Stability of leptonic self-complementarity

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Abstract – We investigate the stability of leptonic self-complementarity such that the sum of three mixing angles in the lepton sector is 90 degrees. Current experimental data of neutrino oscillation indicate that the self-complementarity can be satisfied within 3σ ranges of each mixing angle. Thus self-complementarity may be the key to study the flavor physics behind the standard model, and it is important to discuss its stability. We analyze renormalization group equations in the context of the minimal supersymmetric standard model for the self-complementarity. It is seen that one of the Majorana phases plays an important role for the stability of self-complementarity. We find some stable solutions against quantum corrections at low energy. An effective neutrino mass for the neutrinoless double-beta decay is also evaluated by the use of neutrino parameters giving rise to the stable solutions.

Neutrino oscillation experiments established that there are two large mixing angles (θ₁₂ and θ₂₃) of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix in the lepton sector. Then, a non-vanishing θ₁₃ in the PMNS has been reported by recent long baseline and reactor neutrino experiments [1]. These results can be interpreted by the three flavor mixing of neutrinos. With regard to neutrino masses mᵢ (i = 1, 2, 3), the neutrino oscillation experiments determine only two mass squared differences, Δm²₂₃ ≡ |m₂|² − |m₁|² and Δm²₃₁ ≡ |m₃|² − |m₁|². Therefore, two types of neutrino mass hierarchy are allowed, i.e. normal hierarchy (NH) m₁ < m₂ < m₃ and inverted hierarchy (IH) m₃ < m₁ < m₂. Further, neutrino experiments have not determined whether the neutrinos are Dirac or Majorana particles. Clearly, the nature of neutrinos would be a key to find physics beyond the standard model (SM).

In the theoretical area of neutrino physics, various approaches have been discussed in order to investigate the hidden flavor structure behind the SM, e.g., introductions of flavor symmetry, mass (matrix) texture analyses, and searches for exotic relations among flavor mixing angles etc. In this work, we focus on the leptonic self-complementarity [2] (see also [3] for related discussions) as

θ₁₂ + θ₂₃ + θ₁₃ = π/2 = 90°. (1)

The current experimental data on neutrino oscillation indicate that the self-complementarity can be satisfied within 3σ ranges of each mixing angle. Therefore, the self-complementarity may be the key to investigate the flavor physics behind the SM, and it is important to discuss its stability.

We start with the effective Yukawa interaction and Weinberg operator at a low-energy scale such as the electroweak (EW) scale Λₑ𝑤 in the context of the minimal supersymmetric standard model (MSSM),

\[ \mathcal{L}_Y = -y_e \bar{L}_L H d_R + \frac{\kappa}{2} (H_u L_R)(H_d L_L) + \text{h.c.}, \] (2)

where \( L_L \) are the left-handed lepton doublets, \( e_R \) are the right-handed charged leptons, \( H_u(H_d) \) is the up(down)-type Higgs, \( y_e \) is the Yukawa matrix of charged leptons, and \( \kappa(H_u L_R)(H_d L_L) \) is the Weinberg operator, which can be effectively induced by integrating out a heavy particle(s). One example to obtain this operator is the seesaw mechanism. The typical scale of the seesaw mechanism is \( O(10^{14}) \) GeV. Therefore, note that the effective coupling \( \kappa \) has mass dimension \(-1\) and \( \kappa^{-1} \sim O(10^{14}) \) GeV. Such a heavy mass scale can realize tiny active neutrino mass scales through the seesaw mechanism. In this work, we utilize a useful parameterization for the PMNS...
matrix as [4]

\[ V_{\text{PMNS}} \equiv V_{\alpha}^\dagger V_{D} = 
\begin{pmatrix}
    c_{12} c_{13} & s_{12} c_{13} & s_{13} \\
    -c_{12} s_{23} s_{13} - s_{12} s_{23} c_{13} e^{-i \delta} & s_{12} s_{23} s_{13} + c_{12} c_{23} e^{-i \delta} & s_{23} c_{13} \\
    c_{12} c_{23} s_{13} + s_{12} s_{23} c_{13} e^{-i \delta} & -s_{12} c_{23} s_{13} c_{13} e^{-i \delta} & c_{23} c_{13}
\end{pmatrix} \times 
\begin{pmatrix}
    e^{i \beta} & 0 & 0 \\
    0 & e^{i \alpha} & 0 \\
    0 & 0 & 1
\end{pmatrix}, \tag{3}
\]

