**ck-means**, a novel unsupervised learning method that combines fuzzy and crispy clustering methods to extract intersecting data

Jean-Sébastien Dessureault, Daniel Massicotte

June 22, 2022

**ABSTRACT**

Clustering data is a popular feature in the field of unsupervised machine learning. Most algorithms aim to find the best method to extract consistent clusters of data, but very few of them intend to cluster data that share the same intersections between two features or more. This paper proposes a method to do so. The main idea of this novel method is to generate fuzzy clusters of data using a Fuzzy C-Means (FCM) algorithm. The second part involves applying a filter that selects a range of minimum and maximum membership values, emphasizing the border data. A $\mu$ parameter defines the amplitude of this range. It finally applies a $k$-means algorithm using the membership values generated by the FCM. Naturally, the data having similar membership values will regroup in a new crispy cluster. The algorithm is also able to find the optimal number of clusters for the FCM and the $k$-means algorithm, according to the consistency of the clusters given by the Silhouette Index (SI). The result is a list of data and clusters that regroup data sharing the same intersection, intersecting two features or more. *ck-means* allows extracting the very similar data that does not naturally fall in the same cluster but at the intersection of two clusters or more. The algorithm also always finds itself the optimal number of clusters.

Keywords: *ck-means, FCM, k-means, fuzzy clustering, unsupervised learning, silhouette index*

**1 Introduction**

Unsupervised learning is mainly used to solve clustering problems and dimensionality reduction. Regarding clustering, different methods are used to discriminate the data, regrouping it into clusters of similar elements. The problem is always the same: Find the best way to regroup similar data relative to their cluster. Different approaches compete to perform the most consistent clusters of data. This *ck-means* method tackles the clustering problem from a different angle. It aims to cluster the data that shares the same intersection between features. In other words, *ck-means* method can be used to extract the data in the border spaces of the features.

Let us see the most common algorithms used to cluster data. They all can be evaluated in terms of scalability, geometry, transductivity, and outliers management.

Affinity propagation [12] can be interesting because the algorithm will choose the best number of clusters for a particular dataset on its own. Although, it is hard to scale with big datasets. Mean-Shift [9] and Spectral clustering [21] are also not very scalable, since they require to find the nearest neighbours relative to some centroids. Hierarchical clustering and Agglomerative Clustering [13] process not only the clusters but also multiple levels of subclusters. It can be visualized using dendrograms. DBScan [27] performs well when the clusters have different shapes but may have some difficulties when the cluster densities are different and are not very scalable. Optics [5] is similar to DBScan, but performs better when the density varies between clusters. It also requires minimal parameter tuning. Birch [20] uses hierarchies and requires few resources. It is one of the best algorithms for large datasets.

This novel method uses a *k-means* algorithm along with a Silhouette Index (SI) to evaluate its performance, combined with a *c-means* algorithm (FCM). From all the known clustering methods, *k-means* algorithm [15], [3] is the most popular. Several declinations of this algorithm are proposed in the literature [23], [20], [19]. It is also used in multiple applications [4]. The *k-means* algorithm is known
to by a crispy algorithm, by opposition to a fuzzy algorithm based in the fuzzy logic principles. This crispy aspect of this *k*-means method is one of its limitation.

Since it is a popular method, several metrics have been developed to evaluate the performance of the *k*-means clustering process. The Silhouette Index (SI) is widely used to evaluate the consistency of every data, in every cluster. It also supply an average for all the clusters. The SI can be represented in a very intuitive way in a graphic.

The strategy to emphasize the intersection data in this research is made with a *FCM* algorithm. This fuzzy variation of a *k*-means algorithm is also reputed to be helpful. The advantage is this method is that each element are not included in only one cluster. Each data may be owned, at a certain level (called ‘membership’), to the clusters. Like the *k*-means algorithms, many variations exist to improve the original algorithm.

The datasets used to validate this method has been generated using the *Scikit-Learn* framework and functions using different parameters. It was used to produce different datasets based on classification problems. When being called with the same parameters, the generated datasets are always the same, being fully reproducible.

The main contribution of this paper is to propose a method that combines the *FCM* and the *k*-means algorithm, along with the SI, aiming to do clustering of intersection data. In other words, it regroups the data sharing the same borders. Also, unlike *k*-means and the *FCM* algorithms, *CK*-means is able to find the best number of cluster, according to the best resulting consistency.

