Cosmological singularity as an informational seed for Everything
June 21, 2021

S.L. Cherkas† and V.L. Kalashnikov‡
† Institute for Nuclear Problems, Bobruiskaya 11, Minsk 220030, Belarus
‡ Facoltà di Ingegneria dell’Informazione, Informatica e Statistica, Sapienza
Università di Roma, Via Eudossiana 18 00189 - Roma, RM, Italia

Abstract. It is shown how to place some amount of matter into the cosmological singularity and to encode its state. A free and massless scalar field is considered as a prototype of matter. Two different but coherent approaches to this issue are presented. The expression for the scalar particles’ spectral energy density, which is initially encoded at the singularity, is deduced. An informational aspect of the problem is discussed.

1. Introduction

The well-known Penrose theorem [1–3] states that, under quite general conditions, the initial point of the universe evolution should be singular. Removing of a singularity on a classical level requires some “bounces” [4, 5], using exotic matter or modified theories of gravity. Such “bounces” take place also in some quantum gravity theories, for instance, in the loop quantum gravity [6, 8]. Another view on a cosmological singularity could be that it is not a problem but a necessary feature, related rather with the concept of time [9, 10] than with the gravity. Although one could imagine some infinite events in time, everything that we could observe experimentally has the beginning and the end. In such a case, the beginning of the universe could be associated with a cosmological singularity.

The universe singularity could remain in the quantum cosmology, as well [11–13]. Moreover, it is appealing to set the initial conditions for the universe evolution at the singularity per se [14].
Cosmological singularity as an informational seed for Everything

This issue includes the origin of matter and information in the universe. The mainstream view suggests inflation [15–17] as the ingredient of the cosmological theory, despite the intensive discussions regarding the inflation (see the so-called “letter of 33th” [18]). At the end of inflation, the matter appears from the inflaton field decay, which occurs when it began to oscillate [19, 20]. It is not an invulnerable point of view because the inflaton field cannot be associated with the known fields of the Standard Model of particle physics. Also in the context of the information, inflation erases all the previous information, up to the scales of so-called trans-Planckian physics [21–23] (see, e.g., [24]), and generates a spectrum of the initial inhomogeneities from vacuum fluctuations of the inflaton field and metric tensor. For the theories without inflation, another explanation for the matter origin in the universe is needed.

Since the most theories without inflation predict “bounces,” i.e., exclude singularity [25], the content of the present paper implies a relatively narrow class of the theories with the singularity. Still, without inflation, otherwise, there would be no sense to talk about information if the inflation rewrites it. The Milne-like universes could be an example of the model [26–34], where a stage of the linear universe expansion in cosmic time solves the problem of the horizon without inflation.

As was shown earlier [14], some finite quantities exist at the singularity, namely, the momentums of the dynamical variables despite the dynamical variables’ infiniteness. For instance, the scalar fields’ amplitudes are infinite at the singularity, but their momentums are finite. In the quantum picture, momentums’ finiteness allows building a wave packet at the singularity and setting an initial condition for the universe evolution. It was shown at the example of the Gowdy model considered in the quasi-Heisenberg picture [14]. Here, in the first section, we demonstrate it using the familiar approach to the quantum fields on the classical background. In the second section, as a bridge to the complete quantization of gravity, we consider a toy model where the scale factor evolution is uncoupled from the universe stuffing. It is shown that both approaches give the same results, i.e., reflect the same physical reality. Finally, we discuss the information context of the initial universe state.

2. Quantum fields on the classical background

The main idea that the field momentums remain finite at the singularity and this allows setting the momentum wave packet defining the evolution of the system could be realized by the the conventional approach to the quantum fields on the classical background [35]. The interval for isotropic, a uniform and the flat universe is

\[ ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = a^2(\eta) \left( d\eta^2 - \tilde{\gamma}_{ij}dx^i dx^j \right), \]

