A complete one-loop description of associated $tW$ production at LHC and a search for possible genuine supersymmetric effects

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Abstract

We compute, in the MSSM framework, the sum of the one-loop electroweak and of the total QED radiation effects for the process $pp \rightarrow tW + X$, initiated by the parton process $bg \rightarrow tW$. Combining these terms with the existing NLO calculations of SM and SUSY QCD corrections, we analyze the overall one-loop supersymmetric effects on the partial rates of the process, obtained by integrating the differential cross section up to a final variable invariant mass. We conclude that, for some choices of the SUSY parameters and for relatively small final invariant masses, they could become relevant under realistic experimental conditions at LHC.

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I. INTRODUCTION

Single top production at LHC has already been emphasized by several authors \cite{1} as one of the processes to be possibly measured with maximal experimental accuracy. The main and known reason is the fact that in the Standard Model from this measurement one would be able to derive the first direct determination of the $Wtb$ coupling $V_{tb}$, assumed to be practically equal to one if unitarity of the CKM matrix is postulated. Since the rates of the three different single top production processes, usually called $t$-channel, $s$-channel, and associated production, are proportional to $|V_{tb}|^2$, an accurate measurement of those rates would correspond, in principle, to a correspondingly accurate measurement of $V_{tb}$. From the preliminary experimental analyses \cite{2} one hopes that a realistic overall accuracy might eventually reach the ten percent size. The corresponding precision of the determination of $V_{tb}$ would be therefore of five percent, if no extra theoretical uncertainties had to be added.

The previous statement requires a description of the status of the existing theoretical calculations. A complete NLO QCD calculation exists for all the three single top processes in the Standard Model \cite{3} with an attached uncertainty that varies with the process. It seems probable that this uncertainty will be essentially reduced by the future LHC related measurements, and we shall return to this point later on in the paper. For the three production channels, SUSY NLO QCD effects have been recently evaluated in the MSSM \cite{4}. The pure electroweak one-loop effects in the MSSM have been computed recently for the associated $tW$ production case \cite{5}. This calculation did not contain, though, the QED radiation corrections. In the case of the total rate, a modest (few percent) relative effect was found. In a subsequent note \cite{6} it was shown that, still at the pure electroweak one-loop level, more interesting effects could be found from the measurement of partial rates, defined as an integral of the differential cross section performed from threshold to a final suitable invariant mass. In particular, for low values of the latter, a positive effect was computed, that obviously could not be due to a violation of the CKM matrix structure leading to smaller values of $V_{tb}$.

Starting from these premises, the purpose of this paper is that of providing a really “complete” calculation of one-loop effects in the MSSM on the partial rates of the $tW$ production process, where for each partial rate the QED effect is also included. Moreover, we shall provide a preliminary evaluation of the overall theoretical uncertainties taking into
account the variation of the final considered invariant mass. The result will be, essentially, an indication of an “optimal” choice of partial rate, i.e. of one for which the two combined requests of maximal electroweak effect and of minimal overall uncertainty might be satisfied. This could lead to a relevant precision test of the MSSM at LHC, whose accuracy would certainly improve with time if preliminary encouraging signals were revealed, motivating more accurate dedicated experimental measurements.

The paper will be organized in the following way: section 2 will contain a review of the electroweak calculation, essentially shortened since all the necessary details have already been given in Ref. [5]. In section 3, the new calculation of the additional QED radiation effect will be described. Section 4 will contain a short discussion of the existing QCD calculations and of their uncertainties, taking into account the considered variation of the final invariant mass. Summarized results will be discussed in Section 5 and a few conclusions will finally appear in Section 6.

II. ELECTROWEAK ONE-LOOP EFFECTS

At the partonic level the associated $tW^-$ production is initiated by the process $bg \to tW$, described at Born level by the two diagrams in Figure 1.

In a previous paper [5], we have analyzed the complete one-loop electroweak corrections to this process treating QED effects in the soft-photon approximation. Here, we summarize briefly the results of Ref. [5] as a background to the complete analysis including hard QED radiation to be discussed in the remaining part of the present paper.

