Complex Relativity: 
Gravity and Electromagnetic Fields

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Abstract

We present new aspects of the electromagnetic field by introducing the natural potentials. These natural potentials are suitable for constructing the first order distortions of the metric tensor of Complex Relativity - the theory combining the General Relativity with the electromagnetic equations. A transition from antisymmetric tensors to the symmetric ones helps to define the natural potentials; their form fits a system of the Dirac matrices and this representation leads to distortion of the metric tensor.

Our considerations have originated from the recent progresses in the asymmetric continuum theories. One version of such theories assumes an existence of the antisymmetric strain and stress fields; these fields originate due to some kind of internal friction in a continuum medium which have elastic bonds related to rotations of the particles.

Key words: Complex Relativity, natural potentials, unified fields.

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1 Introduction

This work, related to the General Relativity and electromagnetic field, was inspired by some recent results in the asymmetric continuum theories including the spin motions; therefore, we shall, first of all, quote the numerous attempts to extend the General Relativity to include the spin motions. The first one, that in the Cartan works [4] was influenced by work by Cosserat brothers [5] in which a moment stress tensor is included in a generalized continuum. A gradual development of the Einstein-Cartan Theory (ECT) started by works of Sciana [18], Kibble [9] and Trautman [35, 36]; for a review see: [7]. Kopczyski [13] has proved that in the ECT the cosmological solutions become free from the singularities leading to the modified Friedmann equation supplemented with the conservation laws for mass and spin [37].

In the XX century, we observed an enormous development of the continuum theories: the micropolar and micromorphic theories were developed basing on the Cosserat brothers’ work (for a review see: [6]); the relations joining the theories of a continuum containing defects (dislocation and disclination densities) with the Riemannian curvature and torsion were considered by Bilby, Bullough and Smith [2], Kondo [10, 12] and followed by Hollander [8], Ben-Abraham [1] and many other authors; (for a review see: [25, 26]); the thermal stresses were found to have the same form as that related to dislocation field [16] and on this basis the thermal effects were included in the continuum with a Riemannian curvature by Kroner [14, 15], Teisseyre [22, 23, 24] and Stojanovic et al. [21]; for a review see: [27]).

Recently the continuum theories have been generalized to the asymmetric form [28, 30, 32] in which an additional constitutive law [19] for the antisymmetric part of stresses, replacing the stress moments, joins the spin motion with a new constant, rotation rigidity modulus, to account for the rotation of the point-grains and propagation of the spin elastic waves. Such waves can exist when the elastic bonds related to the rotation motions of particles are postulated. When, in such a continuum, the material bonds for the displacement neglecting originated deformations, there remain only the rotation fields of spin and twist types; the respective equations appear to have exactly the form of electromagnetic equations. On this basis, the degenerated continuum theory (in which there exist only the spin and twist axial motions) has been considered in the last papers by Teisseyre [30] and Teisseyre, Białecki and Górski [31].
Influenced by these results, we introduce in this paper the natural potentials, as defined in the way suitable for constructing the first order distortions in the metric tensor. A transition from antisymmetric tensors to the symmetric ones helped us to define these natural electromagnetic potentials; their form fits a system of the Dirac matrices and this representation leads to distortions of the metric tensor.

The definitions helped us to propose a generalization of the General Relativity; the new theory - the Complex Relativity - includes, beside gravitation, the electromagnetic equations in a first order approximation.

2 Analogies to Asymmetric Continuum

We consider the physical rotation fields which can be related to the curvature deformations of the complex space. However, we shall mention that an inspiration for this idea has its source in considering the rotation and twist motion in the asymmetric elastic continuum; of course, such motions in an elastic continuum are bounded to some constitutive relations describing the elastic bonds. This is main difference in comparison with motion in the space.

In our approach [34, 29, 3] a homogeneous elastic continuum with the rotation nuclei - of spin and twist type - is supplemented, beside the classical ideal elasticity constitutive law for the symmetric strain-stress relation, by the relation between the rotation and asymmetric stresses; such stresses appear when including in a medium the rotation nuclei. By this supplementary constitutive law for the anti-symmetric fields, we can evade an influence of the Hook law, which, when used as the unique law in the ideal elasticity, rules out an existence of rotation waves. Thus, it comes out that the rotation vibrations can propagate and are not attenuated, unlike as the elastic waves in the ideal elastic continuum.

