Research on thermal stiffness of machine tool spindle bearing under different initial preload and speed based on FBG sensors

Yanfang Dong$^{1,2,3}$ · Feifan Chen$^{1,2}$ · Tuanliang Lu$^{1,2}$ · Ming Qiu$^{1,2}$

Received: 30 June 2021 / Accepted: 1 November 2021 / Published online: 12 November 2021
© The Author(s), under exclusive licence to Springer-Verlag London Ltd., part of Springer Nature 2021

Abstract
As a key parameter, stiffness determines the dynamic characteristics of both the bearing and the spindle unit. However, interactions between the spindle speed, initial preload, and cutting force caused by the thermal expansion of spindle unit components alter the bearing stiffness characteristics during the running process. Aiming to carry out real-time online monitoring of the thermal characteristics and their effect on the machine tool spindle bearing stiffness, a fiber Bragg grating (FBG) sensors network was proposed. Further, the effects were considered under various axial loads, combined axial, and specific radial load to study bearing temperature increase, thermally induced preload, and thermal stiffness. The results have shown that the bearing thermal stiffness of the bearing decreases with the increase of the spindle speed. Moreover, an optimal initial preload can be selected with respect to the speed to maximize the thermal stiffness. Finally, the thermally induced preload additionally pre-tightened the bearing.

Keywords Machine tool spindle bearing · Temperature rise · Thermal stiffness · Fiber Bragg grating (FBG)

1 Introduction
As a core spindle unit component, angular contact bearing is widely used in high-speed spindle units, mostly since it can simultaneously withstand high speed, axial force, and radial force. The high speed, axial and radial loads introduce a great deal of heat into the bearing. The actual working and the non-working states are significantly different, which is especially true for the bearing stiffness. The bearing is thus often defined as the key parameter for evaluating bearing running status. Moreover, excessive heat generation in the spindle bearing induces uneven thermal expansion in various spindle elements, causing uneven thermal deformation of bearing rings. Therefore, there is an urgent need to investigate the thermal stiffness of bearings aiming to evaluate their running status, ensuring the reliable operation of high-speed spindle units.

Bearing stiffness refers to the ability of a bearing structure to resist elastic deformation when subjected to force. Exploring the bearing thermal stiffness characteristics should consider the spindle speed, cutting loads, and initial preload, as they are necessary for angular contact bearing. Chao et al. [1] established a heat generation model of ball bearing based on the internal load distribution. Zhao et al. [2] studied the load characteristics and friction coefficient of angular contact ball bearing working at high speed. Wenwu et al. [3, 4] studied the influences of nonuniform loads on the bearing thermal performance under different preloads. Further, the stiffness function for calculating the local contact deformation of angular contact ball bearings with uneven preload was established through analysis, aiming to predict the bearing stiffness. Huang et al. [5] established a comparative model for calculating bearing stiffness for various preload mechanisms by analyzing the preloading process. Additionally, the effects of spindle speed and radial force on the bearing dynamic characteristics and stiffness under different preload mechanisms were discussed.

However, in actual working conditions, the elastic deformation and thermal expansion of the machine tool spindle bearing will affect each other. For this reason,
several bearing thermal stiffness concepts were proposed, that is, the temperature rise causes thermoelastic coupling deformation of the parts. Mingyao et al. [6] found that cutting force changes the distribution of the spindle bearing temperature field, especially after applying radial cutting force. The bearing zone temperature increase of the ball bearing was higher compared to the non-bearing area. Zhang et al. [7] estimated the heat generation fluctuations in the bearing local contact zone, proving that the bearing has an uneven heat generation while working. Truong et al. [8, 9] computed the five-by-five bearing stiffness matrices based on the thermo-mechanical parameters, found that the bearing axial stiffness decreases with the increase of bearing temperature rise. Further, the stiffness coefficients at a specific speed increased significantly caused by the thermal effects. Hao et al. [10] found that the influence of the rotational speed on stiffness is opposite in the thermal balance state.

