A Structured Prediction Approach for Conditional Meta-Learning

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Abstract

Optimization-based meta-learning algorithms are a powerful class of methods for learning-to-learn applications such as few-shot learning. They tackle the limited availability of training data by leveraging the experience gained from previously observed tasks. However, when the complexity of the tasks distribution cannot be captured by a single set of shared meta-parameters, existing methods may fail to fully adapt to a target task. We address this issue with a novel perspective on conditional meta-learning based on structured prediction. We propose task-adaptive structured meta-learning (TASML), a principled estimator that weighs meta-training data conditioned on the target task to design tailored meta-learning objectives. In addition, we introduce algorithmic improvements to tackle key computational limitations of existing methods. Experimentally, we show that TASML outperforms state-of-the-art methods on benchmark datasets both in terms of accuracy and efficiency. An ablation study quantifies the individual contribution of model components and suggests useful practices for meta-learning.

1 Introduction

State-of-the-art learning algorithms such as neural networks typically require vast amounts of data to generalize well. This is a concrete issue for applications with limited data availability (e.g. drug discovery AltAE-Tran et al., 2017). Meta-learning methods are often employed to tackle the lack of training data Finn et al. (2017); Vinyals et al. (2016); Ravi & Larochelle (2017). They are designed to learn new concepts from only a handful of examples by leveraging knowledge accrued from previous tasks Thrun (1996); Vilalta & Drissi (2002). They are applied to settings such as learning-to-optimize Li & Malik (2016) and few-shot learning Fei-Fei et al. (2006); Lake et al. (2011). Meta-learning methods could be broadly categorized into metric-learning (e.g. Vinyals et al., 2016; Snell et al., 2017; Oreshkin et al., 2018), black-box (e.g. Li & Malik, 2016; Hochreiter et al., 2001; Ha et al., 2016), and optimization-based (e.g. Finn et al., 2017; Rusu et al., 2019; Nichol et al., 2018).

We focus on optimization-based approaches, which cast meta-learning as a bi-level optimization Finn et al. (2017); Rajeswaran et al. (2019); Antoniou et al. (2019). At the single-task level, an “inner” algorithm performs task-specific optimization from a set of meta-parameters shared across all tasks. At the “outer” level, a meta learner accrues experiences from observed tasks to learn the aforementioned meta-parameters. These methods aim to learn a single meta-parametrization that can be effectively adapted to all tasks. Unfortunately, the shared parametrization may fail to capture the complexity of tasks distribution. In particular, Rusu et al. (2019) showed that task-specific initialization of meta-parameters improves model performance. In addition, optimization-based meta-learning methods often requires evaluations of higher-order derivatives, which are computationally expensive, and cause potentially training instability Rajeswaran et al. (2019); Antoniou et al. (2019).

To address these challenges, we propose Task-adaptive Structured Meta-Learning (TASML). We offer a novel perspective on conditional meta-learning based on structured prediction Bakir et al. (2007). We interpret the inner algorithm as structured output that needs to be predicted, conditioned on the target task. We derive a principled estimator that minimizes a task-specific meta-learning objective, which weighs known training tasks based on their similarity with the target task. This task-conditional objective captures the local tasks distribution to improve model performance. Informally, the proposed model aims to address the target task by leveraging only the most relevant experiences from the training tasks. The relevance of each training task with
respect to the target one is measured via structured prediction.

We introduce a practical and efficient algorithm for TASML, along with several algorithmic modifications aimed at improving model efficiency and performance, including: representation pre-training, optimization as a layer Amos & Kolter (2017); Bertinetto et al. (2019), and least-squares relaxation of classification loss. We show experimentally that TASML outperforms state-of-the-art methods on two competitive few-shot classification benchmarks, mini- and tieredIMAGENET; and significantly improves computational efficiency. Further ablation study directly quantifies the individual contribution of model components to suggests useful practices for meta-learning.

Our main contributions include: i) a new perspective on conditional meta-learning based on structured prediction. ii) TASML, a conditional meta-learning method inspired by this perspective. iii) a practical algorithm with general efficiency and performance improvements to meta-learning. iv) a thorough evaluation of the proposed approach on benchmarks, outperforming state-of-the-art methods.

2 Background and Notation

For clarity, in this paper we focus on meta-learning for supervised learning tasks. However, the discussion below applies also to general learning settings.

Supervised learning. In supervised learning, given a probability distribution $\rho$ over two spaces $\mathcal{X} \times \mathcal{Y}$ and a loss $\ell : \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}$ measuring prediction errors, the goal is to find $f : \mathcal{X} \to \mathcal{Y}$ that minimizes the expected risk

$$\min_{f : \mathcal{X} \to \mathcal{Y}} \mathcal{E}(f) \quad \text{with} \quad \mathcal{E}(f) = \mathbb{E}_\rho \ell(f(x), y),$$

with $(x, y)$ sampled from $\rho$. The distribution $\rho$ is unknown and instead a finite training set $D = (x_j, y_j)_{j=1}^m$ of i.i.d samples is given. A learning algorithm typically finds $f \in \mathcal{F}$ within a prescribed set $\mathcal{F}$ of candidate models (e.g. neural networks, reproducing kernel Hilbert spaces). This is done by performing empirical risk minimization on $D$ or adopting online strategies such as stochastic gradient methods (SGD) (see e.g. Shalev-Shwartz & Ben-David, 2014, for an overview on statistical learning). Hence, a learning algorithm may be seen as a function $\text{Alg} : \mathcal{D} \to \mathcal{F}$ that maps an input dataset $D$ to a model $f$, where $\mathcal{D}$ is the space of datasets on $\mathcal{X} \times \mathcal{Y}$.

2.1 Meta-learning

While in supervised settings $\text{Alg}(\cdot)$ is chosen a-priori, meta-learning aims to learn a learning algorithm suitable for a family of tasks. Formally, we consider $\text{Alg}(\theta, \cdot) : \mathcal{D} \to \mathcal{F}$ and aim to minimize

$$\mathcal{E}(\theta) = \mathbb{E}_\mu \mathbb{E}_\rho \mathcal{L}(\text{Alg}(\theta, D^{tr}), D^{val}),$$

over a suitable space of meta-parameters $\theta \in \Theta$. Here $\mu$ is a meta-distribution over the possible tasks, $\rho$ a task distribution sampled from $\mu$, and $D^{tr}$ and $D^{val}$ respectively a training and validation set of i.i.d samples $(x, y)$ from $\rho$. The task loss $\mathcal{L} : \Theta \times \mathcal{D} \to \mathbb{R}$ is usually of the form

$$\mathcal{L}(f, D) = \frac{1}{|D|} \sum_{(x,y) \in D} \ell(f(x), y),$$

with $|D|$ the cardinality of $D$. We aim to find the best $\theta^*$ such that applying $\text{Alg}(\theta^*, \cdot)$ on $D^{tr}$ achieves lowest generalization error on $D^{val}$, among all algorithms parametrized by $\theta \in \Theta$. In practice, we have only access to a finite meta-training set $S = (D_i^{tr}, D_i^{val})_{i=1}^N$ and the meta-parameters $\theta$ are often learned by (approximately) minimizing

$$\hat{\theta} = \arg\min_{\theta \in \Theta} \frac{1}{N} \sum_{i=1}^N \mathcal{L}(\text{Alg}(\theta, D_i^{tr}), D_i^{val}).$$
Meta-learning methods address (4) via first-order methods such as (e.g. SGD), which requires access to $\nabla_\theta \text{Alg}(\theta, D)$, the (sub)gradient of the inner algorithm over its meta-parameters. Below, we review several meta-learning algorithms implementing this strategy.

