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A. A. Markov
Mkhitar A. Hobosyan
The University of Texas Rio Grande Valley
Karen S. Martirosyan
The University of Texas Rio Grande Valley

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Slip-Jump Model for Carbon Combustion Synthesis of Complex Oxide Nanoparticles

A.A. Markov¹, M.A. Hobosyan², and K.S. Martirosyan²*

¹IPMech RAS, Moscow 119526, Russia
²Department of Physics, University of Texas at Rio Grande Valley, Brownsville, TX 78520, USA

Abstract

Carbon Combustion Synthesis of Oxides (CCSO) is a promising method to produce submicron- and nano-sized complex oxides. The CCSO was successfully utilized for producing several complex oxides, a complete theoretical model including the sample porosity, flow parameters and reaction energetics is needed to predict the combustion parameters for CCSO. In this work, we studied the ignition temperature and combustion wave axial temperature distribution, activation energy, combustion heat and thermal losses for a typical CCSO synthesis for cylindrical samples of Ni-Zn ferrites with high (>85%) porosity. We developed a two level combustion model of chemically active nano-dispersed mixture, using the experimentally measured ignition temperature and combustion parameter values utilizing the slip-jump method for high Knudsen numbers. The theoretical predictions of highly porous samples when the flow resistivity is small and the gas can easily flow through the cylindrical sample are in good agreement with the experimental data. The calculation of combustion characteristics for the lower porosity values demonstrated that the surface combustion was dominated due to high gas flow resistivity of the sample. Finger combustion features were observed at this combustion mode.

1. Introduction

Recent experimental studies of convective and diffusion transfer in carbon nanotubes [1–3] showed acceleration of transfer on more than 2 orders of magnitude compared to the estimations based on theory of continuum media [4–7]. The intensification of transfer can be partially explained by direct modeling of motion of ensemble of molecules using the method of molecular dynamics. The length of mean free path of the gas molecules inside the pores of the carbon is exceeding the diameter of the pores. The reflection of the gas molecules from the nanotube boundaries dominates the rate of the reflection of gas molecules between each other and the Knudsen numbers may reach 10–70 values. However, the measured flows of the gas exceed the values estimated by the theory of Knudsen layers well over 10 to 100 times.

The experimental results mentioned above and the limitation of theoretical models stimulated the foundation of theory for the processes of thermal and mass transfer based on the effects of gas slip and concentration and temperature jumps in the Knudsen layer [8]. The boundary conditions on the wall of the tubes taking into account the slip and jump of temperature for the large Knudsen numbers are given in the work by Cheng [9], where the focus of discussion is the case of a rarefied homogenous gas flow without chemical reactions near the macro-scale objects. In addition it was confirms the solutions for the stagnation region based on a single dissociation-recombination kinetic model show a substantial concentration jump across the shock, and extensive influence of surface cooling low Reynolds number [9]. The further development of the theory for supersonic flows of absolutely rarefied viscous gas around objects in the transitional regime of continuous to free molecular flow resulted to wide range of Knudsen number of incoming flow using both continuous and kinetic approaches [10]. A two-dimensional analysis with a first-order slip-velocity boundary condition in the Navier-Stokes equations displays that both compressibility and rarefied effects are present in long microchannels [11]. The continuous approach is
based on the usage of specified conditions for the temperature jump on the surface of object.

In contrast to the macro-scaled combustion processes of carbon for the large Knudsen numbers presented in the work [8] gives the generalized approach of Cheng [9] for the combustion of nanoscale carbon particles. With the conditions of temperature jump of gas phase, also the boundary conditions of concentration jumps are introduced for the reacting gas components. The calculations of carbon combustion for the transitional regimes from the continuum to molecular flow regimes are performed, using the methods of molecular dynamics. The theory of slip and jump of temperature and concentration of gaseous mixture O₂ and CO₂ helped to understand the features of complex gas dynamics and thermal processes in the combustion wave for sub-micron scales, which is utilized in the synthesis of nano sized particles.

