Response of the two-dimensional kinetic Ising model under a stochastic field

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Received 7 June 2013
Accepted 31 October 2013
Published 27 November 2013

Online at stacks.iop.org/JSTAT/2013/P11015
doi:10.1088/1742-5468/2013/11/P11015

Abstract. We study, using Monte Carlo dynamics, the time ($t$) dependent average magnetization per spin $m(t)$ behavior of the 2D kinetic Ising model under a binary ($\pm h_0$) stochastic field $h(t)$. The time dependence of the stochastic field is such that its average over each successive time interval $\tau$ is assured to be zero (without any fluctuation). The average magnetization $Q = (1/\tau) \int_0^\tau m(t) \, dt$ is considered as an order parameter of the system. The phase diagram in ($h_0$, $\tau$) plane is obtained. Fluctuations in the order parameter and their scaling properties are studied across the phase boundary. These studies indicate that the nature of the transition is Ising like (static Ising universality class) for field amplitudes $h_0$ below some threshold value $h^*_0(\tau)$ (dependent on $\tau$ values; $h^*_0 \to 0$ as $\tau \to \infty$ across the phase boundary). Beyond these $h^*_0(\tau)$, the transition is no longer continuous.

Keywords: classical Monte Carlo simulations, finite-size scaling, kinetic Ising models
1. Introduction

The study of the kinetic Ising system under a time varying magnetic field has already been an important research area of non-equilibrium statistical physics [1]–[3]. In this context, many studies were made where a periodic time varying magnetic field was applied in the kinetic Ising system and it was observed that a symmetry breaking dynamic transition (in the response magnetization) takes place depending upon the magnitudes of frequency and amplitude of the applied field [4, 5]. Extensive Monte Carlo simulations have been done [6, 3, 7, 8] to estimate the different critical exponents for this dynamic transition. It appears (see e.g., [3]) that the nature of dynamic transition is Ising like (continuous) up to a certain value of field amplitude and frequency after which the transition becomes first order. Of course, some of the later studies suggested [9]–[11] this to be a finite size effect. The same model was also studied by applying field pulses [12]–[15].

In this paper we will discuss the behavior of the kinetic Ising model under a stochastic field (random field in time; \( h(t) \)). The kinetic Ising model under a random field has also been addressed earlier by taking different types of distributions of external magnetic field [3, 16, 17], though its transition behavior has not been analyzed systematically. Here we will investigate the response of the kinetic Ising model under a binary stochastic field \( \pm h_0 \). We study the model by taking the stochastic field such that within every successive time interval \( \tau \), the total field applied on the system is zero (without any fluctuation: \( \int_{n\tau}^{(n+1)\tau} h(t) \, dt = 0 \) for any integer value of \( n \)). We did Monte Carlo studies for different values of \( \tau \). We observe that a continuous dynamic phase transition takes place for small values of field amplitude \( h_0 \) and small \( \tau \) values with Ising-like scaling behavior and exponent values. However, for higher values of \( \tau \) or field amplitude \( h_0 \), the nature of transition does not remain continuous.

doi:10.1088/1742-5468/2013/11/P11015
Specifically, we have studied the response magnetization \( m(t) \) of the system for stochastic fields \( h(t) \) and define an order parameter \( Q = (1/\tau) \int_0^\tau m(t) \, dt \) (where \( m(t) \) is the average magnetization per spin). We study the fluctuation behavior and scaling properties for different ranges of \( \tau \) values. In particular, we study the variation of the Binder cumulant \( U_L = 1 - (\langle Q^4 \rangle_L/3\langle Q^2 \rangle_L^2) \), where \( \langle \cdots \rangle \) denotes the thermal average in the steady state) and the fluctuation or susceptibility \( \chi_L = (\langle Q^2 \rangle_L - \langle Q \rangle_L^2)/T \); \( T \) denotes the temperature and \( k_B \) is the Boltzmann factor) at different values of system size \( L \) and different values of \( h_0 \) and \( \tau \).

