Heat and mass transfer effect on MHD natural convection flow past a moving vertical plate

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Abstract. The present study considers the effect of heat and mass transfer on MHD natural convective flow of an incompressible electrically conducting fluid past a moving vertical plate. The dimensionless governing non-linear partial differential equations for this investigation are solved numerically using finite element method. The effect of the various dimensionless parameters entering into the problem on the velocity, temperature and concentration profiles across the boundary layer is investigated through graphs.

1. Introduction
The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Possible applications of this type of flow can be found in many industries like power industry and chemical process industries. Many natural phenomena and engineering applications are susceptible to magnetohydrodynamic (MHD) analysis. From technological point of view, magnetohydrodynamic flow finds application in the fields of stellar and planetary magneto-spheres, aeronautics, meteorology, solar physics, cosmic fluid dynamics, chemical engineering, electronics and induction flow metry, MHD generators, MHD accelerators, construction of turbine and other centrifugal machines. Very often, along with the free convection currents caused by the temperature difference, the flow is also affected by the difference in concentration of material constituents. In engineering application, the concentration differences are created either by injecting foreign gases or by coating a substrate with a material, and subsequently heating it, so that the material evaporates. Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years.

The problem of free convection under the influence of the magnetic field has attracted the interest of many researchers in view of its applications in geophysics and astrophysics. The problem under consideration has important applications in the study of geophysical formulations, in the explorations and thermal recovery of oil, and in the underground nuclear waste storage sites. The unsteady natural convection flow past a semi-infinite vertical plate was first solved by Soundalgekar and Ganesan [1]. Elbashbeshy [2] studied the heat and mass transfer along a vertical plate under the combined buoyancy effects of thermal and species diffusion, in the
presence of the magnetic field. Ganesan and Palani [3] obtained a numerical solution of the unsteady MHD flow past a semi infinite isothermal vertical plate using the finite difference method. Das et al. [4] investigated the hydromagnetic convective flow past a vertical porous plate through a porous medium with suction and heat source. Palani and Kim [5] analyzed the effect of thermal radiation on convection flow past a vertical cone with surface heat flux. Sivaiah et al [6-8] studied the finite element solution of heat and mass transfer past an impulsively started infinite vertical plate.

In this paper we studied that the heat and mass transfer effect on magneto hydrodynamics natural convection flow past a moving vertical plate. The problem is governed by system of coupled non linear partial differential equations whose exact solution is difficult to obtain, so, the problem is solved by using Galerkin finite element method which is more economical from computational view point.

2. Mathematical formulation
An unsteady two-dimensional hydro magnetic laminar mixed convective boundary layer flow of a viscous, incompressible, electrically conducting and chemically reacting fluid in an optically thin environment, past a semi infinite vertical permeable moving plate embedded in a uniform porous medium, in the presence of thermal radiation is considered. The $x'$ axis is taken in the upward direction along the plate and $y'$ axis normal to it. A uniform magnetic field is applied in the direction perpendicular to the plate. Now, under the usual Boussinesqs approximation, the governing boundary layer equations are:

Continuity Equation:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_0'(\text{constant}).$$

Momentum Equation:

$$\frac{\partial u'}{\partial t'} + v \frac{\partial u'}{\partial y'} = \rho g \beta (T' - T'_{\infty}) + \rho \beta^* (C' - C'_{\infty}) + v \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' - \frac{v}{k} u'.$$

Energy Equation:

$$\frac{\partial T'}{\partial t'} + v \frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2}.$$  

Concentration Equation:

$$\frac{\partial C'}{\partial t'} + v \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2}.$$  

The boundary conditions of the problem are:

$$t' \leq 0 : u' = 0, v' = 0, T' = 0, C' = 0, \text{ for all } y'.$$

$$t' > 0 : u' = 0, v' = -v_0', T' = T'_w + \epsilon (T'_w - T'_\infty) e^{i n' t'}, C' = C'_w + \epsilon (C'_w - C'_\infty) e^{i n' t'} \text{ at } y' = 0,$$

$$u' \to 0, T' \to T'_\infty, C' \to C'_\infty \text{ as } y' \to \infty.$$  

Introducing the following non dimensional variables and parameters,

$$y = \frac{y' v_0'}{v}, t = \frac{t' v_0'^2}{v}, n = \frac{v n'}{v_0'^2}, u = \frac{u'}{v_0'}, M = \left( \frac{\sigma B_0^2}{\rho} \right) \frac{v}{v_0'^2}, K = \frac{K}{v_0'^2}, Sc = \frac{v}{D}, Pr = \frac{v}{k'},$$

$$Gr = \frac{vg \beta (T'_w - T'_\infty)}{v_0'^3}, Gc = \frac{v \beta^* (C'_w - C'_\infty)}{v_0'^3}, \theta = \frac{(T' - T'_{\infty})}{(T'_w - T'_\infty)}, C = \frac{(C' - C'_{\infty})}{(C'_w - C'_\infty)}.$$  

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Substituting (8) and (9) in equations (2), (3) and (4) under boundary conditions (5) to (7), we get

\[
\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = (Gr) \theta + (Gc)C + \frac{\partial^2 u}{\partial y^2} - \left( M + \frac{1}{K} \right) u, \tag{10}
\]

\[
\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2}, \tag{11}
\]

\[
\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2}. \tag{12}
\]

The corresponding boundary conditions are:

\[
\begin{align*}
&u = 0, \theta = 1 + \epsilon e^{nt}, C = 1 + \epsilon e^{nt}ay = 0, \quad \text{at} \quad y = 0, \\
&u \to 0, \theta \to 0, C \to 0 \text{ as } y \to \infty. \tag{13}
\end{align*}
\]

3. Method of solution

The systems of coupled nonlinear equations (10), (11) and (12) with the relevant boundary conditions (13) and (14) are solved numerically for the velocity, temperature and concentration distributions, by Bathe [9] and Reddy [10]. The finite element method is used to obtain an accurate and efficient solution to the boundary value problem under consideration. The fundamental steps comprising the method are as follows:

3.1. Step 1: Discretization of the domain into elements:

The whole domain is divided into finite number of sub-domains, a process known as discretization of the domain. Each sub-domain is termed a finite element. The collection of elements is designated the finite element mesh.

