Effects of Polaron Formation in Semiconductor Quantum Dots on Transport Properties

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We theoretically examine the effects of polaron formation in quantum dots on the transport properties. When a separation between two electron-levels in a quantum dot matches the energy of the longitudinal optical (LO) phonons, the polarons are strongly formed. The Rabi splitting between the levels is observable in a peak structure of the differential conductance $G$ as a function of the bias voltage. The polaron formation suppresses the peak height of $G$, which is due to the competition between the resonant tunneling (resonance between a level in the dot and states in the leads) and the polaron formation (Rabi oscillation between two levels in the dot). $G$ shows a sharp dip at the midpoint between the split peaks. This is attributable to the destructive interference between bonding and anti-bonding states in a composite system of electrons and phonons.

KEYWORDS: quantum dot, polaron, resonant tunneling, LO phonon, Rabi splitting, self-consistent Born approximation

1. Introduction

In semiconductor quantum dots, the relaxation of electrons between discrete energy-levels is an important issue from a viewpoint of application to quantum-dot lasers. The electron-phonon interaction should be much less efficient for the relaxation than in the bulk. This is because the number of phonons is very limited whose energies match the separation between the discrete levels, which is in contrast to the case of bulk where the electron levels form a continuous spectrum. Related to this “phonon bottleneck,” which is not always observed in actual quantum dots, several theoretical works have been reported on the relaxation mechanisms in quantum dots.

A remarkable exception of the inefficiency of the electron-phonon interaction is seen when the energy of the longitudinal optical (LO) phonons matches the level separation. Then the interaction can be significantly strong, reflecting a large density of states of the LO phonons with little dispersion. The electron-LO-phonon interaction results in the formation of “polarons.” The polarons in quantum dots are coherent states consisting of the discrete electron-levels and LO phonons, which were predicted by Inoshita and Sakaki. The polarons have been found in self-assembled InAs quantum dots embedded in GaAs, using the far-infrared spectroscopy. When a spacing between the electron levels is tuned to match the energy of LO phonons by applying magnetic fields, the Rabi splitting of the coherent states has been observed in the absorption spectra.

In the present paper, we theoretically examine a situation to observe the polarons by the electric transport, considering a quantum dot connected to external leads. The conductance takes place by the resonant tunneling through electron levels in the quantum dot. We show that the polaron formation is observable in a peak structure of the differential conductance $G$ as a function of the bias voltage. Recently the transport properties of one of the self-assembled InAs quantum...
dots can be investigated in vertically fabricated samples of heterostructures including a layer of such quantum dots.\textsuperscript{13, 14} Hence our theoretical results are expected to be observed experimentally. Compared with the optical experiments, the transport measurement has the advantage to be able to examine the polaron formation in a single quantum dot. Hence the measurement could reveal its characters in more details.

From a viewpoint of the fundamental research, this is a simple example of important problems for the coupling between electrons and bosonic fields in the environment. We find new interference effects on the conductance in a composite system of electrons and phonons. (i) The polaron formation suppresses the peak height of $G$, which is due to the competition between the resonant tunneling (resonance between a level in the dot and states in the leads) and the polaron formation (Rabi oscillation between two levels in the dot). (ii) $G$ shows a sharp dip at the midpoint between the split peaks. This is attributable to the destructive interference between bonding and antibonding states in the composite system. This is analogous to the transport through double quantum dots connected in parallel\textsuperscript{15} and Fano resonance.\textsuperscript{16}

The organization of the present paper is as follows. In section 2, we explain our models and calculation methods. In section 3, we examine an exactly solvable model. By the calculations using Green’s function, we show that the polaron formation is observable in a peak structure of the differential conductance. In section 4, we consider a more general model. The Green’s function is calculated in the self-consistent Born approximation to take into account the electron-phonon interaction to infinite orders. The new interference effects on the conductance are discussed. The last section (section 5) is devoted to the conclusions and discussion.

