The CP-asymmetry in resonant leptogenesis

A. Anisimov\textsuperscript{a}, A. Broncano\textsuperscript{b} and M. Plüninger\textsuperscript{b}

\textsuperscript{a} Institut de Théorie des Phénomènes Physiques, Ecole Polytechnique
Fédérale de Lausanne, CH-1015 Lausanne, Switzerland

\textsuperscript{b} Max Planck Institute for Physics, Föhringer Ring 6, 80805 Munich, Germany

March 26, 2022

Abstract

We study the resonantly enhanced CP-asymmetry in the decays of nearly mass-degenerate heavy right-handed Majorana neutrinos for which different formulae have been presented in the literature, depending on the method used to calculate it. We consider two different techniques and show that they lead to the same result, thereby reconciling the different approaches.

1 Introduction

Leptogenesis\textsuperscript{11} offers a simple and elegant explanation for the matter-antimatter asymmetry of the universe and relates the observed baryon asymmetry to properties of neutrinos. In particular, a lower bound on the mass of the heavy neutrino whose decays create the baryon asymmetry of $\sim 10^9\text{ GeV}$ has been derived in the simplest scenario of thermal leptogenesis with hierarchical right-handed neutrinos\textsuperscript{23}. Hence, the required reheating temperature for successful leptogenesis in such a scenario cannot be much lower\textsuperscript{4}, which, in supersymmetric scenarios, may be in conflict with upper bounds on the reheating temperature from the gravitino problem\textsuperscript{5}.

Resonant leptogenesis\textsuperscript{6,9} has been proposed as a way to evade this bound. If the heavy right-handed neutrinos are nearly degenerate in mass, self-energy contributions to the CP-asymmetries in their decays may be resonantly enhanced. This in turn would make thermal leptogenesis viable at much lower temperatures in the early universe. Self-energy contributions to the CP-asymmetry in leptogenesis have been considered numerous times in the past\textsuperscript{6,16}. However, different formulae for the CP-asymmetry can be found in the
literature depending on the methods and approximations used to derive it. Indeed, the correct treatment of self-energy contributions for a decaying particle is not obvious.

$CP$-violation in the decays of heavy neutrinos arises due to the interference of the imaginary phases of the couplings with the absorptive parts of one-loop diagrams. A popular technique to calculate the contribution of self-energy diagrams to the $CP$-asymmetry is the use of an effective Hamiltonian, similar to that applied in the $K_0 - \bar{K}_0$ system \[6\]-\[12\]. Due to the unstable nature of the heavy neutrinos, this approach suffers from several shortcomings \[8\]-\[15\]. It is well-known that unstable particles cannot be described as asymptotic free states, i.e. they cannot appear as in- or out-states of S-matrix elements \[17\]. Refs. \[9\]-\[16\] tackle the problem using a field-theoretical approach where the $CP$-asymmetry is extracted from the resonant contributions of heavy neutrinos to stable particle scattering amplitudes. There, the unstable nature of the right-handed neutrinos is taken into account by the resummation of self-energy diagrams. Starting from the same resummed propagator, the authors of Refs. \[9\]-\[16\] and those of Ref. \[14\] obtain different final expressions for the $CP$-asymmetries in the decays of heavy neutrinos. The main difference between these papers is the way that the contributions from different heavy neutrinos are inferred from the scattering amplitudes of stable particles. In Refs. \[9\]-\[16\], the contributions from different neutrino mass eigenstates are identified by means of an expansion of the resummed propagator around its poles, whereas in Ref. \[14\] the resummed propagator is diagonalized in order to identify one-loop contributions to the decay amplitudes of the heavy neutrinos.

In this paper, we compare the diagonalization and the pole expansion methods and show that, using the same renormalization scheme, they lead to the same result for the $CP$-asymmetry, consistent with the one obtained in Ref. \[14\]. We also discuss the range of validity of this perturbative resummation approach. In section 2 we start by introducing some notation and discuss the resummed heavy neutrino propagator. In section 3 we then introduce scattering amplitudes of stable particles from which properties of the unstable right-handed neutrinos can be extracted and compute the $CP$-asymmetries in their decays both in the pole expansion and the propagator diagonalization methods, showing that both approaches yield the same results for physical quantities, e.g. decay widths and $CP$-asymmetries.

2 Self-energy corrections to the heavy neutrino propagator

Leptogenesis is based on the type I seesaw model \[18\], which supplements the standard model with $n'$ right-handed neutrinos. The corresponding Yukawa couplings and masses of charged leptons and neutrinos are then given by the following Lagrangian:

$$L_Y = l_L h_l^* e_R + l_L \tilde{h}_l^* N_R - \frac{1}{2} N_R^C M N_R + \text{h.c.}.$$ \hspace{1cm} (1)

The matrices $h_l$, $h_\nu$ and $M$ are, respectively, $3 \times 3$, $3 \times n'$ and $n' \times n'$ complex matrices. Without loss of generality, one can always choose a basis where $h_l$ and $M$ are diagonal with real and positive eigenvalues, whereas $h_\nu$ depends on $3 + n' + 3n'$ real quantities and $3(n' - 1)$ imaginary phases \[20\]-\[21\]. The physical mass eigenstates are then the Majorana neutrinos $N_i = N_{Ri} + N_{Ci}^*$ with mass eigenvalues $M_i$. At tree level their inverse propagator matrix reads

$$D_{ii}(p) \equiv S^{-1}_{ii}(p) = \not{p} - M_i , \hspace{1cm} (2)$$
which has poles at $p^2 = M_i^2$, corresponding to stable particles. The finite lifetime of the physical Majorana neutrinos is taken into account by resumming self-energy diagrams. At one loop, these diagrams introduce flavour non-diagonal elements in the inverse propagator

$$D_{ij}(p) = (S^{-1}(p))_{ij} = \not{p} - M_i - \Sigma_{ij}(p),$$

where

$$\Sigma_{ij}(p) = \not{p} P_R \Sigma^R_{ij}(p^2) + \not{p} P_L \Sigma^L_{ij}(p^2),$$

are the bare self-energies and $P_{R,L} = \frac{1 \pm \gamma_5}{2}$ are the usual chiral projectors.

The self-energies can be written in terms of a complex function $a(p^2)$ and a hermitian matrix $K$,

$$\Sigma_{ij}^R(p^2) = a(p^2) K_{ij}, \quad K_{ij} \equiv (h^\dagger h)_{ij}.$$  

In dimensional regularization, with $n = 4 - 2\epsilon$ dimensions, $a(p^2)$ is given by

$$a(p^2) = \frac{1}{16\pi^2} \left( -\Delta + \ln\left( \frac{|p^2|}{\mu^2} \right) - 2 - i\pi \theta(p^2) \right),$$

where the ultraviolet divergence is contained in

$$\Delta = \frac{1}{\epsilon} - \gamma_E + \ln(4\pi).$$

In order to identify the physical states, the one-loop resummed propagator has to be renormalized. We will use the on-shell (OS) scheme, since in it the particle masses are renormalized so as to represent the physical masses at the poles of the propagators. We will follow the formalism for mixing renormalization worked out in Ref. [19] and the detailed computation of the renormalization in our case can be found in Appendix A.

