Random phase approximation to neutrino energy losses from a relativistic electron-positron plasma.

L. B. Leinson\textsuperscript{1,2} and A. Pérez\textsuperscript{2}
\textsuperscript{1}Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation
RAS, 142190 Troitsk, Moscow Region, Russia
\textsuperscript{2}Departamento de Física Teórica and IFIC,
Universidad de Valencia-CSIC,
Dr. Moliner 50, 46100–Burjassot, Valencia, Spain

26th March 2022

Abstract

The process of $\nu\bar{\nu}$ radiation from a relativistic plasma of electrons and positrons is studied within the Random Phase Approximation.

The neutrino emission from a relativistic electron-positron plasma plays an important role in many astrophysical scenarios, including the processes in degenerate helium cores of red giant stars, cooling of neutron stars and pre-white dwarf interiors. Up to now, the corresponding neutrino energy losses were considered as a simple sum of those caused by plasmon\textsuperscript{1} decays \cite{1}-\cite{8}, electron-positron annihilation \cite{9}-\cite{12}, neutrino photoproduction processes \cite{12}-\cite{14}, and $\nu\bar{\nu}$ bremsstrahlung from electrons \cite{15}-\cite{22}.

Except for the plasmon decay, the above calculations have been performed neglecting electromagnetic correlations among electrons and positrons in the medium. The first attempt to incorporate the collective effects in the annihilation processes was made by Braaten \cite{23}, by including plasma corrections to the electron dispersion relation. It was shown, however, that the plasma effects give no noticeable modification to the neutrino emissivity from the plasma at temperatures and densities where the annihilation processes dominate.

It is of interest, however, to take into account electromagnetic interactions, since the interference between some of the above particular processes, caused by the plasma polarization, can lead to non-trivial phenomena \cite{21}. To incorporate the collective plasma effects, it is convenient to use the formalism of correlation functions, instead of considering particular neutrino emitting processes. In this case, the differential probability of the neutrino-pair emission is given by the imaginary part of the forward scattering amplitude of the neutrino pair, as shown by the following diagram

\textsuperscript{1}For brevity we use the term "plasmons" both for the transverse and longitudinal eigen modes of the plasma oscillations.
This diagram is the lowest order in the weak interaction, but the internal (electron) part represents the sum of all polarization graphs, which begin and end at the weak vertices, and include all possible electromagnetic interactions inside.

In what follows we use the Standard Model of weak interactions, the system of units $\hbar = c = 1$ and the Boltzmann constant $k_B = 1$. The fine-structure constant is $\alpha = e^2/4\pi = 1/137$.

For the electron energies under consideration, the in-vacuum weak interaction of electrons with the neutrino field can be written, in a point-like current-current approach, as $H_{\text{eff}} = -\left(G_F/\sqrt{2}\right) j^\mu J_\mu$, where $G_F$ is the Fermi coupling constant, and $j^\mu = \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu$ is the neutrino current. The vacuum weak current of the electrons is of the standard form, $J_\mu = \bar{\psi} \Gamma_\mu \psi$, where, $\psi$ represents the electron field, and the weak electron vertex,

$$\Gamma_\mu \equiv C_V \gamma_\mu - C_A \gamma_\mu \gamma_5,$$

includes the vector and axial-vector terms; $C_V = \frac{1}{2} + 2\sin^2 \theta_W$, $C_A = \frac{1}{2}$ stand for electron neutrinos, whereas $C_V' = -\frac{1}{2} + 2\sin^2 \theta_W$, $C_A' = -\frac{1}{2}$ are to be used for muon and tau neutrinos; $\theta_W$ is the Weinberg angle.

