POLARIZATION OF TAU LEPTONS IN SEMILEPTONIC B DECAYS

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Abstract

Analytic formulae for the $\alpha_s$ order QCD corrections to the differential width of the semileptonic $b$ decay are given with the $\tau$ polarization taken into account. Thence the polarization of $\tau$ is expressed by its energy and the invariant mass of the $\tau + \bar{\nu}$ system. The non-perturbative corrections by Falk et al. are incorporated in the calculation.

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I. INTRODUCTION

The semileptonic decays of B mesons are now well described with the aid of the heavy quark effective theory (HQET) \( \) and the QCD perturbative corrections. On the other hand, the nonleptonic processes still suffer from lack of satisfactory theoretical description\(^\dagger\). With this in mind, the former can be successfully employed in the determination of the parameters of the Standard Model (SM). Indeed they have been used to this end, recently yielding the Cabibbo-Kobayashi-Maskawa matrix element \( |V_{cb}| \) \[8\]. Moments of lepton spectra can be used in the determination of \( \alpha_s \), \( m_b \) and \( m_c \) \[9–11\]. The particular processes involving the \( \tau \) lepton make this field yet more interesting as they are affected by the mass of the charged lepton, now comparable with the masses of the involved quarks \( m_b \) and \( m_c \).

The first order QCD correction to the differential decay width of the \( b \) quark has been found analytically \[12\] with the \( \tau \) polarization summed over. Now the polarization itself, which is no more suppressed by heavy quark masses, provides information on the SM parameters in its own right. It does not depend on the \( |V_{CKM}| \) elements, for instance, but can still yield the quark masses. This is all the more important due to the fact that the polarization turns out to be only weakly dependent on the coupling constant \( \alpha_s \). It should be borne in mind that the first order QCD correction to the decay width itself is important, amounting to as much as 20% of the Born approximation. Thus the results of this paper render the \( \tau \) polarization especially applicable for use in evaluation of the quark masses.

What we calculate here is the longitudinal polarization of the charged lepton. It may be added that the longitudinal polarization is transferred to the lepton system via the intermediating \( W^- \) boson whose longitudinal polarization in turn takes root in the Higgs mechanism, so that the process may shed light on physics beyond SM.

We give the formulae for the decay of \( b \) quark into the charmed quark, \( \tau \) lepton and \( \tau^- \)-antineutrino in terms of the charged lepton energy and the squared four-momentum of the intermediating \( W^- \) boson, or, equivalently, the invariant mass of the \( \tau + \bar{\nu} \) system. These formulae, combined with the ones for the unpolarized lepton case, readily give the polarization of the \( \tau \) lepton. This expression is then integrated to give \( \tau \) energy distribution. The correction to the polarization is shown together with the Born-approximated result. Then moments of energy distribution are evaluated for the polarized case.

The sectioning of the paper goes as following. Sec.2 is devoted to the kinematical

\(^\dagger\)The accuracy of HQET for inclusive processes is still under debate, as the assumption of quark–hadron duality in the final state might introduce 1/\( m_c \) corrections not seen in the operator product expansion. Phenomenological analyses suggest that these dangerous terms are absent or small for two–quark processes like hadronic decays of \( \tau \) leptons and semileptonic decays of heavy quarks, see e.g. \[\] and references therein.
variables. In Sec.3 the ideas behind the present calculation of the polarization are discussed. Then in Sec.4 QCD corrections are briefly described. The final analytic result is given in Sec.5 and in the following Sec.6 the moments of energy distribution are given.

II. KINEMATICS

A. Kinematical variables

In this section we define the kinematical variables used throughout the article as well as the constraints on those in both cases of a 3- and 4-body decay. The calculation is performed in the rest frame of the decaying $b$ quark. In order to include the first-order QCD corrections to the decay, one must take into account both the 3-body final state with a produced quark $c$, lepton $\tau$ and an antineutrino $\bar{\nu}_\tau$ and the 4-body state with an additional real gluon. The four-momenta of the particles are denoted as following: $Q$ for the $b$ quark, $q$ for the $c$ quark, $\tau$ for the charged lepton, $\nu$ for the corresponding antineutrino and $G$ for the real gluon. All the particles assumed to be on-shell, their squared four-momenta equal their masses:

$$Q^2 = m_b^2, \quad q^2 = m_c^2, \quad \tau^2 = m_\tau^2, \quad \nu^2 = G^2 = 0. \tag{1}$$

The four-vectors $P=q+G$ and $W=\tau + \nu$ characterize the quark-gluon system and the virtual intermediating $W$ boson, respectively. The employed variables are scaled in the units of the decaying quark mass $m_b$:

$$\rho = \frac{m_c^2}{m_b^2}, \quad \eta = \frac{m_\tau^2}{m_b^2}, \quad x = \frac{2E_\tau}{m_b}, \quad t = \frac{W^2}{m_b^2}, \quad z = \frac{P^2}{m_b^2}. \tag{2}$$

Henceforth we scale all quantities so that $m_b^2 = Q^2 = 1$. The charged lepton is described by the light-cone variables:

$$\tau_\pm = \frac{1}{2}(x \pm \sqrt{x^2 - 4\eta}). \tag{3}$$

Thus, the system of the $c$ quark and the real gluon is described by the following quantities:

$$P_0 = \frac{1}{2}(1 - t + z) \tag{4}$$
$$P_3 = \sqrt{P_0^2 - z} = \frac{1}{2}\sqrt{[1 + t^2 + z^2 - 2(t + z + tz)]^{\frac{1}{2}}}, \tag{5}$$
$$P_\pm(z) = P_0(z) \pm P_3(z), \tag{6}$$
$$\mathcal{Y}_p = \frac{1}{2} \ln \frac{P_+(z)}{P_-(z)} = \ln \frac{P_+(z)}{\sqrt{z}}. \tag{7}$$
where $P_0(z)$ and $P_3(z)$ are the energy and the length of the momentum vector of the system in the $b$ quark rest frame, $\mathcal{Y}_p(z)$ is the corresponding rapidity. Similarly for the virtual boson $W$:

$$W_0(z) = \frac{1}{2}(1 + t - z),$$  \hspace{1cm} (8)$$
$$W_3(z) = \sqrt{W_0^2 - t} = \frac{1}{2}(1 + t^2 + z^2 - 2(t + z + tz))^{1/2},$$  \hspace{1cm} (9)$$
$$W_\pm(z) = W_0(z) \pm W_3(z),$$  \hspace{1cm} (10)$$
$$\mathcal{Y}_w(z) = \frac{1}{2} \ln \frac{W_+(z)}{W_-(z)} = \ln \frac{W_+(z)}{\sqrt{t}}. $$  \hspace{1cm} (11)$$

