Queue-Aware STAR-RIS Assisted NOMA Communication Systems

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Abstract—Simultaneously transmitting and reflecting reconfigurable intelligent surfaces (STAR-RISs) are gaining great attention for their ability to achieve full-space coverage. In this paper, the queue-aware STAR-RIS assisted non-orthogonal multiple access (NOMA) communication system is investigated to ensure system stability. To tackle the challenge of infinite time periods for stability, the long-term stability-oriented problem is reformulated as a per-slot queue-weighted sum rate (QWSR) maximization problem using Lyapunov drift theory. Particularly, the allocated rate weight for each user is determined by the corresponding data queue at the base station (BS). By jointly optimizing the NOMA decoding order, the active beamforming coefficients at the BS, and the passive transmission and reflection coefficients at the STAR-RIS, three STAR-RIS operating protocols are considered, namely energy splitting (ES), mode switching (MS), and time switching (TS). An equivalent-combined channel gain based scheme is proposed to obtain the desired decoding order. For ES, the highly coupled and non-convex problem is solved iteratively and alternatively by invoking the blocked coordinate descent and the successive convex approximation methods. This approach is further expanded to a penalty-based two-loop algorithm to solve the binary amplitude constrained problem for MS. For TS, the problem is decomposed into two subproblems, each of which is solved similarly as ES. Simulation results show that: i) our proposed STAR-RIS assisted NOMA communication achieves superior performance to the conventional schemes; ii) the reformulated QWSR maximization problem is proven to ensure the system stability; and iii) TS performs best in both the QWSR and the average queue length.

Index Terms—Lyapunov drift theory, NOMA, queue stability, STAR-RISs.

I. INTRODUCTION

RECONFIGURABLE intelligent surfaces (RISs) [2], which are also known as intelligent reflecting surfaces (IRSs) [3], [4], or passive holographic MIMO surfaces (HMIMOS) [5], have emerged as a promising and effective technology in the development of wireless communications. Connected with intelligent controllers (e.g., field-programmable gate array (FPGA)), the two-dimensional (2D) RIS is able to adaptively adjust the phase and even the amplitude of the incident signals. By changing the responses of the reconfigurable elements on the surface, the RIS reconfigures the propagation of incident wireless signals and realizes the smart radio environment (SRE) [6]. However, the conventional RIS in most existing studies is reflecting-only [7], [8], which leads to half-space coverage. To tackle this problem, a new technique called simultaneously transmitting and reflecting RIS (STAR-RIS) [9], [10] has received extensive attention from both academia and industry. With STAR-RISs, the wireless signals are not only reflected into the same side of the incident signals, but also transmitted into the opposite side. Consequently, the STAR-RISs extend half-space coverage into full-space coverage. The unique differences between the STAR-RISs and conventional reflecting-only RISs have been discussed in [11] from the perspectives of hardware design, physics principles, and communication system design. Regarding practical implementations, some STAR-RIS-like prototypes have been created utilizing metasurfaces [12], [13]. Each element in [12] is equipped with a parallel resonant LC tank and tiny metallic loops to achieve the necessary electric and magnetic surface reactance. Reconfigurable elements in [13] supporting magnetic currents are like ice cubes in water, while the relayed substrates of STAR-RISs are transparent for wireless signals at the operating frequency.

On the other hand, the non-orthogonal multiple access (NOMA) technique has been already proposed for the third-generation partnership projects long-term evolution advanced (3GPP-LTE-A) [14]. It constitutes a promising technology for addressing the large-scale access challenges in 5G and beyond networks by allowing several users to access the wireless network within the same orthogonal resource block (RB). By doing this, more significant bandwidth efficiency is enhanced and more massive connections are provided than the conventional orthogonal multiple-access (OMA) techniques [15]. The core idea of power-domain NOMA is to ensure that multiple users with different power levels can be served within a given time/frequency RB, by employing superposition coding (SC) techniques at the transmitter and the successive interference cancellation (SIC) at the receiver. The

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employed NOMA scheme will achieve more significant gains if the paired users have distinct channel conditions [16]. Fortunately, on the one hand, a flexible NOMA can be achieved by employing STAR-RISs since STAR-RISs have the capability of adjusting wireless channel gains for different NOMA users. On the other hand, STAR-RIS assisted communications also benefit from the NOMA technique since NOMA can achieve high spectral efficiency and provide massive connections for STAR-RIS communications. Thus, by leveraging the potential benefits of effectively integrating the STAR-RIS with NOMA-assisted communications in this paper, we could explore the performance improvement offered by the aforementioned advantages of these two technologies.

A. Related Works

1) RIS-Enabled NOMA Communications: There have been multiple studies focused on conventional reflecting-only RIS-enabled NOMA communications. A simple design of RIS assisted NOMA downlink transmission was proposed in [17]. In [18], the RIS-aided multiple-input single-output (MISO) NOMA system was investigated to maximize the weighted sum-rate (WSR) of both the primary and the IoT transmissions. In [21], the authors compared the RIS-assisted NOMA scheme and the RIS-assisted zero-forcing beamforming (ZFBF) transmission scheme and then identified the best scenarios to adopt NOMA or ZFBF. In contrast to the alternating optimization techniques, the authors in [22] conceived a novel smart reconfigurable terahertz (THz) multiple-input multiple-output (MIMO)-NOMA framework and proposed a novel multi-agent deep reinforcement learning algorithm by exploiting the decentralized partially-observable Markov decision process. As a step further, in [23], both a deep learning approach and a reinforcement learning approach were developed for the RIS-Assisted NOMA networks to maximize the effective throughput of the entire transmission period.

2) STAR-RIS Assisted Communications: Motivated by the full coverage of STAR-RISs, employing STAR-RISs into wireless communications has attracted some initial research interest. As stated in [10] and [24], there are three STAR-RIS operating protocols including energy splitting (ES) protocol, mode splitting (MS) protocol, and time switching (TS) protocol. In [10], a penalty-based iterative algorithm was proposed for the ES protocol of the STAR-RIS under a two-user downlink MISO scenario, which was also extended to the MS protocol. In [24], a path-following based technique was first developed to handle the non-convex problem and design the beamforming and the transmitting and reflecting coefficients (TARCs) in an alternating manner. Moreover, the integration of STAR-RISs and NOMA has also drawn much attention from researchers. For example, both NOMA and OMA cases incorporating the STAR-RIS technology were considered in [9] and [25]. In [9], a two-user sum coverage

B. Motivation and Contributions

Note that most existing research focused on sum rate maximization problems [18], [25] or the constant WSR maximization problems [20], [24], and ignored the system stability issues. It is a fact that in some practical situations in wireless communications, there would be such a case that a large amount of data packets arrive at the same node suddenly. However, limited by the constrained communication resources, these data cannot be transmitted instantaneously and suddenly. As a result, some data have to remain at the BS and be buffered in the data queue, waiting for the transmission opportunity [27], [28]. In addition, the length of data queue reveals the amount of accumulated data waiting for transmission, which mirrors the transmission urgency. That is, a longer data queue is expected to contribute to a large delay for the data transmission, thus a higher priority to be delivered. Service unavailability due to large delay caused by the unstable data queues will lead to severe consequences. Therefore, it is of vital importance to concentrate on the stability of the queueing system [29], which requires that the data queue length cannot go to infinity over a long period of time. However, the aforementioned studies cannot be applied to the queueing scenarios since they may cause that some queues grow unexpectedly infinitely while other queues are always empty. This consequence reflects the unreasonable allocation and underutilization of wireless resources. In fact, both NOMA and STAR-RIS technologies are supposed to help serve more users and maximize the utilization of scarce resources. To the best of our knowledge, there are no efforts devoted to the system stability problem when employing the STAR-RIS into wireless communications. The stability-oriented STAR-RIS assisted NOMA communication problem is non-trivial to be solved owing to the following challenges.

- First, for the considered stability-oriented problem, it requires that the communication system evolves over infinite time periods, and the adjacent time periods are coupled together by the changes in data queues, hence imposing a high level of difficulty.
- Second, due to the introduction of the transmission coefficients, the optimization of the STAR-RIS-aided system becomes much more challenging than that of the conventional reflecting-only system. The difficulty lies in that STAR-RIS requires optimizing both passive transmission and reflecting beamforming coefficients, which are coupled together by energy conservation. Resource allocation is therefore further complicated.
- Third, the NOMA decoding order among the users is determined by not only the active beamforming
coefficients (ABCs) at the BS, but also the passive transmission and reflection coefficients (PTRCs) at the STAR-RIS, which leads to a highly-coupled problem.

Against the above challenges, we jointly investigate the ABCs at the BS and the PTRCs at the STAR-RIS to stabilize the considered queueing communication system. Note that the conference version [1] of this work has obtained some preliminary results, where only the ES protocol was considered and the direct link between the BS and users was blocked. In this paper, a general scenario with three operating protocols is studied. The main contributions of this paper are summarized as follows.

- We consider a queue-aware STAR-RIS assisted downlink MISO-NOMA communication system, where a couple of data queues maintained at the BS are pending to be sent to the users via the STAR-RIS-aided transmission and reflection links. To handle the challenge of the infinite time duration involved in the system stability, we reformulate the long-term stability-oriented problem to maximize the per-slot queue-weighted sum rate of users, where three operating protocols of the STAR-RIS, namely ES, MS, and TS, are considered.
- For acquiring the desired NOMA decoding order, we propose an equivalent-combined channel gain based scheme, which avoids the factorial exhaustive search required to obtain the optimal decoding order.
- For the ES protocol, to handle the intrinsically coupled non-convex problem, we explore the blocked coordinate descent (BCD) method and the successive convex approximation (SCA) method to iteratively optimize the lower bound of the original problem. We also prove that the rank of the obtained active beamforming vector always satisfies the rank-one constraint.
- For the MS protocol, we extend the algorithm for ES into a two-loop penalty-based iterative algorithm to deal with the binary amplitude constrained problem. For the TS protocol, we decompose the optimization problem into two subproblems, each of which can be solved with the same method adopted for ES.
- The numerical results reveal that: i) the STAR-RIS assisted NOMA communications outperform the conventional reflection-only RIS-enabled NOMA communications and the STAR-RIS enabled OMA communications, which verifies the effectiveness of integrating the STAR-RIS with NOMA techniques; ii) the reformulated QWSR maximization problem is able to ensure the system stability; and iii) among three protocols, TS achieves superior performance with respect to both the QWSR and the average queue length performance.

