Higgs Properties and Fourth Generation Leptons

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It is possible that there are additional vector-like generations where the quarks have mass terms that do not originate from weak symmetry breaking, but the leptons only get mass through weak symmetry breaking. We discuss the impact that the new leptons have on Higgs boson decay branching ratios and on the range of allowed Higgs masses in such a model (with a single new vector-like generation). We find that if the fourth generation leptons are too heavy to be produced in Higgs decay, then the new leptons reduce the branching ratio for $h \to \gamma\gamma$ to about 30% of its standard-model value. The dependence of this branching ratio on the new charged lepton masses is weak. Furthermore the expected Higgs production rate at the LHC is very near its standard-model value if the new quarks are much heavier than the weak scale. If the new quarks have masses near the cutoff for the theory, then for cutoffs greater than $10^{15}$ GeV, the new lepton masses cannot be much heavier than about 100 GeV and the Higgs mass must have a value around 175 GeV.

I. INTRODUCTION

We have observed three generations of quarks and leptons, however, there is no convincing prediction for the number of generations that exist. Hence examining the physics of extensions of the standard model with additional generations of quarks and leptons is worthwhile. Experimental constraints on fourth generation of quark masses are very strong, $m_{u',d'} \gtrsim 330$ GeV \cite{1,2}. To be consistent with these constraints, an extension of the minimal standard model with a chiral fourth generation of quarks and leptons (and no other degrees of freedom) must be low energy effective theory with a cutoff not far from the TeV scale. This is because the large fourth generation quark Yukawa couplings grow with energy scale and one encounters a Landau pole after only a modest amount of renormalization group evolution. Furthermore there are issues with stability of the Higgs potential in such a model. Experimental limits on the masses of fourth generation leptons, on the contrary, are much less stringent. Heavy charged lepton masses must be larger than about 100 GeV, while stable (unstable) heavy neutral leptons must be heavier than about 45 GeV (90 GeV) \cite{3}.

In this paper an additional vector-like fourth generation (i.e., a chiral fourth generation plus its mirror) is considered. In this framework, one can construct scenarios where the fourth generation quarks get a mass term that does not require weak symmetry breaking, but the leptons are forbidden from getting such a mass term. The model constructed in Ref. \cite{4} where baryon and lepton number are gauged and spontaneously broken is an example of this. In such models over most of the parameter space fourth generation quarks acquire masses much greater than the weak scale, nonetheless, their Yukawa couplings to the Higgs doublet can be small. On the other hand, fourth generation leptons cannot have masses far above the weak scale and fourth generation lepton masses around 100 GeV are reasonable. Hence, the problems associated with Landau poles not far from the weak scale and vacuum stability do not occur over a wide range of the allowed parameter space.

Even if the fourth generation quarks are very heavy, the new leptons have a dramatic effect on the decays of the Higgs boson. (For a study of Higgs physics in four generation models, see Ref. \cite{5}.) Since the fourth generation quarks have mass terms that do not require weak symmetry breaking, they decouple as their masses increase. If the new quarks are much heavier than the weak scale then the Higgs production rate at the LHC is near its standard-model value, but we find that the $h \to \gamma\gamma$ branching ratio is reduced to about 30% of its standard-model value. This reduction depends weakly on the charged lepton masses and so it is a signature for this scenario.

Although the small fourth generation quark Yukawa couplings do not develop Landau poles below the GUT or Planck scale, the new leptons may give rise to Landau poles in coupling constants or a vacuum instability in Higgs potential. We study the impact that these leptons have on the renormalization group evolution of the Yukawa couplings and the Higgs self-coupling. The Higgs mass squared is proportional to its self-coupling $\lambda$. There are upper and lower bounds on the Higgs mass from the requirement that $\lambda(\mu) < \infty$ (no Landau pole) and $\lambda(\mu) > 0$ (vacuum stability) for scales $\mu$ less than the cutoff of the theory. We find the Higgs mass (denoted as $m_h$) must be around 175 GeV and the fourth generation lepton masses cannot be greater than about 100 GeV when the cutoff $\Lambda_c \approx 10^{15}$ GeV, assuming the fourth generation quark masses are at the cutoff. With a low cutoff of 10 TeV, the Higgs mass must be in the range $120 \text{ GeV} \lesssim m_h \lesssim 400$ GeV and roughly speaking the fourth generation lepton masses should be smaller than the Higgs mass.