Kinematically, the three body decay is a special case of the four body one, with the four-momentum of the gluon set to zero, thus resulting in simply replacing $z = \rho$. The following variables are then useful:

$$p_0 = P_0(\rho) = \frac{1}{2}(1 - t + \rho), \quad p_3 = P_3(\rho) = \sqrt{p_0^2 - \rho},$$  \hspace{1cm} (12)$$
$$p_\pm = P_\pm(\rho) = p_0 \pm p_3, \quad w_\pm = W_\pm(\rho) = 1 - p_\mp,$$  \hspace{1cm} (13)$$
$$Y_p = \mathcal{Y}_p(\rho) = \frac{1}{2} \ln \frac{p_+}{p_-}, \quad Y_w = \mathcal{Y}_w(\rho) = \frac{1}{2} \ln \frac{w_+}{w_-}. $$  \hspace{1cm} (14)$$

We also express the scalar products in terms of the variables used above, so in the units of the $b$ quark mass one gets:

$$Q \cdot P = \frac{1}{2}(1 + z - t), \quad \tau \cdot \nu = \frac{1}{2}(t - \eta),$$
$$Q \cdot \nu = \frac{1}{2}(1 - z - x + t), \quad \tau \cdot P = \frac{1}{2}(x - t - \eta),$$
$$Q \cdot \tau = \frac{1}{2}x, \quad \nu \cdot q = \frac{1}{2}(1 - x - z + \eta). $$  \hspace{1cm} (15)$$

**B. Kinematical boundaries**

The phase space is divided into two regions. The first of them, henceforth called A, is available for both three- and four-body decay, while the remaining part B corresponds to pure four-body decay. Region A is defined as following:

$$2\sqrt{\eta} \leq x \leq 1 + \eta - \rho = x_m, \hspace{1cm} (16)$$
$$t_1 = \tau_- (1 - \frac{\rho}{1 - \tau_-}) \leq t \leq \tau_+(1 - \frac{\rho}{1 - \tau_+}) = t_2. $$  \hspace{1cm} (17)$$

The additional region B of the phase space, where only 4-body decay is allowed, has the following boundaries:

$$2\sqrt{\eta} \leq x \leq x_m, \quad \eta \leq t \leq t_1. $$  \hspace{1cm} (18)$$
Conversely, if $x$ should vary at a fixed value of $t$, the boundaries read:

$$\eta \leq t \leq (1 - \sqrt{\rho})^2, \quad w_- + \frac{\eta}{w_-} \leq x \leq w_+ + \frac{\eta}{w_+},$$

(19)

for region A, and

$$\eta \leq t \leq \sqrt{\eta}(1 - \frac{\rho}{1 - \sqrt{\eta}}), \quad 2\sqrt{\eta} \leq x \leq w_- + \frac{\eta}{w_-}.$$

(20)

for region B. The upper limit of the mass squared of the $c$-quark–gluon system is in both regions given by

$$z_{\text{max}} = (1 - \tau_+)(1 - t/\tau_+),$$

(21)

whereas the lower limit depends on the region:

$$z_{\text{min}} = \begin{cases} \rho & \text{in Region A} \\ (1 - \tau_-)(1 - t/\tau_-) & \text{in Region B} \end{cases}$$

(22)

III. POLARIZATION AT TREE LEVEL

In order to calculate the longitudinal polarization we must find the differential decay width for a given polarized final state of the $\tau$ lepton. According to the definition

$$P = \frac{\Gamma^+ - \Gamma^-}{\Gamma^+ + \Gamma^-} = 1 - 2\frac{\Gamma^-}{\Gamma},$$

(23)

where $\Gamma = \Gamma^+ + \Gamma^-$. Once we know the width with the polarization summed over, we only need to find, for instance, the width for the negatively polarized $\tau$ lepton. Before going to discuss the QCD corrections to the longitudinal polarization of the $\tau$ lepton, we stop for a while to look at the tree level situation where the calculation is easy to follow. Once we choose the $b$ quark rest frame and decide to look for the longitudinal polarization, we can express the lepton polarization four-vector $s$ in terms of the four-momenta $Q$ and $\tau$ of the $b$ quark and the $\tau$ lepton, respectively:

$$s = A\tau + BQ$$

(24)

This is due to the fact that now only the temporal component of $Q$ does not vanish, whereas the spatial parts of $s$ and $\tau$ are parallel. The coefficients $A, B$ appearing in the formula above can be evaluated using the conditions defining the polarization four-vector $s$:

$$s^2 = -1$$

(25)

$$s \cdot \tau = 0.$$  

(26)
Upon this one arrives at the following expressions:

\[ A^\pm = \pm \frac{1}{\sqrt{\eta}} \frac{x}{\tau_+ - \tau_-}, \]  
\[ B^\pm = \pm \frac{2\sqrt{\eta}}{\tau_+ - \tau_-}. \]  

(27)  
(28)

where the superscripts at \( A, B \) denote the polarization of the lepton.

This observation combines with another one to make the whole calculation simpler. The total decay width \( d\Gamma_0 \) at the tree level for the unpolarized case reads:

\[ d\Gamma_0 = G_F^2 M_0^5 |V_{CKM}|^2 \mathcal{M}_{0,3}^{un} d\mathcal{R}_3(Q; q, \tau, \nu)/\pi^5 \]  

(29)

where the matrix element amounts to

\[ \mathcal{M}_{0,3}^{un}(\tau) = q \cdot \tau Q \cdot \nu. \]  

(30)

With this kind of linear dependence on the four-momentum \( \tau \), it is worth noting that the matrix element with the \( \tau \) polarization taken into account is,

\[ \mathcal{M}_{0,3}^{pol}(\tau) = \frac{1}{2} \mathcal{M}_{0,3}^{un}(K = \tau - ms) = \frac{1}{2} (q \cdot K)(Q \cdot \nu), \]  

(31)

where \( m \) stands for the lepton’s mass and we have introduced the four-vector \( K \)

\[ K = \tau - ms. \]  

(32)

Applying now the representation (24) of the polarization \( s \) we readily obtain the following useful formula for the matrix element with the lepton polarized:

\[ \mathcal{M}_{0,3}^{\pm}(\tau) = \frac{\tau_\pm}{\tau_+ - \tau_-} \mathcal{M}_{0,3}^{un}(\tau) \pm \frac{\eta}{\tau_+ - \tau_-} \mathcal{M}_{0,3}^{un}(Q). \]  

