Research on Calculation of Port Optimal Anchorage Demand Based on Queuing Theory

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Abstract. In view of the contradiction between the supply and demand of anchorage resources, the research on the optimal anchorage demand of the port is carried out. The M/M/c model is constructed by the queuing theory method combined with the main characteristic indicators and main parameter indicators. Taking the coal port area of the Huanghua Port as an example, the Matlab software is used to calculate and determine the optimal anchorage demand, which is 14, according to the restriction that the anchorage guarantee rate is greater than or equal to 95%. And the conclusion shows that the port anchorage resources are not tight. The calculation result shows that the model has certain application value in anchorage planning.

1. Introduction
With the strong support of “The Belt and Road” strategy, the port construction scale has become larger and larger. The anchorage, as the waters in the port for the safe berthing of vessels, shelter from the wind, customs border inspection, quarantine, loading and unloading of goods and carrying out the lighterage operations, is not only a very important port resource, but also an indispensable part of the implementation of “The Belt and Road” initiative.

After the vessel arrives at the port, it is generally not possible to berth immediately and needs to wait at the anchorage. As the arrival rate of vessels continues to rise, the need for anchorage construction is becoming more and more urgent. A key issue in the construction is how to determine the optimal anchorage demand. Under ideal conditions, the optimal anchorage demand in a port should refer to the demand when the anchorage can always be fully utilized. However, this situation can’t be achieved due to the randomness of the vessel’s arrival and other uncontrollable factors. Although the increase in the number of anchor positions can lead to an increase in the anchorage utilization rate, this will inevitably cause unnecessary waste. If the number of anchor positions is too small and the vessel cannot be anchored and fixed, it is easy to cause the channel to be blocked and thus there is a collision hazard. Therefore, it’s necessary to research on the optimal anchorage demand.

Since the berth service time is a continuous random variable and the vessel’s arrival process is dynamic and random, whose law obeys the Poisson distribution, the queuing theory is used to calculate the anchorage scale of the port and determine the optimal anchorage demand. The research ideas of this paper are as follows. On the basis of berth grade division, the queuing theory is used to build anchorage demand calculation model. And then, the optimal anchorage demand is calculated by the restriction of the anchorage guarantee rate.
2. Analysis of factors affecting port anchorage demand
The influencing factors of port anchorage demand are determined by the analysis of the vessel's inbound workflow. After the vessel arrives at the port, if there is no berth free, then it will wait at the anchorage. If there is no anchorage, the vessel will be refused entry to the port. If the berth is idle and there is no waiting vessel at the anchorage, the arriving vessel can berth directly.

Figure 1. Workflow chart of vessel entering the port.
As we can see from Figure 1, the factors affecting port anchorage demand are as follows:
- Number of vessels arriving at the port per unit time.
- Number of berths in the port.
- Berth handling efficiency.

These factors are directly related to the analysis of indicators in the construction of the queuing theory model below, which is beneficial to the selection of parameters and the statistics of features.

3. Construction of calculating model for port optimum anchorage demand
The construction of the calculation model for port optimal anchorage demand can be based on the analysis of the workflow of the vessel entering the port and the determination of the main characteristics and parameters of ship-anchorage-berth stochastic service system according to the influencing factors of port anchorage demand.

3.1. Main characteristic indicators
Queueing Theory, also known as stochastic service system theory, mainly studies the congestion caused by random factors. It consists of three parts, including the input process, the queuing rule, and the service agency.

The work flow after arrival of a ship can be regarded as a stochastic service system of ship-anchorage-berth. Among them, the ship can be regarded as the customer receiving the service, while the anchorage is the lobby where the customer waits for the service, and the berth is the service desk. The system capacity should be the sum of the berth number and the anchorage number. As is shown in Figure 2.

Figure 2. Schematic diagram of ships anchoring queue.

The input process of this system is to investigate the law of arrival of ships. Besides, the analysis above shows that the ship arrives at the port with randomness, and the process forms a Poisson flow, while the queuing rule is "first come, first served". Moreover, the service agency, i.e. berth, has
similar service time for the same type of vessel, and the service time is a random variable, which is generally subject to a negative exponential probability distribution.

### 3.2. Main parameter indicators

It can be seen that the arrival of ships obeys Poisson distribution. According to the nature of Poisson flow, the probability of the number of vessels arriving in the time interval can be expressed as follows:

\[ P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, \quad t > 0, n = 0, 1, 2, \ldots \]  

(1)

The \( \lambda \) in Formula (1) is the average number of vessels arriving in a unit time, i.e. the average arrival rate of the vessel. In the process of evaluating \( \lambda \), the annual operation days of the berth are taken as a period of time, and the ratio of the number of vessels arriving at the port and the number of operation days of berths in the whole year is taken as \( \lambda \).

