A SOLUTION TO THE PROBLEM OF THE SKIN EFFECT WITH A DISPLACEMENT CURRENT IN THE MAXWELL PLASMA BY THE METHOD OF EXPANSION IN EIGENFUNCTIONS

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A problem of the skin effect in the Maxwell plasma is solved analytically by the method of expansion in eigenfunctions based on the Vlasov-Maxwell kinetic equation with a self-consistent electric field. Specular electron reflection from the boundary is used as a boundary condition.

Keywords: skin effect, discrete and continuous spectra, Vlasov-Maxwell equations, characteristic equation, impedance.

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I. INTRODUCTION

The skin effect is caused by the electron gas response to an external variable electromagnetic field tangential to the surface [1]. This classical problem has been studied by many authors (for example, see [13]). The present work develops an analytical method of solving boundary problems for systems of equations describing the behavior of electrons and an electric field in the half-space of weakly ionized plasma. This method is extremely convenient, because it allows the sought-after distribution function to be derived in an explicit form. The method being developed is based on the idea of expansion of the function to be derived in an explicit form. The method is extremely convenient, because it allows the sought-after distribution function to be derived in an explicit form. The method being developed is based on the idea of expansion of the function to be derived in an explicit form. 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where \( l \) is the free path of the electron, \( \delta = \frac{e^2}{2\pi \omega_0 \sigma_0} \), \( \delta \) is the classical depth of the skin layer, \( \sigma_0 = \frac{e^2 \mu}{m v} \), \( \sigma_0 \) is the electric conductance, \( \alpha = \frac{2l^2}{\delta^2} \), \( \alpha \) is the anomaly parameter.

Let us formulate conditions for the distribution function and field on the plasma boundary:

\[
\begin{align*}
    h(0, \mu) &= h(0, -\mu), \quad 0 < \mu < +\infty, \quad e(0) = e_s. & (4) \\
    \text{We search for a distribution function and field that decay with increasing distance from the surface:} & \\
    h(+\infty, \mu) &= 0, \quad -\infty < \mu < +\infty, \quad e(\infty) = 0. & (5) \\
    \text{Without loss of generality, we further set} & \quad e_s = 1.
\end{align*}
\]

II. EIGENFUNCTIONS AND EIGENVALUES

Separation of variables (see \([2]\))

\[
    h_\eta(x, \mu) = \exp\left(-\frac{x}{\eta}\right) \Phi(\eta, \mu),
\]

\[
    e_\eta(x) = \exp\left(-\frac{x}{\eta}\right) E(\eta),
\]

where \( \eta \) is a complex spectral parameter, reduces system of equations (2) and (3) to the characteristic system

\[
    (\eta - \mu) \Phi(\eta, \mu) = \frac{\eta}{z_0} E(\eta),
\]

\[
    [z_0^2 + Q^2 \eta^2] E(\eta) = \frac{i\alpha \eta^2}{\sqrt{\pi}} n(\eta),
\]

where

\[
    n(\eta) = \int_{-\infty}^{\infty} e^{-\mu^2} \Phi(\eta, \mu) d\mu. & (7)
\]

From Eqs. (6) and (7) we find the eigenfunctions of the continuous spectrum in the class of generalized functions [3]:

\[
    \Phi(\eta, \mu) = \frac{a}{\sqrt{\pi}} \eta^3 e^{-\eta^2} \frac{1}{\eta - \mu} + \lambda(\mu) \delta(\eta - \mu), & (8)
\]

\[
    E(\eta) = \frac{a \omega_0}{\sqrt{\pi}} \eta^2 e^{-\eta^2}, \quad a = -i \frac{\alpha}{z_0^2}. & (9)
\]

Taking into account the decrease of the distribution function and electric field far from the boundary, the positive real semiaxis \( 0 < x < +\infty \) is taken to mean the continuous spectrum of the boundary problem. The eigenfunctions of the continuous spectrum \( h_\eta(x, \mu) \) and \( e_\eta(x) \) are decreasing functions of the variable \( x \) for \( \Re z_0 > 0 \).

