Chiral spin symmetry and QCD at high temperature

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Abstract. It has been found very recently on the lattice that at high temperature at vanishing chemical potential QCD is increasingly SU(2)_CS and SU(2_NF) symmetric. We demonstrate that the chemical potential term in the QCD Lagrangian is the only term that can impose the SU(2)_CS symmetry. Consequently the quark condensate vanishes, should have approximate SU(2)_CS and SU(2_NF) symmetry.

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1 Introduction

A structure of the QCD phase diagram as well as a nature of the strongly interacting matter in different regimes attracts enormous experimental and theoretical efforts. It is established in QCD calculations on the lattice that there is a transition to the chiral symmetric regime at high temperatures and low densities, where the quark condensate gets restored [1,2,3]. Very recently the SU(2)_CS symmetry has been observed earlier in dynamical lattice simulations with the domain-wall Dirac operator [4,5] at increasing temperature [6]. These symmetries remain approximate below the chiral restoration line on the T–µ plane, where the quark condensate vanishes, should have approximate SU(2)_CS and SU(2_NF) symmetric.

2 SU(2)_CS and SU(2_NF) symmetries [4,5]

The SU(2)_CS chiral spin transformations, defined in the Dirac spinor space are

\[
\psi \rightarrow \psi' = e^{ie\Sigma/2} \psi ,
\]

with the following generators

\[
\Sigma = \{ \gamma_k, -i\gamma_5\gamma_k, \gamma_5 \},
\]

k = 1, 2, 3, 4. Different k define different irreducible representations of dim=2. U(1)_A is a subgroup of SU(2)_CS. The su(2) algebra [\Sigma_i, \Sigma_j] = 2i\epsilon_{ijk}\Sigma_k is satisfied with any Euclidean gamma-matrix, obeying the following anticommutation relations

\[
\gamma_i\gamma_j + \gamma_j\gamma_i = 2\delta^{ij}; \quad \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 \] .

The SU(2)_CS transformations mix the left- and right-handed fermions. The free massless quark Lagrangian does not have this symmetry.

An extension of the SU(2)_CS × SU(2_NF) product leads to a SU(2_NF) group. This group has the chiral symmetry of QCD SU(N_F)_L × SU(N_F)_R × U(1)_A as a subgroup. Its transformations and generators are given by

\[
\psi \rightarrow \psi' = e^{ie\tau/2} \psi ,
\]

\[
\{ (\tau_a \otimes I_D), (I_F \otimes \Sigma_i), (\tau_a \otimes \Sigma_i) \}
\]

where \( \tau \) are flavour generators with flavour index a and \( i = 1, 2, 3 \) is the SU(2)_CS index.

The fundamental vector of SU(2_NF) at N_F = 2 is

\[
\psi = \begin{pmatrix} u_L \\ u_R \\ d_L \\ d_R \end{pmatrix} .
\]
The $SU(2N_F)$ transformations mix both flavour and chirality.

3 Symmetries of different parts of the QCD Lagrangian and $SU(2)_{CS}$, $SU(2N_F)$ emergence at high temperatures.

The interaction of quarks with the gluon field in Minkowski space-time can be splitted into a temporal and a spatial part:

$$\overline{\Psi} \gamma^\mu D_\mu \Psi = \overline{\Psi} \gamma^0 D_0 \Psi + \overline{\Psi} \gamma^i D_i \Psi. \quad (7)$$

The first (temporal) term includes an interaction of the color-octet quark charge density $\Psi(x)\gamma^\mu \lambda \Psi(x) = \Psi(x)\gamma^0 \lambda \Psi(x)$ with the chromo-electric part of the gluonic field ($\lambda$ are color Gell-Mann matrices). It is invariant with respect to any unitary transformation that can be defined in the Dirac spinor space, in particular it is invariant under the chiral transformations, the $SU(2)_{CS}$ transformations (1) as well as the transformations (4).

The spatial part contains a quark kinetic term and an interaction of the chromo-magnetic field with the color-octet spatial current density. This spatial part is invariant only under chiral $SU(N_F)_L \times SU(N_F)_R \times U(1)_A$ transformations and does not admit higher $SU(2)_{CS}$ and $SU(2N_F)$ symmetries. Consequently the QCD Lagrangian has, in the chiral limit, only the $U(N_F)_L \times U(N_F)_R$ chiral symmetry.

It was found on the lattice with chirally-invariant fermions in $N_F = 2$ dynamical simulations that truncation of the near-zero modes of the Dirac operator results in emergence of the $SU(2)_{CS}$ and $SU(4)$ symmetries in hadrons [7,8,9,10].

The emergence of the $SU(2)_{CS}$ and $SU(4)$ symmetries upon truncation of the lowest modes of the Dirac operator means that the effect of the chromo-magnetic interaction in QCD is located exclusively in the near-zero modes. At the same time the confining chromo-electric interaction, which is $SU(2)_{CS}$- and $SU(4)$-symmetric, is distributed among all modes of the Dirac operator.

