Constraining neutrino mass in dark energy dark matter interaction and comparison with 2018 Planck results

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Abstract  In this paper, we investigate the constraints on the total neutrino mass \( \sum m_\nu \) in a cosmological model in which dark energy and neutrinos are coupled such that the mass of the neutrinos and potentials are function of the scalar field as \( m_\nu = m_0 \exp(\frac{\alpha \phi}{m_{pl}}) \) and \( V(\phi) = m_{pl}^4 \exp(\frac{-\lambda \phi}{m_{pl}}) \) respectively. The observational data used in this work include the type Ia supernovae (SN) observation (Pantheon compilation), CC, CMB and BAO data. We find that the neutrino mass is tightly constrained to \( \sum m_\nu < 0.125 \) eV 95% Confidence Level (C.L.) and the effective extra relativistic degrees of freedom to be \( N_{eff} = 2.95^{+0.11}_{-0.12} \) 68% C.L in agreement with the Standard Model prediction \( N_{eff} = 3.046 \), matter-radiation equality, \( z_{eq} = 3389^{+24}_{-28} \) (68% C.L). These results are in good agreement with the results of Planck 2018 where the limit of the total neutrino mass is \( \sum m_\nu < 0.12 \) eV (95% C.L., TT, TE, EE + lowE + lensing + BAO), \( N_{eff} = 2.99^{+0.17}_{-0.18} \) (68% C.L., TT, TE, EE + lowE + lensing + BAO) and \( z_{eq} = 3387^{+21}_{-21} \) (68% C.L TT, TE, EE + lowE + lensing + BAO).

1 Introduction

The accelerating expansion of the universe [1–4] is one of the must surprising discoveries in cosmology. Also the observations of Cosmic Microwave Background (CMB) anisotropies indicate that the universe is flat and the total energy density is very close to the critical one [5].

According to General Relativity, the dynamic of the universe is dominated by a new (dark) energy form with negative pressure. There are prominent candidates for DE such as the cosmological constant [6,7] in which dark energy takes the form of a cosmological constant and dark matter is taken to be cold, in other words having an equation of state equal to zero. While \( \Lambda \) CDM fits the available data very well, it suffers from a number of issues that motivate the study of alternatives. These include the fine-tuning [7] and coincidence [8] problems. In addition, there are certain tensions between early and late-universe observations in \( \Lambda \) CDM. The present-day expansion rate of the universe, \( H_0 \) and the growth of structure, quantified by \( \sigma_8 \), can be calculated using the best-fit \( \Lambda \) CDM parameters to cosmological data, including the CMB. This gives rise to a smaller \( H_0 \) and a larger \( \sigma_8 \) than the results of local, late-universe measurements (for a recent discussion see Ref. [9]).

A popular class of modifications to \( \Lambda \) CDM is quintessence [10,11], in which the cosmological constant \( \Lambda \) is set to zero and a scalar field \( \phi \) is introduced whose dynamical properties produce a negative equation of state giving rise to the observed late-time accelerated expansion of the universe. Normally it is assumed that the scalar field does not interact with dark matter. However there is no reason why this must be the case, and the consequences of relaxing this assumption have been widely studied. See Ref. [12] with references therein and [13–17] for a discussion of recent research on interacting dark energy. The other candidates are phantom (field with negative energy) [18] that explains the cosmic accelerating expansion. Meanwhile, the accelerating expansion of universe can also be obtained through modified gravity [19], brane cosmology and so on [20–43]. On the other hand, to explain the early and late-time acceleration of the universe. It is most often the case that such fields interact with matter; directly due to a matter Lagrangian coupling, indirectly through a coupling to the Ricci scalar or as the result of quantum loop corrections [44–48]. If the scalar field self-interactions are negligible, then the experimental bounds on such a field are very strong; requiring it to either couple to matter much more weakly than gravity does, or to be very heavy [49–52]. Unfortunately, such a scalar field is usually very light and its coupling to matter should be tuned to extremely to small values in order to not be a conflict with the Equivalence Principal [53]. The discovery of the accelerated expansion of the universe is...
also the main challenge for particle physics [54]. It requires new physics for the explanation of dark energy. Neutrinos were first shown to have mass in observations of neutrino flavour oscillations [55,56], the presence of which demands that at least two of the neutrino states are massive [57]. While the attempts of the laboratory experiments of particle physics to measure the absolute masses of neutrinos, have always been facing great challenges [58–68], the cosmological observations are more prone to be capable of measuring the absolute masses of neutrinos [65–67], since massive neutrinos can leave rich signatures on the cosmic microwave background (CMB) anisotropies and the large-scale structure (LSS) formation at different epochs of the cosmic evolution [68]. Recently some studies have attempted to constrain the total neutrino mass \( \sum m_\nu \) as well as the effective number of relativistic degrees of freedom \( N_{\text{eff}} \) using cosmological observations [69–126] Also the cosmological consequences of interacting dark energy and dark matter have been widely studied [127–166], however, we are interested in consideration that the role of neutrino is prominent. In this work, we implement a cosmological model which proposed by [167] and extend by [168] to constrain total neutrino mass with observations. In this model dark energy and neutrinos are coupled such that the mass of the neutrinos is a function of the scalar field \( m_\nu = m_0 \exp \left( \frac{\alpha \phi}{m_{Pl}} \right) \). The scalar field plays the role of dark energy and drives the late time accelerated expansion of the universe. The motivation of such consideration has been investigated by [168–177]. Here we consider a generalized model of [167] which allows both dark matter and neutrino interact with dark energy with different interacting couplings \( \beta \) and \( \alpha \). We constrain on \( ( \sum m_\nu, N_{\text{eff}}, h, \Omega_m, \Omega_b h^2, \Omega_r h^2, \alpha, \beta, \lambda, \omega ) \) using observational data data. The structure of the paper is as follows. Section 2 introduces the cosmological model explored here while Sect. 3 describes the methodology and the measurements exploited in our data analyses. Section 4 presents our results and we conclude the article in Sect. 5

