CLUSTER BARYON FRACTION AND STRUCTURE FROM THE CONVERGENCE/SZ EFFECT DIAGRAM
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ABSTRACT

The cross-correlation of Sunyaev-Zel’dovich effect (SZ) and weak-lensing imaging surveys can be used to test how well hot baryons trace dark matter in clusters of galaxies. We examine this concept using mock SZ and weak-lensing surveys based on the forthcoming AMiBA experiment and generated from a pre-heated cosmological N-body/hydrodynamic simulation. A cross-correlation diagram between matched lensing convergence and Compton y maps exhibits butterfly-wing-like structures, corresponding to individual clusters, that encode rich information about the distributions of hot gas and dark matter. When the cluster redshift and temperature are available, the slope of a wing reveals the cluster gas fraction and the width of the wing indicates how badly the hot gas traces dark matter. On the basis of simulated data we discuss systematic errors in the projected gas fraction estimates that would be obtained from such survey comparisons.

Subject headings: cosmic microwave background — cosmology: theory — dark matter — galaxies: clusters: general — gravitational lensing

1. INTRODUCTION

Clusters of galaxies, the largest virialized systems known, are key tracers of the matter distribution in the Universe. Clusters are composed of the hot, diffuse intracluster medium (ICM), luminous galaxies (stars), and a dominant mass component in the form of the unseen dark matter. The mass ratio of hot gas to dark matter in clusters (the gas fraction) is believed to be a fair estimate of the universal baryon fraction (White et al. 1993). However, non-gravitational processes associated with cluster formation, such as radiative gas cooling or pre-heating, would break the self-similarities in cluster properties and hence cause the gas fraction to acquire some mass dependence (Kravtsov et al. 2005; Bialek et al. 2001).

Weak gravitational lensing on background galaxies probes directly the projected cluster mass distribution (Bartelmann & Schneider 2001). The thermal Sunyaev-Zel’dovich (SZ) effect on the cosmic microwave background spectrum, on the other hand, measures the projected thermal electron pressure (Birkinshaw 1999). Therefore, a combination of weak-lensing and SZ measurements will allow us to measure directly the cluster gas fractions when the ICM temperature is available (Myers et al. 1997; Holder, Carlstrom, & Evrard 2000).

Both blind SZ surveys, such as that to be done with AMiBA (Ho et al. 2004), and gravitational weak lensing surveys, such as the CFHT Legacy Survey4, aim to detect massive clusters of \( \gtrsim 10^{15} M_\odot \) to study the formation of structure through the abundance and properties of the detected clusters. A comparison of the two types of survey should reveal much about the different structures of hot gas and dark matter in clusters of different mass. Here we investigate this possibility using N-body simulations.

2. CLUSTER GAS FRACTIONS FROM WEAK-LENsING AND SZ OBSERVATIONS

The lensing convergence \( \kappa \) is essentially the surface mass density projected on the sky,

\[
\kappa = \frac{\Sigma_m}{\Sigma_{crit}(z_d, z_s)},
\]

where \( \Sigma_m \) is the surface mass density of a halo acting as gravitational lens at redshift \( z_d \), and \( \Sigma_{crit} \) is the lensing critical surface mass density as a function of lens redshift \( z_d \) and source redshift \( z_s \) for a given background cosmology (e.g., Bartelmann & Schneider 2001).

The SZ effect is described by the Compton y parameter defined as a line-of-sight integral of the temperature-weighted thermal electron density (Birkinshaw 1999):

\[
y = \frac{\sigma_T}{m_e c^2} \int d\ln n_e k_B T_e \approx \frac{1 + X \sigma_T k_B T_e}{2 m_p m_e c^2} \Sigma_g,
\]

where \( n_e \) is the electron number density, \( T_e \) the gas temperature, \( m_e \) the electron mass, \( \sigma_T \) the Thomson cross section, \( \Sigma_g \) the surface gas mass density, and \( X = 0.76 \) the hydrogen abundance. Note that we have assumed isothermality in the second equality in Eq. (2). The presence of temperature gradients in the ICM still remains controversial (see Piffaretti et al. 2005 and references therein). In the paper, we assume the ICM is isothermal as a zeroth-order approximation; we found a temperature variation within the virial radius by a factor two at most from our sample of massive halos (see §4).

