Method for simplified simulation of the pseudo-viscosity of the flows with solid particles

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Abstract. The short review of the solid phase pseudo-viscosity CFD-simulation methods is presented. The simplified method based on Einstein’s equation, experimental data approximation, and the multiphase mixture model for the solid phase pseudo-viscosity simulation is proposed. The results of the experimental verification of the proposed method are presented, its terms of use are determined.

1. Introduction
A lot of industrial water and gas treatment processes are related to the processing of the flows with solid particles. The use of modern computational methods for the design and scaling of industrial equipment requires a mathematical description of the physical properties of the technological flow. One of the main mathematical problems is the description of the solid phase influence on the gas or liquid flow viscosity.

Lagrangian approach requires the use of the conservation equations for each solid particle and allows simulating the motion of the solid particles in the liquid or gas flow well [1,2]. This approach needs additional models for description of flow-particles friction and particle-particle interaction. Such models and methods usually use semi-empirical equations and coefficients [3-5]. The main disadvantages of Lagrangian approach are the low limit of solid phase volume fraction (about 12 %) and high computational cost.

For Eulerian approach, conservation equations are solved for primary continuous fluid phase and secondary bulk pseudo-continuous solid phase [1,6]. One of the main difficulties of the Eulerian approach for solid-phase simulation is the solid phase pseudo-viscosity description. Gidaspow model [7] is widely used in practice to simulate pseudo-viscosity of the solid phase in the fluid-solid flow systems. This model describes the solid phase pseudo-viscosity as a sum of kinetic and collisional viscosities by the semi-empirical equations. For high solid flows, Gidaspow-like models may be added by solid-solid frictional terms [8]. Their main disadvantages for low-power computers, in our opinion, are relatively high computational costs and high requirements for density of a finite element mesh. As alternative methods, we propose using simple semi-empirical equations for suspension viscosity (like the Einstein’s equation [9] or experimental data approximations). But this approach needs additional verification.

2. Numerical simulation
For multiphase flow simulation, we used the multiphase mixture model [6]. The momentum conservation equation in this model takes form:
\[
\frac{\partial}{\partial t}(\rho_m u_{mi}) + \left[ (\rho_m u_{mj}) \frac{\partial u_{mi}}{\partial x_j} - \sum_{\varphi=1}^{n}(\alpha_\varphi \rho_\varphi u_{dr i,\varphi}) \frac{\partial u_{dr i,\varphi}}{\partial x_j} \right] =
\]
\[
= -\frac{dp}{dx} + \frac{\partial}{\partial x_j} \left[ \mu_m \left( \frac{\partial u_{mj}}{\partial x_i} + \frac{\partial u_{mi}}{\partial x_j} - \frac{2}{3} \delta_{ij} \frac{\partial u_{mi}}{\partial x_i} \right) \right] + \rho_m g_i,
\]  
\tag{1}
\]
where \(m\) is the mixture parameters index; \(\varphi\) is the phase parameters index; \(i\) and \(j\) are the longitudinal and transverse Cartesian direction indicies; \(t\) is the time, s; \(\rho\) is the density, kg/m\(^3\); \(u\) is the velocity, m/s; \(x\) is the Cartesian coordinate, m; \(p\) is the pressure, Pa; \(\mu\) is the viscosity, Pa\(\cdot\)s; \(\delta_{ij}\) is the metric tensor; \(g\) is the gravity force acceleration, m/s\(^2\); \(\alpha\) is the volume fraction.

For the solid phase pseudo-viscosity calculation, we used Gidaspow model [7], Einstein’s equation [9], and the experimental data polynomial approximation.

Gidaspow model describes the solid-phase pseudo-viscosity as:

\[
\mu_s = \mu_{s\text{kin}} + \mu_{s\text{col}},
\]  
\tag{6}
\]
\[
\mu_{s\text{kin}} = \frac{5 d_s \rho_s \sqrt{\theta_s}}{\eta_\varphi \psi} \left[ 1 + \frac{1}{8} \psi \gamma \alpha_s \right],
\]  
\tag{7}
\]
\[
\mu_{s\text{col}} = \frac{8}{5} \alpha_s \rho_s d_s \psi \gamma \sqrt{\frac{\theta_s}{\pi}},
\]  
\tag{8}
\]
\[
\psi = \frac{1+\alpha_s}{2},
\]  
\tag{9}
\]
\[
\gamma = \left( 1 - \frac{3}{\sqrt{\alpha_s}} \left( \frac{\alpha_s}{\alpha_{s\text{max}}} \right)^{-1} \right),
\]  
\tag{10}
\]
\[
\frac{3}{2} \theta_s = \frac{1}{2} u_s^2,
\]  
\tag{11}
\]
where \(s\) is the solid phase index; \(\text{kin}\) is the kinetic terms index; \(\text{col}\) is the collisional terms index; \(d_s\) is the particle average size; \(\Theta\) is the granular temperature, \(\text{m}^2/\text{s}^2\); \(\alpha_{s\text{max}}\) is the solid phase maximal volume fraction in the layer.

According to the Einstein’s equation, the fluid-solid mixture viscosity was calculated as:
where $\mu_f$ is the fluid phase viscosity, Pa·s.

The approximation equation form was determined by the experimental data character.