The eigenfunctions in equalities (8) and (9) have been normalized by the condition

\[
    \int_{-\infty}^{\infty} e^{-\mu^2} \Phi(\eta, \mu) d\mu = \left[1 + \left(\frac{\omega l}{c}\right)^2 \eta^2\right]^{-\eta^2},
\]

and the dispersion function

\[
    \lambda(z) = 1 + \left(\frac{Q}{z_0}\right)^2 z^2 + \frac{a^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu^2} d\mu \frac{\mu}{\mu - z},
\]

has been introduced.

Let us designate \( b = \frac{Q^2}{z_0^2} \) and express the dispersion function of the problem in terms of the dispersion function of the Van Kampen plasma \( \lambda_0(z) \):

\[
    \lambda(z) = 1 + (b - a) z^2 + a^2 \lambda_0(z),
\]

where

\[
    \lambda_0(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{\mu e^{-\mu^2} d\mu}{\mu - z}.
\]

For the dispersion function in the vicinity of the point at infinity, the asymptotic expansion

\[
    \lambda(z) = (b - a) z^2 + \left(1 - \frac{a}{2}\right) - \frac{3a}{4z^2} - \frac{15a}{8z^4} + \ldots, \quad z \to \infty,
\]

is fulfilled.

We now elucidate the structure of the discrete spectrum by the method developed in \([2, 3]\). By definition, this spectrum consists of zeros of the dispersion function laying outside of the cut \((-\infty, \infty)\).

Let \( N \) be the number of zeros. Since the dispersion function has a double pole at the point \( z = \infty \), the number of its zeros is

\[
    N = 2 + \frac{1}{2\pi} [\arg \lambda(z)]_{\gamma_\infty}, & (10)
\]

where \( \gamma_\infty \) is a contour passing clockwise over the cut \((-\infty, \infty)\) at distance \( \varepsilon \) and having no zeros inside.

Taking the limit in Eq. (10) when \( \varepsilon \to 0 \), we obtain

\[
    N = 2 + \frac{1}{2\pi} \left[ \arg \frac{\lambda^+(\tau)}{\lambda^-(\tau)} \right]_{(-\infty, \infty)} = 2 + \frac{1}{\pi} \left[ \arg \frac{\lambda^+(\tau)}{\lambda^-(\tau)} \right]_{(0, \infty)}.
\]

Here \( \lambda^+ = \lambda(\mu) + i\pi a \theta^2 e^{-\mu^2} \) are the maximum and minimum values of the function \( \lambda(z) \) in the cut.

Let us consider the region \( D^+ \) (we designate by \( D^- \) its external boundary) in the \( \alpha \) plane whose boundary is set by the parametric equations

\[
    \partial D^+ = \{ \alpha = \alpha_1 + i\alpha_2 : \quad \Re \lambda^+(\mu) = 0, \}
\]
\[ \exists \lambda^+ (\mu) = 0, \quad -\infty < \mu < \infty. \]

By analogy with [2], we can demonstrate that 1) if \( a \in D^+, N = 4 \) and 2) if \( a \in D^-, N = 2 \). The mode with \( a \in \partial D \) is not considered here, since it has already been studied in detail in [3].

Let us write down (discrete) eigenfunctions corresponding to the obtained discrete spectrum \( \{ \pm \eta_k : \lambda(\eta_k) = 0, k = 0, 1 \} : \)

\[ \Phi(\eta_k, \mu) = \frac{a}{\sqrt{\pi}} \sum_{k=0}^1 \eta_k^2 e^{-\eta_k^2} \]

\[ E(\eta_k) = \frac{a \zeta_a}{\sqrt{\pi}} \sum_{k=0}^1 \eta_k^2 e^{-\eta_k^2} \quad (k = 0, 1). \]

We note that in the last formulas, \( k = 0 \) when \( a \in D^- \) and \( k = 0, 1 \) when \( a \in D^+ \).

