Adaptable materials via retraining

Daniel Hexner
Faculty of Mechanical Engineering, Technion, 320000 Haifa, Israel
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Elastic metamaterials are often designed for a single permanent function. We explore the possibility of altering a material’s function repeatedly through a self-organization, “training” process, controlled by applied strains. We show that the elastic function can be altered numerous times, though each new trained task imprints a memory. This ultimately leads to material degradation through the gradual reduction of the frequency gap in the density of states. We also show that retraining adapts previously trained low energy modes to a new function. As a result consecutive trained responses are realized similarly. We show how retraining can be exploited to attain a response that would otherwise be difficult.

The ability to fabricate precise structures along with recent design algorithms has allowed to construct metamaterials with new exotic properties that are not commonly found in nature. Though successful, this results in mechanical devices with specialized functions. Altering functionality after fabrication is usually impractical since it necessitates to alter the structure. This is particularly difficult if the degrees of freedom are on a microscopic scale and buried in the bulk of a three dimensional material.

Nonetheless, altering a device’s function could be highly advantageous, allowing a “general purpose” material or device. That is, the function can be adjusted for the task at hand, enabling a single programmable device to replace multiple single function devices. This could potentially be very important when weight of volume are very costly, as in the case that they are launched into space. We note that programmable materials perform a single function that can be altered in contrast to multifunctional materials that are able to perform simultaneously multiple functions.

In this letter we consider the viability of reprogramming the function of a material, using a recent approach of material training. Training is based on the self-organization of the microstructure through plastic deformations in response to external fields – particularly, imposed strains. By driving the system with carefully choreographed protocols, the evolution of a material can be directed towards a desired elastic response. Recent work has demonstrated that a generic disordered solid can acquire highly complex non-linear functionality in both simulations and experiments. The advantage of self-organization, is that it does not necessitate a direct manipulation of the microstructure. Training can also be thought of as a physical realization of a learning rule.

We show that a material can be trained repeatedly, attaining each time a different predefined elastic response. We first study how one function evolves into another. Towards this goal we perform a normal mode analysis. A strain response corresponds to a low frequency mode, which is gapped from the remaining of the spectrum. Retraining, does not introduce a new low frequency mode but rather adapts the previous one. As a result there is a large overlap in how consecutive functions are encoded in the system wide response. We show that this adaptability can be exploited to achieve functions that are difficult to train.

We also show that each training task imprint a memory, which accumulates with the number of trained functions. Ultimately, this leads to failure to perform the trained task. This degradation is revealed in the density of states, where as a material is retrained the frequency gap between the lowest energy mode and the remaining of the spectrum slowly closes. This implies that material with a larger gap are more robust. We also find that degradation is accelerated by training at large strains. The proposed scenario suggested retraining fails through the formation of spurious low frequency modes that compete with the desired function.

Model and training protocol: We model an amorphous material as a random network of springs that are derived from disordered packings of repulsive soft spheres at zero temperature. The center of each sphere corresponds to a node and each overlapping pair of spheres is attached with a spring of unit stiffness. The coordination number, $N$, is easily tuned by varying the the imposed pressure. Here, $N_b$ is the number of bonds and $N$ is the number of nodes. In the limit of zero pressure the networks are isostatic, and have an anomalously long range elastic responses.

We consider bonds that evolve through plastic deformation in response to the internal stresses. We assume that the rest length of the bond evolves in proportion to the stress it experiences:

$$\partial_t \ell_b \propto k_i (\ell_b - \ell_b^0). \quad (1)$$

Here, $\ell_i$ is the bond length, $\ell_i^0$ is the rest length and $k_i$ is the spring constant. This model can be considered the Maxwell model for viscoelasticity in the limit where the elastic relaxation is fast with respect to the viscous dynamics.

We focus on training the simplest task – the so called allostery inspired response, where squeezing a “source” pair of nodes yields a similar strain at a far away “target” site. Both the source and target are chosen to
be at least half the length of the system. We define the strain on a pair of nodes as the fractional change in distance with respect to its unstrained value. Examples of pairs of allostery sites are shown in Fig. 1.

We follow the procedure of Ref. [18] to train elastic responses. Both the source and target sites are repeatedly strained quasistatically up to an amplitude of $\epsilon_{Age}$ as the system evolves through Eq. 1. This is implemented numerically by discritizing time to small increments; at every time step we vary the strain, minimize the energy to reach force balance and then alter the rest lengths.

