Research Article

Wave Propagation in X-Section Piles for Low Strain Integrity Testing: Three-Dimensional Effects

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X-section cast-in-place concrete pile (referred to as XCC pile) has a different velocity response compared with circular section pile in the low strain testing due to the special cross section. Full-scale model tests of XCC pile were conducted to reveal the velocity response characteristics. The time-domain velocity responses on the pile top were obtained, which showed obvious three-dimensional effects because of the different high-frequency interferences. The test results were compared with the numerical results to validate the numerical model. Furthermore, numerical simulations were conducted to investigate the propagation characteristic of velocity waves along the longitudinal direction in the pile. The results indicated that the wave propagation was complicated as a result of the superposition of the incident wave and the reflected wave. The effects of the geometrical parameters of cross section on the three-dimensional effects of velocity responses were also studied. Three-dimensional effects would be more significant with a larger arc distance. However, the effects of arc angle were not obvious.

1. Introduction

X-section cast-in-place concrete pile (referred to as XCC pile) is a new type of abnormal section pile developed in geotechnical research institute of Hohai University and patented in China [1–4]. In view of its larger bearing capacity and use of less concrete than the common circular section pile, XCC pile has been successfully applied in highway construction as embankment pile and municipal engineering for soft foundation treatment. The quality control during the process of the pile driving is a major problem encountered in XCC pile foundation construction. Low strain integrity test has been widely applied in pile integrity test for its simplicity, speed of execution, and low cost [5, 6]. This method was developed on the basis of the one-dimensional elastic rod longitudinal wave theory, and the propagation of stress wave in the pile should satisfy plane-section assumption. Traditional one-dimensional wave theory is mainly applicable for small-diameter circular section pile. However, during low strain integrity test, the propagation characteristics of stress wave in large diameter pile and abnormal section pile are a three-dimensional problem [7, 8].

Based on the theory of three-dimensional wave, low strain dynamic tests, theoretical and numerical analyses have been conducted by many researchers. Kolsky [9] proposed an analytical expression for the longitudinal vibration response of semi-infinite solid cylinder. Gazis [10, 11] analyzed the dynamic problem of the infinitely long cylinder and obtained the expression of the displacement solution. Liao and Roeset [12] and Chow et al. [13] studied the three-dimensional effects of low strain tests through one-dimensional and three-dimensional finite element methods. Ding et al. [14] derived an analytical solution of low strain dynamic response of finite-length solid piles. With regard to the circular pipe piles, Ding et al. [7, 15] proposed a
simplified analytical method for dynamic response of intact and defective PCC piles under low strain transient concentrated loads and further studied the high-frequency interference problem systematically. Ding et al. [16] investigated the mechanism of velocity wave propagation for a concrete filled steel tubular column through the three-dimensional finite element method.

At present, the studies on the three-dimensional effects of piles in low strain dynamic tests mainly focus on the circular section pile and concrete filled steel tubular column. Despite large applications of XCC piles in practice, the three-dimensional effects of pile in low strain dynamic tests were rarely considered. Due to irregularity of the section of XCC pile, the velocity response on the pile top is different from the circular section pile and concrete filled steel tubular column. The response characteristics of the velocity wave are still unclear. Therefore, the research on the three-dimensional effects of XCC pile in low strain dynamic tests is needed. This paper explores the three-dimensional effects of XCC pile in low strain dynamic testing through full-scale model tests and numerical simulations.

2. Full-Scale Model Tests

2.1. X-Section Model Pile. The XCC pile was cast in a pre-fabricated wood mold. The elasticity modulus of concrete was $3 \times 10^4$ N/mm² after demoulding and curing for 28 days. The section size of XCC pile can be determined by three parameters: the diameter of circumscribed circle $a$ and the distance between arc endpoint $b$ and the arc angle $\theta$, as shown in Figure 1(a). The size of XCC pile in this test is $a = 94$ cm, $b = 14$ cm, and $\theta = 90\degree$. The pile length is 5 m; the model pile is shown in Figure 1(b).

In Figure 1(a) the convex direction is denoted by $i$, and the concave direction is denoted by $j$. The distance from the pile center to the boundary of convex direction is $R$. The distance from the center of the pile to the boundary of the concave direction is $r$. The distance from any measuring point to the center of the pile is $d$. The measuring points were arranged at different positions along the convex direction and the concave direction.

