Prediction of static friction coefficient in rough contacts based on the junction growth theory

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Abstract. The classic approach to the slip-stick contact is based on the framework advanced by Mindlin, in which localized slip occurs on the contact area when the local shear traction exceeds the product between the local pressure and the static friction coefficient. This assumption may be too conservative in the case of high tractions arising at the asperities tips in the contact of rough surfaces, because the shear traction may be allowed to exceed the shear strength of the softer material. Consequently, the classic frictional contact model is modified in this paper so that gross sliding occurs when the junctions formed between all contacting asperities are independently sheared. In this framework, when the contact tractions, normal and shear, exceed the hardness of the softer material on the entire contact area, the material of the asperities yields and the junction growth process ends in all contact regions, leading to gross sliding inception. This friction mechanism is implemented in a previously proposed numerical model for the Cattaneo-Mindlin slip-stick contact problem, which is modified to accommodate the junction growth theory. The frictionless normal contact problem is solved first, then the tangential force is gradually increased, until gross sliding inception. The contact problems in the normal and in the tangential direction are successively solved, until one is stabilized in relation to the other. The maximum tangential force leading to a non-vanishing stick area is the static friction force that can be sustained by the rough contact. The static friction coefficient is eventually derived as the ratio between the latter friction force and the normal force.

1. Introduction
Static friction impedes the relative motion between two solids subjected to a force compressing the two parallel bounding surfaces together and having its direction perpendicular to the surfaces, i.e. a normal load. The classic laws of friction for rigid bodies were developed by Amontons and Coulomb, stating that the static friction is proportional to the normal force and independent of the apparent contact area. In the case of deformable bodies having inherent surface topography, the mechanism of static friction can be explained by the interfacial junctions that are formed at the asperity level. Whereas the most important concepts regarding the friction mechanisms were introduced by Bowden and Tabor [1], Tabor [2] established three elements playing a chief role in the friction of dry contacts: (1) the true contact area (as opposed to the apparent one), (2) the strength of the junctions formed between individual asperities, and (3) the manner in which these junctions are sheared.
The frictional contact problem of smooth spheres or cylinders under combined normal and tangential loading was solved by Cattaneo [3], and independently by Mindlin [4]. Their solution involves a contact area in partial slip, composed of a central stick zone and a peripheral slip region. The latter increases with the static friction force, and full sliding begins when the stick region vanishes completely. The contact model previously advanced for the Cattaneo-Mindlin contact problem [5] is here modified and adapted to accommodate the junction growth theory. According to the latter, full sliding begins when material yielding under combined normal and shear tractions occurs at the interface, leading to shearing of junctions established at the asperity level. The junction growth theory is expected to relieve two conservative assumptions present in the Cattaneo-Mindlin model. Firstly, the contact shear tractions are computed as the product between the local normal pressure and the static friction coefficient, according to the Coulomb theory. With this assumption, shear tractions larger than the local shear strength of the softer material were allowed. Secondly, the static friction coefficient is assumed independent of the surface roughness or of the normal load magnitude.

This invariance is contradicted by numerous experimental studies on friction in lubricated or dry contact interfaces. The influence of the normal load on the measured friction coefficient for various magnetic recording media was expressed by Rabinowicz [6] as a power-law decaying trend. Liu et al. [7] investigated the effect of roughness skewness and kurtosis on the friction coefficient. It was measured experimentally [8,9] that the friction coefficient increases with surface roughness in dry contact conditions and decreases in lubricated interfaces. This results encourage the reformulation of the contact model to allow computation of the friction coefficient based on material properties and surface topography, and independent of a localized Coulomb-type friction law. It is clear that the latter can be unrealistic in case of deformable bodies, as it cannot account for the variation of the friction coefficient with the roughness parameters or the normal load magnitude as observed experimentally.

