ON THE VERY HIGH ENERGY SPECTRUM OF THE CRAB PULSAR

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ABSTRACT

In the present paper, we construct a self-consistent theory interpreting the observations from the MAGIC Cherenkov Telescope of the very high energy (VHE) pulsed emission from the Crab pulsar. In particular, on the basis of Vlasov’s kinetic equation, we study the process of quasi-linear diffusion (QLD) developed by means of the cyclotron instability. This mechanism provides simultaneous generation of low (radio) and VHE (0.01–25 GeV) emission on light cylinder scales in one location of the pulsar magnetosphere. A different approach to the synchrotron emission is considered, giving the spectral index of the VHE emission ($\beta = 2$) and the exponential cutoff energy (23 GeV) in good agreement with the observational data.

Key words: instabilities – plasmas – pulsars: individual (PSRB0531+21) – radiation mechanisms: non-thermal

1. INTRODUCTION

Recent observations from the MAGIC Cherenkov Telescope (Aliu et al. 2008) reveal several characteristic features of the very high energy (VHE) emission of the Crab pulsar. In particular, pulsed $\gamma$-rays above 25 GeV were detected, showing a relatively high-energy cutoff, which indicates that emission happens far out in the magnetosphere (Aliu et al. 2008). In general, for explaining $\gamma$-ray production in the pulsar magnetosphere, two different models are applied: the so-called polar cap and outer gap models. In the former, it is assumed that the VHE emission is generated at the polar cap (e.g., Daugherty & Harding 1982), but it cannot coincide in phase with the entire radio emission. In the outer gap model, the generation of VHE radiation happens in the outer gap region (e.g., Romani & Yadigaroglu 1995), but to our knowledge there is no mechanism to generate the radio emission. The pulsar emission model that underlies our work principally differs from the polar cap and outer gap models. To explain the origin of the VHEs observed by the MAGIC Cherenkov Telescope (Aliu et al. 2008), we rely on the polar emission model first developed by Machabeli & Usov (1979) and Lominadze et al. (1979). According to these works, in the electron–positron plasma of a pulsar magnetosphere, the low-frequency cyclotron modes on the quasi-linear evolution stage create conditions for generating the high-energy synchrotron emission. Special interest is reserved for the coincidence of signals from different frequency bands ranging from radio to X-ray (Manchester & Taylor 1980). Investigations of the last decade have shown that the aforementioned coincidence takes place in the VHE domain (0.01 MeV–25 GeV) as well (Aliu et al. 2008). In the framework of the present paper, generation of the low- and high-frequency waves is a simultaneous process and takes place in one location of the magnetosphere, which explains the observed phase-coincidence of the low and VHE signals. Consequently, we suppose that generation of phase-aligned signals from different frequency bands is a simultaneous process and takes place in one location of the pulsar magnetosphere. This consideration automatically excludes the inverse Compton scattering and curvature radiation mechanisms, respectively, which are not localized (Machabeli & Osmanov 2009, 2010). It is worth noting that a coincidence of pulse phases might be achieved by means of caustic effects (Morini 1983; Romani & Yadigaroglu 1995; Dyks et al. 2004).

It is well known that close to the pulsar surface, due to very strong magnetic fields, magnetospheric particles emit efficiently and the corresponding cooling timescale is short compared to the typical kinematic timescales of particles. Therefore, transversal energy loss becomes extremely efficient, and consequently, electrons and protons lose their perpendicular momenta and very rapidly transit to their ground Landau states, and the distribution function becomes one dimensional. This means that one needs a certain mechanism leading to the creation of the pitch angles restoring synchrotron radiation. The main mechanism of wave generation in plasmas of the pulsar magnetosphere is the cyclotron instability (Kazbegi et al. 1992). During the quasi-linear stage of the instability, a diffusion of particles arises along and across the magnetic field lines. Therefore, the resonant electrons acquire transverse momenta and, as a result, start to radiate in the synchrotron regime.

