Infinite Maxwell fisheye inside a finite circle

Yangjié Liu\textsuperscript{1,2} and Huanyang Chen\textsuperscript{2,3}

\textsuperscript{1} 356 Engineering Building, Antennas Group, School of Electronic Engineering and Computer Science, Queen Mary University of London, 327 Mile End Road, E1 4NS Mile End, London UK
\textsuperscript{2} College of Physics, Optoelectronics and Energy & Collaborative Innovation Center of Suzhou Nano Science and Technology, Soochow University, No.1 Shizi Street, 215006 Suzhou, People’s Republic of China
\textsuperscript{3} Author to whom any correspondence should be addressed.

E-mail: yangjie.liu@qmul.ac.uk and chy@suda.edu.cn

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Abstract
This manuscript proposes a two-dimensional heterogeneous imaging medium composed of an isotropic refractive index. We exploit conformal-mapping to transfer the full Maxwell fisheye into a finite circle. Unlike our previous design that requires a mirror of Zhukovski airfoil shape, this approach can work without a mirror, while offering a comparable imaging resolution. This medium may also be used as an isotropic gradient index lens to transform a light source inside it into two identical sources of null interference. A merit of this approach is reduction of the near-zero-index area from an infinite zone into a finite one, which shall ease its realization.

Keywords: geometrical optics, optical system design, imaging and optical processing, edge and boundary effects, gradient-index lens

1. Introduction

Transformation optics and metamaterial have been giving birth to a plethora of novel electromagnetic devices, including perfect lensing that requires negative refractive index materials\,[1–3]. These negative refractive index materials suffer from material loss and dispersion issues\,[4, 5] and thus a different approach to achieve subwavelength imaging using positive refractive index was proposed\,[6] to ease their realization. This approach is based on a refractive index profile of the mirrored Maxwell fisheye, or the truncated profile of the full Maxwell fisheye, which is based on manipulating curved geometry of light\,[7]. The full Maxwell fisheye profile is of interest as all light rays emanating from any point within converge at its conjugate, to serve as an absolute instrument\,[8]. To ensure a finite structure a mirrored truncation is taken\,[6]. Recently, it has been demonstrated from calculation\,[9] that a drain in a mirrored Maxwell fisheye does not give subwavelength resolution in sense of capturing image distribution simultaneously, whereas a passive drain in resonant can still give infinitesimal fine resolution only when the drain is scanning across the imaging region\,[10].

Moreover, an extension to combine coordinate transformation (herein conformal mapping) with a known spatial profile (mirrored Maxwell fisheye) showed an interesting duality of simultaneous cloaking and imaging\,[11]. Its exterior refractive index in far field reduces to unity, which is amicable to free space environment and thus easy to implement. However, an obvious merit of conformal map is not fully used: its capability to shrink an infinite zone into a finite one, i.e. a unit circle therein\,[12, 13]. Thus in this manuscript we choose to transform the full Maxwell fisheye by conformal mapping while achieving a comparable imaging resolution to the mirrored alternative. We also investigate the effect of varying parameters on the imaging resolution, including the position, the size of profile and the operating wavelength, which was not covered in previous simulations\,[11].

The reason we choose to map the full fisheye is two fold: on one hand, the combined purpose of cloaking and imaging in\,[11] results in the use of the mirrored fisheye. This necessitates a forbidden cavity region of light delineated by the Zhukovski airfoil curve. As this curved mirror is...
redundant for imaging purpose, we remove it for ease of realization. The implementation of full Maxwell fisheye automatically adds a degree of freedom to shift the centre of the full profile, which can be used to investigate resolutions of imaging under scaling of geometrical parameters. On the other hand, our scheme is backed by the equivalence of their optical path between the full and the mirrored Maxwell fisheye [6]. Therefore, our proposed new heterogeneous medium can serve as another stigmatic imaging device, similar to other known absolute instrument profiles.

2. The conformally-mapped full Maxwell fisheye

Let us start from the full Maxwell fisheye profile as refractive index,

\[ n(r) = \frac{2}{1 + (r/r_w)^2}, \quad r \in [0, \infty) \] (1)

where all light rays make circles and \( r_w \) is a characteristic radius. It was Luneberg who realized that Maxwell fisheye profile can be generated by stereographic projection from a sphere surface to a plane [14, 15]. Based on the idea that light travels along geodesics on the sphere in virtual space, an imaging device follows: all rays emitted from any point in a full Maxwell fisheye will converge at its conjugate point—antipole to form an image [6], as shown by the converging circles in figure 1(a). An obvious fault for the full profile of 2D Maxwell fisheye is its infiniteness in area and henceforth the truncation trick along with mirror is employed at the circle \( r = r_w \) [6]. Accordingly, a mirror of Zhukovski airfoil is used in [11]. However, here we attempt to sidestep this obstacle of infiniteness of full profile by applying the conformal-map method [12].