The starting point in this problem reformulation is the Chang–Etsion–Bogy (CEB) frictional model [10], which defines static friction as the minimum shear force leading to a single plastic yielding point inside an asperity. Kogut and Etsion [11] advanced a semi-analytical model in which sliding inception in the sphere-on-flat contact is treated as the moment when yielding occurs on the entire contact region. This single asperity models, relating gross sliding to yielding in the softer material

2. Contact model

The contact model employed in this paper is based on the general contact model developed by Johnson [12]. This model comprises equations for the normal and for the tangential direction, and therefore can be divided into two submodels having the same structure. The contact equations will be repeated here for clarity, then the newly imposed constraints will be discussed in detail.

The resolution of the contact problem with arbitrary (e.g. rough) contact geometry requires numerical methods. The numerical formulation is based on considering all problem parameters as piece-wise constant on a \( N_1 \times N_2 \) uniformly spaced rectangular grid, also referred to as the computational domain \( A_p \), established in the common plane of contact and expected to enclose the contact area \( A_c \) at any point on the loading curve. Problem discretization is performed in the space domain along the \( x_1 \) and \( x_2 \)-axes of a Cartesian coordinate system aligned with the grid sides. The main advantage consists in replacing the analytical integration of known but otherwise arbitrary functions over irregular domains, with summation expressed as discrete cyclic convolution products, which can be computed efficiently using DCFFT technique [13]. It should also be noted that the computation of displacement fields require the contribution of contact tractions in both normal and tangential directions. Consequently, the submodels in the normal and in the tangential direction are not independent, unless the contacting materials have similar elastic properties. The fact that a 2D mesh can be employed to assess the contact stresses is the main advantage of this approach over the finite element analyses, which requires discretization of the whole body (i.e. a 3D mesh).
Even when the constitutive law of the contacting material is linear elastic, the irreversibility of friction as a dissipative process makes the state achieved at one point on the loading curve dependent on all previous states, as opposed to the frictionless elastic case, in which the final state is independent of the loading path. Consequently, the loading history must be simulated by applying the load in small increments and by superimposing the contribution of each increment to the previous state. To conveniently conduct the numerical analysis, all contact parameters are expressed as functions of integers indexing the grid cells, \( i = 1, N_i \), \( j = 1, N_j \), as well as the imposed loading steps, \( k = 1, N_k \), e.g. \( p(i, j, k) \) denotes the elementary pressure at the intersection of the line \( i \) with the column \( j \) of the rectangular spatial grid, resulted after the application of \( k \) loading increments. Parameters having two indexes do not vary with the loading level, e.g. the grid nodes coordinates \( x_s(i, j), n = 1, 2 \). Parameter varying with the loading level only have one index, e.g. rigid-body translations \( \omega_s(k), n = 1, 2, 3 \); when no index is present, the denoted quantity is constant during contact process simulation, e.g. the grid steps \( \Delta_1, \Delta_2 \), or the elementary cell area \( \Delta = \Delta_1 \Delta_2 \).

The static force equilibrium relates the normal force \( W \) to the pressure distribution \( p \) in the normal direction, as shown in equation (1), as well as the tangential force \( T(T_1, T_2) \) to the shear tractions \( q(q_1, q_2) \), as shown in equation (2). Tilting or torsional moments can also be considered, but are omitted here for brevity. It should be noted that the contact tractions corresponding to matching points on two contacting surfaces are the same in magnitude but opposite in direction. The model is valid under the assumption that the normal force is applied first, and the tangential load is subsequently varied:

\[
W(k) = \Delta \sum_{(i,j) \in A_{(k)}} p(i,j,k); \quad k = 1 \ldots N_k;
\]

\[
T_n(k) = \Delta \sum_{(i,j) \in A_{(k)}} q_n(i,j,k); \quad n = 1, 2; \quad k = 1 \ldots N_k.
\]

