Electron and Muon $g - 2$ Contributions from the $T'$ Higgs Sector

Chiu Man Ho$^{1,*}$ and Thomas W. Kephart$^{1,†}$

$^1$Department of Physics and Astronomy,
Vanderbilt University, Nashville, Tennessee 37235, USA

(Dated: April 25, 2019)

Abstract

We study the experimental constraints from electron and muon $g - 2$ factors on the Higgs masses and Yukawa couplings in the $T'$ model, and thereby show that the discrepancy between the standard model prediction and experimental value of muon $g - 2$ factor can be easily accommodated.

$^*$Electronic address: chiuman.ho@vanderbilt.edu
$^†$Electronic address: tom.kephart@gmail.com
I. INTRODUCTION

The electron anomalous magnetic moment has been measured to an extremely high precision and agrees with the theoretical prediction calculated from the standard model (SM) \[1\], with the result

\[ \Delta a_e = |a_{e}^{\text{SM}} - a_{e}^{\text{Expt}}| < 1 \times 10^{-10}. \]  

(1.1)

On the other hand, the most recent theoretical calculation of the muon anomalous magnetic moment gives \[2\]:

\[ a_{\mu}^{\text{SM}} = (11659183.4 \pm 4.9) \times 10^{-10}, \]  

(1.2)

where the errors are dominated by the hadronic contribution. The corresponding most updated experimental value is \[3\]:

\[ a_{\mu}^{\text{Expt}} = (11659208.0 \pm 5.4 \pm 3.3) \times 10^{-10}. \]  

(1.3)

This implies that \( a_{\mu}^{\text{SM}} \) differs from \( a_{\mu}^{\text{Expt}} \) by 3.1\( \sigma \), and suggests that a contribution beyond standard model may be required. As we will show, this discrepancy between the theoretical and experimental values can be easily accommodated in the \( T' \) model \[4–6\] due to the existence of a new and unique Higgs coupling to the muon. While many authors have developed models that resolve this discrepancy \[7\], only a few have invoked a discrete flavor symmetry.

II. HIGGS CONTRIBUTIONS TO \( g-2 \) FACTORS IN THE \( T' \) MODEL

The \( T' \) model \[4–6\] relates quarks and electrons through a discrete flavor symmetry, the binary tetrahedral group \( T' \), whose irreducible representations are three singlets, three doublets and a triplet. The renormalizable \( T' \) model has led to successful predictions of the tribimaximal neutrino mixing matrix as well as the Cabibbo angle \[5, 6\]. More details about the \( T' \) model, its variants and other related models can be found in the literature \[8\].

In the \( T' \) model, electrons and muons couple to the different components of the triplet Higgs \( H_3' \) through the interaction terms \( Y_e \bar{e} H_{3,e}' e \) and \( Y_\mu \bar{\mu} H_{3,\mu}' \mu \). To compute the contribution of a virtual Higgs to the electron and muon \( g-2 \) factors, we need to study its
contribution to the electron/muon-photon vertex. For $f = e, \mu$, the vertex function is given by

$$-i e \bar{u}(p') \Lambda_f^\nu(p', p) u(p)$$

$$= (-i e)(-i Y_f)^2 \int \frac{d^4 k}{(2\pi)^4} \bar{u}(p') \frac{i}{k^2 - M_{H_f}^2 + i\epsilon} \frac{i(p' - k + m)}{(p' - k)^2 - m^2 + i\epsilon} \gamma^\nu \frac{i(p - k + m_f)}{(p - k)^2 - m_f^2 + i\epsilon} u(p).$$

(2.1)

where $\bar{u}(p')$ and $u(p)$ are the spinors obeying the equation of motions $\bar{u}(p')(p'^\mu - m_f) = (p - m_f)u(p) = 0$, and $M_{H_f}$ is the mass of the Higgs which couples to the electron or muon whose mass is denoted by $m_f$.