**Model Agnostic Meta-Learning.** Model-agnostic meta-learning (MAML) Finn et al. (2017) and its variants (see e.g. Antoniou et al., 2019; Li et al., 2017; Rajeswaran et al., 2019) cast meta-learning as a bi-level optimization. In MAML, $\theta$ parametrizes a model $f_\theta : \mathcal{X} \rightarrow \mathcal{Y}$ (e.g. a neural network), and the inner algorithm $\text{Alg}(\theta, D)$ performs one (or more) steps of gradient descent minimizing the empirical risk of $\theta$ on $D$. Formally, given a step-size $\eta > 0$,

$$f_{\theta'} = \text{Alg}(\theta, D) \quad \text{with} \quad \theta' = \theta - \eta \nabla_\theta \mathcal{L}(f_\theta, D).$$

The meta-gradient $\nabla_\theta \text{Alg}(\theta, D)$ involves the second order derivatives of $\mathcal{L}(f_\theta, D)$ with respect to $\theta$, which may be expensive to compute, and cause potential training instability. Several MAML variants have focused on mitigating such issues Bertinetto et al. (2019); Rajeswaran et al. (2019).

**Meta-representation Learning.** Inspired by MAML, Meta-representation learning methods formulate meta-learning as the process of finding a shared representation to be fine-tuned for each task. Formally, they model the task predictor as a composite function $f_W \circ \psi_\theta : \mathcal{X} \rightarrow \mathcal{Y}$, with $\psi_\theta : \mathcal{X} \rightarrow \mathbb{R}^p$ a shared feature extractor meta-parametrized by $\theta$, and $f_W : \mathbb{R}^p \rightarrow \mathcal{Y}$ a map parametrized by $W$. The parameters $W$ are learned for each task as a function $W(\theta, D)$ via the inner algorithm

$$f_{W(\theta, D)} \circ \psi_\theta = \text{Alg}(\theta, D).$$

For example, in CAVIA Zintgraf et al. (2019) $\text{Alg}(\theta, D)$ performs one (or more) steps of gradient descent from an initial $W_0$, while keeping the meta-representation fixed

$$W(\theta, D) = W_0 - \eta \nabla_W \mathcal{L}(f_W \circ \psi_\theta, D)|_{W=W_0}.$$ 

Bertinetto et al. (2019) proposed $\text{Alg}(\theta, D)$ to perform empirical risk minimization of $f_W$ over $D = (x_i, y_i)^m_{i=1}$ with respect to the least-squares loss $\ell(y, y') = \|y - y'\|^2$. Assuming $\mathcal{Y} = \mathbb{R}^C$ and a linear model for $f_W$, this corresponds to performing ridge-regression on the features $\psi_\theta$, yielding the closed-form solution

$$W(\theta, D) = X_\theta^\top (X_\theta X_\theta^\top + \lambda I)^{-1} Y,$$  

where $\lambda_1 > 0$ is a regularizer. $X_\theta \in \mathbb{R}^{m \times p}$ and $Y \in \mathbb{R}^{m \times C}$ are matrices with $i$-th row corresponding to the $i$-th training input $\psi_\theta(x_i)$ and output $y_i$ in the dataset $D$, respectively. The closed-form solution (6) has the advantage of being i) efficient to compute and ii) suited for the computation of meta-gradients with respect to $\theta$. Indeed, $\nabla_\theta W(\theta, D)$ can be computed in closed-form or via automatic differentiation.

### 3 Conditional Meta-Learning

Although remarkably efficient in many applications, MAML and its variants implicitly assume that learning a single set of meta-parameters $\theta$ is sufficient for the entire family of tasks sampled from $\mu$. For example, the original MAML assumes the existence of a network $f_\theta$ such that all tasks sampled from $\mu$ can be solved by performing only one (or a few) steps of gradient descent from $\theta$. Such assumption may not hold in settings involving more complex meta-distributions $\mu$ (e.g. a multi-modal distribution). To address this, we consider *conditional* meta-learning as a potential solution, and our structured prediction perspective to implement it. Fig. 1 (Left column) illustrates this issue.

**Conditional Meta-learning.** Intuitively, meta-learning algorithms might solve a new task $D$ better if they...
Figure 1: **Conditional vs Unconditioned Meta-learning.** (Top) Example of a meta-learning problem where the tasks parameters (blue dots) that can be obtained by one application of Alg(\(\hat{\theta}, \cdot\)) from the learned meta-parameters \(\hat{\theta}\) (red dot). (Bottom) Meta-learning problem where tasks parameters are grouped in three “clusters”. Meta-learning is not able to get there with one application of Alg(\(\hat{\theta}\)). Conditional meta-learning chooses \(\hat{\theta} = \tau(D)\) depending on the target task and is able to handle also this setting.

were to “recall” and leverage the experiences most relevant to it. We formalize this by adapting (or condition) the meta-parameters \(\theta\) on \(D\). In particular, we consider a parametrization Alg(\(\tau(D), \cdot\)) with \(\tau(D) \in \Theta\) a meta-parameter valued function. We formulate conditional meta-learning as a generalization of (2) where we aim to minimize the risk

\[
E(\tau) = \mathbb{E}_\mu \mathbb{E}_\rho \mathcal{L} \left( \text{Alg}(\tau(D^{tr}), D^{tr}), D^{val} \right),
\]

over a suitable space of functions \(\tau: \mathcal{D} \to \Theta\) mapping datasets \(D\) to algorithms Alg(\(\tau(D), \cdot\)). Fig. 1 (Right) illustrates this idea. Note that (7) uses the same \(D^{tr}\) to both condition the meta-parameters and for the inner algorithm. More broadly, \(\tau\) may also depend on a separate conditioning dataset \(\tau(D^{con})\) or on contextual information, similar to settings such as collaborative filtering with side-information (see e.g. Abernethy et al., 2009). Below we review Meta-learning with latent embedding optimization (LEO) Rusu et al. (2019) as an example of conditional meta-learning.

**Latent Embedding Optimization.** Within our notation, LEO models \(\tau(D)\) as a relational network mapping a task to a latent space \(\Theta\). The predictor \(f_{\delta(\tau(D))}: \mathcal{X} \to \mathcal{Y}\) is obtained by mapping the latent meta-parameters into the parameter space via a learned decoder network \(\delta\). The inner Alg(\(\theta, \cdot\)) performs a few gradient steps in both the latent space \(\Theta\) and in the parameter space \(\delta(\Theta)\) to complete adaptation.

Conditional meta-learning leverages a finite number of meta-training task to learn \(\tau: \mathcal{D} \to \Theta\). While it may be feasible to address this problem in a standard supervised learning fashion, we stress that meta-learning poses unique challenges from both modeling and computational perspectives. A critical difference is the output set: in standard settings this is usually a linear space (namely \(\mathcal{Y} = \mathbb{R}^k\)), for which there exists several methods to parametrize suitable spaces of hypotheses \(f: \mathcal{X} \to \mathbb{R}^k\). In contrast, when the output space \(\Theta\) is a complicated, “structured” set (e.g. a space of deep learning architectures), it is less clear how to find a space of hypotheses \(\tau: \mathcal{D} \to \Theta\) and how to perform optimization over them. These settings however are precisely what the literature of structured prediction aims to address.
3.1 Structured Prediction for Meta-learning

Structured prediction methods are designed for learning problems where the output set is not a linear space but rather a set of structured objects such as strings, rankings, graphs, 3D structures Bakir et al. (2007); Nowozin et al. (2011). For conditional meta-learning, the output space is a set of inner algorithms parametrized by $\theta \in \Theta$. Directly modeling $\tau : \mathcal{D} \rightarrow \Theta$ can be challenging. A well-established strategy in structured prediction is therefore to first learn a joint function $T : \Theta \times \mathcal{D} \rightarrow \mathbb{R}$ that, in our setting, measures the quality of a model $\theta$ for a specific dataset $D$. The structured prediction estimator $\tau$ is thus defined as the function choosing the optimal $\tau(D) \in \Theta$ given the input dataset $D$

$$\tau(D) = \arg\min_{\theta \in \Theta} T(\theta, D). \quad (8)$$

Several strategies have been proposed to address the central question of how to model and learn the joint function $T$ (e.g. SVMStruct Tschantaridis et al. (2005), Maximum Margin Markov Networks Taskar et al. (2004)). Within this family of methods, Ciliberto et al. (2019) proposed an estimator with strong theoretical guarantees, such as consistency and learning rates. We propose to adopt this strategy to address conditional meta-learning. In Sec. 3.2 we will characterize the theoretical properties of the proposed estimator.

