THERMAL EFFECTS FOR QUARK AND GLUON DISTRIBUTIONS IN HEAVY-ION COLLISIONS

G.I.Lykasov, A.N.Sissakian, A.S.Sorin, O.V.Teryaev
JINR, Dubna, 141980, Russia

Abstract

In-medium effects for distributions of quarks and gluons in central A+A collisions are considered. We suggest a duality principle, which means similarity of thermal spectra of hadrons produced in heavy-ion collisions and inclusive spectra which can be obtained within the dynamic quantum scattering theory. Within the suggested approach we show that the mean square of the transverse momentum for these partons grows and then saturates when the initial energy increases. It leads to the energy dependence of hadron transverse mass spectra which is similar to that observed in heavy ion collisions.

1 Introduction

Searching for a new physics in heavy-ion collisions at AGS, SPS and RHIC energies has led to intense theoretical and experimental activities in this field of research [1]. In this respect the search for signals of a possible transition of hadrons into the QCD predicted phase of deconfined quarks and gluons, quark-gluon plasma (QGP), is of particular interest. One of these signals can be the recent experimental observation of the transverse-mass spectra of kaons and pions from central Au+Au and Pb+Pb collisions which revealed ”anomalous” dependence on the incident energy. The inverse slope parameter of the transverse mass distribution (the so called effective transverse temperature) at the mid-rapidity rather fast increases with incident energy in the AGS domain [2], then saturates at the SPS [3] and RHIC energies [4].

In this paper we would like to discuss the physical meaning of the so-called thermal spectra of hadrons produced in heavy-ion collisions, see for example [5, 6], and try to understand the dynamic reason of such inclusive spectra. Then we focus on a possible theoretical interpretation of the nontrivial energy dependence for the inverse slope parameter of the transverse mass spectra of mesons produced in central heavy-ion collisions.

2 Duality principle

According to many experimental data, inclusive spectra of hadrons produced in heavy-ion collisions can be fitted by the Fermi-Dirac distribution, corresponding to the thermodynamic equilibrium (LE) for the system of final hadrons, see for example [5, 6]

\[ f_h^A = C_T^A \{\exp((\epsilon_h - \mu_h)/T) \pm 1\}^{-1}, \quad (1) \]
where $+$ is for fermions and $-$ is for bosons, $\epsilon^h$ and $\mu^h$ are the kinetic energy and the chemical potential of the hadron $h$, $T$ is the temperature, $C^A_T$ is the normalization coefficient depending on $T$. Actually, the parameter $T$ depends on the incident energy $\sqrt{s}$ in the $N-N$ c.m.s. For mesons simplifying this case we can assume that $\mu^h \simeq 0$, (in fact, it generally cannot be strictly zero [7]); then Eq.\((1)\) is usually presented in the form

\[ f_h^A \simeq C^A_T \exp(-\epsilon^h/T) . \]  

On the other hand, according to the Regge theory and the $1/N$ expansion in QCD, the inclusive spectrum of hadrons produced, for example in $N-N$ collisions at high energies, has the scaling form, e.g., it depends only on $M_X^2/s$, where $M_X$ is the missing mass of produced hadrons, $s$ is the initial energy squared in the $N-N$ c.m.s., and $M_X^2/s = 1-x_r$, where $x_r = 2E^*_h/\sqrt{s}$ is the radial Feynman variable, $E^*_h$ is the energy of the hadron $h$ in the $N-N$ c.m.s. For example, the quantum scattering theory and the fit of the experimental data for inclusive meson spectra at low $x_r$ results in

\[ \rho_{m}^{NN}(x_r) \sim C_N(1-x_r)^{d_N} \]  

If $x_r << 1$, Eq.\((3)\) can be presented in the exponential form

\[ \rho_{m}^{NN} \sim C_N \exp(-d_Nx_r) \]  

Inserting the form for $x_r$ in Eq.\((4)\), we get the inclusive spectrum of mesons in the form similar to that of the thermal spectrum given by Eq.\((2)\)

\[ \rho_{m}^{NN} = C_N \exp(-d_Nx_r) \equiv C_N \exp(-E^*_h/T^N_s) , \]  

where $T^N_s = \sqrt{s}/2d_N$. However, in contrast to Eq.\((2)\), the form of the inclusive spectrum of mesons produced in $N-N$ collisions given by Eq.\((5)\) does not assume introduction of any temperature of mesons like $T$. Figure 1 illustrates the approximate equivalence between $\rho_{m}^{NN}$ given by Eq.\((3)\) and $\rho_{m}^{NN}$ given by Eq.\((5)\). One can see from Fig.1 that at high energies these two forms for the meson spectrum are very similar to each other to the meson energies about a few GeV. Therefore, $\rho_{m}^{NN}$ can be presented in the exponential form at low and even moderate energies $E^*_h$.

