A combinatorial optimization solution for activity prioritizing problem

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Abstract. This paper discusses a particular situation in project management in which an analyst attempts to prioritize several independent activities to handle all of them one by one in such a way that there would be no precedence relationships over the activities. The novelty of this research is that the structure of prioritized activities is linear in arrangement which can be considered as a combinatorial optimization problem. The paper formulates a mathematical model and applies it to two real cases in the oil and gas industry. In addition, a row generation procedure is developed to solve large-scale problems and the computational results for the problem instances of size up to 300 activities are reported. The results demonstrate the applicability and efficiency of the proposed methodology.

1. Introduction

As Gerardi [1] states in his book, project managers generally list several explanations as to why they love project management: ‘they get to learn new things’, ‘work with new people’, ‘gain new skills’, and ‘are on the leading edge of innovation’. In line with this view, this paper concentrates on an interesting challenge in project planning named Activity Prioritizing Problem (APP), which is a branch of project scheduling problem. Project scheduling has attracted considerable attention because of its critical role in project resource management [2,3]. Consider a situation in which there are project activities with no precedence relationships over them. For instance, after scheduling a lot of project activities, a subset of them (with no interrelated dependencies) should be done in a limited period of time. In this situation, the problem is simply to assign priorities to such activities. It is clear that this challenge is not affected by duration of activities. Instead, several factors likely affect the priorities such as project owner view, program and project manager ideas, risk factors [4], etc. In this regard, the role of a project manager and project experts has been addressed as fundamental dimensions for the project success [5], which appears to be neglected in the literature [6,7]. On the other hand, Greek and Pullin [8] argued that many project managers did not focus on the criticality and urgency of project activities. However, a major concern in project management is the determination of a priority ranking for all activities that are candidates for assignment [9].

With the APP, there are many descriptive solutions and a few analytical models in the relevant literature. Among various descriptive guidelines (6-step process, Eisenhower matrix, Brian Tracy’s method,
ABCDE method, Bubble sort technique, Moscow method, Pareto principle, Zen habit, Rag rating system, Scrum prioritization, 1-3-9 method, and so on), we will have minor explanation on only two methods. According to the Zen habit method [10], as a really simple prioritization guideline, the manager writes down one to three most important activities each time to be completed in a limited period of time and gets on with completing them. The Eisenhower matrix, popularized by Covey [11], works by dividing separate activities into four quadrants based on urgency and importance: ‘urgent and important’, ‘important but not urgent’, ‘urgent but not important’, and ‘neither urgent nor important’.

Despite the importance of weighting and prioritizing project activities, analytical models have been rarely addressed in the related literature [7,12]. In addition, most of the analytical studies use Multiple Attribute Decision Making (MADM) techniques to get a rank or weight vector of activities. Chang and Ibbs [9] introduced an expert system that uses possibility theory and the generalized modus ponens logic inference rule to prioritize activities in building construction projects. Alencar et al. [13] employed ELECTRE III technique to prioritize activities in a real-world construction project. Jha and Misra [14] conducted a survey to identify major factors in classifying and ranking construction coordination activities. An analytical effort was made by Mota et al. [15] to present a model for supporting project managers to focus on the main activities of a project network using a MADM technique, taking into account several, often contradictory, points of view. The mentioned model aims to assign project activities into three classes of managerial practice. In an article by Baykasoglu et al. [12], a practical fuzzy rating and ranking technique was proposed to prioritize project activities with fuzzy attributes. They considered prioritization of activities instead of their classification. In addition, their technique was in some aspects similar to that of Mota et al. [15], with substantial differences. Vanhoucke [16] introduced a scheme to measure sensitivity to the activities and network topological information to monitor the project time performance. Mota and de Almeida [17] proposed an ELECTRE-based model to help project managers make decisions about the key activities of a project. Golpin [18] made a comparison between conventional MADM and fuzzy MADM to determine weights and priorities of activities using Analytical Hierarchy Process (AHP) and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) methods. Hadad et al. [19] prioritized the attributes of project activities using an MADM model. These attributes included activities’ expected time, standard deviation, expected cost, standard cost deviation, and the time and cost obtained through simulation. Hadad [20] proposed a technique to prioritize project activities using AHP and Data Envelopment Analysis (DEA) as research tools to select proper influencing factors. Kalayathankal et al. [21] applied a fuzzy technique to prioritize activities in a software development project with analogous activities in the electrical substation construction studied by Mota et al. [15]. They achieved better project output in terms of finance, personnel, equipment, space, time, etc. Berjis et al. [7] proposed an approach to determine the weights of project activities using DEA. They identified the parameters affecting the importance of activities through a review of the related literature and experts’ opinions. The parameters included activity duration, activity cost, activity importance, activity difficulty, safety, communication rate, intellectual effort, physical effort, unfavourable work conditions, and work-related hazards.

In the current article, a novel optimization solution is proposed to deal with a particular situation of the APP. The idea in the proposed approach is completely different from previous works. The organization of the paper is as follows. Section 2 points out the problem characterization and related definitions. Section 3 develops a mathematical model to solve the problem. Section 4 includes two real cases studied from oil and gas industry. Section 5 discusses how to deal with large-scale problems followed by experimental analysis in Section 6. Finally, Section 7 summarizes the results achieved and provides future efforts and possibilities.

