VARIOUS REALIZATIONS OF LEPTOGENESIS AND NEUTRINO MASS CONSTRAINTS

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Seven types of leptogenesis models which can lead to a successful explanation of baryogenesis are presented. Emphasis is put on the conditions which need to be fulfilled by the neutrino masses as well as by the heavy state masses. The model dependence of these conditions is discussed.

1. Introduction

Following the recent convincing evidence for neutrino oscillations, leptogenesis\(^1\) has become a well motivated possible explanation of the origin of the baryon asymmetry of the universe. In these proceedings, after a short introduction on the three basic ingredients of leptogenesis, we will extend the discussion of Ref.\(^2\) along two main directions. First, in the framework of the usual seesaw model with three right-handed neutrinos (“type-I” seesaw model)\(^3\), we will study the neutrino mass constraints, in particular the neutrino mass upper bound, which exist in order that leptogenesis can successfully explain the baryon asymmetry of the universe. We will show how this neutrino mass upper bound can be relaxed by no longer assuming that the right-handed neutrinos have a hierarchical mass spectrum. Secondly, other models of leptogenesis which might also be attractive for various reasons will be introduced: “type-II” seesaw model with a scalar Higgs triplet, “type-III” seesaw model with fermion triplets, and combinations of them such as the “type-I” plus “type-II” model which is motivated by left-right or SO(10) models. The corresponding neutrino mass bounds will be discussed for all these models. Finally, we will also briefly consider the case of radiative neutrino mass models.
2. The Three Basic Ingredients of Leptogenesis

There are essentially three ingredients to take into account for leptogenesis to work. In this section we will discuss only the case of the “type-I” seesaw model with three heavy right-handed neutrinos $N_i$ which is based on the following Lagrangian

$$L = L_{\text{SM}} + \bar{N}_i i\gamma^5 N_i + (\lambda^{ij} H^\dagger N_i L_j + \frac{M_{N_i}}{2} N_i N_i + \text{h.c.}), \quad (1)$$

with $L_j = (\nu_{jL}, l_{jL})^T$, $H = (H^0, H^-)^T$. In Eq. (1) we made the choice to work in the basis where the $N_i$ mass matrix is real and diagonal. This model has 18 parameters; 9 combinations of them enter in the neutrino mass matrix $M_{\nu} = -\lambda^T M_{N_i}^{-1} \lambda v^2$, and nine of them decouple from it. A very useful parametrization (in terms of an orthogonal complex $R$ matrix) which allows to span easily all the parameter space of this model in agreement with the data, and which is very useful to determine the neutrino mass bounds below, can be found in Ref.4,5. For the following we order the neutrino masses as: $m_{\nu_3} > m_{\nu_2} > m_{\nu_1} \geq 0$ and $M_{N_3} \geq M_{N_2} \geq M_{N_1} \geq 0$.

2.1. The CP Asymmetry

At a temperature well above their masses one can expect the right-handed neutrinos to be in thermal equilibrium in the universe thermal bath, due to Yukawa interactions or other possible interactions (such as gauge interactions in the right-handed sector). However, once the temperature drops below their masses the right-handed neutrinos disappear from the thermal bath by decaying to leptons and Higgs bosons. The crucial quantity for leptogenesis is the CP-asymmetry, that is to say the averaged $\Delta L$ which is produced each time one $N_i$ decays. At lowest order, that is to say at one-loop order, for $N_1$ (and similarly for $N_{2,3}$), it is given by

$$\varepsilon_{N_i} = \frac{\Gamma(N_i \rightarrow lH^*) - \Gamma(N_i \rightarrow l\bar{H})}{\Gamma(N_i \rightarrow lH^*) + \Gamma(N_i \rightarrow l\bar{H})} = -\sum_{j=2,3} \frac{3 M_{N_j}}{2 M_{N_j}} \Gamma_{N_j} I_j \frac{2 S_j + V_j}{3} \quad (2)$$

where

$$I_j = \frac{\text{Im}}{\lambda^{|\lambda^T|_{jj}}} \left[ (\lambda^{|\lambda^T|_{jj}}) \right] M_{N_j} \quad \Gamma_{N_j} = \frac{|\lambda^{|\lambda^T|_{jj}}|}{8\pi} \equiv \frac{\bar{m}_j M_{N_j}}{8\pi v^2}, \quad (3)$$

and

$$S_j = \frac{M_{N_j}^2 \Delta M_{ij}^2}{(\Delta M_{ij}^2)^2 + M_{N_i}^2 \Gamma_{N_j}} \quad V_j = \frac{2 M_{N_j}^2}{M_{N_i}^2} \left[ (1 + \frac{M_{N_j}^2}{M_{N_i}^2}) \log \left( 1 + \frac{M_{N_j}^2}{M_{N_i}^2} \right) - 1 \right]. \quad (4)$$
with $\Delta M_{ij}^2 = M_{N_j}^2 - M_{N_i}^2$. The factors $S_j (V_j)$ come from the one-loop self-energy (vertex) contribution to the decay widths, Fig. 1. The $I_j$ factors are the CP-violating coupling combinations entering in the asymmetry.