The twist motion differs from the pure rotation; formally, it is a motion composed of the rotation and the mirror reflection. It presents the simultaneously occurring opposite rotation motions like shear oscillations (some analogy, in a world of the linear displacements, presents a thermal expansion/compression motion differing from the simple displacements), but it can be also related to the axial motions like those of the polarization type, or, when assuming a possibility of material-space curvature, to a bending of the 3D space, (by analogy to the situation of a flat jellyfish with the bending motions (pulsating motions) leading from 2D form to 3D one).
Our approach to the asymmetry of fields follows from the antisymmetric stresses introduced by \[19, 20\] and related to the internal friction caused by the grain motions under friction forces. Note that in the asymmetric continuum also the related asymmetric incompatibility tensors split into symmetric and anti-symmetric parts.

In our former papers \[26, 28, 32\] we have also analyzed the theory of asymmetric continuum with defect distribution (with the dislocation and disclination densities and the densities of rotation nuclei). Special consideration was paid to rotation and twist motions related to the definition of the twist-bend tensor.

The dislocation-stress relations and the equations of motion for symmetric and asymmetric parts of stresses were derived. The obtained relations for elastic fields, given by difference of the total and self-fields, can be split into the self-parts prevailing on the fracture plane and the total parts describing seismic radiation field in a surrounding space. Some applications were shortly discussed.

Finally, we shall note that a more complex deformation field, like that with the dislocation and disclination densities, leads, when applying the material coordinate system, to description of a deformed state in terms of the Riemannian geometry, or even non-Riemannian one \[2, 11\]. This remark applies also to thermal deformation field \[24, 26\].

3 Natural electromagnetic potentials

We have tested many variants of definition for the EM potentials, and finally we propose the following one. We introduce the 3D vector potentials: \(\tilde{A}_s\), \(\hat{A}_s\), and charge-current potentials \(\varphi_s\), \(\psi_s\) instead of the standard 4D vector potential \(A_\mu\). We call them the natural potentials and assume they fulfill the following equations:

\[
\begin{align*}
B_k &= \epsilon_{bks}\tilde{A}_{s,b} \quad \tilde{A}_{s,s} = 0 \\
E_k + \varphi_{,k} &= \epsilon_{bks}\hat{A}_{s,b} \quad \hat{A}_{s,s} = 0 \\
\frac{4\pi}{c} J_k &= \frac{1}{c} \varphi_{,k} + \frac{1}{c} \epsilon_{bks}\psi_{s,b} \\
\varphi_{,kk} &= -4\pi \rho,
\end{align*}
\]

