Outage Probability Analysis of IRS-Assisted Systems Under Spatially Correlated Channels

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Abstract—This paper investigates the impact of spatial channel correlation on the outage probability of intelligent reflecting surface (IRS)-assisted single-input single-output (SISO) communication systems. In particular, we derive a novel closed-form expression of the outage probability for arbitrary phase shifts and correlation matrices of the indirect channels. To shed light on the impact of the spatial correlation, we further attain the closed-form expressions for two common scenarios met in the literature, where the outage probability becomes independent of the phase shifts when the large-scale fading coefficients are expressed by the loss over a propagation distance. Numerical results validate the tightness and effectiveness of the closed-form expressions. Furthermore, the spatial correlation offers significant decreases in the outage probability as the direct channel is blocked.

Index Terms—Intelligent reflecting surface, outage probability, spatial correlation.

I. INTRODUCTION

Intelligent reflecting surface (IRS), which relies on the recent advancements of meta-materials, is gradually being recognized as a promising technology in the future wireless networks not only because of its significant gains in spectral and energy efficiency but with the ultimate goal for the smart control of the propagation environment [1]. Specifically, an IRS consists of low-cost, nearly passive reflecting elements that can reconfigure the interaction with impinging electromagnetic waves. Owing to its promising benefits, among the last year, IRS has received substantial attention as the literature reveals. Most works have focused on the phase shift matrix design with/without the transmit beamforming using the instantaneous channels with different communication objectives as [2]–[5] and reference therein.

While the majority of previous works have improved our knowledge of IRS-assisted systems, they have relied on the common assumption of tractable independent and identically distributed (i.i.d.) Rayleigh fading channel model and/or in the asymptotic regime regarding the size of IRS (the number of its elements goes to infinity) for a tractable performance analysis [6]. To address this unrealistic conjecture, few works have taken into account the spatial correlation among the phase shifts, being inevitable in practice, for instance [7]. Recently, this consideration was further grounded by showing that i.i.d. Rayleigh fading only appears in rare cases and by deriving a spatially correlated Rayleigh fading model following the IRS-design principles [7]. In parallel, although IRSs are suggested for coverage improvement, most existing works concern the study of common performance metrics such as the achievable rate and the bit error rate while the important outage probability has been neglected. As far as the authors are aware, the outage probability in IRS-enhanced single-input single-output (SISO) scenarios has been investigated only in [8]–[10] without accounting for the spatial correlation.

Motivated by these research gaps, in this paper, we obtain the outage probability for SISO systems for an arbitrary (finite) number of phase shifts by focusing on the impact of channel correlation while including the presence of both direct and indirect propagation channels. Our main contributions are summarized as follows: (i) By focusing on practical aspects including the effect of spatially correlated Rayleigh fading and the consideration of a finite number of IRS elements, we formulate the system model and derive the closed-form expression on the outage probability conditioned on the phase-shift matrix, which is a function only of the channel statistics. Our proposed closed-form expression allows the study of the impact of different spatial correlation structures on the outage probability. We further prove that the equal phase-shift selection provides the minimum outage probability at the asymptotic regime regarding the number of elements. (ii) We provide analytical expressions of the outage probability under either equal or random phase shifts. These scenarios concretely unveil the impact of the spatial correlation. (iii) Our closed-form expressions are verified by Monte-Carlo simulations and confirm that the IRS phase-shift selection and its correlation are of paramount importance to the system performance when the direct channel is blocked. Furthermore, the equal phase-shift design is close to the optimal one based on perfect channel state information (CSI).

Notation: Upper and lower bold letters denotes matrices and vectors. A diagonal matrix is diag(x) with x in the diagonal and tr(·) is the trace of a matrix. CN(·, ·) denotes the circularly symmetric Gaussian distribution while U(a, b) is the uniform distribution in the range [a, b]. In is the identity matrix of size $N \times N$. The expectation and variance of a random variable are $E\{\cdot\}$ and $\text{Var}\{\cdot\}$. The Euclidean norm is $\|\|$, the superscript $^H$ is the Hermitian transpose, and $\odot$ is the Hadamard product. mod (·) is the modulus operation and $\lfloor \cdot \rfloor$ is the floor function. $\Gamma(m, n) = \int_0^{\infty} t^{m-1} e^{-t} dt$ and $\Gamma(n) = \int_0^{\infty} t^{m-1} e^{-t} dt$ are the upper incomplete Gamma function and the Gamma function. Finally, $\text{sin}(x) = \sin(\pi x)/\pi x$ is the sinc function.