This novel method might be helpful in a wide variety of domains: smart cities, health, manufacturers and sports statistics, to name a few.

The next sections of this paper are organized with the following structure: Section describes the proposed methodology. Section presents the results. Section discusses about the results and their meaning and Section concludes this research.

## 2 Methodology

### 2.1 Preprocessing of dataset

Section mentions that the datasets are generated by the *datasets.make_classification()* of the *Scikit-learn* framework. The parameters are used to select the different characteristics of the dataset, like the number of rows and features. Here is a definition of the *datasets.make_classification()* parameters.

- `n_sample`: The number of rows or generated data.
- `n_features`: The number of columns or features.
- `n_classes`: The number of fields that targets the classification data.
- `shuffle`: When equal to true, it randomly changes the order of the data.
- `random_state`: The seed is used to generate the data randomly. The same seed means the same generated data. This is important to have better reproducibility.

### 2.2 Architecture

The Figure shows the architecture of the *ck*-means method.

![Figure 1: Architecture of *ck*-means method](image)

This figure shows that the first part is a standard clustering using the *k*-mean algorithm. It finds the optimal number of clusters for the dataset. It is followed by a fuzzy clustering process using the *FCM* algorithm. As in every *FCM* process, the output is a list of clusters, where every data is identified to every cluster at a certain level called ‘membership’. This output is the input of the next step, consisting of a filter that keeps only the data located in the intersections values in the dataset. The size of the intersection range is defined by a parameter called $\mu$. The filtered data are finally sent to a *k*-means algorithm.
clustering. The results are presented in the form of a list of values/clusters and some graphics.

2.3 FCM: c-mean algorithm

Like the k-means algorithm, FCM (Fuzzy c-means) is a unsupervised learning algorithm to cluster data. The difference between those two techniques is that FCM uses principles of fuzzy logic, while k-means use traditional logic (also called "crispy logic" in opposition to "fuzzy"). The main difference is that a k-means algorithm assign a cluster number to each data, while the FCM assign a membership level for each data, for each cluster. For instance, if the algorithm divided data into 3 clusters, each data will have 3 resulting values explaining the membership level (a normalized value between 0 and 1). In this 3 clusters context, a data could have for instance membership values of 0.4, 0.1, and 0.5 for cluster 1, cluster 2 and cluster 3, respectively. Equations (1), (2) and (3) defines the FCM algorithm.

\[
\mu_{ij} = \frac{1}{\sum_{k=1}^{c} (d_{ij}/d_{ik})^{2/(m-1)}} \tag{1}
\]

\[
v_j = \frac{\sum_{i=1}^{n} (\mu_{ij})^{m} x_i}{\sum_{i=1}^{n} (\mu_{ij})^{m}}, \forall j \in [1, c] \tag{2}
\]

Where \( n \) is the number of data elements, \( m \) is the fuzziness index in domain \( m \in [1, \infty] \), \( \mu_{ij} \) is membership of the \( i \)th data to \( j \)th cluster centroid. \( v_j \) is the \( j \)th cluster centroid. \( c \) is the number of centroids and \( d_{ij} \) represents the Euclidean distance between \( i \)th data and \( j \)th cluster centroid.

\[
J(U, V) = \sum_{i=1}^{n} \sum_{j=1}^{c} (\mu_{ij})^{m} \| x_i - v_j \|^2 \tag{3}
\]

FCM aims to minimize the value of (3). \( \| x_i - v_j \| \) is the Euclidean distance between \( i \)th data and \( j \)th cluster centroid.

2.4 k-means algorithm and SI metric

This process aims to create some clusters using an unsupervised learning technique called k-means. It is necessary to use an unsupervised learning technique since there is no label for each input data. The k-means process will assign to each data a reference cluster according to the similarity level of their features. In this case, the input of this algorithm is not the raw data but the output coming from the filtered FCM algorithm that was executed first. This FCM output is the membership value of the data for each cluster. In other words, a membership level for each data, for each cluster.

