Modelling Flat Spring Performance Using FEA

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Abstract. This paper reports how the stiffness of a Flat Spring can be predicted using nonlinear Finite Element Analysis (FEA). The analysis of a Flat Spring is a nonlinear problem involving contact mechanics, geometric nonlinearity and material property nonlinearity. Research has been focused on improving the accuracy of the model by identifying and exploring the significant assumptions contributing to errors.

This paper presents results from some of the models developed using FEA software. The validation process is shown to identify where improvements can be made to the model assumptions to increase the accuracy of prediction. The goal is to achieve an accuracy level of ±10 % as the intention is to replace practical testing with FEA modelling, thereby reducing the product development time and cost. Results from the FEA models are compared with experimental results to validate the accuracy.

1. Introduction
The term Flat Spring is commonly applied to a wide range of shapes made out of flat strip materials that, when deflected by an external load, store and release energy. Flat Springs are usually manufactured from high carbon spring steel, nickel-silver, high-nickel alloys, stainless steel, and phosphor-bronze. In addition to performing as springs they are frequently used as stops, connectors, hinges, braces, and fasteners. A Flat Spring can either be in the form of a cantilever spring or in a more complex form (Fig 1). The cantilever springs are produced by bending while the complex form springs are produced by bending and forming processes.

Fig 1: Flat Springs: cantilever and complex form
The stiffness of a Flat Spring depends mainly on the geometry, material properties and the load boundary conditions. Depending on the particular application, these factors can be varied to produce the desired spring stiffness.

2. Design and Performance Evaluation

Currently, Flat Spring design relies on simplified models and trial and error methods of development. These simplified models are based upon simple beam theory and curved beam theory. By using these models the performance of simple cantilever Flat Springs can be reliably predicted. However, this is not the case for complex Flat Spring geometries; hence there is a requirement to model the performance of these springs using FEA.

![Complex formed Flat Spring geometry](image)

**Fig 2: Complex formed Flat Spring geometry**

The evaluation of Flat Spring performance primarily involves a compression analyses in order to obtain the stiffness of the spring and the force generated at a specified displacement. This paper focuses on creating an FEA model for spring compression of a complex form Flat Spring. The stiffness of a spring has both linear and nonlinear characteristics (Fig 4). The goal is to replace some of the initial prototype evaluation with an FEA model, thereby reducing development cost. The prototype evaluation involves iterative design, manufacture and testing using a Zwick (compression) machine (Fig 4). In order to confidently replace some of this practical process with simulation, an accuracy of $\pm 10\%$ is required. Flat springs are mostly designed to perform within elastic region and the boundary between the elastic and plastic range. The performance within the elastic region can be accurately predicted within an error range of $\pm 20\%$ using equations derived from a database of empirical results.

![Flat Spring Compression](image)

**Fig 4: Flat Spring Stiffness obtained from a Zwick compression machine**
3. Finite Element Model

The application of FEA usually involves making a number of assumptions and simplifications through discretisation while trying to denote a real system with a mathematical model. These simplifications are essential as they do resolve some of the challenges resulting from modelling complex nonlinear problems. The simplifications also reduce the simulation time required to solve these models. However, these assumptions can also be one of the main sources of error between the theoretical model and the real system. Errors in FEA are either due to critical assumptions/simplifications that affect the accuracy of the model or incorrect assumptions made while constructing the model.

Flat Spring compression analysis involves loading a spring between two flat rigid plates (Fig 5). The analysis can be simplified as a static nonlinear analysis. This is because the expected stresses will exceed the limit of proportionality of the material during compression. There will also be major changes in geometry and boundary conditions due to the application of load (contact) and the direction of load application changes with deformation (pressures forces). The critical model assumptions that will influence the accuracy of the model are those of the contact model, material properties and definition of the geometry. Each one of these factors will be explained in the following sections.

![Fig 5: Flat Spring compression between two rigid plates](image)

3.1. Contact

Contact problems are common sources of boundary nonlinearity in stress analysis as the contact region will change during the loading cycle. Using FEA, many contact problems can be solved with high accuracy. There are, however, a number of limitations to contact analysis using FEA due to the capabilities and fidelity of the numerical solvers. The numerical technique for solving contact involves identification of the points on the boundaries of the bodies in contact, then application of appropriate boundary conditions or models to prevent penetration and to simulate the interaction between the surfaces.

Contact interaction has been setup between the top plate and the top surface of the strip and the bottom plate and the bottom surface of the strip as shown in Fig 5. The model defined to govern the interaction is considered to be frictionless for tangential sliding while a contact pressure constraint is applied in the normal direction. There is no limit in the contact formulation on the magnitude of contact pressure that can be transmitted between the surfaces. A frictionless model was chosen over a classical isotropic Coulomb friction model as it is assumed that the shear force transmitted across the interface will not significantly affect the normal stiffness of the model. The specified constraints are applied when the nodes on the flat plate surface penetrates the surface of the indentation. The surfaces separate when the contact pressure between them becomes zero or negative, when the constraint is removed. In order to validate this assumption, a sensitivity test was carried out to see the effect of friction on the FEA results (Fig 6).
3.2. Material Properties
The material properties were modelled using both elastic and plastic material parameters. These were obtained from a tensile test of the spring (stainless) steel used to manufacture the spring. Within the FEA software the elastic property was modelled by the standard Young’s modulus and Poisson ratio, while the plastic property was defined by its yield point and its post-yield hardening. Two points were used to define the post yield hardening, which were interpolated linearly to obtain intermediate values. The material model was assumed to be homogeneous and isotropic. This assumption ignores the residual stress in the spring, which may be generated during the forming process. The spring was modelled as a shell structure. Shell structures assume a linear stress distribution across the defined thickness.

