A generalized resistive force theory for rate-dependent intrusion phenomena in granular media

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Recent studies reveal that resistive forces in granular intrusion can be explained with rate-independent drag laws when intrusion is slow, but rate effects can occur in situations like dynamic impact. It is challenging to determine the various roles of inertia at the bulk (macro) and grain (micro) scales, as well as the role of rate in boundary interactions, and to reconcile the addition of these effects with the rate-independent force response in reduced-order models like granular Resistive Force Theory (RFT). Many studies measure the role of micro-inertia through a dimensionless shear-rate known as the inertial number and use it to add strain-rate-dependence into the constitutive relations. We demonstrate with data from rapid plate drag tests, freely locomoting rigid wheel experiments, and legged runners that rate-dependent dynamics emerge even when the inertial number is negligibly small. While a velocity-squared correction is often used to describe micro-inertial effects in rapid plate drag tests, this correction is insufficient to reproduce the dynamics observed in more complex intrusions like that of rigid wheel locomotion. We find a frictional flow continuum model replicates all observed behaviors without inclusion of micro-inertial effects. Based on the characteristics of the observed flows, we propose a modified rate-dependent RFT for arbitrary intruders. The form reconciles all considered cases by adding a geometry-dependent modification and a macro-inertial force to RFT. Our results reveal how rate-dependent mechanisms can emerge in rate-insensitive granular media, and highlight the ability of simple physical corrections in enhancing the design capabilities of reduced-order models like RFT.

Granular media | Intrusion | Resistive Force Theory | Continuum modeling | Rate-effects | Locomotion

Locomotion methods for traversing soft granular substrates can vary greatly (see Figure 1). A common scenario involves intruding a rigid or flexible solid into granular media (GM) and using the resistive force to propel the system. Such intrusions into GM create flow and force responses in the media, where the media can exhibit both solid-like and fluid-like characteristics. GM will deform elastically under stress like a solid, but begin to flow like a fluid once a friction-based yield criterion is exceeded. If the body intrudes slowly, a granular stress arises independent of the intrusion rate, and the resistive force on the intruding body remains in the quasistatic limit (1–3).

However, various intrusion scenarios can arise which deform the media fast enough that the net force response is affected. Examples of such intrusions include ballistics, meteor impacts, rapid locomotion, and many industrial processes (4–7). Once the intruder has significant inertia, the momentum transfer into and throughout the system causes the intrusion force and granular stresses to become rate-dependent, which has significant effects on locomotion performance. An example of this is rigid wheel locomotion, which rapidly shears the granular substrate to generate a propulsive force. Unlike the engineered deformation of some rubber tires, rigid wheels like those found in planetary rovers (8) continuously shear and deform the local GM (9) to locomote in loosely consolidated terrain. This coupled system of intruder and media can become complex and difficult to model especially when rate-dependence becomes a factor. To model such locomotion accurately, we must first understand the force response of the actively deforming GM.

In recent years, an empirical, reduced-order methodology called granular Resistive Force Theory (RFT) has been successful in estimating the force response for arbitrary intruding geometries, permitting direct simulation of locomotion in granular volumes (14–16). RFT models these complex intrusions presuming the resistive forces in GM are localized (17) and can be directly decomposed over a set of surface elements as we shall now describe. The components of the granular traction \( t \) on each element in a planarized 2D coordinate system, where \( z \) is the vertical axis parallel to gravity and \( x \) is a chosen horizontal axis perpendicular to \( z \), can be written as \( t_{x,z} = \alpha_{x,z}(\beta, \gamma) H(z|x) \), dependent on the element’s orientation angle \( \beta \), velocity angle \( \gamma \), effective vertical depth from the free surface \( z \), and area \( dA \), with \( H \) being the Heaviside function. The empirical traction-per-depth functions \( \alpha_{x,z} \).

