Prediction for the Cosmological Constant and Constraints on Susy GUTS in Resummed Quantum Gravity

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Abstract

Working in the context of the Planck scale cosmology formulation of Bonanno and Reuter, we use our resummed quantum gravity approach to Einstein’s general theory of relativity to estimate the value of the cosmological constant as $\rho_\Lambda = (0.0024\text{eV})^4$. We show that susy GUT models are constrained by the closeness of this estimate to experiment. We also address various consistency checks on the calculation. In particular, we use the Heisenberg uncertainty principle to remove a large part of the remaining uncertainty in our estimate of $\rho_\Lambda$.

1 Introduction

The calculations in Refs. 1–6 have given considerable support to Weinberg’s suggestion 7 that the general theory of relativity may be asymptotically safe, with an S-matrix that depends only on a finite number of observable

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parameters, due to the presence of a non-trivial UV fixed point, with a finite dimensional critical surface in the UV limit. The former authors, using Wilsonian field-space exact renormalization group methods, obtain results which support the existence of Weinberg’s UV fixed-point for the Einstein-Hilbert theory. Independently, in Refs. 9–12 we have shown that the extension of the amplitude-based, exact resummation theory of Refs. 13, 14 to the Einstein-Hilbert theory leads to UV-fixed-point behavior for the dimensionless gravitational and cosmological constants. The attendant resummed theory, which we have called resummed quantum gravity, is actually UV finite. In Refs. 15, we note that causal dynamical triangulated lattice methods have been used to show more evidence for Weinberg’s asymptotic safety behavior.

The results in Refs. 1–6, which are quite impressive, however, involve cut-offs and some dependence on gauge parameters which remain in the results to varying degrees even for products such as that for the UV limits of the dimensionless gravitational and cosmological constants. Accordingly, we continue to refer to the approach in Refs. 1–6 as the ‘phenomenological’ asymptotic safety approach. Because the above noted dependencies are mild, the non-Gaussian UV fixed point found in these latter references is probably a physical result. But, until it is corroborated by a rigorously cut-off independent and gauge invariant calculation, the result cannot be considered final. Such a calculation is possible to do in resummed quantum gravity. As the results from Refs. 15 involve lattice constant-type artifact issues, to be considered final, they too need to be corroborated by a rigorous calculation without such issues. Again, a possible answer is resummed quantum gravity. Thus in what follows, we try to make contact with experiment on a stage that has been prepared for us.

More specifically, the authors in Refs. 17, 18 have applied the attendant phenomenological asymptotic safety approach in Refs. 1–6 to quantum gravity to provide an inflatonless realization of the successful inflationary model of cosmology: the standard Friedmann-Walker-Robertson classical descriptions are joined smoothly onto Planck scale cosmology developed from the attendant UV fixed point solution. In this way, the horizon, flat-

1At the expense of violating Lorentz invariance, the model in Ref. 16 realizes many aspects of the effective field theory implied by the anomalous dimension of 2 at the Weinberg UV-fixed point.

2The authors in Ref. 19 also proposed the attendant choice of the scale $k \sim 1/t$ used in Refs. 17, 18.
ness, entropy and scale free spectrum problems are solved with quantum mechanical arguments.

The properties as used in Refs. 17, 18 for the UV fixed point of quantum gravity are reproduced in Ref. 12 using the new resummed theory 9–11 of quantum gravity with the bonus of “first principles” predictions for the fixed point values of the respective dimensionless gravitational and cosmological constants. In what follows, the analysis in Ref. 12 is carried forward 22 to an estimate for the observed cosmological constant \( \Lambda \) in the context of the Planck scale cosmology of Refs. 17, 18. We comment on the reliability of the result and present arguments 25 showing that the uncertainty of the estimate of is at the level of a factor of \( O(10) \), as the estimate will be seen already to be relatively close to the observed value 23,24. The closeness to the observed value of our estimate allows us to constrain susy GUT models given that this closeness is now put on a more firm basis 25. The closeness of our estimate to the experimental value again gives, at the least, some more credibility to the new resummed theory as well as to the methods in Refs. 1–6,15,17.

An important point of contact for our approach to quantum gravity is the pioneering result of Weinberg 26 on summing soft gravitons. Weinberg showed that, in an on-shell \( \alpha \rightarrow \beta \) process with transition rate \( \Gamma_{\beta\alpha}^0 \) without soft graviton effects, inclusion of the virtual soft graviton effects results in the transition rate

\[
\Gamma_{\beta\alpha} = \Gamma_{\beta\alpha}^0 \left( \frac{\lambda}{U} \right)^B,
\]

where \( \lambda \) is the infrared cutoff and \( U \) is the Weinberg 26 soft cutoff which is used define what is meant by infrared. Here, \( B \) is given by

\[
B = \frac{G_N}{2\pi} \sum_{n,m} \eta_n \eta_m m_n m_m \frac{1 + \beta_{nm}^2}{\beta_{nm}(1 - \beta_{nm}^2)^{1/2}} \ln \left( \frac{1 + \beta_{nm}}{1 - \beta_{nm}} \right)
\]

