Implications of texture zeros for a variant of tribimaximal mixing

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We study the phenomenological implications of the presence of two texture zeros in the neutrino mass matrix assuming that the neutrino mixing matrix has its first column identical to that of the tribimaximal mixing matrix. Only two patterns of this kind are compatible with the experimental data. These textures have definite predictions for the neutrino observables that are testable in future neutrino experiments.

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In the recent years, considerable efforts have been put towards determining the structure of the neutrino mass matrix \( M_\nu \) \([1]\). The non-zero value of the reactor mixing angle \( \theta_{13} \), recently determined in various neutrino oscillation experiments \([2]\), has called many neutrino mass models predicting \( \theta_{13} = 0 \) into question. The models based upon the Tribimaximal (TBM) mixing, that predicted the reactor, atmospheric and solar mixing angles as \( \theta_{13} = 0 \), \( \theta_{23} = \frac{\pi}{4} \), and \( \theta_{12} = \arctan(1/\sqrt{2}) \), respectively, need modifications in the light of a non-zero \( \theta_{13} \). The TBM mixing matrix is given as

\[
U_{\text{TBM}} = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix}.
\]  

(1)

Many ways have been proposed to modify the TBM ansatz to accommodate a non-zero \( \theta_{13} \). A simple possibility is to keep one column of TBM mixing matrix unchanged while modifying its other two columns within unitarity constraints \([3]\). This gives rise to three Trimaximal (TM) mixing patterns, viz., TM1, TM2, and TM3, that have their first, second and third columns identical to TBM matrix, respectively \([4]\). These three mixing schemes contain TBM mixing as a special case that enlarges the symmetry of these mixing patterns. Hence, they could have been named TM1, TM2, and TM3.

TM1 mixing is given as

\[
U_{\text{TM}1} = \begin{pmatrix}
\frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \cos \theta - \frac{e^{i\phi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} \sin \theta \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \cos \theta + \frac{e^{i\phi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} \sin \theta + \frac{e^{i\phi} \cos \theta}{\sqrt{2}} \\
-\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \cos \theta + \frac{e^{i\phi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} \sin \theta - \frac{e^{i\phi} \cos \theta}{\sqrt{2}}
\end{pmatrix}.
\]  

(2)

The mixing scheme reduces to the TBM scheme in the special case \( \theta = 0 \) and \( \phi = 0 \).

TM2 mixing is given as

\[
U_{\text{TM}2} = \begin{pmatrix}
\frac{\sqrt{2}}{3} \cos \theta + \frac{e^{-i\phi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{2}{3} \sin \theta \\
\frac{\sqrt{2}}{3} \cos \theta - \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{2}{3} \sin \theta \\
-\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{3}
\end{pmatrix}.
\]  

(3)

The mixing scheme reduces to the TBM scheme in the special case \( \theta = \arctan(1/\sqrt{2}) \) and \( \phi = 0 \). This mixing scheme corresponds to the magic symmetry.

TM3 mixing is given as

\[
U_{\text{TM}3} = \begin{pmatrix}
\cos \theta & \frac{\sin \theta}{\sqrt{2}} & 0 \\
\frac{\sqrt{2}}{2} \sin \theta & \frac{-\sqrt{2}}{2} \cos \theta & \frac{1}{\sqrt{2}} \\
-\frac{\sqrt{2}}{2} \sin \theta & \frac{-\sqrt{2}}{2} \cos \theta & \frac{1}{\sqrt{2}}
\end{pmatrix}.
\]  

(4)

The mixing scheme reduces to the TBM scheme in the special case \( \theta = \arctan(1/\sqrt{2}) \) and \( \phi = 0 \). This mixing scheme is equivalent to \( \mu - \tau \) symmetry.

Another simple assumption that can accommodate a non-zero \( \theta_{13} \) is the presence of texture zeros in the neutrino mass matrix \([5, 6]\). Texture zeros induce relations between mixing matrix elements and neutrino masses. Considering neutrinos to be Majorana fermions and working in a basis where the charged lepton mass matrix \( M_l \) is diagonal, there are in total fifteen different patterns of two texture zeros in the neutrino mass matrices. Out of these fifteen possible patterns, only seven can satisfy the present neutrino oscillation data \([5, 6]\). These seven patterns are classified in three classes A, B and C corresponding to the normal, quasi-degenerate and inverted mass hierarchies of neutrinos \([Table I]\).