After some calculations, we obtain

$$\bar{u}(p') \Lambda_f^\nu(p, p') u(p) = F_f(q^2) \bar{u}(p') \frac{i\sigma^\nu\alpha}{2m_f} q_\alpha u(p) + \cdots,$$

(2.2)

where $F_f(q^2)$ is the form factor associated with the electron or muon, and $\sigma^\nu\alpha = \frac{i}{2}[\gamma^\nu, \gamma^\alpha]$. The contributions from the $T'$ Higgs sector to electron or muon anomalous magnetic moment is given by

$$\Delta a_f = \Delta \left(\frac{g_f - 2}{2}\right) = F_f(q^2 = 0)$$

$$= \frac{Y_f^2 m_f^2}{8\pi^2 M_{H_f}^2} \int_0^1 dx \left(\frac{1}{x + (1 - x)^2}\right) \frac{m_f^2}{M_{H_f}^2}.$$  

(2.3)

(2.4)

For $m_f \ll M_{H_f}$, which is likely to be the case, there is a logarithmic divergence in the above integral as $x \to 0$. This divergence can be extracted by setting $1 - x \to 1$ and $1 - x^2 \to 1$ in the integrand. As a result, we obtain

$$\Delta a_f \approx \frac{Y_f^2}{4\pi^2} \left(\frac{m_f}{M_{H_f}}\right)^2 \ln \left(\frac{M_{H_f}}{m_f}\right).$$

(2.5)

Note that for a given value of $Y_f$, $\Delta a_f$ is strictly decreasing when the ratio $M_{H_f}/m_f$ increases.

The condition (1.1) implies that any combinations of $Y_e$ and $M_{H_e}$ must be such that

$$|\Delta a_e| < 1 \times 10^{-10},$$

(2.6)
which imposes the following constraint

\[ Y_e \lesssim 21.4 \lambda_e \frac{M_{H_e}/m_e}{\sqrt{\ln (M_{H_e}/m_e)}}, \]  

(2.7)

where \( \lambda_e \sim 3 \times 10^{-6} \) is the corresponding electron Yukawa coupling in SM. We required the ratio \( M_{H_e}/m_e \gg 1 \) when we were deriving (2.5), but otherwise a free parameter. To have an assessment on the allowed range of \( Y_e \), we need to have some experimental bounds on \( M_{H_e} \). Apparently, we would have hoped that the LEP [9] bound on Higgs mass may help — due to the non-observation of the “Higgs-strahlung” process \( e^+ e^- \to H Z \) at LEP, a lower bound has been given to the SM Higgs, namely \( M_{H_{SM}} \geq 114.5 \) GeV. However, in the \( T' \) model, all the Higgs singlets and triplets couple to \( Z \). Thus, the LEP bound does not apply directly to any of the masses of the Higgs singlets and triplets. If we simply assume that \( M_{H_e} \geq 100 \) GeV, then we require \( Y_e \lesssim 3.5 \) in order to satisfy the condition (1.1). In this case, the upper bound on the Yukawa coupling \( Y_e \) is very loose and any value of \( Y_e \) that is perturbatively small would be allowed.

For the muon anomalous magnetic moment, the discrepancy between the theoretical and experimental values can be accounted for easily in the \( T' \) model if

\[ \Delta a_\mu \sim |a_\mu^{SM} - a_\mu^{Expt}| = (24.6 \pm 8.0) \times 10^{-10}, \]  

(2.8)

leading to the constraint

\[ Y_\mu \sim 0.52 \lambda_\mu \frac{M_{H_\mu}/m_\mu}{\sqrt{\ln (M_{H_\mu}/m_\mu)}}, \]  

(2.9)

where \( \lambda_\mu \sim 0.0006 \) is the corresponding muon Yukawa coupling in SM. It is obvious that \( Y_\mu \gg \lambda_\mu \), for any choice of \( M_{H_\mu}/m_\mu \gg 1 \). For instance, if we assume that \( M_{H_\mu} \geq 100 \) GeV, then in order to satisfy (2.9), we require \( Y_\mu \gtrsim 0.13 \).