The bearing thermal characteristics are significantly affected by external factors. However, available tests are very limited; it is very difficult to monitor the bearing statuses directly via traditional measurement methods. Jiandong et al. [11, 12] developed an improved thermo-mechanical model for studying the effects of the various factors on the transient preload. However, the dedicated test rig interfered with the spindle unit operation. Furthermore, electrical sensors have been used to monitor the temperature field of spindle units housing surfaces [13–15]. It should be added that there is a serious lack of test methods embedded in the spindle unit to ensure the real-time monitoring of the bearing state.

Currently, temperature field and thermal stress state detection methods in spindle bearings are rather limited. Furthermore, carrying out online monitoring of bearing thermal stiffness characteristics to reflect the true service information of the bearing is hard. Thus, aiming to solve this problem, in this paper the measurement advantages of FBG “one line and multiple points” [16] were used. Further, a real-time online monitoring system for the service status of the FBG embedded spindle bearing was built. It was used to complete the spindle unit thermal characteristic test, and explore the variations in the spindle bearing thermal stiffness under the action of spindle speed, initial preload, and cutting force.

2 The real-time online monitoring system of spindle bearing service status

2.1 FBG measurement principle

The FBG sensing technology was used to build an embedded real-time online network to monitor the spindle bearing service statuses and measure the bearing temperature and strain fields.

The FBG sensor can reflect specific light signal wavelengths; the first-order Bragg wavelength can be expressed as

\[ \lambda_B = 2 \cdot n_0 \cdot 1 \]  

(1)

where \( n_0 \) is the effective refractive index of the grating and \( l \) is the grating interval.

It can be seen from formula (1) that the reflected wavelength of the FBG depends on the grating interval and effective refractive index. Changes in either the external environment temperature or strain will change the wavelength of the reflected wave. The demodulation device derives the external temperature or strain by detecting the changes of wavelength. When the external temperature, stress, etc. are measured, the change in the center wavelength of the FBG can be expressed as

\[ \Delta \lambda_B = K_T \cdot \Delta T \cdot \lambda_B \]  

(2)

where \( \Delta T \) is the temperature difference before and after the test (°C) and \( K_T \) is the temperature sensitivity coefficient of the optical fiber.

\[ K_T = \alpha_f + \zeta \]  

(3)

where \( \alpha_f \) is the thermal expansion coefficient of the optical fiber material and \( \zeta \) is the thermo-optical coefficient of the optical fiber material.

Due to the thermal expansion effect of the matrix material, when the expansion is completely transferred to the fiber grating, the reflection wavelength change can be calculated as

\[ \Delta \lambda_B = (1 - P_e) \cdot \Delta \varepsilon \cdot \lambda_B \]  

(4)

where \( P_e \) represents the elastic-optical coefficient of the optical fiber and \( \Delta \varepsilon \) is the structural strain of the optical fiber.

2.2 FBG embedded bearing service status real-time online monitoring network

The real-time online monitoring network of the spindle bearing service status built in this paper has a distributed measurement system based on the FBG sensors, which are connected in one fiber. Figure 1 shows the layout of the front spindle bearings and the service statuses testing system. By using the demodulator and PC, the wavelength changes of the FBG sensors under the action of axial preload and temperature rise were measured in real time.

A new structure spacer was designed and installed between the front and rear tandem bearings (see Fig. 2).
The improved outer bearing spacer was composed of two rings and four constant cross-section cantilever beams. Furthermore, eight FBG sensors were pasted on the constant cross-section cantilever beam, each beam having two FBG sensors. The cantilever beam measures the axial strain change $\Delta \varepsilon$ introduced by the thermal expansion of spindle unit components. Therefore, the measured axial force of a single cantilever beam is $EA \cdot \Delta \varepsilon_1$, while the axial force born by entire spacer is $EA \cdot (\Delta \varepsilon_1 + \Delta \varepsilon_2 + \Delta \varepsilon_3 + \Delta \varepsilon_4)$. The latter expression represents the bearing preload force of axial load, where $E$ is the spacer ring material elastic modulus and $A$ is the cross-sectional area of the cantilever beam with constant cross-section.

