Exact solution for surface wave to space wave conversion by periodical impenetrable metasurfaces

Svetlana N. Tcvetkova, Stefano Maci, and Sergei A. Tretyakov

Abstract—We show that there exists an exact solution for a lossless and reciprocal periodic surface impedance which ensures full conversion of a single-mode surface wave propagating along the impedance boundary to a single plane wave propagating along a desired direction in free space above the boundary. In contrast to known realizations of leaky-wave antennas, the optimal surface reactance modulation which is found here ensures the absence of evanescent higher-order modes of the Floquet wave expansion of fields near the radiating surface. Thus, all the energy carried by the surface wave is used for launching the single inhomogeneous plane wave into space, without accumulation of reactive energy in higher-order modes. The results of the study are expected to be useful for creation of leaky-wave antennas with the ultimate efficiency, and, moreover, can potentially lead to novel applications, from microwaves to nanophotonics.

Index Terms—Metasurfaces, surface waves, leaky wave antennas, boundary conditions, surface impedance

I. INTRODUCTION

Surface-to-propagating wave conversion (or vice versa) is one of the classical problems in antenna theory and techniques, as well as in plasmonics and nanophotonics. Conventionally, such transformation is performed by using leaky-wave antennas [1], [2], which can be realized as periodically modulated open or partially open waveguides as well as other structures with periodic perturbations [3]–[6]. The field structure in the vicinity of open modulated waveguides or reactive surfaces contains higher-order Floquet harmonics, which store reactive fields, limiting the antenna bandwidth.

It is of fundamental importance to find out if an exact solution for the properties of radiating apertures can be found, such that the surface (assuming that it is infinite) would ideally convert a single surface wave into a single propagating plane wave. Some attempts to achieve perfect conversion between space and guided modes are known. A periodic metasurface engineered using the generalized law of reflection [7], which leads to the requirement of a linear reflection phase is reported in [8]. Power conversion efficiency of nearly 100% was claimed in that paper. However, the surface wave is not an eigensolution of the designed metasurface, therefore the structure does not support propagation of the surface wave along the surface. As a result, the reported infinitely long gradient converter operates as a nearly perfect absorber at the steady state. For periodic finite-length supercell-based gradient converters [9] which suffer from the decoupling effect taking place at the interface between supercells, the power conversion efficiency of 78% was predicted. Further, transformation of a propagating wave into a surface wave using passive and lossy periodic structures was discussed in [10], but the reported power conversion efficiency of such metasurface is quite low (≈ 7%). In [11], [12], transformation of surface wave from a beam wave and back into a beam wave using transparent metasurfaces was discussed. However, the propagating beam launched by the structure appears to be generated by the active constituents of the metasurface instead of being converted from the surface wave propagating along the metasurface. In [13], the efficiency of a modulated metasurface antenna with non-uniform modulation is found for a finite circular-aperture antenna. The conversion is obtained by changing the local attenuation parameter to synthesize a specific shape of the aperture amplitude distribution. This allows practical aperture efficiency of 80%, including control of the spill-over effect at the edge.

Recently, new means for the power conversion efficiency enhancement have been proposed for the case of homogeneous plane wave to surface wave conversion using metasurfaces [14]. The authors assumed a TE-polarized homogeneous normally incident plane wave as an illumination and transformed it into one TM-polarized surface wave with a linear power growth along the surface. Converting the incident propagating wave into an orthogonally polarized surface wave helps to minimize generation of unwanted evanescent Floquet harmonics, but among the found solutions the only solution of the boundary problem which satisfies Maxwell’s equations exactly (still imposing essential limitations on the size of the radiating structure) is nonreciprocal and active/lossy. The authors claimed that the most desirable lossless and reciprocal solution can be found approximately, and the power conversion efficiency in this case can reach up to 95% for a 20λ-long surface.

