On the DoF Region of the K-user MISO Broadcast Channel with Hybrid CSIT

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Abstract

An outer bound for the degrees of freedom (DoF) region of the K-user multiple-input single-output (MISO) broadcast channel (BC) is developed under the hybrid channel state information at transmitter (CSIT) model, in which the transmitter has instantaneous CSIT of channels to a subset of the receivers and delayed CSIT of channels to the rest of the receivers. For the 3-user MISO BC, when the transmitter has instantaneous CSIT of the channel to one receiver and delayed CSIT of channels to the other two, two new communication schemes are designed, which are able to achieve the DoF tuple of \((1, \frac{1}{3}, \frac{1}{3})\), with a sum DoF of \(\frac{5}{3}\), that is greater than the sum DoF achievable only with delayed CSIT. Another communication scheme showing the benefit of the alternating CSIT model is also developed, to obtain the DoF tuple of \((1, \frac{4}{7}, \frac{4}{7})\) for the 3-user MISO BC.

Index Terms

Hybrid CSIT, Degrees of freedom, MISO BC.
I. INTRODUCTION

The degrees of freedom (DoF) region, defined as the pre-log factor of the capacity in the high SNR regime, of the broadcast channel (BC) has received considerable attention in the past few years. Under the idealized assumption of perfect CSIT, the capacity of the Gaussian multiple input multiple output (MIMO) BC was obtained in [1], which showed that the sum-DoF of a MIMO BC with $M$ antennas at the transmitter and $N_i$ antennas at the $i$th receiver is $\min(M, \sum_i N_i)$. Later, the DoF region of the MIMO BC with no CSIT was characterized for the 2-user MIMO BC with i.i.d. Rayleigh fading in [2] and for the K-user MIMO BC for a general class of fading distributions (that includes i.i.d. Rayleigh fading as well as the more general isotropic fading model) under which the transmit directions to different receive antennas are statistically indistinguishable in [3], where it was established that the complete absence of any channel state information at the transmitter leads to a collapse of the DoF to that achievable through only time-sharing.

Investigations by Maddah-Ali and Tse in [4], into the DoF region of the MISO BC under the assumption of delayed CSIT, showed that even delayed CSIT can lead to significant gains in DoF beyond that with no CSIT, through the exploitation of the side information available at the various receivers. While further extensions of this result to the MIMO BC have been made, notably in [5] and [6], it has also sparked interest in other CSIT models, which combine both perfect CSIT and delayed CSIT in various ways. The mixed CSIT model was investigated in [7], [8], [9], where the transmitter has the same combination of delayed CSIT and an imperfect version of the instantaneous CSIT from each receiver, and the DoF region for the 2-user MISO BC was completely characterized under this model. The alternating CSIT model was introduced in [10], where the CSIT state for each receiver is allowed to alternate between no CSIT, delayed CSIT and instantaneous CSIT states, and the DoF region of the 2-user MISO BC was characterized under this model in [10].

The DoF region of the 2-user MIMO BC and the 2-user MIMO IC were respectively characterized in [11] and [12], under the hybrid CSIT model, where the CSIT about one receiver is delayed and the CSIT about the other receiver is instantaneous. In this paper, we prove an outer bound for the DoF region of the K-user MISO BC in its most general hybrid CSIT setting, with an arbitrary number of receivers in the instantaneous CSIT state and the rest of the receivers in the delayed CSIT state, and then apply this outer bound to the 3-user MISO BC, where the transmitter has instantaneous CSIT about one receiver and delayed CSIT about the other two. Although we are unable to prove that this outer bound is tight,
we provide two communication schemes for the 3-user MISO BC which achieve a DoF tuple of \((1, \frac{1}{3}, \frac{1}{3})\) for a total sum-DoF of \(\frac{5}{3}\), which exceeds the sum-DoF of \(\frac{18}{11}\) achievable with only delayed CSIT. We also illustrate another communication scheme that uses alternating CSIT to achieve the DoF tuple \((1, \frac{4}{9}, \frac{4}{9})\), which is a corner point of the DoF outer bound region derived for the hybrid CSIT model.

The DoF region of the K-user MISO BC under the alternating CSIT model was also independently investigated by [13], as was discovered by the authors after the completion of this work.

Although the hybrid CSIT model is applied to the 2-user MIMO BC in [11], the 3-user MISO BC problem is more difficult. To tackle this problem, the communication schemes developed in this paper for the 3-user BC illustrate two new ideas, layer peeling and serial interference alignment. In layer peeling, we create a structured auxiliary interference symbol which is a combination of two other interference symbols, such that this auxiliary symbol is useful in its entirety at one receiver, which does not need to decipher the constituent symbols, while simultaneously allowing another receiver to use the layered structure to peel off an already known layer and access the constituent interference symbols. Its usefulness, as is apparent from the communication schemes, lies in simultaneously providing detailed information to one receiver while hiding unnecessary details from another receiver. The communication schemes in this paper also utilize the idea of serial interference alignment, a scenario in which there exists a series of interference alignment by interference symbols, with each aligned interferencesymbol allowing the receiver to decipher the next symbol in the series, culminating in the decoding of a desired symbol.

The channel model is given in the next section, followed by theorems and proofs of the K-user and 3-user outer bounds, in Section III. Section IV explains all the communication schemes in detail, and the paper concludes with Section V.

### II. CHANNEL MODEL

We start with the hybrid CSIT model for the the 3-user MISO BC, before generalizing it to the K-user MISO BC. In the hybrid CSIT model considered here for the 3-user BC, transmitter \(T\) has 3 transmitting antennas, and the 3 receivers \(R_1, R_2\) and \(R_3\) have a single antenna each. The received outputs at the receivers \(R_1, R_2\) and \(R_3\) are given, respectively, by the following equations:

\[
Y_1(t) = H_1(t)X(t) + Z_1(t),
\]
\[
Y_2(t) = H_2(t)X(t) + Z_2(t),
\]
\[
Y_3(t) = H_3(t)X(t) + Z_3(t),
\]
where, at time $t$, $Y_i(t) \in \mathbb{C}^{1 \times 1}$ is the output at receiver $R_i$, $X(t) \in \mathbb{C}^{3 \times 1}$ is the transmitted signal, $Z_i(t)$ is the additive complex Gaussian noise at $R_i$, and $H_i(t) \in \mathbb{C}^{1 \times 3}$ is the channel from the transmitter to receiver $R_i$, $i \in \{1, 2, 3\}$. The transmitted signal has a power constraint of $P$ i.e $E(||X_i(t)||^2) \leq P$. The channel matrices $H_i(t)$ are assumed to be i.i.d. over time and independent of each other. Since, additive noise does not affect the DoF region, we disregard the noise henceforth.

The main scope of this paper is the investigation of the DoF region of the 3-user BC specified above, under the following CSI assumptions:

- Transmitter $T$ learns the channels $H_2(t)$ and $H_3(t)$ with a unit delay i.e., $H_2(t)$ and $H_3(t)$ are known at the transmitter only at time $t + 1$.
- Receivers have global CSI i.e., all receivers know all channels.

The above CSIT model is known as the hybrid CSIT model.

Let $\mathcal{M}_1, \mathcal{M}_2$ and $\mathcal{M}_3$ be the three independent messages to be sent from the transmitter to $R_1$, $R_2$ and $R_3$ respectively. A rate tuple $(R_1(P), R_2(P), R_3(P))$ is said to be achievable if there exists a codeword spanning $n$ channel uses, with a power constraint of $P$, such that the probability of error at all receivers goes to zero as $n \to \infty$, where $R_i(P) = \log(|\mathcal{M}_i|)/n$. The capacity region $C(P)$ of the BC is the region of all such achievable rate tuples, and the DoF is defined as the pre-log factor of the capacity region as $P \to \infty$ i.e.,

$$D = \left\{ (d_1, d_2, d_3) \mid d_i \geq 0 \text{ and } \exists ((R_1(P), R_2(P), R_3(P)) \in C(P)
$$

such that $d_i = \lim_{P \to \infty} \frac{\mathcal{R}_i(P)}{\log(P)}$, $i \in \{1, 2, 3\}\right\}.$

This notion can be extended to multiple-order messages i.e., common messages intended for multiple receivers, and multiple-order DoF can consequently be defined, following the approach in [4]. Let $\mathcal{M}_{ij}$ (with $(i, j) \in \{(1, 2), (1, 3), (2, 3)\}$) be an order-2 message intended for both $R_i$ and $R_j$, at a rate of $\mathcal{R}_{ij}(P)$. We define the order-2 DoF $d_{ij}$ for the receiver pair $R_i, R_j$ as

$$d_{ij} = \lim_{P \to \infty} \frac{\mathcal{R}_{ij}(P)}{\log(P)}, (i, j) \in \{(1, 2), (1, 3), (2, 3)\}.$$

The order-3 DoF $d_{123}$ is similarly defined, as the common DoF for the receivers $(R_1, R_2, R_3)$.

We now define the hybrid CSIT model for the K-user MISO BC, where the transmitter has $K$ antennas and each of the receivers $R_i$, $i \in \{1, \ldots, K\}$ has a single antenna, with the following CSI assumptions:

- Transmitter $T$ learns the channels to the first (WLOG) $K_p$ receivers instantaneously i.e., $H_1(t), \ldots, H_{K_p}(t)$
are known at the transmitter at time $t$.

- Transmitter $T$ learns the channels to the remaining $K-K_P$ receivers with unit delay i.e., $H_{K_P+1}(t), \ldots, H_K(t)$ are known at the transmitter only at time $t+1$.
- Receivers have global CSI i.e., all receivers know all channels.

Global CSI at the receivers is assumed for the remainder of the paper, and shall not be mentioned again for the sake of brevity.

