Collective Properties of Excitons in Presence of a Two-Dimensional Electron Gas

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We have studied the collective properties of two-dimensional (2D) excitons immersed within a quantum well which contains 2D excitons and a two-dimensional electron gas (2DEG). We have also analyzed the excitations for a system of 2D dipole excitons with spatially separated electrons and holes in a pair of quantum wells (CQWs) when one of the wells contains a 2DEG. Calculations of the superfluid density and the Kosterlitz-Thouless (K-T) phase transition temperature for the 2DEG-exciton system in a quantum well have shown that the K-T transition temperature increases with increasing exciton density and that it might be possible to have fast long range transport of excitons. The superfluid density and the K-T transition temperature for dipole excitons in CQWs in the presence of a 2DEG in one of the wells increases with increasing inter-well separation.

Key words: A. Quantum wells; A. Semiconductors; D. Electron-electron interactions; D. Phase transitions.

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The great interest to the dipole excitons in coupled quantum wells (CQWs) with spatially separated electron QW and hole QW interest has been stimulated by the possibility of Bose-Einstein condensation (BEC) and the superfluidity of dipole excitons formed from electron-hole (e-h) pairs. These may result in persistent electrical currents in each QW or coherent optical properties and Josephson junction phenomena \cite{1,2}. The great experimental success was achieved now in this field \cite{8,9,10}.

The coupled QW system is conceptually simple: negative electrons are trapped in a two-dimensional plane, while an equal number of positive holes is trapped in a parallel plane a distance $D$ away. In this system, the electron and hole wavefunctions have very little overlap, so that the excitons can have very long lifetime ($> 100$ ns), and therefore they can be treated as metastable particles to which quasiequilibrium statistics apply. Also, when $D$ is large enough,
the interactions between the excitons are entirely dipole-dipole repulsive.

In this paper, we consider the collective properties of (a) excitons within one QW in the presence of a 2D electron gas (2DEG) and (b) dipole excitons with spatially separated electrons and holes in CQWs when the number of electrons in the electron well is much larger than the number of holes in the hole well. In case (b), the system can be treated as a system of dipole excitons in the presence of a 2DEG in one well. The photoluminescence (PL) spectra caused by the recombination of 2D electrons in a single heterojunction quantum well (SHQW) result in the experimental observation of the excitons coexisting with a degenerate 2DEG in the same subband [12]. As shown by time-resolved PL measurements, an increase in the integrated PL intensity and the anomalously fast occurrence of PL from the region around the filament is induced by the screening response of the 2DEG-exciton system to the abrupt appearance of a remote photocurrent filament in the 2DEG [13]. These observations in conjunction with theory reported in this paper suggest the occurrence of fast long-range transport of excitons.

We emphasize, that although the K-T transition has been investigated for 2D excitons [7, 8], the key result of our Paper is that we include the 2DEG in our considerations. Consequently, the resulting formulas for the superfluid density K-T transition temperature are a novel result which we present below. We then try to connect our theory with recent experimental results as an alternative explanation of the data.

We now describe the theoretical calculations for determining the superfluid density and K-T transition temperature in a QW containing 2D excitons along with the 2DEG. There are two types of interaction in this system: exciton-exciton \((X - X)\) and exciton-electron \((X - e)\) interaction, while in the absence of 2DEG there is only \(X - X\) interaction. In a QW in the presence of 2DEG the \(X - e\) interaction plays an important role, since \(X - e\) scattering is twice as efficient as \(X - X\) scattering [14]. The spectrum of collective excitations (quasiparticles) in a QW containing the 2D excitons has the sound branch. This sound branch on the spectrum of collective excitations is indicative of the interaction strength of exciton-exciton \((X - X)\) scattering and the energy spectrum \(\varepsilon_{X-X}(p)\) in the framework of weakly non-ideal dilute Bose gas provided by Bogoliubov approximation [15, 16] is given by \(\varepsilon_{X-X}(p) = c_s p\), where \(c_s = U_0 n/M\), \(p\) is momentum, \(c_s\) is the sound velocity of collective branch due to \(X - X\) scattering, \(n\) is the exciton density, \(M = m_e + m_h\) is the exciton mass \((m_e\) and \(m_h\) are the effective masses of electron and hole, respectively), \(U_0\) is the zero-order Fourier-component (at \(p = 0\)) of the exciton-exciton repulsion potential assuming a hard sphere approximation, given by [17] \(U_0 = 6 \epsilon^2 a_{2D} / \epsilon\), where \(\epsilon\) is the electron charge, \(a_{2D} = \hbar^2 / (2 \mu_{e-h} \epsilon^2)\), \(\mu_{e-h} = m_e m_h / (m_e + m_h)\), \(\epsilon\) is the dielectric constant.