2.2. The Efficiency Factor

Once the averaged $\Delta L$ produced per decay has been calculated, the second ingredient to consider is the efficiency factor $\eta$. This factor allows to calculate the lepton asymmetry produced from the CP-asymmetry,

$$\frac{n_L}{s} = \varepsilon_{N_i} Y_{N_i} |\Delta L| > M_{N_i} \eta,$$

where $Y_{N_i} = n_{N_i}/s$ is the number density of $N_i$ over the entropy density, with $Y_{N_i} |\Delta L| > M_{N_i} = 135(3)/(4\pi^4 g_*)$ where $g_* = 112$ is the number of degrees of freedom in thermal equilibrium in the “type-I” model before the $N_i$ decayed. If all right-handed neutrinos decay out-of-equilibrium, the lepton asymmetry produced is just given by the CP asymmetry times the number of $N_i$ over the entropy density before the $N_i$ decayed, i.e. $\eta = 1$. However, the efficiency factor can be much smaller than one, if they are not fully out-of-equilibrium while decaying, and/or if there are at this epoch L-violating processes partly in thermal equilibrium. The processes which can put the $N_i$ in thermal equilibrium and/or violate L are the inverse decay process and $\Delta L = 1, 2$ scatterings. To avoid a large damping effect, it is necessary that these processes are not too fast with respect to the Hubble constant.

For the inverse decay process (which is the most dangerous process, see e.g. the discussion of Ref.6), this gives the condition: $\Gamma_{N_i}/H(T \simeq M_{N_i}) \leq 1$ with $H(T) = \sqrt{4\pi^3 g_*/45 T^2}/M_{\text{Planck}}$. In practice to calculate $\eta$ we need to put all these processes in the Boltzmann equations7,8 which allow a precise calculation of the produced lepton asymmetry as a function of the temperature $T$. The corresponding efficiency factor including finite temperature effects can be found in Ref.8 in the limit where the right-handed neutrinos have a hierarchical spectrum $M_{N_i} \ll M_{N_{2,3}}$. In this limit only the

![Figure 1. One-loop diagrams contributing to the asymmetry from the $N_i$ decay.](image-url)
asymmetry produced by the decay of the lightest right-handed neutrino \( N_1 \) survives and is important, which simplifies greatly the calculations.

2.3. The B to L Asymmetry Conversion

Once the \( L \) asymmetry \( n_L/s \) has been produced and determined, the last step is to calculate the corresponding baryon asymmetry produced due to the partial conversion of the \( L \) asymmetry to a \( B \) asymmetry by the Standard Model non-perturbative sphaleron processes. The conversion factor is 
\[
   n_B/s = -(28/79)n_L/s. \tag{9}
\]

It comes from taking into account the fact that these processes conserve \( B-L \), violate \( B+L \) and are fast (i.e. in thermal equilibrium) above the electroweak scale. From this conversion factor we finally get the total baryon asymmetry produced 
\[
   n_B/s = (8.7 \pm 0.4) \cdot 10^{-11}. \tag{11}
\]

3. The Neutrino Mass Constraints

In addition to the three ingredients above there is a fourth crucial ingredient which makes the all interest of the leptogenesis mechanism: the neutrino mass constraints. These constraints come from the fact these are the same interactions which produce both the neutrino masses and leptogenesis. There are two types of neutrino mass constraints on leptogenesis.

3.1. The Neutrino Mass Constraint on the Efficiency

The first constraint is the neutrino mass constraint on the size of the washout, that is to say on \( \eta \). It comes from the fact that, in full generality, the ratio \( \Gamma_{N_i}/H \), which has to be smaller than unity to have no washout suppression, is always larger than the ratio of the lightest neutrino mass \( m_{\nu_1} \) over the \( m_{\nu_1} \) in the \( 10^{-3} \) eV scale:
\[
   \frac{\Gamma_{N_1}}{H} \geq \frac{m_{\nu_1}}{10^{-3} \text{eV}}. \tag{6}
\]

This inequality comes simply from Eq. (3) and the \( v^2|\lambda\lambda^\dagger|_{jj}/M_{N_1} \geq m_{\nu_1} \) inequality. It means that, if \( m_{\nu_1} > 10^{-3} \) eV, there will be some washout and the larger is \( m_{\nu_1} \) above this scale the larger is the washout, i.e. the smaller is \( \eta \). For example for \( m_{\nu_1} = 0.05 \) eV the value of \( \eta \) is \( \sim 0.01 \) for \( M_{N_1} \leq 10^{14} \) GeV. For \( m_{\nu_1} = 0.5 \) eV this value becomes few \( 10^{-4} \) for \( M_{N_1} \leq 10^{12} \) GeV and decreases for larger values of \( M_{N_1} \). The
fact that $m_\ast$, which is a function of the electroweak and Planck scales, is of order the neutrino masses is a quite remarkable fact since it means that for leptogenesis the $N_i$ are naturally only slightly in thermal equilibrium or out-of-equilibrium. Note that Eq. (6) is often rewritten in term of the so-called effective masses of Eq. (3) as $\tilde{m}_i \geq 10^{-3}\text{eV}$.

### 3.2. The Neutrino Mass Constraint on the Size of $\varepsilon_{N_i}$

The second neutrino mass constraint is on the size of the CP-asymmetries $\varepsilon_{N_i}$. It generally applies but, contrary to the first constraint above, it do not always applies. This depends on the type of mass spectrum the right-handed neutrino have: very hierarchical, “normally” hierarchical or quasi-degenerate.