The DoF region for the $K$-user MISO BC is defined analogous to the 3-user MISO BC. Let $\mathcal{M}_1, \ldots, \mathcal{M}_K$ be $K$ independent messages to be sent to $R_1, \ldots, R_K$ respectively. A rate tuple $(\mathcal{R}_1(P), \mathcal{R}_2(P), \ldots, \mathcal{R}_K(P))$ is said to be achievable if there exists a codeword with a power constraint $P$ spanning $n$ channel uses, such that the probability of error goes to zero as $n \to \infty$, with $R_i(P) = \log(|\mathcal{M}_i|)/n$. The capacity region $\mathcal{C}(P)$ is the region of all such achievable rate tuples, and the DoF is defined as the pre-log factor of the capacity region as $P \to \infty$ i.e.,

$$D = \left\{ (d_i)_{i=1}^K \middle| d_i \geq 0 \text{ and } \exists \left((\mathcal{R}_1(P), \mathcal{R}_2(P), \ldots, \mathcal{R}_K(P)) \in \mathcal{C}(P) \right) \text{ such that } d_i = \lim_{P \to \infty} \frac{\mathcal{R}_i(P)}{\log(P)}, i \in \{1, \ldots, K\} \right\}.$$  

We also define multiple-order messages e.g., $\mathcal{M}_S$ intended for all the users in the subset $S \subseteq \{1, 2, \ldots, K\}$, at a rate $\mathcal{R}_S(P)$, and the consequent multiple-order DoF $d_S$, in the same manner as before i.e.,

$$d_S = \lim_{P \to \infty} \frac{\mathcal{R}_S(P)}{\log(P)}, S \subseteq \{1, 2, \ldots, K\}.$$

### III. Outer Bound

We define the notation for the ensuing theorem here. Of the $K$ receivers in the BC, $\mathcal{E}_P := \{1, 2, \ldots, K_P\}$ is the set of all users about whose channels the transmitter has instantaneous CSIT.Now, $\{K_P+1, \ldots, K\}$ are the remaining users about whose channels the transmitter has only delayed CSIT, and $\mathcal{E}_D$ is defined to be any non-empty subset of this set i.e., $\mathcal{E}_D \subseteq \{K_P+1, \ldots, K\}, \mathcal{E}_D \neq \emptyset$. We stress a subtle notational incongruity here; while $\mathcal{E}_P$ is defined as the set of all receivers for which the transmitter has instantaneous CSIT, $\mathcal{E}_D$ is defined as a non-empty subset of the set of all receivers with delayed CSIT. Also, we shall use $\mathcal{E}_P$ to denote both the set of users $\{1, \ldots, K_P\}$ and their corresponding receivers $\{R_1, \ldots, R_{K_P}\}$, with a similar abuse of notation for $\mathcal{E}_D$ and $S \subseteq \{1, 2, \ldots, K\}$. We trust that the context will make the usage clear, without causing any confusion. $\pi_P$ and $\pi_D$ are permutation functions, that permute the sets $\mathcal{E}_P$ and
The sequences of users in $\mathcal{E}_P$ and $\mathcal{E}_D$ after applying permutation $\pi_P$ and $\pi_D$ respectively are $\{\pi_P(1), \pi_P(2), \ldots, \pi_P(K_P)\}$ and $\{\pi_D(1), \pi_D(2), \ldots, \pi_D(|\mathcal{E}_D|)\}$. For any subset $\mathcal{S}$ of receivers, the multiple-order DoF $d_{\mathcal{S}}$ has already been defined in Section II.

**Theorem 1.** An outer-bound for the DoF region of the $K$-user MISO BC with hybrid CSIT, with a combination of private (order-1) and common (multiple-order) messages, is

$$D_{\text{outer}}^{h-CSIT} = \left\{ \langle d_{\mathcal{S}} \rangle_{\mathcal{S} \subseteq \{1,2,\ldots,K\}} \right\},$$

where $\pi_P$ is any permutation of the set $\mathcal{E}_P := \{1,2,\ldots,K_P\}$ and $\pi_D$ is any permutation of the set $\mathcal{E}_D \subseteq \{K_P + 1, K_P + 2, \ldots, K\}$, $\mathcal{E}_D \neq \emptyset$.

While we provide a complete and detailed proof of the theorem later on in this section, an explanation of the above theorem and a sketch of the proof is necessary at this point. For each of the bounds described in Theorem 1, we consider a subset of receivers of the original BC, which contains $\mathcal{E}_P$ i.e., all receivers for which the transmitter has instantaneous CSIT, and a non-empty subset $\mathcal{E}_D$ of the receivers for which the transmitter has delayed CSIT. The subsets $\mathcal{E}_P$ and $\mathcal{E}_D$ are separately permuted using the permutation functions $\pi_P$ and $\pi_D$ respectively, and the resultant BC is then enhanced, by giving appropriate side information at each receiver, to create the physically degraded BC $T \rightarrow R_{\pi_P(1)} \rightarrow R_{\pi_P(2)} \rightarrow \cdots \rightarrow R_{\pi_P(K_P)} \rightarrow R_{\pi_D(1)} \rightarrow \cdots \rightarrow R_{\pi_D(|\mathcal{E}_D|)}$. In this physically degraded BC, message $\mathcal{M}_S$ is a common message meant for a subset $\mathcal{S} \subseteq \mathcal{E}_P \cup \mathcal{E}_D$. Let $R_{\pi(i^*)}$ ($\pi_P$ or $\pi_D$ as the case might be) be the receiver in the subset $\mathcal{S}$ that is last in the degraded chain, such that any message decodable at $R_{\pi(i^*)}$ is decodable by all the receivers in $\mathcal{S}$, by the degradedness of the channel. In particular, if $\mathcal{S}$ contains any receivers from $\mathcal{E}_D$ separately. The sequences of users in $\mathcal{E}_P$ and $\mathcal{E}_D$ after applying permutation $\pi_P$ and $\pi_D$ respectively are $\{\pi_P(1), \pi_P(2), \ldots, \pi_P(K_P)\}$ and $\{\pi_D(1), \pi_D(2), \ldots, \pi_D(|\mathcal{E}_D|)\}$. For any subset $\mathcal{S}$ of receivers, the multiple-order DoF $d_{\mathcal{S}}$ has already been defined in Section II.

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\( \mathcal{E}_D \), we consider the largest integer \( i^* \) such that \( R_{\pi_D(i^*)} \in S \), or else if \( S \) contains only receivers from \( \mathcal{E}_P \), we consider the largest integer \( i^* \) such that \( R_{\pi_P(i^*)} \in S \), and for simplicity, we denote this receiver as \( R_{\pi(i^*)} \). Because of the degradedness of the channel, any message that is decodable at \( R_{\pi(i^*)} \) is decodable by all the receivers in the set \( S \), thus allowing us to convert the common message \( M_S \) into a private message for the receiver \( R_{\pi(i^*)} \). For example, receiver \( R_{\pi_D(|\mathcal{E}_D|)} \) decodes all messages \( M_S \) such that \( R_{\pi_D(|\mathcal{E}_D|)} \in S \) and \( S \subseteq \mathcal{E}_P \cup \mathcal{E}_D \), \( R_{\pi_D(|\mathcal{E}_D|-1)} \) decodes all \( M_S \) such that \( R_{\pi_D(|\mathcal{E}_D|-1)} \in S \) and \( S \subseteq \mathcal{E}_P \cup \mathcal{E}_D \setminus \{ \pi_D(|\mathcal{E}_D|) \} \), and so on for all the other receivers. We use the fact, from [14], that feedback does not increase the capacity of a physically degraded BC, to remove the delayed CSIT feedback from the receivers \( \mathcal{E}_D \), and then use newly created auxiliary receivers (the details are postponed until the complete proof) to remove instantaneous CSIT about the rest of the receivers i.e., \( \mathcal{E}_P \). Thus, we end up with a physically degraded channel without any feedback, with the common messages being converted into private messages as described above. It is well known that the capacity region of a BC without feedback depends only on the marginal distributions of its outputs, and we can thus remove the coupling between the receivers in the enhanced BC, so that the receiver \( R_{\pi_P(i)} \), \( i \in \{1, 2, \ldots, K_P\} \) now has \( K_P + |\mathcal{E}_D| - i + 1 \) antennas and receiver \( R_{\pi_D(i)} \), \( i \in \{1, 2, \ldots, |\mathcal{E}_D|\} \) has \( |\mathcal{E}_D| - i + 1 \) antennas. A direct application of the results from [2] and [3], for the DoF region of the K-user MIMO BC with no CSIT, gives the inequality shown in (I). Permuting over all the possible subsets \( \mathcal{E}_D \) and permutation functions \( \pi_P \) and \( \pi_D \) gives the complete outer bound region in Theorem [I]

Remark 1. We mentioned before the incongruity in defining \( \mathcal{E}_P \) to be the whole set receivers with instantaneous CSIT while defining \( \mathcal{E}_D \) to be a subset of the receivers with delayed CSIT. While it is possible to derive additional inequalities similar to the ones shown in Theorem [I] by considering only a subset of the receivers with instantaneous CSIT instead of the whole set, a careful check will show those inequalities to be redundant. In fact, all such inequalities can be derived from the ones we already have in Theorem [I] by substituting zero for the appropriate DoF symbols.

We now illustrate the kind of DoF region obtained by applying Theorem [I] to the 3-user MISO BC with hybrid CSIT. In this model, defined previously in Section [II] the transmitter knows the channel to user 1 instantaneously and the channels to users 2 and 3 with a unit delay. We now obtain the following corollary.

Corollary 1. The DoF region of the 3-user MISO BC with hybrid CSIT, with a combination of private
(order-1) and common (order-2 and order-3) messages, is bounded by the following inequalities:

\[ 0 \leq d_1 \leq 1, \quad (2) \]
\[ 0 \leq d_2 \leq 1, \quad (3) \]
\[ 0 \leq d_3 \leq 1, \quad (4) \]
\[ 0 \leq d_{12}, d_{13}, d_{23}, d_{123} \leq 1, \quad (5) \]
\[ \frac{d_1}{2} + d_{12} + d_2 \leq 1, \quad (6) \]
\[ \frac{d_1}{2} + d_{13} + d_3 \leq 1, \quad (7) \]
\[ \frac{d_1}{3} + \frac{d_{12} + d_2}{2} + d_{123} + d_{13} + d_{23} + d_3 \leq 1, \quad (8) \]
\[ \frac{d_1}{3} + d_{123} + d_{12} + d_{23} + d_2 + \frac{d_{13} + d_3}{2} \leq 1. \quad (9) \]

We also observe that the conventional DoF region of the 3-user BC with only private messages i.e., order-1 symbols, is obtained from Theorem [1] by setting order-3 symbol i.e., \( d_{123} \) and order-2 symbols \( d_{12}, d_{13} \) and \( d_{23} \) to zero. Thus, we have the following characterization of the outer bounds for the DoF region of the 3-user MISO BC with purely private messages.