The total energy which is emitted into neutrino pairs per unit volume and time is given by the following formula:

$$Q = \frac{G_F^2}{2} \sum_\nu \int \frac{2Im \left[ Tr \left( j^\mu j^{\mu*} \right) \Pi_{\mu\nu} (-\omega, -k) \right]}{e^{\omega/T} - 1} \frac{d^3 k_1}{2\omega_1 (2\pi)^3} \frac{d^3 k_2}{2\omega_2 (2\pi)^3}, \quad (2)$$

where $\Pi_{\mu\nu} (\omega, k)$ represents the exact retarded weak polarization tensor of the plasma, and the integration goes over the phase space volume of neutrinos and antineutrinos of total energy $\omega = \omega_1 + \omega_2$ and total momentum $k = k_1 + k_2$. The symbol $\sum_\nu$ indicates that summation over the three neutrino types has to be performed, with the corresponding values of $C_V$ and $C_A$, as explained above.

One can simplify this equation by inserting $\int d^4K \delta^{(4)} (K - k_1 - k_2) = 1$, and making use of the Lenard’s integral

$$\int \frac{d^3k_1}{2\omega_1} \frac{d^3k_2}{2\omega_2} \delta^{(4)} (K - k_1 - k_2) \text{Tr} (j_\mu j^\mu) = \frac{4\pi}{3} (K_\mu K_\nu - K^2 g_{\mu\nu}) \theta (K^2) \theta (\omega), \quad (3)$$

where $\theta (x)$ is the Heaviside step function. We then obtain

$$Q = \frac{G_F^2}{48\pi^5} \sum_\nu \int \omega \frac{(K_\mu K_\nu - K^2 g_{\mu\nu}) \text{Im} \Pi^{\mu\nu}}{\exp \left( \frac{\omega}{T} \right) - 1} \theta (K^2) \theta (\omega) d\omega d^3k \quad (4)$$

with $K = (\omega, k)$. 

The relevant input for this calculation is, thus, the weak polarization tensor, and the desired degree of accuracy is determined by the approximations made in evaluating this quantity. In contrast to quantum electrodynamics, where there is only one parameter in the perturbation series, $\alpha$ (or $e^2/v$, $v$ being the particle velocity), the plasma is characterized by a few additional parameters depending on the temperature and the density. For the problem under consideration, the most important parameter is the ratio of the plasma frequency to the temperature $\omega_p/T$. In the high-temperature limit, $\omega_p^2/T^2 \ll 1$, the plasma polarization effects can be neglected, while at moderate and low temperatures, $\omega_p^2/T^2 \gtrsim 1$, the plasma polarization must be necessarily taken into account. In what follows we show that the standard calculation of the neutrino energy losses due to $e^+e^-$ annihilation, as described before, are valid, strictly speaking, only in the high-temperature limit, while the intermediate and low temperatures are also typical for applications. To include the plasma effects we use the Random Phase Approximation (RPA) to the weak polarization tensor of the plasma. As we will see, this approach substantially improves the energy losses due to $e^+e^-$ annihilation and correctly reproduces the neutrino energy losses caused by plasmon decays, but it does not allow to describe neither the photoproduction of neutrino pairs nor the neutrino bremsstrahlung from electrons because the latter processes are of the next to RPA order in the fine structure constant.

To introduce some notations to be used further, we begin with the lowest (zero) order in $\alpha$ approximation to the polarization tensor, which is given by the one-loop diagram:

\[ \Gamma^R \Gamma^V \approx \Gamma^R \Gamma^V \]

This approach assumes that the net negative electric charge in a hot relativistic plasma of electrons is cancelled by a uniform background of ions.

By using the above expression for the weak vertex $\Gamma^\mu$ of the electron, one can write the lowest order polarization tensor as follows:

\[ \Pi_{\mu\nu}^0 (\omega, \mathbf{k}) = C^2_V \Pi_{VV}^{\mu\nu} (\omega, \mathbf{k}) + C^2_A \Pi_{AA}^{\mu\nu} (\omega, \mathbf{k}) + 2C_V C_A \Pi_{AV}^{\mu\nu} (\omega, \mathbf{k}). \quad (5) \]

Here $\Pi_{VV}^{\mu\nu}$, $\Pi_{AV}^{\mu\nu}$, and $\Pi_{AA}^{\mu\nu}$ are the retarded vector-vector, axial-vector, and axial-axial one-loop polarizations, respectively, calculated for a fixed temperature $T$ and chemical potential $\mu$ of the plasma. We use the exact one-loop polarization functions, obtained with the Matsubara’s technique. To save place in this short letter, we do not show these well-known expressions.

