Equivalence of the measures of non-Markovianity for open two-level systems

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Abstract

In order to depict the deviation of quantum time evolution in open systems from Markovian processes, different measures have been presented. We demonstrate that the measure proposed by Breuer, Laine and Piilo [Phys. Rev. Lett. 103, 210401 (2009)] and the two measures proposed by Rivas, Huelga and Plenio [Phys. Rev. Lett. 105, 050403 (2010)] have exactly the same non-Markovian time-evolution intervals and thus are really equivalent each other when they apply to open two-level systems coupled to environments via Jaynes-Cummings or dephasing models. This equivalence implies that the three measures in different ways capture the intrinsical characters of non-Markovianity of quantum evolulational processes. We also show that the maximization in the definition of the first measure can be actually removed for the considered models without influencing the sensibility of the measure to detect non-Markovianity.

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I. INTRODUCTION

The evolution of open quantum systems can be divided into two basic types, Markovian and non-Markovian processes [1]. For Markovian processes, the correlation time between the system and environment is considered to be infinitesimally small so that the dynamical map does not carry any memory effects, leading to a monotonic flow of the information from system to environment. In contrast, non-Markovian processes with memory have different dynamical traits which give rise to the back flow of information from environment to the system [2, 3]. Recently, people found that non-Markovian processes can lead to distinctly different effects on decoherence and disentanglement [4, 5] of open systems compared to Markovian processes. Many relevant physical systems, such as quantum optical system [1], quantum dot [6], color-core spin in semiconductor [7], could not be described simply by Markovian dynamics. In addition, quantum chemistry [8] and excitation transfer of biological system [9] also need to be treated as non-Markovian processes. Because of these distinct properties and extensive applications, more and more attention and interest have been devoted to the study of non-Markovian processes of open systems, including the measures of non-Markovianty [2, 3, 10–13], the positivity [14–16], and some other dynamical properties [17–20] and approaches [21, 22] of non-Markovian processes. Experimentally, the simulation [23, 24] of non-Markovian environment has been realized.

The measure of non-Markovianty of quantum evolution is a fundamental problem which aims to detect whether a quantum process is non-Markovian and how much degrees it deviates from a Markovian one. So far, almost all measures can only serve to settle the former, i.e., serve as the sufficient (not necessary) condition for the emergence of non-Markovianty. Therefore, the problem for measuring the non-Markovianty of quantum processes still remains elusive and, in some sense, controversial. Based on the distinguishability of quantum states, Breuer, Laine and Piilo (BLP) [2] proposed a measure to detect the non-Markovianty of quantum processes which is linked to the flow of information between system and environment. Alternatively, Rivas, Huelga and Plenio (RHP) [10] also presented two measures of non-Markovianty by exploiting the dynamical behavior of quantum entanglement under the local trace-preserving CP maps. Other measures of non-Markovianty include the one proposed by Wolf et al. [11] based on the breakdown of the semigroup property, the one proposed by Lu et al. [13] using quantum Fisher information flow, and the ones proposed...
by Usha Devi *et al.* [12] using relative entropy difference and fidelity difference. It is worthwhile to stress that these measures of non-Markovianity are generally not the same. Thus, studying and exposing the relations among all these measures under some specific models becomes very important. Haikka *et al.* [25] studied the links between the BLP measure and one of the RHP measures very recently, for a laser-driven qubit system embedded in a structured Lorentzian environment. By algebraic plus numerical calculations, they showed that, in the nonsecular regime, the two measures agree. But for other more cases, no definite result can be presented.

In this paper, we study the equivalence of the three measures of non-Markovianity recently proposed by BLP [2] and by RHP [10] in the case that a two-level system coupling to its environment via damped Jaynes-Cummings or dephasing model. Two important results are found: Firstly, the three measures are exactly equivalent in the aspect of detecting non-Markovianity for the involved models. And secondly, for our considered models, we find that the maximization in the definition of the BLP measure may be actually removed. This solves the problem that is being explored extensively by many researchers [26, 27]. Our researches have the following features. First, the interaction models involved here are most fundamental in the theoretical studies of dynamics of open quantum systems, and we need not assuming any specific spectral density for the structured environment. Thus, our results possess good adaptability. Next, our operations are completely analytical and hence the results are more convincing.