### 2 The model

The expansion rate of the Universe is given by Hubble parameter \( H = \frac{\dot{a}}{a} \), where \( a \) is scale factor and an overdot denote cosmic time derivative. We assume a spatially-flat Friedman–Robertson–Walker Universe filled with baryons (\( b \)), radiation (\( r \)), dark energy (\( \phi \)), dark matter (\( dm \)) and neutrinos (\( \nu \)). The baryons and radiation are regarded as non-interaction fields. The Friedmann equations which follow from Einstein field equations are as

\[ 3H^2 + \rho = p \]

\[ \dot{H} + 3H^2 = -p \quad (1) \]

Where, \( \rho = \rho_b + \rho_r + \rho_{dm} + \rho_\nu + \rho_\phi \) and \( p = p_b + p_r + p_{dm} + p_\nu + p_\phi \). From the conservation of the energy momentum tensor it follows the evolution equation for the total energy density:

\[ \dot{\rho} + 3H\rho = 0 \quad (2) \]

The baryons are treated like dust (\( p_b = 0 \)) and the barotropic equation of state for the radiation field is (\( p_r = \frac{1}{3} \rho_r \)). Once both of them have no interaction with other components, the evolution equations for their energy densities are:

\[ \dot{\rho}_b + 3H\rho_b = 0 \quad (3) \]

\[ \dot{\rho}_r + 4H\rho_r = 0 \quad (4) \]

respectively. Since the dark energy is modeled as a scalar field \( \phi \) its energy density and pressure are given by

\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi), \quad p_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad (5) \]

where \( V(\phi) \) denotes the potential of the scalar field. In this paper, we consider the interactions between dark matter and dark energy as the following evolution equation

\[ \dot{\rho}_{dm} + 3H\rho_{dm} = \beta \rho_{dm} \dot{\phi} \quad (6) \]

where \( \beta \) stands for the coupling constant between dark matter and dark energy. The neutrinos are understood to be massless and relativistic particles in the past, but with the coupling to the dark energy, they have acquired mass and became non-relativistic, having an oscillating mass behavior at low redshifts. In this paper we follow the idea that proposed by [167] and extended by [168]. In the cosmological context, neutrinos cannot be described as fluid. Instead, one must solve the distribution function \( f(x^i, p^i, \tau) \) in phase space (where \( \tau \) is the conformal time). Considering the case that neutrinos are collisionless, the distribution function \( f \) does not depend explicitly on time. Solving the Boltzmann equation, one can then calculate the energy density stored in neutrinos (\( f_0 \) is the background neutrino distribution function):

\[ \rho_\nu = a^{-4} \int q^2 \sqrt{q^2 + m_\nu(\phi)^2} \dot{f}_0(q) dq d\Omega \quad (7) \]

\[ p_\nu = \frac{1}{3} a^{-4} \int \frac{q^2}{\sqrt{q^2 + m_\nu(\phi)^2} \dot{f}_0(q) dq d\Omega} \quad (8) \]

The evolution equation for its energy density according to [168]

\[ \dot{\rho}_\nu + 3H (\rho_\nu + p_\nu) = \alpha \dot{\phi} (\rho_\nu - 3p_\nu) \quad (9) \]

where \( \alpha \) denotes coupling constant which can be related neutrino mass \( m_\nu \) with relation \( \alpha = \frac{\partial \ln m_\nu}{\partial \phi} \). Furthermore,
from the resulting equations it is possible to obtain evolution equations for the scalar field as
\[
\dot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} + \alpha \dot{\phi} (\rho_\nu - 3p_\nu) - \beta \rho_{dm} \dot{\phi} = 0 \tag{10}
\]

For most of the Universe’s history, the neutrinos are highly relativistic and \((\rho_\nu - 3p_\nu) \approx 0\) such that the scalar field and the neutrinos are effectively uncoupled, here only the coupling parameter \(\beta\) is important. After the neutrinos become non-relativistic hence the coupling parameter \(\alpha\) also becomes important. We consider an exponential potential \(V = M_p^4 \exp(-\lambda \frac{\phi}{M_p})\), where \(\lambda\) is a dimensionless parameter that determines the slope of the potential. The motivation for choosing these functions have been investigated in [169].

Also we define \(\omega = \frac{\rho_\nu}{\rho_r}\). In order to simplify the field equations, we introduce following new variables,
\[
x_1 = \frac{\rho_b}{3H^2}, x_2 = \frac{\rho_\nu}{3H^2}, x_3 = \frac{\rho_r}{3H^2}, \quad x_4 = \frac{\rho_{dm}}{3H^2}, \quad x_5 = \frac{\dot{\phi}}{\sqrt{6}H}, \quad x_6 = \frac{V(\phi)}{3H^2} \tag{11}
\]

Hence, the equations of the autonomous dynamical system can be derived as,
\[
\begin{align*}
\frac{dx_1}{dN} &= -3x_1 - 2 \frac{\dot{H}}{H^2} x_1 \\
\frac{dx_2}{dN} &= -x_2 \left(3(1 + \omega) - \sqrt{6}x_5(1 - 3\omega)\right) - 2 \frac{\dot{H}}{H} x_2 \\
\frac{dx_3}{dN} &= -4x_3 - 2 \frac{\dot{H}}{H} x_3 \\
\frac{dx_4}{dN} &= -x_4 \left(3 + \sqrt{6}x_5\beta + 2 \frac{\dot{H}}{H^2}\right) \\
\frac{dx_5}{dN} &= \frac{3\alpha}{\sqrt{6}} x_5 - \frac{3\omega(1 - 3\omega)}{\sqrt{6}} x_2 + \frac{9\beta}{\sqrt{6}} x_4 - \left(3 + \frac{\dot{H}}{H^2}\right) x_5 \\
\frac{dx_6}{dN} &= -\sqrt{6}\lambda x_6 x_5 - 2 \frac{\dot{H}}{H^2} x_6 
\end{align*}
\tag{12}
\]

