Innovation Feedback Kalman Filter

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Abstract. The conventional Kalman filter demands complete knowledge of the actual filter model, which is usually inaccessible in practical problems. When dealing with the filtering problem with uncertainty, any improper prediction of the uncertainty may degrade the filtering performance and even lead to divergence phenomenon. In this paper, the innovation feedback Kalman filter, which introduces the innovation feedback controller to the Kalman filter equations, is newly proposed on the basis of automatic control theory to address the filtering problem with uncertainty and is different from other filtering methods. By studying the estimate error equations, the estimate bias is extracted and its propagation mechanism is formulated. The estimate bias propagation equations reveal that eliminating the estimate bias essentially equivalent to an output regulation problem with uncertain exosystem. And a concise yet effective innovation feedback controller is subsequently given as an example. The proposed method is applied to one-dimension target tracking scenario and its high estimation accuracy performance and good stability are simulated through a comparative analysis with the ideal filtering results.

1. Introduction

As the optimal linear filter under minimum mean-square error criterion, Kalman filter [1] and its subsequent variants [2]-[4] are widely used in the area of state estimation over the past five decades. However, conventional Kalman filter requires complete dynamic and statistic information about the real system, which is actually impossible in reality. One reason is that the existent of uncertainty in state equation is usually inevitable. The sources of the uncertainty are various, e.g., modeling inaccuracies, discretization errors, high-order terms after linearization, unknown input, etc. This filtering problem can be rather intractable because ignoring the uncertain parameters completely or inaccurate estimation may degrade the filtering performance even lead to “divergence” phenomenon [5]-[6]. A famous example is target tracking, the state equation of which is excited by an unknown and time-varying forcing function. Most of the current target tracking methods [7]-[11] develop approximate state models of the real system and encounter difficulties when tracking highly maneuvering targets. In some demanding cases, multi-model method is applauded because the filtering error using one single model is unacceptable [11].

Significant research efforts have been devoted to solve the filtering problem with uncertainty. In one class, the Kalman filter gain matrix is modified adaptively to estimate the uncertain parameters and the noise statistics at the same time, see, e.g., [13]-[16], etc. An “optimal” gain matrix is usually deduced under a suitable estimation criterion [17] (e.g., Bayes criterion, maximum likelihood criterion, etc.). However, the “optimal” gain matrix needs complete knowledge of the uncertain parameters, which is in fact inaccessible. In another class, the uncertain parameters are estimated directly to
compensate the estimate bias, see, e.g., [18]-[20], etc. This is realized by taking the uncertain parameters as part of the system state or suitable curve-fitting algorithm. For the former, the filtering performance declined dramatically when the uncertain parameters are time-varying. For the latter, the algorithm has to deal with vast of most recent measurements and may destabilize the filtering system. Therefore, debiasing and stabilizing are the key in establishing a desirable filtering method.

Intriguingly, both theoretical analysis and simulations reveal that once the uncertain parameters are not well predicted, the mean of innovation in Kalman filter will be nonzero. If one can find a control input to vanish the mean of innovation, the estimate bias will be eliminated as much as possible at the same time. Inspired by this, the innovation feedback Kalman filter is newly proposed. By studying the estimate bias propagation mechanism with control input, this filtering method converts the intractable filtering problem with uncertainty into an output regulation problem, which is one of the famous research interests in modern control theory. Therefore, the intensive research achievements on output regulation problem (see, e.g., [21]-[27], etc.) can be utilized for developing debiased and stable innovation feedback Kalman filters to solve the concerned filtering problem. In this paper, a simple yet effective version is proposed as an example, and its excellent filtering performance is tested in its application to one-dimension target tracking scenario.

Remainder of this paper is organized into the following sections: Section II states the concerned filtering problem. Section III derives the innovation feedback Kalman filter. Section IV simulates the application case of innovation feedback Kalman Filter in a one-dimension target tracking scenario. Finally, a conclusion is provided in Section V.