We use conformal-map to transform the imaging behaviour of the full Maxwell fisheye. In principle, one can implement any conformal map [16] to the full fisheye profile to create a new heterogeneous medium to reserve the imaging behaviour of Maxwell fisheye (1), as illustrated in figure 1. A conformal map creates a virtual space of light which is made of a stack of Riemann sheets in general [12, 13]. In this manuscript, Zhukovski map⁴

\[ w(z) = z + \frac{a^2}{z}, \quad (a > 0) \] (2)

is chosen in order to allow a fair comparison of our present approach with that of [11]. Under Zhukovski map, a physical space in two-dimensional space is mapped onto two Riemann sheets, of which the lower and upper sheets are geometrically connected by a segment (branch cut) from \(-2a\) to \(2a\). The two sheets labelled by Cartesian \((u, v)\) and \((u', v')\) shown in figure 2(a) are mapped to the exterior and the interior zone in physical space \((x, y)\) in figure 2(c), which are delineated by the purple unit circle \(|z| = a\) therein.

The full and mirrored Maxwell fisheyes preserve the same optical path (see figure 1(a) and virtual space in figure 3(a) of [6]). In our present work we remove the mirror in [11] so as to dismantle the constraint of the curved mirror, by simply implementing the full fisheye on the lower sheet of virtual space. We will see that the removal of the mirror does not deteriorate the imaging resolution. Performing ray-tracing in the full fisheye profile shows that a source at \((-b, 0)\) will be conjugated by its image at \((r_w/b, 0)\), which is different from what occurs in its mirrored profile, as illustrated in figure 1(a). One can also write the wave solution \(E(w)\) in the full Maxwell fisheye profile according to

\[
E(w) = E_0 \left( \frac{r^2 - 1}{r^2 + 1} \right) = \frac{w/r_w \cos \gamma - e^{i\chi} \sin \gamma}{w/r_w e^{-i\chi} \sin \gamma + \cos \gamma}, \quad (3)
\]
Figure 2. (a) Illustration of the lower \((u', v')\) and the upper \((u, v)\) Riemann sheet. On the lower sheet, red dashed circle indicates the coordinate \(r = r_w\), gray contours for \(\theta\) and \(\phi\) in (8) and (9). The red dot \(O_w\) indicates the centre of the Maxwell fisheye put on the lower sheet. Thick green circles (including a straight line through the purple branch cut) are five light trajectories contributing to form an image at the origin point in hollow black circle \(O_u\) from a source point in blue cross \(S_w\). The light trajectories deflect at the branch cut and transmit onto the upper sheet. The wave distribution \(E(u')\) according to (3) and (4) and the refractive index \(n(u')\), both for the branch cut on the lower sheet are plotted on the upper. Parameters: \(b = a, r_w = 4a, l = 24\). (b) Virtual curved space \((\theta, \phi)\) on a sphere, for the lower sheet in (a) under stereographic projection. On the sphere light travels in geodesics, in green circle, diverging from the blue source and converging at the black image (geodesics equation based on equation (19.9) [15]). (c) Physical space \(z = x + iy\). Five green rays are plotted corresponding to (a). In physical space, the branch cut becomes a purple circle, the unit circle on lower sheet becomes a red dashed curve, and the image \(O_u\) and the fisheye centre \(M_w\) are doubly degenerate. Panels (b)–(c) take the same parameters as (a).

\[
E_l(\zeta) = \begin{cases} 
\frac{P_l(\zeta) - e^{i\chi}P_{-l}(\zeta)}{4 \sin(l\gamma)} & \text{for } l \notin \mathbb{N}^+ \\
\frac{e^{i\chi}Q_l(\zeta)}{2\pi} & \text{for } l \in \mathbb{N}^+ 
\end{cases} 
\]

in which \(\gamma\) and \(\chi\) are defined according to the source position \(w_0 = e^{i\chi} \tan \gamma\), and \(P_l(\zeta)\) and \(Q_l(\zeta)\) are the first and the second Legendre function [6, 17]. Eigennumber \(l\) in (3) and (4) is linked to the wavelength

\[
\lambda(w) = \frac{2\pi r_w}{n(w)\sqrt{l(l + 1)}} \quad (l > 0).
\]

The wave solution of a full Maxwell fisheye is plotted in figure 1(b), which corresponds to the ray-tracing diagram in figure 1(a).