The renormalized inverse propagator is then given by

$$\hat{D}(p) = \hat{S}^{-1}(p)$$

$$= \not{p} P_R \left( 1 - \hat{\Sigma}^R(p^2) \right) + \not{p} P_L \left( 1 - (\hat{\Sigma}^R(p^2))^\dagger \right) - P_R \left( \hat{M} + \hat{\Sigma}^M \right) - P_L \left( \hat{M} + \hat{\Sigma}^{M^\dagger} \right).$$

The off-diagonal ($i \neq j$) renormalized self-energies read

$$\hat{\Sigma}^R_{ij}(p^2) = \frac{K_{ij}}{16\pi^2} \left[ \ln\left( \frac{|p^2|}{M_i M_j} \right) - \frac{1}{2} \frac{\hat{M}_j^2 + \hat{M}_i^2}{\hat{M}_j^2 - \hat{M}_i^2} \ln\left( \frac{\hat{M}_j^2}{\hat{M}_i^2} \right) - i\pi \theta(p^2) \right],$$

$$\hat{\Sigma}^M_{ij} = \frac{1}{16\pi^2} \frac{\hat{M}_j \hat{M}_i}{\hat{M}_j^2 - \hat{M}_i^2} \ln\left( \frac{\hat{M}_j^2}{\hat{M}_i^2} \right) \left[ \hat{M}_i K_{ij} + \hat{M}_j K_{ij} \right],$$

whereas the flavour-diagonal ones are given by

$$\hat{\Sigma}^R_{ii}(p^2) = \frac{K_{ii}}{16\pi^2} \left[ \ln\left( \frac{|p^2|}{M_i^2} \right) - 2 - i\pi \theta(p^2) \right],$$

$$\hat{\Sigma}^M_{ii} = \frac{\hat{M}_i K_{ii}}{8\pi^2}. $$
In order to compute the elements of the renormalized propagator \( \hat{S}_{ij} \) it is useful to decompose \( \hat{S}(p) \) into its four chiral components,

\[
\hat{S}(p) = P_R \hat{S}^{RR}(p^2) + P_L \hat{S}^{LL}(p^2) + P_L \not p \hat{S}^{LR}(p^2) + P_R \not p \hat{S}^{RL}(p^2).
\]

These chiral parts of the propagator are obtained by inserting this decomposition into the identity

\[
\hat{D}(p) \hat{S}(p) = I,
\]

and multiplying from the left and the right with chiral projectors \( P_L, R \). The solution reads

\[
\hat{S}^{RR}(p^2) = \left( 1 - \tilde{\Sigma}^L(p^2) \right) \left( 1 - \tilde{\Sigma}^R(p^2) \right)^{-1} \frac{p^2}{M + \Sigma^M} \left( 1 - \hat{\Sigma}^M \hat{\Sigma}^M^* \right) \hat{S}^{LL}(p^2)
\]

\[
\hat{S}^{LL}(p^2) = \left( 1 - \tilde{\Sigma}^R(p^2) \right) \left( 1 - \tilde{\Sigma}^L(p^2) \right)^{-1} \frac{p^2}{M + \Sigma^M} \left( 1 - \hat{\Sigma}^M \hat{\Sigma}^M^* \right) \hat{S}^{RR}(p^2),
\]

\[
\hat{S}^{RL}(p^2) = \frac{1}{M + \Sigma^M} \left( 1 - \tilde{\Sigma}^R(p^2) \right) \hat{S}^{LL}(p^2),
\]

\[
\hat{S}^{LR}(p^2) = \frac{1}{M + \Sigma^M} \left( 1 - \tilde{\Sigma}^L(p^2) \right) \hat{S}^{RR}(p^2).
\]

For simplicity, we will restrict ourselves, in the following, to the case of two right-handed neutrinos, \( n' = 2 \). However, the generalization to more than two generations is straightforward.

Only one-loop self-energy diagrams have been taken into consideration here, i.e. in a consistent computation the chiral parts of the propagator have to be linearized in the couplings \( K_{ij} \). For future use, we introduce here an expansion parameter \( \alpha \) related to the largest of the couplings \( K_{ij} \),

\[
\alpha = \text{Max} \left( \frac{K_{ij}}{16\pi^2} \right).
\]

In the interesting case that the masses of the right-handed neutrinos are quasi-degenerate, i.e. \( \tilde{M}_2 - \tilde{M}_1 \ll \tilde{M}_1 \), one can define an additional small expansion parameter

\[
\Delta = \frac{\tilde{M}_2 - \tilde{M}_1}{\tilde{M}_1}.
\]

Our results, to be presented in the following, will only be valid as long as \( \Delta \gg \alpha \), since otherwise perturbation theory breaks down.

To leading order in \( K \), the \( RR \) part of the propagator is then given by

\[
\hat{S}^{RR}(p^2) = \left( \begin{array}{cc}
\frac{(1+\Sigma^R_{11})\sqrt{s_1}}{p^2-s_1} & \frac{(\tilde{M}_2 \Sigma^R_{21} + \tilde{M}_1 \Sigma^R_{11} + \Sigma^M_{11}^*) p^2 + \tilde{M}_2 \tilde{M}_2 \Sigma^M_{11}}{(p^2-s_1)(p^2-s_2)} \\
\frac{(\tilde{M}_2 \Sigma^R_{11} + \tilde{M}_1 \Sigma^R_{21} + \Sigma^M_{21}) p^2 + \tilde{M}_1 \tilde{M}_2 \Sigma^M_{21}}{(p^2-s_1)(p^2-s_2)} & \frac{(1+\Sigma^R_{22})\sqrt{s_2}}{p^2-s_2}
\end{array} \right),
\]

(21)
where \( s_{1,2} \) are the poles of the propagator. These poles are given by the zeroes of the determinants of the inverse propagators, e.g. by solving \( \det (S^{RR})^{-1} = 0 \). They are the same for all four chiral propagator elements and to leading order in \( K \) they read

\[
s_i(p^2) = \hat{M}_i^2 + 2 \hat{M}_i \hat{\Sigma}^M_{ii} + 2 \hat{M}_i^2 \hat{\Sigma}^{R}_{ii}(p^2) = \hat{M}_i^2 \left( 1 + \frac{K_{ii}}{8\pi^2} \left[ \ln \left( \frac{|p^2|}{\hat{M}_i^2} \right) - i \pi \theta(p^2) \right] \right)
\]

and, therefore,

\[
\sqrt{s_i} \equiv \hat{M}_i + \hat{\Sigma}^M_{ii} + \hat{M}_i \hat{\Sigma}^R_{ii}(p^2).
\]

Note that on-shell, i.e. setting \( p^2 = \hat{M}_i^2 \), the poles have the familiar Breit-Wigner form

\[
s_i(M_i^2) = \hat{M}_i^2 - i \hat{M}_i \Gamma_i,
\]

where \( \Gamma_i \equiv \frac{\hat{M}_i K_{ii}}{8\pi} \) are the decay widths of the right-handed neutrinos.