C. Organizations

The rest of this paper is organized as follows. The system model and problem formulation are demonstrated in Section II. Then the Lyapunov drift based stability-driven optimization problem is reformulated in Section III. In Section IV, an efficient algorithm is developed for determining the active beamforming and passive transmission and reflection coefficients for ES. Section V extends the proposed solution for the MS and TS protocols. Following this, the simulation results are provided in Section VI. Finally, this paper is concluded in Section VII.

Notations: Scalars, vectors, and matrices are denoted by lower-case letters, bold-face lower-case letters, and upper-case letters, respectively. Real-valued and complex-valued matrices with the dimension of \( N \times M \) are denoted by \( \mathbb{R}^{N \times M} \) and \( \mathbb{C}^{N \times M} \), respectively. \( \mathbf{I}_N \) is an \( N \times N \) identity matrix. The rank and the trace of matrix \( \mathbf{A} \) are denoted by \( \text{Rank}(\mathbf{A}) \) and \( \text{Tr}(\mathbf{A}) \). The diagonal elements of matrix \( \mathbf{A} \) are denoted by \( \text{Diag}(\mathbf{A}) \). The positive semidefinite matrix \( \mathbf{A} \) is represented by \( \mathbf{A} \succeq 0 \). Besides, \( \mathbf{A}^T \), \( \mathbf{A}^H \), and \( \text{diag}(\mathbf{a}) \) denote the transpose, the conjugate transpose, and the diagonal matrix of vector \( \mathbf{a} \), respectively.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a STAR-RIS assisted downlink communication scenario, where an \( N \)-antenna BS is sending data to \( K \) single-antenna users, with the aid of an \( M \)-element STAR-RIS. A data server is installed on the BS to buffer the pending-transmission data. Let \( \mathcal{M} = \{1, 2, \ldots, M\} \) denote the set of the STAR-RIS elements. All the users are denoted by the set \( \mathcal{K} = \{1, 2, \ldots, K\} \). Meanwhile, the users are separated into two groups according to their locations. Denote the group located on the one side of the STAR-RIS by the reflection set \( \mathcal{R} \), and denote the other group located on the opposite side of the STAR-RIS by the transmission set \( \mathcal{T} \).

As depicted in Fig. 1, the direct communication links between the BS and the users are considered. Note that the case where some users have direct links while other users do not can be treated as a special case of this work. The time is split into time slots and \( t \) is the duration of one time slot. Let \( t \) denote the slot index. The channel from the BS to the STAR-RIS is denoted by \( \mathbf{G}(t) \in \mathbb{C}^{M \times N} \), while the channel from the STAR-RIS to the users is denoted by \( \mathbf{v}_k(t) \in \mathbb{C}^{1 \times M}, \forall k \in \mathcal{K} \). Furthermore, the channel of the direct link from the BS to user \( k \) is denoted by \( \mathbf{r}_k(t) \in \mathbb{C}^{1 \times N}, \forall k \in \mathcal{K} \). The users located within both sides of the STAR-RIS

![Fig. 1. Queue-aware STAR-RIS assisted NOMA communication system.](image-url)
are assumed to be static or moving slowly in the considered communication systems [18]. In this case, the channels would remain unchanged within the channel coherent time containing several slots then vary in the next coherent time. Thus, it is supposed that all these channels are narrow-band quasi-static fading. Here, all channel state information is assumed to be acquired perfectly [30]. It should be highlighted that numerous effective channel estimate strategies for traditional RISs have been put forth [31], [32], which can also be applied to STAR-RISs. For instance, for TS protocol, the CSI of transmission user and reflection users can be consecutively obtained via existing methods. Moreover, CSI estimation for ES protocol of STAR-RISs has been proposed in [30]. Indeed, the development of more efficient techniques for MS protocol to simultaneously acquire the CSI of users is an interesting topic for future works.

### A. Signal Model of the STAR-RIS

For the STAR-RIS, let

\[ \Theta_\tau(t) = \text{diag}(\sqrt{\beta_1^t(t)}e^{j\phi_1^t(t)}, \sqrt{\beta_2^t(t)}e^{j\phi_2^t(t)}, \ldots, \sqrt{\beta_M^t(t)}e^{j\phi_M^t(t)}) \]

be the reflection-coefficient matrix, and

\[ \Theta_\tau(t) = \text{diag}(\sqrt{\beta_1^t(t)}e^{j\phi_1^t(t)}, \sqrt{\beta_2^t(t)}e^{j\phi_2^t(t)}, \ldots, \sqrt{\beta_M^t(t)}e^{j\phi_M^t(t)}) \]

be the transmission-coefficient matrix. Both the amplitude and the phase shift of each STAR-RIS element are assumed to be adjusted continuously to establish the best performance,\(^1\) that is, \(\theta_m^t(t) \in [0, 2\pi]\), \(\beta_m^t(t) \in [0, 1]\), \(\forall s \in \{r, t\}, m \in \mathcal{M}\). Considering the energy conservation law, it should be met that \(\beta_m^r(t) + \beta_m^t(t) = 1\). Note that the independent phase-shift model is practically suitable for the semi-passive STAR-RIS, while the coupled phase-shift model is applicable for the purely passive STAR-RIS [19]. The results obtained in this work could give an upper bound to the associated coupled phase-shift model for the purely passive STAR-RIS.

In addition, according to the different constraints on the amplitude of each STRA-RIS element, there are three protocols for operating the STAR-RIS in wireless communication systems [10], [11]:

1. **Energy Splitting (ES) protocol**, where all elements operate in the simultaneous reflection and transmission mode, that is, the feasible set for the amplitude coefficients

\[ \mathcal{F}_\beta^E = \{\beta_m^r(t), \beta_m^t(t) : \beta_m^r(t), \beta_m^t(t) \in [0, 1], \beta_m^r(t) + \beta_m^t(t) = 1\} \]

Since all PTRCs are optimized continuously, a high degree of flexibility is enabled to design the wireless communication system. But what follows is the relatively high overhead to exchange the configuration information between the BS and the STAR-RIS, owing to numerous design variables.

2. **Mode Switching (MS) protocol**, where some of the elements operate in the full reflection mode, while the other elements operate in the full transmission mode, namely

\[ \mathcal{F}_\beta^M = \{\beta_m^r(t), \beta_m^t(t) : \beta_m^r(t), \beta_m^t(t) \in [0, 1], \beta_m^r(t) + \beta_m^t(t) = 1\}. \]

Notice that this protocol can be regarded as a special case of the ES protocol, with the amplitude coefficients of transmission and reflection restricted as binary values. Thus, this scheme is much easier to be implemented though at the sacrifice of some performance loss in comparison with the ES protocol.

3. **Time Switching (TS) protocol**, where all elements are switched to operate in the full reflection mode and the full transmission mode periodically in different time periods, denoted by the R period and T period. Denote the feasible amplitude set of the R period by

\[ \mathcal{B}_\beta^R = \{\beta_m^r(t), \beta_m^t(t) : \beta_m^r(t) = 1, \beta_m^t(t) = 0, \forall m \in \mathcal{M}\} \]

and that of the T period by

\[ \mathcal{B}_\beta^T = \{\beta_m^r(t), \beta_m^t(t) : \beta_m^t(t) = 0, \beta_m^r(t) = 1, \forall m \in \mathcal{M}\}. \]

We have

\[ \mathcal{F}_\beta^{TS} = \{\beta_m^r(t), \beta_m^t(t) : \beta_m^r(t), \beta_m^t(t) \in \mathcal{B}_\beta^R \cup \mathcal{B}_\beta^T\}. \]

There are constraints on the time percentage allocated to the R period, \(\alpha^r(t)\) and to the T period, \(\alpha^t(t)\), shown as

\[ \alpha^r(t) + \alpha^t(t) = 1, \alpha^r(t), \alpha^t(t) \in [0, 1]. \]

The optimization for the TS protocol is much simpler, since the coupled constraint on PTRCs \((\beta_m^r + \beta_m^t = 1)\) disappears. However, more precise synchronization is needed to perform the switching, leading to a higher implementation complexity.

### B. STAR-RIS Assisted NOMA Communication Model

As illustrated in Fig. 2, we adopt a simple transmission protocol proposed in [33], where each channel coherent time is divided into a channel estimation phase, a computing and control phase, and a subsequent data transmission phase. At first, channel estimation is performed in the first several slots. Then, based on the estimated channels, the BS computes the optimal transmit beamforming of the BS, as well as the reflection and transmission coefficients of the STAR-RIS, which are fed back to the STAR-RIS controller from the BS. Finally, data is transmitted to each other using the optimized results, with the duration of the data transmission phase.

For users who are static or moving slowly, if the Doppler shift is 1 Hz, the channel coherent time is 1 s. From [32],

\[ K + M + \max(N - 1, [(K - 1)M]/N) \]

pilot symbols are required for perfectly recovering all channel coefficients. Since 14 symbols can be transmitted within 1 ms in 5G [34], the overhead of channel estimation for RIS is

\[ (K + M + \max(N - 1, [(K - 1)M]/N))/14, \]

which is computed as 10 ms when \(K = 10, M = 40, \) and \(N = 4\). In addition, according to existing research results, the time duration required for the reconfiguration is 0.22 ms - 7 ms depending on the number of RIS elements [35]. Therefore, it is practical to assume that the STAR-RIS reflection and transmission coefficients are reconfigured only once at the beginning of each channel coherent time and then remain fixed until next coherent time. Since they are relatively smaller than channel coherent time, thus are neglected when performing the data transmission.