**Corollary 2.** The DoF region of the 3-user MISO BC with hybrid CSIT and only private messages is bounded by the following inequalities:

\[ d_1 \leq 1, \quad (10) \]
\[ d_2 \leq 1, \quad (11) \]
\[ d_3 \leq 1, \quad (12) \]
\[ \frac{d_1}{2} + d_2 \leq 1, \quad (13) \]
\[ \frac{d_1}{2} + d_3 \leq 1, \quad (14) \]
\[ \frac{d_1}{3} + \frac{d_2}{2} + d_3 \leq 1, \quad (15) \]
\[ \frac{d_1}{3} + d_2 + \frac{d_3}{2} \leq 1. \quad (16) \]

As a special case, the DoF region, with only private messages, of the 3-user MISO BC with hybrid CSIT
and the constraint $d_1 = 1$, is bounded by the following inequalities:

\begin{align}
    d_2 & \leq \frac{1}{2}, \\
    d_3 & \leq \frac{1}{2}, \\
    \frac{d_2}{2} + \frac{d_3}{2} & \leq \frac{2}{3}, \\
    d_2 + \frac{d_3}{2} & \leq \frac{2}{3}.
\end{align}

The $(d_1, d_2, d_3)$ outer bound is a 3-dimensional polyhedron in $\mathbb{R}^3$ space, more specifically in the cubical space bounded by the origin $(0, 0, 0)$ and the planes $d_1 = 1$, $d_2 = 1$ and $d_3 = 1$, shown in Fig. 1. To simplify matters, we focus our attention on the shape of this region in the $d_1 = 1$ plane, which is obtained by fixing $d_1 = 1$. The shape of this region is illustrated in Fig. 2.

Remark 2. Focusing on the $d_1 = 1$ plane of the DoF outer bound region translates to giving priority to receiver $R_1$ over the other two receivers, since the transmitter has the maximum information about $R_1$. In other words, we bound the DoF region while providing the maximal DoF $d_1 = 1$ to $R_1$, whose channel is known instantaneously at the transmitter.
Proof of Corollary 1

To illustrate the techniques used in the proof of the converse, we first give the proof of Corollary 1 for the 3-user case, and then generalize the proof to the K-user case to prove Theorem 1. We start with the detailed proof of the inequality (8) i.e.,

\[
\frac{d_1}{3} + \frac{d_{12} + d_2}{2} + d_{123} + d_{13} + d_{23} + d_3 \leq 1,
\]

and then outline how the rest of the bounds in Theorem 1 can be derived using a similar technique, which follows closely that of the converse proof in [11].

We now create two additional auxiliary receivers \( R' \) and \( R'' \) with 3 and 2 antennas respectively. The channels to these auxiliary receivers are \( H'(t) \) and \( H''(t) \), and the corresponding outputs are \( Y'(t) \) and \( Y''(t) \) respectively, where \( Y'(t) \in \mathbb{C}^{3 \times 1} \), \( H'(t) \in \mathbb{C}^{3 \times 3} \), \( Y''(t) \in \mathbb{C}^{2 \times 1} \) and \( H''(t) \in \mathbb{C}^{2 \times 3} \). The channels \( H'(t) \) and \( H''(t) \) are known at all the receivers, but are not known at the transmitter. The new channels are mutually independent of each other as well as the rest of the channels to the original users. The outputs for the two auxiliary receivers are given by

\[
Y'(t) = H'(t)X(t) + Z'(t)
\]
\[
Y''(t) = H''(t)X(t) + Z''(t).
\]
The additive noise $Z'(t)$ and $Z''(t)$ are complex normal and are independent of all other variables, and since additive noise does not affect the DoF region, we shall ignore it henceforth. We now impose the additional restriction on $H'(t)$ and $H''(t)$ that

$$\text{span}(H'(t)) = \text{span}[H_1(t), H_2(t), H_3(t)],$$
$$\text{span}(H''(t)) = \text{span}[H_2(t), H_3(t)].$$

We start by giving a concise description of our original BC (OBC):

- Channel outputs: $Y_1$ at $R_1$, $Y_2$ at $R_2$ and $Y_3$ at $R_3$.
- CSIT: $H_1$ is known instantaneously, $H_2$, $H_3$ are known with delay.

We enhance the OBC by providing side information $(Y_2, Y_3)$ to $R_1$ and $Y_3$ to $R_2$. This creates a physically degraded BC, described below:

- Channel outputs: $(Y_1, Y_2, Y_3)$ at $R_1$, $(Y_2, Y_3)$ at $R_2$ and $Y_3$ at $R_3$.
- CSIT: $H_1$ is known instantaneously, $H_2, H_3$ are known with delay.

In this $T \to R_1 \to R_2 \to R_3$ physically degraded BC, we consider the common/order-2 message $\mathcal{M}_{13}$. Because of the degraded nature of the BC, $R_1$ can now decode any message that $R_3$ is able to decode. Thus it is sufficient that $\mathcal{M}_{13}$ be decodable at $R_3$. Similarly, it is sufficient that the common messages $\mathcal{M}_{12}$ be decipherable at $R_2$ and $\mathcal{M}_{123}$ and $\mathcal{M}_{23}$ be decipherable at receiver $R_3$. Hence, we can now convert the common message $\mathcal{M}_{12}$ into a private message for $R_2$, and the common messages $\mathcal{M}_{13}, \mathcal{M}_{23}$ and $\mathcal{M}_{123}$ into private messages for receiver $R_3$.

It is known from [14] that feedback does not improve the capacity region of a physically degraded BC, a fact which allows us to remove the delayed feedback links from our degraded BC, to obtain the following BC:

- Channel outputs: $(Y_1, Y_2, Y_3)$ at $R_1$, $(Y_2, Y_3)$ at $R_2$ and $Y_3$ at $R_3$.
- CSIT: $H_1$ is known instantaneously, $H_2, H_3$ are unknown.

We now provide our auxiliary output $Y'$ to $R_1$ and $Y''$ to $R_2$, thus enhancing our BC to:

- Channel outputs: $(Y_1, Y_2, Y_3, Y')$ at $R_1$, $(Y_2, Y_3, Y'')$ at $R_2$ and $Y_3$ at $R_3$.
- CSIT: $H_1$ is known instantaneously, $H_2, H_3, H', H''$ are unknown.

The constraint we imposed earlier that $\text{span}(H'(t)) = \text{span}[H_1(t), H_2(t), H_3(t)]$ shows that it is possible for receiver $R_1$ to calculate the outputs $(Y_1, Y_2, Y_3)$ from its knowledge of $Y'$ and $H_1, H_2, H_3, H'$. Similarly,
$R_2$ can calculate outputs $(Y_2, Y_3)$ from its knowledge of $Y''$ and $H_2, H_3$ and $H''$. Thus, our enhanced BC is now equivalent to the following BC:

- Channel outputs: $Y'$ at $R_1$, $Y''$ at $R_2$ and $Y_3$ at $R_3$.
- CSIT: $H_1$ is known instantaneously, $H_2, H_3, H', H''$ are unknown.

We now have an equivalent BC with three outputs $(Y', Y'', Y_3)$, none of whose channel gains are known at the transmitter. The transmitter thus has no need of the knowledge of $H_1$, and our enhanced BC becomes:

- Channel outputs: $Y'$ at $R_1$, $Y''$ at $R_2$ and $Y_3$ at $R_3$.
- CSIT: $H', H'', H_3$ are unknown.

Thus, in our final version, we have a 3-user MIMO BC with no CSIT, 3 antennas at the transmitter and 3 antennas (from our construction of $Y'$) at $R_1$, 2 antennas (our construction of $Y''$) at $R_2$ and 1 antenna at $R_3$. Also, as per the discussion above, in addition to their original private messages, $T_x$ has a private message $M_{12}$ for $R_2$ and private messages $M_{13}, M_{23}$ and $M_{123}$ for $R_3$. From [2] and [3], we know that the DoF region of this MIMO BC with no CSIT is given by

\[
\frac{d_1}{\min(3, 3)} + \frac{d_{12} + d_2}{\min(3, 2)} + \frac{d_{123} + d_{13} + d_{23} + d_3}{\min(3, 1)} \leq 1,
\]

thus proving inequality (8) i.e.,

\[
\frac{d_1}{3} + \frac{d_{12} + d_2}{2} + d_{123} + d_{13} + d_{23} + d_3 \leq 1.
\]

Similarly, by creating the physically degraded BC $T \rightarrow R_1 \rightarrow R_3 \rightarrow R_2$, and converting the common messages $M_{123}, M_{12}, M_{23}$ into private messages for $R_2$ and $M_{13}$ into a private message for $R_3$, we prove inequality (9),

\[
\frac{d_1}{3} + d_{123} + d_{12} + d_{23} + d_2 + \frac{d_{13} + d_3}{2} \leq 1.
\]

The proof of the other inequalities in Theorem 1 follows a similar reasoning. Inequalities (2)-(4) are just the MIMO outer bounds for a single antenna receiver, while inequalities (6)-(7) are proven by using the above technique on the receiver pairs $(R_1, R_2)$ and $(R_1, R_3)$ respectively. Since the reasoning is so similar, we do not mention it explicitly. We note that the 2-user outer bound obtained by considering only receivers $R_2$ and $R_3$ is redundant, and is obtained from inequalities (8)-(9) by setting $d_1, d_{12}, d_{13}, d_{123} = 0$. 

We now prove Theorem 1 in all its generality. As defined in the Section (III), \( \mathcal{E}_P := \{1, 2, \ldots, K_P\} \) is the set of users about whom the transmitter has instantaneous CSIT, while \( \{K_P + 1, \ldots, K\} \) are the remaining users about whom the transmitter has only delayed CSIT available, and \( \mathcal{E}_D \) is now defined to be any non-empty subset of this latter set i.e., receivers with delayed CSIT. \( \pi_P \) and \( \pi_D \) are permutation functions, that permute the sets \( \mathcal{E}_P \) and \( \mathcal{E}_D \) respectively. We now consider the BC consisting of receivers \( \mathcal{E}_P \cup \mathcal{E}_D \) i.e., a combination of ALL receivers for which instantaneous CSIT exists and the subset \( \mathcal{E}_D \) of receivers for which the transmitter has delayed CSIT, and give below a concise description of this BC:

- Channel outputs: \( Y_i \) at \( R_i, i \in \mathcal{E}_P \cup \mathcal{E}_D \),
- CSIT: \( H_i \forall i \in \mathcal{E}_P \) known instantaneously , \( H_j \forall j \in \mathcal{E}_D \) known with delay.

