Nonlinear dynamo in obliquely rotating stratified electroconductive fluid in an uniformly magnetic field

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1 Introduction

As is known, the problems of generation of magnetic fields of planets, stars, galaxies and other cosmic objects are studied within the framework of dynamo theory. For the first time, the term «dynamo» in the scope of origin of magnetic fields was proposed by Larmor [1]. In his opinion, hydrodynamic motion of an electroconductive fluid can generate a magnetic field by acting as a dynamo. In the linear theory or kinematic dynamo, where magnetic energy is small in comparison with the kinetic energy of motion of the medium, magnetic forces hardly influence flow of the medium. By now the kinematic theory of dynamo has been practically built [2]-[11]. In this theory a significant role belongs to rotational motion of cosmic bodies that gives rise to various wave (e.g. of Rossby or inertial waves) and vortex motions (geostrophic, etc. [12]-[19]). In particular, under the influence of the Coriolis force the initial mirror-symmetric turbulence turns into helical one characterized by breakdown of the mirror symmetry of turbulent fluid motion. A significant topological characteristic of helical turbulence is the invariant $J_s = \vec{\text{rot}}\vec{v}$, the measure of knottedness.
of vortex field force lines [20]. In [21] it was shown, that generation of a large-scale field occurs under the action of a turbulent e.m.f. proportional to the mean magnetic field \( \mathbf{E} = \alpha \mathbf{H} \). Coefficient \( \alpha \) is proportional to the mean helicity of the velocity field \( \alpha \sim \text{rot} \mathbf{v} \) and has got a definition of \( \alpha \)-effect. The generation properties of helical turbulence were considered not only in magnetohydrodynamics or in electroconductive media, but also in conventional hydrodynamics. For the first time the hypothesis concerning the ability of helical turbulence to generate large-scale vortices was reported in [22]. It was based on the formal similarity of the equations of magnetic field induction \( \mathbf{H} \) and those for vorticity \( \mathbf{\Omega} = \text{rot} \mathbf{v} \). However, as proved in [22], \( \alpha \)-effect cannot occur in an incompressible turbulent fluid due to the symmetry of the Reynolds tensor of stresses in the averaged Navier–Stokes equations. Thus, helical turbulence per se is not sufficient for the onset of hydrodynamic (HD) \( \alpha \)-effect, other factors are also necessary for symmetry breakdown in a turbulent flow. As shown in [24] and [25], such factors are compressibility and temperature gradient in a gravitational field, respectively. The effect of generation of large-scale vortex structures (LSVS) by helical turbulence is called vortex dynamo. The mechanisms of vortex dynamo were worked out with reference to turbulent atmosphere and ocean. In particular, there was developed the theory of convective vortex dynamo [25]-[31]. According to this theory, helical turbulence gives rise to large-scale instability leading to the formation of a convective cell interpreted as a huge vortex of tropical cyclone type. Moreover, there are known many papers devoted to LSVS generation taking into account the effects of rotation [32]-[37]. Just another \( \alpha \)-effect is reported in [38], where turbulent fluid motion is modelled by means of an external small-scale force \( \mathbf{F}_0 \). This model is characterized by parity violation (at zero helicity: \( \mathbf{F}_0 \text{rot} \mathbf{F}_0 = 0 \)). The effect of generation of large-scale disturbances by such a force is called anisotropic kinetic \( \alpha \)-effect, or AKA-effect [38]. In the mentioned paper there is considered large-scale instability in an incompressible fluid by means of the method of asymptotic multi-scale expansions. In this method the Reynolds number \( R = \frac{v_0 t_0}{\lambda_0} \ll 1 \) is used as a small parameter for small-scale pulsations of the velocity \( v_0 \) caused by the small-scale force.

It is evident, that applicability of kinematic theory of magnetic and vortex dynamos is limited. During rather long period of time amplified fields (vortex and magnetic ones) start influencing the flows. In this case the behavior of the magnetic field and the motion of the substance must be considered concordantly, i.e. in the scope of nonlinear theory. The observed magnetic fields of real objects obviously exist just in nonlinear regime and this testifies to significance of nonlinear theory [39]. In the mentioned paper the nonlinear theory of magnetic dynamo is based on generalization of the theory of mean field (see e.g. [7]) taking into account nonlinear effects. However, this theory does not allow us to distinguish strictly from the whole hierarchy of perturbations the principal order at which instability occurs. Therefore, an alternative for construction of a nonlinear dynamo theory is the method of multi-scale asymptotic expansions [38]. The use of this method makes it possible to develop nonlinear theories of vortex dynamo for compressible media [40]-[41], as well as for convective media with a helical external force [30]-[31]. The asymptotic multi-scale method was used to reveal large-scale instability in a thermally stratified conductive medium in the case of helicity of small-scale velocity and magnetic fields [42]-[43]. Development of such a large-scale instability in a convective electroconductive medium results in generation of both vortex and magnetic fields. Self-consistent or nonlinear theory of magneto-vortex dynamo in a convective electroconductive medium with small-scale helicity was built in [43]. In this work, the possibility of the formation of stationary chaotic large-scale structures in magnetic and vortex fields was shown for the first time.
Moreover, there was considered the particular case of the formation of large-scale stationary magnetic structures. These structures were classified as stationary solutions of three types: nonlinear waves, solitons and kinks. Qualitative estimations of the linear stage [42] for solar conditions made it possible to establish a good agreement of the characteristic scales and times of the formed hydrodynamic structures with those of the structures revealed experimentally [44].

In the above-mentioned papers helical turbulence was considered either to be known, or the problem of its generation was considered independently [45]. Naturally, this prompts to study the possibility of generation of large-scale vortex and magnetic fields in rotating media under the action of a small-scale force with zero helicity $\vec{F}_0 \text{rot} \vec{F}_0 = 0$. Such an example of LSVS generation in a rotating incompressible fluid is reported in [46]. As shown in the said paper, development of large-scale instability in obliquely rotating fluid gives rise to nonlinear large-scale helical structures of Beltrami vortex type, or to localized kinks with internal helical structure. In [47] the new HD $\alpha$-effect revealed in [46] was generalized to the case of electroconductive fluid. This allowed to reveal the large-scale instability leading to generation of LSVS and magnetic fields. Thereat, the nonlinear stage was shown to be characterized by the presence of chaotic localized vortex and magnetic structures. As is known [48]-[49], a large-scale motion caused by nonuniform heating in a gravitation field (free convection) exists in convective zones of the Sun and other stars, as well as in the core of the Earth and other planets. The convection in which the rotation axes of the medium and uniform magnetic field coincide with the direction of gravitation vector, was studied in detail in [49]. However, for astrophysical problems it is most significant to consider the case when the directions of the rotation axes and of magnetic fields are perpendicular, or do not coincide with one another. The role of azimuthal magnetic field essentially increases for convective fluid layers located in the equatorial region of the rotating object. As known from the theory of magnetic dynamo [2]-[7], the toroidal magnetic field in the Earth crust or in the solar atmosphere exceeds the poloidal field by an order of magnitude. Starting from this fact, here we will consider generation and nonlinear evolution of vortex and magnetic fields in a rotating stratified electroconductive fluid in an external uniform
2 Basic equations and formulation of the problem

Consider the dynamics of perturbed state of the electroconductive fluid located in the constant gravitation $\vec{g}$ and magnetic $\vec{B}$ fields with the constant temperature gradient $\nabla \Theta$ in the system of rotating coordinates:

$$\frac{\partial v_i}{\partial t} + v_k \frac{\partial v_i}{\partial x_k} = \nu \frac{\partial^2 v_i}{\partial x_k^2} - \frac{1}{\rho} \frac{\partial P}{\partial x_i} + 2\varepsilon_{ijk} v_j \Omega_k + \frac{\varepsilon_{ijk} \varepsilon_{jlm}}{4\pi \rho} \frac{\partial B_l}{\partial x_m} (B_k + \overline{B}_j) + g e_i \beta \Theta + F_0^i \quad (1)$$

$$\frac{\partial B_i}{\partial t} = \varepsilon_{ijk} \varepsilon_{kmn} \frac{\partial}{\partial x_j} (v_n (B_p + \overline{B}_p)) + \nu_m \frac{\partial^2 B_i}{\partial x_k^2} \quad (2)$$

$$\frac{\partial \Theta}{\partial t} + v_k \frac{\partial \Theta}{\partial x_k} - A e_k v_k = \chi \frac{\partial^2 \Theta}{\partial x_k^2} \quad (3)$$

$$\frac{\partial v_i}{\partial x_i} = \frac{\partial B_i}{\partial x_i} = 0 \quad (4)$$

Here $v_i$, $P$, $B_i$, $\Theta$ are the perturbances of the velocity, pressure, magnetic field induction and temperature of the fluid ($i = x, y, z$); $\overline{B}_i = \text{const}$, the induction of the external homogeneous magnetic field; $\rho$, the equilibrium density of the medium; $\overline{g} = \text{const}$; $\nu$, $\chi$, the fluid viscosity and thermal conductivity coefficients, respectively; $\nu_m = \frac{c^2}{4\pi \sigma_c}$, the magnetic viscosity coefficient; $\sigma_c$, the coefficient of electrical conductivity of the medium; $\beta$, the thermal expansion coefficient. The system of magnetic hydrodynamic Eqs. (1)-(4) is
written in the Boussinesq approximation \cite{48} and describes the evolution of disturbances relative to the equilibrium state, specified by the constant temperature gradient $\nabla T = -A\vec{e}$ ($A > 0$) and the hydrostatic pressure: $\nabla \left( P + \frac{\rho_0^2}{8\pi} \right) = \rho_0 \vec{g}$. Here we neglect the centrifugal forces, since the condition $g \gg \Omega^2 r$, where $r$ is the characteristic radius of fluid rotation, is considered to be satisfied.

Now let us formulate the following problem which geometry is shown in Fig. 1. Consider a thin layer (with the thickness $h$ ) of a rotating electoconductive fluid in which the lower and the upper surfaces have the temperatures $T_1$ and $T_2$, respectively, there at $T_1 > T_2$, i.e. heating from below. In this case the direction of the temperature gradient $\nabla T = \vec{A}$ coincides with the one of the gravitation field $\vec{g} = -g\vec{e}_z$, where $\vec{e} = (0, 0, 1)$ is the unit vector in the direction of the axis $Z$. The temperature profile $T$ linearly depends on the vertical coordinate $z$: $T(z) = T_1 - \frac{T_1 - T_2}{h} \cdot z$. The vector of angular rotation velocity $\vec{\Omega} = (\Omega_1, \Omega_2, \Omega_3)$ is considered to be constant (solid-body rotation) and inclined with respect to the plane $(X, Y)$ where the vector of homogeneous magnetic field $\vec{B} = (B_1, B_2, 0)$ lies. Eq. (1) contains the external force $\vec{F}_0$, which models the source of external excitation in the medium of small-scale and high-frequency fluctuations of the velocity field $\vec{v}_0$ with the small Reynolds number $R = \frac{\lambda_0}{\lambda_0} \ll 1$. Here we will consider the non-helical external force $\vec{F}_0$ with the following properties:

$$\text{div} \vec{F}_0 = 0, \quad \vec{F}_0 \text{rot} \vec{F}_0 = 0, \quad \text{rot} \vec{F}_0 \neq 0, \quad \vec{F}_0 = f_0 \vec{F}_0 \left( \frac{x}{\lambda_0}; \frac{t}{t_0} \right)$$

(5)

where $\lambda_0$ is the characteristic scale, $t_0$ is the characteristic time, $f_0$ is the characteristic amplitude of the external force. Now choose the external force in a rotating coordinate system in the form:

$$F_0^x = 0, \quad F_0^y = f_0 \left( \tilde{r} \cos \phi_2 + \tilde{j} \cos \phi_1 \right),$$

$$\phi_1 = \vec{\kappa}_1 \vec{x} - \omega_0 t, \quad \phi_2 = \vec{\kappa}_2 \vec{x} - \omega_0 t, \quad \vec{\kappa}_1 = \kappa_0 (1, 0, 0), \quad \vec{\kappa}_2 = \kappa_0 (0, 1, 0).$$

(6)
It is evident that this external force satisfies all the conditions (5). Let us consider the dimensionless variables in Eqs. (1)-(4) which notations preserve the ones of the dimensional variables (for convenience):

\[
\vec{x} \rightarrow \lambda_0 \vec{x}, \quad t \rightarrow \frac{t}{t_0}, \quad \vec{v} \rightarrow \frac{\vec{v}}{v_0}, \quad \vec{F}_0 \rightarrow \frac{\vec{F}_0}{f_0}, \quad \vec{B} \rightarrow \frac{\vec{B}}{B_0}, \quad \vec{B} \rightarrow \frac{\vec{B}}{B_0}, \quad \Theta \rightarrow \frac{\Theta}{\lambda_0 A}.
\]

Here \(v_0, B_0, P_0\) are the characteristic values of small-scale pulsations of the velocity, magnetic field and pressure. In the dimensionless variables Eq. (1)-(3) will have the form:

\[
\frac{\partial v_i}{\partial t} + R v_k \frac{\partial v_i}{\partial x_k} = \frac{\partial^2 v_i}{\partial x_k^2} - \frac{\partial P}{\partial x_i} + \varepsilon_{ijk} v_j D_k + \frac{Q}{R \mu m} \varepsilon_{ijk} \varepsilon_{jml} \frac{\partial B_l}{\partial x_m} (B_k + \overline{B}_k) + \varepsilon_i \frac{Ra}{R \mu P} \Theta + F_0^i \quad (7)
\]

\[
\frac{\partial B_i}{\partial t} - P m^{-1} \frac{\partial^2 B_i}{\partial x_k^2} = R \varepsilon_{ijk} \varepsilon_{jml} \frac{\partial}{\partial x_j} (v_n (B_p + \overline{B}_p)) \quad (8)
\]

\[
\frac{\partial \Theta}{\partial t} + R v_k \frac{\partial \Theta}{\partial x_k} - R \varepsilon_{ijk} v_k = P r^{-1} \frac{\partial^2 \Theta}{\partial x_k^2} \quad (9)
\]

When going to new temperature \(\Theta \rightarrow \Theta / R\) and magnetic field \(B \rightarrow B / R\), finally obtain:

\[
\frac{\partial v_i}{\partial t} + R v_k \frac{\partial v_i}{\partial x_k} = \frac{\partial^2 v_i}{\partial x_k^2} - \frac{\partial P}{\partial x_i} + \varepsilon_{ijk} v_j D_k + R \overline{Q} \varepsilon_{ijk} \varepsilon_{jml} \frac{\partial B_l}{\partial x_m} B_k + \overline{Q} \varepsilon_{ijk} \varepsilon_{jml} \frac{\partial B_l}{\partial x_m} \overline{B}_k + \varepsilon_i \frac{Ra}{R \mu P} \Theta + F_0^i \quad (10)
\]

\[
\frac{\partial B_i}{\partial t} - P m^{-1} \frac{\partial^2 B_i}{\partial x_k^2} = R \varepsilon_{ijk} \varepsilon_{jml} \frac{\partial}{\partial x_j} (v_n B_p) + \varepsilon_{ijk} \varepsilon_{jml} \frac{\partial}{\partial x_j} (v_n \overline{B}_p) \quad (11)
\]

\[
\frac{\partial \Theta}{\partial t} - P r^{-1} \frac{\partial^2 \Theta}{\partial x_k^2} = -R v_k \frac{\partial \Theta}{\partial x_k} + \varepsilon_k v_k \quad (12)
\]
\[ \frac{\partial v_i}{\partial x_i} = \frac{\partial B_i}{\partial x_i} = 0 \]  

Here we use the following dimensionless parameters: \( \widetilde{Ra} = \frac{Ra}{Pr} \), \( Ra = \frac{g \beta A \lambda_0^4}{\nu \chi} \) is the Rayleigh number in the scale \( \lambda_0 \); \( D_i = \frac{2 \Omega_i \lambda_0^2}{\nu} \) - the rotation parameter in the scale \( \lambda_0 \) (\( i = 1, 2, 3 \)) connected with the Taylor number \( Ta_i = D_i^2 \); \( \widetilde{Q} = \frac{Q}{Pm} \), \( Q = \frac{\sigma c B_0^2 \lambda_0^2}{c_0^2 \rho \nu} \) - the Chandrasekhar number; \( Pm = \frac{\nu}{\nu_m} \) - the magnetic Prandtl number; \( Pr = \frac{\nu}{\chi} \) - the Prandtl number.