**Significance Statement**

Modeling granular intrusion is critical to understanding biolocomotion and vehicle mobility on earth and beyond. Resistive Force Theory (RFT) allows rapid calculation of force distributions on generic intruders but is restricted to quasi-static substrate behaviors and does not capture rate effects. We demonstrate in laboratory experiments that rate effects arise in nontrivial ways in locomotion settings. Interestingly, a friction-based constitutive model with no shear-rate dependence captures the observed phenomena, enabling discovery of two key effects—one momentum-based, the other structural—which generate intrusive rate dependence. Incorporation of these effects creates "Dynamic" RFT, which accurately and rapidly models penetration, drag, and wheeled and limb locomotion. The success of Dynamic RFT points to new methods to model movement in diverse flowable materials.

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are measured with small plate intrusion experiments which vary \( \beta \) and \( \gamma \). By summing these locally-defined tractions, RFT predicts the net resistive force and moment on the entire intruder. For example, RFT gives the following intrusion force formula:

\[
F_{x,z} = \int_S \alpha_{x,z}(\beta, \gamma) H(z)|z| \, dA
\]

Standard RFT is limited to quasistatic intrusion, however, rate effects are known to emerge in faster granular flows. For instance, grain inertia (i.e. micro-inertia) begins to affect the medium’s effective strength when the flow rate becomes large. This is captured by a dependence of stress on the dimensionless “inertial number”, \( I = \gamma \sqrt{d^2 \rho_s / P} \), where \( \gamma \) is the shear-rate, \( d \) the mean grain diameter, \( \rho_s \) the solid particle density, and \( P \) the local pressure (1, 2). While \( I \) captures micro-inertial effects, there also exists a macro-inertia associated with the granular bulk. Macro-inertia adds resistive reaction forces to the intruder in accord with the total momentum change of the material.

In this paper, we propose a generic RFT that incorporates the role of rate in the granular intrusion of arbitrary shapes. Because simple intruders may mask certain physical effects, we first consider a complex granular intruder comprising a groused wheel locomotor (grousers are short protrusions perpendicular to the cylindrical wheel that help shear the GM), which propels itself with cylindrical shearing of the granular media. It also provides a good case to study inertial effects as it circumferentially accelerates the grains under pressure to create a flow which in turn influences the dynamics of wheel locomotion. By combining existing literature, continuum modeling, and experimental verification, we identify the relevant physics and its interpretation as corrections to the quasistatic RFT model. Two key rate effects are identified, and, once incorporated, the new Dynamic RFT (DRFT) model is able to maintain a reduced-order form like standard RFT. Two additional test cases of submerged lateral plate intrusion and a “four-flap runner” are used for verification. While the plate tests provide a macro-inertia-dominant granular intrusion case, the runner provides a visually similar but dynamically contrasting case to groused wheel locomotion, with which the efficacy of our proposed form can be checked. The final form of DRFT reconciles various inertial effects and effectively models all the considered intrusion cases.

**Results and Discussion**

**Wheel Locomotion Experiments.** Figure 2A shows a representative CAD model of the single grousered-wheel free-locomotion experimental setup used for this study. Figure 2B shows a snapshot of quasi-static RFT modeling of the groused locomotion, and Figure 2C and D indicate the data collection methodology used. More details of the experimental setup and RFT simulations are provided in the Materials and Methods section (and Movie S3 in the Supplemental Information).

Figure 2E and F show the trends of steady state translation velocity (E) and sinkage (F) with increasing angular velocity for a groused wheel’s free locomotion. Experiments indicate the emergence of a rate dependent effect in wheel locomotion, as an increase in slipping accompanied by an increase in the sinkage of the wheels breaks the linear trend seen in the quasi-static domain of \( \omega < 30 \text{RPM} \) (corresponding to \( \omega / \omega_0 < 0.46 \) in Fig 2(E)).

**Role of inertia in simple intrusions.** A fundamental approach for examining rate effects during granular intrusion is a conventional momentum balance-based control volume analysis. Using a control volume fixed in the intruder frame, the resistive force on a moving intruder can be expressed as the sum of the momentum flux along the control volume boundaries, the net traction along boundaries, and the body forces within the control volume (Supplementary Information, Section S1). In the simplest case, where stress along the far-field boundaries and the body forces in the bulk can be assumed to remain constant, only the macroscopic momentum flux \( \int_S \rho v^2 \) changes when intruder speed increases. This effect is approximately proportional to \( \rho Av^2 \) in geometries (including all of our studied scenarios) where the far-field motion is zero in the lab frame and the plate deflects material in accord with its area; here, \( A \) is the intruder area, \( \rho \) is the material density, and \( v \) is the normal intruder velocity.

