The resonant modes associated with engineered photonic structures of different spatial-dimension are essential to obtain bright on-demand single photon sources for quantum technologies. Negatively-charged nitrogen-vacancy (NV-) center in diamond is proposed as an excellent single-photon source at room temperature. The possible optical readout of spin states in diamond makes it a good candidate for the spin-photon interface. However, poor light collection, feeble zero-phonon line, low emission rate, and broad phonon-induced emission limit the use of NV-centers in quantum technologies. Here, a feasible, easy-to-fabricate, asymmetric Tamm structure coupled with a single NV-center to enhance the emission rate of zero phonon line (ZPL) with better light collection efficiency is presented. The asymmetric Tamm structure shows dual resonant mode with one of the modes coinciding with NV-ZPL wavelength of 640 nm. The mode quality factor ($Q$-factor) is 140, which is greater than the $Q$-factor of conventional Tamm structure and hence, improved field intensity localization. The authors achieved four times Purcell enhancement and five times enhanced light collection efficiency at 640 nm. The proposed asymmetric Tamm structure helps to generate bright single photons using NV-center and improves the efficacy of the spin-photon interface.

1. Introduction

On-demand generation of single photons with high emission rates and an efficient spin-photon interface is crucial for developing various quantum technologies such as quantum computing, repeaters, and quantum communication. This motivates the coupling of single quantum emitters to the sub-wavelength photonic cavities.[1-3] The emitter coupled with the cavity experiences a modified electromagnetic environment due to an increased local density of optical states (LDOS) at a specific wavelength (frequency) compared to a vacuum. The LDOS specifies the total number of available electromagnetic modes per unit volume per unit frequency. The decay rate is directly related to LDOS through Fermi’s golden rule; hence, for any reliable control of emission intensity and rate, LDOS engineering is an essential requirement.[4] LDOS can be engineered using several sub-wavelength photonic structures to enhance or suppress the emission from an embedded emitter.[5,6] Photonic structures such as photonic crystal cavities,[6-8] plasmonic cavities,[9] Mie resonances,[10] and resonant metasurfaces[11] show the cavity mode with a high-quality factor ($Q$-factor) and small mode volume to achieve control over the emission properties of an embedded emitter.

Recent advances in solid-state-based quantum emitters promote negatively-charged nitrogen-vacancy (NV-) center in nanodiamond as a promising quantum emitter due to its excellent optical and spin properties at room temperature. NV-center is formed by a substitutional nitrogen atom adjacent to a carbon vacancy in a diamond crystal. It finds applications in quantum communication, quantum sensing, magnetometry, and bio-markers.[12-16] The NV-center emission spectra consist of pure electronic transition at 640 nm, known as the zero phonon line (ZPL), assisted with broad phonon sideband (PSB) emission spanning up to 750 nm. The PSB transitions induce decoherence with a limited Debye–Waller factor of 3% emission at ZPL. Therefore, enhancing the emission intensity and rate at ZPL with simultaneous suppression of PSB is required to efficiently use NV-center in quantum technologies.[8,14] In one approach, the phonon-mediated transitions are suppressed by cooling the sample to cryogenic temperatures. However, this results only in a slight increase in the Debye-Waller factor to 4%. Hence an alternate approach to enhance emission at ZPL is required. The idea proposed is to trap the NV-center in a photonic cavity structure to achieve ZPL enhancement with simultaneous PSB suppression. The photonic crystal cavities could offer a high $Q$-factor but with a large mode volume, and complex fabrication procedures are involved.[17] The photonic crystal cavities are fabricated on a monocrystalline diamond and integrated with NV-center with cavity mode at ZPL.[8] Plasmonic cavities show a low $Q$-factor of $\approx 10$ but possess an ultra-small adequate mode volume resulting in a high Purcell
factor and enhanced coupling rate.\[18\] Purcell-enhanced emission from dye molecules and NV centers coupled to the plasmonic cavity system has been experimentally reported.\[19,20\] However, the fabrication of plasmonic structures and then coupling NVs to them is always a challenge, in addition to the inherent losses associated with plasmonic materials.

Hence, searching for low-cost and easy-to-fabricate photonic cavity structures with moderate $Q$-values is a contemporary research topic. The metal-dielectric layered structures have gained extensive attention due to their capability to enhance light-matter interactions. Hyperbolic metamaterials are metal-dielectric systems having hyperbolic dispersion, which are used to obtain Purcell enhancement of $\approx 250$ at $650$ nm.\[21\] To achieve maximum coupling between the emitter and cavity mode, the emitter must be positioned at a site of maximum localized field intensity inside the cavity. The cavity mode coupling with the emitter ensures a high emission rate and extraction efficiency.\[22\] The optical Tamm structure is another interesting layered cavity structure with moderate $Q$-values. The Tamm structure consists of a distributed Bragg reflector (DBR), spacer layer, and metal layer, forming a metal-dielectric layered structure. The fabrication of the Tamm structure is relatively simple compared to a nanopatterned plasmonic structure which requires a modern micromachining/lithography technique. In contrast, the Tamm structure can be fabricated using layer-by-layer deposition of dielectric material by sol-gel synthesis or an e-beam evaporation method.\[23,24\] The structure is characterized by a Tamm mode, which originates due to the total reflection induced by the DBR and the metal film. The Tamm mode is localized within the spacer layer, termed Tamm plasmon resonance (TPR). The TPR has an in-plane wavevector smaller than the light wavevector in a vacuum, allowing direct TPR excitation in contrast to surface plasmon polaritons. The TPR can be excited using both transverse electric (TE) and transverse magnetic (TM) polarized light at any angle of incidence.\[25\] The TPR occurs primarily in the non-absorbing spacer layer; hence, it is insensitive to dissipative losses in the metal layer. The TPR is a suitable alternative to conventional surface plasmons in a wide range of applications, such as extraction of single photon emission,\[9\] low threshold lasers,\[26\] sensors,\[27\] and optical switches.\[28\] As mentioned earlier, it is imperative to suppress the PSB emission and enhance the ZPL to implement NV-center in many quantum-based applications.\[12,13\] Thus, coupling an NV-center to the TPR will increase the brightness and emission rate at ZPL relative to phonon-mediated broadband emission. The anticipated increase in emission brightness and the rate is a collective consequence of LDOS enhancement and further depends on the emitter’s position in the cavity structure and the orientation of the emitter’s dipole moment.\[29\]