We find that up to a threshold value of \( h_0 \) (\( h_0^c(\tau) \), dependent on \( \tau \) the crossing points of \( U_L \) for different \( L \) values match with the extrapolated peak position in \( \chi \) and the crossing point value \( U^* \) of the Binder cumulant compares well with that of the pure static Ising value, indicating a continuous transition with identical universality class. Scaling behavior of \( \chi \) also suggests that. Beyond the \( h_0^c(\tau) \) value, however, a clear crossover takes place. The phase diagram, giving \( h_0^c(\tau) \), is obtained (\( h_0^c(\tau) \to 0 \) as \( \tau \to \infty \)). Beyond the crossover, the Binder cumulant scaling behavior suggests an immediate drop from its complete order value seeming to indicate a first order like transition. The same is suggested by measurement of the susceptibility peak values \( \chi_{max} \) for different system sizes \( (L) \), giving \( \chi_{max} \sim L^d \) \((d \text{ denotes dimension of the lattice)}\). However the estimated value of correlation length exponent \( \nu \) seems to be quite high, indicating this discontinuous transition from the ordered phase, beyond \( h_0^c(\tau) \), to be a ‘glass like’ dynamically frozen phase \([17, 18] \).

2. The model and Monte Carlo simulation

We consider a two-dimensional \((L \times L \text{ on a square lattice)}\) kinetic Ising model with periodic boundary conditions. The Hamiltonian of the system can be written as

\[
H = - \sum_{\langle ij \rangle} J_{ij} s_i s_j - h(t) \sum_i s_i
\]

where \( J_{ij} \) is the interaction strength between the \( i \)th and \( j \)th spins (here we take \( J_{ij} = 1 \)), \( s_i = \pm 1 \) for any \( i \)th spin, \( \langle ij \rangle \) indicates the nearest-neighbor pairs and \( h(t) \) is the field applied on the system. We consider time dependent stochastic field \( h(t) \) which varies stochastically over time, which in our case takes the values \( \pm h_0 \) with same probability. In order to avoid the case with a net average of \( h(t) \) due to fluctuation, we choose the field amplitudes in such a way that within a certain time interval \( \tau \) the total field applied in the system is zero: in each period \( \tau \), a series of \( \tau/2 \) number of \( h_0 \) and \( -h_0 \) values are first chosen and then called randomly from the set. This ensure that \( \int_{n\tau}^{(n+1)\tau} h(t) \, dt = 0 \) for all integer values of \( n \). The order parameter of the system can be defined as

\[
Q = (1/\tau) \int_{t=n\tau}^{(n+1)\tau} m(t) \, dt,
\]

averaged over \( n \) \((= 0, 1, 2, \ldots,)\), where \( m(t) = (1/L^2) \sum_i s_i(t) \). In our Monte Carlo simulation, we selected any spin randomly and then it was flipped with rate \( \min[1, \exp\{-\Delta E/k_B T\}] \), where \( \Delta E \) is the change in energy due to the spin flip, \( T \) is the temperature of the system and \( k_B \) is the Boltzmann factor. One Monte Carlo (MC) step is defined as \( L^2 \) spins updated randomly.

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In our simulation, we have taken $L = 32, 64, 128$ and 256 (with periodic boundary conditions) and the initial conditions were either all spins up or down. To check the steady state, we took the averages for some initial time (typically $10^5$ MC steps) and checked if the averages match at least for three to five such successive time intervals. After that, the order parameter, and other parameters, discussed later, were averaged over more than $10^6$ time steps.

3. Results

In figure 1 we show the typical time variations of magnetization $m(t)$ for different values of field amplitude $h_0$ and time interval $\tau$. Transition from the ordered state (with $Q \neq 0$) to the disordered state (with $Q = 0$) as temperature changes is clearly seen. We first study the temperature variation of the order parameter $Q$ for a fixed value of $h_0$ (= 0.4) but for different $\tau$ values. In figure 2(a), the order parameter variations with temperature are plotted for different $L$ values for $\tau = 8$. The same for $\tau = 16$, $\tau = 24$ and $\tau = 100$ are shown in figures 2(b), (c) and (d) respectively.

3.1. Binder cumulant behavior

To investigate the transition, we study the Binder cumulant [1] from the fluctuation of $Q$ for different $L$ values at fixed $h_0$ and $\tau$ values:

$$ U_L = 1 - \frac{\langle Q^4 \rangle_L}{3(\langle Q^2 \rangle_L)^2}. $$

Figure 1. Time variations of average magnetization $m(t)$ in the ordered phase and the disordered phase are shown here. (a) $\tau = 16$ and $h_0 = 0.4 (\langle h_0^2(\tau) \rangle)$; $T = 2.20$ corresponds to the disordered phase while $T = 2.10$ corresponds to the ordered phase. (b) $\tau = 64$ and $h_0 = 0.4 (\langle h_0^2(\tau) \rangle)$; $T = 2.20$ corresponds to the disordered phase while $T = 1.90$ corresponds to the ordered phase. The simulations were done for $L = 128$. 

