One-Dimensional Photon Transport Through a Two-Terminal Scattering Cluster: Tight-Binding Formalism

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Employing tight-binding approximation we derive a transfer matrix formalism for one-dimensional single photon transport through a composite scattering center, which consists of parallel connected resonator optical waveguides. By solving the single-mode eigenvectors of the Hamiltonian, we investigate the quantum interference effects of parallel couplings on the photon transport through this parallel waveguide structure. We find a perfect reflection regime determined by the number of coupled resonator waveguides. Numerical analysis reveals that by changing atom transition frequency, the window of perfect reflection may shift to cover almost all incoming photon energy, indicating the effective control of single photon scattering by photon-atom interaction.

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I. INTRODUCTION

Photon transport in one dimensional coupled waveguide is an important model system for exploring quantum information processing and manipulation mechanism. The coupling of atoms or quantum emitters to the optical waveguide offers feasible control schemes to achieve quantum switching, routing, photon storages and other quantum information operations[1–6]. Currently there are several important theoretical approaches being used to study the photon transport in one dimension, including the real space Bethe ansatz approach[7, 8], Input-output formalism [9, 10], and Lippman-Schwinger scattering-theory approach [11–14]. Recently a tight-binding formalism has been employed to show that the scattering of a single photon inside a one-dimensional resonator waveguide can be controlled by the coupled two-level quantum system[15]. It is demonstrated that a finite-bandwidth spectrum of perfect reflection appears as the detuning between the photon and atomic frequency varies. It is obvious that when a single photon transport properties can be influenced by the number of atoms that interact with the waveguide as a result of multiple scattering between the propagating photon and the quantum emitters[16–24]. In addition to the effects of the collective interaction with the multiple emitters, the coupling modes between atom and the one-dimensional waveguide also affect the photon transport and result in quite involved, nontrivial dispersion relations that can lead to strong reduction of the group velocity of photons[16, 17, 25], as a consequence of a finite bandwidth. Since the coupling configuration between atoms and waveguide play an important role in determining the transport properties of photons, in this paper we will focus on the photon transfer through a black-box like scattering cluster, by employing the tight-binding theory[26].

This paper is organized as follows. In Sec. II we derive a general theory for photon transport through a two-terminal interaction box, where multiple atoms are coupled in series as well as in parallel. We then discuss the simple case of coupled cavity array based on the Jaynes-Cummings model for single cavities, in Sec. III. Finally we give a summary in Sec. VI.

II. GENERAL THEORY

The model system discussed in this work consists of two leads sandwiched by a tight-binding network in the parallel configurations, as illustrated in Fig.1.

In this section we derive the transmission and reflection amplitudes in the Tight-Binding (TB) model for parallel connected transmission lines. The typical TB equations, resulting from the Schrodinger equation $H\Psi = E\Psi$ can
be written as
\[(\epsilon_j - E)\psi(j) = \sum_n J_{j,n} \psi(j + n) \quad (1)\]

For purposes of illustration of our method, we consider here only the simplest case where the transfer integral between the nearest neighbors \( J_{j,n} = 1 \) and the lattice constant \( a = 1 \). We also assume that the scattering is elastic and the traveling wave functions may take the following form
\[\psi(j) = Ae^{ikj} + Be^{-ikj}, \quad (2)\]

with the dispersion relation \( \epsilon_j - E_k = 2\cos(k) \). Suppose that there are \( n_j \) sites at the \( j \)-th channel, and the total number of the TB sites in the central scattering network is \( N = \sum_{j=1}^m n_j \). Here \( m \) represents the number of the parallel channels that converge into the left and right terminals.

The TB equations of the parallel configuration can be written as
\[
(\epsilon_j - E_k)\psi(-1) = \psi(-2) + \sum_i^{n_1} \psi_i(0),
\]
\[
(\epsilon_j - E_k)\psi(N) = \psi(N + 1) + \sum_i^{n_1} \psi_i(n_i - 1). \quad (3)
\]

By taking the standard expressions for the left incoming and the right outgoing waves,
\[
\psi_L(j) = e^{ikj} + re^{-ikj}, \quad j < 0,
\]
\[
\psi_R(j) = te^{ikj}, \quad j > N, \quad (4)
\]
we find
\[
1 + r = \sum_{i=1}^m \psi_i(0),
\]
\[
t_N = \sum_{i=1}^m \psi_i(n_i - 1), \quad (5)
\]
where \( t_N = te^{ik(N-1)} \), and \( \psi_i(j) \) is the wave function at the site \( j \) of the \( i \)-th channel.