While we were working on this paper, Ref. \cite{6} appeared. The research presented here is similar to that in Ref. \cite{6}, however this paper is focused on a particular class of models.

II. THE MODEL

We consider a model with a vector-like fourth generation of quarks and leptons in addition to the standard-model particles. This fourth generation has the $SU(2)$ left-handed quark doublet, $Q'_L = (u'_L, d'_L)$, right-handed up- and down-type quark singlets, $u'_R$ and $d'_R$, left-handed lepton doublet,
$L'_L = (\nu'_L, e'_L)$, and right-handed charged and neutral lepton singlets, $e'_R$ and $\nu'_R$. The mirror fourth generation particles are: the $SU(2)$ right-handed quark doublet, $Q''_R = (u''_R, d''_R)$, left-handed up- and down-type quark singlets, $u'_L$ and $d'_L$, the right-handed lepton doublet, $L''_R = (\nu'_R, e'_R)$, and the left-handed charged and neutral leptons singlets, $e''_L$ and $\nu''_L$. Their $U(1)_Y$ charges are the same as the existing fermions in the standard model.

In addition to their gauge invariant kinetic terms, the new quarks have the following mass terms and Yukawa couplings to the Higgs doublet denoted as $H$,

\[
\Delta L_q = -M_Q Q'_R Q''_R - M_U u'_R u''_L - M_D d'_R d''_L \\
- \frac{1}{2} \epsilon^* L'_L H^* e'_R - \frac{1}{2} \epsilon^* Q''_R H^* u'_L \\
- \frac{1}{2} \epsilon^* D'_L H^* d'_R - \frac{1}{2} \epsilon^* Q''_R H^* d''_L + \text{h.c.},
\]

where $\epsilon$ is the antisymmetric $2 \times 2$ matrix (in weak $SU(2)$ space) with non zero components, $\epsilon_{12} = 1$ and $\epsilon_{21} = -1$. Here we ignore terms which mix the fourth generation quarks with the familiar quarks for simplicity. However, we imagine that there are small couplings of this type that allow the new quarks to decay. (For constraints on the mixings in quark sector, as well as lepton sector, from experiments, see e.g., Ref. [5].) Such mixings are forbidden if there is a global $U(1)$ symmetry where the fourth generation quarks and the ordinary quarks have different charge. If the breaking of this symmetry is small, then the mixings are expected to be small.

For the new leptons, the analogous couplings are given by

\[
\Delta L_{\tilde{l}} = -\frac{1}{2} \epsilon^* L'_L H^* e'_R - \frac{1}{2} \epsilon^* Q''_R H^* u'_L \\
- \frac{1}{2} \epsilon^* D'_L H^* d'_R - \frac{1}{2} \epsilon^* Q''_R H^* d''_L + \text{h.c.}.
\]

The crucial difference between the quark and lepton sectors is the absence of bare lepton mass terms. The bare mass terms, in addition to the terms which mix the new leptons and ordinary leptons, can be forbidden if one assumes a global $U(1)$ symmetry where the primed leptons, double primed leptons and ordinary leptons have different charge. When the charge of the ordinary leptons under this symmetry is zero, then the ordinary neutrinos can have Majorana masses. If there is no mixing between the fourth generation and ordinary leptons, one can have an acceptable scenario with stable fourth generation neutrinos.