In this paper, we show that in fact an exact solution exists if the single propagating plane wave in space is an inhomogeneous plane wave. Within this assumption, the required local power balance can be realized exactly in the system of only one surface-bound mode and one propagating plane wave. An infinite-area lossless and reciprocal surface with the properties found here operates as an ideal leaky-wave antenna, launching only one plane wave in a given direction, not storing any reactive energy in higher-order Floquet modes. The conversion efficiency is predicted to be theoretically ideal and numerically tested to be above 99%. The found solution can be considered as a reference solution for modulated impedance leaky-wave antennas.

S. N. Tcvetkova and S. A. Tretyakov are with the Department of Electronics and Nanoengineering, Aalto University, P.O. Box 15500, FI-00076 Aalto, Finland (e-mail: svetlana.tcvetkova@aalto.fi).
S. Maci is with Department of Information Engineering and Mathematics, University of Siena, Via Roma 56, 53100, Siena, Italy.
II. PROBLEM STATEMENT

Let us assume that an infinite lossless boundary (defined by its surface impedance) is illuminated by a plane wave, for example, at normal incidence. Our goal is engineering the surface impedance of the boundary so that all the incident power carried by the plane wave is converted into a single surface wave maintained by the impedance boundary and propagating into a desired direction. While we study the limiting case of an infinite surface, neglecting scattering by the edges, in practice, the size of the receiving surface is finite and the received energy is collected by a matched load at the edge of the receiving surface. We assume that the surface is reciprocal, thus, it can be also seen as converting a surface wave into a leaky-wave propagating in space.

There are two main difficulties in realizing ideal converters. Similarly to the anomalous reflectors, one of the problems is to avoid active regions on impenetrable surfaces [15]. Because the propagating plane wave and the surface wave in general interfere, the required scalar surface impedance has both real and imaginary parts, which corresponds to a lossless in the average, but locally active and lossy converting surface. It is known that for anomalous reflectors this problem can be overcome if the incident and anomalously reflected waves are orthogonally polarized [15], and we adopt the same approach for the thought ideal converter between propagating and surface waves.

Another problem is to ensure power balance between the power carried by the incident propagating plane wave and the power accepted by the surface mode at every point of the boundary. Since we want to realize wave conversion using a locally responding and lossless impedance surface, the normal component of the real part of the total Poynting vector must be equal to zero at every point of the surface. However, the Poynting vector of the incident plane wave is uniform over the surface, but all possible surface modes supported by periodic impedance boundaries have exponential field attenuation (or growth), which does not match the distribution of the incident power over the receiving surface. Therefore, the exact power balance cannot be satisfied within this scenario (see detailed discussion in [14]). We expect that exact conversion between a single surface mode and a single inhomogeneous propagating wave may be possible, because in this case the fields of both surface and space waves have exponential coordinate dependence. Here we explore this possibility and show that an exact solution indeed exists.

The ideal conversion scenario (no dissipation or scattering losses) of one TM-polarized surface wave into a single TE-polarized inhomogeneous plane wave is illustrated in Fig. 1. The surface wave is launched along an impenetrable reactive metasurface characterized by a periodic tensor surface impedance $Z(x) = \mathbf{X}(x)$ with period $d$. For simplicity, we assume that both fields are invariant with respect to the $z$-axis. The design goal is to find the periodic distribution of $Z(x)$ that allows to perform perfect conversion, while the fields of both waves satisfy Maxwell’s equation in the upper half-space (which is considered to be free space).

A complete eigen-mode basis set for the 2D problem in the absence of sources is given by Floquet-wave modes. The modes are inhomogeneous plane waves with $x$-components of the complex wavenumbers given by

$$ k_{xn} = \beta_x - j\alpha_x + \frac{2\pi n}{d}, \quad (1) $$

where $n$ is the Floquet mode index, $\beta_x$ is the propagating constant, and $\alpha_x$ is the attenuation constant. Let us set the surface wave as the 0-indexed mode of the Floquet expansion. The TM-polarized surface wave field components are given by

$$ H_z = H_0 e^{-(\alpha_x + j\beta_x x)} e^{-(\alpha_y + j\beta_y y)}, \quad (2) $$

$$ E_x = -H_0 \frac{(\beta_y - j\alpha_y)\eta_0}{k_0} e^{-(\alpha_x + j\beta_x x)} e^{-(\alpha_y + j\beta_y y)}, \quad (3) $$

$$ E_y = H_0 \frac{(\beta_x - j\alpha_x)\eta_0}{k_0} e^{-(\alpha_x + j\beta_x x)} e^{-(\alpha_y + j\beta_y y)}. \quad (4) $$