where \( a \) is a scale factor, \( \eta \) is a conformal time and \( \tilde{\gamma}_{ij} = \text{diag}\{1,1,1\} \) is an Euclidean 3-metric. A scalar field \( \phi(\eta, r) \) on this background can be represented in the form of the Fourier series \( \phi(\eta, r) = \sum_k \varphi_k(\eta)e^{ikr} \).
The Hamiltonian describing an evolution of the $k$-modes [35] is written as
\[ H = \sum_k \frac{\pi_k \pi_{-k}}{2a^2} + \frac{1}{2} a^2 k^2 \varphi_k \varphi_{-k}. \] (2)
The creation and annihilation operators can be introduced as
\[ \hat{\varphi}_k = \hat{a}^+_{-k} u_k^*(\eta) + \hat{a}_{-k} u_k(\eta). \] (3)
The functions $u_k$ satisfy
\[ u_k'' + \frac{2a'}{a} u_k' + k^2 u_k = 0, \] (4)
\[ a^2(\eta)(u_k(\eta) u_k''(\eta) - u_k'(\eta) u_k'(\eta)) = i, \] (5)
where the last equation (5) is a consequence of the canonical commutators $[\hat{\pi}_k, \hat{\varphi}_q] = -i\delta_{k,q}$ for the field (3) and the momentum
\[ \hat{\pi}_k = a^2 \varphi'_{-k} = a^2(\eta)(\hat{a}_{-k} u_k(\eta) + \hat{a}^*_{-k} u_k'(\eta)). \] (6)
Let us show that the momentum (6) is finite at $\eta = 0$. In the vicinity of singularity, the function $u_k(\eta)$ satisfies Eq. (4) asymptotically without the last term. This equation can be converted into the form
\[ \frac{d}{d\eta} (a(\eta)^2 u_k'(\eta)) = 0. \] (7)
From Eqs. (6) and (7) one may conclude that the momentums $\hat{\pi}_k$ are asymptotically some constant operators in the vicinity of singularity. The assumption, that the kinetic terms $u''_k + \frac{2a'}{a} u'_k$ are dominant over $k^2 u_k$ in the vicinity of singularity, is valid, e.g., for the dependencies $a(\eta) \sim \eta^n$, when $u_k \approx \eta^{1-2n}$. In particular, these dependencies include $a(\eta) \sim \eta$ (radiation background) and $a(\eta) \sim \eta^2$ (matter background). The creation $\hat{a}_k^+$ and annihilation $\hat{a}_k$ operators are very convenient instruments for the description of the quantum fields at the late times, when the field oscillators oscillate and the term $k^2 \hat{\varphi}_k$ exceeds $a' a \hat{\varphi}'_k$ one. However, at early times, it is more convenient to describe fields in terms of the eigen functions of the operators $\hat{\pi}_k(0) = \hat{P}_k$. Let us define the time-independent operator
\[ \hat{\dot{P}}_k = \alpha_k \hat{a}_{-k} + \alpha_k^* \hat{a}^+_{-k}, \] (8)
where the complex-valued constants are
\[ \alpha_k = a^2(\eta) u_k'(\eta) \big|_{\eta \to 0}, \] (9)
and let’s introduce an additional constant operator
\[ \hat{X}_k = b_k \left( \hat{a}_k + \hat{a}^+_{-k} \right), \] (10)
where $b_k$ is some real constant, which must be defined. The requirements of the fulfilment of the commutation relations
\[ [\hat{P}_k, \hat{X}_q] = -i\delta_{k,q}, \] (11)
with the taking into account of \([\hat{a}_k, \hat{a}_k^+] = 1\) allows obtaining the real constants \(b_k\)

\[
b_k = -\frac{i}{\alpha_k - \alpha_k^*}.
\]  

The phases of the quantities \(\alpha_k\) are determined by the phases of the functions \(u_k(\eta)\). These phases are not formal parameters but influence a quantum state of a system [36]. Then, the creation and annihilation operators can be expressed with the help of Eqs. (8), (10) through \(\hat{X}_k\) and \(\hat{P}_k\) as

\[
\hat{a}_k = \frac{\hat{P}_k^+}{\alpha_k - \alpha_k^*} - i\alpha_k^* \hat{X}_k,  \\
\hat{a}_k^+ = \frac{\hat{P}_k^+}{\alpha_k^* - \alpha_k} + i\alpha_k \hat{X}_k.  
\]  

Substitution (13) into (3) allows writing the field operator in the terms of operators \(\hat{P}_k\), \(\hat{X}_k\):

\[
\hat{\varphi}_k(\eta) = \frac{\hat{P}_k^+}{\alpha_k - \alpha_k^*} (u_k(\eta) - u_k^*(\eta)) + i\hat{X}_k (\alpha_k u_k^*(\eta) - \alpha_k^* u_k(\eta)).
\]  

The momentum operator \(\hat{\pi}_k(\eta)\) can be expressed analogously

\[
\hat{\pi}_k(\eta) = a(\eta)^2 \left( \frac{\hat{P}_k}{\alpha_k - \alpha_k^*} (u'_k(\eta) - u''_k(\eta)) + i\hat{X}_k (\alpha_k u''_k(\eta) - \alpha_k^* u'_k(\eta)) \right),
\]  

where it is taken into account that \(\hat{X}_- = \hat{X}_k^*, \hat{P}_- = \hat{P}_k^+\). From (5), (9), (14) it follows that \(\hat{X}_k\) is a nonsingular part of \(\hat{\varphi}_k(\eta)\) at \(\eta = 0\).

The mean value of an arbitrary operator over a wave packet is written as

\[
| C | \hat{A} \left( \eta, \left\{ P_k, i \frac{\partial}{\partial P_k}, P_k^*, i \frac{\partial}{\partial P_k} \right\} \right) | C > =
\int (C(\{P_k, P_k^*\}))^* \hat{A} C(\{P_k, P_k^*\}) \mathcal{D}P_k \mathcal{D}P_k^*,
\]  

where \(\mathcal{D}P_q \equiv dP_k, dP_k, \ldots\) and the following realization of the operators is implied:

\[
\hat{P}_k = P_k, \quad \hat{P}_k^+ = P_k^*, \quad \hat{X}_k = i \frac{\partial}{\partial P_k}, \quad \hat{X}_k^+ = i \frac{\partial}{\partial P_k^*}.
\]  

In particular, the mean value of \(\hat{\varphi}_k(\eta)\) over a wave packet looks as

\[
| C | \hat{\varphi}_k(\eta) | C > = \frac{u_k(\eta) - u_k^*(\eta)}{(\alpha_k - \alpha_k^*)} \int (C(\{P_q, P_q^*\}))^* P_k^* C(\{P_q, P_q^*\}) \mathcal{D}P_q \mathcal{D}P_q^* -

(\alpha_k u_k^*(\eta) - u_k(\eta) \alpha_k^*) \int (C(\{P_q, P_q^*\}))^* \frac{\partial}{\partial P_k} C(\{P_q, P_q^*\}) \mathcal{D}P_q \mathcal{D}P_q^*.
\]  