The small parameter of asymptotic expansion is the Reynolds number \( R = \frac{\nu t_0}{\lambda_0} \ll 1 \), the parameters \( D, \widetilde{Q} \) and \( \widetilde{Ra} \) being arbitrary. Due to the presence of the small parameter \( (R \ll 1) \) in the system of Eqs. (10)-(13), we can apply the theory of multi-scale asymptotic expansions (see e.g. [30]-[31], [38]). In contrast to the theory of mean field [2]-[7], we can sequentially those in each order by \( R \), see the dynamics of perturbations for different space and time scales. In particular, in the zero order of \( R \), small-scale and high-frequency oscillations of the velocity \( \vec{v}_0 \) are excited by the external force \( \vec{F}_0 \) acting at the equilibrium state. Naturally, the dynamics of small-scale fields depends on external factors such as rotation and stratification of the medium, magnetic and gravitation fields, etc. Such oscillations are characterized by zero average values, however, nonlinear interactions in some orders of the perturbation theory give rise to the terms which do not vanish at averaging. The method of finding the solvability conditions for multiscale asymptotic expansion which define the evolution equations for large-scale perturbations will be considered in more detail in the next section.

3 Equations for large-scale fields

In accordance with the method [30]-[31], [38] of construction of asymptotic equations let us present the derivatives with respect to space and time in Eqs. (10)-(13) in the form of
the asymptotic expansion:

\[ \frac{\partial}{\partial t} \rightarrow \partial_t + R^4 \partial_T, \quad \frac{\partial}{\partial x_i} \rightarrow \partial_i + R^2 \nabla_i \]  \quad (14) 

where \( \partial_t \) and \( \partial_i \) denote the derivatives of/with respect to the fast variables \( x_0 = (\bar{x}_0, t_0) \), whereas \( \nabla_i \) and \( \partial_T \) are the derivatives of/with respect to the slow variables \( X = (\bar{X}, T) \). The variables \( x_0 \) and \( X \) may be referred to as small- and large-scale variables, accordingly. While constructing the nonlinear theory, the variables \( \tilde{V}, \tilde{B}, P \) are to be presented in the form of the asymptotic series:

\[ \tilde{V}(\bar{x}, t) = \frac{1}{R} \tilde{W}_{-1}(X) + v_0(x_0) + R \bar{v}_1 + R^2 \bar{v}_2 + R^3 \bar{v}_3 + \cdots \]

\[ \tilde{B}(\bar{x}, t) = \frac{1}{R} \tilde{B}_{-1}(X) + B_0(x_0) + R B_1 + R^2 B_2 + R^3 B_3 + \cdots \]  \quad (15) 

\[ \Theta(\bar{x}, t) = \frac{1}{R} T_{-1}(X) + T_0(x_0) + RT_1 + R^2 T_2 + R^3 T_3 + \cdots \]

\[ P(x) = \frac{1}{R^3} P_{-3} + \frac{1}{R^2} P_{-2} + \frac{1}{R} P_{-1} + P_0(x_0) + R(P_1 + \bar{P}_1(X)) + R^2 P_2 + R^3 P_3 + \cdots \]

Let us substitute the expansions (14)-(15) into the system of Eqs. (10)-(13), then select the terms of the same orders in \( R \) up to the degree \( R^3 \) inclusively and obtain the equations of multi-scale asymptotic expansion. The algebraic structure of the asymptotic expansion of Eqs. (10)-(13) of different orders in \( R \) is presented in Appendix I. In the latter it is shown that the basic secular equations, i.e. those for large-scale fields, are obtained in the order \( R^3 \)

\[ \partial_t W_{-1} - \nabla^2_k W_{-1} + \nabla_k (v_0^i v_0^j) = -\nabla_i \bar{P}_1 + \bar{Q} \varepsilon_{ijk} \varepsilon_{jml} \left( \nabla_m (B_0^i B_0^k) \right) \quad (16) \]

\[ \partial_t B_{-1} - P m^{-1} \nabla^2_k B_{-1} = \varepsilon_{ijk} \varepsilon_{knp} \nabla_j (v_0^i B_0^p) \quad (17) \]

\[ \partial_T T_{-1} - P r^{-1} \nabla^2_k T_{-1} = -\nabla_k \left( \frac{v_0^i T_0}{B_0^j} \right) \quad (18) \]

Using the convolution of the tensors \( \varepsilon_{ijk} \varepsilon_{jml} = \delta_{km} \delta_{il} - \delta_{ml} \delta_{kl}, \varepsilon_{ijk} \varepsilon_{knp} = \delta_{in} \delta_{jp} - \delta_{ip} \delta_{jn} \) and the denotations \( \tilde{W} = \tilde{W}_{-1}, \tilde{B} = \tilde{B}_{-1} \) obtain Eqs. (16)-(17) in the form:

\[ \partial_T W_i - \nabla^2_k W_i + \nabla_k (v_0^i v_0^j) = -\nabla_i \bar{P}_1 + \bar{Q} \left( \nabla_k (B_0^i B_0^j) - \frac{1}{2} (B_0^k)^2 \right) \]  \quad (19) 

\[ \partial_T H_i - P m^{-1} \nabla^2_k H_i = \nabla_j (v_0^i B_0^k) - \nabla_j (v_0^i B_0^k) \]  \quad (20) 

Eqs. (16)-(18) are supplemented with the secular equations derived in Appendix I:

\[ -\nabla_i P_{-3} + \varepsilon_{ijk} W_j D_k + \varepsilon_i \tilde{R} a T_{-1} = 0, \quad W_{-1} = 0, \]

\[ W^k_1 \nabla_k W^i_1 = -\nabla_i P_1 + \tilde{Q} \varepsilon_{jklm} \left( \nabla_m B^l_1 B^k_{-1} + \nabla_m B^l_1 \bar{P}_1 \right) \]

\[ -\varepsilon_{ijk} \varepsilon_{jml} \left( \nabla_j W^m_1 B^k_{-1} + \nabla_j W^m_1 \bar{B}_p \right) = 0, \]

\[ W^k_1 \nabla_k T_{-1} = 0, \quad \nabla_i W^i_1 = 0, \quad \nabla_i B^i_{-1} = 0. \]

To obtain the system of Eqs. (16)-(18) describing the evolution of large-scale fields we had to reach the third order of the perturbation theory. Such a phenomenon is typical of the
use of the method of multi-scale expansions. As seen from Eqs.\((16)-(17)\), the large-scale temperature \(T_{-1}\) does not influence the dynamics of the large-scale field of the velocity \(\vec{W}_{-1}\) and the magnetic field \(\vec{B}_{-1}\), therefore let us confine ourselves to investigation of Eqs.\((16)-(17)\). These equations acquire a closed form after calculation of the correlation functions, i.e. the Reynolds stresses \(\nabla_k (v^j_0 v^i_0)\), the Maxwell stresses \(\nabla_k (B^j_0 B^k_0)\) and the turbulent e.m.f. \(E_n = \varepsilon_{nij} v^i_0 B^j_0\). Calculation of these correlation functions is essentially simplified due to the «quasi-two-dimensional» approximation often used for description of large-scale vortex and magnetic fields in a number of astrophysical and geophysical problems \([3, 14, 30, 31]\). In the framework of this approximation, in the present study we consider the large-scale derivative with respect to \(Z\) more preferable, i.e.

\[
\nabla_Z \equiv \frac{\partial}{\partial Z} \gg \frac{\partial}{\partial X}, \frac{\partial}{\partial Y},
\]

thereat the geometry of large-scale fields has the following form:

\[
\vec{W} = (W_1 (Z), W_2 (Z), 0), \vec{H} = (H_1 (Z), H_2 (Z), 0)
\]

(21)

In the scope of «quasi-two-dimensional» problem the system of Eqs.\((14)-(15)\) is simplified:

\[
\partial_T W_1 - \nabla_Z^2 W_1 + \nabla_Z (v^j_0 v^i_0) = \tilde{Q} \nabla_Z (B^j_0 B^i_0)
\]

(22)

\[
\partial_T W_2 - \nabla_Z^2 W_2 + \nabla_Z (v^j_0 v^j_0) = \tilde{Q} \nabla_Z (B^j_0 B^j_0)
\]

(23)

\[
\partial_T H_1 - P m^{-1} \nabla_Z^2 H_1 = \nabla_Z (v^j_0 B^j_0) - \nabla_Z (v^j_0 B^j_0)
\]

(24)

\[
\partial_T H_2 - P m^{-1} \nabla_Z^2 H_2 = \nabla_Z (v^j_0 B^j_0) - \nabla_Z (v^j_0 B^j_0)
\]

(25)

\[
\partial_T T_{-1} - Pr^{-1} \nabla_Z^2 T_{-1} + \nabla_Z (v^j_0 T^j_0) = 0
\]

(26)
To derive Eqs. (22)-(26) in a closed form we will use the solutions of the equations for small-scale fields in the zeroth order of \( R \) obtained in Appendix II. Then it is necessary to calculate the correlators contained in the system (22)-(26). The technical aspect of this problem is considered in detail in Appendix III. The calculations performed here make it possible to obtain the following closed equations for large-scaler fields of the velocity \((W_1, W_2)\) and the magnetic fields \((H_1, H_2)\):

\[
\partial_T W_1 - \nabla^2_Z W_1 + \nabla_Z \left( \alpha_{(2)} \cdot (1 - W_2) \right) = 0 \quad (27)
\]

\[
\partial_T W_2 - \nabla^2_Z W_2 - \nabla_Z \left( \alpha_{(1)} \cdot (1 - W_1) \right) = 0 \quad (28)
\]

\[
\partial_T H_1 - P m^{-1} \nabla^2_Z H_1 + \nabla_Z \left( \alpha_{H}^{(2)} \cdot H_2 \right) = 0 \quad (29)
\]

\[
\partial_T H_2 - P m^{-1} \nabla^2_Z H_2 - \nabla_Z \left( \alpha_{H}^{(1)} \cdot H_1 \right) = 0 \quad (30)
\]

where the nonlinear coefficients \( \alpha_{(1)}, \alpha_{(2)}, \alpha_{H}^{(1)}, \alpha_{H}^{(2)} \) have the form:

\[
\alpha_{(1)} = \frac{f_0^2}{2} \cdot \frac{D_1 q_1 Q_1 (1 - W_1)^{-1}}{4 (1 - W_1)^2 q_1^2 \tilde{Q}_1^2 + \left[ D_1^2 + W_1 (2 - W_1) + \mu_1 \right]^2 + \xi_1},
\]

\[
\alpha_{(2)} = \frac{f_0^2}{2} \cdot \frac{D_2 q_2 Q_2 (1 - W_2)^{-1}}{4 (1 - W_2)^2 q_2^2 \tilde{Q}_2^2 + \left[ D_2^2 + W_2 (2 - W_2) + \mu_2 \right]^2 + \xi_2},
\]

\[
\alpha_{H}^{(1)} = \frac{f_0^2}{2} \cdot \frac{D_1 (1 - W_1) P m \tilde{Q}_1 (1 + \tilde{B}_1/H_1)}{(1 + P m^2 (1 - W_1)^2) \left[ 4 (1 - W_1)^2 q_1^2 \tilde{Q}_1^2 + \left[ D_1^2 + W_1 (2 - W_1) + \mu_1 \right]^2 + \xi_1 \right]}.
\]

\[
\alpha_{H}^{(2)} = \frac{f_0^2}{2} \cdot \frac{D_2 (1 - W_2) P m \tilde{Q}_2 (1 + \tilde{B}_2/H_2)}{(1 + P m^2 (1 - W_2)^2) \left[ 4 (1 - W_2)^2 q_2^2 \tilde{Q}_2^2 + \left[ D_2^2 + W_2 (2 - W_2) + \mu_2 \right]^2 + \xi_2 \right]}.
\]

The expressions which denote \( q_{1,2}, Q_{1,2}, \tilde{Q}_{1,2}, \mu_{1,2}, \sigma_{1,2}, \chi_{1,2}, \xi_{1,2} \) are also presented in Appendix III. The coefficients \( \alpha_{(1)}, \alpha_{(2)} \) and \( \alpha_{H}^{(1)}, \alpha_{H}^{(2)} \) correspond to the nonlinear
Рис. 8: a) is the plot of the dependence of the Λ-effect on the stratification parameter of the medium $Ra$ (the Rayleigh number); b) is the plot of the dependence of the Λ-effect on the parameter of rotation of the medium $D$; c) is the plot of the dependence of the Λ-effect on the external magnetic field $B$.

HD $α$-effect and the nonlinear MHD $α$-effect, respectively. Thus, we have obtained the self-consistent system of nonlinear evolution equations for large-scale perturbations of the velocity and magnetic field that will be further called the equations of nonlinear magneto-vortex dynamo. It should be noted that the mechanism of dynamo works only due to the effect of rotation of the medium. If such a rotation is absent ($Ω = 0$), then there occurs conventional diffuse spreading of large-scale fields. In the absence of heating ($∇T = 0$) and external magnetic field ($B = 0$) Eqs. (27)-(28) coincide with the results reported in [47]. In the case of non-electroconductive fluid ($σ = 0$) with the temperature gradient ($∇T ≠ 0$) we will have the results obtained in [50]. In the limit of non-electroconductive ($σ = 0$) and homogeneous fluid ($∇T = 0$) there will be obtained the results of [46]. For more illustrative representation of the physical mechanism of the said dynamo model, it is necessary at first to consider the evolution of small perturbations and then to start studying the nonlinear effects.