We now consider a permutation \( \pi_P \) of the set \( \mathcal{E}_P \) and a permutation \( \pi_D \) of the set \( \mathcal{E}_D \), and enhance the BC by providing the output of receiver \( R_{\pi_D(i)} \) to receivers \( R_{\pi_D(i-1)}, R_{\pi_D(i-2)}, \ldots, R_{\pi_D(1)}, R_{\pi_P(K_P)}, \ldots, R_{\pi_P(1)} \), \( \forall i, 1 \leq i \leq |\mathcal{E}_D| \), and the output of receiver \( R_{\pi_P(i)} \) to receivers \( R_{\pi_P(i-1)}, R_{\pi_P(i-2)}, \ldots, R_{\pi_P(1)} \), \( \forall i, 1 \leq i \leq K_P \). This creates a physically degraded BC, described below:

- Channel outputs: \( (Y_{\pi_P(1)}, \ldots, Y_{\pi_P(K_P)}, Y_{\pi_D(1)}, \ldots, Y_{\pi_D(|\mathcal{E}_D|)}) \) at \( R_{\pi_P(1)}, \ldots, (Y_{\pi_D(1)}, \ldots, Y_{\pi_D(|\mathcal{E}_D|)}) \) at \( R_{\pi_D(1)}, \ldots, (Y_{\pi_D(|\mathcal{E}_D|)}) \) at \( R_{\pi_D(|\mathcal{E}_D|)} \),
- CSIT: \( H_{\pi_P(i)} \forall i \in \mathcal{E}_P \) are known instantaneously , \( H_{\pi_D(j)} \forall j 1 \leq j \leq |\mathcal{E}_D| \) are known with delay.

In this \( T \rightarrow R_{\pi_P(1)} \rightarrow R_{\pi_P(2)} \rightarrow \cdots \rightarrow R_{\pi_P(K_P)} \rightarrow R_{\pi_D(1)} \rightarrow \cdots \rightarrow R_{\pi_D(|\mathcal{E}_D|)} \) physically degraded BC, let \( \mathcal{M}_S \) be the multiple-order message intended for all receivers in a set \( S \subseteq \mathcal{E}_P \cup \mathcal{E}_D \). If \( S \) contains any receivers from \( \mathcal{E}_D \), we consider the largest integer \( i^* \) such that \( R_{\pi_D(i^*)} \in S \), or else if \( S \) contains only receivers from \( \mathcal{E}_P \), we consider the largest integer \( i^* \) such that \( R_{\pi_P(i^*)} \in S \). For simplicity, we denote this receiver as \( R_{\pi(i^*)} \). Because of the degradedness of the channel, any message that is decodable at \( R_{\pi(i^*)} \) is decodable by all the receivers in the set \( S \), thus allowing us to convert the common message \( \mathcal{M}_S \) into a private message for the receiver \( R_{\pi(i^*)} \). For example, receiver \( R_{\pi_D(|\mathcal{E}_D|)} \) decodes all messages \( \mathcal{M}_S \) such that \( R_{\pi_D(|\mathcal{E}_D|)} \in S \) and \( S \subseteq \mathcal{E}_P \cup \mathcal{E}_D \), \( R_{\pi_D(|\mathcal{E}_D|-1)} \) decodes all \( \mathcal{M}_S \) such that \( R_{\pi_D(|\mathcal{E}_D|-1)} \in S \) and \( S \subseteq \mathcal{E}_P \cup \mathcal{E}_D \setminus \{\pi_D(|\mathcal{E}_D|)\} \) and so on for all the other receivers.

It is known from [14] that feedback does not improve the capacity region of a physically degraded BC. This allows us to remove the delayed feedback links from our degraded BC, obtaining the following “enhanced” BC:

- Channel outputs: \( (Y_{\pi_P(1)}, Y_{\pi_P(2)}, \ldots, Y_{\pi_P(K_P)}, Y_{\pi_D(1)}, \ldots, Y_{\pi_D(|\mathcal{E}_D|)}) \) at \( R_{\pi_P(1)}, \ldots, (Y_{\pi_D(1)}, \ldots, Y_{\pi_D(|\mathcal{E}_D|)}) \)
at $R_{\pi D(1)}$ \ldots, \(Y_{\pi D(1)}\)) at $R_{\pi D(1)}$,

- CSIT: $H_{\pi P(i)} \forall i \in \mathcal{E}_P$ known instantaneously, $H_{\pi D(j)} \forall j \leq |\mathcal{E}_D|$ unknown.

We now create $K_P + |\mathcal{E}_D| - 1$ additional auxiliary receivers $R_{\pi P(1)}', \ldots, R_{\pi P(K_P)}', R_{\pi D(1)}', \ldots, R_{\pi D(|\mathcal{E}_D|-1)}'$, the first receiver $R_{\pi P(1)}'$ with $K_P + |\mathcal{E}_D|$ antennas, and each consecutive receiver having one less antenna than its predecessor. The outputs at the receivers are denoted as $Y_{\pi P(1)}', \ldots, Y_{\pi P(K_P)}', Y_{\pi D(1)}', \ldots, Y_{\pi D(|\mathcal{E}_D|-1)}'$, and the corresponding channels are similarly labeled $H'_{\pi P(1)}', \ldots, H'_{\pi P(K_P)}', H'_{\pi D(1)}', \ldots, H'_{\pi D(|\mathcal{E}_D|-1)}$, the time indices being suppressed for brevity. The new channels are mutually independent of each other as well as the rest of the channels to the original receivers, and are known instantaneously at all the receivers, but are unknown at the transmitter. We now impose the following restriction on each of the newly created channels:

\[
\begin{align*}
\text{span} \left( H'_{\pi P(1)}(t) \right) &= \text{span} \left[ H_{\pi P(1)}(t), H_{\pi P(2)}(t), \ldots, H_{\pi P(K_P)}(t), H_{\pi D(1)}(t), \ldots, H_{\pi D(|\mathcal{E}_D|)}(t) \right] \quad (21) \\
\text{span} \left( H'_{\pi P(2)}(t) \right) &= \text{span} \left[ H_{\pi P(2)}(t), \ldots, H_{\pi P(K_P)}(t), H_{\pi D(1)}(t), \ldots, H_{\pi D(|\mathcal{E}_D|)}(t) \right], \quad (22) \\
&\vdots \quad (23) \\
\text{span} \left( H'_{\pi D(1)}(t) \right) &= \text{span} \left[ H_{\pi D(1)}(t), \ldots, H_{\pi D(|\mathcal{E}_D|)}(t) \right], \quad (24) \\
&\vdots \quad (25) \\
\text{span} \left( H'_{\pi D(|\mathcal{E}_D|-1)}(t) \right) &= \text{span} \left[ H_{\pi D(|\mathcal{E}_D|-1)}(t), H_{\pi D(|\mathcal{E}_D|)}(t) \right]. \quad (26)
\end{align*}
\]

We provide the auxiliary output $Y'_{\pi P(i)}(t)$ to the receiver $\pi_P(i), \forall i \in \mathcal{E}_P$, and the auxiliary output $Y'_{\pi D(i)}(t)$ to the receiver $R_{\pi D(i)}, \forall i \in \mathcal{E}_D \setminus \{\pi_D(1)\}$. Thus, the BC is now enhanced to the following state:

- Channel outputs: \(\left( Y_{\pi P(1)}, \ldots, Y_{\pi D(1)}, \ldots, Y_{\pi D(|\mathcal{E}_D|)}, Y'_{\pi P(1)} \right)\) at $R_{\pi P(1)}$, \ldots, \(\left( Y_{\pi D(1)}, \ldots, Y_{\pi D(|\mathcal{E}_D|)}, Y'_{\pi D(1)} \right)\) at $R_{\pi D(1)}$, \ldots, \(\left( Y_{\pi D(|\mathcal{E}_D|-1)}, Y_{\pi D(|\mathcal{E}_D|)}, Y'_{\pi D(|\mathcal{E}_D|-1)} \right)\) at $R_{\pi D(|\mathcal{E}_D|)}, \left( Y_{\pi D(|\mathcal{E}_D|)} \right)$ at $R_{\pi D(|\mathcal{E}_D|)}$,

- CSIT: $H_{\pi P(i)} \forall i \in \mathcal{E}_P$ are known instantaneously, $H_{\pi D(j)} \forall j \leq |\mathcal{E}_D|$ unknown, $H'_{\pi P(i)} \forall i \in \mathcal{E}_P$ are unknown, $H'_{\pi D(j)} \forall j \leq |\mathcal{E}_D| - 1$ are unknown.

The constraints we imposed earlier on the span of the auxiliary channels in equations $21, 26$ show that the output at the $K_P + |\mathcal{E}_D|$ antennas of $Y'_{\pi P(1)}$ allow receiver $\pi_P(1)$ to calculate \(\left( Y_{\pi P(1)}, \ldots, Y_{\pi D(1)}, \ldots, Y_{\pi D(|\mathcal{E}_D|)} \right)\) from its knowledge of $Y'_{\pi P(1)}$ and $H'_{\pi P(1)}(t), H'_{\pi P(2)}(t), \ldots, H'_{\pi P(K_P)}(t), H'_{\pi D(1)}(t), \ldots, H'_{\pi D(|\mathcal{E}_D|)}(t)$ and $H'_{\pi P(1)}$. A similar reasoning follows for the rest of the receivers in the enhanced BC, and the enhanced BC takes the form:
• Channel outputs: \( Y'_{\pi_P(1)} \) at \( R_{\pi_P(1)} \), \( Y'_{\pi_P(2)} \) at \( R_{\pi_P(2)} \), \ldots, \( Y'_{\pi_D(1)} \) at \( R_{\pi_D(1)} \), \ldots, \( Y'_{\pi_D(|E_D|-1)} \) at \( R_{\pi_D(|E_D|)} \) at \( R_{\pi_D(|E_D|)} \).

• CSIT: \( H_{\pi_P(i)} \) \( \forall i \in E_P \) known instantaneously , \( H_{\pi_D(j)} \) \( \forall j \leq |E_D| \) unknown, \( H'_{\pi_P(i)} \) \( \forall i \in E_P \) unknown, \( H'_{\pi_D(j)} \) \( \forall j \leq |E_D| - 1 \) unknown.