where \( s_{ij} \equiv \sin \theta_{ij}, \ c \equiv \cos \theta_{ij} \ (i, j = 1, 2, 3; i < j), \ \delta \) is the Dirac phase, and \( D_{\rho} \) is the diagonal phase matrix including two Majorana phases, \( \rho \) and \( \sigma \). A neutrino mass matrix \( M_{\nu} \) can be diagonalized as \( V_{\alpha}^\dagger M_{\nu} V_{\alpha}^\dagger = M_{\nu}^{\text{diag}} \equiv \text{Diag} \{ m_{1}, m_{2}, m_{3} \} \) with \( m_{i} \equiv \kappa_{i} v_{a}^{2} \) where \( v_{a} \) is the vacuum expectation value of up-type Higgs.

Next, we consider renormalization group equations (RGEs) in the MSSM. The RGE of \( \kappa \) is given by

\[ 16\pi^{2} (d \kappa / dt) = \alpha e + [(y_{\mu} y_{\nu}^{\dagger})^{2} + \kappa (y_{\mu} y_{\nu}^{\dagger})^{T}] \] with \( \alpha \equiv 6 \{ -g_{1}^{2} / 5 - g_{2}^{2} + 2 g_{3}^{2} \} \),

where \( g_{i} \) are gauge coupling constants, \( t \) is an arbitrary renormalization scale as \( t \equiv \ln (\mu / \Lambda) \), and \( \Lambda \) is a high-energy scale such as the seesaw scale [5,6]. One can also obtain RGEs of \( \theta_{ij} \) in a diagonal basis of \( y_{s} \), as \( d \theta_{ij} / dt = F_{ij} (\theta_{12}, \theta_{23}, \theta_{13}, \kappa, \delta, \rho, \sigma, y_{r}; t) \) where the right-hand side (RHS) of this equation is given in [6] (also see [7] for other discussions of mixing angles under the RGEs). Now we turn to the self-complementarity relation (1) and investigate the following equation:

\[ \frac{d}{dt} \sum_{ij} \theta_{ij} = \sum_{ij} F_{ij} (\theta_{12}, \theta_{23}, \theta_{13}, \kappa, \delta, \rho, \sigma, y_{r}, t) = F (\theta_{12}, \theta_{23}, \theta_{13}, \kappa, \delta, \rho, \sigma, y_{r}, t), \tag{4} \]

where \( ij \) is summed over 12, 23, and 13. The function \( F \) is described by 3 mixing angles, 3 effective couplings for the light neutrino masses (or equivalently light neutrino masses \( m_{i} \)), 3 CP-phases, a Yukawa coupling of \( \tau \), and renormalization scale. Then once we impose (1) on (4) at an energy scale \( t_{0} \), one of the mixing angles in \( F \) is removed as, e.g., \( \bar{F} (\theta_{12}, \theta_{23}, \kappa, \delta, \rho, \sigma, y_{r}, t_{0}) = 0 \). We now focus on the equation

\[ \bar{F} (\theta_{12}, \theta_{23}, \kappa, \delta, \rho, \sigma, y_{r}, t_{0}) = 0. \tag{5} \]

This equation means that once the equation is satisfied at an energy scale \( t_{0} \), the self-complementarity is also satisfied at all other energy scales \( t \), i.e. the self-complementarity is stable against quantum corrections, if the running effects of the parameters except for mixing angles are tiny. In fact, we can find consistent solutions of (5) with experiments for both NH and IH cases. According to the latest experimental data of neutrino oscillation [8]

\[ 31.3^\circ \leq \theta_{12} \leq 37.5^\circ, \tag{6} \]

at 3σ level for the NH(IH), eq. (1) can be satisfied.