2.2. Velocity Response Characteristics along the Radial Direction. A set of measuring points were arranged in the direction of convex on the pile top. As shown in Figure 1, measuring points P1, P2, P3, P4, and P5 were located at 1/7R, 1/3R, 1/2R, 4/5R, and R. Hammering tests were conducted on the top and the bottom of the pile, and the velocity responses of the pile top and pile bottom were obtained. Figure 2(a) shows the time-domain velocity response in the direction of convex on the pile top. It could be seen that each measuring point was subjected to different degrees of high-frequency interference. The high-frequency wave is superimposed on the incident and reflected waves, which resulted in the fact that the incident and reflected waveform oscillated and the wave peak was difficult to recognize. In order to avoid the influence of high-frequency wave, the measured data was low-pass filtered, and the filtered data is shown in Figure 2(b). The incident and reflected wave peak could be clearly observed. However, the velocity responses on the pile top showed distinct three-dimensional effects. The waveform of point P1 presented obvious difference from other points, and the incident wave showed a larger peak velocity, but a smaller width of velocity. Moreover, an inverted peak could be observed after the incident wave. This was due to the three-dimensional effect [17]. The main three-dimensional effect in intact piles lies in the initial velocity response that becomes negative after the first positive peak in the cross section of the pile [13]. The incident wave waveforms of other measuring points were similar, but the arrival time and peak value of the incident wave for each measuring point were different. Along the convex direction, the arrival time of the incident wave peak would lag more obviously with a larger distance from pile center. However, the peak value of the wave first decreased and then increased. The peak amplitude of the incident wave was slightly larger at $d = 1/3R$ and $R$, while it was the smallest at $d = 1/2R$. This was because the points near the pile center ($d = 1/7R$ and 1/3R) were influenced largely by the hammer test which was conducted at the pile center. While the peak value of the wave for the points at $d = 4/5R$ and $R$ was affected by the wave reflected from the pile boundary, the peak of the reflected wave for each point was basically coincident. This was because as the wave propagation the three-dimensional effects would be more insignificant, and the plane-section assumption would be satisfied.

2.3. Velocity Response Characteristics along the Circumference Direction. Figure 3 shows the measuring points along the circumference direction. The points in one circumference have the same distance to the pile center, while showing different distances to the boundary of the pile. For example, point C is located at the concave arc boundary and point B is slightly away from the boundary, but point A is far away from both concave and convex boundaries. The pile top could be considered as an infinite half space domain. According to the three-dimensional elastic wave theory, as the distances to the excitation point (pile center) were the same for these three points, the response should be the same. However, due to the asymmetry of each point to the boundary of the pile, inconsistent responses of the three points would be produced.

Figure 4 shows the time-domain velocity responses of points A, B, and C. In Figure 4(a), it could be observed that the high-frequency interference was different for the three points. Point C was located at the boundary and suffered the maximum high-frequency interference, while point A was the farthest from the boundary and suffered the minimum high-frequency interference. It could also be seen from the figure that the high-frequency interference wave shape of measuring point C was regular and the amplitude is unique, while the wave shapes of the other two measuring points were more complex. Figure 4(b) shows the results of the filtered measuring data. The curves of the three measurement points were basically consistent after eliminating the effects of high-frequency waves.

Figure 5 shows the time-domain velocity responses of other measuring points along the circumference direction. It
should be noted that measuring data was filtered, and the high-frequency interference was eliminated. The velocity responses with the same distance from the pile center were basically identical. This was because the path of the incident wave to each point was the same and the effects of velocity waves reflecting from the pile boundary surface were eliminated.

3. Numerical Analysis of Three-Dimensional Effects

3.1. Numerical Model. The finite element software ABAQUS was adopted to establish a three-dimensional numerical model of the XCC pile. The low strain dynamic response was simulated and analyzed. The pile strain caused by the hammering load during low strain dynamic testing is low. Thus, the pile material was simulated as linear elasticity. The elasticity modulus \( E_c = 30 \text{ GPa} \), Poisson’s ratio \( v = 0.2 \), and the density \( \rho = 2400 \text{ kg/m}^3 \). The elastic modulus of the pile was several orders of magnitude larger than that of the soil. The damping of the soil mainly affects the response of the reflected waves. However, the soil around the pile has negligible impacts on the formation of the incident wave and the three-dimensional effects. Thus, the influence of soil was not considered in the numerical model. Furthermore, the pile was assumed to be completely free compared with the test results conveniently. The cross section is shown in Figure 1. In order to guarantee the calculation accuracy, eight-node isoparametric unit was adopted in the finite element model, and the finite element length was less than \( 1/10 \)–\( 1/8 \) of the wavelength corresponding to the maximum frequency of the exciting force [18, 19]. The finite element mesh is shown in Figure 6.