Analogously, the other chiral parts of the propagator are evaluated to be

\[
\hat{S}^{LL}(p^2) = \begin{pmatrix}
\frac{(1+\hat{\Sigma}^R_{11})\sqrt{s_1}}{p^2-s_1} & \frac{(\hat{M}_2 \hat{\Sigma}^R_{12} + \hat{M}_1 \hat{\Sigma}^{L}_{11} + \hat{\Sigma}^M_{12})}{(p^2-s_1)(p^2-s_2)} \\
\frac{(\hat{M}_2 \hat{\Sigma}^R_{12} + \hat{M}_1 \hat{\Sigma}^{L}_{11} + \hat{\Sigma}^M_{12})}{(p^2-s_1)(p^2-s_2)} & \frac{(1+\hat{\Sigma}^R_{22})\sqrt{s_2}}{p^2-s_2}
\end{pmatrix},
\]

\[
\hat{S}^{LR}(p^2) = \begin{pmatrix}
\frac{1+\hat{\Sigma}^R_{11}}{p^2-s_1} & \frac{\hat{M}_1 \hat{M}_2 \hat{\Sigma}^R_{12} + \hat{M}_1^2 \hat{\Sigma}^{M}_{12} + \hat{M}_2^2 \hat{\Sigma}^{M}_{12}}{(p^2-s_1)(p^2-s_2)} \\
\frac{\hat{M}_1 \hat{M}_2 \hat{\Sigma}^R_{12} + \hat{M}_1^2 \hat{\Sigma}^{M}_{12} + \hat{M}_2^2 \hat{\Sigma}^{M}_{12}}{(p^2-s_1)(p^2-s_2)} & \frac{1+\hat{\Sigma}^R_{22}}{p^2-s_2}
\end{pmatrix},
\]

and \( S^{RL} = (S^{LR})^T \).

### 3 Two-body scatterings

Since the right-handed neutrinos are unstable, they cannot be treated as asymptotic free states, i.e. they cannot appear as in- or out-states of S-matrix elements. Their properties can, however, be inferred from transition matrix elements of scatterings of stable particles \[17\].

Here, we will only consider one-loop self-energy contributions to these scattering processes. Effects from one-loop corrections to the vertices will be neglected in the following, since their contribution to the \( CP \)-asymmetry is well known \[11,22\] and not controversial.

For the case at hand, the resummed right-handed neutrino propagator appears in the following four lepton-Higgs scattering processes \[13\]:

- **Lepton-number conserving scatterings**: The process \( l_\alpha \phi \rightarrow l_\beta \phi \) and its charge conjugate \( l_\alpha^c \phi^c \rightarrow l_\beta^c \phi^c \) are mediated by heavy neutrinos. The contribution of the resummed neutrino propagator to the amplitude for \( l_\alpha \phi \rightarrow l_\beta \phi \) can be written as

\[
i M = \overline{\nu}_\beta P_R \hat{h}^*_{\beta i} \hat{S}_{ij}(p^2) \hat{h}_{\alpha j} P_L u_\alpha = \overline{\nu}_\beta P_R \hat{h}^*_{\beta i} \phi \hat{S}^{RL}_{ij}(p^2) \hat{h}_{\alpha j} P_L u_\alpha,
\]
where \( \hat{h}_{\alpha}^* \) denote the renormalized Yukawa couplings of right-handed neutrinos to light lepton and Higgs doublets\(^1\). Note that only the chiral part \( \hat{S}^{RL} \) of the full propagator contributes to this amplitude.

Analogously, the contribution of \( N_R \) to the amplitude for the process \( l^c_\alpha \phi^* \rightarrow l^c_\beta \phi^* \) is given by

\[
i \mathcal{M} = \bar{\nu}_\beta P_L \hat{h}_{\beta i} \hat{S}_{ij}(p^2) \hat{h}_{\alpha j}^* P_R u_\alpha = \bar{\nu}_\beta P_L \hat{h}_{\beta i} \hat{S}^{LR}_{ij}(p^2) \hat{h}_{\alpha j}^* P_R u_\alpha , \tag{28}\]

i.e. only \( \hat{S}^{LR} \) contributes to this amplitude.

- **Lepton-number violating scatterings:**

The \( \Delta L = 2 \) scatterings \( l^c_\alpha \phi^* \rightarrow l^c_\beta \phi \) and \( l_\alpha \phi \rightarrow l^c_\beta \phi^* \) again result from right-handed neutrino exchange.

The amplitude for the process \( l^c_\alpha \phi^* \rightarrow l^c_\beta \phi \) reads

\[
i \mathcal{M} = \bar{\nu}_\beta P_R \hat{h}_{\beta i} \hat{S}_{ij}(p^2) \hat{h}_{\alpha j}^* P_R u_\alpha = \bar{\nu}_\beta P_R \hat{h}_{\beta i} \hat{S}^{RR}_{ij}(p^2) \hat{h}_{\alpha j}^* P_R u_\alpha . \tag{29}\]

Similarly, the amplitude for the \( CP \)-conjugated process \( l_\alpha \phi \rightarrow l^c_\beta \phi^* \) is

\[
i \mathcal{M} = \bar{\nu}_\beta P_L \hat{h}_{\beta i} \hat{S}_{ij}(p^2) \hat{h}_{\alpha j}^* P_L u_\alpha = \bar{\nu}_\beta P_L \hat{h}_{\beta i} \hat{S}^{LL}_{ij}(p^2) \hat{h}_{\alpha j}^* P_L u_\alpha , \tag{30}\]

i.e. only \( \hat{S}^{RR} \) and \( \hat{S}^{LL} \) contribute to these amplitudes.

Hence, each of the chiral parts of the propagator participates in a different scattering process. In the following, we will analyze the contributions of heavy neutrinos to these scattering processes and attempt to identify those of each mass eigenstate. This will allow us to define effective couplings of the heavy neutrinos to light lepton and Higgs doublets and, therefore, lead to a consistent computation of the self-energy contribution to the \( CP \)-asymmetry in heavy neutrino decays.