2. Characterization of the problem

Herein, a special situation of the APP is introduced and it is an interesting issue that the authors encountered in real projects. Indeed, real cases (described in Section 4) were the root of the idea of the problem discussed in this paper. As a result, the need for the proposed model (described in Section 3) originates from real-world projects and it, thus, can be applied to actual challenges.

Suppose a situation in which there are a variety of project activities and the aim is to prioritize activities to handle all of them one after another. Indeed, all activities need a unique resource available only for one activity at any point of time. Taking this fact into account, we seek to sequentially perform activities from the most important to the least important. Thus, an arrangement structure of activities needs to be formed. That special resource may be a unique montage platform, a unique consultant, a unique working front, etc. The solution is influenced by some different “adjacency factors”. An adjacency factor indicates a reason according to which some activities need to be assigned locations in the arrangement structure close.
to each other. Obviously, any linear permutation including all activities is a valid solution to the problem. Thus, the idea of the current paper arises from this point of view and it is a linear arrangement in the prioritizing challenge explained above; therefore, we have a combinatorial optimization problem in which activities should be arranged from the most important to the least important.

Let us define the problem in a general form. The problem has two groups of elements: “activities” and “adjacency factors” (henceforth referred to as factors). There are $m$ activities $T_1, T_2, \ldots, T_m$ to be prioritized and $n$ factors having positive weights $\gamma_1, \gamma_2, \ldots, \gamma_n$. In view of an assumed factor, the relevant activities should be placed one after another in the overall arrangement structure. The activities respecting the $j$th factor are shown by subset $f_j \subseteq \{T_1, \ldots, T_m\}$. An assumed factor may be proposed by any of $\alpha$ experts ($\alpha = 1, 2, \ldots, \alpha$). In addition, each expert may suggest various factors. The factors are actually derived from similarity of the complexity of activities, relationships among activities, equality in urgency of activities, etc. The weight of a given factor is calculated via Eq. (1), which is a weighted geometric mean stressed by many authors to aggregate the individual weights [22–25]. In this equation, $w_{jk}$ is the weight of the $j$th factor preferred by the $k$th expert. This individual weight is a number between 1 and 10. If an expert has no idea about a factor, the respected items should be dropped out from Eq. (1). The power $u_j$, a number between 0 and 1, indicates the importance of the $j$th expert’s comments. This power depends on the expert’s skills, experiences, abilities, or his/her area of educations:

$$\gamma_j = \left( \prod_{k=1}^{\alpha} w_{jk}^{u_j} \right)^{1/\sum_{k=1}^{\alpha} u_j}, \quad j = 1, \ldots, m. \tag{1}$$

In view of the $j$th factor, the best case is comprised of a contiguous chain of all the activities in $f_j$. In this case, complete weight $\gamma_j$ will be added to the objective value of the mathematical model; otherwise, a proportion of $\gamma_j$ would be met. By taking this into consideration, the problem is to arrange activities so as to maximize the sum of proportions of factors’ weights. The value of each proportion depends on the adjacency of relevant activities in the overall arrangement structure.

Truly, the introduced challenge falls into the combinatorial optimization problem. Although a wide range of models and algorithms exist to deal with famous graph-theoretic and combinatorial optimization problems, the introduced issue is not like any well-known problems. In particular, it is completely different from cell formation problem [26,27], job-shop scheduling problem [28], project scheduling problem [2], single-row facility layout problem [29], linear ordering problem [30], minimum linear arrangement problem [31,32], adjacency problem [33], MADM problem [22,34], and linear assignment problem [35].

3. Modeling the problem

First, let us define the following elements:

- $T_i$ refers to the $i$th activity ($i = 1, \ldots, m$);
- $f_j$ stands for the set of activities respecting the $j$th factor ($j = 1, \ldots, n$);
- $\gamma_j$ indicates the weight of the $j$th factor ($j = 1, \ldots, n$);
- $\theta_{pq}$ ($p < q$) shows that a binary variable equals one if activity $T_p$ is placed exactly close to activity $T_q$;
- $s()$ shows size of a set.

Now, the formulation of the problem is written as in Model (2)-(5):

$$\max \sum_{j=1}^{n} \gamma_j \tag{2}$$

s.t.: \begin{align*}
\sum_{i=1}^{m} \theta_{ip} & + \sum_{p=1}^{m} \theta_{pq} \leq 2, \quad i = 1, \ldots, m, \\
\sum_{T_i, T_j \in B} \theta_{pq} & \leq s(B) - 1, \quad B \{T_1, \ldots, T_m\}, \quad 2 < s(B), \quad \forall \theta_{pq} \in (0, 1). \tag{3}
\end{align*}

To ensure a better understanding, an explicit formulation for a small problem with the corresponding LINGO code was provided in Appendix A.