The components of the complex wavenumber are

$$ k_x = \beta_x - j\alpha_x, \quad (5) $$

$$ k_y = \beta_y - j\alpha_y, \quad (6) $$

where $\beta_x$ and $\beta_y$ are the propagation constants of the surface wave, $\alpha_x$ and $\alpha_y$ are the attenuation constants of the surface wave ($x$- and $y$-components, respectively). Complex wavenumbers satisfy the free-space dispersion relation

$$ (\beta_x - j\alpha_x)^2 + (\beta_y - j\alpha_y)^2 = k_0^2, \quad (7) $$

where $k_0$ is the free-space wavenumber. Separation of the real and imaginary parts leads to

$$ \beta_x = -\beta_y \alpha_x \alpha_y, \quad (8) $$

$$ (\beta_x)^2 - \alpha_x^2 + (\beta_y)^2 - (\alpha_y)^2 = k_0^2. \quad (9) $$

Any combination of the values of the four parameters that satisfy (8) and (9) corresponds to fields which satisfy Maxwell’s equations above the boundary, at $y > 0$. An evanescent in the $y$-direction and decaying along the $x$-direction wave is associated with the conditions $\beta_x > k_0$ and $\alpha_x > 0$. Thus, $\beta_y < 0$ and $\alpha_y > 0$.

If the impedance modulation period $d$ is smaller than the free-space wavelength at the operation frequency, the propagation constant of the $-1$-indexed mode is smaller than $k_0$,
thus the wave becomes a propagating one. In other words, the energy is leaking from the surface. The complex wavenumber of the $-1$-indexed mode is corresponding to a leaky-wave mode, which exponentially decays along the surface:

$$k_{lw}^x = \beta_{lw}^x - j\alpha_{lw} - \frac{2\pi}{d} = \beta_{lw}^x - j\alpha_{lw}. \quad (10)$$

Here, $\alpha_{lw}$ is identical for both the surface and the leaky waves. Let us consider the case when the phase constant of the leaky wave has negligibly small tangential component: $\beta_{lw}^x \approx 0$. Therefore, from (10), $\beta_{lw}^x \approx 2\pi/d$. Thus, the TE-polarized leaky wave field components are expressed as

$$E_{z_{lw}} = E_{0_{lw}} e^{-\alpha_{lw} z} e^{-j\beta_{lw}^y y}, \quad (11)$$

$$H_{x_{lw}} = \frac{E_{0_{lw}}}{\eta_0 k_0} e^{-\alpha_{lw} z} e^{-j\beta_{lw}^y y}, \quad (12)$$

$$H_{y_{lw}} = \frac{jE_{0_{lw}}}{\eta_0 k_0} e^{-\alpha_{lw} z} e^{-j\beta_{lw}^y y}. \quad (13)$$

with the free-space dispersion relation given by

$$(\beta_{lw}^y - j\alpha_{lw})^2 + (\beta_{lw}^y - j\alpha_{lw})^2 = k_0^2 \quad (14)$$

or, in case of leakage along the normal direction,

$$(\beta_{lw}^y)^2 - \alpha_{lw}^2 = k_0^2. \quad (15)$$

The wave coupler as a point-wise lossless impenetrable metasurface can be realized when the superposition of the decaying surface wave and the leaky wave propagating away from the surface satisfy zero net power density penetration in the $xz$-plane everywhere \cite{14, 16}. Therefore, the normal component of the total power density is required to be equal zero on the boundary ($y=0$):