We have thus created an equivalent BC with outputs \( \{ Y'_{\pi_P(1)} ; \ldots, Y'_{\pi_P(K_P)} ; Y'_{\pi_D(1)} ; \ldots, Y'_{\pi_D(|E_D|-1)} ; Y_{\pi_D(|E_D|)} \} \), none of whose channel gains are known at the transmitter. In this setting, the transmitter thus has no need for its instantaneous knowledge of \( H_{\pi_P(i)} \), \( i \in E_P \), thus making out enhanced BC equivalent to the following final BC:

• Channel outputs: \( Y'_{\pi_P(1)} \) at \( R_{\pi_P(1)} \), \( Y'_{\pi_P(2)} \) at \( R_{\pi_P(2)} \), \ldots, \( Y'_{\pi_D(1)} \) at \( R_{\pi_D(1)} \), \ldots, \( Y'_{\pi_D(|E_D|-1)} \) at \( R_{\pi_D(|E_D|)} \), \( Y_{\pi_D(|E_D|)} \) at \( R_{\pi_D(|E_D|)} \).

• CSIT: \( H'_{\pi_D(j)} \) \( \forall j \leq |E_D| \) unknown, \( H'_{\pi_P(i)} \) \( \forall i \in E_P \) unknown.

In this final version, we have a \( K_P + |E_D| \)-user MIMO BC with no CSIT, \( K \) antennas at the transmitter, with \( K_P + |E_D| - i + 1 \) antennas at receiver \( \pi_P(i) \), \( i \in E_P \) and \( |E_D| - i + 1 \) antennas at receiver \( \pi_D(i) \), \( i \in E_D \), with private messages \( M_S \) as described above. From [2] and [3], we know that the DoF region of this MIMO BC with no CSIT is given by

\[
\sum_{i=1}^{K_P} \min(K, K_P + |E_D| - i + 1) \sum_{\mathcal{S} \subseteq E_P \setminus \{ \pi_P(i+1), \ldots, \pi_P(K_P) \} \pi_P(i) \in \mathcal{S}} d_S + \sum_{i=1}^{|E_D|} \min(K, |E_D| - i + 1) \sum_{\mathcal{S} \subseteq E_P \cup E_D \setminus \{ \pi_D(i+1), \ldots, \pi_D(|E_D|) \} \pi_D(i) \in \mathcal{S}} d_S \leq 1. \quad (27)
\]

Allowing for all possible non-empty subsets \( E_D \) of \( \{ K_P + 1, \ldots, K \} \) and all possible permutations \( \pi_P \) and \( \pi_D \) of \( E_P \) and \( E_D \), respectively, in inequality (27) gives Theorem [1].

IV. NEW COMMUNICATION SCHEMES

A. A Scheme achieving \( \left( 1, \frac{1}{3}, \frac{1}{3} \right) \) DoF

In Fig. 3, we present an interference alignment scheme that achieves the \( \left( 1, \frac{1}{3}, \frac{1}{3} \right) \) DoF tuple for the 3-user MISO BC under the hybrid CSIT model. We show the achievability of this DoF tuple by coding over
Note: $\perp$ denotes zero-forcing at $R_1$ of the adjoining data symbol.

Figure 3. Interference alignment scheme for achieving $(1, \frac{1}{3}, \frac{1}{3})$ DoF tuple for the 3-user MISO BC under the Hybrid CSIT model.

6 time slots, during which the transmitter sends 6 independent data symbols (DSs) $u_1, u_2, u_3, u_4, u_5, u_6$ to receiver $R_1$, 2 independent DSs $v_1$ and $v_2$ to $R_2$ and 2 independent DSs $w_1$ and $w_2$ to $R_3$, with all the data symbols being successfully decoded at their intended receivers. In the following paragraphs, we explain the transmission and decoding strategy at each time slot in detail. For the sake of clarity, we divide the complete scheme into two phases, an initial data dissemination phase and a subsequent data disambiguation phase.

**Data Dissemination Phase**

At $t = 1$, the transmitter transmits 3 DSs $u_1, u_2$ and $u_3$ intended for $R_1$ and 2 DSs $v_1$ and $v_2$ intended for $R_2$. Since the transmitter has perfect knowledge of the channel to $R_1$, it utilizes this knowledge to zero-force $v_1$ and $v_2$ at $R_1$. More precisely, $T_x$ transmits the following signal

$$X(1) = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + B(1) \begin{bmatrix} 0 \\ v_1 \\ v_2 \end{bmatrix},$$

where $B(1)$ is the pre-coding matrix that performs transmit beamforming in the null space of $H_1(1)$ i.e., $H_1(1)B(1) = 0$. In Fig. 3, this zero-forcing at $R_1$ is denoted by a $\perp$ sign besides the symbol that is
zero-forced, in this case $v_1$ and $v_2$. The outputs at the three receivers are as follows:

$$ Y_1(1) = H_1(1) \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \triangleq l_1(u_1, u_2, u_3), $$

$$ Y_2(1) = H_2(1) \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + H_2(1)B(1) \begin{bmatrix} 0 \\ v_1 \\ v_2 \end{bmatrix} \triangleq l_2(u_1, u_2, u_3) + m_1(v_1, v_2), $$

$$ Y_3(1) = H_3(1) \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + H_3(1)B(1) \begin{bmatrix} 0 \\ v_1 \\ v_2 \end{bmatrix} \triangleq l_3(u_1, u_2, u_3) + m_2(v_1, v_2). $$

At $t = 2$, the transmitter transmits DSs $u_4, u_5$ and $u_6$ intended for $R_1$ and $w_1, w_2$ intended for $R_3$, again using a zero-forcing transmission strategy similar to the one used in the previous time slot. $w_1$ and $w_2$ are zero-forced at $R_1$ using a beamforming matrix $B(2)$, such that $H_1(2)B(2) = 0$. The outputs at the three receivers are as follows:

$$ Y_1(2) = l_4(u_4, u_5, u_6), $$

$$ Y_2(2) = l_5(u_4, u_5, u_6) + n_1(w_1, w_2), $$

$$ Y_3(2) = l_6(u_4, u_5, u_6) + n_2(w_1, w_2). $$

We note here that Fig. 3 depicts the same linear combinations (LCs) $l_1, \ldots, l_6$, $m_1, m_2$ and $n_1, n_2$, without mentioning their underlying DSs $u_i, v_i$ and $w_i$ respectively. By creating the convention that the symbol $l$ e.g. $l_1, l_2$ etc. shall henceforth denote linear combinations of DSs $u_i$; $m_1, m_2$ shall denote linear combinations of $v_i$ and $n_1, n_2$ shall denote linear combinations of $w_i$, we dispense with the need to mention the underlying DSs while mentioning the relevant linear combinations. We stress the fact that all the linear combinations are almost surely independent, owing to the generic and independent nature of the channel matrices assumed at the outset. For the sake of brevity, we will not mention the channel or beamforming matrices henceforth, and directly deal with the linear combinations created at the receivers.
At the end of the Data Flooding phase, there are 6 LCs i.e., \( l_1, \ldots, l_6 \) of the 6 DSs \( u_1, \ldots, u_6 \) intended for \( R_1 \), of which \( R_1 \) has knows only \( l_1 \) and \( l_4 \), without any interference. Thus, \( R_1 \) needs to obtain the remaining four LCs in the next phase, so that it can have six independent linear combinations of \( u_1, \ldots, u_6 \), which would be sufficient to decode all of its intended DSs. Sending the LCs \( l_2, l_3, l_5 \) and \( l_6 \) to \( R_1 \) shall therefore be one of the goals of the next phase.

The situation at \( R_2 \) is a bit more complicated. Of the two LCs \( m_1 \) and \( m_2 \) that \( R_2 \) can use to decode its intended DSs, \( R_2 \) observes only \( m_1 \) in this phase, at \( t = 1 \), but with an added interference \( l_2 \). Thus, if \( R_2 \) learns the LC \( l_2 \) in the next phase, it can cancel out this interference and consequently obtain its desired LC \( m_1 \). It is to be noted that the LC \( l_2 \) therefore plays a dual role, helping \( R_1 \) to decode its intended DSs while simultaneously aiding \( R_2 \) to cancel out interference from its received signal. \( R_2 \) also needs to be learn its second LC \( m_2 \) in the next phase. Thus, another goal of the next phase shall be to ensure that \( R_2 \) learns both \( l_2 \) and \( m_2 \). Similarly, in the next phase, \( R_3 \) needs to learn \( l_6 \) (to cancel out the interference from \( Y_3(2) \) and obtain \( n_2 \)) as well as the LC \( n_1 \).

**Data Disambiguation Phase**

At \( t = 3 \), the transmitter sends two symbols \( l_2 \) and \( l_5 + n_1 \) (the knowledge of which it has at \( t = 3 \) due to delayed CSIT of the channel to \( R_2 \) at times 1 and 2). \( l_5 + n_1 \) is transmitted in the null space of the channel to \( R_1 \) at \( t = 3 \) (made possible by the instantaneous CSIT from \( R_1 \)), thus allowing \( R_1 \) to learn \( l_2 \) without any interference. \( R_2 \) gets a linear combination of \( l_2 \) and \( l_5 + n_1 \), from which it cancels the contribution of \( l_5 + n_1 \)(of which it has prior knowledge from \( t = 2 \)) and acquires \( l_2 \). \( R_3 \), on the other hand, sees a linear combination of \( l_2 \) and \( l_5 + n_1 \), from which we group the interference due to user 1 DSs into the the auxiliary symbol \( k_1 = \alpha_1 l_2 + \beta_1 l_5 \) (with \( \alpha_1 \) and \( \beta_1 \) depending on the channel from transmitter to \( R_3 \) at \( t = 3 \)), so that \( Y_3(3) = k_1 + \beta_1 n_1 \), as shown in Fig. 3.

