Fourier analysis of He 4471/Mg 4481 line profiles for separating rotational velocity and axial inclination in rapidly-rotating B-type stars

Y. Takeda, S. Kawanomoto, and N. Ohishi

National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan

ABSTRACT
While the effect of rotation on spectral lines is complicated in rapidly-rotating stars because of the appreciable gravity-darkening effect differing from line to line, it is possible to make use of this line-dependent complexity to separately determine the equatorial rotation velocity ($v_e$) and the inclination angle ($i$) of rotational axis. Although line-widths of spectral lines were traditionally used for this aim, we tried in this study to apply the Fourier method, which utilizes the unambiguously determinable first-zero frequency ($\sigma_1$) in the Fourier transform of line profile. Equipped with this technique, we analyzed the profiles of He I 4471 and Mg II 4481 lines of six rapidly-rotating ($v_e \sin i \sim 150–300 \text{ km s}^{-1}$) late B-type stars, while comparing them with the theoretical profiles simulated on a grid of models computed for various combination of ($v_e$, $i$). According to our calculation, $\sigma_1$ tends to be larger than the classical value for given $v_e \sin i$. This excess progressively grows with an increase in $v_e$, and is larger for the He line than the Mg line, which leads to $\sigma_{1\text{He}} > \sigma_{1\text{Mg}}$. It was shown that $v_e$ and $i$ are separately determinable from the intersection of two loci (sets of solutions reproducing the observed $\sigma_1$ for each line) on the $v_e$ vs. $i$ plane. Yet, line profiles alone are not sufficient for their unique discrimination, for which photometric information (such as colors) needs to be simultaneously employed.

Key words: line: profiles – stars: atmospheres – stars: early-type – stars: rotation – stars: individual (17 Tau, $\alpha$ Leo, $\beta$ CMi, $\eta$ Aqr, $\eta$ Tau, $\zeta$ Peg)

1 INTRODUCTION
In the widely used conventional treatment assuming the invariant line profile over the disk along with the circular-symmetric brightness distribution of limb-darkening (valid for slow rotators), the effect of stellar rotation on a spectral line is quite simple, which can be universally expressed by convolution of the rotational broadening function with the intrinsic profile (see, e.g., Gray 2005). In this case, the equatorial rotational velocity ($v_e$) and the projection factor ($\sin i$, where $i$ is the inclination angle of the rotation axis relative to the line of sight) always appears as both product ($v_e \sin i$), and their separation is impossible.

Meanwhile, for the case of rapid rotators (e.g., $v_e \sim 100–400 \text{ km s}^{-1}$) commonly seen in early-type stars, such an approximation is no more valid in the practical sense, because the gravity-darkening effect (i.e., temperature inhomogeneity on the surface) causes an appreciable variation in the strength and profile of spectral lines on the stellar disk, the extent of which is considerably line-dependent. Accordingly, the effect of rapid rotation on a spectral line is so complicated as to be treated only by detailed numerical calculations based on a properly designed gravity-darkened stellar model. In compensation for this complexity, however, separated determination of $v_e$ and $i$ may be possible by simultaneously analyzing the profiles of two or more lines of different temperature sensitivity.

Actually, several investigators challenged this delicate and difficult task of $v_e$–$i$ separation by carefully analyzing the spectral lines: Stoeckley (1968a) studied five rapidly rotating B and A-type stars by using He I 4471, Mg II 4481, and Ca II 3934 lines. Hutchings & Stoeckley (1977) examined the UV lines based on the Copernicus data (and also lines in the visual region) for 20 stars of mostly B-type. Rushton (1982) computed the half-width of He I 4471 and Mg II 4481 lines for various model parameters and applied...
to 19 early B-type stars. Stoeckley & Buscombe (1987) extended the previous work of Stoeckley (1968a) and analyzed the He 4471 and Mg ii 4481 lines of 19 rapidly-rotating B-type main-sequence stars, where they also tried to detect the degree of differential rotation (in addition to independent determination of $v_e$ and $i$).

Somewhat surprisingly, observational investigations of this kind seem to have been barely done since then over these 30 years, as far as we know.\footnote{Vinicius, Townsend & Leister (2007) reported their preliminary results for the rotational inclination of five early B-type (Be) stars determined from the strengths (not the widths) of various spectral lines, though they do not appear to have published the details of their analysis.} When we review these old studies from the aspect of current knowledge, some points are noticed, which may have room for further improvement.

First, rather rough interpolation appears to have been adopted in the simulation of spectral line profiles in gravity-darkened stellar photospheres, which requires integrations of local spectra at many points over the visible stellar disk. Presumably, this is due to the limitation of computational capacity at the time. It may be possible nowadays to implement more realistic computations; e.g., by using our code developed for simulating the gravity-darkened spectrum of Vega (Takeda, Kawanomoto & Ohishi 2008).

Second, these previous studies employed “line widths” (most half-width at the half maximum of the line depth) for their analysis. However, precisely measuring the line width suffers practical difficulties for the relevant case of rapid rotators characterized by very wide and shallow profiles of merged line features, because it critically depends on the continuum level, for which exact determination is not easy. Given this situation, it occurred to us to apply the Fourier method, which utilizes the frequencies corresponding to the zero amplitude in the Fourier transform of a line profile, because these zero-frequencies are easily determinable from the amplitude vs. frequency diagram.

