Learning obstacles on linear equations concept in junior high school students: analysis of intellectual need of DNR-based instructions

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Abstract. The purpose of this study was analysis the learning obstacle by students in solving the problems of linear equations system based on students intellectual need. Two problems about linear equations was given to two students of 8th Grade as subjects who had the same initial mathematical abilities. The data collection was using the test of mathematical concept comprehension, and interview with subjects. Interviews was conducted by asking open-ended questions related to the mathematical concept comprehension test that they had been working on. This data were transcribed then analysis in depth to see the learning obstacle and intellectual need. This research finally finds three types of learning obstacle students related to intellectual need. These learning obstacle are ontogenic obstacle, epistemological obstacle, and didactical obstacle. Ontogenic obstacles are where students didn’t have of basic mathematical knowledge Epistemological obstacles were where students couldn’t translate problems into mathematical models, miscalculate and also couldn’t provide an explanation of the answers obtained. This indicates that the students did not have need for certainty, need for causality, need for computation, need for communication, and need for structure. Didactical obstacle occurs when the teachers are unable to create learning that accommodates the intellectual need of students.

1. Introduction
The study in Mathematical Thinking and Learning, state that generally, learning obstacle is the barriers of the way of thinking and how to understand [1]. These barriers can be both epistemological and didactic. Didactical obstacles according to Harel are more easily recognizable and probably easier to overcome rather than epistemological obstacles. The result of the Duroux study in Brousseau [2] about obstacle learning states that the epistemological obstacle is a conception or knowledge which is correct in a particular context, and this conception becomes invalid in another context, this obstacle relates to the truth of knowledge that is limited to a particular concept. Therefore, it is not misconception or lack of knowledge. Another learning obstacle is an ontogenic obstacle that is happening when the given lesson is not suitable for student's thinking level, Learning obstacle can also be an ontogenic obstacle that is when the given lesson is not suitable for the level of student thinking, and also occur because of less precise learning by teachers. One of research of Learning Obstacle was
conducted by Sulistiawati [3] who gives problems to see the ability of mathematical reasoning and learning obstacle to the material of geometry to the students, then the obstacle is designed a didactic design used in the learning. The results of her study is that there is an increase in mathematical reasoning of students who obtain learning by using didactic design. The study was conducted by giving a test of the ability of the respondent to the students who had studied the material of Equation and Linear Inequality one variable, then saw their learning obstacle. The results of her study found three types of Learning Obstacle namely ontogenic obstacle, epistemological obstacle, and Didactical obstacle [4].

Research on Learning Obstacle has been widely researched on different materials, with the same type of Learning Obstacle. But no one has studied Learning obstacle associated with Intelectual Need on understanding mathematical concepts. This study describes Learning obstacle faced by students associated with intellectual Need which must be fulfilled by the student in solving a problem related to System of Linear Equation Two Variable. Intellectual need appears from DNR instructional principle in mathematics (about DNR) [5-9]. The DNR instruction is an abbreviation of D, N, and R which means Duality, Necessity, and Repeated Reasoning. The principle of DNR is the Necessity principle which states that "for students who learn what the teacher wants to teach them, they must have a need for it which is related to intellectual need, not the social or economic needs."

The four premises underlying intellectual need in mathematical learning: the premise of knowledge of mathematics, knowing, knowledge-knowing linkage (based on Piaget's theory [10] and modified by Brousseau [2] and subjectivity. Harel [8] divides intellectual needs into five parts: 1) need for certainty; 2) need for causality; 3) need for computation; 4) need for communication and 5) need for structure.

Need for certainty is the desire to dispel doubts over unrecognized math statements. Need for causality is a desire to explain and to determine the cause of a phenomenon. Need for computation is the desire to determine the missing or unknown quantities of a series of quantity limits. Need for communication refers to formulating and formalizing. Need for the structure is the need to organize the learned knowledge into a logical structure. Intellectual Need is important to be presented in learning mathematics because learning is one-way student’s process to understand new material. where students as individuals (epistemophilia) have the ability to develop a desire to be confused and learn to do mental actions to solve the confusion they create (Aristotle, in Lawson-Tancred) [11]. And knowing is a process of development that moves through the continuing tension between assimilation and accommodation directed at a Piaget equilibrium [10]. These situations are most likely done in the classroom.

Research on intellectual need should be developed because it is very useful to the teacher and also students' mathematical thinking process. Although it is realized that there are still many teachers who teach mathematics in results-oriented schools and are still fixated on the syllabus that lacks the presence of intellectual needs in it. The same thing as the research of Fuller E, Rabin M & Harel [12] entitled Intellectual Need and Problem-Free Activity in the Mathematics Classroom. This study, comparing a learning situation that includes Intellectual Need and Free Activity. The results found that most students feel intellectually helpless in the math class because teachers fail to help fulfill their intellectual needs.