Let \(w_k(t) \in \mathbb{C}^{N \times 1}\) and \(x_k(t)\) denote the active beamforming vector and the information-bearing symbol for user \(k \in \mathcal{K}\) at the BS, respectively. Let \(P_{\text{max}}\) denote the maximum

![Fig. 2. Illustration of the transmission protocol.](image-url)
transmitted power of the BS, then we have
\[ \sum_{k \in \mathcal{K}} \mathbf{w}_k^H(t) \mathbf{w}_k(t) \leq P_{\text{max}}. \] (1)

All users are grouped together to form NOMA pairs. The received signal at user \( k \in \mathcal{K} \) is
\[ y_k(t) = (\mathbf{v}_k(t) \Theta_{s_k}(t) \mathbf{G}(t) + \mathbf{g}_k(t)) \sum_{i \in \mathcal{K}} \mathbf{w}_i(t) x_i(t) + n_k(t), \] (2)
where \( s_k \in \{r, t\} \) indicates one of the half spaces of the STAR-RIS where user \( k \) is located, and \( s_k = r \) if \( k \in \mathcal{R} \) while \( s_k = t \) if \( k \in \mathcal{T} \). \( \mathbb{E}[|x_i(t)|^2] = 1 \), and \( n_k(t) \in \mathcal{CN}(0, \sigma^2) \) is the additive white Gaussian noise at user \( k \).

To eliminate the interference efficiently, SIC is utilized at each user according to the NOMA principle. Let \( o_k(t) \) denote the decoding order of user \( k \in \mathcal{K} \). The smaller the decoding order \( o_k(t) \) is, the earlier its signal is decoded, and the more interference this user will suffer. By treating the signals of users with a larger decoding order as interference, the achievable signal-to-interference-plus-noise ratio (SINR) for user \( k \) decoding its own signal is
\[ \text{SINR}_{kk}(t) = \frac{\| (\mathbf{v}_k(t) \Theta_{s_k}(t) \mathbf{G}(t) + \mathbf{g}_k(t)) \mathbf{w}_k(t) \|^2}{\sum_{i:o_k(t) < o_j(t)} \| (\mathbf{v}_i(t) \Theta_{s_i}(t) \mathbf{G}(t) + \mathbf{g}_i(t)) \mathbf{w}_i(t) \|^2 + \sigma^2}. \] (3)

Moreover, the SINR achieved by decoding the signal of user \( k \) at user \( j \), the one with a larger decoding order (i.e., \( o_k(t) < o_j(t) \)), is
\[ \text{SINR}_{kj}(t) = \frac{\| (\mathbf{v}_j(t) \Theta_{s_j}(t) \mathbf{G}(t) + \mathbf{g}_j(t)) \mathbf{w}_k(t) \|^2}{\sum_{i:o_k(t) < o_j(t)} \| (\mathbf{v}_i(t) \Theta_{s_i}(t) \mathbf{G}(t) + \mathbf{g}_i(t)) \mathbf{w}_i(t) \|^2 + \sigma^2} \] (4)

Define \( R_k \) as the achievable rate of user \( k \) decoding its own signal. For successful SIC operations, it is crucial that \( R_k \) is limited by the minimum of the achievable rates at which user \( j \) as well as user \( k \) can decode the signal of user \( k \) [36]. To this end, the following conditions should be satisfied for the SIC to be applied successfully, given by
\[ R_k(t) \leq \min \{ \log_2(1 + \text{SINR}_{kk}(t)), \log_2(1 + \text{SINR}_{kj}(t)) \}, \] (5)

In addition, more wireless resources should be allocated to the user with a lower decoding order to keep itself a reasonable communication rate, since this user is suffering more interference than the one with a higher order. Thus, for a given decoding order, the following \( K - 1 \) rate fairness conditions should be satisfied,
\[ \| (\mathbf{v}_i(t) \Theta_{s_i}(t) \mathbf{G}(t) + \mathbf{g}_i(t)) \mathbf{w}_i(t) \|^2 \geq \| (\mathbf{v}_j(t) \Theta_{s_j}(t) \mathbf{G}(t) + \mathbf{g}_j(t)) \mathbf{w}_j(t) \|^2, \] if \( o_k(t) < o_j(t) \), \( \forall i \in \mathcal{K} \), (6)

where \( s_i = r \) if \( i \in \mathcal{R} \) and \( t \), otherwise.

C. Queue Dynamics and Stability

Different from the existing studies ignoring the data arrival process, we consider a bursty data arrival at the BS. Let \( \mathbf{A}(t) = \{ A_k(t), \forall k \in \mathcal{K} \} \) be the random arrival from the data servers. Assume that \( \mathbf{A}(t) \) is independent and identically distributed over time slots, where \( E[A_k(t)] = \lambda_k \) and \( \lambda_k \) is the average arrival rate of the data transmitted to user \( k \). A data queue is maintained at data server of the BS for sending to the corresponding user with the serving rate being \( R_k(t) \). Let \( Q_k(t) \) denote the queue length for user \( k \) at the current slot, then we have the following queue dynamics for user \( k \) at the next slot:
\[ Q_k(t+1) = [Q_k(t) - R_k(t) \tau]_+ + A_k(t) \tau, \forall k \in \mathcal{K}, \] (7)
where \([ \cdot ]_+ \) is equal to the number enclosed in the parenthesis if this number is nonnegative and to 0, otherwise.
The queue vector in slot \( t \) for all users is denoted by \( \mathbf{Q}(t) = \{ Q_k(t), \forall k \in \mathcal{K} \} \).

Definition 1: (Queue Stability): A queue \( Q_k(t) \) is strongly stable if
\[ \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}(Q_k(t)) < \infty. \] (8)
The system is said to be stable if all the queues in the system are strongly stable. To ensure that the system is stabilizable, the capacity region following [37] is defined as follows:

Definition 2 (Capacity Region): The capacity region is defined as the closure of the set of all input rate vectors \( \lambda \triangleq \{ \lambda_k \} \) stabilizable under some rate allocation algorithm.

D. Problem Formulation

Our goal is to stabilize the system for any arrival rate vector \( \lambda \) strictly interior to the capacity region, by jointly optimizing the active beamforming coefficients (ABCs) at the BS, the passive transmission and reflection coefficients (PTRCs) at the STAR-RIS, and the NOMA decoding order. From this viewpoint, this problem can be formulated as
\[ \text{find } \{ \mathbf{w}_k(t), \Theta_{s_k}(t), o_k(t), \alpha^*(t) \} \] (9)
s.t. \[ Q_k(t), \forall k \in \mathcal{K} \] is strongly stable, \[ \sum_{k \in \mathcal{K}} \mathbf{w}_k^H(t) \mathbf{w}_k(t) \leq P_{\text{max}}, \] (11) \[ \beta^r_{m}(t), \beta^t_{m}(t) \in \mathcal{F}_\beta^X, \] (12) \[ \theta^r_{m}(t), \theta^t_{m}(t) \in [0, 2\pi], \] (13) \[ R_k(t) \leq \min \{ \log_2(1 + \text{SINR}_{kk}(t)), \log_2(1 + \text{SINR}_{kj}(t)) \}, \] (14) \[ \| (\mathbf{v}_i(t) \Theta_{s_i}(t) \mathbf{G}(t) + \mathbf{g}_i(t)) \mathbf{w}_i(t) \|^2 \geq \| (\mathbf{v}_j(t) \Theta_{s_j}(t) \mathbf{G}(t) + \mathbf{g}_j(t)) \mathbf{w}_j(t) \|^2, \] if \( o_k(t) < o_j(t), \forall k, j \in \mathcal{K}, \) (15) \[ \alpha^t(t) + \alpha^r(t) = 1, \alpha^t(t), \alpha^r(t) \in [0, 1], \] (16)
where \( X \in \{ \text{ES, MS, TS} \} \) represents the operating protocols of the STAR-RIS. Constraint (10) guarantees that all data queues are strongly stable to stabilize the system.
Constraint (11) is the total transmission power limited at the BS. Constraint (12) indicates the restriction for energy conservation. Constraint (13) is the reflection and transmission phase shift for each element of the STAR-RIS. Moreover, constraint (14) provides the decoding order conditions for SIC, and constraint (15) ensures the rate fairness among users. Constraint (16) is related to the time allocation variables for TS, which is invalid for ES and MS.

Remark 1: Since the STAR-RIS is nearly-passive, the optimizations are conducted at the BS. The derived reflection and the transmission coefficients are fed back to the STAR-RIS through a smart controller, which enables communication and coordination with the BS via a separate wireless link. In this case, STAR-RISs can be realized simply enough by only needing to receive the configuration signals and set up the network accordingly [6].

Remark 2: Note that the problem above is non-trivial to be solved since the stability requirement in constraint (10) for each queue demands the evolution over an infinite time horizon, and every two adjacent time slots are coupled by the data queue length revealed by (7).

III. STABILITY-DRIVEN REFORMULATED OPTIMIZATION

To address the infinite-horizon time-coupled problem, in this section, we eliminate the coupling between adjacent time slots by exploiting the Lyapunov drift approach [37], and then divide the queue stability into the effort of minimizing the drift in each time slot so that the stability constraints are met in the long term.

To be more specific, we adopt a widely-used quadratic Lyapunov function [38], which increases quadratically with the queue length, given by

\[ L(Q(t)) = \sum_{k \in K} (Q_k(t))^2, \tag{17} \]

and then the Lyapunov drift in slot \( t \) is given by

\[ \Delta(Q(t)) = \mathbb{E}(L(Q(t+1)) - L(Q(t))). \tag{18} \]

Taking the square operation on both sides of (7), we have

\[ (Q_k(t+1))^2 \leq (Q_k(t))^2 + (R_k(t))^2 + (A_k(t))^2 \]
\[ + 2Q_k(t)A_k(t) + 2Q_k(t)R_k(t) \tau. \tag{19} \]

Sum over \( k \) on both sides of (19) and rearrange the items, then combine with (18), we have

\[ \Delta(Q(t)) \leq B_o + 2\sum_{k \in K} Q_k(k)A_k(t)\tau - 2\sum_{k \in K} Q_k(t)R_k(t)\tau, \]
\[ \leq B_o + 2\sum_{k \in K} Q_k(k)\lambda_k\tau - 2\sum_{k \in K} Q_k(t)R_k(t)\tau, \tag{20} \]

where \( B_o \) is a bounded constant, given by

\[ B_o = \sum_k \mathbb{E}\left((R_{\max}(t))^2\right) + \sum_k \mathbb{E}\left((A_k(t))^2\right) \]
\[ \text{with } R_{\max}(t) = \max_k R_k(t). \]

We give the following theorem to guarantee the queue stability.