In this work we provide the theoretical and experimental study of the synthesis of Ni-Zn ferrite nanoparticles using the CCSO method [12–14]. Currently there is a lack of theory predicting combustion parameters for the synthesis of nanoparticles using CCSO. The combustion initiation temperature and the axial temperature distribution during the combustion, as well as the activation energy and reaction heat values were estimated experimentally. These values were used to develop a two level model of chemically active nano-dispersed mixture [15–19] which can be applied to the synthesis of nano-sized particles of Ni₀.₃₅Zn₀.₆₅Fe₂O₄ based on kinetics [14] and processes of mass and thermal transfer in the isolated pore (meso-scale), which is a procedure of averaging the micro-scaled mass and thermal flows in the calculations of characteristic mixture in macro scale. The theoretical predictions and the experimental values of the combustion axial temperature distribution and combustion wave propagation characteristics are in good agreement for the highly porous (>85%) samples when the oxidizer can easily flow through the sample. In addition, the theoretical calculations for lower porosity values were performed, when the flow resistivity is high and surface combustion takes place, and the combustion parameters and regimes were predicted.

The defining equations in macro scale use the Euler formulations – the principles of mass, impulse and energy conservation for the leading phase, including the averaged micro-scaled flows, as well as equations for fractions of particles.

The effect of slip decelerates the combustion process when the slip parameters are small and the heating time is short. For this case, we observed a decrease of temperature (more than twice), significant differences in the pressure distribution, and deceleration of carbon combustion. The intensive outflow of the gas from the combustion zone and from the sample in general results in the thermal losses and decelerates the combustion process during the slippage. The combustion transforms from the diffusion to the convective-diffusion regime.

In the case of intensive slip parameter, a non-monotonous dependence of temperature on the parameter of slippage values A_u is observed. In addition to the thermal outflow from the combustion zone in some cases increase of combustion efficiency during the slip increase was observed, due to the increased inflow of the oxidizer to the synthesis zone. In the case of intensive slip parameter values we have a non-monotonous dependence of temperature on the value of the A_u. In some cases, the thermal outflow from the combustion zone results in slowing down the synthesis and decrease in temperature while increasing the A_u parameter.

However, in other cases, in parallel to the thermal outflow from the combustion zone, we observed an effect of intensified combustion process, which was due to the increased inflow of the oxidizer to the combustion zone. This intensification of combustion results in increase of temperature, during which the sample is intensively heated through the extreme gas convection. Thus, the decrease of temperature while increasing the A_u parameter transforms into the increase of temperature due to the intensive heating of the whole sample during the extreme gas convection. An analogous effect of temperature increase takes place when the heating time of the sample and the oxygen supply rate is increased.

2. Experimental System and Procedure

The CCSO synthesis of the Ni₀.₃₅Zn₀.₆₅Fe₂O₄ ferrite was via the reaction:

\[ 0.35\text{NiO} + 0.65\text{ZnO} + \text{Fe}_2\text{O}_3 + \alpha(\text{C}+\text{O}_2) \rightarrow \text{Ni}_0.35\text{Zn}_0.65\text{Fe}_2\text{O}_4 + \alpha\text{CO}_2 \]

where 2<α<14 [14]. Reagents were purchased from Sigma Aldrich and were used as received. The reagents included NiO (nano powder, < 50 nm), Fe₂O₃ (nano powder, < 50 nm), ZnO (< 100 nm), and activated carbon (water content < 30 wt.%, 100 mesh).

The reagents were mixed in rotary ball mill in stainless steel jar with ZrO₂ balls for 3 h (ball to mixture weight ratio was 6:1). The mixture was loosely filled in vertical quartz tube with 14 mm inner diameter, which had alumina foam to support the mixture and allow the oxygen flow (Fig. 1). The
height of the sample was 4 mm and the relative density ~0.12. The oxygen (99.994% purity, Air-gas) was fed by the flow rate 3 L/min, and the flow rate was measured both in the inlet and outlet of the reaction tube. The oxygen flow direction and the combustion wave direction were the same.