3.2. Step 2: Derivation of the element equations:

The derivation of finite element equations i.e. algebraic equations among the unknown parameters of the finite element approximation, involves the following three steps: a) Construct the variational formulation of the differential equation. b) Assume the form of the approximate solution over a typical finite element. c) Derive the finite element equations by substituting the approximate solution into variational formulation.

3.3. Step 3: Assembly of element equations:

The algebraic equations so obtained are assembled by imposing the inter-element continuity conditions. This yields a large number of algebraic equations, constituting the global finite element model, which governs the whole flow domain.

3.4. Step 4: Impositions of boundary conditions:

The physical boundary conditions defined in equation (13) and (14) are imposed on the assembled equations.

3.5. Step 5: Solution of the assembled equations:

The final matrix equation can be solved by a direct or indirect (iterative) method. Numerical solutions for these equations are obtained by MATLAB. In order to prove the convergence and stability of finite element method, the same MATLAB was run with slightly changed values of u, \( \theta \) and C, no significant change was observed in the values of u, \( \theta \) and C. This process is repeated until the desired accuracy of 0.0005 is obtained. Hence, the finite element method is stable and convergent.
4. Results and discussion
To understand the physical meaning of the problem, we have computed the expression for $u$, $\theta$ and $C$ for different values of thermal Grashof number $Gr$, solutal Grashof number $Gc$, magnetic field parameter $M$, permeability parameter $K$, Prandtl number $Pr$, Schmidt number $Sc$. The purpose of the numerical result given here is to assess the effects of different parameters upon the nature of the flow, temperature and concentration. The values of the Prandtl number are chosen $Pr = 7$ (water) and $Pr = 0.71$ (air). The values of the Schmidt number are chosen to represent the presence of species by hydrogen (0.22), water vapor (0.60) and ammonia (0.78). In the present study we have chosen $Gr = 2.0$, $Gc = 2.0$, $M = 1.0$, $K = 0.5$, $Pr = 0.71$, $Sc = 0.22$, $\varepsilon = 0.2$, $n = 1.0$, $t = 1.0$ while $Gr, Gc, M, K, Pr, Sc$ are varied over a range, which are listed in the figures.

![Velocity profiles for different values of $Gr$](image1)

![Velocity profiles for different values of $Gc$](image2)

Figure 1: Velocity profiles for different values of $Gr$ and $Gc$.

![Velocity profiles for different values of $M$](image3)

![Velocity profiles for different values of $K$](image4)

Figure 2: Velocity profiles for different values of $M$ and $K$.

The temperature and the species concentration are coupled to the velocity via thermal Grashof number $Gr$ and solutal Grashof number $Gc$ as seen in equation (10). For various values of thermal Grashof number and solutal Grashof number, the velocity profiles are plotted in figures 1.(a) and (b). The thermal Grashof number $Gr$ signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is
observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Also, as $Gr$ increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity. The solutal Grashof number $Gc$ defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. It is noticed that the velocity increases with increasing values of the solutal Grashof number. The effect of the Hartmann number $M$ is shown in figure 2(a). It is observed that the velocity of the fluid decreases with the increase of the magnetic field number values. The decrease in the velocity as the Hartmann number $M$ increases is because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This resistive force slows down the fluid velocity component as shown in figure 2(a). For different values of the permeability parameter $K$, the velocity on the porous wall are plotted in figure 2(b). It is obvious that the increased values of $K$ tend to increasing of velocity and on the porous wall and so enhance the momentum boundary layer thickness. Figure 3(a) depicts the effect of Prandtl number $Pr$ against $y$ on the temperature field keeping other parameters of the flow field constant. The Prandtl number is a dimensionless number; the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. It is observed that an increase in the prandlt number decreases the temperature of the flow field at all points. In Figure 3(b), we analyze the effect of Schmidt number $Sc$ on concentration profiles of the flow field. Schmidt number is a dimensionless number defined as the ratio of momentum diffusivity (kinematic viscosity) and mass diffusivity. It is observed that a growing Schmidt number decreases the concentration boundary layer thickness of the flow field at all points.

5. Conclusion

In this work, we have studied the effect of heat and mass transfer on magneto hydrodynamics natural convection flow past a moving vertical plate. The governing equations are solved by using finite element method. The results are shown graphically for different values of the parameters considered in the analysis. The following conclusions can be made from the present investigation:

(i) The effect of increasing Grashof number for heat and mass transfer is to accelerate velocity
of the flow field at all points.

(ii) A growing Hartmann number retards the velocity of the flow field at all points.

(iii) The velocity of the flow field increases with an increase in permeability parameter.

(iv) The Prandtl number increases the temperature is reduce of the flow field at all points.

(v) The effect of increasing Schmidt number is to reduce the concentration boundary layer thickness of the flow field at all points.

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