Mass spectra of neutrinos at low energy are defined by \( m_{1} \equiv \sqrt{m_{2}^{2} - | \Delta m_{21}^{2} |} \) and \( m_{2} \equiv \sqrt{m_{3}^{2} - | \Delta m_{31}^{2} | + | \Delta m_{21}^{2} |} \), with best-fit values \( | \Delta m_{31}^{2} | = 7.62 \times 10^{-5} \text{eV}^{2} \) and \( \Delta m_{31}^{2} = 2.53 \times 10^{-3} \text{eV}^{2} \) for the NH, and \( m_{1} \equiv \sqrt{m_{2}^{2} - | \Delta m_{31}^{2} |} \) and \( m_{3} \equiv \sqrt{m_{3}^{2} - | \Delta m_{31}^{2} | - | \Delta m_{21}^{2} |} \), with \( | \Delta m_{31}^{2} | = 2.40 \times 10^{-3} \text{eV}^{2} \) for the IH. Therefore, the largest neutrino mass \( m_{3} (m_{2}) \) of the NH(IH) case is a free parameter in our analyses. We analyze in the range of \( \sqrt{| \Delta m_{31}^{2} |} \leq m_{3} \leq 0.2 \text{eV} \) (\( \sqrt{| \Delta m_{31}^{2} |} + | \Delta m_{21}^{2} | \leq m_{2} \leq 0.2 \text{eV} \)). The case of \( m_{3} (m_{2}) \) is bounded by a cosmological constraint on the sum of neutrino mass as \( m_{3} \leq 0.6 \text{eV} \) [9], and thus as \( m_{3} (m_{2}) \) can be approximated at a low energy as \( y_{r} (y_{\text{EW}}) = 10^{-2} \) in our analyses.

There are 7 parameters (3 mixing angles, 3 CP-phases, and 1 neutrino mass, \( m_{3} \) or \( m_{2} \)) and 2 imposed equations (1) and (5)). Therefore, the number of free parameters is 5. Since it is however intricate to deal with all 5 parameters as completely free ones, we make numerical analyses at some fixed neutrino masses as examples. According to our analyses, a CP-phase is important to give solutions of (5) with (1); we numerically found that there is no solution to satisfy (5) in cases of \( (\delta = 0, \sigma = 0) \) and \( (\delta = 0, \rho = 0) \) if \( (\rho = 0, \delta = 0) \) can give solutions of (5) in some cases of NH. In the case of NH with the minimal \( m_{3} \) (i.e. \( m_{3} = \sqrt{| \Delta m_{31}^{2} |} \)) and in all cases of IH, neither \( (\sigma = 0, \delta = 0) \) nor \( (\rho = 0, \delta = 0) \) can give the solution. Therefore, in the following, we focus on the cases of \( (\rho = 0, \delta = 0) \) and \( (\sigma = 0, \delta = 0) \) for other cases of NH in detail. Now we have 2 free parameters (one of Majorana phases and one of mixing angles) in order to look for the solution, i.e. once we fix one of the Majorana phases and one of the mixing angles, all the values of our parameters are uniquely determined, as we will explain below.

We have scanned over \( 0 \leq |\rho| \leq |\sigma| \leq \pi \). Some results of our numerical analyses are shown in fig. 1 as examples. Vertical and horizontal axes are \( \theta_{23} \) and \( \theta_{12} \), respectively, and the shaded regions mean that the self-complementarity (1) is correlatively satisfied within 3σ ranges of mixing angles (6)–(8). Lower (upper) slanting and right (left) sides are bounded by maximal (minimal) \( \theta_{12} \) and \( \theta_{23} \) at 3σ level, respectively. Note that an allowed range of \( \theta_{23} \) becomes narrow as \( 41.6^\circ \leq \theta_{23} \leq 51.7^\circ \) compared to (7) due to (1). Both two lines in the
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2 lines corresponding to $|\rho| \simeq 124.0^\circ$ and $143.0^\circ$ cross the corners of the shaded region, which is described at the upper left by point A and at the lower right by point B, respectively (there are 2 (different sign) solutions of $\rho$ or $\sigma$ in all cases, i.e. the solutions are symmetric for reflection with respect to $\pi$). Therefore, the values of all the 7 parameters in our analyses are uniquely determined at, e.g., point A or B. Further, the points A and B in all cases determine the maximal and minimal values of the parameters for the stability of self-complementarity. Therefore, the self-complementarity is stable in the shaded regions of $124.0^\circ \leq |\rho| \leq 143.0^\circ$, $31.3^\circ \leq \theta_{12} \leq 37.5^\circ$, $7.0^\circ \leq \theta_{13} \leq 10.9^\circ$, and $\text{Min}[\theta_{23}] \leq \theta_{23} \leq \text{Max}[\theta_{23}]$ where $\text{Max}(\text{Min})[\theta_{ij}]$ is the maximal(minimal) value of $\theta_{ij}$.