The combustion temperatures were measured with K type micro-thermocouples (Omega) placed inside the sample near the center and at the wall of the quartz tube. The thermocouples signals were recorded with the rate of 1 kHz and processed by an Omega data acquisition board connected to a PC. The combustion wave was initiated with a short (~1 s) laser pulse (1.5 W) and the combustion process was recorded by high-speed camera (1000 Hz) to estimate combustion front motion behavior and velocity. The thermo-gravimetric (TG) and differential thermal analysis (DTA) was performed by using Differential Scanning Calorimeter (DSC) with the sensitivity of 0.1 μg (Q-600, TA Instruments). The measurements were made in oxygen atmosphere (99.994% purity).

Fig. 1. Schematics of the experimental setup.

3. Experimental results

Figure 2a is representing the TG/DTA results for the carbon at heating rate 20 °C/min. The blue line represents the weight change, the green line is the heat flow, and the red line indicates the points through which the heat flow peaks are integrated for the thermal effects. The evaporation of absorbed water completes at around 100 °C with the energy consumption of about 822 J/g, when the mass change due to evaporation was 23.64 wt.%. This graph clearly shows that the ignition of carbon combustion starts at ~405 °C and the combustion releases 7645 J/g energy. Figure 2b is representing the same results for the system, where α = 10. Due to smaller fraction of carbon, the absorbed water is about 9 wt.% of the overall mixture with evaporation energy ~208 J/g before 100 °C. The ignition temperature was 366 °C and the combustion energy estimated 5470 J/g. Thus, the formation of oxides is endothermic process that consumes ~2175 J/g energy.

We estimated the activation energy from the DTA data by using the isoconversional method suggested by Starink [20], which was shown to provide a more accurate value than the Kissinger and Ozawa methods. The Starink method determines the activation energy from the equation:

\[
\ln\left(\frac{\beta}{T^{1.92}}\right) = -1.0008 \frac{E_a}{RT} + \text{const},
\]

where \(E_a\) is the apparent activation energy (in kJ/mol), \(\beta\) is the heating rate in thermal analysis (in K/min), \(T\) is the peak temperature of the exothermic curve (in K), and \(R\) the universal gas constant. \(E_a\) is estimated from the slope of the graph of \(\ln(\beta/T^{1.92})\) vs. \(1/T\).

![Fig. 2. a) DTA-TG curves for the carbon combustion at 20 °C/min heating rate, b) DTA-TG curves for the mixture with \(\alpha = 10\) at 20 °C/min heating rate.](image-url)
We apply the time and length scales for combustion proposed by Frank-Kamenetskii [21]. The characteristic time scale $t_0$ and length scale $l_0$ are as follows:

$$
\lambda_0 \text{ and } R_0 \text{ are the referred thermal conductivity, }
R \text{ is the universal gas constant. Below we will use the transformed non-dimensional variables marked tilde above the letters.}
$$

For Cartesian coordinates the variables follow:

$$
\text{where } M \text{ denotes the molar mass, and } c \text{ denotes the thermal capacity of the corresponding component. We will use the effective density values denoted by } \bar{\rho}:
$$

$$
\bar{\rho}_g = \bar{\rho}_{g1} + \bar{\rho}_{g2} \quad (4)
$$

The simulation of zinc nickel ferrite synthesis is based on following kinetics:

$$
0.35B_2(s) + 0.65B_3(s) + B_4(s) \rightarrow B_5(s),
$$

where $B_2(s) = \text{NiO}(s)$, $B_3(s) = \text{ZnO}(s)$, $B_4(s) = \text{Fe}_2\text{O}_3(s)$, $B_5(s) = \text{Ni}_{0.35}\text{Zn}_{0.65}\text{Fe}_2\text{O}_4(s)$

The molar masses of reagents, products and their thermal capacities satisfy the following relations:

$$
0.35M_{2x} + 0.65M_{3x} + 1 = 0, c_{4x} = \left( c_{2x} \frac{0.35M_{2x} + 0.65M_{3x}}{M_{4x}} \right)
$$

We apply the time and length scales for combustion proposed by Frank-Kamenetskii [21].