2. Models and Calculation Methods

We consider a quantum dot with two levels, $g$ and $e$, for electrons, as shown in Fig. 1(a). The level spacing, $\Delta \varepsilon = \varepsilon_e - \varepsilon_g$, is tuned. The levels are coupled to two external leads, $L$, $R$, through tunnel barriers. The Hamiltonian of electrons, $H_e + H_T$, is given by

\begin{equation}
H_e = \sum_{i=g,e} \varepsilon_i d_i^\dagger d_i + \sum_{\alpha=L,R} \sum_k \varepsilon_k a_{\alpha,k}^\dagger a_{\alpha,k},
\end{equation}

\begin{equation}
H_T = \sum_{i=g,e} \sum_{\alpha=L,R} \sum_k (V_{\alpha}^2 d_i^\dagger a_{\alpha,k} + \text{h.c.}),
\end{equation}

where $d_i^\dagger$, $a_{\alpha,k}^\dagger$ ($d_i$, $a_{\alpha,k}$) are the creation (annihilation) operators for an electron in the dot and in lead $\alpha$, respectively. The spins of electrons are disregarded since they are not relevant in this problem. The electron-electron interaction is not taken into account in this paper. The tunnel couplings, $H_T$, broaden the level $i$ in the dot with $\Delta^i = \Delta^i_L + \Delta^i_R$.

\begin{equation}
\Delta^i_\alpha = \pi \nu |V^i_\alpha|^2,
\end{equation}

where $\nu$ is the density of states in the leads.\textsuperscript{17}

For phonons, we consider an LO phonon mode,

\begin{equation}
H_{ph} = \sum_q \hbar \omega_q b_q^\dagger b_q,
\end{equation}

where $b_q^\dagger$ ($b_q$) creates (annihilates) an LO phonon with momentum $q$. Since only phonons with long wavelength, $|q| \lesssim 2\pi/($dot size$)$, interact with electrons significantly,\textsuperscript{10} we can regard the phonon
mode as dispersionless; $\omega_q = \omega_{\text{LO}}$. Other modes of phonons are neglected since they do not effectively couple to electrons in the quantum dot when $\Delta \varepsilon$ is of the order of $10\text{meV}^{11,12}$ (Appendix A).

The electron-LO-phonon interaction is described by the Fröhlich Hamiltonian $H_{\text{e-ph}}$ (Appendix A). For the phonon-absorption process, the matrix element between an electron at level $e$ with $N-1$ phonon, $|e, (N-1)q\rangle$, and an electron at $g$ with $N$ phonon, $|g, N_q\rangle$, is written as

$$\langle e, (N-1)q | H_{\text{e-ph}} | g, N_q \rangle = \sqrt{N_q} v_q.$$  \hfill (5)

We denote

$$v^2 = \sum_q |v_q|^2$$ \hfill (6)

which characterizes the strength of the electron-phonon interaction. We set $v = 0.2\hbar \omega_{\text{LO}}$, considering the experimental situations.$^{11,12}$

The total Hamiltonian is

$$H_{\text{tot}} = H_e + H_T + H_{\text{ph}} + H_{\text{e-ph}}.$$ \hfill (7)

To examine the polaron formation, we have to include $H_{\text{e-ph}}$ to all orders. For the purpose, (i) we study an exactly solvable model in which only level $e$ in the dot couples to the leads by the tunneling; $V^0_L = V^0_R = 0$. We present the Green’s function in an analytical form in section 3. (ii) In general situations, we take $H_e + H_T + H_{\text{ph}}$ as an unperturbed Hamiltonian, taking account of the tunnel couplings between the dot and leads exactly. Then we consider $H_{\text{e-ph}}$ as a perturbation, using the self-consistent Born approximation$^{18}$ (section 4).

3. Calculated Results with Solvable Model

To illustrate the effects of polaron formation on the transport properties, we begin with a simple model with $V^0_L = V^0_R = 0$. This model is exactly solvable if we restrict the states in the quantum dot to $|e, 0_{\text{ph}}\rangle$ (an electron at level $e$ and no phonon), $|g, 1_q\rangle$ (an electron at level $g$ and one phonon), and no electron nor phonon (Fig. 1(b)).