Where, \(N = \ln a\). In term of the new dynamical variable, we also have,
\[
\frac{\dot{H}}{H^2} = \frac{1}{2} \left(-3 - x_3 - 3\omega x_2 - 3x_1^2 + 3x_6\right) \tag{13}
\]

In term of new variable the Friedmann equation (1) puts a constraint on new variables as
\[
x_1 + x_2 + x_3 + x_4 + x_5^2 + x_6 = 1 \tag{14}
\]

We demonstrate that for the flat Friedmann–Robertson–Walker model the dynamics can be reduced to the form of the six dimensional autonomous dynamical system where by exerting the constraint (14) it reduces to five dimensional dynamical system. The parameters \(\alpha, \beta, \lambda, \omega\) are the free parameters of the model. While the main advantage of the dynamical system methods is that without knowledge of an exact solution it is possible to investigate the properties of the solutions as well as their stability. But this method is not necessarily to check the stability of the system. Rather, even if our goal is to solve equations numerically, this method is a useful method. Because in the solution of the original equations (Friedman and field equations), we are faced with differential equations of order 2 and higher, which are relatively more complex where not only the initial conditions but also the first and second order initial conditions must be determined for any dynamical variable. For example, because of advent of the \(\dot{\phi}\) in field equation, we need \(\phi(0), \dot{\phi}(0)\) and \(\ddot{\phi}(0)\) and due to \(\dot{H}\) we need \(a(0), H(0)\) and \(\dot{H}(0)\) for numerical solutions, however, when the equations are introduced in terms of a set of the first order equations, only the initial condition for the new variable need to be determined \(x_1(0)...x_5(0)\), this makes numerical solution of the equations easier. On the other hand the variables in the dynamical system usually are dimensional less and in most cases, or at list in the case of Friedman equation we have Pre information about the range of initial conditions. For example, we get \(x_4 = \frac{\rho_{dm}}{3H^2}\) which is \(\Omega_m\). Hence in numerical solution, it is not necessary to cover a large area of \(\Omega_m (0)\) but we focus on a small area, approximately between 0.2 and 0.4. It makes the analysis easier. The importance of this issue becomes more reveal in observational cosmology where the initial condition play important role in the evolution of the universe.

However, the question that may arise is that why we don’t implement this method for any set of high order differential equations. The answer is that although any second-order differential equation is equivalent to two first-order equations, however the suitable choice of new variables and converting equations to the first order may not be easy except in special cases, but if we could do that then we are faced with a set of first order equations which are more easier to solve.

## 3 Observational data, analysis and results

In what follows, first, we briefly describe the observational data sets used to constrain the parameters of the models under consideration.

- **Pantheon:** The use of type Ia supernovae (SNe) as standard candles has been of critical importance to cosmology, leading to the discovery of cosmic acceleration [54,178]. In this paper, we use the new “Pantheon” sample of Scolnic et al. [179], which adds 276 supernovae from the Pan-STARRS1 Medium Deep Survey at 0.03 < \(z\) < 0.65 and various low-redshift and HST
The luminosity distance $d_L$ can be calculated by

$$d_L = (1 + z) \int \frac{dz}{H(z)}$$  \hspace{1cm} (15)$$

In order to incorporate the Eq. (15) with the dynamical system equations of (12), it can be rewritten in terms of the following differential equations

$$\frac{ddL}{dN} = -d_L - \frac{e^{-2N}}{H}$$  \hspace{1cm} (16)$$

$$\frac{dH}{dN} = H \left( \frac{\dot{H}}{H^2} \right)$$  \hspace{1cm} (17)$$

Where, since $1 + z \equiv \frac{1}{x}$, then $(1 + z) \equiv e^{-N}$, $dz \equiv -e^{-N}dN$ and $dN \equiv H dt$. Hence by defining new variables $x_d = d_L$ and $x_h = H$ we can write the Eq. (16) as

$$\frac{dx_d}{dN} = -x_d - \frac{e^{-2N}}{x_h}$$

$$\frac{dx_h}{dN} = -x_h \left( \frac{\dot{H}}{H^2} \right)$$  \hspace{1cm} (18)$$

The Eq. (18) are related with Eq. (12) by $\frac{\dot{H}}{H^2}$ which have been obtained in terms on new variables. It is also important to note that $\frac{\dot{H}}{H^2}$ play the important role in cosmology, since important cosmological parameters such as deceleration parameters $q$ an effective equation of state (EoS) $w_{eff}$ can be expressed in terms of this parameter as

$$q = -1 - \frac{\dot{H}}{H^2}$$

$$w_{eff} = -1 - \frac{2}{3} \frac{\dot{H}}{H^2}.$$ Hence in order to find $x_d$ and $x_h$ the set of Eqs. (18) and (12) must be coupled and solved simultaneously. Hence the distance modulus also can be obtained as $\mu_{th}(z) = 5 \log(x_d) + 42.38$. we can compute the $\chi^2$-statistics for each case. Therefore, we proceed to define the following quantities:

$$\chi^2_{\text{Pantheon}} = \sum_i \frac{(\mu(z)_{\text{obs}} - \mu(z)_{\text{th}})^2}{\sigma(z)_{\text{obs, Pantheon}}}$$  \hspace{1cm} (19)$$

where $N$ is the number of data points, $\sigma_i$ is the uncertainty associated with each measurement.