Since the FBG is sensitive to the simultaneous action of temperature and strain. Thus, when measuring the initial preload and the thermal deformation-induced axial force $F_{a1}$ acting on the components and ambient temperature change. Hence, the FBG1 center wavelength change $\Delta \lambda_{B1}$ can be expressed as

$$\Delta \lambda_{B1} = \left[ -(1 - P_e) \cdot \frac{F_{a}}{Ewh} + (\alpha_f + \xi) \cdot \Delta T \right] \cdot \lambda_{B1}$$  \hspace{1cm} (5)

The FBG2 was pasted in the vertical direction, while the cantilever beam is compressed in the axial direction; therefore, the strain was also generated in the vertical axial direction. Besides the environmental temperature change, the FBG2 center wavelength change $\Delta \lambda_{B2}$ can be expressed as

$$\Delta \lambda_{B2} = \left[ \nu \cdot (1 - P_e) \cdot \frac{F_{a}}{Ewh} + (\alpha_f + \xi) \cdot \Delta T \right] \cdot \lambda_{B2}$$  \hspace{1cm} (6)

where $\lambda_{B1}$ and $\lambda_{B2}$ are the initial center wavelengths of FBG1 and FBG2, respectively, $E$ is the elastic modulus of the spacer material, $\nu$ is the Poisson’s ratio of the spacer material, $w$ is the beam width, and $h$ is the beam thickness.

By combining Eqs. (5) and (6), it is possible to eliminate the influence of the ambient temperature $\Delta T$ on the central wavelength of FBG sensors. Further, the influence of the ambient temperature on FBG1 can be compensated by the data collected for FBG2.

Since the FBG sensor was pasted close to the tested bearing, it was assumed that the measured temperature rise is equal to the bearing temperature rise; therefore, the spindle bearing temperature rise is
\[
\Delta T = \frac{\Delta \lambda_{B2} \cdot \lambda_{B1} - v \Delta \lambda_{B1} \cdot \lambda_{B2}}{\lambda_{B1} \cdot \lambda_{B2}(1 - v)(\alpha_f + \xi)} \tag{7}
\]

Given the structural characteristics of the spindle unit studied in this paper, the thermal displacement of a single bearing can be expressed as

\[
2\Delta \epsilon_b = \Delta \epsilon 1 + 2\Delta B + \Delta l \tag{8}
\]

where \(\Delta \epsilon 1\) is the deformation of the spacer ring measured by the fiber grating, \(\Delta B\) is the axial linear expansion of the bearing, and \(\Delta l\) is the axial linear expansion of the spacer. The values are calculated using formulas (9) to (11), respectively.

\[
\Delta \epsilon 1 = \frac{F_{a1}}{Ew}\hat{h} = \frac{\Delta \lambda_{B2} \cdot \lambda_{B1} - \Delta \lambda_{B1} \cdot \lambda_{B2}}{2 \lambda_{B1} \cdot \lambda_{B2}(1 - v)(1 - P_e)} \tag{9}
\]

\[
\Delta B = \alpha_b B \cdot \Delta T \tag{10}
\]

\[
\Delta l = \alpha_l l \cdot \Delta T \tag{11}
\]

where \(\alpha_b\) and \(\alpha_l\) are the linear expansion coefficients of the bearing and spacer materials, respectively, \(l\) is the beam length, \(B\) is the beam width.

The comparison shows that \(\Delta \epsilon 1 > \Delta B + \Delta l\), meaning that the spindle bearing thermal displacement can be approximated as

\[
\Delta \epsilon_b = \frac{1}{2} \Delta \epsilon 1 = \frac{F_{a1}}{2Ew}\hat{h} = \frac{\Delta \lambda_{B2} \cdot \lambda_{B1} - \Delta \lambda_{B1} \cdot \lambda_{B2}}{2 \lambda_{B1} \cdot \lambda_{B2}(1 - v)(1 - P_e)} \tag{12}
\]

