

The rotation speed corresponds to a linear speed of 100 km/h at the undeformed tire periphery. The measured tire contact angle is 22°. The corresponding car speed is \( U_0 = 99.39 \) km/h. The reference frame is attached to the car. The air density and kinematic viscosity are 1.205 kg/m\(^3\) and 1.512×10\(^{-5}\) m\(^2\)/s.
Fig. 6 Car body model

Fig. 7 Tire (blue), wheel (green), and disk rotor (red) models. (Color figure online)

Table 1 Model data for the tire, wheel, and disk rotor

|                         | mm |
|-------------------------|----|
| Tire diameter           | 632|
| Tire width              | 211|
| Groove depth            | 8.9 |
| Groove (center) width   | 3.5 |
| Groove (side) width     | 5.4 |
| Wheel diameter          | 371|
| Disk rotor diameter     | 320|
| Disk rotor width        | 80 |

Fig. 8 Tire model

3.3 Boundary conditions

Figure 9 shows the computational domain and the car body and the tires. The domain has a length 13 times the body length, width 20 times the body width, and height 18 times the body height. The car center is located at 5 times the body length from the inflow plane. The velocity is specified on the inflow and bottom boundaries, at $U_0$. The outflow boundary condition is stress-free, and the lateral and top boundary conditions are slip. The car body is a no-slip boundary. The tires, wheels, and disk rotors are also no-slip boundaries, and the meshes around them are rotating with the tire. The velocity of the rotating surfaces is obtained from the motion.

3.4 Mesh

We generate the mesh in five subdomains, separately, and connect them with SIs. Figure 10 shows the subdomains. The outer domain (“O”) is of the simplest shape. Inside that, is the body domain (“B”), which excludes the tire domains. Each of the four tire domains consists of three subdomains: tire rotating domain (“TR”), tire stationary domain (“TS”), and the wheel domain (“W”). In this way, the complexities of the car and tire geometries, further complicated by the contact, can be treated without a strong mutual dependence between them.

Remark 6 Since the tire does not have transverse grooves, we can do the computation without rotating the TR meshes. However, we still rotate them with the ST-SI-TC. With that, we show that we will be able to handle more complicated tread patterns in the future, using the mesh generation method described in Sect. 2.4.

The O mesh is generated with the two-surface version (see Remark 3) of the NSVGMG, followed by a mesh relaxation, as described in Sect. 2.3. Figure 11 shows the coarse B mesh, obtained with the two-surface version of the NSVGMG. This is followed by a mesh relaxation. After that, we refine the
mesh by knot insertion, followed by a mesh relaxation on that. As explained in Sect. 2.3, we control the layers of elements near the car body. We use 13 layers of elements in the normal direction, with the first-layer thickness 0.7 mm. Figure 12 shows the fine B mesh. The TR mesh is generated with the method in Sect. 2.4. There are 10 layers in the radial direction. The first-layer thickness is about 1.5 mm. There are 6 layers in the axial direction, with the first-layer thickness about 5.0 mm. Figure 13 shows the TR mesh. Figure 14 shows how the TR mesh rotates in the ST-SI-TC. The TS and W meshes are generated with the two-surface version of the NSVGMG. The TS meshes for the front and rear tires have the same number of control points and elements, however the domain shapes are different. The number of control points and elements for all five meshes are shown in Table 2.

### 3.5 Computational conditions

There are 144 time steps per rotation, equivalent to a time-step size of $4.96 \times 10^{-4}$ s. The method is the ST-VMS, and the stabilization parameters are those given by Eqs. (4)–(9) in [12]. The number of nonlinear iterations per time step is 3, and the number of GMRES iterations per nonlinear iteration...
is 300. The initial condition is $U_0$ everywhere except for the car body, tire, wheel, and disk rotor.

4 Results

We measure time from a point reached after a sufficiently long computation with the full Reynolds number. The computation is carried out for $5T_{TR}$ after that, where $T_{TR}$ is the tire rotation period.

4.1 Flow visualization near the car body

Figure 15 shows the velocity magnitude at a cross-section placed centrally in the width direction. We can see the boundary layer near the top surface of the car body. Figure 16 shows the pressure distribution over the car body and tires, averaged from $4T_{TR}$ to $5T_{TR}$, positional-averaged in the case of the tires. The pressure distribution looks reasonable. Figure 17 shows the flow patterns during the last $T_{TR}/36$. We look at multiple instants in that duration to see how the flow evolves in a short time interval. We see vortex sheets around the hood and windshield. We can see the vortex collapse near the side of the front tire contact part. The vortices generated by the front tire are advected downstream and collapse. The vortices seen here might be too large, which we think is due to the coarseness of the mesh around the car body.

