Low-intensity pressure waves in a stratified bubbly liquid

U O Agisheva and M N Galimzyanov
Mavlyutov Institute of Mechanics, Ufa Federal Research Centre, Russian Academy of Sciences, 71 Prospect Oktjabrya, Ufa, Russia
E-mail: agisheva_u@mail.ru, monk@anrb.ru

Abstract. In this work, the features of the dynamics of two-dimensional pressure waves of weak intensity in a layered bubbly liquid are investigated. A bubbly liquid, piecewise-inhomogeneous in volume content, was used as the calculated medium. The influence of the width of the bubble layer on the propagation dynamics of a pulse signal is analyzed. The possibility of the formation of pressure peaks near the boundary between the layers is shown.

1. Introduction

The main studies on the dynamics of one-dimensional waves in two-phase vapor-gas-droplet media are described in detail in [1–3]. In the first papers on the modeling of two-dimensional waves in bubble liquid [4, 5] several statements of problems on the effect of a pulsed signal on a bubble liquid and a bubble layer in a pure liquid were considered. Based on numerical studies, criteria for strengthening and damping of the signal by means of a bubble region of finite dimension were established.

In contrast to the first papers, works where a wide-range equation of state of water and vapor was used [6] appeared later. The authors considered the effect of pressure, vapor-gas content on the speed of sound in the gas-liquid mixture. On the basis of the Rankine-Hugoniot relations, the parameters of incident and reflected shock waves in the gas-liquid medium are obtained for the cases of isothermal, adiabatic and shock compression of the gas component.

In the series of works [7–10] the dynamics of nonlinear waves in a tube in an axisymmetric formulation is studied under various conditions for the distribution of bubbles over a cross section in the form of a ring or an axial tube; the features of the propagation of pressure waves in the ring layer with various volume contents of bubbles were studied; the process of a soliton rise when exposed to a rigid impactor at the end of the ring layer is described; the influence of the initial exposure method and the volumetric content of bubbles at the time of detonation is investigated.

The dynamics and damping of pressure waves in a vertical shock tube with a nonuniform distribution of bubbles over its cross section were studied experimentally in [11] and it was shown that the nonuniformity bubble distribution leads to an improvement in the damping properties of the bubble zone.

Work [12] consider the problems of initial wave strengthening by a cloud of the bubbles in the with further wave reemission. Further in the work [13] when researching the structure and dynamics of the pressure waves generated by a bubble system of a ”cord” form was shown that
excitation of oscillations of the bubble zone results in formation of a quasi-steady shock wave in the cord and in the ambient liquid.

In this study we will consider waves in a stratified bubbly liquid taking the two-dimensional effects into account. We present the results for the two-dimensional-wave dynamics in a bubble zone with a piecewise nonuniform bubble volume fraction.

2. Governing equations and dispersive analysis

Let a the bubble region bounded be located in a channel filled with a fluid (the bubble-zone lengthwise dimension is much greater than the transverse dimension) (Figure 1). We will consider two-dimensional wave disturbances. These disturbances may result, for example, from the action of a planar shock on the fluid with a finite bubble zone or from the action of a boundary pressure, nonuniform in the y coordinate ($p = p_0(t, y)$ for $x = x_0$).

In describing the bubbly fluid motion, we will assume that (i) in each elementary volume all the bubbles are spherical and monodisperse, (ii) viscosity and heat conduction are important only in the phase interaction, and (iii), in particular, bubble fragmentation and coalescence are absent.