From our discussion earlier, we recall that \( l_2 \) is useful at both \( R_1 \) and \( R_2 \), simultaneously providing a desired LC to \( R_1 \) and allowing \( R_2 \) to acquire its required DS \( m_1 \) by interference cancellation. Another way of interpreting this is to observe that \( l_2 \), while directly useful as a LC at \( R_1 \), aligns with the interference already present at \( R_2 \) at \( t = 1 \). We use another layer of interference alignment to motivate our use of the “composite” interference symbol \( l_5 + n_1 \) at \( t = 3 \). The symbol \( l_5 + n_1 \) provides \( R_3 \) with its desired LC \( n_1 \)(albeit with an interference \( k_1 \)), and at the same time aligns with the interference seen at \( R_2 \) at \( t = 2 \). This interference alignment of \( l_5 + n_1 \) at \( R_2 \) allows \( R_2 \) to acquire \( l_2 \) (by canceling out the interference \( l_5 + n_1 \) at \( t = 3 \)), which in turn is now aligned with the interference at \( R_2 \) at \( t = 1 \). We thus have a series
of interference alignment by symbols, in this case \( l_5 + n_1 \rightarrow l_2 \rightarrow m_1 \), where each aligned interference symbol allows the receiver (here \( R_2 \)) to decipher the next symbol in the series, which in turn is also aligned at the same receiver, finally culminating in the decoding of a desired symbol, here \( m_1 \), at that receiver. We call this idea serial interference alignment.

The idea of serial interference alignment is repeated at \( t = 4 \), this time focusing on aligning the interference at \( R_3 \). The transmitter sends \( l_6 \) and \( l_3 + m_2 \), the latter in the null space of the channel to \( R_1 \) at \( t = 4 \). This allows \( R_1 \) to learn \( l_6 \) without any interference, while \( R_3 \) uses its prior knowledge of \( l_3 + m_2 \) (from the output at \( t = 1 \)) to subtract out its contribution and learn \( l_6 \). \( R_2 \) sees a linear combination of \( l_6 \) and \( l_3 + m_2 \), which we write in terms of the auxiliary symbol \( k_2 = \alpha_2 l_6 + \beta_2 l_3 \), once again combining the interference due to user 1 DSs and \( \alpha_2 \) and \( \beta_2 \) depending on the channel to \( R_2 \) at \( t = 4 \), so that \( Y_2(4) = k_2 + \beta_2 m_2 \). The serial interference alignment chain \( l_3 + m_2 \rightarrow l_6 \rightarrow n_2 \) allows \( R_3 \) to successfully obtain its desired LC \( n_2 \).

At time \( t = 5 \), the transmitter sends \( k_1 \) and at time \( t = 2 \) it sends \( k_2 \), both of which are linear combinations of symbols of user 1. Upon receiving these, \( R_1 \) is able to cancel the contribution of \( l_2 \) in \( k_1 \) and \( l_6 \) in \( k_2 \) to obtain the two remaining LCs \( l_5 \) and \( l_3 \) for a total of 6 linear combinations \( l_1, \ldots, l_6 \), thus allowing it to decode all its DSs \( u_1, \ldots, u_6 \). Because \( R_2 \) obtains \( k_2 \) at \( t = 6 \) it can subtract its contribution to \( Y_2(4) \) and obtain the second linear combination \( m_2 \) it desires, so that along with its knowledge of \( m_1 \) it can decode \( v_1 \) and \( v_2 \). Similarly, by obtaining \( k_1 \) at \( t = 5 \), \( R_3 \) can subtract its contribution to \( Y_3(3) \) and obtain the second linear combination \( n_1 \) it desires so that along with its knowledge of \( n_2 \) it can decode all its DSs \( w_1 \) and \( w_2 \).

The motivation behind grouping the interference from user 1 DSs into auxiliary interference symbol \( k_1 \) and \( k_2 \) becomes clear now. We observe that at \( t = 5 \), \( R_3 \) needed only a particular linear combination of \( l_2 \) and \( l_5 \) i.e., \( k_1 \), without incurring the additional expense of learning \( l_2 \) and \( l_5 \) individually, to cancel out the interference \( k_1 \) from \( Y_3(3) \) and obtain its desired LC \( n_1 \). At the same time, \( R_1 \) also utilized \( k_1 \) by using its previous knowledge of \( l_2 \) (from \( Y_1(3) \)) to peel off \( l_2 \) from \( k_1 \) and access its desired LC \( l_5 \). Thus, the motivation behind creating the auxiliary interference symbol i.e., \( k_1 \) was to have a structured symbol which is useful in its entirety at one receiver (\( R_3 \)), but whose additional layered structure simultaneously allows another receiver (\( R_1 \)) to peel off already known layers (\( l_2 \)) and access more detailed useful information (\( l_5 \)). We name this idea as layer peeling. Its usefulness, as is apparent from this communication scheme, lies in simultaneously providing detailed information to one receiver while hiding unnecessary details.
## B. Alternate Scheme for achieving $(1, \frac{1}{3}, \frac{1}{3}) \text{ DoF}$

Figure 4 depicts another communication scheme for the same $(1, \frac{1}{3}, \frac{1}{3}) \text{ DoF}$ tuple. Because of its similarity to the previous communication scheme, we give here a brief description, emphasizing only the aspects in which it differs from the former scheme.

As before, we show the achievability of this DoF tuple by coding over 6 time slots, by sending 6 independent DSs $u_1, u_2, u_3, u_4, u_5, u_6$ to receiver $R_1$, 2 independent DSs $v_1$ and $v_2$ to $R_2$ and 2 independent DSs $w_1$ and $w_2$ to $R_3$, such that all DSs are successfully decoded at their intended receivers. Once again, we divide the complete scheme into two phases, an initial data dissemination phase and a subsequent data disambiguation phase.

### Data Dissemination Phase

The major difference between the present scheme and the previous one lies in the data dissemination phase. While previously the DSs intended for $R_2$ ($v_1, v_2$) and the DSs intended for $R_3$ ($w_1, w_2$) were transmitted during separate time slots $t = 1$ and $t = 2$ respectively, this scheme transmits $v_1, v_2, w_1, w_2$ along with $u_1, u_2, u_3$ at $t = 1$, thus causing a different interference pattern at the receivers. The transmitter uses its instantaneous knowledge of the channel to $R_1$ to send $v_1, v_2, w_1, w_2$ in the null space of $H_1$. More
precisely, the following signal is transmitted,

\[ X(1) = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + B(1) \begin{bmatrix} 0 \\ v_1 \\ v_2 \end{bmatrix} + C(1) \begin{bmatrix} 0 \\ w_1 \\ w_2 \end{bmatrix}, \]

where \( B(1) \) and \( C(1) \) are the pre-coding matrices that perform transmit beamforming in the null space of \( H_1(1) \) i.e., \( H_1(1)B(1) = 0 \) and \( H_1(1)C(1) = 0 \). The outputs at the three receivers are denoted as follows:

\[ Y_1(1) = H_1(1) \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \]

\[ \triangleq l_1(\mathbf{u}), \]

\[ Y_2(1) = H_2(1) \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + H_2(1)C(1) \begin{bmatrix} 0 \\ w_1 \\ w_2 \end{bmatrix} + H_2(1)B(1) \begin{bmatrix} 0 \\ v_1 \\ v_2 \end{bmatrix}, \]

\[ \triangleq (l_2(\mathbf{u}) + n_1(\mathbf{w})) + m_1(\mathbf{v}), \]

\[ Y_3(1) = H_3(1) \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + H_3(1)B(1) \begin{bmatrix} 0 \\ v_1 \\ v_2 \end{bmatrix} + H_3(1)C(1) \begin{bmatrix} 0 \\ w_1 \\ w_2 \end{bmatrix}, \]

\[ \triangleq (l_3(\mathbf{u}) + m_2(\mathbf{v})) + n_2(\mathbf{w}). \]

For further simplification, the interference terms are grouped in brackets at the receivers \( R_2 \) and \( R_3 \) in Fig. 4. We stick to the previous notation of denoting the linear combinations (LCs) of \( u_i \) by the symbol \( l \), \( v_i \) by the symbol \( m \) and \( w_i \) by the symbol \( n \) respectively. We also note that all the linear combinations are almost surely independent, owing to the generic and independent nature of the channel matrices.

At \( t = 2 \), the transmitter transmits the symbols \( u_4, u_5 \) and \( u_6 \), thus creating the independent linear combinations \( l_4, l_5 \) and \( l_6 \) at receivers \( R_1, R_2 \) and \( R_3 \) respectively. In the next phase, \( R_1 \) needs the LCs \( l_2, l_3, l_5 \) and \( l_6 \), while \( R_2 \) needs to learn the LC \( m_2 \) and the composite interference term \( l_2 + n_1 \). Similarly, \( R_3 \) needs \( n_1 \) and the composite interference term \( l_3 + m_2 \) in the next phase.

**Data Disambiguation Phase**

At \( t = 3 \), the transmitter sends two symbols \( l_2 + n_1 \) (the knowledge of which it has at \( t = 3 \)
due to delayed CSIT of channel to $R_2$ at time 1), the latter in the null space of the channel from the transmitter to $R_1$ at $t = 3$ (since the receiver has instantaneous knowledge of the channel to $R_1$). $R_1$ acquires its desired linear combination $l_5$, while $R_2$ sees a linear combination of $l_5$ and $l_2 + n_1$, from which it removes the contribution of $l_5$ (which it learnt at $t = 2$) to acquire $l_2 + n_1$. $R_3$ on the other hand observes a linear combination of $l_5$ and $l_2 + n_1$, which we write in terms of an auxiliary symbol $k_1 = \alpha_1 l_5 + \beta_1 l_2$ (i.e., the interference due to user 1 DSs, $\alpha_1$ and $\beta_1$ dependent on the channel to $R_3$ at $t = 3$) so that $Y_3(3) = k_1 + \beta_1 n_1$.

The transmission strategy is similar at $t = 4$, where the symbols $l_6$ and $l_3 + m_2$ are transmitted, the latter in the null space of the channel to $R_1$. $R_1$ acquires $l_6$, and $R_3$ uses its previous knowledge of $l_6$ to obtain $l_3 + m_2$. The situation at $R_2$ parallels that of $R_3$ at $t = 3$, where we let $Y_2(4) = k_2 + \beta_2 m_2$, with $k_2 = \alpha_2 l_6 + \beta_2 m_2$ (i.e., interference due to user 1’s DSs with $\alpha_2$ and $\beta_2$ depending on channel from transmitter to $R_2$ at $t = 4$).

Thus, at $t = 3, 4$, the transmitter sends 2 useful linear combinations $l_5$ and $l_6$ to $R_1$, the composite interference symbols $l_2 + n_1$ and $l_3 + m_2$ to receivers $R_2$ and $R_3$ respectively, which then use this knowledge to cancel out the interference from $Y_2(1)$ and $Y_3(1)$ to obtain their respective desired linear combinations $m_1$ and $n_2$. At the same time, $R_2$ and $R_3$ are also provided with their other desired linear combination i.e., $m_2$ and $n_1$, albeit with additional interference $k_2$ and $k_1$ respectively.