II. SYSTEM MODEL AND CHANNEL CAPACITY

This paper considers a system with one single-antenna source sending signals to one single-antenna destination. An
IRS with $N$ phase-shift elements is deployed between source and destination to enhance communication reliability.

A. Channel Modelling

Even though the propagation channels vary over time and frequency, we assume a block-fading channel model, where the channels are static and frequency flat in each coherence interval. We denote $h_{sd} \in \mathbb{C}$ the channel between the source and the destination, $h_{sr} \in \mathbb{C}^{N}$ the channel vector between the source and the IRS, and $h_{rd} \in \mathbb{C}^{N}$ the channel vector between the IRS and the destination. Notably, we take into account for correlated Rayleigh channel model. Mathematically, the channels are described as

$$h_{sd} \sim \mathcal{CN}(0, \beta_{sd}), h_{sr} \sim \mathcal{CN}(0, \mathbf{R}_{sr}), h_{rd} \sim \mathcal{CN}(0, \mathbf{R}_{rd}),$$

where $\beta_{sd} \in \mathbb{C}$, $\mathbf{R}_{sr} \in \mathbb{C}^{N \times N}$, and $\mathbf{R}_{rd} \in \mathbb{C}^{N \times N}$ are the large-scale fading coefficient and the covariance matrices, respectively. Note that for the sake of clarity, we have incorporated the large-scale fading coefficients $\beta_{sd} \in \mathbb{C}$ and $\beta_{rd} \in \mathbb{C}$ of the assisted link inside $\mathbf{R}_{sr}$ and $\mathbf{R}_{rd}$. The channel model (1) is aligned with an infinitesimal small source, which radiates isotropically in the IRS [11]. Henceforth, $h_{sd}$ is also called the direct channel and $h_{sr}, h_{rd}$ comprise the indirect or else the cascaded channel [1]. The considered channels in (1) are of practical interest for the performance analysis of IRS-assisted systems, where IRSs are fabricated as a planar array [7].

B. Data Transmission

The received complex baseband signal at the destination, formulated under a first-order IRS reflection assumption, is given by $y = \sqrt{h_{sd}}^\dagger \mathbf{h}_{sd}^H \Theta \mathbf{h}_{rd}^H s + \sqrt{\beta_{sd}} n + \eta$, where $\rho$ is the transmit power allocated by the source, $s$ is the transmit data symbol with $\mathbb{E}\{|s|^2\} = 1$, and $n \sim \mathcal{CN}(0, \sigma^2)$ is the additive Gaussian noise. Also, $\Theta = \text{diag}(e^{j\theta_1}, \ldots, e^{j\theta_N})$ is the phase-shift matrix, where $\theta_n \in [\pi, \pi]$, $\forall n$, is the phase shifts induced by the IRS. By assuming coherent combination as in [2], [10], we obtain the channel capacity for arbitrary phase shifts as

$$C = \log_2 \left(1 + \frac{\rho}{\bar{\sigma}^2} |h_{sd} + h_{sr}^H \Theta \mathbf{h}_{rd}|^2\right), \quad [\text{b/s/Hz}].$$

The channel capacity in (2) is a function of the instantaneous channels varying upon the time and frequency plane, which can be perfectly known when the coherence intervals are sufficiently long. We subsequently use this capacity expression to analyze the outage probability and obtain a closed-form expression, which will depend only on channel statistics as will be shown.