To specify the components of the polarization tensors, we select a basis constructed from the following orthogonal four-vectors $h^\mu \equiv (\omega, \mathbf{k})/\sqrt{K^2}$ and $l^\mu \equiv (k, \mathbf{n})/\sqrt{K^2}$, where the space-like unit vector $\mathbf{n} = \mathbf{k}/k$ is directed along the space component $k$ of the transferred 4-momentum $K$. Then the transversal (with respect to $\mathbf{k}$) basis tensor can be chosen as $L^{\rho\mu} \equiv -l^\rho h^\mu$. The transverse components of $\Pi^{\rho\mu}$ have a tensor structure proportional to the tensor $T^{\rho\mu} \equiv (g^{\rho\mu} - h^\rho h^\mu - L^{\rho\mu})$, where $g^{\rho\mu}$ is the signature tensor.

In this basis, the vector-vector polarization tensor has the following form

\[ \Pi_{VV}^{\rho\mu} (K) = \pi_l (K) L^{\rho\mu} + \pi_t (K) T^{\rho\mu}, \quad (6) \]
where the longitudinal polarization function is defined as 
\[ \pi_l = (1 - \omega^2/k^2) \Pi_{nV}^{(0)} \]
and the transverse polarization function is found to be 
\[ \pi_t = (g_{\mu\nu}\Pi^{(0)}_{\mu\nu} - \pi_l)/2. \]

The axial-vector polarization has to be an antisymmetric tensor. In the rest frame of the plasma, it can be written as
\[ \Pi^{\rho\mu}_{AV}(K) = \Pi^{\rho\mu}_{VA}(K) = \pi_{AV}(K) iK_{\lambda} \epsilon^{\rho\mu\lambda0}, \]
where \( \epsilon^{\rho\mu\lambda0} \) is the completely antisymmetric tensor \( (\epsilon^{0123} = +1) \) and \( \pi_{AV}(K) \) is the axial-vector polarization function of the medium. As for the axial term, it must be a symmetric tensor. The most general expression for this tensor is, therefore
\[ \Pi^{\mu\nu}_{A}(K) = \pi_l(K) L^{\mu\nu} + \pi_t(K) T^{\mu\nu} + \pi_A(K) g^{\mu\nu}. \]

Using Eqs. (5 - 8) one can easily obtain from Eq. (4) the zero order (in \( \alpha \)) expression for the neutrino emissivity:
\[ Q_0 = -\frac{G_F^2}{48\pi^3} \sum_{\nu} \int_{2m}^{\infty} \frac{d\omega}{\omega} \exp \left( \frac{\omega}{\omega - 4m^2} \right) \delta^3 k K^2 \times \left[ C_{V}^2 (\text{Im} \pi_l + 2\text{Im} \pi_t) + C_{A}^2 (\text{Im} \pi_t + 2\text{Im} \pi_t + 3\text{Im} \pi_A) \right]. \]

In this expression, integration goes over the domain of time-like momentum transfer, in agreement with the total energy and momentum of the outgoing neutrino pair. In this case, the imaginary part of the lowest-order polarization functions is caused by the creation and annihilation of the \( e^+e^- \) pairs in the plasma, and exists only for \( K^2 > 4m^2 \).