Now let us derive a relation between the wave functions \( \psi_L(0) \) and \( \psi_L(n_i - 1) \). To this end, we resort to the transfer matrix expression of the TB equations,
\[
\begin{bmatrix}
\psi_i(j + 1) \\
\psi_i(j)
\end{bmatrix} = \begin{pmatrix} \alpha_{i,j} & -1 \\ 1 & 0 \end{pmatrix} \begin{bmatrix} \psi_i(j) \\
\psi_i(j - 1)
\end{bmatrix}, \quad (6)
\]
with \( \alpha_{i,j} = \epsilon_{i,j} - E_k \). Here \( \epsilon_{i,j} \) stands for the energy of the \( j \)-th site of the \( i \)-th waveguide channel. It is straightforward that
\[
\begin{bmatrix} \psi_i(n_i) \\
\psi_i(n_i - 1) \end{bmatrix} = M_i \begin{bmatrix} \psi_i(0) \\
\psi_i(-1) \end{bmatrix}, \quad (7)
\]
Taking into account of the continuity condition of the wave functions at any TB sites, we have that \( \psi_i(-1) = 1 + r \) and \( \psi_i(n_i) = \psi(N) = t_N \). Introducing that
\[
M_i = \begin{pmatrix} \alpha_{i,n_i-1} & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} a_{i,n_i-1} & b_{i,n_i-1} \\ e_{i,n_i-1} & d_{i,n_i-1} \end{pmatrix}, \quad (8)
\]
where the matrix elements can be obtained from the following recursion relations
\[
a_{i,n} = \alpha_{i,n-1}a_{i,n-1} - c_{i,n-1},
\]
\[
b_{i,n} = \alpha_{i,n-1}b_{i,n-1} - d_{i,n-1},
\]
\[
c_{i,n} = a_{i,n-1},
\]
\[
d_{i,n} = b_{i,n-1}, \quad (9)
\]
with \( a_{i,0} = \alpha_{i,0} \), \( b_{i,0} = -1 \), \( c_{i,0} = 1 \) and \( d_{i,0} = 0 \). By inserting (7)-(9) into (5) we get the transmission and reflection amplitudes for the scattering cluster formed by parallel coupled oupled cavity waveguides,
\[
r = \frac{(P'e^{ik} - 1)(1 - Q'e^{-ik}) + PQ'}{(Q'e^{ik} - 1)(P'e^{ik} - 1) - PQ'e^{2ik}}, \quad (10)
\]
\[
t_N = \frac{-2iQ'\sin(k)}{(Q'e^{ik} - 1)(P'e^{ik} - 1) - PQ'e^{2ik}},
\]
with
\[
P = \sum_{i=1}^m \frac{1}{a_{i,n_i-1}}, \quad Q = -\sum_{i=1}^m \frac{b_{i,n_i-1}}{a_{i,n_i-1}}, \quad (11)
\]
\[
P' = \sum_{i=1}^m \frac{c_{i,n_i-1}}{a_{i,n_i-1}}, \quad Q' = \sum_{i=1}^m \frac{(d_{i,n_i-1} - b_{i,n_i-1}c_{i,n_i-1})}{a_{i,n_i-1}}. \quad (12)
\]

This is the main result of this work. It is worth to point out that the physical relavance of the site energy \( \epsilon_i(j) \) may be quite general, including the PT-symmetric potentials. It is remarkable that we can evaluate the photon transport properties without assuming the detailed wave functions in the scattering zone, provided that the tight-binding approximation is valid, in sharp contrast with the current approaches used in the literature.