The thermally induced preload \(F_{a1}\) of a cantilever beam during the spindle unit assembly thermal expansion is as follows:

\[
F_{a1} = \frac{(\Delta \lambda_{B2} \cdot \lambda_{B1} - \Delta \lambda_{B1} \cdot \lambda_{B2})Ew h}{\lambda_{B1} \cdot \lambda_{B2}(1 - v)(1 - P_e)} \tag{13}
\]

Similarly, data for the remaining three temperature rise (\(\Delta T2, \Delta T3, \Delta T4\)), strain change (\(\Delta \epsilon 2, \Delta \epsilon 3, \Delta \epsilon 4\)), and thermally induced preload (\(F_{a2}, F_{a3}, F_{a4}\)) can be obtained. The environmental temperature compensation method is used as well, enabling the accurate monitoring of the spindle bearing service status of the spindle bearing through the FBG network. Further, and the bearing service status information such as the temperature field of the spindle bearing and the thermally induced preload can be collected online in real-time, enabling the authors to obtain the bearing thermal stiffness. The values are calculated using formulas (14) for bearing stiffness without thermal influence and formulas (15) for measures points thermal stiffness with thermal influence, respectively.

\[
K = \frac{F_a}{8\Delta \epsilon 1 l} \tag{14}
\]

\[
K_{thermal} = \frac{(F_a/8 + F_{a1})}{2(\Delta \epsilon 1 + \Delta \epsilon_b)l} \tag{15}
\]

### 2.3 The calibration of FBG sensors

Taking the influence of the sensor substrate and pasting effect on the FBG sensors sensitivity into account, the FBG sensors on one beam were calibrated by applying axial force by the hydraulic station when the spindle is stationary. The test was carried out in three groups, and the test results are given in Fig. 3. It is evident that there is a good linear relationship between the pasted FBG sensors and the axial force change. When the axial force was 300 N, the maximum test deviation was 12.4 N, and the relative error was 4.13%.

### 3 Experimental research on the spindle bearing thermal stiffness

#### 3.1 Testing platform structure

In this paper, the TS30-70 high-speed machine tool spindle bearing test platform was used to carry out the spindle bearing thermal stiffness characteristics test. The test platform can achieve the maximum speed of 36,000 rpm with bearing lubricated with oil, apply up to 1500 N axial loads and 500 N radial loads. The test platform composition is shown in Fig. 4. The front and rear spindle ends include two tandem bearings arranged...
in “O” layout, shown in Fig. 5. When the initial preload \( F_a \) is applied, the front end bearings of the spindle are preloaded, and the spindle moves backward to preload the rear end bearings.

Using the high-speed machine tool spindle bearing test platform and the proposed real-time online monitoring system of the spindle bearing service statuses, the thermal characteristics test of the spindle bearing under only the axial load and the combined axial and radial load was carried out. Each test lasted for 3 h and the next set of tests was performed when the test machine cooled to room temperature.

### 3.2 Uniform spindle bearing thermal stiffness under axial load

As the spindle speed and axial force change during the operation of the machine tool spindle, the spindle bearing temperature field, thermally induced preload, and thermal stiffness will also change. In this paper, the bearing thermal characteristic test was carried out at the spindle speeds ranging from 2000 to 12,000 rpm and the initial preload between 350 and 550 N.

As can be seen from Fig. 6, at the initial preload 350 and 400 N, the bearing temperature rise changes with the spindle speed in a similar manner. As the spindle speed increases, the bearing temperature rise also increases. It can be noticed that once the speed exceeds 10,000 rpm, the bearing temperature rises rapidly, reaching the maximum value during the test early stage, which is followed by a slow decrease until it finally stabilizes. This is caused by the rate of heat generation which is much higher than the rate of heat lost, resulting in the accumulation of heat inside the bearing. Further, as the bearing temperature increases, the lubricant viscosity decreases, resulting in the decrease of the viscous friction torque and bearing heat generation rate. However, the heat conduction is increased due to the temperature increase, meaning that bearing temperature rise decreases slowly after reaching the maximum value.