Usually one does not impose global symmetries since quantum gravity effects will violate them. However, it is possible that there are underlying gauge symmetries that leave Eqs. (1) and (2) as the low energy effective theory after they spontaneously break. In fact baryon and lepton number could be such gauge symmetries. By adding a vector-like fourth generation one can gauge baryon and lepton number provided the difference in baryon number between the fourth generation and mirror generation is $-1$ and the difference in lepton number between the fourth generation and mirror generation is $-3$ [4]. The quarks in these families have mass terms that do not require weak symmetry breaking provided we introduce a scalar, $S_B$, with baryon number 1 that gets a vacuum expectation value (VEV). Similarly fourth generation lepton masses that do not require weak symmetry breaking arise if a scalar, $S_L$, with lepton number 3 gets a VEV. However this charge for $S_L$ is not preferred since then one cannot generate light neutrino masses through the seesaw mechanism [7]. If we use $S_L$ with lepton number 2 to break lepton number, Majorana neutrino masses for the light neutrinos are generated through the seesaw mechanism and furthermore proton decay is forbidden since the field that breaks lepton number has even charge. In that case the breaking of lepton number does not give rise to mass terms for the fourth generation leptons.

It is appropriate to keep the fourth generation quarks in the low energy effective theory if their masses are well below the scale of baryon number symmetry breaking. However, there is no particular reason for this to be the case and the generic situation is that one would only be left with just the fourth generation leptons and the standard-model particles in the low energy effective theory.

### III. PHENOMENOLOGY OF THE HIGGS BOSON

Here we discuss phenomenology of the Higgs boson in the model with the vector-like fourth generation. In this model the fourth generation quark and lepton masses have different origins. Thus they have different impact on the properties of the Higgs boson. In this section we discuss Higgs boson production and decay at the LHC.

#### A. Fourth generation quarks and the Higgs production

Here we study the impact of vector-like fourth generation on Higgs production at the LHC. In the usual chiral fourth generation scenario, the Higgs production rate, which is dominated by the gluon fusion process, is increased by about a factor of nine and this result is almost independent of the chiral fourth generation quark masses. (See Ref. [5] and references therein.) Contrary to such a result, we will see that the production rate rapidly approaches to the standard-model value as the vector-like fourth generation quark masses get larger than a TeV.

Let us derive the interaction Lagrangian of fourth generation mass eigenstate quarks with the Higgs boson. It is convenient to introduce the four-component fourth generation quark fields: $\psi'_L$, $\psi'_R$, $\psi''_L$, and $\psi''_R$. Their left and right components are $\psi'_UL = u'_L$, $\psi'_UR = u''_L$, $\psi''_UL = u'_L$, $\psi''_UR = u''_L$ and similarly for the down-type quarks. In the basis $\Psi = (\psi'_L, \psi'_R)$ (and including the effects of weak symmetry breaking) the quark-type mass terms are from Eq. (1) as

\[
\Delta L^{(u)}_{\text{mass}} = -\bar{\Psi}_{UL} M_U \Psi_{UR} + \text{h.c.},
\]

where

\[
M_U = \begin{pmatrix} M_Q & m_{U} \\ m_{U}^* & M_U \end{pmatrix},
\]
and $m_U' = h_U'v/\sqrt{2}$ and $m_U'' = h_U''v/\sqrt{2}$. Here $\langle H^0 \rangle = v/\sqrt{2}$ with $v \approx 246$ GeV, the VEV of the neutral component of the Higgs doublet. The up-type quark mass matrix is diagonalized by making unitary transformations $V_L(u)$ and $V_R(u)$ on the left and right-handed up-type quark fields so that,

$$V_L(u)^\dagger M_U V_R(u) = \begin{pmatrix} M_{U_1} & 0 \\ 0 & M_{U_2} \end{pmatrix}.$$ \hfill (5)