Recently, we applied the method of the quasi-linear diffusion (QLD) to the Crab pulsar to explain the VHE emission observed by the MAGIC Cherenkov Telescope (Aliu et al. 2008). In Machabeli & Osmanov (2009), we found that on the light cylinder (a hypothetical zone where the linear velocity of rigid rotation exactly equals the speed of light) length scales the cyclotron modes are generated, provoking the re-creation of pitch angles and the subsequent synchrotron radiation in the VHE (>25 GeV) domain. QLD guarantees the observationally evident fact of coincidence of signals in the low and VHE bands (Machabeli & Osmanov 2009), resulting in the simultaneous generation of these radiation domains in one location of the pulsar magnetosphere. As it has been shown, neither curvature radiation nor inverse Compton scattering may provide the above-mentioned coincidence. This particular problem was considered in Machabeli & Osmanov (2010), in which analyzing inverse Compton scattering, demonstrated that, for reasonable physical parameters, even very energetic electrons are unable to produce the photon energies of the order of 25 GeV. Studying curvature radiation, we found that the curvature drift instability (CDI; Osmanov et al. 2008, 2009) efficiently rectifies the magnetic field lines, making the role of the curvature emission process negligible (Machabeli & Osmanov 2010).

According to the theory of synchrotron emission (Bekefi & Barrett 1977; Ginzburg 1981), the spectral index is usually less than 1 (in most cases it equals 1/2), although it is observationally evident that for the VHE pulsed radiation from the Crab pulsar, the corresponding value is of the order of two (Aliu et al.
which, in the framework model, provides all necessary conditions to re-create the pitch angles, consequently restricting their values. In the framework of the paper, we study the spectral index, which, for reasonable magnetospheric parameters, is of the order of two for the VHE domain.

The paper is organized as follows. In Section 2, we consider the emission model; in Section 3, we study the synchrotron radiation spectrum; in Section 4, we discuss our results; and in Section 5, we summarize them.

2. EMISSION MODEL

According to the works of Sturrock (1971) and Tademaru (1973), due to the cascade processes of pair creation, a pulsar’s magnetosphere is filled by electron–positron plasma with an anisotropic one-dimensional distribution function (see Figure 1 in Arons 1981) and consists of the following components: the bulk of plasma, with an average Lorentz factor $\gamma \sim \gamma_p$; a tail $-\gamma$; and the primary beam, with $\gamma \sim \gamma_b$. Considering that the eigenmodes of electron–positron plasma have small inclination angles with respect to the magnetic field, we have three branches, two of which are mixed longitudinal-transversal waves ($\ell l_1, \ell l_2$). The high-frequency branch on the diagram $\omega(k)$ begins with the Langmuir frequency, and for longitudinal waves ($k_\perp = 0$), $\ell l_1$ reduces to the pure longitudinal Langmuir mode. The low-frequency branch, $\ell l_2$, is similar to the Alfvén wave. The third $t$ mode is the pure transversal wave, the electric component of which, $E^t$, is perpendicular to the plane of the wavevector and the magnetic field, $(\mathbf{k}, \mathbf{B})$. The vector of the electric field $E^{\ell l_1, \ell l_2}$ is located in the plane $(\mathbf{k}, \mathbf{B})$. When $k_\perp = 0$, the $t$ mode merges with the $\ell l$ waves, and the corresponding spectra are given by (Kazbegi et al. 1992)

$$\omega_t \approx k c (1 - \delta), \quad \delta = \frac{\omega_p^2}{4 \omega_B^2 \gamma_p^2},$$

where $k$ is the modulus of the wavevector; $c$ is the speed of light; $\omega_p \equiv \sqrt{4\pi n_e e^2/m}$ is the plasma frequency; $e$ and $m$ are the electron’s charge and rest mass, respectively; $n_e$ is the plasma density; $\omega_B \equiv eB/\gamma mc$ is the cyclotron frequency; and $B$ is the magnetic field induction.