(33)

The first term on the right hand side of (33) can be calculated immediately once we know the result for the unpolarized case. Thus the problem reduces to performing this calculation again with the only difference amounting to replacing the four-momentum \( \tau \) of the lepton with that of the decaying quark, \( Q \). The Born-approximated distribution can be written explicitly as

\[ \frac{d\Gamma_{0, \pm}}{dx} = 12\Gamma_0 f_{0, \pm}(x), \]  

(34)

where

\[ \Gamma_0 = \frac{G_F^2 m_s^5}{192\pi^5} |V_{CKM}|^2. \]  

(35)

The Born level function \( f_{0}(x) \) reads, for the unpolarized case,

\[ f_{0}(x) = \frac{1}{6} \zeta^2 \tau_3 \left\{ \zeta [x^2 - 3x(1 + \eta) + 8\eta] + (3x - 6\eta)(2 - x) \right\}, \]  

(36)
while the polarized cases are obtained using the function $\Delta f_0$:

$$\Delta f_0(x) = \frac{1}{12} \tau_3^2 \zeta^2 \{ \zeta(3 - x - \eta) + 3(x - 2) \}$$  \hspace{1cm} (37)$$

in the following way:

$$f_0^\pm(x) = \frac{1}{2} f_0(x) \pm \Delta f_0(x).$$  \hspace{1cm} (38)$$

In the formulae above,

$$\tau_3 = \sqrt{x^2 - 4\eta}, \quad \zeta = 1 - \frac{\rho}{1 - x + \eta}.$$  \hspace{1cm} (39)$$

At the tree level, one can express the polarization integrated over the energy of the charged lepton as well:

$$P = 1 - 1/18 \left\{ -24\eta^2 + (x_m^3 - 8\eta^{3/2})(3 + \eta - 3\rho) + 
12\eta(x_m^2) - 3x_m^4/2 + 3(1 - \eta)^3(\rho - \rho^2/s^4) + 
3(-1 + \eta)(1 - \rho/s^2)[3(1 - \eta)^2 + \rho(3 + 5\eta)] + 3T_3 S/2 + 
3(x_m - 2\sqrt{s})(12\eta - 4\eta^2 + 12\eta\rho - 3\rho^2 + 3\eta\rho^2 + \rho^3) - 
12\eta\rho^3 \ln(s^2/\rho) - 18(2\eta^2 - \rho^2 - \eta^2\rho^2) \ln[2\sqrt{s}/(x_m + T_3)] + 
18(1 - \eta^2)\rho^2 \ln \{2s^2\sqrt{s}/[(1 - \eta)(1 - \eta - T_3) - \rho - \eta\rho]\} \right\} 
/ \left\{ T_3 S/12 + (2\eta^2 - \rho^2 - \eta^2\rho^2) \ln[(x_m + T_3)/(2\sqrt{s})] 
- (1 - \eta^2)\rho^2 \ln \{2\sqrt{s}\rho/[(1 - \eta)(1 - \eta + T_3) - \rho - \eta\rho]\} \right\},$$  \hspace{1cm} (40)$$

where

$$s = 1 - \sqrt{\eta}, \quad T_3 = \sqrt{x_m^2 - 4\eta},$$  \hspace{1cm} (41)$$

$$S = 1 - 7[(1 + \eta)(\eta + \rho^2) + \rho(1 + \eta^2)] + \eta^3 + \rho^3 + 12\eta\rho.$$  \hspace{1cm} (42)$$

Anticipating the subsequent discussion of the QCD corrections, let us already note that the specific linear dependence as featured in (30,31) goes back to the tensorial structure of the matrix element $\mathcal{M}_{0,3}$:

$$\mathcal{M}_{0,3}^{un}(\tau) = \mathcal{L}_{\mu\nu}^{un}\mathcal{H}^{\mu\nu},$$  \hspace{1cm} (43)$$

where $\mathcal{L}$ and $\mathcal{H}$ stand for the leptonic and hadronic tensors, respectively. It is of course the leptonic tensor $\mathcal{L}$ where the linearity derives from:

$$\mathcal{L}_{\mu\nu}^{\pm} = \mp \frac{\tau_+}{\tau_+ - \tau_-} \mathcal{L}_{\mu\nu}^{un}(\tau) \pm \frac{\eta}{\tau_+ - \tau_-} \mathcal{L}_{\mu\nu}^{un}(Q).$$  \hspace{1cm} (44)$$

It is not surprising then that the corrections to the hadronic tensor $\mathcal{H}$ will not affect this property.
IV. CALCULATION OF QCD CORRECTIONS

The QCD-corrected differential rate for the $b \to c + \tau^- + \bar{\nu}$ reads:

$$d\Gamma^\pm = d\Gamma^\pm_0 + d\Gamma^\pm_{1,3} + d\Gamma^\pm_{1,4},$$

where

$$d\Gamma^\pm_0 = G_F^2 m_b^5 |V_{CKM}|^2 M^\pm_{0,3} dR_3(Q; q, \tau, \nu) / \pi^5$$

is the Born approximation, while

$$d\Gamma^\pm_{1,3} = \frac{2}{3} \alpha_s G_F^2 m_b^5 |V_{CKM}|^2 M^\pm_{1,3} dR_3(Q; q, \tau, \nu) / \pi^6$$

comes from the interference between the virtual gluon and Born amplitudes. Then,

$$d\Gamma^\pm_{1,4} = \frac{2}{3} \alpha_s G_F^2 m_b^5 |V_{CKM}|^2 M^\pm_{1,4} dR_4(Q; q, G, \tau, \nu) / \pi^7$$

is due to the real gluon emission, $G$ denoting the gluon four-momentum. $V_{CKM}$ is the Cabibbo-Kobayashi-Maskawa matrix element corresponding to the $b$ to $c$ or $u$ quark weak transition. The Lorentz invariant $n$-body phase space is defined as

$$dR_n(P; p_1, ..., p_n) = \delta^{(4)}(P - \sum p_i) \prod_i \frac{d^3p_i}{2E_i}.$$ (49)

The superscript $\pm$ refers to the fact that now the polarization of the charged lepton is taken into account. In order to evaluate the appropriate rates it is convenient to take advantage of the decomposition (44) which led to the formula for the Born-approximated matrix element (33). As the QCD corrections influence only the hadronic tensor, the leptonic tensor and thus its linear dependence on $\tau$ is left intact. This allows us to represent all the involved matrix elements in an analogous way and, in fact, the whole decay width is of the very same form:

$$d\Gamma^\pm = \mp \tau_+ \tau_- - \tau_+ - \tau_- d\Gamma^{un}(\tau) + \eta d\Gamma^{un}(Q).$$ (50)