The current neutrino data are consistent with the possibility of keeping first or second column of the mixing matrix unmodified (TM1 or TM2 mixing) while modifying other columns within unitarity constraints. The experimental data is also consistent with the presence of two texture zeros in the neutrino mass matrix. If we combine both approaches together by having texture zeros in a mass matrix corresponding to TM1 or TM2 mixing, we
ized as.

In an earlier work [9], we studied the implications of two texture zeros in a magic mass matrix giving $\text{TM}_2$ mixing. In the present work, we study the phenomenological implications of the presence of two texture zeros in a neutrino mass matrix giving $\text{TM}_1$ mixing.

A mass matrix giving $\text{TM}_1$ mixing can be parameterized as

$$
M_{\text{TM}_1} = \begin{pmatrix}
2a & 2b & 2c \\
2b & b - c + d & 2a + 2b - d \\
2c & 2a + 2b - d & -3b + 3c + d
\end{pmatrix}.
$$

(5)

We can obtain the form of the neutrino mass matrix giving $\text{TM}_1$ mixing for the seven patterns of two texture zeros by substituting the respective constraints from Table I in Eq. (5).

| Type | Constraining Equations |
|------|------------------------|
| A    | $M_{ee} = 0, M_{e\mu} = 0$ |
| A    | $M_{ee} = 0, M_{e\tau} = 0$ |
| B    | $M_{e\tau} = 0, M_{e\mu} = 0$ |
| B    | $M_{e\mu} = 0, M_{e\tau} = 0$ |
| B    | $M_{\mu\tau} = 0, M_{e\mu} = 0$ |
| B    | $M_{\mu\tau} = 0, M_{e\tau} = 0$ |
| C    | $M_{e\mu} = 0, M_{e\tau} = 0$ |

TABLE I. Seven allowed mass matrices with two zeros classified into three classes.

are bound to get very predictive neutrino mass matrices. In an earlier work [9], we studied the implications of two texture zeros in a magic mass matrix giving $\text{TM}_2$ mixing. In the present work, we study the phenomenological implications of the presence of two texture zeros in a neutrino mass matrix giving $\text{TM}_1$ mixing.

The neutrino mass matrix of type $A$ giving $\text{TM}_1$ mixing is

$$
M_{\text{TM}_1} = \begin{pmatrix}
0 & 0 & 2c \\
0 & -2c + \Delta & c - \Delta \\
2c & c - \Delta & 2c + \Delta
\end{pmatrix}.
$$

(6)

where $\Delta = d + b$. Similarly, the neutrino mass matrix of type $A_2$ giving $\text{TM}_1$ mixing is

$$
M_{\text{A}_2} = \begin{pmatrix}
0 & 2b & 0 \\
2b & b + \Delta & b - \Delta \\
0 & b - \Delta & -2b + \Delta
\end{pmatrix},
$$

(7)

where $\Delta = d - c$.

The four mass matrices giving $\text{TM}_1$ mixing for class $B$ are

$$
M_{\text{B}_1} = \begin{pmatrix}
2a & 2b & 0 \\
2b & 0 & 2a + 3b \\
0 & 2a + 3b & -4b
\end{pmatrix},
$$

(8)

$$
M_{\text{B}_2} = \begin{pmatrix}
2a & 0 & 2c \\
0 & -4c & 2a + 3c \\
2c & 2a + 3c & 0
\end{pmatrix},
$$

(9)

$$
M_{\text{B}_3} = \begin{pmatrix}
2a & 0 & 2c \\
0 & 0 & 2a - c \\
2c & 2a - c & 4c
\end{pmatrix},
$$

(10)

and

$$
M_{\text{B}_4} = \begin{pmatrix}
2a & 2b & 0 \\
2b & 4b & 2a - b \\
0 & 2a - b & 0
\end{pmatrix}.
$$

(11)

We will show that all these mass matrices of type class $B$ giving $\text{TM}_1$ mixing are not allowed by the experimental data.