$$S_y(x, y = 0) = S_{y_{lw}} + \mathcal{S}_{y_{lw}}^0 + \frac{1}{2} \frac{\beta_{lw}^y \eta_0}{k_0} H_{0_{lw}}^y e^{-2\alpha_{lw} x} + \frac{1}{2} \frac{\beta_{lw}^y}{k_0 \eta_0} E_{0_{lw}} e^{-2\alpha_{lw} x} = 0, \quad (16)$$

where $S_{y_{lw}}$ and $\mathcal{S}_{y_{lw}}^0$ are the normal components of the Poynting vector associated with the surface wave and leaky wave, respectively. This condition defines the following magnetic field magnitude of the surface wave:

$$H_{0_{lw}}^y = \frac{E_{0_{lw}}}{\eta_0} \sqrt{-\frac{\beta_{lw}^y}{\beta_{lw}^y}} \quad (17)$$

where $\beta_{lw}^y$ is a negative value, as mentioned above. Fulfilling this condition ensures that all the power from the surface wave is entirely converted in the single radiated leaky wave. Reciprocally, the power density transported by the incident propagating wave with the same parameters is entirely received by the surface wave.

### III. Solution and Results

To find the type of the surface needed for ideal conversion, the impedance boundary condition at $y=0$ is considered. The total tangential fields on the surface are given by

$$\mathbf{H}_t = zH_{t_{xz}} + xH_{t_{xz}} = zH_{0_{lw}} e^{-(\alpha_{lw} + \beta_{lw}^y) x} + xE_{0_{lw}} \frac{\beta_{lw}^y}{\eta_0 k_0} e^{-\alpha_{lw} x}, \quad (18)$$

$$\mathbf{E}_t = zE_{t_{xz}} + xE_{t_{xz}} = zE_{0_{lw}} e^{-\alpha_{lw} x} - \frac{x H_{0_{lw}}^y}{k_0} (\beta_{lw}^y - j\alpha_{lw}) \eta_0 e^{-(\alpha_{lw} + \beta_{lw}^y) x}, \quad (19)$$

where $E_{t_{xz}}$, $H_{t_{xz}}$, and $H_{t_{xz}}$, $H_{t_{xz}}$ are the tangential components of the electric and magnetic fields, respectively.

The surface located at $y = 0$ can be characterized by a surface impedance tensor $\mathbf{Z}$, which relates the total tangential electric and magnetic fields as

$$\mathbf{E}_t = \mathbf{Z} \cdot (\mathbf{y} \times \mathbf{H}_t) = \begin{bmatrix} Z_{xx} & Z_{xz} \\ Z_{zx} & Z_{zz} \end{bmatrix} \cdot \begin{bmatrix} y \\ x \end{bmatrix} \cdot \mathbf{H}_t, \quad (20)$$

where $Z_{xx}$, $Z_{xz}$, $Z_{zx}$, and $Z_{zz}$ are components of the anisotropic impedance tensor, which generally can be complex numbers. Since there are two complex-valued equations in (20) for four complex-valued impedance elements, the solution is not unique. Thus, it gives some freedom to choose the values of the impedance elements. The most desirable and practically implementable case of lossless and reciprocal system is considered further. All the elements of the impedance matrix for such boundaries are purely imaginary and the matrix is skew-Hermitian: $Z_{xx} = jX_{xx}$, $Z_{xz} = Z_{zx} = jX_{xz}$, and $Z_{zz} = jX_{zz}$. Inspection of (18), (19) along with the use of (17) allow to recognize the four components of the impedances as

$$\mathbf{Z} = \frac{j}{k_0} \begin{bmatrix} \eta_0 (\alpha_{lw}^y - \beta_{lw}^y \cot \beta_{lw}^y x) & \eta_0 \frac{\alpha_{lw}^y}{\sin \beta_{lw}^y x} \sqrt{-\frac{\beta_{lw}^y}{\beta_{lw}^y}} \\ \eta_0 \frac{\alpha_{lw}^y}{\sin \beta_{lw}^y x} \sqrt{-\frac{\beta_{lw}^y}{\beta_{lw}^y}} & \eta_0 \frac{\beta_{lw}^y}{\beta_{lw}^y} \cot \beta_{lw}^y x \end{bmatrix} \quad (21)$$

Figure 2 shows the surface reactance values $\Re \mathbf{Z}$ for some chosen values of the wave parameters.

