VORTICES AND CONFINEMENT

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Abstract. We review recent developments in the vortex picture of confinement. We discuss numerical simulations demonstrating that the entire asymptotic string tension is due to vortex-induced fluctuations of the Wilson loop. Analytical and numerical results concerning the presence of vortices as the necessary and sufficient condition for confinement at arbitrarily weak coupling in SU(N) gauge theories are also discussed.\footnote{Presented by E.T. Tomboulis at the workshop “Lattice fermions and structure of the vacuum”, 5-9 October 1999, Dubna, Russia.}

1. Introduction

The proposal that extended vortex configurations are responsible for maintaining confinement at arbitrarily weak coupling in SU(N) gauge theories has a long history [1] - [6]. Several important results were established by the early eighties. Over the last two years, the vortex picture of confinement has undergone very substantial development by a series of numerical investigations as well as new analytical results [7]-[20].

It was originally conjectured that thick vortices occur with nonvanishing measure contribution in the path integral at arbitrarily large beta, and that this provides a sufficient mechanism for confinement. In light of recent developments, it now appears that not only is this contribution sufficient but also necessary: it is responsible for the full string tension of large Wilson loops in SU(N) gauge theories. After briefly recalling basic features of
the vortex picture, we discuss some of the recent numerical and analytical results underlying these developments. Conclusions are presented in section 7.

2. Physical Picture

A vortex configuration of the gauge field may be characterized by a multi-valued singular $SU(N)$ gauge transformation function $V(x)$. The multi-valuedness ambiguity lies in the center $Z(N)$, so the transformation is single-valued in $SU(N)/Z(N)$. If one attempts to extend such a gauge transformation $V(x)$ throughout spacetime, it becomes singular on a closed surface $V$ of codimension 2 (i.e. a closed loop in $d = 3$, a closed 2-dimensional sheet in $d = 4$) forming the topological obstruction to a single-valued choice of $V(x)$ throughout spacetime. Generic vortex configurations of the gauge potentials then consist of a pure-gauge long-range tail given by $V(x)$, and a core enclosing the region where $V$ would be if one were to try to smoothly extend $V$ everywhere. Equivalently, the configuration cannot be smoothly deformed to pure gauge everywhere without encountering a topological obstruction $V$. Note that it is only the existence of this obstruction, and not its precise location, that is relevant; the location may always be moved around by a regular gauge transformation. The asymptotic pure-gauge part provides then a topological characterization of the configurations irrespective of the detailed structure of the core.

Assume now that two gauge field configurations $A_\mu(x)$ and $A'_\mu(x) = VAV^{-1} + V\partial_\mu V^{-1}$ differ by such a singular gauge transformation $V(x)$, and denote the path ordered exponentials of $A_\mu$ and $A'_\mu$ around a loop $C$ by $U[C]$ and $U'[C]$, respectively. Then $\text{tr} U'[C] = z \text{tr} U[C]$, where $z \neq 1$ is a nontrivial element of the center, whenever $V$ has obstruction $V$ linking with the loop $C$; otherwise, $z = 1$. Conversely, changes in the value of $\text{tr} U[C]$ by elements of the center can be undone by singular gauge transformations on the gauge field configuration linking with the loop $C$. This means that vortex configurations are topologically characterized by elements of $\pi_1(SU(N)/Z(N)) = Z(N)$. This topological $Z(N)$ flux is of course conserved only mod $N$, and, hence, the number of vortices in a given gauge field configuration in a given spacetime region can be defined only mod $N$.

Vortex configurations, if sufficiently spread out, can be present at any nonzero coupling. This is because, by spreading the flux over a sufficiently ‘thick’ core, one can incur sufficiently small cost in local action so that the configuration is not energetically suppressed at large beta; while at the same time there is very substantial disordering over long distances. In this way UV asymptotic freedom can coexist with IR confinement. The crucial question then is whether this class of configurations contributes with enough
weight in the path integral measure to provide a sufficient mechanism for confinement at large beta. One may suspect that this is so since it is easily seen that, given one vertex configuration, an enormous number of others may be produced by fluctuations that do not alter the vorticity content.