At time $t = 5$ the transmitter sends $k_1$ and at time $t = 6$ it sends $k_2$. Upon receiving these, $R_1$ is able to cancel the contribution of $l_5$ in $k_1$ and $l_6$ in $k_2$ to obtain the two remaining linear combinations $l_2$ and $l_3$ for a total of 6 linear combinations $l_1, \ldots, l_6$ so that it can obtain its DSs $u_1, \ldots, u_6$. Because $R_2$ obtains $k_2$ at $t = 6$ it can subtract its contribution to $Y_2(4)$ and obtain the second linear combination $m_2$ it desires, so that along with its knowledge of $m_1$ it can decode $v_1$ and $v_2$. Similarly, because $R_3$ obtains $k_1$ at $t = 5$ it can subtract its contribution to $Y_3(3)$ and obtain the second linear combination $n_1$ it desires so that along with its knowledge of $n_2$ it can decode its DSs $w_1$ and $w_2$.

1) Compactness of Communication Scheme in Fig. 4: We refer back to Fig. 2 which shows $(1, \frac{1}{2}, \frac{1}{3})$ as one of the extreme points of the DoF region, which translates to $(6, 3, 2)$ DSs sent over 6 time slots. Since the communication scheme presented in Fig. 4 successfully sends $(6, 2, 2)$ DSs over 6 time slots, it is tempting to try and transmit an extra DS to either $R_2$ or $R_3$ using the same scheme in the hope of achieving the aforementioned DoF extreme point. We argue in the following paragraphs against such a possibility for the present scheme, with the help of the outer bounds proved in Theorem 1.
At time $t = 1$, the transmitter transmits 3 independent DSs intended for $R_1$, which is the maximum number of independent DSs that can be transmitted simultaneously over 3 antennas. Visualizing the 3-antenna system at $T_x$ as a 3-dimensional vector space, the null space of $H_1$ is a 2-dimensional subspace of this vector space. To zero-force $v_1$ and $v_2$ (intended for $R_2$) at $R_1$, we need to transmit the DSs using transmit vectors in this 2-dimensional null space. More precisely, the last two rows of the beamforming matrix $B$ should lie in this 2-dimensional null space, which means that we can send a maximum of two independent DSs in the same time slot to $R_2$. The same holds true for the DSs $w_1, w_2$ intended for $R_3$. Hence, we see that the transmission strategy at $t = 1$, under the constraint of zero-forcing $v_i$ and $w_i$ at $R_1$, is tight and can not be improved upon.

At the end of $t = 1$, we have the following requirements for the next 5 time slots, $t = 2, ..., 6$:

\[
\begin{align*}
R_1 & \leftarrow l_2, l_3, u_4, u_5, u_6, \\
R_2 & \leftarrow (l_2 + n_1), (l_3 + m_2), l_3, \\
R_3 & \leftarrow (l_3 + m_2), (l_2 + n_1), l_2.
\end{align*}
\]

We reformulate these requirements in terms of order-1 and order-2 DSs, to be sent in 5 time slots, as

\[
\begin{align*}
R_1 & \leftarrow u_4, u_5, u_6, \\
R_{1,2} & \leftarrow l_3, \\
R_{1,3} & \leftarrow l_2, \\
R_{2,3} & \leftarrow (l_2 + n_1), (l_3 + m_2).
\end{align*}
\]

We denote the cardinality of the order-1 symbols as $d'_1, d'_2, d'_3$ and the order-2 symbols as $d'_{12}, d'_{13}, d'_{23}$. From the requirements mentioned above, we obtain the following values for the cardinality of the various symbols:

\[
\begin{align*}
d'_1 &= 3, \quad (28) \\
d'_{12} &= 1, \quad (29) \\
d'_{13} &= 1, \quad (30) \\
d'_{23} &= 2. \quad (31)
\end{align*}
\]

We now reformulate inequalities (8) and (9) in terms of the cardinality of the symbols, keeping in mind
that all the symbols need to be transmitted in the next 5 time slots, we have

\[
\frac{d_1}{3} + \frac{d_{12}}{2} + d_{13} + d_{23} + d_3 \leq 5,
\]

\[
\frac{d_1}{3} + d_{12} + d_{23} + d_2 + \frac{d_{13} + d_3}{2} \leq 5.
\]

Substituting the values of the cardinality from equations (28)-(31) in the above inequalities, we obtain the following inequalities:

\[
\frac{d_2}{2} + d_3 \leq \frac{1}{2},
\]

\[
d_2 + \frac{d_3}{2} \leq \frac{1}{2}.
\]

Any attempt at sending an extra DS to \( R_2 \) in these 5 time slots i.e., setting \( d_2 = 1 \), would violate inequality (33). Similarly, sending an extra DS to \( R_3 \) would violate inequality (32). This proves the impossibility of sending an extra DS to either \( R_2 \) or \( R_3 \) to achieve the DoF extreme point \((1, \frac{1}{2}, \frac{1}{3})\) using the communication scheme depicted in Fig. 4.

C. Communication Scheme for achieving \((1, \frac{4}{9}, \frac{4}{9})\) DoF under the Alternating CSIT Model

In Fig. 5, we illustrate an communication scheme that achieves \((1, \frac{4}{9}, \frac{4}{9})\) DoF for the 3-user MISO BC under the alternating CSIT model, where we show the achievability of the DoF tuple by coding over 9 time slots. Under the alternating CSIT model considered for this particular achievable scheme, the BC remains in the original hybrid CSIT state for 7 time slots, during which the transmitter has instantaneous CSIT about the channel to receiver \( R_1 \) and delayed CSIT about the channel to receivers \( R_2 \) and \( R_3 \), and for the remaining 2 time slots, the transmitter switches to a new CSIT state, where it has perfect CSIT about the channels to receivers \( R_2 \) and \( R_3 \) and no CSIT about the channel to receiver \( R_1 \), a CSIT state we abbreviate as NPP (no CSIT, perfect CSIT, perfect CSIT).

Over the 9 time slots, the transmitter sends 9 independent DSs \( u_1, u_2, \ldots, u_9 \) to receiver \( R_1 \), 4 independent DSs \( v_1, v_2, v_3 \) and \( v_4 \) to \( R_2 \) and 4 independent DSs \( w_1, w_2, w_3 \) and \( w_4 \) to \( R_3 \), with all the DSs being successfully decoded at their intended receivers, using a transmission and decoding strategy that we explain in detail in the following paragraphs. We divide the scheme into 3 different phases, \textit{data dissemination phase}, \textit{data disambiguation phase} and a new \textit{NPP phase}. We note that in both the data dissemination and data disambiguation phase, the transmitter is in the original hybrid state, but switches to the NPP CSIT state in the NPP phase.
Note: ⊥ denotes zero-forcing at \( R_1 \) of the adjoining data symbol.

**Data Dissemination Phase**

At time \( t = 1 \), the transmitter sends 3 DSs \( u_1, u_2 \) and \( u_3 \) for \( R_1 \), \( v_1 \) and \( v_2 \) for \( R_2 \) and \( w_1 \) and \( w_2 \) for \( R_3 \), using its instantaneous knowledge of the channel to \( R_1 \) at \( t = 1 \) to send \( v_1, v_2, w_1, w_2 \) in the null space of \( H_1 \). More precisely, the following signal is transmitted,

\[
X(1) = \begin{bmatrix} u_1 \\
 u_2 \\
 u_3 \end{bmatrix} + B(1) \begin{bmatrix} 0 \\
 v_1 \\
 v_2 \end{bmatrix} + C(1) \begin{bmatrix} 0 \\
 w_1 \\
 w_2 \end{bmatrix},
\]

where \( B(1) \) and \( C(1) \) are the pre-coding matrices that perform transmit beamforming in the null space of \( H_1(1) \) i.e., \( H_1(1)B(1) = 0 \) and \( H_1(1)C(1) = 0 \). In Fig. 4, this zero-forcing is denoted by a ⊥ sign.
besides the symbols that are zero-forced at $R_1$. The outputs at the three receivers are precisely as follows:

\[
Y_1(1) = H_1(1) \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \\
\triangleq l_1(u_1, u_2, u_3),
\]

\[
Y_2(1) = H_2(1) \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + H_2(1)C(1) \begin{bmatrix} 0 \\ w_1 \\ w_2 \end{bmatrix} + H_2(1)B(1) \begin{bmatrix} 0 \\ v_1 \\ v_2 \end{bmatrix} \\
\triangleq (l_2(u_1, u_2, u_3) + n_1(w_1, w_2)) + m_1(v_1, v_2),
\]

\[
Y_3(1) = H_3(1) \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + H_3(1)B(1) \begin{bmatrix} 0 \\ v_1 \\ v_2 \end{bmatrix} + H_3(1)C(1) \begin{bmatrix} 0 \\ w_1 \\ w_2 \end{bmatrix} \\
\triangleq (l_3(u_1, u_2, u_3) + m_2(v_1, v_2)) + n_2(w_1, w_2).
\]

We again use the symbol $l$ for linear combinations of $u_i$, $m$ for linear combinations of $v_i$ and $n$ for linear combinations of $w_i$. Also, all the linear combinations are almost surely linearly independent, owing to the generic nature of the channel matrices.

At $t = 2$, the transmitter uses the exact same strategy as in the first time slot, but now the DSs it sends are $u_4, u_5$ and $u_6$ for $R_1$ and $v_3, v_4$ for $R_2$ and $w_3, w_4$ for $R_3$, with $v_3, v_4, w_3, w_4$ being transmitted in the null space of the channel to $R_1$ at $t = 2$. Thus, the outputs at each of the receivers are

\[
Y_1(2) = l_4
\]

\[
Y_2(2) = (l_5 + n_3) + m_3,
\]

\[
Y_3(2) = (l_6 + m_4) + n_4,
\]

where we have grouped the interference terms separately. At $t = 3$, the transmitter transmits the remaining DSs $u_7, u_8$ and $u_9$, thus creating the LCs $l_7, l_8$ and $l_9$ at $R_1, R_2$ and $R_3$ respectively. At the end of this time slot, all the DSs have been transmitted, and the rest of the scheme focuses on canceling out the interference to allow each of the receivers to decode their respective DSs.