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The temperature variations of the Binder cumulant for different sizes show a crossing point $U^*(T_c)$ independent of system sizes at the critical point $T_c(h_0, \tau)$ (see e.g., [1, 19]). In figure 3(a) we plot $U_L$ for $L = 32, 64, 128$ and $256$ with $\tau = 8$. From figure 3(a) we see that there exists a crossing point (for different $L$ values) with the value of the cumulant $U^* \simeq 0.61$ at a critical temperature ($T_c \simeq 2.18$ for $h_0 = 0.4$ and $\tau = 8$). Figures 3(b) and (c) correspond to the same results for $\tau = 16$ and $\tau = 24$ respectively. It is again observed that though the critical points change, the Binder cumulant ($U^*$) does not change. Indeed this value of $U^*$ compares well with that for equilibrium Ising model, in 2D. In figure 3(d), we have increased the $\tau$ value further ($\tau = 100$), and here we could not detect a precise crossing point anywhere other than at the cumulant value for perfect order ($U_L = 2/3$). The indication of transition after this perfectly ordered phase suggests a crossover to first order transition in this large $\tau$ limit. Next, we consider a fixed value of $\tau$ but different values of field amplitude $h_0$. For example, we take four different field amplitude values $h_0 = 0.1, 0.4, 0.7$ and $1.0$ with same $\tau$ (=$20$) value. Figure 4 shows the Binder cumulant values for system sizes $L = 32, 64, 128$ and $256$. It is clear that the value of $U^*$ is around 0.61 for smaller field amplitude (for $h_0 \leq 0.4$). But in figure 4(d), where the field amplitude ($h_0$) is taken as $1.0$, the value of $U^*$ is around $2/3$. Therefore, it is an indication that the static Ising universality like transition disappears for higher values of field amplitude $h_0$ as well as time period $\tau$.

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Figure 3. (a) Temperature variations of the Binder cumulant for \( h_0 = 0.4 \) with different \( \tau \) values are shown. (In (a) \( \tau = 8 \), (b) \( \tau = 16 \), (c) \( \tau = 24 \), (d) \( \tau = 100 \).) From the plots, we observe that \( U^* \) is very much dependent on the \( \tau \) values: at higher \( \tau \) values the Binder cumulant crossing point \( U^* \) deflects from the value 0.61 and eventually assumes a value 0.66.

3.2. Phase boundary

We have shown that there are certain ranges of field amplitude \( h^c_0(\tau) \) for fixed values of \( \tau \) below which the nature of the transition is the static Ising universality class. But for \( h_0 > h^c_0(\tau) \) the nature of transition is completely different from static Ising universality. Here we give an effective phase boundary from ordered to disordered transition. Figure 5 shows the phase diagram of the \((h_0, T)\) plane for different \( \tau \) values. For any fixed value of \( \tau \) (=16), there is a range of \( h_0 \) \((h^c_0(\tau) = 0.6)\) values below which the critical Binder cumulant \((U^*)\) is approximately 0.61. For \( h_0 > h^c_0 \), \( U^* \) has a much larger value (than 0.61), seeming to be 2/3, corresponding to complete order. This part of the phase boundary is represented by dashed lines, across which a first order phase transition may occur. We find \( h^c_0(\tau) \to 0 \) as \( \tau \to \infty \) (practically \( h^c_0 \to 0 \) for \( \tau > 60 \)).

3.3. Susceptibility and correlation length behaviors

As mentioned earlier, we also investigated the behavior of fluctuations \((\chi)\) in \( Q \) for different system sizes. We define the susceptibility as

\[
\chi_L = \left( \frac{L^2}{k_B T} \right) \left( \langle Q^2 \rangle_L - \langle |Q| \rangle_L^2 \right)
\]

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Figure 4. Binder cumulants for the same $\tau$ value (=20) but different $h_0$ values are shown in the figures ((a) $h_0 = 0.1$, (b) $h_0 = 0.4$, (c) $h_0 = 0.7$ and (d) $h_0 = 1.0$).