The vortex picture of confinement may then be summarized as follows. Confinement at arbitrarily weak coupling in $SU(N)$ gauge theories is the result of nonzero vorticity in the vacuum over sufficiently large scales. In other words, in any sufficiently large spacetime region, the expectation for the presence of a spread out vortex is nonzero at all large $\beta < \infty$. This is strikingly demonstrated by a recent lattice computation which shows that the relative probability for vortex excitation in fact approaches unity (section 6). Configurations carrying sufficiently thick vortices contribute with essentially the same weight as ones without vortices. In this sense one has a ‘condensate’ of vortex configurations.

On the lattice a discontinuous (singular) gauge transformation introduces a thin vortex. The topological obstruction $\mathcal{V}$ is regulated to a coclosed set of plaquettes (in $d = 4$ this is a closed 2-dimensional surface of dual plaquettes on the dual lattice). This represents the core of the thin vortex, each plaquette in $\mathcal{V}$ carrying flux $z \in \mathbb{Z}(N)$.

Thick vortex configurations can be constructed by perturbing the bond variables $U_b$ in the boundary of each plaquette $p$ in $\mathcal{V}$ so as to cancel the flux $z$ on $p$, and distribute it over the neighboring plaquettes. Continuing this process by perturbing bonds in the neighboring $p$’s one may distribute the flux over a thickened core in the two directions transverse to $\mathcal{V}$. Beyond the thickness of the core, the vortex contribution reduces to the original multivalued pure gauge. If the original thin vortex is long enough, it may be made thick enough, so that each plaquette receives a correspondingly tiny portion of the original flux $z$ that used to be on each $p$ in $\mathcal{V}$. Long thick vortices may therefore be introduced in $\{U_b\}$ configurations having $\text{tr}U_p \sim \text{tr}1$ for all $p$. (Here $U_p$ denotes the product of the $U_b$’s around the plaquette boundary.) Thus they may survive at weak coupling where the plaquette action becomes highly peaked around $\text{tr}U_p \sim \text{tr}1$. Long vortices may link with a large Wilson loop anywhere over the area bounded by the loop, thus potentially disordering the loop and leading to confining behavior. Thin vortices, on the other hand, necessarily incur a cost proportional to the size of $\mathcal{V}$, and only short ones can be expected to survive at weak coupling. These can link then only along the perimeter of a large loop generating only perimeter effects.
3. Isolation of Vortices and their Contribution - Simulation Results

Considerable activity has been devoted over the last two years to the isolation of vortices, and the computation of their contribution to the heavy quark potential and other physical quantities on the lattice. Several approaches have been pursued:

- Computation of the expectation of the Wilson loop fluctuation solely by elements of the center on smoothed configurations which remove short distance fluctuations.
- Center projection in maximal center gauge (MCG) and isolation of P-vortices.
- Direct computation of excitation probability of a vortex in the vacuum (magnetic-flux free energy).

Of these, the third is the most direct and physically transparent, and closely connected to the formulation of rigorous analytical results on the necessity and sufficiency of vortices for confinement. It also is computationally the most expensive. The result of a recent computation is presented in section 6 below. In this section we discuss the first two.

3.1. Quark Potential from Center Fluctuations on Smoothed Configurations

We saw above that the fluctuation in the value of $\text{tr}U[C]$ by elements of $Z(N)$, parametrizing the different $\pi_1(SU(N)/Z(N))$ homotopy sectors, expresses the changes in the number (mod $N$) of vortices linked with the loop over the set of configurations for which it is evaluated. One is not, however, interested in fluctuations produced by small thin vortices, which will still be present even at large beta but are irrelevant to the long distance physics. One would like to isolate only fluctuations by elements of the center due to extended configurations that can reflect long distance dynamics. To ensure this one performs local smoothing on the configurations which is constructed so that it removes short distance fluctuations but preserves long distance physics. There is another, in fact essential reason for employing smoothing: it ensures good topological representation of extended fluctuations on the lattice. (According to rigorous theorems, only for lattice configurations with sufficiently small variations of the plaquette function $U_p$ from its maximum is it possible to unambiguously define a continuum interpolation assignable to a topological sector.)