where \(k, b, s \in \{1, 2, 3\}\). An index after a comma denotes differentiation and the summation convention for repeated indices is used. These new natural
potentials yield
\[ \epsilon_{kbs} \tilde{A}_{s,b} - \frac{1}{c} \frac{\partial}{\partial t} \tilde{A}_k = \frac{1}{c} \psi_k \quad \tilde{A}_{b,b} = 0 \]
(5)
\[ \epsilon_{kbs} \hat{A}_{s,b} + \frac{1}{c} \frac{\partial}{\partial t} \hat{A}_k = 0 \quad \hat{A}_{b,b} = 0. \]
(6)
When applying to (5)-(6) the operator \( \epsilon_{ndk} \frac{\partial}{\partial x^d} \), we arrive at the Maxwell equations:
\[ \epsilon_{ndk} B_{k,d} - \frac{1}{c} \frac{\partial}{\partial t} E_n = \frac{4\pi}{c} J_k \quad B_{k,k} = 0 \]
(7)
\[ \epsilon_{ndk} E_{k,d} + \frac{1}{c} \frac{\partial}{\partial t} B_n = 0 \quad E_{k,k} = 4\pi \rho \]
(8)
Now, we construct the complex antisymmetric tensor \( A_{\alpha\beta} \) \((\alpha, \beta \in \{1, 2, 3, 4\})\) for potentials \( \tilde{A}, \hat{A} \). We define \( A_{\alpha\beta} = \tilde{A}_{\alpha\beta} + i\hat{A}_{\alpha\beta} \) by
\[ A_{\alpha\beta} = \begin{bmatrix} 0 & \tilde{A}_3 & -\tilde{A}_2 & -i\hat{A}_1 \\
-\tilde{A}_3 & 0 & \tilde{A}_1 & -i\hat{A}_2 \\
-\tilde{A}_2 & -\tilde{A}_1 & 0 & -i\hat{A}_3 \\
i\hat{A}_1 & i\hat{A}_2 & i\hat{A}_3 & 0 \end{bmatrix} + i \begin{bmatrix} 0 & \hat{A}_3 & -\hat{A}_2 & i\hat{A}_1 \\
-\hat{A}_3 & 0 & \hat{A}_1 & i\hat{A}_2 \\
-\hat{A}_2 & -\hat{A}_1 & 0 & i\hat{A}_3 \\
-i\hat{A}_1 & -i\hat{A}_2 & -i\hat{A}_3 & 0 \end{bmatrix} \]
(9)
or
\[ A_{\alpha\beta} = \begin{bmatrix} 0 & \tilde{A}_3 & -\tilde{A}_2 & -\tilde{A}_1 \\
-\tilde{A}_3 & 0 & \tilde{A}_1 & -\tilde{A}_2 \\
-\tilde{A}_2 & -\tilde{A}_1 & 0 & -\tilde{A}_3 \\
\tilde{A}_1 & \tilde{A}_2 & \tilde{A}_3 & 0 \end{bmatrix} \]
(10)
where \( \tilde{A}_k = \tilde{A}_k + i\hat{A}_k \). The form of equation (9) follows the fact, that these potentials are constructed in a similar manner as the tensor \( f_{\alpha\beta} \) of the EM field is constructed from the EM vector fields \( B_s \) and \( E_s \).
According to (5)-(6) the tensor \( A_{\alpha\beta} \) fulfils the condition
\[ A_{\alpha\beta\beta} = \frac{1}{c} \psi_\alpha \quad \psi_\alpha = \{ \psi_k , 0 \} \quad k = 1, 2, 3. \]
(11)
Comparing this relation with equations for the potentials \( \tilde{A}_s, \hat{A}_s \) we obtain
\[ \Box \tilde{A}_n = -\frac{1}{c} \epsilon_{ndk} \psi_{k,d} \quad \tilde{A}_{b,b} = 0 \]
(12)
\[ \Box \hat{A}_n = \frac{1}{c^2} \frac{\partial}{\partial t} \psi_n \quad \hat{A}_{b,b} = 0 \]
(13)
and when defining
\[ -\frac{1}{c}\epsilon_{ndk}\psi_{k,d} = \tilde{J}_n, \quad \frac{1}{c^2}\frac{\partial}{\partial t}\psi_k = \dot{J}_k \]  
we arrive at
\[ \Box\tilde{A}_n = \tilde{J}_n, \quad \Box\hat{A}_n = \hat{J}_n \]  
(15)

Then, we define tensor \( J_{\alpha\beta} \):
\[ J_{\alpha\beta} = \begin{pmatrix}
0 & \bar{J}_3 & -\bar{J}_2 & -\bar{J}_1 \\
-\bar{J}_3 & 0 & \bar{J}_1 & -\bar{J}_2 \\
\bar{J}_2 & -\bar{J}_1 & 0 & -\bar{J}_3 \\
\bar{J}_1 & \bar{J}_2 & \bar{J}_3 & 0
\end{pmatrix} \]  
(16)

where \( J_n = \tilde{J}_n + i\hat{J}_n \) and we arrive to relation
\[ \Box A_{\alpha\beta} = J_{\alpha\beta}. \]  
(17)

Applying the operator \( \epsilon_{sbn}\frac{\partial}{\partial x_b} \) we get, with the help of (12)-(13),
\[ \Box B_s = \frac{1}{c}\psi_{s,dd} = \frac{4\pi}{c}\epsilon_{spk}J_{k,p} \]  
(18)
\[ \Box E_s = \frac{1}{c^2}\epsilon_{sbn}\frac{\partial}{\partial t}\psi_{n,b} - \Box\varphi_s = -\frac{4\pi}{c^2}\bar{J}_s + 4\pi\rho_s. \]  
(19)

### 4 Natural EM Potential — the Symmetric Tensor

To include the EM field into the Riemannian or non-Riemannian geometry we search for a way how to build a metric tensor which in the first order approximation could describe the gravity and EM fields.

