Tuning the Non-local Spin-Spin Interaction between Quantum Dots with a Magnetic Field

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We describe a device where the non-local spin-spin interaction between quantum dots (QDs) can be turned on and off with a small magnetic field. The setup consists of two QDs at the edge of two two-dimensional electron gases (2DEGs). The QDs’ spins are coupled through a RKKY-like interaction mediated by the electrons in the 2DEGs. A magnetic field $B_z$ perpendicular to the plane of the 2DEG is used as a tuning parameter. When the cyclotron radius is commensurate with the interdot distance, the spin-spin interaction is amplified by a few orders of magnitude. The sign of the interaction is controlled by finely tuning $B_z$. Our setup allows for several dots to be coupled in a linear arrangement and it is not restricted to nearest-neighbors interaction.

Quantum information processing requires control and operation of interacting quantum mechanical objects \cite{1}. One possibility is to produce systems with localized spins in atomic impurities, molecules or quantum dots (QDs) and manipulate the spin-spin interaction by engineering the electronic wave functions of the surrounding material \cite{2,3,4}. This would allow for the non-local control of spins opening new possibilities for the fast developing field of spintronics \cite{5}. An important step in this direction was reported very recently by Craig et al. \cite{6}, who coupled two QDs through a confined 2DEG (a larger QD) and controlled the magnitude of the interaction by closing or opening up the QDs. Besides its relevance for spintronics, this experiment also opens up the possibility to study the interplay between two competing many-body effects: the Kondo effect and the RKKY-interaction \cite{7,8,9}.

In this work, we propose a different device where the magnitude (and sign) of the spin-spin interaction between two QDs can be tuned by an external field. Our setup consists of two QDs at the edge of two semi-infinite two dimensional electron gases (2DEGs). When each dot is gated to have an odd number of electrons—this is controlled by the gate voltages \cite{11,12,13} or QD \cite{14}. The control mechanism is based on the possibility of focusing the electrons that interact through a polarization of the 2DEG as in the usual tight binding model on a square lattice, \cite{26}. This interaction can be controlled by applying a perpendicular magnetic field \cite{27}, being maximum when the cyclotron orbit matches the distance between the two QD. A third (optional) gate \cite{15} can be used to interrupt one of the electrons’ path and cancel out the interaction.

\[ \hat{H} = \hat{H}_{\text{QD}} + \hat{H}_{\text{2DEG}} + \hat{H}_{\text{T}}, \]

where the first term is the Hamiltonian of the QDs

\[ \hat{H}_{\text{QD}} = \sum_{\alpha, \sigma} E_{\alpha\sigma} d_{\alpha\sigma}^\dagger d_{\alpha\sigma} + U_{\alpha} d_{\alpha\uparrow}^\dagger d_{\alpha\downarrow}^\dagger d_{\alpha\downarrow} d_{\alpha\uparrow} \].

Here $\alpha = 1$ and 2 refer to the left and right QD respectively, $d_{\alpha\sigma}^\dagger$ creates an electron with spin $\sigma$ and energy $E_{\alpha\sigma}$ in the QD labeled by $\alpha$. $U_{\alpha}$ is the Coulomb energy defined by the capacitances of the system \cite{28}. The single particle energies $E_{\alpha\sigma}$ (measured from the Fermi energy, $E_F$) can be varied with a gate voltage $V_{\alpha}$ as shown in Fig. 1. The second term in the Hamiltonian \cite{29} describes the electrons in the two half planes. To describe these 2DEGs, we discretize the space and use a tight binding model on a square lattice,

\[ \hat{H}_{\text{2DEG}} = \sum_{\gamma n\sigma} \varepsilon_{\gamma n\sigma} c_{\gamma n\sigma}^\dagger c_{\gamma n\sigma} - \sum_{< n,m> \sigma} t_{nm} c_{\gamma n\sigma}^\dagger c_{\gamma m\sigma} + \text{h.c.} \]

where $c_{\gamma n\sigma}^\dagger$ creates an electron with spin $\sigma$ at site $\mathbf{n} = (n_x, n_y)$ of the upper ($\gamma = 1$) and lower ($\gamma = 2$) half planes. The hopping matrix element $t_{nm}$ connects nearest neighbors and

\[ B \]