Now we proceed to determine the position of light source in our proposed profile. As the two sheets in virtual space are juxtaposed vertically in figure 2(a), a degree of freedom arises naturally to shift the centre of the fisheye profile: we shift its centre \(M_w\) away from the origin \(O_w\) by an arbitrary distance \(b\) on the lower sheet along \(u'\) axis\(^5\). To make use of the conformal map [11], we hereby transfer the imaging point from the lower sheet to the upper one via the branch cut \([-2a, 2a]\), which means the image point must fall on the branch cut. Without losing the generality above, we put the image point at the origin \(O_u\). Due to the geometrical relation of source and image in a full Maxwell fisheye profile, we know that the source \(S_w\) must lie on the opposite side to the image \(O_u\) with respect to the centre, as shown in figure 1 and 2(a). This means that the source should fall on the branch cut or its extension line. Here the source point \(S_w\) is intentionally positioned off the branch cut, as illustrated in figure 2(a). The reason is as follows: if it also fell on the branch cut alike the image, the source in physical space \(z\) would in general

\(^5\text{We choose this direction of shifting the centre } M_w \text{ for simplicity. One could shift the centre along other directions (e.g. perpendicular to the branch cut) to achieve essentially the same results in this manuscript.}\)
Figure 3. (a), (b) Electric field distribution (in arbitrary unit) in z direction for $b = 0$ and $a$ (COMSOL simulation). (c), (d) Ray trajectories computed according to Hamilton’s equations for $b = 0$ and $a$, where rays in different directions and profiles of refractive indices $n(z)$ are coloured. (a)–(d) Unit circle is marked black to indicate the boundary between two Riemann sheets in virtual space. (e), (f) Far field norm patterns. Note that the ticks are re-scaled to emphasize the relative contrast of the field norm. Parameters: (a)–(f) $r_w = 4a$, $l = 24$, (a), (c), (e) $b = 0$ and (b), (d), (f) $b = a$. 
degenerate into two sources on the unit circle, when parameter $b$ is swept over arbitrary positive values under the condition $r_w \leq a$. This condition would obliterate the merit of Zhukovski map (2) to transfer the image point underneath onto the upper sheet\(^6\). For this reason we restrict the fisheye size under the condition $r_w > a$ so that $b$ can sweep over the whole positive range to maintain the source off the branch cut. To prove that any positive value of $b$ guarantees that if $r_w > a$, $O_w S_w > 2a$ i.e. the source falls out of the branch cut, one can write $O_w S_w = O_d M_w + M_w S_w = b + r_w^2/b \in [2r_w, \infty)$, $\forall b > 0$. Thus equivalently in physical space, the source $S_w$ lies inside the unit circle and is transformed into two identical images $O_w (\pm z_{1,2} = \pm i a)$ upon the circle, as displayed in figure 2(c).

Finally, based on the optical conformal mapping method \cite{12, 15}, the refractive index profile in physical space of our conformally-mapped full Maxwell fisheye shown in figure 3(c)–(d) is written as

$$n(z) = n(w) \frac{dw(z)}{dz} = \begin{cases} 1 - \frac{a^2}{z^2}, & \text{for } |z| > a; \\ \frac{2}{|z^2 - \alpha| + 1}, & \text{for } |z| \leq a. \end{cases}$$

This is of a relatively large range of refractive index, for instance $[0, 33]$ for $b = a$ in figure 3(d).

Considering TE polarization where electric field directs perpendicular to the complex $z$ plane, the scalar Helmholtz equation applies for the wavelength $\lambda$ therein given by \cite{6}

$$\lambda = \frac{2\pi r_w}{n(z)\sqrt{i(l + 1)}} \quad (l > 0).$$

In all the simulations in this manuscript we follow the previous simulation \cite{11} to assign integer values to eigenvalues $l$ in (7) to compare the imaging resolutions; although $l$ can be non-integer, which would vary the imaging phase.