### III. CONCLUSIONS

In this article, we have computed the contributions to electron and muon \( g - 2 \) factors from the Higgs sector in the \( T' \) model. We then used the experimental data to constrain the \( T' \) model Higgs masses and Yukawa couplings.
If we assume that $M_{H_e} \gtrsim 100$ GeV, then the upper bound on the electron Yukawa coupling $Y_e$ would be very loose and any value of $Y_e$ consistent with the perturbation theory would be allowed.

Our main result is the demonstration that the discrepancy between the standard model and experimental values of muon anomalous $g-2$ factor can be accounted for easily in the $T'$ model. Assuming $M_{H_\mu} \gtrsim 100$ GeV, we found that the Yukawa coupling $Y_\mu$ should be much larger than the corresponding SM value in order to explain the discrepancy.

Acknowledgments

We thank Shinya Matsuzaki for useful comments. This work was supported by US DOE grant DE-FG05-85ER40226.

[1] D. Hanneke, S. Fogwell and G. Gabrielse, Phys. Rev. Lett. 100, 120801 (2008).
[2] M. Davier, A. Hoecker, B. Malaescu, C. Z. Yuan and Z. Zhang, arXiv:0908.4300 [hep-ph].
[3] G. W. Bennett et al [Muon g-2 Collaboration], Phys. Rev. D 73, 072003 (2006).
[4] P. H. Frampton and T. W. Kephart, Int. J. Mod. Phys. A 10 (1995) 4689 arXiv:hep-ph/9409330.
[5] P. H. Frampton and T. W. Kephart, JHEP 0709 (2007) 110 arXiv:0706.1186 [hep-ph]]
[6] P. H. Frampton, T. W. Kephart and S. Matsuzaki, Phys. Rev. D 78 (2008) 073004 arXiv:0807.4713 [hep-ph]]
[7] E. Ma and M. Raidal, Phys. Rev. Lett. 87 (2001) 011802 [Erratum-ibid. 87 (2001) 159901] arXiv:hep-ph/0102255; T. W. Kephart and H. Pas, Phys. Rev. D 65 (2002) 093014 arXiv:hep-ph/0102243; Z. H. Xiong and J. M. Yang, Phys. Lett. B 508 (2001) 295 arXiv:hep-ph/0102259; Z. z. Xing, Phys. Rev. D 64 (2001) 017304 arXiv:hep-ph/0102304; T. Ibrahim, U. Chattopadhyay and P. Nath, Phys. Rev. D 64 (2001) 016010 arXiv:hep-ph/0102324; J. R. Ellis, D. V. Nanopoulos and K. A. Olive, Phys. Lett. B 508 (2001) 65 arXiv:hep-ph/0102331; X. Calmet, H. Fritzsch and D. Holtmannspotter, Phys. Rev. D 64 (2001) 037701 arXiv:hep-ph/0103012; K. Choi, K. Hwang, S. K. Kang, K. Y. Lee and W. Y. Song, Phys. Rev. D 64 (2001) 055001 arXiv:hep-ph/0103048; S. Ra-
[90x710]jpoot, arXiv:hep-ph/0103069; C. A. de S.Pires and P. S. Rodrigues da Silva, Phys. Rev. D 64 (2001) 117701 [arXiv:hep-ph/0103083]; E. O. Iltan and H. Sundu, Acta Phys. Slov. 53 (2003) 17 [arXiv:hep-ph/0103105]; S. Baek, P. Ko and H. S. Lee, Phys. Rev. D 65 (2002) 035004 [arXiv:hep-ph/0103218]; M. Raidal, Phys. Lett. B 508 (2001) 51 [arXiv:hep-ph/0103224]; A. Dedes and H. E. Haber, arXiv:hep-ph/0105014; E. O. Iltan, JHEP 0305 (2003) 065 [arXiv:hep-ph/0304097]; H. Chavez, C. N. Ferreira and J. A. Helayel-Neto, Phys. Rev. D 74 (2006) 033006 [arXiv:hep-ph/0410373]; A. Mondragon, M. Mondragon and E. Peinado, J. Phys. A 41, 304035 (2008) [arXiv:0712.1799 [hep-ph]].