FIG. 1: Schematic representation of the proposed device. Both QDs are setup to have an odd number of electrons—this is controlled by the gate voltages $V_1$ and $V_2$. The QDs spins are then coupled through a RKKY-like interaction mediated by the electrons in the two 2DEGs. This interaction can be tuned by a perpendicular magnetic field $B_z$, being maximum when the cyclotron orbit matches the distance between the two QD. A third (optional) gate $V_3$ can be used to interrupt one of the electrons’ path and cancel out the interaction.

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includes the effect of the diamagnetic coupling through the Peierls substitution. To avoid any zone boundary effects we take a lattice parameter $a_0 = 5\text{nm}$ much smaller than the characteristic Fermi wavelength ($\lambda_F \sim 50\text{nm}$). We use the Landau gauge for which $t_{n(x)} = te^{-in_\phi/\phi_0}$ and $t_{n(y)} = t$, where $\phi = a_0^2 B_z$ is the magnetic flux per plaquette, $\phi_0 = \hbar c / e$ is the flux quantum and $t = \hbar^2 / 2 m^* a_0^2$ with $m^*$ is the carriers effective mass. The third term in the Hamiltonian describes the coupling between the QD and the 2DEGs

$$H_1 = -\sum_{\gamma \sigma} t_{\gamma \sigma} (c_{\gamma \alpha \sigma}^\dagger d_{\alpha \sigma} + d_{\alpha \sigma}^\dagger c_{\gamma \alpha \sigma}), \quad (4)$$

where $c_{\gamma \alpha \sigma} = N^{-\frac{1}{2}} \sum_n c_{\gamma \alpha \sigma}^n$ creates an electron at the half plane $\gamma$ in a linear combination of $N_0$ sites next to the QD $\alpha$. We are interested in the range of magnetic fields that produce a cyclotron radius $r_c$ of the order of the interdot distance $R$, defined as the average distance between the sites connected to dot 1 and dot 2. For these small fields, the Zeeman splitting can be neglected restoring the spin rotational symmetry. In what follows we take $E_{\alpha \uparrow} = E_{\alpha \downarrow}$. The generalization to the case of a large Zeeman energy is straightforward. We assume that the QDs are gated to be in the strong Coulomb blockade regime, $U + E_{\alpha \sigma} \approx -E_{\alpha \sigma}$, so that the charge fluctuations can be eliminated by the Schrieffer-Wolf transformation [17].

The spin dynamics is then described by a Kondo Hamiltonian where $H_{\text{QD}} + H_1$ is replaced by [17]

$$\hat{H}_K = \sum_{\alpha} J_\alpha \hat{S}_\alpha \cdot (c_{1 \alpha \sigma}^\dagger c_{2 \alpha \sigma}) \sigma / 2 \hat{\sigma}_{\sigma \sigma}, \quad (5)$$

where $\hat{S}_\alpha$ is the spin operator associated to the QD $\alpha$ and

$$J_\alpha = 2 |t_\alpha|^2 \left( \frac{1}{U_{\alpha \sigma} + E_{\alpha \sigma}} - \frac{1}{E_{\alpha \sigma}} \right) \approx \frac{4 |t_\alpha|^2}{U_{\alpha \sigma}} \frac{\Gamma_\alpha}{\pi \rho}, \quad (6)$$

with $\Gamma_\alpha$ the level width and $\rho$ the local density of states per spin at $E_F$. $\Gamma_\alpha$ and $U_{\alpha \sigma}$ can be measured by transport experiments [18][19]. For simplicity, we take $t_{\alpha \sigma} \equiv t_\alpha$ and neglect a potential scattering term which is not relevant for the present work [17]. Finally, usual perturbative procedures give an effective exchange interaction between the QDs’ spins mediated by electrons in the 2DEG. The interdot exchange Hamiltonian $\hat{H}_J$ contains the non-local exchange which can be written in terms of one-particle propagators. With a negligible Zeeeman splitting, the propagators are spin independent and the Hamiltonian reduces to $\hat{H}_J = J \hat{S}_1 \cdot \hat{S}_2$ with