Owing to the fact that for the six potentials \( \tilde{A}_s, \hat{A}_s \) we can introduce the two additional conditions, we define the other set of four potentials:
\[ \bar{N}_n = \{\bar{N}_1, \bar{N}_2, 0\} \]  
and \[ \hat{N}_n = \{\hat{N}_1, \hat{N}_2, 0\}. \]  
(20)
With these conditions we introduce the natural symmetric tensor of potentials:

\[
N_{\alpha\beta} = \tilde{N}_{\alpha\beta} + i \hat{N}_{\alpha\beta} \\
N_s = \tilde{N}_s + i \hat{N}_s
\]  

(21)

\[
N_{\alpha\beta} = \begin{bmatrix}
0 & 0 & -N_2 & -N_1 \\
0 & 0 & N_1 & -N_2 \\
-N_2 & N_1 & 0 & 0 \\
-N_1 & -N_2 & 0 & 0 
\end{bmatrix}
\]

(22)

The bridge relations. We demand that first bridge relation is satisfied:

\[
\epsilon_{kbs} \tilde{N}_{s,b} = \epsilon_{kbs} \tilde{A}_{s,b} , \quad \epsilon_{kbs} \hat{N}_{s,b} = \epsilon_{kbs} \hat{A}_{s,b}
\]

(23)

and with the conditions

\[
N_s = \epsilon_{skn} \Omega_{n,k} , \quad \Omega_1 = \Theta_1 , \quad \Omega_2 = \Theta_2
\]

(24)

we have

\[
N_{1,1} + N_{2,2} = 0 , \quad N_3 = 0.
\]

(25)

The tensor \( N_{\alpha\beta} \) (i.e. symmetric EM potential) has changed the signs of the terms below the diagonal of the matrix when comparing to tensor \( A_{\alpha\beta} \) (i.e. antisymmetric EM potential).

To connect the antisymmetric potentials with the symmetric ones we shall introduce also a new definition for current potential

\[
\eta_\alpha = \{ \eta_1, \eta_2, 0, 0 \} , \quad \eta_{s,s} = 0
\]

(26)

We demand the validity of the second bridge relations:

\[
\epsilon_{kbs} \eta_{S,b} = \epsilon_{kbn} \psi_{n,b} ; \quad S = 1, 2.
\]

(27)

Remark. A class of these EM potentials which can be transformed into the form described above will be called two-component EM potentials. We claim such potentials have proper form to contribute to metric tensor as described in section 5.
The field relations. Having these bridge relations we return to symmetric tensors. Due to this fact we get the equations for new symmetric potentials, similar to (12)-(13),

\[ \Box \bar{\mathcal{N}}_K = -\frac{1}{c} \epsilon_{KdN} \eta_{N,d} \quad \bar{\mathcal{N}}_{S,S} = 0 ; \quad \Box \hat{\mathcal{N}}_K = \frac{1}{c^2} \frac{\partial}{\partial t} \eta_K \quad \hat{\mathcal{N}}_{S,S} = 0. \] (28)

Instead of relation (14) for currents \( \bar{\mathcal{J}}_n \) and \( \hat{\mathcal{J}}_k \) we shall introduce a new definition of currents \( \bar{Y}_N \) and \( \hat{Y}_N \) and tensor \( Y_{\alpha\beta} \):