Separate out the $Z(N)$ part of the Wilson loop observable by writing $\arg(\text{tr}U[C]) = \varphi[C] + \frac{2\pi}{N} n[C]$, where $-\pi/N < \varphi[C] \leq \pi/N$, and $n[C] =$
0, 1, ..., N - 1. Thus, with \( \eta[C] = \exp(i \frac{2\pi}{N} n[C]) \in Z(N) \),

\[
W[C] = \langle \text{tr} U[C] \rangle = \langle \text{tr} U[C] | e^{i\varphi[C]} \eta[C] \rangle \\
= \langle \text{tr} U[C] | \cos(\varphi[C]) \cos(\frac{2\pi}{N} n[C]) \rangle,
\]

using the fact that the expectation is real by reflection positivity, and that it is invariant under \( n[C] \to (N - n[C]) \). Next define

\[
W_{Z(N)}[C] = \langle \cos(\frac{2\pi}{N} n[C]) \rangle.
\]

One then compares the string tension extracted from the full Wilson loop \( W[C] \), eq. (1), to the string tension extracted from \( W_{Z(N)}[C] \), eq. (2), on sets of progressively smoothed configurations [9]-[10]. Results have been obtained for \( N = 2 \) and \( N = 3 \) using the smoothing procedure in [21]. Typical results for the heavy quark potential for \( N = 3 \) [10] from six times smoothed lattices are shown in figure 1. There is striking coincidence of

![Figure 1](image)

*Figure 1.* The heavy quark potential at \( \beta = 6.0 \) on a set of 112 \( 12^3 \times 16 \) lattices extracted at time slice \( T=5 \) from 6 times smoothed lattices.
different number of smoothings. This is a very stringent test, as configurations subjected to different degrees of smoothings are vastly different, and indicates that this is an actual long-distance physics effect.

3.2. CENTER PROJECTION, MCG AND P-VORTICES

Most of the numerical simulations follow this approach. As almost all results in MCG are for \( N = 2 \), we discuss only this case. The method [11] consists of the following steps.

1. Fix the gauge by maximizing the quantity: \( \sum_b |\text{tr} U_b|^2 \). This is the MCG.

2. Make the center projection: \( U_b \rightarrow Z_b \) by replacing each \( SU(2) \)-valued bond variable by the closest center element \( Z_b \).

3. The excitations of the resulting \( Z(2) \) bond configurations are coclosed sets of plaquettes each carrying \( -1 \) flux, i.e. \( Z(2) \) vortices. These are the projection vortices (P-vortices).

The string tension extracted from loops of the center projected \( Z(2) \) variables is then found [11], [12] to reproduce the full asymptotic string tension of the \( SU(2) \) LGT.

The rational for the method is as follows. Consider two configurations of the bond variables \( U \) and \( U' \) that differ by a discontinuous (singular) gauge transformation introducing a vortex. The corresponding configurations in the adjoint representation \( U_A \) and \( U'_A \) are then gauge equivalent by a regular gauge transformation. Now go to MCG. Then \( U_A \) and \( U'_A \) go to the same MCG-fixed adjoint configuration \( \bar{U}_A \); whereas \( U \) and \( U' \) are transformed to \( \bar{U} \) and \( \bar{U}' \) corresponding to the same adjoint \( \bar{U}_A \). Hence \( U \) and \( U' \) can only differ by a discontinuous plus possibly regular \( Z(2) \) gauge transformations. Upon center projection, the projected \( Z \) and \( Z' \) configurations will also differ by the same discontinuous \( Z(2) \) transformation (plus possibly regular transformations), i.e one P-vortex (mod \( N \)) reflecting the vortex introduced by the singular transformation by which \( U \) and \( U' \) differ.