Here, we demonstrate the enhanced emission rate and brightness at ZPL emission from a single NV-center coupled to an asymmetric Tamm structure. The proposed easy-to-fabricate asymmetric Tamm structure generates dual TPR at normal incidence, with the first resonance appearing at ZPL of NV-center offering a high $Q$-factor compared to the conventional Tamm structure. The enhancement in the decay rate of NV-center and far-field light collection efficiency is studied using finite difference time domain and Green’s function method, respectively. We discuss Purcell and light collection efficiency enhancement, from the coupled NV-center, at $640$ nm compared to NV-center in a homogeneous dielectric slab.

2. Results and Discussions

Figure 1a represents the schematic of the proposed asymmetric Tamm structure made up of two separate sets of DBRs (DBR$_1$ and DBR$_2$) consisting of different bi-layer pairs composed of titanium dioxide (TiO$_2$) and silicon dioxide (SiO$_2$). The optical thickness of dielectric layers is set according to $\lambda_0 = 2(n_a d_a + n_b d_b)$, where $\lambda_0$ is the central wavelength of DBR’s stop-gap, $n_a$, $n_b$ are the refractive indices of TiO$_2$ and SiO$_2$ layers having thicknesses $d_a$ and $d_b$, respectively. The $d_a$ value is initially fixed at $60$ nm, and the calculated value of $d_b$ is $\approx 110$ nm for $\lambda_0 = 640$ nm, which corresponds

Figure 1. a) Schematic of the asymmetric Tamm structure consists of two DBRs comprised of TiO$_2$ (yellow) and SiO$_2$ (light blue) bilayers and a spacer layer (red); the spacer layers are separated by a thin Ag layer (black) with $d_{sp} = 40$ nm. b) The complex refractive index ($n = n + ik$) of the Ag layer was obtained from the Drude model (line with star) and the Palik handbook (line with square).
to the ZPL of the NV-center used in the present study. A silver (Ag) layer of thickness $d_{\text{Ag}} = 40\text{nm}$ is sandwiched between DBR$_{1}$ and DBR$_{2}$, having different layer pairs. The layers adjacent to the Ag layer on either side are the two spacer layers with thicknesses $d_{1}$ and $d_{2}$. TPR resonance is susceptible to $d_{1}$ or $d_{2}$ values, and for the feasibility of structure, these thicknesses are approximated to be an integral multiple of 5.

The condition for the appearance of eigenmodes in conventional Tamm structure is given by $r_{\text{BR}} r_{\text{Ag}} e^{i\phi} = 1$, with $r_{\text{BR}}$, $r_{\text{Ag}}$ being the reflection coefficients of metal, DBR, and $\phi$ is phase change across the spacer layer. The noble metal Ag follows the behavior of a free-electron model described by the Drude model.$^{[30]}$ The frequency-dependent Ag dielectric function is given by $\varepsilon(\omega) = 1 - \frac{\omega_{p}^{2}}{\omega(\omega + i\gamma)}$, $\Gamma$ is the plasma collision rate related to absorption in the Ag layer. In the limit $\omega \ll \omega_{p}$; for negligible absorption, the eigenfrequency $\omega$ with $n_{a} > n_{i}$ can be approximated as$^{[25,31]}$

$$\omega = \frac{\omega_{p}}{1 + \frac{2\hbar}{m_{e} \omega_{p} n_{a} - n_{i}}} \quad (1)$$

where $\omega_{p}$ is the central frequency of the DBR stop-gap and $\omega_{p}$ is the bulk plasma frequency of the Ag layer. The Drude model offers analytical values of the Ag refractive index, comparable to the index values given in the Palik handbook, as shown in Figure 1b.$^{[32]}$ It shows the comparison between analytical values of refractive index ($\tilde{n}$) with real ($n$) and imaginary ($k$) parts obtained using the Drude model (line with stars) and compared with those values given in the Palik handbook (line with squares). The dotted and solid line shows the $n$ and $k$ values of $\tilde{n}$, respectively. In the region of our interest (400 to 1000 nm), the Drude model and the Palik handbook show nearly identical values. Hence index profile given by the Palik handbook is used in the following calculations.