#### 3.2.1. Very hierarchical right-handed neutrinos: $M_{N_1} << M_{N_{2,3}}$

If right-handed neutrino masses differ by several orders of magnitude, the size of the $\varepsilon_{N_i}$ asymmetry is quite constrained by the size of the neutrino masses. There exists an upper bound\textsuperscript{11,12,13} which in its exact form was given by Ref.\textsuperscript{13}:

$$|\varepsilon_{N_i}| \leq \frac{3}{16\pi} \frac{M_{N_i}}{v^2} (m_{\nu_3} - m_{\nu_1}) = \frac{3}{16\pi} \frac{M_{N_i}}{v^2} \frac{\Delta m^2_{\text{atm}}}{m_{\nu_3} + m_{\nu_1}}. \quad (7)$$

Since $\Delta m^2_{\text{atm}} \simeq 2 \cdot 10^{-3}\text{eV}^2$ is fixed experimentally, this bound decreases as the neutrino masses increase. Therefore as the neutrino masses increase there are two suppression effects arising: the washout effect increases and the upper bound on the asymmetry decreases. Successful leptogenesis leads therefore to the upper bound\textsuperscript{14,8,5}

$$m_{\nu_3} < 0.12 - 0.15\text{ eV}. \quad (8)$$

For more details see Ref.\textsuperscript{2} in these proceedings. Note that in order to derive this bound Eq. (7) is not sufficient. As the final produced lepton asymmetry depends not only on $m_{\nu_3}$ and $M_{N_1}$ but also, through the efficiency factor, on $\Gamma_{N_1}$ (or $\tilde{m}_1$), it is necessary\textsuperscript{14,5} to have an upper bound for fixed values of these 3 parameters and not only as a function of $m_{\nu_3}$ and $M_{N_1}$ as in Eq. (7). This bound can be found in Ref.\textsuperscript{5}. Note also that, due to the fact that the upper bound on the CP-asymmetry is proportional to $M_{N_1}$, successful leptogenesis with hierarchical right-handed neutrinos implies also a lower bound on this mass:\textsuperscript{13,11,12,8}

$$M_{N_1} > 5 \times 10^8\text{ GeV}. \quad (9)$$
This result holds for the case where the $N_1$ are in thermal equilibrium before decaying. Starting instead (due to inflation dynamics) from a universe with no (with only) right-handed neutrinos at a temperature above their mass, this bound becomes: $M_{N_1} > 2 \times 10^9 \text{ GeV}$, $(2 \times 10^7 \text{ GeV})$.

### 3.2.2. “Normally” hierarchical right-handed neutrinos

If right-handed neutrinos have a hierarchy similar to the ones of the charged leptons or quarks, that is to say if $M_{N_1} \simeq (10 - 100)M_{N_2}$ with $M_{N_3} > M_{N_2}$, the L-asymmetry production is still dominated by the decays of the lightest right-handed neutrino $N_1$. In this case the upper bound on the CP-asymmetry is the same as for very-hierarchical neutrinos, except that there are extra corrections in $M_{N_2}^2 / M_{N_{2,3}}^2$ to be added in Eq. (7):

$$\delta \varepsilon_1 \simeq \frac{3}{16\pi} \frac{M_{N_1}}{v^2} \tilde{m}_{2,3} \frac{M_N^2}{M_{N_{2,3}}^2}.$$  \hspace{1cm} (10)

These corrections generically are small so that all the bounds obtained in the previous section are still valid, but not always. There exist configurations of the Yukawa couplings which lead to large corrections. The point is that, contrary to the leading term in Eq. (7), the corrections do not decrease when the neutrino masses decrease and they do not necessarily vanish for degenerate light-neutrino masses. As a result, for special configurations giving large $\tilde{m}_{2,3}$ but small neutrino masses, one can have successful leptogenesis with $M_{N_1}$ well below the lower bound of Eq. (9) and with neutrino masses well above the bound of Eq. (8). An explicit example of such Yukawa coupling configuration leading to successful leptogenesis with $M_{N_1} \simeq 10^6 \text{ GeV}$ has been recently considered in Ref.\textsuperscript{15}.

### 3.2.3. Quasi-degenerate right-handed neutrinos

If at least two right-handed neutrinos have masses very close to each other, $M_{N_1} \sim M_{N_2}$, the situation is changing drastically with respect to the two previous cases. This is due to the fact that the one loop self-energy diagram of Fig.1 displays in this case a resonance behaviour\textsuperscript{16,5,17} coming from the propagator of the virtual right-handed neutrino in this diagram. This effect can be seen from the $S_j$ factors in Eq. (2). Since in the seesaw model the decay widths of the $N_i$ are generically much smaller than their masses, this resonance effect can lead to a several order of magnitude enhancement of the asymmetry. At the resonance, that is to say for $M_{N_2} - M_{N_1} = \Gamma_{N_2}/2$, the $S_2$ factor, which is unity in the $M_{N_1} << M_{N_{2,3}}$ limit, is as large as
\( \frac{1}{2} M_{N_2}/\Gamma_{N_2} \). In this case, up to a 1/2 factor, the \( S_2 \) factor compensates the \( \Gamma_{N_2}/M_{N_2} \) prefactor in Eq. (2) which gives \( \varepsilon_{N_1} = \frac{1}{2} I_2 \). Moreover in this case one can show that the asymmetry is not bounded anymore by an expression depending on the neutrino masses. In particular the upper bound is not proportional to \( \Delta m^2_{\text{atm}}/(m_{\nu_3} + m_{\nu_1}) \) as in the hierarchical case,\(^a\) Eq. (7). The Yukawa coupling factor \( I_2 \) turns out to be bounded just by unity so that the upper bound on \( \varepsilon_{N_1} \) is just \( \frac{1}{2} \) independently of light and heavy neutrino masses\(^5\). Together with \( \varepsilon_{N_2} \), which is equal to \( \varepsilon_{N_1} \) in this case and has also to be taken into account, CP-violation is just bounded by unity. As a result, since neither the maximal asymmetry nor the washout effect (coming from inverse decay\(^b\)) depend on \( M_{N_1} \), successful leptogenesis can be obtained at any scale except that the \( L \) to \( B \) conversion from sphalerons still needs to be effective. This requires \( M_{N_1} \) to be above the electroweak scale (i.e. typically above \( \sim 1 \) TeV). Moreover since the upper bound on the CP asymmetry is independent of neutrino masses, there is no more suppression of the asymmetry for large neutrino masses.