**Cosmic Microwave Background (CMB):** The observations of temperature anisotropies in the CMB provide a valuable independent test for the reality of dark energy at the recombination epoch $z = 1090$. The photons were coupled to baryons and electrons before that red shift and decoupled right after. Due to the fact that in the Boltzmann and Einstein equations all the components of the universe are coupled, in order to extract information from the full spectrum, demanding numerical simulations are needed. A convenient and efficient way to summarize information from the CMB data, without using the full spectrum, is by employing the so called CMB shift parameters or distance priors. The CMB shift parameter $R$, given by [180,181]

$$R = \Omega_m^2 \int_0^{z_{rec}} \frac{dz}{E(z)}$$  \hspace{1cm} (20)$$

where $E(z) = \frac{H(z)}{H_0}$ and $z_{rec}$ is the redshift of recombination $z_{rec} = 1090$ [182]. The parameter $R$ ties up the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at $z = 1091.3$ and the angular scale of the first acoustic peak in CMB power spectrum of temperature fluctuations $[180, 181]$. The updated value of $R$ from WMAP5 is $R_{\text{obs}} = 1.710 \pm 0.019$ [183]. The $\chi^2_{CMB}$ for the CMB data is

$$\chi^2_{CMB} = \frac{(R - R_{\text{obs}})^2}{\sigma_R}$$  \hspace{1cm} (21)$$

where the corresponding $1 \sigma$ errors is $\sigma_R = 0.019$.

**Baryon oscillations BAO data**

For BAO data, from the measurement of the BAO peak in the distribution of SDSS luminous red galaxies, we define parameter $A$ as [184]

$$A = \Omega_m \int_{z_b}^{1} \left[ \frac{1}{z_b} \int_0^{z_b} \frac{dz}{E(z)} \right]^2$$  \hspace{1cm} (22)$$

where $z_b = 0.35$. The SDSS BAO measurement [184] gives $A_{\text{obs}} = 0.469(n_s/0.98).0.35 \pm 0.017$, where the scalar spectral index is taken to be $n_s = 0.965$ as measured by Planck 2018 [185]. The parameter $A$ is nearly model-independent and imposes the robust constraint as complement to SNIa data. The $\chi^2$ for the BAO data is

$$\chi^2_{BAO} = \frac{(A - A_{\text{obs}})^2}{\sigma_A}$$  \hspace{1cm} (23)$$

where the corresponding $1 \sigma$ errors is $\sigma_A = 0.017$. Table 1 shows the best fitted model parameters and initial conditions in both power law and exponential cases. We use the cosmic chronometers (CC) data set comprising of 36 measurements spanning the redshift range.
\[ z \leq 2.36, \text{recently compiled in} [186] \]

\[ \chi^2_{CC} = \sum_i \frac{(H(z_i)_{\text{obs}} - H(z_i)_{\text{th}})^2}{\sigma(z_i)^2_{\text{obs,CC}}} . \tag{24} \]

In order to put constraints on the parameters of the model we must note that the model has five independent variables \((x_1, x_2, x_3, x_4, x_5)\) which according to 11 are equivalent to \((\Omega_k, \Omega_\nu, \Omega_\nu, \Omega_{dm}, \Omega_\phi^{1/2})\) as well as four free parameters of the model \((\alpha, \beta, \lambda, \omega)\). Hence in order to solve the equation numerically the five initial conditions \((x_1(0), \ldots, x_5(0))\) and value of the parameters must be known. In observational measurements one or more parameter are added to the free parameters. For example in numerical analysis using Pantheon data the two new variables \(x_4 = d_L\) and \(x_5 = H\) and for CC, CMB and BAO data the variable \(x_3 = H\) are added to the free parameters. The other parameters are expressed in terms of the main parameters and can be constrained indirectly. For example \(\sum m_\nu\) can be related to the main parameters \((h, \Omega_\nu)\) as

\[ \Omega_\nu = \frac{\sum m_\nu}{94 h^2 \text{eV}} \tag{25} \]

where \(h\) is the reduced Hubble constant (the Hubble constant \(H_0 = 100h\) km/s/Mpc. Hence if the parameters \((h, \Omega_\nu)\) are constrained then the parameter \(\sum m_\nu\) is constrained automatically.

The relativistic energy density in the early universe include the contributions from photons and neutrinos, and possibly other extra relativistic degrees of freedom, called dark radiation. The effective number of relativistic species, including neutrinos and any other dark radiation, is defined by a parameter, \(N_{\text{eff}}\), for which the standard value is 3.046 corresponding to the case with three-generation neutrinos and no extra dark radiation [187]. If the value of \(N_{\text{eff}}\) is beyond 3.046, it indicates that there is some dark radiation other than three-generation active neutrinos. The behaviour of dark radiation is exactly equivalent to massless neutrinos. Thus, the total radiation energy density in the Universe is given by

\[ \rho_r = \rho_r \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{\frac{1}{3}} N_{\text{eff}} \right] \tag{26} \]

where \(\rho_r\) is the energy density of photons. We parametrize the relativistic degrees of freedom using the effective number of neutrino species, \(N_{\text{eff}}\). This quantity can be written in terms of the matter density, \(\Omega_m h^2\), and the redshift of matter-radiation equality \(z_{eq}\) as [182]

\[ N_{\text{eff}} = 3.04 + 7.44 \left( \frac{\Omega_m h^2}{0.1308} \right) \left( \frac{3139}{1 + z_{eq}} - 1 \right) \tag{27} \]

4 Results

Throughout this section we will present the results obtained within the two different IDE scenario

4.1 IDE+ \(\sum m_\nu\)

The results for the cosmological parameters within this interacting dark energy model are shown in Table 1. Figure 1 also show the parametric space at 68 %CL and 95%CL for some selected parameters for the different observational data sets. In order to compare our results with those obtained by Planck 2018 [185], we have listed some of the Planck 2018 results [185] in Table 2. For this case the free parameters are \((\sum m_\nu, h, \Omega_\nu h^2, \Omega_\phi h^2, \alpha, \beta, \lambda, \omega)\).