In general, the lensing \( \kappa \) and the Compton \( y \) reconstructed from noisy observations are smoothed by a certain window function in the map making process. In interferometric observations, the synthesized beam (or PSF) is well approximated by a Gaussian window. In weak lensing surveys, Gaussian smoothing is often adopted to find clusters as local maxima in the reconstructed convergence map (e.g., Miyazaki et al. 2002). We hence express the convergence field smoothed with a Gaussian window \( W_G(\theta) = \exp(-\theta^2/\theta_G^2)/\pi \theta_G^2 \) as \( \kappa_G(\theta) \), where \( \kappa_G \) is related to FWHM of the Gaussian window \( \theta_{G,\text{FWHM}} \) by \( \theta_G = \theta_{G,\text{FWHM}}/\sqrt{\ln(2)} \). Similarly, we define the Gaussian smoothed Compton y parameter, \( y_G(\theta) \). We then compare the reconstructed \( \kappa_G \) and \( y_G \) values at a certain point \( \theta \). Taking the ratio of smoothed \( \kappa_G \) and \( y_G \), we have

\[
\eta_k \equiv \frac{y_G}{\kappa_G} = 8.9h \times 10^{-4} \left( \frac{k_B T_e}{10\text{keV}} \right) \left( \frac{\Sigma_{crit}}{1h^2\text{g/cm}^2} \right) \left( \frac{f_e}{0.1} \right).
\]
where \( f_g(\theta) = \frac{\Sigma_{\text{g,C}}(\theta)}{\Sigma_{\text{m,C}}(\theta)} \) is the local gas fraction defined with the Gaussian smoothed surface mass densities. In order to extract information on the cluster gas fraction from weak lensing and SZ observations, we thus need additional information on the ICM temperature \( T_g \), the cluster redshift \( z_d \), and the redshift distribution of background galaxies. Such information can be in principle available from X-ray observations and photometric redshift measurements of background galaxies.

We can define a global estimator for the cluster gas fraction based on weak-lensing and SZ observation as

\[
\langle \eta_g \rangle = \frac{\sum_i w_i \eta_{g,i}}{\sum_i w_i} \propto \frac{\Sigma_{\text{crit}}(z_d, z_s)}{\langle f_g(T_g) \rangle}
\]

where \( \eta_{g,i} = \eta_{g}(\theta_i) \) and \( w_i = w(\theta_i) \) is the weight for the \( i \)th pixel \( \theta_i \). Accordingly, the global gas fraction of a halo can be estimated from \( k \) and \( y \) maps as \( \langle f_g(T_g) \rangle \propto \langle \eta_g \rangle T_g^{-1} \Sigma_{\text{crit}} \) if one assumes the isothermality of the ICM. The case with \( w_i = \delta(\theta_i) \) corresponds to a commonly used definition of the gas fraction, \( \eta_g^{(1)} \equiv \langle y \rangle / \langle \kappa \rangle \propto \langle \Sigma_g \rangle / \langle \Sigma_m \rangle \), as the ratio of the mean surface mass densities. However, the mean convergence \( \langle \kappa \rangle \) is ill constrained from weak-lensing observations: Weak lensing \( \kappa \) reconstructions based solely on image distortions suffer from the mass sheet degeneracy (Schneider & Seitz 1995) unless additional information such as the magnification bias is taken into account (Broadhurst, Taylor & Peacock 1995; Broadhurst et al. 2005). On the other hand, interferometric observations are insensitive to the DC signal, and hence do not constrain the total flux in the field-of-view (FoV). Bolometers also suffer from a high level of contamination by atmospheric and other environmental signals, and hence rely on the differencing scheme to extract the sky signals. We therefore consider an alternative weight \( w_i = \kappa G(\theta_i) \) that weights high density regions more strongly than \( w_i = \kappa G(\theta_i) \). Then we have the gas fraction estimator of the form:

\[
\eta_g^{(2)} = \frac{\sum_i \kappa G(\theta_i) \eta_{g,i}(\theta)}{\sum_i \kappa G(\theta_i) \kappa G(\theta_i)} \propto \frac{\langle \kappa y \rangle}{\langle \kappa^2 \rangle} \tag{5}
\]