The results for the bearing steady-state temperature increase under various spindle speeds and initial preloads are shown in Fig. 7. Given the same initial preload, the bearing steady-state temperature rise increases with the increase of the spindle speed. In particular, at a certain speed, the bearing steady-state temperature rise has an inflection point, and the higher the speed, the lower the inflection point. This phenomenon shows that there is an optimal preload for minimizing the bearing temperature rise for any speed. As the speed increases, the bearing optimal initial preload increases accordingly.

Thermally induced preload induced by the thermal expansion of the spindle unit components is one of the
dominant influence parameters affecting the spindle bearing dynamic performance. As shown in Fig. 8, the thermally induced bearing preload change with time under different spindle speeds and initial preloads. For the same initial preload, the thermally induced bearing preload increases with the spindle speed, especially at the beginning of the test—the higher the spindle speed, the greater the increase. Once the rotation speed reaches 10,000 rpm, the thermally induced preload rapidly increases to the maximum value firstly at the start, which is followed by a slow decrease. Finally, the value tends to stabilize. This is mostly due to the expansion rate of the rolling elements and the inner ring, which is higher than the expansion rate of the outer ring and the bearing housing. Furthermore, the bearing clearance reduces at the beginning of the test. With the progress of heat transfer, the temperature of the rolling elements and the inner ring tend to be stable. The thermally induced bearing preload tends to stabilize at the thermal equilibrium state of the whole spindle unit.

The change curve of the steady-state thermally induced bearing preload with the initial preload at various spindle speeds is shown in Fig. 9. It is clear that, given the same initial preload, the thermally induced preload reaches a larger value. At higher speeds, the thermally induced preload reaches a larger value. This is caused by the high speed, which causes the spindle unit to generate more heat, ultimately resulting in a greater thermal expansion of the spindle unit assembly. In turn, spindle unit assembly thermal expansion increases the thermally induced bearing preload. Similar to the steady-state temperature rise of the bearing, at the same speed, there is an inflection point found on the curve. Lastly, it also can be seen that the thermally induced preload is equivalent to the initial preload and thus cannot be ignored.

Bearing stiffness is a critical parameter determining the spindle unit dynamic characteristics. The initial bearing preload is usually used to achieve the high machine tool spindle stiffness. At the same time, as the speed and working conditions change, the generation of bearing friction heat will further change the thermal stiffness characteristics of the bearing and produce thermal instability due to

![Fig. 6](image)

The influence of spindle speed and initial preload on the bearing temperature rise. (a) Initial preload of 350 N; (b) initial preload of 550 N

![Fig. 7](image)

Fig. 7 Steady-state bearing temperature increase for various spindle speeds and initial preloads
the positive feedback of the preload thermal closed-loop. Figure 10 shows the influence of spindle speeds and initial preloads on the bearing stiffness. It can be found that under a given spindle speed condition, there is an optimal initial preload needed to maximize the bearing stiffness. Either too high or too low initial preload will significantly reduce the bearing stiffness.

By comparing Fig. 10a, b under the same working conditions, depending on if the thermally induced preload is considered, the maximum difference was $0.32 \times 10^7$ N/m.

This is mainly because the thermally induced preload increases the stiffness of the bearing. And it should be noted that the spindle speed was 8000 rpm and the initial preload was 350 N, while the bearing stiffness was $1.25 \times 10^7$ N/m when the thermally induced preload was not considered. On the other hand, the bearing stiffness was $1.57 \times 10^7$ N/m when the thermally induced preload was considered. Therefore, the effect of thermally induced preload on the operating state of the spindle unit should be fully considered both when applying and optimizing the initial preload.

### 3.3 Nonuniform spindle bearing thermal stiffness under combined axial and radial load

The test condition was selected as follows: initial preload of 550 N, the radial load of 100 N, and spindle speed of 12,000 rpm. The degree of heterogeneity was thus significant, and the nonuniform thermal characteristics of the spindle bearing were monitored at four measuring points (A, B, C, and D). The positions of the measuring points are shown in Fig. 11.