It appears that this Fourier approach has not been so popularly applied to early-type stars, which is presumably related to the problem involved with rapid rotators commonly seen in hot stars. As well known, its merit for rotational velocity study is especially manifest when the emer-

Given this situation, we decided to carry out a feasibility study to examine whether it is possible to separately determine $v_e$ and $i$ based on the Fourier analysis of line profiles for selected six rapidly-rotating late B-type stars, since such a trial seems to have been barely made to our knowledge.\footnote{We note that several researchers have recently conducted sophisticated modeling of theoretical line profiles for rapidly-rotating B-type stars under the effect of gravity darkening, and also investigated the possibility of Fourier transform method for extracting in-depth information of stellar rotation such as the nature of differential rotation (e.g., Domiciano de Souza et al. 2004, Zorec et al. 2011, 2017). However, these studies placed emphasis on simulations of model profiles with particular intention of applying to Be stars based on combined interferometric and spectroscopic observations, which are thus somewhat different from the topic under question.} For this purpose, we calculated a set of theoretical profiles on a grid of gravity-darkened models for various combinations of ($v_e$, $i$). Here, we focus only on He 4471 and Mg ii 4481 lines, which are suitable for the present aim and actually have been widely used so far. Our strategy is to compare the zeros in the Fourier transforms of the observed and the numerically simulated profiles directly to each other. In this sense, our analysis (except for the first preparatory analysis following the conventional procedure) is irrelevant to the concept of rotational broadening function (whichever classical or special) used to model the line profile by convolution. Yet, we also examine how the zeros in the Fourier transforms of the realistically computed line profiles of rapid rotators are compared with those of the classical rotational broadening, which would be worthwhile for evaluating the accuracies of apparent $v_e$ sin $i$ values derived from the conventional treatment.

The remainder of this paper is organized as follows. Our observational data of six target stars are described in Sect. 2. In Sect. 3 are presented the results of our preparatory analysis following the standard procedure (parameter determination from colors, examination of positions on the HR diagram, spectrum-fitting analysis by using the classical rotational broadening function, evaluation of equivalent widths, etc.). Profile simulations of He 4471 and Mg 4481 lines based on gravity-darkened models and Fourier transform calculations of observed as well as theoretical profiles are explained in Sect. 4. Sect. 5 is the discussion section, where the characteristics of first-zero locations are examined, and we try separate determinations of $v_e$ and $i$ by comparing the observed first-zero frequency with the theoretical grid. The summary of this investigation is given in Sect. 6.
2 OBSERVATIONAL DATA

As the targets of this study, we selected 6 apparently bright ($V \sim 1$–4) late B-type stars (spectral type of B6–B9 and luminosity classes of III–V; cf. Table 1), all of which are known to be rapid rotators with $v\sin i \sim 100$–300 km s$^{-1}$ (e.g., Abt, Levato & Grosso 2002).

Note that 3 stars ($\eta$ Tau, $\beta$ CMi, 17 Tau) out of these 6 program stars are classified as classical Be stars according to Jaschek & Egret’s (1982) catalogue. Actually, we can confirm the existence of appreciable emission component in H$\alpha$ in the spectra of these 3 stars (especially for $\eta$ Tau and $\beta$ CMi). Accordingly, we should keep in mind a possibility that the spectra may be contaminated by circumstellar Be disk for these stars. However, we consider that such an effect (even if any exists) is presumably not significant for the Be disk for these stars. However, we consider that such an effect (even if any exists) is presumably not significant for the Be disk for these stars.

The observations were carried out on 2006 November 1 ($\beta$ CMi, $\eta$ Tau), 2 ($\zeta$ Peg), and 4 (17 Tau, $\alpha$ Leo, and $\eta$ Aqr) by using the High-Dispersion Echelle Spectrograph (HIDES; Izumiya 1999) at the conde focus of the 188 cm reflector of Okayama Astrophysical Observatory (OAO). Equipped with a 4K$\times$2K CCD detector at the camera focus, the HIDES spectrograph enabled us to obtain an echellogram covering a wavelength range of 3860–4630 Å with a resolving power of $R \sim 70000$ (case for the normal slit width of 200 $\mu$m) in the mode of blue cross-disperser. The total exposure time for each star ranged from 6 min to 100 min depending on the stellar brightness or the weather condition.

The reduction of the spectra (bias subtraction, flat-fielding, scattered-light subtraction, spectrum extraction, wavelength calibration, co-addition of frames to increase the S/N ratio, and continuum normalization) was performed by using the “echelle” package of the software IRAF$^3$ in a standard manner. The S/N ratios of the finally resulting spectra turned out to be sufficiently high for the present purpose ($\geq 500$ for most stars at the 4450–4500 Å region).

3 CONVENTIONAL MODEL ATMOSPHERE ANALYSIS

Before going into the Fourier analysis of spectral line profiles, we first conducted a preparatory study following the conventional manner, in order to grasp the basic characteristics of program stars.

3.1 Stellar parameters

The effective temperature ($T_{\text{eff}}$) and the surface gravity ($\log g$) of each star were determined from the colors of Strömgren’s $uvby/\beta$ photometric system with the help of Napiwotzki, Scönbner, and Wenske’s (1993) uvbybetanew program$^4$, where the observational data of $b - y$, $c_1$, $m_1$, and $\beta$ were taken from Hauck and Mermilliod (1998) via the SIMBAD database. The resulting $T_{\text{eff}}$ and $\log g$ are summarized in Table 1.

We also estimated the luminosity ($L$) of these stars by using the Hipparcos parallaxes (ESA 1997) along with the extinction correction (Arenou, Grenon & Gomez 1992) and bolometric correction (Flower 1996), as done by Takeda et al. (2010). These log $L$ values are plotted against log $T_{\text{eff}}$ in Fig. 1, where Ekström et al.’s (2012) theoretical evolutionary tracks for both rotating and non-rotating models are also depicted. We can see from this figure that the masses of our sample stars are in the range between $\sim 3$ $M_{\odot}$ and $\sim 5$ $M_{\odot}$.

