LETTER TO THE EDITOR

Strangeness Production at RHIC in the Perturbative Regime

Daphne Y. Chang†, Steffen A. Bass‡†, and Dinesh K. Srivastava§,
† Department of Physics, Duke University, Durham, North Carolina 27708-0305, USA
‡ RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA
§ Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700 064, India
E-mail: dyc3@phy.duke.edu

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Abstract. We investigate strange quark production in Au-Au collisions at RHIC in the framework of the Parton Cascade Model(PCM). The yields of (anti-) strange quarks for three production scenarios – primary-primary scattering, full scattering, and full production – are compared to a proton-proton baseline. Enhancement of strange quark yields in central Au-Au collisions compared to scaled p-p collisions increases with the number of secondary interactions. The centrality dependence of strangeness production for the three production scenarios is studied as well. For all production mechanisms, the strangeness yield increases with $(N_{\text{part}})^{4/3}$. The perturbative QCD regime described by the PCM is able to account for up to 50% of the observed strangeness at RHIC.
The investigation of strangeness production in relativistic heavy ion collisions has been proven to be a powerful tool for the study of highly excited nuclear matter, both in terms of the reaction dynamics and in terms of its hadrochemistry. Furthermore, strangeness has been suggested as a signature for the creation of a Quark-Gluon-Plasma (QGP): in the final state of a heavy-ion collision, subsequent to the formation and decay of a QGP, strangeness has been predicted to be enhanced relative to the strangeness yield in elementary hadron-hadron collisions. Strangeness production is suppressed in purely hadronic collisions due to the large constituent strange quark mass, whereas (anti-)strange quarks are nearly massless in a QGP due to the restoration of chiral symmetry and are therefore produced in abundance. The chemical equilibration times for strange and multistrange particles have been shown to be considerably shorter in the QGP phase than in a thermally equilibrated hadronic fireball. The dominant strangeness production mechanism, i.e. $gg \rightarrow s\overline{s}$, should allow for equilibration times similar to the expected QGP lifetime.

Recent data from the Relativistic Heavy-Ion Collider (RHIC) at Brookhaven Lab have provided strong evidence for the existence of a transient QGP – among the most exciting findings are strong (hydrodynamic) collective flow, the suppression of high-$p_T$ particles and evidence for parton recombination as hadronization mechanism at intermediate transverse momenta.

However, the dynamics of initial strangeness production in the deconfined phase at RHIC, how it relates to the data, and what we can learn from it, has not received much attention lately. In this letter, we address this topic and investigate strangeness production in the framework of a microscopic transport model – the Parton Cascade Model – which is well suited for the investigation of the pQCD driven early deconfined reaction phase of a ultra-relativistic heavy-ion collision.

### The Parton Cascade Model

The parton cascade model (PCM) provides a detailed space-time description of nuclear collisions at high energy, from the onset of hard interactions among the partons of the colliding nuclei up to the moment of hadronization. The PCM is best suited for the description of the early, pre-equilibrium reaction phase, which is dominated by perturbative hard scattering processes. Due to the introduction of a momentum cut-off, needed to regularize the IR-divergence of the perturbative parton-parton cross sections, the PCM is not equipped to account for soft (re)scatterings, which would dominate a thermalized ensemble of quarks and gluons.

The PCM is based on the relativistic Boltzmann equation for the time evolution of the parton density due to perturbative QCD interactions:

$$p^\mu \frac{\partial}{\partial x^\mu} F_i(x, p) = C_i[F].$$  \hspace{1cm} (1)

The collision term $C_i$ is a nonlinear functional of the phase-space distribution function.
Strangeness Production at RHIC in the Perturbative Regime

$F(x,p)$, containing the matrix elements which account for the following processes:

\[
\begin{align*}
    gg & \rightarrow gg \\
    gg' & \rightarrow gg' \\
    q\bar{q} & \rightarrow q\bar{q} \\
    q\bar{q} & \rightarrow gg \\
    q\bar{q} & \rightarrow q\gamma \\
    q\bar{q} & \rightarrow g\gamma
\end{align*}
\]

