DETECTING OUTER PLANETS IN EDGE-ON ORBITS: COMBINING RADIAL VELOCITY AND ASTROMETRIC TECHNIQUES

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ABSTRACT

The astrometric and radial velocity techniques of extrasolar planet detection attempt to detect the periodic reflex motion of the parent star by extracting this periodic signal from a time-sampled set of observations. The extraction is generally accomplished using periodogram analysis or the functionally equivalent technique of least-squares fitting of sinusoids. In this paper, we use a frequentist approach to examine the sensitivity of the least-squares technique when applied to a combination of radial velocity and astrometric observations. We derive an analytical expression for the sensitivity and show that the combined approach yields significantly better sensitivity than either technique on its own. We discuss the ramifications of this result to upcoming astrometric surveys with the Full-Sky Astrometric Mapping Explorer, the Keck Interferometer, and the Space Interferometry Mission.

Subject headings: astrometry — planetary systems — techniques: radial velocities

1. INTRODUCTION

Radial velocity (RV) surveys of nearby stars have been employed in the search for extrasolar planets for nearly two decades (see Marcy, Cochran, & Mayor 2000). As these efforts continue into the next decade, they will be supplemented by precision astrometric searches, e.g., by the Full-Sky Astrometric Mapping Explorer (FAME), the Keck Interferometer, and the Space Interferometry Mission (SIM) (Horner et al. 2000; van Belle et al. 1998; Danner, Unwin, & Allen 1999). In previous papers, we examined the RV and astrometric techniques in detail, paying particular attention to the regime where the time baseline of the observations is shorter than the orbital period of the extrasolar companion (Eisner & Kulkarni 2001a, 2001b, hereafter EK01a, EK01b). This regime is interesting because one expects giant planets to form in the colder regions of the proto–planetary nebula, and thus one expects such objects to possess periods of many years to centuries (Boss 1995). In EK01a and EK01b we demonstrated that one can achieve a significant improvement in sensitivity (over current techniques) if the orbital amplitude and phase are included in the analysis.

Here we examine the sensitivity of combined astrometric and RV observations. Specifically, we consider the simple case of edge-on circular orbits and compare the sensitivity of combined RV + astrometry analysis with the sensitivities of either RV or astrometry alone. In § 2 we lay out the basic equations describing RV and astrometric observations. In § 3 we simulate large numbers of hypothetical data sets containing (1) noise only and (2) signal and noise and determine the frequentist type I and II errors. As in EK01a and EK01b, we acknowledge that a frequentist approach is not as rigorous as a Bayesian approach. However, this approach is simple enough that it is amenable to deriving analytical estimates of the sensitivity—a principal goal of the paper. We conclude by discussing the parameter space opened up by combining Keck or Palomar adaptive optics, FAME, Keck Interferometer, or SIM astrometric surveys with ongoing precision RV studies.

2. BASIC EQUATIONS

We will assume edge-on circular orbits throughout this discussion. The astrometric signature of an edge-on circular orbit is given by

$$\theta(t) = A \sin \left(\frac{2\pi t}{\tau} + \phi\right) + \lambda t + \mu,$$  \hspace{1cm} (1)

where \(\tau\) is the orbital period, \(\phi\) is the phase, \(\lambda\) and \(\mu\) are the proper motion and parallax of the system, respectively, and

$$A = \frac{M_p}{D} \left(\frac{G \tau^2}{4\pi^2 M_p^2}\right)^{1/3}.$$  \hspace{1cm} (2)

Here \(D\) is the distance to the system, \(M_\star\) is the mass of the star, and \(M_p\) is the mass of the planet. We ignore the annual parallax. However, annual parallax should be included in modeling of planets with periods around 1 yr.

The RV signature of this orbit is given by the derivative of the orbital position along the line of sight

$$v(t) = v^c \sin \left(\frac{2\pi t}{\tau} + \phi\right) + \gamma.$$  \hspace{1cm} (3)

Here \(\gamma\) is the systemic radial velocity of the planetary system, and

$$v^c = M_p \left(\frac{2\pi G}{M_\star \tau}\right)^{1/3} = \frac{2\pi D}{\tau} A.$$  \hspace{1cm} (4)

Thus, we can express the sensitivity (defined as the minimum-mass planet that can be detected) of the RV and astrometric techniques in terms of \(A\):

$$M_p = D A \left(\frac{4\pi^2 M_\star^2}{G^2 \tau^2}\right)^{1/3}. $$  \hspace{1cm} (5)

As discussed in EK01a and EK01b, it is more difficult to identify planets with long periods than equation (5) might suggest. In the so-called long-period regime, defined as...
\( T_0 \ll \tau \), where \( T_0 \) is the duration of the survey, we observe a fraction of the orbit, and as a result, the sensitivity is expected to depend critically on the orbital phase. The reflex velocity is covariant with \( \gamma \), and thus the RV technique is most sensitive when \( 2\pi t/\tau + \phi = n\pi \) (EK01a). In contrast, the astrometric signal of an edge-on orbit is covariant with \( \lambda t + \mu \), and thus the astrometric technique is sensitive when \( 2\pi t/\tau + \phi = (n+1/2)\pi \) (EK01b). Thus, the RV and astrometric techniques achieve their maximal sensitivities for different orbital phases, and we expect, on general grounds, that combining the two techniques should yield a substantial benefit in the long-period regime.