2. Filtering Problem Statement
This paper focuses on the filtering problem with uncertainty that the discrete-time state equation and measurement equation of the filtering model are given as

\[
\begin{align*}
    x_k &= A_{k-1}x_{k-1} + B_{k-1}f_{k-1} + w_{k-1} \\
    y_k &= H_kx_k + n_k
\end{align*}
\]  

where \( x_k \in \mathbb{R}^n \) is the state vector, \( f_{k-1} \in \mathbb{R}^l \) is the uncertainty vector, \( w_{k-1} \sim N(0, Q_{k-1}) \) is the zero-mean Gaussian process noise, \( y_k \in \mathbb{R}^m \) is the measurement vector, \( n_i \sim N(0, R_k) \) is the zero-mean Gaussian measurement noise, \( A_{k-1} \in \mathbb{R}^{n \times n} \) is the state transition matrix, \( B_{k-1} \in \mathbb{R}^{n \times l} \) is the uncertainty distribution matrix, \( H_k \in \mathbb{R}^{m \times n} \) is the measurement matrix. Note that \( f_{k-1} \) is unknown and time-varying. In addition, note that \( \mathbb{E}\{w_k r_i^T\} = 0 \) holds for all \( k \) and \( i \).

The major challenge of the concerned filtering problem is that there may be bias in state estimate if the effect of the uncertainty (namely, \( f_{k-1} \)) is not well predicted. What’s more, for time-varying \( f_{k-1} \), the estimate bias may potentiality propagate as time goes by.

3. Innovation Feedback Kalman Filter
As its name suggests, innovation feedback Kalman filter (IFKF) consists of two major parts, feedback controller and standard Kalman filter. The main task of Kalman filter is to generate gain matrix thus diminish the random error, while the core function of feedback controller is to estimate the effect of state modelling uncertainty and then diminish the estimate bias. Therefore, this kind of division of work simplifies the concerned filtering problem and is more targeted.

The objective of this section is to investigate estimate bias propagation mechanism and design the innovation feedback controller, thus establish IFKF algorithm.

3.1. Estimate Bias Propagation
In IFKF, \( f_{k-1} \) is replaced by innovation feedback controller \( u_{k-1} \). And correspond to the filtering model depicted in (1), the Kalman filter equations [28] are given as:

\[
\]
\[ \begin{align*}
    m_k &= A_{k-1}m_{k-1} + B_{k-1}u_{k-1} \\
    P_k &= A_{k-1}P_{k-1}A_{k-1}^T + Q_{k-1} \\
    v_k &= y_k - H_k m_k \\
    S_k &= H_k P_k H_k^T + R_k \\
    K_k &= P_k H_k S_k^{-1} \\
    m_k &= m_k^e + K_k v_k \\
    P_k &= P_k - K_k S_k K_k^T
\end{align*} \]

where \( m_k \) is the predicted mean, \( m_k^e \) is the mean, \( u_{k-1} \) is the innovation feedback controller (IFC), \( P_k \) is the predicted covariance, \( P_k \) is the covariance, \( K_k \) is the gain matrix, \( v_k \) is the innovation, \( S_k \) is the innovation covariance. Note that the recursion is started from the prior mean \( m_0 \) and covariance \( P_0 \).

Let \( \delta m_k = m_k - x_k \) be the estimate error. Comparing the Kalman filter equations with the filtering model depicted in (1) yields:

\[ \begin{align*}
    \delta m_k &= (I_n - K_k H_k)(A_{k-1}\delta m_{k-1} - B_{k-1}f_{k-1} + B_{k-1}u_{k-1}) \\
    &- (I_n - K_k H_k)w_{k-1} + K_k r_k \\
    v_k &= -H_k(A_{k-1}\delta m_{k-1} - B_{k-1}f_{k-1} + B_{k-1}u_{k-1}) + H_k w_{k-1} + r_k
\end{align*} \]  

(3)

The estimate error \( \delta m_k \) is composed of two parts. The first is the systematic error, or, the estimate bias, which is mainly excited by \( (u_{k-1} - f_{k-1}) \) and can be quantified by \( \text{E}\{\delta m_k\} \). The second is random error, which is mainly generated by \( w_{k-1} \) and \( r_k \) and can be quantified by \( \text{Var}\{\delta m_k\} \).

The main task of Kalman filter gain \( K_k \) is to minimize the random error (namely, minimize \( \text{Var}\{\delta m_k\} \)). While the core function of IFC \( u_{k-1} \) is to compensate the effect of state modeling uncertainty \( f_{k-1} \) thus diminish the estimate bias (namely, diminish \( \text{E}\{\delta m_k\} \)). This kind of division of work is a major innovation comparing IFKF with some of the other adaptive Kalman filter approaches.