The intuition behind this strategy is that strain-response follows low energy directions that couple the source and target, and therefore the goal of training to create such a low energy "valley". Aging at a constant strain decreases the energy at that imposed strain value. This is seen by identifying the right hand side of Eq. 1 as the gradient of the energy with respect to $\ell_{i,0}$. Straining periodically lowers the energy along the entire strain range, creating a low energy mode that couples the source and target.

Since we focus on training allosteric response that couples distant sites, we resort to networks with a small deviation from isostaticity, $\Delta Z \equiv Z - Z_{iso} \ll 1$, characterized by anomalously long range elasticity [25, 26].

**Elastic response, and density of states:** We sequentially train task after task by straining periodically both the source and target sites up to a strain of $\epsilon_{Age}$. Each task corresponds to a different source and target sites both selected randomly. Our goal is to train a system such that an imposed strain on the source results in the same strain on the target. The convergence of the trained response is characterized by straining the source up the training strain $\epsilon_{Age}$ while measuring the strain on the target. The error squared, $(\delta \epsilon)^2$, is defined as the squared difference between the strain on the target and its desired value, averaged over a measurement cycle.

**Overlap between consecutive responses:** The adaptation of one function into another suggest that two consecutive training tasks. As noted the response to an applied strain is associated with a low energy valley. Fig. 2(a) shows $\delta \epsilon$ as a function of the number training cycles for twenty different tasks, each depicted in a different shade. Initially, the error for the newly assigned training task is large, but the error decreases with training cycles, until the task is switched. Overall, the system is able to adapt to a new task multiple times, however, the error slightly grows with the number of altered task.

As noted the response to an applied strain is associated with a low energy valley. Fig. 2(b) shows the energy measured along each of the training paths, by straining both the source and target. As expected, when a given task is trained the energy decreases. Interestingly, even after the training task is changed the energy does not return to its value prior to training, and remains substantially smaller. As further tasks are trained that energy slowly increases.

These findings suggest that the system retains a memory of previously trained tasks. To probe the fate of low energy modes we study the density of states, $D(\omega)$. This characterizes the high dimensional energy landscape within linear response. We first compute Hessian, the matrix of second derivatives of the energy with respect to the node locations, and then diagonalize to find the eigen-frequencies; these are then binned to compute $D(\omega)$. In Fig. 2(d) we show its evolution as the system is retrained. The low frequency regime is of particular interest and shows a gap which we discuss below. To better visualize the low frequency regime we also compute the integrated density of states, $I(\omega) = \int_{0}^{\omega} d\omega' D(\omega')$, which allows to enumerate the low energy modes.

Fig. 2(e), shows that prior to training $I(\omega)$ has two zero modes that are associated with the two independent translations in two dimensions (due to the periodic boundary conditions). After training the first task, another low energy mode is created that is gapped from the remaining of the modes; namely, $I(\omega \ll 1) = 3$. In our initial networks the gap scales as, $\Delta \omega \propto \Delta Z$ [23, 31]. Even after retraining several times, $I(\omega \ll 1) = 3$, implying that there always a single extra low energy mode. This suggest that the system adapts the previously trained mode to a new function, rather than forming an additional mode.
is measured when task $r$ is trained. We normalize $V$ so that it is bounded by unity, $C_{o}^{-1} = \sqrt{\sum_{i} \delta x_{i}^{(r-1)} \delta x_{i}^{(r)}}$. If the motion is uncorrelated then the overlap will vanish for large systems as $1/\sqrt{N}$ ($N$ is the number of nodes).

Fig. 2(c) shows $V^{(r)}(t)$ as function of time for each of the training tasks. The overlap is substantial even though different sets of source and targets are actuated. That value does not decrease with system size, indicating that this is not a finite size effect (see Supplemental material). We also consider the effect of varying the training strain in Fig. 3(d). We find that at small training strain the overlap is large. Training at large strains necessitates larger changes to the structure [18] yielding a smaller overlap. The finite overlap indicated a similarity in the way that the system realizes both tasks, supporting the assertion that one function transforms into another.

We understand the large overlap as follows. Prior to retraining, there is a single low energy mode corresponding to the function of the old task. Elastic response follows the low energy modes and therefore during training the new function, initially the motion couples strongly to that mode. As a result, the new function shares a similarity between the previous function. Similar behavior was found in Ref. [32] in the context of epistasis.

**Exploiting retraining to attain difficult responses:** Up to now we have focused on training networks that are nearly isostatic, whose long range elasticity was crucial. Straining only the source and target do not couple distant sites in highly coordinated networks. Ref. [18] introduced a strategy to access this regime, that is based on additional “repeater” sites that are strained in a similar manner to the source and target. These additional sites are chosen randomly with a density that insures that repeater sites are not too distant to couple. As a result the source and target are able to couple through these intermediate repeaters.