where \( G_N \) is Newton’s constant, \( \eta_n = +1(-1) \) when particle \( n \) is outgoing (incoming), respectively, and \( \beta_{nm} \) is the relative velocity

\[
\beta_{nm} = \left[ 1 - \frac{m_n^2 m_m^2}{(p_n p_m)^2} \right]^{1/2}
\]

for particles \( n \) and \( m \) with masses \( m_n, m_m \) and four momenta \( p_n, p_m \), respectively. In the 2-to-2 case where 1 and 2 are incoming, 3 and 4 are outgoing, \footnote{We do want to continue to caution against overdoing this closeness to the experimental value.}
and all masses have the same value $m$, we see that (2) shows a growth of the damping represented by $B$ with large values of $U$ as the exponential of $-(4G_Ns/\pi)\ln(2\ln(U/\lambda))$ for large values of the cms energy squared $s$ for the wide-angle case with the scattering angle at $90^\circ$ in the center of momentum system. We will see in our discussion below that we recover this same type of growth of the analog of $B$ with large invariant squared masses in the context of resumming the large IR regime of quantum gravity.

We present the discussion as follows. In Section 2 we give a brief review of the Planck scale cosmology presented phenomenologically in Refs. [17,18]. In Section 3 we review our results in Ref. [12] for the dimensionless gravitational and cosmological constants at the UV fixed point. In Section 4, using our results in Section 3 in the context of the Planck scale cosmology scenario in Refs. [17,18], we estimate the observed value of the cosmological constant $\Lambda$ and we use the attendant estimate to constrain susy GUT’s. We also address consistency checks on the analysis. Specifically, in Section 5, we use the consistency between the Heisenberg uncertainty principle and the solutions of Einstein’s equations to argue that the error on our estimate of $\Lambda$ is $O(10)$. Section 6 contains our summary remarks.

## 2 Review of Planck Scale Cosmology

The Einstein-Hilbert theory with which we work is defined by the Lagrangian

$$L(x) = \frac{1}{2\kappa^2}\sqrt{-g}(R - 2\Lambda),$$

where $R$ is the curvature scalar, $g$ is the determinant of the metric of spacetime $g_{\mu\nu}$, $\Lambda$ is the cosmological constant and $\kappa = \sqrt{8\pi G_N}$. The authors in Ref. [17,18], using the phenomenological exact renormalization group for the Wilsonian [8] coarse grained effective average action in field space, have argued that the attendant running Newton constant $G_N(k)$ and running cosmological constant $\Lambda(k)$ approach UV fixed points as $k$ goes to infinity in the deep Euclidean regime. Accordingly, we have $k^2G_N(k) \to g_*$, $\Lambda(k) \to \lambda_*k^2$ for $k \to \infty$ in the Euclidean regime.

One may use a connection between the momentum scale $k$ characterizing the coarseness of the Wilsonian graininess of the average effective action and the cosmological time $t$ to make contact with cosmology. Using a phenomenological realization of this latter connection, the authors in Refs. [17,18] show
that the standard cosmological equations admit of the following extension:

\[
\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{1}{3} \Lambda + \frac{8\pi}{3} G_N \rho,
\]

\[\dot{\rho} + 3(1 + \omega) \frac{\dot{a}}{a} \rho = 0,\]

\[\dot{\Lambda} + 8\pi \rho G_N = 0,\]

\[G_N(t) = G_N(k(t)), \quad \Lambda(t) = \Lambda(k(t)).\]  

(4)

Here, \(\rho\) is the density and \(a(t)\) is the scale factor with the Robertson-Walker metric representation given as

\[
ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right)
\]

(5)

where \(K = 0, 1, -1\) correspond respectively to flat, spherical and pseudospherical 3-spaces for constant time \(t\). We take the equation of state \(p(t) = \omega \rho(t)\), where \(p\) is the pressure. Proceeding phenomenologically, the attendant functional relationship between the respective momentum scale \(k\) and the cosmological time \(t\) is determined via \(k(t) = \xi t\) for some positive constant \(\xi\) determined from constraints on physically observable predictions.

The authors in Refs. 17, 18, using the UV fixed points as discussed above for \(k^2 G_N(k) \equiv g_*\) and \(\Lambda(k)/k^2 \equiv \lambda_*\) obtained from their phenomenological, exact renormalization group (asymptotic safety) analysis, solve the cosmological system given above. For \(K = 0\), they find a solution in the Planck regime where \(0 \leq t \leq t_{\text{class}}\), with \(t_{\text{class}}\) a “few” times the Planck time \(t_{\text{Pl}}\), which joins smoothly onto a solution in the classical regime, \(t > t_{\text{class}}\), which coincides with standard Friedmann-Robertson-Walker phenomenology but with the horizon, flatness, scale free Harrison-Zeldovich spectrum, and entropy problems all solved purely by Planck scale quantum physics.