In this paper, a half sine concentrated load acting on the pile center was adopted to simulate the hammering load, expressed as

\[
p(t) = \begin{cases} 
  p_0 \sin\left(\frac{\pi t}{T_d}\right), & 0 \leq t \leq T_d, \\
  0, & t > T_d, 
\end{cases}
\]

where \( p_0 \) is the peak hammering load and \( T_d \) is duration of the hammering load. The half sine function waveform of the exciting force is shown in Figure 7.

Figure 1: The cross section of X-section pile: (a) schematic diagram; (b) model pile.

Figure 2: The time-domain velocity responses in the direction of convex on pile top: (a) the measured data; (b) the filtered data.
incident and reflected waves (test results. Figure 9 shows the ratio of peak velocity of observed that the numerical results were consistent with the k%_hus, increases, the arrival time of the incident wave gradually lags.

Generally, as the distance to the pile center increases, the velocity waves in the pile could be observed more clearly. In order to validate the finite element results, the numerical model of the tested pile was established, and the test results of P1, P2, P3, P4, and P5 were selected for comparison. Figure 8 shows the time difference Δt between incident and reflected waves along convex direction on the pile top. Generally, as the distance to the pile center increases, the arrival time of the incident wave gradually lags. Thus, Δt gradually decreased as d/R increased. It could be observed that the numerical results were consistent with the test results. Figure 9 shows the ratio of peak velocity of incident and reflected waves (V_i and V_r) along the convex direction. As d/R increased, the ratio decreased sharply, reaching minimum value in the range of d/R = 0.4–0.5, and then increased slowly. The numerical results were consistent with the test results, which indicated that the numerical results had high reliability, and the numerical model was applicable.

3.2. The Propagation Characteristic of Velocity Waves along the Longitudinal Direction in the Pile. To study the propagation characteristic of velocity wave in the X-section pile, the cloud picture of longitudinal velocity propagating along the pile was drawn. Figure 10 shows the manifestation and the scale of the velocity time-domain curve. A series of color segments from dark blue to crimson in the velocity cloud picture represented velocity waves. The wavefront in the time-domain velocity curve appeared as a pale yellow segment in the cloud picture. As the amplitude of the velocity increased, the color gradually changed, and the vicinity of the wave peak appeared to be crimson. The blue segment represented the reversed-phase wave and the dark blue referred to the peak of the reversed-phase wave. The velocity cloud picture combines velocity waves, time, and coordinates in one picture, through which the propagation of the velocity waves in the pile could be observed more clearly.

Figure 11 shows the schematic diagram of the cross section of the cloud picture. In order to reflect the three-dimensional effects, the cross section of the cloud picture was selected as the surface formed by the longitudinal extension of the connection between the midpoints of the convex and concave and the pile center.

Figure 12 shows the velocity cloud picture of X-section pile subjected to transient concentrated excitation at the pile center. In Figure 12(a) I-III, the velocity wave was generated on the pile top after the excitation. The wavefront appeared to be arc-shaped before the velocity wave reached the boundary. It could be inferred that the radiation spread in the form of spherical surface in view of the three-dimensional model of the pile. The velocity wave would reflect after reaching the boundary surface. Simultaneously, part of the velocity wave propagated along the pile. In the process of propagation, it could be observed that the shape of the wavefront gradually changed from arc to straight line, as shown in Figure 12(a) III-VIII. In Figure 12(a) V-VIII, the velocity wave first reached the concave direction and then reflected. The wavefront in the concave direction changed from arc shape to straight-line shape firstly, and then the wavefront in the convex direction propagated as the same style. Eventually, the wavefront in the two directions propagated to the pile bottom in straight-line shape uniformly. At about 0.75 ms, the peak of the incident wave reached the convex boundary, as shown in Figure 12(a) VIII.