The characteristic time scale $t_0$ and length scale $l_0$ are as follows:

$$
\frac{t_0 = \exp \left( \frac{E}{RT_0} \right)}{k} \quad l_0 = \sqrt{\frac{\lambda_0 t_0}{c_r \rho_0}} \quad (5)
$$

$\lambda_0$ is the referred thermal conductivity, $R$ is the universal gas constant. Below we will use the transformed non-dimensional variables marked tilde above the letters. For Cartesian coordinates the variables follow:

$$
\bar{x}_k = \frac{x_k}{l_0}, \bar{t}_k = t_{0k}, \bar{u}_k = \frac{u_k}{u_0}, k = 1,2,3; u_0 = \frac{l_0}{t_0} \quad (6)
$$

$$
\bar{p}_p = \frac{p}{p_0}, \bar{\rho}_g = \frac{\rho_g}{\rho_0}, \bar{\rho}_s = \frac{\rho_s}{\rho_0}, j = 1,2; \bar{\rho}_s = \frac{\rho}{\rho_0} \quad (7)
$$

### Table 1

| System | $T_{ign}, ^\circ C$ | $E_{act}, \text{kJ/mol}$ | $E_{heate}, \text{kJ/g}$ | Porosity, vol.% | $T_{c,max}, ^\circ C$ | $T_{wall}, ^\circ C$ | Normalize O$_2$ flow rate, L/min$^2$cm$^{-2}$ |
|--------|-------------------|--------------------------|------------------------|-----------------|---------------------|--------------|--------------------------|
| Carbon combustion in oxygen | 405 | 114±3.4 | 7645 | 88 | 1108 | 570 | 1.95 |
| Mixture with $\alpha = 10$ in oxygen | 336 | 73±2.2 | 5770 | 88 | 780 | 520 | 1.95 |
Here, the characteristic pressure is
\[
P_0 = \frac{R \rho_0 T_0}{M_0}
\]
where \( T_0 = 1273 \text{ K} \) is the characteristic temperature. The main similarity parameters will be
\[
\beta = \frac{RT_0}{E}, \quad \text{and} \quad \gamma = \frac{c_p T_0 \beta}{Q}
\]
Where \( Q \) is the combustion heat and \( E \) is the activation energy. The non-dimensional temperature is expressed as
\[
\tilde{T} = \frac{(T - T_0) E}{RT_0^2}, \quad \frac{T}{T_0} = 1 + \beta \tilde{T}
\]

The non-dimensional variables are applied for numerical simulation.

For non-dimensional variables
\[
\frac{\rho_0 C_v}{T_0} = \frac{\lambda_0}{T_0}, \quad T = T_0 (1 + \beta \tilde{T})
\]

The solid-gas concentration flux jump equation reads: the mass fluxes caused by chemical reaction are as follows:
\[
J_{s-g}^{\text{macro}} = \rho_s \rho_g k \exp \left( \frac{T_s}{\beta T_g + 1} \right), \quad J_{s-g}^{\text{macro}} = \frac{M_s}{M_g} J_{s-g}^{\text{micro}}, l = 1, 2
\]
\[
J_{s-g}^{\text{micro}} = \rho_s \rho_g k \exp \left( \frac{T_s}{\beta T_g + 1} \right), \quad J_{s-g}^{\text{micro}} = \frac{M_s}{M_g} J_{s-g}^{\text{micro}}, l = 1, 2
\]
Here,
\[
\rho_s = (1 - \chi) \rho_1, \rho_g = \chi \rho_1, \rho_{g,2} = \chi \rho_{2,2}
\]
where \( \chi \) is the porosity. Mass fractions \( C_1 \) and \( C_2 \) are as follows
\[
\rho_{g,1} = \rho_{g,1} C_1, C_2 = 1 - C_1
\]
The equation of state reads as
\[
P_g = \rho_g T_g \frac{R}{M_g}
\]