$$J = -\frac{J_1 J_2}{2\pi} \int d\omega f(\omega) \text{Im} [G_\sigma(1, 2) G^\dagger(2, 1)], \quad (7)$$

where $\text{Im}$ denotes the imaginary part and $G_\sigma(\alpha, \alpha') = \langle \langle c_{1 \alpha \sigma} c_{2 \alpha' \sigma} \rangle \rangle$ is the Fourier transform of the retarded Green function [20] and $f(\omega)$ is the Fermi function. In the following we take $kF T \ll J$—in this regime $f(\omega) \approx \Theta(E_F - \omega)$. To lowest order in $J_\alpha$, the one-particle propagators are calculated with $\hat{H}_{\text{2DEG}}$ only. Then, we have

$$G_\sigma(\alpha, \alpha') = g_{1\sigma}(\alpha, \alpha') + g_{2\sigma}(\alpha, \alpha') \quad \text{and} \quad J = J(R \sim \lambda_F, B_z = 0).$$

with $g_{\sigma}(\alpha, \alpha') = \langle \langle c_{1 \alpha \sigma} c_{2 \alpha' \sigma} \rangle \rangle$ and $J$ as a function of $B_z$ for interdot distances $R = 0.5$, 1 and $1.5 \mu m$. In all cases, the interaction presents large oscillations whenever the magnetic field is such that twice the cyclotron radius $r_c = \hbar c / e B_z$ is commensurable with $R$. We refer to this fields as the focusing fields since in this situation electrons that interact with one dot are focused into the other
FIG. 3: (Color online). Exchange interaction as a function of the interdot distance $R$ for $B_z = 0$ (a), 231 mT (b) and 241 mT (c). These values of $B_z$ are indicated in Fig. 2a with arrows. The magnitude of $J$ increases a few orders of magnitude when $R \approx 2r_c \approx 500 nm$ while its sign depends on the precise value of $B_z$. The solid and dashed lines correspond to $N_0 = 1$ and 7, respectively.

by the action of the field. As $R$ increases the characteristic period of the oscillations and their amplitude decreases (the nature of these oscillations is discussed below). The exchange coupling $J$ as a function of the interdot distance is shown in Fig. 3 for fixed fields. For $B_z = 0$ the dominant contribution is $J \propto \cos(2k_F R)/(k_F R)^2$, in contrast to the conventional 2D behavior $J \propto \cos(2k_F R)/(k_F R)^2$. The power law decay $R^{-4}$ is due to the structure of the states near the edge of the 2DEG. Namely, $\rho$ depends linearly on energy (it is constant in bulk). Since in a semiclassical picture the contribution to the propagator Eq. 8 arises only from the classical trajectories near the boundary, one could argue that the effective density is $\rho(\varepsilon) \propto \varepsilon$, so this case “mimics” a 4D one (thus the $R^{-4}$ decay). This is confirmed by both a quantum and a semiclassical analytical calculation. For $B_z \neq 0$, a large amplification of $J$ is observed for distances such that $R = n 2r_c$, where $n$ is an integer. Comparison of Figs. 3 and 4 shows that at distances of the order of 0.5 $\mu m$, an increase of the field from zero to its focusing value increases the coupling by more than two orders of magnitude (notice $J/J_0 \sim 1$ for $n = 1$). Additionally, the sign of the interaction is controlled by finely tuning the magnetic field around the focusing value (see arrows in Fig. 3). Note that a larger $N_0$ (tunneling region) enhances the effect. This is due to a reduction of diffraction effects, which leads to a better definition of the classical cyclotron orbit [13].