The phenomenological nature of the analyses in Refs. 17, 18 is made manifest by the dependencies of the fixed-point results \(g_*, \lambda_*\) on the cut-offs used in the Wilsonian coarse-graining procedure, for example. We point out that the key properties of \(g_*, \lambda_*\) used for their analyses are that the two UV limits are both positive and that the product \(g_* \lambda_*\) is only mildly cut-off/threshold function dependent. In what follows, we review the predictions of resummed quantum gravity (RQG) [9–11] in Refs. 12 for these UV limits and we show how to use the limits to predict [22] the current value of \(\Lambda\). For completeness,
a brief review of the basic principles of RQG theory is given at the beginning of the next section.

3 Review of Resummed Quantum Gravity Prediction for $g_*$ and $\lambda_*$

We start with the prediction for $g_*$ as it is presented in Refs. 10–12,22. Here, we recapitulate the main steps in the calculation. The graviton couples to an elementary particle in the infrared regime which we shall resum independently of the particle’s spin [26,27]. This means that we may develop the required calculational framework using a scalar field. We extend that framework to spinning particles straightforwardly. We start with the Lagrangian density for the basic scalar-graviton system already given by Feynman in Refs. 28,29:

$$\mathcal{L}(x) = -\frac{1}{2\kappa^2} R\sqrt{-g} + \frac{1}{2} \left( g^{\mu\nu}\partial_\mu\varphi \partial_\nu\varphi - m_o^2 \varphi^2 \right) \sqrt{-g}$$

$$= \frac{1}{2} \left\{ h^{\mu\nu}\lambda\tilde{h}_{\mu\nu,\lambda} - 2\eta^{\mu\nu} \eta^{\lambda\lambda'} h_{\mu\nu,\lambda'} \eta^{\sigma\sigma'} \tilde{h}_{\sigma\sigma'} \right\} + \frac{1}{2} \left\{ \varphi_\mu\varphi^\mu - m_o^2 \varphi^2 \right\} - \kappa \frac{1}{2} h_{\lambda\rho}\tilde{h}^{\rho\lambda} \left( \varphi_\mu\varphi^\mu - m_o^2 \varphi^2 \right) - 2\eta^{\mu\nu} h^\mu\varphi^\nu \varphi_\mu \varphi_\nu \right\} + \cdots \tag{6}$$

Here, $\varphi(x)$ can be identified as the physical Higgs field as our representative scalar field for matter, $\varphi(x)_\mu \equiv \partial_\mu \varphi(x)$, and $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$ where we follow Feynman and expand about Minkowski space so that $\eta_{\mu\nu} = \text{diag}\{1, -1, -1, -1\}$. We have introduced Feynman’s notation

$$\bar{y}_{\mu\nu} \equiv \frac{1}{2} \left( y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu} y_\rho \rho \right)$$

for any tensor $y_{\mu\nu}^4$. The bare (renormalized) scalar boson mass here is $m_o(m)$ and we set presently the small observed [23,24] value of the cosmological constant to zero so that our quantum graviton, $h_{\mu\nu}$, has zero rest mass. We return to the latter point, however, when we discuss phenomenology. Feynman [28,29] has essentially worked out the Feynman rules for (6), including
Figure 1: Graviton loop contributions to the scalar propagator. $q$ is the 4-momentum of the scalar.

the rule for the famous Feynman-Faddeev-Popov \cite{28,30,31} ghost contribution required for unitarity with the fixing of the gauge (we use the gauge of Feynman in Ref. \cite{28}, $\partial^\mu \bar{h}_{\nu\mu} = 0$). For more details of this material we refer to Refs. \cite{28,29}. We turn now directly to the quantum loop corrections in the theory in (6).

We have shown in Refs. \cite{9,11} that the large virtual IR effects in the respective loop integrals for the scalar propagator in Fig. 1 in quantum general relativity can be resummed to the exact result

\begin{equation}
\left. i\Delta'_F(k) \right|_{\text{resummed}} = i \frac{k^2 - m^2 - \Sigma_s(k) + i\epsilon}{ie^{\gamma_e}(k)} \equiv i\Delta'_F(k) |_{\text{resummed}}
\end{equation}

for \((\Delta = k^2 - m^2)\)

\begin{equation}
\begin{aligned}
B''_g(k) &= -2i\kappa^2 k^4 \int \frac{d^4 \ell}{16\pi^4} \frac{1}{\ell^2 - \lambda^2 + i\epsilon} \\
&= \kappa^2 |k^2| \left[ \ln \left( \frac{m^2}{m^2 + |k^2|} \right) + \frac{1}{8\pi^2} \ln \left( \frac{m^2}{m^2 + |k^2|} \right) \right].
\end{aligned}
\end{equation}

The latter form holds for the UV (deep Euclidean) regime, so that \(\Delta'_F(k) |_{\text{resummed}}\) falls faster than any power of \(|k^2|\) – by Wick rotation, the identification
$-|k^2| \equiv k^2$ in the deep Euclidean regime gives immediate analytic continuation to the result in the last line of (8) when the usual $-i\epsilon$, $\epsilon \downarrow 0$, is appended to $m^2$. See Ref. [9] for the analogous result for $m = 0$. Here, $-i\Sigma_s(k)$ is the 1PI scalar self-energy function so that $i\Delta_F'(k)$ is the exact scalar propagator. As the residual $\Sigma_s$ starts in $O(\kappa^2)$, we may drop it in calculating one-loop effects. When the respective analogs of $i\Delta_F'(k)|_{\text{resummed}}$ are used for the elementary particles, one-loop corrections are finite. In fact, the use of our resummed propagators renders all quantum gravity loops UV finite [9–11]. We have called the attendant representation of the quantum theory of general relativity the theory of resummed quantum gravity (RQG).