The rule to formulate Function (2) is that “if two activities of $f_j$ are placed right beside each other in the overall string of activities, then “1” is added to objective function coefficient of $\gamma_j$”. Let us consider some cases. For instance, suppose that there are 4 activities and 3 factors and we want to determine the objective function coefficients of $\gamma_1$, $\gamma_2$, and $\gamma_3$ for the given string $T_4 T_1 T_2 T_3$. The set of activities respecting the factors includes $f_1 = \{T_1, T_3, T_4\}$, $f_2 = \{T_2, T_4\}$, and $f_3 = \{T_1, T_2, T_4\}$. For a given factor $f_j$, in the best case in which each activity $f_j$ has at least one adjacent activity from $f_j$, $\sum_{T_i, T_j \in f_j} \theta_{pq} = 2$ and coefficient of $\gamma_j$ equals 1. In the worst case, none of activities of $f_j$ is placed right next to another activity of $f_j$; consequently, $\sum_{T_i, T_j \in f_j} \theta_{pq} = 0$. Thus, the objective function
we need to determine the direction of the arranged structure of activities from the most important to the least important. This objective can be reached by comparing two tail-end activities of the structure using the expert’s opinions.

$$\sum_{p=1}^{m} \sum_{q=1}^{m} \theta_{pq} = m - 1. \quad (6)$$

### 4. Cases studied

#### 4.1. Transformer oil commercialization

The focus of this case concerns a project of the know-how commercialization of a technology known in the industry as transformer oil. This technology is a highly-refined mineral oil that is stable at high temperatures and has excellent electrical insulating properties. It is used in oil-filled transformers, some high-voltage capacitors, fluorescent lamp ballasts, and some types of high-voltage switches and circuit breakers. Transformer oil is used in oil-immersed transformers, high-voltage capacitors, tap changers, fluorescent lamp ballasts, and some switches and circuit breakers.

One of the project modules requires prioritizing seven major independent activities, as given in Table 1. Two experts were involved in this case. The importance of the experts was identified as \( u_1 = 1 \) and \( u_2 = 0.5 \). The experts recommended five factors, \#1 to \#5. The first expert proposed the factors’ weights \( 8, 8, 5, 4 \), and 7, respectively. The latter only conferred Factors \#2 and \#5 with \( w_{22} = 3.5 \) and \( w_{52} = 9 \), respectively. The final weights of the factors were obtained as \( \gamma_1 = (8^1)^{1/1} = 8 \), \( \gamma_2 = (8^1 \times 3.5^{0.5})^{1/1.5} = 6.07 \), \( \gamma_3 = (5^1)^{1/1} = 5 \), \( \gamma_4 = (4^1)^{1/1} = 4 \), and \( \gamma_5 = (7^1 \times 9^{0.5})^{1/1.5} = 7.61 \). Model (2)-(5) were formulated with 21 variables and 105 constraints. This model was solved by means of the LINGO release 8.0 on a personal computer. The optimal arrangement of

| Activities                          | Factors |
|-------------------------------------|---------|
|                                     | 1 2 3 4 5 |
| **T1** Specifying electrically insulate requirements. | * * - - - |
| **T2** Specifying suppressed corona and arcing equipment. | - * * * - |
| **T3** Studying coolant features | - * * - * |
| **T4** Determining resist oxidation items | * * - - - |
| **T5** Determining deposit formation items | * - - - - |
| **T6** Studying the overall system performance | - * - * * |
| **T7** Creating a manual for installation of the system | * - - - - |

![Figure 1. Infeasible patterns containing (a) unacceptable tree, (b) cycle, and (c) non-conneced tree.](image-url)
the activities was obtained as $T_1$ $T_5$ $T_4$ $T_1$ $T_2$ $T_3$ $T_6$ corresponding to the objective value 26.68. Therefore, Factor #4 is the only one whose activities include a gap. The experts determined $T_6$ as the most important activity.

### 4.2. Planning at NIOC
National Iranian Oil Company (NIOC), according to its mission on oil production from the country’s reservoirs, compiles five-year productions and investment plans for all oil and gas fields in the country. This task is the responsibility of the project planning management deputy of the NIOC. In this regard, the deputy receives final reports of several studies made by other deputies (such as deputy for hydrocarbon reservoirs, deputy for economic feasibility and investment, deputy for oil and gas projects, and so on) and they are analyzed to make investment plans.

The current case reports a real project at the NIOC in which the output of eight studies is received for analysis by the relevant team. Therefore, eight activities are defined as follows:

- $T_1$: Analyzing reservoir study;
- $T_2$: Analyzing surface and downstream study;
- $T_3$: Analyzing the study of target markets;
- $T_4$: Analyzing the study of energy capital financing;
- $T_5$: Analyzing the study of the type of development and maintenance contracts;
- $T_6$: Analyzing oil transfer and transportation study;
- $T_7$: Analyzing similar projects;
- $T_8$: Analyzing the production of crude oil.

Due to the limited human resources in the team (i.e., technical and planning experts), the activities must be considered sequentially. Moreover, based on the four factors defined by the planning management of the NIOC, the following groups of activities were determined to be done continuously:

- Activities $T_1$ and $T_2$ (due to engineering and reservoir factor, with weight 100);
- Activities $T_3$, $T_6$, and $T_8$ (due to international market factor, with weight 40);
- Activities $T_4$ and $T_5$ (due to economic feasibility factor, with weight 70);
- Activities $T_4$ and $T_7$ (due to technology factor, with weight 40).

