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Explicit Rieffel induction module for quantum groups. (English) [Zbl 07597891]

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Summary: For $\mathbb{G}$ an algebraic (or more generally, a bornological) quantum group and $\mathbb{B}$ a closed quantum subgroup of $\mathbb{G}$, we build in this paper an induction module by explicitly defining, on the convolution algebra of $\mathbb{G}$, an inner product which takes its value in the convolution algebra of $\mathbb{B}$, as in the original approach of Rieffel. In this context, we study the link with the induction functor defined by Vaes. In the last part, we illustrate our result with parabolic induction of complex semisimple quantum groups. We first show that our induction functor coincides with the one already defined in the case of parabolic induction. Then we use the tools developed in this paper to give a geometric interpretation to the parabolic induction functor, following the approach suggested by Clare in the classical case.

MSC:

20G42 Quantum groups (quantized function algebras) and their representations

16T05 Hopf algebras and their applications

46L65 Quantizations, deformations for selfadjoint operator algebras

46L51 Noncommutative measure and integration

Keywords:

induction; quantum groups; bornological algebras; algebraic quantum groups; locally compact quantum groups; semisimple quantum groups

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