A number of previous studies modeled the rate-dependent force response of granular materials to simple rigid intruders by separating the forces into depth-dependent and rate-dependent force terms (18–26). The rate-dependent momentum flux in these scenarios is representable with a \( v^2 \) term akin to the aforementioned inertial force \( \int_S \rho v^2 \), which is added to a depth-dependent model to model the force response. These models were used for a variety of flat plate-like intrusions (3, 18, 20, 27). Examination of experimental data in (20, 27) using critically packed media (i.e. whose flow induces no volume change), agrees with a rate-dependent force addition taking the form \( \lambda \rho Av^2 \) in simple vertical and horizontal intrusions, where \( \lambda \) is a \( O(1) \) scalar fitting constant that accounts for the extent of flow deviation of the media around the intruding surface (see Figure S1 and S2, and Movie S1 and S2 of the Supplementary Information for details). Physically, these findings suggest that in the intruder’s frame, incoming material is deflected away as it nears the intruder, resulting in a reaction force on the intruder roughly equal to the size of the deflection zone (order of the intruder width) times the momentum flux of the material.

**Investigating a direct inertial modification to RFT.** Motivated by these results, we consider whether a local inertial force addition to the quasi-static RFT relations can explain the
We use a ‘leading edge hypothesis’ to ensure resistive forces would experience if it were moving on its own as an isolated plate in a semi-infinite domain. Figure 2 E and F show that granular media used in our experiments (14), RFT was applied outward normal and velocity make a positive inner product (n · v) > 0). Using the established RFT functions αx,z for the granular media used in our experiments (14), RFT was applied to the spinning wheel geometry. Figure 2 E and F show that while RFT captures the speed vs ω trends at low ω, at higher ω it does not predict the wheel locomotion kinematics. RFT predicts a linear relation between the angular and translation speeds, but diverges as ω increases. The insufficiency of quasi-static RFT alone (λ = 0 in Figure 2 D and E) is expected in this domain due to the rate-insensitive nature of RFT’s solutions (see Section 2 of the Supplementary Information for more details).

The introduction of the inertial force term (λ > 0) adds a new force contribution, which has net force components opposite to the downward direction of gravity and opposite to the horizontal direction of wheel translation. This upward force results in a decrease of wheel sinkage, which is opposite to the experimental observation. The magnitude of these extra forces is also very small; the pre-factor λ was varied from 1 to 100 in an attempt to match experiment, but this has little effect on the outcome and the trends for both velocity and sinkage could not be matched (Figure 2 E and F). It seems a phenomenon beyond a simple additive momentum...
flux appears to dominate the rate effects present in wheeled locomotion, suggesting different physics are at play in complex intrusions like wheeled locomotion. Owing to the failure of the momentum flux correction term on its own, we next consider a more fundamental model of the material, a continuum model, to discern a broader rationale for what causes rate-dependent intruder phenomenology.

Continuum modeling analyses of granular intrusion. We consider a simple granular continuum model, which has been shown to capture intruder dynamics well in previous studies (16, 28). The poppy seeds (PS) are modeled as a granular continuum with a Ducker-Prager (rate-independent friction-based) yield criterion, incompressible plastic shear flow, and a criterion that the material separates into a stress-free media when sinkage with rotation rate. It is worth emphasizing that no I-dependent rheology is assumed; the granular constitutive model is rate-insensitive. Below the yield criterion, the grains act like a linear-elastic solid. The input material properties in Table S1 (in the Supplementary Information) are used. We use the Material Point Method (MPM) algorithm described in Dunatunga and Kamrin (29, 30) to implement a set of constitutive equations representing granular volumes assuming 2D plane-strain motion. A schematic representation of an explicit-time integration MPM-step is shown in Figure 3A; the material points carry the continuum data and are moved each step with the help of a background grid. Figure 3B shows a sample wheel-locomotion using MPM, plotting the displacement magnitude.

