Large scales space-time waves from inflation with time dependent cosmological parameter

Juan Ignacio Musmarra*, 1,2 Mauricio Bellini. †

1 Departamento de Física, Facultad de Ciencias Exactas y Naturales,
Universidad Nacional de Mar del Plata, Funes 3350, C.P. 7600, Mar del Plata, Argentina.
2 Instituto de Investigaciones Físicas de Mar del Plata (IFIMAR),
Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Mar del Plata, Argentina.

We study the emission of large-scales wavelength space-time waves during the inflationary expansion of the universe, produced by back-reaction effects. As an example, we study an inflationary model with variable time scale, where the scale factor of the universe grows as a power of time. The coarse-grained field to describe space-time waves is defined by using the Levy distribution, on the wavenumber space. The evolution for the norm of these waves on cosmological scales is calculated, and it is shown that decreases with time.

I. INTRODUCTION

The study of the General Relativistic dynamics is a very important topic that has been subject of research since many years ago[1, 2]. However, during many time, the study of this problem has not evolved too much. The strategy used in [2], generalized in [3] for null surfaces, consisted of adding an appropriate term to the original action, so that, when the action was changed, the boundary terms become null. The original problem resides in that, when we consider the Einstein-Hilbert action $I$, which describes gravitation and matter

$$I = \int_V d^4x \sqrt{-g} \left[ \frac{R}{2\kappa} + L_m \right],$$

the variation of the action

$$\delta I = \int d^4x \sqrt{-g} \left[ \delta g^{\alpha\beta} (G_{\alpha\beta} + \kappa T_{\alpha\beta}) + g^{\alpha\beta} \delta R_{\alpha\beta} \right] = 0,$$

includes some boundary terms that cannot be arbitrarily neglected to obtain the Einstein’s equations. These terms are the last inside the brackets and must be studied in detail. In some earlier works, we have studied some physical consequences and applications of this terms[4–9]. In this work we shall study the possible emission of space-time waves during the inflationary evolution of the universe. Cosmic inflation describes a primordial era in which the universe growth quasi-exponentially and provides a solution to some cosmological problems that cannot be explained otherwise[10–15]. During inflation the equation of state $\omega = P/\rho$ remained close to a vacuum expansion: $\omega \simeq -1$, so that the universe becomes spatially flat and the energy density was very close to who of a critical one: $\rho \simeq \frac{3H^2}{8\pi G}$. Because of this the universe becomes, at cosmological scales, very isotropic and homogeneous. For this reason it is possible to describe the universe at large-scales with a Friedmann-Lamaitre-Robertson-Walker (FLRW) metric. However, the physical origin of such that scales remains unknown. A possible explanation was done in[16].

II. RELATIVISTIC QUANTUM GEOMETRY WITH NONZERO FLUX

We consider a flux $\delta \Phi$ of $\delta W^\alpha$ across the 3D closed hypersurface $\partial M$ when we variate an Einstein-Hilbert (EH) action

$$\delta \Phi = \left[ \delta W^\alpha \right]_{\alpha} - \left( g^{\alpha\beta} \right)_{\beta} \delta \Gamma^\alpha_{\alpha\beta} + \left( g^{\alpha\beta} \right)_{\beta} \delta \Gamma^\alpha_{\alpha\beta},$$

where 4-vector $\delta W^\alpha$ is given in terms of the varied connections: $\delta \Gamma^\alpha_{\beta\epsilon}$

$$\delta W^\alpha = \delta \Gamma^\alpha_{\beta\epsilon} g^{\beta\alpha} - \delta \Gamma^\alpha_{\beta\gamma} g^{\beta\gamma},$$

* jmusmarra@mdp.edu.ar
† Corresponding author: mbellini@mdp.edu.ar
and $\delta R_{\alpha\beta}$ is defined by using the extended Palatini identity [21]

$$\delta R_{\beta\gamma\alpha} = \delta R_{\beta\gamma} = (\delta \Gamma^\alpha_{\beta\gamma})_{\alpha} - (\delta \Gamma^\alpha_{\beta\gamma})_{\alpha},$$

(5)

where ”" denotes the covariant derivative on the extended manifold.