Let us note that we consider the pore surface values \( \overline{P}_{gS,1}, \overline{P}_{gS,2} \) of gas components along with the values \( \overline{P}_{g1}, \overline{P}_{g2} \) of these components in side the pores. Note that the surface is denoted with capital “S” while the solid phase components are denoted with lower case “s”. The basic equations for carbon combustion at macro level in non-dimensional variables are presented by using the flux jump values and the equation of mass, energy and momentum conservation as follow (tilde denoting non dimensional value hereafter is omitted for brevity):
\[
\frac{\partial \chi \rho_g}{\partial t} + \nabla \cdot (\chi \rho_g u) = (1 - \chi) \left( J_{g-s}^{\text{macro}} + J_{g-s}^{\text{jump},1} + J_{g-s}^{\text{jump},2} \right)
\]

Here, \( D_f \) is the thermal diffusivity coefficient, \( P_e \) are the Peclet numbers \( P_e = \frac{u_{ref}}{D_{ref}} \), \( A_T \) and \( A_{C,l} \) are temperature and concentration jump parameters \( A_T = \frac{2 \chi b_T}{r_1} \frac{\rho_0 P_0}{\rho_g} \) and \( A_{C,l} = B_{C,l} \frac{2 \rho_0}{r_1} b_l \) where \( b_l \) is the slip parameter:
\[
b_l = \frac{2 - \alpha}{\alpha} \sqrt{\frac{\pi}{2 \gamma + 1}}, \quad \alpha \text{ is the thermal accommodation coefficient, bl denotes the wall constant for the species jump that is similar to the value of bu for the velocity slip: } b_u = \frac{\theta}{2 - \theta} \frac{\rho_0 P_0}{\rho_g}
\]
Here, \( \rho_{ref} \approx 10^{-5} \) m, using \( \sqrt{\rho_{ref}^2 \approx 10^{-5} \cdot 10^{-5} = 10, \rho_0 \approx 10^{-5}, T_0 \approx 10^5, \mu_g = 5 \cdot 10^{-5} \) we estimated \( A_u \approx 200 \chi b_u, A_{C,l} \approx 200 \chi b_u, \) and \( A_T \approx 10^5 \chi b_u \).

For nanoscale pores \( r_i \approx 5 \cdot 10^{-8} \) m the intensities of velocity slip and concentration and temperature jump increase to one order of magnitude. The relative concentration of oxygen and carbon dioxide
\[
\frac{\partial \chi \rho_g C_1}{\partial t} + \nabla \cdot (\chi \rho_g C_1 u) = \chi \left( \frac{D_{g1}}{P_e \rho_g} \frac{\nabla C_1}{M_g} \right) - \frac{M_{g,1}}{M_g} J_{g-s}^{\text{macro},1,2} C_2 = 1 - C_1
\]
We note that

\[ \rho_{1g} = \rho_{O_2(g)}, \rho_{2g} = \rho_{CO_2(g)}, \rho_{3g} = \rho_{C(s)}, \] and \( C_i = \frac{\rho_{1g}}{\rho_{g}} \)

\[ J_{C,\text{jump}}^{\text{macro}} = A_{C,i} \left( -\rho_{g} C_{g,i} + \rho_{gs,i} \right) + D_f \rho_{g} Q_{\text{jump}}, l = 1,2 \]

The equation for gas concentrations at the pore surface

\[ \frac{\partial \rho_{gS,1}}{\partial t} = -J_{gS,1}^{\text{macro}} J_{gS,2}^{\text{macro}} = \rho_{gs,1} \exp \left( T_{i} T_{T} + 1 \right) \]  (18)

\[ \frac{\partial \rho_{gS,2}}{\partial t} = -J_{gS,2}^{\text{macro}} J_{gS,1}^{\text{macro}} = \frac{M_{gs}}{M_{gS}} J_{gS,1}, l = 1,2, \]  (19)

where \( \rho_{gs,1}, \rho_{gs,2} \) are the values of gas densities \( \rho_{1g}, \rho_{2g} \) at the surface of pores and \( T_i \) is the solid carbon temperature. The carbon concentration denotes:

\[ \frac{\partial \rho_{c}}{\partial t} = -J_{c,\text{jump}} \]  (20)