The only remaining suppression effect arising for large neutrino masses is the one of section 3.1 above coming from the washout. Therefore the upper bound on the neutrino masses in this case gets considerably relaxed. This is shown in Fig.2.a where is plotted, as a function of \( m_{\nu_3} \), the level of degeneracy which is needed to have successful leptogenesis. Values far above the eV are possible which means that \textit{in full generality there is no more relevant upper bound on neutrino masses coming from leptogenesis.} The value \( m_{\nu_3} \simeq 1 \) eV can lead to successful leptogenesis with a level of degeneracy of order \( (M_{N_2} - M_{N_1})/M_{N_2} \simeq 4 \cdot 10^{-2} \) which is quite moderate.

One could argue that such a degeneracy of right-handed neutrinos is unnatural and that we would anyway generally expect a hierarchical spectrum for the right-handed neutrino by analogy with charged leptons or quarks. However, since the bound on neutrino masses from leptogenesis is relevant only for a quasi-degenerate spectrum of light neutrinos, when calculating this bound one can wonder what would be the most natural right-handed neutrino mass spectrum to explain such correlations between the light-neutrino masses. Presumably, three quasi-degenerate right-handed neutrino masses.

\(^a\)One can check\(^1\) that this neutrino mass factor comes from the fact that for hierarchical right-handed neutrinos \( S_2 = S_3 \). This equality has no reason to be true anymore with quasi-degenerate right-handed neutrinos.

\(^b\)Note that there is a \( M_{N_1} \) mass dependence in the washout coming from \( \Delta L = 2 \) scatterings. This effect however can be large only for large values of the \( M_{N_1} \) close to the GUT scale.
Figure 2. Fig.2.a shows how much the maximal value of neutrino mass compatible with thermal leptogenesis increases when right-handed neutrinos are allowed to be quasi-degenerate. The dashed line is what we obtain if we take into account only the resonance enhancement effect, not the neutrino mass effect, see text and Ref.\textsuperscript{5}. Fig.1.b holds assuming Eqs. (11)-(12) as a generic example that everything is as degenerate as neutrinos, considering the natural possibility that the $\tilde{m}_1$ should be quasi-degenerate with the neutrino masses, taking $\tilde{m}_{1,2,3} < m_{\nu_1} + n(m_{\nu_3} - m_{\nu_1})$.

To complete this section note that there exists nevertheless one specific pattern, which is probably the most realistic one, which leads to more stringent bounds.\textsuperscript{5} In fact, if neutrinos were quasi-degenerate, the degeneracy would presumably not be accidental but due to some reason: a broken SO(3) flavour symmetry is probably the simplest possibility. One expects that in such framework all quantities, and not only neutrino masses, are close to the ideal limit where three degenerate right-handed neutrinos give equal masses to three orthogonal combinations of left-handed neutrinos. Therefore one expects something like $\tilde{m}_i - \tilde{m}_j \approx m_{\nu_i} - m_{\nu_j}$ and

$$\begin{align*}
\frac{M_{N_3} - M_{N_1}}{M_{N_1}} &\sim \frac{m_{\nu_2} - m_{\nu_1}}{m_1} \approx \frac{\tilde{m}_2 - \tilde{m}_1}{m_{\nu_1}} \approx \frac{\Delta m^2_{\text{sun}}}{2m^2_{\nu_1}} \approx 0.5 \times 10^{-4} \left( \frac{\text{eV}}{m_{\nu_2}} \right)^2 \\
\frac{M_{N_3} - M_{N_2}}{M_{N_2}} &\sim \frac{m_{\nu_3} - m_{\nu_2}}{m_{\nu_2}} \approx \frac{\tilde{m}_3 - \tilde{m}_2}{m_2} \approx \frac{\Delta m^2_{\text{atm}}}{2m^2_{\nu_3}} \approx 10^{-3} \left( \frac{\text{eV}}{m_{\nu_2}} \right)^2.
\end{align*}$$

Right-handed neutrinos can be more degenerate than in the above estimates if only the neutrino Yukawa couplings deviate from the symmetric limit, and can be less degenerate only if there are accidental cancellations between non-universal Yukawa couplings and non-degenerate $M_{N_{1,2,3}}$ in the see-saw prediction for neutrino masses.