The distribution function is one dimensional and anisotropic, and plasma becomes unstable, which can lead to excitation of the aforementioned waves. The beam particles undergo drifting perpendicularly to the magnetic field due to the curvature, $\rho$, of the field lines. The corresponding drift velocity is given by

$$u_\parallel \equiv \frac{eV_s \gamma_{res}}{\rho \omega_B},$$

where $V_s$ is the component of velocity along the magnetic field lines and $\gamma_{res}$ is the Lorentz factor of the resonant particles. Both of these factors (the one dimensionality of the distribution function and the drift of particles) might cause generation of eigenmodes in the electron–positron plasma if the following resonance condition is satisfied (Kazbegi et al. 1992):

$$\omega - k_\parallel V_\parallel - k_\perp u_\parallel + \frac{s \omega_B}{\gamma_{res}} = 0,$$

where $k_\parallel$ is the wavevector’s component along the drift and $s = 0, \pm 1, \pm 2, \ldots$

For $s = 0$, one has a hollow cone of the modified Cherenkov radiation (Kazbegi et al. 1992; Lyutikov et al. 1999; Shapakidze et al. 2003). For the Crab pulsar, one has the core emission, a result of the anomalous Doppler effect ($s = -1$). Our aim is to interpret the results of the MAGIC Cherenkov Telescope (Aliu et al. 2008); therefore, in the present paper we consider the aforementioned resonance condition $s = -1$.

During the generation of $t$ or $\ell l$ modes by resonant particles, one also has simultaneous feedback of these waves on the electrons (Vedenov et al. 1961). This mechanism is described by QLD, leading to the diffusion of particles along and across the magnetic field lines. The process of QLD in the external magnetic field is examined by Melrose & McPhedran (1991) and Akhiezer (1967). Generally speaking, at the pulsar surface, relativistic particles efficiently lose their perpendicular momenta via synchrotron emission in very strong ($B \sim 10^{12}$ G) magnetic fields, therefore, they very rapidly transit to their ground Landau state (pitch angles are vanishing). Contrary to this process, QLD leads to the creation of pitch angles by resonant particles, and as a result they start to radiate in the synchrotron regime. To explain the observed VHE emission of the Crab pulsar, it is supposed that the resonant particles are the primary beam electrons, with $\gamma_b \sim 10^8$, giving the synchrotron emission in the VHE domain.

When emitting in the synchrotron regime, the resonant particles undergo the radiation reaction force $\mathbf{F}$, having longitudinal and transversal components (Landau & Lifshitz 1971):

$$\mathbf{F}_\perp = -\alpha_1 p_\perp \left(1 + \frac{p_\parallel^2}{m^2 c^2}\right), \quad \mathbf{F}_\parallel = -\frac{\alpha_1}{m^2 c^2} p_\parallel^2,$$

where $\alpha_1 = 2e^2 \omega_B^2/3c^2$.

The wave excitation leads to a redistribution process of the particles via QLD. The kinetic equation for the distribution function of the resonant particles can be written as (Machabeli & Usos 1979; Malov & Machabeli 2002)

$$\frac{\partial f_0}{\partial t} + \frac{\partial}{\partial p_\parallel} \left(\frac{1}{\rho} \left(\frac{2}{\gamma} \frac{\partial f_0}{\partial \rho} \right) \right) + \frac{1}{p_\perp} \frac{\partial}{\partial p_\perp} \left(\frac{2}{\gamma} \frac{\partial f_0}{\partial \rho} \right) = 0,$$

where $\gamma = \frac{\omega_B^2}{\rho c^2}$ and $\rho \equiv \gamma m c^2/e$. For the Crab pulsar, one has the core emission, and as a result they start to radiate in the synchrotron regime. To explain the observed VHE emission of the Crab pulsar, it is supposed that the resonant particles are the primary beam electrons, with $\gamma_b \sim 10^8$, giving the synchrotron emission in the VHE domain.

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where $\alpha_1 = 2e^2 \omega_B^2/3c^2$.