2. Method
This study aims to analyze students’ obstacle learning associated with their intellectual need in solving problems related to two-variable linear equations. Based on that purpose, this study is used qualitative analysis. The research procedure was chosen by two 8th grade students with the same initial ability from one of the junior high schools in Ternate to answer two problems. Then the two students were interviewed with open questions related to the concept comprehension test they had to do it. The interview process of the students was then transcribed and in-depth analysis to see their obstacle and need intellectual learning [13].
Two problems related to the linear equations of two variables are made referring to the conceptual ability indicator that is able to link a concept with other concepts, represents the mathematical situation in various ways, and able to determine a more appropriate representation for certain situations. The problems can be seen in Table 1.

Table 1. Problem description.

| No | Problem about linear equations |
|----|---------------------------------|
| 1  | A rectangular object with a circumference of 22 cm. If the length is made to three times the original length and width is made twice the original width, then the circumferences of the object become 58 cm. Find the area of the rectangle and explain the answers you get |
| 2  | Aksa has 14 head of cattle consisting of goats and chickens. The number of legs of cattle is 36 feet. Aksa asked friends at his school to guess how many chickens and goats he has? Can you help find answers for his friends Aksa? Describe every answer you get |

3. Result and discussion

3.1. Analysis of student work and interview on problem 1

In this section will be presented the results of student work. The discussion is associated with DNR learning consisting of duality, necessity and repeated reasoning. The following will show the results of student work and interview results to confirm their answers.

Figure 1. Nurul representation on problem 1.

Figure 1 shows that Nurul is able to relate problems using rectangular circumferences, but she has an obstacle in translating the statement if the length is made "three times the original length" and "the width is made twice the original width" into the mathematical model. Nurul translates by adding 3 units on the original length and 2 units on the original width (note figure step 5). To know the learning barrier experienced Nurul, conducted interview related to the settlement of the problem 1. Here are excerpts of the interview (interviewer (I) with Nurul (N)).

I: "Does Nurul feel there is something wrong with the work? (Without showing any correction results)
N: "I misunderstood the language on the matter".
I: "Why not re-review the work before submitting it to the teacher"?
N: "I have checked, and I write what I know"
I: "What should be like?"
N: "Smiling shyly ... she said I do not know yet"
I: "While pushing the question towards Nurul," try to understand this sentence. "The length is made to three times the original length, Pause for a moment ... how long before?"
N: "Original length = (2p). Means (happy smiley expression, maybe because have understood the sentence while writing the answer sheet) ... means it should be 3 (2p)? And if the width is made twice the width, then 2 (2l)"

From this interview revealed that the barrier of learning faced by Nurul is the lack of ability to interpret the problem to the form of the mathematical model, and for the algebra process Nurul slightly
shows the ability of her understanding. So these obstacles are categorized as **ontogenies obstacle and epistemologies obstacle**. Learning obstacle arises because Nurul does not have all the categories of intellectual need that is 1) need for certainty, namely the need to eliminate doubts on the way of thinking, and not challenge herself to find other alternatives of the solution. 2) need for communication, where Nurul has not been able to formulate the writing language into algebraic statements or model, 3) need for computation is unable to find the quantity in question / which is not yet known on this problem. 4) Need for causality, because Nurul is unable to show the cause of his way of thinking and 5) need for structure, because Nurul does not have the ability to organize his knowledge. Next will be shown the work of Iksan. The work of Iksan can be seen in figure 2.

![Figure 2. Iksan representations on problem 1.](image)

In general, Iksan is able to work on the problem given although it is not perfect as shown in figure 2. However, he is unable to give a logical explanation when confirmed in the interview session. Iksan when asked: "why write" \( k = 2(p + 1) \). Iksan replied "square circumference formula. The answer given is not wrong, Iksan understands and able to connect the problem with other mathematical concepts, but he has not been able to show the correct formula in other ways (such as images), in addition, to memorize/remember what teachers have taught/exemplified to him. Iksan has not found the area of the rectangle. Iksan when asked about the change from \(-p = -7\) to \(p = 7\) answers is "that is my teacher taught". Here it appears that the teacher has not been able to apply the multiplication properties \((-) \cdot (-) = (+)\). Iksan also uses only one way to solve the problem 1. This is because Iksan felt the easiest to remember. Based on the analysis of the results of work and interview results concluded that the barriers to learning experienced Iksan on problem 1 is the ontogenic obstacle, epistemological obstacle and also because of the didactical obstacle taught by the teacher.