Equation (4) defines the k-means clustering equation where \( J \) is a clustering function, \( k \) is the number of clusters, \( n \) is the number of features, \( x_i \) is the input (feature \( i \) in cluster \( j \)) and \( c \) is the centroid for cluster \( j \). To find the centroids values, the algorithm must try some random values and select the ones who minimize the inertia value. This inertia is a standard metric to evaluate the cluster consistency with k-means.

\[
J = \sum_{j=1}^{k} \sum_{i=1}^{n} \left\| x_i - c_j \right\|^2 \tag{4}
\]

Along with inertia, there are several other metrics to measure clustering performance. Each metric is not compatible with every clustering algorithm. The SI metric is widely used to evaluate the consistency of clusters generated with k-means. This SI metric is documented in the work of [17] and [14]. Three equations define the SI metric. First, the distance between each point and the center of its cluster is defined by (5). Then, the distance between the center of each cluster is shown in (6). At last, (7) uses the result of (5) and (6) to calculate the final SI score that indicates the quality (the consistency) of the clustering process.

For data point \( i \in C_l \):

\[
a(i) = \frac{1}{|C_l| - 1} \sum_{j \in C_l, i \neq j} d(i, j) \tag{5}
\]

\( a(i) \) is the mean distance between \( i \) and other data points in the same cluster. The number of point in the cluster \( i \) is \( |C_l| \). \( d(i, j) \) is the distance between data points \( i \) and \( j \) in the cluster \( C_l \).

Then, for data point \( i \in C_l \):

\[
b(i) = \min_{k \neq l} \frac{1}{|c_k|} \sum_{j \in c_k} d(i, j) \tag{6}
\]

\( b(i) \) is the minimal mean distance of \( i \) to all points in any other cluster, of which \( i \) is not a member. This cluster having minimum mean dissimilarity is the "neighboring cluster" of \( i \) because it is the next best fit cluster for point \( i \). Having defined \( a(i) \) and \( b(i) \), the final equation defines \( s(i) \) which is the final silhouette index.

\[
s(i) = \frac{b(i) - a(i)}{\max(a(i), b(i))}, \text{ if } |C_l| > 1 \tag{7}
\]

SI domain of values is defined from -1 to +1. Values from -1 to 0 indicate a wrong classification. SI values from 0 to 1 indicate the points associated with
a good cluster. The higher the value, the better the cluster consistency [17].

2.5 Intersection filter

The "intersection filter" goal is to remove data identified in only one cluster, having a membership too low to be considered in the intersection. For instance, data with membership values of 0.97, 0.03, or 0.0 would be removed since this data would not be considered at an intersection. The filter keeps data within a threshold defined by the user-defined $\mu$ parameter. This $\mu$ parameter defines the minimum and the maximum values of the thresholds when $ck$-means algorithm is called. Using $\mu$, the minimum value kept by the filter will be $0.5 - (\mu/2)$ and the maximum will be $0.5 + (\mu/2)$. For instance, having $\mu = 0.4$, the minimum would be 0.3, and the maximum would be 0.7.

Hence, the data scientist can narrow or enlarge the amount of data included in the intersection.

3 Results

The following cases test some different aspects of this novel method. For each case, different parameters and a different dataset. For each dataset, 'Nb.samples' is the number of rows in the dataset. $Nb.features$ is the number of different features and $Nb.Centroids$ is the number of natural clusters generated. The Cluster standard deviation is a metric of consistency. The higher this value, the less the cluster will be consistent, and the points might be mixed up with others points of other clusters. Finally, the Seed generates the data randomly. Having the same seed means that the same data sequence will be generated each time. This is very useful for reproducibility purposes.

3.1 Case 1

This first case is a basic example of the intersection extraction using only 2 clusters and a 2 features dataset. A 2 features dataset can be easily presented using 2d graphics. Table 1 shows the dataset used for this first case.

![Figure 2: Distribution of the data, emphasizing the intersection data (in red, using $\mu = 0.4$)](image)

Figure 2: Distribution of the data, emphasizing the intersection data (in red, using $\mu = 0.4$)

The same intersection data are filtered ($\mu = 0.4$) and magnified in Fig. 3. The colour intensity represents the level of membership in their respective cluster.

![Figure 3: Intersection data magnified and colored according to their cluster membership level.](image)

Figure 3: Intersection data magnified and colored according to their cluster membership level.

In Fig. 4 we can see the distribution of the membership value that has been produced by the $FCM$
algorithm. Since 2 clusters have been found, the result is linear because the sum of the 2 memberships is always 1. If the first membership value is 0.2, the other value will automatically be 0.8.

Figure 4: Distribution of the membership of the data in the intersection.