The trends of steady state translation velocity and sinkage with varying ω obtained using continuum modeling are plotted in Figure 3C and D. Continuum modeling successfully captures the experimental trends for wheel locomotion; in particular the model captures the turnover in the normalised v−ω curve at the correct rotation speed, and correctly predicts increased sinkage with rotation rate. It is worth emphasizing that we obtained these results with a rate-insensitive constitutive model and rate-insensitive surface interaction, which suggests that the observed rate-effects in these data do not have micro-inertia origins nor origins related to complex wheel-grain interactions. The rate effects observed must therefore originate from macro-inertia, i.e. the presence of ρvI in the momentum balance. To check robustness, we also applied small changes to the initial state of the experimental and
simulated systems — including minor variations in initial wheel depth, initial wheel velocity, and ramp-rate of the wheel — and observed that the steady-state results in both systems were insensitive to these variations.

To further validate the model predictions, we conducted experiments to visualize subsurface flow fields and compare to the model. The experiments place the wheel adjacent to a plexiglass plate so PIV can capture the underlying grain motion (3). Velocity fields in grains for 30 and 60 RPM cases from continuum modeling and experimental PIV analysis are plotted Figure 4A. Wall drag from the plexiglass plate likely causes the granular flows in experiment to be slower overall than the model, however the key structural features of the flow under the wheel agree between the experiments and model. Importantly, both show a zone of material ahead of the wheel being pushed forward and a wide zone under and behind the wheel being pushed to the rear. The rear flow zone also grows with increasing RPM due to higher flow entrainment and material movement at higher RPM.

Figure 4B shows the plastic strain-rate magnitude from continuum modeling simulations for low and high ω cases. The plots make it possible to visualize how different portions of the wheel derive their resistive forces from different zones of the granular media. While the strain-rate profiles change as angular velocities increase, the basic patterns of shearing remain similar. The sheared material reaches the free surface in the zone affecting the traction at the trailing rear face of the wheel. The remaining flow-lines extend to the free surface on the leading front face of the wheel. Importantly, the height of the free-surface on the rear side of the wheel decreases with increasing ω; qualitatively, as ω grows, the wheel expels out the material on the rear side. The reduction in rear free surface height suggests a reduction in the pressure head and consequent weakening of the material in the rear shear zone.

**Dynamic RFT.** We now propose a more general RFT covering slow to rapid intrusion in granular media, which we shall refer to as ‘Dynamic RFT’ (DRFT). DRFT modifies the quasi-static RFT in two ways. As mentioned before, there is the momentum flux contribution, which we term the dynamic inertial correction ($\sim \rho v^2 \lambda$), which is required for the transfer of momentum to the granular material surrounding the intruder. The second modification, which we now propose, describes the way in which increased bulk inertia can change the free-surface geometry. A change to the free-surface geometry then feeds back onto the resistive forces through the depth-dependence of RFT. We denote this modification as the dynamic structural correction. Taken together, DRFT imposes the following formula for the traction on a surface element:

$$t_{x,z} = \alpha_{x,z}(\beta, \gamma) H(\tilde{z})|\tilde{z}| + \lambda \rho v^2 \rho_{x,z}$$  \[6\]

where $\tilde{z}$ indicates the effective depth of the surface element. That is, $\tilde{z} = z - \delta h$ where $\delta h$ is the height decrease of the free surface in the zone affecting the traction at $(x, z)$. To use DRFT, one must determine the appropriate $\delta h$ for each
surface element of the intruder as a function of the intruder motion.

Considering the DRFT relation together with the observations from Figure 4, Figure 3 C and D, and Figure 2 E and F, for the case of wheels, a net decrease in the effective free surface height (δh) behind the wheel occurs. While a dynamic inertial correction still exists and increases the traction in the forward direction of wheel translation, the decrease in height (δh) is expected to produce a dominating dynamic structural correction. Since the decrease in height (δh) is greater in the higher velocity range, the latter effect is also expected to be larger in such cases, overtaking any net effect due to the dynamic inertial correction. As we shall show, this explanation matches the trends observed in the continuum modeling simulations as well as experiments.