3. MONTE CARLO ANALYSIS

The signal analysis for the astrometric and RV techniques consists of fitting the observations to the models specified in equations (1) and (3). As noted by several authors (e.g., Scargle 1982; Nelson & Angel 1998; EK01a), the most optimal fitting is obtained by using the technique of least-squares fitting. First, we convert the physical model specified by equations (1) and (3) to equations linear in the unknowns

\[
\theta(t) = A_c \cos(\omega t) + A_s \sin(\omega t) + \lambda t + \mu, \quad (6)
\]

\[
v(t) = V_c \cos(\omega t) + V_s \sin(\omega t) + \gamma. \quad (7)
\]

Here \( A_c = A \sin \phi \), \( A_s = A \cos \phi \), \( V_c = V \sin \phi \), \( V_s = V \cos \phi \), and \( \omega = 2\pi/\tau \). In EK01a and EK01b we discuss the importance of the \( \gamma \), \( \lambda \), and \( \mu \) terms. These three variables are not directly relevant in detecting or characterizing a companion planet, but they are unknown and in the long-period regime are covariant with some of the orbital parameters (Black & Scargle 1982). Thus, the three variables must be solved for in order to correctly model the observations.

Using equations (6) and (7) as our physical model, we perform the following analysis. First, we simulate a large number of data sets containing only Gaussian noise (i.e., no signal). For each of these data sets, we perform a least-squares fit to three models: a model using only astrometric measurements (eq. [6]), a model using only RV measurements (eq. [7]), and a model that utilizes both astrometric and RV measurements. In each case, for each simulated data set we fitted for amplitude and phase. We note here that for the RV + astrometry model, since the two measurements have different variances we minimize the \( \chi^2 \) (where \( \chi \) is the difference between the model and the rms-weighted measurements).

Specifically, we simulate \( N = 1000 \) data sets, sampled at 1 month intervals for \( T_0 \) yr (with no loss of generality, we take the time interval to go from \(-T_0/2\) to \(T_0/2\)). We also perform simulations in which we allow different durations for RV and astrometric surveys (\( T_{0,\text{RV}} \) and \( T_{0,\text{ast}} \), respectively). We assume that the measurement noise in both the RV and astrometric surveys is characterized by Gaussian noise with rms of \( \sigma_{\text{RV}} \) and \( \sigma_{\text{ast}} \), respectively.

Following EK01a and EK01b, we determine the ellipse (in \( A_c-V_s \) space) within which 99% of the fitted amplitudes and phases lie. This ellipse, denoted by \( \epsilon_1 \), describes the “type I” errors (also known as false detection probabilities) of the detection technique. Thus, the inferred \( A_c \) and \( \phi \) have a 1% chance of being outside the \( \epsilon_1 \) ellipse (in the absence of a signal).

As discussed earlier (§ 2), we expect RV and astrometric models to show orthogonal sensitivity. Indeed, as can be seen from Figure 1, the \( \epsilon_{1,\text{RV}} \) and \( \epsilon_{1,\text{ast}} \) are 90° out of phase.

\[ \frac{A_c}{M/M_\odot} \]

\[ \frac{\sigma_{A_c}}{A_c} \]

![Figure 1](image1.png)

**Figure 1.**—Plot of the 1% error ellipses for the astrometric and RV techniques for a 90 yr orbit (\( \tau \approx 9 T_0 \)). The error ellipse for combined RV + astrometry technique is also shown. These are the ellipses, \( \epsilon_1 \), for which 99% of least-squares fits to simulated Gaussian noise produce fitted amplitudes and phases that lie within \( \epsilon_1 \) (§ 3). We have scaled the ellipses to units of companion mass via eq. (5).

\[ \frac{\log(A_c/M_\odot)}{\log(T/\text{yr})} \]

![Figure 2](image2.png)