### 2.1. The Dual Tamm Plasmon Resonance

The substantial dip in the reflectivity spectra of DBR signifies the excitation of TPR states and photonic cavity states.$^{[33]}$ Hence, the modes can be distinguished based on field intensity localization within the structure. The cavity mode localizes the intensity in the cavity layer embedded inside the DBR, while the TPR mode localizes the intensity at the interface between DBR and metal film.$^{[25]}$ In our calculation, a broadband source emitting light in the wavelength range from 400 to 1000 nm is incident on the top of the DBR$_{1}$, as shown in Figure 1a. The normal-incidence reflectivity spectra are calculated using the transfer matrix method (TMM), which involves the calculation of incident and reflected field amplitudes at each interface of the structure.$^{[34]}$ The plane wave excitation creates the TPR (black symbols) within the asymmetric Tamm structure, which appears as a dip in the reflectivity spectra of the DBR stop-gap (solid red line), as seen in Figure 2a. When the Ag layer is absent, an asymmetric cavity mode is manifested at 570 nm (dotted vertical line) for a cavity thickness $d_{1} = 95\text{nm}$. Light enters the cavity, reflecting back and forth between two DBRs. The number of bilayer pairs in DBR$_{1}$ is less than that of DBR$_{2}$. Thus, 100% light is reflected by the DBR$_{2}$, whereas the DBR$_{1}$ reflects minimal light into the cavity layer, resulting in a shallow reflectivity dip with a very low Q value. Here, the confinement is relatively weak, and most of the incident light is reflected toward the incident medium. The weak confinement is due to the asymmetric nature of the photonic cavity with a shallow reflectivity dip at 570 nm.

The inclusion of an Ag layer in between the two spacer layers ($d_{1}$ and $d_{2}$) forms an asymmetric Tamm structure. In contrast to the single reflectivity dip in conventional Tamm structure, we observe two spectrally separated reflectivity dips for asymmetric Tamm structure, as shown in Figure 2a. The reflectivity dip at a lower wavelength is labeled TPR$_{1}$, while the higher wavelength reflectivity dip is called TPR$_{2}$. In our calculation, we use the Ag layer thickness ($d_{\text{Ag}}$) of 40 nm, and the spacer layer thickness $d_{1} = 45\text{nm}$ and $d_{2} = 50\text{nm}$ to tune TPR$_{1}$ to 640 nm and TPR$_{2}$ to 700 nm. The coupled TPRs originate when the Ag layer is introduced within the structure, which alters the electric field distribution along the growth direction. Kaliteevski et al. also propose a similar hybrid state$^{[35]}$ where TPRs are resonantly coupled to the cavity exciton-polaritons modes.

Due to the periodic variation of refractive index in DBR and intrinsic negative dielectric constant of Ag, both DBRs and Ag layer behave as highly reflecting mirrors. The peak reflectivity
wavelength of DBR is tuned to overlap with the reflectivity spectra of the Ag mirror by engineering DBR structural parameters. Accordingly, we optimize structural parameters to obtain field localization at the TPR wavelength compared to an off-resonance wavelength, as shown in Figure 2b. The refractive index profile (solid line) of the asymmetric Tamm structure is presented in Figure 2b for better visualization of the structure. For TPR, at 640 nm, the field intensity is more confined at the interface between the spacer-1 and Ag layer. The corresponding field intensity at 640 nm is estimated to be 55 times (red circles) compared to the incident field intensity. At the TPR2 wavelength of 700 nm, the field confinement (blue squares) is achieved between the spacer-2 and Ag interface with a low enhancement factor of 10. The $Q$-factor is higher for TPR1 than TPR2 and hence achieves better field confinement at 640 nm. Further, the field intensity at an off-resonance wavelength of 550 nm (green triangles) is mostly back-reflected into the incident medium and hence, does not show any significant intensity enhancement inside the structure. Therefore, we emphasize that only TPR wavelengths show substantial field intensity localization, and the asymmetric Tamm structure acts as a perfect reflector for off-resonant wavelength.

### 2.2. The Metal and Spacer Thickness-Dependent Tamm Mode

We used optimized values of $d_m$, $d_n$, $d_m$, $d_n$, and $d_g$ to obtain dual TPR with one of the resonances at 640 nm. However, the appropriate choices of these values would enable the achievement of the TPR at any desired wavelength. The changes in the layer thicknesses modify the phase relations and thus induce the shift in TPR wavelength. As we have seen earlier, when the $d_m$ value is set to zero with $d_n = 45$ nm and $d_g = 50$ nm, the proposed structure behaves like an asymmetric photonic cavity with a cavity layer thickness of 95 nm, as shown in Figure 2a (solid red line). The emergence of two resonances (TPR1 and TPR2) depends on $d_m$ value, and with an increase in $d_m$ value, the resonances start appearing from lower and higher spectral regions, as shown in Figure 3a. An increase in $d_m$ value redshifts the TPR1, while the TPR2 is blue-shifted within the DBR stop gap. Thus, TPR1 and TPR2 evolve and move toward each other as $d_m$ value increases. Moreover, it is seen that both resonances are halted near a critical $d_m$ value of 65 nm, which is due to the phase shift being almost constant at higher $d_m$ values. Hence, TPR is insensitive to a further increase in $d_m$ values. For $d_m > 65$ nm, the TPR1 starts disappearing as light cannot propagate to the other side (DBR1) of the asymmetric Tamm structure due to significant light absorption induced by the Ag layer. This makes the asymmetric Tamm structure behave like a conventional Tamm structure with a single Tamm mode at 640 nm.