## III. NUMERICAL ANALYSIS

To illustrate our method, in the following, we consider the problem of the single photon transport in an one-dimensional coupled resonator waveguides. To be more specific, we assume that photon propagates along the left waveguide and enters the multiple parallel coupled waveguides through the splitter point \( O \). And then it is transfered to the left waveguide via the converter \( O' \).
As part of the scattering cluster those one-dimensional waveguide are coupled with two-level atoms each. Let us denote by $a^\dagger$ the single mode in the $j$-th cavity, with frequency $\omega$. The Hamiltonian of the CRW is given by

$$H_{cv} = \omega \sum_j a_j^\dagger a_j - \sum_{j,j'} (J_{j,j'} a_j^\dagger a_{j'}^\dagger + H.c.)$$

(13)

where $J_{j,j'}$ is the inter-cavity photon hopping constant among the connecting cavities. For uniform hopping constant $J_{j,j'} = J$, the Hamiltonian (1) can be readily diagonalized to yield the dispersion relation $E_k = \omega - 2J \cos(k)$. Here we assume that $\hbar = 1$ and the lattice constant (inter-cavity distance) $a = 1$.

We further assume that the Hamiltonian of the atom is given by $H_A = \sum_j \omega_j |e_j><e_j|$ and the interaction of the single photon with a two-level atom inside the $j$-th cavity, is then described by the Jaynes-Cummings Hamiltonian

$$H_j = g_j (a_j^\dagger g_j + H.c.) \quad 0 \leq j \leq N - 1$$

(14)

Here $|g_j>$ and $|e_j>$ are the ground and excited state of the $j$-th two-level atom, respectively. $\omega_j$ is the transition energy between the two energy levels. $g_j$ is the photon-atom coupling strength. The total Hamiltonian of the whole system is $H = H_{cv} + H_A + H_{int}$. Thus the stationary eigenstate may be expressed as

$$|\psi_k> = \sum_j |\psi(j)\rangle (a_j^\dagger g_j |0> + |e_j> |e,0>$$

(15)

where $|0>$ stands for the vacuum state the photon in the cavities coupled to the waveguide, and $|\psi_j$ $(0 \leq j \leq N - 1)$, gives the probability amplitude of the atom in the excited state.

A. N atoms coupled in series

The photon transport in one-dimensional coupled resonator waveguide has been extensively in recent years studied with the use of scattering theory based on the quantum Lippman-Schwinger formalism, by transfer matrix method, and the Bethe ansatz approach in the context of the real continuous and discrete space. The dynamics of the photon transport has been investigated by the coupled mode theory as well as the input-output theory. Here we derive a formalism based on the tight-binding approximation. We show that this formalism is a practical and efficient method for the calculation of the transmission and reflection amplitudes for arbitrary atoms and coupling modes.

Suppose that the scattering cluster is formed by $N$ cavities with identical two-level atoms inside. The TB equations for this central coupled lattice is described by

$$\omega \psi(j) - J \psi(j - 1) - J \psi(j + 1) + g \phi_j = E_k \psi(j),$$

(16)

where $\omega_j$ stands for the transition energy of the coupled atoms. Taking into account the fact of identical atoms, i.e., $\omega_j = \omega_0$ for $0 \leq j \leq N - 1$, we find

$$\alpha = 2\cos(k) + \frac{g^2}{\Delta_k},$$

(18)

where $\Delta_k = \omega - 2\cos(k) - \omega_0$ is the detuning. Then it follows immediately,

$$r = -\frac{a_{N-1} - b_{N-1} e^{-ik} + c_{N-1} e^{ik} + d_{N-1}}{a_{N-1} + (b_{N-1} - c_{N-1}) e^{ik} - d_{N-1} e^{2ik}},$$

$$t_N = -\frac{-2i(a_{N-1} b_{N-1} - b_{N-1} c_{N-1}) \sin(k)}{a_{N-1} + (b_{N-1} - c_{N-1}) e^{ik} - d_{N-1} e^{2ik}}.$$

(19)