\[
Y_{\alpha\beta} = \begin{bmatrix}
0 & 0 & -\bar{Y}_2 & -\bar{Y}_1 \\
0 & 0 & \bar{Y}_1 & -\bar{Y}_2 \\
-\bar{Y}_2 & \bar{Y}_1 & 0 & 0 \\
-\bar{Y}_1 & -\bar{Y}_2 & 0 & 0 \\
\end{bmatrix} + i \begin{bmatrix}
0 & 0 & -\hat{Y}_2 & -\hat{Y}_1 \\
0 & 0 & \hat{Y}_1 & -\hat{Y}_2 \\
-\hat{Y}_2 & \hat{Y}_1 & 0 & 0 \\
-\hat{Y}_1 & -\hat{Y}_2 & 0 & 0 \\
\end{bmatrix} \] (29)

\[
-\frac{1}{c} \eta_{1,3} = \bar{Y}_2 , \quad \frac{1}{c} \eta_{2,3} = \bar{Y}_1 ; \quad \frac{1}{c^2} \frac{\partial}{\partial t} \eta_N = \hat{Y}_N \] (30)

where similarly to (15) we have:

\[ \Box \bar{\mathcal{N}}_N = \bar{Y}_N , \quad \Box \hat{\mathcal{N}}_N = \hat{Y}_N \] (31)

The following relations for the potentials are to be noted

\[ N_{\alpha\gamma,\gamma} = \frac{1}{c} \eta_\alpha , \quad \Box N_{\alpha\beta} = Y_{\alpha\beta} \] (32)

We will show, further on, that all these symmetric matrices are the tensors related to the Dirac tensors.

Complex metric tensor and EM natural potentials. The natural tensor of potentials \( \mathcal{N}_{\alpha\beta} \) can be presented as follows:

\[ N_{\alpha\beta} = \bar{\mathcal{N}}_1 \epsilon^1 + \bar{\mathcal{N}}_2 \epsilon^2 + i \bar{\mathcal{N}}_1 \epsilon^1 + i \bar{\mathcal{N}}_2 \epsilon^2 = N_1 \epsilon^1 + N_2 \epsilon^2 \] (33)

where

\[
\epsilon^1 = \begin{bmatrix}
0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
\end{bmatrix} , \quad \epsilon^2 = \begin{bmatrix}
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
\end{bmatrix} \] (34)
The matrices $\epsilon^\nu$ fulfil the conditions for the Dirac’s matrices:

$$\epsilon^\alpha \epsilon^\beta + \epsilon^\beta \epsilon^\alpha = 2\eta^{\alpha\beta} \quad (35)$$

with

$$\epsilon^3 = \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, \quad \epsilon^4 = \begin{bmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix}. \quad (36)$$

However, when disturbing these matrices in the way indicated below

$$\gamma^1 = (1 + N_1)\epsilon^1, \quad \gamma^2 = (1 + N_2)\epsilon^2, \quad \gamma^3 = \epsilon^3, \quad \gamma^4 = \epsilon^4 \quad (37)$$

we obtain the relation for the Dirac’s matrices in non-Euclidean space:

$$\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha = 2g^{\alpha\beta} \quad (38)$$

This relation justifies our approach in which we have assumed that the natural potentials (22) can be used as the disturbances to metric tensor in any reference system.

These complex disturbances to metric tensor can be combined with the gravity disturbances $h_{\alpha\beta}^G$. However let us first discuss another possible fields constructed similarly as above.

**Comment: Field V.** Thus, our next question is related to what would happen when one disturbes the $\epsilon^3$ and $\epsilon^4$ matrices. To this aim, we will consider the complex tensor potential $V_{\alpha\beta}$ defined as disturbances to $\epsilon^3$, $\epsilon^4$ similarly as $N_{\alpha\beta}$ disturbs $\epsilon^1$, $\epsilon^2$ in (33):

$$V_{\alpha\beta} = i(V_3\epsilon^3 + V_4\epsilon^4). \quad (39)$$

The disturbances related to $V_3$ are not symmetric hence we put $V_3 = 0$.