The Green’s function $\hat{G}(\varepsilon)$, which is defined by

$$(\varepsilon - H_{\text{tot}} + i\delta)\hat{G}(\varepsilon) = 1,$$ \hfill (8)

is obtained in an analytical way, as shown in Appendix B. The diagonal elements for the states in the dot are

$$G_{(e,0_{\text{ph}})}(\varepsilon) = \frac{\langle e, 0_{\text{ph}} | \hat{G}(\varepsilon) | e, 0_{\text{ph}} \rangle}{\varepsilon - \varepsilon_e + i\Delta \varepsilon - \Sigma_{(e,0_{\text{ph}})}(\varepsilon)},$$ \hfill (9)

$$\Sigma_{(e,0_{\text{ph}})}(\varepsilon) = \frac{v^2}{\varepsilon - (\varepsilon_g + \hbar \omega_{\text{LO}}) + i\delta},$$ \hfill (10)
and

\[
G_{(g,1q)}(\varepsilon) = \langle g,1q|\hat{G}(\varepsilon)|g,1q\rangle = \frac{1}{\varepsilon - (\varepsilon_g + \hbar\omega_{LO}) - \Sigma_{(g,1q)}(\varepsilon)},
\]

\[
\Sigma_{(g,1q)}(\varepsilon) = \frac{\hbar}{\varepsilon - \varepsilon_e + i\Delta_e - \varepsilon - (\varepsilon_g + \hbar\omega_{LO}) + i\delta}.
\]

In terms of the Green’s function, the density of states in the dot is written as

\[
D(\varepsilon) = -\frac{1}{\pi} \text{Im} \left[ G_{(e,0\text{ph})}(\varepsilon) + \sum_q G_{(g,1q)}(\varepsilon) \right].
\]

The electric current from lead \( L \) to \( R \) is

\[
I = e^2 \frac{2\pi}{\hbar} \sum_{k,k'} \left| V_{e,k}^* G_{(e,0\text{ph})}(\varepsilon_k) V_{e,k'}^2 \delta(\varepsilon_{k'} - \varepsilon_k) \right| \times \left[ f(\varepsilon_k - \mu_L) - f(\varepsilon_{k'} - \mu_R) \right],
\]

where \( \mu_\alpha \) is the Fermi level in lead \( \alpha \) and \( f(\varepsilon) \) is the Fermi distribution function. At \( T = 0 \), the differential conductance is given by

\[
G = e \frac{dI}{d\mu_L} = e^2 \left. \frac{4\Delta_e^L \Delta_e^R}{(\varepsilon - \varepsilon_e - \text{Re}\Sigma_{(e,0\text{ph})}(\varepsilon))^2 + (\Delta_e - \text{Im}\Sigma_{(e,0\text{ph})}(\varepsilon))^2} \right|_{\varepsilon = \mu_L},
\]

when \( \mu_R \) is fixed.

In Fig. 2, we show the calculated results of (a) the density of states in the quantum dot and (b) differential conductance \( G \) at \( T = 0 \). The level spacing, \( \Delta \varepsilon = \varepsilon_e - \varepsilon_g \), is changed gradually. In (a), two Lorentzian peaks appear corresponding to the states \( |g,1q\rangle \) and \( |e,0\text{ph}\rangle \). The peak of the former \( (\varepsilon \approx \varepsilon_g + \hbar\omega_{LO}) \) is sharp and high, reflecting large density of states of the LO phonons. The peak of the latter \( (\varepsilon \approx \varepsilon_g + \Delta \varepsilon) \) is more broadened owing to the tunnel coupling to the leads. Around \( \Delta \varepsilon = \hbar\omega_{LO} \), an anti-crossing between the states is seen, which is ascribed to the formation of polarons. The electron-phonon interaction mixes the states \( |g,1q\rangle \) and \( |e,0\text{ph}\rangle \), making the bonding state with lower energy and the anti-bonding state with higher energy. Hence this anti-crossing of the two peaks is due to the Rabi splitting caused by the electron-phonon interaction.

The differential conductance \( G \) shows a similar peak structure, as a function of the bias voltage, \((\mu_L - \mu_R)/e\), to that in the density of states \( D(\varepsilon) \). The current flows by the resonant tunneling through the electronic states in the dot. Because of the mixture between \( |g,1q\rangle \) and \( |e,0\text{ph}\rangle \), \( G \) has a peak even at the position of states \( |g,1q\rangle \) in spite of the absence of the tunnel coupling of level \( g \). In conclusion, we clearly observe the Rabi-splitting by the polaron formation in the transport properties of the quantum dots.