Let one has some fixed set of the functions \(u_k\) (in the paper it was used a set in which quantities \(\alpha_k\) is pure imaginary) and intends to proceed to another set \(u_k e^{-i\theta_k}\). Then formula (14) will look as

\[
\hat{\varphi}_k(\eta) = \frac{\hat{P}_k^+ \left( e^{-i\theta_k} u_k(\eta) - e^{i\theta_k} u_k^*(\eta) \right)}{e^{-i\theta_k \alpha_k} - \alpha_k^* e^{i\theta_k}} + i \hat{X}_k (\alpha_k u_k^*(\eta) - \alpha_k^* u_k(\eta)),
\]
where $\dot{P}_k = P_k$ and $\dot{X}_k = i \frac{\partial}{\partial P_k}$. The mean value of $\hat{\varphi}_k(\eta)$ over wave packet takes the form

\[
<C | \hat{\varphi}_k(\eta) | C> = \frac{e^{-i\theta_k u_k(\eta)} - e^{i\theta_k u_k^*(\eta)}}{e^{-i\theta_k \alpha_k} - e^{i\theta_k \alpha_k^*}} \int (C(P_q))^* P^*_k C(P_q) D P_q D P^*_q
\]

\[
- (\alpha_k u^*_k(\eta) - u_k(\eta) \alpha_k^*) \int (C(P_q))^* \frac{\partial}{\partial P_k} C(P_q) D P_q D P^*_q,
\]

instead of the expression (18).

Let us consider the transformation of the wave packet $C(P_k) \rightarrow C(P_k) \exp \left(-i \sum_q g_q P^*_q P_q \right)$ in the Eq. (18), where $g_k$ is some real constant. It could be seen from (18),(20) that the transformation above is equivalent to the appearance the phase $\theta_k$ satisfying to the equation

\[
e^{i\theta_k} e^{-i\theta_k \alpha_k} - e^{i\theta_k \alpha_k^*} = 1 \alpha_k - \alpha_k^* - ig_k \alpha_k^*.
\]

That is, the phases $\theta_k$ have to be considered twofold: mathematically, they are the phases of the basis functions $u_k(\eta)$, but physically, they are the property of the quantum state, because they are equivalent to the constants $g_k$.

### 3. Quasi-Heisenberg picture for matter appearance

As a gravity quantization is a too complicated issue, let us consider a toy quantum model based on an analogy with the Gowdy cosmological model [14], where the Hamiltonian contains the gravitational degrees of freedom in the first order on momentums. To exclude the quantum field back-reaction to the universe expansion, the heuristic Hamiltonian could be written as

\[
H = -p_a f(a) + \sum_k \frac{\pi_k \pi_{-k}}{2a^2} + \frac{1}{2} a^2 k^2 \varphi_k \varphi_{-k},
\]

where $f(a)$ is some arbitrary function of the scale factor, $\pi_k$ is the momentums corresponding to $\varphi_k$. Let us consider Eq. (22) not only as a Hamiltonian but simultaneously as a constraint $H = 0$, again, by analogy with the Gowdy model.

As one can see, the Hamiltonian (22) contains momentum $p_a$ corresponding to the universe scale factor $a$, as it occurs in the typical minisuperspace models. However, it has a first-order degree like the Gowdy model [14]. Note that the Hamiltonian (22) is purely heuristic and differs from that coming from the scalar constraint of GR, which is second-order in momentums.

The Hamilton equations result in the equation of motion for the scale factor

\[
a' = -\frac{\partial H}{\partial p_a} = f(a).
\]

The scale factor is classical and given by the solution of Eq. (23)

\[
\int_{a_0}^{a(\eta)} \frac{d\varsigma}{f(\varsigma)} = \eta,
\]
so that \( a(0) = a_0 \). For instance, \( f(a) = \text{const} = \mathcal{H} \) gives
\[
a(\eta) = \mathcal{H}\eta + a_0, \tag{25}\]
where \( a_0 \) is the initial value of the scale factor at \( \eta = 0 \). Namely, this particular case corresponding to the radiation domination universe will be considered below for illustration.

Let us sketch the quantization scheme in the quasi-Heisenberg picture described in [11–14]. It consists of the quantization of the classical equations of motion, i.e., one should write “hats” under every quantity in the equations of motion. Then, one has to define the commutation relations for the operators, to chose the operator ordering in the equations, and finally to define the Hilbert space, where the operators act.

The equations of motion for the scalar field is written as
\[
\varphi'_{\Omega} = \frac{\partial H}{\partial \pi_{\Omega}^*} = \frac{\pi_{\Omega} - a^2 \varphi_{\Omega}}{a^2}, \quad \pi_{\Omega}' = -\frac{\partial H}{\partial \varphi_{\Omega}} = -a^2 k^2 \varphi_{-\Omega}. \tag{26}\]
The resulting equation of motion originates from (26):
\[
\varphi''_{\Omega} + \frac{2a'}{a} \varphi'_{\Omega} + k^2 \varphi_{\Omega} = 0. \tag{27}\]