The final step is to cluster the filtered resulting membership using a \textit{k-means} algorithm, as shown in Fig. 5. This process may seem useless in these 2 clusters’ context since data are already quite regrouped. Although, the clusters will be distinct when the number of clusters is in higher dimensions. Also, even with only two clusters, there might be some natural clusters in the intersection that was not obvious before the filtering process.

Figure 5: Final \textit{k-means} clustering of the intersection data

The last graphic (Fig. 6) is the proof of the consistency of the intersection clusters. It shows a SI of 0.7199, which means a very consistent clustering. The grey and green bands are the clusters composed of the data. X axe is the value of the clustering consistency (a normalized distance from each point to their cluster centroid). We can see no misplaced value (negative values on X axe). The red dotted line defines the SI value of the whole clustering process.

Figure 6: Consistency proof of the intersection data using SI, after a \textit{k-means} clustering.

This first basic case showed a good consistency of the extracted intersection data using a \textit{FCM} algorithm and clustered using a \textit{k-means} algorithm.

3.2 Case 2

This second example shows the frequent case of no data in the intersection. In this case, the utility of the \textit{ck-means} algorithm for the data scientist is to learn that there is no intersection data corresponding to a certain filter range (here: $\mu = 0.4$). Table 2 presents the used dataset.

Table 2: Datasets for case 2, using $\mu=0.4$

| Dataset          | Value |
|------------------|-------|
| Nb.samples       | 500   |
| Nb.features      | 2     |
| Nb.centroids     | 3     |
| Clusters standard deviation | 0.4   |
| Seed             | 12    |

The Fig. 7 shows 3 very consistent clusters. There is no intersection data. It would have been the red points. In this case, the \textit{ck-means} algorithm returns this graphic and a message saying, "No data are found in the intersection corresponding to the $\mu$ parameter.".
This case showed that the *ck-means* algorithm behaves correctly when no data is located at the clusters’ intersections.

### 3.3 Case 3

This case is similar to case 1 but adds a feature to obtain 3 dimensions data. There are also only 2 centroids, suggesting that the *FCM* algorithm will work with 2 clusters. The dataset used for is shown in Table 3.

Table 3: Datasets for case 3, using $\mu=0.4$

| Dataset                  | Value  |
|--------------------------|--------|
| Nb.samples               | 500    |
| Nb.features              | 3      |
| Nb.centroids             | 2      |
| Clusters standard deviation | 3.5    |
| Seed                     | 15     |

The Fig. 8 presents the distribution of the data. As foreseen by the number of centroids, 2 clusters have been found. The orange points represent the intersection data extracted with the *FCM* algorithm and a filter of $\mu = 0.4$.

Fig. 9 shows the distribution of the membership value that has been produced by the *FCM* algorithm. As in case 1, the result is linear because the sum of the 2 memberships is always 1. Although, in this case, the *k-means* algorithm found that those membership values are more consistently clustered in 3 groups, according to the SI of 0.7057. The result of this process is shown in Fig. 10.
This case tests the *ck*-means algorithm using 3 dimensions data. It proves that the intersection data are correctly extracted, filtered, and clustered.

### 3.4 Case 4

This 4th case aims to extract intersection data when there are multiple centroids in the data. The $\mu$ parameter has been increased to 0.6 for this case, meaning a membership function filtered between 0.3 to 0.7. The final *k*-means clustering process will be particularly significant in this case since the data are located in distinct parts of the graphic. Table 4 presents the parameter used to generate the dataset.

Table 4: Datasets for case 4, using $\mu=0.6$

| Dataset               | Value  |
|-----------------------|--------|
| Nb.samples            | 500    |
| Nb.features           | 2      |
| Nb.centroids          | 5      |
| Clusters standard deviation | 1.0   |
| Seed                  | 2      |

Fig. 11 displays the data distribution. Although there were 5 centroids in the data, 4 clusters have been found due to a significant standard deviation and 2 close centroids. The red points represent the intersection data using parameter $\mu = 0.6$. Those filtered data are shown in Fig. 12. The colour intensity explains the membership level of their cluster.

As in the other cases, a *k*-means algorithm (Fig. 13) processed the data to rebuild new clusters from the intersection data. In this case, 4 clusters was found to maximize a SI of 0.6492 (Fig. 14).
Figure 13: Final $k$-means clustering of the intersection data.

Even with this higher number of clusters, the $ck$-means algorithm succeeds in extracting the intersection data and in clustering the results based on the membership values generated by the $FCM$ algorithm.