We note that if the impedance boundary conditions for the reciprocal and lossless system are satisfied by one surface wave and a single leaky wave, the exact solution of the
boundary value problem for an impedance surface $\mathbb{X}$ does not include any higher-order Floquet modes of the field expansion. In the following, a numerical verification is presented.

A. Visualization of the exact solution

We consider a rectangular cross-section area in the $xy$-plane with the length $L = 20\lambda$ and the height $H = 5\lambda$, where $\lambda$ is the free-space wavelength. Impenetrable impedance boundary condition is used to model the surface by impressing electric currents specified in terms of electric fields and a surface admittance matrix. The propagating constants of the surface wave for the case of $20\lambda$-long surface are chosen to be $\beta_x = 1.1k_0$, $\beta_y = -0.1k_0$, therefore, from (8), (9) and (15) all other parameters of the waves can be found, and they read $\alpha_x = 0.0425k_0$, $\alpha_y = 0.4671k_0$, $\beta_x = 0$, $\beta_y = 1.0009k_0$, and $\alpha_{yw} = 0$. The operating frequency is $f = 10$ GHz, but the results can be scaled to any frequency.

Figures 3(a) and (b) visualize the exact field solutions for the $z$-components of the magnetic field of the TM-polarized surface wave and of the electric field of the TE-polarized inhomogeneous plane wave, respectively. The results are obtained using MATLAB [17]. The next step is to verify how close the conversion of the surface wave into a leaky wave using the surface impedance to the ideal picture.

B. Numerical full-wave analysis

By using full-wave simulations with COMSOL Multiphysics [18], the conversion efficiency [12] of the metasurface can be found. The parameters are identical to the MATLAB simulations presented above. On the top and on the right boundaries of the computation domain the perfectly matched layer condition is specified by the box with the thickness of $\lambda/2$. Double-port condition is set on the left side of the box to allow the surface wave excitation with the specified settings, while not perturbing the converted inhomogeneous plane wave (set in the software manually). The general mesh of the model has the maximum size of the element equal to $\lambda/20$, while finer mesh elements required along the impedance boundary are $\lambda/300$. Figures 4(a) and (b) show the $z$-component of the total electric and magnetic fields, respectively, for the lossless and reciprocal system. The conversion efficiency is determined as the ratio of the power carried by the TE-polarized inhomogeneous plane wave, which is detected at the distance of $5\lambda$ away from the surface to the power launched by the port and carried by TM-polarized surface wave. For the considered case the conversion efficiency of the surface with the length of $20\lambda$ is around 99.8%. Next, a possibility of shortening the converting surface length is considered.

C. Surface length reduction

Figure 5 shows the model with a rectangular cross-section box in the $xy$-plane with the length $L = 6\lambda$ and the height $H = 5\lambda$. The new mesh used is the following: the whole model is divided into $900$ elements with the maximum size of the element equal to $\lambda/20$, while finer mesh elements required along the impedance boundary are $\lambda/300$. Figures 6(a) and (b) show the $z$-component of the total electric and magnetic fields, respectively, for the lossless and reciprocal system. The conversion efficiency is determined as the ratio of the power carried by the TE-polarized inhomogeneous plane wave, which is detected at the distance of $5\lambda$ away from the surface to the power launched by the port and carried by TM-polarized surface wave. For the considered case the conversion efficiency of the surface with the length of $6\lambda$ is around 99.8%. Next, a possibility of shortening the converting surface length is considered.
$H = 5\alpha$. The surface wave propagation constants are chosen

$$\begin{align*}
\text{Fig. 5: Numerically computed distributions of (a) TM-polarized magnetic field } &\text{Re}(H_z) \text{ of the single surface wave and (b) TE-polarized electric field } \text{Re}(E_z) \text{ of the inhomogeneous plane wave with the following wave parameters: } \\
\alpha_x &= 0.0844k_0, \quad \alpha_y = 0.6748k_0, \quad \beta_x = 1.2k_0, \quad \beta_y = -0.15k_0, \\
\beta_w &= 0, \quad \alpha_{yw} = 1.0036k_0, \text{ and } \alpha_{yw} = 0.
\end{align*}$$