Specifically, we use our resummed propagator results, extended to all the particles in the SM Lagrangian and to the graviton itself, working with the complete theory

$$L(x) = \frac{1}{2\kappa^2} \sqrt{-g} (R - 2\Lambda) + \sqrt{-g} L^G_{SM}(x)$$  \hspace{1cm} (9)

where $L^G_{SM}(x)$ is SM Lagrangian written in diffeomorphism invariant form as explained in Refs. [9,11] to show in the Refs. [9,11] that the denominator for the propagation of transverse-traceless modes of the graviton becomes $(M_{Pl})$  

$$q^2 + \Sigma^T(q^2) + i\epsilon \cong q^2 - q^4 c^2_{\text{eff}} \frac{360\pi}{M^2_{Pl}}, \hspace{1cm} (10)$$

where we have defined

$$c_{\text{eff}} = \sum_{\text{SM particles } j} n_j I_2(\lambda_c(j)) \cong 2.56 \times 10^4$$  \hspace{1cm} (11)

with $I_2$ defined [9,11] by

$$I_2(\lambda_c) = \int_0^\infty dx x^3 (1 + x)^{-4 - \lambda_c x}$$  \hspace{1cm} (12)

and with $\lambda_c(j) = \frac{2m^2_j}{\pi M_{Pl}^2}$ and $n_j$ equal to the number of effective degrees of freedom [9,11] of particle $j$. We refer the reader to Refs. [9] for the details of

\footnote{These follow from the observation [9,26,27] that the IR limit of the coupling of the graviton to a particle is independent of its spin.}
the derivation of the numerical value of $c_{2,\text{eff}}$. In this way, we identify (we use $G_N$ for $G_N(0)$)

$$G_N(k) = G_N / (1 + \frac{c_{2,\text{eff}} k^2}{360\pi M_P^2})$$  (13)

and we compute the UV limit $g_*$ as

$$g_* = \lim_{k^2 \to \infty} k^2 G_N(k^2) = \frac{360\pi}{c_{2,\text{eff}}} \approx 0.0442.$$  (14)

For the prediction for $\lambda_*$, we use the Euler-Lagrange equations to get Einstein’s equation as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa^2 T_{\mu\nu}$$  (15)

in a standard notation where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$. $R_{\mu\nu}$ is the contracted Riemann tensor, and $T_{\mu\nu}$ is the energy-momentum tensor. Working then with the representation $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$ for the flat Minkowski metric $\eta_{\mu\nu} = \text{diag}(1,-1,-1,-1)$ we see that to isolate $\Lambda$ in Einstein’s equation (15) we may compute the VEV (vacuum expectation value) of both sides of that equation. When we do this computation as described in Ref. [22] we see that a scalar makes the contribution to $\Lambda$ given by

$$\Lambda_s = -8\pi G_N \int \frac{d^4k}{(2\pi)^4} (2k_0^2) e^{-\lambda_c (k^2/(2m^2)) \ln(k^2/m^2 + 1)}$$

$$\approx -8\pi G_N \left[ \frac{1}{G_N^2 64 \rho^2} \right],$$  (16)

where $\rho = \ln \frac{2}{\lambda_c}$ and we have used the calculus of Refs. [9,11]. The standard methods [22] then show that a Dirac fermion contributes $-4$ times $\Lambda_s$ to $\Lambda$. The deep UV limit of $\Lambda$ then becomes, allowing $G_N(k)$ to run,

$$\Lambda(k) \to k^2 \lambda_*,$$

$$\lambda_* = -\frac{c_{2,\text{eff}}}{2880} \sum_j (-1)^j n_j / \rho_j^2$$  (17)

$$\approx 0.0817$$

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*We note the use here in the integrand of $2k_0^2$ rather than the $2(k^2 + m^2)$ in Ref. [12] to be consistent with $\omega = -1$ [22] for the vacuum stress-energy tensor.*
where $F_j$ is the fermion number of particle $j$, $n_j$ is the effective number of degrees of freedom of $j$ and $\rho_j = \rho(\lambda_c(m_j))$. In an exactly supersymmetric theory $\lambda_*$ would vanish.