The entire measurement process lasted 3 h. As shown in Fig. 12, the bearing temperature first raised sharply with time and then dropped before finally stabilizing. After reaching the thermal equilibrium, the temperature of the measuring points A, B, C, and D were 14.92 °C, 17.84 °C, 15.02 °C, and 13.25 °C, respectively.

It was clear that a significant difference existed in the distribution temperature field along the bearing circumferential direction under the action of the radial load.
Thus, this phenomenon also led to uneven thermal expansion of the bearing rolling elements, inner and outer rings, and the bearing seat. As shown in Fig. 13, the curve of measuring point B is steeper than other measuring points while rising, which is also directly related to the radial load effect.

The stiffness of the spindle bearing is closely related to the bearing temperature rise. Figure 14 shows the local thermal stiffnesses of the bearing at different measuring points.

From Fig. 14, it can be seen that the local stiffness of the B measuring point was the smallest, while the D measuring point had the largest stiffness. And the stiffness of the A and C measuring points was not much different from each other. This is due to the effect of radial load, the spin friction torque between each rolling element and the ring of the bearing is unevenly distributed. The bearing temperature in the bearing zone is higher than that in the non-loading zone, resulting in an uneven thermal expansion of the bearing inner and outer rings. Therefore, there are differences in bearing local stiffness at different measuring points. This
will cause unbalanced thermal deformation of the spindle, resulting in spindle vibration and reducing the machining accuracy of the machine tool.

Additionally, the spindle bearing temperature rise was also compared to the ANSYS simulation results, aiming to validate the test accuracy based on FBG sensors. The system heat generation and sinks were both calculated in detail [17], and the finite model was shown in Fig. 15.

The specific comparison results are shown in Fig. 16. From Fig. 16, it can be seen that the maximum difference between the simulation and test results is 1.61 °C, verifying the accuracy of the test results.
4 Conclusion

In this paper, the thermal stiffness characteristics of machine tool spindle bearings under the interaction of spindle speed, initial preload, and cutting force were explored. A spindle unit thermal characteristics test system based on the FBG sensors network was proposed, and the bearing thermal characteristics tests were carried out. Based on the analysis of test and simulation results, the following conclusions can be drawn:

1. Considering that the FBG is sensitive to temperature and strain, at the same time, a method of arranging two adjacent FBG sensors, pasting them perpendicular to each other, was proposed to compensate for the effects of temperature change.
2. The temperature rise and thermally induced bearing preload both increased with the increase of the spindle speed. Especially when the spindle speed was above 10,000 rpm. The bearing temperature rise and the thermally induced preload both appeared to sharply increase at first, then slowly drop before finally reaching a steady state.
3. There is an optimal initial preload for each spindle speed, which minimizes the spindle bearing temperature rise and the thermally induced preload. In general, the higher the spindle speed, the greater the optimal initial preload.
4. The effect of cutting force divides the bearing into a load-bearing area and a non-load-bearing area. The bearing thermal stiffness in the load-bearing area is 30% lower compared to the non-load-bearing area. Additionally, the thermally induced preload also has an important influence on the bearing thermal stiffness.

Acknowledgements The authors would like to acknowledge funding support from the National Key R&D Program of China (No. 2018YFB2000504), National science foundation (No. 51805151), and Henan Provincial Department of Education Project (No. 19A460018), as well as the contributions from all collaborators within the projects mentioned.

Author contribution Methodology and writing—original draft preparation, DONG Yanfang and CHEN Feifan; writing review and editing, LU Tuanliai; supervision, QIU Ming.

Funding This research was funded by The National Key R&D Program of China, grant number 2018YFB2000504, National science foundation, grant number 51805151, and Henan Provincial Department of Education Project, grant number 19A460018.

Availability of data and materials The data sets supporting the results of this article are included within the article.

Code availability Not applicable.

Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent for publication Not applicable.

Competing interests The authors declare no competing interests.