From the analyses of the Pantheon data alone, as shown in Table 1, we find that

\[ \sum m_\nu < 0.253 \text{ eV} \quad 95\% \text{CL} \tag{28} \]

This result is very close to the result of [121], the case IDE+ \(\sum m_\nu\) using (CMB + Pantheon + CC data) with \(\sum m_\nu < 0.255 \text{ eV} \) at 95% CL. Both the model is similar to ours and the data used includes Pantheon data. The result is also closed to the result of[122], the case interacting vacuum scenario (IVS)+ \(\sum m_\nu\), with \(\sum m_\nu < 0.277 \text{ eV} \) at 95% CL and the result of Planck 2018 [185] the case (TT, TE, EE + lowE + lensing) with \(\sum m_\nu < 0.241 \text{ eV} \) at 95% CL. Using CC data, we find that

\[ \sum m_\nu < 0.199 \text{ eV} \quad 95\% \text{CL} \tag{29} \]

which is close to the result of [121], the case IDE+ \(\sum m_\nu\) using (CMB+Pantheon+CC data) with \(\sum m_\nu < 0.159 \text{ eV} \) at 95% CL and the result of Planck 2018 [185] the case (TT, lowE + BAO) with \(\sum m_\nu < 0.16 \text{ eV} \) at 95% CL.

Using CMB+BAO, we find that

\[ \sum m_\nu < 0.33 \text{ eV} \quad 95\% \text{CL} \quad \text{CMB + BAO} \tag{30} \]

This result is very close to the result of [121], the case IDE+ \(\sum m_\nu\), using (CMB data) with \(\sum m_\nu < 0.313 \text{ eV} \) 95% CL and comparable with and comparable with results of [185] the case TT, TE, EE + lowE[CamSpec] with \(\sum m_\nu < 0.38 \text{ eV} \) 95% CL.
Fig. 1 The constraints at the 68% and 95% CL two-dimensional contours for selected cosmological parameters and parameters of the model in IDE+∑mν scenario for the Pantheon, CC, CMB + BAO and Pantheon + CC + CMB + BAO dataset.

For combination of full data, Pantheon + CC + CMB + BAO, we find

\[ \sum m_\nu < 0.125 \text{ eV} \quad 95\% \text{CL} \]  

(31)

The result is very close to results of [185] TT, TE, EE + lowE + lensing + BAO with \( \sum m_\nu < 0.12 \text{ eV} \) at 95% CL and case TT, TE, EE + lowE + BAO with \( \sum m_\nu < 0.13 \text{ eV} \) at 95% CL. It is also comparable with that obtained by [121] with \( \sum m_\nu < 0.156 \text{ eV} \) at 95% CL using same model IDE+∑mν and same data (Pantheon + CC + CMB + BAO).

We also put constraint on coupling parameters (\( \alpha, \beta, \lambda, \omega \)). The lower panel of Fig. 1 shows (68.3%, 95.%) confidence levels for the parameters (\( \alpha, \beta \)) and (\( \lambda, \omega \)) for Pantheon, CC and CMB + BAO and combination of the data Pantheon + CC + CMB + BAO. The results also have been listed in Table 1. Constraining on parameter \( \lambda \), we have obtained \( \lambda = 11.85_{-4.35}^{+4.35}, \lambda = 10.35_{-1.55}^{+1.55}, \lambda = 8.6_{-6.70}^{+6.70} \) and \( \lambda = 12.8_{-0.90}^{+0.90} \) at 68% CL for Pantheon, CC, CMB + BAO and Pantheon + CC + CMB + BAO respectively. Since we have considered \( V(\phi) = m_\text{pl}^4 \exp(\frac{-\lambda \phi}{m_\text{pl}}) \), the positive value of \( \lambda \) indicates that potential \( V(\phi) \) is a monotonically decreasing function of \( \phi \) and has a negative gradient. It is also important to note that the best fitted values of \( \lambda \) for both individual and combination data values are very close to that deter-
Table 1  Observational constraints at 68% on main and derived parameters of the IDE+ \( \sum m_v \) scenario. The parameter \( H_0 \) is in the units of km/s/Mpc, whereas \( \sum m_v \) reported in the 95% CL, is in the units of eV