A. Minimum action with flux $\delta R_{\alpha\beta} = \lambda(t) \delta g_{\alpha\beta}$

In this work we shall consider the case where $\delta R_{\alpha\beta}$ is related to the variation of the metric tensor

$$\delta R_{\alpha\beta} = \lambda(t) \delta g_{\alpha\beta}.$$  

(6)

Here, $\lambda(t)$ is the called cosmological parameter, which it is observed as a decaying function of time [19, 20]. This means that the varied action (2), will can be written as

$$\delta I = \int \! d^4x \sqrt{-g} \left[ \delta g^{\alpha\beta} \left( G_{\alpha\beta} - \lambda(t) g_{\alpha\beta} + \kappa T_{\alpha\beta} \right) \right] = 0,$$

(7)

where we have made use of the fact that

$$\delta g^{\alpha\beta} g_{\alpha\beta} = -\delta g_{\alpha\beta} g^{\alpha\beta}.$$  

(8)

As in previous works [7, 18], we shall consider the connections given by Levi-Civita symbols plus a variation that describes the displacement with respect to the metric tensor

$$\Gamma^\alpha_{\beta\gamma} = \left\{ \begin{array}{c} \alpha \\ \beta \\ \gamma \end{array} \right\} + \delta \Gamma^\alpha_{\beta\gamma} = \left\{ \begin{array}{c} \alpha \\ \beta \\ \gamma \end{array} \right\} + b \sigma_\gamma g_{\beta\gamma}.$$  

(9)

Here, $\sigma_\gamma = \sigma_\alpha$ is the ordinary partial derivative of $\sigma$ with respect to $x^\alpha$. On the Riemann manifold it is required that non-metricity to be null: $\Delta g_{\alpha\beta} = g_{\alpha\beta,\gamma} dx^\gamma = 0$. However, on the extended manifold the variation of the metric tensor is

$$\delta g_{\alpha\beta} = g_{\alpha\beta,\gamma} dx^\gamma = -\frac{1}{3} \left( \sigma_\beta g_{\alpha\gamma} + \sigma_\alpha g_{\beta\gamma} \right) dx^\gamma,$$

(10)

where $g_{\alpha\beta,\gamma}$ is the covariant derivative on the extended manifold given by (9).

For $b = 1/3$, it is obtained in [4] that $\delta W^\alpha = -\sigma^\alpha$, and hence the expression (3) can be written as

$$g^{\alpha\beta} \delta R_{\alpha\beta} - \delta \Phi = [\delta W^\alpha]_{\alpha} - (g^{\alpha\beta})_{\beta} \delta \Gamma^\alpha_{\alpha\beta} + (g^{\nu\rho})_{\rho} \delta \Gamma^\alpha_{\nu\rho} = \nabla_\alpha \delta W^\alpha = g^{\alpha\beta} \left[ \square \delta \Psi_{\alpha\beta} - \lambda(t) \delta g_{\alpha\beta} \right] = -\nabla_\alpha \sigma^\alpha \equiv -\square \sigma = 0,$$

(11)

where $\delta \Psi_{\alpha\beta}$ can be interpreted as the components of gravitational waves. Notice that this is true only in the case with $b = 1/3$, where the flux can be written in terms of a 4-divergence for $\delta W^\alpha$ defined in terms of covariant derivatives in the Riemann manifold. To calculate the flux $\delta \Phi$, we must know $\delta R_{\alpha\beta}$. The variation of the Ricci tensor on the extended manifold is

$$\delta R_{\alpha\beta} = (\delta \Gamma^\epsilon_{\alpha\beta})_{\beta} - (\delta \Gamma^\epsilon_{\alpha\beta})_{\epsilon} = \frac{1}{3} \left[ \nabla_\beta \sigma_\alpha + \frac{1}{3} \left( \sigma_\alpha \sigma_\beta + \sigma_\beta \sigma_\alpha \right) - g_{\alpha\beta} \left( \nabla_\epsilon \sigma^\epsilon + \frac{2}{3} \sigma_\epsilon \sigma^\nu \right) \right],$$

(12)

so that, in agreement with the equation (6), it is possible to write the following equation:

$$\frac{1}{3} \left[ \nabla_\beta \sigma_\alpha + \frac{1}{3} \left( \sigma_\alpha \sigma_\beta + \sigma_\beta \sigma_\alpha \right) - g_{\alpha\beta} \left( \nabla_\epsilon \sigma^\epsilon + \frac{2}{3} \sigma_\epsilon \sigma^\nu \right) \right] - \lambda(t) \delta g_{\alpha\beta} = 0,$$

(13)

where the last term in (13) is due to the flux that cross the closed 3D-hypersurface: $\delta \Phi = \lambda(t) g^{\alpha\beta} \delta g_{\alpha\beta}$.