With the undisturbed matrices $\bar{\gamma}^1 = \epsilon^1$, $\bar{\gamma}^2 = \epsilon^2$, $\bar{\gamma}^3 = \epsilon^3$, we define the disturbed matrices $\bar{\gamma}^4$:

$$\bar{\gamma}^4 = i(1 + V_4)\epsilon^4 \quad (40)$$

Further on, we obtain the related metric tensor:

$$\bar{g}_{\mu\nu} = \frac{1}{2}(\bar{\gamma}_\mu \bar{\gamma}_\nu + \bar{\gamma}_\nu \bar{\gamma}_\mu) \quad (41)$$
Both relations correspond to those given in (37) and (38) and, with Einstein equations, can lead us to \( V^{\alpha\beta},_\beta = Z^\alpha, \), \( V^{\alpha\beta},_\gamma = U^{\alpha\beta} \) where \( Z^\alpha, \) \( U^{\alpha\beta} \) would represent some source fields.

Further on, we neglect the field \( V^{\alpha\beta}. \)

**Remark.** We shall note that, instead of the 4D presentation, it was also possible to present our relations in the 2D forms with the tensor

\[
N_{AB} = \begin{bmatrix} N_2 & N_1 \\ N_1 & -N_2 \end{bmatrix}
\]

and with the help of the Pauli 2D tensors.

## 5 Complex metric and curvature tensors

Our formulation of the Complex Relativity is based on the symmetric form of perturbations introduced into the metric tensor.

The respective metric tensor can be constructed when considering the coordinates \( X^\alpha \) \( (X^4 =ict) \) for which the numerical values, ascribed to the points \( x^\alpha \) of the Minkovski space, do not change under deformation (for example see [20]):

- before deformation we can write

\[
ds^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta
\]

- while after deformation the first order disturbances into the metric tensor are:

\[
dS^2 = g_{\alpha\beta}dX^\alpha dX^\beta, \quad h_{\alpha\beta} \approx g_{\alpha\beta} - \eta_{\alpha\beta}
\]

For weak fields this formalism can lead us to equations for electromagnetic and gravitation fields, when assuming that the disturbances \( h_{\alpha\beta} \) can be related to the following fields:

- \( h_{\alpha\beta}^G \) — the classical disturbances related to gravity
- \( h_{\alpha\beta}^N = N_{\alpha\beta} \) — the disturbances related to EM field.

We propose the metric tensor of the following form

\[
g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}^N + h_{\alpha\beta}^G.
\]
Introducing such first order disturbances, we can consider the complex Riemann $R_{\alpha\beta}$ and Einstein tensors

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}$$

(45)

and the related basic field equation:

$$G_{\alpha\beta} = 0$$

(46)

Now we consider (46) up to the first order terms.

**Contributions from the non-diagonal terms** $h^N_{\alpha\beta}$. For non-diagonal terms we perturb the metric by the potentials $h^N_{\alpha\beta} = N_{\alpha\beta}$ fulfilling the relation

(28).

Considering the first order contributions to the Einstein tensor we obtain (an index after $|$ denotes differentiation)

$$G^N_{\alpha\beta} \approx -\left(\frac{1}{2}N_{\mu\beta} |^\mu_{\alpha} + \frac{1}{2}N_{\mu\alpha} |^\mu_{\beta}\right) + \frac{1}{2}N_{\alpha\beta} |^\nu_{\nu} = -\frac{1}{2}Y_{\alpha\beta}$$

(47)

where according to (32) we have

$$\frac{1}{2}N_{\mu\beta} |^\mu_{\alpha} + \frac{1}{2}N_{\mu\alpha} |^\mu_{\beta} = \frac{1}{2c}(\eta_{\alpha,\beta} + \eta_{\beta,\alpha}).$$

**Contributions from the $h^G_{\alpha\beta}$ terms** The classical General Relativity relations

$$G^G_{\alpha\beta} = R_{\alpha\beta} - 1/2g_{\alpha\beta} = -\frac{8\pi G}{c^4} T_{\alpha\beta}$$

(48)

with the matter-energy tensor

$$T_{\alpha\beta} = c^2 \rho_0 v_\alpha v_\beta, \quad v_\alpha = \{v_s/c, i\}$$

(49)

can be included into a new complex form, with the assumption that the elements $h^G_{k4}$ present the imaginary values:

$$\frac{1}{2}h^G_{\beta\alpha} |^\nu_{\nu} + \frac{1}{2}\eta^{\mu\nu}h^G_{\mu\nu} |_{\alpha\beta} \approx \frac{8\pi G}{c^4} T_{\alpha\beta}$$