In Figure 12(b) I-III, a blue reversed-phase wave was generated in the pile center, which was induced by the inertial reverse vibration after excitation. Meanwhile, the velocity wave propagating down along the pile had not completely left the pile top. At about 1.5 ms, the velocity wave gradually left the pile top and continued to propagate to the pile bottom. The wavefront maintained plane propagation to the pile bottom along the pile. Meanwhile velocity wave still propagated on the pile top. This was caused by the surface wave generated by excitation reflecting back and forth on the top surface of the pile, which produced the high-frequency interference wave in the velocity response curve as shown in Figure 12(b) IV-VIII. It could also be observed that the velocity waves propagating downward along the pile in the convex and concave direction were consistent. However, the propagation of the high-frequency waves on the pile top was inconsistent in these two directions. In Figure 12(b) VI-VIII, there were still a series of velocity waves following the pile passing by the incident wave. At about 2.8 ms, the wavefront of the velocity wave reached the pile bottom, as shown in Figure 12(b) VIII.

At about 3.6 ms, the incident wave peak reached the pile bottom, as shown in Figure 12(c) I. Once the incident wave reached the pile bottom, it would reflect and then propagate to the pile top, as shown in Figure 12(c) II-VIII. As a result, the reflected wave from the pile bottom and the velocity wave propagating downward in the pile would be superimposed. The wavefront of the reflected wave was no longer keeping plane propagation at this moment. When the reflected wave from the pile bottom approached the pile top, it was separated from the velocity wave propagating downward in the pile, and the wavefront gradually returned to
plane propagation. In Figure 12(c) VI-VII, it could be seen that the wavefront of the reflected wave in the convex and concave direction reached the pile top at the same time at about 5.7 ms. In addition, the wavefront of the reflected wave was superimposed on the high-frequency interference wave when it reached the pile top. At about 6.4 ms in Figure 12(c) VIII, the peak of the reflected wave reached the pile top.

According to the above research, the three-dimensional effects of the velocity responses of the X-section pile were observed obviously, which was mainly reflected in the
incident wave response and high-frequency interference. The study of the velocity response in the convex and concave direction indicated that the peak of the incident wave near the pile center was larger. The incident wave width was narrower, and it was followed by a reversed-phase wave. When it is farther away from the pile center, the arrival time of the peak incident wave lagged more obviously. In addition, the peak value of the incident wave first decreased and

![Figure 6: Three-dimensional finite element mesh of X-section pile.](image)

![Figure 7: Sketch of the exciting force.](image)

![Figure 8: The time difference between incident and reflected waves of the measuring points along the convex direction.](image)

![Figure 9: The ratio of peak velocity of incident and reflected waves \((V_i) \) and \((V_r)\) along the convex direction.](image)

![Figure 10: The manifestation and scale of the velocity time-domain curve.](image)

![Figure 11: The schematic diagram of the cross section of the cloud picture.](image)
then increased. The research on the circumferential measuring points indicated that the incident and reflected wave response at the same distance from the pile center were basically identical. The measuring points on the pile top were subjected to different degrees of high-frequency interference, and the waveforms were complicated. Since the distance to the pile edge was not the same, the high-frequency interference was different for the circumferential measuring points. The closer the pile boundary is, the higher the degree of high-frequency interference would be.

4. Parametric Analysis

4.1. Calculation Cases. In order to further analyze the velocity response on the pile top, the models considering diameter of circumcircle \(a\), arc distances \(b\), arc angles \(\theta\),

| Case  | \(a\) (m) | \(b\) (m) | \(\theta\) (°) | \(L\) (m) | \(Ec\) (GPa) | \(Td\) (ms) |
|-------|-----------|-----------|---------------|-----------|-------------|-------------|
| Case 1| 0.8       | 0.2       | 90            | 10        | 30          | 1.5         |
| Case 2| 0.8       | 0.2       | 110           | 10        | 30          | 1.5         |
| Case 3| 0.8       | 0.2       | 130           | 10        | 30          | 1.5         |
| Case 4| 0.8       | 0.1       | 90            | 10        | 30          | 1.5         |
| Case 5| 0.8       | 0.3       | 90            | 10        | 30          | 1.5         |

Figure 12: The velocity cloud picture of the X-section pile.
elasticity modulus ($E_c$), and the pulse width of excitation force ($T_d$) were calculated and analyzed. The calculation cases and parameters are shown in Table 1. Figure 13 shows the cross section of XCC pile.