**Task-adaptive Structured Meta-Learning.** The approach in Ciliberto et al. (2019) builds upon a kernel Aronszajn (1950) $k : \mathcal{D} \times \mathcal{D} \rightarrow \mathbb{R}$ on the space of datasets (see Sec. 4.2 for an example). Given a meta-training dataset $S = (D^t_i, D^v_i)_{i=1}^N$ the general structured prediction estimator from (8) is formulated as

$$\tau(D) = \arg\min_{\theta \in \Theta} \sum_{i=1}^N \alpha_i(D) \mathcal{L}(\text{Alg}(\theta, D^t_i), D^v_i) \quad \text{with} \quad \alpha(D) = (K + \lambda_2 I)^{-1} v(D) \in \mathbb{R}^N,$$

where $\lambda_2 > 0$ is a regularizer, $\alpha_i(D)$ denotes the $i$-th entry of the vector $\alpha(D)$ while $K \in \mathbb{R}^{N \times N}$ and $v(D) \in \mathbb{R}^N$ are the kernel matrix and evaluation vector with entries $K_{i,j} = k(D^t_i, D^t_j)$ and $v(D)_i = k(D^t_i, D)$, respectively. We note that (9) is an instance of (8), where the joint functional $T$ is modelled according to Ciliberto et al. (2019) and learned on the meta-training set $S$.

We refer to the estimator in (9) as *Task-adaptive Structured Meta-Learning (TASML)*. It consists in solving a weighted meta-learning problem, where the $\alpha : \mathcal{D} \rightarrow \mathbb{R}^N$ can be interpreted as a “scoring” function that identifies the training tasks more relevant to the target one and encourages the candidate $\theta$ to fit them. The structured prediction process is therefore divided into two distinct phases: a *learning* phase for estimating the scoring function $\alpha$ and ii) a *prediction* phase for where we obtain $\tau(D)$ by solving (9) on $D$.

**Remark 1 (Connection with MAML).** The objective in (9) recovers the empirical risk minimization for meta-learning introduced in (4), if we set a constant function $\alpha_i(D) \equiv 1$. This implies that the methods from Sec. 2 – such as MAML – can be interpreted as conditional meta-learning algorithms that assume all tasks to be equally related to one other.

3.2 Theoretical Properties

We characterize TASML’s learning rate in Thm. 1. These, indicate how fast we can expect the generalization error of $\tau$ to decrease as the number $N$ of meta-training tasks grows.

**Theorem 1 (Informal – Learning Rates for TASML).** Let $S = (D^t_i, D^v_i)_{i=1}^N$ be sampled from a meta-distribution $\mu$ and $\tau_N$ the estimator in (9) trained with $\lambda_2 = N^{-1/2}$ on $S$. Then, with high probability with respect to $\mu$,

$$\mathcal{E}(\tau_N) - \inf_{\tau : D \rightarrow \Theta} \mathcal{E}(\tau) \leq O(N^{-1/4}). \quad (10)$$
The result above shows in particular that the proposed algorithm asymptotically yields the best possible task-conditional estimator for the family of tasks identified by $\mu$. The proof of Thm. 1 leverages recent results from the literature on structured prediction Luise et al. (2018) combined with standard regularity assumptions on the meta-distribution $\mu$. A formal proof and further discussion on the relation between TASML and general structured prediction is available in Appendix A.

4 A Practial Algorithm for TASML

In this section we introduce a practical and efficient algorithm for TASML, followed by the specific model and implementation details used for the experiments in Sec. 5.

4.1 Model Modification

The proposed TASML estimator $\tau : D \rightarrow \Theta$ offers a principled approach to conditional meta-learning. However, As observed in Remark 1, the cost of solving (9) once is similar to learning a un-conditioned meta-learning model for existing methods like MAML. Having to repeatedly solve (9) for each target task $D$ could be computationally expensive, in particular when the number $N$ of meta-training tasks is large. To mitigate this, below we discuss several adjustments to TASML to yield a significant speed-up in practice.

Initialization by Meta-Learning. Following the observation in Remark 1, we propose to learn an “agnostic” $\hat{\theta} \in \Theta$ as the initial meta-parameters to be used for each subsequent application of TASML. Specifically, we obtain $\hat{\theta}$ by applying a standard (un-conditioned) meta-learning method solving (4) (see Sec. 2.1 or Sec. 4.2). To minimize (9), we first initialize the inner algorithm with $\hat{\theta}$, followed by gradient descent over the meta-parameters. We observed that, in practice, this significantly improves the speed of convergence to a stationary point of (9).

Top-$M$ Filtering. The weight $\alpha_i(D)$ from TASML measure the relevance of the $i$-th meta-training task $D_{tr}^i$ to the target task $D$. To improve the computational efficiency of minimizing (9), we propose to keep only the top-$M$ values from $\alpha(D)$, with $M$ a hyperparameter, and set the others to zero. This filtering effectively constrains task-conditioning to only those meta-training tasks $D_{tr}^i$ most relevant to $D$. In practice, we have observed that when the total number $N$ of training tasks is large (e.g. $N > 10K$), setting $M$ around 1% of $N$ offers a good trade-off between speed and accuracy (see also Sec. 5.2).

Task-adaptation. The structured output $\tau(D)$ depends on the task $D$ only indirectly, via the weights $\alpha(D)$. To make $\tau$ use $D$ more directly, we propose the following variant of (9),

$$
\tau(D) = \arg\min_{\theta \in \Theta} \sum_{i=1}^{N} \alpha_i(D) \mathcal{L}(\text{Alg}(\theta, D_{tr}^i), D_{val}^{i}) + \lambda_3 \mathcal{L}(\text{Alg}(\theta, D), D),
$$

where we added the “regularizer” $\mathcal{L}(\text{Alg}(\theta, D), D)$ to encourage a candidate $\theta$ to achieve small empirical error also on the target task $D$. In principle, this additional term may promote overfitting on $D$, which is undesirable since the goal of meta-learning is to generalize well on validation set $D'$ different from $D$ (albeit sampled from the same distribution). However, in our experiments, we found that there exists usually a wide range of values of $\lambda_3 > 0$ for which this additional term actually grants a significant boost in performance without overfitting.

Alg. 1 outlines the implementation of TASML combined with the improvements presented in this section. During an initialization phase, given a meta-training set $S = (D_{tr}^1, D_{val}^{i=1})_1^N$, we compute the meta-parameters $\hat{\theta}_i$ by applying SGD to solve the (un-conditioned) meta-learning problem in (4). We also learn the weighting function $\alpha$ according to (9), which consists of inverting the training kernel matrix $K$. While in principle this is an expensive step of up to $O(N^3)$ in complexity, sketching methods can be adopted to significantly speed-up this process without loss of accuracy Rudi et al. (2017). When a new target task $D$ is presented thereafter, the weights $\alpha(D)$ are evaluated and the top-$M$ tasks $S_M \subset S$ with largest $\alpha_i(D)$ are kept. A first-order method is
Algorithm 1 Task-adaptive Structured Meta-learning

Require: meta-training set $S = (D_i^{tr}, D_i^{val})_{i=1}^N$, Filter size $M$, batch size $B$, number of steps $J$, step-size $\eta$.

Initialization:
\[ \hat{\theta} = \text{MetaLearning}(S, \eta) \quad // \text{solving (4).} \]
Learn $\alpha : D \to \mathbb{R}^N$ // as in (9).

Function TASML(Dataset $D$):
Compute $\alpha(D) \in \mathbb{R}^N$.
Take $S_M \subset S$ the $M$ tasks with highest scores $\alpha(D)$.
Let $\theta \leftarrow \hat{\theta}$
For $j = 1, \ldots, J$:
Sample uniformly a mini-batch $S_B \subset S_M$ of size $B$.
Compute the meta-gradient $\nabla_\theta$ of (11) over $S_B$.
$\theta \leftarrow \theta - \gamma \nabla_\theta$ // or via e.g. ADAM.
\[ \text{return } \theta \]

then applied to minimize (11) over the reduced set $S_M$, starting from the meta-parameter $\hat{\theta}$ learned during initialization.