All these results can be put together in a single density plot as shown in Fig. 4. The lines are hyperbolas corresponding to different focusing fields

$$B_z^{(n)} = 2n(hc/\pi eR) = 2n\phi_0/\lambda_F R.$$  \hspace{1cm} (9)

Along these lines, $ie$. when field and distance are simultaneously varied to keep the focusing condition, the exchange coupling oscillates with large amplitude. A simple fitting of the numerical results shows that the dominant contribution to $J$ along the hyperbolas is given by

$$J \propto \frac{\cos(k_F \pi R/2 + \varphi)}{R^2}.$$  \hspace{1cm} (10)

Note that the interaction decays as $R^{-2}$ as in the usual 2D case—this is consistent with the semiclassical picture since now the classical trajectories are not restricted to be close to the boundary. The argument of the cosine can be written as $k_F R_{\text{eff}}$ where $R_{\text{eff}} = \pi R/2$ is the length of the classical trajectory (see Fig. 4). The period of the oscillations is then equal to the period at which the Landau levels cross $E_F$. In fact, we have $k_F R_{\text{eff}} = 2\pi R_{\text{eff}}/\lambda_F = 2\pi E_F/\hbar \omega_c$. These oscillations of the non-local susceptibility are the analog of the de Haas-van Alphen oscillations of the magnetization. The complex oscillating pattern observed in Fig. 2 corresponds to a cut in Fig. 4 along a vertical line. The inset in Fig. 4 shows the coupling $J$ when the field and the interdot distance are varied along the first hyperbola ($n = 1$). The solid line is a fitting to Eq. 10.

So far we have presented results obtained by fixing the chemical potential. However, in a 2DEG with a fixed charge density, $E_F$ is pinned at the energy of the partially filled Landau level and presents periodic jumps when plotted as a function of $1/B_z$. Figures 5 shows the coupling $J$ versus the external field for constant electron density at the bulk of the 2DEG. Now again, the spin-spin interaction presents a large enhancement at the focusing fields. There are, however, some differences with the previous case: as the external field is varied
around the focusing values, the sign of the interdot interactions tends to be preserved. Both ferromagnetic and antiferromagnetic couplings are obtained. The dominant sign of \( J \) at the different focusing field depends on the parameters, in particular on the particle density. The jumps of the Fermi energy overestimate some charge redistribution at the edges. Including the electron-electron interactions in a self-consistent approximation would tend to preserve local charge neutrality. This may generate an intermediate situation where the effective coupling changes sign as \( B_z \) sweeps the focusing values.

It is worth pointing out that in the presence of a magnetic field, the exchange interaction between two spins in the bulk of a 2DEG also shows some structure: there is a small enhancement of \( J \) for \( R \sim 2r_c \) while it decays exponentially for \( R \geq 2r_c \). In the proposed geometry, however, the focusing effect produces an amplification of \( J \) much larger than what is obtained in an homogeneous 2DEG. Moreover, with present technologies, it is possible to built a device like the one schematically shown in Fig. 5, where the contacts used to control the QD parameters \( (E_o, t_o) \) are also used to divide the 2DEG in two halves. An interesting advantage of our setup is that it allows to change the magnitude of the exchange coupling between QDs without modifying their coupling to the 2DEG. Therefore, transport measurements through the coupled and decoupled system can be easily compared.

Since the exchange mechanism requires the two half planes, interrupting the particle propagation in one of them decouples the QDs. This provides an alternative way to act on the effective coupling, which can be implemented with a gate voltage on a side contact indicated as \( V_3 \) in Fig. 5. Also, three or more QDs could be built along the central gate with the same or different interdot distances, allowing for a variety of alternatives in which, with the help of side contacts and the external magnetic field \( B_z \), the different couplings could be varied in sign and magnitude. It is important to emphasize that the interaction between QDs is not restricted to nearest neighbors.

In summary, we showed that two QDs at the edge of a 2DEG interact with an exchange coupling \( J \) that can be controlled with a small magnetic field perpendicular to the 2DEG. When the cyclotron radius \( r_c \) becomes commensurable with the interdot distance \( R \) there is a large amplification of the interdot interaction. This condition, \( 2nr_c = R \), defines the focusing fields. As the external field is varied around this values, the enhanced interaction changes sign allowing for a fine tuning of a ferromagnetic or an antiferromagnetic coupling.

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