In Born approximation the contribution to the decay rate into the three-body final state is proportional to the expression

$$M_{0,3}^- = \frac{1}{4} F_0^-(x, t) = \frac{1}{2} q \cdot KQ \cdot \nu =$$

$$\frac{(1 - \rho - x + t)}{4(\tau_+ - \tau_-)} \left[ \tau_+ (x - t - \eta) - \eta(1 + \rho - t) \right]$$

(51)

The three-body phase space is parametrized by Dalitz variables:

$$dR_3(Q; q, \tau, \nu) = \frac{\pi^2}{4} dxd\tau dt.$$ (52)
The evaluation of the virtual gluon exchange matrix element yields:

\[ \mathcal{M}_{1,3}^{un}(\tau) = - [H_0 q \cdot \tau Q \cdot \nu + H_+ \rho Q \cdot \nu Q \cdot \tau + H_- q \cdot \nu q \cdot \tau + \frac{1}{2} \rho (H_+ + H_-) \nu \cdot \tau + \frac{1}{2} \rho (H_+ - H_- + H_L)] (\tau \cdot (Q - q - \nu)) (Q \cdot \nu) - \frac{1}{2} H_L (\tau \cdot (Q - q - \nu)) (q \cdot \nu), \]  

\[ \text{(53)} \]

where

\[ H_0 = 4(1 - \frac{Y_p}{p_0/p_3}) \ln \lambda_G + (2p_0/p_3) [Li_2 (1 - \frac{p_+w_-}{p_+ w_+}) - Li_2 (1 - \frac{w_-}{w_+}) - Y_p (Y_p + 1) + 2 (\ln \sqrt{\rho} + Y_p) (Y_w + Y_p)] + \left[ 2p_3 Y_p + (1 - \rho - 2t) \ln \sqrt{\rho} / t + 4, \right] \]

\[ H_\pm = \frac{1}{2} (1 \pm (1 - \rho) / t) Y_p / p_3 \pm \frac{1}{t} \ln \sqrt{\rho}, \]

\[ H_L = \frac{1}{t} (1 - \ln \sqrt{\rho}) + \frac{1 - \rho}{t^2} \ln \sqrt{\rho} + \frac{2}{t^2} Y_p p_3 + \frac{\rho Y_p}{t p_3}. \]

(54)

and then the polarized case requires

\[ \mathcal{M}_{1,3}^- = \frac{\tau_+}{\tau_+ - \tau_-} \mathcal{M}_{1,3}^{un}(\tau) - \frac{\eta}{\tau_+ - \tau_-} \mathcal{M}_{1,3}^{un}(Q). \]

\[ \text{(57)} \]

After renormalization, the virtual correction \( \mathcal{M}_{1,3}^\pm \) is ultraviolet convergent. However, the infrared divergences are left. They are regularized by a small mass of gluon denoted as \( \lambda_G \). In accordance with the Kinoshita-Lee-Nauenberg theorem, this divergence cancels out when the real emission is taken into account. The rate from real gluon emission is evaluated by integrating the expression

\[ \mathcal{M}_{1,4}^{un}(\tau) = \frac{B_1(\tau)}{(Q \cdot G)^2} - \frac{B_2(\tau)}{Q \cdot GP \cdot G} + \frac{B_3(\tau)}{(P \cdot G)^2}, \]

\[ \text{(58)} \]

where

\[ B_1(\tau) = q \cdot \tau [Q \cdot \nu (Q \cdot G - 1) + G \cdot \nu - Q \cdot \nu Q \cdot G], \]

\[ B_2(\tau) = q \cdot \tau [Q \cdot \nu - q \cdot \nu Q \cdot G + Q \cdot \nu (q \cdot G - Q \cdot G - 2q \cdot Q)] + Q \cdot \nu (Q \cdot \tau q \cdot G - G \cdot \tau q \cdot Q), \]

\[ B_3(\tau) = Q \cdot \nu (G \cdot \tau q \cdot G - \rho \tau \cdot P). \]

\[ \text{(59)} \]  

\[ \text{(60)} \]  

\[ \text{(61)} \]

Taking account of polarization amounts to substituting \( K \) for \( \tau \) in the coefficients \( B_{1,2,3} \):

\[ \mathcal{M}_{1,4}^- = \frac{\tau_+}{\tau_+ - \tau_-} \mathcal{M}_{1,4}^{un}(\tau) - \frac{\eta}{\tau_+ - \tau_-} \mathcal{M}_{1,4}^{un}(Q). \]

\[ \text{(62)} \]
The four-body phase space is decomposed as follows:

\[ dR_4(Q; q, G, \tau, \nu) = dzdR_3(Q; P, \tau, \nu)dR_2(P; q, G). \] (63)

After employing the Dalitz parametrization of the three-body phase space \( R_3 \) and integration, we arrive at an infrared-divergent expression. The method used in these calculations is the same as the one employed in the previous ones [13,14,12]. The infrared-divergent part is regularized by a small gluon mass \( \lambda_G \) which enters into the expressions as \( \ln(\lambda_G) \). When the three- and four-body contributions are added, the divergent terms cancel out, and then the limit \( \lambda_G \to 0 \) is performed. This procedure yields well-defined double-differential distributions of lepton spectra as described below.

V. ANALYTICAL RESULTS

The following formula gives the differential rate of the decay \( b \to \tau\bar{\nu}X, \) \( X \) standing for a \( c \) quark or a pair of \( c \) and a gluon, once the lepton is taken to be negatively polarized:

\[
\frac{d\Gamma^-}{dx\, dt} = \begin{cases} 
12\Gamma_0 \left[ F_0^-(x, t) - \frac{2\alpha_s}{3\pi} F_{1,A}^-(x, t) \right] & \text{for } (x, t) \text{ in A} \\
12\Gamma_0 \frac{2\alpha_s}{3\pi} F_{1,B}^-(x, t) & \text{for } (x, t) \text{ in B}
\end{cases}
\] (64)

\( F_1^- \) differs according to which region (A or B) it belongs. Region A is available for the 3- and 4-body decay, while Region B with a gluon only. The following formulae are given for the negative polarization of the lepton, that is, we take

\[
A^- = -\frac{1}{\sqrt{\eta}} \frac{x}{\tau_+ - \tau_-},
\] (65)

\[
B^- = \frac{2\sqrt{\eta}}{\tau_+ - \tau_-}.
\] (66)