(50)
First order contributions to $G_{\alpha\beta}$. For the disturbances given by equation (44) we obtain

$$G_{\alpha\beta} \approx -\frac{1}{2} N_{\alpha\beta} |_{\nu} - \frac{1}{2} h_{\beta\alpha}^G |_{\nu} - \frac{1}{2} \eta^{\mu\nu} h_{\mu\nu}^G |_{\alpha\beta}$$  \hspace{1cm} (51)

or if we define

$$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} h_{\alpha}^\alpha$$  \hspace{1cm} (52)

we arrive at

$$G_{\alpha\beta} \approx -\frac{1}{2} N_{\alpha\beta} |_{\nu} - \frac{1}{2} \bar{h}_{\beta\alpha} |_{\nu} \approx \frac{1}{c} \eta_{(\alpha,\beta)} - \frac{1}{2} Y_{\alpha\beta} - \frac{8\pi G}{c^4} T_{\alpha\beta}.$$  \hspace{1cm} (53)

The matrices $Y_{\alpha\beta}$ is built also with the help of the $\epsilon$ matrices

$$Y_{\alpha\beta} = \tilde{Y}_1 \epsilon_1 + \tilde{Y}_2 \epsilon_2 + i \hat{Y}_1 \epsilon_1 + i \hat{Y}_2 \epsilon_2$$  \hspace{1cm} (54)

while the tensor $\frac{1}{2c}(\eta_{(\alpha,\beta)} + \eta_{(\beta,\alpha)})$ with the condition $\eta_1 = \theta_1$ and $\eta_2 = -\theta_2$ ($\eta_{1,2} + \eta_{2,1} = 0$ and $\eta_{1,2} - \eta_{2,1} = 2\theta_{1,2}$) becomes

$$\frac{1}{2c}(\eta_{(\alpha,\beta)} + \eta_{(\beta,\alpha)}) = \begin{bmatrix} 0 & 0 & \frac{1}{2} \eta_{1,3} & \frac{1}{2} \eta_{1,4} \\ 0 & 0 & \frac{1}{2} \eta_{2,3} & \frac{1}{2} \eta_{2,4} \\ \frac{1}{2} \eta_{1,4} & \frac{1}{2} \eta_{2,4} & 0 & 0 \end{bmatrix}.$$  \hspace{1cm} (54)

Finally, we obtain

$$G_{\alpha\beta} \approx \frac{1}{c} \eta_{(\alpha,\beta)} - \frac{1}{2} Y_{\alpha\beta} - \frac{8\pi G}{c^4} T_{\alpha\beta} = \begin{bmatrix} 0 & 0 & i\frac{1}{2} \tilde{Y}_2 & \frac{1}{2} \tilde{Y}_1 \\ 0 & 0 & -i\frac{1}{2} \hat{Y} & 0 \\ i\frac{1}{2} \hat{Y} & 0 & 0 & 0 \\ \frac{1}{2} \tilde{Y}_1 & i\frac{1}{2} \hat{Y}_1 & \frac{1}{2} \tilde{Y}_2 & 0 \end{bmatrix} - \frac{8\pi G}{c^4} T_{\alpha\beta}$$

Notice: It is worth noticing a complimentary structures of the amplitude related tensors $T_{\alpha\beta}$ and $E_{\alpha\beta} = (\frac{1}{2c}(\eta_{(\alpha,\beta)} + \eta_{(\beta,\alpha)}) - \frac{1}{2} Y_{\alpha\beta} )$:

$$T_{\alpha\beta} = \begin{bmatrix} \text{Re} & \text{Re} & \text{Re} & \text{Im} \\ \text{Re} & \text{Re} & \text{Re} & \text{Im} \\ \text{Re} & \text{Re} & \text{Re} & \text{Im} \\ \text{Im} & \text{Im} & \text{Im} & \text{Re} \end{bmatrix}, \quad E_{\alpha\beta} = \begin{bmatrix} 0 & 0 & \text{Im} & \text{Re} \\ 0 & 0 & \text{Im} & \text{Re} \\ \text{Im} & \text{Im} & 0 & 0 \\ \text{Re} & \text{Re} & 0 & 0 \end{bmatrix}$$  \hspace{1cm} (55)

which implies a possibility of separation of the EM and gravity fields on a linear level.
**Remark: Meaning of natural potentials.** Let us confine ourselves to the EM fields and let us consider the 3D curvilinear complex space (or 6D real space) with coordinates

$$\bar{X}_s = \tilde{X}_s + i\hat{X}_s.$$  
(56)