For the system of the spatially separated excitons and electrons in coupled quantum wells (CQWs) it was shown
that the effective $X - X$ interaction pair potential becomes weaker due to screening by electrons $[18]$. We take into account this screening effect by introducing the following phenomenological form of the zero-order Fourier-component of the exciton-exciton repulsion potential in the presence of the 2DEG: $\tilde{U}_0 = \beta(n, n_e)U_0$, where $\beta(n, n_e) < 1$ is the coefficient reflecting the electron screening effects, and $n_e$ is the electron density. This substitution causes the electron screening renormalization of the sound velocity $\tilde{c}_s = \beta(n, n_e)c_s$. Note that $\beta(n, n_e)$ can depend on the exciton density $n$ and electron density $n_e$. The calculation of a more accurate phenomenological relationship will be addressed in future work.

The superfluid-normal phase transition in the 2D exciton system is the K-T transition $[19]$. The transition temperature $T_c$, for the K-T transition to the superfluid state in a 2D exciton system is determined by the equation $[19]$

$$T_c = \frac{\pi}{8\hbar^2 n_s(T_c)}$$

where $n_s(T)$ is the superfluid density of the exciton system as a function of temperature $T$ and $k_B$ is Boltzmann’s constant.

The function $n_s(T)$ has been calculated from the relation $n_s = n - n_n$ ($n$ is the total exciton density, $n_n$ is the normal component density). We determine the normal component density by means of the usual procedure $[15, 20]$. Let us suppose that the exciton system moves with velocity $u$. At finite temperature $T$, dissipating quasiparticles will appear in this system. Since their density is small at low temperatures, one can assume that the gas of quasiparticles is an ideal Bose gas. To calculate the superfluid component density, we find the total current of quasiparticles in a frame in which the superfluid component is at rest. Then we obtain the mean total current of 2D excitons in the coordinate system, moving with a velocity $u$:

$$\langle J \rangle = \frac{1}{M} \langle p \rangle = \frac{1}{M} s \int \frac{dp}{(2\pi \hbar)^2} p f \left[ \varepsilon_{X-X}(p) - pu \right],$$

where $f \left[ \varepsilon_{X-X}(p) \right] = (\exp \left[ \varepsilon_{X-X}(p)/(k_B T) \right] - 1)^{-1}$ is the Bose-Einstein distribution function, $s$ is the level degeneracy (equal to 4 for excitons in GaAs quantum wells). Expanding the expression inside the integral in the first order by $pu/(k_B T)$, we have:
\[
\langle J \rangle = -\frac{u}{2M^8} \int \frac{dp}{(2\pi\hbar)^2} p^2 \frac{\partial f[\varepsilon_{X-X}(p)]}{\partial \varepsilon_{X-X}} \int dp (2\pi\hbar)^2 p^2 \frac{\partial f[\varepsilon_{X-X}(p)]}{\partial \varepsilon_{X-X}}
= \frac{3\zeta(3) k_B^3 T^3}{2\pi \hbar^2} \frac{1}{c_s^4} u \, ,
\]

where \( \zeta(z) \) is the Riemann zeta function (\( \zeta(3) \approx 1.202 \)). Then we define the normal component density \( n_n \) as

\( \langle J \rangle = n_n u \). Comparing this definition of \( n_n \) and (2), we obtain the expression for the normal density \( n_n \). Consequently, we have for the superfluid density:

\[
n_s = n - n_n = n - \frac{3s\zeta(3) k_B^3 T^3}{2\pi \hbar^2} \frac{1}{c_s^4} \, .
\]

In a 2D system, superfluidity of excitons in a 2DEG appears below the K-T transition temperature, where only coupled vortices are present. Making use of Eq. (3) for the density \( n_s \) of the superfluid component, we obtain an equation for the Kosterlitz-Thouless transition temperature \( T_c \). Its solution is given by

\[
T_c = \left[ \left( 1 + \sqrt{\frac{32}{27} \left( \frac{Mk_B T_c^0}{\pi \hbar^2 n_s} \right)^3 + 1} \right) - \frac{32}{27} \left( \frac{Mk_B T_c}{\pi \hbar^2 n_s} \right)^3 + 1 \right]^{1/3} \frac{T_c^0}{2^{1/3}} \, .
\]

Here \( T_c^0 \) is an auxiliary quantity, equal to the temperature at which the superfluid density vanishes in the mean-field approximation (i.e., \( n_s(T_c^0) = 0 \)),

\[
T_c^0 = \frac{1}{k_B} \left( \frac{2\pi \hbar^2 n c_s^4 M}{3s \zeta(3)} \right)^{1/3} \, .
\]