Note that the projected configurations \( Z \) and \( Z' \) give for any Wilson loop linking with the P-vortex values differing by a sign (nontrivial element of \( Z(2) \)). But, as we saw in the previous section, the value of the Wilson loop in the original \( U \) and \( U' \) will indeed differ by a sign if they differ by a discontinuous gauge transformation. In this way contact is made with the method in 3.1 above.

The above, however, relies on certain assumptions. It assumes that: a) \( U_A \) and \( U'_A \) are gauge equivalent everywhere; b) the gauge fixing of the adjoint links by the MCG is complete everywhere leaving only a residual \( Z(2) \) symmetry. It turns out, however, that, for the purpose of associating a P-vortex with a thick vortex, these assumptions cannot hold in general.
First it is evident that a) applies strictly only if the singular gauge transformation corresponds to a thin vortex; for a thick vortex it cannot apply in the thick core, hence there is no unique $\bar{U}_A$ everywhere. Furthermore, there is a pronounced Gribov copies problem. The MCG gauge fixing functional has in fact many local maxima, and in practice only a local maximum can be achieved. The result after projection can be strongly dependent on the chosen maximum.

All this is in fact inexorably connected with the physics of the problem. General non-Abelian configurations can have $U_p \sim 1$ everywhere, but the bond variables gradually wandering all over the group over long distances. In fact, as we argued, this is precisely why smooth extended vortices can survive at large beta. The MCG attempts to put every bond as closely to an element of the center as possible, and largely compress a thick vortex to a thin. It is to be expected that in general this cannot be achieved everywhere without gauge fixing ambiguities and Gribov problems.

Numerical demonstration of the Gribov copies problem was given in [18]. Starting from configurations fixed in the Lorentz gauge and then going to the MCG produces on average a maximum higher than starting from random gauge and then going to MCG. The resulting picture in the two cases is dramatically different. In the latter case the results of [11] are reproduced. In the former case there is essentially complete loss of string tension, while there is a drop of only about 40% in the density of P-vortices indicating that they cannot be associated with thick vortices contributing to the string tension, but only with short thin vortices.

It is also known [11] that slight local smoothing of the $SU(2)$ configurations also causes considerable decrease in the center projected string tension. This again indicates ambiguities in associating thick vortices with P-vortices that are introduced with expanded cores and additional smoothness over longer distances in the configurations.

In conclusion, gauge fixing can be a useful way of isolating vortices, but clearly further work is needed. A more sophisticated approach is called for which exhibits the vortex cores (topological obstructions) as an intrinsic property of the gauge field, and hence independent of the particular gauge fixing procedure adopted. A promising proposal along these lines was made in [20]. (Further discussion can be found in [8].)

4. Necessary Condition for Confinement at Weak Coupling

The numerical results above indicate that only the fluctuations between different $\pi_1(SU(N)/Z(N))$ homotopy sectors are responsible for the asymptotic string tension. Fluctuations among the same sector become irrelevant for large enough loops. Numerically, a rather delicate near cancellation be-
tween the different sectors occurs, resulting in area-law for the Wilson loop expectation (as opposed to an exponentially larger perimeter-law result) at weak coupling. This suggests that eliminating fluctuations between different sectors will result in vanishing string tension, i.e. loss of confinement at weak coupling. Since thick vortices are precisely the configurations allowing jumps between different sectors as the continuum limit is approached, this implies that their presence is a necessary condition for confinement.

A relevant rigorous result was in fact already obtained long ago in Ref. [4]. There it was shown that in the presence of constraints eliminating thick vortices completely winding around the lattice with periodic boundary conditions, the electric-flux free energy order parameter [1] in $SU(N)$ LGT exhibits non-confining behavior at arbitrarily weak coupling. The electric-flux free energy gives an upper bound on the Wilson loop [22]. To exhibit non-confining behavior for the Wilson loop itself, one needs a lower bound. In [17] we recently obtained the following rigorous result. Consider the expectation of the Wilson loop in the presence of constraints that eliminate from the functional measure all configurations that can represent thick vortices linking with the Wilson loop, i.e. allow the Wilson loop to fluctuate into the nontrivial $\pi_1$ homotopy sectors.