**Theorem 1:** The queue stability condition in (8) holds if the Lyapunov drift \( \Delta(Q(t)) \) satisfies the inequality in (20).

**Proof:** Please refer to Appendix A.

To stabilize the system, we minimize the upper bound of the Lyapunov drift (the right-hand side (RHS) of (20)) in each time slot [29], which is equivalent to maximizing the term \( \sum_k Q_k(t)R_k(t) \), i.e., the queue-weighted sum rate (QWSR) in each time slot. In this case, the stability-driven optimization for the STAR-RIS assisted NOMA communication can be cast as the following QWSR-optimal designs. We omit the time index \( t \) in what follows for notation simplicity.

**Problem 1:** (QWSR-optimal STAR-RIS-assisted NOMA Communication Problem)

\[
\begin{align*}
\max_{\{\Theta_s, w_k, o_k\}} & \sum_{k \in K} Q_k R_k \quad \max_{\{\Theta_s, w_k, o_k, \alpha\}} & \sum_{k \in K} \alpha^s Q_k R_k^s \tag{21a} \\
\text{s.t.} & \quad (11) - (16), \tag{21b}
\end{align*}
\]

where the term \( R_k^s \), the parameter \( \alpha^s \), \( \forall s \in \{r, t\} \) and the constraint (16) are only valid for the TS protocol. Furthermore, the achievable rate of user \( k \), \( R_k^s, \forall s \in \{r, t\} \) depends on the time period in which the STAR-RIS is working in under the TS protocol. Here, \( s = r \) if the STAR-RIS is in the R period, and \( s = t \) otherwise.

Remark 3: The queuing system is stable as long as the average arrival rate vector is within the system stability region. Moreover, the positive queue length based weight can be regarded as the urgency/priority of the user in resource allocation, which is formed in the media access control (MAC) layer to achieve certain fairness purposes.

However, the optimization problem (21) is still challenging to be solved directly for the following reasons. Firstly, there are multiple highly coupled variables (i.e., \( \{w_k\}, \{\Theta_s\} \)) and the non-convex objective and constraint (15). Furthermore, the decoding order needs to be determined, which is influenced by not only the ABCs at the BS, but also the PTRCs at the STAR-RIS. In the following, we propose an efficient approach to address this challenge.

IV. PROPOSED SOLUTION FOR THE ES PROTOCOL

In this section, we focus on solving problem (21) for the ES protocol. For a given SIC decoding order, we first transform the problem into a solvable form. Then, the intrinsically coupled problem is decomposed into two subproblems, which are optimized separately and alternatively.

For the \( K \) users, \( K! \) times exhaustive searching are required to obtain the optimal decoding order, which is reasonable for a few users, but is not scalable for a large number of users. To avoid the exhaustive search, we propose an equivalent-combined channel gain based scheme to acquire the desired NOMA decoding order.

**Lemma 1:** Under the given ABCs \( \{w_k\} \) at the BS and the PTRCs \( \{\Theta_s\} \) at the STAR-RIS, sorting the users according to

\[ O_1 \leq O_2 \leq \cdots \leq O_K, \tag{22} \]

where \( O_k \) is the equivalent-combined channel gain from the BS to user \( k \), given by

\[ O_k = \left( \frac{[w_k \Theta_s G + g_s] w_k}{|w_k|^2 \sigma^2} \right)^2, \tag{23} \]

then the optimal decoding order is determined by \( o_k = k \).
Thus, for a given decoding order \( o_k \), problem (21) for ES can be detailed as
\[
\begin{align*}
\max_{\{d_k, w_k, \beta_k\}} & \quad \sum_{k \in K} Q_k R_k \\
\text{s.t.} & \quad \beta_m + \beta_n = 1, \beta_m, \beta_n \in [0, 1], \quad (11), (13) - (15). 
\end{align*}
\] (24a)

To facilitate the design, we define the reflection- and transmission-coefficient vectors as \( d_k = [\sqrt{\beta_1} e^{j\theta_1}, \sqrt{\beta_2} e^{j\theta_2}, \ldots, \sqrt{\beta_M} e^{j\theta_M}]^T, \forall s \in \{t, r\} \), which means \( \Theta_{sk} = \text{diag}(d_{sk}) \). In this case, we have
\[
\|v_k \Theta_{sk} G + g_k w_k\|^2 = |d_{sk}^o H_k w_k|^2,
\]
where \( H_k = [\text{diag}(v_k) G] \). Furthermore, we define
\[
D_s = d_s d_s^H, \forall s \in \{t, r\}, \text{which satisfies } D_s \succeq 0 \text{ and } \text{Rank}(D_s) = 1. \text{ The diagonal elements of } D_s \text{ are chosen to satisfy}
\]
\[
\begin{align*}
d_{sk}^o &= \beta^* \text{, } \forall s \in \{t, r\} \text{,}\quad H_{sk} = \text{diag}(d_{sk}).
\end{align*}
\]
The diagonal elements of \( D_s \) are chosen to satisfy
\[
\begin{align*}
1 &= \text{Tr}(W_k H_k^H D_s H_j), \\
I_{kj} &= \sum_{i : o_i < o_k} \text{Tr}(W_i H_i^H D_s H_j) + \sigma^2,
\end{align*}
\] (25)
\[
\begin{align*}
1 &= \sum_{i : o_i > o_k} \text{Tr}(W_i H_i^H D_s H_j) + \sigma^2,
\end{align*}
\] (26)

we can rewrite the SINR as
\[
\text{SINR}_{kj} = \frac{1}{S_{kj} I_{kj}}, \quad o_k \leq o_j. \quad (27)
\]

Till now, problem (24) for ES can be reformulated as
\[
\begin{align*}
\max_{\{d_k, w_k, \beta_k\}} & \quad \sum_{k \in K} Q_k R_k \\
\text{s.t.} & \quad \frac{1}{S_{kj}} \leq \text{Tr}(W_k H_k^H D_s H_j), \quad \text{if } o_k \leq o_j, \forall k, j \in K, \\
& \quad I_{kj} \geq \sum_{i : o_i < o_k} \text{Tr}(W_i H_i^H D_s H_j) + \sigma^2, \\
& \quad \text{if } o_k \leq o_j, \forall k, j \in K, \\
& \quad R_k \leq \min\left\{\log_2 (1 + \frac{1}{S_{kk} I_{kk}}), \log_2 (1 + \frac{1}{S_{kj} I_{kj}})\right\}, \\
& \quad \text{if } o_k < o_j, \forall k, j \in K, \\
& \quad \sum_{k \in K} \text{Tr}(W_k) \leq P_{max}, \\
& \quad \text{Tr}(W_k H_k^H D_s H_i) \geq \text{Tr}(W_j H_j^H D_s H_i), \\
& \quad \text{if } o_k < o_j, \forall i \in K, \\
& \quad \beta_m + \beta_n = 1, \beta_m, \beta_n \in [0, 1], \forall m \in M, \\
& \quad \text{Diag}(D_s) = [\beta^*, 1], \forall s \in \{t, r\}, \\
& \quad \text{Rank}(D_s) = 1, \forall s \in \{t, r\}, \\
& \quad \text{Rank}(W_k) = 1, \forall k \in K, \\
& \quad D_s \succeq 0, \forall s \in \{t, r\}, \\
& \quad W_k \succeq 0, k \in K.
\end{align*}
\] (28a)

In the problem above, the equality of constraints (28b) and (28c) can be always guaranteed to make the problems (28) and (24) equivalent. When \( k = j \), if any strict inequality in constraints (28b) and (28c) holds, we can adjust it by decreasing the values of \( S_{kj} \) and \( I_{kj} \) so that the equality is attained, which increases the value of the objective function. When \( k \neq j \), based on constraint (28d), we can do the same operations but without changing the objective function’s value if initially \( \log_2 (1 + \frac{1}{S_{kk} I_{kk}}) \leq \log_2 (1 + \frac{1}{S_{kj} I_{kj}}) \), or with increasing the objective function’s value if initially \( \log_2 (1 + \frac{1}{S_{kk} I_{kk}}) > \log_2 (1 + \frac{1}{S_{kj} I_{kj}}) \). As a result, the equivalence between problems (28) and (24) has been established.

However, it is still a non-convex optimization problem due to the non-convex objective and constraints (28d), (28h)-(28i), as well as the non-convex highly coupled terms in constraints (28b)-(28c) and (28f). In this case, to overcome the challenge, we decompose this problem into two subproblems in terms of the active beamforming optimization at the BS and the passive beamforming optimization at the STAR-RIS, which are optimized separately and iteratively.