4.2 Flow visualization near the tires

Figure 18 shows the velocity magnitude on two planes at $5T_{TR}$. The velocity distributions around the front and rear tires are quite different. For both the front and rear tires, the velocities are higher in the inner groove than in the outer groove. Figures 19 and 20 show the flow patterns for the four tires at the same instants as in Fig. 17. We see that the vortices near the contact area for the front and rear tires are different. For the front tire, the vortex patterns have some level of symmetry between the two sides of the car, for the rear tire, they do not. Near the contact area, we see that the flow around a front tire is faster than it is around a rear tire. Figures 21 and 22 show, for the front and rear left tires, the positional-averaged wall shear stress over the last $T_{TR}$. We can see how the cover part of the car body affects the flow around the tires. Comparing the front and rear tires, the shear stress on the front tire below the cover is higher and the shear stress on the rear tire sidewall is higher. Near the contact area, we see some differences in the shear stress on the sidewall.
Table 2  Number of control points (nc) and elements (ne)

| Mesh | nc     | ne     |
|------|--------|--------|
| O    | 57,960 | 41,928 |
| B    | 246,388| 158,052|
| TR   | 4×144,448| 4×89,856|
| TS   | 4×34,696| 4×18,432|
| W    | 4×43,164| 4×18,560|
| Total| 1,193,580| 707,372|

Fig. 15  Velocity magnitude (km/h) at a cross-section placed centrally in the width direction, at 5\(T_{TR}\).

Fig. 16  Pressure (kPa) distribution over the car body and tires, averaged from 4\(T_{TR}\) to 5\(T_{TR}\), positional-averaged in the case of the tires.

Although we see in Figs. 18, 19 and 20 that the flow patterns are different between the front and rear tires, the shear stresses are not that different. This means that the tire deformation and contact have significant effect on the flow near the tire.

Fig. 17  Isosurfaces corresponding to a positive value of the second invariant of the velocity gradient tensor, colored by the velocity magnitude (km/h), at 5 uniformly spaced instants during the last \(T_{TR}/36\). (Color figure online)
5 Concluding remarks

We have presented an ST isogeometric analysis framework for car and tire aerodynamics with road contact and tire deformation and rotation. In the computation we have presented, the geometries of the computational models for the car body and tires are close to the actual geometries. The computational challenges include (1) the complexities of these geometries, (2) the tire rotation, (3) maintaining accurate representation of the boundary layers near the tire while being able to deal with the flow-domain topology change created by the road contact, (4) the turbulent nature of the flow, (5) the aerodynamic interaction between the car body and the tires, and (6) NURBS mesh generation for model geometries close to the actual geometries.

The computational framework is made of the ST-VMS, ST-SI, ST-TC, ST-IGA, integrated combinations of these ST methods, NSVGMG, and the EBMR. The ST context provides higher-order accuracy in general, the VMS feature of the ST-VMS addresses the challenge created by the turbulent nature of the flow, and the moving-mesh feature of the ST context enables high-resolution flow computation near the moving fluid–solid interfaces. The ST-SI enables moving-mesh computation with the tire rotating. The mesh covering the tire rotates with it, and the SI between the rotating mesh and the rest of the mesh accurately connects the two sides of the solution. The ST-TC enables moving-mesh computation even with the TC created by the contact between the tire and the road. It deals with the contact while maintaining high-resolution flow representation near the tire. Integration of the ST-SI and ST-TC enables high-resolution representation even though parts of the SI are coinciding with the tire and road surfaces. It also enables dealing with the tire–road contact location change and contact sliding. By integrating the ST-IGA with the ST-SI and ST-TC, in addition to having a more accurate representation of the tire geometry and increased accuracy in the flow solution, the element density in the tire grooves and in the narrow spaces near the contact areas is kept at a reasonable level. The NSVGMG enables NURBS mesh generation for the complex car and tire geometries, and the EBMR improves the quality of the meshes.

The mesh used in the computation has five subdomains, generated separately and connected with SIs. The five subdomains are the outer domain, body domain, tire rotating domain, tire stationary domain, and the wheel domain. With this way of mesh generation, the complexities of the car and tire geometries, further complicated by the contact, can be treated without a strong mutual dependence between them. Each subdomain mesh was generated by using one or more methods selected from a set that includes the NSVGMG, knot insertion, EBMR, and manual mesh generation. The mesh has controlled layers near the car and tire surfaces.

The car and tire aerodynamics computation presented show the effectiveness of the analysis framework we have built, including effectiveness in generating NURBS meshes for car and tire models with complex, realistic geometries.
Fig. 19 Isosurfaces corresponding to a positive value of the second invariant of the velocity gradient tensor, colored by the velocity magnitude (km/h), viewed from the bottom, at the same instants as in Fig. 17. Front right tire and front left tire. The free-stream flow is from left to right. The dark gray zones are the contact areas. The top frames show the tires viewed, with the arrows indicating the viewing direction. (Color figure online)
Fig. 20 Isosurfaces corresponding to a positive value of the second invariant of the velocity gradient tensor, colored by the velocity magnitude (km/h), viewed from the bottom, at the same instants as in Fig. 17. Rear right tire and rear left tire. The free-stream flow is from left to right. The dark gray zones are the contact areas. The top frames show the tires viewed, with the arrows indicating the viewing direction. (Color figure online)
Fig. 21 Positional-averaged shear stress (Pa) over the last $T_{FR}$. Front left tire. The free-stream flow is from left to right.

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