Figure 3b shows the calculated reflectivity spectra at various $d_m$ values while $d_n$ and $d_g$ are fixed at 50 and 40 nm, respectively. When $d_m$ = 0 nm, the adjacent layer on top of the Ag layer becomes quarter-wave thick with an index $n_g$ associated with DBR1. Therefore, it induces a phase change of $\pi$ into the incoming light; hence, DBR does not contribute to the formation of dual TPR modes, and the structure becomes a conventional Tamm structure. Thus, we naively expect a single TPR resonance originated...
due to the Ag layer and DBR₂ in the reflectivity spectra as seen at 686 nm ($d_1 = 0\text{nm}$) in Figure 3b. When the value of $d_1$ increases, a new resonant mode emerges from the short-wavelength side, and with an increase in the $d_1$ value, an overall phase is acquired that shifts this mode to the long-wavelength side. Interestingly, both modes exhibit an avoided crossing for $d_1 = 52\text{nm}$, as expected. When the $d_1$ value is < 20 nm, the coupling between TPR₁ and TPR₂ is weak. For such cases, the TPR resonances appear near the resonant mode exhibited by the conventional Tamm structure having spacer thickness values of $d_1$ and $d_2$. The higher value of $d_1$ enhances the coupling between TPR₁ and TPR₂, forming a hybrid state characteristically distinct from a single TPR.

The TPR can be excited using either TE or TM polarization state at any angle of incidence. Figure 3c shows the angular dispersion of the TPR mode in the asymmetric Tamm structure. The negative angles correspond to TM polarization, while the positive angles correspond to TE polarization. We have used structural parameters such as $d_1 = 45\text{nm}$, $d_2 = 50\text{nm}$, and $d_m = 40\text{nm}$ to achieve TPR₁ at 640 nm for normal incidence as seen in Figure 2. The dual modes show angular dependence similar to the conventional Tamm mode. The spectral width of TPR₁ is relatively low compared to TPR₂, hence a higher Q-factor for TPR₁. The Q-factor is calculated as $\lambda/\Delta\lambda$, where $\Delta\lambda$ is the full width at half maximum of the reflectivity dip at TPR wavelength $\lambda$. The $\Delta\lambda$ at normal incidence for TPR₁ is 4.6 nm which is relatively less compared to $\Delta\lambda = 13.8\text{nm}$ for conventional Tamm structure at the same TPR wavelength. Figure 3d shows the variation of the Q-factor with the angle of incidence for the asymmetric Tamm structure. The Q-factor is estimated to be 140 at normal incidence for TPR₁, and it increases (decreases) for TE (TM) polarization as the angle of incidence increases seen in Figure 3d. The Q-factor for TPR mode in a conventional Tamm structure is $\approx 45$ at a TPR wavelength of 640 nm. Thus the proposed asymmetric Tamm structure provides high Q modes that enable significant modulation of emission properties of an embedded quantum emitter. As shown earlier, the asymmetric Tamm structure supports resonances with localization of the field intensity at the upper and the lower spacer layers-Ag interface. We can tune specific structural parameters to vary the spatial confinement of field intensity in the structure. Here we discuss three different cases: a) strong field confinement at the spacer-1-Ag interface, b) strong field confinement at the spacer-2-Ag interface, and c) an equalized field confinement at the interface between both spacer layers and the Ag layer. The light is incident at normal incidence on the structure along the z-direction, and the corresponding index profile (dotted line) is shown in Figure 4a. The electric field profiles at TPR₁ (640 nm) and TPR₂ (700 nm) show different propagation characteristics within the structure. TPR₁ changes sign within the Ag layer while TPR₂ attains a minimum value. The field confinement associated with TPR₁ resonance is not evanescent, even in the presence of the Ag layer. This indicates that there would be a reduction in absorption at 640 nm, which results in a high Q-factor for TPR₁. The NV- center coupled with such a resonant mode helps to radiate higher fields out of the structure as the absorption gets mitigated and results in higher collection efficiency.

Figure 4a shows the field intensity variation through the asymmetric Tamm structure with $d_m = 40\text{nm}$, $d_1 = 45\text{nm}$, and $d_2 = 50\text{nm}$. It is seen that the field confinement is more prominent at the interface between spacer-1 and Ag at TPR₁ of 640 nm (solid black line). However, the field confinement is less noticeable and appears at the interface between spacer-2 and Ag for TPR₂ (solid red line). Figure 4b depicts the field intensity variation for $d_m = 40\text{nm}$, $d_1 = 50\text{nm}$, and $d_2 = 45\text{nm}$, with prominent field confinement achieved at the interface between spacer-2 and Ag for TPR₂. However, minimal confinement occurs at the interface between spacer-1 and Ag for TPR₁. In the case of equal spacer thickness ($d_1 = d_2 = 47\text{nm}$), nearly the same field confinement is obtained at both interfaces for TPR₁. However, the field confinement is less prominent, with almost equal strength for TPR₂. Thus by varying the thickness of $d_1$ and $d_2$, we can tune the field confinement at the TPR, which appears in one of the interfaces or both interfaces simultaneously, as shown in Figure 4. Moreover, the fields corresponding to TPR₁ and TPR₂ are in the phase before they reach the Ag layer. After crossing the Ag layer, the fields become out of phase in all the above three cases, as shown in Figure 4. This eventually indicates that both resonances could show an avoided-crossing characteristic at a particular wavelength. The out-of-phase wavelength obtained from the field analysis appears at 680 nm, which matches the avoided crossing wavelength in the TPR modes, shown in Figure 3b. At 680 nm, the field amplitudes of both resonances cancel each other and hence cannot propagate inside the structure, resulting in higher reflectivity, as seen in Figure 3b. TPR₁ is tuned so that the asymmetric Tamm structure can enhance the NV- center decay rate at ZPL wavelength. In addition, TPR₂ exhibits a higher Q-factor, and hence higher light collection efficiency (LCE) can be achieved at 640 nm, as discussed in the following.
2.3. Spontaneous Emission Decay Rate Enhancement

