Stresses in Circular Plates with Rigid Elements

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Abstract. Calculations of residual stress fields are carried out by numerical and static methods, using the flat cross-section hypothesis. The failure of metal when exposed to residual stresses is, in most cases, brittle. The presence in the engineering structures of rigid elements often leads to the crack initiation and structure failure. This is due to the fact that rigid elements under the influence of external stresses are stress concentrators. In addition, if these elements are fixed by welding, the residual welding stresses can lead to an increase in stress concentration and, ultimately, to failure. The development of design schemes for such structures is a very urgent task for complex technical systems. To determine the stresses in a circular plate with a welded circular rigid insert under the influence of an external load, one can use the solution of the plane stress problem for annular plates in polar coordinates. The polar coordinates of the points are the polar radius and the polar angle, and the stress state is determined by normal radial stresses, tangential and shearing stresses. The use of the above mentioned design schemes, formulas, will allow more accurate determination of residual stresses in annular welded structures. This will help to establish the most likely directions of failure and take measures at the stages of designing, manufacturing and repairing engineering structures to prevent these failures. However, it must be taken into account that the external load, the presence of insulation can lead to a change in the residual stress field.

1. Introduction
Circular plates reinforced with rigid elements are widely used as elements of various engineering constructions. However, the presence of such elements often leads to the crack initiation and structure failure. This is due to the fact that rigid elements under the influence of external stresses are stress concentrators. In addition, in the process of their manufacture, mechanical and thermal technological operations are used, creating residual stress fields in the metal, which can lead to an increase in stress concentration and ultimately to failure [1–9].

At present, residual stress fields are calculated by various methods: numerical; static; cross-section hypothesis, etc. [10–18].

For example, in [10] a mechanical distortion of an aircraft engine disk is modeled under the influence of residual stresses. The initial workpiece for the disc had residual stresses distributed in its volume as a result of previous thermomechanical processing. During machining by turning and milling operations, the disc is distorted as the material is removed, while the residual stress fields develop to achieve a new equilibrium. It is proposed to predict disc distortion when removing a material using the developed automated procedure implemented in the application package of computer programs.

In work [11], the stresses of the entire part in the manufacturing process and after cooling were predicted by a simplified numerical model based on individual layers of finite elements.

Determination of residual deformations and stresses arising during the pressing process in thin-walled engineering constructions is considered in [12]. For this purpose, two main procedures were
proposed: optimization of design in a sequential order and taking into account the influence of the manufacturing process on the final results.

Estimation of residual stresses arising from laser hardening of surface microstructures of stainless steel specimens was investigated in [13]. The surface residual stresses and microhardness of the specimens were recorded before and after hardening, using an optical microscope and a scanning electron microscope.

Despite the existing research in this area, the development of design schemes for determining residual stresses is a very urgent task. In particular, in determining the residual stresses in ship hull structures: bulk seals in the shaft line passages through the cross bulkheads; welded bitts covers; welded pipe flanges; strengthening of pipes in places of pipes passage through bulkheads; constructions of bottom stub-pipes, etc.

2. Design scheme
To determine the stresses in a circular plate with a rigid insert, installed by welding, under the influence of an external load, one can use the solution of the plane stress problem for annular plates in polar coordinates [19–20]. The polar coordinates of the points are polar radius $r$ and polar angle $\theta$, and the stress state is determined by tangents $\tau_{r\theta}$, normal radial $\sigma_r$, and tangential $\sigma_\theta$ stresses.

Let us determine the residual stresses in a circular plate of radius $R_1$ with an absolutely circular rigid insert of radius $R$ (figure 1) in the absence of external forces. In this case, the displacements of the points of the plate on the boundary with the insert in the Cartesian coordinate system are:

$$g_1 = u_i + iv_1,$$  

where $u_i = \Delta_1 \cos \theta$ – the displacements of points in the longitudinal (along the axis OX) direction; $v_1 = \Delta_2 \sin \theta$ – the displacements of points in the transverse (along the axis OY) direction; $\Delta_1, \Delta_2$ – the longitudinal and transverse shrinkage of the welded seam.

![Figure 1. The design scheme for a circular plate of finite dimensions with a circular insert under uniaxial tension.](image)

Representing radial $u$ and tangential $v$ displacements in polar coordinates as:

$$\begin{align*}
u &= u_i \cos \theta + v_1 \sin \theta; \\
v &= -u_i \sin \theta + v_1 \cos \theta; \\
\end{align*}$$

let us obtain the following:
\[
\begin{cases}
  u = N + M \cos 2\theta \\
  v = -M \sin 2\theta.
\end{cases}
\]  

(3)

Here:

\[
\begin{cases}
  N = 0.5(\Delta + \Delta_2) \\
  M = 0.5(\Delta - \Delta_2).
\end{cases}
\]  

(4)

Boundary conditions on contour \( r = R \) will be written as:

\[
\begin{cases}
  u(R, \theta) = N + M \cos 2\theta \\
  v(R, \theta) = -M \sin 2\theta.
\end{cases}
\]  

(5)

Boundary conditions on contour \( r = R_1 \) will be written as:

\[
\begin{cases}
  \sigma_r(R_1, \theta) = 0 \\
  \tau_{r\theta}(R_1, \theta) = 0.
\end{cases}
\]  

(6)

For annular plates the general solution is known in the form of an infinite trigonometric series in \( \sin(n\theta) \) and \( \cos(n\theta) \) (\( n \) – the series term number).