III. OUTAGE PROBABILITY ANALYSIS

From (2), we now consider the outage probability of the network, which is defined as $P = \Pr(C < \xi)$, where $\xi$ [b/s/Hz] is the target rate. By setting $z = \sigma^2 (2\xi - 1) / \rho$, the outage probability is recast to an equivalent signal-to-noise ratio requirement as

$$P = \Pr \left(|h_{sd} + h_{sr}^H \Theta \mathbf{h}_{rd}|^2 < z\right).$$

In this paper, the moment-matching method is used to manipulate the outage probability as follows.

**Theorem 1.** For a given value $z$, the outage probability (3) is obtained in closed form as a function of the phase-shift matrix $\Theta$ as

$$P(\Theta) = 1 - \Gamma(k_a, z/w_a)/\Gamma(k_a),$$

where the shape parameter $k_a$ and the scale parameter $w_a$ are respectively given by

$$k_a = \frac{(\beta_{sd} + \text{tr}(\Theta))^2}{\beta_{sd}^2 + 2\beta_{sd} \text{tr}(\Theta) + (\text{tr}(\Theta))^2 + 2\text{tr}(\Theta^2)},$$

$$w_a = \frac{2\beta_{sd} + \text{tr}(\Theta) + 2\text{tr}(\Theta^2)}{\beta_{sd} + \text{tr}(\Theta)}$$

with $\Theta = \mathbf{R}_{sr} \Theta^H \mathbf{R}_{rd} \Theta$.

**Proof.** Let us define a new random variable $X = h_{sd} + h_{sr}^H \Theta \mathbf{h}_{rd}^H$. Its mean value is computed by the independence of the direct and indirect channels as

$$\mathbb{E}(X) = \mathbb{E}(|h_{sd}|^2) + \mathbb{E}(|h_{sr}^H \Theta \mathbf{h}_{rd}^H|^2) = \beta_{sd} + \mathbb{E}(|h_{sr}^H \Theta \mathbf{h}_{rd}|^2)$$

$$= \beta_{sd} + \mathbb{E}(h_{sr}^H \Theta \mathbf{h}_{rd} h_{sr}^H \Theta^H \mathbf{h}_{sd}) = \beta_{sd} + \text{tr}(\mathbf{R}_{sr} \Theta \mathbf{R}_{rd} \Theta).$$

Its second moment, denoted by $\mathbb{E}(X^2)$, is computed as

$$\mathbb{E}(X^2) = \mathbb{E}(h_{sd}^2 + h_{sr}^H \Theta \mathbf{h}_{rd}^H h_{sd} + h_{sr}^H \Theta \mathbf{h}_{rd}^H h_{sr} + h_{sr}^H \Theta \mathbf{h}_{rd}^H h_{sr} + |h_{sr}^H \Theta \mathbf{h}_{rd}|^2)$$

$$= \beta_{sd}^2 + \mathbb{E}(h_{sr}^H \Theta \mathbf{h}_{rd} h_{sr}^H \Theta^H \mathbf{h}_{sd}) = \beta_{sd}^2 + \text{tr}(\mathbf{R}_{sr} \Theta \mathbf{R}_{rd} \Theta)$$

Let us define $a = |h_{sd}|^2$, $b = h_{sr}^H \Theta \mathbf{h}_{rd}$, $c = h_{sr}^H \Theta h_{sd}$, and $d = h_{sr}^H \Theta h_{sr}$. Thus, (3) is recast as

$$\mathbb{E}(X^2) = \mathbb{E}\{|a|^2\} + \mathbb{E}\{|b|^2\} + \mathbb{E}\{|c|^2\} + 2\mathbb{E}\{ad\} + \mathbb{E}\{|d|^2\},$$

where $\mathbb{E}\{|a|^2\} = 2\beta_{sd}$ by [12] Lemma 9. Also, we obtain

$$\mathbb{E}\{|b|^2\} = \mathbb{E}\{ad\} = \mathbb{E}\{|c|^2\} = \beta_{sd} \text{tr}(\mathbf{R}_{sr} \Theta \mathbf{R}_{rd} \Theta),$$

due to the independence among the channels. The last expectation of (9) is computed as

$$\mathbb{E}\{|d|^2\} = \mathbb{E}\left(\left\|\mathbf{R}_{sr}^{1/2} \Theta \mathbf{h}_{rd}^H\right\|^4 \left|\frac{h_{sr}^H \Theta \mathbf{h}_{rd}^H}{\mathbf{R}_{sr}^{1/2} \Theta \mathbf{h}_{rd}^H} \frac{h_{sr}^H \Theta h_{sd}^H}{\mathbf{R}_{sr}^{1/2} \Theta h_{sd}^H} \right|^2\right)\right)$$