Our starting observable for this process was the invariant mass distribution defined as

$$\frac{d\sigma(pp \to tW^- + X)}{dM_{tW}} = \int dx_1 dx_2 d\cos\theta \left[ b(x_1, \mu) g(x_2, \mu) + g(x_1, \mu) b(x_2, \mu) \right] \times \frac{d\sigma_{bg \to tW^-}}{d\cos\theta} \delta\left(\sqrt{x_1 x_2 S} - M_{tW}\right),$$

(1)

where $\sqrt{S}$ is the proton-proton c.m. energy, $M_{tW}$ is the $tW$ invariant mass, $\theta$ is the top-quark scattering angle in the partonic c.m. frame, and $i(x_i, \mu)$ are the distributions of the parton $i$ inside the proton with a momentum fraction $x_i$ at the scale $\mu$.

The invariant mass distribution $d\sigma/dM_{tW}$ has been evaluated for a number of SUSY benchmark points with a wide variation of mass spectra and values of the mixing parameter $\tan \beta$. In particular, the considered MSSM points have been the standard ATLAS DC2 SU1
and SU6 and two generic mSUGRA points with light spectrum. The calculation has been performed with a kinematical cut on the $W$ or top transverse momentum $p_{T,\text{min}} = 15$ GeV, and we have included a QED soft-photon contribution, computed assuming an upper value of the soft-photon energy $\Delta E = 0.1$ GeV. The main results of our detailed numerical investigation can be summarized as follows. The electroweak one-loop effect in the MSSM is the sum of the pure SM and of the genuine SUSY components. These two terms have a fundamentally different dependence on the invariant mass of the final state. More precisely, the SM effect varies from positive values of approximately 5% in the lowest invariant mass range to larger negative values in the high invariant mass sector. One actually expects from general considerations such negative effects, coming from a logarithmic contribution of Sudakov kind to the asymptotic value of the scattering amplitude. As a consequence of the change of sign of the SM contribution, the overall effect in the total rate is practically vanishing. The genuine SUSY effect has a rather different nature. It remains systematically positive in all the invariant mass range (realistically considered up to a final value of 1 TeV) for all the considered SUSY benchmark points, assuming essentially the same relative effect of 3-4% in all cases, with a maximum value reached in the SU6 point. The lack of a negative large energy effect can be understood as a consequence of the fact that in the considered benchmark points there are relatively large SUSY masses that appear in the virtual loops, which hides the appearance of asymptotic logarithmic Sudakov effects. A possibility that was considered in the note is that of considering partial rates, in particular low energy partial rates, obtained by integrating the differential cross section from threshold to a given final invariant mass. If the latter is fixed at small but meaningful values (say, 400 GeV), the relative SM and genuine SUSY effects sum up. Also, the effect will be positive, therefore not possibly due to those violations of the CKM matrix structure which would decrease the value of $V_{tb}$. This idea is confirmed by the detailed numerical calculation, shown in the next Figure taken from Ref. and computed with the same kinematical and infrared cuts as in Ref. One sees that for values of the final invariant mass in the range mentioned above the MSSM electroweak effect is not negligible, reaching values of 6-7%, that appear possibly relevant.

The previous considerations miss two important points. The first one is an original calculation of the complete (soft and hard) QED effect, which has never been performed before. The second one is a summary of the QCD NLO calculations, including both SM and
SUSY QCD (the latter one will be in fact rather useful for our purposes). These two topics will be discussed in the forthcoming sections.