The factor \( \Gamma_0 \) is defined in Eq.(35), while

\[
F_0^-(x, t) = \frac{(1 - \rho - x + t)}{\tau_+ - \tau_-} \left[ \tau_+(x - t - \eta) - \eta(1 + \rho - t) \right]
\] (67)

and

\[
F_{1,A}^-(x, t) = F_0^- \Phi_0 + \sum_{n=1}^{5} D_n^A \Phi_n + D_6^A,
\] (68)

\[
F_{1,B}^-(x, t) = F_0^- \Psi_0 + \sum_{n=1}^{5} D_n^B \Psi_n + D_6^B.
\] (69)
The factor 12 in the formula (64) is introduced to meet the widely used \[\text{[10,15,11]}\] convention for $F_0(x)$ and $\Gamma_0$. The symbols present in (68) are defined as follows:

$$
\Phi_0 = \frac{2p_0}{p_3} [Li_2(1 - \frac{1 - \tau^+}{p_+}) + Li_2(1 - \frac{1 - t/\tau^+}{p_+}) - Li_2(1 - \frac{1 - \tau^+}{p_-}) - Li_2(1 - \frac{1 - t/\tau^+}{p_-}) + Li_2(w_-) - Li_2(w_+) + 4Y_p \ln \rho] + 4(1 - p_0 p_3 Y_p) \ln(z_{max} - \rho) - 4 \ln z_{max},
$$

$$
\Phi_1 = Li_2(w_-) + Li_2(w_+) - Li_2(\tau_+) - Li_2(t/\tau_+),
$$

$$
\Phi_2 = \frac{Y_p}{p_3},
$$

$$
\Phi_3 = \frac{1}{2} \ln \sqrt{\rho},
$$

$$
\Phi_4 = \frac{1}{2} \ln(1 - \tau_+),
$$

$$
\Phi_5 = \frac{1}{2} \ln(1 - t/\tau_+),
$$

$$
\Psi_0 = 4\left(\frac{p_0 Y_p}{p_3} - 1\right) \ln\left(\frac{z_{max} - \rho}{z_{min} - \rho}\right) + 4 \ln\left(\frac{z_{max}}{z_{min}}\right) + 2p_0 [Li_2(1 - \frac{1 - \tau^+}{p_-}) + Li_2(1 - \frac{1 - t/\tau^+}{p_-}) - Li_2(1 - \frac{1 - \tau^+}{p_+}) - Li_2(1 - \frac{1 - t/\tau^+}{p_+}) + Li_2(1 - \frac{1 - \tau^-}{p_+}) + Li_2(1 - \frac{1 - t/\tau^-}{p_+}) - Li_2(1 - \frac{1 - \tau^-}{p_-}) - Li_2(1 - \frac{1 - t/\tau^-}{p_-})],
$$

$$
\Psi_1 = Li_2(\tau_+) + Li_2(t/\tau_+) - Li_2(\tau_-) - Li_2(t/\tau_-),
$$

$$
\Psi_2 = \frac{1}{2} \ln(1 - \tau_-),
$$

$$
\Psi_3 = \frac{1}{2} \ln(1 - t/\tau_-),
$$

$$
\Psi_4 = \frac{1}{2} \ln(1 - \tau_+),
$$

$$
\Psi_5 = \frac{1}{2} \ln(1 - t/\tau_+).
$$

We introduce $C_1...C_5$ to simplify the formulae for $D^n_A$ and $D^n_B$:

$$
C_1 = \frac{1}{\sqrt{x^2 - 4\eta}} \left[\rho(-\eta x + 4\eta t - \eta \tau_+ - 4\eta + x\tau_+ - t\tau_+) - 2\rho^2 \eta + \eta (xt + x)
$$

10
\[ C_2 = \rho (\eta - t - 2\tau_+) + (xt^2 - 2t^2\tau_+)/{(2\eta)} + \eta (-x - 2t + 2\tau_+ + 30)/2 \\
+ (t(1 + t - x) + \tau_+(-2x + 2t + 6); \] 

\[ C_3 = \frac{1}{\sqrt{x^2 - 4\eta}} \left[ \rho(-10\eta x + 18\eta t - 7\eta\tau_+ + xt + 7x\tau_+ - 11t\tau_+ + 4\tau_+) \\
+ \rho^2(-9\eta + \tau_+) + (2xt^2\tau_+ - x^2t^2/2 - t^2\tau_+^2)/\eta + \eta^2(-\tau_+ + 10) \\
+ xt(x - t - 1)\eta(4xt - x\tau_+ - 2x - x^2/2 + t\tau_+ + 4t \\
- 4t^2 + 11\tau_+ - \tau_+^2 - 12) + \tau_+(-6xt - x + 2x^2 - 3t + 5t^2 - 5) \right]; \] 

\[ C_4 = \frac{1}{\sqrt{x^2 - 4\eta}} \left[ \rho(2\eta x\tau_+/t - 8\eta x + 8\eta\tau_+/t + 18\eta t - 11\eta\tau_+ - 9 - 4\eta^2\tau_+/t^2 \\
- xt + 5\tau_+ x - 7t\tau_+) + \rho^2(2\eta\tau_+/t - 9\eta - \eta^2\tau_+/t^2) + xt(1 + t - x) \\
+ (-2xt^2\tau_+ + x^2t^2/2 + t^2\tau_+^2)/\eta + \eta^2(5\tau_+/t^2 - 6\tau_+/t - 16/t + 6) \\
+ \eta(2x\tau_+/t + 2xt - 3x\tau_+ - 4x + x^2/2 - 10\tau_+/t + 3t\tau_+ + 12t \\
- 4t^2 + 3\tau_+ + \tau_+^2 + 4) + \tau_+(-4xt - 3x + 2x^2 + 11t + 2t^2) \right]; \] 

\[ C_5 = [\rho/(x - t - \eta/t)](-\rho\eta^2\tau_+/t^2 + \eta\tau_+/t - \eta - \eta^2\tau_+/t^2 + \eta^2/t + \rho\eta\tau_+/t \\
- \rho\eta + \rho\eta^2/t)/2 + [\rho/(1 - x + \eta)](-\eta - \eta\tau_+ + \eta^2 + t\tau_+)/2 \\
+ \rho(3\eta\tau_+/t + 9\eta - 9t - 3\tau_+)/2 + (xt^2/4 - t^2\tau_+)/\eta + \eta(3x - 10\tau_+/t \\
- 12t - 2\tau_+ - 24 + 2\eta)/4 + (3t + 5)\tau_+/2 - xt + 6t + 5t^2/2; \] 

