In this paper we suggest a new algorithm for determination of signal-to-noise ratio (SNR). SNR is a quantitative measure widely used in science and engineering. Generally, methods for determination of SNR are based on using of experimentally defined power of noise level, or some conditional noise criterion which can be specified for signal processing. In the present work we describe method for determination of SNR of chaotic and stochastic signals at unknown power levels of signal and noise. For this aim we use information as difference between unconditional and conditional entropy. Our theoretical results are confirmed by results of analysis of signals which can be described by nonlinear maps and presented as overlapping of harmonic and stochastic signals.

Introduction

The main characteristics of communication electronic systems are SNR and bit error rate (BER) defined via SNR [1].

As usual, SNR can be defined as signal power to noise power ratio [2-6]. This approach is developed in many works. For example, [7] contains a description of using of multiple linear regression with coefficients chosen for different types of noise for defining of SNR.

Estimation of SNR can be made via relation between autocorrelation functions of a signal and noise shifted by time [8], but noise and signal levels have arbitrary chosen values. Authors of [9] use wavelet signal filtering with certain chosen coefficients, and define SNR as relation between variance of signal and variance of noise. It was specified that in this case the SNR calculation is relatively fast. However, due to the limited bandwidth of wavelet functions spectrum, this method is not applicable to analyze very noisy signals. Effective SNR estimation defined as difference between standard SNR and signal quality indicator which also must be given previously is described in [10]. The single distinction between modified segmental SNR and standard SNR is in the fact that for calculation of modified SNR it is necessary to summarize SNR for separated time intervals [11].

SNR is used in many areas of science and engineering such as in wireless telecommunications systems [12-15], medicine [16-17], nuclear physics [18], neuroscience [19], sound technique [20-22], optoelectronics [23-24], nanotechnology [25-26], astrophysics [27-28], etc.

However, generally accepted methods and original algorithms for calculation of SNR used in the mentioned above papers have the following limitations:

- necessity to set value of noise level according to experimental details or conditional criteria;

- absence of standard algorithms for SNR definition;

- absence of a universal theoretical approach for SNR definition for signals with unknown noise level.

So, the problem follows from the described above: is it possible to define SNR as a relation between information and entropy? We formulate this problem by the following way because information is a universal measure of determinacy of a signal, and entropy is a measure of its uncertainty (noise). For solving of this problem we can accept that information is not a
local characteristic, but an averaged value defining via difference between unconditional and conditional entropy [29]. Information entropy is often used in different research. For example, comparison of composite, refined and multi-scale cross-sample entropy of complex signals is described in [30]. Entropic analysis can be applied for the description of such complex signals as multi-fractal signals, financial time series, etc. Informational entropy can be used for signal filtering [31]. Entropy can be also applied for classification of infrasound signals [32]. Cognitive state of human subjects on the base of entropic analysis of physiological signal is described in [33].

Relationship between SNR and entropy is specified in [34], in this work value of maximum entropy of probability density function for convolutional noise is used for the description of modulation and SNR. An image filtering based on calculation of entropy is suggested in [35]. Seismic signal filtering based on wavelet transform and application of Shannon and Tsallis entropy for determination of SNR is described in [36]. Entropy analysis can be useful for the description of dynamical systems with chaotic behavior [37-38]. Unfortunately, in spite of the fact that relation between entropy and SNR has been described in these works, a ratio between information and entropy hasn’t been used, and described above limitations for definition of SNR remain valid.

The aim of this work is to define value of SNR as information to entropy ratio (IER) for various signals (mixture of harmonic signal and noise, chaotic signals from dynamical systems [39-41]).