III. QED RADIATION

The $\mathcal{O}(\alpha)$ electroweak corrections include contributions from virtual and from real photon emission. The virtual photon exchange diagrams belong to the complete set of electroweak virtual corrections, and are necessary for the gauge invariance of the final result. The singularities associated with the massless nature of the photon have been regularized by introducing a small photon mass $m_\gamma$. The real radiation contribution has been split into a soft part, derived within the eikonal approximation, where the photon energy has been integrated from the lower bound $m_\gamma$ to a maximum cut off $\Delta E$, and into a hard part, evaluated by means of a complete calculation of the diagrams shown in Figure 3. The soft real contribution contains explicitly the photon mass parameter $m_\gamma$. The logarithmic terms containing $m_\gamma$ cancel exactly in the sum of virtual and soft real part, leaving only polynomial spurious terms, which approach zero at least as $m_\gamma^2$. The hard contribution, integrated from the minimum photon energy $\Delta E$ to the maximum allowed kinematical value, can be calculated with a massless external photon. The complete matrix element for real radiation, including fermion mass effects, has been calculated analytically with the help of FORM [9], in order to handle efficiently the traces of strings of Dirac gamma matrices. As an internal check, the complete matrix element has been verified to recover (in the limit $k \rightarrow 0$, where $k$ stands for the photon momentum) the analytical expression of the eikonal approximation, factorized over the tree-level amplitude.

The integration over the phase space has been performed numerically with Monte Carlo methods. In order to treat efficiently the regions related to the infrared and collinear singularities, the importance sampling technique has been adopted. In particular the photon energy is generated in the partonic center of mass system according to the distribution $1/E_\gamma$; the photonic angular variables are generated with a multichannel strategy according to the distributions $1/(1 - \beta_i \cos \vartheta_{\gamma i})$, where $i = t, b, W$, $\beta_i$ represent their velocities, and $\vartheta_{\gamma i}$ are the relative angle between the photon and the charged particles $i$.

The final cross section has to be independent of the fictitious separator $\Delta E$, for sufficiently small $\Delta E$ values. This has been checked numerically to hold at the level of few 0.01% for
\( \Delta E \leq 1 \text{ MeV} \), as shown in Figure 4 (right panel), despite the strong sensitivity to \( \Delta E \) of the soft plus virtual and of the hard cross section separately, as shown in Figure 4 (left panel). The leading logarithmic dependence on \( \Delta E \) (with opposite coefficients) in the soft plus virtual and in the hard cross sections is manifest in the logarithmic scale plot.

IV. QCD EFFECTS

A. NLO SM corrections

The NLO SM QCD corrections to the \( tW \) signature have been calculated by various authors \([10, 11, 12]\) within the on-shell approximations for \( W \) boson and top quark. In Ref. \([12]\) the corrections considering also the decay of the top quark have been evaluated. Such a calculation is implemented in the fixed-order Monte Carlo program MCFM \([13]\), v5.1, which we use to estimate the QCD uncertainties associated with the integrated \( tW \) mass distribution. In our simulation we adopt the on-shell approximation for \( W \) and top quark (available as an option in the MCFM code), consistently with the electroweak calculation presented in this study. The input parameter values of the program have been tuned with the ones adopted in the electroweak calculation. For internal perturbative consistency we used in the NLO calculation the CTEQ6M set for PDFs. In addition, at NLO there is the need to fix the factorization and renormalization scales, \( \mu_F \) and \( \mu_R \), respectively. The typical way of studying the remaining QCD theoretical uncertainty of the predictions, as due to missing higher order terms, is to vary \( \mu_F \) and \( \mu_R \) around the relevant scale of the process, which, in the case under study, would be given by \( m_t + m_W \). However, the presence of a \( b \) quark in the initial state introduces two subtleties, which have been addressed in Ref. \([12]\). The real \( \mathcal{O}(\alpha_s) \) radiation contribution contains diagrams where an initial state gluon splits into a \( b \bar{b} \) pair giving rise to the \( Wtb \) final state. The collinear \( g \rightarrow b\bar{b} \) splitting is already accounted for in the \( b \)-quark distribution function, used in the lowest order calculation. Therefore the net contribution from the \( gg \rightarrow Wtb \) diagrams should be approximately zero, including appropriate counter-terms and integrating over all \( b \)-quark transverse momenta up to \( \mu_F \). In Ref. \([12]\) this has been checked to happen for \( \mu_F \leq 65 \text{ GeV} \). An additional problem associated with the \( gg \rightarrow Wtb \) diagrams arises in the portion of the phase space where the \( Wb \) system crosses over the pole of the virtual top-quark propagator. Actually
this contribution represents the doubly resonant $t\bar{t}$ production process, and it is preferable to exclude it from the NLO corrections to the $tW$ process \[12\]. This is achieved in MCFM by applying a veto on the $p_T$ of the additional $b$ quark that appears at next-to-leading order. For consistency, the maximum allowed $p_T$ of the $b$ quark should be chosen of the same order of $\mu_F$.