The diffusion coefficients in Equation (5) are evaluated in the momentum space as (Melrose & McPhedran 1991)

$$\left(\begin{array}{c} D_{\ell l_1} \\ D_{\ell l_2} \\ D_{\parallel} \end{array} \right) = \int \frac{d^3k}{(2\pi)^3} \frac{\pi^2 e^2 \gamma^2 \psi}{4\pi} |E_k| \frac{\hbar^2 \omega^2}{4\pi} \times \left(\begin{array}{c} \delta(\omega(k) - k_\parallel V_\parallel - k_\perp u_\parallel) + \omega_B(\gamma) \\ \left(\frac{\Delta p_\perp}{\Delta p_\parallel}\right)^2 \\ \left(\frac{\Delta p_\perp}{\Delta p_\parallel}\right)^2 \end{array} \right),$$

where $|E_k|^2/4\pi$ is the density of electric energy in the excited waves and

$$\Delta p_\perp = -\frac{\hbar \omega_B}{\gamma \nu \sin \psi}, \quad \Delta p_\parallel = \hbar k_\parallel.$$
The evaluation in our case gives

$$\begin{align*}
D_{\perp\perp} - D_{\parallel\parallel} = \left(\begin{array}{c}
D_{\perp\parallel} = D_{\parallel\perp}
\end{array}\right) = \left(\begin{array}{c}
D\psi \left| E_{k} \right|_{k=k_{\text{res}}}^{2} \\
-D\psi \left| E_{k} \right|_{k=k_{\text{res}}}^{2} \\
D\psi_{\perp} \left| E_{k} \right|_{k=k_{\text{res}}}^{2}
\end{array}\right),
\end{align*}$$

(8)

where $D = e^{2}/8c$.

The pitch angle acquired by resonant particles during the process of QLD satisfies $\psi = p_{\perp}/p_{\parallel} \ll 1$. Thus, one can assume $\partial / \partial p_{\perp} \gg \partial / \partial p_{\parallel}$, which reduces Equation (5) to the following form:

$$\begin{align*}
\frac{\partial f^{0}}{\partial t} + \frac{1}{p_{\perp}} \frac{\partial}{\partial p_{\perp}} \left( p_{\perp} F_{\perp} f^{0} \right) &= \frac{1}{p_{\perp}} \frac{\partial}{\partial p_{\perp}} \left( p_{\perp} D_{\perp\perp} \frac{\partial f^{0}}{\partial p_{\perp}} \right).
\end{align*}$$

(9)

The transversal diffusion leads to the isotropization of the one-dimensional distribution function, whereas the force $F_{\perp}$ works against the diffusion. The dynamical process saturates when these effects balance each other. Considering the quasi-stationary case ($\partial f / \partial t = 0$), one finds

$$f(p_{\perp}) = C \exp \left( \int \frac{F_{\perp}}{D_{\perp\perp}} dp_{\perp} \right) = Ce^{-\left(\frac{p_{\perp}}{p_{\parallel}}\right)^{4}},$$

(10)

where

$$p_{\perp} \approx \frac{\pi^{1/2}}{B \gamma^{4/3} \epsilon_{m}} \left( \frac{3 \beta e^{11/8} \gamma_{5}^{6}}{32 \epsilon_{m}^{17/8} P_{2n}^{5}} \right)^{1/4}.$$

(11)

Taking into account the above relation, we can find that the mean value of the pitch angle, $\psi_{0} \approx p_{\perp0}/p_{\parallel0}$, is of the order of $10^{-6}$.