(2)

with $q$ and $q'$ indicating different quark flavors. The processes in parentheses can be optionally disabled to exclude production due to them (see later). The corresponding scattering cross sections are expressed in terms of spin- and color-averaged amplitudes $|M|^2$ [26]:

\[
\left(\frac{d\hat{\sigma}}{dQ^2}\right)_{ab\rightarrow cd} = \frac{1}{16\pi s^2} \langle |M|^2 \rangle
\]

(3)

The total cross section, necessary for the transport calculations, is obtained from (3):

\[
\hat{\sigma}_{ab}(s) = \sum_{c,d} \int_{(p_T^{\text{min}})^2}^s \left(\frac{d\hat{\sigma}}{dQ^2}\right)_{ab\rightarrow cd} dQ^2 \]

(4)

The low momentum-transfer cut-off $p_T^{\text{min}}$ regularizes the infrared divergence of the parton-parton cross section. For the initial parton distribution, we choose the GRV-HO parameterization [27] and sample the distribution functions at the initialization scale $Q_0^2$. For our calculation we choose $Q_0^2 = (p_T^{\text{min}})^2 = 0.5 \text{ GeV}^2$. Initial state partons are virtual, i.e. their momentum distribution is governed by the parton distribution function, yet they propagate with the velocity of their parent hadron.

Additionally, we include the branchings $q \rightarrow qg$, $g \rightarrow gg$ and $g \rightarrow q\bar{q}$ [28]. The soft and collinear singularities in the showers are avoided by terminating the branchings when the virtuality of the time-like partons drops below $\mu_0 = 1 \text{ GeV}$. The present work is based on the thoroughly revised, corrected, and extensively tested implementation of the original parton cascade model [29], called VNI/BMS [30].

The PCM distinguishes between two types of strangeness – intrinsic strangeness and produced strangeness. Intrinsic strangeness arises from strange quarks that are already contained in the Dirac sea of the initial parton distribution, and are released when they are put on-shell through scattering. These scattering processes can either be elastic (anti-)quark – (anti-)quark (e.g. $qs \rightarrow qs$) or elastic (anti-)quark – gluon (i.e. $gs \rightarrow gs$) interactions. Produced strangeness refers to $s\bar{s}$ pairs newly created in binary collisions, such as $q\bar{q} \rightarrow s\bar{s}$ and $gg \rightarrow s\bar{s}$ as well as in time-like branching processes, e.g. $g \rightarrow s\bar{s}$. Strange quarks produced from these processes constitute the enhancement of strangeness in the system.

Dynamics of Strangeness Production

Our analysis is set up to investigate the following scenarios:
• **primary-primary scattering:** all partons are allowed to scatter only once. Interactions are limited to elastic scattering, no annihilation or pair production is permitted (i.e. the processes in parentheses in table 2 are disabled), and time-like branching processes are also disabled. This calculation will allow for the analysis of the release of strangeness from the initial state through “first-chance” energetic initial state parton scattering.

• **full scattering:** partons are allowed to rescatter multiple times. Interactions are again limited to elastic scattering without time-like branching (i.e. the processes in parenthesis in table 2 remain disabled). Analysis of this calculation will provide an estimate for the amount of strangeness released from the initial state in secondary interactions.

• **full production:** partons may rescatter both elastically as well as inelastically, and undergo time-like branchings in the final state. This calculation allows for both the initial state release as well as the additional production of $s\bar{s}$ pairs, and therefore provides an estimate of the total strangeness production in the pQCD regime accessible by the PCM.

None of the above described calculations contain hadronization. We evaluate the $s\bar{s}$ distributions at the level of quarks — these can then be compared to the $s$ and $\bar{s}$ valence quark content of measured hadron distributions.

Figure 1 shows the rapidity distribution for all three scenarios as well as for the $s\bar{s}$ distribution of the initial state. Central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV are shown with full symbols, whereas the open symbols refer to a sample of minimum bias p-p collisions at the same energy. The p-p yield has been scaled by 197: if the Au-Au calculations were just an independent superposition of 197 nucleon + 197 nucleon collisions, the two curves would fall on top of each other.