Using the proposed model, the optimal solution was obtained including three non-connected trees $T_5$ $T_4$ $T_7$, $T_1$ $T_2$ and $T_6$ $T_3$ $T_8$. Finally, according to the expert’s opinions, a linear arrangement of the activities was determined as $T_1$ $T_2$ $T_7$ $T_4$ $T_5$ $T_6$ $T_3$ $T_8$ with the best objective value 250.

### 5. Dealing with large scale problems
Model (2)-(5) include $m^2/2 - m/2$ binary variables and $2^m - m^2/2 + m/2$ constraints ($m > 2$). Hence, the constraints exponentially grow in number by increasing $m$. For example, there are 22 constraints for $m = 5$, 979 constraints for $m = 10$, and 32063 constraints for $m = 15$. The problem was modeled as binary programming which is a subset of integer programming. Since integer programming is NP-hard [36,37], the problem must be hard. Due to the involved NP-hardness of the problem, heuristic and meta-heuristic approaches may strike us. However, the current paper aims at looking for a method that provides a global optimal solution. To solve large-scale models, this paper suggests a relaxation that removes Constraint (4) from Model (2)-(5), resulting in a relaxed model with only $m$ constraints. Needless to say, the solution pattern of the relaxed model may include cycles and non-connected sub-graphs. Definitively, the output pattern may be one of the two cases: (a) the solution pattern, i.e., a linear layout of all activities, or (b) a non-connected graph which includes at least one cycle. Accordingly, at first, we should identify any cycles, if these exist. With this purpose in mind, a pseudo code is designed as in Figure 2. This code, in each round, arbitrarily chooses an initial vertex named “root” and then, tries to search for the next vertex connected to the initial vertex. The algorithm continues to search for the next vertex connected to the previous one found until getting the root or failing to find the next connected vertex. The former shows a cycle, while the latter indicates a chain.

Obviously, if there are no cycles (i.e., $t = 0$), the current pattern is the optimal solution; otherwise, there will be one or more cycles. Therefore, in order to prevent generating cycles, by using a row generation approach, new Constraint (7) will be added to Model (2)-(5), where $V_i$ is the set of vertices (i.e., activities) for the $r$th cycle. This approach is often called branch-and-cut algorithm. Moreover, it should be noted that the row generation approach is a dual form of the column generation approach [38]:

$$\sum_{T_{pq} \in V_i, \theta_{pq} < t} \theta_{pq} \leq s(V_i) - 1,$$

$$\forall r = 1, 2, \ldots, t. \quad (7)$$

To sum up, let us review the proposed procedure. At the first iteration, Model (2)-(5) are solved without Constraint (4) (i.e., $t = 0$). Now, subject to the presence of any cycle in the generated pattern, Constraint (7) would be added to the model. Admittedly, this version of the model includes Constraint (7) instead of Constraint (4). This model should be solved and rechecked in the presence of cycles again. This iterative process continues until no cycle can be found.
6. Experimental evaluation

As a preliminary assumption, there are three parameters affecting running time: the number of activities \(m\), the number of factors \(n\), and the size of factors \(s\). Consequently, a typical problem is addressed by \((m,n,s)\) i.e., a problem with \(m\) activities and \(n\) factors by size \(s\). Given that the current study is the first research to consider the introduced problem, there is naturally no benchmark instance in the literature. Hence, the instance problems (i.e., data set) were randomly generated by a macro in LINGO software. LINGO software version 8.0 was employed to solve the mathematical models along with an Excel macro to identify cycles. The procedure was run on an ASUS Core (i7-7500) 2.7 GHz with 12.00 GB memory.

6.1. Sensitivity analysis

For the purpose of sensitivity analysis, a study is conducted on several instance problems with the number of activities ranging from \(m = 25\) to \(m = 300\), the number of factors ranging from \(n = 100\) to \(n = 2500\), and the sizes of factors ranging from \(s = 3\) to \(s = 75\). Figure 3(a)-(f) portray running time plots resulting from solving instance problems. All the reported times are average of ten times that the algorithm runs on each level of the parameters.

Figure 3(a), (b), and (c) show the running time variation with \(s\), each curve under a fixed \(m\) and \(n\). These figures indicate that increasing \(m\) often results in excessive increase in the running time. Interestingly, we found that at \(n = 500\) and \(n = 1000\), a peak occurs for \(s\) around 10. Figure 3(d) and (e) present how the running time varies as \(n\) is grown. These figures are made with fixed \(m\) (i.e., \(m = 100\) and \(m = 150\)) and three levels \(s = 10\), \(s = 20\), and \(s = 30\). The curves in Figure 3(d) show the slowly incremental effect of \(n\) on the running time of instance problems, even up to 2500 factors. On the contrary, Figure 3(e) does not follow the same trends as curves in Figure 3(d). The curves have an increasing trend up to around \(n = 1550\) followed by an abrupt drop, such that there is a rather uniform trend after \(n = 1600\). Figure 3(b) confirms this phenomenon, in which the curve of \(n = 2000\) is quite below the curve of \(n = 1500\). Figure 3(f) shows the running time variation with \(n\). This figure is drawn based on instance problems by \(n = 500\) and four levels of \(s\). The results of analysis illustrate that the curves as in Figure 3(f) are the only ones with exponential growth. Almost all the curves show an exponential growth in the running time as \(m\) increases. Except curve with \(s = 3\), on the other cases there were large scale instance problems (about \(m = 250\) to \(m = 300\) activities) with a huge running time, even more than 24 hours. Note that these long running times were achieved only at the first iteration of the solving procedure.