Calculating the mathematical expectation of every term in (3) yields the estimate bias propagation equations as

\[ \begin{align*}
    \text{E}\{\delta m_k\} &= (I_n - K_k H_k)(A_{k-1}\text{E}\{\delta m_{k-1}\} - B_{k-1}f_{k-1} + B_{k-1}u_{k-1}) \\
    \text{E}\{v_k\} &= -H_k(A_{k-1}\text{E}\{\delta m_{k-1}\} - B_{k-1}f_{k-1} + B_{k-1}u_{k-1})
\end{align*} \]  

(4)

Comparing the two equations in (4) yields:

\[ \text{E}\{v_k\} = -H_k(I_n - K_k H_k)^{-1}\text{E}\{\delta m_k\} \]

(5)

Obviously, \( \text{E}\{v_k\} \neq 0 \) is a very important mark for the existence of the estimate bias.

The ideal IFC input is \( u_{k-1} = f_{k-1} \), which cannot be acquired since \( f_{k-1} \) is uncertain. If one can find a set of \( u_{k-1} \) to track \( f_{k-1} \), the estimate bias will be maximally eliminated. Regard \( \text{E}\{\delta m_k\} \) as the state, \( f_{k-1} \) the exogenous signal, \( u_{k-1} \) the input, and \( \text{E}\{v_k\} \) the output, the filtering problem with uncertainty is converted into an output regulation problem with uncertain exosystem, which is described by the estimate bias propagation equations. And the key point is to design a suitable IFC to solve the output regulation problem.

For a desirable IFC, three points should be considered: 1) the major challenge in IFC designing is that the exosystem is unknown; 2) the actual output \( v_k \) is noisy when considering its relation with \( \text{E}\{v_k\} \); 3) the output feedback approach is appreciated because \( v_k \) is accessible while \( \delta m_k \) is not.

### 3.2. An IFC Design Approach

In real-life applications, IFKF is usually used as state observer, which generates state estimate information for the controller to process. Therefore, the delay of the observer certainly will influence
the control performance, and a nearly real-time IFC is applauded. In this paper, a concise and representative IFC design approach is proposed on the basis of the assumption that the change of the uncertain vector $f_{k-1}$ from time step $k-1$ to time step $k$ is very small. This assumption is universal because usually $f_{k-1}$ is sampled from a continuous function $f(t)$. Consequently, at time step $k$, the IFC input $u_{k-1}$ is similar to that of the previous step, namely, let

$$\tilde{u}_{k-1} = u_{k-2}$$  \hspace{1cm} (6)

Then, the corresponding innovation is

$$\tilde{v}_k = y_k - H_k (A_{k-1} m_{k-1} + B_{k-1} u_{k-2})$$  \hspace{1cm} (7)

According to (5), $\tilde{v}_k$ carries the current estimate bias information when $u_{k-1}$ is set as $u_{k-2}$. To restrain steady-state error, the sum signal of $\tilde{v}_k$ is also introduced:

$$q_k = q_{k-1} + \tilde{T} \tilde{v}_k$$  \hspace{1cm} (8)

Note that $q_{k-1}$ can be regarded as the discrete-time integration of $v_{k-1}$, so that it contains the past estimate bias information. Use the linear combination of $\tilde{v}_k$ and $q_{k-1}$ to construct feedback correcting signal, $u_{k-1}$ is updated in the following recursive way

$$u_{k-1} = u_{k-2} + K_1 \tilde{v}_k + K_2 q_{k-1}$$  \hspace{1cm} (9)

where $K_1 \in \mathbb{R}^{hm}$ and $K_2 \in \mathbb{R}^{hm}$ are the feedback gain matrices. Combining (7) with (9) yields

$$u_{k-1} = (I_l - K_1 H_k B_{k-1}) u_{k-2} + K_1 (y_k - H_k A_{k-1} m_{k-1}) + K_2 q_{k-1}$$  \hspace{1cm} (10)

Since then, the basic structure of the proposed IFC is determined. The designing focus is shifted to selecting $K_1$ and $K_2$ to stabilize the filtering system, that is to say, substitute the IFC recursive equation (10) into the estimate bias propagation equations (4) and stabilize the derived closed-loop system.