\[ D_1^A = C_1; \]

\[ D_2^A = \frac{1}{4\sqrt{x^2 - 4\eta}} \left\{ \rho\eta(34 - 6x\tau_+/t^2 - 4x/t^2 + 3x/t + 5xt - 2x\tau_+ + 2x \\
+ 3x^2/t^2 + x^2 + 8\tau_+/t^2 + 18\tau_+/t - 2/t + 2t\tau_+ + 38t - 14t^2 + 20\tau_+) \\
+ \rho\eta^2(16 - 2x/t^2 - 5\tau_+/t^2 - 2\tau_+/t + 16/t - \tau_+) + \rho\tau_+(-4xt - 4x + 6t \\
+ 7t^2 + 11) + \rho^2 \left[ \eta(-10 + 6x\tau_+/t^2 + 6x/t^2 + 4x\tau_+/t - 3x - 3x^2/t^2 \\
- 2x^2/t^2 - 12\tau_+/t^2 - 18\tau_+/t + 6/t + 18t - 6\tau_+) + \eta^2(x/t^2 + \tau_+/t^2 \\
- \tau_+/t - 2/t) + \tau_+(2x - 5t - 7) + \rho\eta(-10 - 2x\tau_+/t^2 - 4x/t^2 - x/t \\
+ x^2/t^2 + 8\tau_+/t^2 + 6\tau_+/t - 6/t) + \rho\eta^2\tau_+/t^2 + \rho\tau_+ + \rho^2 \eta(x/t^2 - 2\tau_+/t^2 \\
+ 2/t)\right] + \tau_+(-4xt + 2xt^2 + 2x + 7t + t^2 - 3t^3 - 5) + \eta(-14 + 2x\tau_+/t^2 \\
+ x/t^2 - 4x\tau_+/t - 2x/t + 4xt - 2xt^2 + 2x\tau_+/t - x - x^2/t^2 + 2x/t - x^2 \\
- 2\tau_+/t^2 - \tau_+^2 - 10\tau_+/t + 32t - 22t^2 + 4t^3 + 18\tau_+ + \eta^2(28 + x/t^2 \\
- 2x/t + x + 3\tau_+/t^2 - 5\tau_+/t - 14/t + t\tau_+ - 14t + \tau_+); \right\}; \]

\[ D_3^A = \frac{1}{\sqrt{x^2 - 4\eta}} \left\{ \rho \left[ \eta(-6 - 4x\tau_+/t^2 - 3x/t^2 - 2x\tau_+/t + x/t - 13x + 2x^2/t^2 \\
+ x^2/t + 6x\tau_+/t^2 + 12\tau_+/t - 2/t + 26t - 10\tau_+ + \eta^2(-x/t^2 - 2\tau_+/t^2 \\
+ x^2/t^2) \right]; \right\}; \]
\(+2/t + \tau_+(6x + 6 - 12t)] + \rho^3 \eta(-x/t^2 + 2\tau_+/t^2 - 2/t) \\
+\rho^2 \left[ \eta(-10 + 2x\tau_+/t^2 + 3x/t^2 - x^2/t^2 - 6\tau_+/t^2 - 4\tau_+/t + 4/t) \\
-\eta^2 \tau_+/t^2 - \tau_+ \right] + \eta^2 (14 + x/t^2 - x/t + 3\tau_+/t^2 - 2\tau_+/t - 14/t - \tau_+) \\
+\eta(-14 + 2x\tau_+/t^2 + x/t^2 - 2x\tau_+/t - x/t + 2xt - 2x - x^2/t^2 + x^2/t \\
-2\tau_+/t^2 - 8\tau_+/t + 18t - 4t^2 + 10\tau_+) + \tau_+(-2xt + 2x + 2t + 3t^2 - 5) \right); \\
\]

\[D_4^A = -C_3 - C_2; \quad (89)\]
\[D_5^A = -C_4 + C_2; \quad (90)\]
\[D_6^A = -\frac{1}{2}C_5 + \frac{1}{4\sqrt{x^2 - 4\eta}} \left\{ \left[ \rho/(1 - x + \eta) \right] \left[ \eta(t\tau_+/t + \tau_+ - \eta^2(1 + t + \tau_+) \\
+\eta^3 - t\tau_+/t \right] + [\rho/(x - t - \eta/t)](1 + \rho) \left[ \eta(t - \tau_+) + \eta^2(-1 + \tau_+/t^2 + \tau_+/t - 1/t) + \eta^3(1/t^2 - \tau_+/t^3) \right] + \rho\eta(43 - x\tau_+/t - 4x/t - 5x + 2x^2/t \\
+9\tau_+/t + 13t + 5\tau_+ + \rho\eta^2(-5 - \tau_+/t^2 + 2\tau_+/t + 1/t) + \tau_+(9xt + 5x + 4t + 6t^2) + 12xt + 5xt^2 - 2x^2t + \rho(-9xt - 3x\tau_+ + 5t\tau_+) + \rho^2\eta(-10 + 2x/t - 3t\tau_+/t) + \rho^2\eta^2(-\tau_+/t^2 + 1/t) + (-6x^2\tau_+ + x^2t^2 \\
+2t^2\tau_+^2)/(2\eta) + \eta(-35 - x\tau_+/t + 2x/t - 2xt + 4xt\tau_+ + 26x \\
-2x^2/t - 3x^2/2 - 4\tau_+/t - - - 2\tau_+ - 34t - t^2 - 22\tau_+ + 3\tau_+^2) \\
+\eta^2(-2 + 2x/t - x + 6\tau_+/t + 2t) \right\}; \quad (92)\]

\[D_1^B = C_1; \quad (93)\]
\[D_2^B = C_2 - C_3; \quad (94)\]
\[D_3^B = -C_2 - C_4; \quad (95)\]
\[D_4^B = C_2 + C_3; \quad (96)\]
\[D_5^B = -C_2 + C_4; \quad (97)\]
\[D_6^B = C_5; \quad (98)\]

Equation (99)