The mass matrix with $\text{TM}_1$ mixing for the class $C$ is

$$
M_{\text{C}} = \begin{pmatrix}
2a & 2b & 2b \\
2b & 0 & 2a + 2b \\
2b & 2a + 2b & 0
\end{pmatrix}.
$$

(12)

This mass matrix has $\mu - \tau$ symmetry and implies $\theta_{13} = 0$. Hence, it is not allowed.

The phenomenology of patterns $A_1$ and $A_2$ is related: one can obtain the predictions for $A_2$ by making the transformations $[8, 7]

$$
\theta_{23} \rightarrow \frac{\pi}{2} - \theta_{23}, \delta \rightarrow \pi - \delta
$$

(13)

on the predictions of $A_1$. Hence, we study the phenomenological implications for pattern $A_1$ only.

Any mass matrix $M$ giving $\text{TM}_1$ mixing can be diagonalized by a mixing matrix $U = U_{\text{TM}_1}$ given in Eq. (4) using the relation

$$
U^T M U = M_{\text{diag}}
$$

(14)

where $M_{\text{diag}}$ is the diagonal mass matrix given as

$$
M_{\text{diag}} = \begin{pmatrix}
m_1 & 0 & 0 \\
0 & e^{2i\alpha} m_2 & 0 \\
0 & 0 & e^{2i\beta} m_3
\end{pmatrix}.
$$

(15)

Here, $m_1, m_2,$ and $m_3$ are the neutrino masses and $\alpha$ and $\beta$ are the two Majorana phases.

Once the mixing matrix $U$ is known, the mixing angles can be calculated using the relations:

$$
s^2_{12} = \frac{|U_{12}|^2}{1 - |U_{13}|^2},
$$

$$
s^2_{23} = \frac{|U_{23}|^2}{1 - |U_{13}|^2},
$$

and

$$
s^2_{13} = \frac{1}{3} \sin^2 \theta.
$$

(16)

We see from Eq. (17) that $\theta_{12}$ is smaller than its TBM value $s^2_{12} = 1/3$. In contrast, the value of $\theta_{12}$ is larger

$$
\theta_{12} \rightarrow \frac{\pi}{2} - \theta_{12}, \delta \rightarrow \pi - \delta
$$

(13)
than the TBM value for TM$_2$ mixing. Since the experimental value of $\theta_{12}$ is towards the lower side of the TBM value, TM$_1$ mixing is more appealing than TM$_2$ mixing.

The CP violating phase $\delta$ can be calculated from the Jarlskog rephasing invariant measure of CP violation \cite{8}

\[ J = Im(U_{11}U_{21}^*U_{22}) \] (20)

using the relation

\[ J = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin \delta. \] (21)

Substituting the elements of the TM$_1$ mixing matrix in Eq. (20), we obtain

\[ J = \frac{1}{6\sqrt{6}} \sin 2\theta \sin \phi. \] (22)

From Eqs. (21) and (22), we get

\[ \cot^2 \delta = \cot^2 \phi - \frac{6 \sin^2 2\theta \cot^2 \phi}{(3 - \sin^2 \theta)^2}. \] (23)

We reconstruct the neutrino mass matrix for TM$_1$ mixing using the relation:

\[ M_\nu = U^* M_{\text{diag}} U^\dagger \] (24)

where $U = U_{\text{TM}_1}$. To obtain the predictions for the neutrino mass matrix of the type A$_1$ given by Eq. (5), we have to solve the two complex equations: $M_{\nu_{11}} = 0$ and $M_{\nu_{12}} = 0$. Solving the equation $M_{\nu_{11}} = 0$, we get

\[ \frac{m_1}{m_2} = \frac{\sin 2(\alpha - \beta) \cos^2 \theta}{2 \sin 2\beta} \] (25)

and

\[ \frac{m_2}{m_3} = -\frac{\sin 2\beta \tan^2 \theta}{\sin 2\alpha}. \] (26)

Using these two equations, we evaluate $m_1/m_3$ and invert the resulting relation to obtain

\[ \cot 2\alpha = \cot 2\beta + \frac{m_1}{m_3} \csc 2\beta \cot^2 \theta. \] (27)

We note that the presence of a zero at (1,1) entry in a mass matrix with TM$_1$ mixing, through Eqs. (25) and (26), implies a beautiful sum-rule on neutrino masses:

\[ \frac{\sin 2(\alpha - \beta)}{2m_1} - \frac{\sin 2\beta}{m_2} + \frac{\sin 2\alpha}{m_3} = 0. \] (28)