According to the unitarity theorem, the diagrams for any particular weak processes, for a given approximation to the weak polarization tensor, can be obtained by cutting the forward scattering amplitude of the neutrino pair across the lines of the intermediate states, as shown by the dashed line:

![Diagram](image)

The matrix elements obtained in this way correspond exactly to those used by previous authors in order to calculate the neutrino energy losses due to annihilation of \( e^+e^- \) pairs. Therefore, there is no need to proceed with the analysis of Eq. (9). From the optical theorem, it is clear that this equation yields the same neutrino emissivity obtained by Dicus [12] with the aid of the above matrix elements:
\[ Q_0 = \frac{G_F^2 m^9}{18\pi^3} \sum_{\nu} \left[ (7C_{V}^2 - 2C_{A}^2) \left( G_0^{-} G_{1/2}^{+} + G_0^{+} G_{-1/2}^{-} \right) \right. \]
\[ + 9C_{V}^2 \left( G_0^{+} G_{1/2}^{+} + G_0^{+} G_{1/2}^{-} \right) + (C_{V}^2 + C_{A}^2) \left( 4G_1^{+} G_{1/2}^{+} + 4G_1^{+} G_{-1/2}^{-} \right) \]
\[ - G_1^{-} G_{1/2}^{+} - G_0^{-} G_{1/2}^{+} - G_1^{+} G_{1/2}^{+} - G_1^{+} G_{-1/2}^{-} \right]. \]

\[ \text{(10)} \]

\[ ^2 \text{We have checked this explicitly.} \]
In this expression, the functions $G_{n}^{\pm} (\lambda, \nu)$ with
\[
\lambda = \frac{T}{m}, \quad \nu = \frac{\mu}{T}
\] (11)
are defined as follows
\[
G_{n}^{\pm} (\lambda, \nu) \equiv \lambda^{3+2n} \int_{\lambda^{-1}}^{\infty} \frac{dx}{x^{2n+1}} \frac{x^{2} - \lambda^{-2}}{\exp(x \pm \nu) + 1}. \tag{12}
\]

How accurate is this approach? If the polarization effects are important, the minimal approximation to the polarization tensor for a Coulomb system of particles requires summation of all the ring diagrams. The well known example to this is the photon propagator in the plasma. It is represented by an infinite sum of ring diagrams and has to be found from the Dyson's equation, depicted graphically as

Here, the thin dashed-line represents $D_{0} (K)$ - the photon propagator in vacuum. The solution to this equation is also well known: it consists on the sum of the longitudinal and transverse terms
\[
D_{\lambda \rho} (K) = D_{l} (K) L_{\lambda \rho} + D_{t} (K) T_{\lambda \rho} \tag{13}
\]
with
\[
D_{l} (K) = \frac{1}{K^{2} - \epsilon^{2} \pi_{l} (K)}, \quad D_{t} (K) = \frac{1}{K^{2} - \epsilon^{2} \pi_{t} (K)}. \tag{14}
\]

With the aid of this in-medium photon propagator, the minimal approach to the weak polarization tensor reduces to the sum of the following two diagrams,

where the thick dashed line stands for the in-medium photon propagator $D^{\rho \lambda} (K)$, as given by Eqs. (13), (14). In this way, we have collected the infinite number of ring diagrams in the weak polarization tensor. As it is well known, this corresponds to RPA. The irreducible polarization insertions are taken here in the one-loop approximation, therefore this weak polarization tensor does not allow to describe neither the photoproduction of neutrino pairs nor the neutrino bremsstrahlung from electrons. The latter processes, caused by the photon exchange among the in-medium particles, appear due to electromagnetic corrections, in the irreducible polarization insertions, as shown in the following diagrams

...
These corrections are proportional to fine structure constant. In contrast, the RPA corrections, described above, contribute through the parameter $\omega_p/T$, and, at low temperatures, can be of the same order as the main terms\(^3\). Therefore, such corrections to the irreducible polarization insertions are out of scope of our consideration. Moreover, in the RPA, we shall omit all extra terms which are simply proportional to $\alpha$.