A. Active Beamforming Optimization at the BS

For any given passive beamforming coefficients \( \{D_s\} \), or saying \( \{d_s\}, \forall \in \{t, r\} \), the active beamforming optimization at the BS can be rewritten as
\[
\begin{align*}
\max_{\{w_k, R_k, I_{kj}\}} & \quad \sum_{k \in K} Q_k R_k \\
\text{s.t.} & \quad (28b) - (28f), (28j), (28l).
\end{align*}
\] (29b)

For the non-convex constraint (28d), it is worth noting that the RHS of constraint (28d) is a joint convex function with respect to \( S_{kj} \) and \( I_{kj} \) since its Hessian function is semidefinite for any \( S_{kj} > 0 \) and \( I_{kj} > 0 \). If given any local point \( \{S_{kj}, I_{kj}\} \), we can get a lower bound at this point by utilizing the first-order Taylor expansion as
\[
\begin{align*}
& \log_2 (1 + \frac{1}{S_{kj} I_{kj}}) \\
& \geq R_{kj}^\text{low} = \log_2 (1 + \frac{1}{S_{kj} I_{kj}}) - \frac{S_{kj} - \tilde{S}_{kj}}{\log_2 (1 + \frac{1}{S_{kj} I_{kj}})} - \frac{I_{kj} - \tilde{I}_{kj}}{\log_2 (1 + \frac{1}{S_{kj} I_{kj}})}.
\end{align*}
\] (30)

Based on (30), the non-convex constraint (28d) is relaxed to be the following inequality
\[
R_k \leq \min\{R_{kk}^\text{low}, R_{kj}^\text{low}\}, \quad \text{if } o_k < o_j, \forall k, j \in K. \quad (31)
\]

We reformulate problem (29) which can be approximated as
\[
\begin{align*}
\max_{\{w_k, R_k\}} & \quad \sum_{k \in K} Q_k R_k \\
\text{s.t.} & \quad (28b), (28c), (28f), (28g), (28i), (31).
\end{align*}
\] (32b)

As for the non-convex rank-one constraint (28j), we have the following theorem.

Theorem 2: The solution \( \{W_k\} \) obtained without the rank-one constraint always satisfies that \( \text{Rank}(W_k) = 1, \forall k \in K \).
Proof: Please refer to Appendix B.

By exploiting this theorem, we can obtain a rank-one solution by ignoring this constraint directly. Since the final relaxed problem without the rank-one constraint is a standard semidefinite program (SDP) [39], it can be efficiently solved by well-known convex optimization tools, such as the CVX [40]. The objective value obtained from problem (32) yields a lower bound of that from problem (29) owing to the relaxation in (31). After the solution is derived, we can get the beamforming vector \( \{w_k\} \) via Cholesky decomposition as \( W_k = w_k w_k^H \).

### B. Passive Beamforming Optimization at the STAR-RIS

For any given active beamforming vectors \( \{w_k\}, \forall k \in \mathcal{K} \), the passive beamforming optimization at the STAR-RIS can be rewritten as

\[
\begin{aligned}
\max_{\{d_s^2, D_r, S_{kj}, I_{kj}\}} & \quad \sum_{k \in \mathcal{K}} Q_k R_k \\
\text{s.t.} & \quad (28b) - (28d), (28f) - (28i), (28k).
\end{aligned}
\]

(33b)

The manipulation for the non-convex constraint (28d) is similar to the methods used in Sec. IV-A, from which they can be approximated as (31). Let \( e_m \) denote an \( (M + 1) \)-dimensional column vector with the \( m \)-th element being 1 and all others being 0. Then, combed constraints (28g) and (28h), we have the equivalent formula below

\[
\begin{aligned}
e_m^H D_r e_m + e_m^H D_r e_m = 1, \forall m \in \mathcal{M}, \\
e_m^H D_r e_m = 1, \forall s \in \{r, t\}.
\end{aligned}
\]

(34)

after which the variable \( \beta^2 \) in problem (33) is eliminated. Then the relaxed problem is given by

\[
\begin{aligned}
\max_{\{d_s^2, D_r, S_{kj}, I_{kj}\}} & \quad \sum_{k \in \mathcal{K}} Q_k R_k \\
\text{s.t.} & \quad (28b), (28c), (28f), (28i), (28k), (31), (34).
\end{aligned}
\]

(35b)

As for the non-convex rank-one constraint (28i), there is no guarantee that the obtained \( \{d_s\} \) by ignoring this constraint can always be rank-one. In this case, the typical SDR approach may fail to work well since the reconstructed rank-one solution via the Gaussian randomization method [21] may not be feasible. To address this challenge, we turn to find a local optimal rank-one solution by applying the sequential rank-one constraint relaxation (SROCR)-based method [18], [41], where the rank-one constraint is replaced with a relaxed convex formula. Then the further relaxed optimization problem is given as

\[
\begin{aligned}
\max_{\{d_s^2, D_r, S_{kj}, I_{kj}\}} & \quad \sum_{k \in \mathcal{K}} Q_k R_k \\
\text{s.t.} & \quad (28b), (28c), (28f), (28i), (28k), (31), (34),
\end{aligned}
\]

(35a)

The parameter \( \gamma^{(i)} \in [0, 1] \) is introduced to manipulate the ratio of the largest eigenvalue of the obtained solution to the trace of \( D_s \). Here, \( \gamma^{(i)} = 0 \) means that the rank-one constraint is ignored directly, and \( \gamma^{(i)} = 1 \) indicates that a rank-one solution can be found. For a given \( \gamma^i \), problem (35) is a convex problem and can be solved via CVX [40]. In this case, a local optimal rank-one solution can be acquired for problem (35) by iteratively increasing \( \gamma^i \) from 0 to 1. Specifically, in each iteration, \( \gamma^i \) is updated as

\[
\gamma^{i+1} = \min \left\{ 1, \frac{\max_i \mathbf{D}^{(i)} \mathbf{u}^{(i)}_s}{\text{Tr}(\mathbf{D}^{(i)}_{opt}) + \delta^{(i)}} \right\},
\]

(37)

where \( \delta^{(i)} \) is the step size. If the pre-defined \( \delta^{(i)} \) makes the problem infeasible, it can be reduced as \( \delta^{(i+1)} = \delta^{(i)}/2 \) until the problem is solvable. The termination condition for the SROCR algorithm is that \( \text{obj}(\mathbf{D}^{(i)}) - \text{obj}(\mathbf{D}^{(i-1)}) \leq \xi_1 \) and \( 1 - \gamma^{(i-1)} \leq \xi_2 \) are reached simultaneously, where \( \text{obj}(\mathbf{D}^{(i)}) \) denotes the objective function value obtained with solution \( \mathbf{D}^{(i)} \), and \( \xi_1 \) and \( \xi_2 \) are the convergence thresholds. For the convergence of the SROCR method, please refer to [41].

After the rank-one solution is found, we can get the passive beamforming coefficients \( \{d_s\} \) via Cholesky decomposition as \( D_s = d_s d_s^H, \forall s \in \{r, t\} \).

### C. Complexity and Convergence Analysis

We summarize the overall algorithm in Algorithm 1, where the ABCs \( \{w_k\} \) at the BS and the PTRCs \( \{d_s\} \) at the STAR-RIS are optimized alternately and iteratively. Random generation is conducted to obtain the initial points in Line 2. If the initial points are not feasible, new points will be generated randomly again until the feasible points are found. Furthermore, the obtained solutions in each iteration serve as the input local points of the next iteration until the algorithm termination conditions are satisfied. Explicitly, the problem complexity of the SDP subproblem for the ABCs design at the BS is \( \mathcal{O}(\max(5K^2 + 1, N)^4 \sqrt{N \log_2(1/\epsilon)}) \) for a given solution accuracy \( \epsilon > 0 \) [42]. Additionally, the complexity of the subproblem for the PTRCs design is \( \mathcal{O}(\sum_{i=0}^{SRO} \max(5K^2 - 2K + 5, M)4 \sqrt{M \log_2(1/\epsilon)}) \), where \( SRO \) is the iteration numbers needed for the SROCR approach. In consequence, the total complexity of Algorithm 1 is \( \mathcal{O}(\sum_{i=0}^{SRO} \max(5K^2 + 2K + 5, M)) \).
where $\eta > 0$ is the actual maximum iteration numbers.

Theorem 3: The convergence of the proposed Algorithm 1 is guaranteed.

Proof: Please refer to Appendix C.

V. EXTENSION TO THE MS AND TS PROTOCOLS

In this section, we extend the proposed solution to the MS protocol and the TS protocol.

A. Optimization for the MS Protocol

The main difference between the problems for MS and ES lies in that the continuous amplitude $\beta_m^s \in [0, 1], \forall s \in \{r, t\}, \forall m \in \mathcal{M}$ in constraint (24b) becomes binary variable $\beta_m^s \in \{0, 1\}, \forall s \in \{r, t\}, \forall m \in \mathcal{M}$. Thus, the formulated problem (21) for MS under a given decoding order $o_k$ is

$$\max_{\{\Theta, w_k\}} \sum_{k \in \mathcal{K}} Q_k R_k \quad (38a)$$

s.t. $\beta_m^t + \beta_m^s = 1, \beta_m^t, \beta_m^s \in \{0, 1\}, \forall m \in \mathcal{M}$

(38b)

(11), (13) - (15),

(38c)

which can be solved in a similar process to that of the ES protocol by alternatively optimizing the ABCs $\{w_k\}$ and the PTRCs $\{d_s\}$. In this case, for the ABCs design at the BS under a given $\{d_s\}, \forall s \in \{r, t\}$, we refer to the same method for the ES protocol to get the solution. In the following, we focus on PTRCs design at the STAR-RIS under a given $\{w_k\}, \forall k \in \mathcal{K}$.

The constraint $\beta_m^s \in \{0, 1\}, \forall s \in \{r, t\}$ in (38b) is equal to the equality constraint below

$$\beta_m^s (1 - \beta_m^s) = 0, \forall s \in \{r, t\}, m \in \mathcal{M}. \quad (39)$$

Inspired by this, we can resort to a penalty-based algorithm proposed in [10] via adding the constraint (39) as a penalty term into the objective function. The optimization subproblem for the PTRCs design is expressed as

$$\max_{\{d_s, w_k, \beta_m\}} \sum_{k \in \mathcal{K}} Q_k R_k - \eta \sum_{s \in \{r, t\}} \sum_{m \in \mathcal{M}} \left(\beta_m^s (1 - \beta_m^s)\right) \quad (40a)$$

s.t. (28b) - (28d), (28f) - (28i), (28k),

(40b)

where $\eta > 0$ is the penalty factor for restricting the value of $\{\beta_m^s\}$ into the set $\{0, 1\}$, otherwise the objective function will be penalized if $\{\beta_m^s\}$ falls into the range $(0, 1)$.

