Opioids Propagation Model and Strategy Analysis Based on SEIR

Xinyu Cui1, Yujing Wang2 and Nan Chen3

1 Digital Media Technology, Beijing University of Posts and Telecommunications, Xitucheng Road10, Haidian District, Beijing, China
Email: 13919062848@163.com
2 Industrial Design, Beijing University of Posts and Telecommunications, Xitucheng Road10, Haidian District, Beijing, China
Email: 739383223@qq.com
3 Industrial Design, Beijing University of Posts and Telecommunications, Xitucheng Road10, Haidian District, Beijing, China
Email: chennanchn@foxmail.com

Abstract. Through the analysis of the underlying data and the creation of the model, we have specific strategies for the transmission of opioids. To begin with, we process and analyse the data by graph theory and integrity principles. Then we find that the relevant variables of drug propagation law. After that, by modelling the results, we continue using the mechanism of the infectious disease SEIR model to analyse and construct the opioid drug propagation SEIR model [ S(Susceptible nodes)- General opioid users; E(Exposed node)-Opioid addicts; I(Infected nodes)-Drug users; R(Recovered nodes)-Drug abusers ]. By setting the corresponding parameters in the differential equations, we get the transmission speed of opioid abuse is positively correlated with the degree of initial infected node. The easier the node is infected, the faster the spread of opioids will be, the earlier it will stabilize, and the more nodes will eventually recover. We analyse the constructive ways to control the spread of opioids in the end, and propose the most valuable coping strategies.

1. Introduction

1.1. Problem Background

Opioids drugs are used to treat and control pain and are considered by the World Health Organization as a routine treatment for three-step cancer pain. However, with the increase in the use of opioids, the problems caused by misuse and abuse are also growing. The United States is experiencing a national crisis regarding the use of synthetic and non-synthetic opioids.

For this problem, we focus on the individual counties located in five U.S. states: Ohio, Kentucky, West Virginia, Virginia, and Pennsylvania. We are asked to analyse the spread and characteristics of the reported synthetic opioid and heroin incidents (cases) in and between the five states and their counties over time. And we are also required to explore its relationship with the U.S. Census socio-economic data provided. We will finally put forward something feasible suggestion for solving the drug problem. Figure 1 is the hot map of five states’ counties annual average growth rate in 2010-2017.
1.2. Research Contents

Synthetic opioids are established on the basis of the infectious disease model because their transmission mechanisms are similar to those of infectious diseases. By using the method of graph theory modelling, the population using synthetic opioids is represented by the nodes in the graph. The nodes are divided into susceptible nodes (S), infected nodes (I), recovered nodes (R) [1], etc. The connecting lines between nodes indicate that the states can be converted.

**Table 1.** The table of node status and population correspondence in SEIR model a.

| The node type       | Abbreviation | Classification of people exposed to opioids |
|---------------------|--------------|--------------------------------------------|
| Susceptible nodes   | S            | General opioid users                       |
| Exposed nodes       | E            | Opioid addicts                             |
| Infected nodes      | I            | Drug users                                 |
| Recovered nodes     | R            | Drug abusers                               |

Note: All of the node types in the following text are used in abbreviations.

2. SEIR Model of Opioid Transmission

2.1. Opioid Transmission Mechanism

In the real society, a person's mental state after abusing opioids will affect the people around him. People exposed to opioids may be transferred between four states, general opioid users(S), opioid addicts(E), drug users(I) and drug abusers(R). Propagation rules are described as follows:

For S, if it is adjacent to I, S becomes E with probability p1.

For E, it changes from v1 to I and begins to spread the drug. Or with probability p2 becomes R, no longer transmits the drug. Paragraphs should be justified.

For I, the probability p3 becomes R, giving up and stopping the transmission of the drug.

For R, if it is adjacent to I, then R becomes E with probability p4.

![Figure 2. Node State Transition Diagram.](image-url)
2.2. SEIR Model of Opioid Drug Transmission

Through data processing and analysis, we know that drug use and propagation is related to time \( t \) and node degree \( k \) (node degree: the number of neighboring nodes that a node has in a network). For a node \( j \) in the SNS network its state may vary between S, E, I, and R.

Suppose that at time \( t \), using \( S(k,t), E(k,t), I(k,t), R(k,t) \) to represent the number of four types of nodes with degree \( k \). In summary, the propagation dynamic equation [2] is:

\[
\begin{aligned}
\frac{\partial S(t)}{\partial t} &= S(t)I(k_1)P_1M \\
\frac{\partial E(t)}{\partial t} &= S(t)I(k_1)P_1M - P_1M - \theta_1E(t) - E(t)I(k_1)P_2M \\
\frac{\partial I(t)}{\partial t} &= -2\theta_1E(t) - E(t)I(k_1)P_2M - I(t)R(k_1)P_3M \\
\frac{\partial R(t)}{\partial t} &= I(t)R(k_1)P_2M - R(t)E(k_1)P_3M - 2\theta_1E(k_1) - E(t)I(k_1)P_3M \\
\end{aligned}
\]

\[ M = \sum_{k_1} p(k_1|k) \]

3. Numerical Simulation and Analysis

3.1. Evolution of Four Types of Nodes Over Time

In the above SEIR model of opioid transmission, \( p_1 \) is the transmission rate of susceptible nodes turning into exposed nodes, \( p_2 \) is the immunity rate of exposed nodes turning into recovered nodes, \( v_1 \) is the rate of exposed nodes turning into infected nodes, \( p_3 \) is the immunity rate of infected nodes turning into recovered nodes, and \( p_4 \) is the transmission rate of recovered nodes turning into exposed nodes.