4 Large-scale instability

Consider the behavior of small perturbations of the field of velocity $(W_1, W_2)$ and the magnetic field $(H_1, H_2)$. Then expand the nonlinear coefficients $α_{(1,2)}$ and $α_{H}^{(1,2)}$ in Eqs. (27)-(30) into the Taylor series with respect to the small values $(W_1, W_2)$, $(H_1, H_2)$:

$$α_{(1,2)} \cdot (1 - W_{1,2}) \approx α_{0}^{(1,2)} - α_{1,2}^{(H)} \cdot H_{1,2} - α_{1,2}^{(W)} \cdot W_{1,2}, \quad α_{0}^{(1,2)} = \text{const},$$

$$α_{H}^{(1,2)} \cdot H_{1,2} \approx α_{0H}^{(1,2)} + \tilde{α}_{1,2}^{(H)} \cdot H_{1,2} - β_{W}^{(1,2)} \cdot W_{1,2}, \quad α_{0H}^{(1,2)} = \text{const}. \quad (31)$$

After substituting (4) into Eqs. (27)-(30) we obtain the linearized system of equations:

$$\partial_T W_1 - ∇^2 Z W_1 - α_2^{(H)} \cdot ∇ Z H_2 - α_2^{(W)} \cdot ∇ Z W_2 = 0 \quad (32)$$

$$\partial_T W_2 - ∇^2 Z W_2 + α_1^{(H)} \cdot ∇ Z H_1 + α_1^{(W)} \cdot ∇ Z W_1 = 0 \quad (33)$$

$$\partial_T H_1 - ∇^2 Z H_1 + \tilde{α}_2^{(H)} \cdot ∇ Z H_2 - β_0^{(2)} \cdot ∇ Z W_2 = 0 \quad (34)$$
Рис. 9: On the left is the plot of the dependence of the Λ-effect on the angle of inclination \( \theta \) of the angular velocity vector \( \vec{\Omega} \); on the right is the plot of the dependence of the instability increment for the Λ-effect on the wave numbers \( K \).

\[
\frac{\partial T H_2}{\partial z} - \nabla^2 H_2 - \alpha^{(1)}_H \cdot \nabla Z H_1 + \beta^{(1)}_W \cdot \nabla Z W_1 = 0, \tag{35}
\]

where the constant coefficients \( \alpha^{(H)}_{1,2}, \alpha^{(W)}_{1,2}, \alpha^{(1,2)}_H, \beta^{(1,2)}_W \) have the following form:

\[
\alpha^{(H)}_{1,2} = \frac{f_0^2 D_{1,2}}{2} \cdot Q B_{1,2} \times \frac{(2 - Ra) (2 - QB_{1,2}^2) (4(D_{1,2}^2 - Ra) + (Ra + 1)^2 + 7)}{4 (4 + (D_{1,2}^2 - Ra)^2)} + \frac{QB_{1,2}^2 - 2(Ra - 1)}{4 (4 + (D_{1,2}^2 - Ra)^2)}, \tag{36}
\]

\[
\alpha^{(W)}_{1,2} = \frac{f_0^2 D_{1,2}}{2} \times \frac{(2 - Ra) (2 - QB_{1,2}^2) (D_{1,2}^2 - Ra - 2)}{4 (4 + (D_{1,2}^2 - Ra)^2)} + \frac{Q B_{1,2}^2 + Ra (1 - QB_{1,2}^2)}{2 (4 + (D_{1,2}^2 - Ra)^2)}, \tag{37}
\]

\[
\alpha^{(1,2)}_H = \frac{f_0^2 D_{1,2}}{4} \times \frac{2 + Ra - QB_{1,2}^2 + B_{1,2}(2 + Ra)}{4 + (D_{1,2}^2 - Ra)^2} - \frac{B_{1,2}(2 + Ra) (4(D_{1,2}^2 - Ra) + (Ra + 1)^2 + 7)}{(4 + (D_{1,2}^2 - Ra)^2)}, \tag{38}
\]

\[
\beta^{(1,2)}_W = \frac{f_0^2}{4} \cdot \frac{D_{1,2} B_{1,2}(2 + Ra) (D_{1,2}^2 - Ra - 2)}{(4 + (D_{1,2}^2 - Ra)^2)^2} - \frac{f_0^2}{4} \cdot \frac{D_{1,2} B_{1,2}Ra}{4 + (D_{1,2}^2 - Ra)^2}. \tag{39}
\]
Рис. 10: In the figures a) and b) shown the Poincaré sections for a trajectory with initial conditions $\tilde{W}_1(0) = 1.25$, $\tilde{W}_2(0) = 1.25$, $H_1(0) = 1.4$, $H_2(0) = 1.4$. This is a regular type of trajectory, which is wound on the tori. The figures c) and d) correspond to Poincaré sections for a trajectory with initial conditions $\tilde{W}_1(0) = 1.398$, $\tilde{W}_2(0) = 1.398$, $H_1(0) = 1.4$, $H_2(0) = 1.4$. These pictures show stochastic layers, to which the corresponding chaotic trajectory belongs. The calculations were carried out for the case $B = 0$. 

While deriving Eqs. (32)-(35) we used the simplification connected with the equality: $Pr = Prm = 1$. As seen from Eqs. (32)-(35), in the presence of external magnetic field the coefficients $\alpha_{1,2}^{(H)}$ and $\beta_{W}^{(1,2)}$ define the positive feedback in the self-consistent dynamics of the fields $W_{1,2}$ and $H_{1,2}$. We will search for the solution of the linear system of Eqs. (32)-(35) in the form of plane waves with the wave vector $\vec{K} \parallel OZ$:

$$\begin{pmatrix} W_{1,2} \\ H_{1,2} \end{pmatrix} = \begin{pmatrix} \dot{W}_{1,2} \\ \dot{H}_{1,2} \end{pmatrix} \exp(-i\omega T + iKZ)$$

(40)

After substituting (40) into the system (32)-(35) we obtain the dispersion equation:

$$\left[(K^2 - i\omega)^2 - K^2 \left(\alpha_{1}(W)\alpha_{2}(W) + \alpha_{2}(H)\beta_{W}^{(1)}\right)\right] \left[(K^2 - i\omega)^2 - K^2 \left(\tilde{\alpha}_{H}^{(1)}\tilde{\alpha}_{H}^{(2)} + \alpha_{1}^{(H)}\beta_{W}^{(2)}\right)\right] + K^4 \left(\tilde{\alpha}_{H}^{(1)}\alpha_{2}^{(H)} - \alpha_{1}^{(H)}\alpha_{2}^{(W)}\right) \left(\alpha_{1}^{(W)}\beta_{W}^{(2)} - \tilde{\alpha}_{H}^{(2)}\beta_{H}^{(1)}\right) = 0$$

(41)

4.1 Analysis of dispersion equation (4) in the absence of external magnetic field $\overline{B}_{1,2} = 0$

It is obvious that without external magnetic field $\overline{B}_{1,2} = 0$ the coefficients $\alpha_{1,2}^{(H)}$ and $\beta_{W}^{(1,2)}$ vanish, and Eq. (4) breaks down into two independent equations:

$$\left[(K^2 - i\omega)^2 - \alpha_{1}^{(W)}\alpha_{2}^{(W)} K^2\right] \left[(K^2 - i\omega)^2 - \tilde{\alpha}_{H}^{(1)}\beta_{H}^{(2)} K^2\right] = 0$$

(42)

where the coefficients $\alpha_{1,2}^{(W)}$, $\tilde{\alpha}_{H}^{(1,2)}$ do not depend on $\overline{B}_{1,2}$. Dispersion Eq. (42) corresponds to the physical situation when small perturbations of vortex and magnetic fields independently gain in intensity due to development of large-scale instability such as $\alpha$-effect. Substituting the frequency $\omega = \omega_{0} + i\Gamma$ from Eq. (42) we find:

$$\Gamma_{1} = Im\omega_{1} = \pm \sqrt{\alpha_{1}^{(W)}\alpha_{2}^{(W)} K - K^2}$$

(43)

$$\Gamma_{2} = Im\omega_{2} = \pm \sqrt{-\tilde{\alpha}_{H}^{(1)}\tilde{\alpha}_{H}^{(2)} K - K^2}$$

(44)

Solutions (43) testify to instability at $\alpha_{1}\alpha_{2} > 0$ for large-scale vortex perturbations with the maximum instability increment $\Gamma_{1\max} = \alpha_{1}\alpha_{2}$ at the wave numbers $K_{1\max} = \frac{\sqrt{\alpha_{1}\alpha_{2}}}{2}$. Similarly, for magnetic perturbations the instability increment $\Gamma_{2\max} = \frac{\tilde{\alpha}_{H}^{(1)}\tilde{\alpha}_{H}^{(2)}}{2}$ reaches its maximum at the wave numbers $K_{2\max} = \frac{\sqrt{\tilde{\alpha}_{H}^{(1)}\tilde{\alpha}_{H}^{(2)}}}{2}$. If $\alpha_{1}\alpha_{2} < 0$ and $\tilde{\alpha}_{H}^{(1)}\tilde{\alpha}_{H}^{(2)} < 0$, then instead of instability there arise damped oscillations with the frequencies $\omega_{01} = \sqrt{\alpha_{1}\alpha_{2}}K$ and $\omega_{02} = \sqrt{-\tilde{\alpha}_{H}^{(1)}\tilde{\alpha}_{H}^{(2)}}K$, respectively.

It is clear that in the considered linear theory the coefficients $\alpha_{1}^{(W)}$, $\alpha_{2}^{(W)}$, $\tilde{\alpha}_{H}^{(1)}$, $\tilde{\alpha}_{H}^{(2)}$ depend not on the amplitudes of the fields, but on the rotation parameters $D_{1,2}$, the Rayleigh number $Ra$ and the amplitude of the external force $f_{0}$. Now analyze the dependence of these coefficients on the dimensionless parameters. For simplicity let us assume that the dimensionless amplitude of the external force $f_{0} = 10$. Fixation of the level of the dimensionless force signifies the choice of a certain level of steady background of small-scale and fast oscillations. It is convenient to replace the Cartesian projections
Рис. 11: The upper part (a), b) shows the dependence of the velocity and magnetic field on the height $Z$ for the numerical solution of equation system (54)-(57) with the initial conditions $\tilde{W}_1(0) = 1.25$, $\tilde{W}_2(0) = 1.25$, $H_1(0) = 1.4$, $H_2(0) = 1.4$. This dependence corresponds to regular motions of the Poincaré section shown on top of Fig. Below (c), d)) a similar dependence is shown for the numerical solution of equation system (54)-(57) with the initial conditions: $\tilde{W}_1(0) = 1.398$, $\tilde{W}_2(0) = 1.398$, $H_1(0) = 1.4$, $H_2(0) = 1.4$. This chaotic dependence corresponds to the Poincaré sections in Fig. 10c-10d shown at the bottom.
$D_1$ and $D_2$ in the coefficients $\alpha_1^{(W)}$, $\alpha_2^{(W)}$, $\tilde{\alpha}_H^{(1)}$, $\tilde{\alpha}_H^{(2)}$ by their projections in the spherical coordinate system $(D, \varphi, \theta)$. The coordinate surface $D = \text{const}$ is a sphere: $\theta$, the latitude: $\theta \in [0, \pi]$, $\varphi$, the longitude: $\varphi \in [0, 2\pi]$ (see Fig. 2). Let us analyze the dependences of the coefficients $\alpha_1$, $\alpha_2$, $\tilde{\alpha}_H^{(1)}$, $\tilde{\alpha}_H^{(2)}$ on the effect of rotation and stratification. For simplicity assume that $D_1 = D_2$, which corresponds to the fixed longitude value $\varphi = \pi/4 + \pi n$, where $n = 0, 1, 2...k$, $k$ being an integer. In this case the coefficients for vortex and magnetic perturbations are:

$$\alpha = \alpha_1^{(W)} = \alpha_2^{(W)} = f_5^2 \sqrt{2} D \sin \theta \cdot \frac{4 \left(D^2 \sin^2 \theta - 2Ra - 4\right) \left(2 - Ra\right) + \frac{Ra}{D^2} \left((D^2 \sin^2 \theta - 2Ra)^2 + 16\right)}{\left(D^2 \sin^2 \theta - 2Ra\right)^2 + 16}, \quad (45)$$

$$\alpha_H = \tilde{\alpha}_H^{(1)} = \tilde{\alpha}_H^{(2)} = \frac{f_5^2 \sqrt{2}}{2} \cdot \frac{D(2 + Ra) \sin \theta}{(D^2 \sin^2 \theta - 2Ra)^2 + 16} \quad (46)$$

As seen from these relations, at the poles ($\theta = 0$, $\theta = \pi$) generation of vortex and magnetic perturbations is inefficient, since $\alpha, \alpha_H \rightarrow 0$, i.e. large-scale instability occurs in the case when the vector of angular rotation velocity $\tilde{\Omega}$ deviates from the axis $Z$. In the case of a homogeneous medium $Ra = 0$, where the generation of large-scale vortex and magnetic disturbances is due to the action of an external small-scale non-spiral force and the Coriolis force $\beta$. The coefficient $\alpha$ of vortex perturbations for a rotating stratified non-electroconductive fluid coincides with the analogous coefficient $\alpha$ for a rotating stratified electroconductive fluid obtained in $[53]$. Therefore, the conclusions made in the said paper concerning gain of vortex perturbations may be applied to the problem considered here. The dependence of the coefficient $\alpha$ on the parameter of fluid stratification (the Rayleigh number $Ra$) at the fixed value of latitude $\theta = \pi/2$ and $D = 2.5$ is presented in the left part of Fig. 3. As is seen, the temperature stratification ($Ra \neq 0$) may give rise to an essential increase of the coefficient $\alpha$ and, consequently, make generation of large-scale vortex perturbations faster in comparison with that in a homogeneous medium. Such an effect is especially explicit at $Ra \rightarrow 5$. With further rise of the Rayleigh numbers the values of the coefficient $\alpha$ are diminishing. Now let us clarify the influence of the rotation of the medium on the coefficient $\alpha$. For this purpose we fix the value of the Rayleigh number $Ra = 5$ at $\theta = \pi/2$. In this case the functional dependence $\alpha(D)$ is presented in the right part of Fig. 3. One can see that at a certain value of the rotation parameter $D$ the coefficient $\alpha$ reaches its maximum $\alpha_{max}$. With further rise of $D$ the coefficient $\alpha$ smoothly tends to zero, i.e. $\alpha$-effect is being suppressed by the rotation of the medium. Now consider the dependence of the coefficient $\alpha_H$ on the parameters of stratification $Ra$ and rotation $D$ at the latitude $\theta = \pi/2$. The dependence of the coefficient $\alpha_H$ on the stratification parameter (the Rayleigh number $Ra$) at the fixed $\theta = \pi/2$ and $D = 2.5$ is shown in the left part of Fig. 4. Here we also see that the presence of temperature stratification ($Ra \neq 0$) essentially increases the coefficient $\alpha_H$, and, consequently, makes generation of large-scale perturbations faster than the one in a homogeneous medium. Magnetohydrodynamic $\alpha$-effect (or $\alpha_H$-effect) also increases at «slow» rotation up to the maximum value $\alpha_{H_{max}}$. Then with the rise of the parameter $D$ the coefficient $\alpha_H$ decreases, but its sign does not change. The analysis of the dependence $\alpha_H(D)$ shows that at «fast» rotation of the medium MHD $\alpha$-effect is also suppressed (see the right part of Fig. 5). Similar phenomenon, i.e. suppression of $\alpha$-effect by the rotation of a turbulent medium is shown in $[51]$.