Suppose that \( \kappa \) and \( y \) satisfy a linear bias relation \( y(\theta) = a \kappa(\theta) \). Then errors in the Compton parameter of the form \( \eta_g^{(1)} \) lead to errors in the two estimators of \( \eta_g \) of \( \Delta \eta_g^{(1)} = \epsilon(\kappa) \) and \( \Delta \eta_g^{(2)} = \epsilon(\kappa)/\langle \kappa^2 \rangle \). The difference between these errors \( |\Delta \eta_g^{(1)}| - |\Delta \eta_g^{(2)}| = \langle \epsilon^2 / \langle \kappa^2 \rangle \rangle (\text{Var}[\kappa]/\langle \kappa^2 \rangle) \geq 0 \), with equality only if variance in \( \kappa \) vanishes. Thus the \( \eta_g^{(2)} \) estimator is always better than the \( \eta_g^{(1)} \) estimator. Similarly, an error on the convergence of the form \( \kappa \rightarrow \kappa + \epsilon \) leads to errors in the two estimators of \( \eta_g \) of \( \Delta \eta_g^{(1)} = \epsilon \) and \( \Delta \eta_g^{(2)} = \epsilon / \langle \kappa^2 \rangle \). The difference between these errors \( |\Delta \eta_g^{(1)}| - |\Delta \eta_g^{(2)}| > 0 \), again showing that the cross-correlation based estimator \( \eta_g^{(2)} \) is more robust than \( \eta_g^{(1)} \).

In practical observations, noise contributions to the estimator \( \eta_g^{(2)} \) must be taken into account. Since the noise properties between the \( \kappa \) and \( y \) maps will not be correlated, the required correction is in the denominator of Eq. \( \eta_g^{(2)} \), where we replace \( \langle \kappa^2 \rangle \) by \( \langle \kappa^2 \rangle - \sigma_{\kappa}^2 \) with \( \sigma_{\kappa}^2 \) being the noise variance in the smoothed convergence field, \( \kappa G \) (§3.3).

3. MOK OBSERVATIONS

We use cosmological simulation data by Lin et al. (2004) to demonstrate what useful information we can extract from the cross correlation between weak-lensing and SZ surveys. Specifically, we simulate the forthcoming AMiBA experiment as an illustration of such combined surveys.

3.1. N-body/hydrodynamic simulations

To make sky maps with realistic SZ and weak lensing signals, we use results from preheating cosmological simulations of a ΛCDM model (\( \Omega_{\text{m}} = 0.34, \Omega_{\Lambda} = 0.66, \Omega_b = 0.044, h = 0.66, \sigma_8 = 0.94 \)) in a 100 h^{-1} Mpc co-moving box generated with an N-body/hydrodynamics code GADGET (Springel et al. 2001), which reproduce the observed cluster M_X-T_g and L_X-T_g relations at \( z = 0 \) (Lin et al. 2004). We construct a total of 29 SZ sky maps, each 1 deg^2 in size and containing 1024^2 pixels, by projecting the electron pressure through the randomly displaced and oriented simulation boxes, separated by 100 h^{-1} Mpc, along a viewing cone out to the redshift of \( z = 2 \). Similarly, 29 \( \kappa \)-maps are constructed by projecting the distance-weighted mass overdensity \( \rho \) out to a source plane at a fixed redshift, \( z = z_s \).