3. SYNCHROTRON RADIATION SPECTRUM

Let us consider the synchrotron emission of the set of electrons. If $p_{\perp0} dp_{\perp} dp_{\perp} dV dQ_{\parallel}$ is the number of emitting particles in the elementary $dV$ volume, with momenta from the intervals $[p_{\perp}, p_{\perp} + dp_{\perp}]$ and $[p_{\parallel}, p_{\parallel} + dp_{\parallel}]$ and with the velocities that lie inside the solid angle $dQ_{\parallel}$ near the direction of $\bar{r}$, then the emission flux of the set of electrons is given by (Ginzburg 1981)

$$F_{\epsilon} = \int I_{\epsilon} p_{\parallel} dp_{\perp} dp_{\parallel} dV dQ_{\parallel},$$

(12)

where $I_{\epsilon}$ is the Stokes parameter, which is additive in this case, as the observed synchrotron radiation wavelength $\lambda$ is much less than the value of $n^{-1/8}$—the average distance between particles—where $n$ is the density of plasma component electrons. Taking into account that

$$\int p_{\perp} f^{0} dp_{\perp} = f_{\|}(p_{\|}),$$

(13)

the integral (16) is easily reduced to

$$F_{\epsilon} \propto \int f_{\parallel}(p_{\|}) B \psi \epsilon_{m} \int_{\epsilon_{m}}^{\infty} K_{5/3}(z) d\bar{z} dp_{\|}.$$

(14)

Here, $\epsilon_{m} \approx 5 \times 10^{-18} B \psi \gamma^{2}$ GeV is the photon energy of the maximum synchrotron spectrum of a single electron, and $K_{5/3}(z)$ is a Macdonald function. After substituting the mean value of the pitch angle in the above expression for $\epsilon_{m}$, we get

$$\epsilon_{m} \approx 5 \times 10^{-18} B \psi \gamma^{2} \frac{3 m_{e}^{5} c^{7} \gamma_{5}^{2}}{4 e^{0} P_{2n}^{5}} \frac{1}{\gamma^{1/4}}.$$

(15)

Accordingly, the beam electrons should have $\gamma_{5} \approx 6 \times 10^{5}$ to radiate the photons with $\sim 10$ GeV energy. This in turn implies that the gap models providing the Lorentz factors $\sim 10^{5}$ are not enough to explain the detected pulsed emission. On the other hand, Aliu et al. (2008) confirmed that their observations indicate that emission happens far out in the magnetosphere. One real scenario could be the centrifugal acceleration of electrons, which takes place in corotating magnetospheres (Machabeli & Rogava 1994; Rogava et al. 2003; Osmanov et al. 2007).

Another alternative mechanism of acceleration could be a collapse (e.g., Artsimovich & Sagdeev 1979; Zakharov 1972) of the centrifugally excited, unstable Langmuir waves (Machabeli et al. 2005) in the pulsar’s magnetosphere.

According to our emission model, the observed radiation comes from a region where the magnetic field lines are practically straight and parallel to each other; therefore, electrons with $\psi \approx \psi_{0}$ efficiently emit in the observer’s direction.

To find the synchrotron flux in our case, we need to know the one-dimensional distribution function of the emitting particles $f_{\perp}$. Let us multiply both sides of Equation (5) by $p_{\perp}$ and integrate it over $p_{\perp}$. Taking into account that the distribution function vanishes at the boundaries of integration, Equation (5) reduces to

$$\frac{\partial f_{\perp}}{\partial t} = \frac{\partial}{\partial p_{\perp}} \left( \frac{\alpha_{s}}{m_{e}^{2} c^{2} \pi^{1/2} P_{2n}^{2} p_{\parallel}^{2} f_{\perp}} \right).$$

(16)

Considering the quasi-stationary case, we find

$$f_{\perp} \propto \frac{1}{p_{\perp}^{1/2} |E_{k}|}.$$  

(17)

For $\gamma \psi \ll 10^{10}$, a magnetic field inhomogeneity does not affect the process of wave excitation. The equation that describes the cyclotron noise level, in this case, has the form (Lominadze et al. 1983)

$$\frac{\partial |E_{k}|^{2}}{\partial t} = 2 \Gamma_{c} |E_{k}|^{2} f_{\perp},$$

