Fujimori, S.; Rossman, W.; Umehara, M.; Yamada, K.; Yang, S.-D.
Spacelike mean curvature one surfaces in de Sitter 3-space. (English) Commun. Anal. Geom. 17, No. 3, 383-427 (2009).

As mentioned in the abstract of this article, the first author studied space-like constant mean curvature one (CMC-1) surfaces in the de Sitter 3-space \( S^3_1 \), when the surfaces have no singularities except within some compact subsets and are of finite total curvature on the complement of this compact subset. This article deals with CMC-1 surfaces whose singular sets are not compact. The authors use two projections, one in the hyperbolic space and the other in the de Sitter 3-space. The first of these projections gives a conformal mean curvature one immersion and, through the second projection, one obtains space-like CMC-1 surfaces that may have singularities. These are called CMC-1 faces. The relationship between these two types of surfaces is analogous to the one between minimal surfaces in Euclidean 3-space \( \mathbb{R}^3 \) and space-like maximal surfaces with singularities in the Lorentz-Minkowski 3-space \( \mathbb{R}^3_1 \).

The two main theorems proved are: Theorem 0.1: A complete end of a CMC-1 face in \( S^3_1 \) is never hyperbolic, so it must be either elliptic or parabolic. Moreover, the total curvature measured over a neighborhood of such an end is finite. (The end of a CMC-1 face is called elliptic, parabolic or hyperbolic if the monodromy matrix of the holomorphic lift \( F : M^2 \to SL_2 \mathbb{C} \) is respectively elliptic, parabolic or hyperbolic.) Theorem 0.2: Suppose a CMC-1 face \( f : M^2 \to S^3_1 \) is complete. Then there exists a compact Riemann surface \( \overline{M}^2 \) and a finite number of points \( p_1, \ldots, p_n \in \overline{M}^2 \), such that \( M^2 \) is holomorphic to \( \overline{M}^2 \setminus \{ p_1, \ldots, p_n \} \), and \( 2 \deg(G) \geq \chi(\overline{M}^2) + 2n \), where \( G \) is the hyperbolic Gauss map of \( f \) and \( \chi(\overline{M}^2) \) is the Euler characteristic of \( \overline{M}^2 \). Furthermore, equality holds if and only if each end is regular and properly embedded.

The article is organized in 6 sections and two appendices: Section 0 is the introduction. Section 1, Preliminaries, includes subsections on the representation formula, completeness, monodromy of ends of CMC-1 faces and the Schwartzian derivative. Section 2, The monodromy of punctured hyperbolic metrics, includes a subsection on lifts of \( PSU_{1,1} \)-projective connections on a punctured disk and the monodromy of punctured hyperbolic metrics. Section 3, Intrinsic behavior of regular ends, includes a subsection on completeness. Section 4, The light-cone Gauss map and extrinsic behavior of ends. Section 5, The Osserman-type inequality. Appendix A, Meromorphicity of the Hopf differential, and Appendix B, Conjugacy classes of \( SU_{1,1} \). In section 5, the authors show many interesting examples.

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MSC:

53C42 Differential geometry of immersions (minimal, prescribed curvature, tight, etc.)
53C50 Global differential geometry of Lorentz manifolds, manifolds with indefinite metrics
53C30 Differential geometry of homogeneous manifolds

Keywords:
constant mean curvature; trinoids; hyperbolic surfaces; de Sitter surfaces; Osserman-type inequality; ends

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