According to Fermi’s golden Rule, the spontaneous emission decay rate ($\gamma$) of an excited quantum emitter depends on the available density of optical states (DOS) at the emission wavelength, which states that:\[6\]

$$\gamma = \frac{2\pi}{\hbar^2} \sum_k \left| \left\langle f | \hat{H}_l | i \right\rangle \right|^2 \delta (\omega_f - \omega_i) \tag{2}$$

where $\hat{H}_l = -\hat{\mu} \cdot \hat{E}$ is the interaction Hamiltonian in the dipole approximation in terms of dipole moment operator ($\hat{\mu}$) and electric field operator ($\hat{E}$). The quantum emitter is approximated as an oscillating electric dipole that acts as a source of electromagnetic radiation. The decay rate of a dipolar emitter depends on its orientation and the local environment, surrounding it through DOS.\[4,6\] As the emission from dipolar emitters is sensitive to orientation and the low frequency component of optical states (PLDOS) is a more relevant quantity to calculate the orientation-dependent decay rate ($\gamma_l$). A dipole aligned along the unit vector $\hat{r}_o$ will interact strongly with those modes for which the electric field is polarized in the same direction. The interaction is expected to be weak for perpendicularly polarized modes. The $\gamma_p$ for a dipole in free space is written in terms of PLDOS as\[6\]

$$\gamma_p = \frac{2\alpha^2}{3\varepsilon_0 \mu_0} \left| \hat{\mu} \right|^2 \sum_k \left[ \hat{n}_p \cdot (\hat{u}_k \hat{u}_p) \right] \delta (\omega_k - \omega_o) \tag{3}$$

$$\gamma_p = \frac{2\alpha^2}{3\varepsilon_0 \mu_0} \rho_p (\hat{r}_o, \omega_o) \tag{4}$$

where $\rho_p (\hat{r}_o, \omega_o)$ is PLDOS which is sensitive to dipole orientation in the direction of dipole moment $\hat{\mu}$, $\hat{u}_k$ is the energy normal modes, the sum over $k$ refers to summation over all modes, and $\omega_o$ denotes the mode frequency $\tilde{k}$. The $\delta (\omega_k - \omega_o)$ accounts for the frequencies of available optical states, and with significant dissipation, the delta function loses its meaning in Equation (3). The mode amplitudes decrease over time due to dissipation, and the eigenfrequencies for such a system must be complex with null real eigenfrequencies. It is more convenient to represent PLDOS in Green’s function formalism. It is a powerful tool for calculating PLDOS and spontaneous emission decay rates.\[6\] The electromagnetic Green’s function is a dyadic quantity that estimates the radiated field due to a point source located at $\hat{r}_o$. To consider all source components, we need Green’s function as a tensor quantity denoted as $G$. It is a compact notation for the three components of Green’s function corresponding to three distinct dipole orientations along the x-y-z-axis. The $\rho_p$ is related to $G$ by the relation:\[6\]

$$\rho_p (\hat{r}_o, \omega_o) = \frac{6\alpha^2}{\pi \varepsilon_0 c^2} \left[ \hat{n}_p \cdot \mathbb{M} \left\{ G (\hat{r}_o, \tilde{r}_o; \omega_o) \right\} \right] \cdot \hat{n}_p \tag{5}$$

Here, $\rho_p$ represents PLDOS when the dipole is placed in air/vacuum. For other dielectric media, Equation (5) needs to be modified to consider the refractive index contribution. Moreover, we also need to calculate the LDOS for the emitter, which emits in all spatial directions. The LDOS is the average of PLDOS over orthogonal dipole orientations along the x-y-z-axis. The spontaneous decay rate $\gamma_l$ is related to LDOS in terms of $G$ as:\[6\]

$$\gamma_l = \frac{2\alpha^2}{3\varepsilon_0 \mu_0} \left| \hat{\mu} \right|^2 \rho_l (\hat{r}_o, \omega_o) \tag{6}$$

$$\gamma_l = \frac{2\alpha^2}{3\varepsilon_0 \mu_0} \left| \hat{\mu} \right|^2 \frac{2\alpha^2}{\pi c^2} \mathbb{M} \left\{ \text{Tr} [G (\hat{r}_o, \tilde{r}_o; \omega_o)] \right\} \tag{7}$$

where $\rho_l (\hat{r}_o, \omega_o) = \frac{2\alpha^2}{\pi c^2} \mathbb{M} \left\{ \text{Tr} [G (\hat{r}_o, \tilde{r}_o; \omega_o)] \right\}$ is the LDOS and $\text{Tr} [G (\hat{r}_o, \tilde{r}_o; \omega_o)]$ denotes the trace of dyadic $3 \times 3$ tensor matrix $G$. The value of LDOS determines how quickly an excited atom decays to its ground state, producing a photon in the process. Thus we need the information of $G$ within a system in which the emitter emits the radiation.