At the end of the data dissemination phase, of the 9 linear combinations $l_1, \ldots, l_9$ of the DSs $u_1, \ldots, u_9$, $R_1$ knows only 3 LCs and needs to acquire the other 6 LCs to be able to decode all its DSs. On the
other hand, the 2 LCs \( m_1 \) and \( m_3 \) that \( R_2 \) sees comes added with the composite interference symbols \( l_2 + n_1 \) and \( l_5 + n_3 \) respectively, both of which need to be learnt in the next phase. \( R_2 \) also needs to learn the remaining of its desired LCS i.e., \( m_2 \) and \( m_4 \). The situation at \( R_3 \) is similar, and it needs to learn the composite interference symbols \( l_3 + m_2 \) and \( l_6 + m_4 \) as well as its own desired LCs \( n_1 \) and \( n_3 \).

**Data Disambiguation Phase**

At \( t = 4 \), the transmitter sends two symbols \( l_8 \) and \( l_2 + n_1 \) (the knowledge of which it has at \( t = 3 \) due to delayed CSIT of channel to \( R_2 \) at time 1), the latter in the null space of the channel from the transmitter to \( R_1 \) at \( t = 3 \) (since the receiver has instantaneous knowledge of the channel to \( R_1 \)). \( R_1 \) acquires its desired linear combination \( l_8 \), while \( R_2 \) sees a linear combination of \( l_8 \) and \( l_2 + n_1 \), from which it removes the contribution of \( l_8 \) (which it learnt at \( t = 3 \)) to acquire \( l_2 + n_1 \). We thus utilize the idea of serial interference alignment introduced earlier, with the alignment chain being \( l_8 \rightarrow l_2 + n_1 \rightarrow m_1 \), culminating in \( R_2 \) obtaining its desired LC \( m_1 \) at the end of the chain. \( R_3 \) on the other hand observes a linear combination of \( l_8 \) and \( l_2 + n_1 \), which we write in terms of an auxiliary symbol \( k_1 = (l_2, l_8) \), where the bracket signifies a linear combination, with the coefficients depending on the channel to \( R_3 \) at \( t = 4 \), so that \( Y_3(3) = (k_1, n_1) \). To simplify notation, we shall henceforth use brackets to denote a linear combination, the coefficients depending on the channel to the receiver in that particular time slot at which the linear combination is created.

The transmission strategy is similar at \( t = 4 \), where the symbols \( l_9 \) and \( l_3 + m_2 \) are transmitted, the latter in the null space of the channel to \( R_1 \). \( R_1 \) acquires \( l_6 \), and \( R_3 \) uses its previous knowledge of \( l_6 \) to obtain \( l_3 + m_2 \). The serial alignment chain at \( R_3 \) is now \( l_9 \rightarrow l_3 + m_2 \rightarrow n_2 \). The situation at \( R_2 \) parallels that of \( R_3 \) at \( t = 3 \), where we let \( Y_2(4) = (k_2, m_2) \) , with \( k_2 = (l_3, l_9) \) (i.e., interference due to user 1’s DSs).

Thus, at \( t = 4, 5 \), the transmitter sends 2 useful linear combinations \( l_8 \) and \( l_9 \) to \( R_1 \), the composite interference symbols \( l_2 + n_1 \) and \( l_3 + m_2 \) to receivers \( R_2 \) and \( R_3 \) respectively, which then use this knowledge to cancel out the interference from \( Y_2(1) \) and \( Y_3(1) \) to obtain their respective desired linear combinations \( m_1 \) and \( n_2 \). At the same time, \( R_2 \) and \( R_3 \) are also provided with their other desired linear combination i.e., \( m_2 \) and \( n_1 \), albeit with additional interference \( k_2 \) and \( k_1 \) respectively.

At time \( t = 6 \), the transmitter transmits \( k_1 \) and \( l_6 + m_4 \), the latter in the null space of the channel to \( R_1 \) at \( t = 6 \), resulting in \( R_1 \) seeing \( k_1 \) and doing layer peeling on it i.e., canceling out \( l_8 \), to obtain its desired LC \( l_2 \). \( R_2 \) also cancels out the contribution of \( l_8 \) (which it knows from \( t = 3 \)) from \( k_1 \), and thus
obtains a linear combination of \(l_2, l_6\) and \(m_4\), in which we group the interference due to user 1 DSs into the auxiliary symbol \(k_3 = (l_2, l_6)\), a linear combination of \(l_2\) and \(l_6\), and thus \(Y_2(6) = (k_3, m_4)\). \(R_3\) sees a linear combination of \(k_1\) and \(l_6 + m_4\), which we have shown separately in Fig. 5 to emphasize the fact that \(R_3\) sees \(l_6 + m_4\) with the same interference \(k_1\) that it encountered at \(t = 4\).

At time \(t = 7\), the transmitter similarly sends \(k_2\) and \(l_5 + n_3\), the latter in the null space of the channel to \(R_1\) at \(t = 6\), resulting in \(R_1\) seeing \(k_2\) and canceling out \(l_9\), to obtain its desired LC \(l_3\). \(R_3\) also cancels out the contribution of \(l_9\) (which it knows from \(t = 3\)) from \(k_2\), and thus obtains a linear combination of \(l_3, l_5\) and \(n_3\), in which we group the interference due to user 1 DSs into the auxiliary symbol \(k_4 = (l_3, l_5)\), and thus \(Y_3(7) = (k_3, n_3)\). \(R_2\) sees a linear combination of \(k_2\) and \(l_5 + n_3\), which we have again shown separately to emphasize that \(R_3\) sees \(l_5 + n_3\) with the same interference \(k_2\) that it encountered at \(t = 5\).

Thus, at \(t = 6, 7\), \(R_1\) obtains two useful linear combinations \(l_2\) and \(l_3\), while both \(R_2\) and \(R_3\) acquire one desired LC each i.e., \(m_4\) and \(n_3\), but with added interference \(k_3\) and \(k_4\) respectively. We note that layer peeling is once again the motivation behind creating the auxiliary interference symbols \(k_3\) and \(k_4\), e.g. in the subsequent NPP phase, \(k_3\) is useful in its entirety at \(R_3\) and also manages to deliver the desired LC \(l_6\) at \(R_1\). The other major idea is to repeat the previously seen interference \(k_1\) at \(R_3\) (at \(t = 5\)) and \(k_2\) at \(R_2\) (at \(t = 6\)), respectively. This sets up the endgame where knowledge of \(k_1\) at \(R_3\) will lead simultaneously to interference cancellation from \(Y_3(4)\) which provides \(n_1\) at \(R_3\) and the unwrapping of the serial alignment chain \(k_1 \rightarrow l_6 + m_4 \rightarrow n_4\) which furnishes \(n_4\) at \(R_3\). A similar logic shows that knowledge of \(k_2\) at \(R_2\) in the subsequent phase allows \(R_2\) to acquire \(m_3\) and \(m_4\). This is our motivation to switch to the NPP CSIT mode for the next 2 time slots, and use the perfect knowledge of channels to \(R_2\) and \(R_3\) to deliver the auxiliary interference symbols \(k_2, k_3\) to \(R_2\) and \(k_1, k_4\) to \(R_3\) without any interference.

**NPP Phase**

At \(t = 8\), the transmitter sends \(k_1\) and \(k_3\), in the null space of the channels to \(R_2\) and \(R_3\) at \(t = 8\) respectively (possible due to the now instantaneous knowledge about the channels from the transmitter to \(R_2\) and \(R_3\)). \(R_1\) sees a linear combination of \(k_1\) and \(k_3\), from which it cancels out the contribution of \(l_5\) (known previously at \(t = 6\)) and \(l_8\) (known at \(t = 4\)) to obtain its required DS \(l_6\). Since \(k_1\) is zero-forced at \(R_2\), \(R_2\) sees only \(k_3\), and uses it to cancel the interference from \(Y_2(6)\) to obtain its desired LC \(m_4\). \(R_3\) is able to acquire \(k_1\) without any interference, because of zero-forcing of \(k_3\) at \(R_3\). As discussed in the previous paragraph, knowledge of \(k_1\) lets \(R_3\) cancel interference from \(Y_3(4)\) to obtain its desired LC \(n_1\) and also allows \(R_3\) to do interference cancellation from \(Y_3(6)\) to obtain \(l_6 + m_4\), which in turn allows
interference cancellation from $Y_3(2)$ to provide $R_3$ with another desired LC $n_4$, a chain of events that is elegantly described by the serial alignment chain $k_1 \rightarrow l_6 + m_4 \rightarrow n_4$.

In the final time slot $t = 9$, the transmitter sends $k_4$ and $k_2$, respectively in the null space of the channels to $R_2$ and $R_3$ at $t = 9$. $R_1$ uses its previous knowledge of $l_3$ and $l_9$ to cancel out their contribution from the linear combination of $k_2$ and $k_4$ to obtain its final required LC $l_5$. $R_2$, which sees only $k_2$ free from interference, uses $k_2$ to cancel interference from $Y_2(5)$ to obtain the desired LC $m_2$ and also to unravel the serial alignment chain $k_2 \rightarrow l_5 + n_3 \rightarrow m_3$ (from $Y_3(9), Y_3(7)$ and $Y_3(2)$) to obtain its final desired LC $m_3$. $R_3$ sees an interference free version of $k_4$, and it cancels its contribution from $Y_3(7)$ to obtain the LC $n_3$.

At the end of the 9 time slots, $R_1$ has the LCs $l_1, \ldots, l_9$, and can thus decode its DSs $u_1, \ldots, u_9$. $R_2$ possesses the LCs $m_1, \ldots, m_4$ which enable it to decode all its desired DSs $v_1, \ldots, v_4$, and similarly, $R_3$ uses the LCs $n_1, \ldots, n_4$ to decode its DSs $w_1, \ldots, w_4$.

V. Conclusion

In this paper, we obtain an outer bound for the DoF region of the K-user MISO BC in the most general hybrid CSIT setting, where an arbitrary number of receivers are in the instantaneous CSIT mode and the rest of the receivers are in the delayed CSIT mode. We specialize these results for the 3-user MISO BC, where the transmitter has instantaneous CSIT about one receiver and delayed CSIT about the other two. We develop new communication schemes for the 3-user BC for the hybrid CSIT model, and demonstrate achievability of a sum-DoF that is more than that obtainable only with delayed CSIT. We also show how an outer bound corner-point for the hybrid CSIT can be achieved using alternating CSIT.

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