4. Calculated Results in Self-Consistent Born Approximation

When both the levels in the quantum dot couple to the leads, we adopt the self-consistent Born approximation to consider the electron-phonon interaction to infinite orders (Appendix C). In this
between bonding and anti-bonding couplings of the discrete level and the continuum. Note that, the resonance, with a dip on one side of the resonance. This dip is due to the destructive interaction interference between a discrete level and a continuum of states results in an asymmetric shape of left and right leads are equivalent. Another analogous phenomenon is the Fano resonance.

\[ G \sim \frac{1}{\varepsilon - \varepsilon_i + i\Delta \varepsilon} \]  

\((i = g, e)\), is determined by the self-consistent equation

\[ \Sigma_i(\varepsilon) = \sum_q |q|^2 \left[ \frac{N_q}{\varepsilon + i\hbar\omega_q - \varepsilon_j + i\Delta_j - \Sigma_j(\varepsilon + \hbar\omega_q)} + \frac{N_q + 1}{\varepsilon - i\hbar\omega_q - \varepsilon_j + i\Delta_j - \Sigma_j(\varepsilon - \hbar\omega_q)} \right]. \]  

(17)

Here, \( j \neq i \) and \( N_q \) is the number of phonons with momentum \( q \). Note that we disregard the inelastic transport processes where an electron enters a level in the dot and goes out from the other level in the dot, to focus on the effects of the polaron formation on the transport. This is a good approximation when \( \Delta^g_{L,R} \ll \Delta^e_{L,R} \). We will discuss this approximation later. The density of states in the dot, \( D(\varepsilon) \), and the differential conductance, \( G \), are calculated in the same way as in the previous section.

Figure 3 presents the calculated results at \( T = 0 \). The density of states in the dot shows three large peaks at \( \varepsilon \approx \varepsilon_g, \varepsilon_g + \hbar\omega_{\text{LO}}, \varepsilon_g + \Delta \varepsilon \), which correspond to 'level \( g \) with no phonon,' 'level \( g \) with one phonon' and 'level \( e \) with no phonon,' respectively. A small peak at \( \varepsilon \approx \varepsilon_g + \Delta \varepsilon + \hbar\omega_{\text{LO}} \) is also observable, corresponding to 'level \( e \) with one phonon.' The peak widths are determined mainly by the level broadening, \( \Delta^g \), due to the tunnel couplings to the leads. An anti-crossing between \( |g, 1\rangle \) and \( |e, 0_{\text{ph}} \rangle \) is seen at \( \Delta \varepsilon \approx \hbar\omega_{\text{LO}} \), as discussed in Fig. 2.

The differential conductance \( G \) through the dot shows a similar peak structure as a function of the bias voltage. Now we discuss two new interference effects of the polaron formation on \( G \). First, the polaron formation suppresses the peak height of \( G \). When \( |\Delta \varepsilon - \hbar\omega_{\text{LO}}| \) is large, the resonant tunneling through level \( g \) or \( e \) makes peaks of \( G \approx e^2/h \) in height. When \( \Delta \varepsilon \approx \hbar\omega_{\text{LO}} \), however, \( G \) is much smaller than \( e^2/h \). This is because the electron-phonon interaction results in a finite life-time of the electron levels, \( \tau (\text{Im} \Sigma \sim -\hbar/\tau) \), which weakens the resonant tunneling by a factor of \( \sim |\Delta^g/(\Delta^e + \hbar/\tau)|^2 \). Second, the differential conductance shows a dip at the midpoint between the two peaks. To see this dip more clearly, we show the conductance at \( T = 0 \) with larger level broadening \( \Delta^e \) in Fig. 4. The level spacing in the dot is fixed to match the energy of the LO phonons; \( \Delta \varepsilon = \hbar\omega_{\text{LO}} \). This dip is sharp and deep when \( \Delta^g \ll \Delta^e \). (In model (a) with \( \Delta^g = 0 \), the analytical solution indicates that \( G = 0 \) exactly at \( \mu_L = \varepsilon_g + \hbar\omega_{\text{LO}} \).) With increasing \( \Delta^g/\Delta^e \), the dip is smeared. This dip of \( G \) is a consequence of the destructive interference between the bonding and anti-bonding states of \( |g, 1\rangle \) and \( |e, 0_{\text{ph}} \rangle \). Kawamura and Aono have proposed a similar dip of \( G \) in the transport through double quantum dots coupled in parallel. They have considered a situation where only one of the dots is connected to the leads by the tunneling. In this case, the differential conductance has two peaks due to the resonant tunneling through bonding and anti-bonding orbitals between the dots. \( G \) has a dip at the midpoint between the peaks (\( G = 0 \) exactly when the tunnel couplings to left and right leads are equivalent). Another analogous phenomenon is the Fano resonance. The interference between a discrete level and a continuum of states results in an asymmetric shape of the resonance, with a dip on one side of the resonance. This dip is due to the destructive interaction between bonding and anti-bonding couplings of the discrete level and the continuum. Note that,
in our model, the dip of $G$ is attributable to the destructive interference in a composite system of electrons and phonons.