For an identical coupled resonator waveguide it is shown that a finite bandwidth of perfect reflection $R = 1$ may be observed for certain values of the atom transition frequency $\omega_0$, even for a single coupled atom. The increase of the number of coupled atom can only lead to a clear cut-off of the band borders. In order to demonstrate how the atom transition frequency controls the photon transport in our model system, we assume that the cavity-waveguide and the cavity-atom coupling strength are constant and taken to be 1, throughout this paper. Obviously much diversified transport properties may emerge varying coupling constants. In general, there are two parameter regimes are considered. One is that the detuning of the incoming photon energy to the atom transition energy is inside the interference range, where the waveguide photon mode is comparable to that of the atom, so that stimulated absorption or emission may occur. Another regime is the photon frequency is far away from the interference range, and so the resonant transmission is usually expected. In the following numerical analysis, we focus on the perfect reflection regime. This phenomenon is related to the manipulation of the photon propagating group velocity as a finite bandwidth makes it possible to acommodate a real wave packet.

Fig.2 shows typical photon transport properties for identical coupled atoms. We plot the reflectance $R$ as a function of the photon wavevector and the detuning $\Delta$. As is shown in Fig.2 (a) and (b), there two typical parameter sets related to $R = 1$. One is defined by $dR/dk = 0$, and another corresponds to $d^2R/dk^2 >> 1$. Our numerical results reveals that as the atom transition energy is changed the position and the bandwidth of the $R=1$ window vary accordingly. As shown in Fig.2(c), when the atom frequency is changes the $R = 1$ band will sweep over all spectrum and the bandwidth increases as the atom frequency increases from $\omega_0 = 0$ to $\omega = 2\pi$. And then it decreases to its minimum at $\omega_0 = 4\pi$. This variation pattern repeats itself periodically as one varies the atom frequency. It looks quite surprising that even when the incoming photon energy is complete out of the
FIG. 2. The reflectance vs. the waveguide photon momentum of two identical atoms coupled in series for the atom frequency \(\omega_0 = 2.9\pi\) (red crosses), \(2\pi\) (blue circles), \(1.1\pi\) (orange pluses); (b) The reflectance vs. the detuning with the same parameter values as (a). (c) The reflectance vs. the detuning for \(N = 50\) atoms, with the atom transition frequency \(\omega_0 = 2\pi\) (red circles), \(1.1\pi\) (orange pluses), \(0.75\pi\) (blue triangles), \(0.45\pi\) (green crosses), \(0.1\pi\) (black squares). The frequency of the waveguide photon mode is \(\omega = 2\pi\).

FIG. 3. Pattern-dependent photon transport. Here we denote by \(A\) the atoms with \(\omega_1\) and \(B\) those with \(\omega_2\). For coupled resonator waveguide with different atom transition frequency, we find that different sequence of serially coupled atom will lead to properties. Fig.3(a) and (b) shows the reflection constant \(R\) of three coupled atoms with sequence \(ABA\) (red line) and \(BAB\) (blue circles) for \(\omega_1 = 5.5\) (the atom A) and \(\omega_2 = 3.2\) (the atom B), the frequencies of both atoms are within the interference zone with the waveguide photon, whose frequency is \(\omega = 5\). Fig3(c) and (d) are for \(\omega_1 = 8.5\) and \(\omega_2 = 2.5\), both fall outside the interaction regime with the propagating photon.

B. \(N\) atoms coupled in parallel

One of the well studied two-terminal parallel connected system is the quantum ring, where two paragonation channels are merged at the left and right junctions, through which they are connected to the left and right lead, respectively. Here we first derive a formalism for an arbitrary number of transmission lines that are joined at their extremes and coupled to two terminals (see Fig.1(b)).

To illustrate the effects of the branched scattering processes in each single coupled waveguide, let us first study the special case where there is only one atom coupled to the single waveguide. Consider now \(N\) resonators with embedded two-level atoms, are connected to the left and the right leads in a parallel way, which are labelled by \(j = 0\) through \(j = N - 1\). Those resonators are coupled to the same 0-th resonator to the left and to the \(N\)-th oscillator to the right lead, respectively. Hence the tight-binding equations for those parallel connected resonators read

\[
\omega\psi(j) - J[\psi(-1) - \psi(N)] + g_j\phi_j = E_k\psi(j),
\]

\[
\omega\psi(-1) - J\psi(-2) - J\sum_{j=0}^{N-1}\psi(j) = E_k\psi(-1),
\]

\[
\omega\psi(N) - J[\psi(N+1) - \psi(1)] = E_k\psi(N),
\]

possible interaction range with the atom, there exists still a small finite \(R = 1\) windows. and this leads to the atomic mirror with all-frequency perfect reflection. It is noticed that for identical coupled atoms perfect reflection intervals can not be created by increasing the number of the atoms, which is against the intuitive or conclusions drawn in other model systems that the accumulated effects of the multiple scattering will result eventually to the complete reflection[1111].