The temperature \( T_c^0 \) may be used as a crude estimate of the crossover region where local superfluid density appears for rare exciton system in 2DEG on a scale smaller or of the order of the mean intervortex separation in the system. The local superfluid density can manifest itself in local optical properties or local transport properties.
The K-T transition temperature $T_c$ obtained in our calculations as a function of the exciton density $n$ in a single quantum well is given by Eq. (4). It is shown that $T_c$ increases when the exciton density $n$ increases. Besides, at the fixed temperature the superfluidity in a single quantum well exists at the exciton densities greater than the critical one. This critical exciton density increases when the temperature increases. The theoretical results also show that a superfluid transition at $T_c = 2$ K requires an exciton density of $7.5 \times 10^{10}$ cm$^{-2}$; which is reasonably close to the theoretical estimate of the exciton density realized in the experiment of Ref. [13].

Besides, we investigate dipole excitons with spatially separated electrons and holes in CQWs when the number of electrons in the electron well is much more than the number of holes in the hole well. In a dilute system when the numbers of electrons and holes are equal, $(na^2_{2D} \ll 1)$ two dipole excitons repel each other like two parallel dipoles with the pair repulsion potential $U(R) = e^2 D^2/(\epsilon R^3)$, where $R$ is the distance between two excitons, and $D$ is the inter-well septation. The collective excitations in this spectrum have the sound spectrum [21]: $\varepsilon_{X-X}(p) = c_s(D)p$. The sound velocity $c_s(D)$, when the electron and hole numbers are equal, is calculated in the ladder approximation assuming the vertex correction is equal the sum of the ladder diagrams and given by [7, 21]

$$c_s(D) = \left( \frac{4\pi \hbar^2 n}{M^2 \log \left( \frac{e^2 e^4}{8\pi^2 n^2 M^2 \epsilon^4 D^4} \right)} \right)^{1/2}. \quad (6)$$

For the case when number of electrons is much greater than number of holes, taking into account the electron screening effects we assume the following phenomenological form of the exciton-exciton dipole repulsion potential in the presence of the 2DEG: $\tilde{U}(R) = \gamma(n, n_e, D)e^2 D^2/(\epsilon R^3)$, where $\gamma(n, n_e, D) < 1$ is the coefficient reflecting the electron screening effects. This substitution results in the electron screening renormalization of the sound velocity:

$$\tilde{c}_s(D) = \left( \frac{4\pi \hbar^2 n}{M^2 \log \left( \frac{e^2 e^4}{8\pi^2 n^2 M^2 \gamma(n, D)e^4 D^4} \right)} \right)^{1/2}. \quad (7)$$

It follows from Eq. (7) that $\tilde{c}_s(D) < c_s(D)$ due to the fact that $\gamma(n, n_e, D) < 1$. Therefore, the sound velocity for the collective excitation spectrum in the CQWs with the number of electrons greater than number of holes can be represented as $\tilde{c}_s(D) < \theta(n, n_e, D)c_s(D)$, where $\theta(n, n_e, D) < 1$ is the phenomenological parameter. The rest of the procedure for the calculation of the superfluid density and Kosterlitz-Thouless temperature is the same as for the single quantum well in the presence of 2DEG. The superfluid density for CQWs can be calculated by applying Eq. (3).
and and Kosterlitz-Thouless temperature can be calculated by using Eq. (4) with the substitution of the renormalized sound velocity from Eq. (7) instead of $c_s = \beta(n, n_e)U_0n/M$ for the single quantum well.

The calculation of the phenomenological parameters $\beta(n, n_e)$ for a single quantum well in the presence of 2DEG and and $\theta(n, n_e, D)$ for CQWs with the number of electrons higher than the number of holes is an open question which should be answered by future work.

The K-T transition temperature $T_c$ obtained in our calculations as a function of the exciton density $n$ in CQWs is presented in Fig. 1. It is shown that $T_c$ increases monotonically with increasing exciton density $n$ and the inter-well separation $D$, which can be explained by the fact that the dipole-dipole repulsion causing the superfluidity increases if $D$ increases. Furthermore, the results of our calculations show that $T_c$ increases when $\beta(n, n_e)$ for a single well or $\theta(n, n_e, D)$ for CQWs increase. According to Fig. 1 analogously to the exciton-electron system a single quantum well at the fixed temperature the superfluidity in CQWs exists at the exciton densities greater than the critical one. This critical exciton density increases, when the temperature increases and the interwell separation $D$ decreases. Increasing $\beta(n, n_e)$ for a single quantum well or $\theta(n, n_e, D)$ for CQWs results in the decreasing critical exciton density.