**III. ANALYTICAL PROBLEM SOLUTION**

Let us represent the general solution of system (2)-(5) in the form of expansion in eigenfunctions of the discrete and continuous spectra, automatically satisfying the boundary conditions at infinity:

\[ h(x, \mu) = \frac{a}{\sqrt{\pi}} \sum_{k=0}^1 \frac{A_k \eta_k^3}{\eta_k - \mu} + \int_0^\infty \exp \left(-z_0 \frac{x}{\eta}\right) A(\eta) \Phi(\eta, \mu) d\eta, \quad (11) \]

\[ e(x) = \frac{a \zeta_a}{\sqrt{\pi}} \sum_{k=0}^1 A_k \eta_k^2 + \int_0^\infty \exp \left(-z_0 \frac{x}{\eta}\right) \eta^2 A(\eta) d\eta, \quad (12) \]

Here \( A_k \) \( (k = 0, 1) \) are unknown coefficients of the discrete spectrum with \( A_1 = 0 \) for \( a \in D^- \). \( A(\eta) \) is unknown function called the coefficient of the continuous spectrum, \( \Re(z_0 / \eta_k) > 0 \quad (k = 0, 1) \), and \( \Re z_0 = 1 \).

Substituting expansions (11) and (12) into the boundary conditions, we obtain the following integral equations:

\[ a \varphi(\mu) + \int_0^\infty A(\eta) \Phi(\eta, \mu) d\eta - \int_0^\infty A(\eta) \Phi(\eta, -\mu) d\eta = 0, \quad (13) \]

\[ \frac{1}{\sqrt{\pi}} \sum_{k=0}^1 A_k \eta_k^3 \exp(-\eta_k^2) + \]

\[ + \frac{1}{\sqrt{\pi}} \sum_{k=0}^1 \eta_k^2 \exp(-\eta_k^2) A(\eta) d\eta = \frac{1}{az_0}. \quad (14) \]

where

\[ \varphi(\mu) = \frac{1}{\sqrt{\pi}} \sum_{k=0}^1 A_k \eta_k^3 \exp(-\eta_k^2) \left( \frac{1}{\eta_k - \mu} - \frac{1}{\eta_k + \mu} \right). \]

Let us transform Eq. (14) setting \( A(-\eta) = -A(\eta) \), that is, expanding the coefficient \( A(\eta) \) to the entire real axis as an odd one. Considering that \( \Phi(-\eta, -\mu) = \Phi(\eta, \mu) \), we reduce Eq. (13) to the form

\[ \varphi(\mu) + \int_{-\infty}^\infty A(\eta) \Phi(\eta, \mu) d\eta = 0, \quad -\infty < \mu < \infty, \]

or after substitution of the eigenfunctions into this equation,

\[ \frac{a}{\sqrt{\pi}} \int_{-\infty}^\infty \eta^3 A(\eta) \exp(-\eta^2) d\eta + \lambda(\mu) A(\mu) + \]

\[ + a \varphi(\mu) = 0, \quad -\infty < \mu < \infty. \quad (15) \]

Let us introduce the auxiliary function

\[ N(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^\infty \eta^3 \exp(-\eta^2) A(\eta) d\eta, \]

whose boundary values, according to the Sokhotskii formulas, obey the equality

\[ N^+(\mu) - N^-(\mu) = 2\sqrt{\pi} \mu^3 \exp(-\mu^2) A(\mu) = \]

\[ = \frac{A(\mu)}{a} [\lambda^+ (\mu) - \lambda^- (\mu)]. \]

With the help of boundary values of the auxiliary function \( N(z) \) and the dispersion function, we reduce the integral equation with Cauchys kernel (15) to the Riemann boundary problem

\[ \lambda^+ (\mu) [N^+(\mu) + \varphi(\mu)] = \lambda^- (\mu) [N^-(\mu) + \varphi(\mu)], \]

whose general solution has the form

\[ N(z) = -\frac{1}{\sqrt{\pi}} \sum_{k=0}^1 A_k \eta_k^3 \exp(-\eta_k^2) \times \]

\[ \int_0^\infty \exp \left(-z_0 \frac{x}{\eta}\right) A(\eta) \Phi(\eta, \mu) d\eta, \]

\[ \int_0^\infty \exp \left(-z_0 \frac{x}{\eta}\right) A(\eta) \Phi(\eta, -\mu) d\eta, \]
\[ \times \left[ \frac{1}{\eta_k - z} - \frac{1}{\eta_k + z} \right] + \frac{C_1 z}{\lambda'(z)} \tag{16} \]