However, the penalty term regarding the $\beta_m^s$ in the objective is non-convex. To deal with this challenge, we adopt the successive convex approximation (SCA) approach to approximate this term by employing the first-order Taylor expansion. Particularly, for the given local point $\{d_s\}$, namely $\{\beta^*\}$, an upper bound for the penalty term can be derived as

$$\sum_{m \in \mathcal{M}} \left(\beta_m^s (1 - \beta_m^s)\right) \leq \sum_{m \in \mathcal{M}} \left(\beta_m^s - (\beta_m^s)^2 + (1 - 2\beta_m^s)(\beta_m^s - \beta_m^s)\right)$$

$$= \sum_{m \in \mathcal{M}} \beta_m^s - 2 \sum_{m \in \mathcal{M}} \beta_m^s \beta_m^s + \sum_{m \in \mathcal{M}} (\beta_m^s)^2$$

$$= \text{Tr}(D_s) - 2\text{Tr}(\tilde{P}_s D_s) + \text{Tr}(\tilde{P}_s \tilde{P}_s)$$

$$\triangleq \Lambda(D_s, \tilde{D}_s), \forall s \in \{r, t\},$$

where $\tilde{P}_s$ is the corresponding diagonal matrix of $D_s$. After that, the objective function in (40) is transformed as an affine function about the variables.

As for the non-convex constraint (28d), we can deal with them via a similar method as described for the ES protocol. After all the operations, this transformed problem is given as

$$\max_{\{d_s, w_k\}} \sum_{k \in \mathcal{K}} Q_k R_k - \eta \sum_{s \in \{r, t\}} \sum_{m \in \mathcal{M}} \Lambda(D_s, \tilde{D}_s) \quad (41a)$$

s.t. (28b), (28c), (28f), (28i), (28k), (31), (34),

(41b)

which is in the same form as problem (35) for ES, and can be solved via the same method.

Different from Algorithm 1 for ES, a two-loop algorithm for MS is summarized in Algorithm 2. In the outer loop, the penalty factor is gradually increased after each iteration, i.e., $\eta = \zeta \eta$ with $\zeta > 1$. The termination criterion in the outer loop is defined as

$$\max \left\{\beta_m^s - (\beta_m^s)^2, \forall s \in \{r, t\}, m \in \mathcal{M}\right\} \leq \tilde{\xi}_2, \quad (42)$$

where $\tilde{\xi}_2$ is the predefined accuracy. In the inner loop, $\{w_k, D_s\}$ are optimized alternately via the blocked coordinate descent (BCD) method similar to the way for ES under the given penalty factor. Since the value of the objective function is non-decreasing in each iteration of the outer loop, the optimal value of the objective function is bounded below. Thus, this penalty-based iterative algorithm is guaranteed to convergence as the factor $\eta$ approaches infinity.

Remark 4 (Complexity Analysis for MS): Let $l_{out}$ denote the iteration number required in the outer loop. Then the total complexity of Algorithm 2 for MS is $O\left(l_{MS} \max\left(\frac{5K^2 + 1}{\sqrt{N} + l_{SRO}} \max(5K^2 - 2K + 5, M^4 \sqrt{M}) \log_2(1/\epsilon)\right)\right)$, where $l_{MS}$ is the actual maximum iteration numbers in the inner loop.

B. Optimization for the TS Protocol

For a given $\alpha^s, \forall s \in \{r, t\}$, this problem can be decomposed into two subproblem depending on whether the STAR-RIS
works in R period or T period as

$$\max_{\{\Theta_k, w_k\}} \sum_{k \in K, s \in \{r, t\}} Q_k R_k^s$$

$$\text{s.t. } \beta_r^m(t), \beta_t^m(t) \in B^T_{\beta} \text{ or } B^R_{\beta},$$

(43a)

(43b)

(43c)

It is worth noting that each of these two subproblems can be solved like the problem (24). Let $R_{\text{sum}}^s, s \in \{r, t\}$ denote the maximal QWSR for the corresponding problem, then problem (21) for TS is equivalent to the following problem

$$\max_{\{\alpha^s\}} \sum_{s \in \{r, t\}} \alpha^s R_{\text{sum}}^s$$

$$\text{s.t. } (16),$$

(44)

(45)

which is the linear programming and can be efficiently solved.

Remark 5: Based on the above analysis, it is worth noting that the optimization for the TS protocol in one time slot results in a per-slot single-surface working style of the STAR-RIS. However, as the data queues evolve with time, the working surface of the STAR-RIS will switch in different time slots to serve different users and keep all queues stable from the perspective of a long term. This is because the employed time-varying queue length-based weights reflect the priority of each user at different moments.

Remark 6 (Complexity Analysis for TS): The total complexity of the algorithm for TS is $O\left(\frac{P_{\text{TS}}^r}{T S^r} \left( \max(\frac{5K_r^2 + 1}{N})^{4N + 1} + l_{TS}^r \max(5K_r^2 - 2K_r + 5, M)^{4N} \right) + \frac{P_{\text{TS}}^t}{T S^t} \left( \max(\frac{5K_t^2 + 1}{N})^{4N} + l_{TS}^t \max(5K_t^2 - 2K_t + 5, M)^{4N} \right) \log_2(1/\epsilon) \right)$, where $P_{\text{max}}^{TS_r}$ and $P_{\text{max}}^{TS_t}$ are the actual maximum iteration numbers when solving the subproblem in T period and R period, respectively. $K_r$ and $K_t$ are the number of users located in the transmission set and the reflection set, respectively. Moreover, $l_{TS}^r$ and $l_{TS}^t$ are the iteration numbers for the SROCR approach when solving the subproblem in T period and R period, respectively.

VI. SIMULATION RESULTS

In this section, numerical results are provided to show the efficiency of the proposed algorithm.

A. Simulation Configuration

In the simulation, we consider a three-dimensional (3D) coordinate system to describe the locations of the transceivers in the STAR-RIS assisted NOMA communications. We assume that the BS is 22 meters (m) tall and the START-RIS is located on a building with a height of 10m, and they are set at the point (250, 0, 22) and the point (0, 250, 10), respectively. In addition, there are five users located on the opposite sides of the STAR-RIS, where the coordinates of three reflection users are (50, 250, 0), (30, 210, 0), and (30, 290, 0), while the coordinates of two transmission users are (−50, 250, 0) and (−30, 290, 0).

For the small scale fading, the Rayleigh fading channel model is assumed for the direct link between BS and users, while the links between the BS and STAR-RIS as well as the links between the STAR-RIS and users are modeled as Rician fading channels, which are given by

$$g_k = \sqrt{P_l(\rho_g) g_{k, NLOS}} \forall k \in K,$$

$$G = \sqrt{P_l(\rho_G) G_{LOS} + \sqrt{\frac{1}{\rho_G + 1}} N_{LOS}},$$

$$v_k = \sqrt{P_l(\rho_v) v_{LOS} + \sqrt{\frac{1}{\rho_v + 1}} N_{LOS}} \forall k \in K,$$

(46)

(47)

(48)

where the first terms on the right hand of the equations above are the distance-dependent path loss, with $\rho_g$, $\rho_G$ and $\rho_v$ being the distance between the BS and the users, between the BS and the STAR-RIS, and between the STAR-RIS and the users. Moreover, $\phi_G$ and $\phi_v$ are the Rician factors with $\phi_G = \phi_v + 3$ dB, $G_{LOS}$ and $v_{LOS}$ are the corresponding deterministic LOS components, while $g_{NLOS}$, $G_{NLOS}$ and $v_{NLOS}$ are the Rayleigh fading components. According to the 3GPP technical report for the urban macro (UMa) scenario [43], the distance-dependent path loss in dB is given by $P_l(\rho) = 28 + 22 \log_{10}(\rho) + 20 \log_{10}(f_c)$, where $\rho$ is the distance between the transmitter and the receiver, and $f_c = 2$ GHz is the carrier frequency. Assume the BS is equipped with a uniform linear array and the STAR-RIS is equipped with a uniform planar array, with the antenna spacing of both arrays being half of the wavelength. Unless otherwise stated, the maximal transmit power at the BS is $P_{\text{max}} = 40$ W, with the signal-to-noise ratio (SNR) being 5dB. The length of one slot is 1ms. The data arrives randomly according to a Poisson distribution with the average arrival rate being randomly chosen from the range of [2, 6] bits/s/Hz. Moreover, the antenna number of the BS is $N = 4$ and the element number of the STAR-RIS is $M = 40$.

B. Convergence of the Proposed Algorithms

Fig. 3 provides the convergence of our proposed algorithms by setting the threshold in Algorithm 1 as $\xi = 10^{-4}$. It can be seen from the figure that the algorithms converge approximately after 12 iterations. Therefore, Theorem 3 is proven...
to be accurate. In addition, we can see that TS has superior QWSR performance than the others. The reason for this is that in TS, one group of users can utilize the full energy, but in ES and MS, the energy is split up by the STAR-RIS and used by two groups of users at the same time with mutual interference. That is, TS benefits from interference-free communications by inefficiently using the communication time. However, due to the periodical switch-over of the elements, the time synchronization for TS involves a great level of complexity in hardware implementation.

C. Quantization Performance

Take the ES protocol as an example, Fig. 4 shows the QWSR performance for different resolutions of the discrete phase shifter, as well as the both discrete phase shifter and discrete amplitude. Specifically, the uniform quantization and the discretization of one-half exponential powers are employed for the phase shifters and the amplitudes, respectively. From the figure we can see, compared with the discrete phase shifter, the performance of the both discrete phase shifter and discrete amplitude scheme deteriorates further with a small resolution bit. However, the performance gap between these two cases almost disappears when the resolution is equal to or greater than 3 bits. Meanwhile, we notice that the performance loss between the discrete cases with a 3-bit resolution quantization and the continuous case can be negligible no matter what the value of the element number is.