Max$(\text{Min})[\theta_{23}]$ is evaluated by $\text{Max}(\text{Min})[\theta_{23}] = 90^\circ - \text{Min}(\text{Max})[\theta_{23}] - \text{Min}(\text{Max})[\theta_{13}]$ due to (1). Note that the value of one of the mixing angle (e.g., $\theta_{23}$) is not independently taken because of (1). In that case, we obtain $\text{Max}(\text{Min})[\theta_{23}] \simeq 51.7^\circ(41.6^\circ)$. The results are summarized in table 1.

Figure 1(b) shows the case of $m_3 = \sqrt{|\Delta m^2_{31}| + \Delta m^2_{21}}$ with $(\sigma \neq 0, \delta = \rho = 0)$. In this case, we obtain values of $|\sigma|$ within the region $124.9^\circ \leq |\sigma| \leq 154.0^\circ$, where the self-complementarity is satisfied, in a similar way to the previous case. Regions of mixing angles for the realization of the self-complementarity are the same as in the previous case.

We have also analyzed the case of $m_3 = 0.2\,\text{eV}$. Results are given in table 1. We cannot obtain any solutions of (5) in the case of $(\rho \neq 0, \delta = \sigma = 0)$ but we can do that in the case of $(\sigma \neq 0, \delta = \rho = 0)$. In this case, values of $|\sigma|$ within the region $101.9^\circ \leq |\sigma| \leq 105.2^\circ$ can give the solutions. Allowed regions of mixing angles are the same as ones in the $m_3 = \sqrt{|\Delta m^2_{31}|}$ case, i.e. contours of solutions can be reached at both points A and B as in the case of $m_3 = \sqrt{|\Delta m^2_{31}|}$. For the $\text{Max}(\text{Min})[\sigma]$, the solution is determined by $\text{Min}(\text{Max})[\theta_{23}]$ in contrast with the above two cases.

Of course, other parameters (CP-phases and neutrino masses) contributing to mixing angles evolve under the RGEs. First, we comment on the running effects of CP-phases. It has been seen that the Majorana phases are important for the stability of self-complementarity. One may worry about the running effects of phases on low-energy solutions for the stability, i.e. whether such effects spoil the solutions at a high-energy scale or not. We have approximated the running effects of $\delta$, $\rho$, and $\sigma$ from the seesaw scale on solutions by using the leading-log estimation in the RGEs. These running effects $(\Delta \delta, \Delta \rho, \Delta \sigma)$ are $(\Delta \delta, \Delta \rho, \Delta \sigma) \sim (\mathcal{O}(0.1^\circ), \mathcal{O}(0.1^\circ), \mathcal{O}(0.1^\circ))$. Therefore, most regions of Majorana phases are stable against such small running effects up to the seesaw scale because of $\text{Max}(|\sigma|) - \text{Min}(|\sigma|) > \Delta \sigma$. In fact, we show complete stable values of $|\rho|$ and $|\sigma|$, which are described by the values without parentheses in table 1, including the above contursof solutions (a) $m_3 = \sqrt{|\Delta m^2_{31}| + \Delta m^2_{21}}$ and (b) $m_3 = \sqrt{|\Delta m^2_{31}|}$ and (c) $m_3 = 0.2\,\text{eV}$ and $(\sigma \neq 0, \rho = \sigma = 0)$.

Fig. 1: (Colour on-line) Examples of solutions of (5). (a) $m_3 = \sqrt{|\Delta m^2_{31}| + \Delta m^2_{21}}$ and (b) $m_3 = \sqrt{|\Delta m^2_{31}|}$ and (c) $m_3 = 0.2\,\text{eV}$ and $(\sigma \neq 0, \rho = \sigma = 0)$.
running effects from CP-phases. Further, a small (but non-vanishing) running effect of $\delta$ does not affect the stability of self-complementarity. Then, one must remember that our analyses are only for the Majorana neutrino case.