Let us define $t = h_{sr}^H \Theta \mathbf{h}_{rd}^H / \|\mathbf{R}_{sr}^{1/2} \Theta \mathbf{h}_{rd}^H\|$, then by conditioning on $h_{rd}$, $t$ is a circularly symmetric Gaussian variable. Furthermore, thanks to the normalization factor $\|\mathbf{R}_{sr}^{1/2} \Theta \mathbf{h}_{rd}^H\|$ and the use of the circular symmetric property, we have $t \sim \mathcal{CN}(0, 1)$. Hence, (11) is manipulated as

$$\mathbb{E}\{|d|^2\} = \mathbb{E}\left(\left|\frac{\mathbf{R}_{sr}^{1/2} \Theta \mathbf{h}_{rd}^H}{\|\mathbf{R}_{sr}^{1/2} \Theta \mathbf{h}_{rd}^H\|}\right|^4 \right) \mathbb{E}\{|t|^4\} \left(=\mathbb{E}\left(\left|\frac{\mathbf{R}_{sr}^{1/2} \Theta \mathbf{h}_{rd}^H}{\|\mathbf{R}_{sr}^{1/2} \Theta \mathbf{h}_{rd}^H\|}\right|^4 \right) \mathbb{E}\{|t|^4\}\right)$$

$$= \mathbb{E}\{|t|^4\} 2 \text{tr}(\mathbf{R}_{sr} \Theta \mathbf{R}_{rd} \Theta) + 2\text{tr}(\mathbf{R}_{sr} \Theta h_{sr}^H \mathbf{R}_{rd}^H \Theta),$$

where $(a)$ is obtained by the fact that $\mathbf{h}_{rd}$ and $t$ are independent; $(b)$ is obtained by the use of [13] Lemma 9 to compute...
the forth moment of random variables. Combining the results of (8)–(12), we obtain

\[ \mathbb{E}\{X^2\} = 2\beta^2 + 2\beta_a \text{tr}\left(R_{rd}\Theta^H R_{rd}\Theta\right) + 2\text{tr}\left(R_{rd}\Theta^H R_{rd}\Theta\right)^2 + 2\text{tr}\left(R_{rd}\Theta^H R_{rd}\Theta\right)^2. \]

By exploiting the identity

\[ \text{Var}\{X\} = \mathbb{E}\{X^2\} - \mathbb{E}^2\{X\} \]

with the results in (7) and (13), we obtain

\[ \text{Var}\{X\} = \beta^2 + 2\beta_a \text{tr}\left(R_{rd}\Theta^H R_{rd}\Theta\right) + \left(\text{tr}\left(R_{rd}\Theta^H R_{rd}\Theta\right)\right)^2 + 2\text{tr}\left(R_{rd}\Theta^H R_{rd}\Theta\right)^2. \]

One can use the moment-matching method to match the random variable \(X\) to a Gamma distribution with the shape and scale parameters as \(k_a = \frac{\mathbb{E}\{X^2\}}{\text{Var}\{X\}}\) and \(w_a = \frac{\text{Var}\{X\}}{\mathbb{E}\{X\}}\). Exploiting (7) and (14), the result is obtained as in the theorem.

We emphasize the advantage of the closed-form expression obtained in (14) since it can be described by different spatial correlation models and allows their study. In the aspects of mathematics, this is indeed a generalized version of (10) concerning uncorrelated Rayleigh channels. Unlike previous works relying on the instantaneous CSI to optimize the phase shifts, e.g., (14), the proposed phase-shift design utilizing (14) as the utility function is independent of the small-scale fading coefficients and is stable for a long period of time. The phase shifts do not need optimization at every coherence interval but only when the large-scale statistics change.