The article is organized as follows. In Sec. II we briefly review the three measures proposed by BLP and RHP respectively. The definition of equivalence between different measures is also presented. And in secs. III and IV we study the equivalence of the three measures for a two-level system interacting with environment via damped Jaynes-Cummings model and dephasing model respectively. A conclusion is arranged in Sec.V.

II. REVIEW OF THE THREE MEASURES

In reference [2], BLP presented a measure of non-Markovianity for quantum processes of open systems which is based on the idea that Markovian processes tend to continuously reduce the trace distance between any two states of a quantum system. Thus, an increase of trace distance during any time intervals implies the emergence of non-Markovianity. The
authors further linked the changes of trace distance to the flow of information between system and its environment, and concluded that the back flow of information from environment to the system is the key feature of a non-Markovian dynamics. Considering that the measure should reveal the total feature of a whole quantum process, they thus suggest the quantity,

\[ \mathcal{N} = \max_{\rho_{1,2}(0)} \int_{\sigma > 0} dt \sigma(t, \rho_{1,2}(0)), \]

as the measure of non-Markovianity of a quantum process. Where \( \sigma(t, \rho_{1,2}(0)) \) denotes the time derivative of trace distance for a pair of dynamical states with initial values \( \rho_{1,2}(0) \) of the considered system. The time integration is extended over all intervals in which \( \sigma \) is positive, and the maximum is taken over all pairs of initial states. For any Markovian process, \( \mathcal{N} = 0 \). And if \( \mathcal{N} > 0 \), the process must be non-Markovian.

Alternatively, RHP \[10\] presented another method to measure the non-Markovianity of a quantum process which is based on the monotonic drop of quantum entanglement between bipartite systems under the influence of local Markovian environments. Suppose a system of interest is initially prepared in a maximally entangled state with an ancillary particle, where only the system is influenced by a noise environment and the ancillary particle is free. Then the quantity for measuring the non-Markovianity of the quantum process is defined as

\[ I(E) = \int_{t_0}^{t_{\max}} \frac{dE[\rho_{SA}(t)]}{dt} dt - \Delta E, \]

where the time derivative in the integrand is for the dynamical entanglement between the system and ancillary particle, \( \Delta E = E[\rho_{SA}(t_0)] - E[\rho_{SA}(t_{\max})] \) denotes the difference of entanglements at the initial time \( t_0 \) and the final time \( t_{\max} \) of the interest quantum process.

The above two measures, \( \mathcal{N} \) and \( I(E) \), are introduced respectively through the monotonicity of trace distance or quantum entanglement under CP (or local CP) maps. Their mathematical forms are similar actually. It is not difficult to find that eq.(2) may be equivalently rewritten as

\[ I(E) = 2 \int_{\dot{E}(t) > 0} \dot{E}(t) dt, \]

with \( \dot{E}(t) \) being the time derivative of dynamical entanglement \( E(t) \). Obviously, the measure \( I(E) \) is very similar to \( \mathcal{N} \) in forms, with the time derivative \( \dot{E}(t) \) of dynamical entanglement replaced by the time derivative \( \sigma = \dot{D} \) of the dynamical trace distance \( D(\rho_1(t), \rho_2(t)) \). The visible difference is: the measure \( \mathcal{N} \) involves a maximum over all pairs of initial states, while \( I(E) \) is defined via a given maximally entangled initial state and thus escapes from
the optimization problem. Actually, the escaping of the optimization is just one of the original intentions for the measure $\mathcal{I}^{(E)}$ to be proposed. In the following sections, we will demonstrate that, for the models considered in this article, the maximization process can be removed actually.

The third quantity for measuring the non-Markovianity of a quantum process is also based on the entanglement dynamics between the system and an ancillary particle. Given the maximally entangled state $|\Phi\rangle$ of the system plus the ancillary particle, a locally complete positive map $\varepsilon$ will keep the positivity of density operator $\rho = |\Phi\rangle\langle\Phi|$ invariable, i.e., $\varepsilon(|\Phi\rangle\langle\Phi|) \geq 0$. Starting from this point of view, a measure of non-Markovianity of quantum processes is then born

$$\mathcal{I} = \int_0^\infty g(t)dt,$$

with

$$g(t) = \lim_{\epsilon \to 0^+} \frac{\| [I + (\mathcal{L}_t \otimes I)\varepsilon])|\Phi\rangle\langle\Phi|\| - 1}{\epsilon}.$$  \hspace{1cm} (5)

Where $\mathcal{L}_t$ is the super operator in the non-Markovian dynamical master equation $\frac{d\rho}{dt} = \mathcal{L}_t(\rho)$ for the open system.