Assuming the relations of Eqs. (11)-(12), the bound on the neutrino masses can be read off from Fig. 2.b where we give the maximal baryon asymmetry we obtain as a function of $m_{\nu_3}$ for values of $\tilde{m}_{1,2,3} < m_{\nu_1} + n(m_{\nu_3} - m_{\nu_1})$ with $n = \{1/2, 1, 2, 4\}$. Taking $\tilde{m}_{1,2,3} < m_{\nu_3}$ ($n = 1$), as the generic example for the case that the $\tilde{m}$ would be precisely of order the neutrino masses, gives the constraint

$$m_{\nu_3} < 0.6 \text{ eV}, \quad (13)$$

which is stronger than in the fully general case of Fig. 2.a. The suppression effect comes essentially from the fact that for values of $\tilde{m}_{1,2,3}$ close to the neutrino masses the Yukawa coupling factors $I_j$ are suppressed\(^5,\)\(^14\). The result is quite sensitive to how these quantities are close to each other. For example taking $n=4$ (which could be considered as a quite moderately fine-tuned case) leads already to an upper bound as large as 1 eV.

In summary without a predictive flavour model which would show how the correlations between the seesaw parameters at the origin of the degenerate spectrum occur, in order to have a safe bound we must consider the fully general case of Fig. 2.a (where $n$ was left as a free parameter in order to maximize the asymmetry). Even in a very constrained situation the neutrino masses can be as large as 0.6 eV, Eq. (13), or 1 eV.\(^6\)

4. Leptogenesis in the Framework of Other Seesaw Models

The type-I seesaw mechanism is probably the most direct extension of the Standard Model we can consider in order to explain the neutrino masses and is in this sense the most attractive. However it is not the only attractive seesaw model. Beside the type I seesaw, one can think about two other basic seesaw mechanisms. The first one is the type-II seesaw model\(^18\) where neutrino masses are due to the exchange of an heavy scalar Higgs triplet. The second one is from the exchange of three heavy self-conjugated $SU(2)_L$ triplets of fermions\(^19\), a model which according to us should be called type-III seesaw model as it induces the neutrino mass from a third type of

\(^6\)Stronger constraints will arise if supersymmetry exists and if right-handed neutrinos lighter than $10^{10}$ GeV will be needed to avoid gravitino overproduction.
heavy particles. In addition to these three seesaw basic mechanisms one can also think about combinations of them. In the following we consider these various alternatives and see whether they can lead to successful leptogenesis and what are the corresponding mass bounds.

4.1. The Type-II Seesaw Model

The type-II seesaw model with just one heavy scalar triplet $\Delta_L$ which couples to 2 leptons doublets and to two Higgs doublets is a particularly minimal model. It is based on the Lagrangian

$$L \ni -M_\Delta^2 \text{Tr} \Delta_L^\dagger \Delta_L - (Y_\Delta)_{ij} L_i \bar{C} i\tau_2 \Delta_L L_j + \mu H^T \tau_2 \Delta_L H + h.c.,$$

(14)

with

$$\Delta_L = \begin{pmatrix} \frac{1}{\sqrt{2}} \delta^+ & \delta^+ \delta^0 \delta^+ 
\delta^0 & -\frac{1}{\sqrt{2}} \delta^+ \end{pmatrix}. \quad (15)$$

It leads to the neutrino mass matrix: $M^{II}_\nu = 2Y_\Delta v_\Delta \simeq 2Y_\Delta \mu^* v^2 / M_\Delta^2$. This model in full generality has only 11 parameters, the triplet mass, its $\mu$ coupling and 6 real parameters plus 3 phases in the $Y_\Delta$ Yukawa coupling matrix. This model has the attractive property that the knowledge of the full low energy neutrino mass matrix $M_\nu$ would allow to determine the full flavour high energy structure in $Y_\Delta$, both matrices being just proportional to each other. However for leptogenesis it turns out that this model is too minimal. As explained in Ref.20,21,5,22, since the triplet is not a self-conjugated particle, there is no vertex diagram and leptogenesis could come only from a self-energy diagram involving two leptons in the final state and two Higgs doublets in the self-energy, third diagram of Fig.3. This diagram with just one triplet is real and therefore doesn’t bring any CP-violation. At two loops the asymmetries are too suppressed. Therefore the standard model extended by just one scalar triplet leads in a “minimal” way to neutrino masses but do not lead to successful leptogenesis.

However, based on the type-II model with just one triplet coupling to leptons, there is one framework which can work. It is the supersymmetric version of this model, i.e. the MSSM model extended by a pair of scalar triplets with opposite hypercharges. In a way similar to the soft leptogenesis mechanism with right-handed neutrinos23,2, the supersymmetry breaking terms involving the triplets can remove the mass degeneracy between the two triplets and lead to resonant leptogenesis from the self-energy diagram.
involving the two triplets. This requires triplet mass between $\sim 10^3 \text{ GeV}$ and $\sim 10^9 \text{ GeV}$.