3.2. AMiBA SZ cluster survey

As an AMiBA specification, we adopt a close-packed hexagonal configuration of \( 19 \times 1.2\) m dishes on a single platform. AMiBA operates at 95 GHz, and this array configuration yields a synthesized beam of \( \theta_{\text{beam}} \approx 2\ (0.6 h^{-1}\text{Mpc at } z = 0.8) \), which is optimized to detect high-z clusters (Zhang et al. 2002). The primary beam has an FoV of FWHM \( \approx 11\). We assume the following telescope system: bandwidth \( \Delta \nu = 20\) GHz, system temperature \( T_{\text{sys}} = 70 \) K, system efficiency of 0.7, and dual polarizations.

AMiBA will operate as a drift scan interferometer to remove ground contamination to high order (Pen et al. 2002; Zhang et al. 2002; Park et al. 2003). We approximate the resulting synthesized beam in the AMiBA map by a Gaussian with \( \theta_{\text{beam}} \approx 2\), and we convolve the simulated Compton y maps with this Gaussian beam to generate a set of noise-free AMiBA yG-maps. The expected rms noise over a sky area of \( \Omega_t \) with a total integration time of \( t_{\text{int}} \) is (Pearson et al. 2003),

\[
\sigma_{y_G} = 2 \times 10^{-6} \left( \frac{T_{\text{sys}}}{70\text{K}} \right) \left( \frac{\Delta \nu}{20\text{GHz}} \right)^{-1/2} \left( \frac{\Omega_t}{1\text{deg}^2} \right)^{1/2} \left( \frac{t_{\text{int}}}{240\text{hrs}} \right)^{-1/2}.
\]

3.3. Weak lensing cluster survey

We assume a weak lensing survey with a mean galaxy number density of \( n_g = 40 \) arcmin^{-2}, an rms amplitude of the intrinsic ellipticity distribution of \( \sigma_e = 0.3 \), and a mean galaxy redshift of \( z_g = 1 \), which are close to typical values for a ground based optical imaging survey with a magnitude limit of \( R_{\text{lim}} \approx 25.5 \) mag in a sub-arcsecond seeing condition (e.g., Hamana, Takada, & Yoshida 2004). The rms noise in a Gaussian-smoothed convergence field, \( \kappa G \), is given as

\[
\sigma_{\kappa_G} = 1.1 \times 10^{-2} \left( \frac{\sigma_e}{0.3} \right) \left( \frac{n_g}{40\text{arcmin}^{-2}} \right)^{-1/2} \left( \frac{\theta_{\text{beam}}}{2'} \right)^{-1} \tag{7}
\]

(e.g., van Waerbeke 2000). We choose the same smoothing scale of \( \theta_{\text{beam}} \approx 2' \) as for the AMiBA SZ survey and convolve the \( \kappa \) maps from N-body simulations (see §3.1) with the Gaussian beam to produce a set of noise-free \( \kappa G \) maps. Note that the adopted value of \( \theta_{\text{beam}} \) is close to the optimal smoothing scale of \( \theta_{\text{beam}} \approx 1' \) (or \( \theta = 1' \)) for an efficient survey of massive halos with \( M \gtrsim 10^{14} h^{-1} \text{M}_\odot \) found by Hamana et al. (2004) based on cosmological N-body simulations.

\[\text{http://www.cita.utoronto.ca/~pen/download/AmiBa}\]
which the hot gas traces dark matter. The wing width serves the cluster core, and the bulk of the wing reveals the degree to surface mass densities of both dark matter and hot gas in the cluster region. Furthermore, we can see a clear disparity between the noise-free result for the most massive halo, which contains rich information on the relative distributions of dark matter and hot gas. Most halos shown in Fig. 1 have significant wing widths. These widths arise because the halo properties such as $M_{200}$ and $T_{200}$ are also indicated within the figure. Also plotted are the $y_G$ and $\kappa_G$ maps smoothed with $\theta_{500m} = 2'$ Gaussian in the left and the right subpanels, respectively. A circle with radius of $\theta_{500}$ is shown in each subpanel.