to be different from the previous case to ensure full leakage, and they are equal to $\beta_x = 1.2k_0, \beta_y = -0.15k_0$. From [8], [9] and [15] all other parameters of the waves can be found: 

$$\begin{align*}
\alpha_x &= 0.0844k_0, \quad \alpha_y = 0.6748k_0, \quad \beta_x = 0, \quad \beta_y = 1.0036k_0, \\
\alpha_{yw} &= 0. \quad \text{Conversion efficiency of the surface with the length of } 6\alpha \text{ with these wave parameters is around } 99\%, \text{ while about } 1\% \text{ of the energy is still not converted and carried by the surface wave.}
\end{align*}$$

D. Suggestions for implementation by eigenvector analysis

One possibility for simplifying the realization of such a surface is to transform the impedance matrix (21) in Cartesian coordinates into a diagonal one written in a new basis. This transformation is equivalent to finding the eigenvalues (diagonal components) and the eigenvectors (new basis vectors) of the impedance matrix. The eigenvalues can be easily calculated solving the equation $|\xi I - Z| = 0$, where $\xi$ is an eigenvalue, $I$ is the identity matrix. Finally, eigenvalues are given by

$$\xi_{1,2} = \frac{1}{2} \left[ j(X_{xx} + X_{zz}) \pm \sqrt{-4X_{zz}X_{xx} - (X_{xx} - X_{zz})^2} \right],$$

Figure 6(a) depicts two eigenvalues of the surface impedance along one period $d$. The eigenvalues are purely imaginary and widely vary within a period spanning from capacitive to inductive values.

Eigenvectors $\nu_1$ and $\nu_2$ correspond to the found eigenvalues $\xi_1$ and $\xi_2$, and consist of two components (projections in the Cartesian coordinates) each, $\nu_{1,1}$, $\nu_{1,2}$ and $\nu_{2,1}$, $\nu_{2,2}$, respectively, that relate as

$$\nu_{1,2} = -\frac{jX_{xx} - \xi_1}{jX_{xz}} \nu_{1,1}, \quad \nu_{2,2} = -\frac{jX_{xx} - \xi_2}{jX_{xz}} \nu_{2,1}. \quad (23)$$

We plot Figure 6(b) assuming $\nu_{1,1} = \nu_{2,1}$. The arrows show the direction for each of the eigenvectors as a function of the coordinate along the surface. Here, the surface is divided into ten equal cells per period and the eigenvalue from the middle of the cell is used for eigenvector calculation.

Eigenvalues and eigenvectors give the information of the resonant inclusions type and its reactive impedance value as well as its orientation on the surface plane. Design and practical realization of such converters is a scope of future
research.

IV. CONCLUSIONS

The problem of ideal conversion of a propagating plane wave into a surface wave has been examined theoretically and numerically. It is important to note, that the found exact solution of the boundary value problem for the reciprocal and lossless system is satisfied by one surface wave and one leaky wave only. Therefore, no higher-order Floquet modes of the asymptotic expansion which store the energy are excited near the radiating modulated reactive surface. It means that all the power carried by the surface wave is used for the creation of the leaky wave without storing reactive energy in higher order Floquet-modes. It is expected that the found exact solution represents a reference for minimal possible Q-factor for any leaky wave antenna, thus establishing a bound of maximum bandwidth. This investigation of maximum bandwidth will be the subject of an incoming paper.