| Dataset                      | \( \Omega_b h^2 \) | \( \Omega_c h^2 \) | \( H_0 \) | \( \Omega_m \) | \( \Omega_m h^2 \) | \( \sum m_v \) | \( \alpha \) | \( \beta \) | \( \lambda \) |
|------------------------------|------------------|------------------|----------|--------------|----------------|----------------|---------|-------|-------|
| Pantheon                     | 0.02226 \( ^{+0.00016}_{-0.00016} \) | 0.1107 \( ^{+0.00076}_{-0.00076} \) | 70.01 \( ^{+0.24}_{-0.20} \) | 0.295 \( ^{+0.0032}_{-0.0032} \) | 0.1356 \( ^{+0.0030}_{-0.0027} \) | < 0.253 | 15.5 \( ^{+9.50}_{-5.45} \) | -24.95 \( ^{+5.45}_{-5.45} \) | 11.85 \( ^{+4.35}_{-4.35} \) |
| CC                           | 0.02187 \( ^{+0.00057}_{-0.00051} \) | 0.1195 \( ^{+0.0045}_{-0.0038} \) | 68.7 \( ^{+0.12}_{-0.08} \) | 0.2995 \( ^{+0.0193}_{-0.0147} \) | 0.1413 \( ^{+0.0049}_{-0.0032} \) | < 0.199 | 7.91 \( ^{+0.0050}_{-0.0050} \) | -19.03 \( ^{+3.50}_{-3.50} \) | 10.35 \( ^{+1.55}_{-1.55} \) |
| CMB + BAO                    | 0.01957 \( ^{+0.00048}_{-0.00047} \) | 0.1079 \( ^{+0.0026}_{-0.0026} \) | 65.7 \( ^{+0.8}_{-0.8} \) | 0.289 \( ^{+0.017}_{-0.015} \) | 0.1248 \( ^{+0.0300}_{-0.0200} \) | < 0.33 | 20.02 \( ^{+0.20}_{-0.20} \) | -33.03 \( ^{+7.05}_{-7.05} \) | 8.6 \( ^{+0.70}_{-0.70} \) |
| CMB + BAO + CC + Pantheon    | 0.01946 \( ^{+0.00048}_{-0.00047} \) | 0.1078 \( ^{+0.0026}_{-0.0026} \) | 69.5 \( ^{+0.3}_{-0.3} \) | 0.288 \( ^{+0.008}_{-0.008} \) | 0.1248 \( ^{+0.0300}_{-0.0200} \) | < 0.125 | 11.1 \( ^{+0.10}_{-0.10} \) | -19.8 \( ^{+1.10}_{-1.10} \) | 12.8 \( ^{+0.90}_{-0.90} \) |

Table 2  Results of Planck 2018 [185]: constraint at 95% CL, using different observational data, whereas \( \sum m_v \), reported in the 95% CL, is in the units of eV

| Dataset                      | \( \Omega_b h^2 \) | \( \Omega_c h^2 \) | \( H_0 \) | \( \Omega_m \) | \( \Omega_m h^2 \) | \( Z_{eq} \) | \( \sum m_v \) | \( N_{eff} \) | \( h \) |
|------------------------------|------------------|------------------|----------|--------------|----------------|--------|----------------|-------------|-------|
| TT + LowE                    | 0.02212 \( ^{+0.00022}_{-0.00022} \) | 0.1206 \( ^{+0.0021}_{-0.0021} \) | 66.88 \( ^{+0.92}_{-0.92} \) | 0.321 \( ^{+0.013}_{-0.013} \) | 0.1434 \( ^{+0.0020}_{-0.0020} \) | 341 \( ^{+48}_{-48} \) | < 0.537 | 3.00 \( ^{+0.53}_{-0.53} \) | 0.66 \( ^{+0.0092}_{-0.0092} \) |
| TE + LowE                    | 0.02249 \( ^{+0.00025}_{-0.00025} \) | 0.1177 \( ^{+0.0020}_{-0.0020} \) | 68.44 \( ^{+0.91}_{-0.91} \) | 0.301 \( ^{+0.012}_{-0.012} \) | 0.1408 \( ^{+0.0019}_{-0.0019} \) | 3349 \( ^{+46}_{-46} \) | - | - | 0.64 \( ^{+0.0091}_{-0.0091} \) |
| EE + LowE                    | 0.0240 \( ^{+0.00012}_{-0.00012} \) | 0.1158 \( ^{+0.0046}_{-0.0046} \) | 69.9 \( ^{+2.7}_{-2.7} \) | 0.289 \( ^{+0.0036}_{-0.0036} \) | 0.1404 \( ^{+0.0034}_{-0.0034} \) | 3340 \( ^{+81}_{-82} \) | - | - | - | 0.69 \( ^{+0.0074}_{-0.0074} \) |
| TT, TE, EE + LowE            | 0.02236 \( ^{+0.00015}_{-0.00015} \) | 0.1202 \( ^{+0.0014}_{-0.0014} \) | 67.27 \( ^{+0.60}_{-0.60} \) | 0.3166 \( ^{+0.0084}_{-0.0084} \) | 0.1432 \( ^{+0.0013}_{-0.0013} \) | 3407 \( ^{+31}_{-31} \) | < 0.257 | 2.92 \( ^{+0.36}_{-0.36} \) | 0.67 \( ^{+0.0060}_{-0.0060} \) |
| TT, TE, EE + LowE + Lensing  | 0.02237 \( ^{+0.00015}_{-0.00015} \) | 0.1200 \( ^{+0.0012}_{-0.0012} \) | 67.36 \( ^{+0.54}_{-0.54} \) | 0.3153 \( ^{+0.0073}_{-0.0073} \) | 0.1430 \( ^{+0.0011}_{-0.0011} \) | 3402 \( ^{+26}_{-26} \) | < 0.241 | 2.89 \( ^{+0.36}_{-0.36} \) | 0.67 \( ^{+0.0054}_{-0.0054} \) |
| TT, TE, EE + LowE + Lensing + BAO | 0.02242 \( ^{+0.00014}_{-0.00014} \) | 0.11933 \( ^{+0.00091}_{-0.00091} \) | 67.66 \( ^{+0.42}_{-0.42} \) | 0.3111 \( ^{+0.0056}_{-0.0056} \) | 0.1424 \( ^{+0.0087}_{-0.0087} \) | 3387 \( ^{+21}_{-21} \) | < 0.120 | 2.9 \( ^{+0.34}_{-0.33} \) | 0.67 \( ^{+0.0042}_{-0.0042} \) |
mined by upper bounds on early dark energy (λ ≥ 10)[161].

In fact, when the potential energy approaches a constant

\( V(\phi) = m_{pl}^4 \exp(\frac{-m_{pl}}{\phi}) \rightarrow V_t = V(\phi) = m_{pl}^4 \exp(\frac{-m_{pl}}{\phi}) \),

where the evolution of the cosmon field stops close to a value \( \phi_t \) which is characteristic for the transition between the two different cosmological epochs, it acts similar to a cosmological constant and causes the accelerated expansion and for \( \frac{m_{pl}}{\phi_t} \simeq 276 \) the cosmological constant has a value compatible with observation. This amount gives \( \lambda \geq 10 \) (for more discussion see [161]).