We denote the two up-type quark mass eigenstates as $U_1$ and $U_2$ and take $M_{U_1} > M_{U_2}$. The mass eigenvalues are,

$$M_{U_{1,2}}^2 = \frac{1}{2} \left( (M_Q^2 + M_U^2 - M_{U_2}^2) \mp \sqrt{X + Y} \right).$$ \hfill (6)

with

$$X = (M_Q^2 + M_U^2 - M_{U_1}^2 - m_{U_1}^2)^2,$$

$$Y = 4 (m_{U_1}^2 M_Q + m_{U_2}^2 M_U)^2.$$ \hfill (7)

For simplicity we assume the up-type quark Yukawas, $h'_U$ and $h''_U$, are real so that the transformations that diagonalize the mass matrix are the real orthogonal matrices,

$$V_L(u) = \begin{pmatrix} \cos \theta_L^{(u)} & \sin \theta_L^{(u)} \\ -\sin \theta_L^{(u)} & \cos \theta_L^{(u)} \end{pmatrix},$$ \hfill (9)

$$V_R(u) = \begin{pmatrix} \cos \theta_R^{(u)} & \sin \theta_R^{(u)} \\ -\sin \theta_R^{(u)} & \cos \theta_R^{(u)} \end{pmatrix}.$$ \hfill (10)

Since $M_Q$ and $M_U$ are larger than the mass terms that arise from weak symmetry breaking, the angles $\theta_L^{(u),R}$ are small and their cosines are positive. The angles are given by,

$$\tan \theta_L^{(u)} = \frac{m_{U_1}^2 M_Q + m_{U_2}^2 M_U}{M_{U_2}^2 - M_{U_1}^2 - m_{U_2}^2},$$ \hfill (11)

and the right-handed angle is given by flipping $m'$ with $m''$,

$$\tan \theta_R^{(u)} = \frac{m_{U_1}^2 M_Q + m_{U_2}^2 M_U}{M_{U_2}^2 - M_{U_1}^2 - m_{U_1}^2}.$$ \hfill (12)

Similar formulae hold for the down-type fourth generation quarks.

For the calculation of the Higgs production rate, we need to know the couplings of the Higgs boson to the fourth generation quark mass eigenstates which do not change the type of heavy quark. Using the above definitions, these are

$$\mathcal{L}_{\text{Higgs}}^Q = -\frac{\mu_{U_1}}{v} h U_1 U_1 - \frac{\mu_{U_2}}{v} h U_2 U_2$$

$$- \frac{\mu_{D_1}}{v} h D_1 D_1 - \frac{\mu_{D_2}}{v} h D_2 D_2,$$ \hfill (13)

where

$$\mu_{U_1} = -\cos \theta_L^{(u)} \cos \theta_R^{(u)} \left( m_{U_1}^2 \tan \theta_L^{(u)} + m_{U_2}^2 \tan \theta_L^{(u)} \right),$$

$$\mu_{U_2} = \cos \theta_L^{(u)} \cos \theta_R^{(u)} \left( m_{U_1}^2 \tan \theta_L^{(u)} + m_{U_2}^2 \tan \theta_L^{(u)} \right),$$

$$\mu_{D_1} = -\cos \theta_L^{(d)} \cos \theta_R^{(d)} \left( m_{D_1}^2 \tan \theta_L^{(d)} + m_{D_2}^2 \tan \theta_L^{(d)} \right),$$

$$\mu_{D_2} = \cos \theta_L^{(d)} \cos \theta_R^{(d)} \left( m_{D_1}^2 \tan \theta_L^{(d)} + m_{D_2}^2 \tan \theta_L^{(d)} \right).$$ \hfill (14)

For comparison we give the coupling of the Higgs boson to the top quark,

$$\mathcal{L}_{\text{Higgs}}^t = -\frac{m_t}{v} h t t,$$ \hfill (15)

where $m_t$ is the top quark mass.