4.2. The Heavy Triplet of Fermion Model

The third seesaw basic way to induce neutrino masses is by adding to the Standard Model 3 self-conjugated $SU(2)_L$ triplets of fermions. The Lagrangian keeps the same structure as the one of the type-I seesaw model but with different $SU(2)_L$ contractions:

$$L = L_{SM} + \bar{N}_a^i D^a N_i^a + (\lambda^{ij} \tau_{a,3} N_i^a L_j^a H^{1\beta} + \frac{M_{N_1}}{2} N_i^a N_i^a + \text{h.c.}),$$

with $a = 1, 2, 3; \alpha, \beta = 1, 2$. As a result it leads to the same seesaw formula than with singlets: $M_\nu = -\lambda^T M_N^{-1} \lambda v^2$. For leptogenesis, the one-loop diagrams are exactly the same as with right-handed neutrinos, Fig. 1, which lead to the same asymmetries up to $SU(2)_L$ factors of order unity,$^5$

$$\varepsilon_{N_1} = \sum_{j=2,3} \frac{3 M_{N_1}}{2 M_{N_j} M_{N_j}^*} \frac{V_j - 2 S_j}{3},$$

and is therefore 3 times smaller in the hierarchical limit where $V_j = S_j = 1$, see Eq. (2). The final amount of baryon asymmetry is given by the CP-asymmetry times the efficiency factor $\eta$ times a numerical coefficient which is 3 times bigger than in the singlet case because now $N_1$ has three components: $n_B = -4.1 \cdot 10^{-3} \varepsilon_{N_1} \eta$. The decay width of each of the 3 components of $N_1$ is given by the same expression as in the singlet case, Eq. (3). Scatterings are same as in the type-I model except for $SU(2)_L$ factors.$^5$ The only important difference is that the triplets have $SU(2)_L \times U(1)$ gauge scatterings the singlets do not have. This reduces the efficiency factor$^5$. As a result, in the hierarchical limit $M_{N_1} \ll M_{N_{2,3}}$, the bounds for successful leptogenesis are slightly more stringent than for singlets:

$$M_{N_1} > 1.5 \cdot 10^{10} \text{ GeV}, \quad m_{\nu_3} < 0.12 \text{eV}.$$ 

In the quasi-degenerate case the discussion is similar to the one of the type-I model, see section 3.2.3 above. A value of $M_{N_1}$ as low as $\sim 1 \text{ TeV}$ is possible, very close to the resonance.

$^d$Note that triplets, unlike right-handed neutrinos have gauge scatterings, which tend to put them in closer thermal equilibrium, reducing the efficiency. The exact efficiency for a scalar triplet is yet to be calculated but the one for a fermion triplet is known.$^5$ All lower bounds on the scalar triplet masses we give here are obtained using the fermion triplet efficiency, assuming that both efficiencies are same, as gauge scatterings are expected to be similar up to factors of order one. This should be checked explicitly.
4.3. The Type-I plus Type-II Model

The case where we add to the standard model three right-handed neutrinos and one scalar triplet is quite interesting because it is the situation of the ordinary left-right models and of the renormalizable $SO(10)$ models such as defined in Ref. 24. In this case the relevant Lagrangian is just the sum of the Lagrangians of Eqs. (1) and (14). To discuss this possibility it is necessary to consider two cases, depending on which particle is the lightest one, the scalar triplet $\Delta_L$ or the lightest right-handed neutrino $N_1$.

4.3.1. The $M_\Delta < M_{N_1}$ case

If the triplet is lighter than $N_1$, the production of the asymmetry will be naturally dominated by the decay of the triplet to two leptons. The CP-asymmetry comes in this case from the difference between the decay width of $\Delta^*_L$ to two leptons and of $\Delta_L$ to two anti-leptons. The leptogenesis one loop diagram is a vertex diagram involving both the decaying triplet and a virtual right-handed neutrino, first graph of Fig. 3. This diagram was first displayed in Ref. 25. Calculating explicitly its contribution we get 26:

$$
\varepsilon_\Delta = 2 \frac{\Gamma(\Delta^*_L \to l + l) - \Gamma(\Delta_L \to \bar{l} + \bar{l})}{\Gamma_{\Delta^*_L} + \Gamma_{\Delta_L}}
$$

(19)

$$
= -\frac{1}{8\pi} \frac{M^2_\Delta}{(\sum_{ij} |(Y_\Delta)_{ij}|^2 M^2_\Delta + |\mu|^2)} \frac{1}{v^2} Im[(M^I_\nu)_{il}(Y_\Delta)_{i\mu} \mu^*],
$$

(20)

where $M^I_\nu$ is the type-I contribution to the neutrino mass matrix (given in section 2). For each of the 3 triplet components the total decay width is:

$$
\Gamma_\Delta = \frac{1}{8\pi} M_\Delta \left( \sum_{ij} |(Y_\Delta)_{ij}|^2 + \frac{|\mu|^2}{M^2_\Delta} \right).
$$

(21)