The equations of the surface of separation between the two contacting bodies yield the geometrical conditions of deformation in the normal direction:

\[
h(i, j, k) = h_i(i, j) + u_s(i, j, k) - \omega_s(k); \quad (i, j) \in A_r; \quad k = 1 \ldots N_k;
\]

where \( h_i \) is the gap between the undeformed (i.e., initial) surfaces, \( h \) the gap between the deformed surfaces, \( u_s \) the relative normal displacement, and \( \omega_s \) the rigid-body approach. The geometry of the contact process in the tangential direction is described by equation (4), relating the relative slip distances \( s_s \), the tangential displacements \( u_s \), and the rigid-body tangential translations \( \omega_s, i = 1, 2 : \)

\[
\begin{bmatrix}
  s_1(i,j,k) - s_1(i,j,k-1) \\
  s_2(i,j,k) - s_2(i,j,k-1)
\end{bmatrix} =
\begin{bmatrix}
  u_s(i,j,k) - u_s(i,j,k-1) \\
  u_s(i,j,k) - u_s(i,j,k-1)
\end{bmatrix} -
\begin{bmatrix}
  \omega_s(k) - \omega_s(k-1) \\
  \omega_s(k) - \omega_s(k-1)
\end{bmatrix} ;
\]

\((i,j) \in A_r; \quad k = 1 \ldots N_k.
\]

The tangential displacements in any cell from the computational domain can be expressed in terms of tractions over the contact area based on the Boussinesq–Cerruti fundamental solutions for the elastic half-space. Equations (3) and (4), respectively, can be reduced to linear system of equations, having as unknowns the nodal pressures on the contact area, and the shear tractions on the stick area, respectively. An additional difficulty arises from the fact that neither the contact area, not the stick area (in other words, the size of each of the two linear systems) are known in advance. An iterative approach is thus required, and the contact model must be completed with additional constraints, i.e. the contact complementarity conditions, which impose the boundary conditions and provide criteria for assessing the status of every cell.
In the normal contact problem, the gap between the deformed contacting surfaces vanishes on the contact area, as no interpenetration of the contacting solids is allowed in the frame of elasticity. On the other hand, the gap must be positive outside the contact area, where there is clearance between the contacting bodies. In the same manner, pressure is positive on the contact area (adhesion is not considered in this contact model), but vanishes outside the contact area. Based on the assumption of an elastic–perfectly plastic response, the contact pressure is constrained to be less than the local normal strength $p_{\text{max}}$. This assumption, while not accounting for the hardening of the elastic-plastic material, allows for computation of pressure without assessing the internal residual state (i.e. the plastic strains developed in the softer material). Thus, the pressure computation can still be performed on the initial 2D surface grid. This assumption is typical [14,15] for rough contact models.

$$h(i, j, k) = 0 \quad \text{and} \quad 0 < p(i, j, k) < p_{\text{max}}(i, j, k); \quad (i, j) \in A_c(k); \quad k = 1 \ldots N_i;$$

(5)

$$p(i, j, k) = 0 \quad \text{and} \quad h(i, j, k) > 0; \quad (i, j) \in A_p - A_c(k); \quad k = 1 \ldots N_i.$$  \hspace{1cm} (6)

The division between the contact and the non-contact zone is not known in advance and therefore must be found by a trial-and-error approach, in which, in a first approximation, the whole computational domain is in contact. Elementary cells for which the nodal pressure results negative are then excluded from the contact area and the associated nodal pressure is set to nil.

In the tangential direction, the complementarity conditions control the boundary between the slip and the stick regions, and their reunion must amount to the whole contact area. The solution of the contact problem in the normal direction (i.e. the contact area established due to the normal loading) is therefore needed to solve the tangential contact problem. The classical formulation of the Cattaneo-Mindlin problem assumes a constant friction coefficient, and shear tractions that cannot exceed a limiting value in the stick region, i.e. $\|q(i, j, k)\| \leq \mu p(i, j, k)$, while in the slip region, $\|q(i, j, k)\| = \mu p(i, j, k)$, with $\|q\| = \sqrt{q_1^2 + q_2^2}$. The formulation requires that the friction coefficient is known in advance before conducting the contact simulation. This course of action allows assessment of the maximum tangential force that can be sustained by a slip-stick contact between dissimilarly elastic materials. It was found numerically [16] that this force is less than the limiting friction force from the general theory (i.e. the Cattaneo-Mindlin framework), as the mismatch in elastic properties generates additional contact tractions which lead to gross slip at smaller than expected tangential forces. This approach, however, can be unrealistic, as the shear stress defined in this way (i.e., from a Coulomb-type law) is allowed to exceed the local shear strength of the softer material.