3.3. Geometry
The geometric definitions required for the model include the 3D spring geometry and the two flat plates required to compress the spring. Initially the spring geometry was created in a CAD package using the tool dimensions and setup to predict the geometry of an indentation. A number of idealisations of the geometry were made in order to model the complex geometry of the spring (Fig 7). The mid-surface of the 3D geometry was extracted to model it a shell structure. The thickness of the spring was assumed to be uniform. The top and the bottom plate were modelled using rigid bodies - assuming infinite stiffness. To ensure consistency the critical dimensions of the spring were measured using a contact measurement system. The dimensions were measured in the x and y axes. Dimensions such as the blend radius could not be measured accurately hence they were simplified as edge radii.
4. Analysis and Results

A total of ten benchmark springs were modelled using the approach previously described. These rings are similar in shape but are of different sizes (i.e. spring height & width). Some spring models gave results within the specified accuracy requirement while others fell outside the accuracy requirement (Fig 8). The complex flat spring with large indentation height to surface area ratio gave results well outside the 10% error bar when compare to experimental results. The main significant difference between the model and the practical setup was in the geometric idealisations made.

![Flat Spring A; Compression](image)

**Fig 8: FEA and practical result (±10% Error range)**

In the cases of the less accurate results, the FEA model reliably predicts the maximum force generated, however, it over-predicts the elastic stiffness of the strip and the inception of plastic variation of stiffness. These are critical points, especially if the FEA model is to be used for design and development purposes.

As expected the geometry of the spring is a critical factor, hence in order to achieve accurate results the geometry must be accurately modelled with minimal abstraction. In other to validate the FEA model with the practical results it is essential to ensure that the critical dimensions on the practical geometry are close to the FEA geometry. Another source of error in the CAD geometry is that it does not account for the effect of factors that influence the final spring geometry during forming or bending e.g. spring back and permanent material deformation. This will affect sheet deformation, thinning, and material flow. In the case of a Flat Spring, even if these critical factors can be accurately measured, modelling them presents another set of challenges. Many simplifications will still be made in order to produce an accurate 3D representation of an indentation. The solution to this is to model the forming process of these springs and then compress the subsequent geometry. The formed model will then account for all of the factors previously mentioned.

The forming model was created using FEA and the CAD geometry used was that of the forming blade and the forming die, which can be accurately represented (Fig 9). The Flat Strip was modelled as
a shell with the thickness and material properties of stainless steel. Frictionless contact was setup between the forming blade and the strip as well as the die and the strip. The forming blade was then displaced by the distance required to achieve the required indentation height taking into consideration the spring back that occurs during the forming process.

The mesh geometry created was imported into the indentation compression model to replace the CAD geometry created initially. Figure 10 shows the new spring geometry, which gives a better representation of the actual geometry for FEA. Blend radii and edge corner radii are better represented in the formed geometry. Another advantage of the formed geometry is the mesh created (Fig 10). Meshing a flat strip before forming will create a better mesh structure (i.e. no element distortion and Hex elements) when compared to the mesh created on a 3D CAD geometry (Tetrahedral elements with distortion).

Using the same setup from the formed model the results showed considerable improvement and fell well within the ±10 % accuracy range (Fig 11).
5. Discussion
Resolving the assumptions due to the geometry has improved the accuracy of the FEA model. This does not eliminate all sources of error as some Flat Spring geometries do have a different structural response to the compression load, for example some springs will buckle under the load. An
understanding of the structural response is required to make the right assumptions while constructing the model. Figure 12 shows an example of a Flat Spring, which was assumed to be a static nonlinear problem. However examination of the practical ring indicated that the spring does exhibit buckling characteristics, which is not accounted for in the model. The formed model predicts the elastic nature of the spring; however, it fails to accurately predict the buckling and post buckling characteristics.

Fig 12: Flat Spring exhibiting buckling behavior under compression load

Future work is intended to use the modelling and analysis technique to develop new products and to improve exiting ones using design of experiments.

6. Conclusion

A study has been undertaking to enable the prediction of Flat Spring stiffness characteristics to be achieved with some degree of confidence. It is shown that assumptions in geometry typically of those from CAD, may lead to significant errors, these errors were overcome by performing a pre-analysis representing the forming of an indentation during manufacture. The resulting geometry was then more representative of the actual geometry and hence force/deflection prediction better matched the measured experimental results – achieving the accuracy requirement of ±10% error.
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