The equations for ferrite synthesis reads:

\[ \frac{\partial \rho_{S}}{\partial t} = -0.35 \frac{M_{2S}}{M_{4S}} J_{S}, \frac{\partial \rho_{S}}{\partial t} = -0.65 \frac{M_{3S}}{M_{4S}} J_{S}, \]

\[ \frac{\partial \rho_{S}}{\partial t} = -J_{S}, \frac{\partial \rho_{S}}{\partial t} = \frac{M_{SS}}{M_{4S}} J_{S}, \]

where

\[ J_{S} = (1-\chi)k_{2} \left( \rho_{23} \right)^{0.35} \left( \rho_{3s} \right)^{0.65} \left( \rho_{43s} \right) \exp \left( T_{S} \right) \]  (26)

The equation of gas mixture velocity:

\[ \frac{\partial \rho_{g} u}{\partial t} + \nabla \left( \chi \rho_{g} u \right) = S_{v} + J_{\text{macro}}^{\text{macro}} - Ma^{-2} \nabla P_{g} + Re^{-1} \nabla \cdot \tau; \]

\[ \tau = \mu \left( \nabla u + (\nabla u)^{T} - \frac{2}{3} \nabla \left( \nabla u \right) \right) \]  (21)

Where the distributed pore resistivity is

\[ (S_{v}) = -u_{i} k_{i}, k_{i} = a_{i} |u| + \zeta_{i}, i = 1,2,3; \]  (22)

\( a_{i} \) and \( \zeta_{i} \) are respectively for non-linear and linear coefficient parts of the distributed pore resistivity. \( Ma \) is the Mach number; \( Re \) is the Reynolds number

\[ Re_{u} = \frac{u_{i} \rho_{u}}{\mu_{o}} \] and the slip flux

\[ J_{\text{slip}}^{\text{macro}} = A_{u} u \sqrt{P_{g}} \rho_{g} \]  (23)

The heat balance for gas

\[ \rho_{g} C_{v} \left( \frac{\partial T_{g}}{\partial t} + u_{i} \nabla T_{g} \right) = \nabla \left( \lambda_{1} \nabla T_{g} \right) + Q_{\text{macro}}^{\text{macro}} + Q_{\text{jump}}^{\text{macro}}, \rho_{g} \]  (24)

The heat balance for solid

\[ \rho_{s} C_{s} \left( \frac{\partial T_{s}}{\partial t} + u_{i} \nabla T_{s} \right) = \nabla \left( \lambda_{1} \nabla T_{g} \right) + Q_{\text{macro}}^{\text{macro}}, \rho_{s} \]  (25)

The temperature-jump flux will be corresponding to be

\[ Q_{\text{jump}}^{\text{macro}} = A_{1} \sqrt{P_{g}} \left( \frac{T_{g} + T_{s} - 0.5 \beta \left( 1 + T_{s} \right) \left( 1 + \beta T_{s} \right)^{b_{w}}} \right) \]  (26)

where \( 0 \leq b_{w} \leq 1 \).

5. Computational results and discussion

We present the numerical modeling of \( \text{Ni}_{0.35}\text{Zn}_{0.65}\text{Fe}_{2}\text{O}_{4} \) synthesis at both high and low porosity values. In the calculation, we observed two distinct modes of synthesis: axial combustion and surface combustion. We studied the characteristics of these synthesis modes for nanoparticle size range between 10–100 nm. The model is presented for the synthesis taking into account the gas slip, concentration and temperature jumps for \( O_{2} \) and \( CO_{2} \) at the surface of pores. We observed some cases of finger combustion for instable temperature fields. We compare the features of synthesis of coarser particles of \( ~10 \text{ micron} \) with the synthesis of nanoparticles of \( ~100 \text{ nm} \) at the porosity value 0.05\%. The synthesis of coarser particles takes place faster, because during the synthesis of coarser particles, there is no gas slippage on the surface of pores, and as a result, there is no thermal loss from the zone of combustion.