Frame (a) shows the rapidity distribution of the initial $s$ and $\bar{s}$ (sea) quarks contained in the parton distributions of the Au-Au and p-p systems, which serves as the baseline for our strangeness enhancement analysis. Since the partons in the initial state are virtual, their rapidities are ill-defined. However, one can relate a parton’s longitudinal momentum $p_z$ before or immediately after the first collision of a parton to the rapidity variable $y = Y + \ln x + \ln(M/Q_s)$, where $Y$ is the rapidity of the fast moving nucleon, $M$ is the nucleon mass, and $Q_s$ denotes the typical transverse momentum scale. Depending on the picture of the initial state, $Q_s$ is either given by the average intrinsic virtuality, often called the saturation scale $[31]$, or by the typical transverse momentum given to the parton in the first interaction which brings it onto the mass shell. In any case, $|\ln(M/Q_s)| < 1/2$ for Au-Au collisions at RHIC. This initial state baseline can now be compared to frame (b), showing the primary-primary scattering calculation, frame (c), showing the full scattering calculation and frame (d), which displays the full production calculation.

The release of intrinsic strangeness via scattering can therefore be studied by comparing frames (b) and (c), and the enhancement of strangeness via $s\bar{s}$ production can
Figure 1. Distribution of (anti-) strange quarks as a function of rapidity $y$ for central Au-Au (full symbols) and $p$-$p$ (open symbols) collisions at $\sqrt{s_{NN}} = 200$ GeV. Proton-proton results are scaled by $N_{\text{part, } Au-Au} / N_{\text{part, } p-p} = 197$. (a) shows the initial state strange (sea) quark distribution. (b) shows a calculation with primary-primary scattering, whereas all scattering processes (primary-primary, primary-secondary, and secondary-secondary) are turned on in (c). (d) shows the full production calculation where in addition to the (re-)scattering processes (c) strangeness production is enabled via inelastic binary collisions and time-like branching.

Figure captions:

\textbf{Figure 1.} Distribution of (anti-) strange quarks as a function of rapidity $y$ for central Au-Au (full symbols) and $p$-$p$ (open symbols) collisions at $\sqrt{s_{NN}} = 200$ GeV. Proton-proton results are scaled by $N_{\text{part, } Au-Au} / N_{\text{part, } p-p} = 197$. (a) shows the initial state strange (sea) quark distribution. (b) shows a calculation with primary-primary scattering, whereas all scattering processes (primary-primary, primary-secondary, and secondary-secondary) are turned on in (c). (d) shows the full production calculation where in addition to the (re-)scattering processes (c) strangeness production is enabled via inelastic binary collisions and time-like branching.

...be seen by comparing frame (d) with frame (c). We find that the rapidity distribution of released strangeness still has the double-hump structure of the initial state, where the positions of the peaks are shifted by several units of rapidity. Approximately 13% of the initial strangeness is released by primary-primary interactions, with another 10% being set free through primary-secondary and secondary-secondary rescatterings. Newly created $s\bar{s}$ pairs, however, are mostly produced around mid-rapidity and exhibit a roughly Gaussian rapidity distribution. For Au-Au collisions, a comparison between frames (c) and (d) shows that about 40% of the total strangeness yield is released from the initial state, while nearly 60% is newly produced. The total strangeness yield ($s + \bar{s}$) of the Au-Au PCM calculation accounts for 55% of STAR’s measured yield at mid-rapidity. This undersaturation of strangeness production in the PCM is most likely due to the limitation of the model to pQCD processes. Previous calculations based on pQCD rate-equations or a very early implementation of the Parton Cascade Model confirm this trend.

The enhancement of strangeness production in Au-Au reactions compared to scaled $p$-$p$ collisions increases from primary-primary scattering (b) to full scattering (c) to full production (d). The enhancement factors for the integrated yield in the three scenarios are, respectively, 4.5, 6.0, and 10.5. We attribute this increase in the strangeness yield enhancement to be primarily caused by the enhanced probability of parton rescattering in Au-Au versus $p$-$p$. 
A common way of comparing strangeness yields and enhancement across different systems is by calculating the Wroblewski factor $\lambda_s$, defined as:

$$\lambda_s \equiv \frac{2\langle ss\rangle}{\langle u\bar{u} \rangle + \langle dd \rangle} \quad (5)$$

In our analysis we only use the multiplicities of those partons which have been released or produced through a binary scattering or a branching process. The Wroblewski factors extracted for the different modes of our calculations can be found in table 1. For the full Au-Au calculation we find a Wroblewski factor of 0.5. However, this result has to be interpreted very cautiously and cannot be directly compared to the experimental findings, since our extraction of the Wroblewski factor neglects all hadronization effects, e.g. the splitting of gluon into quark-anti-quark pairs at hadronization. Furthermore, it remains unclear whether all released or produced quarks in the PCM will be converted into valence quarks of hadrons at hadronization.