References

1. Chao J, Jun Bo Hu, Youmin (2012) Heat generation modeling of ball bearing based on internal load distribution. Tribol Int 45:8–15. https://doi.org/10.1016/j.triboint.2011.08.019
2. Zhao CI, Yu XK, Huang QX, Ge SD, Gao X (2015) Analysis on the load characteristics and coefficient friction of angular contact ball bearing at high speed. Tribol Int 87:50–60. https://doi.org/10.1016/j.triboint.2015.02.012
3. Wenwu Wu, Li Xiaohu Xu, Feng HJ, Yang Li (2014) Investigating effects of non-uniform preload on the thermal characteristics of angular contact ball bearings through simulations. Proc IMechE Part J: J Engineering Tribology 228(6):667–681
4. Wenwu Wu, Jun H, Yang Li, Xiaohu Li (2017) Investigation of non-uniform preload effect on stiffness behavior of angular contact ball bearings. Adv Mech Eng 9(3):1–19
5. Huang WD, Gan CB, Yang SX, Xu LH (2017) Analysis on the stiffness of angular contact ball bearings and its effect on the critical speed of a high speed motorized spindle. J Shock Vib 36(10):19–25
6. Mingyao L, Weijian Z, Han S, Yanfang D, Wenzhi W, Shiguang Z (2018) Study of the temperature distribution of a machine tool spindle bearing based on FBG quasi-distributed sensing. Int J Adv Manuf Technol 98:263–274. https://doi.org/10.1007/s00170-018-2215-3
7. Zhang Y, Xiao hu Li, Jun Hong, Ke Yan, Sen Li, (2018) Uneven heat generation and thermal performance of spindle bearings. Tribol Int 126:324–335. https://doi.org/10.1016/j.triboint.2018.04.035
8. Truong DS, Kim B-S, Ro S-K (2021) An analysis of a thermally affected high-speed spindle with angular contact ball bearings. Tribol Int 157:106881. https://doi.org/10.1016/j.triboint.2021.106881
9. Truong DS, Kim B-S, Park J-K (2019) Thermally affected stiffness matrix of angular contact ball bearings in a high-speed spindle system. Adv Mech Eng 11:1–17
10. Hao Xu, Jingyu Z, Junshuai L, Yitong C, Qingkai H (2020) Time-varying stiffness characteristics of roller bearing influenced by thermal behavior due to surface frictions and different lubricant oil temperatures. Tribol Int 144:106125. https://doi.org/10.1016/j.triboint.2019.106125
11. Jiandong Li, Yongsheng Z, Ke Y, Jun H, Xiaoyun Y (2019) An improved thermo-mechanical model for spindle transient preload analysis. Proc IMechE Part J: J Engineering Tribology 233:1698–1711
12. Ke Y, Bei Y, Yatai W, Jun H, Jinhua Z (2016) Study on thermal induced preload of ball bearing with temperature compensation based on state observer approach. Int J Adv Manuf Technol 94:3029–3040. https://doi.org/10.1007/s00170-016-9469-4
13. Yanlei Li, Wang Mingyan Hu, Bo YW (2016) Thermal error prediction of the spindle using improved fuzzy-filtered neural networks. Proc IMechE Part B: J Engineering Manufacture 230:770–778
14. Liu K, Liu Y, Sun MJ, Wu YL, Zhu TJ (2017) Comprehensive thermal growth compensation method of spindle and servo axis error on a vertical drilling center. Int J Adv Manuf Technol 88:2507–2516. https://doi.org/10.1007/s00170-016-8972-y

15. Jun Y, Shi Hu, Bin F, Liang Z, Chi Ma, Xuesong M (2015) Thermal error modeling and compensation for a high-speed motorized spindle. Int J Adv Manuf Technol 77:1005–1017. https://doi.org/10.1007/s00170-014-6535-7

16. Li T, Tan Y, Zhou Z (2017) String-type based two-dimensional fiber Bragg grating vibration sensing principle and structure optimization. Sens Actuators, A 259:85–95. https://doi.org/10.1016/j.sna.2017.03.031

17. Chi Ma, Jun Y, Liang Z, Xuesong M, Shi Hu (2015) Simulation and experimental study on the thermally induced deformations of high-speed spindle system. Appl Therm Eng 86:251–268. https://doi.org/10.1016/j.applthermaleng.2015.04.064

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.