Now we are ready to calculate the Higgs production rate at the LHC. As we mentioned, the production rate is dominated by the gluon fusion process. It is induced by a quark loop. In the standard model a top quark loop is the main contribution. In our model, fourth generation quarks are additional contributions. Thus the Higgs production rate in our model divided by its standard-model value is given by

$$\frac{\sigma(gg \rightarrow h)}{\sigma_{\text{SM}}(gg \rightarrow h)} = 1 + \sum_{i=1,2} \left[ \frac{\mu_{U_i}}{M_{U_i}} I(r_{U_i}) + \frac{\mu_{D_i}}{M_{D_i}} I(r_{D_i}) \right]/I(r_i),$$ \hfill (16)

where $r_i \equiv m_i^2/4m_{D_i}^2$, $r_{U_i,D_i} \equiv m_h^2/4M_{U_i,D_i}^2$, and the function $I(x)$ is

$$I(x) = 2[x + (x-1)f(x)]/x^2,$$ \hfill (17)

with

$$f(x) = \begin{cases} \arcsin^2(\sqrt{x}) & 0 < x \leq 1 \\ -4 \sqrt{\frac{\log \frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} - i\pi}{i\pi}} & 1 < x \end{cases}.$$ \hfill (18)

We neglect the other light quarks, $b, c, s, d$ and $u$.

In Fig. 1 we plot this ratio of the cross sections for $m_U' = 2m_U'' = 60$ GeV, $m_D' = 2m_D'' = 60$ GeV and $M_Q = M_U = M_D = 300$ GeV, 600 GeV and 1 TeV as a function of $m_h$. The production rate rapidly approaches the standard-model rate as $M_{U,D,Q}$ increase. This is because the contribution of the fourth generation quarks is suppressed by $m_{U,D}^2/M_{U,D}^2$, $m_{D,D}^2/M_{D,D}^2$, which can be seen from Eqs. (11), (12) and (13). In usual fourth generation scenario (i.e., not vector-like scenario), on the contrary, the Higgs production rate by gluon fusion process is increased by about a factor of nine over the standard-model production rate and its dependence on the chiral fourth generation quark masses is weak. Therefore, the exclusion of the Higgs with mass in the range, $131$ GeV $\leq m_h \leq 204$ GeV, by the Tevatron in the usual fourth generation scenario [8], is not applicable to our model.

**B. Fourth generation leptons and the Higgs decay**

So far we have discussed the impact of a fourth vector-like generation on the Higgs production rate at the LHC. Next, we study the Higgs decay branching ratios. The partial decay widths of each mode in our model are the same as those in standard model, except for the modes $h \rightarrow gg, \gamma Z$ and...
\[ \Gamma(h \rightarrow \gamma\gamma) = \alpha^2 G_F m_h^3 \text{Im} J_{\gamma\gamma} \],

where \( \alpha \) and \( G_F \) are fine structure constant Fermi constant and

\[ J_{\gamma\gamma} = \left( \frac{2}{3} \right)^2 N_e I(r_t) + I(r_{E'}) + I(r_{E''}) + K(r_W) + \frac{2}{3} \sum_{i=1,2} N_c \frac{\mu_{U_i}}{M_{U_i}} I(r_{U_i}) + \frac{1}{3} \sum_{i=1,2} N_c \frac{\mu_{D_i}}{M_{D_i}} I(r_{D_i}). \]
The partial decay width of the mode $h \to \gamma Z$ does not exhibit this dramatic effect. We have checked that the ratio of the partial decay width of this mode to its standard-model value is $\approx 1$. Finally it is obvious that partial decay width for $h \to gg$ does not change at all in the limit $M_{Q,U,D} \to \infty$. Therefore, although the Higgs production rate is almost unchanged from the value predicted in standard model, the branching ratio for $h \to \gamma \gamma$ is reduced significantly. This is the outstanding feature of this model and can be tested at the LHC.