3.5 Case 5

This case uses higher dimensions of features (6), so there is graphically a challenge to illustrate the validity of the process and the final clustering consistency. Parameters used to generate the dataset are shown in Table 5.

Table 5: Datasets for case 5, using $\mu=0.6$

| Dataset                  | Value   |
|--------------------------|---------|
| Nb.samples               | 500     |
| Nb.features              | 6       |
| Nb.centroids             | 5       |
| Clusters standard deviation | 1.5     |
| Seed                     | 1       |

The first $FCM$ algorithm generates membership data that can be displayed in a 3d space (Fig. 15). It clearly shows a future $k$-means process of 3 distinct clusters.

Figure 14: Consistency proof of the intersection data using SI, after a $k$-means clustering.

Figure 15: Membership values of the initial $FCM$ 3 clusters.

To illustrate the validity of the $ck$-means algorithm, Fig. 16, 17 and 18 present the 3 generated clusters resulting from the $FCM$ and the $k$-means algorithm. Those are stacked radar graphics of the clustered and filtered intersection data. The data has been normalized using a MinMax function to fit on the same scale radar graphic.
Although there was a challenge in visually presenting the validity of the process, this case showed that the \textit{ck-means} works well in a higher dimension context.

### 3.6 Case 6

This final case tests the scalability of the \textit{ck-means} algorithm. The dimensionality of the samples (100,000) and of the features (30) is significantly higher as presented in Table 6. Also, the filter parameter will keep most of the data using a $\mu$ value of 0.8. This case aims to see if, at this scale, the algorithm still works in a reasonable time.

#### Table 6: Datasets for case 1, using $\mu=0.8$

| Dataset                  | Value |
|--------------------------|-------|
| Nb.samples               | 100000|
| Nb.features              | 30    |
| Nb.centroids             | 5     |
| Clusters standard deviation | 1.5   |
| Seed                     | 1     |

The whole process was executed within minutes (5:36). The SI index was excellent at 0.9840. To illustrate the complexity of the final clustering process, Fig. 19 shows an example of one cluster (Cluster A).

Even using a significantly larger dataset, the \textit{ck-means} passed the scalability test. The reason is that the algorithm does not add an extra embedded loop for the clustering process. There are only two embedded loops in the filtering process. This algorithm may scale without difficulty.
3.7 Comparison between this novel method and original methods

There are already multiple methods to extract clusters from datasets. In the introduction, several are referenced like Affinity propagation [12], MeanShift [9], Spectral clustering [21]. There are also hierarchical clustering methods, like Agglomerative Clustering [13] and Birch [29] that can process multiple levels of subclusters, being visualized using dendrograms. DBScan [27] can handle the clusters having different shapes. Finally the very known \textit{k-means} algorithm [15] and its fuzzy equivalent: the FCM algorithm [8]. Those are useful methods when it is time to extract data in clusters. Using those methods, the problem is that they can not extract specific data at the intersection of two clusters or more. In some situations, it is useful to address this specific data.

To compare those methods with this novel \textit{ck-means}, we must refer to the mathematics field called the 'set theory'. While existing methods aim to determine data included in a set, \textit{ck-means} aims to determine data included in the intersection of two or more sets.

Also, at the opposite of the \textit{k-means} and the FCM algorithms, \textit{CK-means} can find alone the best number of clusters according to the best resulting SI metric.

Using \textit{ck-means}, the metric to evaluate the resulting cluster consistency is the known SI metric. The clusters can be evaluated the same way as with a \textit{k-means} algorithm, for instance.

Table 7 compares a sample of 20 data the \textit{k-means}, the FCM and the \textit{ck-means} methods. A novel unsupervised machine learning method to extract intersection data between clusters has been developed in this paper. The method has been validated using different datasets generated with the \textit{make_classification()} function available in the scikit-learn framework. The parameters used to gen-