The above three measures $\mathcal{N}$, $\mathcal{I}^{(E)}$ and $\mathcal{I}$, for the non-Markovianity of a quantum process are introduced through different ways. The physical meanings are different. A question naturally arises: They are equivalent each other? In this paper, we demonstrate that the three measures are equivalent each other when they apply to open two-level systems with damped Jaynes-Cummings or dephasing models. This equivalence implies that the measures, from different sides, well capture the intrinsical characters of non-Markovianity of quantum evolitional processes.

As the end of this section, let us elaborate the implication for different measures to be equivalent. We say two measures $M_1$ and $M_2$ to be equivalent means that: For a given quantum process, if by measure $M_1$, the non-Markovianity emerges, then it also emerges by measure $M_2$; Conversely, if by measure $M_1$, the non-Markovianity does not emerge, it also does not by measure $M_2$. The exact logics are: $M_1 > 0 \iff M_2 > 0$, $M_1 = 0 \iff M_2 = 0$. 

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III. THE CASE OF JAYNES-CUMMINGS MODEL

In this section, we will demonstrate the equivalence of the three measures of non-Markovianity for the case of a two-level system coupled to its environment via Jaynes-Cummings model. For this end, we should first derive out the expressions of the three measures given the specific interaction model. For the calculation of $\mathcal{N}$, let us consider a two-level atom interacting with its environment via a Jaynes-Cummings model. The environment is assumed to be initially in the vacuum state, but its spectral density is arbitrary. This case can be solved exactly. For any pair of initial atomic states, $\rho_1(0)$ and $\rho_2(0)$, one can obtain the time derivative of trace distance of the corresponding dynamical states as

$$\sigma(t, \rho_{1,2}(0)) = \frac{2|G(t)|^2 a^2 + |b|^2}{\sqrt{|G(t)|^2 a^2 + |b|^2}} \frac{d}{dt} |G(t)|,$$

where $a = \langle 1|\rho_1(0)|1 \rangle - \langle 1|\rho_2(0)|1 \rangle$ and $b = \langle 1|\rho_1(0)|0 \rangle - \langle 1|\rho_2(0)|0 \rangle$ are the differences of populations and of coherences respectively between the two given initial states. The function $G(t)$ is defined as the solution of the integro-differential equation,

$$\frac{d}{dt} G(t) = -\int_0^t d\tau f(t-\tau) G(\tau),$$

with initial condition $G(0) = 1$. The kernel $f(t-\tau)$ denotes the two-point reservoir correlation function which is the Fourier transformation of the spectral density. Introducing the time-dependent decay rate,

$$\gamma(t) = -2\text{Re}\left[\frac{\dot{G}(t)}{G(t)}\right] = -\frac{2}{|G(t)|} \frac{d}{dt} |G(t)|,$$

then the derivative of the trace distance may be rewritten as,

$$\sigma(t, \rho_{1,2}(0)) = -\gamma(t) F(t).$$

Here the completely positive real function $F(t)$ is defined as,

$$F(t) = \frac{a^2 e^{-\frac{3}{2} \Gamma(t)} + |b|^2 e^{-\frac{1}{2} \Gamma(t)}}{\sqrt{a^2 e^{-\Gamma(t)} + |b|^2}}.$$

with $\Gamma(t) = \int_0^t dt' \gamma(t')$. According to eq.(1), the non-Markovian measure $\mathcal{N}$ hence may be expressed as,