Different techniques of identifying the contributions of each neutrino mass eigenstate have been advocated in the literature. In Ref. \[16\] a decomposition into partial fractions in \( p^2 - s_i \) was proposed and the contributions of each mass eigenstate were identified with those associated with the corresponding poles. Alternatively, one can diagonalize the different chiral parts of the propagator and identify the eigenstates of the propagator with the effective couplings of the right-handed neutrinos, as proposed in Ref. \[14\]. In the following we will consider both methods and will show that they lead to consistent results.

### 3.1 Propagator pole expansion

Each chiral part of the resummed propagator can be decomposed into partial fractions,

\[
S^{AB} = \frac{X^{AB}}{p^2 - s_1} + \frac{Y^{AB}}{p^2 - s_2} , \quad \text{where } A, B = R, L , \tag{31}
\]

\(^1\)For simplicity, we drop the subscript \( \nu \) from the renormalized neutrino Yukawa couplings. Further, Greek indices \( \alpha, \beta, \ldots \) denote the generation indices of SM lepton doublets, whereas Latin indices \( i, j, \ldots \) are flavour indices of the right-handed neutrinos.
s_1 and s_2 are the poles and X and Y are the matrices which contain the coefficients of the expansion. Note, that they are matrices in flavour space and have no spinorial structure. The diagonal elements of X and Y can be abbreviated by

\begin{align}
x_{11} &= 1 + \frac{1}{2} \hat{\Sigma}_{11}^R (p^2), \\
y_{22} &= 1 + \frac{1}{2} \hat{\Sigma}_{22}^R (p^2),
\end{align}

whereas the non-diagonal elements are given by

\begin{align}
x_{12} &= -\frac{\hat{M}_1 \hat{M}_2 \hat{\Sigma}_{21}^R + \hat{M}_1 \hat{\Sigma}_{12}^M + \hat{M}_2 \hat{\Sigma}_{12}^M}{s_2 - s_1}, \\
x_{21} &= -\frac{\hat{M}_1 \hat{M}_2 \hat{\Sigma}_{12}^R + \hat{M}_1 \hat{\Sigma}_{12}^M + \hat{M}_2 \hat{\Sigma}_{12}^M}{s_2 - s_1}, \\
y_{12} &= \frac{\hat{M}_2 \hat{\Sigma}_{12}^R + \hat{M}_1 \hat{M}_2 \hat{\Sigma}_{21}^R + \hat{M}_2 \hat{\Sigma}_{12}^M + \hat{M}_1 \hat{\Sigma}_{12}^M}{s_2 - s_1}, \\
y_{21} &= \frac{\hat{M}_2 \hat{\Sigma}_{21}^R + \hat{M}_1 \hat{M}_2 \hat{\Sigma}_{12}^R + \hat{M}_2 \hat{\Sigma}_{12}^M + \hat{M}_1 \hat{\Sigma}_{12}^M}{s_2 - s_1}.
\end{align}

With these abbreviations the coefficient matrices of the partial fraction decomposition have a rather simple structure. From Eqs. (21) and (31) for example, one obtains for the RR part of the propagator

\begin{align}
X^{RR} &= \sqrt{s_1} \begin{pmatrix} (x_{11})^2 & x_{12} \\ x_{12} & 0 \end{pmatrix} \quad \text{and} \quad Y^{RR} = \sqrt{s_2} \begin{pmatrix} 0 & y_{12} \\ y_{12} & (y_{22})^2 \end{pmatrix}.
\end{align}

It is clear that these results, derived in perturbation theory, are only valid as long as the non-diagonal elements are small, i.e. as long as \( \Delta \gg \alpha \), as mentioned in section 2. From Eqs. (25) and (31), one analogously finds for the LL part

\begin{align}
X^{LL} &= \sqrt{s_1} \begin{pmatrix} (x_{11})^2 & x_{21} \\ x_{21} & 0 \end{pmatrix} \quad \text{and} \quad Y^{LL} = \sqrt{s_2} \begin{pmatrix} 0 & y_{21} \\ y_{21} & (y_{22})^2 \end{pmatrix}.
\end{align}

Further, Eqs. (26) and (31) yield

\begin{align}
X^{LR} &= \begin{pmatrix} (x_{11})^2 & x_{12} \\ x_{21} & 0 \end{pmatrix} \quad \text{and} \quad Y^{LR} = \begin{pmatrix} 0 & y_{21} \\ y_{12} & (y_{22})^2 \end{pmatrix}.
\end{align}

Finally, the RL part of the propagator is just given by the transpose of the LR part, \( X^{RL} = (X^{LR})^T \) and \( Y^{RL} = (Y^{LR})^T \).

The flavour structures appearing in the scattering matrix elements (27)-(30) can then be decomposed into different contributions from each right-handed neutrino mass eigenstate. For example, the flavour structure appearing in Eq. (30) can be written as

\begin{align}
\hat{h}_{\beta i} \hat{S}_{ij}^{LL} (p^2) \hat{h}_{\alpha j} \equiv \frac{\sqrt{s_1}}{p^2 - s_1} \lambda_{\alpha 1} \lambda_{\beta 1} + \frac{\sqrt{s_2}}{p^2 - s_2} \lambda_{\alpha 2} \lambda_{\beta 2},
\end{align}
where we have introduced an effective one-loop coupling $\lambda_{ai}$ of the right-handed neutrino $N_i$ to the lepton doublet $l_i^\alpha$ and the Higgs doublet $\phi^*$,

\[
\lambda_{ai}(p^2) = \hat{h}_{ai} x_{11} + \hat{h}_{ai} x_{21} = \hat{h}_{ai} \left( 1 + \frac{1}{2} \hat{S}_{R1}^i(p^2) \right) - \hat{h}_{ai} \frac{\hat{M}_1^2 \hat{\Sigma}_{12}^R(p^2) + \hat{M}_1 \hat{M}_2 \hat{\Sigma}_{21}^R(p^2) + \hat{M}_1 \hat{\Sigma}_{12}^M \hat{\Sigma}_{12}^M}{s_2(p^2) - s_1(p^2)},
\]

\[
\lambda_{a2}(p^2) = \hat{h}_{a2} y_{22} + \hat{h}_{a2} y_{21} = \hat{h}_{a2} \left( 1 + \frac{1}{2} \hat{S}_{R2}^2(p^2) \right) + \hat{h}_{a2} \frac{\hat{M}_2^2 \hat{\Sigma}_{21}^R(p^2) + \hat{M}_1 \hat{M}_2 \hat{\Sigma}_{22}^R(p^2) + \hat{M}_2 \hat{\Sigma}_{12}^M \hat{\Sigma}_{12}^M}{s_2(p^2) - s_1(p^2)}.
\]