| Data | F.1 | F.2 | F.3 | C.1 | C.2 | C.3 |
|------|-----|-----|-----|-----|-----|-----|
| 2.79 | 9.88| 2.49| -6.67| 1.23 | -7.48| -1.28 |
| -6.84| -2.05| -2.00| -6.68| -6.69| 1.16 | 5.21 |
| 0.90 | 0.76 | 0.77 | 0.10 | 0.94 | 0.06 | 0.12 |
| 0.09 | 0.23 | 0.22 | 0.89 | 0.05 | 0.93 | 0.87 |
| 1 | N.A. | 1 | N.A. | 0 | N.A. | 0 |
| 9.88 | 9.50 | 9.76 | 1.05 | 7.41 | 5.48 | -8.29 |
| -2.05 | -1.00 | -8.29 | -4.29 | -5.13 | -2.88 | -9.95 |
| 0.76 | 0.95 | 0.97 | 0.95 | 0.71 | 0.93 | 0.04 |
| 0.23 | 0.22 | 0.02 | 0.04 | 0.28 | 0.06 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | N.A. |
| -6.67 | 1.28 | -2.00 | -4.29 | -5.13 | -7.48 | 1.16 |
| -6.64 | 5.21 | -6.68 | -9.95 | -5.13 | 1.16 | 0.12 |
| 0.10 | 0.87 | 0.89 | 0.93 | 0.71 | 0.93 | 0.87 |
| 0.89 | 0.22 | 0.02 | 0.04 | 0.28 | 0.06 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | N.A. |
| -1.28 | 9.76 | 5.21 | 4.00 | 1.16 | 0.16 | 0.83 |
| 2.79 | 9.88 | -6.84 | -2.05 | -6.67 | 1.23 | -7.48 |
| 9.88 | 9.50 | 9.76 | 1.05 | 7.41 | 5.48 | -8.29 |
| -2.05 | -1.00 | -8.29 | -4.29 | -5.13 | -2.88 | -9.95 |
| 0.76 | 0.95 | 0.97 | 0.95 | 0.71 | 0.93 | 0.04 |
| 0.23 | 0.22 | 0.02 | 0.04 | 0.28 | 0.06 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | N.A. |
| -6.67 | 1.28 | -2.00 | -4.29 | -5.13 | -7.48 | 1.16 |
| -6.64 | 5.21 | -6.68 | -9.95 | -5.13 | 1.16 | 0.12 |
| 0.10 | 0.87 | 0.89 | 0.93 | 0.71 | 0.93 | 0.87 |
| 0.89 | 0.22 | 0.02 | 0.04 | 0.28 | 0.06 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | N.A. |

Feature 1 (F.1) and Feature 2 (F.2) in this table identify the data. The result of the FCM clustering is presented as a membership value in cluster 1 (C.1) and cluster 2 (C.2). The cluster number associated with the \textit{k-means} (k-m.) and the \textit{ck-means} ck-m algorithms are also presented to compare the output of each one. Looking at the FCM values, it can be seen that the \textit{ck-means} results are defined only for the values lying inside the range for the filter $\mu = 0.4$. Hence, values between 0.3 and 0.7 are kept by the filter and clustered again using a \textit{k-means} method. Note that all the values are trunked at 2 decimals.

This table shows how the 3 methods are complementary. \textit{k-means} is used for crispy clustering, FCM is used for fuzzy clustering, and \textit{ck-means} uses both \textit{k-means} and FCM to cluster data at the intersection of the clusters.

4 Discussions

A novel unsupervised machine learning method to extract intersection data between clusters has been developed in this paper. The method has been validated using different datasets generated with the \textit{make_classification()} function available in the scikit-learn framework. The parameters used to gen-
erate the data differed for each of the 6 use cases. The seed parameter made the datasets fully reproducible.

Note that every figure in this paper was generated by the ck-means algorithm, making this method more explainable.

The first case is the basic one. Fig. 2 shows the data distribution, emphasizing the filtered data (using $\mu=0.4$) located at the intersection of the 2 clusters. Fig. 3 shows the filtered data after this first clustering process using FCM algorithm, using colour tones according to their membership level to the clusters. Fig. 4 is shown to explain the basic data (the membership level) used to do the next clustering ($k$-means). The result of the $k$-means clustering is shown in Fig. 5. The consistency of this final clustering process is displayed in Fig. 6 showing a good cluster consistency (SI=0.7199). This first basic example displays all the available graphics for the whole process.

Case 2 proves that the algorithm still works without intersection data. This can happen due to the structure of the data or a too-small $\mu$ parameter. Fig. 7 shows the distribution of the data. Since there is no red dot in this graphic, we can conclude that the filter has eliminated all the data. This case may happen often. There is nothing to do if the data is very regrouped around the centroids. Nonetheless, the $\mu$ parameters may be augmented to a higher value, preventing the filter from dropping too much data.