Let us assume that in central $A-A$ collisions at high energies in the first $N-N$ interaction at some time the mini-jet consisting of $(q\bar{q})$ pairs is created and then pions are produced in the central rapidity region. We also suggest that the distribution of these pairs in the $(q\bar{q})$ mini-jet has the form similar to the one given by Eq.\((5)\)

\[ f_{qq}^{jet} = C_A(1-x_r)^{d_A} \simeq C_A \exp(-E^*_h/T^A_s) , \]  

where $T^A_s = \sqrt{s}/2d_A$. In the general case, the parameter $d_A$ is not the same as $d_N$ which can be found from the quantum scattering theory or fitting the experimental data on inclusive spectra of mesons produced in $N-N$ collisions. Let us call the assumption corresponding to Eq.\((6)\) the **Dynamic ansatz (DA)**. One can suggest the **duality principle** which is the similarity of thermal spectra given by Eq.\((2)\) and the dynamical spectra given by Eq.\((6)\).
To find the form for $d_A$ one can use the approach suggested by Kuti, Weiskopf [8] for the calculation of parton distribution in a nucleon. One can show that $d_A$ is proportional to the number $n$ of $q\bar{q}$ pairs in the mini-jet, e.g., $d_A \sim n$. To estimate the in-medium effects we replace $n$ by the mean multiplicity $< n >_{NN}$ of pions produced in $NN$ collision, e.g., $d_A \simeq d_0 < n >_{NN}$.

3 Parton distribution in medium

Recently the parton distribution in a medium was analyzed on the assumption of the local thermodynamic equilibrium for quark objects like hadrons produced in heavy ion central collisions [9]. It was shown that, for example, the valence quark distribution in the quark object like the hadron $h$, which is in local thermodynamic equilibrium with surrounding nuclear matter, can be calculated by the following equation:

$$f_A(x_h, p_{ht}) = \int_{0}^{1} dx_1 \int_{0}^{1} dx_h \int d^2p_{1t} d^2p_{ht} q_v^h(x, p_t) q_r^h(x_1, p_{1t}) \times$$

$$f_A(x_h, p_{ht}) \times \delta(x + x_1 - x_h) \delta^{(2)}(p_t + p_{1t} - p_{ht}),$$

where $f_A^h(x_h, p_{ht})$ is the distribution of quark objects like hadrons locally equilibrated in a medium (LE); $q_v^h$, $q_r^h$ are the probabilities to find the valence quark and other partons (valence, sea quarks (antiquarks) and gluons) in $h$; $x_1, x_h, x$ are the Feynman variables, $p_t, p_{1t}, p_{ht}$ are the transverse momenta. The thermodynamic distribution like Eq.(2) was assumed in [9] for $f_A^h(x_h, p_{ht})$. The same form for $f_A^q$ can be obtained suggesting the dynamic distribution (DA) for $f_A^h$ given by Eq.(8) instead of Eq.(2). Assuming the factorized form for $f_A^h(x, p_t) = f_q(x) g_q(p_t)$ we have approximately [9] the following form for the mean transverse momentum squared of the valence quark in a medium:

$$< p_{q,t}^2(x \simeq 0) >_{q,appr.} \simeq \frac{< p_{t}^2 >_q^h + \tilde{T} \sqrt{m_h^2 + s/4}}{1 + \tilde{T} \sqrt{m_h^2 + s/4}/(2 < p_{t}^2 >_q^h)},$$

Figure 1: $f_1(x_r) = \exp(-dx_r)$ and $f_2(x_r) = (1 - x_r)^d$ as functions of $E_h$. 