Shown in Fig. 5 is the graph which represents the influence of rotation and stratification on $\alpha$ and $\alpha_H$-effects in the plane $(D, Ra)$. Here the regions of instability $\alpha > 0, \alpha_H > 0$
Рис. 12: Показан график зависимости автокорреляционной функции $K_{\tilde{W}_1\tilde{W}_1}$ от времени для траектории с начальными условиями хаотическое движение). The plot of the dependence of the autocorrelation function $K_{\tilde{W}_1\tilde{W}_1}$ on time $\tau$ for a trajectory with initial conditions $\tilde{W}_1(0) = 1.398$, $\tilde{W}_2(0) = 1.398$, $H_1(0) = 1.4$; $H_2(0) = 1.4$ (chaotic motion).

are marked with grey color. Having fixed the values of the rotation and stratification parameters $D$ and $Ra$ for the latitudinal angles $\theta = \pi/2$ we will plot the dependences of the growth rate of the vortex $\Gamma_1$ and magnetic $\Gamma_2$ perturbations on the wave numbers $K$.

These graphs have the form typical of $\alpha$-effect (see Fig. 6).

4.2 Analysis of dispersion equation (4) in the presence of external magnetic field $B_{1,2} \neq 0$

Let us study Eq.(4) at $B_{1,2} \neq 0$. In this case it is transformed into the biquadratic equation:

$$\left(K^2 - i\omega\right)^4 - b(K^2 - i\omega)^2 + a = 0,$$

(47)

where

$$b = K^4 \left(\alpha_1^{(W)} \alpha_2^{(W)} + \alpha_2^{(H)} \beta_1^{(1)} + \tilde{\alpha}_{H}^{(1)} \tilde{\alpha}_{H}^{(2)} + \alpha_1^{(H)} \beta_2^{(2)}\right) = K^2\tilde{b},$$

$$a = K^4 \left(\tilde{\alpha}_{H}^{(2)} \tilde{\alpha}_{H}^{(W)} + \tilde{\alpha}_{H}^{(W)} \alpha_1^{(H)} \beta_1^{(1)} + \alpha_2^{(H)} \beta_2^{(2)} + \alpha_2^{(H)} \beta_1^{(1)} + \tilde{\alpha}_{H}^{(1)} \alpha_1^{(W)}\right) = K^4\tilde{a}.$$

The solution of Eq.(47) has the form:

$$K^2 - i\omega = \pm K \sqrt{\frac{\tilde{b}}{2} + \frac{1}{2} \sqrt{\tilde{b}^2 - 4\tilde{a}}}$$

(48)

Since we are interested in increasing solutions, easily find the increment of large-scale instability from Eq.(48):

$$\Gamma = \text{Im}\omega = \Lambda K - K^2,$$

(49)

where $\Lambda = \sqrt{\frac{\tilde{b}}{2} + \frac{1}{2} \sqrt{\tilde{b}^2 - 4\tilde{a}}}$ is the coefficient for vortex and magnetic perturbations which has a positive value at $\tilde{b}^2 > 4\tilde{a}$. The maximum increment of instability $\Gamma_{\text{max}} = \Lambda^2/4$ corresponds to the wave numbers $K_{\text{max}} = \Lambda/2$. Shown in the right part of Fig. 9 is the dependence of the increment $\Gamma$ of large-scale instability (49) on the wave numbers $K$ for fixed values of the inclination angle $\theta = \pi/2$, the amplitude of the external force $f_0 = 10$.
The time interval $\tau$.

A rectilinear dependence of the autocorrelation function $K_{\overline{W}_1\overline{W}_1}$ in logarithmic scales from the time interval $\tau$ for strongly chaotic motion.

and the dimensionless parameters $D = 2.5$, $Ra = 5$, $Q = 10$, $\overline{B} = 0.5$. The form of this graph is analogous to that of graph $\alpha$-effect (see Fig. 6).

As in the previous Section, it is convenient to replace the Cartesian projections $D_{1,2}$ and $\overline{B}_{1,2}$ by their projections in the spherical coordinate system (see Fig. 7). Now let us analyze the dependences of the gain coefficient $\Lambda$ on the effects of rotation ($D$), stratification ($Ra$) and the external magnetic field ($\overline{B}$). For simplicity assume that $D_1 = D_2$ and $\overline{B}_1 = \overline{B}_2$, and this corresponds to the fixed value of the angle $\varphi = \pi/4 + \pi n$, where $n = 0, 1, 2, \ldots k$, $k$ is an integer. In such a case the coefficients $\alpha_1^{(W)}$, $\alpha_2^{(H)}$, $\alpha_H^{(1,2)}$, $\beta_W^{(1,2)}$ which enter into the gain coefficient $\Lambda$ for the vortex and magnetic perturbations will acquire the form:

$$A = \alpha_1^{(W)} = \alpha_2^{(W)} = f_0^2 \sqrt{2} D \sin \theta \left[ \frac{(D^2 \sin^2 \varphi - 2Ra - 4)(2 - Ra)(4 - Q\overline{B}^2)}{(D^2 \sin^2 \varphi - 2Ra)^2 + 16} \right] +$$

$$+ \frac{Ra(2 - Q\overline{B}^2) + Q\overline{B}^2}{4(D^2 \sin^2 \varphi - 2Ra)^2 + 16},$$

$$B_H = \alpha_1^{(H)} = \alpha_2^{(H)} = \frac{f_0^2}{8} DQ\overline{B} \sin \theta \times$$

$$\times \left[ \frac{4(4 - Q\overline{B}^2)(2 - Ra)(2(D^2 \sin^2 \varphi - 2Ra) + (Ra + 1)^2 + 7)}{(16 + (D^2 \sin^2 \varphi - 2Ra)^2)(4 - Q\overline{B}^2 - 4(Ra - 1)) + 16 + (D^2 \sin^2 \varphi - 2Ra)^2} \right].$$

$$A_H = \alpha_1^{(1)} = \alpha_2^{(2)} = \frac{f_0^2 \sqrt{2}}{2} \cdot \frac{D(2 + Ra - 2Q\overline{B}^2 + \sqrt{2}(2 + Ra)\overline{B}) \sin \theta}{(D^2 \sin^2 \varphi - 2Ra)^2 + 16} -$$

$$- f_0^2 \cdot \frac{2D\overline{B}(2 + Ra) \sin \theta}{(D^2 \sin^2 \varphi - 2Ra)^2 + 16} \cdot \left[ (2(D^2 \sin^2 \varphi - 2Ra) + (Ra + 1)^2 + 7) \right],$$

$$B_W = \beta_1^{(1)} = \beta_2^{(2)} = f_0^2 \cdot \frac{4D\overline{B}(2 + Ra) \sin \theta}{(D^2 \sin^2 \varphi - 2Ra)^2 + 16} \cdot \left[ (D^2 \sin^2 \varphi - 2Ra - 4) -$$

$$- f_0^2 \cdot \frac{D\overline{B}Ra \sin \theta}{(D^2 \sin^2 \varphi - 2Ra)^2 + 16} \right].$$

Fig. 8a shows the dependence of the coefficient $\Lambda$ on the Rayleigh number $Ra$ at the fixed latitude values $\theta = \pi/2$ and the dimensionless numbers $D = 2.5$, $Q = 10$, $\overline{B} = 0.2$. As
before, assume that the amplitude of the external force \( f_0 = 10 \). In Fig. the value of the coefficient \( \Lambda \) at \( Ra = 0 \) (homogeneous medium) is denoted by dashes. As one can see, with the increase of the Rayleigh number \( Ra \to 5 \) the coefficient \( \Lambda \) considerably exceeds its value for a homogeneous medium, i.e. reaches its peak magnitude. Further rise of the parameter \( Ra \) leads to a drop of the value of \( \Lambda \) and, consequently, to less intense generation of the magneto-vortex perturbations. Let us fix the Rayleigh number e.g. on the level of \( Ra = 5 \) and find the dependence of the coefficient \( \Lambda \) on the rotation parameter \( D \) at the external magnetic field \( \mathbf{B} = 0.2 \) and \( Q = 10 \). The graph presented in Fig. shows the dependence \( \Lambda(D) \). Here we observe the increase of the Rayleigh number \( Ra \to \) leads to a drop of the value of \( \Lambda \) and, consequently, to less intense generation of the magneto-vortex perturbations. «Fast» rotation of the medium also suppresses the considered \( \Lambda \)-effect. To clarify the influence of the homogeneous magnetic field \( \mathbf{B} \) on \( \Lambda \)-effect, let us fix the following parameters: \( D = 2.5, Ra = 5, Q = 10 \). Fig. presents the dependence \( \Lambda(B) \). The upper dashed line denotes the level \( \Lambda_0 \) corresponding to the case when the external magnetic field is absent: \( \mathbf{B} = 0 \). As seen from this figure, the growth of the magnetic field value provides intensification of the magneto-vortex perturbations up to a certain level \( \Lambda_{max} \approx 100 \). The lower dashed line in Fig. shows the minimum level of the coefficient \( \Lambda_{min} \approx 9.63 \) which corresponds to the value of the magnetic field \( B \approx 0.72 \) for the given parameters \( D, Ra \) and \( Q \). From here it follows that «strong» external magnetic field suppresses the considered \( \Lambda \)-effect. For the fixed parameters \( D = 2.5, Ra = 5, Q = 10 \) and \( B = 0.5 \) one can find the dependence of the coefficient \( \Lambda \) on the angle \( \theta \) of deviation for the vector of the angular rotation velocity \( \Omega \) from the vertical direction \( OZ \). This dependence \( \Lambda(\theta) \) is presented in the left part of Fig. As is seen, generation of magneto-vortex perturbations does not occur \( (\Lambda \to 0) \) at \( \theta \to 0 \) and \( \theta \to \pi \) \((\text{the pole})\), whereas at \( \theta \to \pi/2 \) \((\text{the equator})\) it is most effective.

5 Nonlinear stationary structures

With the growth of the amplitude of the perturbations \( W_{1,2} \) and \( H_{1,2} \) due to development of large-scale instability, the linear theory considered in the previous Section becomes inapplicable. The evolution of these perturbations will be described by the nonlinear system of \((27)-(30)\). Now study the regime of instability saturation which leads to the formation of nonlinear stationary structures. To describe such structures, let us put \( \partial_T W_1 = \partial_T W_2 = \partial_T H_1 = \partial_T H_2 = 0 \) in the system of Eqs. \((27)-(30)\), and then integrate these equation over to \( Z \):

\[
\frac{d\tilde{W}_1}{dZ} = -\frac{f_0^2}{16W_0^2q_2^2Q_2^2} \cdot \frac{\sqrt{2}Dq_2Q_2}{\left[ D^2 + 2(1 - \tilde{W}_2^2) + 2\mu_2 \right]^2 + 4\xi_2} + C_1 \tag{54}
\]

\[
\frac{d\tilde{W}_2}{dZ} = \frac{f_0^2}{16W_0^2q_1^2Q_1^2} \cdot \frac{\sqrt{2}Dq_1Q_1}{\left[ D^2 + 2(1 - \tilde{W}_1^2) + 2\mu_1 \right]^2 + 4\xi_1} + C_2 \tag{55}
\]

\[
\frac{dH_1}{dZ} = \frac{f_0^2}{(1 + \tilde{W}_2^2)} \cdot \frac{\sqrt{2}D\tilde{W}_2Q_2(2H_2 + B\sqrt{2})}{16W_0^2q_2^2Q_2^2 + \left[ D^2 + 2(1 - \tilde{W}_2^2) + 2\mu_2 \right]^2 + 4\xi_2} + C_3 \tag{56}
\]
In the figures a) and b) shown the Poincaré sections for a trajectory with initial conditions \( \tilde{W}_1(0) = 1.31, \tilde{W}_2(0) = 1.31, H_1(0) = 1.4, H_2(0) = 1.4 \). This is a regular type of trajectory, which is wound on the tori. The figures c) and d) correspond to Poincaré sections for a trajectory with initial conditions \( \tilde{W}_1(0) = 1.31, \tilde{W}_2(0) = 1.31, H_1(0) = 1.8, H_2(0) = 1.8 \). These pictures show stochastic layers, to which the corresponding chaotic trajectory belongs. The calculations were carried out for the case \( \overline{B} = 0.1 \).
\[
\frac{dH_2}{dZ} = -f_0^2 \frac{\sqrt{2D\tilde{W}_1}\tilde{Q}_1(2H_1 + B\sqrt{2})}{(1 + \tilde{W}_1^2)} \left[ \frac{16\tilde{W}_1^2q_1^2\tilde{Q}_1^2}{D^2 + 2(1 - \tilde{W}_1^2) + 2\mu_1} + 4\xi_1 \right] + C_4 \tag{57}
\]

Here \(\tilde{W}_1 = 1 - W_1\), \(\tilde{W}_2 = 1 - W_2\); \(C_1\), \(C_2\), \(C_3\) and \(C_4\) are arbitrary integration constants.

While obtaining Eqs. (54)-(57) we put the Prandtl numbers \(Pr = Pm = 1\), and replaced the Cartesian projections for \(D_{1,2}\) and \(\bar{B}_{1,2}\) in the coefficients \(\alpha_{(1,2)}\), \(\alpha_H^{(1,2)}\) by their projections in the spherical coordinate system (see Fig. [7]). For simplicity we fixed the values of the angles \(\varphi_\Omega = \varphi_B = \pi/4\) and \(\theta = \pi/2\).