We quantify the amount of parton rescattering and production enhancement in figure 2: the upper frame shows the average number of collisions quarks undergo as a function of rapidity for Au-Au as well as for p-p, whereas the lower frame displays the
quark production enhancement factor, namely the ratio of the $u, d$ and $s$ quark rapidity distributions of Au-Au over the scaled p-p distributions. As can be seen, strangeness enhancement is largest at mid-rapidity, which correlates to (anti-)strange quarks that have scattered the most number of times. We also find that (anti-) up and down quarks scatter less often than (anti-) strange quarks in Au-Au collisions, and that $u\bar{u}$ and $d\bar{d}$ production has a smaller dependence on rescattering than $s\bar{s}$ production.

The lower frame of figure 3 shows the number distribution of binary parton-parton center of mass collision energies (i.e. $\sqrt{\hat{s}}$) for the process $gg \rightarrow s\bar{s}$ and $q\bar{q} \rightarrow s\bar{s}$ for central Au-Au (circles) and p-p (squares) collisions at 200 GeV. p-p results are scaled with the ratio of the number of elastic $gg \rightarrow gg$ and $q\bar{q} \rightarrow gg$ reactions in Au-Au relative to p-p, thus allowing us to determine in which regime (anti-)strange quark pair-production in Au-Au is enhanced (or suppressed) compared to p-p. The scaling with respect to elastic $gg \rightarrow gg$ reactions quantifies this enhancement beyond the trivial increase due to the larger parton density in Au-Au vs. p-p. A significant enhancement of pair production is observed for for small $\sqrt{\hat{s}}$ up to 2.5 GeV followed by a suppression with increasing $\sqrt{\hat{s}}$ beyond 4 GeV. This behavior can be explained by analyzing the $\sqrt{\hat{s}}$ dependence of the respective elastic and inelastic $gg$ scattering cross sections, which are shown in the upper frame of figure 3: the cross section for (anti-) strange quark production decreases strongly with increasing $\sqrt{\hat{s}}$, while the elastic gluon-gluon cross section remains constant. Since the inelastic $gg \rightarrow s\bar{s}$ process dominates strangeness production, a decrease in $gg \rightarrow s\bar{s}$ cross section with increasing $\sqrt{\hat{s}}$ suppresses strangeness production for parton-parton collisions at high $\sqrt{\hat{s}}$. At the same time, partons in Au-Au collisions scatter and fragment much more frequently than in p-p collisions, and therefore lose energy relatively quickly. This ensures that partonic collisions at smaller $\sqrt{\hat{s}}$ are more likely in nucleus-nucleus interactions. The implication of this finding is broad: strangeness production is driven by the amount of quasi-thermal rescattering at small parton-parton $\sqrt{\hat{s}}$ rather than by a large amount of available energy.

Finally, we wish to address the centrality dependence of strangeness production in the perturbative domain. In our analysis, we varied the centrality of the system by two different methods: (a) by varying the target/projectile size for central collisions and (b) by varying the impact parameter for Au-Au collisions. The results of this study for the

|               | Au-Au | p-p  |
|---------------|-------|------|
| primary-primary scattering | 0.26  | 0.25 |
| full scattering         | 0.31  | 0.28 |
| full production            | 0.52  | 0.36 |

Table 1. Wroblewski factor $\lambda_s$, calculated on the basis of released/produced (anti-)strange quarks in the PCM.
Strangeness Production at RHIC in the Perturbative Regime