In this work we shall consider an expanding universe that is isotropic and homogenous. Due to this fact, it is possible to define the redefined background Einstein equations

$$\bar{G}_{\alpha\beta} = G_{\alpha\beta} - \lambda(t) g_{\alpha\beta} = \kappa T_{\alpha\beta},$$

(14)

with the redefined boundary conditions [13]. Notice that these set of equations, given by (13) and (14), comply with the minimum action principle given in the equation (7). Notice that the transformation (14) preserves the EH
action, and the flux that cross the 3D-gaussian hypersurface, \( \delta \Phi \), is related to the cosmological parameter \( \lambda(t) \), and the variation of the scalar field, \( \delta \sigma \)

\[
\delta \Phi = -\frac{2}{3} \lambda(t) \delta \sigma.
\]  

(15)

The field \( \chi(x^\alpha) \equiv g^{\mu\nu} \chi_{\mu\nu} \) is a classical scalar field, such that \( \chi_{\mu\nu} = \frac{\delta \Phi}{\delta S} \) describes the waves of space-time produced by the source through the 3D-Gaussian hypersurface

\[
\Box \chi = \frac{\delta \Phi}{\delta S},
\]  

(16)

where \( \delta S = U_\alpha dx^\alpha \). Here, \( U^\alpha = \frac{dx^\alpha}{dS} \) are the components of the 4-velocity, given as a solution of the geodesic equation on the Riemann manifold:

\[
\frac{dU_\alpha}{dS} + \left\{ \begin{array}{c} \alpha \\ \beta \gamma \end{array} \right\} U_\beta U_\gamma = 0,
\]  

(17)

with unity squared norm: \( U_\alpha U^\alpha = 1 \). The differential operator \( \Box \) that acts on \( \chi \) in (16), is written in terms of the covariant derivatives defined on the background metric:

\[ \Box \chi \equiv g^{\alpha\beta} \nabla_\alpha \nabla_\beta, \]

such that \( \nabla_\alpha \) give us the covariant derivative on the Riemann background manifold.

**B. Quantum space-time**

If we deal with a space-time which is quantum in nature, we can describe it as a Fourier expansion in terms of the modes

\[
\delta \hat{\chi}^\alpha(t, \vec{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \hat{e}^\alpha \left[ b_k \hat{x}_k(t, \vec{x}) + b_k^\dagger \hat{x}_k^* (t, \vec{x}) \right],
\]

where \( b_k^\dagger \) and \( b_k \) are the creation and annihilation operators of space-time, that comply with the algebra \( \hat{B} = \delta^{(3)}(\vec{k} - \vec{k}') \) and \( \hat{e}^\alpha = e^{\alpha\beta\gamma\delta} \hat{e}^\beta \hat{e}^\gamma \hat{e}^\delta \). The operators of creation \( \delta \hat{x}^\alpha(x^\beta) \), applied to a background state \( |B\rangle \), return an eigenvalue \( dx^\alpha |B\rangle = U^\alpha dS |B\rangle = \delta \hat{x}^\alpha(x^\beta) |B\rangle \),

(18)

so that the background line element results to be

\[
dS^2 \delta_{BB'} = (U_\alpha U^\alpha) dS^2 \delta_{BB'} = \langle B | \delta \hat{x}_\alpha \delta \hat{x}^\alpha | B' \rangle,
\]  

(19)

and the quantum states are described by a Fock space.