In this case the expressions for \(q_{1,2}\), \(Q_{1,2}\), \(\tilde{Q}_{1,2}\), \(\mu_{1,2}\), \(\sigma_{1,2}\), \(\chi_{1,2}\), \(\xi_{1,2}\) will be also simplified:

\[ q_{1,2} = 1 + \frac{QH_{1,2}(2H_{1,2} + B\sqrt{2})}{2(1 + \tilde{W}_{1,2}^2)} - \frac{Ra}{1 + \tilde{W}_{1,2}^2}, \quad Q_{1,2} = 1 - \frac{Q(2H_{1,2} + B\sqrt{2})^2}{4(1 + \tilde{W}_{1,2}^2)}, \]

\[ \mu_{1,2} = QH_{1,2}(2H_{1,2} + B\sqrt{2}) + Q^2H_{1,2}^2(2H_{1,2} + B\sqrt{2})^2 \cdot \frac{1 - \tilde{W}_{1,2}^2}{4(1 + \tilde{W}_{1,2}^2)} \]

\[ - Ra \cdot \frac{1 + \tilde{W}_{1,2}^2 + QH_{1,2}(2H_{1,2} + B\sqrt{2})}{1 + \tilde{W}_{1,2}^2}, \]

\[ \xi_{1,2} = 2\Xi_{1,2} + \tilde{W}_{1,2}^2\Pi_{1,2} - 2\tilde{W}_{1,2}^2(1 - \tilde{Q}_{1,2}^2)\Pi_{1,2} - 2(1 - \tilde{q}_{1,2}^2)\Xi_{1,2} + \Xi_{1,2}\Pi_{1,2} + \Xi_{1,2}^2 + \xi_{1,2}^2W_{1,2}^2 + \chi_{1,2}(1 - \sigma_{1,2}), \]

\[ \Xi_{1,2} = \frac{4\tilde{W}_{1,2}^2\tilde{Q}_{1,2}Ra}{1 + \tilde{W}_{1,2}^2} + \frac{2\tilde{W}_{1,2}^2Ra^2}{(1 + \tilde{W}_{1,2}^2)^2} + Ra \cdot \frac{1 + \tilde{W}_{1,2}^2 + QH_{1,2}(2H_{1,2} + B\sqrt{2})}{1 + \tilde{W}_{1,2}^2}, \]

\[ \Pi_{1,2} = \frac{4q_{1,2}Ra}{1 + \tilde{W}_{1,2}^2} + \frac{2Ra^2}{(1 + \tilde{W}_{1,2}^2)^2} - Ra \cdot \frac{1 + \tilde{W}_{1,2}^2 + QH_{1,2}(2H_{1,2} + B\sqrt{2})}{1 + \tilde{W}_{1,2}^2}, \]

\[ \sigma_{1,2} = \frac{QH_{1,2}(2H_{1,2} + B\sqrt{2})}{4(1 + \tilde{W}_{1,2}^2)} \cdot \left[ 4(1 + \tilde{W}_{1,2}^2) + QH_{1,2}(2H_{1,2} + B\sqrt{2}) \right], \]

\[ \chi_{1,2} = \frac{Ra}{1 + \tilde{W}_{1,2}^2} \cdot \left[ Ra - \left( 2(1 - \tilde{W}_{1,2}^2) + QH_{1,2}(2H_{1,2} + B\sqrt{2}) \right) \right], \]

\[ \tilde{Q}_{1,2} = 1 - \frac{QH_{1,2}(2H_{1,2} + B\sqrt{2})}{2(1 + \tilde{W}_{1,2}^2)} + \frac{Ra}{1 + \tilde{W}_{1,2}^2}. \]

Eqs. (54)-(57) constitute the nonlinear dynamic system in 4-dimensional phase space in which phase flow divergence is (equal to) zero. Therefore, the system of Eqs. (54)-(57) is conservative. Search for the Hamiltonian of this system is a very difficult task, as the integration is complicated by the dependence of the nonlinear coefficients \(\alpha_{(1,2)}\), \(\alpha_H^{(1,2)}\) on the fields \(\tilde{W}, \tilde{H}\), that takes it beyond the class of elementary functions. A complete qualitative analysis of this system is extremely complicated due to a high dimension of the phase space, as well as to a large number of the parameters included in the system.
Рис. 15: The upper part (a), b)) shows the dependence of the velocity and magnetic field on the height $Z$ for the numerical solution of equation system (54)-(57) with the initial conditions $\tilde{W}_1(0) = 1.31$, $\tilde{W}_2(0) = 1.31$, $H_1(0) = 1.4$, $H_2(0) = 1.4$. This dependence corresponds to regular motions of the Poincaré section shown on top of Fig. 14a-14b. Below (c), d)) a similar dependence is shown for the numerical solution of equation system (54)-(57) with the initial conditions: $\tilde{W}_1(0) = 1.31$, $\tilde{W}_2(0) = 1.31$, $H_1(0) = 1.8$, $H_2(0) = 1.8$. This chaotic dependence corresponds to the Poincaré sections in Fig. 14c-14d shown at the bottom.
Proceeding from general ideas, it is to be expected that this system of conservative equations may contain structures of resonance and non-resonance tori in the phase space and, consequently, chaotic stationary structures of hydrodynamic and magnetic fields. The considered system of nonlinear Eqs. (54)-(57) can be studied e.g. using the Poincaré cross-section method.

6 Stationary chaotic structures in the absence of external magnetic field $\overline{B} = 0$.

Using the standard Mathematica programs, let us build the Poincaré cross-sections for the trajectories in the phase space for the case of rotating electroconductive fluid stratified with respect to temperature ($Ra \neq 0$) not taking into account the external magnetic field $\overline{B} = 0$. All the numerical calculations will be performed for the following parameters: $f_0 = 10$, $D = 2$, $Q = 1$, $Ra = 0.1$ and the constants $C_1 = 1, C_2 = -1, C_3 = -0.5, C_4 = 0.5$. For the initial conditions $\tilde{W}_1(0) = 1.25, \tilde{W}_2(0) = 1.25, H_1(0) = 1.4, H_2(0) = 1.4$ the Poincaré cross-sections presented in Fig. 10a-10b, demonstrate regular trajectories for the velocity and magnetic fields. With the rise of the initial perturbation velocity $\tilde{W}_1(0) = 1.398, \tilde{W}_2(0) = 1.398, H_1(0) = 1.4, H_2(0) = 1.4$ the regular trajectories become chaotic. They correspond to the Poincaré cross-sections shown in Fig. 10c-10d. Fig. 11a-11d present the dependence of the stationary large-scale fields on the altitude $Z$. The latter was obtained numerically for the initial conditions corresponding to the Poincaré cross-sections presented in Fig. 10a-10d. These figures also show the emergence of stationary chaotic solutions for magnetic and vortex fields. To prove the emergence of chaotic regime of stationary large-scale fields, we will use not only the Poincaré cross-sections, but also the notion of autocorrelated function. As is known (see e.g. [52]), the autocorrelated function $K(\tau)$ is the value which characterizes the intensity of chaos. It is defined as averaging of the product of random functions $P(t)$ and $P(t + \tau)$ at the moment of time $t$ and $t + \tau$, 

$K(\tau) = \frac{1}{T} \int_0^T P(t)P(t+\tau)dt$.
respectively, over «long» interval of time $\Delta t: K(\tau) = \lim_{\Delta t \to \infty} \frac{\Delta t}{\Delta t} \int_0^{\Delta t} P(t)P(t + \tau)dt$. In the case we consider the coordinate $Z$ acts as the time $t$, whereas the product $P(t)P(t + \tau)$ consists of 16 components:

$$P(t)P(t + \tau) = \begin{bmatrix} \widetilde{W}_1(t) & \widetilde{W}_2(t) & H_1(t) & H_2(t) \\ \widetilde{W}_1(t + \tau) & \widetilde{W}_2(t + \tau) & H_1(t + \tau) & H_2(t + \tau) \end{bmatrix}.$$

The plot of the dependence of the autocorrelated function for the component $K_{\widetilde{W}_1W_1}$ on the time $\tau$ is presented in Fig. 12. The case of chaotic motion corresponds to the section of the trajectory with the exponential decay of the function $K_{\widetilde{W}_1W_1}$. It is evident that the said section in the logarithmic scale of the autocorrelated function $K_{\widetilde{W}_1W_1}$ is approximated by a straight line (see Fig. 13). The data presented in Fig. 13 make it possible to determine the characteristic correlation time $\tau_{cor} \approx 1324$ of the stationary random process $P_{W_1}$. To take into account the introduced to the above definition of «time», it becomes clear that we have found the estimated value of the altitude $Z_{cor} \approx 1324$ corresponding to the onset of chaotic motion of the large-scale fields. Shown in Fig. 13 are the chaotic solutions for the velocity and magnetic fields of height $Z \approx 90$ which considerably less than $Z_{cor}$. However, even in this case one can see the start of the intricate trajectory for the large-scale fields at the increase of the altitude $Z$. Therefore, such trajectories cannot be plotted. Thus, with the increase of the altitude $Z$ up to a critical value $Z_{cor}$, quasi-periodic motion of the stationary large-scale fields becomes chaotic.

7 Stationary chaotic structures in the presence of external magnetic field $\overline{B} \neq 0$.

Now let us build the Poincaré cross-sections of the trajectories in the phase space for the nonlinear system of Eqs. (53)- (57) taking into account external homogeneous magnetic field, by means of the standard Matemathica programs. For this purpose all the numerical calculations will be carried out for the following dimensionless parameters: $f_0 = 10, D = 2, Q = 1, Ra = 0.1, \overline{B} = 0.1$ and the constants $C_1 = 1, C_2 = -1, C_3 = -0.5, C_4 = 0.5$. Shown in Fig. 14a-14d are the regular trajectories of the velocity and magnetic fields built at the numerical solutions of Eqs. (53)- (57) with the following initial conditions: $\widetilde{W}_1(0) = 1.31, \overline{W}_2(0) = 1.31, H_1(0) = 1.4, H_2(0) = 1.4$. This type of trajectories corresponds to quasi-periodic character of motion for large-scale perturbations of the velocity ($\overline{W}_{1,2}$) and magnetic fields ($H_{1,2}$). By increasing only the amplitudes of the initial values of perturbations for the magnetic field $\widetilde{W}_1(0) = 1.31, \overline{W}_2(0) = 1.31, H_1(0) = 1.8, H_2(0) = 1.8$ we find that the quasi-periodic motion transforms into chaotic. This case demonstrates the Poincaré cross-sections shown in Fig. 14a-14f. Using the initial data for the regular ($\widetilde{W}_1(0) = 1.31, \overline{W}_2(0) = 1.31, H_1(0) = 1.4, H_2(0) = 1.4$) and chaotic ($\overline{W}_1(0) = 1.31, \overline{W}_2(0) = 1.31, H_1(0) = 1.8, H_2(0) = 1.8$) trajectories, one can numerically build the dependence of the stationary large-scale fields on the altitude $Z$ (see Fig. 14a-14f). The emergence of stationary chaotic solutions for the magnetic and vortex fields is also observed in Fig. 15a-15d. To confirm the onset of chaotic regime of the stationary large-scale fields we will plot the dependence of the autocorrelated function for the component $K_{\overline{W}_1\overline{W}_1}$ on
Рис. 17: A rectilinear dependence of the autocorrelation function $K_{\tilde{W}_1\tilde{W}_1}$ in logarithmic scales from the time interval $\tau$ for strongly chaotic motion on condition $\overline{B} = 0.1$.

the time $\tau$ (see Fig. 16). The trajectories of chaotic motion correspond to the to the section of the graph with the exponential decay of the function $K_{\tilde{W}_1\tilde{W}_1}$ in Fig. 16. In the logarithmic scale of the autocorrelated function $K_{\tilde{W}_1\tilde{W}_1}$ this section is approximated by a straight line (see Fig. 17). Using this graph it is easy to find the estimated value of the characteristic correlation time for a stationary random process: $\tau_{cor} \approx 2000$. The obtained value of the correlation time corresponds to the altitude $Z_{cor} \approx 2000$, above which there arise strongly chaotic stationary structures of the large-scale fields. Shown in Fig. 15c-15d are chaotic solutions for the velocity and magnetic fields at the altitude $Z \approx 50$ which is considerably less than $Z_{cor}$. It is evident that for $Z$ tending to a critical value $Z_{cor}$ the motion trajectories become more intricate and, finally, completely chaotic.

8 Conclusion

There is obtained the closed system of nonlinear equations for vortex and magnetic large-scale perturbations (magneto-vortex dynamo) in an obliquely rotating stratified electroconductive fluid in external uniformly magnetic field. At the initial stage small amplitudes of large-scale perturbations increase due to the average helicity $\langle \vec{v}_0 \cdot \vec{\Omega} \rangle \neq 0$ of small-scale motion in a rotating stratified electroconductive fluid excited by the external non-helical force $\vec{F}_0 \cdot \vec{\Omega} = 0$. The mechanism of the amplification of the large-scale perturbations is bound up with development of large-scale instability of $\alpha$-effect type. Thereat, in the absence of external magnetic field ($\overline{B} = 0$) the linear equations of magneto-vortex dynamo are split into two subsystems: vortex and magnetic ones. In this case the large-scale vortex and magnetic perturbations are generated owing to development of instability such as HD $\alpha$-effect and MHD $\alpha$-effect, respectively. Both types of instability occur when the vector of angular rotation velocity $\vec{\Omega}$ is deflected from the vertical axis $OZ$. Unlike the case of a homogeneous medium [46]-[47], the combined effects of rotarion and stratification of the medium (at heating from below) give rise to an essential amplification of the large-scale perturbations. Such a phenomenon becomes especially noticeable at the parameters of the medium $D \to 3$ and $Ra \to 5$ (see Fig. 3). In this case there arises the regime of maximal generation of the small-scale helical motion caused by
the action of the Coriolis force and inhomogeneity of the medium with respect to the temperature. In the presence of the external magnetic field ($\vec{B} \neq 0$) the evolution of the vortex and magnetic perturbations is characterized by a positive feedback due to which the rates of the growth of the vortex and magnetic large-scale perturbations coincide. Thereat, «weak» external magnetic field favours generation of the said perturbations, whereas «strong» field suppresses them (see Fig. 8c). Generation of the large-scale vortex and magnetic perturbations also depends on the angle of deflection of the vector of angular rotation velocity $\vec{\Omega}$. It is minimal at $\theta \to 0$ or $\theta \to \pi$ (nearby the poles) and maximal at $\theta \to \pi/2$ (nearby the equator) (see the left part of Fig. 9). The performed analysis of the influence of rotation on the growth of the vortex and magnetic perturbations shows that at «fast» rotation they are being suppressed. With the rise of the perturbation amplitude the instability is stabilized and then becomes stationary. Under such conditions there arise nonlinear stationary vortex and magnetic structures. The dynamical system of equations describing these structures is Hamiltonian in the four-dimensional phase space. The possibility of the existence of the large-scale chaotic vortex and magnetic fields in stationary regime is proved by numerical methods. In the absence of external magnetic field ($\vec{B} = 0$) stationary chaotic structures arise in a rotating stratified electroconductive fluid at the increase of the initial velocity of perturbations $\tilde{W}_1, 2(0)$. In the presence of external magnetic field ($\vec{B} \neq 0$) these structures are formed at the rise of the initial values of the perturbed field $H_1, 2(0)$.