4. CLUSTER GAS FRACTIONS FROM CROSS CORRELATION BETWEEN WEAK-LENSEING AND SZ SIGNALS

We carried out a statistical analysis on the simulated weak lensing $\kappa_G$ and SZ Compton $y_G$ maps of $29 \times 1$ deg$^2$ for the AMiBA experiment. Figure 1 shows the cross correlation between the noise-free $\kappa_G$ and $y_G$ sky maps of the entire 29 deg$^2$ as a 2D scatter plot histogram. The scatter plot is presented in units of rms dispersions $\kappa_*$ and $y_*$ of the smoothed noise-free fields $\kappa_G(\theta)$ and $y_G(\theta)$, respectively; $\kappa_* = 1.4 \times 10^{-2}$ and $y_* = 2.2 \times 10^{-6}$. Halo peaks with $\kappa > 4.5 \kappa_*$ and $y < 6 y_*$, are indicated with markers according to their redshifts: $\z < 0.3$ (circle), $0.3 \leq \z < 0.5$ (triangle), $0.5 \leq \z < 1$ (square).

Another important feature in Fig. 1 is a number of butterfly-wing-shaped structures extending from the origin to points corresponding to the halo peaks. Each blade of a butterfly wing corresponds to an individual cluster. Hereafter, we call the $\kappa/y$ cross-correlation diagram the butterfly diagram. For illustrative purposes, we show in Fig. 2 a butterfly diagram from the noise-free result for the most massive halo, which has $M_{200} = 7.0 \times 10^{14} h^{-1} M_\odot$ at $\z = 0.142$. The SZ signal for this halo is more extended than the weak lensing signal in this cluster region. Furthermore, we can see a clear disparity between the shapes of the contours in the $\kappa$ and $y$ maps.

The butterfly diagram of a single cluster shows a wing that contains rich information on the relative distributions of dark matter and hot gas. The tip of a butterfly wing shows the surface mass densities of both dark matter and hot gas in the cluster core, and the bulk of the wing reveals the degree to which the hot gas traces dark matter. The wing width serves as an indicator of the difference between projected distributions of dark matter and hot gas. Most halos shown in Fig. 1 have significant wing widths. These widths arise because the baryon distribution is closer to axisymmetric than the dark matter distribution due to collisional relaxation of the baryons. The local slope $\eta_g(\theta)$ of a wing in conjunction with the cluster redshift and temperature can be an indicator of the local gas fraction $f_g(\theta)$ (see Eq. 3). Note that the estimator $\eta_g^2$ defined in Eq. 3 can be interpreted as a global slope of a wing in the least-squares sense.

We use the halo sample with $\kappa_G > 4.5 \kappa_*$ and $y_G > 6 y_*$ derived from our noise-free sky maps of 29 deg$^2$ to compare the halo-based cumulative gas fractions $f_g(\theta) = M_\Delta(r)/M(r)$ with their projected estimates $f_g^2(\z) = \eta_5^2(\z) T_g < \z \Sigma_{\rm crit}^{-1}(\z) \z / D_A(\z) / D_A(z)$ from cross correlation between $\kappa$ and $y$; $\theta = r / D_A(z)$ being the angular diameter distance to redshift $\z$. We make this comparison inside $r_{500}$, and calculate...
for each halo a gas fraction using Eqs. 4 and 5 within an aperture of projected radius $\theta_{500} = r_{500}/D_{A(z)}$. We assume cluster isothermality and take the emission-weighted mean temperature $T_{500}$ inside $r_{500}$ as the observable X-ray temperature. The weight $w_i$ in Eq. 4 is set to zero if $\kappa_i < 0$ or $y_i \leq 0$ to exclude low-signal regions where cross correlation measurements are of low significance due to projection effects and/or observational noise. Further, we remove those halos whose centers are located outside the inner $50^\prime \times 50^\prime$ region of a $1^\circ$ sky map, in order to avoid systematics in the gas fraction estimates due to the boundaries. The effective survey area is thus about $20 \, \text{deg}^2$