**Figure 2.**—Plot of \( A_c/M_\odot \) vs. orbital period (solid line). The analytic expression given by eq. (8) is also plotted (dashed line). \( A_j \) is the value of \( |A_c| \) that is exceeded in 1% of least-squares fits to Gaussian noise and describes the radius of the type I error ellipse (actually a circle in this case).
Fig. 3.—Plots of $\log(M_{99})$, in units of Jupiter masses, vs. $\log(\tau/T_0)$. $M_{99}$ is $A_{99}$ expressed in units of companion mass, assuming $M_{99} = M_\star$ (eq. [5]). The positions of Jupiter, Saturn, and Uranus in this parameter space are also indicated: (a) Both the RV and astrometric surveys have a duration of $T_0 = 10$ yr, (b) shows the simulated sensitivity for FAME + RV, (c) shows Keck Interferometer + RV, (d) shows SIM + RV, and (e and f) show potential sensitivities for 2 yr astrometric measurements on large adaptive optics (AO) systems (e.g., Keck AO, Palomar AO) combined with RV. References for the various survey parameters are as follows: FAME (Horner et al. 2000), Keck Interferometer (van Belle et al. 1998), SIM (Danner et al. 1999), and AO (Dekany 1994; Colavita 1994).
in the long-period regime. On a basic level, we can understand the benefit of combining RV and astrometric observations by noting that the intersection of $\epsilon_{1, RV}$ and $\epsilon_{1, ast}$ is much smaller than either of the individual ellipses, and thus it is easier to detect signals over the level of the noise.

Analytic expressions for the type I errors for RV or astrometric techniques are given in EK01a and EK01b, respectively. The analytic expression for the type I errors in the case of combined astrometric + RV technique is essentially given by the intersection of $\epsilon_{1, RV}$ and $\epsilon_{1, ast}$. For example, for $\sigma_{RV} = 3 \text{ m s}^{-1}$ and $\sigma_{ast} = 100 \mu\text{as}$, the semimajor axes for $\epsilon_{1, RV}$ and $\epsilon_{1, ast}$ are approximately equal (assuming $D = 10$ pc), and thus $\epsilon_{1}$ for the combined analysis will be a circle whose radius is given by

$$A_{c1} = \begin{cases} 2^{-1/2} \min \left( A_{1s}, \frac{\tau}{2\pi D} v_{1s} \right) & \text{for } \tau < T_0, \\ \frac{2A_{1s}}{1 - \cos(\pi T_0/\tau)} & \text{for } \tau > T_0. \end{cases}$$

Here $A_{c1}$ is the value of $|\mathcal{A}^1_r|$ that is exceeded in 1% of simulations, $A_{1s} = 3.69 \sigma_{ast} h_0^{1/2}$ is the corresponding quantity in the short-period regime, $v_{1s} = 3.69 \sigma_{RV} h_0^{-1/2}$, and the factor of $2^{-1/2}$ reflects the fact that in the short-period regime there are essentially twice as many measurements. As illustrated in Figure 2, this analytic function provides an excellent fit to the data.

Next, we evaluate ”type II” errors for the three models. Type II errors describe the probability of failing to detect a genuine signal because of contamination by noise (i.e., the sensitivity). To understand the type II statistics, we simulate a large number of data sets consisting of a simulated signal and noise (see EK01a and EK01b for further details). The signal is a sinusoidal wave with an amplitude $\mathcal{A}_0$ (astrometry), and the corresponding velocity amplitude is $2\pi D \mathcal{A}_0/\tau$ (we set $D = 10$ pc); the phase, $\phi$, is randomly chosen from the interval $[0, 2\pi]$ (uniform distribution). For each model, we increment the amplitude(s) until 99% of the fitted orbital parameters lie outside of the appropriate $\epsilon_1$ ellipse; this amplitude is denoted by $A_{99}$ (for each method). Figure 3 shows $A_{99}$ for RV, astrometry, and RV + astrometry for various survey parameters.

4. DISCUSSION

The benefit of combining RV and astrometric analysis accrues mainly from the fact that the error ellipses for the two techniques in $A - \phi$ parameter space are perpendicular to each other (Fig. 1). RV + astrometry analysis will be most useful in cases where the error ellipses for the two techniques are roughly the same size (otherwise, one error ellipse might lie entirely within the other, and no additional benefit would arise from combining the two techniques). The best achieved $\sigma_{RV} = 3 \text{ m s}^{-1}$ (Butler et al. 1996), which means that for a 10 yr survey, we must use astrometric measurements with $\sim 100 \mu\text{as}$ precision (for a system at $D = 10$ pc) to reap the maximal benefit from RV + astrometry technique (Fig. 3a). This is approximately the sensitivity that will be obtained by future instruments like the Keck Interferometer and FAME (van Belle et al. 1998; Horner et al. 2000; see also our Figs. 3b and 3c). When $\sigma_{ast} \ll 100 \mu\text{as}$ (e.g., using SIM; Danner et al. 1999), the main benefit of RV + astrometry is that optimal sensitivity is attained over a wider range of inclination angles (Fig. 3d).

It is also worth noting that RV + astrometry yields valuable gains in the short-period regime ($\tau < T_0$). When $\sigma_{RV}$ is comparable to $\sigma_{ast}$, the sensitivity of RV + astrometry is better by $2^{-1/2}$ over RV or astrometry alone. Furthermore, noting that the sensitivity of astrometry to face-on orbits is $2^{-1/2} \sigma_{ast}$ (EK01b), we see that the short-period sensitivity of RV + astrometry is approximately independent of orbital inclination.

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