Finally, we discuss the differential conductance $G$ at finite temperature. When $k_B T \gg \Delta_{g,e}$, the peak heights of $G$ are much smaller than $e^2/h$, whereas the peak widths are determined by the thermal energy. Figure 5 shows $G$ at $k_B T = 0.05 \hbar \omega_{LO}$ ($T \approx 18 K$), $\Delta_{g,L}^e = \Delta_{g,R}^e = 0.005 \hbar \omega_{LO}$ and $\Delta_{e,L}^c = \Delta_{e,R}^c = 0.01 \hbar \omega_{LO}$. Although the peaks of $G$ are thermally broadened, the evidence of the polaron formation is still observable. We can also see the above-mentioned characters of $G$, the suppression of the peak heights at $\Delta \varepsilon \approx \hbar \omega_{LO}$ and the dip of $G$ between the resonant peaks.

5. Conclusions and Discussion

We have investigated the effects of polaron formation in a quantum dot on the transport properties. When a separation between two levels in the dot, $g$ and $e$, matches the energy of the LO phonons, the polarons are strongly formed. The Rabi splitting between states $|g, 1_q\rangle$ and $|e, 0_{ph}\rangle$ is observable in a peak structure of the differential conductance $G$, as a function of the bias voltage. We have found new interference effects on the transport in this composite system of electrons and phonons. The polaron formation suppresses the peak height of $G$, which is attributable to the competition between the resonant tunneling (resonance between a level in the dot and states in the leads) and polaron formation (Rabi oscillation between two levels in the dot). $G$ shows a sharp dip at the midpoint between the split peaks. This is due to the destructive interference between bonding and anti-bonding states of $|g, 1_q\rangle$ and $|e, 0_{ph}\rangle$. These theoretical results are expected to be observed experimentally. The measurement of the electric current would enable to observe the polaron formation in a single quantum dot and reveal its characters in more details.

In the calculations with the self-consistent Born approximation, we have disregarded inelastic transport processes: When an electron is injected to a level in the dot, say $e$, from lead $L$, the electron is always ejected from the same level, $e$, to lead $R$. This is a good approximation when $\Delta_{L,R}^g \ll \Delta_{L,R}^e$. (Indeed the upper level usually couples to the leads more strongly than the lower level in quantum dots.) Otherwise, the relaxation from level $e$ to $g$ could take place in transport processes, which would broaden the peaks of the differential conductance. To consider the inelastic processes, we have to develop the self-consistent Born approximation in the nonequilibrium Green’s function method. This is a challenging problem and requires the further study. Besides, we have not considered the coupling of electrons to longitudinal acoustic (LA) phonons nor electron-electron interaction (Auger processes for the relaxation of electrons, etc.) which might cause the decay of polarons and, as a result, broaden the conductance peaks further. Although the quantitative estimation of these effects is beyond the scope of the present study, the polaron formation should not be influenced by them qualitatively.

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Appendix A: Electron-Phonon Interaction in Quantum Dots

The interaction between electrons and LO phonons is described by the Fröhlich Hamiltonian.\(^\text{(18)}\) In quantum dots, it is given by

\[
H_{e-\text{ph}} = \frac{1}{\sqrt{V}} \sum_{q,ij} M_{q,ij} d_i^\dagger d_j (b_q + b_q^\dagger), \tag{A.1}
\]

\[
M_{q,ij} = i \sqrt{4 \alpha_{e-\text{ph}} \hbar \omega_q \left( \frac{\hbar}{2m^* \omega_q} \right)^{1/4}} \langle i | e^{i\mathbf{q} \cdot \mathbf{r}} | j \rangle, \tag{A.2}
\]

\[
\alpha_{e-\text{ph}} = \frac{e^2}{\hbar} \sqrt{\frac{m^*}{2\hbar \omega_q}} \left( \frac{1}{\varepsilon(\infty)} - \frac{1}{\varepsilon(0)} \right), \tag{A.3}
\]

where \(|i\rangle\) is the envelope function for electron level \(i\) in the dot and \(m^*\) is the effective mass of electrons. \(\varepsilon(\infty)\) and \(\varepsilon(0)\) are the dielectric constants at high and low frequencies, respectively. We assume that phonons are the same as those in the bulk GaAs \((V\) is the volume of the bulk system) in which the InAs quantum dots are embedded. The dimensionless coupling constant \(\alpha_{e-\text{ph}}\) should be 0.15 to explain the experimental results,\(^\text{11,12}\) whereas \(\alpha_{e-\text{ph}} = 0.06\) in bulk InAs.