Then, instead of Eq. (5), we have

$$\Pi^{\mu
\nu}_{\text{RPA}} = C_V^2 \left( \Pi^{\mu
\nu}_{VV} + e^2 \Pi^{\mu
\lambda}_V D_{\lambda\rho} \Pi^{\rho
\nu}_V \right) + C_A^2 \left( \Pi^{\mu
\nu}_{AA} + e^2 \Pi^{\mu
\lambda}_A D_{\lambda\rho} \Pi^{\rho
\nu}_A \right) + 2C_V C_A \left( \Pi^{\mu
\nu}_{AV} + e^2 \Pi^{\mu
\lambda}_V D_{\lambda\rho} \Pi^{\rho
\nu}_A \right).$$  \hspace{1cm} (15)

In RPA we deal with both weak interactions of the electrons with the neutrino field, and electromagnetic interactions of the electrons with the plasma background. Therefore it is convenient to introduce short notations for the electromagnetic polarization functions:

$$\Pi_L = 4\pi\alpha k^2 / \omega^2 - k^2 \pi_l(\omega, k),$$  \hspace{1cm} (16)

$$\Pi_T = 4\pi\alpha \pi_t(\omega, k),$$  \hspace{1cm} (17)

$$\Pi_{AV} = 4\pi\alpha \pi_{AV}(\omega, k)$$  \hspace{1cm} (18)

By inserting Eq. (15) into Eq. (4), and after a lengthy (although straightforward) calculation, we obtain the RPA formula for the $\nu\bar{\nu}$ emissivity from the plasma, as consisting on the axial and vector contributions:

$$Q_{\text{RPA}} = Q_A + Q_{LV} + Q_{TV} + Q_{TA},$$  \hspace{1cm} (19)

where the axial term $Q_A$ is given by the integral over the domain $\omega^2 > k^2 + 4m^2$, compatible to the kinematics of creation and annihilation of the $e^+e^-$ pair of total energy $\omega$ and total momentum $k$.

$$Q_A = \frac{G_F^2}{12\pi^4} \sum_\nu C_A^2 \int_{2m}^{\infty} \frac{d\omega}{\exp \left( \frac{\omega}{T} \right) - 1} \int_0^{\sqrt{\omega^2 - 4m^2}} dk \times k^2 (\omega^2 - k^2) \left( \text{Im}\pi_l + 2\text{Im}\pi_t + 3\text{Im}\pi_A \right).$$  \hspace{1cm} (20)

The remaining contribution to the neutrino energy losses consists on the longitudinal and transverse parts, as given by the following integrals

$$Q_{LV} = -\frac{G_F^2}{48\pi^3\alpha} \sum_\nu C_V^2 \int_0^{\infty} \frac{d\omega}{\exp \left( \frac{\omega}{T} \right) - 1} \int_0^\omega dk \times k^4 (\omega^2 - k^2)^2 \times \frac{\text{Im}\Pi_L}{(k^2 - \text{Re}\Pi_L)^2 + (\text{Im}\Pi_L)^2},$$  \hspace{1cm} (21)