Figure 4C shows δh as measured from the continuum model simulations by identifying the lowest point making rear contact with the wheel for which hydrostatic-pressure 0. Indeed, the faster the wheel spins, the deeper this point descends. Given the paucity of parameters in the continuum model, dimensional analysis is useful; for a given substrate material it suggests the form δh = r · ψ(rω²/g) for some function ψ. Surprisingly, we find that ψ is well-approximated by the identity. The fit of δh = r (rω²/g) and the continuum modeling results in Figure 4D show good agreement. Combined with the understanding developed in the previous section, the form of the effective free surface is approximated using a simple partition as shown in Figure 4C, with the rear zone of the wheel set to have a constant free-surface height reduction h_{back} differing from the initial free-surface height (undisturbed medium height) by a term δh = r (rω²/g). To select the dividing angle delineating the front and rear affected zones of flow, we choose to evenly divide along the terramechanical contact angle for driven wheels (31). This new model only changes the effective free-surface heights for elements on the rear part of the intruding wheel surface. We implement this DRFT model using the same implicit RFT code framework discussed in the material and methods section, using λ = 1 and ρ ≈ ρc = 638kg/m³.

The trends of translation velocity and sinkage with respect to ω now show good agreement between experiment, continuum modeling, and DRFT (Figure 3 C and D).

The agreement with DRFT suggests that the low-to-high slip transition in wheeled locomotion (where slip=1 − v/rω for v is the translational velocity, r is the nominal radius, and ω is the angular velocity of the wheel) occurs largely because faster spinning wheels remove material from behind the wheel, which reduces the pressure in the rear zone, thereby weakening the base of material that would otherwise provide a scaffold off of which the wheel pushes. Updating RFT by accounting for this effect has appropriately captured the dynamics of the complex wheel locomotion scenario in a reduced-order modeling framework.

Additional verification studies for Dynamic RFT. The wheel tests provide a complex intrusion scenario, and have a dynamic structural correction that is much larger than the inertial correction. To check the robustness of Eq 6, we now examine the converse situation with two additional sets of simulations where, based on the arguments of the previous section, we expect the dynamic structural correction to be small and the inertial correction to dominate.

Submerged horizontal intrusion: Thin plates submerged in a granular media (PS) at various fixed depths (20–40mm) are dragged horizontally at different speeds. The continuum MPM model runs in plane strain, where the plate has a length of 0.016m and the effective medium density is 900kg/m². The filled circles in Figure 5A show the calculated drag force with respect to the drag speed. Figure 5B shows the deformation profiles around the plate at two selected speeds (which differ by about an order of magnitude). The profiles in Figure 5B of high and low speed intrusion (which differ by about an order of magnitude). The filled circles in Figure 5A show the calculated drag force with respect to the drag speed. Figure 5B shows the deformation profiles around the plate at two selected speeds (which differ by about an order of magnitude). The profiles in Figure 5B of high and low speed intrusion suggest that the intruder tractions arise from pressing the granular material in front toward the same free surface height. In the context of the proposed DRFT formulation, δh = 0 in both the slow and fast cases. For this reason, we expect no dynamic structural correction from DRFT and
for rate-effects to emerge solely from the inertial correction, i.e. \( F_{\text{drag}} = K|z| + \lambda \rho Av^2 \). In the slowest cases \( (v \sim 0) \), we obtain a linear force versus depth relation, \( F_{\text{drag}} = K|z| \) for \( K = 580 \text{N/m} \). As speed grows, we find the DRFT prediction matches the data well at three different depths, for \( \lambda = 1.1 \). The same value of \( \lambda \) also matches the rate dependence observed in experiments for horizontally driven intruders at the free surface (27). These results concur with our hypothesis and confirm the DRFT prediction for submerged sideways intrusion. For similar reasons as just discussed, we expect symmetric vertical intrusion of plates to also invoke a negligible structural correction; see Supplementary Information (Figure S2) for details and confirmation against DRFT.