Then for sufficiently large $\beta$, and dimension $d \geq 3$ the so constrained Wilson loop expectation $W[C]$, exhibits perimeter law, i.e. there exist constants $\alpha, \alpha_1(d), \alpha_2(d)$ such that

$$W[C] \geq \alpha \exp \left( -\alpha_2 e^{-\alpha_1 \beta |C|} \right).$$

Here $|C|$ denotes the perimeter length of the loop $C$. In other words, the potential between two external quark sources is nonconfining at weak coupling. In [17] only the $SU(2)$ case is treated explicitly. The result is proven for a variety of actions. One class of actions considered is given by:

$$A_p(U) = \beta \text{tr} U_p + \lambda \text{sign}(\text{tr} U_p),$$

where $0 \leq \lambda < \infty$ extrapolates between the standard Wilson action ($\lambda = 0$) and the ‘positive plaquette action’ model ($\lambda \to \infty$). Another choice is:

$$A_p(U_p) = \beta \text{tr} U_p + \ln(\theta(|\text{tr} U_p| - k)),$$

i.e. Wilson action with an excised small ‘equatorial’ strip in $SU(2)$ of width $k$ such that $k/\beta$ large as $\beta$ becomes large; e.g. $k$ a small constant, or $k \sim 1/\beta^{1/2}$. All these actions have the same naive continuum limit and expected to be in the same universality class [23]. Choice of different actions serves to emphasize that the result is independent of the particular choice of YM action latticization.
It should be emphasized that the constraints do not eliminate thin vortices. This is achieved by employing the $SO(3) \times Z(2)$ formulation [3], [5] of the $SU(2)$ LGT. All constraints depend only on the $SO(3)$ coset variables, and thus any $Z(2)$ plaquette fluxes on thin vortices remain unaffected. We refer to Ref. [17] which contains detailed explicit derivations.

5. Sufficiency Condition - Lower Bound on the String Tension by the Excitation Probability for a Vortex

As already mentioned an upper bound on the Wilson loop is given by the electric-flux free energy order parameter [22]. This quantity is the $Z(N)$ Fourier transform of the magnetic-flux free energy [1]. The magnetic flux free energy order parameter is defined as the ratio $Z(z)/Z$ of the partition function with a ‘twisted’ action to that with the original (untwisted) action. The ‘twist’ inserts a nontrivial element $z \in Z(N)$, i.e. a discontinuous gauge transformation, in the action on every plaquette of a $(d-2)$-dim topologically nontrivial coclosed set of plaquettes $S^*$ (closed 2-dim surface of dual plaquettes on the dual lattice in $d = 4$) with periodic boundary conditions. Thus $\ln(Z(z)/Z)$ gives the free energy cost for exciting a vortex completely winding in $(d-2)$ spacetime directions around the lattice, and is also referred to as the vortex free energy. (Alternatively, one may consider appropriate fixed boundary conditions in the remaining two spacetime directions so that the winding vortex again remains trapped [3].) The upper bound on the Wilson loop in terms of the $Z(N)$ Fourier transform of such ‘vortex containers’ [22], [3], [6] implies area-law only if the vortex free energy remains finite in the large volume limit (in the Van Hove sense). Now to cancel a cost proportional to $L^{(d-2)}$ ($L$ lattice linear length), the system must respond by spreading the discontinuous gauge transformation on $S^*$ in the two transverse directions, i.e. the vortex free energy will remain finite only if the expectation for exciting an arbitrarily long, thick vortex remains finite. This provides then a sufficiency condition for confinement.