The texture zero at (1,1) entry in a mass matrix with TM$_1$ mixing also gives a prediction for the ratio $r = \Delta m^2_{31}/\Delta m^2_{21}$. From Eqs. (25) and (26), we obtain

\[ r = \frac{-\sin^2 2(\alpha - \beta) + 4 \sec^4 \theta \sin^2 2\beta}{-\sin^2 2(\alpha - \beta) + 4 \sec^4 \theta \sin^2 2\beta}. \] (29)

We solve the second equation $M_{\nu_{12}} = 0$ by equating its real and imaginary parts to zero. We eliminate $m_1$ and $m_2$ from the resulting equations using Eqs. (25) and (26). Equating the imaginary part of $M_{\nu_{12}}$ to zero, we obtain

\[ \sin^2 \theta = \frac{-\sin 2\alpha \sin(2\beta - \phi)}{\sin(2(\alpha - \beta) \sin \phi)} \] (30)

We get a quadratic equation in $\tan^2 \theta$ on equating the real part of $M_{\nu_{12}}$ to zero:

\[ \frac{\sqrt{6} \sin 2\beta \cos(2\alpha - \phi)}{\sin 2(\alpha - \beta)} \tan^2 \theta + 3 \tan \theta + \frac{\sqrt{6} \sin 2\beta \cos(2\beta - \phi)}{\sin 2(\alpha - \beta)} = 0. \] (31)

Solving this equation by substituting $\alpha$ from Eq. (27), we obtain

\[ \tan \theta = \frac{\sqrt{3} \sin(2\beta - \phi)}{2 \sin 2\beta}. \] (32)

The value of $\theta$ calculated in Eqs. (30) and (32) must be identical. This requirement gives

\[ \cot 2\alpha = \cot \phi + \frac{2 \sin 2\beta}{3 \sin(2\beta - \phi) \sin \phi}. \] (33)

Equations (25), (26), (32) and (33) are the four predictions for the neutrino mass matrix of type A$_1$. We can express these four predictions as expressions for $\alpha$, $\beta$, $m_3$, and $m_2$ as functions of $\theta$ and $\phi$. Inversion of Eq. (32) gives

\[ \cot 2\beta = \cot \phi - \frac{\sqrt{3}}{3} \csc \phi \tan \theta. \] (34)

Substituting $\beta$ from Eq. (34) in Eq. (33) gives

\[ \cot 2\alpha = \cot \phi + \frac{\sqrt{3}}{3} \csc \phi \cot \theta. \] (35)

Equations (25) and (26) after substituting values of $\alpha$ and $\beta$ give

\[ \frac{m_2}{m_1} = 5 \sec^2 \theta + 4 \sqrt{6} \tan \theta \cos \phi + \tan^2 \theta - 1 \] (36)

and

\[ \frac{m_3}{m_1} = 6 \sec^2 \theta - 4 \sqrt{6} \cos \theta \cos \phi - 2. \] (37)

We can use these ratios to calculate $r = \frac{\Delta m^2_{31}}{\Delta m^2_{21}}$ as a function of $\theta$ and $\phi$ or we can calculate it directly from Eq. (29). We obtain

\[ r = \frac{3 - 6 \sec^2 \theta - 4 \sqrt{6} \tan \theta \cos \phi}{3 - \csc \theta + 4 \sqrt{6} \cot \theta \cos \phi}. \] (38)

For TM$_2$ mixing, we have $r = \tan^2 \theta \left[ \frac{3}{2} \right]$. In contrast, $r$ is function of both $\theta$ and $\phi$ for TM$_1$ mixing. By demanding
that $r$ satisfies its experimental value, one can calculate experimentally allowed values of $\phi$.

Once the experimentally allowed values of $\theta$ and $\phi$ are known, the observables $\theta_{12}, \theta_{23}, \theta_{13}$ and $\delta$ can be calculated from Eqs. (17), (18), (19) and (23) as they are functions of $\theta$ and $\phi$.