To understand the impact of the fourth generation on Higgs decay further, we give the Higgs decay branching ratios in Figs. 3 and 4. Here we take $m_E' = m_E'' = 100$ GeV and $m_N' = m_N'' = 100$ GeV in Fig. 3 and $m_E' = m_E'' = 100$ GeV and $m_N' = m_N'' = 70$ GeV in Fig. 4. In the calculation, we take the limit $M_{Q,U,D} \to \infty$ as in the previous plot, and utilize HDECAY package [9]. Here off-shell decays of fourth generation leptons are not considered. For comparison, we also show the branching ratios in standard model in Fig. 5. The decay channels of the standard-model fermion pairs ($tt$, $bb$ and $\tau^+\tau^-$) are shown in solid lines, those of gauge bosons ($W^+W^-$, $ZZ$, $\gamma\gamma$, $\gamma Z$ and $gg$) are in dashed lines and those of the fourth generation leptons pairs ($e^+e^-$, $e'^+e'^-$, $\nu'\bar{\nu}'$ and $\nu''\bar{\nu}''$) are in dot-dashed lines. The line shows the branching ratio for $e^+e^- \to \nu'\bar{\nu}' + e'^+e'^-$ (or $\nu''\bar{\nu}''$) with the sum denoted just by $e^+e^- \to \nu'\bar{\nu}'$ (or $\nu''\bar{\nu}''$). We omit $c\bar{c}$ for simplicity of presentation in those plots. In Fig. 3 the branching ratios are very similar to those in the standard model when $m_h < 200$ GeV, except for $h \to \gamma \gamma$. In the mass parameter region $m_h > 200$ GeV, we find that the branching ratios for $h \to W^+W^-$ and $ZZ$, which are the main decay modes, are reduced due to the appearance of the new decay.
channels, $h \rightarrow e^+ e^-$, $e'^+ e''^-$, $\nu' \nu'$ and $\nu'' \nu''$. For example, the branching ratio for $h \rightarrow W^+ W^-$ ($ZZ$) is 71% (72%) of the standard-model value for $m_h = 300$ GeV. In Fig. 4 it is seen that branching ratios become quite different from those in the standard model especially around $m_h \sim 150$ GeV. Because of the new decay channels, the branching ratio for $h \rightarrow W^+ W^-$ turns out to be 27%, 74% and 73% of the standard-model value for $m_h = 150$ GeV, 200 GeV and 300 GeV, respectively. When $m_h \sim 150$ GeV, the Higgs decays mostly to the fourth generation neutral lepton pairs. Such neutral lepton pairs are observed as large missing transverse momentum when $\nu'$ or $\nu''$ does not decay in the detector. Otherwise, they would be followed by decay to the standard-model leptons. The decay channels $e^+ e^-$ and $e'^+ e''^-$ are also interesting. These leptons subsequently decay to the off-shell $W$ boson or $\nu'$ (or $\nu''$). (The case where the Higgs decays to a stable chiral fourth generation neutrino pair is previously studied, e.g., Ref. [10]. See also recent work [11].)

IV. HIGGS MASS BOUNDS

As we described in the Introduction, the new generation fermions affect the running coupling constants in the Yukawa terms and the Higgs potential. It is well known that the Yukawa coupling of a new fermion has a Landau pole not very far above the weak scale when the additional fermion gets its mass from the Higgs VEV and it is much heavier than the top quark. Also it is known that such a Yukawa coupling may cause an instability of the Higgs potential or Landau pole of the Higgs self-coupling [12][13]. In this section we take the fourth generation quarks to have large masses of order the cutoff of the theory and we evaluate the running of Yukawa coupling constants for the fourth generation leptons, the top quark Yukawa coupling, and the Higgs self-coupling. We use this to discuss the lower and upper Higgs mass bounds which arise, respectively from avoiding instability of the vacuum and from the Landau pole in the Higgs potential.