4.2 Implementation Details

TASML is a general algorithm applicable to a wide range of (meta-parameterized) inner algorithms $\text{Alg}(\theta, \cdot)$. In this section we describe the specific implementation of Alg. 1 used for our experiments in Sec. 5.

Model Architecture. We consider a meta-representation learning model of the form $f_W \circ \psi_\theta : \mathcal{X} \to \mathcal{Y}$ introduced in Sec. 2.1. In particular:

- The meta-representation architecture $\psi_\theta : \mathcal{X} \to \mathbb{R}^p$ is a two-layer fully-connected network with residual connection He et al. (2016).
- The task predictor $f_W : \mathbb{R}^p \to \mathcal{Y}$ is a linear model $f_W(\psi_\theta(x)) = W \psi_\theta(x)$ with $W \in \mathbb{R}^{C \times p}$ the model parameters. We assume $\mathcal{Y} = \mathbb{R}^C$ (e.g. one-hot encoding of $C$ classes in classification settings).
- The inner algorithm is $f_{W(\theta, D)} \circ \psi_\theta = \text{Alg}(\theta, D)$, where $W(\theta, D)$ is the least-squares closed-form solution introduced in (6).
- The task loss $\mathcal{L}$ is also induced by the least-squares $\ell(y, y') = \|y - y'\|^2$, according to (3).

Similar to Bertinetto et al. (2019), we chose the least-squares empirical risk minimizer as our inner algorithm. However, we note that Bertinetto et al. (2019) uses the cross-entropy $\ell$ to induce $\mathcal{L}$. Consequently, when optimizing the meta-parameters $\theta$, the performance of $W(\theta, D)$ are measured on a validation set $D'$ with respect to a loss function (cross-entropy) different from the one used to learn it (least-squares). This incoherence between inner- and meta-problems can lead to sub-optimal performance in practice (see Sec. 5.2). We note that while least-squares minimization is not a standard approach in classification settings, it is theoretically principled (see e.g. Bartlett et al., 2006; Mroueh et al., 2012).

Reproducing Kernel on Datasets. A key ingredient for TASML is the positive definite kernel $k : \mathcal{D} \times \mathcal{D} \to \mathbb{R}$, which is essential in (9) to learn the score function $\alpha : \mathcal{D} \to \mathbb{R}^N$. In this work we take $k$ to be the Gaussian kernel of the maximum mean discrepancy (MMD) Gretton et al. (2012) of two datasets. The MMD is a commonly used distance on datasets or distributions. More precisely, given a dataset $D = (x_j, y_j)_{j=1}^m$ and a
feature map \( \varphi : \mathcal{X} \to \mathbb{R}^p \), denote the mean embedding of \( D \) in \( \mathbb{R}^p \) as
\[
\varphi(D) = \frac{1}{m} \sum_{j=1}^{m} \varphi(x_j),
\]
(12)

Then, given a bandwidth parameter \( \sigma > 0 \), for any two datasets \( D_1, D_2 \in \mathcal{D} \), we define our kernel as
\[
k(D_1, D_2) = \exp \left( -\frac{\| \varphi(D_1) - \varphi(D_2) \|^2}{\sigma^2} \right).
\]
(13)

The map \( \varphi \) plays a central role. It can be either fixed a-priori or learned, depending on the application. Below, we describe the feature map \( \varphi \) used in our experiments.

**Pre-trained Feature Map.** In general, having an expressive representation of input data is key to good performance. This is feasible in some meta-learning settings via a suitable auxiliary process. For instance, Rusu et al. (2019); Qiao et al. (2018) learned an image representation of by pre-training a state-of-the-art ResNet classifier He et al. (2016) using only meta-training data.

We adopt a similar approach in our experiments. Specifically, we chose \( \varphi \) for the kernel in (13) to be the publicly available pre-trained feature maps on mini and tieredImageNet from Rusu et al. (2019). Additionally, we also employed the same feature maps as a pre-processing step to TASML. Our model architecture is thus defined as \( f_W \circ \psi \circ \varphi : \mathcal{X} \to \mathcal{Y} \), with the pre-trained \( \varphi \) fixed throughout Alg. 1. The ablation study in Sec. 5.2 quantifies the impact of this pre-processing step on performance.

5 Experiments

We perform experiments on \( C \)-way-\( K \)-shot learning within the episodic formulation of Vinyals et al. (2016). In this setting, train-validation pairs \( (D^{tr}, D^{val}) \) are sampled as described in Sec. 2.1. \( D^{tr} \) is a \( C \)-class classification problem with \( K \) examples per class. \( D^{val} \) contains samples from the same \( C \) classes for estimating model generalization and training meta learner. We evaluate the proposed method against a wide range of meta-learning algorithms on two few-shot learning benchmarks: the miniImageNet and tieredImageNet datasets. We consider the commonly used 5-way-1-shot and 5-way-5-shot settings. For training, validation and testing, we sample three separate meta-datasets \( S^{tr}, S^{val} \) and \( S^{ts} \), each accessing a disjoint set of classes (e.g. no class in \( S^{ts} \) appears in \( S^{tr} \) or \( S^{val} \)). To ensure fair comparison with the other methods, we adopted the same training and evaluation setup as Rusu et al. (2019). Further experimental details including network specification and hyperparameter choices are available in Appendix B.

5.1 Performance Comparison

Table 1 report TASML’s performance compared with a representative set of meta-learning methods: MAML Finn et al. (2017), iMAML Rajeswaran et al. (2019), REPTILE Nichol et al. (2018), BerTinetto et al. (2019); Qiao et al. (2018), CAVIA Zintgraf et al. (2019), LEO Rusu et al. (2019) and META-SGD Li et al. (2017) with LEO’s feature maps \( \varphi \) as input (from Rusu et al., 2019). We include results from our local replication of LEO, using the official implementation with a sparser grid search for hyperparameters. Indeed, LEO appeared sensitive to hyperparameter choices, and obtaining the values recommended in the original work was beyond our computational budget.

All results, except for LEO (local) and TASML, are cited directly from their respective papers. The tables report the average accuracy and standard deviation of the tested methods over 50 runs, with each run containing 200 random test tasks. We observe that TASML outperforms the baselines in three out of the four settings. In the remaining one, our method only lags behind LEO and outperforms the rest. The results suggest the efficacy of the proposed method. Further, We performed very limited tuning for our model, suggesting room for further improvements.

To exclude the effects of the pre-trained feature maps, we highlight the comparison with methods using the same feature maps, namely META-SGD and LEO variants. This shows that some improvements can be
further attributed to algorithmic and model choices, such as the structured prediction formulation, and the least-square objective. In the ablation study below, we study the respective contributions of individual model components in more details.