D. Baseline Schemes

To verify the good performance offered by integrating the STAR-RIS and NOMA, we denote our proposed algorithm as STAR-ES/MS/TS and adopt the following three baselines:

- **Conventional RIS-assisted NOMA system (Conv-RIS):** This scheme employs one reflecting-only RIS and one transmitting-only RIS, each consisting of $M/2$ elements to achieve full-space coverage. It can be seen as a special case of the STAR-RIS, where half of the elements operate to reflect signals only and the other elements transmit signals only. Thus, this problem can be solved by setting $\beta = [1_{1 \times M/2}, 0_{1 \times M/2}]$ and $\beta' = [0_{1 \times M/2}, 1_{1 \times M/2}]$.

- **Uniform energy splitting (STAR-UES):** In this case, the same amplitude coefficients (namely $\beta_m = 0.5, \forall s \in \{r, t\}, m \in \mathcal{M}$) are assumed to be employed among all elements of the STAR-RIS for transmission and reflection, respectively. It can be viewed as another special case of the STAR-RIS that utilizes a surface-by-surface design for amplitudes.

- **STAR-ES/MS/TS-OMA:** In this case, instead of NOMA, the BS transmit signals to the users through the orthogonal frequency/time resources [9]. Let $\beta_k \in [0, 1]$ denote the proportion of resource blocks allocated to user $k$, which satisfies $\sum_{k \in \mathcal{K}} \beta_k \leq 1$. Then, the achievable rate of user $k$ in this case is $R_k^o = \beta_k \log_2 (1 + \frac{|v_k \mathbf{g}_{k} |^2}{\omega_k \sigma^2})$, $\forall k \in \mathcal{K}$.

E. QWSR Performance Comparison

Fig. 5 reveals the impact of deploying STAR-RIS. Fig. 5(a) illustrates the QWSR performance versus different STAR-RIS element numbers. The QWSR increases with the growing $M$ for the reason that more elements contribute to a higher array gain. In addition, the STAR-RIS schemes achieve a significant performance gain compared with conventional RIS though NOMA is employed in both schemes, referred to as the “STAR gain for NOMA”, which verifies the superiority of the proposed STAR-RIS. In addition, due to the binary value restrictions for the amplitude coefficients of transmission and reflection, MS suffers from some performance loss as compared to the ES and TS protocols. Nevertheless, MS is more attractive since that the on-off attribute of each element is easier to implement. Fig. 5(b) compares the QWSR performance versus different transmitting power. As shown in the figure, the QWSR performance increases as the SNR grows. It can be observed that compared with the Conv-RIS scheme, a smaller transmitting power (i.e., less power) is needed for the STAR schemes to achieve the same QWSR. In addition, it should be highlighted that the unique characteristic of interference-free communications contributes to boosting the performance of TS no matter what the values of the transmitting power. Fig. 5(c) presents the QWSR performance versus different antenna numbers of the BS. It implies from the figure that the performance can be improved if more antennas are assembled at the BS. This is because the system benefits from achieving more active beamforming gain with more antennas involved. Fig. 5(d) investigates the QWSR performance versus different user numbers. It is observed from the figure that the performance keeps at an almost constant value no matter what the number of users is. Though the number of users poses negligible impact on the QWSR performance, a larger number of users results in a reduced average rate that a single user is able to achieve.

By comparing with OMA schemes, Fig. 6 depicts the NOMA gain for STAR-RISs in terms of the QWSR performance. With STAR-RIS, the BS serves all users via the time-division multiple access under OMA. As observed from Figs. 6(a) and 6(b), the NOMA schemes outperform the
Fig. 5. QWSR comparison for the impact of STAR-RIS.

(a) QWSR versus the number of STAR-RIS elements $M$ for $N = 4$, $K = 5$, $P_{\text{max}} = 40$W.

(b) QWSR versus the transmitting power $P_{\text{max}}$ for $N = 4$, $M = 40$, $K = 5$.

(c) QWSR versus the number of antennas $N$ for $M = 40$, $K = 5$, $P_{\text{max}} = 40$W.

(d) QWSR versus the number of users $K$ for $N = 4$, $M = 40$, $P_{\text{max}} = 40$W.

Fig. 6. QWSR comparison for the impact of NOMA.

(a) QWSR versus the number of STAR-RIS elements $M$ for $N = 4$, $K = 5$, $P_{\text{max}} = 40$W.

(b) QWSR versus different SNR for $N = 4$, $M = 40$, $K = 5$.

OMA schemes for all operating protocols. This is because the NOMA schemes allow all users to be served simultaneously no matter which protocol, thus using the communication efficiently compared with the OMA schemes.
F. Average Queue Length Comparison

Fig. 7 demonstrates the stability of the ES and TS protocols via comparing the average queue length over a long time with the schemes optimized by maximizing the sum rate, i.e., throughput-optimal cases without considering the queue-based weight. These cases are referred to as “STAR-ES-ThruputOpt” and “STAR-TS-ThruputOpt”, respectively. Note that since MS is a special case of ES, we omit it here for conciseness. It can be observed that the throughput-optimal cases both for ES and TS have growing queues as time involves, which implies that the system cannot be stable since the average queue length is unbounded after an infinite horizon of time. This is indeed expected, since the inconsideration of the urgency of users results in an improper rate allocation, causing that queues of some users keep accumulating. By contrast, the proposed QWSR-based schemes (i.e., ES, TS, and UES) achieve the state of dynamic equivalence, i.e., stable queues with limited oscillations, which proves that the reformulated QWSR maximization problem is able to guarantee the system stability in the long run.

In Fig. 8, we compare the average queue length among the STAR-RIS schemes and the best performing baseline UES over 10 slots. Fig. 8(a) compares the average queue length performance versus different element numbers of the STAR-RIS. It can be observed from the figure that a large element number leads to a lower queue length. This is reasonable since numerous elements contribute to a higher beamforming gain and thus a higher rate, which in turn reduces the amount of data in the queue. Fig. 8(b) demonstrates the average queue length performance versus different arrival rates of the data pending to be transmitted. Note that the performance gap almost disappears among all algorithms when the arrival rate is very small, but becomes more noticeable with a higher rate. The reason for this trend is explained as follows. When the data arrives at a very low rate, fewer of them can be accumulated in the queue, thus the average queue length is almost zero for all algorithms. As the rate increases, the queue length grows accordingly due to the constrained transmission rate restricted by the limited energy budget. Accordingly, the advantages of the proposed algorithms show up.

VII. Conclusion

In this paper, the stability for the queue-aware STAR-RIS assisted NOMA communication system has been studied, which was reformulated to maximize the per-slot QWSR of users. The employed rate weights were determined by the length of the data queues kept at the BS. More explicitly, by jointly optimizing the NOMA decoding order, the ABCs at the BS, and the PTRCs at the STAR-RIS, three operating protocols for the STAR-RISs, including ES, MS, and TS, were considered. We first proposed an equivalent-combined channel gain based scheme to acquire the desired NOMA decoding order. For ES, the BCD and the SCA methods were exploited to iteratively handle the intrinsically coupled non-convex problem. Then for MS, the proposed iterative algorithm was further extended to exploit the penalty-based method. For TS, the formulated problem was decomposed into two subproblems, each of which can be handled in the same manner as introduced for ES. Simulation results confirmed the queue stability under the reformulated QWSR maximization problem and revealed that our proposed STAR-RIS assisted NOMA communication system achieves better performance compared with the conventional schemes. Furthermore, the simulations also showed that the TS protocol has superior performance among the three protocols in terms of both the QWSR and the average queue length.

APPENDIX A: PROOF OF THEOREM 1

First of all, (20) can be rearranged as

\[
\Delta(Q(t)) \leq B_0 - 2 \sum_{k \in K} \mathbb{E}(Q_k(t))(R_k(t)\tau - \lambda_k(t)\tau). \quad (49)
\]

Let \( \zeta = \max_k \{2(R_k(t)\tau - \lambda_k(t)\tau)\} \), yielding

\[
\Delta(Q(t)) \leq B_0 - \zeta \sum_{k \in K} \mathbb{E}(Q_k(t)). \quad (50)
\]

The above inequality holds for all time slots \( t \in \{0, 1, 2, \ldots\} \). Summing the inequality over time slots \( t \in \{0, \ldots, T-1\} \) yields a telescoping series on the left hand side, resulting in

\[
\mathbb{E}(L(Q(T-1))) - \mathbb{E}(L(Q(0))) \leq B_0 T - \zeta \sum_{t=0}^{T-1} \sum_{k \in K} \mathbb{E}(Q_k(t)). \quad (51)
\]

Dividing (51) by \( T \), shifting terms, and using the fact that \( L(Q(t)) \geq 0, \forall t \), we have

\[
\frac{1}{T} \sum_{t=0}^{T-1} \sum_{k \in K} \mathbb{E}(Q_k(t)) \leq \frac{B_0 + \mathbb{E}(L(Q(0)))}{T\zeta}. \quad (52)
\]