Analogously, the flavour structure in the scattering amplitude [20] can be decomposed as

\[
\hat{h}_{\beta i}^* \hat{S}_{Lij}^R(p^2) \hat{h}_{\alpha j} = \frac{\sqrt{s_1}}{p^2 - s_1} \bar{\lambda}_{\beta 1} \lambda_{\alpha 1} + \frac{\sqrt{s_2}}{p^2 - s_2} \bar{\lambda}_{\alpha 2} \lambda_{\beta 2},
\]

where the effective one-loop couplings $\bar{\lambda}_{\alpha i}$ of $N_i$ to the lepton doublet $l_\alpha$ and the Higgs doublet $\phi$ read

\[
\bar{\lambda}_{\alpha 1}(p^2) = \hat{h}_{\alpha 1}^* x_{11} + \hat{h}_{\alpha 1} x_{12} = \hat{h}_{\alpha 1} \left( 1 + \frac{1}{2} \hat{S}_{L1}^i(p^2) \right) - \hat{h}_{\alpha 1} \frac{\hat{M}_1^2 \hat{\Sigma}_{12}^L(p^2) + \hat{M}_1 \hat{M}_2 \hat{\Sigma}_{21}^L(p^2) + \hat{M}_1 \hat{\Sigma}_{12}^M \hat{\Sigma}_{12}^M}{s_2(p^2) - s_1(p^2)},
\]

\[
\bar{\lambda}_{\alpha 2}(p^2) = \hat{h}_{\alpha 2} y_{22} + \hat{h}_{\alpha 2} y_{21} = \hat{h}_{\alpha 2} \left( 1 + \frac{1}{2} \hat{S}_{L2}^2(p^2) \right) + \hat{h}_{\alpha 2} \frac{\hat{M}_2^2 \hat{\Sigma}_{21}^L(p^2) + \hat{M}_1 \hat{M}_2 \hat{\Sigma}_{22}^L(p^2) + \hat{M}_2 \hat{\Sigma}_{12}^M \hat{\Sigma}_{12}^M}{s_2(p^2) - s_1(p^2)}.
\]

Note that $\bar{\lambda}_{\alpha i} \neq \lambda_{a i}^*$ as a consequence of $CP$-violation.

The above effective one-loop couplings were derived from the lepton-number violating scattering amplitudes [29] and [30]. In order for the above decomposition to be consistent, the lepton number conserving scattering amplitudes [27] and [28] must be recovered from the effective couplings $\lambda$ and $\bar{\lambda}$. Indeed, it is easy to see that the corresponding flavour structures can be written as

\[
\hat{h}_{\beta i}^* \hat{S}_{Lij}^R(p^2) \hat{h}_{\alpha j} = \frac{1}{p^2 - s_1} \bar{\lambda}_{\beta 1} \lambda_{\alpha 1} + \frac{1}{p^2 - s_2} \bar{\lambda}_{\alpha 2} \lambda_{\beta 2},
\]

\[
\hat{h}_{\beta i}^* \hat{S}_{Lij}^R(p^2) \hat{h}_{\alpha j} = \frac{1}{p^2 - s_1} \lambda_{\beta 1} \bar{\lambda}_{\alpha 1} + \frac{1}{p^2 - s_2} \lambda_{\alpha 2} \bar{\lambda}_{\beta 2}.
\]

Hence, the effective couplings in Eqs. [42], [43], [45], and [46] consistently take the one-loop self-energy contributions to couplings of right-handed neutrinos to light lepton and Higgs doublets into account. Correspondingly, the self-energy contributions to heavy neutrino decay widths can be written as

\[
\Gamma(N_i \rightarrow l \phi) = \frac{\hat{M}_i}{16\pi} \sum_{\alpha} \bar{\lambda}_{\alpha i}(p^2) \bar{\lambda}_{\alpha i}(p^2),
\]

\[
\Gamma(N_i \rightarrow l^c \phi^*) = \frac{\hat{M}_i}{16\pi} \sum_{\alpha} \lambda_{ai}(p^2) \lambda_{ai}(p^2).
\]
The partial decay widths of $N_1$ are then evaluated to be

$$\Gamma(N_1 \to l \phi) = \frac{\hat{M}_1}{16\pi} \left\{ K_{11} |x_{11}|^2 + 2 \text{Re} (K_{21} x_{12}) \right\}, \quad (51)$$

$$\Gamma(N_1 \to l^c \phi^*) = \frac{\hat{M}_1}{16\pi} \left\{ K_{11} |x_{11}|^2 + 2 \text{Re} (K_{12} x_{21}) \right\}. \quad (52)$$

Analogously, the partial decay widths of $N_2$ read

$$\Gamma(N_2 \to l \phi) = \frac{\hat{M}_2}{16\pi} \left\{ K_{22} |y_{22}|^2 + 2 \text{Re} (K_{21} y_{12}) \right\}, \quad (53)$$

$$\Gamma(N_2 \to l^c \phi^*) = \frac{\hat{M}_2}{16\pi} \left\{ K_{22} |y_{22}|^2 + 2 \text{Re} (K_{21} y_{21}) \right\}. \quad (54)$$

It is now straightforward to compute the self-energy contributions to the $CP$-asymmetry $\varepsilon_i$ in the decay of $N_i$. From Eqs. (53) and (54) one finds

$$\varepsilon_i(p^2) \equiv \frac{\Gamma(N_i \to l \phi) - \Gamma(N_i \to l^c \phi^*)}{\Gamma(N_i \to l \phi) + \Gamma(N_i \to l^c \phi^*)} = \frac{\sum_\alpha \left[ |\tilde{\chi}_{\alpha i}(p^2)|^2 - |\lambda_{\alpha i}(p^2)|^2 \right]}{\sum_\alpha \left[ |\tilde{\chi}_{\alpha i}(p^2)|^2 + |\lambda_{\alpha i}(p^2)|^2 \right]} \quad (55)$$

Using Eqs. (12), (13), (15), and (16) for the effective couplings and going on-shell, one finally obtains for the $CP$-asymmetries

$$\varepsilon_1(M_1^2) = \frac{\text{Im} \left( K_{12}^2 \right)}{8\pi K_{11}} \frac{\hat{M}_1 \hat{M}_2 \left( \hat{M}_2^2 - \hat{M}_1^2 \right)}{\left( \hat{M}_2^2 - \hat{M}_1^2 - \frac{1}{2} \hat{M}_2 \hat{M}_1 \hat{\Gamma}_1 \ln \left( \frac{\hat{M}_2^2}{\hat{M}_1^2} \right) \right)^2 + \left( \hat{M}_2 \hat{\Gamma}_2 - \hat{M}_1 \hat{\Gamma}_1 \right)^2}, \quad (56)$$

$$\varepsilon_2(M_2^2) = \frac{\text{Im} \left( K_{12}^2 \right)}{8\pi K_{22}} \frac{\hat{M}_1 \hat{M}_2 \left( \hat{M}_2^2 - \hat{M}_1^2 \right)}{\left( \hat{M}_2^2 - \hat{M}_1^2 - \frac{1}{2} \hat{M}_2 \hat{M}_1 \hat{\Gamma}_1 \ln \left( \frac{\hat{M}_2^2}{\hat{M}_1^2} \right) \right)^2 + \left( \hat{M}_2 \hat{\Gamma}_2 - \hat{M}_1 \hat{\Gamma}_1 \right)^2}. \quad (57)$$

The logarithmic terms in the denominators of Eqs. (55) and (56) describe, e.g. the running of $\hat{M}_2$ to the scale $p^2 = \hat{M}_1^2$ and vice-versa. In the resonance regime, $\Delta \ll 1$, these terms are $O(\alpha \Delta)$ and, therefore, they can be neglected. We have just included them for completeness.