For most of the Universe’s history, the neutrinos are highly relativistic and \( (\rho_\nu - 3p_\nu) \approx 0 \) such that the scalar field and the neutrinos are effectively uncoupled, here only the coupling parameter β is important. After the neutrinos become non-relativistic hence the coupling parameter α also becomes important. Constraining on parameter α, we have obtained \( \alpha = 5.5^{+9.50}_{-9.50} \alpha = 7.01^{+10.05}_{-10.05} \alpha = 20.02^{+10.20}_{-10.20} \) and \( \lambda = 11.1^{+3.60}_{-3.60} \) at 68% CL for Pantheon, CC, CMB + BAO and Pantheon + CC + CMB + BAO respectively. As investigated by [188], the following conditions must be met to give rise to growing neutrino quintessence:

- \( V(\phi) \) must have a negative gradient in order to cause the value of the scalar field to increase with time. This gradient must be sufficiently steep that \( \phi \) reaches large enough values in the late Universe to act as dark energy.
- \(|\alpha| \) must be sufficiently large when the neutrinos become non-relativistic that \( \beta(\rho_\nu - 3p_\nu) \) is able to act as a strong enough restoring force to stop the evolution of \( \phi \) in Eq. (10).

The best fitted of \( (\alpha, \lambda) \) satisfy the above condition. Despite the small value of \( \Omega_\nu \) the neutrinos are important for the evolution of the cosmon due to their large coupling \( \alpha \)

For both individual and combination of the dataset, we find that the large value for coupling parameter \( \beta \). We find |β| > 19 at (95% CL). This indicates there is a strong interaction between dark matter and dark energy.

### 4.2 IDE+ \( \sum m_\nu + N_{\text{eff}} \)

The results for the cosmological parameters within this interacting dark energy model are shown in Table 3. Figure 2 Also show the parametric space at 68 %CL and 95%CL for some selected parameters for the different observational data sets. For this case the free parameters are \((\sum m_\nu, N_{\text{eff}}, h, \Omega_m, \Omega_b h^2, \Omega_c h^2, \alpha, \beta, \lambda, \omega)\).

Although the results for this case, IDE+ \( \sum m_\nu + N_{\text{eff}} \), are consistent with those in the previous case IDE+ \( \sum m_\nu \) and the presence of \( N_{\text{eff}} \) does not significantly shift the result,

| Dataset | \( \Omega_m h^2 \) | \( \Omega_b h^2 \) | \( \Omega_c h^2 \) | \( \sum m_\nu \) (GeV) | \( N_{\text{eff}} \) |
|---|---|---|---|---|---|
| Pantheon | 0.02179(10)0.0035 | 0.1219(10)0.0034 | 0.2921(10)0.013 | 0.0014 | 0.0014 |
| CC | 0.02179(10)0.0035 | 0.1219(10)0.0034 | 0.2921(10)0.013 | 0.0014 | 0.0014 |
| CMB + BAO | 0.02179(10)0.0035 | 0.1219(10)0.0034 | 0.2921(10)0.013 | 0.0014 | 0.0014 |
| CMB + BAO + CC + Pantheon | 0.02179(10)0.0035 | 0.1219(10)0.0034 | 0.2921(10)0.013 | 0.0014 | 0.0014 |

Table 3: Observational constraints at 68% and on main and derived parameters of the IDE+ \( \sum m_\nu + N_{\text{eff}} \) scenario. The parameter \( H_0 \) is in the units of km/sec/Mpc, whereas \( \Omega_m h^2 \), reported in

- \( \beta \) the neutrinos are effectively uncoupled, here only the coupling parameter β is important. After the neutrinos become non-relativistic hence the coupling parameter α also becomes important. Constraining on parameter α, we have obtained \( \alpha = 5.5^{+9.50}_{-9.50} \alpha = 7.01^{+10.05}_{-10.05} \alpha = 20.02^{+10.20}_{-10.20} \) and \( \lambda = 11.1^{+3.60}_{-3.60} \) at 68% CL for Pantheon, CC, CMB + BAO and Pantheon + CC + CMB + BAO respectively. As investigated by [188], the following conditions must be met to give rise to growing neutrino quintessence:

- \( V(\phi) \) must have a negative gradient in order to cause the value of the scalar field to increase with time. This gradient must be sufficiently steep that \( \phi \) reaches large enough values in the late Universe to act as dark energy.
- \(|\alpha| \) must be sufficiently large when the neutrinos become non-relativistic that \( \beta(\rho_\nu - 3p_\nu) \) is able to act as a strong enough restoring force to stop the evolution of \( \phi \) in Eq. (10).

The best fitted of \( (\alpha, \lambda) \) satisfy the above condition. Despite the small value of \( \Omega_\nu \) the neutrinos are important for the evolution of the cosmon due to their large coupling α

For both individual and combination of the dataset, we find that the large value for coupling parameter β. We find |β| > 19 at (95% CL). This indicates there is a strong interaction between dark matter and dark energy.
Fig. 2 The constraints at the 68% and 95% CL two-dimensional contours for selected cosmological parameters and parameters of the model in IDE+ $\sum m_{\nu} + N_{\text{eff}}$ scenario for the Pantheon, CC, CMB + BAO and Pantheon + CC + CMB + BAO dataset however the bounds on some of the parameters have been changed slightly.