**III. INFLATIONARY UNIVERSE WITH TIME DEPENDENT COSMOLOGICAL PARAMETER**

In order to study an inflationary model where the time scale is variable, we shall consider the line element

\[
dS^2 = e^{-2 \int \Gamma(t) dt} dt^2 - a_0^2 e^{2 \int H(t) dt} \delta_{ij} dx^i dx^j.
\]  

(20)

If the expansion of the universe is driven by the inflaton field \( \phi \), the action will be

\[
I = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G} - \frac{\dot{\phi}^2}{2} - e^{2 \int \Gamma(t) dt} - V(\phi) \right),
\]  

(21)

and the dynamics for the background inflaton field, that describes an universe, which at large scales is globally isotropic, spatially flat and homogenous, is

\[
\ddot{\phi} + 3H(t) \dot{\phi} + \frac{\delta V}{\delta \phi} = 0.
\]  

(22)
The relevant Einstein equations with the time dependent cosmological parameter included, are

\[ 3H(t)^2 + \lambda(t) = 8\pi G \rho, \tag{23a} \]

\[- [3H(t)^2 + 2\dot{H}(t) + 2\Gamma(t)H(t) + \lambda(t) = 8\pi G P, \tag{23b} \]

where \( P \) and \( \rho \) are respectively the pressure and energy density. The equation of state describes the ratio between both scalar quantities

\[ \omega = \frac{P}{\rho} = -\left( 1 + \frac{2(\dot{H}(t) + \Gamma(t)H(t))}{3H^2(t) + \lambda(t)} \right), \tag{24} \]

such that \( \omega \) must remain close to a vacuum dominated state during the inflationary expansion of the universe.

Furthermore, from the Einstein equations, we obtain that the redefined scalar potential \( \bar{V}(\phi) \) and \( \dot{\phi} \), are respectively given by

\[ \bar{V} = e^{-\frac{2}{4\pi G} \int \Gamma(t) dt} \left( 3H(t)^2 + \dot{H}(t) + \Gamma(t)H(t) + \lambda(t) \right), \tag{25a} \]

\[ \dot{\phi} = \sqrt{-\left( \dot{H} + \Gamma H \right)} e^{-\int \Gamma(t) dt} = \sqrt{\frac{p(1-q)}{4\pi G}} \frac{1}{t^{1+q}}. \tag{25b} \]

Using this in (22), we obtain that the following equation must be fulfilled:

\[ \dot{\lambda}t^3 - 2\lambda qt^2 - 12p^2q - 2pq^2 + 2pq = 0. \tag{26} \]

The solution for \( \lambda(t) \) is

\[ \lambda(t) = C_1 t^{2q + \frac{pq(1 - q - 6p)}{q + 1} \frac{1}{t^2}}. \tag{27} \]

where \( C_1 \) is a constant. If \( q = 0, \lambda(t) = C_1 \) and we can recover a traditional power-law expansion[7, 22]. Furthermore, we shall consider that if \( C_1 = 0 \) and \( p = \frac{(1-q)q}{3(3q+1)} \), then \( \lambda(t) = 3H(t)^2 = \frac{3p^2}{t^2} \). On the other hand, for \( p > 0 \) we have \( q < 1 \), which is consistent with a real \( \dot{\phi} \) in (25b).

**IV. WAVES OF SPACE-TIME FROM BACK-REACTION EFFECTS**

With the choice \( b = 1/3 \), by using the expression (11), we obtain that

\[ g^{\alpha\beta} \delta R_{\alpha\beta} = -\left[ \Box \sigma + \frac{2}{3} \sigma_\nu \sigma^\nu + \delta \Phi \right] = 0, \tag{28} \]

and, on the other hand, we have that

\[ \nabla_\alpha \delta W^\alpha = -\Box \sigma = 0. \tag{29} \]

In order to make resoluble the system of equations, we shall consider the gauge \( \sigma_\nu \sigma^\nu = \lambda(t) \delta \sigma \), where for a co-moving observer \( U^0 = \sqrt{g^{00}} \), so that

\[ \delta \sigma = U^\alpha \sigma_\alpha = U^0 \sigma_0. \tag{30} \]

With this choice, and using the equations [3] and [16], the dynamics for \( \sigma \) and \( \chi \), results to be

\[ \Box \sigma = 0, \tag{31a} \]

\[ \Box \chi = -\frac{2}{3} U^0 \lambda(t) \delta \sigma, \tag{31b} \]
where $\Gamma(t) = \frac{q}{t}$, $H(t) = \frac{t}{q}$. Therefore, in order to resolve the dynamics we must obtain the solution of (31a), and use this solution in order to resolve (31b), with (33).