For the above reasons we have selected $\mu = \mu_F = \mu_R = 50$ GeV and $p_T^b \, \text{veto} = 50$ GeV, as in Ref. \[12\], and studied the variation of the resulting $K$-factor (defined as $\sigma_{NLO}/\sigma_{LO}$) in the range $25$ GeV $\leq \mu \leq 255$ GeV. The $K$-factor ranges from 1.26 for $\mu = 25$ GeV to 1.12 for $\mu = 50$ and 100 GeV (1.17 for $\mu = 255$ GeV). The inclusive NLO cross section varies from $36.07(1)$ pb for $\mu = 25$ GeV to $34.84(1)$ pb for $\mu = 50$ GeV, $35.09(1)$ pb for $\mu = 100$ GeV and $35.86(1)$ pb for $\mu = 255$ GeV, thus showing a stability at the level of few per cent, in agreement with Figure 7 of Ref. \[12\].

At this point we can study the stability of the QCD NLO predictions on the partial rates. In Figure 5, we quantify the size of the NLO QCD corrections showing the differential $K$-factor, with $p_T^b \, \text{veto} = 50$ GeV. The curves correspond to the values $\mu = 25$ (solid line), 50 (dot-dashed line) and 100 GeV (dashed line) in the NLO calculation, while the scale of LO calculation is kept fixed at the value $\mu_0 = 50$ GeV. The size of the QCD corrections is of the order of 20%, decreasing by about 10% when $M_{tW}$ ranges from threshold to 1 TeV. The scale uncertainties are lower than about 4%, being maximal at the highest $M_{tW}$.

B. SUSY QCD corrections

The one-loop SUSY QCD effects to the three channels of single top production have been computed at LHC in Ref. \[4\]. These are radiative corrections with propagation of virtual gluinos in the quantum loop. The numerical analysis of Ref. \[4\] is performed in the constrained MSSM within mSUGRA. The MSSM parameters are the five inputs at grand unification scale

$$M_{1/2}, \ M_0, \ A_0, \ \tan \beta, \ \text{sign} \mu, \ (2)$$

where $M_{1/2}, M_0, A_0$ are the universal gaugino mass, scalar mass, and the trilinear soft breaking parameter in the superpotential. This last parameter has been set to $A_0 = -200$ GeV. The sign of $\mu$ is positive.

The effects in the associated production channel are studied by evaluating the $K$ factor
defined as the ratio of the SUSY QCD corrected cross sections to LO total cross sections, calculated using the CTEQ6M PDFs set. The dependence of the $K$ factor on the various MSSM parameters is analyzed in great detail.

The dependence on the gluino mass $M_{\tilde{g}} (M_{1/2})$ can be studied at various values of $\tan \beta = 5, 20$ and 35. The $K$ factor increases with $M_{\tilde{g}}$ for small $M_{\tilde{g}} (\lesssim 160 \text{GeV})$, while it decreases with $M_{\tilde{g}}$ for large $M_{\tilde{g}} (\gtrsim 160 \text{GeV})$. In general the dependence on $\tan \beta$ is rather mild. The typical values are about $K = 1.06$.

For example, assuming $\tan \beta = 5$ and computing the $K$ factor as functions of $M_{\tilde{g}} (M_{1/2})$ for different $M_0$, one finds that there are large variations in $K$ when $M_{\tilde{g}}$ becomes small. It decreases with $M_{\tilde{g}}$ and especially rapidly when $M_{\tilde{g}} \lesssim 150 \text{GeV}$ at least for $M_0 = 150 \text{GeV}$. Nevertheless, as soon as $M_{\tilde{g}}$ becomes large, the decoupling of heavy gluinos ($M_{\tilde{g}} \gtrsim 450 \text{GeV}$) gives saturated stable values of $K$. Again $K \simeq 1.06$ for $M_{\tilde{g}} \lesssim 500 \text{GeV}$.