1. A new algorithm for determination of SNR

By the definition given in [29] value of full information of a signal \( x = x(t) \) transmitted over a communication channel can be define as difference between one-dimensional Shannon entropy and conditional entropy as

\[
I(x, y) = S(x) - S(x \mid y),
\]

where \( y(t) \) is characteristic of a receiver. Unconditional Shannon entropy can be defined as

\[
S(x) = -\sum_{i=1}^{N} p_i \ln(p_i),
\]

where \( p_i \) is a probability of detecting of variable \( x \) in a i-th cell characterized by size \( \delta \), \( S(x \mid y) \) is conditional entropy given as

\[
S(x \mid y) = -\sum_{i=1}^{N} \sum_{j=1}^{M} P(x_i, y_j) \ln(P(x_i \mid y_j)).
\]

Here \( P(x_i \mid y_j) \) is conditional probability. Defining of information according to Eq. (1) is possible if we have an empirical set of probabilities for time series of \( x(t) \) and \( y(t) \). For the description of dynamical systems we can accept \( y(t) = x'(t) \), so, we consider the derivative of \( x(t) \) as a second variable. Instead of one-dimensional Shannon entropy \( S(x) \) we use a two-dimensional full entropy \( S(x, y) \) . So, we can rewrite Eq.(1) as
\[
\begin{aligned}
I(x,y) &= S(x,y) - S(x \mid y) > 0, \\
S(x,y) &= -\sum_{i=1}^{N} \sum_{j=1}^{M} P_{ij} \ln(P_{ij}),
\end{aligned}
\]

where \( P_{ij} \) is probability of detecting of a point in a cell of phase space \((x,y)\).

Values of two-dimensional information and conditional entropy can be normalized to full entropy according to the following relations:

\[
\tilde{I} = \frac{I(x,y)}{S(x,y)},
\]

\[
\tilde{S} = \frac{S(x \mid y)}{S(x,y)}.
\]

We determine value of IER as ratio of normalized information and conditional entropy as

\[
IER = \frac{\tilde{I}}{\tilde{S}} = 1/\tilde{S} - 1
\]

Eq. (7) defines SNR with unknown noise level if values of time series \(x(t)\) are defined.

2. Information to entropy ratio for signals of chaotic dynamical systems

Let us consider different types of dynamical systems described by

- one-dimensional logistic map [39]
  \[
x_{i+1} = r x_i (1 - x_i),
  \]
  where \( r \) is a control parameter;

- two-dimensional Henon map [40]
  \[
x_{i+1} = 1 - ax_i^2 - by_i, \quad y_{i+1} = x_i,
  \]
  where \( a \) and \( b \) are control parameters;

- "bursting" map [41]
  \[
x_{i+1} = \left(\frac{1}{C + \mu_i}\right)x_i \left(\frac{1}{\gamma}\right), \quad \mu_{i+1} = -\frac{1}{\gamma} \left(\frac{1}{C + \mu_i}\right)x_i \left(\frac{1}{\gamma}\right)^{-1},
  \]
  where \( \gamma \) is a fractional part of fractal dimension of a considered physical quantity, \( C \) is a parameter similar to compression gain of a fractal signal, \( \mu_i \) is a multiplier [41].

Dependence of IER on control parameters calculated by use of Eq. (7) and bifurcation diagrams built with taking into account maximum \((X_{\text{max}})\) and minimum \((X_{\text{min}})\) local values of the signal described by Eqs. (9), (10) and (11) are shown in Figure 1.

Value of IER decreases in case of appearance of complex cycles corresponding to doubling S2 and quadruplicating S4 of the period, as can be seen in Figure 1(a, b). IER is minimal in case of transition to chaos (Figure 1(c)). Chaotization of a process (decreasing of ordering of a signal) can be also seen from the bifurcation diagram shown in Figure 1(b).
Figure 1 - Time series at $r = 4$ (a), bifurcation diagram (b) of the logistic map (9) at iteration step $\delta = 0.01$ and dependence of IER on control parameter $r$ (c)

Figure 2 - Time series at $a = 1.4$, $b = 0.1$ (a), bifurcation diagram of Henon map at $b = 0.1$ (b);
\[ \delta = 0.01 \] and the dependence of IER on control parameter \( a \) (c).

The same regularities are also observed for two-dimensional Henon map (Figure 2) and "bursting" map (Figure 3). For periodic and quasi-periodic regimes value of IER is greater than for chaotic regimes. Figure 3 shows that peaks of IER can be observed in “windows” of intermittency.