To conclude, the low-lying modes of the Dirac operator are responsible not only for chiral symmetry breaking, as it is seen from the Banks-Casher relation [11], but also for the $SU(2)_{CS}$ and $SU(4)$ breaking via the magnetic effects. The magnetic effects are linked exclusively to the near-zero modes.

Given this insight one could expect emergence of the $SU(2)_{CS}$ and $SU(4)$ symmetries at high temperatures, because at high temperature the near-zero modes of the Dirac operator are suppressed. This expectation has been confirmed very recently in lattice simulations with chiral fermions [12]. It was found that indeed above the critical temperature at vanishing chemical potential the approximate $SU(2)_{CS}$ and $SU(4)$ symmetries are seen in spatial correlation functions and by increasing the temperature the $SU(2)_{CS}$ and $SU(4)$ breaking effects decrease rapidly; at the highest available temperature 380 MeV these breaking effects are at the level of 5%.

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4 Symmetries of the quark chemical potential.

Will a non-zero chemical potential break this symmetry? Consider the quark part of the Euclidean QCD action at a temperature $T = 1/\beta$ in a medium with the quark chemical potential $\mu$:

$$S = \int_0^\beta d\tau \int d^3x \overline{\Psi} \gamma_\mu D_\mu + \mu \gamma_4 \Psi, \quad (8)$$

where $\Psi$ and $\overline{\Psi}$ are independent integration variables. The field $\overline{\Psi}$ is defined such that it transforms like $\Psi^\dagger \gamma_4$, i.e. like Minkowskian $\overline{\Psi}$.

This means that the quark chemical potential term

$$\mu \overline{\Psi} \gamma_4 \Psi \quad (9)$$

transforms under chiral, $SU(2)_{CS}$ and $SU(2N_F)$ transformations as

$$\mu \Psi^\dagger \overline{\Psi}, \quad (10)$$

i.e. it is invariant under all these unitary groups. In other words, the dense QCD matter not only does not break the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries, but in a sense imposes them since the chemical potential $\mu$ is an external parameter that can be arbitrary large. The chemical potential term is a color-singlet. Consequently this term can only reinforce the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries at high temperature and zero chemical potential arising from the chromo-electric color-octet term and a compensation is impossible.

We conclude that at high temperature $T \sim 400$ MeV at any chemical potential the QCD matter is approximately $SU(2)_{CS}$- and $SU(4)$-symmetric. The $SU(2)_{CS}$ and $SU(4)$ symmetries emerge due to yet unknown microscopic dynamics. This dynamics suppresses (screens)
the chromo-magnetic field while the chromo-electric interaction between quarks is still active.

The elementary objects in the high temperature QCD matter are chiral quarks connected by the chromo-electric field, without any magnetic effects, a kind of a string. These objects cannot be described as bound states in some nonrelativistic potential. With the nonrelativistic Schrödinger equation appearance of chiral as well as of $SU(2)_{CS}$ and $SU(4)$ symmetries is impossible. Consequently the QCD matter at high temperature and low chemical potential could be named a “stringy matter”, see Fig. 1.

5 Conclusions

The main new insight of this short note is that the approximate $SU(2)_{CS}$ and $SU(2N_F)$ symmetries emerge at a temperature $\sim 2T$, on the $T - \mu$ phase diagram and their breaking decreases with increased chemical potential. So we can consider the QCD matter at these temperatures as at least approximately $SU(2)_{CS}$- and $SU(2N_F)$- symmetric. These symmetries rule out the asymptotically free deconfined quarks: free quarks are incompatible with these symmetries. Note that these symmetries cannot be obtained in perturbation theory which relies on a symmetry of a free Dirac equation, i.e. on chiral symmetry. The elementary objects in the QCD matter at these temperatures are chiral quarks connected by the chromo-electric field. Such a matter is not a quark-gluon plasma (the plasma notion is defined in physics as a system of free charges with Debye screening of the electric field) and could be more adequately named as a stringy fluid.

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1. A plausible microscopic explanation of this phenomenon could be related to suppression at high $T$ of the local topological fluctuations of the gluonic field, like instantons, monopoles etc. According to the Atiyah-Singer theorem difference of the number of the left- and right-handed zero modes of the Dirac operator is related to the topological charge $Q$ of the gauge configuration. Consequently with $|Q| \geq 1$ amount of the right- and left-handed zero modes is not equal which manifestly breaks the $SU(2)_{CS}$ symmetry since the $SU(2)_{CS}$ transformations mix the left- and right-handed components of quarks. The topological configurations contain the chromo-magnetic field. What would be exact zero modes become the near-zero modes of the Dirac operator in the global gauge configuration that contain local topological fluctuations, like in the Shuryak-Diakonov-Petrov theory of chiral symmetry breaking in the instanton liquid. Consequently all effects of the chromo-magnetic field are localised in the near-zero modes, while confining chromo-electric field is distributed among all modes. At $T > T_c$ the local topological fluctuations are melt what leads first to restoration of chiral symmetry and then to $SU(2)_{CS}$ emergence.