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REFERENCES

[1] A. A. Oliner and D. R. Jackson, “Leaky-wave antennas,” in Antenna Engineering Handbook, 4th ed., J. L. Volakis, Ed. New York: McGraw-Hill, 2007, ch. 11.
[2] F. Monticone and A. Alù, “Leaky-wave theory, techniques, and applications: from microwaves to visible frequencies,” Proc. IEEE, vol. 103, no. 5, pp. 793–821, May 2015.
[3] B. B. Tierny and A. Grbic, “Arbitrary leaky-wave antenna patterns with stacked metasurfaces,” in Proc. 2015 IEEE Antennas Propag. Soc. Int. Symp., Vancouver, Canada, Jul. 2015, pp. 1088–1089.
[4] G. Minatti, F. Caminita, E. Martini, M. Sabbadini, and S. Maci, “Synthesis of modulated-metasurface antennas with amplitude, phase, and polarization control,” IEEE Transactions on Antennas and Propagation, vol. 64, no. 9, pp. 3907–3919, Sept 2016.
[5] G. Minatti, F. Caminita, E. Martini, and S. Maci, “Flat optics for leaky-waves on modulated metasurfaces: Adiabatic floquet-wave analysis,” IEEE Transactions on Antennas and Propagation, vol. 64, no. 9, pp. 3896–3906, Sept 2016.
[6] J. Martinez-Ros, J. L. Gómez-Tornero, and G. Goussetis, “Planar leaky-wave antenna with flexible control of the complex propagation constant,” IEEE Trans. Antennas Propag., vol. 60, no. 3, pp. 1625–1630, Mar. 2012.
[7] N. Yu, P. Genevet, M. A. Kats, F. Aieta, J.-P. Tetienne, F. Capasso, and Z. Gaburro, “Light propagation with phase discontinuities: Generalized laws of reflection and refraction,” Science, vol. 334, no. 6054, pp. 333–337, 2011. [Online]. Available: http://science.sciencemag.org/content/334/6054/333.
[8] S. Sun, Q. He, S. Xiao, Q. Xu, X. Li, and L. Zhou, “Gradient-index meta-surfaces as a bridge linking propagating waves and surface waves,” Nat. Mater., vol. 11, pp. 426–431, May 2012.
[9] C. Qu, S. Xiao, S. Sun, Q. He, and L. Zhou, “A theoretical study on the conversion efficiencies of gradient meta-surfaces,” EPL (Europhysics Letters), vol. 101, no. 5, p. 54002, 2013. [Online]. Available: http://stacks.iop.org/0295-5075/101/i=5/a=54002.
[10] N. Mohammadi Estakhri and A. Alù, “Wave-front transformation with gradient metasurfaces,” Phys. Rev. X, vol. 6, p. 041008, Oct 2016. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevX.6.041008.
[11] K. Achouri and C. Caloz, “Surface wave routing of beams by a transparent birefringent metasurface,” in Proc. 10th Int. Congress Adv. Electromagn. Mater. Microw. Opt. (Metamaterials 2016), Crete, Greece, Sep. 2016, pp. 13–15.
[12] K. Achouri and C. Caloz, “Space-Wave Routing via Surface Waves Using a Metasurface System,” ArXiv e-prints, Dec. 2016.
[13] G. Minatti, E. Martini, and S. Maci, “Efficiency of metasurface antennas,” IEEE Transactions on Antennas and Propagation, vol. 65, no. 4, pp. 1532–1541, April 2017.
[14] S. N. Tsvetkova, D.-H. Kwon, A. Díaz-Rubio, and S. A. Tretyakov, “Near-perfect conversion of a propagating plane wave into a surface wave using metasurfaces,” Phys. Rev. B, vol. 97, p. 115447, Mar 2018. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevB.97.115447.
[15] V. S. Asadchy, M. Albooyeh, S. N. Tsvetkova, A. Díaz-Rubio, Y. Ra‘dî, and S. A. Tretyakov, “Perfect control of reflection and refraction using spatially dispersive metasurfaces,” Phys. Rev. B, vol. 94, p. 075142, Aug 2016. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevB.94.075142.
[16] D.-H. Kwon and S. A. Tretyakov, “Perfect reflection control for impenetrable surfaces using surface waves of orthogonal polarization,” Phys. Rev. B, vol. 96, p. 085438, Aug 2017. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevB.96.085438.
[17] https://www.mathworks.com/.
[18] https://www.comsol.com/.