$$\mathcal{N} = -\int_{\gamma(t) < 0} \gamma(t) F(t) dt.$$
Where we assume that the pair of initial states \( \rho_{1,2}(0) \) just maximizes \( \mathcal{N} \) so that the maximization symbol is removed. Actually, due to the completely positivity of function \( F(t) \), the time intervals in which the trace distance increases monotonously, or equivalently the intervals in which non-Markovianity emerges (we call them non-Markovian intervals below), are uniquely determined by the condition \( \gamma(t) < 0 \). The change of initial states \( \rho_{1,2}(0) \) would not alter the positions and lengths of these non-Markoviany intervals, i.e., would not alter the distribution of them. No matter how the initial states change, \( \mathcal{N} \) would not shift from positive to zero, or vice versa. In this sense, the maximization to eq.(11) may be removed without influencing the sensibility of \( \mathcal{N} \) to detect non-Markovianity. This is an important result which solves the problem being explored by many researchers [26, 27].

Next, we begin with the calculation of the second measure \( \mathcal{I}^{(E)} \). Consider a maximally entangled state of two two-level atoms,

\[
|\Phi\rangle = \frac{1}{\sqrt{2}}(|10\rangle_{SA} + |01\rangle_{SA}),
\]

where atom 1 is our considered system which couples to an environment via Jaynes-Cummings interaction and atom 2 is an ancillary particle which remains isolated from environment. The whole Hamiltonian reads,

\[
H = H_0 + H_I,
\]

\[
H_0 = \sum_{i=1}^{2} \omega_0 \sigma_+^{(i)} \sigma_-^{(i)} + \sum_k \omega_k a_k^\dagger a_k,
\]

\[
H_I = \sum_k g_k \sigma_+^{(1)} a_k + h.c.,
\]

where \( \sigma_+^{(i)} \) and \( \sigma_-^{(i)} \) denote ladder operators for the \( i \)-th atom, \( \omega_k \) and \( a_k \) are the frequency and annihilation operator for the \( k \)-th harmonic oscillator of the environment, \( g_k \) is the coupling constant. We assume the two atoms have the same transition frequency \( \omega_0 \). Introducing to the interaction picture with respect \( H_0 \), one has,

\[
\tilde{H}_I = \sigma_+^1 \sum_k g_k a_k \exp[(\omega_0 - \omega_k)t] + h.c.
\]

Assume that the two atoms are initially in the entangled state \( |\Phi\rangle \) and environment is initially in vacuum state \( |0\rangle \), then the dynamical wave function for the compound system including both atoms and environment may be expressed as,
The dynamical reduced state for the two atoms thus reads,

\[ |\psi(t)\rangle = c_1(t)|10\rangle_{SA}|0\rangle_E + c_2(t)|01\rangle_{SA}|0\rangle_E + \sum_k c_k(t)|00\rangle_{SA}|1k\rangle_E. \]  (15)

where

\[ \rho_{SA}(t) = (1 - |c_1(t)|^2 - |c_2(t)|^2)|00\rangle\langle 00| + |c_1(t)|^2|10\rangle\langle 10| + |c_2(t)|^2|01\rangle\langle 01| 

+ c_1(t)c_2^*(t)|10\rangle\langle 01| + c_1^*(t)c_2(t)|01\rangle\langle 10|, \]  (16)

where the time-dependant coefficients are governed by Schrödinger equation,

\[ -i\frac{\partial}{\partial t} |\psi(t)\rangle = \tilde{H}_I(t)|\psi(t)\rangle. \]  (17)

After a straightforward deduction, we find

\[ c_2(t) = c_2(0) = 1/\sqrt{2} \text{ and } c_1(t) = G(t)/\sqrt{2} \]

with \( G(t) \) being determined by eq.(7). If we use concurrence to describe the entanglement between the system and ancillary particle, we have from eq.(16) that,

\[ C(\rho_{SA}(t)) = 2|c_1^*(t)c_2(t)| = |G(t)|, \]  (18)

and thus,

\[ \frac{d}{dt}C(\rho_{SA}(t)) = -\frac{1}{2}\gamma(t)e^{-\Gamma(t)/2}, \]  (19)

here the definition of \( \gamma(t) \) in eq.(8) is used. Eq.(5) then immediately produces,

\[ \mathcal{I}^{(E)} = -\int_{\gamma(t)<0} \gamma(t)e^{-\Gamma(t)/2}dt. \]  (20)

It again shows that the non-Markovian intervals for measure \( \mathcal{I}^{(E)} \) are uniquely determined by condition \( \gamma(t) < 0 \).