Let us compare values of IER with values of SNR defined by the following relation [4]

\[
SNR = 10 \times \log_{10}(S/P) = 10 \times \log_{10} \left( \frac{\sum_{i=1}^{n} S^2[i]}{\sum_{i=1}^{n} N^2[i]} \right),
\]

(11)

where \( S \) and \( N \), \( P_S \) and \( P_N \) are amplitudes and powers of signal and noise, respectively.

\[ \begin{align*}
0 & 100 & 200 & 300 \\
0 & 20 & 40 & 60 & 80 \\
0 & 10 & 20 & 30 & 40 \\
\end{align*} \]

Figure 3 - Time series at \( \gamma = 4.4 \) (a), bifurcation diagram of "bursting" map at \( C = 2.806 \) (b); \( \delta = 0.01 \) and dependence of IER on control parameter \( \gamma \) (c).

For this aim it is necessary to describe a signal and noise separately. We use a sine signal as an informational signal, and signals described by mentioned above maps and Gaussian noise we consider as noise \( \xi(t) \). So, we have

\[ s(t) = \sin(t) + A \cdot \xi(t) \]

(12)

We gradually increase amplitude of noise \( A \) and define IER and SNR for different signals (Figure 4). Increasing of noise amplitude \( A \) leads to decreasing of SNR for all described above dynamical systems (uncolored marks on the graph). Values of IER defined via Eq. (7) (colored marks) decrease also, but by use of these data we can specify a signal waveshape. Therefore, the new characteristic IER can be used can be used for the description of topological singularities of signals.
Conclusions

We developed a new universal method for SNR definition on the base of informational-entropic analysis. We suggested the new characteristic (IER) which is the information to entropy ratio. Our approach has the following advantages. As opposed to the well-known methods for SNR definition, IER can be defined even if noise level is unknown. Estimation of IER is possible for short time ranges less than 10 sec in real time.

The new method for SNR estimation via IER based on the theoretical approach which can be used for different applications. We developed algorithms for determination of normalized values for information to entropy ratio.

Results of numeric analysis describing mixtures of regular, chaotic, and stochastic signals demonstrate that using of our method let us to define SNR correctly, and our approach can be used for specification of a signal waveshape.

As known, entropy depends on minimal scale of measurement, and for continuous processes entropy is unlimited. We solved the problem of normalization of entropy by using of information defined via difference between conditional and unconditional entropy normalized on unconditional entropy.

References

1. Walid M. Aldeeb, A study on the channel and BER-SNR performance of ultra wide band systems applied to commercial vehicles, University of Michigan-Dearborn, 2010, 202 p.
2. Chao Gong; Bangning Zhang; Aijun Liu; Daoxing Guo, A Highly Accurate and Low Bias SNR Estimator: Algorithm and Implementation//Radioengineering, Proceedings of Czech and Slovak Technical Universities and URSI Committees. Dec2011, Vol. 20 Issue 4, p. 976-981.
3. Arun Narayanan, DeLiang Wang, A CASA-Based System for Long-Term SNR Estimation// IEEE Trans. Audio, Speech, Lang. Process., vol. 20, no. 9, pp. 2518-2527, November 2012.
4. M. Vondrasek and P. Pollak, Methods for speech SNR estimation: Evaluation tool and analysis of VAD dependency// Radio Engineering, vol. 14, NO. 1, pp. 6–11, Jan. 2005.
5. Shao H., Wu D., Li Y., Liu W., Chu X., Improved signal-to-noise ratio estimation algorithm for asymmetric pulse-shaped signals// IET Communications, Vol. 9 (14), 1788-1792 pp. 2015.
6. Elias Nemer, Rafik Goubran, Samy Mahmoud, SNR estimation of speech signals using subbands and fourth-order statistics// IEEE SIGNAL PROCESSING LETTERS, VOL. 6, NO. 7, JULY 1999, pp. 171-174.