From the analyses of the Pantheon data alone, as shown in Table 3, we find that

$$\sum m_{\nu} < 0.45 \text{ eV} \quad 95\% \text{ CL} \quad (32)$$

Which in comparison of previous model, IDE+ $\sum m_{\nu}$, with $\sum m_{\nu} < 0.253 \text{ eV}$ have been changed significantly, however it is close to the result of [189] where using same model, interacting scenario IDE1p + $\sum m_{\nu} + N_{\text{eff}}$ and using Planck 2018 data, have been obtained $\sum m_{\nu} < 0.438 \text{ eV}$ at 95% CL. The interesting result of our analysis is that for this model, the most stringent upper limit we have on this parameter is obtained for the data set combination $CMB + BAO + \text{Pantheon}$. 
Pantheon + CC in which

\[ \sum m_v < 0.125 \text{ eV} \quad 95\% CL \]  

(33)

This result is in good agreement with the results of Planck 2018 [185], where the limit of the total neutrino mass is \( \sum m_v < 0.12 \text{ eV} \) at (95% C.L. using TT, TE, EE + lowE + lensing + BAO data) and close to the result of [121], where they also find the most stringent upper limit on this parameter for the same model, IDE+ \( \sum m_v \) and the same data (CMB + Pantheon + CC data) with \( \sum m_v < 0.15 \text{ eV} \) at 95% CL. For combination data we also find

\[ N_{\text{eff}} = 2.955_{-0.12}^{+0.11} \quad 68\% CL \]  

(34)

which is very close to the result of Planck 2018 [185] with \( N_{\text{eff}} = 2.96_{-0.03}^{+0.04} \) at 68% CL, the case TT,TE,EE,LowE + lensing+BAO and close to result of [121] with \( N_{\text{eff}} = 3.05_{-0.33}^{+0.34} \) using the same data and same model.

For this case, the best fitted values for \( \lambda \), have obtained as \( \lambda = 9.71_{-0.3}^{+0.8} \), \( \lambda = 11.1_{-1.05}^{+1.05} \), \( \lambda = 7.9_{-3.3}^{+3.3} \) and \( \lambda = 10.25_{-1.08}^{+1.05} \) at 68% CL for Pantheon, CC, CMB + BAO respectively. Also for combination of dataset, CMB + Pantheon + CC data we have found

\[ \lambda = 10.25_{-1.08}^{+1.05} \]  

(35)

The results are close to those obtained in previous case, IDE+ \( \sum m_v \). Hence same as the previous case, the positive values of \( \lambda \) indicate that potential \( V(\varphi) \) is a monotonically decreasing function of \( \varphi \) and has a negative gradient as well as these value are consistency with that predicted by observation.

5 Conclusion

In this paper, we have explored possible extensions of the Interacting Dark Energy, where the dark energy and the dark matter fluids interact with each other.

We have considered a cosmological model in which dark energy and neutrinos are coupled such that the mass of the neutrinos and potentials are function of the scalar field as \( m_v = m_0 \exp(\frac{-\varphi}{m_p}) \) and \( V(\varphi) = m_p^4 \exp(\frac{-\varphi}{m_p}) \) respectively. While the theoretical aspects of the neutrino-dark energy interaction have been studied in previous studies [167–169,188], here we have extended the model such that not only neutrino but also dark matter interact with dark energy with different coupling terms \( Q_{de-\Delta m} = \beta \frac{\dot{\varphi}}{m_p} \rho_{dm} \) and \( Q_{de-\nu} = \frac{\omega}{m_p} (\rho_{\nu} - 3 p_{\nu}) \dot{\varphi} \) and focused on observational aspect of the model. We have exploited the most recent publicly available cosmological observations, which include the Supernovae Type Ia Pantheon data and measurements of the Hubble parameter from Cosmic Chronometers. CMB data, Baryon Acoustic Oscillations data (BAO) to put constrain on parameters on \( \sum m_v, N_{\text{eff}}, \varepsilon_{eq}, \Omega_m, \Omega_c, \Omega_b, \alpha, \beta, \lambda, \omega \).

We find that while the results of individuals and combination of datasets are closed, the most stringent upper limit we have on this parameter is obtained for the data set combination CMB + BAO + Pantheon + CC in which \( \sum m_v < 0.125 \text{ eV} \quad 95\% CL \) This result is in good agreement with the results of Planck 2018 [185], where the limit of the total neutrino mass is \( \sum m_v < 0.12 \text{ eV} \) at (95% C.L. using TT,TE,EE,LowE+lensing+BAO data) and close to the result of [121], where they also find the most stringent upper limit on this parameter ..for the same model, IDE+ \( \sum m_v \) and same data (CMB + Pantheon + CC data) with \( \sum m_v < 0.15 \text{ eV} \) at 95% CL.

This study also investigated one of the main problem in growing neutrinos and cosmological selection. As point out by Amendola and Wetterich [190], the most crucial observational issues can be understood by understanding on constant parameters (\( \lambda, \alpha, \beta \)) it will be a challenge to measure them or to falsify the growing matter scenario. For neutrino growing matter a determination of \( \lambda \) and \( \alpha \) would fix the neutrino mass, allowing for an independent test of this hypothesis by comparing with laboratory experiments. The values obtained for these parameters in this study are close to those require for growing matter mechanism.

For both model IDE + \( \sum m_v \) and IDE+ \( \sum m_v + N_{\text{eff}} \) and for all dataset, Both individual and combination, we find the mean value of \( \lambda \) as \( \lambda \approx 10 \). This value is very close to that determined by upper bounds on early dark energy (\( \lambda \geq 10 \)) [161]. As also point out by \( V(\varphi) \) must have a negative gradient in order to cause the value of the scalar field to increase with time. This gradient must be sufficiently steep that \( \varphi \) reaches large enough values in the late Universe to act as dark energy. Since we get \( V(\varphi) = m_p^4 \exp(\frac{-\varphi}{m_p}) \), the best fitted values of \( \lambda \) which is positive satisfies this condition.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: The paper has no external data.]

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