Next, we consider running effects of neutrino masses. We have also evaluated the effects from neutrino masses by the use of the leading-log approximation in corresponding RGEs for the neutrino masses, which are also given in [6]. These running effects from the seessaw scale to the electroweak one are

\[
\langle \Delta m^2_{31}, \Delta m^2_{32}, \Delta m^2_{33} \rangle \sim \begin{cases} \langle O(10^{-8}), O(10^{-7}), O(10^{-6}) \rangle eV, \\ \langle O(10^{-6}), O(10^{-6}), O(10^{-6}) \rangle eV, \end{cases}
\]

for the cases of $m_3 = \sqrt{\Delta m^2_{31} + \Delta m^2_{32}}$ and 0.2 eV, respectively, where $\Delta m^2_{3i}$ affects only evolutions of mixing angles and CP-phases and not absolute values of neutrino mass eigenvalues, \textit{i.e.} overall (flavor mixing independent) contributions from the running of Yukawa couplings (top Yukawa gives the dominant contribution) are omitted. Even with these running effects of neutrino masses, the solutions given in the table 1 are stable, \textit{i.e.} we can also obtain solutions of (5) within the almost same range of $|\sigma|$ as the ones in table 1. We have also numerically checked evolutions of mixing angles and their sum in order to make sure that the one of solutions in the NH is stable. In the calculation, we take

\[
m_1 = 8.73 \times 10^{-3} eV,  \hspace{1cm} (10)
m_2 = 1.23 \times 10^{-2} eV,  \hspace{1cm} (11)
m_3 = \sqrt{\Delta m^2_{31} + \Delta m^2_{32}},  \hspace{1cm} (12)
d = \sigma = 0,  \hspace{1cm} \rho = 133.5^\circ,  \hspace{1cm} (13)
\]

at low energy as an example and

\[
\theta_{12} = 33.5^\circ,  \hspace{1cm} \theta_{23} = 47.7^\circ,  \hspace{1cm} \theta_{13} = 8.8^\circ,  \hspace{1cm} (14)
\]

as low-energy boundary conditions for the RGEs. The running effects of the mixing angles from the secessaw scale to the electroweak one are

\[
\langle \Delta \theta_{12}, \Delta \theta_{23}, \Delta \theta_{13} \rangle \sim \langle -O(10^{-4}), O(10^{-3}), -O(10^{-4}) \rangle \text{ (degrees)}.  \hspace{1cm} (15)
\]

Since the deviation of the leptonic self-complementarity from 90$^\circ$ is $O(10^{-4})$ degrees, the self-complementarity relation can be still stable.

Finally, we evaluate the effective mass term of the neutrinoless double-beta decay (0$\nu\beta\beta$),

\[
\langle m_{ee} \rangle \equiv \langle \sum_{i=1}^{3} (V_{PMNS})^2_{ei} m_i \rangle,  \hspace{1cm} \text{in our parameter space. It is written down as}
\]

\[
\langle m_{ee} \rangle = \langle m_1 c_{12} c_{13} e^{2i\rho} + m_2 s_{12} c_{13} e^{2i\sigma} + m_3 s_{13}^2 \rangle,  \hspace{1cm} (16)
\]

in our notation. The phenomenon of 0$\nu\beta\beta$ can distinguish whether neutrinos are Dirac or Majorana particles. The results at some sample points are given in table 1.

The magnitude of $\langle m_{ee} \rangle$ strongly depends on the scale of $m_1$ or $m_2$ rather than on mixing angles and CP-phases in these cases. In the NH with $m_3 = \sqrt{\Delta m^2_{31}}$ and $\sqrt{\Delta m^2_{31} + \Delta m^2_{32}}$, the dominant contribution comes from the second term on the RHS of (16) because of the small $s_{13}$ and vanishing $m_1$. We predict 6.08 meV $\lsim \langle m_{ee} \rangle \lsim 97.3$ meV for the NH within the parameter space to make the self-complementarity stable. The Heidelberg-Moscow experiment [10] for 0$\nu\beta\beta$ gives the most severe bound on $\langle m_{ee} \rangle$, which is $\langle m_{ee} \rangle \lsim 210$ meV. The CUORE