Figure 4 demonstrates the comparison modeling with experiment, when no reagents of ferrite synthesis were considered. The temperature of gas at the axis of symmetry and at the outer boundary of the sample is shown in Fig. 4a and 4b, respectively. The slip jump intensities are as follows \( A_{u} = 100, \) \( A_{1} = A_{02} = A_{\text{co2}} = 10. \) The satisfactory agreement can be seen in the Fig. 4c that presents the comparison of simulation of nickel zinc ferrite synthesis via carbon combustion with the experimental data. The discrepancy of data is less than 15\%. The heat loss during the ferrite synthesis was evaluated using the experimental investigation. We estimated the thermal parameter of synthesis as follows \( Q_{2} = -0.01. \)

The slip and jump intensities were \( A_{u} = 100, A_{1} = A_{02} + A_{\text{co2}} + 10. \)
The simulation of nickel zinc ferrite synthesis for fast and slow heat exchange between gas and solid in the sample of large porosity 40% is shown in Figs. 5 and 6. The ferrite concentrations at subsequent moments of time are presented in Fig. 5, which illustrates the actual rate of ferrite synthesis and the non-uniformity of ferrite distribution. The synthesis of ferrite nanoparticles was modeled via axial carbon combustion $J_T = \text{const}$, $Q_2 = -0.01$. Firstly, the modeling of fast thermal exchange between gaseous and solid phases during synthesis is considered. The distribution of gas and solid temperature and gas velocity at time $t = 0.2$ is shown in Fig. 6 a-c. In spite of rather fast heat exchange between gas and solid, the temperature of solid phase is four times less than the temperature of gas. According these results fast heat transfer occurs between gas and solid, when the heat transfer coefficient is $\kappa = 1500$. The slippage takes place at the pore surface only, i.e. no slippage at the outer wall is considered. The slip and jump intensities are as follows $A_u = 100, A_T = A_{O_2} = A_{CO_2} = 10$. The interfacial interactions among the surface and gas molecules lead to inelastic reflections, due to the long-range interaction forces between the gas and surface molecules.

In Fig. 5 the slow heat transfer between gas and solid is shown, when the heat transfer coefficient is $k = 1$. The slip and jump parameters are the same as in Fig. 5. The nickel zinc ferrite synthesis was simulated taking into account the activation energy and heat loss values obtained during our experimental study. The temperature of gas and solid differs more significantly for slow heat transfer between gas and solid. We can see the formation of local high temperature zones presented in Fig. 6 (a) caused by combustion instability due to outer boundary heat loss. These local regions of high gas temperature can be explained by a finger-like instability of combustion [15, 16]. The finger-like structure of temperature was observed experimentally [12] during the CCSO for ferrite synthesis. The non-uniformity of gas temperature causes a non-uniformity of ferrite density distribution and the process of synthesis as well as shown in the Fig. 7. The rate of ferrite synthesis for slow heat transfer between gas and solid can be seen. Let us note that fingers mode are not found at $t = 0.1$ (see Fig. 8 (c), however are structured later at $t = 0.2$. The comparison of Fig. 5 (a) and Fig. 6 (c) allows us to conclude that slower heat transfer between gas and solid lead to faster synthesis of nanoparticles.
In Fig. 8 the nickel zinc ferrite synthesis is presented for small porosity of 0.05%, when large resistance for oxygen flux takes place and surface combustion occurs. The slip and jump intensities are the same as in previous case. The large particles (with several microns in diameter) of nickel zinc ferrite synthesis were simulated neglecting the slip and jump effects, since the Knudsen number was sufficiently small. These results demonstrate that the synthesis of large particles runs faster compared to the nanoscale particles, because the slippage results in heat loss from the combustion domain.

The performed systematical study of the slip and jump behavior reveals the effects of Knudsen layers on the thermal and mass transfers in the large interval of jump and slip parameters. The intensities of temperature and gas component concentration jumps are determined by the thermal diffusion as well as by the thermal coefficient of accommodation of molecules. The value of slippage depends on the coefficient of reflection of molecules from the wall of the tube and the pores. There is slowing down of the synthesis when the parameter of slippage grows from $A_u = 1$ to $A_u = 10$.