The solution of Eq. (27) is convenient to write through the functions \( u_k \) satisfying Eqs. (4) and (5):
\[
\hat{\varphi}_k(\eta) = i \left( u_k(\eta) \left( \hat{P}_k^+ u_k^*(0) - a^2 \hat{\Phi}_k u_k^*(0) \right) - u_k^*(\eta) \left( \hat{P}_k^+ u_k(0) - a^2 \hat{\Phi}_k u_k^*(0) \right) \right), \tag{28}\]
where the operators \( \hat{P}_k \) and \( \hat{\Phi}_k \) do not depend on time and satisfy the commutation relations \([\hat{P}_k, \hat{\Phi}_q] = -i \delta_{k,q}\). They are the initial values of the operators \( \hat{\pi}_k(\eta) \) and \( \hat{\varphi}_k(\eta) \) at \( \eta = 0 \). In the momentum representation \( \hat{P}_k = P_k, \hat{\Phi}_k = i \frac{\partial}{\partial P_k} \). It is not difficult to see that \( \hat{\pi}_k(\eta) \) and \( \hat{\varphi}_k(\eta) \) have the commutation relation \([\hat{\pi}_k, \hat{\varphi}_q] = -i \delta_{k,q}\) due to Eq. (5).

Momentum \( p_a \) is expressed from the constraint \( H = 0 \) as
\[
\hat{p}_a = \frac{1}{f(a)} \left( \sum_k \hat{\pi}_k \hat{\pi}_k^* + \frac{1}{2} a^2 k^2 \varphi_{-k} \varphi_{-k}^* \right), \tag{29}\]
where it is taken into account that \( \varphi_{-k} = \varphi_{k}^* \).

According to the quasi-Heisenberg scheme, the next step is to build the Hilbert space where the quasi-Heisenberg operators act. That can be done with the help of the Wheeler-DeWitt equation in the vicinity of the small scale factors \( a \sim a_0 \to 0 \). At this stage anew, we quantize the Hamiltonian constraint canonically [11–14], which gives in the momentum representation
\[
if(a) \partial_a \psi(a, \{ \hat{P}_k, \hat{P}_k^* \}) = \left( \frac{1}{a^2} \sum_{q, q_0 > 0} P_q P_{q_0}^* \right) \psi(a, \{ \hat{P}_k, \hat{P}_k^* \}), \tag{30}\]
where only half-space of the wave vectors \( q \) is taken by condition \( q_z > 0 \) to avoid a double counting originating from \( P_{-k} = P_k^* \). The curly bracket in (30) denotes a set of \( P_{k_1}, P_{k_1}^*, P_{k_2}, P_{k_2}^*, \ldots \). The solution of Eq. (30) is
\[
\psi(a, \{ \hat{P}_k, \hat{P}_k^* \}) = \exp \left( -i \sum_{q, q_0 > 0} P_q P_{q_0}^* \left( \int \frac{da}{a^2 f(a)} + \Theta_q \right) \right) C(\{ \hat{P}_k, \hat{P}_k^* \}), \tag{31}\]
where $\Theta_k$ is an arbitrary real constant arising as a result of integration and $C(\{P_k, P_k^*\})$ is the momentum wave packet defining the quantum state of the model and containing all the information about it.

The calculation of the mean values \cite{14} includes the integration over $\mathcal{D}P_k, \mathcal{D}P_k^*$ on the hypersurface $a = a_0$ and proceeding the limit $a_0 \to 0$:

$$
< \psi | \hat{A} \left( \eta, a, \left\{ P_k, i \frac{\partial}{\partial P_k}, P_k^*, i \frac{\partial}{\partial P_k^*} \right\} \right) | \psi > = \\
\int \psi^*(a, \{P_k, P_k^*\}) \hat{A}(a, \{P_k, P_k^*\}) \mathcal{D}P_k \mathcal{D}P_k^* \bigg|_{a=a_0 \to 0}.
$$

(32)

In particular, the mean value calculation $\hat{\varphi}_k(\eta)$ leads to

$$
< \psi | \hat{\varphi}_k(\eta) | \psi > = i (a_0^2 (I(a_0) + \Theta_k) (u_k'(0)u_k^*(\eta) - u_k(\eta)u_k'^*(0)) \\
+ u_k'(0)u_k(\eta) - u_k(0)u_k^*(\eta)) \int (C(\{P\})^*)^* P_k C(\{P\}) \mathcal{D}P_q \mathcal{D}P_q^*
\\
- a_0^2 (u_k'(0)u_k^*(\eta) - u_k(\eta)u_k'^*(0)) \int (C(\{P\})^*) \frac{\partial}{\partial P_k} C(\{P\}) \mathcal{D}P_q \mathcal{D}P_q^* \bigg|_{a=a_0 \to 0},
$$

(33)

where the indefinite integral is defined as $I(a) = \int \frac{1}{\sqrt{f(a)}} da$ and $C(\{P_q, P_q^*\})$ is implied under $C(\{P\})$.

It should be noted that, generally, the quasi-Heisenberg quantization scheme holds also in a more general case, when the background geometry is quantum \cite{11–13} and the Klein-Gordon “current” scalar product appears instead of the Schrödinger-like product in (32). However, Eq. (32) is not a pure Schrödinger “density” scalar product (i.e., more exactly, an expression for the mean value calculation) even in the present case, but also it includes hyperplane $a = a_0 \to 0$ explicitly.

As may see, the resulting Eq. (33) is analogous to Eq. (18). Considering the functions $u_k(\eta)$ in (18) as a limit $a_0 \to 0$ of the functions $u(\eta, a_0)$ in (33) allows concluding that these formulas coincide exactly if the constants $\Theta_k$ are defined as

$$
\Theta_k = \frac{i}{2} \frac{1 + 2 u_k'(0, a_0) \alpha_k}{|\alpha_k|^2} - I(a_0) \bigg|_{a_0 \to 0},
$$

(34)

where we conventionally define the functions $u_k(\eta, a_0)$ in such a way that $\alpha_k/|\alpha_k| = i$.