In order to describe the fields $\chi$ and $\sigma$, we can expand these fields as Fourier series

$$\chi(x^\alpha) = \frac{1}{(2\pi)^{\frac{d}{2}}} \int d^d k \left[ A_k e^{i k \cdot x} \Theta_k(t) + c.c. \right], \quad (32a)$$

$$\sigma(x^\alpha) = \frac{1}{(2\pi)^{\frac{d}{2}}} \int d^d k \left[ B_k e^{i k \cdot x} \xi_k(t) + c.c. \right], \quad (32b)$$

where $\xi_k$ are the time dependent modes of the field $\sigma$, which once normalised, are $\xi_k(t) = \frac{\pi}{\sqrt{4(p+q-1)}} t^{-\frac{3}{2} (q+3p-1)} \mathcal{H}_{\nu}^{(2)}[y(t)]. \quad (33)$

Here, $\mathcal{H}_{\nu}^{(2)}[y(t)]$ is the second kind Hankel function, with

$$\nu = \frac{q + 3p - 1}{2(p+q-1)}, \quad (34a)$$

$$y(t) = \frac{k t^{p+q} t^{-(p+q-1)}}{a_0 (p+q-1)}. \quad (34b)$$

The solution can be obtained using the expressions (32a), (32b) and (33) in (31b), with the general solution for $\Theta_k(t)$:

$$\Theta_k(t) = \Theta_k^{(h)}(t) + \Theta_k^{(p)}(t). \quad (35)$$

Here, the homogeneous part of the solution for the modes of $\chi$, is

$$\Theta_k^{(h)}(t) = C_2 t^{-\frac{1}{2} (q+3p-1)} J_{-\nu}[y(t)] + C_3 t^{-\frac{1}{2} (q+3p-1)} Y_{-\nu}[y(t)]. \quad (36)$$

The general solution finally results to be

$$\Theta_k^{(p)}(t) = \frac{1}{k^d} \sqrt{\frac{\pi}{9(p+q-1)}} \left[ h_{1,k}(t) \int \mathcal{H}_{\nu}^{(2)}[y(t)] \frac{\lambda(t) f_{1,k}(t)}{g_k(t)} dt + h_{2,k}(t) \int \mathcal{H}_{\nu}^{(2)}[y(t)] \frac{\lambda(t) f_{2,k}(t)}{g_k(t)} dt \right]. \quad (37)$$

where $J_{-\nu}$ and $Y_{-\nu}$ are the first and second kind Bessel functions with parameter $-\nu$. Furthermore, $h_{1,k}(t)$, $h_{2,k}(t)$, $f_{1,k}(t)$, $f_{2,k}(t)$ and $g_k(t)$ are functions given by the expressions

$$h_{1,k}(t) = -t^{-\frac{1}{2} (p-q+1)} \frac{a_0}{t^{p+q}} (p-q+1) Y_{\nu_1}[y(t)] - t^{-\frac{1}{2} (q+3p-1)} k Y_{\nu_2}[y(t)], \quad (38a)$$

$$f_{1,k}(t) = \frac{a_0}{t^{p+q}} t^{p+q-1} (p-q+1) J_{\nu_1}[y(t)] + k J_{\nu_2}[y(t)], \quad (38b)$$

$$h_{2,k}(t) = t^{-\frac{1}{2} (p-q+1)} \frac{a_0}{t^{p+q}} (p-q+1) J_{\nu_1}[y(t)] + t^{-\frac{1}{2} (q+3p-1)} k J_{\nu_2}[y(t)], \quad (38c)$$

$$f_{2,k}(t) = \frac{a_0}{t^{p+q}} t^{p+q-1} (p-q+1) Y_{\nu_1}[y(t)] + k Y_{\nu_2}[y(t)], \quad (38d)$$

$$g_k(t) = Y_{\nu_2}[y(t)] J_{\nu_1}[y(t)] - Y_{\nu_1}[y(t)] J_{\nu_2}[y(t)], \quad (38e)$$