Recently, we have obtained an alternative lower bound on the string tension for $SU(2)$ which can be expressed directly in terms of the ‘t Hooft loop expectation (magnetic-disorder parameter) [1]. The ‘t Hooft operator amounts to a source exciting a $Z(2)$ monopole current on a coclosed set of cubes (closed loop of dual bonds on the dual lattice), and forming the coboundary of a set of plaquettes $S^*$ (forming the boundary of a $(d-2)$-dim surface of dual plaquettes) representing the attached Dirac sheet. The operator inserts a twist $(-1) \in Z(2)$ on each plaquette in $S^*$. In our case the monopole ‘loop’ is taken to be the minimal coclosed set of cubes consisting of the $2(d-2)$ cubes sharing a given plaquette $p$. The set $S^*$ attached to it winds around the lattice in the $(d-2)$ perpendicular directions. Again, to
cancel a cost proportional to $L^{(d-2)}$, the system must respond by spreading a discontinuous gauge transformation on $S^*$ in the two transverse directions, i.e. the operator gives the expectation for exciting a thick vortex ‘punctured’ by a short monopole loop. Now the presence of the ‘puncture’ by the small monopole loop (site of the 't Hooft loop source) is a purely local effect that can be extracted with fixed action cost (at finite lattice spacing). Shrinking the monopole loop to a point gives then the magnetic flux free-energy observable $Z(z)/Z$.

It is known that, for $SU(N)$, individual configurations exist giving vanishing vortex energy cost. The non-Abelian nature of the group is crucial for their existence. No construction of a finite measure contribution, hence no proof at the nonperturbative level is available though.

6. Measurement of the Vortex Free Energy

In the absence of an analytical proof, we have resorted to numerical evaluation of the magnetic flux free energy (vortex free energy) for $N = 2$.

Measurement was performed by combining Monte Carlo simulation with the multihistogram method of Ref. [24]. The method was used in [16] to compute the free energy of a $Z(2)$ monopole pair as a function of the pair’s separation. The method consists roughly of looking at the probability distribution of the energy along the twist (all other variables integrated out). This probability is reconstructed by combining histograms of the energy along the twist obtained from several simulations at different values of the coupling along the twist. The method tends to be computationally expensive. The result of our computation is shown in figure 2. The lattice spacings...
are $a = 0.119$ fm and $a = 0.085$ fm for $\beta = 2.4$ and $\beta = 2.5$, respectively. As expected by physical reasoning, not only does the vortex free energy cost remain finite as the lattice volume grows, but it tends to zero, i.e. the weighted probability for the presence of a vortex goes to unity for sufficiently large lattice. This reflects the exponential spreading of color-magnetic flux in a confining phase.

7. Conclusions

Simulations show that the full string tension is accounted for by center fluctuations of the Wilson loop insensitive to short distance details. The result is robust under local smoothings of configurations, consistent with the picture of thick vortex configurations in the vacuum being responsible for confinement at weak coupling.

The method of gauge fixing for associating vortices in the full theory with vortices in center-projected $Z(N)$ configurations (P-vortices) is, upon closer inspection, a rather tricky proposition. The common implementation (maximal center gauge) suffers from pronounced Gribov and smoothing problems. Clearly a more sophisticated approach is needed that manifestly does not depend on the particular gauge fixing procedure adopted. Work along these lines is being currently pursued.

Elimination of thick vortex configurations in the vacuum allowing the Wilson loop to fluctuate into different $\pi_1(SU(N)/Z(N))$ homotopy sectors has recently been rigorously shown to lead to loss of confinement at arbitrarily weak coupling. It is an old result that this also holds true for the electric-flux free energy order parameter. In other words, the presence of vortices is a necessary condition for confinement.

The numerical simulations indicate that in fact the presence of vortices is both a necessary and sufficient condition. Again analytical arguments relate the existence of nonzero string tension directly to the nonvanishing of the excitation probability of a sufficiently thick vortex in the vacuum. A numerical evaluation of the vortex free energy presented here shows that this probability is indeed equal to unity. This indicates that the vacuum indeed exhibits a ‘condensate’ of thick vortices.