In this case leptogenesis works in a way similar to the type-I model apart from 3 important differences. The first is that, like for triplet of fermions above, there are gauge scatterings which tend to put the $\Delta_L$ closer to
thermal equilibrium. The effect is similar to the one encountered for triplet of fermions above. The second difference is that here there is no one-loop self-energy diagram, so there is no possible resonance effect. The third one is that the interplay between the neutrino masses and the size of the washout and the size of the asymmetry is completely different from the type-I case. The decay width is proportional to the triplet couplings but the asymmetry is essentially proportional to the right-handed neutrino couplings (taking into account the fact that there are two triplet couplings in both numerator and denominator of the asymmetry). As a result one may consider the possibility that the type-II contribution to neutrino masses is small enough to avoid large washout and that the type-I contribution to neutrino masses is large. By increasing in this way the type-I neutrino mass contribution, the washout remains unchanged but the asymmetry increases proportionally to the neutrino masses. Therefore, there is no upper bound on neutrino masses in this model to have successful leptogenesis. Note that there is nevertheless a lower bound on the triplet mass because the asymmetry is proportional to this mass. For a hierarchical spectrum of light neutrinos (i.e. $m_{\nu_3} \sim \sqrt{\Delta m^{2}_{\odot}}$) the bound is essentially the same as for the triplet of fermions, Eq. (18). As there is no possible resonance effect this bound is an absolute bound. For larger values of $m_{\nu_3}$ this bound decreases because in this case the asymmetry increases. However, assuming $m_{\nu_3}$ below 1 eV, this bound cannot decrease by more than one order of magnitude.

4.3.2. The $M_\Delta > M_{N_1}$ case

If the triplet is sizeably heavier than at least one right-handed neutrino, then it is the decay of the right-handed neutrino to a lepton and a Higgs boson which dominates the production of the L asymmetry. In this case leptogenesis can be produced from the pure type-I model just as in sections 2 and 3. However here, in addition to this pure type-I contribution, there is a new contribution to leptogenesis coming from a diagram with a real right-handed neutrino and a virtual triplet, second diagram of Fig. 3. Disregarding the pure type-I contribution (assuming that it has a small contribution because the type-I Yukawa couplings and/or their phases are small) this new diagram can perfectly lead to successful leptogenesis alone. For the asymmetry defined in Eq. (2) one obtains from this diagram

$$\varepsilon_{N_1}^\Delta = \frac{3}{16\pi} \frac{M_{N_1}}{v^2} \sum_i \frac{\text{Im}[\lambda_{1i} \lambda_{1i}^*(M_{I_1}^{I*})_{il}]}{\sum_i |\lambda_{1i}|^2} ,$$ (22)
where $M_{\nu}^{II}$ is the neutrino mass matrix induced by the type-II contribution (given in section 4.1 above). The discussion is similar to the one of the $M_{\Delta} < M_{N_1}$ case above inverting the role of type-I and type-II two contributions. It is now the decay of the right-handed neutrino to lepton and Higgs, induced by the type-I couplings, Eq. (3), which essentially determines the washout. To increase the asymmetry without increasing the washout, one can then consider the possibility of keeping the type-I contribution small, increasing the type-II contribution which increases the asymmetry but not the washout. As a result, here too, there is no more upper bound on neutrino masses for leptogenesis. For hierarchical light neutrinos the lower bound on the lightest right-handed neutrino mass is to a good approximation the same as in the pure type-I model, Eq. (9). As the asymmetry is linear in both $M_{N_1}$ and the neutrino masses, for larger values of $m_{\nu_3}$ the asymmetry linearly increases, so the bound on $M_{N_1}$ linearly decreases. But here too assuming $m_{\nu_3}$ below 1 eV, this bound cannot decrease by more than one order of magnitude. The precise bound is given in Ref. as a function of the efficiency factor $\eta$.

Note that, if instead of considering a dominant type-II contribution, we consider the case where both type-I and type-II contributions are important this lower bound on $M_{N_1}$ doesn’t get relaxed. In this case both contributions appear to be just proportional to their respective contributions to the neutrino masses so that, baring a possible cancellation of CP-violating phases, leptogenesis is expected to be dominated by the contribution which dominates the neutrino masses. From Eq. (2) the pure type-I contribution to leptogenesis for hierarchical right-handed neutrinos turns out to be given by Eq. (22) replacing $M_{\nu}^{II}$ by $M_{\nu}^{I}$, a fact which can be nicely understood by using effective dimension five neutrino mass operators.

For a quasi-degenerate spectrum of right-handed neutrino the pure type-I contribution to leptogenesis is expected dominant and the triplet contribution which doesn’t display any resonant behaviour is in this case negligible.

### 4.4. The Multiple Type-II Models

If there is more than one heavy scalar triplet, leptogenesis can be easily induced by the decay of the triplets to two leptons with a one-loop self-energy diagram involving two different triplets, third diagram of Fig.3. The asymmetry in this case is given by

$$\varepsilon_{\Delta_i} = -\frac{1}{\pi} M_{\Delta_i} \frac{Im[\mu_i \mu_j (Y_{\Delta_i})_{kl} (Y_{\Delta_j})_{kl}]}{|(Y_{\Delta_i})_{kl}|^2 M_{\Delta_i}^2 + |\mu_i|^2 (\Delta M_{ij})^2 + M_{\Delta_i}^2 \Gamma_{\Delta_j}^2},$$  \hfill (23)
where there is now a triplet scalar index on the couplings of Eqs. (14) and (21). For hierarchical triplets, under the assumption of footnote d above, successful leptogenesis leads to a triplet mass bound similar to the one of Eq. (18) for hierarchical fermion triplets, in accordance with the estimate of Ref. 21. This bound is higher than for right-handed neutrinos due to gauge scatterings. Similarly for quasi-degenerate triplets, one can go down to 1-10 TeV (very close to the resonance). And, here too, even with hierarchical triplets, there is no more upper bound on neutrino masses for successful leptogenesis because one can always increase the neutrino mass contribution of the virtual triplet in the self-energy diagram leading to a larger asymmetry without increasing the washout.