\(^3\)Just for the same reason, the well known Eqs. (14), as obtained in RPA, perfectly describe the plasma polarization in the photon propagator.
\[ Q^T = \frac{G_F^2}{24 \pi^3 \alpha} \sum_\nu C^2_\nu \int_0^\infty \frac{d\omega}{\exp \left( \frac{\omega}{T} \right) - 1} \int_0^\omega dk k^2 (\omega^2 - k^2)^3 \]
\[ \times \frac{\Im \Pi_T}{(\omega^2 - k^2 - \Re \Pi_T)^2 + (\Im \Pi_T)^2}, \]
\[ Q^L = -\frac{2G_F^2}{3\pi^3} \sum_\nu C^2_\nu \int_0^\infty d\omega \frac{\omega}{\exp \left( \frac{\omega}{T} \right) - 1} \int_0^\omega dk k^2 (\omega^2 - k^2)^3 \]
\[ \times \left[ \left( (\Re \pi_{AV})^2 - (\Im \pi_{AV})^2 \right) \frac{\Im \Pi_T}{(\omega^2 - k^2 - \Re \Pi_T)^2 + (\Im \Pi_T)^2} \right. \]
\[ + 2 \Re \pi_{AV} \Im \pi_{AV} \frac{(\omega^2 - k^2 - \Re \Pi_T)}{(\omega^2 - k^2 - \Re \Pi_T)^2 + (\Im \Pi_T)^2} \right], \]
\[ \text{The integrand in these contributions is proportional to the spectral function of the in-medium longitudinal and transverse photons} \]
\[ A_L (\omega, k) = -\frac{1}{\pi} \frac{\Im \Pi_L}{(k^2 + \Re \Pi_L)^2 + (\Im \Pi_L)^2}, \]
\[ A_T (\omega, k) = -\frac{1}{\pi} \frac{\Im \Pi_T}{(\omega^2 - k^2 - \Re \Pi_T)^2 + (\Im \Pi_T)^2}, \]
\[ \text{and the integration goes over the domain } \omega^2 > k^2. \]
\[ \text{To analyze this part of the neutrino energy losses it is reasonable to divide this domain of } \omega \text{ and } k \text{ into two parts.} \]
\[ \text{The first one corresponds to } 0 < \omega^2 - k^2 < 4m^2, \text{ where the imaginary part of the polarizations vanishes. In this case, the spectral functions of the in-medium photons reduce to } \delta \text{-functions} \]
\[ \lim_{\Im \Pi_L \to 0} A_L (\omega, k) = \frac{1}{k^2} \delta \left( 1 - \frac{1}{k^2} \Pi_L(\omega, k) \right), \]
\[ \lim_{\Im \Pi_T \to 0} A_T (\omega, k) = \delta (\omega^2 - k^2 - \Pi_T(\omega, k)). \]
\[ \text{By performing the integral over } d\omega, \text{ one can easily show (see the Appendix in [22]), that the integration over the domain } 0 < \omega^2 - k^2 < 4m^2 \text{ yields the neutrino-pair emissivity due to the decay of the transverse and longitudinal plasmons as considered in [3]. We shall not consider these particular terms.} \]
\[ \text{Consider now contributions from the second domain, } \omega^2 - k^2 > 4m^2, \text{ where the imaginary part of the polarizations does not vanish. The axial-vector contribution [23] represents a small } \alpha \text{ correction, which has to be omitted since we do not include the corresponding corrections in the irreducible polarization insertions. Thus, in RPA, the neutrino energy losses due to the annihilation processes are the sum of the axial and vector contributions} \]
\[ Q_{\text{RPA}}^{\text{annih}} = Q_A + Q_V^L + Q_V^T. \]
\[ \text{In RPA, the neutrino energy losses due to the annihilation processes are the sum of the axial and vector contributions} \]
\[ \left[ \tilde{C}^L_\nu (\omega, k) \Im \pi_l + 2 \tilde{C}^T_\nu (\omega, k) \Im \pi_t + C^A_\nu (\Im \pi_l + 2 \Im \pi_t + 3 \Im \pi_A) \right]. \]
with
\[
\tilde{C}_L^V (\omega, k) = \frac{k^4}{(k^2 - \text{Re}\Pi_L)^2 + (\text{Im}\Pi_L)^2} C^2_V 
\]
(30)
\[
\tilde{C}_T^V (\omega, k) = \frac{(\omega^2 - k^2)^2}{(\omega^2 - k^2 - \text{Re}\Pi_T)^2 + (\text{Im}\Pi_T)^2} C^2_V 
\]
(31)