(18)

where

$$\Gamma_{c} = \frac{\pi^{2} e^{2}}{k_{\parallel}} f_{\perp}(p_{\text{res}})$$

(19)

is the growth rate of the instability. Here, $k_{\parallel}$ can be found from the resonance condition (3):

$$k_{\text{res}} \approx \frac{\omega_{\parallel}}{c} \gamma_{\text{res}}.$$

(20)

Combining Equations (16) and (18), one finds

$$\frac{\partial}{\partial t} \left[ f_{\perp} - \frac{\alpha}{p_{\parallel}^{2}} \left( \frac{|E_{k}|^{2}}{p_{\perp}^{2}} \right) \right] = 0,$$

(21)

where

$$\alpha = \left( \frac{4}{3} \frac{e^{2}}{\pi^{2} c^{3} \gamma_{5}^{2}} \right)^{1/4}.$$  

(22)
which reduces to
\[ f_1 - \alpha \frac{\partial}{\partial p_\parallel} \left( \frac{|E_k|}{p_\parallel^{3/2}} \right) = \text{const.} \] (23)

Taking into account that for the initial moment, the major contribution of the left-hand side of Equation (23) comes from \( f_{\text{lo}} \), the corresponding expression can be written as
\[ f_1 - \alpha \frac{\partial}{\partial p_\parallel} \left( \frac{|E_k|}{p_\parallel^{3/2}} \right) = f_{\text{lo}}. \] (24)

If the distribution function \( f \) is proportional to \( n \sim 1/r^3 \) (here, \( r \) is the distance from the pulsar), then one should neglect \( f_1 \) in comparison with \( f_{\text{lo}} \). Consequently, the above equation reduces to
\[ \alpha \frac{\partial}{\partial p_\parallel} \left( \frac{|E_k|}{p_\parallel^{3/2}} \right) + f_{\text{lo}} = 0. \] (25)

As we can see, the function \( E_k(p_\parallel) \) drastically depends on the initial distribution of the primary beam electrons. According to the work of Goldreich & Julian (1969), a spinning, magnetized neutron star generates an electric field that extracts electrons from the star’s surface and accelerates them to form a low-density \((n_0 \approx B/Pce)\) and energetic primary beam. We only know the scenario of creating the primary beam, but nothing can be said about its distribution, which drastically depends on the neutron star’s surface properties and temperature. To our knowledge, there is no convincing theory that could predict the form of the distribution function of the beam electrons. Thus, we can only assume that the beam electrons have the power-law distribution
\[ f_{\text{lo}} \propto p_\parallel^{-n}, \] (26)
and for the energy density of the waves we get
\[ |E_k|^2 \propto p_\parallel^{3-2n}. \] (27)

The effective value of the pitch angle depends on \( |E_k|^2 \) as follows:
\[ \psi_0 = \frac{1}{2\pi \omega_k} \left( \frac{3n^2c^2\omega_k^2}{p_\parallel^{3/2} \gamma_p^2} |E_k|^2 \right)^{1/4}. \] (28)

Using expressions (17), (27), and (28), and replacing the integration variable \( p_\parallel \) with \( x = \epsilon/\epsilon_m \) from Equation (14), we get
\[ F_\epsilon \propto \epsilon^{-3/4} \int x^{-3/4} \left[ \int_x^{\infty} K_{5/3}(z)dz \right] dx. \] (29)

According to Aliu et al. (2008), the observed high-energy pulsed emission of the Crab pulsar is best described by the power-law spectrum \( F(\epsilon) \propto \epsilon^{-2.022} \) in the energy domain 0.01–5 GeV. At \( \epsilon = 25 \text{ GeV} \), a measured flux is several times lower, which requires a spectral cutoff somewhere between 5 and 25 GeV.