On the basis of these assumptions, in the single-velocity approximation we can formulate the system of macroscopic equations for the mass, bubble number, momentum, and the pressure in the bubble [2]:

$$\begin{aligned}
&\frac{d\rho_l}{dt} + \rho_l \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \quad (i = l, g), \\
&\frac{dn}{dt} + n \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0,
\end{aligned}$$

$$\begin{aligned}
&\rho \frac{du}{dt} + \frac{\partial \rho_l}{\partial x} = 0, \\
&\rho \frac{dv}{dt} + \frac{\partial \rho_l}{\partial y} = 0, \\
&\rho = \rho_g + \rho_l, \\
&\frac{dp_l}{dt} = -\frac{3\gamma p_g}{a} w - \frac{3(\gamma - 1)}{a} q,
\end{aligned}$$

(1)

Here, $a$ is the bubble radius, $\gamma$ is the gas specific heat ratio, $p_i$ are the phase pressures, $\rho_i^0$ are the real phase densities, $\alpha_i$ are the phase volume fractions, $q$ is the heat transfer rate, $n$ is the bubble number per unit volume, and $w$ is the bubble radial velocity. The velocity components $u$ and $v$ correspond to the motion in the $x$ and $y$ coordinates. The subscripts $i = l, g$ denote the parameters of the liquid and gaseous phases.

In describing the radial motion, in accordance with the correction proposed in [14] we assume that $w = w_R + w_A$, where $w_R$ is described by the Rayleigh-Lamb equation and $w_A$ is determined

![Figure 1](image_url)

**Figure 1.** The calculation domain: $l_y$ is the bubble channel width, $L_x$ is the channel length and $L_y$ is the channel width.
from the solution of the problem of spherical unloading for a sphere of radius $a$ in a carrier fluid in the acoustic approximation:

$$\frac{a dw_R}{dt} + \frac{3}{2} w_R^2 + 4\nu \frac{w_R}{a} = \frac{(p_g - p_l)}{\rho_i^0}, \quad w_A = \frac{p_g - p_l}{\rho_i^0} \frac{1}{\sqrt{3}}.$$

(2)

Here, $\nu$ is the fluid viscosity.

We assume that the fluid is acoustically compressible and the gas is calorically perfect:

$$p_l = p_0 + C_l^0 (\rho_i^0 - \rho_l^0), \quad p_g = \rho_g^0 R T_g.$$

(3)

Here, $R$ is the gas constant. Here and in what follows, the subscript (0) denotes the parameters relating to the initial undisturbed state.

The heat flux is determined from the approximate formula:

$$q = Nu \lambda_g \frac{T_g - T_0}{2a}, \quad T_g = \frac{p_g (\frac{a}{\rho_0})^3}{\rho_l^0 \lambda_g \rho_0^0},$$

$$Nu = \sqrt{Pe}, \quad Pe \geq 100, \quad Nu = 100, \quad Pe < 100,$$

$$Pe = 12(\gamma - 1) \frac{T_0}{|T_g - T_0|} \frac{a |w|}{\kappa_g}, \quad \kappa_g = \frac{\lambda_g}{c_g \rho_g}.$$

Here, $T_0 = const$ is the fluid temperature, $\lambda_g$ is the thermal conductivity, and $Nu$ is the Nusselt number.

The system of equations formulated makes possible the adequate description of the dynamics of waves with fairly “steep” segments, when the bubble compression is determined not only by the carrier-fluid radial inertia but also by the acoustic unloading on the bubbles and, hence, the fluid compressibility. In addition, in the particular case $\alpha_g = 0$ this model reduces to the wave equation for an acoustically compressible fluid. In studying the wave interaction in a “pure” fluid with a bubble region, this makes it possible to use shock-capturing calculation methods.

3. Numerical method

For the numerical analysis of the problem of wave evolution in a fluid with a bubble zone, it is convenient to use the system of equations (1)–(3), written in Lagrangian variables. In particular, this is because in the Lagrangian coordinates the bubble zone is fixed. From equations, after transformations we obtain the following system in the Lagrangian variables:

$$\frac{du}{dt} = -\frac{1}{T \rho} \left( \frac{\partial p_l}{\partial x} \frac{\partial y}{\partial y} - \frac{\partial p_l}{\partial y} \frac{\partial y}{\partial x} \right), \quad \frac{\partial x}{\partial t} = u,$$

$$\frac{dv}{dt} = -\frac{1}{T \rho} \left( \frac{\partial p_l}{\partial y} \frac{\partial x}{\partial x} - \frac{\partial p_l}{\partial x} \frac{\partial x}{\partial y} \right), \quad \frac{\partial y}{\partial t} = v,$$