Similar results are obtained by varying the stop mass $M_{\tilde{t}_1} (M_0)$, assuming $\tan \beta = 5$, and $M_{1/2} = 40, 70$ and $100 \text{GeV}$, respectively. The $K$ factor is about 1.06 for most values of $M_{\tilde{t}_1}$, decreasing slowly with $M_{\tilde{t}_1}$.

In conclusion, the typical SUSY QCD correction to the process of associated production is about $+6\%$ for most values of the explored parameters values.

\section*{C. PDF uncertainties}

Another source of theoretical uncertainty is given by the contribution of the parametric errors associated with the parton densities. This could become particularly relevant for single top channels, due to the presence of an initial state $b$ quark, whose distribution function is strictly related to the gluon distribution. We have studied the impact of such uncertainties by using the NLO PDF sets MRST2001E and CTEQ61 as in the LHAPDF package \cite{14}. The results are shown in Figure 6. The spread of the predictions obtained with the MRST set displays a relative deviation of about 1\%, while the CTEQ set gives a larger uncertainty reaching the 3\% level for high $tW$ mass. This is due to different values of the tolerance parameter \cite{15}, the latter being defined as the allowed maximum of the $\Delta \chi^2$ variation w.r.t. the parameters of the best PDFs fit. Conservatively, we can associate to our predictions an uncertainty due to the present knowledge of parton densities of about 3\%. However it is worth noting that the uncertainties obtained according to such a procedure are of purely
experimental origin only \(i.e.\) as due to the systematic and statistical errors of the data used in the global fit), leaving aside other sources of uncertainty of theoretical origin.

V. RESULTS

Coming back to the genuine electroweak corrections, we present in this section our results for the distribution \(d\sigma/dM_{tW}\) and for the integrated cross section in the Standard Model and in the MSSM. The numerical results shown in the following have been obtained in terms of the LO PDF set CTEQ6L with \(\mu = m_t + M_W\), the latter being the top-quark and W-boson mass, respectively.

We begin with the invariant mass distribution \(d\sigma/dM_{tW}\). We show in Figure 7 the electroweak effect in the Standard Model case. The two lines allow to appreciate the effect of the hard photon radiation. It adds a positive contribution to the effect that is uniformly positive over most of the explored energy range. The effects in the MSSM are shown in Figure 8 at the two benchmark points SU1 and SU6. The pattern is similar to that of the Standard Model, in agreement with the general partial decoupling of the genuine SUSY components.

The behaviour of the integrated cross section is shown in Figure 9, for the Standard Model, and in Figure 10 for the MSSM. Again, the pattern is similar. The cross section is integrated from threshold up to a maximum invariant mass \(M_{tW}^{\text{max}}\). The electroweak effect is always positive and is maximum for small \(M_{tW}^{\text{max}}\). This is a consequence of a coherent sum of positive one-loop effects coming from the electroweak sector of the MSSM and from the complete QED contribution. For larger \(tW\) invariant masses, the electroweak SM contribution decreases, and the overall effect is weakened. For what concerns the electroweak SUSY effect, it remains of the order of a few (positive) percent in all the considered benchmark points, the largest effect being obtained in the SU6 case. This is not, though, the total genuine SUSY effect. In fact, to obtain the latter, one still has to add the SUSY QCD contribution of Ref. [4]. The corresponding effect on the integrated partial rates is shown by the dashed curves in the right panels of Figure 10 in a qualitative, but essentially correct, way, simulating the SUSY QCD component by a constant 6\% shift, consistently with the analysis of Ref. [4]. From an inspection of Figure 10 one can conclude that, for final invariant masses of the 400 GeV size, an overall one loop effect of approximately 13-14\% is produced.
in the MSSM by the positive sum of electroweak and SUSY QCD contributions. The size of the genuine one-loop SUSY effect is of, roughly, ten percent. To obtain the complete value of the rates it is now sufficient to add to the values of Figure 10 the remaining SM QCD effect, exhaustively illustrated in section 4. In this way, the complete one-loop expression of the rates can be obtained. In fact, from the calculations that we have performed, the values of other possibly interesting observable quantities can be easily obtained. The reason why we have limited our presentation to the partial rates will be summarized in the final conclusions.