By comparing $M$ and $\log g$, we find that the radius values are $R \sim 3$–6 $R_{\odot}$ for most stars (excepting $\eta$ Tau, for which $R \sim 10$ $R_{\odot}$).

3.2 Synthetic spectrum fitting

Then, spectrum-synthesis analysis was carried out for the He 4471/Mg 4481 line feature in the same manner as described in Sect. 4.2 of Takeda et al. (2010). That is, while applying the optimization algorithm described in Takeda (1995), we determined the solutions of $A$(He) (He abundance), $A$(Mg) (Mg abundance), $v\sin i$ (projected rotational velocity), and $\Delta\lambda$ (radial velocity shift) accomplishing the best fit between the theoretical and observed spectra, where the rotational broadening was treated by convolving the intrinsic flux profile with the rotational broadening function (limb-darkening coefficient of $c = 0.5$). Neither the instrumental broadening nor the macroturbulence broadening was taken into account, which are negligible compared to the rotational broadening.

The model atmosphere for each star used for this analysis was constructed by two-dimensionally interpolating Kurucz’s (1993) ATLAS9 model grid (solar-metallicity models) in terms of $T_{\text{eff}}$ and $\log g$. Regarding the He 1 4471 line (multiplet No. 14; $\chi_{\text{low}} = 20.96$ eV), we adopted the total $gf$ value (summed over components) of 1.13 (cf. Table 2b of Takeda 1994) and the opacity profile (with forbidden components) broadened by Stark damping was treated according to Barnard, Cooper & Smith (1974). As to the atomic line data for Mg II 4481 (Multiplet No. 4: $\chi_{\text{low}} = 8.86$ eV), we consulted the VALD database (Ryabchikova et al. 2015), which gives log $gf = +0.740$, $-0.560$, and $+0.590$ for the components at 4481.126, 4481.150, and 4481.325 Å, respectively. The microturbulence velocity was fixed at 2 km s$^{-1}$.

We assumed LTE throughout this study. The non-LTE effect for the He 4471 and Mg 4481 lines is not likely to be important as far as late-B stars are concerned, even though it should act in the direction of intensifying the strengths of lines for both cases. According to Auer & Mihalas (1973), the non-LTE increase in the equivalent width for the He 4471 line is only $\sim 5\%$ for the model of $T_{\text{eff}} = 15000$ K and $\log g = 4.0$, which becomes even less significant as $T_{\text{eff}}$ is lowered. Regarding the Mg 4481 line, the non-LTE abundance correction for the A0V star Vega ($T_{\text{eff}} \approx 9500$ K and $\log g \approx 4.0$) was reported to be $\sim -0.1$ dex (Gigas 1988) or

\footnotesize
3 See, e.g., the spectral collection of Be stars available at \texttt{http://www.astrosurf.com/bull/us/becat.htm}.

4 IRAF is distributed by the National Optical Astronomy Observatories, which is operated by the Association of Universities for Research in Astronomy, Inc. under cooperative agreement with the National Science Foundation.

\footnotesize
5 \texttt{http://www.astro.le.ac.uk/~rn38/uvbybeta.html}
We also computed the equivalent widths (Mg 4481) inversely from the converged solutions of non-LTE intensification of equivalent width (4.1 Transform of line profiles)

As such, in order to compute a spectrum, it is necessary to specify five parameters: $M$, $R_p$, $T_{\text{eff},p}$, $v_\infty$, and $i$.

According to this formula, $\beta$ becomes equal to von Zeipel's value of 0.25 at the higher $T_{\text{eff}}$ region of $\log T_{\text{eff}} \geq 0.9$ ($T_{\text{eff}} \geq 7943 \, K$). Since most of our models fall in this $T_{\text{eff}}$ range (though the minimum $T_{\text{eff}}$ is 7204 K in the exceptional case), we may say that the simple von Zeipel’s law was adopted for almost all models. However, we should keep in mind a possibility that actual $\beta$ may deviate from the von Zeipel value (0.25) even at such a higher $T_{\text{eff}}$ range where the radiative equilibrium holds in the stellar envelope. That is, according to Espinosa Lara & Rieutord's (2011) calculation, $\beta$ is a function of rotational flattening in the sense that $\beta$ progressively decrease from $\sim 0.25$ (no rotation) to $\sim 0.15$ (near to the rotational break-up limit). If this is really the case, the simple application of $\beta = 0.25$ to rapidly rotating stars may lead to an overestimation of gravity darkening.
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5 DISCUSSION

5.1 Characteristics of first-zero frequencies

We first discuss the trend of theoretical \( \sigma_1 \) modeled in Sect. 4.2. Since this quantity is expressed as \( \sigma_1 = 0.660/ (\lambda v_\text{obs} \sin i/c) \) in the classical approximation, the projected rotational velocity (or the line-width for the case of rotation-dominated broadening) plays the most significant role, though its precise value is affected also by the profile shape. The resulting values of \( \sigma_1 \) corresponding to the theoretical line profiles simulated on the gravity-darkened models are plotted against \( v_\text{eq} \sin i \) in Fig. 7, where the classical \( \sigma_1 \) vs. \( v_\text{eq} \sin i \) relation is depicted by a slanted solid line and the observed \( \sigma_1 \) values for 6 stars are indicated by horizontal dashed lines.