Figure 3. (a) Center of mass energy dependence of the g-g elastic and inelastic cross sections. The inelastic cross section may lead to the pair production of (anti-) strange quarks. The cross section for (anti-) strange quark production decreases with $\sqrt{s}$, while the elastic gluon cross section remains constant. (b) Binary parton-parton center of mass collision energy distribution for the reaction $gg \rightarrow s\bar{s}$ and $q\bar{q} \rightarrow s\bar{s}$ as a function of $\sqrt{s}$ for central Au-Au (circles) and p-p (squares) collisions at 200 GeV. p-p results are scaled with the ratio of the number of elastic $gg \rightarrow gg$ reactions in Au-Au relative to p-p. A significant enhancement of pair production is observed for for small $\sqrt{s}$ followed by a suppression with increasing $\sqrt{s}$.

three production scenarios are shown in figure 4. Full symbols represent calculations of varying target/projectile size, whereas stars represent calculations with varying impact parameter. For each of the three production scenarios, the total integrated strangeness multiplicity increases with $(N_{\text{part}})^{4/3}$ (fitted curves), indicative of a scaling with binary collisions, which should not come as a surprise given the pQCD input of our PCM calculation.

Summary and outlook

We have studied the production of strangeness in Au-Au collisions at RHIC in the framework of the Parton Cascade Model (PCM). The yields of (anti-) strange quarks for three production scenarios — primary-primary scattering, full scattering, and full production — have been compared to a proton-proton baseline. In Au-Au collisions at $\sqrt{s}_{NN} = 200$ GeV, about 40% of the strangeness yield is released from the initial state, and 60% of the yield is newly produced through binary parton-parton interactions and final state radiation. We find an enhancement of strange quark yields in central Au-Au collisions compared to scaled p-p collisions — this enhancement rises with the number of secondary interactions of the respective partons. Strangeness production
Figure 4. Total integrated multiplicity of (anti-) strange quarks as a function of the number of participants $N_{\text{part}}$ for collision systems at $\sqrt{s_{NN}} = 200$ GeV. Full symbols represent central p-p ($N_{\text{part}}=2$), S-S ($N_{\text{part}}=64$), Cu-Cu ($N_{\text{part}}=128$), Ag-Ag ($N_{\text{part}}=216$), and Au-Au ($N_{\text{part}}=394$) collisions. Stars are the results of Au-Au collisions at various impact parameters. Full production (circles), full scattering (squares), and primary-primary scattering (triangles) processes are the same as previously defined. The yields of all three scenarios increase with $(N_{\text{part}})^{4/3}$ (fitted curves).

therefore seems to be sensitive to quasi-thermal rescattering. A comparison with STAR’s measurement shows that the PCM can account for 55% of the observed strangeness yield at mid-rapidity. The underprediction of the measured strangeness yield can be attributed to the limitation of the PCM to the dynamics of the early pQCD driven pre-equilibrium phase of the heavy-ion reaction. This limitation is largely due to the momentum cut-off in leading order pQCD calculation for binary cross sections in the PCM, which prohibits soft non-perturbative scatterings. Naturally one would expect a significant amount of strangeness to be produced in the later thermalized QGP phase of the reaction. A more comprehensive treatment of parton-parton interactions, e.g. via the introduction of a screening mass instead of a hard momentum cut-off may allow the PCM to account for a higher fraction of the experimentally observed strangeness. In the future we shall therefore turn our attention to the investigation of strangeness balance functions, strangeness equilibration in infinite partonic matter and the estimation of soft, nonperturbative contributions to strangeness production.