Generally speaking, there are many different coupling patterns by which the atoms are connected to form an array of coupled resonator waveguide. In addition to the uniform coupled atoms, the next important pattern is the so-called super array of coupled resonator waveguide, where two different kinds of atoms are connected alternatively, symbolized as \(ABABABA\)... Here we conduct numerical analysis on the simplest combination case, that is, two two-level atoms with different atom transition energies. Fig.3 shows the transport features for \(N = 3\) atoms with connection patterns symbolized by \(ABA\) and \(BAB\). This type of coupling resembles the model systems studied in [16, 18], where it has been shown that such kind of systems support photon quasibound states and offers a mechanism for photon storage and control of photon group velocity. For this kind of super-array system, by simply changing the position order, one may obtain an ideal atomic mirror with perfect reflection.
\[ \omega_j \phi_j + g_j \psi(j) = E_k \psi(j), \quad (23) \]

Using the standard traveling wave function
\[ \psi(-1) = e^{-ik} + re^{ik}, \quad (24) \]
\[ \psi(N) = te^{ikN}. \quad (25) \]

and defining that
\[ \gamma_N = \sum_{j=0}^{N-1} \frac{J}{\omega - E_k + g_j G_j}, \quad (26) \]
\[ G_j = \frac{g_j}{E_k - \omega_j}, \quad 0 \leq j \leq N - 1 \quad (27) \]
we obtain the corresponding transmission and reflection amplitudes,
\[ r = \frac{2\gamma_N \cos(k) - 1}{1 - 2\gamma_N e^{ik}}, \quad (28) \]
\[ t_N = te^{ik(N-1)} = -2i\gamma_N \sin(k) \frac{1}{1 - 2\gamma_N e^{ik}}, \quad (29) \]

It is an easy matter to verify that if \( \Delta_j = \omega - \omega_j - 2\cos(k) = 0 \) or \( \omega - E_k + g_j G_j = 0 \), which gives rise immediately to \( R = 1 \), and \( T = 0 \). Moreover, when \( N \) goes to infinity, we have \( R = \cos^2(k) \) and \( T = \sin^2(k) \).

Fig.4 demonstrates the reflectance of single photon through a ring with one atom (a) and two atoms (b) on each branch. An interesting feature of the scattering spectrum is coexistence of perfect transmission and reflection bands as illustrated in Fig.4(a). This result suggests that coupled resonator ring waveguide behaves just like an energy filter that allows photons of certain energy passes freely and reflects others completely.

We now turn to calculate the transport property for general parallel connected coupled resonator waveguides. In this case the scattering box is composed by \( N \) identical parallel coupled resonator waveguides, where the left single waveguide is split into \( N \) identical sub-waveguides. In this case we have \( \psi_j(0) = \phi(0) \) and \( \psi_j(n_j - 1) = \phi(n-1) \). Assume that there are \( N_0 \) parallel connected waveguides, and each of them coupled with \( n \) identical two-level atoms. Thus, (5) becomes
\[ 1 + r = \sum_{i=1}^{N_0} \psi_i(0) = N_0 \phi(0), \quad (30) \]
\[ t_N = \sum_{i=1}^{N_0} \psi_i(n_i - 1) = N_0 \phi(n-1). \]