Note that (52) holds for all positive integers \( T \). As \( Q(0) \) has an initially bounded value, \( \mathbb{E}(L(Q(0))) \) is bounded. Taking a limsup as \( T \to \infty \) yields (8), then the network is strongly stable.
of problem (35) is

\[ L = \sum_{k \in K} \sum_{j: a_k \leq \sigma_j} a_{kj} \left( \frac{1}{S_{kj}} - \text{Tr}(W_k H_j^H D_j H_j) \right) \]
\[ + \sum_{k \in K} \sum_{j: a_k \leq \sigma_j} b_{kj} \left( \sum_{i: i \leq \sigma_i} \text{Tr}(W_k H_i^H D_i H_i) + \sigma^2 - I_{kj} \right) \]
\[ + \sum_{i \in K} \sum_{k \in K: j: a_k \leq \sigma_j} c_{x_{kj}} \left( \text{Tr}(W_k H_i^H D_i H_i) - \text{Tr}(W_k H_i^H D_i H_i) \right) \]
\[ + c \left( \sum_{k \in K} \text{Tr}(W_k) - P_{\max} \right) - \sum_{k \in K} \text{Tr}(W_k X_k) + L_0, \quad (53) \]

where \( L_0 \) is the terms independent of \( W_k, k \in K \). Moreover, the terms \( a_{kj}, b_{kj}, c, x_{kj} \), and \( X_k \) are the Lagrange multipliers associated with the corresponding constraints. The Karush-Kuhn-Tucker (KKT) conditions for the optimal \( W_k^* \) are displayed as follows

\[ a_{kj}, b_{kj}, c, x_{kj}^* \geq 0, X_k \geq 0, X_k^* W_k^* = 0, \nabla_{W_k} L = 0, \quad (54) \]

where \( a_{kj}^*, b_{kj}^*, c^*, x_{kj}^* \), and \( X_k^* \) stand for the optimal Lagrange multipliers, and \( \nabla_{W_k} L \) is the gradients of \( L \) with respect to \( W_k \). Then, we have the following equations,

\[ \nabla_{W_k} L = - \sum_{j: a_k \leq \sigma_j} a_{kj} \left( H_j^H D_j H_j \right)^T \]
\[ + \sum_{m:o_m < a_k} \sum_{j: a_k \leq \sigma_j} b_{mj} \left( H_j^H D_j H_j \right)^T \]
\[ + \sum_{i \in K} \sum_{m:o_m < a_k} x_{mk}^* - \sum_{j: a_k \leq \sigma_j} x_{kj}^* \left( H_i^H D_i H_i \right)^T \]
\[ + c^* I - (X_k^*)^T, \]
\[ = c^* I - (X_k^*)^T - \sum_{i \in K} y_{ki}^* \left( H_i^H D_i H_i \right)^T = 0, \quad (55) \]

with \( y_{ki}^* \) defined as

\[ y_{ki}^* = \mathbb{1}(a_k \leq o_i)a_{ki}^* - \mathbb{1}(a_k \leq o_i) \sum_{m:o_m < a_k} b_{mi}^* \]
\[ - \left( \sum_{m:o_m < a_k} x_{mk}^* - \sum_{j:o_j < a_k} x_{kj}^* \right), \quad (56) \]

where \( \mathbb{1}(\cdot) \) is the indicator function, whose value is 1 if the condition in (\cdot) is true, and 0 if not.

Let \( A_k = \sum_{i \in K} y_{ki}^* (H_i^H D_i H_i) \), and rearrange the order of the items in (55), we have

\[ X_k^* = c^* I - A_k, \forall k \in K. \quad (57) \]

Let \( z_k \) denote the maximal eigenvalue of \( A_k \) and \( f_k \) be the algebraic multiplicity of \( z_k \). Reviewing (54), we know that \( c^* \geq 0 \) and \( X_k^* \geq 0 \). Next, we try to compare the value of \( c^* \) and \( z_k \). If \( c^* < z_k \), it contradicts the condition \( X_k^* \geq 0 \). If \( c^* > z_k \), the smallest eigenvalue of \( X_k^* \) is \( c^* - z_k > 0 \), which indicates that \( X_k^* \) is a full rank positive-semidefinite matrix and its null space is zero, i.e., \( \text{Rank}(W_k^*) = 0 \) and \( W_k^* \) is a zero matrix. However, zero matrix is not an effective solution for \( W_k \) since \( \sum_{k \in K} \text{Tr}(W_k) \leq P_{\max} \). Thus, \( c^* \) can only be equal to \( z_k \), i.e., \( c^* = z_k \).

Since \( z_k \) is the largest eigenvalue of \( A_k \), the other eigenvalues are less than \( c^* \). Combing with (57), we can know that \( X_k \) is a positive-semidefinite matrix with \( f_k \) zero eigenvalues and \( N - f_k \) positive eigenvalues, which means \( \text{Rank}(X_k^*) = N - f_k \). Recalling that \( X_k^* W_k^* = 0 \), the rank of \( W_k^* \) satisfies \( \text{Rank}(W_k^*) \leq \text{Rank}(\text{the null space of } X_k^*) = N - \text{Rank}(X_k^*) = f_k \). In fact, due to the randomness of the channels, the probability that \( A_k \) has multiple eigenvalues with the same value (i.e., \( f_k > 1 \)) is very low. As such, we can say that \( \text{Rank}(W_k^*) = 1 \).

**APPENDIX C: PROOF OF THEOREM 3**

Let \( R_{\text{sum}}(\{w_k^l\}, \{d_k^l\}) \) denote the objective value of problem (24) in the \( l \)-th iteration, then

\[ R_{\text{sum}}(\{w_k^l\}, \{d_k^l\}) \stackrel{(a)}{=} R_{\text{sum}}(\{w_k^{l+1}\}, \{d_k^l\}) \]
\[ \leq R_{\text{sum}}(\{w_k^{l+1}\}, \{d_k^{l+1}\}), \quad (58) \]
where \( (a) \) holds since that the optimal objective value for the ABC design we get is the lower bound of that of problem \( (29) \) for the given PTRCs \( \{d_i\} \). Similarly, \( (b) \) holds for the reason that optimal objective value for the PTRCs design serves as the lower bound of that of problem \( (33) \) for the given ABC value \( \{w_i^+\} \).

Moreover, it is suggested from Eq. \( (58) \) that the objective value of problem \( (24) \) is no-decreasing after each iteration. Meanwhile, owing to the finite value that the system sum rate value of problem \( (24) \) is no-decreasing after each iteration.

**REFERENCES**

[1] N. Zhang, Y. Liu, X. Mu, and W. Wang, “Stability-oriented STAR-RIS aided MISO-NOMA communication systems,” in *Proc. IEEE Global Conf. (GLOBECOM)*, Dec. 2022, pp. 3120–3125.

[2] Y. Liu et al., “Reconfigurable intelligent surfaces: Principles and opportunities,” *IEEE Commun. Surveys Tuts.*, vol. 23, no. 3, pp. 1546–1577, 3rd Quart., 2021.

[3] Q. Wu, S. Zhang, B. Zheng, C. You, and R. Zhang, “Intelligent reflecting surface-aided wireless communications: A tutorial,” *IEEE Trans. Commun.*, vol. 69, no. 5, pp. 3313–3351, May 2021.

[4] X. Pang, M. Sheng, N. Zhao, J. Tang, D. Niyato, and K.-K. Wong, “When UAV meets IRS: Expanding air-ground networks via passive reflection,” *IEEE Wireless Commun.*, vol. 28, no. 5, pp. 164–170, Oct. 2021.

[5] C. Huang et al., “Holographic MIMO surfaces for 6G wireless networks: Opportunities, challenges, and trends,” *IEEE Wireless Commun.*, vol. 27, no. 5, pp. 118–125, Oct. 2020.

[6] M. Di Renzo et al., “Smart radio environments empowered by reconfigurable intelligent surfaces: How it works, state of research, and the road ahead,” *IEEE J. Sel. Areas Commun.*, vol. 38, no. 11, pp. 2450–2525, Nov. 2020.

[7] C. Huang, A. Zappone, G. C. Alexandropoulos, M. Debbah, and C. H. Papadias, “Reconfigurable intelligent surfaces for energy efficiency in wireless communication,” *IEEE Trans. Wireless Commun.*, vol. 18, no. 8, pp. 4157–4170, Aug. 2019.

[8] Z. Mao, W. Wang, Q. Xia, C. Zhong, X. Pan, and Z. Ye, “Element-grouping intelligent reflecting surface: Electromagnetic-compliant model and geometry-based optimization,” *IEEE Trans. Wireless Commun.*, vol. 21, no. 7, pp. 5362–5376, Jul. 2022.

[9] C. Cui, Y. Liu, X. Mu, X. Gu, and O. A. Dobre, “Coverage characterization of STAR-RIS networks: NOMA and OMA,” *IEEE Commun. Lett.*, vol. 25, no. 9, pp. 3036–3040, Sep. 2021.

[10] X. Mu, Y. Liu, L. Guo, J. Lin, and R. Schober, “Simultaneously transmitting and reflecting (STAR) RIS aided wireless communications,” *IEEE Trans. Wireless Commun.*, vol. 21, no. 5, pp. 3083–3098, May 2022.

[11] Y. Liu et al., “STAR: Simultaneous transmission and reflection for 360° coverage by intelligent surfaces,” *IEEE Wireless Commun.*, vol. 28, no. 6, pp. 102–109, Dec. 2021.

[12] B. O. Zhu et al., “Dynamic control of electromagnetic wave propagation with the equivalent principle inspired tunable metasurface,” *Sci. Rep.*, vol. 4, no. 1, p. 4971, May 2014.

[13] NTT DOCOMO. (Jun. 2020). *Docomo Conducts World’s First Successful Trial of Transparent Dynamic Metasurface*. [Online]. https://www.docomoresearch.info/news/20200117_00.html

[14] B. Makki, K. Chitti, A. Behravan, and M.-S. Alouni, “A survey of NOMA: Current status and open research challenges,” *IEEE Open J. Commun. Soc.*, vol. 1, pp. 179–189, 2020.

[15] N. Zhao et al., “Joint trajectory and precoding optimization for UAV- assisted NOMA networks,” *IEEE Trans. Commun.*, vol. 67, no. 5, pp. 3723–3735, May 2019.

[16] L. Bariah, S. Mushtaq, P. C. Sofotasios, F. E. Bouanani, O. A. Dobre, and W. Hamouda, “Large intelligent surface-assisted nonhomogeneous multiple access for 6G networks: Performance analysis,” *IEEE Internet Things J.*, vol. 8, no. 7, pp. 5129–5140, Apr. 2021.

[17] Z. Ding and H. Vincent Poor, “A simple design of IRS-NOMA transmission,” *IEEE Commun. Lett.*, vol. 24, no. 5, pp. 1119–1123, May 2020.

[18] X. Mu, Y. Liu, L. Guo, J. Lin, and N. Al-Dhahir, “Exploiting intelligent reflecting surfaces in NOMA networks: Joint beamforming optimization,” *IEEE Trans. Wireless Commun.*, vol. 19, no. 10, pp. 6884–6898, Oct. 2020.
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