4.5. The Multiple Type-I Seesaw Case

To add extra fermion singlets $S_i$ on top of the 3 right-handed neutrinos $N_i$ is also a possibility one might consider, especially if these extra $SU(2)_L \times U(1)$ singlets are also singlets of $SO(10)$ in case one can build a renormalizable $SO(10)$ model without a 126 scalar multiplet to give mass to the $N_i$’s. In this case, as there is no 126, there is no triplet. This may lead to successful leptogenesis just from the same diagrams as in the type-I model, Fig. 1, except that there are now more than 3 singlets to be put in these diagrams. There are diagrams where both real and virtual singlets are $N_i$’s or are $S_i$’s. Clearly, for example if the lightest heavy particle is a $S$, one can increase the Yukawa couplings of the $N$ leaving the $S$ Yukawa couplings unchanged. In this way the neutrino masses increase but not the washout which is due the $S$ couplings, so that, here too, there is no relevant upper bound on neutrino masses for successful leptogenesis.

Note that extra diagrams with a real $N$ and a virtual $S$ in Fig.1 (or vice versa) are also possible if they couple to the same scalar multiplets. In the $SO(10)$ models of Ref. 29,30 the $S$ and $N$ do not couple to the same multiplets because the $S$ which is a singlet of $SO(10)$ couples to the 16 of matter $\psi_{16}$ and a scalar $H_{16}$ but the $N$ which is in $\psi_{16}$ couples to an other $\psi_{16}$ and a scalar $H_{10}$. However through mixing of the $H_{16}$ and $H_{10}$ from a coupling involving the vev of another $H_{16}$ such a diagram may exist, a possibility still to consider. It is straightforward to check that such a diagram can lead to successful leptogenesis and here too without leading to a relevant upper bound on the neutrino masses.

Note also that there is an other possibility with extra singlets 31, simply by assuming more than three generations of fermions and by assuming a
huge hierarchy of Yukawa couplings (to have $m_{\nu_1} > 45$ GeV). This may also lead to successful leptogenesis (e.g. at a low scale).

5. The Radiative Models

An other class of models one might consider to explain the neutrino masses and mixings is the one where neutrino masses are induced by loop diagrams. This is quite interesting as in most of these models the scale where they are generated is not far beyond the reach of present particle accelerators. However to generate leptogenesis at the $\sim$ TeV scale in this framework turns out to be a quite hard task for various reasons. A first reason is that in these models, such as the ones based on violation of R-parity or based on the presence of a charged scalar $SU(2)_L$ singlet (Zee model), the heavy states whose decays could generate the lepton asymmetry are not gauge singlets. As a result there is a large washout suppression coming from gauge scatterings involving these heavy states which are very fast at low scale. A second reason is that at the TeV scale the Hubble constant is much smaller than for example at $10^{10}$ GeV since it is proportional to the square of the temperature. Since the decay width is only linear in the mass, the condition $\Gamma < H$ requires therefore couplings much smaller at the TeV scale than at $10^{10}$ GeV. Since the asymmetry is proportional to these tiny couplings, this leads in general to too small lepton asymmetry, see Ref. 12.

One solution to these problems is to consider three body decays instead of two-body decays. An other solution is to consider the seesaw extended MSSM, that is the say the MSSM extended by three right-handed (s)neutrinos. In this framework it has been shown that a large enough asymmetry can be obtained from the L-violating soft supersymmetry breaking terms involving the right-handed sneutrinos. With sneutrino masses of order a few TeV, successful leptogenesis together with light neutrino masses (induced radiatively from the same soft terms) can be induced.

6. Summary

In summary there are quite a few models which from the same interactions can lead to successful generation of neutrino masses and leptogenesis: type-I model with right-handed neutrinos, type-II model (although only if involving the soft terms), type-III model with triplet of fermions, type-I plus type-II model, multiple type-I or type-II models, or even a radiative model (in the seesaw extended MSSM from soft terms).

The upper bound on neutrino masses is quite sensitive to the model con-
sidered as well as on the assumptions made on the heavy mass spectrum in each model. In the type-I model (and similarly in the type-III model) a stringent bound, Eq. (8), can be found only assuming a hierarchical spectrum of right-handed neutrinos. However this assumption doesn’t appear to be the most natural assumption one can make when considering this bound, for which the light neutrinos have a quasi-degenerate spectrum. A probably more natural quasi-degenerate spectrum of right-handed neutrinos leads instead in full generality to an upper bound on neutrino masses far beyond the eV scale. Even in a highly constrained model, such as the one based on a SO(3) symmetry in section 3.2.3, a value around 1 eV appears to be possible for successful leptogenesis. Only with extra assumptions (such as low reheating temperature) one might get a more stringent bound. In the other models (multiple type-I and/or type-II) the upper bound is far above the eV scale even for a hierarchical spectrum of heavy states.

Similarly the lower bound on the scale of leptogenesis is sensitive to the model and/or the heavy mass spectrum considered. However for this scale there is at least one firm conclusion one can draw: in all the seesaw models we considered here, if the masses of the heavy states differ by orders of magnitude, this scale has to be orders of magnitude above the TeV scale, that is to say above $10^7 - 10^{10}$ GeV depending on the model.

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