The LDOS calculations are performed using a 3D finite-difference time-domain method using Ansys Lumerical. A single NV-center emitting at a ZPL wavelength of 640 nm is positioned within the spacer-1 layer, wherein maximum electric field intensity is obtained for TPR, as seen in Figure 2b. The simulation is performed within a 2.25 $\mu$m$^2$ region in the x-y plane with a mesh size of 4 nm using perfectly matching boundary conditions in x-y-z directions. The resulting radiated power is detected by a 3D transmission box that calculates net outward power flow from the dipole within the simulation region. The resulting electric field at the dipole location within spacer-1 layer computes Green’s dyadic $G:

$$\mathbb{E} (\tilde{r}) = \frac{n^2}{\varepsilon_n} \mathbb{G} (\tilde{r}_o, \tilde{r}_o) \cdot \hat{\mu}$$

\[8\]

where the electric field $\mathbb{E}$ is calculated at the position $\tilde{r}$, $n = \sqrt{\mu_n \varepsilon_n}$ is the refractive index of material with relative permeability and permittivity as $\mu_n$ and $\varepsilon_n$, respectively. The dipole is placed within the spacer-1 layer, which has an index of $n_o$.

2.4. Purcell Enhancement and Light Collection Efficiency

The interaction of a single NV-center with an enhanced localized field at TPR is analyzed using the Purcell factor calculations. When the local environment is not homogeneous, Equation (3) suggests that $\gamma$ is discrete for parallel and perpendicular orientations of the emitter. The Purcell enhancement is defined as $\frac{\gamma}{\gamma_p}$, where $\gamma_p$ and $\gamma$ are the spontaneous decay rate when the NV-center is embedded in a dielectric slab and an asymmetric Tamm structure, respectively. The Purcell enhancement is averaged for emitters oriented in parallel and perpendicular directions to the plane of the structure (x-y plane) to account for all distinct emitter orientations. The emission properties of the NV-center are studied using radiating point dipole source. The dipole is positioned in spacer-1 at a distance of 30 nm away from the Ag layer, as shown in Figure 5a. If the separation distance < 30 nm, DOS sharply rises due to the excitation of surface plasmon modes.
Figure 5.  
a) The electric field intensity variation associated with an emitted field for the emitter placed in the spacer layer. The dipole is placed in the high magnitude region (red spot above the Ag layer). 
b) Far-field patterns of light emission intensity at various wavelengths. 
c) The wavelength-dependent Purcell enhancement for NV-center located in the spacer-1 layer. The shaded region corresponds to TPR1 with maximum Purcell enhancement of 4. 
d) Purcell enhancement and light collection efficiency at various wavelengths for the dipole oriented parallel to the structure with $d_m = 40\, \text{nm}$.

at the Ag-TiO$_2$ interface. However, these states include radiative and non-radiative modes induced by the environment. The non-radiative decay channels dominate at a smaller distance, resulting in a significant emission quenching. Thus positioning the emitter at a distance of 30 nm from the Ag layer in the spacer-1 layer ensures that the dipole emission quenching due to excited plasmon mode is avoided. Figure 5a shows the electric field intensity of the emitter positioned in the dielectric spacer-1 layer and emitting at 640 nm. The dashed lines in Figure 5a separate the SiO$_2$-TiO$_2$ layers, and the grey area at $z = 0$ nm encloses the Ag layer. We observe localized field intensity at the interface between spacer-1 and Ag layers due to field confinement induced by TPR1. Moreover, the field decays off along the z-axis with high intensity in the SiO$_2$ and low intensity in the TiO$_2$ layers. The far-field emission pattern at the resonance wavelength of 640 nm and different off-resonance wavelengths is shown in Figure 5b. The on-resonance emission pattern shows four times higher emission intensity at the far field compared to the off-resonance wavelength, which indicates the off-resonant emission intensity suppression.

The asymmetric Tamm structure shows dual TPR in the reflectivity spectra with distinct field localization in the spacer layer, as seen in Figure 2a. The resonance wavelength has a strong dependence on the $d_m$ value, as seen in Figure 3a, and thus, we should expect Purcell enhancement also depends on $d_m$ value at different TPR wavelengths. Figure 5c shows the Purcell enhancement factors, estimated for different $d_m$ values. For $d_m \geq 40\, \text{nm}$, we observe Purcell enhancement for both modes, with TPR1 mode showing a higher Purcell enhancement with a factor of 4 in contrast to TPR2 mode, as shown in Figure 5c. Moreover, the Purcell enhancement factor is comparatively low for smaller $d_m$ values at the TPR1. This is due to thin Ag film; below a specific $d_m$ value, TPR mode cannot sustain and thus results in weak emitter-TPR coupling. The wavelength-dependent Purcell enhancement averaged over all dipole orientations (parallel and perpendicular) shows a strong dependence on the $d_m$ value. With an increase in its value, the Purcell factor increases and saturates to 4 for $d_m \geq 40\, \text{nm}$. It should be noted that the Purcell enhancement peak shows a redshift with an increase in $d_m$ value, which follows from Figure 3a. The presence of a thick Ag layer (high $d_m$ value) increases absorption, thus inhibiting an effective field that can escape the structure. However, a minimum $d_m$ value is necessary to sustain the TPR mode. Therefore, there is a trade-off between the required Purcell enhancement and the amount of light escaping from the structure through the judicious choice of $d_m$ value. The optimum $d_m$ value is 40 nm, corresponding to a high Purcell factor at TPR1 helps in the acquisition of fast-emitted photons from the asymmetric Tamm structure.