Let’s come to a concrete example of (25), where $I(a_0) = \frac{1}{\hbar a_0}$ and the functions are

$$
\begin{align*}
& u_k(\eta, a_0) = - \frac{i \sqrt{a_0^2 k^2 + \mathcal{H}^2}}{\sqrt{2k(a_0 + \eta \mathcal{H})(\mathcal{H} + ia_0 k)}} e^{-ik\eta}, \\
& u_k^*(\eta, a_0) = \frac{i \sqrt{a_0^2 k^2 + \mathcal{H}^2}}{\sqrt{2k(a_0 + \eta \mathcal{H})(\mathcal{H} - ia_0 k)}} e^{ik\eta},
\end{align*}
$$

(35)

that are in the limit of $a_0 \to 0$

$$
\begin{align*}
& u_k(\eta) = - \frac{i}{\sqrt{2k \eta \mathcal{H}}} e^{-ik\eta}, \\
& u_k^*(\eta) = \frac{i}{\sqrt{2k \eta \mathcal{H}}} e^{ik\eta},
\end{align*}
$$

(36)
and \( \alpha_k = i\mathcal{H}^2/\sqrt{2k} \).

The evaluation of (34) gives \( \Theta_k = 0 \). Further calculations show that two formalisms are fully equivalent not only for the mean value of \( \hat{\varphi}_k(\eta) \), but for other operators mean values, as well.

4. Energy density spectrum of the created particles

Although the field oscillators do not oscillate near \( \eta \sim 0 \), the information about a matter is encoded at the singularity, whereas the particle density becomes well-defined later when the notion of a “particle” takes shape.

Let us consider a Gaussian wave packet

\[
C(\{P\}) = N \prod_q \exp\left(-\Delta_q P_q^* P_q\right),
\]

where \( N \) is a normalizing constant and the function \( \Delta_q \) has both real and complex parts \( \Delta_q = \Delta'_q + i\Delta''_q \). The quantities \( \Delta'_q \) should be positive to provide the integral convergence.

The mean energy density of the created scalar particles can be defined as

\[
\bar{\rho} = \langle C(\{P\})|\hat{\rho}|C(\{P\})\rangle - \langle 0|\hat{\rho}|0 \rangle,
\]

where

\[
\hat{\rho} = \frac{1}{V} \int_V \left(\frac{\hat{\varphi}'^2}{2a^2} + \frac{(\nabla \hat{\varphi})^2}{2a^2}\right) d^3r = \frac{1}{2a^2} \sum_k \hat{\varphi}_k^* \hat{\varphi}_k' + k^2 \hat{\varphi}_k \hat{\varphi}_k' \equiv \sum_k \hat{\varphi}_k
\]

and the vacuum energy density is

\[
\tilde{\rho} \equiv \langle 0|\hat{\rho}|0 \rangle = \frac{1}{a(\eta)^4} \left(\sum_k \frac{k^2}{2} + \frac{1}{4k^2}\eta^2\right) \equiv \sum_k \hat{\varphi}_k.
\]

The definition (38) includes both some state \(|C\rangle\) set at singularity and a late time vacuum state \(|0\rangle\). The last one is an ordinary vacuum state \( a_k|0\rangle = 0 \), but, as shown later, this state corresponds to some particular wave packet defined at the singularity that does not produce particles in the future. It could be said that everything is planted at the singularity. However, to understand what will grow from this state, one needs to have a late times “measure,” i.e., a well-defined vacuum. Here, a purely classical background is considered, which allows finding an analog of the vacuum state in the form of the wave packet defined at the singularity. In the case of the common quantum gravity, it is possible to define only the approximate vacuum state at the late time when the background becomes approximately classical.

According to Eq. (39), the mean energy density consists of the sum over every wave mode, and the quantity \( \varphi_k \) can be considered as the spectral energy density of the created particles. By turning from summation to integration, one comes to

\[
\bar{\rho} \equiv \sum_k \tilde{\varphi}_k = \frac{1}{2\pi^2} \int \tilde{\varphi}_k k^2 dk,
\]
Cosmological singularity as an informational seed for Everything

Figure 1. Spectral density of the particles $\bar{\rho}_k = \bar{\rho}_k/k$ appeared at the different moments of the conformal time for some model functions $\Delta''_k$ and $\Delta'_k$ given by (44). (a) $L = 10^{-2}/\mathcal{H}$, $\Omega = 1$, (b) $L = 10^{-2}/\mathcal{H}$, $\Omega = 10^4$, where $\Omega$ and $L$ are parameters describing wave packet (44). Dashed, dashed-dotted lines and solid line corresponds to $\eta = 0.05$, $\eta = 0.1$ and $\eta = 1$, respectively.

Let us expose the results on the mean value calculation of the energy density (39) for the wave packet (37) and the dependence $a(\eta) = \mathcal{H}/\eta$. The method of the second section suggests using Eqs. (14), (16), (36). Since the wave packet (39) represents the product over $k$-terms, the calculation of the energy density, which is a sum over the modes, can be done for each $k$-mode separately. The integration over $dP_k d\bar{P}_k$ is understood like that in the holomorphic representation [37]:

$$dP_k d\bar{P}_k \equiv \rho_k d\rho_k d\theta_k,$$

where $P_k = \rho_k e^{i\theta_k}$.