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Figure 1: $f_1(x_r) = \exp(-dx_r)$ and $f_2(x_r) = (1 - x_r)^d$ as functions of $E_h$.
where $\tilde{T} = T$ for Eq. (2) (LE) and $\tilde{T} = T_s = \sqrt{s}/2d_A$ for Eq. (6) (DA). As is seen from Eq. (8), $\langle p_{q,t}^2 \rangle^A_{q,\text{appr.}}$ grows when $\sqrt{s}$ increases and then saturates, its more careful calculation is presented in [9]. Note that in the LE case $\sqrt{s}$ is some scale energy which cannot be equal to the initial energy $\sqrt{s_{NN}}$ [9], whereas in the DA case it is the same as $\sqrt{s_{NN}}$. For mesons produced in central $A - A$ collisions we have similar broadening for the hadron $p_t$-spectrum [9].

$$\langle p_{h,t}^2 \rangle^A_{\text{appr.}} \approx \frac{\langle p_{h,t}^2 \rangle^{NN}_{q} / (1 + r) + \tilde{T} \sqrt{m_h^2 + s/4}}{1 + \tilde{T} \sqrt{(m_h^2 + s/4)(1 + r)} / (2 \langle p_{h,t}^2 \rangle^{NN}_{q})} + \frac{\langle p_{h,t}^2 \rangle^{NN}_{q}}{r},$$

where $\langle p_{h,t}^2 \rangle$ is the mean value for the transverse momentum squared of the meson $h_1$ produced in the central heavy-ion collision, $\langle p_{l}^2 \rangle^m_{q}$ is the same quantity for a quark in a medium, $r = \gamma_c/\gamma_q$, $\tilde{T} = T$ (LE) or $\tilde{T} = \sqrt{s}/2d_A$. Here $\gamma_q$ and $\gamma_c$ are the slopes in the Gauss form of the $p_t$ dependence for the quark distribution in the hadron $h$ and its fragmentation function, see details in [9]. As is seen from Eqs. (8,9), the saturation properties for $\langle p_{q,t}^2 \rangle^A_{q}$ and $\langle p_{h,t}^2 \rangle^A_{\text{appr.}}$ at high $\sqrt{s}$ do not depend on the values of $\tilde{T}$, whereas the growth of these quantities at $\sqrt{s} \leq 20 - 30 \text{ GeV}$ is very sensitive to the value of $d_A$. To describe the experimental data on the transverse momentum squared of $K$-mesons produced in central $A - A$ collisions we took $d_0 = 0.5$ and the energy dependence for $\langle n \rangle^{NN}_{\pi}$ from [10]. We included also the energy dependence for the mean squared of the transverse momentum of kaons produced in the $N - N$ collision, see [11] and references there in.

In Fig.2 we present our estimation for the mean transverse momentum squared of the $K^+$-meson produced in the A-A collision. One can see from Fig.2 that this quantity increases when the incident energy increases to the AGS energies and then saturates at higher energies. Note that this calculation is very approximate and we need to improve it including standard nuclear effects like rescattering and others. The suggested approach results in the saturation of the effective slope $T_{\text{eff}}$ for the transverse mass spectrum of mesons produced in central heavy-ion collisions that is directly related to the quantity presented in Fig.2. In contrast to this the thermodynamic models predict the increase in $T_{\text{eff}}$ when $\sqrt{s_{NN}}$ increases even to very high energies [12]. Therefore, the presented results can be verified by more careful measurements at the SPS energy and future experiments at the LHC.

### 4 Conclusion

We suggest the duality principle. Thermal spectra of hadrons produced in central A-A collisions can have a dynamical nature. Similar spectra can be obtained within the quantum scattering theory without introducing the temperature. One can assume that hadron jets consisting of colorless quark objects are produced in central A-A collisions. Then we get broadening for the mean transverse momentum squared of quarks in a medium respective to the incident energy. Similar effects can be obtained for transverse momentum spectra of mesons produced in central $A - A$ collisions. The mean transverse momentum squared of these mesons as a function of the incident energy grows and then saturates at high energies.

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Figure 2: The mean transverse momentum squared of the $K^+$-meson produced in the central A-A collision. The solid line corresponds to the DA, when the effective slope for the transverse mass spectrum of kaons produced in the $p-p$ collision $T_{eff}^{pp}$ depends on $\sqrt{s}$ [11]. The long dashed line corresponds to the DA, when $T_{eff}^{pp} = constant$, whereas the short dashed line corresponds to the LE, when $T_{eff}^{pp} = constant$ and the temperature $T = 150$ MeV. Experimental data were taken from [2, 3, 4].

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