Simulation results based on the SEIR model of opioid transmission are presented below. To study the changes of susceptible nodes, exposed nodes, infected nodes and recovered nodes in the opioid transmission network over time. Simulation when the parameters used for: \( N=10000, p_1=0.6, p_2=0.2, p_3=0.05, p_4=0.002, v_1=0.25. \) Assuming that there is only one propagation node in the initial state, \( I(0) = 1, S(0)=9999, E(0)=R(0)=0, \) set the model parameter as \( M = 15. \)

![Figure 3](image-url)

**Figure 3.** Changes in the density of the four types of nodes over time.

As can be seen from figure 3, the density of \( S \) decreases rapidly to 0 in the initial stage, indicating that the opioids abuse spreads extremely fast among people exposed to opioids. The density of \( E \) increases rapidly at the beginning, reaches the highest point, and then decreases rapidly to 0. Initially, large Numbers of people are exposed to opioids, but they all follow the doctor's advice or use small amounts of opioids without immediate opioid addiction. The curve of \( E \) increases rapidly, and when the peak is reached, there are no new drug abusers, and after the use of opioid addiction, the curve of \( E \)
gradually migrates to I and starts to take drugs, or to R and refuses to abuse drugs, and the curve of E drops rapidly. The density of I also increases and then decreases to zero. At the initial stage of opioid transmission, the opioid addicts gradually increased their dependence on opioids and began to take drugs. The nodes in the model changed from E to I, and the curve of I increased. After reaching the peak, there were no new drug users, and the former drug users also gradually stopped taking drugs, so the curve decreased. The density of R increases rapidly in the initial stage until it approaches 1. Due to the low probability of taking drugs again after drug withdrawal, all people exposed to opioids will eventually refuse to abuse drugs after exposure to opioids.

3.2. Influence of Degree of Initial Infected Node on Opioid Transmission

Assuming that the degree of the initial infected node is M=7, 15 and 30 respectively, the change of the probability of each node in the SEIR model in these three cases is studied.

![Figure 4](image)

**Figure 4.** The density of the four types of nodes changes with time under different M values.

As can be seen from figure 4, the transmission speed of opioid abuse is positively correlated with the degree of initial infected node. When M=7, the speed of drug transmission is the slowest, and when M=30, the speed of drug transmission is the fastest. The greater the degree of the initial infected node, the more friends the drug addict has at the initial moment, and the more people he comes into contact with, the faster the scale of transmission increases. So, the size of the initial infected node would have a big impact on the spread of opioids.

3.3. Influence of Nodal Infection Rate $p_1$ on the Transmission Process

If the susceptible node is adjacent to the infected node, it will be infected with probability $p_1$. $p_1$ was set at 0.1, 0.2, 0.4, 0.6, 0.8, respectively. The influence of infection rate $p_1$ on the process of opioid transmission is shown in the figure 5.
Figure 5. Density of the four types of nodes changes with time under different $p_1$ values.

As can be seen from figure 5, the easier the node is infected, the faster the spread of opioids will be, the earlier it will stabilize, and the more nodes will eventually recover. For people exposed to opioids, if a drug is more addictive, the more people end up abusing opioids, and the faster opioid transmission reaches a stable state. Figure 5 also shows that $p_1$ is critical to the extent of opioid transmission. When $p_1$ value is small, the scope of opioid transmission is more sensitive to the change of $p_1$, and the increase of $p_1$ value will cause a larger area of opioid diffusion.

4. Conclusion

In this paper, we make an analysis and prediction on opioid use to set realistic and proper strategies for establishing a drug-use rational society.

Firstly, we selected and aggregated variables from the data provided to create an opioid drug propagation model called SEIR to characterize the evolution of propagation over time, degrees, and probability. In these four states, human-based points were analysed to predict drug use in the future.

Secondly, based on the previous analysis, we constructed propagation hypotheses and immune strategies to predict the status of each state. We successfully obtained the target curve by setting independent variable variables. And, the size of the initial infected node would have a big impact on the spread of opioids. Therefore, in order to control the spread of opioids as soon as possible, nodes with high degree should be selected as the primary target of regulation. In addition, if the spread of opioids is to be widely suppressed, consideration should be given to how to effectively reduce the probability of opioid addiction and make the management of opioids both rapid and extensive.

Of course, there may be some issues with the linear model assumptions, but we do not believe them to be so serious as to seriously threaten the model’s power.
5. References
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[2] Lei W, Hao L, Kai W, et al 2012 Globally Asymptotical Stability of a Delayed SEIR Epidemic Model with Saturation Incidence Rate and Vaccination[J]. Mathematics in Practice and Theory