For the calculation of the third measure \( \mathcal{I} \), we need the dynamical master equation of the open two-level system

\[ \frac{d\rho}{dt} = -\frac{i}{2}S(t)[\sigma_+\sigma_-\rho] + \gamma(t)[\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-\},\rho], \]  (21)

here \( S(t) = -2\text{Im} \left( \frac{G(t)}{G(t)} \right) \) with \( G(t) \) determined by eq.(7) and \( \gamma(t) \) is defined by eq.(8). After a straightforward calculation, one find from eq.(4) that

\[ g(t) = \begin{cases} 
0 & \text{for } \gamma(t) \geq 0 \\
-\gamma(t) & \text{for } \gamma(t) < 0 \end{cases} \]  (22)
and thus
\[ \mathcal{I} = -\int_{\gamma(t) < 0} \gamma(t) dt. \] (23)

Once again, the non-Markovian intervals for this measure are determined by \( \gamma(t) < 0 \).

Through the above arguments, we conclude that, for an open two-level system with damped Jaynes-Cummings model, the distributions of non-Markovian intervals for the three measures, \( N, \mathcal{I}^{(E)} \) and \( \mathcal{I} \), are exactly the same, which are determined uniquely by the condition \( \gamma(t) < 0 \). According to the viewpoint of equivalence for different measures mentioned in Sec.II, we thus conclude that the three measures are equivalent for detecting the existence of non-Markovianity of a two-level system interacting with environment via Jaynes-Cummings model. In the arguing process, we did not assume any specific structure of spectral density. Thus the result has good adaptability. They are valid for arbitrary structured environment (Lorentzian, Ohmic, waveguide spectrum etc.), arbitrary detunings (including resonance) between system and its environment, and arbitrary coupling strengths.

IV. THE CASE OF DEPHASING MODEL

As the second example that we use to demonstrate the equivalence of measures of non-Markovianity, we consider a two-level atom which is coupled to a reservoir of harmonic oscillators via dephasing model. The Hamiltonian in Schrödinger picture is taken to be

\[ H = \frac{\omega_0}{2} \sigma_z + \sum_k \omega_k b_k^\dagger b_k + \sum_k \sigma_z (\lambda_k b_k^\dagger + \lambda_k^* b_k) \] (24)

where \( \omega_0 \) is transition frequency and \( \sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0| \) the Pauli operator of the atom. \( \omega_k, b_k \) are respectively the frequency and annihilation operator for the \( k \)-th harmonic oscillator of the reservoir. The coupling strength \( \lambda_k \) is assumed to be complex in general. This dephasing model, which is extensively used to simulate the decoherence of a qubit coupling to its environment in quantum information science, can be solved exactly. For the initial state of the total system

\[ \rho_{SB}(0) = \rho(0) \otimes \rho_B, \] (25)

with \( \rho(0) \) the initial state of the atom and \( \rho_B \) the initial thermal equilibrium state of the reservoir, the evolution for the elements of the reduced density matrix of the atom may be
\[ \rho_{11}(t) = \rho_{11}(0), \rho_{00}(t) = \rho_{00}(0), \]
\[ \rho_{10}(t) = \rho_{01}^*(t) = \rho_{10}(0)e^{\Gamma_p(t)}. \]  

(26)

Here the negative dephasing function \( \Gamma_p(t) \) is defined as

\[ \Gamma_p(t) = -\int_0^\infty d\omega J(\omega) \coth(\omega/2k_B T) \frac{1 - \cos(\omega t)}{\omega^2}, \]  

(27)

with \( J(\omega) \) the spectral density of the reservoir, \( k_B \) the Boltzmann constant and \( T \) the reservoir temperature in thermal equilibrium. For any pair of initial states \( \rho_{11}(0) \) and \( \rho_{22}(0) \) of the atom, one can easily obtain the dynamical states \( \rho_{11}(t) \) and \( \rho_{22}(t) \), and then get the dynamical trace distance as,

\[ D(t, \rho_{12}(0)) = \sqrt{a^2 + |b|^2 e^{2\Gamma_p(t)}}. \]  

(28)