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References

[1] J. Rafelski and B. Müller, Phys. Rev. Lett. 48, 1066 (1982).
[2] P. Koch, B. Müller, J. Rafelski, Phys. Rep. 142, 167 (1986).
[3] P. Koch, B. Müller, H. Stöcker, W. Greiner, Mod. Phys. Lett. A3, 737 (1988).
[4] P. Braun-Munzinger, J. Stachel, J. P. Wessels and N. Xu, Phys. Lett. B 344, 43 (1995) arXiv:nucl-th/9410026.
[5] J. Letessier, A. Tounsi and J. Rafelski, Phys. Lett. B 389, 586 (1996).
[6] S. E. Vance and M. Gyulassy, Phys. Rev. Lett. 83, 1735 (1999) [arXiv:nucl-th/9901009].
[7] S. Soff, S. A. Bass, M. Bleicher, L. Bravina, E. Zabrodin, H. Stöcker and W. Greiner, Phys. Lett. B 471, 89 (1999) arXiv:nucl-th/9907026.
[8] P. Braun-Munzinger, I. Heppe and J. Stachel, Phys. Lett. B 465, 15 (1999) arXiv:nucl-th/9903010.
[9] J. Rafelski, J. Letessier and G. Torrieri, Phys. Rev. C 64, 054907 (2001) [Erratum-ibid. C 65, 069902 (2002)] arXiv:nucl-th/0104042.
[10] P. Huovinen, P. F. Kolb, U. W. Heinz, P. V. Ruuskanen and S. A. Voloshin, Phys. Lett. B 518, 41 (2001) arXiv:hep-ph/0105229.
[11] P. Braun-Munzinger, I. Heppe and J. Stachel, Phys. Lett. B 465, 15 (1999) arXiv:nucl-th/9903010.
[12] D. Teaney, J. Lauret and E. V. Shuryak, arXiv:nucl-th/0110037.
[13] S. S. Adler et al. [PHENIX Collaboration], Phys. Rev. Lett. 91, 182301 (2003) arXiv:nucl-ex/0305013.
[14] S. Esumi (for the PHENIX collaboration), Nucl. Phys. A 715, 599 (2003).
[15] C. Adler et al. [STAR Collaboration], Phys. Rev. Lett. 90, 032301 (2003); ibid. 89 132301 (2002); ibid. 87 182301 (2001).
[16] J. Adams et al. [STAR Collaboration], nucl-ex/0306007.
[17] X. N. Wang, Phys. Rev. C 63, 054902 (2001).
[18] M. Gyulassy, I. Vitev, X. N. Wang, Phys. Rev. Lett. 86, 2537 (2001).
[19] K. Adcox et al. [PHENIX Collaboration], Phys. Rev. Lett. 88, 022301 (2002).
[20] J. Adams et al. [STAR Collaboration], Phys. Rev. Lett. 91, 172302 (2003) arXiv:nucl-ex/0305015.
[21] R. J. Fries, B. Müller, C. Nonaka and S. A. Bass, Phys. Rev. Lett. 90, 202303 (2003).
[22] V. Greco, C. M. Ko, and P. Lévai, Phys. Rev. Lett. 90, 202303 (2003).
[23] D. Molnar and S. A. Voloshin, Phys. Rev. Lett. 91, 092301 (2003) arXiv:nucl-th/0307014.
[24] C. Nonaka, R. J. Fries and S. A. Bass, Phys. Lett. B in print, arXiv:nucl-th/0308051.
[25] K. Geiger and B. Müller, Nucl. Phys. B 369, 600 (1992).
[26] R. Cutler and D. W. Sivers, Phys. Rev. D 17, 196 (1978); B. L. Combridge, J. Kripfganz and J. Ranft, Phys. Lett. B 70, 234 (1977).
[27] M. Glück, E. Reya and A. Vogt, Z. Phys. C 67, 433 (1995).
[28] M. Bengtsson and T. Sjöstrand, Phys. Lett. B 185, 435 (1987); Nucl. Phys. B 289, 810 (1987).
[29] K. Geiger, Comput. Phys. Commun. 104, 70 (1997).
[30] S. A. Bass, B. Müller and D. K. Srivastava, Phys. Lett. B 551, 277 (2003).
[31] L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rept. 100, 1 (1983); D. Kharzeev and M. Nardi, Phys. Lett. B 507, 121 (2001).
[32] Helen Caines, private communication.
[33] T. Biro, E. van Doorn, B. Müller, M. H. Thoma and X. N. Wangr., Phys. Rev. C 48 (1993) 1275; P. Levai and X. -N. Wang, arXiv:hep-ph/9504214. D. Pal, A. Sen, M. G. Mustafa, and D. K. Srivastava, Phys. Rev. C 65, 034901, 2002.
[34] K. Geiger and J. I. Kapusta, Phys. Rev. D 47 (1993) 4905.
[35] A. Wroblewski, Acta Phys. Polon. B 16, 379 (1985).