**Four-flap runner:** While the dragged plates are forced to move at set speeds, we also study a self-propelling locomotor, a four-flap runner, whose locomotion speed is determined via the interactions of the locomotor’s self-actuated limbs (flap motion) and the substrate dynamics (geometric details are in Table S1 of the Supplementary Information). The low number of flaps, along with the large flap length to inner radius ratio minimizes the interaction between neighboring flap intrusions of the runner’s resultant granular flow.

The outer diameter and dimensionless mass ratio, \( (\text{mass of wheel})/\rho uk^2 w \), of the runner were set to be in the same range as the corresponding twenty grousseor wheel of the earlier section (190mm vs 212mm and \( \approx 6 \), respectively). The runners had their angular velocity varied over a range of 10 RPM to 300 RPM. We used a characteristic angular velocity of \( \omega_0 = \sqrt{g/\ell} \) to set the tested range, varying from \( \omega_0 \omega_0 = 0 \) to \( \omega_0 \omega_0 = 4.5 \). MPM solutions of the continuum model generated baseline reference solutions for these locomotion scenarios for comparison with DRFT. Figure 6A shows the variation of equivalent plastic strain for four different angular velocities in the continuum model. No visible self-interaction of the granular material between limb strikes suggests a minimal role of the dynamic structural correction, as the free surface height remains unchanged. We then modeled these scenarios using DRFT with small, \( O(1) \), \( \lambda \) values \( (\lambda = 0.2, 4) \) and the dynamic structural correction set to zero. Figure 6B and C show the resulting steady-state sinkage and translation velocity at various angular velocities, showing that DRFT captures the kinematic trends of the reference solution. It is interesting that the runners and grousseowheels display qualitatively reversed behaviors — as spin increases, runners sink less and move faster whereas wheels sink more and travel slower. This difference is reconciled in DRFT.

Furthermore, using this approach, we can now explain experimental observations of running robots made in Li et al. (14). Similar to our four-flap runners, they observed decreasing slips with increasing angular velocities in experiments with C-legged robots (similar to Fig 1c). Their robotic runner was driven with dimensionless spin ratios \( (\omega_0 \omega_0) \) ranging over \( 0 - 1.25 \) \( (\omega_{\text{max}} = 240 \text{RPM} \text{ and } \omega_0 = (g/\ell)^{0.5} = 190 \text{RPM} \) and the authors found quasistatic RFT sufficient to describe their experimental findings. Our four-flap runner study covers a larger spin ratio range, \( 0 - 4.5 \), and from comparing behaviors, the quasistatic RFT modeling in Li appears to have been sufficient because the *dynamic inertial correction* is still small in their tested range; as in our results, the dynamic inertial correction starts becoming noticeable only above \( \omega_0 \)-ratios of \( \sim 1.2 \). Another C-legged robot study by Zhang et al. (32), tested locomotion over a larger \( \omega_0 \omega_0 \) range of \( 0 - 3.8 \) \( (\omega_{\text{max}} = 12 \text{Hz} = 720 \text{RPM} \text{ and } \omega_0 = (g/\ell)^{0.5} = 9.8/0.025)^{0.5} = 190 \text{RPM} \). In the higher range of spins, the sinkage and velocity in their experiments breaks away from quasistatic RFT and matches the trends we find with our DRFT runner simulations — e.g. elevating above the rest depth and slipping less at faster spins — further verifying our hypothesis.
Conclusions