9 Appendix I. Multi-scale asymptotic expansions

Let us consider the algebraic structure of the asymptotic expansion of Eqs. (10)-(13) in different orders in $R$ starting with the lowest of them. In the order $R^{-3}$ we have only one equation:

$$\partial_1 P_{-3} = 0 \Rightarrow P_{-3} = P_{-3}(X)$$

(58)

In the order $R^{-2}$ there is the following equation:

$$\partial_1 P_{-2} = 0 \Rightarrow P_{-2} = P_{-2}(X)$$

(59)

Eqs. (58) and (59) are satisfied automatically, since $P_{-3}$ and $P_{-2}$ are the functions of slow variables only. In the order $R^{-1}$ we obtain the system of equations:

$$\partial_1 W_{-1}^i + W_{-1}^k \partial_k W_{-1}^i = -\partial_1 P_{-1} - \nabla_i P_{-3} + \partial_1^2 W_{-1}^i + \varepsilon_{ijk} W_j D_k +$$

$$+ \tilde{Q} \varepsilon_{ijk} \varepsilon_{jml} \partial_m B_{-1}^k B_{-1}^l + \tilde{Q} \varepsilon_{ijk} \varepsilon_{jml} \partial_m B_{-1}^k T_{-1} + e_i \tilde{Ra} T_{-1}$$

$$\partial_1 B_{-1}^i - P m^{-1} \partial_1^2 B_{-1}^i = \varepsilon_{ijk} \varepsilon_{knp} \partial_j W_{-1}^n B_{-1}^p + \varepsilon_{ijk} \varepsilon_{knp} \partial_j W_{-1}^n T_{-1}$$

$$\partial_1 T_{-1} - P r^{-1} \partial_1^2 T_{-1} = -W_{-1}^k \partial_k T_{-1} + W_{-1}^z$$

$$\partial_1 W_{-1}^i = 0, \quad \partial_1 B_{-1}^i = 0$$

(60)

The averaging of Eqs. (60) over the «fast» variables gives the secular equation:

$$-\nabla_i P_{-3} + \varepsilon_{ijk} W_j D_k + e_i \tilde{Ra} T_{-1} = 0, \quad W_{-1}^z = 0$$

(61)
which corresponds to geostrophic equilibrium.

In the zero order in $R$ we have the following system of equations:

$$
\partial_t v_0^i + W_{-1}^k \partial_k v_0^i + v_0^k \partial_k W_{-1}^i = -\partial_t P_0 - \nabla_i P_{-2} + \partial_k^2 v_0^i + \varepsilon_{ijk} v_0^j D_k + \\
+ \tilde{Q} \varepsilon_{ijk} jml \left( \partial_m B_{-1}^j B_{-1}^k + \partial_m B_0^l B_{-1}^k \right) + \tilde{Q} \varepsilon_{ijk} jml \partial_m B_0^l \nabla_k + \varepsilon_{ijk} v_0^j D_k
$$

$$
\partial_t B_0^i - P m^{-1} \partial_k^2 B_0^i = \varepsilon_{ijk} \varepsilon_{jml} \left( \partial_j W^n_{-1} B_0^p + \partial_j v_0^m B_{-1}^p \right) + \varepsilon_{ijk} v_0^j \sum B_0^p 
$$

$$
\partial_t T_0 - P r^{-1} \partial_k^2 T_0 = -W_{-1}^k \partial_k T_0 - \partial_k (v_0^k T_{-1}) + v_0^z
$$

$$
\partial_t v_0^i = 0, \quad \partial_t B_0^i = 0
$$

These equations give only one secular term:

$$
\nabla P_{-2} = 0 \quad \Rightarrow \quad P_{-2} = const
$$

Now consider the first-order approximation $R^1$:

$$
\partial_t v_1^i + W_{-1}^k \partial_k v_1^i + v_0^k \partial_k v_0^i + v_1^k \partial_k W_{-1}^i + W_{-1}^k \nabla_k W_{-1}^i = -\nabla_i P_{-1} - \\
- \partial_t (P_1 + \sum B_{-1}^i) + \partial_k^2 v_1^i + 2 \partial_k \nabla_k W_{-1}^i + \varepsilon_{ijk} v_1^j D_k + \\
+ \tilde{Q} \varepsilon_{ijk} jml \left( \partial_m B_{-1}^j B_{-1}^k + \partial_m B_0^l B_{-1}^k + \partial_m B_1^l B_{-1}^k + \nabla_m B_{-1}^l B_{-1}^k \right) + \\
+ \tilde{Q} \varepsilon_{ijk} jml \left( \partial_m B_1^l \nabla_k + \nabla_m B_{-1}^l \nabla_k \right) + \varepsilon_{ijk} v_1^j \sum B_1^p 
$$

$$
\partial_t B_1^i - P m^{-1} \partial_k^2 B_1^i - P m^{-1} 2 \partial_k \nabla_k B_{-1}^i = \varepsilon_{ijk} \varepsilon_{jml} \left( \partial_j W^n_{-1} B_1^p + \partial_j v_0^m B_{-1}^p + \partial_j v_0^m B_{-1}^p \right) + \\
+ \nabla_j W^n_{-1} B_{-1}^p \sum + \varepsilon_{ijk} v_1^j \sum \left( \partial_j v_0^m \sum + \nabla_j W^n_{-1} \sum \right) 
$$

$$
\partial_t T_1 - P r^{-1} \partial_k^2 T_1 - P r^{-1} 2 \partial_k \nabla_k T_{-1} = -W_{-1}^k \partial_k T_1 - W_{-1}^k \nabla_k T_{-1} - v_0^k \partial_k T_0 - v_1^k \partial_k T_{-1} + v_1^z
$$

$$
\partial_t v_1^i + \nabla_k W_{-1}^i = 0, \quad \partial_t B_1^i + \nabla_k B_{-1}^i = 0
$$

This system yields the following secular equations:

$$
W_{-1}^k \nabla_k W_{-1}^i = -\nabla_i P_{-1} + \tilde{Q} \varepsilon_{ijk} jml \left( \nabla_m B_{-1}^l B_{-1}^k + \nabla_m B_{-1}^l \sum \right) 
$$

$$
\varepsilon_{ijk} \varepsilon_{jml} \left( \nabla_j W^n_{-1} B_{-1}^p + \nabla_j W^n_{-1} \sum \right) = 0
$$

$$
W_{-1}^k \nabla_k T_{-1} = 0, \quad \nabla_i W_{-1}^i = 0, \quad \nabla_k B_{-1}^k = 0
$$

For the second order $R^2$ we obtain the equations:

$$
\partial_t v_2^i + W_{-1}^k \partial_k v_2^i + v_0^k \partial_k v_2^i + W_{-1}^k \nabla_k v_0^i + v_0^k \nabla_k W_{-1}^i + v_1^k \partial_k v_0^i + v_2^k \partial_k W_{-1}^i = \\
= -\nabla_i P_2 - \nabla_i P_0 + \partial_k^2 v_2^i + 2 \partial_k \nabla_k v_2^i + \varepsilon_{ijk} v_2^j D_k + \\
+ \tilde{Q} \varepsilon_{ijk} jml \left( \partial_m B_{-1}^j B_{-1}^k + \partial_m B_0^l B_{-1}^k + \partial_m B_1^l B_{-1}^k + \nabla_m B_{-1}^l B_{-1}^k + \nabla_m B_{-1}^l B_{-1}^k \right) + \\
+ \tilde{Q} \varepsilon_{ijk} jml \left( \partial_m B_1^l \sum + \nabla_m B_{-1}^l \sum \right) + \varepsilon_{ijk} v_2^j \sum
$$
\[ \partial_t B_2^i - Pr^{-1} \partial_k^2 B_2^i - Pr^{-1} 2 \partial_k \nabla_k B_0^i = \varepsilon_{ijk} \varepsilon_{knp} \left( \partial_j W_{\nu m}^n B_p^i + \partial_j v_0^n B_1^p + \partial_j v_0^n B_0^p + \partial_j v_0^n B_0^0 + \partial_j v_0^n B_0^0 + \partial_j v_0^n B_0^0 + \partial_j v_0^n B_0^0 \right) + \partial_j v_0^n B_0^0 + \partial_j v_0^n B_0^0 + \varepsilon_{ijk} \varepsilon_{knp} \left( \partial_j v_0^n B_0^0 + \partial_j v_0^n B_0^0 \right) \] (67)

\[ \partial_t T_2 - Pr^{-1} \partial_k^2 T_2 - Pr^{-1} 2 \partial_k \nabla_k T_0 = -W_{\nu m}^n \partial_k T_2 - W_{\nu m}^n \nabla_k T_0 - v_0^n \partial_k T_1 - v_0^n \nabla_k T_1 - v_0^n \nabla_k T_1 - v_0^n \nabla_k T_1 = -v_0^n \partial_k T_0 - v_0^n \partial_k T_0 + v_0^n \partial_k T_0 + v_0^n \partial_k T_0 = 0 \]

As seen after the averaging of the system of Eqs. (9) over the «fast» variables, in the order \( R^2 \) secular terms are absent.

Finally, let us consider the most significant order \( R^3 \). Here the equations have the following form:

\[ \partial_t v_3^i + \partial_t W_{-1}^i + W_{-1}^k \partial_k v_3^i + v_0^n \partial_k v_2^i + W_{-1}^k \nabla_k v_3^i + v_0^n \nabla_k v_3^i + v_0^n \nabla_k v_3^i + v_0^n \nabla_k v_3^i + v_0^n \nabla_k v_3^i + v_0^n \nabla_k v_3^i = -\partial_t P_3 - \nabla_i (P_1 + \overline{P}_1) + \partial_k^2 v_3^i + 2 \partial_k \nabla_k v_3^i + \Delta W_{-1}^i + \varepsilon_{ijk} v_3^j D_k + \] + \tilde{Q} \varepsilon_{ijk} \varepsilon_{jml} \left( \partial_m B_{-1}^l B_3^j + \partial_m B_0^l B_2^j + \partial_m B_1^l B_3^j + \partial_m B_1^l B_2^j + \partial_m B_2^l B_3^j \right) + \varepsilon_{ijk} \varepsilon_{jml} \left( \nabla_m B_3^j B_k + \nabla_m B_1^j B_k \right) + e_i \tilde{R} a T_3

\[ \partial_t B_3^i + \partial_t B_{-1}^i - P m^{-1} \partial_k^2 B_3^i - 2 P m^{-1} \partial_k \nabla_k B_3^i - P m^{-1} \Delta B_{-1}^i = \varepsilon_{ijk} \varepsilon_{knp} \left( \partial_j W_{-1}^n B_3^p + \partial_j v_0^n B_2^p + \partial_j v_0^n B_1^p + \partial_j v_0^n B_1^p + \partial_j v_0^n B_1^p + \partial_j v_0^n B_1^p + \partial_j v_0^n B_1^p \right) + \partial_j v_0^n B_1^p + \partial_j v_0^n B_1^p + \partial_j v_0^n B_1^p + \partial_j v_0^n B_1^p + \partial_j v_0^n B_1^p + \partial_j v_0^n B_1^p \] + \nabla_j v_0^n B_1^p + \nabla_j v_0^n B_1^p + \nabla_j v_0^n B_1^p + \varepsilon_{ijk} \varepsilon_{knp} \left( \partial_j v_0^n B_1^p + \partial_j v_0^n B_1^p \right) + e_i \tilde{R} a T_3

\[ \partial_t T_3 + \partial_t T_{-1} - Pr^{-1} \partial_k^2 T_3 - Pr^{-1} 2 \partial_k \nabla_k T_1 - Pr^{-1} \Delta T_{-1} = -W_{-1}^k \partial_k T_3 - W_{-1}^k \nabla_k T_1 - W_{-1}^k \nabla_k T_1 - W_{-1}^k \nabla_k T_1 - W_{-1}^k \nabla_k T_1 - W_{-1}^k \nabla_k T_1 = -v_0^n \partial_k T_3 - v_0^n \partial_k T_3 - v_0^n \nabla_k T_3 - v_0^n \nabla_k T_3 - v_0^n \nabla_k T_3 - v_0^n \nabla_k T_3 = 0 \]

By averaging this system of equations over the «fast» variables we obtain the basic secular equations which describe the evolution of the large-scale perturbations:

\[ \partial_t W_{-1}^i - \Delta W_{-1}^i + \nabla_k \left( \overline{v_0^i} \overline{v_0^j} \right) = -\nabla_i \overline{P}_1 + \tilde{Q} \varepsilon_{ijk} \varepsilon_{jml} \nabla_m \left( B_0^j B_0^l \right) \] (69)

\[ \partial_t B_{-1}^i - Pr^{-1} \Delta B_{-1}^i = \varepsilon_{ijk} \varepsilon_{knp} \nabla_j \left( v_0^n B_0^p \right) \] (70)

\[ \partial_t T_{-1} - Pr^{-1} \Delta T_{-1} = -\nabla_i \left( v_0^n T_0 \right) \] (71)
10 Appendix II. Small-scale fields

In Appendix I we have obtained the equations of asymptotic expansion in the zero-order approximation. Taking into account the new denotations $\tilde{W} = \tilde{W}_{-1}$, $\tilde{H} = \tilde{B}_{-1}$ they can be written in the following form:

$$\hat{D}_W v_0^i = -\partial_t P_0 + \varepsilon_{ijk} v_0^j D_k + \tilde{Q} H_k \left( \partial_t B_0^i - \partial_i B_0^k \right) + \tilde{Q} B_k \left( \partial_t B_0^i - \partial_i B_0^k \right) + \epsilon_i \widetilde{\text{Ra}} T_0 + F_0^i \tag{72}$$

$$\hat{D}_H B_0^i = \left( H_p \partial_p + |\mathbf{B}_k| \partial_k \right) v_0^i \tag{73}$$

$$\hat{D}_\theta T_0 = \epsilon_i v_0^k \tag{74}$$

$$\partial_i v_0^i = \partial_t B_0^i = \partial_t F_0^i = 0 \tag{75}$$

where the operators are denoted as:

$$\hat{D}_W = \partial_t - \partial^2 + W_k \partial_k, \quad \hat{D}_H = \partial_t - P m^{-1} \partial^2 + W_k \partial_k, \quad \hat{D}_\theta = \partial_t - Pr^{-1} \partial^2 + W_k \partial_k.$$ 

The small-scale oscillations of the magnetic field and the temperature are easily found from Eqs. (73)-(74):

$$B_0^i = \frac{(H_p + \mathbf{B}_p)}{D_H} \partial_p v_0^i, \quad T_0 = \frac{v_0^i}{D_\theta} \tag{76}$$

Now we substitute (76) into (72) and find the pressure $P_0$ using the condition of field solenoidality (75):

$$P_0 = \varepsilon_{ijk} \frac{\partial_i v_0^j}{\partial^2} D_k + \varepsilon_i \varepsilon_k \widetilde{\text{Ra}} \frac{\partial_i v_0^k}{\partial^2 D_\theta} - \tilde{Q} \frac{(H_p + \mathbf{B}_p)}{D_H} \partial_p (H_k \partial^2 v_0^k) \tag{77}$$