Define $\delta u_{k-1} = f_{k-1} - u_{k-1}$, it can be derived from (10) that

$$\delta u_{k-1} = (I_l - K_1 H_k B_{k-1}) \delta u_{k-2} - K_1 (y_k - H_k A_{k-1} m_{k-1}) + K_2 q_{k-1}$$  \hspace{1cm} (11)

Let $z_k = [E\{\delta m_k\}, E\{q_k\}, \delta u_{k-1}']^T$ be the augmented state, the closed-loop state equation is

$$z_{k+1} = G_{k-1} z_k + M_{k-1} (f_{k-1} - f_{k-2}) + N_{k-1} (H_k w_{k-1} + r_k)$$  \hspace{1cm} (12)

where,

$$G_{k-1} = \begin{bmatrix} (I_n - K_1 H_k) (I_n - B_{k-1} K_1 H_k) A_{k-1} & (I_n - K_1 H_k) B_{k-1} K_2 & (I_n - K_1 H_k) B_{k-1} (I_l - K_1 H_l B_{k-1}) \\ - T H_k (I_n - B_{k-1} K_1 H_k) A_{k-1} & I_m - T H_k B_{k-1} K_2 & - T H_k B_{k-1} (I_l - K_1 H_l B_{k-1}) \\ - (I_n - K_1 H_k) B_{k-1} (I_l - K_1 H_l B_{k-1}) & K_2 & (I_l - K_1 H_l B_{k-1}) \end{bmatrix}$$

$$M_{k-1} = \begin{bmatrix} - (I_n - K_1 H_k) B_{k-1} (I_l - K_1 H_l B_{k-1}) \\ T H_k B_{k-1} (I_l - K_1 H_l B_{k-1}) \\ - (I_l - K_1 H_l B_{k-1}) \end{bmatrix}$$

$$N_{k-1} = \begin{bmatrix} (I_n - K_1 H_k) B_{k-1} K_1 \\ - T H_k B_{k-1} K_1 \\ K_1 \end{bmatrix}$$

System (12) is stable if $K_1$ and $K_2$ are selected to make

$$\lambda_i (G_{k-1}) < 1, \ i = 1, 2, \ldots, n$$  \hspace{1cm} (14)

where $\lambda_i (*)$ denotes the $i^{th}$ eigenvalue of $*$.  

Suppose system (12) is stable, solving its Z-transform yields

$$Z \left ( E \{ \delta m_{k-1} \} \right ) = (z-1)^2 E (z) (I_l - K_1 H_l B_{k-1}) Z (f_{k-1})$$  \hspace{1cm} (15)
where \( E(1) \neq 0 \), \( F(1) \neq 0 \), \( Z(*) \) represent the \( Z \)-transform of *. Note that in (15) only the influence of \( (f_{k-1} - f_{k-2}) \) is considered. It can be concluded that (15) is transfer function of type 2 system. Therefore, according to final value theorem, the steady state error of \( E(\delta m_{k-1}) \) will be eliminated if:

\[
\Delta^2 f_{k-1} = 0, \quad i = 1, 2, \ldots, n
\]

where \( \Delta^2(*) \) represents the second order difference of *.

3.3. IFKF Algorithm

In the end, IFKF algorithm are given as the following filtering equations:

1) Innovation feedback step:

\[
u_{k-1} = (I - K_k H_k B_{k-1}) u_{k-2} + K_1 (y_k - H_k A_{k-1} m_{k-1}) + K_2 q_{k-1}
\]

2) Predict step:

\[
\begin{align*}
m_{k} &= A_{k-1} m_{k-1} + B_{k-1} u_{k-1} \\
P_{k} &= A_{k-1} P_{k-1} A_{k-1}^T + Q_k
\end{align*}
\]

3) Update step:

\[
\begin{align*}
q_k &= q_{k-1} + T (y_k - H_k m_{k}) \\
K_k &= P_k H_k (H_k P_k H_k^T + R_k)^{-1} \\
m_k &= m_{k} + K_k (y_k - H_k m_{k}) \\
P_k &= P_k - K_k (H_k P_k H_k^T + R_k) K_k^T
\end{align*}
\]

4. Application to One-Dimension Target Tracking

In this section, a one-dimension target tracking scenario is taken into account to testify the filtering performance of IFKF:

\[
\begin{align*}
x_k &= A x_{k-1} + B j_{k-1} + w_{k-1} \\
y_k &= H x_k + r_k
\end{align*}
\]

where \( x_k = [x_k, v_k, a_k]^T \) is the vector of position, velocity, and acceleration, the uncertain parameter \( j_k \) is the derivative of \( a_k \), namely, jerk, and

\[
A_k = \begin{bmatrix} 1 & T & T^2 / 2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \quad B_k = \begin{bmatrix} T^3 / 6 \\ T^2 / 2 \\ T \end{bmatrix},
\]

\[
Q_k = \begin{bmatrix} T^5 q / 20 & T^4 q / 8 & T^3 q / 6 \\ T^4 q / 8 & T^3 q / 3 & T^2 q / 2 \\ T^3 q / 6 & T^2 q / 2 & T q \end{bmatrix},
\]

\[
H_k = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \quad R_k = \sigma_r^2
\]

where \( T = 0.001 \) is the time interval, \( q \) is the power spectral density of process noise, \( \sigma_r \) is the standard deviation of measurement noise. And the initial conditions used in simulations are shown in Table 1. Note that \( j_k \) is sampled from \( j(t) \).
Table 1 Initial conditions of simulations.