One may compare the UV fixed-point calculated here,

$$(g_*, \lambda_*) \cong (0.0442, 0.0817),$$

with the estimate

$$(g_*, \lambda_*) \approx (0.27, 0.36)$$

in Refs. 17, 18. Here, one must keep in mind that the analyses in Refs. 17, 18 did not include the specific SM matter action and that there is definitely cutoff function sensitivity to the results in the latter analyses. Qualitatively, the two sets of results are similar in that in both of them $g_*$ and $\lambda_*$ are positive and are less than 1 in size. Further discussion of the relationship between our $\{g_*, \lambda_*\}$ predictions and those in Refs. 17, 18 is given in Refs. 9.

4 Review of the RQG Estimate of $\Lambda$ and its Constraints on Susy GUTS

When taken together with the results in Refs. 17, 18 the results given above allow us to estimate the value of $\Lambda$ today. We start from the normal-ordered form of Einstein’s equation

$$: G_{\mu\nu} : + \Lambda : g_{\mu\nu} : = -\kappa^2 : T_{\mu\nu} : . \tag{18}$$

If we use the coherent state representation of the thermal density matrix we can write the Einstein equation in the form of thermally averaged quantities with $\Lambda$ given by our result in (16) summed over the degrees of freedom as specified above in lowest order. The Planck scale cosmology description of inflation in Ref. 18 gives the transition time between the Planck regime and the classical Friedmann-Robertson-Walker (FRW) regime as $t_{tr} \sim 25t_{Pl}$. (We discuss in Ref. 22 the uncertainty of this choice of $t_{tr}$ and we present more on this uncertainty below.) Starting with the quantity

$$\rho_\Lambda(t_{tr}) \equiv \frac{\Lambda(t_{tr})}{8\pi G_N(t_{tr})} = \frac{-M_{Pl}^4(k_{tr})}{64} \sum_j \frac{(-1)^F n_j}{\rho_j^2} \tag{19}$$
and employing the arguments in Refs. [33] (\(t_{\text{eq}}\) is the time of matter-radiation equality) we get the first principles field theoretic estimate

\[
\rho_\Lambda(t_0) \approx \frac{-M_{Pl}^4(1 + c_{2, \text{eff}} k^2_r/(360 \pi M_{Pl}^2))}{64} \sum_j \frac{(-1)^F n_j}{\rho_j^2} \times \frac{t_{\text{tr}}^2}{t_{\text{eq}}^2} \times \left(\frac{t_{\text{eq}}}{t_0^2}\right)^3
\]

\[
\approx -M_{Pl}^2(1.0362)^2(-9.194 \times 10^{-3}) (25)^2 \frac{t_0^2}{64}
\]

\[
\approx (2.4 \times 10^{-3} \text{eV})^4.
\]

Here, \(t_0\) is the age of the universe and we take it to be \(t_0 \approx 13.7 \times 10^9\) yrs.

In the estimate in (20), the first factor in the second line comes from the radiation dominated period from \(t_{\text{tr}}\) to \(t_{\text{eq}}\) and the second factor comes from the matter dominated period from \(t_{\text{eq}}\) to \(t_0\).\(^7\) The estimate in (20) is close to the experimental result \(\rho_\Lambda(t_0)|_{\text{expt}} \approx ((2.37 \pm 0.05) \times 10^{-3}) \text{eV}^4\)\(^8\).

We do believe our estimate of \(\rho_\Lambda(t_0)\) represents some amount of progress in the long effort to understand its observed value in relativistic quantum field theory. We do not consider the estimate to be a precision prediction, as hitherto unseen degrees of freedom, such as a high scale GUT theory, may exist that have not been included in the calculation.

One may ask what would happen to our estimate if there were a GUT theory at high scales? The main viable approaches in this regard involve susy GUT’s. For definiteness and purposes of illustration, we will use the susy SO(10) GUT model in Ref. [35] to illustrate how such a theory might affect our estimate of \(\Lambda\).

In this model, the break-down of the GUT gauge symmetry to the low energy gauge symmetry occurs with an intermediate stage with gauge group \(SU_{2L} \times SU_{2R} \times U_1 \times SU(3)^c\) where the final break-down to the Standard Model \(SU_2 \times U_1 \times SU(3)^c\), occurs at a scale \(M_R \gtrsim 2\text{TeV}\) while the breakdown of global susy occurs at the (EW) scale \(M_\phi\) which satisfies \(M_R > M_\phi\). For our analysis the key observation is that susy multiplets do

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\(^7\)The method of the operator field forces the vacuum energies to follow the same scaling as the non-vacuum excitations.