We can see from this figure that the \( \sigma_1 \) values of the simulated spectra closely follow the classical relation (i.e., inversely proportional to \( v_\text{eq} \sin i \)), as long as \( v_\text{eq} \sin i \) is small (e.g., up to several tens km s\(^{-1}\)). \( \sigma_1 \), though appreciable deviations (\( \sigma_1 \) tends to be larger than the classical value) reflecting the gravity-darkening effect begin to be recognized as \( v_\text{eq} \sin i \) becomes larger. A closer inspection of the figure reveals that (i) the extent of deviation is essentially determined by \( v_\text{eq} \sin i \), (ii) this discrepancy is more manifest for He 4471 than for Mg 4481, and (iii) no significant difference exists regarding the trend of \( \sigma_1 \) between the \( T_{\text{eff},p} = 12000 \) K and 15000 K cases (though the deviation being slightly larger for the former).

The fact that \( \sigma_1 \) tends to be in excess of the classical value is reasonably explained by considering the physical effect caused by gravity darkening. In the rotationally-broadened spectral lines, the line width is mainly determined by maximum Doppler velocity caused by rotation (i.e., \( v_\text{eq} \sin i \)), to which the low-latitude region near to the equator makes most significant contribution because the absolute rotational velocity is largest there. In the case of rapid rotators with large \( v_\text{eq} \), these equatorial regions are considerably darkened and thus their contribution to line broadening becomes lessened. Accordingly, the width becomes narrower compared to the classical case, which eventually shifts \( \sigma_1 \) to a larger value. Moreover, this effect is further amplified for the case of He 4471, because the strength of this line quickly drops as \( T_{\text{eff}} \) is lowered (cf. Fig. 3). Consequently, the inequality relation \( \sigma_1^\text{He} > \sigma_1^\text{Mg} \) holds for rapid-rotator models, which is actually observed in our program stars (cf. Fig. 4).

Another important implication read from Fig. 7 is that \( \sigma_1 \) tends to be stabilized at the high-rotation limit \((\gtrsim 300 \text{ km s}^{-1})\), in the sense that \( \sigma_1 \) does not effectively decrease (or line width does not effectively increase) any more no matter how the rotational velocity is increased, which is because two effects (increase of rotation and enhanced gravity darkening) on the width counteract with each other and tend to be cancelled. Actually, this effect was first pointed out by Stoeckley (1968b) and also confirmed by Townsend, Owoc & Howarth (2004); i.e., rotational velocities of rapidly-rotating stars (especially those close to the limit) may be significantly underestimated if simply determined from the apparent line widths.

5.2 Separation of rotation and inclination

Now that we have theoretical \( \sigma_1 \) values as functions of \( v_\text{eq} \) and \( i \) for He 4471 as well as Mg 4481 lines, we can compare them with the observed \( \sigma_1 \). Then, a possible set of \((v_\text{eq}, i)\) solutions is expressed as a locus on the \( v_\text{eq} \) vs. \( i \) plane by solving the equation \( \sigma_1^\text{theo}(v_\text{eq}, i) = \sigma_1^\text{obs} \) for each line, from which \((v_\text{eq}, i)\) may be established from the intersection of two loci on the plane. Such constructed loci (displayed in Fig. 8), and the estimation of \((v_\text{eq}, i)\) values (summarized in Table 1) are discussed below star by star.

5.2.1 ζ Peg

Fig. 8a and Fig. 8a’ represent the characteristic difficulty involved with this analysis. Since the loci derived for He and Mg lines are rather similar in shape, the intersection is not clearly defined. To our embarrassment, this similarity brings about a marked difference of \((v_\text{eq}, i)\) solution depending on \( T_{\text{eff},p} \), despite that \( \sigma_1 \) is not so sensitive to it (cf. Sect. 4.2). That is, \((\sim 150–200 \text{ km s}^{-1}, \sim 50–60^\circ)\) for 12000 K and \((\sim 250–300 \text{ km s}^{-1}, \sim 30–40^\circ)\) for 15000 K. Fortunately, however, the former (more equator-on-like slower rotation) and the latter (more pole-on-like faster rotation) yield distinctly different \( T_{\text{eff},p} \): i.e., \( \sim 11200–11500 \) K and \( \sim 13000–13700 \) K according to Table 2. Comparing these with \( T_{\text{eff},p}^{\text{color}} \) of \( \sim 12700 \) K, we can guess that the actual solution would be almost the intermediate between these two as \((\sim 200–250 \text{ km s}^{-1}, \sim 40–50^\circ)\). Stoeckley & Buscombe (1987) derived \( (195 (158–246) \text{ km s}^{-1}, 54 (40–90)^\circ) \) for this star, nearly consistent with our result.

5.2.2 η Aqr

This star has the largest \( v_\text{eq} \sin i \) of \( \sim 290 \text{ km s}^{-1} \) among our 6 sample stars. Unfortunately, we were unable find the solution for this star, because the \( \sigma_1^\text{obs} \) values (0.172 for He and 0.153 for Mg) are too small to be covered by the grid of \( \sigma_1^\text{theo} \) (see Fig. 7). The reason for this incompatibility is not clear. It might be possible that the physical condition can not be adequately described by our modeling for such case of very rapid rotator. Alternatively, there might be systematic errors in the measurement of \( \sigma_1 \) for such case of especially large \( v_\text{eq} \sin i \), because He and Mg lines tend to somewhat merge in-between (Fig. 2). In this context, the ambiguity in the measurement of \( \sigma_1^\text{He} \) would be larger than \( \sigma_1^\text{Mg} \), because the cuspy feature near to the zero amplitude is less sharp.
for the former than the latter (cf. Fig. 4b). As a test, we increased the original $\sigma_{\text{He}}$ arbitrarily by 10% ($0.172 \rightarrow 0.189$) and made a retry. In this case, we could find a solution for the $T_{\text{eff,p}} = 15000$ K case (but failed again for the 12000 K case) as shown in Fig. 8b, which suggests ($\sim 300$ km s$^{-1}$, $\sim 80-90^{\circ}$). In any event, we may conclude that this star is a very rapidly rotating star ($v_\tau \sim 300-350$ km s$^{-1}$) or even somewhat larger) seen nearly equator-on. In such cases, solution search becomes especially difficult, because $i = 90^{\circ}$ is a singularity point, in the sense that it can not be encompassed by $i$ values of the grid. As such, the feasibility of establishing the solution is quite vulnerable to errors in $\sigma_i$ or modeling inadequacies.