In the newly proposed framework, the shear stress magnitude is set to be less than the local shear strength $q_{\text{max}}$ in the stick zone, and reaches this magnitude in the slip zone. With this formulation, the friction coefficient is no longer needed to compute the shear tractions. Slip $s$ vanish in the stick zone and is positive in the regions where microslip occurs between corresponding points on the contacting surfaces. It should be noted that all corresponding points in a stick region undergo the same tangential displacement. This complementarity conditions for the tangential direction can be expressed as:

$$\|q(i, j, k)\| \leq q_{\text{max}}(i, j, k) \quad \text{and} \quad \|s(i, j, k) - s(i, j, k - 1)\| = 0; \quad (i, j) \in A_s(k); \quad k = 1 \ldots N_i;$$

(7)

$$\|q(i, j, k)\| = q_{\text{max}}(i, j, k) \quad \text{and} \quad \|s(i, j, k) - s(i, j, k - 1)\| > 0; \quad (i, j) \in A_s(k) - A_s(k); \quad k = 1 \ldots N_i.$$  \hspace{1cm} (8)

As in the case of the normal contact problem, the boundary between the slip and the stick regions is not known a priori, and must be found by trial. In a first approximation, it is convenient to assume that the whole contact area is in stick. Slip is then likely to occur in the regions where the computed shear traction exceeds its limiting value $q_{\text{max}}$. Moreover, the stick or slip status of specific points on the contact area is expected to change with the application of subsequent tangential load increments.
originally included in the stick zone conserve their status if no relative motion occurs between matching particles on the two contacting surfaces, i.e. the increment of relative slip distance in the considered timeframe is nil, as in equation (7). Points for which this condition is not met pass into the slip zone. It appears clearly that the existence of slip is intrinsically conditioned by the variation of the tangential load, and therefore a purely static, time-independent model, is not appropriate.

An additional constraint is that the direction of the shear traction must oppose that of slip:

$$\frac{\mathbf{q}(i, j, k)}{\|\mathbf{q}(i, j, k)\|} = -\frac{\mathbf{s}(i, j, k)}{\|\mathbf{s}(i, j, k)\|}; \quad (i, j) \in A_r(k) - A_s(k); \quad k = 1 \ldots N_r. \quad (9)$$

Finally, the relation between the local normal strength and the local shear strength is established by the Tabor equation [17] in the form:

$$p_{\text{max}}^2 + \alpha q_{\text{max}}^2 = H^2, \quad (10)$$

where $\alpha$ is the Tabor constant, generally determined empirically from experimental measurements, and $H$ is the hardness of the softer material. For elastic–perfectly-plastic materials, $H \approx 2.8\sigma_y$, where $\sigma_y$ is the uniaxial yield strength. In this study, the value of $\alpha$ was set at $\alpha = 23.52$, as indicated in [18] for cases with light load and small roughness.

The contact model (1) - (10) can be solved for arbitrary contact geometry as described in the following section.