From the plots, we observe that $U^*$ is very much dependent on field values: at higher field values the Binder cumulant crossing point $U^*$ deflects from the value 0.61 and eventually assumes a value 0.66, corresponding to completely ordered phase and seeming to indicate a first order transition to the disordered phase (from complete order).

and fit the data to the scaling form

$$
\chi_L = L^{\gamma/\nu} \chi^0(\epsilon L^{1/\nu}).
$$

where the scaling function $\chi^0$ is an asymptotically defined function and $\epsilon = (T - T_c)/T_c$. We estimated $\chi_L$ for $L = 32, 64, 128$ and 256 for $\tau = 8$ (figure 6(a)), and $\tau = 100$ (figure 6(b)) with the same field amplitude $h_0 = 0.4$. The peak point temperatures compare well with the estimates of $T_c(h_0, \tau)$ obtained from the Binder cumulant crossing point. To estimate the critical exponents of the model for different values of $\tau$, we investigate the scaling behavior of $\chi_L$. As $\chi \sim \xi^{\gamma/\nu}$, where $\xi$ denotes the correlation length which is bounded by a maximum value of $L$ for finite systems, the maximum value of $\chi$ varies as $L^{\gamma/\nu}$.

Figure 7(a) shows a log–log plot of the susceptibility peaks as a function of system sizes for $\tau = 8$ and 100. It is observed that for $\tau = 8$, the $\gamma/\nu$ fits with value $1.75 \pm 0.05$ which is very close to equilibrium Ising exponent value $\gamma/\nu$ [19]. This result again supports that the critical exponents of the transition for smaller values of $\tau$ is close to Ising universality class. However, for larger values of $\tau \geq 100$, the slope fits to different values (around 2.0;
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Figure 5. Dynamic phase boundaries are shown in the figure for $\tau = 8, 16, 32$ and 64. The nature of phase transition across the boundary represented by solid line is the static Ising universality class. For the phase boundary represented by dashed lines, the nature of the transition crosses over (from continuous Ising like) to a discontinuous dynamically frozen spin-glass like phase. The inset shows the crossover regions more clearly. The phase boundaries are obtained by measuring the Binder cumulant crossing point for system sizes $L = 32, 64, 128$ and 256.

Figure 6. The temperature variations of susceptibility for different system sizes are plotted for fixed field amplitude $h_0 = 0.4$ but different $\tau$ values. (a) $\tau = 8$, (b) $\tau = 100$.

not comparing well with Ising universality class value). Again, this perhaps indicates a crossover to a first order transition for large $\tau$ values.

To estimate the correlation length exponent value of $\nu$ independently, we assume the scaling form as

$$T_c(L) = T_c - aL^{-1/\nu} \quad (6)$$

where $T_c$ is the critical point in thermodynamic limit and $T_c(L)$ is the critical point of system size $L$ and $a$ is any constant value. In the figure 7(b) we plot $T_c(L)$ versus $a/L^\nu$ for $\tau = 8$ and 100. We observe that $\nu \simeq 1$ (same as equilibrium 2D Ising exponent value) fits for $\tau = 8$ and $h_0 = 0.4$ very well, while for $\tau = 100$ and $h_0 = 0.4$, we get $\nu \simeq 2.5$ as the best fit value. The estimated values of $\nu$ and $\gamma/\nu$ suggest both that nature of phase
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3.4. Scaling collapse for $h_0 < h_0^c(\tau)$

Our study here indicates that for field amplitude $h_0$ less than a critical value of field amplitude $h_0^c(\tau)$, dependent on $\tau$, the transition belongs to the static Ising universality class. To confirm this, we looked for scaling collapse of data for $Q$ the order parameter near critical point. We assume that the order parameter $Q$ and susceptibility $\chi$ scale near critical point as

$$Q = L^{-\beta/\nu} Q^0(\epsilon L^{1/\nu}), \quad \chi = L^{\gamma/\nu} \chi^0(\epsilon L^{1/\nu}),$$

where the scaling functions $Q^0$ and $\chi^0$ are asymptotically defined functions. We measured the temperature variation of the average values of $Q$ for different system sizes for a given $\tau = 16$ and $h_0 = 0.4$ ($< h_0^c(\tau)$) and looked for a scaling fit. From the data collapse, the estimated exponent values are $\beta/\nu = 0.125 \pm 0.005$ and $\nu = 1.00 \pm 0.02$ (see figure 8(a)). These exponent values fit well with the Ising universality class. We also found data collapse for susceptibility and our estimated exponent values are $\gamma/\nu = 1.75 \pm 0.05$ and $\nu = 2.5 \pm 0.5$ (see figure 8(b)), which are again close to those for the static Ising universality class.