\(^8\)See also Ref. [34] for an analysis that suggests a value for \(\rho_\Lambda(t_0)\) that is qualitatively similar to this experimental result.
not contribute to our formula for $\rho_\Lambda(t_{tr})$ when susy is not broken – there is exact cancellation between fermions and bosons in a given degenerate susy multiplet. Only the the broken susy multiplets can contribute. In the model at hand, these are just the multiplets associated with the known SM particles and the extra Higgs multiplet required by susy in the MSSM [38]. In the light of recent LHC results [39], we take for illustration the values $M_R \approx 4M_S \sim 2.0\text{TeV}$ and set the following susy partner values:

$$m_{\tilde{g}} \approx 1.5(10)\text{TeV}, \ m_{\tilde{G}} \approx 1.5\text{TeV}, \ m_{\tilde{q}} \approx 1.0\text{TeV}, \ m_{\tilde{\ell}} \approx 0.5\text{TeV},$$

$$m_{\tilde{\chi}_i^0} \approx \begin{cases} 0.4\text{TeV}, & i = 1 \\ 0.5\text{TeV}, & i = 2, 3, 4 \end{cases},$$

$$m_{\tilde{\chi}_i^\pm} \approx 0.5\text{TeV}, \ i = 1, 2, \ m_S = .5\text{TeV}, \ S = A^0, \ H^\pm, \ H_2,$$

where we use a standard notation for the susy partners of the known quarks ($q \leftrightarrow \tilde{q}$), leptons ($\ell \leftrightarrow \tilde{\ell}$) and gluons ($G \leftrightarrow \tilde{G}$), and the EW gauge and Higgs bosons ($\gamma, Z^0, W^\pm, H, A^0, H^\pm, H_2 \leftrightarrow \tilde{\chi}$) with the extra Higgs particles denoted as usual [38] by $A^0$(pseudo-scalar), $H^\pm$(charged) and $H_2$(heavy scalar). $\tilde{g}$ is the gravitino, for which we show two examples of its mass for illustration. From these particles we get the extra contribution

$$\Delta W_{\rho, \text{GUT}} = \sum_{j \in \{\text{MSSM low energy susy partners}\}} \frac{(-1)^F n_j}{\rho_j^2} \approx 1.13(1.12) \times 10^{-2}$$

(22)

to the factor $W_\rho \equiv \sum_j \frac{(-1)^F n_j}{\rho_j^2}$ on the RHS of our equation for $\rho_\Lambda(t_{tr})$ for the two respective values of $m_{\tilde{g}}$ indicated by the parentheses. The attendant values of $\rho_\Lambda$ are $-(1.67 \times 10^{-3}\text{eV})^4-(1.65 \times 10^{-3}\text{eV})^4$, respectively. Due to their signs, these results would appear to be in conflict with the positive observed value quoted above by many standard deviations. This last conclusion holds even when we allow for the considerable uncertainty in the various other factors, all positive, multiplying $W_\rho$ in our formula for $\rho_\Lambda(t_{tr})$. To remedy this situation, we may either add new particles to the model, approach (A), or allow a near GUT scale soft susy breaking mass term for the gravitino, where the GUT scale $M_{\text{GUT}}$ is $\sim 4 \times 10^{16}\text{GeV}$ here [35], approach (B). Approach (A) doubles the number of quarks and leptons, but inverts the mass hierarchy between susy partners, so that the new squarks and sleptons are lighter than the new quarks and leptons. This also requires that we increase $M_R, M_S$
so that we have the new quarks and leptons at $M_{\text{High}} \sim 3.4(3.3) \times 10^3 \text{TeV}$ while leaving their partners at $M_{\text{Low}} \sim .5 \text{TeV}$. Approach (B) sets the mass of the gravitino soft breaking term to $m_{\tilde{g}} \sim 2.3 \times 10^{15} \text{GeV}$. What these results demonstrate is that our estimate in (20) can be used as a constraint of general susy GUT models and we hope to explore such in more detail elsewhere.

As we explain in Ref. [22] we stress that we actually do not know the precise value of $t_{tr}$ at this point in the discussion to better than a couple of orders of magnitude. This translates to an uncertainty at the level of $10^4$ on our estimate of $\rho_\Lambda$. We return to this issue in the next Section.

We have not mentioned the effect of the various spontaneous symmetry vacuum energies on our $\rho_\Lambda$ estimate. From the standard methods we know for example that the energy of the broken vacuum for the EW case contributes an amount of order $M_W^4$ to $\rho_\Lambda$. If we consider the GUT symmetry breaking we expect an analogous contribution from spontaneous symmetry breaking of order $M_{\text{GUT}}^4$. The RHS of our equation for $\rho_\Lambda(t_{tr})$ is

$$\sim (-1.0362)^2 W_\rho / 64) M_{Pl}^4 \simeq \frac{10^{-2}}{64} M_{Pl}^4.$$

It follows that including these broken symmetry vacuum energies would make relative changes in our results at the level of

$$\frac{64}{10^{-2}} \frac{M_W^4}{M_{Pl}^4} \simeq 1 \times 10^{-65}$$

and

$$\frac{64}{10^{-2}} \frac{M_G^4}{M_{Pl}^4} \simeq 7 \times 10^{-7},$$

respectively, where we use our value of $M_{\text{GUT}}$ given above in the latter evaluation for definiteness. We ignore such small effects here.