For simplicity we assume $b_E^i = b_E^N = h_E$ and $h_N = b_N^N = h_N$. Then the renormalization group equations (RGEs) for the lepton Yukawas and the top Yukawa are

$$\frac{16\pi^2 \mu}{\partial \mu} \frac{\partial h_E}{\partial \mu} = -h_E \left( \frac{9}{4} g_2^2 + \frac{15}{4} g_1^2 \right) + \frac{7}{2} h_E^3 + \frac{1}{2} \lambda v \left( \sum_{i=1}^{4} \frac{3y_i^2 + \frac{1}{2} h_N^2}{2} \right),$$  \quad (24)

$$\frac{16\pi^2 \mu}{\partial \mu} \frac{\partial h_N}{\partial \mu} = -h_N \left( \frac{9}{4} g_2^2 + \frac{3}{4} g_1^2 \right) + \frac{7}{2} h_N^3 + \frac{1}{2} \lambda v \left( \sum_{i=1}^{4} \frac{3y_i^2 + \frac{1}{2} h_E^2}{2} \right),$$  \quad (25)

$$\frac{16\pi^2 \mu}{\partial \mu} \frac{\partial y_i}{\partial \mu} = -y_i \left( \frac{8g_2^2}{3} + \frac{9}{4} g_1^2 + \frac{17}{12} g_1^2 \right) + \frac{9}{2} y_i^3 + \frac{1}{2} \lambda v \left( \sum_{i=1}^{4} \frac{3h_E^2 + 2h_N^2}{2} \right).$$  \quad (26)

(for formulae we refer to Ref. [18]. See also Refs. [19, 20].)

Here $g_3$, $g_2$ and $g_1$ are gauge coupling constants of $SU(3)_c$, $SU(2)$ and $U(1)_Y$, respectively, and $\mu$ is the renormalization scale. In our evaluation we neglect all other quark and lepton Yukawa couplings. RGEs of the gauge couplings are,

$$\frac{16\pi^2 \mu}{\partial \mu} \frac{\partial g_i}{\partial \mu} = -b_i g_i^3,$$  \quad (27)

with

$$b_1 = \frac{2}{3} \left( \frac{3}{2} n_L + \frac{11}{6} n_Q \right) - \frac{1}{6} n_H,$$  \quad (28)

$$b_2 = \frac{2}{3} - \left( \frac{1}{3} n_L + n_Q \right) - \frac{1}{6} n_H,$$  \quad (29)

$$b_3 = 11 - \frac{4}{3} n_Q.$$  \quad (30)

Here $n_L$ and $n_Q$ are the number of generations of leptons and quarks, and $n_H$ is the number of the Higgs doublets. In our model $n_H = 1$, $n_L = 5$ (three generation plus fourth generation and its mirror) and $n_Q = 3$, assuming that heavy fourth generation quarks have masses of order the cutoff of the theory so they do not contribute to the running of the gauge couplings. For the Higgs sector we write Higgs potential as

$$V^H = -\mu_H^2 |H|^2 + \lambda |H|^4,$$  \quad (31)

so that the Higgs mass is,

$$m_h = \sqrt{2\lambda v}.$$  \quad (32)

In this convention, RGE for $\lambda$ is given by

$$16\pi^2 \mu \frac{\partial \lambda}{\partial \mu} = 24 \lambda^2 - 3\lambda(3g_2^2 + g_1^2)$$

$$+ 4\lambda \left[ 3y_t^2 + 2(h_E^2 + h_N^2) \right] - 2 \left[ 3y_t^2 + 2(h_E^2 + h_N^2) \right]$$

$$+ \frac{3}{8} \left[ 2g_2^2 + (g_2^2 + y_1^2)^2 \right].$$  \quad (33)