with parameters $\nu_1 = \frac{q-p-1}{2(p+q-1)}$ and $\nu_2 = \frac{p+3q-3}{2(p+q-1)}$, such that they are related by the expression $\nu_2 = \nu_1 + 1$. 
In order to obtain a solution to the physical problem in which the waves are produced by the source, the homogeneous solution must be null, so that we impose \( C_2 = C_3 = 0 \). For an analytical expression of the particular solution, we must approach in the limit case where \( y(t) \ll 1 \), corresponding to long wavelengths. So we obtain:

\[
\Theta_k^{(p)}(t)|_{y(t)\ll 1} \simeq i \Gamma(\nu_1) \sqrt{\frac{\pi}{4(p + q - 1)}} \left[ (p + q - 1)^{\nu_1} \left( \frac{a_0}{k} \right)^{\nu_1 - 1} \left( \frac{A_1}{t^{3p-2}} - \frac{A_2}{t^{3p+2q}} \right) + (p - q + 1)^{\nu_1} \left( \frac{a_0}{k} \right)^{\nu_1 + 1} \left( \frac{B_1}{t^{p-2}} - \frac{B_2}{t^{p+2}} \right) \right],
\]

(39)

where the constants \( A_1, A_2, B_1 \) and \( B_2 \) are given by the expressions

\[
A_1 = C_1 (p - q + 1) \left( \frac{4t_0^{-(\nu_1 - 2)(p + q)}}{3\pi(3q + p - 3)(5q + 3p - 5)(p - 2q)} + \frac{2^\nu_1 t_0^{3(p + q)}(p - q + 1)}{(q + 2p - 1)(p - q + 1)} \right),
\]

(40a)

\[
A_2 = pq(6p + q - 1) \left( \frac{4t_0^{-(\nu_1 - 2)(p + q)}}{3\pi(3q + p - 3)(5q + 3p - 5)(p + 2)} + \frac{2^\nu_1 t_0^{3(p + q)}(2p + q - 3)}{(q + 2p - 1)(p + q - 1)} \right),
\]

(40b)

\[
B_1 = C_1 \left( \frac{4t_0^{\nu_1(p + q)}(p + q + 1)^{\nu_1}(p + q - 1)}{3\pi(3q + p - 3)(p - 2q)(q + 1)} + \frac{t_0^{\nu_1(p + q)}(3q + p - 3)p^2 - (q + 1)^2}{(q + 1)(p^2 - (q - 1)^2)} \right),
\]

(40c)

\[
B_2 = pq(6p + q - 1) \left( \frac{4t_0^{\nu_1(p + q)}(p + q)(p - q + 1)^{\nu_1}(p + q - 1)}{3\pi(3q + p - 3)(p + 2)} + \frac{t_0^{\nu_1(p + q)}(3q + p - 3)p^2 - (q + 1)^2}{(q + 1)(p^2 - (q - 1)^2)} \right).
\]

(40d)

### A. Redefined fields

The dynamics of \( \sigma \) and \( \chi \) fields are described respectively by the equations

\[
\dot{\sigma} + [3H(t) + \Gamma(t)]\sigma - \frac{e^{-2 f(H(t) + \Gamma(t))dt}}{a_0^2} \nabla^2 \sigma = 0,
\]

(41a)

\[
\dot{\chi} + [3H(t) + \Gamma(t)]\chi - \frac{e^{-2 f(H(t) + \Gamma(t))dt}}{a_0^2} \nabla^2 \chi = -\frac{2 U^0 \lambda(t)}{3 e^{f(H(t) + \Gamma(t))dt}}.
\]

(41b)

Notice that the right side of the equation (41b) is originated in the flux \( \delta \Phi \) of \( 3W^a \equiv \sigma^a \), that cross the 3D-Gaussian hypersurface. In our case, because the relativistic observer is in a co-moving frame the unique nonzero relativistic velocity is \( U^0 \), so that only contributes \( \dot{\sigma} \) in (41b). In order to simplify the structure of the equations (41a) and (41b), we make the changes of variables \( \sigma = e^{-\frac{1}{2} f(H(t) + \Gamma(t))dt} u \) and \( \chi = e^{-\frac{1}{2} f(H(t) + \Gamma(t))dt} v \), and we obtain:

\[
\ddot{u} + \frac{[\nabla^2 - k_0^2(t)]}{a_0^2 e^{2 f(H(t) + \Gamma(t))dt}} u = 0,
\]