In discussing the impact of our approach to $\Lambda$ on the phenomenology of big bang nucleosynthesis (BBN) [40], we proceed as follows. We observe that the authors in Ref. [18] have noted that, when one passes from the Planck era to the FRW era, a gauge transformation (from the attendant diffeomorphism invariance) is necessary to maintain consistency with the solutions of the system (4) (or of its more general form as given below) at the transition time $t_{tr}$ at the boundary between the two regimes. From continuity of the Hubble parameter at $t_{tr}$ the authors in Ref. [18] arrive at the gauge transformation on
the time for the FRW era relative to the Planck era \( t \rightarrow t' = t - t_{as} \). It follows that continuity of the Hubble parameter at the boundary gives

\[
\frac{\alpha}{t_{tr}} = \frac{1}{2(t_{tr} - t_{as})}
\]

when \( a(t) \propto t^\alpha \) in the (sub-)Planck regime. This implies

\[
t_{as} = (1 - \frac{1}{2\alpha})t_{tr},
\]

In our analysis, we have from Ref. [18] the generic case \( \alpha = 25 \), so that \( t_{as} = 0.98t_{tr} \). Here, we use the diffeomorphism invariance of the theory to choose another coordinate transformation for the FRW era, namely,

\[
t \rightarrow t' = \gamma
\]

as a part of a dilatation where \( \gamma \) now satisfies the boundary condition required for continuity of the Hubble parameter at \( t_{tr} \):

\[
\frac{\alpha}{t_{tr}} = \frac{1}{2\gamma t_{tr}}
\]

so that \( \gamma = \frac{1}{2\alpha} \). According to the model in Ref. [18], for \( t > t_{tr} \), one has the time \( t' \) and an effective FRW cosmology with such a small value of \( \Lambda \) that it may be treated as zero. Here, we extend this by retaining \( \Lambda \neq 0 \) so that we may estimate its value. With our diffeomorphism transformation between the (sub-)Planck regime and the FRW regime, we see that, at the time of BBN, the ratio of \( \rho_\Lambda \) to \( \frac{3H^2}{8\pi G_N} \) is

\[
\Omega_\Lambda(t_{BBN}) = \frac{M^2_{Pl}(1.0362)^29.194 \times 10^{-3}(25)^2/(64t_{BBN}^2)}{(3/(8\pi G_N))(1/(2\gamma t_{BBN}^2))}
\]

\[
\approx \frac{\pi 10^{-2}}{24} = 1.31 \times 10^{-3}.
\]

At \( t_{BBN} \) our \( \rho_\Lambda \) has a negligible effect on the standard BBN phenomenology. Note that, in contrast to what happens in [20], the uncertainty in the value of \( \alpha \) does not affect the estimate in (23) because the factors of \( \alpha^2 = 25^2 \) cancel between the numerator and the denominator on the RHS in the first line of (23).
Turning next to the issue of the covariance of the theory when $\Lambda$ and $G_N$ depend on time, Eqs. (4) follow the corresponding realization of the improved Friedmann and Einstein equations as given in Eqs. (3.24) in Ref. 17. The more general realization of (4) is given in Eqs. (2.1) in Ref. 18 and it is this latter realization which our discussions in this Section effectively followed. The two realizations differ in the solution of the Bianchi identity constraint:

$$D^\nu (\Lambda g_{\nu\mu} + 8\pi G_N T_{\nu\mu}) = 0;$$

for, this identity is solved in (4) for a covariantly conserved $T_{\mu\nu}$ whereas, in Eqs. (2.1) in Ref. 18, one has the modified conservation requirement

$$\dot{\rho} + 3\frac{\dot{a}}{a}(1 + \omega)\rho = -\frac{\dot{\Lambda} + 8\pi \rho \dot{G}_N}{8\pi G_N}.$$  

In (4) the RHS of this latter equation is set to zero. The phenomenology from Ref. 17 is qualitatively unchanged by the simplification in (4) but the attendant details, such as the (sub-)Planck era exponent for the time dependence of $a$, etc., are affected, as is the relation between $\dot{\Lambda}$ and $\dot{G}_N$ in (4). We observe that (4) contains a special case of the more general realization of the Bianchi identity requirement when both $\Lambda$ and $G_N$ depend on time and in this Section we use that more general realization. Note that covariant conservation of matter in the current universe is guaranteed only when

$$\dot{\Lambda} + 8\pi \rho \dot{G}_N = 0$$

holds and that the case without such guaranteed conservation is possible provided the attendant deviation is small. See Refs. 41–43 for detailed studies of such deviation, including its maximum possible size.

5 Einstein-Heisenberg Consistency Condition and the Uncertainty of $t_{tr}$

Given the closeness of our estimate of $\rho_\Lambda$ to its observed value and the potential constraints on Beyond the Standard Model physics scenarios this closeness would obtain, it is appropriate to address the theoretical error of our estimate. This we do in this section using consistency arguments [25] based on the Heisenberg uncertainty principle and the solutions of Einstein’s equation for the general theory of relativity.