\( \mu \) is used to express the number of customers completed by a service desk in a unit time, i.e. the average service rate of the vessel. In the process of evaluating \( \mu \), the time difference in visa registration between the arrival and the departure of the vessel is recorded as \( d \), the reciprocal of whose average is taken as \( \mu \).

\( c \) denotes the number of service desks and \( n \) denotes the system capacity. When there are \( n \) vessels in the system, the new vessel will not be able to enter. Meanwhile, the system reaches a stable state, which is recorded as \( \rho \leq 1 \). \( \rho \) is called the service intensity and its calculation formula is:

\[ \rho = \frac{\lambda}{c\mu} \]  

(2)

### 3.3. Model construction

Since the state of the system is constantly changing with the arrival and departure of the vessel, the Markov state flow as shown in Figure 3 can be obtained.

![State Transition Diagram of M/M/c Queuing Model.](image)

State \( k (0 \leq k \leq n) \) denotes that there are \( k \) ships in the system and the probability when the system is under statistical equilibrium is \( P_k \). Then the transition from state 1 to state 0 indicates that there was a ship in the system at first, and after the ship was served, it left the port. Its state transition rate is \( \mu P_1 \). Furthermore, the transition from state 2 to state 1 indicates that one of the two vessels in the system being served in the berth was completed by service and left the port. Its state transition rate is \( 2\mu P_2 \).

By analogy, when state \( c \) is transferred to state \( c-1 \), it is considered in the following two cases:

- When \( k < c \), the state transition rate is \( k\mu P_k \);  
- When \( k \geq c \), since the number of service desks, i.e. berth, in the system is \( c \), there are \( c \) vessels being served at a time in the system at most. The state transition rate is \( c\mu P_c \).

According to the above analysis combined with Figure 3, the K-algebraic equation is obtained as follows when the system reaches a steady state:

\[ \mu P_0 = \lambda P_0 \]  

(3)

\[ (k+1)\mu P_{k+1} + \lambda P_{k-1} = (\lambda + k\mu)P_k, 1 \leq k < c \]  

(4)
\[ c \mu P_{k+1} + \lambda P_{k-1} = (\lambda + c \mu)P_k, \quad c \leq k < n \]  \hspace{1cm} (5)

\[ \lambda P_{n-1} = c \mu P_n, \quad k = n \]  \hspace{1cm} (6)

\[ \sum_{i=0}^{n} P_i = 1 \]  \hspace{1cm} (7)

It can be known from formula (7) that by using recursive method to solve the above difference equation, the steady-state probability equation can be obtained as follows:

\[ P_k = \frac{1}{k!} \left( \frac{\lambda}{\mu} \right)^k P_0, \quad 0 \leq k < c \]  \hspace{1cm} (8)

\[ P_k = \frac{1}{c!e^{k-c}} \left( \frac{\lambda}{\mu} \right)^k P_0, \quad c \leq k \leq n \]  \hspace{1cm} (9)

When \( \rho = 1 \),

\[ P_0 = \left[ \sum_{k=0}^{c} \frac{c^k}{k!} + \rho^{n-c+1} \right]^{-1} \]  \hspace{1cm} (10)

When \( \rho \neq 1 \),

\[ P_0 = \left[ \sum_{k=0}^{c} \frac{c^k}{k!} \right]^{-1} + \frac{1}{c!} \frac{1-\rho^{n-c+1}}{1-\rho} \left( \frac{\lambda}{\mu} \right)^{c-1} \]  \hspace{1cm} (11)

If the number of vessels is \( n \), the corresponding probability is:

\[ Q_n = \sum_{i=0}^{n} P_i \]  \hspace{1cm} (12)

Anchorage demand is determined by the restriction of the anchorage guarantee rate. When the evaluated value of \( Q_n \) is greater than the anchorage guarantee rate, the optimal anchorage demand obtained is:

\[ N_a = n - c \]  \hspace{1cm} (13)

4. Application

Since the coal port area of Huanghua Port has the characteristics such as single channel, suitable weather and single cargo type, it is easy to collect data. This paper mainly calculates and studies the optimum anchorage demand of the coal port area of Huanghua Port based on the vessel arrival situation in 2018.