7. Soojeong Lee, Chungsoo Lim, Joon-Hyuk Chang, A new a priori SNR estimator based on multiple linear regression technique for speech enhancement// Digital Signal Processing, Volume 30, July 2014, Pages 154–164.

8. Aulis Telle and Peter Vary, A new method for SNR- estimation in impulse response measurements//17th European Signal Processing Conference (EUSIPCO 2009) Glasgow, Scotland, August 24-28, 2009.

9. Boscaro, S. Jacquir, K. Sanchez, P. Perdu, S. Binczak, Improvement of signal to noise ratio in electro optical probing technique by wavelets filtering//Microelectronics Reliability, Volume 55, Issues 9–10, August–September 2015, Pages 1585-1591

10. Fei Qin, Xuewu Dai, John E. Mitchell, Effective-SNR estimation for wireless sensor network using Kalman filter// Ad Hoc Networks Volume 11, Issue 3, May 2013, Pages 944–958.

11. Sunhyun Yook, Kyoung Won Nam, Heepyung Kim, See Youn Kwon, Dongwook Kim, Sangmin Lee, Sung Hwa Hong, Dong Pyo Jang, In Young Kim, Modified segmental signal-to-noise ratio reflecting spectral masking effect for evaluating the performance of hearing aid algorithms//Speech Communication, Volume 55, Issue 10, November–December 2013, Pages 1003-1010

12. Shahid Manzoor, Varun Jeoti, Nidal Kamel, Muhammad Asif Khan, Novel SNR Estimation Technique In Wireless OFDM Systems// Int. J. of Future Generation Communication and Networking.- 2011.-Vol. 4, No. 4.-P.1-18.

13. Bhavani Shankar Y., Chandrasekhar Rao J., Anil Babu B., An improved SNR estimation approach for OFDM system// IJERA.- 2012.- Vol. 2, Issue 3.- P.2561-2563.

14. Kun Wang, Xianda Zhang, Blind noise variance and SNR estimation for OFDM systems based on information theoretic criteria// Signal Processing.- 2010.- Vol. 90, Issue 9.- P. 2766-2772.

15. Di Wu, Huaiizong Shao, Fan Yang, Linli Cui, An improved SNR estimator for wireless OFDM systems// Procedia Engineering.-2012.-Vol. 29.- P. 3124-3131.

16. Monisha Chakrabortya, Shreya Dasb, Determination of signal to noise ratio of electrocardiograms filtered by band pass and Savitzky-Golay filters//Procedia Technology.- 2012.- V.4.- P.830 – 833.

17. Jie Liu, Dongwen Ying, Ping Zhou, Wiener filtering of surface EMG with a priori SNR estimation toward myoelectric control for neurological injury patients// Medical Engineering & Physics.- 2014.-Vol. 36, I. 12.- P. 1711-1715.

18. Eungi Min, Mincheol Ko and etc, Identification of radionuclides for the spectroscopic radiation portal monitor for pedestrian screening under a low signal-to-noise ratio condition// NIMPR Section A: Accelerators, Spectrometers, Detectors and Associated Equipment.- 2014.-Vol. 758, 11.-P. 62-68.

19. Alicia Gonzalez-Moreno and etc, Signal-to-noise ratio of the MEG signal after preprocessing//Journal of neuroscience methods.- 2014.-Vol. 222, 30.-P. 56-61.

20. Rajesh Patil, Noise reduction using wavelet transform and singular vector decomposition// Procedia Computer Science.- 2015.-V.54.- P. 849 – 853.

21. Saiful Islama, Hyungseob Hana and etc, Small target detection and noise reduction in marine radar systems//IERI Procedia.- 2013.- V.4.-P. 168 – 173.
22. Elias Nemer, Rafik Goubran, Samy Mahmoud, SNR estimation of speech signals using subbands and fourth-order statistics // SIGNAL PROCESSING LETTERS. - 1999. - VOL. 6, NO. 7. - P. 171-174.