5.2.3 $\eta$ Tau

The ($v_\tau, i$) for this star appears to be comparatively easier to embrace. Since the intersection of He and Mg loci lies at ($\sim 300$ km s$^{-1}$, $\sim 35^{\circ}$) for $T_{\text{eff,p}} = 12000$ K (Fig. 8c) and at ($\sim 350$ km s$^{-1}$, $\sim 30^{\circ}$) for $T_{\text{eff,p}} = 15000$ K (Fig. 8c') and not much different from each other, we may regard that this star is a very rapid rotator with $v_\tau \sim 300-350$ km s$^{-1}$ seen from a rather low inclination angle ($i \sim 30-35^{\circ}$).

5.2.4 $\beta$ CMi

We could not find a reasonable solution for this star. First, $\sigma_{\text{He}}$ is so small (as is the case for $\eta$ Aqr) and no He-locus exists for $T_{\text{eff,p}} = 12000$ K (Fig. 8d) and at ($\sim 350$ km s$^{-1}$, $\sim 30^{\circ}$) for $T_{\text{eff,p}} = 15000$ K (Fig. 8d') and not much different from each other, we may regard that this star is a very rapid rotator with $v_\tau \sim 250-300$ km s$^{-1}$ and $i \sim 60-80^{\circ}$ as the possible ranges (i.e., rather high inclination angle), any conclusion should be withheld. On the other hand, Stockeckley (1968a) reported (as a tentative conclusion) the aspect angle of this star to be $i \sim 30-50^{\circ}$.

5.2.5 $\alpha$ Leo

Almost the same situation holds for this star as the case of $\eta$ Aqr, since the $\sigma_1$ values are indiscernibly similar to each other (cf. Fig. 7), which means that no solution exists because $\sigma_1$ is outside of the range of the grid. Again, we increased $\sigma_{\text{He}}$ by 10% ($0.172 \rightarrow 0.189$), as a trial and searched for the ($v_\tau, i$) solution. Naturally, the results are essentially the same as the case for $\eta$ Aqr (compare Fig. 8e' with Fig. 8b'), suggesting ($\sim 300$ km s$^{-1}$, $\sim 80-90^{\circ}$) as a rough estimate. We may thus at least state that $\alpha$ Leo is a very rapidly-rotating star seen nearly equator-on. This is consistent with the previous determinations: (300 km s$^{-1}$, $90^{\circ}$) estimated by Hutchings & Stockeckley (1977) from the widths of UV lines, [285 (270–309) km s$^{-1}$, 90 (67–90)]$^{\circ}$ determined by Stockeckley & Buscombe (1987) from the widths of He 4471 and Mg 4481 lines, and [317 $\pm 3$ km s$^{-1}$, 75–90$^{\circ}$] concluded by McAllister et al. (2005) based on interferometric observations.

5.2.6 17 Tau

The behavior of loci on the $v_\tau$ vs. $i$ plane for this star is similar to the case of $\zeta$ Peg. The intersection of He and Mg loci depends on $T_{\text{eff,p}}$ as ($\sim 200$ km s$^{-1}$, $\sim 60^{\circ}$) for 12000 K and ($\sim 300$ km s$^{-1}$, $\sim 30-40^{\circ}$) for 15000 K, each of which correspond to $T_\text{rot} = 11000$ K and $\sim 13000$ K according to Table 2. Considering that $T_{\text{rot}}$ is $\sim 12700$ K, we conclude that the latter solution is relevant, which means that this star is very rapidly-rotating with $v_\tau \sim 300$ km s$^{-1}$ and seen with a rather low aspect angle ($i \sim 30-40^{\circ}$).

5.2.7 Comparison with Zorec et al.’s (2016) results

Zorec et al. (2016) recently studied the rotational velocity distribution of a large sample of 233 Be stars. They also separated rotation and inclination for each star by comparing the observed stellar parameters affected by rotation (effective temperature, surface gravity, luminosity, projected rotational velocity) with extensive theoretical calculations based on gravity-darkened stellar models (cf. Appendix E therein). Since three of our targets are included in their sample, it is interesting to compare our profile-based results of axial inclination with their determinations. Comparing their $i$ values ($62 \pm 15^{\circ}$, $66 \pm 16^{\circ}$, and $45 \pm 11^{\circ}$ for $\eta$ Tau, $\beta$ CMi, and 17 Tau, respectively) with our results presented in Table 1 ($\sim 30-35^{\circ}$, $\sim 60-80^{\circ}$, and $\sim 30-40^{\circ}$), we notice an appreciable disagreement for $\eta$ Tau, although a reasonable consistency is seen for $\beta$ CMi and 17 Tau. This discrepancy for $\eta$ Tau may be attributed to the fact that this is an exceptional case of evolved subgiant ($R \sim 10R_\odot$) among our sample (cf. Sect. 3.2). Our result for this star would be less reliable, because theoretical calculations corresponding to main-sequence B stars ($R = 4R_\odot$) were applied to it like others.