In the 3rd case, 3-dimensional data show that the algorithm can also work in multiple dimensions. 3d graphics like Fig. 8 and 10 are used to represent the data. Case 4 processes higher cardinality in terms of number of centroids (5), showing the utility to cluster one more time the resulting data using a $k$-means algorithm, as shown in 11 12 13. Once again, a SI of 0.6492 proves the result is consistent 14. Case 5 aims to show the possibility of representing the data of 3d and higher in multiple clusters. There are 6 features distributed around 5 centroids for this case. There is a challenge in representing the data. It has been made using some stacked radar graphics, after the feature has been normalized using a MinMax function (Fig. 15 17 and 18). The very high value of the SI (0.9374) is explained clearly by Fig. 19.

The last case (case 6) is the one that proves that the ck-means algorithm can scale at a certain point. It uses 100000 samples and 30 features. It means that the intersection data were very isolated, without overlapping. It took less than 6 minutes to execute, showing that there are no combinatorial explosions. The result was good, showing a very high SI of 0.9840. It is also challenging to graphically represent a cluster consistency with 30 features. Nevertheless, Fig. 19 succeeds in doing so, using a complex graphical structure, but showing an evident result. Without any doubt, this figure shows a good consistency for this cluster A.

Myriads of applications use this ck-means algorithms. For instance, it can emphasize the intersection data in a smart city context. It can be used with geographical coordinates to extract data at the borders and frontiers. It can be used in sports statistics. Some players may be very similar, but they are never regrouped together. For baseball statistics, the classical clustering method would not necessarily regroup players sharing a similar home run/stolen base/walk ratio. $k$-means allows extracting those similar data that do not naturally fall in the same cluster. It is easy to foresee other applications for this novel algorithm. The domains of marketing, health and agriculture would be attractive, to name a few.

5 Conclusion

To resume this paper, this novel method is about clustering the membership results of a FCM algorithm, emphasizing the intersection data. It aims to put in evidence data that are usually not in the same cluster while being very similar. The FCM algorithm and the $k$-means algorithm are used sequentially, and the final cluster consistency is evaluated with a SI metric. A filter is applied to discard the data that does not fall in any cluster intersection, allowing emphasis on intersection data. This filter uses a parameter called $\mu$, having a domain between 0 and 1. It represents the amplitude of the membership value (resulting from FCM algorithm) the filter has to keep. The higher the value, the wider will be the size of the intersection. 6 cases are processed to show the accuracy of this new model. This method implements several types of graphics to ensure a better explainability of the results.

Future works may include testing this algorithm on higher dimensionality. It would be interesting to test more systematically the scalability of this method. This method is a fruit of this fundamental research, and it would be interesting to apply it to a concrete domain in the future. Some domain ideas are mentioned in sec. 4.
6 Acknowledgment

This work has been supported by the 'Cellule d'expertise en robotique et intelligence artificielle' of the Cégep de Trois-Rivières.

References

[1] sklearn.datasets.make_classification.

[2] Towards improving cluster-based feature selection with a simplified silhouette filter. 181(18):3766–3782. Publisher: Elsevier.

[3] Mohiuddin Ahmed, Raihan Seraj, and Syed Mohammed Shamsul Islam. The k-means algorithm: A comprehensive survey and performance evaluation. 9(8):1295. Number: 8 Publisher: Multidisciplinary Digital Publishing Institute.

[4] Md Shahariar Alam, Md Mahibubur Rahman, Mohammad Amazad Hossain, Md Khaqirul Islam, Kazi Mowdud Ahmed, Khandaker Takdir Ahmed, Bikash Chandra Singh, and Md Sipon Miah. Automatic human brain tumor detection in MRI image using template-based k means and improved fuzzy c means clustering algorithm. 3(2):27. Number: 2 Publisher: Multidisciplinary Digital Publishing Institute.

[5] Mihael Ankerst, Markus M. Breunig, Hans-Peter Kriegel, and Jörg Sander. OPTICS: ordering points to identify the clustering structure. 28(2):49–60.

[6] Jyoti Arora, Kiran Khatter, and Meena Tushir. Fuzzy c-means clustering strategies: A review of distance measures. In M. N. Hoda, Naresh Chauhan, S. M. K. Quadri, and Praveen Ranjan Srivastava, editors, Software Engineering, Advances in Intelligent Systems and Computing, pages 153–162. Springer.

[7] Salar Askari. Fuzzy c-means clustering algorithm for data with unequal cluster sizes and contaminated with noise and outliers: Review and development. 165:113856.