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References

1. G. ’t Hooft, Nuc. Phys. B138 (1978) 1; ibid B153 (1979) 141.
2. J.M. Cornwall, Phys. Rev. D 26, 1453 (1979); in “Workshop on Non-perturbative QCD”, K. A. Milton and M. A. Samuel, eds., Birkhauser, Boston (1983).
3. G. Mack and V.B. Petkova, Ann. Phys. (NY) 123, 442 (1979); ibid 125, 117 (1980); Z. Phys. C 12, 177 (1982).
4. L.G. Yaffe, Phys. Rev. D21 (1979) 1574.
5. E.T. Tomboulis, Phys. Rev. D 23, 2371, (1981); in “Proceedings of the Brown Workshop on Nonperturbative Studies in QCD”, A. Jevicki and C-I. Tan, eds., (1981); Phys. Lett. B 303, 103 (1993).
6. T. Yoneya, Nucl. Phys. B 205 [FS5], 130 (1982).
7. C.T.H. Davies et al (eds), Lattice 97 Proceedings (Edinburgh), Nucl. Phys. B (Proc. Suppl.) 63 (1998); T. DeGrand et al (eds), Lattice 98 Proceedings (Boulder), Nucl. Phys. B (Proc. Suppl.) 73 (1999).
8. Lattice 99 Proceedings (Pisa), to appear in Nucl. Phys. B (Proc. Suppl.).
9. T.G. Kovács and E.T. Tomboulis, Phys. Rev. D57 (1998) 4045; Nucl. Phys. B (Proc. Suppl.) 63, 534 (1998); ibid 53 (Proc. Suppl.), 509 (1997).
10. T. G. Kovács, E. T. Tomboulis, Phys. Lett. B443, 239 (1998).
11. L. Del Debbio, M. Faber, J. Giedt, J. Greensite, and Š. Olejník, Phys. Rev. D 55, 2298 (1997); Nucl. Phys. B (Proc. Suppl.) 63, 552 (1998); Phys. Rev. D58 (1998) 094501.
12. K. Langfeld, H. Reinhardt, and O. Tennert, Phys. Lett. B419, 317 (1998); M. Engelhardt, K. Langfeld, H. Reinhardt, and O. Tennert, Phys. Lett. B431, 141 (1998).
13. M. Engelhardt, K. Langfeld, H. Reinhardt, and O. Tennert, hep-lat/9904004; Phys. Lett. B452, 301 (1999).
14. J.M. Cornwall, Phys. Rev. D 57, 7589 (1998).
15. J.D. Stack and W. Tucker, Nucl. Phys. B (Proc. Suppl.) 73, 563 (1999).
16. C. Hoelbing, C. Rebbi, and V.A. Rubakov, Nucl. Phys. B (Proc. Suppl.) 73 (1999) 527, hep-lat/9809113.
17. T.G. Kovács and E.T. Tomboulis, Jour. Math. Phys., 40, 4677 (1999), hep-lat/9806030.
18. T.G. Kovács and E.T. Tomboulis, Phys. Lett. B463, 104 (1999).
19. A. Montero, Phys. Lett. B467, 106 (1999).
20. C. Alexandrou, M. D’Elia, and Ph. de Forcrand, hep-lat/9907028; hep-lat/9909005.
21. T. DeGrand, A. Hasenfratz, and T.G. Kovács, Nucl. Phys. B 505, 417 (1997).
22. E.T. Tomboulis and L.G. Yaffe, Commun. Math. Phys. 100, 313 (1985).
23. J. Fingberg, U.M. Heller, and V. Mitroyushkin, Nucl. Phys. B 435, 311 (1995); V.G. Bornyakov, M. Creutz, and V. Mitroyushkin, Phys.Rev. D44, 3918 (1991).
24. A. Ferrenberg and R. Swendsen, Phys. Rev. Lett. 63 (1989) 1195.