Figure 3 shows the ratio of the distribution of the ratio $f_g^{(2)}(< \theta_{500})/f_g(< r_{500})$ obtained from the noise-free maps as a function of $M_{200}$ for three redshift subsamples: (a) $z \leq 0.3$ (where we find 8 halos); (b) $0.3 \leq z < 0.5$ (16 halos); and (c) $0.5 \leq z < 1$ (13 halos). The sample means and their standard errors of $f_g^{(2)}(< \theta_{500})/f_g(< r_{500})$ are (a) $0.96 \pm 0.03$, (b) $1.10 \pm 0.04$, and (c) $0.95 \pm 0.05$. Note that the quoted errors represent the intrinsic scatter of the estimator $f_g^{(2)}$ from the noise-free signal maps. On the whole, the estimator $f_g^{(2)}(< \theta_{500})$ for the gas fraction is close to unbiased, but there is a slight ($\sim 2\sigma$) over-estimation bias of $f_g$ in $0.3 \leq z < 0.5$. We found that most halos with overestimated values of $f_g$ are associated with mergers. The shock heating of gas in mergers boosts the temperature in the region between the merging halos. However, we ascribe the lower temperature $T_{500}$ to this gas, based on the emission-weighted temperature of the entire halo, and so overestimate the value of $f_g$ for the system as a whole. This overestimate of $f_g$ appears to be smaller at higher $z$ because such halos have smaller angular sizes and so suffer larger beam smearing effects. If the 5 interacting/merger systems in subsample (b) are rejected, then the sample mean $f_g^{(2)}(< \theta_{500})/f_g(< r_{500}) = 1.01 \pm 0.03$, showing that the bias is eliminated. Such a debias should be possible based on X-ray observations of the survey field.

On the other hand, the projection effect in $\kappa$ due to intervening matter (Metzler et al. 2001) also affects cluster $f_g$ estimates made using weak lensing data. This is more severe for higher-$z$ clusters due to their lower geometrical lensing efficiency and smaller angular sizes. If an intervening mass concentration is projected onto a cluster, then the gas fraction of that cluster could be under-estimated. Furthermore, the gradients in 3D cluster gas-fraction and temperature profiles (Ettori et al. 2004) can also induce a systematic bias in projected $f_g^{(2)}$ estimates, especially for spatially-resolved low-$z$ clusters. Intrinsic gradients in $T_e$ and $f_g$ are manifest as a curvature of the wing in a butterfly diagram, and they cannot be distinguished unless detailed information is available on the temperature distribution in the ICM.

The error bars in Fig. (3) are based on 50 realizations of Monte-Carlo simulated noise maps which are linearly added to the Gaussian smoothed signal maps of $\kappa_G$ and $\gamma_G$. Following van Waerbeke (2000), a correlated noise map of $\kappa_G$ is generated for each realization using the noise auto-correlation function specified by the smoothing scale $\theta_{	ext{binm}}$ and the observational noise rms $\sigma_{\kappa}$ (Eq. 7). A similar correlated noise map for $\gamma_G$ uses the noise rms $\sigma_{\gamma}$ (Eq. 6). For a fixed halo mass, the random error in $f_g^{(2)}$ is smaller for lower-$z$ halos with larger angular scales, $\theta_{500}$, over which pixel noise is averaged, but systematic errors will be dominant for these clusters.

5. CONCLUSIONS

By simulating the AMiBA experiment, we have shown that the butterfly diagram that emerges from a comparison of the SZ and weak-lensing signals contains rich information on the distribution of the cluster gas fraction within an individual cluster and between different clusters (§4). The cross-correlation based estimator $f_g^{(2)}(< \theta)$ should be used for a robust estimate of the cluster gas fraction. Overall, this estimator is close to unbiased, but some halos show intrinsic scatter that exceeds the observational errors (§3) due to various sources of systematics (§4). The bias for the intermediate-$z$ subsample of halos can be eliminated by removing interacting/merger systems. Careful selection of clusters should minimize systematic uncertainties in $f_g$. Detailed information regarding the temperature distribution in clusters, derived from X-ray observations, improves the calibration of the gas fraction estimator, and yields valuable information on the thermodynamic history of the ICM.

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