We assume that the energy of the beam electrons varies between \( \gamma_{\text{min}} \sim 10^6 \) and \( \gamma_{\text{max}} \sim 10^7 \), in which case we have \( (\epsilon/\epsilon_m)_{\text{max}} \ll 1 \) and \( (\epsilon/\epsilon_m)_{\text{min}} \gg 1 \). Under such conditions, the integral (29) can be approximately expressed by the following function:
\[ F_\epsilon \propto \epsilon^{-3/4} \exp \left[ -\left( \frac{\epsilon}{23} \right)^{1.6} \right]. \] (30)

When \( n = 6 \), the spectral index, \( \beta \), of the synchrotron emission equals 2, and the flux \( F_\epsilon \propto \epsilon^{-3/4} \exp \left[ -\left( \frac{\epsilon}{23} \right)^{1.6} \right] \). As we can see, our emission scenario predicts the exponential cutoff, with the cutoff energy of 23 GeV.

4. DISCUSSION

One of the interesting observational features of the Crab pulsar is that its multi-wavelength emission pulses from the low-frequency radio waves to hard \( \gamma \)-rays \((\epsilon > 25 \text{ GeV})\) are coincident in phase (Manchester & Taylor 1980; Aliu et al. 2008), which implies that generation of these waves occurs in the same location of the pulsar magnetosphere. According to the generally accepted point of view, the VHE emission is produced either by inverse Compton upscattering or curvature radiation, although it is clear that the aforementioned processes cannot provide the observationally evident coincidence of signals, because they do not restrict the spatial location of emission (the area in the pulsar magnetosphere where the corresponding radiation is produced). This particular problem has been studied by Machabeli & Osmanov (2010). Considering curvature radiation, we have shown that the CDI (Osmanov et al. 2008, 2009) makes the magnetic field lines rectify very efficiently. It has been shown that the increment of the instability is given by
\[ \Gamma \approx \left( -\frac{3 \omega_k^3 k_z u_b}{2 \gamma_b k_c} \right)^{1/2} \left| J_0 \left( \frac{k_z u_b}{4\Omega} \right) \right| \left( \frac{k_z c}{\Omega} \right), \] (31)
where \( \omega_k \) is the plasma frequency of the beam component; \( k_z \) and \( k_c \) are the wavevector’s radial component and the component along the rotation axis, respectively; \( u_b \) is the drift velocity of the beam component; and \( J_0 \) denotes the Bessel function of the zeroth order. By considering the perturbation corresponding to \( \lambda_s \sim R_k \), \( \lambda_r \sim 10^5 R_k \), and the initial curvature of the magnetic field lines of the order of \( R_k \), where \( R_k \sim 10^{-6} \text{cm} \) is the light cylinder lengthscale, one can show that the timescale, \( t_{\text{CDI}} \sim 1/\Gamma \), of the CDI for \( \gamma_b \sim 10^6 \) equals 1.6 s. On the other hand, the instability does its job (amplifies the toroidal magnetic field) until the excited mode escapes the magnetosphere. This happens in the characteristic timescale \( t_{\text{esc}} \sim R_k/(\nu_{\text{ph}} \sin \theta) \), where \( \nu_{\text{ph}} \equiv \omega/k \) is the phase velocity of the curvature drift wave and \( \theta \approx k_z/k_c \) is the inclination of the wavevector with respect to the rotation axis. After taking into account the dispersion relation of the curvature drift mode, \( \omega = k_z u_b / 2 \) (Osmanov et al. 2008, 2009), it is straightforward to show that \( t_{\text{esc}} \approx 2.5 \times 10^5 \text{ s} \). Therefore, the timescales satisfy the condition \( t_{\text{CDI}} \ll t_{\text{esc}} \), implying that the CDI is efficient enough to rectify the magnetic field lines (the curvature radius tends to infinity), leading to a negligible role of the curvature emission process in the observed VHE domain. By analyzing inverse Compton scattering, we have found that for the Crab pulsar’s magnetospheric parameters, even very energetic electrons are unable to produce the observed photon energies.