The experimentally allowed value of $\theta$ can be calculated from Eq. (19) using the experimental value of $\sin^2 \theta_{13} = 0.0216 \pm 0.00075$ [10]. We get $\theta = 14.78 \pm 0.26$ degrees. We could have also used the experimental value of $\sin^2 \theta_{12} = 0.306 \pm 0.012$ [10] to constrain $\theta$. However, this gives a larger range of $\theta$. We plot $r$ as a function of $\phi$ in Fig. 1. From the experimental values $\Delta m^2_{21} = (7.50 \pm 0.19) \times 10^{-5}$ eV$^2$ and $\Delta m^2_{31} = (2.524 \pm 0.04) \times 10^{-3}$ eV$^2$ [10], we get the experimental value $r = 0.0297 \pm 0.0003$. Here, all the experimental errors are at one standard deviation. We find that the predicted and experimental values of $r$ are consistent only for two regions of $\phi$ depicted in Fig. 1. In this way, we can constrain both $\theta$ and $\phi$. Then, all other observables can be obtained as they are functions of $\theta$ and $\phi$.

A better approach is to constrain $\theta$ and $\phi$ by minimizing the $\chi^2$-function

$$\chi^2(\theta, \phi) = \frac{3}{i=1} \left[ \frac{v_i(\theta, \phi) - v_i^{exp}}{\sigma^i_{exp}} \right]^2$$

where the variables $v_i = (\sin^2 \theta_{12}, \sin^2 \theta_{13}, r); v_i^{exp}$ are experimental values of $v_i$; and $\sigma^i_{exp}$ are the standard deviations in the experimental values of $v_i$. The $\chi^2(\theta, \phi)$ is minimum at $\theta = 14.8$ degrees and $\phi = 103.2$ degrees or $\phi = 256.8$ degrees. The minimum value is $\chi^2_{min} = 1.8$. The contours of $\Delta \chi^2 = \chi^2 - \chi^2_{min}$ corresponding to $1\sigma$, $2\sigma$, and $3\sigma$ confidence level are shown in Fig. 2. The predictions for $\theta_{23}$ and $\delta$ for $\Delta M^2_{12}$ mixing in pattern A1 have been shown in Fig. 3 at various confidence levels. The predictions for $\Delta M^2_{13}$ mixing in pattern A2 can be obtained by using the transformations given in Eq. (13). We find that $\delta$ is predicted either around $100^\circ$ or around $260^\circ$ with a spread of around $10^\circ$. Table II shows the $3\sigma$ ranges of $\delta$ and $\theta_{23}$ for these two solutions obtained from our analysis. The mixing angle $\theta_{23}$ lies below (above) $45^\circ$ for pattern A1 (A2).

Recently, long baseline neutrino oscillation experiments like MINOS and T2K [11] are showing a preference for the CP violating phase $\delta$ to be around $270^\circ$. In particular, a recent global analysis in Ref. [10] rules out $\delta$ from about $55^\circ$ to $120^\circ$ at $3\sigma$ CL for inverted mass ordering. A certain portion of $\delta$ is also ruled out if $\theta_{23} < 45^\circ$ for the normal mass ordering (see Fig. 11 in Ref. [10]). In our $\chi^2$ analysis, we do not put any experimental constraints on $\delta$ and $\theta_{23}$ as these parameters are not precisely measured and their distributions are not Gaussian. If we take into consideration the limits on $\delta$ as given in Ref. [10], the Solution - I in Table II for pattern A1 is ruled out and only Solution - II, where $\delta$ lies around $260^\circ$, remains compatible. Since the exclusion region of $\delta$ with respect to $\theta_{23}$ for normal mass spectrum (Fig. 11 of Ref. [10]) is not symmetric around $\theta_{23} = 45^\circ$, both solutions are still allowed for pattern A2.

The predictions for Majorana phases $\alpha$ and $\beta$ are shown in Fig. 4. We also predict the lines for the effective electron neutrino mass for $\beta$-decay $m^e_1 = m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 + m_3 |U_{e3}|^2$ and the sum of neutrino masses $\sum m_i = m_1 + m_2 + m_3$ in Fig. 5. Since the $(1,1)$ element of the neutrino mass matrix vanishes for patterns A1 and A2, this leads to a vanishing effective Majorana neutrino mass ($|M_{ee}| = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2|$) for these patterns.