6 SUMMARY AND CONCLUSION

It is known that the gravity-darkening effect causes an appreciable latitudinal inhomogeneity on the surface of a rapidly-rotating star; i.e., lower $T_{\text{eff}}$ as well as $g$ at lower latitude (in short, cool/dark equator and hot/bright pole). Owing to this effect, the impact of rotation on the shape of spectral lines is complicated and different from line to line, and the simple classical treatment (convolution of the intrinsic profile with the rotational broadening function determined by $v_\tau \sin i$) is no more valid.

From the converse point of view, it is possible to make advantage of this line-dependent complexity to separately determine the equatorial rotational velocity ($v_\tau$) and the inclination angle ($i$) of rotational axis, which is impossible within the framework of the conventional approximation. Although several investigators challenged this task several decades ago, those old studies appear rather outdated as viewed from the present-day standard, especially in terms of their policy of simply invoking line-widths and seemingly insufficient accuracy in simulating line-profiles.

We thus tried to examine in this study whether the Fourier method, which is a comparatively modern technique utilizing the unambiguously determinable first-zero frequency of the Fourier transform of a line profile, is applicable to this problem of spectroscopically separating $v_\tau$ and $i$. Conveniently, since we already have a computer code of simulating line profiles for a rapidly-rotating star with distorted surface of inhomogeneous brightness, we can make...
use of it. As to the lines to be analyzed, we chose He I 4471 and Mg II 4481 lines, which have been often used for this purpose because they are strong and have markedly different temperature sensitivity.

Toward this aim, six rapidly-rotating late B-type stars (ζ Peg, η Aqr, η Tau, β CMi, α Leo, 17 Tau) were selected as our targets, for which high-dispersion spectra of sufficient quality are available. We first evaluated T_eff and log g from uvbyβ colors, and then carried out a conventional spectrum-fitting analysis on the He I 4471+Mg II 4481 line feature based on plane-parallel model atmospheres to estimate v_e sin i and the equivalent widths of two lines.

The theoretical line profiles of gravity-darkened rapid rotators were simulated for a grid of models for various combinations of v_e (100–350 km s^{-1}) and i (0–90°), while typical values were assumed for other parameters (T_{eff,p} = 12000 K and 15000 K, M_p = 4.0 M_⊙, R_p = 4.0 R_⊙). We then computed their Fourier transforms and measured the frequencies corresponding to the first zero (σ_1).

These modeled σ_1 values revealed the characteristic trends, reflecting the gravity-darkening effect and the different temperature susceptibility of these lines. (i) They tend to be in excess of the classical value σ_1^(obs) = 0.660/(λ v_e sin i/c) when compared at a given v_e sin i, and this difference progressively grows with an increase in v_e. (ii) The amount of this excess is larger for He 4471 than for Mg 4481, resulting in an inequality relation σ_1^He > σ_1^Mg.

Then, by comparing the σ_1 of the observed profile with the theoretical grid of σ_1(v_e, i) for each of the He 4471 and Mg 4481 line, we can define two loci on the v_e vs. i plane, from which (v_e, i) may be separately determined from their intersection. We tried this solution search for 6 program stars, and confirmed that the intersection point is measurable, except for the difficult case of largest v_e sin i (η Aqr and α Leo; equator-on case with considerably large v_e). However, since the position of this intersection tends to depend upon T_{eff,p}, its adequate specification was often necessarily, for which we compared (T_{eff}) of the theoretical grid with T_{eff,p}. That is, line profiles alone are not sufficient but photometric information needs to be simultaneously employed.

To conclude, we could show in this investigation that the Fourier method using the first-zero frequencies in the transforms of line profiles is effectively applicable to separation of v_e and i. Although this approach is not essentially different from the traditional method using line widths, it has a merit that characteristic zero point of the transform is comparatively easier to measure even for the case of rapid rotators.

Yet, such a rough feasibility study as attempted in this paper still has room for further improvements. We enumerate below several issues to be considered in the future.

- Although we employed only He 4471 and Mg II 4481 lines, simultaneously using more lines of different parameter sensitivity would surely improve the reliability of the solutions.
- Our adopted grid of theoretical gravity-darkened models was rather rough, especially in terms of the parameters other than v_e and i (i.e., T_{eff,p}, R_p), for which much finer mesh would be required.
- Since the assumption of rigid rotation is not necessarily guaranteed, additional parameter describing the degree of differential rotation would desirably be included.
- Similarly, the possibility of rotation-dependence in the exponent β (see footnote 6), which determines the degree of gravity darkening, should be seriously investigated.
- In our Fourier analysis of line profiles, we made use of only σ_1, which is nothing but one of the various quantities characterizing the Fourier transform. It may be possible to utilize other observables such as the second-zero frequency or the amplitude of the side lobe.

When advanced analysis has been carried out by adequately taking account of these points, more realistic and trustworthy results would be expected.

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Table 1. Basic parameters and observed data of program stars.

| Name   | HR#  | HD#   | Sp.Type | V  | T_{\text{eff}} | log g_{color} | v_{\text{e} sin i} | W_{\text{He I} 4471} | W_{\text{Mg II} 4481} | \sigma_{\text{He I} 4471} | \sigma_{\text{Mg II} 4481} | \left(v_{\text{e}}, i\right) |
|--------|------|-------|---------|----|---------------|---------------|---------------------|-------------------|-------------------|-------------------|-------------------|------------------|
| \zeta\ Peg | 8634 | 214923 | B8 V    | 3.41 | 11182        | 3.65          | 153                 | 329               | 0.305             | 0.283             | (~200–250, ~40–50)   |
| \eta\ Aqr | 8597 | 213998 | B9 IV-Vn | 4.03 | 11458        | 3.91          | 289                 | 450               | 0.357             | 0.172*            | 0.153             | (~300, ~80–90)     |
| \eta\ Tau | 1165 | 23639 | B7 III  | 2.87 | 11599        | 2.50          | 158                 | 546               | 0.340             | 0.269             | (~300–350, ~30–35)  |
| \beta\ CMi | 2840 | 58715 | B8 V    | 2.89 | 11696        | 3.42          | 231                 | 431               | 0.203             | 0.191             | (~250–300, ~60–80) |
| \alpha\ Leo | 3982 | 87901 | B8 IV-Vn | 1.49 | 12223        | 3.54          | 276                 | 511               | 0.172*            | 0.152             | (~300, ~80–90)     |
| 17 Tau | 1142 | 23302 | B6 II/IIe | 3.70 | 12698        | 3.28          | 164                 | 851               | 0.287             | 0.254             | (~300, ~30–40)     |