(42a)

\[
\ddot{v} + \frac{[\nabla^2 - k_0^2(t)]}{a_0^2 e^{2 f(H(t) + \Gamma(t))dt}} v = -\frac{2 U^0 \lambda(t)}{3 e^{f(H(t) + \Gamma(t))dt}} \dot{\sigma}.
\]

(42b)

### B. Coarse-grained

The coarse-grained approach provides a description of the dynamics for a desirable part of the spectrum. In our case this part is the long-wavelength sector of the spectrum, which is described by wavelengths much bigger than the size of the Hubble horizon. This wavelengths variate with time, because the horizon is expanding. The wavenumber
related to the horizon wavelength in a co-moving frame is \( k_0(t) \), so that we shall be interested in wavenumbers \( k \), which are smaller than \( k_0 \) in order to describe the cosmological sector (infrared sector), of the spectrum during inflation

\[
k \ll k_0(t) \equiv a_0 e^{\int (H(t)+\Gamma(t))dt} \left[ \frac{3H(t)+\Gamma(t)}{4} + \frac{3\dot{H}(t)+\dot{\Gamma}(t)}{2} \right]^{1/2}.
\]  

(43)

At this point, we will define the coarse-grained fields\[15], by using a suppression factor \( f(k,t) \) to select the desirable wavenumbers of the infrared sector of the spectrum:

\[
u_{cg} = \frac{1}{(2\pi)^3} \int d^3k f(k,t) \left[ A_k e^{i\vec{k}\cdot\vec{r}} \xi_k(t) + c.c. \right], \tag{44a}
\]

\[
u_{cg} = \frac{1}{(2\pi)^3} \int d^3k f(k,t) \left[ B_k e^{i\vec{k}\cdot\vec{r}} \tilde{\xi}_k(t) + c.c. \right], \tag{44b}
\]

where the suppression factor \( f(k,t) \) is given by a Levy distribution,

\[
f(k,t) = \left( \frac{\epsilon k_0(t)}{2\pi} \right) \frac{e^{\epsilon k_0(t)-\epsilon k(t)}}{(k-\epsilon k_0(t))^3/2}.
\]  

(45)

The square fluctuations for the coarse-grained fields are\[13]

\[
\langle B|u_{cg}^2|B\rangle = \int_{0}^{\infty} \frac{dk}{k} P_{u_{cg}}(k) = \frac{1}{2\pi^2} \int_{0}^{k_0} dk \ k^2 |\xi_k(t)|^2 f(k,t)^2, \tag{46a}
\]

\[
\langle B|v_{cg}^2|B\rangle = \int_{0}^{\infty} \frac{dk}{k} P_{v_{cg}}(k) = \frac{1}{2\pi^2} \int_{0}^{k_0} dk \ k^2 |\tilde{\xi}_k(t)|^2 f(k,t)^2. \tag{46b}
\]

In the figures (1) and (2) we show the power spectrums of \( P_{\sigma_{cg}}(k) \) and \( P_{\chi_{cg}}(k) \), for different times (during the inflationary era — time scale is in Planckian times), with \( q = 0.9 \) and \( p = 1.5 \). In both cases the peaks move toward higher \( k \)-values, while their intensities decrease with time. Notice that the intensities of \( P_{\chi_{cg}}(k) \) are weakest than the \( P_{\sigma_{cg}}(k) \) ones.

V. FINAL COMMENTS

We have shown that back-reaction effects in the primordial universe act as sources of space-time waves that propagate in all directions. These sources would be homogeneously and isotropically distributed in cosmological scales, which is the scale that concern us. They do not be the standard gravitational waves, but rather space-time waves originated by local space-time fluctuations which have a quantum origin. It is expected that these waves would came from all directions, as cosmic background radiation, but its intensity is so low to be detected with the instrumentation available today. We have calculated the spectrums for the squared fluctuations of \( \sigma_{cg} \) and \( \chi_{cg} \). In both cases, it is shown that the amplitudes are decreasing with time and the distributions are dispersed along the large-scale \( k \)-spectrum. In this work we have supposed that these sources are scalar fluctuations. All these sources can be viewed on the background metric as a decaying cosmological parameter \( \lambda(t) \), due to the fact we have supposed the universe as globally isotropic and homogenous in the distribution of the sources. However, if we loss homogeneity, this parameter would be a function of \( r \) and \( t \) [i.e., a \( \lambda(r,t) \)]. This topic will be studied in a future work.