| Case | initial state vector | \(j(t)\) | \(q\) | \(\sigma_x\) |
|------|----------------------|----------|------|---------|
| 1    | \([1110, 10, 2]\)    | \(1+0.1t+0.05t^2\) | 25    | 1       |
| 2    | \([1110, 10, 2]\)    | \(20\sin(0.2t)\)     | 25    | 1       |

4.1. IFC Design Results

The eigenvalue with the maximum module in (14) (denoted by \(\lambda_d\)) can be regarded as the dominant pole. The strategy of selecting feedback gain is to minimize \(|\lambda_d|\). The design results are shown in Figure 1. Note that \(\omega_d\) is the eigenfrequency of \(\lambda_d\) and is computed by \(\exp(\omega_d T) = |\lambda_d|\). (b) shows positive correlation between \(q\) and \(\omega_d\), while negative correlation between \(\sigma_r\) and \(\omega_d\). It can be inferred that larger \(\sigma_r\) may deteriorate the stability and fast responsiveness of IFKF, while increasing \(q\) seems to have positive effects in these two aspects considering that \(\omega_d\) generally quantifies the stability margin of a system.

4.2. Tracking Performance Analysis

The tracking performance comparisons are carried out between IFKF and Kalman filter using ideal filtering model (denoted by IF). Note that the filtering performance of IF is optimal because \(kj\)'s are set as the ideal value. The quadratic and sine signals are used. And the initial simulation conditions are shown in table 1. Note that \(j_k\) is sampled from \(j(t)\). For tracking accuracy performance analysis, the root-mean-square error (RMSE) is used as the criteria.

The results of Case 1 are shown in Figure 2, while Case 2 are shown in Figure 3. In case one, it can be concluded that the IFC input (value of jerk in IFKF) can well track the ideal one. The filtering accuracy performance of IFKF is good and is only slightly inferior to the ideal one, say, the average position RMSE of IFKF after convergence is 0.0090 while that of IF is 0.0089. In Case 2, when tracking the sine signal, the filtering accuracy performance of IFKF is a little degraded, say, the average acceleration RMSE of IFKF after filtering convergence is increased from 1.73 in Case 1 to 1.85 in Case 2. This degradation is probably because the exosystem is uncertain and the compensation of the proposed IFC is not perfectly accuracy. But the gap between IFKF and IF is still very small, say,
the position accuracy performance penalty is no more than 6%. In a word, IFKF performs well in the two cases.

![Graph](image1.png)

**Figure 2.** Filtering results of Case 1.

![Graph](image2.png)

**Figure 3.** Filtering results of Case 2.

5. Conclusion

In this paper, a novel innovation feedback Kalman filter (IFKF) is newly proposed to solve the filtering problem with uncertainty. By introducing innovation feedback controller (IFC) into the filtering equations of IFKF and exploring the estimate bias propagation mechanism, the intractable filtering problem with uncertainty is transformed into a well-known output regulation problem with uncertain exosystem, which is the major creativeness of IFKF. Therefore, the progresses in solving the output regulation problem can also be used to develop IFCs with higher tracking accuracy and better stability performance in IFKF and the solutions to the concerted filtering problem is broadened as well. As an example, the IFC with universal applicability is proposed subsequently, which uses the linear
combination of innovation and its sum to compose feedback correction signal. Take standard Kalman filter as framework, IFKF with the proposed IFC adds only two equations in filtering algorithm to deal with the filtering problem with uncertainty. That is rather abbreviated and effective, especially for the demanding cases that require fast state estimating with less computing resources. After that, the application of the proposed IFKF to one-dimension target tracking is realized and studied. And simulations reveal its significant outperformance when tracking quadratic and sine signals. In addition, IFKF can also be applied to other filtering or state estimation problems with uncertain parameters, and other appropriate IFCs are applauded as well.

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