4.2. Influences of the X-Section Geometry. The cross-sectional dimension of the X-section pile was controlled by three parameters: the diameter of circumcircle ($a$), the arc distance ($b$), and the arc angles ($\theta$). The section size was controlled by the diameter of the outsourcing circles, and the section shape was controlled by the arc distance and the arc angles. The results of the traditional circular section pile show that, as the diameter increased, the three-dimensional effects would be more serious. The X-section pile had the same law and the effects of the diameter were not investigated in this paper.

Figure 14 shows the effects of the arc distance on the arrival time of the peak of incident wave in the convex direction. The one-dimensional theoretical solution of the arrival time of incident wave peak is $T_d/2$. However, the propagation of velocity wave in the shallow part of X-section pile top is a three-dimensional problem in practice. The arrival time of the peak of incident wave changes with the position of measuring points. In order to facilitate the comparison of different cases, the deviation degree of incident wave caused by three-dimensional effect from one-dimensional solution was studied, and the difference between arrival time of the peak of incident wave and $T_d/2$ was taken as the vertical coordinate. The one-dimensional solution was shown using red dotted line. It could be observed that the trend of each curve was similar, but the position close to one-dimensional solution was not the same. As the arc distance decreased, the position having the same results to one-dimensional solution was closer to the pile center. Before approaching the one-dimensional solution, the results were closer to the one-dimensional solution as the arc distance decreased. This was because the smaller arc distance would induce a smaller cross-sectional area. As a result, the three-dimensional effects would be smaller. After reaching the one-dimensional solution, the difference between the three curves decreased gradually. This was because changing the arc distance did not greatly change the length in the convex direction. Eventually, the path of the incident wave reaching each point was similar. Figure 15 shows the effects of arc distance on the ratio of the peak of incident and reflected waves in the convex direction. It could be seen that the overall trend of each curve was consistent. As the arc distance decreased, the curve would be closer to the one-dimensional solution, indicating a smaller three-dimensional effect.

Figure 16 shows the effects of arc angles on the arrival time of the peak of incident wave.
positions close to one-dimensional solution were basically the same, in the range of 0.4 \( R \)−0.5 \( R \). This was because the change of arc angles induced only small change of the shape and the area of the cross section. The difference of the cross-sectional areas for \( \theta = 90^\circ \) and \( \theta = 130^\circ \) was only 16.7%. In addition, the change of arc angles would not affect the farthest propagation distance of the velocity wave. Figure 17 shows the effects of arc angles on the ratio of the peak of incident and reflected waves in the convex direction. The curves were almost coincident, indicating that the arc angles had little effects on the peak of incident wave. They were all close to the one-dimensional solution in the region of 0.4 \( R \)−0.5 \( R \).

5. Conclusion

XCC pile is considered as a special-shaped cross section pile, whose stress wave propagation law is different from the traditional circular section pile in the low strain testing. Based on the full-scale model tests and numerical simulations, this paper studies the three-dimensional effects of wave propagation in the XCC pile. The conclusions that can be drawn from this work are as follows.

(1) The velocity response on the pile top showed significant three-dimensional effects and high-frequency interference. The peak of the incident wave near the pile center was the largest, while the width was narrower. From the pile center to the boundary of the convex, the arrival time of the incident wave peak lagged gradually, and the peak value of the wave first decreased and then increased. The reflected wave peak from the pile bottom of each measuring point was basically coincident.

(2) The responses of incident and reflected waves for the points with the same distance from the pile center were basically consistent, while the high-frequency interference showed obvious difference due to the different distances to the boundary. The closer the pile edge is, the higher the frequency of high-frequency interference would be.

(3) The velocity wave propagated in the form of spherical surface before reaching the pile boundary. After the velocity wave reached the concave and concave boundary, the propagation of wavefront would change to the straight-line shape. In addition, the plane propagation at a certain depth would be changed due to the superposition of the incident wave and the reflected wave.
(4) The incident peak arrival time and the incident-reflected wave peak ratio closest to the one-dimensional solution were in the region of $d = 0.4 R - 0.5 R$. As the arc distance decreased, the velocity response of the pile would be closer to the one-dimensional solution, indicating the smaller three-dimensional effects. The effects of the arc angle on the three-dimensional effect were not obvious.

Data Availability
The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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