Table 1: Accuracy comparison on miniImageNet and tieredImageNet datasets. TASML outperforms other baselines.

| Accuracy (%) | miniImageNet | tieredImageNet |
|--------------|--------------|----------------|
|              | 1-shot       | 5-shot         | 1-shot       | 5-shot         |
| MAML         | 48.70 ± 1.84 | 63.11 ± 0.92  | 51.67 ± 1.81 | 70.30 ± 0.08  |
| iMAML        | 49.30 ± 1.88 | -              | -            | -              |
| Reptile      | 49.97 ± 0.32 | 65.99 ± 0.58  | -            | -              |
| Bertinetto et al. (2019) | 51.90 ± 0.20 | 68.70 ± 0.20  | -            | -              |
| CAVIA        | 51.82 ± 0.65 | 65.85 ± 0.55  | -            | -              |
| Qiao et al. (2018) | 59.60 ± 0.41 | 73.74 ± 0.19  | -            | -              |
| Meta-SGD (LEO feat.) | 54.24 ± 0.03 | 70.86 ± 0.04  | 62.95 ± 0.03 | 79.34 ± 0.06  |
| LEO (local)  | 60.37 ± 0.74 | 75.36 ± 0.44  | 65.11 ± 0.72 | 79.70 ± 0.59  |
| LEO (orig.)  | 61.76 ± 0.08 | 77.59 ± 0.12  | 66.33 ± 0.05 | 81.44 ± 0.09  |
| TASML        | 62.04 ± 0.72 | 78.22 ± 0.47  | 65.87 ± 0.69 | 82.92 ± 0.61  |

Table 2: Accuracy of the three main components (Sec. 5.2) of TASML (bottom three rows) compared with meta-learning baselines.

| Accuracy (%) | miniImageNet | tieredImageNet |
|--------------|--------------|----------------|
|              | 1-shot       | 5-shot         | 1-shot       | 5-shot         |
| MAML         | 48.70 ± 1.84 | 63.11 ± 0.92  | 51.67 ± 1.81 | 70.30 ± 0.08  |
| Bertinetto et al. (2019) | 51.90 ± 0.20 | 68.70 ± 0.20  | -            | -              |
| Meta-SGD (LEO feat.) | 54.24 ± 0.03 | 70.86 ± 0.04  | 62.95 ± 0.03 | 79.34 ± 0.06  |
| LEO (local)  | 60.37 ± 0.74 | 75.36 ± 0.44  | 65.11 ± 0.72 | 79.70 ± 0.59  |
| LEO (orig.)  | 61.76 ± 0.08 | 77.59 ± 0.12  | 66.33 ± 0.05 | 81.44 ± 0.09  |
| Pre-trained Features | 51.37 ± 0.39 | 69.91 ± 0.21  | 57.23 ± 0.35 | 78.48 ± 0.27  |
| LS Meta-Learning | 60.19 ± 0.65 | 76.76 ± 0.43  | 64.32 ± 0.65 | 81.43 ± 0.55  |
| TASML        | 62.04 ± 0.72 | 78.22 ± 0.47  | 65.87 ± 0.69 | 82.92 ± 0.61  |

5.2 Ablation Study

Conceptually, TASML is structured in three distinct phases:

i) Pre-Trained Features. Data representation is learned for the family of tasks considered.

ii) LS Meta-Learning. Shared meta-parameters $\hat{\theta}$ are learned for all tasks.

iii) TASML. Given a target task $D$, $\tau(D)$ is learned via TASML by solving (11) starting from $\hat{\theta}$.

In Table 2 (three bottom rows), we report an ablation study isolating the performance of the three phases outlined above. For instance, the pre-trained features achieve 69.91% on miniImageNet’s 5-way-5-shot setting,
NS Meta-Learning brings it to 76.76% and finally, the structured prediction phase improves performance further to 78.22%. We discuss each phase in detail below.

Pre-trained Features. To assess the quality of the pre-trained feature maps, we solve each target task in isolation via (6). This corresponds to fixing the meta-representation to the identity $\psi_\theta(x) = x$. PRE-TRAINED FEATURES in Table 2 shows that the pre-trained features $\varphi$ from Rusu et al. (2019) already provide an expressive representation for few-shot learning. Surprisingly, this algorithm significantly outperforms the original MAML architecture, without relying on meta-learning at all. These results indicate that it may be beneficial, when possible, to learn a good data representation prior to performing meta-learning, as optimization-based meta-learning is often computationally expensive and sensitive to the model architectures Antoniou et al. (2019).

LS Meta-Learning. We study the effect of using the least-squared closed-form solution $W(\theta, D)$ from (6) for the inner algorithm in our work. LS Meta-Learning in Table 2 reports the performance of the model in Sec. 4.2 before the structured prediction stage. We compare it with Meta-SGD and LEO, since all three methods use the same pre-trained features $\varphi$. We note that LS Meta-Learning is comparable or outperforms its competitors, even if it uses a significantly simpler architecture (the two-layer residual block $\psi_\theta$ introduced in Sec. 4.2). This suggests that the inner algorithm performing least-squares minimization is indeed very beneficial.

Table 2 also offers a comparison between LS Meta-Learning and Bertinetto et al. (2019). As discussed in Sec. 4.2 the two methods use same inner algorithm (empirical risk minimization with respect to the least-square loss) but different task loss functions (least squares for ours and cross-entropy for Bertinetto et al. (2019)). We note that our approach performs significantly better$^2$. A possible explanation is that, by employing the same loss on both training and validation data, we obtain more coherent models.

Structured Prediction. The structured prediction phase consistently improves the overall performance. Specifically, Table 2 shows that TASML yields an absolute gain of more than 1.5% classification accuracy across all datasets (e.g. from 60.16% to 62.04% for 5-way-1-shot on miniImageNet). While a potential limitation of this phase is additional computational cost, we show in Sec. 5.3 that our proposed implementation is reasonably efficient when compared with its closest competitors.

5.3 Model Efficiency

We compared the sample and computational efficiency of LEO and TASML, which share same experimental setup and achieve competitive performance across all tasks.

Sample Efficiency. We use model validation performance as a proxy for sample efficiency. Fig. 2 shows the validation performance of LEO and the LS META-LEARNING phase in TASML with respect to number training steps on the ImageNet benchmarks, averaged over 5 runs. In all settings, TASML is very efficient to train, as it achieves highest validation accuracy within a few thousand steps. We note that TASML’s overfitting to the training tasks on miniImageNet is not an issue since the validation performance is precisely used for model selection. Indeed, the test performance reported in Sec. 5.1 were obtained with the models yielding highest validation accuracy. In contrast, LEO takes much longer to converge, with the figure only showing the first 30K out of 100K steps. We attribute the sample efficiency of our model mainly to the fast adaptation provided by the least-squares closed-form solution, combined with the relatively simpler network $\psi_\theta$.

Computational Efficiency. To quantify computational efficiency of the two methods, Fig. 3a reports the average number of meta-gradient steps per second performed by our method and LEO. Experiments were performed on a commodity desktop machine with a single Nvidia GTX 2080. We note that TASML is at least twice as fast as LEO since the model is both simpler and admits efficient meta-gradient computation with

$^2$We tested our method with a cross entropy meta loss and achieved results similar to Bertinetto et al. (2019).
Figure 2: Validation performance between TASML and LEO. TASML obtains best validation performance significantly faster.

(a) Meta-gradient steps per second on 5-shot tasks

| steps/sec | miniIMAGE NET | tieredImageNET |
|-----------|--------------|----------------|
| LEO       | 7.52 ± 0.19  | 6.95 ± 0.47    |
| TASML     | 17.82 ± 0.27 | 14.71 ± 0.34   |

Figure 3

(b) Average test task performance over 500 structured prediction steps. Performances improve in all settings.

Structured Prediction. Sec. 5.3 reports the classification accuracy of TASML when minimizing the structured prediction functional in (11), with respect to the number of training steps $J$. The initial point ($J = 0$) of each curve corresponds to the performance of LS Meta-Learning.

Aside from the slight decrease in performance on 1-shot miniIMAGE NET after 50 steps, TASML shows a consistent and stable performance improvements over the 500 steps via structured prediction. This suggests that depending on the computational constraints of an application, it is possible to choose the desired trade-off between performance improvement and the number $J$ of structured prediction steps. In particular we note that 100 structured prediction steps – after which we observe the largest improvement in general – take about 6 seconds on average (see Fig. 3a).

Useful Practices for Meta-learning. From the results above, we summarize several generally useful practices for meta-learning: i) feature pre-training for improving model performance and training stability, ii) inner algorithms with closed-form solutions for model efficiency, and iii) consistent objectives between inner-
and meta-problems.

6 Conclusion

We proposed a novel perspective conditional on meta-learning based on structured prediction. Within this context, we presented a novel algorithm for task-adaptive structured meta-learning, that could better solves a new task by leveraging the most relevant experiences. Differing from most previous methods, TASML can both learn a set of meta-parameters generally useful for a family of tasks, and adapt to each target task at test time. Experimental evaluation over two benchmarks demonstrated the efficacy and effectiveness of our method compared with the state-of-the-art. Possible future works include meta-learning of task signatures to improve the scoring function, and investigating inner algorithms more powerful than the least-squares solver.