The calculated wavelength-dependent Purcell enhancement shows a lower value for shorter wavelengths, increases to its maximum value at 640 nm for $d_m = 40\, \text{nm}$, and has a sudden drop after 640 nm. This enhancement results from excitations of surface plasmon polaritons and TPR within the structure. The emitted light from a single NV- center is coupled to the evanescent field, and surface plasmon polaritons are excited accordingly. However, the TPR excitations are primarily dominant at higher wavelength regions. The evanescent field penetration is known to be proportional to the wavelength.$^{[6]}$ Thus, in the
shorter wavelength regime (< 450 nm), the evanescent field penetration is minimal for the dipole placed at a distance of 30 nm above the Ag layer. This results in reduced near-field coupling and, thus, an insignificant Purcell enhancement factor. However, for the spectral region between 450 to 500 nm, the increase in the Purcell enhancement factor is mainly due to emitter coupling with surface plasmon polaritons. For the wavelength above 570 nm, the Purcell enhancement is primarily due to emitter coupling with TPRs, which can be understood using the parabolic dispersion of TPR.\[^{[25]}\]

The $\gamma$ can also be given in terms of group velocity dispersion ($\frac{d\omega}{dk}$) such that $\gamma \propto \frac{d\omega}{dk}$.\[^{[39]}\] In the spectral range between 570 to 640 nm, $\frac{d\omega}{dk}$ decreases gradually, increasing Purcell enhancement (dipole oriented parallel to the structure). At 640 nm, the $\frac{d\omega}{dk}$ reaches its minimum value, and accordingly, the maximum Purcell enhancement of 4 is obtained. There is a steep slope above 640 nm, which suggests a decrease in coupling strength above 640 nm, which follows from the angle-dependent reflectivity contour map shown in Figure 3c. This emphasizes that the high Purcell enhancement at the TPR wavelength is due to emitter coupling with TPR, and above TPR, the Purcell enhancement factor decreases rapidly. In the above analysis, we have considered [111] oriented nanodiamond surface containing NV- center. Suppose we consider [100] diamond surface with tilted NV- centers such that dipole orientation is perpendicular to the surface of a structure; the emitter would not be able to couple Tamm plasmons effectively. This results in low Purcell enhancement and light collection efficiency at 640 nm. We have taken the average Purcell enhancement that includes both kinds of dipole orientations. However, the dipole would interact strongly with those modes whose electric field is polarized in the same direction as the dipole orientation. Thus the main contribution to Purcell enhancement shown in Figure 5c arises from the parallel-oriented dipole.

In addition to the Purcell enhancement, LCE is an essential parameter for quantum emitter trapped in photonic cavities.\[^{[40]}\] The LCE quantifies the amount of light collected at the far field using a suitable detector from a single emitter coupled to a photonic structure. Even if the emission rate or Purcell enhancement is high, the LCE must be enhanced to achieve maximum collection of fast photons. Figure 5d shows Purcell enhancement (black circle) and LCE (red triangle) for parallel-oriented dipole as a function of wavelength. The emitted light is collected in the far field within a cone of half-angle 60° above the structure to estimate the LCE. This angular range corresponds to a microscopic objective of numerical aperture 0.85, typically used in experiments involving single quantum emitters. The LCE is calculated as $LCE = \frac{\text{FFC}_{60}}{\text{FCC}_{60}}$, where FCC$_{60}$ is the far field enhancement of emission intensity collected within 60° cone above the metal layer. It is obtained as the ratio between emission intensity contained within a light cone of 60° from an asymmetric Tamm structure to the emission from a dielectric slab (TiO$_2$). The $P_i$ is the Purcell enhancement for the dipole aligned parallel to the structure. The results show a large LCE for the emitted light corresponding to TPR. It manifests that the TPR plays a significant role in extracting the emitted photons compared to the light collection using surface plasmon polaritons. The NV- center coupled to an asymmetric Tamm structure provides maximum Purcell enhancement and higher LCE at 640 nm. The structure shows the maximum LCE factor of 5 and 2.5 at 640 and 700 nm, respectively. The light collection is improved five times compared to the emitter emitting in a dielectric slab at 640 nm. It is found that after 640 nm, there is a sharp decrease in LCE compared to the Purcell enhancement factor. The drop in Purcell enhancement is due to the absence of TPR above 640 nm (Figure 3b), and surface plasmon polaritons induce remaining enhancement. Moreover, for the LCE, only TPRs excitations provide the maximum extraction of emitted photons, and the absence of TPRs causes a sudden decrease in LCE. Further, the $Q$-factor of TPR, is high, which also helps eliminate emissions above 640 nm. Thus, for NV- center, the PSB emission near ZPL can be efficiently suppressed, providing an enhanced ZPL emission at 640 nm. The coupling of NV-center with TPR in the asymmetric Tamm structure ensures an enhanced spontaneous emission decay rate at 640 nm. Furthermore, the high $Q$-factor and low dissipative losses give high collection efficiency of the emitted photons. The resonances can be controlled by varying the thickness of the metal layer and the adjacent dielectric (spacer) layer. Thus, the structure can be tuned to enhance the emission from other quantum emitters like Si vacancy center and h-BN integrated with asymmetric Tamm structures.