**Remark 1.** The closed-form expression in Theorem 1 can be applied in the case of no direct channel between the source and the destination. Especially, the closed-form expression is still computed by (10), but the shape and scale parameters are obtained from (5) and (6) with \(\beta_d = 0\). Moreover, the methodology used in this paper can be easily extended to multi-user multiple-input multiple-output scenarios with imperfect CSI as long as the channel estimate and estimation error are uncorrelated and their moments are bounded (5).

Below, we provide a specific example with the real correlation matrices that describes properly the IRS structure.

**Example 1.** Under the assumption in (7), i.e., for a rectangular phase-shift array with \(N = N_V N_H\) where \(N_V\) and \(N_H\) are the elements to per row and per column, respectively, and under isotropic Rayleigh fading, we express \(R_{sr}\) and \(R_{rd}\) as

\[ R_{sr} = \beta_d d_H d_V R \quad \text{and} \quad R_{rd} = \beta_a d_H d_V R, \]

where the size of each phase shift element is \(d_H \times d_V\), where \(d_V\) is the vertical height and \(d_H\) is the horizontal width. The matrix \(R\) represents the spatial correlation at the IRS with the \((n,m)\)-th coefficient given by \(r^{nm} = \text{sin}(2\pi |u_n - u_m|/\lambda)\), where \(u_n = \{n\} \mod (\alpha - 1)/N_H\) and \(d_V\) is the wavelength of a plane wave. Consequently, we obtain that \(\text{tr}(\Theta) = \beta_a \beta_d d_H^2 \sum_{\beta=1}^N \text{tr}(R\Theta^H R\Theta)\) and \(\text{tr}(\Theta^2) = \beta_a \beta_d d_H^2 \sum_{\beta=1}^N \text{tr}(R\Theta^H R\Theta)\).

For rich scattering environments and strict assumptions on the IRS structure in order to have uncorrelated Rayleigh fading channels [7], [10], i.e., \(R_{sr} = \beta_d d_H d_V I_N\) and \(R_{rd} = \beta_a d_H d_V I_N\), one obtains that \(\text{tr}(\Theta) = N\beta_a \beta_d d_H^2 d_V\) and \(\text{tr}(\Theta^2) = N\beta_a \beta_d d_H^2 d_V^2\).

The example shows that Theorem 1 can be applied for various covariance matrices by adjusting the inputs of the incomplete/complete Gamma functions. For uncorrelated Rayleigh fading, the closed-form expression is directly proportional to the array gain, which can be easily observed by neglecting the spatial correlation, which is however inevitable in practical systems enabled by IRSs. Since the shape and scale parameters are always non-negative, we now introduce the optimal phase-shift design at an asymptotic regime as in Corollary 1.

**Corollary 1.** For the spatial correlation channel model (15), the equal phase-shift selection minimizes the outage probability as \(N \to \infty\).

**Proof.** We first take the first-order derivative of the outage probability with respect to \(w_a\) as

\[ \frac{\partial P(\Theta)}{\partial w_a} = \frac{-1}{\Gamma(k_a)} \frac{\partial \Gamma(k_a, z/w_a)}{\partial w_a} = -\frac{z}{\Gamma(k_a)} w_a e^{-z/w_a} < 0, \]

by exploiting (16). For a given shape parameter \(k_a\), the obtained result in (16) indicates that the outage probability is a decreasing function of the scale parameter \(w_a\), which is positive based on (6). As \(N\) grows, both the numerator and denominator of the shape parameter scale up with the same order, i.e., \((\text{tr}(\Theta))^2\), while the scale parameter is dominated by \(\text{tr}(\Theta)\). Thus,

\[ k_a \to 1, w_a \to \text{tr}(\Theta) \quad \text{as} \quad N \to \infty. \]

Moreover, by utilizing the spatial structure in (15), one obtains

\[ \text{tr}(\Theta) = \beta_a \beta_d d_H^2 d_V \sum_{n=1}^N \sum_{m=1}^N r^{nm} r_{nm} e^{j(\theta_n - \theta_m)} \]

\[ \leq \beta_a \beta_d d_H^2 d_V \sum_{n=1}^N \sum_{m=1}^N r^{nm} r_{nm}, \]

which is maximized when \(\theta_n = \theta_m, \forall n, m\). By combining (16)–(18), we conclude the proof.