Owing to the factor of \(\langle i | e^{i\mathbf{q} \cdot \mathbf{r}} | j \rangle\), the electron-phonon interaction is negligibly small when \(|\mathbf{q}| > 2\pi/L\) with \(L\) being the dot size (~ 20nm\(^\text{11,12}\)). Hence we can regard the LO phonons as dispersionless; \(\hbar \omega_q = \hbar \omega_{\text{LO}} = 36\text{meV}\).

In this paper, we consider two electron levels \((g,e)\). The coupling constant for the interlevel interaction is denoted by \(v_q = M_{q,e,g}/\sqrt{V}\) in the text, Eq. (5). The strength of the couplings is characterized by

\[
v^2 = \sum_{|\mathbf{q}| \leq 2\pi/L} |v_q|^2 = \frac{1}{V} \sum_{|\mathbf{q}| \leq 2\pi/L} |M_{q,e,g}|^2 = \frac{4\alpha_{e-\text{ph}}}{\pi} (\hbar \omega_{\text{LO}})^2 \sqrt{\frac{\hbar^2}{2m^* L^2 \hbar \omega_{\text{LO}}}}. \tag{A.4}
\]

In the above-mentioned situation, \(v/(\hbar \omega_{\text{LO}}) \approx 0.2\). We neglect the intralevel interaction, \(M_{q,g,g}\), \(M_{q,e,e}\), assuming that these effects are involved in the self-energies of electrons.

We disregard the LA phonons for the following reasons. (i) LA phonons with \(|\mathbf{q}| > 2\pi/L\) hardly couple to the electrons for the same reasons as LO phonons. (ii) LA phonons with \(|\mathbf{q}| \leq 2\pi/L\) are not relevant for the polaron formation since the energies (<2meV) are much smaller than the level spacing of electrons \((\Delta \varepsilon \sim 10\text{meV})\).

Appendix B: Calculations of Green’s Function (1)

In the exactly solvable model with \(V_L^q = V_R^g = 0\) (Fig. 1(b)), we divide the total Hamiltonian as \(H_{\text{tot}} = H_0 + V\), where \(H_0 = H_e + H_{\text{ph}}\) and \(V = H_T + H_{e-\text{ph}}\). Using Eq. (8),

\[
\langle e, 0_{\text{ph}} | (\varepsilon - H_0 + i\delta) \hat{G} | e, 0_{\text{ph}} \rangle = 1 + \langle e, 0_{\text{ph}} | V \hat{G} | e, 0_{\text{ph}} \rangle, \tag{B.1}
\]
which yields

\[(\varepsilon - \varepsilon_e + i\delta)G_{e,0\text{ph}}(\varepsilon) = 1 + \sum_q v_q g, 1_q |\hat{G}|e, 0\text{ph} + \sum_{\alpha=L,R} \sum_k V_{\alpha}^e (\alpha k, 0_q |\hat{G}|e, 0\text{ph}). \tag{B.2}\]

Similarly, using Eq. (8), we obtain

\[\left[\varepsilon - (\varepsilon_g + \hbar\omega_q) + i\delta\right] g, 1_q |\hat{G}|e, 0\text{ph} = v_q G_{e,0\text{ph}}(\varepsilon) \tag{B.3}\]

\[\left(\varepsilon - \varepsilon_k + i\delta\right) (\alpha k, 0_q |\hat{G}|e, 0\text{ph}) = V_{\alpha}^e G_{e,0\text{ph}}(\varepsilon). \tag{B.4}\]