(1) Star name. (2) HR number. (3) HD number. (4) Spectral type from SIMBAD. (5) Apparent visual magnitude from SIMBAD (in mag). (6) Effective temperature from $uvby\beta$ (in K). (7) Logarithmic surface gravity from $uvby\beta$. (8) Fitting-based projected rotational velocity (in km s$^{-1}$). (9) Fitting-based equivalent width of He I 4471 (in m\AA). (10) Fitting-based equivalent width of Mg II 4481 (in m\AA). (11) First-zero frequency of He I 4471 (in \AA$^{-1}$). (12) First-zero frequency of Mg II 4481 (in \AA$^{-1}$). (13) Estimated solution of $(v_{\text{e}}, i)$, where $v_{\text{e}}$ is in km s$^{-1}$ and $i$ is in degree (see Sect. 5.2 for the details).

*Actually, an arbitrarily increased value by 10% (0.189) was used for the $(v_{\text{e}}, i)$ solution search.

Figure 1. Plots of the 6 program stars on the log($L/L_\odot$) vs. log $T_{\text{eff}}$ diagram, where the effective temperature ($T_{\text{eff}}$) was determined from $uvby\beta$ photometry (Sect. 2) and the bolometric luminosity ($L$) was evaluated from the apparent visual magnitude (Table 1), Hipparcos parallax (ESA 1997), Arenou, Grenon & Gómez’s (1992) interstellar extinction map, and Flower’s (1996) bolometric correction. Theoretical evolutionary tracks corresponding to the solar metallicity computed by Ekström et al. (2012) for six different initial masses (1.7, 2, 2.5, 3, 4, and 5 $M_\odot$) are also depicted for comparison, where solid lines and dashed lines correspond to the non-rotating models and the rotating models (with 40% of the critical break-up velocity), respectively.
Table 2. Computed models of rotating stars and the first-zero frequencies of Fourier transforms.

| Code         | $R_p$  | $R_s$  | $T_{	ext{eff}, p}$ | $T_{	ext{eff}, s}$ | $v_{	ext{rot}} \sin i$ | $\sigma_{1p}$ | $\sigma_{1p}/\sigma_{1s}$ | $\sigma_{1p}$ | $\sigma_{1s}$ | $\sigma_{1p}/\sigma_{1s}$ |
|--------------|--------|--------|---------------------|---------------------|--------------------------|---------------|-----------------------------|---------------|--------------|-----------------------------|
| m40r40t120v350i70 | 11759  | 98.5   | 0.479               | 1.065               | 0.444                    | 0.991         |                              |               |              |                              |
| m40r40t120v300i70 | 11755  | 100.0  | 0.472               | 1.068               | 0.439                    | 0.993         |                              |               |              |                              |
| m40r40t120v250i70 | 11900  | 170.0  | 0.624               | 1.135               | 0.657                    | 1.027         |                              |               |              |                              |
| m40r40t120v200i70 | 11972  | 176.0  | 0.676               | 1.171               | 0.706                    | 1.039         |                              |               |              |                              |
| m40r40t120v150i70 | 12023  | 182.0  | 0.711               | 1.200               | 0.749                    | 1.052         |                              |               |              |                              |
| m40r40t120v100i70 | 12061  | 188.0  | 0.745               | 1.230               | 0.792                    | 1.065         |                              |               |              |                              |
| m40r40t120v50i70  | 12099  | 194.0  | 0.772               | 1.257               | 0.833                    | 1.077         |                              |               |              |                              |