The emission model proposed in previous works (Machabeli & Osmanov 2009, 2010) and developed in the present paper ensures the simultaneous generation of the low- and high-frequency waves in the same area of the magnetosphere. The distribution function of relativistic particles is one dimensional at the pulsar surface, but the plasma with an anisotropic distribution function is unstable, which inevitably leads to wave excitation. The main mechanism of wave generation in plasmas of the pulsar magnetosphere is the cyclotron instability, which develops near the light cylinder. During the quasi-linear stage of the instability, a diffusion of particles arises along and across the magnetic field lines. Therefore, plasma particles acquire transverse momenta and, as a result, the synchrotron mechanism is switched on. If the resonant particles are the primary beam electrons with \( \gamma_b \sim 6 \times 10^6 \), their synchrotron emission occurs in the high-energy domain (~10 GeV). The frequency
of the original waves excited during cyclotron resonance can be estimated from Equation (3) as follows: \( \omega_{0} \approx \omega_{B}/\delta \gamma_{b} \). Estimations show that for the beam electrons with the Lorentz factor from the interval \( \gamma_{b} \sim 10^{6}–10^{8} \), radio waves are excited. Consequently, we explain the coincidence of the radio and \( \gamma \)-ray signals.

We provide theoretical confirmation of the measured power-law spectrum \( F_{\nu} \propto \nu^{-\beta} \) with \( \beta = 2 \) in the energy domain \( \epsilon = 0.01–25 \text{ GeV} \). In contrast to the standard theory of synchrotron emission (Ginzburg 1981), which only explains the spectral index, \( \beta < 1 \), our approach affords the possibility of obtaining values of the spectral index that are much higher than 1. The main reason for this is that we take into account the mechanism of creating pitch angles and obtain a certain distribution function of the emitting particles from their perpendicular momenta (see Equation (10)), which restricts the possible values of the pitch angles. The emission comes from a region of the pulsar magnetosphere where the magnetic field lines are practically straight and parallel to each other. But in the standard theory of synchrotron emission (Ginzburg 1981), it is supposed that the observed radiation is collected from a large spatial region, in various parts of which the magnetic field is oriented randomly. Thus, it is supposed that along the line of sight, the magnetic field directions are chaotic, and when finding emission flux, Equation (14) is averaged over all directions of the magnetic field (which means that integration over \( \psi \) varies from 0 to \( \pi \)). The measured decrease of the flux at \( \epsilon = 25 \text{ GeV} \) is also explained. Our theoretical spectrum \( F_{\nu} \propto \nu^{-2} \exp[-(\epsilon/23)^{1.6}] \) yields the exponential cutoff, with the cutoff energy 23 GeV.

5. SUMMARY

1. Constructing a self-consistent theory, we interpret the observations from the MAGIC Cherenkov Telescope of the pulsed emission, \( (0.01–25 \text{ GeV}) \), from the Crab pulsar.

2. It is emphasized that, due to very small cooling timescales, particles rapidly transit to the ground Landau state, with the subsequent synchrotron radiation vanishing to zero. The situation changes via the cyclotron instability, which efficiently develops on the light cylinder scales and creates non-vanishing pitch angles, leading to an efficient synchrotron process.

3. The observational fact of the coincidence of signals in the low (radio) and VHE domains is explained. The original waves excited during the cyclotron instability occur in the radio band. It is shown that the resonant electrons interact with the aforementioned waves via QLD, acquire pitch angles, and start to radiate in the synchrotron regime.

4. Considering a new approach to synchrotron theory based on our emission model, we find the spectral index, \( \beta \), of VHE emission to be equal to 2 and the exponential cutoff (characterized by energy 23 GeV) in good agreement with the observational data (Aliu et al. 2008).

In the present paper, we have shown the spatial coincidence of the region of generation of the radio and VHE emissions of the Crab pulsar, although the aforementioned coincidence of signals is detected for broad frequency ranges (from radio waves to hard \( \gamma \)-rays). Thus, we suppose that the generation of the multi-wavelength radiation of the Crab pulsar takes place in one location of the pulsar magnetosphere. In particular, if the cyclotron resonance occurs for the tail electrons, the above described processes might cause the simultaneous generation of waves in different energy domains, which is the topic of our future work.

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