[8] M. Schmaltz, Phys. Rev. D 52 (1995) 1643 [arXiv:hep-ph/9411383]; L. J. Hall and H. Murayama, Phys. Rev. Lett. 75 (1995) 3985 [arXiv:hep-ph/9508296]; C. D. Carone and R. F. Lebed, Phys. Rev. D 60 (1999) 096002 [arXiv:hep-ph/9905275]; P. H. Frampton and A. Rasin, Phys. Lett. B 478 (2000) 424 [arXiv:hep-ph/9910522]; R. Dermisek and S. Raby, Phys. Rev. D 62 (2000) 015007 [arXiv:hep-ph/9911275]; G. Altarelli and F. Feruglio, New J. Phys. 6 (2004) 106 [arXiv:hep-ph/0405048]; K. S. Babu and J. Kubo, Phys. Rev. D 71 (2005) 056006 [arXiv:hep-ph/0411226]; N. Haba and K. Yoshioka, Nucl. Phys. B 739 (2006) 254 [arXiv:hep-ph/0511108]; Y. Kajiyama, E. Itou and J. Kubo, Nucl. Phys. B 743 (2006) 74 [arXiv:hep-ph/0511268]; C. Hagedorn, M. Lindner and R. N. Mohapatra, JHEP 0606 (2006) 042 [arXiv:hep-ph/0602244]; T. Kobayashi, H. P. Nilles, F. Ploger, S. Raby and M. Ratz, Nucl. Phys. B 768 (2007) 135 [arXiv:hep-ph/0611020]; M. C. Chen and K. T. Mahanthappa, Phys. Lett. B 652 (2007) 34 [arXiv:0705.0714 [hep-ph]]; M. Frigerio and E. Ma, Phys. Rev. D 76 (2007) 096007 [arXiv:0708.0166 [hep-ph]]; S. Sen, Phys. Rev. D 76 (2007) 115020 [arXiv:0710.2734 [hep-ph]]; G. Altarelli, [arXiv:0711.0161 [hep-ph]]; N. Kifune, J. Kubo and A. Lenz, Phys. Rev. D 77 (2008) 076010 [arXiv:0712.0503 [hep-ph]]; M. Honda and M. Tanimoto, Prog. Theor. Phys. 119 (2008) 583 [arXiv:0801.0181 [hep-ph]]; G. Altarelli, F. Feruglio and C. Hagedorn, JHEP 0803 (2008) 052 [arXiv:0802.0090 [hep-ph]]; F. Plentinger, G. Seidl and W. Winter, JHEP 0804 (2008) 077 [arXiv:0802.1718 [hep-ph]]; C. Luhn, Phys. Lett. B 670 (2009) 390 [arXiv:0807.1749 [hep-ph]]; P. H. Frampton and S. Matsuzaki, Mod. Phys. Lett. A 24 (2009) 2081 [arXiv:0810.1029 [hep-ph]]; D. A. Eby, P. H. Frampton and S. Matsuzaki, Phys. Lett. B 671 (2009) 386 [arXiv:0810.4899 [hep-ph]]; F. Bazzocchi and S. Morisi, Phys. Rev. D 80 (2009) 096005 [arXiv:0811.0345 [hep-ph]]; P. H. Frampton and S. Matsuzaki, Phys. Lett. B 679 (2009) 347 [arXiv:0902.1140 [hep-ph]]; D. A. Eby, P. H. Frampton and S. Matsuzaki,
Phys. Rev. D 80 (2009) 053007 [arXiv:0907.3425 [hep-ph]]; M. C. Chen, K. T. Mahanthappa and F. Yu, [arXiv:0907.3963 [hep-ph]]; M. C. Chen and K. T. Mahanthappa, Phys. Lett. B 681, 444 (2009) [arXiv:0904.1721 [hep-ph]]; F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, Nucl. Phys. B 809, 218 (2009) [arXiv:0807.3160 [hep-ph]]; F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, Nucl. Phys. B 775, 120 (2007) [arXiv:hep-ph/0702194].

[9] R. Barate et al. [LEP Collaborations], Phys. Lett. B 565, 61 (2003).