Since the boundary conditions (5, 6) are homogeneous or contain only expansion terms with numbers \( n=0 \) and \( n=2 \), then in general expressions for stresses it is sufficient to leave the terms with the indicated numbers and only those that determine the expansion of displacements \( u \) by cosines, and \( v \) – by sinuses.

Design schemes for circular plates reinforced with circular rigid elements at uniform pressure acting along their contours are shown in figure 2. The design scheme of this problem is reduced to the plane stress problem of the elastic ring of internal radius \( R_1 \) (figure 2,a), in which an elastic circular plate is embedded, which initially has a larger radius in comparison with \( R_1 \). The plate has an undeformable insert of radius \( R \), rigidly connected with it (figure 2,b).

Figure 2. The design schemes for circular plates at uniform pressure acting along their contour.

Let us assume that there is no friction between the ring and the plate. Then the entire interaction of these objects is reduced to normal pressure \( p \) on the inner ring rim and the outer plate rim. The pressure will be constant along the rims because of its full symmetry. Therefore, the final solution of the problem can be made up of a solution for the plate that is under the action of external pressure and
having the undeformable insert and solution for the ring under the influence of internal pressure on the basis of dependencies (1–6).

3. Numerical Results and Discussion
Calculation of residual stresses $\sigma_{r1}, \sigma_{\theta1}$ was carried out under the assumption that the stresses in the welding seam reached the yield point of the plate material, $\sigma_T = E\varepsilon_T$ ($\varepsilon_T$ – relative extension).

The results of stress calculations are based on the dependencies (1-6). Figure 3 shows the curves characterizing the residual welding stresses in a circular plate of finite dimensions. The values of these stresses are given in a dimensionless form obtained by assigning their current value to yield point $\sigma_T$ by line $\theta=0$: $\sigma_{r1}/\sigma_T$ and $\sigma_{\theta1}/\sigma_T$.

![Figure 3](image)

**Figure 3.** Diagrams of relative residual weld stresses: 1 and 2 – $\sigma_{r1}/\sigma_T$ and $\sigma_{\theta1}/\sigma_T$ (variant $\Delta_1 = R\varepsilon_T$, $\Delta_2 = 0$); 3 and 4 – $\sigma_{r1}/\sigma_T$ and $\sigma_{\theta1}/\sigma_T$ (variant $\Delta_1 = R\varepsilon_T$, and $\Delta_2 = 0$).

It should be noted that radial stresses $\sigma_{r1}$ are compressive almost along the entire length of the annular plate, and tangential stresses $\sigma_{\theta1}$ are stretching. Hence in the construction under consideration (an annular plate with a rigid circular insert) in the considered zone ($\theta=0$) in the presence of structural or technological stress concentrators, the initiation and propagation of cracks in the radial direction are less likely and more likely along the arc of the circle. However, it must be taken into account that the external load, the presence of insulation can lead to a change in the residual stress field.

4. Conclusion
The use of the proposed design schemes and dependences (1)–(6) allows more accurate determination of residual stresses in annular welded structures. This will help to establish the most likely directions of failure and take measures to prevent them at the design, manufacturing and repair of engineering structures.

**References**
[1] Bertini L, Frendo F and Marulo G 2016 *International journal of fatigue* 90 78–86
[2] Gornostajev D, Aryassov G and Penkov I 2016 *International Review of Mechanical Engineering* 10(2) 115–124
[3] Hsieh C-F 2014 *Mechanism and Machine Theory* 80 1–16
[4] Keller A and Aliukov S 2015 SAE Technical Paper 2015-01-2788
[5] Shi G, Ban H Y, Shi Y J and Wang Y Q 2012 12th International Symposium on Structural Engineering (ISSE-12) 1 583–586
[6] Taratorkin I A, Derzhansky V B and Taratorkin A I 2017 AIP Conference Proceedings 1915(1) 040060
[7] Ohms C, Wimpory R C, Katsareas D E and Youtsos A G 2009 International journal of pressure vessels and piping 86(1) 63–72
[8] Filippi S and Lazarrin P 2004 International Journal of Fatigue 26(4) 377–391
[9] Penkov I and Aleksandrov D 2011 International Review of Mechanical Engineering 5(7) 1213–1218
[10] Barrett T J, Savage D J, Ardeljan M and Knezevic M 2018 Computational materials science 141 269–281
[11] An K, Yuan L, Dial L, Spinelli I, Stoica A D and Gao Y 2017 Materials & design 135 122–132
[12] Sun G Y, Zhang H L, Wang R Y, Lv X J and Li Q 2017 Structural and multidisciplinary optimization 56(6) 1571–1587
[13] Agyenim-Boateng E, Huang S, Sheng J, Yuan G, Wang Z W, Zhou J Z and Feng A X 2017 Surface & coatings technology 328 44–53
[14] Seo S, Huang E W, Woo W and Lee S Y 2017 International journal of fatigue 104 408–415
[15] Pritykin A I and Lavrova A S 2017 Mechanika 23(4) 488–494
[16] Hossain S, Zheng G, Truman C E and Smith D J 2017 Experimental techniques 41(5) 483–503
[17] Huang H, Tsutsumi S, Wang J D, Li L Q and Murakawa H 2017 Finite elements in analysis and design 135 1–10
[18] Velikanov N L, Koryagin S I and Sharkov O V 2016 IOP Conference Series: Materials Science and Engineering 124 012094
[19] Sadd M H 2014 Elasticity: Theory, Applications, and Numerics (Amsterdam-Tokyo: Academic Press) p 600
[20] Pytel A and Kiusalaas J 2012 Mechanics of Materials (Boston: Cengage Learning) p 570