The method of the third section includes Eqs. (25), (28), (31), (32), (35) and the limit of $a_0 \to 0$. It turns out to be that the different terms, which diverge at $a_0 \to 0$, cancel each other in this limit. This topic was discussed earlier in [12].

Both methods give the same result:

$$a^4 \bar{\rho} = \frac{1}{8} \eta^{-2} \mathcal{H}^{-2} \sum_k k^{-2} \Delta'^{-1}_k \left( (2\eta^2 k^2 + 1) \left( \Delta''_k \mathcal{H}^4 + (k - \Delta'_k \mathcal{H}^2)^2 \right) + 2k \sin(2\eta k) \left( \eta k^2 - \mathcal{H}^2 (\eta \mathcal{H}^2 |\Delta_k|^2 + \Delta''_k) \right) \right) \cos(2\eta k) \left( k^2 (4\eta \Delta''_k \mathcal{H}^2 + 1) - \mathcal{H}^4 |\Delta_k|^2 \right).$$

(42)

At the late times $\eta \to \infty$, the oscillations decay and

$$a^4 \bar{\rho} \equiv a^4 \sum_k \bar{\rho}_k \approx \sum_k \frac{\mathcal{H}^4 |\Delta_k|^2 - 2\mathcal{H}^2 k \Delta'_k + k^2}{4 \Delta''_k \mathcal{H}^2},$$

(43)

which turns to zero at $\Delta'_k = k/\mathcal{H}^2$, $\Delta''_k = 0$ in the agreement with (42). Thus, the particles do not appear this value of $\Delta_k$, and, thereby, it corresponds to a vacuum state.

Let us consider some numerical example and take

$$\Delta'_k = \sqrt{k^2 + \mathcal{H}^4 \Delta''_k^2}, \quad \Delta''_k = \frac{\Omega k}{\mathcal{H}^2} \exp (-kL),$$

(44)

where $\Omega$ and $L$ are some constants. Parameter $\Omega$ increases oscillatory component of the wave packet, which leads to increasing of the number of the created particles. The spectral density of the particle number for this illustrative example is shown in Fig. 1. The number of the created particles increases with the increasing of $\Omega$ as it is shown in Fig. 1.
The spectral density, as can be seen, is not always positive at the early time, because the notion of “particle” is not well-defined yet, but this quantity becomes positive later.

The mathematical aspect of the calculations requires some additional comments. Integration $\mathcal{D}P_q\mathcal{D}P^*_q \equiv dP_{k_1}dP^*_{k_1}dP_{k_2}dP^*_{k_2}...$ represents, in fact, a continual integration without a rigorously defined measure‡. There are two possibilities to proceed with these integrals. The first possibility is to consider these integrals as finite-dimensional formally, i.e., look at $|k|$ as a quantity confined by some $k_{\text{max}}$. That allows calculating the quantities, which are zero in a vacuum state by definition, and non-zero in the “excited” states. The density of the created particles given by (38) is such a quantity. No dependence on $k_{\text{max}}$ appears for mean values of such quantities because this consideration is, in fact, equivalent to that using the Fock space and the normal operators ordering.

The second opportunity supported by cosmological arguments [43] is to consider that $k_{\text{max}}$ exists really and is of the order of the Planck mass. That allows calculating the arbitrary quantities, which are nonzero in a vacuum state. The UV cut-off will figurate in the final result of these calculations [43].

5. Informational content of a singularity

Although the word “information” is popular in different science branches, the notion of information is not purely physical. The most interrelated notion is the “entropy,” which is defined for an assembly of the systems and equals zero for a pure quantum state [44]. Saying about “Everything,” we imply that only one single “Everything” exists, and it is a single universe being in a single quantum state, i.e., its entropy equals zero. Thus, other definition for the “information content” is required.

A basic assumption could be accepted that a vacuum state contains no information. The next point for introducing the information could be the formula by Kullback-Leibler [45] comparing two probability distributions of a random variable in the information theory. In fact, it is a measure in the functional space of the functions $\bar{\varphi}_k$.

One may use the mean value of the spectral energy density $\bar{\varphi}_k$ (43) of the created particles and, from the other hand, use the vacuum value (see (38), (40) and (41)) $\tilde{\varphi}_k$ for the normalization. Thus, the following prescription for the information density can be taken:

$$I = \sum_k \frac{\bar{\varphi}_k}{k} \ln \left(1 + \frac{\bar{\varphi}_k}{\tilde{\varphi}_k}\right) = \frac{1}{2\pi^2} \int_0^\infty \frac{\bar{\varphi}_k}{k} \ln \left(1 + \frac{\bar{\varphi}_k}{\tilde{\varphi}_k}\right) k^2 dk.$$  (45)

As can see, it is the distance in a functional space between the function $\bar{\varphi}_k$ and some singled out function $\tilde{\varphi}_k = 0$, whereas the $\bar{\varphi}_k$ given by Eq. (40) does not correspond to the real particles, but rather to virtual ones. A logarithmic scale is used in (45) like the star luminosity in astrophysics or the decibels in acoustics, and besides, the definition of the entropy in statistical physics. Having the density of created particles

‡ Nevertheless, that does not prevent to deal formally with the Gaussian continual integrals [37].
Figure 2. Amount of information per one particle, depending on the parameter of the wave packet (44).

\[ \bar{n} = \sum_{k} \frac{\bar{\varphi}_k}{k}, \]  