[8] James C. Bezdek, Robert Ehrlich, and William Full. FCM: The fuzzy c-means clustering algorithm. 10(2):191–203.

[9] Yizong Cheng. Mean shift, mode seeking, and clustering. 17(8):790–799. Conference Name: IEEE Transactions on Pattern Analysis and Machine Intelligence.

[10] Keh-Shih Chuang, Hong-Long Tzeng, Sharon Chen, Jay Wu, and Tzong-Jer Chen. Fuzzy c-means clustering with spatial information for image segmentation. 30(1):9–15.

[11] Andrzej Dudek. Silhouette index as clustering evaluation tool. In Krzysztof Jajuga, Jacek Batóg, and Marek Walesiak, editors, Classification and Data Analysis, Studies in Classification, Data Analysis, and Knowledge Organization, pages 19–33. Springer International Publishing.

[12] Delbert Dueck and Brendan J. Frey. Nonmetric affinity propagation for unsupervised image categorization. In 2007 IEEE 11th International Conference on Computer Vision, pages 1–8. ISSN: 2380-7504.

[13] Alberto Fernández and Sergio Gómez. Solving non-uniqueness in agglomerative hierarchical clustering using multidendrograms. 25(1):43–65.

[14] Natacha Gueorguieva, Iren Valova, and George Georgiev. M&MFCM: Fuzzy c-means clustering with mahalanobis and minkowski distance metrics. 114:224–233.

[15] J. A. Hartigan and M. A. Wong. Algorithm AS 136: A k-means clustering algorithm. 28(1):100–108. Publisher: [Wiley, Royal Statistical Society].

[16] Timothy C. Havens, James C. Bezdek, Christopher Leckie, Lawrence O. Hall, and Marimuthu Palaniswami. Fuzzy c-means algorithms for very large data. 20(6):1130–1146. Conference Name: IEEE Transactions on Fuzzy Systems.

[17] Leonard Kaufman and Peter Rousseeuw. Finding Groups in Data: An Introduction to Cluster Analysis. 09 2009.

[18] Oliver Kramer. Scikit-learn. pages 45–53.

[19] K. Krishna and M. Narasimha Murty. Genetic k-means algorithm. 29(3):433–439. Conference Name: IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics).

[20] Aristidis Likas, Nikos Vlassis, and Jakob J. Verbeek. The global k-means clustering algorithm. 36(2):451–461.

[21] Andrew Ng, Michael Jordan, and Yair Weiss. On spectral clustering: Analysis and an algorithm. In Advances in Neural Information Processing Systems, volume 14. MIT Press.
[22] N.R. Pal, K. Pal, J.M. Keller, and J.C. Bezdek. A possibilistic fuzzy c-means clustering algorithm. 13(4):517–530. Conference Name: IEEE Transactions on Fuzzy Systems.

[23] D T Pham, S S Dimov, and C D Nguyen. Selection of k in k-means clustering. 219(1):103–119. Publisher: IMECHE.

[24] Mohammad Rawashdeh and Anca Ralescu. Crisp and fuzzy cluster validity: Generalized intra-inter silhouette index. In 2012 Annual Meeting of the North American Fuzzy Information Processing Society (NAFIPS), pages 1–6.

[25] Peter J. Rousseeuw. Silhouettes: A graphical aid to the interpretation and validation of cluster analysis. 20:53–65.

[26] Artur Starczewski and Adam Krzyżak. Performance evaluation of the silhouette index. In Leszek Rutkowski, Marcin Korytkowski, Rafal Scherer, Ryszard Tadeusiewicz, Lotfi A. Zadeh, and Jacek M. Zurada, editors, Artificial Intelligence and Soft Computing, Lecture Notes in Computer Science, pages 49–58. Springer International Publishing.

[27] Thanh N. Tran, Klaudia Drab, and Michal Daszykowski. Revised DBSCAN algorithm to cluster data with dense adjacent clusters. 120:92–96.

[28] Weina Wang, Yunjie Zhang, Yi Li, and Xiaona Zhang. The global fuzzy c-means clustering algorithm. In 2006 6th World Congress on Intelligent Control and Automation, volume 1, pages 3604–3607.

[29] Tian Zhang, Raghu Ramakrishnan, and Miron Livny. BIRCH: an efficient data clustering method for very large databases. 25(2):103–114.