$$\frac{\partial p_l}{\partial t} = \frac{C_l^2 \rho_l^0}{1 - \alpha_g} \left[ \frac{3\alpha_g}{a} w - \frac{(\alpha_g + \rho_l^0 / \rho_0^0)}{J} \frac{\partial J}{\partial t} \right],$$

$$\frac{\partial \alpha_g}{\partial t} = \frac{3\alpha_g}{a} w - \frac{\alpha_g J}{J} \frac{\partial J}{\partial t}, \quad \frac{\partial \rho_l}{\partial t} = -\frac{3\gamma p_g}{a} w - \frac{3(\gamma - 1)}{a} q,$$

$$\frac{\partial \frac{a}{\partial t}}{\partial t} = w = w_R + w_A,$$

$$\frac{\partial w_R}{\partial t} = \left[ \frac{p_g - p_l}{\rho_i^0} - \frac{3}{2} w_R^2 - 4\nu \frac{w_R}{a} \right] \frac{1}{a}, \quad w_A = \frac{p_g - p_l}{\rho_i^0} \frac{1}{\sqrt{3}}.$$

(4)
\[ J = \frac{\partial x}{\partial x_0} \frac{\partial y}{\partial y_0} - \frac{\partial x}{\partial x_0} \frac{\partial y}{\partial y_0}, \quad \frac{\partial J}{\partial t} = \frac{\partial u}{\partial x_0} \frac{\partial y}{\partial y_0} - \frac{\partial u}{\partial x_0} \frac{\partial y}{\partial y_0} + \frac{\partial x}{\partial x_0} \frac{\partial u}{\partial y_0} - \frac{\partial x}{\partial x_0} \frac{\partial u}{\partial y_0}. \]

Here, \( x_0 \) and \( y_0 \) are the Lagrangian variables, which are taken equal to the initial values of the Eulerian coordinates, and \( J \) is the Jacobian of the transformation from Lagrangian to Eulerian coordinates.

The system (4) is solved numerically using an implicit scheme. It is not necessary to introduce an artificial viscosity since, due to the taking into account of the interphase heat transfer and acoustic unloading, the system above is naturally dissipative.

The calculation domain is assumed to be rectangular and, in the calculations, the boundary conditions on the domain boundaries (\( x_0 = 0, x_0 = L_x, 0 < y_0 < L_y; y_0 = 0, y_0 = L_y, 0 < x_0 < L_x \)) are the same as on a solid wall.

4. Numerical results

We illustrate below the results of a numerical experiment on the evolution of a wave pulse given in the form

\[ p^0(t, y) = p_0 + \Delta p. \]

in bubble liquid (Figure 1). For all the figures presented, the calculations were performed for the following parameters of the mixture and the pulse: \( 0 < l_y < 0.5 \text{ m}, \alpha_0 = 10^{-3} \text{ m}, p_0 = 0.1 \text{ MPa}, T_0 = 300 \text{ K}, \text{ and } \Delta p = 0.3 \text{ MPa}, \lambda_g = 2.6 \cdot 10^{-2} \text{ J/(K-s-kg)}, \rho_0 = 1000 \text{ kg/m}^3, c_g = 1006 \text{ J/(K-kg)}, \rho_g^0 = 1.29 \text{ kg/m}^3, \mu_l = 2 \cdot 10^{-6} \text{ m}^2/\text{sec}, \gamma = 1.4 \) [15]. The pulse impinges on the Lagrangian boundary of the fluid \( x_0 = 0 \).

Figure 2 shows the numerical results for the evolution of a wave pulse in a bubble region with a piecewise nonuniform bubble volume fraction in the \( y \) direction, located between two plane parallel walls. The bubble region consists of two layers (in the \( y \) direction) with different initial bubble volume fractions: \( \alpha_g(1) = 10^{-3} \text{ and } \alpha_g(2) = 10^{-4} \). The length of the bubble layer \( l_y \) varied from 0 to 0.5. The calculation scheme are given in Figure 1.