By employing the tight-binding equations, we obtain the following results,
\[ r = \frac{-a_{n-1} - N_0 b_{n-1} e^{-ik} + N_0 c_{n-1} e^{ik} + N_0^2 d_{n-1} e^{-ik}}{a_{n-1} + N_0 e^{ik}(b_{n-1} - c_{n-1} - N_0 d_{n-1} e^{ik})}, \]
\[ t_N = \frac{-2iN_0(a_{n-1}d_{n-1} - b_{n-1}c_{n-1} - N_0 d_{n-1} e^{ik})}{a_{n-1} + N_0 e^{ik}(b_{n-1} - c_{n-1} - N_0 d_{n-1} e^{ik})}. \quad (31) \]

For \( N_0 = 1 \) we return to the case of a single waveguide with serially coupled atoms. There are several remarkable features of our model system. One is the apparently surprising conclusion that when \( N_0 \) is sufficiently large, the parallel coupled system behaves like a total perfect reflection mirror with \( R = |t|^2 = 1 \) and \( T = |t|^2 = 0 \), independent of the the detuning. Particularly when the detuning is such that no apparent radiation interaction between the photon and the atom, in which case the resonant transmission is expected. The possible implication is directly related to destructive interference among the photon states propagating along different channels. It is obvious that the mechanism behind the perfect reflection revealed here seems to be different from the serially coupled waveguide as illustrated in the literature[12, 19], where the total reflection is accumulation process due to multiple consecutive scatterings. Nevertheless, as will be shown later, this phenomena indeed is closely related to the characteristic features of the single coupled waveguide.

In Fig.5 we show that the reflectance for single coupled resonator waveguide (a) and (b), and the parallel connected multiple coupled resonator waveguides. we find that by increasing the number of connected waveguides the perfect reflection windows expand into all spectrum, while the the bandwidth and spectrum position are almost independent of the number of serial coupled atoms along the waveguide.

This seems quite contra-intuitive, since the resistance in serial circuit is expected to grow and reduces in parallel connections.

It should be emphasized that the mechanism of atomic mirror observed here is different than those discussed in
The photon frequency is \( \omega = 5 \), and the atom energies are \( \omega_1 = 2 \) and \( \omega_2 = 3 \). The reflection constant as a function of photon wavevector (a) and detuning (b) for the single waveguide coupled with 3 atoms, while (c) and (d) are for 11 coupled atoms in a single channel. One of the most notable features is that the parallel connection tends to block out the propagation of the single photon for almost all the incoming photon frequency, while the increase of coupled atoms in a single waveguide can generate a fine border of the already existing perfect reflection window. Note that here both atoms are outside the interference interval.

![Graphs showing reflectance R as a function of photon wave vector k and detuning \( \Delta \).](image)

Ref. [12], where the perfect reflections are attributed to the multiple collisions with serially coupled atom-contained cavities. Numerical calculations reveal that in contrast to the serially coupled resonator waveguide, by incorporating more cavities into the system destroy the above-mentioned perfect transmission and reflections windows, leaving the usual transmission patterns. This can be derived directly from Eq.(28). That is, for identical cavities, when \( N \to \infty \), \( R = \cos^2(k) \).

VI. CONCLUSIONS

In conclusion, we develop a simple tight-binding formalism for both serially and parallel connected coupled resonator waveguides, and conduct a series of numerical calculation of single photon transport properties. Our numerical results reveal the following novel features of the coupled resonator waveguide: (i) For single waveguide with serially coupled cavities with embedded tow-level quantum system, the waveguide photon mode with arbitrary wavevector can be completely relected by properly chosen atom transition energy. A similar atomic control of photon transfer can be achieved by the so-called super-array waveguide, where two types of atoms are connected alternatively. (ii) In the case of parallel coupled resonator waveguides, an all-frequency perfection reflection regime is reached when the number of coupled waveguides are sufficiently large. A photon filter can be obtained when the single photon propagate through a ring of coupled resonator waveguide, in which case the resonant transmission occurs for certain values of photon energies, while perfect reflection is imposed on single photons with different wavevectors.

It is obvious that our method provide a straightforward means for investigating one-dimensional photon transport through a scattering clusters with certain regular internal structures, both in theoretic discussion and numerical analysis. It is expected that more real, complicated parameter regimes can be studied with the use of the tight-binding formalism presented in this work, and much novel transport properties can be revealed.

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