23. Yin-Ping Yao and etc, Effect of turbulence on visibility and signal-to-noise ratio of lensless ghost imaging with thermal light // Optik - IJLEO. - 2013. - Vol. 124, I. 24. - P. 6973-6977.

24. Jingjing Ai and etc, Signal-to-noise ratio of the spatially modulated imaging spectrometer based on modified Sagnac interferometer // Optics Communications. - 2013. - Vol. 298–299. - P. 46-53.

25. Maria Claesson and etc, Improved QCM-D signal-to-noise ratio using mesoporous silica and titania // Sensors and Actuators B: Chemical. - 2012. - Vol. 166–167. - P. 526-534.

26. Nina Bondre, Yongxia Zhang, Chris D. Geddes, Metal-enhanced fluorescence based calcium detection: Greater than 100-fold increase in signal/noise using Fluo-3 or Fluo-4 and silver nanostructures // Sensors and Actuators B: Chemical. - 2011. - Vol. 152, Issue 1. - P. 82-87.

27. A.S. Bosh and etc, Signal-to-noise ratios for possible stellar occultations by pluto // Icarus. - 1986. - V. 66, I. 3. - P. 556-560.

28. M. Ameri and etc, A photo-detector for UHECR observation from space // NIMPRS A: Accelerators, Spectrometers, Detectors and Associated Equipment. - 2006. - V. 567, I. 1. - P. 107-109.

29. Stone J.V., Information theory: a tutorial introduction. - Sebtel Press, 2015. - 260 p.

30. Yi Yin, Pengjian Shang, Guochen Feng, Modified multiscale cross-sample entropy for complex time series // Applied Mathematics and Computation 289 (2016) pp. 98–110

31. Xiao Zhang, Changlin Mei, Degang Chen, Jinhai Li, Feature selection in mixed data: A method using a novel fuzzy rough set-based information entropy // Pattern Recognition Volume 56, August 2016, Pages 1–15

32. Mei Li, Xueyong Liu, Xu Liu, Infrasound signal classification based on spectral entropy and support vector machine // Applied Acoustics Volume 113, 1 December 2016, Pages 116–120

33. Zhengxiang Cai, Qi Wu, Dan Huang, Lu Ding, Biting Yu, Rob Law, Jiayang Huang, Shan Fu, Cognitive state recognition using wavelet singular entropy and ARMA entropy with AFPA optimized GP classification // Neurocomputing Volume 197, 12 July 2016, Pages 29–44.

34. Adiel Freiman, Monika Pinchas, A Maximum Entropy inspired model for the convolutional noise PDF // Digital Signal Processing Volume 39, April 2015, Pages 35–49

35. Yang Lei, Technique for image de-noising based on non-subsampled shearlet transform and improved intuitionistic fuzzy entropy // Optik - International Journal for Light and Electron Optics Volume 126, Issue 4, February 2015, Pages 446–453.

36. M. Beenamol, S. Prabavathy, J. Mohanalin, Wavelet based seismic signal de-noising using Shannon and Tsallis entropy // Computers & Mathematics with Applications Volume 64, Issue 11, December 2012, Pages 3580–3593.

37. Ned J. Corron, Roy M. Cooper, Jonathan N. Blakely, Entropy rates of low-significance bits sampled from chaotic physical systems // Physica D: Nonlinear Phenomena Volume 332, 1 October 2016, Pages 34–40.
38. Shaobo He, Kehui Sun, Huihai Wang, Multivariate permutation entropy and its application for complexity analysis of chaotic systems// Physica A: Statistical Mechanics and its Applications Volume 461, 1 November 2016, Pages 812–823.

39. Steven Henry Strogatz, Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering// Westview Press, 1994. 498 p.

40. M. Hénon, A two-dimensional mapping with a strange attractor//Communications in Mathematical Physics, 1976, V. 50 (1), pp. 69–77.

41. Zhanabaev Z.Zh., Akhtanov S.N., New method for investigating of bifurcation regimes by use of realizations of a dynamical system//Eurasian Physical Technical Journal, 2015, Vol.12, No.2(24), pp.10-16, ISSN 1811-1165.