3. Algorithm description

It is convenient to divide the contact model in two sets of equations and inequalities, whose individual solution is easier to obtain. This separation is suggested by the existence of a numerical solution [19] for the problem of the frictionless normal contact. Therefore, relations (1), (3), (5) and (6) are grouped in a submodel describing the contact process in the normal direction, that will be referred from now on as the NC. Similarly, the set of equations and inequalities (2), (4), (7), (8) and (9) are isolated as the submodel TC for the tangential direction. Considering the similarity between NC and TC, it was proven [20] that the same type of algorithm can solve either model considered independently. Both the NC and the TC can be reduced to essentially linear system of equations, resulting from equations (3) and (4), respectively, having as unknowns the nodal pressure on the contact area and the tangential tractions on the stick area, respectively. For a detailed description of the algorithm to solve the uncoupled TC or NC, the reader is referred to [21].

The latter algorithm can be used to solve the NC when the displacement field induced by the tangential tractions is known but otherwise arbitrary, and also the TC when the displacement field due to pressure is known but otherwise arbitrary.

In the newly advanced contact model, the coupling of the NC and TC is due to the following dependencies: (1) computation of displacements fields requires the contribution of all contact tractions (i.e. both normal and tangential), and (2) the local normal and shear strength are related through equation (10), showing that the local normal strength is expected to decrease with the increase in the magnitude of the tangential force. Consequently, the NC and the TC cannot be solved independently, and the simulation of the contact process requires an iterative approach, in which the NC and the TC are solved successively, until the solution of one is stabilized with respect to the solution of the other.

The main algorithm steps to reach this stabilised state are summarized below.

1. Acquire the input of the contact simulation: the contact geometry, the mechanical properties of the materials (i.e. Young modulus, Poisson’s ratio, the hardness of the softer material), the loading history.
2. Establish the computational domain and the mesh parameters. Compute the influence coefficients for the displacements induced by the contact tractions, as discussed in [21].
3. Apply the normal load and solve the NC contact model using the algorithm for the frictionless contact problem. Obtain the contact area and the pressure distribution.
4. Apply an increment of the tangential load. The first increment should be small enough so that gross slip does not occur with its application.
5. Solve the TC using the existing pressure distribution. Obtain the stick area and the distribution of shear tractions.
6. Solve the NC again, but this time with a displacement field accounting for the contribution of the shear tractions estimated in the previous step. Obtain a new, more precise, pressure distribution.
7. Repeat steps 5 and 6, until the pressure distributions obtained in two iterations vary within an imposed precision. Alternatively, the stabilization between the NC and the TC (i.e. the algorithm convergence) could be estimated from the discrepancy in the shear tractions distribution between two iterations, but the magnitude of the shear tractions is generally lower than that of pressure and could be more easily affected by numerical oscillations.
8. Resume algorithm execution from step 4, until full sliding occurs. Numerically, the gross sliding regime initiates when no elementary cells are predicted to have a stick status.
9. Compute the static friction coefficient as the ratio between the static friction force and the pre-applied normal load. In the framework advanced in this paper, the static friction force is therefore the maximum tangential force that the preloaded normal contact can sustain at sliding inception:

$$\mu = \frac{\|T\|}{W}.$$ (11)

The flowchart of the iterative process in steps 5 – 9 in presented in figure 1.

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**Figure 1.** Algorithm flowchart for the stabilisation between the NC and the TC.

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4. **Numerical simulations and discussions**

In order to test the algorithm behavior on a simpler contact geometry, the single asperity spherical contact is simulated first. The elastic parameters of the contacting materials were chosen as follows: the Young moduli $E_{steel} = 210$ GPa, $E_{copper} = E_{steel} / 2$, the Poissons ratio $\nu_{steel} = \nu_{copper} = 0.3$, the hardness of the softer material $H_{copper} = 1$ GPa. A steel ball of radius $R = 18$ mm is pressed against a smooth copper half-space by a normal load five times greater than the one required for the initiation of plastic yielding inside the softer material (i.e. copper). The latter load, also referred to as a critical load $W_{cr}$, was computed by gradually increasing the force magnitude until the maximum of the von Mises equivalent stress exceeds the yield strength of the softer material, as discussed in [22]. An increasing tangential force is subsequently applied in small increments, until the predicted stick area vanishes, indicating the transition from slip-stick to full sliding. The contact radius predicted at full sliding inception was 107% of that achieved prior to tangential load application. This behavior shows that the
maximum normal strength is reduced with the development of shear stresses, as expected from equation (10). This further leads to an increase of the contact area, as the same normal load has to be accommodated with a smaller normal strength.

