Characterization of metal-forming processes with respect to non-monotonity

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Abstract. The method of severe plastic deformation (SPD) is often used to produce bulk ultra-fine grained materials. SPD is a procedure, whereby the structure of the material changes from the initial coarse grained state to ultra-fine grained structure. For these techniques shear deformation as well as the so-called non-monotonic deformation are characteristic. The aim of this paper is to compare different metal-forming techniques with respect to non-monotony of deformation. A quantity and its calculation for the characterization of non-monotony are introduced. Study of different metal-forming processes (simple shear, ECAP, upsetting) concerning non-monotony and the comparison of these results are presented.

1. Introduction
The method of severe plastic deformation is often used to produce bulk ultra-fine grained (UFG) materials, whereby the structure of the material changes from the initial coarse grained state to UFG structure. By this microstructure special material properties can be obtained. Essentially, for these SPD techniques the shear deformation as well as the so-called non-monotonic deformation is characteristic. The paper introduces the calculation of a degree of non-monotony (DNM), which can be a measure of technology planning of SPD processes. On the other hand, the mechanical study of different metal-forming techniques will be presented, with calculation and comparison of DNM.

2. Monotonic deformation
The mechanical concept of monotonity was introduced by Smirnov-Aljajev [1]. A forming process develops monotonically if no component of the rate of deformation tensor $\mathbf{d}$ changes its sign, namely the eigenvectors of the rate of deformation tensor are parallel to the same material lines during the whole deformation process and the Lode parameter remains constant. The two necessary conditions for a monotonic deformation that the relative state of the material lines and the eigenvectors of the rate of deformation tensor are not changing during the process, and the Lode parameter remains constant. Investigating the monotony of the forming processes, can be distinguished which are monotonic processes and which are further or closer to the monotonic state. Through the inspection of the first condition (the type of the deformation) can be ascertained more about the evolving microstructure, than through the analysis of the stress state (second condition). Through the investigation of the type of the deformation, a measure can be created, which is appropriate to describe the degree of non-monotonity of a forming process.
3. Degree of non-montonity

In this chapter the definition of a degree of non-monotony (DNM) is shown. To define the monotony, or rather the non-monotony, the separate analysis of the rigid body rotation of a material particle and the rotation of the eigenvectors of \( \mathbf{d} \) is necessary. Both values can deduced from the velocity gradient tensor, hence an appropriate description of the velocity field of the forming process is required (equations (1)(2)). The velocity gradient tensor \( \mathbf{L} \) can be decomposed into a symmetric and an anti-symmetric part using equation (3). The first describe \( \mathbf{d} \), the second shows the rigid body rotation of a material point. The vorticity tensor \( \mathbf{\omega} \) can transcribe to vector-form using equation (4).

\[
\mathbf{v}_{(x_1, x_2, x_3)} = \left[ \mathbf{v}_1(x, t), \mathbf{v}_2(x, t), \mathbf{v}_3(x, t) \right]
\]

\[
\mathbf{d}_{kl} = \frac{1}{2} (\mathbf{L}_{kl} + \mathbf{L}_{lk})
\]

\[
\mathbf{\omega}_{kl} = \frac{1}{2} (\mathbf{L}_{kl} - \mathbf{L}_{lk})
\]

\[
\mathbf{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}
\]

The \( \mathbf{\omega} \) vector specified defines the axis and the angular velocity of the rotation of a material point at the given point of space and at the given time. Furthermore the rotation of the eigenvectors of the rate of deformation tensor is required. The eigenvectors \( \mathbf{n}_i \) can calculated from \( \mathbf{d} \). The relation between the eigenvector \( \mathbf{n}_i \) and its rate of change \( \dot{\mathbf{n}}_i \) with respect to time can be written as follows

\[
\dot{\mathbf{n}}_i = \mathbf{\Omega} \times \mathbf{n}_i
\]