The main result of this computation is that the regulator of the mass singularity at $\hat{M}_1 = \hat{M}_2$ is the difference of the masses times the decay widths of the neutrinos, i.e. $\hat{M}_2 \hat{\Gamma}_2 - \hat{M}_1 \hat{\Gamma}_1$, in perfect analogy, e.g. with the $CP$-asymmetry in the $K_0-\bar{K}_0$ system [23]. This result is consistent with the one obtained in Ref. [14] but deviates from the expression for the $CP$-asymmetry given in Ref. [16], where only one of the decay widths appears as regulator.

### 3.2 Diagonalization of the propagator

An alternative approach to determining the contributions of the different right-handed neutrino mass eigenstates to the scattering amplitudes [27]-[30] consists in diagonalizing the
various chiral parts of the propagators and identifying the diagonal elements with the contributions from the neutrino mass eigenstates \[^14\].

The propagators \( S^{RR} \) and \( S^{LL} \) are complex symmetric matrices, which are diagonalized by complex orthogonal matrices \( V \) and \( U \), respectively,

\[
\hat{S}^{RR}(p^2) = Z(p^2) V^T(p^2) \hat{S}^{\text{diag}}(p^2) V(p^2) Z(p^2), \\
\hat{S}^{LL}(p^2) = Z(p^2) U^T(p^2) \hat{S}^{\text{diag}}(p^2) U(p^2) Z(p^2).
\]

Here, \( Z \) is a diagonal normalization matrix,

\[
Z = \begin{pmatrix}
\left[\sqrt{s_1} \left(1 + \hat{\Sigma}_{11}^R\right)\right]^{1/2} & 0 \\
0 & \left[\sqrt{s_2} \left(1 + \hat{\Sigma}_{22}^R\right)\right]^{1/2}
\end{pmatrix},
\]

chosen in such a way as to give the canonical normalization for the diagonal propagator

\[
\hat{S}^{\text{diag}} = \begin{pmatrix}
\frac{1}{p^2 - s_1} & 0 \\
0 & \frac{1}{p^2 - s_2}
\end{pmatrix},
\]

where \( s_i \) are the propagator poles of Eq. \((22)\).

Let us first consider the diagonalization of \( \hat{S}^{LL} \). By choosing

\[
U = \begin{pmatrix}
\cos \theta_U & -\sin \theta_U \\
\sin \theta_U & \cos \theta_U
\end{pmatrix},
\]

the complex mixing angle \( \theta_U \) can be determined from Eqs. \((25)\) and \((58)\)

\[
\tan(2\theta_U) = \frac{2}{(s_1(p^2)s_2(p^2))^{1/4}} \frac{p^2 \left( \hat{M}_2 \hat{\Sigma}_{21}^R(p^2) + \hat{M}_1 \hat{\Sigma}_{11}^R(p^2) + \hat{\Sigma}_{12}^M + \hat{M}_1 \hat{M}_2 \hat{\Sigma}_{12}^M\right)}{s_2(p^2) - s_1(p^2)}.
\]

Note, that Eq. \((62)\) is of order

\[
\tan(2\theta_U) = \frac{\mathcal{O}(\alpha)}{\mathcal{O}(\Delta) + \mathcal{O}(\alpha)}.
\]

Thus, if the condition for the applicability of perturbation theory, \( \Delta \gg \alpha \), is fulfilled, Eq. \((62)\) can be then expanded and the elements of the mixing matrix \( U \) read

\[
\cos \theta_U \simeq 1, \\
\sin \theta_U \simeq \frac{1}{(s_1(p^2)s_2(p^2))^{1/4}} \frac{p^2 \left( \hat{M}_2 \hat{\Sigma}_{21}^R(p^2) + \hat{M}_1 \hat{\Sigma}_{11}^R(p^2) + \hat{\Sigma}_{12}^M + \hat{M}_1 \hat{M}_2 \hat{\Sigma}_{12}^M\right)}{s_2(p^2) - s_1(p^2)}.
\]

In perfect analogy to the procedure followed in the pole expansion method, the flavour structure in the scattering amplitude \((30)\) can now be written as

\[
\hat{h}_{\beta i} \hat{S}_{ij}^{LL}(p^2) \hat{h}_{\alpha j} = \sum_i \left(\hat{h} Z U^T\right)_{\beta i} s_i^{-1/4} \sqrt{s_i} \hat{S}_{ii}^{\text{diag}} \left(\hat{h} Z U^T\right)_{\alpha i} s_i^{-1/4},
\]

10
Hence, we can define an effective one-loop coupling $\xi_{\alpha i}$ of the right-handed neutrino $N_i$ to the lepton doublet $l^c_\alpha$ and the Higgs doublet $\phi^*$,

$$\xi_{\alpha i} \equiv (\hat{h}^* Z U^T)_{\alpha i} s_i^{-1/4}. \tag{67}$$

From Eqs. (59), (61) and (64) explicit expressions for these effective couplings are given by

$$\xi_{\alpha 1}(p^2) = \hat{h}_{\alpha 1} \left(1 + \frac{1}{2} \hat{\Sigma}^R_{11}(p^2)\right) - \hat{h}_{\alpha 2} \frac{p^2 \left(\hat{M}_1 \hat{\Sigma}^R_{21}(p^2) + \hat{M}_2 \hat{\Sigma}^R_{12}(p^2) + \hat{\Sigma}^M_{12}\right)}{M_1 (s_2(p^2) - s_1(p^2))}, \tag{68}$$

$$\xi_{\alpha 2}(p^2) = \hat{h}_{\alpha 2} \left(1 + \frac{1}{2} \hat{\Sigma}^R_{22}(p^2)\right) + \hat{h}_{\alpha 1} \frac{p^2 \left(\hat{M}_1 \hat{\Sigma}^R_{21}(p^2) + \hat{M}_2 \hat{\Sigma}^R_{12}(p^2) + \hat{\Sigma}^M_{12}\right)}{M_2 (s_2(p^2) - s_1(p^2))}. \tag{69}$$