It can be seen from the study of the arrival vessels provided by Huanghua Port Authority that nearly 95% of the arrival vessels in the coal port area of Huanghua Port in 2018 are bulk carriers. Therefore, this paper mainly takes the bulk carrier as an example.

4.1. Berth grade division

According to the situation of vessels arriving in 2018, the tonnage of berthing vessels varies from 10,000 tons to 100,000 tons. Obviously, vessels can’t berth arbitrarily, and berths must be reasonably selected in accordance with the ship plan and the arrangements of the maritime department. So before calculating the optimum anchorage demand, this paper must classify the berths according to the tonnage distribution of vessels arriving and the berthing capacity of the port, and then calculate the parameters separately.

According to the wharf information provided by the Port Authority and the tonnage distribution of vessels arriving, the berth grades can be classified as follows: 10000-ton; 15000-ton; 35000-ton; 50000-ton; 70000-ton and 100000-ton. Then, the number of vessels arriving at the port in 2018 is distinguished according to the corresponding grade of tonnage distribution.
4.2. Determination of parameters

According to Article 46 of the Port Regulations of Huanghua Port, CangZhou Maritime Safety Administration has the right to take compulsory measures such as restricting navigation, stopping navigation, and stopping operation under one of the following meteorological conditions:

- The wind power is at level 6 or above.
- Visibility is less than 1000 meters.

According to the historical weather data obtained from the website of Weather Post-Report, we can calculate that the number of days affected by the windy weather of level 6 or above is 48, and the number of days when visibility is affected by weather such as haze, wind and heavy rain is 19. Hence, the number of operation days of berths in the whole year can be calculated as 298.

It can be known from the collation and analysis of the arrival vessel data in 2018 provided by the Port Authority that the number of bulk ships arriving at the port is 4,151. What’s more, we can get the statistics of total service days, i.e. \( d \), as shown in Table 1.

| Berth grade/t | Berth number | Arrival times | Berth operation days | \( \lambda \) | \( \mu \) | \( \rho \) |
|---------------|--------------|---------------|----------------------|--------------|----------|--------|
| 10000         | 1            | 213           | 298                  | 0.7148       | 0.9958   | 0.7178 |
| 15000         | 2            | 41            | 298                  | 0.1376       | 0.9811   | 0.0701 |
| 35000         | 2            | 523           | 298                  | 1.7550       | 0.9005   | 0.9745 |
| 50000         | 11           | 2707          | 298                  | 9.0839       | 0.8265   | 0.9991 |
| 70000         | 2            | 486           | 298                  | 1.6309       | 0.8168   | 0.9983 |
| 100000        | 2            | 181           | 298                  | 0.6074       | 0.6784   | 0.4477 |

4.3. Calculation of optimum anchorage demand

In this paper, The optimum anchorage demand of the port is determined by linear optimization method. After obtaining the parameters of each level above, this paper uses Matlab software to calculate the distribution probability through the input parameters, as shown in Table 3, and then uses it combined with the constraint conditions to calculate the final result.
We still take 10000-ton class as an example to determine the optimum anchorage demand. It is known that the anchorage guarantee rate of bulk carriers should be not less than 95%. According to formula (12), we can obtain as follows:

When \( n = 9 \), \( Q_n = P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 = 96.38\% > 95\% \). According to formula (13), we can finally get the optimal anchorage demand, which is 8. The optimal anchorage demand determination steps of other grades are the same as above, and the final result is shown in Table 4.

Table 4. Optimal anchorage demand of each level in Coal Port Area of Huanghua Port.

| Berth grade/t | 10000 | 15000 | 35000 | 50000 | 70000 | 100000 |
|---------------|-------|-------|-------|-------|-------|--------|
| \( n \)       | 9     | 2     | 4     | 11    | 4     | 4      |
| \( c \)       | 1     | 2     | 2     | 11    | 2     | 2      |
| \( N_a \)     | 8     | 0     | 2     | 0     | 2     | 2      |
| Total         |       |       |       |       |       | 14     |

According to the existing anchorage information, there are three anchorages in Coal Port Area of the Huanghua Port, namely 1#, 2#, and 3# anchorages, with an anchorage area of 53.41 square kilometers and an anchorage elevation of -9 meters, -12 meters and -15 meters, which can accommodate 18-27 ships to anchor. Therefore, the existing anchorage resources in Huanghua Port are relatively abundant, which can meet the anchorage demand.

Acknowledgments
Thanks to the support of the Innovation Training Program of the College of Innovation and Entrepreneurship of Zhejiang Ocean University in 2019.

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