For $h_0 > h_0^c(\tau)$, no such good fit could be obtained (and that too for different exponent values if fitted) and the Binder cumulant data indicated (see e.g., figure 4(d)) a discontinuous drop of the cumulant value from about 0.66 (complete order) to zero as temperature is increased beyond the critical value.

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Figure 8. (a) Data collapse for the order parameter for different system sizes. (b) Data collapse for susceptibility for different system sizes. Both figures are simulated by taking $\tau = 16$ and $h_0 = 0.4 (< h_0^c (\tau))$.

Figure 9. (a) The susceptibilities for different system sizes are plotted. From the figure it is clear that $\gamma/\nu \geq 2$ (assuming $\chi_{\text{max}} \sim L^{\gamma/\nu}$). (b) Here we plot the Binder cumulant for different system sizes. Again from the figure it is clear that the Binder cumulant crossing point $U^* \simeq 0.66$ (which is much different from the static Ising value; it may be a signature of a first order transition).

3.5. Absence of continuous transition for $h_0 > h_0^c (\tau)$

Here we have done the system size analysis for larger sizes where the systems are claimed to be showing first order or glass like transitions ($h_0 = 1.5$ and $\tau = 100$). We plot susceptibility for different system sizes ($L = 32, 64, 128, 256$ and $512$) as shown in figure 9(a) and from the figure it is clear that $\gamma/\nu > 2$. We also plot the Binder cumulant correspondingly and from figure 9(b) a different behavior from a second order transition case is clearly seen.

4. Summary and discussion

Here we have investigated, using Monte Carlo dynamics, a 2D Ising spin system (on a square lattice with periodic boundary condition) under a stochastically varying field (with binary values $\pm h_0$) and the spins are flipped according to Glauber dynamics. Such systems...
were already considered earlier [3, 16, 17]. While the numerical study [16] was not very conclusive, the mean field study [17] indicated several intriguing phases and transitions in the model. In order to set a time scale for the stochastically varying external field, we have time ordered the field in such a way that after every time interval \( \tau \) the total field applied in the system is zero, i.e., \( \int_{t=n\tau}^{(n+1)\tau} h(t) = 0 \) where \( n = 0, 1, 2, \ldots \) and corresponding order parameter is defined as \( Q = (1/\tau) \int_{t=n\tau}^{(n+1)\tau} m(t) \, dt \). To locate the critical point of the system precisely, we have measured the Binder cumulant for different system sizes. We have obtained the phase diagram in the \( h_0, T \) plane for different \( \tau \) values and found out the crossover point between the static Ising transition and non-static Ising transition. For a truly stochastic field \( (\tau \to \infty) \), we find \( h_0^c \) goes to zero. It has been observed that for \( h_0 \) values less than \( h_0^c \), the Binder cumulant \( (U^*) \) is approximately 0.61 (which fits well with corresponding value for static Ising universality class). For \( h_0 \) values greater than \( h_0^c \), the Binder cumulant for different system sizes cross each other at a larger value, which is close to the value for the completely ordered state, indicating a discontinuous transition after that. We have also made a scaling analysis of the fluctuations of the order parameter \( Q \) for this model. We have seen that for \( h_0 \) values less than \( h_0^c(\tau) \), the maximum susceptibility \( (\chi_{\max}) \) scales with \( L^{\gamma/\nu} \) with the scaling exponent \( \gamma/\nu = 1.75 \pm 0.05 \) for different system sizes \( L \), indicating Ising universality behavior. But for \( h_0 > h_0^c(\tau) \) the scaling exponent \( \gamma/\nu \) fits well to a value close to 2.0 = \( d \), seeming to indicate again a first order transition [20, 21]. We also find that the correlation length exponent for the discontinuous transition has an unusually high value \( (\nu \simeq 2.5) \), seeming to indicate a ‘spin-glass’ like [18] ‘frozen’ dynamical [17] phase.

As suggested earlier [10, 11] in the periodic field case [3]–[6], this first order transition behavior (for \( h_0 > h_0^c(\tau) \), \( h_0^c \to 0 \) as \( \tau \to \infty \)) may be a finite size effect. Our investigation, so far, does not indicate of course any such finite size effect. For \( \tau \to \infty \), however, the successive fluctuations will destroy any order and the system is always in a disordered phase.

Acknowledgment

We thank A Chatterjee for useful comments and suggestions. We would also like to thank S Biswas for discussions and critical reading of the manuscript.

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