VI. CONCLUSIONS

We have performed in this paper the first (to our knowledge) complete calculation of the one-loop effect on the process of $tW$ production, including a discussion of the overall size of the theoretical uncertainties. Our interest has been concentrated on the particular quantities that we have defined as partial rates, with special emphasis on the low (400 GeV) final invariant mass. The reason of this interest is actually twofold, since with this choice the related quantity meets, at the same time, two conditions on the purely theoretical side. In fact, it maximizes the overall one-loop electroweak (including QED) effect, that can reach the 8% size, a value that should not be neglected. With the addition of SUSY QCD one-loop terms, the genuine SUSY contribution reaches the 10% size. At the same time, it maximizes the theoretical uncertainties that we have considered in the paper. On a purely experimental side, we do not have yet at our disposal a rigorous experimental analysis of the $tW$ production process, which seems to us extremely relevant and necessary. It is well known that the largest background for associated $tW$ production is top quark pair production. From the point of view of signal identification, the region with small $tW$ invariant mass, near the $t\bar{t}$ threshold, has possibly a chance to be optimal. Waiting for a dedicated effort we can though rely on the fact that a measurement of the rate at the ten percent level should be considered as a “must” project for the study of single top production. Should this result be met, the measurement of our partial rates could indeed represent a relevant and original test of genuine SUSY effects in the MSSM at LHC.

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FIG. 1: Born diagrams for the process $bg \rightarrow tW^−$. 
FIG. 2: Integrated cross section (from threshold up to $M_{tW}^{\text{max}}$) in the soft-photon approximation.
FIG. 3: Diagrams for the radiative process $bg \to tW^-\gamma$. 
FIG. 4: Left: dependence of the $\mathcal{O}(\alpha)$ soft plus virtual and hard cross sections on the soft-hard separator $\Delta E$. Right: independence of the sum of $\mathcal{O}(\alpha)$ soft plus virtual and hard cross sections of the separator $\Delta E$, checked numerically up to an accuracy of the order of 0.01%.

FIG. 5: QCD K-factor ($\sigma_{NLO}(\mu)/\sigma_{LO}(\mu_0 = 50 \text{ GeV})$) as a function of $M_{tW}^{\text{max}}$. Solid line, $\mu = 25 \text{ GeV}$, dot-dashed line, $\mu = 50 \text{ GeV}$, dashed line, $\mu = 100 \text{ GeV}$. 
FIG. 6: Left: Integrated cross section from threshold up to $M_{tW}^{\text{max}}$ (LO calculation) with the MRST2001E PDF set. For each bin, the minimum value (solid line), the maximum (dashed), the average (dot-dashed) and the value corresponding to the best fit parton density (dotted) are shown. In the lower panel the relative deviation w.r.t. the best fit PDF is shown. Right: the same as in the left panel, obtained with the set CTEQ61.
FIG. 7: Electroweak one-loop effect on the distribution $d\sigma/dM_{tW}$ in the Standard Model. The histogram without hard photon includes the soft-photon contributions only.
FIG. 8: Electroweak one-loop effect on the distribution $d\sigma/dM_{tW}$ in the MSSM at the two benchmark points SU1, SU6.
FIG. 9: Integrated cross section (from threshold up to $M_{tW}^{\text{max}}$) and electroweak one-loop effect in the Standard Model.
FIG. 10: Integrated cross section (from threshold up to $M_{tW}^{\text{max}}$) and electroweak one-loop effect in the MSSM at the two benchmark points SU1, SU6. SUSY QCD corrections have been inserted in the dashed line of right panel and simulated by a $+6\%$ shift, consistently with the analysis of Ref. [4].