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Supplementary Material: A Structured Prediction Approach for Conditional Meta-Learning

The Appendix is organized in two main parts:

- Appendix A Where we provide additional details on the connection between structured prediction and conditional meta-learning investigated in this work.
- Appendix B Where we provide additional details on the model hyperparameters and additional experimental evaluation.

A Structured Prediction for Conditional Meta-learning

We first recall the general formulation of the structured prediction approach in Ciliberto et al. (2019) and then show how the conditional meta-learning problem introduced in Sec. 3 can be cast within this setting.

A.1 General Structured Prediction

In this section we borrow from the notation of Ciliberto et al. (2019). Consider \( \mathcal{X}, \mathcal{Y}, \) and \( \mathcal{Z} \) three spaces, respectively the input, label and output sets of our problem. We make a distinction between label and output space since conditional meta-learning can be formulated within the setting described below by taking \( \mathcal{Z} \) to be the meta-parameter space \( \Theta \) and \( \mathcal{Y} \) the space \( \mathcal{D} \) of datasets.

Structured prediction methods address supervised learning problems where the goal is to learn a function \( f: \mathcal{X} \to \mathcal{Z} \) taking values in a “structured” space \( \mathcal{Z} \). Here, the term structured is general and essentially encompasses output sets of strings, graphs, points on a manifold, probability distributions, etc. Formally, these are all spaces that are not linear or do not have a canonical embedding into a linear space \( \mathbb{R}^k \).

As we will discuss in the following, the lack of linearity on \( \mathcal{Z} \) poses concrete challenges on modeling and optimization. In contrast, formally, the target learning problem is cast as a standard supervised learning problem of the form (1). More precisely, given a distribution \( \rho \) on \( \mathcal{X} \times \mathcal{Y} \)

\[
\min_{f: \mathcal{X} \to \mathcal{Z}} \mathcal{E}(f) \quad \text{with} \quad \mathcal{E}(f) = \int \Delta(f(x), y|x) \, d\rho(x, y),
\]

where \( \Delta: \mathcal{Z} \times \mathcal{Y} \times \mathcal{X} \to \mathbb{R} \) is a loss function measuring prediction errors. Note that \( \Delta(z, y|x) \) does not only compare the predicted output \( z \in \mathcal{Z} \) with the label \( y \in \mathcal{Y} \), but does that also depending or conditioned on the input \( x \in \mathcal{X} \) (hence the notation \( \Delta(z, y|x) \) rather than \( \Delta(z, y, x) \)). These conditioned loss functions were originally introduced to account for structured prediction settings where prediction errors depend also on properties of the input. For instance in ranking problems or in sequence-to-sequence translation settings, as observed in Ciliberto et al. (2019).

Structured Prediction Algorithm\(^3\). Given a finite number \( n \in \mathbb{N} \) of points \( (x_i, y_i)_{i=1}^n \) independently sampled from \( \rho \), the structured prediction algorithm proposed in Ciliberto et al. (2019) is an estimator \( \hat{f}: \mathcal{X} \to \mathcal{Z} \) such that, for every \( x \in \mathcal{X} \)

\[
\hat{f}(x) = \arg\min_{z \in \mathcal{Z}} \sum_{i=1}^n \alpha_i(x) \Delta(z, y_i|x_i).
\]

\(^3\)We note that in the original work, the authors considered a further parametrization of the loss \( \Delta \) leveraging the concept of locality and parts. This led to the derivation of a more general (and involved) characterization of the estimator \( \hat{f} \). However, for the setting considered in this work we consider a simplified scenario (see Appendix A.2 below) and we can therefore restrict to the case where the loss does not assume a factorization into parts, namely the set of parts \( P \) corresponds to \( P = \{1\} \) the singleton, leading to the structured prediction estimator (15).

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where, given a reproducing kernel $k : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$, the weights $\alpha$ are obtained as

$$
\alpha(x) = (\alpha_1(x), \ldots, \alpha_n(x))^\top \in \mathbb{R}^n \quad \text{with} \quad \alpha(x) = (\mathbf{K} + \lambda \mathbf{I})^{-1} \, v(x),
$$

where $\mathbf{K} \in \mathbb{R}^{n \times n}$ is the empirical kernel matrix with entries $K_{ij} = k(x_i, x_j)$ and $v(x) \in \mathbb{R}^n$ is the evaluation vector with entries $v(x)_i = k(x, x_i)$, for any $i, j = 1, \ldots, n$ and $\lambda > 0$ is a hyperparameter.

The estimator above has a similar form to the TASML algorithm proposed in this work in (9). In the following we show that the latter is indeed a special case of (15).

### A.2 A Structured Prediction perspective on Conditional Meta-learning

In the conditional meta-learning setting introduced in Sec. 3 the goal is to learn a function $\tau : \mathcal{D} \to \Theta$ where $\mathcal{D}$ is a space of datasets and $\Theta$ a space of learning algorithms. We define the conditional meta-learning problem according to the expected risk (7) as

$$
\min_{\tau : \mathcal{D} \to \Theta} \mathcal{E}(\tau) \quad \text{with} \quad \mathcal{E}(\tau) = \int \mathcal{L} \left( \text{Alg}(\tau(D^{\text{tr}}), D^{\text{val}}), D^{\text{val}} \right) \, d\pi(D^{\text{tr}}, D^{\text{val}}),
$$

where $\pi$ is a probability distribution sampling the pair of train and validation datasets $D^{\text{tr}}$ and $D^{\text{val}}$. We recall that the distribution $\pi$ samples the two datasets according to the process described in Sec. 2.1, namely by first sampling $\rho$ a task-distribution (on $\mathcal{X} \times \mathcal{Y}$) from $\mu$ and then obtaining $D^{\text{tr}}$ and $D^{\text{val}}$ by independently sampling points $(x, y)$ from $\rho$. Therefore $\pi = \pi_{\mu}$ can be seen as implicitly induced by $\mu$. In practice, we have only access to a meta-training set $S = \{(D^{\text{tr}}_1, D^{\text{val}}_1), \ldots, (D^{\text{tr}}_{N}, D^{\text{val}}_{N})\}$ of train-validation pairs sampled from $\pi$.

We are ready to formulate the conditional meta-learning problem within the structured prediction setting introduced in Appendix A.1. In particular, we take the input and label spaces to correspond to the set $\mathcal{D}$ and choose as output set the space $\Theta$ of meta-parameters. In this setting, the loss function is of the form $\Delta : \Theta \times \mathcal{D} \times \mathcal{D} \to \mathbb{R}$ and corresponds to

$$
\Delta(\theta, D^{\text{val}}|D^{\text{tr}}) = \mathcal{L} \left( \text{Alg}(\theta, D^{\text{tr}}), D^{\text{val}} \right).
$$

Therefore, we can interpret the loss $\Delta$ as the function measuring the performance of a meta-parameter $\theta$ when the corresponding algorithm $\text{Alg}(\theta, \cdot)$ is trained on $D^{\text{tr}}$ and then tested on $D^{\text{val}}$. Under this notation, it follows that (17) is a special case of the structured prediction problem (14). Therefore, casting the general structured prediction estimator (15) within this setting yields the TASML estimator proposed in this work and introduced in (9), namely $\tau_N : \mathcal{D} \to \Theta$ such that, for any dataset $D \in \mathcal{D}$,

$$
\tau_N(D) = \arg\min_{\theta \in \Theta} \sum_{i=1}^N \alpha_i(D) \, \mathcal{L} \left( \text{Alg}(\theta, D^{\text{tr}}_i), D^{\text{val}}_i \right),
$$

where $\alpha : \mathcal{D} \to \mathbb{R}^N$ is learned according to (16), namely

$$
\alpha(x) = (\alpha_1(x), \ldots, \alpha_N(x))^\top \in \mathbb{R}^N \quad \text{with} \quad \alpha(x) = (\mathbf{K} + \lambda \mathbf{I})^{-1} \, v(D),
$$

with $\mathbf{K}$ and $v(D)$ defined as in (9). Hence, we have recovered $\tau_N$ as it was introduced in this work.