15
Specifically, the basic physical idea which we wish to apply here is the known property of a de Sitter universe, which we describe here with the metric
\[
g_{\mu\nu} dx^\mu dx^\nu = dt^2 - e^{2t/b} [dw^2 + w^2 (d\theta^2 + \sin^2 \theta d\phi^2)]
\]
in an obvious notation, with \( b = \sqrt{3/\Lambda} \): if a light ray starts at the origin (\( w = 0 \) here) and travels uninterruptedly, it never gets past the point \( w = w_0 \equiv b \) along its geodesic. Because we treat quantum mechanics as truly interwoven with the fabric of space-time, as it most certainly should be, according to Einstein’s general theory of relativity, quantum mechanics must know about the latter limit for the quantum wave function of the photons in this light ray. According to the Heisenberg uncertainty principle, the uncertainty associated with the momentum conjugate variable to the coordinate distance \( w \) is correspondingly bounded in the quantum theory of general relativity. To get a manifestation of the respective constraint, we use the results in Refs. 45–48 to check for the consistency of this bound with the effective scale \( k \) associated to the running values of \( G_N(k) \), \( \Lambda(k) \) as we discussed above.

More precisely, we start from the basic formulation of the Heisenberg uncertainty principle,
\[
\Delta p \Delta q \geq \frac{1}{2},
\]
where we define \( \Delta A \) as the quantum mechanical uncertainty of the observable \( A \) and \( p \) is the momentum conjugate to the observable coordinate \( q \). In our case, we have \( q = w \cos \theta \) where \( \theta \) is the polar angle when the direction of \( \vec{k} \) is taken along the \( \hat{z} \) direction and we may identify \( \Delta p \) as our effective \( k \), as \( k \) represents the size of the mean squared momentum fluctuations in the universe that are effective for the running of the universe observables \( G_N(k) \), \( \Lambda(k) \).

For the universe in the Planck regime, from the explicit solutions of the field equations in Refs. 49–48, we see that the solutions of the scalar field equations, in an appropriate set of coordinates, are spanned by plane waves in 3-space with Bessel/Hankel function-related dependence on time. We thus arrive at the estimate, at any given time, again using an obvious notation,
\[
(\Delta q)^2 \approx \frac{\int_0^{w_0} dww^2 \cos^2 \theta}{\int_0^{w_0} dww^2} = \frac{1}{5} w_0^2.
\]
\(^9\)Spin continues to be an inessential complication here 49.
From this estimate, we get the Einstein-Heisenberg consistency condition

\[ k \geq \sqrt{\frac{5}{2w_0}} = \sqrt{\frac{5}{2}} \frac{1}{\sqrt{3/\Lambda(k)}} \]  

(26)

where \( \Lambda(k) \) follows from \(^{(20)}\) (see Eq.(52) in Ref. \(^{22}\)):

\[
\Lambda(k) = \frac{-\pi M_{Pl}^2(k)}{8} \sum_j \frac{(-1)^{F_j} n_j}{\rho_j^2} \\
= \frac{-\pi M_{Pl}^2(1 + c_{2,eff} k^2/(360 \pi M_{Pl}^2))}{8} \sum_j \frac{(-1)^{F_j} n_j}{\rho_j^2} \\
\approx \frac{\pi M_{Pl}^2(1 + c_{2,eff} k^2/(360 \pi M_{Pl}^2)) \times 9.194 \times 10^{-3}}{8}.
\]

(27)

We argue that the Planck scale inflation must end when \( k \) becomes too small to satisfy the condition \(^{(26)}\). Accordingly, we estimate the transition time, \( t_{tr} = \alpha/M_{Pl} = 1/k_{tr} \), from the Planck scale inflationary regime to the Friedmann-Robertson-Walker regime via the value of \( \alpha \) for which equality holds in \(^{(26)}\). On our solving for \( \alpha \) we get

\[ \alpha \approx 25.3, \]  

(28)

which is in agreement with the value \( \alpha \approx 25 \) implied by the numerical studies in Ref. \(^{17,18}\). We conclude that the error on our estimate of \( t_{tr} \) is at the level of a factor \( O(3) \) or less so that the uncertainty on our estimate of \( \Lambda \) is now reduced from a factor of 100 \(^{22}\) to a factor of \( O(10) \).

6 Summary

We have presented what amounts to a status report on the theory of resummed quantum gravity. We have reviewed the elements of the foundations of the theory, its predictions for the running of the Newton’s constant and of the cosmological constant, and its prediction for the current value of the cosmological constant, where the latter prediction uses the Planck scale cosmology model of Bonanno and Reuter. Our prediction for the current value of the cosmological constant is close enough to experiment that it allows us
to put constraints on possible susy GUT models, such as that presented in Ref.\textsuperscript{35}. We have discussed as well the status of our RQG theory with respect to various other phenomenological and theoretical cosmological constraints. We have finally reviewed the use\textsuperscript{25} of the Heisenberg uncertainty principle and the properties of the solutions of Einstein’s equation for general relativity to constrain the main uncertainty in our estimate for the cosmological constant. In this way, we have reduced the latter uncertainty to a factor $O(10)$. In principle, further improvement in this error should be possible.

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