Where \( \mathbf{\Omega} \) is the vector of rotation and \( \mathbf{\Omega} \) is the anti-symmetric tensor form of \( \mathbf{\Omega} \), similar to equation (4). \( \mathbf{\Omega} \) can be calculated from the system of equations (5). The \( \dot{\mathbf{n}}_i \) vector is a material derivative of the eigenvector \( \mathbf{n}_i \), which can be expressed with the formula (6). One of the necessary conditions of monotony is, that the eigenvectors of the rate of deformation coincide with the same material line through the whole deformation. This condition is fulfilled if \( \mathbf{\Omega} \) and \( \mathbf{\omega} \) vectors are equal. If the difference between the vectors \( \beta = \mathbf{\omega} - \mathbf{\Omega} \) differ from 0 then the process is non-monotonic in the given time for the given material point. In order to characterize the deformation of a chosen material particle through the forming process, the integration of \( \beta \) over the time and along a particle trajectory is needed. The DNM \( \mu \) is the integral of the absolute value of \( \beta \) according to equation (7).

\[
\dot{\mathbf{n}}_i = \frac{dn_i}{dt} + v_k n_{ik}
\]

\[
\mu = \int_0^t |\beta| dt
\]

4. Calculation of degree of non-montonity

The starting point of the calculation of the DNM is the known velocity field. For that purpose an appropriate model of the forming process is necessary, which describes the material flow. Analytical models are often used to modeling a forming process. In this paper analytical models were also used for the calculations. These contain mostly simplification in order of faster and easier calculus. They are not so accurate as a finite element computation, but provide a continuous velocity field.

Discontinuity surfaces are often used to simplify the flow model. The velocity of a particle which is passing through this surface has a stepwise change (figure 1). The normal component to the surface \( A_f \)
remains continuous \((v^* = v_n)\) but the tangential changes are abrupt. The material particle is deformed infinitely rapid; hence the calculation of the DNM on these surfaces is not trivial. Assume that the tangential velocity near to the surface \(A_f\) in the \(\delta\) wide region between \(A_f^+\) and \(A_f^-\) changes continuously. The non-zero components of \(L_1, \omega_1\) and \(d_1\) are

\[
\frac{21}{21} \frac{v_1}{\delta} = \frac{23}{23} \frac{v_1}{\delta} = \frac{25}{25} \frac{v_1}{\delta} = \frac{27}{27} \frac{v_1}{\delta} \quad (8)
\]

This shows that a rigid body rotation occurs in the \(\delta\) wide region, without any rotation of the eigenvectors of the rate of deformation. The value of the expression \(\Delta v_1 / \delta\) becomes infinitesimally high if \(\delta \to 0\). If a material particle goes through this region in \(t_0\) time, then \(\delta = v_n \cdot t_0\). From this calculation results that the DNM and equivalent strain are the following

\[
\mu = \int_0^{t_0} \frac{\Delta v_1 dt}{2 \delta} = \int_0^{t_0} \frac{\Delta v_1}{2v_n t_0} dt = \int_0^{t_0} \frac{\Delta v_1}{2v_n} t_0 = \frac{\Delta v_1}{2v_n} \quad (9)
\]

5. Characteristic deformation of metal forming processes

5.1. Simple shear

The velocity field for simple shear is given in a Cartesian coordinate system using equation (10) with a parameter \(k\). The DNM is increasing with the deformation, according to the equation (11). Simple shear (figure 2) is the way of deformation - strongly non-monotonic - in the idealized process of ECAP and HPT.

\[
\mathbf{v}(x_1,x_2,x_3) = \begin{bmatrix} k \cdot x_2 & 0 & 0 \end{bmatrix} \quad (10)
\]

\[
\mu = \frac{\sqrt{3}}{2\varphi} \quad (11)
\]

5.2. Forward rod extrusion

The material flow in a constricted channel (figure 4) can be described in a spherical coordinate system using equation (14). Five points of the cross section at \(\theta/\alpha=0;0,3;0,5;0,8;1\) were investigated during the extrusion from the initial to the final diameter in four steps (\(\phi_{10}\) to \(\phi_{08}\), \(\phi_{08}\) to \(\phi_{06}\), \(\phi_{06}\) to \(\phi_{05}\), \(\phi_{05}\) to \(\phi_{04}\)).