Before looking into the wave behaviour in virtual space, it helps to look at the coordinates on the lower sheet. The full Maxwell fisheye profile bends Cartesian coordinates $u$, $v$ on lower $w$ sheet into curvilinear ones ($\theta$, $\phi$ spherical in figure 2(b)), as plotted in gray coordinates both on the lower sheet in figure 2(a) and inside the unit circle in figure 2(c). This is in accord to the inverse stereographic projection taking into account which quadrant the point $(u' - b, v')$ lies in:

$$\theta = \arccos \left( \frac{|u' - b + iv'|^2 - r_w^2}{|u' - b + iv'|^2 + r_w^2} \right) \in [0, \pi],$$

$$\phi = \arctan (u' - b, v') \in [0, 2\pi).$$

An imaging process in our profile consists of the light flow from any source point towards its antipodal and afterwards the leakage into the upper sheet, illustrated by five representative light paths in thick green curves in figure 2(a)–(c). It is interesting to notice that all rays emanating from the source do not refract on the upper sheet since total reflection occurs for a fraction of them. These reflected rays can constitute to a laser-like resonator mechanism inside the unit circle in physical space however, this is beyond the scope of this work.

3. Wave simulations and ray-tracing diagrams

In this section, we shall perform numerical simulations to investigate the resolutions of our proposed medium–conformally-mapped full Maxwell fisheye, when three parameters are swept to find the optimal resolution in section 3.1. In section 3.2 ray-tracing is performed to illustrate the light flow to image.

3.1. Wave simulation

Firstly, we perform two-dimensional numerical simulation to investigate the imaging resolution of the medium of mapped full Maxwell fisheye. We feed an active point source on point $S(z)$ on $z$ plane (1A as input) and solve for the TE polarization distribution $E_i(x, y)$ induced by the indicated source via COMSOL Multiphysics \cite{19}.

In figure 3, we demonstrate two examples of our proposed medium: $b = 0$ and $a$ respectively, from perspective of wave solution (a)–(b), ray-tracing (c)–(d) and far field pattern (e)–(f). As a preliminary demonstration, we plot the TE field distribution in figure 3(a)–(b). In such simulations, truncation is taken beyond a radius of $5r_w$ away from the origin in the physical space since the refractive index of the medium therein can be approximated as unity similar to [11] ($|dw/dz| \sim 1$ when $|z| \to \infty$ \cite{12, 20}). This feature of approximate unity refractive index when $r > 5r_w$ with respect to the origin, allows one to position two image detectors in a free space environment, which is easy to realize in experiment. The discontinuous pattern at the branch circle $|z| = a$ in figures 3(a) and (b) can be attributed to the slight jump between their refractive indices of the two Riemann sheets. This discontinuity is also present in previous simulation work (see figure 2 in [11]), which can be improved by adding a layer of continuous gradient index \cite{21} but here we stick to profile (6) for simplicity.

The wave flow from the source to image is contained in the wave picture as seen from figures 3(a) and (b). Moreover, this is more pronounced as seen from our ray-tracing diagrams, shown in figures 3(c) and (d) respectively. Therein the light diverges from the source inside the unit circle and follows two distinctive paths upward and downward, until they converge at the two symmetric points $\pm ia$ (note that the zero points of refractive index occur at $z = \pm a$, which aligns with the doom of trajectory-tracing from Hamilton’s equations at these two points). This is similar to the wave picture of
mapped mirrored fisheye and aligns with our design in section 2 to position the image at the origin in virtual space. The far field norm patterns in figures 3(e) and (f) also reveal the almost isotropic radiation pattern from our proposed medium despite slightly higher main lobes in side directions (0° and 180°).

In order to seek the best resolution of imaging for our proposed medium, we sweep over three parameters below to calibrate spatial distribution of average energy flowing out of the unit circle: the distance between the origin and the fisheye centre \( b := \Omega_{\text{u}} M_w \), the characteristic radius \( r_w \) and the eigenvalue \( l \). We define the resolution by the full width at half maximum (FWHM) for the flowing energy distribution, scaled up to the wavelength \( \lambda \) in (7).

Firstly, to find an optimal image resolution for the considered profile, we sweep \( b \), i.e. shift the centre of the fisheye along \( u' \)-axis on the lower \( w \) sheet. Due to the geometrical symmetry of the medium, varying \( b \) within positive regime is sufficient to disclose the resolution scaling. In figure 4(a), the average energy flowing out along the upper half circle (branch cut) is plotted versus radiant angle, under different values of \( b \). It is found that as \( b \) decreases until zero, the resolution becomes sharper until a lower limit of 0.24\( \lambda \), as shown in figure 4(a) inset. We may clarify this by the observation that, in virtual space in figure 2(a) light will converge from complex infinity on the lower sheet in omni-direction into the origin, which is corroborated by the ray-tracing diagram in figure 3(c).