The algorithm was then applied to a deterministic roughness sample consisting in \(512 \times 512\) individual heights measured in a rectangle of sides \(0.1 \times 0.1\) mm. The resulting composite surface topography is presented in figure 2, with the convention that the points with the smaller height touch the counter surface (i.e., a smooth half-space) first. The loading history was the same as for the smooth spherical contact. The contact area maps before the application of the tangential force and at the moment of full slide inception (when the stick area drops to zero) are plotted in figures 3 and 4, respectively. Their comparison confirm qualitatively the trend of an increasing contact area identified for the smooth contact.

![Figure 2. Composite initial contact geometry (hi) in the contact between a smooth steel ball and a rough copper half-space. The roughness sample was submitted to the The Tribology Letters Contact Mechanics Challenge 2016.](image)

![Figure 3. Contact area map before tangential load application, \(W/W_o = 5\).](image)

![Figure 4. Contact area map at full sliding inception, \(W/W_o = 5\).](image)

The influence of the \(\text{rms}\) index on the predicted static friction coefficient was studied by scaling the peaks and valleys of the original sample by a factor of two with respect to the mean centerline, leading
to an increase with the same factor of the rms value $R_q$. The comparison of the two simulations data in figure 5 suggests that the predicted static friction coefficient for the rougher surface is smaller when the same normal load level is imposed. This behavior can be attributed the fact that, in case of the rougher surface, the smaller contact area established between higher asperities leads to higher local pressure and thus to smaller shear strength, as resulting from equation (10). Consequently, the resulting tangential force will be smaller, leading to a smaller static friction coefficient. The decaying trend of the friction coefficient with the increase of the normal load is consistent with the tendency observed by Rabinowicz [6].

![Figure 5. Influence of normal load level and of roughness rms on the static friction coefficient.](image)

### 5. Conclusions

The classical approach (i.e. based on the Cattaneo-Mindlin formalism) to the frictional contact of rough surfaces has difficulty with explaining the trends of static friction coefficient variation with the normal load level, the material properties or the surface topography, as measured in numerous experimental findings.

This may be attributed to two conservative assumptions: (1) the friction coefficient is an invariant regardless the normal load and the surface roughness, and (2) the maximum shear stress is defined by a Coulomb-type friction law, allowing the shear stress to exceed the strength of the softer material. These assumptions are released in the numerical model advanced in this paper, in which slip occurs when the entire contact area undergoes yielding due to the joint effect of normal and tangential tractions, according to the junction growth theory.

In this formulation, the static friction force, regarded as the maximum tangential force that the contact can accommodate before gross slip, can be predicted from the contact process simulation by limiting the contact stresses to the strength of the softer material. The contact process is simulated with a gradually increasing tangential force, until the predicted shear tractions exceed the local shear strength of the softer material, leading to shearing of junctions at the interface and to full sliding inception.

The predictions of the developed computer program follow qualitatively the trends reported in the literature of experimental frictional rough contacts: the friction coefficient decreases with the increase in normal load magnitude, as well as with the increase in rms roughness. However, the rigorous quantitative characterisation of the static friction coefficient may be challenged by the fact that the Tabor constant, empirically determined from experimental measurements, is required in the equation relating the normal and the shear strength to the hardness of the softer material.
Acknowledgement
This work was partially supported from the project “Integrated Center for Research, Development and Innovation in Advanced Materials, Nanotechnologies, and Distributed Systems for Fabrication and Control”, Contract No. 671/09.04.2015, Sectoral Operational Program for Increase of the Economic Competitiveness co-funded from the European Regional Development Fund.

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