Using the above-derived formula (77) we exclude the pressure from Eq. (72) and obtain the equation for the velocity field of the zeroth-order approximation:

$$\left[ \left( \hat{D}_W - \frac{\tilde{Q}(H_k \partial_k)^2 + H_k \mathbf{B}_l \partial_k \partial_l}{D_H} \right) \delta_{ij} - \left( \frac{\widetilde{\text{Ra}} \varepsilon_{ijp} D_k}{D_\theta} + \varepsilon_{ijk} \mathbf{D}_k \right) \hat{P}_p \right] v_0^i = F_0^i, \tag{78}$$

where $\hat{P}_p = \delta_{ip} - \frac{\partial_i \partial_p}{\partial^2}$ is the projection operator. For finding the small-scale field $\tilde{v}_0$ it is convenient to present Eq.(78) in the coordinate form:

$$\begin{cases}
\hat{d}_{11} v_0^x + \hat{d}_{12} v_0^y + \hat{d}_{13} v_0^z = \hat{F}_0^x \\
\hat{d}_{21} v_0^x + \hat{d}_{22} v_0^y + \hat{d}_{23} v_0^z = \hat{F}_0^y \\
\hat{d}_{31} v_0^x + \hat{d}_{32} v_0^y + \hat{d}_{33} v_0^z = \hat{F}_0^z
\end{cases} \tag{79}$$

The components of the tensor $\hat{d}_{ij}$ have the following form:

$$\hat{d}_{11} = \hat{D}_W - \frac{\tilde{Q}(H_k \partial_k)^2 + H_k \mathbf{B}_l \partial_k \partial_l}{D_H} + \frac{D_2 \partial_{\alpha} \partial_{z} - D_3 \partial_{\alpha} \partial_{y}}{\partial^2}, \quad \hat{d}_{12} = \frac{D_3 \partial_{z}^2 - D_1 \partial_{\alpha} \partial_{z}}{\partial^2} - D_3,$$

$$\hat{d}_{13} = D_2 + \frac{D_1 \partial_{\alpha} \partial_{\alpha} - D_2 \partial_{\alpha}^2}{\partial^2} + \frac{\widetilde{\text{Ra}} \partial_{\alpha} \partial_{z}}{\partial^2 D_\theta}, \quad \hat{d}_{21} = D_3 + \frac{D_2 \partial_{\alpha} \partial_{z} - D_3 \partial_{\alpha}^2}{\partial^2},$$

$$\hat{d}_{22} = \hat{D}_W - \frac{\tilde{Q}(H_k \partial_k)^2 + H_k \mathbf{B}_l \partial_k \partial_l}{D_H} + \frac{D_3 \partial_{\alpha} \partial_{\alpha} - D_1 \partial_{\alpha} \partial_{\alpha}}{\partial^2},$$

$$\hat{d}_{31} = \frac{D_3 \partial_{\alpha} \partial_{z}}{\partial^2}, \quad \hat{d}_{32} = \frac{D_2 \partial_{\alpha} \partial_{\alpha} - D_3 \partial_{\alpha}^2}{\partial^2}, \quad \hat{d}_{33} = \frac{\widetilde{\text{Ra}} \partial_{\alpha} \partial_{z}}{\partial^2 D_\theta}. $$
\[ \tilde{d}_{23} = \frac{\tilde{R} a}{\partial^2 \tilde{D}_\theta} \partial_y \partial_z + \frac{D_1 \partial_y^2 - D_2 \partial_y \partial_x}{\partial^2} - D_1, \quad \tilde{d}_{31} = \frac{D_2 \partial_z^2 - D_3 \partial_z \partial_y}{\partial^2} - D_2, \]
\[ \tilde{d}_{32} = \frac{D_3 \partial_z \partial_x - D_1 \partial_x^2}{\partial^2} + D_1, \quad \tilde{d}_{33} = \tilde{D}_W - \tilde{D}_W - \tilde{D}_H \]
\[ + \frac{D_1 \partial_z \partial_y - D_2 \partial_z \partial_x}{\partial^2} - \frac{\tilde{R} a}{\partial^2 \tilde{D}_\theta} + \frac{\tilde{R} a}{\partial^2 \tilde{D}_\theta}. \]

As is known, the solution for the system \( (79) \) can be found by means of the Cramer rule:

\[ v_0^\theta = u_0 = \frac{1}{\Delta} \left\{ \left( \tilde{d}_{22} \tilde{d}_{33} - \tilde{d}_{32} \tilde{d}_{23} \right) F_0^x + \left( \tilde{d}_{13} \tilde{d}_{32} - \tilde{d}_{12} \tilde{d}_{33} \right) F_0^y + \left( \tilde{d}_{12} \tilde{d}_{23} - \tilde{d}_{13} \tilde{d}_{22} \right) F_0^r \right\} \]
\[ v_0^\eta = v_0 = \frac{1}{\Delta} \left\{ \left( \tilde{d}_{23} \tilde{d}_{31} - \tilde{d}_{21} \tilde{d}_{33} \right) F_0^x + \left( \tilde{d}_{11} \tilde{d}_{33} - \tilde{d}_{13} \tilde{d}_{31} \right) F_0^y + \left( \tilde{d}_{13} \tilde{d}_{21} - \tilde{d}_{11} \tilde{d}_{23} \right) F_0^r \right\} \]
\[ v_0^\varphi = w_0 = \frac{1}{\Delta} \left\{ \left( \tilde{d}_{21} \tilde{d}_{32} - \tilde{d}_{22} \tilde{d}_{31} \right) F_0^x + \left( \tilde{d}_{12} \tilde{d}_{31} - \tilde{d}_{11} \tilde{d}_{32} \right) F_0^y + \left( \tilde{d}_{11} \tilde{d}_{22} - \tilde{d}_{12} \tilde{d}_{21} \right) F_0^r \right\} \]

Here \( \Delta \) is the determinant of the system of equations \( (79) \) which in the open form is the following:

\[ \Delta = \tilde{d}_{11} \tilde{d}_{22} \tilde{d}_{33} + \tilde{d}_{21} \tilde{d}_{32} \tilde{d}_{13} + \tilde{d}_{12} \tilde{d}_{23} \tilde{d}_{31} - \tilde{d}_{13} \tilde{d}_{22} \tilde{d}_{31} - \tilde{d}_{32} \tilde{d}_{23} \tilde{d}_{11} - \tilde{d}_{21} \tilde{d}_{12} \tilde{d}_{33} \]

Now let us present the external force \( F_0 \) in the complex form:

\[ F_0^r = \frac{\tilde{f}_0^r}{2} e^{i \phi_2} + \frac{\tilde{f}_0^r}{2} e^{i \phi_1} + c.c. \]

Then all the operators contained in \( (80)-(83) \) act on the eigenfunctions from the left:

\[ \tilde{D}_{W,H,\eta} e^{i \phi_1} = e^{i \phi_1} \tilde{D}_{W,H,\eta} \tilde{\kappa}_1, -\omega_0), \quad \tilde{D}_{W,H,\eta} e^{i \phi_2} = e^{i \phi_2} \tilde{D}_{W,H,\eta} \tilde{\kappa}_2, -\omega_0), \]
\[ \Delta e^{i \phi_1} = e^{i \phi_1} \Delta \tilde{\kappa}_1, -\omega_0), \quad \Delta e^{i \phi_2} = e^{i \phi_2} \Delta \tilde{\kappa}_2, -\omega_0) \]

To simplify the formulae, assume that \( \kappa_0 = 1, \omega_0 = 1 \) and introduce the new denotations:

\[ \tilde{D}_W (\tilde{\kappa}_1, -\omega_0) = \tilde{D}_{W1} = 1 - i (1 - W_1), \quad \tilde{D}_W (\tilde{\kappa}_2, -\omega_0) = \tilde{D}_{W2} = 1 - i (1 - W_2) \]
\[ \tilde{D}_H (\tilde{\kappa}_1, -\omega_0) = \tilde{D}_{H1} = P m^{-1} - i (1 - W_1), \quad \tilde{D}_H (\tilde{\kappa}_2, -\omega_0) = \tilde{D}_{H2} = P m^{-1} - i (1 - W_2) \]
\[ \tilde{D}_\theta (\tilde{\kappa}_1, -\omega_0) = \tilde{D}_{\theta1} = P r^{-1} - i (1 - W_1), \quad \tilde{D}_\theta (\tilde{\kappa}_2, -\omega_0) = \tilde{D}_{\theta2} = P r^{-1} - i (1 - W_2) \]

Here and further the complex-conjugate terms are marked by asterisk. In subsequent calculations some components in the tensors \( \tilde{d}_{ij} (\tilde{\kappa}_1, -\omega_0) \) and \( \tilde{d}_{ij} (\tilde{\kappa}_2, -\omega_0) \) vanish, and there remain the non-zero components:

\[ \tilde{d}_{11} (\tilde{\kappa}_1, -\omega_0) = \tilde{D}_{W1}^* \frac{\tilde{Q} H_1 (H_1 + P T_1)}{D_{H1}}, \quad \tilde{d}_{12} (\tilde{\kappa}_1, -\omega_0) = 0, \quad \tilde{d}_{13} (\tilde{\kappa}_1, -\omega_0) = 0, \]
\[ \tilde{d}_{21} (\tilde{\kappa}_1, -\omega_0) = D_3, \quad \tilde{d}_{22} (\tilde{\kappa}_1, -\omega_0) = \tilde{d}_{11} (\tilde{\kappa}_1, -\omega_0), \quad \tilde{d}_{23} (\tilde{\kappa}_1, -\omega_0) = -D_1, \]
\[ \tilde{d}_{31} (\tilde{\kappa}_1, -\omega_0) = -D_2, \quad \tilde{d}_{32} (\tilde{\kappa}_1, -\omega_0) = D_1, \quad \tilde{d}_{33} (\tilde{\kappa}_1, -\omega_0) = \tilde{d}_{22} (\tilde{\kappa}_1, -\omega_0) - \frac{\tilde{R} a}{\tilde{D}_{\theta1}} \]
the expressions (76) and (89)-(91): velocity be used while calculating the correlation functions.

\[ \tilde{d}_{11}(\tilde{k}_2, -\omega_0) = \tilde{D}_{W2} + \frac{QH_2(H_2 + B_2)}{D_{H2}^*}, \quad \tilde{d}_{12}(\tilde{k}_2, -\omega_0) = -D_3, \quad \tilde{d}_{13}(\tilde{k}_2, -\omega_0) = D_2, \]

\[ \tilde{d}_{21}(\tilde{k}_2, -\omega_0) = 0, \quad \tilde{d}_{22}(\tilde{k}_2, -\omega_0) = \tilde{d}_{11}(\tilde{k}_2, -\omega_0), \quad \tilde{d}_{23}(\tilde{k}_2, -\omega_0) = 0, \quad (88) \]

\[ \tilde{d}_{31}(\tilde{k}_2, -\omega_0) = -D_2, \quad \tilde{d}_{32}(\tilde{k}_1, -\omega_0) = D_1, \quad \tilde{d}_{33}(\tilde{k}_2, -\omega_0) = \tilde{d}_{22}(\tilde{k}_2, -\omega_0) - \frac{\tilde{R}_a}{D_{b2}^*}. \]

Taking into account the expressions (87)-(88) we find the velocity fields in the zero-order approximation:

\[ u_0 = \frac{f_0}{2} \frac{\tilde{B}_2^*}{A_2^*B_2^* + D_2^*} e^{i\phi_2} + c.c. = u_{03} + u_{04} \quad (89) \]

\[ v_0 = \frac{f_0}{2} \frac{\tilde{B}_1}{A_1^*B_1^* + D_1^*} e^{i\phi_1} + c.c. = v_{01} + v_{02} \quad (90) \]

\[ w_0 = -\frac{f_0}{2} \frac{D_1}{A_1^*B_1^* + D_1^*} e^{i\phi_1} + \frac{f_0}{2} \frac{D_2}{A_2^*B_2^* + D_2^*} e^{i\phi_2} + c.c. = w_{01} + w_{02} + w_{03} + w_{04} \quad (91) \]

where

\[ \tilde{A}_{1,2}^* = \tilde{D}_{W1,2} + \frac{QH_{1,2}}{D_{H1,2}^*}(H_{1,2} + B_{1,2}), \quad \tilde{B}_{1,2} = \frac{\tilde{A}_{1,2}^*}{D_{b1,2}^*} - \frac{\tilde{R}_a}{D_{b1,2}^*}. \quad (92) \]

The velocity components satisfy the following relations: \( w_{02} = (w_{01})^*, \quad w_{04} = (w_{03})^*, \quad v_{02} = (v_{01})^*, \quad v_{04} = (v_{03})^*, \quad u_{02} = (u_{01})^*, \quad u_{04} = (u_{03})^*. \) In the limiting case of non-electroconductive fluid (\( \sigma = 0 \)), in the absence of temperature gradient (\( \nabla T = 0 \)) and external magnetic field (\( B_{1,2} = 0 \)) the formulae (89)-(91) coincide with the results obtained in [11]. Now we will calculate the small-scale oscillations of the magnetic field \( \tilde{B}_0 \) using the expressions (76) and (89)-(91):

\[ B_0^x = \tilde{u}_0 = \frac{f_0}{2} \frac{i(H_2 + B_2)\tilde{B}_2^*}{D_{H2}^*A_2^*B_2^* + D_2^*} e^{i\phi_2} + c.c. = \tilde{u}_{03} + \tilde{u}_{04} \quad (93) \]

\[ B_0^y = \tilde{v}_0 = \frac{f_0}{2} \frac{i(H_1 + B_1)\tilde{B}_1^*}{D_{H1}^*A_1^*B_1^* + D_1^*} e^{i\phi_1} + c.c. = \tilde{u}_{03} + \tilde{u}_{04} \quad (94) \]

\[ B_0^z = \tilde{w}_0 = -\frac{f_0}{2} \frac{i(H_1 + B_1)D_1}{D_{H1}^*A_1^*B_1^* + D_1^*} e^{i\phi_1} + \frac{f_0}{2} \frac{i(H_2 + B_2)D_2}{D_{H2}^*A_2^*B_2^* + D_2^*} e^{i\phi_2} + c.c. = w_{01} + w_{02} + w_{03} + w_{04} \quad (95) \]

In the expressions for the small-scale oscillations (\( \tilde{u}_0, \tilde{B}_0, \tilde{T}_0 \)) the component of the angular velocity \( D_3 \) is absent due to the choice of the external force. Further Eqs. (89)-(10) will be used while calculating the correlation functions.
11 Appendix III. Calculation of the Reynolds stresses, Maxwell stresses and turbulent e.m.f.