In contrast to the wave patterns in uniform bubbly fluids (Figure 2, 1a–1c and 6a–6c), for nonuniform distributions of the gas volume fraction the pulse-disturbance propagation is accompanied by the formation of transverse pressure profiles with peaks located near the interfaces of the layers (in graphs are indicated by the dashed line). These peaks exceed the original-signal amplitude by about 0.5 MPa (Figure 3, 2–4). This is attributable to the difference in the wave velocities in the layers with different gas volume fractions. Indeed, in the middle layer with \( \alpha_g(1) = 10^{-3} \) the pulse has a velocity \( \approx 420 \text{ m/s} \), which is approximately 3.5 times less than the velocity in the near-wall layers with \( \alpha_g(2) = 10^{-4} \). Hence, due to the expansion of the bottom layer, the wave propagating in it pre-compresses the top layer. Accordingly, when the main wave pulse propagates, in the “pre-compressed” wall layer near the boundaries between the layers the amplitudes are superimposed.

For a complete understanding of the calculation results, figure 3 presents volumetric patterns of pressure distribution in the entire computational domain.

5. Conclusion

The propagation of two-dimensional waves through a bubbly fluid with a piecewise-nonuniform bubble volume fraction is studied numerically.

Pulse propagation through a region with a piecewise-nonuniform bubble volume fraction is accompanied by the formation of transverse pressure profiles with peaks near the interfaces between the layers. This is attributable to the difference in the wave velocity in layers with different gas volume fractions.
Figure 2. Wave pulse evolution in a bubble region with a piecewise-nonuniform bubble volume fraction. The pressure distributions correspond to the instants $t = 0.5$ ms (a), 1.0 ms (b) and 1.5 ms (c). Here 1 – $l_y = 0.5$ m, 2 – 0.4 m, 3 – 0.3 m, 4 – 0.2 m, 5 – 0.1 m, 6 – 0 m
Figure 3. Pressure distribution to the instant $t = 1.0 \text{ ms}$. Here 1 – $l_y = 0.5 \text{ m}$, 2 – 0.4 m, 3 – 0.3 m, 4 – 0.2 m, 5 – 0.1 m, 6 – 0 m

Acknowledgments
The study of Galimzyanov M.N. supported by the state budget for the state task for 2019-2022 (No. 0246-2019-0052). The study of Agisheva U.O. performed by a grant from the Russian Science Foundation (project No. 17-41-020582-r_a)

References
[1] S S Kutateladze and V E Nakoryakov 1984 Heat and Mass Transfer in Gas-Liquid Systems (Novosibirsk: Nauka) (in Russian)
[2] R I Nigmatulin 1990 Dynamics of Multiphase Media (Washington: Hemisphere)
[3] V E Nakoryakov, B G Pokusaev and I R Shreiber 1993 *Wave Propagation in Gas-Liquid Media* (London: CRC Press)
[4] R I Nigmatulin et al 2001 *Doklady Physics* **46** (6) 445–51
[5] M N Galimzyanov et al 2002 *Fluid Dynamics* **37** (2) 139–47
[6] U O Agisheva et al 2013 *Fluid Dynamics* **48** (2) 151–62
[7] A R Bayazitova et al 2013 *Fluid Dynamics* **48** (2) 201–10
[8] A R Bayazitova et al 2013 *Tyumen St. Univ. Herald* **7** 25–32
[9] V S Shagapov et al 2009 *High Temperature* **47** (3) 424–31
[10] R I Nigmatulin et al 2005 *Doklady Physics* **50** (8) 405–8
[11] V E Dontsov et al 2003 *J. Appl. Mechanics and Tech. Physics* **44** (4) 538–42
[12] V K Kedrinskii et al 2001 *Doklady Physics* **46** (12) 856–9
[13] V K Kedrinskii et al 2005 *J. Appl. Mechanics and Tech. Physics* **46** (5) 652–7
[14] R I Nigmatulin et al 1989 *Dokl. Akad. Nauk SSSR* **304** (5) 1077–88
[15] N B Vargaftik 2006 *Ref. Book on Thermophysical Proper. of Gases and Liquids* (Moscow: Nauka) (in Russian)