In the high-temperature regime \(\omega_p^2 / T^2 \ll 1\), or in the low density regime \(\omega^2 / m^2 \ll 1\), when the plasma polarization effects can be neglected, one has \(\omega^2 - k^2 \sim T^2, m^2\) and \(\Pi_L, T \sim \omega_p^2\). Eqs. (29) and Eqs. (30)-(31) completely reproduce the one-loop energy losses, as given by Eq. (9). Indeed, in the above limits, we have
\[
\lim_{\omega^2 / m^2 \to 0} \tilde{C}_L^V (\omega, k) = \lim_{\omega^2 / m^2 \to 0} \tilde{C}_T^V (\omega, k) = C^2_V. 
\]

We have considered the Random Phase Approximation to neutrino energy losses from a relativistic electron-positron plasma. The main results of our approach are Eqs. (29), (30) and (31), representing the corrected neutrino energy losses due to annihilation of electrons and positrons. The well-known energy losses due to the decay of real (on-shell) photons are also well reproduced in the RPA.

Although the RPA is a good approximation to the neutrino energy losses due to annihilation of electrons and positrons, and reproduces accurately the neutrino energy losses caused by the plasmon decays, it is not yet sufficient for a complete description of the neutrino energy losses from a hot plasma. Indeed, the RPA does not allow to describe, for example, the photoproduction of neutrino pairs, nor the neutrino bremsstrahlung from electrons, which require the next (in \(\alpha\)) approach to the weak polarization tensor of the medium.

**Acknowledgments**

This work has been supported by Spanish Grants AYA2004-08067-C01, FPA2005-00711 and GV2005-264.

**References**

[1] J. B. Adams, M. A. Ruderman, & C.-H. Woo, Phys. Rev. 129 (1963) 1383.
[2] E. Braaten, Phys. Rev. Lett. 66 (1991) 1655.
[3] E. Braaten and D. Segel, Phys.Rev. D48 (1993) 1478.
[4] H. Munakata, Y. Kohyama, & N. Itoh, ApJ 296 (1986) 197.
[5] Y. Kohyama, N. Itoh, & H. Munakata, ApJ 310 (1986) 815.
[6] P. J. Schinder, D. N. Schramm, P. J. Wiita, S. H. Margolis, & D. L. Tubbs, ApJ 313 (1987) 531.
[7] N. Itoh, T. Adachi, M. Nakagawa, Y. Kohyama, & H. Munakata, ApJ 339 (1989) 34; 360 (1990) 741.
[8] S. Ratkovic, S. I. Dutta, and Maddapa Prakash, Phys. Rev. C67:123002, 2003
[9] M. Y. Chiu, & P. Morrison, Phys. Rev. Lett. 5 (1960) 573.
[10] M. Y. Chiu, & R. C. Stabler, Phys. Rev. 122 (1961) 1317.
[11] M. Y. Chiu, Phys. Rev. 123 (1961) 1040.
[12] D. A. Dicus, Phys. Rev. D6 (1972) 941.
[13] G. Beaudet, V. Petrosian and E. E. Salpeter, Phys. Rev. 154 (1966) 1445.
[14] G. Beaudet, V. Petrosian and E. E. Salpeter, ApJ 150 (1967) 979.
[15] Georg G. Festa and Malvin A. Ruderman, Phys. Rev., 180 (1969) 1227.
[16] E. Flowers, ApJ, 180 (1973) 911.
[17] N. Itoh, Y. Kohyama, N. Matsumoto, M. Seki, ApJ, 285 (1984) 304.
[18] N. Itoh, H. Hayashi, A. Nishikawa, Y. Kohyama, ApJ Suppl., 102 (1996) 411.
[19] C. J. Pethick and V. Thorsson, Phys. Rev. Lett., 72 (1994) 1964.
[20] P. Haensel, A. D. Kaminker, and D. G. Yakovlev, Astron. Astrophys. 314 (1996) 328.
[21] L. B. Leinson, Phys. Lett. B469, (2000) 166.
[22] L. B. Leinson, A. Perez, Nucl. Phys. B 597 (2001) 279.
[23] E. Braaten, ApJ 392 (1992) 70.