Table 1: Examples of solutions, and minimal and maximal values of neutrino parameters in the corresponding regions: values of $|\sigma|$ in parentheses are the maximal ones giving solutions of (5) but the solutions realized by these maximal values are not stable against the running effects of CP-phases. The values of $|\rho|$ and $|\sigma|$ without parentheses are complete stable ones against the running effects of CP-phases.

| Parameter | NH | MMC1 | MMC2 |
|-----------|----|------|------|
| $m_1$     | $\sqrt{\Delta m^2_{31} + \Delta m^2_{32}}$ | 0.2 eV |
| $m_2$     | $1.23 \times 10^{-2}$ eV | 0.194 eV |
| $m_3$     | $8.73 \times 10^{-3}$ eV | 0.194 eV |
| Phases    | $\rho \neq 0, \delta = \sigma = 0$ | $\sigma \neq 0, \delta = \rho = 0$ | $\sigma \neq 0, \delta = \rho = 0$ |
| $\Min[\theta_{12}(\sin^2 \theta_{12})]$ | 31.3$^\circ$(0.27) |
| $\Max[\theta_{23}(\sin^2 \theta_{23})]$ | 37.5$^\circ$(0.37) |
| $\Min[\theta_{31}(\sin^2 \theta_{31})]$ | 41.6$^\circ$(0.44) |
| $\Max[\theta_{32}(\sin^2 \theta_{32})]$ | 51.7$^\circ$(0.62) |
| $\Min[\theta_{31}(\sin^2 \theta_{31})]$ | 7.0$^\circ$(0.015) |
| $\Max[\theta_{32}(\sin^2 \theta_{32})]$ | 10.9$^\circ$(0.036) |
| $\Min[|\rho|$ or $|\sigma|]$ | $|\rho| = 124.0^\circ$ | $|\sigma| = 124.9^\circ$ | $|\sigma| = 101.9^\circ$ |
| $\Max[|\rho|$ or $|\sigma|]$ | $|\rho| = 143.0^\circ$ | $|\sigma| = 153.9^\circ(154.0^\circ)$ | $|\sigma| = 104.6^\circ(105.2^\circ)$ |
| $\Min[\langle m_{ee} \rangle]$ (meV) | 6.08 | 6.33 | 97.3 |
| $\Max[\langle m_{ee} \rangle]$ (meV) | 9.13 | 9.42 | 72.7 |
of the effective neutrino mass can be predicted as $\langle m \rangle_{ee} = (24-93) \text{meV}$ in the future. Therefore, a part of our prediction may be checked in future experiments.

We have investigated the stability of the leptonic self-complementarity relation in the PMNS sector against quantum corrections by considering RGEs in the MSSM. The current experimental data on neutrino oscillation experiments indicates that the self-complementarity can be satisfied at $3\sigma$ ranges of each mixing angle. This motivates us to study the self-complementarity and its stability as the key to find the physics behind the SM. As a result of the analyses, we have found solutions stabilizing the self-complementarity by using low-energy data of neutrino oscillation experiments. It is seen that the Majorana phases play an important role to give the solutions. The self-complementarity relation can be satisfied up to an arbitrary high-energy scale if neutrino parameters are correlatively within $31.3^\circ \lesssim \theta_{12} \lesssim 37.5^\circ$, $7.0^\circ \lesssim \theta_{13} \lesssim 10.9^\circ$, and $\max(\min) [\theta_{23}] \simeq 51.7^\circ (41.6^\circ)$ with $124.0^\circ \lesssim |\rho| \lesssim 143.0^\circ$ or $124.9^\circ \lesssim |\sigma| \lesssim 153.9^\circ$ for $m_3 = \sqrt{|\Delta m_{31}^2| + |\Delta m_{21}^2|}$, and $101.9^\circ \lesssim |\sigma| \lesssim 104.6^\circ$ for $m_3 = 0.2\text{eV}$ of NH at a low energy. These solutions and the leptonic self-complementarity relation are stable against the running effects of CP-phases and neutrino masses. With regard to the $0\nu\beta\beta$, the effective neutrino mass can be predicted as $6.08\text{meV} \lesssim \langle m \rangle_{ee} \lesssim 97.3\text{meV}$ for the stable solution in the NH case of $m_3 = \sqrt{|\Delta m_{31}^2|}$.

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