Eliminating the first-order poles at points \( \eta_k \), we obtain

\[ C_1 = -\frac{1}{\sqrt{\pi}} A_k \eta_k^2 \exp(-\eta_k^2) \lambda'(\eta_k) \quad (k = 0, 1). \]

Substituting general solution (16) into the Sokhotskii formula, we obtain the coefficient for the continuous spectrum:

\[ \eta^2 \exp(-\eta^2) A(\eta) = \frac{C_1}{2 \sqrt{\pi i}} \left[ \frac{1}{\lambda^+(\eta)} - \frac{1}{\lambda^-(\eta)} \right]. \]

We now return to Eq. (14) and write it in the form

\[ -\frac{1}{\lambda'(\eta_0)} - \frac{1}{\lambda'(\eta_1)} + \frac{1}{2 \pi i} \int_0^\infty \left[ \frac{1}{\lambda^+(\eta)} - \frac{1}{\lambda^-(\eta)} \right] d\eta = \frac{1}{a z_0 C_1}. \tag{17} \]

After integration of Eq. (17) by the methods of contour integration, we transform the last equation and calculate first the constant \( C_1 \):

\[ C_1 = \frac{1}{a z_0 J(a)}, \quad J(a) = \frac{1}{2 \pi} \int_{-\infty}^{\infty} \frac{dr}{\lambda(i r)} = \frac{1}{\pi} \int_0^{\infty} \frac{d\tau}{\lambda(i \tau)}, \]

and then constants \( A_k \) with the help of Eq. (14): \( A_k = \frac{\sqrt{\pi} \exp(\eta_k^2)}{a z_0 J(a) \eta_k^2 \lambda'(\eta_k)} \quad (k = 0, 1). \)

To calculate the impedance, we consider the electric field derivative

\[ e'(0) = a z_0^2 C_1. \]

\[ \left[ \frac{1}{\eta_0 \lambda'(\eta_0)} + \frac{1}{\eta_1 \lambda'(\eta_1)} - \frac{1}{2 \pi i} \int_0^\infty \left[ \frac{1}{\lambda^+(\eta)} - \frac{1}{\lambda^-(\eta)} \right] d\eta \right]. \]

To integrate this expression, we take advantage of the representation

\[ \frac{1}{\lambda(z)} = \frac{1}{2 \pi i} \int_{-\infty}^{\infty} \left[ \frac{1}{\lambda^+(\eta)} - \frac{1}{\lambda^-(\eta)} \right] \frac{d\eta}{\eta - z} - \sum_{k=1}^{\infty} \frac{2 \eta_k}{(\eta_k^2 - z^2) \lambda'(\eta_k)}. \tag{18} \]

From equality (18) for \( z = 0 \), we obtain

\[ I = -\sum_{k=0}^{\infty} \frac{2 \eta_k}{\eta_k \lambda'(\eta_k)} + \frac{1}{2 \pi} \int_{-\infty}^{\infty} \left[ \frac{1}{\lambda^+(\eta)} - \frac{1}{\lambda^-(\eta)} \right] d\eta. \]

Taking into account the evenness of the integrand, we obtain

\[ \frac{1}{2 \pi} \int_{-\infty}^{\infty} \left[ \frac{1}{\lambda^+(\eta)} - \frac{1}{\lambda^-(\eta)} \right] d\eta = \frac{1}{2} + \frac{1}{\eta_0 \lambda'(\eta_0)} + \frac{1}{\eta_0 \lambda'(\eta_0)}. \]

Now it is clear that the derivative of the electric field is

\[ e'(0) = \frac{a z_0^2}{2} C_1 = \frac{z_0}{2 J(a)} \quad \text{and the expression for the surface impedance is} \]

\[ Z = \frac{8 \pi i \omega l}{c^2 z_0} \left[ \frac{1}{\pi} \int_0^{\infty} \frac{d\tau}{\lambda(i \tau)} \right]^{-1}. \tag{19} \]

Let us express all constants in Eq. (19) in terms of \( \gamma = \frac{\omega}{\omega_p} \) and \( \varepsilon = \frac{\nu}{\omega_p} \), where \( \omega_p = \frac{4 \pi n e^2}{m} \) is the plasma frequency, \( b = \frac{\gamma^2}{(\varepsilon - i \gamma)^2} v_c^2, \ a = -i \frac{\gamma}{(\varepsilon - i \gamma)^2} v_c^2, \) and \( v_c = \frac{1}{\sqrt{\varepsilon}}. \)