3.2. Analysis of student work and interview on problem 2
Nurul's work on problem no.2 can be seen in figure 3 of the following:

![Figure 3. Nurul representation on problem 2.](image)
In figure 3, it can be seen that Nurul's work is very different from most students' answers. Nurul's answer uses a representation of images. When asked in the interview, why choose to answer like this? (While showing the result of her work).

Nurul replied: "I imagine 14 heads of animals". It means Nurul uses his intuition to answer mathematical problems. Intuition by Parselle translates experience to produce quick action. Here are excerpts of the interview

"I imagined the fourteen heads of animals and drew them (pointing round the headed circle). Then make the legs of the animal (referring to the line made). Because there are goats and chickens, then there are legs that I give a line of four and two. But at first, I gave them two lines, then counted. Because it is still less than 36 feet, then I add two more lines on each head up to the amount equal to 36 feet. So four-legs showed a number of goats and two-legged showed the number of chickens.

Nurul's way of thinking based on her experience is namely the premise of subjectivity (Harel) [8] that the knowledge gained by each individual is the connection of her mind to her environment. Analytically, Nurul probably can not translate the problems into a mathematical model, as teachers usually teach math at school. However, Nurul's way of thinking produces a representative way of understanding. Nurul's way of solving problem number 2 is a way that may only apply to specific problems, but still, be appreciated. The type of learning obstacle is an ontogenic obstacle, epistemological obstacle. In general, intellectual needs that Nurul does not yet have are 1) Need for certainty, namely the need to convince herself that what she does is right. For example by showing different alternative solutions to test the answers obtained. Nurul has not shown the need for all the problems given; 2) Need for causality. Although in problem number 2, Nurul was able to explain the answers that were done, but in general Nurul has not been able to explain the cause of the correct answers obtained; 3) Need for communication, where Nurul still have to be given repetitive exercise to be able to interpret the problem into mathematical model; 4) Need for structure, where Nurul has not been able to reconstruct the knowledge she has gained into new knowledge for herself.

Next, describing the results of the problem solving done by Iksan. Figure 4 below is the result of Iksan's work on a problem no. 2.

![Figure 4. Iksan representation on problem 2.](image)

On the problem number 2, Iksan was unable to translate the problem into a mathematical model. In the interview session, the answer then is confirmed to Iksan. The following excerpt of interviews with Iksan

Q: "Why write an answer like this (while pointing to Iksan's work).
I: "I just write down from problem’s hint, the number of goats and chickens is 14 and the number of legs is 36."
Q: Yes it's the key word, which should be changed to a mathematical model. Meaning writing \(K + A = 14\) meaning goat + chicken = 14?
I: "yes... But I cannot do for the number of legs goats and chicken = 36. I do not understand"
Q: (with my curiosity asking), why? Does not the goat have 4 legs and the chicken has 2 legs? And can it be written like this? \(4k + 2a = 36\)?
I: (Smile while scratching his head) I do not think that way that is so easy.
Q: If you have obtained a mathematical model like this, can you solve it?
I: I will try.
Challenges given can be completed by Iksan. But Iksan has a psychological barrier that does not want to ask his peers. He is more confident if the explanation is given by the teacher. This is common for students who have an unconfident character and closed tendency. From the interview description obtained that the obstacles are owned Iksan similar to that owned by Nurul. Both on the first and the second. That is, the ontogenic obstacle, epistemological obstacle, and it is "suspected" because of the way the teaching teacher does not pay attention to the needs of the students. One form of abandonment of students' needs is that teachers over-dominate the learning process so students assume that the correct way of completion is introduced by the teacher.

4. Conclusion
Some of the more specific learning obstacle encountered in this study, but generally, there are three factors of student learning barriers in solving the given problem of an ontogenic obstacle, epistemological obstacle, and didactical obstacle caused by lack of intellectual needs of students so that students have no goals in learning mathematics. The intellectual needs that students do not have are the need for certainty, causal needs, calculating needs, communication needs, and structural needs. This study is still limited to two samples and has not involved teachers. For further research, it is necessary to look at the aspects of students and teachers, as well as to see more specific types of learning barriers that are not included in the ontogenic obstacle, and epistemological obstacle categories. For example, the psychological barriers that students have. Our belief that intellectual research needs still should be developed in subsequent studies both on the number of samples and on the involvement of teachers. The results of this study help teachers to recognize the various intellectual needs of students who must be filled in the teacher in teaching.

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