Similarly, $S^{RR}$ is diagonalized by the normalization matrix $Z$ given in Eq. (59) and the orthogonal matrix

$$V = \begin{pmatrix} \cos \theta_V & -\sin \theta_V \\ \sin \theta_V & \cos \theta_V \end{pmatrix}, \tag{70}$$

where the complex mixing angle $\theta_V$ is given by

$$\tan (2\theta_V) = \frac{2 \left(\hat{M}_1 \hat{\Sigma}^R_{21}(p^2) + \hat{M}_2 \hat{\Sigma}^R_{12}(p^2) + \hat{\Sigma}^M_{12}\right) + \hat{M}_1 \hat{M}_2 \hat{\Sigma}^M_{12}}{2 \left(s_2(p^2) - s_1(p^2)\right)^{1/4}}. \tag{71}$$

Again, at leading order, one finds

$$\cos \theta_V \simeq 1, \tag{72}$$

$$\sin \theta_V \simeq \frac{1}{(s_1(p^2) s_2(p^2))^{1/4}} \frac{p^2 \left(\hat{M}_1 \hat{\Sigma}^R_{21}(p^2) + \hat{M}_2 \hat{\Sigma}^R_{12}(p^2) + \hat{\Sigma}^M_{12}\right) + \hat{M}_1 \hat{M}_2 \hat{\Sigma}^M_{12}}{s_2(p^2) - s_1(p^2)}. \tag{73}$$

The flavour structure in the amplitude (24) can be then written as

$$\hat{\hat{h}}_{ji}^* \hat{S}_{ij}(p^2) \hat{h}_{\alpha 3}^* = \sum_i \left(\hat{h}^* Z V^T\right)_{\beta i} s_i^{-1/4} \sqrt{s_i} S^{\text{diag}}_{\alpha i} \left(\hat{h}^* Z V^T\right)_{\alpha i} s_i^{-1/4}. \tag{74}$$

We can again define an effective one-loop coupling $\overline{\xi}_{\alpha i}$ of $N_i$ to the lepton doublet $l_\alpha$ and the Higgs doublet $\phi$,

$$\overline{\xi}_{\alpha i} \equiv (\hat{h}^* Z V^T)_{\alpha i} s_i^{-1/4}. \tag{75}$$

Explicitly, these effective couplings read

$$\overline{\xi}_{\alpha 1}(p^2) = \hat{h}_{\alpha 1}^* \left(1 + \frac{1}{2} \hat{\Sigma}^R_{11}(p^2)\right) - \hat{h}_{\alpha 2}^* \frac{p^2 \left(\hat{M}_1 \hat{\Sigma}^R_{21}(p^2) + \hat{M}_2 \hat{\Sigma}^R_{12}(p^2) + \hat{\Sigma}^M_{12}\right)}{\hat{M}_1 (s_2(p^2) - s_1(p^2))}, \tag{76}$$

$$\overline{\xi}_{\alpha 2}(p^2) = \hat{h}_{\alpha 2}^* \left(1 + \frac{1}{2} \hat{\Sigma}^R_{22}(p^2)\right) + \hat{h}_{\alpha 1}^* \frac{p^2 \left(\hat{M}_1 \hat{\Sigma}^R_{21}(p^2) + \hat{M}_2 \hat{\Sigma}^R_{12}(p^2) + \hat{\Sigma}^M_{12}\right)}{\hat{M}_2 (s_2(p^2) - s_1(p^2))}. \tag{77}$$
Note that on-shell, i.e. for $p^2 = \hat{M}_i^2$, the effective couplings in Eqs. (68), (69), (76) and (77), agree with the effective couplings (42), (43), (45) and (46), derived in the pole expansion method:

$$\xi_{\alpha i}^{(\hat{M}_i^2)} = \lambda_{\alpha i}^{(\hat{M}_i^2)},$$  \hspace{1cm} (78)

$$\overline{\xi}_{\alpha i}^{(\hat{M}_i^2)} = \overline{\lambda}_{\alpha i}^{(\hat{M}_i^2)}.$$ \hspace{1cm} (79)

Thus, the effective resummed one-loop couplings of the different right-handed neutrino mass eigenstates derived in the pole expansion and the diagonalization methods are identical on-shell. In particular, this means that physical quantities computed on-shell, e.g. decay widths and $CP$-asymmetries, in the propagator diagonalization method will agree with those obtained in the pole expansion method, thereby confirming our results from section 3.1.

4 Conclusions

In this paper we have studied the resonantly enhanced $CP$-asymmetry in the decays of nearly mass-degenerate heavy right-handed neutrinos, a regime known as resonant leptogenesis. Such a scenario is phenomenologically interesting since it allows to evade the rather stringent lower limit on the reheating temperature that can be obtained in the simplest scenario of thermal leptogenesis with hierarchical right-handed neutrino masses. Further, it may open the possibility of directly observing right-handed neutrinos at future colliders [24].

Unfortunately, different formulae for the $CP$-asymmetry in resonant leptogenesis had previously been proposed in the literature. Obviously, this hampers phenomenological investigations of resonant leptogenesis, since it was not clear which of these formulae one should use.

We have clarified the situation by computing the $CP$-asymmetry with two independent methods, the pole expansion and the propagator diagonalization method. We showed that, within the same renormalization scheme, both methods give identical results for physical quantities, such as decay widths and, in particular, $CP$-asymmetries in the decays of heavy right-handed neutrinos. Furthermore, we have also discussed the range of validity of the resulting formulae. Due to the limitations of the perturbative approach, the degree of degeneracy of heavy neutrinos must be restricted to be much larger than the expansion parameter, determined by the neutrino Yukawa couplings.

Acknowledgments

We would like to thank P. Di Bari, S. Dittmaier and B. Gavela for for useful discussions and comments on the manuscript. A. B. and M. P. acknowledge support by the Deutsche Forschungsgemeinschaft within the Emmy-Noether program.
A On-shell renormalization with particle mixing

In this appendix we will briefly summarize the renormalization procedure we use, following the formalism developed in Ref. [19]. In the seesaw model described by the Lagrangian in Eq. (1), the bare one-loop self-energy of the right-handed neutrinos has the structure given in Eq. (4).