Corollary 1 gives a simple and effective phase-shift selection at the asymptotic regime. By means of the channel hardening...
where the outage probability is obtained by computing the expectation in (a) via considering all the values of \( \theta_m, \beta_m, \beta_m', \) and \( \theta_m' \) such that \( \theta_m - \theta_m + \theta_m' = 0 \). The result in (25) is obtained after some algebraic manipulations of the last equation of (29).

Moreover, \( \delta = \mathbb{E}\left(\text{tr}\left(\mathbf{R}_{sd}^H\mathbf{R}_{rd}\right)^2\right)\) is obtained similarly as

\[
\delta = \sum_{n=1}^{N} \sum_{m=1}^{N} \sum_{n'=1}^{N} \sum_{m'=1}^{N} \sum_{n'\neq n} \sum_{m'\neq m} r_{rd}^m r_{rd}^{n'} r_{mn} r_{mn'} \mathbb{E}\left(e^{j(\theta_m - \theta_m' + \theta_m - \theta_m')}\right).
\]

By inserting (24)–(26) into (14), we obtain

\[
\text{Var}(X) = \beta_{sd}^2 + 2\beta_{sd} \nu + \eta + 2\delta.
\]

Plugging (28) and (31) into the expressions of the shape and scale parameters, we obtain the result. \( \square \)

Unlike the deterministic setup in Corollary 2, the result exhibited by Corollary 3 presents the outage probability when the phase shifts follow the common uniform distribution. Notably, the same methodology can be apparently extended to other phase shifts distributions with i.i.d. elements.
are formulated as with Monte-Carlo simulations. For a target rate of both direct and indirect channels. All the analytical results are formulated as with $d_H = d_V = \lambda/40$.

Figure 2 depicts the outage probability with the existence of both direct and indirect channels. All the analytical results match well with Monte-Carlo simulations. For a target rate less than 4 [b/s/Hz], the outage probability is lower than 0.6. Interestingly, we observe that the outage probability varies slightly with the different selections of the phase shifts due to the dominance of the direct channel, which meets much better large-scale fading than that of the indirect one. In other words, both the uniformly random and equal phase shifts do not have any impact on the outage probability. Hence, the IRS does not improve coverage under these conditions. In addition, the optimal phase-shift design by exploiting the entire information on perfect CSI [2], [10] has a slight gain compared to the suboptimal in Corollary 1 based on the channel statistics only.

Figure 2 shows the outage probability when the direct channel is blocked. Compared to Fig. 2a, we notice a significant growth of the outage probability. The system is also very sensitive to phase shift selection. The outage probability becomes quite high when the phase shifts are uniformly distributed since the random phase shifts can produce either constructive or destructive combinations of the received signals. The gap between the equal and optimal phase shifts is large, so an IRS is quite advantageous by phase optimization.

Figure 2 illustrates the impact of correlated fading on the outage probability. The results correspond to the scenario of equal phase shifts ($\theta_p = \pi/4, \forall n$). When a direct channel exists, the channel correlation does not affect significantly the outage probability because the existence of the direct channel has a stronger effect and does not allow the cascaded channel to exhibit its contribution. However, in the case of direct signal blockage, the correlation is clearly shown in terms of a severe impact on the outage probability. The spatial correlation has high impacts when only the indirect channel exists.

V. CONCLUSION

We derived the outage probability of IRS-supported SISO communication systems under spatially correlated Rayleigh fading. Actually, for a given phase-shift matrix, we achieved to obtain its expression in closed-form by means of its channel statistics. Hence, we shed light on the impact of IRS correlation and its reflect beamforming matrix. The tightness of the analytical results was verified by Monte-Carlo simulations. The outage probability is more sensitive to the phase-shift matrix when the direct channel is blocked while the correlation has a significant impact under the same conditions.

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