Acknowledgements

The authors acknowledge CONICET, Argentina (PIP 11220150100072CO) and UNMdP (EXA852/18) for financial support.

[1] J. W. York, Phys. Rev. Lett. 16: 1082 (1972).
[2] G. W. Gibbons, S. W. Hawking, Phys. Rev. D10: 2752 (1977).
[3] K. Parattu, S. Chakraborty, T. Padmanabhan, Eur. Phys. J. C76: 129 (2016).
[4] L. S. Ridao, M. Bellini, Astrophys. Space Sci. 357: 94 (2015).
[5] L. S. Ridao, M. Bellini, Phys. Lett. B751: 565 (2015).
[6] M. R. A. Arcodia, L. S. Ridao, M. Bellini, Astrophys. Space Sci. 361: 296 (2016).
[7] J. I. Musmarra, M. Anabitarte, M. Bellini, Phys. Dark Univ. 24: 100273 (2019).
[8] J. I. Musmarra, M. Anabitarte, M. Bellini, Eur. Phys. J. C79 no.1, 5 (2019).
[9] J. Mendoza Hernández, M. Bellini, C. Moreno González. Phys. Dark Univ. 23: 100251 (2019).
[10] A. A. Starobinsky, Phys. Lett. B91: 99 (1980).
[11] A. H. Guth, Phys. Rev. D23: 347 (1981).
[12] A. D. Linde, Phys. Lett. B129: 177 (1983).
[13] A. R. Liddle, D. H. Lyth, Phys. Rep. 231 (1998).
[14] M. Bellini, H. Casini, R. Montemayor, P. D. Sisterna, Phys. Rev. D54: 7172 (1996).
[15] M. Bellini, Nucl. Phys. B563 245-258 (1999).
[16] J. E. Madriz Aguilar, C. Moreno, M. Bellini, Phys. Lett. B728: 244 (2014).
[17] S. W. Hawking, G. F. R. Ellis. The large scale structure of space-time. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Cambridge, UK (1973).
[18] M. Bellini, J. E. Madriz Aguilar, M. Montes, P. A. Sánchez, Phys. Dark Univ. 25: 100309 (2019).
[19] I. Dymnikova, M. Khlopov, Eur. Phys. J. C20: 139 (2001).
[20] M. Bellini, Phys. Lett. B632: 610(2006).
[21] A. Palatini. Deduzione invariants delle equazioni gravitazionali dal principio di Hamilton, Rend. Circ. Mat. Palermo 43: 203-212 (1919). [English translation by R. Hojman and C. Mukku in P. G. Bergmann and V. De Sabbata (eds.) Cosmology and Gravitation, Plenum Press, New York (1980)].
[22] S. Kumar, Mon. Not. R. Astron. Soc. 422: 2532 (2012).
FIG. 1: $P_{w_g}(k)$ for $p = 1.5$ and $q = 0.9$, with parameters $t_0 = G^{1/2}$, $a_0 = 0.66 G^{1/2}$ and $\epsilon = 10^{-3}$. The red, blue and green lines correspond respectively to the cases $t = 1.0 \times 10^7 G^{1/2}$, $t = 1.2 \times 10^7 G^{1/2}$ and $t = 1.4 \times 10^7 G^{1/2}$.

FIG. 2: $P_{w_g}(k)$ for $p = 1.5$ and $q = 0.9$, with parameters $C_1 = 0$, $t_0 = G^{1/2}$, $a_0 = 0.66 G^{1/2}$ and $\epsilon = 10^{-3}$. The red, blue and green lines correspond respectively to the cases $t = 1.0 \times 10^7 G^{1/2}$, $t = 1.2 \times 10^7 G^{1/2}$ and $t = 1.4 \times 10^7 G^{1/2}$. 