Where \( a = \langle 1|\rho_{11}(0)|1 \rangle - \langle 1|\rho_{22}(0)|1 \rangle \) and \( b = \langle 1|\rho_{11}(0)|0 \rangle - \langle 1|\rho_{22}(0)|0 \rangle \) are the differences of the populations and of coherences for the two given initial states respectively. Inserting this trace distant into eq.(1), one immediately get the non-Markovian measure for this dephasing model as

\[ N_p = -2 \int_{\gamma_p(t)<0} dt \gamma_p(t) \frac{|b|^2 e^{2\Gamma_p(t)}}{\sqrt{a^2 + |b|^2 e^{2\Gamma_p(t)}}}, \]  

(29)

where the dephasing rate is defined as \( \gamma_p(t) = -\frac{1}{2} \dot{\Gamma}_p(t) \), and based on the same reason as before, we neglect the maximization to \( N_p \). It shows that the non-Markovian intervals are uniquely determined by \( \gamma_p(t) < 0 \).

In order to calculate the expression of the second measure \( I^{(E)} \), we introduce, in like manner as before, an isolated ancillary atom which is initially prepared in a maximally entangled state \( |\Phi\rangle \) in eq.(12) with the system atom. According to the evolution of eq.(26) and employing the trick in reference [28], one can easily obtain the dynamical density matrix for the system and ancillary atoms,

\[ \rho_{SA}(t) = \frac{1}{2}[[10\rangle\langle 10| + e^{\Gamma_p(t)}|01\rangle\langle 01| + e^{\Gamma_p(t)}|10\rangle\langle 10| + |01\rangle\langle 01|]. \]  

(30)

Where the first state refers to the system and the second one to the ancillary atom. It is easy to find that the concurrence for this state is \( C(t) = \exp[\Gamma_p(t)] \), and thus eq.(5) produces

\[ I^{(E)}_p = -4 \int_{\gamma_p(t)<0} \gamma_p(t)e^{\Gamma_p(t)} dt. \]  

(31)
Again, the non-Markovian intervals for measure $\mathcal{I}^{(E)}_p$ is also uniquely determined by $\gamma_p(t) < 0$.

Finally, let us calculate the measure $\mathcal{I}$ of eq.(3). It is not difficult from eq.(26) to verify that the dynamical master equation for the dephasing two-level atom can be written as

$$\frac{d\rho}{dt} = \gamma_p(t)[\sigma_z \rho \sigma_z - \rho],$$

(32)

where we again have used the relation $\dot{\Gamma}_p(t) = -2\gamma_p(t)$. Following the same deduction for eqs.(21)-(23), we obtain

$$\mathcal{I}_p = -2 \int_{\gamma_p(t) < 0} \gamma_p(t) dt.$$

(33)

Once again, the non-Markovian intervals for this measure are uniquely determined by $\gamma_p(t) < 0$.

All in all, for a two-level system with dephasing model, we again draw the conclusion that the distributions of non-Markovian intervals for the three measures are exactly the same and uniquely determined by the condition $\gamma_p(t) < 0$. Thus the three measures are equivalent.

V. CONCLUSION

In conclusion, we have compared the three measures of non-Markovianity proposed in references [2] and [10] respectively for two-level systems interacting with environments via damped Jaynes-Cummings or dephasing model. The results show that the three measures have exactly the same distribution of non-Markovian intervals, and hence are actually equivalent each other in the aspect of detecting non-Markovianity of quantum processes. This equivalence implies that these measures, in different ways, well capture the intrinsical characters of non-Markovianity of quantum processes. We have also found that the maximization in the BLP measure can be removed for the considered models, without influencing the sensibility of the measure to detect non-Markovianity. This result is important which extremely avoids the complicated mathematical calculations. The dynamical models considered here, i.e., damped Jaynes-Cummings and dephasing models, represent two kinds of fundamental coupling forms in studying problems of open systems. And we did not assume any specific spectral density for the structured environment. Thus our results have good adaptability. They apply to arbitrary structured environment, arbitrary detunings between system and its environment, and arbitrary coupling strengths. Exposing the relations between different
measures of non-Markovianity in different physical systems and different models is important. We expect similar researches for many more measures could be expanded for more general models or in more complicated systems.

VI. ACKNOWLEDGMENTS

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