Equations (B.2), (B.3) and (B.4) yield

\[G_{e,0\text{ph}}(\varepsilon) = \frac{1}{\varepsilon - \varepsilon_e - \Sigma_T(\varepsilon) - \Sigma_{e,0\text{ph}}(\varepsilon)}, \tag{B.5}\]

where

\[\Sigma_T(\varepsilon) = \sum_{\alpha=L,R} \sum_k \frac{|V_{\alpha}^e|^2}{\varepsilon - \varepsilon_k + i\delta} \]

\[= \sum_{\alpha=L,R} |V_{\alpha}^e|^2 \int \nu d\varepsilon_k \left[P \frac{1}{\varepsilon - \varepsilon_k} - i\pi \delta(\varepsilon - \varepsilon_k)\right] \]

\[= -i(\Delta^e_L + \Delta^e_R), \tag{B.6}\]

and

\[\Sigma_{e,0\text{ph}}(\varepsilon) = \sum_q \frac{|v_q|^2}{\varepsilon - (\varepsilon_g + \hbar\omega_q) + i\delta} \]

\[= \frac{v^2}{\varepsilon - (\varepsilon_g + \hbar\omega_{\text{LO}}) + i\delta}. \tag{B.7}\]

(Eqs. (9) and (10) in the text). The real part of \(\Sigma_T(\varepsilon)\) can be included in the renormalization of the energy level \(\varepsilon_e\), and hence it is omitted here.

In the same way, we obtain \(G_{g,1_q}(\varepsilon)\) in an analytical form of Eqs. (11) and (12). The electric current is written as Eq. (14) because

\[\langle R\hat{k}', 0\text{ph} |\hat{T}(\varepsilon) |L\hat{k}, 0\text{ph}\rangle = V_{R}^{e*} G_{e,0\text{ph}}(\varepsilon) V_{L}^e, \tag{B.8}\]

where the T-matrix is given by \(\hat{T}(\varepsilon) = H_T + H_T \hat{G}(\varepsilon) H_T\).

**Appendix C: Calculations of Green’s Function (2)**

When both the levels in the quantum dot couple to the leads, the Green’s function cannot be obtained analytically. Then we divide the Hamiltonian as \(H_{\text{tot}} = H_0 + H_{e-\text{ph}}\) with \(H_0 = H_e + H_{\text{ph}} + H_T\). First, we calculate the unperturbed Green’s function, \((\varepsilon - H_0 + i\delta)\hat{G}^0(\varepsilon) = 1\), in the same way as in Appendix B:

\[G^0_{e}(\varepsilon) = \frac{1}{\varepsilon - \varepsilon_i + i\Delta^e} \tag{C.1}\]

\((i = g, e)^{17}\) Next, the electron-phonon interaction, \(H_{e-\text{ph}}\), is taken into account to infinite orders, by the self-consistent Born approximation.\(^{18}\) The calculation yields Eqs. (16) and (17).
At $T = 0$, $N_q = 0$ in Eq. (17). $\Sigma_i(\varepsilon)$ is expressed by $\Sigma_i(\varepsilon - \hbar \omega_{\text{LO}})$, which is expressed in turn by $\Sigma_i(\varepsilon - 2\hbar \omega_{\text{LO}})$, and so on. These yield the expression of $\Sigma_i(\varepsilon)$ as a continued fraction. We find that the continued fraction converges with the truncation at $\Sigma_i(\varepsilon - 6\hbar \omega_{\text{LO}})$. At $T \neq 0$, $\Sigma_i(\varepsilon)$ is coupled to $\Sigma_i(\varepsilon \pm \hbar \omega_{\text{LO}})$ in Eq. (12), $\Sigma_i(\varepsilon \pm \hbar \omega_{\text{LO}})$ are coupled to $\Sigma_i(\varepsilon \pm 2\hbar \omega_{\text{LO}})$ and $\Sigma_i(\varepsilon)$, and so on. These are simultaneous nonlinear equations. We solve the equations self-consistently in the approximation of $\Sigma_i(\varepsilon \pm 6\hbar \omega_{\text{LO}}) = 0$ to obtain $\Sigma_i(\varepsilon)$.

After the calculations of the Green’s functions, we evaluate the density of states of electrons by

$$D(\varepsilon) = -\frac{1}{\pi} \text{Im} [G_g(\varepsilon) + G_e(\varepsilon)]. \quad (C.2)$$

The electric current is expressed as

$$I = \frac{2e}{h} \sum_{i=g,e} \sum_{k,k'} |V^*_i R | V^*_i L |^2 \delta(\varepsilon_k - \varepsilon_{k'}) [f(\varepsilon_k - \mu_L) - f(\varepsilon_{k'} - \mu_R)]. \quad (C.3)$$

The differential conductance is obtained by $G = e dI/d\mu_L$. As mentioned in the text, we do not take account of inelastic transport processes where an electron enters a level in the dot and goes out from the other level in the dot.