The neutrino mass matrix of type B1 has zeros at (1,3) and (2,2) entries. This implies following expression of ratio $r$ in the presence of $\Delta M^2_{13}$ mixing:

$$r = \frac{1}{6} \left( 2\sqrt{6} \sin(2\theta) \cos(\phi) + 9 \cos(2\theta) - 3 \right).$$

We can express $\theta$ in terms of $\theta_{13}$ using Eq. (19) in the above relation expressing $r$ as a function of $\theta$ (Fig. 6). It is clear that we cannot have both $r$ and $\theta_{13}$ in their experimentally allowed ranges simultaneously. Hence, this pattern is inconsistent with the experimental data when combined with $\Delta M^2_{13}$ mixing. The neutrino mass matrix of type B2 is related to the neutrino mass matrix of type B1 by a $\mu$-$\tau$ exchange [12] and has identical predictions for $r$ and $\theta_{13}$. Hence, neutrino mass matrix of type B2 with $\Delta M^2_{13}$ mixing is also incompatible with the recent experimental data.

The neutrino mass matrix of type B3 has zeros at (1,2) and (2,2) entries. The expression of ratio $r$ in the presence of $\Delta M^2_{13}$ mixing is given by

$$r = \frac{1}{10} \left( 2\sqrt{6} \sin(2\theta) \cos(\phi) - \cos(2\theta) - 5 \right).$$

In case of pattern B3, the parameter $r$ always remains larger than 0.4 whereas the experimental range of this parameter lies well below the value 0.4 (see Fig. 6). Thus pattern B3 is also incompatible with the experimental data. Since pattern B3 is related to pattern B4 by $\mu$-$\tau$ exchange symmetry, the pattern B3 is also incompatible with the experimental data when combined with $\Delta M^2_{13}$ mixing.

In conclusion, we have studied the phenomenological implications of two texture zeros in the presence of $\Delta M^2_{13}$ mixing. There are seven allowed patterns for the presence of two texture zeros in the neutrino mass matrix. The

| Type | $\theta_{23}$ | Solution - I | Solution - II |
|------|---------------|--------------|--------------|
| A1   | 41.18° - 44.02° | 94.6° - 106.5° | 253.5° - 265.5° |
| A2   | 45.98° - 48.82° | 73.5° - 85.5° | 274.6° - 286.5° |

TABLE II. Allowed 3σ ranges of $\theta_{23}$ and $\delta$ for patterns A1 and A2. Solution - I and Solution - II are two allowed solutions for $\delta$. |
FIG. 1. The ratio \( r = \frac{\Delta m_{12}^2}{\Delta m_{13}^2} \) as a function of \( \phi \) (degrees). The horizontal line depicts the experimental value of \( r \). The inner solid bands depict the 1\( \sigma \) range and the outer gray bands depict the 3\( \sigma \) range.

presence of TM\(_1\) mixing rules out five out of the seven patterns of two texture zeros. The neutrino mass matrix having two texture zeros and TM\(_1\) mixing simultaneously can only belong to patterns A\(_1\) and A\(_2\). The Dirac CP violating phase \( \delta \) is restricted to two narrow regions around 100° and 260° for these patterns. For TM\(_1\) mixing, \( \theta_{12} \) is smaller than its TBM value and moves towards its best fit value with the increase in \( \theta \). The imposition of TM\(_1\) mixing on two zeros make these classes very predictive and these predictions can be tested in future neutrino oscillation experiments.

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FIG. 2. The allowed region on \((\theta - \phi)\) parameter space. The contours are at 1, 2, and 3 \(\sigma\) confidence levels (CL) and all the angles are in degrees.

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FIG. 3. The predictions for $\theta_{23}$ and $\delta$ at 1, 2, and 3 $\sigma$ confidence levels where all the angles are in degrees.

FIG. 4. Predictions for Majorana phases $\alpha$ and $\beta$ at 1, 2, and 3 $\sigma$ confidence levels where all the angles are in degrees.

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FIG. 5. Predictions for effective electron neutrino mass for $\beta$-decay and sum of neutrino masses at 1, 2, and 3 $\sigma$ confidence levels.

FIG. 6. The inconsistency between $r$ and $\theta_{13}$ (degrees) for patterns B$_1$ (left) and B$_3$ (right).

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