Regarding unusual variations shown in Figure 3(c) and (e), we could not find a definite interpretation except that they may result from intrinsic connections among factors. It should be noted that all the solved instance problems obtained an optimal solution through 3 to 12 iterations. Further, as shown in the figures, running times range between 1 and 3500 seconds. In sum, the running times are very reactive (i.e., alert) to \(m\) and slightly sensitive to \(s\) and \(n\).

6.2. Efficiency analysis

In order to demonstrate the efficiency of the developed
row generation algorithm, the employed approach attempts to (a) solve the test problems using the un-relaxed Model (2)–(5) as much as possible depending on running time or computer memory limitations and then, (b) to solve the same test problems using the row generation algorithm. This will indicate the effectiveness of the row generation algorithm over the un-relaxed Model (2)–(5). In order to determine the values of such parameters as \( m \), \( n \), and \( s \) to generate the instance problems, the major result of the previous section, i.e., sensitivity analysis, was considered. In this respect, the number of activities \( (m) \) is considered as the main parameter that differs from other parameters at two levels of low \((n = m \text{ and } s \leq 0.25 \text{ m})\) and high \((n = 3 \text{ m and } s \leq 0.75 \text{ m})\). Table 2 presents the characteristics and results of 10 test problems with the number of activities ranging from \( m = 5 \) to \( m = 19 \) for the un-relaxed Model (2)–(5), compared with the relaxed version. It was not possible to bring in the instances with more activities due to computer memory limitations (after verifying that the application of many activities did not affect the obtained results). In comparison, the two last columns in Table 2 were considered. The former, i.e., reduction of constraints,
Table 2. The characteristics and results of 10 instances for both un-relaxed and relaxed forms.

| Problem | (m,n,s) | Variables | Constraints | Optimal objective function value | Running time (seconds) | Problem | Constraints | Added constraints for loops | Final objective function value | Running time (seconds) | Comparison |
|---------|---------|-----------|-------------|---------------------------------|------------------------|---------|-------------|----------------------------|-----------------------------|------------------------|------------|
| P1      | (5,5,3) | 10        | 22          | 237.5                           | 0.06                   | P1      | 10          | 5 1                                      | 237.5                      | 0.05                   | 72.727% 16.667% |
| P2      | (5,15,4)| 10        | 22          | 692.66                          | 0.07                   | P2      | 10          | 5 3                                      | 692.66                      | 0.05                   | 63.636% 28.571% |
| P3      | (10,10,3)| 45        | 979         | 355                             | 0.1                    | P3      | 45          | 10 0                                     | 355                         | 0.05                   | 98.979% 50.000% |
| P4      | (10,30,7)| 45        | 979         | 1319.5                          | 0.1                    | P4      | 45          | 10 1                                     | 1319.5                      | 0.08                   | 98.876% 20.000% |
| P5      | (15,15,4)| 105       | 32663       | 536.66                          | 3.32                   | P5      | 105         | 15 4                                     | 536.66                      | 0.07                   | 99.942% 97.802% |
| P6      | (15,45,11)| 105      | 32663       | 1949.5                          | 6.04                   | P6      | 105         | 15 3                                     | 1949.5                      | 0.07                   | 99.945% 98.841% |
| P7      | (17,17,4)| 136       | 130936      | 467.33                          | 66.13                  | P7      | 136         | 17 9                                     | 467.33                      | 0.1                    | 99.980% 99.849% |
| P8      | (17,51,13)| 136     | 130936      | 2290.583                        | 23.9                   | P8      | 136         | 17 4                                     | 2290.583                    | 0.09                   | 99.984% 99.623% |
| P9      | (19,19,5)| 171       | 524117      | 627                             | 121.3                  | P9      | 171         | 19 2                                     | 627                         | 0.08                   | 99.996% 99.934% |
| P10     | (19,57,14)| 171     | 524117      | 2259                            | 603                    | P10     | 171         | 19 4                                     | 2259                        | 0.12                   | 99.996% 99.983% |

is defined as one minus the ratio of the total number of constraints in the un-relaxed Model (2)-(5) to that in the relaxed model. The latter, i.e., reduction of running time, is similar to the former, except that it concerns the running time instead of the total number of constraints. Unquestionably, the results confirm the efficiency of the proposed algorithm.

6.3. Running time analysis

This section answers this question: “What size of the problems can be solved using the proposed branch-and-cut algorithm within a logical time?” Among many solved problems, the results for the average running time of 45 typical tested problems are shown in Table 3 and sorted in terms of running time. Problems P43 to P45 are typical instances with a very long running time. It can be concluded that the proposed procedure is undeniably essential to get the optimal solution to the problems including even up to 250, or some more activities, in 90 minutes at most. However, when the number of activities is 300 or more, the running time may unexpectedly jump to even more than 24 hours.