### 2.5. Practical Implementation of the Proposed Structure

The discussion so far considered precise thicknesses of the different layers of the structure. It is expected to have thickness variations while fabricating the proposed structure; hence, we discuss the thickness tolerance of various layers. Figure 3a shows the dependence of Tamm resonance wavelength on the $d_m$ value. Additionally, Figure 5c indicates that a minimum $d_m$ value is required to achieve high Purcell enhancement with an optimum $d_m = 40$nm, so maximum light collection efficiency is also achievable. Hence, a $d_m$ value in the 40–45 nm range is feasible, keeping the Tamm mode at 640 nm while fabricating the proposed structure. The thickness accuracy of the Ag layer can be ensured by using modern metal deposition methods such as the thermal evaporation method using the Ag palette. In the case of DBR, which has a large number of layers and variations in the layer thickness can affect the spectral position of resonance. Figure 6a shows the photonic bandgap of DBR for various thicknesses of the layers that are used to make DBR. The thickness of each layer of TiO$_2$ and SiO$_2$ is randomized between 55–65 nm and 105–110 nm, respectively, for 500 iterations (counts). The randomization of dielectric thickness is performed by generating random numbers within a range of thickness variation for each dielectric layer and then obtaining reflectivity using TMM.

Figure 6a shows that the photonic bandgap from 550 nm to 750 nm remains intact for any thicknesses values in the range of 55–65 nm for TiO$_2$ and 105–110 nm for SiO$_2$ layers. However, the bandgap begins to collapse as thicknesses are randomized between 45–75 nm and 95–125 nm for TiO$_2$ and SiO$_2$ layers, as shown in Figure 6b. Thus, DBR can have 10 nm of thickness tolerance during fabrication from the optimized $d_m = 60$nm for TiO$_2$ and $d_m = 110$nm for SiO$_2$. The fabrication of DBR can be done using an economical sol-gel method and can be done at scale with a few initial optimizations. However, the
DBR₁ (having few layers) needs to be more precise in terms of thickness values and thus requires more sophisticated physical deposition techniques such as e-beam evaporation deposition and RF sputtering. This way, DBR₂ can have a more considerable tolerance to the fabrication variations than DBR₁.

The following approach can realize the whole structure incorporating nanodiamonds containing a single NV-center. The proposed structure synthesis begins by fabricating DBR₂ using the sol-gel synthesis method. The Ag layer on top of DBR₂ can be made using the thermal evaporation deposition method, which can provide reasonable control over Ag thickness values. For DBR₁, the spacer-1 layer needs to be fabricated in three stages. Firstly, 30 nm of TiO₂ layer would be deposited on top of the Ag layer with high precision and the nanodiamonds containing a single NV-center are then spin-coated on top of it. Lastly, the remaining TiO₂ deposition completes the spacer-1 layer as an active layer with NV-centers as emitters. The last part of fabrication is fabricating DBR₁ on top of spacer-1 using sputtering methods, which completes the proposed asymmetric Tamm structure. The layered structure fabrication is relatively easy compared to other plasmonic structures, which involve nano-size holes and grating structures using e-beam lithography and patterns. The proposed structure contains multiple bilayers of dielectric (TiO₂/SiO₂) material that can be fabricated by the sol-gel synthesis/sputtering techniques or an e-beam evaporation method. The advantage of implementing this approach is that the emitter’s position and tuning of resonance are possible by changing a few structural parameters. Besides, the proposed structure can be made over larger areas, such as on a one-inch Si wafer or quartz substrate. The total number of bilayers required for the combined structures is 13, which is possible using current fabrication facilities.

The positioning of the quantum emitter at the correct location in the spacer layer is an important metric to achieve high Purcell enhancement. In our work, a single NV-center is placed at a distance of 30 nm from the Ag layer, providing the best possible coupling scheme to achieve high Purcell enhancement and light collection efficiency. However, Purcell enhancement decreases very slowly as the emitter position is shifted away from the Ag layer due to a reduction in quantum emitter coupling with structure. For the emitter with a distance > 40 nm from the Ag layer, the Purcell enhancement reduces by a factor of 1. So, we can have an acceptable variation of ±5 nm in the NV-center position near the 30 nm from the Ag layer and still obtain high Purcell enhancement and light collection efficiency.

3. Conclusion

The asymmetric Tamm structure offers dual resonances, one having a high-quality factor that can be used to tune the emission rate of a single NV-center. The structure provides high-quality resonance with Δλ = 4.6nm compared to the conventional Tamm structure (Δλ = 13.8nm). The coupling of NV-center, emitting at a ZPL of 640 nm, shows the enhancement in the decay rate up to a factor of 4 compared to NV-center emitting in a dielectric slab. This arises from strong light confinement at metal-dielectric interfaces induced by the Tamm plasmon resonance mode. We have obtained four times enhanced Purcell factor due to the modification in LDOS caused by Tamm plasmon resonance. The proposed asymmetric Tamm structure provides five times more light collection at 640 nm compared to light collection from NV-center embedded in a dielectric slab. The structure can be fabricated over a large area which is possible using the current sample fabrication tools. The proposed asymmetric Tamm structure offers a paradigm shift in achieving bright single photon emission with an enhanced rate and improved collection efficiency.

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Conflict of Interest

The authors declare no conflict of interest.
Data Availability Statement
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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Purcell effect, quantum photonics, single NV-center, single photons, Tamm plasmons

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