6. Conclusions

For the modeling of nickel zinc ferrite synthesis, the macro fluxes caused by the gas slip, temperature and concentration jumps at the submicron pore surface are crucial. The synthesis can be simulated via the carbon combustion in a sample using the mechanics of continuum media supplemented with the effect of Knudsen layer. Thus, the effect of slippage slows down the combustion process. The lowering of temperature causes significant difference in the pressure distribution. The intensive outflow of the gas from the combustion zone and from the
sample in general results in the thermal losses and slows down the combustion in case of slippage. The combustion transforms from the diffusion regime to convective-diffusion regime. At the high slippage parameter value there is non-monotonic dependence of temperature on the value of Au. The stage of temperature decreasing while the parameter increases, transforms to increasing of temperature due to intensive heating of the whole sample at the extreme gas convection. The simulation result shows good agreement with the experimental investigation of nickel zinc ferrite synthesis.

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### List of notations

| Notation | Description |
|----------|-------------|
| $T_0 = 1273$ K | Referred temperature |
| $T'$ | Dimensional temperature |
| $T = \frac{T'}{T_0}$ | Dimensionless temperature |
| $\bar{T} = \frac{T - 1}{\beta}$ | Dimensionless normalized temperature |
| $T_gT_s$ | Gas temperature and solid phase temperature |
| $T_{init}$ | Ignition temperature |
| $T_{initial} = - \frac{1}{\gamma}$ | Initial temperature of the sample |
| $C_{pk}$ | Specific capacity at constant pressure for $k$-specie |
| $n_p$ | Numerical density of solid particles |
| $\rho_{lg}$, $1 = 2$ | Densities of gas species |
| $\rho_g = \rho_{lg} + \rho_{2g}$ | Density of gas mixture |
| $\overline{\rho}_g, \overline{\rho}_s$ | Effective values of gas density $\rho_g$ and solid density $\rho_s$ |
| $\rho_s = (1 - \chi)\overline{\rho}_s, \overline{\rho}_g = \overline{\rho}_g(s)$ | Effective density of gas species |
| $\overline{\rho}_{lg} = \overline{\rho}_{O2(g)}, \overline{\rho}_{2g} = \overline{\rho}_{CO2(g)}$ | |
| $\beta = \frac{RT_0}{E}$ | Dimensionless parameter |
| $\gamma = \frac{c_v T_0 \beta}{Q}$ | Dimensionless parameter (Zeldovich number) |
| $Pe_i = \frac{n_i D_i}{D_{i0}}$, $i = 1, 2$ | Peclet numbers |
| $u_0 = \frac{t_0}{t_u}$ | Referred velocity |
| $Re_0 = \frac{u_0 D_{i0} \rho_h}{\mu_0}$ | Reynolds number |
| $(S_i)_i = -u_i \chi, k_i = a_i \nu | + \zeta_i, i = 1, 2, 3$ | Porous resistance coefficient |

### Equations

\[
J_{slip}^{macro} = A_s u \sqrt{p \rho}
\]
Macro momentum flux due to velocity slip at the boundaries of pores

\[
J_{c,j}^{macro}
\]
Macro mass flux of specie component due to its concentration jump at the pores boundary

\[
\chi
\]
Porosity coefficient

\[
M_j
\]
Molar mass of specie

\[
P_0 = \frac{R \rho T_g}{M_0}
\]
Referred pressure

\[
\mu_0
\]
Referred gas viscosity

\[
\lambda_0
\]
Referred thermal conductivity

\[
Ma_0
\]
Mach number

\[
V(x, y, z)
\]
Mesovolume

\[
Q_i
\]
Thermal flux

\[
Q_g, Q_{jump}^{macro}
\]
Heat fluxes due to chemical reaction, and temperature jump

\[
b_u;\]
Slip parameter

\[
\theta, (0 \leq \theta \leq 1)
\]
Reflection coefficient

\[
A_1; A_C
\]
Temperature and concentration jump parameters

\[
a', (0 \leq a' \leq 1)
\]
Thermal accommodation coefficient

\[
A_{CO2}; A_{O2}
\]
Concentration jump of CO2 and O2 parameters

\[
D_T
\]
Thermal diffusivity coefficient

\[
R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}
\]
Gas constant

\[
t_0 = \frac{\exp(E/RT_0)}{k}
\]
Referred time and length scales for Frank-Kamenetskii variables

\[
t_{init}
\]
Ignition time