The renormalized masses and fields, denoted by a hat in the following, are related to the bare ones by the counterterms:

\[ \hat{M}_i = M_i + \delta M_i , \]
\[ \hat{N}_{Ri} = (Z_{Rij}^{1/2}) N_{Rj} = \left( \delta_{ij} + \frac{1}{2} \delta Z_{Rij}^R \right) N_{Rj} , \]
\[ \hat{N}_{Ri}^c = (Z_{Rij}^{R*1/2}) N_{Rj}^c = \left( \delta_{ij} + \frac{1}{2} \delta Z_{Rij}^{R*} \right) N_{Rj}^c . \]

From the counterterm Lagrangian, one then obtains the following relations between renormalized and bare self-energies:

\[ \hat{\Sigma}_{Rij}^{\text{dis}}(p^2) \hat{M}_j = \Sigma_{Rij}^{\text{dis}}(p^2) + \frac{1}{2} \left( \delta Z_{Rij}^R + \delta Z_{Rji}^{R*} \right) , \]
\[ \hat{\Sigma}_{ij}^{M}(p^2) = -\frac{1}{2} \left( \hat{M}_j \delta Z_{Rji}^R + \hat{M}_i \delta Z_{Rij}^R \right) - \delta_{ij} \delta M_i . \]

From the counterterm Lagrangian, one then obtains the following relations between renormalized and bare self-energies:

\[ \hat{\Sigma}_{ij}(p^2) = \Sigma_{ij}(p^2) + \frac{1}{2} \left( \delta Z_{ij}^R + \delta Z_{ji}^{R*} \right) , \]
\[ \hat{\Sigma}_{ij}(p^2) = -\frac{1}{2} \left( \hat{M}_j \delta Z_{ji}^R + \hat{M}_i \delta Z_{ij}^R \right) - \delta_{ij} \delta M_i . \]

In the OS scheme, the counterterms are determined from the following renormalization conditions:

\[ \hat{\Sigma}_{ij}^{\text{dis}}(p^2) u_j(p) \big|_{p^2 = \hat{M}_j^2} = 0 , \]
\[ \frac{1}{p^2 - \hat{M}_i} \hat{\Sigma}_{jj}^{\text{dis}}(p^2) u_i(p) \big|_{p^2 \rightarrow \hat{M}_i^2} = 0 , \]

where the subscript dis refers to the dispersive part of the self-energy, since absorptive parts cannot contribute to the renormalization without spoiling the required hermiticity of the counterterm Lagrangian. The first renormalization condition, Eq. (85), yields the following two equations:

\[ \hat{\Sigma}_{ij}^{R\text{dis}}(\hat{M}_j^2) \hat{M}_j + \hat{\Sigma}_{ij}^{M\text{dis}}(\hat{M}_j^2) = 0 , \]
\[ \hat{\Sigma}_{ij}^{R\text{dis}}(\hat{M}_j^2) \hat{M}_j + \hat{\Sigma}_{ij}^{M\text{dis}}(\hat{M}_j^2) = 0 . \]

Since \( \Sigma_{ij}^{R\text{dis}} = \Sigma_{ji}^{R\text{dis}} \) these two equations are equivalent. From Eqs. (86) and (87) one then obtains the mass counterterms as well as the non-diagonal elements of \( \delta Z_R \),

\[ \delta M_i = \hat{M}_i \Sigma_{ij}^{R\text{dis}}(\hat{M}_j^2) , \]
\[ \delta Z_{ij}^R = \frac{2}{\hat{M}_i^2 - \hat{M}_j^2} \left[ \hat{M}_j^2 \Sigma_{ij}^{R\text{dis}}(\hat{M}_j^2) + \hat{M}_j \hat{M}_i \Sigma_{ij}^{R\text{dis}}(\hat{M}_j^2) \right] , \text{ for } i \neq j . \]
Similarly, Eq. (86) yields
\[ \hat{\Sigma}_{R \text{ dis}}^{ii}(\hat{M}^2_i) + 2 \hat{M}_i \frac{\partial}{\partial p^2} \left( \hat{M}_i \hat{\Sigma}_{R \text{ dis}}^{ii}(p^2) + \hat{\Sigma}_{M \text{ dis}}^{ii}(p^2) \right) \bigg|_{p^2 = \hat{M}_i^2} = 0, \] (91)

from which the flavour diagonal counterterms \( \delta Z_{ii} \) can be obtained,

\[ \delta Z_{R \text{ dis}}^{ii} = -\hat{\Sigma}_{R \text{ dis}}^{ii}(\hat{M}^2_i) - 2 \hat{M}_i \frac{\partial}{\partial p^2} \left[ \hat{\Sigma}_{R \text{ dis}}^{ii}(p^2) \right] \bigg|_{p^2 = \hat{M}_i^2}. \] (92)

Substituting these counterterms into Eqs. (83) and (84) gives rise to the renormalized self-energies in Eqs. (9)–(12).

References

[1] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45
[2] S. Davidson and A. Ibarra, Phys. Lett. B 535 (2002) 25
[3] W. Buchmüller, P. Di Bari and M. Plümacher, Nucl. Phys. B 643 (2002) 367.
[4] W. Buchmüller, P. Di Bari and M. Plümacher, Ann. Phys. 315 (2005) 305.
[5] J. Ellis, D. V. Nanopoulos, S. Sarkar, Nucl. Phys. B 259 (1985) 175.
[6] M. Flanz, E.A. Paschos, U. Sarkar and J. Weiss, Phys. Lett. B 389, 693 (1996).
[7] L. Covi and E. Roulet, Phys. Lett. B 399 (1997) 113.
[8] A. Pilaftsis, Nucl. Phys. B 504 (1997) 61.
[9] A. Pilaftsis, Phys. Rev. D 56 (1997) 5431.
[10] J. Liu and G. Segre, Phys. Rev. D 49 (1994) 1342.
[11] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B 384, 169 (1996).
[12] M. Flanz, E.A. Paschos and U. Sarkar, Phys. Lett. B 345 (1995) 248; Phys. Lett. B 384 (1996) 487 (erratum).
[13] M. Flanz and E.A. Paschos, Phys. Rev. D 58 (1998) 113009.
[14] W. Buchmüller and M. Plümacher, Phys. Lett. B 431 (1998) 354.
[15] R. Rangarajan and H. Mishra, Phys. Rev. D 61 (2000) 043509
[16] A. Pilaftsis and T. E. J. Underwood, Nucl. Phys. B 692 (2004) 303
[17] M. Veltman, Physica 29 (1963) 186.
[18] P. Minkowski, Phys. Lett. B 67 (1977) 421; M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, edited by P. van Nieuwenhuizen and D. Freedman, (North-Holland, 1979), p. 315; T. Yanagida, in Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe, edited by O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, 1979), p. 95; R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44, 912 (1980).

[19] B. A. Kniehl and A. Pilaftsis, Nucl. Phys. B 474, 286 (1996)

[20] G. C. Branco, T. Morozumi, B. M. Nobre and M. N. Rebelo, Nucl. Phys. B 617 (2001) 475.

[21] A. Broncano, M. B. Gavela and E. Jenkins, Phys. Lett. B 552 (2003) 177.

[22] M. Plümmacher, Z. Phys. C 74 (1997) 549.

[23] T.P. Cheng and L.F. Li, Gauge theory of elementary particle physics, Clarendon Press, Oxford (1994)

[24] A. Pilaftsis and T. E. J. Underwood, arXiv:hep-ph/0506107