7. Concluding remarks

This study defined a particular problem of prioritizing activities, named Activity Prioritizing Problem (APP) with the aim of handling activities one after another by a serial form. Later, the notion of making a solution to the problem via a mathematical model was explained. The results of the state-of-the-art optimization and decision analysis illustrated that the proposed idea has not been studied before. By taking this fact into consideration, the first contribution of the research is the introduction of the idea of prioritizing activities as an optimization problem. In addition, the challenge of large-scale problems was discussed; next, a branch-and-cut procedure to deal with such situations was developed. Thereby, another contribution of the research is demonstrating how a row generation
Table 3. The running time (in seconds) of the selected instances.

| Problem | \(m\) | \((m, n, s)\) | Running time | Average |
|---------|-------|---------------|--------------|---------|
| P1      |       | (150,100,30)  | 98           |         |
| P2      |       | (150,500,30)  | 126          |         |
| P3      |       | (150,500,10)  | 567          |         |
| P4      |       | (150,1800,10) | 651          |         |
| P5      |       | (150,1900,10) | 798          |         |
| P6      |       | (150,200,20)  | 806          |         |
| P7      | 150   | (150,2500,10) | 917          | 946.61  |
| P8      |       | (150,2000,20) | 1064         |         |
| P9      |       | (150,2200,10) | 1099         |         |
| P10     |       | (150,1500,30) | 1134         |         |
| P11     |       | (150,2100,10) | 1295         |         |
| P12     |       | (150,2500,10) | 1526         |         |
| P13     |       | (150,2000,30) | 2135         |         |
| P14     |       | (200,500,10)  | 105          |         |
| P15     |       | (200,500,3)   | 112          |         |
| P16     |       | (200,500,20)  | 280          |         |
| P17     | 200   | (200,500,30)  | 294          | 443     |
| P18     |       | (200,500,30)  | 427          |         |
| P19     |       | (200,500,20)  | 623          |         |
| P20     |       | (200,500,10)  | 1260         |         |
| P21     |       | (225,1500,5)  | 91           |         |
| P22     |       | (225,500,5)   | 189          |         |
| P23     |       | (225,2000,5)  | 217          |         |
| P24     |       | (225,1000,10) | 266          |         |
| P25     |       | (225,2000,5)  | 322          |         |
| P26     |       | (225,500,40)  | 378          |         |
| P27     | 225   | (225,1000,20) | 476          | 17.36   |
| P28     |       | (225,1000,40) | 910          |         |
| P29     |       | (225,1500,20) | 920          |         |
| P30     |       | (225,2000,20) | 1715         |         |
| P31     |       | (225,500,5)   | 2065         |         |
| P32     |       | (225,2000,30) | 2163         |         |
| P33     |       | (225,1000,10) | 6636         |         |
| P34     |       | (225,1000,10) | 7777         |         |
| P35     |       | (250,500,3)   | 105          |         |
| P36     |       | (250,500,20)  | 252          |         |
| P37     | 250   | (250,500,10)  | 693          | 1300.83 |
| P38     |       | (250,500,20)  | 1267         |         |
| P39     |       | (250,500,10)  | 2492         |         |
| P40     |       | (250,500,20)  | 2996         |         |
| P41     |       | (300,500,3)   | 413          |         |
| P42     |       | (300,500,30)  | 651          |         |
| P43     | 300   | (300,500,10)  | More than 24 hours |         |
| P44     |       | (300,500,20)  | More than 24 hours |         |
| P45     |       | (300,500,30)  | More than 24 hours |         |
approach could be used to get the optimal solution of the APP. The evaluations and discussions presented the efficiency and usability of the methodology for large-scale problems, even for instances up to 250 activities. In conclusion, the proposed methodology has a promising future and enjoys practical applications in project decision-making challenges in a productive manner. Further research can be undertaken in (I) Applying the proposed method to complex and large-size real-world cases and (II) Developing heuristic algorithms to solve the problem.