The potentials $\bar{A}_s = \tilde{A}_s + i\hat{A}_s$ can be identified with such frames of the complex space $\bar{X}_s = \tilde{X}_s + i\hat{X}_s$; same holds for the respective tensors: $A_{\alpha\beta} \equiv \bar{X}_{\alpha\beta}$:

$$\bar{X}_s = \tilde{X}_s + i\hat{X}_s; \quad A_{\alpha\beta} \equiv \bar{A}_{\alpha\beta}  \; (57)$$

We assume that at each point of this 6D complex space continuum there can appear the independent spin and twist motions\(^1\) and that complex space is combined to the point of such continuum

$$\epsilon_{abs}\bar{X}_s = \epsilon_{abs}\tilde{X}_s + i\epsilon_{abs}\hat{X}_s; \quad \bar{A}_{s,s} = \tilde{A}_{s,s} = \hat{X}_{s,s} + i\hat{X}_{s,s} = 0.$$  
(58)

**Conclusion and perspectives**

In this initial work we have presented the Complex Relativity theory as inspired by recent progress in asymmetric continuum approach to material sciences. The goal of this work is to propose the common relativity framework for electromagnetic and gravity fields as close as possible to the classical General Relativity formulation. For another classical approach see \[17\].

We mention here possible ways of further development of our approach.

The relations for the symmetric tensors as expressed by the $\gamma$ -tensors remain valid in any reference system, but we might also return to the system with the antisymmetric potentials ($\bar{A}^3$ different from zero; see relations between natural potentials in both systems) in order to get notation comparable with the gravity part. We shall be also aware that in such a case we return to antisymmetric natural potential tensor; we would obtain

$$G^{\text{ANTISYM}}_{\alpha\beta} = \frac{1}{2c} \times$$

\(^1\)It can be realized as "internal" motions of points in such "grained" (micropolar) space based on the Planck length.
\[
\times \begin{bmatrix}
0 & \epsilon_3 d_k \psi_{k,d} - i \frac{\partial}{\partial t} \psi_3 & -\epsilon_2 d_k \psi_{k,d} + i \frac{\partial}{\partial t} \psi_2 & -\epsilon_1 d_k \psi_{k,d} + i \frac{\partial}{\partial t} \psi_1 \\
-\epsilon_3 d_k \psi_{k,d} + i \frac{\partial}{\partial t} \psi_3 & 0 & \epsilon_1 d_k \psi_{k,d} - i \frac{\partial}{\partial t} \psi_1 & -\epsilon_2 d_k \psi_{k,d} + i \frac{\partial}{\partial t} \psi_2 \\
\epsilon_2 d_k \psi_{k,d} - i \frac{\partial}{\partial t} \psi_2 & -\epsilon_1 d_k \psi_{k,d} + i \frac{\partial}{\partial t} \psi_1 & 0 & -\epsilon_3 d_k \psi_{k,d} + i \frac{\partial}{\partial t} \psi_3 \\
\epsilon_1 d_k \psi_{k,d} - i \frac{\partial}{\partial t} \psi_1 & \epsilon_2 d_k \psi_{k,d} - i \frac{\partial}{\partial t} \psi_2 & \epsilon_3 d_k \psi_{k,d} - i \frac{\partial}{\partial t} \psi_3 & 0
\end{bmatrix}
\]

(59)

where \( G_{\alpha\beta}^{\text{ANTISYM}} \) is no longer the complex Einstein tensor, but stands for the corresponding antisymmetric expression.

In the above considerations we tried to preserve the symmetric property when constructing the Complex Relativity, however, it seems more natural to admit the possibility that the perturbations into metric and to Einstein tensor can be asymmetric or even antisymmetric.

These problems will be discussed in our next paper.

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