Secondly, the geometrical size of Maxwell fisheye on the lower sheet is also varied via \( r_w \). Although the imaging resolution shrinks as \( r_w \) increases, it never shrinks below 0.3\( \lambda \) in figure 4(b). The sharper trend with increasing \( r_w \) may imply the refractive index jump along unit circle \( |z| = a \), ignored by the previous study [11]. As \( r_w \) increases, the discontinuity in the refractive index \( n(z) \) at the unit circle diminishes and thus better resolution is achieved. This discontinuity in the refractive index may also clarify the scattered colour pattern in figure 3.

Thirdly, when one tunes integer values of \( l \), a similar trend can be observed in figure 4(c) where the imaging resolution reduces as \( l \) increases. Yet no shaper resolution than \( \sim 0.2 \) wavelengths (corresponding to the least value \( l = 2 \)) can be achieved herein, which is comparable to the resolution previously obtained in [11]. As mentioned in section 2 on (7), only integer values of \( l \) are chosen for simplicity but non-integer values also apply for imaging.

From these analyses, the optimal parameter according to our simulation is: \( b = 0, r_w = 8a \) and \( l = 2 \) and the optimal resolution achievable in our scenario is \( \sim 0.2 \lambda \) according to our definition of FWHM for average flown-out energy. We demonstrate that the removal of mirror does not affect the imaging resolution compared to mapped mirrored fisheye profile. Our simulation results align with theoretical computation [9] that the mirrored Maxwell fisheye does not give super-resolution from the traditional process of imaging, if one captures the image distribution from its source simultaneously in the whole profile.

Figure 4. Average energy flowing out along the upper half circle when one sweeps over (a) \( b \) (in unit of \( a \)), (b) \( r_w \) and (c) \( l \). Note that the \( b \)-\( c \) main panels and insets \( a \)-\( c \) are all normalized to compare with the operating wavelength in physical space \( \lambda_{\text{u}} = \frac{1}{24} \) (this corresponds to the same wavelength 0.5130a in [11]) according to (6) and (7).

Parameters: (a) \( r_w = 4a, l = 24 \); (b) \( b = 4a, l = 24 \); (c) \( b = 0, r_w = 4a \).

3.2. Ray-tracing diagrams

The bifurcating behaviour of light wave, inside the unit circle in figure 3(a)–(b) leads one to trace the propagation scenario of light within. One simple method is to compute the ray-tracing diagrams. It should be clarified that in virtual space the light rays form circles on the lower sheet and straight lines on the upper (see figure 2(a), if one neglects reflection at the branch cut\(^8\)). However, it is more relevant in physical space, \( \approx 0.2 \) wavelengths in figure 3.

Note that the discussion for null reflection from an incidence wave from the upper Riemann sheet (on equations (11)–(13), p4 [22]) does not apply in our scenario since herein, the incidence wave originates from the lower sheet instead of the upper as in [22].
to peep through the branch cut into the ray-tracing of the transformed fisheye.

To compute the ray trajectories in the profile of refractive index distribution (6), we use Hamilton’s equations \( \partial_t r = \partial \omega / \partial k \) and \( \partial_t k = - \partial \omega / \partial r \) [15, 23] by Mathematica [18], in which \( k \) indicates the momentum, \( r \) the position, \( \omega \) the Hamiltonian. Two instances are plotted in figure 3(c)–(d) to illustrate the wave flow of imaging in figure 3(a)–(b) one-to-one. In figure 3(c)–(d): light rays originate from source point \( z = 0.0590a \) and \( z = 0 \) respectively, converge at two image points \( z = \pm ia \) and then propagate outward. The ray-tracing diagrams align remarkably with the wave solutions in figure 3(a)–(b).

4. Summary

In this manuscript, we demonstrate a new heterogeneous, isotropic optical medium from perspective of wave simulation, by making use of full Maxwell fisheye as well as conformal-mapping method [12, 24]. The merit of this idea lies on making use of the full profile of Maxwell fisheye and the removal of the mirror, which still maintains the same imaging resolution as the mirrored Maxwell fisheye under conformal mapping [11]. By conformal map, the area of near-zero-index is transferred from an infinite zone to beside a unit circle. It can also be used for source-transforming purpose to create illusion of two coherent light sources without interference, similar to [25]. Future work may investigate the mutual interference of multiple sources in our medium and their role in improving its functionality.

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Author contributions

H C proposed this starting idea and co-designed it with L Y. L Y performed COMSOL simulation, completed all the computation, and prepared the manuscript draft. H C revised the manuscript.

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