References

1. Gerardi, B., No-Drama Project Management: Avoiding Predicative Problems for Project Success, 1st Edn., pp. 2–3, Apress, N.Y., USA (2011).
2. Birjandi, A., Mousavi, S.M., and Vahdani, B. “Designing a resource-constrained project scheduling model considering multiple routes for flexible project activities: Meta-heuristic algorithms”, *Scientia Iranica, E*, 27(5), pp. 2572–2591 (2020).
3. Namazian, A., Haji Yakhchali, S., and Rabbani, M. “Integrated bi-objective project selection and scheduling using Bayesian networks: A risk-based approach”, *Scientia Iranica, E*, 26(6), pp. 3695–3711 (2019).
4. Dorfshian, Y., Mousavi, S.M., Vahdani, B., et al. “Determining project characteristics and critical path by a new approach based on modified N Wrath method and risk assessment under an interval type-2 fuzzy environment”, *Scientia Iranica, E*, 26(4), pp. 2579–2600 (2019).
5. Pheng, L.S. and Chuan, Q.T. “Environmental factors and work performance of project managers in the construction industry”, *Int. J. Project Management*, 24, pp. 24–37 (2006).
6. Turner, J.R. and Muller, R. “The project manager’s leadership style as a success factor on projects: a literature review”, *Project Management J.*, 36(1), pp. 49–61 (2005).
7. Berjis, N., Shirouyehzad, H., and Jouzdani, J. “A new approach to determine the weights of project activities using data envelopment analysis: a case of Moharakhe steel company”, *Int. J. Managing Projects in Business*, 13(6), pp. 1187–1217 (2020).
8. Greek, D. and Pullin, J. “Overrun, overspent, overlooked”, *Professional Engineering*, 12(3), pp. 27–8 (1999).
9. Chang, T.C. and Ihbs, C.W. “Priority ranking: A fuzzy expert system for priority decision making in building construction resource scheduling”, *Building and Environment*, 25, pp. 253–267 (1990).
10. Babauta, L., Zen To Done: The Ultimate Simple Productivity System, 1st Edn., LLC, Platbatts, UK (2012).
11. Covey, S.R., *7 Habits of Highly Effective People*, 25th Edn., Pocket Books, UK (1999).
12. Baykasoglu, A., Gocen, T., and Kaplanoglu, V. “A practical approach to prioritize project activities through fuzzy ranking”, *Cybernetics and Systems: An International Journal*, 42(3), pp. 165–179 (2011).
13. Alencar, L.H., Mota, C.M.M., and Almeida, A.T. “Activities prioritization in a construction project management based on multi-criteria methods”, *Third International Conference on Production Research, Americas’ Region, Fedecho da Industrias do Estado do Paraná*, Brazil, July 30–August 2 (2006).
14. Jia, K.N. and Misra, S. “Ranking and classification of construction coordination activities in Indian projects”, *Construction Management and Economics*, 25, pp. 409–421 (2007).
15. Mota, C.M.M., Almeida, A.T., and Alencar, L.H. “A multiple criteria decision model for assigning priorities to activities in project management”, *Int. J. Project Management*, 27, pp. 175–181 (2009).
16. Vanhoucke, M. “Using activity sensitivity and network topology information to monitor project time performance”, *Omega*, 38(5), pp. 339–370 (2010).
17. Mota, C.M.M. and de Almeida, A.T. “A multi-criteria decision model for assigning priority classes to activities in project management”, *Annals of Operations Research*, 199(1), pp. 361–372 (2012).
18. Golpira, H. “Making comparison between FMCMD and MCDM to define project activities weights”, *First International Scientific Conference on Project Management in the Baltic Countries, February 8–9*, Riga, University of Latvia, 35 (2012).
19. Hadad, Y., Keren, B., and Laslo, Z. “Applications of ranking indexes of project activities for project management tasks”, *Challenges of IT Project Portfolio Selection*, 1392, 1422 (2014).
20. Hadad, Y. “Multi-criteria methods for ranking project activities”, *Yugoslav J. Operations Research*, 26(2), pp. 201–219 (2016).
21. Kalayathankal, S.J., Varghese, J., and Abraham, J.T. “A modified fuzzy approach to prioritize project activities”, *Int. J. Engineering and Technology*, 7(2), pp. 158–162 (2018).
22. Hwang, C.L. and Yoon, K., *Multiple Attribute Decision Making: Methods and Applications: A State-of-the-Art Survey*, 1st Edn., Springer, Berlin (1981).
23. Zavadskas, E.K. and Podvezko, V. “Integrated determination of objective criteria weights in MCDM”, *Int. J. Information Technology and Decision Making*, 15(2), pp. 267–283 (2016).
24. Vinogradova, I., Podvezko, V., and Zavadskas, E.K. “The recalculation of the weights of criteria in MCDM methods using the Bayes approach”, *Symmetry*, 10(205), pp. 2–18 (2018).
25. Hatfli, M.A. “A model for measuring success of an organisational management system”, *Int. J. Advanced Operations Management*, 9(2), pp. 73–87 (2017).
26. Pinheiro, R.G.S., Martins, I.C., Protti, F., et al. “A matheuristic for the cell formation problem”, Optimization Letters, 12(2), pp. 335-346 (2018).

27. Rabali, M., Habibnejad-Ledari, H., Raifei, H., et al. “A bi-objective mathematical model for dynamic cell formation problem considering learning effect, human issues, and worker assignment”, Scientia Iranica. E., 23(5), pp. 2341-2354 (2016).

28. Leunis, M.E., Frazzon, E.M., Maklesado, M.U., et al. “Solving the Job-Shop Scheduling Problem in the Industry 4.0 Era”, Technologies, 6(107), pp. 2–27 (2018).

29. Palubeckis, G. “Single row facility layout using multi-start simulated annealing”, Computers and Industrial Engineering, 103, pp. 1-16 (2017).

30. Celero, J., Mendiburu, A., and Lozano, J.A. “The linear ordering problem revisited”, Eur. J. Operational Research, 241(3), pp. 686–696 (2015).

31. Horton, S.B., Gary Parker, R., and Borie, R.B. “On minimum cuts and the linear arrangement problem”, Discrete Applied Mathematics, 103, pp. 127–139 (2000).