We consider a highly coordinated network, where an extended low energy mode is trained using repeaters. We then alter the material’s function by retraining it by actuating only the source and target. Note that the set of repeaters does not include the source and target sites. Training via repeaters requires the actuation of a potentially large number of sites, however these sites are non-specific. Namely, both the sites are chosen randomly as well as the sign of the strain. We believe, that this can be realized in experiment by having a small fraction of the sites with an electric dipole moment; these are actuated by applying a time varying electric field. Because the parameters are chosen randomly the specific details of the driving is unimportant.

The allosteric function is then trained by actuating only the specific source and target sites. Fig. 3(a) compares the success in training allostery with and without prior training with repeaters. Adapting the old function trained with repeaters appears to converge to the desired response, while in the system without prior train-

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**Degradation:** Fig. 2(d) shows that retraining reduces the gap separating the mode associated with the function and the remaining of the spectrum. Thus, the system softens as modes move to lower energy. More importantly, the decrease of the frequency gap affects the low energy spectrum which dictates the quasistatic response. We argue, that this can be considered degradation since the additional low energy modes compete with the trained function, leading to a larger training error, as shown in Fig. 2(a). Ultimately when the gap closes we expect that the system fails to attain the trained function.

Such a scenario suggests that the gap plays an important role in the ability to retrain the system. The gap prior to training depends on the coordination number, $\Delta \omega \propto \Delta Z$. We therefore expect that for small $\Delta Z$, where the gap is small, degradation occurs more quickly. Indeed Fig. 3(b) shows that networks with a small $\Delta Z$ can be retrained fewer times. Thus, there is a trade-off between large $\Delta Z$ where the system can be retrained many times, and small $\Delta Z$ where distant sites are easily coupled.

**Strain amplitude dependence:** We also consider the effect of varying the training amplitude, $\epsilon_{Age}$. Fig. 3(c) shows the response error at the end of each training set as a function of the number of tasks. For the two smallest $\epsilon_{Age}$ degradation is barely visible after 10 tasks. At larger $\epsilon_{Age}$ the error grows with the number of trained tasks. This is consistent with the diminished frequency gap in the density of states at large $\epsilon_{Age}$, seen in Fig. 3(e) and (f).

As noted, the overlap is large at small training strains, and decreases with the strain amplitude (see Fig. 3(d)).

**Conclusions:** We have demonstrated that the function of a material can be altered numerous time by retraining a disordered dashpot-spring networks, enabling a material that can be programmed. Our approach does not require the direct manipulation of the microstructure, but is rather based on the autonomous evolution of the structure due to the imposed strains. Our results suggest that retraining, repurposes an old function rather than creating a new function. This, we argued, is the result of the system following, during training, the already present low energy mode of the previous trained function. Retraining can be exploited to train functions that are otherwise difficult to attain, such as, allostery in network with high connectivity. Training a non-specific extended mode with repeaters, can then be retrained to a specific allosteric response by just actuating the source and target. This approach can potentially allow to couple distant sites in experiments.

While repurposing of low energy modes, has been beneficial, there are cases where it is not. Particularly, if one would like to train a number of different coexisting low energy modes. Our results suggest this is challenging since training two set of different degrees of freedom.
Figure 2. Sequentially training 20 allostery pairs, each denoted by a different color. (a) The error in the trained response, slightly grows with the number of tasks. (b) The energy for straining both the source and target as a function of training cycles. Training reduces the energy cost of the corresponding trained set. Note, the energy does not return to its pretrained value when the task is switched. (c) The overlap of the system-wide response between consecutive training tasks. (d) The evolution of the density of states with number of trained tasks. (e) The integrated density of states. Prior to training there are the two zero modes associated with translations. Training adds an third low energy mode (the dashed line marks $I(\omega) = 3$). Here, $N = 512$, $\epsilon_{Age} = 0.2$, $\Delta Z = 0.038$.

creates a single low energy mode.

We have also shown that each trained function imprints a memory, which ultimately leads to degradation. This is revealed by the normal mode analysis, where retraining reduces the frequency gap between the low energy mode associated with the function and the remaining of the spectrum. We note that degradation is of a geometric origin since the bond stiffnesses do not change. Ultimately the creation of competing spurious low energy modes leads to failure. Systems with a larger gap are more robust to retraining, while large strain amplitudes increase the rate of degradation. We speculate that this could have relevance to biological proteins that over the course of evolution alter their function via mutations. Perhaps, the elastic behavior of a protein encodes memories of their previous functionality.
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