To close the system of Eqs. (17)-(20) which describe the evolution of the large-scale fields, it is necessary to calculate the correlators of the types

\[ T^{31} = \overline{w_0 u_0} = \overline{w_{03} (u_{03})^*} + (w_{03})^* u_{03} \]  \hspace{1cm} (96)

\[ T^{32} = \overline{w_0 v_0} = \overline{w_{01} (v_{01})^*} + (w_{01})^* v_{01} \]  \hspace{1cm} (97)

\[ S^{31} = \overline{w_0 u_0} = \overline{\tilde{w}_{03} (\tilde{u}_{03})^*} + (\tilde{w}_{03})^* \tilde{u}_{03} \]  \hspace{1cm} (98)

\[ S^{32} = \overline{w_0 v_0} = \overline{w_{01} (\tilde{v}_{01})^*} + (w_{01})^* \tilde{v}_{01} \]  \hspace{1cm} (99)

\[ G^{13} = \overline{u_0 w_0} = \overline{u_{03} (\tilde{w}_{03})^*} + (u_{03})^* \tilde{w}_{03} \]  \hspace{1cm} (100)

\[ G^{31} = \overline{w_0 u_0} = \overline{w_{03} (\tilde{u}_{03})^*} + (w_{03})^* \tilde{u}_{03} \]  \hspace{1cm} (101)

\[ G^{23} = \overline{v_0 w_0} = \overline{v_{01} (\tilde{w}_{01})^*} + (v_{01})^* \tilde{w}_{01} \]  \hspace{1cm} (102)

\[ G^{32} = \overline{w_0 v_0} = \overline{w_{01} (\tilde{v}_{01})^*} + (w_{01})^* \tilde{v}_{01} \]  \hspace{1cm} (103)

At first let us calculate the Reynolds stresses (96)-(97). For this purpose we will use the expressions for the small-scale velocity fields (89)-(91). Their substitution into (96)-(97) gives:

\[ T^{31} = f_0^2 \frac{D_2 q_2}{2} \left| \hat{A}_2 \hat{B}_2 + D_2^2 \right|^2, \]  \hspace{1cm} (104)

\[ T^{32} = -f_0^2 \frac{D_1 q_1}{2} \left| \hat{A}_1 \hat{B}_1 + D_1^2 \right|^2, \]  \hspace{1cm} (105)

where

\[ q_{1,2} = 1 + \frac{Q H_{1,2}(H_{1,2} + \overline{B}_{1,2})}{1 + P m^2 (1 - W_{1,2})^2} \frac{Ra}{1 + P r^2 (1 - W_{1,2})^2}. \]

To calculate the correlators of the magnetic field or the Maxwell stresses \( S^{31} \) and \( S^{32} \), we will use the expressions (92)-(94). By substituting (92)-(94) into (98), (99) obtain:

\[ S^{31} = \frac{H_2^3}{|\hat{D}_{H_2}|} T^{31}, \hspace{1cm} S^{32} = \frac{H_1^3}{|\hat{D}_{H_1}|} T^{32} \]  \hspace{1cm} (106)

The differences \( T^{31} - Q S^{31} \) and \( T^{32} - Q S^{32} \) contained in the right sides of Eqs. (17)-(18) can be easily found using the expressions (105)-(106):

\[ T^{31} - Q S^{31} = T^{31} \left( 1 - \frac{Q (H_2 + \overline{B}_2)^2}{P m |\hat{D}_{H_2}|^2} \right) = T^{31} Q_2 \]  \hspace{1cm} (107)
\[ T^{32} - \tilde{Q} S^{32} = T^{32} \left( 1 - \frac{Q(H_1 + B_1)^2}{P_m D_{H_1}^2} \right) = T^{32} Q_1 \]  

(108)

To calculate the group of oscillators \((100)-(103)\) we will use the expressions for the small-scale velocity field \((89)-(91)\) and the magnetic field \((92)-(94)\). Simple mathematical operations yield:

\[ G^{13} = \frac{f_0^2}{4} \frac{i(H_2 + B_2)D_2}{|A_2 B_2 + D_2|^2} \cdot \left( \frac{\tilde{B}_2}{D_{H_2}} - \frac{\tilde{B}_2^*}{D_{H_2}} \right) \]  

(109)

\[ G^{31} = \frac{f_0^2}{4} \frac{i(H_2 + B_2)D_2}{|A_2 B_2 + D_2|^2} \cdot \left( \frac{\tilde{B}_2^*}{D_{H_2}} - \frac{\tilde{B}_2}{D_{H_2}} \right) \]  

(110)

\[ G^{23} = \frac{f_0^2}{4} \frac{i(H_1 + B_1)D_1}{|A_1 B_1 + D_1|^2} \cdot \left( \frac{\tilde{B}_1}{D_{H_1}} - \frac{\tilde{B}_1^*}{D_{H_1}} \right) \]  

(111)

\[ G^{32} = \frac{f_0^2}{4} \frac{i(H_1 + B_1)D_1}{|A_1 B_1 + D_1|^2} \cdot \left( \frac{\tilde{B}_1^*}{D_{H_1}} - \frac{\tilde{B}_1}{D_{H_1}} \right) \]  

(112)

To close the equations for the large-scale magnetic field \((24), (25)\), it is necessary to calculate the differences \(G^{13} - G^{31}\) and \(G^{23} - G^{32}\) corresponding to the turbulent e.m.f. components \(\mathcal{E}_2 = \mathcal{E}_y\) and \(\mathcal{E}_1 = \mathcal{E}_x\). In view of the expressions \((109)-(112)\) we obtain:

\[ \mathcal{E}_2 = G^{13} - G^{31} = \frac{f_0^2}{4} \frac{i(H_2 + B_2)D_2}{|A_2 B_2 + D_2|^2} \cdot \left( \frac{\tilde{B}_2 - \tilde{B}_2^*}{D_{H_2}} \right) \cdot (\tilde{D}_{H_2}^* + \tilde{D}_{H_2}) \]  

(113)

\[ \mathcal{E}_1 = G^{23} - G^{32} = -\frac{f_0^2}{4} \frac{i(H_1 + B_1)D_1}{|A_1 B_1 + D_1|^2} \cdot \left( \frac{\tilde{B}_1 - \tilde{B}_1^*}{D_{H_1}} \right) \cdot (\tilde{D}_{H_1} + \tilde{D}_{H_1}) \]  

(114)

Using the expressions \((86)\) and \((92)\) let us write several relations:

\[ |\tilde{A}_{1,2} B_{1,2} + D_{1,2}^2| = |\tilde{A}_{1,2}|^2 \cdot |\tilde{B}_{1,2}|^2 + D_{1,2}^2 \cdot (\tilde{A}_{1,2} B_{1,2}^* + \tilde{A}_{1,2} B_{1,2}) + D_{1,2}^4, \]

\[ |\tilde{A}_{1,2}|^2 = |\tilde{D}_{W_{1,2}}|^2 + \tilde{Q} H_{1,2}(H_{1,2} + B_{1,2}) \left( \frac{\tilde{D}_{W_{1,2}}}{D_{H_{1,2}}} + \frac{\tilde{D}_{W_{1,2}}}{D_{H_{1,2}}} \right) + \frac{\tilde{Q}^2 H_{1,2}^2(H_{1,2} + B_{1,2})^2}{|D_{H_{1,2}}|^2}, \]

\[ |\tilde{B}_{1,2}|^2 = |\tilde{A}_{1,2}|^2 - \tilde{R} a \left( \frac{\tilde{A}_{1,2}}{D_{\theta_{1,2}}} + \frac{\tilde{A}_{1,2}^*}{D_{\theta_{1,2}}} \right) + \frac{\tilde{R} a^2}{|D_{\theta_{1,2}}|^2}, \]

\[ \tilde{A}_{1,2} B_{1,2}^* + \tilde{A}_{1,2} B_{1,2} = (\tilde{A}_{1,2})^2 + (\tilde{A}_{1,2})^2 - \tilde{R} a \left( \frac{\tilde{A}_{1,2}}{D_{\theta_{1,2}}} + \frac{\tilde{A}_{1,2}^*}{D_{\theta_{1,2}}} \right), \]

(115)

\[ |\tilde{D}_{W_{1,2}}|^2 = 1 + (1 - W_{1,2})^2, \quad |\tilde{D}_{H_{1,2}}|^2 = P_m^{-2} + (1 - W_{1,2})^2, \quad |\tilde{D}_{\theta_{1,2}}|^2 = P \tau^{-2} + (1 - W_{1,2})^2, \]
\[ \tilde{D}_{H_{1,2}} + \tilde{D}_{H_{1,2}}^* = 2Pm^{-1}, \quad \tilde{D}_{W_{1,2}} \tilde{D}_{H_{1,2}} + \tilde{D}_{W_{1,2}}^* \tilde{D}_{H_{1,2}}^* = 2(Pm^{-1} - (1 - W_{1,2})^2), \]
\[ \tilde{D}_{\theta_{1,2}} \tilde{D}_{H_{1,2}} + \tilde{D}_{H_{1,2}} \tilde{D}_{\theta_{1,2}}^* = 2(Pm^{-1}Pr^{-1} + (1 - W_{1,2})^2), \quad \left( \tilde{D}_{W_{1,2}} \right)^2 + \left( \tilde{D}_{W_{1,2}}^* \right)^2 = 2(1 - (1 - W_{1,2})^2), \]
\[ \tilde{D}_{W_{1,2}} \tilde{D}_{\theta_{1,2}} + \tilde{D}_{\theta_{1,2}} \tilde{D}_{W_{1,2}}^* = 2(Pr^{-1} + (1 - W_{1,2})^2), \quad \tilde{D}_{W_{1,2}}^* \tilde{D}_{H_{1,2}} + \tilde{D}_{W_{1,2}} \tilde{D}_{H_{1,2}}^* = 2(Pm^{-1} + (1 - W_{1,2})^2). \]

These relations substituted into (107)–(108) make it possible to obtain the difference of the Reynolds and Maxwell stresses:

\[
T^{31} - \tilde{Q}S^{31} = \frac{f_0^2}{2} \cdot \frac{D_2 q_2 Q_2}{4(1 - W_2)^2 q_2^2 Q_2^2 + \left[ D_2^2 + W_2 (2 - W_2) + \mu_2 \right]^2 + \xi_2},
\]
\[
T^{32} - \tilde{Q}S^{32} = \frac{f_0^2}{2} \cdot \frac{D_1 q_1 Q_1}{4(1 - W_1)^2 q_1^2 Q_1^2 + \left[ D_1^2 + W_1 (2 - W_1) + \mu_1 \right]^2 + \xi_1},
\]

with the following denotations:

\[
q_{1,2} = 1 + \frac{QH_{1,2}(H_{1,2} + \overline{B}_{1,2})}{1 + Pm^2 (1 - W_{1,2})^2} - \frac{Ra}{1 + Pr^2(1 - W_{1,2})^2}, \quad Q_{1,2} = 1 - \frac{QPrm(H_{1,2} + \overline{B}_{1,2})}{1 + Pm^2 (1 - W_{1,2})^2},
\]
\[
\tilde{Q}_{1,2} = 1 - \frac{QPrmH_{1,2}(H_{1,2} + \overline{B}_{1,2})}{1 + Pm^2 (1 - W_{1,2})^2} + \frac{RaPr}{1 + Pr^2(1 - W_{1,2})^2},
\]
\[
\nu_{1,2} = 2QH_{1,2}(H_{1,2} + \overline{B}_{1,2}) \cdot \frac{1 + Prm(1 - W_{1,2})^2}{1 + Pm^2 (1 - W_{1,2})^2} + Q^2 H_{1,2}^2(1 + \overline{B}_{1,2})^2 \cdot \frac{1 - Pm^2 (1 - W_{1,2})^2}{1 + Pm^2 (1 - W_{1,2})^2}
\]
\[
- Ra \cdot \frac{1 + Prm(1 - W_{1,2})^2 + 2QH_{1,2}(H_{1,2} + \overline{B}_{1,2}) \cdot \frac{1 - Prm(1 - W_{1,2})^2}{1 + Pm^2 (1 - W_{1,2})^2}}{1 + Pm^2 (1 - W_{1,2})^2},
\]
\[
\xi_{1,2} = 2\Xi_{1,2} + 2(1 - W_{1,2})^2 \Pi_{1,2} - 2(1 - W_{1,2})^2 (1 - \tilde{Q}_{1,2}) \Pi_{1,2} - 2(1 - q_{1,2}^2) \Xi_{1,2} + \Xi_{1,2} \Pi_{1,2} +
\]
\[
+ \chi_{1,2} (1 - W_{1,2})^2 + \chi_{1,2} (1 + \sigma_{1,2}),
\]
\[
\Xi_{1,2} = - \frac{4(1 - W_{1,2})^2 \tilde{Q}_{1,2} RaPr}{1 + Pr^2(1 - W_{1,2})^2} + \frac{2(1 - W_{1,2})^2 Ra^2 Pr}{\left( 1 + Pr^2(1 - W_{1,2})^2 \right)^2} +
\]
\[
+ Ra \cdot \frac{1 + Prm(1 - W_{1,2})^2 + 2QH_{1,2}(H_{1,2} + \overline{B}_{1,2}) \cdot \frac{1 - Prm(1 - W_{1,2})^2}{1 + Pm^2 (1 - W_{1,2})^2}}{1 + Pm^2 (1 - W_{1,2})^2},
\]
\[
\Pi_{1,2} = \frac{4q_{1,2} Ra}{1 + Pr^2(1 - W_{1,2})^2} + \frac{2 Ra^2}{\left( 1 + Pr^2(1 - W_{1,2})^2 \right)^2} -
\]
\[
- Ra \cdot \frac{1 + Prm(1 - W_{1,2})^2 + 2QH_{1,2}(H_{1,2} + \overline{B}_{1,2}) \cdot \frac{1 - Prm(1 - W_{1,2})^2}{1 + Pm^2 (1 - W_{1,2})^2}}{1 + Pm^2 (1 - W_{1,2})^2},
\]
\[
\sigma_{1,2} = \frac{QH_{1,2}(H_{1,2} + \overline{B}_{1,2})}{1 + Pm^2 (1 - W_{1,2})^2} \cdot \left[ 2 \left( 1 + Pm^2 (1 - W_{1,2})^2 \right) + QH_{1,2}(H_{1,2} + \overline{B}_{1,2}) \right],
\]
\[
\chi_{1,2} = \frac{2 Ra}{1 + Pr^2(1 - W_{1,2})^2} \cdot \left[ \frac{Ra}{2} - (1 - Pr(1 - W_{1,2})^2 +
\]

34
By substituting the relations (115) into (113)-(114) we find the expressions for a turbulent e.m.f. $\mathcal{E}_{1,2}$ in the explicit form:

$$
\mathcal{E}_1 = f_0^2 \cdot \frac{D_1 (1 - W_1) P_m \tilde{Q}_1 (H_1 + \overline{B}_1)}{(1 + P m^2 (1 - W_1)^2) \left[ 4 (1 - W_1)^2 q_1^2 \tilde{Q}_1^2 + \left[ D_1^2 + W_1 (2 - W_1) + \mu_1 \right]^2 + \xi_1 \right]},
$$

$$
\mathcal{E}_2 = f_0^2 \cdot \frac{D_2 (1 - W_2) P_m \tilde{Q}_2 (H_2 + \overline{B}_2)}{(1 + P m^2 (1 - W_2)^2) \left[ 4 (1 - W_2)^2 q_2^2 \tilde{Q}_2^2 + \left[ D_2^2 + W_2 (2 - W_2) + \mu_2 \right]^2 + \xi_2 \right]}.
$$

(117)

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