Our study of rigid intrusion into granular media indicates that the force response upon intrusion consists of two primary rate-dependent modifications: (1) a dynamic inertial correction, and (2) a dynamic structural correction. The dynamic inertial correction accounts for the momentum transfer to the surrounding material, whereas the dynamic structural correction describes how a rapidly moving intruder can change the pressure head by modifying the free surface. Both effects are related to the macro-inertia of the media. For the scenarios considered here, micro-inertial effects are not significant even if the motion appears ‘fast’; previous work on rapid projectile penetration (30) indicates that the high pressures that develop around rapid intruders keep I small. Dynamic RFT has enough generality to explain two opposing scenarios: weakening of the GM during groused wheel locomotion, as well as strengthening of the GM during rapid running. We have shown that DRFT accurately predicts the GM system behaviors in the limiting cases, i.e. when one of the two dynamic effects is dominant. Further studies will be required to fully test the model for mixed cases where both dynamic corrections are significant. We have assumed additivity, in line with previous notions of a ‘static’ component and an inertial component of the intrusion force (3, 18, 20, 27). However, it is possible a more complicated functional combination may arise.

Although our study mainly focused on dry non-cohesive granular media, the formulation of DRFT in granular flows suggests the existence of other similar reduced-order models in other materials. A combination of experiments and continuum modeling proved vital in this study for verifying the underlying physics. Future work may explore faster methods of predicting flows, along with various complex intruders to systematically determine the dynamic structural correction. Further studies could also explore the existence of similar reduced-order models for related classes of materials like non-critical state GM, cohesive sands/muds, and fluid-saturated sands.

Materials and Methods

Experiments (Wheel Locomotion). To perform systematic experiments of free-wheel locomotion, we built a simple, automated ‘terrain mechanics tested’. A powerful gear motor (capable of providing up to 70RPM at 14.1 Nm ) is mounted in a carriage (Figure 2A) which moves freely along vertical and horizontal linear bearings. We control the effective vertical loading of the wheels through a wheels (Figure 4A, right), we also perform PIV analysis of the wheel which moves freely along vertical and horizontal linear bearings. 8 inches of the flow fields due to the friction experienced by the material flowing next to the sidewall. The open-source PIVLab package was used in MATLAB for the analysis.

RFT modeling. To capture the experimental dynamics of the wheel locomotion trials with a reduced-order model, we implement RFT simulations using independent experimental variables and an implicit iterative scheme. A sample simulation diagram is shown in Figure 2B. Utilizing the rigid wheel assumption, the wheel surfaces are discretized into smaller sub-elements which as a whole approximate the total geometry. The orientation angle (β), velocity angle (γ), effective depth from the free surface (z), and areas (dA) of each sub-element is used along with RFT-assumptions of locality and additivity of granular resistive forces, and a ‘leading edge hypothesis’ (discussed earlier) to find net the resistive force and moment. In doing so, equation 1 is evaluated using established RFT coefficients from (14) and the associated scaling coefficients from Table S1 of the Supplementary Information. A momentum balance in the x and z coordinates then models wheel motion in the horizontal and vertical direction. The effective heights of wheel grousers were also taken to be one-third of their true physical length (based on experimental PIV data) to account for the shadowing effect (34). Convergence studies of the force response determined the discretization fineness of the wheel shape. Each inner-circumferential subsurface lug was divided into 14 elements and each of the lug surfaces (1 normal and 2 side-wise) was divided into 8 elements. Thus, wheel had 570 surface-elements in total. For the Dynamic RFT implementation, only the effective heights experienced by surface-elements on the rear side of the wheel were modified. This height modification was based on the formulation shown in figure 4(D). The rear region was taken as the region of rear half of the contact area between sand and wheel (see figure 4(C)). The division was based on the angle subtended by the contact region at the wheel center.

Continuum modeling. We use the material point method (MPM) to carry out the continuum modeling of the system. In MPM, material is discretized as a set of material point tracers that carry the full continuum state. These tracers, representing a chunk of material around their position, are ‘cast’ onto a background simulation grid where equations of motion are solved. Thus, material point tracers act as quadrature points for solving the weak form of the momentum balance equations on a static background simulation grid. A forward-Euler time integration method was used to update the material position and properties. A representative schematic for a MPM time-step update is given in figure 3A. We model the wheel as a high-stiffness elastic solid with a fixed angular velocity, which is instantaneously enforced on the wheel. In terms of accuracy, we use a 200x200 grid representing a domain size of 1m x 1m with initial seeding of 2x2 linear material points per grid cell.

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