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17) We assume two channels in the leads; one channel couples to level $g$ in the dot only and the other couples to level $e$. This assumption avoids the mixing between levels $g$ and $e$ through the tunnel couplings to the leads.
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19) In section 4, we calculate the Green’s function for electrons. The states for the phonons are summed up in the self-energy. This is in contrast to the self-energies in Eqs. (10), (12) in section 3 where we consider the Green’s function for the complex system of electrons and phonons.
20) Equation (15) in section 3 implies the suppression of the peak height. In Fig. 2(b), however, this suppression of $G$ is not observed since $\text{Im} \Sigma_{(e,0\rho \omega)} = 0$ at the peak positions.
Fig. 1. (a) We consider a quantum dot with two levels, \( g \) and \( e \), in the presence of electron-LO-phonon interaction. The level spacing, \( \Delta \varepsilon = \varepsilon_e - \varepsilon_g \), is tuned. The levels are coupled to two external leads (\( \alpha = L, R \)) through tunnel barriers with \( V_{g \alpha} \) and \( V_{e \alpha} \). (b) In a solvable model with \( V_{g L} = V_{g R} = 0 \), the states in the dot are restricted to \( |e, 0_{ph}\rangle \) (an electron at level \( e \) and no phonon; middle panel), \( |g, 1_q\rangle \) (an electron at level \( g \) and one phonon; right panel), or no electron nor phonon (left panel).
Fig. 2. The calculated results of an exactly solvable model with $V_L^p = V_R^p = 0$. The broadening of level $e$ by the tunnel coupling to the leads is $\Delta e_L = \Delta e_R = 0.01 \hbar \omega_{LO}$. (a) The density of states in the quantum dot, $D(\varepsilon)$, with changing the level spacing $\Delta \varepsilon = \varepsilon_e - \varepsilon_g$. (b) The differential conductance $G$ through the dot at $T = 0$, as a function of the Fermi level in lead $L$, $\mu_L$, whereas $\mu_R$ is fixed.

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Fig. 3. The calculated results using the self-consistent Born approximation. The broadenings of levels \( g \) and \( e \) by the tunnel coupling to the leads are \( \Delta_L^g = \Delta_R^g = 0.005\hbar\omega_{LO} \) and \( \Delta_L^e = \Delta_R^e = 0.01\hbar\omega_{LO} \), respectively. (a) The density of states in the quantum dot, \( D(\varepsilon) \), with changing the level spacing \( \Delta \varepsilon = \varepsilon_e - \varepsilon_g \). (b) The differential conductance \( G \) through the dot at \( T = 0 \), as a function of the Fermi level in lead \( L, \mu_L \), whereas \( \mu_R \) is fixed.

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Fig. 4. The differential conductance $G$ through the quantum dot at $T = 0$, as a function of the Fermi level in lead $L$, $\mu_L$ ($\mu_R$ is fixed). The level spacing in the dot matches the energy of the LO phonons; $\Delta \varepsilon = \hbar \omega_{LO}$. The broadening of the upper level $e$ is fixed at $\Delta^e_L = \Delta^e_R = 0.2\hbar \omega_{LO}$, whereas that of the lower level $g$ is $\Delta^g_L = \Delta^g_R = 0.02\hbar \omega_{LO}$ (solid line) and $0.05\hbar \omega_{LO}$ (dotted line). The case without the tunnel coupling of the lower level ($V^R_L = V^R_R = 0$) is shown by broken line.

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Fig. 5. The differential conductance $G$ through the quantum dot at $k_B T = 0.05 \hbar \omega_{LO}$ ($T \approx 18$K), as a function of the Fermi level in lead $L$, $\mu_L$ ($\mu_R$ is fixed). The level spacing $\Delta \varepsilon = \varepsilon_e - \varepsilon_g$ in the dot is gradually changed. The level broadenings are the same as in Fig. 3 ($\Delta \varepsilon_L = \Delta \varepsilon_R = 0.005 \hbar \omega_{LO}$, $\Delta \varepsilon_L = \Delta \varepsilon_R = 0.01 \hbar \omega_{LO}$).

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