### A.3 Theoretical Analysis

In this section we prove Thm. 2. Our result can be seen as a corollary of (Thm.5 Ciliberto et al., 2016) applied to the generalized structured prediction setting of Appendix A.1. The result hinges on two regularity assumptions on the loss $\Delta$ and on the meta-distribution $\pi$ that we introduce below.

**Assumption 1.** The loss $\Delta$ is of the form (18) and admits derivatives of any order, namely $\Delta \in C^\infty(\mathcal{Z} \times \mathcal{Y} \times \mathcal{X})$. 

Recall that by (18) we have

$$\mathcal{L}(\theta, D^{val}, D^{tr}) = \frac{1}{|D^{val}|} \sum_{(x,y) \in D^{val}} \ell\left( \left[ \text{Alg}(\theta, D^{tr}) \right](x), y \right). \quad (19)$$

Therefore, sufficient conditions for Assumption 1 to hold are: i) the inner loss function $\ell$ is smooth (e.g. least-squares, as in this work) and ii) the inner algorithm Alg$(\cdot, \cdot)$ is smooth both with respect to the meta-parameters $\theta$ and the training dataset $D^{tr}$. For instance, in this work, Assumption 1 is verified if the meta-representation network $\psi_\theta$ is smooth with respect to the meta-parametrization $\theta$. Indeed, $\ell$ is chosen to be the least-squares loss and the closed form solution $W(\theta, D^{tr}) = X_0^\top (X_0X_0^\top + \lambda I)^{-1}Y$ in (6) is smooth for any $\lambda > 0$.

The second assumption below concerns the regularity properties of the meta-distribution $\pi$ and its interaction with the loss $\triangle$. The assumption leverages the notion of Sobolev spaces. We recall that for a set $K \subset \mathbb{R}^d$ the Sobolev space $W^{s,2}(K)$ is the Hilbert space of functions from $K$ to $\mathbb{R}$ that have square integrable weak derivatives up to the order $s$. We recall that if $K$ satisfies the cone condition, namely there exists a finite cone $C$ such that each $x \in K$ is the vertex of a cone $C_x$ contained in $K$ and congruent to $C$ (Adams & Fournier, 2003, Def. 4.6), then for any $s > d/2$ the space $W^{s,2}(K)$ is a RKHS. This follows from the Sobolev imbedding theorem (Adams & Fournier, 2003, Thm. 4.12) and the properties of RKHS (see e.g. Berlinet & Thomas-Agnan, 2011, for a detailed proof).

Given two Hilbert spaces $\mathcal{H}$ and $\mathcal{F}$, we denote by $\mathcal{H} \otimes \mathcal{F}$ the tensor product of $\mathcal{H}$ and $\mathcal{F}$. In particular, given two basis $(h_i)_{i \in \mathbb{N}}$ and $(f_j)_{j \in \mathbb{N}}$ for $\mathcal{H}$ and $\mathcal{F}$ respectively, we have

$$\langle h_i \otimes f_j, h_{i'} \otimes f_{j'} \rangle_{\mathcal{H} \otimes \mathcal{F}} = \langle h_i, h_{i'} \rangle_{\mathcal{H}}, \langle f_j, f_{j'} \rangle_{\mathcal{F}},$$

for every $i, i', j, j' \in \mathbb{N}$. We recall that $\mathcal{H} \otimes \mathcal{F}$ is a Hilbert space and it is isometric to the space $HS(\mathcal{F}, \mathcal{H})$ of Hilbert-Schmidt (linear) operators from $\mathcal{F}$ to $\mathcal{H}$ equipped with the standard Hilbert-Schmidt $\langle \cdot, \cdot \rangle_{HS}$ dot product. In the following, we denote by $T : \mathcal{H} \otimes \mathcal{F} \rightarrow HS(\mathcal{F}, \mathcal{H})$ the isometry between the two spaces.

We are ready to state our second assumption.

**Assumption 2.** Assume $\Theta \subset \mathbb{R}^{d_\Theta}$ and $\mathcal{D} \subset \mathbb{R}^{d_\mathcal{D}}$ compact sets satisfying the cone condition and assume that there exists a reproducing kernel $k : \mathcal{D} \times \mathcal{D} \rightarrow \mathbb{R}$ with associated RKHS $\mathcal{F}$ and $s > (d_\mathcal{D} + 2d_\Theta)/2$ such that the function $g^* : \mathcal{D} \rightarrow \mathcal{H}$ with $\mathcal{H} = W^{s,2}(\Theta \times \mathcal{D})$, characterized by

$$g^*(D^{tr}) = \int_\triangle \delta \cdot D^{val} \ | \cdot \rangle \ d\tau(D^{val}|D^{tr}) \quad \forall D^{tr} \in \mathcal{D}, \quad (20)$$

is such that $g^* \in \mathcal{H} \otimes \mathcal{F}$ and, for any $D \in \mathcal{D}$, we have that the application of the operator $T(g^*) : \mathcal{F} \rightarrow \mathcal{H}$ to the function $k(D, \cdot) \in \mathcal{F}$ is such that $T(g^*) k(D, \cdot) = g^*(D)$.

The function $g^*$ in (20) can be interpreted as capturing the interaction between $\triangle$ and the meta-distribution $\pi$. In particular, Assumption 2 imposes two main requirements: i) for any $D \in \mathcal{D}$ the output of $g^*$ is a vector in a Sobolev space (i.e. a function) of smoothness $s > (d_\mathcal{D} + 2d_\Theta)/2$, namely $g^*(D) \in W^{s,2}(\Theta \times \mathcal{D})$ and, ii) we require $g^*$ to correspond to a vector in $W^{s,2}(\Theta \times \mathcal{D}) \otimes \mathcal{F}$. Note that the first requirement is always satisfied if Assumption 1 holds. The second assumption is standard in statistical learning theory (see e.g. Shalev-Shwartz & Ben-David, 2014; Caponnetto & De Vito, 2007, and references therein) and can be interpreted as requiring the conditional probability $\pi(\cdot|D^{tr})$ to not vary dramatically for small perturbations of $D^{tr}$.

We are ready to state and prove our main theorem, whose informal version is reported in Thm. 1 in the main text.

**Theorem 2 (Learning Rates).** Under Assumptions 1 and 2, let $S = (D_i^{tr}, D_i^{val})_{i=1}^N$ be a meta-training set of points independently sampled from a meta-distribution $\pi$. Let $\tau_N$ be the estimator in (9) trained with $\lambda_2 = N^{-1/2}$ on $S$. Then, for any $\delta \in (0, 1]$ the following holds with probability larger or equal than $1 - \delta$,

$$\mathcal{E}(\tau_N) - \inf_{\tau : \mathcal{D} \rightarrow \Theta} \mathcal{E}(\tau) \leq c \log(1/\delta) \ N^{-1/4}, \quad (21)$$

where $c$ is a constant depending on $\kappa^2 = \sup_{D \in \mathcal{D}} k(D, D)$ and $\|g^*\|_{\mathcal{H} \otimes \mathcal{F}}$ but independent of $N$ and $\delta$. 

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Proof. Let $\mathcal{H} = W^{s,2}(\Theta \times \mathcal{D})$ and $\mathcal{G} = W^{s,2}(\mathcal{D})$. Since $s > (d_1 + 2d_2)/2$, both $\mathcal{G}$ and $\mathcal{H}$ are reproducing kernel Hilbert spaces (RKHS) (see discussion above or Berlinet & Thomas-Agnan, 2011). Let $\psi : \Theta \times \mathcal{D} \to \mathcal{H}$ and $\varphi : \mathcal{D} \to \mathcal{G}$ be two feature maps associated to $\mathcal{H}$ and $\mathcal{G}$ respectively. Without loss of generality, we can assume the two maps to be normalized.