![Fig. 1. Modulus of the impedance.](image1)

![Fig. 2. Argument of the impedance.](image2)

Let us now represent dispersion function (10) in the form

\[ \lambda(z) = 1 + \frac{\gamma^2 v_c^2}{(\varepsilon - i \gamma)^2} z^2 + i \frac{\gamma v_c^2}{(\varepsilon - i \gamma)^2} p(z) = \]
An analysis of plots drawn in Figs. 1–4 demonstrates that near the plasma resonance, the modulus of the impedance has a sharp maximum which is not observed in the low-frequency limit or in the theory of normal skin effect, and the argument of the impedance changes abruptly near the resonance.

IV. CONCLUSIONS

The expression for the impedance can be represented as \( Z = RZ_0 \), where \( R = 2\pi\omega\delta c^{-2} \) is the magnitude of the normal skin effect and \( Z_0 \) is the dimensionless impedance. The behavior of the dimensionless impedance modulus is shown in Figs. 1 and 3, and the behavior of its argument is illustrated by Figs. 2 and 4 for \( \varepsilon = 10^{-3} \) and \( \nu_c = 10^{-3} \). The plots in Figs. 3 and 4 are drawn near the resonance, that is, when the parameter \( \gamma \) passes through the value \( \gamma = 1 \) for \( \omega = \omega_p \).

\[
Z = \frac{1}{(\varepsilon - i\gamma)^3} \left[ (\varepsilon - i\gamma)^3 + (\varepsilon - i\gamma)\gamma^2v_c^2z^2 + i\gamma v_c^2\gamma^2v_c^2p(z) \right],
\]

where \( p(z) = \frac{z^3}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-\mu^2)}{\mu - z} d\mu. \)

After substitution of the expression obtained into formula (19) for the surface impedance, we have

\[
Z = \frac{8\pi i\omega l}{c^2 \delta_0} \times \left[ \frac{1}{\pi} \int_0^{\infty} \frac{(\varepsilon - i\gamma)^3 dt}{(\varepsilon - i\gamma)^3 + (\varepsilon - i\gamma)\gamma^2v_c^2t^2 + i\gamma v_c^2\gamma(t)} \right]^{-1}.
\]

[1] A.F. Alexandrov, I.S. Bogdankevich and A.A. Rukhadze. Principles of Plasma Electrodynamics (Springer–Verlag, New York, 1984).

[2] A.V. Latyshev and A.A. Yushkanov. Analytical solutions in the skin effect theory. Monograph. Moscow State Regional University, Moscow, 2008. P. 285. (in Russian).

[3] A.V. Latyshev and A.A. Yushkanov. Analytical solutions of the boundary problem of the kinetic theory. Monograph. Moscow State Regional University, Moscow, 2004. P. 263. (in Russian).

[4] M. Opher, G.J. Morales and J.N. Leboeuf. Krook collisional models of the kinetic susceptibility of plasmas// Phys. Rev. E. 2002 66 (1), 016407, pp. 66 – 75.

[5] I.D. Kaganovich, O.V. Polomarov and C.E. Theodosiou. Resisting the anomalous rf field penetration into a warm plasma// ArXiv: physics/0506135

[6] M. Dressel and G. Grüner. Electrodynamics of Solids. Optical Properties of Electrons in Matter. Cambridge university press. 2002. P. 474.

[7] V.S. Vladimirov and V.V. Zharinov. Equations of mathematical physics. Fizmatlit, Moscow, 2001. P. 400. (in Russian).

[8] K.M. Case and P.M. Zweifel. Linear Transport Theory. Addison – Wesley. 1967.

[9] F.D. Gakhov. Boundary – Value Problems [in Russian]. Nauka, Moscow, 1977.

[10] Zimbovskaya N.A. Fermi–liquid theory of the surface impedance of a metal in a normal magnetic field (2006) Phys. Rev. B 74 035110.

[11] Zimbovskaya N.A. Local geometry of the Fermi surface and the skin effect in layered conductors (1998) JETP 86 6.