An experimental observation [13] has been reported on a sharp increase in the average photocurrent at the critical voltage $V_g = -36$ V and hysteresis in the I-V characteristics with subsequent variation of $V_g$ which is indicative of the formation of a photocurrent filament from the undoped GaAs layer into the 2DEG in the SHQW. It has been observed in Ref. [13] that there is concurrent, significant changes in the time-integrated excitonic PL at $V_g = -36$ V. The data show that shortly after excitation at $V_g = -36$ V, the PL intensity at the filament position is approximately a factor of 8 larger than the PL intensity observed at the same point and time with $V_g = 0$ V even though the residual laser excitation intensity at the filament position is unchanged. TR PL profile data [13] shows that there is no observed delay between the PL peaks in the photoexcitation and the filament regions indicating that no in-plane transport of excitons from the excitation region to the filament region is observed. However, the transport of excitons in a time less than 0.1 ns (the experimental resolution time) cannot be ruled out. The data suggests that an exciton excitation energy transport mechanism is responsible for the fast observation of 1550 meV PL from the filament region. One possibility is that in response to the filament-induced potential, excitons undergo a Kosterlitz-Thouless superfluid transition with a long-range transport of exciton excitation energy. Theory points out that exciton superfluidity does not involve transport of mass; instead it involves a transport of excitation energy. The filament-induced increase in exciton density and lifetime along with efficient electron-exciton scattering in the presence of the 2DEG are conducive to exciton condensation. Using theoretical results we estimate that the peak exciton density
in the SHQW is approximately $4 \times 10^{10}$ cm$^{-2}$. There is a large uncertainty in calculating and measuring the actual exciton density in a QW with a 2DEG-exciton system. Nevertheless, the observed appearance of high-intensity PL at a point 25 $\mu$m from the edge of the excitation region less than 0.1 ns after excitation is a possible manifestation of a long range transport of exciton excitation energy. Further research is needed to understand this exciton excitation energy transport mechanism and its potential application to optoelectronic devices.

The experimental results obtained from measurement of the exciton dephasing rate show that electron-exciton scattering is twice as efficient as exciton-exciton scattering. It is important to note that the enhanced electron-exciton scattering also leads to efficient relaxation of exciton energy in agreement with previous theoretical results [24]. Efficient exciton energy relaxation involves collective plasmon-phonon interactions in the exciton/two-dimensional electron gas (exciton/2DEG) system which could result in the formation of the second sound branch. Experimental results [13] clearly show that the collective screening response of the exciton/2DEG system to a photo-induced potential results in fundamental changes in exciton characteristics including the possibility of condensation and transport of excitons. The 2DEG density in our experiment is $2 \times 10^{11}$ cm$^{-2}$.

Negatively charged excitons (trions), which consist of two electrons bound to a single hole with a theoretical [26] and observed [27–29] binding energy relative to the exciton of approximately 1 meV have been observed in modulation-doped GaAs QWs. A photoluminescence peak from trions has been observed from the exciton 2DEG system in our structure around 1 meV below the exciton PL peak only when the exciton linewidth has been minimized using slight variations of the applied gate voltage [30]. In the present experiments, broadening of the PL linewidths at $V_g = -36$ V and the short exciton lifetime of 0.5 ns at $V_g = 0$ precluded the resolution of a PL peak from trions.

In conclusion, calculations of the superfluid density and the K-T transition temperature for the 2DEG-exciton system show that the K-T transition temperature increases with increasing exciton density and support the possible occurrence of a fast transport of exciton excitation energy. The superfluid density and the K-T transition temperature for dipole excitons in CQWs in the presence of a 2DEG in one of the wells increases with increasing inter-well separation $D$. The superfluid density and the K-T transition temperature in CQWs increase when the phenomenological screening parameters $\beta(n, n_e)$ for a single well or $\theta(n, n_e, D)$ for CQWs in the presence of 2DEG increase.

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FIG. 1: (Color on line) Kosterlitz-Thouless transition temperature $T_c$ as a function of the exciton density $n$ in CQWs for different inter-well separations $D$ and phenomenological parameter $\theta(n_n, D)$: $D = 0.5$ nm, $\theta(n_n, D) = 0.5$ – solid curve; $D = 0.5$ nm, $\theta(n_n, D) = 0.94$ – dotted curve; $D = 5$ nm, $\theta(n_n, D) = 0.5$ – dashed curve; $D = 5$ nm, $\theta(n_n, D) = 0.94$ – dashed-dotted curve.