\[
\mathbf{v}_{(r,\theta,z)} = \begin{bmatrix} v_0^2 \cos \theta \cdot x_1^2 & 0 & 0 \end{bmatrix} \quad (12)
\]

5.3. ECAP

The flow model of [2] was used (figure 3). According to this model, material particles in the die's deformation zone are moving in a circular way between two discontinuity surfaces. The velocity field is given in a cylindrical coordinate system according to equation (13). The DNM on the entry and exit surface is \(\mu = \sqrt{3}/2\varphi_{A_{en}} = 1/2 \tan \theta_0\). Between the surfaces the equivalent strain and DNM are \(\mu = \pi/4 \theta_0\) and \(\varphi_{channel} = (\pi/2-2\theta_0)/\sqrt{3}\). The relation between the equivalent strain and DNM is \(\mu = \sqrt{3}/2\varphi\), which shows that the deformation is simple shear-like non-monotonic.

\[
\mathbf{v}_{(r,\theta,z)} = \begin{bmatrix} 0 & v_0 \cdot \cos \theta & 0 \end{bmatrix} \quad (13)
\]

\[
\mathbf{v}_{(r,\theta,z)} = \begin{bmatrix} v_0^2 \cos \theta \cdot x_1^2 & 0 & 0 \end{bmatrix} \quad (14)
\]

5.4. Simultaneous torsion, tension and frictionless upsetting of a cylindrical specimen

The velocity of the deformation can be described in a cylindrical coordinate system as follows
\[ v_{r(\hat{r},z)} = \left[ \frac{\omega_0 T}{2h} \frac{ZV_0}{h} - \frac{ZV_0}{h} \right] \]  

where \( h = h_0 \) is the actual height of the specimen and \( \omega_0 \) the angular velocity of torsion. By uniaxial tension or frictionless upsetting (\( v_0 = \text{const} \), \( \omega_0 = 0 \)) \( \mu \) remains zero and the process is pure monotonic (the lode parameter is also constant). If no tension (or upsetting) occurs (\( v_0 = 0 \), \( \omega_0 = \text{const} \)) then the type of deformation is simple shear, the relationship described by equation (11) is valid. If both types of forming are present, then the DNM can change between simple shear type and pure monotonic type deformation (figure 5).

6. Results

In figure 5 the results of the DNM calculation are shown for the above presented processes. As it is shown in the diagram, the simple shear, ECAP, and the torsion differ most from a monotonic deformation. By forward extrusion, the DNM is not constant in the cross section: in the symmetry axis the deformation is monotonic, and with the distance from the centre the DNM is increasing, but remains far from ECAP or simple shear. By uniaxial tension and frictionless upsetting the value of the DNM remains zero according to the requirements.

7. Conclusion

Several authors made experiment and investigation about the grain refinement by the mentioned forming processes (ECAP[4][5], HPT[6], extrusion[7][8]). The experiments show that through the procedures like ECAP and HPT stronger grain refinement occurs than through the other forming processes. The results show that the DNM for the former processes is higher than for the latter. For processes which differ more from pure monotonic deformation, an UFG microstructure evolves sooner. The above mentioned method can be suitable for planning of production of UFG metals, enhancement of existing methods, or planning new technologies.

Reference

[1] G. A. Smirnov-Aljajev 1978 Сопротивление Материалов Пластическому Деформированию, Машиностроение (Leningrad)
[2] A.R. Eivani, A.K. Taheri, J. Mater. Process. Technol. 182 (2007) 555–563
[3] B. Avitzur 1968 Metal Forming (McGraw-Hill Book Comp)
[4] J.-Y. Chang, A. Shan Mater. Sci. Eng. A 347 (2003) 165-170
[5] U. Chakkingal, A. B. Suriadi, P.F. Thomson Scripta Mater. 39 (1998) 677-684
[6] C. Xu, Z. Horita, T. G. Langdon Acta Mater. 56 (2008) 5168–5176
[7] S. Kanekoa, K. Murakamib, T. Sakaic Mater. Sci. Eng. A 500 (2009) 8-15
[8] J. Swiostek, J. Goken, D. Letzig, K.U. Kainer Mater. Sci. Eng. A 424 (2006) 223–229