32. Glover, F. and Rego, C. “New relationships for multi-neighbourhood search for the minimum linear arrangement problem”, J. Discrete Algorithms, 4647, pp. 16–24 (2017).

33. Seppanen, J. and Moore, J.M. “Facilities planning and graph theory”, Int. J. Management Science, 17, pp. 242–253 (1970).

34. Hatifi, M.A. “BRAW: block-wise rating the attribute weights in MADM”, Computers and Industrial Engineering, 156, 107274 (2021).

35. Hillier, F. and Lieberman, G., Introduction to Operations Research, 11th Edn., McGraw-Hill Education, NY, USA (2020).

36. Garey, M.R. and Johnson, D.S. “Computers and intractability: A guide to the theory of NP completeness”, WH Freeman, NY, USA (1979).

37. Page, D.R. “Approximation algorithms for problems in make-span minimization on unrelated parallel machines”, Ph.D. Thesis, University of Western Ontario (2019).

38. Hatifi, M.A. “Developing column generation approach to solve the rectangular two-dimensional single knapsack problem”, Scientia Iranica. E., 24(6), pp. 3287–3296 (2017).

Appendix A

In order to illustrate how Model (2)-(5) are written, this appendix provides a numerical example for a small problem. This problem includes five activities, $T_1$ to $T_5$, and six factors with relevant sets as

- $f_1 = \{T_3, T_4\}$,
- $f_2 = \{T_1, T_2\}$,
- $f_3 = \{T_2, T_3\}$,
- $f_4 = \{T_1, T_2, T_3\}$,
- $f_5 = \{T_1, T_3, T_4, T_5\}$, and
- $f_6 = \{T_1, T_2, T_4\}$.

The weights of factors are $\gamma_1 = 9$, $\gamma_2 = 7$, $\gamma_3 = 8$, $\gamma_4 = 2$, $\gamma_5 = 7$, and $\gamma_6 = 3$. The mathematical Model (2)-(5) for this problem are as follows:

$$\max 9(\theta_{34}) + 7(\theta_{13}) + 8(\theta_{15}) + 2(\theta_{12})$$

$$+ \frac{7}{3}(\theta_{13} + \theta_{14} + \theta_{15} + \theta_{34} + \theta_{35} + \theta_{45})$$

$$+ \frac{3}{2}(\theta_{12} + \theta_{14} + \theta_{24})$$

$$\theta_{12} + \theta_{13} + \theta_{14} + \theta_{15} \leq 2,$$

$$\theta_{12} + \theta_{23} + \theta_{24} + \theta_{25} \leq 2,$$

$$\theta_{13} + \theta_{23} + \theta_{34} + \theta_{35} \leq 2,$$

$$\theta_{14} + \theta_{24} + \theta_{34} + \theta_{45} \leq 2,$$

$$\theta_{15} + \theta_{25} + \theta_{35} + \theta_{45} \leq 2,$$

$$\theta_{12} + \theta_{13} + \theta_{23} \leq 2,$$

$$\theta_{12} + \theta_{14} + \theta_{24} \leq 2,$$

$$\theta_{12} + \theta_{15} + \theta_{25} \leq 2,$$

$$\theta_{13} + \theta_{14} + \theta_{34} \leq 2,$$

$$\theta_{13} + \theta_{15} + \theta_{35} \leq 2,$$

$$\theta_{14} + \theta_{15} + \theta_{45} \leq 2,$$

$$\theta_{23} + \theta_{24} + \theta_{34} \leq 2,$$

$$\theta_{23} + \theta_{25} + \theta_{35} \leq 2,$$

$$\theta_{24} + \theta_{35} + \theta_{45} \leq 2,$$

$$\theta_{34} + \theta_{35} + \theta_{45} \leq 2,$$

$$\theta_{12} + \theta_{13} + \theta_{14} + \theta_{23} + \theta_{24} + \theta_{34} \leq 3,$$

$$\theta_{12} + \theta_{13} + \theta_{15} + \theta_{23} + \theta_{25} + \theta_{35} \leq 3,$$

$$\theta_{12} + \theta_{14} + \theta_{15} + \theta_{24} + \theta_{25} + \theta_{35} \leq 3,$$

$$\theta_{13} + \theta_{14} + \theta_{15} + \theta_{34} + \theta_{35} + \theta_{45} \leq 3,$$

$$\theta_{23} + \theta_{24} + \theta_{25} + \theta_{34} + \theta_{35} + \theta_{45} \leq 3,$$

$$\theta_{12} + \theta_{13} + \theta_{14} + \theta_{15} + \theta_{23} + \theta_{24},$$

$$+ \theta_{25} + \theta_{34} + \theta_{35} + \theta_{45} \leq 4$$

$$\theta_{12}, \theta_{13}, \theta_{14}, \theta_{15}, \theta_{23}, \theta_{24}, \theta_{25}, \theta_{34}, \theta_{35}, \theta_{45} \in (0, 1).$$
In this example, the LINGO code and the solution are presented as in Figure A.1, leading to the optimal string as T_5 T_1 T_3 T_4 T_2. This corresponds to the objective value 32.5.

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