To test the method, we used 3d simulation of the pressure drop of the suspension laminar flow in a capillary with a diameter of 1 mm and length of 1000 mm. The flow velocity varied in the range of 10-50 mm/s. As a model fluid, we used CaCO$_3$ water suspensions. The solid phase volume fraction varied in the range of 0.0-6.0 %. All simulations were performed using ANSYS Fluent pressure-based solver with and the 1.5·10$^5$ prismatic elements mesh. The mathematical model of the process was described by equations (1) – (11) for the steady-state isothermal conditions. Equations (12) and (13) were added to the model performed by compiled user define functions (UDF) as the Dynamic-link Library (DLL) for Windows 10 operation system.

3. Experimental

For experimental verification of the Einstein’s equation and determination of the regression coefficients of the experimental approximation equation, we measured the CaCO$_3$ water suspension density and kinematical viscosity. The CaCO$_3$ mass was measured by the digital analytical scales. The volumes of water and suspension were measured by the measuring cup with a graduation of 5 ml. The water volume for all suspensions was 700 ml. To measure the kinematical viscosity, we used Zahn cup #1 viscosity meter. The measurements of all physical properties were repeated five times. The average measured values and the total measuring errors were calculated by Student method (t-test method) for the accepted error probability of no more than 10 %.

Using the measured densities and kinematical viscosities we calculated the dynamical viscosities of the suspensions.

Table 1 shows the suspensions properties and the measurement average errors. The experimental data approximation equation for the suspension viscosity takes form:

$$\mu = (4.813\alpha_s + 0.973) \cdot 10^{-3} \pm 5.1 \%.$$  

| Properties             | Average measurement error | Sample number |
|------------------------|---------------------------|---------------|
|                        |                           | #1 | #2 | #3 | #4 | #5 |
| Water volume, ml       | 5.1 %                     | 700| 700| 700| 700| 700|
| CaCO$_3$ mass, g        | 4.8 %                     | 35.73 | 59.56 | 83.38 | 104.22 | 125.07 |
| CaCO$_3$ volume phase   | 9.9 %                     | 0.019 | 0.031 | 0.042 | 0.052 | 0.062 |
| Suspension density, kg/m$^3$ | 9.9 %                     | 1032 | 1052 | 1075 | 1089 | 1106 |
| Suspension viscosity, mPa·s | 12.3 %                   | 1.15 | 1.23 | 1.23 | 1.27 | 1.35 |

4. Results and discussion

Figure 1 shows both experimental and calculated by Einstein’s equation dependences of the CaCO$_3$ dynamical viscosity on the solid phase volume fraction. It is shown that maximum and average divergences of the experimental data and the calculation results are 12.4 % and 5.7 %, respectively. The divergences are lower than measurement errors by t-test. Thus, Einstein’s equation may be used for the relatively low-solid fluid-solid flows CFD-simulation.
Figure 1. Dependences of the suspension viscosity on the solid phase volume fraction.

Figure 2 shows the CFD-simulated dependences of the pressure drop in a capillary with diameter of 1 mm and length of 1000 mm. The simulation result shows that models based on the Einstein’s equations and the experimental approximation can be used as alternative for Gidaspow-based models for the low-solid phase mixtures with Newtonian rheology to reduce computing power and time. For these cases, the divergence of these models does not exceed 12-15 %. The divergence grows with the flow velocity and solid phase volume fraction growth due to the impact of non-Newtonian effects.

Figure 2. CFD-simulated pressure drops in in the capillary.

5. Conclusion
Experimental verification of Einstein’s equations for the suspension viscosity is performed. It is shown that divergence of the experimental data and calculations is lower than measurement errors by t-test. The results of the pressure drop CDF-simulation for the laminar flow in the capillary are shown. It is found that the divergence of simulations with Gidaspow model together with the modified multiphase mixture model with Einstein’s equations and experimental data approximation equation does not exceed 12-15 % for the relatively low-solid Newtonian flows. The discrepancy between the simulation results increases with an increase in flow velocity and volume fraction of the solid phase due to the influence of non-Newtonian effects. Thus, methods based on Einstein’s equation and those based on the experimental data approximation may be used for CFD-simulation of the Newtonian rheology flows.
References

[1] Crowe C T, Schwarzkopf J D, Sommerfeld M and Yutaka T 2012 *Multiphase Flows with Droplets and Particles* (Boca Raton: CRC Press)

[2] Ranade V V 2002 *Computational Flow Modelling for Chemical Reactor Engineering* (London: Academic Press)

[3] Morsi S A and Alexander A J 1972 *J. Fluid Mech.* 55 193

[4] Lister J D, Smit D J and Hounslow M J 1995 *AIChE J.* 41 591

[5] Hounslow M J, Ryall R L and Marshall V R 1988 *AIChE J.* 34 1821

[6] Manninen M, Taivassalo V and Kallio S 1996 *On the Mixture Model for Multiphase Flow* (Espoo: VTT Publications)

[7] Gidaspow D 1994 *Multiphase Flow and Fluidization: Continuum and Kinetic Theory Description* (London: Academic Press)

[8] Schaeffer D G 1987 *J. Differ. Equ.* 66 19

[9] Leal G L 2007 *Advanced Transport Phenomena: Fluid Mechanics and Convective Transport Processes* (Cambridge: Cambridge University Press)