We are in the hypotheses\(^1\) of (Thm. 7 Luise et al., 2018), which guarantees the existence of a Hilbert-Schmidt operator $V : \mathcal{G} \to \mathcal{H}$, such that $\Delta$ can be characterized as

$$\triangle(\theta, D^{val}|D^{tr}) = \langle \psi(\Theta, D^{tr}), V \varphi(D^{val}) \rangle_{\mathcal{H}}$$

(22)

for any $D^{tr}, D^{val} \in \mathcal{D}$ and $\theta \in \Theta$. Since the feature maps $\varphi$ and $\psi$ are normalized Berlinet & Thomas-Agnan (2011), this implies also $\|V\|_{\text{HS}} = \|\triangle\|_{s,2} < +\infty$, namely that the Sobolev norm of $\Delta$ in $W^{s,2}(\Theta \times \mathcal{D} \times \mathcal{D})$ is equal to the Hilbert-Schmidt norm of $V$.

The result in (22) corresponds to the definition of Structure Encoding Loss Function (SELF) in (Ciliberto et al., 2019, Def. 1). Additionally, if we denote $\tilde{\varphi} = V \varphi$, we obtain the equality

$$g^*(D^{tr}) = \int \tilde{\varphi}(D^{val}) \, d\pi_{\imath}(D^{val}|D^{tr}) = \int \triangle(\cdot, D^{val} | \cdot) \, d\pi_{\imath}(D^{val}|D^{tr}),$$

(23)

for all $D^{tr} \in \mathcal{D}$, where $g^* : \mathcal{D} \to \mathcal{H}$ is defined as in Assumption 2, we are in the hypotheses of the comparison inequality theorem (Ciliberto et al., 2019, Thm. 9). In our setting, this result states that for any measurable function $g : \mathcal{D} \to \mathcal{H}$ and the corresponding function $\tau_g : \mathcal{D} \to \Theta$ defined as

$$\tau_g(D) = \arg\min_{\theta \in \Theta} \langle \psi(\theta, D), g(D) \rangle_{\mathcal{H}} \quad \forall D \in \mathcal{D},$$

(24)

we have

$$\mathcal{E}(\tau_g) - \inf_{\tau : \mathcal{D} \to \Theta} \mathcal{E}((\tau)) \leq \sqrt{\int \|g(D) - g^*(D)\|^2_{\mathcal{H}} \, d\pi_{\imath}(D)},$$

(25)

where $\pi_{\imath}(D^{tr})$ denotes the marginal of $\pi_{\imath}(D^{val}, D^{tr})$ with respect to training data. Note that the constant $c_\Delta$ that appears in the original comparison inequality is upper bounded by 1 in our setting since $c_\Delta = \sup_{\mathcal{D}, \theta} \|\psi(\theta, D)\|$ and the feature map $\psi$ is normalized.

Let now $g_N : \mathcal{D} \to \Theta$ be the minimizer of the vector-valued least-squares empirical risk minimization problem

$$g_N = \arg\min_{g \in \mathcal{H}_{\Theta \otimes \mathcal{F}}} \frac{1}{N} \sum_{i=1}^{N} \|g(D^{tr}_i) - \tilde{\varphi}^{val}_i\|^2_{\mathcal{H}} + \lambda_2 \|g\|^2_{\mathcal{H}_{\Theta \otimes \mathcal{F}}}.$$  

This problem can be solved in closed form and it can be shown (Ciliberto et al., 2016, Lemma 17) that $g_N$ is of the form

$$g_N(D) = \sum_{i=1}^{n} \alpha_i(D) \, \tilde{\varphi}^{val}_i,$$

(26)

for all $D \in \mathcal{D}$, where $\alpha_i(D)$ is defined as in (9). Due to linearity (see also Lemma 8 in Ciliberto et al., 2019) we have

$$\tau_{g_N}(D) = \arg\min_{\theta \in \Theta} \langle \psi(\theta, D), g_N(D) \rangle_{\mathcal{H}}$$

(27)

$$= \arg\min_{\theta \in \Theta} \sum_{i=1}^{N} \alpha_i(D) \, \mathcal{L} \left( \text{Alg}(\theta, D^{tr}), D^{val} \right)$$

(28)

$$= \tau(D),$$

(29)

\(^1\)The original theorem was applied to the case where $\mathcal{Z} \times \mathcal{X} = \mathcal{Y}$ was the probability simplex in finite dimension. However the proof of such result requires only that $\mathcal{H}$ and $\mathcal{G}$ are RKHS and can therefore be applied to the general case where $\mathcal{Z} \times \mathcal{X}$ and $\mathcal{Y}$ are different from each other and they do not correspond to the probability simplex but are rather subset of $\mathbb{R}^k$ (possibly with different dimension for each space) and satisfy the boundary condition Berlinet & Thomas-Agnan (2011). Therefore in our setting we can take $\mathcal{Z} = \Theta$ and $\mathcal{X} = \mathcal{Y} = \mathcal{D}$ to obtain the desired result.
which corresponds to the estimator $\tau_N(D)$ studied in this work and introduced in (9). The comparison inequality (25) above, becomes

$$\mathcal{E}(\tau_N) - \inf_{\tau: D \to \Theta} \mathcal{E}(f) \leq \sqrt{\int \|g_N(D) - g^*(D)\|^2_H d\pi_D(D)}.$$  

(30)

Therefore, we can obtain a learning rate for the excess risk of $\tau_N$ by studying how well the vector-valued least-squares estimator $g_N$ is approximating $g^*$. Since $g^* \in \mathcal{H} \otimes \mathcal{F}$ from hypothesis, we can replicate the proof in (Ciliberto et al., 2016, Thm. 5) to obtain the desired result. Note that by framing our problem in such context we obtain a constant $c$ that depends only on the norm of $g^*$ as a vector in $\mathcal{H} \otimes \mathcal{F}$. We recall that $g^*$ captures the “regularity” of the meta-learning problem. Therefore, the more regular (i.e. easier) the learning problem, the faster the learning rate of the proposed estimator.

\[ \square \]

B Model and Experiment Details

We provide additional details on the model architecture, experiment setups, and hyperparameter choices. We performed only limited mode tuning, as it is not the focus on the work.

B.1 Model Architecture

Given the pre-trained representation $\varphi(x) \in \mathbb{R}^{640}$, the proposed model is $f_{\theta}(\varphi(x)) = \varphi(x) + g_{\theta}(\varphi(x))$, a residual network with fully-connected layers. Each layer of the fully-connected network $g_{\theta}(\varphi(x))$ is also 640 in dimension.

We added a $\ell_2$ regularization term on $\theta$, with a weight of $\lambda_\theta$ reported below.

For top-$M$ values from $\alpha(D)$, we normalize the values such that they sum to 1.

B.2 Experiment Setups

We use the same experiment setup as LEO Rusu et al. (2019) by adapting its official implementation\(^5\). For both 5-way-1-shot and 5-way-5-shot settings, we use the default environment values from the implementation, including a meta-batch size of 12, and 15 examples per class for each class in $D_{val}$ to ensure fair comparison.

B.3 Model Hyperparameters

Models across all settings share the same hyperparameters, listed in Table 3.

Table 3: Hyperparameter values used in the experiments

| SYMBOL | DESCRIPTION | VALUES |
|--------|-------------|--------|
| $\lambda_1$ in (6) | REGULARIZER FOR THE LEAST-SQUARE SOLVER | 0.1 |
| $\lambda_2$ in (9) | REGULARIZER FOR LEARNING $\alpha(D)$ | $10^{-8}$ |
| $\lambda_3$ in (11) | REGULARIZER FOR THE ADDITIONAL TERM | 1 |
| $\lambda_\theta$ | $\ell_2$ REGULARIZER ON $\theta$ | $10^{-6}$ |
| $\sigma$ in (13) | KERNEL BANDWIDTH | 50 |
| $\eta$ | META LEARNING RATE | $10^{-4}$ |
| $N$ | TOTAL NUMBER OF META-TRAINING TASKS | 30,000 |
| $M$ | NUMBER OF TASKS TO KEEP IN Alg. 1 | 500 |

\(^5\)https://github.com/deepmind/leo