One can perform the limit \(\rho \to 0\), which corresponds to the decay of the bottom quark to an up quark and leptons. The formulae, which are much simpler in this case, are presented in the same manner as the full results:

\[
\frac{d\Gamma}{dx dt} = \begin{cases} 
12\Gamma_0 \left[ \bar{F}_0^-(x, t) - \frac{2\alpha_s}{3\pi} \bar{F}_{1,A}(x, t) \right] & \text{for } (x, t) \text{ in } \text{A} \\
12\Gamma_0 \frac{2\alpha_s}{3\pi} \bar{F}_{1,B}(x, t) & \text{for } (x, t) \text{ in } \text{B} 
\end{cases} \quad (100)
\]

with

\[
\bar{F}_0^-(x, t) = \frac{(1 - x - t)}{\tau_+ - \tau_-} \left[ \tau_+(x - t - \eta) - \eta(1 - t) \right] \quad (101)
\]
and

\[ \bar{F}_{1,A}(x,t) = \bar{F}_0 \bar{\Phi}_0 + \sum_{n=1}^{5} D_n^A \bar{\Phi}_n + D_6^A, \]  
(102)

\[ \bar{F}_{1,B}(x,t) = \bar{F}_0 \bar{\Psi}_0 + \sum_{n=1}^{5} D_n^B \bar{\Psi}_n + D_6^B. \]  
(103)

where

\[ \bar{\Phi}_0 = 2 \left[ Li_2(\frac{\tau_+ - t}{1 - t}) + Li_2(\frac{1}{\tau_+ - 1}) + Li_2(t) \right] + \frac{1}{2} \pi^2 \]

\[ + \ln^2(1 - \tau_+) \ln^2(1 - t) + \ln^2(1 - \tau_+) \ln(1 - t) \ln z_{max}, \]  
(104)

\[ \bar{\Phi}_1 = \frac{\pi^2}{12} + Li_2(t) - Li_2(\tau_+) - Li_2(\tau_+), \]

\[ \bar{\Phi}_{2,03} = \frac{2 \ln(1 - t)}{1 - t}, \]  
(105)

\[ \bar{\Phi}_4 = \Phi_4, \]

\[ \bar{\Phi}_5 = \Phi_5, \]  
(106)  

and

\[ \bar{\Psi}_0 = 2 \left[ Li_2(\frac{\tau_+ - t}{1 - t}) + Li_2(\frac{1}{\tau_- - 1}) - Li_2(\frac{\tau_+ - t}{1 - t}) \right] \]

\[ - Li_2(\frac{1}{\tau_- - 1}) \right] + \ln(1 - t) \ln \left( \frac{z_{max}}{z_{min}} \right) \]

\[ - \ln(1 - \tau_+) \ln [(1 - \tau_+)(1 - \tau_-)] \]

\[ - \ln(1 - \tau_-) \ln [(1 - \tau_+)(1 - \tau_-)], \]  
(108)

\[ \bar{\Psi}_n = \Psi_n, \quad (n = 1...5). \]  
(109)

\[ C_1^A = \frac{1}{\sqrt{x^2 - 4\eta}} \left[ \eta(6 + xt - x\tau_+ + x + t\tau_+ + 4t - 2t^2 + 5\tau_+) + 2\eta^2 + \right] \]

\[ \tau_+(x - 2xt + t^2) \right]; \]  
(110)

\[ C_2^A = \frac{(xt^2 - 2t^2\tau_+)/2\eta + \eta(30 - x - 2t + 2\tau_+)/2 - xt - 2x\tau_+ + 2t\tau_+ + t + t^2 + 6\tau_+}; \]  
(111)

\[ C_3^A = \frac{1}{\sqrt{x^2 - 4\eta}} \left[ (4xt^2\tau_+ - x^2t^2 - 2t^2\tau_+^2)/2\eta + \eta^2(10 - \tau_+) \right] \]

\[ \eta(-24 + 8xt - 2x\tau_+ - 4x - x^2 + 2t\tau_+ + 8t - 8t^2 + 22\tau_+ - 2\tau_+^2)/2 \]  
(112)
\[ R(x) = \frac{f_1(x)}{f_0(x)} - \frac{f_1^-(x)}{f_0^-(x)} \]
FIG. 1. Polarization of \( \tau \) lepton in the Born approximation (dashed line) and including the first order QCD correction (solid line) as functions of the scaled \( \tau \) energy \( x \). The mass of the \( b \) quark taken at 4.75 GeV, \( c \) quark 1.35 GeV and the coupling constant \( \alpha_s = 0.2 \)

thus its meaning is how the radiative corrections differ with respect to the state of polarization they act on. In (121) \( P_0 \) denotes the zeroth order approximation to the polarization. The function \( f_1^{-}(x)/f_0^{-}(x) \) has been presented in Fig. 2, while Fig. 3 shows the function \( R(x) \), so that one can immediately see how small the correction is in comparison to the correction to the energy distribution.

Integrating over the charged lepton energy one can obtain its total polarization as well as the corrections to which it is subject. If we take \( m_b = 4.75 \) GeV as the central value for the decaying quark mass and \( m_b = 4.4 \) and \( m_b = 5.2 \) for the limits, we arrive at the following (the mass difference \( m_b - m_c = 3.4 \) GeV everywhere):

\[
1 - P = (1 - P_0) \left( 1 + \frac{2\alpha_s}{3\pi} R_s + \frac{\lambda^2_1}{m_b^2} R_{np}^1 + \frac{\lambda^2_2}{m_b^2} R_{np}^2 \right)
\]

with

\[
\begin{align*}
P_0 &= -0.7388^{+0.0105}_{-0.0109} \\
R_s &= -0.016^{+0.023}_{-0.017} \\
R_{np}^1 &= 0.421^{+0.027}_{-0.025}
\end{align*}
\]
FIG. 2. The ratio $f_1^-(x)/f_0^-(x)$ representing the radiative correction for the negatively polarized state as dependent on the scaled $\tau$ lepton energy $x$.

$$R_{np}^2 = -2.28^{-0.22}_{+0.16}$$

VI. MOMENTS OF $\tau$ ENERGY DISTRIBUTION

The moments of $\tau$ energy distribution, which are useful sources of information on the physical parameters regarding the discussed decay can be evaluated according to the formula:

$$M_n^\pm = \int_{E_{min}}^{E_{max}} E \frac{d\Gamma^\pm}{dE_\tau} dE_\tau,$$

$$r_n^\pm = \frac{M_n^\pm}{M_0^\pm},$$

where $E_{min}$ and $E_{max}$ are the lower and upper limits for $\tau$ energy and $M_n$ include both perturbative and nonperturbative QCD corrections to $\tau$ energy spectrum. The superscripts denote the polarization states. Since one obviously has

$$M_n = M_n^+ + M_n^-$$

where $M_n$ stands for the unpolarized momenta, we only give the values of the momenta for the negative polarization case. The unpolarized distributions were given in [12].
FIG. 3. The QCD-correction function $R(x)$ for the pole mass values of the $b$ quark taken to be 4.4 GeV (dashed), 4.75 GeV (solid) and 5.2 GeV (dash-dotted) as dependent on the scaled $\tau$ lepton energy $x$.