(46)

the information \( I/\bar{n} \) per one created particle can be calculated. Substitution of the values (44) into (43) results in

\[ a^4 \bar{\varphi}_k = \frac{k}{2} \left( \sqrt{1 + \Omega^2 e^{-2kL}} - 1 \right). \]  

(47)

Using (47) and the asymptotic vacuum value of \( a^4 \bar{\varphi}_k \approx k/2 \) (see Eq. (40)) gives the particle density and the information density in a final form

\[ a^4 \bar{n} = \frac{1}{4\pi^2} \int_{0}^{\infty} \left( \sqrt{1 + \Omega^2 e^{-2kL}} - 1 \right) k^2 dk, \]  

(48)

\[ a^4 \mathcal{I} = \frac{1}{8\pi^2} \int_{0}^{\infty} \left( \sqrt{1 + \Omega^2 e^{-2kL}} - 1 \right) \ln \left( 1 + \Omega^2 e^{-2kL} \right) k^2 dk. \]  

(49)

Roughly, the particle density is of the order of \( \bar{n} \sim \frac{\Omega}{a^4 L^3} \), whereas the information density is \( \mathcal{I} \sim \frac{\Omega \ln \Omega}{a^4 L^3} \). In the example considered, an informational content per created particle increases only logarithmically with \( \Omega \) as shown in Fig. 2. Although the particles’ density could be increased by decreasing \( L \), this does not increase information content per particle.

Here we consider only some simple class of the quantum states and a straightforward definition of the information. In principle, other definitions of information are of interest. Let us propose to Reader the information formula accounting for quantum states explicitly:

\[ \mathcal{I} = \int \left( \frac{C(\{P\}) \ln \frac{C(\{P\})}{\mathcal{C}(\{P\})}}{C(\{P\})} \right)^{\star} \sum_{k} \frac{\hat{\varphi}_k}{k} C(\{P\}) \ln \frac{C(\{P\})}{\mathcal{C}(\{P\})} \mathcal{D}_P \mathcal{D}_q \mathcal{D}_q^{\star}, \]  

(50)

where \( \mathcal{C}(\{P\}) \) is the wave packet (37) with \( \Delta'_{k} = k/\mathcal{H}^2 \), \( \Delta''_{k} = 0 \) producing no real particles in a future asymptotically. On the one hand, the formula (50) uses the Kullback-Leibler idea but compares the quantum states with the state \( \mathcal{C}(\{P\}) \) giving no particles. On the other hand, it contains features of the expression for the appeared
particles' mean density:
\[ \bar{n} = \int \left( (C\{P\})^* \sum_k \hat{\phi}_k \bar{C}\{P\} - \left( \hat{C}\{P\} \right)^* \sum_k \hat{\phi}_k \hat{C}\{P\} \right) \mathcal{D}P_q \mathcal{D}P_q^*. \] (51)

The calculations, along with the definition (50) seem more complex, and we defer the examination of this issue to a later date.

6. Conclusions and Discussion

To summarize, we considered an illustrative example within a comparatively simple approach illustrating how the information about “Everything” could be stored at the singularity. We demonstrated that the momentum wave packet could be defined at the cosmological singularity so that: 1) some amount of matter can be “placed” at the singularity, and, thereby, 2) some information can be encoded into it. It is not creating the particles from vacuum [38–41], which is widely considered at 1960th. This phenomenon produces a very low amount of matter for the power-law of universe expansion, including the linear expansion in cosmic time [42]. On the other hand, a vacuum could be defined only after the moment when the field oscillators begin to oscillate, which is relatively far from the singularity. In contrast, in the approach considered, it seems evident that one could place any amount of matter and information into the singularity.

In the second section, the pure Heisenberg picture was considered in which a Heisenberg operator evolves under some time-independent state. This state has been constructed (or “enumerated”) in terms of momentum operators existing at the singularity. It allows claiming that the state is set at the singularity and determines the universe’s matter and informational content.

The quasi-Heisenberg picture of the third section is more drastic because the hyperplane \( a = 0 \), figuring in the mean values’ definition, points that a state is set directly at the singularity. It occurs that both formalisms are equivalent in the framework of the simplified model defined on the classical background. In the general quasi-Heisenberg picture of quantum gravity, the quantum state would inevitably be set at the singularity due to the hyperplane \( a = 0 \) in the Klein-Gordon scalar product.

In this work, the problem of the source of matter and information is considered for the relatively narrow class of the theories with singularity and without inflation. It is interesting to discuss a broader class of theories. For instance, the loop-quantum gravity (LQG) is based on the version of the Wheeler-DeWitt equation and operates mainly in a timeless manner [46, 47], i.e., the time is recovered only quasi-classically, or not used at all by considering the transition amplitudes. Besides, the “bounces” arise on the quasi-classical level, which excludes singularity. In such a case, a state determining matter and information in the universe is not connected with the singularity or with some moment generally, and exists ”elsewhere.” Due to the combinatorial features of LQG, it could be interesting to consider some definitions of the information for such a
Cosmological singularity as an informational seed for Everything

state of the universe. It should be noted that a similar problem of the entropy of a black hole is was solved in LOQ [47], considering a black hole as an open system connected with the environment through the horizon.