Solving those RGEs, we derive the Higgs mass bounds by imposing $0 < \lambda(\mu) < 2\pi$ [21], with the Higgs mass determined by Eq. (32) using the coupling $\lambda$ evaluated at the Higgs mass. At some scale the condition $0 < \lambda(\mu) < 2\pi$ cannot be satisfied and we interpret this scale as a cutoff for the model, $\Lambda_c$. Thus, the Higgs mass bounds are given as a function of the cutoff. The condition $\lambda(\mu) > 0$ gives the lower bound for the Higgs mass, while the condition $\lambda(\mu) < 2\pi$ gives the upper bound. Numerical results are given in Fig. 6 and 7. Here we take $m_E^i = m_E^N = m_N^i = m_N^N = 100$ GeV (150 GeV) in Fig. 6 (Fig. 7). In Fig. 6 we also plot the result when $m_E^i = m_E^N = 100$ GeV and $m_N^i = m_N^N = 70$ GeV are chosen using a dot-dashed line. In the plots, we also give the result in standard model using a dotted line. In the first case (i.e., Fig. 6) we have checked that the Yukawa couplings do not have Landau poles up to the Planck scale, while in the second case (i.e., Fig. 7) the top quark Yukawa has a Landau pole around $\mu \sim 10^{10}$ GeV. In Fig. 6 $m_h \sim 180$ (170-180 GeV) is indicated for $m_N^i = m_N^N = 100$ (70) GeV when the cutoff
of the theory is about $10^{15}$ GeV. When the Higgs is lighter, the cutoff is significantly reduced. From the numerical calculations, we find $\Lambda_c \simeq 4.3 (6.2)$ TeV, $26 (54)$ TeV and $1.2 (8.6) \times 10^3$ TeV for lower bounds, $m_h = 115$ GeV, $130$ GeV and $150$ GeV when $m'_E = m''_E = m'_N = m''_N = 100$ (70) GeV. In Fig. 7 $m_h \sim 210$ GeV is implied when cutoff of the theory is near the Landau pole of the Yukawa couplings, i.e., $\sim 10^{10}$ GeV. Similarly to the previous result, we obtained $\Lambda_c \simeq 8.3 \times 10^2$ GeV, $1.8$ TeV and $8.2$ TeV for lower bounds $m_h = 115$ GeV, $130$ GeV and $150$ GeV. In Fig. 8 the allowed region for the Higgs mass vs. fourth generation lepton masses is given for a fixed cutoff of $10$ TeV. Here we take $m'_E = m''_E = m'_N = m''_N \equiv m_L$. We found $120$ GeV $\lesssim m_h \lesssim 400$ GeV and $m_L \lesssim m_h$ is the allowed region. This is consistent with what is expected from previous works where a similar analysis was performed for chiral fourth generation scenario [18].

Finally we note that evaluation of the Higgs mass bound has theoretical uncertainty coming, for example, from matching conditions of fermion and the Higgs sector at the low energy boundary [17]. The allowed Higgs mass region may change due to this; however it is shown in Ref. [17] that this uncertainty is less than $\sim 10$ GeV in the standard model. We do not estimate this kind of uncertainty, expecting a similar order of uncertainty in our case.

V. CONCLUDING REMARKS

Although we observe three chiral generations of quarks and leptons, there is no established physical principal that fixes the number of generations. In this paper we consider an additional vector-like generation. Within this framework, we focus on a scenario where fourth generation quarks gets large masses without the Higgs VEV, while fourth generation lepton masses are determined by weak symmetry breaking. Then quark sector Yukawa couplings do not develop Landau poles near the weak scale. We have studied Higgs properties in this scenario. We found that the new leptons reduce the branching ratio for $h \to \gamma\gamma$ to about $30\%$ of its standard-model
value. Furthermore the Higgs production rate at the LHC is very near its standard-model value if the new fourth generation quarks are much heavier than the weak scale. We have also examined the upper and lower limits on the Higgs mass in this model from the condition that all the Yukawa coupling constants and the Higgs self-coupling are free of Landau poles and that the familiar weak symmetry breaking vacuum is stable. We found when cutoff of the theory is about $10^{15}$ GeV then $m_h \sim 175$ GeV and fourth generation lepton masses should not be greater than about 100 GeV. When cutoff is around 10 TeV, $120$ GeV $\lesssim m_h \lesssim 400$ GeV with fourth generation lepton masses being roughly less than $m_h$ in the allowed region.

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