\[ R_p, R_s, T_{\text{eff}, p}, T_{\text{eff}, s}, v_{\text{rot}} \sin i, \sigma_{1p}, \sigma_{1p}/\sigma_{1s}, \sigma_{1p}, \sigma_{1s}, \sigma_{1p}/\sigma_{1s} \]
Frequency of classical rotational broadening function (see, e.g., Gray 2005), which is related to continuum intensity. (7) Product of $v_i$ (1) Model code ($m**r**t***v***i**), which describes the adopted stellar mass ($M_{\odot}$), polar radius ($R_p$), polar effective temperature ($T_{eff,p}$), equatorial rotation velocity ($v_o$), and inclination angle ($i$). For example, "m40r40t150v300i30" is the case of $M=4.0M_{\odot}$, $R_p=4.0R_p$, $T_{eff,p}=15000$ K, $v_o=300$ km s$^{-1}$, and $i=30^\circ$. (2) Polar radius (in $R_p$). (3) Equatorial radius (in $R_p$). (4) Polar effective temperature (in K). (5) Equatorial effective temperature (in K). (6) Mean effective temperature in K (which was derived by averaging the local effective temperature over the observed stellar disk while weighting according to the local continuum intensity). (7) Product of $v_o$ and $\sin i$ (in km s$^{-1}$). (8) First-zero frequency of He 4471 (in $\lambda^{-1}$). (9) Ratio of $\sigma^{He}_{1}$ to $\sigma_{1}$, where $\sigma_{1}$ is the first-zero frequency of classical rotational broadening function (see, e.g., Gray 2005), which is related to $v_o \sin i$ as $\sigma_{1} = 0.660/(\lambda v_o \sin i / c)$ ($\lambda$: wavelength, $c$: velocity of light). (10) First-zero frequency of Mg 4481 (in $\lambda^{-1}$). (11) Ratio of $\sigma_{v}^{He}$ to $\sigma_{1}$.
Figure 2. Synthetic spectrum fitting in the 4460–4490 Å region comprising He \textsc{i} 4471 and Mg \textsc{ii} 4481 lines, which was accomplished based on the conventional modeling (using the plane-parallel model atmosphere and the classical rotational broadening function) by adjusting four parameters [$A$(He), $A$(Mg), $v_s \sin i$, $\Delta \lambda$; cf. Sect. 3.2]. The observed data are plotted by red symbols, while the best-fit theoretical spectra are shown by blue solid lines. The spectra are arranged (from top to bottom) in the ascending order of $T_{\text{eff}}$ as in Table 1, and a vertical offset of 0.1 is applied to each spectrum relative to the adjacent one. The green crosses indicate the reference points of pseudo-continuum (cf. Sect. 4.1), based on which the normalized line-depth profiles (to be used for calculating the Fourier transforms) were computed.
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Figure 3. Equivalent widths (W) of He i 4471 (blue) and Mg ii (red) plotted against the effective temperature. The theoretical W values (computed for models with $T_{\text{eff}} = 9000, 10000, 11000, 12000$, and $13000$ K, and $\log g = 2.5, 3.0, 3.5$, and $4.0$) are depicted by lines (solid lines: solar abundance, dashed lines: overabundance by +0.5 dex), while the observed values for 6 program stars are shown by symbols (open circles for He 4471 and crosses for Mg 4481). Note the markedly large dependence of $W$(He) upon $T_{\text{eff}}$ (increasing with $T_{\text{eff}}$), while $W$(Mg) is only weakly $T_{\text{eff}}$-dependent (decreasing with $T_{\text{eff}}$). Regarding the gravity dependence, $W$(He) tends to decrease with an increase in $\log g$ (indicated in the figure), while the tendency is reversed for $W$(Mg).
Figure 4. Fourier transform amplitudes $|f(\sigma)|$ plotted against $\sigma$, for the observed profiles of He 4471 (blue) and Mg 4481 (red). The positions of the first zero ($\sigma_1$) are indicated by vertical dashed lines. (a) $\zeta$ Peg, (b) $\eta$ Aqr, (c) $\eta$ Tau, (d) $\beta$ CMi, (e) $\alpha$ Leo, and (f) 17 Tau.
Figure 5. Theoretically synthesized spectra in the 4463–4487 Å region computed for six representative models of v100i90, v200i30, v300i90, v100i20, v300i40, and v300i90 (the code “v***i**” means \( v_e = *** \) km s\(^{-1}\) and \( i = **\)\(^\circ\)), which were so chosen as to demonstrate the three \( v_e \sin i \) cases of \( \sim 100, \sim 200, \) and 300 km s\(^{-1}\). The results for \( v_e = 100, 200, \) and 300 km s\(^{-1}\) are depicted in brown, green, and blue, respectively. The upper panel (a) shows the case for \( T_{\text{eff,p}} = 12000 \) K, while the lower panel (b) is for \( T_{\text{eff,p}} = 15000 \) K.
Figure 6. Fourier transform amplitudes $|f(\sigma)|$ plotted against $\sigma$, for the theoretical profiles of He 4471 (left panels) and Mg 4481 (right panels) computed for six representative models (same as those shown in Fig. 5). The upper panels (a) and (b) show the results for $T_{\text{eff},p} = 12000$ K, while the lower panels (c) and (d) are for $T_{\text{eff},p} = 15000$ K.
Figure 7. The values of theoretical $\sigma_1$, measured from the Fourier transforms of simulated spectra for gravity-darkened models, are plotted against $v_\text{e} \sin i$. The results for He 4471 (left panels) and Mg 4481 (right panels) are shown in open and filled circles, respectively, where those corresponding to different $v_\text{e}$ are discriminated by their size and color (larger symbol for higher $v_\text{e}$). The solid line shows the classical relation $\sigma_1 = 0.660/(\lambda v_\text{e} \sin i/c)$. The observed $\sigma_1$ values for each of the six program stars are indicated by horizontal dashed lines. The upper panels (a) and (b) show the results for $T_{\text{eff},p} = 12000$ K, while the lower panels (c) and (d) are for $T_{\text{eff},p} = 15000$ K.
Figure 8. Graphical display of contours in the \((v_e, i)\) plane, which were derived as the solution satisfying the equation 
\(\sigma_{\text{obs}}^i = \sigma_{\text{th}}^i(i, v_e)\). The results for He 4471 and Mg 4481 are shown in thinner blue traces and thicker red traces, respectively. From top to bottom are arranged the panels for \(\zeta\) Peg, \(\eta\) Aqr, \(\eta\) Tau, \(\beta\) CMi, \(\alpha\) Leo, and 17 Tau, where each of the left-hand and right-hand panels are for \(T_{\text{eff}, p} = 12000\) K and \(T_{\text{eff}, p} = 15000\) K, respectively. Note that, in deriving the results for \(\eta\) Aqr and \(\alpha\) Leo, the observed \(\sigma_{\text{He}}^i\) values for He 4471 were arbitrarily increased by 10\%, in order to bring the solution to a determinable level.