The discussion of inflation within the above context is also interesting because the initial inhomogeneities determining the universe’s history originate from vacuum fluctuations. At the same time, the enormous expansion during inflation hides the previous universe’s history. In some sense, inflation is analogous to singularity because field oscillators do not oscillate near a singularity, at the same time, when the scale of inhomogeneity becomes greater horizon during inflation, the field oscillators cease to oscillate also. Then, during the inflationary universe expansion, the inhomogeneities cross the horizon (i.e., enter into the causality-connected region) and begin to oscillate again. It is interesting to note, that in the Milne-like cosmologies [26–34] the events always remain within the horizon.

References

[1] Penrose R 1965 Phys. Rev. Lett. 14 57–59
[2] Geroch R 1968 Ann. Phys. NY 48 526–540
[3] Hawking S and Penrose R 1970 Proc. Roy. Soc. Lond. A 314 529–548
[4] Minkevich A V 2006 Grav. Cosmol. 12 11–20
[5] Brandenberger R and Peter P 2017 Found. Phys. 47 797–850
[6] Ashtekar A, Pawlowski T and Singh P 2006 Phys. Rev. D73 124038
[7] Bojowald M 2008 Gen. Rel. Grav 40 2659–2683
[8] Singh P 2009 Class. Quant. Grav. 26 125005
[9] Reichenbach H 1999 The Direction of Time (Dover: Dover Publications)
[10] Cherkas S L and Kalashnikov V L 2020 Illusiveness of the problem of time (Preprint 2005.06917)
[11] Cherkas S L and Kalashnikov V L 2006 Grav. Cosmol. 12 126–129
[12] Cherkas S L and Kalashnikov V L 2012 Gen. Rel. Grav. 44 3081–3102
[13] Cherkas S L and Kalashnikov V L 2015 Nonlin. Phen. Compl. Syst. 18 1–14
[14] Cherkas S L and Kalashnikov V 2017 Theor. Phys. 2 124
[15] Starobinsky A A 1980 Phys. Lett. 91B 99–102
[16] Guth A 1981 Phys. Rev. D 23 347–356
[17] Liddle A R and Lyth D H 2000 Cosmological inflation and large-scale structure (Cambridge: Univ. Press, Cambridge)
[18] Lerner E 2004 New Scientist 2448 20
[19] Linde A 1990 Particle physics and inflationary cosmology (Claur: Harwood Academic Publishers)
[20] Mukhanov V 2005 Physical foundations of cosmology (Cambridge: Univ. Press)
[21] Brandenberger R H and Martin J 2001 Mod. Phys. Lett. A 16 999–1006
[22] Martin J and Brandenberger R 2001 Phys. Rev. D 63 123501
[23] Danielsson U H 2002 Phys.Rev. D 66 023511
[24] Agullo I 2018 Gen. Rel. Grav. 50 91
[25] Brandenberger R 2008 Physics Today 61 44
[26] Milne E 1935 Relativity, Gravitation and World-Structure (Oxford: The Clarendon Press)
[27] John M V and Joseph K B 1996 Phys. Lett. B387 466–470
[28] Dev A, Safonova M, Jain D and Lohiya D 2002 Phys.Lett. B548 12–18
[29] Cherkas S L and Kalashnikov V L 2008 Universe driven by the vacuum of scalar field: VFD model Proc. Int. conf. “Problems of Practical Cosmology”, Saint Petersburg, Russia, June 23 - 27, 2008 pp 135–140
Cosmological singularity as an informational seed for Everything

[30] Benoit-Lévy A and Chardin G 2014 *Int. J. Mod. Phys.: Conf. Ser* **30** 1460272
[31] Melia F 2015 *MNRAS* **446** 1191–1194
[32] Lewis G F, Barnes L A and Kaushik R 2016 *MNRAS* **460** 291–296
[33] Singh G and Lohiya D 2018 *MNRAS* **473** 14–19
[34] Cherkas S L and Kalashnikov V L 2018 Plasma perturbations and cosmic microwave background anisotropy in the linearly expanding Milne-like universe *Fractional Dynamics, Anomalous Transport and Plasma Science* ed Skiadas C H (Cham: Springer) chap 9, pp 181–201
[35] Birrell N D and Davis P 1982 *Quantum fields in curved space* (Cambridge: Univ. Press)
[36] Cherkas S L and Kalashnikov V 2017 *Proc. Natl. Acad. Sci. Belarus, Ser. Phys.-Math.* **4** 88–97
[37] Faddeev L D and Slavnov A A 1987 *Gauge fields: an introduction to quantum theory* (London & New York: Addison-Wesley)
[38] LP 1969 *Phys. Rev.* **183** 1057–1068
[39] Sexl R U and Urbantke H 1969 *Phys. Rev.* **179** 1247–1250
[40] Zel’dovich Y and Starobinsky A 1972 *Sov. Phys.-JETP* **34** 1159–1166
[41] Grib A A and Mamaev H 1970 *Sov. J. Nucl. Phys* **10** 722
[42] Anischenko S V, Cherkas S L and Kalashnikov V 2010 *Nonlin. Phen. Compl. Syst* **13** 315–319
[43] Cherkas S L and Kalashnikov V L 2007 *J. Cosmol. Astropart. Phys.* **01** 028
[44] Kak S 2007 *Int. J. Theoretical Phys* **46** 860
[45] Kullback S and Leibler R 1951 *The Annals of Mathematical Statistics* **22** 79–86
[46] Rovelli C and Vidotto F 2014 *Covariant Loop Quantum Gravity* (Cambridge: Cambridge Monographs on Mathematical Physics, Cambridge University Press)
[47] Casares P 2018 *An review on Loop Quantum Gravity*. Ph.D. thesis Kellogg College University of Oxford Oxford