The nonperturbative corrections to the charged lepton spectrum from semileptonic $B$ decay have been derived in the HQET framework up to order of $1/m_b^2$ [15–17]. The corrected heavy lepton energy spectrum can be written in the following way:

$$
\frac{1}{12\Gamma_0} \frac{d\Gamma}{dx} = f_0(x) - \frac{2\alpha_s}{3\pi} f_1(x) + \frac{\lambda_1}{m_b^2} f^{(1)}_{np}(x) + \frac{\lambda_2}{m_b^2} f^{(2)}_{np}(x),
$$

(127)

where $\lambda_1$ and $\lambda_2$ are the HQET parameters corresponding to the $b$ quark kinetic energy and the energy of interaction of the $b$ quark magnetic moment with the chromomagnetic field produced by the light quark in the meson $B$. The functions $f^{(1,2)}_{np}$ can be easily extracted from the formula (2.11) in [15]. The formula (127) looks identically if one considers definite polarization state of the final $\tau$ lepton. The appropriate calculation within the HQET scheme has also been performed [15], see formula (2.12) therein.

Following ref. [11] we expand the ratios $r_n$:

$$
r^{-}_n = r^{(0)-}_n - \left( 1 - \frac{2\alpha_s}{3\pi} \delta^{(p)-}_n + \frac{\lambda_1}{m_b^2} \delta^{(1)-}_n + \frac{\lambda_2}{m_b^2} \delta^{(2)-}_n \right),
$$

(128)
where \( r_n^{(0)} \) is the lowest approximation of \( r_n \),

\[
r_n^{(0)} = \left( \frac{m_b}{2} \right)^n \frac{f_n^{(1+n)}}{f_n^{(0)}} \frac{f_n^{-}(x) x^n dx}{f_n^{-}(x) dx}.
\]  

(129)

Each of the \( \delta_n^{(i)} \) is expressed by integrals of the corresponding correction function \( f^{(i)}(x) \) and the tree level term \( f_0(x) \)

\[
\delta_n^{(i)} = \frac{f_n^{(1+n)}}{f_n^{(0)}} \frac{f_n^{-}(x) x^n dx}{f_n^{-}(x) dx} - \frac{f_n^{(1+n)}}{f_n^{(0)}} \frac{f_0^{-}(x) x^n dx}{f_0^{-}(x) dx},
\]  

(130)

where the index \( i \) denotes any of the three kinds of corrections discussed above. The coefficients \( \delta_n^{(i)} \) depend only on the two ratios of the charged lepton and the \( c \) quark to the mass \( m_b \). Following ref. [12] we employ the functional dependence of the form

\[
\delta_n^{(i)}(m_b, m_c, m_\tau) = \delta_n^{(i)} \left( \frac{m_b}{m_\tau}, \frac{m_c}{m_b} \right).
\]  

(131)

The quark masses are not known precisely so we have calculated the coefficients in a reasonable range of the parameters, that is, \( 4.4 \text{ GeV} \leq m_b \leq 5.2 \text{ GeV} \) and \( 0.25 \leq m_c/m_b \leq 0.35 \) and then fitted to them functions of the following form:

\[
\delta(p, q) = a + b(p - p_0) + c(q - q_0) + d(p - p_0)^2 \\
= +e(p - p_0)(q - q_0) + f(q - q_0)^2,
\]  

(132)

where \( p = m_b/m_\tau, p_0 = 4.75 \text{ GeV}/1.777 \text{ GeV} = 2.6730, q = m_c/m_b, q_0 = 0.28 \) and the polynomial coefficients can be fitted for each of the \( \delta_n^{(i)} \) separately with a relative error of less than 2%. Our choice of the central values reflects the realistic masses of quarks: \( m_b = 4.75 \text{ GeV} \) and \( m_c = 1.35 \text{ GeV} \), for which \( \delta_n^{(i)} = \delta_n^{(i)} \).

To bring out the difference in the extent to which the corrections affect the two different polarization states it is useful to compare these coefficients with the ones obtained with the polarization summed over. This can be done along the lines suggested by the treatment of the polarization itself, see Eq. 127. The corresponding expansion takes the form

\[
\frac{r_n^{-}}{r_n} = \frac{r_n^{-(0)}}{r_n^{(0)}} \left[ 1 - \frac{2\alpha_s}{3\pi} (\delta_n^{(v)} - \delta_n^{(v)}) + \frac{\lambda_1}{m_b^2} (\delta_n^{(1)} - \delta_n^{(1)}) + \frac{\lambda_2}{m_b^2} (\delta_n^{(2)} - \delta_n^{(2)}) \right].
\]  

(133)

With these, one can readily find the actual relative correction of each kind, assuming reasonable values of \( \alpha_s, \lambda_1 \) and \( \lambda_2 \). Here we take \( \alpha_s = 0.2, 0.15 \text{ GeV}^2 \leq -\lambda_1 \leq 0.60 \text{ GeV}^2 \), \( \lambda_2 = 0.12 \text{ GeV}^2 \), keep the \( b \) quark mass fixed at 4.75 GeV and the mass of the \( c \) quark equal to 1.35 GeV. The corrections then read for \( n = 1(n = 5) \)

| Type            | \( \frac{r_n^{-}/r_n^{(0)}}{r_n^{(0)}} \) | \( \frac{r_n/r_n^{(0)}}{r_n^{(0)}} \) |
|-----------------|----------------------------------------|---------------------------------------|
| perturbative    | -0.00084(-0.0048)                      | -0.0009(-0.0052)                      |
| kinetic energy  | 0.008 ± 0.005(0.06 ± 0.04)             | 0.008 ± 0.005(0.06 ± 0.04)            |
| chromomagnetic  | -0.0097(-0.053)                        | -0.0092(-0.0511)                     |
VII. SUMMARY

The first order perturbative QCD correction to the polarization of the charged lepton in semileptonic $B$ decays has been found analytically. It is expressed by the charged lepton energy and the invariant mass of the lepton system. The polarization correction has turned out to be very small as it does not exceed 1% taking common values for the parameters occurring in the formulae. This makes the polarization a very useful quantity in determining the quark masses. The moments of $\tau$ lepton energy distribution have been evaluated for the case of a polarized lepton and the correction has again been found to be little different from the one in the case of an unpolarized lepton. The nonperturbative HQET corrections to the moments have also been calculated using the formulae from [15].
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